Total internal reflection of orbital angular momentum beams

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Abstract

We investigate how beams with orbital angular momentum (OAM) behave under total internal reflection. This is studied in two complementary experiments: in the first experiment, we study geometric shifts of OAM beams upon total internal reflection (Goos–Hänchen and Imbert–Fedorov shifts, for each the spatial and angular variant), and in the second experiment we determine changes in the OAM mode spectrum of a beam, again upon total internal reflection. As a result we find that, in the first case, the shifts are independent of OAM and beam focusing, while in the second case, modifications in the OAM spectrum occur which depend on the input OAM mode as well as on the beam focusing. This is investigated by experiment and theory. We also show how the two methods, beam shifts on the one hand, and OAM spectrum changes on the other, are related theoretically.

Keywords: orbital angular momentum, beam shifts

(Some figures may appear in colour only in the online journal)

1. Introduction

Simple optical reflection at planar interfaces still offers new surprises. More than 60 years ago, Goos and Hänchen found that a realistic optical beam experiences an in-plane displacement, with respect to the geometric optics expected path, under total internal reflection (TIR) \cite{1}. This is caused by the fact that each of the plane-wave components, which constitute the beam, picks up a slightly different reflection coefficient. In other words, even if a beam has a flat wavefront (which is approximately true for well-collimated beams), beams are not plane waves and have a finite wavevector spread that needs to be taken into account during reflection. Nevertheless, beams are the best approximation of a geometric-optics ray, and it turns out that beam shifts are the first-order diffractive corrections to geometric optics. Beam shifts are polarization dependent. In-plane Goos–Hänchen shifts appear for \textit{p} and \textit{s} linear polarization while the (transverse) Imbert–Fedorov shifts appear for circular polarization \cite{2, 3}. In recent years, the study of beam shift phenomena has received a strong boost from (amongst other effects) the discovery of the spin-Hall effect of light \cite{4–6} and angular shifts \cite{7–9}. In the meantime, shifts have been discovered not only for light, but also for matter waves \cite{10, 11}.

Beam shifts occurring under partial (external) reflection, such as angular beam shifts \cite{9}, are known to be sensitive to the spatial structure of the beam \cite{12}. In particular, these shifts are sensitive to the beam’s orbital angular momentum \cite{13}, which appears, for instance, in Laguerre–Gauss (LG) laser modes. We have recently shown that one can describe the diffractive corrections appearing upon reflection also as a change in the transverse mode spectrum of the beam; notably a pure LG input mode was found to acquire sidebands \cite{14} upon external reflection. These results were obtained by external reflection of the input beam. How does this change
if we investigate total internal reflection? Specifically, how does the orbital angular momentum influence beam shifts, and how is the OAM spectrum modified by TIR? This has not been studied yet and we shed light on this by theory and experiment.

2. Shifts of beams with OAM under total internal reflection

We start with the general case of optical reflection, i.e. we do not yet specialize to the TIR case. Consider an incoming paraxial, monochromatic and homogeneously polarized ($\lambda = 1, 2 \equiv p, s$), but otherwise arbitrary, optical field $U^i(x, y, z) = \sum a_i U_i(x, y, z)$ propagating along $\hat{x}_i^l$ (z coordinate), where $(a_1, a_2)$ is the polarization Jones vector of the incoming beam. We use dimensionless quantities in units of $1/\hbar\omega_0$, where $\hbar=\lambda$ is the vector. The coordinate systems and their unit vectors $\hat{x}_i^l$ are attached to the incoming (i) and reflected (r) beam, respectively. After reflection at a dielectric interface, the polarization and spatial degree of freedom are coupled by the Fresnel reflection coefficients $r_{p,s}$ as [12]

$$U^r(x, y, z) = \sum a_i r_i U_i^r(x, y, z) = \sum a_i r_i U_i^r,$$

(1)

$X_i$ and $Y_i$ are the polarization-dependent dimensionless beam shifts ($\theta$ is the angle of incidence):

$$X_i = -i a_i [\ln r_i(\theta)], \quad Y_i = i a_i \left(1 + \frac{r_i}{r_i^*}\right) \cot \theta$$

(2a)

and

$$X_2 = -i a_2 [\ln r_2(\theta)], \quad Y_2 = -i a_1 \left(1 + \frac{r_1}{r_2^*}\right) \cot \theta.$$  

(2b)

Their real parts yield the spatial beam shifts, and their imaginary parts the angular beam shifts. They can appear as longitudinal Goos–Hänchen type shifts [1] $X_\lambda$ (along $\hat{x}$ coordinate), or as transverse Imbert–Fedorov type shifts [3, 4] $Y_\lambda$ along $\hat{y}$. Transverse shifts $Y_\lambda$ require that both $a_1$ and $a_2$ are finite, such as present in circularly polarized light; this is not necessary for the longitudinal shifts $X_\lambda$. To make the step from these dimensionless shifts to observable shifts, we have to find the centroid of the reflected intensity $I(z)$ at position $z$ from the beam waist,

$$I(z) = \sum a_i a_i^* |U_i(z)|^2$$

(3)

where $a_i = |r_i a_i^2|/|\sum a_i a_i^*|^2$ is the fraction of the reflected intensity with polarization $\lambda$, and $R = x\hat{x}_1^r + y\hat{x}_2^r$. The shift of the centroid depends on the structure of the field, while the dimensionless beam shifts (equation (2)) are independent of the exact form of $U$. Equation (3) can be calculated straightforwardly by first-order Taylor expansion around zero shift ($X_\lambda = Y_\lambda = 0$). We obtain for the centroid, which is the expectation value for the 2D position vector $R$ at distance $z$

$$\langle R \rangle(z) = \sum a_i a_i^* \int_{-\infty}^{\infty} U(-x + X_i, y - Y_i, z) |z|^2 dx dy$$

(4)

where $\bar{M} = \langle R \rangle(z)$ is the polarization-independent 2D position vector of the reflected beam position. The 2D position vector $R$ at distance $z$

$$\bar{M}(z) = 2 \frac{\int U^r(x, y) U^r* dy dx}{\int U^r* U dx dy}$$.  

(5)

with $i,j = 1, 2, x_1 \equiv x$, and $x_2 \equiv y$. The $z$-dependent diagonal elements of $\bar{M}(z)$ describe how angular shifts influence the apparent position $\langle R \rangle(z)$, and the off-diagonal elements effectively mix transverse angular shift into the longitudinal spatial shift, and the longitudinal angular shift into the transverse spatial shift. Evaluating (equation (3)) for Laguerre–Gauss beams with $p = 0$ and an OAM of $\ell \hbar$, and using the dimensionless Rayleigh range $\Lambda = 2/\theta_0^2$, we obtain [12]:

$$\bar{M}(z) = \sum a_i a_i^* \int_{-\infty}^{\infty} U(x, y) U^* dx dy$$

(6)

Finally, we specialize to the case of total internal reflection, where the Fresnel coefficients are imaginary. Hence, the dimensionless beam shifts $X_\lambda$ and $Y_\lambda$ (equations (2)) become real, which means that mixing via $\bar{M}(z)$ does not occur; therefore the occurring shifts are expected to be of purely spatial nature and independent of the orbital angular momentum $\ell$.

This brings us to our first experiment as shown in figure 1, where we investigate if the OAM mode influences spatial Goos–Hänchen and Imbert–Fedorov shifts under TIR, and if angular shifts disappear as expected. The OAM beam is prepared with a custom HeNe laser and mode conversion. In the laser cavity, a wire (40 µm diameter) is introduced to enforce a Hermite–Gaussian (HG$_{nm}$) fundamental mode with $m = 0$. This mode is sent through an astigmatic mode converter.
were accurate: under TIR, only spatial shifts, here in Figure 2 demonstrates that our theoretical expectations detector), and lock-in techniques (for details, see [12]). polarization-differential beam displacement using a quadrant detector (which is binned to act effectively as a split converter [13] consisting of two cylindrical lenses. The nodal line of the HG mode is oriented at 45° relative to the common axis of the mode converter, such that a HG mode is transformed into an LG mode with 0 = n and p = 0. Additional lenses (L1 and L2) are used to ensure mode matching; the final beam after L2 has a beam waist of 775 µm. After polarization modulation with a liquid-crystal variable matching; the common axis of the mode converter, such that a HG mode is oriented at 45° line of the HG mode is oriented at 45° BK7 prism. We measure the polarization-differential beam displacement using a quadrant detector (which is binned to act effectively as a split detector), and lock-in techniques (for details, see [12]). Figure 2 demonstrates that our theoretical expectations were accurate: under TIR, only spatial shifts, here in the polarization-differential form $\Delta_{\text{GH}} \equiv \text{Re}[X_1 - X_2]$ and $\Delta_{\text{IF}} \equiv \text{Re}[Y_1 - Y_2]$, occur, and are independent of the OAM $\ell$. The angular shifts $\Theta_{\text{GH}} \equiv \text{Im}[X_1 - X_2]$ and $\Theta_{\text{IF}} \equiv \text{Im}[Y_1 - Y_2]$ are identically zero. The shifts are also independent of the collimation properties of the beam as determined by the beam opening angle $\theta_0$ (data not shown), which we expect from the discussion above.

3. Appearance of OAM sidebands under TIR

We now come to the second experiment, which addresses changes in the OAM spectrum upon total internal reflection, following [14]. We start by now evaluating equation (1) directly. A first-order Taylor expansion around zero shift ($X_\lambda = Y_\lambda = 0$) results in

$$U(-x + X_\lambda, y - Y_\lambda, z) \simeq U(-x, y, z) + \mathbf{R}_\lambda \cdot \frac{\partial}{\partial \mathbf{R}} U(-x, y, z),$$

with $\mathbf{R} = (x, y)$ and $\mathbf{R}_\lambda = (X_\lambda, Y_\lambda)$. We substitute $U$ for the well-known normalized Laguerre–Gauss functions, $U \rightarrow U_{p,\ell} \rightarrow \text{LG}_{p,\ell}$, where $\ell$ and $p$ are the azimuthal and radial-mode indices, respectively. The spatial Fresnel coefficients $c_{p,\ell,p',\ell'}$ describe the scattering amplitude for an incoming LG$_{p,\ell}$ mode into the LG$_{p',\ell'}$ output channel. These coefficients are obtained by OAM decomposition of the shifted reflected beam from equation (7)

$$c_{p,\ell,p',\ell'} = \int d^2R \text{LG}_{p,\ell}(\mathbf{R})U_{p,\ell}(-x + X_\lambda, y - Y_\lambda, z).$$

If we consider only the OAM part of the Laguerre–Gauss modes (by setting $p = p' = 0$), we find the following simple coefficients (upper and lower signs refer to the case $\ell \geq 0$, and $\ell < 0$, respectively):

$$c_{p,\ell,p',\ell'} = \begin{cases} \pm Z_\ell^Z \sqrt{|\ell + 1|} & \text{for } \ell' = -\ell \mp 1 \\ \pm Z_\ell^Z \sqrt{|\ell|} & \text{for } \ell' = -\ell \pm 1 \\ (-1)^\ell & \text{for } \ell' = -\ell \\ 0 & \text{otherwise}. \end{cases}$$

We see that (in our first-order approximation, see equation (7)) these coefficients couple ‘neighboring’ OAM modes with $\ell' = -\ell \pm 1$, where the minus sign stems from image reversal upon reflection. In other words, the OAM spectrum acquires sidebands upon reflection. The complex-valued parameters

$$Z_\ell^Z = \frac{\theta_0}{2^{3/2}} (-1)^\ell (X_\lambda \pm i Y_\lambda)$$

combine all dimensionless shifts. The intensity which appears in a specific sideband after reflection is $C_{p,\ell}^2 = |c_{p,\ell}|^2$; it depends on the strength of the shifts via equation (10), further it is proportional to $\ell$ and to the square of the beam opening angle $\theta_0^2$. In contrast to the previous case of beam shifts (equation (6)), the OAM sideband method does not discriminate real (spatial) and imaginary (angular) components of the dimensionless shifts $X_\lambda$ and $Y_\lambda$, which is surprising.

Figure 2. Experimental beam shifts for total internal reflection: we see that the spatial Goos–Hänchen shift $\Delta_{\text{GH}}$ (a) and Imbert–Fedorov shift $\Delta_{\text{IF}}$ (b) are present, but independent of the orbital angular momentum $\ell$ of the beam. Angular Goos–Hänchen shift $\Theta_{\text{GH}}$ (c) and angular Imbert–Fedorov shift $\Theta_{\text{IF}}$ (d) shifts do not occur. Shown are the polarization-differential beam shifts (the relative shift between $p$ and $s$ polarization) as a function of the angle of incidence $\theta$ in real, non-dimensionless quantities as measured in the lab.
To demonstrate experimentally the appearance of OAM sidebands under total internal reflection, we check these dependences by varying $\ell$ and $\theta_0$. The setup is shown in figure 3, where we use again total internal reflection in a 45°–90°–45° prism (BK7). We collimate a fiber-coupled 635 nm diode laser with a 20× objective, then use a spatial light modulator to imprint the incoming beam helical phase $\exp(i\ell\phi)$. This method produces, in terms of Laguerre–Gauss modes, a superposition of modes with the same azimuthal index $\ell$, but with many radial modes of different $\ell'$; this superposition depends on the magnitude of $\ell$. This OAM beam is then reflected internally at the prism hypotenuse. We analyze the reflected beam by its OAM spectrum with another combination of SLM and a single-mode fiber, which in turn is connected to a photo-diode. We align the setup for best mode matching between the single-mode fibers of the laser and the detector for the case of $\ell = \ell' = 0$. To vary $\theta_0$, we introduce two microscope objectives (10×, 0.25 NA, underfilled aperture). To compensate for small residual alignment errors, we use polarization modulation by $\lambda/2$ wave plates on rotation stages, and determine the total polarization-differential OAM sideband intensity $I_{\text{pd}}(\ell)$, see [14]. To obtain a theoretical prediction, we use numerical modeling of the experiment: this is required because (i) our setup has a transmission which depends strongly on the selected OAM mode (i.e., transmission for $\ell' = \ell$, see supplementary information [14]); and (ii) the radial-mode superposition as produced by the SLMs has to be taken into account. In any case, the OAM part of the modes is well defined throughout our setup.

In figure 4(a) we compare the OAM sideband intensity from experiment and theory at an angle of 45°, for a collimated ($\theta_0 = 0.0002$) and a focused ($\theta_0 = 0.05$) beam. We can confirm that the sideband intensity depends on the input OAM $\ell$ and on the beam opening angle $\theta_0$, and agrees with the simulated data. Figure 4(b) shows the theoretically calculated total sideband intensity for an incoming $\ell = 4$ OAM beam over a larger range of incident angles. In the case of a focused beam ($\theta_0 = 0.05$), the sideband intensity exceeds 1% for the $s$ polarization over the whole range of total internal reflection. This has to be taken into account if total internal reflection is used in, e.g., beam steering applications.

4. Discussion: OAM beam shifts and sidebands

We compare now our two experiments: in the first case, we found that in total internal reflection, beam shifts are independent of the OAM of the beam, and independent of beam focusing. However, if we measure the OAM sidebands appearing during total reflection, both properties influence the result (i.e., the sideband strength). As is obvious from the similarity of the experiments, both effects describe the same underlying physical phenomenon: diffractive corrections to geometric optics for total internal reflection. We want to explore this relation briefly theoretically. We start by writing the spatial part (for polarization $\lambda$) of the reflected field in a quantum-like notation based on the spatial Fresnel coefficients.
orthogonality relations of the LG modes, and by recognizing
Equation (12) can easily be evaluated by using the known
p
the sidebands for pure azimuthal LG input modes with
To be able to compare this to equation (6), we need to find

follows:

The real space representation of this leads to the centroid as

\[
\langle R \rangle (z) = \frac{\langle \text{out} | R | \text{out} \rangle}{\langle \text{out} | \text{out} \rangle}.
\]

To be able to compare this to equation (6), we need to find
the sidebands for pure azimuthal LG input modes with
Equation (12) can easily be evaluated by using the known
orthogonality relations of the LG modes, and by recognizing
that the position operator \( R \) can be written as

\[
R = r (\hat{x}_1 \cos \phi + \hat{x}_2 \sin \phi)
\]

\[
= \frac{r}{2} [e^{i\phi} (\hat{x}_1 - i\hat{x}_2) + e^{-i\phi} (\hat{x}_1 + i\hat{x}_2)].
\]

We see that this operator couples modes with \( \Delta \ell = \pm 1 \) in
\( \langle \text{out} | R | \text{out} \rangle \). Straightforward evaluation of equation (12) leads
then exactly to equation (6), i.e., the dependency of \( \ell \) and \( \theta_0 \)
(which was implicit in the coefficients \( c_{\ell', \ell', p', p} \)) disappears for
total internal reflection.

Can we give an intuitive explanation why \( \theta_0 \)-dependent
OAM sidebands occur for TIR, while angular beam shifts
are absent? In short, the measurement method is different.
Spatial beam shifts are absolute, they do not depend on the
beam waist. The OAM spectrum of a displaced beam, however,
depends on the ratio of the displacement to the beam
waist [16, 17]. If the beam is collimated, the Goos–Hänchen
and Imbert–Fedorov shifts are usually very small compared to
the beam waist, and the OAM spectrum is not modified; if the
beam is focused, appreciable sidebands appear, in agreement
with our experimental results in figure 4(a).

In conclusion, we have studied the behavior of OAM beams
under total internal reflection. We have investigated
this by two complementary methods: firstly, by the analysis
of optical beam shifts, and secondly by the observation of
modifications in the OAM spectrum of a probe beam by the
spatial Fresnel coefficients. In the first case, we found that
OAM does not modify the spatial beam shifts under total
reflection. In the other case, the opposite is true: the OAM
spectrum of a beam is modified under TIR, and the strength
of these modifications increase with the OAM of the beam as
well as its focusing. To resolve this issue, we have shown how
to derive the beam shifts from the spatial Fresnel coefficients
in the short didactical discussion at the top of this section.

Very recently, a third method to study such diffractive
corrections was found: it turns out that physical reflection
induces a splitting of a high-order vortex into spatially
separated first-order vortices, and the splitting is characteristic
for a given experimental condition [18]. Based upon our
experimental result here, we would expect that such splitting
will occur also under TIR: vortex splitting can be explained
by coherent background fields [19], and the OAM sidebands,
as found here, are of course an example of such a coherent
background field.

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