Hybrid PUF: A Novel Way to Enhance the Security of Classical PUFs

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Abstract. Physical unclonable functions (PUFs) provide a unique ‘fingerprint’ to a physical entity by exploiting the inherent physical randomness. However, most of the current day PUFs are vulnerable against sophisticated machine learning based attacks. With the help of quantum information theory, this paper proposes solutions to protect PUFs against machine learning-based attacks. Here, based on the querying capability, we first divide the adversaries into two classes, namely adaptive and weak adversaries. We also modify an existing security notion, universal unforgeability, to capture the power of those two classes of adversaries. We then introduce the notion of a hybrid PUF (HPUF), using a classical PUF and quantum conjugate coding. This construction encodes the output of a classical PUF in non-orthogonal quantum states (namely BB84 states). We show that the indistinguishability of those non-orthogonal states can significantly enhance the security of the classical PUFs against weak adversaries. Moreover, we show that learning the underlying classical PUF from the outputs of our HPUF construction is at least as hard as learning the classical PUF from its random noisy outputs. To prevent the adversaries from querying the PUFs adaptively, we borrow ideas from a classical lockdown technique and apply them to our hybrid PUF construction. We show that the hybrid PUFs, together with the lockdown technique, termed as hybrid locked PUF (HLPUF), can provide a secure client authentication protocol against adaptive adversaries and are implementable with the current day quantum communication technology. Moreover, we show that HLPUF allows the server to reuse the challenges for further client authentication, providing an efficient solution for running a PUF-based client authentication protocol for a long period while maintaining a small-sized challenge-response pairs database on the server-side. Finally, we explore the lockdown technique with quantum PUF and show that the direct adaptation of the classical lockdown technique will not work with the fully quantum PUFs, highlighting the vulnerabilities of such primitives against adaptive quantum adversaries.

Keywords: Hardware Security · Physical Unclonable Functions · Quantum Cryptography

1 Introduction

With the advancement of digital communication technology and intelligent electronic devices, the internet of things (IoT) is becoming ubiquitous [ARJ19, GBMP13]. From the healthcare systems to the homes, and smart cities [GBMP13, KRM17] we observe that

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the interconnected networks of smart IoT devices, e.g., integrated circuits (IC), are automating human activities, and in many applications, these devices need to perform some security-sensitive tasks while providing secrecy and privacy [ZCW+14]. Device authentication or device identification is a fundamental requirement in many applications and a challenging yet unsolved task in IoT. PUFs provide a resource-efficient solution for device authentication. The recent machine learning based attacks provide a serious threat to most of the current day PUFs. Designing a secure and efficient PUF is an active topic of research. In this paper, with the help of quantum information theory, we address the security issues, and the challenge re-usability issues of the classical PUFs, without increasing the implementation overhead. Moreover, the proposed solution can be implemented using the current day classical PUF and quantum key distribution (QKD) technology.

All the conventional solutions for device authentication are based on shared unique digital keys. Digital signatures that are generated from the shared key can solve the device authentication problem, but the shared keys need to be stored in a non-volatile memory (NVM). Maintaining the security of the key over the lifetime of an embedded entity is non-trivial [AK96, AK97, Sko05] and non-economic for many resource-constrained IoT devices. Moreover, if the stored key is not inherently bound with the physical device, it is vulnerable to being copied.

In this context, Physically Unclonable Functions (PUFs) are a promising technology that can establish trust in embedded systems without using any NVM [GCVDD02a, GCVDD02b, LLG+04]. A PUF derives volatile secret keys on the fly by exploiting the inherent random variations introduced by manufacturing processes of the ICs. Here the generated volatile secret is unique and internal to the PUF. It only exists in a digital form when the PUF is powered on and running. A slight variation in the manufacturing process can produce a different PUF. In practice, even if the random variation is measurable, due to the impossibility of having complete control over the micro and nanoscale fabrication variations, it is still infeasible to create an identical physical ‘clone’ of a PUF [RDK12]. Hence, the PUF produces a copy-proof, cost-efficient unique hardware fingerprint. Usually, one can generate such fingerprints just by querying the PUF physically. In the literature, we refer to the queries and response pairs as challenge-response pairs (CRPs). Due to the uniqueness property, we get different CRPs for different PUFs.

Client authentication is one of the most important applications for PUFs. In those applications, a server first gets a PUF from a manufacturer and creates a database of random challenge-response pairs by querying that PUF. Later, it sends the physical device to a client. For the authentication purposes, the server sends a random challenge from its database to the client. The client gets the response corresponding to that challenge from the PUF device and sends back the response to the server. The authentication process succeeds if the client’s response is the same as the response stored at the server’s database.

The literature of the classical PUFs (CPUFs) is rich, and there is a multitude of constructions available based on many different hardware technologies [GCVDD02b, GKST07, KL18]. We refer to [Roe12] for a detailed review of the constructions of classical PUFs. Although all of those constructions provide unique and inexpensive hardware fingerprints, they all suffer from providing good randomness. As a result, most of the existing CPUF constructions are vulnerable against the machine learning modeling-based attacks [Bec15, Bec14, Del19, RSS+13a, RSS10]. In these types of attacks, the attacker first collects a sufficient number of CRPs just adaptively querying the PUF and then use that data to derive a numerical model using the tools from machine learning. Here, the goal of the model is to predict the response of the PUF for an arbitrary challenge. These attacks open multiple new research directions on designing machine learning-based attack resilient PUFs [NSJ+19, SMCN17, YHD+16]. In the classical domain, to prevent the PUFs from such sophisticated attacks, often their output interfaces are protected by some other cryptographic primitives like cryptographic hash functions. In such proposals, the adversary
has only access to the hash of the response of the PUF. Note that this combination of the PUFs with the other building blocks can cause a significant resource overhead. In some of the proposals, to prevent the adversary from querying the PUF with arbitrary challenges, the input interfaces are also protected by a lock [YHD+16]. In the literature, this technique is known as lockdown technique. Usually, in the lockdown technique, the challenger needs to query with a challenge \( x \) and some part of the response \( y_1 \) as a query to the locked PUF. The locked PUF returns the rest of the response if and only if \( y_1 \) is a valid response corresponding to \( x \). In addition to the lockdown technique, in many proposals, multiple PUFs are combined to increase the unpredictability of the response [AP19, SMCN17]. For example, in one of such proposals, a PUF is generated by combining two different PUFs [SMCN17]. If two different PUFs produce two outputs \( y_a, y_b \) corresponding to a challenge \( x \), then the output of the combined PUF is \( y_a \oplus y_b \). It is still a very active research area. For more detailed information about the current-day classical PUF technology, we refer to [GASA20].

On the other hand, some of the proposals exploit the exciting features of the quantum information theory to design a secure PUF [ADDK21, GKB20, KMK21, Ško12, Ško16, ŠMP13]. In general, for these kinds of PUFs, the challenge-response pairs are quantum states and, they are called quantum PUFs (QPUFs), or in some cases quantum readout of PUF. Note that, for the quantum PUFs, the adversary can make adaptive superposition queries to the QPUFs and use the tools from quantum machine learning to predict the outcomes of the QPUF [ADDK21, ML16]. Though there are some candidates of the QPUFs that provide security against such quantum machine learning-based attacks, they are difficult to realise with the current day quantum technologies [KMK21]. Moreover, the server needs to store the challenges in a quantum memory for running any QPUF-based client authentication protocol. Such non-volatile quantum memories are far from today’s implementation, making the QPUF-based authentication protocols a futuristic technology.

In this paper, we are interested in enhancing the security of the existing CPUFs by protecting their input and output interfaces using the commercially available tools from the quantum information theory. Here for the first time, we show that by encoding the output of the classical PUFs into non-orthogonal qubits one can enhance the security of the PUFs against a class of adversaries, called weak adversaries. We refer to such PUFs with classical challenge and quantum response as hybrid PUFs (HPUFs). To make those PUFs secure against more powerful adversaries like the adaptive adversaries, we borrow the ideas from the classical lockdown technique and give a construction of hybrid locked PUF (HLPUF). We show that classical PUF together with quantum encoding and the lockdown can boost the security of the classical PUF without adding too much overhead. Note that, for both HPUF and HLPUF, the server only needs to store the classical challenge-response pairs from the classical PUF. For authentication purposes, the server/client encodes the response bit string on the fly into non-orthogonal quantum states. The client/server can verify the response just by performing simple single-qubit measurements on the received qubits.

One significant drawback of the PUF-based authentication protocol is that the server cannot use the same challenge multiple times to authenticate a client due to the man-in-the-middle attacks. Therefore, the server exhausts all the challenges from its database after running the authentication protocol certain number of rounds. There is no way to avoid this limitation for the classical PUFs. This paper shows that due to the measurement uncertainty principle of the quantum information theory, with our HLPUF construction, the server can reuse a challenge as long as it successfully authenticates the client using that challenge in the previous rounds, overcoming the major drawback of the CPUF-based client authentication protocols. However, if the server couldn’t authenticate a client in one round, the challenge corresponding to that round will not be reusable. In the following
subsection, we summarise our contributions.

1.1 Our Results in a Nutshell

- **HPUF construction based on conjugate-coding**: In this part, we focus on enhancing the security of the classical PUFs against weak adversaries with the idea to protect the output interface of the classical PUFs by encoding the classical outcomes in non-orthogonal states. Hence, for this kind of PUFs, which we name it Hybrid PUF, the input is a classical string, and the output is a quantum state. In this construction, we use a classical PUFs $f_i : \{0,1\}^n \rightarrow \{0,1\}^{2m}$ and define an two-to-one mapping the tuple $(y_i,(2j−1),y_i,2j) (1 \leq j \leq m)$ of $f_i$’s outcome to a qubit $|\psi_{\text{out}}^{i,j}\rangle \in \{ |0\rangle, |1\rangle, |+\rangle, |-\rangle \}$. Here, $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

- **Security of HPUF**:
  1. Here we show that if any weak adversary needs to have $q$ challenge-response pairs from a classical PUF $f : \{0,1\}^n \rightarrow \{0,1\}^{2m}$ to break it with a non-negligible (in $n$) probability then with the same number of queries the weak-adversaries can break the corresponding hybrid PUF with negligible probability (in $q$).
  2. Later, we show that for the weak-adversaries, learning about our constructed hybrid PUF is at least as hard as learning the classical PUF from an erroneous database of challenge-response pairs.

- **Hybrid Locked PUF (HLPUF) and authentication**: We apply the idea of the lockdown technique on an HPUF and construct HLPUF. Further, we show that if the HPUFs are secure against weak adversaries then the HLPUF-based authentication protocol is secure against adaptive adversaries.

- **Challenge Re-usability with HLPUF**: In this part, we show that for the HPUF, one CRP can be potentially used several times for HLPUF-based authentication protocol due to the unclonability of the quantum state as a response, while previous authentications are accomplished with the same pair. It overcomes the constraint that each CRP can only be used once with classical communication.

- **Infeasibility of lockdown technique on quantum PUF**: In the last part of this paper, we formalise the model of lockdown technique on quantum PUF and observe that such lockdown technique is incapable of quantum PUF since the response of QPUF is not always a separable state and makes the verification algorithm impractical to implement in the authentication protocol.

1.2 Structure of the Paper

In Section 2 we talk about all the tools and concepts that we use in the rest of this paper. Later, in Section 3 we talk about the notion of the weak and adaptive adversary. In the same section, we also introduce the modified notion of unforgeability against both the weak and adaptive adversaries. Here we use these notions to prove the security of our constructions. In Section 4, we introduce the notion of the lockdown technique with the classical setting and talk about the security of the client authentication protocol using the lockdown technique. In Section 5 we introduce the notion of HPUF and discuss the security of such PUFs against weak adversaries. We also give the construction of HLPUF.
and an HLPUF-based authentication protocol and prove its security in the same section.
In Section 6, we show the re-usability of the challenge-response pair for authentication with HLPUF. In Section 7, we formalise the model of lockdown technique on full quantum PUF and show the constraints of this. We finish this paper with a conclusion and discussion in Section 8.

2 Preliminaries

In this section, we discuss some of the main concepts and definitions that we rely upon in the paper.

2.1 Quantum information and quantum tools

Quantum states are denoted as unit vectors in a Hilbert space $\mathcal{H}$. Any $d$-dimensional Hilbert space is equipped with a set of $d$ orthonormal bases. We say a quantum state is pure if it deterministically describes a vector in Hilbert space. On the other hand, a mixed quantum state is described as a probability distribution over different pure quantum states, represented as a density matrix $\rho \in \mathcal{H}_d$. If a quantum state can be written as the tensor product of all its subsystems, we say that the state is separable, otherwise, it is referred to as an entangled state.

If a quantum resource takes an input $\rho_{in} \in \mathcal{H}_A^{in}$ and produces an output $\rho_{out} \in \mathcal{H}_B^{out}$, we use a completely positive and trace preserving (CPTP) map $E$ to describe the general quantum transformation $E : \mathcal{H}_A^{in} \rightarrow \mathcal{H}_B^{out}$.

The measurement of a quantum state is defined by a set of operators $\{ M_i \}$ satisfying $\sum_i M_i^\dagger M_i = I$ with its conjugate transpose operator $M_i^\dagger$. The probability of getting measurement result $i$ on quantum state $|\psi\rangle$ is:

$$ P(i) = \langle \psi | M_i^\dagger M_i | \psi \rangle = \langle \psi | M_i | \psi \rangle. $$

Furthermore, we define the set $\{ E_i \}$ a POVM (Positive Operator-Valued Measure) with positive operators $E_i = M_i^\dagger M_i$, where $\sum_i E_i = I$.

An important property of the quantum states is the impossibility of creating perfect copies of general unknown quantum states, known as the no-cloning theorem [WZ82]. This is an important limitation imposed by quantum mechanics which is particularly relevant for cryptography. A variation of the same feature makes it impossible to obtain the exact classical description of quantum states by having a single or very few copies, therefore, there exists a bound on how much classical information can be extracted from quantum states, known as Holevo bound [Hol73]. Moreover, distinguishing between two unknown quantum states is also a probabilistic procedure known in the literature of quantum information as quantum state discrimination. The distinguishability of the quantum states depends on their distance. There exist several distance measures for quantum states and quantum processes [NC10], although, for the purpose of this paper, we introduce the fidelity, the trace distance and the diamond norm. The trace distance between two quantum states $\rho$ and $\sigma$ is defined as:

$$ D_{tr}(\rho, \sigma) = \frac{1}{2} \| \rho - \sigma \|_1 = \frac{1}{2} Tr[\sqrt{(\rho - \sigma)^2}] $$ (1)

The fidelity of mixed states $\rho$ and $\sigma$ is defined by the Uhlmann fidelity [NC10]:

$$ F(\rho, \sigma) = [Tr(\sqrt{\sqrt{\rho \sigma} \sqrt{\rho} \sigma})]^2 $$ (2)

which will become $| \langle \psi | \phi \rangle|^2$ the following expression for two pure quantum states $|\psi\rangle$ ($\rho = |\psi\rangle \langle \psi|$) and $|\phi\rangle$ ($\sigma = |\phi\rangle \langle \phi|$). The fidelity is bounded between 0 and 1, $0 \leq F(\rho, \sigma) \leq 1$. 


$F(\rho, \sigma) = 0$ when two states $\rho$ and $\sigma$ are orthogonal and $F(\rho, \sigma) = 1$ when $\rho$ and $\sigma$ are identical.

In this paper, we denote all the verification algorithms for checking equality of two quantum states by distance as a CPTP map $\text{Ver} : \mathcal{H}^d \otimes \mathcal{H}^d \to \{0, 1\}$. For any two states $\rho_1, \rho_2 \in \mathcal{H}^d$, this mapping is defined below.

$$\text{Ver}(\rho_1, \rho_2) := \begin{cases} 1 & \text{if } \|\rho_1 - \rho_2\|_1 \leq \epsilon, \\ 0 & \text{otherwise}. \end{cases} \quad (3)$$

This general verification also includes measurements of quantum states as verification algorithms since it has been defined as a general CPTP map. Finally, we mention the notion of SWAP test as a quantum circuit for implementing the verification algorithm $\text{Ver}(\cdot)$ above. The swap test’s circuit uses the controlled version of a swap gate, that swaps the order of two quantum states if the control qubit is $|1\rangle$. The circuit outputs $|0\rangle$ with probability $\frac{1}{2} + \frac{1}{2}F(|\psi\rangle, |\phi\rangle)$ and it outputs $|1\rangle$ with probability $\frac{1}{2} - \frac{1}{2}F(|\psi\rangle, |\phi\rangle)$. As can be seen, the success probability of this test depends on the fidelity of the states. This occurs because of the quantum nature of these states and measurements in quantum mechanics.

### 2.2 Models for PUF

A Physical Unclonable Function is a secure hardware cryptographic device that is, by assumption, hard to clone or reproduce. Here we give the mathematical model for the classical PUFs first, and then we also briefly mention the quantum analogue of them known as quantum PUF (QPUF) as defined in [ADDK21]. As classical PUFs are usually defined with probabilistic functions, due to their inherent physical randomness, we first define the notion of probabilistic functions as follows.

**Definition 1** (Probabilistic Function). A probabilistic function is a mapping $f : \mathcal{R} \times \mathcal{X} \to \mathcal{Y}$ with an input space $\mathcal{X}$, an random coin space $\mathcal{R}$, and an output space $\mathcal{Y}$.

For a fixed input $x \in \mathcal{X}$, and a random coin (or key) $R \leftarrow \mathcal{R}$, we define the probability distribution of the output random variable $f(x) := f(R, x)$ over all $y \in \mathcal{Y}$ as,

$$p_y^x := \Pr[f(x) = y|x] = \sum_{r : f(r, x) = y} \Pr[R = r]. \quad (4)$$

A classical PUF can be modelled as a probabilistic function $f : \mathcal{R} \times \mathcal{X} \to \mathcal{Y}$ where $\mathcal{X}$ is the input space, $\mathcal{Y}$ is the output space of $f$ and $\mathcal{R}$ is the identifier. The creation of a classical PUF is formally expressed by invoking a manufacturing process $f \leftarrow \mathcal{MP}_C(\lambda)$, where $\lambda$ is the security parameter.

To model classical PUF $f$ in terms of security primitives, Armknecht et al. [AMSY16] define some requirements which are parameterized by some threshold $\delta_1$ and a negligible function $\epsilon(\lambda) \leq \lambda^{-c}$, where $c > 0$ and $\lambda$ is large enough. Note that the requirements in our paper correspond to the requirements of intra and inter distances of PUF $f$.

**Definition 2.** The classical PUF $f : \mathcal{R} \times \mathcal{X} \to \mathcal{Y}$ with $(\mathcal{MP}_C, \delta_1, \delta_2, \delta_3, \epsilon, \lambda)$ satisfies the requirements defined below:

**Requirement 1 ($\delta_1$-Robustness).** Whenever a single classical PUF is repeatedly evaluated with a fixed input, the maximum distance between any two outputs $y_i \leftarrow f(x)$ and $y_j \leftarrow f(x)$ is at most $\delta_1$. That is for a created PUF $f$ and $x \in \mathcal{X}$, it holds that:

$$\Pr \left[ \max(\text{Dist}(y_i, y_j)_{i \neq j}) \leq \delta_1 \right] = 1 - \epsilon(\lambda). \quad (5)$$
Requirement 2 ($\delta_2$-Collision Resistance). Whenever a single classical PUF is evaluated on different inputs, the minimum distance between any two outputs $y_i \leftarrow f(x_i)$ and $y_j \leftarrow f(x_j)$ is at least $\delta_2$. That is for a created PUF $f$ and $x_i, x_j \in \mathcal{X}$, it holds that:

$$\Pr \left[ \min_{i \neq j} \text{Dist}(y_i, y_j) \geq \delta_2 \right] = 1 - \epsilon(\lambda).$$

(6)

Requirement 3 ($\delta_3$-Uniqueness). Whenever any two classical PUFs are evaluated on a single, fixed input, the minimum distance between any two outputs $y_i \leftarrow f_i(x)$ and $y_j \leftarrow f_j(x)$ is at least $\delta_3$. That is for a created PUF $f$ and $x \in \mathcal{X}$, it holds that:

$$\Pr \left[ \min_{i \neq j} \text{Dist}(y_i, y_j) \geq \delta_3 \right] = 1 - \epsilon(\lambda)$$

(7)

where $\text{Dist}(\cdot, \cdot)$ is a general notion of distance between the responses.

We also introduce the notion of randomness for the classical PUF $f$. It says the maximal probability of $p_f(x_j)$ with an input $x_j \in \mathcal{X}$ on PUF $f_i$ where $i \in R$, conditioned on the residual output space. A formal definition is as follows.

Definition 3 ($p$-Randomness). We define the $p$-randomness of a classical PUF $f : \mathcal{R} \times \mathcal{X} \rightarrow \mathcal{Y}$ as

$$p := \max_{x \in \mathcal{X}} \max_{y \in \mathcal{Y}} p_f^i(y).$$

(8)

For a correct valid modeling of PUF, $\delta_1 < \delta_2$ and $\delta_1 < \delta_3$ are necessary conditions to allow for a clear distinction between different input and different PUFs.

A quantum PUF, is again a hardware primitive that is unclonable by assumption which also utilises the properties of quantum mechanics. Similar to a classical PUF, a QPUF is assessed via challenge and response pairs (CPR). However, in contrast to a classical PUF where the CRPs are classical states, the QPUF CRPs are quantum states. Moreover, the evaluation algorithm of a QPUF is modelled by a general quantum transformation that is a CPTP map that produces an output in the form of a quantum state. A quantum transformation needs to have few requirements such as robustness, collision resistance and uniqueness to be considered a QPUF, similar to its classical counterpart. The focus of this paper is not on full quantum PUFs, although for the Section 7, where we discuss the feasibility of lockdown technique for general quantum PUFs, we use the QPUF as defined in [ADDK21].

3 Unforgeability against Adaptive and Weak Adversaries

3.1 Models for Adaptive and Weak Adversaries

In this paper, we only consider the network adversarial model, i.e., the adversary has only access to the communication channel. Moreover, we assume that the manufacturer of the PUF is honest. The network adversaries can get the challenge-response pairs just by intercepting the messages that are exchanged between the server and the clients. They can also pretend to be the server and make queries to the PUF on the client side with a challenge and get the response.

Any network adversary that tries to predict the response of a PUF $E : \mathcal{D}^{in} \rightarrow \mathcal{D}^{out}$, can be modelled as an interactive algorithm. Here we consider Quantum Polynomial-Time (QPT) adversaries that have $q$-query classical access to the evaluation of the PUF, namely $E$ where $q$ is polynomial in the security parameter. An adaptive adversary can choose and issue any arbitrary query (up to $q$-query) which could also depend on the previous responses received from the PUF. On the other hand, a weak non-adaptive adversary, cannot choose the queries and instead receives $q$ CRPs of $E$. In this case, the queries are being picked at random from a uniform distribution by an honest party and sent to the adversary.
3.2 Unforgeability with Game-based Security

Unforgeability is the main security property of PUFs. Unforgeability means that given a subset of challenge-response pairs of the target PUF, the probability of correct estimation of a new challenge-response pair is negligible in terms of the security parameter. The unforgeability for Classical PUFs has been defined in [AMSY16], and for Quantum PUFs in [ADDK21] as a game-based definition. Moreover, a general game-based framework for quantum unforgeability has been defined in [DDKA21] for both quantum and classical primitives in an abstract way. Following the previous works, here in this paper, we present a game-based unforgeability definition for PUFs, emphasizing the adversary’s capabilities in the learning phase, and capturing both adaptive and weak adversaries as defined in the previous section. We define the unforgeability of PUF as a formal game between two parties: a challenger (C) and an adversary (A). The game is divided with 4 phases: Setup, Learning, Challenge and Guess. A formal description is given as follows:

Game 1 (Universal Unforgeability of PUF\(^1\)). Let \(MP\) be the manufacturing process, \(Ver(.\) be a verification algorithm for checking the responses, and \(\lambda\) the security parameter. We define the following game \(G_{PUF}(A, \lambda)\) running between an adversary \(A\) and a challenger \(C\):

- **Setup phase.**
  - \(C\) selects a manufacturing process \(MP\) and security parameter \(\lambda\). Then \(C\) creates a PUF by \(E \leftarrow MP(\lambda)\), which is described by a CPTP map. The challenge and response domain and range \(D^{in}\) and \(D^{out}\) are shared between \(C\) and \(A\).

- **Learning phase.**
  - If the adversary is adaptive, \(A = A_{ad}\):
    * \(A_{ad}\) selects any desired challenge \(c_i \in D^{in}\), and issues to \(C\) (up to \(q\) queries).
    * \(C\) queries the PUF with each challenge \(c_i\) and sends the response \(r_i = E(c_i) \in D^{out}\) back to \(A_{ad}\).
  - If the adversary is weak (non-adaptive), \(A = A_{weak}\):
    * \(C\) selects a challenge \(c_i \in D^{in}\) uniformly at random from \(D^{in}\) independent of \(i\).
    * \(C\) queries the PUF with \(c_i\) and produces the response \(r_i = E(c_i)\).
    * \(C\) issues to \(A_{weak}\) the set of random challenges and their respective responses \(\{(c_i, r_i)\}_i^{q}\).

- **Challenge phase.**
  - \(C\) chooses a challenge \(\tilde{c}\) uniformly at random from challenge domain \(D^{in}\).
  - \(C\) issues \(\tilde{c}\) to \(A\).

- **Guess phase.**
  - For the challenge \(\tilde{c}\), \(A\) produces his forgery \(\sigma^r \leftarrow A(1^\lambda, \tilde{c}, \{(c_i, r_i)\}_i^{q})\) and sends to \(C\).
  - \(C\) runs a verification algorithm \(b \leftarrow Ver(\sigma^r, \tilde{r})\), where \(\tilde{r} = E(\tilde{c})\) is the correct output and \(b \in \{0, 1\}\), to check the fidelity or equality of the responses.

\(^1\)We use the term Universal Unforgeability as defined in [DDKA21], to avoid confusion with a stronger security model. Nevertheless, in the PUF literature, this level of security is also called Selective Unforgeability as also was used in [ADDK21].
The above game is the abstract version of the unforgeability game that can be used for different classical or quantum PUFs and with different challenge types. For instance, the learning phase challenges \( c_i \) can be classical bit-strings or quantum states and in that case, the domain \( D^\text{in} \) will be a Hilbert. Here we mostly focus on the notion of classical and Hybrid PUFs. As a result, we do not need the full generalization to the quantum setting. Nevertheless, for the sake of completeness, we also give a full quantum version of this game-based definition in Appendix B.

Note that the adversary could not choose arbitrarily the challenges in the challenge phase in this game. So it is so-called universal unforgeability. Relatively, there are different notions of unforgeability e.g., unconditional unforgeability and existential unforgeability [ADDK21]. Unconditional unforgeability models the PUF against an unbounded adversary with unlimited queries during the learning phase, which is the strongest notion of unforgeability. The difference between existential unforgeability and universal unforgeability is that the adversary could choose the challenges during the challenge phase with existential unforgeability instead of choosing the challenges by the challenger. Even though the universal unforgeability is the weaker one compared with the rest of the two, it is sufficient for most PUF-based applications.

Finally, we define game-based security in terms of universal unforgeability in this setting:

**Definition 4** (Universal Unforgeability against Adaptive Adversary). A PUF with manufacturing process \( MP \) and verification algorithm \( \verb(.). \) provides \((\epsilon, \lambda)\)-universal unforgeability against adaptive adversary \( A_{\text{ad}} \) in winning the game \( G_{\text{PUF}}(A_{\text{ad}}, \lambda) \) is at most \( \epsilon(\lambda) \).

\[
Pr[1 \leftarrow G_{\text{PUF}}(A_{\text{ad}}, \lambda)] \leq \epsilon(\lambda) \tag{9}
\]

**Definition 5** (Universal Unforgeability against Weak Adversary). A PUF with manufacturing process \( MP \) and verification algorithm \( \verb(.). \) provides \((\epsilon, \lambda)\)-universal unforgeability against weak (non-adaptive) adversary if the success probability of any weak QPT adversary \( A_{\text{weak}} \) in winning the game \( G_{\text{PUF}}(A_{\text{weak}}, \lambda) \) is at most \( \epsilon(\lambda) \).

\[
Pr[1 \leftarrow G_{\text{PUF}}(A_{\text{weak}}, \lambda)] \leq \epsilon(\lambda) \tag{10}
\]

### 4 Lockdown Technique for the Classical PUFs

For a basic PUF-based authenticated protocol with classical CRPs, the scenario between a server and a client with PUF \( f \) is always given as follows: The server with a database \( D \) with all \( d \) CRPs: \( D := \{(x_i, y_i)\}_{1 \leq i \leq d} \) authenticates the client with PUF \( f \) by issuing a challenge \( x_i \) chosen uniformly at random in \( D \). The client obtains a response \( y_i \) by querying \( f \) with \( x_i \) and sends \( y_i \) back to the server. Finally, the server can do an equality test between responses from the client and from database \( D \) by measuring their distance. Authentication aborts if the distance exceeds a predefined threshold. Here, we denote a equality test algorithm for any two classical string \( c_1 \) and \( c_2 \) with threshold \( \delta_c \) as follows:

\[
\text{Equal}(c_1, c_2) = \begin{cases} 
1 & \text{if } \text{Dist}(c_1, c_2) \leq \delta_c, \\
0 & \text{otherwise.} 
\end{cases} \tag{11}
\]

However, the unilateral authentication protocol is vulnerable against the adversary such that he chooses the queries \( x_i \) adaptively and emulates the challenge-response behaviours of PUF \( f \) with machine learning attacks [Bec14, Bec15, RSS10, RSS+13b]. To address this problem, a lockdown technique introduced in [YHD+16] upper-bounds the
adversary’s capability of querying CRPs by converting the adaptive adversary into a weak one. We refer to Figure 1 for the construction of classical locked PUF (CLPUF).

![Figure 1: Classical PUF f with Lockdown Technique](image)

The main characteristic of lockdown technique construction with classical setting is that every response \( y_i \) of PUF \( f \) is split into \( y^1_i || y^2_i \). Here, \( y^1_i \) and \( y^2_i \) should be independent of each other. CLPUF accepts \( x_i \), as well as the first part of response \( \tilde{y}^1_i \), and uses \( x_i || \tilde{y}^1_i \) to query \( f \) to obtain the response \( y^1_i || y^2_i \). CLPUF performs the equality check by measuring the distance between \( \tilde{y}^1_i \) and \( y^1_i \). In an absence of adversary knowing the input/output behavior of \( f \), the distance measure \( \text{Dist}(\tilde{y}^1_i, y^1_i) \leq \delta_1 \). Otherwise, CLPUF outputs an abort state \( \bot \). Then, CLPUF sends back \( y^2_i \) to the server, and the server follows the same procedure to finish the authentication.

**Protocol 1** Classical CRPs PUF-based Authentication Protocol with Lockdown Technique

1. **Server’s Resource:**
   (a) A specification of PUF: \( f : \mathcal{X} \rightarrow \mathcal{Y} \) with fabrication process \( f \leftarrow \mathcal{MP}_C(\lambda) \) and secure parameter \( \lambda \).
   (b) A database \( D := \{ (x_i, y_i) \}_{i=1}^d \) with all \( d \) CRPs of \( f \).
2. **Client’s Resource:**
   (a) The device of PUF \( f \) with no compromise during the fabrication process \( \mathcal{MP}_C \).
3. **Authentication:**
   (a) The server randomly chooses a challenge \( x_i \) from \( D \), splits the response \( y_i \) into \( y^1_i \) and \( y^2_i \), and issues \( x_i || y^1_i \) to the client.
   (b) The client accepts \( x_i || \tilde{y}^1_i \) and queries PUF \( f \) with challenge \( x_i \) and obtains the response \( y^1_i || y^2_i \) from \( f \).
   (c) The client firstly measures the distance between \( y^1_i \) and \( \tilde{y}^1_i \). If \( \text{Dist}(y^1_i, \tilde{y}^1_i) \geq \delta_1 \), the authentication aborts; Otherwise, the client issues the second part of response \( \tilde{y}^2_i \) to the server.
   (d) The server receives and measure the distance between \( y^2_i \) and \( \tilde{y}^2_i \). If \( \text{Dist}(y^2_i, \tilde{y}^2_i) \geq \delta_1 \), the authentication aborts. Otherwise, the authentication passes.

A protocol of classical PUF-based authentication with lockdown technique is shown in Protocol 1. In the protocol, \( x_i || y^1_i \) prevents the adaptive queries from an adversary since the adversary could not issue \( x_i || y^1_i \) to the client if this packet is not yet issued by the server previously. Meanwhile, with a classical setting, genuine authentications made by the server is limited by the number of CRPs since every CRP could not be used repeatedly.
Otherwise, an adversary can still copy CRPs passively in-between the server and the client and finally collects enough CRPs to emulate the input/output behaviour of \( f \).

## 5 Hybrid Locked PUF

In this section, we give the first construction for lockdown mechanics in the quantum setting. First, we propose a construction, that we call Hybrid PUF (HPUF), combining the classical PUFs with quantum conjugate coding to increase the security of the classical PUFs against quantum adversaries. Then we define the quantum locking mechanism on such PUFs and construct a Hybrid Locked PUF (HLPUF) that resists powerful quantum adaptive adversaries. We then give a PUF-based authentication based on HLPUF, and we analyse its security.

### 5.1 Construction for Hybrid PUF

For our construction, we start with a classical PUF (CPUF) that has a certain amount of randomness (also denoted as min-entropy). To increase the min-entropy further, we encode the output of the CPUF into non-orthogonal quantum states and send the qubits through the communication channel. We refer to the entire system, i.e., CPUF together with a quantum encoding as hybrid PUF (HPUF). The hybrid PUF receives a classical query and produce a quantum state as a response. In Construction 1 we give a simple design of a HPUF based on conjugate coding [Wie83].

**Construction 1** (Hybrid PUF). Suppose \( f : \{0,1\}^n \rightarrow \{0,1\}^{4m} \) be a classical PUF, that maps an \( n\)-bit string \( x_i \in \{0,1\}^n \) to an \( 4m\)-bit string output \( y_i \in \{0,1\}^{4m} \). We denote the \( j \)-th bit of \( y_i \) as \( y_{i,j} \in \{0,1\} \). From the \( 4m\)-bit string, we prepare the set of \( 2m \)-tuples \( \{(y_{i,(2j-1)},y_{i,2j})\}_{1 \leq j \leq 2m} \). The hybrid PUF encodes each of the tuples \( (y_{i,(2j-1)},y_{i,2j}) \) into a single qubit \( |\psi^{i,j}\rangle_{S_{i,j}} \) (also known as BB84 states). The exact expression of the encoding is defined in the following way,

\[
|\psi^{i,j}\rangle_{S_{i,j}} := \begin{cases} 
|0\rangle_{S_{i,j}}|0\rangle & \text{if } y_{i,(2j-1)} = 0 \text{ and } y_{i,2j} = 0 \\
|1\rangle_{S_{i,j}}|1\rangle & \text{if } y_{i,(2j-1)} = 1 \text{ and } y_{i,2j} = 0 \\
|+\rangle_{S_{i,j}}|+\rangle & \text{if } y_{i,(2j-1)} = 0 \text{ and } y_{i,2j} = 1 \\
|\rangle_{S_{i,j}}\langle-| & \text{if } y_{i,(2j-1)} = 1 \text{ and } y_{i,2j} = 1.
\end{cases}
\]  

(12)

For any \( x_i \in \{0,1\}^n \), the mapping of the HPUF \( \mathcal{E}_f : \{0,1\}^n \rightarrow (\mathbb{H}^2)^{\otimes 2m} \) is defined as follows.

\[
x_i \rightarrow |\psi^{i}_\text{out}\rangle_{S_i} \langle \psi^{i}_\text{out}| (\text{or } |\psi_{f(x_i)}\rangle\langle \psi_{f(x_i)}|)
\]  

(13)

where \( |\psi^{i}_\text{out}\rangle_{S_i} \langle \psi^{i}_\text{out}| = \bigotimes_{j=1}^{2m} |\psi_{S_{i,j}}^{i,j}\rangle_{S_{i,j}} \langle \psi_{S_{i,j}}^{i,j}|.\)

Intuitively, if the adversary would like to extract the information about the \( i,2j \)-th bit of the out of the classical PUF corresponding to a challenge \( x_i \), then it needs to guess whether the state is prepared in \( Z = \{ |0\rangle, |1\rangle \} \) basis or in \( X = \{ |+\rangle, |-\rangle \} \) basis, as well as knowing the encoded bit. In Theorem 2 we estimate the number of required queries the weak adversaries need to make to the HPUF for winning the universal unforgeability game.

### 5.2 Lockdown technique for Hybrid PUF

In construction 2 we show how to apply the lockdown technique on a hybrid PUF. We refer to such HPUFs with the lockdown technique as the hybrid locked PUFs (HLPUFs).

Similar to the classical lockdown technique, we subdivide the output of the HPUF \( \mathcal{E}_f : \{0,1\}^n \rightarrow (\mathbb{H}^2)^{\otimes 2m} \):
\[ \{0,1\}^n \rightarrow (\mathbb{H}^2)^{\otimes 2m} \] corresponding to a classical PUF \( f : \{0,1\}^n \rightarrow \{0,1\}^{4m} \) into two different parts. The first part contains the first \( m \) qubits, and the second half contains the last \( m \) qubits of the outcome of the HPUF \( \mathcal{E}_f \). Note that, the first \( m \) qubits of the HPUF’s outcome comes from the first \( 2m \) bits outcome of the corresponding classical PUF \( f \). For any challenge \( x \in \{0,1\}^n \) we can write the outcome of the classical PUF as \( f(x) = f_1(x) || f_2(x) \), where the mapping \( f_1 : \{0,1\}^n \rightarrow \{0,1\}^{2m} \) denotes the first \( 2m \) bits of \( f \) and \( f_2 : \{0,1\}^n \rightarrow \{0,1\}^{2m} \) denotes the last \( 2m \) bits of \( f \). Similarly, we can rewrite the HPUF \( \mathcal{E}_f \) as a tensor product of two mappings \( \mathcal{E}_{f_1} : \{0,1\}^n \rightarrow (\mathbb{H}^2)^{\otimes m} \), and \( \mathcal{E}_{f_2} : \{0,1\}^n \rightarrow (\mathbb{H}^2)^{\otimes m} \), where for any challenge \( x \in \{0,1\}^n \), \( \mathcal{E}_{f_1}(x) \) denotes the first \( m \) qubits of \( \mathcal{E}_f(x) \), and \( \mathcal{E}_{f_2}(x) \) denotes the last \( m \) qubits of \( \mathcal{E}_f(x) \).

The hybrid locked PUF, takes the classical input \( x_i \) and a quantum state \( \tilde{\rho}_{S_1} \) and produces the second half of the response of the hybrid PUF, \( |\psi_{\text{out}}^{i}\rangle_{S_2}\langle\psi_{\text{out}}^{i}| \), as an output if \( \tilde{\rho}_{S_1} \) is equal to the first half of the output of the hybrid PUF \( |\psi_{\text{out}}^{i}\rangle_{S_1}\langle\psi_{\text{out}}^{i}| \). We formalise the construction as follows:

**Construction 2 (HLPUF).** Suppose we have a hybrid PUF \( \mathcal{E}_f \) where \( f : \{0,1\}^n \rightarrow \{0,1\}^{4m} \) is a CPUF. The mapping of the HLPUF \( \mathcal{E}_f^L : \mathcal{D}^m \times \mathcal{H}^{\text{out}_1} \rightarrow \mathcal{H}^{\text{out}_2} \otimes \mathcal{H}^l \) corresponding to a hybrid PUF \( \mathcal{E} \) is defined as follows:

\[
(x_i, \tilde{\rho}_{S_1}) \rightarrow \begin{cases} 
|\psi_{\text{out}}^{i}\rangle_{S_2}\langle\psi_{\text{out}}^{i}| & \text{if } \text{Ver}(|\psi_{\text{out}}^{i}\rangle_{S_1}\langle\psi_{\text{out}}^{i}|, \tilde{\rho}_{S_1}) = 1 \\
\perp & \text{otherwise.} 
\end{cases}
\]  

(14)

where \( \text{Ver}(..) \) is verification algorithm that checks the equality of the first half of the response based on the classical response \( y_i^1 \).

![Figure 2: Hybrid Locked PUF (HLPUF) \( \mathcal{E}_f^L \) with Construction 2](image)

Similarly to the classical setting, we propose Protocol 2 a hybrid PUF-based authentication protocol with lockdown technique between a server and a client. Note that we use \( \tilde{\rho}_{S_1} \) and \( \tilde{\rho}_{S_1}^b \) to denote the actual quantum state received by the client/server respectively.

### 5.3 Security Analysis

Now, we give a comprehensive security analysis of the previously proposed constructions. First, we show that using hybrid construction will exponentially improve security. More precisely, it will exponentially decrease the success probability of a quantum adversary in the universal unforgeability game, compared to a classical PUF with the same number of learning queries. Further, we show that how much quantum communication can improve the security of a weaker classical PUF and as a result propose an efficient and secure construction that can be build using existing classical PUFs. Finally, we analyse the completeness and security of hybrid PUF-based device authentication protocol and show that under the assumption that the inherent classical PUF resist against the weak quantum adversary, the HLPUF-based protocol will be secure against an adaptive adversary.
Protocol 2 Hybrid PUF-based Authentication Protocol with Lockdown Technique

1. The Server’s Resource:
   (a) A specification of hybrid PUF: \( E_f : \{0,1\}^n \rightarrow (\mathcal{H}^2)^{\otimes 2m} \) constructed upon a classical PUF \( f : \mathcal{X} \rightarrow \mathcal{Y} \). Here, the classical PUF \( f \) maps an \( n \)-bit string \( x_i \in \{0,1\}^n \) to an \( 4m \)-bit string output \( y_i \in \{0,1\}^{4m} \).
   (b) A database \( D := \{(x_i,y_i)\}_{i=1}^d \) with all \( d \) CRPs of \( f \).

2. The Client’s Resource:
   (a) The device of PUF \( f \) with no compromise during the fabrication process \( MPC \) and necessary quantum apparatus for encoding and measuring qubits.

3. Authentication:
   (a) The server randomly choose a CRP \( (x_i,y_i) \) and split the response equally into two partition \( y_i = f_1(x_i)||f_2(x_i) = y_i^1||y_i^2 \) with length \( 2m \).
   (b) The server encodes the first partition of response into \( |\psi_i^{\text{out}}\rangle_{S_1^i} \langle \psi_i^{\text{out}}| := \bigotimes_{j=1}^m |\psi_i^{1,j}\rangle_{S_1^i,j} \langle \psi_i^{1,j}| \) and issues the joint state \( |x_i,|\psi_i^{\text{out}}\rangle_{S_1^i} \langle \psi_i^{\text{out}}| \) to the client.
   (c) The client receives the joint state \( (x_i,\hat{\rho}_{S_1^i}) \) and queries classical PUF \( f \) with challenge \( x_i \) and obtains the response in the set of \( m \)-tuples \( \{(y_i,(2j-1),y_i,2j)\}_1 \leq j \leq m \) as Construction 1 and measures every single qubit of received quantum state with basis according to value of \( y_i,2j \) for each tuple. If \( y_i,2j = 0 \), the client measures every corresponding qubit with computational basis \( Z = \{|0\rangle,|1\rangle\} \). Otherwise he measures with Hadamard basis \( X = \{|+\rangle,|-\rangle\} \). For each measurement with single qubit, the measurement result \( r_j = 0 \) if outcome state of measurement \( |\psi_i^{1,j}\rangle_{S_1^i,j} \langle \psi_i^{1,j}| \) is either \( |0\rangle / |+\rangle \). Otherwise, \( r_j = 1 \).
   (d) The client obtains \( \tilde{y}_i \), which consists of \( m \) tuples \( \{(\tilde{y}_i,(2j-1),y_i,2j)\}_1 \leq j \leq m \) recording the measurement result of each qubit of \( \tilde{\rho}_{S_1^i} \), where \( \tilde{y}_i,(2j-1) = r_j \). Authentication aborts if \( Dist(\tilde{y}_i,\hat{\rho}_{S_1^i}) \geq \delta_1 \) with the first \( 2m \) bits of \( f \). Otherwise the client encodes the second partition of response into states \( |\psi_i^{2,j}\rangle_{S_2^i} \langle \psi_i^{2,j}| := \bigotimes_{j=m+1}^{2m} |\psi_i^{2,j}\rangle_{S_2^i,j} \langle \psi_i^{2,j}| \) and sends back to the server.
   (e) The server receives the quantum state \( \tilde{\rho}_{S_2^i} \), measures every qubit with the same process as described in Step (3c) and obtains \( \tilde{y}_i^2 \) as Step (3d). Authentication aborts if \( Dist(y_i^2,\tilde{y}_i^2) \geq \delta_1 \) with the last \( 2m \) bits of \( f \). Otherwise the authentication passes.

5.3.1 Assumptions on the CPUFs

For the security analysis of our constructions we consider the following assumptions of the CPUFs \( f : \{0,1\}^n \rightarrow \{0,1\}^{4m} \).

1. For any input \( x \in \{0,1\}^n \) the probability distributions of the \( 4m \) output bits \( f(x)_1, \ldots, f(x)_4m \) are independent and identically distributed (i.i.d).

2. The output distributions \( \{p_f(y)\}_{y \in \{0,1\}^{4m}} \) for all the inputs \( x \) are independent and identically distributed (i.i.d).
5.3.2 Security of the HPUFs, and HLPUFs:

Intuitively the security of our HPUF comes from the indistinguishability property of the non-orthogonal quantum states. In Lemma 1, first we give an upper bound on the adversaries guessing probability of the response \( f(x_i) \) corresponding to a challenge \( x_i \) and a single copy of the quantum response state \( |\psi_{f(x_i)}\rangle \). The complete proof can be found in Appendix C.

**Lemma 1.** Suppose \( f: \{0,1\}^n \rightarrow \{0,1\}^{4m} \) be a CPUF with the following property,

\[
\forall \ x_i \in \{0,1\}^n, \forall \ 1 \leq j \leq 4m, \quad p_{x_i}^j(y_{i,j} = 0) = \frac{1}{2} + \delta_r,
\]

(15)

with a biased distribution \( \frac{1}{2} + \delta_r \), where \( 0 \leq \delta_r \leq \frac{1}{4} \), and \( \mathcal{E}_f \) be a HPUF corresponding to \( f \) that we construct using Construction 1. Let \( A_{\text{guess}}^{i,j} \) be a quantum adversary that guesses the value of the tuple \( (y_{i,(2j-1)}, y_{i,2j}) \) corresponding to a random challenge \( x_i \). If all the output bits of the CPUF are independent and identically distributed, then for any quantum adversary \( A_{\text{guess}}^{i,j} \), and \( \forall \ x_i \in \{0,1\}^n \),

\[
\Pr[A_{\text{guess}}^{i,j}(x_i, |\psi_{\text{out}}\rangle \langle \psi_{\text{out}}|) = (y_{i,(2j-1)}, y_{i,2j})] \leq \frac{1}{2} + \delta_r.
\]

(16)

And the probability of guessing response \( f(x) \) from single copy of the quantum response state \( |\psi_{f(x_i)}\rangle \) is thus:

\[
\Pr[A_{\text{guess}}^{i,j}(x_i, |\psi_{f(x_i)}\rangle) = f(x_i)] \leq \left(\frac{1}{2} + \delta_r\right)^{2m}.
\]

(17)

Using lemma 1, in Theorem 1, and Theorem 2 we address the security of our HPUF construction against the weak adversaries. In Theorem 1 we prove that the if any q-query weak adversary wins the universal unforgeability game for a classical PUF \( f \) with a non-negligible probability, then with the same number of queries any weak adversary can break the HPUF with exponentially (in \( q \)) low probability.

**Theorem 1.** Suppose \( f: \{0,1\}^n \rightarrow \{0,1\}^{4m} \) be a classical PUF with \( p \)-randomness. For any fixed \( q \), if any weak adversary, wins the universal unforgeability game for the CPUF \( f \) with probability \( P(m,p) \) with at least \( q \)-queries, then a \( q \)-query weak adversary can win the universal unforgeability game for the HPUF \( \mathcal{E}_f \) with probability at most \( P(m,p)^q \).

Proof. For breaking a hybrid PUF \( \mathcal{E}_f \), corresponding to a classical PUF \( f: \{0,1\}^n \rightarrow \{0,1\}^{4m} \) any \( q \)-query weak adversary has to win the universal unforgeability game from a set of random classical challenges \( x_i \in \{0,1\}^n \) and the corresponding responses \( |\psi_{f(x_i)}\rangle \).

If any weak adversary requires at least \( q \)-queries to win the universal unforgeability game for a CPUF \( f: \{0,1\}^n \rightarrow \{0,1\}^{4m} \) with probability \( P(m,p) \) which is non-negligible, then with any \( q' < q \) queries the weak adversaries cannot win the universal unforgeability game with desired probability. This implies, to win the universal unforgeability game for the the hybrid PUFs \( \mathcal{E}_f \), the weak adversaries need to get the information about at least \( q \) responses. Therefore, to break the HPUF, the weak adversaries need to extract \( f(x_i) \) correctly from \( |\psi_{f(x_i)}\rangle \) for all the responses of the \( q \)-queries. Note that, according to construction 1, \( |\psi_{f(x_i)}\rangle \langle \psi_{f(x_i)}| = \bigotimes_{j=1}^{2m} |\psi_{\text{out}}^{i,j}\rangle \langle \psi_{\text{out}}^{i,j}|_{S_i,j} \), where \( |\psi_{\text{out}}^{i,j}\rangle_{S_i,j} \in \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\} \). From lemma 1 we get that, the adversary can guess the value of \( f(x_i) \) from a single copy of \( |\psi_{f(x_i)}\rangle \) with probability at most \( p = (\frac{1}{2} + \delta_r)^{2m} \). This implies, the \( q \)-query weak adversaries can win the universal unforgeability game with at most \( (\frac{1}{2} + \delta_r)^{2mq} = P(m,p)^q \). This concludes the proof.

\[\square\]
The above result is a general statement for any fixed number of queries and compares the success probability of a weak adversary in breaking the unforgeability of CPUF and HPUF. We can also easily state the following corollary that ensures the universal unforgeability of an HPUF constructed from a CPUF.

**Corollary 1.** Let the success probability of any QPT weak-adversary in the universal unforgeability game with a CPUF $f : \{0,1\}^n \rightarrow \{0,1\}^{4m}$ with $p$-randomness, be at most $P$, where $0 \leq P \leq 1 - \text{non-negl}(2m)$. Then, the success probability of any QPT adversary in the universal unforgeability game for the HPUF $E_f$, is at most $\epsilon(2m)$, which is a negligible function in the security parameter. Hence such HPUFs are universally unforgeable.

In Theorem 2 we prove that, for the weak adversaries, predicting the outcome of an HPUF $E_f$ from a random database of quantum responses is at least as hard as predicting the outcome of the CPUF $f$ from a random classical database of erroneous responses.

**Theorem 2.** For any weak adversary, learning about the HPUF $E_f$ corresponding to a CPUF $f : \{0,1\}^n \rightarrow \{0,1\}^{4m}$ with $p$-randomness is at least as hard as learning about the CPUF $f$ from the random outcomes of a noisy implementation $\tilde{f} : \{0,1\}^n \rightarrow \{0,1\}^{4m}$ of $f$. We define $\tilde{f}$ as follows.

$$\tilde{f}(x) := f(x) + Z,$$

where $Z$ is a random variable such that for any $z \in \{0,1\}^{4m}$, $\Pr[Z = z] \leq p$, and $\Pr[Z = 0] = p$.

Here, we give a brief proof sketch of this theorem. For the detailed security analysis we refer to the Appendix C.

**Proof sketch.** In case of HPUF $E_f$, the weak adversaries can extract information about the underlying CPUF $f$ only by performing a CPTP map on the collected random responses $\{|\psi_f(x_i)\rangle\}_{1 \leq i \leq q}$. From Lemma 1, we get that due to the encoding of the classical outcomes of $f$ into non-orthogonal quantum states, the adversary can guess the value of $f(x)$ from $|\psi_f(x_i)\rangle$ with probability at most $p$. In other words, if $f(x)$ denotes the adversary’s guess about $f(x)$ then $\tilde{f}(x) = f(x)$ with probability at most $p$. This scenario is equivalent to the case where the adversary gets a classical string $\tilde{f}(x)$ as a response from the PUF for a random challenge $x$. The function $\tilde{f}$ is already defined in Equation (18), i.e.,

$$\tilde{f}(x) = f(x) + Z,$$

where $Z$ is random variable such that for any $z \in \{0,1\}^{4m}$, $\Pr[Z = z] \leq p$. Therefore, $Z = 0$ with probability at most $p$. This implies $\tilde{f}$ is a noisy version of the CPUF $f$. For the weak-adversaries learning about $f$ from the quantum, responses are at least as hard as learning about $f$ from a noisy response $\tilde{f}$. This concludes the proof sketch. 

Theorem 2 implies, if the noisy version $\tilde{f}$ of the classical PUF $f$ is secure against any weak adversary then the corresponding hybrid PUF $E_f$ is secure against any weak adversary. Note that, in general, it is hard to learn about a function from such erroneous responses [Pie12, BKW03, BLP+13, KS09, Reg09]. This implies, by exploiting the indistinguishability property of the non-orthogonal quantum states we can gain a secure HPUF.

In the last two theorems, we analyse the security of the HPUFs against only weak adversaries. In Theorem 3 we show that if the HPUFs are secure against the weak adversaries then with the lockdown technique we can make the HLPUs secure against the adaptive adversaries.
Theorem 3. Let $E_f: \{0,1\}^n \rightarrow (\mathcal{H}^2)^{\otimes m} \otimes (\mathcal{H}^2)^{\otimes m}$ be a hybrid PUF that we construct from a classical PUF $f: \{0,1\}^n \rightarrow \{0,1\}^{2^m} \otimes \{0,1\}^{2^m}$ and let $E_f^L: \{0,1\}^n \rightarrow (\mathcal{H}^2)^{\otimes m}$ denotes the HLPUF that we construct from $E_f$ using the Construction 2. If $E_f = E_{f_1} \otimes E_{f_2}$ and if each of the mappings $E_{f_1}, E_{f_2}$ has $(\epsilon, m)$-universal unforgeability against the $q$-query weak adversaries, then the corresponding HLPUF $E_f^L$ is $(\epsilon, m)$-secure against the $q$-query adaptive adversaries.

Proof. At the $i$-th round, the HLPUF $E_f^L$ receives the queries of the form $(x_i, \hat{\rho}_{S_i})$, where the classical string $x_i \in \{0,1\}^n$, and $\hat{\rho}_{S_i} \in (\mathcal{H}^2)^{\otimes m}$. The HLPUF returns $E_{f_2}(x_i)$ if $\text{Ver}(\hat{\rho}_{i}, E_{f_1}(x_i)) = 1$, otherwise it returns an abort state $|\perp\rangle$ corresponding to $\perp$. Hence, to get any non-abort state $|\perp\rangle$ from the HLPUF, the adaptive adversaries $A_{ad}$ need to produce a query of the form $(x_i, E_{f_1}(x_i))$. As the adversary doesn’t have any direct access to the mapping $E_{f_1}$, the only way it can get any information about $E_{f_1}(x_i)$ by intercepting the challenges that are sent by the server to the client. Suppose that the adaptive adversary has access to a set of $q$ queries $X_{[q]} := \{X_i\}_{1 \leq i \leq q}$ and the corresponding responses $\Psi_{[q]} := \{E_{f_1}(x_i)\}_{1 \leq i \leq q}$. Here each $X_i$ follows a uniform distribution over the challenge set $\{0,1\}^n$. Hence, for the mapping $E_{f_1}$, the power of the adaptive adversary reduces to the power of a weak adversary. As $E_{f_1}$ has the universal unforgeability property against any $q$-query weak adversary, hence we get, for any random challenge $X \not\in X_{[q]}$:

$$
\Pr_{X,X_{[q]}}[1 \leftarrow \mathcal{G}^{E_{f_1}}(A_{ad}, m, X, X_{[q]}))] = \Pr_{X,X_{[q]}}[1 \leftarrow \mathcal{G}^{E_{f_1}}(A_{weak}, m, X, X_{[q]})] \leq \epsilon(m).
$$

(20)

This implies, using the set of challenges $X_{[q]}$ and responses $\Psi_{[q]}$, the adversary cannot produce the response corresponding to a random challenge $X \not\in X_{[q]}$. Suppose from the query set $X_{[q]}$ and the corresponding, the adaptive adversary successfully generates a set $X'_{[q']}$ of $q'$ adaptive queries, and corresponding responses $\Psi'_{[q']}$ for the HLPUF $E_f^L$. Without any loss of generality we assume that for all of the queries $X'_{i} \in X'_{[q']}$ the HLPUF returns a non-abort state.

We assume that the adaptive adversary wins the universal unforgeability game using the query set $X_{ad} = X_{[q]} \cap X'_{[q']}$). This implies,

$$
\Pr_{X,X'_{[q']}}[1 \leftarrow \mathcal{G}^{E_{f_1}}(A_{ad}, m, X, X_{ad})] \geq \text{non-negl}(m).
$$

(21)

From the construction of our HLPUF in Construction 2 we get that winning the universal unforgeability game with the HLPUF $E_f^L$ implies winning the universal unforgeability with $E_{f_2}$. Hence, we can rewrite Equation (21) in the following way,

$$
\Pr_{X,X_{ad}}[1 \leftarrow \mathcal{G}^{E_{f_1}}(A_{ad}, m, X, X_{ad})] \geq \text{non-negl}(m).
$$

(22)

Note that, if the adaptive adversary manages to get non-abort outcomes from the HLPUF corresponding to all $X'_{i} \in X_{ad}$ then from the Construction 2 we get, $1 \leftarrow \mathcal{G}^{E_{f_1}}(A_{ad}, m, X'_{i}, X_{ad})$. Due to the unforgeability assumption of Equation (20) we get,

$$
\Pr_{X,X_{[q]}}[1 \leftarrow \mathcal{G}^{E_{f_1}}(A_{weak}, m, X, X_{[q]})] = \Pr_{X,X_{ad}}[1 \leftarrow \mathcal{G}^{E_{f_1}}(A_{ad}, m, X, X_{ad})] \leq \epsilon(m).
$$

(23)

Note that, the main difference between adaptive and weak adversary lies in the choice of the query set. If we fix the query set $X_{ad}$, then the both adaptive $A_{ad}$ and weak
adversary can extract the same amount of information from the responses corresponding to the query set $X_{\text{ad}}$. Therefore, their winning probability of the universal unforgeability game become equivalent. This implies, we can rewrite Equation (23) in the following way,

$$
\Pr_{X,X_{\text{ad}}} [1 \leftarrow G^{E_{f_1}}(A_{\text{ad}}, m, X, X_{\text{ad}})] = \Pr_{X,X_{\text{ad}}} [1 \leftarrow G^{E_{f_1}}(A_{\text{weak}}, m, X, X_{\text{ad}})] \leq \epsilon(m). \quad (24)
$$

By combining Equation (23) and Equation (24) we get, both the random variables $X_{[q]}$ and $X_{\text{ad}}$ are equivalent. From the universal unforgeability property of the PUF $E_{f_2}$ against any $q$-query weak adversary, we get

$$
\Pr_{X,X_{[q]}} [1 \leftarrow G^{E_{f_2}}(A_{\text{weak}}, m, X, X_{[q]})] \leq \epsilon(m). \quad (25)
$$

As both of the random variables $X_{[q]}$ and $X_{\text{ad}}$ are equivalent, so we get,

$$
\Pr_{X,X_{[q]}} [1 \leftarrow G^{E_{f_2}}(A_{\text{weak}}, m, X, X_{[q]})] = \Pr_{X,X_{\text{ad}}} [1 \leftarrow G^{E_{f_2}}(A_{\text{ad}}, m, X, X_{\text{ad}})] \leq \epsilon(m). \quad (26)
$$

The second equality follows from the fact that for a fixed query set $X_{\text{ad}}$ the adaptive adversary $A_{\text{ad}}$ and weak adversary $A_{\text{weak}}$ becomes equivalent. Note that, only one of Equation (22) and Equation (26) is true. The Equation (26) is true because of the unforgeability of $E_{f_2}$. Hence, our assumption of Equation (22) is wrong. Therefore, Equation (21) is also not true. Hence, with the proof by contradiction we get,

$$
\Pr_{X,X_{\text{ad}}} [1 \leftarrow G^{E_{f}}(A_{\text{ad}}, m, X, X_{\text{ad}})] \leq \epsilon(m). \quad (27)
$$

This concludes the proof.

5.3.3 Security of the HLPUF-based Authentication Protocol:

In this section, we firstly define the completeness and security property of Protocol 2. Later, in Theorem 4 we prove its completeness and security.

**Definition 6** (Completeness of HLPUF-based Authentication Protocol 2). We say the HLPUF-based authentication protocol 2 satisfies completeness if in the absence of any adversary, an honest client and server generating $|\psi_{\text{out}}^{i}\rangle_{S_1}\langle\psi_{\text{out}}^{i}|$ and $|\psi_{\text{out}}^{i}\rangle_{S_2}\langle\psi_{\text{out}}^{i}|$ with a valid HLPUF for any selected challenge $x_i$, can pass the verification algorithms with overwhelming probability:

$$
\Pr \left[ \text{Ver}(|\psi_{\text{out}}^{i}\rangle_{S_1}\langle\psi_{\text{out}}^{i}|, \tilde{\rho}_{S_1}) = \text{Ver}(|\psi_{\text{out}}^{i}\rangle_{S_2}\langle\psi_{\text{out}}^{i}|, \tilde{\rho}_{S_2}) = 1 \right] \geq 1 - \epsilon(\lambda) \quad (28)
$$

Now, we also define the security of our HLPUF-based authentication protocol, in relation with the universal unforgeability game as follows:

**Definition 7** (Security of the HLPUF-based Authentication Protocol 2). We say the HLPUF-based authentication protocol 2 is secure if the success probability of any QPT adaptive adversary $A_{\text{ad}}$ in winning the universal unforgeability game to forge an output
of HLPUF according to Construction 2, for any randomly selected challenge of the form \( \tilde{c} = (x, |\psi_{\text{out}}\rangle_{S1}, \langle \psi_{\text{out}}|) \) is at most negligible in the security parameter:

\[
Pr[1 \leftarrow G^{\text{HLPUF}}(A_{\text{ad}}, \lambda)] \leq \epsilon(\lambda)
\] (29)

where the verification algorithm of the universal unforgeability game checks the adversary’s output \( \sigma_{S2} \), with the output of the HLPUF, \( \tilde{r}_{S2} = |\psi_{\text{out}}\rangle_{S2}, \langle \psi_{\text{out}}| \), belonging to the second subsystem \( S2 \).

**Theorem 4.** If the HLPUF \( E_f^L \) is constructed from a hybrid PUF \( E_f \) using the Construction 2 then the locked PUF-based authentication Protocol 2 satisfies both the completeness and security conditions.

**Proof.** In Protocol 2 with hybrid PUF \( E_f = E_{f_1} \otimes E_{f_2} \), the server chooses the classical input \( x_i \in X \), encodes the quantum state corresponding to \( 2m \) bits of \( f_1(x_i) \) and issues the joint state to the client. If there is no adversary, the client receives the joint state and queries \( E_f \) with \( x_i \) and \( \tilde{\rho}_{S_1} \), where \( \tilde{\rho}_{S_1} = E_{f_1}(x_i) = |\psi_{\text{out}}^i\rangle_{S1}, \langle \psi_{\text{out}}^i| \) for the first \( m \) qubits of \( E_f(x_i) \). Hence we have:

\[
Pr \left[ \text{Ver}(|\psi_{\text{out}}^i\rangle_{S1}, \langle \psi_{\text{out}}^i|, \tilde{\rho}_{S_1}) = 1 \right] = 1
\] (30)

On the client side, since the verification algorithm of HLPUF \( E_f^L \) always passes with \( \text{Ver}(|\psi_{\text{out}}^i\rangle_{S1}, \langle \psi_{\text{out}}^i|, \tilde{\rho}_{S_1}) = 1 \), he returns the quantum state \( E_{f_2}(x_i) = |\psi_{\text{out}}^i\rangle_S, \langle \psi_{\text{out}}^i| \) corresponding to \( 2m \) bits of \( f_2(x_i) \) to the server. Without the presence of adversary, the server always receives the state with \( \tilde{\rho}_{S_2} = |\psi_{\text{out}}^i\rangle_S, \langle \psi_{\text{out}}^i| \), and we obtain the equation similarly to Equation (30). Therefore, we can say the locked PUF-based authentication protocol satisfies the completeness condition with

\[
Pr \left[ \text{Ver}(|\psi_{\text{out}}^i\rangle_{S1}, \langle \psi_{\text{out}}^i|, \tilde{\rho}_{S_1}) = 1 \right] = 1
\] (31)

On the other hand for the security, we rely on Theorem 3 that the HLPUF \( E_f^L \) is \((\epsilon, m)\)-secure against any QPT adaptive adversary (a \( q \)-query adaptive adversary for any \( q \) polynomial in the security parameter). For both \( E_{f_1} \) and \( E_{f_2} \) of HPUF \( E_f \), we show that the power of adaptive adversary can be reduced to the power of a weak adversary, due to the lockdown technique. Also since \( E_{f_1} \) has the universal unforgeability against weak adversary by definition, for any adaptive query of the form \((x_i, \sigma_{S1})\) that an adaptive adversary issues to the HLPUF, the following applies:

\[
Pr \left[ \text{Ver}(|\psi_{\text{out}}^i\rangle_{S1}, \langle \psi_{\text{out}}^i|, \sigma_{S1}) = 1 \right] \leq \epsilon(m)
\] (32)

Where \( |\psi_{\text{out}}^i\rangle_S(\langle \psi_{\text{out}}^i| \) is the correct response constructed from CPUF according to HPUF construction. Thus the power of the adaptive adversary reduces to the power of weak adversary and we have:

\[
Pr[1 \leftarrow G^{\text{HLPUF}}(A_{\text{ad}}, m)] \approx Pr[1 \leftarrow G^{\text{HLPUF}}(A_{\text{weak}}, m)]
\] (33)

Now given the fact that the adaptive adversary cannot boost from the weak-learning phase to the HPUF, producing a forgery \( \sigma_{S2} \) for the HLPUF that passes the verification \( \text{Ver}(|\psi_{\text{out}}^i\rangle_{S2}, \langle \psi_{\text{out}}^i|, \sigma_{S2}) \), reduces to forging the HPUF \( E_{f_2} \). Again by assumption, \( E_{f_2} \) has the universal unforgeability against weak adversary, hence we have:

\[
Pr[1 \leftarrow G^{\text{HLPUF}}(A_{\text{ad}}, m)] = Pr[1 \leftarrow G^{\text{HPUF}}(A_{\text{weak}}, m)] \leq \epsilon(m)
\] (34)

This concludes the proof.
6 Challenge Re-usability

In any PUF-based protocol relying on the classical communication of challenges and responses of the PUF, each challenge can only be used once as the adversary can simply copy and record the challenges and responses and have a perfect copy of the challenger’s database which later they can use to falsely identify themselves. This is an important limitation of the classical PUFs [SD07, HYKD14]. Quantum communication can solve this issue due to the unclonability of quantum states. In this section, we discuss how our hybrid construction can allow for challenge states to be used several times during the authentication, under the circumstances of previous successful authentication rounds. This will resolve an important practical issue as the challenger can avoid storing a big database or renewing the database of challenge-responses frequently.

First, we need to clarify the condition under which the challenge can be reused. We assume the challenger’s database to only include $q$ number of challenge-response pairs such that $q$ is polynomial in the security parameter. We also need to recall that in our hybrid construction, the challenges are still being sent as classical bit-strings over the public channel and hence the adversary after polynomial rounds of communication can have the same challenge set as the server’s database. Due to this fact, we should emphasize that the adversary does not get any physical access to the internal classical PUF in the HLPUF construction during the authentication and, no query can be directly issued to the CPUF by the adversary. This condition is satisfied using our lockdown technique. Thus, the adversary has access to the following information: a pre-learnt polynomial-size local database of challenge-responses of the CPUF, a set of classical challenges used during the protocol, and the set of quantum states that encode the first and second half of the response, in the BB84 states.

Now, it is a straightforward observation that the challenges for which the verification test has failed should never be used again. A trivial attack, in this case, would be that the adversary intercepts the communication and stores the response state, and later when the same challenge has been queried again, will re-send the stored correct response state to pass the verification. As a result, all the challenges in the failed rounds should be discarded.

Nonetheless, we argue that in the events of successful authentication, the challenges can be re-used. Here, by successful identification, we mean that the received response state passes the verification on the client and server sides and, the prover identifies an honest party. Even though the events of false identification of an adversary, is still possible (for example, if the challenge is the same as one of the challenges that previously exists in the adversary’s local database), but the unforgeability of PUF and our security proof for the hybrid construction, ensures that these events occur only with negligible probability.

We are thus interested in the eavesdropping attacks by the adversary on the first and second half of the response states that are of the form $|\psi_{\text{out}}^i\rangle S_1^i \langle \psi^i_\text{out}| = \bigotimes_{j=1}^{m} |\psi^{i,j}_\text{out}\rangle S_1^{i,j} \langle \psi^{i,j}_\text{out}|$ and $|\psi_{\text{out}}^j\rangle S_2^j \langle \psi^j_\text{out}| = \bigotimes_{j=1}^{m} |\psi^{j,i}_\text{out}\rangle S_2^{j,i} \langle \psi^{j,i}_\text{out}|$. Note that eavesdropping on the states that encode the first part of the response will lead to breaking the locking mechanism while eavesdropping on the second half will lead to an attack on the identification. Without loss of generality, we only consider one of the cases where the adversary want to eavesdrop on the first (or second) half to break the protocol in the upcoming rounds where the challenge is re-used. The arguments will hold equivalently for both cases since the states and verification are symmetric.

Given all these considerations, the challenge re-usability problem will reduce to the optimal probability of the eavesdropping attack on $|\psi_{\text{out}}^i\rangle S_1^i \langle \psi^i_\text{out}| = \bigotimes_{j=1}^{m} |\psi^{i,j}_\text{out}\rangle S_1^{i,j} \langle \psi^{i,j}_\text{out}|$ which is in fact $m$ qubit states encoded in conjugate basis same as BB84 states. In the most general case, the adversary can perform any arbitrary quantum operation on the state $\bigotimes_{j=1}^{m} |\psi^{i,j}_\text{out}\rangle S_1^{i,j} \langle \psi^{i,j}_\text{out}|$ or separately on each qubit state $|\psi^{i,j}_\text{out}\rangle S_1^{i,j}$, together with a
local ancillary system and sends a partial state of this larger state to the verifier to pass the verification test, and keep the local state to extract the encoded response bits. Let $\rho_{SEC}$ be the joint state of the server, the eavesdropper and the client. Since the states used in the protocol are from Mutually Unbiased Basis (MUB) states i.e. from either $Z = \{0, 1\}$ or $X = \{+1, -1\}$, in order to show the optimal attack, we can rely on the entropy uncertainty relations that have been used for the security proof of QKD. The measurements for verification are also performed in the $\{Z, X\}$ bases accordingly. We use the entropy uncertainty relations from [CBTW17] where the security criteria for QKD has been given in terms of the conditional entropy for MUBs measurements. Using these results we show that the entropy of Eve in guessing the correct classical bits for the response is very high if the state sent to the verification algorithm passes the verification with a high probability. Intuitively this is due to the uncertainty that exists related to the commutation relation between $X$ and $Z$ operators in quantum mechanics. Hence we conclude that the success probability of Eve in extracting information from the encoded halves of the response is relatively low. Also, we show that this uncertainty increases linearly with $m$ similar to the number of rounds for QKD. This argument results in the following theorem which we will formally describe and prove in Appendix C.3 where we also introduce the uncertainty relations.

**Theorem 5** (informal). In Protocol 2, if the client (or server for the second half of the state) verification does not abort for a challenge $x$, then Eve’s uncertainty on respective response of the CPUF, denoted by $H_{Eve}^{min}$ is greater than $m - \epsilon(m)$.

Now, we first define the re-usability in relation with the unforgeability game and then using Theorem 5, we prove the challenge re-usability of the HLPUF-based Protocol 2.

**Definition 8** (Challenge $(k)$-re-usability in the universal unforgeability game). Let $G_{re}(\lambda, A, x_{k+1})$ be a special instance of the universal unforgeability game, where a challenge $x$, picked uniformly at random by the challenger, has been previously used $k$ times. We are interested in the events where the same challenge is used in the $(k + 1)$-th round, which we denote by $x_{k+1}$. We say the challenge $x$ is $(k)$-re-usable if the success probability of any QPT adversary in winning $G_{re}(\lambda, A, x_{k+1})$, i.e, in forging message $x_{k+1}$, is negligible in the security parameter:

$$Pr_{forge}(A, x_{k+1}) = Pr[1 \leftarrow G_{re}(\lambda, A, x_{k+1})] \leq \epsilon(\lambda) \quad (35)$$

**Theorem 6** (Challenge re-usability of HLPUF-based Authentication Protocol 2). A challenge $x$ can be reused $k$ times during the Protocol 2 as long as the received respective response $\sigma$ for each round passes the (client’s or server’s) verification with overwhelming probability. In other words, under the successful verification, the success probability of the adversary in passing the $(k + 1)$-th round with the same challenge $x$ is bounded as follows:

$$Pr_{forge}(A, x_{k+1}) \leq k2^{-m} \approx \epsilon(m). \quad (36)$$

**Proof.** To prove this theorem, we use the Theorem 5 directly. First, we assume that $x$ has been used one time before in a previous round. Given the assumption that the verification is passed with probability $1 - \epsilon(m)$, and this theorem, we conclude that the uncertainty of the adversary in guessing the encoded response of the HLPUF is larger than $m - \epsilon(m)$. In our case, the joint quantum state between the server and the adversary is a classical-quantum state (server has the classical description of $f(x)$, and the adversary has the quantum state $|\psi_f(x)\rangle$). For such states, Eve’s uncertainty, $H_{min}^{Eve}$ is same as $-\log P_{guess}$, where $P_{guess}$ is Eve’s guessing probability of the classical information encoded in the quantum state [KRS09]. Therefore,

$$P_{guess}^{Eve} = 2^{-H_{min}^{Eve}} \leq 2^{-m + \epsilon(m)} \quad (37)$$
This probability is negligible in the security parameter, which means that after performing any arbitrary quantum operations, the adversary’s local state includes at most, a negligible amount of information on the response of \( x \), each round that the state \( x \) is reused. Now, we can use the union bound to show that this success probability only linearly scales with \( k \):

\[
P_{\text{Eve},k} \text{guess} = P\left(\bigcup_{i=1}^{k} E_{\text{guess}}^i\right) \leq \sum_{i=1}^{k} P(E_{\text{guess}}^i) \approx k2^{-m}\tag{38}
\]

where \( E_{\text{guess}}^i \) are the events where Eve correctly guesses the response and \( P(E_{\text{guess}}^i) = (P_{\text{Eve}}\text{guess})^i \) is the success probability of Eve in guessing in the \( i \)-th round. Finally, let the success probability of an adversary in the universal unforgeability game for the HLPUF be upper-bounded by \( \epsilon_1(m) \) which is a negligible function in the security parameter since we assume that the HLPUF satisfies the universal unforgeability. This is the same as the success probability of the adversary in passing the verification for a new challenge, chosen at random from the database. Now in the \((k+1)\)-th round, where the same \( x \) is reused, the success probability is at most boosted by the guessing probability over the previous \( k \)-th rounds, hence we will have:

\[
Pr_{\text{forge}}(A, x_{k+1}) \leq \epsilon_1(m) + k2^{-m} = \epsilon(m)\tag{39}
\]

As long as \( k \) is polynomial in the security parameter, the second term is also a negligible function and since the sum of two negligible probabilities will be also negligible. This concludes the proof. \( \square \)

7 Limitations of Lockdown Technique for Generic Quantum PUFs

In this section, we study for the first time, the possibility of exploiting the lockdown technique for quantum PUFs, and we demonstrate the mathematical model for it. It is also worth mentioning that implementing quantum PUFs in practice is challenging and subject to current researches. Some constructions have been proposed for constructing fully secure unitary quantum PUFs such as [KMK21], but they are usually resourceful quantum constructions. Also, some other classes of quantum PUFs, namely quantum-readout PUFs [Ško12], have been defined in weaker attack models and under restricted quantum adversaries. Apart from the theoretical aspect of the problem, it is also interesting to see whether the lockdown technique can help to reduce the adversarial power in the quantum case. One of the main problems in the case of a quantum PUF is that if the adversaries manage to query the PUF with the same input multiple times, then it can get multiple copies of the same output state. This allows the adversaries to use the tools from the quantum state tomography [DLP01], and quantum emulation algorithm to emulate the outputs [ML16] of the PUF. Obviously, one possible way to protect the PUF from such sophisticated attacks is to use the lockdown technique. The main goal of such a lockdown technique is to prevent the adversary from querying in an adaptive manner, and with desired challenges.

Similarly to the hybrid PUF setting, an important feature of the lockdown technique on quantum PUFs is the equality test of unknown quantum states for verification. As introduced previously, the verification algorithm can be efficiently implemented by SWAP test [BCWdW01, CDM+18] if two states \( \rho_1, \rho_2 \) are two pure states. With this constraint in mind, we prove that only very restricted quantum PUFs can be efficiently constructed to a quantum locked PUF (QLPUF) with verification algorithm.

**Theorem 7.** The construction of QLPUF with verification algorithm can be achieved if and only if the input/output mapping of the targeted quantum PUF \( \mathcal{E} : \mathcal{H}^{d_{\text{in}}} \rightarrow \mathcal{H}^{d_{\text{out}}} \)
\( \mathcal{H}^{d_{out1}} \otimes \mathcal{H}^{d_{out2}} \) is of the form \( |\psi_{in}\rangle \langle \psi_{in}| \rightarrow |\psi_{out}\rangle_{S^2} \langle \psi_{out}| \otimes |\psi_{out}\rangle_{S^2} \langle \psi_{out}| \). Otherwise, such lockdown technique is incapable for quantum PUFs.

**Proof.** The proof is twofold. For a quantum PUF \( \mathcal{E} : \mathcal{H}^{d_{in}} \rightarrow \mathcal{H}^{d_{out1}} \otimes \mathcal{H}^{d_{out2}} \) that maps an input state \( |\psi_{in}\rangle_{S^1} \langle \psi_{in}| \in \mathcal{H}^{d_{in}} \) to an output state \( |\psi_{out}\rangle_{S^1} \langle \psi_{out}| \in \mathcal{H}^{d_{out1}} \otimes \mathcal{H}^{d_{out2}} \) with subsystems \( S^1 \) and \( S^2 \). The mapping of the QLPUF \( \mathcal{E}_L : \mathcal{H}^{d_{in}} \otimes \mathcal{H}^{d_{out1}} \rightarrow \mathcal{H}^{d_{out2}} \otimes \mathcal{H}^{d_{out1}} \) corresponding to a quantum PUF \( \mathcal{E} \) is defined as follows:

\[
|\psi_{in}\rangle_{S^1} \langle \psi_{in}| \rightarrow \left\{ \begin{array}{ll}
\rho_{S^2} \quad & \text{if } \text{Ver}(\rho_{S^1}, \tilde{\rho}_{S^1}) = 1 \\
\perp & \text{otherwise}.
\end{array} \right.
\]

where \( \rho_{S^1} = \text{Tr}_{S^2} [|\psi_{out}\rangle_{S^1} \langle \psi_{out}| |\psi_{out}\rangle_{S^2} \langle \psi_{out}|] \) and \( \rho_{S^2} = \text{Tr}_{S^1} [|\psi_{out}\rangle_{S^1} \langle \psi_{out}| |\psi_{out}\rangle_{S^2} \langle \psi_{out}|] \).

According to such construction, the QLPUF takes the input \( |\psi_{in}\rangle_{S^1} \langle \psi_{in}| \otimes \tilde{\rho}_{S^1} \). Among the two input states, the QLPUF uses \( |\psi_{in}\rangle_{S^1} \langle \psi_{in}| \) to get an output state \( |\psi_{out}\rangle_{S^1} \langle \psi_{out}| |\psi_{out}\rangle_{S^2} \langle \psi_{out}| \). The QLPUF outputs a state \( \rho_{S^2} \) if \( \rho_{S^1} \) is same as the state \( \tilde{\rho}_{S^1} \). Otherwise, it outputs an abort state \( \perp \). We refer to Figure 3 for the circuit of the QLPUF. Note that, here the QLPUF needs to check internally whether \( \rho_{S^1} = \tilde{\rho}_{S^1} \) or not. If \( \rho_{S^1} \) is a pure state then we can use SWAP test to check the equality of two pure states. The circuit of the SWAP test makes the circuit of entire QLPUF efficient.

![Figure 3: Construction of QLPUF \( \mathcal{E}_L \) with quantum PUF \( \mathcal{E} : \mathcal{H}^{d_{in}} \rightarrow \mathcal{H}^{d_{out1}} \otimes \mathcal{H}^{d_{out2}} \)](image)

On the other hand, however, in the case when the quantum channel \( \mathcal{E} \) of the quantum PUF can have entangling power and hence the subsystems \( S^1 \) and \( S^2 \) that represent the different parts of the response, may be entangled. Let’s start from the simple situation with 2-qubit entangled state as \( |\psi_{out}\rangle \langle \psi_{out}| \). i.e., for a quantum PUF \( \mathcal{E} \) that maps an input state \( |\psi_{in}\rangle_{S^1} \langle \psi_{in}| \) to an entangled output state \( |\psi_{out}\rangle \langle \psi_{out}| := (|a_1\rangle |b_1\rangle + \beta |a_2\rangle |b_2\rangle) \otimes (|a_1\rangle |b_1\rangle + \beta^* |a_2\rangle |b_2\rangle) \) where \(||a_1|^2 + |\beta|^2 = 1, |a_1\rangle \) and \(|a_2\rangle \) are any two vectors in the space of subsystem \( S^1 \), and \(|b_1\rangle \) and \(|b_2\rangle \) are any two vectors in the space of subsystem \( S^2 \). Consider a POVM measurement on the subsystem \( S^1 \) with \( m \) elements \( \{E_m\} \) where \( \Sigma_n E_n = I \), the reduced density operator of \( S^2 \) after tracing out \( S^1 \) is:

\[
\rho_{S^2} = \sum_m \text{Tr}_{S^1} [\text{Tr}(|\psi_{out}\rangle_{S^1} \langle \psi_{out}| E_m)]
= \sum_m \text{Tr}_{S^1} [|\psi_{out}\rangle_{S^1} \langle \psi_{out}| E_m]
= |\alpha|^2 |b_1\rangle \langle b_1| + |\beta|^2 |b_2\rangle \langle b_2|
\]

The state of subsystem \( S^2 \) is clearly a mixed state. However, checking the equality between two mixed states is difficult, sometimes not possible. For example, we have two different mixed states:

\[
|\psi_i\rangle = \begin{cases} 
|b_1\rangle & \text{with probability } |\alpha|^2 \\
|b_2\rangle & \text{with probability } |\beta|^2 
\end{cases}
\]
and

\[ |\psi_i^2\rangle = \begin{cases} \alpha |b_i^1\rangle + \beta |b_i^2\rangle & \text{with probability } \frac{1}{2} \\ \alpha |b_i^1\rangle - \beta |b_i^2\rangle & \text{with probability } \frac{1}{2} \end{cases} \]  

(43)

The density operators of both mixed states are represented as Equation (41). That is to say, these two mixed states are unequal, but totally indistinguishable. This can be trivially extended to the n-qubit situation. So the lockdown technique is not implementable with generic quantum PUFs.

For the case of quantum PUFs, our study shows that some quantum mechanical properties of quantum PUFs such as entanglement generation, makes it challenging to use the straightforward quantum analogue of the classical lockdown technique. However, this is still an interesting observation, because we do not need this kind of condition on encoding the output of the classical PUF to construct an HPUF with lockdown technique.

8 Conclusions and Discussions

In this paper, we propose a new practical way to enhance the security of PUFs using quantum communication technology. First, we classify the adversaries into adaptive and weak adversaries based on the querying capability of a PUF device. In the PUF-based authentication protocol, one important issue (in the case of both classical and quantum PUF) is that an adaptive adversary can query the PUF with arbitrary input challenges. It permits such an adversary to learn efficiently and emulate the input/output behaviour of the targeted PUF. By harnessing the power of quantum information theory, here we propose a construction of hybrid PUF with classical challenge and quantum response. The main idea is to encode the output of classical PUF into non-orthogonal quantum states. In Theorem 2, we show that learning the responses of the underlying classical PUF from the outputs of such hybrid PUF is at least as hard as learning about the classical PUF from the random samples of \( f(x) + Z \), where \( Z \) is a random error term. Note that, in general, learning about a function from its noisy sample is a hard problem even if the actual function \( f \) is a simple linear function [Pie12, BKW03, BLP+13, KS09, Reg09]. This implies, if learning of a CPUF from its noisy sample is hard, then it is hard to learn about the CPUF from our HPUF. Note that, here, we get the noise term \( Z \) without any pre-shared key. It comes from the impossibility of distinguishing the non-orthogonal quantum states. We further propose a hybrid PUF-based authenticated protocol with the lockdown technique and prove the security against the adaptive adversaries. The advantage is twofold: On one hand, the probability of knowing information about a quantum state is upper-bounded compared to a classical PUF due to the quantum information theory. On the other hand, the implementation of hybrid PUF is practical nowadays with quantum communication technology.

Another advantage of hybrid PUF construction with lockdown technique is that for each challenge-response pair used for a successful authentication in Protocol 2, it can be used several times for authentication due to the unclonability of response quantum state. Therefore, with our solution, a server can continue the client authentication protocol for a longer period without exhausting its CRP database. This result overcomes the fundamental drawbacks of the existing classical PUF-based authentication protocol and provides a novel and practical use case of our HLPUF construction.

Here we also explore the use of the lockdown technique for the full quantum PUFs. However, we show that such lockdown technique is incapable with generic quantum PUFs due to the quantum entanglement. In general, if the quantum PUFs produce an entangled state as an outcome, then we cannot check the consistency of some of the response qubits without disturbing the rest of them. This negative result makes the design of secure quantum PUF challenging.
QKD technology is one of the most mature quantum technologies. The long-distance QKD networks are already implemented and used in many different countries like the USA, UK, China, EU, Japan, [SFI+11, SLB+11, PPM08, WCY+14, Cou16] etc. Many commercially available QKD infrastructures provide almost 300kb/s secret key rate over optical fibre links of length 120km [FLD+17]. Moreover, the availability of the mature QKD on-chip technology [SEG+17, SSH+20, BLL+18] makes all the proposed constructions in this paper implementable inside the IoT devices. Our results show that picking the classical PUF technology and QKD technology off the shelf can solve all the shortcomings of the device authentication problem.

Furthermore, more sophisticated constructions of hybrid PUF can be extended. For example, we can exploit another classical PUF with the same size of output as the classical PUF of hybrid construction to perfectly encrypt the quantum outcome states of hybrid PUF with one-time pad, where $4m$ classical bits of entropy are necessary and sufficient for the transmission of $m$ quantum bits in an information-theoretically secure way [MTW00]. However, one of the main limitation of such construction is the imperfection of randomness based on classical PUF device in the real world. Another direction of future work of us will extend the construction of hybrid PUF with the study of trade-off between minimum classical entropy and approximate encryption [KN06].

Finally, to further study the feasibility and practicality of hybrid PUF constructions, an important future direction would be towards experimental implementation of our proposal and the HLPUF-based authentication protocol.
Appendix A  Appendix Overview

The structure of the appendix is as follows: In Appendix B we give a detailed description of an adaptive and weak quantum adversary, in the most general case of the unforgeability game where all the learning queries are density matrices. Then, we also give a more detailed version of the quantum unforgeability game, with adaptive and weak adversaries. In Appendix C, we present the full proof of some of the main results of the paper, including the proof of Lemma 1 and Theorem 2. Finally, in Appendix C.3, we first give a brief introduction of the entropic uncertainty relations that have been used in the literature of quantum information for different purposes like security proof of QKD protocols. Then, we establish a formal version of Theorem 5, in terms of the described uncertainty quantities, and finally, we give a full detailed proof of this theorem which we will use to establish the challenge re-usability property for our HLPUF-based protocol.

Appendix B  Unforgeability game for QPUF against adaptive and weak adversary

In this appendix, we introduce the full quantum unforgeability game against adaptive and weak (non-adaptive) adversaries. Any adversary that tries to predict the response of a PUF $E : \mathcal{H}^{d_{in}} \to \mathcal{H}^{d_{out}}$, can be modelled as an interactive algorithm. Here we consider Quantum Polynomial-Time (QPT) adversaries that have $q$-query access to the evaluation of the PUF, namely $E$ where $q$ is polynomial in the security parameter. An adaptive adversary can choose and issue any arbitrary query which could also depend on the previous responses received from the PUF. On the other hand, a weak non-adaptive adversary, cannot choose the queries and will instead receive $q$ input/output pairs states of $E$. In the case that all the queries are quantum, the post-learning phase database of a weak adversary can be easily modelled by the definition. However, an adaptive quantum adversary is likely to consume the quantum state of the response to be able to pick the next query adaptively. Hence modelling the post-query database of an adaptive quantum adversary is more challenging. In what follows we give a $q$-query mathematical model for adaptive and weak adversaries.

Definition 9 (Adaptive and Weak Adversary). Let $q$ be a positive integer, and $E : \mathcal{H}^{d_{in}} \to \mathcal{H}^{d_{out}}$ be a PUF. We model a probabilistic adversary as a CPTP map $A : R \times (\mathcal{H}^{d_{in}})^{\otimes q} \otimes (\mathcal{H}^{d_{out}})^{\otimes q} \to (\mathcal{H}^{d_{in}})$. Such an adversary is called an adaptive adversary $A_{ad}$ if for all random coin $r \in R$ and for any $\otimes_{i=1}^q \rho_{in}^{i} \in (\mathcal{H}^{d_{in}})^{\otimes q}$ and for $\otimes_{i=1}^q \rho_{out}^{i} \in (\mathcal{H}^{d_{out}})^{\otimes q}$ (where $\rho_{out}^{i} := E(\rho_{in}^{i})$), the mapping $\otimes_{i=1}^q (\rho_{in}^{i} \otimes \rho_{out}^{i}) \mapsto A_{ad} (\otimes_{i=1}^q (\rho_{in}^{i} \otimes \rho_{out}^{i}))$ is dependent on the $\rho_{in}^{i} \otimes \rho_{out}^{i}$. For a weak adversary $A_{weak}$ the mapping $\otimes_{i=1}^q (\rho_{in}^{i} \otimes \rho_{out}^{i}) \mapsto A_{weak} (\otimes_{i=1}^q (\rho_{in}^{i} \otimes \rho_{out}^{i}))$ is independent of $\otimes_{i=1}^q (\rho_{in}^{i} \otimes \rho_{out}^{i})$. Moreover, the adversary has no choice over the query, i.e., all the queries $\otimes_{i=1}^q \rho_{in}^{i}$ are chosen following a distribution $R$, and a third party chooses the distribution.

Intuitively, an adaptive adversary $A : R \times (\mathcal{H}^{d_{in}})^{\otimes q} \otimes (\mathcal{H}^{d_{out}})^{\otimes q} \to (\mathcal{H}^{d_{in}})$ captures the strategy to choose the query input $\rho_{in}^{i+1} \in \mathcal{H}^{d_{in}}$ to the PUF $E$. The adversary can use these query response pairs to predict the output of the PUF. We call the pair $(\otimes_{i=1}^q \rho_{in}^{i}, \otimes_{i=1}^q \rho_{out}^{i})$ that is generated after the $q$-round of interaction between an adversary $A$ and a PUF $E$, as a transcript. Note, that the transcripts depend on the choice of the random coins of $A$.

B.1 Unforgeability with Game-based Security

Similar to Game 1, We define the unforgeability of PUF as a formal game between two parties: a challenger ($C$) and an adversary ($A$). The difference here is that our adversaries
are defined according to Definition 9. A formal description is given as follows:

**Game 2** (Universal Unforgeability of PUF). Let $MP$ be the manufacturing process, $Ver(.)$ be a verification algorithm for checking the responses, and $\lambda$ the security parameter. We define the following game $G_{PUF}(A, \lambda)$ running between an adversary $A$ and a challenger $C$:

- **Setup phase.**
  - $C$ selects a manufacturing process $MP$ and security parameter $\lambda$. Then $C$ creates a PUF by $E \leftarrow MP(\lambda)$, which is described by a CPTP map. The challenge and response domain $H^{in}$ and $H^{out}$ are shared between $C$ and $A$.

- **Learning phase.**
  - If the adversary is adaptive, $A = A_{ad}$:
    * $A_{ad}$ selects and prepares an initial state $\rho^i_{0} \in H^{in}$, while having full access to the preparation algorithm.
    * $A_{ad}$ issues to $C$ the initial challenge state $\rho^i_{0} \otimes \rho_{anc}$ where $\rho_{anc}$ is an initially blank state.
    * $C$ queries the PUF with $\rho^i_{0}$ and sends the response $(E \otimes I)\rho^i_{0} \otimes \rho_{anc}$ back to $A_{ad}$
    * for the next challenges ($i \neq 0$), the adaptive adversary $A_{ad}$ produces a new challenge for next query as $\rho^i_{0} = A^i_{ad}((E \otimes I)\rho^i_{0} \otimes \rho_{anc})$ and issues to $C$.
    * $C$ queries the PUF with $\rho^i_{0}$ and sends the response to $A$. Recursively, $A$ obtains the CPRs with challenge $\rho^i_{0} = A^i_{ad}(E \otimes I)A^{i-1}_{ad}(E \otimes I) \ldots A^{1}_{ad}(E \otimes I)\rho^0_{0}$ and corresponding response $\rho^i_{out} = (E \otimes I)(\rho^i_{0} \otimes \rho_{anc})$
  - If the adversary is (weak) non-adaptive, $A = A_{weak}$:
    * $C$ selects a challenge $\rho^i_{0}$ uniformly at random from $H^{in}$ and independent of $i$, while being able to prepare arbitrary copies of each challenge.
    * $C$ queries the PUF with $\rho^i_{0}$ and produces the response $E(\rho^i_{0})$.
    * $C$ issues to $A_{weak}$ the set of random challenges $\otimes_{i=1}^{q} \rho^i_{0}$ and their respective responses $\otimes_{i=1}^{q} \rho^i_{out}$.

- **Challenge phase.**
  - $C$ chooses a challenge $\rho^{c}$ uniformly at random from challenge domain $H^{in}$. $C$ can produce multiple copies of the challenge, and the respective response locally.
  - $C$ issues $\rho^{c}$ to $A$.

- **Guess phase.**
  - For the challenge $\rho^{c}$, $A$ produces his forgery $\sigma^{c} \leftarrow A(1^{\lambda}, \rho^{c}, \{(\rho^{i}_{0}, \rho^{i}_{out})\})$ and sends to $C$.
  - $C$ runs a verification algorithm $b \leftarrow Ver(\sigma^{c}, \rho^{c}, \rho_{C})$, to check the fidelity of the responses. Where $\rho^{c} = E(\rho^{c})$ is the correct output, $\rho_{C}$ is the local register of the challenger that can include extra copies of correct output if necessary for the verification, and $b \in \{0,1\}$.
  - $C$ outputs $b$. $A$ wins if $b = 1$.

Finally, the security definitions can be defined based on this game, similar to definitions 4 and 5.
Appendix C  Detailed Security Analysis

C.1  Proof of Lemma 1

Here, we give a detailed proof for Lemma 1.

**Proof of Lemma 1.** According to Construction 1, for a given \( x_i \), we use the \( 2j \)-th bit \( y_{i,2j} \in \{0,1\} \) of the outcome of the CPUF to choose the basis (either \( \{|0\}, |1\} \)-basis or \( \{|+,|\rangle \} \)-basis) of the \( j \)-th qubit output of the HPUF. Further, we use the \( y_{i,(2j-1)} \in \{0,1\} \) to choose a state from the chosen basis. Here, if \( y_{i,2j} = 0 \) then from an adversarial point of view, the output state is \( \rho_0 := (\frac{1}{2} + \delta_r)|0\rangle\langle 0| + (\frac{1}{2} - \delta_r)|1\rangle\langle 1| \). Similarly, if \( y_{i,2j} = 1 \) then from an adversarial point of view, the output state is \( \rho_1 := (\frac{1}{2} + \delta_r)|+\rangle\langle +| + (\frac{1}{2} - \delta_r)|-\rangle\langle -| \). The adversary has only knowledge about the priori distribution of \( y_{i,2j} \). This implies, for the adversary, the probability of correctly guessing \( \{ y_{i,2j}, y_{i,(2j-1)} \} \) is the same as distinguishing the two states \( \rho_0, \rho_1 \) with priori probability \( \frac{1}{2} + \delta_r \) and \( \frac{1}{2} - \delta_r \). Here

\[
\Pr[A_{\text{guess}}^{ij}(x_i, |\psi_{\text{out}}^{ij}\rangle\langle \psi_{\text{out}}^{ij}|) = (y_{i,(2j-1)}, y_{i,2j})] = \frac{1}{2} + \delta_r. \tag{44}
\]

And the probability of guessing probability of response \( f(x) \) from single copy of the quantum response state \( |\psi_{f(x_i)}\rangle \) is thus:

\[
\Pr[A_{\text{guess}}^{ij}(x_i, |\psi_{f(x_i)}\rangle\langle \psi_{f(x_i)}|) = f(x_i)] \leq \left( \frac{1}{2} + \delta_r \right)^{2m}. \tag{45}
\]

This concludes the proof. \( \square \)

C.2  Proof of Theorem 2

Here, we give a detailed proof for Theorem 2. However, before going to the roof, we introduce some concepts that are related to the accessible information from a quantum state.

Consider a scenario where Alice prepares a pure quantum state drawn from the ensemble \( \{ p_y, |\psi_y\rangle \} \) with the density matrix \( \rho_{AB} \), where

\[
\rho_{AB} = \sum_y p_y |y\rangle_A \langle y| \otimes |\psi_y\rangle_B \langle \psi_y|. \tag{46}
\]

Bob knows the ensemble i.e., the mixed state \( \rho_{AB} \), but not the particular state that Alice chose. He wants to acquire as much information as possible about \( y \). Bob collects his information by performing a generalized measurement, the POVM \( M_{\bar{y}} \). On average Bob’s state is of the form \( \rho_{AB} := \text{Tr}_A(\rho_{AB}) \).

If Alice chose preparation \( y \), Bob will obtain the measurement outcome \( \bar{y} \) with conditional probability \( p(\bar{y}|y) = \langle \psi_y|M_{\bar{y}}|\psi_y\rangle \). For this kind of classical-quantum state \( \rho_{AB} \), the amount of information Bob can extract from this measurement and the given state is given by the mutual information \( I(Y; \bar{Y})_\rho \) between \( Y, \bar{Y} \). It is defined as follows.

\[
I(Y; \bar{Y}) := h(Y) - h(\bar{Y}|Y), \tag{47}
\]

where \( h(.) \) denotes the Shannon entropy. The average accessible information from a classical quantum state \( \text{Acc}(\rho_{AB}) \) is given by the following equation.
In our case, if we construct a HPUF from a CPUF $f$ with $p$-randomness, then $h(\tilde{Y}|Y) \geq -\log p$. By substituting this relation in Equation (48) we get,

$$\text{Acc}(\rho_{AB}) \leq h(\tilde{Y}) + \log p. \quad (49)$$

We use the Equation (50) for proving Theorem 2.

**Proof of Theorem 2.** For the HPUF $E_f$ corresponding to a CPUF $f : \{0,1\}^n \rightarrow \{0,1\}^{4m}$, any weak adversary can have access to a set of random classical challenge strings and response quantum states. Note that, the adversary has no information about the exact value of $f(x)$ for a given challenge $x$. Therefore, the adversary only knows that the response state $\rho$ is chosen from an ensemble $\{p'_f(y)|\psi_y\}_{y\in\{0,1\}^{4m}}$. We refer to the distribution $\{p'_f(y)\}_{y\in\{0,1\}^{4m}}$ as $Y$. Due to the i.i.d property of the CPUF we have that all $Y$’s are identically distributed, and we denote each of them by $Y$. This implies, the $\rho$’s are also identical. Therefore, after $q$ queries the weak adversary gets a joint state of the form $\rho^{\otimes q}$. To extract information about the outcome of CPUF $f(x)$, the adversary performs a measurement on $\rho$, and gets $\tilde{Y}$ as outcome. From lemma 1 we get that for any such measurements $\Pr[\tilde{Y} = f(x)] \leq p$. In general, the adversary will get $\tilde{Y} = f(x) + \tilde{Z}$ as an outcome. Moreover, as the adversary has only one state $|\psi_f(x)\rangle$, it can not get the exact value of $\tilde{Z}$, other than the situation when $\tilde{Z} = 0$. From Equation 49, we get the following bound on the accessible information $\text{Acc}(\rho)$ of $f(x)$ from the ensemble $\{p'_f(y)\}_{y\in\{0,1\}^{4m}}$.

$$\text{Acc}(\rho) \leq h(Y) + \log p. \quad (50)$$

On the other hand, if the adversary has only access to random samples of $\tilde{f}$, then for each $x$ the mutual information between $Y$ and $\tilde{f}(x)$ is given by,

$$I(Y; \tilde{f}(x)) = h(Y) - h(Y|\tilde{f}(x)). \quad (51)$$

From the definition of the conditional Shannon entropy we get,

$$h(Y|\tilde{f}(x)) := - \sum_{y\in\{0,1\}^{4m}, f(x)\in\{0,1\}^{4m}} p(y, \tilde{f}(x)) \log p(y|\tilde{f}(x)). \quad (52)$$

Note that, according to Equation (18) we get $p(y|\tilde{f}(x)) = p$. By substituting this relation (with simple algebraic manipulation) in Equation (52) we get,

$$h(Y|\tilde{f}(x)) = - \log p. \quad (53)$$

Substituting the value of $h(Y|\tilde{f}(x))$ from Equation (53) to Equation (51) we get,

$$I(Y; \tilde{f}(x)) = h(Y) + \log p. \quad (54)$$

By combining Equation (50) and Equation (54) we get,

$$I(Y; \tilde{f}(x)) \geq \text{Acc}(\rho). \quad (55)$$

Equation (55) suggests that the amount of information the weak adversaries can extract about $f(x)$ from a random sample of $\tilde{f}(x)$ is at least as large as the accessible information of $f(x)$ from a random sample of the outcome of the HPUF $E_f$. This implies, from a database of the outcomes of the HPUF, the weak adversaries can extract less (or equal) information than the database of the samples from $\tilde{f}$. Therefore, if an adversary can learn about the CPUF from the random samples of the outcome of the HPUF $E_f$, then it can learn about $f$ from the random samples of $\tilde{f}$. This concludes the proof. \qed
C.3 Challenge re-usability Proof

In this subsection, we give a detailed security analysis and proof for the challenge re-usability discussed in Section 6. First, we introduce the tools and uncertainty relation that we need for the proof mostly from [CBTW17], then we give the formal statement and proof for Theorem 5.

Heisenberg’s uncertainty principle is one of the most important fundamental properties of quantum mechanics which is mathematically speaking due to the non-commuting property of some observables like Pauli $X$ and $Z$ measurements. Reformulating these relations in terms of entropic quantities has been very useful in the foundations of quantum information and has also been widely used in the security proofs of different quantum communication protocols such as QKD. The most well-known uncertainty relation for these operators was given by Deutsch [Deu83] and later improved [MU88] as follows:

$$H(X) + H(Z) \geq \log_2 \left( \frac{1}{c} \right)$$

where $c$ denotes the maximum overlap between any two eigenvectors of $X$ and $Z$.

Usually, a quantum system $A$ is considered where the state is described with the density matrix $\rho_A$ on a finite-dimensional Hilbert space. If the measurement is performed in a $X$ and $Z$ basis (or equivalently any other MUB bases), then the measurements are just projective operators that project the state into the subspace spanned by those bases. In the most general case, the measurements are a set of POVM operators on system $A$ denoted as \{${M^x}$\}$_x$ and \{${N^z}$\}$_z$ where the general Born rule states that the probability of obtaining outcomes $x$ and $z$ to be as follows:

$$P_X(x) = \text{tr}[\rho_A M^x], \quad P_Z(z) = \text{tr}[\rho_A N^z]$$

In this case, the Equation (56) still gives the generalised uncertainty relation with the difference that the $c$ is defined as follows:

$$c = \max_{x,z} c_{xz}, \quad c_{xz} = \| \sqrt{M^x} \sqrt{N^z} \|^2$$

where $\| \cdot \|$ denotes the operator norm (or infinity norm). The above uncertainty relation can be extended to conditional entropy as well in the context of guessing games [CBTW17]. Assume two parties, Alice and Bob, where Bob prepares a state $\rho_A$ and Alice randomly performs the $X$ and $Z$ measurements leading to a bit $K$. Then Bob wants to guess $K$ given the basis choice $R = \{0, 1\}$. The conditional Shannon entropy is defined as follows:

$$H(K|R) := H(KR) - H(R)$$

Thus one can get the same uncertainty relation with the conditional entropy as:

$$H(K|R = 0) + H(K|R = 1) \geq \log_2 \left( \frac{1}{c} \right)$$

We also have the quantum equivalent of Shannon entropy for mixed quantum state called von Neumann entropy, that is defined as $H(\rho) = \text{tr}[\rho \log \rho] = - \sum \lambda_i \log_2(\lambda_i)$ where $\lambda_i$ are the eigenvalues of $\rho$. Similar, to the classical case, for a bipartite system $\rho_{AB}$ the conditional von Neumann entropy is defined as follows:

$$H(A|B) := H(\rho_{AB}) - H(\rho_B)$$

Furthermore, this can be generalised to any tripartite quantum system with state $\rho_{ABC}$. An interesting property here is an inequality referred to as data processing inequality [CBTW17] which states that the uncertainty of $A$ conditioned on some system $B$ never goes down if $B$ performs a quantum channel on the system. In other words for
any tripartite system $\rho_{ABC}$ where system $C$ will perform a quantum operation on the quantum state in order to extract some information, we have the following:

$$H(A|BC) \leq H(A|B)$$

(62)

Given the above inequality leads to the general uncertainty relations between any tripartite system including two parties Alice and Bob, and an eavesdropper Eve:

$$H(K|ER) + H(K|BR) \geq \log_2 \left( \frac{1}{c} \right)$$

(63)

Where $K$ is the measurement output and $R$ is the basis bit. This imposes a fundamental bound on the uncertainty in terms of von Neumann entropy, in other words, the amount of information that an eavesdropper can extract from the joint quantum systems shared between the three parties. These inequalities can also be extended to the case where $n$ bits are encoded in $n$ quantum states where $R^n$ and $K^n$ are bit-strings denoting the basis random choices for the qubits and measurement outputs respectively, and $B^n$ denotes Bob’s bit-string. Also $E$ denotes Eve’s system that is a general quantum system operating on $n$-qubit messages and any arbitrary local system. We have the following inequality which is the main result that we will use in the proof of the next theorem:

$$H(K^n|ER^n) + H(K^n|B^nR^n) \geq n \log_2 \left( \frac{1}{c} \right)$$

(64)

Now we are ready to give a more formal version of the Theorem 5 and the proof.

**Theorem 8.** In Protocol 2, let $x$ be a challenge and $(y_1,\ldots,y_{2m})$ be the response of a classical PUF used inside the HPUF construction, with randomness bias $p = (\frac{1}{2} + \delta)_2^{2m}$ in generating the random classical responses. If the verification algorithm for a state $\tilde{\rho}$ passes with probability $1 - \epsilon(m)$, then the Eve’s conditional min-entropy $H_{min}^{Eve}$ in terms of von Neumann entropy over the server’s (or client’s) classical responses, satisfies the following inequality:

$$H_{min}^{Eve} = H_{min}(S^m|ER^m) \geq m - \epsilon(m)$$

(65)

Proof. We prove this theorem based on the first half of the state used in Protocol 2, i.e. the state $|\psi_{out}^{1,3}\rangle = \otimes_{j=1}^{m-1} |\psi_{out}^{1,j}\rangle_{S_j} |\psi_{out}^{1,j}\rangle_{S_{j+1}}$ that is being sent by the Server (S) and received and measured by the Client (C). Nevertheless, the same proof applies for the second state due to the symmetry of the states and the protocol.

Let $R^m = (R_1,\ldots,R_m)$ be the randomness bitstring showing the choice of the basis encoding of the response, $S^m = (S_1,\ldots,S_m)$ be the server’s bit encoded in the $R^m$ bases. Note that both $R^m$ and $S^m$ are produced according to the bitstring $(y_1,\ldots,y_{2m})$ which is the first half of the response of CPUF to a given challenge $x$. Also, let $C^m = (C_1,\ldots,C_m)$ be the client’s correct bit string. We denote the arbitrary joint state of three systems by $\rho_{S=EC^n}$ where $E$ denotes any arbitrary quantum system held by the eavesdropper. Now, let the the Client’s measurement outcomes, after the verification be $Y^m = (Y_1,\ldots,Y_m)$ which shows the estimated bits by the Client. Now we can write the tripartite uncertainty principle, in terms of the von Neumann entropy, for MUB measurements and MUB states as follows:

$$H(X_1Z_2X_3\ldots Z_{m-1}X_m|E) + H(Z_1Z_2X_3\ldots Z_{m-1}X_m|C) \geq \log_2 \left( \frac{1}{c} \right)^m$$

(66)

where $c = \max_{x,z} e_{xz}$ and $e_{xz} = \| \sqrt{M^x} \sqrt{N^z} \|^2$ for an arbitrary POVM sets $M = \{M^x\}_x$ and $N = \{N^z\}_z$. We note that if the CPUF creates perfect random bitstring for $R^m$ then states are perfect MUB states and $c = \frac{1}{2}$. Nonetheless we consider a weaker CPUF with a biased distribution of $p = (\frac{1}{2} + \delta)_2^{2m}$ in creating 0s and 1s in the response. Hence, we can translate this imperfectness into a disturbance in the measurement bases. Let
$M^0 = |0\rangle \langle 0|$ and $M^1 = |1\rangle \langle 1|$ be the usual measurement in the computational basis but let the $N$ measurements be a slightly shifted version of the measurements in the $X$ basis. Consider the following states:

$$|\psi_N\rangle = \sqrt{\frac{1}{2} + \delta_r}|0\rangle + \sqrt{\frac{1}{2} - \delta_r}|1\rangle$$

$$|\psi_N^\perp\rangle = \sqrt{\frac{1}{2} - \delta_r}|0\rangle + \sqrt{\frac{1}{2} - \delta_r}|1\rangle$$

(67)

We define the new $N$ projective operators according to the following states as $N^0 = |\psi_N\rangle \langle \psi_N|$ and $N^1 = |\psi_N^\perp\rangle \langle \psi_N^\perp|$. Now we calculate the operator norm for all the pairs of measurements and we have:

$$\| \sqrt{M^0N^0} \|^2 = \frac{1}{2} + \delta_r, \quad \| \sqrt{M^0N^1} \|^2 = \frac{1}{2} - \delta_r$$

$$\| \sqrt{M^1N^0} \|^2 = \frac{1}{2} + \delta_r, \quad \| \sqrt{M^1N^1} \|^2 = \frac{1}{2} - \delta_r$$

(68)

Thus we conclude that $c = \frac{1}{2} + \delta_r$ and the Equation (66) can be re-written as follows:

$$H(X_1X_2X_3X_4…X_{m-1}Z_m|E) + H(Z_1Z_2X_3Z_4…Z_{m-1}X_m|C) \geq m - \log_2(1 - 2\delta_r)$$

(69)

Now, as mentioned at the beginning of the section, using the data processing inequality [CBTW17], we have got the following security criteria that show Eve’s uncertainty (in terms of the von Neumann entropy) of the actual response bits $S^m$:

$$H(S^m|ER^m) + H(S^m|\tilde{Y}^m) \geq m - \log_2(1 - 2\delta_r)$$

(70)

We can get the same inequality in terms of smooth min and max entropy [CBTW17, TR11], which is more appropriate for ensuring the security in the finite size, for min and max entropy we equivalently have:

$$H^e_{\min}(S^m|ER^m) \geq m - H^e_{\max}(S^m|\tilde{Y}^m) - \log_2(1 - 2\delta_r)$$

(71)

In order to calculate the above bound we need to find the bound on the $H^e_{\max}(S^m|\tilde{Y}^m)$. Here we use another result from [TR11] where it states that for any bitstring $X$ of $n$ bit and the respective measurement outcome $X'$, which at most a fraction $\zeta$ of them disagree according to the performed statistical test, then the smooth max entropy is bounded as follows:

$$H^e_{\max}(X|X') \leq nh(\zeta)$$

(72)

where $h(.)$ denotes the classical binary Shannon entropy. Now we can use this result and our assumption of successful verification together. Given the assumption that the verification is passed with a probability $1 - \epsilon(m)$, and the verification algorithm consists of measuring the states in the $Z$ and $X$ bases, we can conclude that they final bits differ in at most a fraction $\zeta = \epsilon(m)$ where $\epsilon(m)$ is a negligible function. As a result we have:

$$H^e_{\max}(S^m|\tilde{Y}^m) \leq nh(\zeta) \approx m\epsilon(m)$$

(73)

Putting Equations (71) and (73) together, we have:

$$H^e_{\min}(S^m|ER^m) \geq m - m\epsilon(m) - \log_2(1 - 2\delta_r)$$

(74)

On the right-hand side of the above inequality, the second term is still a negligible function and the third term depends on the CPUF bias probability distribution. We assume the CPUF satisfies $p$-Randomness, as defined in the Definition 3, thus the $\delta_r$ is a small value and hence the term $(1 - 2\delta_r)$ is negligibly close to 1, which means that the third term, is negligibly close to 0 in the security parameter which is $m$. Finally, we conclude that:

$$H^E_{\min} = H^e_{\min}(S^m|ER^m) \geq m - \epsilon'(m)$$

(75)

where $\epsilon'(m)$ is a negligible function and the proof is complete.
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