Production of axial-vector mesons at $e^+e^-$ collisions with double-tagging as a way to constrain the axial meson LbL contribution to muon g-2 and/or hyperfine splitting of muonic hydrogen

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Abstract

We calculate cross sections for production of axial-vector $f_1(1285)$ mesons for double-tagged measurements of the $e^+e^- \rightarrow e^+e^- f_1(1285)$ reaction. Different $\gamma^*\gamma^* \rightarrow f_1(1285)$ vertices from the literature are used. Both integrated cross section as well as differential distributions are calculated. Predictions for a potential measurement at Belle II are presented. Quite different results are obtained for the different vertices proposed in the literature. Several observables are discussed. The distribution in photon virtuality asymmetry is especially sensitive to the $\gamma^*\gamma^* \rightarrow f_1$ vertex. Future measurements at $e^+e^-$ colliders could test and/or constrain the $\gamma^*\gamma^* \rightarrow f_1(a_1, f_1')$ vertices and associated form factors, known to be important ingredients for calculating contributions to anomalous magnetic moment of muon and hyperfine splitting of levels of muonic atoms.

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I. INTRODUCTION

The coupling of neutral mesons to two photons is an important ingredient of mesonic physics. In Ref. [1] tensorial coupling was discussed for different types of mesons (pseudoscalar, scalar, axial-vector and tensor). In general, the amplitudes can be expressed in terms of functions of photon virtualities often called transition form factors. They were tested in details for pseudoscalar mesons ($\pi^0, \eta, \eta'$). Recently there was discussion how to calculate such objects for pseudoscalar [2] and scalar [3] quarkonia.

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The anomalous magnetic moment of muon is one of the most fundamental quantities in particle physics (see e.g. [11, 12]). A first calculation of QED corrections to anomalous magnetic moment was performed long ago [13]. Recent state of art can be found e.g. in [11, 12, 14]. The current precision of QED calculation is so high that hadronic contributions to muon anomalous moment must be included. The so-called light-by-light (LbL) contributions are very important but rather uncertain. The coupling $\gamma^*\gamma^* \rightarrow f_1(1285)$ is one of the most uncertain ingredients. Different couplings have been suggested in the literature.

Recently the contribution of the $\gamma^*\gamma^* \rightarrow f_1(1285)$ coupling was identified and included in calculating hyperfine splitting of levels of muonic hydrogen, and turned out to be quite sizeable [15]. These are rather fundamental problems and better constraints on $\gamma^*\gamma^*$ coupling are badly needed.

In calculating $\delta a_{\mu}^{f_1}$ one often writes:

$$\delta a_{\mu}^{f_1} = \int dQ_1^2 dQ_2^2 \, \rho_{\mu}^{f_1}(Q_1^2, Q_2^2), \quad (1.1)$$

where $\rho_{\mu}^{f_1}(Q_1^2, Q_2^2)$ is the density of the $f_1$ contribution to the muon anomalous magnetic moment. The integrand of (1.1) (called often density for brevity) peaks at $Q_1^2, Q_2^2 \sim 0.5 \text{ GeV}^2$ and gives almost negligible contribution for $Q_1^2, Q_2^2 > 1.5 \text{ GeV}^2$, see e.g. [7].

The $\gamma^*\gamma^* f_1(1285)$ coupling can be also quite important for hyperfine splitting of levels of muonic hydrogen [15]. It is also very important to calculate rare decays such as $f_1(1285) \rightarrow e^+e^-$ [16, 17]. There both space-like and time-like photons enter corresponding loop integral(s) so one tests both regions simultaneously. The corresponding branch-
FIG. 1: Possible tests of the $\gamma^*\gamma^* \to AV$ vertex in the $(Q_1^2, Q_2^2)$ space: contribution to $g - 2$, hyperfine splitting of muonic hydrogen, EIC, $f_1 \to e^+e^-$ or $e^+e^- \to f_1$ and DT in $e^+e^-$ collisions discussed in the present paper in extent.

The branching fraction is very small ($BF \sim 10^{-8}$). The same loop integral enters the production of $f_1$ in electron-positron annihilation [16, 17]. There is already a first evidence of such a process from the SND collaboration at VEPP-2000 [20]. The $f_1(1285)$ was also observed in $\gamma p \to f_1(1285)p$ reaction by the CLAS collaboration [21]. The experimental results do not agree with theoretical predictions [22–24].

Fig. 1 illustrates how different regions of the vertex functions are tested in different processes. The square $(0, Q_0^2) \times (0, Q_0^2)$ close to the origin shows the region where the dominant contributions to $g - 2$ comes from. The square $(Q_0^2, \infty) \times (Q_0^2, \infty)$ marked in red represents the region which can be tested in double-tagging experiments. The short diagonal ($Q_1^2 = Q_2^2$) line represents region important for hyperfine splitting of levels of muonic hydrogen. The narrow strips along the $x$ and $y$ axis shows a possibility to study production of $f_1(1285)$ in $e^+A$ collisions at EIC. Marked is also the region of photon virtualities which contributes to $f_1 \to e^+e^-$ or to the production of $f_1(1285)$ in $e^+e^-$ annihilation.

In the present paper we suggest how to limit the behaviour of the $\gamma^*\gamma^* \to f_1(1285)$...
coupling(s)\textsuperscript{1} at somewhat larger photon virtualities accessible at double-tagged $e^+e^-\rightarrow e^+e^-f_1(1285)$ measurements, where typically $Q_1^2, Q_2^2 > Q_0^2 = 2\text{ GeV}^2$.

II. SOME DETAILS OF THE MODEL CALCULATIONS

Fig.2 shows the Feynman diagram for axial-vector meson production in $e^+e^-$ collisions. The small circle in the middle represent the $\gamma^*\gamma^*\rightarrow AV$ vertex tested in double-tagging experiment.

A. $\gamma^*\gamma^* \rightarrow f_1(1285)$ vertices

In the formalism presented e.g. in \cite{4} the covariant matrix element for $\gamma^*\gamma^* \rightarrow f_1(1285)$ is written as: where

$$R_{\mu\nu} = -g^{\mu\nu} + \frac{1}{X} \left[ (q_1q_2^\prime) (q_1^\mu q_2^\nu + q_2^\mu q_1^\nu) - q_1^2 q_2^2 - q_2^2 q_1^2 q_1^\nu \right]$$

where

$$X = (q_1q_2) - q_1^2 q_2^2 = \frac{M_f^4}{4} \left( 1 + \frac{2(q_{1t}^2 + q_{2t}^2)}{M_f^2} + \frac{(q_{1t}^2 - q_{2t}^2)^2}{M_f^4} \right).$$

DKMMR2019 vertex

FIG. 2: The generic diagram for $e^+e^-\rightarrow e^+e^-AV$ and kinematical variables used in this paper.

\textsuperscript{1}The same is true for other axial-vector $(a_1,f_1')$ mesons.
In Ref. [15] the vertex was written as:

\[
T^{\mu\nu}_{\alpha} = 4\pi\alpha_{em}\epsilon_{\rho\sigma\tau\alpha} \left\{ R^{\mu\rho}(q_1, q_2) R^{\nu\sigma}(q_1, q_2) (q_1 - q_2)^\tau \nu F^0(q_1^2, q_2^2) \\
+ R^{\mu\rho}(q_1, q_2) \left(q_1^\rho - \frac{q_1^2}{v} q_2^\rho\right) q_1^\tau q_2^\nu F(1)(q_1^2, q_2^2) \\
+ R^{\mu\rho}(q_1, q_2) \left(q_2^\rho - \frac{q_2^2}{v} q_1^\rho\right) q_2^\tau q_1^\nu F(1)(q_2^2, q_1^2) \right\}, \tag{2.3}
\]

where

\[
v = (q_1 q_2) = \frac{1}{2} \left((q_1 + q_2)^2 - q_1^2 - q_2^2\right). \tag{2.4}
\]

In the nonrelativistic model

\[
F^{(0)}(0, 0) = -F^{(1)}(0, 0). \tag{2.5}
\]

We use the normalization of form factors

\[
F^{(0)}(0, 0) = 0.266 \text{ GeV}^{-2}. \tag{2.6}
\]

In Ref. [15] the vertex was supplemented by the following factorized dipole form factor

\[
F_{DKMR}(Q_1^2, Q_2^2) = \frac{\Lambda_D^4}{(\Lambda_D^2 + Q_1^2)^2} \frac{\Lambda_D^4}{(\Lambda_D^2 + Q_2^2)^2}. \tag{2.7}
\]

The \( \Lambda_D \approx 1 \text{ GeV} \) was suggested as being consistent with the L3 collaboration data [29].

We will ascribe also the name NQM (nonrelativistic quark model) to this vertex.

**OPV2018 vertex**

In Ref. [6] the vertex function for \( \gamma^*\gamma^* \rightarrow f_1 \) was constructed based on an analysis of the \( f_1(1285) \rightarrow \rho^0\gamma \) decay and vector meson dominance picture. The corresponding vertex for two-photon coupling there reads

\[
T^{\mu\nu\alpha} = i C_{OPV} \left\{ \epsilon^{\mu\nu\rho\sigma}(q_1, (q_1 q_2) + 2q_1^2) - q_2, \nu((q_1 q_2) + 2q_2^2)) \\
+ \epsilon^{\rho\sigma\nu\alpha} q_2, \rho q_1, \sigma(q_2 + 2q_1)\nu \\
+ \epsilon^{\rho\sigma\mu\alpha} q_1, \rho q_2, \sigma(q_1 + 2q_2)\nu \right\}. \tag{2.8}
\]

Above

\[
C_{OPV} = \frac{5\alpha_{em} g_\rho}{36\pi M_{f_1}^2}. \tag{2.9}
\]
The value of $g_\rho$ is explicitly given in [6]. We supplemented this vertex with one common for all terms form factor of the VDM type:

$$F(Q_1^2, Q_2^2) = \frac{M_V^2}{M_V^2 + Q_1^2} \frac{M_V^2}{M_V^2 + Q_2^2}.$$  \hspace{1cm} (2.10)

consistent with the philosophy there.

**LR2019 vertex**

Finally we consider also the vertex used very recently in [9]. In this approach the vertex is

$$T^{\mu\nu\rho\sigma} \propto \varepsilon^{\alpha\beta\rho\sigma} \left\{ (q_1^2 \delta_\alpha^\mu - q_1, \alpha q_1^\mu)q_2^\sigma \delta_\beta^\nu A(Q_1^2, Q_2^2) \\ - (q_2^2 \delta_\beta^\nu - q_2, \beta q_2^\nu)q_1^\sigma \delta_\alpha^\mu A(Q_2^2, Q_1^2) \right\}.$$  \hspace{1cm} (2.11)

The normalization was also given there. It was pointed out that the $A(Q_1^2, Q_2^2)$ function does not need to be symmetric under exchange of $Q_1^2$ and $Q_2^2$. Actually asymmetric form factors calculated from the hard wall and Sakai-Sugimoto models were used there. In our evaluation here we will use Hard Wall (HW2) form factors as well as factorized dipole symmetric/asymmetric form factors as specified below to illustrate the effect of the holographic approach. The HW2 form factor can be sufficiently well represented as:

$$A(Q_1^2, Q_2^2) \approx A(0, 0) F_S(Q_1^2) F_L(Q_2^2),$$

$$A(Q_2^2, Q_1^2) \approx A(0, 0) F_L(Q_1^2) F_S(Q_2^2),$$  \hspace{1cm} (2.12)

where

$$F_S(Q^2) = \left( \frac{\Lambda_S^2}{\Lambda_S^2 + Q^2} \right)^2,$$

$$F_L(Q^2) = \left( \frac{\Lambda_L^2}{\Lambda_L^2 + Q^2} \right)^2,$$  \hspace{1cm} (2.13)

where $\Lambda_L > \Lambda_S$. We show the HW2 form factor and its factorized dipole approximate representation as a function of $(\log_{10}(Q_1^2), \log_{10}(Q_2^2))$ in Fig.3.

**RS2019 vertex**

In Ref. [8] a vertex based on $R \chi T$ approach was considered. In this approach one gets:

$$T^{\mu\nu\alpha} = e^2 F_{RS}(q_1, q_2) \left\{ i e^{\nu\tau\alpha\rho} q_{1, \rho}(q_2^\nu q_{2, \tau} - g_\tau^\nu q_2^2) - i e^{\nu\tau\alpha\rho} q_{2, \rho}(q_1^\nu q_{1, \tau} - g_\tau^\nu q_1^2) \\ + i e^{\nu\rho\sigma} q_{1, \rho} q_{2, \sigma}(q_1^\nu - q_2^\nu) \right\}.$$  \hspace{1cm} (2.14)

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FIG. 3: Maps of the original (left panel) and parametrized (right panel) HW2 form factor $A(Q_1^2, Q_2^2)/A(0,0)$ as a function of $(\log_{10}(Q_1^2), \log_{10}(Q_2^2))$. In the latter case $\Lambda_S = 0.8$ GeV and $\Lambda_L = 1.2$ GeV.

Above we have denoted:

$$F_{RS}(q_1, q_2) = \frac{2c_A}{M_A (q_1^2 - M_V^2)(q_2^2 - M_V^2)} \cdot$$

(2.15)

The $c_A$ is defined in [8]. $M_V \approx m_\rho \approx m_\omega = 0.8$ GeV. The reader is asked to note vanishing of $F_{RS}$ at $Q_1^2 = Q_2^2$. This, as will be discussed below, has important consequences for the double tagged measurements.

The form factor used in RS2019 are antisymmetric. Additional symmetric form factors arising at higher order were discussed in a revised version of [8] (see Appendix C there). In the following we will use the lower order result to illustrate the situation.

It was ascertained recently in [19] that the $R\chi T$ approach provides only purely transverse axial-vector meson contributions.

**MR2019 vertex**

In Ref. [17] the following vertex was used (we change a bit notation to be consistent with our previous formulae)

$$T^{\mu \nu \alpha} = \frac{i}{m_{f_1}^2} \epsilon^{\mu \nu \rho \sigma} \left\{ F(q_1^2, q_2^2) q_2 \rho q_1 \sigma (q_1 - q_2)^\alpha 
- q_2^2 G(q_1^2, q_2^2) \delta_\rho^{\alpha} q_1, \sigma + q_1^2 G(q_1^2, q_2^2) \delta_\rho^{\alpha} q_2, \sigma \right\}$$

(2.16)

to the production of $f_1(1285)$ in the $e^+ e^-$ annihilation. Since in this case both space-like and time-like virtualities enter the calculation of the relevant matrix element the form...
factors had to be generalized. In [17] the form factors were parametrized in the spirit of vector meson dominance approach as:

\[
G(q_1^2, q_2^2) = \frac{g_2 M_f^5}{q(q_1^2 - m_\rho^2 + i m_\rho \Gamma_\rho)(q_2^2 - m_\rho^2 + i m_\rho \Gamma_\rho)}, \quad (2.17)
\]

\[
F(q_1^2, q_2^2) = \frac{g_1 M_f^3 (q_2^2 - q_1^2)}{q(q_1^2 - m_\rho^2 + i m_\rho \Gamma_\rho)(q_2^2 - m_\rho^2 + i m_\rho \Gamma_\rho)}. \quad (2.18)
\]

One can see the characteristic \( \rho \) meson propagators. The \( F(q_1^2, q_2^2) \) form factor is asymmetric with respect to \( q_1^2 \) and \( q_2^2 \) exchange to assure Bose symmetry of the amplitude. An extra \( q \) in the denominator was attached to the VDM-like vertex to assure "correct" behaviour of the form factors at large photon virtualities [1]. Of course, it is not obvious that such a correction should enter in the multiplicative manner. The coupling constant

\[
g_2 = (2.9 \pm 0.4) \cdot 10^{-4} \quad (2.19)
\]

was found in [17]. It was allowed in [17] for \( g_2 \) to be complex. It was argue that \( |g_1| \sim g_2 \) to describe the first \( e^+e^- \rightarrow f_1(1285) \) data from VEPP-2000 [20]. We shall show in this paper how important is the interference of both terms in the DT case.

### B. General requirements

Any correct formulation of the \( \gamma^*\gamma^* \rightarrow f_1(1285) \) vertex must fulfill at least three general requirements:

- **Gauge invariance** requires:

  \[
  q_{1\mu} T^{\mu\nu\alpha} = q_{2\nu} T^{\mu\nu\alpha} = 0 , \quad (2.20)
  \]

- **Landau-Yang theorem** [25] requires:

  \[
  T^{\mu\nu\alpha} \rightarrow 0 \text{ when } q_1^2 \rightarrow 0 \text{ and } q_2^2 \rightarrow 0 . \quad (2.21)
  \]

- **Bose symmetry** implies

  \[
  T^{\mu\nu\alpha}(q_1, q_2) = T^{\nu\mu\alpha}(q_2, q_1) \quad (2.22)
  \]
which for our reaction means e.g.:

$$\frac{d\sigma(t_1, t_2; y, \phi)}{dt_1 dt_2 dy d\phi} = \frac{d\sigma(t_2, t_1; y, \phi)}{dt_1 dt_2 dy d\phi}$$

(2.23)

for each $y, \phi$.

Some vertices fulfil also

$$T^{\mu\nu} p_\mu ,$$

(2.24)

where $p$ is four-momentum of the axial-vector meson. This automatically guarantees that only spin-1 particle $f_1$ is involved and unphysical states are ignored. A related discussion can be found e.g. in [26].

C. Form factors

Some of the $F(Q^2_1, Q^2_2)$ form factors can be constraint from the so-called decay width into transverse and longitudinal photon, some are poorly know as they can not be obtained as they do not enter the formula for the radiative decay width. The radiative decay width is known [27] and is

$$\tilde{\Gamma}_{\gamma\gamma} = 3.5 \text{ keV} .$$

(2.25)

Then some of the form factors are parametrized as:

$$F(Q^2_1, Q^2_2) = \left( \frac{\Lambda_M^2}{\Lambda_M^2 + Q^2_1} \right) \left( \frac{\Lambda_M^2}{\Lambda_M^2 + Q^2_2} \right),$$

(2.26)

$$F(Q^2_1, Q^2_2) = \left( \frac{\Lambda_D^2}{\Lambda_D^2 + Q^2_1} \right)^2 \left( \frac{\Lambda_D^2}{\Lambda_D^2 + Q^2_2} \right)^2,$$

(2.27)

$$F(Q^2_1, Q^2_2) = \left( \frac{\Lambda_M^2}{Q^2_1 + Q^2_2 + \Lambda_M^2} \right),$$

(2.28)

$$F(Q^2_1, Q^2_2) = \left( \frac{\Lambda_D^2}{Q^2_1 + Q^2_2 + \Lambda_D^2} \right)^2.$$

(2.29)

Both monopole and dipole parametrizations of form factors will be used in the following. We will call the first two as factorized Ansätze and the next two as pQCD inspired power-like parametrizations.
In general, the form factors in Eqs. (2.11) do not need to be symmetric with respect to $Q_1^2$ and $Q_2^2$ exchange [9]. For example in Ref. [9] asymmetric form factor $A(Q_1^2, Q_2^2)$ obtained in Hard Wall and Sakai-Sugimoto models were used to calculate contribution to anomalous magnetic moment of muon. Here we shall take a more phenomenological approach and try to parametrize the form factors in terms of simple functional forms motivated by physical arguments such as vector dominance model or asymptotic pQCD behaviour of transition form factors (see e.g. [28]).

The behaviour of transition form factors at asymptotia may be another important issue [18]. Where the pQCD sets in is interesting but still an open issue. It was discussed in [2] that for $\gamma^* \gamma^* \eta_c$ coupling this happens at very high virtualities. We leave this issue for the $\gamma^* \gamma^* f_1$ coupling for a future study.

D. $e^+e^- \rightarrow e^+e^- f_1$ reaction

The amplitude for the $e^+e^- \rightarrow e^+e^- f_1$ reaction (see Fig.2) in high-energy approximation can be written as:

$$\mathcal{M}^a = e \left( p_1 + p'_1 \right)^{\mu_1} \left( \frac{ig_{\mu_1\nu_1}}{t_1} \right) T_{\gamma^*\gamma^* \rightarrow f_1} \left( p_2 + p'_2 \right)^{\mu_2} \left( \frac{ig_{\mu_2\nu_2}}{t_2} \right) , \quad (2.30)$$

Above $e^2 = 4\pi\alpha_{em}$. The four-momenta are defined in Fig.2. The $T_{\gamma^*\gamma^* \rightarrow f_1}$ vertex function responsible for the $\gamma^*\gamma^* \rightarrow f_1$ coupling was discussed in detail in the previous subsection.

The square of the matrix element, summed over polarizations of $f_1$, can be obtained as:

$$|\mathcal{M}|^2 = \sum_{a_1, a_2} \mathcal{M}^{a_1} \mathcal{M}^{a_2} P_{a_1 a_2} (p_{f_1}) , \quad (2.31)$$

where $P$ is spin-projection operator for spin-1 massive particle:

$$P_{a_1 a_2} = -g_{a_1, a_2} + \frac{p_{a_1} p_{a_2}}{M_{f_1}^2} . \quad (2.32)$$

The cross section for the 3-body reaction $e^+e^- \rightarrow e^+e^- f_1(1285)$ can be written as

$$d\sigma = \frac{1}{2s} |\mathcal{M}|^2 \cdot d^3PS . \quad (2.33)$$

The three-body phase space volume element reads

$$d^3PS = \frac{d^3p'_1}{2E'_1(2\pi)^3} \frac{d^3p'_2}{2E'_2(2\pi)^3} \frac{d^3p_M}{2E_M(2\pi)^3} \cdot (2\pi)^4 \delta^4(p_1 + p_2 - p'_1 - p'_2 - p_M) . \quad (2.34)$$
The phase-space for the $pp \rightarrow pp f_1$ reaction has four independent kinematical variables. In our calculation we integrate over $\xi_1 = \log_{10}(p_{1t})$, $\xi_2 = \log_{10}(p_{2t})$, azimuthal angle between positron and electron and rapidity of the produced axial-vector meson (four-dimensional integration). Here $p_{1t}$ and $p_{2t}$ are transverse momenta of outgoing positron and electron, respectively.

In the case of holographic approach first the $A(Q_1^2, Q_2^2)$ form factor entering the central vertex function (see Eq. (2.30)) is calculated on a two-dimensional grid and then the grid is used for interpolation for each phase space point (see (2.33)).

III. NUMERICAL PREDICTIONS

A. Low $Q_1^2, Q_2^2$ region

In Fig. 4 we show a two-dimensional distribution ($\xi_1, \xi_2$) of the full phase space cross section. Quite large cross sections are obtained for small $\xi_1$ and/or $\xi_2$. In addition, the different models of the $\gamma^* \gamma^* f_1$ couplings lead to very different results for the total cross section. The measurement of the total cross section is, however, rather difficult.

In Fig. 5 we show distributions in $(t_1, t_2)$ (four-momenta squared of the virtual photons as shown in Fig. 2). Clearly some couplings generate strongly enhanced cross section at small $t_1, t_2$.

Clearly those different vertices lead to different cross sections even for very small photon virtualities where the cross section is relatively large. Could one measure inclusive cross section for production of axial-vector meson without tagging? Is then $\gamma^* \gamma^* \rightarrow f_1(1285)$ the dominant mechanism? If yes, such measurements would verify the different vertices used in calculating $\delta a_\mu$ (axial-vector meson contribution to $a_\mu$). Small $Q_1^2$ and $Q_2^2$ means small transverse momenta of $f_1(1285)$. Can one then identify $f_1(1285)$. Which channel is the best? This requires further Monte Carlo studies. The resonant $e^+ e^- \rightarrow f_1(1285)$ production is very small [17] and important only at resonance energies ($\sqrt{s} \sim m_{f_1}$). We are not aware about other competitive reaction mechanisms in $e^+ e^-$ collisions.

In general, one observes a strong enhancement of the $e^+ e^- \rightarrow e^+ e^- f_1(1285)$ cross section at $Q_1^2, Q_2^2 \rightarrow 0$ which is dictated by the singular behaviour of photon propagators
FIG. 4: Distributions in $\xi_1$ and $\xi_2$ for $\sqrt{s} = 10.5$ GeV. Here the OPV, NQM, LR and RS vertices were used.

In (2.30). To illustrate and explore the effect of Landau-Yang vanishing of $T^{\mu\nu\alpha}$ vertex function for $\gamma^*\gamma^* \to f_1$ in Fig[6] we plot the following quantity:

$$\Omega_{LY}(Q_1^2, Q_2^2) = \frac{Q_1^3 Q_2^3}{M_0^4 M_0^4} \frac{d\sigma(Q_1^2, Q_2^2)}{dQ_1^2 dQ_2^2}.$$  \hspace{1cm} (3.1)

The arbitrary scale $M_0$ is chosen to be $M_0 = 1$ GeV in the following.

One can clearly see vanishing of the special quantity (3.1) at $Q_1^2 \to 0$ and $Q_2^2 \to 0$ which reflects Landau-Yang theorem. Slightly different approach patterns to zero can be observed for the different couplings. For the RS coupling we observe deep valley around $Q_1^2 = Q_2^2$ which is a direct consequence of the specific form factor used there. In this case $\Omega_{LY}$ is much smaller than for other vertices in the limited range of $Q_1^2$ and $Q_2^2$ shown in the figure.
B. Double-tagging case

In Table 1 we show integrated cross sections in nb for different couplings discussed in the previous section. Here we imposed only Lorentz invariant cuts $Q_1^2, Q_2^2 > 2 \text{ GeV}^2$. Quite different values are obtained with different couplings which show huge uncertainties of our predictions. Surprisingly small cross sections are obtained with the MR2019 couplings, where we show results with different sign of the second term. Therefore we show also contributions of individual terms for some couplings from the literature. They give contributions of similar order of magnitude.

The results are also strongly dependent on the form factor used in the calculation which is discussed below. In Table 2 we show integrated cross section for a simple LR2019 coupling \cite{9} supplemented by the pQCD or factorized dipole form factor with different
values of the form factor parameter $\Lambda$. The results dramatically depend on the value of $\Lambda$. In addition for the same $\Lambda$ the pQCD and factorized dipole Ansätze give cross section for double tagged case differing by an order of magnitude. In contrast for single tagged case they give almost the same result.

Now we wish to show several differential distributions for the double-tagged mode. In Fig. 7 we show distributions in rapidity and transverse momentum of $f_1(1285)$, $t_1$ or $t_2$, azimuthal angle between outgoing electrons, averaged virtuality $Q^2_a = (Q^2_1 + Q^2_2)/2$ (3.2) and the asymmetry parameter

$$\omega = \frac{Q^2_1 - Q^2_2}{Q^2_1 + Q^2_2}.$$  (3.3)
TABLE I: Integrated cross section in nb for the double-tagging case with $Q_1^2, Q_2^2 > 2\text{GeV}^2$. The MR+, MR- below show the effect of interference due to sign changing of a “subleading” contribution.

| vertex | cross section | comment |
|--------|---------------|---------|
| LR     | 0.6892(-04)   | fact. dipole, $\Lambda = 1\text{ GeV}$ |
|        | 0.3715(-04)   | HW2 form factor |
| OPV    | 0.9212(-04)   | pQCD dipole, $\Lambda = M_{f_1}$ |
| NQM    | 0.4905(-07)   | factorized dipole $\Lambda = 1\text{ GeV}$ |
| RS     | 0.2138(-02)   | antisymmetric form factor, $\Lambda = 0.8\text{ GeV}$ |
| MR +   | 0.4327(-07)   | symmetric and antisymmetric form factors |
| MR -   | 0.7410E(-07)  | symmetric and antisymmetric form factors |
| MR first | 0.3432E(-07)  | antisymmetric form factors |
| MR second | 0.2435E(-07)  | symmetric form factor |

TABLE II: Integrated cross section in nb for $e^+ e^- \rightarrow e^+ e^- f_1(1285)$ at $\sqrt{s} = 10.5\text{ GeV}$ for the vertex used in [9] for arbitrarily changed form factors. We present results for different values of form factor parameter.

| pQCD dipole $\Lambda$ (GeV) | $\sigma$ (nb) | factorized dipole $\Lambda$ (GeV) | $\sigma$ (nb) |
|-----------------------------|---------------|----------------------------------|---------------|
| 0.8                         | 0.4477(-3)    | 0.8                              | 0.4292(-5)    |
| 1.0                         | 0.2236(-2)    | 1.0                              | 0.6892(-4)    |
| 1.2                         | 0.7867(-2)    | 1.2                              | 0.5432(-3)    |

The Bose symmetry requires that:

$$\frac{d\sigma}{d\omega}(\omega) = \frac{d\sigma}{d\omega}(-\omega). \quad (3.4)$$

Quite different distributions are obtained for the different vertices used recently in the literature. Especially interesting are distribution in relative azimuthal angle between outgoing electrons and distribution in virtuality asymmetry $\omega$. For the RS2019 vertex [8], the vanishing of the cross section for $\omega = 0$ is a consequence of the asymmetric form
factor which goes to 0 for $Q_1^2 = Q_2^2$. With the RS2019 vertex axial vector mesons do not contribute to the hyperfine splitting of muonic atoms. It is obvious that the DT measurements of distributions shown in Fig.7 would provide strong limitations on the vertices used in calculating fundamental quantities such as muon anomalous magnetic moment $a_\mu$ and/or hyperfine splitting of muonic hydrogen.
FIG. 7: Several distributions for production of $f_1(1285)$ in double-tagging mode with $Q_1^2, Q_2^2 > 2 GeV^2$. The solid line is for LS, the dashed line for NQM, the dotted line for OPV and the dash-dotted line for RS vertices.
IV. CONCLUSIONS

In this paper the results of calculations of cross sections and differential distributions for the $e^+e^- \rightarrow e^+e^- f_1(1285)$ have been performed using different $\gamma^*\gamma^* \rightarrow f_1(1285)$ couplings known from the literature. These couplings were used previously to calculate hadronic light-by-light axial meson contributions to anomalous magnetic moment of muon as well as for hyperfine splitting of the muon hydrogen.

We have presented predictions relevant for future double-tagged experiments for Belle II. The results strongly depend on the details of calculation (type of tensorial coupling and/or form factors used). The form factor cannot be reliably calculated at present. We have presented several differential distributions in photon virtualities, transverse momentum of $f_1(1285)$, distribution in azimuthal angle between outgoing electron and positron and so-called asymmetry of virtualities ($\omega$). Especially the latter observable (asymmetry) seems promising for verifying the quite different models of the $\gamma^*\gamma^* AV$ coupling. The results strongly depend on details of the coupling(s). The double tagged measurement would therefore be very valuable to constrain the couplings and form factors and in a consequence would help to decrease uncertainties of their contribution to anomalous magnetic moment of muon and hyperfine splitting of muonic hydrogen.

Both $\eta \pi^+\pi^-$ (as in [29]) as well as $\pi^+\pi^-\pi^+\pi^-$ (used recently at the LHC [30]) channels could be applied experimentaly to identify the $f_1(1285)$ meson. The $\eta \pi^+\pi^-$ option is dangerous as there is another meson close by which decays to the same decay channel: $\eta(1295) \rightarrow \eta \pi\pi$ [31]. This meson may be also abundantly produced in $\gamma\gamma$ fusion as $\Gamma_{\eta'\rightarrow\gamma\gamma} = 4.27$ keV [32]. $f_1(1285) \rightarrow \rho^0\gamma$ with BR = 5.3 % [27] would be another possible choice. The decays of light axial vector mesons were discussed e.g. in [33–35].

In the present paper we concentrated on production of $f_1(1285)$ meson. A similar analysis could be performed for other axial-vector mesons such as $a_1(1260)$ or $f_1(1420)$. Then coupling constants and some form factors must be changed in the calculation. On the experimental side, decay channels specific for a given meson must be selected.

The production of isoscalar axial-vector mesons is very interesting also in the context of central exclusive processes $pp \rightarrow pfpf_1$. There the unknown ingredient is pomeron-pomeron-$f_1$ vertex. This will be discussed elsewhere [36].
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