On the light-curve anomalies of radio pulsars

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Abstract. The wave propagation theory (Beskin and Philippov 2012 \textit{MNRAS} 425 814; Hakobyan et al 2017 \textit{MNRAS} 469 2704) allowed us to understand the main polarization properties of mean profiles. However, some radio pulsars indicate clear deviation from the theory predictions. Below we show how such anomalies can be explained within our theory without additional assumptions about orthogonal modes mixing.

1. Introduction
Quantitative theory of the radio waves propagation \cite{1, 2} based on Kravtsov & Orlov approach allows us to describe general properties of mean profiles such as the position angle of the linear polarization \( p.a. \) and the circular polarization for the realistic structure of the magnetic field in the pulsar magnetosphere. The main theoretical prediction is the correlation of signs of the circular polarization, \( V \), and derivative of the position angle with respect to pulsar phase, \( d/p.a./d\phi \): for extraordinary X-mode the signs should be the same, and for ordinary O-mode the signs should be opposite. In most cases it gives us the possibility to recognize the orthogonal mode, ordinary or extraordinary, playing the main role in the formation of the mean profile. On the other hand, there are some pulsars for which observations are in disagreement with above prediction. Below we demonstrate that complex behaviour of pulsar light curves and polarization profiles can be explained by including aberration effects into consideration.

2. Changeover effect
According to \cite{1}, the theory predicts O-X-O sequence where two orthogonal modes are detected in the three-component mean profile. However, some pulsars show polarization profiles that poorly fit into this simplified model. E.g., \( p.a. \) curve of pulsar PSR J2048$^\circ$1616 shows one orthogonal mode whereas Stokes parameter \( V \) changes sign along the mean profile.

To understand this discrepancy, remember that according to \cite{1} the sign of the circular polarization \( V \) is determined by the sign of the derivative \( d(\beta_B + \delta)/dr \) along the ray \( r \) in the vicinity of escape radius \( r_{esc} \)

\[ r_{esc} \sim 10^3 R \cdot \lambda_4^{2/5} \cdot \nu_{GHz}^{-2/5} \]  

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Figure 1. The plot of $\beta_B + \delta$ along the propagation ray. If the polarization is formed below the extremum, the sign is governed by the derivative of $\beta_B$, and when it’s formed above — the sign is determined by the derivative of $\delta$.

Figure 2. Changeover radius $r_{ch}$ as a function of period $P$ for different parameters. Black line corresponds to start parameters, red line corresponds to smaller radiation radius $r_{rad}$, blue line corresponds to another inclination angle $\chi$. Dashed lines correspond to another phase $\phi$.

where the polarization characteristics cease to depend on the distance $r$ from the neutron star. Here $\lambda = n_\text{e}/n_{\text{GJ}}$ (in units $10^4$) is the multiplicity parameter and the angle $\beta_R$ determines the orientation of the external magnetic field in the picture plane. Finally, aberration angle $\delta$ results from electric drift motion of particles in the pulsar magnetosphere:

$$\tan \delta = \frac{\cos \theta U_y/c}{U_x/c - \sin \theta},$$

where $U_x$ and $U_y$ are two components of the $E \times B$ drift velocity, and $\theta$ is the angle between wave vector $\mathbf{k}$ and external magnetic field $\mathbf{B}$.  

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Figure 3. Simulated profiles for O-mode pulsar at two distinct frequencies 430 MHz (left) and 1414 MHz (right).

It is important to stress that the mode classification presented above corresponds to the case when it is the aberration angle $\delta$ that plays the leading role in the derivative $d(\beta_B + \delta)/dl$. As is shown on figure 1, it takes place at large enough distances $r > r_{ch}$ where $r_{ch}$ is so-called changeover radius. As is shown on figure 2, it weakly depends on pulsar’s parameters: $r_{ch} \approx (30 - 60)R$.

But as the escaping radius $r_{esc}$ (1) depends on frequency $\nu$ and, via the multiplicity parameter $\lambda$, on the number density $n_e$, at high frequencies and at that part of the directivity pattern where in the vicinity of the escaping radius the number density of the outflowing plasma is low, the Stokes parameter V can change sign at some parts of the mean profile although radio emission corresponds to the same orthogonal mode (see figure 3). Thus, in reality the circular polarization of a given mode can switch its sign, without supposing the existence of another radiation mode or other effects.

3. Width of the emitting region

Analyzing the width of the p.a. curve obtained in [3] one can formulate a method which allows us to determine the radius-to-frequency mapping. Indeed, assuming that the width of this curve results from different radiation radii for the given frequency, we can evaluate the height and characteristic depth of the radiation region. To do that, we compare the results of our simulation with the corresponding observational data obtained in [3].

In figure 3 we show simulated mean profile corresponding to O-mode double profile. As one
can see, the width of the p.a. curve is slimmer in the center of a profile and wider near the pulse edges. This is in a good agreement with observational data.

4. ‘Shift to the right’ and O-mode directivity pattern
Finally, let us remember that aberration results in so-called ‘shift to the right’ effect, i.e., the $\delta \phi = 4 \Omega r_{\text{rad}}/c$ shift between the position of the maximum derivative $\text{d} p.a./\text{d} \phi$ and the center of the mean profile [4]. As was shown in [5], this effect can help us to determine the generation level $r_{\text{rad}}$ which gives quite reasonable values.

Besides, knowing generation level $r_{\text{rad}}$, one can evaluate the width of the mean profile as $W_{r} \approx 2(\Omega r_{\text{rad}}/c)^{1/2}$ and compare it with observable $W_{\text{obs}}$. As is shown on figure 4, for X-mode pulsars the distribution of the ratio $W_{\text{obs}}/W_{\text{cal}}$ has a sharp maximum near unity, while for O-mode pulsars these ratios are much larger. This difference can be easily explained as the width of the O-mode pulsars is to be larger due to its refraction.

5. Concluding Remarks
In this work we demonstrate that complex behavior of pulsar light curves and polarization profiles can be explained by propagation theory developed in [1]. In particular, one can understand different signs $V$ at different phases for the same polarization mode. In addition, our method provides direct information about the width of the radiating region.

Acknowledgments
This research was partially supported by the government of the Russian Federation (agreement No. 05.Y09.21.0018) and by Russian Foundation for Basic Research (Grant no. 17-02-00788).

References
[1] Beskin V S and Philippov A A 2012 Mon. Not. Royal Astron. Soc. 425 814–40
[2] Hakobyan H L, Beskin V S and Philippov A A 2017 Mon. Not. Royal Astron. Soc. 469 2704–19
[3] Hankins T H and Rankin J M 2010 Astron. J. 139 168–75
[4] Blaskiewicz M, Cordes J M and Wasserman I 1991 Astrophys. J. 370 643–69
[5] Rookyard S C, Weltevrede P and Johnston S 2015 Mon. Not. Royal Astron. Soc. 446 3367–88