Effect of temperature and bias voltage on the conductance distribution of disordered 1d quantum wires

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Abstract. – The statistical properties of the conductance of one dimensional disordered systems are studied at finite bias voltage $V$ and temperature $T$, in an independent-electron picture. We calculate the complete distribution of the conductance $P(G)$ in different regimes of $V, T$ within a statistical model of resonant tunneling transmission. We find that $P(G)$ changes from the well-known log-normal distribution at $T = 0$ in the linear response regime to a Gaussian distribution at large $V, T$. The dependence on $T$ and $V$ of average quantities such as $\langle G \rangle$, $\langle \ln G \rangle$ is analyzed as well. Our analytical results are confirmed by numerical simulations. We also discuss the limits of validity of the model and conclude that the effects of finite $T, V$ presented here should be observable.

Quantum electronic transport in one-dimensional disordered systems has been widely studied ever since the concept of Anderson localization has been introduced in the sixties [1]. In particular it is known that within the model of noninteracting electrons all states are localized in 1d, with average extension $\xi$, the localization length. For systems of finite length $L$, the typical conductance decreases exponentially with increasing length. However, there are large sample to sample fluctuations and a statistical analysis of the conductance $G$ is required. For a 1d wire of length $L \gg \xi$, the full distribution of linear conductance at $T = 0$ has been determined [1-5]: it is log-normal with a large variance of the order of the mean.

Finite temperature $T$ and a finite bias voltage $V$ have a dramatic effect on the conductance distribution, as we show in this letter. The conductance is given by an energy integral of the transmission coefficient $g(E)$. A window of energies within which electron transport is possible is determined by the value of $T$ and $V$. Since $g(E)$ is a strongly fluctuating function of $E$, the result depends sensitively on the width of the energy window.

In the linear conductance regime (zero bias voltage), Azbel and Vincenzo [6] calculated the $T$ dependence of the averages of the conductance and its logarithm, at low temperatures. They employed a model of $g(E)$ as a sum of resonances with vanishing width ($\delta$-functions)

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caused by resonance tunneling through localized states situated at random positions, and decaying exponentially over the distance $\xi$. They did not consider the full distribution of conductances. Moreover, they did not take into account the fluctuations of the localization length. Their model therefore does not recover the log-normal distribution of $G(T)$ at $T = 0$. More recently, it has been shown numerically that the average resistance decreases strongly with increasing temperature [7].

The statistical properties of the nonlinear conductance $G(T, V) = J(T, V)/V$, in particular its distribution $P(G)$ have not been considered before (here $J(T, V)$ is the charge current), to our knowledge.

One may ask whether interaction effects will preempt the behavior found here within the independent particle model. The phase coherence required for resonant tunneling is limited to a finite length $L_0$ by phase relaxation at finite temperature/voltage. We will argue below that for sufficiently short wires $L \ll L_0$, even at the values of $T, V$ of interest and phase coherence is preserved. Furthermore, at finite $T, V$ transport will occur by way of inelastic processes, e.g. thermally activated hopping [8–10]. As shown below, these processes will dominate the transport in a regime of $T, V$ beyond the one we will consider here. Our estimates show that the model we consider should be applicable to realistic systems.

In this letter we study the effect of bias voltage and temperature on the statistical properties of transport in a one-dimensional disordered system, within the model of independent electrons. We consider transmission through the wire by way of resonant tunneling, and employ a statistical model of individual resonances with Breit-Wigner line shapes [11]. We calculate the full distribution of the conductance in various regimes of $T, V$, as well as average quantities such as $\langle G \rangle$ and $\langle \ln G \rangle$.

We show that the distribution of the conductance develops from a log-normal distribution in the limit of zero voltage and temperature to a Gaussian distribution in the high $V$ and $T$ regime. We also find that the conductance average $\langle G \rangle$ is independent of $V, T$, while $\langle \ln G \rangle$ decreases with $V, T$ from its zero temperature and voltage value up to $-\ln(G)$, at high values of $V, T$.

Our analytical results are supported by numerical simulations of a disordered one-dimensional multibarrier system. We calculate numerically the total scattering matrix of the system by combining the scattering matrices associated to each barrier. We consider all the barriers to have the same fixed height, but random widths and separations [12].

The model. – The nonlinear conductance $G(T, V)$ in units of $G_0 = 2e^2/h$ is given by [13, 14]

$$G(T, V) = \frac{1}{eV} J_{\mu} \int_{-eV/2}^{eV/2} dE' \int_{-\infty}^{\infty} dE g(E) \frac{\partial f(E - E')}{\partial E}, \quad (1)$$

where $f(E) = (\exp[(E - \mu)/kT] + 1)^{-1}$ is the Fermi function; $k$, the Boltzmann constant; and $\mu$, the chemical potential. $g(E)$ is the transmission probability, equal to the zero temperature linear conductance for $E = E_F = \mu(T = 0)$. In a wire of length $L \gg \xi$, where $\xi$ is the average localization length, transmission through the wire takes place via localized states with random energies $E_\nu$, located at random positions $z_\nu$ in the wire. The wave function of any of these localized states decays exponentially over a random length $\xi_\nu$.

The transmission probability associated with each localized state is given by a Breit-Wigner formula, with width $\Gamma_\nu = \Gamma_\nu^{(l)} + \Gamma_\nu^{(r)}$, $\Gamma_\nu^{(l,r)} = \Delta \exp[-(L \pm 2z_\nu)/\xi_\nu]$. $\Gamma_\nu^{(l,r)}$ are the partial widths induced by coupling to the left/right lead, and $\Delta$ is the mean-level spacing of resonant states. We will assume $\Gamma_\nu \ll \Delta$ in the following, corresponding to $L/\xi \gg 1$. Within this
model, the conductance \( g(E) \) is given by

\[
g(E) = \sum_{\nu} \frac{\Gamma^{(l)}_{\nu} \Gamma^{(r)}_{\nu}}{(E - E_{\nu})^2 + \Gamma^2_{\nu}}. \tag{2}
\]

On resonance \( (E = E_{\nu}) \), the terms in the sum eq. (2), \( t_{\nu} \), depend on the location of the state,

\[
t_{\nu} = [2 \cosh(2z_{\nu} / \xi_{\nu})]^{-2} \tag{3}
\]

and are maximum for \( z_{\nu} = 0 \) (center of wire).

We assume \( E_{\nu} \) and \( z_{\nu} \) to be uniformly distributed in the intervals \( |z| \leq L/2 \) and \( (\nu - \frac{1}{2}) \Delta < E_{\nu} < (\nu + \frac{1}{2}) \Delta \), respectively. The inverse localization lengths \( \xi_{\nu}^{-1} \) (more conveniently \( x_{\nu} = 2L/\xi_{\nu} \)) are assumed Gaussian distributed

\[
p(x_{\nu}) = C \exp \left[ -\frac{(x_{\nu} - \langle x_{\nu} \rangle)^2}{4 \langle x_{\nu} \rangle} \right], \tag{4}
\]

where \( \langle x_{\nu} \rangle = 2L/\xi_{\nu} \) and \( C \) is a normalization constant.

The above model of a disordered 1d system leads to the well-known log-normal distribution for the linear conductance at \( T = 0 \) and \( L \gg \xi \) [15]:

\[
P(\ln g) = C' \exp \left[ -\frac{1}{2} \left( \frac{4L/\xi}{2} \right) - 1 \ln(1/g) - 2L/\xi \right]^2; \tag{5}
\]

\( C' \) is a normalization constant. In this case the resonance closest to the Fermi level, \( E_F \), dominates and one may safely neglect additional resonances.

At finite \( T, V \), the nonlinear conductance is given by

\[
G = -2\pi kT \text{Re} \left[ \sum_{\nu} \sum_{n=0}^{\infty} \frac{t_{\nu} \Gamma_{\nu}}{(E_{\nu} - \mu + i\Gamma_{\nu} + i\omega_n)^2 - (\frac{V}{2})^2} \right], \tag{6}
\]

where we have performed the integrals in eq. (1) by using the expansion

\[
\partial f / \partial E = 2kT \text{Re} \{ \sum_{n=0}^{\infty} (E - \mu - i\omega_n)^{-2} \} \text{ with } \omega_n = (2n + 1)\pi kT.
\]

**Gaussian distribution at high temperatures/voltages.** – Let us first consider the regime of not too low temperatures defined by \( \Gamma_{\nu} \ll kT \). Neglecting \( \Gamma_{\nu} \) in the denominator of eq. (6) yields

\[
G(T, V) = \frac{\pi}{2} \sum_{\nu} \frac{t_{\nu} \Gamma_{\nu}}{eV} \left[ \frac{\tanh(\widetilde{E}_{\nu} + \frac{eV}{2kT}) - \tanh(\widetilde{E}_{\nu} - \frac{eV}{2kT})}{2kT} \right], \tag{7}
\]

where \( \widetilde{E}_{\nu} = E_{\nu} - \mu \).

Now, for \( kT \) or \( eV \gg \Delta \), many terms in the sum over \( \nu \), with \( |\widetilde{E}_{\nu}| < E_{\text{max}} \), will contribute, where \( E_{\text{max}} = eV \coth(eV/2kT) \simeq \max(eV, 2kT) \). Consider \( E_{\text{max}} = eV \), for example. In this case, \( G(T, V) \) is given by

\[
G(T, V) \approx \frac{\pi}{2} \frac{\Delta}{eV} \sum_{\nu} g_{\nu} \Theta(E_{\text{max}} - \widetilde{E}_{\nu}), \tag{8}
\]

where \( g_{\nu} = t_{\nu} \Gamma_{\nu} / \Delta \) and \( \Theta(x) \) is the step function. Since the quantities \( g_{\nu} \) are statistically independent, we may apply the central limit theorem to find a Gaussian distribution for the conductance :

\[
P(G) = C \exp \left[ -\frac{(G - \langle G \rangle)^2}{2\sigma^2} \right], \tag{9}
\]
where \( \langle G \rangle = \langle g_0 \rangle \propto \exp \left( -L/2\xi \right) \) is essentially the zero temperature and voltage result, while the variance \( \sigma^2 = \frac{\Delta}{E_{\text{max}}} (\langle g_0 - \langle g_0 \rangle \rangle)^2 \) decreases as a function of the inverse voltage. \( C \) is a normalization constant. A similar analysis may be done for the linear conductance regime. In this case \( E_{\text{max}} = kT \) and the average of the Gaussian distribution is given by \( \langle G \rangle \) (independent of \( T \)), while \( \sigma^2 \) decreases with the temperature as \( T^{-1} \). In fig. 1 we show an example of the central limit theorem at work: \( P(G) \) from our simulation of a 1d system follows a Gaussian distribution and its variance (inset) dependent on the temperature as \( 1/kT \), as predicted above [16].

We have shown that the average conductance at high temperature/voltage is given by its corresponding zero temperature result. In fact, our model gives a very smooth temperature/voltage dependence for \( \langle G(T,V) \rangle \) at all regimes of \( T,V \). From eq. (6), averaging over disorder we find for \( L \gg \xi \)

\[
\langle G(T,V) \rangle \approx \frac{2kT}{eV} \langle g \rangle_0 \ln \frac{\cosh w_{N_+} \cosh w_{1_-}}{\cosh w_{N_-} \cosh w_{1_+}},
\]

where \( \langle g \rangle_0 = \sqrt{\xi/2\pi L} \exp \left( -L/2\xi \right) \) is the average linear conductance at zero temperature, \( w_{N_+, -} = (\Delta/2kT)(N \pm eV/2\Delta) \), and \( w_{1_+, -} = (\Delta/2kT)(1 \pm eV/2\Delta) \), with \( N \) the total number of resonances that contributes to the sum in eq. (6) (\( N \sim E_{\text{max}}/\Delta \), at high temperatures/voltages). eq. (10) depends weakly on \( T,V \): for example, we can see that in both limit cases \( eV/kT \gg 1 \) or \( eV/kT \ll 1 \), \( \langle G(T,V) \rangle \approx \langle g \rangle_0 \). In contrast, \( \langle \ln G(T,V) \rangle \) depends strongly on \( T,V \). At high temperatures/voltages, we have seen that the width of the Gaussian distribution \( P(G) \) decreases with the inverse temperature/voltage, keeping constant its average, i.e. \( P(G) \) becomes a narrow function around \( \langle G \rangle \). As a consequence, \( \langle \ln G \rangle \) approaches to \( \ln \langle G \rangle \) at large values of \( T/V \). As a verification of the above results, in fig. 2 we show the results from the numerical simulation: the value \( \langle G \rangle \) at different finite temperatures lies on the corresponding zero temperature result (solid line). In the inset, \( -\ln G = 2L/\xi \) at \( T = 0 \), while \( \langle \ln G \rangle \) approaches to \( \ln \langle G \rangle \) (solid line), at high \( T \).
Fig. 2 – \( \langle G(T) \rangle \) from the numerical simulation as a function of the length of the system (in units of \( \xi \)) at different temperatures. The solid line is the zero temperature result \( \langle g \rangle_0 \) (below eq. 10). Similarly, the inset shows \( -\ln G \) at several temperatures, here the solid line correspond to \( \ln \langle g \rangle_0 \) (the dotted lines are to guide the eye).

Intermediate temperatures/voltages. – When \( \Gamma_{\nu} \ll E_{\text{max}} \leq \Delta \), only the resonance closest to the Fermi level, say \( \nu = 0 \), will contribute appreciably to \( G \), and

\[
G(T, V) = \frac{\pi t_0 \Gamma_0}{2 eV} \left[ \tanh \frac{E_0 - eV/2}{2kT} - \tanh \frac{E_0 + eV/2}{2kT} \right].
\] (11)

The distribution of conductances is then given by

\[
P(G) = C \int_0^\infty dx p(x) \int_{-L/2}^{L/2} dz \int_{-\Delta/2}^{\Delta/2} d\tilde{E}_0 \delta (G - G(T, V)),
\] (12)

with \( p(x) \) given by eq. (4). \( C \) is a normalization constant. Performing the integral over \( \tilde{E}_0 \) with the help of the \( \delta \)-function we find

\[
P(G) = C' \int_{x_1}^{x_2} dx \int_{x}^{s_c} ds \frac{g_0}{\sqrt{\left( \frac{\pi \Delta g_0}{eV} - \coth v \right)^2 - \csch^2 v}},
\] (13)

where \( g_0 = 2e^{-x/2} \csch(x/2), v = eV/2kT, s_c = \arccosh(b \exp(-x/2))/x, x_1 = \ln(2b - 1), x_2 = 2 \ln b, \) and \( b = (\pi \Delta/eV G) \sinh v/(1 + \cosh v) \).

In fig. 3 (insets) we compare \( P(\ln G) \) as calculated from eq. (13) \( P(\ln G) = GP(G) \) to the numerical simulation data, in the linear and nonlinear conductance regime. The agreement is seen to be good. Compared to the \( T \) and \( V \) zero result, the peak of \( P(\ln G) \) is shifted to larger values of \( G \) (smaller values of \( \ln G \)), and is clearly narrower.

Typical conductance at low temperatures/voltages. – When \( E_{\text{max}} \ll \Gamma_{\nu} \ll \Delta \), there is no significant effect of finite temperatures/voltages at the level of the bulk distribution of the conductance: \( P(\ln G) \) is seen to be well described by the zero temperature linear conductance distribution, as it is shown in Fig 4. To discuss the correction to this result we now calculate \( \langle \ln G \rangle \). At small values of \( V \) and \( T \) we find, from eq. (9),

\[
\langle -\ln G(V, T) \rangle \approx \frac{2L}{\xi} \left[ 1 - \frac{\pi}{6\Delta} \left[ (eV)^2 + (2\pi kT)^2 \right] \right],
\] (14)
The first term in eq. (14) corresponds to the zero temperature result for the linear conductance regime, as one may expect. In figure 4 (insets) we compare our expression (14) to the numerical simulations.

Observability of effects and conclusion. – We now estimate the effect of inelastic processes at temperatures $kT \sim \Delta$. At temperatures $kT \ll (\hbar v_F/\alpha \xi)$, where $\alpha < 1$ is a dimensionless interaction constant, the phase relaxation length $L_\phi \sim L_0^2 \left(\frac{E_F}{\Delta}\right)^{2\alpha} \left(\frac{\Delta}{kT}\right)^{2+2\alpha} \gg L$ [17], provided...
$kT \ll \sqrt{\frac{\xi}{L}} (\frac{E_F}{\Delta})^2 \Delta$, where $E_F/\Delta$ is very large; thus for $kT > \Delta$ phase coherence is seen to be preserved. The contribution of variable range hopping has been estimated in [9,10]. It is negligible compared to resonance tunneling at temperatures $kT < \Delta(\xi/L)$. This means that in the regime $kT > \Delta$ thermally activated hopping processes dominate, and the narrowing of $P(G)$ in the linear response regime will not be accessible. At $kT \ll \Delta$ and $eV \gg \Delta$, however, thermal activation will not be relevant. Also, the phase coherence should be maintained as long as $eV \ll (\frac{E_F}{\Delta})^2 \Delta$, which would allow to access the Gaussian regime of $P(G)$. In fig. 3 (main frame) we show $P(\ln G)$ for values of $T, V$, as discussed above. A narrower distribution is clearly seen, compared to the linear conductance regime. On the other hand, some of the effect of inelastic scattering may be included in the present model by adding an inelastic component to the Breit-Wigner resonance width [9].

To conclude, we expect that the statistical effect of averaging over the resonance structure of the transmission coefficient at large $V, T$ should lead to large observable changes in the conductance distributions of quantum wires. Experimental observation of this phenomenon should be of considerable interest [18].

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