Abstract

Data on the nucleon spin structure suggests that the $u$ (and $d$) quark distributions in the $\Lambda$ hyperon may be polarized. If this correlation carries over into fragmentation, then the study of polarized $\Lambda$'s in the current fragmentation region of deep inelastic lepton scattering will be a sensitive probe of nucleon spin structure. $\Lambda$ production by polarized electrons from unpolarized targets can search for this correlation. If it is significant, $\Lambda$ production by unpolarized electrons from longitudinally and transversely polarized targets can probe the $u$-quark helicity and transversity distributions in the nucleon. We review what is known about quark polarization in the $\Lambda$, summarize electroproduction of polarized $\Lambda$s, and estimate the sensitivity to quark polarizations in the nucleon. We also describe polarization phenomena associated with vector meson electroproduction that can be observed in the same experimental configuration.
In the nonrelativistic quark model all the spin of the Λ resides on the $s$-quark. The $u$ and $d$ quarks are supposed to be paired to spin and isospin zero. The same model predicts that all the spin of the nucleon is carried by its quarks. Data on hyperon β-decays and deep inelastic scattering from polarized nucleons shows that the latter is not true. The latest published estimates of the spin fraction carried by quarks in the nucleon is $\Sigma(10\text{GeV}^2) = 0.2 \pm 0.1$. Applied to the Λ, the same data indicate that about 60% of the Λ spin is on $s$ (and $\bar{s}$) quarks, while $-40\%$ is on $u$ (and $\bar{u}$) and $d$ (and $\bar{d}$) quarks. These values are as reliable as the values of quark spin fractions in the nucleon.

Λ’s are unique among light hadrons in that their polarization can be easily reconstructed from the non-leptonic decay $\Lambda \rightarrow p\pi$. Other hyperons are too rare to be of much interest. Other hadrons with spin do not preserve polarization information in their decay products because the decays conserve parity. With the advent of modern deep inelastic spin physics many authors have examined the potential role of Λ’s as a probe of nucleon spin substructure. Generally these papers focus only on the $s$-quark polarization within the Λ. Since polarized $s$-quarks are relatively rare in the nucleon and their squared charge is only $1/9$, the prospect for using Λ’s to probe polarized $s$-quarks in the nucleon is not too good. On the other hand, polarized $u$-quarks are abundant in the nucleon and their squared charge is $4/9$. It is easy to see that even a small correlation between the spins of the $u$-quarks and the Λ’s into which they fragment would make them dominant over $s$-quarks and potentially useful as probes of the polarized $u$-quark distributions in the nucleon.

In this Report I first update what is known about the (integrated) quark helicity distributions in the Λ. Next, following the work of Artru and Mekhfi, I summarize the opportunities for exploring and exploiting Λ polarization in electroproduction. I use a helicity density matrix formalism which is particularly simple in the approximation that current fragments are produced at small angles in the target rest frame. In the last section of this Report I summarize this formalism and use it explain the spin-dependent effects accessible through the electroproduction of vector mesons.

My principal conclusions are as follows:

- Production of Λ’s in the current fragmentation region by a longitudinally polarized electron beam scattering off an unpolarized target has a large analyzing power for the Λ helicity difference fragmentation function $\Delta \hat{u}_\Lambda(z,Q^2)$. $10^4$ events in the current fragmentation region would be sensitive to $\Delta \hat{u}_\Lambda(z,Q^2) \approx 0.03$ for the (assumed dominant) $u$-quark. If the Λ’s are found to have significant polarization in this experiment then the $u \rightarrow \Lambda$ longitudinal spin transfer is likely to be the source.

- If the longitudinally polarized $u \rightarrow \Lambda$ fragmentation function, $\Delta \hat{u}_\Lambda(z,Q^2)$, is sizeable, then production of Λ’s in the current fragmentation region by an unpolarized electron

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1 Although $\Sigma^0$’s are common enough to generate a small depolarizing background via the decay $\Sigma^0 \rightarrow \Lambda\gamma$. A precision experiment should veto Λ’s secondary to $\Sigma^0$ decay.

2 Note that polarized $s \rightarrow \Lambda$ fragmentation functions can play a major role in $e^+e^- \rightarrow \Lambda + X$ on the $Z^0$ peak where strange quarks are both copiously produced and strongly polarized.
beam scattering off a transversely polarized target may have a significant sensitivity to the $u$-quark transversity distribution in the nucleon.

There are many if’s in this project, but the Λ’s come for free at any DIS experiment sensitive to the hadronic final state, such as the HERMES experiment now underway at DESY [10] and there are no candidate experiments now running with greater potential sensitivity to the nucleon’s tranversity.

I. THE QUARK SPIN STRUCTURE OF THE Λ

The quark distribution and fragmentation functions that figure in this analysis are defined as follows:

- $q(x,Q^2)$ is the spin average quark distribution function for flavor $q$, which contributes to $f_1(x,Q^2) = \frac{1}{2} \sum_q e_q^2 q(x,Q^2)$.
- $\Delta q(x,Q^2)$ is the helicity difference quark distribution for flavor $q$, which contributes to $g_1(x,Q^2) = \frac{1}{2} \sum_q e_q^2 \Delta q(x,Q^2)$.
- $\delta q(x,Q^2)$ is the transversity difference quark distribution, which contributes to the transversity structure function, $h_1$, best known for its role in Drell-Yan processes [11]. $\delta q$ is identical to the distribution called $\Delta T q$ by Artru.

We denote antiquark distributions as $\bar{q}$, $\Delta \bar{q}$, and $\delta \bar{q}$ respectively. The fragmentation functions with the same spin structure are denoted with a caret: $\hat{q}$, $\Delta \hat{q}$, and $\delta \hat{q}$. Quark distribution or fragmentation functions in the nucleon or Λ are denoted $q_N$ or $q_\Lambda$, respectively, and likewise for fragmentation functions. Finally, deep inelastic scattering from polarized nucleons measures the sum of quark and antiquark helicity difference distributions which we denote as follows: $\Delta Q \equiv \Delta q + \Delta \bar{q}$, when necessary.

The integrated polarized quark distributions in any octet baryon state are determined by the $F$ and $D$ constants from hyperon $\beta$-decay and the $g_1$ sum rules measured in deep inelastic scattering from nucleons. The analysis rests on the assumption of $SU(3)$ symmetry for hyperon $\beta$-decay, which gives $F + D = 1.2573 \pm 0.0028$ and $F/D = 0.575 \pm 0.016$. The errors are dominated by the $g_1$ sum rules which we summarize by the quark spin fraction, $\Sigma(10\text{Gev}^2) = 0.20 \pm 0.11$ [1]. The Λ spin fractions are given by

$$\Delta U_\Lambda = \Delta D_\Lambda = \frac{1}{3}(\Sigma - D)$$
$$\Delta S_\Lambda = \frac{1}{3}(\Sigma + 2D)$$

(1)

Naive quark model results are obtained by setting $D = 1$, $F = 2/3$, and $\Sigma = 1$. A somewhat more sophisticated estimate is obtained by combining the measured $F$ and $D$ values with the assumption of no polarized $s$-quarks in the nucleon, which we call the $\Delta S_N = 0$ model [12]. Table I summarizes the information now available on the integrated polarized quark distributions in the nucleon and the Λ. For comparison we tabulate the expected values in the naive quark model and in the $\Delta S_N = 0$ approximation. These entries are for pedagogical purposes only — the important entries come from the data.
TABLE I. Light quark spin fractions in the nucleon and \( \Lambda \) as predicted by the naive quark model, by baryon \( \beta \)-decay plus the assumption that there are no polarized \( s \)-quarks in the nucleon, and as measured.

| Distribution | Case I | Case II |
|--------------|--------|---------|
| \( \Delta u_\Lambda \) | \(-0.14 \pm 0.04\) | \(-0.09 \pm 0.03\) |
| \( \Delta d_\Lambda \) | \(-0.14 \pm 0.04\) | \(-0.09 \pm 0.03\) |
| \( \Delta s_\Lambda \) | \(0.66 \pm 0.04\) | \(0.66 \pm 0.04\) |

TABLE II. Polarized quark and antiquark content of the \( \Lambda \).

The message from Table I is that the \( u \)- (and \( d \))-quarks in the \( \Lambda \) are polarized. One might suppose, however, that this has little to do with the \( \Lambda \) structure and more to do with the sea of \( Q\bar{Q} \) pairs present in all baryons. To analyze this possibility we consider two different parameterizations of the sea quarks in the \( \Lambda \). In both cases we assume that the sea quark polarization distribution in the nucleon and \( \Lambda \) are identical. This is a crude approximation necessary to obtain a first order estimate. The failure of the Gottfried relation shows that antiquark distributions depend on valence quark content \[13\]. In Case I we go further and assume that the sea is \( SU(3) \) flavor symmetric: \( \Delta s_N = \Delta \bar{u}_\Lambda = \Delta \bar{d}_\Lambda = \Delta \bar{s}_\Lambda \), etc. In particular, \( \Delta \bar{u}_\Lambda = \Delta s_N = \frac{1}{2} \Delta S_N = -0.06 \pm 0.02 \) (Case I). Alternatively we suppose that polarized \( s \)-quarks are suppressed in the baryon sea. We choose a suppression factor of \( 1/2 \) from the neutrino data on the \( s \)-quark momentum distribution in the nucleon \[14\]. Then \( \Delta \bar{u}_\Lambda = \Delta \bar{d}_\Lambda = \Delta \bar{u}_N = \cdots = 2\Delta s_N = \Delta S_N \), or \( \Delta \bar{u}_\Lambda = \Delta S_N = -0.12 \pm 0.04 \) (Case II).

Combining the data from Table I with the two cases we arrive at estimates of the polarized quark and (separately) antiquark content of the \( \Lambda \). These are summarized in Table II. In both cases, the \( u \)-quark’s spin is (anti)correlated with the \( \Lambda \) spin. The correlation is relatively small \((-0.14 \pm 0.04 \) [Case I] or \(-0.09 \pm 0.04 \) [Case II]), but if it carries over into the polarized fragmentation function, \( \Delta \hat{u}_\Lambda \), then the \( u \)-quark will dominate polarized \( \Lambda \) production in deep inelastic scattering. To illustrate this, consider the product \( \Pi_q \equiv \Delta q_N \times e_q^2 \times \Delta \hat{q}_\Lambda \), which roughly measures the importance of the quark of flavor \( q \) in polarized \( \Lambda \) production from nucleons. Taking \( \Delta \hat{q}_\Lambda \propto \Delta q_\Lambda \), for Case I, \( \Pi_u/\Pi_s \approx 12 \) and for Case II, \( \Pi_u/\Pi_s \approx 8 \). The crucial question remains whether the correlation between the \( u \)-quark spin and the \( \Lambda \) spin persists in fragmentation. Experiment will tell.
II. POLARIZATION TRANSFER IN DEEP INELASTIC SCATTERING

The analysis of polarization transfer in deep inelastic scattering is made considerably simpler by the fact that the electron scattering angle is so small. At $E_e \approx 30\text{GeV}$, $x \approx 0.1$, and $Q^2 \approx 3\text{GeV}^2$, $\theta \approx 0.08$. Complexities in the analysis of fragment polarization turn out to be proportional to $\sin^2 \theta$ and can be ignored at fixed target facilities of interest.

In this approximation the production of $\Lambda$’s can be viewed as an essentially collinear process. For longitudinally polarized electrons and an unpolarized target, the crucial question is how effectively is the electron polarization transferred to the $\Lambda$ fragment. The answer is

$$\vec{P}_\Lambda = \hat{e}_3 P_e \frac{y(2-y)}{1 + (1-y)^2} \sum_q e_q^2 q_N(x, Q^2) \Delta \hat{q}_\Lambda(z, Q^2),$$

(2)

where, by convention, the electron beam defines the $\hat{e}_3$ axis, and $y$ is the usual DIS variable, $y \equiv (E - E')/E$.

For a polarized target and unpolarized beam the calculation is somewhat more complicated but the results simplify considerably in the $\sin \theta \approx 0$ limit. When the target is polarized along the electron beam, we find

$$\vec{P}_\Lambda = \hat{e}_3 P_N \frac{\sum_q e_q^2 \Delta q_N(x, Q^2) \Delta \hat{q}_\Lambda(z, Q^2)}{\sum_q e_q^2 q_N(x, Q^2) \hat{q}_\Lambda(z, Q^2)}$$

(3)

and when the target is polarized transverse to the electron beam, we find

$$\vec{P}_\Lambda = \vec{P}_N \frac{2(1-y) \sum_q e_q^2 \delta q_N(x, Q^2) \delta \hat{q}_\Lambda(z, Q^2)}{1 + (1-y)^2 \sum_q e_q^2 q_N(x, Q^2) \hat{q}_\Lambda(z, Q^2)},$$

(4)

as first derived by Artru and Mekhfi. Eqs. (2), (3), and (4) are the fundamental results here. They are accurate to leading twist in the small $\theta$ limit. Also, $R = \sigma_L/\sigma_T$ was set to zero in the derivation.

Returning to eq.(2) assuming $u$-quark dominance, we find

$$\vec{P}_\Lambda = \hat{e}_3 P_e \frac{y(2-y)}{1 + (1-y)^2} \Delta \hat{u}(z, Q^2) \hat{u}(z, Q^2).$$

(5)

At $E_e \approx 30\text{GeV}$, $x = 0.1$ and $Q^2 = 3\text{GeV}^2$, $y = 0.53$. With a beam polarization of 50% we find $P_\Lambda(z, Q^2) = 0.3 \Delta \hat{u}_\Lambda(z, Q^2)$. $10^4$ $\Lambda$’s in the current fragmentation regions would be sensitive to $\Delta \hat{u}_\Lambda/ \hat{u}_\Lambda$ as small as 0.03.

If existing data do show a significant longitudinal $q \to \Lambda$ spin transfer in the current fragmentation region, then the next step would be to orient the target spin transverse to an (unpolarized) electron beam and look for a transverse $\Lambda$ polarization, which would provide the first measurement of the $u$-transversity distribution in the nucleon. In the $u$-quark dominance approximation eq.(2) reduces to

$$\vec{P}_\Lambda = \vec{P}_N \frac{2(1-y) \delta u_N(x, Q^2) \delta \hat{u}_\Lambda(z, Q^2)}{1 + (1-y)^2 u_N(x, Q^2) \hat{u}_\Lambda(z, Q^2)}.$$

(6)
Neither the transverse $N \to u$ polarization transfer nor the transverse $u \to \Lambda$ polarization transfer are known. A dedicated run with transversely polarized target would open the opportunity for measurement of both of these novel quark spin distributions.

It should be noted that the polarization described in eq. (6) is leading twist. Inclusive transverse polarization phenomena in DIS are twist-three and consequently difficult to observe. The dominant, twist-two polarization distribution, $\delta q$, decouples from inclusive DIS because it is chiral-odd [5]. In eq. (6) the chiral-odd transverse fragmentation function $\delta \hat{q}$ combines with the chiral-odd transverse distribution function $\delta q$ to conserve net quark chirality [3]. So observation of the final $\Lambda$ transverse spin acts as a filter that selects a piece of the transverse spin asymmetry that does not fall like $1/\sqrt{Q^2}$. At twist three there are other processes which may provide a measure of $\delta q$. In particular, inclusive pion production from a transversely polarized nucleon is sensitive to $\delta q$ [5]. Since pions dominate the final state in the current fragmentation region it is possible that their abundance may compensate for the suppression of twist-three by $1/\sqrt{Q^2}$. In any case the pion and $\Lambda$ data could be accumulated simultaneously in a dedicated run with a transversely polarized target.

III. DENSITY MATRIX FORMALISM AND APPLICATION TO VECTOR MESON PRODUCTION

The formalism used for the calculations summarized in the previous section is simple and general. It is based on a helicity formalism that renders the parton formulation of spin dependent processes at leading twist essentially trivial. This formalism matches easily onto the helicity amplitude formalism in which perturbative QCD calculations are already known to simplify considerably [15].

At leading twist quarks are described by two-component spinor field built out of the so-called “good” light-cone fields. We use a basis of helicity eigenstates for these fields [3]. The various quark (and gluon) distribution and fragmentation functions can be related to helicity amplitudes, and transcribed as helicity density matrices. If the hard scattering cross sections are transcribed to the same basis, then the extraction of spin dependent observables reduces to multiplication of helicity matrices. Parts of this formalism were developed in Ref. [8] and a review of the underlying light-cone physics can be found in Ref. [16].

The basic ingredients are the $N \to q$ distribution function, the $q \to \Lambda$ fragmentation function and the hard partonic cross section, all as density matrices in the helicity basis. We discuss the $N \to q$ distribution function in detail first.

The distribution function $\mathcal{F}$ is a function of $x$ and $Q^2$ carries both quark ($h_1 h'_1$) and nucleon ($H' H$) helicity labels. It describes the emission of a helicity $h_1$ quark with momentum fraction $x$ by a nucleon of helicity $H$, followed by reabsorption of the quark, with helicity $h'_1$ forming a nucleon of helicity $H'$. The process is shown at the bottom of Fig. [1].

By convention all quark helicities are lower case, all baryon helicities are upper case, and

3 At next-to-leading twist, a second two-component field, built of “bad” light-cone components, enters and may be treated with similar methods.
helicity matrices carry outgoing particle indices on the left and incoming on the right. Thus this distribution function is labelled, $\mathcal{F}_{H'H,h_1h_1'}$.

\[ \mathcal{F}_{H'H,h_1h_1'} = \mathcal{F}_{-H'-h_1-h_1'} \quad \text{(parity)} \\
\mathcal{F}_{H'H,h_1h_1'} = \mathcal{F}_{HH',h_1h_1'} \quad \text{(T-reversal)} \]

Only three independent amplitudes remain,

\[ \mathcal{F}_{++,++} = \mathcal{F}_{--,--} = q + \Delta q \]
\[ \mathcal{F}_{++,--} = \mathcal{F}_{--,,} = q - \Delta q \]
\[ \mathcal{F}_{+-,-} = \mathcal{F}_{++,-} = \delta q \]

and they can be identified with the conventional quark distributions, $q(x, Q^2)$, $\Delta q(x, Q^2)$ and $\delta q(x, Q^2)$ on the basis of the known helicity structure of the distributions. To preserve clarity, the flavor, $Q^2$, $x$, and target labels on eq. (9) have been suppressed. The helicity $\pm \frac{1}{2}$ states have been denoted $\pm$ respectively.

It is now trivial to encode the information in eq. (9) in a (double) density matrix notation. By inspection [5,8],

\[ \mathcal{F} = q \ I \otimes I + \Delta q \ \hat{\sigma} \cdot \hat{e}_3 \otimes \hat{\sigma} \cdot \hat{e}_3 + \delta q \ \sum_{j=1,2} \hat{\sigma} \cdot \hat{e}_j \otimes \hat{\sigma} \cdot \hat{e}_j. \]

\[ \text{FIG. 1. Hard scattering diagram for } \Lambda \text{ production in the current fragmentation region of lepton scattering from a target nucleon. In perturbative QCD the diagram factors into the products of distribution function (lower), hard scattering (middle), and fragmentation function (upper part of diagram). Helicity density matrix labels are shown explicitly.} \]
The \( \{\sigma^k\} \) are the usual Pauli matrices. The first matrix in the direct product \( M \otimes N \) is in the nucleon helicity space, the second is in the quark helicity space. Thus \( I \otimes I \) denotes \( \delta_{H'H_1} \delta_{h_1h'_1} \). The dependence of the distributions \( (q, \text{ etc.}) \) on \( x \) and \( Q^2 \) is suppressed. The remarkably simple form of \( \mathcal{F} \) displays the analogy between longitudinal and transverse spin effects at leading twist in pQCD.

The fragmentation function is defined analogously. A quark of helicity \( h_2 \) fragments into a \( \Lambda \) of helicity \( H_1 \), followed by reabsorbing the \( \Lambda \) of helicity \( H'_1 \) to reform a quark of helicity \( h'_2 \). The process is shown at the top of Fig. [8]. The fragmentation function is a distribution differential in the momentum fraction of the observed hadron: \( [d\mathcal{D}/dz]_{h'_2, h_1, H'_1} \), and is given by [8]:

\[
\frac{d\mathcal{D}}{dz} = \frac{1}{2} \hat{q} I \otimes I + \frac{1}{2} \Delta \hat{q} \cdot \hat{\sigma} \cdot \hat{\sigma}'_3 \otimes \hat{\sigma}'_3 + \frac{1}{2} \delta \hat{q} \sum_{j=1,2} \hat{\sigma} \cdot \hat{\sigma}'_j \otimes \hat{\sigma} \cdot \hat{\sigma}'_j. \tag{11}
\]

The notation in eqs. (10) and (11) requires a further word of explanation. In general the axis along which the helicity of the quark distribution function is defined \( \langle \hat{\sigma} \rangle \) does not coincide with the axis along which the helicity of the fragmentation function is defined \( \langle \hat{\sigma}'_3 \rangle \). Thus it is important to distinguish the helicity basis vectors in the two. The hard scattering cross section must be a double density matrix with one “leg” in the unprimed basis and the other in the primed basis. When the distinction between the two can be ignored \( (\text{e.g. } \sin \theta \approx 0) \) then \( \hat{\sigma} \cdot \hat{\sigma}'_3 = \hat{\sigma} \cdot \hat{\sigma}'_3 = \sigma_3 \), etc. can be substituted to simplify eqs. (10) and (11).

The cross section for the hard QCD process of interest is obtained by combining these distribution functions with the appropriate hard parton scattering density matrix. In the case of interest here the process is forward virtual Compton scattering in which an incoming quark of momentum \( p \) and helicity \( h_1 \) absorbs a virtual photon of momentum \( q \) to become an outgoing quark of momentum \( p' = p + q \) and helicity \( h_2 \). To allow for the most general possible spin structure we must consider the process in which the conjugate amplitude has different helicities, \( h'_1 \) and \( h'_2 \). This is shown in the middle of Fig. [8]. The resulting hard density matrix is denoted \( [d\mathcal{M}/dx dy d\phi]_{h_2, h'_2, h'_1, h_1} \) where \( \phi \) is the azimuthal angle of the scattering plane.

Up to inessential kinematic factors \( [d\mathcal{M}/dx dy d\phi]_{h_2, h'_2, h'_1, h_1} \) is given by

\[
\left[ \frac{d\mathcal{M}^\pm}{dx dy d\phi} \right]_{h_2, h'_2, h'_1, h_1} = \bar{u}(p, h'_1) \gamma_\mu u(p', h'_2) \bar{u}(p', h_2) \gamma_\nu u(p, h_1) \left( k^{\mu} k'^{\nu} + k'^{\mu} k^{\nu} - g^{\mu\nu} k \cdot k' \mp i \varepsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \right), \tag{12}
\]

where \( k \) and \( k' \) are the initial and final electron momenta respectively, \( u \) and \( \bar{u} \) are Dirac spinors and the \( \pm \) sign refers to the initial electron helicity. The resulting density matrix has four terms,

\[
\frac{d\mathcal{M}^\pm}{dx dy d\phi} = A I \otimes I + B \hat{\sigma} \cdot \hat{\sigma}'_3 \otimes \hat{\sigma} \cdot \hat{\sigma}'_3 + \sum_{j, t=1,2} C_{j t} \hat{\sigma} \cdot \hat{\sigma}_j \otimes \hat{\sigma} \cdot \hat{\sigma}'_t \pm D (I \otimes \hat{\sigma} \cdot \hat{\sigma}'_3 + \hat{\sigma} \cdot \hat{\sigma}_3 \otimes I) \tag{13}\]

The coefficients \( A, B, C_{j t}, D \) are easily calculated once one has an expression for the Dirac spin density matrix in a helicity basis. Let
Using the familiar definition of the spin projection operator,
\[ \mathcal{P}(p, s) \equiv u(p, s)\bar{u}(p, s) = \frac{\gamma \cdot p + m}{2m} \frac{m + \gamma_5\gamma \cdot s}{2} \]  
(15)
and taking the \( m \rightarrow 0 \) limit carefully (maintaining \( s^2 = -m^2 \) and \( s \cdot p = 0 \)), we find
\[ U(p) = \frac{1}{2} \frac{\gamma \cdot p}{\gamma_5} \left[ I - \gamma_5 (\hat{\sigma} \cdot \hat{e}_3 + \sum_{j=1,2} \hat{\sigma}_j \cdot \hat{e}_j \hat{\sigma}_j \cdot \hat{e}_j) \right]. \]  
(16)
With the aid of eq. (16) we find
\[ A = \frac{e^4}{32\pi^2 Q^2} \frac{1 + (1 - y)^2}{2y}, \]
\[ B = A, \]
\[ D = \frac{y(2 - y)}{1 + (1 - y)^2} A, \]
\[ C_{j\ell} = \left\{ 2(1 - y) \hat{e}_j \cdot \hat{e}_\ell' + \frac{2}{Q^2} \hat{p}_j' \cdot \hat{e}_j (y \hat{k}_j \cdot \hat{e}_\ell' + y(1 - y) \hat{k} \cdot \hat{e}_\ell') \right. \]
\[ - \frac{4y^2}{Q^2} \hat{k}_j' \cdot \hat{e}_j \hat{k} \cdot \hat{e}_\ell' \} \frac{A}{1 + (1 - y)^2}. \]  
(17)
Finally, when \( \sin \theta \) can be ignored \( C_{j\ell} \) simplifies to
\[ \lim_{\sin \theta \rightarrow 0} C_{j\ell} = \frac{2(1 - y)}{1 + (1 - y)^2} \delta_{j\ell} A, \]  
(18)
which is the result quoted by Artru and Mekhfi [3].

To obtain the \( \Lambda \) production density matrix in the current fragmentation region for electrons of a given helicity incident on a target of given spin orientation, it remains only to multiply the ingredients,
\[ \frac{d\mathcal{M}^\pm}{dxdydzd\phi} = \frac{e^4}{4\pi^2 Q^2} \left\{ \frac{1 + (1 - y)^2}{2y} \sum_q e_q^2 q(x, Q^2) \hat{q}(z, Q^2) I \otimes I \right. \]
\[ + \frac{1 + (1 - y)^2}{2y} \sum_q e_q^2 \Delta q(x, Q^2) \Delta \hat{q}(z, Q^2) \sigma^3 \otimes \sigma^3 \]
\[ + \frac{2(1 - y)}{2y} \sum_q e_q^2 \delta q(x, Q^2) \delta \hat{q}(z, Q^2) \sum_{j=1,2} \sigma^j \otimes \sigma^j \]
\[ \pm \frac{2 - y}{2} \sum_q e_q^2 [q(x, Q^2) \Delta \hat{q}(z, Q^2) I \otimes \sigma^3 \]
\[ + \Delta q(x, Q^2) \hat{q}(z, Q^2) \sigma^3 \otimes I] \right\} \]  
(19)
\( \mathcal{M} \) is a matrix in the helicity space of both the nucleon and the \( \Lambda \). The first (second) matrix lies in the nucleon (\( \Lambda \)) helicity basis. Note that the \( \hat{e}_j \) and \( \hat{e}_j' \) bases have become identical in
the $\sin \theta \rightarrow 0$ limit. Given $\mathcal{M}$, the $\Lambda$ polarization is defined by $\frac{\text{Tr} \{ M \sigma \} \cdot \text{Tr} \{ M I \}}{\text{Tr} \{ M \mathcal{M} \}}$ for a given nucleon and electron helicity configuration. An elementary calculation yields the results quoted in §2.

As an exercise to illustrate the usefulness of these methods (and to prepare for the appearance of data on vector meson production in the final state of deep inelastic electron and muon scattering) I calculate the helicity information that can be extracted in production of vector mesons by polarized leptons scattering from unpolarized nucleons.

Since the decays of vector mesons such as the $\rho$, $K^*$ and $J/\psi$ conserve parity, it is impossible to measure their polarization from the decay angular distribution alone. However, the decay angular distribution of a spin-one particle depends on the absolute value of its helicity. Consider, for example, the $\rho$-meson decaying into $\pi\pi$. In the $\rho$ rest frame (with the helicity defined along the $\hat{e}_3$-axis) the helicity zero state decays with a $\cos^2 \theta$ distribution while the helicity $\pm 1$ states decay with a $\sin^2 \theta$ distribution.

When classifying the independent fragmentation helicity amplitudes of a vector meson [17], one discovers several more beyond those present for spin-$\frac{1}{2}$. The fragmentation (double) density matrix can be expressed in terms of the direct product of a $2 \times 2$ quark density matrix and a $3 \times 3$ meson density matrix with components along $(+, 0, -)$ helicity. The result mimics eq.(11) with two exceptions. (1) Time reversal allow two distinct helicity flip fragmentation functions [14]; and (2) there is a “quadrupole” fragmentation function analogous to the quadrupole distribution function called $b_q(x, Q^2)$ in Ref. [18] which measures the difference in the quark fragmentation into the helicity zero and $\pm 1$ states of the meson. It is this quadrupole fragmentation function that can be measured by scattering electrons from an unpolarized nucleon target and observing the angular distribution of the vector meson decay products. Ignoring Ji’s possible T-violating fragmentation function (which requires nontrivial final state interactions and does not contribute to scattering from unpolarized targets), the fragmentation double density matrix for vector meson production is given by

$$\frac{d\mathcal{D}}{dz} = \frac{1}{3} \hat{q} I \otimes I - \frac{1}{4} \hat{b}_q I \otimes Q + \frac{1}{2} \Delta \hat{q} \cdot \hat{e}_3 \otimes \hat{S} \cdot \hat{e}_3' + \frac{1}{2} \delta \hat{q} \sum_{j=1,2} \hat{\sigma} \cdot \hat{e}'_j \otimes \hat{S} \cdot \hat{e}'_j$$

(20)

where $\hat{b}_q$ is the quadrupole fragmentation function for a quark of flavor $q$ to fragment to the vector meson, and $Q$ is the diagonal member of the quadrupole helicity basis for spin-one, $Q = \text{diag}(1, -2, 1)$. $\hat{S}$ are the spin-matrices for a spin-one particle: $S_3 = \text{diag}(1, 0, -1)$, etc. As in the simple case of spin-1/2, the structure of $\mathcal{D}$ can be read off the definitions of the quark fragmentation functions in a helicity basis.

Combining this fragmentation density matrix with the distribution and hard scattering density matrices we find that $b_q$ can contribute to scattering of unpolarized electrons from an unpolarized target as well as scattering of polarized electrons from a polarized target. The relevant observable is the number of helicity-0 $\rho$’s minus the number of helicity-$\pm 1$ $\rho$’s divided by the sum. For the case of unpolarized leptons and unpolarized target,

$$A_T = \frac{N_0 - N_+ - N_-}{3(N_0 + N_+ + N_-)} = \frac{\sum_q e_q^2 q_N(x, Q^2) \hat{b}_q(x, Q^2) \hat{b}_q(z, Q^2)}{\sum_q e_q^2 q_N(x, Q^2) \hat{b}_q(x, Q^2) \hat{b}_q(z, Q^2)}.$$  \hspace{1cm} (21)

A similar expression describes the production of $\rho$’s by a polarized beam from a polarized target.
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