Nucleon structure at large $x$: nuclear effects in deuterium

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Abstract. I review quark momentum distributions in the nucleon at large momentum fractions $x$. Particular attention is paid to the impact of nuclear effects in deuterium on the $d/u$ quark distribution ratio as $x \to 1$. A new global study of parton distributions, using less restrictive kinematic cuts in $Q^2$ and $W^2$, finds strong suppression of the $d$ quark distribution once nuclear corrections are accounted for.

Keywords: neutron structure function; nuclear effects; deuteron

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Introduction

The momentum space distributions of quarks and gluons (partons) in the nucleon provide fundamental characterizations of the nucleon’s bound state nature. Considerable progress has been made in mapping out the parton distribution functions (PDFs) of sea quarks and gluons in recent years from deep inelastic scattering (DIS) and other high energy processes, particularly at small values of the parton momentum fraction $x$. In this region, however, the large fluctuation length of a virtual photon into $q\bar{q}$ pairs means that it is not always clear whether one is probing the structure of the target or the structure of the probe itself.

At large values of $x$, where sea quarks and gluons play a negligible role, the momentum distributions of valence quarks can be more directly related to the nonperturbative structure of the nucleon. The ratio of $d$ to $u$ quark distributions, for example, is very sensitive to the mechanisms of spin-flavor symmetry breaking in the nucleon [1]. The large-$x$ region is also unique in allowing perturbative QCD predictions to be realized for the $x$ dependence of PDFs in the limit $x \to 1$ [2]. Knowledge of PDFs at large $x$ is also important for searches of new physics signals in collider experiments, where uncertainties in PDFs at large $x$ and low $Q^2$ percolate through $Q^2$ evolution to affect cross sections at smaller $x$ and larger $Q^2$ [3], as well as in neutrino oscillation experiments.

From high energy measurements involving proton targets one has obtained a rather precise determination of the $u$ quark distribution, which dominates the proton’s valence structure due to its larger charge weighting compared with the $d$. Constraining the $d$ distribution, on the other hand, requires in addition data on neutron structure functions. However, because of the absence of free neutron targets, neutron structure is usually extracted from a combination of deuteron and proton data, which necessitates understanding of the nuclear corrections in deuterium. As a result, knowledge of PDFs at large $x$, and especially the $d$ quark distribution, has been severely limited beyond $x \sim 0.6$ [4, 5].
In this talk, I will briefly review the status of nucleon structure at large $x$, focusing in particular on nuclear effects in deuterium and finite-$Q^2$ corrections, and present results from a new global analysis [4] which attempts to place stronger constraints on large-$x$ PDFs. My personal interest in large-$x$ physics began around 16 years ago with a 1994 paper [6] with Tony Thomas and Andreas Schreiber on DIS from off-shell nucleons. It has been a pleasure to collaborate with Tony on this and many other problems over the years. I am also delighted to have Andreas, who has since moved on to bigger and better things, present at this workshop.

**Nuclear effects in deuterium**

Because the deuteron is a very weakly bound nucleus, most analyses have assumed that it can be treated as a sum of a free proton and neutron. On the other hand, it has long been known from experiments on nuclei that a nontrivial $x$ dependence exists for ratios of nuclear to deuteron structure functions. These effects include nuclear shadowing at small values of $x$ [7], anti-shadowing at intermediate $x$ values, $x \sim 0.1$, a reduction in the structure function ratio below unity for $0.3 \lesssim x \lesssim 0.7$, known as the European Muon Collaboration (EMC) effect, and a rapid rise as $x \to 1$ due to Fermi motion.

The conventional approach to describing nuclear structure functions in the intermediate- and large-$x$ regions is the nuclear impulse approximation, in which the virtual photon scatters incoherently from the individual bound nucleons in the nucleus [8]. Furthermore, since quarks at large momentum fractions $x$ are most likely to originate in nucleons carrying large momenta themselves, the effects of relativity will be ever more important as $x \to 1$. A relativistic description of the process therefore required the development of a formalism for DIS from bound, off-shell nucleons, which was pioneered in Ref. [6]. (Actually, the original motivation for that study was the quest for a consistent description of pion cloud corrections to nucleon PDFs, in particular the $\bar{d}/\bar{u}$ ratio, through the coupling of the photon to an off-shell nucleon dressed by a pion [9].)

The off-shell DIS analysis [6] identified the conditions under which usual convolution model [8] of nuclear structure functions holds, and found that in general these are not satisfied within a relativistic framework. In a follow-up study [10] (referred to as “MST”), it was found that one can however isolate a convolution component from the total deuteron structure function, together with calculable off-shell corrections. The general expression for the deuteron $F_2$ structure function can then be written as [10]

$$F_2^d(x, Q^2) = \sum_{N=p,n} \int dy \ f_{N/d}(y, \gamma) \ F_2^N \left( \frac{x}{y}, Q^2 \right) + \delta^{\text{off}} F_2^d(x, Q^2)$$  \hspace{1cm} (1)

where $F_2^N$ is the nucleon structure function, and $f_{N/d}$ gives the relativistic light-cone momentum distribution of nucleons in the deuteron (also referred to as the nucleon “smearing function”). The scaling variable $y = (M_d/M)(p \cdot q/p_d \cdot q)$ is the deuteron’s momentum fraction carried by the struck nucleon, where $q$ is the virtual photon momentum, and $p(p_d)$ and $M(M_d)$ are the nucleon (deuteron) four-momentum and mass.
In the Bjorken limit the distribution function \( f_{N/d} \) is a function of \( y \) only and is limited to \( y \leq M_d/M \). At finite \( Q^2 \), however, it depends in addition on the ratio \( \gamma = |\mathbf{q}|/q_0 = \sqrt{1 + 4x^2M^2/Q^2} \) [11], which can have significant consequences when fitting large-\( x \) deuteron data [12]. Furthermore, at finite \( Q^2 \) the lower limit of the \( y \) integration is given by \( y_{\text{min}} = x(1 - 2M\varepsilon_d/Q^2) \), where \( \varepsilon_d \) is the deuteron binding energy, while the upper limit is in principle unbounded [13].

The relativistic nucleon momentum distribution derived by MST [10] (written here for simplicity in the \( \gamma \rightarrow 1 \) limit) is given by

\[
 f_{N/d}(y) = \frac{M_d}{32\pi^2} y \int \frac{dp^2}{(M_d/E_p - 1)} |\Psi_d(p)|^2 \theta(p_0), \tag{2}
\]

where \( E_p = \sqrt{M^2 + p^2} \) and \( p_0 = M_d - E_p \) are the recoil and struck nucleon energies, respectively, and \( p^2 = p_0^2 - p^2 \) the struck nucleon’s virtuality. The deuteron wave function \( \Psi_d(p) \) contains the usual nonrelativistic \( S \) - and \( D \) -states, as well as the small \( P \) -state contributions in relativistic treatments, and is normalized according to \( \int d^3p |\Psi_d(p)|^2/(2\pi)^3 = 1 \).

Since the deuteron binding energy \( \varepsilon_d = -2.2 \text{ MeV} \) is \( \approx 0.1\% \) of its mass and the typical nucleon momentum in the deuteron is \( |p| \sim 130 \text{ MeV} \), the average nucleon virtuality \( p^2 \) will be \( \sim 4\% \) smaller than the free nucleon mass. For \( x \) not too close to 1 one can therefore expanded the deuteron scattering amplitude in powers of \( p/M \), using the so-called weak binding approximation (WBA) [11, 12, 14]. To order \( \mathcal{O}(p^2/M^2) \) one can then show explicitly that the relativistic smearing function in Eq. (2) reduces to the nonrelativistic WBA smearing function [11, 12],

\[
 f_{N/d}(y) \mathcal{O}(p^2/M^2) \int \frac{d^3p}{(2\pi)^3} \left(1 + \frac{p_z}{N}\right) |\Psi_D(p)| \delta \left(y - 1 - \frac{\varepsilon + p_z}{M}\right) \equiv f_{N/d}^{\text{WBA}}(y), \tag{3}
\]

where \( \varepsilon = M_d - M - E_p \approx \varepsilon_d - p^2/2M \). The resulting distribution function is sharply peaked around \( y \approx 1 \), with the width determined by the amount of binding (in the limit of zero binding it would be a \( \delta \)-function at \( y = 1 \)). At finite \( Q^2 \) (or \( \gamma \)) the function becomes somewhat broader, effectively giving rise to more smearing for larger \( x \) or lower \( Q^2 \).

Finally, the convolution-breaking, off-shell correction \( \delta^{\text{(off)}} F_2^d \) in Eq. (1) receives contributions from explicit \( p^2 \) dependence in the quark–nucleon correlation functions, and from the relativistic \( P \) -state components of the deuteron wave function. This correction was estimated within a simple quark–spectator model [10], with the parameters fitted to proton and deuteron \( F_2 \) data, and leads to a reduction in \( F_2^d \) of \( \approx 1 - 2\% \) compared to the on-shell approximation.

The overall effect on the ratio \( F_2^d/F_2^N \) is a \( \sim 2 - 3\% \) depletion relative to the free case at intermediate \( x (x \sim 0.5) \), with a steep rise at larger \( x (x \gtrsim 0.6 - 0.7) \) due to Fermi motion, as illustrated in Fig. 1 for \( Q^2 = 5 \text{ GeV}^2 \). Here the result for the WBA distribution (3), with relativistic kinematics, is shown with and without the off-shell correction from Ref. [10], and including finite-\( Q^2 \) target mass corrections (TMCs) [15]. In both cases the EMC effect is larger than that obtained within a light-cone approach [16], in which one assumes on-shell kinematics and no binding. The depletion at large \( x \) is smaller,
FIGURE 1. Ratio of $F^d_2/F^N_2$ structure functions for the WBA smearing function with relativistic kinematics with (dashed) and without (solid) TMCs at $Q^2 = 5$ GeV$^2$. For comparison the ratio in the light-cone (dotted) and nuclear density extrapolation (dot-dashed) models are shown.

however, than that predicted by the nuclear density extrapolation model [17], in which the $F^d_2/F^N_2$ ratio is taken to scale with nuclear density.

Consequently, using binding/off-shell models one will extract a larger neutron structure function from $F^d_2$ than with the on-shell light-cone model [1]. On the other hand, the extracted neutron will be smaller compared to that obtained assuming the density model, or no nuclear effects at all (see vertical arrows in Fig. 1). Note that while a few global PDF analyses have attempted to incorporate nuclear effects in the deuteron using smearing functions, most studies simply neglect nuclear corrections altogether. Although the extension of the density model to deuterium is problematic [18], it is included here for reference since it is also used sometimes to analyze deuterium data.

Finally, a word of caution against taking the ratios in Fig. 1 too literally. From Eq. (1) it is clear that the deuteron structure function depends on both the details of the nuclear physics embodied in $f_N/d^*$ and on the shape of the input nucleon structure functions. While the input proton structure function can be taken from experiment, the neutron $F^n_2$ is unknown at large $x$, and generally a harder $F^n_2$ input will lead to a larger EMC effect, pushing the rise of $F^d_2/F^N_2$ above unity to larger $x$. The practical solution is to perform an iteration procedure to eliminate the dependence on the input $F^n_2$, or implement the smearing directly in a global analysis, which is discussed next.

**New CTEQ6X distributions from large-$x$, low-$Q^2$ data**

Recently a global NLO analysis (referred to as “CTEQ6X”) was performed using an extended set of proton and deuteron data from DIS, from $pp$ and $pd$ Drell-Yan cross sections, $W^\pm$ asymmetry data, and jet cross sections (see Ref. [4] for details). The standard DIS cuts in previous global fits have excluded data with $Q^2 < 4$ GeV$^2$ and $W^2 < 12.25$ GeV$^2$, effectively rendering PDFs unconstrained above $x \approx 0.7$. In the CTEQ6X fit the kinematical coverage was extended to larger $x$ by relaxing the $Q^2$ and
FIGURE 2. CTEQ6X $u$ and $d$ distributions relative to the earlier CTEQ6.1 PDFs with no nuclear corrections [4]. The vertical lines indicate the upper limits of validity of the fits.

$W^2$ cuts to $Q^2 > 1.69$ GeV$^2$ and $W^2 > 3$ GeV$^2$, which approximately doubles the number of DIS data points.

In any analysis of data extending into the low-$Q^2$ region, it is imperative to account for kinematical target mass corrections associated with finite values of $M^2/Q^2$ [15], as well as dynamical $1/Q^2$-suppressed higher twist (HT) effects arising from long distance multi-parton correlations. For the CTEQ6X global analysis [4] different prescriptions for TMCs were considered, including the usual operator product expansion approach, as well as a more recent formalism based on collinear factorization. For the HT correction a phenomenological parametrization was applied, $F_2 = F_2^{LT+TMC}(1 + C/Q^2)$, with the coefficient $C$ determined empirically. The fit was found to be stable with respect to the reduction of the $Q^2$ and $W^2$ cuts, which is a rather nontrivial result given the expanded kinematical coverage. Remarkably, the leading twist PDFs turn out to be independent of the TMC prescription adopted, provided the phenomenological HT term is included. This reveals an important interplay between the TMC and HT corrections, which tend to compensate each other in the fitting procedure; in contrast, without TMCs the HT alone cannot accommodate the full $Q^2$ dependence of the data.

The inclusion of nuclear corrections in deuterium has profound effects for the $d$ quark distribution. Using the WBA finite-$Q^2$ smearing model, the $d$ distribution in the CTEQ6X fit was found to be suppressed by up to 40% for $x \approx 0.8$ relative to previous fits with no nuclear corrections, as Fig. 2 illustrates. The $u$ distribution, which is strongly constrained by proton data, is relatively unaffected by the nuclear corrections. This trend is already clear from a comparison of the $F_2^d/F_2^N$ ratios in Fig. 1, where the ratio in the nuclear smearing model is $\gg 1$ at $x \approx 0.8$, so that the corresponding neutron $F_2^d$ structure function (and hence the $d$ distribution) will be smaller. Not accounting for nuclear smearing in deuterium will therefore lead to a significant overestimate of the
distribution for $x \gtrsim 0.6$. This will be the case for a wide range of nuclear smearing models, and regardless of the details of the deuteron wave function.

The implication of a smaller $d/u$ ratio for nucleon structure is that nonperturbative QCD physics, which generally predicts $d/u \to 0$ as $x \to 1$ [1], is still dominant at the currently accessible $x$ and $Q^2$. The behavior expected from perturbative QCD-inspired models, which predict a finite $d/u$ ratio in the $x \to 1$ limit [2], is not observed; whether this behavior will be revealed at even larger $x$ remains to be seen.

**Outlook**

The fact that nuclear effects in deuterium play a vital role in determining the structure of the neutron at large $x$ has been known for some time. As the focus of global PDF studies extends to larger values of $x$ and lower $Q^2$, with the availability of high-precision data from Jefferson Lab and elsewhere, the need to incorporate deuterium corrections is becoming paramount. The CTEQ6X NLO fit has illustrated the significant impact of these corrections on the $d$ quark distribution, which is found to be suppressed by up to $\sim 40\%$ at the highest accessible value of $x$ ($x \approx 0.8$) compared with earlier analyses with no nuclear effects. Constraining the $d$ distribution at $x \gtrsim 0.8$ from inclusive $F^d_2$ data will be challenging given the increasing uncertainty in the nuclear corrections at larger nucleon momenta in the deuteron.

Further progress will be made with the help of several key experiments planned at Jefferson Lab with 12 GeV. This includes a novel idea of using the ratio of mirror symmetric $^3$He and $^3$H nuclear structure functions, in which the nuclear effects cancel to within $\sim 1\%$, to extract the $F^u_2/F^p_2$ ratio up to $x \approx 0.85$ [19]. Another program already under way uses measurements of DIS on a deuterium target with low-momentum spectator protons in the backward region to isolate an almost free neutron in the deuteron [20]. Avoiding the use of nuclei altogether, yet another proposal utilizes the weak interaction to probe the $d$ quark through parity-violating electron DIS on a hydrogen target [21, 22]. Here the asymmetry between left- and right-hand polarized electrons selects the interference between $\gamma$ and $Z$-boson exchange, which depends on the $d/u$ ratio weighted by electroweak charges, and the expected 1% asymmetry measurements would strongly constrain $d/u$ up to $x \sim 0.8$ [21].

An exciting time lies ahead, with the expectation that the planned program of measurements should finally close the book on one of the longest-standing puzzles in the structure of the nucleon.

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