Evidence Against an Association Between Gamma-Ray Bursts and Type I Supernovae

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We present a rigorous method, based on Bayesian inference, for calculating the odds favoring the hypothesis that any particular class of astronomical transients produce gamma-ray bursts over the hypothesis that they do not. We then apply this method to a sample of 83 Type Ia supernovae and a sample of 20 Type Ib-Ic supernovae. We find overwhelming odds against the hypothesis that all Type Ia supernovae produce gamma-ray bursts, whether at low redshift ($10^9 : 1$) or high-redshift ($10^{12} : 1$), and very large odds ($6000 : 1$) against the hypothesis that all Type Ib, Ib/c, and Ic supernovae produce observable gamma-ray bursts. We find large odds ($34 : 1$) against the hypothesis that a fraction of Type Ia supernovae produce observable gamma-ray bursts, and moderate odds ($6 : 1$) against the hypothesis that a fraction of Type Ib-Ic supernovae produce observable bursts. We have also re-analyzed both a corrected version of the Wang & Wheeler sample of Type Ib-Ic SNe and our larger sample of 20 Type Ib-Ic SNe, using a generalization of their frequentist method. We find no significant evidence in either case of a correlation between Type Ib-Ic SNe and GRBs, consistent with the very strong evidence against such a correlation that we find from our Bayesian analysis.

*Subject headings:* gamma-rays: bursts—methods: statistical—supernovae: general—supernovae: individual (SN 1998bw)
1. Introduction

The discovery that the sky distribution of faint gamma-ray bursts (GRBs) is isotropic, coupled with the confirmation of a roll-over in the cumulative brightness distribution of the bursts, suggested that the bursts lie at cosmological distances (Meegan et al. 1992). About one year ago, the rapid dissemination of arcminute-sized GRB error circles provided by the Wide-Field Camera (WFC) on BeppoSAX (Costa et al. 1997a) led to the discovery of fading X-ray (Costa et al. 1997b) and optical (Groot et al. 1997) counterparts to the bursts. The subsequent measurement of absorption lines at \( z = 0.835 \) in the spectra of the optical afterglow of GRB 970508 (Metzger et al. 1997) firmly established the extra-galactic nature of this burst, and presumably, of most or all GRBs. Redshifts are now known for the apparent host galaxies of two other bursts: \( z = 0.965 \) in the case of GRB 980703 (Djorgovski et al. 1998) and \( z = 3.42 \) in the case of GRB 971214 (Kulkarni et al. 1998a).

GRB980425 has complicated this simple “cosmological” picture of GRBs. Following the detection of this burst by the BeppoSAX Gamma-Ray Burst Monitor and WFC (Soffitta, P. et al. 1998; see also Kippen et al. 1998a), X-ray follow-up observations were made using the BeppoSAX Narrow Field Instrument (NFI) (Pian et al. 1998a,b; Piro et al. 1998). The initial observation revealed a faint X-ray source (detected at the \( 5\sigma \) level) that was not seen in several subsequent observations. Optical follow-up observations led to the discovery of a supernova, SN 1998bw, within the 8 arcminute radius of the BeppoSAX WFC error circle for the GRB but not coincident with the fading X-ray source (Galama et al. 1998). No other optically variable object was detected within the BeppoSAX WFC error circle for the burst. SN 1998bw was subsequently found to be of Type Ic (Galama et al. 1998) and very bright in the radio (Kulkarni et al. 1998b). The supernova is coincident with a galaxy (presumably the host galaxy) that lies at \( z = 0.008 \) (Tinney et al. 1998).

An association between GRB 980425 and SN 1998bw is an intriguing possibility, made
more so by the recent heightened interest in “collapsar” or “hypernova” models of GRBs (Woosley 1993; Woosley, Eastman & Schmidt 1998; Paczyński 1998; Höflich, Wheeler & Wang 1998). The principal argument in favor of an association between GRB 980425 and SN 1998bw is the positional and temporal coincidence between the two events. Given a supernova rate of \( \sim 2 \) per \( L_\star \) galaxy per century (Strom 1995), a density of \( L_\star \) galaxies of \( 0.01 \) Mpc\(^{-3}\), and that approximately \( 2/7 \) of these are SNe of Types Ib, Ib/c and Ic (Woosley & Weaver 1986, Strom 1995), the chance probability of such a spatial-temporal coincidence for a Type Ib-Ic SN with \( z \leq 0.008 \) is \( \sim 10^{-7} \). But would an association between GRB980425 and SN 1998bw be acceptable SN 1998bw were at \( z = 1 \) rather than at \( z = 0.008 \)? Almost certainly. If so, the appropriate value for the chance probability of the positional and temporal coincidence becomes \( \sim 0.1\% \). This illustrates how difficult it is to evaluate a posteriori statistical arguments.

And there are specific reasons to be cautious in this case. Assuming a power-law decay with time, and connecting the 2-10 keV X-ray flux detected by the BeppoSAX WFC during and immediately following the burst, and the 2-10 keV X-ray flux of the fading X-ray source detected 10 hours later by the BeppoSAX NFI, yields a power-law index of \( \sim 1.2 \) (Pian et al. 1998b), which is similar to the power-law indices of the X-ray afterglows of other BeppoSAX bursts. Thus GRB 980425 is more plausibly associated with this fading X-ray source than with SN 1998bw.

Also, if the association between GRB 980425 and SN 1998bw were true, the luminosity of this burst would be \( \sim 10^{46} \) erg s\(^{-1}\) and its energy would be \( \sim 10^{47} \) erg. Each would therefore be five orders of magnitude less than that of other bursts, and the behavior of the X-ray and optical afterglow would be very different from those of the other BeppoSAX bursts, yet the burst itself is indistinguishable from other BeppoSAX and BATSE GRBs with respect to duration, time history, spectral shape, peak flux, and and fluence (Galama...
et al. 1998).

In view of the difficulty in assessing the significance of any association between SNe and GRBs on the basis of this single event, the safest procedure is to regard the association as a hypothesis that is to be tested by searching for correlations between SNe and GRB in catalogs of SNe and GRBs, excluding SN 1998bw and GRB980425. Wang & Wheeler (1998) have performed such a study, and find evidence for a significant (at the $10^{-5}$ level) correlation between Type Ib-Ic SNe and GRBs detected by BATSE.

While the results of Wang & Wheeler (1998) seem promising, their study suffers from several deficiencies. The number (six) of Type Ib-Ic SNe in their sample is small, and one of these events is mis-classified [SN 1992ad is a Type II SN (McNaught 1992, Filippenko 1992), not a Type Ic SN], which eliminates one of their SN-GRB associations. The range of possible explosion dates that we derive for another event (SN 1997X) is much smaller than the range they allow, which rules out another of their associations. Furthermore, two other SN-GRB associations are ruled out by Interplanetary Network positions (Hurley et al. 1998, Kippen et al. 1998a). Moreover, Wang & Wheeler’s methodology is somewhat arbitrary, in the sense that they increase the size of the BATSE GRB positional error circles by a large, arbitrary factor. Finally, Wang & Wheeler’s methodology makes no provision for the fact that the BATSE temporal exposure is less than unity. In fact, their result (six of six “Type Ib-Ic” SNe correlated with GRBs) is unlikely, even if the proposed association between Type Ib-Ic SNe and GRBs were real, since BATSE has on average a probability of 0.48 of detecting any given GRB because of Earth blocking and other effects (Hakkila et al. 1998).

Here we carry out an analysis that overcomes these deficiencies. We correct the “Type Ib-Ic” SN sample of Wang & Wheeler (1998) and supplement it with 15 additional Type Ib-Ic SNe, so that we can study a larger sample. Further, we develop an alternative method,
based on Bayesian inference and therefore using the likelihood function, that incorporates information about the BATSE position errors in a non-arbitrary way and that is free of the ambiguities of \textit{a posteriori} statistics. The method also accounts the fact that the BATSE temporal exposure is less than unity.

Applying this method to a sample of 83 Type Ia SNe and a sample of 20 Type Ib-Ic SNe, we find overwhelming odds against the hypothesis that all Type Ia SNe produce observable gamma-ray bursts, irrespective of whether the SNe are at low- or high-redshift, and very large odds against the hypothesis that all Type Ib, Ib/c, and Ic SNe produce observable gamma-ray bursts. We find large odds against the hypothesis that a fraction of Type Ia supernovae produce observable gamma-ray bursts, and moderate odds against the hypothesis that a fraction of Type Ib, Ib/c, and Ic supernovae produce observable bursts.

We have also re-analyzed both a corrected version of the Wang & Wheeler sample of Type Ib-Ic SNe and our larger sample of 20 Type Ib-Ic SNe, using a generalization of their frequentist method. We find no significant evidence in either case of a correlation between Type Ib-Ic SNe and GRBs, consistent with the very strong evidence against such a correlation that we find from our Bayesian analysis.

The plan of this paper is as follows. In §II we present a rigorous method, based on Bayesian inference, for calculating the odds favoring the hypothesis that any particular class of astronomical transients produces GRBs over the hypothesis that they do not. In §III we apply this method to various subclasses of Type I SNe. In §IV we discuss our results, and compare them with other work. We present our conclusions in §V.

2. Statistical Methodology
2.1. Bayesian Odds

We denote the data by \( D = \{ D_i | i = 1, \ldots, N_{SN} \} \), where \( N_{SN} \) is the number of observed SNe. For the \( i \)th observed SN, the data consists of the SN position \( \mathbf{x}_i \) (a unit vector), the earliest time \( t_i \) at which the SN explosion could have occurred, the duration \( \tau_i \) of the period of time during which the SN explosion could have occurred, the number \( N_i \) of GRBs that occurred during the time interval \( [t_i, t_i + \tau_i] \), and the list \( (\mathbf{y}_{ij}, \sigma_{ij}) \), \( j = 1, \ldots, N_i \) of BATSE positions and error parameters for those bursts. Thus, \( D_i = \{ \mathbf{x}_i, t_i, \tau_i, N_i, \{ (\mathbf{y}_{ij}, \sigma_{ij}) | j = 1, \ldots, N_i \} \} \).

Note that the \( \sigma_{ij} \) are Fisher distribution parameters, not the BATSE-style 68\% error-circle radii. They enter the odds calculation through the assumption that an observed burst position \( \mathbf{y} \) is distributed around its true position \( \mathbf{x} \) according to the Fisher distribution

\[
P(\mathbf{y}|\mathbf{x}, \sigma) = \kappa \exp \left[ (\mathbf{y} \cdot \mathbf{x} - 1)/\sigma^2 \right] \quad (1a)
\]

\[
\kappa \equiv \left[ \frac{2\pi \sigma^2 (1 - e^{-2/\sigma^2})}{\left(1 - \frac{1}{2\sigma^2}ight)} \right]^{-1} \quad (1b)
\]

(see, for example [Mardia 1972, p. 228]). The \( \sigma \) are related to the 68\% total errors (including correction for systematic error) by the linear relation \( \sigma_{tot}^{68\%} = 1.52\sigma \).

We compare two hypotheses:

\( H_1 \): The association between SNe and GRBs is real. If a SN is observed, there is a chance \( \epsilon \) that BATSE sees the associated GRB, where \( \epsilon \) is the average BATSE temporal exposure. While \( \epsilon \) varies with Declination, the variation is modest and we neglect it. The probability density for the time of occurrence of the \( i \)th supernova is assumed uniform in the interval \( [t_i, t_i + \tau_i] \), so that all GRBs that occur in that interval have an equal prior probability of being associated with the SN.

\( H_2 \): There is no association between SNe and GRBs.
We wish to calculate the odds favoring $H_1$ over $H_2$, given the data. That is, we want

$$O = \frac{P(H_1|D, I)}{P(H_2|D, I)} = \frac{P(D|H_1, I) P(H_1|I)}{P(D|H_2, I) P(H_2|I)} = \frac{P(D|H_1, I)}{P(D|H_2, I)} = \prod_{i=1}^{N_{\text{obs}}} \frac{P(D_i|H_1, I)}{P(D_i|H_2, I)},$$

(2)

where we have set the prior probabilities $P(H_1|I) = P(H_2|I) = 1/2$, and we have assumed the statistical independence of all the $D_i$. The symbol $I$ is shorthand for all the available prior information.

From equation (2), it is apparent that $O$ is equal to the likelihood ratio. A simplification that occurs here is that our hypotheses $H_1$ and $H_2$ are simple — they are not parametrized families of models. As a consequence, the likelihoods $P(D|H_i, I)$ are not “global” likelihoods, averaged over parameter space weighted by a prior density, as is common in odds ratio calculations \cite{Loredo_Lamb_1992, Graziani_et_al_1992}. Rather, they are genuine likelihoods, the computation of which requires no prior probability density over parameter space.

### 2.2. Simple Model

We now compute the likelihoods for a model in which all Type Ib-Ic SNe produce GRBs. Under the no-association hypothesis $H_2$, we have

$$P(D_i|H_2, I) = P(x_i, t_i, \tau_i|H_2, I) \times P(N_i, \{y_{ij}, \sigma_{ij}\}|\tau_i, H_2, I)$$

$$= f(x_i, t_i, \tau_i) \times \frac{e^{-R\tau_i}(R\tau_i)^{N_i}}{N_i!} \times \left[ \prod_{j=1}^{N_i} g(\sigma_{ij}) \right] \times \left( \frac{1}{4\pi} \right)^{N_i},$$

(3)
where \( f(x_i, t_i, \tau_i) \) \( d^2x_i dt_i d\tau_i \) is the differential rate for observing such SNe, \( g(\sigma) d\sigma \) is the differential rate for observing a BATSE GRB positional error \( \sigma \), \((4\pi)^{-N_i} d^2y_{i1} \ldots d^2y_{iN_i} \) is the differential probability of \( N_i \) isotropic GRB positions, and \( R \) is the time rate at which BATSE observes GRBs. It is unnecessary to specify \( f \) and \( g \) in greater detail, since they are the same under \( H_1 \) as under \( H_2 \), so that they cancel in the odds.

Under the association hypothesis \( H_1 \), we must take into account the possibility that the GRB associated with the \( i \)-th SN may not have been detected by BATSE as a consequence of incomplete temporal exposure. We denote by \( E \) the proposition that BATSE was exposed to the direction of the SN when it occurred, and by \( \neg E \) the negation of \( E \). Then,

\[
P(D_i|H_1, I) = P(D_i, E|H_1, I) + P(D_i, \neg E|H_1, I)
\]

\[
= P(E|H_1, I)P(D_i|E, H_1, I) + P(\neg E|H_1, I)P(D_i|\neg E, H_1, I)
\]

\[
= P(\neg E|H_1, I)P(D_i|\neg E, H_1, I) + \epsilon P(D_i|E, H_1, I).
\]

(4)

Now, if BATSE was not exposed to the SN, then the \( N_i \) observed GRBs are purely coincidental, and the probability for observing them is the same as it would be if \( H_2 \) held instead of \( H_1 \):

\[
P(D_i|\neg E, H_1, I) = P(D_i|H_2, I).
\]

(5)

The second term in equation (4) is

\[
P(D_i|E, H_1, I) = P(x_i, t_i, \tau_i|H_1, I) \times P(N_i|\tau_i, E, H_1, I) \times P(\{\sigma_{ij}\}|N_i, E, H_1, I)
\]

\[
\times P(\{y_{ij}\}|\{\sigma_{ij}\}, N_i, x_i, E, H_1, I)
\]

\[
= f(x_i, t_i, \tau_i) \times \frac{e^{-R\tau_i}(R\tau_i)^{N_i-1}}{(N_i - 1)!} \times \left( \prod_{j=1}^{N_i} g(\sigma_{ij}) \right)
\]

\[
\times P(\{y_{ij}\}|\{\sigma_{ij}\}, N_i, x_i, E, H_1, I).
\]

(6)

Denoting by \( A_{ij} \) the proposition that the \( j \)-th GRB is associated with the \( i \)-th SN, we
have

\[
P(\{y_{ij}\}|\{\sigma_{ij}\}, N_i, x_i, \tau_i, E, H_1, I) = \sum_{j=1}^{N_i} P(A_{ij}|N_i, E, H_1, I) \times P(\{y_{ij}\}|A_{ij}, \{\sigma_{ij}\}, N_i, x_i, E, H_1, I)
\]

\[
= \sum_{j=1}^{N_i} \frac{1}{N_i} \left( \frac{1}{4\pi} \right)^{N_i-1} \times \frac{\exp \left[ (y_{ij} \cdot x_i - 1)/\sigma_{ij}^2 \right]}{2\pi\sigma_{ij}^2 \left( 1 - e^{-2/\sigma_{ij}^2} \right)},
\]

where we have used the assumed equality of the prior probabilities \(P(A_{ij}|N_i, E, H_1, I) = 1/N_i\), as well as the Fisher distribution for the position of the GRB associated with the SN.

Combining equations (7) and (8), we obtain

\[
P(D_i|E, H_1, I) = f(x_i, t_i, \tau_i) \times \frac{e^{-R\tau_i} (R\tau_i)^{N_i-1}}{(N_i - 1)!} \times \left( \prod_{j=1}^{N_i} g(\sigma_{ij}) \right) \times \frac{1}{N_i} \left( \frac{1}{4\pi} \right)^{N_i-1}
\]

\[
\times \sum_{j=1}^{N_i} \exp \left[ (y_{ij} \cdot x_i - 1)/\sigma_{ij}^2 \right] \frac{1}{2\pi\sigma_{ij}^2 \left( 1 - e^{-2/\sigma_{ij}^2} \right)}
\]

\[
= P(D_i|H_2, I) \times \frac{1}{R\tau_i} \times \sum_{j=1}^{N_i} \frac{\exp \left[ (y_{ij} \cdot x_i - 1)/\sigma_{ij}^2 \right]}{\frac{1}{2\sigma_{ij}^2 \left( 1 - e^{-2/\sigma_{ij}^2} \right)}}.
\]

Finally, inserting equations (9) and (5) into equation (4), and combining the result with equation (2), we obtain the following expression for the odds:

\[
\mathcal{O} = \prod_{i=1}^{N_{SN}} \left\{ (1 - \epsilon) + \epsilon \frac{1}{R\tau_i} \sum_{j=1}^{N_i} \frac{\exp \left[ (y_{ij} \cdot x_i - 1)/\sigma_{ij}^2 \right]}{\frac{1}{2\sigma_{ij}^2 \left( 1 - e^{-2/\sigma_{ij}^2} \right)}} \right\}
\]

\[
\equiv \prod_{i=1}^{N_{SN}} \mathcal{O}_i.
\]

Some of the properties of this expression for \(\mathcal{O}\) are worth pointing out:

The term proportional to the average temporal exposure \(\epsilon\) contains a sum over candidate GRB counterparts to the SN. Each term in the sum consists of an exponential
term that can penalize a candidate GRB counterpart for excessive angular distance from
the position of the SN, and a denominator that can reward a candidate counterpart for
having a small error circle. Thus, a GRB with a small error circle that is not far from the
position of the SN can produce a large term in the sum. A GRB whose error circle is very
far from the SN will produce an inconspicuous term in the sum, as will a GRB with a very
large error circle, irrespective of its position.

The term proportional to $\epsilon$ is also inversely proportional to $R_{\tau} \equiv \bar{N}_{i}$, the expected
number of GRBs observed by BATSE during the interval $\tau_{i}$. This term prevents the
expression for $O_{i}$ from becoming large as a consequence of a large $\bar{N}_{i}$ resulting in one or
more GRBs coinciding with the SN position purely by chance. In fact, we see that each
term in the sum is inversely proportional to $\bar{N}_{i} \sigma_{ij}^{2}/2$, a quantity that estimates the number
of bursts whose error circles bracket the SN by chance.

Finally, there the term $(1 - \epsilon)$. This term is an “escape hatch”, allowing for the
possibility that all of the candidate GRB counterparts are terrible fits because the actual
counterpart was missed due to incomplete temporal exposure. This term has an interesting
consequence: while a well-correlated individual SN-GRB pair may produce a large $O_{i}$, the
lack of such a pair cannot in general produce a tiny $O_{i}$ if $\epsilon$ is not close to unity. Thus,
no individual SN can rule out the hypothesized association. However, $H_{1}$ may still be
convincingly be ruled out if we have a collection of many SNe, the great majority of which
have no plausible GRB counterpart, since in that case we will have $O \approx (1 - \epsilon)^{N_{sn}}$, which
can be small.

The interpretation of the numerical value of $O$ is straightforward: if $O \gg 1$ then the
evidence favors $H_{1}$. If $O \ll 1$, the evidence favors $H_{2}$. If $O \sim 1$, then the evidence is
insufficient to make a decisive case either way.

Note that $O \to 1$ as $\epsilon \to 0$. In other words, in the limit of no GRB observations
at all, the evidence becomes insufficient to budge the odds from the assigned prior value
$P(H_1|I) / P(H_2|I) = 1$. A similarly plausible limiting behavior of the odds is $\lim_{\tau \to \infty} \mathcal{O} = 1$, which may be derived by considering the expected number of GRBs whose error circles bracket the SN by chance in the long run. Thus, in the limit of a total lack of knowledge about the epoch of the SN explosion, the proliferation of candidate GRB counterparts introduces noise that swamps our ability to distinguish between the two hypotheses.

It is also worth pointing out that this expression for the odds bears some resemblance to the odds favoring the association of GRBs with host galaxies derived by Band & Hartmann (1998). That work also compared two “simple” hypotheses — either GRBs have (intensity-redshift correlated) host galaxies, or they don’t. Their expression for the odds (equation [5] of Band & Hartmann 1998) bears a structural resemblance to our equation (10), including a sum over possible counterpart galaxies and a term accounting for the possibility that the host galaxy was not observed because its luminosity was below the detection threshold. The main difference is that in their study, GRBs play the role that SNe play in ours, with their GRB error circles replacing our uncertainty in the time of the SN explosion, and with the fraction of galaxies above the detection threshold replacing the BATSE exposure.

There are two useful generalizations of this method: we can add Interplanetary Network (IPN) annuli to the data when they are available, and we can consider a more general model in which not all SNe produce detectable GRBs.

2.3. IPN Annuli

The inclusion of IPN annuli in the data is straightforward. The IPN catalog ([Hrley et al. 1998]) gives the orientation of the line joining two burst-detecting spacecraft, the
angle $\theta$ between this direction and the direction to the burst (which is the angular radius of the IPN annulus), and a 3-$\sigma$ error in this angle, which we denote by $\alpha$. Thus $\theta$ is the angular radius and $\alpha$ is the angular width of the IPN annulus. Analogously, we define $\theta_i$ as the angle between the line joining two burst-detecting spacecraft and the $i$th SN, and $\theta_{ij}$ as the angle between the line joining two burst-detecting spacecraft and the $j$th burst possibly associated with this SN. We assume that under $H_1$, $\cos \theta_{ij}$ has a Gaussian distribution with mean $\cos \theta_i$ and error $\sigma_{ij} = \sin \theta_{ij} \alpha_{ij}/3$ (with $\alpha_{ij}$ in radians). We also assume that under $H_2$, $\cos \theta_{ij}$ is distributed uniformly in the range $[-1, 1]$. When an IPN annulus is available, we replace the BATSE position by the annulus in the data set. It is not difficult to show that the odds for an individual SN then become

$$O_i = (1 - \epsilon) + \epsilon \frac{1}{\bar{R} \bar{t}_i} \sum_{j=1}^{N_i} L_{ij},$$

where

$$L_{ij} = \begin{cases} 
\frac{\exp \left[ -{(\cos \theta_{ij} - \cos \theta_i)^2 / 2\sigma_{ij}^2} \right]}{\sqrt{2\pi \sigma_{ij}^2}} ; & \text{IPN annulus available}, \\
\frac{\exp \left[ \frac{\left( \mathbf{y}_{ij} \cdot \mathbf{x}_i - 1 \right)}{\sigma_{ij}^2} \right]}{\frac{1}{2} \sigma_{ij}^2 \left( 1 - e^{-2/\sigma_{ij}^2} \right)} ; & \text{Otherwise} 
\end{cases}$$

(12)

The overall odds are still given by the product of the individual odds.

### 2.4. More Complicated Model

We may generalize $H_1$ to a model $H'_1$ in which only a fraction $f$ of SNe produce observable GRBs. This may be due to beaming, or for some other reason. The necessary modification of the above formulas is straightforward. We assume a uniform prior density for $f$ in the range $[0, 1]$. The odds favoring model $H'_1$ over model $H_2$ are denoted by $O'$, and given by the expression

$$O' = \int_0^1 df \; O(f),$$

(13)
where
\[ O(f) \equiv \prod_{i=1}^{N_{\text{SN}}} \left\{ (1 - \epsilon f) + \epsilon f \frac{1}{R_{\tau_i}} \sum_{j=1}^{N_i} L_{ij} \right\}. \] (14)

In other words, \( O(f) \) is constructed by setting the effective probability of observing a GRB associated with a SN to \( \epsilon f \), rather than to \( \epsilon \).

The quantity \( O' \) can help us decide whether hypothesis \( H'_1 \) or \( H_2 \) is favored by the evidence. If we were to find that \( H'_1 \) is strongly favored, we could then attempt to estimate likely values for \( f \). For this purpose, we may use the posterior probability density for \( f \), given by
\[ P(f|H'_1, I) = O(f)/O' \] (15)

We may construct a point estimate for \( f \) by locating the maximum of \( P(f|H'_1, I) \), and we may obtain interval estimates for \( f \) by finding intervals that contain a prescribed amount of probability — 68%, say — as calculated by integrating \( P(f|H'_1, I) \).

The quantity \( O' \) is subject to an ambiguity: it is dependent upon our choice of prior probability density for \( f \). If instead of a uniform prior density for \( f \) in the range \([0, 1]\) we had chosen, for example, a uniform prior density in the range \([0, f_{\text{SN}}^0]\) (with \( 0 < f_{\text{SN}}^0 < 1 \)), then the expression for \( O' \) given in Equation (13) would be increased by a factor of \( f_{\text{SN}}^0 - 1 \). Thus, a model that predicts small values of \( f \) might find the comparison with data less damaging than a model that is agnostic about the value of \( f \).

However, with \( P(f|H'_1, I) \) in hand, we may, if we wish, take a different approach to the assessment of the plausibility of \( H'_1 \). Instead of calculating the odds, we may calculate a \( 3 - \sigma \) upper bound for \( f \). The dependence of this upper bound on the number of SNe in the sample may be calculated approximately as follows:

Assuming \( f \) is in fact small, so that not many coincidences are observed, then the
dependence of $P(f|H'_1, I)$ on $f$ is seen from Equation (14) to be approximately

$$P(f|H'_1, I) \sim (1 - \epsilon f)^{N_{\text{SN}}}.$$  

(16)

After normalizing this expression, we may integrate it to produce the cumulative probability:

$$Q(f^0_{\text{SN}}) = \int_0^{f^0_{\text{SN}}} df P(f|H'_1, I) \approx \frac{1 - (1 - \epsilon)^{N_{\text{SN}}+1}}{1 - (1 - \epsilon)^{N_{\text{SN}}+1}}.$$  

(17)

The quantity $Q(f^0_{\text{SN}})$ is the significance level of our upper limit, say 99.73%. We may solve Equation (17) for $f^0_{\text{SN}}$, obtaining

$$f^0_{\text{SN}} = \frac{1}{\epsilon} \left\{ 1 - \left[ 1 - Q \times (1 - (1 - \epsilon)^{N_{\text{SN}}+1}) \right]^{1/N_{\text{SN}}+1} \right\}.$$  

(18)

The dependence of $f^0_{\text{SN}}$ on $N_{\text{SN}}$ is plotted in Figure 5. It is evident from the figure that even assuming maximum exposure, the 3-σ upper limit on $f$ can only be expected to decrease very slowly with $N_{\text{SN}}$. It is straightforward to show that for large $N_{\text{SN}}$, the behavior of $f^0_{\text{SN}}$ is $f^0_{\text{SN}} \approx -\ln(1 - Q)/\epsilon(N_{\text{SN}} + 1)$. Given the form of this dependence on $N_{\text{SN}}$, and given the relatively low rate ($\sim 10 \text{ yr}^{-1}$) at which Type Ib-Ic SNe are currently being discovered, it does not seem likely that observational evidence can constrain $f$ in a significant manner anytime soon.

3. Results

We now apply the above methodology to the question of whether or not the odds favor the hypothesis that a particular class of Type I SNe produce GRBs over the hypothesis that they do not. We first discuss the samples of GRBs and Type I SNe that we use to address this question.
3.1. GRB and SN Samples

The sample of GRBs that we use in our analysis consists of the BATSE 4B catalog [Meegan et al. 1998, http://www.batse.msfc.nasa.gov/data/grb/4bcatalog/], and BATSE bursts that occurred subsequent to the 4B catalog but before 1 May 1998 [http://www.batse.msfc.nasa.gov/data/grb/catalog/]. The BATSE 4B catalog consists of 1637 bursts, while the online archive contains an additional 497 bursts through 1 May 1998. We also use the Ulysses supplement to the BATSE 4B catalog, which contains 219 BATSE bursts for which 3rd IPN annuli have been determined [Hurley et al. 1998]. Hurley (private communication, 1998) has kindly made available at our request 3rd IPN annuli for an additional 9 BATSE bursts that occurred subsequent to the period of the BATSE 4B catalog but before 1 May 1998.

We have compiled three Type I SNe samples. The first is a sample of 37 Type Ia SNe (see Table 1) at low redshift ($z < 0.1$). The data for most of these events were kindly provided to us by the CfA SN Search Team (Riess 1998, private communication). The second is a sample of 46 moderate redshift ($0.1 < z < 0.830$) Type Ia SNe (see Table 2). The Supernova Cosmology Project (SCP) kindly supplied the data for nearly all of these events (Perlmutter 1998, private communication). The third sample consists of 20 Type Ib, Ib/c, and Ic SNe (see Table 3). We have compiled the data for these last events from information available in the IAU Circulars and in various SNe catalogs (see the footnotes to Table 3). The procedure we use to estimate the range of possible explosion dates $\Delta T$ for each SN event depends on the type of SN and on the information available.

For the low-redshift Type Ia SNe, estimate the explosion date using the formula:

$$ T = T_{\text{max}} - 18.8d \times (1 + z), $$

where $T_{\text{max}}$ is the estimated or observed date of maximum light and $z$ is the redshift of
the SN. The uncertainty in the date of maximum light is taken to be $\pm 1d$ for the CfA data supplied by Riess. In the sixteen cases where the date of maximum light has been interpreted from spectra of the SN, we assign a greater uncertainty to the date of maximum light. When the language associated with the spectral dating describes the observation as having been made near maximum light, we adopt $\pm 6d$ for the uncertainty in the date of maximum light. We expand the uncertainty to $\pm 10d$ when the date of maximum light is estimated to be more than ten days prior to the date on which the spectrum was taken or when the language of the Circular suggests additional uncertainties. We reject events for which our evaluation of the uncertainty in the date of maximum light exceeds $\pm 10d$. The total uncertainty in the explosion date that we assign is the linear sum of the uncertainty in the date of maximum light and an additional $\pm 2d$ for the uncertainty in the rise time predicted by Type Ia explosion models. The range of possible explosion dates $\Delta T$ is given by adding and subtracting the total uncertainty to/from the estimated explosion date $T$.

For the moderate $z$ Type Ia sample, the data provided by Perlmutter include the SCP’s best estimate of the explosion date, which was computed using the formula:

$$T = T_{\text{max}} - 18.8d \times (1 + z) \times s,$$

where $T_{\text{max}}$ and $z$ are the same as before and $s$ is stretch factor, determined from the rate of decline of the Type Ia light curve and applied to the rising light curve (Perlmutter, et al. 1998). The uncertainty assigned by the SCP to the explosion date is $\pm 2.5d$, which we have rounded up to $\pm 3d$ for simplicity (see Table 2).

Four additional SNe events are included in this sample. The data for these are taken from the Circulars and the estimated explosion dates are calculated in the same way as were those for the low-$z$, Type Ia sample.

For the sample of Type Ib, Ib/c, and Ic SNe, when it was possible to estimate the date
of maximum light, we calculated the estimated explosion date using the formula:

$$T = T_{\text{max}} - T_{\text{rise}} \times (1 + z),$$

(21)

where $T_{\text{max}}$ and $z$ are the same as before, and $T_{\text{rise}}$ is taken to be 15d for Type Ib, 13d for Type Ib/c, and 12d for Type Ic events. The uncertainty in the date of maximum light and the range of possible explosion dates $\Delta T$ are found by the same procedure as for the low-$z$, Type Ia sample.

When this method yields a range of possible explosion dates that extends beyond the discovery date of the SN event, we take the end of the range to be the discovery date. Similarly, we limit the beginning of the range of possible explosion dates when it extends to a date earlier than the latest date on which an image was taken that does not show the SN event. We make the conservative assumption the the explosion occurred no earlier than two days prior to the image date. This two-day allowance provides for the possibility that the brightness of the SN may have been less than the limiting magnitude of the observation on the date the image was taken (see Table 3).

There are three SNe in Table 3 (SN 1997C, 1998T, and 1998cc) for which reliable information about the date of maximum light was unavailable. In these three cases, we were able to use other information to estimate the range of possible explosion dates.

For SN 1997C, we take the earliest possible explosion date to be two days prior to the date of an image that shows no evidence of the SN. As before, the two-day allowance provides for the possibility that the brightness of the SN may have been less than the limiting magnitude of the observation on the date the image was taken. There is also a spectrum of this SN which shows it to be a Type Ic event 21 to 29 days past maximum light (Li, et al. 1997). As a conservative estimate, we use that date as the latest possible date of maximum light, and subtract 14 days ($T_{\text{rise}} = 12d$ for a Type Ic SN plus 2d for the uncertainty in the rise time predicted by explosion models), to find the latest possible
SN 1998T increased in brightness between two successive photometric observations (Li, Li, & Wan 1998), while a spectrum taken of SN 1998cc showed features indicating that the SN had not yet reached maximum light (Jha, Garnavich, & Kirshner 1998). Again, we are conservative and take the date of these observations to be the earliest possible date of maximum light. Consequently, the earliest possible explosion date for each SN is 17 days prior to the pre-maximum observation ($T_{\text{rise}} = 15d$ for a Type Ib SN plus 2d for the uncertainty in the rise time predicted by explosion models). The latest possible explosion date for these events is taken to be the discovery date (see Table 3).

Our sample of twenty Type Ib, Ib/c and Ic SNe includes five of the six SNe considered by Wang & Wheeler (1998); the sixth (SN 1992ad) is a Type II SN (McNaught 1992, Filippenko 1992) that was mis-classified as a Type Ic SN by Wang & Wheeler. In two of the remaining five cases, the range of possible explosion dates $\Delta T$ that we derive agrees closely with theirs; in the other three cases they do not. In the case of SN 1996N, the beginning of the range of possible explosion dates that we adopt is similar to that of Wang & Wheeler (1998), but the end of the range is two weeks later. In the case of SN 1997ei, the range of possible explosion dates that we adopt is of the same duration as that of Wang & Wheeler (1998), but shifted later by one month. The differences between our and Wang & Wheeler’s ranges of possible explosion dates for SN 1996N and SN 1997ei do not affect our results. However, in the third case (SN 1997X), our range of possible explosion dates begins 14$^d$ later than theirs because it is limited by the existence of an image that shows no evidence of the SN (Nakano & Aoki 1997); as a result, the GRB that they associate with SN 1997X is excluded by the revised range of possible explosion dates, and this makes a modest difference in our results (see below).

Finally, Wang & Wheeler (1998) list SN 1997ef in their Table 1, but do not classify
it and therefore do not include it in their analysis. We are able to classify it as a Type Ic SN (Iwamoto, et al. 1998, Garnavich, et al. 1997) and we include it in our analysis (see Tables 3 and 4); the range of possible explosion dates we derive for this SN is shifted later by about one month relative to that given by Wang & Wheeler (1998).

3.2. Type Ia Supernovae

We first apply our methodology to the 83 events in our sample of Type Ia SNe. Since it is not expected that Type Ia SNe can produce GRBs, these events constitute a “control” sample. The results we find for this sample illustrate the power of the methodology. We find overwhelming odds ($10^{21}:1$) against the hypothesis that all Type Ia SNe produce observable GRBs (see Table 5). Dividing our sample of Type Ia SNe into two subsamples, a low-$z$ ($z \leq 0.1$) subsample and a moderate-$z$ ($z > 0.1$) subsample, we find overwhelming odds against the hypotheses either that all low-$z$ Type Ia SNe or all moderate-$z$ Type Ia SNe produce observable GRBs (again, see Table 5).

We also find large odds (34 : 1) against the hypothesis that some fraction $f_{SN}$ of Type Ia SNe produce observable GRBs (see Table 5). Again dividing our sample of Type Ia SNe into two subsamples, a low-$z$ ($z \leq 0.1$) subsample and a moderate-$z$ ($z > 0.1$) subsample, we find moderate odds against the hypotheses that some fraction $f_{SN}$ of either low-$z$ Type Ia SNe or moderate-$z$ Type Ia SNe produce observable GRBs (again, see Table 5).

These results are not unexpected, given that an association between Type Ia SNe is deemed unlikely on theoretical grounds and no observational evidence has been reported linking the two. Thus, the Type Ia SNe constitute a control sample, validating the methodology we have developed, and illustrating that, even with a BATSE mean temporal exposure efficiency of $\epsilon = 0.48$, a SNe sample of moderate size is sufficient to provide a
severe test of the hypothesis that all Type Ia SNe produce observable GRBs, and a strong
test of the hypothesis that a fraction $f_{SN}$ do.

3.3. Type Ib, Ib/c and Ic Supernovae

Applying our methodology to our sample of 20 Type Ib-Ic SNe, we find very strong
odds (6000 : 1) against the hypothesis that all Type Ib-Ic SNe produce observable GRBs
(see Table 5). We find modest odds (6 : 1) against the hypothesis that some fraction $f_{SN}$
of Type Ia SNe produce observable GRBs (see Table 5). If we nevertheless assume that
this hypothesis is correct, we find that the fraction $f_{SN}$ of Type Ib, Ib/c and Ic SNe that
produce observable GRBs must be less than 0.17, 0.42, and 0.70 with 68%, 95%, and 99.6%
probability, respectively. These limits are relatively weak because of the modest size (20
events) of our sample of Type Ib-Ic SNe.

In order to verify that our results are insensitive to the range of possible explosion
dates $\Delta T$ that we have derived, we repeated our analysis of the sample of Type Ib, Ib/c and
Ic SNe with $\Delta T$ increased by $\pm$ 1, 2, 3, 5, and 10 days, except when the beginning of the
range is limited by an image that does not show the SN or the end of the range is limited
by the SN discovery date. The resulting odds vary little (see Figure 1), demonstrating the
robustness of our conclusions.

4. Discussion

4.1. Implications of Our Results

We have applied a methodology based Bayesian inference to a sample of 83 Type Ia
SNe. We find overwhelming odds against the hypothesis that all Type Ia SNe produce
observable gamma-ray bursts, irrespective of whether the SNe are at low- or high-redshift, and large odds against the hypothesis that a fraction of Type Ia SNe produce observable GRBs.

Applying this methodology to a sample of 20 Type Ib-Ic SNe, we find very large odds against the hypothesis that all Type Ib-Ic SNe produce observable GRBs, and modest odds against the hypothesis that some fraction $f_{SN}$ of Type Ia SNe produce observable GRBs (see Table 5). If we nevertheless assume that this hypothesis is correct, we find that the fraction $f_{SN}$ of Type Ib, Ib/c and Ic SNe that produce observable GRBs is less than 0.70 with 99.7% probability. This limit is relatively weak because of the modest size (20 events) of our sample of Type Ib-Ic SNe.

Type Ib, Ib/c and Ic SNe are now being found at a rate of about eight a year, so that the size of the sample of known Type Ib-Ic SNe should double within three years and triple within about five years. One might hope that future analyses, using the statistical methodology that we have presented here, could either show that the association between Type Ib-Ic SNe and GRBs is rare, or confirm the proposed association. Unfortunately, equation (18) and Figure 2 show that achieving the former will be difficult: the limit on the fraction $f_{SN}$ of Type Ib-Ic SNe that produce observable GRBs scales like $N_{SN}^{-1}$ for large $N_{SN}$, and therefore tripling the size of the sample of known Type Ib-Ic SNe without observing an additional possible SN–GRB association would only reduce the 99.7% probability upper limit on $f_{SN}$ to 0.24.

An alternative approach is to use the upper limit on the fraction of BATSE bursts that can have come from a homogeneous, isotropic distribution to place an upper limit on the fraction of nearby Type Ib-Ic SNe that can have produced observable GRBs. We infer from Smith & Lamb (1993) that no more than roughly 20% of the BATSE burst can have come from such a population, corresponding to $\approx 400$ bursts during the $\approx 7$ years covered by our
GRB 980425 had a 1024 msec peak flux $F_{\text{peak}}^{1024} = 0.9$ photons cm$^{-2}$ s$^{-1}$, which is less than a factor of four above the BATSE threshold. Consequently, assuming that the association between SN 1998bw and GRB 980425 is real and that the GRBs produced by Type Ib-Ic SNe are standard candles, BATSE only detects Type Ib-Ic SNe that lie at redshifts smaller than $z \approx 2z_{\text{SN1998bw}} = 0.016$. This is a sampling distance of $\sim 48h^{-1}$ Mpc.

Given a supernova rate of $\sim 2$ per $L_\star$ galaxy per century [Strom 1995], a density of $L_\star$ galaxies of $0.01h^3$ Mpc$^{-3}$, and that approximately $2/7$ of these are SNe of Types Ib, Ib/c and Ic (Woosley & Weaver 1986, Strom 1995), we find that during the $\approx 7$ years covered by our study, the number of SNe that occurred and could have produced GRB detectable by BATSE is 185. Given the BATSE average temporal exposure $\epsilon = 0.48$, this implies that at most $\sim 90$ such SNe could have been detected by BATSE. Comparing this number with the number $\approx 400$ of GRBs that can have come from such a population, we derive an upper limit on the fraction of Type Ib-Ic SNe that can have produced an observable GRB of $F_{\text{peak}}^{\text{SN}} \approx 400/90 > 1$. This means that no constraint may be placed on $F_{\text{peak}}^{\text{SN}}$ by this method, and shows that the method used in the present study, as opposed to modeling of the BATSE angular and brightness distributions, provides the more stringent constraint.

One can also approach the proposed association between SNe and GRBs from the opposite direction. The interesting question, from this point of view, is what fraction $f_{\text{GRB}}$ of the GRBs detected by BATSE could have been produced by Type Ib-Ic SNe? The above discussion indicates that $f_{\text{GRB}}$ can be no more than $f_{\text{GRB}}^0 \lesssim (90/2000)f_{\text{SN}}^0 = 0.045f_{\text{SN}}^0 \lesssim 0.03$, where in the last step we have used the 99.7% probability upper limit derived from our Bayesian analysis.

This question can also be addressed by examining relatively accurate ($\lesssim 1'$) GRB positional error circles for the presence of Type Ib-Ic SNe. BeppoSAX observations have
already placed a weak limit on this fraction: Setting aside GRB 980425, none of the remaining 14 BeppoSAX WFC GRB error circles has been found to contain a Type Ib-Ic SN. The HETE-II mission is expected to place somewhat stronger limits on this fraction (or possibly confirm the proposed association between GRBs and Type Ib-Ic SNe), since it is expected to provide a larger number of relatively accurate positions for GRBs.

The limit \( f_{\text{GRB}}^0 \) that can be placed on the fraction \( f_{\text{GRB}} \) of GRBs that can have been produced by SNe is given by equation (18), with the number of SNe, \( N_{\text{SN}} \), replaced by the number of GRBs, \( N_{\text{GRB}} \), and \( \epsilon \) set equal to one (we assume that the efficiency of detecting a Type Ib-Ic SN in a relatively accurate GRB positional error circle is 100%). Thus the dashed curve in Figure 2 shows this limit as a function of \( N_{\text{GRB}} \). Equation (18) and Figure 2 show that this limit scales as \( N_{\text{GRB}}^{-1} \) for large \( N_{\text{GRB}} \), as one intuitively expects. Consequently, placing a 99.7% upper limit on \( f_{\text{GRB}} \) that is more stringent than the rough limit derived above, or confirming the proposed association between GRBs and Type Ib-Ic SNe if such associations are rare, will require a mission that produces relatively accurate (\( \leq 1' \)) positions for a very large number (\( \geq 1000 \)) of GRBs.

### 4.2. Comparison with Other Work

Wang & Wheeler (1998) have reported that analysis of a sample of six Type Ib-Ic SNe using an \emph{a posteriori} frequentist statistic rules out at the \( 10^{-5} \) significance level the null hypothesis that the positions on the sky of Type Ib, Ib/c, and Ic SNe and GRBs are uncorrelated. The statistic they used is the probability that at least one of the BATSE positional error circles for the GRBs occurring within the range of possible explosion dates \( \Delta T \) of a Type Ib-Ic SN includes the position of the SN.

In order to understand the apparent discrepancy between our results and those of
Wang & Wheeler (1998), we have re-analyzed their sample of Type Ib-Ic SNe, except for the elimination of SN1992ad (which is a Type II SN, not a Type Ic as Wang & Wheeler assumed), and corrections to their ranges of possible explosion dates. We use a generalization of their methodology that is applicable to a sample in which not all of the SNe are bracketed by a GRB error circle. We have also applied this generalization of their methodology to our larger sample of 20 Type Ib-Ic SNe.

Following Wang & Wheeler (1998), we ascribe to the $i$th SN a probability $f_i$ that its position should be “bracketed” by chance by at least one of the positional error circles of the GRBs that occurred during its range of possible explosion dates $\Delta T$. We assume the power-law model of BATSE systematic errors (Graziani & Lamb 1996) and combine these systematic errors with the BATSE statistical errors to produce a total 1-$\sigma$ error circle radius for each burst. We then multiply that error circle radius by 3 to produce the bracketing circle radius $\mu_i$. Note that $\mu$ is larger than the “3-$\sigma$” (99.7% probability) error circle radius, which would be obtained by multiplying the 68.3% radius by 2.27 - as may be inferred from the $\chi^2$ distribution with two degrees of freedom. We multiply by 3 in order that our procedure agree as closely as possible with that of Wang & Wheeler (1998).

Following Wang & Wheeler (1998), the $f_i$ are given by

$$f_i = 1 - \prod_{j=1}^{N_i} \left[ 1 - \frac{1}{2} (1 - \cos \mu_j) \right]. \quad (22)$$

Wang & Wheeler (1998) asserted that the positions of all six of the SNe in their sample were bracketed by the positional error circles of at least one GRB, and employed a probability that was simply the product of all their $f_i$. In the more general case in which not all the SN are bracketed, one cannot apply their procedure, or even merely multiply by $f_i$ for each bracketed SN and by $1 - f_i$ for each non-bracketed SN. The reason this latter procedure fails is that it is guaranteed to produce a small number — irrespective of whether or not many SN were bracketed — because the product of many numbers that lie between
zero and one can be small, even if most of them lie near one.

The problem is that by calculating the probability that this SN should be bracketed and that one should not be, we are inquiring after a peculiar state of the data, rather than a generic one. The distinction is analogous to the distinction between microstates and macrostates in statistical mechanics. We should instead calculate the probability of some generic feature of the observed data. We choose the number \( N_b \) of bracketed SN as our generic property, and calculate the probability that the observed number of bracketed SN should have been \( N_b^{(\text{obs})} \) or higher. Obviously, there are many contributing configurations \( C(N_b) \equiv \{q_i; i = 1, \ldots, N_{\text{sn}}\} \) in which \( N_b \) of the \( q_i \) assume the values \( f_i \), and the remainder assume the value \( 1 - f_i \). We must sum over the contribution of all such configurations. Our significance is then given by

\[
S = \sum_{N_b = N_b^{(\text{obs})}}^{N_{\text{sn}}} \sum_{C(N_b)} \prod_{i=1}^{N_{\text{sn}}} q_i.
\]

(23)

For the modified Wang & Wheeler sample (SN1994I, SN1996N, SN1997X, SN1997ei, and SN1998T), we find that all but SN1997X are bracketed by a GRB in the explosion time window. The resulting significance is \( S = 2.6\% \). By comparison, Wang & Wheeler (1998) found \( S = 1.5 \times 10^{-5} \). For the full sample of Type Ib-Ic SNe, we find 9 bracketed SNe out of 20, with a significance \( S = 35\% \).

Thus, upon re-analyzing the corrected sample of Type Ib-Ic SNe studied by Wang & Wheeler (1998), we find no significant evidence for an association between GRBs and Type Ib, Ib/c and Ic SNe. Moreover, using a sample of Type Ib, Ib/c and Ic SNe that is four times larger, we find that the significance becomes even weaker — not stronger, as would be expected if the association were real. We conclude that a frequentist analysis similar to that performed by Wang & Wheeler (1998) shows no evidence for an association between Type Ib-Ic SNe and GRBs, consistent with the very strong evidence against such a correlation that we find from our Bayesian analysis.
Kippen et al. (1998) have approached the proposed association between SNe and GRBs from the opposite direction. They have asked the question, what fraction of BATSE bursts can have been produced by known SNe? They find no evidence of any correlation between SNe and BATSE bursts, and derive a 3-σ (99.7%) limit on any such fraction of 1.5%, which corresponds to ≈ 18 bursts. Unfortunately, this result is not very interesting because known SNe comprise such a small fraction (≲ 10^{-5}) of the SNe that occurred during the time interval they study. This, together with our earlier result that the fraction of BATSE bursts that could be produced by SNe is ≲ 3%, implies that a negligible fraction (≈ 10^{-7}) of BATSE bursts could be produced by known SNe. Kippen et al.‘s result is further weakened by the fact that their study does not distinguish among Type II, Type Ia, and Type Ib-Ic SNe, whereas it is only the last type of SNe that are thought possibly to produce GRBs.

5. Conclusions

We find very large odds against the hypothesis that all Type Ib-Ic SNe produce observable GRBs, and moderate odds against the hypothesis that a fraction of Type Ib-Ic supernovae produce observable GRBs. We have also re-analyzed a corrected version of the Wang & Wheeler (1998) sample of Type Ib-Ic SNe, as well as our larger sample of 20 Type Ib-Ic SNe, using a generalization of their frequentist method. We find no significant evidence of a correlation between Type Ib-Ic SNe and GRBs in either case, consistent with the very strong evidence against such a correlation that we find from our Bayesian analysis.

While these statistical studies cannot address the question of whether a particular GRB is produced by a particular Type Ib-Ic SNe (i.e., GRB 980425 by SN 1998bw), they show not only that there is no evidence of an association between Type Ib-Ic SNe and GRBs, but that the odds against the hypothesis that all Type Ib-Ic SNe produce observable GRBs are very large, and the odds against the hypothesis that a fraction of them do are
moderate. These results suggest that considerable caution is warranted before accepting the association between GRB 980425 and SN 1998bw. This is particularly the case, given that there exists in the BeppoSAX WFC error circle a fading X-ray source whose behavior is consistent with the power-law temporal decline observed for the X-ray afterglows of other BeppoSAX bursts (Pian et al. 1998b). This fading X-ray source might well be the counterpart to the GRB, rather than SN 1998bw.

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Fig. 1.— Dependence of the 3σ upper limit for $f$ on the number of observed supernovae, assuming no excess of GRB-SN associations is observed above what is expected by chance. The upper curve was calculated using the average BATSE exposure $\epsilon = 0.48$ (Hakkila et al. 1998), while the lower curve was calculated assuming complete exposure.
Fig. 2.— Variation of the odds favoring the association hypothesis $H_1$ with added uncertainty in the time window of occurrence of the supernova explosion. The added uncertainty is the number of days added (linearly) to the half-width of the time window.
Table 1. Estimated Explosion Dates and Positions for Low-z Type Ia Supernovae

| SN    | z     | Discovery | Max. Light | Range Exp. Dates | RA     | DEC     |
|-------|-------|-----------|------------|------------------|--------|---------|
| 1993B | .0690 | 930117    | 930118 ±6\textsuperscript{a} | 921221 – 930106 | 10 34 51.38 | −34 26 30.0 |
| 1993Q | .0300 | 930528    | 930513 ±10\textsuperscript{b} | 930412 – 930506 | 20 35 46.94 | −42 47 33.4 |
| 1993ac| .0493 | 931013    | 931010 ±1\textsuperscript{c} | 930918 – 930924 | 05 46 23.55 | +63 22 07.0 |
| 1993ae| .0190 | 931107    | 931031 ±1\textsuperscript{c} | 931009 – 931015 | 01 29 48.92 | −01 58 37.2 |
| 1994B | .0899 | 940116    | 940123 ±1\textsuperscript{d} | 931231 – 940106 | 08 20 40.92 | +15 43 48.4 |
| 1994C | .0515 | 940305    | 940227 ±1\textsuperscript{d} | 940205 – 940211 | 07 56 40.27 | +44 52 19.3 |
| 1994M | .0230 | 940429    | 940430 ±1\textsuperscript{c} | 940408 – 940414 | 12 31 08.61 | +00 36 19.9 |
| 1994Q | .0290 | 940602    | 940526 ±1\textsuperscript{c} | 940504 – 940510 | 16 49 51.14 | +40 25 55.8 |
| 1994S | .0152 | 940604    | 940614 ±1\textsuperscript{c} | 940523 – 940529 | 12 31 21.86 | +29 08 04.2 |
| 1994T | .0347 | 940611    | 940607 ±1\textsuperscript{c} | 940516 – 940522 | 13 18 56.16 | −01 53 15.0 |
| 1994U | .0043 | 940627    | 940705 ±1\textsuperscript{d} | 940614 – 940620 | 13 04 56.13 | −07 56 51.5 |
| 1994ae| .0043 | 941114    | 941128 ±1\textsuperscript{c} | 941107 – 941113 | 10 47 01.95 | +17 16 31.0 |
| 1995D | .0065 | 950210    | 950219 ±1\textsuperscript{c} | 950129 – 950204 | 09 40 54.75 | +05 08 26.2 |
| 1995E | .0116 | 950220    | 950224 ±1\textsuperscript{c} | 950202 – 950208 | 07 51 56.75 | +73 00 34.6 |
| 1995M | .0531 | 950422    | 950415 ±1\textsuperscript{d} | 950324 – 950330 | 09 38 41.78 | −12 20 07.9 |
| 1995ac| .0500 | 950922    | 951002 ±1\textsuperscript{c} | 950910 – 950916 | 22 45 34.14 | −08 45 04.7 |
| 1995ae| .0677 | 950922    | 950922 ±1\textsuperscript{d} | 950850 – 950922 | 23 16 55.65 | −02 04 36.4 |
| 1995ak| .0230 | 951027    | 951030 ±1\textsuperscript{c} | 951008 – 951014 | 02 45 48.83 | +03 13 50.1 |
| 1995al| .0051 | 951101    | 951107 ±1\textsuperscript{c} | 951017 – 951023 | 09 50 55.97 | +33 33 09.4 |
| 1995bd| .0160 | 951219    | 960102 ±1\textsuperscript{c} | 951211 – 951217 | 04 45 21.24 | +11 04 02.5 |
| 1996C | .0296 | 960215    | 960212 ±1\textsuperscript{c} | 960121 – 960127 | 13 50 48.60 | +49 19 07.1 |
| 1996X | .0068 | 960412    | 960417 ±1\textsuperscript{c} | 960327 – 960402 | 13 18 01.13 | −26 50 45.3 |
| 1996V | .0247 | 960328    | 960330 ±1\textsuperscript{d} | 960308 – 960314 | 11 21 31.23 | +02 48 40.4 |
Table 1—Continued

| SN   | z     | Discovery | Max. Light | Range Exp. Dates | RA          | DEC          |
|------|-------|-----------|------------|------------------|-------------|--------------|
| 1996Z | .0076 | 960516    | 960513 ±1<sup>c</sup> | 960422 − 960428  | 09 36 44.82 | −21 08 51.7  |
| 1996ai| .0032 | 960616    | 960620 ±1<sup>c</sup> | 960530 − 960605  | 13 10 58.13 | +37 03 35.4  |
| 1996bk| .0068 | 961012    | 961009 ±1<sup>c</sup> | 960918 − 960924  | 13 46 57.98 | +60 58 12.9  |
| 1996bl| .0068 | 961011    | 961020 ±1<sup>c</sup> | 960929 − 961005  | 00 36 17.97 | +11 23 40.5  |
| 1996bo| .0173 | 961018    | 961030 ±1<sup>c</sup> | 961008 − 961014  | 01 48 22.80 | +11 31 15.8  |
| 1996bv| .0167 | 961103    | 961116 ±1<sup>c</sup> | 961025 − 961031  | 06 16 13.00 | +57 03 08.9  |
| 1996by| .0137 | 961214    | 961220 ±6<sup>e</sup> | 961125 − 961207  | 05 58 24.96 | +68 27 12.1  |
| 1996ca| .0167 | 961215    | 961220 ±6<sup>e</sup> | 961125 − 961207  | 22 30 59.26 | −13 59 50.9  |
| 1997Y | .0162 | 970202    | 970209 ±6<sup>e</sup> | 970115 − 970127  | 12 45 31.40 | +54 44 17.0  |
| 1997bp| .0077 | 970406    | 970408 ±1<sup>d</sup> | 970318 − 970324  | 12 46 53.75 | −11 38 33.2  |
| 1997dg| .0340 | 970927    | 970930 ±6<sup>e</sup> | 970905 − 970917  | 23 40 14.21 | +26 12 11.8  |
| 1997dt| .0073 | 971122    | 971123 ±6<sup>e</sup> | 971030 − 971111  | 23 00 02.93 | +15 58 50.9  |
| 1998V | .0176 | 980310    | 980304 ±6<sup>e</sup> | 980210 − 980222  | 18 22 37.40 | +15 42 08.4  |
| 1998bu| .0033 | 980509    | 980515 ±6<sup>e</sup> | 980421 − 980503  | 10 46 46.03 | +11 50 07.1  |

<sup>a</sup>Phillips 1993<sup>b</sup>  <sup>b</sup>Della Valle 1993<sup>c</sup>  <sup>c</sup>Adam G. Riess 1998, private communication<sup>d</sup>Riess, et al. 1998<sup>e</sup>CfA SN Team Website 1998
Table 2. Estimated Explosion Dates and Positions for High-z Type Ia Supernovae

| SN       | z    | Discovery | Max. Light | Range Exp. Dates | RA(2000) | DEC(2000) |
|----------|------|-----------|------------|------------------|----------|-----------|
| 1992bi   | 0.458| 920421    | na         | 920313 − 920319a | 16 10 12.74 | +39 47 12.7 |
| 1994F    | 0.354| 940109    | na         | 931226 − 940101a | 11 49 59.53 | +10 42 59.5 |
| 1994G    | 0.425| 940213    | na         | 940124 − 940130a | 10 19 16.72 | +50 52 16.7 |
| 1994H    | 0.374| 940108    | na         | 931225 − 931231a | 02 40 04.60 | −01 34 04.6 |
| 1994al   | 0.420| 940108    | na         | 931221 − 931227a | 03 06 22.41 | +17 18 22.4 |
| 1994am   | 0.372| 951022    | na         | 940106 − 940112a | 02 40 02.06 | −01 37 02.1 |
| 1994an   | 0.378| 941031    | na         | 941005 − 941011a | 22 44 18.79 | +00 06 18.8 |
| 1995K    | 0.478| 950330    | 950401 ±1b | 950228 − 950308 | 10 50 47.00 | −09 15 07.4 |
| 1995ao   | 0.240| 951118    | 951123 ±6c | 951023 − 951108 | 02 57 30.70 | −01 41 19.8 |
| 1995ap   | 0.300| 951118    | 951123 ±6c | 951022 − 951107 | 03 12 28.13 | +00 41 43.4 |
| 1995ay   | 0.480| 951120    | na         | 951030 − 951105a | 03 01 07.49 | +00 21 07.5 |
| 1995aq   | 0.453| 951119    | na         | 951021 − 951027a | 00 29 04.22 | +07 51 04.2 |
| 1995ar   | 0.497| 951119    | na         | 951028 − 951103a | 01 01 20.37 | +04 18 20.4 |
| 1995as   | 0.498| 951119    | na         | 951019 − 951025a | 01 01 35.26 | +04 26 35.3 |
| 1995at   | 0.655| 951120    | na         | 951024 − 951030a | 01 04 50.90 | +04 33 50.9 |
| 1995aw   | 0.400| 951119    | na         | 951103 − 951109a | 02 24 55.50 | +00 53 55.5 |
| 1995ax   | 0.615| 951119    | na         | 951020 − 951026a | 02 26 25.77 | +00 48 25.8 |
| 1995az   | 0.450| 951120    | na         | 951107 − 951113a | 04 40 33.56 | −05 30 33.6 |
| 1995ba   | 0.388| 951120    | na         | 951017 − 951023a | 08 19 06.45 | +07 43 06.4 |
| 1996aj   | 0.105| 960615    | 960604 ±10d| 960502 − 960526 | 13 29 06.82 | −29 14 02.0 |
Table 2—Continued

| SN    | z    | Discovery | Max. Light | Range Exp. Dates | RA(2000) | DEC(2000) |
|-------|------|-----------|------------|------------------|----------|-----------|
| 1996cf| 0.570| 960317    | na         | 960220 − 960226^a| 10 48 50.96 | +00 03 51.0 |
| 1996cg| 0.460| 960317    | na         | 960215 − 960221^a| 08 24 13.33 | +03 24 13.3 |
| 1996ci| 0.495| 960317    | na         | 960218 − 960224^a| 13 45 56.16 | +02 26 56.2 |
| 1996ck| 0.656| 960317    | na         | 960216 − 960222^a| 12 48 35.19 | +00 46 35.2 |
| 1996cl| 0.828| 960317    | na         | 960216 − 960220^a| 10 56 59.13 | −03 37 59.1 |
| 1996cm| 0.450| 960318    | na         | 960227 − 960302^a| 13 48 27.22 | +02 27 27.2 |
| 1997F | 0.580| 970105    | na         | 961221 − 961227^a| 04 55 14.25 | −05 51 14.2 |
| 1997G | 0.763| 970105    | na         | 961130 − 961206^a| 04 58 30.21 | −03 16 30.2 |
| 1997H | 0.526| 970105    | na         | 961205 − 961211^a| 04 59 36.56 | −03 09 36.6 |
| 1997I | 0.172| 970105    | na         | 961223 − 961229^a| 04 59 37.30 | −03 09 37.3 |
| 1997J | 0.619| 970105    | na         | 961205 − 961211^a| 07 41 17.82 | +09 33 17.8 |
| 1997K | 0.592| 970106    | na         | 961205 − 961211^a| 07 54 55.07 | +04 19 55.1 |
| 1997L | 0.550| 970105    | na         | 961219 − 961225^a| 08 21 57.12 | +03 53 57.1 |
| 1997N | 0.180| 970105    | na         | 961130 − 961206^a| 08 23 50.01 | +03 28 50.0 |
| 1997O | 0.374| 970106    | na         | 961223 − 961229^a| 08 24 02.49 | +04 07 02.5 |
| 1997P | 0.472| 970106    | na         | 961206 − 961212^a| 10 55 55.90 | −03 56 55.9 |
| 1997Q | 0.430| 970106    | na         | 961207 − 961213^a| 10 56 51.45 | −03 58 51.4 |
| 1997R | 0.657| 970106    | na         | 961220 − 961226^a| 10 57 19.20 | −03 54 19.2 |
| 1997S | 0.612| 970106    | na         | 961202 − 961208^a| 10 57 51.57 | −03 45 51.6 |
Table 2—Continued

| SN    | z   | Discovery | Max. Light | Range Exp. Dates | RA(2000)  | DEC(2000) |
|-------|-----|-----------|------------|------------------|-----------|-----------|
| 1997ac| 0.320 | 960317    | na         | 970131 – 970206\(^a\) | 08 24 05.21 | +04 11 05.2 |
| 1997af| 0.579 | 960317    | na         | 970221 – 970227\(^a\) | 08 23 52.68 | +04 08 52.7 |
| 1997ai| 0.450 | 970305    | na         | 970202 – 970208\(^a\) | 10 48 57.62 | +00 31 57.6 |
| 1997aj| 0.581 | 970305    | na         | 970218 – 970224\(^a\) | 10 55 52.98 | −03 59 53.0 |
| 1997am| 0.416 | 970305    | na         | 970127 – 970202\(^a\) | 10 57 31.52 | −03 13 31.5 |
| 1997ap| 0.830 | 970305    | na         | 970207 – 970213\(^a\) | 13 47 09.90 | +02 23 09.9 |

\(^a\)Perlmutter, et al. 1998 and private communication  \(^b\)Riess, et al. 1998  \(^c\)Kirshner, et al. 1995  
\(^d\)Garnavich, Challis, & Kirshner 1996
| SN   | Type | z#  | Discovery | Max. Light | Range Exp. Dates | W<sup>2</sup>Range | RA(2000) | DEC(2000) |
|------|------|-----|-----------|------------|------------------|---------------------|----------|-----------|
| 1992ar | Ic  | .1450 | 920727 | 920802±6<sup>a</sup> | 920712–920727* | na | 23 17 28.40 | −44 38 53.8 |
| 1993P | Ic  | .0480 | 930518 | 930521±6<sup>b</sup> | 930501–930517 | na | 13 29 25.80 | −30 24 47.4 |
| 1994I | Ic  | .0015 | 940402 | 940411±2<sup>c</sup> | 940326–940402* | 940329–949402 | 13 29 54.01 | +47 11 31.7 |
| 1994ai | Ic  | .0050 | 941220 | 941224±6<sup>d</sup> | 941204–941220 | na | 02 23 06.17 | −21 13 58.3 |
| 1995F | Ic  | .0051 | 950210 | 950207±10<sup>e</sup> | 950114–950207 | na | 09 04 57.40 | +59 55 58.7 |
| 1996D | Ic  | .0159 | 960209 | 960218±6<sup>f</sup> | 960129–960209* | na | 04 34 00.00 | −08 35 00.0 |
| 1996N | Ib/c | .0047 | 960312 | 960309±10<sup>g</sup> | 960214<sup>†</sup>–960308 | 960210–960224 | 03 38 55.31 | −26 20 04.1 |
| 1996aq | Ic  | .0055 | 960817 | 960820±6<sup>h</sup> | 960731–960816 | na | 14 22 22.73 | −00 23 24.3 |
| 1996cd | Ib/c | .0480 | 961216 | 961224±10<sup>ijk</sup> | 961129–961216<sup>†</sup> | na | 07 57 20.73 | +11 12 23.7 |
| 1997B | Ic  | .0104 | 970114 | 970104±6<sup>k</sup> | 961215–961231 | na | 05 53 02.97 | −17 52 23.5 |
| 1997C | Ic  | na | 970114 | na | 961216<sup>±</sup>–970104<sup>†</sup> | na | 10 13 56.18 | +38 49 00.5 |
| 1997X | Ic  | .0037 | 970201 | 970125±6<sup>mkkn</sup> | 970114<sup>†</sup>–970121 | 961231–970121 | 12 48 14.28 | −03 19 58.5 |
| 1997dc | Ib  | .0116 | 970805 | 970811±6<sup>o</sup> | 970719–970804 | na | 23 28 28.41 | +22 25 23.0 |
| 1997dq | Ib  | .0033 | 971102 | 971105±6<sup>p</sup> | 971013–971029 | na | 11 40 55.90 | +11 28 45.7 |
| 1997ef | Ic  | .0117 | 971125 | 971206±6<sup>qkr</sup> | 971116–971125<sup>†</sup> | 971113–971125 | 07 57 02.87 | +49 33 41.3 |
| 1997ei | Ic  | .0106 | 971223 | 971225±10<sup>s</sup> | 971201–971223<sup>†</sup> | 971103–971203 | 11 54 59.98 | +58 29 26.4 |
Table 3—Continued

| SN   | Type | $z^#$ | Discovery | Max. Light | Range Exp. Dates | $w^2$Range | RA(2000) | DEC(2000) |
|------|------|-------|-----------|------------|------------------|------------|----------|-----------|
| 1998T | Ib   | 0.0101 | 980303 | na         | 980214–980303$^t$ | 980208–980303 | 11 28 33.16 | +58 33 43.7 |
| 1998bo | Ic   | 0.0161 | 980422 | 980412±10$^u$ | 980329$^t$–980412 | na         | 19 57 22.55 | −55 08 18.4 |
| 1998cc | Ib   | 0.0134 | 980515 | na         | 980429–980515$^v$ | na         | 13 29 19.31 | +17 02 42.4 |
| 1998cv | Ic   | 0.0272 | 980624 | 980627±6$^w$ | 980607–980623 | na         | 22 09 46.29 | −49 47 43.0 |

$^z$z values computed from recession velocities found in the Sternberg SN Catalog

*Latest estimated explosion date limited by supernova discovery date

$^t$Earliest estimated explosion date limited by absence of supernova on image taken two days after this date

$^a$Williams, Hamuy, & Phillips 1992 $^b$Phillips 1993b $^c$Iwamoto, et al. 1994 $^d$Benetti 1994
$^e$Filippenko & Barth 1995 $^f$Cappellaro & Pata 1996 $^g$Germany, et al. 1996 $^h$Benetti, Turatto, & Augusteijn 1996
$^i$Pollas 1997 $^j$Filippenko, et al. 1997 $^k$Benetti & Lidman 1997 $^l$Li, et al. 1997 $^m$Nakano & Aoki 1997 $^n$Benetti, Turrato, & Perez 1997 $^o$Piemonte, Benetti, & Turatto 1997
$^p$Jha, et al. 1997 $^q$Iwamoto, et al. 1998 $^r$Garnavich, et al. 1997 $^s$Wang, Howell, & Wheeler 1998
$^t$Li, Li, & Wan 1998 $^u$Patat & Maia 1998 $^v$Jha, Garnavich, & Kirshner 1998 $^w$Phillips 1998
Table 4. Type Ib-Ic Supernovae and GRB Odds

| SN/GRB   | Dates       | RA  | DEC  | $\sigma_{\text{total}}$ | $\Delta\theta$ | Likelihood | Odds   |
|----------|-------------|-----|------|-------------------------|----------------|------------|--------|
| SN 1992ar | 920712−920727 | 23  | 17  | 28.40                   | −44 38 53.8    | 0.5200     |
| GRB 920721 |            | 00  | 02  | 33                       | −28 10         | 4.36       | 18.76  | $5.551 \times 10^{-7}$ |
| SN 1993P  | 930501−930517 | 13  | 29  | 25.80                   | −30 24 47.4    | 0.5200     |
| GRB 930510 |            | 10  | 02  | 40                       | +55 00         | 13.09      | 96.20  | $5.953 \times 10^{-20}$ |
| SN 1994I  | 940326−940402 | 13  | 29  | 54.01                   | +47 11 31.7    | 0.5236     |
| GRB 940331 |            | 10  | 31  | 14                       | +57 31         | 10.79      | 28.55  | $4.941 \times 10^{-2}$ |
| SN 1994ai | 941204−941220 | 02  | 23  | 06.17                   | −21 13 58.3    | 0.5200     |
| GRB 941217 |            | 08  | 40  | 40                       | −80 22         | 9.84       | 69.81  | $1.100 \times 10^{-20}$ |
| SN 1995F  | 950114−950207 | 09  | 04  | 57.40                   | +59 55 58.7    | 0.5200     |
| GRB 950118 |            | 06  | 51  | 52                       | +58 17         | 4.36       | 16.97  | $2.470 \times 10^{-5}$ |
| SN 1996D  | 960129−960209 | 04  | 34  | 00.00                   | −08 35 00.0    | 0.5200     |
| GRB 960129 |            | 06  | 02  | 55                       | −38 36         | 10.02      | 36.06  | $8.628 \times 10^{-5}$ |
| SN 1996N  | 960214−960308 | 03  | 38  | 55.31                   | −26 20 04.1    | 1.285      |
| GRB 960229 |            | 04  | 02  | 26                       | −15 15         | 13.01      | 12.36  | $3.174 \times 10^{1}$  |
| SN 1996aq | 960731−960816 | 14  | 22  | 22.73                   | −00 23 24.3    | 0.5210     |
| GRB 960731 |            | 13  | 18  | 07                       | −18 17         | 8.52       | 23.86  | $2.908 \times 10^{-2}$ |
| SN 1996cd | 961129−961216 | 07  | 57  | 20.73                   | +11 12 23.7    | 0.5200     |
| GRB 961216 |            | 04  | 38  | 50                       | −21 28         | 6.48       | 58.66  | $1.498 \times 10^{-35}$ |
| SN 1997B  | 961215−961231 | 05  | 53  | 02.97                   | −17 52 23.5    | 1.901      |
| GRB 961218 |            | 06  | 31  | 00                       | −21 43         | 15.31      | 9.72   | $4.052 \times 10^{1}$  |
Table 4—Continued

| SN/GRB     | Dates       | RA   | DEC   | $\sigma_{\text{total}}$ | $\Delta \theta$ | Likelihood | Odds      |
|------------|-------------|------|-------|--------------------------|----------------|------------|-----------|
| SN 1997C   | 961216−970104 | 10 13 56.18 | +38 49 00.5 |                           |                |            | 0.5200    |
| GRB 961218 |             | 06 31 00 | −21 43 | 15.31                    | 79.90          |            | $1.981 \times 10^{-10}$ |
| SN 1997X   | 970114−970121 | 12 48 14.28 | −03 19 58.5 |                           |                |            | 0.5200    |
| GRB 970116 |             | 08 13 48 | −11 09 | 6.66                     | 68.38          |            | $7.724 \times 10^{-45}$ |
| SN 1997dc  | 970719−970804 | 23 28 28.41 | +22 25 23.0 |                           |                |            | 0.5200    |
| GRB 970725 |             | 00 51 47 | +36 12 | 6.04                     | 22.72          |            | $4.517 \times 10^{-5}$ |
| SN 1997dq  | 971013−971029 | 11 40 55.90 | +11 28 45.7 |                           |                |            | 1.718     |
| GRB 971013 |             | 11 08 07 | +02 39 | 11.62                    | 12.00          |            | $3.299 \times 10^1$ |
| SN 1997ef  | 971116−971125 | 07 57 02.87 | +49 33 41.3 |                           |                |            | 0.5288    |
| GRB 971120 |             | 10 23 02 | +76 24 | 12.71                    | 30.42          |            | $1.517 \times 10^{-1}$ |
| SN 1997ei  | 971201−971223 | 11 54 59.98 | +58 29 26.4 |                           |                |            | 0.8908    |
| GRB 971220 |             | 13 41 45 | +58 50 | 9.44                     | 13.79          |            | $1.475 \times 10^1$ |
| SN 1998T   | 980214−980303 | 11 28 33.16 | +58 33 43.7 |                           |                |            | 0.5200    |
| GRB 980223 |             | 14 39 23 | +27 56 | 10.44                    | 44.78          |            | $2.706 \times 10^{-7}$ |
| SN 1998bo  | 980329−980412 | 19 57 22.55 | −55 08 18.4 |                           |                |            | 0.5200    |
| GRB 980404 |             | 04 36 57 | −49 57 | 12.22                    | 66.91          |            | $4.882 \times 10^{-12}$ |
| SN 1998cc  | 980429−980515 | 13 29 19.31 | +17 02 42.4 |                           |                |            | 0.8349    |
| GRB 980501 |             | 14 55 28 | +23 21 | 17.33                    | 21.15          |            | $9.258 \times 10^0$ |
| SN 1998cv  | 980607−980623 | 22 09 46.29 | −49 47 43.0 |                           |                |            | 0.5200    |
| GRB 980609 |             | 21 22 04 | −18 38 | 3.83                     | 32.58          |            | $7.241 \times 10^{-33}$ |
Table 5. Summary Of Results of Bayesian Analysis

| SN Sample | Model | Odds       |
|-----------|-------|------------|
| All Ia    | $f = 1$ | $2.23 \times 10^{-22}$ |
| Ia ($z \leq 0.1$) | $f = 1$ | $3.01 \times 10^{-10}$ |
| Ia ($z > 0.1$) | $f = 1$ | $7.41 \times 10^{-13}$ |
| All Ia    | $f \leq 1$ | $2.90 \times 10^{-2}$ |
| Ia ($z \leq 0.1$) | $f \leq 1$ | $6.24 \times 10^{-2}$ |
| Ia ($z > 0.1$) | $f \leq 1$ | $5.22 \times 10^{-2}$ |
| Ib, Ib/c, Ic | $f = 1$ | $1.76 \times 10^{-4}$ |
| Ib, Ib/c, Ic (W&W) | $f = 1$ | $1.62 \times 10^{-1}$ |
| Ib, Ib/c, Ic | $f \leq 1$ | $1.59 \times 10^{-1}$ |