On the orbital period modulation of RS CVn binary systems

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ABSTRACT

The Applegate hypothesis proposed to explain the orbital period modulation of RS Canum Venaticorum (RS CVn) close binaries (Applegate 1992) is considered in the framework of a general model to treat the angular momentum exchanges within the convective envelope of a magnetically active star. This model assumes that the convection zone is strictly adiabatic and that the Taylor–Proudman balance holds, leading to an internal angular velocity constant over cylindrical surfaces co-axial with the rotation axis. It turns out that the angular velocity perturbations, whatever their origin, can be expressed in terms of the eigenfunctions of the equation of angular momentum conservation with stress-free boundary conditions. Moreover, a lower limit for the energy dissipation rate in a turbulent convection zone can be set, thanks to the extremal properties of the eigenfunctions. This approach allows to apply precise constraints on the amplitude and the radial profile of the angular velocity variations that are required to explain the observed orbital period changes in classical RS CVn binaries (i.e. with orbital period longer than 1–2 d and a subgiant secondary component). It is found that an angular velocity change as large as 10 per cent of the unperturbed angular velocity at the base of the stellar convection zone is needed. Such a large change is not compatible with the observations. Moreover, it would produce an energy dissipation rate much larger than the typical luminosities of the active components of RS CVn systems, except in the case that fast rotation and internal magnetic fields reduce the turbulent viscosity by at least 2 orders of magnitude with respect to the value given by the mixing-length theory. Therefore, the model proposed by Applegate should be rejected, at least in the case of classical RS CVn close binaries. Possible alternative models are briefly discussed, emphasizing the effects of intense magnetic fields (∼10 T) on the internal structure of magnetically active stars and the dynamics of close binary systems.

Key words: MHD – stars: activity – binaries: close – stars: magnetic fields – stars: rotation.

1 INTRODUCTION

The close binaries belonging to the RS Canum Venaticorum (RS CVn) class are detached systems with component stars of late spectral types and typical orbital periods between 1 and 15 d. The secondary components are subgiants with sizeable outer convective envelopes forced to rotate quasisynchronously with the orbital motion by intense tidal forces. This implies a strong influence of the Coriolis force on the convective turbulence as parametrized by values of the Rossby number significantly smaller than unity and provides the basic conditions for an intense solar-like magnetic activity maintained by a powerful hydromagnetic dynamo (see, e.g. Parker 1979; Hall 1991; Strassmeier 2001). The magnetic fields generated by the dynamo action produce spatially inhomogeneous and transient phenomena in the atmospheres of the active component stars, as observed in different domains of the electromagnetic spectrum, and represent the analogous of solar activity phenomena, although involving magnetic field energies at least 2 or 3 orders of magnitude larger (e.g. Rodonò 1992; Guinan & Gimenez 1993; Strassmeier et al. 1993; García-Alvarez et al. 2003 and references therein).

The long-term timing of the eclipses or of the radial velocity curves has shown that the orbital periods of RS CVn binary systems are oscillating with typical relative amplitudes of \( \Delta P / P \sim (1–3) \times 10^{-5} \) and time-scales of about 30–50 yr. For the systems with the longest observational records, long-term oscillations on time-scales of the order of a century have also been detected, with an amplitude comparable to that of the oscillations on the time-scale of decades (see Lanza & Rodonò 2004 and references therein). Hall (1989, 1991) noticed the connection between orbital period modulation and the presence of a late-type fast-rotating secondary in RS CVn binary systems, as well as in Algols and other binaries, and concluded that the modulation of the orbital period is likely to be related to their magnetic activity. RS CVn systems are particularly suited to investigate the correlation between magnetic activity and orbital dynamics because the dynamical effects of mass exchange or of
mass loss are negligible and light-time effects due to a third body may be excluded in most of the cases (e.g. Lanza & Rodonò 2004; Frasca & Lanza 2005).

Applegate (1992), Lanza, Rodonò & Rosner (1998) and Lanza & Rodonò (1999) presented theoretical models aimed at relating the orbital period modulation to the operation of a hydromagnetic dynamo in the active component star. The Applegate model assumes that a few per cent of the internal angular momentum of the active component is cyclically exchanged between an inner and an outer convective shell due to a varying internal magnetic torque versus the activity cycle phase. This modifies the oblateness and the gravitational quadrupole moment of the active star, which oscillates around its mean value. When the quadrupole moment is maximum, the companion star feels a stronger gravitational acceleration, so that it is forced to move closer and faster along its orbit, thus attaining the minimum orbital period. Conversely, when the quadrupole moment is minimum, the orbital period attains its maximum value.

Lanza et al. (1998) and Lanza & Rodonò (1999) extended Applegate’s considerations by including the effects of internal magnetic fields on the hydrostatic equilibrium of the active component and exploring the energy balance of the hydromagnetic dynamo by means of the virial theorem. Their considerations were mostly illustrative and did not include a detailed and consistent model of angular momentum transfer and magnetic field variations. Rüdiger et al. (2002) presented results on the gravitational quadrupole moment variations produced by a quite specific dynamo model. Therefore, more work is required to reach general and sound results. In the present work, we revisit the original Applegate model with the assumption that the angular velocity of the active component star is a function only of the distance from its rotation axis, as it is predicted by the Taylor–Proudman balance for an adiabatic rapidly rotating stellar convection zone. This assumption will allow us to provide a general treatment of the angular momentum transport within a stellar convection zone, which is independent of the specific processes responsible for its redistribution along the activity cycle. In such a way, we find that the original Applegate’s model fails to explain the observed orbital period modulations in a typical RS CVn binary system. Therefore, we discuss other possible models to explain the suggested connection between orbital period modulation and magnetic activity.

2 THE MODEL

2.1 Hypotheses and basic equations

We developed a simplified dynamical model in the framework of the Taylor–Proudman balance for a strictly polytropic, i.e. adiabatic, stellar convection zone. In the momentum equation, the Coriolis force is assumed to be balanced by the gradients of the pressure and a turbulent dynamical viscosity, \( \tilde{\rho} \), which is the magnetic permeability, \( \rho \), the density and \( v \) the velocity perturbation induced by the Lorentz force. By assuming \( B = 1 \) T, \( \Omega \) = 2.5 \( \times \) 10\(^{-3} \) s\(^{-1} \), \( \rho \) = 10\(^7\) kg m\(^{-3}\), \( R = 4 \), \( v = 100 \) m s\(^{-1}\), we find \( E = 2.8 \times 10^{-4} \), confirming that the perturbations produced by the Lorentz force should not invalidate our approximation.

Because we assume that our convection zone is strictly adiabatic, the energy equation reduces to the constancy of the specific entropy, and only the mass continuity equation and the equation of conservation of the angular momentum need to be considered. We assume an inertial reference frame with the origin at the stellar baricentre and the \( \xi \) axis along the rotation axis; moreover, we consider a cylindrical polar coordinate system (\( s, \zeta, \varphi \)) with \( s \) distance from the \( \xi \) axis, \( \zeta \) the coordinate along the rotation axis and \( \varphi \) the azimuthal coordinate. Adopting a mean-field description, we write the velocity \( \mathbf{v} = \mathbf{V} + \mathbf{v}' \), where \( \mathbf{v} \) is the mean velocity and \( \mathbf{v}' \) its fluctuation with respect to the mean value at a given point and time. The averaging operation giving the mean quantities is indicated by an overbar, viz.: \( \mathbf{v} = \overline{\mathbf{V}} \).

By neglecting the density perturbations (the so-called anelastic approximation), the equation of mass continuity becomes

\[
\nabla \cdot (\rho \mathbf{v}) = 0,
\]

where \( \rho \) is the unperturbed density. The equation of conservation of the angular momentum can be written as (cf. Rüdiger 1989, section 4.1)

\[
\frac{\partial}{\partial t}(\rho s \mathbf{v}_s) + \nabla \cdot \mathbf{\Theta} = 0,
\]

where \( t \) is the time and \( \mathbf{\Theta} \) is the angular momentum flux vector defined in terms of the stress tensor \( T_{ik} \) as

\[
\mathbf{\Theta}_i = s T_{ik} v_k.
\]

The expression of the stress tensor in the mean-field description is

\[
T_{ik} = \rho v_i v_k + \Lambda_{ik} - \eta_i \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) + \frac{1}{\tilde{\rho}} (B_i B_k + M_{ik})
\]

\[
+ \delta_{ik} \left( \mu + \frac{B_i^2}{2\tilde{\rho}} \right),
\]

where \( \Lambda_{ik} = \rho \overline{v_i v_k}' \) is the turbulent Reynolds stress tensor, \( \eta_i \) the turbulent dynamical viscosity, \( B \) the magnetic field, \( M_{ik} = B_i' B_k' \) the Maxwell stress tensor arising from the average of the fluctuations of the magnetic field, \( \mu \) the pressure and \( \delta_{ik} \) the Kronecker delta. The molecular viscosity of the plasma has been neglected in comparison to the turbulent viscosity.
It is convenient to define the average of a generic scalar quantity \( f \) over the lateral surface of the cylinder of radius \( s \) co-axial with the rotation axis \( \xi \):

\[
(f) = \frac{1}{C(s)} \int_{0}^{2\pi} \int_{\sqrt{s^2 - \xi^2}}^{s} f(s, z, \varphi) \, dz \, d\varphi,
\]

where \( R \) is the star radius and \( C(s) = 4\pi s \sqrt{s^2 - \xi^2} \) is the area of the lateral surface of the cylinder. The average of a quantity that can be expressed as the azimuthal gradient of a single-valued function, i.e. \( f(s, z, \varphi) = \frac{\partial s}{\partial \varphi} \) vanishes, whereas the average of the divergence of a vector that vanishes over the surface of the star is given by

\[
(\nabla \cdot \mathbf{u}) = \frac{1}{C(s)} \frac{\partial}{\partial s} \left[ C(s) \langle u_s \rangle \right].
\]

The proof of equation (6) can be obtained by applying equation (5) to the divergence of a vector expressed in cylindrical polar coordinates.

By applying equation (6) to equation (11) and keeping into account that the component of the flow velocity orthogonal to the stellar surface is bound to vanish, we find that

\[
\langle \rho v_s \rangle = 0,
\]

because \( v_s \) is a function of \( s \) only, thanks to the Taylor–Proudman approximation, and therefore it is not affected by the average operation. In a similar way, the average of equation (2) over a cylindrical surface yields

\[
\frac{\partial s}{\partial t} + \frac{1}{C(s)} \frac{\partial}{\partial s} \left[ C(s) \langle \Theta \rangle \right] = 0,
\]

where \( \langle \Theta \rangle \), in view of equation (7), is given by

\[
\langle \Theta \rangle = -s \langle \eta \rangle \left( \frac{\partial v_s}{\partial s} - \frac{v_s}{s} \right) + s \langle \Lambda \rangle,
\]

Introducing the angular velocity \( \omega(s, t) \equiv v_s(s, t)/s \), equation (8) can be written in the form

\[
C(s)(\rho)^2 \frac{\partial \omega}{\partial t} + \frac{\partial}{\partial s} \left[ C(s) \langle \eta \rangle \frac{\partial \omega}{\partial s} \right] = 0,
\]

where we put the terms containing \( \omega \) in the left-hand side and all the other terms, representing the source of variation of \( \omega \), in the right-hand side of the equation. The solution of equation (10) must verify the stress-free boundary conditions at the extreme points of the interval \([s_1, s_2]\) within which it is defined, i.e.

\[
\left. \frac{\partial \omega}{\partial s} \right|_{s_1, s_2} = 0.
\]

This ensures that the angular momentum flux outside of the interval \([s_1, s_2]\) vanishes, i.e. the total angular momentum is conserved.

\[ \]
2.3 Variation of the gravitational quadrupole moment

The outer gravitational potential of the secondary component of an RS CVn binary can be written as

$$\Phi(x) = \frac{GM}{r} + \frac{3}{2} GQ_{ki} \frac{x_i x_k}{r^2} + O(r^{-5})$$  \hspace{1cm} (20)

where $x$ is the position vector, $r$ the radial distance from the barycentre of the star, $M$ its mass, $G$ the gravitational constant, $Q_{ki}$ the quadrupole moment tensor, which is defined as the traceless part of the inertia tensor $I_{ik}$:

$$Q_{ki} = I_{ki} - \frac{1}{3} \delta_{ik} Tr I,$$  \hspace{1cm} (21)

with the inertia tensor defined by

$$I_{ki} = \int_V \rho x_i x_k \, dx,$$  \hspace{1cm} (22)

where $V$ is the volume of the star (cf. Chandrasekhar 1961). Considering a Cartesian reference frame with the $\hat{x}$ axis in the direction of the line joining the barycentres of the two component stars, only the $Q_{xx}$ component is important for the variation of the orbital period and its relative perturbation can be written as

$$\frac{\Delta P}{P} = -9 \frac{\Delta Q}{Ma^2},$$  \hspace{1cm} (23)

where $\Delta Q$ indicates the variation of $Q_{xx}$ and $a$ is the semi-major axis of the relative orbit assumed to be circular (cf. Applegate 1992; Lanza & Rodonò 1999). Because the masses of the active component stars in classical RS CVn systems are quite comparable ($M \sim 1.0-1.3 \, M_\odot$), equation (23) shows that the orbital period variation mainly depends on the quadrupole moment change $\Delta Q$ and the orbital period $P$. Considering equation (11) in Lanza et al. (1998) and the fact that the ratio $a/R \approx 4$ in RS CVn systems, it follows that $\Delta P$ scales linearly with $P$ if the relative variations of the kinetic and magnetic energies are similar for different systems (see Lanza & Rodonò 1999). Such a dependence is in a general agreement with presently available observations (cf., for example, fig. 1 in Rädiger et al. 2002).

The variation of the gravitational quadrupole moment of the secondary component of a typical RS CVn binary is computed following the approach of Ulrich & Hawkins (1981), as described in detail in Appendix A. In the framework of the original Applegate model, we assume that only the variation of the internal rotation affects $Q$. Its variation is computed with respect to the state of rigid rotation with the same total angular momentum. This assumption corresponds to the hypothesis that the unperturbed star has the minimum kinetic energy of rotation compatible with its total angular momentum.

Because the relative variation of the angular velocity is at most a few per cent of its unperturbed value, $\Delta Q$ depends linearly on the angular velocity change and can be expressed as

$$\Delta Q = \sum_k \delta Q_k,$$  \hspace{1cm} (24)

where $\delta Q_k$ is the quadrupole moment change associated with the term $\alpha_k \xi_k$ in equation (15).

2.4 Kinetic energy variation and dissipation

The variation of the kinetic energy of the secondary star with respect to the unperturbed state of rigid rotation is

$$\Delta T = \sum_k \delta T_k = \frac{1}{2} \sum_k E_k \alpha_k^2,$$  \hspace{1cm} (25)

where $\delta T_k$ is the variation associated with $\alpha_k \xi_k$ and the terms of the first order in $\alpha_k$ vanish owing to the conservation of the angular momentum. In the Applegate model, the angular momentum is exchanged back and forth between an inner and an outer shell in the stellar convection zone producing a periodic change of the kinetic energy of the convection zone itself. However, the process is not reversible because turbulent convection introduces energy dissipation whenever angular velocity gradients are present. A fraction of the kinetic energy is injected into the turbulent Kolmogorov cascade and is eventually dissipated at the length-scales at which molecular viscosity becomes important. The amount of kinetic energy dissipated per unit time can be derived from the turbulent stress tensor (e.g. Landau & Lifshitz 1959; Chandrasekhar 1961):

$$\frac{d\mathcal{T}}{dt} = -\int_{\xi_1}^{\xi_5} C(s) \langle \eta_i \rangle s^2 \left( \frac{\partial \omega}{\partial s} \right)^2 \, ds.$$  \hspace{1cm} (26)

An important result on the minimum dissipated energy for a given kinetic energy variation can be derived taking into account the extramal properties of the eigenfunctions $\xi_k$ (see Morse & Feshbach 1953). More precisely, the minimum value of $\frac{d\mathcal{T}}{dt}$ is attained when the radial profile of the angular velocity variation is proportional to the eigenfunction of the lowest order, i.e. $\omega(s, t) = \alpha(t) \xi_1(s)$:

$$\left( \frac{d\mathcal{T}}{dt} \right)_{\min} = -2\lambda_1 \delta T_1 = -E_1 \alpha^2(t).$$  \hspace{1cm} (27)

This result will be applied in the next section to evaluate the minimum amount of energy dissipated during one cycle of the quadrupole moment variation in a typical RS CVn secondary component.

3 APPLICATION TO A STELLAR MODEL

The catalogue by Strassmeier et al. (1993) lists the basic parameters of the components of RS CVn binaries. Considering systems with orbital periods longer than 1–2 d, a typical secondary component is a subgiant star with a mass $M = 1.0-1.3 \, M_\odot$, a radius $R = 3.0-4.0 \, R_\odot$ and a luminosity of $L \sim 4-8 \, L_\odot$. As a typical angular velocity, we assumed $\omega_0 = 2.569 \times 10^{-5}$ s$^{-1}$, which corresponds to a synchronous orbital period of 2.83 d, that of the very active RS CVn system HR 1099 (Frasca & Lanza 2005). An internal structure model for a non-rotating star was obtained by means of the Dartmouth Stellar Evolution Web Server\(^1\), which allowed us to run a specialized version of the stellar evolution code described by Charbonneau et al. (2001) and Guenther et al. (1992). Specifically, the present model was obtained starting from an initial mass of $M = 1.3 \, M_\odot$ with chemical abundances $X = 0.705$, $Y = 0.275$, $Z = 0.020$ and allowing the star to evolve for 4.583 Gyr. At the considered evolutionary stage, the model radius and luminosity are $R = 4.047 \, R_\odot$ and $L = 8.298 \, L_\odot$, $L = 3.20 \times 10^{27}$ W, respectively, giving an effective temperature of 4871 K. The model quantities are shown in Fig. 1. The base of the convection zone is located at a fractionary radius $r_h/R = 0.181$ or, in terms of the mass coordinate, at $m_h/M = 0.240$. The velocity of convective motions $u_c$ is computed from the mixing-length theory, by assuming that the convective flux is given by $F_{\text{conv}} = (10/\alpha_m) \rho u_c^3$, where $\alpha_m = 1.5$ is the ratio of the mixing-length to the local pressure scaleheight, and that the energy is transported through the envelope only by convection (cf. Kippenhahn & Weigert 1990). This yields

\(^1\)http://stellar.dartmouth.edu/
as anticipated in Section 2, the eigenvalues increase monotonically with the radial order \( k \) and the number of nodes of the corresponding eigenfunctions in \([s_1, s_2]\) is equal to \( k \). The first eigenvalue is the smallest and thus corresponds to the longest characteristic time-scale for angular momentum transfer through the convection zone under the action of the turbulent dynamical viscosity. It is \( \lambda_{k1}^{-1} \approx 0.6 \text{ yr} \), i.e. much smaller than the time-scale for quadrupole moment variation, which is equal to the length of the cycle of the orbital period modulation (cf. Section 1). Therefore, equation (18) gives \( \alpha_\ell \sim \lambda_{k1}^{-1} \beta_\ell \).

In order to estimate the effects of the terms that are responsible for the angular momentum redistribution within the convective zone, it is useful to consider equation (19) together with Fig. 4 showing the plots of the \( \xi_k \). Only the derivative of the eigenfunction of order \( k = 1 \) has no radial nodes in \([s_1, s_2]\). The derivative of \( \xi_2 \) has \( k - 1 \) nodes for \( k \geq 2 \), which implies \( k - 1 \) sign changes that, generally speaking, are expected to average out the value of \( \beta_\ell \) for sufficiently high values of \( k \). Hence, we may roughly estimate the order of magnitude of the Maxwell stress \( B_\ell B_\phi \) required to produce a variation of the angular velocity of 1 per cent (for an unperturbed angular velocity \( \Omega_0 = 2.569 \times 10^{-3} \text{ s}^{-1} \)) by considering \( k = 1 \) in equation (19). We find \( B_\ell B_\phi \approx (2 - 3) \times 10^{-2} \text{T} \) giving a minimum magnetic field strength \( B_\ell \sim B_\phi \sim (1 - 1.7) \times 10^{-1} \text{T} \), comparable with Applegate’s estimate and verifying a posteriori that the Lorentz force cannot perturb significantly the Taylor–Proudman balance.

In order to evaluate the quadrupole moment variation and the associated energy change, we assume that the unperturbed state corresponds to rigid rotation with angular velocity \( \Omega_0 = 2.569 \times 10^{-3} \text{ s}^{-1} \) and we consider separately the angular velocity perturbations whose radial profiles are proportional to each of the eigenfunctions, respectively. The amplitude of the angular velocity perturbation is fixed at 1 per cent of \( \Omega_0 \) at \( s = s_1 \) in all the cases. The absolute value of the quadrupole moment variation \( |\delta Q_k| \), the corresponding orbital period change \( (\delta P/P)_k \), the kinetic energy change \( \delta T_k \) and the maximum dissipated power \( 2\lambda_k \delta T_k \) for the eigenfunctions of order \( k = 1, \ldots, 6 \) are listed in Table 1, in the columns from the third to the sixth, respectively. The orbital period change is derived from equation (23) in the case of \( \alpha = 4R \). We notice that the time average of the dissipated power is \((\sqrt{2})^{-1}\) times the maximum value listed in Table 1 in the case that the variation of the angular velocity is sinusoidal versus time.

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The important result coming from the application of our model is that the quadrupole moment variation associated with the eigenfunction of order \( k = 1 \) dominates over the variations associated with the eigenfunctions of higher order by at least a factor of 20. This is due to the fact that \( \xi_1 \) has only one node and one sign change, whereas \( \xi_k \) for \( k > 1 \) has \( k \) nodes and sign changes that average out the effects of the angular velocity perturbation on the gravitational quadrupole moment. The cancellation effect is particularly pronounced for the radial order of the angular velocity perturbation on the gravitational quadrupole moment.

The variation is expressed by the following equation:

\[
\Delta Q \equiv Q(t) - Q_0 = q_1 \alpha(t),
\]

where \( q_1 \) is a factor of proportionality that can be easily derived from Table 1. The corresponding kinetic energy variation and the minimum dissipated power are given by

\[
\Delta T = \frac{1}{2} E_1 \alpha_i^2(t) \quad \text{(31)}
\]

and

\[
P_{\text{diss}} \sim \lambda_1 E_1 \alpha_i^2(t) = \frac{\lambda_1 E_1}{q_1^2} (\Delta Q)^2, \quad \text{(32)}
\]

where we have used equation (30) to express the instantaneous power in terms of the quadrupole moment change that, in turn, can be related to the observed orbital period variation by equation (23).

Specifically, by inserting the numerical coefficients, equation (32) can be recast as

\[
P_{\text{diss}}(t) \simeq 5.16 \times 10^{38} \left( \frac{\Delta P}{P} \right)^2 \text{W}, \quad \text{(33)}
\]

which gives the minimum dissipated power for a given orbital period change for the assumed stellar model.

The typical relative amplitude of the orbital period variations in RS CVn binaries is of the order of \((1–3) \times 10^{-5}\), i.e. 1 order of magnitude larger than the variation found from our model for a perturbation of the angular velocity with an amplitude of 1 per cent and a radial profile proportional to the eigenfunction of order \( k = 1 \). Assuming that the variation is sinusoidal in time, the average dissipated power is \( \sim 1.37 \times 10^{29} \text{W}\), i.e. \( \sim 43 \) times the stellar luminosity.

In consideration of these results, the conclusion is that the Ap-plegate hypothesis must be rejected, at least in the case of classical RS CVn binaries: the required angular velocity variations are of the order of 10 per cent, i.e. 1 or 2 orders of magnitude larger than the variations inferred from the observations (e.g. Collier Cameron & Donati 2002; Donati, Collier Cameron & Petit 2003; Lanza & Rodonò 2004 and references therein) and the kinetic energy dissipated during a cycle of the variation is too large to be supported by the stellar luminosity.
4 DISCUSSION

We apply a general, although simplified, model for the angular momentum transfer within the convective envelope of a rapidly rotating star. The key result is that the angular velocity profile can be expressed in terms of the eigenfunctions of the angular momentum conservation equation, thus allowing us to draw general conclusions on the variation of the gravitational quadrupole moment of a star produced by temporal fluctuations of its differential rotation profile. The eigenfunctions are entirely specified by the internal stratification via $\langle \rho \rangle (s)$, the average turbulent dynamical viscosity ($\eta_t$) $s$ and the boundary conditions. In previous illustrative approaches (Applegate 1992; Lanza et al. 1998; Lanza & Rodonò 1999), the variation of the internal angular velocity was only subject to the general constraint set by the conservation of the total angular momentum, but its radial dependence was left substantially free. Therefore, it was possible to assume the radial profile maximizing the quadrupole moment variation and explaining the observed orbital period changes by the Applegate mechanism with a relative amplitude of the angular velocity variation of a few per cent, without energy changes exceeding the energy made available by the stellar luminosity along a cycle of the variation. This freedom is now impossible, because our model fixes the radial profile of each eigenfunction and the minimum dissipated energy, thanks to the extremal properties of the eigenfunctions themselves. Thus, we can apply a tight energetic constraint limiting the amplitude of the internal angular velocity variation to $\approx 1$ per cent because the average dissipated power cannot exceed the stellar luminosity.

It is important to note that, in a rapidly rotating convective envelope, our estimate of the turbulent dynamical viscosity may not be correct. Unfortunately, no fully consistent theory of the interaction between turbulence, rotation and magnetic fields in stars is presently available. Considering the heuristic approach proposed by Stevenson (1979), it is possible to obtain a rough estimate in the case of our model convection zone for which the average Rossby number is about 0.0035 and the toroidal magnetic field can be assumed to be of the order of the field giving equipartition between magnetic and turbulent kinetic energy densities in the absence of rotation. Stevenson’s estimates imply a reduction of the turbulent kinematic viscosity $\eta_k$ by a factor of about 20 with respect to a non-rotating convection zone without magnetic fields. The theory based on a linearized treatment of the interaction between the turbulent velocity field and the rotation proposed by Kichatinov, Pipin & Rüdiger (1994) supports this result. It shows that $\eta_k$ is indeed a tensor and that the value of the component mediating the viscous stresses in the $s$ direction may be reduced by $\approx 30$ times for Rossby numbers smaller than 0.1 with respect to the scalar value given by the mixing-length theory. Therefore, a reduction by a factor of about 50–100, needed to reduce the power dissipated in the Applegate model to a sustainable value, cannot be excluded in the convection zones of rapidly rotating stars such as the secondary components of RS CVn systems.

In any case, the observations indicate that the relative amplitude of the unperturbed surface differential rotation in fast-rotating active close binaries and single stars does not exceed 2–3 per cent (Hall 1991; Messina & Guinan 2003) which is smaller than the required variation of $\approx 10$ per cent. Such a large variation can be rejected because it should lead to a violation of the Rayleigh criterion on the distribution of the angular momentum inside the convection zone, inducing a dynamical instability (Chandrasekhar 1961; Kippenhahn & Weigert 1990).

In principle, it could be possible to reduce the amplitude of the angular velocity variation by changing the outer boundary condition or by assuming a linear superposition of modes of different order $k$. However, the typical time-scale of the variation of the angular velocity required to explain orbital period modulation is much smaller than the typical time-scale of angular momentum loss due to a magnetized stellar wind and at least 2 or 3 orders of magnitude smaller than the tidal time-scale in RS CVn binaries (cf. DeCampli & Baliunas 1979). Therefore, it is unlikely that the angular momentum loss due to the stellar wind or the tidal torque can change the outer boundary condition. A superposition of modes of different radial orders is also unlikely to significantly reduce the amplitude of the differential rotation variation because of the energetic constraint. In order to give $\omega = 0$ at $s = s_0$, a linear combination of, for example, the modes with $k = 1$ and $k = 2$ would require comparable amplitudes for both modes leading to an increase of the dissipated power by a factor of $\approx 2–3$.

Recent non-linear dynamo models, including the back-reaction of the Lorentz force on the differential rotation, give amplitudes of the temporal variation of the differential rotation of $\approx 0.1$ per cent for stars with a deep convective envelope like those considered in the present paper (Covas, Moss & Tavakol 2005). Such amplitudes produce an energy dissipation rate well below the stellar luminosity taking into account that they are not very extended in latitude.

As a matter of fact, the differential rotation is in general a function also of the $z$ coordinate in the Sun and solar-like stars. This dependence leads to a centrifugal force that cannot be expressed as the gradient of a potential and drives a meridional circulation. From a qualitative point of view, the gravitational quadrupole moment is mainly affected by the potential component of the centrifugal force and the meridional circulation is usually neglected in its computation (cf., for example, Ulrich & Hawkins 1981). Therefore, our assumption of a strictly potential centrifugal force field is expected to lead to an upper limit for the quadrupole moment variation for a given kinetic energy change.

In consideration of the difficulties encountered by the original Applegate hypothesis, a new model is needed to explain the orbital period modulation in RS CVn binaries. Lanza et al. (1998) and Lanza & Rodonò (1999) pointed out the role of the Lorentz force as a possible source of perturbation of the quadrupole moment, thus reducing the amplitude of the angular velocity variations. Rüdiger et al. (2002) moved along the same line, trying to build a consistent model by considering an $r^2$ dynamo with an antisymmetric and inhomogeneous $\alpha$ tensor to produce oscillatory magnetic fields inducing quadrupole moment fluctuations. The main problem with these models is the large amplitude of the magnetic field required to affect the quadrupole moment of the star. From the virial theorem, an average field strength of at least 10 T can be estimated (Lanza & Rodonò 1999). Azimuthal fields with such a large strength cannot be stably stored within a convective envelope and the only reasonable possibility is that the magnetic field is directed vertically, as it is assumed for the subsurface magnetic fields of sunspot groups. In this case, the most important contribution to the quadrupole moment perturbation should come from an anisotropic magnetic pressure, i.e. depending on the latitude. Specifically, an estimate of the effects of a radially directed mean field can be easily derived for a polytropic convection zone by equation (A7), including the magnetic pressure term in the perturbation of the total potential.

Several questions remain to be addressed by future work on models including the dynamical effects of magnetic fields. An important one is the explanation of the observed relationship between the
orbital period cycle and the spot activity cycle in RS CVn binaries, as described in detail by Lanza et al. (1998) and Lanza & Rodonò (2004). Alternative models invoking the thermal effects of the magnetic fields to change the structure of the convection zone (and thus the apsidal motion constant k2 in equation 10 of Lanza et al. 1998) have been proposed, but a detailed analysis has not been presented yet. The main difficulty with such models is the large heat capacity of the stellar convection zone that would not allow the required changes to take place on time-scales of a few decades, as required by the observations (cf. Spruit 1982; Spruit & Weiss 1986).

It is important to acquire simultaneous data on orbital period changes and star-spot activity to test the proposed models, not only in the case of RS CVn binaries but also in the case of other classes of close binaries showing similar phenomena. In particular, the dynamics of overcontact binaries, although significantly affected by mass exchange, may shed light on the role of magnetic activity on the orbital period modulation (cf., for example, Qian & Yang 2005).

5 CONCLUSIONS

A general model for the angular momentum transport within a stellar convective envelope under the action of turbulent viscosity and the Maxwell and Reynolds stresses has been discussed. It is based on the simplifying assumptions that the convection zone is strictly adiabatic and that the Taylor–Proudman balance holds, as it is expected for rapidly rotating convection zones.

In the framework of this model, we consider the Applegate hypothesis to explain the orbital period variations observed in classical RS CVn close binary systems and find that the required variation of the internal differential rotation is too large to agree with the observations and to be supported against turbulent dissipation.

Therefore, a reconsideration of the hypothesis, including the effects of the Lorentz force on the gravitational quadrupole moment, or an entirely new theoretical framework is needed to interpret the observed orbital period variations in magnetically active RS CVn binaries.

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APPENDIX A: CALCULATION OF THE GRAVITATIONAL QUADRUPOLE MOMENT

In order to compute the gravitational quadrupole moment of a star for different internal angular velocity profiles, we applied the perturbative approach described by Ulrich & Hawkins (1981). Taking into account that ω depends only on the distance from the rotation axis x, we generalize the treatment of Ulrich & Hawkins and obtain the equation for the quadrupole component of the gravitational potential:

$$\frac{d^2 \Phi_{12}}{dr^2} + \frac{2}{r} \frac{d \Phi_{12}}{dr} - \frac{6}{r^2} \Phi_{12} = \frac{4\pi \sigma}{m(r)} \left( \frac{d \omega}{dr} \right) \left[ \Phi_{12} - r^2 b_2(r) \right], \quad (A1)$$
where $\Phi_{12}(r)$ is the quadrupole component of the interior gravitational potential, $r$ is the radial distance from the baricentre of the star, $m(r)$ is the mass of the star inside the radius $r$, $\rho$ is the density of the unperturbed star and $b_2(r)$ is the function appearing in equation (A18) of Lebovitz (1970). It can be expressed in terms of the functions $a_0$ and $a_2$, also defined in equation (18) of Lebovitz (1970), as

$$b_2(r) = \frac{5}{6}a_0(r) + \frac{1}{3}a_2(r), \quad (A2)$$

where:

$$a_0(r) = \frac{1}{2} \int_{-1}^{1} (1 - \mu^2)\mu^2(r\sqrt{1 - \mu^2})d\mu, \quad (A3)$$

$$a_2(r) = \frac{5}{4} \int_{-1}^{1} (1 - \mu^2)\mu^2(r\sqrt{1 - \mu^2})(3\mu^2 - 1)d\mu, \quad (A3)$$

where $\mu \equiv \cos\theta$ and $\theta$ is the colatitude, i.e., $s = r\sin\theta = r\sqrt{1 - \mu^2}$ and $\Omega(s) = \Omega_0 + \omega(s,t)$ is the angular velocity at a given time $t$. The solution of equation (A1) must satisfy the matching condition with the outer gravitational potential at the surface $r = R$, i.e.,

$$\Phi_{12}(R) + 3\Phi_{12}(R)/R = 0, \quad (A4)$$

while close to the centre $\Phi_{12}(r) = Cr^2$. In order to solve equation (A1) with the appropriate boundary condition, we apply a shooting method by integrating outwards from the centre of the star, varying the trial constant $C$ until equation (A4) is satisfied. The gravitational quadrupole moment $Q$ appearing in equation (23) is then given by

$$Q = -R^3 \Phi_{12}(R)/3G, \quad (A5)$$

where $G$ is the universal gravitational constant.

We checked the correctness of our code for the integration of equation (A1) by computing the gravitational quadrupole moment in the case of rigid rotation and comparing the result with equation (10) in Lanza et al. (1998), where the apsidal motion constant for our stellar model, computed by integrating the Radau equation (cf. Kopal 1978), was found to be $k_1 = 0.069$. Other tests were made by considering arbitrary internal rotation profiles for polytropic stellar models with indexes $n = 1.5$ (adiabatic convection zone) and $n = 2$. In all the cases, the values of the gravitational quadrupole moment $Q$ as derived by our code agreed with those expected within 1–1.5 per cent.

More precisely, in the case of a polytropic model, the density perturbation $\delta\rho$ is linked to the perturbation of the total potential $\delta\omega$ (gravitational plus centrifugal) by a simple algebraic equation, obtained by differentiating equation (10) of Ostriker & Mark (1968) (cf. also Chandrasekhar & Lebovitz 1962):

$$\delta\rho = \frac{n}{K(n+1)}\rho \frac{\omega}{\rho} \delta\omega,$$  

where $n$ is the polytropic index and $K$ the polytropic constant appearing in the polytropic relation between pressure and density $p = K\rho^{n+1}$. The component of the Poisson equation to be integrated in order to obtain the internal quadrupole potential reads (following Ulrich & Hawkins 1981)

$$\frac{d^2\Phi_{12}}{dr^2} + \frac{2}{r} \frac{d\Phi_{12}}{dr} - \frac{6}{r^2} \Phi_{12} = -4\pi G \rho_{12}(r), \quad (A6)$$

where the quadrupole component of the perturbation of the density is given by

$$\rho_{12}(r) = \frac{5n}{2K(n+1)} \int_{-1}^{1} \left[ \rho(r) \right]^{n+1} \delta\omega(r, \mu) P_2(\mu) d\mu \quad (A7)$$

and $P_2(\mu)$ is the second-order Legendre polynomial. The perturbation of the total potential can be written as

$$\delta\omega(r, \mu) = \delta\Phi + \int_{0}^{1} \sigma^2(\sigma) d\sigma, \quad (A8)$$

where the first term is the perturbation of the gravitational potential and the second term is the centrifugal potential in the case of an angular velocity depending only on $s$. By combining together the above equations, it is possible to compute the function $\Phi_{12}(r)$ for a polytrope and compare it with the result obtained by the general perturbative approach of Ulrich & Hawkins (1981) in order to test the respective integration codes.