INTRODUCTION

Quantum chromodynamics (QCD) provides the fundamental understanding of the strong nuclear force. It describes a vast range of hadrons and their properties in terms of just the quark masses and a gauge coupling. The recent discoveries [1] of several tetraquark candidates are reminders of the richness still remaining to be understood within QCD.

Lattice gauge theory is the first-principles nonperturbative theoretical tool for studying QCD. Emerging quantum computers will allow lattice studies to access new topics within QCD, such as real-time evolution [2, 3]. In this work, we use real-time evolution to present the first study of a tetraquark on a quantum computer. The calculations use SU(3) gauge theory with a single fermion flavor. The theory is written in one spatial dimension and, to match the available quantum hardware, our computations focus on the basic building block (i.e., minimal lattice length).

Previous quantum computations within U(1) gauge theory [4–11] showed electron-positron pair production. Moving from this Abelian case to a non-Abelian theory reveals qualitatively new phenomena. For example, in addition to quark-antiquark pair production (and the existence of a meson), there is also a gauge-singlet particle having valence quarks without valence antiquarks (i.e., the baryon). A recent paper [12] presented the first quantum computation of a baryon mass in a SU(2) gauge group performed on superconducting quantum computers. Our experiment contributes to this quest by taking the crucial step to arrive at the simulation of quarks using the SU(3) gauge group relevant for hadron physics experiments. All these experiments appear in a timely manner to benchmark and max out the current noisy intermediate scale quantum devices to eventually address the open questions in high energy and particle physics. In fact, upon completion of our manuscript an independent but related work has appeared forty-eight hours earlier [19], underlining the ongoing demand for techniques that allow quantum simulations of lattice gauge theories.

RESULTS

SU(3) gauge theory. Our calculations use the Hamiltonian approach where time is not discretized, and the lattice is purely spatial. We consider a one-dimensional (1D) lattice with open boundary conditions, where each site $n$ hosts a fermionic field with three color components, $\hat{\phi}_n = (\phi_n^1, \phi_n^2, \phi_n^3)^T$. We choose to work with staggered fermions [20] with the convention that odd sites host antimatter while even sites host matter.

Real-time evolution of SU(3) hadrons on a quantum computer

Yasar Y. Atas,¹,² J. Jan F. Haase,¹,²,³ Jinglei Zhang,¹,²,⁷ Victor Wei,¹,⁴ Sieglinde M.-L. Pfendler,⁵ Randy Lewis,⁶ and Christine A. Muschik¹,²,⁷

¹Institute for Quantum Computing, University of Waterloo, Waterloo, ON, Canada, N2L 3G1
²Department of Physics & Astronomy, University of Waterloo, Waterloo, ON, Canada, N2L 3G1
³Institut für Theoretische Physik und IQST, Universität Ulm, Albert-Einstein-Allee 11, D-89069 Ulm, Germany
⁴Department of Physics, McGill University, Montreal, QC, Canada, H3A 2T8
⁵IBM Deutschland Research & Development GmbH, Schönjaicher Str. 220, D-71032 Böblingen, Germany
⁶Department of Physics and Astronomy, York University, Toronto, ON, Canada, M3J 1P3
⁷Perimeter Institute for Theoretical Physics, Waterloo, ON, Canada, N2L 2Y5

(Dated: July 8, 2022)
The gauge fields $\hat{U}_n$ are defined on the link between sites $n$ and $n + 1$, and mediate the interaction between color degrees of freedom. The gauge-invariant lattice Hamiltonian in natural units ($\hbar = c = 1$) reads

$$\hat{H}_l = \frac{1}{2\kappa} \sum_{n=1}^{N-1} (\hat{\phi}_n^+ \hat{U}_n \hat{\phi}_{n+1} + \text{H. c.})$$

$$+ m \sum_{n=1}^{N} (-1)^n \hat{\phi}_n^+ \hat{\phi}_n + \frac{g^2}{2} \sum_{n=1}^{N-1} \hat{L}_n^2,$$  \hspace{1cm} (1)

where H. c. denotes the Hermitian conjugate, $N$ is the number of lattice sites with spacing $a$, $m$ is the bare mass, and $g$ is the bare coupling. The alternating sign appearing in the mass term is the signature of the staggered formulation. The first term in the Hamiltonian describes the creation of particle-antiparticle pairs. The last term encodes the color electric energy of the system and is expressed in terms of the left electric field $\hat{L}_n$ on the link $n$. Furthermore, it is convenient to introduce the non-Abelian charges at site $n$, $\hat{Q}^n_\alpha = \sum_{i,j=1}^{8} \hat{\phi}_n^{+i}(T^n)_{ij} \hat{\phi}_n^j$, where $T^n = \lambda^n/2$, and $\lambda^n, a = 1, \ldots, 8$, are the Gell-Mann matrices [21]. These charges appear in the non-Abelian version of the Gauss law that physical states must satisfy [22]. We work in the sector with zero external charges and zero total non-Abelian charge, i.e. a colour singlet state must satisfy $\hat{Q}^n_{\text{tot}}, \langle \Psi \rangle = \sum_n \hat{Q}^n_\alpha |\Psi\rangle = 0$. Besides the eight non-Abelian charges, the Hamiltonian also conserves the baryon number, $B$, which measures the matter-antimatter imbalance (see Methods).

Quantum simulation of time evolutions. To simulate and study the rich physics of the SU(3) theory, we encode Eq. (1) in a Hamiltonian suitable for quantum simulations. In a first step, a gauge transformation is applied that eliminates the gauge degrees of freedom from the Hamiltonian and allows us to express the Hamiltonian in terms of fermions only [23]. This first step saves resources (as gauge fields are not stored explicitly in the qubit register) at the expense of introducing long-range interactions. In a second step, a Jordan-Wigner transformation [24] translates fermionic matter degrees of freedom into spin 1/2, i.e. qubit degrees of freedom (see Fig. 1). After rescaling by the lattice spacing $a$, the resulting Hamiltonian reads

$$\hat{H} = \hat{H}_{\text{kin}} + \hat{m}\hat{H}_m + \frac{1}{2x^2} \hat{H}_e, \hspace{1cm} (2)$$

where $\hat{m} = am$ and $x = 1/g^2a^2$ are the dimensionless mass and coupling constant respectively. In the spin formulation, the kinetic term is given by

$$\hat{H}_{\text{kin}} = \frac{1}{2}\sum_{n=1}^{N-1} (-1)^n \left( \hat{\sigma}_3^{2n-2}\hat{\sigma}_z^{3n-1}\hat{\sigma}_3^{3n+1} - \hat{\sigma}_3^{3n-1}\hat{\sigma}_3^{2n+1}\hat{\sigma}_3^{3n+2} + \hat{\sigma}_3^{3n}\hat{\sigma}_3^{3n+1}\hat{\sigma}_3^{2n+2} \hat{\sigma}_3^{3n+3} \right) + \text{H. c.}, \hspace{1cm} (3)$$

and the mass term reads

$$\hat{H}_m = \frac{1}{2}\sum_{n=1}^{N} \left[ (-1)^n (\hat{\sigma}_3^{2n-2} + \hat{\sigma}_z^{3n-1} + \hat{\sigma}_z^{3n}) + 3 \right]. \hspace{1cm} (4)$$

The electric field Hamiltonian takes the form

$$\hat{H}_e = \sum_{n=1}^{N-1} \left( \sum_{m<n} \hat{Q}_m \right)^2, \hspace{1cm} (5)$$

where $\hat{Q}_m$ is a vector with eight components given by the non-Abelian charges. The exact expression of the non-Abelian charges in terms of qubits can be found in the Methods section. In Eq. (3)-(4), the operators $\hat{\sigma}^z = (\hat{\sigma}^- + \hat{\sigma}^+), \hat{\sigma}^y = i(\hat{\sigma}^- - \hat{\sigma}^+)$ and $\hat{\sigma}^x$ are the usual Pauli matrices.

In the following, we consider and study the basic building block consisting of $N = 2$ discretized lattice sites. A convenient basis is the strong coupling one given by $\hat{m} \to \infty$ and $x \to 0$ (i.e. in the limit in which $\hat{H}_m$ and $\hat{H}_e$
FIG. 2. **Strong coupling states.** The energy eigenstates in the strong coupling limit form a convenient basis for $N = 2$ discretised lattice sites. For the basic building block of the lattice (see Fig. 1), these are given by the bare vacuum $|\text{vac}\rangle$, meson $|m\rangle$, tetraquark $|T\rangle$, and baryon-antibaryon (baryonium) $|BB\rangle$. We resort to a two column representation for the states in the fermion occupation number basis, where the first and second column indicates the state of the antimatter and matter respectively.

dominate over the pair-creation term $\hat{H}_{\text{kin}}$). The gauge-invariant basis states in that limit can be constructed by successively applying the kinetic term to the vacuum state. The different basis states obtained are depicted in Fig. 2 in the fermion occupation picture, where the first and second column of the ket describe the antimatter and matter content of the state respectively. While all of them possess the same quantum baryon number $B = 0$, the number of particle-antiparticle pairs contained in them differ. These states are all eigenstates of the mass Hamiltonian in Eq. (4), which counts the number of particles and antiparticles in a state, with eigenvalues 0, 2, 4 and 6 for the vacuum, meson, tetraquark and baryon-antibaryon states respectively. The meson state consists of a color-singlet superposition of particle-antiparticle pairs. By contrast the tetraquark state is a color superposition of diquark-antidiquark pairs. Like a single antiquark, the diquark is a color antitriplet state.

At finite coupling $x$ and mass $\tilde{m}$, the eigenstates of the Hamiltonian are given by a superposition of the strong coupling basis states. By studying time evolution under the Hamiltonian in Eq. (2), we can probe the transitions between the different eigenstates. In particular, by choosing the initial state as the strong coupling limit baryon-antibaryon state (containing six particles and antiparticles in total) and in the regime where $x/\tilde{m} \leq 1$, we can probe a single transition between the baryon-antibaryon and the tetraquark state. When the parameters are chosen outside of this regime, more than one transition becomes involved in the time evolution, which makes the dynamics richer and more complex.

The time evolution $e^{-i\hat{H}t}$ is obtained from a Trotter decomposition [20] that we optimize for minimal gate depth. Fig. 4 in the Methods shows the Trotter circuit for a basic building block ($N = 2$). While this minimal lattice is described by six spins (compare Fig. 4), we can simulate the $B = 0$ sector using only three qubits, due to a particle-antiparticle symmetry in this case (see Methods). We are interested in tracking the particle number

$$
\langle n(t) \rangle = \langle \Psi_0 | e^{i H (3) t} \hat{H}_m (3) e^{-i H (3) t} | \Psi_0 \rangle ,
$$
as we evolve the system in time starting from an initial state $|\Psi_0\rangle$ with $\hat{H}(3)$ the three qubits Hamiltonian derived in Methods. We focus here on $|\Psi_0\rangle = |\Psi_{BB}\rangle$, the baryon-antibaryon state in the strong coupling limit (see Fig. 2). In terms of spins, the strong coupling baryon-antibaryon state is given by $|\Psi_0\rangle = |↓↓↓⟩|↑↑↑⟩$ where the first ket refers to antiquarks and the second to quarks (note that only the first ket is implemented in the quantum simulation and the second is implied).

**Error mitigation.** We are using self-mitigation as introduced in Ref. [17]. The basic idea is to use our quantum circuit in two ways. A “physics run” applies the desired number of Trotter steps, $N_T$, forward in time to reach the final time of interest. A “mitigation run” applies $N_T/2$ steps forward in time followed by $N_T/2$ steps backward in time, which results in a noisy experimental determination of the known initial state. Randomized compiling is used to surround the CNOT gates with Pauli gates that turn coherent errors into incoherent errors. We find that the number of physics and mitigation runs can be as low as 40 each, and up to 560 depending on the quantum computing device chosen. Throughout this work we always collect 2048 shots from a single circuit execution. As described in Ref. [17], the information gleaned from the mitigation runs provides an excellent error mitigation for the physics runs. As in [12], each separate calculation is further accompanied by a set of $2^3$ calibration circuits to estimate the transfer map mixing the true outcome probabilities into the observed ones.

**Experiment.** We perform two Trotter time evolution experiments on a universal superconducting quantum computer [21] using up to eight Trotter time steps. In both experiments, the system is initialized in the strong coupling baryon-antibaryon state $|BB\rangle$ and evolved in time under the gauge-invariant three-qubit Hamiltonian in the Methods. Since the Hamiltonian preserves the baryon number of the initial state, the observed evolution remains within the $B = 0$ sector.

The first experiment is carried out for the Hamiltonian parameters $x = 0.8$ and $\tilde{m} = 1.2$ [see Eq. (3)]. This quark mass is large enough to organize the hadron spectrum in an intuitive way: the vacuum, meson, tetraquark, and baryon-antibaryon states have energies near 0, $2\tilde{m}$, $4\tilde{m}$ and $6\tilde{m}$ respectively. For reference, Fig. 3(a) shows the spectrum and the composition of physical hadron states.
The energy spectra on the left are shown to scale and reflect the proportion to which the strong coupling states in Fig. 2 contribute to the individual energy eigenstates. The data obtained on three different IBM quantum computers [panel b ibm_peekskill, panel e ibm_geneva for \(N_T = 4\) and ibm_lima for \(N_T = 8\)] is shown in the middle column, where the different markers denote different Trotter steps and hence different circuit lengths. The circuit is heavily optimized (see Methods) and contains \(N_T \cdot 10\) CNOT gates and a total circuit depth of \(N_T \cdot 25 + 1\) after transpilation to the employed native gates. For the data points shown in the figure, the error mitigation has already been applied (see Supplemental Information). The dashed lines mark the exact Trotter evolution obtained via a numerical exponentiation. We further composed an expected graph for the evolution obtained from the Trotter protocol, which is plotted in a solid black line. The error bars shown here originate via bootstrapping of the error mitigation method \[25\]. Error bars corresponding to the quantum projection noise are small due to the 2048 performed shots and would be hidden by the size of the markers and hence not shown. To obtain the energy differences indicated by the arrows in the left column (panel a and d), we resort to Bayesian inference. The results are shown in panels c and f respectively, where the solid lines denote the mean of 5000 samples drawn from the posterior predictive distribution (see Methods). From these samples we also compute the highest density interval (HDI) equivalent, i.e. the grey area marks the interval between the 2.5 and 97.5 percentiles. The point estimate for the energy gap between \(|BB⟩\) and \(|T⟩\) [panel a] is given by \(ω_0 = (2π) \cdot 0.181/[\tilde{mt}]\) with an HDI of \((2π) \cdot [0.153, 0.204]/[\tilde{mt}]\). For the second case in panel d, we find that \(ω_1 = (2π) \cdot 0.482/[\tilde{mt}]\) with HDI = \((2π) \cdot [0.409, 0.545]/[\tilde{mt}]\) and \(ω_2 = (2π) \cdot 0.427/[\tilde{mt}]\) where HDI = \((2π) \cdot [0.297, 0.502]/[\tilde{mt}]\). Values for other parameters in our probabilistic model can be found in the Supplemental Information.

in terms of strong coupling states. Since we choose the strong-coupling baryon-antibaryon as initial state, quark-antiquark annihilation provides a direct connection to the tetraquark state, and indeed the collected data allows to observe this oscillation to dominate the time evolution as shown in Fig. 3(b). The experiment has been performed on the ibm_peekskill device, where for each data point we run 280 (140, 140) repetitions in the case of \(N_T = 4\) (\(N_T = 2\), \(N_T = 6\)) for physics and mitigation runs respectively, where \(N_T\) is the number of Trotter steps. We perform a Bayesian analysis \[28\] to extract the frequency of the oscillation and thus calculate the mass gap between the baryon-antibaryon state and the tetraquark state (see Methods for more details). We identify one frequency, where the point estimate is given by \(ω_0 = (2π) \cdot 0.181/[\tilde{mt}]\) with the highest density interval (HDI) of \((2π) \cdot [0.153, 0.204]/[\tilde{mt}]\), which corresponds to the energy gap between \(|BB⟩\) and \(|T⟩\). The HDI, is the interval where we find 95% of the values during the sampling procedure. In Fig. 3(c) we plot 5000 samples from the posterior predictive distribution which within the 2.5 and 97.5 percent quantile agrees well with the collected data that we have observed.
data.

The second experiment is carried out on ibm_geneva (N_T = 4, 560 repetitions) and ibmq_lima (N_T=8, 40 repetitions), employing a smaller quark mass \( m = 0.45 \) but the same coupling constant \( x = 0.8 \). In this case, the hadron spectrum is not ordered simply by counting the number of quarks in each state. Instead, our quantum calculation of the real-time dynamics is able to reveal two dominant energies that produce a beat frequency between them which is easily visible in Fig. 3. Applying the same Bayesian inference techniques, we extract two frequency components \( \omega_1 = (2\pi) \cdot 0.482/[\tilde{m}t] \) with HDI \( (2\pi)\cdot[0.409, 0.545]/[\tilde{m}t] \) and \( \omega_2 = (2\pi) \cdot 0.427/[\tilde{m}t] \) where HDI \( (2\pi) \cdot [0.297, 0.502]/[\tilde{m}t] \), which confirms the underlying physics. In particular, the strong-coupling \(|BB\rangle \) initial state is once again mixing with the strong-coupling tetraquark, but this tetraquark is now a significant percentage of two physical eigenstates, as shown in Fig. 3.

**DISCUSSION**

Recent observations of tetraquarks and other hadrons beyond the traditional mesons and baryons have sparked a great deal of theoretical activity \[1\], with lattice gauge theory playing a central role in the determination of static properties. To access time-dependent dynamics, we turn to the Hamiltonian approach on quantum computers.

Building on previous proposals \[29–40\] and demonstrations in simpler gauge theories \[12, 14–18\], we have constructed a one-dimensional lattice gauge theory for QCD itself, with one flavor of dynamical matter coupled to SU(3) gauge fields. Real-time oscillations of the tetraquark with other hadron states are observed by running on IBM Quantum hardware \[27\]. Specifically, we begin from a strong-coupling baryon-antibaryon state at time \( t = 0 \) and see how the tetraquark emerges. The success of these simulations required the use of recent advances in error mitigation \[17\].

Our approach is based on an elimination of the gauge degrees of freedom, with that physics being re-expressed as nonlocal interactions among matter fields. Future work will extend this methodology to two (and ultimately three) spatial dimensions using the methods developed in \[11\].

Another important route for generalisations is the extension of wider classes of simulated time evolutions to extract truly dynamical quantities. Interesting applications include time correlation functions and the ongoing quest to simulate particle collisions with quantum computers. Our simulation of SU(3) hadrons on a quantum computer accomplishes a key step on the path toward accessing increasingly relevant quantum computations for QCD.

**ACKNOWLEDGEMENTS**

We are immensely grateful to Thomas Fernholz and Christian Sommer for sharing their important scientific insights and invaluable input to our project. We thank John Watrous for his support, and we are grateful for the IBM Quantum Researchers Program Access Award enabling the use of IBM Quantum services for this work. The views expressed are those of the authors, and do not reflect the official policy or position of IBM or the IBM Quantum team. This work has been supported by Transformative Quantum Technologies Program (CFREF), NSERC and the New Frontiers in Research Fund. CM acknowledges the Alfred P. Sloan foundation for a Sloan Research Fellowship.

\* These authors contributed equally.
\[\dagger\] jan.frhaase@gmail.com These authors contributed equally.
\[\ddagger\] jingleizl@gmail.com

\[1\] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, arXiv:2204.02649
\[2\] M. C. Bañuls, R. Blatt, J. Catani, A. Celi, J. I. Cirac, M. Dalmonte, L. Fallani, K. Jansen, M. Lewenstein, S. Montangero, C. A. Muschik, B. Reznik, E. Rico, L. Tagliacozzo, K. Van Acoleyen, F. Verstraete, U.-J. Wiese, M. Wingate, J. Zakrzewski, and P. Zoller, Eur. Phys. J. D **74**, 165 (2020)
\[3\] C. W. Bauer, Z. Davoudi, A. B. Balantekin, T. Bhattacharya, M. Carena, W. A. de Jong, P. Draper, A. El-Khadra, N. Gemelke, M. Hanada, D. Kharzeev, H. Lamm, Y.-Y. Li, J. Liu, M. Lukin, Y. Meurice, C. Monroe, B. Nachman, G. Pagano, J. Preskill, E. Rinaldi, A. Roggero, D. I. Santiago, M. J. Savage, I. Siddiqi, G. Siopsis, D. Van Zanten, N. Wiebe, Y. Yamauchi, K. Yeter-Aydeniz, and S. Zorzetti, arXiv:2204.03381
\[4\] E. A. Martinez, C. A. Muschik, P. Schindler, D. Nigg, A. Erhard, M. Heyl, P. Hauke, M. Dalmonte, T. Monz, P. Zoller, and R. Blatt, Nature **534**, 516 (2016)
\[5\] N. Klco, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage, Phys. Rev. A **98**, 032331 (2018)
\[6\] C. Kokail, C. Maier, R. van Bijnen, T. Brydges, M. K. Joshi, P. Jurcevic, C. A. Muschik, P. Silvi, R. Blatt, C. F. Roos, and P. Zoller, Nature **569**, 355 (2019)
\[7\] H.-H. Lu, N. Klco, J. M. Lukens, T. D. Morris, A. Bansal, A. Ekström, G. Hagen, T. Papenbrock, A. M. Weiner, M. J. Savage, and P. Lougovski, Phys. Rev. A **100**, 012320 (2019)
\[8\] A. Mil, T. V. Zache, A. Hegde, A. Xia, R. P. Bhatt, M. K. Oberthaler, P. Hauke, J. Berges, and F. Jendrzejewski, Science **367**, 1128 (2020)
\[9\] F. M. Surace, P. P. Mazza, G. Giudici, A. Lerose, A. Gambassi, and M. Dalmonte, Phys. Rev. X **10**, 021041 (2020)
\[10\] B. Yang, H. Sun, R. Ott, H.-Y. Wang, T. V. Zache, J. C. Halimeh, Z.-S. Yuan, P. Hauke, and J.-W. Pan, Nature **587**, 392 (2020)
In a second step, we triple the size of the lattice to define the single

- In order to simulate time dynamics in the baryon sector $B = 0$ on a quantum computer, we first transform the fermionic Hamiltonian in Eq. (1) into one involving only quibits degrees of freedom. The transformation is achieved in two steps. We first eliminate the gauge fields by following the methods developed in [12], the dimensionless Hamiltonian reads

$$
\hat{H} = \frac{1}{2} \sum_{n=1}^{N-1} 3 \left( \hat{\phi}_{n+1}^{i} \hat{\phi}_{n+1}^{\dagger} + \text{H.c.} \right) + \hat{m} \sum_{n=1}^{N} \left[ \left( -1 \right)^{n} \hat{\phi}_{n}^{i} \hat{\phi}_{n}^{\dagger} + \frac{1}{2z} \sum_{n=1}^{N-1} \left( \sum_{m \leq n} \hat{Q}_{m} \right)^{2} \right],
$$

where $\hat{m} = am$ and $x = 1/g^{2}a^{2}$. The last term represents the color electric energy of the system and is expressed in terms of the non-Abelian charges at site $n$

$$
\hat{Q}_{n}^{a} = \sum_{i,j=1}^{3} \hat{\phi}_{n}^{i} (T^{a})_{ij} \hat{\phi}_{n}^{\dagger}.
$$

In a second step, we triple the size of the lattice to define $3N$ new sites and distribute the color components of the fermionic field among them by defining the single

**METHODS**

**Gauge elimination and qubit formulation.** Due to gauge invariance, the Hamiltonian in Eq. (1) commutes with the Gauss' law operators (which generate the local gauge transformations) $\hat{G}^{a}_{n} = \hat{L}^{a}_{n} - \hat{R}^{a}_{n-1} - \hat{Q}^{a}_{n}$, where $\hat{L}^{a}_{n}$ and $\hat{R}^{a}_{n-1}$ are the $a$-component (with $a = 1, \ldots, 8$) of the left and right electric field living on the link $n$ respectively. For a non-Abelian gauge group, the right and left color electric field are related via the adjoint representation $\hat{R}^{a}_{n} = (\hat{U}^{adj}_{n})_{ab} \hat{L}^{b}_{n}$ with $(\hat{U}^{adj}_{n})_{ab} = 2 \text{Tr} \left[ \hat{U} \hat{T}^{a} \hat{U}^{\dagger} \hat{T}^{b} \right]$, where $T^{a} = \lambda^{a}/2$, and $\lambda^{a}$ ($a = 1, \ldots, 8$) are generators of the SU(3) Lie algebra and are given by the Gell-Mann matrices [21]. The Hamiltonian also commutes with the redness operator $\hat{R} = \sum_{n=1}^{N} \phi_{n}^{\dagger} \phi_{n} - N/2$, the greenness $\hat{G} = \sum_{n=1}^{N} \phi_{n}^{\dagger} \phi_{n}^{2} - N/2$ and bluesness operator $\hat{B} = \sum_{n=1}^{N} \phi_{n}^{\dagger} \phi_{n}^{3} - N/2$ which measures the matter-antimatter imbalance of a specific color. It is however more convenient to combine these three operators into a single one which measure the matter-antimatter imbalance irrespective of the color. We therefore define the baryon number operator as

$$
\hat{B} = \frac{1}{3} \left( \hat{R} + \hat{G} + \hat{B} \right).
$$

In order to simulate time dynamics in the baryon sector $B = 0$ on a quantum computer, we first transform the fermionic Hamiltonian in Eq. (1) into one involving only quibits degrees of freedom. The transformation is achieved in two steps. We first eliminate the gauge fields by following the methods developed in [12], the dimensionless Hamiltonian reads

$$
\hat{H} = \frac{1}{2} \sum_{n=1}^{N-1} 3 \left( \phi_{n}^{i} \phi_{n+1}^{\dagger} + \text{H.c.} \right) + \hat{m} \sum_{n=1}^{N} \left[ \left( -1 \right)^{n} \phi_{n}^{i} \phi_{n}^{\dagger} + \frac{1}{2z} \sum_{n=1}^{N-1} \left( \sum_{m \leq n} \hat{Q}_{m} \right)^{2} \right],
$$

where $\hat{m} = am$ and $x = 1/g^{2}a^{2}$. The last term represents the color electric energy of the system and is expressed in terms of the non-Abelian charges at site $n$

$$
\hat{Q}_{n}^{a} = \sum_{i,j=1}^{3} \hat{\phi}_{n}^{i} (T^{a})_{ij} \hat{\phi}_{n}^{\dagger}.
$$

In a second step, we triple the size of the lattice to define $3N$ new sites and distribute the color components of the fermionic field among them by defining the single

- In order to simulate time dynamics in the baryon sector $B = 0$ on a quantum computer, we first transform the fermionic Hamiltonian in Eq. (1) into one involving only quibits degrees of freedom. The transformation is achieved in two steps. We first eliminate the gauge fields by following the methods developed in [12], the dimensionless Hamiltonian reads

$$
\hat{H} = \frac{1}{2} \sum_{n=1}^{N-1} 3 \left( \phi_{n}^{i} \phi_{n+1}^{\dagger} + \text{H.c.} \right) + \hat{m} \sum_{n=1}^{N} \left[ \left( -1 \right)^{n} \phi_{n}^{i} \phi_{n}^{\dagger} + \frac{1}{2z} \sum_{n=1}^{N-1} \left( \sum_{m \leq n} \hat{Q}_{m} \right)^{2} \right],
$$

where $\hat{m} = am$ and $x = 1/g^{2}a^{2}$. The last term represents the color electric energy of the system and is expressed in terms of the non-Abelian charges at site $n$

$$
\hat{Q}_{n}^{a} = \sum_{i,j=1}^{3} \hat{\phi}_{n}^{i} (T^{a})_{ij} \hat{\phi}_{n}^{\dagger}.
$$

In a second step, we triple the size of the lattice to define $3N$ new sites and distribute the color components of the fermionic field among them by defining the single

**METHODS**

**Gauge elimination and qubit formulation.** Due to gauge invariance, the Hamiltonian in Eq. (1) commutes with the Gauss’ law operators (which generate the local gauge transformations) $\hat{G}^{a}_{n} = \hat{L}^{a}_{n} - \hat{R}^{a}_{n-1} - \hat{Q}^{a}_{n}$, where $\hat{L}^{a}_{n}$ and $\hat{R}^{a}_{n-1}$ are the $a$-component (with $a = 1, \ldots, 8$) of the left and right electric field living on the link $n$ respectively. For a non-Abelian gauge group, the right and left color electric field are related via the adjoint representation $\hat{R}^{a}_{n} = (\hat{U}^{adj}_{n})_{ab} \hat{L}^{b}_{n}$ with $(\hat{U}^{adj}_{n})_{ab} = 2 \text{Tr} \left[ \hat{U} \hat{T}^{a} \hat{U}^{\dagger} \hat{T}^{b} \right]$, where $T^{a} = \lambda^{a}/2$, and $\lambda^{a}$ ($a = 1, \ldots, 8$) are generators of the SU(3) Lie algebra and are given by the Gell-Mann matrices [21]. The Hamiltonian also commutes with the redness operator $\hat{R} = \sum_{n=1}^{N} \phi_{n}^{\dagger} \phi_{n} - N/2$, the greenness $\hat{G} = \sum_{n=1}^{N} \phi_{n}^{\dagger} \phi_{n}^{2} - N/2$ and bluesness operator $\hat{B} = \sum_{n=1}^{N} \phi_{n}^{\dagger} \phi_{n}^{3} - N/2$ which measures the matter-antimatter imbalance of a specific color. It is however more convenient to combine these three operators into a single one which measure the matter-antimatter imbalance irrespective of the color. We therefore define the baryon number operator as

$$
\hat{B} = \frac{1}{3} \left( \hat{R} + \hat{G} + \hat{B} \right).
$$

In order to simulate time dynamics in the baryon sector $B = 0$ on a quantum computer, we first transform the fermionic Hamiltonian in Eq. (1) into one involving only quibits degrees of freedom. The transformation is achieved in two steps. We first eliminate the gauge fields by following the methods developed in [12], the dimensionless Hamiltonian reads

$$
\hat{H} = \frac{1}{2} \sum_{n=1}^{N-1} 3 \left( \phi_{n}^{i} \phi_{n+1}^{\dagger} + \text{H.c.} \right) + \hat{m} \sum_{n=1}^{N} \left[ \left( -1 \right)^{n} \phi_{n}^{i} \phi_{n}^{\dagger} + \frac{1}{2z} \sum_{n=1}^{N-1} \left( \sum_{m \leq n} \hat{Q}_{m} \right)^{2} \right],
$$

where $\hat{m} = am$ and $x = 1/g^{2}a^{2}$. The last term represents the color electric energy of the system and is expressed in terms of the non-Abelian charges at site $n$

$$
\hat{Q}_{n}^{a} = \sum_{i,j=1}^{3} \hat{\phi}_{n}^{i} (T^{a})_{ij} \hat{\phi}_{n}^{\dagger}.
$$

In a second step, we triple the size of the lattice to define $3N$ new sites and distribute the color components of the fermionic field among them by defining the single
component fields $\hat{\psi}_n^i = \hat{\psi}_{3n-3+i}$ with $n = 1, 2, \ldots, N$ and $i = 1, 2, 3$ (see Fig. 1).

We then perform a generalised Jordan-Wigner transformation on the single component fermionic field $\hat{\psi}_n$ [24]

$$\hat{\psi}_n = \left( \prod_{i < n} s_i \sigma_i^x \right) \hat{\sigma}_n^- \quad \hat{\psi}^\dagger_n = \hat{\sigma}_n^+ \left( \prod_{i < n} s_i \sigma_i^x \right)$$

(10)

where $s_i$ are phase factors that we choose equal to $+1$ on antimatter cells and $-1$ on matter cells. This choice is convenient because it matches the standard colour notation, such as $(\bar{r}r + \bar{g}g + \bar{b}b)/\sqrt{3}$. After the Jordan-Wigner transformation, the kinetic Hamiltonian in terms of qubits is given by Eq. (3) while the mass Hamiltonian is given by Eq. (4). In the qubit formulation, the non-Abelian charges defined in Eq. (9) are given by

$$\hat{Q}_n^1 = \frac{(-1)^n}{2} (\sigma_{3n-2}^+ \sigma_{3n-1}^- + \text{H.c.}),$$

(11)

$$\hat{Q}_n^2 = \frac{i(-1)^n}{2} (\sigma_{3n-1}^+ \sigma_{3n-2}^- - \text{H.c.}),$$

(12)

$$\hat{Q}_n^3 = \frac{1}{4} (\sigma_{3n-2}^- \sigma_{3n-1}^- - \sigma_{3n-2}^+ \sigma_{3n-1}^+),$$

(13)

$$\hat{Q}_n^4 = -\frac{1}{2} (\sigma_{3n-2}^- \sigma_{3n-1}^- + \sigma_{3n-2}^+ \sigma_{3n-1}^+ + \text{H.c.}),$$

(14)

$$\hat{Q}_n^5 = \frac{i}{2} (\sigma_{3n-2}^+ \sigma_{3n-1}^- - \text{H.c.}),$$

(15)

$$\hat{Q}_n^6 = \frac{(-1)^n}{2} (\sigma_{3n-1}^- \sigma_{3n}^- + \text{H.c.}),$$

(16)

$$\hat{Q}_n^7 = \frac{i(-1)^n}{2} (\sigma_{3n-1}^+ \sigma_{3n}^- - \text{H.c.}),$$

(17)

$$\hat{Q}_n^8 = \frac{1}{4\sqrt{3}} (\sigma_{3n-2}^- + \sigma_{3n-1}^- - 2\sigma_{3n}^-).$$

(18)

The electric field Hamiltonian can be obtained by injecting the expressions of the non-Abelian charges (11)-(18) in the color electric term in Eq. (8). We obtain

$$\hat{H}_e = \frac{1}{3} \sum_{n=1}^{N-1} (3 - \hat{\sigma}_{3n-2}^- \hat{\sigma}_{3n-1}^- - \hat{\sigma}_{3n-2}^+ \hat{\sigma}_{3n}^- - \hat{\sigma}_{3n-1}^+ \hat{\sigma}_{3n}^+)$$

\[+ \sum_{n=1}^{N-2} \sum_{m=n+1}^{N-1} \left[ (N - m) (\sigma_{3n-2}^+ \sigma_{3n-1}^+ \sigma_{3m-1}^+ \sigma_{3m-2}^- + \sigma_{3n-1}^- \sigma_{3n}^- \sigma_{3m-1}^- \sigma_{3m-2}^+ + \sigma_{3n-2}^- \sigma_{3n}^+ \sigma_{3m-2}^- \sigma_{3m-1}^+ + \text{H.c.})\right] \]

\[- \frac{1}{12} (N - m) \sigma_{3n-2}^- \sigma_{3n-2}^- + \hat{\sigma}_{3n}^- + \hat{\sigma}_{3n-2}^- - 2 \hat{\sigma}_{3n}^-)$$

\[- \frac{1}{12} (N - m) \sigma_{3n}^- \sigma_{3n-1}^- + \hat{\sigma}_{3n}^- + \hat{\sigma}_{3n-2}^- - 2 \hat{\sigma}_{3n}^-)$$

\[- \frac{1}{12} (N - m) \sigma_{3n}^- (\hat{\sigma}_{3n}^- - \hat{\sigma}_{3n-2}^- - 2 \hat{\sigma}_{3n}^-),$$

(19)

which exhibits long range spin-spin interaction as a direct consequence of the gauge elimination. The baryon number operator is proportional to the total magnetization of the system in the qubit encoding

$$\hat{B} = \frac{1}{6} \sum_{n=1}^{3N} \hat{\sigma}_n^z.$$

(20)

We are interested in a basic building block consisting of $N = 2$ discretized lattice sites. The model is then described by a chain with 6 qubits and the terms in the Hamiltonian read

$$\hat{H}_{\text{kin}} = -\frac{1}{2} (\hat{\sigma}_1^+ \hat{\sigma}_2^+ \hat{\sigma}_3^+ \hat{\sigma}_4^- - \hat{\sigma}_2^+ \hat{\sigma}_3^+ \hat{\sigma}_4^- \hat{\sigma}_5^+ + \hat{\sigma}_3^+ \hat{\sigma}_4^+ \hat{\sigma}_5^+ \hat{\sigma}_6^+ + \text{H.c.}),$$

(21)

$$\hat{H}_m = \frac{1}{2} (6 - \hat{\sigma}_1^- \hat{\sigma}_2^- - \hat{\sigma}_3^- + \hat{\sigma}_4^- + \hat{\sigma}_5^- + \hat{\sigma}_6^-),$$

(22)

$$\hat{H}_e = \frac{1}{3} (3 - \hat{\sigma}_1^- \hat{\sigma}_2^- - \hat{\sigma}_1^- \hat{\sigma}_3^- - \hat{\sigma}_2^- \hat{\sigma}_3^-).$$

(23)

In the sector with baryon number $B = 0$ (i.e. as much matter as antimatter), the three terms composing the Hamiltonian commute with the following operator

$$\hat{C} \hat{P} = \prod_{n=1}^{3} \hat{\sigma}_n^+ \hat{\sigma}_n^- \hat{W}_{n+n'},$$

(24)

where $\hat{W}_{n+n'}$ is the SWAP unitary operator between qubit $n$ and $n'$. This symmetry corresponds to the composition of a spatial reflection with respect to the middle of the chain ($\hat{P}$) followed by a charge conjugation operation ($\hat{C}$) which flips the spins. Local spin operators $\hat{\sigma}_0^x$ transforms as $(\hat{C} \hat{P})^\dagger \hat{\sigma}_0^x \hat{C} \hat{P} = (\hat{\sigma}^x \hat{\sigma}^z \hat{\sigma}^x)_{7-n}$ under the $\hat{C} \hat{P}$ operation with $a = x, y, z$ and $n = 1, 2, \ldots, 6$. It is thus clear that a convenient basis is the one spanned by states of the form $|\Psi\rangle = \sum c_{ijk}|i\rangle_1|j\rangle_2|k\rangle_3 \otimes \hat{\sigma}_4^z \hat{\sigma}_5^z \hat{\sigma}_6^z |i\rangle_4 |j\rangle_5 |k\rangle_6$, which are invariant under the $\hat{C} \hat{P}$ operation and have $B = 0$. When one works with this basis, the state of the last three qubits is completely determined by the state of the first three. As a direct consequence, we can encode the states by using only the first 3 qubits (i.e. the state of the antimatter) rather than 6. The reduced three-qubit Hamiltonian reads

$$\hat{H}_{\text{kin}}^{(3)} = \frac{1}{2} (\hat{\sigma}_1^- \hat{\sigma}_2^- \hat{\sigma}_3^- + \hat{\sigma}_1^+ \hat{\sigma}_2^- \hat{\sigma}_3^+ + \hat{\sigma}_1^- \hat{\sigma}_2^+ \hat{\sigma}_3^-),$$

(25)

$$\hat{H}_m^{(3)} = 3 - \hat{\sigma}_1^- - \hat{\sigma}_2^- - \hat{\sigma}_3^-,$$

(26)

$$\hat{H}_e^{(3)} = \frac{1}{3} (3 - \hat{\sigma}_1^- \hat{\sigma}_2^- - \hat{\sigma}_1^+ \hat{\sigma}_3^- - \hat{\sigma}_2^+ \hat{\sigma}_3^-),$$

(27)

and the time evolution is obtained using the Hamiltonian

$$\hat{H}^{(3)} = \hat{H}_{\text{kin}}^{(3)} + \hat{m} \hat{H}_m^{(3)} + \frac{1}{2x} \hat{H}_e^{(3)}$$

(28)

Trotter evolution Although the Hamiltonian of Eqs. (25),(27) is expressed in terms of Pauli X and Z gates, a simple rotation to Y and Z gates allows for more cancellations among CNOT gates. This is especially valuable on hardware that does not provide all-to-all connectivity.
among the qubits. The first half of our first-order Trotter step is displayed in Fig. 4 and, to match the available hardware, it does not use any entangling gates directly between \(q_1\) and \(q_3\). The second half of the Trotter step is the same except for a relabeling of \(q_1 \leftrightarrow q_3\).

\[
q_1 \xrightarrow{RZ(-2\tilde{m}t)} q_3 \xrightarrow{RZ(-2\tilde{m}t)} q_1
\]

\[
q_2 \xrightarrow{RY(-t)} q_3 \xrightarrow{RY(-t)} q_1 \xrightarrow{RZ(-t)} q_2
\]

\[
q_3 \xrightarrow{RZ(-2\tilde{m}t)} q_1 \xrightarrow{RZ(-2\tilde{m}t)} q_3 \xrightarrow{RZ(-2\tilde{m}t)} q_1
\]

**Bayesian inference analysis.** We model the obtained data \(D\) as draws from a normal distribution with variance \(\sigma\), where the mean \(S_K\) is given by a cosine series

\[
S_K = \sum_{i=1}^{K} A_i \cos(\omega_i t + \phi_i) + \xi, \tag{29}
\]

with uniform priors on the frequencies \(\omega_i\) and the amplitudes \(A_i\), while the priors for the phases \(\phi_i\) and the offset \(\xi\) are normal distributions located at zero. The uniform priors are adjusted to include the frequencies that are visible from the data by eye. The prior on \(\sigma\) is given by the maximum of 0.3 and the value of the largest errorbar in the dataset. Note that each dataset stemming from different \(N_T\) is modeled as a separate likelihood while the parameters are shared. After obtaining a representation of the posterior distribution for the parameters \(\theta = \{A_i, \omega_i, \phi_i, \xi, \sigma\}\), \(P(\theta|D)\), via Monte Carlo sampling from Bayes’ rule, we sample the posterior predictive distribution \(P(D'|D) = \int_{\Theta} P(\Theta = \theta|D)P(D'|\Theta = \theta)\), where \(\Theta\) denotes the collective random variables for all parameters.