The frontal Flow Analogy Research on ACP Material in Horizontal Well Annular

Xiong Qingshan\textsuperscript{a}, Li Yikun\textsuperscript{b}, Zhang Zhiquan\textsuperscript{a}, Wei Falin\textsuperscript{b} and Zhao Zhicheng\textsuperscript{a*}

\textsuperscript{a}College of Petroleum Engineering, Yangtze University, Jingzhou Hubei, 434023, China
\textsuperscript{b}PetroChina Exploration and Development Research Institute, Beijing 100083, China

Abstract

At later stages of the horizontal well production, water output restricted development effectiveness. The technique of ACP can shut off the water in horizontal well whose well completion system was slotted screen pipe completion. The rheological equations of various flow patterns fluid were unified into one general equation. Based on the N-S equation, the relationship between the velocity and the pressure drop was found and handled non-dimensionally, the pressure drop and the velocity were solved according to the boundary condition and the numerical analysis. The software whose integrated development environment is VB was developed to calculate velocity and simulate flow. The results indicate the velocity in a narrow place is low and there exists the flow nuclear in the annulus. In order to improve quality of ACP material injection, the centralizer must be adopted more.

© 2011 Published by Elsevier Ltd.
Selection and/or peer-review under responsibility of Society for Resources, Environment and Engineering

Keywords: Horizontal well; Frontal velocity distribution; Flow analogy; ACP

1. Introduction

The horizontal well has advantages of accelerating the construction productivity speed, improving oil recovery efficiency, increasing recoverable reserves, etc. However, with the comprehensive application of horizontal well development technique, the problem of water output at later stages of the horizontal well production becomes increasingly serious, and restricts horizontal well development effectiveness, as...
shown in figure 1. The above problems can be solved by the annulus chemistry plug (ACP) technique that can shut off the water in horizontal well whose well completion system is slotted screen pipe completion.

Figure 1. The water output of production formation in horizontal well

The ACP technique which can shut off the water is mainly used to the horizontal well whose well completion system is slotted screen pipe completion. The solidifiable fluid is injected into annulus between slotted casing and well wall, and form chemical packer layer. Because the packer layer is impermeable and it strength is high, it can prevent formation fluid into the hole to achieve the purpose of directly shutting off water, as shown in figure 2. It also can be injected into different places in annulus to form stemming to make plugging agent inject into formation directionally to achieve shutting off water[1], as shown in figure 3.

ACP material distribution is in osculating connection with effectiveness of water plugging. And it is also connection with frontal velocity distribution, namely, the connection that the solidifiable fluid flows in annulus. So it is necessary to study on solidifiable fluid flow in annulus.

2. Rheological model

The rheological model of ACP material is as follow:

\[
\frac{1}{\tau^m} = \frac{1}{\tau_0^m} + \frac{1}{\eta_h^m} \dot{\gamma}^n
\]  \hspace{1cm} (1)

Where \( \tau \) = shear stress; \( \tau_0 \) = yield shearing stress; \( \eta_h \) = structural viscosity; \( \dot{\gamma} \) = shear rate; \( m \) = rheological constant and \( n \) = rheological constant.

The rheological model actually includes the model of binghamien fluid, power law fluid, Casson fluid and Herschel-Bulkley fluid.

Moreover,

\[\tau = \eta \dot{\gamma}\]  \hspace{1cm} (2)
Where \( \eta \) = apparent viscosity. According to formula (1) and formula (2), the \( \eta \)-expression is as follow:

\[
\eta = \eta_0 \left( \frac{\tau}{\tau^2_0 - \tau^2_0} \right)^n
\]

(3)

3. Physical model

The physical model of eccentric annulus is shown in the Fig.4. Its corresponding geometric parameters are as follows:

![Figure 4. The geometrical parameters figure of eccentric annulus section](image)

- \( R_1 \) = The outer radius of screen pipe, namely, the inner circle radius;
- \( R_2 \) = Wellbore radius, namely, the excircle radius;
- \( c \) = Eccentricity, namely, the distance between inner circle center and excircle center. For concentric annulus, \( c = 0 \);
- \( r \) = The distance from screen pipe center to any point in annulus;
- \( R(\theta) \) = The distance from screen pipe center to shaft lining, its calculation is as follows:

\[
R(\theta) = \sqrt{R^2_2 - c^2 \sin^2 \theta + c \cos \theta}
\]

- \( \theta \) = The angle between vertical center line and radius of any point in annulus, namely, inclined angle;
- \( d \) = Annulus ratio, namely, the ratio of outer radius of screen to wellbore radius, its calculation is as follows:

\[
d = \frac{R_1}{R_2}
\]

- \( e \) = Excentricity rate, namely, the ratio of eccentricity to the difference between outer radius of screen and wellbore radius. For concentric annulus, \( e = 0 \). Its calculation is as follows:

\[
e = \frac{c}{(R_2 - R_1)}
\]
4. Mathematical model

4.1. Fluid velocity equation [2], [3], [4].

According to the N-S equation, the velocity equation of any point in annulus can be derived and it is as follow.

\[
    u(r,\theta) = \frac{\Delta p}{2L} \int_{R_i}^{R_o} \left( -r + \frac{A(\theta)}{r} \right) \frac{dr}{\eta} \tag{4}
\]

Where \( u \) = the velocity of any point in annulus; \( \Delta p \) = pressure drop; \( L \) = flow distance and \( A \) = integral constant.

According to boundary condition, When \( r = R_i \) and \( r = R(\theta) \), \( u = 0 \), and the formula (4) can be written as follow:

\[
    u(\theta) = \frac{\Delta p}{2L} \int_{R_i}^{R(\theta)} \left( -r + \frac{A(\theta)}{r} \right) \frac{dr}{\eta} = 0 \tag{5}
\]

Namely:

\[
    F(\theta) = \frac{\Delta p}{2L} \int_{R_i}^{R(\theta)} \left( -r + \frac{A(\theta)}{r} \right) \frac{dr}{\eta} = 0 \tag{6}
\]

4.2. Outflow equation

The annulus outflow is calculated as follow:

\[
    Q = \pi (R_o^2 - R_i^2) \bar{V}_{cp} = \int_0^{2\pi} \int_{R_i}^{R(\theta)} u(r,\theta) \cdot dr \cdot r d\theta \tag{7}
\]

According to formula (7), the following conclusion can be drawn:

\[
    \int_0^{2\pi} \int_{R_i}^{R(\theta)} u(r,\theta) \cdot dr \cdot r d\theta - \pi (R_o^2 - R_i^2) \bar{V}_{cp} = 0 \tag{8}
\]

5. Dimensionless process

In order to solve velocity distribution of various flow patterns expeditiously, the dimensionless process was necessary.

The dimensionless radius of screen pipe: \( \rho_1 = \frac{R_o}{R_o - R_i} = \frac{d}{1-d} \);

The dimensionless radius of borehole: \( \rho_2 = \frac{R_o}{R_o - R_i} = \frac{1}{1-d} \);

The dimensionless distance from any point in annulus to screen pipe center: \( \rho = \frac{r}{R_o - R_i} \);

The dimensionless distance from any point in shaft lining to screen pipe center:

\[
    \rho(\theta) = \frac{R(\theta)}{R_o - R_i} = \frac{\sqrt{R_o^2 - c^2 \sin^2 \theta + c \cos \theta}}{R_o - R_i} = \sqrt{\left( \frac{1}{1-d} \right)^2 - e^2 \sin^2 \theta + e \cos \theta}
\]
The dimensionless velocity at any point in annulus: \( \nu = \frac{\overline{u}}{V_{cp}} \)

Where \( \overline{V}_{cp} \) = axial average velocity in annulus,

Dimensionless shear stress: \( \tau' = \tau \left( \frac{R_s - R_i}{\eta_s V_{cp}} \right)^{m} \)

Dimensionless dynamic shear stress: \( \tau'_0 = \tau_0 \left( \frac{R_s - R_i}{\eta_s V_{cp}} \right)^{m} \)

Dimensionless apparent viscosity: \( \eta' = \frac{\tau'}{\left( \tau^{\infty} - \tau_0^{\infty} \right)^{s}} \)

Dimensionless velocity equation:

\[
\nu(\rho, \theta) = \int_{\rho}^{\rho} \overline{F} \left( -\rho + \frac{a(\theta)}{\rho} \right) \frac{d\rho}{\eta'}
\]  

(9)

Where \( \overline{F} = \frac{\Delta P}{2L} \left( \frac{R_s - R_i}{\eta_s V_{cp}} \right)^{m} (R_s - R_i) \); \( a(\theta) \) = the undetermined dimensionless integral constant from different angles.

Dimensionless boundary velocity equation is as follow:

\[
F(a(\theta)) = \int_{\rho_i}^{\rho} \overline{F} \left( -\rho + \frac{a(\theta)}{\rho} \right) \frac{d\rho}{\eta'} = 0
\]  

(10)

Dimensionless flow equation in annulus:

\[
f(\overline{P}) = \frac{1 - d}{2\pi(1 + d)} \int_{0}^{2\pi} d\theta \int_{\rho_i}^{\rho} \rho^2 \overline{F}(\rho - \frac{a(\theta)}{\rho}) \frac{d\rho}{\eta'} - 1 = 0
\]  

(11)

6. Solving velocity [5]

6.1. Solving \( a \)

Assuming \( \overline{P}_0 \) was given and \( a = a_0 + \Delta a \), if \( \Delta a \) was the infinitesimal, the \( F(a_0 + \Delta a) \) was written as follow:

\[
F(a_0 + \Delta a) = F(a_0) + \left( \frac{\partial F}{\partial a} \right)_{a_0} \Delta a
\]

and,

\[
\Delta a = \left( \frac{\partial F}{\partial a} \right)_{a_0}
\]
If $a_0$ was an approximate solution, Then $a_0 + \Delta a = a$ was much closer to the exact solution than $a_0$.

6.2. Solving $\bar{P}$

Assuming $a_0$ was given and $\bar{P} = \bar{P}_0 + \Delta \bar{P}$, if $\Delta \bar{P}$ was a the infinitesimal, then $f(\bar{P}_0 + \Delta \bar{P})$ was written as follow:

$$f(\bar{P}_0 + \Delta \bar{P}) = f(\bar{P}_0) + \left( \frac{\partial f}{\partial \bar{P}} \right)_{\bar{P}_0} \Delta \bar{P}$$

and $\Delta \bar{P}$-expression was as follow:

$$\Delta \bar{P} = \frac{f(\bar{P}_0)}{\left( \frac{\partial f}{\partial \bar{P}} \right)_{\bar{P}_0}}$$

If $\bar{P}_0$ was an approximate solution, the solution $\bar{P}_0 + \Delta \bar{P} = \bar{P}$ was much closer to the exact solution than $\bar{P}_0$.

6.3. Solving $\psi$

First, the initial $\bar{P}$-value, namely $\bar{P}_0$, was given. $\bar{P}_0$ was put into formula (12), so the corresponding $\alpha$ could be solved by iteration method. Then $\alpha$ was put into formula (11) and $\bar{P} = \bar{P}_0 + \Delta \bar{P}$ was solved. Similarly, $\bar{P} = \bar{P}_i + \Delta \bar{P}$ was put into formula (12), and the improved value $\alpha$ was solved. The improved value $\alpha$ was put into formula (11), and the better $\bar{P}$-value was solved. This process repeated until the values of $\alpha$ and $\bar{P}$ were not changed. At this time, the $\alpha_i$ and $\bar{P}_i$ were the most appropriate approximate solution. Once $\alpha_i$ and $\bar{P}_i$ were solved, according to formula (10), $\psi$ at any point could be worked out.

7. Dimensionless calculation examples

According to actual working condition, taking VB as a development platform and applying object-oriented design way, the software is developed, which is based on the physical model, mathematical model, and dimensionless process above. The boundary conditions and corresponding results are shown in figure 5.
8. Conclusion

- In eccentric annulus, the flow velocity in wide place is faster than that in narrow place.
- If dynamic shear force exists, the flow nuclear will appear in the annulus.
- In order to improve the injection effectiveness of ACP materials, more centralizers should be used to the tubular column in horizontal section as far as possible.
- The process of solving the flow velocity is the process of solving the circulation pressure drop.
- The solution of eccentric annulus is also fit for concentric annulus.
- Dimensionless process brings great convenience for calculation.

References

[1] Zhang, Z.Q., Li, Y.K., Wei, F.L., Numerical simulation research on ACP water plugging in the Horizontal Well [R]. JingZhou: Yangtze University. 1-3.

[2] Wang, H.G., Su, Y.N., (1998). Flow of Robertson-Stiff Fluids Through an Eccentric Annulus [J]. Applied Mathematics and Mechanics. 19(10): 931-939.

[3] Li, Z.M., etc. (2004). Study of the velocity and temperature profiles for the annulus flow of H-B fluid [J]. Journal Of Hydrodynamics, 1:32-37.

[4] Fan, H.H, Liu, X.S., (1993). Flowing and pressure drop of Herschel-Bulkley fluids in drilling well concentric annuli[J]. Journal of China University of Petroleum (Edition of Natural Science), 17(6): 28-34.

[5] Wang, Y.P., Zheng, X.H., Xia, B.r., Li, C.B., Li, Y.D., (2008). Studies of the Laminar Flow of Herschel-Bulkley Fluids In Pipes[J]. Drilling Fluid and Completion Fluid, 25(1): 28-32.