Determination of latent heat at the finite temperature phase transition of SU(3) gauge theory

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Introduction

• First order phase transitions are expected in many interesting systems of lattice field theories. e.g. high density QCD, many flavor etc..
  
  → important to study first order phase transitions.

• The latent heat (energy gap) the most basic quantity.

• The gap of pressure must be vanish.
  Reliability of the calculation can be confirmed.

In this talk,

• We study the equation of state at the first order phase transition of SU(3) gauge theory.

• Gaps of energy density and pressure are measured using the derivative method.
  • Volume dependence is investigated
  • Continuum extrapolation is performed. [H. Suzuki, 2013]

• We tested the gradient flow method for the calculation of EoS.
Thermodynamic quantities by the derivative method

energy density
\[ \epsilon = -\frac{1}{V} \frac{\partial \ln Z}{\partial T^{-1}} \bigg|_V \]

pressure
\[ p = T \frac{\partial \ln Z}{\partial V} \bigg|_T \]

temperature
\[ \frac{1}{T} = N_t a_t \]

volume
\[ V = (N_s a_s)^3 \]

\[ Z = \int DU \ e^{-S} \]

For the SU(3) gauge theory,
\[ S = -3N_{\text{site}}(\beta_s P_s + \beta_t P_t) \]
\[ (P_{s(t)} \text{ space-like (time-like) plaquette}) \]

\[ \epsilon = -\frac{3N_t^4 T^4}{\xi^3} \left\{ \left( a_t \frac{\partial \beta_s}{\partial a_t} - \xi \frac{\partial \beta_s}{\partial \xi} \right) \langle P_s \rangle - \langle P \rangle_0 \right\} - \left\{ a_t \frac{\partial \beta_t}{\partial a_t} - \xi \frac{\partial \beta_t}{\partial \xi} \right\} \langle P_t \rangle - \langle P \rangle_0 \right\} \]

\[ p = \frac{N_t^4 T^4}{\xi^3} \left\{ \frac{\partial \beta_s}{\partial \xi} \langle P_s \rangle - \langle P \rangle_0 + \frac{\partial \beta_t}{\partial \xi} \langle P_t \rangle - \langle P \rangle_0 \right\} \]

Independent variables:
\[ a_t, \ \xi = \frac{a_s}{a_t} \]

\[ \langle P \rangle_0: \text{The expectation value at } T = 0 \]

\[ \frac{\Delta \epsilon}{T^4} = -3N_t^4 \left\{ \left( a_t \frac{\partial \beta_s}{\partial a_t} - \frac{\partial \beta_s}{\partial \xi} \right) \langle P_s \rangle_{\text{hot}} - \langle P_s \rangle_{\text{cold}} + \left( a_t \frac{\partial \beta_t}{\partial a_t} - \frac{\partial \beta_t}{\partial \xi} \right) \langle P_t \rangle_{\text{hot}} - \langle P_t \rangle_{\text{cold}} \right\} \]

\[ a_t \frac{\partial \beta_s}{\partial a_t}, a_t \frac{\partial \beta_t}{\partial a_t}, \frac{\partial \beta_s}{\partial \xi}, \frac{\partial \beta_t}{\partial \xi} \]

These 4 coefficients must be determined.
Determination of the anisotropy coefficients at $\xi = a_s / a_t = 1$

Isotropic lattice ($\beta = \beta_s = \beta_t$): $\left( a_t \frac{\partial \beta_s}{\partial a_t} \right)_{\xi=1} \left( a_t \frac{\partial \beta_t}{\partial a_t} \right)_{\xi=1} = a \frac{d\beta}{da}$

$a \frac{d\beta}{da}$ is determined by the data of the critical $\beta$ ($\beta_c(N_t)$)

String tension is independent of $\xi = \frac{a_s}{a_t}$

$$\left( \frac{\partial \beta_s}{\partial \xi} + \frac{\partial \beta_t}{\partial \xi} \right)_{a_t: fixed, \xi=1} = \frac{3}{2} a \frac{d\beta}{da}$$

Data: Francis, Kaczmarek, Laine, Neuhaus, Ohno, Phys. Rev. D 91, 096002 (2015) and our data for $N_t = 4 \sim 22$

[F. Karsch, Nucl. Phys. B205 (1982) 285]
Ratio of the anisotropy coefficients

The slope of the phase transition line in the \((\beta_s, \beta_t)\) plane: \(r_t\)

[ Ejiri, Iwasaki, Kanaya, Phys.Rev.D 58,094505 (1998) ]

Along the phase transition line, \(a_t\) is constant because \(\frac{1}{T_c} = N_t a_t\).

When one changes \((\beta_s, \beta_t) \rightarrow (\beta_s + d\beta_s, \beta_t + d\beta_t)\),

\[
d\alpha_t = \frac{\partial \alpha_t}{\partial \beta_t} d\beta_s + \frac{\partial \alpha_t}{\partial \beta_t} d\beta_t = 0
\]

The slope of the transition line

\[
r_t = \frac{d\beta_s}{d\beta_t} = - \frac{\left(\frac{\partial \alpha_t}{\partial \beta_t}\right)_{\xi=1}}{\left(\frac{\partial \alpha_t}{\partial \beta_s}\right)_{\xi=1}} = \frac{\left(\frac{\partial \beta_s}{\partial \beta_t}\right)_{\xi=1}}{\left(\frac{\partial \beta_s}{\partial \beta_t}\right)_{\xi=1}}
\]

Using the reweighting method, \((\beta_s, \beta_t)\)-dependence of the Polyakov loop susceptibility is measured.
Anisotropy coefficients

From

\[
\left( \frac{\partial \beta_s}{\partial \xi} + \frac{\partial \beta_t}{\partial \xi} \right)_{a_t:\text{fixed}, \xi=1} = \frac{3}{2} a \frac{d\beta}{da}
\]

\[
r_t = \left( \frac{\partial \beta_s}{\partial \xi} \right)_{\xi=1} = \frac{3}{2(1 + r_t)} a \frac{d\beta}{da}
\]

Conventional combinations of the energy density and pressure

\[
\frac{\Delta(\epsilon + p)}{T^4} = 3N_t^4 a \frac{d\beta}{da} \frac{r_t - 1}{r_t + 1} \left\{ (\langle P_s \rangle_{\text{hot}} - \langle P_s \rangle_{\text{cold}}) - (\langle P_t \rangle_{\text{hot}} - \langle P_t \rangle_{\text{cold}}) \right\}
\]

\[
\frac{\Delta(\epsilon - 3p)}{T^4} = 3N_t^4 a \frac{d\beta}{da} \left\{ (\langle P_s \rangle_{\text{hot}} - \langle P_s \rangle_{\text{cold}}) - (\langle P_t \rangle_{\text{hot}} - \langle P_t \rangle_{\text{cold}}) \right\}
\]
Simulation details

Pure SU(3) gauge theory

Standard plaquette action is used.

Pseudo-heat bath algorithm + over-relaxation.

| Lattice size | # β | No. of Conf. |
|--------------|-----|-------------|
| $48^3 \times 6$ | 1   | 201200      |
| $64^3 \times 6$ | 4   | 442000      |
| $48^3 \times 8$ | 6   | 1220000     |
| $64^3 \times 8$ | 5   | 4585000     |
| $64^3 \times 12$ | 3  | 624000      |
| $96^3 \times 12$ | 5   | 1558000     |

Parts of the results in Ejiri, Iwasaki, Kanaya, Phys.Rev.D 58,094505 (1998) are used in this analysis.

Simulations are performed at 1–6 β points near transition point.

High statistics data: \( \sim O(10^6) \)

The multi-point reweighting method is used for the measurements.
Measurement of the slope of the transition line $r_t$

We used the reweighting method. The slope $r_t$ can be determined with sufficient accuracy.

Order parameter: Polyakov loop $\Omega(x, t)$

Transition point:
Peak position of Polyakov loop susceptibility

$$
\chi_\Omega(\beta_s, \beta_t) = N_s^3 (\langle \Omega^2 \rangle_{(\beta_s, \beta_t)} - \langle \Omega \rangle^2_{(\beta_s, \beta_t)})
$$

| Lattice size | $r_t$   |
|--------------|---------|
| $48^3 \times 6$ | -1.2020(39) |
| $64^3 \times 6$ | -1.2022(52) |
| $48^3 \times 8$ | -1.209(33) |
| $64^3 \times 8$ | -1.255(37) |
| $64^3 \times 12$ | -1.16(61) |
| $96^3 \times 12$ | -1.204(53) |
Separation of the hot and cold phases

- We identify the phase by the Polyakov loop.
- Two peaks in the histogram.
- Flip-flops between two phases.
- Mixed configurations are rare. (We omit mixed configurations.)
Vanishing pressure gap $\Delta p = 0$

$$\frac{\Delta p}{T^4} = N_t^4 \left\{ \frac{\partial \beta_s}{\partial \xi} ((P_s)_{\text{hot}} - (P_s)_{\text{cold}}) + \frac{\partial \beta_t}{\partial \xi} ((P_t)_{\text{hot}} - (P_t)_{\text{cold}}) \right\} = 0$$

Condition for $\Delta p = 0$

$$\frac{\partial \beta_s}{\partial \xi} = r_t = - \frac{(P_t)_{\text{hot}} - (P_t)_{\text{cold}}}{(P_s)_{\text{hot}} - (P_s)_{\text{cold}}}$$

| lattice    | $r_t$     | $\frac{(P_t)_{\text{hot}} - (P_t)_{\text{cold}}}{(P_s)_{\text{hot}} - (P_s)_{\text{cold}}}$ |
|------------|-----------|-----------------------------------------------------------------------------------|
| $48^3 \times 6$ | -1.2020(39) | 1.216(50)                                                                         |
| $64^3 \times 6$ | -1.2022(52) | 1.2053(38)                                                                         |
| $48^3 \times 8$ | -1.209(33) | 1.204(14)                                                                          |
| $64^3 \times 8$ | -1.255(37) | 1.2344(66)                                                                         |
| $64^3 \times 12$ | -1.16(61) | 1.324(84)                                                                           |
| $96^3 \times 12$ | -1.204(53) | 1.283(53)                                                                           |

The pressure gap is zero on each finite lattice.
Volume-dependence of $\Delta \varepsilon$

Because the correlation length is finite at the first order transition, $\Delta \varepsilon$ should be constant for large volume (> correlation length). We did a constant fit as a function of $N_s/N_t$ for each $N_t$. 
Continuum extrapolation of the latent heat

Fit the data at $N_t = 6, 8, 12$ with a linear function of $1/N_t^2$ assuming $O(a^2)$ error.

\[
\frac{\Delta \epsilon}{T^4} = 0.75 \pm 0.17
\]

\[
\frac{\Delta \epsilon - 3 \Delta \rho}{T^4} = 0.623 \pm 0.056
\]
EoS by the Gradient Flow (H. Suzuki, 2013)

[H. Suzuki, Prog. Theor. Exp. Phys. 2014, 083B03 (2013); FlowQCD, Phys. Rev. D90, 011501(2014)]

• Smeared field strength: \( F_{\mu\nu} \xrightarrow{\text{Gradient Flow}} G_{\mu\nu} \)

• Dim. 4 operators:
  \[
  E(t, x) = \frac{1}{4} \delta_{\mu\nu} G_{\rho\sigma}(t, x) G_{\rho\sigma}(t, x) \\
  U_{\mu\nu}(t, x) = G_{\mu\rho}(t, x) G_{\nu\rho}(t, x) - \frac{1}{4} \delta_{\mu\nu} G_{\rho\sigma}(t, x) G_{\rho\sigma}(t, x)
  \]

• Energy momentum tensor
  \[
  T_{\mu\nu}^R = \lim_{t \to 0} \left\{ \frac{1}{\alpha_U(t)} U_{\mu\nu}(t, x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} \left[ E(t, x) - \langle E(t, x) \rangle_0 \right] \right\}
  \]
  \[
  \alpha_U(t) = g^2 [1 + 2b_0 s_1 g^2 + \cdots] \\
  \alpha_E(t) = \frac{1}{2b_0} [1 + 2b_0 s_2 g^2 + \cdots]
  \]
  \( g \): running coupling constant with \( \overline{\text{MS}} \) scheme based on the 4-loop beta function.

• Energy density and Pressure
  \[
  \epsilon = \langle T_{00} \rangle \\
  p = \frac{1}{3} \sum_i \langle T_{ii} \rangle
  \]
EoS by gradient flow method (preliminary)

- $\Delta \varepsilon / T^4$ becomes constant at long flow time.
- As flow time proceeds, $\Delta \rho$ vanishes. ($\Delta \varepsilon - 3\Delta \rho \approx \Delta \varepsilon + \Delta \rho$)

$G_{\mu \nu}^2$ defined by plaquette

$G_{\mu \nu}^2$ defined by clover tem
EoS by the gradient flow method (preliminary)

• In the $t=0$ limit, the results by the gradient flow method and the derivative method are consistent within the error.

are the results by the derivative method on $96^3 \times 12$ lattice.
Conclusions and Outlook

• We study the equation of state at the first order phase transition of SU(3) gauge theory.

• Gaps of energy density and pressure are measured using the derivative method.
  • We confirmed that the pressure gap is zero on each finite lattice.
  • The result of the latent heat in the continuum limit is
    \[ \frac{\Delta \varepsilon}{T^4} = 0.75 \pm 0.17 \quad \frac{\Delta (\varepsilon - 3p)}{T^4} = 0.623 \pm 0.056 \]

• We tested the gradient flow method for the calculation of EoS.
  • As flow time increases, \( \Delta \varepsilon / T^4 \) becomes constant and \( \Delta p \) vanishes.
  • In the \( t=0 \) limit, the latent heat is consistent with that by the derivative method on the finite lattice.