The Space Filling Dirichlet 3-Brane in $N = 2$, $D = 4$ Superspace

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We discuss a four–dimensional Volkov–Akulov supersymmetric theory on a D3–brane with $N = 2$ supersymmetry broken down to $N = 1$.

1. INTRODUCTION

In [1] Volkov and Akulov proposed the first four–dimensional field theoretical model which possessed space–time supersymmetry. The model was constructed in such a way that (as we assume to be realized in nature) supersymmetry is broken spontaneously with a “neutrino” playing the role of the associated fermionic Goldstone particle. This was the first example of a mechanism of spontaneous breaking of global supersymmetry which was generalized by Volkov and Soroka [2] to the super–Higgs effect in the first supergravity model with spontaneously broken local supersymmetry [3].

Later on it has been realized [4] that models of the Volkov–Akulov type describe supersymmetric effective field theories exhibiting partial supersymmetry breaking on the worldvolumes of branes. This subject has recently faced a significant revival of interest due to the extensive study of various aspects of brane physics (see e.g. [5–10]).

The aim of this work is to study peculiarities of the superembedding description [11, 12] of a space filling D(irichlet)3–brane propagating in an $N = 2$, $D = 4$ superspace and to establish the relationship of this covariant geometrical formulation with a Goldstone superfield formulation of an $N = 1$, $D = 4$ supersymmetric Dirac–Born–Infeld theory [13–16] based on methods of non–linear realizations first applied to supersymmetry by Volkov and Akulov.

We shall show that the superembedding conditions and worldvolume gauge field constraints do not put D3–brane dynamics in $D = 4$ on the mass shell, in contrast, for example, to the case of a D3–brane [14, 17] and a D9–brane [18] in type IIB D=10 supergravity. A geometrical consequence of these conditions is the Grassmann analyticity of both the $N = 1$, $D = 4$ superworldvolume and $N = 2$, $D = 4$ target superspace. This is an extension to the D3–brane of results on the relationship of the superembedding condition with Grassmann analyticity properties of supermanifolds observed previously in the cases of an $N = 1$, $D = 4$ superparticle and an $N = 1$, $D = 4$ superstring [19, 20].

The fact that the superembedding conditions are off–shell constraints will allow us to construct a worldvolume superfield action for the space filling D3–brane which we briefly discuss in the end of this contribution.
Notation. We use the formalism of twodimensional Weyl spinors both in $N = 1, D = 4$ superworldvolume and in $N = 2, D = 4$ target superspace. Since the $D3$–brane in $D = 4$ is a space filling brane we can always gauge fix local Lorentz rotations in the worldvolume in such a way that they coincide with Lorentz transformations in the target superspace. So there is no need to distinguish between the vector and spinor indices corresponding to the tangent spaces of superworldvolume and target superspace. Small letters of the Greek and Latin alphabet stand, respectively for spinor and vector indices, e.g. $\alpha, \dot{\alpha} = 1, 2$; $a, b = 0, 1, 2, 3$. Capital Latin letters denote both the spinor and vector indices. The curved target superspace indices denoted by the letters from the second half of the alphabets are underlined to indicate that the worldvolume and target space superdiffeomorphism groups are a priori independent.

2. SUPEREMBEDDING CONDITIONS

In the case of the space filling superbranes the basic superembedding condition reads \[ (4) \] that the superembedding of the brane superworldvolume into a target superspace is carried out in such a way that (using local Lorentz transformations on the worldvolume and in target superspace) it is always possible to choose the vector component $e^a$ of a worldvolume supervielbein

\[ (1) \]

\[ e^A(z) = (e^a, e^\alpha, \bar{e}^\dot{\alpha}) \]

\[ z^M = (\xi^m, \eta^\mu, \bar{\eta}^{\dot{\mu}}) \]

to coincide with the pullback of the vector component $E^a$ of a target space supervielbein

\[ (2) \]

\[ E^A(Z) = (E^a, E^I\alpha, \bar{E}^{I\dot{\alpha}}), \]

\[ Z^M = (X^a, \Theta^I\mu, \bar{\Theta}^{I\dot{\mu}}), \quad I = 1, 2 \].

Namely,

\[ e^a = E^a(Z(z)) \].

Note that by imposing \[ (4) \] we have identified the group of local Lorentz rotations in the tangent space of the superworldvolume with that of the target space Lorentz group, while the worldvolume superdiffeomorphisms still remain an independent group of transformations, and can be used (as we will do in the final stage of our analysis), to impose a physical gauge

\[ \xi^m = X^m, \quad \eta^\mu = \Theta^I\mu, \quad \bar{\eta}^{\dot{\mu}} = \bar{\Theta}^{I\dot{\mu}}. \]

In this gauge the theory remains manifestly invariant under $N = 1, D = 4$ supersymmetry associated with the supertranslations along $\eta^\mu$ and $\bar{\eta}^{\dot{\mu}}$, while the second target space supersymmetry associated with the $\Theta^2$ translations is realized nonlinearly in the transformation law of $\Theta^{2\mu}(z)$, which implies its spontaneous breaking,

\[ \delta \Theta^2 = \epsilon^2 + i(\epsilon^2 \sigma^a \bar{\Theta}^2 + \Theta^2 \sigma^a \epsilon^2 )\partial_\mu \Theta^2. \]

Thus $\Theta^{2\mu}(z)$ is the Volkov–Akulov Goldstone fermion associated with the half of $N = 2, D = 4$ supersymmetry spontaneously broken by the $D3$–brane.

As a consequence of \[ (4) \] the pullback of $E^a$ along the Grassmann directions \[ (4) \] of the superworldvolume is zero. If the target superspace is flat (which is the case of our interest)

\[ (6) \]

\[ E^a = dX^a - i d\Theta^I\alpha \sigma^a_{\alpha\beta} \bar{\Theta}^{I\dot{\beta}} + i \Theta^{I\alpha} \sigma^a_{\alpha\beta} d\bar{\Theta}^{I\dot{\beta}}, \]

eq \text{and eq. } (4) \text{ implies}

\[ (7) \]

\[ E^a = \bar{D}_a X^a - i \bar{D}_a \Theta^I\alpha \sigma^a \bar{\Theta}^I - i \Theta^I\alpha \bar{D}_a \bar{\Theta}^I = 0, \]

where $D_\alpha$ and $\bar{D}_{\dot{\alpha}}$ are worldvolume covariant derivatives.

As far as the spinor components of the worldvolume supervielbein \[ (4) \] are concerned, by a simple redefinition they can always be chosen to coincide with the pullback of one of the two spinor components of the target space supervielbein \[ (8) \].

For instance, we can choose

\[ (8) \]

\[ e^a = E^{1\alpha}, \quad \bar{e}^{\dot{\alpha}} = \bar{E}^{1\alpha}. \]

(Such a choice reflects the possibility of gauge fixing kappa–symmetry by putting $\Theta^{1\alpha}_{\alpha=0} = 0$ in the conventional Green–Schwarz formulation of the Dirichlet brane dynamics \[ (4) \]).

Then the generic expression for the pullbacks of the fermionic one–forms $E^{2\alpha}$ and $\bar{E}^{2\dot{\alpha}}$ in the worldvolume local frame \[ (4) \] and \[ (8) \] is

\[ (9) \]

\[ E^{2\alpha} = E^{1\beta} h^{\alpha\beta}(z) + \bar{E}^{1\dot{\alpha}} C_{\alpha}^{\dot{\alpha}}(z) + E^\alpha \psi^\alpha_2(z), \]
\[ E^{2\alpha} = E^{1\beta} h_{\beta}^{\alpha}(z) + E^{1\alpha} \tilde{C}_{\alpha}^\beta(z) + E^{\alpha} \tilde{\psi}^{\alpha}(z). \]

Note that in the flat target superspace
\[ E^{1\alpha} = d\Theta^{1\alpha}, \quad E^{1\tilde{\alpha}} = d\tilde{\Theta}^{1\tilde{\alpha}}, \]
and
\[ h_{\beta}^{\alpha}(z) = D_\beta \Theta^{2\alpha}, \quad \tilde{\psi}^{\alpha}(z) = D_\alpha \Theta^{2\alpha}, \]
\[ C_{\alpha}^\beta(z) = D_\alpha \Theta^{2\beta}, \]
the analogous expressions being valid for the complex conjugate superfields $h, \tilde{C}$ and $\tilde{\psi}$.

The possibility of identifying the worldvolume supervielbein with the "pulled back" components $\tilde{h}$ and $\tilde{C}$ of the target space supervielbein implies that in flat target superspace the induced superworldvolume geometry is also flat, and that the worldvolume spin connection is zero. This is natural, since the brane worldvolume completely fills in (or coincides with) the bosonic core of the flat target superspace. We should note that though the superworldvolume is flat the supervielbein $e^A$ defined by (3) and (8) differs from the standard flat superspace basis
\[ e_0^A = (d\xi^a - id\eta^a \tilde{b} + i\eta^\alpha d\tilde{\eta}^\alpha, d\eta^a, d\tilde{\eta}^\alpha). \]
This, in particular, implies that the superworldvolume covariant derivatives $D_A$ in (3) and (8) associated with the basis $\tilde{h}$ and $\tilde{C}$ differ from conventional flat covariant derivatives and form a more complicated superalgebra, which we shall present a bit later.

The integrability of (3) and (10) requires some differential relations between the components $h(z), C(z)$ and $\tilde{\psi}(z)$ of the superforms (3), however the integrability of the superembedding condition (3), which implies that the worldvolume torsion is the pullback of the target space torsion
\[ de^a = dE^a \equiv T^a = -2iE^{a1} \wedge E^{\tilde{a}1}\sigma_{\alpha\tilde{\alpha}}^{\alpha\tilde{\alpha}} - 2iE^{a2} \wedge E^{\tilde{a}2}\sigma_{\alpha\tilde{\alpha}}^{\alpha\tilde{\alpha}}, \]
does not put any further restrictions on $h, C,$ and $\tilde{\psi}$, and these superfields are still too general to be associated with the physical modes of the D3-brane, which form a gauge vector supermultiplet.

The situation when the basic superembedding condition is not enough to determine the dynamics of the brane even off the mass shell is generic for the space–filling [18] and codimension one [22] branes. In such cases, for the superembedding to describe superbrane dynamics, an additional constraint should be imposed 1. In our case this is a constraint on an 'extended' field–strength two–form
\[ F_2 = dA - B_2 \]
of a worldvolume gauge field $dz^A A_A(z)$ living on the D3-brane, the two–form $B_2$ being the pullback of an “NS–NS” gauge superfield of an $N = 2, D = 4$ supergravity which the D3 brane couples to.

2.1. Worldvolume gauge field constraints

We assume the worldvolume superfield constraint on $F_2$ to be
\[ F_2 = dA - B_2 = \frac{1}{2} E^a \wedge E^b F_{ab}, \]
which implies that $F_2$ has nonzero components only along bosonic directions (3) of the superworldvolume.

To argue that the constraint (15) is relevant to the description of the Born–Infeld gauge field and of the D3–brane as a whole, we note that it follows from a generalized action [13] for super–Dp–branes constructed and analyzed in [17][18] and in a linear approximation it reduces to standard $N = 1, D = 4$ super–Maxwell constraints, as we shall demonstrate in Subsection 2.3 upon analyzing the integrability condition of (15)
\[ -H_3 = E^a \wedge T^b F_{ab} + \frac{1}{2} E^a \wedge E^b \wedge dF_{ab}, \]
where $H_3 = dB_2$ and $T^b$ are the pullbacks of, respectively, the $B_2$ field strength and the torsion of the target superspace which we will further consider to be flat (see eqs. (3), (10), and (13)). In flat target superspace $H_3$ has the following form
\[ H_3 = 2iE^a \wedge (E^{1\alpha} \wedge E^{1\tilde{\alpha}} - E^{2\alpha} \wedge E^{2\tilde{\alpha}})\sigma_{\alpha\tilde{\alpha}a\tilde{a}}, \]
and from (10) and (13) we get
\[ -2iE^a \wedge (E^{a1} \wedge E^{\tilde{a}1})\sigma_{a\tilde{a}b}(\eta - F)_{ba}, \]
1Note that these additional constraints are reproduced by the generalized action [13][18] on the same footing as the basic superembedding conditions and the dynamical equations of motion.
\[
-E^{\alpha 2} \wedge E^{\beta 2} \sigma^b_{\alpha\beta} (\eta + F)_{ba} = \frac{1}{2} E^a \wedge E^b \wedge dF_{ba}.
\] (18)

Our goal is to show that when the constraints (3) and (14) are imposed the components of the superfields \(F_2(z), h(z), C(z)\) and \(\psi(z)\) either vanish or are expressed through the worldvolume chiral spinor superfield \(\Theta_\alpha(z)\) describing the gauge vector supermultiplet. To this end we analyze the integrability condition (15).

Substituting (3) into (18) and taking its \(E^{\alpha 1} \wedge E^{\beta 1}\) and \(E^{\alpha 2} \wedge E^{\beta 2}\) components one finds that
\[
\begin{align*}
    &h_{\alpha}^\gamma \sigma_{\alpha\beta}^\gamma \bar{C}^\beta_\beta (\eta - F)_{ba} = 0, \\
    &\bar{h}_{\beta}^\gamma \sigma_{\alpha\beta}^\gamma C^\gamma_\gamma (\eta - F)_{ba} = 0,
\end{align*}
\] (19)

where \(\eta_{ab}\) is the \(D = 4\) Minkowski metric.

If the matrix \((\eta - F)\) is non–degenerate (which is the general assumption of the Born–Infeld–like models) the equations (19) are satisfied if and only if
\[
\begin{align*}
    &\bar{C}^\gamma_\beta = 0, \quad C^\gamma_\beta = 0 \quad \text{(20)}
\end{align*}
or
\[
\begin{align*}
    &h_{\alpha}^\beta = 0, \quad \bar{h}_{\alpha}^\beta = 0. \quad \text{(21)}
\end{align*}

As one can verify the second choice (eq. (21)) leads to a trivial solution of the superembedding conditions (which does not describe any physical dynamical system), so we shall analyze the nontrivial consequences of the first solution (20).

Then the spinor supervielbein pullbacks (4) take the form
\[
\begin{align*}
    &E^{2\alpha} = E^{1\beta} h_{\beta}^\alpha + E^a \psi^a_{\alpha}, \\
    &E^{2\alpha} = E^{1\beta} \bar{h}_{\beta}^\alpha + E^a \bar{\psi}^a_{\alpha}.
\end{align*}
\] (22)

Now consider the \(E^{1\alpha} \wedge E^{1\alpha}\) component of (18).

In view of (22) it reduces to
\[
\begin{align*}
    &h^\beta_{\alpha} \sigma^\beta_{\alpha\beta} \bar{h}^\beta_{\alpha} = (h \sigma^a_{\alpha})_{\alpha} = \sigma^b_{\alpha\alpha} k^a_{\beta}, \quad \text{(23)}
\end{align*}

where the matrix \(k^a_{\beta}\) takes values in the (pseudo)orthogonal group \(SO(1, 3)\) which follows from its definition
\[
k^a_{\beta} = (\eta - F)^{-1} (\eta + F)^{-1} c^a
\]

Then the relation (23) implies that (up to a \(U(1)\) rotation) \(h^\alpha_{\beta}\) belongs to a spinor representation of \(SO(1, 3)\)
\[
h^\alpha_{\beta} \in SL(2, C) \times U(1)
\]

and
\[
|\text{det}(h)| = 1 \rightarrow \text{det}(h) = e^{2i\alpha(z)}, \quad \text{(25)}
\]

where the real superfield \(a(z)\) takes values on the circle \(S^1\).

We have thus found the relationship between the spin–tensor superfield \(h_{\alpha}^\beta(z)\) (which in the flat target superspace is \(h_{\alpha}^\beta(z) = D^\beta \Theta^\alpha\)) and the field strength \(F_{ab}(z)\) of the worldvolume gauge field. Namely, from (23) and (24) it follows that
\[
F_{a}^b = H_{a}^b - \delta_{a}^b,
\] (26)

where \(H_{a}^b = \frac{1}{2} \{ \frac{1}{2} \text{tr}(h \sigma b \bar{h} a) + \delta_{a}^b \} \).

The equation (23) implies that the superfield \(h^\alpha_{\beta}\) satisfies the nonlinear constraint
\[
\text{det}(h^\alpha_{\beta}) \cdot \text{det}(\bar{h}^\beta_{\alpha}) = 1. \quad \text{(27)}
\]

This is the exact form of the nonlinear generalization of the Maxwell superfield constraint (see eq. \(\text{(18)}\) below) which was found in (3) to order \(O(\Theta^4)\).

Because of the group theoretical properties (24), (25) and (26) of \(h^\alpha_{\beta}\) and \(k^a_{\beta}\) they also satisfy the following relation
\[
h_{\alpha}^\gamma dh^\beta_{\gamma} = \frac{1}{2} (k^{-1} dk)^{ab} \sigma_{\alpha\beta} + i da(z) \delta_{\alpha}^\beta. \quad \text{(28)}
\]

Eq. (28) implies, in particular,
\[
h_{\beta}^\gamma dh^\beta_{\gamma} = 2 i da(z). \quad \text{(29)}
\]

To conclude the analysis of the consequences of the superembedding condition (3) and of the gauge field constraint (15) we shall now demonstrate that they do not put the theory on the mass shell.

The dynamical fermionic equation of motion of the D3–brane which is obtained by varying the
Green–Schwarz–like \cite{23} or the generalized action \cite{17} for the D3–brane with respect to $\Theta^2$ is

$$\sigma^b_{a\dot{\alpha}}(\eta - F)^{-1} b^a D_a \Theta^{2a} \equiv \sigma^b_{a\dot{\alpha}}(\eta - F)^{-1} b^a \psi^a_{\dot{\alpha}} = 0, \quad (30)$$

(plus its complex conjugate).

We should, therefore, check that eq. (32) does not follow from the constraints (3) and (15). To this end let us note that in view of (3), (8), (13), (21), (22) and (23) the algebra of the worldvolume covariant derivatives $D_A$ is

$$\{D_\alpha, \bar{D}_\dot{\alpha}\} = -T^a_{\alpha\dot{\alpha}} D_a = 2i \sigma^b_{a\dot{\alpha}} (\delta^a_b + \tilde{a}^a_{\dot{\alpha}}) D_a = 4i \sigma_{b\dot{\alpha}} (\eta + F)^{-1} b^a D_a, \quad (31)$$

$$\{D_\alpha, D_\beta\} = 0 = \{\bar{D}_\dot{\alpha}, \bar{D}_\dot{\beta}\}, \quad (32)$$

$$\{D_\alpha, D_\dot{\beta}\} = -T^b_{\alpha\dot{\beta}} D_a = -2i (\delta^b_a)_{\alpha\dot{\beta}} \bar{\psi}^a_{\dot{\beta}} D_a, \quad (33)$$

$$\{D_\alpha, D_\dot{\beta}\} = -T^b_{\alpha\dot{\beta}} D_a = -2i (\delta^b_a)_{\alpha\dot{\beta}} \psi^a_{\dot{\beta}} D_a, \quad (34)$$

Then applying $\bar{D}_\dot{\beta}$ to $h^\beta_{\dot{\beta}}$ of (11), and taking into account (29) and (31) we find that

$$\bar{D}_\dot{\beta} h^\alpha_{\dot{\beta}} = 4i \sigma^b_{b\beta} (\eta + F)^{-1} b^a \psi^a_{\dot{\alpha}}, \quad (35)$$

which relates $\psi^\alpha_{\dot{\alpha}}$ with $h^\alpha_{\dot{\beta}}$ and $F_{ab}$.

Now multiplying eq. (35) by $(h^{-1})_{\dot{\alpha}} \tilde{h}^{-1})_{\dot{\beta}}$, and using the relations (23), (24) and (25) we get

$$\sigma_{b\alpha}(\eta - F)^{-1} b^a \psi^a_{\dot{\alpha}} = -\frac{1}{4} \tilde{h}^{-1} \tilde{h}^{-1} a^a \bar{D}_\dot{\beta} a(z), \quad (36)$$

where the left hand side is the same as in eq. (30), but it is non–zero, since $a(z)$ is generically non–constant. Thus, in the case of the space–filling D3–brane the superembedding conditions and the field strength constraint does not produce dynamical equations of motion and, therefore, leave the theory off the mass shell. The equations of motion arise only if in addition we put $da(z) = 0$ or $a(z) = const$. Then, on the mass shell, the spin tensor $h$ becomes an $SL(2, C)$ valued matrix (c.f. 18) for a D=10 super-D9-brane

$$\det h^\alpha_{\dot{\beta}} = 1. \quad (37)$$

Eq. (37) can be regarded as the nonlinear superfield equation of motion of the D3-brane, which generalizes the linear super–Maxwell equation of motion (see Subsection 2.3).

### 2.2. Grassmann analyticity

As we have already mentioned, the superembedding conditions (3), (8), (13), and (15) result in double analyticity, i.e. Grassmann analyticity both in the worldvolume and in target superspace, the phenomenon which was declared in \cite{17} as a principle for some types of superembeddings, describing for instance certain superparticles and superstrings \cite{17, 21}. Indeed, since the integrability of the constraints requires eq. (21), from (3)–(1) it follows that $\Theta^I^\alpha$ are chiral worldvolume superfields, i.e.

$$\bar{D}_\dot{\beta} \Theta^I^\alpha = 0, \quad D_\alpha \Theta^I^\alpha = 0. \quad (38)$$

Then eqs. (3) take the form

$$D_\alpha (X^a - i \Theta^I \sigma^a \bar{\Theta}^I) = 0, \quad \bar{D}_\dot{\beta} (X^a + i \Theta^I \sigma^a \bar{\Theta}^I) = 0, \quad (39)$$

or

$$X^a = \frac{1}{2} (X^a_R + X^a_L), \quad X^a_L - X^a_R = 2i \Theta^I \sigma^a \bar{\Theta}^I = 0, \quad (40)$$

where $X^a_R = X^a - i \Theta^I \sigma^a \bar{\Theta}^I = (X^a_L)$ are complex conjugate chiral worldvolume superfields

$$D_\alpha X^a_L = 0, \quad D_\alpha X^a_R = 0. \quad (41)$$

The equations (40) are nothing but the definition of complex coordinates $Z^M_L = (X^a_L = X^a + i \Theta^I \sigma^a \bar{\Theta}^I, \Theta^I)$ of a chiral subspace of the $N = 2, D = 4$ superspace, which in their turn are chiral superfields in the $N = 1, D = 4$ superworldvolume.

We have thus obtained that the conditions imposed on the embedding of the D3–brane imply that the superembedding is performed in such a way that the chiral subsuperspace of the superworldvolume gets mapped into the chiral subsuperspace of the target superspace.

### 2.3. Linearized limit

We shall now demonstrate that in the physical gauge (3) and in a linearized limit in worldvolume superfields the gauge field constraint (15) gives rise to standard constraints on the field strength of the Maxwell field supermultiplet.
Upon imposing the physical gauge \( \Theta^\alpha(z) \) the only independent (chiral) variable which remains in the model is the Volkov–Akulov Goldstone superfield \( \Theta^{2\alpha}(z) \), to which the gauge field constraint \( F_{ab}(z) \) is related via eq. (23).

To be able to perform a correct linearization limit we should choose \( \Theta^{2\alpha}(z) \) in the form
\[
\Theta^{2\alpha}(z) = \eta^\alpha + W^\alpha(z),
\]
where \( W^\alpha(z) \) is a chiral worldvolume superfield.

This choice can be understood with the following reasoning. When there is no a gauge field on the D3–brane worldvolume \( F_{ab}(z) = 0 \). Then the integrability (18) of the gauge field constraint (15) reduces to
\[
2i E^a \wedge (E^{1a} \wedge E^{1\bar{a}} - E^{2a} \wedge E^{2\bar{a}}) \sigma_{a_0 \bar{a}_0} = 0,
\]
which is satisfied if we choose \( E^{1a} = E^{2\alpha} \) along superworldvolume. Hence, in the static gauge this “vacuum” configuration of the D3–brane can be associated with the map \( \eta^\alpha = \Theta^{1\alpha} = \Theta^{2\alpha} \), and fluctuations around this solution are described by the chiral superfield \( W^\alpha(z) \) of eq. (42).

We shall now analyze, in the static gauge (4), the consequences of the superembedding (chirality) conditions (38), (39) and the integrability condition (18) in the linear order in the fields \( W^\alpha(z) \) and \( F_{ab}(z) \). From (38), (39) and (18) we find that in the linear approximation
\[
\begin{align*}
D_\alpha &= D_\alpha + i(\sigma^a \hat{W}_a)_{\bar{a}0} \partial_\alpha + iD_\alpha W^{\bar{a}0} \bar{\eta} \partial_{\bar{a}0}, \\
\bar{D}_{\bar{a}} &= \bar{D}_{\bar{a}} + i(W^{\sigma}_a)_{\bar{a}0} \partial_\alpha + i\eta^a \bar{D}_{\alpha} \hat{W}_a, \\
\end{align*}
\]
where
\[
\begin{align*}
D_\alpha &= \frac{\partial}{\partial \eta^\alpha} + 2i(\sigma^a \bar{\eta})_{\bar{a}0} \frac{\partial}{\partial \xi^a}, \\
\bar{D}_{\bar{a}} &= \frac{\partial}{\partial \bar{\eta}^\alpha} + 2i(\eta^a \bar{\eta})_\alpha \frac{\partial}{\partial \bar{\xi}^a} 
\end{align*}
\]
are flat covariant derivatives.

Note that in the linear approximation \( W^\alpha(z) \) satisfies the flat superspace chirality condition
\[
\bar{D}_{\bar{a}} W^\alpha = 0, \quad D_\alpha W^\alpha = 0.
\]

Finally eq. (23) reduces to
\[
F_{ab} = \frac{1}{4} \bar{D}^{\alpha\bar{a}} (\sigma_{\alpha\bar{a}} D_\alpha W^{\bar{a}} + \sigma_{\bar{a}0} \bar{D}_{\alpha} W^\alpha),
\]
which, in particular, implies that
\[
D^\alpha W_\alpha + \bar{D}_{\bar{a}} W^{\bar{a}} = 0.
\]

In equations (40) and (48) one can recognize the standard constraints on the field strength superfield of a Maxwell supermultiplet. They arise as the linear approximation of the Goldstone superfield constraints (38) and (27).

The Maxwell superfield equations of motion
\[
D^\alpha W_\alpha - \bar{D}_{\bar{a}} W^{\bar{a}} = 0.
\]
arise as the linearized approximation of the D3-brane superfield equation (57).

We have thus demonstrated that the choice of the basic superembedding condition (4) and the gauge field constraint (15) is consistent with the linearized limit of the D3–brane model which is \( N = 1, D = 4 \) supersymmetric Maxwell theory.

3. The D3–brane action

We now present a worldvolume superfield action which we assume to produce upon integrating over Grassmann coordinates and solving for the auxiliary fields the standard action \([23]\) for the D3–brane coupled to an \( N = 2 \) supergravity.

The D3–brane couples to supergravity fields via the worldvolume pullback of the Wess–Zumino form \([23]\)
\[
\hat{C} = C_4 + F_2 \wedge C_2 + \frac{1}{2} F_2 \wedge F_2 C_0,
\]
where \( F_2 \) is defined in (14) and \( C_p \) (p=0,2,4) are ‘Ramond-Ramond’ p–form fields.

Since, as we have shown in Subsection 2.2, the superembedding conditions imply chirality of the worldvolume superfields we assume the action to be an integral over \( N = 1, D = 4 \) chiral superspace \( Z_L = (\xi^\alpha, \eta^\alpha) \) of an appropriate pullback component of \( \hat{C} \). Such a structure is prompted by the form of the worldvolume superfield actions for a heterotic string \([24]\) and a supermembrane \([23]\). Because of the dimensional reasons the Lagrangian is constructed with the use of \( \hat{C}_{\alpha\beta a} \), and the action (accompanied by the superembedding condition (14)) has the following formally simple form
\[
S = \int d^2 \xi_L d^2 \eta \xi_L \sigma^{a\bar{a}\bar{b}\bar{c}} \hat{C}_{\alpha\beta a} + h.c.
\]
where $E_L = s\text{det}(e_{A}^{\beta}) \ det(\eta_{ab} - F_{ab}) \ det^{-1}h_{\alpha}^{\beta}$ is the chiral measure $D_{A}E_L = 0$, and $\sigma^{ab}$ is the antisymmetrized product of the Pauli matrices.

Upon integration over $\eta$ (51) should produce both the Dirac–Born–Infeld and the Wess–Zumino term of the standard D3–brane action. In the static gauge (4) and in the linearized limit (41), (46) and (48) the action (51) reduces to the superfield Maxwell action.

4. CONCLUSION

Using the superembedding approach we have shown that the off–shell dynamics of the D3–brane in $N = 2$, $D = 4$ target superspace is described by the worldvolume superfield (superembedding) conditions (3) and (15). In the static gauge they reduce to nonlinear off–shell constraints on the spinor (Goldstone) superfield strength of the Dirac–Born–Infeld supermultiplet which generalize the Maxwell superfield linear constraints. This establishes the link of the superembedding formulation of the D3–brane with the nonlinear realization method used by Bagger and Galperin [5] to construct the $N = 1$, $D = 4$ superfield formulation of the Dirac–Born–Infeld theory as a Volkov–Akulov–type model exhibiting partial breaking of $N = 2$ supersymmetry down to $N = 1$.

The detailed analysis of the D3–brane superworldvolume action (51) and its relation to the Goldstone–Maxwell superfield action and equations of motion of [5] will be given elsewhere.

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