Classification of gauge theories of gravity

\[ \begin{align*}
\text{PG} & = \text{Poincaré gauge theory (of gravity)}, \\
\text{EC} & = \text{Einstein–Cartan (–Sciama–Kibble) theory (of gravity)}, \\
\text{GR} & = \text{general relativity (Einstein’s theory of gravity)}, \\
\text{TG} & = \text{translation gauge theory (of gravity) aka teleparallel theory (of gravity)}, \\
\text{GR}_{||} & = \text{a specific TG known as teleparallel equivalent of GR (spoken “GR teleparallel”)}, \\
\text{WG} & = \text{Weyl (–Cartan) gauge theory (of gravity)}, \\
\text{MAG} & = \text{metric-affine gauge theory (of gravity)}, \\
\text{CG} & = \text{conformal gauge theory (of gravity)}, \\
\text{AdSG} & = \text{(anti-)de Sitter gauge theory (of gravity)}, \\
\text{SuGra} & = \text{supergravity (super-Poincaré gauge theory of gravity)}.
\end{align*} \]

The symbols in the figure have the following meaning: rectangle \( \square \rightarrow \text{class of theories}; \)
circle \( \circ \rightarrow \text{definite viable theories}; \)
nonmetricity \( Q = \mathcal{Q} + \frac{1}{4}(\text{tr} \mathcal{Q})\mathcal{I}, \) torsion \( T, \) curvature \( R. \)
Contents

Foreword by T. W. B. Kibble ix
Preface xi
Acknowledgments xiii
List of Useful Books xv

Part A The Rise of Gauge Theory of Gravity up to 1961 1

1. From Special to General Relativity Theory 3

Commentary
1.1 A. Einstein, The foundation of the general theory of relativity, *Annalen der Physik* 49, 769–822 (1916); extract

2. Analyzing General Relativity Theory 17

Commentary
2.1 E. Cartan, On a generalization of the notion of Riemann curvature and spaces with torsion (in French), *Comptes Rendus Acad. Sci. (Paris)* 174, 593–595 (1922)
2.2 E. Cartan, Space with a Euclidean connection, in: E. Cartan, *Riemannian Geometry in an Orthogonal Frame*, Lectures given at the Sorbonne 1926–27 (World Scientific, River Edge, NJ, 2001); extract
2.3 H. Weyl, Electron and gravitation. I, *Zeitschrift für Physik* 56, 330–352 (1929)
2.4 E. Stueckelberg, A possible new type of spin-spin interaction, *Phys. Rev.* 73, 808–808 (1948)
2.5 H. Weyl, A remark on the coupling of gravitation and electron, *Phys. Rev.* 77, 699–701 (1950)

3. A Fresh Start by Yang–Mills and Utiyama 71

Commentary
3.1 C. N. Yang and R. Mills, Conservation of isotopic spin and isotopic gauge invariance, *Phys. Rev.* 96, 191–195 (1954)
3.2 R. Utiyama, Invariant theoretical interpretation of interactions, *Phys. Rev.* 101, 1597–1607 (1956)
3.3 J. J. Sakurai, Theory of strong interactions, *Ann. Phys. (N.Y.)* **11**, 1–48 (1960); extract

3.4 S. L. Glashow and M. Gell-Mann, Gauge theories of vector particles, *Ann. Phys. (N.Y.)* **15**, 437–460 (1961); extract

3.5 R. Feynman, F. B. Morinigo, and W. G. Wagner, *Feynman Lectures on Gravitation*, Lectures given 1962/63, B. Hatfield (ed.) (Addison–Wesley, Reading, MA, 1995); extract

### Part B Poincaré Gauge Theory

4. Einstein–Cartan(–Sciama–Kibble) Theory as Viable Gravitational Theory

Commentary

4.1 D. W. Sciama, On the analogy between charge and spin in general relativity, in: *Recent Developments in General Relativity*, Festschrift for Infeld (Pergamon Press, Oxford; PWN, Warsaw, 1962), pp. 415–439

4.2 T. W. B. Kibble, Lorentz invariance and the gravitational field, *J. Math. Phys.* **2**, 212–221 (1961)

4.3 P. von der Heyde, The equivalence principle in the $U_4$ theory of gravitation, *Nuovo Cim. Lett.* **14**, 250–252 (1975)

4.4 W.-T. Ni, Searches for the role of spin and polarization in gravity, *Rep. Prog. Phys.* **73**, 056901 (2010) [24 pages]; extract

4.5 A. Trautman, The Einstein–Cartan theory, in: J.-P. Françoise et al. (eds.), *Encyclopedia of Mathematical Physics, vol. 2* (Elsevier, Oxford, 2006), pp. 189–195

5. General Structure of Poincaré Gauge Theory (Including Quadratic Lagrangians)

Commentary

5.1 F. W. Hehl, J. Nitsch, and P. von der Heyde, Gravitation and Poincaré gauge field theory with quadratic Lagrangian, in: A. Held (ed.), *General Relativity and Gravitation—One Hundred Years after the Birth of Albert Einstein, vol. 1* (Plenum, New York, 1980), pp. 329–355

5.2 K. Hayashi and T. Shirafuji, Gravity from Poincaré gauge theory of fundamental interactions, *Prog. Theor. Phys.* **64**, 866–882 (1980); extract

5.3 P. Baekler, F. W. Hehl, and J. M. Nester, Poincaré gauge theory of gravity: Friedmann cosmology with even and odd parity modes. Analytic part, *Phys. Rev. D* **83**, 024001 (2011) [23 pages]; extract

6. Translational Gauge Theory

Commentary

6.1 G. D. Kerlick, Spin and torsion in general relativity: foundations, and implications for astrophysics and cosmology, Ph.D. Thesis (Princeton University, Princeton, NJ, 1975); extract

6.2 J. M. Nester, Gravity, torsion and gauge theory, in: H. C. Lee (ed.), *An Introduction to Kaluza–Klein Theories* (World Scientific, Singapore, 1984); extract
6.3 Y. Itin, Energy-momentum current for coframe gravity, *Class. Quantum Grav.* **19**, 173–190 (2002); extract

7. Fallacies About Torsion

Commentary

7.1 B. Gogala, Torsion and related concepts: an introductory overview, *Int. J. Theor. Phys.* **19**, 573–586 (1980); extract

**Part C Extending the Gauge Group of Gravity**

8. Poincaré Group Plus Scale Transformations: Weyl–Cartan Gauge Theory of Gravity

Commentary

8.1 W. Kopczyński, J. D. McCrea, and F. W. Hehl, The Weyl group and its currents, *Phys. Lett. A* **128**, 313–317 (1988)

8.2 J. M. Charap and W. Tait, A gauge theory of the Weyl group, *Proc. R. Soc. Lond. A* **340**, 249–262 (1974)

8.3 H. T. Nieh, A spontaneously broken conformal gauge theory of gravitation, *Phys. Lett. A* **88**, 388–390 (1982)

9. From the Poincaré to the Affine Group: Metric-Affine Gravity

Commentary

9.1 F. W. Hehl, G. D. Kerlick, and P. Von der Heyde, On a new metric affine theory of gravitation, *Phys. Lett. B* **63**, 446–448 (1976)

9.2 Y. Ne’eman, Gravitational interaction of hadrons: Band-spinor representations of $GL(n, R)$, *Proc. Nat. Acad. Sci. (USA)* **74**, 4157–4159 (1977)

9.3 E. A. Lord, The metric-affine gravitational theory as the gauge theory of the affine group, *Phys. Lett. A* **65**, 1–4 (1978)

9.4 F. W. Hehl, J. D. McCrea, E. W. Mielke, and Y. Ne’eman, Progress in metric-affine gauge theories of gravity with local scale invariance, *Foundations of Physics* **19**, 1075–1100 (1989)

9.5 Y. Ne’eman and Dj. Šijački, $SL(4, R)$ world spinors and gravity, *Phys. Lett. B* **157**, 275–279 (1985)

10. Conformal Gauge Theory of Gravity*

Commentary

10.1 D. J. Gross and J. Wess, Scale invariance, conformal invariance, and the high-energy behavior of scattering amplitudes, *Phys. Rev. D* **2**, 753–764 (1970); extract

10.2 E. A. Lord and P. Goswami, Gauging the conformal group, *Pramana – J. Phys.* **25**, 635–640 (1985)
11. (Anti-)de Sitter Gauge Theory of Gravity

Commentary

11.1 S. W. MacDowell and F. Mansouri, Unified geometric theory of gravity and supergravity, *Phys. Rev. Lett.* **38**, 739–742 (1977)

11.2 K. S. Stelle and P. C. West, Spontaneously broken de Sitter symmetry and the gravitational holonomy group, *Phys. Rev. D* **21**, 1466–1488 (1980); extract

11.3 E. A. Lord, Gauge theory of a group of diffeomorphisms. II. The conformal and de Sitter group, *J. Math. Phys.* **27**, 3051–3054 (1986)

12. From the Square Root of Translations to the Super Poincaré Group

Commentary

12.1 S. Deser and B. Zumino, Consistent supergravity, *Phys. Lett. B* **62**, 335–337 (1976)

12.2 A. H. Chamseddine and P. C. West, Supergravity as a gauge theory of supersymmetry, *Nucl. Phys. B* **129**, 39–44 (1977)

12.3 P. Townsend, Cosmological constant in supergravity, *Phys. Rev. D* **15**, 2802–2804 (1977)

12.4 J. Isenberg, J. M. Nester, and R. Skinner, Massive spin 3/2 field coupled to gravity, in: *GR8 – Abstracts of Contributed Papers*, 8th International Conference on General Relativity and Gravitation, August 7–12, 1977, University of Waterloo, Waterloo, Ontario, Canada, p. 196.

Part D Specific Subjects of Metric-Affine Gravity and Poincaré Gauge Theory

13. Hamiltonian Structure

Commentary

13.1 M. Blagojević and M. Vasilić, Asymptotic symmetry and conserved quantities in the Poincaré gauge theory of gravity, *Class. Quantum Grav.* **5**, 1241–1257 (1988)

13.2 H. Chen, J. M. Nester and H.-J. Yo, Acausal PGT modes and the nonlinear constraint effect, *Acta Physica Polonica B* **29**, 961–970 (1998)

13.3 J. M. Nester, A covariant Hamiltonian for gravity theories, *Mod. Phys. Lett. A* **6**, 2655–2661 (1991)

14. Equations of Motion for Matter

Commentary

14.1 P. B. Yasskin and W. R. Stoeger, Propagation equations for test bodies with spin and rotation in theories of gravity with torsion, *Phys. Rev. D* **21**, 2081–2094 (1980)

14.2 D. Puetzfeld and Yu. N. Obukhov, Probing non-Riemannian spacetime geometry, *Phys. Lett. A* **372**, 6711–6716 (2008)
15. Cosmological Models

Commentary
15.1 M. Tsamparlis, Cosmological principle and torsion, *Phys. Lett. A* **75**, 27–28 (1979)
15.2 A. V. Minkevich, Generalised cosmological Friedmann equations without gravitational singularity, *Phys. Lett. A* **80**, 232–234 (1980)
15.3 K.-F. Shie, J. M. Nester, and H.-J. Yo, Torsion cosmology and the accelerating universe, *Phys. Rev. D* **78**, 023522 (2008) [16 pages]

16. Exact Solutions

Commentary
16.1 P. Baekler, A spherically symmetric vacuum solution of the quadratic Poincaré gauge field theory of gravitation with Newtonian and confinement potentials, *Phys. Lett. B* **99**, 329–332 (1981)
16.2 J. D. McCrea, P. Baekler, and M. Gürses, A Kerr-like solution of the Poincaré gauge field equations, *Nuovo Cim. B* **99**, 171–177 (1987)
16.3 A. García, A. Macías, D. Puetzfeld, and J. Socorro, Plane fronted waves in metric affine gravity, *Phys. Rev. D* **62**, 044021 (2000) [7 pages]
16.4 T. Dereli, M. Önder, J. Schray, R. W. Tucker, and C. Wang, Non-Riemannian gravity and the Einstein–Proca system, *Class. Quant. Grav.* **13**, L103–L109 (1996)

17. Poincaré Gauge Theory in Three Dimensions

Commentary
17.1 E. W. Mielke and P. Baekler, Topological gauge model of gravity with torsion, *Phys. Lett. A* **156**, 399–403 (1991)
17.2 M. Blagojević and B. Čvetković, Black hole entropy in 3D gravity with torsion, *Class. Quantum Grav.* **23**, 4781–4795 (2006)

18. Dislocations and Torsion*

Commentary
18.1 E. Kröner, Continuum theory of defects, in: R. Balian et al. (eds.), *Physics of Defects, Proceedings of the Les Houches Summer School, Session XXXV* (North-Holland, Amsterdam, 1981), pp. 215–315; extract
18.2 R. A. Puntigam and H. H. Soleng, Volterra distortions, spinning strings, and cosmic defects, *Class. Quant. Grav.* **14**, 1129–1149 (1997); extract

19. The Yang Episode: A Historical Case Study*

Commentary
19.1 C. N. Yang, Integral formalism for gauge fields, *Phys. Rev. Lett.* **33**, 445–447 (1974)
19.2 Chen Ning Yang, *Selected Papers 1945–1980*, with commentary (World Scientific, Singapore, 2005); extract
19.3 C. W. F. Everitt, Gravity Probe B: I. The scientific implications, in: *Sixth Marcel Grossmann Meeting on General Relativity, Part B* (World Scientific, Singapore, 1992), pp. 1632–1644; extract

19.4 J.-P. Hsu and D. Fine (eds.), *100 Years of Gravity and Accelerated Frames, The Deepest Insights of Einstein and Yang–Mills* (World Scientific, Hackensack, NJ, 2005); extract

19.5 W.-T. Ni, Yang’s gravitational field equations, *Phys. Rev. Lett.* 35, 319–320 (1975)
Symmetry has always played a big role in physics. Advancing understanding has time and again revealed previously unknown symmetries. Isaac Newton abandoned the idea of a preferred origin of space, revealing the underlying translational symmetry; Albert Einstein uncovered an unexpected symmetry between time and space.

A key innovation of the twentieth century was Hermann Weyl’s invention of gauge theory, in which a global physical symmetry is replaced by a local one; the arbitrary phase in the quantum wave-function becomes a function of space and time, a change that requires the existence of the electromagnetic field. This proved to be an astonishingly fruitful idea. Today, all the components of the “standard model” of particle physics that so accurately describes our observations are gauge theories. Weyl’s “gauge principle”, that global symmetries should be promoted to local ones, applied to the standard-model symmetry group $SU(3) \times SU(2) \times U(1)$, is enough to yield the strong, weak and electromagnetic interactions.

Only gravity is missing from this model. But it too shows many of the same features. Going from special to general relativity involves replacing the rigid symmetries of the Poincaré group—translations and Lorentz transformations—by freer, spacetime dependent symmetries. So it was natural to ask whether gravity too could not be described as a gauge theory. Is it possible that starting from a theory with rigid symmetries and applying the gauge principle, we can recover the gravitational field? The answer turned out to be yes, though in a subtly different way and with an intriguing twist. Starting from special relativity and applying the gauge principle to its Poincaré-group symmetries leads most directly not precisely to Einstein’s general relativity, but to a variant, originally proposed by Élie Cartan, which instead of a pure Riemannian spacetime uses a spacetime with torsion. In general relativity, curvature is sourced by energy and momentum. In the Poincaré gauge theory, in its basic version, there is also torsion, sourced by spin.

As someone who was involved in the early stages of this development, I am astonished and intrigued by how the theory has developed over the last half century. Reading this book makes it clear how wide its ramifications have spread. Over the years, Poincaré gauge theory has been put on a much firmer mathematical base. In its simplest form, it gives predictions that are in almost all observational situations identical with those of general relativity, but in situations of extremely high density there are significant differences. These differences
may be of profound importance for the physics of the very early universe and of black holes, and could one day be subject to observational test.

Moreover, Poincaré gauge theory is not necessarily the end of the story. There are several possible extensions, in which the basic symmetry group is even larger; the Poincaré group may be augmented by the inclusion of dilatations or even enlarged to the full group of affine transformations. The resulting theories, the Weyl–Cartan theory and the metric-affine gravity theory, have some very attractive features. Only time will tell whether any of these intriguing theories is correct and which of the hypothesized hidden symmetries is actually realized in nature. For anyone interested in pursuing these ideas, this book certainly provides a fascinating and very valuable resource.

London, March 2012

Professor Tom Kibble, FRS
Imperial College London
Preface

We have been both fascinated by gauge theories of gravity since the 1960s and the 1970s and have followed the subject closely through our own work. In this reprint volume with commentaries we would like to pass over our experience to the next generation of physicists. We have tried to collect the established results and thus hope to prevent double work and to focus new investigations on the real loopholes of the theory.

The aim of this reprint volume with commentaries is to introduce graduate students of theoretical physics, mathematical physics or applied mathematics, or any other interested researcher, to the field of classical gauge theories of gravity. We assume that our readers are familiar with the basic aspects of classical mechanics, classical electrodynamics, special relativity (SR), and possibly elements of general relativity (GR). Some knowledge of particle physics, group theory, and differential geometry would be helpful.

Why gauge theory of gravity? Because all the other fundamental interactions (electroweak and strong) are described successfully by gauge theories (of internal symmetries), whereas the established gravitational theory, Einstein’s GR, seems to be outside this general framework, even though, historically, the roots of gauge theory grew out of a careful analysis of GR. A full clarification of the gauge dynamics of gravity might be the last missing link to the hidden structure of a consistent unification of all the fundamental interactions at both the classical and the quantum level.

Our book is intended not just to be a simple reprint volume, but more a guide to the literature on gauge theories of gravity. The reader is expected first to study our introductory commentaries and become familiar with the basic ideas, then to read specific reprints, and after that to return again to our text, explore the additional literature, etc. The interaction is expected to be more complex than just starting with commentaries and ending with reprints. A student, guided by our commentaries, can get self-study insight into gauge theories of gravity within a relatively short period of time.

The underlying structure of gravitational gauge theory is the group of motions of the spacetime in SR, namely the Poincaré group $P(1,3)$. If one applies the gauge-theoretical ideas to $P(1,3)$, one arrives at the Poincaré gauge theory of gravity (PG). Therein, the conserved energy-momentum current of matter and the spin part of the conserved angular momentum current of matter both act as sources of gravity. The simplest PG is the
Einstein–Cartan theory, a viable theory of gravity that, like GR, describes all classical experiments successfully. On the other hand, if one restricts attention to the translation subgroup of $P(1, 3)$, one ends up with the class of translation gauge theories of gravity, one of which, for spinless matter, can be shown to be equivalent to GR. The developments that led to PG are presented in Part A of our book; in Part B, definite and enduring results of PG are displayed. The content of Parts A and B should be considered as a mandatory piece of the general education for all gravitational physicists, while the remaining two parts cover subjects of a more specialized nature.

Since SR is such a well-established theory, from a theoretical as well as from an experimental point of view, the gauging of $P(1, 3)$ rests on a very solid basis. Nevertheless, there arise arguments as to why an extension of PG seems desirable; they are presented in Part C. As a finger exercise, we gauge the group of Poincaré plus scale transformations. Then, we extend $P(1, 3)$ to the general real linear group $GL(4, R)$, thus arriving at metric-affine gauge theory of gravity (MAG). This general framework leads to a full understanding of the role of a non-vanishing gradient of the metric (nonmetricity). Several other extensions treated in Part C appear to be rather straightforward tasks.

The gauge theory of gravity, since 1961, when it first had been definitely established, has had a broad development. Therefore, in Part D we display the results on several specific aspects of the theory, like the Hamiltonian structure, equations of motion for matter, cosmological models, exact solutions, three-dimensional gravity with torsion, etc. These subjects could be starting points for research projects for our prospective readers.

Clearly, making a good choice of reprints is a very demanding task, particularly if we want to take care of the historical justice and authenticity. But we also wanted to take care of another aspect—that our collection of reprints should be a useful guide to research-oriented readers without too many historical detours. These two aspects are not always compatible, and we tried to ensure a reasonable balance between them. To what extent these attempts were successful is to be judged by our readers.

- Chapters of the book that can be skipped at a first reading are marked by the star symbol *.

March 2012

Milutin Blagojević (Belgrade)
Friedrich W. Hehl (Cologne and Columbia, Missouri)
mb@ipb.ac.rs, hehl@thp.uni-koeln.de
We are very grateful to the people who looked over early versions of our manuscript, helping us with detailed comments to improve the final form of the text: Peter Baekler (Düsseldorf), Giovanni Bellettini (Rome), Yakov Itin (Jerusalem), David Kerlick (Seattle), Claus Kiefer (Cologne), Bahram Mashhoon (Columbia, MO), Eckehard Mielke (Mexico City), Milan Mijić (Los Angeles), James M. Nester (Chungli), Yuri N. Obukhov (Moscow & Cologne), Hans Ohanian (Burlington), Dirk Puetzfeld (Bremen), Lewis Ryder (Canterbury), Tilman Sauer (Pasadena), Erhard Scholz (Wuppertal), Thomas Schücker (Marseille), Djordje Šijački (Belgrade), Andrzej Trautman (Warsaw) and Milovan Vasilić (Belgrade). The frontispiece on the classification of gauge theories was jointly created with Yuri Obukhov. One of us (MB) was supported by two short-term grants from the German Academic Exchange Service (DAAD), and the other one (FWH) is most grateful to Maja Burić (Belgrade) for an invitation to a workshop that took place in Divčibare, Serbia. FWH was partially supported by the German–Israeli Foundation for Scientific Research and Development (GIF), Research Grant No. 1078–107.14/2009. We also thank Ms. Hochscheid, Ms. Wetzels (both of Cologne), and Ms. Mihajlović (Belgrade) for technical support.

We wish to express our sincere gratitude to the publishing companies and the individuals who kindly granted us permission to reproduce the material for which they hold copyrights: Acta Physica Polonica B, American Institute of Physics, American Physical Society, Peter Baekler, Caltech, Dover Publications, Elsevier, French Academy of Sciences, Indian Academy of Sciences, Institute of Physics, David Kerlick, Tom W. B. Kibble, Gertrud Kröner, Eric A. Lord, Dvora Ne’eman, James M. Nester, Wei-Tou Ni, Progress of Theoretical Physics, Dirk Puetzfeld, Royal Society of London, Ken Sakurai, Lidia D. Sciama, Società Italiana di Fisica, Springer Science+Business Media, Kellogg S. Stelle, William R. Stoeger, Paul K. Townsend, Andrzej Trautman, Paul von der Heyde, World Scientific, Chen Ning Yang and Hwei-Jang Yo.

We thank Professor Kibble, one of the founders of the gauge theory of gravity, who honored us by writing a foreword to this book.
List of useful books

Here is a chronologically ordered list of books, in which readers can find useful information on the subject of gauge theories of gravity. The selection is made by requiring at least some mentioning of the EC theory.

- V. N. Ponomariev, A. O. Barvinsky, and Yu. N. Obukhov, *Geometrodynamical Methods and the Gauge Approach to the Theory of Gravitational Interactions* (Energoatomizdat, Moscow, 1985) (in Russian)
- W. Thirring, *A Course in Mathematical Physics 2: Classical Field Theory, 2nd ed.*, translated by E. M. Harrell (Springer, New York, 1986)
- E. W. Mielke, *Geometrodynamics of Gauge Fields—On the Geometry of Yang–Mills and Gravitational Gauge Theories* (Akademie-Verlag, Berlin, 1987)
- M. Göckeler and T. Schücker, *Differential Geometry, Gauge Theories and Gravity* (Cambridge University Press, Cambridge, 1987)
- P. Ramond, *Field Theory: A Modern Primer, 2nd ed.* (Addison–Wesley, Redmond City, CA, 1989)
- W. Kopczyński and A. Trautman, *Spacetime and Gravitation* (PWN, Warsaw; Wiley, Chichester, 1992)
- M. Blagojević, *Gravitation and Gauge Symmetries* (IoP, Bristol, 2002)
- T. Ortín, *Gravity and Strings* (Cambridge University Press, Cambridge, 2004)
- L. Ryder, *Introduction to General Relativity* (Cambridge University Press, Cambridge, 2009)
Gauge theory gravity (GTG) is a theory of gravitation cast in the mathematical language of geometric algebra. To those familiar with general relativity, it is highly reminiscent of the tetrad formalism although there are significant conceptual differences. Most notably, the background in GTG is flat, Minkowski spacetime. The equivalence principle is not assumed, but instead follows from the fact that the gauge covariant derivative is minimally coupled. As in general relativity, equations structurally Gauge-theoretic formalism (universal principle of the local invariance and the mechanism of spontaneous breaking of the gauge symmetry) forms the basis for the modern understanding of fundamental physical interactions and is successfully confirmed by the experimental discoveries of the gauge bosons and the Higgs particle. The carefully selected material of the book provides a minimal but sufficient mathematical introduction to the methods of the gauge gravitational theory, and gives a concise but exhaustive description of all specific physical consequences. In order to describe the Gravitation, gauge theories and differential geometry. Tohru EGUCHI Stanford Linear Accelerator Center, Stanford, California 94305, USA and The Enrico Fermi Institute and. Department of Physics, The University of Chicago, Chicago, Illinois, USA Peter B. GILKEY. Fine Hall, Box 37. Department of Mathematics, Princeton University, Princeton, New Jersey 08544, USA and Department of Mathematics, University of Southern California, Los Angeles, California 90007, USA. and Andrew J. HANSON Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720, USA and P.O. Box 11693A, Palo