Is there a maximum observable redshift in an open universe?

Julio A. Gonzalo
Facultad de Ciencias
Universidad Autónoma de Madrid.
Cantoblanco, 28049 Madrid
Spain

An estimate of the maximum observable redshift is obtained using only \( t_0 \cong (14 \pm 3) \times 10^9\) years, \( H_0 \cong 65 \pm 10\) \( Km sec^{-1} Mpc^{-1} \) (\( t_0H_0 \cong 0.91 \pm 0.08/0.18 \)) assuming \( \Lambda \cong 0 \). The resulting maximum redshift \( z_+ \cong 10 \) appears to give a reasonable upper limit to the highest actually observed redshifts. Some implications are discussed.

As pointed out about one quarter of a century ago by Beatriz M. Tinsley [Tinsley 1977], the dimensionless product \( H_0t_0 \) (Hubble’s constant times the time elapsed since the big bang) could turn out to be the most tractable method to elucidate the actual large scale cosmic dynamics, rather than using the dimensionless ratio \( \Omega_0 = \frac{\rho_c}{\rho_m} \), where \( \rho_c = \frac{3H_0^2}{\pi G} \). She pointed out that, although the problems to find well defined distance scales to get \( H_0 \), and to estimate the ages of the oldest stars to get \( t_0 \), were (and still are) quite formidable, they were probably not as difficult as those encountered to determine small redshift differences for nearby and for distant galaxies.

Her warning [Tinsley 1977] that the test of \( H_0t_0 \) ”should be kept in mind when estimates of \( H_0 \) and \( t_0 \) are refined” should be remembered now that we have improved observational data [APS News 1999]. The test requires only local data, as she noted, which implies a definite advantage.

It is well known that at very early times Einstein’s cosmological equations for \( k < 0 \) (open), \( k = 0 \) (flat) and \( k > 0 \) (closed) universes are alike in predicting \( Ht = 2/3 \) and \( \Omega \cong 1 \). This is true for a zero and a vanishingly small cosmological constant, \( \Lambda = 0 \). In this work we will use refined recent values [Astroph 1999] for \( H_0 = R_0/H_0 \cong 65 \pm 10\) \( Km sec^{-1} Mpc^{-1} \) and \( t_0 \cong (15 \pm 2) \times 10^9\) years to specify global cosmic dynamics from the open solutions of Einstein’s equations.

The total mass density can be given by \( \rho = \rho_m + \rho_\gamma + \rho_\Lambda \), where \( \rho_m \) is the matter mass density, \( \rho_\gamma \) the radiation mass density (now \( \rho_\gamma \ll \rho_m \) ) and \( \rho_\Lambda = \Lambda/8\pi G > 0 \) (moderately repulsive) is the mass density associated with small but nonvanishing cosmological constant \( \Lambda \).

For \( k < 0 \) (open universe) and \( \Lambda = 0 \) Einstein’s cosmological equation can be written as

\[
\dot{R} = R^{-1/2} \left\{ \frac{8\pi G}{3} \rho R^3 + |k| c^2 R \right\}^{1/2} =
R^{-1/2} \left\{ \frac{8\pi G}{3} \rho_\Lambda R_+^3 + |k| c^2 R \right\}^{1/2}
\]

where \( R_+ \) is defined precisely at \( \frac{8\pi G}{3} \rho_\Lambda R_+^3 = |k| c^2 R_+ \). Then, for \( R < R_+ \), the term involving \( |k| \) within the square root in \( 1 \) becomes negligible, i.e. we are in the early (Einstein-de Sitter) phase of the expansion. For \( R \geq R_+ \), on the other hand, both terms within the square root become important, and we enter into the explicitly open phase of the expansion.

The parametric solutions of \( 1 \) can be given [Cereceda et al] as a function of \( R_+ \), which has the meaning of a Schwarzschild radius for the total mass \( M_+ = \frac{4\pi}{3}\rho_\Lambda R_+^3 \) of the universe, assumed to be conserved. They are given by

\[
t = \frac{R_+}{|k|^{1/2} c} (\sinh y \cosh y - y), \quad R = R_+ \sinh^2 y
\]

It is easy to check from \( 2 \) that the dimensionless product \( Ht = (R/R)t \) is given by

\[
Ht = \frac{\{\sinh y \cosh y - y\} \cosh y}{\sinh^3 y} \geq \frac{2}{3}
\]

Present observational constraints [Astroph 1999] on \( H_0 \) and \( t_0 \) result in \( H_0t_0 \cong 0.91 > 2/3 \), with

\[
(\text{H}_0t_0)_{\text{min}} \geq 0.73 > 2/3
\]

and

\[
\Omega_0 \cong 0.2 \pm 0.1, \quad \Omega_0 = \Omega_{m0} + \Omega_{\gamma 0} + \Omega_{\Lambda 0}
\]

The data on \( H_0(65Km/sMpc) \) and \( t_0(13.7 \times 10^9\) years) which imply \( H_0t_0 \cong 0.91 \), are sufficient, together with \( 3 \) to specify the redshift \( z_+ \) corresponding to \( R = R_+ \) (Swarzschild radius), by means of

\[
y_0 = \sinh^{-1}(R_0/R_+)\frac{1}{2} \equiv \sinh^{-1}\left(\frac{1 + z_+}{1 + 0}\right) \cong 1.92
\]

taking into account that the scale factor \( R \) is related [Peebles 1993] to the redshift by means of

\[
R = R_0/(1 + z), \quad \rightarrow \quad R_+ = R_0/(1 + z_+)
\]

At \( R < R_+ \) no protogalaxies (no early quasars) could have become gravitationally bound yet. Then one is entitled to expect that \( z(R_+) \cong 10.1 \) (from \( 3 \)) is the highest
possible observable redshift in our open universe. Estimates of galaxy formation epoch redshifts [Peebles 1993] are not inconsistent with \( z_+ = z(R_+) \equiv 10 \) as an upper observable redshift limit.

After decoupling/atom formation (\( \rho_+ = \rho_m \)) taking place \[ \text{Cereceda et al.} \] at \( T_{af} \approx 3880 \text{K} \) the universe became transparent. The data on \( H_0, t_0 \) together with data on \( T_0 \) and \( T_{af} \) indicate that radiation/matter mass density equality (\( \rho_+ \equiv \rho_m \)) and atom formation (\( \rho_+(T_{af}) \)) with \( T_{af} \) as given by Saha’s law \[ \text{Cereceda et al.} \] occur at the same cosmic time.

Table I gives the Hubble’s constant, the density parameter and the redshift corresponding to atom formation density (\( \rho_{af} \)), protogalactic density (\( \rho_g \)), Schwarzschild (\( \rho_+ = \frac{M}{4\pi R_+^2} \)) and present density (\( \rho_0 \)) determined at the respective cosmic temperature. It may be noted that \( \Omega_0 \approx 0.082 \) is below the generally estimated value \( \Omega_0 \approx 0.2 \pm 0.1 \), but not very far. A non-zero \( \Lambda \) may alter the effective values of \( R_+ \) and \( T_+ \), but we may hope that changes in \( H_+, \Omega_+ \) and \( z_+ \) are not too drastic.

In summary, a direct estimate of the maximum observable redshift \( z_+ \equiv 10 \) is obtained for an open universe with negligible cosmological constant which appears to be compatible with observed redshifts for the oldest galaxies and quasars. It may be noted that recent evidence [Peerlmutter 1999, Schmidt 1998] in favor of an accelerated expansion of the universe from type Ia supernovae is consistent with \( \rho_0 \approx 1.33 \times 10^{27} \text{cm}^2 \) which is somewhat larger but of the order of \( R_+ \approx 2GM/c^2 \). This might be taken as an indirect indication that \( z_+ \approx 10 \) may be not too drastically affected by a \( \Lambda \) of this order.

In any event our expectation of \( (z_{obs})_{max} \approx 10 \) is ”testable” by comparison with new incoming data from the more modern and powerful telescopes.

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Table I. Redshifts at various cosmic events

| Event                        | $T(K)$ | $y = \sinh^{-1}(T/\gamma)^{1/2}$ | $H(y)$(Km/secMpc) | $\Omega(y)$(dimensionless) | $z$(dimensionless) |
|------------------------------|--------|----------------------------------|-------------------|-----------------------------|-------------------|
| Atom formation ($\rho_m = \rho_\gamma$) | 3880   | 0.0884                           | $1.00 \times 10^6$| 0.9922                      | 1422              |
| Protogalaxies ($\rho \cong \rho_\gamma$) | 300    | 0.3131                           | $2.26 \times 10^4$| 0.9080                      | 109               |
| Schwarzschild ($\rho_+ = M_+ / 4\pi R_+^3$) | 30.4   | 0.8813                           | 982               | 0.5000                      | 10.1              |
| Present ($\rho_0 \cong 6.54 \times 10^{-31}g/cm^3$) | 2.726  | 1.9206                           | 65                | 0.0823                      | 0                 |