Primordial Black Holes and the String Swampland

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The “swampland conjectures” have been recently suggested as a set of criteria to assess if effective field theories (EFTs) are consistent with a quantum gravity embedding. Such criteria, which restrict the behavior of scalar fields in the theory, have strong implications for cosmology in the early universe. As we demonstrate, they will also have direct consequences for formation of primordial black holes (PBHs) and dark matter (DM).

I. INTRODUCTION

Primordial black holes can form in the early universe and can account for all or part of the dark matter (e.g. [1–17]). They have also been linked to a variety of topics in astronomy, including the recently discovered [18–20] gravitational waves [21–28], formation of supermassive black holes [22–29, 30], cosmic infrared background fluctuations [31] as well as r-process nucleosynthesis [32], gamma-ray bursts and micro-quasars [33] from compact star disruptions.

The vast “landscape” of string theory vacua is believed to result in EFTs consistent with quantum gravity. On the other hand, the “swampland” contains EFTs for which this is not the case [34]. Recently, two conditions have been proposed, the so-called “swampland conjectures”, to discriminate between these two classes:

- \textit{SC1} [34]: scalar field excursion, measured in Planck units in the field space, is bounded from above
  \[ |\Delta \phi| \lesssim d \sim \mathcal{O}(1) \]  

- \textit{SC2} [34]: the gradient of the potential of a canonically normalized scalar field satisfies
  \[ \frac{|V'|}{V} \gtrsim c \sim \mathcal{O}(1) \]  

Here, \( c, d \) are constants of order unity. We take the Planck mass to be \( M_{\text{pl}} = 2.4 \times 10^{18} \text{ GeV} \) = 1 throughout. As discussed in [37], the above criteria have profound implications for the early universe cosmology, as follows. The general features of inflationary physics can be parameterized by the slow-roll parameters \( \epsilon, \eta \), which in terms of the scalar inflaton potential are given by [38]

\[ \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 , \quad \eta = \left( \frac{V''}{V} \right) . \]  

For a successful period of inflation one requires \( \epsilon, |\eta| \ll 1 \).

The first slow-roll parameter \( \epsilon \) is related to the elapsed number of expansion e-folds \( N \), with \( dN = H dt \) and \( H \) denoting the Hubble parameter, as \( |d\phi/dN| = \sqrt{2\epsilon} \). Taking that inflation has lasted at least 60 e-folds to address the problems with the Big Bang cosmology, one obtains \( 60 < d/c, \) which is in mild tension with \( d,c \sim \mathcal{O}(1) \). The tensor-to-scalar ratio \( r = 16\epsilon \), constrained by the cosmic microwave background (CMB) B-modes as \( r < 0.07 \) at the comoving wave-number pivot scale of \( k_0 = 0.05 \text{ Mpc}^{-1} \) by the Planck-2018 satellite data [39], leads to \( \Delta \phi \lesssim 6 \), which approaches the bound implied by SC1. The precise values of \( c,d \) depend on the details of string compactification and can deviate from strict unity [40]. As has also been noted in [37] and further explored in e.g. [41, 42], the swampland conjectures will also have strong implications for dark energy.

PBHs form when density fluctuations are comparable to \( \mathcal{O}(1) \) at the horizon crossing. Hence, for PBHs to constitute dark matter, one requires a large amplification of the inflationary power spectrum between the cosmic microwave background (CMB) and the PBH mass scales. As we demonstrate, the swampland criteria have direct consequences for formation of PBHs and dark matter.

II. PRIMORDIAL BLACK HOLE FORMATION

The power spectrum of primordial curvature perturbations is given by (e.g. [43])

\[ \Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} = A_s \left( \frac{k}{k_0} \right)^{n_s - 1} , \]  

where \( A_s = (2.105 \pm 0.030) \times 10^{-9} \) is the scalar power spectrum amplitude and \( n_s = 0.9665 \pm 0.0038 \) is the scalar spectral index, evaluated from the Planck-2018 measurements at \( k_0 \) [50]. The PBH mass is defined to be \( M = \gamma M_H \), where \( \gamma \) is an \( \mathcal{O}(1) \) parameter specify-
ing efficiency of overdensity collapse to a black hole and 
$M_H = 1/2GH$ is the horizon mass. The corresponding 
scale $k_M = a_H H = a_{\text{exit}} H_{\text{inf}}$ has exited $N$ e-folds after the 
CMB scale $k_0 = a_0 H_{\text{inf}}$, where $H_{\text{inf}}$ is the Hubble 
parameter value during inflation. Taking $H_{\text{inf}} \approx \text{const}$, 
one obtains 44

$$N = 18.4 - \frac{1}{12} \log \left( \frac{g_s}{g_{*0}} \right) + \frac{1}{2} \log \gamma - \frac{1}{2} \log \left( \frac{M}{M_\odot} \right), \quad (5)$$

where $g_s$ denotes the effective degrees of freedom in the 
energy density, with $g_{*0} = 3.36$ being their number today.

For PBHs to constitute dark matter, the minimal mass 
that is necessary in order to survive Hawking evaporation 
to the present day is given by

$$M_{\text{min}} = 1.5 \times 10^{-21} \left( \frac{\Omega_m h^2}{0.14} \right)^{-2/3} M_\odot, \quad (6)$$

where $\Omega_m h^2 = 0.14240 \pm 0.00087$ from Planck-2018 43. 
Hence, this scale left the horizon $N \simeq 42$ e-folds after the 
CMB, where in Eq. 5 the values of $g_s = 106.75$ as 
in the Standard Model and $\gamma = 1$ have been assumed, 
conservatively.

Starting from the usual Press-Schechter formalism 45 
for PBH formation during the radiation-dominated era, 
for all of the DM to reside in PBHs of mass $M > M_{\text{min}}$ 
one needs 44

$$\Delta^2(M_{\text{min}}) \simeq 2.1 \times 10^{-2}. \quad (7)$$

Modification of $\Delta^2(M_{\text{min}})$ by an order of magnitude, as 
suggested by the recent analysis of 46, will not have a 
drastic effect on our conclusions.

At the leading order in the slow-roll, the curvature and 
tensor perturbations, respectively, are given by

$$\Delta^2 \simeq \frac{H^2_{\text{inf}}}{8\pi^2 \epsilon}, \quad \Delta^2 \simeq \frac{H^2_{\text{inf}}}{\pi^2}. \quad (8)$$

Using the observed value of $\Delta^2(k_0) \simeq 2.1 \times 10^{-9}$ 47, the 
tensor-to-scalar ratio can be parametrized as

$$r = \frac{\Delta^2}{\Delta^2_\epsilon} \simeq 9.6 \times 10^7 H^2_{\text{inf}}. \quad (9)$$

Eliminating $H_{\text{inf}}$ from Eq. 8 and substituting the required 
perturbation amplification for PBHs, as given by 
Eq. 4, we obtain

$$\epsilon = 6.3 \times 10^{-9} r. \quad (10)$$

Hence, taken together with the constraint from Planck- 
2018 of $r < 0.07$ 47, PBH formation consistent with the 
CMB measurements restricts the first slow-roll parameter 
$\epsilon$ to be

$$\epsilon < 4.4 \times 10^{-10}. \quad (11)$$

The required amplification for PBHs to constitute DM 
also leads to $O(1)$ violation of the slow-roll parameter 
combination, irrespective of the inflationary model 
details. Namely, given the required amplification of curvature 
perturbations to form PBHs for DM over $N = 42$ 
e-folds after the CMB, using Eq. 44, one obtains 44

$$\left| \frac{\Delta \log \epsilon}{\Delta N} \right| > 0.4. \quad (12)$$

Since the horizon-flow equations 48, 49 give

$$\frac{d \log \epsilon}{dN} = 2 \left[ \frac{V'}{V} \right] \frac{V''}{V} = 4\epsilon - 2\eta, \quad (13)$$

from Eq. 12 we have

$$|2\epsilon - \eta| > 0.2. \quad (14)$$

Together with Eq. 11, this can viewed as a restriction on 
the second slow-roll parameter $\eta$ for PBH DM, consistent 
with CMB observations.

As discussed, significant number of long-lived PBHs re-
quire power enhancement on smaller scales, correspond-
ing to large wave-number $k$. This demands that the spectral 
index is running and is “blue-tilted”, with $n_s > 1$ at 
relevant scales (e.g. 50, 51). In terms of the slow-roll 
parameters, this translates to

$$n_s - 1 = 2\eta - 6\epsilon > 0. \quad (15)$$

Here we comment on the validity of Eq. 3. In deriving 
Eq. 3 we have used the slow-roll approximation, which 
may not be applicable, as suggested by Eq. 12. In fact, 
44 shows that naive use of Eq. 3 leads to some errors 
in PBH formation models. However, the errors are not 
so large as to affect our argument.

We note, in passing, that PBHs can also form in 
matter-dominated era (e.g. 53), which requires that the 
collapsing regions are sufficiently spherically symmetric.

### III. SWAMPLAND RESTRICTION

From the first slow-roll parameter $\epsilon$, combining $\epsilon \gtrsim c^2/2$ from $SC2$ and Eq. 11 for PBH formation, one obtains

$$\epsilon \lesssim 3.0 \times 10^{-5}. \quad (16)$$

The swampland conjectures will also constrain the sec-
ond slow-roll parameter $\eta$. Since PBH formation implies 
that the spectrum is blue-tilted at the relevant scales, we 
restrict ourselves to $V'' > 0$ potential, resulting in $\eta > 0$. 
Then, $SC2$ leads to 52

$$\eta \gtrsim c^2. \quad (17)$$

From the blue-tilted spectrum requirement of Eq. 15, 
combined with $SC1$, one obtains a slightly stronger re-
The proposed criteria themselves, see [55, 60] for potential suggestions.

IV. CONCLUSIONS

We have shown that the swampland conjectures, as originally proposed, are incompatible with formation of PBHs that can constitute DM in the context of single-field inflation. This highlights that placing restrictions on the behavior of the scalar fields in EFTs can have significant implications for structure formation in the early universe as well as dark matter.

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[1] Y. B. Zel’dovich and I. D. Novikov, Sov. Astron. 10, 602 (1967).
[2] S. Hawking, Mon. Not. Roy. Astron. Soc. 152, 75 (1971).
[3] B. J. Carr and S. W. Hawking, Mon. Not. Roy. Astron. Soc. 168, 399 (1974).
[4] J. Garcia-Bellido, A. D. Linde, and D. Wands, Phys. Rev. D54, 6040 (1996), arXiv:astro-ph/9602094 [astro-ph].
[5] M. Yu. Khlopov, Res. Astron. Astrophys. 10, 495 (2010) arXiv:0809.1016 [astro-ph].
[6] P. H. Frampton, M. Kawasaki, F. Takahashi, and T. T. Yanagida, JCAP 1004, 023 (2010) arXiv:1001.2308 [hep-ph].
[7] M. Kawasaki, A. Kusenko, Y. Tada, and T. T. Yanagida, Phys. Rev. D94, 083523 (2016) arXiv:1606.07631 [astro-ph.CO].
[8] E. Cotner and A. Kusenko, Phys. Rev. Lett. 119, 031103 (2017) arXiv:1612.02529 [astro-ph.CO].
[9] B. Carr, F. Kuhnel, and M. Sandstad, Phys. Rev. D94, 083504 (2016) arXiv:1607.00777 [astro-ph.CO].
[10] K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada, and T. T. Yanagida, (2016) arXiv:1611.06130 [astro-ph.CO].
[11] K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada, and T. T. Yanagida, (2017) arXiv:1701.02544 [astro-ph.CO].
[12] J. Garcia-Bellido, M. Peloso, and C. Unal, JCAP 1709, 013 (2017) arXiv:1707.02441 [astro-ph.CO].
[13] Y. Inoue and A. Kusenko, JCAP 1710, 034 (2017) arXiv:1705.00791 [astro-ph.CO].
[14] J. Georg and S. Watson, (2017), arXiv:1703.04825 [astro-ph.CO].
[15] K. Inomata, M. Kawasaki, K. Mukaida, and T. T. Yanagida, (2017), arXiv:1711.06129 [astro-ph.CO].
[16] B. Kocsis, T. Suyama, T. Tanaka, and S. Yokoyama, (2017), arXiv:1709.09007 [astro-ph.CO].
[17] K. Ando, K. Inomata, M. Kawasaki, K. Mukaida, and T. T. Yanagida, (2017), arXiv:1711.08956 [astro-ph.CO].
[18] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 116, 061102 (2016) arXiv:1602.03837 [gr-qc].
[19] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 116, 241103 (2016) arXiv:1606.04855 [gr-qc].
[20] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 118, 221101 (2017) arXiv:1706.01812 [gr-qc].
[21] T. Nakamura, M. Sasaki, T. Tanaka, and K. S. Thorne, Astrophys. J. 487, L139 (1997) arXiv:astro-ph/9708060 [astro-ph].
[22] S. Clesse and J. Garcia-Bellido, Phys. Rev. D92, 023524 (2015) arXiv:1501.07565 [astro-ph.CO].
[23] S. Bird et al., Phys. Rev. Lett. 116, 201301 (2016) arXiv:1603.00464 [astro-ph.CO].
[24] M. Raidal, V. Vaskonen, and H. Veermäe, (2017), arXiv:1707.01480 [astro-ph.CO].
[25] Yu. N. Eroshenko, (2016), arXiv:1604.04932 [astro-ph.CO].
[26] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, Phys. Rev. Lett. 117, 061101 (2016).
