A Simple Formula for Predicting the Compressive Strength of Circular CFT Stub Columns

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Abstract

In this paper, the predictions of ultimate strength of circular concrete-filled steel tube (CFT) stub columns using current design codes such as AIJ-2001, AISC-2005, DL/T-1999, and Eurocode 4-2004, are examined and compared with published experimental tests of CFT short columns made with normal- and high-strength steel and concrete. Based on the investigations, a simple formula for predicting the ultimate strength of CFT stub columns is proposed. The concrete confinement, which depends mainly on the ratio of the external diameter of the steel tube to the plate thickness, the yield stress of the steel tube, and the unconfined compressive strength of the filled concrete, was reasonably achieved in the proposed formula. From comparisons with the published experimental data for CFT short columns of normal and high strength steel and concrete, the present formula is shown to give a good representation of the ultimate strength of circular CFT columns.

Keywords: composite columns; concrete; steel; tubes; compressive strength

1. Introduction

Concrete-filled steel tubular (CFT) columns are being more widely used in the construction of high-rise buildings, bridges, subway platforms, and barriers. Their usage provides excellent static and earthquake-resistant properties, such as high strength, high ductility, high stiffness, and large energy-absorption capacity. CFT columns provide the benefits of both steel and concrete: a steel tube surrounding a concrete column not only assists in carrying the axial load but also confines the concrete. Furthermore, it eliminates the permanent formwork, which reduces construction time, while the concrete core takes the axial load and prevents or delays local buckling of the steel tube.

There are various shapes of CFT columns, as shown in Fig.1. Of interest here are short circular CFT columns, which are considered to offer much more post-yield axial ductility than rectangular, square, and octagonal tube sections (Schneider 1998; Susantha et al. 2001), and which are the more commonly used type in many modern structures. Research on circular CFT stub columns has been ongoing worldwide for decades, and significant contributions have been made by many researchers. A series of tests on the behavior of circular thin-walled steel tubes have been carried out by O'Shea and Bridge (1997, 2000), who concluded that a steel tube with a diameter-to-thickness ratio greater than 55 and filled with 110-120 MPa high strength concrete provides insignificant confinement to the concrete. Braun (1999) stated that the effect of confinement exists at high stress levels when the structural steel acts in tension and the concrete in compression. Fourteen short CFT columns subjected to axial loads were tested by Schneider (1998) to investigate the effect of the tube shape and steel tube plate thickness on the composite column strength. Huang et al. (2002) tested 17 CFT columns specimens with a higher diameter-to-thickness ratio. Sakino et al. (2004) studied the effect of steel tube tensile strength and concrete strength on the behavior of composite columns. Giakoumelis and Lam (2004) carried out 15 tests on circular CFT columns and investigated the effects of the steel tube plate thickness, the bond between the steel tube and concrete, and the concrete confinement on the behavior of these columns.

On the other hand, success has been achieved in developing accurate models for concrete confinement and the interaction between the steel tube and the concrete core. Schneider (1998) and Hu et al. (2003) developed a three-dimensional (3D) nonlinear finite element model (FEM) for CFT circular columns, in which the ABAQUS program was used. The concrete confinement was achieved by matching the numerical
The accuracy of these analytical methods has been verified by the results of the tests. However, a simple relationship between the concrete confinement and the diameter-to-thickness ratio, the strength of the steel tube and filled concrete has not been established till now. Recently, a 3D FEM simulation of fracture behavior of CFT in axial compression was conducted by Shibata and Tachibana (2006).

In general, these studies have shown that with the advent of high strength steel and the production of high strength concrete using conventional materials with careful quality control, high strength CFT columns are both technically and economically feasible. However, they are scarcely adopted in the construction industry, mainly due to the lack of understanding of their structural behavior and reliable design recommendations (Liu 2005). The present design codes, such as AIJ (2001), AISC (2005), DL/T (1999), and Eurocode 4 (2004) have some limitations in applications concerning material strength. In order to expand the applications of high strength CFT columns, it is necessary to have a simple and accurate formula to understand the concrete confinement and predict the axial capacity of CFT stub columns with not only normal strength but also high strength concrete and steel.

The intentions of the present paper are to clarify the effects of the diameter-to-thickness ratio, the strength of the concrete and the strength of the steel tube on the concrete confinement, and to present a simple and accurate formula to predict the axial capacity of circular CFT stub columns with normal and high strength steel and concrete. A comparison with the published experimental results (Gardner and Jacobson 1967; Sakino and Hayashi 1991; Kato 1995; Saisho et al. 1999; Yamamoto et al. 2002; Zhang and Wang 2004; Giakoumelis and Lam 2004; Sakino et al. 2004; Han et al. 2005; Tan 2006) indicates that the present formula can effectively predict the ultimate capacity of circular CFT stub columns with normal and high strength steel and concrete.

2. Review of Current Code Provisions for Predicting Capacity of Circular CFT Stub Columns

For completeness, a brief review of the prediction of axial capacity of circular CFT stub columns using methods described in the codes is provided as follows. In all the design calculations, the resistance factors and material partial factors are set to one.

2.1 The AIJ code (2001)

\[ N_{AIJ} = 0.85 f_{cyl,100} A_e + 1.27 f_c A_c \]  

where

- \( f_{cyl,100} \) = concrete compressive strength with 100 x 200 mm cylinder tests;
- \( A_c \) = the cross-sectional area of the steel tube;
- \( f_c \) = the yield stress of the steel tube.

2.2 The AISC code (2005)

(a) When \( P_e \geq 0.44 P_0 \)

\[ N_{AISC} = P_0 \left( 0.658 \frac{f_{cyl}}{f_y} \right) \]  

(b) When \( P_e < 0.44 P_0 \)

\[ N_{AISC} = 0.877 P_e \]  

where

- \( P_0 = (f_y A_s + 0.95 f_{cyl}) A_e) \)  

\[ P_e = \frac{\pi^2 (EI)_{eff}}{(KL)^2} \]  

\[ (EI)_{eff} = E_s I_s + C_I E_c I_c \]  

\[ C_I = 0.6 + 2 \left( \frac{A_s}{A_e + A_c} \right) \leq 0.9 \]  

where \( f_{cyl,150} \) = concrete compressive strength with 150 x 300 mm cylinder tests; \( KL \) = effective length of the column; \( E_s \) = the elasticity of the steel modulus = 2 \times 10^5 \text{ MPa}; \( E_c \) = elastic modulus of concrete = 4730 \( (f_{cyl,150})^{1.2} \) (MPa); \( (EI)_{eff} \) = effective moment of inertia rigidity of composite section; and \( I_s \) and \( I_c \) = moment of inertia of steel tube and concrete core, respectively.

2.3 The Chinese Code DL/T (1999)

\[ N_{DL/T} = f_{cyl} A_{sc} \]  

in which

\[ A_{sc} = A_s + A_c \]
short columns. The data was collected from:

- Sakino and Hayashi (1991), 12 tests;
- Saisho et al. (1995), 12 tests;
- Yamamoto et al. (2002), 13 tests;
- Zhang and Wang (2004), 36 tests;
- Giakoumelis and Lam (2004), 8 tests;
- Sakino et al. (2004), 36 tests;
- Gardner and Jacobson (1967), 12 tests;
- Sakino and Wang (2004), 36 tests;
- Yamamoto (1999), 29 tests;
- Han et al. (2005), 26 tests;
- Tan (2006), 16 tests.

All the specimens were short with length-to-



diameter ratios ranging from 1.99 to 3.50 to



avoid the effects of overall buckling and were loaded



simultaneously with the concrete and steel tube.



The main parameters varied in these tests were:

- diameter-to-thickness ratio \( D/t \) from 16.7 to 152.0;
- the compressive strength of concrete \( f_{\text{cly},100} \) from 25.4 to 135.6 MPa; and
- the yield strength of the steel tube \( f_y \) from 232 to 853 MPa.

The conversion relations between \( f_{\text{cly},150} \) and \( f_{\text{cly},100} \) that were proposed by Eurocode 2 (2004) are seen in Table 1. The conversion relationship between \( f_{\text{cly},150} \) and \( f_{\text{cly},100} \) can be expressed as (Rashid et al. 2002):

\[
f_{\text{cly},150} = 0.96 f_{\text{cly},100}
\]

For overall comparisons, the mean, standard deviation, maximum, minimum, and difference between maximum and minimum of the ratio of \( N_{\text{cu}}/N_a \) for the different design methods are shown in Table 2., in which \( N_{\text{cu}} \) = predicted section capacities using the different methods and \( N_{\text{cu}} \) = test results. The \( N_{\text{cu}}/N_a \) ratios of the 200 experiment tests are depicted in Fig.2. As can be seen from Table 1 and Fig.2., AIJ (2001) and AISC (2005) provide conservative results with sectional capacities about 13.1% and 24.4% lower than the measured ultimate strengths. Although DL/T (1999) gives a section capacity about 6.3% lower than the measured ultimate strengths, the scatterness of the predictions is quite large. Eurocode 4 (2004) gives the best results among the four methods with a mean and standard deviation of the \( N_{\text{cu}}/N_a \) ratio of 1.009 and 0.097, respectively. However, the computation is relative complex.

In view of the foregoing, it is necessary to propose a simple analytical formulation to predict the axial capacity of circular CFT stub columns made with not only normal strength but also with high strength steel and concrete.

### 3. A Simple Formula for Compressive Strength of Circular CFT Stub Columns

The stress state for a circular CFT stub column is shown in Fig.3. When the CFT section is under the ultimate compressive force \( N_{\text{cu}} \), the concrete in a circular CFT section is subjected to axial stress \( \sigma_c \), and lateral pressure \( \sigma_s \), and the steel tube is subjected to axial stress \( \sigma_{st} \) and ring tension stress \( \sigma_{rt} \). From Fig.3., one can easily understand that the axial compressive strength \( N_{\text{cu}} \) can be given by

\[
N_{\text{cu}} = A_c \sigma_c + A_s \sigma_{st}
\]

where \( \sigma_c \) = the strength of the confined concrete; and
Table 2. Comparison between Predicted Section Capacities ($N_{cu}$) and Test Results ($N_{test}$)

|            | AIJ (2001) | AISC (2005) | DL/T (1999) | Eurocode 4 (2004) | The present |
|------------|------------|-------------|-------------|-------------------|-------------|
| $N_{test}/N_{cu}$ | 1.131  | 1.244       | 1.063       | 1.009             | 1.013       |
| Mean       | 1.131  | 1.244       | 1.063       | 1.009             | 1.013       |
| Standard deviation | 0.109 | 0.131       | 0.126       | 0.100             | 0.097       |
| Maximum    | 1.488  | 1.689       | 1.603       | 1.420             | 1.353       |
| Minimum    | 0.905  | 0.898       | 0.829       | 0.800             | 0.810       |
| Maximum-Minimum | 0.583 | 0.791       | 0.774       | 0.620             | 0.543       |

Fig. 2. Comparisons of Experimental Tests with Results Predicted by Some Design Codes

Fig. 3. Stress State for a Circular CFT Column in a Limited State
\( \sigma_{sz} \) = the ultimate value of the axial stress of steel.

If \( \sigma_{cc} \) and \( \sigma_{sz} \) are determined, the axial strength of the CFT columns will be known. First, the strength of the confined concrete \( \sigma_{cc} \) is assumed to be given by

\[
\sigma_{cc} = f_{c,150} + k \sigma_r
\]

where \( k = \) confinement coefficient = 4.1 (Richart et al. 1928).

According to Fig. 3., the equilibrium of \( \sigma_r \) and \( \sigma_{sz} \) gives

\[
(D-2t) \cdot \sigma_r = -2t \cdot \sigma_{sz}
\]

where \( D \) is the external diameter of the steel tube and \( t \) is the plate thickness of the steel tube.

Rewriting Eq. (11), one obtains

\[
\sigma_r = \frac{-2t}{(D-2t)} \cdot \sigma_{sz}
\]

Substituting Eq. (12) in Eq. (10), one obtains

\[
\sigma_{ce} = f_{c,150} - \frac{2tk}{D-2t} \cdot \sigma_{sz}
\]

Then, \( N_{cu} \) is expressed as

\[
N_{cu} = A_c f_{c,150} - \frac{2tk}{D-2t} A_s \sigma_{sz} + A_s \sigma_{sz}
\]

Suppose the axial stress \( \sigma_r \) and ring tension stress \( \sigma_{sz} \) of the steel tube under ultimate load are expressed by

\[
\sigma_r = \alpha f_y, \quad \sigma_{sz} = \beta f_y
\]

where \( \alpha, \beta \) = coefficients.

Substituting Eq. (15) in Eq. (14) leads to

\[
N_{cu} = A_c f_{c,150} - \frac{2tk}{D-2t} A_s \beta f_y + A_s \alpha f_y
\]

Eq. (16) can be rewritten as

\[
N_{cu} = A_c f_{c,150} \left[ 1 + (\alpha - 1) \frac{A_s}{A_c} - \frac{2tk}{D-2t} f_y \right] f_{c,150} + A_s f_y
\]

The ratio of the cross-sectional area of the steel tube to that of concrete is given by

\[
\frac{A_c}{A_r} = \frac{\pi D^2 / 4 - \pi (D-2t)^2 / 2}{\pi (D-2t)^2 / 2} = \frac{4(D-t)t}{(D-2t)^2}
\]

Substituting Eq. (18) in Eq. (17) produces

\[
N_{cu} = A_c f_{c,150} \left[ 1 + \frac{4(\alpha - 1)(D-t-1) - 2k\beta(D/t-2)}{(D/t-2)^2} \right] f_y + A_s f_y
\]

Denoting

\[
\eta_c = \frac{4(\alpha - 1)(D/t-1) - 2k\beta(D/t-2)}{(D/t-2)^2} \frac{f_y}{f_{c,150}}
\]

Then, the ultimate strength of circular CFT short columns can be obtained as

\[
N_{cu} = A_c f_{c,150} (1 + \eta_c) + A_s f_y
\]

Here, \( \eta_c \) is called the coefficient of confinement for the concrete core.

In order to determine the coefficient of confinement for the concrete core \( \eta_c \), expressed in Eq. (21), one needs to determine the values of \( \alpha \) and \( \beta \) in Eq. (20).

According to the work of Hatzigeorgiou (2008), for circular CFT stub columns, \( \beta \) is empirically given by

\[
\beta = -\exp[\ln(D/t) + \ln(f_y) - 1] \leq 1.0
\]

According to Eq. (22), the coefficient of ring tension stress \( \beta \) will change as the diameter-to-thickness ratio or the yield stress of the steel tube change. For simplicity, \( \beta \) is taken as -0.311 in this paper. This value results from regression analysis of the 200 experimental tests described previously, as shown in Fig. 4.

Based on the assumption that steel stresses at the limit state given by Eq. (15) satisfy the Von Mises yield criterion, the relationship between \( \alpha \) and \( \beta \) is given by

\[
\alpha^2 - \alpha \beta + \beta^2 = 1
\]

Since \( \beta = -0.311 \), according to Eq. (23) the value of \( \alpha \) is obtained as 0.807.

Substituting the values of \( \alpha, \beta, \) and \( k \) in Eq. (20), the coefficient of confinement for concrete core \( \eta_c \) becomes

\[
\eta_c = \frac{1.78(D/t-2.43)}{(D/t-2)^2} \frac{f_y}{f_{c,150}}
\]

Simplifying Eq. (24), an approximating formula is given as

\[
\eta_c = \frac{1.8t}{D} \frac{f_y}{f_{c,150}}
\]
The comparison of Eq. (24) and Eq. (25) is illustrated in Fig. 5. (a, b, and c) for $\frac{f_y}{f_{cl,150}} = 5, 10,$ and 15, respectively. One can clearly see that the coefficient of confinement for concrete core $\eta_c$ obtained from Eq. (25) is in good agreement with that calculated by using Eq. (24).

From Eq. (25), it is not difficult to understand that the coefficient of confinement for concrete core $\eta_c$ is dependent on the column $D/t$ ratio, the yield stress of the steel tube ($f_y$), and the unconfined compressive strength of the concrete core ($f_{cl,150}$). Because the coefficient of confinement for the concrete core is proportional to the strength ratio $f_y/f_{cl,150}$ between the steel and concrete, if $D/t$ is fixed, simultaneously increasing of $f_y$ and $f_{cl,150}$ will not increase the confinement effect.

The variations of the confinement coefficient ($\eta_c$) with respect to $D/t$, $f_y$, and $f_{cl,150}$ are depicted in Fig. 6. As can be seen from Fig. 6(a), there is a sharp decrease of $\eta_c$ when the diameter-to-thickness ($D/t$) ratio is small and that $\eta_c$ tends to be moderate when $D/t > 60$. This is similar to O'Shea and Bridge's (1997, 2000) conclusions that a steel tube with a $D/t$ ratio greater than 55 provides insignificant confinement for the concrete core. From Fig. 6(b and c), one can see that the $\eta_c$ decreases when $f_{cl,150}$ increases and increases when $f_y$ increases, which means that CFT columns with high strength concrete and low strength steel tubes do not provide good confinement efficiency for the CFT columns.

### 3.1 Verification of the Proposed Formula

The 200 experimental tests of circular CFT short columns were again used to verify the efficiency of the present methods. For overall comparisons, the mean, standard deviation, maximum, minimum, and the difference between maximum and minimum of the ratio of $N_{test}/N_{cu}$ for the different design methods are also listed in Table 2. The $N_{test}/N_{cu}$ ratios of the 200 experimental tests are depicted in Fig. 7.

As can be seen from Table 2., Fig. 2. and Fig. 7., the proposed method gives the best representation of the ultimate strength of circular CFT stub columns with a mean of 1.013 and a standard deviation of 0.097.
4. Conclusions
A simple formula for predicting the compressive strength of circular CFT stub columns with normal and high strength steel and concrete is proposed. From the investigations of this paper, the following conclusions can be drawn:

1. Generally, AISC (2005) and AIJ (2001) predict conservative results compared with the published experimental data, while the predictions of DL/T (1999) show considerable scatterness. Eurocode 4 (2004) gives the best outcome, but it is relatively complex.

2. A practical design formula for predicting the axial capacity of circular CFT short columns is proposed. A comparison with the published experimental results indicates that the present formula can effectively predict the ultimate capacity of circular CFT stub columns made with normal- and high-strength steel and concrete.

3. A simple expression for the concrete confinement is presented. A sharp decrease of the concrete confinement is found when the diameter-to-thickness ratio is small, and it tends to be moderate when the diameter-to-thickness ratio is greater than 60. The concrete confinement increases linearly when the yield stress of the steel tube increases, and it decreases as the unconfined compressive strength of the concrete core increases, respectively.

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