Metric and edge-metric dimensions of bobble-neighbourhood-corona graphs

Rinurwati\(^1\) and R E Nabila\(^1\)

\(^1\)Department of Mathematics, Institut Teknologi Sepuluh Nopember, Indonesia

E-mail: rinur@matematika.its.ac.id, rida.eka16@mhs.matematika.its.ac.id

Abstract. Resolving set in a graph \(G=(V(G), E(G))\) is an ordered subset \(W\) of \(V(G)\) such that every vertex in \(V(G)\) has distinct representation with respect to \(W\). Resolving set of \(G\) of minimum cardinality is called basis of \(G\). Cardinality of basis of \(G\) is called metric dimension of \(G\), \(\dim(G)\). An ordered set \(W\) is called edge resolving set of \(G\) if every edge in \(E(G)\) has distinct edge-representation with respect to \(W\). Edge-resolving sets in a graph \(G\) of minimum cardinality is called edge-metric basis of \(G\). Cardinality of edge-basis of \(G\) is called as edge-metric dimension of \(G\), \(\text{edm}(G)\). Neighbourhood corona of \(G\) and \(H, G \ast H\), is a graph obtained by taking graph \(G\) and \(|V(G)|\) graph \(H_i\), with \(H_i, i = 1, 2, ..., |V(G)|\) is copy of \(H\), then all vertices in \(H\), are connected with neighbouring vertex of vertex \(v_i\) in \(V(G)\). In this paper, we determine and analyse metric and edge-metric dimension of bobble-neighbourhood-corona, that is metric and edge-metric dimension of neighbourhood-corona of \(G\) and \(H, G \ast H\), with \(H\) is trivial graph \(K_1\), and \(G \in \{C_n, K_n\}\).

1. Introduction

The metric dimension is ne of the areas of studies in graph theory. Metric dimension of graphs first was introduced by Slater in 1975, then Harary and Melter in 1976 [1]. Metric dimension of graphs that resulted from graph operations were also discovered. For metric dimension of corona graphs, Iswadi et al. have found them in 2011 [2]. They found corona graph \(G \odot H\), where \(H\) was a trivial graph \(K_1\). One of variants of corona graphs is edge-corona graph. In 2017, Rinurwati et al. have determined metric dimension of edge-corona graph \(G \odot H\) where \(H\) is trivial graph \(K_1\) [3]. Beside metric dimension, Rinurwati et al. have also found local metric dimension [3]. For local adjacency metric dimension, Rinurwati et al. have found it for some wheel related graphs with pendant points [4]. The other variant of corona graphs is neighbourhood-corona graph. The graph was introduced by Gopalapillai [5]. The neighbourhood corona graph in [5] was studied for spectrum of the graph.

Another study about graph dimension is edge-metric dimension. Edge-metric dimension of graphs were introduced by Kelenc et al. [6]. To determine edge-metric dimension of graphs, Kelenc et al. used distance between two edges. On another research, Nasir et al. determined metric dimension of line graph of graph \(G\) first to determined edge-metric dimension of \(G\) [7]. Nasir et al. have found edge-metric dimensions of \(n\)-sunlet and prism graphs.
Hence, in this paper we result and analyse metric and edge-metric dimensions of bobble-neighbourhood-corona graphs, which is neighbourhood corona operation of connected graphs $G$ and $H$, where $H$ is a trivial graph.

2. Preliminaries

Definition of metric dimension was remarked in [1], also edge-metric dimension was defined in [6].

**Definition 2.1.** [6] Let $G$ be a connected graph with vertices set $V(G) = \{v_1, v_2, \ldots, v_k\}$ and edges set $E(G) = \{e_1, e_2, \ldots, e_k\}$. An ordered set $W = \{w_1, w_2, \ldots, w_k\}$ is subset of $V(G)$. $u, v$, and $w$ are belong to $V(G)$, edge $e = uv$ is an edge of $E(G)$. Distance from vertex $u$ to $v$ is denoted by $d(u, v)$. Distance between vertex $v$ and edge $e = uw$, $d_E(e, v)$, is $\min\{d(u, v), d(w, v)\}$. The edge representation $r_E(e|S)$, of edge $e$ to $W$ is k-tuple $(d_E(e, w_1), d_E(e, w_2), \ldots, d_E(e, w_k))$. Set $W$ is called edge-resolving set of $G$ if every two different edges $e, f \in E(G)$, $r_E(e|W) \neq r_E(f|W)$. Edge resolving set $W$ with minimum number of elements is called edge basis for $G$. Cardinality of edge basis of $G$, $|W|$, is called edge-metric dimension of $G$, is denoted by $edim(G)$. Neighbourhood corona has mentioned in [5] for spectrum of the graph, but there is no result for metric and edge-metric dimension of graphs.

**Definition 2.2.** [5] Neighbourhood-corona of $G$ and $H$ graphs, $G \ast H$, is a graph obtained by taking $G$ and $|V(G)|$ copies of $H$, with $H_i, i \in \{1, 2, 3, \ldots, |V(G)|\}$, then joining all vertices in $H_i$ with all neighbours of vertex $i$ in $G$. The bobble neighbourhood corona graph of $G$ and $H$, is the neighbourhood corona graph of $G$ and $H$ denoted by $G \ast H$, with $H$ is trivial graph.

**Example 2.3.** Let $G$ be a cycle $C_5$ and $H$ be a trivial graph $K_1$. The neighbourhood corona graph $G \ast H$ is illustrated in Figure 1.

![Figure 1. $C_5 \ast K_1$.](image)

3. Main results

In this part, we determine and analyse value of metric and edge-metric dimensions of $G \ast H$ where $H$ is trivial graph $K_1$.

3.1 Metric dimension of bobble-neighbourhood-corona graphs

In this section, here is the result of metric dimension of $G \ast K_1$ where $G$ is belong to $\{K_n, C_n\}$ which $n$ is odd.

3.1.1 Metric dimension of $K_n \ast K_1$

Given a complete graph $K_n$ with vertices set $V(K_n) = \{v_1, v_2, \ldots, v_n\}$, and a trivial graph $K_1$ with vertices set $V(K_1) = \{u\}$. The $i$-th copy of $K_1$ denoted by $H_i$ and vertices set $V(H_i) = \{u_i\}$.

The neighbourhood corona graph of $K_n$ and $K_1$ denoted by $K_n \ast K_1$. Graph $K_n \ast K_1$ obtained by taking $K_n$ and $H_1, H_2, \ldots, H_n$, then joining vertex $u_i$ with neighbours of $v_i$. Graph $K_n \ast K_1$ has vertices
Let $G$ be a complete graph $K_n$ with $n \geq 3$, and $H$ be a trivial graph $K_1$. Metric dimension of neighbourhood-corona of $G$ and $H$, $\text{dim}(G \ast H)$, is $\text{dim}(K_n * K_1) = n - 1$.

Proof. Choose a set $D = \{u_1, u_2, \ldots, u_{n-1}\} \subseteq V(K_n * K_1)$. Representation of vertices in $V(K_n * K_1)$ are

\[
\begin{align*}
    r(v_1|D) &= (2,1,1,\ldots,1,1) \\
    r(v_2|D) &= (1,2,1,\ldots,1,1) \\
    \vdots & \\
    r(v_{n-1}|D) &= (1,1,\ldots,1,2) \\
    r(v_n|D) &= (1,1,\ldots,1,1) \\
    r(u_1|D) &= (0,2,2,\ldots,2,2) \\
    r(u_2|D) &= (2,0,2,\ldots,2,2) \\
    \vdots & \\
    r(u_{n-1}|D) &= (2,2,\ldots,2,0) \\
    r(u_n|D) &= (2,2,\ldots,2,2)
\end{align*}
\]

Because all vertex above have distinct representations to $D$, then $D$ is the resolving set of $K_n * K_1$.

Set $C \subseteq V(K_n * K_1)$ with $|C| = n - 2$ is not resolving set, because:

(a) Let $C = \{u_1, u_2, \ldots, u_{n-2}\}$, then $d(u_{k-1}, u_i) = d(u_k, u_i) = 2$ and $d(v_k, u_i) = 1$; $u_i \in C$, such that $r(u_{k-1}|C) = r(u_k|C) = (2,2,\ldots,2,2)$ and $r(v_k|C) = (1,1,\ldots,1,1)$.

(b) Let $C = \{v_1, v_2, \ldots, v_{n-2}\}$, then $d(u_k, v_i) = d(v_k, v_i) = 1$; $v_i \in C$, such that $r(u_k|C) = (1,1,\ldots,1,1)$.

(c) Let $C = \{v_1, \ldots, v_q, u_1, \ldots, u_p\}; q = n - p - 2$, then $d(v_k, v_i) = d(v_{k-1}, v_i) = d(v_{k-2}, v_i) = 1$ and $d(u_k, v_i) = d(u_{k-1}, v_i) = 2$; $v_i \in C$, such that $r(v_k|C) = r(v_{k-1}|C) = r(v_{k-2}|C) = r(v_{k-2}|C) = (1,1,\ldots,1,1)$ and $r(u_k|C) = r(u_{k-1}|C) = r(u_{k-2}|C) = \left(\frac{p-\text{term}}{q-\text{term}}, \frac{p-\text{term}}{q-\text{term}}\right)$.

(d) Let $C = \{v_1, \ldots, v_q, u_1, \ldots, u_p\}; q = n - p - 2$, then $d(v_k, v_i) = d(v_{k-1}, v_i) = d(v_{k-2}, v_i) = 1$ and $d(u_k, v_i) = d(u_{k-1}, v_i) = d(u_{k-2}, u_i) = 2$; $v_i \in C$, such that $r(v_k|C) = r(v_{k-1}|C) = r(v_{k-2}|C) = \left(\frac{p-\text{term}}{q-\text{term}}, \frac{p-\text{term}}{q-\text{term}}\right)$.

From (a) to (d), we know that there are some vertices of $K_n * K_1$ have same representation to $C$. Thus $D = \{u_1, u_2, \ldots, u_{n-1}\}$ is a basis for $K_n * K_1$. 

3.1.2 Metric dimension of $C_n * K_1$

Given a cycle graph $C_n$ with vertices set $V(C_n) = \{v_1, v_2, \ldots, v_n\}$, and a trivial graph $K_1$ with vertices set $V(K_1) = \{\}$, the i-th copy of $K_1$ denoted by $H_i$ and vertices set $V(H_i) = \{i\}$. The neighbourhood corona graph of $C_n$ and $K_1$ denoted by $C_n * K_1$. Graph $C_n * K_1$ obtained by taking $C_n$ and $H_1, H_2, \ldots, H_n$ then joining vertex $u_i$ with neighbours of $v_i$. Graph $C_n * K_1$ has vertices set $V(C_n * K_1) = \bigcup_{i=1}^{n} (V(C_n) \cup V(H_i))$. 


\( K_1 \) = \( V(C_n) \cup V(H_2) \cup V(H_3) \cup \ldots \cup V(H_n) = V(C_n) \cup \bigcup_{i=1}^{n} V(H_i) = \{ v_1, v_2, \ldots, v_n \} \cup \{ u_1, u_2, \ldots, u_3 \} \).

**Theorem 3.1.2.** Let \( G \) be a cycle graph \( C_n \) with \( n \geq 3 \), and \( H \) be a trivial graph \( K_1 \). Metric dimension of neighbourhood-corona of \( G \) and \( H \), \( \dim(G \ast H) \), is

\[
\dim(C_n \ast K_1) = \begin{cases} 
2, & n = 3 \\
3, & n = 5 \\
\lfloor \frac{n}{3} \rfloor, & n \geq 7 
\end{cases}
\]

**Proof.** In this paper, we will prove the theorem for \( n \geq 7 \).

Choose \( Z \subseteq \{ u \in H_i | i = 3t + 1, t \in I = \{ 0, 1, 2, \ldots, \lfloor \frac{n-1}{3} \rfloor \} \} = \{ u_1, u_4, u_7, \ldots, u_p \} \subseteq V(C_n \ast K_1) \)

with \( p = \begin{cases} 
 n-1, & n = 3t + 2 \\
 n-2, & n = 3t 
\end{cases} \).

By direct observation on \( C_n \ast K_1 \), we will show that \( \dim(C_n \ast K_1) = \lfloor \frac{n}{3} \rfloor \) for \( n \geq 7 \), with the following steps:

I. Create a distance matrix of \( C_n \ast K_1 \) (the elements of the matrix are distance between two distinct vertices of \( C_n \ast K_1 \)). The elements of each row of the matrix, in order from left to right represents 1st, 2nd, \ldots, nth coordinates of vertex representation \( v \in C_n \ast K_1 \).

II. Observe a single value on:

1) First column of the distance matrix of \( C_n \ast K_1 \), then we obtain some vertices representations with single value from 1st coordinate. The vertices with single representations are collected into set \( A \).

2) First two columns of the distance matrix of \( C_n \ast K_1 \), then we obtain some vertices representations with single value from first two coordinates. The vertices with single representations are collected into set \( B \).

So on with the same steps, the observation is continues until first \( \lfloor \frac{n}{3} \rfloor \) columns of the distance matrix. The vertices with single representations are collected into set \( M \).

By \( \lfloor \frac{n}{3} \rfloor \)- observation above, we obtain a set \( A_1 \cup A_2 \cup \ldots \cup A_{\lfloor \frac{n}{3} \rfloor} = V(C_n \ast K_1) \) with all of vertices representation to \( Z \) are distinct. So, \( Z \) isa resolving set of \( C_n \ast K_1 \). \( \lfloor \frac{n}{3} \rfloor \) is a minimum cardinality of \( Z \).

Since a set \( D_1 \subseteq Z \) with cardinality less than \( \lfloor \frac{n}{3} \rfloor - 1 \) is not a resolving set of \( C_n \ast K_1 \). Because, let choose \( D_1 = \{ u_1, u_4, u_7, \ldots, u_{p-1} \} \) and \( p = n \), then representation \( r(u_{n-1}|D_1) = r(v_{n-1}|D_1) \). So, \( Z \) is basis of \( C_n \ast K_1 \).

As an example of direct observation on the process of determine basis for \( C_n \ast K_1 \), choose \( n = 7 \). The elements of vertex distance matrix of \( C_7 \ast K_1 \) displayed on Table 1.

**Table 1.** Vertex distance matrix of \( C_7 \ast K_1 \).

| \( d(a, b) \) | \( u_1 \) | \( u_4 \) | \( u_7 \) |
|-------------|--------|--------|--------|
| \( u_1 \)    | 0      | 3      | 3      |
| \( u_2 \)    | 3      | 2      | 2      |
| \( u_3 \)    | 2      | 3      | 3      |
| \( u_4 \)    | 3      | 0      | 3      |
Table 1 information:

|   | 3 | 3 | 2 |
|---|---|---|---|
| $u_5$ | 3 | 2 | 2 |
| $u_6$ | 2 | 2 | 3 |
| $u_7$ | 3 | 3 | 0 |
| $v_1$ | 2 | 3 | 1 |
| $v_2$ | 1 | 2 | 2 |
| $v_3$ | 2 | 1 | 3 |
| $v_4$ | 3 | 2 | 3 |
| $v_5$ | 3 | 1 | 2 |
| $v_6$ | 2 | 2 | 1 |
| $v_7$ | 1 | 3 | 2 |

$Z := \{u_1, u_4, u_7\}$

distance between vertex $a \in C_n * K_1$ and vertex $b \in Z$.

$r(u_1[Z]) := (0, 3, 3)$ is the first row of Table 1.

The result of observation from:

1. First column of vertex distance matrix of $C_7 * K_1$: We obtain the representation of $u_1$ as single representation from first coordinate observation. $A_1 = \{u_1\}$.
2. First two columns of vertex distance matrix of $C_7 * K_1$: We obtain the representation of $u_1, u_2, v_1, v_3, v_5$ and $v_7$ as single representation from first two coordinate observation. $A_2 = \{u_1, u_2, v_2, v_3, v_5, v_7\}$.
3. First three columns of vertex distance matrix of $C_7 * K_1$: We obtain the representation of all vertices of $C_7 * K_1$ as single representation from first three coordinate observation. $A_3 = M = V(C_7 * K_1)$.

3.2 Edge metric dimension of bobble-neighbourhood-corona graphs

In this section, here is the result of the metric dimension of $G * K_1$ where $G$ is belong to $\{K_n, C_n\}$ which $n$ is odd.

3.2.1 Edge metric dimension of $K_n * K_1$

Given complete graph $K_n$ with vertices set $V(K_n) = \{v_1, v_2, \ldots, v_n\}$, and trivial graph $K_1$ with vertices set $V(K_1) = \{u\}$.

The neighbourhood corona graph of $K_n$ and $K_1$ denoted by $K_n \ast K_1$. Graph $K_n \ast K_1$ obtained by taking $K_n$ and $H_1, H_2, \ldots, H_n$, then joining vertex $u_i$ with neighbours of vertex $v_i$. Graph $K_n \ast K_1$ has vertices set $V(K_n \ast K_1) = V(K_n) \cup V(H_1) \cup V(H_2) \cup \ldots \cup V(H_n) = V(K_n) \cup \bigcup_{i=1}^{n} V(H_i) = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\}$ and edges set $E(K_n \ast K_1) = \{v_i u_k | a, b = 1, 2, \ldots, n\} \cup \{v_b u_c | b, c = 1, 2, \ldots, n\}$ and $b \neq c$.

Theorem 3.2.1. Let $n$ be an odd integer number. If $G$ is complete graph $K_n$ with $n \geq 3$, and $H$ is trivial graph $K_1$, then edge-metric dimension of neighbourhood corona of $G$ and $H$, $\text{dim}(G \ast H)$, is $\text{edim}(K_n \ast K_1) = 2(n - 1)$.

Proof. Choose set $S = \{v_1, v_2, \ldots, v_{n-1}, u_1, u_2, \ldots, u_{n-1}\} \subseteq V(K_n \ast K_1)$. There are 5 cases, i.e.

1. If an edge $e_1 = v_i v_j ; e_1 \in E(K_n \ast K_1)$; $v_i, v_j \in V(K_n \ast K_1)$, and $i, j \in \{1, 2, \ldots, n - 1\}$ with $i \neq j$, then $d_G(e_1, v_i) = d_G(e_1, v_j) = 0$ and distance between $e_1$ and other vertices in $S$ are 1.
2. If an edge $e_2 = v_i u_j ; e_2 \in E(K_n \ast K_1)$; $v_i, u_i, u_j \in V(K_n \ast K_1)$, and $i, j \in \{1, 2, \ldots, n - 1\}$ with $i \neq j$, then $d_G(e_2, v_i) = d_G(e_2, u_j) = 0$, $d_G(e_2, u_i) = 2$ and distance between $e_2$ and other vertices in $S$ are 1.
3. If an edge $e_3 = v_i v_j$; $e_3 \in E(K_n \ast K_1)$; $v_i, v_j \in V(K_n \ast K_1)$, and $i \in \{1, 2, \ldots, n-1\}$, then $d_G(e_3, v_i) = 0$ and distance between $e_3$ and other vertices in $S$ are 1.

4. If an edge $e_4 = v_i u_k$; $e_4 \in E(K_n \ast K_1)$; $v_i, u_k \in V(K_n \ast K_1)$, and $i \in \{1, 2, \ldots, n-1\}$, then $d_G(e_4, v_i) = 0$ and distance between $e_4$ and other vertices in $S$ are 1.

5. If an edge $e_5 = v_i u_k$; $e_5 \in E(K_n \ast K_1)$; $v_i, u_k \in V(K_n \ast K_1)$, and $i \in \{1, 2, \ldots, n-1\}$, then $d_G(e_5, u_k) = 0$ and distance between $e_5$ and other vertices in $S$ are 1.

From 5 cases above, we can see clearly that there are no edge which have same representation to $S$, so $S$ is a edge resolving set for $K_n \ast K_1$.

Otherwise, let $T$ be a vertices set with cardinality $2n-3$, then $T$ is not a edge resolving set for $K_n \ast K_1$, because:

(a) Let $T = \{v_1, v_2, \ldots, v_{n-2}, u_1, u_2, \ldots, u_{n-2}\}$ and edges $g = v_{n-1} v_n$ and $h = v_{n-1} u_n$; $g, h \in E(K_n \ast K_1)$, then $d_G(g, u_i) = d_G(h, u_i) = 1$, $d_G(g, v_{n-1}) = d_G(h, v_{n-1}) = 0$ and $d_G(g, v_j) = d_G(h, v_j) = 1; 1 \leq j \leq n-2$, where $v_j, v_{n-1}, u_i \in T$, such that $r(g|T) = r(h|T) = (1, 1, 0, 1, \ldots, 1)$.

(b) Let $T = \{v_1, v_2, \ldots, v_{n-2}, u_1, u_2, \ldots, u_{n-2}\}$ and $e = v_1 v_{n-1}$ and $s = v_1 v_n$; $e, s \in E(K_n \ast K_1)$, then $d_G(e, u_i) = d_G(s, u_i) = 1$, $d_G(e, v_1) = d_G(s, v_1) = 0$ and $d_G(e, v_j) = d_G(s, v_j) = 1; 2 \leq j \leq n-2$ where $v_1, v_j, u_i \in T$, such that $r(e|T) = r(s|T) = (0, 1, 1, \ldots, 1)$.

From (a) and (b), we know that in a set $T$ with cardinality $2n-3$, there are edges of $K_n \ast K_1$ that have same representation to $T$. Therefore, $S = \{v_1, v_2, \ldots, v_{n-1}, u_1, u_2, \ldots, u_{n-1}\}$ is a minimum edge resolving set for $K_n \ast K_1$.

### 3.2.2 **Edge metric dimension of $C_n \ast K_1$**

Given a complete graph $C_n$ with vertices set $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and a trivial graph $K_1$ with vertices set $V(K_1) = \{u\}$. The $i$-th copy of $K_1$ denoted by $H_i$ and vertices set $V(H_i) = \{u_i\}$. The neighbourhood corona of $C_n$ and $K_1$ denoted by $C_n \ast K_1$. Graph $C_n \ast K_1$ obtained by taking $C_n$ and $H_1, H_2, \ldots, H_n$, then joining each vertex $u_i, i \in \{1, 2, \ldots, n\}$ with adjacent vertices to $v_i$. Graph $C_n \ast K_1$ has vertices set $V(C_n \ast K_1) = V(C_n) \cup V(H_1) \cup V(H_2) \cup \ldots \cup V(H_n) = V(C_n) \cup \bigcup_{i=1}^{n} V(H_i) = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\}$ and edges set $E(C_n \ast K_1)$.

**Theorem 3.2.2.** Let $G$ be a cycle graph $C_n$ with $n \geq 3$, and $H$ be a trivial graph $K_1$. Edge metric dimension of neighbourhood-corona of $G$ and $H$, $\text{dim}(G \ast H)$, is

$$\text{edim}(C_n \ast K_1) = \left\lceil \frac{n}{3} \right\rceil, n = 3, 5, 7, 9, n \geq 11$$

**Proof.** In this paper, we will proof the theorem for $n \geq 11$.

Choose $L = \{u_i \in H_i | i = 3t + 1, t \in I = \left\{0, 1, 2, \ldots, \left\lceil \frac{n}{3} \right\rceil - 1 \right\}\} = \{u_1, u_4, u_7, \ldots, u_p\} \subseteq V(C_n \ast K_1)$

with $p = \left\lceil \frac{n - 1}{3} \right\rceil$, $n = 3t + 2$

and $p = \left\lceil \frac{n - 2}{3} \right\rceil$, $n = 3t + 3$.

By direct observation on $C_n \ast K_1$, we will show that $\text{edim}(C_n \ast K_1) = \left\lceil \frac{n}{3} \right\rceil$, for $n \geq 11$, with the following steps:

I. Create a distance matrix of $C_n \ast K_1$ (the elements of the matrix are distance between edges and vertex of $C_n \ast K_1$). The elements of each row of the matrix, in order from left to right represents 1st, 2nd, ..., nth coordinates of edge representation $e \in C_n \ast K_1$.

II. Observe a single value on:
1) First column of the distance matrix of $C_n * K_1$, then we obtain some edge representations with single value from 1st coordinate. The edges with single representations are collected into set $F_1$.

2) First two columns of the distance matrix of $C_n * K_1$, then we obtain some edge representations with single value from first two coordinates. The edges with single representations are collected into set $F_2$.

So on with the same steps, the observation is continues until first $\left\lceil \frac{n}{3} \right\rceil$ columns of the distance matrix. The edges with single representations are collected into set $T$.

By $\left\lceil \frac{n}{3} \right\rceil$- observation above, we obtain a set $F_1 \cup F_2 \cup ... \cup F_{\left\lceil \frac{n}{3} \right\rceil} = E(C_n * K_1)$ with all of vertices representation to L are distinct. So, L is edge resolving set of $C_n * K_1$. $\left\lceil \frac{n}{3} \right\rceil$ is minimum cardinality of L.

Since set $W_1 \subseteq L$ with cardinality less than $\left\lceil \frac{n}{3} \right\rceil - 1$ is not a edge resolving set of $C_n * K_1$. Because, let choose $W_1 = \{u_1, u_4, u_7, ..., u_{p-1}\}$ and $p = n$, then the edge representation $r(v_{n-1}v_n|W_1) = r(v_nu_{n-1}|W_1)$. Hence, L is edge basis for $C_n * K_1$.

As an example of direct observation on the process of determine edge basis for $C_n * K_1$, choose $n = 11$. The elements of edge distance matrix of $C_{11} * K_1$ displayed on Table 2.

### Table 2. Edge distance matrix of $C_{11} * K_1$.

| $d_G(e, b)$ | $u_1$ | $u_4$ | $u_7$ | $u_{10}$ |
|-------------|-------|-------|-------|---------|
| $v_1v_2$    | 1     | 2     | 5     | 2       |
| $v_1u_2$    | 2     | 2     | 5     | 2       |
| $v_2u_{11}$ | 2     | 3     | 4     | 2       |
| $v_2v_3$    | 1     | 1     | 4     | 3       |
| $v_2u_4$    | 0     | 2     | 5     | 2       |
| $v_2u_5$    | 1     | 2     | 4     | 3       |
| $v_3v_4$    | 2     | 1     | 3     | 4       |
| $v_3u_2$    | 2     | 1     | 3     | 3       |
| $v_3u_4$    | 2     | 0     | 3     | 4       |
| $v_4v_5$    | 3     | 1     | 2     | 5       |
| $v_4u_3$    | 2     | 2     | 3     | 4       |
| $v_4u_5$    | 3     | 2     | 2     | 5       |
| $v_5v_6$    | 4     | 1     | 1     | 4       |
| $v_5u_4$    | 3     | 0     | 2     | 5       |
| $v_5u_6$    | 4     | 1     | 2     | 4       |
| $v_6v_7$    | 5     | 2     | 1     | 3       |
| $v_6u_5$    | 4     | 2     | 1     | 4       |
| $v_6u_7$    | 5     | 3     | 0     | 3       |
| $v_7v_8$    | 4     | 3     | 1     | 2       |
| $v_7u_6$    | 5     | 2     | 2     | 3       |
| $v_7u_9$    | 4     | 3     | 2     | 2       |
| $v_8v_9$    | 3     | 4     | 1     | 1       |
| $v_8u_7$    | 4     | 3     | 0     | 2       |
| $v_8u_9$    | 3     | 4     | 1     | 2       |
| $v_9v_{10}$ | 2     | 5     | 2     | 1       |
| $v_9u_8$    | 3     | 4     | 2     | 1       |
We have obtained the representation of

\[ \mathbf{C}_{11} \cap K_1 \] as single edge representation from first coordinate observation.

4. Conclusion
We have discussed about metric and edge-metric dimension of neighbourhood-corona of \( G \) and \( K_1 \), with \( G \in \{K_n, C_n\} \). Research about metric and edge-metric dimension of neighbourhood-corona of \( G \) and \( H \) can be continued for another graph \( G \), for example \( G \) is bipartite graph.

Acknowledgments
The author thanks the reviewers for their valuable thoughts and comments so that this paper becomes better.

References
[1] Chartrand G, Eroh L, Johnson M A and Oellermann O R 2000 Resolvability in graphs and the metric dimension of a graph Discrete Applied Mathematics 105 99–113
[2] Iswadi H, Baskoro E T and Simanjuntak R 2011 On the metric dimension of corona product of graphs Far East Journal of Mathematical Sciences (FJMS) 52 155-170
[3] Rinurwati, Slamin and Suprajitno H 2017 On (local) metric dimension of graphs with \( m \)-pendant points Journal of Physics: Conference Series 855 01235 1–8
[4] Rinurwati, Suprajitno H dan Slamin 2017 On local adjacency metric dimension of some wheel related graphs with pendant points Proc. of AIP Conf. 1867 p 00265

| \( d_G(e, b) \) | \( u_1 \) | \( u_4 \) | \( u_7 \) | \( u_{10} \) |
|----------------|-------|-------|-------|-------|
| \( v_3u_{10} \) | 2     | 5     | 2     | 0     |
| \( v_{10}v_1 \) | 1     | 4     | 3     | 1     |
| \( v_{10}u_9 \) | 2     | 5     | 2     | 2     |
| \( v_{10}u_{11} \) | 2     | 4     | 3     | 2     |
| \( v_{11}v_4 \) | 1     | 3     | 4     | 1     |
| \( v_{11}u_4 \) | 1     | 4     | 3     | 0     |
| \( v_{11}u_{10} \) | 0     | 3     | 4     | 1     |

Table 2 information:

\( L := \{u_1, u_4, u_7, u_{10}\} \)

\( d_G(e, b) := \text{distance between edge } e \in C_n \cap K_1 \text{ and vertex } b \in L. \)

\( r(v_1v_2|L) := (1,2,5,2) \) is the first row of Table 2.
[5] Gopalapillai I 2011 The spectrum of neighbourhood corona of graphs *Kragujevac journal of mathematics* **35** 493-500

[6] Kelenc A, Tratnik N and Yero I 2018 Uniquely identifying the edges of a graph: the edge metric dimension *Discrete Applied Mathematics* **251** 204-220

[7] Nasir R, Zafar S and Zahid Z 2018 Edge metric dimension of graphs *Ars Combin* **147** 143-156