Spin transparency for an interface of an ultrathin magnet within the spin dephasing length

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We examine a modified drift-diffusion formalism to describe spin transport near a ultrathin magnet whose thickness is similar to or less than the spin dephasing length. Previous theories on spin torque assume the transverse component of a injected spin current dephases perfectly thus are fully absorbed into the ferromagnet. However, in the state-of-art multilayer systems under consideration of recent studies, the thicknesses of ferromagnets are on the order of or less than a nanometer, thus one cannot safely assume the spin dephasing to be perfect. To describe the effects of a finite dephasing rate, we introduce an effective spin transparency, which determines the spin torque efficiency. Interestingly, for an ultrathin magnet with a finite dephasing rate, the spin transparency can be even enhanced and there arises a nonnegligible field-like spin-orbit torque even in the absence of the imaginary part of the spin mixing conductance. The effective spin transparency provides a simple extension of the drift-diffusion formalism which is accessible to experimentalists analyzing their results.

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I. INTRODUCTION

Spin torque [1–3] has been a central concept in magnetism for a few decades, as it allows electrical control of magnetism. When a spin current is injected to a ferromagnet, its transverse component to magnetization dephases rapidly, thus its angular momentum is transferred to the magnetization, giving rise to a torque [4–7]. It is typically assumed that the spin dephasing in the ferromagnetic bulk is infinitely fast, thus the spin current right at the interface solely determines the total angular momentum transfer to the ferromagnet [8]. Indeed, the spin dephasing length is on the order of or less than a nanometer, thus one cannot safely assume the spin dephasing to be perfect. Therefore, in the considered interaction, the two interfaces do not communicate with each other, the properties of the normal metal in the other side of NM1, the spin transparency determines how effectively a given spin chemical potential profile $\mu_s$ in Eq. (1) generates a spin torque. More explicitly, denoting the spin transparency for the interface 1 in Fig. 1 by $T_1$ and assuming a nonequilibrium spin chemical potential $\mu_s(0\, \parallel \, z = 0\, - \,)$ at $z = 0\, - \,$, the resulting spin torque is determined by $T_1 \mu_s(0\, - \,)$, not $\mu_s(0\, - \,)$ itself. In this sense, the spin transparency can be understood by the absorption efficiency of a transverse spin current at the given interface. Because of reflection of spin current at the interfaces of NM1, the spin transparency depends on the spin mixing conductance of the interface and the properties of the normal metal [See Eq. (6c) for an explicit expression]. Note that, for a thick film where the two interfaces hardly communicate with each other, the properties of the normal metal in the other side (NM2) does not affect the transparency.

However, recent interest of researches in magnetism has moved to ultrathin magnetic films, which not only allow high density spintronic applications but also result in much richer physics originating from broken symmetry such as spin-orbit torque (SOT) [31–32], the Dzyaloshinskii-Moriya interaction [33–35], and other chiral phenomena [36–41]. The typical order of magnitudes of thicknesses of ferromagnetic layers under consideration is a few angstroms [40–46], which cannot be assumed to be sufficiently larger than the spin dephasing length.

\[G_1^{\parallel i} = \text{Re}[G_1^{\parallel i}]\quad \text{and} \quad \text{Im}[G_1^{\parallel i}]\] are the coefficients for the Slonczewski-like torque $[m \times (m \times \mu_s)]$ and the field-like torque $[m \times \mu_s]$, respectively. The drift-diffusion formalism with the BC in Eq. (1) has been used for numerous theories [13–24] and experiments [25–30].

A relatively intuitive way to understand the effects of spin diffusion is introducing the spin transparency [28] of a given interface $i$. The spin transparency determines how effectively a given spin chemical potential profile $\mu_s$ in Eq. (1) generates a spin torque. More explicitly, denoting the spin transparency for the interface 1 in Fig. 1 by $T_1$, and assuming a nonequilibrium spin chemical potential $\mu_s(0\, - \,)$ at $z = 0\, - \,$, the resulting spin torque is determined by $T_1 \mu_s(0\, - \,)$, not $\mu_s(0\, - \,)$ itself. In this sense, the spin transparency can be understood by the absorption efficiency of a transverse spin current at the given interface. Because of reflection of spin current at the interfaces of NM1, the spin transparency depends on the spin mixing conductance of the interface and the properties of the normal metal [See Eq. (6c) for an explicit expression]. Note that, for a thick film where the two interfaces hardly communicate with each other, the properties of the normal metal in the other side (NM2) does not affect the transparency.

In Eq. (1), the presence of the minus sign in front of $\mu_s(0\, - \,)$ is because $\Delta \mu_s(0\, - \,) = -\mu_s(0\, - \,)$. 2 Here, we denoted the reported transparency by the spin transparency, to emphasize that it is irrelevant for charge transport. For instance, the spin transparency is not necessarily zero for a magnetic insulator, through which a charge current cannot flow. Still, it is worth noting that the spin transparency considered throughout this paper is the one for the transverse spin transport, not the longitudinal one.
length. Moreover, there are recent experimental reports on a ferrimagnetic multilayer with an extremely long spin coherence length $\sim 10$ nm \cite{47} and a direct experimental evidence that the two interfaces of an ultrathin ferromagnet is no longer independent \cite{30}. Therefore, to correctly analyze the magnetic multilayers of contemporary research interest, it is desirable to construct a formal theory taking into account a finite dephasing rate. In this work, the spin dephasing is characterized by a thickness-dependent function $\xi(t)$, whose features for various materials are discussed below. The third process is the spin dephasing in the bulk of the magnetic layer. In this work, the spin dephasing is characterized by a certain dephasing length $\lambda_{dp}$. In previous papers, the effect of $G_{T}^{\gamma\nu}$ has been examined in the ballistic regime \cite{49} or focused on a specific experimental regime under consideration \cite{30} because of the complexity of the general solution. In this paper, we introduce an effective spin transparency, which provides a clear physical generalization of the conventional spin transparency, as well as significantly simplifies the complicated solution of the drift-diffusion equation \cite{30} in general cases. Interestingly, we explicitly demonstrate that the spin transparency can be even enhanced when a finite dephasing length is concerned.

This paper is organized as follows. In Sec. II, we present a modified BC to consider a finite dephasing rate. In Sec. III, we solve the drift-diffusion equation and calculate various physical quantities such as SOT, the inverse spin Hall current, and the spin pumping effect. To express our result in simple forms, we introduce an effective spin transparency. In Sec. IV, we summarize the paper. Appendices include mathematical information that is not crucial for the main flow of the paper.

II. TRANSMITTED MIXING CONDUCTANCE

When the thickness of the magnet is not much larger than the spin dephasing length, the spin chemical potential at the interface 1 can generates the spin current at the interface 2 (and vice versa) [Fig. 1(b)]. In this case, it is necessary to introduce another conductance $G_{T}^{\gamma\nu}$, called the transmitted mixing conductance \cite{30} and whose properties are discussed below. As illustrated in the upper part of Fig. 2(a), the transmitted mixing conductance connects $\mu_{\nu}^{s}(0^{+})$ and $j_{s}(0^{+})$ (and vice versa), giving the following modified BC.

$$
e\mu_{\nu}^{s}(0^{+}) = \text{Re}[ -G_{T}^{\gamma\nu} M \mu_{\nu}^{s}(0^{+}) ]$$

Equation (2) gives a simple extension of the conventional BC [Eq. (1)] to allow for a finite dephasing rate. Note that the Onsager reciprocity \cite{50} guarantees that the transmitted conductances in Eqs. (2a) and (2b) are identical.

There are three physical processes behind the transmitted mixing conductance [lower part of Fig. 2(a)]. First, when a transverse spin is injected to and passing through the interface 1, there arise the interfacial spin filtering and the interfacial spin rotation, which make the spin current discontinuous at the interface $j_{s}(0^{+}) \neq j_{s}(0^{-})$. The details of the interfacial spin filtering and rotation are substantially discussed in Ref. \cite{7}. In Fig. 2(a), we denote this process by $G_{1}^{\gamma\nu}_{T}$. The second process is the spin dephasing in the bulk of the magnetic layer. In this work, the spin dephasing is characterized by a thickness-dependent function $\xi(t)$, whose features for various materials are discussed below. The third process is the additional spin filtering and rotation at the interface 2 denoted by $G_{2}^{\gamma\nu}_{T}$ in Fig. 2(a). Now, we may write the transmitted mixing conductance by the following form.

$$G_{T}^{\gamma\nu}(t) = G_{T}^{\gamma\nu}(0) \xi(t),$$

where $t$ is the thickness of the magnet. $G_{T}^{\gamma\nu}(0)$ is the interfacial contribution determined by $G_{1}^{\gamma\nu}$ and $G_{2}^{\gamma\nu}$. $\xi(t)$ is the bulk
FIG. 2. (a) (upper part) Illustration of the definitions of the conventional ($G_{T,i}^{ij}$) and the transmitted ($G_{T,i}^0$) mixing conductances. (lower part) The transmitted mixing conductance is determined by the product of the interface discontinuity at each interface ($G_{T,i}^{ij}$ and $G_{T,i}^{ij}$) and the bulk contribution $\xi(t)$, which refers to the spin dephasing and is thickness dependent. (b) The decaying and oscillatory nature of the spin dephasing, as a result of the rotation of the transverse spin around the magnetization. The plot is generated for a real $G_{T,i}^0(0)$.

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spin diffusion length would be the relevant length scale. For a magnet showing spin superfluidity [52], the spin current decays algebraically rather than exponentially.

Two remarks are in order. First, although we only consider trilayers for writing Eq. (2), generalization of our theory to multilayer is straightforward. This is because normal metals are typically in the regime where the drift-diffusion equation is valid. Thus, one can write down the drift-diffusion equation in each layer and apply the modified BC [Eq. (2)] for all embedded magnetic layers. Second, consideration of the effects of interfacial spin-orbit coupling [22, 23, 53] goes beyond the scope of this paper. An additional consideration of the interface-generated spin current [54] would be a way to generalize the formalism.

III. PHYSICAL CONSEQUENCES

A. Transparency for injecting a spin Hall current

One of the most frequently performed experiments with ultrathin ferromagnets is injecting a spin Hall current to a ferromagnet to generate SOT [28, 32, 43, 55]. In Fig. 2(a), we consider a magnetic trilayer consisting of a ferromagnetic layer (FM) sandwiched by two normal metal layers (NM1 and NM2): NM1([-d1, 0])/FM(0, t)/NM2([t, t + d2]). When an electrical current is applied along the $x$ direction in NM1, the spin Hall effect [56] generates a torque to FM. The injection efficiency is determined by the spin transparency proposed in Ref. [28]. As demonstrated in the previous paper and discussed in Sec. I if the ferromagnet is thick enough, the two interfaces do not communicate with each other; thus the transparency of the interface 1 is solely determined by the properties of NM1 and the spin mixing conductance of the interface 1. However, if the spin dephasing is not perfect, the situation is no longer as simple as the previous result.

To properly take into account the interaction between each interface, we solve the drift-diffusion equation for the trilayer. Taking notations in Ref. [19], the spin chemical potential satisfies the spin drift-diffusion equation [3]

$$\partial^2 \mu_s = \frac{\mu_s}{\lambda_s},$$

where $\lambda_s$ is the spin diffusion length for each normal metal ($i = 1, 2$). The spin currents flowing along $z$ in the normal metals are

$$\mathbf{j}_s(z) = \begin{cases} \frac{\sigma_1}{2e} \partial_z \mu_s - \sigma_{SH,1} E_x \hat{y} & \text{in NM1} \\ \frac{\sigma_2}{2e} \partial_z \mu_s & \text{in NM2} \end{cases},$$

where $\sigma_{SH,i}$ are the electrical conductivity and the spin Hall conductivity of each normal metal, and $E_x$ is the applied

3 In this paper, the longitudinal part of the equation is ignored and it does not affect the calculation of spin torque and spin pumping at all.
electric field in NM1 along the x direction. The gradient terms are the diffusion current and the additional electric field contribution is the spin Hall current. The spin Hall conductivity in NM2 contributes to the inverse spin Hall current discussed in Sec. III B. The set of BCs for the drift-diffusion equation is given by \( j_s(-d_1) = j_s(t + d_2) = 0 \) in addition to Eq. (2).

After solving Eq. (5), the SOT per unit area is calculated by the angular momentum conservation, \( \tau = (\hbar/2e) [j_s(0(t)) - j_s(t+)] \). From the explicit solution available in Appendix A, one can obtain

\[
\tau = \frac{\hbar E_x}{2e} \sigma_{SH,1} E_z \text{Re}[\text{Tr}_{eff,1} M \hat{y}],
\]

\[
T_{eff,1} = \frac{G_{T_1}^{-1}(t)T_2/T_2}{1 - G_{T_1}^{-1}(t)T_2/1}.
\]

where \( T_{eff,1} \) is the effective spin transparency for the interface \( 1 \), \( G_{T_1}^{-1}(t) = G_{T_1}^{-1}(t)/\sqrt{G_{T_1}^{-1}G_{T_2}^{-1}} \) is the normalized transmitted mixing conductance (dimensionless), \( T_i \) is the conventional spin transparency for the interface \( i = 1, 2 \),

\[
T_i = \frac{G_{T_i}^{-1} \tanh d_2/2\lambda_i}{G_{T_i}^{-1} \coth d_2/2\lambda_i}
\]

and the other transparency-like quantities are given by

\[
T_i' = \frac{G_{T_i}^{-1} \tanh d_2/2\lambda_i}{G_{T_i}^{-1} \coth d_2/2\lambda_i},
\]

\[
T_{12} = T_1 T_2 \coth d_2/2\lambda_i \tanh d_2/2\lambda_i.
\]

The effective spin transparency is the central result of this paper. In the expression of \( T_{eff} \), \( \xi(t) \) appears indirectly through \( G_{T_1}^{-1}(t) \). Note that Eq. (5b) restores the previously reported result \( T_{eff} = T_1 \) [28] for \( t \to \infty \) where \( \xi(t) \to 0 \). For later purpose, we also define \( T_{eff,2} \) and \( T_{eff,1} \) by the same way as Eq. (5b) except the exchange between subscripts 1 and 2.

For simplicity of analysis, we assume that \( G_{T_1}^{-1} \) are positive real numbers and \( |G_{T_1}^{-1}| < G_{T_2}^{-1} \) as considered in most experimental situations [48]. One can easily prove that \( |T_{eff}| \) is always smaller than \( T_1 \) (thus SOT cannot be enhanced) if \( G_{T_1}^{-1} \) is a positive real number. To mathematically show this, we use Eq. (6b) and verify that \( T_{eff,1} < T_1 \) holds if and only if \( T_2/T_2 > T_{12} \) (See Appendix [B] for proof). In addition, it is also easy to show that \( T_{12} < T_2/T_2 \) if \( G_{T_1}^{-1} \) is positive and real (See Appendix [C] for proof). This gives \( T_{eff,1} < T_1 \), concluding the proof. Therefore, SOT is unlikely to be enhanced for a positive and real \( G_{T_1}^{-1} \).

However, in more general cases that \( G_{T_1}^{-1} \) is not a positive real number, SOT can be enhanced. For metallic cases described by Eq. (4a), \( \text{Im}[G_{T_1}^{-1}] \) is on the same order of magnitude as \( \text{Re}[G_{T_1}^{-1}] \), thus SOT cannot be enhanced if \( G_{T_1}^{-1} \) is a positive real number. To mathematically show this, we use Eq. (6b) and verify that \( T_{eff,1} < T_1 \) holds if and only if \( T_2/T_2 > T_{12} \) (See Appendix [B] for proof). In addition, it is also easy to show that \( T_{12} < T_2/T_2 \) if \( G_{T_1}^{-1} \) is positive and real (See Appendix [C] for proof). This gives \( T_{eff,1} < T_1 \), concluding the proof. Therefore, SOT is unlikely to be enhanced for a positive and real \( G_{T_1}^{-1} \).

The enhancement of spin torque can be understood by Fig. 3(b). It clearly shows that, for some regions (\( t < \lambda_{dp} \)), the spin torque can be enhanced (\( \text{Re}[T_{eff,1} / T_1] > 1 \)) and there arises a nonnegligible field-like component of SOT (\( \text{Im}[T_{eff,1}] \neq 0 \)) even for \( \text{Im}[G_{T_1}^{-1}] = 0 \), which makes a qualitative difference from thick film cases.

The enhancement of spin torque can be understood by Fig. 3(b). For \( \lambda_{dp} < t < 2\lambda_{dp} \), Eq. (4a) has a negative real part, thus \( s_2(z = t) \) in Fig. 1(b) can be negative. Thus, the angular momentum transfer to the ferromagnet...
\[ s_x(z = 0) - s_x(z = t) \] is larger than \( s_x(z = 0) \). A recent experiment \[50\] also suggests that the negativity of \( G_T^{\pm} \) may enhance the SOT. In that experiment, the spin flip at \( z = t \) may result in \( s_x(z = t) \) being negative. This is an interfacial contribution (\( \text{Re} \{ G_T^{\pm} \} < 0 \) in our convention), while the enhanced spin transparency in Fig. 3(b) originates from the bulk contribution (\( \text{Re} \{ \xi(t) \} < 0 \)) not requiring such a special interface.

**B. Inverse spin Hall effect from NM2**

One of physical consequences that are absent for \( G_T^{\pm} = 0 \) but present for \( G_T^{\pm} \neq 0 \) is the inverse spin Hall current in NM2. As depicted in Fig. 3(a), when an electric field is applied in NM1, the injected spin Hall current from NM1 can reach \( z = t+ \) (blue) since the dephasing in the ferromagnetic bulk is not perfect. The nonzero spin current at \( z = t+ \) may give rise to an inverse spin Hall current along \( x \) in NM2 (green). To calculate the resulting charge current along \( x \), we assume that \( \mathbf{m} \) is perpendicular to the injected spin current \( \sigma_{SH,1} E_x \mathbf{y} \) since transport of a longitudinal spin in the ferromagnet is beyond the scope of this paper. The total inverse spin Hall current in NM2 is given by \( I_{\text{ISHE,2}} = W \int_{t^+}^{z_+} \sigma_{SH,2} E_x \mathbf{y} \cdot \hat{j}_x(z) dz \), where \( W \) is the width of the wire and \( \theta_{SH,i} = \sigma_{SH,i}/\sigma_i \) is the spin Hall angle, and \( \sigma_{SH,2} \) is the spin Hall conductivity of NM2. Using the solution in Appendix A for \( \hat{j}_x(z) \), we obtain

\[
I_{\text{ISHE,2}} = -\frac{\sigma_{SH,1} \sigma_{SH,2} E_x W}{2} \text{Re} \left[ \frac{G_T^{\pm} G_T^{\pm}}{G_T^{\pm} G_T^{\pm} - G_T^{\pm} G_T^{\pm}} \right].
\]

In Fig. 4, we plot \( I_{\text{ISHE,2}} \) as a function of thickness with using the ansatz Eq. 4a. It changes the sign at \( t = \lambda_{d_{ip}} \), since the dephasing in the ferromagnetic bulk is not perfect. The nonzero spin current at \( z = t+ \) may give rise to an inverse spin Hall current along \( x \) in NM2 (green). To calculate these currents, one needs to take into account the spin pumping currents as additional BCs. Taking the theory of spin pumping \[57\], we add

\[
e_j^{s, SP}(0-) = \frac{\hbar}{2} \text{Re}[\{G_T^{\pm} - G_T^{\pm}\} \mathbf{M} \times \partial_z \mathbf{m}],
\]

\[
e_j^{s, SP}(t+) = \frac{\hbar}{2} \text{Re}[\{G_T^{\pm} - G_T^{\pm}\} \mathbf{M} \times \partial_t \mathbf{m}],
\]

to Eqs. (2a) and Eq. (2b), respectively. Solving the same drift-diffusion equation [Eq. (5)] without the external electric field \( (E_x = 0) \), one obtains the spin current profile \( j_x(z) \) and the resulting inverse spin Hall currents in NM1 and NM2 given by \( I_{SP,i} = W \int_{t_0}^{z_m} \theta_{SH,i} \mathbf{y} \cdot j_x(z) dz \). After some algebra,

\[
I_{SP,i} = (-1)^i \frac{W \hbar}{4e} \sigma_{SH,i} \mathbf{m} \times \partial_t \mathbf{m} \cdot \text{Re} \{ T_{\text{eff},i}^* \mathbf{y} \}.
\]

The appearance of the same \( T_{\text{eff},i} \) is understood by the Onsager reciprocity of spin pumping and spin torque. The factor \((-1)^i\) is also understandable by Fig. 5 where the direction of the spin pumping current to NM1 and NM2 are opposite. The inverse spin Hall measurement of the spin pumping effect can give \( T_{\text{eff},i} \) separately.

**C. Spin pumping**

Spin pumping \[57, 58\] is another physical phenomenon in which the mixing conductances play an important role. It is frequently used for measuring the spin transparency \[59\], the spin Hall angle \[25, 60, 61\], and the spin diffusion length \[61, 62\]. Here, we examine the effect of a nonzero \( G_T^{\pm} \) on spin pumping for the geometry depicted in Fig. 5. In the presence of magnetization dynamics \( \partial_t \mathbf{m} \neq 0 \), angular momentum is pumped to both normal metals, as so-called the spin pumping currents (blue). These pumped currents generate measurable inverse spin Hall currents along the \( x \) direction in each normal metal, which are denoted by \( I_{SP,i} \) (green). To

![Fig. 5. Geometry of the spin pumping calculation considered in Eq. (8)]. In the presence of magnetization dynamics (black), the spin pumping currents are generated at both interfaces [denoted by \( j_x^{s, SP}(0-) \) and \( j_x^{s, SP}(t+) \), blue], which further generates the inverse spin Hall current in each layer (denoted by \( I_{SP,i} \)).
However, the enhanced Gilbert damping [57] from the spin pumping effect requires more careness. This is because the Gilbert damping enhancement $\Delta \alpha_{\text{SP}}$ is not strictly given by the Onsager reciprocity when the system consists of multiple sources (interfaces 1 and 2) of angular momentum pumping. To calculate $\Delta \alpha_{\text{SP}}$, we calculate the total angular momentum transfer per unit area as $\tau = \langle h/2e \rangle [j_s(0^-) - j_s(t^+)]$ and project $\tau$ to $m \times \partial_t m$ to obtain its coefficient. Neglecting the renormalization of the gyromagnetic ratio $[57]$, we obtain

$$\Delta \alpha_{\text{SP}} = \frac{\gamma \hbar^2}{8 M_s e^2 t} \text{Re} \left[ \sum_{i=1,2} \frac{\sigma_i T_{\text{eff},i} \coth \left( \frac{d_i}{2\lambda_i} \right)}{\lambda_i} \right], \quad (10)$$

where $\gamma$ is the gyromagnetic ratio and $M_s$ is the saturation magnetization. Note that $\Delta \alpha_{\text{SP}}$ is given by the sum of $T_{\text{eff}}$ for each interface with some weighting factors. Since the weighting factors $[(\sigma_i/\lambda_i) \coth(d_i/2\lambda_i)]$ for each interface are not identical, extracting $T_{\text{eff},i}$ from measurement of $\Delta \alpha_{\text{SP}}$ requires more experimental information.

### IV. SUMMARY

In summary, we consider the effects of a nonzero transmitted mixing conductance in the drift-diffusion formalism to allow for the finite rate of the spin dephasing in a ultrathin ferromagnetic whose thickness is not much larger than the spin dephasing length. Solving the drift-diffusion equation with a modified BC, we demonstrate that spin torque can be enhanced in thin films, because of rotation of an injected spin current in ferromagnetic metals. Moreover, a nonnegligible field-like SOT can arise even in the absence of the imaginary part of the conventional spin mixing conductance. We demonstrate these by simply introducing an effective spin transparency, which also appears in the expression of the spin pumping current and the resulting Gilbert damping enhancement. The effective spin transparency obtained here provides a simple and straightforward extension of the conventional BC of the drift-diffusion formalism.

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### Appendix A: Explicit solution of Eq. (5)

After solving Eq. (5), one obtains the chemical potential,

$$\mu_s(z) = \left\{ \begin{array}{ll}
2eE_x \lambda_1 \theta_{\text{SH,1}} \text{Re} \left[ \frac{\mathcal{T}_1}{G_1} \left( \frac{G_1}{2\lambda_1} \sinh \frac{z}{2\lambda_1} \cosh \frac{d_1}{2\lambda_1} + \frac{\sigma_1}{2\lambda_1} \sinh \frac{z + (d_1/2)}{2\lambda_1} \right) - \tilde{G}_T^{\uparrow\downarrow} T_{12} \frac{\sinh \frac{z}{2\lambda_1}}{\cosh \frac{d_1}{2\lambda_1}} \hat{M} \hat{y} \right] \right. & \text{in NM1,} \\
\frac{1}{2} e \sigma_{\text{SH,1}} E_x \cosh \frac{z - (t + d_2)}{\lambda_2} \text{csch}^2 \frac{d_2}{2\lambda_2} \text{Re} \left[ \frac{G_1^{\uparrow\downarrow} T_1 T_2}{G_1^{\uparrow\downarrow} G_2^{\downarrow\uparrow} - G_1^{\downarrow\uparrow} T_{12}} \hat{M} \hat{y} \right] & \text{in NM2,}
\end{array} \right. \quad (A1a)$$

and the current,

$$j_s(z) = \left\{ \begin{array}{ll}
\sigma_{\text{SH,1}} E_x \text{Re} \left[ \left( \frac{\mathcal{T}_1}{G_1} \left( \frac{G_1}{2\sinh^2 \frac{z}{2\lambda_1}} + \frac{\sigma_1}{2\lambda_1} \cosh \frac{z + (d_1/2)}{2\lambda_1} \right) - \tilde{G}_T^{\uparrow\downarrow} T_{12} \frac{\cosh \frac{z}{2\lambda_1}}{\sinh \frac{d_1}{2\lambda_1}} \right) \hat{M} \hat{y} \right] & \text{in NM1,} \\
-\frac{1}{4\lambda_2} \sigma_{\text{SH,1}} \sigma_2 E_x \sinh \frac{z - (t + d_2)}{\lambda_2} \text{csch}^2 \frac{d_2}{2\lambda_2} \text{Re} \left[ \frac{G_1^{\uparrow\downarrow} T_1 T_2}{G_1^{\uparrow\downarrow} G_2^{\downarrow\uparrow} - G_1^{\downarrow\uparrow} T_{12}} \hat{M} \hat{y} \right] & \text{in NM2.}
\end{array} \right. \quad (A1b)$$

### Appendix B: Condition for $|T_{\text{eff},i}| < T_1$ for a real $G_T^{\uparrow\downarrow}$

Provided that all the mixing conductances are real, all transparencies defined in Eq. (6) are real. We first look at the denominator of

$$\frac{T_{\text{eff,1}}}{T_1} = \frac{1 - \tilde{G}_T^{\uparrow\downarrow} (T_2/T_2')}{1 - \tilde{G}_T^{\uparrow\downarrow} T_{12}}. \quad (B1)$$
Note that we assume $|\tilde{G}_T^{1+}| < 1$ and $0 < T_{12} < 1$ (see Appendix [C]), we obtain

$$0 < \tilde{G}_T^{1+2}T_{12} < 1,$$

implying that the denominator is positive. Then we look at the numerator. By noting that

$$\left| \frac{G_T^{1+2}T_2}{T_2'} \right| < \left| \frac{G_T^{1+} \coth \frac{d_1}{\lambda_k} + \frac{\sigma_2}{2\lambda_k}}{G_T^{1+} \coth \frac{d_2}{\lambda_k} + \frac{\sigma_2}{2\lambda_k}} \right| < 1. \quad (B3)$$

Thus the numerator is also positive and $T_{\text{eff,1}} > 0$. Now we calculate

$$1 - \frac{T_{\text{eff,1}}}{T_1} = \frac{\tilde{G}_T^{1+2} \left( \frac{T_2}{T_2'} - T_{12} \right)}{1 - \tilde{G}_T^{1+2}T_{12}}. \quad (B4)$$

Since $T_{\text{eff,1}}$ is positive, $|T_{\text{eff,1}}| < T_1$ if and only if $1 - T_{\text{eff,1}}/T_1 > 0$. Since the numerator is positive, the sign of $1 - T_{\text{eff,1}}/T_1$ is determined by that of $T_2/T_2' - T_{12}$. Hence, we conclude that

$$T_{\text{eff,1}} > T_1 \quad \text{if and only if} \quad \begin{cases} T_{12} > T_2/T_2' \text{ for a positive real } G_T^{1+} \\ T_{12} = T_2/T_2' \quad \text{if } G_T^{1+} \text{ is positive and smaller than } G_T^{1+}. \end{cases} \quad (B5)$$

As a result, we obtain

$$T_{12} < 1 < T_2/T_2'$$

under our assumptions.

**Appendix C: Proof of $T_{12} < T_2/T_2'$ for a positive real $G_T^{1+}$**

First we define

$$\tilde{T}_i \equiv T_i \frac{\coth \frac{d_1}{\lambda_k} + \frac{\sigma_2}{2\lambda_k}}{\coth \frac{d_2}{\lambda_k} + \frac{\sigma_2}{2\lambda_k}} \quad (C1)$$

then $T_{12} = \tilde{T}_1 \tilde{T}_2$. Since $\sigma_i$ and $\lambda_i$ are positive, Eq. (C1) implies that $\tilde{T}_i < 1$ if $G_T^{1+}$ is positive and real. Therefore, we obtain $T_{12} < 1$.

Next we consider

$$\frac{T_2}{T_2'} = \frac{\coth \frac{d_1}{\lambda_k} + G_T^{1+} \frac{\sigma_2}{2\lambda_k}}{\coth \frac{d_2}{\lambda_k} + G_T^{1+} \frac{\sigma_2}{2\lambda_k}} > 1, \quad (C2)$$

if $G_T^{1+}$ is positive and smaller than $G_T^{1+}$.
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