The Virtual Photon Structure Functions and AdS/QCD Correspondence

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We study the virtual photon structure functions from gauge/string duality. If the Bjorken variable $x$ is not small, supergravity approximation becomes good in dual string theory. We calculate the virtual photon structure functions at large 't Hooft coupling in a moderate $x$-region and determine $x$-behavior of the structure functions. We also show that the Callan-Gross relation $F_L = 0$ is satisfied to a good approximation in gravity calculation.

Subject Index: 161, 168

§1. Introduction

The structure functions are important objects in quantum chromodynamics (QCD). For example, the nucleon structure functions control the cross section of deep inelastic scattering and they are related to the parton densities inside the initial hadrons. The nucleon structure functions however are defined by nucleon matrix elements of the electromagnetic currents, so they cannot be calculated by the perturbative method. We can only calculate energy scale evolution on the basis of the perturbation theory.

AdS/CFT correspondence relates $\mathcal{N} = 4$ super Yang-Mills theory at large 't Hooft coupling in four dimensions to weakly coupled string theory in $AdS_5 \times S^5$. The authors of Ref. 2) have studied the dual gravity description of nucleon structure functions and have calculated nucleon structure functions. This is called the hard-wall model. In this model, the AdS space has a cutoff at the infrared region and conformal invariance is broken. This cutoff scale corresponds to the infrared mass scale of the gauge theory. At small coupling, probe photons scatter off partons inside hadrons. However, at large 't Hooft coupling, the situation is completely different; probe photons scatter off entire hadrons and do not destroy hadronic states. Hadronic states are dual to (massless) string states in AdS space and we can calculate structure functions from supergravity interactions. Usually, the leading-twist operators have the leading contribution to OPE of currents correlators at small coupling. However, at large 't Hooft coupling, these operators have large anomalous dimensions and do not dominate in OPE.

In this article, we consider the virtual photon structure functions and study their property from bulk dynamics in AdS space. The photon structure functions are defined by the absorptive part of the four quark currents correlator. Naively, they can be obtained by replacing the initial hadronic states in hadronic tensors by the target photon states. Thus, their properties are similar to nucleon structure functions;
we can apply the standard OPE technique and calculate the anomalous dimensions of twist-two operators and coefficient functions perturbatively. A crucial difference in the virtual photon structure functions from the nucleon structure functions is that one can determine the $x$-behavior of the virtual photon structure functions perturbatively. However, from the theoretical viewpoint, it is still interesting to consider QCD objects at strong coupling, which are observed at small coupling.4)

§2. Review of photon structure functions

We review photon structure functions briefly.5) We take the Lorentz metric $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$. For definiteness, we consider virtual photon scattering on $e^+e^- \to e^+e^- + \text{hadrons}$ (Fig. 1). Four-dimensional momenta of two virtual photons are $q^\mu$ and $p^\mu$ ($p^2 \leq q^2$). We call the photon whose momentum is $q^\mu$ ($p^\mu$) the probe (target) photon. If the invariant mass of the target photon is close to on-shell $p^2 \simeq 0$, then vector meson dominance is realized. This process is better described by vector meson-photon coupling than by photon-photon scattering. Dual gravity descriptions of such interactions are studied in Refs. 12) and 14). If the invariant mass of the target photon is far off-shell $\Lambda_{QCD}^2 \ll p^2$, this process is described by virtual photon-virtual photon scattering.

We define the tensor $T^{\mu\nu\alpha\beta}$ as

$$T^{\mu\nu\alpha\beta}(p, q) = i \int d^4x d^4y d^4z e^{iq\cdot x} e^{ip\cdot (y-z)} \langle 0 | T(J^\alpha(y) J^\mu(x) J^\nu(0) J^\beta(z)) | 0 \rangle,$$  \hspace{1cm} (2.1)

where $J^\mu$ denotes quark currents that couple to the photons. The structure tensor of virtual photons is defined by the absorptive part of $T^{\mu\nu\alpha\beta}$.

$$W^{\mu\nu\alpha\beta}(p, q) = \frac{1}{\pi} \text{Im} T^{\mu\nu\alpha\beta}(p, q).$$  \hspace{1cm} (2.2)

This tensor has eight independent components. If the initial photon states are unpolarized, we average the target photon helicities and we obtain

$$W^{\mu\nu}(p, q) = \frac{1}{2} \sum_\lambda \epsilon^{(\lambda)*}_\alpha(p) W^{\mu\nu\alpha\beta}(p, q) \epsilon^{(\lambda)}_\beta(p)$$

$$= \frac{1}{2} \eta_{\alpha\beta} W^{\mu\nu\alpha\beta}(p, q).$$  \hspace{1cm} (2.3)

$W^{\mu\nu}$ can be decomposed into two structure functions $F_1(x, q^2, p^2)$ and $F_2(x, q^2, p^2)$ as

$$W^{\mu\nu}(p, q) = \left( \eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) F_1(x, q^2, p^2) + \left( p^\mu + \frac{q^\mu}{2x} \right) \left( p^\nu + \frac{q^\nu}{2x} \right) \frac{4x}{q^2} F_2(x, q^2, p^2),$$  \hspace{1cm} (2.4)

where $x = \frac{q^2}{2p^2}$. $x$ takes $0 \leq x \leq 1/(1 + \frac{q^2}{p^2})$.

The structure functions $F_i(x, q^2, p^2)$ are called the real photon structure functions in the region $p^2 \simeq 0$ and are called the virtual photon structure functions in the region $\Lambda_{QCD}^2 \ll p^2$. 

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Fig. 1. $e^- + e^+ \rightarrow e^- + e^+ + X$. $X$ represents the final hadron states. Wavy lines denote virtual photons that couple to quark currents. $q^2$ ($p^2$) is invariant mass-squared of the probe (target) photon.

To apply OPE to the product of the quark currents that couple to probe photons, the condition $p^2 \ll q^2$ is required. This means that the distance between quark currents correlators that couple to the probe photons is much shorter than that of the others. Thus, in the region $\Lambda_{QCD}^2 \ll p^2 \ll q^2$, we can calculate the virtual photon structure functions perturbatively at small coupling.

Inserting the final hadronic states $\sum_X |X\rangle\langle X| = 1$, we can represent the structure tensor $W^{\mu\nu}$ as

$$W^{\mu\nu}(p, q) = \frac{1}{2} \langle 0 | \bar{J}^\alpha(-p) \tilde{J}^\mu(q) | X \rangle \langle X | J^\nu(0) \tilde{J}_\alpha(p) | 0 \rangle$$

$$= \frac{1}{2} (2\pi)^4 \sum_X \delta^{(4)}(p + q - P_X) \langle 0 | \bar{J}^\alpha(-p) J^\mu(0) | X \rangle \langle X | J^\nu(0) \tilde{J}_\alpha(p) | 0 \rangle,$$

(2.5)

where we denote $P_X$ as the momentum of the final state hadron $X$ and $\tilde{J}^\mu$ as the Fourier transformation of $J^\mu$.

§3. Virtual photon structure functions in AdS/QCD

In this section, we calculate the virtual photon structure functions from supergravity interaction. The scattering process is caused by gauged $U(1)$ R-symmetry, which is a subgroup of $SO(6)$ R-symmetry in $\mathcal{N} = 4$ super Yang-Mills theory. Although $\mathcal{N} = 4$ super Yang-Mills theory does not have a quark nor a photon, hadron properties such as meson form factors are well described using a gauged $U(1)$ R-current. We use this current to mimic electromagnetic current.

We treat the process to the lowest order in $U(1)$ coupling constant and leading order in large 't Hooft coupling. Indices $m, n, \cdots$ denote the AdS$_5$ space and $\mu, \nu, \cdots$ denote four-dimensional Minkowski space. Indices $m, n, \cdots$ are raised with...
the curved metric $g^{mn}$ and $\mu, \nu, \cdots$ are raised with $\eta^{\mu\nu}$. We define $q^2 = \eta_{\mu\nu} q^\mu q^\nu$ and $q = \sqrt{q^2}$. In the hard-wall model, the AdS space has an infrared cutoff at $r_0 = \Lambda R^2$, and $\Lambda$ corresponds to the infrared mass scale of the gauge theory.

The metric of $AdS_5 \times S^5$ is

$$ds^2 = \frac{r^2}{R^2} \eta_{\mu\nu} dy^\mu dy^\nu + \frac{R^2}{r^2} d^2r + R^2 ds^2_{S^5}, \quad (3.1)$$

where $R = (4\pi g_s N_c)^{1/4} \alpha'$ is the $AdS_5 \times S^5$ radius and $ds^2_{S^5}$ is the metric of $S^5$ with unit radius. The final state hadron is dual to a string state in ten-dimensional space.² In the hard-wall model, scalar (vector) meson fluctuations are described by normalizable modes of radial KK excitation of ten-dimensional dilaton (KK gauge bosons of ten-dimensional metric) in dual gravity. Some of the spin 1/2 and spin 3/2 baryon states are modeled by dilatino $\Psi$ and gravitino $\Psi_\mu$ in $AdS_5 \times S^5$.⁷ In large $N_c$ approximation, multihadron states and two currents correlator is

$$\langle \Phi_F \rangle$$

from the fermion number conservation, a single fermion that satisfies

$$\Phi_5 = q\bar{\Phi}_5 \frac{\eta_{\nu\mu}}{q^2} \Phi_5 \frac{J_{k+2}(m_X R^2/r)}{Y_k(\Omega)}, \quad (3.2)$$

where $c_X$ is a dimensionless constant and $m_X = \sqrt{P_X^2}$. $J_{k+2}$ is a Bessel function. $Y_k(\Omega)$ is a scalar spherical harmonics of $S^5$ that satisfies

$$\Box_{S^5} Y_k(\Omega) = -k(k+4) Y_k(\Omega). \quad (k = 0, 1, \cdots) \quad (3.3)$$

In four dimensions, the different values of mass-squared $m_X^2 = P_X^2$ correspond to the different hadrons. In the hard-wall model, the mass spectra $m_X^2$ is determined so that $\Phi_X$ satisfies the boundary condition $\Phi_X = 0$ at the hard cutoff $r_0 = \Lambda R^2$. This is given by the zeros of a Bessel function $J_{k+2}(m_n R^2/r_0) = 0$. Thus, for each $k$, the solution Eq. (3.2) contains a tower of hadron states labeled by $m_n$.

The Kaluza-Klein gauge field $A_m$ couples to the R-current at the $AdS$ boundary.⁶,⁸ The boundary condition of the gauge field is imposed by

$$\lim_{r \to \infty} A_\mu(y, r) = \epsilon_\mu(q) e^{iqy}, \quad (3.4)$$
where \( \epsilon_\mu \) denotes the polarization vector of the gauge field in four dimensions. The bulk gauge field satisfies the Maxwell equation in the AdS\(_5\) space. We take the Lorentz gauge in five dimensions, then the nonnormalizable modes of the gauge field are

\[
A_\mu(y, r) = \epsilon_\mu(q)e^{iqy}qR^2_rK_1(qR^2/r),
\]

\[
A_r(y, r) = i(\epsilon(q) \cdot q)e^{iqy}R^4_{r^3}K_0(qR^2/r),
\]

where \( K_0 \) and \( K_1 \) are modified Bessel functions of the second type. The field strength tensors of \( A_\mu \) and \( A_r \) are

\[
F_{\mu\nu}(q) = i[q_\mu\epsilon_\nu(q) - \epsilon_\mu(q)q_\nu]qR^2_rK_1(qR^2/r)e^{iqy},
\]

\[
F_{\mu r}(q) = [\epsilon_\mu(q)q^2 - q_\mu(\epsilon(q) \cdot q)]R^4_{r^3}K_0(qR^2/r)e^{iqy}.
\]  

The insertion of R-symmetry currents excites the metric perturbation \( \delta g_{ma} = A_m(y, r)v_a(\Omega) \) with Killing vector \( v^a(\Omega)\partial_a \) associated with a \( U(1) \)-isometry of \( S^5 \). The correlation function of two R-symmetry currents and a scalar corresponds to the supergravity interaction\(^9\)

\[
\epsilon_\mu(q)\epsilon_\nu(p)\langle X|\tilde{J}^\mu(q),\tilde{J}^\nu(p)|0\rangle = \frac{1}{4} \int d^{10}x\sqrt{-g}F_{mn}F^{mn}v^av_a.
\]  

From Eq. (3-6), Eq. (3-7) becomes

\[
\int d^{10}x\sqrt{-g}F_{mn}(q)F^{mn}(p)v^av_a = \frac{2\pi}{4}(2^4\delta^{(4)}(p + q - P_X)c_XC_ks^{1/4}A^{1/2}/r^3 \int dr \left( \frac{R^4}{r^4}F_{\mu\nu}(q)F^{\mu\nu}(p) + 2F_{\mu r}(q)F^\rho_r(p) \right)J_{k+2}(s^{1/2}R^2/r),
\]

where \( C_k = \int_{S^5} Y_k(\Omega)v^av_a \). The correlation function becomes

\[
\epsilon_\mu(q)\epsilon_\nu(p)\langle X|J^\mu(0),J^\nu(p)|0\rangle
\]

\[
= \frac{1}{4}c_XC_ks^{1/4}A^{1/2} \int dr \frac{r}{R^3} \left( \frac{R^4}{r^4}F_{\mu\nu}(q)F^{\mu\nu}(p) + 2F_{\mu r}(q)F^\rho_r(p) \right)
\]

\[
= \frac{1}{4}c_XC_ks^{1/4}A^{1/2}\left\{ -2\left[ (q \cdot p)(\epsilon(q) \cdot \epsilon(p)) - (q \cdot \epsilon(p))(p \cdot \epsilon(q)) \right]
\]

\[
\times q_\rho A_k(x, q_2^2, p_2^2) + 2\left[ (\epsilon(q) \cdot \epsilon(p))q_2^2p_2
\]

\[
- (p \cdot \epsilon(q))(p \cdot \epsilon(p))q_2^2 - (q \cdot \epsilon(q))(q \cdot \epsilon(p))p_2
\]

\[
+ (q \cdot p)(p \cdot \epsilon(q))(p \cdot \epsilon(p)) \right] B_k(x, q_2^2, p_2^2) \right\}.
\]  

In the above equation, \( A_k(x, q_2^2, p_2^2) \) and \( B_k(x, q_2^2, p_2^2) \) are defined by

\[
A_k(x, q_2^2, p_2^2) = \int_{AR}^\infty dr \frac{R^5}{r^5}K_1(qR^2/r)K_1(pR^2/r)J_{k+2}(s^{1/2}R^2/r),
\]

\[\text{Eq. (3.8)}\]
\[
B_k(x, q^2, p^2) = \int_{\Lambda R^2}^{\infty} \frac{dr}{r^5} R^5 \frac{K_0(q R^2 / r) K_0(p R^2 / r) J_{k+2}(s^{1/2} R^2 / r)}{p^5},
\]
where \( s = -(p + q)^2 = q^2(1/x - 1 - p^2/q^2) \).

We have to sum up the final-state hadrons to obtain the structure tensor \( W^{\mu\nu} \). We can estimate the density of radial KK states by a hard cutoff at a radius \( r_0 = \Lambda R^2 \), so that the spacing of the zeros of the Bessel function Eq. (3.2) gives \( m_X = m_n = n \pi \Lambda \).

\[
\sum_n \delta(m_n^2 - s) \sim (\partial m_n^2 / \partial n)^{-1} \sim (2\pi s^{1/2} \Lambda)^{-1},
\]
where the spacing of the zeros is close to \( \Lambda \ll q \). We also have to sum KK states of \( S^5 \) labeled by \( k \). We can show that the coefficient \( C_k(k \neq 0, 2) \) vanishes. To see this, we examine the properties of scalar spherical and of the Killing vectors on \( S^5 \). We can describe the unit sphere as the surface \( x^A x^A = 1 \) in \( R^6 \), where \( x^A \) denotes the Cartesian coordinates in \( R^6 \). Let \( C_{A_1 \cdots A_k} \) be symmetric and traceless constants in \( A_1 \cdots A_k \). Then, the scalar spherical harmonics of \( S^5 \) are expressed as \( Y^k = C_{A_1 \cdots A_k} x^{A_1} \cdots x^{A_k} \). The integration of monominal of \( x^A \) on \( S^5 \) is \( 10) \)

\[
\int_{S^5} x^{A_1} \cdots x^{A_k} \propto \left\{ \begin{array}{ll}
(\text{All possible contractions}), & (k : \text{even}) \\
0, & (k : \text{odd})
\end{array} \right.
\]
where “all possible contractions” denotes \( \delta^{A_1 A_2} \) for \( k = 2, \delta^{A_1 A_2} \delta^{A_3 A_4} + \delta^{A_1 A_3} \delta^{A_2 A_4} + \delta^{A_1 A_4} \delta^{A_2 A_3} \) for \( k = 4 \) and so on. A basis of the Killing vectors on \( S^5 \) is given by

\[
K_{AB} := x^A \delta_B - x^B \delta_A.
\]
The inner product between Killing vectors \( K_{AB} \) and \( K_{CD} \) with respect to the metric of \( S^5 \) is \( 11) \)

\[
g(K_{AB}, K_{CD}) = \delta_{AC} x_B x_D + \delta_{BD} x_A x_C - \delta_{AD} x_B x_C - \delta_{BC} x_A x_D.
\]
From Eqs. (3.12), (3.14) and the tracelessness of \( C_{A_1 \cdots A_k} \), we can see \( \int_{S^5} Y_k g(K_{AB}, K_{CD}) = 0, (k \neq 0, 2) \) for each \( A, B, C, \) and \( D \). Any inner product of the Killing \( v_a \) can be expressed by a linear combination of \( g(K_{AB}, K_{CD}) \). Thus, we have shown the equation

\[
\int_{S^5} Y_k(\Omega) v^a v_a = 0. \quad (k \neq 0, 2)
\]
Using Eqs. (3.9) and (3.11) and taking the average for the polarization vectors of the target photon, we obtain the structure tensor Eq. (2.5),

\[
W^{\mu\nu}(p, q) = \sum_{k=0,2} \frac{|C_k|^2}{24\pi} \left[ \left( \eta^\mu - \eta^\nu \frac{q^2}{q^2} \right) p^2 q^4 \left( \frac{A_k(x, q^2, p^2)}{2x} + \frac{p}{q} B_k(x, q^2, p^2) \right)^2 \right. \\
\left. + \left( p^\mu + \frac{q^\mu}{2x} \right) \left( p^\nu + \frac{q^\nu}{2x} \right) p^2 q^4 \left( A_k(x, q^2, p^2)^2 - B_k(x, q^2, p^2)^2 \right) \right].
\]
Hence, we have the following structure functions,

$$F_1(x, q^2, p^2) = \sum_{k=0,2} \frac{|c_x C_k|^2}{2^4 \pi} p^2 q^6 \left( \frac{A_k(x, q^2, p^2)}{2x} + \frac{p}{q} \frac{B_k(x, q^2, p^2)}{2x} \right)^2,$$

$$F_2(x, q^2, p^2) = \sum_{k=0,2} \frac{|c_x C_k|^2}{2^6 \pi x} p^2 q^6 \left( \frac{A_k(x, q^2, p^2)}{2x} - \frac{B_k(x, q^2, p^2)}{2x} \right)^2.\quad (3.17)$$

We stress that the condition $p^2 \ll q^2$ is not used in the supergravity calculation of the structure functions. This is quite different from pQCD analysis in which the condition $p^2 \ll q^2$ is necessary for applying OPE to the product of quark currents. We analyze $F_i(x, q^2, p^2)$, $(i = 1, 2)$ in the kinematical region $p^2 \ll q^2$. $K_i(qR^2/r)$ damps exponentially in the small $r$ region. Thus, the region that contributes to integrals in $A_k(x, q^2, p^2)$ and $B_k(x, q^2, p^2)$ is $qR^2 \leq r$.

To begin with, we evaluate $F_1(x, q^2, p^2)$. We show later that the dominant contribution to $F_1(x, q^2, p^2)$ comes from $A_k(x, q^2, p^2)$ and we can neglect the second term in Eq. (3.17) for $p^2 \ll q^2$. When $p^2 \ll q^2$ is satisfied, we can also use the leading behavior $K_1(pR^2/r) \simeq r/pR^2$. Then, with an approximation $1/\Lambda \sim \infty$, this integral can be performed and we obtain

$$F_1(x, q^2, p^2) \simeq \int_0^\infty dr \frac{R^3}{pr^4} K_1(qR^2/r) J_{k+2}(s^{1/2} R^2/r),$$

$$= \frac{2}{pq^3 R^3} \frac{1}{x - 1 - \frac{p^2}{q^2}} \Gamma\left( \frac{k+6}{2} \right) \Gamma\left( \frac{k+4}{2} \right) \Gamma\left( k + 3 \right),$$

$$= \frac{2}{pq^3 R^3} x^{-2} \left[ 1 - \left( 1 + \frac{p^2}{q^2} \right) x \right]^{\frac{k+1}{2}} \left[ 1 - \frac{p^2}{q^2} x \right]^{-\frac{k+4}{2} - \frac{1}{x} \Gamma\left( \frac{k+6}{2} \right) \Gamma\left( \frac{k+4}{2} \right) \Gamma\left( k + 3 \right)}, \quad (3.19)$$

where $2F_1(a; b; c; z)$ is a hypergeometric function. From the second line to the third line, we use the identity $2F_1(a; b; c; z) = (1 - z)^{-a} 2F_1(a; c - b; c; z/(z - 1))$. Then, $F_1$ becomes

$$F_1(x, q^2, p^2) \simeq \sum_{k=0,2} \frac{|c_x C_k|^2}{2^6 \pi x^2} p^2 q^6 A_k(x, q^2, p^2)^2$$

$$\simeq \frac{|c_x C_0|^2}{2^4 \pi R^6} x^2 \left[ 1 - \left( 1 + \frac{p^2}{q^2} \right) x \right]^2$$

$$+ \frac{|c_x C_2|^2}{2^8 \pi R^6} x^4 \left[ 1 - \left( 1 + \frac{p^2}{q^2} \right) x \right]^4 \left( 1 - \frac{p^2}{q^2} x \right)^{-4}$$

$$\times 2F_1 \left( 4, 2; 5; \frac{1 - (1 + \frac{p^2}{q^2}) x}{1 - \frac{p^2}{q^2} x} \right). \quad (3.20)$$
One can see that $F_1(x, q^2, p^2)$ increases with $x$ in the small $x$-region and decreases near $x_{\text{max}} = 1/(1 + p^2/q^2)$. The structure functions should vanish at $x_{\text{max}}$ and Eq. (3.20) actually satisfies this requirement. From Eq. (3.20), we have found that $F_1(x, q^2, p^2)$ scales as $F_1(x)$ in the limit $p^2/q^2 \to 0$. This is a crucial difference between the photon structure functions and the nucleon structure functions in gravity calculation. If we define $\Delta_i$ as the conformal dimension that is dual to the initial state hadron, the momentum dependence of the nucleon structure functions always behaves as $(\lambda/q)^{2\Delta_i - 2}$.

The nucleon structure functions do not have the scaling property in general $\Delta_i$. The nucleon structure functions have the scaling property when the conformal dimension is a special value $\Delta_i = 1$.\(^{14}\)

Next, we evaluate $F_2$. We transform $w = qR^2/r$ in Eq. (3.10); $B_k(x, q^2, p^2)$ can be written as

$$\frac{1}{R^4q^4} \int_0^\infty dw w^3 K_0(wp/q)K_0(w)J_{k+2}(ws^{1/2}/q). \quad (3.21)$$

When the condition $p^2 \ll q^2$ is satisfied, we can use the asymptotic form $K_0(wp/q) \simeq \log(2q/wp)$. Then,

$$B_k(x, q^2, p^2) = \frac{1}{R^4q^4} \lim_{\epsilon \to 0} \int_0^\infty dw w^3 \frac{(2q/wp)^\epsilon - 1}{\epsilon} K_0(w)J_{k+2}(ws^{1/2}/q).$$

$$\approx \begin{cases} 
\frac{2^2 \log q/p}{R^4q^4} \left[ 1 + 4(1-x) + (1-x)^2 \right] + O(q^{-4}), & (k = 0) \\
\frac{2^3 \log q/p}{R^4q^4} x^2(1-x)^2 F_1(4, 1; 3; 1-x) + O(q^{-4}), & (k = 2)
\end{cases}$$

where $\log x = \lim_{\epsilon \to 0} (x^\epsilon - 1)/\epsilon$. Therefore, $B_k(x, q^2, p^2)^2$ is a subleading contribution to $F_2$. From the expression for $F_2$, we calculate the longitudinal structure function defined as $F_L = F_2 - x F_1$.

$$F_L(x, q^2, p^2) \simeq \sum_{k=0,2} \frac{|c_x C_k|^2}{2^6 \pi x} p^2 q^6 B_k(x, q^2, p^2)^2$$

$$\simeq \frac{|c_x C_0|^2}{2^2 \pi R^6 x} \log^2(q/p) \left[ 1 + 4(1-x) + (1-x)^2 \right]^2$$

$$+ \frac{3^2 |c_x C_k|^2}{2^4 \pi R^6} \log^2(q/p) x^3(1-x)^2 F_1(4, 1; 3; 1-x)^2. \quad (3.22)$$

From the above equation, in the limit $p^2/q^2 \to 0$, one can see that the Callan-Gross relation $F_L = 0$ holds in the gravity calculation. In QCD, the Callan-Gross relation implies that the partons involved in the scattering process are spin 1/2 particles. This relation is weakly broken by higher order corrections in real QCD, and $F_L$ is smaller than $F_1$ and $F_2$. In our calculation, the function $B(x, q^2, p^2)$ originates from the radial gauge field $A_r$; thus, we have found that the Callan-Gross relation is weakly broken by the effect of the radial gauge field $A_r$ in Eq. (3.22).
§4. Summary

In this article, we have calculated the virtual photon structure functions in the hard-wall model. The structure functions $F_i(x, q^2, p^2)$, ($i = 1, 2$) that we have obtained possess the following properties which we have in QCD.

1) $F_i$'s increase at small $x$-region, decrease at large $x$-region, and vanish at $x = x_{\text{max}}$.

2) $F_L$ is small compared with $F_i$'s. It goes to zero when the limit $p^2 \ll q^2$ as $q^2 \to \infty$.

We simply comment on some extension of the virtual photon structure functions at strong coupling. In the hard-wall model, the mass of higher excitations behaves as $m_n \sim n^2$, but actually the mass of higher excitations should behave as $m_n^2 \sim n^2$ (Regge trajectory). A background that reproduces the Regge trajectory is proposed in Ref. 13). This is called the soft-wall model. Deep inelastic scattering in the soft-wall model is studied, and the leading order results in the soft-wall model are the same as those in the hard-wall model.\(^\text{15}\) We treat the virtual photon structure functions in the not so small $x$-region. At the small $x$-region, we have to take into account excited string states.\(^\text{2}, 16\) Then, we cannot use the insertion of ten dimensional dilaton states and have to treat the four quark currents correlator directly. The analysis would become much more complicated. It is interesting to investigate the virtual photon structure functions in such situations.

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