Improved Approximation for Orienting Mixed Graphs*

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\textbf{Abstract.} An instance of the maximum mixed graph orientation problem consists of a mixed graph and a collection of source-target vertex pairs. The objective is to orient the undirected edges of the graph so as to maximize the number of pairs that admit a directed source-target path. This problem has recently arisen in the study of biological networks, and it also has applications in communication networks.

In this paper, we identify an interesting local-to-global orientation property. This property enables us to modify the best known algorithms for maximum mixed graph orientation and some of its special structured instances, due to Elberfeld et al. (CPM ’11), and obtain improved approximation ratios. We further proceed by developing an algorithm that achieves an even better approximation guarantee for the general setting of the problem. Finally, we study several well-motivated variants of this orientation problem.

\section{Introduction}

An instance of the \textit{maximum mixed graph orientation} problem consists of a \textit{mixed} graph $G = (V, E_D \cup E_U)$ with $n$ vertices, such that $E_D$ and $E_U$ indicate the sets of directed and undirected edges, respectively. An additional ingredient of the input is a collection $P \subseteq V \times V$ of source-target vertex pairs. A source-target vertex pair $(s, t) \in P$ is called a \textit{request}. The objective is to orient $G$ in a way that maximizes the number of satisfied requests. An \textit{orientation} of $G$ is a directed graph $G = (V, E_D \cup E_U)$, where $E_U$ is a set of directed edges obtained by choosing a single direction for each undirected edge in $E_U$. A request $(s, t)$ is said to be \textit{satisfied} under an orientation $G$ if there is a directed path from $s$ to $t$ in $G$.

One may assume without loss of generality that the mixed graph $G$ is \textit{acyclic}, that is, a graph that has no cycles. This assumption holds since any instance of maximum mixed graph orientation can be reduced to another instance in which the underlying mixed graph is acyclic without affecting the number of requests that can be satisfied \cite{16,6}.

* Due to space limitations, some proofs are omitted from this extended abstract. We refer the reader to the full version of this paper (available online at http://arxiv.org/abs/1204.0219), in which all missing details are provided.

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Indeed, if the input graph contains cycles, one can sequentially contract them one after the other. In each step, the undirected edges of an arbitrary cycle are all oriented in the same direction. In particular, if this cycle contains directed edges then the undirected edges are oriented in a consistent way with those edges. As a result, every pair of vertices on this cycle admits a directed path between them, and thus, the cycle can be contracted. One can easily validate that the resulting mixed acyclic graph consists of undirected components, each of which must be an undirected tree, and those components are connected by directed edges in a way that does not produce cycles. The maximum mixed graph orientation problem draws its interest from applications in network biology and communication networks:

**Network Biology.** Recent technological advances, such as yeast two-hybrid assays [8] and protein co-immunoprecipitation screens [11], enable detecting physical interactions in the cell, leading to protein-protein interaction (PPI) networks. One major caveat of those PPI measurements is that they do not reveal information about the directionality of the interactions, namely, the directions in which the signal flows. Since PPI networks serve as the skeletons of signal transduction in the cell, inferring the hidden directionality information may provide insights to the inner working of the cell. Such an information may be inferred from causal relations in those networks [17]. One such source of causal relations is perturbation experiments, in which a gene is perturbed (cause) and as a result, other genes change their expression levels (effects). A change of expression of a gene suggests that the corresponding proteins admit a path in the network, and in particular, it is assumed that there must be a directed path from the causal gene to the affected gene.

Up until this point in time, the above-mentioned scenario can be modeled as a special instance of the maximum mixed graph orientation problem in which one is interested to orient the edges of an undirected network in a way that maximizes the number of cause-effect pairs that admit a directed path from the causal gene to the affected gene. However, in the more accurate biological variant, there are several interactions whose directionality is known in advance. For instance, protein-DNA interactions are naturally directed from a transcription factor to its regulated genes, and some PPIs, like kinase-substrate interactions, are known to transmit signals in a directional fashion. Therefore, in general, the input network is a mixed graph.

**Communication Networks.** A unidirectional communication network consists of communication links that allow data to travel only in one direction. One main benefit of such communication links is that the data of the device on one side is kept confidential while it may still access the data of the device on the other side. As a consequence, unidirectional networks are most commonly found in high security environments, where a connection may be made between devices with differing security classifications. For example, unidirectional communication links can be used to facilitate access to a vulnerable domain such as the Internet to devices storing sensitive data. The maximum mixed graph orientation problem captures the interesting scenario in which one is interested to design a unidirectional network that maximizes the number of connection requests that can be satisfied in a secure way. We remark that unidirectional networks have also been studied in distributed and wireless ad hoc settings (see, e.g., [21][14] and the references therein), where a common focus is on algorithmic questions that arise in