PERFORMANCE OF SHARIAH-COMPLIANT EQUITY PORTFOLIO USING MODEL-BASED RETURN AND RISK ESTIMATION

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ABSTRACT

Establishing optimal allocation for different stocks in a portfolio via modern portfolio theory is highly depended on the accuracy of the return and risk estimation. For retail investors, technological advancement has made it possible for them to apply the complex estimation procedure for decision making. Therefore, this study aims to assess the mean-variance Shariah-compliant portfolio performance with model-based return and risk estimation. The methodology adopted is based on the implementation of ARMA and GARCH model, focused on the daily stock prices from the year 2011 until 2018. Further, we used one-step-ahead forecast for the best ARMA-GARCH model as well as an arithmetic mean and variance estimation to prepare the composition of diversified portfolio weights for top 10 constituent companies listed in FBM Hijrah Shariah (FBMHS) Index. We also measure out of sample performance in a constructed portfolio using Sharpe, Treynor and Jensen’s measures. The result shows that the stock allocation for the model-based portfolio is less diversified as compared to the non-model-based portfolio. The composition of the model-based portfolio weight is capable of achieving high annual returns which can compensate for high risk. The out of sample portfolio performance of both techniques is capable to outperform the FBMHS Index.

INTRODUCTION

Investment is an act of putting money into an asset with the hope to get the future return. However, take into account that these potential returns on investment usually come together with the risks that vary depending on the type of investment. Considering that both return and risk are strongly correlated, this implies that the greater the ability of an investment to achieve higher returns, the greater the risk associated with it. Hence, understanding the tradeoff between return and risk becomes a crucial part of investors in attaining the fine balance between the highest possible return and lowest possible risk. Although it is very essential to comprehend the return and risk tradeoff, evidently there is still a lot of misinterpretation among Malaysian investors in terms of their expectations of prospective annual returns and risk. As it shows that they have very poor financial literacy, so at that moment there
will be an initiative to improve this case, especially when Malaysians would like to encourage retail investors to be active in the stock market.

Above all, the first step that investors will take is the need to develop their knowledge of investment in order to get engaged in the stock market. This is certainly because involvement in investment activities without knowledge can give a negative impact to the portfolio return and risk. While the basic part is estimating return and risk because when investors want to allocate stocks in a portfolio, the return and risk first need to be understood as it is the input of portfolio allocation. Generally, the input estimation involves estimating the portfolio expected return and standard deviation. However, Bielstein and Hanauer (2019) claimed the difficulty to assess the expected return and its covariance matrix because both inputs are highly vulnerable to estimation errors. Thus, apart from concerning how much risk they are willing to bear due to the outcome uncertainty in the stock market, the investors also need to use an appropriate approach to improve the accuracy of the estimation in constructing the optimal portfolio. One approach that can be employed is by applying model-based estimation. A model is typically built in model-based estimation, where some dependent variables are represented as certain independent variables series. The model could then be used according to some assumptions to estimate the missing values of the dependent variables. This shows that the model-based solutions may have added value to fix measurement errors.

On the other hand, investors are advised to concentrate on the Islamic fund as it introduces lower portfolio risk (Shah, Iqbal, & Malik, 2012). However, the Shariah-compliant equity fund performance in Malaysia is said to be significantly underperforming their Islamic benchmarks (Hayat & Kraeussl, 2011). Thus, other than capturing the problem of return and risk estimations, we interested to utilize Shariah-compliant equities in examining the underperformance of Shariah-compliant equity funds in Malaysia as well as to help the Shariah-compliant investors to improve oneself selectivity skills. Mainly the objectives of this study are:

1. Modeling the mean and variance of Shariah-compliant stocks return using an Autoregressive Moving Average (ARMA) model and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model.

2. Acquiring the optimal Shariah-compliant portfolio weights using the model-based return and risk estimation of ARMA-GARCH model and the non-model-based of arithmetic mean-variance.

3. Comparing the out of sample performance in a constructed Shariah-compliant portfolio.

This paper has been designed as follows. The second part after this first introduction highlights a short review of the literature from prior studies. The methodology for this study is presented in the third part. Then, the fourth part provides the findings along
with the assessment of all modeling and estimation process. Lastly, the fifth part concludes this study.

**LITERATURE REVIEW**

**Mean-Variance Portfolio Optimization**

Mean-variance approach is broadly used among investors in analyzing their portfolio investment due to its simplicity and ease of derivation (Shalit & Yitzhaki, 2005). Therefore, many studies had been conducted using this concept to construct several optimal portfolios. Some had successfully come out with the optimal portfolio in yielding the most favorable return on investment for a specified amount of risk, such as the constructed portfolio by Ivanovic, Baresa, and Bogdan (2013) that comprised of two common assets sort (cash funds and equities) in the Croatian stock market. Similarly, Ivanova and Dospatliev (2017) had figured out the efficient portfolio that performs better than any domestic individual security traded on the Bulgarian Stock Exchange. However, the mean-variance portfolio optimization is not always possible to be used, especially in a limited number of accessible stocks and very low liquidity of emerging financial markets. García, González-Bueno, and Oliver (2015) analyzed the maximum portfolio performance using the Colombian stock market, each of the efficient frontiers indicated a major difference between the risk and return. On the other hand, Sun (2010) encourages Indonesia domestic investors in the constructed portfolio of Indonesian stock market because it poses a lower risk for a given return amount that suited to the risk-averse investor.

While the theory is intuitive and compelling, there are many practical issues related to applying this method, especially in the input estimation such as the expected return and risk. Early studies have already criticized the practice of using the historical average return and standard deviation as a proxy for the expected stock return and risk due to an estimation error (Best & Grauer, 1991; Chopra & Ziemba, 2013). Therefore, previous researchers proposed some techniques that can minimize the effect of large measurement errors on historical data returns. Some studies have focused on the minimum variance portfolio (MVP), however, is only effective for mean-variance if it is assumed that the expected return is the same for all stocks, which is unlikely to hold. To address this shortcoming, Bielstein and Hanauer (2019) used the implied cost of capital (ICC) as a proxy for expected returns. The resulting expected return is used to measure in a maximum approximation of the Sharpe ratio, which then allows the portfolio to outperform the MVP robustly. Regardless for some limitation in the study, at each point in time, the need to numerically solve a polynomial equation may cause the fund manager hard to implement the ICC for a large investment and the optimized portfolio can be too concentrated and involve quite so much turnover for certain fund managers. Therefore, pursuing other approaches to estimate the expected return and risk could be noteworthy.
MODELLING STOCK RETURN AND VOLATILITY

The accuracy of the Box–Jenkins ARMA model in empirical tests was debated for out-sample time series forecasting due to the high level of randomness and unconvinced pattern constancy. Hence, Makridakis and Hibon (1997) examined an Autoregressive Integrated Moving Average (ARIMA) model out-sample forecasting precision and found that the process of constructing the series stationary in its mean become the major problem. This is because ARMA models perform better than the corresponding time-series methods if alternative techniques are used in removing and extrapolating the trend in the data. The application of the ARIMA model is supported by Mondal, Shit, and Goswami (2014) that studied the ARIMA model efficiency for various Indian stock sectors and found that the precision of the ARIMA model in the equity prices prediction is exceeding 85%, means that ARIMA model provides sound predictability. Furthermore, the GARCH model is widely used by a large number of researchers for estimating volatility in considering various characteristics of the data (Akhtar & Khan, 2016; Fabozzi, Tunaru, & Wu, 2004). Abdalla (2012) used the GARCH model to analyze the variability in Saudi equity returns and it provides good evidence of the time persistence differing volatility. This is supported by Aktan, Korsakiené, and Smaliukiene (2010) where the GARCH models can also be used to model Baltic stock market daily returns.

PERFORMANCE OF SHARIAH-COMPLIANT EQUITIES

Previously, there have been many studies that concentrate not only on conventional equity fund performance but also on Islamic one. Such works are worth acknowledging, for example, research done by Kamil, Alhabshi, Bacha, and Masih (2014) that used FBM Shariah Index and FBM Hijrah Index to set a reference point for Islamic equity funds. The finding indicates that Islamic equity fund does not reach market benchmarks in term of the risk-adjusted performance. This is because only 1.95% of the fund managers have the skill to manage the portfolio, while luck is considered to be as much as 47%. However, the evidence of Islamic underperformance indices in the emerging market somehow drawn the attention of several academics with contradictory findings. Examples include Hoepner, Rammal, and Rezec (2011), which suggested the Shariah-compliant funds’ performance against equity market benchmark should not be concluded as underperforming anymore, especially in the emerging market. The Gulf Cooperation Council (GCC) countries and Malaysia, which are the major Islamic financial centers had Islamic funds that significantly outperformed international equity market benchmarks with competitive performance.

METHODOLOGY

TYPE AND SOURCES OF DATA

We use the daily stock prices of top 10 constituent companies with different sectors listed in the FBM Hijrah Shariah (FBMHS) index as following name: TENA, AXIA,
DSOM, MXSC, IOIB, PGAS, MISC, PEPT, KLKK and HTHB. The data have been partitioned into six subsections over the entire study period, on rolling forward after the first six months from January 2011 to December 2018. This measures the portfolio performance by rebalancing it semi-annually within five years, which is in line with the study made by Eaker and Grant (2002). Suppose the value of the stock return is formulated as the following equation:

\[ R_{it} = \ln \left( \frac{P_{it}}{P_{it-1}} \right) \]  

(1)

where,

\[ R_{it}: \text{stock } i \text{ return at time } t, \]

\[ P_{it}: \text{stock } i \text{ price at time } t. \]

**ARMA Model**

ARMA (p,q) model initially introduced by Box and Jenkins (1976) is an autoregressive, AR (p) and a moving average, MA(q) processes, which are combined in a general form as follows:

\[ R_{it} = \phi_1R_{it-1} + \cdots + \phi_p R_{it-p} + u_{it} + \theta_1 u_{it-1} + \cdots + \theta_q u_{it-q} \]  

(2)

where,

\[ q: \text{lag length of the moving average}, \]

\[ \phi_1...p: \text{coefficient of } R_{it-1...p}, \]

\[ \theta_1...q: \text{coefficient of } u_{it-1...q}. \]

The stock return in equation (2) has a future value that is the linear composition of its past and previous residuals. Sequences of residual, \( u_{it} \) is normally distributed with white noise (WN) that has zero mean and variance \( \sigma_i^2 \) as \( u_{it} \sim \text{WN}(0, \sigma_i^2) \).

However, most economic, and financial time series is trended and not stationary, which indicates non-constant mean in the series. Thus, according to Pindyck and Rubinfeld (2008), the raw data can be differentiated one or two times in order to induce stationarity, which then followed the ARMA model. The general ARMA model comprises ARMA (p, d, q) whereas d represents the degree of differentiation needed to render the series stationary.

**GARCH Model**

GARCH (p,q) model that was originally proposed by Bollerslev (1986) is the generalized form of the ARCH model. It includes the lagged conditional variance term as an autoregressive term. The GARCH (p, q) model satisfies the following general form:

\[ \sigma_{it}^2 = \omega_i + \sum_{j=1}^{q} \gamma_j u_{it-j}^2 + \sum_{k=1}^{p} \delta_k \sigma_{it-k}^2 + \varepsilon_t \]  

(3)

where,
\( \sigma^2_{it} \): conditional variance,
\( u^2_{it} \): squared of residual,
\( \omega_i \): constant coefficient,
\( \gamma_{ij} \): coefficient of \( u^2_{it-j} \),
\( \delta_{ik} \): coefficient of \( \sigma^2_{it-k} \),

The value of conditional variance, \( \sigma^2_t \), depends on the past value of the stocks, the lagged squared residual terms, and on the past value of itself. Sequences of residual, \( \varepsilon_t \), are independent and identically distributed (i.i.d) with mean zero and variance one.

**One-Step Ahead Estimation**

The ARMA and GARCH models are used to estimate the expected mean, \( \hat{\mu}_{it} = \hat{R}_i(k) \) and the expected variance, \( \hat{\sigma}^2_{it} = \hat{\sigma}^2_i(k) \) from the point prediction \( l \) for \( k \) period ahead. In the case of this study, a one-step ahead prediction of a day ahead was chosen, where \( k = 1 \).

**Portfolio Optimization Model**

The portfolio return \( R_p = x'R \), where \( x'e = 1 \) and the portfolio mean \( \mu' = (\mu_1t, ..., \mu_Nt) \) can be expressed by:

\[
\mu_p = E(R_p) = x'\mu
\]

(7)

where,
\( \mu_p \): portfolio mean,
\( E(R_p) \): expected of portfolio return,
\( \mu \): stock mean vector.

The covariance matrix \( \Sigma = (\sigma_{ij}) \) for \( i,j = 1, ... N \), where \( \sigma_{ij} = Cov(R_{it}, R_{jt}) \) and the portfolio variance can be expressed by:

\[
\sigma^2_p = x'\Sigma x
\]

(8)

where,
\( \sigma^2_p \): portfolio variance,
\( \Sigma \): covariance matrix.

The objective function is to maximize the return for a certain degree of risk, which is then used to achieve an efficient portfolio. The objective function is defined as:

\[
\max (x'\mu)
\]

(9)

subject to \( x'e = 1 \) and \( x'\Sigma x = \sigma^2_p \)

With an increase of 0.0001, the risk value is set by a specified level of risk that ranges within the lowest value of the stock standard deviation and its highest value in a
portfolio. All points will then be plotted in a combination of \((\mu_p, \sigma_p^2)\) to produce an efficient frontier.

**Out of Sample Portfolio Performance Evaluation**

A set of the market-adjusted, risk-adjusted rate of return measures is readily accessible to be used in the portfolio performance assessment. Apart from that, beta is calculated first as a portfolio’s systematic risk measurement relative to the entire market. Beta is computed as a variable represented below in the CAPM:

\[
\mu_p = R_f + [\beta_p (R_m - R_f)]
\]

(10)

where,

\(R_f\): risk free rate,
\(\beta_p\): portfolio beta.
\(R_m\): market return.

For the risk-free rate of Malaysian index, we use 3-month Kuala Lumpur Interbank Offered Rate (KLIBOR).

**Sharpe’s Measure**

Sharpe’s measure was developed by Sharpe (1966). In general, it would be preferable in such a way the larger the value of the Sharpe’s measure, which means the larger the risk premium for each risk unit. Sharpe’s measure can be defined as the formulation below:

\[
Sharpe's\ measure = \frac{\mu_p - R_f}{\sigma_p}
\]

(11)

where,

\(\sigma_p\): portfolio standard deviation.

**Treynor’s measure**

The measure of Treynor was suggested by Treynor (1965). The higher value of Treynor’s measure is preferred, which means the higher the risk premium for each unit of systematic risk. The Treynor’s measure is described as shown in the following formula:

\[
Treynor's\ measure = \frac{\mu_p - R_f}{\beta_p}
\]

(12)

**Jensen’s Measure (Jensen’s Alpha)**

Jensen (1968) proposed Jensen’s measure of portfolio performance particularly designed according to the CAPM by calculating the portfolio’s excess return. Jensen’s measure is formulated as shown in the equation:

\[
Jensen's\ measure = (\mu_p - R_f) - [\beta_p (R_m - R_f)]
\]

(13)
Jensen’s measure illustrates the relationship between both the actual return of the portfolio and its expected return. The preferred positive values suggest that the portfolio has received a return above its market-adjusted, risk-adjusted return.

**Holding Period Return (HPR)**

HPR is employed to calculate return measuring both periodic income and value changes. Nevertheless, we focus only on the capital appreciation component and therefore the dividend income is not included in the calculation. Then, the equation for determining the holding period return was as follows:

$$HPR = \frac{P_N - P_0}{P_0}$$

where,

- $P_N$: ending investment value,
- $P_0$: beginning investment value.

**RESULTS AND FINDINGS**

**ARMA Model Result**

Box and Jenkins introduced a three-stage technique to select a suitable (parsimonious) ARMA model. Firstly, in model identification, the data’s autocorrelation function (ACF) and partial autocorrelation function (PACF) are drawn to capture the time series stationary or non-stationary. To achieve stationarity, we found that most stock series need to be differentiated up to the degree of two and the study must, therefore, use the ARIMA model. This works according to the results of Ariyo, Adewumi, and Ayo (2014) where the ARIMA model was used to transform non-stationary data of Nokia and Zenith bank stock price index for the first difference. Secondly, model estimation gives a significant parameter for all stocks and Table I indicates the estimated ARIMA model parameters.

**Table I: Estimated ARIMA Model Parameters**

| Period          | TENA       | AXIA       | DSOM      | MXSC       | IOIB       |
|-----------------|------------|------------|-----------|------------|------------|
| Jan 2011–Dec 2015 | (0,2,2)    | (0,2,2)    | (0,1,1)   | (1,1,1)    | (1,0,1)    |
|                 | [-1.96, 0.96] | [-1.97, 0.97] | [-1.00]   | [-0.17, -1.00] | [0.66, -0.75] |
|                 | (0,1,1)    | (0,1,1)    | (0,1,1)   | (1,1,1)    | (1,0,1)    |
| Jul 2011–Jun 2016 | [-1.00]    | [-1.00]    | [-1.00]   | [-0.10, -1.00] | [-1.00]    |
|                 | (0,1,1)    | (0,2,2)    | (0,1,1)   | (1,1,1)    | (1,0,1)    |
|                 | (0,1,1)    | (0,2,2)    | (0,1,1)   | (2,1,2)    | (0,2,2)    |
| Jan 2012–Dec 2016 | [-1.00]    | [-1.97, 0.97] | [-1.00]   | [-0.11, -1.00] | [-1.00]    |
|                 | (1,1,1)    | (2,1,2)    | (1,2,2)   | (1,1,1)    | (1,1,1)    |
| Jul 2012–Jun 2017 | [-1.00]    | [-1.97, 0.97] | [-1.00]   | [-0.11, -1.97, 0.97] | [-1.98, 0.98] |
|                 | (1,1,1)    | (2,1,2)    | (2,1,2)   | (1,1,1)    | (1,1,1)    |
| Jan 2013–Dec 2017 | [0.05, -1.00] | [-1.99, 0.99] | [-0.08, -1.95, 0.95] | [-0.11, -1.00] | [-0.06, -1.00] |
|                 | (0,1,1)    | (0,1,1)    | (1,1,1)   | (1,1,1)    | (1,1,1)    |
| Jul 2013–Jun 2018 | [-1.00]    | [-1.00]    | [-1.00]   | [-0.12, -1.00] | [-0.05, -1.00] |

*Note:* ( ) is the ARIMA model structure, [ ] is the ARIMA model parameters coefficient.
Table I: Estimated ARIMA Model Parameters, Continued

| PGAS     | MISC   | PEPT   | KLKK   | HTHB   |
|----------|--------|--------|--------|--------|
| (0,2,2)  | (0,2,2)| (0,2,2)| (0,1,1)| (0,1,1)|
| [-1.96, 0.96] | [-1.91, 0.91] | [-1.98, 0.98] | [-1.00] | [-1.00] |
| (0,1,1)  | (0,2,2) | (0,2,2) | (0,1,1) | (0,1,1) |
| [-1.00]  | [-1.95, 0.95] | [-1.99, 0.99] | [-1.00] | [-1.00] |
| (1,1,1)  | (0,2,2) | (1,1,1) | (0,1,1) | (0,2,2) |
| [-0.06, -1.00] | [-1.93, 0.93] | [-0.06, -1.00] | [-1.00] | [-1.99, 0.99] |
| (1,1,1)  | (0,1,1) | (1,1,2) | (2,1,1) | (0,1,1) |
| [-0.08, -1.00] | [-1.00] | [-0.07, -1.00] | [-0.06, -0.06, -1.00] | [-1.00] |
| (1,1,2)  | (0,1,1) | (2,1,1) | (2,0,2) | (1,1,1) |
| [0.66, -1.77, 0.77] | [-1.00] | [-0.10, -0.06, -1.00] | [0.35, 0.53, -0.41, -0.55] | [0.05, -1.00] |
| (1,1,2)  | (0,2,2) | (1,0,1) | (1,1,2) | (0,1,1) |
| [0.63, -1.75, 0.76] | [-1.97, 0.97] | [0.61, -0.73] | [0.63, -1.69, 0.69] | [-1.00] |

The elements can be described as follows. For example, ARMA (0,1,1) of DSOM shows that the present value of its stock return is conditional on the value of the immediate past error in lagged 1. Thus, the best ARIMA model’s equation for DSOM in the first sub-period can be expressed as below:

\[ Y_t = -1.00 \theta_1 u_{it-1} \] (15)

Remember that a parsimonious model is needed, so there might be a problem of overfitting here. So thirdly, the models are tested whether or not they meet the diagnostic test. The model diagnostic checking is tested by the Q-statistics of the correlograms of residuals for lag 8, 16 and 24 and Ljung-Box test hypotheses. The result shows that all stocks have insignificant lags for 5% significant level indicating that the residuals are not correlated, and the selected ARIMA model's residuals are white noise. After all the steps in assessing the appropriate ARIMA models have been accomplished, the models are used to forecast one-step ahead of the expected mean.

**GARCH Model Result**

One of the most important issue before applying GARCH is to first examine the heteroskedasticity phenomenon in the return series residuals using ARCH Lagrange Multiplier test. Rejecting the null hypothesis is proof of the existence of ARCH effects at the mean equation in the residual sequence and therefore the variation in the sequence of returns for all stocks is inconsistent.

GARCH (1,1) model is used to model the stock returns volatility. Table II displays the estimation of GARCH (1,1) model regarding variance parameters of all stocks.

Table II: Estimated GARCH(1,1) Model Parameters

| Period   | Element | TENA | AXIA | DSOM | MXSC | IOIB | PGAS | MISC | PEPT | KLKK | HTHB |
|----------|---------|------|------|------|------|------|------|------|------|------|------|
| Jan 2011–2012 | ARCH   | 0.45* | 0.18* | 0.03* | 0.34* | 0.11* | 0.34* | 0.47* | 0.39* | 0.09* | 0.15* |
| Dec 2015  | GARCH  | 0.29* | 0.79* | 0.95* | 0.16* | 0.84* | 0.52* | 0.40* | 0.20* | 0.83* | 0.75* |
| Jul 2011–2012 | ARCH | 0.06* | 0.06* | 0.03* | 0.09* | 0.17* | 0.16* | 0.30* | 0.39* | 0.31* | 0.16* |
| Jun 2016  | GARCH  | 0.91* | 0.93* | 0.95* | 0.60* | 0.81* | 0.74* | 0.46* | 0.08* | 0.35* | 0.75* |
|           | ARCH   | 0.12* | 0.11* | 0.02* | 0.37* | 0.13* | 0.15* | 0.30* | 0.11* | 0.11* | 0.42* |
Tocks are relatively persistent parameters. After the model has only three estimated parameters, the predictor variables are constant with ARCH (1) and GARCH (1). This makes, for instance, the variance equation for GARCH (1,1) of DSOM be displayed as follows:

\[ \sigma_t^2 = 0.00 + 0.03u_{t-1}^2 + 0.95\sigma_{t-1}^2 \] (16)

Besides, the number of the estimates for both ARCH and GARCH is rather equivalent to one, suggesting that the volatility stocks are relatively persistent. After the estimation of GARCH (1,1) parameters, it is used to predict one step ahead of the expected variance.

**PORTFOLIO OPTIMIZATION RESULT**

With the goal of maximizing return by creating optimal portfolios, weight heavily those Shariah-compliant stocks that generate the highest return. The best portfolio weight for both model-based and non-model-based portfolio is shown in Table III and IV accordingly.

| Period     | TENA | AXIA | DSOM | MXSC | IOIB | PGAS | MISC | PEPT | KLKK | HTHB |
|------------|------|------|------|------|------|------|------|------|------|------|
| Jan 2012–  | 12.70| 0.00 | 0.00 | 26.09| 31.28| 1.84 | 0.00 | 0.00 | 28.09|      |
| Dec 2015   |      |      |      |      |      |      |      |      |      |      |
| Jul 2011–   | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 26.32| 0.00 | 0.00 | 73.68|      |
| Jun 2016   |      |      |      |      |      |      |      |      |      |      |
| Jan 2012–   | 0.00 | 0.00 | 22.11| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 77.89|      |
| Dec 2016   |      |      |      |      |      |      |      |      |      |      |
| Jul 2012–   | 0.00 | 0.00 | 0.00 | 0.00 | 17.77| 0.00 | 0.00 | 0.00 | 82.23|      |
| Jun 2017   |      |      |      |      |      |      |      |      |      |      |
| Jan 2013–   | 8.41 | 84.57| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 7.02 |      |
| Dec 2017   |      |      |      |      |      |      |      |      |      |      |
| Jul 2013–   | 88.76| 11.24| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |      |
| Jun 2018   |      |      |      |      |      |      |      |      |      |      |

With the total allocation of 100%, Table III reveals that the stock distribution is less diversified and is most likely to have a concentrated percentage of stock. Most
portfolios assign only two stocks over the entire period, with the exception of the first and fifth sub-period. This shows that there will be high risk in holding the portfolio and makes the aggressive investor expecting more returns. Similarly, the finding made by Ivanovic et al. (2013) shows that high-risk tolerance investors may choose a portfolio with a greater risk level that brings a better return as well. The plot points in Fig. 1 illustrate the efficient frontier and graphically define the model-based portfolio during the first sub-period. The line curve denotes the portfolios that provide the best risk-return tradeoff.

![Efficient Frontier](image)

**Fig. 1:** Efficient Frontier for Model-Based Portfolio

The non-model-based estimation obtained different outcome from the model-based portfolio, which is presented in Table IV.

**Table IV:** Non-Model-Based Portfolio Weight (%)

| Period     | TENA | AXIA | DSOM | MXSC | IOIB | PGAS | MISC | PEPT | KLKK | HTHB |
|------------|------|------|------|------|------|------|------|------|------|------|
| Jan 2011–Dec 2015 | 12.38 | 5.76 | 20.91 | 21.79 | 0.00 | 13.06 | 0.00 | 0.00 | 0.95 | 25.15 |
| Jul 2011–Jun 2016   | 26.64 | 4.66 | 22.78 | 4.80 | 0.00 | 12.02 | 0.00 | 0.00 | 0.65 | 28.44 |
| Jan 2012–Dec 2016   | 29.02 | 0.00 | 13.25 | 13.61 | 3.16 | 11.56 | 3.68 | 0.00 | 5.02 | 20.70 |
| Jul 2012–Jun 2017   | 32.71 | 0.96 | 11.20 | 1.61 | 2.59 | 7.65 | 7.70 | 5.90 | 7.19 | 22.51 |
| Jan 2013–Dec 2017   | 32.61 | 0.00 | 6.47 | 7.69 | 5.77 | 1.46 | 8.57 | 10.27 | 5.62 | 21.55 |
| Jul 2013–Jun 2018   | 36.94 | 0.00 | 0.00 | 0.00 | 6.53 | 0.00 | 3.15 | 15.05 | 10.77 | 27.56 |

As can be seen, the stock allocation is more diversified as compared to the model-based portfolio and the amount of weight is less concentrated. The number of stocks allocated varies from six to ten stocks throughout the entire period. This ensures that
it can reduce market-related risks and create a more diversified portfolio in an attempt to minimize overall investment risk. Fig. 2 presents the efficient frontier for the non-based-model portfolio for the first sub-period.

![Efficient Frontier for Non-Model-Based Portfolio](image)

**Fig. 2: Efficient Frontier for Non-Model-Based Portfolio**

However, the optimum portfolio does not include stocks with the highest possible returns or stocks at low risk, this seeks to combine stocks with the highest potential returns with an adequate level of risk. Comparing Fig. 1 and 2, one can infer that the efficient frontier for the non-model-based portfolio reaches the highest return of 0.12% with a risk of 0.018%, and above this level, the return cannot be enhanced. Unless the investor disagrees with optimizing efficient frontier returns for the non-model-based portfolio, the investor might want an efficient frontier portfolio for the model-based portfolio, because it extends to the portfolio which has the highest return of 0.55%, and risk of 0.013%.

**OUT OF SAMPLE PORTFOLIO PERFORMANCE RESULT**

The returns from FBMHS are used as a benchmark market index to assess which portfolios are doing better than the market. Table V presents the constructed *Shariah*-compliant portfolios and the market performance measure.

| Portfolio                        | Model-Based | Non-Model-Based | FBMHS  |
|----------------------------------|-------------|-----------------|--------|
| Annualized Return (%)            | 9.76        | 2.33            | -2.61  |
| Annualized Standard Deviation (%)| 21.58       | 10.57           | 9.17   |
| Beta                             | 1.22        | 0.79            | 1.00   |
| Sharpe’s Measure                 | 0.29        | -0.12           | -0.67  |
| Treynor’s Measure (%)            | 5.09        | -1.54           | -6.16  |
| Jensen’s Measure (%)             | 0.05        | 0.01            | -      |

Based on Table V, the beta of the model-based portfolio is 1.22 and this shows that the model-based portfolio return is more responsive towards fluctuations in market returns. In comparison to the beta of 0.79 for the non-model-based portfolio, the
model-based portfolio’s higher beta indicates that its portfolio is riskier and moves in the same direction as the market with positive beta. Moreover, the model-based portfolio has a 9.76% higher return than a 2.33% non-model-based portfolio, but also a higher standard deviation of 21.58%. The non-model-based portfolio reported smaller expected returns, yet lesser standard deviation of 10.57%.

The risk-free rate is calculated as 3.56%. Here, the Sharpe’s measure of 0.29 for the model-based portfolio is higher than the measure of -0.67 for the FBMHS market portfolio, thus the model-based portfolio has superior performance. The same for the Sharpe’s measure of non-model-based portfolio of -0.12 that is larger than the market, however, the negative value indicates that it has a bad performance towards the negative risk premium. This result reflects that perhaps the risk premium for each risk unit of model-based portfolios looks better than the non-model-based portfolio and that of the market.

The fact that Treynor’s measure of 5.09% for the model-based portfolio is greater than the FBMHS market portfolio measure of -6.16% suggests that the model-based portfolio gives outperformance return and the portfolio would be considered superior to market performance. Besides, when compared with the measure of -1.54% for the non-model-based portfolio, the model-based portfolio has a higher value of the Treynor’s measure, indicating the higher the risk premium per unit of non-diversifiable risk that is better.

Even the market itself exhibited the negative measure of Sharpe and Treynor, which made the market index to be underperformed. In fact, some of the previous researchers had stated that the risk-adjusted measures would demonstrate outperformance of Islamic indices in developed markets while underperformance in emerging markets. Since Malaysia is part of the emerging market, such a result may be considered. This result contradicts with the finding made by Banani and Hidayatun (2017) where the Islamic indices performance has shown an outperformance and this be illustrated due to the reduction of non-systematic risk and systematic risk.

Furthermore, Table V gives 0.05% value of Jensen’s measure for the model-based portfolio. Despite its systematic risk as calculated by beta, the model-based portfolio achieved an excess return of 0.05 percentage points beyond that expected return. Apparently, on a risk-adjusted basis, the model-based portfolio has outperformed the market. Besides, the non-model-based portfolio also generates a positive value of 0.01% Jensen’s measure, but a little less than the model-based portfolio. But then regardless of the positive Jensen’s measure for these two portfolios, the values are too small and nearly to zero. This means that the portfolios had almost acquired exactly its required return.

**Holding Period Return (HPR)**

Table VI indicates the percentage of HPR.
The model-based portfolio had an annualized HPR of 10.96% higher than the non-model-based portfolio with annualized HPR of 3.90%. However, it is not possible to ensure that the return of the model-based portfolio is better than the non-model-based portfolio. This is because the HPR for the model-based portfolio is quite volatile, although it can generate up to 42.25% of the HPR, it must be wary to lose 24.40%. In the meantime, the HPR for the non-model-based portfolio is more stable over the six-month holding period, whereas the highest HPR it can achieve is 12.79% while the lowest loss it can bear is 10.69%.

Besides, using the transaction costs imposed by Elias, Razak, and Kamil (2014) at a rate of 0.60% of overall transaction value, the annualized HPR directly obtained by the investors is shown in the last row of Table VI. Even after considering transaction costs, a reasonable return can still be made by both portfolios with a positive return. This suggests that the transaction cost for the semi-annual rebalancing had no major effect on portfolio return. The constructed portfolio return is bigger than the transaction costs incurred for semi-annual rebalancing.

**CONCLUSION**

In this paper, we suggested the application of model-based return and risk estimation in portfolio optimization, which are using the ARMA and GARCH model. The ARIMA model showed the best mean model with a residual white noise and the GARCH (1,1) model get to explain the daily volatility of the FBMHS constituent stock returns. Furthermore, applying the one-step ahead of expected mean and variance from the ARMA-GARCH model, the model-based portfolio is found to be less diversified and most likely to have a concentrated stock weight as compared to the non-model-based portfolio using the arithmetic mean and variance estimation. Moreover, the portfolio performance evaluation is important for continuous monitoring and rebalancing of the constructed portfolio due to the uncertainty in the stock market. Thus, we apply three classical measurements of Sharpe, Treynor and Jensen’s measure to conduct an analysis of the out of sample performance. The finding reveals that the model-based portfolio has outperformed the non-model-based portfolio and market performance. Meanwhile, the holding period return (HPR) for model-based portfolio gives an extremely positive return as well as a deep downturn.

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**Table VI: Holding Period Return (%)**

| Portfolio                  | Model-Based | Non-Model-Based |
|----------------------------|-------------|-----------------|
| Jan 2016 – Jun 2016        | -9.17       | -10.69          |
| Jul 2016 – Dec 2016        | 8.06        | 2.15            |
| Jan 2017 – Jun 2017        | 42.25       | 9.90            |
| Jul 2017 – Dec 2017        | 37.01       | 12.79           |
| Jan 2018 – Jun 2018        | -24.40      | -0.56           |
| Jul 2018 – Dec 2018        | -6.41       | -0.60           |
| Annualized HPR             | 10.96       | 3.90            |
| Annualized HPR – Transaction Cost | 8.56  | 1.50            |
for a certain sub-period. This concludes that the model-based portfolio had given a riskier portfolio than the non-model-based portfolio, but then the risk taken is compensated with a higher return. The constructed Shariah-compliant portfolio using model-based return and risk estimation is therefore worthy of being practiced by the risk taker investors, while the risk-averse investor may remain loyal to the arithmetic mean and variance through its simplicity and stability.

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