In this paper we study and compare susy unification using two different approaches in order to take into account the effect of light particle thresholds on the evolution of gauge couplings: the step–function approximation, on the one hand, and a mass dependent procedure, which gives a more accurate description of the dependence of the results on the masses, on the other. We also include the effect of heavy thresholds, when $SU(5)$ is chosen as the unifying group. We find that the mass–dependent procedure excludes scenarios where all susy masses are below 1 $TeV$, and favors a value of $\alpha_3(m_Z)$ near its upper experimental bound, contrary to the results obtained with the step–function approximation. We underline the dependence of the results on the procedure chosen to deal with light thresholds.

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Experimental data on $\sin^2 \theta$ and $\alpha_e$ measured at $m_Z$ can be used to test supersymmetric grand unification theories (SGUT). Use of 2–loop RGE’s to go from $M_X$ (the unification scale) down to $m_Z$, gives a value for $\sin^2 \theta$ in good agreement with experiment \[1\]. It can also be checked that when the three gauge couplings $\alpha_3^{-1}, \alpha_2^{-1}$ and $\alpha_1^{-1}$ of $SU(3) \times SU(2) \times U(1)$ run from low to high energies, they become equal at a scale $M_X$ high enough to satisfy experimental lower limits on proton decay \[2\]. Further work has been done on this problem, making use not only of limits on proton decay \[2,3\], but of cosmological arguments on the relic abundance of the light susy particle (LSP) \[4\] to constrain the susy spectrum. In general, the susy spectrum obtained from this scenario is below 1 TeV, and available in the next generation of particle accelerators. Their detection would be the best test of susy theories.

In studying SGUT’s one has to deal with both light thresholds (associated with the susy masses), and heavy thresholds (associated with the heavy masses) arising from the specific unification group $G$. When supersymmetry is broken at the Planck scale by a “hidden” sector, the large number of susy masses can be determined at the weak scale in terms of a small number of parameters at the unification scale: $m_{1/2}$ (universal gaugino masses), $m_0$ (universal scalar masses), $A$ (cubic scalar couplings) and $m_4$ (bilinear scalar coefficient) \[7\]. On the other hand, for the minimal choice $G = SU(5)$, there are three basic heavy mass parameters: $M_Y$ (heavy gauge bosons), $M_{\Phi}$ (heavy colored scalars), and $M_{\Sigma}$ (heavy adjoint scalar multiplet). At issue is how to handle these thresholds.

In general, thresholds are included in the evolution of the gauge couplings by a step–function, given logarithmic corrections. For light thresholds, instead of using specific susy masses one can used an effective scale $M_{\text{susy}}$ to summarize the effect of the degenerate spectrum \[3\]. Corrections due to heavy thresholds in general increase $M_{\text{susy}}$, so that taking a null correction will give a lower bound on $M_{\text{susy}}$. Nevertheless, heavy threshold corrections are constrained if one takes into account limits on proton decay via dimension-five operators (allowed in SGUT’s, via the $\Phi$ field) \[8\], and the theoretical requirement that Yukawa couplings of heavy scalars do not blow up below the Planck scale \[9\].
In this paper we use a different approach (different from the step–function approximation) to include threshold effects in the study of SGUT’s. In a recent paper [9] we have defined effective charges [10] for the three gauge couplings, calculated with a mass dependent subtraction procedure [11], which include a complete dependence on the masses. With this method, one includes, for example, threshold contributions due to massive gauge bosons $W^\pm$ and $Z^0$, which are missed in the step–function approximation. In the step–function approximation each particle with mass $m \geq m_Z$ contributes to the running coupling $\alpha_i^{-1}(\mu)$ with a logarithmic term $\ln \frac{m^2}{\mu^2}$; on the other hand, the mass dependent method gives a more precise description of the physics via a function, for both $m \geq m_Z$ and $m < m_Z$, which can be approximated by [11,9]:

$$f(\mu, m) = \ln \frac{m_Z^2 + cm^2}{\mu^2 + cm^2},$$

where “$c$” is a constant of order 1–10. If $m^2 \ll m_Z^2$ we recover the usual term $\ln \frac{m^2}{\mu^2}$, and when $m^2 \gg \mu^2$ there are no contributions: the particle decouples from the theory [12]. The constant “$c$” is chosen so that it “matches” the logarithm with the exact function for intermediate scales, $m_Z < m < \mu$, and its value depends on the type of particle running inside the loop. When one has two mass degenerated fermions or two scalars, “$c$” can be obtained from the leading term of a power expansion of the exact threshold function when $m^2/\mu^2$ goes to infinity. In other cases, the coefficient given by the power expansion has to be slightly modified to get a best fitting for intermediate scales.

To see the basic features of the mass dependent procedure versus the step–function, we first study the evolution of the coupling constants from $m_Z$ to $M_X$ without any reference to heavy thresholds. Thus, we impose the unification condition:

$$\alpha_1^{-1}(M_X) = \alpha_2^{-1}(M_X) = \alpha_3^{-1}(M_X) = \alpha_G^{-1},$$

with $M_X \geq 10^{16}$ GeV to prevent fast proton decay. The effective charges $\alpha_3^{-1}(M_X)$, $\alpha_2^{-1}(M_X)$, $\alpha_1^{-1}(M_X)$ calculated at one–loop order are given by:

$$\alpha_i^{-1}(M_X) = \alpha_i^{-1}(m_Z) + \frac{1}{4\pi} \sum_k b_i^{(k)} f^{(k)}(M_X, m_k),$$
where we sum over all the particles in the SSM, and \( f^{(k)}(M_X, m_k) \) is given by (1). The values of \( m_W \) and \( m_Z \) are well known, and we have as arbitrary mass parameters: \( m_t \) (top quark), and \( m_h \) (light Higgs particle of the SM); \( m_{\tilde{w}} \) (winos) and \( m_{\tilde{g}} \) (gluinos); \( m_{\tilde{q}} \) (squarks) and \( m_{\tilde{l}_L} \), \( m_{\tilde{l}_R} \) (sleptons left and right); \( m_\mu \) (higgsinos); \( m_{H} \), \( m_+ \) and \( m_a \) (scalar masses from the second Higgs doublet needed by susy). For the susy masses we take a simplified parametrization in terms of \( m_{1/2} \) and \( \xi_0 = (m_0/m_{1/2})^2 \), neglecting the mixing between winos and higgsinos, and the s–top left and right:

\[
\begin{align*}
    m_{\tilde{w}} &= m_{1/2} & m_{\tilde{g}} &= 3 m_{1/2} \\
    m_{\tilde{q}} &= m_{1/2} \sqrt{7 + \xi_0} & \text{(for all squarks, including s–top)} \\
    m_{\tilde{l}_L} &= m_{1/2} \sqrt{0.5 + \xi_0} & m_{\tilde{l}_R} &= m_{1/2} \sqrt{0.15 + \xi_0}
\end{align*}
\]

we also take \( m_+ \simeq m_a \simeq m_H \). Thus, we are left with six arbitrary parameters: \( m_t, m_h, m_{1/2}, \xi_0, m_\mu \) and \( m_H \). These parameters have lower bounds \[14\] derived from experimental searches for the top, Higgs and susy particles:

\[
\begin{align*}
    m_t &\geq 91 \text{ GeV} & m_h &\geq 60 \text{ GeV} \[13\] \\
    m_{1/2} &\geq 45 \text{ GeV}.
\end{align*}
\]

Moreover, perturbative bounds on Yukawa top couplings and quartic Higgs couplings yield the theoretical upper bound \( m_t, m_h \leq 200 \text{ GeV} \[14\] \); and no extreme fine–tuning on the susy parameters gives \( m_{1/2}, m_0 \leq 1 \text{ TeV} \).

We also need the initial values \( \alpha_i^{-1}(m_Z) \). The values for \( \alpha_1^{-1}(m_Z) \) and \( \alpha_2^{-1}(m_Z) \) are derived from the experimental data on \( \sin^2 \theta \) and \( \alpha_e^{-1} \),

\[
\alpha_e^{-1}(m_Z) = 127.9 \pm 0.2 \[16\] , \quad \sin^2 \theta(m_Z) = 0.2327 \pm 0.0007 \[17\].
\]

but for \( \alpha_3(m_Z) \) there is no agreement between different measurements \[18\]: The latest LEP data average to \( \alpha_3(m_Z) = 0.122 \), while data from low energy measurements average to

\[1\] The remaining fermions in the Standard Model are taken as massless.
\( \alpha_3(m_Z) = 0.109 \). Because of this discrepancy we will not take \( \alpha_3(m_Z) \) as the initial data, instead we will derive it from the unification condition. In this way susy masses can be bounded by requiring that \( \alpha_3(m_Z) \) be in the range \((0.108, 0.125)\).

In Fig. (1) we have plotted \( \alpha^{-1}_3(m_Z) \) versus \( \log_{10}(m_{1/2}) \), for different values of the remaining free parameters, and including the experimental and theoretical constraints on \( m_{1/2}, \alpha^{-1}_3 \) and \( M_X \) mentioned above:

\[
m_{1/2} > 45 \text{ GeV} \quad , \quad M_X > 10^{16} \text{ GeV} \quad , \quad 0.108 \leq \alpha_3(m_Z) \leq 0.125 .
\] (4)

Since \( m_\mu, m_H \) contribute with the same sign, we have simply taken \( m_\mu = m_H \) in order to check the basic features of the mass dependent method. Furthermore, since both \( \alpha^{-1}_3(m_Z) \) and \( M_X \) depend very slightly on \( \xi_0 \), for this plot we have fixed \( \xi_0 = 1 \). Varying the susy masses, we observe a trend similar to what happens when using a step–function: the higher the susy masses, the higher \( \alpha^{-1}_3(m_Z) \) and the lower \( M_X \). However, while with the step–function (Fig. 2) one obtains the limit \( \alpha_3(m_Z) < 0.116 \) (this procedure does not distinguish masses lower than \( m_Z \)), with the mass dependent procedure the full range of \( \alpha^{-1}_3(m_Z) \) can be covered for adequate values of the mass parameters. Thus the limiting values \( \alpha_3(m_Z) = 0.125 \) and \( M_X = 10^{16} \) give us a lower bound on \( m_\mu = m_H \) and an upper limit on \( m_{1/2} \). These bounds depend on \( m_t, m_h \) and \( \xi_0 \); both of them decrease with \( m_t \) and \( m_h \), while the bound on \( m_{1/2} \) increases with \( \xi_0 \) and the bound on \( m_\mu \) decreases, remaining practically unchanged for \( \xi_0 \geq 10^4 \). Therefore, if we allow a maximum difference of two orders of magnitude between \( m_0 \) and \( m_{1/2} \), we get the following upper bounds (using the central values of \( \alpha_e \) and \( \sin^2 \theta \))

\[
m_{1/2} \leq 2.5 \text{ TeV} \quad (m_h = 60 \text{ GeV} , \quad m_t = 91 \text{ GeV}) ,
\]
\[
m_\mu \geq 338 \text{ GeV} \quad (m_h = m_t = 200 \text{ GeV} , \quad m_\mu = m_H) .
\] (5)

The one–loop calculation already shows the differences between the step–function and the mass dependent procedure. The first favors \( \alpha_3(m_Z) \) to be in the range of the low energy experimental data; in addition, the lower data excludes susy masses greater 1 \( \text{ TeV} \). On the
other hand, when we include a more precise treatment of thresholds, the naturalness bound of 1 TeV favors $\alpha_3(m_Z)$ to be in the range of the last LEP data.

When we improve the accuracy, calculating at two–loop order, the value of $\alpha_3(m_Z)$ diminishes by about 10%. In the case of the step–function, this effect drives the upper bound obtained at one–loop towards the experimental upper value, while the naturalness bound on susy masses gives a lower bound $\alpha_3(m_Z) \geq 0.118$. With the mass dependent procedure, a 10% decrease in the one–loop result puts the values of $\alpha_3^{-1}(m_Z)$ obtained with $m_\mu = m_H = 1$ TeV away from the experimental band, being now the lower bound on $m_\mu = m_H$ around 10 TeV. In Fig. (3) we have plotted the upper bound on $m_{1/2}$ and the lower bound on $m_\mu = m_H$ obtained at two–loop order. The behavior of these bounds differs slightly from one–loop: the upper bound on $m_{1/2}$ is a maximum for $\xi_0 \approx 60$ (the value plotted), nearly independently of $m_t$ and $m_h$, and the lower bound on $m_\mu$ decreases even for $\xi_0 > 10^4$. We would need $\xi_0 > 10^{10}$ to have $m_\mu$ in the range of TeV, and then squark and slepton masses of order of PeV. Thus, this simple scenario of perturbative unification at two–loop order is not compatible with experimental data on $\alpha_3(m_Z)$ and the theoretical naturalness bound on the susy spectrum. We need at least a heavy higgsino or a heavy Higgs (or both of them) beyond this bound. We also note that the remaining susy masses (gauginos, squarks and sleptons) are allowed to have values below 1 TeV.

Up to now we have not included the effects of heavy thresholds, but a correct picture of perturbative unification needs to include them. Thus, the unification condition (2) reverts to:

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2 As was pointed in Ref. [19], this bound is consistent with the one obtained including the impact of the evolution of gaugino masses (EGM) in the step–function approximation [20]; the lower bound on $\alpha_3(m_Z)$ (upper bound on $\alpha_3^{-1}(m_Z)$) comes from taking $m_{1/2} \approx 45$ GeV, for which the EGM effect is small. This does not occur with the “mass dependent” effect, which gives differences of order 13%, no matter the region of masses we take.
where $M_j$ are the heavy masses, and $\mu$ is a mass scale satisfying $m_i \ll \mu \ll M_j$, i.e., it is far away from both light and heavy thresholds \[21,22\]. The unification condition is obtained taking into account the decoupling of the heavy degrees of freedom from the low energy theory; integrating out these fields from the action one gets an effective field theory in terms of low energy parameters $(\alpha_i, m_i)$, which are related to the high energy parameters $(\alpha_G, M_i)$ through Eq. (6). The functions $\lambda_i$ at one–loop include logarithmic terms due to heavy degrees of freedom, and constant terms due to light degrees of freedom, \[21\]:

$$\lambda_i(\mu, M_j) = \lambda_i^{(l)} + \lambda_i^{(H)}(\mu, M_j) ,$$

(7)

$$\lambda_i^{(l)} = c_i \left( \frac{10s}{3\sqrt{3}} - \frac{76}{9} \right) + T_h^{i} \left( \frac{8}{9} \right) + T_f^{i} \left( \frac{10}{9} \right) ,$$

(8)

$$\lambda_i^{(H)}(\mu, M_j) = 7c_i \ln \frac{M_V}{\mu} - \frac{2}{3} T_h^{i} \ln \frac{M_H}{\mu} - \frac{4}{3} T_f^{i} \ln \frac{M_F}{\mu} .$$

(9)

Here $h, f$ refer to light scalars and fermions respectively, and $H, F$ to the heavy ones; $c_i = C_2(G_i)$ for $G_i = SU(3), SU(2), U(1)$, and $\bar{c}_i = C_2(G) - C_2(G_i)$; and $T_a^i$ are representation–dependent coefficients\[3\]. Thus, if we make the reasonable assumption that the members of each heavy supermultiplet are degenerate we get:

$$\lambda_1(\mu, M_j) = \frac{66}{5} + \frac{96}{5} \ln \frac{M_V}{\mu} - \frac{4}{5} \ln \frac{M_\Phi}{M_V} - \frac{20}{3} \ln \frac{M_\Sigma}{M_V} ,$$

(10)

$$\lambda_2(\mu, M_j) = \frac{20s}{3\sqrt{3}} - \frac{2}{3} + 8 \ln \frac{M_V}{\mu} - 8 \ln \frac{M_\Sigma}{M_V} ,$$

(11)

$$\lambda_3(\mu, M_j) = \frac{10s}{\sqrt{3}} - 10 - \ln \frac{M_\Phi}{M_V} - \frac{26}{3} \ln \frac{M_\Sigma}{M_V} .$$

(12)

The couplings $\alpha_i^{-1}(\mu)$ in Eq.(3) are the same as given by (3), where $M_X$ is now replaced by $\mu \ll M_X$. Since heavy masses are typically of order $10^{16} GeV$, and light masses are

\[3\] The constant term $-\bar{c}_i/3$ is not present in $\lambda_i^{(H)}$ when one uses the $\overline{DR}$ subtraction procedure \[23\], as it occurs in supersymmetric theories \[24\].
expected to be less than 1 TeV, we choose $\mu = 10^7 GeV$. Eliminating $\alpha^{-1}_G(\mu)$ from (3), we obtain $\alpha^{-1}_3(m_Z)$ and $\ln M_V$:

$$\alpha^{-1}_3(m_Z) = \frac{1}{2} \left( 3\alpha^{-1}_2(m_Z) - \alpha^{-1}_1(m_Z) \right) + \frac{1}{8\pi} F_3(m_i, \mu) - \frac{3}{5\pi} \ln M_\Phi - \frac{3}{5\pi}, \quad (13)$$

$$\ln M_V = \frac{3\pi}{8} \left( 3\alpha^{-1}_1(m_Z) - \alpha^{-1}_2(m_Z) \right) + \frac{3}{32} F_V(m_i, \mu)$$

$$- \frac{3}{40} \ln M_\Phi - \frac{1}{8} \ln M_\Sigma - \frac{13}{10} + \frac{5s}{8\sqrt{3}}, \quad (14)$$

The dependence of $\alpha^{-1}_3(m_Z)$ and $\ln M_V$ with the light masses (given by the functions $F_3(m_i, \mu), F_V(m_i, \mu)$) is qualitatively the same as before, i.e., $\alpha^{-1}_3(m_Z)$ increases with the susy mass parameters while $\ln M_V$ decreases. We now focus our attention on the heavy mass parameters $M_\Phi$ and $M_\Sigma$. We see from Eq. (13) that $\alpha^{-1}_3(m_Z)$ depends only on $M_\Phi$, so the limits on $M_\Phi$ will put a bound on $\alpha_3(m_Z)$. The lower bound comes from the experimental limits on proton decay via dimension–five operators. Using a chiral Lagrangian technique, the lifetime obtained for the dominant mode is $[3]$:

$$\tau(p \rightarrow K^+ \bar{\nu}_\mu) = 6.9 \times 10^{31} \left| \frac{0.003 \sin 2\beta_H M_\Phi}{\beta + y^{tK} f(m_\tilde{q}, m_\tilde{l}, m_\tilde{w})} \right|^2 \text{yrs}, \quad (15)$$

where yet three more unknown parameters have popped–in: the hadron matrix element parameter $\beta$, which ranges from 0.003 to 0.03 $GeV$; the ratio of vacuum expectation values of two Higgs doublets $\tan \beta_H$; and the parameter $y^{tK}$, which represents the ratio of the contribution of the third generation relative to the second. To allow an $M_\Phi$ as low as possible, we take $\beta = 0.003$, $\sin 2\beta_H = 1$, and $[3] \left| 1 + y^{tK} \right| = 1$. The experimental limit for this mode is $\tau(p \rightarrow K^+ \bar{\nu}_\mu) > 1.0 \times 10^{32} \text{yrs}$ $[29]$, so we get:

$$M_\Phi > 1.2 \times 10^{20} f(m_\tilde{q}, m_\tilde{l}, m_\tilde{w}) = M^{\text{min}}_\Phi. \quad (16)$$

The function $f(m_\tilde{q}, m_\tilde{l}, m_\tilde{w})$, with the parametrization we have adopted for the susy masses, is given by:

---

4 In this case we take the simplest choice, because of our experimental ignorance about the value of $y^{tK}$. In fact, $y^{tK}$ could be negative, giving $\left| 1 + y^{tK} \right| < 1$. 

8
\[ f(m_q, m_t, m_w) = \frac{1}{6.5m_{1/2}} \left( \frac{\xi_0 + 13.5}{\xi_0 + 6} \ln(\xi_0 + 7) - \frac{\xi_0 + 0.5}{\xi_0 - 0.5} \ln(\xi_0 + 0.5) \right), \] (17)

and therefore the lower bound only depends on \( m_{1/2} \) and \( \xi_0 \), decreasing with \( m_{1/2} \) and \( m_0 \).

On the other hand, both \( M_\Phi \) and \( M_\Sigma \) can be bounded from above by requiring that the Yukawa couplings involving these fields do not blow up below the Planck scale. This leads to \( M_\Phi < 2M_V \), \( M_\Sigma < 1.8M_V \), and from (14) we get the upper bound on \( M_\Phi \) in terms of the light masses as well as \( M_\Sigma \):

\[
\ln M_\Phi < \frac{15\pi}{37} \left( \alpha^{-1}_1(m_Z) - \alpha^{-1}_2(m_Z) \right) + \frac{15}{148} F_V(m_i, \mu) - \frac{5}{37} \ln M_\Sigma - \frac{52}{37} + \frac{200s}{111\sqrt{3}} + \frac{40}{37} \ln 2 = \ln M_{\Phi}^{max}. \] (18)

With \( M_{\Phi}^{min}(m_{1/2}, \xi_0) \), \( M_{\Phi}^{max}(m_i, M_\Sigma) \) we get upper and lower bounds on \( \alpha^{-1}_3(m_Z) \), which have to be within the range of experimental data. Therefore, we can play with the expressions (13), (14), (15), and the limits on the susy masses trying to check whether or not the perturbative unification scenario is compatible with all the constraints.

In order to compare, we first examine the case of the step–function approximation for \( F_3(m_i, \mu) \) and \( F_V(m_i, \mu) \). With the constraint of susy masses below 1 TeV there is no problem in having \( \alpha_3 \) inside its experimental range. As it can be seen in Fig.(4), the upper limit on \( \xi_0 \) gives us the minimum allowed value for \( M_\Phi \) and the maximum for \( M_\Sigma \); furthermore, the lower limit on \( M_\Sigma \) would give us the lower allowed value for \( \xi_0 \), which is just reached for the maximum allowed values of \( m_{1/2} \) and \( m_0 \), and the upper one for \( M_\Phi \). In principle, we do not have any constraint on the lower bound for \( M_\Sigma \), except the requirement that there is no large splitting between the heavy masses. As \( M_\Phi \), \( M_V \) will be around \( 10^{16} \) GeV, we use \( M_\Sigma \geq 10^{13} \) GeV to give the results. Increasing \( M_\Sigma \) reduces both \( M_{\Phi}^{max} \) and \( \alpha_3^{max} \).

These results are summarized in Table I, where we have considered limiting susy masses of 1 TeV and 2 TeV respectively. For both examples, the allowed range on \( \alpha_3(m_Z) \) is inside the experimental bounds; raising the limiting mass enlarges it, the same happens with the allowed range for \( M_\Phi \) and \( m_{1/2} \). It is seen that if we require all light masses below 1 TeV, and \( M_\Sigma \geq 10^{13} \) GeV, we get \( m_{1/2} < m_Z \), near it lower experimental bound, and also
strong constraints for $m_0$, $M_\Phi$, $M_\Sigma$, so that practically, $m_0 \approx 1\,TeV$, $M_\Phi \approx 10^{16.6}\,GeV$, and $M_\Sigma \approx 10^{13}\,GeV$. There are no further constraints on $m_\mu$, $m_H$ except, of course, the common upper limit we chose for the susy masses.

As we have printed out without taking into account heavy thresholds, the mass dependent procedure gives us values of $\alpha_3^{-1}(m_Z)$ lower than those obtained with the step-function approximation. And now, to satisfy the constraints on $M_\Phi$ and $\alpha_3^{-1}(m_Z)$ we will need to have $m_0$, $m_\mu$ or $m_H$ beyond $1\,TeV$. The lower $\xi_0$ ($m_0$), the higher will be $M_\Phi$, and higher values of $m_\mu$, $m_H$ will be needed to raise $\alpha_3^{-1}(m_Z)$ above its minimum bound (Fig. 5). The results obtained in this case are given in Table II: there are no solutions if we maintain all susy masses below $1\,TeV$.

We have seen that the use of a mass dependent procedure and the requirement of having susy masses not too high, favor a value of $\alpha_3^{-1}(m_Z)$ ($\alpha_3(m_Z)$) near its lower (upper) experimental bound. This is easily understood, since the mass dependent procedure has the effect of raising the values of $\alpha_i^{-1}(\mu)$ at high energies, with respect to those obtained with other methods ($\overline{MS}$, step–function), and their different results tend to merge as susy masses are increased. When the maximum allowed value for $\alpha_3^{-1}(m_Z)$ is not enough to unify the couplings we will have to increase the masses; sometimes above its naturalness bound, as it happens in the two–loop calculation without heavy thresholds. Improving our scenario of coupling constant unification with the effect of heavy thresholds does not improve the situation about the light masses. In principle, taking $M_\Phi$ low enough we will get $\alpha_3(m_Z) \leq 0.125$. But experimental data on proton decay do not allow to freely decrease the value of $M_\Phi$ without adequately increasing the squark and slepton masses. Thus, in the end, we conclude that it would be necessary to have some of the susy light masses, (either squarks and sleptons, or higgsinos or heavy Higgs), heavier than the naturalness bound of $1\,TeV$ commonly taken, contrary to the results obtained using the simplest treatment of the light thresholds given by the step–function approximation.

We do not want to emphasize the numerical results, which depend on details of the model (such as susy mass parametrization) as well as on the experimental data. We would specially
like to advert to their dependence on the procedure chosen to compute the thresholds, which becomes relevant when running the couplings over such a huge range of scales: from $m_Z$ to $M_X$ (all light thresholds are crossed). It is clear that the step-function approximation gets worse as the number of crossed thresholds begins to proliferate; this is the case, for example, in susy theories, where the mass dependent procedure gives the exact contribution for each massive degree of freedom, independently of their number, and avoids the presence of cumulative errors that afflicts threshold crossing in a less complete treatment. Therefore, in models with intermediate mass scales or with many heavy–matter degrees of freedom, the use of mass dependent procedures is mandatory before reaching conclusions on unification and allied phenomena.
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FIGURES

FIG. 1. Values of $\alpha_3^{-1}(m_Z)$, compatible with the unification condition (Eq. 3), calculated with a mass dependent procedure at 1–loop order, for different values of $m_\mu = m_H$: $m_Z$, 1 TeV, 10 TeV, 100 TeV. Dotted lines are the experimental limits on $\alpha_3^{-1}(m_Z)$ and $m_{1/2}$; solid lines are for $m_t = m_h = 200$ GeV, and dashed lines for $m_t = 91$ GeV and $m_h = 60$ GeV. The straight lines (solid for $m_t = m_h = 200$ GeV and dashed for $m_t = 91$ GeV, $m_h = 60$ GeV) are the upper limit obtained for $\alpha_3^{-1}(m_Z)$ when imposing $M_X = 10^{16}$ GeV ($m_\mu = m_H$ increase along these lines from bottom to top). The allowed region for $\alpha_3^{-1}(m_Z)$ are to the left of the straight lines, and between the dotted lines, $8 \leq \alpha_3^{-1}(m_Z) \leq 9.2$ and $m_{1/2} \geq 45$ GeV.

FIG. 2. Same as Fig. 1, but with $\alpha_3^{-1}(m_Z)$ calculated with the step–function approximation.

FIG. 3. Lower bound on $m_\mu = m_H$ (bottom curves), and upper bound on $m_{1/2}$ (straight lines), calculated with a mass dependent procedure at 2–loop order. The lower bound on $m_\mu$ is obtained for the lower experimental limit on $\alpha_3^{-1}(m_Z)$ and the upper limit on $\xi_0$ (for the plot we choose $\xi_0 = 10^4$, see text). The upper bound on $m_{1/2}$ is obtained for the upper limit on $M_X$, and $\xi_0 \simeq 60$. Solid and dashed lines follow the same convention as Fig. 1 and 2.

FIG. 4. $\alpha_3^{-1}(m_Z)$ versus $\log_{10}(M_\Phi)$ at 1–loop order with the step–function approximation, for limiting susy masses of 1 TeV, $m_{1/2} = 45$ GeV (solid lines) and $m_{1/2}^{\text{max}} = 53$ GeV (dashed lines), and satisfying different constraints: (i) The less slope bottom lines are obtained with $M_\Phi^{\text{min}}(m_{1/2}, \xi_0)$ (Eq. 16), and fixing $m_\mu = m_H = m_Z$; $\xi_0$ increases along these lines, from right to left. We have marked the points $\xi_0^{\text{max}}(m_{1/2}^{\text{min}} = 45$ GeV), and $\xi_0^{\text{min}}(m_{1/2}^{\text{max}} = 53$ GeV). Dotted lines set $M_\Phi^{\text{min}}$ and $M_\Phi^{\text{max}}$. (ii) The most slope lines are obtained with $M_\Phi^{\text{max}}(m_{1/2}, \xi_0, m_\mu)$ (Eq. 18), with $\xi_0$ fixed (really depend very slightly on this parameter), $m_\mu = m_H$ increasing from bottom to top, and limiting values for $M_\Sigma$, $M_\Sigma^{\text{min}} = 10^{13}$ GeV to the right, and $M_\Sigma^{\text{max}} = 10^{13.4}$ GeV to the left.
FIG. 5. Same as Fig. 4 now with a mass dependent procedure and limiting susy masses of 2 $TeV$ (there is no solution for 1 $TeV$); $m_{1/2}^{\text{min}} = 45$ $GeV$ in solid lines and $m_{1/2}^{\text{max}} = 86$ $GeV$ in dashed lines. Dotted lines for $M_{\Phi}^{\text{min}}, M_{\Phi}^{\text{max}},$ and lower experimental value for $\alpha_3^{-1}(m_Z) (\alpha_3^{-1}(m_Z) \geq 8)$.
## TABLES

| $m_0^{\text{max}}$ | $\xi_0^{\text{max}}$ | $M_\Phi^{\text{min}}$ | $M_\Sigma^{\text{max}}$ | $\alpha_3^{\text{min}}$ | $m_1^{\text{max}}$ | $\xi_0^{\text{min}}$ | $M_\Sigma^{\text{min}}$ | $M_\Phi^{\text{max}}$ | $\alpha_3^{\text{max}}$ |
|---------------------|-----------------------|------------------------|-------------------------|-------------------------|---------------------|------------------------|-------------------------|-------------------------|-------------------------|
| 1000                | 494                   | $10^{16.58}$           | $10^{13.4}$             | 0.112                   | 53                  | 357                    | 936                     | $10^{13}$               | $10^{16.63}$            | 0.119                  |
| 2000                | 1975                  | $10^{16.06}$           | $10^{16.2}$             | 0.108                   | 221                 | 82                     | 936                     | $10^{13}$               | $10^{16.63}$            | 0.122                  |

**TABLE I.** 1–Loop with Step-function\(^a\)

\(^a\) Mass values in GeV

| $m_0^{\text{max}}$ | $\xi_0^{\text{max}}$ | $m_{\mu}^{\text{min}}$ | $M_\Phi^{\text{min}}$ | $M_\Sigma^{\text{max}}$ | $\alpha_3^{\text{min}}$ | $m_1^{\text{max}}$ | $\xi_0^{\text{min}}$ | $M_\Sigma^{\text{min}}$ | $M_\Phi^{\text{max}}$ | $\alpha_3^{\text{max}}$ |
|---------------------|-----------------------|------------------------|------------------------|-------------------------|-------------------------|---------------------|------------------------|-------------------------|-------------------------|-------------------------|
| 1492                | 1100                  | 1492                   | $10^{16.29}$           | $10^{16.34}$            | 0.125                   | 45                  | 1100                   | $10^{13}$               | $10^{16.29}$            | 0.125                  |
| 2000                | 1975                  | 805                    | $10^{16.06}$           | $10^{16.37}$            | 0.123                   | 86                  | 540                    | $10^{13}$               | $10^{16.39}$            | 0.125                  |
| 3000                | 4444                  | 358                    | $10^{15.75}$           | $10^{16.40}$            | 0.120                   | 333                 | 81                     | $10^{13}$               | $10^{16.54}$            | 0.125                  |
| 4000                | 7901                  | 179                    | $10^{15.53}$           | $10^{16.41}$            | 0.118                   | 4000                | 0                      | $10^{13}$               | $10^{16.64}$            | 0.125                  |
| 5000                | 12348                 | 106                    | $10^{15.35}$           | $10^{16.42}$            | 0.116                   | 5000                | 0                      | $10^{13}$               | $10^{16.72}$            | 0.125                  |
| 6084                | 18278                 | $m_Z$                  | $10^{15.20}$           | $10^{16.44}$            | 0.115                   | 6084                | 0                      | $10^{13}$               | $10^{16.79}$            | 0.125                  |

**TABLE II.** 1–Loop with MDSP\(^a\)

\(^a\) Mass values in GeV