Resolution of the Landau pole problem in QED*

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We present new numerical results for the renormalized mass and coupling in non-compact lattice QED with staggered fermions. Implications for the continuum limit and the role of the Landau pole are discussed.

1. INTRODUCTION

In the 1950s Landau investigated the relation between the bare charge $e$ and the renormalized charge $e_R$ in QED. He found

$$\frac{1}{e_R^2} - \frac{1}{e^2} = \frac{N_f}{6\pi^2} \ln \frac{\Lambda}{m_R},$$

(1)

where $\Lambda$ is the momentum cutoff, $m_R$ is the renormalized mass of the electron and $N_f$ is the number of flavours (for staggered fermions $N_f = 4$). It is well known that (1) implies two potential problems when $\Lambda \to \infty$:

- When $e$ is fixed the theory has a trivial continuum limit, i.e., $e_R \to 0$.

- When $e_R > 0$ is fixed $e$ becomes singular at $\Lambda_{\text{Landau}} = m_R \exp (6\pi^2/N_f e_R^2)$. This singularity is called the Landau pole.

These problems do not affect the phenomenological success of QED because in practical perturbative calculations the cutoff can be chosen to be large compared with experimental energies. Finding a solution to them is of fundamental theoretical interest and requires non-perturbative methods. We have therefore extensively studied QED on the lattice using non-compact gauge-fields and dynamical staggered fermions\textsuperscript{[2,3]}.

In this talk we report on new measurements of the renormalized mass and charge. This was started in\textsuperscript{[2]} and is now extended to lattices of size $16^4$ and bare masses $am$ down to 0.005 ($a$ denotes the lattice spacing). Using our measurements we have determined functions $am_R(e^2, am)$ and $e_R^2(e^2, am)$. These functions imply that the theory is trivial. They also give a resolution of the Landau pole problem.

2. THE RENORMALIZED MASS

The renormalized mass was obtained from fits to the fermion propagator as explained in\textsuperscript{[2,3]}. To get $am_R$ as a function of $am$ and $e$ we need to know the equation of state which relates the bare parameters to the chiral condensate $\sigma \equiv a^3 \langle \bar{\chi} \chi \rangle$. In\textsuperscript{[3]} we have found that the $\sigma$ data obey a mean field equation of state with logarithmic corrections

$$am = A_0 \frac{\sigma^3}{\ln^{p_0}(1/\sigma)} + A_1 \left( \frac{1}{e^2} - \frac{1}{e_R^2} \right) \frac{\sigma}{\ln^{p_1}(1/\sigma)}.$$

(2)

We fitted this expression to $\sigma$ data that were extrapolated to infinite lattice size and obtained $1/e_R^2 = 0.19040(9)$, $A_0 = 1.798(5)$, $p_0 = 0.324(15)$, $A_1 = 6.76(3)$, $p_1 = 0.485(7)$. 

*Talk presented by H. Stüben
\[ \chi^2 / \text{d.o.f.} = 7.6. \]

We have observed that \( \sigma \) can be well described by a polynomial in \( am_R \)
\[ \sigma = A_1 am_R + A_3 a^3 m_R^3 + A_5 a^5 m_R^5 + A_7 a^7 m_R^7 \tag{3} \]
where the first parameter \( A_1 \equiv 0.6197 \) can be taken from perturbation theory and a fit to data from 12\(^4\) and 16\(^4\) lattices gave \( A_3 = -0.321(5), A_5 = +0.169(13), A_7 = -0.040(7), \chi^2 / \text{d.o.f.} = 2.1. \) Because the results of both lattice sizes fall on a universal curve we conclude that the polynomial (3) is also valid on an infinite lattice.

3. THE RENORMALIZED CHARGE

The determination of the renormalized charge has been improved since [2]. The method can only be sketched here. It consists of making a global fit to all gauge field propagators \( D(k) \) that we have measured
\[ \frac{1}{e^2} D(k) - \frac{1}{e^2} = -\Pi(k, am_R, L) \tag{4} \]
where \( L \) is the linear lattice size. The ansatz for the fit function \( \Pi \) was taken from [4] to be
\[ \Pi = U - \frac{V}{U} \ln(1 - e^2 U) \tag{5} \]
where \( U \) is given by 1-loop lattice perturbation theory (see [2]) and where we have set
\[ V(k, am_R, L) = v_0 + v_1 U(k, am_R, L). \tag{6} \]
We then find \( e^2_R(e^2, am_R) \) using \( e^2_R = Z_3 e^2 \) and
\[ Z_3 = \lim_{k \to 0} \lim_{L \to \infty} D(k) \] from
\[ \frac{1}{e^2_R} - \frac{1}{e^2} = -\Pi(0, am_R, \infty). \tag{7} \]
A simultaneous fit of (4) to the gauge field propagators at our 52 values \( (e^2, am, L) \) gave \( v_0 = -0.00207(2), v_1 = -0.0328(7), \chi^2 / \text{d.o.f.} = 1.7. \) Since \( U(0, am_R, \infty) \approx (N_f/6\pi^2)(-0.31 + \ln am_R) \) we only find small corrections to the old result (1).

4. DISCUSSION

We can now discuss the mapping \( (e^2, am) \leftrightarrow (e^2_R, am_R) \). A global qualitative view of this mapping is shown in Figure 1, while a quantitative plot of \( (1/e^2, am) \leftrightarrow (1/e^2_R, am_R) \) is shown in Figure 2.

In both figures accessible regions are plotted in grey. The whole plane of bare parameters is accessible but this plane is mapped only onto a part of the plane of the renormalized parameters.
The border of the accessible region is shown as a thick line. It is the image of the corresponding thick line on the $am = 0$ line starting/ending at the critical value of the coupling constant.

On the line $am_R = 0$ in Figure 3 the only accessible point is the origin. This reflects triviality. In Figure 2 triviality is expressed by the fact that no line of (finite) constant $1/e^2_R$ flows into the critical point.

The dotted line in both figures is the position of the Landau pole, i.e., the line of pairs $(e^2_R, am_R)$ with $1/e^2 = 0$ from (7) with $\Pi \equiv U$. This line is well separated from the border for all finite $e^2_R$.

5. CONCLUSIONS

From the presented analysis we conclude that non-compact lattice QED with staggered fermions has a trivial continuum limit. In addition our analysis implies a resolution of the Landau problem. The resolution is that for given $e$ and $am$ the theory does not allow arbitrary choices of $am_R$. Instead through chiral symmetry breaking the theory itself provides a minimal lattice spacing or maximal cutoff which is below $\Lambda_{\text{Landau}}$. Non-perturbatively the function $\Pi$ is very close to what one finds in perturbation theory, but there is no Landau pole problem because the pole always lies in the forbidden region.

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