ADAPTIVE NEURO-FUZZY VIBRATION CONTROL OF A SMART PLATE

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(Communicated by Cedric Yiu)

ABSTRACT. In the present paper, the vibration suppression of a smart plate with the use of ANFIS (Adaptive Neuro-Fuzzy Inference System) is investigated. The whole system consists of a nonlinear mechanical model, which is an extension of the von Kármán plate model with control. The structure is subjected to external disturbances and generalized control forces. Initial and boundary conditions are set up. The initial boundary value problem is spatially-discretized by a time spectral method. The obtained discretized model is a system of nonlinear ordinary differential equations (ODEs) with respect to time. A neuro-fuzzy inference system is built and tested in order to create a nonlinear controller for the vibration suppression of the plate. More specifically, a Sugeno-type fuzzy inference system is employed and trained through ANFIS. The inputs of the controller are the displacement and the velocity and the output is the control force. An effective optimization procedure is proposed and numerical results are presented.

1. Introduction. In this paper an optimized procedure of suppression of vibrations of a rectangular plate is proposed. The plate is subjected to external disturbances and generalized control forces, which are produced by the electromechanical coupling effects. The behaviour of the structure is described by a nonlinear control model for large deflections [12]. The mechanical model is an extension of the von Kármán plate system ([2, 5] etc.) with control. Thus, a system of two nonlinear partial differential equations is derived. Initial and boundary conditions are set up for the system. The equations are spatially-discretized by a time spectral method ([10]). The obtained system of nonlinear ordinary differential equations (ODEs) is solved by the implicit iterative Newmark-β method [12]. The nonlinear controller is designed using the Sugeno-type Fuzzy Inference System (FIS) and optimized by using Adaptive Neuro-Fuzzy Inference System (ANFIS). The input data of the controller are the displacement and the velocity and output data is the control force. The ANFIS procedure is employed for the training of the system.

2010 Mathematics Subject Classification. Primary: 74K20, 35Q74, 34H05, 93C42; Secondary: 74S25, 65T40.

Key words and phrases. Nonlinear problem, von Kármán plate, time spectral method, vibration control, ANFIS, optimization procedure.

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Classical mathematical theories of control work fine for linear systems, however their effectiveness deteriorates, as many restrictions are involved. On the other hand, nonlinear controllers, based on fuzzy logic with built-in smart computational methods, can describe well nonlinear structures, nonlinear feedback and nonlinear behaviour ([17]).

Smart structures incorporate some control schemes against disturbances. Active vibration control of smart elastic structures has been considered by many authors. An active vibration control incorporating active piezoelectric actuator and self-learning control for a flexible plate structure with the use of a finite difference method is presented in [18]. In [6] hybrid control strategies (active and semi-active vibration suppression) are discussed. A brief survey on industrial applications of fuzzy control is given in [13]. The control systems are classified in three groups; control systems with Mamdani fuzzy controllers, control systems with Takagi-Sugeno (Sugeno) fuzzy controllers, and adaptive and predictive control systems. A review of active structural control has been done in [8].

Fuzzy control is a suitable tool for the systematic development of nonlinear active control strategies in several problems, especially when the system parameters are known. Moreover, the whole system can be fine-tuned in various ways in case of control schemes of high complexity or if no experience exists (e.g. [4,14,19,20,16]). Two effective computational procedures with the use of Mamdani-type FIS in Fuzzy Logic Toolbox of MATLAB are outlined in [11] and [12], respectively. An active vibration control of a simply supported rectangular plate using fuzzy logic rules have also been considered in [16], where the comparison of the results with the classical proportional integral derivative (PID) controller demonstrates the efficiency of the fuzzy controller.

The present paper is organized as follows. In Section 2 the initial nonlinear mechanical model and its spatial discretization are formulated. Section 3 is devoted to the spatially-discretized system. Section 4 briefly describes the fuzzy strategies. Section 5 focuses on the construction of the Sugeno-type FIS and the optimization procedure with the use of ANFIS. Numerical examples are illustrated in Section 6 while in Section 7 a comparison with a classical LQR controller is performed. Finally, the main results along with the conclusions are reported in Section 8.

2. Formulation of the problem.

2.1. Governing equations. A nonlinear mathematical model in-plane vibrations of an isotropic, homogeneous elastic plate in the presence of active control, and allowance for the rotational inertia of the plate elements and viscous damping is written as (12, 2, 1, 8 etc.)

\[ \rho h w_{tt} - \rho \frac{h^3}{12} \Delta w_{tt} + h c w_t + D \Delta^2 w - h[w, \psi] = Q + [Z], \]  
\[ \Delta^2 \psi + \frac{E}{2} [w, w] = 0, \quad (t, x, y) \in \Omega. \]

Here the following notations are used:
- \([w, \psi] = \partial_1 w \partial_2 \psi + \partial_1 \psi \partial_2 w - 2 \partial_1 w \partial_1 \psi \) (Monge-Ampère’s form).
- \(w\) is the deflection (displacement) of the plate.
- \(\psi(t, x, y)\) is the Airy stress potential describing internal stresses, which appear due to the deformation of the plate (e.g., [1] [2] [5]).
\( \Omega = (0, T] \times G, \) \( T \) is the final time and \( G = (0, l_1) \times (0, l_2) \) is the shape of the plate (\( l_1 \) and \( l_2 \) are the lengths of the plate).

- \( \rho \) is the density of the material.
- \( h \) is the thickness of the plate.
- \( c \) is the viscous damping coefficient.
- \( D \) is the flexural rigidity of the plate.
- The function \( Q = Q(t, x, y) \) describes the external disturbance forces (loadings). In the numerical examples presented below \( Q(t, x, y) \) is an harmonic exciting pressure load.
- \([Z]\) are the control forces, which are given as the output of the controler and obtained on each time-step of simulation.

2.2. Initial and boundary conditions. The displacement and velocity at the initial time are defined as

\[
w(0, x, y) = u(x, y), \quad w_t(0, x, y) = \nu(x, y) \quad \text{in} \quad G,
\]

where the functions \( u, \nu \in L^2(G) \).

We consider simply supported, partially and totally clamped plates. The simply supported boundary conditions are written as

\[
w = \Delta w = 0, \quad \psi = \Delta \psi = 0 \quad \text{in} \quad (0, T] \times \partial G,
\]

or

\[
\partial_\nu \psi = \partial_\nu \Delta \psi = 0 \quad \text{in} \quad (0, T] \times \partial G,
\]

where \( \partial_\nu \) denotes the normal derivative. If the plate is simply supported at the ends \( y = \{0, l_2\} \) and clamped on the sides \( x = \{0, l_1\} \) then the partially clamped boundary conditions read

\[
w = \begin{cases} 
\partial_\nu w = 0 & \text{for} \ x = \{0, l_1\}, \\
\Delta w = 0 & \text{for} \ y = \{0, l_2\},
\end{cases}
\]

and the Airy potential \( \psi \) on the boundaries is defined as before \( [4] \) or \( [5] \).

For the totally clamped plate we have

\[
w = 0, \quad \partial_\nu w = 0, \\
\psi = 0, \quad \partial_\nu \psi = 0 \quad \text{in} \quad (0, T] \times \partial G.
\]

3. Spatially-discretized system. We consider an approximate analytical solution of \([1, 2]\) in the form of partial sums of double Fourier series with the time-dependent coefficients, \([10]\)

\[
W(t, x, y) = \sum_{i,j=1}^{N} w_{ij}^N(t) \omega_{ij}(x, y), \quad W_t(t, x, y) = \sum_{i,j=1}^{N} w_{ij}^N(t) \omega_{ij}(x, y),
\]

\[
\Psi_N(t, x, y) = \sum_{i,j=1}^{N} \psi_{ij}^N(t) \varphi_{ij}(x, y), \quad (t > 0),
\]
where the global basis functions \( \omega_{ij} \) and \( \varphi_{ij} \) are chosen to match the boundary conditions. For the initial conditions we assume

\[
W(0, x, y) = u_N(x, y) = \sum_{i,j=1}^{N} w_{ij}^{(0)} \omega_{ij}(x, y), \tag{10}
\]

\[
W_t(0, x, y) = \nu_N(x, y) = \sum_{i,j=1}^{N} w_{i,N}^{(0)} \omega_{ij}(x, y). \tag{11}
\]

The application of Galerkin’s projections to (1), (2) yields the following system of equations of motion:

\[
M\ddot{w}_N(t) + C\dot{w}_N(t) + K_1 w_N(t) = A_{1,N}(w_N(t), \psi_N(t)) + Pq_N(t) + [z_N(t)], \tag{12}
\]

\[
K_2 \psi_N(t) = A_{2,N}(w_N(t), w_N(t)). \tag{13}
\]

Here \( w_N(t) \) and \( \psi_N(t) \) are the vectors with components, which are the Fourier coefficients in the expansion for the displacement \( \omega \) and the Airy stress function \( \psi \). The number \( N \times N \) is the total number of the degrees of freedom of the system \( (12), (13) \), a number of independent elements of the vectors \( w_N(t) \) and \( \psi_N(t) \). We call them characteristics of the displacement and the Airy stress function, respectively. Furthermore, \( M = \rho h (H + (h^2/12)B) \) is the mass matrix \( (B \) is an approximation of the Laplacian), \( C = hcH \) is the viscous damping matrix, \( K_1 \) is the stiffness matrix (the result of approximation of the biharmonic operator, multiplied by the rigidity of the plate \( D \)), \( K_2 \) is the approximation of the biharmonic operator, \( A_{1,2} \), \( A_{2,N} \) are nonlinear approximations of the Monge-Ampère forms, \( P \) is the matrix and \( q_N \) is the vector, obtained after the approximations of the external exciting pressure. Finally, \( z_N(t) \) is the control vector, produced on each time step of the control simulation. The operators \( H, B, P \) and \( F \) take different forms depending on the boundary conditions. The expressions for them and for \( A_{1,2} \) and \( A_{2,N} \) can be found in \[10\].

Substituting (13) into (12) we obtain

\[
M\ddot{w}_N(t) + C\dot{w}_N(t) + K_1 w_N(t) = A_{1,N}(w_N(t), \psi_N(t)) + Pq_N(t) + [z_N(t)].
\]

For the components of (14) we have

\[
(M\ddot{w}_N(t))_{mn} + (C\dot{w}_N(t))_{mn} + (K_1 w_N(t))_{mn} \\
= (A_{1,N}(w_N(t), K_2^{-1} A_{2,N}(w_N(t), w_N(t))))_{mn} \\
+ (Pq_N(t))_{mn} + [z_N^{mn}(t)], \quad m, n = 1, 2, \ldots, N. \tag{15}
\]

Thus, the control of \( W(t, x, y) \) in (8) is executed through the characteristics of the displacement, i.e. the Fourier coefficients \( w_{ij}^{(0)}(t) \) and \( w_{i,N}^{(0)}(t) \). For simplicity we will omit the denomination “characteristics” for \( w_{ij}^{(0)}(t) \) and \( w_{i,N}^{(0)}(t) \), and index \( N \) below.

For solving the resulting equations (15) we apply the implicit Newmark-\( \beta \) technique with iterations (12) and for the linear model the Newmark formulas.

4. Fuzzy control strategy. The fuzzy inference systems, also known as fuzzy rule based systems, are based on a set of verbal rules which in turn can be formed by deterministic statements (e.g. velocity is high), if-then condition statements (e.g. if velocity is high, then the control force is maximum) or even by statements without any condition. It is rather common that a fuzzy system is usually described by a
set of rules, rather than a single one. The summarizing process which is followed in order to obtain an overall conclusion is called aggregation. If the rules are associated with the AND operator, the aggregation is done by the conjunction of the rules, that is, by taking the intersection of the individual rules. On the other hand, when the OR operator is used, the determination of the final result is obtained by the disjunction of the rules, which is provided by the union of the involved rules. These methods are also known as methods of “min” and “max”, respectively, borrowing their names from the respective operators usually applied for their calculation.

In general, a fuzzy inference system consists of a rule base, i.e. a set of if-then rules, a data base which include a set of membership functions, a decision-making system (DMS) where the inference process takes place, a fuzzification interface and a defuzzification interface. The inference procedure has as follows. The inputs are turned into fuzzy variables through the fuzzification process. Subsequently, the set of rules, which forms the database is created. The fuzzy output is calculated by the implication of rules and the final decision is obtained by the defuzzification process. The whole process is depicted in Figure 1.

**Figure 1.** The structure of a fuzzy inference system.

The two most widely known fuzzy inference methods are the Mamdani and the Sugeno method. The two methods are quite similar. The main difference between them lies in the type of membership functions of their outputs. In the Mamdani method, the membership functions are fuzzy sets. Instead, in the Sugeno method the outputs are either linear functions or constant values.

The Mamdani inference is a widely accepted intuitive method, which adapts well to real problems. It is relatively simple in application and functions perfectly without sacrificing accuracy.

In the present investigation, the fuzzy rules have been formulated considering the known behaviour of a simple pendulum. This technique is only one of the possible approaches for the formulation of the fuzzy rules and can be used for simple fuzzy inference systems, i.e. a Mamdani controller, and for collocated sensors and actuators. More specifically, the behaviour of the structure is predicted like if it was a pendulum, and thus it is possible to propose a suitable control force for a given set of the input parameters. The shape of the membership functions (trapezoidal and triangular) have been chosen using the trial and error method, while the ranges of these functions were based on the uncontrolled dynamic model. Of course, all or
some of these parameters can be obtained by a global optimization method, such as genetic algorithms [9].

On the other hand, the Sugeno method is computationally accurate and can be combined with linear techniques, as well as with several optimization techniques. Moreover, it presents a guaranteed continuous output, which is susceptible to mathematical analysis. However, the most significant advantage of the controllers which are built by using the Sugeno inference method lies to the fact that they can be efficiently trained using adaptive neuro-fuzzy inference (ANFIS) techniques, based of course on an adequate set of training data, as it will be described in the next section.

5. Procedure with ANFIS. The design of nonlinear controllers based on classical fuzzy inference systems and artificial neural networks, i.e. adaptive neuro-fuzzy inference systems, can provide very satisfactory results in terms of vibration suppression, even when the examined system is partially known. However, in problems with high-order nonlinear equations, adaptive neuro-fuzzy inference system can give desirable results only in special cases, which are analyzed below.

In this section, a nonlinear controller, which uses the Sugeno FIS optimized by ANFIS is considered. The initial fuzzy inference system can be formulated based on an appropriate set of input and output data of the examined system. The occurring FIS, which is considered in the present investigation, has two inputs, i.e. the displacement and the velocity and one output, i.e. the control force. The procedure can be described in the following steps.

Step 1. On the first step of simulation, a set of discrete values of the displacement and velocity without control are computed. The dynamic system provides the discrete values of the displacement \( w^{mn}(t_k) \) and the velocity \( w'^{mn}(t_k) \), \( k = 1, 2, ..., K \) without control. These values can be used as data for training. A Sugeno FIS can be constructed, using \( w^{mn}(t_k) \) and \( w'^{mn}(t_k) \) with fixed \( m, n \). (Below in Section 6 we consider \( m = n = 1 \) for constructing the Sugeno FIS.)

Step 2. A Sugeno-type controller is built and optimized, by using the training data obtained in the previous step. The output of this controller is a control force which can be used at each time step of the simulation for vibration suppression. The input data are the displacement \( w^{mn}(t_k) \) and the velocity \( w'^{mn}(t_k) \) at a given point (sensor), and the output data are the control forces, which are applied by the actuator. Subtractive clustering is chosen for the creation of the membership functions of the inputs and then the FIS is generated. Let \( m = m_0, n = n_0 \) and suppose \( v(t_k) = w^{m_0n_0}(t_k), v'(t_k) = w'^{m_0n_0}(t_k), k = 1, 2, ..., K \). A typical set of rules is used. For inputs \( v(t_k), v'(t_k) \) the output is assumed to be a linear function

\[
 f_k = av(t_k) + bv'(t_k) + c, \tag{16}
\]

where \( a, b \) and \( c \) are created by the system. For a zero-order Sugeno model, the output level \( f_k \) is a constant (\( a = b = 0 \)). Each rule \( i \) weights its output level \( f^i_k \), by the firing strength of the rule \( \delta^i_k \),

\[
 \delta^i_k = \text{AndMethod} (F_1(v(t_k)), F_2(v'(t_k))), \tag{17}
\]

where \( F_1, F_2 \) are the membership functions for the inputs \( v(t_k), v'(t_k) \). In the present investigation the “AND” method for the combination of the involved rules is used. The final output of the system is calculated and then defuzzified by using...
the weighted average of all the outputs resulting from the activated rules as

\[ \bar{f}(t_k) = \frac{\sum_{i=1}^{M} \delta_i f_i}{\sum_{i=1}^{M} \delta_i}, \]  

(18)

where \( M \) is the number of rules for Input 1: \( v(t_k) \) and Input 2: \( v'(t_k) \).

**Step 3.** For the formulation of the Sugeno FIS, the training data are loaded in ANFIS. Then, the membership functions of the inputs (clusters) are formulated using subtractive clustering, and are associated with each other using a set of if-then rules. After the creation of the initial Sugeno FIS, the system is trained, that is, optimized by using a hybrid method, which is based on the least square and the backpropagation method. Finally, the system optimizes, i.e. corrects the rules and all the characteristics of the Sugeno controller, based on the training data and finds the optimal \( a, b \) and \( c \), i.e. the best fitting for \( f_k \).

**Step 4.** After the training process, an optimized controller for the suppression of the displacement and velocity (i.e. all the coefficients in (8)) is available. In some cases scaling is required.

As will be shown in the following Section 6, the same FIS can be used even for external sinusoidal loading functions with higher frequency. In order to optimize the procedure, each of the coefficients can be controlled separately, which means that \( N \times N \) Sugeno systems are required. It is obvious that this option is computationally expensive, however the results are usually much better.

6. **Numerical examples.** Some numerical examples using Sugeno-ANFIS and Mamdani FIS controllers for the simply supported plate (4) with \( l_1 = l_2 = 1 \), \( E = 2 \), \( \nu = 0.5 \), \( c = 0 \), \( D = 1 \), \( \rho = 1000 \) are presented below. The external loading force is given as \( Q(t) = 10 \sin \omega t \) and it is uniformly distributed on the surface of the plate. Hence, \( P_{Q_N}(t_k)_{mn} = 80\sqrt{l_1 l_2}/(\pi^2 mn) \sin \omega t_k \) (see (15)). In the examples below the number of basis functions is \( N = 4 \) and the final time which is needed for the simulation is set equal to \( T = 25 \).

The Sugeno-ANFIS and Mamdani FIS control schemes are generated for input data \( w^{11}(t_k), w^{11}_1(t_k) \) (displacement and velocity) and output data (control forces) \( P_{Q_N}(t_k)_{11} \) with frequency \( \omega = 5\pi \).

In the first two examples \( \omega = 10\pi \) in the loading function has been considered, and the controllers are based on the previously explained Sugeno-ANFIS and Mamdani FIS. For the Mamdani FIS, the membership functions are plotted in Figures 2-4 and the displacement \( w^{11}(t_k) \) and velocity \( w^{11}_1(t_k) \) without control and with control are presented in Figures 5-6. The external loading \( z^{11}(t_k) \) and the control forces are shown in Figure 7. For the Sugeno-ANFIS, the membership functions of the two inputs are the clusters shown in Figures 8 and 9 as described in Section 5. The membership function of the output is a linear function (see Step 2 in Section 5). The coefficients \( w^{11}(t_k), w^{11}_1(t_k) \) and the control forces \( z^{11}(t_k) \) with the external loadings are presented in Figures 10-12.

Through a comparison of the results of the previous scenario with the ones of the case when \( \omega = 5\pi \) (see Figures 13-15), one can observe that the vibration suppression is greater for \( \omega = 10\pi \). Very good results can also be reached when a constant loading function and therefore a constant control force is applied. In this case, ANFIS performs better because of the existence of stable vibrations.
**Figure 2.** Displacement (input 1) membership functions.

**Figure 3.** Velocity (input 2) membership functions
Figure 4. Control force (output) membership functions

Figure 5. Displacement before and after control with Mamdani FIS ($\omega = 10\pi$).
Figure 6. Velocity before and after control with Mamdani FIS ($\omega = 10\pi$).

Figure 7. External and Control forces with Mamdani FIS ($\omega = 10\pi$).
Figure 8. Clusters of input 1 (Displacement).

Figure 9. Clusters of input 2 (Velocity).
Figure 10. Displacement before and after control with Sugeno FIS ($\omega = 10\pi$).

Figure 11. Velocity before and after control with Sugeno FIS ($\omega = 10\pi$).
Figure 12. External and Control forces with Sugeno FIS ($\omega = 10\pi$).

Figure 13. Displacement before and after control with Sugeno FIS ($\omega = 5\pi$).
**Figure 14.** Velocity before and after control with Sugenoi FIS ($\omega = 5\pi$).

**Figure 15.** External and Control forces with Sugenoi FIS ($\omega = 5\pi$).
7. Comparison with the classical controllers. Most techniques for vibration control in nonlinear systems are based on a linearization of nonlinear models by approximating them by a linear system, which in turn is obtained by expanding the nonlinear solution in a series. Then, linear techniques can be used (7 etc.). Because of complexity in behaviour of the solution of some high-order nonlinear equations, a large error of computations from the linearization can be expected. Deep learning techniques, as well as neural networks can provide better approximation of the solution, however this occurs only in some special cases. The reason is the presence of nonlinear behaviour into the final solution. Moreover, tuning of the training data cannot always guarantee a good approximation of the output function.

On the other hand, the Mamdani type FIS (fuzzy controller) is quite good for the other cases. An important thing is that the fuzzy techniques either with Mamdani-type, as well as with Sugeno-type FIS, do not require linearization and can be easily performed.

Here, an adaptive neuro-fuzzy approach with Sugeno type FIS for control problems under harmonic-type loadings with high frequency is investigated. The idea is based on the use of an appropriate set of training data for the inputs and the output, along with an optimization method, in order to give the necessary feedback (approximation of control function) which is used for the control forces.

Below we compare one of the classical controllers, designed/adapted for linear problems with the FIS. In particular, we consider Linear Quadratic Regulator for vibration suppression of the linearized problem of (14). If the nonlinear part (the first term in the summation) in the right hand side of (14) is not present, the equations (14) correspond to the case of spatial discretization of the Kirchhoff-Love linear mechanical model (11, 15),

\[
M \ddot{w}_N(t) + C \dot{w}_N(t) + K_1 w_N(t) = Pq_N(t) + z_N(t). \tag{19}
\]

Let us now suppose \( v_1 = w, v_2 = \dot{w} \) and \( K = K_1 \). Then from (19) we obtain

\[
\dot{v}_1 = v_2, \\
\dot{v}_2 = M^{-1} [-Cv_2 - Kv_1 + Pq_N + Fz_N]. \tag{20}
\]

Hence we can write down the model (20) in the state space representation

\[
\dot{v} = \tilde{A}v + B\tilde{F}, \\
w = \tilde{C}v + D\tilde{F}, \tag{21}
\]

where \( v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \), \( \tilde{A} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \), \( B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \), \( \tilde{C} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \), \( D = [0] \) and \( \tilde{F} = Pq_N + Fz_N \). The size of the matrices \( \tilde{A}, B, \tilde{C} \) and \( D \) is \( 2xN^2 \times 2xN^2 \). The identity matrix \( I \) and the matrices \( M, K, C \) have the size \( N^2 \times N^2 \). Further, the vectors \( \tilde{F} \) and \( v \) have the size \( 2N^2 \).

At this point, the LQR function can be used in order to obtain the gain matrix \( G \) and the corresponding control \( z = -Gw \). For a continuous time system, the state-feedback law \(-Gw\) minimizes the quadratic cost function, which subjects to the dynamic system. After using the LQR, the system is again simulated and the output suppressed displacement is obtained.

Both methods, i.e. the classical LQR and the Fuzzy Inference System, have been tested. The results of vibration suppression are shown by an example. In the example the following physical parameters for the plate are taken: \( h = 0.5 \),

\[ \text{Here and below the index } N \text{ is omitted for the convenience.} \]
\( \rho = 1000, \ l_1 = l_2 = 1, \ N = 2, \) the final time \( T = 25 \) and the number of time steps is 1000. The periodic loading function has been considered, \( Q = q \sin(\omega t) \). Figures 16, 20, 17 and 21 show the displacement before and after the control by ANFIS and LQR, respectively. The Sugeno FIS has been generated with \( D = 1 \) and \( \omega = 5\pi \). In Figures 18, 22, 19 and 23 the external and the control forces after using the LQR and the ANFIS control respectively, are presented.

It is worth noting that the control forces, produced by the neuro-fuzzy controller, are lower than the ones produced by LQR. Another important aspect is that once the Sugeno FIS is generated, it can be used for different physical parameters of the plate. In Figures 16, 21 the displacement and the control forces for different values of the rigidity of the plate, \( D = 10 \) and \( D = 50 \) are presented. The results not only remain acceptable, but they are better than the ones obtained by the application of the LQR, indicating that the proposed controller can be also used even for larger values of \( \omega \).

8. **Conclusions.** Vibration suppression of a smart rectangular plate with the use of a Sugeno-ANFIS has been investigated. An effective procedure has been developed. A detailed analysis of an application of the proposed computational technique have been presented. The numerical results obtained by a trained neuro-fuzzy controller (Sugeno-type FIS) have been compared with the ones of a simple fuzzy controller (Mamdani-type FIS), as well as with the ones of a classical LQR controller. More specifically, structures with different rigidity have been tested with both methods. The results have shown that LQR control works only for linear problems and it is more expensive, as it requires the knowledge of the whole state-space. Moreover, the numerical experiments have shown that the Sugeno-type controller, which was optimized through ANFIS, works better for high frequency vibrations, for constant loading, and, in general, it shows its optimal behaviour when the solution of the problem is stable or close to stable. This can be explained by the fact that the least square, nonlinear regression and backpropagation methods cannot provide an adequate feedback when the solution has a non-stable complex behaviour. In these cases the Mamdani FIS is quite more suitable for the control of such nonlinear smart structures, when the direct fuzzy logic rules are applied on each time step.
Figure 16. Displacement before and after ANFIS with $\omega = 10$, $D = 10$ (the linear problem).

Figure 17. Displacement before and after using LQR with $\omega = 10$, $D = 10$. 
Figure 18. Loading and control forces with ANFIS with $\omega = 10$, $D = 10$ (the linear problem).

Figure 19. Loading and control forces with using LQR with $\omega = 10$, $D = 10$. 
Figure 20. Displacement before and after ANFIS with $\omega = 10$, $D = 50$ (the linear problem).

Figure 21. Displacement before and after using LQR with $\omega = 10$, $D = 50$. 
Figure 22. Loading and control forces with ANFIS with $\omega = 10$, $D = 50$ (the linear problem).

Figure 23. Loading and control forces with using LQR with $\omega = 10$, $D = 50$. 
REFERENCES

[1] Ph. G. Ciarlet, *Mathematical Elasticity, V. II: Theory of Plates*, Elsevier, Amsterdam, 1997.
[2] P. Ciarlet and P. Rabier, *Les Equations de von Kármán*, Springer-Verlag, Berlin, Heidelberg, New York, 1980.
[3] Ph. Destuynder and M. Salaun, *Mathematical Analysis of Thin Plate Models*, Mathématiques & Applications (Berlin) [Mathematics & Applications], Springer, 1996.
[4] D. Driankov, H. Hellendoorn and M. Reinfrank, *An Introduction to Fuzzy Control, 2nd edition*, Springer-Verlag, Berlin, Heidelberg, New York, 1996.
[5] G. Duvaut and J. L. Lions, *Les Inequations en Meccaniques et en Physiques*, Dunod, 1972.
[6] N. R. Fisco and H. Adel, Smart structures: Part II: Hybrid control systems and control strategies, *Scientia Iranica*, 18 (2011), 285–295.
[7] A. Isidori, *Nonlinear Control Systems* 3rd edition, Springer Verlag, London, 1995.
[8] S. Korkmaz, A review of active structural control: challenges for engineering informatics, *Comput. and Struct.*, 89 (2011), 2113–2132.
[9] P. Koutsianitis, G. K. Tairidis, G. A. Drosopoulos, G. A. Foutsitzi and G. E. Stavroulakis, Effectiveness of optimized fuzzy controllers on partially delaminated piezocomposites, *Acta Mechanica*, 228 (2017), 1373–1392.
[10] A. D. Muradova, A time spectral method for solving the nonlinear dynamic equations of a rectangular elastic plate, *J. Eng. Math.*, 92 (2015), 83–101.
[11] A. D. Muradova and G. E. Stavroulakis, Fuzzy vibration control of a smart plate, *Int. J. Comput. Meth. Eng. Sci. Mech.*, 14 (2013), 212–220.
[12] A. D. Muradova and G. E. Stavroulakis, Hybrid control of vibrations of smart von Kármán, *Acta Mechanica*, 226 (2015), 3463–3475.
[13] R. E. Precup and H. Hellendoorn, A survey on industrial applications of fuzzy control, *Computers in Industry*, 62 (2011), 213–226.
[14] A. Preumont, *Vibration Control of Active Structures* Springer, 2002.
[15] J. N. Reddy, *Theory and Analysis of Elastic Plates and Shells*, CRC Press, Taylor & Francis, 2007.
[16] A. H. N. Shirazi, H. R. Owji and M. Rafeeyan, Active vibration control of an FGM rectangular plate using fuzzy logic controllers, *Procedia Engineering*, 14 (2011), 3019–3026.
[17] G. K. Tairidis, G. E. Stavroulakis, D. G. Marinova and E. C. Zacharenakis, Classical and soft robust active control of smart beams, *Computat. Struct. Dynamics and Earthquake Engineer. (eds. Papadrakis, M., Charmpis, D. C. Lagaros and N. D., Tsompanakis)*, CRC Press/Balkema and Taylor & Francis Group, London, UK., Ch. 11 (2009), 165–178.
[18] A. R. Tavakolpour, M. Mailah, I. Z. M. Darus and O. Tokhi, Self-learning active vibration control of a flexible plate structure with piezoelectric actuator, *Simul. Model. Prac. and Theory*, 18 (2010), 516–532.
[19] Q. Wangzong, S. Jincai and Q. Yang, Active control of vibration using a fuzzy control method, *J. of Sound and Vibration*, 275 (2004), 917–930.
[20] I. J. Zeinoun and F. Khorrami, An adaptive control scheme based on fuzzy logic and its application to smart structures, *Smart Mater. Struct.*, 3 (1994), 266–276.