Optimal Spatial Camera Orientation Control

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Abstract. Well-design spatial camera orientation is desired for camera movement in many applications. In this paper, the camera orientation control for camera roam along the given path in real or virtual environments is studied. A general adapted frame for camera orientation control is constructed to describe the problem of camera movement. Rotation-equilibrating adapted frame based camera orientation is proposed to smooth the variation of the camera orientation so as to minimize the variation of images. Compared with the existing rotation-minimizing adapted frame, the rotation-equilibrating frame is able to obtain more favourable visions.

1. Introduction

The motion of a rigid body in 3D space is widely studied by many researchers which is related to computer animation [1][2], construction of swept surface [3][4], computer numerical control machining [5][6], virtual endoscopy in medicine [7] and so on. The motion of a rigid body mainly contains two aspects: one is the trajectory of a distinguished point of the body (e.g., the center of mass) which is referred to the translational motion, the other is the orientation of the body along the motion path which can be considered as the rotational motion and determined by an orthonormal motion frame. Researches are mainly focused on the latter as it is nonlinear and more complex[8][9][10].

When a rigid body rotates in the motion frame along the track path, one component of the frame is generally constrained by the path for different applications, then one is often required to specify one free component of the frame to have a desirable rotary movement. The adapted frame on space curves is the most widely studied case which is a natural choice for specifying[11][12]. However when the Frenet adapted frame (FAF) is applied for motion planning or swept surface construction, it will have “unnecessary” rotation about the tangent vector for the basic vectors in the normal plane, therefore the rotation-minimizing frame (RMF) was first noted by Klok[3]. Then through taking the spatial camera orientation control as the typical application context for RMF, Farouki [13] systematically discussed the rotation-minimizing adapted frame (RMAF) and the rotation-minimizing directed frame (RMDF) for the rigid body orientation plan. In this paper, the optimization of the spatial orientation control for camera roam is explored further and new solution is developed to obtain the least apparent rotation of the image through minimizing the camera orientation variation.

The paper is organized as follow. In section 2, the general adapted frame for the camera orientation control is presented with relative geometry, and a simplification operation is conducted on the first-order derivative equation of the frame. Based on the simplified equation, rotation-equilibrating adapted frame (REAF) is proposed to control the variation of camera orientation in section 3. Experiments are...
implemented in section 4 to demonstrate the efficiency of REAF compared with FAF and REAF. In the end, a conclusion is given.

Fig. 1 The camera roam along the given path

2. Camera orientation by the general adapted frame
The movement of the camera roam is showed in Fig. 1. Given a path curve \( r(\xi) = S(u(\xi), v(\xi)) \) for roaming where \( \xi \) is the path parameter and \( \sigma \) is the parameter speed, the camera is moving from one side to the other side of the curve, the optical axis of camera is parallel with the tangent vector \( t \) so that the camera can go forward along the path to execute the task of roaming in real or virtual circumstance. Obviously, the optical axis is fixed when the path curve is given. Therefore there is only one undetermined rotary axis on the camera which is defined as the free rotary axis \( T_a \). Set \( m = T_a \times t \), the movement of the camera roam can be represented by \( G(t, T_a, m) \) which is an adapted frame. When the camera is rotating along with the free rotary axis, the appearance of images obtained by the viewer will be influenced. To obtain the favourable apparent of images for the viewer, it is desirable to smooth the variation of the free rotary axis. Supposing a point \( C \) on the roam path, the \( t, p \) and \( b \) are the tangent vector, the principle normal vector and the bi-normal vector.

\[
t = \frac{r'}{|r'|}, \quad p = \frac{r' \times r''}{|r' \times r''|} \times t, \quad b = \frac{r' \times r''}{|r' \times r''|}
\]

(1)

The relationship of their derivatives can be expressed by the well-known Frenet-Serret equations.

\[
\begin{bmatrix}
t' \\
p' \\
b'
\end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ \sigma & -\kappa & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} t \\
p \\
b \end{bmatrix}
\]

(2)

where \( \kappa \) and \( \tau \) are the curvature and the torsion respectively, and the angular velocity \( \omega \) is \( \kappa t + \tau b \).

The free rotary axis \( T_a \) can be expressed by

\[
T_a = b \cos \theta + p \sin \theta
\]

(3)

\( \theta \) is positive in the direction of rotation of \( b \) onto \( t \). the derivative of \( T_a \) can be calculated with equation (3) and equation (4).

\[
T_a' = \cos \theta(\theta' - \sigma \tau)p - \sin \theta(\theta' - \sigma \tau)b - \sigma \kappa \sin \theta t
\]

(4)

As \( T_a \cdot T_a' = 0 \), therefore the derivative of \( T_a \) is on the plane determined by \( t \) and \( m, m = T_a \times t = p \cos \theta - b \sin \theta \). then equation (5) can be rewritten.

\[
T_a' = (\theta' - \sigma \tau)m + (-\sigma \kappa \sin \theta)t
\]

(5)

Then the general adapted frame \( G(T_a, m, t) \) can be obtained. Similarly with well-known Frenet frame, \( G \) is described by the angular velocity and the derivatives of components.
Then set $\theta - \sigma t = \sigma \eta$, then $T_s' = -\sigma k(m + \sin \theta t)$, $m' = \sigma k(T_s + t \cos \theta)$, therefore the angular velocity and the relationship of the derivatives of $G$ are rewritten as follow.

$$\omega = \kappa(b - \eta t)$$

$$T_s' = \begin{bmatrix}
0 & \sin \theta & -\cos \theta \\
-\sigma k\sin \theta & 0 & -(\theta - \sigma t) \\
\sigma k\cos \theta & (\theta - \sigma t) & 0 \\
\end{bmatrix}
\begin{bmatrix}
t' \\
m' \\
\end{bmatrix}
$$

(6)

Compared with equation (7), the expression of equation (8) is more succinct, so the problem of the camera orientation control can be concentrated on the plan of parameter $\eta$. With different assignment for the parameter $\eta$, different measures can be developed.

3. Camera orientation by rotation-equilibrating adapted frame

As the components of FAF have the significant geometric meaning, FAF is the first method applied to control the camera orientation which is widely studied and used in many applications. However, the principle normal vector is always pointing to the center of the curvature circle, therefore when the camera is rotating along with $p$ roaming in the space, it will encounter unreasonable twist which can lead to the perturbation of images. To address the problem in FAF, the rotation-minimizing adapted frame is proposed for eliminating unnecessary rotation to smooth the variation of the free rotary axis of the camera. The condition for FAF is $\eta = -\tau / \kappa$. Thus, the component of $T_s'$ in the $t$-direction is supressed and only the component in the $m$-direction is retained. This implies $T_s$ is bound to only rotate about $m$ which is rather limiting. The condition for RMAF is $\eta = 0$, therefore the component of $T_s'$ in the $m$-direction is supressed and only the component in the $t$-direction is retained. This implies $T_s$ is bound to only rotate about $t$ which is rather limiting. Therefore it is suggested that a more equilibrating approach would retain both components of $T_s'$. An obvious strategy is to give equal weighing to both components. This leads to the condition of

$$\eta = \pm \sin \theta$$

(8)

Then the integration of $\theta$ can be calculated according to equation (9).

$$\theta = \int^t_0 \sigma t \pm \sigma k \sin \theta d\xi$$

(9)

The derivative of $T_s$ becomes

$$T_s' = (\sigma k \sin \theta)(\pm m - t)$$

(10)

Therefore a rotation-equilibrating adapted frame $(t, T_s, m)$ is defined, and the derivative of $t$ can be expressed as

$$t' = \sigma k p = \sigma k m \cos \theta + \sigma k T_s \sin \theta$$

(11)

The angular velocity of the REAF and the relationship of their derivatives are given as follow.

$$\omega = \kappa(b \mp \sin \theta t)$$

$$T_s' = \begin{bmatrix}
0 & \sin \theta & -\cos \theta \\
-\sigma k \sin \theta & 0 & \mp \sin \theta \\
\sigma k \cos \theta & \pm \sin \theta & 0 \\
\end{bmatrix}
\begin{bmatrix}
t' \\
m' \\
\end{bmatrix}
$$

(12)

The instantaneous rotary speed of $T_s$ and $m$ are

$$||T_s'||_{\text{REAF}} = (\sigma k \sqrt{\eta^2 + (\sin \theta)^2})_{\text{REAF}} = \sigma k \sqrt{2} ||\sin \theta||_{\text{REAF}}$$

$$m'_{\text{REAF}} = \sigma k$$

(13)
One can consider this dynamic scene in camera roam. Images of new things coming into view along the free rotary axis \( T_a \) are supposed to vary slowly for the detail observation, therefore the instantaneous rotary speed of \( T_a \) should be as small as possible. Relatively, images observed will be hoped to leave the viewer field of vision along the axis \( m \) fast for accelerating information filtering. Obviously, the instantaneous rotary speed of \( m \) in REAF is larger than RMAF and as the different definition of \( \eta \), \((| T'_a |)_{REAF} \) can be in the orange area and \((| T'_a |)_{RMAF} \) can be in blue area in Fig. 2, then the change rate of \( T_a \) in REAF will be smaller than the one in RMAF. Under this condition, REAF will be more suitable to control the camera orientation for this requirement compared with REAF. Notice that \( T_a \) of REAF is not likely with \( p \) of FAF which has unreasonable twist along the path curve.

4. Experiments
Two examples are exhibited to validated the efficiency of REAF. Obviously experiments should focus on the comparison of the instantaneous rotary speed of the free rotary axis \( T_a \). The numerical method in [5] is adopted for the computation of the integrations in this paper and the grid size for all the path curves is set \( \Delta \xi = 0.01 \). The path curve is marked with green colour, RMAF and REAF are marked with black, blue and red colour respectively.

4.1. Example 1
A path curve is defined on a torus by
\[
r(u, v) = ((8 + \cos v) \cos u, (8 + \cos v) \sin u, 2 \sin v) \quad (14)
\]
with \((u, v) = (0.5 \pi \xi, 0.5 \pi \xi)\), \( \xi \in [0,1] \), the initial angle \( \theta_0 \) is \( \frac{\pi}{6} \). An ellipsoid is provided as the object of the camera roam in Fig. 3.

\[
Ob = (2 \cos(v) \cos(u) + 4, 2 \cos(v) \sin(u) + 16, 4 \sin(v)) \quad (15)
\]

As showed in Fig. 4, in the small interval \( \xi \in [0,0.21] \), \(| T'_a |\) of REAF is larger than the one of RMAF. Then in right part, the curve of REAF method is declining rapidly, and \(| T'_a |\) of RMAF is no less than 0.88. As the camera is going forward along the tangent vector, it is also rotating along the direction of
\( T_a \). A small \( |T_a'| \) will reduce the fluctuation from the rotation movement of camera and enhance the stability of the adjacent images with smooth appearance. Since \((\sum |T_a'|)_{RMAF} = 67.27 < (\sum |T_a'|)_{REAF} = 97.94\), REAF is more efficiency in reducing \( |T_a'| \), then when the camera is rotating along \( T_a \), the variation of images of ellipsoid will be smoother. In Fig. 5 and Fig. 6 the different strategy of RMAF and REAF can be traced clearly that the variation of images of the ellipsoid coincide with that \( T_a \) of RMAF has no instantaneous rotation speed about \( t \) and \( T_a \) of REAF has equal weighing on \( t \) and the image plane. However, one can also see that the appearance of images of the ellipsoid are more stability as the reduction of REAF on \( |T_a'| \) is distinct.

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**Fig. 3** The path curve of the camera roam on a torus with an Ellipsoid.

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**Fig. 4** The instantaneous rotary speed of \( T_a \).
4.2. Example 2

A path curve on the demo is defined by

\[ s(u, v) = (8u, 8v, 8(1 - u^2 - v^2)) \]  

(16)

with \((u, v) \in [\xi, \xi], \xi \in [0, 1]\). The initial angle \(\theta_0 = \frac{\pi}{6}\). An ellipsoid is provided as the object of camera roam in Fig. 7.

\[ Ob = (2\cos(v)\cos(u) - 10, 2\cos(v)\sin(u) - 10, 4\sin(v) + 10) \]  

(17)

As showed in Fig. 8, \(|T'_e|\) of REAF and RMAF fall off from 2.00 and 1.41 respectively and \(|T'_e|\) of REAF is larger than the one of RMAF in the interval \(\xi \in [0, 0.13]\), then \(|T'_e|\) of REAF is always smaller than the one of RMAF and \(\sum|T'_e|_{\text{REAF}} = 52.53 < \sum|T'_e|_{\text{RMAF}} = 62.33\). When the camera is rotating along \(T_e\), the instantaneous appearance of images received by the viewer is definitely influenced by the change rate of \(T_e\). As a whole, the variation of \(|T'_e|\) in REAF is definitely smaller and smoother than the one in RMAF so that the viewer can obtain smoother images. In Fig. 9 and Fig. 10, the different directions of rotation strategy in RMAF and REAF can be traced. However, their curves of \(|T'_e|\) are similar and approximate, therefore the variation of images of ellipsoid in the image plane are not so distinctive between REAF and RMAF as Example 1 exhibited.
Fig. 7 The path curve of the camera roam on a torus with an Ellipsoid.

Fig. 8 The instantaneous rotary speed of $T_\xi$.

Fig. 9 The variation of images of the ellipsoid with RMAF in 3-D view.
5. Conclusion
Based on the analysis of the kinematic geometry of the camera movement, a general adapted frame for the spatial camera orientation control in camera roam is established. For the convenience to develop more alternative technicals in different applications, a simplification operation is applied on the first-order derivative equation of the frame through replacing the complex formula with one parameter. Then camera orientation based on rotation-equilibrating adapted frame is proposed in order to minimize the instantaneous rotary speed of the free rotary axis of the camera for the requirement of observation. Experiments exhibits the advantage of REAF on minimizing the variation of images over RMAF. As the camera roam is a typical movement of the rigid body, the simplified model can be extended into robotic motion, swept surface construction, computer animation and more alternative measures for different requirements can be developed.

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