Flavor-Tuned 125 GeV SUSY Higgs Boson at the LHC:
MSSM and NATURAL SUSY TESTS

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Abstract

We show that an enhanced two-photon signal of the Higgs boson, $h$, observed with 125 GeV mass by the ATLAS and CMS collaborations, can be obtained if it is identified principally with the neutral $H_0^u$ of the two Higgs doublets of minimal Supersymmetry. We focus on sparticles and the pseudoscalar Higgs $A$ at the TeV scale. The off-diagonal element of the $(H_0^u,H_0^d)$ mass matrix in the flavor basis must be suppressed, and this requires both a large Higgsino mass parameter, $\mu \sim$TeV, and large $\tan\beta$. A MSSM sum rule is derived that relates $\gamma\gamma$ and $b\bar{b}$ rates, and a $\gamma\gamma$ enhancement relative to the SM predicts the $b\bar{b}$ reduction. On the contrary, Natural SUSY requires $|\mu| \lesssim 0.5$ TeV, for which $\gamma\gamma$ is reduced and $b\bar{b}$ is enhanced. This conclusion is independent of the $m_A$-value and the SUSY quantum correction $\Delta_b$. Relative $\tau\bar{\tau}$ to $b\bar{b}$ rates are sensitive to $\Delta_b$. 

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A $\gamma\gamma$ enhancement of the 125 GeV Higgs boson signal relative to the Standard Model (SM) expectation has been reported by the ATLAS and CMS experiments at the LHC[1, 2]. We investigate this in the Minimal Supersymmetric Standard Model (MSSM) in the region of large $m_A \sim$TeV by flavor-tuning of the mixing angle $\alpha$ between two neutral CP-even Higgs flavor states $H^0_u$ and $H^0_d$, with the 125 GeV Higgs signal identified principally with $H^0_u$. Then, the $b\bar{b}$ decay, which is predicted to be dominant decay of the SM Higgs boson, is reduced and the cross sections of the other channels are correspondingly enhanced, except possibly the $\tau\tau$ channel. We relate the cross-section enhancements/suppressions in $\gamma\gamma/b\bar{b}/\tau\tau$ channels compared with those of the SM Higgs boson. We also consider the consequences for Natural SUSY[3]. Our focus is on a heavy pseudoscalar $A$ and large $\tan\beta \equiv \langle H^0_u \rangle / \langle H^0_d \rangle$, a region that has not yet been constrained by LHC experiments[5]. Light stau[6, 7] and light stop[8] scenarios that have been considered are outside of our purview.

**Ratios of the SUSY Higgs couplings to those of the SM Higgs** The SUSY Higgs mechanism is based on the two Higgs doublet model of type II[9–11] with the $H_u$ doublet coupled to up-type quarks and the $H_d$ doublet coupled to down-type quarks. After spontaneous symmetry breaking, the physical Higgs states are two CP-even neutral Higgs $h, H$, one CP-odd neutral pseudo-scalar $A$ and the charged Higgs $H^\pm$.

We focus on the CP-even neutral Higgs boson $h$ and $H$, which are related to the flavor eigenstates $H^0_u$ and $H^0_d$ by

$$\frac{h}{\sqrt{2}} = c_\alpha H^0_u - s_\alpha H^0_d, \quad \frac{H}{\sqrt{2}} = s_\alpha H^0_u + c_\alpha H^0_d,$$

(1)

where $H^0_{u,d}$ is the shorthand for the real part of $H^0_{u,d} - \langle H^0_{u,d} \rangle$. We use the notation $s_\alpha = \sin \alpha$, $c_\alpha = \cos \alpha$, and $t_\alpha = \tan \alpha$. Our interest is in large $\tan\beta$, $\tan\beta \gtrsim 20$, and in the decoupling regime with large $m_A$ for which $\alpha \simeq \beta - \pi/2$.

The ratios of the $h$ and $H$ couplings to those of the SM Higgs $h_{SM}$, denoted as $r^{h,H}_{PP} (\equiv g^{h,H}_{PP} / g_{h_{SM}PP})$, are given by

$$r^h_{VV} = s_{\beta-\alpha}, \quad r^h_{tt} = r^h_{cc} = \frac{c_\alpha}{s_\beta}, \quad r^h_{\tau\tau} = \frac{-s_\alpha}{c_\beta}, \quad r^h_{bb} = \frac{-s_\alpha}{c_\beta} \left[ 1 - \frac{\Delta_b}{1 + \Delta_b} \left( 1 + \frac{1}{t_\alpha t_\beta} \right) \right],$$

$$r^H_{VV} = c_{\beta-\alpha}, \quad r^H_{tt} = r^H_{cc} = \frac{s_\alpha}{s_\beta}, \quad r^H_{\tau\tau} = \frac{c_\alpha}{c_\beta}, \quad r^H_{bb} = \frac{c_\alpha}{c_\beta} \left[ 1 - \frac{\Delta_b}{1 + \Delta_b} \left( 1 - \frac{t_\alpha}{t_\beta} \right) \right],$$

(2)

where we include the 1-loop contribution $\Delta_b$ to the $b\bar{b}$ coupling. It is the $b$-quark mass
correction factor \[\Delta_b = \frac{\mu t_\beta}{2 \alpha_s m_\tilde{g}} \left[ \frac{2\alpha_s}{3\pi} \tilde{m}_\tilde{g} I(\tilde{m}_\tilde{g}^2, \tilde{m}_{\tilde{b}_1}^2, \tilde{m}_{\tilde{b}_2}^2) + \frac{h_t^2}{16\pi^2} a_t I(\tilde{\mu}^2, \tilde{m}_{\tilde{t}_1}^2, \tilde{m}_{\tilde{t}_2}^2) \right] \] (3)

The first(second) term of \(\Delta_b\) is due to the sbottom-gluino(stop-chargino) loop. We take \(M_{\text{susy}} = 1\) TeV and sparticle masses \(\tilde{m}\) in units of \(M_{\text{susy}}\). The top Yukawa coupling is \(h_t = \tilde{m}_t/v_u = \tilde{m}_t/(v s_\beta)\) and \(\tilde{m}_t = m_t(\tilde{m}_t) = 163.5\) GeV is the running top quark mass. \[14\]

We consider \(m_Q = m_U = m_D = M_{\text{susy}}\) for the squark masses in the third generation.

The off-diagonal element of the stop squared mass matrix is \(\tilde{m}_t X_t\) where the stop mixing parameter \(X_t\) is given by \(X_t = A_t - \mu/t_\beta\). These quantities are also defined in units of \(M_{\text{susy}}\) as \(a_t \equiv A_t/M_{\text{susy}}, \tilde{\mu} \equiv \mu/M_{\text{susy}},\) and \(x_t \equiv X_t/M_{\text{susy}} = a_t - \tilde{\mu}/t_\beta\). Our sign convention for \(\mu\) and \(A_t\) is the same as \[15\], which are opposite sign convention of \[16\]. We fix \(\tilde{m}_\tilde{g} = 2\), well above the current LHC reach, \(\tilde{m}_{\tilde{b}_1} = \tilde{m}_{\tilde{b}_2} = 1\), and \(\tilde{m}_{\tilde{t}_1} = 0.8, \tilde{m}_{\tilde{t}_2} = 1.2\). A stop mass difference \(m_{\tilde{t}_2} - m_{\tilde{t}_1} \geq 0.4\) TeV is chosen in accord with Natural SUSY prediction \[24\]. Then \(\Delta_b\) is well approximated numerically by

\[\Delta_b \simeq \frac{\mu t_\beta}{20} \left[ 0.26 + \left( \frac{0.09}{|\tilde{\mu}| + 0.6} - 0.003 \right) a_t \right],\] (5)

where the first and the second terms in the square bracket are the numerical values from the gluino and the chargino contributions respectively.

The chargino and neutralino masses have no special role except possibly in \(b \rightarrow s\gamma\) decay, but consistency with Natural SUSY has been found there \[14\]. Large \(m_A\) implies large charged Higgs \(H^+\) mass that suppresses the \(H^+\) loop contribution to \(b \rightarrow s\gamma\).

The \(gg, \gamma\gamma\) coupling ratios \(r_{gg, \gamma\gamma}^\phi\) for \(\phi = h, H, A\) relative to those of \(h_{\text{SM}}\) are \[17\]

\[r_{gg}^\phi = \frac{I_t^\phi r_{tt}^h + I_{bb}^\phi r_{bb}^h}{I_{tt}^t + I_{bb}^b}, \quad r_{\gamma\gamma}^\phi = \frac{7}{4} I_{WW}^\phi r_{VV}^h - \frac{4}{9} I_{tt}^\phi r_{tt}^h - \frac{1}{9} I_{bb}^\phi r_{bb}^h,\] (6)

where \(I_{WW,tt,bb}^\phi\) represent the triangle-loop contributions to the amplitudes normalized to the \(m_h \rightarrow 0\) limit \[18\].

The \(XX \rightarrow h \rightarrow PP\) cross section ratios \[17\] relative to \(h_{\text{SM}}\) are obtained from

\[\sigma_P \equiv \frac{\sigma_{PP}}{\sigma_{SM}} = \frac{\sigma_{XX \rightarrow PP}}{\sigma_{XX \rightarrow h_{\text{SM}} \rightarrow PP}} = \frac{|r_{XX} h_{PP}|^2}{R_h},\] (7)

\[R_h = \frac{\Gamma_h^{\text{tot}}}{\Gamma_{h_{\text{SM}}}^{\text{tot}}} = 0.57 |r_{bb}^h|^2 + 0.06 |r_{rr}^h|^2 + 0.25 |r_{VV}^h|^2 + 0.09 |r_{gg}^h|^2 + 0.03 |r_{cc}^h|^2,\] (8)
where $R^h$ is the ratio of the $h$ total width to that of $h_{SM}$, $\Gamma_{h_{SM}}^{\text{tot}} = 4.14$ MeV$^2$ for $m_h = 125.5$ GeV. The coefficients in Eq. (8) are the SM Higgs branching fractions. Here we have assumed no appreciable decays to dark matter.

**Sum rule of cross-section ratios** In the large $m_A$ region close to the decoupling limit, $\alpha$ takes a value

$$\alpha = \beta - \frac{\pi}{2} + \epsilon$$

with $|\epsilon| < \frac{\pi}{2} - \beta$. Then, the $r^h_{XX}$ of Eq. (2) are well approximated by

$$r^h_{VV} = 1, \quad r^h_{tt,cc} = 1 + \epsilon/t_\beta, \quad r^h_{\tau\tau} \simeq 1 - \epsilon t_\beta, \quad r^h_{bb} \simeq 1 - \frac{1}{1 + \Delta_b} \epsilon t_\beta.$$ 

through first order in $\epsilon$. The $r^h_{tt,cc}$ are close to unity because those deviations from SM are $t_\beta$ suppressed. Thus,

$$r^h_{gg} \simeq r^h_{\gamma\gamma} \simeq 1,$$

since the bottom triangle loop function $I^h_{bb}$ is negligible in Eq. (6). Only $r^h_{bb}, r^h_{\tau\tau}$ can deviate sizably from unity for large $m_A$ and large $\tan\beta$. Following Eqs. (7) and (8), the $\sigma_P \equiv \sigma_{PP}/\sigma_{SM}$ of the other channels are commonly reduced(enhanced) in correspondence with $r^h_{bb} > 1$ ($r^h_{bb} < 1$). We predict the cross sections relative to their individual SM expectations

$$\sigma_\gamma = \sigma_W = \sigma_Z = \frac{1}{0.6(r^h_{bb})^2 + 0.4},$$

and

$$0.4\sigma_\gamma + 0.6\sigma_b = 1$$

where the SM $bb$ branching fraction$^{22}$ is approximated as 60%. Equation (12) holds independently of the production process. Enhanced $\sigma_\gamma$ implies reduced $\sigma_b$, as well as enhanced $\sigma_W$ and $\sigma_Z$.

**Flavor-Tuning of mixing angle $\alpha$** Note that $r^h_{bb,\tau\tau} = 1$ in the exact decoupling limit $m_A \to \infty$ for which $\epsilon = 0$. Flavor-tuning of $\epsilon$ to be small but non-zero is necessary to obtain a significant variation of $r^h_{bb}$ from unity. Positive(negative) $\epsilon$ gives $bb$-reduction(enhancement).

The mixing angle $\alpha$ is obtained by diagonalizing the squared-mass matrix of the neutral Higgs in the $u, d$ basis. Their elements at tree-level are

$$(M^2_{ij})^{\text{tree}} = M^2_Z s^2_\beta + m^2_A c^2_\beta; \quad M^2_Z c^2_\beta + m^2_A s^2_\beta; \quad -(M^2_Z + m^2_A)s_\beta c_\beta$$  

(14)
for $ij = 11; 22; 12$, respectively, which gives $\epsilon < 0$ in all region of $m_A$. Thus, in order to get $b\bar{b}$-reduction, it is necessary to cancel $(M_{ij}^2)_{\text{tree}}$ by higher order terms $\Delta M_{ij}^2$.

In the 2-loop leading-log (LL) approximation the $\Delta M_{ij}^2$ are given by

$$M_{ij}^2 = (M_{ij}^2)_{\text{tree}} + \Delta M_{ij}^2$$

(15)

where

$$\Delta M_{11}^2 = \frac{3\bar{m}_t^4}{4\pi^2 v^2 s^2_\beta} \left[ t(1 - G_{\frac{1}{2}}^t) + a_t x_t (1 - \frac{a_t x_t}{12}) (1 - 2G_{\frac{1}{2}}^t) \right] - M_{Z}^2 s^2_\beta (1 - F_3)$$

$$\Delta M_{22}^2 = -\frac{\bar{m}_t^4}{16\pi^2 v^2 s^2_\beta} \left[ (1 - 2G_{\frac{1}{2}}^t) (x_t \bar{\mu})^2 \right]$$

$$\Delta M_{12}^2 = -\frac{3\bar{m}_t^4}{8\pi^2 v^2 s^2_\beta} \left[ (1 - 2G_{\frac{1}{2}}^t) (x_t \bar{\mu}) (1 - \frac{a_t x_t}{6}) \right] + M_{Z}^2 s_\beta c_\beta (1 - F_2)$$

(16)

where $F_l = 1/(1 + \frac{h_l^2}{8\pi^2} t)$ with $l = 3, \frac{3}{2}, \frac{9}{2}$ and $G_l = -\frac{1}{16\pi^2} (l h_l^2 - 32\pi\alpha_s)$ with $l = \frac{15}{2}, \frac{9}{2}$. The $F_l$ are due to the wave function (WF) renormalization of the $H_u$ field and the index $l$ is numbers of $H^0_u$ fields in the effective potential of the two Higgs doublet model. $F_3 \xi^4 \simeq F_2 \xi^3 \simeq F_{\frac{3}{2}} \xi^2 \simeq 1$ where $\xi$ is defined by $H_u(M_s) = H_u(\bar{m}_t) \xi$ where $\xi = F_4^{-1}$.

The parameter $\tan\beta = v_u / v_d$ is defined in terms of the Higgs vacuum expectation values $v_{u,d} = \langle H^0_{u,d} \rangle$ at the minimum of the 1-loop effective potential at the weak scale $\mu = \bar{m}_t$ and $v = \sqrt{v_u^2 + v_d^2} \simeq 174$ GeV, while $a_t, x_t, \bar{\mu}$ have scale $\mu = M_{\text{susy}}$. The relation $\cot\beta(\bar{m}_t) = \cot\beta(M_s) \xi^{-1}$ will be used in the following calculation.

Numerically $\alpha_s = \alpha_s(\bar{m}_t) = 0.109$ giving $-32\pi\alpha_s = -10.9$, while $h_t = \bar{m}_t / v = 0.939$ is small. $G_{\frac{3}{2}}, x_t = 0.0274, 0.0442$ and $t = \log(\frac{\sqrt{2} \text{TeV}}{\bar{m}_t})^2 = 3.62$; thus, $G_{\frac{3}{2}} t = 0.099$ and $2G_{\frac{1}{2}} t = 0.320$, and $F_3 = 0.892$.

In large $m_A$ limit, the $m_h^2$ expression is

$$m_h^2 = M_{Z}^2 c^2_\beta + \frac{3\bar{m}_t^4}{4\pi^2 v^2} \left[ t(1 - G_{\frac{1}{2}}^t) + (1 - 2G_{\frac{1}{2}}^t) (x_t^2 - \frac{x_t^4}{12}) \right] - M_{Z}^2 s^2_\beta (1 - F_3) - 2s^2_\beta c^2_\beta (1 - F_2)$$

(17)

where the Higgs WF renormalization factor $\xi$ is retained in the denominator of $F_3$. This $F_3$ factor is usually expanded to the numerator in 2LL approximation, and correspondingly $G_{\frac{3}{2}}$ and $G_{\frac{1}{2}}$ are replaced by $G_{\frac{3}{2}}$: $m_h^2 = M_{Z}^2 (1 - F_3) + 3\bar{m}_t^4 [t(1 - G_{\frac{3}{2}} t) + (1 - 2G_{\frac{3}{2}} t) (x_t^2 - \frac{x_t^4}{12})] - M_{Z}^2 s^2_\beta (1 - F_2)$. However, numerically Eq. (17) significantly increases $m_h$ at large $M_{\text{susy}}$ as shown in Fig. 11. Eq. (17) gives increasing $m_h$ as $M_{\text{susy}}$ increases up to $\sim 7$ TeV, while the usual formula with
FIG. 1. $M_{\text{susy}}$ dependence of Higgs mass $m_h$ by the improved formula Eq. (17) (solid black) in comparison with the one by the usual 2LL approximation (dashed blue) where the first $F_3$ is removed and $G_{15}^2$ and $G_{18}^2$ are replaced by $G_{33}^2$. In this illustration $x_t$ is taken to be $\sqrt{6}$ following the ”maximal-mixing” condition, and $\tan\beta = 20$.

The expansion approximated for $F_3$ gives decreasing $m_h$ when $M_{\text{susy}} > 1.3$ TeV and is not applicable at large $M_{\text{susy}}$.

The experimental $m_h$ determinations from the LHC experiments are [1, 2]

$$m_h = 125.3 \pm 0.4 \pm 0.4, \quad 126.0 \pm 0.4 \pm 0.4 \text{ GeV}$$

(18)

It seems unlikely that the central $m_h$ determination will change much with larger statistics because of the excellent mass resolution in the $\gamma\gamma$ channel. The experimental $m_h$ value is near the maximum possible value of $m_h$ in Eq. (17) and this constrains the value of $x_t$ to $|x_t| \simeq \sqrt{6}$, to maximize the term $x_t^2 - \frac{x_t^4}{12}$. This is known as ”maximal-mixing” in the stop mass-matrix [24]. In Eq. (17) we require $m_h \geq 124$ GeV. This implies

$$1.95(\equiv x_{t\text{min}}) < |x_t| < 2.86(\equiv x_{t\text{max}}),$$

(19)

where we should note that the positive $x_t$ branch is favored by the SUSY renormalization group prediction [24].
By using Eq. (15) the Higgs mixing angle $\alpha$ is determined from

$$
t_{2\alpha} = \frac{2M_{12}^2}{M_{22}^2 - M_{11}^2} \approx \frac{(m_\gamma^2 + M_{Z}^2)s_{2\beta} - 2\Delta M_{12}^2}{(m_\gamma^2 - M_{Z}^2)c_{2\beta} + (\Delta M_{11}^2 - \Delta M_{22}^2)} ,
$$

$$
\Delta M_{12}^2 \approx -\frac{\bar{\mu}}{\tan^2 \beta} x_t (1 - \frac{x_t^2}{6}) \frac{558 \text{GeV}^2 + 24 \cdot \frac{20}{\tan \beta} \text{GeV}^2}{}. \tag{20}
$$

Defining $z(= M_Z^2/m_A^2)$, $\delta(= \Delta M_{12}^2/m_A^2)$, and $\eta(= \frac{1}{2} (\Delta M_{11}^2 - \Delta M_{22}^2)/m_A^2)$, $\epsilon$ is simply given in the first order of $z$, $\delta$, and $\eta$ by

$$
\epsilon = -2z + \frac{\eta}{\tan \beta} + \delta . \tag{22}
$$

We note that $r_{bb}^h$ is related to $\epsilon$ through Eq. (15). With the $x_t$ constraint in Eq. (19), we can derive the allowed region of $r_{bb}^h$ for each $\bar{\mu}$-value. Correspondingly, the allowed regions of $\sigma_{\gamma} (= \sigma_{\gamma\gamma}/\sigma_{\text{SM}} = \frac{1}{0.6(r_{bb}^h)^2 + 0.4})$, $\sigma_b (= \sigma_{bb}/\sigma_{\text{SM}} = \frac{(r_{bb}^h)^2}{0.6(r_{bb}^h)^2 + 0.4})$ and $\sigma_{\tau} (= \sigma_{\tau\tau}/\sigma_{\text{SM}} = \frac{(r_{\tau\tau}^h)^2}{0.6(r_{bb}^h)^2 + 0.4})$

are given respectively by the two curves in Fig. 2 where we take $\tan \beta = 50$.

The condition $r_{bb}^h = 1$, or equivalently $\epsilon = 0$, $t_{2\alpha} = t_{2\beta}$, defines the boundary that separates the $\gamma\gamma$ enhancement and suppression in the parameter space.

$$
r_{bb}^h = 1 \leftrightarrow \epsilon = 0 \leftrightarrow \Delta M_{12}^2 = M_Z^2 s_{2\beta} - \frac{\Delta M_{11}^2 - \Delta M_{22}^2}{2} t_{2\beta} , \tag{23}
$$

This condition is independent of $m_A$ and the quantum correction $\Delta_b$.

$\Delta M_{12}^2 > M_Z^2 s_{2\beta} - \frac{\Delta M_{11}^2 - \Delta M_{22}^2}{2} t_{2\beta}$ gives $b\bar{b}$ reduction. Flavor-tuning (FT) with small $\alpha$ requires a cancellation of $(M_{12}^2)_{\text{tree}}$ by the loop-level $\Delta M_{12}^2$ contribution, which requires rather large values of $\bar{\mu}$ and $\tan \beta$. This possibility was raised in ref. [25].

The region of $\gamma\gamma$ enhancement does not overlap with the region $|\bar{\mu}| < 0.5$ of Natural SUSY for any value of $\tan \beta$ from 20 to 60. For $\tan \beta = 20$, $|\bar{\mu}| \gtrsim 2$ is necessary for $\gamma\gamma$ enhancement.

We give a benchmark point of the FT model in the MSSM (FT1) and two benchmark points of Natural SUSY (NFT1, NFT2).

$$
\begin{array}{cccccccc}
\bar{\mu} & \tan \beta & x_t & m_h(\text{GeV}) & \sigma_{\gamma} & \sigma_b & \sigma_{\tau} \\
\hline
\text{FT1} & -3 & 20 & -2.86 & 124 & 1.17 & 0.89 & 1.05 \\
\text{NFT1} & -0.5 & 20 & 2.70 & 125 & 0.84 & 1.11 & 1.04 \\
\text{NFT2} & -0.15 & 20 & 2.70 & 125 & 0.87 & 1.08 & 1.07 \\
\end{array}
$$

(24)

where $M_{\text{susy}} = 1 \text{ TeV}$ and $m_A = 0.5 \text{ TeV}$. The relevant sparticle masses are taken commonly with the values given above Eq. (5). The $m_h$ value is predicted by Eq. (17). We also note
that the predicted BF($B_s \rightarrow \mu^+\mu^-$) values of these benchmark points are consistent with the experimental measurement\cite{26} $\text{BF}(B_s \rightarrow \mu^+\mu^-) = \left(3.2^{+1.4}_{-1.2} \text{stat}^{+0.5}_{-0.3} \text{syst}\right) \times 10^{-9}$ within 2\sigma.

Natural SUSY predictions  
Natural SUSY always predicts $b\bar{b}$-enhancement and $\gamma\gamma$ reduction.\cite{29}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$m_A$ & $\sigma_\gamma$ & $\sigma_b$ & $\sigma_\tau$ \\
\hline
$m_A \geq 500$ GeV & 0.82 $\sim$ 0.91 & 1.06 $\sim$ 1.12 & 1.04 $\sim$ 1.08 \\
$m_A \geq 1000$ GeV & 0.95 $\sim$ 0.98 & 1.01 $\sim$ 1.03 & 1.01 $\sim$ 1.02 \\
\hline
\end{tabular}
\caption{}
\end{table}

Here we have taken $|\mu| \leq 500$GeV and the other parameters are fixed with the values given above Eq. (5).

Concluding remarks
We have explored the $\gamma\gamma$, $b\bar{b}$ and $\tau\tau$ signals in the MSSM, relative to SM, and also in Natural SUSY. In MSSM an enhancement in the diphoton signal of the 125 GeV Higgs boson relative to the SM Higgs can be obtained in a flavor-tuned model with $h = H_u$ provided that $|\mu|$ is large( TeV) and negative. A $\gamma\gamma$ enhancement is principally due to the reduction of the $b\bar{b}$ decay width compared to $h_{SM}$. The ratios of $WW^*$ and $ZZ^*$ to their SM values are predicted to be the same as that of $\gamma\gamma$. There is also a corresponding reduction of the $h$ to $\tau\tau$ signal. The Tevatron evidence of a Higgs to $b\bar{b}$ signal in $W +$ Higgs production\cite{28} does not favor much $b\bar{b}$ reduction. The flavor-tuning of the neutral Higgs mixing angle $\alpha$ requires a large $\mu \sim$TeV and large $\tan\beta$. For small $|\mu| \lesssim 0.5$ TeV of Natural SUSY, $\gamma\gamma$-suppression relative to the SM is predicted. Thus, precision LHC measurements of the $\gamma\gamma$, $W^*W$, $Z^*Z$ and $b\bar{b}$ signals of the 125 GeV Higgs boson can test MSSM models.

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[29] Here we note that as $\bar{\mu} = 0$, then $r_{bb} = 1 + 2M_Z^2/m_A^2 \to \sigma_{\gamma\gamma}/\sigma_{SM} = 1 - 2.4M_Z^2/m_A^2$, independently of $\tan\beta$. The smaller $m_A$ gives the larger suppression of $\sigma_{\gamma\gamma}$ [27]. Then, from the $\gamma\gamma$ deviation from unity, the $CP$-odd state mass $m_A$ could be estimated.
FIG. 2. $\bar{\mu}$ dependence of $\sigma_\gamma = \sigma_{\gamma\gamma}/\sigma_{\text{SM}}$ (upper panel), $\sigma_b = \sigma_{b\bar{b}}/\sigma_{\text{SM}}$ (middle panel), and $\sigma_b = \sigma_{b\bar{b}}/\sigma_{\text{SM}}$ (lower panel) for $m_A = 500$ GeV: Their allowed values are between the solid red curve (corresponding to $|x_t| = x_t^\text{max}$) and the dashed blue curve (corresponding to $|x_t| = x_t^\text{min}$). Left(Right) panels show negative(positive) $x_t$ region. Deviations from unity are enlarged for a large negative $\bar{\mu}$, but there the perturbative calculation is unreliable due to a large quantum correction.