Non-universal transport mechanisms in vertical natural convection with dispersed light droplets

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We present results on the effect of dispersed droplets in vertical natural convection (VC) using direct numerical simulations based on a two-way fully coupled Euler–Lagrange approach with a liquid phase and a dispersed droplets phase. For increasing thermal driving, characterised by the Rayleigh number, $Ra$, of the two analysed droplet volume fractions, $\alpha = 5 \times 10^{-3}$ and $\alpha = 2 \times 10^{-2}$, we find non-monotonic responses to the overall heat fluxes, characterised by the Nusselt number, $Nu$. The $Nu$ number is larger when the droplets are thermally coupled to the liquid. However, $Nu$ retains the effective scaling exponents that are close to the $1/4$-laminar VC scaling, suggesting that the heat transport is still modulated by thermal boundary layers. Local analyses reveal the non-monotonic trends of local heat fluxes and wall-shear stresses: Whilst regions of high heat fluxes are correlated to increased wall-shear stresses, the spatio-temporal distribution and magnitude of the increase is non-universal, implying that the overall heat transport is obscured by competing mechanisms. Most crucially, we find that the transport mechanisms inherently depend on the dominance of droplet driving to thermal driving that can quantified by (i) the bubblance parameter $b$, which measures the ratio of energy produced by the dispersed phase and the energy of the background turbulence, and (ii) $Ra_d/Ra$, where $Ra_d$ is the droplet Rayleigh number, which we introduce in this paper. When $b \lesssim O(10^{-1})$ and $Ra_d/Ra \lesssim O(100)$, the $Nu$ scaling is expected to recover to the VC scaling without droplets, and comparison with $b$ and $Ra_d/Ra$ from our data supports this notion.

1. Introduction

Bubbles are ubiquitous. Within a liquid, they can play an important role in the transport of mass and heat. Such complex interactions of bubbles and liquids can be found in various applications and process technologies, for example in cooling systems of power plants, metallurgical industries, catalytic reactions and in the mixing of chemicals (Brennen 2005; Balachandar & Eaton 2010; Mathai \textit{et al.} 2020). One commonly studied class of bubble-liquid interaction is the bubble column (Mudde 2005), where liquid

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Turbulence is generated and sustained by a rising swarm of bubbles. This form of turbulence is typically referred to as pseudo-turbulence (Lance & Bataille 1991; van Wijngaarden 1998; Mercado et al. 2010) or bubble-induced agitation (Risso 2018).

Various parameters can be controlled to modulate heat transport in a bubbly flow. For instance, one can use microbubbles to increase heat transport in the boundary layer (Kitagawa & Murai 2013) or by inclining the domain (Piedra et al. 2015). The fluid properties can also be varied. For example, Deen & Kuipers (2013) studied the effects of bubble deformability and found localised increase of heat fluxes when bubble coalescence prevails in the near-wall region, whereas Dabiri & Tryggvason (2015) showed that nearly spherical bubbles tend to aggregate at the walls, which in turn agitate the thermal boundary layers and result in higher heat transport than for the case with deformable bubbles. From these studies, one key observation that can be made is that heat transport enhancement has been largely linked to boundary layer effects, e.g. thinning of the thermal boundary layers or ejection of thermal plumes. On the other hand, a recent experimental campaign using a homogeneous bubble column found that the heat transport, characterised by the Nusselt number $Nu$, not only increases by up to 20 times, but also becomes insensitive to the thermal driving of the background flow, characterised by the Rayleigh number $Ra$ (Gvozdić et al. 2018, 2019). The $Ra$-insensitivity persists across a range of bubble volume fractions $\alpha$ between $5 \times 10^{-3}$ and $5 \times 10^{-2}$, implying that bubble-induced liquid agitation overwhelmingly dominates the heat transport mechanism across the thermal boundary layers. Indeed, the multifold enhancement in $Nu$ is consistent with engineering estimates in the design of bubble column gas-liquid reactors (Deckwer 1980).

Is there, however, any link between bubbly flows that directly influence the boundary layers versus bubble column experiments? And if any, are the boundary layers affected by the dispersed phase in a universal manner when $\alpha > 0$? In this paper, we ask the question of how other parameters, specifically the density ratio of the dispersed phase to liquid phase, influence heat transport. Inspired by the water column experiments in Gvozdić et al. (2018) and to make contact with recent studies, we selected a setup of natural convection in a rectangular cell containing a dispersed phase consisting of freely rising and deformable light droplets. The model setup of the flow is thermal natural convection, in particular, a flow sustained by applying a temperature difference between two opposing walls. Classical examples of thermal natural convection include Rayleigh–Bénard convection (Ahlers et al. 2009), where the hot wall is at the bottom and the cold wall at the top, and horizontal convection (Hughes & Griffiths 2008; Shishkina et al. 2016), where heating and cooling is applied at the same horizontal level. When the flow is confined between a hot vertical wall and a cold vertical wall, gravity acts orthogonal to the heat flux and this setup is referred to as vertical natural convection (VC). For confined VC, the bulk flow is quiescent (see mean profiles in figure 1a and visualisation in 1b) and at low $Ra$, the laminar-like boundary layers are expected to dominate heat and momentum transport (Shishkina 2016). This flow is unlike the unconfined, doubly-periodic VC (Ng et al. 2015, 2017) where a mean shear is present and determines heat transport in the bulk flow region (Ng et al. 2018). Hereinafter, we refer to the rectangular VC cell setup as VC, for simplicity.

When light droplets are introduced into VC, we ask two specific questions:

- Are the heat and momentum transport statistics universal for droplets (i.e. when the density of droplets are close to the density of the liquid)?
- How important is the role of thermal coupling between the droplets and the liquid?

To answer these questions, we perform direct numerical simulations (DNS) of VC
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Figure 1. Visualisations of instantaneous temperature fields for VC at Rayleigh number of $2 \times 10^8$. In (a), the red and blue curves correspond to mean velocity and temperature profiles at $x = 0.25L_x$, $0.5L_x$ and $0.75L_x$, respectively, at $\alpha = 0$. Volume fractions shown are (b) $\alpha = 0$, (c) $5 \times 10^{-3}$, and (d) $2 \times 10^{-2}$. Rendered flow fields are for droplets with mechanical coupling.

with droplets where we have control over the density ratio and thermal coupling of the droplet phase to the liquid phase. The droplets are fully coupled to the liquid phase DNS using the immersed boundary method (IBM) and the interaction potential approach, both of which are versatile numerical methodologies to simulate fully-coupled fluid flows with deformable interfaces (e.g. Spandan et al. 2018b; Meschini et al. 2018; Viola et al. 2020). Furthermore, IBM offers some computational advantages over existing numerical methods for multiphase flows (e.g. volume-of-fluid, level-set and front tracking), for instance, the underlying discretised grid is fixed and no sharp interfaces need to be resolved (Spandan et al. 2017). Recent advancements in the numerical methodology have allowed the use of sparser discretisations of the deformable interface relative to the underlying grid (Spandan et al. 2018a) without compromising numerical accuracy, further easing the computational requirements for large-scale multiphase flows. The disadvantage of IBM, however, is that droplet coalescence or splitting is hard to model and correspondingly in this paper we refrain from attempting to do so.

Our paper is organised as follows: in §2, we first describe the flow setup and numerical details for the fluid and dispersed phase. In §3, the numerical results are examined in detail. By analyzing the near-wall heat fluxes (§4) and wall-shear stresses (§5), we relate the droplet driving dynamics to changes in the near-wall statistics. In §6, we discuss and compare the influence of our selected parameters and the experimental parameters as reported in Gvozdić et al. (2018). Finally, in §7, we summarise our results and provide an outlook.

2. Flow setup

Our reference setup is the single-phase VC flow (figure 1b), which is a buoyancy driven flow confined between two differentially heated vertical walls and two adiabatic horizontal walls. This reference flow will be referred to as the liquid carrier phase. The flow is governed by mass conservation, balances of momentum and energy conservation which
within the Boussinesq approximation read,
\[
\begin{align*}
\partial_t u_i &= 0, \\
\partial_t u_i + u_j \partial_j u_i &= -\frac{1}{\rho_{\text{ref}}} \partial_i p + \delta_{ij} g \beta (\theta - \theta_{\text{ref}}) + \nu \partial^2_j u_i + f_i, \\
\partial_t \theta + u_j \partial_j \theta &= \kappa \partial^2 \theta + q_i,
\end{align*}
\]
where \( \partial_t \equiv \partial / \partial t \), \( \partial_i \equiv \partial / \partial x_i \), \((i, j) = 1, 2, 3 \) and repeated indices imply summation. In (2.2), \( f_i \) is the back-reaction forces of the dispersed phase on the fluid and for single-phase VC, \( f_i = 0 \). The thermal analogue to \( f_i \) in (2.2) is \( q_i \) in (2.3), which we selectively enable or disable in the present study. We define \( \rho_{\text{ref}} \) as the reference density, \( \theta_{\text{ref}} \) as the reference temperature, and \( \beta \) is the thermal expansion coefficient of the fluid, \( \nu \) the kinematic viscosity and \( \kappa \) the thermal diffusivity, all assumed to be independent of temperature. The unit length is defined as the distance between the heated plates, \( L_z \), and the streamwise and spanwise domain lengths are \( L_x = 2.4L_z \) and \( L_y = 0.25L_z \), respectively. Hereinafter, all length scales are non-dimensionalized by \( L_z \). No-slip and no-penetration boundary conditions are imposed on the velocity at all four walls, whereas periodic boundary conditions are imposed in the \( y \)-direction. The left and right walls are imposed with temperatures hotter and cooler than the reference temperature \( \theta_{\text{ref}} \equiv (\theta_h + \theta_c) / 2 \). The control parameters are the Rayleigh and Prandtl numbers, which are respectively defined as
\[
Ra \equiv \frac{g \beta \Delta L_z^3}{\nu \kappa^2}, \quad Pr \equiv \frac{\nu}{\kappa},
\]
where \( \Delta \equiv \theta_h - \theta_c \). The aspect ratio can also be an additional control parameter for confined thermal convection problems (van der Poel et al. 2011; Zwirner & Shishkina 2018), but at present, we restrict our analyses to a fixed value. Our simulations cover the values of \( Ra = 1.3 \times 10^8 - 1.3 \times 10^9 \) and for \( Pr = 7 \), corresponding to water.

The typical flow response is described by the Nusselt and Reynolds numbers,
\[
Nu \equiv \frac{f_w L_z}{\Delta \kappa}, \quad Re \equiv \frac{U_s L_z}{\nu},
\]
which quantify the dimensionless heat flux and degree of turbulence, respectively. In (2.4a), \( f_w \equiv -\kappa (\langle \partial \theta / \partial z \rangle)_{\text{wall}} \) is the wall heat flux and \( \langle \cdot \rangle_{\text{wall}} \) denotes the wall value. \( U_s \) is the \('wind\)-based velocity scale for VC (Ng et al. 2015) and accordingly, we set \( U_s \equiv \overline{u_{\text{max}}} \), which is the maximum mean vertical velocity. The notation \( \langle \cdot \rangle \) denotes time and \( xy \)-averaged quantities, and the notation \( \langle \cdot \rangle' \) denotes the fluctuating component, e.g. \( u' = u - \overline{u} \). With the addition of the thermal forcing term, \( q_i \), in (2.3), a different definition for \( Nu \) becomes necessary because \( \langle \partial \theta / \partial z \rangle_{z=0} \neq \langle \partial \theta / \partial z \rangle_{z=L_z} \) and the mean temperature equation now obeys \( \overline{w' \theta'}(z) - \kappa \overline{\partial \theta / \partial z} = \text{const.} \). To overcome this difficulty, we employ the dissipation rate-based definition for the Nusselt number:

\[
Nu \equiv \frac{\varepsilon_{\theta}}{\kappa (\Delta / L_z)^2} = \frac{\langle \theta_h (\partial \theta / \partial z)_h - \theta_c (\partial \theta / \partial z)_c \rangle}{\Delta^2 / L_z} + \frac{\langle \theta \cdot q_i \rangle}{\kappa (\Delta / L_z)^2},
\]
where \( \varepsilon_{\theta} \) is the volume-averaged thermal dissipation due to turbulent fluctuations and \( \langle \cdot \rangle \) denotes time- and volume-averaged quantities. When \( q_i = 0 \), (2.6) equals to (2.5a). The definition in (2.6) is also a direct analogue to the drag reduction calculations for multiphase Taylor–Couette flows (e.g. Sugiyama et al. 2008; Spandan et al. 2018b), making it convenient when comparing heat transport at matched \( Ra \) (discussed in §3.5). Throughout this paper, we will use (2.6) when reporting values of \( Nu \), unless defined otherwise.
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\[ Ra \times 10^9 \quad (\times 10^{-2}) \quad \Delta x^+ \quad \Delta y^+ \quad \Delta z_w^+ \quad \Delta z_c^+ \quad T_s/(L_z/U_\Delta) \quad T_s/\langle t_d \rangle \]

| Ra \times 10^9 | \alpha | \Delta x^+ | \Delta y^+ | \Delta z_w^+ | \Delta z_c^+ | T_s/(L_z/U_\Delta) | T_s/\langle t_d \rangle |
|---------------|-------|------------|------------|-------------|-------------|------------------|------------------|
| 0.1           | -     | 0.8       | 0.8       | 0.3         | 1.1         | 400              | -                |
| 0.2           | -     | 1.0       | 1.0       | 0.3         | 1.4         | 570              | -                |
| 0.4           | -     | 1.3       | 1.3       | 0.4         | 1.7         | 570              | -                |
| 0.7           | -     | 1.3       | 1.3       | 0.3         | 1.8         | 480              | -                |
| 1.3           | -     | 1.6       | 1.6       | 0.4         | 2.2         | 470              | -                |
|               |       | 0.1       | 0.5       | 0.9         | 0.9         | 1.2             | 250              |
|               |       | 0.2       | 0.5       | 1.1         | 1.1         | 1.5             | 230              |
|               |       | 0.4       | 0.5       | 1.4         | 1.3         | 0.4             | 230              |
|               |       | 0.7       | 0.5       | 1.3         | 1.3         | 0.4             | 230              |
|               |       | 1.3       | 0.5       | 1.6         | 1.6         | 0.4             | 320              |
| Mech. coupling|       | 0.1       | 2.0       | 0.9         | 0.9         | 0.3             | 230              |
|               |       | 0.2       | 2.0       | 1.1         | 1.1         | 0.3             | 220              |
|               |       | 0.4       | 2.0       | 1.4         | 1.3         | 0.4             | 240              |
|               |       | 0.7       | 2.0       | 1.4         | 1.4         | 0.4             | 300              |
|               |       | 1.3       | 2.0       | 1.7         | 1.7         | 0.5             | 390              |
| Mech.+therm. coupling| | 0.1 | 2.0 | 0.9 | 0.9 | 0.3 | 1.3 | 220 |
|               |       | 0.2       | 2.0       | 1.1         | 1.1         | 0.3             | 210              |
|               |       | 0.4       | 2.0       | 1.4         | 1.3         | 0.4             | 260              |
|               |       | 0.7       | 2.0       | 1.4         | 1.4         | 0.4             | 320              |
|               |       | 1.3       | 2.0       | 1.7         | 1.7         | 0.5             | 410              |
|               |       | 0.1       | 2.0       | 0.9         | 0.9         | 0.3             | 200              |
|               |       | 0.2       | 2.0       | 1.1         | 1.1         | 0.3             | 200              |
|               |       | 0.4       | 2.0       | 1.4         | 1.3         | 0.4             | 230              |
|               |       | 0.7       | 2.0       | 1.4         | 1.4         | 0.4             | 290              |
|               |       | 1.3       | 2.0       | 1.7         | 1.7         | 0.5             | 400              |

The droplets are fully resolved using IBM for deformable interfaces and the interaction potential approach (de Tullio & Pascazio 2016; Spandan et al. 2017, 2018a). The simulations are also coupled in a so-called four-way manner, i.e. the simulation is capable to handle droplet-fluid forcing, fluid-droplet forcing, droplet-droplet collisions and droplet-wall collisions. Our numerical methodology differs from point-particle-type simulations with heat transport (e.g. Oresta et al. 2009): Since the droplets with diameter \( D \) (at the point of injection) are significantly larger than the turbulent Kolmogorov length-scale \( \eta \), we therefore fully resolve the inhomogeneous hydrodynamic forces acting at the droplet interface. To illustrate this point, we wish to stress that \( D/\eta \approx 7–19 \) in our simulations. Here, \( \eta \equiv (\nu^3/\varepsilon)^{1/4} \), where \( \varepsilon \equiv \nu \langle (\partial u_i/\partial x_j)^2 \rangle \) is the volume-averaged turbulent kinetic energy dissipation rate. The key points of our IBM are summarised in §2.1.

### 2.1. Numerical details

The liquid phase is solved using DNS by a staggered second-order accurate finite difference scheme and marched in time using a fractional-step approach (Verzicco & Orlandi 1996). We employ equal grid spacings in the \( x \) and \( y \) directions, whereas the
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The dispersed phase is simulated with the IBM using a fast moving-least-squares algorithm (Spandan et al. 2017, 2018a). Two volume fractions are simulated: \( \alpha = 5 \times 10^{-3} \) and \( 2 \times 10^{-2} \) (see table 1). In addition to the sampling intervals for fluid simulations, \( T_s/(L_z/U_{\Delta}) \), where \( U_{\Delta} \equiv (g\beta\Delta L_z)^{1/2} \) is the free-fall velocity, the number of droplet flow-through cycles, \( T_s/(\langle t_d \rangle) \) where \( \langle t_d \rangle \) is the time-averaged droplet rise time, is also an important parameter. At least 20 droplet rise intervals are recorded for each simulation. Droplets that rise close to the top of the domain are removed and re-injected randomly at the bottom of the domain. This procedure ensures a constant droplet volume fraction and a uniform droplet spatial distribution. In addition, the interfacial temperature is set as the mean temperature of the immersed fluid.

For the droplet boundary conditions, we assume that the droplets have negligible thermal inertia and are surfactant-laden. The first assumption implies a small droplet Biot number, defined by \( \text{Bi} \equiv hD/k \) where \( h \) is the heat transfer coefficient and \( k \) is the thermal conductivity of the droplet interface, so that the internal droplet temperature can be approximated by a uniform temperature in accordance with the lumped-capacitance model (Wang et al. 2017). Owing to deformation, individual droplet volumes can vary
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slightly throughout the simulation, but fluctuate about a constant reference volume—this is the underlying model of the interaction potential approach. Effectively, the droplet boundary conditions are no-slip and impermeable for velocity, with a homogeneous time-dependent temperature at the interface (The thermal boundary conditions are discussed in § 2.2). Indeed, for physical systems with surface-active impurities, droplet interfacial dynamics may be closely approximated by a no-slip interface (Jenny et al. 2004; Duineveld 1995). These simplified boundary conditions also have the added benefit that they can be handled easily from a numerical point-of-view, and hence, are computationally efficient given the size of the flow problem.

To quantify the droplet deformability, we define the Weber number, $\text{We} \equiv \rho_{\text{ref}} U^2 D / \sigma$, which yields the ratio of inertia to capillary forces, where $\sigma$ is the surface tension. In our simulation strategy, $\sigma$ is not prescribed explicitly. Rather, an additional tuning step is performed to obtain a set of interaction potential constants such that $\text{We} \approx 3 \times 10^{-2}$, in accordance with the tuning criteria described in Spandan et al. (2017). It is emphasised that in order to simplify existing continuum models, this tuning step is a necessary and felicitous step in the implementation of our numerical model. After extensive precursor simulations and checks, we decided to simulate droplets at half the density of the fluid, i.e. $\hat{\rho} \equiv \rho_d / \rho_{\text{ref}} = 0.5$, which is within the numerical stability limit for explicit IBM time integration schemes (Schwarz et al. 2015). Another reason why the explicit formulation is typically favoured over implicit (i.e. strongly coupled) approaches for the fluid-structure interaction is also because of its computationally inexpensive nature. The detailed explanation of the methodology is, however, beyond the scope of this paper. For an in-depth discussion of the formulation, we refer readers to the paper of Spandan et al. (2017).

2.2. Model for thermally coupled droplets

For the lumped-capacitance model, two simplifying assumptions are made: (i) the droplets do not generate heat, and (ii) the internal temperature fields (and therefore interfacial temperature) of the droplets are uniform. Based on these assumptions, the interfacial droplet temperature is updated at every timestep according to

$$\frac{\partial \theta_b}{\partial t} = \left\langle -\frac{\kappa}{V_d} \oint_{S_d} \frac{\partial \theta}{\partial n} \cdot n \, dS \right\rangle_{S_d},$$

where $\theta_b$ is the mean interfacial droplet temperature, $V_d$ is the volume of the droplet, $S$ is the droplet surface area and $n$ is the outwardly-directed unit normal. $\langle \cdot \rangle_S$ denotes the surface-averaged quantity. The droplet surface temperature is initialised as the mean surface temperature at its injected location. After injection, the droplets rise and deform with respect to their original state (a sphere with diameter $D$), but do not significantly change in volume. Our model is therefore simpler than other numerical models with thermal coupling, for instance, studies that consider droplet growth at the boiling limit (e.g. Oresta et al. 2009; Lakkaraju et al. 2011) or models that rely on droplets with a constant geometry (e.g. Wang et al. 2017). Our code was extensively validated in a recent study for convection-dominated dissolution of droplets (Chong et al. 2019).

2.3. Derivation of the droplet Rayleigh number

In addition to the control parameters defined in (2.4), we introduce the droplet Rayleigh number, $Ra_d$, to quantify the droplet driving. It is defined as

$$Ra_d \equiv \frac{\alpha g L^3}{\hat{\rho} \nu \kappa},$$

(2.9)
which is conveniently derived from scaling arguments of the governing equations.

Following the definition of IBM for deformable interfaces/fluid-structure interaction, the droplet interface is represented by a network of nodes evolved by the interaction potential model (de Tullio & Pascazio 2016; Spandan et al. 2017). The equation of motion for each node, $e$, moving with velocity $\mathbf{u}_e$ is

$$\frac{d\mathbf{u}_e}{dt} = \mathbf{F}^h + \mathbf{F}^g + \mathbf{F}^i. \tag{2.10}$$

In (2.10), the terms are made dimensionless with the unit length $L_z$ and free-fall velocity $U_\Delta$. The forces contributing to the right-hand-side of (2.10) are the hydrodynamic loads $\mathbf{F}^h$, buoyancy $\mathbf{F}^g$ and internal forces $\mathbf{F}^i$, where

$$\mathbf{F}^h = \frac{L_z}{\rho V_e U^2_\Delta} \int_S \mathbf{\tau} \cdot \mathbf{n} \, dS \quad \text{and} \quad \mathbf{F}^g \equiv \left(1 - \frac{1}{\hat{\rho}}\right) \frac{L_z}{U^2_\Delta} \mathbf{g}. \tag{2.11a,b}$$

In (2.11a), $V_e$ is the volume of the node, but lacks a physical definition because the definition of the thickness of a liquid-liquid interface is not straightforward. To overcome this, following Spandan et al. (2017), we treat $V_e$ as a free parameter and fix $V_e = 1$. The model parameters for the internal forces $\mathbf{F}^i$ are then correspondingly tuned. $\mathbf{F}^i$ represents the surface forces acting on the nodes of the discretised droplet surface and is based on the principle of minimising the potential energy of immersed interface. Under external hydrodynamic loads, the network of nodes deform and stores potential energy into the system. The potential energy is subsequently converted to surface forces by differentiating the potentials with respect to the displacements of each node. The details of the individual potentials are described in Spandan et al. (2017).

We focus on the second term $\mathbf{F}^g$ in (2.11b). Since (2.11b) represents the contribution from an isolated droplet and we are interested to define a parameter for collective droplet effects, it would be reasonable to include the volume fraction parameter, $\alpha$. Therefore, for $\hat{\rho} < 1$, we define,

$$\mathbf{F}^g_\alpha \sim \frac{\alpha g L_z}{\hat{\rho} U^2_\Delta} =: \frac{Ra_d}{Ra}, \tag{2.12}$$

which quantifies the relative dominance of droplet driving to thermal driving.

Other dimensionless parameters similar to $Ra_d/Ra$ have also been proposed for different flow configurations, but these require a priori knowledge of the dispersed phase dynamics and/or flow statistics. For example, Climent & Magnaudet (1999) proposed the Rayleigh number expression, $Ra_{CM} \equiv \rho g \alpha H^3/(\nu U_b)$ ($H$ is the height of the liquid layer and $U_b$ is the relative rise velocity of the bubble), to quantify bubble-induced convection. Based on the notion of pseudo-turbulence (Lance & Bataille 1991), which is defined as the fluctuating energy induced by the passage of bubbles under non-turbulent conditions, van Wijngaarden (1998) proposed the so-called bubblance parameter $b \equiv (1/2) U^2_b \alpha/u_0^2$ ($u_0$ is the vertical velocity fluctuations of background turbulence). Since $Ra_d/Ra$ is a natural control parameter for VC with light droplets, we therefore use this ratio as input for our simulations. Note that $Ra_d$ is constant for a given $\alpha$ and therefore $Ra_d/Ra$ reduces with increasing $Ra$ (this is equivalent to an increase in Froude number with increasing $Ra$). To make the simulations of the fluid-structure interaction tractable, we also run the simulations at $g/200$. The resulting $Ra_d/Ra$ is $5 \times 10^{-4} - 5 \times 10^{-5}$ for $\alpha = 5 \times 10^{-3}$ and $2 \times 10^{-3} - 2 \times 10^{-4}$ for $\alpha = 2 \times 10^{-2}$.
3. Droplet influence on flow statistics and profiles

In this section, we analyse the results for $0 \leq \alpha \leq 2 \times 10^{-2}$, starting with a discussion of the droplets statistics.

3.1. Distribution of droplet aspect ratio versus bubble Reynolds number

From our simulations, the maximum droplet Reynolds number is $Re_d \approx 220$ and its time-averaged value is, $(Re_d) \approx 100$. As the droplets rise, they undergo deformation from the interfacial hydrodynamic loads. In figure 2, we characterise the deformation of the droplets in our simulations using the aspect ratio, $\Gamma$, of the major to minor axes, which are determined by fitting two-dimensional Fourier descriptors (Duineveld 1995; Lunde & Perkins 1998) to the projected droplet outlines in the $xy$- and $xz$-plane. The joint probability density distribution in figure 2 shows that the droplets undergo moderate deformation between $\Gamma \approx 1$ to $\Gamma \approx 1.3$, agreeing with the relatively small $We$ values. Visual inspections of the instantaneous shapes (insets of figure 2) show that the spherical droplet loses its fore-aft symmetry, with the front of the droplet becoming flatter than the back. Due to the relatively moderate $Re_d$ values, we do not observe droplet path instabilities throughout our simulations.

3.2. Profiles of mean vertical velocity and temperature

Now, we turn our focus to the flow statistics. To establish a baseline, we first analyse the influence of the droplets on the mean flow profiles of VC. Figure 3 shows the mean vertical velocity and temperature profiles plotted versus $z$ (Note that all length scales have been made dimensionless with $L_z$). Without droplets, the mean profiles are anti-symmetric about the channel centre-line (figure 3a,d). The cell centre is stably stratified (figure 4d) with $d\theta/dz|_{z=0.5} = 0$ and $\overline{u}|_{z=0.5} = 0$. Therefore, unlike the doubly-periodic VC setup (Ng et al. 2015, 2017), there is no persistent mean shear in the bulk of the flow. For $\alpha > 0$ and for both coupling cases, the mean vertical velocity profiles are asymmetric with a much stronger downward velocity magnitudes near the cooler walls (figures 3b,c). The difference between the maxima and minima of $\overline{u}$ is largest for the smallest $Ra$, indicating that the droplet forcing is strongest.
Figure 3. Mean profiles as a function of horizontal location $z$: (a-c) vertical velocity, (d-f) temperature. (a,d) $\alpha = 0$, (b,e) $\alpha = 5 \times 10^{-3}$, and (c,f) $\alpha = 2 \times 10^{-2}$. For $\alpha = 0$, only the left half of the profiles are shown since the profiles are antisymmetric about the vertical centreline. Dashed grey curves represent mechanical coupling only. Solid red curves represent mechanical and thermal coupling. Darker curves represent higher $Ra$.

The mean temperature profiles (figures 3e,f) also exhibit asymmetries. For $\alpha = 5 \times 10^{-3}$ and at the lowest $Ra$, the temperature profiles for both coupling cases are relatively constant and do not exhibit any undershoot, which is observed for $\alpha = 0$ in figure 3(d) at $z \approx 0.04$. However, at higher $Ra$, the profiles now bear some resemblance to the cases when $\alpha = 0$, corroborating the notion that thermal driving increasingly dominates. Here, we note that although $\bar{\theta} > \theta_{ref}$ in the bulk, the globally averaged temperature field $\langle \theta \rangle_t$ is statistically stationary within 0.5% for all cases. In the bulk region ($0.2 \leq z \leq 0.8$), we obtain $\left. d\bar{\theta}/dz \right|_{\text{bulk}} \approx 0$. Based on these results, the influence of the light droplets is seemingly most pronounced at the vertical boundaries as compared to the bulk.

3.3. Profiles of mean horizontal velocity and temperature

Figure 4 shows the mean horizontal velocity and temperature profiles plotted versus $x$. When $\alpha = 0$, the velocity profiles are antisymmetric (figure 4a) and the temperature profiles are constant for all $Ra$ (figure 4d). When $\alpha > 0$, the antisymmetries are destroyed: for $\alpha = 0.5 \times 10^{-2}$, the horizontal velocities are larger at the top wall (figure 4b), whereas for $\alpha = 2 \times 10^{-2}$, the horizontal velocities are larger at the bottom wall (figure 4c). The source for the asymmetry can be traced to passage of droplets entering the bottom or leaving the top of the domain: at the lower boundary, the droplets which have near zero velocity block the horizontal flow causing the fluid to accelerate around the droplets. At
the upper boundary, the droplets exit the domain at terminal velocity, and the entrained fluid impinges on the upper wall. Both mechanisms trigger intermittent intrusions of hotter and colder fluid at the upstream corners of the thermal boundary layers at the vertical walls. Since the blockage factor is higher for the $\alpha = 2 \times 10^{-2}$ cases, the magnitude of the mean horizontal velocities are larger at $x \lesssim 0.3$ as compared to the $\alpha = 5 \times 10^{-3}$ cases.

For the temperature profiles, we note an overall weakening of the stable stratification at higher $\alpha$ (figures 4e and f), with the bulk mean temperatures $\bar{\theta} \to \theta_{ref}$. The relatively uniform value of $\bar{\theta}$ for the most part of $x$ indicates strong mixing of the thermal field with increasing $\alpha$.

### 3.4. r.m.s. profiles of vertical velocity and temperature

The root-mean-square (r.m.s.) of the fluctuating quantities are plotted in figure 5 for all cases as a function of horizontal distance $z$. Here, we define $\langle \cdot \rangle_{rms} \equiv [(\bar{u}')^2]^{1/2}$. When $\alpha = 0$ (figure 5a,d), both $u'_{rms}$ and $\theta'_{rms}$ exhibit near wall peaks and are symmetrical about the channel-centreline.

When $\alpha > 0$, the bulk velocity fluctuations $u'_{bulk,rms} > 0$ as a direct result of droplet induced liquid fluctuations. Interestingly, $u'_{bulk,rms}$ at lower $Ra$ values are much larger than at higher $Ra$, which highlights the larger influence of droplet forcing on the flow at
lower $Ra$. The $u'_\text{rms}$ profiles also exhibit slight asymmetry with values tending to be larger closer to the colder wall as compared to the hotter wall. This asymmetry is consistent with the notion of a more intermittent colder downwards flow caused by the disruption of the large-scale circulation by the droplet passage, as discussed in §3.2.

For $\theta'_\text{rms}$, the magnitudes in the bulk for $\alpha > 0$ (figures 5e,f) tend to be lower than for the case when $\alpha = 0$ (figure 5d), where $\theta'_\text{rms,bulk} \approx 0.2$. With thermal coupling, the $\theta'_\text{rms}$ profiles are typically slightly larger than without thermal coupling and counteracts the mechanical agitation by the droplets. This effect can be explained by the thermal exchange of the droplet and the surrounding liquid which induces local thermal fluctuations. Therefore, both the mechanical agitation at larger $\alpha$ and the thermal coupling of the droplets contribute to the bulk mixing of the thermal field.

3.5. Scaling of Nusselt and Reynolds numbers versus Rayleigh number

In figure 6, we present the scaling of the $Nu$ and $Re$ versus $Ra$. Here, we employ the wind-based Reynolds number, $Re \equiv \bar{u}_{\text{max}} L_z / \nu$ as a measure of the large scale circulation.

When $\alpha = 0.0$ (solid circles, figure 6), we find that $Nu \sim Ra^{0.25 \pm 0.003}$ and $Re \sim Ra^{0.50 \pm 0.002}$ which are in agreement with the $Nu \sim Ra^{1/4}$ and $Re \sim Ra^{1/2}$ analytical predictions for laminar boundary layer-dominated VC (Shishkina 2016). For pure mechanical coupling (open triangles), the $Nu$ trends exhibit steeper slopes and from a least-square fit to a power-law, we obtain $Nu \sim Ra^{0.29 \pm 0.02}$ for $\alpha = 5 \times 10^{-3}$ and $Nu \sim Ra^{0.26 \pm 0.04}$ for $\alpha = 2 \times 10^{-2}$. In contrast, the $Re$ slopes are less steep and from a least-square fit
to a power-law, we obtain $Re \sim Ra^{0.38 \pm 0.03}$ for $\alpha = 5 \times 10^{-3}$ and $Re \sim Ra^{0.37 \pm 0.02}$ for $\alpha = 2 \times 10^{-2}$. When both mechanical and thermal coupling are enabled (filled triangles), we obtain $Nu \sim Ra^{0.29 \pm 0.02}$ and $Re \sim Ra^{0.38 \pm 0.03}$ for $\alpha = 5 \times 10^{-3}$, and $Nu \sim Ra^{0.29 \pm 0.04}$ and $Re \sim Ra^{0.36 \pm 0.06}$ for $\alpha = 2 \times 10^{-2}$. When comparing the coupling cases, the effective scaling for $Nu$ and $Re$ is largely unaffected. However, by including thermal coupling, the temperature field is distributed more efficiently and so, the magnitude of the heat transport is increased.

As a direct comparison for $Nu$, the ratio $Nu/Nu_{\alpha=0}$ is shown in the inset of figure 6(a) and the values range from 0.95 to 1.1. Some caution is warranted here when interpreting the ratios. Because of the rather large variations of $NuRa^{-1/4}$ as shown in the figure, we cannot conclusively claim that there exist a decrease in $Nu$ at low $Ra$. However, we can link the variations of the ratios to the different manner in which the droplets locally influence the wall heat fluxes and wall shear stresses. The local influences are quantified and discussed in § 4 and in § 5.

Now, we focus on the $Re$ trends. For $\alpha > 0$, the $Re$ values tend to be larger than for the $\alpha = 0$ case and this is consistent with the response of the VC flow due to the passage of the droplets across the top and bottom boundary layers. As the droplets cross the horizontal boundary layers, the large-scale circulation of the background VC flow is continuously disrupted, triggering horizontal intrusions of warmer fluid at the top wall and cooler fluid at the bottom wall (peaks in mean horizontal velocities in figures 4b and c), similar to the intrusions observed in transient VC in a square cavity (Patterson & Imberger 1980; Arnfield & Patterson 1991). For $\alpha = 5 \times 10^{-3}$, at the higher $Ra$-values, the $Re$-values tend to approach the $Re$ values for $\alpha = 0$. This incipient trend suggests that the droplet driving is no longer dominant at this part of the parameter space as compared to the $\alpha = 2 \times 10^{-2}$ case.
4. Droplet influence on local Nusselt number

In this section, we link the $Nu$ versus $Ra$ variations discussed in §3.5 to the changes in the local Nusselt number evaluated at the hot and cold walls. We define the local Nusselt number as $Nu_{loc} = f_{w, loc} L_z / (\Delta \kappa) = [\partial \theta(x) / \partial z]_{w} / (\Delta L_z)$, which is the local dimensionless temperature gradient evaluated at $z = 0$ and $L_z$. The trends are shown in figure 7 as function of $x$.

From figure 7, $Nu_{loc}$ are larger in the upstream of the vertical boundary layers, that is $x \lesssim 1.2$ for figures 7(a-c) and $x \gtrsim 1.2$ for figures 7(d-f). Here, the larger values of $Nu_{loc}$ simply reflect the thinner thermal boundary layers developing from the corners of the domain. For $\alpha = 0$, $Nu_{loc}$ monotonically decreases as the boundary layer develops and is consistent across the $Ra$ range. However, the trends vary considerably for $\alpha > 0$. For example, relative to the $\alpha = 0$ cases, (i) $Nu_{loc,h}$ becomes lower for $x \lesssim 1.2$, and (ii) for $\alpha = 2 \times 10^{-2}$, both $Nu_{loc,h}$ and $Nu_{loc,c}$ are roughly constant for $0.6 \lesssim x \lesssim 1.8$. Since these changes directly reflect the thermal boundary layer thicknesses, we can conclude that the droplets not only influence the bulk statistics as shown in §3, but would also influence the local thermal boundary layers.

To emphasise the changes in $Nu_{loc}$, we plot the ratio of $Nu_{loc}$ and $Nu_{loc,0}$ in figure 8. ($Nu_{loc,0}$ is $Nu_{loc}$ computed for the $\alpha = 0$ cases). The corresponding wall-areas for $Nu_{loc}$ are also shown in the insets, with reduced-$Nu_{loc}$ values denoted by left-pointing open triangles, and increased-$Nu_{loc}$ values denoted by right-pointing solid triangles.
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$\alpha = 5 \times 10^{-3}$

\[
\begin{array}{cccc}
\text{Hot wall} & \text{Cold wall} \\
\text{Hot wall} & \text{Cold wall}
\end{array}
\]

$\alpha = 2 \times 10^{-2}$

\[
\begin{array}{cccc}
\text{Hot wall} & \text{Cold wall} \\
\text{Hot wall} & \text{Cold wall}
\end{array}
\]

$\alpha = 5 \times 10^{-3}$, the decreased $N_{\text{loc},h}$ can be clearly seen for all $Ra$ and $x/L_x \lesssim 1.2$ (figures 8a,c). This decreasing behaviour can also be observed for $\alpha = 2 \times 10^{-2}$, although the corresponding wall-area with decreased $N_{\text{loc},h}$ is smaller for the mechanically coupled case (see figure 8c and the inset plot). The decreased $N_{\text{loc},h}$ for the $\alpha = 5 \times 10^{-3}$ case overwhelms the increased $N_{\text{loc},c}$ for $x/L_x \lesssim 1.2$, with the lowest $Ra$ cases being most strongly influenced, as previously shown in figure 6. In contrast, $N_{\text{loc},c}$ is significantly increased for $\alpha = 2 \times 10^{-2}$ and $x \lesssim 1.2$ by roughly a factor of 1.5 times (figures 8d,h). Based on the much stronger droplet driving for $\alpha = 2 \times 10^{-2}$, $N_{\text{loc}|\alpha=2\times10^{-2}}$ is increased by about 5% for the lowest $Ra$ relative to $N_{\text{loc}|\alpha=5\times10^{-3}}$.

For the different distributions of $N_{\text{loc}}$ in figures 7 and 8, we emphasize that they are non-universal phenomena for the flow problem considered since the mechanism of the changes rely on the strength of the droplet driving, $Ra_d$. These changes therefore cannot be trivially determined \textit{a priori}. What can be discerned from the current results is that the droplets influence the bulk flow (as seen in the mean and r.m.s. statistics in figures 3 to 5), the near-wall flow and the large-scale circulation of VC. Different mechanisms in these regions compete and the prevailing mechanism(s) would presumably determine the heat transport of the setup.
5. Droplet influence on local skin-friction coefficient

Unlike Rayleigh–Benard convection, the thermal boundary layers in VC are sheared by a mean wind with a constant direction that is predetermined by the boundaries (Ng et al. 2015). Therefore, to quantify the influence of the droplets on wind-shearing, we plot the local skin-friction coefficient $C_{f,\text{loc}}$ versus $x$ in figure 9. Here, $C_{f,\text{loc}} \equiv 2\tau_w(x)/U^2\Delta$, where $\tau_w(x) \equiv \sqrt{\mu \partial u(x)/\partial z|_w}$ is the wall shear stress. Similar to the idea of figure 8, the relative changes in the local skin-friction coefficients are plotted in figure 10.

For $\alpha = 0$, $C_{f,\text{loc}}$ is largest at wall heights that are close to the upstream of the developing boundary layer. However, when $\alpha > 0$, $C_{f,\text{loc}}$ is roughly constant for the most part of $x$ at low $Ra$. Two points can be made from the distributions of $C_{f,\text{loc}}$. First, the roughly uniform distribution of $C_{f,\text{loc}}$ at low $Ra$ for $\alpha > 0$ imply that the droplet driving dominates the mean wind of VC and, on a mean sense, homogenizes the viscous boundary layer particularly at the hot wall. Second, the distributions of $C_{f,\text{loc}}$ are not symmetric at the hot and cold walls (for example, $\max[C_{f,\text{loc},c}] > \max[C_{f,\text{loc},h}]$) as compared to the $\alpha = 0$ case (figure 8a and d). One possible explanation of this asymmetry can be made by observing the rising direction of the droplets: At the cold wall, the droplets oppose the downwards flow whereas at the hot wall, the droplets aid the upwards flow. Coupled with the asymmetry of the mean horizontal velocity profiles in figure 4, the resulting viscous boundary layer becomes thinner at the cold wall, and a larger $C_{f,\text{loc}}$ results.

Figure 9. Similar to figure 7, but now for the local skin-friction coefficient $C_{f,\text{loc}}$. Colour legends are the same as figure 3.
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\[ \alpha = 5 \times 10^{-3} \]

\[ \alpha = 2 \times 10^{-2} \]

Hot wall Cold wall

\( C_{f,\text{loc},c}/C_{f,\text{loc},c,0} \)

\( C_{f,\text{loc},h}/C_{f,\text{loc},h,0} \)

\( C_{f,\text{loc},c}/C_{f,\text{loc},c,0} \)

\( C_{f,\text{loc},h}/C_{f,\text{loc},h,0} \)

\text{Mech. coupling}

\text{Mech. + therm. coupling}

Figure 10. Similar to figure 8, but now for the local skin-friction coefficient \( C_{f,\text{loc}} \). The corresponding wall-areas of the ratios versus \( Ra \) are shown in the inset: wall-areas with reduced \( C_{f,\text{loc}} \) are denoted by left-pointing open triangles, and increased \( C_{f,\text{loc}} \) by right-pointing solid triangles. Colour legends are the same as figure 8.

However, this conjecture may not hold at higher \( Ra \) cases because the viscous boundary layers eventually become much thinner and closer to the walls. As a result, at sufficiently high \( Ra \), the influence of droplets presumably diminishes with increasing distance from the edge of the viscous boundary layers, eventually yielding to the dynamics of thermal driving.

When compared with \( C_{f,\text{loc},0} \) (figure 10), we find larger values of \( C_{f,\text{loc}} \) in concomitant regions with larger values of \( \text{Nu}_{\text{loc}} \) in figure 8. Interestingly, whilst \( \text{Nu}_{\text{loc}} \) is relatively insensitive to \( Ra \) (see figure 8), the wall-height distributions of \( C_{f,\text{loc}} \) exhibit a strong non-monotonic behaviour which depends on \( Ra, \alpha \) and whether the cold or hot wall is considered. Therefore, it appears that \( C_{f,\text{loc}} \) is more sensitive to the droplets induced agitation as compared to \( \text{Nu}_{\text{loc}} \). These results provide a strong indication that the light droplets interact with VC flow in a non-universal manner.
6. Light droplets versus bubbles - a comparison to experiments by Gvozdić et al. (2018)

In this section, we discuss several aspects of the physical parameters in our simulations, which distinguish our findings from the laboratory results of bubbly VC by Gvozdić et al. (2018).

A crucial difference between our investigation and the experiments is that $\hat{\rho} = 0.5$ in our simulations (corresponding to light droplets) whereas $\hat{\rho} \approx 10^{-3}$ in their experiments (corresponding to air bubbles in water). Clearly, the large differences in the density ratios play a role and this is reflected in our simulations. For example, the mean temperature in the bulk region of our simulations have approximately zero gradient (figure 3), whereas the mean temperature in the bulk region of the experiments have a finite gradient (see figure 9(a) of their paper), indicating a much stronger mixing of the thermal field by the bubbles as compared to light droplets. Furthermore, the values of $Nu$ for VC with light droplets is within 10% of the $Nu$ values without droplets (figure 6), whereas in the laboratory experiments of Gvozdić et al. (2018), $Nu$ can be larger by up to 20 times with bubbles than without and remain $Ra$-independent for their investigated parameter range. Therefore, we conclude that the background VC flow remains relatively dominant even with influence of light droplets, and this is reflected in the non-universal distributions of the local heat transport and skin-friction coefficients shown in figures 8 and 10.

The strength of the bubble-induced agitation versus droplet-induced agitation can also be quantified a posteriori using the bubblance parameter,

$$b \equiv U_b^2 \alpha / u_0^2 ,$$

(cf. Lance & Bataille 1991; van Wijngaarden 1998; Rensen et al. 2005; Alméras et al. 2017), which defines the ratio of energy produced by a bubble swarm, i.e. $U_b^2 \alpha$, and the energy of the background turbulence without bubbles, i.e. $u_0^2$. Note that a prefactor of 1 is chosen for (6.1), which is different to previous definitions which employ a prefactor of 1/2 (based on the added mass coefficient, cf. Rensen et al. 2005), however the present
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Since we have the full information from our DNS, the calculation of $b$ is straightforward. For the laboratory experiments, $u_0$ was not recorded and so, invoking dynamic similarity, we estimate the values using our DNS results at matched $Ra$. $U_b$ is assumed to be 0.34 m s$^{-1}$ for the laboratory experiments. The values of $b$ are plotted in figure 11(a). From the figure, we find that $O(10^{-3}) \lesssim b \lesssim O(10^{-1})$ for our DNS whereas $O(10^{1}) \lesssim b \lesssim O(10^{3})$ for the experiments. The much smaller magnitude of $b$ for our DNS clearly indicates that light droplets produce much lower kinetic energy compared bubbles. Also, $b$ decreases with increasing $Ra$ and implies that the kinetic energy of the background flow will eventually dominate the (constant) injection of kinetic energy by the dispersed phase. Based on the same idea, we compare the ratio of $Ra_d/Ra$ for our DNS and the experiments in figure 11(b). From the figure, we observe a similar scale separation and decreasing trend with increasing $Ra$: The values are $O(10^{-5}) \lesssim Ra_d/Ra \lesssim O(10^{-3})$ for our DNS and $O(10^{3}) \lesssim Ra_d/Ra \lesssim O(10^{5})$ for the experiments, confirming that the bubble driving is indeed a stronger driving mechanism than light droplets.

It is useful for applications such as in chemical mixing to have an estimate of the parameter space for $b$ or $Ra_d/Ra$ where the driving by background turbulence eventually dominates bubble driving. For the laboratory experiments with bubbly VC, Gvozdić et al. (2018) estimated this parameter space by defining a critical Rayleigh number, $Ra_c$, as follows: First, an effective power-law trend of $Nu \sim Ra^{0.33}$ is obtained from the single phase experiments. Then, observing that the $Nu$ trends are insensitive to $Ra$ for $5 \times 10^{-3} \lesssim \alpha \lesssim 5 \times 10^{-2}$ (cf. figure 12 of Gvozdić et al. 2018), the $Nu \sim Ra^{0.33}$ and constant $Nu$ trends are extrapolated to higher $Ra$ values. The intersection of these curves are defined as $Ra_c$, where $7 \times 10^{10} \lesssim Ra_c(\alpha) \lesssim 2 \times 10^{12}$ for the $\alpha$ values investigated. The range of $Ra_c$ values are marked in figure 11(c).

We can now directly extrapolate the trends of $b$ and $Ra_d/Ra$ to the $Ra_c$ values. From least-square fits, the effective power laws are $b \sim Ra^{-1}$ and $Ra_d/Ra \sim Ra^{-1}$. Therefore, the extrapolated values are $b \sim Ra_c^{-1}$ and $Ra_d/Ra \sim Ra_c^{-1}$, visually marked by the blue patches in figure 11. For illustration purposes, only the $\alpha = 5 \times 10^{-3}$ and $5 \times 10^{-2}$ are drawn and an allowance of $Ra_c \pm 10\%$ was employed to compute the extrapolation. The corresponding values are $(b, Ra_d/Ra)|_{\alpha=5 \times 10^{-3}} \approx (0.2, 60)$ and $(b, Ra_d/Ra)|_{\alpha=5 \times 10^{-2}} \approx (0.06, 18)$. These values suggest that the VC flow will dominate bubble-induced liquid agitation at $b \lesssim O(10^{-1})$ and $Ra_d/Ra \lesssim O(100)$. We note that our dataset for $\alpha = 2 \times 10^{-2}$ coincide with this regime for $b|_{\alpha=5 \times 10^{-2}}$ (lower horizontal blue line in figure 11a), however, since the boundary layer dynamics are still dominant for our configuration, it suggests that $\hat{\rho}$ is an additionally important parameter when characterising bubbly turbulence. Interestingly, for bubbles rising in grid-generated turbulence (or incident turbulence), Alméras et al. (2017) determined a slightly larger value for $b (\approx 0.7)$, where bubble-induced agitation appears to dominate. The mechanism was related to an increase in development length of the secondary bubble wake, which significantly enhances liquid velocity fluctuations. Indeed, the values of $b$ from our DNS are smaller which is consistent with the notion that the background flow remains dominant for our parameter space considered.
7. Conclusions and outlook

In this study, we simulated the VC flow with dispersed light droplets between $Ra = 1.3 \times 10^8$ and $1.3 \times 10^9$ and $Pr$-value of 7. The liquid phase is simulated using DNS whereas the dispersed phase is simulated using an IBM with the interaction potential method for deformable interfaces. Our approach extends the IBM of Spandan et al. (2017) and Spandan et al. (2018a), where now the dispersed phase is fully coupled both mechanically and thermally to the flow. In addition, two datasets are simulated with and without thermal coupling to investigate its influence on the heat transport. Although $Nu$ is slightly larger when the droplets are thermally coupled, we found that the VC flow with light droplets exhibits a non-monotonic change in heat transport with increasing $Ra$ and largely retains the laminar-like VC scaling. We reason that a significant enhancement of heat transport depends crucially on a sufficiently strong droplet driving, which we show can be characterised by the relative strength of $Ra_d$ to $Ra$ and the bubblance parameter, $b$.

When light droplets are introduced, the mean velocity and temperature profiles are highly skewed with the lowest $Ra$ being most sensitive (figures 3 to 5). However, this sensitivity is masked by the $Nu$ versus $Ra$ trend, where we observe a non-monotonic behaviour with increasing $Ra$ (figure 6a). This suggests the presence of competing mechanisms in the flow that contribute to the net heat transport. In contrast, the decreasing $Re$ versus $Ra$ trends are commensurate with the higher sensitivity at lower $Ra$, i.e. mechanical stirring is strongest at lowest $Ra$ and higher $\alpha$ (figure 6b).

Based on analyses of the near-wall regions, we found that regions with higher values of local heat fluxes, $Nu_{loc}$, correspond to concomitant regions with higher values of skin-friction coefficient, $C_{f,loc}$, which is consistent with the notion that the local wind has influence over the local heat transport (figures 7 and 9). In turn, the strength of the local wind is related to whether the direction of the rising droplets aids or opposes the flow (figure 10). However, the trends of $Nu_{loc}$ and $C_{f,loc}$ remain spatially non-monotonic and is sensitive to $\alpha$. Based on these observations, we deduce that the results are specific to our selected simulation parameters and stress that the trends are non-universal.

The $Nu$ versus $Ra$ trend in figure 6 is different from recent experimental results by Gvozdić et al. (2018) for bubbly flow. Whilst $Nu$ exhibits some $Ra$-dependency for our simulations with light droplets, Gvozdić et al. (2018) reported that $Nu$ is largely insensitive to $Ra$ for various volume fractions of droplets. The key distinction between our DNS and the experiments by Gvozdić et al. (2018) becomes readily apparent when we quantify the bubblance parameter $b$ and the droplet driving parameter $Ra_d/Ra$ (cf. §6). Both $b$ and $Ra_d/Ra$ have a large separation in scales between the laboratory experiments and our DNS. More specifically, at $b \gtrsim O(10^{-1})$ and $Ra_d/Ra \gtrsim O(100)$, we anticipate that the dynamics of the dispersed phase-induced liquid agitations become overwhelmed by the dynamics of the background VC flow. For light droplets, both $b$ and $Ra_d/Ra$ are significantly lower. Therefore the local heat fluxes and skin friction coefficients exhibit non-universal behaviour, which reflects the dominance of the background VC flow.

Our results collectively indicate a non-universal heat transport behaviour for light droplets. Locally, the near-wall trends of heat fluxes and wall-shear stresses suggest the presence of competing mechanisms that, in concert, govern heat transport. One question that arises naturally here is: Can a universal trend be eventually obtained by increasing $b$ and $Ra_d/Ra$ for fixed $Ra$? The answer to this question may provide some clues on disentangling the competing heat transport mechanisms in multiphase VC and is a subject for our future investigations.
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Declaration of Interests
The authors report no conflict of interest.

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