Supersymmetric Hybrid Inflation: Explaining the Spectrum of Cosmological Perturbations through a Multiple-Stage Inflationary Model

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**Abstract.** We explore the possibility that a multiple-stage inflationary scenario based on supersymmetric GUT models, can account for the break at $k_b \simeq 0.05 \, h \, \text{Mpc}^{-1}$ in the power spectrum of galaxy clustering, while it reproduces the angular power spectrum of cosmic microwave background anisotropies.

**INTRODUCTION**

High energy physics may provide the necessary mechanisms to explain some of the open questions on the early stages in the history of our universe. Conversely, the early universe provides an adequate environment for testing ideas on fundamental physics. A useful example of the interplay between high energy physics and cosmology is presented here [1].

The inflationary paradigm was proposed in order to explain the shortcomings of the Big Bang cosmological model. In addition, it offers a scenario for the generation of the primordial density perturbations, which can lead to the formation of the observed large-scale structure. Within the wide variety of inflationary models, we consider a rather natural candidate, a scenario which arises within the context of supersymmetric hybrid inflation [2]. The necessary superpotential can arise in supersymmetric GUT models. Since the observable part of inflation takes place for field values below the Planck scale, supergravity corrections can be made in terms of controlled expansions in powers of fields in units of $m_{Pl}$ ($m_{Pl}$ denotes the reduced Planck scale $m_{Pl} = M_{Pl}/\sqrt{8\pi}$, $M_{Pl} = 1.22 \times 10^{19} \, \text{GeV}$). Such inflationary models [3] have been shown to survive the supergravity corrections [4].

The origin of the measured anisotropies in the cosmic microwave background (CMB) and the generation and evolution of large-scale structure in the universe are burning questions of modern cosmology. The COBE-DMR measurements of the anisotropies of the CMB lead to the power spectrum of density perturbations at large scales. This spectrum has to be matched with the power spectrum at small scales one deduces from galaxy surveys. The otherwise rather successful standard cold dark matter (CDM) model with a scale-invariant initial spectrum of adiabatic perturbations, predicts too much power at small scales, once normalized to the COBE data at large scales. This problem can be resolved within a scenario where the power spectrum has a break at position indicated by the observed clustering in the galaxy distribution. At $k_b \simeq 0.05 \, h \, \text{Mpc}^{-1}$ the APM galaxy survey data [5–8] indicate a sharp drop of the spectral index $n$, which persists even after the effects of the non-linear evolution of matter fluctuations have been removed [8,9].

We examine [1] the possibility of a multiple-stage inflationary scenario, with a sequence of scales starting near $m_{Pl}$ and continuing down to the scale implied by COBE. We are interested in examining whether the COBE data can be made compatible with the data of galaxy surveys, employing the smallest number of cosmological parameters. The COBE-DMR measurements require an inflationary stage with $n \simeq 1$ and at least 5 e-foldings, while the APM galaxy survey data support the possibility of a second stage of inflation with $n \simeq 0.6$ that generates $\sim 3$ e-foldings. The subsequent $\sim 50$ e-foldings, needed to resolve the horizon and flatness problems,
generate density perturbations at scales that are strongly affected by non-linear evolution, and therefore can not be constrained from current observational and experimental data.

SUPERSYMMETRIC HYBRID INFLATION MODEL

Our model \cite{1} is described by the superpotential

\[ W = S_1 \left( -\mu_1^2 + \lambda_1 \bar{\Phi}_1 \Phi_1 + g \bar{\Phi}_2 \Phi_2 \right) + S_2 \left( -\mu_2^2 + \lambda_2 \bar{\Phi}_2 \Phi_2 \right) , \tag{1} \]

where the superfields \( S_1, S_2 \) are gauge singlets; \( S_2 \) is the linear combination of gauge singlets which does not couple to \( \Phi_1 \Phi_1 \) \cite{3}. The scalar components of the various superfields are real. Staying along the \( D \)-flat directions, we define canonically normalized scalar fields according to

\[ S_1 = \frac{\sigma_1}{\sqrt{2}} , \quad S_2 = \frac{\sigma_2}{\sqrt{2}} , \quad \Phi_1 = \Phi_1 = \frac{\phi_1}{2} , \quad \Phi_2 = \Phi_2 = \frac{\phi_2}{2} . \tag{2} \]

The potential \( V(\sigma_1, \sigma_2, \phi_1, \phi_2) = \sum_i |\partial W/\partial \Phi_i|^2 \) is then given by

\[ V = \left( \mu_1^2 - \frac{\lambda_1}{4} \phi_1^2 \right)^2 - \frac{g}{2} \mu_1^2 \phi_2^2 + \frac{g^2}{16} \sigma_2^4 + \frac{g \lambda_1}{8} \phi_1^2 \phi_2^2 + \frac{\lambda_2^2}{4} \sigma_2^2 \phi_1^2 \]
\[ + \left( \mu_2^2 - \frac{\lambda_2}{4} \phi_2^2 \right)^2 + \frac{1}{4} \left( g \sigma_1 + \lambda_2 \sigma_2 \right)^2 \phi_2^2 , \tag{3} \]

where the mass scales \( \mu_1, \mu_2 \) are chosen to satisfy the inequality \( \mu_1 > \mu_2 \).

During the first stage of inflation \( \phi_1 = \phi_2 = 0 \), and the vacuum energy density is \( V = \mu_1^4 + \mu_2^4 \) independent of \( \sigma_{1,2} \). Along this flat direction, the mass terms of the \( \phi_{1,2} \) fields are

\[ M_{\phi_1}^2 = -\lambda_1 \mu_1^2 + \frac{\lambda_1^2}{2} \sigma_1^2 , \quad M_{\phi_2}^2 = -\lambda_2 \mu_2^2 - g \mu_1^2 + \frac{1}{2} \left( g \sigma_1 + \lambda_2 \sigma_2 \right)^2 \sigma_2^2 . \tag{4} \]

The mass term \( M_{\phi_1}^2 \) becomes negative for \( \sigma_1^2 < \sigma_{1,\text{ins}}^2 = 2 \mu_1^2 / \lambda_1 \), indicating the presence of an instability which can lead to the growth of the \( \phi_1 \) field.

During the second stage of inflation, \( \sigma_1 = 0, \phi_1^2 = 4 \mu_1^2 / \lambda_1, \phi_2 = 0; \) the vacuum energy density is \( V = \mu_2^4 \) independent of \( \sigma_2 \). Along this flat direction, the mass term of the \( \phi_2 \) field is

\[ M_{\phi_2}^2 = -\lambda_2 \mu_2^2 + \frac{\lambda_2^2}{2} \sigma_2^2 . \tag{5} \]

An instability appears for \( \sigma_2^2 < \sigma_{2,\text{ins}}^2 = 2 \mu_2^2 / \lambda_2 \), which can lead to the growth of the \( \phi_2 \) field.

The flatness of the potential is lifted by radiative corrections. During the first stage of inflation and for \( \sigma_{1,2} \) far above the instability points, the one-loop contribution to the effective potential, assuming that the coupling \( g \) is sufficiently small, is

\[ \Delta V(\sigma_1, \sigma_2) \simeq \frac{M_1}{16 \pi^2} \lambda_1^2 \mu_1^4 \left[ \ln \left( \frac{\lambda_1^2 \sigma_1^2}{2 \Lambda_1^2} \right) + \frac{3}{2} \right] + \frac{M_2}{16 \pi^2} \left( \lambda_2 \mu_2^2 + g \mu_1^2 \right)^2 \left[ \ln \left( \frac{\left( g \sigma_1 + \lambda_2 \sigma_2 \right)^2}{2 \Lambda_2^2} \right) + \frac{3}{2} \right] , \tag{6} \]

where \( M_{1,2} \) are the dimensionalities of the representations of the groups to which the superfields \( \Phi_{1,2} \) belong. The above contribution provides the slope that leads to the slow rolling of the \( \sigma_{1,2} \) fields during the first stage of inflation.

During the second stage of inflation the slope for the \( \sigma_2 \) field is provided by the radiative correction

\[ \Delta V(\sigma_1, \sigma_2) \simeq \frac{M_2}{16 \pi^2} \lambda_2^2 \mu_2^4 \left[ \ln \left( \frac{\lambda_2^2 \sigma_2^2}{2 \Lambda_2^2} \right) + \frac{3}{2} \right] . \tag{7} \]

Based on the existence of these two flat directions, with a small slope generated by logarithmic radiative corrections, we will construct \cite{1} a model for a multiple-stage inflation, in the context of supersymmetry and supergravity.
THE INFLATIONARY STAGES

The first stage of inflation starts at \( t_{1i} \) and lasts until \( t_{1f} \), when \( \sigma_1^2(t_{1f}) \equiv \sigma_{1f}^2 \simeq \frac{M_1 \lambda_1^2 m_{pl}^2}{(8\pi^2)} \ll \sigma_{1i}^2 \) and the “slow-roll” conditions for \( \sigma_1 \) cease to be satisfied. The Hubble parameter during this period \( H_1^2 \simeq \frac{(\mu_{1f}^2 + \mu_{2i}^2)}{(3m_{pl}^2)} \) is almost constant, and the total number of e-foldings is [1]

\[
N_{1{i\text{tot}}} = \frac{4\pi^2}{M_1 \lambda_1^2} \frac{\mu_{1f}^4 + \mu_{2i}^4 - \sigma_{1i}^2}{\mu_{1f}^4} \frac{\sigma_{1f}^2}{m_{pl}^2},
\]

where \( \sigma_{1i}^2 \equiv \sigma_1^2(t_{1i}) \). We can choose the parameters in our model [1], so that the evolution of \( \sigma_2 \) during the first stage of inflation can be neglected.

Then it follows an intermediate stage, lasting between times \( t_{1f} \) and \( t_{2i} \), during which the scale factor increases by [1]

\[
N_{1{\text{int}}} = \frac{2}{3(1 + w)} \ln \left( \frac{H_1}{H_2} \right),
\]

where \( w \) characterizes the equation of the state of the universe \((p = w\rho)\). Assuming the massive fields \( \sigma_1, \phi_1 \) have fast decay channels into lighter species that eventually thermalize, the fields \( \sigma_2, \phi_2 \) remain constant during the intermediate stage and the universe is in the radiation-dominated era [1].

At time \( t_{2i} \), the second inflationary stage starts and lasts until \( t_{2f} \), when \( \sigma_2^2(t_{2f}) \equiv \sigma_{2f}^2 \simeq \frac{M_2 \lambda_2^2 m_{pl}^2}{(8\pi^2)} \ll \sigma_{2i}^2 \) and the “slow-roll” conditions for \( \sigma_2 \) cease to be satisfied. The Hubble parameter during this period \( H_2^2 \simeq \frac{\mu_{2f}^4}{(3m_{pl}^2)} \) is almost constant, and the total number of e-foldings is [1]

\[
N_2(t) = \frac{4\pi^2}{M_2 \lambda_2^2} \frac{\sigma_{2i}^2 - \sigma_{2f}^2}{m_{pl}^2},
\]

where \( \sigma_{2i}^2 \equiv \sigma_2^2(t_{2i}) \).

Subsequent inflationary stages are needed in order to resolve the flatness and horizon problems of standard cosmology. We can easily extend our model [1] to accommodate additional inflationary stages which can provide the remaining required number of \( \sim 50 \) e-foldings. Choosing the parameters of our model, the above discussion of the first two inflationary stages remains unchanged.

THE PRIMORDIAL SPECTRUM

The primordial spectrum of density inhomogeneities has its origin in the quantum fluctuations that crossed outside the horizon during inflation [10]. For \( k < k_2 = a_2 t H_2 \), or \( k > k_1 = a_1 t H_1 \), the Hubble radius is crossed only once, during the first, or second, stage of inflation respectively. On the other hand, the scales \( k_2 < k < k_1 \) cross the Hubble radius three times: they exit the horizon during the first stage, re-enter during the intermediate stage and exit again during the second stage of inflation.

Assuming that the field fluctuations are random gaussian variables, the spectrum of adiabatic density perturbations during the first stage of inflation is [1]

\[
\delta_H(k) \simeq \frac{8\pi}{5M_2} \left( \frac{\mu_{2i}^2}{\lambda_2 \mu_{2i}^2 + g\mu_1^2} \right)^2 \frac{H_1 \sigma_{2i}}{m_{pl}^2}.
\]

The predicted spectrum is scale invariant with a spectral index \( n = 1 \) to a very good accuracy [1].

The spectrum of adiabatic density perturbations during the second stage of inflation is [1]

\[
\delta_H(k) \simeq \frac{8\pi}{5M_2 \lambda_2^2} \frac{H_2 [\sigma_{2i}^2]_{k=H_2}}{m_{pl}^2},
\]

while the spectral index is given by [1]

\[
n - 1 = \frac{M_2 \lambda_2^2}{4\pi^2} \frac{m_{pl}^2}{\sigma_2^2(k)} \neq 1.
\]

We believe [1] that for scales \( k_2 < k < k_1 \) there is a smooth interpolation between the two parts of the spectrum.
COMPARISON WITH OBSERVATIONS

We compare the predictions of our model with the experimental data from COBE and the observational data from the APM galaxy survey. We consider [1] a CDM model with $\Omega_\Lambda = 0$, $h = 0.5$ ($H_0 \equiv h 100 \, \text{km/s/Mpc}$), and $\Omega_{\text{matter}} = 1$. We assume $\Omega_b = 0.05$, $\Omega_{\text{CDM}} = 0.95$, $Y_{\text{He}} = 0.24$, while we do not consider any massive neutrinos. The values of the parameters for our model are [1]: $\lambda_1 = 1$, $\lambda_2 = 0.1$, $g = 7.1 \times 10^{-3}$, $\mu_1/m_{\text{Pl}} = 3.9 \times 10^{-3}$, $\mu_2/m_{\text{Pl}} = 8.8 \times 10^{-4}$, $\sigma_{1i}/m_{\text{Pl}} = 0.36$, $\sigma_{2i}/m_{\text{Pl}} = 0.025$. We obtain 5 e-foldings with an almost scale-invariant spectrum of perturbations for the first stage of inflation, and 3 e-foldings with a spectrum having a spectral index $n \simeq 0.6$ for the second stage. The intermediate stage affects the scales $k_2 < k < k_1$ with $k_1/k_2 \simeq 4.4$. The values of the power spectrum for $k_2$ and $k_1$ are determined by Eqs. (11), (12) and satisfy $P(k_2)/P(k_1) \simeq 11.8$.

The primordial spectrum of fluctuations arising in our model generates matter and radiation perturbations, which, after amplification, give rise to the observed large scale structure and the measured anisotropies of the CMB. Thus, to test our model, we confront its theoretical predictions with the measured anisotropies of CMB and the observed clustering in the galaxy distribution. For the comparison with the CMB data, we assume [1] a form of the power spectrum in the range $k_2 < k < k_1$ that interpolates smoothly between the spectrum in the ranges $k > k_1$ and $k < k_2$.

In order to calculate the predictions of our model for the angular power spectrum of CMB anisotropies, we have used [1] the code CMBFAST of Seljak and Zaldarriaga [11]. Fig. 1 illustrates [1] our theoretical predictions for the angular power spectrum of CMB anisotropies against the most recent CMB flat-band power measurements.

![FIGURE 1. Theoretical predictions of our model against the most recent CMB flat-band power measurements.](image)
spectrum is scale invariant, while the intermediate and second stages affect only the highest multipoles. Better agreement with the data is obtained [1] for a CDM model with $\Omega_\Lambda = 0.5$, $h = 0.5$, $\Omega_b = 0.05$ and $\Omega_{CDM} = 0.45$ within our multiple-stage inflationary scenario.

We then turn to the comparison of our theoretical predictions [1] with the power spectrum of galaxy clustering derived from the angular APM galaxy survey [12]. In Fig. 2 we plot [1] the power spectrum generated in our multiple-stage model [1], together with the APM power spectrum and the linear spectrum as predicted by Refs. [8,9]. The spectrum is flat for $k \leq 0.06 \, h{\text{Mpc}}^{-1}$ and tilted with $n = 0.6$ for $k \geq 0.26 \, h{\text{Mpc}}^{-1}$.

![Figure 2. Theoretical power spectrum together with the APM power spectrum, and linear spectrum.](image)

Finally, calculating the parameter $\sigma_8$, we find [1] $\sigma_8$ near 0.75, both for $\Omega_\Lambda = 0$ and $\Omega_\Lambda = 0.5$, which is in good agreement with the values deduced from the abundances of rich clusters of galaxies: $\sigma_8 \simeq 0.6 \pm 0.2$ for $\Omega_{\text{matter}} = 1$, $\Omega_\Lambda = 0$ [13], and $\sigma_8 \simeq 0.85 \pm 0.3$ for $\Omega_{\text{matter}} = 0.5$, $\Omega_\Lambda = 0.5$ [14].

**CONCLUSIONS**

In the context of supersymmetric hybrid inflation, we presented [1] a multiple-stage inflationary scenario which could account for some of the characteristics of the current observational and experimental data. We proposed a rather natural mechanism to generate a primordial spectrum of adiabatic density perturbations with a break at $k_b \simeq 0.05 \, h{\text{Mpc}}^{-1}$. The existence of this break in the power spectrum of galaxy clustering was suggested by the angular APM galaxy survey. In addition, our model reproduces successfully the angular power spectrum of CMB anisotropies.

More precisely, we described a scenario of multiple short bursts of inflation. The first two observable inflationary stages, for which we have constraints from current observational and experimental data, generate $\sim 8$ e-foldings. The subsequent stages, providing the required additional $\sim 50$ e-foldings, generate perturbations at scales that are strongly affected by non-linear evolution. It is thus very hard to extract any relevant information. The first inflationary stage generates $\sim 5$ e-foldings and a scale invariant spectrum for scales $k \lesssim 0.06 \, h{\text{Mpc}}^{-1}$. 
The second inflationary stage generates $\simeq 3$ e-foldings and a spectrum with spectral index $n \simeq 0.6$ for scales $k \gtrsim 0.26 \, h \, \text{Mpc}^{-1}$. Between these two inflationary stages there is a rather complicated intermediate stage of normal expansion, and therefore we are unable to calculate the spectrum for $0.06 \, h \, \text{Mpc}^{-1} \lesssim k \lesssim 0.26 \, h \, \text{Mpc}^{-1}$. However, we believe that in this intermediate stage there is no significant new feature and we can interpolate smoothly between the two parts of the spectrum that we have computed.

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