Pionic contribution to relativistic Fermi Liquid Parameters

Kausik Pal, Subhrajyoti Biswas, Abhee K. Dutt-Mazumder.

*High Energy Physics Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, INDIA.*

**Abstract**

We calculate pionic contribution to the relativistic Fermi Liquid parameters (RFLPs) using Chiral Effective Lagrangian. The RFLPs so determined are then used to calculate chemical potential, exchange and nuclear symmetry energies due to \( \pi N \) interaction. We also evaluate two loop ring diagrams involving \( \sigma \), \( \omega \) and \( \pi \) meson exchanges and compare results with what one obtains from the relativistic Fermi Liquid theory (RFLT).

**Key words:** Pion, Fermi Liquid Parameters, Exchange energy.

**PACS:** 21.65.-f, 13.75.Cs, 13.75.Gx, 21.30.Fe

Fermi liquid theory (FLT) provides us with one of the most important theoretical schemes to study the properties of strongly interacting Fermi systems involving low lying excitations near the Fermi surface [1]. Although developed originally in the context of studying the properties of \(^3\)He, it has widespread applications in other disciplines of many body physics like superconductivity, super fluidity, nuclear and neutron star matter etc. [2].

In nuclear physics FLT was first extended and used by Migdal [1] to study the properties of both unbound nuclear matter and Finite nuclei [3]. FLT also provides theoretical foundation for the nuclear shell model [5] as well as nuclear dynamics of low energy excitations [2,4]. Particularly, ref. [4] reveals the connection between Landau, Brueckner-Bethe and Migdal theories, ref. [5,7], on the other hand, calculates the Migdal parameters using one-boson-exchange models of the nuclear force and shows how these parameters are modified if nuclear matter is considered within the context of Relativistic Brueckner-Hartree-Fock (RBHF) model. Some of the recent work that examines Fermi-liquid properties of hadronic matter also incorporates Brown-Rho (BR) scaling which is very important for the study of the properties of hadrons in dense nuclear matter [10,11].

Most of the earlier nuclear matter calculations that involved Landau theory were done in a non-relativistic framework. The relativistic extension of the FLT was first developed by Baym and Chin [3] in the context of studying the properties of dense nuclear matter (DNM). In [3] the authors invoked Walecka model to calculate various interaction parameters (\( f pp \)) but did not consider mean fields (MF) for the \( \sigma \) and \( \omega \) meson i.e. there the FLPs are calculated perturbatively.

Later Matsui revisited the problem in [12] where one starts from the expression of energy density in presence of scalar and vector meson MF and takes functional derivatives to determine the FLPs. The results are qualitatively different than the perturbative results as may be seen from [3,12]. A comparison of relativistic and non-relativistic calculations have been made in [8,9] which also discusses how the FLPs are modified in presence of the \( \sigma \) and \( \omega \) MF and contrast those with perturbative results.

Besides \( \sigma \) and \( \omega \) meson, ref. [12] also includes the \( \rho \) and \( \pi \) meson and the model adopted was originally proposed by Serot to incorporate pion into the Walecka model. It is to be noted, however, that the FLPs presented in [12] are independent of \( \pi \) meson. This is because \( \pi \), being a pseudoscalar, fails to contribute at the MF level. Hence to estimate the pionic contribution to FLPs it is necessary to go beyond MF formalism.

It is to be noted that, to our knowledge, such relativistic calculations including \( \pi \) exchange does not exist, despite the fact that pion has a special status in nuclear physics as it is responsible for the spin-isospin dependent long range nuclear force. Furthermore, there are various non-relativistic calculations including the celebrated work of Migdal which shows that the pionic contribution to FLPs are important
and most dominant for low energy excitations. It might be mentioned here that [11] discusses how can one incorporate relativistic corrections to $F^\pi_1$ in the static potential model calculation. We, however, take the approach of [3] where all the fields are treated relativistically.

The other major departure of the present work from ref. [12] resides in the choice of model for the description of the many body nuclear system. The straightforward incorporation $\pi$ meson into the Wealecka model as was done in ref. [13] has serious difficulties. In particular, at the MF level it gives rise to tachyonic mode in matter at densities as low as $0.1\rho_0$, where $\rho_0$ is the nuclear saturation density [13]. Inclusion of exchange diagrams removes such unphysical mode but makes the effective mass unrealistically large. Replacing pseudoscalar coupling with the pseudovector interaction avoids these difficulties, however, it turns the theory non-renormalizable [16]. The theoretical challenge therefore was then to construct a model with $\pi N$ interaction which preserves renormalizibility of the theory and at the same time yields realistic results for the pion dispersion relations in matter. This was accomplished in [15]. But this model was also not found to be trouble free, particularly it had several shortcomings in describing $\pi N$ dynamics in matter which we do not discuss and refer the reader to [17,18,19,20,21,22].

These apart, Walecka model itself has several problems in relation to convergence which forbids systematic expansion scheme to perform any perturbative calculations. This was first exposed in ref. [23]. Since then a lot of theoretical efforts have been directed to circumvent these problems. The most recent model that cure all these maladies and provides us with a systematic scheme to study the dense nuclear system is provided by Chiral Effective Field theory ($\chi$EFT) [21,23,24]. $\chi$EFT, apart from $\sigma$ and $\omega$ mesons, also includes pion and therefore is best suited for the present purpose.

In this letter we estimate contributions of pion exchange to the FLPs within the framework of RFLT and subsequently use the parameters so determined to calculate various quantities like pionic contribution to the chemical potential, energy density, symmetry energy ($\beta$) etc. For completeness and direct comparison with the two loop results we also calculate here the exchange energy due to the interaction mediated by the $\sigma$, $\omega$ and $\pi$ mesons. The pionic contribution to the effective chemical potential, as we shall see, is significantly large. This, therefore, might alter the $\beta$ equilibrium condition than what one obtains in MF calculations [12]. This we plan to study in a separate work [26].

Now we quickly outline the formalism. In FLT total energy $E$ of an interacting system is the functional of occupation number $n_p$ of the quasi-particle states of momentum $p$. The excitation of the system is equivalent to the change of occupation number by an amount $\delta n_p$. The corresponding energy of the system is given by [23],

$$E = E^0 + \sum_s \left[ \frac{1}{2} \sum_{ss'} \int \frac{d^3p}{(2\pi)^3} \epsilon_p^0 \delta n_{ps} \right],$$

where $E^0$ is the ground state energy and $s$ is the spin index, and the quasi-particle energy can be written as,

$$\epsilon_p = \epsilon_p^0 + \sum_s \int \frac{d^3p'}{(2\pi)^3} \delta n_{p's} \delta n_{ps'},$$

where $\epsilon_p^0$ is the non-interacting single particle energy. It is to be remembered, that, although the interaction ($f_{ps,p's'}$) between the quasiparticles is not small, the problem is greatly simplified because it is sufficient to consider only pair collisions between the quasiparticles [3].

Since quasi-particles are well defined only near the Fermi surface, one assumes

$$\epsilon_p = \mu + v_\theta (p - p_f)$$

and $p \simeq p' \simeq p_f$. Then LPs $f_l$s are defined by the Legendre expansion of $f_{ps,p's'}$ as [23],

$$f_l = \frac{2l + 1}{4} \sum_{ss'} \int \frac{d\Omega}{4\pi} f_l(\cos \theta) f_{ps,p's'}$$

where $\theta$ is the angle between $p$ and $p'$, both taken to be on the Fermi surface, and the integration is over all directions of $p$. We restrict ourselves for $l \leq 1$ i.e. $f_0$ and $f_1$, as higher $l$ contribution decreases rapidly [12].

Now the Landau Fermi liquid interaction $f_{ps,p's'}$ is related to the two particle forward scattering amplitude via [3],

$$f_{ps,p's'} = \frac{M}{\epsilon_p^0} \frac{M}{\epsilon_{p'}^0} M_{ps,p's'},$$

where the Lorentz invariant matrix $M_{ps,p's'}$ consists of the usual direct and exchange amplitude, which may be evaluated directly from the relevant Feynman diagrams. The spin averaged scattering amplitude ($f_{pp'}$) is given by [3],

$$f_{pp'} = \frac{1}{4} \sum_{ss'} \frac{M}{\epsilon_p} \frac{M}{\epsilon_{p'}} M_{ps,p's'}.$$
\[
\begin{aligned}
&\frac{1}{2} \partial_\mu \Phi_\sigma \partial_\nu \Phi_\sigma - \frac{1}{2} m_\sigma^2 \Phi_\sigma^2 - \frac{1}{2} \omega^\mu \omega_\nu - \frac{1}{2} m_\omega^2 \omega_\mu + \\
&\frac{1}{2} \partial_\mu \tilde{\Phi} \cdot \partial_\nu \tilde{\Phi} - \frac{1}{2} m_\omega^2 \Phi_\mu + L_{NL} + \delta L,
\end{aligned}
\]

where \( \omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \), \( \overline{\tau} = \tau_1 \tau_2 + \tau_1 - \tau_2 - + \tau_0 \tau_20 \), and \( \tau_+ \) and \( \tau_- \) are the isospin raising and lowering operators, respectively, and \( \tau_0 \) represents the third component of \( \tau \) [13].

The effective nucleon mass \( M^* \) is determined self-consistently from the following equation [12],

\[
M^* = M - \frac{g_\sigma^2}{m_\sigma} \sum_i \frac{M^*}{(p_i^2 + M^*)^{1/2}}.
\]

Using Eq. (11) and Eq. (1) we can derive isovector LPs \( f_0^* \) and \( f_1^* \),

\[
f_{0,1}^{\text{ex},\pi} = \frac{3g_\sigma^2 M^*}{2f_\pi^2} \left[ -2 + \frac{m_\pi^2}{2p_f^2} \ln \left( 1 + \frac{4p_f^2}{m_\pi^2} \right) \right]
\]

and

\[
\frac{1}{3} f_{1}^{\text{ex},\pi} = -\frac{3g_\sigma^2 M^* m_\pi^2}{2f_\pi^2 p_f^2} \times \left[ -2 + \left( \frac{m_\pi^2}{2p_f^2} + 1 \right) \ln \left( 1 + \frac{4p_f^2}{m_\pi^2} \right) \right].
\]

Using Eqs. (12) and (13) we find that

\[
f_{0}^{\text{ex},\pi} - \frac{1}{3} f_{1}^{\text{ex},\pi} = \frac{3g_\sigma^2 M^*}{8f_\pi^2} \times \left[ \frac{m_\pi^4}{8p_f^4} \ln \left( 1 + \frac{4p_f^2}{m_\pi^2} \right) - \frac{m_\pi^2}{2p_f^2} + 1 \right].
\]

It is this combination \( i.e. f_0 - \frac{1}{3} f_1 \), which appears in the calculation of chemical potential and other relevant quantities. For the massless pion, Eq. (14) turns out to be finite,

\[
\left( f_0^{\text{ex},\pi} - \frac{1}{3} f_1^{\text{ex},\pi} \right) \mid_{m_\pi \to 0} = \frac{3g_\sigma^2 M^*}{8f_\pi^2}. \tag{15}
\]

It is to be noted that, in the massless limit for \( \sigma \) and \( \omega \) meson, \( f_0^{\text{ex}} \) and \( f_1^{\text{ex}} \) diverge as shown in [3], in contrast for pion, even in the massless limit, these are finite. This is due to the presence of \((1 - \cos \theta)\) in the numerator of Eq. (9), unlike \( \sigma \) and \( \omega \) meson.

The dimensionless LPs can be determined by equating the equation \( F_0^* = N(0) f_0^* \) and \( F_1^* = N(0) f_1^* \), where \( N(0) \) is the density of states at the Fermi surface defined in Eq. (7). Thus the dimensionless parameters are

\[
F_0^{\text{ex},\pi} = -g_\epsilon g_\pi \frac{3g_\sigma^2 M^2}{32\pi^2 f_\pi^4} \left[ -2 + \frac{m_\pi^2}{2p_f^2} \ln \left( 1 + \frac{4p_f^2}{m_\pi^2} \right) \right]. \tag{16}
\]

and

\[
\frac{1}{3} F_1^{\text{ex},\pi} = -g_\epsilon g_\pi \frac{3g_\sigma^2 M^2}{64\pi^2 f_\pi^6} \times \left[ -2 + \left( \frac{m_\pi^2}{2p_f^2} + 1 \right) \ln \left( 1 + \frac{4p_f^2}{m_\pi^2} \right) \right]. \tag{17}
\]

![Fig. 1. Dimensionless Landau parameters in symmetric nuclear matter for pion exchange in relativistic theory. Solid and dashed line represent \( F_0^{\text{ex}} \) and \( F_1^{\text{ex}} \) respectively.](image)

In Fig. (1) we show the density dependent of \( F_0^{\text{ex}} \) and \( F_1^{\text{ex}} \) due to pionic interaction. Numerically at nuclear matter density \( (\rho_0 = 0.148 \text{fm}^{-1}) \), \( F_0^{\text{ex},\pi} = 1.35 \) and \( F_1^{\text{ex},\pi} = -0.40 \). This might be contrasted with what one obtains in various other models. For example, at \( \rho = \rho_0 \) [10] and [13] report \( F_1^{\text{ex},\pi} = -0.45 \) and \(-0.35 \) respectively.

We now proceed to calculate the chemical potential due to the exchange terms denoted by \( \mu^{\text{ex}} \). As in ref. [3] we have

\[
\mu d\mu = \left[ p_f + g_\pi g_\pi \frac{\mu p_f^2}{2\pi^2} (f_0 - \frac{1}{3} f_1) \right] dp_f. \tag{18}
\]

To calculate \( \mu \), it is sufficient to let \( \mu = \varepsilon_f \) in the right hand side of Eq. (15). With the constant of integration adjusted so that at high density \( p_f \simeq \varepsilon_f \), Eq. (15) upon integration together with Eq. (14) yield
\[ \mu_{\pi}^{ex} = \varepsilon_f - g_{\rho}g_{\pi} \frac{3g_{A}^{2}M^{4}}{128\pi^{2}f_{\pi}^{2}\varepsilon_f} \times \]
\[ \left[ -2y_{\pi}^{4}\sqrt{4 - y_{\pi}^{2}} + 4x\tan^{-1}\left( \frac{x\sqrt{4 - y_{\pi}^{2}} + 4}{y_{\pi}\sqrt{1 + x^{2}}} \right) \right. \]
\[ \left. + \frac{y_{\pi}^{4}}{x}\sqrt{1 + x^{2}}\ln\left( 1 + \frac{4x^{2}}{y_{\pi}^{2}} \right) - 4x\sqrt{1 + x^{2}} \right] \]
\[ -2(\frac{y_{\pi}^{4}}{4} - 2y_{\pi}^{2} - 2)\ln(x + \sqrt{1 + x^{2}}) \right], \tag{19} \]
where \( x = p_{f}/M^{*} \) and \( y_{\pi} = m_{\pi}/M^{*} \).

The calculations of LPs and exchange chemical potential for other mesons is straightforward. However, for brevity, we do not present corresponding expressions for \( \sigma \) and \( \omega \) mesons but quote their numerical values in Table 1. The numbers cited above are relevant for normal nuclear matter density \( \rho_{0} = 0.148\text{fm}^{-3} \). For the coupling constants we adopt the same parameter set as designated by M0A in 25.

Interestingly, individual contribution to LPs of \( \sigma \) and \( \omega \) meson are large while sum of their contribution to \( F_{0}^{\text{tot}} \) is small due to the sensitive cancelation of \( F_{0}^{\sigma} \) and \( F_{0}^{\omega} \), as can be seen from Table 1. Such a cancellation is responsible for the nuclear saturation dynamics 39]. Numerically, \( F_{0}^{\sigma+\omega} \) is three times smaller than \( F_{0}^{\sigma} \).

Once the \( \mu_{\pi}^{ex} \) is determined, one can readily calculate its contribution to the energy density 32,27.

\[ E_{\pi}^{ex} = \int d\rho(\mu_{\pi}^{ex} - \varepsilon_f) \]
\[ = -g_{\rho}g_{\pi} \frac{3g_{A}^{2}M^{6}}{512\pi^{4}f_{\pi}^{4}} \times \]
\[ \left[ I_{\pi} + \frac{y_{\pi}^{4}}{2} \left\{ -\frac{x^{2}}{2} + \frac{y_{\pi}^{2} + 4x^{2}}{8} \ln\left( 1 + \frac{4x^{2}}{y_{\pi}^{2}} \right) \right\} - x^{4} \right. \]
\[ \left. + \left( \frac{y_{\pi}^{4}}{2} - y_{\pi}^{2} - 1 \right) \left( \{\eta x - \ln(x + \eta)\}^{2} - x^{4} \right) \right], \tag{20} \]
where \( \eta = \sqrt{1 + x^{2}} \) and
\[ I_{\pi} = -2y_{\pi}^{4}\sqrt{4 - y_{\pi}^{2}} \int \frac{x^{2}}{\eta}\tan^{-1}\left( \frac{x\sqrt{4 - y_{\pi}^{2}}}{y_{\pi}\eta} \right) dx. \tag{21} \]

For the massless pion this reads as
\[ E_{\pi}^{ex}|_{m_{\pi} \rightarrow 0} = \frac{3g_{A}^{2}M^{6}}{32f_{\pi}^{4}}\{\eta x - \ln(x + \eta)\}^{2}. \tag{22} \]

The contribution arising from pion exchange from the direct evaluation of Fig 2(c) reads as 23.

---

Table 1

| Meson | \( F_0 \) | \( F_1 \) | \( \mu_{\pi}^{ex} \) (MeV) |
|-------|---------|---------|---------------------|
| \( \sigma \) | -5.04   | 0.875   | 731.89              |
| \( \omega \) | 5.44    | -0.93   | 501.82              |
| \( \pi \)  | 1.35    | -0.40   | 635.78              |

---

Fig. 2. Two-loop contributions to the nuclear matter energy density. The solid line represents the baryon propagator, \( \sigma \), \( \omega \) and \( \pi \) mesons are denoted by dotted, wavy and dashed line respectively.

Table 2

| Meson | \( E_{\pi}^{ex} \) | \( E_{\pi}^{ex} \) |
|-------|-----------------|-----------------|
| \( \sigma \) | 42.18 | 40.48 |
| \( \omega \) | -28.17 | -23.41 |
| \( \pi \)  | 13.66 | 12.49 |

\[ E_{\pi}^{ex} = g_{\rho}g_{\pi} \frac{3g_{A}^{2}M^{6}}{128\pi^{4}f_{\pi}^{2}} \int_{0}^{p_{f}} \frac{|p^{I}|^{2}dp^{I}}{\varepsilon_{p}} \int_{0}^{p_{f}} \frac{|p^{I'}|^{2}dp^{I'}}{\varepsilon_{p'}} \, \frac{1}{1 - \frac{\varepsilon_{p}\varepsilon_{p'} - |p^{I}|\varepsilon_{p'}|\cos \theta - M^{*+2}}{2\varepsilon_{p}\varepsilon_{p'} - 2|p^{I'}|\varepsilon_{p'}|\cos \theta - 2M^{*2} + m_{\pi}^{2}}}. \tag{23} \]

Similarly from Fig 2((a),(b)) one can determine exchange energy due to \( \sigma \) and \( \omega \) meson interaction 25. In Table 2 we compare the exchange energy results obtained from the direct evaluation of two loop diagrams with those calculated from RFLPs.

Knowing the isovector LPs, to which here only the pion contributes, one can calculate nuclear symmetry energy. The symmetry energy is defined as the difference of energy between the neutron matter and symmetric nuclear matter is given by the following expression 12,28

\[ \beta = \frac{1}{2} \frac{\partial^{2}E}{\partial \rho_{3}^{2}}|_{\rho_{3}=0}. \tag{24} \]

In terms of LPs, the symmetry energy can be expressed as 12,28

\[ \beta = \frac{y_{\pi}^{2}}{6\varepsilon_f}(1 + F_{0}^{\rho}). \tag{25} \]

Using Eq. 10 and Eq. 25, numerically at saturation density \( (\rho = \rho_{0}) \) we obtain \( \beta = 44.36 \text{MeV} \).

In this letter, we calculate RFLPs within the framework of RFLT. For the description of dense nuclear system \( \chi \text{EFT} \) is invoked. Although our main focus was to estimate the contribution of pions to RFLPs, for comparison and completeness we also present results for the \( \sigma \) and \( \omega \) meson. It is seen that the pionic contribution to the RFLPs are significantly larger compared to the combined contributions of \( \sigma \) and \( \omega \) meson. Thus any realistic relativistic calculation for
the FLPs should include $\pi$ meson which necessarily implies going beyond the MF calculations. The LPs what we determine here are subsequently used to calculate exchange and symmetry energy of the system. Finally we evaluate two loop ring diagrams with the same set of interaction parameters and show that the numerical results are consistent with those obtained from the FLT.

Acknowledgements

The authors would like to thank Prof. G. Baym for his valuable comments. We also wish to thank P. Roy, J. Alam and S. Sarkar for their critical reading of the manuscript.

References

[1] C. Song, Phys. Rept. 347 (2001) 289.
[2] G. Baym and C. Pethick, *Landau Fermi liquid Theory: Concepts and Applications*, United States Of America (1991).
[3] G. Baym and S. A. Chin, Nucl. Phys. A 262 (1976) 527.
[4] A. B. Migdal, Rev. Mod. Phys. 50 (1978) 107.
[5] A. B. Migdal, *Theory of Finite Fermi Systems* (1967) (Wiley, New York) (Russ. ed. 1965), A. B. Migdal, *Nuclear Theory: The Quasiparticle Method* (1968) (Benjamin, New York).
[6] S. Krewald, K. Nakayama and J. Speth, Phys. Rept. 161 (1988) 103-170.
[7] G. E. Brown, Rev. Mod. Phys. 43 (1971) 1.
[8] L. S. Celenza and C. M. Shakin, *Relativistic Nuclear Physics* (World Scientific, USA).
[9] M. R. Anastasio, L. S. Celenza, W. S. Pong and C. M. Shakin, Phys. Rept. 100 (1983) 327-392.
[10] B. Friman and M. Rho, Nucl. Phys. A 606 (1996) 303-319.
[11] B. Friman, M. Rho and C. Song, Phys. Rev. C 59 (1999) 3357-3370.
[12] T. Matsui, Nucl. Phys. A 370 (1981) 365.
[13] G. E. Brown and M. Rho, Nucl. Phys. A 338 (1980) 269.
[14] B. D. Serot, Phys. Lett. B 86 (1979) 146; Erratum, Phys. Lett. B 87 (1979) 403.
[15] J. I. Kapusta, Phys. Rev. C 23 (1981) 1648.
[16] T. Matsui and B. D. Serot, Annals of Physics 144 (1982) 107-167.
[17] R. J. Furnstahl, C. E. Price and G. E. Walker, Phys. Rev. C 36 (1987) 2590.
[18] R. J. Furnstahl and B. D. Serot, Phys. Lett. B 316 (1993) 12.
[19] R. J. Furnstahl, H. B. Tang and B. D. Serot, Phys. Rev. C 52 (1995) 1368.
[20] R. J. Furnstahl, B. D. Serot and H. B. Tang, Nucl. Phys. A 598 (1996) 539.
[21] B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6 (1997) 515.
[22] Subhrayojit Biswas and Abhee K. Dutt-Mazumder, arXiv: 0704.0318 [nucl-th].
[23] R. J. Furnstahl, R. J. Perry, B. D. Serot, Phys. Rev. C 40 (1989) 321.
[24] R. J. Furnstahl, B. D. Serot and H. B. Tang, Nucl. Phys. A 615 (1997) 441, R. J. Furnstahl, B. D. Serot and H. B. Tang, Nucl. Phys. A 640 (1998) 505, Erratum.
[25] Y. Hu, J. McIntire, B. D. Serot, Nucl. Phys. A 794 (2007) 187.
[26] Kausik Pal, Subhrayojit Biswas and Abhee K. Dutt-Mazumder (in preparation).
[27] S. A. Chin, Annals of Physics 108 (1977) 301.
[28] V. Greco, M. Colonna, M. Di Toro and F. Matera, Phys. Rev. C 67 (2003) 015203.