Simulating chiral magnetic effect and anomalous transport phenomena in the pre-equilibrium stages of heavy-ion collisions

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Abstract

We present a first principles approach to study the Chiral Magnetic Effect during the pre-equilibrium stage of a heavy-ion collision. We discuss the dynamics of the Chiral Magnetic Effect and Chiral Magnetic Wave based on real-time lattice simulations with dynamical (Wilson and Overlap) fermions simultaneously coupled to color and electromagnetic fields. While for light quarks we observe a dissipation-less transport of charges as in anomalous hydrodynamics, we demonstrate that for heavier quarks the effects of explicit chiral symmetry breaking lead to a significant reduction of the associated currents.

Keywords: CME, magnetic fields, real-time lattice simulations

1. Introduction

Novel transport phenomena associated with the interplay of quantum anomalies and magnetic fields and vorticity have created excitement across the physics community [1, 2, 3]. Experimental searches for such anomalous transport phenomena in heavy-ion collisions have revealed intriguing hints at possible signals, however the situation remains unclear because of potentially large background effects [4, 5, 6]. Despite the fact that various theoretical techniques [7, 8, 9, 10, 11, 12, 13] have been developed to study anomalous transport processes, a robust theoretical prediction of the signal remains challenging. One of the key difficulties in this regard is due to the short lifetime (∼1 fm/c) of the magnetic field in off-central heavy-ion collisions [14, 15], forcing anomalous transport processes to take place predominantly during the pre-equilibrium phase. Based on the recent progress in understanding the early-time dynamics of high-energy heavy-ion collisions [16], a compact summary of the space-time evolution in the pre-equilibrium phase is given in Fig. 1. While a combination of different theoretical techniques is required to describe the entire evolution, the early-time dynamics (a-c) can be accurately described using a classical-statistical field theory description, which can be solved using first principles lattice techniques.
In this proceeding, we report first steps towards a quantitative description of anomalous transport phenomena during the non-equilibrium stage from classical-statistical real-time lattice simulations. Based on a brief introduction to the formalism in Sec. 2, we highlight the key results of our present studies [17, 18] in Sec. 3 and conclude with an outlook to future applications in heavy-ion collisions and towards a quantitative understanding of anomalous transport phenomena in high-energy heavy-ion collisions.

2. Non-equilibrium lattice simulations

Based on a classical-statistical description of the bosonic gauge-field degrees of freedom, we study the non-equilibrium production of fermions without any further approximations. Most importantly, the fermion degrees of freedom are treated fully quantum mechanically by discretizing the theory on a lattice and numerically solving the operator Dirac equation [25, 26],

\[ i\gamma^0 \frac{\partial}{\partial t} \hat{\psi}_x = (-i\mathcal{D} + m) \hat{\psi}_x . \]  

Here \( \mathcal{D} \) is either formulated in terms of Wilson-Dirac (see also [25, 26, 30] for related studies) or Overlap fermion (see [18]) discretization schemes. In the case of Wilson fermions, we employ tree-level operator improvements to explicitly cancel lattice artifacts of order \( O(a^2 n^{-1}) \) (in practice we use a scheme accurate to order \( O(a^3) \)), where \( a \) is the lattice spacing

\[ -i\mathcal{D} \hat{\psi}_x = \frac{1}{2} \sum_{n,i} C_n \left[ (-i\gamma^\nu - nr_w)U_{x+\nu} \hat{\psi}_{x+\nu} + 2nr_w \hat{\psi}_x - (-i\gamma^\nu + nr_w)U_{x-\nu} \hat{\psi}_{x-\nu} \right] . \]  

The operator valued fermion field, \( \hat{\psi} \), is decomposed into a complete set of operators acting on the initial state, each multiplying a time dependent complex valued wavefunction, making the system of equations tractable on a computer. We then solve the time evolution of each wave function in the background of a single topological transition [27, 28], that we construct explicitly. Details of the implementation, such as the definitions of observables are described in [17, 18].

3. Non-equilibrium dynamics of anomalous effects

Based on the setup described in Sec. 2, we will now discuss the dynamics of axial charge generation during a sphaleron transition and the subsequent transport of axial and vector charge densities via the Chiral Magnetic Effect (CME) and Chiral Separation Effect (CSE). Our results are compactly summarized in the left panel of Fig. 2, where we plot profiles of the axial and vector charges during and after a sphaleron
transition in the presence of a strong magnetic field and very light quarks. By means of the axial anomaly,
\[ \partial_\mu j^\mu_a = -\left(\frac{g^2}{8\pi^2}\right) tr F_{\mu\nu} \tilde{F}^{\mu\nu}, \]
the sphaleron transition leads to production of axial charge concentrated at the position of the sphaleron. In the presence of a magnetic field, the CME generates a vector current, resulting in a dipole like separation of vector charges along the direction of the magnetic field. As the system evolves, the vector charge imbalance at the edges of the dipole in turn creates an axial current due to the CSE. Ultimately, the simultaneous excitation of CME and CSE leads to the formation of a Chiral Magnetic Wave (CMW) [29], continuing to transport vector and axial charges along the magnetic field direction.

In the center panel of Fig. 2 the magnetic field dependence of the vector charge separation \[ \Delta J^0_v = \int_{z \geq 0} d^2x_{\perp} j^0_v(t, x) \] is shown, and as predicted by the CME, charge separation rises linearly for small magnetic field. At large magnetic fields the behavior is asymptotic, as the vector charge separations saturates to unity.

Since a finite quark mass leads to an explicit breaking of chiral symmetry, the axial anomaly equation is modified for massive quarks to include an explicit violation term, resulting in dissipation of axial charge. Indeed, we find that the explicit chiral symmetry breaking drastically reduces the amount of axial charge produced during the sphaleron transition. As the CME is approximately proportional to the axial charge imbalance, explicit chiral symmetry breaking resulting in a significant reduction of the vector charge separation observed in the right panel of Fig. 2. Drastic differences between heavy and light fermions emerge already for modest fermion masses and anomalous transport essentially ceases to exist for \[ m_{\text{sph}} \gtrsim 0.75 \] (\( r_{\text{sph}} \) is the size of the sphaleron), i.e. as soon as the quark mass becomes comparable to the other relevant scales in the problem. Even though such effects appear to be irrelevant for the two light flavors over the lifetime of a heavy-ion collision, our results suggest that strange quarks should not contribute significantly to anomalous transport processes.

Our microscopic simulations enable an explicit study of the constitutive relations for currents in anomalous hydrodynamics, \[ j^\mu_a = n_{\text{CME}} \partial^\mu + \sigma^\mu_a B_\mu, \]
by extracting the ratios between vector (axial) densities and axial (vector) currents at sufficiently late times. Interestingly we find that these ratios are not time-independent constants and away from the strong field limit differ significantly from unity [17, 18]. Most importantly, we observe that the CME and CSE currents are not generated instantaneously. Given that the lifetime of the magnetic field in heavy ion collisions is short, this finite relaxation time must be taken into account for macroscopic descriptions of anomalous transport phenomena.
4. Conclusions and Outlook

We studied the non-equilibrium dynamics of axial and vector charges and currents in the presence of non-Abelian and Abelian fields using real time lattice simulations. Our microscopic approach has enabled us to study anomalous transport from first principles. In comparison to analytic results for strong magnetic fields [1], we find significant alterations at weak and moderate magnetic field strength, where ratios between vector (axial) densities and axial (vector) currents clearly differ from unity, indicating only a partial alignment of spins with the magnetic field. Our findings also suggest that the onset of the CME and CSE is not instantaneous, and a finite relaxation time for the generation of the CME and CSE should be taken into account. Similarly, we find that for massive fermions significant dissipation effects exists, which effectively prohibit anomalous transport via the CME and CSE.

Since the lifetime of the magnetic field in a heavy ion collision is expected to be very short, it is essential to study the early time dynamics of the aforementioned phenomena. Our findings and technical progress in terms of novel algorithmic techniques form the basis for future studies towards a quantitative understanding of the CME at early times. Besides the direct importance to early-time dynamics, the results of these microscopic studies can interface with macroscopic descriptions of anomalous transport, e.g. chiral kinetic theory or anomalous hydrodynamics. These macroscopic studies should include as an initial condition pre-equilibrium axial and vector charges and currents, which can be provided from real-time lattice simulations. Moreover, we can study the electromagnetic response of the dynamically created medium at earliest time and thus investigate the lifetime of the magnetic field by including fermionic back-coupling [30], a crucial input for phenomenology.

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