Non-linear corrections to the cosmological matter power spectrum and scale-dependent galaxy bias: implications for parameter estimation

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Received 30 April 2008
Accepted 27 June 2008
Published 23 July 2008

Online at stacks.iop.org/JCAP/2008/i=07/a=017
doi:10.1088/1475-7516/2008/07/017

Abstract. We explore and compare the performances of two non-linear correction and scale-dependent biasing models for the extraction of cosmological information from galaxy power spectrum data, especially in the context of beyond-ΛCDM (CDM: cold dark matter) cosmologies. The first model is the well known $Q$ model, first applied in the analysis of Two-degree Field Galaxy Redshift Survey data. The second, the $P$ model, is inspired by the halo model, in which non-linear evolution and scale-dependent biasing are encapsulated in a single non-Poisson shot noise term. We find that while the two models perform equally well in providing adequate correction for a range of galaxy clustering data in standard ΛCDM cosmology and in extensions with massive neutrinos, the $Q$ model can give unphysical results in cosmologies containing a subdominant free-streaming dark matter whose temperature depends on the particle mass, e.g., relic
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thermal axions, unless a suitable prior is imposed on the correction parameter. This last case also exposes the danger of analytic marginalization, a technique sometimes used in the marginalization of nuisance parameters. In contrast, the $P$ model suffers no undesirable effects, and is the recommended non-linear correction model also because of its physical transparency.

**Keywords:** dark matter, surveys of galaxies, semi-analytic modelling, power spectrum

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1. Introduction

The past decade saw an explosion of precision cosmological measurements. Foremost amongst these is the observation of temperature and polarization fluctuations in the cosmic microwave background (CMB) by a range of experiments [1]–[6]. The distribution of large-scale structures (LSS) has also been mapped to unprecedented breadths and depths by galaxy redshift surveys such as the Two-degree Field Galaxy Redshift Survey.
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(2dFGRS) [7] and the Sloan Digital Sky Survey (SDSS) [8, 9]. Together with observations of distant type Ia supernovae [10, 11], these measurements have fostered the emergence of a benchmark framework—the adiabatic, nearly scale-invariant, ‘vanilla’ ΛCDM model—based on which one can test for evidence of new physics.

Clustering statistics of galaxies as probed by surveys like 2dF and SDSS are particularly well suited for the exploration of physics that introduces new effects on length scales $\mathcal{O}(10) \rightarrow \mathcal{O}(100) \, h^{-1} \, \text{Mpc}$. A classic example is the possibility of detecting a subdominant component of free-streaming hot dark matter (HDM) [12], notably massive neutrinos [13, 14] and variants such as thermal axions [15]–[19], light gravitinos [20], etc. The sensitivity of these surveys at small length scales also lends a greater lever arm to the search for features in the primordial density perturbation power spectrum, possible remnants of inflationary physics [21, 22]. Last but not least, since the power spectrum of large-scale structures probes uniquely the parameter combination $\Omega_m h$, it helps to lift the degeneracy between—and hence tighten the constraints on—the physical matter density $\Omega_m h^2$ and the Hubble parameter $h$ from CMB observations alone.

Usage of data from galaxy clustering surveys is based on the premise that one can reliably predict the distribution of galaxies, at least on a statistical basis, from theory. This is complicated by a number of factors. First, galaxies are necessarily collapsed objects, i.e., they have undergone a phase of non-linear evolution. Using them as tracers of the underlying matter field implicitly assumes that we know how to relate the two distributions to one another. A reasonable assumption is that on sufficiently large scales the power spectra of galaxies and of the matter field are identical up to a constant normalization, or bias, factor. But this bias relation is expected to become scale dependent when the dimensionless power spectrum of the galaxies $\Delta_{\text{gal}} \equiv k^3 P_{\text{gal}} / 2\pi^2$ exceeds unity [23, 24]. Indeed, the apparent tension between the 2dF and the SDSS galaxy power spectra is now believed to have originated from a more strongly scale-dependent bias factor for the red galaxies dominating the SDSS galaxy catalogue [25, 26]. On the theoretical front, a good deal of recent effort has also been devoted to understanding the origin of scale-dependent biasing (e.g., [27, 28]).

Second, galaxy positions are inferred from their redshifts. However, the peculiar motions of the galaxies, particularly when amplified by virialization, induce additional Doppler shifts that can potentially obscure the inference. This is known as redshift space distortion, and on small length scales requires corrections beyond linear perturbation theory. Third, the clustering of the underlying dark matter field itself becomes non-linear on scales $k \gtrsim 0.2h \, \text{Mpc}^{-1}$. Thus, how reliably one can extract cosmological information from galaxy clustering statistics depends crucially on how well one can model these three non-linear effects.

While all three effects can in principle be modelled by numerical simulations, these simulations, and indeed our understanding of galaxy formation, are not yet at a stage where one can reliably predict the power spectrum of galaxies as a function of galaxy type and redshift given some underlying cosmological model. In the meantime, non-linear evolution and scale-dependent biasing must be modelled empirically as a systematic effect and the associated nuisance parameters marginalized when extracting cosmological information from galaxy clustering surveys, especially in beyond-ΛCDM cosmologies.

In this connection, Cole et al [7] recently proposed a correction formula which maps directly between the matter power spectrum calculated from linear perturbation theory
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$P_{\text{lin}}(k)$ and the power spectrum expected for the galaxies $P_{\text{gal}}(k)$,

$$P_{\text{gal}}(k) = b^2 \frac{1 + Q_{\text{nl}} k^2}{1 + A_{\text{nl}} k} P_{\text{lin}}(k).$$  \hfill (1.1)

The formula is partially calibrated against ΛCDM-based semi-analytic galaxy formation
simulations ($A_{\text{nl}} = 1.4$ for redshift space, and 1.7 for real space), and contains two free
parameters ($b$ and $Q_{\text{nl}}$) to be fixed by observational data. Equation (1.1) has been applied
to the galaxy power spectra of 2dF [7] and the SDSS luminous red galaxy (LRG) sample [8]
to test standard vanilla cosmology. However, there is a priori no guarantee that its
usefulness extends also to cosmologies beyond ΛCDM. Indeed, as we shall show below,
equation (1.1) can be highly pathological when applied to certain classes of cosmological
models containing a subdominant component of free-streaming dark matter.

In comparison, a conceptually more appealing framework in which to discuss non-
linear corrections is the halo model [29]–[32]. Building on the assumptions (i) of
hierarchical clustering, and (ii) that galaxies form only inside dark matter halos, halo
model-based non-linear corrections can in principle be made applicable to all hierarchical
CDM cosmologies. The minimal model proposed in [33]–[35], for example,

$$P_{\text{gal}}(k) = b^2 P_{\text{lin}}(k) + P_{\text{shot}},$$ \hfill (1.2)

where $b$ and $P_{\text{shot}}$ are free parameters, does not demand vanilla ΛCDM as the sole input
cosmology.

In the present work, we explore and compare the performance of these non-linear
correction models in some detail. We confront them with various observed galaxy power
spectra, especially in the context of beyond-ΛCDM cosmologies. We discuss parameter
degeneracies and the role of priors on the nuisance parameters.

The paper is organized as follows. We describe first in section 2 the galaxy clustering
data sets used in the analysis. Section 3 contains a more detailed discussion of the two
non-linear models that we wish to explore. In sections 4, 5 and 6 we test the non-linear
models against data for standard vanilla cosmology, vanilla with massive neutrinos, and
vanilla with thermal axions respectively. We conclude in section 7.

2. Data sets

We use the publicly available galaxy power spectra from the following galaxy catalogues.

2dF. This data set comes from the final data release of the Two-degree Field Galaxy
Redshift Survey [7]. We use up to 36 data bands, corresponding to redshift space power
spectrum data for wavenumbers $0.02 \lesssim k/(h \text{ Mpc}^{-1}) \lesssim 0.18$.

SDSS-2 main. The real space power spectrum of the main galaxy sample from the
Sloan Digital Sky Survey data release 2 [36]. We use up to 19 data bands, i.e.,
$0.016 \lesssim k/(h \text{ Mpc}^{-1}) \lesssim 0.2$.

SDSS-4 LRG. The real space power spectrum of the luminous red galaxies from the Sloan
Digital Sky Survey data release 4 [8]. The 20 data bands correspond to wavenumbers
$0.012 \lesssim k/(h \text{ Mpc}^{-1}) \lesssim 0.2$.

WMAP-3. We also use at times CMB data from the Wilkinson Microwave Anisotropy
Probe experiment after three years of observations [37]–[39], mainly for the construction
of priors on certain cosmological parameters. This calculation is performed using version 2 of the likelihood package provided by the WMAP team on the LAMBDA web page [40].

3. Two non-linear models

3.1. The Q model

We refer to the correction formula (1.1) as the Q model. For ΛCDM cosmologies, galaxy formation simulations suggest that the parameter $A_{nl}$ can be held fixed at $A_{nl} = 1.4$ in redshift space and at $A_{nl} = 1.7$ in real space [7]. The parameter $Q_{nl}$, however, exhibits a strong dependence on the galaxy type. Fitting the Q model to the 2dF galaxy power spectrum at $0.02 \lesssim k/(h\text{ Mpc}^{-1}) \lesssim 0.3$, Cole et al found $Q_{nl} = 4.6 \pm 1.5$ for vanilla cosmology [7]. Tegmark et al [8] applied the same model to SDSS-4 LRG, for which $Q_{nl} = 30 \pm 4$ provides a good fit.

Note that the case of $Q_{nl} = 0$ is not equivalent to no non-linear correction, since a non-zero $A_{nl}$ parameter also modulates the power spectrum in a scale-dependent way. For $Q_{nl} \lesssim 7$, the Q model suppresses the power spectrum at $k \lesssim 0.2h\text{ Mpc}^{-1}$. However, for $Q_{nl}$ values as large as 30 such as required by the SDSS-4 LRG, the main role of the Q model is to add power at $k \gtrsim 0.07h\text{ Mpc}^{-1}$.

3.2. The P model

The functional form of the Q model (1.1) was recently criticized in reference [28] for its incorrect dependence on $k$. Specifically, it lacks a constant term needed to account for the presence of non-Poisson shot noise, a generic consequence of the assumption that galaxies form exclusively in halos. Indeed, from the halo model [29]–[32], one should expect a correction formula with the skeletal form

$$P_{\text{gal}}(k) = P_{2h}(k) + P_{1h}(k).$$

Here, the two-halo term, $P_{2h}(k) = b^2(k)P_{\text{lin}}(k)$, arises from correlations between galaxies in two different halos, and approximates the familiar linear bias relation on large scales,

$$P_{2h}(k) \approx b^2 P_{\text{lin}}(k).$$

The one-halo term

$$P_{1h}(k) \approx \text{const} \equiv P_{\text{shot}}$$

accounts for correlations within the same halo and is approximately independent of the exact spatial distribution of the galaxies within a halo provided $k$ is not too large. Combining equations (3.2) and (3.3) we recover the minimal model (1.2).

5 Tegmark et al [8] used $A_{nl} = 1.4$, although strictly speaking $A_{nl} = 1.7$ is the more appropriate value for the real space power spectrum of SDSS-4 LRG. However, at the present level of precision, our tests show that adopting the correct $A_{nl} = 1.7$ only causes a statistically insignificant upward shift in the best-fit $Q_{nl}$ ($\Delta Q_{nl} \sim 2$), while the estimates of other cosmological parameters remain unaffected. Henceforth we shall use exclusively $A_{nl} = 1.4$ for both real and redshift space power spectra.
The role of the shot noise term $P_{\text{shot}}$ is to add power at small length scales, so that the ratio $P_{\text{gal}}(k)/P_{\text{lin}}(k)$ is effectively scale dependent at large $k$ values. This is in contrast with the $Q$ model (1.1), which for some values of the non-linear parameter $Q_{\text{nl}}$ suppresses the power spectrum on the observable scales. Interestingly, taken at face value, the $P$ model also predicts a significant $k$-dependence for $P_{\text{gal}}(k)/P_{\text{lin}}(k)$ as $k \to 0$ when $P_{\text{shot}}$ once again rises above $b^2 P_{\text{lin}}(k) \sim k$ [29]. It should be noted however that this small-$k$ behaviour is not present for dark matter clustering, since momentum conservation demands that the dark matter power spectrum falls off faster than $P(k) \propto k^4$ as $k \to 0$ [41], a behaviour also observed in numerical simulations [42,43]. On the other hand, a non-vanishing shot noise on large scales is in principle not forbidden for galaxy clustering, since tracers need not conserve momentum. Whether or not this is so remains to be understood.

In the present work we take the view that since galaxy clustering has not been observed on scales where the shot noise behaviour may become problematic ($k \ll 0.01h$ Mpc$^{-1}$), equation (1.2) constitutes a sufficient phenomenological model for describing the galaxy power spectrum at $k \gtrsim 0.01h$ Mpc$^{-1}$.

Additional modifications to the basic $P$ model (1.2) to account for the damping of baryon acoustic oscillations and other non-linear mode coupling effects have been discussed in the literature [28,43]–[45]. These generally lead to non-linear models of the form

$$P_{\text{gal}}(k) = b^2 A(k) P_{\text{lin}}(k) + P_{\text{shot}},$$

(3.4)

where $A(k)$ is some function of $k$ that depends also on the galaxy type. It is also possible to extend the halo model to include redshift space distortion [35,45,46]. We ignore these additional corrections in the present analysis in order to keep the number of extra fit parameters to a minimum. However, we emphasize that the effects encapsulated by $A(k)$ will become increasingly important as more precise data from future galaxy redshift surveys become available.

4. Test 1: vanilla

4.1. Set-up

To compare the performance of the two non-linear models, we test them against galaxy clustering data in three minimal parameter spaces:

(i) $\Omega_m h, \Omega_b h^2, h, n_s, \ln(10^{10} A)$,
(ii) $\Omega_m h, \Omega_b h^2, h, n_s, \ln(10^{10} A), P_{\text{shot}}$, and
(iii) $\Omega_m h, \Omega_b h^2, h, n_s, \ln(10^{10} A), Q_{\text{nl}}$.

We take the geometry of the universe to be flat, and the initial conditions adiabatic. The parameter $A \equiv b^2 A_s$ accounts for the normalization of the galaxy power spectrum, and incorporates both the amplitude of the primordial scalar perturbations $A_s$ and the constant galaxy bias factor $b$. We do not use any non-linear correction for parameter space (i), i.e., $P_{\text{gal}}(k) = b^2 P_{\text{lin}}(k)$. For parameter spaces (ii) and (iii), we use the $P$ model (1.2) and the $Q$ model (1.1) respectively.

Note the definition of the matter density parameter $\Omega_m h$. We choose this parameterization because the turning point of the matter power spectrum is sensitive to the comoving Hubble radius at matter–radiation equality, which, for fixed values of $h$,
Table 1. Priors used in our vanilla and vanilla + massive neutrinos analyses. Top: WMAP priors based on the three-year data release. These are approximated as perfect Gaussians, and we give here their respective mean and standard deviation. Bottom: top-hat priors for the remaining parameters.

| Parameter          | Vanilla | Vanilla + neutrinos |
|--------------------|---------|----------------------|
| WMAP-3 priors      |         |                      |
| $\Omega_b h^2$    | 0.02229 ± 0.00073 | 0.02158 ± 0.00082 |
| $h$                | 0.732 ± 0.032     | 0.651 ± 0.054       |
| $n_s$              | 0.958 ± 0.016     | 0.941 ± 0.022       |
| Top-hat priors     |         |                      |
| $\Omega_m h$      | 0.05–0.72        |                      |
| $\ln(10^{10} A_s)$| 1–5             |                      |
| $P_{\text{shot}}$ | 0–min($P_{\text{obs}}$) |                 |
| $Q_{\text{nl}}$   | 0–50            |                      |

depends on $\Omega_m h$, not the physical matter density $\Omega_m h^2$. For fixed values of $n_s$ and $h$, $\Omega_m h$ alone determines the broad shape of the matter power spectrum within the ΛCDM framework.

We use standard Bayesian inference techniques and the Markov chain Monte Carlo package CosmoMC [47,48] to explore the posterior hypersurfaces as functions of the model parameters and the galaxy clustering data sets of section 2. Here, we note that within the vanilla framework, the physical baryon density $\Omega_b h^2$, the Hubble parameter $h$, and the scalar spectral index $n_s$ can be individually well constrained by CMB observations. This information is encapsulated in a set of ‘WMAP-3 priors’ in table 1, which we apply when varying these three parameters. This approach differs slightly from the more common practice of fixing the parameter values $h = 0.72$ and $n_s = 1$ adopted in, e.g., references [7,26]. Reference [8] further fixes $\Omega_b h^2 = 0.0223$. Our approach has the advantage that it properly takes into account the uncertainties on these parameters and thus avoids two inherent dangers of fixed parameter analyses: biased parameter estimates and underestimated errors. Additionally, it permits a consistent comparison of our $\Omega_m h$ constraints not only between different galaxy clustering data sets, but also with those obtained from CMB observations alone.

Broad, top-hat priors are imposed on the remaining parameters (table 1). For the parameter $Q_{\text{nl}}$, we choose the range 0–50 for all three data sets. In the absence of additional information from, e.g., galaxy formation simulations, the upper limit of this prior is somewhat arbitrary. We will discuss this point in more detail in section 6. For the $P_{\text{shot}}$ prior, the upper limit min($P_{\text{obs}}$) denotes the minimum clustering power measured by a survey. In other words, the linear matter power spectrum $P_{\text{lin}}(k)$ must not be negative anywhere. For 2dF, SDSS-2 main, and SDSS-4 LRG, min($P_{\text{obs}}$) = \{4000, 1200, 9300\}, respectively.

4.2. The internal test: do the individual data sets call for non-linear correction?

Figures 1–3 show the 1D marginal constraints on the parameters \{$\Omega_m h, P_{\text{shot}}, Q_{\text{nl}}$\} and the corresponding minimum $\chi^2$ and numbers of degrees of freedom (d.o.f.) as functions of the maximum wavenumber $k_{\text{max}}$ included in the analysis. The number of d.o.f. is defined
Figure 1. 1D marginal 68% MCIs for $\Omega_m h$ and 1σ intervals (see the main text for definitions) for the non-linear parameters $P_{\text{shot}}$ and $Q_{\text{nl}}$ as functions of $k_{\text{max}}$ using the SDSS-4 LRG power spectrum (green/shaded regions). Left: no non-linear correction. Centre: correction with the $P$ model. Right: correction with the $Q$ model. In each case the black dotted line indicates the 1D mode. The top plot in each column shows the minimum $\chi^2$ (red/solid) and the number of degrees of freedom in the fit (blue/dashed).

For $\Omega_m h$, the green/shaded regions correspond to the 1D marginal 68% minimum credible intervals (MCI), while the 1D modes are indicated by black/dotted lines. On the other hand, the marginalized posterior distributions in $P_{\text{shot}}$ and $Q_{\text{nl}}$ are often extremely flat, especially at small values of $k_{\text{max}}$. While it is technically possible to construct credible intervals in these cases, quoting such an interval would distract from the fact that the parameters are essentially unconstrained. Therefore, instead of Bayesian intervals, we give in figures 1–3 for $P_{\text{shot}}$ and $Q_{\text{nl}}$ the parameter regions satisfying

$$-2(\ln P - \ln P_{\text{max}}) < 1,$$

where $P$ is the 1D marginal posterior and $P_{\text{max}}$ denotes its value at the 1D mode. We loosely label this the ‘1σ’ interval. For large $k_{\text{max}}$ values, especially $k_{\text{max}} \sim 0.2 h \text{ Mpc}^{-1}$, where the posterior distributions are approximately Gaussian, this 1σ interval is identical to the 68% MCIs. See [49] for more detailed discussion of the various statistical quantities.

4.2.1. Changes in the $\Omega_m h$ estimates. The effects of non-linear correction are most evident when we include data beyond $k_{\text{max}} \sim 0.1 h \text{ Mpc}^{-1}$. Here, non-linear correction generally
shifts the $\Omega_m h$ estimates to lower values relative to the case with no correction, irrespective of the non-linear model used. For SDSS-4 LRG, this shift is very dramatic: at $k_{\text{max}} \sim 0.2 h \text{ Mpc}^{-1}$ the 68% MCI moves down by an amount comparable to six or seven times its half-width (figure 1). In contrast, the shifts induced for 2dF and SDSS-2 main by either non-linear model are mild: at $k_{\text{max}} \sim 0.2 h \text{ Mpc}^{-1}$ a small overlap between the 68% MCIs before and after correction can still be seen in figures 2 and 3.
4.2.2. The χ² test. The need for non-linear correction in the case of SDSS-4 LRG is corroborated by a comparison of the minimum χ² values. Between correction and no correction, figure 1 shows that at \( k_{\text{max}} \sim 0.2h \text{ Mpc}^{-1} \), \( \chi^2_{\text{min}} \) changes from \( \sim 60 \) for 18 d.o.f. to \( \sim 20 \) for 17 d.o.f., again irrespective of the non-linear model used. On the other hand, similar comparisons for 2dF and for SDSS-2 main in figures 2 and 3 do not indicate any urgent need for an extra correction parameter: for 2dF, \( \Delta \chi^2_{\text{min}} \sim 4 \); for SDSS-2 main, \( \Delta \chi^2_{\text{min}} \) is virtually negligible.

4.2.3. Internal consistency. We consider a data set internally consistent if the \( \Omega_m h \) credible intervals agree for all choices of \( k_{\text{max}} \). For SDSS-4 LRG, figure 1 clearly demonstrates that non-linear correction is necessary in order to achieve some semblance of internal consistency. A small discrepancy remains after correction: at \( k_{\text{max}} \sim 0.03 \) and \( 0.2h \text{ Mpc}^{-1} \) the 68% MCIs do not quite touch each other. However, this discrepancy is not statistically significant. We bring up this point here nonetheless as a caution against over-interpretation of credible intervals.

Non-linear correction also helps to improve consistency in the \( \Omega_m h \) estimates from 2dF in figure 2. As for SDSS-2 main, the large uncertainties in \( \Omega_m h \) in figure 3 mean that internal consistency is not an issue, with or without non-linear correction.

4.2.4. How much correction? Focusing on the cases with non-linear correction, we see in figures 1–3 that while SDSS-4 LRG clearly prefers a non-zero correction parameter—be it \( P_{\text{shot}} \) or \( Q_{\text{nl}} \)—at \( k_{\text{max}} \gtrsim 0.1h \text{ Mpc}^{-1} \), the 2dF and the SDSS-2 main data sets do not exhibit the same strong preference. For SDSS-2 main and most choices of \( k_{\text{max}} \) for 2dF, the 1σ regions include \( P_{\text{shot}} = 0 \) and \( Q_{\text{nl}} = 0 \) (although, as mentioned before, the case of \( Q_{\text{nl}} = 0 \) is not equivalent to no non-linear correction). At \( k_{\text{max}} \lesssim 0.1h \text{ Mpc}^{-1} \), the parameters \( P_{\text{shot}} \) and \( Q_{\text{nl}} \) cannot be constrained by data.

In terms of the \( P \) model, roughly half of the small-scale (\( k \gtrsim 0.1h \text{ Mpc}^{-1} \)) power in SDSS-4 LRG can be attributed to the shot noise term. For 2dF and SDSS-2 main, the shot noise contribution is about a quarter according to the 1D mode.

4.2.5. Which non-linear model? Perhaps the most striking feature as regards the two non-linear models considered here is that their corrective effects appear to be identical. This is particularly apparent in figures 1 and 2 at \( k_{\text{max}} \gtrsim 0.1h \text{ Mpc}^{-1} \), where the preferred values of the correction parameters \( P_{\text{shot}} \) and \( Q_{\text{nl}} \) exhibit virtually the same dependence on \( k_{\text{max}} \). At \( k_{\text{max}} \sim 0.2h \text{ Mpc}^{-1} \) the \( \Omega_m h \) estimates also show little if any dependence on the priors imposed on the non-linear correction parameters. Thus as far as vanilla cosmology is concerned, there is no preference for either non-linear model from a phenomenological standpoint, although the transparency of the \( P \) model still makes it the more attractive of the two.

4.3. The external test: are all data sets consistent with each other?

Figure 4 shows how the constraints on \( \Omega_m h \) obtained from different galaxy clustering data sets and with different non-linear correction methods compare with each other. For good comparison we indicate in the figure also the corresponding estimate from WMAP-3. Similar information is available in table 2, in which we give the 1D marginal 68% and 95% MCIs at \( k_{\text{max}} \sim 0.2h \text{ Mpc}^{-1} \) (\( k_{\text{max}} \sim 0.18h \text{ Mpc}^{-1} \) for 2dF).
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Figure 4. 1D marginal 68% MCIs for $\Omega_m h$ as functions of $k_{\text{max}}$ using galaxy clustering data from SDSS-4 LRG (red/shaded region), 2dF (blue/dashed line) and SDSS-2 main (green/solid line). Top: no non-linear correction is used. Middle: correction with the $P$ model. Bottom: correction with the $Q$ model. For comparison we show also the corresponding 68% MCI from WMAP-3 (grey/dotted line).

For both SDSS-4 LRG and SDSS-2 main, non-linear correction clearly leads to better agreement with 2dF and with WMAP-3 at $k_{\text{max}} \gtrsim 0.1 h$ Mpc$^{-1}$. In the case of SDSS-2 main, although the correction is not sufficient to cause the 68% region to overlap with that from 2dF (using the same correction method), the two 95% regions are certainly consistent. Importantly, the level of disagreement between SDSS-2 main and 2dF after correction is no worse than the small internal inconsistency between the low and the high $k_{\text{max}}$ constraints on $\Omega_m h$ from SDSS-4 LRG already discussed in section 4.2. Thus if we were to accept that the non-linear models \((1.1)\) and \((1.2)\) offer sufficient correction for SDSS-4 LRG, logically we must also consider them adequate for SDSS-2 main.

Lastly, we observe in figure 4 that the 2dF preferred values of $\Omega_m h$ tend in any case to be on the low side of SDSS-2 main, even at values of $k_{\text{max}}$ well below those at which
non-linearity nominally sets in. This suggests that the residual inconsistency after non-linear correction can rather be put down to a statistical aberration at small k values, than is indicative of a failure of either non-linear model. Indeed, Sanchez and Cole [26] compared directly the power spectra of red galaxies (believed to be the main source of scale-dependent biasing) from 2dF and the main galaxy sample of SDSS data release 5, and found a similar discrepancy in the raw power spectrum data (see figure 7 of their paper). Our SDSS-2 main data set is but a subsample of SDSS data release 5; that it contains the same small fluctuation should be no surprise.

5. Test 2: vanilla + massive neutrinos

It is well known that a subdominant component of massive neutrino HDM in the matter content slows down the growth of density perturbations on small length scales. In terms of the matter power spectrum, we expect a suppression of power of order \( \Delta P_{\text{lin}}/P_{\text{lin}} \sim 8\Omega_\nu/\Omega_m \) at \( k > k_{\text{FS}} \), where \( k_{\text{FS}} \) is the neutrinos' free-streaming wavenumber, and \( \Omega_\nu \) the neutrino energy density. Since this suppression effectively changes the shape of the matter power spectrum at large wavenumbers, some amount of degeneracy between the neutrino energy density and the non-linear correction parameters could conceivably exist. In this section we investigate the possible existence of such a degeneracy, and if so, its effects on massive neutrino cosmology.

5.1. Set-up

We consider three parameter spaces:

(i) \( f_\nu, \Omega_m, h, \Omega_b h^2, n_s, \ln(10^{10} A) \),

(ii) \( f_\nu, \Omega_m, h, \Omega_b h^2, n_s, \ln(10^{10} A), P_{\text{shot}} \), and

(iii) \( f_\nu, \Omega_m, h, \Omega_b h^2, n_s, \ln(10^{10} A), Q_{\text{nl}} \).
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Table 3. 1D marginal 68% (95%) MCIs for $\Omega_m h, f_\nu$, and the non-linear parameters $P_{\text{shot}}$ and $Q_{\text{nl}}$ from SDSS-4 LRG, and the associated minimum $\chi^2$ values. These are obtained using (i) no non-linear correction, (ii) correction with the $P$ model, and (iii) correction with the $Q$ model. We quote here also the WMAP-3 preferred regions for comparison.

| Model       | $\Omega_m h$ | $f_\nu$ | $P_{\text{shot}}$ or $Q_{\text{nl}}$ | $\chi^2_{\text{min}}$ |
|-------------|--------------|--------|------------------------------------|------------------------|
| No correction | 0.246–0.287 (0.232–0.314) | 0.01–0.047 (<0.065) | — | 63.2 |
| $P$ model   | 0.168–0.216 (0.154–0.256) | <0.057 (<0.112) | 4070–5270 (3430–5810) | 19.7 |
| $Q$ model   | 0.171–0.226 (0.158–0.274) | <0.061 (<0.135) | 23.6–36.9 (17.5–47.5) | 20.6 |
| WMAP-3      | 0.178–0.245 (0.151–0.270) | <0.080 (<0.127) | — | — |

Here, the neutrino fraction $f_\nu \equiv \Omega_\nu / \Omega_m$ is defined as the ratio of the neutrino energy density $\Omega_\nu$ to the total matter density $\Omega_m$. The former is given by the well known expression

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{93 \text{ eV}},$$

where $\sum m_\nu$ denotes the sum of the neutrino masses. We assume 3.04 degenerate neutrino species, which should be a good approximation since neither CMB nor galaxy clustering measurements are at present sufficiently sensitive to $\sum m_\nu \lesssim 0.3 \text{ eV}$, where one might expect some small effects due to the neutrino mass hierarchy.

We fit these three cases to SDSS-4 LRG up to $k_{\text{max}} \sim 0.2 h \text{ Mpc}^{-1}$, using no non-linear correction in case (i), and correction with the $P$ and the $Q$ model respectively for cases (ii) and (iii). As in the previous section, we impose WMAP-3 priors on the parameters $h$, $n_s$, and $\Omega_b h^2$ tabulated in table 1. Note that these priors differ from those used previously, since CMB constraints are model dependent. We adopt a top-hat prior on $f_\nu$, 0–0.5, while for $Q_{\text{nl}}$ we use 0–100, in anticipation that more non-linear correction may be required to offset the higher $\Omega_m h$ values usually inferred in cosmologies with massive neutrinos.

5.2. Results and discussion

Table 3 shows the 1D 68% and 95% MCIs for $\Omega_m h$, $f_\nu$, and the non-linear parameters $P_{\text{shot}}$ and $Q_{\text{nl}}$ for the three cases considered. We also give the minimum $\chi^2$ values as a measure of the goodness of fit.

As in the case for vanilla cosmology, fitting the SDSS-4 LRG data without non-linear correction leads to a very poor goodness of fit; the $\chi^2_{\text{min}}$ value is 63.2, for approximately $20 + 3 – 6 = 17$ degrees of freedom (20 data points of SDSS-4 LRG, 3 for the priors on $n_s$, $h$ and $\Omega_b h^2$, and 6 for 6 free parameters). Adding non-linear correction reduces $\chi^2_{\text{min}}$ by more than 40 units at the expense of only one extra parameter. Again, the two non-linear models offer very similar corrections to the power spectrum in terms of the inferred $\Omega_m h$ and $f_\nu$ values, although the $P$ model appears to provide a slightly better fit to the data judging by its slightly smaller $\chi^2_{\text{min}}$. Interestingly, despite the dramatic decrease in the minimum $\chi^2$ between correction and no correction, the resulting shifts in the $\Omega_m h$ and $f_\nu$ estimates are deceptively small, so the 95% MCIs remain compatible, before and after correction.
In the absence of non-linear correction, the 95% upper limit on $f_\nu$ is about a factor of two too tight compared with the corrected case. This situation is reminiscent of the overly constraining bounds on $\sum m_\nu$ derived in some recent combined analyses of WMAP-3 and the flux power spectrum of the Lyman-\(\alpha\) forest [50]. Too much power at large wavenumbers—either because of uncorrected non-linearities in the galaxy power spectrum or an unusually large normalization in the case of the Lyman-\(\alpha\) forest—leads to the appearance of an overly flat matter power spectrum, which in turn favours a smaller neutrino fraction and hence a smaller neutrino mass. The difference between the two corrected $f_\nu$ bounds is about 20%. The corresponding 95% limits on $\sum m_\nu$, <1.76 eV for the $P$ model and <1.83 eV for the $Q$ model, differ by even less. Thus cosmological neutrino mass determination is at present unaffected by our choice of non-linear correction model.

Finally, we see in figure 5 that no strong degeneracy exists between the neutrino fraction $f_\nu$ and the non-linear correction parameters $P_{\text{shot}}$ and $Q_{\text{nl}}$. For small values of $f_\nu$, the changes induced in the linear matter power spectrum by massive neutrino dark matter can be approximately mimicked by a redefinition of the apparent $\Omega_m h$ parameter [7],

\[
(\Omega_m h)_{\text{apparent}} = (\Omega_m h)_{\text{true}} - 1.2 f_\nu,
\]

assuming $h \sim 0.7$. Here, $(\Omega_m h)_{\text{apparent}}$ is the parameter that is actually constrained by power spectrum data, while $(\Omega_m h)_{\text{true}}$ denotes the true value of $\Omega_m h$. This expression also encapsulates the well known (approximate) degeneracy between $\Omega_m h$ and $f_\nu$. Larger $f_\nu$ values induce more complicated changes in the power spectrum, and it is reassuring to see that these changes are not degenerate with non-linear correction using either model.

6. The pathology of the $Q$ model: vanilla + thermal axions

While both non-linear models work very well for vanilla and for vanilla + massive neutrino cosmologies, and we may be tempted to conclude that they are for all purposes phenomenologically identical, we provide in this section a counter-example in which the incorrect functional form of the $Q$ model can lead to some misleading results.
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The case in point is a class of models containing a possible subdominant HDM component due to relic thermal axions with mass $m_a$. These models differ from those with massive neutrino HDM in that the temperature $T_a = [\zeta(3)/\pi^2]T^3_a$ of the axions are functions of $m_a$, since $m_a$ determines when the particle species should decouple from the primordial plasma. Here, the function $g^*_D(m_a)$ denotes the effective number of thermal degrees of freedom at the time of decoupling, and must be calculated by carefully tracking the freeze-out process (e.g., [16]).

Thus, although qualitatively thermal axion HDM exhibits free-streaming features very similar to those of massive neutrinos, quantitatively the suppression of small-scale power in the matter power spectrum has a non-linear dependence on the axion mass. We show in this section how this non-trivial dependence can cause some problems for the $Q$ model.

6.1. Set-up

We consider three parameter spaces:

(i) $m_a, \Omega_m h, \Omega_b h^2, h, n_s, \ln(10^{10}A_s), b, Q_{nl}$, with a top-hat prior 0–100 on $Q_{nl}$,
(ii) $m_a, \Omega_m h, \Omega_b h^2, h, n_s, \ln(10^{10}A_s), b, Q_{nl}$, with a top-hat prior 0–200 on $Q_{nl}$, and
(iii) $m_a, \Omega_m h, \Omega_b h^2, h, n_s, \ln(10^{10}A_s), b, P_{shot}$.

Cases (i) and (ii) both use the non-linear model (1.1), and differ only in the priors imposed on the correction parameter $Q_{nl}$. Case (iii) uses the non-linear model (1.2), with the usual prior on $P_{shot}$ (see table 1). We fit each case to the combined data set WMAP-3 +SDSS-4 LRG, using data up to $k_{max} \sim 0.2 h$ Mpc$^{-1}$ for the latter.

6.2. Results and discussion

Figure 6 shows the 2D marginal 68% and 95% MCIs for $\{\Omega_m h, m_a\}$, $\{\Omega_m h, Q_{nl}\}$, and $\{m_a, Q_{nl}\}$, as well as the 1D marginal posteriors for the same three parameters for cases (i) and (ii).

Consider first the 1D marginal posteriors for $Q_{nl}$. We see in figure 6 that the posteriors in both cases (i) and (ii) remain finite all the way up to the upper limit of the prior imposed on $Q_{nl}$. This is also reflected in the relevant 2D contours, which are abruptly cut off at $Q_{nl} = 100$ and $Q_{nl} = 200$ respectively. Data alone do not constrain $Q_{nl}$ in this class of cosmological models.

While this does not affect the $\Omega_m h$ estimates, the inference of the axion mass $m_a$ depends crucially on how well we can constrain $Q_{nl}$ because of a persisting degeneracy between the two parameters. Indeed, if we do not cut off $Q_{nl}$ by hand at some sufficiently small value, a second peak begins to appear in the posterior at $\{m_a \sim 6 \text{ eV}, Q_{nl} \sim 200\}$, besides the one at $\{m_a \sim 0 \text{ eV}, Q_{nl} \sim 30\}$. From figure 7 we see that the power spectra for the two peaks after non-linear correction are almost perfectly degenerate up to (and beyond) $k \sim 0.2 h$ Mpc$^{-1}$, even though the corresponding linear matter power spectra differ markedly already at $k \sim 0.04 h$ Mpc$^{-1}$.

We may be inclined to regard a 6 eV axion and, by implication, a large $Q_{nl}$ value as unphysical because the former runs in conflict with constraints on $m_a$ derived from stellar energy loss arguments and from telescope searches for axion radiative decays [51].
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Figure 6. 2D marginal 68\% and 95\% MCIs for \( \{\Omega_m h, m_a\} , \{\Omega_m h, Q_{nl}\} \), and \( \{m_a, Q_{nl}\} \) from WMAP-3 +SDSS-4 LRG. In the diagonal are the 1D marginal posteriors for the three named parameters. Black/solid lines correspond to case (i) of section 6.1 with a top-hat prior 0–100 on \( Q_{nl} \), while the blue/shaded regions and blue/dashed lines denote case (ii) with \( Q_{nl} \) prior 0–200.

Furthermore, the \( m_a, Q_{nl} \)-degeneracy exists only in a limited \( k \) range: as shown in figure 7, the corrected power spectra for the two peaks begin to deviate at \( k \gtrsim 0.3h \text{ Mpc}^{-1} \). This suggests that the exact location of the high \( m_a \) peak in \( \{m_a, Q_{nl}\} \)-space may in fact depend on our choice of \( k_{\text{max}} \). Naturally we could have avoided this second peak simply by demanding consistency with other astrophysical constraints on \( m_a \). However, in the absence of, e.g., galaxy formation simulations for this very class of cosmological models, we have a priori no reason to reject \( Q_{nl} \sim 200 \) or any other higher or lower value besides our own prejudices. Moreover, even if we manage to avoid the second peak with a finely tuned prior on \( Q_{nl} \), the \( m_a, Q_{nl} \)-degeneracy means that the constraint thus obtained on \( m_a \) will still depend sensitively on exactly what we choose to be the upper limit of that prior.

This exercise also highlights the danger of analytic marginalization \cite{47}, a technique sometimes used on nuisance parameters in CosmoMC to shorten the computation time. Here, analytic integration of the posterior in the direction of a nuisance parameter is made possible by taking the lower and upper limits of the parameter’s top-hat prior to \(-\infty\) and \(\infty\) respectively. Using this technique on the marginalization of \( Q_{nl} \), we find a unimodal 1D posterior for \( m_a \) whose 95\% MCI of 4.95 \(\lesssim m_a/\text{eV} \lesssim 7.17 \) favours unambiguously the high \( m_a \) region. Thus, analytic marginalization can be very useful if the likelihood function itself sufficiently constrains the nuisance parameters. Otherwise, as we have seen here, the end results are potentially misleading.
Figure 7. Matter power spectra for the two peak values of $m_a$. The black/solid lines are for $\{m_a = 0.6 \text{ eV}, Q_{nl} = 31\}$, and the red/dashed lines for $\{m_a = 5.9 \text{ eV}, Q_{nl} = 170\}$. The upper panel shows the linear matter power spectra, while the lower panel includes non-linear correction with the $Q$ model. The vertical line indicates the maximum value of $k$ used in the analyses.

Finally, we note that the $P$ model does not suffer from these problems. Figure 8 shows the 2D marginal 68% and 95% MCIs for $\{\Omega_m h, m_a\}$, $\{\Omega_m h, P_{\text{shot}}\}$, and $\{m_a, P_{\text{shot}}\}$, and the corresponding 1D marginal posteriors for case (iii). Here, although the correction parameter $P_{\text{shot}}$ is slightly degenerate with the axion mass $m_a$, it is independently well constrained by data. Importantly, even if data fail to constrain $P_{\text{shot}}$, we have a simple and well defined way to choose our prior on $P_{\text{shot}}$. Thus we conclude that the $P$ model is at present superior to the $Q$ model for non-linear correction in non-vanilla cosmologies.

7. Conclusions

In this paper we have explored two non-linear correction and scale-dependent bias models—the $Q$ model of [7] and the halo model-inspired $P$ model—in some detail. We have confronted them with a range of galaxy clustering data and cosmologies to determine the strengths and weaknesses of the models.
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Figure 8. 2D marginal 68% and 95% MCIs for $\{\Omega_m h, m_a\}$, $\{\Omega_m h, P_{\text{shot}}\}$, and $\{m_a, P_{\text{shot}}\}$ from WMAP-3 +SDSS-4 LRG for case (iii) of section 6.1. 1D marginal posteriors for the same three parameters are shown in the diagonal.

In the context of standard $\Lambda$CDM cosmology, we find that the two models perform equally well on present galaxy power spectrum data, in the sense that their corrective effects on the matter power spectrum are essentially identical. The case for non-linear correction is very strong for the SDSS-4 LRG power spectrum. An indicative figure of merit is the minimum $\chi^2$: fitting data up to $k_{\text{max}} \sim 0.2h$ Mpc$^{-1}$, we find that $\chi^2_{\text{min}}$ changes from $\sim 60$ for 18 degrees of freedom without correction to $\sim 20$ for 17 degrees of freedom with correction, irrespective of the non-linear model used. For 2dF and SDSS-2 main, however, the need for correction is marginal and subsists primarily because their respective $\Omega_m h$ estimates show better agreement with than without non-linear correction. The preferred values of $\Omega_m h$ from SDSS-4 LRG, SDSS-2 main, and 2dF after correction, as well as from WMAP-3, can all be reconciled at 95% confidence, contrary to the case without correction.

Similar results are obtained for $\Lambda$CDM cosmologies extended with a subdominant component of massive neutrino hot dark matter. Data again show no strong preference for either non-linear correction model; nor do we find any detrimental degeneracy between the neutrino fraction $f_\nu$ and either non-linear correction parameter. Cosmological neutrino mass determination is at the time being unaffected by our choice of non-linear correction model.

However, if the subdominant free-streaming dark matter is in the form of relic thermal axions, a non-trivial degeneracy between the axion mass $m_a$ and the correction parameter $Q_{\text{nl}}$ renders the $Q$ model highly pathological, so that our inference of $m_a$ depends sensitively on the prior that we impose on $Q_{\text{nl}}$. In contrast, the $P$ model, whose
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functional form is based on well motivated physics, does not suffer from this problem, and is arguably superior to the $Q$ model. Note that we have used relic thermal axions here as an example. But our results may also be relevant for other light thermal relics whose temperature and hence abundance depend on the mass of the particle.

More precise data from future galaxy redshift surveys will eventually render these simplistic models inadequate to describe non-linear evolution and scale-dependent biasing. For example, the damping of baryon acoustic oscillations due to non-linear mode coupling will need to be factored in at some stage [28,43,44]. The advent of wide- and deep-field weak gravitational lensing surveys in the next decade as an alternative probe of the large-scale structure distribution will circumvent some of these non-linearity issues. But galaxy redshift surveys will remain an important tool for the observation of baryon acoustic oscillations, and non-linear evolution/scale-dependent bias modelling will continue to constitute an important aspect of the cosmological analysis machinery.

Acknowledgments

We thank Alexia Schulz and Robert E Smith for useful comments on the manuscript. We acknowledge use of computing resources from the Danish Centre for Scientific Computing (DCSC).

References

[1] Hinshaw G et al, Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: data processing, sky maps, and basic results, 2008 Preprint 0803.0732 [astro-ph]
[2] Nolta M R et al, Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: angular power spectra, 2008 Preprint 0803.0593 [astro-ph]
[3] Reichardt C L et al, High resolution CMB power spectrum from the complete ACBAR data set, 2008 Preprint 0801.1491 [astro-ph]
[4] Jones W C et al, A measurement of the angular power spectrum of the CMB temperature anisotropy from the 2003 flight of boomerang, 2006 Astrophys. J. 647 823 [SPIRES] [astro-ph/0507494]
[5] Piazzentini F et al, A measurement of the polarization–temperature angular cross power spectrum of the cosmic microwave background from the 2003 flight of BOOMERANG, 2006 Astrophys. J. 647 833 [SPIRES] [astro-ph/0507507]
[6] Montroy T E et al, A measurement of the CMB spectrum from the 2003 flight of BOOMERANG, 2006 Astrophys. J. 647 813 [SPIRES] [astro-ph/0507514]
[7] Cole S et al (The 2dFGRS Collaboration), The 2dF Galaxy Redshift Survey: power-spectrum analysis of the final dataset and cosmological implications, 2005 Mon. Not. R. Astron. Soc. 362 505 [astro-ph/0501174]
[8] Tegmark M et al, Cosmological constraints from the SDSS luminous red galaxies, 2006 Phys. Rev. D 74 123507 [SPIRES] [astro-ph/0608632]
[9] Percival W J et al, The shape of the SDSS DR5 galaxy power spectrum, 2007 Astrophys. J. 657 645 [SPIRES] [astro-ph/0608636]
[10] Riess A G et al (Supernova Search Team Collaboration), Observational evidence from supernovae for an accelerating universe and a cosmological constant, 1998 Astron. J. 116 1009 [SPIRES] [astro-ph/9805201]
[11] Perlmutter S et al (Supernova Cosmology Project Collaboration), Measurements of omega and lambda from 42 high-redshift supernovae, 1999 Astrophys. J. 517 565 [SPIRES] [astro-ph/9812133]
[12] Hu W, Eisenstein D J and Tegmark M, Weighing neutrinos with galaxy surveys, 2004 Phys. Rev. Lett. 80 5255 [SPIRES] [astro-ph/9712057]
[13] Hannestad S, Primordial neutrinos, 2006 Ann. Rev. Nucl. Part. Sci. 56 137 [SPIRES] [hep-ph/0602058]
[14] Lesgourgues J and Pastor S, Massive neutrinos and cosmology, 2006 Phys. Rep. 429 307 [SPIRES] [astro-ph/0603494]
[15] Hannestad S and Raffelt G, Cosmological mass limits on neutrinos, axions, and other light particles, 2004 J. Cosmol. Astropart. Phys. JCAP04(2004)008 [SPIRES] [hep-ph/0312154]
[16] Hannestad S, Mirizzi A and Raffelt G, New cosmological mass limit on thermal relic axions, 2005 J. Cosmol. Astropart. Phys. JCAP07(2005)002 [SPIRES] [hep-ph/0504059]
Non-linear corrections to the cosmological matter power spectrum

[17] Hannestad S, Mirizzi A, Raffelt G G and Wong Y Y Y, *Cosmological constraints on neutrino plus axion hot dark matter*, 2007 J. Cosmol. Astropart. Phys. JCAP08(2007)015 [SPIRES] [0706.4198] [astro-ph]
[18] Hannestad S, Mirizzi A, Raffelt G G and Wong Y Y Y, *Cosmological constraints on neutrino plus axion hot dark matter: update after WMAP-5*, 2008 J. Cosmol. Astropart. Phys. JCAP04(2008)019 [SPIRES] [0803.1585] [astro-ph]

[19] Melchiorri A, Mena O and Slosar A, *An improved cosmological bound on the thermal axion mass*, 2007 Phys. Rev. D 76 041303 [SPIRES] [0705.2695] [astro-ph]

[20] Viel M, Lesgourgues J, Mena O and Slosar A, *'Eppur Si Muove': on the motion of the acoustic peak in the cos expansion*, 2007 Phys. Rev. D 76 041303 [SPIRES] [0705.2695] [astro-ph]

[21] Covi L, Hamann J, Melchiorri A and Slosar A, *Time evolution of galaxy formation and dark matter halos*, 2005 Phys. Rev. D 71 063534 [SPIRES] [astro-ph/0501562]

[22] Covi L, Hamann J, Melchiorri A, Slosar A and Sorbera I, *Inflation and WMAP three year data: features have a future*, 2006 Phys. Rev. D 74 083509 [SPIRES] [astro-ph/0606452]

[23] Benson A J, Cole S, Frenk C S, Baugh C M and Lacey C G, *The nature of galaxy bias and clustering*, 2000 Mon. Not. R. Astron. Soc. 311 793 [astro-ph/9903343]

[24] Blandford R D, Lesgourgues J, Mena O and Slosar A, *Warm dark matter candidates including sterile neutrinos and light gravitinos with WMAP and the Lyman-alpha forest*, 2005 Phys. Rev. D 71 063534 [SPIRES] [astro-ph/0501562]

[25] McDonald P, *Clustering of dark matter tracers: renormalizing the bias parameters*, 2006 Phys. Rev. D 74 103512 [SPIRES] [astro-ph/0609413]

[26] McDonald P, *Analytic model for galaxy and dark matter clustering*, 2006 Phys. Rev. D 74 129901 (erratum)

[27] Smith R E, Scoccimarro R and Sheth R K, *Systematic effects in the sound horizon scale measurements*, 2007 Phys. Rev. D 75 063512 [SPIRES] [astro-ph/0609547]

[28] Seljak U, *Analytic model for galaxy and dark matter clustering*, 2000 Mon. Not. R. Astron. Soc. 318 203 [astro-ph/0001493]

[29] Peacock J A and Smith R E, *Halo occupation numbers and galaxy bias*, 2000 Mon. Not. R. Astron. Soc. 318 1144 [astro-ph/0005010]

[30] Cooray A and Sheth R, *Halo models of large scale structure*, 2002 Phys. Rep. 372 1 [SPIRES] [astro-ph/0206508]

[31] Schulz A E and White M J, *Scale-dependent bias and the halo model*, 2006 Mon. Not. R. Astron. Soc. 375 1329 [astro-ph/0605594]

[32] Peacock J A and Smith R E, *Halo occupation numbers and galaxy bias*, 2000 Mon. Not. R. Astron. Soc. 318 1144 [astro-ph/0005010]

[33] Seljak U, *Halo occupation numbers and galaxy bias*, 2000 Mon. Not. R. Astron. Soc. 318 1144 [astro-ph/0005010]

[34] Cooray A and Sheth R, *Halo models of large scale structure*, 2002 Phys. Rep. 372 1 [SPIRES] [astro-ph/0206508]

[35] Schulz A E and White M J, *Scale-dependent bias and the halo model*, 2006 Mon. Not. R. Astron. Soc. 375 1329 [astro-ph/0605594]

[36] Seljak U, *Redshift space bias and beta from the halo model*, 2001 Mon. Not. R. Astron. Soc. 325 1359 [astro-ph/0009016]

[37] Seljak U, *Halo occupation numbers and galaxy bias*, 2000 Mon. Not. R. Astron. Soc. 318 1144 [astro-ph/0005010]

[38] Cooray A and Sheth R, *Halo models of large scale structure*, 2002 Phys. Rep. 372 1 [SPIRES] [astro-ph/0206508]

[39] Schulz A E and White M J, *Scale-dependent bias and the halo model*, 2006 Mon. Not. R. Astron. Soc. 375 1329 [astro-ph/0605594]

[40] Seljak U, *Redshift space bias and beta from the halo model*, 2001 Mon. Not. R. Astron. Soc. 325 1359 [astro-ph/0009016]

[41] Seljak U, *Halo occupation numbers and galaxy bias*, 2000 Mon. Not. R. Astron. Soc. 318 1144 [astro-ph/0005010]

[42] Seljak U, *Halo occupation numbers and galaxy bias*, 2000 Mon. Not. R. Astron. Soc. 318 1144 [astro-ph/0005010]

[43] Seljak U, *Halo occupation numbers and galaxy bias*, 2000 Mon. Not. R. Astron. Soc. 318 1144 [astro-ph/0005010]

[44] Seljak U, *Halo occupation numbers and galaxy bias*, 2000 Mon. Not. R. Astron. Soc. 318 1144 [astro-ph/0005010]
Non-linear corrections to the cosmological matter power spectrum

[45] Huff E, Schulz A E, White M J, Schlegel D J and Warren M S, Simulations of baryon oscillations, 2007 Astropart. Phys. 26 351 [SPIRES] [astro-ph/0607061]

[46] White M J, The redshift space power spectrum in the halo model, 2001 Mon. Not. R. Astron. Soc. 321 1 [astro-ph/0005085]

[47] Lewis A and Bridle S, Cosmological parameters from CMB and other data: a Monte Carlo approach, 2002 Phys. Rev. D 66 103511 [SPIRES] [astro-ph/0205436]

[48] Lewis A, Homepage http://cosmologist.info

[49] Hamann J, Hannestad S, Raffelt G G and Wong Y Y Y, Observational bounds on the cosmic radiation density, 2007 J. Cosmol. Astropart. Phys. JCAP08(2007)021 [SPIRES] [0705.0440] [astro-ph]

[50] Seljak U, Slosar A and McDonald P, Cosmological parameters from combining the Lyman-alpha forest with CMB, galaxy clustering and SN constraints, 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)014 [SPIRES] [astro-ph/0604335]

[51] Raffelt G G, Astrophysical axion bounds, 2008 Lect. Notes Phys. 741 51 [hep-ph/0611350]