On the theory of the relativistic cross sections for stimulated bremsstrahlung on an arbitrary electrostatic potential in the strong electromagnetic field.

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On the base of relativistic generalized eikonal approximation wave function the multiphoton cross sections of a Dirac particle bremsstrahlung on an arbitrary electrostatic potential and strong laser radiation field are presented. In the limit of the Born approximation the ultimate analytical formulas for arbitrary polarization of electromagnetic wave have been obtained.

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I. INTRODUCTION

In the previous papers [1], [2] so called generalized eikonal approximation (GEA) has been developed in the relativistic quantum theory of elastic scattering of Dirac particle on an arbitrary electrostatic field ([1]) and for the stimulated bremsstrahlung (SB) in the presence of an external strong electromagnetic (EM) radiation field ([2]). These wave functions enable to leave the framework of ordinary eikonal approximation as for elastic as well as for inelastic scattering ([3] and [4]), that is not applicable beyond the interaction region \( z \ll p a^2 / \hbar \), where \( z \) is the coordinate along the direction of initial momentum \( \vec{p} \) of the particle and \( a \) is the characteristic size of the interaction region. It means, that such a wave function is applicable in both quantum and quasiclassical limits, i.e., connects the particle wave functions of the Born and ordinary eikonal approximations.

The first nonrelativistic treatment of SB in the Born approximation has been carried analytically by Bunkin and Fedorov ([3]) and then that approach has been extended to the relativistic domain by Denisov and Fedorov ([4]). Further the multiphoton cross sections of SB process have been obtained in the low-frequency [5] and eikonal [6] approximations. In the present paper the relativistic cross sections of SB in the scope of the above-mentioned GEA approximation are obtained and comparably simple formulas for the transition amplitudes and cross sections for the Dirac-particle scattering in the presence of an arbitrary polarized plane electromagnetic wave in the limit of the Born approximation are obtained. Note, that recently ([7]) the complicate formulas for the relativistic cross sections of SB in the case of circularly polarized monochromatic wave corresponding to the relativistic first Born approximation have been obtained.

The organization of the paper is as follows. In Sec. II the analytic expressions for differential cross sections of the SB on an arbitrary electrostatic potential taking into account the spin interaction as well are obtained with the help of the dynamic GEA wave function and in the limit of the Born approximation. In Sec. III we consider the multiphoton cross sections of SB on screening Coulomb potential.

II. MULTIPHOTON CROSS SECTIONS OF STIMULATED BREMSSTRAHLUNG

The knowledge of the solution of the evolution equation for a Dirac particle interacting with the electrostatic and electromagnetic fields makes possible to calculate the scattering amplitude that takes into account the interaction with the both potential and EM fields simultaneously. When the wave function describes the particle states only in the region where the potential energy \( U(\vec{r}) \) is not zero then determination of the scattering amplitude by the asymptote of the wave function [10] is impossible. Although the scattering amplitude can be defined by the Green function formalism [11]. As far as the wave function in the GEA describes the particle states either within the range of the scattering field or at asymptotic large distances, both approaches with such wave function are applicable. In this paper we shall consider both approaches making an emphasis on asymptotic one.

We assume the EM wave to be quasimonochromatic and of an arbitrary polarization with the vector potential

\[
\vec{A}(\varphi) = A_0(\varphi)(\hat{\vec{e}}_1 \cos \varphi + \hat{\vec{e}}_2 \sin \varphi),
\]

where \( A_0(\varphi) \) is a slow varying amplitude of the vector-potential of the plane EM wave \( \vec{A}(t, \vec{r}) \) with the phase \( \varphi = k x \), \( k = (\omega, \vec{k}) \) is the four-wave vector of EM field with frequency \( \omega \), \( \hat{\vec{e}}_1 \cdot \hat{\vec{e}}_2 = \hat{\vec{e}}_1 \cdot \vec{k} = \hat{\vec{e}}_2 \cdot \vec{k} = 0 \), and \( |\hat{\vec{e}}_1| = |\hat{\vec{e}}_2| = 1 \).
and \( \arctan \zeta \) is the polarization angle.

The state of the particle in EM wave field is characterized by the average kinetic momentum ("quasimomentum") \( \Pi = (\Pi_0, \Pi \vec{T}) \) defining via free electron four-momentum \( p = (\varepsilon, \vec{p}) \) and relativistic invariant parameter of the wave intensity \( Z \) by the following equation

\[
\Pi = p + kZ(1 + \zeta^2); \quad Z = \frac{e^2 \mathcal{A}_0}{4k \cdot \vec{p}},
\]

\((\mathcal{A}_0)\) is the averaged value of the \( A_0(\varphi) \) with corresponding Volkov wave function

\[
\Psi_{\Pi, \mu}^{V} = \frac{1}{\sqrt{2\Pi_0}} f_{\nu}(\varphi) \exp \left[ iS_{\nu}(x) \right],
\]

where

\[
S_{\nu}(x) = -\Pi x + \alpha \left( \frac{\Pi}{k} \right) \sin[\varphi - \theta(\Pi)] - \frac{Z(1 - \zeta^2)}{2} \sin 2\varphi
\]

is the classical action of charged particle in EM wave field \((2.1)\) and

\[
f_{\nu}(\varphi) = u^\mu_p - \frac{eA_0(\varphi)(\gamma k)}{2(k\Pi)} \left[ (\vec{\gamma} \cdot \vec{\gamma}) \cos \varphi + \zeta (\vec{\gamma} \cdot \vec{\gamma}) \sin \varphi \right] u^\mu_p.
\]

is the bispinor amplitude, where \( \gamma = (\gamma_0, \vec{\gamma}) \) are the Dirac matrices, \( u^\mu_p \) is the bispinor amplitude of a free particle with polarization \( \mu \) and four-momentum \( p \) and mass \( m \), \( \bar{\pi}^0 u^\mu_p = 2m \), \( \bar{\pi}_p = u^\dagger_p \gamma_0 \). Here the quantities \( \alpha(\vec{p}) \), \( \theta(\vec{p}) \) as a functions of any vector \( \vec{p} \) are defined by the relations

\[
\alpha(\vec{p}) = e\mathcal{A}_0 \sqrt{\left( \vec{p} \cdot \vec{\gamma}_1 \right)^2 + \zeta^2 \left( \vec{p} \cdot \vec{\gamma}_2 \right)^2},
\]

\[
\theta(\vec{p}) = \arctan \left( \frac{\vec{p} \cdot \vec{\gamma}_2}{\vec{p} \cdot \vec{\gamma}_1} \zeta \right).
\]

where \( e \) is the particle charge, and the products like \( \rho x, k\rho, \gamma k \) are relativistic scalar products:

\[
\rho x = \rho^0 x^0 - \vec{\rho} \cdot \vec{x}.
\]

The wave function of Dirac particle in generalized eikonal approximation describing induced scattering on an arbitrary electrostatic field in the presence of strong EM radiation field \((2.1)\) has the following form \((2.2)\)

\[
\Psi_{\Pi, \mu} = \frac{1}{\sqrt{2\Pi_0}} \left( f_{\nu}(\varphi) + f_1(x) \right) \exp \left[ iS_{\nu}(x) + iS_1(x) \right],
\]

where

\[
S_1(t, \vec{q}) = \frac{i}{4\pi^3} \sum_{n=-\infty}^\infty \int \frac{\bar{U}(\vec{q})}{\vec{q}^2 + 2\vec{\rho} \cdot \vec{q} + 2Z \vec{k} \cdot \vec{q} - 2n(k \cdot p - \vec{k} \cdot \vec{q}) - i0} \times \exp \left( i \{ -n\varphi + \vec{q} \cdot \vec{p} + \alpha_1(\vec{q}) \sin[\varphi - \theta_1(\vec{q})] - \alpha_2(\vec{q}) \sin 2\varphi + \theta_1(\vec{q})n \} \right) d\vec{q},
\]

and

\[
f_1(t, \vec{q}) = \frac{1}{(2\pi)^3} \sum_{n=-\infty}^\infty \int \frac{F_n(\varphi, \vec{q}) d\vec{q}}{\vec{q}^2 + 2\vec{\rho} \cdot \vec{q} + 2Z \vec{k} \cdot \vec{q} - 2n(k \cdot p - \vec{k} \cdot \vec{q}) - i0}
\]

are the action and bispinor amplitude which describe the impact of both the scattering and EM radiation fields on the particle state simultaneously. Here

\[
F_n(\varphi, \vec{q}) = \bar{U}(\vec{q}) \exp \left( i \{ -n\varphi + \vec{q} \cdot \vec{p} + \alpha_1(\vec{q}) \sin[\varphi - \theta_1(\vec{q})] - \alpha_2(\vec{q}) \sin 2\varphi + \theta_1(\vec{q})n \} \right)
\]

\((2.10)\)
\[ \times \left\{ D_n(\vec{\gamma} \cdot \vec{q})\gamma_0 + Z D_{2,n}(\frac{\vec{k} \cdot \vec{q}}{k \cdot p - \vec{k} \cdot \vec{q}})(\vec{\gamma} \cdot k)\gamma_0 - \omega(\vec{\gamma} \cdot k)(\vec{\gamma} \cdot \vec{q}) + \frac{(\vec{\gamma} \cdot k)(\vec{\gamma} \cdot \vec{D})(\vec{\gamma} \cdot \vec{q})\gamma_0}{2(k \cdot p - \vec{k} \cdot \vec{q})} \right\} \]

\[ + \frac{1}{2k \cdot p} \left[ \frac{\omega\left[ \vec{q}^2 + 2\vec{p} \cdot \vec{q} - 2 \vec{\gamma} \cdot \vec{k} \cdot \vec{q} \right]}{k \cdot p - \vec{k} \cdot \vec{q}} - (\vec{\gamma} \cdot \vec{q})\gamma_0 \right] (\vec{\gamma} \cdot k)(\vec{\gamma} \cdot \vec{D}) \]

\[ + \frac{\omega e \alpha(\vec{q} \cdot \vec{p})}{k \cdot p - \vec{k} \cdot \vec{q}} D_{1,n}(\theta(\vec{q})) \]

\[ - \frac{e \omega}{2(k \cdot p - \vec{k} \cdot \vec{q})} \left( \vec{q}^2 + 2\vec{p} \cdot \vec{q} - 2 \vec{\gamma} \cdot \vec{k} \cdot \vec{q} \right) (\vec{\gamma} \cdot k)(\vec{\gamma} \cdot \vec{A}(\varphi)) D_n \left\{ u_p^o \right\}, \tag{2.11} \]

where \( \vec{U}(\vec{q}) = \int U(\vec{r}) \exp(-i\vec{q} \cdot \vec{r}) d\vec{r} \) is the Fourier transform of the function \( U(\vec{r}) \), and \( \alpha_1(\vec{q}), \alpha_2(\vec{q}) \) are dynamic parameters of the interaction defining by expressions

\[ \alpha_1(\vec{q}) = \alpha \left( \vec{k} \cdot \vec{q} \right) / \left( k \cdot p + \vec{q} \right), \quad \alpha_2(\vec{q}) = \frac{\vec{k} \cdot \vec{q}}{2(k \cdot p - \vec{k} \cdot \vec{q})} Z(1 - \zeta^2), \tag{2.12} \]

and \( \theta_1(\vec{q}) \) is the phase angle

\[ \theta_1(\vec{q}) = \theta \left( \vec{k} \cdot \vec{q} \right) / \left( k \cdot p + \vec{q} \right). \tag{2.13} \]

The functions \( D_n, D_{1,n}(\theta(\vec{p})), D_{1,n}(\theta(\vec{q})), \) and \( D_{2,n} \) are defined by relations

\[ D_n = J_n(\alpha_1(\vec{q}), -\alpha_2(\vec{q}), \theta_1(\vec{q})), \tag{2.14} \]

\[ D_{1,n}(\theta(\vec{q})) = \frac{1}{2} \left[ J_{n-1}(\alpha_1(\vec{q}), -\alpha_2(\vec{q}), \theta_1(\vec{q}))) e^{-i(\theta_1(\vec{q}) - \theta(\vec{q}))} + J_{n+1}(\alpha_1(\vec{q}), -\alpha_2(\vec{q}), \theta_1(\vec{q}))) e^{i(\theta_1(\vec{q}) - \theta(\vec{q}))} \right], \tag{2.15} \]

\[ D_{2,n} = (1 + \zeta^2) D_n + \frac{(1 - \zeta^2)}{2} \left[ J_{n-2}(\alpha_1(\vec{q}), -\alpha_2(\vec{q}), \theta_1(\vec{q}))) e^{-i\theta_1(\vec{q})} + J_{n+2}(\alpha_1(\vec{q}), -\alpha_2(\vec{q}), \theta_1(\vec{q}))) e^{i\theta_1(\vec{q})} \right], \tag{2.16} \]

\[ \vec{D} = eA_0 \left\{ \frac{\vec{\gamma} \cdot \vec{q}}{2} J_{n-1}(\alpha_1(\vec{q}), -\alpha_2(\vec{q}), \theta_1(\vec{q}))) e^{-i\theta_1(\vec{q})} + \frac{\vec{\gamma} \cdot \vec{q}}{2} J_{n+1}(\alpha_1(\vec{q}), -\alpha_2(\vec{q}), \theta_1(\vec{q}))) e^{i\theta_1(\vec{q})} \right\}. \tag{2.17} \]

In the denominator of the integrals in the expression (2.9) and (2.10) \( -i\theta \) is an imaginary infinitesimal, which shows how the path around the pole in the integrand should be chosen to obtain a certain asymptotic behavior of the wave function, i.e. the outgoing spherical wave. Note that the wave functions is normalized for the one particle in the unit volume.

Let us determine the scattering amplitude by the Green function formalism in GEA (for the elastic scattering see [13]). For the transition amplitude in the EM wave field from the state with the "quasimomentum" \( \Pi \) and the polarization \( \nu \) to the state with the "quasimomentum" \( \Pi' \) and the polarization \( \mu \) we have the expression
\[
T^{\mu\nu}(\Pi \rightarrow \Pi') = \int \overline{\Psi}_{\Pi',\mu}^V(x)\gamma_0\Psi_{\Pi',\nu}(x)U(\varpi)\,d^4x
\]  
(2.18)

where \(x\) is the four-radius vector, \(\overline{\Psi} = \Psi^\dagger\gamma_0\), \(\Psi^\dagger\) denotes the transposition and complex conjugation of \(\Psi\). According to (2.3) and (2.8) the transition amplitude (2.18) can be expressed in the following form

\[
T^{\mu\nu}(\Pi \rightarrow \Pi') = \int e^{i(\Pi'_{\mu} - \Pi_{\mu})t} B(t, \varpi)\,dt,
\]  
(2.19)

where

\[
B(t, \varpi) = \int e^{-i(\Pi'^{\mu}_0 - \Pi_{\mu})t}\overline{\Psi}_{\Pi',\mu}^V(x)\gamma_0\Psi_{\Pi',\nu}(x)U(\varpi)d\varpi
\]  
(2.20)

is the periodic function of time. So making a Fourier transformation of the function \(B(t, \varpi)\) over \(t\) by the relations

\[
B(t, \varpi) = \sum_{n=-\infty}^{\infty} \widetilde{B}_n \exp(-int),
\]  
(2.21)

\[
\widetilde{B}_n = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} B(t, \varpi) \exp(int)\,dt,
\]  
(2.22)

and carrying out the integration over \(t\) in the formula (2.19) we obtain

\[
T^{\mu\nu}(\Pi \rightarrow \Pi') = 2\pi\widetilde{B}_0 \delta\left(\Pi'_0 - \Pi_0 - n\omega\right).
\]  
(2.23)

The differential probability of SB process per unit time in the phase space \(d\Pi'/(2\pi)^3\) (space volume \(V = 1\) in accordance with normalization of electron wave function) is

\[
dW_{\Pi \rightarrow \Pi'} = \lim_{t \to \infty} \frac{1}{t} \left| T^{\mu\nu}(\Pi \rightarrow \Pi') \right|^2 \left| \frac{\Pi'}{\Pi_0'} \right| \frac{d\Omega}{(2\pi)^3}
\]  
(2.24)

where \(d\Omega\) is the differential solid angle.

Dividing the differential probability of SB process \(dW_{\Pi \rightarrow \Pi'}\) by initial flux density \(\left| \frac{\Pi'}{\Pi_0} \right|\) and summing over the particle final states and averaging over initial polarization states, and integrating over \(\Pi'_0\) we obtain the differential cross section of SB process for the non-polarized particles

\[
\frac{d\sigma}{d\Omega} = \sum_n \frac{d\sigma^{(n)}}{d\Omega},
\]  
(2.25)

where

\[
\frac{d\sigma^{(n)}}{d\Omega} = \frac{1}{32\pi^2} \left| \frac{\Pi'}{\Pi} \right|^2 \sum_{\mu\nu} \left| \widetilde{B}_n \right|^2 \delta\left(\Pi'_0 - \Pi_0 + n\omega\right)
\]  
(2.26)

is the partial differential cross section which describes \(n\)-photon SB process. Because of very complicated analytical expressions for multiphoton cross sections of SB in considering approximation (GEA) the ultimate results require numerical investigations which will be presented elsewhere.

Now let us proceed to the asymptotic approach to construct the multiphoton cross sections of SB. Note that at asymptotic large distances \(r \to +\infty\) the GEA wave function coincides with the Born approximation one when \(|S_1(\varpi, t)| \ll 1\) (2.10). As far as we consider an inelastic scattering the wave function of the particle at large distances \(r \to +\infty\) will be the sum of spherical convergent waves with the superposition of a plane wave

\[
\lim_{r \to +\infty} \Psi(\varpi, t) = u_p e^{i\varpi \cdot \bar{t}} - ie^{i\mu t} \sum_{n=-\infty}^{\infty} G_n(\varpi) e^{i|\Pi_n| r} - \Pi_n t \frac{r}{r}.
\]  
(2.27)
where $G_n(\vec{\tau})$ is a bispinor depending on $\vec{\tau} = \vec{r}/r$. Each term of sum describes $n$ photon SB process and the partial inelastic scattering amplitude will be defined as

$$f_{n}^{\mu\nu}(\vec{\Pi} \rightarrow \vec{\Pi}') = \frac{1}{2m} \Pi_{\mu}^{\nu}, G_n(\vec{\tau}),$$

and for the partial differential cross section of SB for the non-polarized particles we have:

$$\frac{d\sigma^{(n)}}{d\Omega} = \frac{1}{2} \frac{1}{|\vec{\Pi} |} \sum_{\mu\nu} \left| f_{n}^{\mu\nu}(\vec{\Pi} \rightarrow \vec{\Pi}') \right|^2$$

As the wave function of the particle for SB process in GEA (2.11) in asymptotic limit of large $r$ has the form

$$\lim_{r \rightarrow +\infty} \frac{\phi(\vec{r}, t)}{\sqrt{2\Pi_0}} = \exp (iS_V(x))$$

$$\times \left\{ F_n(\varphi) + \frac{\exp [-i\vec{\Pi} \cdot \vec{r}]}{4\pi r} \sum_{n = n_0}^{\infty} e^{i(\Pi_n \vec{\tau} - n \vec{k}) \cdot \vec{r}} F_n(\varphi, \vec{q}_n) \right\},$$

where $\vec{q}_n = \Pi_n \vec{\tau} - \vec{\Pi} - n \vec{k}$,

$$\Pi_n = \sqrt{\vec{\Pi}^2 + n\omega (2\Pi_0 + n\omega)},$$

$F_n(\varphi, \vec{q}_n)$ defines by relation (2.11) at the $r \rightarrow +\infty$ and $\vec{q}_n = \Pi_n \vec{\tau} - \vec{\Pi} - n \vec{k}$, then from the Eqs. (2.28), (2.30) follows that the bispinor $G_n(\vec{\tau})$ is the function $F_n(r \rightarrow +\infty, \vec{q})$ (the unessential phase corrections are neglected):

$$G_n(\vec{\tau}) = F_n(r \rightarrow +\infty, \vec{q}_n) = \frac{1}{4\pi} \frac{\sqrt{\vec{\tau}^2}}{n\omega} \frac{1}{1 + \frac{n\alpha}{2\Pi_0}}$$

$$\times \left\{ D_n(\vec{\tau} \cdot \vec{q}_n) \frac{\gamma_0 + ZD_{2,n}}{kp - \vec{k} \cdot \vec{q}_n} \right\}$$

$$\times \left\{ \frac{(\vec{k} \cdot \vec{q}_n)(\gamma k)\gamma_0 - \omega(\gamma k)(\vec{\gamma} \cdot \vec{q}_n)}{2(\gamma k \cdot \vec{k} \cdot \vec{q}_n)} + \frac{(\gamma k)(\vec{\gamma} \cdot \vec{D})(\vec{\gamma} \cdot \vec{q}_n)\gamma_0}{2(\gamma k \cdot \vec{k} \cdot \vec{q}_n)} \right\}$$

$$+ \frac{1}{2k \cdot p} \left[ \frac{\omega [\vec{q}_n^2 + 2\vec{\tau} \cdot \vec{q}_n - 2\vec{k} \cdot \vec{q}_n]}{kp - \vec{k} \cdot \vec{q}_n} \right] - \frac{(\gamma k)(\vec{\gamma} \cdot \vec{D})}{2(\gamma k \cdot \vec{k} \cdot \vec{q}_n)}$$

$$- \frac{2}{2k \cdot p} \left[ \varepsilon D_{n} - \omega(\vec{\tau} \cdot \vec{p}) \right] D_{1,n} \theta(\vec{\tau}) + \omega(\gamma k \cdot \vec{q}_n) \right\} u_{p},$$

Here the functions $D_n, D_{1,n}, D_{2,n}$ and $\vec{D}$ are defined by the expressions (2.14)-(2.17), and

$$\alpha_1(\vec{q}_n) = \alpha_1 \left( \frac{\vec{p}'}{kp'} - \frac{\vec{p}}{kp} \right), \quad \alpha_2(\vec{q}_n) = \frac{Z' - Z}{2}(1 - \zeta^2),$$

$$\theta_1(\vec{q}_n) = \theta \left( \frac{\vec{p}'}{kp'} - \frac{\vec{p}}{kp} \right) = \theta \left( \frac{\vec{\Pi}'}{k\Pi'} - \frac{\vec{\Pi}}{k\Pi} \right).$$

In addition, taking into account that $\Pi_{\mu}^{\nu}(p_0'\gamma_0 - \vec{\gamma} \cdot \vec{p}' - m) = 0$ and $(p_0'\gamma_0 - \vec{\gamma} \cdot \vec{p}' - m) u_p = 0$ and known relations between the $\gamma$-matrix elements

$$\gamma_{\mu\nu} - \gamma_{\nu\mu} = 2\delta_{\mu\nu}$$

we reduce the transition amplitude to the following form

$$\frac{d\sigma^{(n)}}{d\Omega} = \frac{1}{2} \frac{1}{|\vec{\Pi} |} \sum_{\mu\nu} \left| f_{n}^{\mu\nu}(\vec{\Pi} \rightarrow \vec{\Pi}') \right|^2$$
\[ f_{\mu\nu}'(\Pi' \rightarrow \Pi) = -\frac{1}{4\pi} \frac{\pi}{\rho'} A_{\rho} U(\mathbf{q}_n'), \]  
\( \text{(2.35)} \)

where

\[ A = \mathbf{E} + \mathbf{k}' \mathbf{D}, \]  
\( \text{(2.36)} \)

\[ \mathbf{E} = \gamma_0 D_n + \frac{\omega Z}{k \rho'} (\gamma k) D_{2,n}, \]  
\[ k' = \left( \frac{k}{k \rho'} + \frac{\mathbf{k}}{k \rho'} \right)/2, \]  
\[ \mathbf{k} = \left( k_0, -\mathbf{k}' \right). \]  
\( \text{(2.37)} \)

Then introducing (2.35) into the (2.29) the scattering cross section will be in the form

\[ \frac{d\sigma^{(n)}}{d\Omega} = \frac{\left| \mathbf{P}' \right| \left| \mathbf{U}(\mathbf{q}_n') \right|^2}{(4\pi)^2 \left| \mathbf{P} \right|^2} 2\Re \left\{ \rho' \mathbf{A} \mathbf{A}^\dagger \right\}, \]  
\( \text{(2.38)} \)

where

\[ \rho = \frac{\mathbf{p} + m}{2}, \quad \rho' = \frac{\mathbf{p}' + m}{2} \]

are the initial and final density matrices.

Taking into account that \( k \rho = k \Pi, k \rho' = k \Pi' \) and using the properties of defined functions \( D_n, D_{1,n}, D_{2,n} \) and \( \mathbf{D} \) (which follow directly from Eqs. (2.14)-(2.17) and relation (A8)) we obtain the following expression for the partial differential cross sections of SB process

\[ \frac{d\sigma^{(n)}}{d\Omega} = \frac{\left| \mathbf{P}' \right| \left| \mathbf{U}(\mathbf{q}_n') \right|^2}{(4\pi)^2 \left| \mathbf{P} \right|^2} \left\{ 4 \left| \mathbf{E} D_n + \omega Z D_{2,n} - \omega \alpha \left( \frac{\mathbf{E} \mathbf{P}}{k \Pi} \right) D_{1,n}(\theta(\mathbf{P})) \right|^2 \right. \]

\[ \left. - \mathbf{q}_n^2 |D_n|^2 + \frac{1}{(k \Pi)(k \Pi') \omega^2 \mathbf{q}_n^2} \left[ (\mathbf{k}' \cdot \mathbf{q}_n)^2 \right] \left( \left| \mathbf{D} \right|^2 - \frac{\mathbf{e}^2 \mathbf{A}_0^2}{2} \Re D_n D_{2,n}^\dagger \right) \right\}, \]  
\( \text{(2.39)} \)

where

\[ \left| \mathbf{D} \right|^2 = \frac{e^2 \mathbf{A}_0^2}{4} \left[ (1 + \mu^2) \left| J_{n-1} \right|^2 + |J_{n+1}|^2 \right] + 2(1 - \mu^2) \left[ \cos 2\theta_1(\mathbf{q}_n) \Re J_{n-1} J_{n+1}^\dagger \right. \]

\[ + \left. 2i \sin 2(\theta_1(\mathbf{q}_n') - \theta(\mathbf{P}')) \Im \left( J_{n-1} J_{n+1}^\dagger \right) \right]. \]  
\( \text{(2.40)} \)

Comparing the cross sections of SB process for spinor and scalar particles we can conclude that the spin interaction is described by the terms in order of the square of the quantum recoil \( \mathbf{q}_n^2 \) and gives a considerable contribution in the SB cross sections only for the large-angle scattering (which is known also for the elastic scattering from the formula of Mott ) and for the relativistic intensities of EM wave at \( K = eA/m \geq 1 \).

### III. DIFFERENTIAL CROSS SECTIONS OF SB ON THE SCREENING COULOMB POTENTIAL FOR THE CIRCULAR AND LINEAR POLARIZATIONS OF EM WAVE

For concreteness we utilize the Eq. 2.38 to obtain the differential cross section of SB on a screening Coulomb potential for which the Fourier transform is

\[ \mathbf{U}(\mathbf{q}_n) = \frac{4\pi Z_n e^2}{\mathbf{q}_n^2 + \mathbf{\chi}^2}, \]  
\( \text{(3.1)} \)
Then taking into account that at $\alpha$ functions, for the partial differential cross section of SB we have

$$J = \text{and} \quad \frac{1}{\chi}$$

where $1/\chi$ is the radius of screening, $Z_{\alpha}$ is the charge number of the nucleus.

For circular polarized EM wave the quantities in (2.33) are given by relations (2.7)-(2.34), (2.14)-(2.17) at $\zeta = 1$. Then taking into account that at $\alpha_z(\varphi) = 0$ the functions $D_n$, $D_{1,n}(\theta(\Pi))$ and $D_{2,n}$ are expressed by ordinary Bessel functions, for the partial differential cross section of SB process we have

$$\frac{d\sigma^{(n)}}{d\Omega} = \frac{(Z\alpha e^2)^2 |\Pi'|}{|\Pi| (\varphi_n^2 + \chi^2)^2} \left\{ J_n^2(\alpha_1(\varphi_n)) \left[ 4 \left( \Pi_0 - \frac{n\omega \alpha(\Pi)}{\alpha_1(\varphi_n)} \cos \left[ \theta_1(\varphi_n) - \theta(\varphi) \right] \right)^2 \right. 

- \varphi_n^2 + \beta^2 \left( \frac{n^2}{\alpha_1^2(\varphi_n)} - 1 \right) \right] + J_n^2(\alpha_1(\varphi_n)) \left( 4\omega^2 \alpha^2(\varphi)\sin^2 \left[ \theta_1(\varphi_n) - \theta(\varphi) \right] + \beta^2 \right) \right\}, \quad (3.2)$$

where

$$\alpha_1(\varphi_n) = \alpha_1 \left( \frac{\Pi'}{k\Pi'} - \frac{\Pi}{k\Pi} \right), \quad (3.3)$$

$$\theta_1(\varphi_n) = \theta_1 \left( \frac{\Pi'}{k\Pi'} - \frac{\Pi}{k\Pi} \right) \quad (3.4)$$

$$\beta^2 = \frac{e^2 \alpha^2}{(kp)(kp')} \left[ \omega^2 \varphi_n^2 - (\vec{k} \cdot \varphi)^2 \right]. \quad (3.5)$$

and $J'_n(\varphi)$ denotes the first derivative of ordinary Bessel function with respect to $\varphi$.

In the case of linearly polarized EM wave all quantities are defined for $\zeta = 0$. As far as $\theta_1(\varphi_n) = \theta(\varphi_n) = 0$, the functions $D_n$, $D_{1,n}(\theta(\Pi))$ and $D_{2,n}$ are defined by the real function $J_n(u, v)$, further called the generalized Bessel function $\Pi$. Making the simple transformations and using the expression (A.8)

$$2\alpha J_n(u, v) = u \left[ J_{n-1}(u, v) + J_{n+1}(u, v) \right] + 2v \left[ J_{n-2}(u, v) + J_{n+2}(u, v) \right]$$

we obtain the partial differential cross section of SB process $d\sigma^{(n)}/d\Omega$ for the linear polarization of the wave

$$\frac{d\sigma^{(n)}}{d\Omega} = \frac{(Z\alpha e^2)^2 |\Pi'|}{|\Pi| (\varphi_n^2 + \chi^2)^2} \left\{ J_n^2(u, v) \left[ e^2 - \varphi_n^2 - \beta^2 \left( \frac{1}{2} + \frac{n}{4v} \right) \right] 

+ \frac{I_n^2(u, v)}{4\omega^2 \alpha'^2 + \beta^2} \right] 

+ J_n(u, v) I_n(u, v) \left[ \frac{u \beta^2}{4v} - 4\omega \alpha' \right] \right\}, \quad (3.6)$$

where

$$I_n(u, v) = \frac{1}{2} \left( J_{n-1}(u, v) + J_{n+1}(u, v) \right),$$

and

$$\epsilon = 2\Pi_0 + \frac{n\omega Z}{v}$$

$$\alpha' = \alpha \left( \frac{\Pi}{k \cdot \Pi} \right) + \frac{uZ}{2v}.$$
The arguments $u$, $v$ are determined by the relations

$$u = e \overrightarrow{A}_{0} \cdot \left( \frac{\overrightarrow{\Pi}'}{k' \Pi} - \frac{\overrightarrow{\Pi}}{k \Pi} \right),$$

$$v = -\alpha_{2}(q_{n}) = \frac{Z - Z'}{2}.$$

Comparing the nonrelativistic cross section $\Pi$ with relativistic one it is easy to see that besides the additional terms, which come from spin-orbital and spin-laser interaction ($\sim q_{n}^{2}$) as well as effect of intensity ($\sim K^{2}$), the relativistic contribution is conditioned by arguments of the Bessel functions. Because of sensitivity of the Bessel function to relationship of its argument and index the most probable number of emitted or absorbed photons will be defined by the condition $|n| \sim |\alpha_{1}(q_{n})|$. By this reason the contribution of relativistic effects on the scattering process, as have been shown in Ref. (7), becomes essential already for $K \sim 0.1$, consequently, the dipole approximation is violated for nonrelativistic parameters of interaction.

In Fig. 1 the numerical calculations are presented for the same parameters as were in Ref. (7) as for the circular polarization of EM wave. In Fig. 1a. the envelopes of partial differential cross sections as a function of the number of emitted or absorbed photons are shown for the deflection angle $\angle \Pi \Pi' = 0.6$ mrad, for an intensity of Neodymium laser $3.5 \times 10^{16}$ W/cm$^2$ (which corresponds to relativistic parameter of intensity $K \simeq 0.17$) and a moderate initial electron kinetic energy $\varepsilon_{k} = 2.7$ keV. The blue and green curves correspond to initial electron momentum parallel and antiparallel to the laser propagation direction $\overrightarrow{k}$ respectively and the red curve gives the nonrelativistic result. Note, that in case of colinear $\overrightarrow{k}$ and $\overrightarrow{p}$ there is an azimuthal symmetry with respect to propagation direction.

In Fig. 1b. the envelopes of partial differential cross sections for linear polarization of EM wave are shown. To emphasize the differences the relativistic parameter of intensity and initial electron kinetic energy are taken the same. In this case there is no azimuthal symmetry and we have taken the final electron momentum in the same plane with $\overrightarrow{A}$ and $\overrightarrow{k}$. The deflection angle is again $\angle \Pi \Pi' = 0.6$mrad.

The energy exchange increases for large deflection angles and for the high intensities of EM wave. In Fig. 2 the envelopes of partial differential cross sections as a function of the number of emitted or absorbed photons for circular polarization of EM wave are shown for the deflection angle $\angle \Pi \Pi' = 6$mrad. The laser parameters and initial electron kinetic energy are taken the same. As is seen from Fig. 2, the differences between the cases of initial electron momentum parallel or antiparallel to the laser propagation direction $\overrightarrow{k}$ on the one hand and between nonrelativistic result on the other hand are notable.

To show the dependence of SB process upon laser intensity in the Fig. 3 the total differential cross sections are plotted as a function of relativistic parameter of intensity $K$. Fig. 3a. and Fig. 3b. correspond to initial electron momentum parallel and antiparallel to the laser propagation direction $\overrightarrow{k}$, respectively, and the Fig. 3 gives the nonrelativistic result.

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APPENDIX A: DEFINITION OF THE FUNCTION $J_{N}(U, V, \triangle)$

A function $J_{n}(u, v, \triangle)$ may be defined by the expression

$$J_{n}(u, v, \triangle) = (2\pi)^{-1} \int_{-\pi}^{\pi} d\theta \exp \left[ i(u \sin(\theta + \triangle) + v \sin 2\theta - n(\theta + \triangle)) \right]$$

(A1)

or by an infinite series representation

$$J_{n}(u, v, \triangle) = \sum_{k=-\infty}^{\infty} e^{-i2k\triangle}J_{n-2k}(u)J_{k}(v).$$

(A2)
Both defining relations are equivalent. From either Eq. (A1) or (A2) it follows that

$$J_n(u, 0, \triangle) = J_n(u),$$

(A3)

and

$$J_n(0, v, \triangle) = \begin{cases} \quad e^{-i\triangle n} J_{2n}(v), & n \text{ even} \\ \\ 0, & n \text{ odd} \end{cases}.$$  

(A4)

Then we have directly relative formulas

$$J_n(-u, v, \triangle) = (-1)^n J_n(u, v, \triangle),$$
$$J_n(u, -v, \triangle) = (-1)^n J_n(u, v, -\triangle),$$
$$J_n(u, -v, -\triangle) = (-1)^n J_n(u, v, \triangle).$$

(A5)

From the well known recurrence relations for the Bessel function s we have

$$J_{n-1}(u, v, \triangle) - J_{n+1}(u, v, \triangle) = 2\partial_u J_n(u, v, \triangle),$$

(A6)

and

$$e^{-i2\triangle} J_{n-2}(u, v, \triangle) - e^{i2\triangle} J_{n+2}(u, v, \triangle) = 2\partial_v J_n(u, v, \triangle),$$

(A7)

that follows directly from Eq. (A1) or (A2).

The integration by parts in Eq. (A1) yields to the following relation

$$2n J_n(u, v, \triangle) = u [J_{n-1}(u, v, \triangle) + J_{n+1}(u, v, \triangle)]$$
$$+ 2v \left[ e^{-i2\triangle} J_{n-2}(u, v, \triangle) + e^{i2\triangle} J_{n+2}(u, v, \triangle) \right].$$

(A8)

Other results can be obtained by combination of Eqs. (A2)-(A8). We perform two important formulas, which can be proved from Eq. (A1). The first is

$$\sum_{n=-\infty}^{\infty} e^{in(\varphi + \triangle)} J_n(u, v, \triangle) = \exp \{i [u \sin(\varphi + \triangle) + v \sin 2\varphi] \},$$

(A9)

and the second is

$$\sum_{k=-\infty}^{\infty} J_{n+k}(u, v, \triangle) J_k(u', v, \pm \triangle) = J_n(u \pm u', v \pm v', \triangle).$$

(A10)

Then the function $J_n(u, v, \triangle)$ at $\triangle = 0$ turns to the generalized Bessel function $J_n(u, v)$, that was induced by Reiss in Ref. [12].

[1] H.K. Avetissian and S.V. Movsissian, Phys. Rev. A 54, 3036 (1996).
[2] H.K. Avetissian, K.Z. Hatsagortsian, A.G. Markossian, and S.V. Movsissian, Phys. Rev. A 59, 549 (1999).
[3] F.V. Bunkin and M.V. Fedorov, Sov. Phys. JETP 22, 844 (1966).
[4] M.M. Denisov and M.V. Fedorov, Sov. Phys. JETP 26, 779 (1968).
[5] N.M. Kroll and K.M. Watson, Phys. Rev. A 8, 804 (1973).
[6] J.L. Gersten and M.H.M. Mittleman, Phys. Rev. A 12, 1840 (1975).
[7] C. Szymanovsky and A. Maquet, Opt. Express 2, 262 (1998).
[8] V.I. Ritus, Trudl Fiz. Inst. Akad. Nauk 111, 141 (1979).
[9] H.R. Reiss, Phys. Rev. A 22, 1786 (1980).
Figure captions

Figure 1. The envelopes of partial differential cross sections $d\sigma^{(n)}/d\Omega$ in atomic units as a function of the number of emitted or absorbed photons, for an intensity of Neodymium laser $3.5 \times 10^{16} W/cm^2$, $\omega = 1.17 eV$. The radius of screening is $1/\chi = 4$ a.u., $Z_a = 1$, the initial electron kinetic energy $\varepsilon_k = 2.7$ keV. a) and b) correspond to circular and linear polarization of EM wave respectively. The deflection angle equals $\angle \Pi_\Pi' = 0.6$ mrad. The blue and green curves correspond to initial electron momentum parallel and antiparallel to the laser propagation direction $\vec{k}$ respectively and the red curve gives the nonrelativistic result.

Figure 2. The same as Fig.1, for the deflection angle $\angle \Pi_\Pi' = 6$ mrad.

Figure 3. The total differential cross sections $d\sigma/d\Omega$ are plotted as a function of relativistic parameter of intensity $K$ in the range $0 < K < 1$. Fig.3a. and Fig.3b. correspond to initial electron momentum parallel and antiparallel to the laser propagation direction $\vec{k}$, respectively, and the Fig.3c. gives the nonrelativistic result.
a) Diff. Cross Section (a.u.)

b) Diff. Cross Section (a.u.)
Diff Cross Section (a.u.)
a) Total Diff. Cross Section (a.u.)

b) Total Diff. Cross Section (a.u.)

c) Total Diff. Cross Section (a.u.)