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Multi-criteria decision making of COVID-19 vaccines (in India) based on ranking interpreter technique under single valued bipolar neutrosophic environment

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A B S T R A C T
COVID-19 is a respiratory infection caused by a coronavirus that spreads from person to person. In the present situation, the COVID-19 pandemic is a swiftly rising phase. Now the time is the second wave ending phase of coronavirus and the third wave coming phase of coronavirus in India. The pandemic situation is moving forward all over India. Nowadays, the worldwide COVID-19 pandemic structure is a very hazardous situation. The COVID-19 vaccine can suppress this situation and gain preventive measures against coronavirus. In producing the COVID-19 vaccine, the Indian medical board plays a significant role. The COVID-19 vaccines have exhibited 90%–95% efficacy in preventing symptomatic COVID-19 infections. Against COVID-19, for emergency purposes, the Indian medical board has approved three vaccines: Covishield, Covaxin, and Sputnik V. Generally, the Indian people are embarrassed about the vaccination of COVID-19. All people are thinking about which vaccine is best for them. This labyrinth can be evaluated effectively using the multi-criteria decision-making (MCDM) technique. Therefore, we have proposed a novel MCDM technique for selecting COVID-19 vaccines. The main aim of this paper is to develop an MCDM technique based on a \( \lambda \)-weighted ranking interpreter \((R_{\lambda^+}, R_{\lambda^-})\). The first time, we have defined positive and negative \( \lambda \)-weighted rank interpreter for the ranking of single-valued bipolar neutrosophic (SVbN) number. Additionally, positive and negative \( \lambda \)-weighted values and positive and negative \( \lambda \)-weighted ambiguity of an SVbN-number are formulated here. Some important, valuable theorems and corollary of SVbN-number are formulated. To show the applicability of the proposed MCDM technique, we have considered a real decision-making problem where ratings of the alternatives are with SVbN-numbers.

1. Introduction
In Dec 2019, the novel coronavirus disease was first detected in the Wuhan city of China, which was known as COVID-19. This coronavirus spread in the worldwide country except in Antarctica. WHO declared COVID-19 a global epidemic in March 2020. As of July 10, 2021, the worldwide confirmed cases around 191,112,014 and confirmed deaths of 4,105,121. India is a piece of the worldwide pandemic of the COVID-19. On January 30, 2020, India reported its first case of COVID-19 disease (2020). Recently, India has the maximum number of COVID-19 confirmed cases in Asia (2020). As of June 12, 2021, India had 29.3 million reported cases and the world’s second-highest number of confirmed cases(cf. Figs. 1 and 3). The third-highest number of COVID-19 deaths in India as of June 12, 2021. Due to the high population density of India, the spreading rate of coronavirus is significantly faster in other countries. Some essential measures are necessary for India to prevent the spread of coronavirus which are (1) wearing a face mask, (2) when you go outside, keep a safe distance from others person, (3) cleaning your hand with a sanitizer, (4) do not touch your face, (5) vaccination of COVID-19. In addition, three protocols are very significant in your life against the COVID-19 virus (cf. Fig. 2), which are (i) test when symptomatic, (ii) track the severity, (iii) treat with vaccination.

Due to the COVID-19 pandemic emergency purpose, the Indian medical government approved the three vaccines: Covishield, Covaxin, and Sputnik V. Because the vaccine can earn immunity against the coronavirus. Also, the vaccine plays a significant role in reducing the spread of COVID-19. On 16 Jan 2020, the Indian administration started the COVID-19 vaccination process. The Indian medical board has approved...
two vaccines for use in emergencies. The first one is Covishield, a brand of Oxford-Astra Zeneca vaccine manufactured by the Serum Institute of India. The second one is Covaxin which Bharat Biotech developed. The third vaccine is Sputnik V, a Russian vaccine, and the Indian government approved it in April 2021 (viz. Table 1). This vaccine will be locally distributed by Dr. Reddy's Laboratories in May 2021. So, these three vaccines are used against the COVID-19 virus. Although, the efficacy rate of the three vaccines is near about the same address by AIIMS. Generally, people are confused about taking the vaccine and which vaccine is better? On 6 June 2021, the All India Institute of Medical Sciences (AIIMS) addressed imprecise doubts concerning the COVID-19 vaccine. AIIMS Director declared that the efficacy of three vaccines is more or less equivalent. According to several criteria, the rate of success is different. We have ranked the three vaccines based on four criteria in this study. The characteristic of the three vaccines is given in Section 5. Here, we defined the decision-making problem in the selection of COVID-19 vaccines. We have introduced the ranking interpreter technique of SVbN-number. A novel MCDM technique has been developed and applied to the COVID-19 vaccine selection problem.
1.1. Literature review

In a short time, a number of studies have been done in this area of pandemic COVID-19. Ebrahim et al. (2020) described the different strategies the society could decrease the spread of coronavirus. Due to the pandemic situation, the government’s economic level is decreasing, and the imprecise measures that develop the economic level are proposed by Anderson et al. (2020). The different government strategies against the COVID-19 pandemic under the fuzzy TOPSIS method were studied by Alkan and Kahraman (2020). The consequence of policy and technological changes during the epidemic of COVID-19 was established by Goel et al. (2021). Petersen et al. (2020) studied an open, transparent data sharing path and avoided several regulatory standards of imprecise COVID-19 vaccines. Many researchers have studied the area of modelling an epidemic of the COVID-19 virus, such as Carli et al. (2020), Fang et al. (2020), Giordano et al. (2020) and Pare et al. (2020), Alkan and Kahraman (2021).

COVID-19 is a coronavirus-caused respiratory disease that spreads from person to person. In a short time, a number of studies have been directed on this pandemic virus. Using the MCDM technique, a limited number of COVID-19 studies exist in this literature. In choosing the best alternative from the set of alternatives, the MCDM technique can play a significant role. Due to the high level of applicability of the MCDM technique to real decision-making problems, in the last years, many researchers studied this area. Majumder et al. (2020) developed an important risk factor for COVID-19 using the TOPSIS method. Mohammed et al. (2020) proposed several diagnostic structures of COVID-19 based on the different criteria under the MCDM approach. Using the weighted Bonferroni mean operator, an MCDM technique was developed by Yang et al. (2020) and applied to antiviral mask selection over the COVID-19. Albahri et al. (2020) proposed a novel MCDM method for identifying the most appropriate convexesplasma. Hezametal et al. (2020) proposed a neutrosophic MCDM technique to determine the priority groups of the COVID-19 vaccine. TOPSIS method for evaluating government strategies against the COVID-19 epidemic under q-rung orthopair fuzzy sets, Alkan and Kahraman (2020). The comparative treatment of coronavirus with MCDM techniques was developed by Yildirim et al. (2021). Some author’s studies in this area like as Carli et al. (2020), Deng and Kong (2021), Ouyang et al. (2021), Pamucar et al. (2021) and Requia et al. (2020), Rahimi et al. (2021), Chen et al. (2020), WHO (2020).

In the decision-making (DM) models, sometimes the information of the decision-maker is uncertain type. Fuzzy sets (FSs) were introduced by Zadeh (1965), and it has a suitable mechanism to represent this kind of information. Later, Atanassov (1986) proposed an intuitionistic fuzzy set (IFS) which is an extension of FSs. After that, Smarandache (1999) represents the neutrosophic set that handles the hesitante, inconsistent, and indeterminate information of the DM problem. The simple representation of neutrosophic sets, which is known as single-valued neutrosophic (SVN) sets proposed by Wang et al. (2010). Zhang (1998) introduced a bipolar fuzzy (BF) set which is an independent extension of FSs. The recent real decision-making problem contains positive characteristics as well as negative characteristics.BF-environment is a suitable tool for handling this DM problem. BF-sets are an extension of FSs whose membership degree range is [−1,1]. A new MCDM method with BF-environment was proposed by Alghamdi et al. (2018). Novel distance measures of BF-number and their application to the MCDM problem were studied by Riaz et al. (2020). Akramy and Arshadz (2019) introduced a ranking method of trapezoidal bipolar fuzzy numbers based on total ordering. Ghanbari et al. (2018) proposed a new method for comparing bipolar LR fuzzy numbers. Chen et al. (2014) introduced a m-polar fuzzy set, an extension of BF-sets. In this study, he has proposed the [0,1]m-set (m-polar set), which is a more generalization of BF-sets, where m is an arbitrary ordinal number. Deli et al. (2015) proposed an MCDM problem under a bipolar neutrosophic (BN) environment. Biswas et al. (2016) used a TOPSIS method in a BN-environment to solve a (multi-attribute decision making) MADM problem. This study used a value and ambiguity index-based ranking method for single-valued trapezoidal neutrosophic numbers. Under the SVN environment, Deli and Subas (2017) proposed a MADM method based on values and ambiguity. Using the TP-based model transformation technique, Hedrea et al. (2021) presented a modelling approach for the nonlinear model of a TCR system. Precup et al. (2012) proposed two TSK fuzzy models to describe the dynamics specific to the position of SMA wire actuators. Precup et al. (2012) proposed a new stability analysis and convergence results for Takagi–Sugeno–Kang proportional–integral–fuzzy controllers’ Iterative Feedback Tuning. Jeong et al. (2010) uses machine-learning-based regression techniques to develop accurate architecture-level on-chip router cost models. Albu et al. (2019) presents some of the results obtained by the Process Control group of the Politehnica University Timioara, Romania, in artificial neural networks (ANNs) applied to modelling, prediction, and decision-making in medical systems. For the problem of sustainable supplier selection, Yazdani et al. (2021) proposed an interval-valued neutrosophic decision-making structure. Kwak et al. (2021) proposes an original fuzzy reasoning method for approximate inference in uncertain fuzzy systems. Garai and Garg (2021) defined the possibilistic ranking method of single-valued bipolar neutrosophic numbers and applied it to water resource management problems. Using the Bonferroni mean, Chiao (2021) proposed an multi-criteria decision making under interval type-2 fuzzy environment. Zakeri et al. (2021) presented a difficult MCDM problem involving the evaluation of suppliers as potential alternatives against a variety of criteria. Other researchers studied various decision-making methods; Dey et al. (2016), Dong et al. (2015), Wan and Xu (2017) and Zeng et al. (2014), Garai et al. (2020a), Garai et al. (2020b). However, no one can study the MCDM technique for the COVID-19 vaccine selection problem in an uncertain environment. Therefore in this paper, we have developed a novel MCDM technique based on rank interpreter under BN-environment and applied it to the selection of the COVID-19 vaccine.

In this study, we have developed the DM technique for the COVID-19 problem based on the interpreter ranking method. The 𝜆-weighted positive and negative values of SVBn-number are defined here. Also, we have proposed the 𝜆-weighted positive and negative ambiguity of SVBn-numbers. Some important, valuable theorems and corollary of SVBn-number are formulated here. The first time we have defined the DM problem of selecting COVID-19 vaccines. The proposed MCDM technique is applied in the selection of COVID-19 vaccines. Finally, this MCDM problem was solved under SVBn-environment. To demonstrate the utility of our proposed method, we have considered one numerical illustration. A comparative analysis of our proposed method with existing methods is studied here.

1.2. Motivation of the research

In Dec 2019, coronavirus was first detected in the Wuhan city of China. The common symptoms of the coronavirus are tiredness,
fever, sore throat, runny nose, dry cough, diarrhoea, etc. The COVID-19 pandemic is a swiftly rising phase in the present day. Due to handling the pandemic race of COVID-19, such effective measures are essential, which are wearing masks, hand hygiene, and staying at home. There is no worldwide approved particular antiviral medicine accessible for COVID-19. Against the COVID-19, many partial treatment processes have been used. Until now, no effective treatment can be found at this stage. The COVID-19 vaccine has a preventive measure against the COVID-19. Indian people are confused about the injection of the COVID-19 vaccine. Some researchers studied on COVID-19 vaccine, most of them using the usual decision-making technique, and anyone cannot draw the negative human mind. (Yildirim et al. (2021) evaluated COVID-19 treatment options using MCDM techniques (namely PROMETHEE and VIKOR). The COVID-19 vaccine alternatives ranking method using a neutrosophic TOPSIS was proposed by Helam et al. (2021). Ouyang et al. (2021) considered a decision Support algorithm for selecting an antivirus mask over COVID-19 under a fuzzy environment. Based on the AHP technique, Abdelwahab et al. (2021) proposed a vaccine selection decision-making method. Using a multi-criteria decision-making method, Sarwar et al. (2021) assessed vaccination willingness in response to COVID-19.

In India, three vaccines (Covishield, Covaxin, and Sputnik V) are approved by the Indian medical board to protect the COVID-19 (viz. Table 1). The efficacy rate of three vaccines is near about the same address by the All India Institute of Medical Sciences. Although, every vaccine has some positive and minor negative side effects on the human body. Due to positive and negative human thoughts, every person thinks about which vaccine is best for them? When human decision-making is based on double-sided or bipolar judgemental thinking on a positive and negative side. At that time, bipolar fuzzy information plays a vital role in human decision-making. Therefore, we have tried to develop a novel MCDM technique based on the ranking interpreters technique under a bipolar neutrosophic environment. Using the proposed MCDM technique, we can determine which is the best vaccine according to the criteria of nature. In this paper, according to the four criteria, we have ranked the three vaccines (Covishield, Covaxin, and Sputnik V) under a bipolar neutrosophic environment.

1.3. Novelty of the paper

Previously, few research works had been published in the area DM on the COVID-19 problem. Most of them proposed the usual decision-making technique for that kind of problem with a crisp environment. Till now, no one considered this kind of COVID-19 decision-making problem. However, many exciting decision-making techniques of COVID-19 decision making is still unknown. Our primary focus is on developing these unknown results, which are organized as follows:

* A novel λ-weighted interpreter ranking method of SVbN-number has been introduced.
* We have formulated the positive and negative λ-weighted values, positive and negative λ-weighted ambiguity for authenticity, hesitate, and falsity membership function of SVbN-number.
* We defined the decision-making problem for the selection of COVID-19 vaccines.
* The proposed MCDM technique was applied to the COVID-19 vaccine selection problem.
* The COVID-19 vaccine selection problem was solved under SVbN-environment.
* Sensitivity analysis was performed according to the value of λ, and one comparative analysis was studied here.

1.4. Structure of the paper

The rest of the paper is organized as follows: In Section 2, we provided some basic definitions and properties of single-valued bipolar neutrosophic (SVbN) numbers. In Section 3, we defined the novel interpreter ranking method of SVbN-numbers. Multi-criteria decision-making methods with SVbN information have been discussed in Section 4. In Section 5, we described the COVID-19 vaccine selection problem. In Section 6, we have exercised the MCDM technique in the selection of the COVID-19 vaccine. In Section 7, we performed the sensitivity analysis w.r.t. λ and comparative analysis. Finally, the conclusion and chance for future work plans are considered in Section 8.

2. Preliminaries

Definition 2.1. Let X be a universal set. A neutrosophic set (NS) (Deli & Subas, 2017) Ĉ over the set X is defined by $\hat{C} = \{(x, (T_C(x), I_C(x), F_C(x))) : x \in X\}$, where $T_C$ is the truth membership function, $I_C$ is the indeterminacy membership function and $F_C$ is the falsity membership function of the NS Ĉ over X. The range of these functions are respectively defined as $T_C : X \rightarrow [0^+, 1^+]$, $I_C : X \rightarrow [0^-, 1^-]$, $F_C : X \rightarrow [0^-, 1^+]$ such that $0^- \leq T_C(x) + I_C(x) + F_C(x) \leq 3^+$. 

Definition 2.2. Let X be a universal set. A single valued neutrosophic (SVNS) (Deli & Subas, 2017) Ĉ over X is a neutrosophic set over X. But $T_{\tilde{C}}(x), I_{\tilde{C}}(x)$ and $F_{\tilde{C}}(x)$ are defined as follow $T_{\tilde{C}} : X \rightarrow [0, 1]$, $I_{\tilde{C}} : X \rightarrow [0, 1]$, $F_{\tilde{C}} : X \rightarrow [0, 1]$ such that $0 \leq T_{\tilde{C}}(x) + I_{\tilde{C}}(x) + F_{\tilde{C}}(x) \leq 3$.

Definition 2.3. Single valued neutrosophic (SVN) number is a special (Deli & Subas, 2017) type NS on $\mathbb{R}$, which is defined as $\tilde{c}^0 = (c_1^0, c_2^0, c_3^0, u_{c_1}, u_{c_2}, u_{c_3}, y_{c_1}, y_{c_2})$ where $u_{c_1}, u_{c_2}, y_{c_1} \in [0, 1]$. The truth membership function $T_{\tilde{c}^0} : \mathbb{R} \rightarrow [0, 1]$, indeterminacy membership function, $I_{\tilde{c}^0} : \mathbb{R} \rightarrow [0, 1]$ and falsity membership function $F_{\tilde{c}^0} : \mathbb{R} \rightarrow [0, 1]$ and all membership functions are defined as:

$$T_{\tilde{c}^0}(x) = \begin{cases} T^*_1(x), & \text{if } c_1^0 \leq x < c_2^0 \\ u_{c_1}, & \text{if } c_2^0 \leq x \leq c_3^0 \\ T^*_2(x), & \text{if } c_3^0 < x < c_4^0 \\ 0 & \text{otherwise} \end{cases}$$

$$I_{\tilde{c}^0}(x) = \begin{cases} I^*_1(x), & \text{if } c_1^0 \leq x < c_2^0 \\ u_{c_1}, & \text{if } c_2^0 \leq x \leq c_3^0 \\ I^*_2(x), & \text{if } c_3^0 < x < c_4^0 \\ 1 & \text{otherwise} \end{cases}$$

$$F_{\tilde{c}^0}(x) = \begin{cases} F^*_1(x), & \text{if } c_1^0 \leq x < c_2^0 \\ y_{c_1}, & \text{if } c_2^0 \leq x \leq c_3^0 \\ F^*_2(x), & \text{if } c_3^0 < x \leq c_4^0 \\ 1 & \text{otherwise} \end{cases}$$

Definition 2.4. Let X be a universal set. A bipolar neutrosophic set (BNs) $\tilde{c}^b$ is an extension of neutrosophic set on $\mathbb{R}$, which is defined as $\tilde{c}^b = ((x; T^b_{\tilde{c}}(x), I^b_{\tilde{c}}(x), F^b_{\tilde{c}}(x), I^b_{\tilde{c}}(x), F^b_{\tilde{c}}(x), F^b_{\tilde{c}}(x)) : x \in X)$, where $T^b_{\tilde{c}} : \mathbb{R} \rightarrow [0, 1], I^b_{\tilde{c}} : \mathbb{R} \rightarrow [-1, 0], F^b_{\tilde{c}} : \mathbb{R} \rightarrow [-1, 0]$ represents the degree of confidence, $I^b_{\tilde{c}} : \mathbb{R} \rightarrow [0, 1], F^b_{\tilde{c}} : \mathbb{R} \rightarrow [-1, 0]$ represents the degree of hesitancy and $F^b_{\tilde{c}} : \mathbb{R} \rightarrow [0, 1], F^b_{\tilde{c}} : \mathbb{R} \rightarrow [-1, 0]$ represents the degree of falseness of the decisions maker.
2.1. Single valued trapezoidal bipolar neutrosophic number

Definition 2.5. A single valued trapezoidal bipolar neutrosophic (SVbN) number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$ is a special type neutrosophic set on $\mathbb{R}$, whose authenticity, hesitation and falsity membership functions (cf. Fig. 4) are defined as follows:

\[
T_{+}^{\tilde{w}}(x) = \begin{cases} 
\frac{x - c_{1}}{c_{2} - c_{1}}, & \text{if } c_{1} \leq x < c_{2} \\
\frac{0}{c_{2} - c_{1}}, & \text{otherwise}
\end{cases}
\]

\[
T_{-}^{\tilde{w}}(x) = \begin{cases} 
\frac{x - c_{2}}{c_{1} - c_{2}}, & \text{if } c_{2} \leq x \leq c_{3} \\
\frac{0}{c_{1} - c_{2}}, & \text{if } c_{3} < x \leq c_{4} \\
\frac{0}{c_{1} - c_{2}}, & \text{otherwise}
\end{cases}
\]

where $\vee = \max$, $\land = \min$.

Definition 2.7. Let $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$ be a single valued bipolar neutrosophic (SVbN) number. Then the positive and negative $a, b, \gamma$-cut sets of a SVbN number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$, denoted by $\tilde{w}_{a,b,\gamma}$, and $\tilde{w}_{a,b,\gamma}$ is defined as

\[
\tilde{w}_{a,b,\gamma}^{+} = \{x : T_{+}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq a, F_{-}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq \gamma, x \in \mathbb{R}\}
\]

and

\[
\tilde{w}_{a,b,\gamma}^{-} = \{x : T_{+}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq a, F_{-}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq \gamma, x \in \mathbb{R}\}
\]

where $a, b, \gamma$ lies in the regions $-1 \leq a \leq w_{\tilde{y}}, -u_{\tilde{y}} \leq b \leq 1$, $-u_{\tilde{y}} \leq \gamma \leq 1$ and $-1 \leq a + b + \gamma \leq 1$.

Definition 2.8. Let $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}} = \langle (c_{1}, c_{2}, c_{3}), \tilde{w}_{\tilde{y},\tilde{x},\tilde{z}} \rangle$ be a SVbN-number. Then, positive and negative $a$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$, denoted by $\tilde{w}_{a}^{+}$ and $\tilde{w}_{a}^{-}$ respectively, are defined as $\tilde{w}_{a}^{+} = \{x : T_{+}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq a, x \in \mathbb{R}\}$ for $a \in [0, w_{\tilde{y}}]$ and $\tilde{w}_{a}^{-} = \{x : T_{-}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq a, x \in \mathbb{R}\}$ for $a \in [-u_{\tilde{y}}, 0]$. Clearly, any positive and negative $a$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$ is a crisp subset of $\mathbb{R}$. Here positive and negative $a$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$ for authenticity membership function is a closed interval, which are denoted by $\tilde{w}_{+}^{a} = [c_{1}, c_{2}^{+}]$ and $\tilde{w}_{-}^{a} = [c_{1}, c_{2}^{-}]$.

Definition 2.9. Let $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}} = \langle (c_{1}, c_{2}, c_{3}), \tilde{w}_{\tilde{y},\tilde{x},\tilde{z}} \rangle$ be a SVbN-number. Then, positive and negative $b$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$, denoted by $\tilde{w}_{b}^{+}$ and $\tilde{w}_{b}^{-}$ respectively, are defined as $\tilde{w}_{b}^{+} = \{x : T_{+}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq b, x \in \mathbb{R}\}$ for $b \in [0, u_{\tilde{y}}]$ and $\tilde{w}_{b}^{-} = \{x : T_{-}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq b, x \in \mathbb{R}\}$ for $b \in [-u_{\tilde{y}}, 0]$. Clearly, any positive and negative $b$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$ is a crisp subset of $\mathbb{R}$. Here positive and negative $b$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$ is for hesitant membership function is a closed interval, which are denoted by $\tilde{w}_{+}^{b} = [c_{1}, c_{2}^{+}]$ and $\tilde{w}_{-}^{b} = [c_{1}, c_{2}^{-}]$.

Definition 2.10. Let $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}} = \langle (c_{1}, c_{2}, c_{3}), \tilde{w}_{\tilde{y},\tilde{x},\tilde{z}} \rangle$ be a SVbN-number. Then, positive and negative $\gamma$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$, denoted by $\tilde{w}_{\gamma}^{+}$ and $\tilde{w}_{\gamma}^{-}$ respectively, are defined as $\tilde{w}_{\gamma}^{+} = \{x : T_{+}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq \gamma, x \in \mathbb{R}\}$ for $\gamma \in [0, y_{\tilde{y}}]$ and $\tilde{w}_{\gamma}^{-} = \{x : T_{-}^{\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}} \geq \gamma, x \in \mathbb{R}\}$ for $\gamma \in [-y_{\tilde{y}}, 0]$. Clearly, any positive and negative $\gamma$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$ is a crisp subset of $\mathbb{R}$. Here positive and negative $\gamma$-cut set of a SVbN-number $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}$ is for falsity membership function is a closed interval, which are denoted by $\tilde{w}_{+}^{\gamma} = [c_{1}, c_{2}^{+}]$ and $\tilde{w}_{-}^{\gamma} = [c_{1}, c_{2}^{-}]$.

Theorem 2.1. Let $\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}} = \langle (c_{1}, c_{2}, c_{3}), \tilde{w}_{\tilde{y},\tilde{x},\tilde{z}} \rangle$ be a SVbN-number. Then \(\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}^{+} = \tilde{w}_{a}^{+} \cap \tilde{w}_{b}^{+} \cap \tilde{w}_{\gamma}^{+}\) and \(\tilde{w}_{\tilde{y},\tilde{x},\tilde{z}}^{-} = \tilde{w}_{a}^{-} \cap \tilde{w}_{b}^{-} \cap \tilde{w}_{\gamma}^{-}\) both are holds.

Proof. The result of theorem is trivial.
Example 2.1. Let $\varepsilon^{\alpha}(\langle c_1, c_2, c_3, c_4; \mu_{\alpha}, \eta_{\alpha}, \gamma_{\alpha} \rangle)$ be a SVbN number. From the above Definitions 2.8–2.10 the $\alpha$-cut, $\beta$-cut and $\gamma$-cut sets of SVbN number $\varepsilon^{\alpha}$ defined as:

\[
e^{\alpha}_{\alpha} = \left\{ c \mid \frac{c - c_1}{w_{\alpha}(c_1)} \leq \frac{1}{\alpha} \right\}
\]
\[
e^{\alpha}_{\beta} = \left\{ c \mid \frac{c - c_1}{w_{\alpha}(c_1)} \leq \frac{1}{\beta} \right\}
\]
\[
e^{\alpha}_{\gamma} = \left\{ c \mid \frac{c - c_1}{w_{\alpha}(c_1)} \leq \frac{1}{\gamma} \right\}
\]

And the negative values of the SVbN-number $\varepsilon^{\alpha}$ denoted as $V_{\alpha^{-}}(\varepsilon^{\alpha})$ for negative $\gamma$-cut set, is defined by

\[
V_{\alpha^{-}}(\varepsilon^{\alpha}) = \int_{-\gamma_{\alpha}}^{-1} (c^{\alpha}_{\beta} + c^{\alpha}_{\gamma}) h(\gamma) d\gamma
\]

where for $\gamma \in [-\gamma_{\alpha}, -1]$, $h(\gamma) = 0$ and $h(\gamma)$ is non-increasing.

Here, all values $V_{\alpha}, V_{\alpha^{-}}$, $V_{\beta}, V_{\beta^{-}}$ and $V_{\gamma}, V_{\gamma^{-}}$ are measure the uncertainties of SVbN-number $\varepsilon^{\alpha}$.

Definition 3.2. Let $\varepsilon^{\alpha}(\langle c_1, c_2, c_3, c_4; \mu_{\alpha}, \eta_{\alpha}, \gamma_{\alpha} \rangle)$ be a SVbN number. If positive $\alpha$-cut set, $\beta$-cut set and $\gamma$-cut set of the SVbN-number $\varepsilon^{\alpha}$ are the $\varepsilon^{\alpha}_{\alpha} = \left\{ c \mid c_1 + c_3 \right\}$, $\varepsilon^{\alpha}_{\beta} = \left\{ c \mid c_1 + c_2 \right\}$ and $\varepsilon^{\alpha}_{\gamma} = \left\{ c \mid c_1 + c_4 \right\}$ respectively, and negative $\alpha$-cut set, $\beta$-cut set and $\gamma$-cut set of the SVbN-number $\varepsilon^{\alpha}$ are the $\varepsilon^{\alpha}_{\alpha} = \left\{ c \mid c_1 - c_3 \right\}$, $\varepsilon^{\alpha}_{\beta} = \left\{ c \mid c_1 - c_2 \right\}$ and $\varepsilon^{\alpha}_{\gamma} = \left\{ c \mid c_1 - c_4 \right\}$ respectively. Then

(i) The positive ambiguities of the SVbN-number $\varepsilon^{\alpha}$ denoted as $A_{\alpha}(\varepsilon^{\alpha})$ for positive $\alpha$-cut set, is defined by

\[
A_{\alpha}(\varepsilon^{\alpha}) = \int_{0}^{w_{\alpha}(c_1)} (c^{\alpha}_{\alpha} - c^{\alpha}_{\beta}) f(a) da
\]

where for $a \in [0, w_{\alpha}(c_1)], f(a) \in [0, 1], f(0) = 0$ and $f(a)$ is non-decreasing.

And the negative values of the SVbN-number $\varepsilon^{\alpha}$ denoted as $V_{\alpha^{-}}(\varepsilon^{\alpha})$ for negative $\alpha$-cut set, is defined by

\[
V_{\alpha^{-}}(\varepsilon^{\alpha}) = \int_{-w_{\alpha}(c_1)}^{0} (c^{\alpha}_{\beta} - c^{\alpha}_{\gamma}) f(-a) da
\]

where for $a \in [-w_{\alpha}(c_1), 0], f(a) \in [0, 1], f(0) = 0$ and $f(a)$ is non-decreasing.

(ii) The positive ambiguities of the SVbN-number $\varepsilon^{\alpha}$ denoted as $A_{\beta}(\varepsilon^{\alpha})$ for positive $\beta$-cut set, is defined by

\[
A_{\beta}(\varepsilon^{\alpha}) = \int_{0}^{w_{\beta}(c_1)} (c^{\alpha}_{\beta} - c^{\alpha}_{\gamma}) g(\beta) d\beta
\]

where for $\beta \in [w_{\beta}(c_1), 1], g(\beta) \in [0, 1], g(0) = 0$ and $g(\beta)$ is non-increasing.

And the negative values of the SVbN-number $\varepsilon^{\alpha}$ denoted as $V_{\beta^{-}}(\varepsilon^{\alpha})$ for negative $\beta$-cut set, is defined by

\[
V_{\beta^{-}}(\varepsilon^{\alpha}) = \int_{-w_{\beta}(c_1)}^{0} (c^{\alpha}_{\gamma} - c^{\alpha}_{\beta}) g(-\beta) d\beta
\]

where for $\beta \in [-w_{\beta}(c_1), 0], g(\beta) \in [-1, 0], g(0) = 0$ and $g(\beta)$ is non-increasing.

(iii) The positive ambiguities of the SVbN-number $\varepsilon^{\alpha}$ denoted as $A_{\gamma}(\varepsilon^{\alpha})$ for positive $\gamma$-cut set, is defined by

\[
A_{\gamma}(\varepsilon^{\alpha}) = \int_{0}^{w_{\gamma}(c_1)} (c^{\alpha}_{\gamma} - c^{\alpha}_{\beta}) h(\gamma) d\gamma
\]

where for $\gamma \in [w_{\gamma}(c_1), 1], h(\gamma) \in [0, 1], h(0) = 0$ and $h(\gamma)$ is non-increasing.

And the negative values of the SVbN-number $\varepsilon^{\alpha}$ denoted as $V_{\gamma^{-}}(\varepsilon^{\alpha})$ for negative $\gamma$-cut set, is defined by

\[
V_{\gamma^{-}}(\varepsilon^{\alpha}) = \int_{-w_{\gamma}(c_1)}^{0} (c^{\alpha}_{\beta} - c^{\alpha}_{\gamma}) h(-\gamma) d\gamma
\]

where for $\gamma \in [-w_{\gamma}(c_1), -1], h(\gamma) \in [-1, 0], h(0) = 0$ and $h(\gamma)$ is non-increasing.

Corollary 3.1. Let $\varepsilon^{\alpha}(\langle c_1, c_2, c_3, c_4; \mu_{\alpha}, \eta_{\alpha}, \gamma_{\alpha} \rangle)$ is an any SVbN number. Then the positive values and ambiguity for $\alpha$-cut set, $\beta$-cut set and $\gamma$-cut set are calculated respectively, as follows:

(i) If we assume that $f(a) = a$, for $a \in [0, w_{\alpha}(c_1)]$ using Eq. (7) & (13) we can find the positive values and ambiguity for authenticity function of
the SVTbn-number $\tilde{c}_b^n$ respectively, as follows:

$$V_{T_+}(\tilde{c}_b^n) = \int_{0}^{\tilde{c}_b^n} \left( c_1 + \frac{a}{w_{j,n}}(c_2 - c_1) + c_2 - \frac{a}{w_{j,n}}(c_4 - c_3) \right) f(a)da$$

$$= \int_{0}^{\tilde{c}_b^n} \left( c_1 + c_2 + \frac{(c_2 - c_1 + c_2 - c_3)\alpha}{w_{j,n}} \right) ada$$

$$= \left[ \left( c_1 + c_2 + (c_2 - c_1 + c_2 - c_3)\alpha \right) \frac{1}{2} \right]_{0}^{w_{j,n}}$$

$$= \frac{1}{6}$$

and

$$A_{T_+}(\tilde{c}_b^n) = \int_{0}^{\tilde{c}_b^n} \left( c_2 - \frac{a}{w_{j,n}}(c_2 - c_1) - c_1 - \frac{a}{w_{j,n}}(c_4 - c_3) \right) f(a)da$$

$$= \int_{0}^{\tilde{c}_b^n} \left( c_2 - c_1 - \frac{(c_2 - c_1 + c_2 - c_3)\alpha}{w_{j,n}} \right) ada$$

$$= \left[ \left( c_2 - c_1 + (c_2 - c_1 + c_2 - c_3)\alpha \right) \frac{1}{2} \right]_{0}^{w_{j,n}}$$

$$= \frac{1}{6}$$

(ii) If we assume that $g(\beta) = 1 - \beta$, for $\beta \in [u_{j,n}, 1]$ using Eq. (9) & (15) we can find the positive values and ambiguity for hesitance function of SVTbn-number $\tilde{c}_b^n$ respectively, as follows:

$$V_{T_+}(\tilde{c}_b^n) = \int_{u_{j,n}}^{1} \left( c_2 - u_{j,n}c_1 - \beta(c_2 - c_1) \right) f(\beta)d\beta$$

$$= \int_{u_{j,n}}^{1} \left( c_2 - u_{j,n}c_1 - \frac{c_2 - u_{j,n}c_1 - \beta(c_2 - c_1)}{1 - u_{j,n}} \right) g(\beta)d\beta$$

$$= \left[ \left( c_2 - u_{j,n}c_1 \right) \frac{1}{1 - u_{j,n}} \right]_{u_{j,n}}^{1}$$

$$= \frac{1}{6}$$

and

$$A_{T_+}(\tilde{c}_b^n) = \int_{u_{j,n}}^{1} \left( c_2 - u_{j,n}c_1 - \beta(c_2 - c_1) \right) f(\beta)d\beta$$

$$= \int_{u_{j,n}}^{1} \left( c_2 - u_{j,n}c_1 - \frac{c_2 - u_{j,n}c_1 - \beta(c_2 - c_1)}{1 - u_{j,n}} \right) g(\beta)d\beta$$

$$= \left[ \left( c_2 - u_{j,n}c_1 \right) \frac{1}{1 - u_{j,n}} \right]_{u_{j,n}}^{1}$$

$$= \frac{1}{6}$$

(iii) If we assume that $h(\gamma) = 1 - \gamma$, for $\gamma \in [y_{j,n}, 1]$ using Eq. (11) & (17) we can find the positive values and ambiguity for falsity function of SVTbn-number $\tilde{c}_b^n$ respectively, as follows:

$$V_{T_+}(\tilde{c}_b^n) = \int_{y_{j,n}}^{1} \left( c_2 - y_{j,n}c_1 - y(\gamma(c_2 - c_1)) \right) f(\gamma)d\gamma$$

$$= \int_{y_{j,n}}^{1} \left( c_2 - y_{j,n}c_1 + \frac{c_2 - y_{j,n}c_1 + y(\gamma(c_2 - c_1))}{1 - y_{j,n}} \right) g(\gamma)d\gamma$$

$$= \left[ \left( c_2 - y_{j,n}c_1 \right) \frac{1}{1 - y_{j,n}} \right]_{y_{j,n}}^{1}$$

$$= \frac{1}{6}$$

and

$$A_{T_+}(\tilde{c}_b^n) = \int_{y_{j,n}}^{1} \left( c_2 - y_{j,n}c_1 - y(\gamma(c_2 - c_1)) \right) f(\gamma)d\gamma$$

$$= \int_{y_{j,n}}^{1} \left( c_2 - y_{j,n}c_1 - \frac{c_2 - y_{j,n}c_1 - y(\gamma(c_2 - c_1))}{1 - y_{j,n}} \right) g(\gamma)d\gamma$$

$$= \left[ \left( c_2 - y_{j,n}c_1 \right) \frac{1}{1 - y_{j,n}} \right]_{y_{j,n}}^{1}$$

$$= \frac{1}{6}$$

Corollary 3.2. Let $\tilde{e}_b^n = ((c_1, c_2, c_3, c_4, u_{j,n}, y_{j,n}, \beta, \gamma, \alpha)\cup \beta)$ is any SVTbn-number. Then the negative values and ambiguity for the $a$-cut set, $\beta$-cut set and $\gamma$-cut set are calculated respectively, as follows:

(i) If we assume that $f(a) = a$, for $a \in [-w_{j,n}, 0]$ using Eq. (8) & (14) we can calculate the negative values and ambiguity for authenticity function of the SVTbn-number $\tilde{c}_b^n$ respectively, as follows:

$$V_{T_+}(\tilde{c}_b^n) = \int_{-w_{j,n}}^{0} \left( c_2 - \frac{a}{w_{j,n}}(c_2 - c_1) + c_4 + \frac{a}{w_{j,n}}(c_4 - c_3) \right) f(-a)da$$

$$= \int_{-w_{j,n}}^{0} \left( c_2 - c_1 + \frac{(c_2 + c_1 + c_2 - c_3)\alpha}{w_{j,n}} \right) (-a)da$$

$$= \left[ \left( -c_2 + c_1 + c_2 - c_3 \right) \frac{1}{2} \right]_{-w_{j,n}}^{0}$$

$$= \frac{1}{6}$$

and

$$A_{T_+}(\tilde{c}_b^n) = \int_{-w_{j,n}}^{0} \left( c_2 - \frac{a}{w_{j,n}}(c_2 - c_1) + c_4 + \frac{a}{w_{j,n}}(c_4 - c_3) \right) f(-a)da$$

$$= \int_{-w_{j,n}}^{0} \left( c_2 - c_1 + \frac{(c_2 + c_1 + c_2 - c_3)\alpha}{w_{j,n}} \right) (-a)da$$

$$= \left[ \left( -c_2 + c_1 + c_2 - c_3 \right) \frac{1}{2} \right]_{-w_{j,n}}^{0}$$

$$= \frac{1}{6}$$

(ii) If we assume that $g(\beta) = 1 - \beta$, for $\beta \in [-1, -u_{j,n}]$ using Eq. (10) & (16) we can calculate the negative values and ambiguity for hesitance function of the SVTbn-number $\tilde{c}_b^n$ respectively, as follows:

$$V_{T_+}(\tilde{c}_b^n) = \int_{-1}^{-u_{j,n}} \left( c_2 - u_{j,n}c_1 + \beta(c_2 - c_1) \right) f(-a)da$$

$$= \int_{-1}^{-u_{j,n}} \left( c_2 - u_{j,n}c_1 - \frac{c_2 - u_{j,n}c_1 + \beta(c_2 - c_1)}{1 - u_{j,n}} \right) g(-\beta)d\beta$$

$$= \left[ \left( -c_2 + u_{j,n}c_1 \right) \frac{1}{1 - u_{j,n}} \right]_{-1}^{-u_{j,n}}$$

$$= \frac{1}{6}$$

and

$$A_{T_+}(\tilde{c}_b^n) = \int_{-1}^{-u_{j,n}} \left( c_2 - u_{j,n}c_1 + \beta(c_2 - c_1) \right) f(-a)da$$

$$= \int_{-1}^{-u_{j,n}} \left( c_2 - u_{j,n}c_1 - \frac{c_2 - u_{j,n}c_1 + \beta(c_2 - c_1)}{1 - u_{j,n}} \right) g(-\beta)d\beta$$

$$= \left[ \left( -c_2 + u_{j,n}c_1 \right) \frac{1}{1 - u_{j,n}} \right]_{-1}^{-u_{j,n}}$$

$$= \frac{1}{6}$$

and

$$\tilde{e}_b^n = ((c_1, c_2, c_3, c_4, u_{j,n}, y_{j,n}, \beta, \gamma, \alpha)\cup \beta)$$

is any SVTbn-number.
positive authenticity function, we can verify that 

\[ V_F(c^{\text{thm}}) > V_F(c^{\text{thm}}). \]

Likewise, from Eq. (10) it is clear that

\[ V_F(c^{\text{thm}}) = \int_{-1}^{-u(c)} e^{-1} g(\beta) d\beta \geq \int_{-1}^{-u(c)} 2c_g(\beta) d\beta \]

and

\[ V_F(c^{\text{thm}}) = \int_{-1}^{-u(c)} e^{-1} g(\beta) d\beta \leq \int_{-1}^{-u(c)} 2e_g(\beta) d\beta \]

For the assume condition \(u(c) = u(c)\), we have \(f^{\text{thm}}(\beta) = f^{\text{thm}}(\beta)\). Combining all these above results, and \(c_1 > e_1\). For the negative hesitate function, we can verify that \(V_F(c^{\text{thm}}) > V_F(c^{\text{thm}})\). From Eq. (11) & (12), we can easily verify that for the positive and negative hesitate function \(V_F(c^{\text{thm}}) > V_F(c^{\text{thm}})\) and \(V_F(c^{\text{thm}}) > V_F(c^{\text{thm}})\).

The proof of the theorem is completed.

**Theorem 3.2.** Let us assume that \(c^{\text{bthm}} = \langle (c_1, c_2, c_3, c_4; u(c), u(c), u(c), u(c)) \rangle\) and \(c^{\text{bthm}} = \langle (c_1, c_2, c_3, c_4; u(c), u(c), u(c), u(c)) \rangle\) are any two SVN-numbers with \(u(c) = u(c)\) and \(u(c) = u(c)\). If \(c^{\text{bthm}} > c^{\text{bthm}}\), then \(c^{\text{thm}} > c^{\text{thm}}\) for any SVN-number \(c^{\text{thm}}\).

**Proof.** According to Eq. (7), we can easily derived for the positive authenticity function \((T^+)\)

\[ V_T(c^{\text{thm}} + d^{\text{thm}}) = \int_{-1}^{u(c)} \langle c^{\text{thm}} + d^{\text{thm}} \rangle f(c) d\alpha \]

and

\[ V_T(c^{\text{thm}} + d^{\text{thm}}) = \int_{-1}^{u(c)} \langle c^{\text{thm}} + d^{\text{thm}} \rangle f(c) d\alpha \]

Here, \(V_T\) is the positive authenticity membership function of SVN-numbers \(c^{\text{thm}} + d^{\text{thm}} + c^{\text{thm}} + d^{\text{thm}}\). According to the assume condition \(u(c) = u(c)\) and \(u(c) = u(c)\), we have

\[ \int_{-1}^{u(c)} \langle c^{\text{thm}} + c^{\text{thm}} \rangle f(c) d\alpha > \int_{-1}^{u(c)} \langle c^{\text{thm}} + c^{\text{thm}} \rangle f(c) d\alpha \]

\[ \Rightarrow V_T(c^{\text{thm}} + d^{\text{thm}}) > V_T(c^{\text{thm}} + d^{\text{thm}}) \]

Similarly from Eq. (8), we can easily derived for the negative authenticity function \((T^-)\)

\[ V_T(c^{\text{thm}} + d^{\text{thm}}) = \int_{-1}^{u(c)} \langle c^{\text{thm}} + c^{\text{thm}} \rangle f(c) d\alpha \]

and

\[ V_T(c^{\text{thm}} + d^{\text{thm}}) = \int_{-1}^{u(c)} \langle c^{\text{thm}} + c^{\text{thm}} \rangle f(c) d\alpha \]

For the assume condition \(u(c) = u(c)\), we have \(f^{\text{thm}}(\beta) = f^{\text{thm}}(\beta) \)

Combining all these above results, and \(c_1 > e_1\). For the positive hesitate function, we can verify that \(V_T(c^{\text{thm}}) > V_T(c^{\text{thm}})\). Likewise, from Eq. (10) it is clear that

\[ V_T(c^{\text{thm}}) = \int_{-1}^{-u(c)} e^{-1} g(\beta) d\beta \geq \int_{-1}^{-u(c)} 2c_g(\beta) d\beta \]

and

\[ V_T(c^{\text{thm}}) = \int_{-1}^{-u(c)} e^{-1} g(\beta) d\beta \leq \int_{-1}^{-u(c)} 2e_g(\beta) d\beta \]

For the assume condition \(u(c) = u(c)\), we have \(f^{\text{thm}}(\beta) = f^{\text{thm}}(\beta) \). Combining all these above results, and \(c_1 > e_1\). For the negative hesitate function, we can verify that \(V_T(c^{\text{thm}}) > V_T(c^{\text{thm}})\). From Eq. (11) & (12), we can easily verify that for the positive and negative hesitate function \(V_T(c^{\text{thm}}) > V_T(c^{\text{thm}})\) and \(V_T(c^{\text{thm}}) > V_T(c^{\text{thm}})\). The proof of the theorem is completed.
Here, $V_{F+}$ is the negative authenticity membership function of SVbN-numbers $\tilde{c}^{bm} + d^{bn}$ and $\tilde{e}^{bm} + d^{bn}$. According to the assume condition $u_{\tilde{c}^{bm}} = u_{\tilde{d}^{bn}}$ and $\tilde{c}^{bm} > \tilde{e}^{bm}$, we have

$$\int_{-\infty}^{\infty} \left( e^{+}_a + e^{-}_a \right) f(-a) da > 0$$

Again from Eq. (9), we can easily derive the positive hesitative function ($F^+$)

$$V_{F^+}(\tilde{e}^{bm} + d^{bn}) = \int_{u_{\tilde{d}^{bn}} > u_{\tilde{e}^{bn}}} \left[ (e^{+}_a + e^{-}_a) (d^{+}_a + d^{-}_a) \right] g(\beta) d\beta$$

and

$$V_{F^+}(\tilde{e}^{bm} + d^{bn}) = \int_{u_{\tilde{d}^{bn}} > u_{\tilde{e}^{bn}}} \left[ (e^{+}_a + e^{-}_a) (d^{+}_a + d^{-}_a) \right] g(\beta) d\beta$$

Here, $V_{F^+}$ is the positive hesitative membership function of SVbN-numbers $\tilde{c}^{bm} + d^{bn}$ and $\tilde{e}^{bm} + d^{bn}$. According to the assume condition $u_{\tilde{c}^{bm}} = u_{\tilde{d}^{bn}}$ and $\tilde{c}^{bm} > \tilde{e}^{bm}$, we have

$$\int_{u_{\tilde{d}^{bn}} > u_{\tilde{e}^{bn}}} \left( e^{+}_a + e^{-}_a \right) g(\beta) d\beta > 0$$

Similarly from Eq. (10), we can easily derived for the negative authenticity function ($F^-$)

$$V_{F^-}(\tilde{e}^{bm} + d^{bn}) = \int_{-\infty}^{\infty} \left[ (e^{+}_a + e^{-}_a) (d^{+}_a + d^{-}_a) \right] g(-\beta) d\beta$$

and

$$V_{F^-}(\tilde{e}^{bm} + d^{bn}) = \int_{-\infty}^{\infty} \left[ (e^{+}_a + e^{-}_a) (d^{+}_a + d^{-}_a) \right] g(-\beta) d\beta$$

Here, $V_{F^-}$ is the negative hesitative membership function of SVbN-numbers $\tilde{c}^{bm} + d^{bn}$ and $\tilde{e}^{bm} + d^{bn}$. According to the assume condition $u_{\tilde{c}^{bm}} = u_{\tilde{d}^{bn}}$ and $\tilde{c}^{bm} > \tilde{e}^{bm}$, we have

$$\int_{-\infty}^{\infty} \left( e^{+}_a + e^{-}_a \right) g(-\beta) d\beta > 0$$

Likewise, from Eq. (11) & (12) it is easily verify that for the positive and negative hesitative function $V_{F^+}(\tilde{c}^{bm} + d^{bn}) > V_{F^+}(\tilde{e}^{bm} + d^{bn})$ & $V_{F^-}(\tilde{c}^{bm} + d^{bn}) > V_{F^-}(\tilde{e}^{bm} + d^{bn})$ holds.

The proof of the theorem is completed.

Definition 3.3. Let $\tilde{e}^{bm} = \{(c_1, c_2, c_3; u_{\tilde{c}^{bm}}, u_{\tilde{d}^{bn}}, y_{\tilde{e}^{bm}})\}$ be a SVbN-number. Then for any $\lambda \in [0, 1]$, we can constructed as:

(i) The $\lambda$-weighted positive and negative values of the SVbN-number $\tilde{c}^{bm}$ are defined as

$$V_{\lambda^+}(\tilde{c}^{bm}) = \lambda V_{F^+}(\tilde{c}^{bm}) + (1 - \lambda) V_{F^-}(\tilde{c}^{bm})$$

and

$$V_{\lambda^-}(\tilde{c}^{bm}) = \lambda V_{F^-}(\tilde{c}^{bm}) + (1 - \lambda) V_{F^-}(\tilde{c}^{bm})$$

(ii) The $\lambda$-weighted positive and negative ambiguity of the SVbN-number $\tilde{c}^{bm}$ are defined as

$$A_{\lambda^+}(\tilde{c}^{bm}) = A_{F^+}(\tilde{c}^{bm}) + (1 - \lambda) A_{F^-}(\tilde{c}^{bm})$$

and

$$A_{\lambda^-}(\tilde{c}^{bm}) = A_{F^-}(\tilde{c}^{bm}) + (1 - \lambda) A_{F^-}(\tilde{c}^{bm})$$

Definition 3.4. Let $\tilde{e}^{bm} = \{(c_1, c_2, c_3; u_{\tilde{c}^{bm}}, u_{\tilde{d}^{bn}}, y_{\tilde{e}^{bm}})\}$ be a SVbN-number. Then for any $\lambda \in [0, 1]$, we can defined the positive and negative $\lambda$ weighted interpreter ranking method as

$$R_{\lambda^+}(\tilde{c}^{bm}) = \frac{V_{\lambda^+}(\tilde{c}^{bm})}{A_{\lambda^+}(\tilde{c}^{bm})} \text{ and } R_{\lambda^-}(\tilde{c}^{bm}) = \frac{V_{\lambda^-}(\tilde{c}^{bm})}{A_{\lambda^-}(\tilde{c}^{bm})}$$

Definition 3.5. Let $\tilde{c}^{bm}$ and $\tilde{e}^{bm}$ be two SVbN-numbers. For $\lambda \in [0, 1]$, the positive and negative $\lambda$ weighted interpreter of SVbN-numbers $\tilde{c}^{bm}$ and $\tilde{e}^{bm}$, the ranking order of $\tilde{c}^{bm}$ and $\tilde{e}^{bm}$ is constructed as:

1. If $R_{\lambda^+}(\tilde{c}^{bm}) > R_{\lambda^+}(\tilde{e}^{bm})$, then $\tilde{c}^{bm}$ is bigger than $\tilde{e}^{bm}$, denoted by $\tilde{c}^{bm} > \tilde{e}^{bm}$.
2. If $R_{\lambda^+}(\tilde{c}^{bm}) < R_{\lambda^+}(\tilde{e}^{bm})$, then $\tilde{c}^{bm}$ is smaller than $\tilde{e}^{bm}$, denoted by $\tilde{c}^{bm} < \tilde{e}^{bm}$.
3. If $R_{\lambda^+}(\tilde{c}^{bm}) = R_{\lambda^+}(\tilde{e}^{bm})$ then
   (i) If $R_{\lambda^-}(\tilde{c}^{bm}) = R_{\lambda^-}(\tilde{e}^{bm})$, then $\tilde{c}^{bm}$ is equal to $\tilde{e}^{bm}$, denoted by $\tilde{c}^{bm} \approx \tilde{e}^{bm}$.
   (ii) If $R_{\lambda^-}(\tilde{c}^{bm}) > R_{\lambda^-}(\tilde{e}^{bm})$, then $\tilde{c}^{bm}$ is bigger than $\tilde{e}^{bm}$, denoted by $\tilde{c}^{bm} > \tilde{e}^{bm}$.
   (iii) If $R_{\lambda^-}(\tilde{c}^{bm}) < R_{\lambda^-}(\tilde{e}^{bm})$, then $\tilde{c}^{bm}$ is smaller than $\tilde{e}^{bm}$, denoted by $\tilde{c}^{bm} < \tilde{e}^{bm}$.

Example 3.1. Let $\tilde{c}^{bm} = ((3, 5, 7, 10); 0, 9, 0.8, 0.6)$ and $\tilde{e}^{bm} = ((2, 4, 6, 9); 0.6, 0.7, 0.5)$ be two SVTN-numbers. Then, we can compared the two SVTN-number $\tilde{c}^{bm}$ and $\tilde{e}^{bm}$.

Proof. Here $\tilde{c}^{bm} = ((3, 5, 7, 10); 0, 9, 0.8, 0.6)$ and $\tilde{e}^{bm} = ((2, 4, 6, 9); 0.6, 0.7, 0.5)$ be two SVTN-numbers. Firstly, we find the positive $\lambda$-weighted values and ambiguity, then we also find the negative $\lambda$-weighted values and ambiguity of SVTN-number $\tilde{c}^{bm}$ respectively. $V_{\lambda^+}(\tilde{c}^{bm}) = 1.23 + 3.75 \lambda$ and $V_{\lambda^-}(\tilde{c}^{bm}) = 3.76 + 1.11 \lambda$ Now the positive and negative $\lambda$-weighted rank interpreter of SVTN-number $\tilde{c}^{bm}$ are calculated as $R_{\lambda^+}(\tilde{c}^{bm}) = 1.23 + 3.75 \lambda$ and $R_{\lambda^-}(\tilde{c}^{bm}) = 3.76 + 1.11 \lambda$. Secondly, we find the positive $\lambda$-weighted values and ambiguity, then we also find the negative $\lambda$-weighted values and ambiguity of SVTN-number $\tilde{e}^{bm}$ respectively. $V_{\lambda^+}(\tilde{e}^{bm}) = 1.75 + 1.29 \lambda$ and $V_{\lambda^-}(\tilde{e}^{bm}) = 3.12 + 1.23 \lambda$. Now the positive and negative $\lambda$-weighted rank interpreter of SVTN-number $\tilde{e}^{bm}$ are calculated as $R_{\lambda^+}(\tilde{e}^{bm}) = 1.75 + 1.29 \lambda$ and $R_{\lambda^-}(\tilde{e}^{bm}) = 3.12 + 1.23 \lambda$. So, $R_{\lambda^+}(\tilde{c}^{bm}) - R_{\lambda^+}(\tilde{e}^{bm}) = 0.62 + 0.45 \lambda > 0$ and hence, the ranking order of the two SVTN-numbers $\tilde{c}^{bm}$ and $\tilde{e}^{bm}$ holds.
4. Multi-criteria decision making method with SVbN informations using interpreter ranking technique

This section presented a multi-criteria decision-making method based on the weighted interpreter ranking technique. Here, the rating of the alternatives of an MCDM model has been assumed by the SVbN-numbers. Let \( C = \{C_1, C_2, \ldots, C_m\} \) be a set of criteria with respect to set of \( m \)-alternatives \( A = \{A_1, A_2, \ldots, A_m\} \). In the decision making systems, every attributes have various importance. Let \( w = (w_1, w_2, \ldots, w_n)^T \) be the weight vector of the criteria \( C_j (j = 1, 2, \ldots, n) \) according to the decision maker and satisfy the condition \( w_i \in [0, 1] \), \( \sum_{i=1}^{n} w_i = 1 \). The rating values of MCDM problem are considered by

\[
\bar{r}_{ij} = (\bar{r}_{ij}^1, \bar{r}_{ij}^2, \bar{r}_{ij}^3, \bar{r}_{ij}^4) \colon u_{ij}^{\lambda}, u_{ij}^{\gamma}, y_{ij}^{\lambda}, y_{ij}^{\gamma}
\]

The graphical presentation of the proposed methodology is depicted in Fig. 5. We have constructed the methodology steps of the MCDM method under SVbN information as follows.

Methodology steps:

Step 1: Formulate the decision matrix \( \bar{U} \) according to the alternatives \( A_i \in A \) and criteria \( C_j \in C \) with SVbN informations \( \bar{a}_{ij} = (\bar{u}_{ij}^{\lambda}, \bar{u}_{ij}^{\gamma}, \bar{u}_{ij}^{+}, \bar{u}_{ij}^{-}) \). Then the decision matrix can be presented as follows:

\[
\bar{U} = \left[ (\bar{a}_{ij})_{i, j}^{\lambda} \right]_{m \times n}
\]

Step 2: The decision matrix \( [\bar{a}_{ij}]^{\lambda} \) transformed to the normalized decision matrix \( [\bar{r}_{ij}]^{\lambda} \). Because different criteria of the decision matrix depend on various units, therefore normalization is need in the decision matrix \( [\bar{a}_{ij}]^{\lambda} \). Here, the normalization technique of the decision matrix is considered by the following rule

\[
\bar{r}_{ij} = \left( \frac{\bar{u}_{ij}^{\lambda}}{\bar{u}_{ij}^{\lambda} + x_{ij}^{-}}, \frac{\bar{u}_{ij}^{\gamma}}{\bar{u}_{ij}^{\gamma} + x_{ij}^{-}}, \frac{\bar{u}_{ij}^{+}}{\bar{u}_{ij}^{+} + x_{ij}^{-}}, \frac{\bar{u}_{ij}^{-}}{\bar{u}_{ij}^{-} + x_{ij}^{-}} \right) \colon u_{ij}^{\lambda}, u_{ij}^{\gamma}, y_{ij}^{\lambda}, y_{ij}^{\gamma}
\]

Hence the normalized decision matrix is given by

\[
\bar{R} = \left[ (\bar{r}_{ij})_{i, j}^{\lambda} \right]_{m \times n}
\]

Step 3: Calculated the weighted normalized decision matrix according to the different weights \( w = (w_1, w_2, \ldots, w_n)^T \) of the alternatives \( A_i (i = 1, 2, \ldots, m) \). The weighted normalized decision matrix can be determined as follows:

\[
\bar{S}_i = \sum_{j=1}^{n} w_j \bar{r}_{ij}^{\lambda} \quad f o r \quad i = 1, 2, \ldots, m
\]

we get \( \bar{S}_i = [\bar{s}_{ij}^{\lambda}]_{m \times 1} \) for \( i = 1, 2, \ldots, m \) and each \( \bar{s}_{ij}^{\lambda} \) are SVbN-numbers

Step 4: First find the positive and negative \( \lambda \)-weighted values of each \( \bar{S}_i \), then find the positive and negative \( \lambda \)-weighted ambiguity of each \( \bar{S}_i \). Thereafter calculated the \( \lambda \)-weighted positive and negative \( \lambda \)-weighted rank interpreter (\( R_{+\lambda}R_{-\lambda} \)) value of each \( \bar{S}_i \). According to value of \( R_{+\lambda} \), \( R_{-\lambda} \), determine the non-increasing order of \( \bar{S}_i \) for \( i = 1, 2, \ldots, m \).

Step 5: Rank the alternatives \( A_i \) according to the \( \lambda \)-weighted positive and negative \( \lambda \)-weighted rank interpreter (\( R_{+\lambda} \) and \( R_{-\lambda} \)) value of \( \bar{S}_i \) for \( i = 1, 2, \ldots, m \). Finally choose the best alternative \( A_i \) from the \( m \) alternatives.

5. Define decision making problem of COVID-19 vaccines selection

In Dec 2019, the origin of coronavirus was first detected in the Wuhan Lab of China. Some common symptoms of COVID-19 (cf. Fig. 6) in infected people are fever, cold, dry cough, tiredness, sore throat, headache, loss of smell, etc. Most of the below 50 year age COVID-19 infected people is recovering without any special treatment. COVID-19 is very dangerous for older people who already have serious illnesses (cancer, diabetes, cardiovascular disease, etc.). In March 2020, the WHO declared that the COVID-19 virus is a global pandemic. The following steps are the best path to prevent infection of COVID-19.

- Clean your hand with alcohol-based hand sanitizer and hand wash.
- Wear a face mask when you go the outside of your home.
- Maintaining the 1-metre social distance from one person to another person.
- Avoid touching your face.
- If you feel your health is unwell then stay at home.

Till now, there has been no specific treatment for infected COVID-19 people. If someone is affected by COVID-19 and has the symptoms mentioned above, take medicine for fever, body aches, and sore throat. Also takes proper rest, drinking warm water and gargling 5 or 6 times a day. Therefore, social distancing and wearing face masks are necessary for public places as preventive measures. Since December 2020 worldwide, many countries have developed several vaccines. Recently, many researchers have been working on medication that suppresses the coronavirus.
Table 2
Comparison between three vaccine Covishield vs. Covaxin vs. Sputnik V.

| Information                      | Covishield         | Covaxin           | Sputnik V         |
|----------------------------------|--------------------|-------------------|-------------------|
| Manufactured Lab                 | Serum Institute, India | Bharat Biotech, India | Gamaleya-Russia and Dr. Reddy's |
| Type of Vaccine                  | Viral Vector       | Inactivated Virus | Viral Vector      |
| Storage & Expiry date            | 2–8 °C and 6 months | 2–8 °C and 24 months | Liquid form: −18.55 °C and Dry form: 2–8 °C and 6 months |
| Time gap between two doses       | 12–16 weeks between two dose | 28 days gap between two dose | 21 days to 3 months between two dose |
| Price                            | Free at government centre, Private hospital: Rs. 300-600 per dose | Free at government centre, Private hospital: Rs. 600-1200 rupee per dose | Global price Rs. 750 |

5.1. Covishield

The Indian government first approved the Covishield vaccine for emergency use. This vaccine was developed by the University of Oxford with the partnership of Astra Zeneca and manufactured by the Serum Institute of India. The current price of the vaccine is Rs. 300 for any state government medical house.

Work Capability of the Covishield: Covishield is a dissipated version of adenovirus, one of the chimpanzees and ChAdOx1 common cold virus. Recently, it has been developed to SARS CoV-2. After completing the vaccination, this vaccine boosts the human immune system to make antibodies against the COVID-19. But this vaccine cannot intent the illness.
Efficiency of the Covishield: The efficacy of this vaccine is 70.40% as per the third phase trial result. Many international volunteers proved this clinical trial result. It was observed that when one man took the first dose and then completed the second dose, the efficacy reached up to 90%.

Side effects of the Covishield: The side effects of this vaccine are minor. Some common ill effects are seen in one out of ten people as joint pain, feeling feverish, redness, nausea, headache, etc. After the vaccination, if a person is feeling unwell, take paracetamol tablets, and the common symptoms are decreased.

Doses and time of interval: This is a two-dose vaccine, and the time gap between two doses is four to twelve weeks. The stored temperature of this vaccine is 2 to 8 °C.

5.2. Covaxin

Covaxin is a two-dose COVID-19 vaccine given in Jan 2021 for emergency use. This vaccine was manufactured by the Bharat Biotech, an Indian Company with the partnership of the National Institute of Virology (NIV) and the Indian Council of Medical Research. The current price of this vaccine is Rs. 1200 for the private sector and Rs. 600 for the State Government.

Work Capability of the Covaxin: Bharat Biotech, and Pharma Companies, produced this vaccine exercising a sample of Coronavirus that the NIV isolated. After being injected into the human body, this vaccine worked to start producing antibodies against COVID-19.

Efficiency of the Covaxin: After the completion of the third phase trial, the efficacy rate of this vaccine is 81.00%. So many national volunteers gave the effectiveness of this trial results.

Side effect of the Covaxin: The side effects of this vaccine are minor. Some common side effects were seen like as fever, joint pain, redness, body ache, nausea, vomiting, weakness in the arm, etc.

Doses and time of interval: This is a two doses vaccine and the time gap between two doses is four weeks. The stored temperature of this vaccine is 2 to 8 °C.

5.3. Sputnik V

Sputnik V is a two-dose vaccine, a Russian vaccine, and the Indian government approved it in April 2021. This vaccine boosted the immune system and developed the body’s resistance power. It’s used a cold-type virus, which is managed to be innocuous.

Work Capability of the Sputnik V: In the human body, the Sputnik V vaccine labours as a porter to inflect a small portion of coronavirus. This vaccine safely discloses the body to a part of the genetic code of the coronavirus. Without any illness, this vaccine worked properly in the human body.

Efficiency of the Sputnik V: As per the clinical trial result, the Sputnik V efficacy rate is 91.60%. In the Indian environment, this vaccine efficacy rate is not exactly 91.6%.

Side effect of the Sputnik V: The side effects of this vaccine are minor. After the vaccination, when the human body is made to boost the immune system, very few cases of slight fever will come. This fever is easily overcome with a paracetamol tablet.

Doses and time of interval: This is a two-dose vaccine, and the time gap between two doses is 21 days. The stored temperature of this vaccine is –18.55 °C.

For emergency purposes against COVID-19 various, the Indian medical board has approved three vaccines: Covishield, Covaxin, and Sputnik V. Generally, people are confused about taking the vaccine and which vaccine is better? On 6 June 2021, the All India Institute of Medical Sciences (AIIMS) addressed imprecise doubts concerning the COVID-19 vaccine. AIIMS Director declared that the efficacy of three vaccines is more or less equivalent. However, here we have ranked the three vaccines (Covishield, Covaxin, and Sputnik V) according to four criteria which are (i) Work capability of the vaccine (ii) Efficiency of the vaccine (iii) Side effects of the vaccine, and (iv) Doses and time interval.

6. MCDM technique on the selection of COVID-19 vaccines under SVTbN-numbers

Here the problem is to find the best vaccine out of three vaccines (alternatives) which are Covishield (A1), Covaxin (A2) and Sputnik V (A3) according four criteria which are (1) Work capability of vaccine (C1) (2) Efficiency of vaccine (C2) (3) Side effect of the vaccine (C3) and (4) Doses and time interval (C4). We ranked to three vaccines using the proposed λ-weighted interpreter ranking technique based on four criteria.

Assume that the weight vector of the four criteria is \( w = (0.25, 0.30, 0.25, .20) \). The three possible alternatives (A1, A2, A3) are to be evaluated with respect to the four criteria (C1, C2, C3, C4) have considered as SVTbN-numbers. Hence the following steps can be considered in evaluating this result.

Step 1: Formulate the decision matrix \( \hat{U} = [\hat{u}_{ij}]^{m}_{o} \) with SVTbN informations as given in Box I.

Step 2: Using Eq. (22), we have normalized the decision matrix \([\hat{u}_{ij}]^{m}_{o}\). The normalized decision matrix can be formulated as given in Box II.

Step 3: The weighted normalized decision matrix can formulate in the following way for weighted \( w = (0.25, 0.30, 0.25, .20) \).

Step 4: Calculate the positive and negative λ-weighted values and ambiguity for the authenticity, hesitancy and falsity membership functions of each alternatives \( ̃ 𝑠_{1}, ̃ 𝑠_{2} \) and \( ̃ 𝑠_{3} \). Then the λ-weighted positive values of each alternatives \( ̃ 𝑠_{1}, ̃ 𝑠_{2} \) and \( ̃ 𝑠_{3} \) are calculated as

\[ V_{\lambda} (̃ 𝑠_{1}) = 0.38 − 0.21λ, \quad V_{\lambda} (̃ 𝑠_{2}) = 0.20 − 0.09λ, \quad V_{\lambda} (̃ 𝑠_{3}) = 0.18 − 0.06λ \]

and the λ-weighted positive ambiguity for each alternatives \( ̃ 𝑠_{1}, ̃ 𝑠_{2} \) and \( ̃ 𝑠_{3} \) are computed as

\[ A_{\lambda} (̃ 𝑠_{1}) = 0.08 − 0.04λ, \quad A_{\lambda} (̃ 𝑠_{2}) = 0.04 − 0.01λ, \quad A_{\lambda} (̃ 𝑠_{3}) = 0.03 − 0.01λ \]

Similarly, the λ-weighted negative values of each alternatives \( ̃ 𝑠_{1}, ̃ 𝑠_{2} \) and \( ̃ 𝑠_{3} \) are calculated as

\[ V_{\lambda} (̃ 𝑠_{1}) = 0.21 − 0.03λ, \quad V_{\lambda} (̃ 𝑠_{2}) = 0.38 − 0.25λ, \quad V_{\lambda} (̃ 𝑠_{3}) = 0.40 − 0.16λ \]

and the λ-weighted negative ambiguity for each alternatives \( ̃ 𝑠_{1}, ̃ 𝑠_{2} \) and \( ̃ 𝑠_{3} \) are computed as

\[ A_{\lambda} (̃ 𝑠_{1}) = 0.21 − 0.03λ, \quad A_{\lambda} (̃ 𝑠_{2}) = 0.10 − 0.03λ, \quad A_{\lambda} (̃ 𝑠_{3}) = 0.11 − 0.02λ \]

Now, the positive λ weighted interpreter rank can be calculated as

\[ R_{\lambda} (̃ 𝑠_{1}) = 0.38 − 0.21λ, \quad R_{\lambda} (̃ 𝑠_{2}) = 0.20 − 0.09λ \& R_{\lambda} (̃ 𝑠_{3}) = 0.18 − 0.06λ \]

\[ R_{\lambda} (̃ 𝑠_{1}) = 0.21 − 0.03λ, \quad R_{\lambda} (̃ 𝑠_{2}) = 0.38 − 0.25λ \& R_{\lambda} (̃ 𝑠_{3}) = 0.40 − 0.16λ \]

Similarly, the negative λ weighted interpreter rank can be calculated as

\[ R_{\lambda} (̃ 𝑠_{1}) = 0.45, \quad R_{\lambda} (̃ 𝑠_{2}) = 5.0 \& R_{\lambda} (̃ 𝑠_{3}) = 7.5 \]

and

\[ R_{\lambda} (̃ 𝑠_{1}) = 1, \quad R_{\lambda} (̃ 𝑠_{2}) = 3.1 \& R_{\lambda} (̃ 𝑠_{3}) = 3.2 \]

Hence for \( λ \in [0,1] \), the non-increasing order of \( ̃ 𝑠_{1}, ̃ 𝑠_{2} \) and \( ̃ 𝑠_{3} \) is \( ̃ 𝑠_{1} > ̃ 𝑠_{2} > ̃ 𝑠_{3} \) (\( A_{1} > A_{2} > A_{3} \)). So the best alternatives \( ̃ 𝑠_{1} \) and worst one is \( ̃ 𝑠_{3} \) for \( λ = 0.5 \).
In the present day, the COVID-19 pandemic is a swiftly rising phase. Due to India’s high population density, the pandemic situation is very dangerous. Therefore, no one is safe until all are safe. A critical condition arises in society due to the rapid transmission of coronavirus. Against the delta variant coronavirus, everybody must inject the COVID-19 vaccine. The coronavirus data curve shows that the COVID-19 vaccine significantly impacts the decreasing infection rate. The COVID-19 data curve of middle-income countries, including India, says that the positive case of COVID-19 cannot decrease without vaccination. The efficacy rate of all vaccines is not equal. For emergency purposes against coronavirus, the Indian medical board has approved three vaccines: Covishield, Covaxin, and Sputnik V. Generally, people are confused about the vaccination. All the Indian people try to establish which vaccine is better for him out of the three vaccines. Here, we have ranked the three vaccines according to the four criteria, which are (1) Work capability of the vaccine (2) Efficiency rate of the vaccine (3) Side effects of the vaccine and (4) Doses and time interval. By the proposed MCDM technique, we ranked the three vaccines which are Covishield (\(\text{i}^6\)), Covaxin (\(\text{i}^7\)) and Sputnik V (\(\text{i}^8\)). Our proposed method is beneficial when human decisions are based on positive and negative thoughts. Sometimes many decision-making problems have a heavy risk factor. For this case, with this proposed method, we will get maximum benefit. For the selection of the COVID-19 vaccine, many negative thoughts have come to the human mind. Therefore, in this case, our decision-making method was beneficial. In addition, our interpreter ranking tool is a straightforward mechanical tool from other automated in the bipolar environment.

Using the \(\lambda\)-weighted positive and negative interpreter ranking method, we have ranked the three vaccine Covishield (\(\text{i}^6\)), Covaxin (\(\text{i}^7\)) and Sputnik V (\(\text{i}^8\)). For \(\lambda \in [0, 1]\), by Eq. (19) and Definition 3.5, we calculated the positive and negative interpreter ranking value and then ranked the alternatives \(\text{i}^6\), \(\text{i}^7\) and \(\text{i}^8\). From Table 3, it is clear that for \(\lambda \in [0, 0.5]\), the best alternatives is \(\text{i}^6\) and worst one is \(\text{i}^8\). For \(\lambda = 0.4\), positive rank interpreter value is same for the alternatives \(\text{i}^6\), \(\text{i}^7\) therefore decision maker goes to negative rank interpreter value. For \(\lambda \in [0.5, 1]\), we observe that the best alternatives is \(\text{i}^6\) and worst one is \(\text{i}^7\) according to both positive and negative rank interpreter value \(R_D\&R_P\). By the \(\lambda\)-weighted positive and negative rank interpreter value the best alternatives are \(\text{i}^6\) and worst one \(\text{i}^7\) or \(\text{i}^8\) (cf. Figs. 10 and 11). From Table 3, we observed that by proposing decision-making techniques, people easily identified which vaccine is better than other vaccines. The decision-maker also removed the negative thought from people’s minds. We have observed that decision-makers can easily characterize which one is better under the bipolar neutrosophic environment due to the formation of bipolar neutrosophic numbers.

### 7.1. Comparative analysis

Many researchers considered the different MCDM methods in the uncertain environments, we have discussed in the existing literature. When one researcher chooses an MCDM method, that time focuses on many factors like knowledge, accuracy, calculation, effort, etc. Here, we have compared our method with other existing methods on the same MCDM problem under the same uncertain environment (Single valued Bipolar neutrosophic). This paper proposed a novel MCDM technique based on the \(\lambda\)-weighted ranking interpreter technique. This technique is straightforward to understand and has a solid computational ability. To show the applicability of our proposed MCDM method, we compared our proposed method with other existing methods based on the different techniques. The comparative studies are
obtain the best alternative is \( \tilde{s} \). Again by the Deli et al. (2015) method, we got the best alternative is \( \tilde{s} \). By the Chakraborty et al. (2019) method, for this problem we got best alternative and its application to the MCDM problem. By the Meena Kumari and Thirucheran (2022) method, we considered conducted with four methods: Meena Kumari and Thirucheran (2022), Chakraborty et al. (2019), Princy and Mohana (2019), and Deli et al. (2015) methods. Meena Kumari and Thirucheran (2022) considered an MCDM based on the score function under a single-valued bipolar neutrosophic environment. Using the De-Bipolarization technique, Chakraborty et al. (2019) proposed an MCDM method under a single-valued bipolar neutrosophic environment. Princy and Mohana (2019) developed an MCDM method based on score values under a single-valued bipolar neutrosophic environment and its application to the MCDM problem. By the Meena Kumari and Thirucheran (2022) method, for this problem we got best alternative is \( \tilde{s} \) and worst one is \( \tilde{s} \). By the Chakraborty et al. (2019) method, we obtain best alternative is \( \tilde{s} \) and worst one is \( \tilde{s} \). According to the Princy and Mohana (2019) method, we got the best alternative is \( \tilde{s} \) and worst one is \( \tilde{s} \). Again by the Deli et al. (2015) method, we get the best alternative is \( \tilde{s} \) and worst one is \( \tilde{s} \). But according our proposed method the best alternative is \( \tilde{s} \) for \( \lambda \in [0, 1] \) and worst one is \( \tilde{s} \) for \( \lambda \in [0, 0.5] \) and \( \tilde{s} \) for \( \lambda \in (0.5, 1] \). Hence, from Table 4, it is clear that our proposed method gives better results than the other two methods for this real problem. Because, by our proposed method, \( \tilde{s} \) is the best alternative, which result is suitable according to the Indian environment and also comfortable with collective data. Hence, by the our MCDM method Sputnik V (alternative \( \tilde{s} \)) is the best vaccine out of three vaccine. Based on clinical trials and analysis of 19,866 volunteers who received both the first and second doses, the Sputnik V Vaccine was found to be 91.6 percent effective. After phase 1 and phase 2 clinical trials, international peer-reviewed data published in The Lancet validated efficacy.

8. Conclusion and future work plan

In Dec 2019, the prevalence of coronavirus virus malady was first indicated, which is known as a COVID-19. COVID-19 has a fast motion propagating rate. In the pandemic race of COVID-19, such effective measures are very significant, which are wearing masks, hand hygiene, and staying at home. There is no worldwide approved particular antiviral medicine accessible for the coronavirus. Against the COVID-19, many partial treatment processes have been used. Till now, no reactive treatment process has been found. There only case emerged enrichment had done partially. As a preventive measure, COVID-19 vaccination plays a significant role. The vaccine can earn immunity against the coronavirus and reduce the spread of COVID-19. For emergency purposes, the Indian medical government approved three COVID-19 vaccines: Covishield, Covaxin, and Sputnik V. In this paper, we have tried to propose which vaccine is best according to four criteria. We have developed a novel MCDM technique for choosing the best vaccine out of three approved vaccines. This MCDM technique is constructed with the proposed rank interpreter method. The first time we have defined the \( \lambda \)-weighted rank interpreter of single-valued bipolar neutrosophic numbers. Using the proposed MCDM technique, we have solved the decision-making problem under SVbN-environment. Rating of the criteria considered as single-valued trapezoidal bipolar neutrosophic numbers. We have also presented the comparative analysis with some existing methods. According to our proposed ranking method, we get 'Sputnik V' is the best vaccine out of three vaccines in the Indian environment.

Although, this paper contributed many positive ideas to this field of decision making. In the future, researchers can apply this DM method to different MCDM problems like hospital selection problems during COVID-19, green supplier selection problems, inventory control management problems, medical diagnoses problems, etc. In the near future, we will try to introduce the new DM technique of advanced fuzzy numbers and employ it in real-life decision-making problems.

Compliance with ethical standards

Ethical approval: This article does not contain any studies with animals performed by any of the authors.

| Authors                | Methods                    | Environment                              | Ranking order                     | Best alternatives |
|------------------------|----------------------------|------------------------------------------|-----------------------------------|-------------------|
| Meena Kumari and Thirucheran (2022) | Score function | Single valued Bipolar neutrosophic | \( \tilde{s} \) > \( \tilde{s} \) > \( \tilde{s} \) | \( \tilde{s} \) |
| Chakraborty et al. (2019) | De-Bipolarization | Single valued Bipolar neutrosophic | \( \tilde{s} \) > \( \tilde{s} \) > \( \tilde{s} \) | \( \tilde{s} \) |
| Princy and Mohana (2019) | Score Values             | Single valued Bipolar neutrosophic      | \( \tilde{s} \) > \( \tilde{s} \) > \( \tilde{s} \) | \( \tilde{s} \) |
| Deli et al. (2015)     | Certainty function       | Single valued Bipolar neutrosophic      | \( \tilde{s} \) > \( \tilde{s} \) > \( \tilde{s} \) | \( \tilde{s} \) |
| Our method             | For \( \lambda \in [0, 0.5] \) | Single valued Bipolar neutrosophic      | \( \tilde{s} \) > \( \tilde{s} \) > \( \tilde{s} \) | \( \tilde{s} \) |
| Our method             | For \( \lambda \in (0.5, 1] \) | Single valued Bipolar neutrosophic      | \( \tilde{s} \) > \( \tilde{s} \) > \( \tilde{s} \) | \( \tilde{s} \) |
Declarations of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Abdelwahab, S. F., Isha, U. H., & Ashour, H. M. (2021). A novel vaccine selection decision-making model (VSDMM) for COVID-19. Vaccines, 9, 1–16.

Akramp, M., & Ardeshir, M. (2019). Ranking of trapezoidal bipolar fuzzy information system based on total ordering. Applied Mathematics E-Notes, 19, 292–309.

Albahari, O., Al-Obaidi, A., Zaidan, A., Albahari, B., Salih, M., Qays, A., Dawood, K., Mohammed, R., Abdulkareem, K., Anesa, A., Alamoodi, A., Chyad, M., & Zulkifli, M. (2020). Helping doctors having COVID-19 treatment: Towards an rescue framework for the transfusion of best convalescent plasma to the most critical patients based on biological requirements via ml and novel MCDM methods. Computer Methods and Programs in Biomedicine, 196, 78–105.

Albu, A., Precup, R. E., & Teban, T. A. (2019). Results and challenges of artificial neural networks based for decision-support and control in medical applications. Faculty Universitas. Series: Mechanical Engineering, 17, 285–308.

Alghamdi, M. A., Alshehri, N. O., & Akram, M. (2018). Multi-criteria decision-making methods in bipolar fuzzy environment. International Journal of Fuzzy System, 18, 1–10.

Alkan, N., & Kabranac, C. (2020). Evaluation of government strategies against COVID-19 pandemic using q-rung orthopair fuzzy TOPSIS method. Applied Soft Computing, 110, Article 107653.

Alkan, N., & Kabranac, C. (2021). Evaluation of government strategies against COVID-19 pandemic using q-rung orthopair fuzzy TOPSIS method. Applied Soft Computing, 110, Article 107653.

Anderson, R., Heesterbeek, H., Klinkenberg, D., & Hollingsworth, T. (2020). How Chong. J., Li, S., Ma, S., & Wang, X. (2014). M-polar fuzzy sets: An extension of bipolar fuzzy sets. Information Science and Technology, 415, 1–20.

Garai, T., Dalapati, S., Garg, H., & Roy, T. K. (2020a). Possibilistic multi-attribute decision making for water resource management problem under single valued bipolar neutrosophic environment. International Journal of Intelligent Systems, 38, 1–28.

Garai, T., Garg, H., & Roy, T. K. (2020b). A ranking method based on possibility mean for multi-attribute decision making with single valued neutrosophic numbers. Journal of Ambient Intelligence and Humanized Computing, 11, 5245–5258.

Garciadiego, A., Moghtader, K., Garg, A., & Nair, M. (2018). A direct method to compare bipolar LR fuzzy numbers. Advances in Fuzzy Systems, 2018, 1–7.

Giori, D., Blanchini, F., Bruno, R., & Colaneri, P. (2020). Modelling the COVID-19 epidemic and implementation of population-wide interventions in Italy. Nature Medicine, 26, 855–860.

Goel, I., Sharma, S., & Kanchanmika, S. (2021). Effects of the COVID-19 pandemic in India: An analysis of policy and technological interventions. Health Policy and Technology, 10, 151–164.

Hedrea, E. L., Precup, R. E., Roman, R. C., & Petriu, E. M. (2021). Tensor product-based model transformation approach to tower crane systems modeling. Asian Journal Control, 21, 1–11.

Hearst, J. M., Nayyeri, M. K., Foul, A., & Alrashrhi, A. F. (2021). COVID-19 vaccine: A neutrosophic MCDM approach for determining the priority groups. Results in Physics, 20, Article 103654.

Jeong, K., Kahng, A. B., Lin, B., & Samadi, K. (2010). Accurate machine-learning-based on-chip router modelling. IEEE Embedded Systems Letters, 2, 62–67.

Kwak, C. J., Ri, R. C., Kwak, S. I., Kim, K. J., Ryu, U. S., Kwon, O. C., & Kim, H. N. (2014). Fuzzy modus ponens and tollens based on moving distance in siso fuzzy environment. Romanian Journal of Information Science and Technology, 24, 257–283.

Majumder, P., Biswas, P., & Majumder, S. (2020). Application of new TOPSIS approach to identify the most significant risk factor and continuous monitoring of death of COVID-19. Electronic Journal of Fuzzy Systems, 9, 131–140.

Meena Kumari, E. R., & Thirumurthy, M. (2022). Bipolar single valued neutrosophic approach to multi criteria decision making problem. Advances and Applications in Mathematical Sciences, 21, 3265–3279.

Mohammed, M., Abdulkareem, K., Al-Waisy, A., Mostafa, S., Al-Fahdawi, S., Dinar, A., Alhakami, W., Baz, A., Al-Mhiqani, M., Alhakami, H., Arbaiz, N., Maahli, M., Mustafa, A., Zajrpin, A., & Tetteh, I. A. (2020). Benchmarking methodology for selection of optimal COVID-19 diagnostic model based on entropy and TOPSIS method. IEEE Access, 8, 99115–99131.

Ouyang, L., Yuan, Y., Cao, Y., & Wang, F. Y. (2021). A novel framework of collaborative early warning for COVID-19 based on block-chain and smart contracts. Information Sciences, 579, 124–143.

Pamucar, D., Zivovic, M., Marinovic, D., Doljanica, D., Jovanovic, S. V., & Buzrakov, P. (2021). Development of a multi-criteria model for sustainable reorganization of a healthcare system in an emergency situation caused by the COVID-19 pandemic. Sustainability, 12, 1–24.

Parr, P., Beck, C., & Basar, T. (2020). Modeling, estimation, and analysis of epidemics over networks: An overview. Annual Reviews in Control, 50, 345–360.

Peterson, E., Wejse, C., & Zumbal, A. (2020). Advancing COVID-19 vaccines avoiding different regulatory standards for different vaccines and need for open and transparent data sharing. International Journal of Infectious Diseases, 98, 501–502.

Precup, R. E., Bojan-Dragos, C. A., Hedrea, E. L., Roman, R. C., & Petriu, E. M. (2021). Evolving fuzzy models of shape memory alloy wire actuators. Romanian Journal of Information Science and Technology, 24, 353–365.

Prett, E. D., Momen, T. M., Tomes, M. L., & Petriu, E. (2012). Stable and convergent iterative feedback tuning of fuzzy controllers for discrete-time SISO systems. Expert Systems with Applications, 40, 1149–1159.

Princy, R., & Mohana, K. (2019). An application of neutrosophic bipolar vague on multi-criteria decision making problems. International Journal of Research in Advent Technology, 7, 1–8.

Rahimi, I., Chen, F., & Gandomi, A. (2021). A review on COVID-19 forecasting models. Neural Computer Application, 6, 1–20.

Requia, W., Kondo, E., Adams, M., Gold, D., & Struchiner, C. (2020). Risk of the Brazilian health care system over 5572 municipalities to exceed health care capacity due to the 2019 novel coronavirus (COVID-19). Science of the Total Environment, 730, 412–450.

Riaz, M., Garg, H., Muhammad, H., Farid, A., & Chinran, R. (2020). Multi-criteria decision making based on bipolar picture fuzzy operators and new distance measures. CMES: Computer Modeling in Engineering & Sciences, 127, 771–800.

Salvar, A., Nazar, N., Nazar, N., & Qadir, A. (2021). Measuring vaccination willingness in response to COVID-19 using a multi-criteria decision-making method. Human Vaccines & Immunotherapeutics, 17, 4865–4872.

Smarandache, F. (1999). A unifying field in logics: neutrosophy: neutrosophic probability, set and logic. American Research Press.

Wan, S. P., & Xu, J. (2017). A method for multi-attribute group decision-making with triangular intuitionistic fuzzy numbers application to trustworthy service selection. Scientia Iranica Transactions E: Industrial Engineering, 24, 794–807.

Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. International Journal of Uncertainty Fuzziness and Knowledge-Based Systems, 18, 285–310.

WHO (2020). Coronavirus disease (COVID-19) India situation report 2020. Available from: https://www.who.int/emergencies/coronavirus-disease-(covid-19)/india-situation-report.

Yang, Z., Li, X., Garg, H., & Qi, M. (2020). Decision support algorithm for selecting antiviral masks under COVID-19 pandemic with spatial normal fuzzy environment. International Journal of Environmental Research Public Health, 17, 1–35.
Yazdani, M., Torkayesh, A., Stević, Z., Chatterjee, P., Ahari, S. A., & Hernandez, V. D. (2021). An interval valued neutrosophic decision-making structure for sustainable supplier selection. *Expert Systems with Applications*, 183, Article 115354.

Yıldırım, F. S., Sayan, M., Sanlidag, T., Uzun, B., Ozsahin, D. U., & Ozsahin, I. (2021). Comparative evaluation of the treatment of COVID-19 with multi criteria decision making techniques. *Journal of Healthcare Engineering*, 2021, 1–11.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8, 338–353.

Zakeri, S., Chatterjee, P., Cheikhrouhou, N., & Konstants, D. (2021). Ranking based on optimal points and win-loss-draw multi-criteria decision-making with application to supplier evaluation problem. *Expert Systems with Applications*, 191, Article 116258.

Zeng, X. T., Li, D. F., & Yu, G. F. (2014). A value and ambiguity-based ranking method of trapezoidal intuitionistic fuzzy numbers and application to decision making. *The Scientific World Journal*, 2014, 1–8.

Zhang, W. R. (1998). Bipolar fuzzy sets. *IEEE International Conference on Fuzzy Systems*, 1, 835–840.