Lattice-Constrained Dispersive Bounds for $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}$ Decays

Laurent Lellouch

Abstract

I present a recent piece of work on semileptonic $B \to \pi$ decays in which lattice results and kinematical and dispersive constraints are combined to obtain model-independent bounds on the relevant form factors and rates.

Key-Words: Semileptonic Decays of $B$ Mesons, Determination of Kobayashi-Maskawa Matrix Elements ($V_{ub}$), Dispersion Relations, Lattice QCD Calculation, Heavy Quark Effective Theory.

Number of figures: 1

September 1996
CPT-96/P.3385

anonymous ftp or gopher: cpt.univ-mrs.fr

* Unité Propre de Recherche 7061
† Talk given at the 28th International Conference on High Energy Physics (ICHEP96), Warsaw, Poland, 25-31 July 1996. To appear in the proceedings.
‡ email: lellouch@cpt.univ-mrs.fr
Lattice-Constrained Dispersive Bounds for $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}$ Decays

Laurent Lellouch

Centre de Physique Théorique (UPR 7061), CNRS-Luminy, Case 907, F-13288 Marseille, France

I present a recent piece of work on semileptonic $B \to \pi$ decays in which lattice results and kinematical and dispersive constraints are combined to obtain model-independent bounds on the relevant form factors and rates.

1 Introduction

The CLEO Collaboration has very recently presented a measurement of the branching ratio for $B \to \pi \ell \bar{\nu}$ decays ($\ell=e, \mu$)$]^{11}$,

$$B(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4},$$

where the errors are statistical, systematic and the estimated model-dependence introduced in determining efficiencies. This measurement represents an excellent opportunity to determine the poorly known CKM matrix element $|V_{ub}|$. This determination requires calculating, over the full kinematical range, the non-perturbative matrix element:

$$\langle \pi(p')|V^\mu|\bar{B}^0(p)\rangle = \left(p + p' - q \frac{M^2-m^2}{q^2}\right)^\mu f^+ + q^\mu \frac{M^2-m^2}{q^2} f^0,$$

where $q=p-p'$, $V^\mu=\bar{u} \gamma^\mu b$, $M$ is the mass of the $B$ and $m$, that of the $\pi$. Though this matrix element can be calculated using lattice QCD, current day lattice simulations, with lattice spacings on the order of $3$ GeV$^{-1}$, cannot cover the full kinematical range. The problem is that the energies and momenta of the particles involved, whose orders of magnitude are set by the $b$ quark mass ($m_b \approx 5$ GeV), are large on the scale of the cutoff over much of phase space. To limit these energies in relativistic lattice quark calculations, one performs the simulation with heavy-quark mass values $m_Q$ around that of the charm ($m_c \approx 1.5$ GeV), where discretization errors remain under control. Then one extrapolates the results up to $m_b$ by fitting heavy-quark scaling relations (HQSR) with power corrections to the lattice results. Another approach is to work with discretized versions of effective theories such as Non-Relativistic QCD (NRQCD) or Heavy-Quark Effective Theory (HQET) in which the mass of the heavy quark is factored out of the dynamics. All these approaches, however, are based on the heavy-quark expansion which is more limited for heavy $\to$ light quark decays, such as the one that concerns us here, than it is for heavy $\to$ heavy quark decays: it imposes no normalization condition on the relevant form factors at the the zero recoil point $q^2=q^2_{\text{max}}$ and only applies in a limited region around $q^2_{\text{max}}$. Furthermore, momentum-dependent discretization errors restrict the momentum of the initial and final state mesons to around $1 \sim 2$ GeV. Thus, all these approaches are constrained to relatively small momentum transfors: one can only reconstruct the $q^2$ dependence of the relevant form factors in a limited region around $q^2_{\text{max}}$ and one is left with the problem of extrapolating these results to smaller $q^2$.

Heavy $\to$ light quark decays are difficult in any theoretical approach. Indeed, they require understanding the underlying QCD dynamics over a large range of momentum transfers from $q^2_{\text{max}}=26.4$ GeV$^2$ for semileptonic $B \to \pi$ decays, where the $\pi$ is at rest in the frame of the $B$ meson, to $q^2=0$ where it recoils very strongly.

2 $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}$ and Dispersive Constraints

The solution to the problem of the limited kinematical reach of lattice simulations of heavy $\to$ light quark decays presented here consists in supplementing lattice results for the relevant form factors around $q^2_{\text{max}}$ with dispersive bound techniques to obtain improved, model-independent bounds for the form factors for all $q^2$. For the case of $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}$ decays, one can use the kinematical constraint, $f^+(0)=f^0(0)$, to further constrain the bounds.

2.1 Dispersive Bounds

The subject of dispersive bounds in semileptonic decays has a long history going back to S. Okubo...
et al., who applied them to semileptonic $K \to \pi$ decays. C. Bourrely et al. first combined these techniques with QCD and applied them to semileptonic $D \to K$ decays. Very recently, C. G. Boyd et al. applied them to $B \to \pi \ell \nu$ decays.

The starting point for $B \to \pi \ell \nu$ decays is the polarization function

$$
\Pi^{\mu\nu}(q) = \int d^4x \ e^{iq\cdot x} \langle 0 | T \left(V^{\mu}(x)V^{\nu}(0)\right) | 0 \rangle
$$

$$
= (q^\mu q^\nu - g^\mu\nu q^2) \Pi_T(q^2) + q^\mu q^\nu \Pi_L(q^2),
$$

(3)

where $\Pi_{T,L}(q^2)$ corresponds to the propagation of a $J^P=1^- (0^\pm)$ particle. The corresponding spectral functions, $\text{Im} \Pi_{T,L}$, are sums of positive contributions coming from intermediate $B^*$ ($J^P=1^-$), $B\pi$ ($J^P=0^+ \text{ and } 1^-$), ... states and are thus upper bounds on the $B\pi$ contributions. Combining, for instance, the bound from $\text{Im} \Pi_L$ with the dispersion relation ($Q^2 = -q^2$)

$$
\chi_L(Q^2) = \frac{\partial}{\partial Q^2} (Q^2 \Pi_L(Q^2))
$$

$$
= \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im} \Pi_L(t)}{(t+Q^2)^2},
$$

(4)

one finds

$$
\chi_L(Q^2) \geq \frac{1}{\pi} \int_{t_+}^\infty dt k(t, Q^2) |f^0(t)|^2
$$

(5)

where $t_\pm=(m_B \pm m_\pi)^2$ and $k(t, Q^2)$ is a kinematical factor. Now, since $\chi_L(Q^2)$ can be calculated analytically in QCD for $Q^2$ far enough below the resonance region (i.e. $-Q^2 \ll m_b^2$), Eq. (3) gives an upper bound on the weighted integral of the magnitude squared of the form factor $f^0$ along the $B\pi$ cut. To translate this bound into a bound on $f^0$ in the region of physical $B \to \pi \ell \nu$ decays is a problem in complex analysis (please see Ref. [5] for details). A similar constraint can be obtained from $\Pi_T$ for $f^+$. There, however, one has to confront the additional difficulty that $f^+$ is not analytic below the $B\pi$ threshold because of the $P^+$ pole.

The beauty of the methods of Ref. [3] is that they enable one to incorporate information about the form factors, such as their values at various kinematical points, to constrain the bounds. For the case at hand, however, these methods must be generalized in two non-trivial ways. In constructing these generalizations, one must keep in mind that the bounds: 1) form inseparable pairs; 2) do not indicate the probability that the form factor will take on any particular value within them.

2.2 Imposing the Kinematical Constraint

The first problem is that Eq. (3) and the equivalent constraint for $f^+$ yield independent bounds on the form factors which do not satisfy the kinematical constraint $f^+(0)=f^0(0)$. The bounds on $f^+$ require $f^+(0)$ to lie within an interval of values $I_+$ and those on $f^0$, within an interval $I_0$. Together with these bounds, however, the kinematical constraint requires $f^+(0)=f^0(0)$ to lie somewhere within $I_+ \cap I_0$. Thus, we seek bounds on the form factors which are consistent with this new constraint.

A natural definition is to require these new bounds to be the envelope of the set of pairs of bounds obtained by allowing $f^+(0)$ and $f^0(0)$ to take all possible values within the interval $I_+ \cap I_0$. In Ref. [3], it is shown how this envelope can be constructed efficiently and that the additional constraint can only improve the bounds on the form factors for all $q^2$. Also, as a by product, one obtains a formalism which enables one to constrain bounds on a form factor with the knowledge that it must lie within an interval of values at one or more values of $q^2$.

2.3 Taking Errors into Account

As they stand, the methods of Ref. [3] can only accommodate exact values of the form factors at given kinematical points and contain no provisions for taking errors on these values into account. Of course, the results given by the lattice do carry error bars. More precisely, the lattice provides a probability distribution for the value of the form factors at various kinematical points. What must be done, then, is to translate this distribution into some sort of probability statement on the bounds. The conservative solution is to consider the probability that complete pairs of bounds lie within a given finite interval at each value of $q^2$. Then, using this new probability, one can define upper and lower $p\%$ bounds at each $q^2$ as the upper and lower boundaries of the interval that contains the central $p\%$ of this probability [6] These bounds indicate that there is at least a $p\%$ probability that the form factors lie within them at each $q^2$.

The density of pairs of bounds increases toward the center of the distribution as long as the distribution of the lattice results does.
2.4 Lattice-Constrained Bounds

To constrain the bounds on \( f^+ \) and \( f^0 \), the lattice results of the UKQCD Collaboration are used, to which a large range of systematic errors is added to ensure that the bounds obtained are conservative. Because of these systematic errors, the probability distribution of the lattice results is not known. The simplifying and rather conservative assumption that the results are uncorrelated and gaussian distributed is made. The required probability is constructed by generating 4000 pairs of bounds from a Monte-Carlo on the distribution of the lattice results. The results for the bounds on the form factors are shown in Fig. 1. The two form factors are plotted back-to-back to show the effect of the kinematical constraint. Without this constraint, the bounds on \( f^+ \) would be looser, especially around \( q^2 = 0 \), where phase space is large. Since \( f^+ \) determines the rate, the kinematical constraint and the bounds on \( f^0 \) are important.

Also shown in Fig. 1 is the light-cone sumrule (LCSR) result of Ref. 6, and the 3-point sumrule (SR3) result of Ref. 7. The latter is parametrized by a pole form with \( f^+(0) = 0.26 \) and \( m_{\text{pole}} = 5.25 \text{ GeV} \). The former has two components: for \( q^2 \) below 15 GeV\(^2\), the \( q^2 \) dependence of \( f^+ \) is determined directly from the sumrule; for larger \( q^2 \), pole dominance is assumed with a residue determined from the same correlator. While agreement of the LCSR result with the bounds is excellent, the SR3 result is quite strongly disfavored. The bounds are compared with the predictions of more authors as well as with direct fits of various parametrizations to the lattice results in Ref. 6.

Though again certain predictions are strongly disfavored, the lattice results and bounds will have to improve before a firm conclusion can be drawn as to the precise \( q^2 \) dependence of the form factors.

The bounds on \( f^+ \) also enable one to constrain the \( B^* B \pi \) coupling \( g_{B^* B \pi} \) which determines the residue of the \( B^* \) pole contribution to \( f^+ \). The constraints obtained are poor because \( f^+ \) is weakly bound at large \( q^2 \), as can be seen in Fig. 1. Fitting the lattice results for \( f^0 \) and \( f^+ \) to a parametrization which assumes \( B^* \) pole dominance for \( f^+ \) and which is consistent with HQS and the kinematical constraint gives the more precise result \( g_{B^* B \pi} = 28 \pm 4 \). However, because

\[ |V_{ub}|10^4 \sqrt{\tau_{\text{PV}}/1.56 \text{ ps}} = (34 \div 49) \pm 8 \pm 6 , \quad (6) \]

where the range given in parentheses is that obtained from the 30% CL bounds on the rate and represents the most probable range of values for \( |V_{ub}| \). The first set of errors is obtained from the 70% CL bounds and the second is ob-
Table 1: Bounds on rate in units of $|V_{ub}|^2 ps^{-1}$ and on $f^+(0)$: top block. Quark model (QM) and light-cone (LCSR), 2pt (SR2) and 3pt (SR3) sumrule predictions:

| $I (B^0 \to \pi^+ \ell^- \nu)$ | $f^+(0)$ | details |
|-------------------------------|---------|---------|
| 2.4 → 28                      | -0.26 → 0.92 | 95% CL |
| 2.8 → 24                      | -0.18 → 0.85 | 90% CL |
| 3.6 → 17                      | 0.00 → 0.68  | 70% CL |
| 4.4 → 13                      | 0.10 → 0.57  | 50% CL |
| 4.8 → 10                      | 0.18 → 0.49  | 30% CL |
| 7.4 ± 1.6                     | 0.33 ± 0.06  | QM     |
| 2.1                           | 0.09       | QM     |
| 9.6                           | 0.26       | QM     |
| 9.6–15.2                      | 0.29 – 0.46 | QM     |
| 7 ± 2                         | 0.20 – 0.29 | QM     |
| 14.5 ± 5.9                    | 0.4 ± 0.1   | SR2    |
| 4.5–9.0                       | 0.27 ± 0.05 | SR2 + 3 |
| 3.60 ± 0.65                   | 0.23 ± 0.02 | SR3    |
| 5.1 ± 1.1                     | 0.26 ± 0.02 | SR2    |
| 8.1                           | 0.24 – 0.29 | LCSR   |
| 7 ± 1                         | 0.21 – 0.27 | LAT    |
| 9 ± 6                         | 0.10 – 0.49 | LAT    |
| 8 ± 4                         | 0.23 – 0.43 | LAT    |

obtained by combining all experimental uncertainties in quadrature and applying them to the average value of $|V_{ub}|$ given by the 30% CL results. This determination of $|V_{ub}|$ has a theoretical error of approximately 37%. Though non-negligible, this error is quite reasonable given that the bounds on the rate are completely model-independent and are obtained from lattice data which lie in a limited kinematical domain and include a conservative range of systematic errors. For comparison, the lattice results of ELC [4] and APE [5] were obtained from the determination of $f^+$ at a single $q^2 \sim 18 - 20$ GeV$^2$ supplemented with the assumption of $B^*$-pole dominance while those of UKQCD [6] rely on constrained model fits to the same values of $f^+$ and $f^0$ (without the added systematics errors) used to obtain the bounds presented here.

3 Conclusion and Outlook

I have presented a new formalism by which lattice results for semileptonic $B \to \pi$ decays, limited to a narrow kinematical range, are combined with dispersive and kinematical constraints to obtain model-independent bounds on the relevant form factors and rates over the full kinematical domain. I have compared these bounds, which have a well defined statistical meaning, to the predictions of other authors.

Though the bounds will benefit from forthcoming, improved lattice results, they would benefit most from an increase in the range of $q^2$.

Finally, the techniques presented here are in principle applicable to limited results obtained by non-lattice means as well as to other processes such as $B \to \rho \ell \nu$ and $B \to K^{*} \gamma$ decays.

1. CLEO Collaboration, CLEO CONF 96-16.
2. L. Lellouch, CPT-95/P.3236 (revised), hep-ph/9509358.
3. S. Okubo and I-Fu Shih, Phys. Rev. D4 (1971) 250.
4. C. Bourrely, B. Machet and E. de Rafael, Nucl. Phys. B189 (1981) 157.
5. C.G. Boyd, B. Grinstein and R.F. Lebed, Phys. Rev. Lett. 74 (1995) 4603.
6. UKQCD, Nucl. Phys. B447 (1995) 425; and D. Burford, private communication.
7. V.M. Belyaev et al., Phys. Rev. D51 (1995) 6177; Z. Phys. C60 (1993) 349; A. Khodjamirian et al., in Proc. “Beauty 95”, Oxford, July 1995.
8. P. Ball, Phys. Rev. D48 (1993) 3190.
9. M. Wirbel, B. Stech and M. Bauer, Z. Phys. C29 (1985) 637.
10. N. Isgur, D. Scora, B. Grinstein and M. B. Wise, Phys. Rev. D39 (1989) 799.
11. D. Scora and N. Isgur, Phys. Rev. D52 (1995) 2783.
12. P.J. O’Donnell, Q.P. Xu and H.K.K. Tung, Phys. Rev. D52 (1995) 3966.
13. C.Y. Cheung et al., IP-ASTP-16-95 and hep-ph/9602304.
14. I.L. Grach, I.M. Narodetskii and S. Simula, INFN-PP-96/4 and hep-ph/9605349.
15. D. Melikhov, Phys. Lett. B380 (1996) 363.
16. C.A. Dominguez and N. Paver, Z. Phys. C41 (1988) 217.
17. A.A. Ovchinnikov, Phys. Lett. B229 (1989) 127.
18. S. Narison, Phys. Lett. B345 (1995) 166.
19. ELC, Nucl. Phys. B416 (1994) 675.
20. APE, Phys. Lett. B345 (1995) 513.