A New Method for Finding Clusters of Galaxies at $z \gtrsim 1$

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ABSTRACT

At large redshifts, a cluster or group may be too distant for the galaxies within the cluster to be detected individually. However, the light from these “undetected” galaxies still modulates the surface brightness of the background sky. Clusters can appear as $10'' - 1.5'$ sized fluctuations in the surface brightness of the EBL. The fluctuations have central surface brightnesses between roughly 26 and 28 mag/arcsec$^2$ (in $V$) for clusters between $z = 1$ and $z = 2$, and are brighter than the fluctuations produced by background field galaxies. While such low surface brightnesses are difficult to achieve with direct high-resolution imaging, we demonstrate that they are easily reached in short exposures through smoothing the sky in very flat CCD images.

For a reasonable extrapolation of the properties and space densities of clusters and groups, we find that for a wide range of cosmological assumptions there should be tens of clusters per square degree visible in the extragalactic background. The detection rate can range between 0.1 and 100 clusters per square degree, for extreme assumptions about the rate of cluster and galaxy evolution. Unfortunately, the effects of galaxy luminosity evolution and cluster mass evolution cannot be easily separated; this limits the current usefulness of this method in discriminating between different cosmological models. Drift scans provide sufficient accuracy of flat-fielding to make the method discussed in this paper an efficient technique for finding candidate high redshift clusters.
1. Introduction

When we observe the night sky, our attention is immediately and understandably drawn to what we can easily see, namely stars and galaxies. We study these high surface brightness, high contrast objects in great detail, and obtain deeper and deeper images to make ever more distant and fainter objects appear above the brightness and noise of the background sky. But what about the sky itself? It is not a simple uniform background; it contains information about all of the distant galaxies that are too faint or too low surface brightness to stand out above the mean rms noise. The light from undetected galaxies does not simply disappear. Instead, the structure of the background sky must be shaped by the structure of the undetected galaxies.

Extremely deep CDD images show that faint galaxies almost uniformly cover the sky (Tyson 1988), so to first order, the approximation that the sky is simply uniform is a good one. The large fraction of galaxies in the field today suggests that the approximately uniform background of galaxies is primarily made of field galaxies, distributed along walls and/or filaments. However, there are also regions of the universe where the distribution of galaxies is far from uniform, namely in rich groups and clusters, where the space density of galaxies is up to factors of $10^4$ times greater than in the field. These pockets of galaxies are in the background sky as well, and while they may be so distant that none of the individual galaxies may be detected, the integrated light from the cluster is still luminous and compact, and will leave a clear signature superimposed upon the extragalactic background light. The bright patch that the cluster imprints on the sky may be detected as a specific feature in the EBL, provided that that cluster is of sufficient richness to be “rare” and thus not cover the sky uniformly, and provided that the surface brightness of cluster rises above random fluctuations in the EBL on the same angular scale. By using these bright spots in the background sky to trace the population of clusters and rich groups, we can easily identify candidate clusters for further study.

The idea that the extragalactic background can be used to glean information about the underlying distribution of galaxies is not a new one. Schectman (1973,1974) first demonstrated that the power spectrum of fluctuations in the background light is shaped by the power spectrum and redshift distribution of the background field galaxies, and measured the power spectrum from photographic plates. Martin & Bowyer (1989) repeated much of this analysis for the far-UV, where the sky is much darker. More recently, Cole, Treyer, & Silk (1992) updated Schectman’s work, using current determinations of the faint galaxy correlation function and redshift distribution to predict the power spectrum of the EBL as a function of wavelength for several cosmological scenarios. All of this work has shown that statistics of the EBL can be a powerful discriminant between different cosmogonies.
However, the EBL can be used as more than a broad statistical tool; it can be used as a tracer of specific features in the galaxy distribution. As can be done with nearby bright galaxies, we can not only measure the global correlation function, but can identify the regions where the local correlation is the greatest, namely groups and clusters.

Using bumps in the EBL to find candidate clusters presents an extremely efficient strategy for identifying clusters at high redshift, especially when compared with the cost in telescope time of identifying all the cluster galaxies individually. Surveys that attempt to find distant clusters through locating overdensities in galaxy number counts must typically image at high resolution and must reach at least the characteristic apparent magnitude of the cluster galaxies \(m^* \approx 24\) in \(V\) for \(z \gtrsim 1\). This limits surveys to small areas unless the observer (and Time Allocation Committee) are willing to invest an extremely large amount of telescope time. Furthermore, imaging more deeply than \(m^*\) does little to increase the contrast of the cluster against the background galaxies, given that the slope of the faint galaxy number counts is typically steeper than the faint-end slope of the cluster luminosity function.

Instead, searching for large scale fluctuations in the background sky requires neither high resolution nor particularly deep data. By smoothing the sky on the scale of the clusters (typically tens of arcseconds), one rapidly gains signal-to-noise at the expense of unnecessary spatial resolution, allowing even short exposures to reveal very low surface brightness fluctuations. The only stringent requirement of this method is extremely accurate flatfielding. With the increasing stability of CCD’s, and the coming-of-age of “shift-and-stare” supersky flats and transit scans, this is a diminishing limitation. For example, an analysis of actual transit scan data shows that one can survey an area of \(\approx 10\) square degrees in a single night on a 4m-class telescope, and detect fluctuations with central surface brightness of \(\lesssim 28\) mag/arcsec\(^2\) (Dalcanton 1995) by smoothing the background over large areas. Furthermore, clusters identified as bumps in the EBL are almost guaranteed to be at moderate to high redshifts; an absence of foreground galaxies implies that all of the cluster galaxies are far enough away to have dropped below the magnitude limit of the image.

Our goals for this paper are three-fold. First, we wish to show that clusters truly appear as easily detected fluctuations in the EBL. We do this in §2 by considering how a rich cluster like Coma would appear if it were at large redshift, and by actually detecting both simulated and real high-redshift clusters. Secondly, we would like to demonstrate that the number of fluctuations due to clusters is likely to be large enough to make surveying for clusters in the EBL feasible. We do so in §3 by calculating \(N(\Sigma, \theta)\), the number density of clusters on the sky as a function of apparent central surface brightness \(\Sigma\) and apparent
angular size $\theta$. We calculate $N(\Sigma, \theta)$ by extrapolating the luminosity and surface brightness distributions of present day clusters backwards to higher redshift, taking into account the growth of the cluster mass with time and the luminosity evolution of the galaxies within the cluster for different cosmological scenarios, and we discuss the resulting distributions in §4 and §5. Finally, we wish to discuss how a survey might actually be carried out. We do this in §6, where we discuss the merits and drawbacks of various strategies, ways to optimize the search for the most distant clusters, and other possible sources of background fluctuations.

2. The Appearance of Clusters at $z = 1 - 2$

First, let us show that it is possible to detect a high-redshift cluster, even when the galaxies within it cannot be detected individually. The necessary ingredient is that the threshold for detecting objects of a given surface brightness is a strong function of angular size. For example, consider a sky-limited image of an extended object (i.e. a galaxy or cluster) with a mean surface brightness $\Sigma$ within an angle $\theta$, and a sky surface brightness of $s$. The signal-to-noise of this object is $\Sigma \pi \theta^2 / \sqrt{s \pi \theta^2}$ which is proportional to $\theta$. Turning this around, at a given limiting signal-to-noise, the minimum surface brightness that can be detected is proportional to $1/\theta$; larger objects may be detected at a fainter surface brightness than smaller objects. This limiting surface brightness as a function of angular size is plotted as the dotted line in Figure 1. Also plotted are the typical mean surface brightness profiles of a CD galaxy and of a Coma-like cluster as they would be observed at $z \approx 1.5$ (neglecting any luminosity evolution). Because of its small angular size, the galaxy never rises above the threshold for detection. The cluster, however, can be detected for large enough apertures, because it has a mean surface brightness within an angular size $\theta$ that falls off more slowly with radius than does the limiting surface brightness.

Next, let us look at the properties of the fluctuations that would be caused by a Coma-like cluster at high redshift. As an approximation, we will consider a cluster with a Hubble profile surface brightness distribution ($\Sigma(r) = \Sigma_0 / [1 + r/r_c]^2$), a core radius $r_c = 0.1$ Mpc, and an Abell luminosity of $100L_V^*$. This cluster would have a central surface brightness of $\mu_0 \approx 26, 27, 28$ mag/arcsec$^2$, if it were at redshifts of $z = 1, 1.5, 2$, respectively (ignoring any evolutionary effects or k-corrections). This corresponds to between two percent and two-tenths of a percent of the sky brightness in $V$. Such low pixel-to-pixel noise can only be reached in a long integration on a large telescope; however, since the cluster is an extended object, we can sacrifice resolution for greater sensitivity by smoothing over
large scales. The angular size of the cluster, defined by the radius at which the surface brightness falls below the limiting isophote, would be roughly $10'' - 1.5'$ for $z = 1 - 2$ and for a limiting isophote $\mu_{lim} = 29.0$. Thus, if the cluster were observed with a CCD with a pixel scale of $0.5''$/pixel, then smoothing the image on the scale of the cluster would increase the signal-to-noise of the large low-surface brightness fluctuation by over a factor of 20 from the single pixel signal-to-noise, provided that the flat-fielding variations are smaller than the amplitude of the fluctuation produced by the cluster. Therefore, although the cluster produces a very low surface brightness fluctuation, by smoothing of the EBL such fluctuations should be easily detectable. The angular size of the cluster, which corresponds to a radius of a few core radii in the frame of the cluster, is much smaller than typical a typical field of view, although the Abell radius ($1.5$ Mpc) of a cluster is typically larger. Because the detectable portion of the cluster easily fits within most CCD cameras, the fluctuation is less prone to being confused with large scale flat-fielding errors. This is even less of a problem for CCD drift scans.

Figure 2 is a more concrete example, showing how a rich, high-redshift cluster could be detected in the EBL by merely smoothing the background light. In order, the panels show an artificial rich cluster at $z = 1$, the same cluster embedded in a uniform background of field galaxies, the cluster and the field galaxies plus the EBL, and finally the result of smoothing the previous image. Notice how after smoothing the cluster stands out prominently above the fluctuations in the background, although it is completely invisible beneath the poisson noise of the EBL in the unsmoothed image.

Figure 3 shows a similar sequence for an actual X-ray selected cluster at $z=0.83$, generously provided by Gioia & Luppino (1994). The sequence of images show how the cluster could be detected as a bright patch in the EBL in an exposure of less than 30 seconds on a two-meter telescope, immediately demonstrating the feasibility of using the EBL to locate rich high-redshift clusters. The cluster is $6\sigma$ above the mean sky level, and obviously stands well above the level of the background fluctuations. The rms fluctuations are only $12\%$ larger than are seen after smoothing comparable uniform Poisson noise, suggesting that variations in the distribution of field galaxies are not responsible for the majority of the fluctuation amplitude at these scales, especially considering the residual flat-fielding errors. This is not surprising in light of the very weak angular correlation of faint galaxies (Koo & Szalay 1984, Stevenson et al. 1985, Efstathiou et al. 1991, Pritchett & Infante 1992, Bernstein et al. 1993).

3. Calculating the Observed Properties of Distant Clusters
We now turn our attention from detecting an individual cluster in the EBL towards understanding the properties of the entire population of clusters that could be revealed in the background light. Can clusters be detected with sufficient frequency to make surveying the EBL practical? With what range of surface brightnesses do clusters appear? What are the richesses of the systems that produce fluctuations of a given amplitude and angular scale? At what redshifts are the detected clusters? We will answer these questions by calculating the apparent size, surface brightness, and number density on the sky of clusters as a function of their redshift and luminosity, using a reasonable extrapolation from cluster properties at low redshift.

First, we wish to calculate the distribution of cluster surface brightnesses. To constrain the apparent surface brightness of high redshift clusters, we may capitalize on a fortunate coincidence uncovered by West, Oemler, & Dekel (1987,1989). After reanalyzing data on 29 Abell clusters in a consistant manner, they concluded that the surface brightness profiles of the clusters in the outer regions \( r > 0.1 \, \text{Mpc} \) were well fit by a deVaucouleur’s profile \( \Sigma'_e(r) = \Sigma'_e \exp(-7.67[(r/R_e)^{1/4} - 1]) \) where \( R_e \) is the half-light radius of the cluster, and \( \Sigma'_e \) is the intrinsic surface brightness (in units of luminosity per area) at \( R_e \). Furthermore, West, Oemler, & Dekel found that \( R_e \) and \( L_{\text{tot}} \) obey a simple scaling equation:

\[
R_e \propto L_{\text{tot}}^{0.51 \pm 0.07} \tag{1}
\]

\( (H_0 = 100 \, \text{km/s/Mpc}, \, q_0 = 0.5) \), immediately implying that clusters over a wide range in luminosity \( (30 \lesssim L_{\text{tot}}/L^{\ast} \lesssim 550) \) have approximately constant characteristic surface brightness. (For a deVaucouleur’s profile the total integrated luminosity of the cluster, \( L_{\text{tot}} \) is 7.22\( \pi R_e^2 \Sigma_e' \).) Assuming that galaxies trace the mass of a cluster, the clusters must have a constant characteristic surface density as well. If the scaling relationship between \( R_e \) and \( L_{\text{tot}} \) holds true with increasing redshift, as we would expect if the growth of groups and clusters is scale-invariant (Kaiser 1987), then the characteristic surface density is not only independent of total mass, but is independent of time as well. All evolution in the intrinsic surface brightness is then due only to luminosity evolution of the galaxies themselves, a process that can be readily modelled with stellar synthesis codes. We are thus in the rather surprising position of being able to estimate the apparent surface brightness of groups and clusters of galaxies as a function of redshift. Letting \( \Sigma' \) be the intrinsic central surface brightness and fitting to the data in West, Oemler, & Dekel (1989), we take the probability of a cluster at redshift \( z \) having an intrinsic central surface brightness \( \Sigma' \) at to be a log-normal distribution
where

\[ \Sigma'_0(z) = \Sigma'_0 \Upsilon(z) \]  

and where

\[ \Upsilon(z) \equiv \left( \frac{M/L_X(0)}{M/L_X(z)} \right) \]  

absorbs the effects of galaxy spectral evolution and of the change in the rest wavelengths observed through filter \( X \) with increasing redshift. We have determined \( \Upsilon(z) \) using the stellar population synthesis code developed by Bruzual & Charlot (1993). We use a galaxy formed at \( z = 2 \) with an exponential star-formation rate of \( \tau = t_{\text{lookback}}/2 \) as a "spiral" galaxy, and use a galaxy formed at \( z = 4 \) with \( \tau = t_{\text{lookback}}/14 \) as an "elliptical" galaxy (see Figure 7 of Bruzual & Charlot (1993)).

To find the specific values of \( \Sigma'_0 \) and \( \sigma_\Sigma \), we choose to make the assumption that the centers of the clusters are better fit by Hubble profiles with core radii than by steeper deVaucouleur profiles \( ^3 \). We take a core radius \( r_c = R_e/9.83 \), which preserves the luminosity within \( R_e \), gives a surface brightness profile that differs by no more than 6\% from the deVaucouleur’s between \( r_c \) and \( R_e \), and reduces the central surface brightness by a factor of 17.96 (or arguably more intuitively, by 3.14 mag/arcsec\(^2\)). Rescaling the distribution of central surface brightnesses in West, Oemler & Dekel by 17.96, we find \( \Sigma'_0 = 1.63 \times 10^3 L^*/\text{Mpc}^2 \) and \( \sigma_\Sigma = 0.55 \). Throughout this paper we use West, Oemler, & Dekel’s value of \( L^*_V = 1.3 \times 10^{10} h_{100}^{-2} L_\odot \) and \( M^*_V = -21.0 \).

We make a luminosity dependent correction to \( \Sigma_0 \) to improve the estimate of the central surface brightness for poorer clusters. Bahcall’s (1980) analysis of the optical properties of

\(^3\)As will be shown later in this paper, only the core of the cluster is typically detectable in the EBL \( (r \lesssim 0.1 \text{ Mpc}) \). Thus the results of this paper will be sensitive to the particular form of the profile chosen to model the core. Choosing a Hubble profile to model the center regions is most likely an improvement over choosing to extrapolate the deVaucouleur profile all the way into the center.
Morgan’s poor clusters (Morgan, Kayser & White 1975, Albert, White, & Morgan 1977) shows that the central surface brightness of poor clusters drops by roughly a factor of 3 from richness class 0 through richness class −3, (which corresponds to 10 galaxies within 1.5 $h^{-1}_{100}$ Mpc of the center – see Bahcall’s Figure 1). We include this effect as a correction to the central surface brightness of clusters with $L_{1.5} < 30 L^*$. We neglect clusters (or more truly, groups) with $L_{1.5} < 10 L^*$, as their profiles and central surface brightnesses are not well determined.

While equation 2 gives the distribution of intrinsic surface brightnesses, at large redshifts the apparent surface brightness will be much smaller due to the different redshift dependences of the luminosity and angular diameter distances. As we are interested in the observable properties of clusters, we must correct equation 2 to account for this “cosmological dimming”. We may relate the apparent surface brightness of a cluster at redshift $z$, $\Sigma(z)$, to the intrinsic surface brightness $\Sigma'(z)$ as

$$\Sigma(z) = \frac{\Sigma'(z)}{4\pi (1 + z)^4}. \quad (5)$$

Note the unfortunate fact that both $\Sigma$ and $\Sigma'$ have the same dimensions, where the former is measured in flux per steradian, and the latter is in luminosity per area. To attempt to reduce confusion between these two, the primed superscript will be used for intrinsic surface brightnesses. With this transformation, the distribution of apparent surface brightnesses is

$$p_\Sigma(\Sigma, z) d\Sigma = p_{\Sigma'}(\Sigma' = 4\pi (1 + z)^4 \Sigma, z) \cdot 4\pi (1 + z)^4 d\Sigma. \quad (6)$$

If one instead wishes to express the apparent surface brightness in terms of $\mu$, in magnitudes/square arcsecond, one may use

$$\mu(z) = M_* + 48.82 - 2.5 \log \left( \frac{\Sigma(z)}{L_*/\text{Mpc}^2/\text{radian}^2} \right). \quad (7)$$

to change variables in equation 6 from $\Sigma$ to $\mu$.

We now have the distribution of apparent central surface brightnesses as a function of redshift. To get the surface density of clusters on the sky, we need to combine the distribution in equation 6 with the number density of clusters as a function of redshift and luminosity, $n_L(L, z) dL$. The surface density on the sky of clusters at $z$ with luminosity $L$ and apparent central surface brightness $\Sigma$ is then
\[ N_L(\Sigma, L, z) \, d\Sigma \, dL \, dz = n_L(L, z) \, dL \, dV(z) \times p(\Sigma, z) \, d\Sigma \]  \hspace{1cm} (8)

where

\[ dV(z) = \left( \frac{c}{H_0} \right)^3 \frac{\chi^2(z)}{(1+z)^3 \sqrt{1+2q_0z}} \, dz \]  \hspace{1cm} (9)

is the comoving volume of one sterradian of a shell between \( z \) and \( z + \delta z \) and

\[ \chi(z) = \frac{1}{q_0} \left[ q_0 z + (q_0 - 1)(-1 + \sqrt{1 + 2q_0z}) \right]. \]  \hspace{1cm} (10)

We must determine the form of \( n_L(L, z) \) to use in equation (8). We do this by taking the number density of clusters at \( z = 0 \), \( n_L(L) \), then using linear theory to estimate how the number density might scale with redshift. Moore, Frenk, & White (1993) have analyzed the CfA redshift survey (Davis et al. 1982, Huchra et al. 1983) to find \( n_L(L) \, dL \), the comoving density of groups and clusters with luminosity between \( L \) and \( L + dL \). They first use a friends-of-friends linking algorithm, fine-tuned through comparisons with simulations, to identify groups and clusters. They then scale the luminosity of the groups to include the missing luminosity of galaxies lying below the magnitude limit of the survey. They find that \( n(L) \) is well fit by a double power-law:

\[ n_L(L) = n_0 \left[ \left( \frac{L}{L_0} \right)^{\beta_1} + \left( \frac{L}{L_0} \right)^{\beta_2} \right]^{-1}, \]  \hspace{1cm} (11)

with \( \beta_1 = 1.34, \beta_2 = 2.89, L_0 = 7L_*, \) and \( n_0 = 1.26 \times 10^{-3} \, h_3 \, L_*/\text{Mpc}^3 \).

While percolation algorithms are a convenient unbiased way to generate group catalogs, they do not offer a particularly well-defined definition of the group luminosity. This makes it difficult to make a sensible comparison between the group luminosity function and any other study. Moore et al. argue that their groups are approximately bounded by equidensity surfaces at 10-100 times the mean galaxy density. The best that we can do to attempt to define our group luminosities in the same manner is to calculate the luminosity within some equi-surface density countour \( \Sigma_{lum} \). To determine \( \Sigma_{lum} \), we use the total luminosity and effective radius of the deVaucouleurs’s profile fit to Coma’s surface brightness profile given in West, Oemler, & Dekel (1987,1989), and calculate the limiting surface brightness within which one measures the luminosity reported by Moore et al. We find \( \Sigma_{lum} = 1.4 \, h_3^2 \, L_*/\text{Mpc}^2 \). This value is uncertain by a factor of ten, given Moore et
al. quoted range in the variation of the bounding equidensity surfaces. Because of the slow fall-off with luminosity in our assumed Hubble-law cluster profile, this yields considerable uncertainty in identifying the correct number density to use for a cluster with a given luminosity within $\Sigma'_\text{lum}$. If we shift our choice of $\Sigma'_\text{lum}$ up by a factor of 10, the peak detection rate for clusters goes up by $\sim40\%$. If we shift our choice down by a factor of 10, the detection rate drops by $\sim25\%$.

We now estimate the $z$-dependence of $n_L$. In bottom-up scenarios of structure formation, the masses of galaxies and clusters continually grow through infall of density perturbations collapsing on progressively larger scales, or through merging of small clumps of mass to form larger ones. For a spectrum of perturbations that has no preferred scale, the masses of clusters grows self-similarly with time. Thus, $n_M(M)$, the number density of objects as a function of mass $M$, should have the same shape at every redshift, but should shift towards larger masses with increasing time.

To calculate the rate at which the characteristic cluster mass grows with time, we use a result from Peebles (1980). In linear theory, the amplitude of density perturbations is closely related to the power spectrum, which takes the form $P(k) = |\delta(k)|^2 D(z)^2$, where $\delta(k)$ gives the relative strength of fluctuations on different scales, and $D(z)$, the growth rate, describes the rate at which the amplitudes of fluctuations grow with time. We may approximate the growth of a cluster as being self-similar with a characteristic mass $M_0$ that grows as some power of the growth rate which depends on $|\delta(k)|^2$:

$$M_0 \propto D(z)^7$$

where

$$D(z) = \begin{cases} (1 + z)^{-1} & \text{if } \Omega = 1 \\ 1 + \frac{3}{x} + \frac{3(1+x)^{1/2}}{x^{3/2}} \ln \left[(1 + x)^{1/2} - x^{1/2} \right] & \text{if } \Omega < 1 \end{cases}$$

and $x(z) = \left| \frac{1}{\Omega_0} - 1 \right| \frac{1}{1+z}$.

For $x << 1$, at high redshift, the growth-rate approaches that of the Einstein-deSitter case. However, at small redshift, or $x >> 1$, the growth-rate is approximately 1, after which point the cluster mass stops growing with increasing time. Thus if $\Omega << 1$, the mass of a cluster is roughly constant after $z \approx 1/\Omega_0 - 2$, (or $z = 3$ if $\Omega = 0.2$, and $z = 0.5$ if $\Omega = 0.4$), while if $\Omega = 1$ the cluster mass continues to increase rapidly until the present day.

Over a wide range of scales, $|\delta(k)|^2$ can be approximated as a power-law, $k^n$, for which
\[ \gamma = \frac{6}{n+3}, \] ranging from 2 to \( \infty \) for \( n = 0, -3 \). For the mass scales in which we are interested, the cluster-cluster correlation function favors \( n = -1.2 \), or \( \gamma = 3.3 \) (Baugh & Efstathiou 1993, Fisher et al. 1993, Vogeley et al. 1992). However, given that the technique described in the paper observes only the highly non-linear cluster cores, we may expect a different rate of evolution than predicted by linear theory alone. Observations of large gravitationally lensed arcs and massive x-ray halos in clusters at high redshift (Gioia & Luppino 1994, Luppino et al. 1994, Dickinson 1993) immediately suggest that there were already massive cluster cores at \( z \gtrsim 0.5 \), and thus favor the smaller values of \( \gamma \). Because of this uncertainty, in \( \S 4 \) we will not restrict ourselves to one particular value for \( \gamma \), and will instead consider models for a range of \( \gamma \).

We now may use equation 12 to calculate the characteristic cluster luminosity \( L_0 \):

\[ L_0(z) = L_0(z = 0) D_0(z)^\gamma \Upsilon(z) \quad (14) \]

where \( D_0(z) \) is the appropriate form for \( D(z) \), normalized to be 1 at \( z = 0 \), and where \( \Upsilon(z) \) absorbs the evolution of the cluster mass-to-light ratio due to galaxy luminosity evolution (equation 4). \( L_0(z) \) absorbs the entire evolution of the cluster luminosity function by simply shifting \( n_L(L, z = 0) \) towards smaller luminosity with increasing redshift, without changing shape.

It is better to recast \( N(\Sigma, L, z) \) (equation 8) in terms of directly observable quantities. To do so, we must transform the cluster luminosity \( L \) into the angular size \( \theta \) of the cluster. The size of the cluster is set by the radius at which the surface brightness of the cluster falls below some limiting isophote, and is therefore sensitive to the surface brightness profile of the cluster. We assume that the central regions of the cluster are fit by a Hubble profile at \( z > 0 \), which allows us to calculate the apparent angular size of the cluster\(^4\). Note that by assuming that the cluster surface density is invariant while the cluster mass is changing, we are implicitly assuming that the core radius of the cluster grows with time. Taking \( \Sigma_l \) to be the surface brightness at the limiting outer isophote, and \( d_a(z) \) to be the angular diameter

\(^4\)This is a strong assumption. If clusters grow through continuous merging of sub-clusters of comparable mass, this picture of simple self-similar growth is an approximation. N-body simulations can possibly test these assumptions, and may easily be incorporated into further refinements of the analytic work presented in this paper; however it is unclear if they have either sufficient resolution or physics to correctly model the evolution of the core. Simulations could also be used to directly measure the contribution of distant clusters to surface brightness fluctuations.
distance at redshift \( z \), the apparent angular size of the cluster is

\[
\theta = \frac{r_c}{d_a(z)} \left[ \left( \frac{\Sigma}{\Sigma_l} \right)^{1/2} - 1 \right]
\]

(15)
or

\[
\theta = \left( 41.3'' \right) \left( \frac{r_c}{0.1 \text{ Mpc}} \right) \left( \frac{10^3 \text{ Mpc}}{d_l(z)} \right) \left( \frac{\left( \frac{\Sigma}{\Sigma_l} \right)^{1/2} - 1}{2} \right).
\]

(16)

which immediately indicates that fluctuations in the extragalactic background light created by high-redshift, unresolved clusters will easily fit within the field of view of most detectors. The physical size of the cluster, \( R_l \), at the limiting isophote is

\[
R_l = r_c \left[ \left( \frac{\Sigma}{\Sigma_l} \right)^{1/2} - 1 \right].
\]

(17)

Therefore, a cluster whose apparent central surface brightness \( \mu \) is 2.5 magnitudes brighter than the limiting isophote \( \mu_l \) will be detected out to roughly two core radii. We should also remark at this point that, as discussed in §2, the limiting isophote is in fact a function of \( \theta \), and would more appropriately be written \( \mu_l(\theta) \). Including this effect would certainly complicate the machinery being developed in this paper, and in practice may be sidestepped by either analyzing a sample of clusters using a fixed \( \mu_l \) set by the limiting isophote of the smallest scale on which clusters were searched for, or by redoing the analysis for several different ranges of \( \theta \), using an approximately correct \( \mu_l \) for each, and then binning the results.

We may now relate \( \theta \) to the luminosity using equation (15) and

\[
L(\Sigma, \theta, z) = 8\pi^2 r_c^2 (\Sigma, \theta, z)(1 + z)^4 \Sigma \left[ \ln (1 + x) - \frac{x}{1 + x} \right],
\]

(18)

the luminosity contained within a radius \( x r_c \) for a Hubble profile, where

\[
x \equiv \sqrt{\frac{4\pi(1 + z)^4 \Sigma}{\Sigma_{lum}}} - 1.
\]

(19)

Equation (17) corresponds to the luminosity used in the Moore, Frenk, & White luminosity function (equation [11]). Figure 3 shows \( L \) as a function of apparent angular size \( \theta \) for several
values of the apparent central surface brightness $\Sigma$. Note the peculiar fact that at a fixed apparent size and redshift, a cluster with a brighter central surface brightness in fact has a smaller total luminosity. This reflects that the cluster has a smaller effective radius (see Equation 17), and thus a more steeply declining surface brightness profile than a cluster of the same apparent size but a fainter central surface brightness.

With equation 18, we may change variables in $N(\Sigma, L, z)\,d\Sigma\,dL\,dz$ to get $N(\Sigma, \theta, z)\,d\Sigma\,d\theta\,dz$, the number of clusters per steradian with apparent central surface brightness $\Sigma$, apparent angular size $\theta$, and redshift $z$. This yields

$$N_z(\Sigma, \theta, z)\,d\Sigma\,d\theta\,dz = n_L(L(\Sigma, \theta, z), z) \left| \frac{dL}{d\theta} \right| \,d\theta \,dV(z) \times p(\Sigma, z)\,d\Sigma$$  \hspace{1cm} (20)

where

$$\left| \frac{dL}{d\theta} \right| = \frac{2L(\Sigma, \theta, z)}{\theta} \left[ 1 - \frac{1}{2} \left( \frac{x}{1+x} \right)^2 \left( \frac{1}{\ln (1+x) - \frac{x}{1+x}} \right) \right]$$  \hspace{1cm} (21)

Equation 20 may be integrated over all redshift to yield $N(\Sigma, \theta)$:

$$N(\Sigma, \theta)\,d\Sigma\,d\theta = \int_0^\infty N_z(\Sigma, \theta, z)\,d\Sigma\,d\theta\,dz.$$  \hspace{1cm} (22)

This may also be recast in terms of $\mu$ using equation 7.

### 4. Detection Rates and the Distribution of Surface Brightnesses

Integrating Equation 22 over $\theta$ and switching variables from $\Sigma$ to $\mu$, we may calculate the distribution of apparent central surface brightnesses, $N(\mu)$. The distributions are plotted in Figure 6 for a variety of cosmological models. We have calculated $N(\mu)$ for $\Omega_0 = 0.2, 1$, $H_0 = 50, 100$ km/s/Mpc, $\gamma = 0, 1, 2, 3$, and for three different assumptions about the spectral evolution of galaxies in the cores of clusters. In one case the cores of clusters are assumed to be entirely populated by elliptical-like galaxies formed at $z = 4$. In the second case the cores are filled with spiral-like galaxies formed at $z = 2$. In the final case, which can more readily used to compare the effects of varying the different cosmological parameters, we assume that there is no evolution in the spectra of the cluster galaxies, and we ignore the change in effective bandpass with redshift. Hopefully the most
appropriate model for galaxy evolution lies somewhere in the space spanned by the models used here. (The models are described in more detail in §3.) We have assumed a limiting isophotal magnitude of 29 mag/arcsec$^2$.

There are a few general points to note about the resulting distributions:

(1) The detection rates can be almost arbitrarily high or low, depending on the adopted form for the cluster mass and luminosity evolution, and on the values of cosmological parameters. Surface densities of tens of clusters per square degree seem common, but can be as high as several hundred clusters per square degree, and as low as one cluster per ten square degrees.

(2) The distribution of surface brightnesses is highly peaked in $\mu$. The peak of the distribution tends to fall between 25 and 28 mag/arcsec$^2$ (for $\mu_{lim} = 29$). If the limiting isophote is moved to a brighter level, the same angular size selects groups and clusters that would appear at larger angular sizes for the fainter limiting isophote. See the next point for a discussion of how this affects $N(\mu)$.

(3) The detection rates are smaller for fluctuations with large angular size. This reflects two fairly simple facts. First, intrinsically large systems tend to have larger angular extents; therefore, at large angular sizes one is picking up richer systems, which are rarer and have a correspondingly lower number density. Second, because clusters that are nearby will have larger angular sizes than more distant ones, by restricting a survey to large angular sizes, one is probing a more local, smaller volume of the universe; this again leads to a smaller surface density on the sky.

(4) Because the systems selected at large angular size are closer on average, they have also suffered less cosmological $(1 + z)^4$ dimming; therefore, the distribution of $N(\mu)$ peaks at brighter surface brightnesses for large angular sizes.

(5) The surface density of clusters increases for smaller $\Omega$ and larger $H_0$. For small $\Omega$, there is both a larger accessible volume and a reduced rate of cluster evolution; this leads to larger surface densities of richer systems. For larger values of $H_0$, the luminosity distance is smaller, which makes clusters visible out to larger distances, increasing the accessible volume.

(6) The effects of galaxy luminosity evolution and cluster mass evolution, (parameterized by $\Upsilon(z)$ and $\gamma$, respectively) are very strong and are the largest uncertainties in the determination of the surface density of fluctuations. As will be shown in §6, clusters must be at redshifts greater than $\approx 0.5$ before their properties enter the range of surface brightness and angular size to which surveys are likely to be sensitive. If there is rapid
evolution of the mass of cluster cores (i.e. large $\gamma$), then at these high redshifts there will be so little mass assembled in what will eventually be the cluster core that the cluster will be too faint to be detected. However, if the galaxies are very young and actively star-forming at high redshifts, then the cluster luminosity and surface brightness can be greatly enhanced by the increased star-formation. Therefore strong galaxy evolution could completely cancel the effects of the cluster evolution. Disentangling these two competing effects will be quite difficult until information is brought to bear from other observations of distant clusters and galaxies. The masking of one effect by the other can limit the usefulness of this method in constraining cosmological scenarios. If one is brave enough to trust one’s models, there is perhaps some differentiation that could be extracted from the location of the peak surface brightness; we consider the required level of trust to be premature at this time.

5. The Redshift and Richness Dependence of $N(\Sigma, \theta, z)$

We may use the distribution of $N(\mu, \ln \theta, z)$ and $L(\Sigma, \theta, z)$ to understand the redshifts and richesses of the clusters that contribute to $N(\mu)$. Both of these functions are shown in Figure 7. First note that for a given surface brightness $N(\mu, \ln \theta, z)$ tends to be peaked at a particular redshift. In the models that include the effects of galaxy evolution, the distribution is much broader in redshift; at high redshift, the cluster galaxies are much younger and are postulated to have much higher star formation rates, and correspondingly larger $e+k$ corrections, which help to compensate for the decrease in flux at large distance. These models suggest that the clusters found could exist at redshifts between 0.5 and 3.

Second, notice the strong effect that increasing $\gamma$ has on pushing $N(\mu, \ln \theta, z)$ towards smaller redshifts. In models which assume that the typical cluster mass evolves rapidly with redshift ($\gamma > 0$), only at $z < 1$ are groups and clusters large enough to produce significant fluctuations in the EBL. The redshift distributions of clusters found in the EBL could potentially be used to constrain the value of $\gamma$.

The top plots in Figure 7 show how the luminosity of clusters detected at a fixed angular size changes with redshift and central surface brightness. The luminosity is given as the luminosity that the cluster would have today, after mass and spectral evolution have taken place. By comparing the top and bottom figures, one can read off the richness of the system that is being detected at a given redshift and surface brightness. Not surprisingly, the largest contribution to $N(\mu, \ln \theta, z)$ usually comes from poorer systems, which are more numerous. We ignore contributions of systems which have present day luminosities below $10L^*$, the luminosity of the smallest of Morgan’s poor clusters (Bahcall 1980, Albert et al.
1977, Morgan et al. 1975); below this luminosity, the radial profiles of poor groups are not well understood, and are probably not well defined for an individual group. With the exception of the dense, but rare, groups typified by Hickson’s compact groups (Hickson 1982), it is not unreasonable to assume that poor groups will be less tightly bound and thus will have low central surface brightnesses, and will be nearly undetectable when placed at a distance such that their component galaxies cannot be seen individually.

Figure 7 also demonstrates how the richness of a group or cluster that we detect in the EBL depends on its redshift. In the absence of galaxy evolution, a system that we detect at high redshift will be richer than a system at lower redshift which has the same observed central surface brightness and angular size. This is due to the larger \((1 + z)^4\) cosmological dimming that sources at high redshift experience, as well as the (typically) larger physical size that corresponds to a given apparent angular size (e.g. Equation 15). If galaxy evolution is included, however, a poor cluster of young galaxies at high redshift may have the same apparent size and surface brightness as a richer, older cluster at lower redshift.

6. Some Thoughts on Applying this Method

We now turn our discussion from rather idealized theoretical concerns to the more grisly, messy issues related to carrying out an actual survey that is optimized to detect the richest systems.

As discussed above, the EBL must show signatures of both distant rich clusters and nearer, poorer groups. The degeneracy that exists between distant rich systems and nearby poor systems is a difficult one to break, especially when all objects of interest are at high enough redshift that the angular diameter distance changes very little. However, there are some steps one may take to maximize the efficiency of finding distant rich clusters.

First, richer clusters dominate at lower surface brightness, for a fixed angular size. This may at first seem counterintuitive; however, a system with a radial surface brightness distribution with a low central surface brightness and a large angular size has a more slowly falling profile, and thus a greater integrated luminosity beyond the central core, than a more centrally peaked system. Therefore, by restricting one’s attention to fluctuations with lower surface brightness, one is more likely to be selecting for richer systems.

Second, we may maximize the contribution from rich clusters by selecting objects of a particular angular size that have few individual galaxies within the area of the EBL
fluctuation. The expression for \( L(\Sigma, \theta, z) \) given in equation 18 shows that the rate at which \( L \) rises with redshift strongly increases with \( \theta \); this simply reflects the rather simplistic rule that nearby objects tend to look larger than distant ones. Fluctuations seen at large angular size will therefore tend to be closer. (As an example of this, for an angular radius of 30'' and a central surface brightness of 26 mag/arcsec\(^2\), a cluster with \( L_{1.5} = 100L^* \) would have these properties at \( z = 1.5 \) in a no-evolution model, but a similar cluster must have the much larger redshift of \( z = 2.5 \) if the detected angular radius were reduced by a factor of 2. A group with the minimum luminosity \( L_{1.5} = 10L^* \), would appear at \( z = 1, 0.5 \) for \( \theta = 15'', 30'' \), respectively.) By properly choosing the angular size, poorer clusters will be brought close enough that the individual galaxies within the cluster could be readily seen, while richer clusters, which tend to be further away (for the same angular size and smoothed surface brightness), will have fewer of their component galaxies above the magnitude limit of the survey.

How close must a cluster be before one could see its component galaxies? We can estimate the redshift beyond which the cluster recedes into the background light in two ways. First, we can estimate the apparent \( m^* \) of the cluster as a function of redshift. A cluster that is far enough away that its \( m^* \) drops below the magnitude limit of the survey will have too few members visible to be detectable as a cluster in the traditional way. The unsmoothed survey data would yield little information on the richness of the cluster. Assuming that \( M^* = -21 \) and a magnitude limit of \( m_{lim} \sim 23 \), \( L^* \) galaxies in the cluster should be visible at redshifts less than 0.5-1, depending on the details of the evolution.

For a more empirical limit on the redshift at which cluster galaxies become visible, we can look at the apparent redshift of the brightest cluster galaxy (BCG) as a function of redshift and richness. (For simplicity, we will ignore the misnomer and take “BCG” to mean the brightest galaxy in a cluster or group). To determine the apparent magnitude of a BCG with redshift, we construct a \( V \)-band Hubble diagram from the high redshift \((0.5 \lesssim z \lesssim 0.9)\) clusters in Aragón-Salamanca et al. (1993), the intermediate redshift \((0.2 \lesssim z \lesssim 0.4)\) clusters in Pickles & van der Kruit (1993), and Abell clusters from Schneider, Gunn, & Hoessel (1983) (See Figure 3). We fit the inverse Hubble diagram for rich clusters with two straight lines

\[
\begin{align*}
z_0(V) &= \begin{cases} 
0.0471 \times V - 0.6587 & \text{if } V < 18 \text{ (roughly } z < 0.2) \\
0.1423 \times V - 2.3260 & \text{if } 18 < V \text{ (roughly } 0.2 < z < 1) 
\end{cases}
\end{align*}
\tag{23}
\]

As very little is known about clusters or BCG’s above \( z = 1 \), it is difficult to extend this relation to higher redshifts. However, radio galaxies, which are often identified as BCG’s, have apparent \( V \) magnitudes of \( \approx 22.5 \) at \( z > 1 \) (Djorgovski, Spinrad, & Dickinson 1987).
If true, then the presence or absence of foreground galaxies is a poor distance discriminant beyond $z \approx 1$. Thus, an angular size should be chosen such that the typical redshifts of the poorest systems one is interested in lie below $z = 1$.

To extend the apparent magnitude of BCG’s to poorer systems, we have estimated the magnitude of the brightest group galaxy as a function of group size using the CfA Group Catalog (Geller & Huchra 1983). The constancy of $M_{BCG}$ with richness seems to extend to clusters with a luminosity of $\sim 0.2$ of the luminosity of the Coma cluster (corresponding to roughly 30 group members in the CfA $m < 14.5$ Catalog). We define this luminosity to be $L_c$. For poorer groups, the typical BCG becomes fainter, as would be expected if merging in the high density cores of clusters is resposible for enriching large central CD galaxies. A reasonable empirical fit to the change in $M_{BCG}$ with group luminosity is, relative to the typical absolute magnitude of a BCG in a rich cluster,

$$\Delta M_{BCG}(L) \approx \begin{cases} 
2.4 \left(1 - \frac{L}{L_c}\right) & \text{if } L < L_c \\
0 & \text{if } L \geq L_c
\end{cases} \quad (24)$$

We fix the value of $L_c$ from Abell’s luminosity of the Coma cluster as used by Bahcall (1979) and find $L_c \approx 20L^*$. There is considerable scatter in this relation, but the general trend of groups having fainter BCG’s than rich clusters seems evident. This shifts the lines in Figure 8 towards smaller redshifts.

6.1. Other Sources of Fluctuations

Distant clusters will of course not be the only source of fluctuations in the EBL. Any source which causes low level surface brightness fluctuations in the background light will contribute to the observed $N(\mu)$. We are fortunate, however, that many of the possible sources are either negligible or easily separated. Preliminary results from a survey for nearby low-surface brightness galaxies (Dalcanton 1995), as well as follow-up observations at Kitt Peak, show that large LSB’s have a very low surface density on the sky, particularly at the faint surface brightnesses that are relevant for detecting clusters. Therefore, they will not be a substantial source of confusion. Likewise, low-surface brightness extensions to disturbed galaxies, such as might be due to tidal interactions or merging, will be fairly easily distinguished by the presence of the disturbed galaxy adjacent to the detected fluctuation. Other instrumental sources of low-surface brightness fluctuations, such as artifacts due to scattered light and flat fielding, are not fixed to the sky, and thus can be
separated by moving the telescope; for example, repeating a drift scan with slight shifts in declination would be one way of constraining instrumental contributions. Scattering and flat-fielding errors are usually either linear features or large scale variations, both of which will have signatures that are distinct from the “bumps” due to distant clusters. The one remaining source of fluctuations that could potentially be confused with clusters is chance superpositions of physically unrelated faint field galaxies. In the Appendix we show that the contribution from a uniform background of field galaxies is small, especially on the large angular scales over which we are interested. Typically the Poisson fluctuations of the EBL itself dominate the fluctuations from the field galaxies. This is supported by the analysis of MS1054 presented in §2.

7. Conclusions

The technique of looking for high-redshift clusters manifested as low-surface brightness fluctuations in the background light presents a powerful new tool for finding one of the most elusive of astronomical objects. The method we have proposed could isolate high-redshift clusters at a significant rate, up to hundreds per square degree, depending on the particular form of cosmology and galaxy evolution that Nature has chosen. Clusters give us a way of isolating a population of galaxies at a particular redshift, and therefore finding the highest-redshift clusters opens a window onto some of the earliest stages of galaxy evolution. The surface density of clusters that this method could find would also be a constraint on various scenarios for structure formation. More accurate simulations of cluster profiles as a function of redshift in different cosmologies and more work on untangling luminosity evolution from mass evolution are required before this method could provide a strong constraint, however.

Looking for clusters in the EBL is a method that we have shown works in principle, and in practice. The technical limitations involved are few and readily surmountable. The extremely accurate flat-fielding required is easily reached with telescopes operating in drift-scan mode. And while searching for low-surface brightness fluctuations requires extremely accurate flat-fielding, it has dramatically less stringent requirements on exposure time than traditional direct imaging. This means that this method could easily survey large areas of the sky in just a few nights. Deeper follow-up imaging would be required to confirm the existence of clusters. As more is learned about the composition of clusters at higher redshift, color information could be incorporated into the cluster selection criteria to increase the efficiency of cluster selection. A region of the sky could be observed through
multiple band passes, and the colors of the bumps in the EBL could be used to help isolate high-redshift clusters, and perhaps shed light on the global behavior of star formation in galaxies at redshifts greater than 1. Similarly, information from other wavelengths could be incorporated. Regions of large fluctuations in the optical EBL could be compared to the smoothed x-ray EBL or to catalogs of radio galaxies to increase the likelihood of identifying rich systems.

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The idea for this paper grew out of a series of ongoing discussions with my thesis advisor, David Spergel. To him goes much of the credit for encouraging me to pursue a nutty idea and turn it into something vaguely sensible. (If it does in the end turn out to be nutty, however, I accept full credit.) It is a pleasure to thank Gerry Lupino and Isabella Gioia for giving me access to their lovely image of MS1054, which provided the first definitive test of the methods proposed in this paper. I also thank Gerry for sharing his image display secrets. Jim Gunn and Dan Rosenthal are also warmly acknowledged for their encouragement and advice, as is Teresa Shaw for assistance with typing.

Appendix A

Contributions of Faint Field Galaxies

Assuming a power law distribution in magnitude for the faint galaxy number counts, we may use Monte Carlo simulations to estimate the surface brightness at which faint field galaxies will begin to contribute significantly to the fluctuations. A high surface brightness fluctuation in the background of field galaxies will be sufficiently rare to make effectively no contribution to the number of fluctuations. At lower surface brightnesses, however, fluctuations due to field galaxies will be more common, and will thus give an appreciable signal. The surface brightness threshold at which this happens will be a strong function of the scale length over which the background sky is smoothed. If the sky is smoothed over large apertures, then many field galaxies will be averaged together, and the level of fluctuations will be smaller than if a small aperture were used.

The aim of this section is to estimate the surface brightness limit (as a function of smoothing length) at which background field galaxies are expected to make a significant contribution to the number of fluctuations. The method for doing so is rather straightforward. First, assume that the number of galaxies per magnitude per area on the sky can be described by a single power law
\[
\log \left( \frac{dn(m)}{dm d\Omega} \right) = \alpha (m - m_0), \tag{25}
\]

as is seen in deep surveys of field galaxies (see Tyson 1994 for a review). Second, choose \( N \) galaxies with magnitudes between \( m_{\text{min}} \) and \( m_{\text{max}} \),

\[
N = \frac{\Omega}{\alpha \ln 10} \left[ \frac{dn(m_{\text{max}})}{d\Omega} - \frac{dn(m_{\text{min}})}{d\Omega} \right], \tag{26}
\]

where \( m_{\text{min}} \) is the magnitude of the faintest galaxy that cannot be detected above the pixel-to-pixel noise of the sky, \( m_{\text{max}} \) is the magnitude at which the faint galaxy number counts fall off from a power law, and \( \Omega \) is the area of the aperture over which the sky is smoothed (i.e., the area of the smoothing kernel). Third, multiply the flux from each galaxy by a value drawn from the two-dimensional smoothing function used within the aperture, and then add the fluxes together to find the surface brightness. (The units of the smoothing function are “probability/area”, so multiplying the fluxes by the smoothing function automatically yields a surface brightness). Finally, repeat these steps many times to generate the probability distribution of surface brightness fluctuations due to field galaxies.

We have performed this exercise using simple gaussian windows of different sizes, truncated at 5\( \sigma \), and for a range of assumptions about the slope of the faint galaxy number counts and possible survey magnitude limits. We have taken 1 “galaxy clump” per square degree to be the fiducial limit at which the contribution from field galaxies becomes significant. We find the surface brightness which corresponds to this detection rate by first assuming that we can treat a square degree of the sky as being \( M = \frac{(1\text{deg}^2)}{\pi \sigma^2} \) independent regions and then finding the surface brightness at which there is only a \( 1/M \) chance of reaching a brighter surface brightness due to chance fluctuations in the surface density of field galaxies. This is obviously not strictly correct given that neighboring regions of the sky are not independent. It is also not correct for very small smoothing lengths, where the background galaxies cannot be treated as point sources. However, it does serve as an estimate of the relevant surface brightness at which we might expect field galaxies to become important.

The results are shown in Figure 9. The contribution of field galaxies to the fluctuations in the smoothed sky background is a strong function of smoothing length, as expected, and
becomes effectively negligible for smoothing lengths greater than 10″. At smaller smoothing
lengths, the field galaxies make a significant contribution at brighter surface brightnesses
(μ ≈ 26.5); however, as shown in Figure 4(b), the angular size at a limiting isophote μ = 29
is never greater than 10″. Therefore, if a survey is limited to smoothing lengths greater
than 10″, or to finding objects with angular size greater than 10″, then we would expect
the contribution from faint field galaxies to be small. These results will of course change if
different functions are chosen for the smoothing window, and should be repeated depending
on the details of a particular survey.
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Figure 1. The mean surface brightness within $\theta$ for a CD galaxy (light solid line) and a Coma-like cluster (dark solid line) at $z \approx 1.5$, neglecting luminosity evolution or k-corrections. Also plotted is an example of the limiting surface brightness within an aperture of radius $\theta$. Note that while there is no aperture size within which the mean surface brightness of the CD galaxy rises above the limiting surface brightness, for large enough apertures the mean surface brightness of the cluster is high enough to be detectable. The CD galaxy is taken to have a deVaucouleur's profile with $r_e = 35$ kpc and a total luminosity of $4L^*$. The cluster has a Hubble profile with $r_{\text{core}} = 0.1$ Mpc and a luminosity within an Abell radius (1.5 Mpc) of $L_{\text{Abell}} = 100L^*$. ($H_0 = 75$ km/s/Mpc, $\Omega = 0.2$).

Figure 2. Detecting an artificial cluster with $L_{\text{Abell}} = 90L^*$ and $r_c = 0.1 Mpc$ at $z \approx 1.5$ ($m^* = 23.5$ and $\theta_c = 15''$). (a) An artificial cluster of galaxies distributed in a Hubble profile with $\theta_c = 15''$ and with magnitudes drawn from a Schecter-function with $m^* = 23.5$ and $\alpha = -1.25$, down to $m = 29$. The entire image is 11.5' across. (b) The same cluster from (a), but embedded in a background of field galaxies with magnitudes drawn from a power-law distribution ($\log \frac{dN}{dm dz} = 0.38(m - 12.13)$) between $m = 22.5$ and $m = 29$. (c) The cluster and field galaxies from (b) embedded in Poisson noise with a surface brightness of 21.3 mag/arcsec$^2$. Note that the galaxies all fall below the detection limit of the image. (d) The image from (c), smoothed with a Gaussian filter with $\sigma = 13.5''$. While there is only the barest hint of the cluster in (c), the smoothed image shows the presence of the cluster quite clearly due to the dramatically lower pixel-to-pixel variation of the noise. The peak is $9\sigma$ above the sky level.

Figure 3. Recovering a known high-redshift cluster using surface brightness fluctuations. (a) A 3-hour $I$-band image of the rich x-ray selected cluster MS 1054.5-0321 ($L_x = 9.3 \times 10^{44}$ ergs/s) at $z = 0.823$, taken with a Tektronics 2048x2048 CCD (0.22'' pixels in 0.8'' seeing) on the UH 88-inch telescope, kindly provided by Gioia & Luppino (see Gioia & Luppino 1994). The image is a composite of 18 600s images, each flattened with a “supersky flat” made from the median of 50 deregistered images. The typical shift between exposures was $10'' - 20''$. There are some obvious low-level, large-scale flat-fielding variations across the image that are visible at much higher contrast. (b) The same cluster, but with Poisson-noise added to reduce the effective exposure of the image by a factor of 400, producing an effective exposure of 27 seconds. Note that almost all of the cluster galaxies now fall below the level of the sky noise. (c) The image from (b) “cleaned” of foreground objects. FOCAS was used to identify all $4\sigma$ peaks. Bright, large objects were masked out and replaced with the median sky level of the entire image. The remaining objects were replaced with values drawn from the sky histogram of a ring around the object. This preserves
large low-surface brightness features that lie below compact high-surface brightness features. The procedure used to generate this image is fully automated and is being used for a survey of low surface brightness galaxies. It is described more fully in Dalcanton (1995). (d) The image from (c), smoothed with a Gaussian filter with $\sigma = 7''$. The cluster is easily detected as the brightest and largest peak on the image, lying $6\sigma$ above the sky with a diameter of roughly 30''. Extended light from the foreground group to the right of the cluster is also apparent, though at lower significance. There are also indications of some of the principle sub-clumps at low significance as well. The largest negative deviations are only $3\sigma$ below sky, and correspond to regions of obviously flat-fielding errors on the original image. The level of these deviations would presumably be smaller in a drift-scanned image, increasing the significance of the cluster detection.

Figure 4. The luminosity and bandpass evolution corrections used in the paper for "elliptical" and "spiral" galaxies (Equation 4. The corrections are derived from the models described in Bruzual & Charlot (1992), using $z_{\text{formation}} \approx 2$, $t_{\text{formation}}/\tau = 2$, 14 for the "spiral" and "elliptical" galaxies, respectively, as viewed through a Gunn g filter. Adding $-2.5 \log \Upsilon(z)$ to $5 \log d_l(z)$ gives the full e+k correction.

Figure 5. The properties of a cluster at $z = 1.5$ as a function of apparent angular size and central surface brightness. (a) The luminosity within an Abell radius (1.5 Mpc) in units of $L^*$ as a function of apparent angular size $\theta$. The lines are for different apparent central surface brightnesses: $\mu = 26$ (solid), $\mu = 27$ (dotted), $\mu = 28$ (dashed), assuming $M^* = -21$. See equation 18. (b) The core radius of the cluster as a function of apparent angular size. The lines correspond to the same central surface brightnesses as in (a). See equation 15.

Figure 6. $N(\mu)$ for various parameters, in units of $#/\text{degree}^2/\text{mag}/\text{arcsec}^2$. The solid and dashed lines are $\Omega = 1$ and $\Omega = 0.2$, respectively. The heaviest line is $N(\mu, \theta)$ integrated between $10''$ and $20''$, the medium weight line is $N(\mu, \theta)$ integrated between $20''$ and $30''$, and the lightest line is $N(\mu, \theta)$ integrated between $30''$ and $40''$. Note that the number of clusters goes up for lower $\Omega$, smaller angular sizes, and smaller values of $\gamma$. Changing $\gamma$ affects the number of cluster more dramatically for $\Omega = 1$ than for low values of $\Omega$. The limiting isophotal magnitude is taken to be $\mu_{\text{lim}} = 29$, and the limiting redshift is taken to be $z_{\text{lim}} = 2$. Only groups that will have luminosities greater than $10 L^*$ at $z = 0$ are included. $h_{100} = 1$ unless otherwise noted.

Figure 7. $N(\mu, \ln \theta, z)$ (in units of $#/\text{degree}^2/\text{mag}/\text{arcsec}^2/\ln(\text{radians})$) and the corresponding luminosity that the detected cluster will eventually have at
$z = 0$. These pairs of plots can be used to understand the redshifts and richnesses of the systems that contribute to surface brightness fluctuations in different cosmological scenarios. For all of these plots, $h_{100} = 1$ and $\Omega = 1$. (a) Models with no luminosity evolution. The different line types correspond to different values of $\mu$: $\mu = 24$ (heavy solid); $\mu = 24.5$ (heavy dot-dashed); $\mu = 25$ (heavy long dashed); $\mu = 25.5$ (heavy short dashed); $\mu = 26$ (heavy dotted); $\mu = 26.5$ (light solid); $\mu = 27$ (light dot-dashed); $\mu = 27.5$ (light long dashed); $\mu = 28$ (light short dashed); $\mu = 28.5$ (light dotted). The columns show models with no cluster mass evolution ($\gamma = 0$) on the left, progressing to more dramatic cluster evolution ($\gamma = 3$) on the right. While for a fixed $\mu$, $\theta$, and $z$, one always detects a system with the same luminosity, for larger $\gamma$’s the system will accrete more mass at late times than if $\gamma$ were smaller; this effect can be seen as a gradual steepening of $L(\mu, \theta, z = 0)$ with increasing $\gamma$. Because the current number density of clusters is fixed, if $L(\mu, \theta, z = 0)$ grows more steeply with redshift, then there will be fewer high redshift systems contributing to the number of surface brightness fluctuations; this can be seen in the behavior of $N(\mu, \ln \theta, z)$, shown in the lower of each pair of plots. The top row of plots shows $N(\mu, 35'' z)$, which can be compared with the integrated function $N(\mu)$ shown as the lightest lines in Figure 6). The bottom row of plots shows $N(\mu, 15'', z)$, which can be compared with the darkest lines in Figure 6). (b) Models with “elliptical” luminosity evolution. (c) Models with “spiral” luminosity evolution.

Figure 8. $V$ magnitude of the brightest cluster galaxy as a function of redshift for three different samples.

Figure 9. (a) The central surface brightness at which the fluctuations due to a Gaussian-smoothed uniform background of field galaxies contributes one peak per square degree, as a function of the size of the Gaussian window. The symbols correspond to different assumptions for the magnitude of the brightest undetected galaxy and for the slope of the faint number counts. (b) The same central surface brightness as in (a), but plotted as a function of the angular size where the surface brightness fluctuation falls below a limiting isophote of $\mu_{lim} = 29$. While at small angular sizes the contribution from randomly distributed field galaxies to surface brightness fluctuations in the background sky can be significant, for angular sizes greater $10''$ in radius, the contribution is negligible.