DNS and the theory of receptivity of a supersonic boundary layer to free-stream disturbances

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Abstract. Direct numerical simulation (DNS) of receptivity of a boundary layer over flat plate is carried out. The free stream Mach number is equal to 6. The following two-dimensional disturbances are introduced into the free-stream flow: fast and slow acoustic waves, temperature spottiness. A theoretical model describing the excitation of unstable waves in the boundary layer is developed using the biorthogonal eigenfunction decomposition method. The DNS results agree with the theoretical predictions.

1. Introduction

Prediction of laminar-turbulent transition is important for aerothermal design and drag calculations of high-speed vehicles. In quiet free streams, transition on aerodynamically smooth surfaces includes receptivity, linear phase and nonlinear breakdown to turbulence. Receptivity refers to the mechanism by which free-stream disturbances enter to the laminar boundary layer and generate unstable waves (Reshotko, 1976). Knowing the initial amplitudes of these waves is an equally critical component of transition prediction as the growth rates.

There are three types of free-stream disturbances: fast and slow acoustic waves, temperature spottiness and vorticity fluctuations. Acoustic waves dominate in wind tunnels because they are naturally radiated by vortices propagating in the turbulent boundary layer on the walls of supersonic wind tunnels (Laufer, 1964). In the case of free flight and so-called quiet wind tunnels, the free-stream acoustic field is negligible. Therefore, receptivity to vorticity or entropy fluctuations can be dominant mechanism. A study of receptivity to all types of free-stream disturbances is necessary for a complete analysis of high-speed boundary layer transition.

A theoretical model of receptivity to acoustic disturbances interacting with the boundary layer flow near the flat-plate leading edge was developed by Fedorov & Khokhlov (2001) and Fedorov (2003). The boundary-layer mode excited near the leading edge by a fast acoustic wave can be referred as mode F, and by a slow acoustic wave as mode S. DNS of receptivity to acoustic, vorticity and entropy waves of flat plate boundary layer was performed by Ma & Zhong (2003, 2005) and Soudakov (2010) for different inclination angles with respect to plate surface. Experimental investigations of the flat plate boundary layer receptivity to acoustic waves were performed by Maslov et al. (2001). An initial value problem for a two-dimensional wave packet induced by a local two-dimensional temperature disturbance in a hypersonic boundary layer was analyzed by Fedorov & Tumin (2003).
In our work, DNS of receptivity to acoustic waves and temperature spottiness is carried out. A theoretical model describing receptivity mechanism is developed using the biorthogonal eigenfunction decomposition method. DNS results are compared with theoretical ones.

2. Problem formulation

Viscous two-dimensional unsteady compressible flows are governed by the Navier-Stokes equations. The flow variables are made nondimensional using the steady-state free-stream parameters: $(u, v) = (u^*, v^*)/U^*_\infty$ are velocity components, $p = p^*/(\rho^*_\infty U^2_\infty)$ is pressure, $\rho = \rho^*/\rho^*_\infty$ is density, $T = T^*/T^*_\infty$ is temperature. Nondimensional coordinates and time are $(x, y) = (x^*, y^*)/L^*$, $t = t^*U^*_\infty/L^*$. Hereafter $U^*_\infty$ is free-stream velocity, $L^*$ is plate length, asterisks denote dimensional quantities, subscript "$\infty$" denotes free-stream values.

The fluid is a perfect gas with the specific heat ratio $\gamma = 1.4$ and Prandtl number $Pr = 0.72$. The viscosity-temperature dependence is approximated by the power law $\mu^*/\mu^*_\infty = (T^*/T^*_\infty)^{0.7}$. Calculations are carried out for hypersonic flow over a flat plate with sharp leading edge at the free-stream Mach number $M^*_\infty = 6$ and the Reynolds number $Re^*_\infty = \rho^*_\infty U^*_\infty L^*/\mu^*_\infty$.

The computational domain is a rectangle with its bottom side corresponding to the plate surface. The no-slip boundary conditions and adiabatic wall condition are imposed on the plate surface. Details on the problem formulation and governing equations are given by Egorov, Fedorov & Soudakov (2006). The problem is solved in two steps. Firstly, the steady-state solution is calculated to provide the mean flow field. Then, unsteady disturbances are induced on the computational domain boundaries at the initial time moment and the unsteady problem is integrated.

Numerical solutions are obtained using the implicit second-order finite-volume method described in Egorov, Fedorov & Soudakov (2006). The Navier-Stokes equations are approximated by the conservative shock-capturing scheme. The advection terms are approximated by the third-order WENO scheme. The computational grid has 2201 × 301 nodes. In the boundary-layer region, the grid nodes are clustered in the direction normal to the body surface.

3. Receptivity to acoustic waves

For modeling of receptivity to small disturbances, a plain monochromatic wave is imposed on the free stream (figure 1) as

$$(u', v', p', T')^T = (|u'|, |v'|, |p'|, |T'|)^T \exp[i(k_x x + k_y y - \omega t)]$$

where $|u'|$, $|v'|$, $|p'|$, $|T'|$ are non-dimensional perturbation amplitudes. For the acoustic wave with zero angle of inclination:

$$|p'| = \varepsilon, |u'| = \pm M^*_\infty |p'|$$

$$|v'| = 0, |T'| = (\gamma - 1) M^2_\infty |p'|$$

the upper (lower) sign corresponds to the fast (slow) acoustic wave. $\varepsilon$ is amplitude of incident wave; $k_x = k_\infty$, $k_y = 0$ are wavenumber components; $\omega = \omega^* L^*/U^*_\infty$ is circular frequency. The wavenumber is expressed as $k_\infty = \omega M^*_\infty/(M^*_\infty \pm 1)$. Herein we consider acoustic waves of small amplitude $\varepsilon = 5 \times 10^{-5}$ at which the receptivity process is linear. The disturbance frequency $\omega = 260$ corresponds to the frequency parameter $F = \omega/Re = 1.3 \times 10^{-4}$. At this frequency the maximum amplitude predicted by linear stability theory (LST) is observed at the section $x \approx 0.9$ and associated with the second mode wave. The temperature disturbance is zero on the plate surface.
Figure 1. Sketch of boundary layer receptivity to free-stream waves.

The DNS results are compared with predictions of the leading-edge receptivity theory coupled with the two-mode approximation (Fedorov, 2003) of the disturbance propagation in the boundary layer. In this theory, the boundary-layer disturbance is expressed as a sum of modes F and S. The amplitude coefficients of these modes are governed by the system of two ordinary differential equations, which accounts for the interaction between modes due to nonparallel effects. The initial values of the amplitude coefficients are determined using the analytical solution (Fedorov, 2003) of the leading-edge receptivity problem. Then, the Cauchy problem for coefficients is solved numerically.

Comparison of the wall pressure disturbance $p_\text{\scriptsize w}'$ (difference between an instantaneous flow field and the mean flow field) predicted by DNS and theory is shown in figure 2 for the case of disturbances generated by the fast (figure 2a) and slow (figure 2b) acoustic wave of zero angle of incidence. The theory predicts well the initial disturbance amplitude and captures nuances of the disturbance evolution from the plate leading edge to the unstable region.

Figure 2. Wall pressure disturbances induced by fast (a) and slow (b) acoustic wave of zero angle of inclination, solid line - DNS, dashed line - theory.

For the case of fast incident wave, at the section $x = 0.3$ the pressure and velocity disturbance profiles predicted by DNS agrees well with the eigenfunction of mode F (figure 3). Similar comparison for a slow incident wave is shown in figure 4 (section $x = 0.3$) and 5 (section $x = 0.9$), where DNS solution agrees with the theoretical eigenfunction of mode S.

Phase speed distribution $c_x(x) = \omega/k_x$ of the disturbance propagation is shown in figure 6. The lines 1 and 2 indicate phase speed of free-stream fast ($c_x = 1 + 1/M_\infty$) and slow
Figure 3. Distributions of pressure (a) and \( u \)-velocity (b) disturbances across the boundary layer at the section \( x = 0.3 \), solid line - disturbance induced by fast acoustic wave (DNS), dashed line - eigenfunction of the mode F (theory); the disturbance is normalized by the wall pressure.

Figure 4. Distributions of pressure (a) and \( u \)-velocity (b) disturbances across the boundary layer at the section \( x = 0.3 \), solid line - disturbance induced by fast acoustic wave (DNS), dashed line - eigenfunction of the mode S (theory); the disturbance is normalized by the wall pressure.

\((c_x = 1 - 1/M_\infty)\) acoustic waves, respectively. Curves 5 and 6 show phase speeds of modes F and S predicted by LST. This terminology was proposed by Fedorov (2003) in connection with the asymptotic behavior of mode phase speed in the leading-edge vicinity. Namely, the phase speed of mode F (fast mode) tends to \( c_x = 1 + 1/M_\infty \) of fast acoustic wave whereas the phase speed of mode S (slow mode) tends to \( c_x = 1 - 1/M_\infty \) of slow acoustic wave. In the case considered herein, the mode S corresponds to the Mack first mode upstream from the synchronization point \( x \approx 0.7 \), and to the Mack second mode downstream from this point. The phase speed distribution predicted by DNS (curve 4) in the case of slow incident acoustic wave is close to the phase speed of the mode S. The phase speed predicted by DNS (curve 3) in the case of fast incident acoustic wave is close to the phase speed of mode F in the region \( x < 0.75 \). Downstream from \( x \approx 0.75 \), the phase speed of DNS solution evolves toward the
Figure 5. Distributions of pressure (a) and $u$-velocity (b) disturbances across the boundary layer at the section $x = 0.9$, solid line - disturbance induced by fast acoustic wave (DNS), dashed line - eigenfunction of the mode S (theory); the disturbance is normalized by the wall pressure.

Figure 6. Disturbance phase speeds: 1 - free-stream fast acoustic wave, 2 - free-stream slow acoustic wave, 3 - disturbances induced in the boundary layer by fast acoustic wave (DNS), 4 - disturbances induced in the boundary layer by slow acoustic wave (DNS), 5 - disturbances induced in the boundary layer by fast acoustic wave (theory), 6 - disturbances induced in the boundary layer by slow acoustic wave (theory).

phase speed of mode S. This behavior indicates that mode S excited somewhere upstream from $x \approx 0.75$ becomes dominant due to its instability.
4. Receptivity to temperature spottiness

For simulation of receptivity to temperature spottiness, the initial disturbances function

\[ T'_0(x, y, t) = \varepsilon \delta(x - x_0) \exp\left[-(y - y_0)^2/\sigma_0^2\right] \sin(\omega t) \]

where \( \varepsilon \) is small parameter representing the disturbance amplitude; \( \delta(x - x_0) \) is the delta-function. The Gaussian distribution versus \( y \) is centered at the point \( y_0 \) and has characteristic width \( \sigma_0 \). Herein we consider the temperature spot of the amplitude \( \varepsilon = 0.1 \) at which the receptivity process is linear. The disturbance frequency is \( \omega = 260 \). The temperature disturbance is zero on the plate surface. Calculations are performed for the following set of spot parameters: \( x_0 = 0.3; y_0 = \delta_0, 1.25\delta_0, 1.5\delta_0, 1.75\delta_0, 2\delta_0; \sigma_0 = 1/8\delta_e, 2/8\delta_e, 3/8\delta_e, 4/8\delta_e \). Hereafter \( \delta_0 = 0.008 \) is the boundary layer thickness at \( x = 0.5 \).

A scheme of the temperature spottiness evolution is presented in figure 7. The spots are induced above the upper boundary-layer edge and propagate downstream along the inviscid flow streamlines. It should be noted, that temperature spots are induced downstream from the shock. This allows us to avoid the spot-shock interaction, which leads to excitation of acoustic waves and vortical disturbances. Thus our study is focused on the receptivity mechanism associated with the only one type of external disturbances - the temperature spottiness.

\[ \text{Figure 7. Scheme of receptivity to temperature spots propagating over the boundary layer.} \]

Phase speed diagram is presented in figure 8a. The curve 6 shows the phase speed for disturbances induced in the boundary layer by the temperature spottiness having \( x_0 = 0.3, y_0 = 2\delta_e, \sigma_0 = 1/8\delta_e \). The DNS results are qualitatively the same for all the cases considered, they correlate well with the LST predictions. Namely, the temperature disturbances propagate downstream with the phase speed \( c_x = 1 \) (line 1 in figure 8a) and weakly interact with the boundary-layer flow. Then, approximately in the section \( x = 0.5 \), they are synchronized with mode F (the line 1 intersects the curve 4) and excite the latter. Because mode F is stable, its amplitude decreases downstream from the synchronization point \( x \approx 0.5 \). This is clearly seen in the wall-pressure disturbance distribution (figure 8b). Further downstream, mode F is synchronized with mode S and excite the latter via the inter-modal exchange mechanism described by Fedorov & Khokhlov (2001). This occurs in the vicinity of \( x \approx 0.7 \). Ultimately, mode S amplifies due to its instability as shown in figure 8b (\( x > 0.7 \)).

The first-cut model of the boundary layer receptivity to temperature spottiness is developed using the biorthogonal eigenfunction decomposition method. The amplitude of mode F, which is excited in the vicinity of the synchronization point \( x \approx 0.5 \), can be estimated using the weight coefficient resulted from an expansion of the temperature-spot disturbance vector into
Figure 8. Phase speeds (a) and wall pressure disturbances (b) induced by temperature spottiness of $x_0 = 0.3$, $y_0 = 2\delta_e$, $\sigma_0 = 1/8\delta_e$.

In figures 9a and 9b, the pressure and temperature disturbances predicted by DNS are compared with the corresponding eigenfunctions of mode F predicted by theory at $x = 0.5$. The temperature distributions are close to each other in the near-wall region $y < 0.005$. For $y > 0.005$, they are essentially different because the DNS solution is affected by the temperature spot. Discrepancies between the pressure distributions in the outer region $y > 0.004$ indicate that the spots generate weak acoustic waves not presented in the LST solution.

Figure 9. Pressure (a) and temperature (b) disturbance eigenfunctions in the boundary layer, $x = 0.5$.

To evaluate the receptivity mechanism and compare the theoretical and DNS results, we introduce the receptivity coefficients $K_T = T'_{\text{max}}/\varepsilon$, where $T'_{\text{max}}$ is the temperature disturbance amplitude measured in the first maximum of its $y$-distribution at $x = 0.5$ (see figure 9b). The receptivity coefficients were calculated for the cases: 1) spots of $\sigma_0 = \delta_e/8$ at various distances from the wall $y_0$; 2) spots of $\sigma_0 = \delta_e/2$ at various distances from the wall $y_0$; 3) spots of fixed
$y_0$ and various sizes $\sigma_0$.

![Figure 10.](image1)

**Figure 10.** Receptivity coefficients versus $y_0$ at $x = 0.5$: $\sigma_0 = \delta_e/8$ (a) and $\sigma_0 = \delta_e/2$ (b).

![Figure 11.](image2)

**Figure 11.** Receptivity coefficients versus spots size $\sigma_0$ at $x = 0.5$, $y_0 = 2\delta_e$; solid line - theory, symbols - DNS.

The receptivity coefficients for cases 1 and 2 are shown in figures 10. As expected, the agreement between DNS and theory improves as $y_0$ increases. For relatively small $y_0$, the spot shape is affected by the boundary layer flow that is not taken into account by the theoretical model. This effect is stronger for spots of larger size $\sigma_0$. The receptivity coefficients for case 3 are shown in figure 11. The theoretical distribution of $K_T(\sigma_0)$ agrees well with DNS. In this case, the spots are located sufficiently far from the boundary-layer edge where the theoretical model works better.
5. Conclusions

DNS of receptivity to fast and slow acoustic waves radiating onto a flat plate in Mach=6 flow agrees well with the theoretical model that combines the leading-edge receptivity theory with the two-mode approximation of the disturbance evolution in the boundary-layer flow.

DNS of receptivity of a flat-plate boundary-layer to two-dimensional temperature spottiness was carried out. The temperature spots were induced downstream from the bow shock in order to avoid the spot-shock interaction leading to excitation of acoustic waves and vortical disturbances. These simulations were focused on the receptivity mechanism associated with only one type of external disturbances: temperature spots. A theoretical model describing excitation of mode F by temperature spots propagating over the boundary-layer edge has been developed. The receptivity coefficients were evaluated using the DNS and the theoretical solutions for different parameters of the temperature spottiness. Comparisons showed that the theory agrees with DNS in the cases when the temperature spots are localized outside the boundary layer.

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