Influence of edge slotting of magnet pole with fixed slot opening width on the cogging torque in inset permanent magnet synchronous machine

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Abstract
The cogging torque in an inset permanent magnet synchronous machine causes some undesirable vibration and noises which should be reduced in the earliest possible stage of design. The influences of edge slotting and slot opening width in the magnet rotor pole on the cogging torque were investigated in this article. The structure of the proposed inset permanent magnet synchronous machine has 8 poles and 24 stator slots with fixed slot opening width and a modified magnet pole in the rotor. The finite element method magnetics (FEMM 4.2) tool was used to investigate the core saturation induced in the stator and rotor of the inset permanent magnet synchronous machine. The simulation results show that the slotting design in the magnet edge can effectively reduce the cogging torque of the inset permanent magnet synchronous machine.

Keywords
Cogging torque, finite element, inset permanent magnet synchronous machine, slotting

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Introduction
Inset permanent magnet synchronous machines (inset PMSMs) have been widely used in many applications because of its compact features, size, simple mechanical construction, easy maintenance, reliability, high torque density, and efficiency.\(^1\) The cogging torque is produced by the magnetic interaction between the mounted permanent magnet of the rotor and the stator teeth.\(^4\) However, the main circumferential component of the attractive force attempts to maintain the alignment between the stator teeth and the permanent magnets,\(^1\) leading to some undesirable vibration and noises. This limits applications to high-precision control systems.

The cogging torque of an inset PMSM can be minimized by changing the shapes and number of poles of the permanent magnet or the slots of the armature core\(^4\) and by modifying the residual magnet flux density of the permanent magnet in air gap. One efficient way to modify the residual magnet flux density of the permanent magnet, and thus reduce the cogging torque, is to slot the surface or edge of the permanent magnet.\(^4\)–\(^7\) Three slots in the permanent magnet surface of the outer rotor system were found to reduce the cogging torque by 40.7% when compared with a non-slotted magnet surface.\(^4\) Scuiller\(^5\) made slots in the permanent magnet surface. The existence of slots increases...
magnetic losses by the corresponding demagnetization effect, particularly in a small air gap machine.

Another significant approach for cogging torque reduction is to use a one-step slot in the edge of a permanent magnet,\(^6\) this results in an 89% cogging torque reduction as compared with a non-lead loaf system. SmCo20Moe permanent magnet was used for the magnet rotor and M-27 for the cores of the stator and rotor, but detailed information regarding the flux density in the machine’s air gap needs to be further investigated. L Wu et al.\(^7\) studied the method of gradually inclining surface edge (GISE) and in this manner achieved 20% cogging torque reduction. The combination of GISE with another auxiliary slot significantly extends this reduction to 25.7% cogging torque.

In this article, a modified two-step slotting with load bread system in the magnet edge is proposed to reduce the cogging torque without changing the stator structure of the inset PMSM. Higher grade materials, NdFe-B and M-19, for the fabrication of the permanent sintered magnets in the core of the stator and rotor are used to gain better performance. The relationships between the slotting of the machines to the normal air gap flux density are investigated, and the resultant cogging torque is compared with another popular structure shown in Figure 1(b) and (c), respectively. The physical parameters and the detailed structure of the three inset PMSM models have the same characteristics.

Basic theories of the cogging torque

The cogging torque is calculated using equation (1) with the magnet pole structure assumed to have radial magnetization\(^3\)

\[
T_c = -\frac{1}{2} \phi_g^2 \frac{dR_g}{d\theta} \tag{1}
\]

where \(\phi_g\) is the magnet flux in the air gap, \(R_g\) is the passed air gap reluctance, and \(\theta\) is the rotor position of the machine.

If the reluctance \(R\) does not vary with the rotor positions, the cogging torque is 0. In addition, the cogging torque is independent of the magnet flux direction in the air gap since the amount of the magnet flux is squared as shown in equation (1). The air gap reluctance varies periodically; it causes the cogging torque to be periodic. The magnetic flux in the air gap, shown in equation (2), is influenced by the normal flux density and the cross-sectional area. The cogging torque can be formulated as a Fourier series, as shown in equation (3)

\[
\phi_g = \int B \cdot dA \tag{2}
\]

\[
T_c = \sum_{k=1}^{\infty} T_{mk} \sin (mk\theta) \tag{3}
\]

where \(m\) is the least common multiple of the number of stator slots \((N_s)\) and the number of poles \((N_p)\), \(k\) is an integer, and \(T_{mk}\) is a Fourier coefficient.

For an inset PMSM with 8 poles and 24 slots, each pole of the machine has a whole number multiple of stator teeth, such that the cogging effects of each magnet are in phase and added.\(^3\) The cogging torque contribution for each magnet can be described by equation (4)

\[
T_c = N_s \sum_{k=1}^{\infty} T_{pN,k} \sin (N_s k\theta) \tag{4}
\]

where \(T_{pN,k}\) is a per magnet cogging torque coefficient. The air gap reluctance in equation (1) can also be determined using equation (5)

\[
R_g = \frac{l_g}{\mu_0 A_g} \tag{5}
\]

where \(R_g\), \(l_g\), and \(A_g\) are the air gap reluctance, air gap length, and air gap cross-sectional area, respectively. \(\mu_0\) is a magnetic permeability in air gap. For practical reasons related to the machine’s design, the reduction in the pole arc increases the cross-sectional area of the air gap; this directly impacts the decrease in the air gap reluctance and cogging torque.

Assuming the magnet to be unity and there is no current in the stator, the flux density in the air gap can be predicted by equation (6)

\[
B_g = \frac{B_m l_m}{(r_r + l_g) \ln \left(\frac{r_r + l_g}{r_r - l_m}\right)} \tag{6}
\]

where \(B_m\), \(l_m\), \(r_r\), and \(l_g\) are the magnetic remanence, magnet length, rotor radius, and air gap length, respectively.

The proposed structure of inset PMSM

To study the effect of the slotting in the magnet edges, three inset PMSM models have been used and compared. These three inset PMSM models have the same stator core structure as the actual machine, such as slot opening width, stator slot number, height of stator teeth, and stator teeth width. It is assumed that the stator of the three inset PMSM models has the same characteristics.

The experimental inset PMSMs with 8 poles and 24 slots are shown in Figure 1. The original model is a conventional popular magnet structure with no slots employed at the end or surface of the magnet pole. Model 1, with a one-step slot in the magnet,\(^6\) and Model 2, with a two-step slot in the magnet edge, are depicted in Figure 1(b) and (c), respectively. The physical parameters and the detailed structure of the three
experimental inset PMSMs are shown and depicted in Table 1 and Figure 2.

Based on the structure of original machine shown in Figure 2(a), the magnet edge slotting is shaped on the width of stator opening slot to minimize the cogging torque. The strategy is to allow as much as possible magnetic flux circulating in rotor teeth instead of flowing into stator slot. The larger and deeper slot of the magnet edge induces more flux in the magnet slot area circulating in the rotor teeth, but decreases the magnet flux flowing into the stator slot because of larger distance between the magnet edge and the stator slot.

To optimize the effect of cogging torque reduction, the magnet height and length of slot in the leading and trailing edge of the permanent magnet of rotor with NdFeB material, shown as $H_1$, $H_2$, $L_1$, and $L_2$ in Figure 2, are selected based on the general knowledge of electrical machine design and were implemented by some experienced parameters. The values of base magnet pole arc ($\alpha_b$) of all machines are considered the same. The presence of slots in the magnet edges in the structure of Model 1 and Model 2 decreases the magnet pole arc ($\alpha_s$) surface shown in Figure 2(b) and (c).

The reduction in the magnet pole arc length and magnet pitch conducts a shorter route in the leading (left) and trailing (right) edge of the magnet. Because the presence of slotting at the magnet increases the air gap cross-sectional area, as indicated by equations (5) and (1), the air gap reluctance and cogging torque can be reduced effectively. Other important factors affecting the peak of the cogging torque, as shown in equations (3) and (4), are magnet side arc ($\theta_1$) and magnet center arc ($\theta_2$). The optimal iterative process for the design of the proposed inset PMSM is described in the following section.

**Simulation results and discussion**

The finite element method magnetics tool, FEMM 4.2, and LUA 4.0 scripting language were used to investigate the inset PMSM characteristics based on finite element analysis. The air gap magnetic flux distribution and cogging torque for the experimental models have been investigated and compared.

To determine the optimum shape of permanent magnets for minimum cogging torque, some dedicated

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**Table 1.** Tested parameters for the experimental inset PMSMs.

| Parameters                              | Original model | Model 1 | Model 2 |
|-----------------------------------------|----------------|---------|---------|
| Stator diameter, $d_{\text{stator}}$ (m) | 0.1213         | 0.1213  | 0.1213  |
| Air gap length, $l_g$ (m)               | 0.002417       | 0.002417| 0.002417|
| Rotor teeth angle, $\alpha_{\text{rot}}$ (°) | 4.03189   | 4.03189 | 4.03189 |
| Slot opening width, $w_{\text{so}}$ (m)  | 0.002          | 0.002   | 0.002   |
| Shaft diameter, $d_{\text{shaft}}$ (m)  | 0.0423         | 0.0423  | 0.0423  |
| Magnet length, $l_m$ (m)                | 0.036889       | 0.036889| 0.036889|
| Magnet height, $h_m$ (m)                | 0.006039       | 0.006039| 0.006039|
| Magnetic radius, $r_m$ (m)              | 0.05822        | 0.05822 | 0.05822 |
| Rotor core radius, $r_c$ (m)            | 0.052186       | 0.052186| 0.052186|
| Inner stator radius, $r_i$ (m)          | 0.060637       | 0.060637| 0.060637|
| Out stator radius, $r_0$ (m)            | 0.095          | 0.095   | 0.095   |
| Magnet cross-sectional area, $A_m$ (m²) | 0.000237       | 0.000224| 0.0002209|
| Air gap cross-sectional area, $A_a$ (m²) | 0.000986     | 0.001087| 0.001196 |
| Stator teeth width, $w_{\text{tw}}$ (m) | 0.009151       | 0.009151| 0.009151|
| Stator teeth height, $w_{\text{th}}$ (m) | 0.015044      | 0.015044| 0.015044|

PMSM: permanent magnet synchronous machine.
optimization algorithm can be used. Iterative procedures between script language, auto computer-aided design (CAD) drawing, and FEMM software package were used to achieve experienced parameters, and much computational time is devoted to approach optimal design parameters. In this study, two significant structure parameters, the height and length of the slot in the rotor’s magnet edge, were tested systematically using some experienced parameters. The magnet slot width from the rotor teeth based on the slot opening width is set to be 2 mm in the article. The magnet side arc and magnet center arc is also considered. To reduce the cogging torque, the effect of magnet edge slotting is not so significant compared to the distance between the magnet edge and the rotor teeth. The materials used for the study are permanent magnet NdFeB and M-19 for stator and rotor core. For this reason, the proposed machine is designed to operate at magnetic flux density of 1.7–1.8 T. The optimal iterative process for the design of the proposed inset PMSM is shown in Figure 3.

The air gap length is constant in the center of the magnet and the flux can normally reach the stator slot; however, the flux amount depends on the arc of the magnet side and arc of the magnet center. The optimal rotor structure can be achieved by adopting the appropriate arc in the magnet side and magnet center based on design experience and finite element simulation results. The optimal value for the other state variables shown in Figure 3 are 0.003 m for the first magnet length ($L_1$) and 0.0025 m for the second magnet length ($L_2$). The magnet flux tended to circulate in the rotor teeth instead of flow into stator slot under the optimal structure. The optimal magnet side arc and magnet center arc are $\frac{\pi}{18}$ and $\frac{\pi}{12}$.

The final results were worth the effort, as the simulation time was dramatically reduced. For comparisons, the influences of various magnet edge heights of $H_1$ and $H_2$ of the proposed Model 2 on the cogging torque are presented in Tables 2–4 and Figure 4. Figure 5 shows the effects of different second magnet edge lengths on the cogging torque of the inset PMSM. The optimal shape parameters of the rotor structure are summarized in Table 5.

Compared with the original model, the cogging torque of the proposed model is quite small. The number of effective digits after the decimal point used in the article is six digits for all the cogging torque values of

![Figure 2](image_url). Details of the structure of the simulated inset PMSMs: (a) original model, (b) Model 1, and (c) Model 2.
The experimental machine. Less effective digits induce unnecessary misinterpretation. For example, most of the peak value for the proposed model was around 0.000554 N m, but the original model was around 0.011731 N m.

Figure 5 shows the effects of different magnet edge lengths ($L_2$) on the cogging torque of the inset PMSM. The second magnet height $H_2$ is tested by the variations from 0.000810 to 0.000850. The final optimal magnet height of $H_1$ is 0.0008 mm and $H_2$ is higher than 0.0008 m. The arc of magnet side is 7° and the arc of magnet center is 15°. The impacts of different $H_2$ on the cogging torque are concluded in Table 6.

No load simulation is implemented in the article. The cogging torque values can be expressed in percent by comparing the maximum of the cogging torque value of the proposed model to the maximum value of the cogging torque of the initial model. For example, the maximum cogging torque value of the proposed model is 0.000553 N m and the maximum cogging torque of the original model is 0.011732 N m, the percentage of the cogging torque is 4.71%, and the decrement of the cogging torque is 95.2%.

Figure 6 shows the simulation results for the magnetic density flux wave in the air gap on all machines; the flux density curves are distorted because of the slot opening in the stator core. The simulation results show that the shape of the magnetic flux distribution in the air gap of the proposed Model 2 is not distorted, and the peak of normal flux density remains constant. The shape and peak of magnetic flux density in the air gap of an electrical machine are the significant parameters predicting the output power. The proposed Model 2 can achieve better power performance and characteristic than the other experimental models.

### Table 2. Comparisons of the cogging torque of Model 2 using different magnet edge heights.

| Magnet edge height (m), $H_1 = H_2$ | Max. cogging torque (N m) | Min. cogging torque (N m) |
|-------------------------------------|----------------------------|---------------------------|
| 0.0012                              | 0.0016348                 | -0.0016712                |
| 0.0011                              | 0.0013178                 | -0.0013248                |
| 0.0010                              | 0.0009929                 | -0.0009743                |
| 0.0009                              | 0.0006609                 | -0.0006982                |
| 0.0008                              | **0.0005374**             | **-0.0005429**            |

Bold values represent minimum cogging torque.

### Table 3. Comparisons of the cogging torque of Model 2 using different magnet edge lengths under the optimal height from Table 2.

| $L_1$ (m) | $L_2$ (m) | $H_1 = H_2$ (m) | Max. cogging torque (N m) | Min. cogging torque (N m) |
|-----------|-----------|-----------------|----------------------------|---------------------------|
| 0.003     | 0.0025    | 0.0008          | **0.000551**               | **-0.000538**             |
| 0.003     | 0.0024    | 0.0008          | 0.000619                   | -0.000640                 |
| 0.003     | 0.0023    | 0.0008          | 0.000813                   | -0.000794                 |
| 0.003     | 0.0022    | 0.0008          | 0.000955                   | -0.000979                 |
| 0.003     | 0.0021    | 0.0008          | **0.00113**                | **-0.001184**             |

Bold values represent minimum cogging torque.

Figure 3. The optimal iterative process.
Table 4. Comparisons of the cogging torque of Model 2 using different magnet trailing edge heights under the optimal length from Table 3.

| $H_1$ (m) | $H_2$ (m) | $L_1$ (m) | $L_2$ (m) | Max. cogging torque (N m) | Min. cogging torque (N m) |
|----------|----------|----------|----------|-------------------------|-------------------------|
| 0.0008   | 0.0008   | 0.0003   | 0.0025   | 0.000537                | -0.000543               |
| 0.0008   | 0.0007   | 0.0003   | 0.0025   | 0.000706                | -0.000669               |
| 0.0008   | 0.0006   | 0.0003   | 0.0025   | 0.000826                | -0.000816               |
| 0.0008   | 0.0005   | 0.0003   | 0.0025   | 0.000920                | -0.000919               |
| 0.0008   | 0.0004   | 0.0003   | 0.0025   | 0.001008                | -0.001004               |

Bold values represent minimum cogging torque.

Table 5. Optimal shape parameters of rotor structure.

| Parameter              | Original model | Model 1 | Model 2 |
|------------------------|----------------|---------|---------|
| Magnet edge height $H_1$ | --             | 0.0023  | 0.0008  |
| Magnet edge height $H_2$ | --             | --      | 0.0008  |
| Magnet edge length $L_1$ | --             | 0.0025  | 0.0025  |
| Magnet edge length $L_2$ | --             | --      | 0.003   |

Figure 4. Influences of different heights of the magnet edge on the cogging torque.

Figure 5. Influences of different lengths of the magnet edge on the cogging torque.
Figure 7 shows the magnetic flux density of the core in the stator and rotor of all three experimental models. Different colors have been used to make it easier to distinguish different magnetic flux densities in the core of the machines. Three marked points, A, B, and C, each with high magnetic flux density have been chosen to investigate the effects of cogging torque reduction. The A and B points are in rotor core, and C is in the stator teeth.

As the simulation results of Figure 7(c) show, the magnetic flux density of the proposed Model 2 at the stator core marked as point C is 1.35215 T, while the values at the rotor core for points A and B are 0.771101 and 0.728327 T, respectively. Compared with the original model and Model 1, the stator’s flux density at point C is 1.36896 and 1.36174 T, while the values of the rotor’s flux density at points A and B are 0.821691 and 0.752841 T for the original model, and 0.850965 and 0.82878 T for Model 1, respectively. The two-step slot in the magnet edge for the proposed Model 2 achieved the lowest magnetic flux density both at the cores of the stator and rotor. The cogging torque can be reduced effectively under the proposed structure of this inset PMSM.

Using finite element analysis, the variations in the cogging torque over 30 geometrical rotation degrees in the three models have been plotted and the results are shown in Figure 8. The maximum cogging torque for the three experimental machines is $-0.011818$, $-0.005893$, and $0.000921$ N·m, respectively. It can be seen that the peak value of the cogging torque for the proposed Model 2 can be reduced by as much as 92.20% compared with the original model and by 50.14% for Model 1.

Increasing the air gap cross-sectional area leads to a decreasing air gap reluctance, as shown in equation (5). In fact, the air gap reluctance value varies when mechanical rotor position changes. The peak of the cogging torque varies swiftly along the mechanical rotor position as the structure of the machine is changed. It is influenced by the magnet interactions in the stator and rotor core. For example, the highest cogging torque occurs at $22^\circ$ mechanical rotor position for the proposed Model 2, but $12^\circ$ for the original model and $10^\circ$ for Model 1. However, the reduction in peak cogging torque dominates the machine’s structural design.

The benefits of this structure of the proposed inset PMSM can be summarized as follows:

1. The proposed two-step slot in the magnet edge directly reduces the cross-sectional area of the magnet without changing the rotor diameter. The presence of the two-step slot in the magnet can be inferred to modify and reduce the magnetic flux strength without any change in the magnetic material of the inset PMSM.

2. The proposed two-step slot in the magnet edge decreases the reluctance of the air gap, an important parameter which influences the cogging torque in PMSM. Meanwhile, the magnetic flux density of the proposed inset PMSM in the stator and rotor cores also decreases when compared with the non-slotting design in the magnet edge.

3. The proposed two-step slot in the magnet edge refers to the variations in the magnet arc length varying, which affects the magnet pitch and surface area of the permanent magnet. In other words, the cogging torque can be reduced.

### Conclusion

In this article, the influences of slotting in the magnet edges with fix slot opening width in inset PMSMs have
Figure 7. Magnetic flux density in the core of stator and rotor for three experimental machines: (a) original model, (b) Model 1, and (c) Model 2.
been investigated. The impacts of different magnet structures on the cogging torque and air gap magnetic flux density are presented and compared. The simulation results show that the proposed two-step slot in the magnet edge achieves a better cogging torque reduction than a one-step slot and without slotting. The simulation accuracy of the cogging torque reduction has been validated using a finite element magnetic tool. The peak value of the cogging torque for the proposed model can be reduced by as much as 92.20% compared with the experimental model. The proposed structure can be applied to design a wind turbine generator operating at the regions of limited wind energy, such as low wind speed.

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