Decoherence Effects in Neutrino Fluxes from the Sun and from Microquasars

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Abstract. We study the change which electron-neutrinos may experience during their trip from solar, galactic and extra-galactic sources. The formalism is based on the interaction between neutrinos and the environment. We calculate the time evolution of the density matrix and set limits on the observability of decoherence for neutrino fluxes produced in the sun, in galactic supernovae and in microquasars.

1. INTRODUCTION

Neutrino fluxes produced in electroweak decays in the stars are the proper tools to gain some knowledge about astrophysical events and their dynamics. Examples of this type of events are supernovae explosions, highly energetic jets from binary systems, burning of stars, etc. The neutrinos produced in these events can travel from very distant sources to the detectors, nearly at the speed of light with practically no interactions, a fact that allows to re-construct the mechanism of the initial emitting process. However neutrinos of a given flavor are combinations of neutrino mass eigenstates with non-zero masses, as the existence of neutrino oscillations have demonstrated, though their values or hierarchies are still unknown, except for upper limits [1].

From the quantum mechanical point of view the neutrinos are emitted as pure states, in any of the flavors and they could evolve into mixed states due to decoherence, a process which is determined by the interactions with environment [2, 3, 4]. One may then ask the question about the structure of the mixed states which result from the time evolution of the initial states. We may also identify the way in which decoherence would manifest [5, 6]. In the next sections we shall present a simple model and discuss the consequences of it upon the identification of neutrinos from different sources. Then, we shall extent our discussion to some systems of astrophysical interest like microquasars.

2. FORMALISM

We shall start with the simplest case by working in the two-flavor basis, where the mass eigenstates $|1\rangle$ and $|2\rangle$ have masses $m_1$ and $m_2$ and the Hamiltonian is defined by the matrix

\[ H_m = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}. \]
The transformed Hamiltonian in the flavor basis is given by the expression

\[ H_f = U H m U^{-1} = \begin{pmatrix} c^2 m_1 + s^2 m_2 & -c s (m_2 - m_1) \\ -c s (m_2 - m_1) & s^2 m_1 + c^2 m_2 \end{pmatrix}, \]

where \( c \) and \( s \) are a short-hand notation for the cosine and sine of the mixing angle \( \theta \). To this Hamiltonian we assign the corresponding form in the spin-matrix representation

\[ H_f = \frac{(m_1 + m_2)}{2} n_\Omega - \frac{\epsilon}{2} \sigma_z + \frac{\Delta}{2} \sigma_x, \]

where \( \epsilon = (c^2 - s^2)(m_2 - m_1) \)

\[ \Delta = -2c s (m_2 - m_1). \]

The diagonalization of the eigenvalue problem leads to the density matrix, at time \( t = 0 \)

\[ \rho(0) = \frac{I}{2} + \frac{1}{\delta} \begin{pmatrix} -\frac{\Delta}{2} & -\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & \frac{\Delta}{2} \end{pmatrix}, \]

with \( \delta = \pm \sqrt{\epsilon^2 + \Delta^2} \). The time evolution of the density is given by the operator [6, 7]

\[ U_n (t) = e^{-iH_n t / \hbar} \]

\[ = I \cos \omega_n t - \frac{i}{2\omega_n} (\epsilon_n \sigma_z - \Delta \sigma_x) \sin \omega_n t \]

\[ = \begin{pmatrix} \cos \omega_n t - \frac{i}{2\omega_n} \epsilon_n \sin \omega_n t & \frac{i}{2\omega_n} \Delta \sin \omega_n t \\ \frac{i}{2\omega_n} \Delta \sin \omega_n t & \cos \omega_n t + \frac{i}{2\omega_n} \epsilon_n \sin \omega_n t \end{pmatrix}, \]

with (hereafter \( \hbar = 1 \)) \( \omega_n = \frac{\sqrt{(\epsilon_n)^2 + \Delta^2}}{2} \) and \( \epsilon_n = \epsilon - B_n \), \( B_n \) being the environment’s energy states. The states of the environment are taken as discrete, time independent, states and denoted by the subindex \( n \). Moreover, they may be taken as continuous variables as explained next. The density matrix at the time \( t \) is [7]

\[ \rho_n (t) = U_n (t) \rho (0) U_n^\dagger (t). \]

The integrated matrix elements, \( n \) is taken as a continuous variable such that \( \sum_n B_n \to \int dB \), read

\[ \rho_{ij} (t) = \int dB \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{B^2}{2\sigma}} [\rho_{ij} (t)]_n. \]

As a result of the trace operation, \( \rho (t) \) represents a mixed state. Diagonalization of this density matrix yields the probabilities \( \pi_\pm \) that the neutrinos are found in the pure states \( \rho_\pm \)

\[ \rho (t) = \pi_+ \rho_+ + \pi_- \rho_- \]

\[ \pi_+ - \pi_- = \left( \cos^2 2\theta + \sin^2 2\theta \exp \left[ -\left( s_N t \right)^2 \right] \right)^{1/2} \]

\[ \rho_\pm = \frac{1}{2} \left( I \pm \frac{\cos 2\theta}{\pi_+ - \pi_-} \sigma_z \right) \]

\[ \pm \frac{\sin 2\theta}{\pi_+ - \pi_-} \exp \left[ \frac{-\left( s_N t \right)^2}{2} \right] \left( \cos (t) \sigma_x - \sin (t) \sigma_y \right). \]
The probability to detect a neutrino with mixing angle $\gamma$ at time $t$ is given by
\[
\pi_{\gamma} = \frac{1}{2} \left( 1 + \cos (2\gamma) \cos (2\theta) + \sin (2\gamma) \sin (2\theta) \exp \left[ -\frac{(s_Nt)^2}{2} \right] \cos (t) \right). \tag{9}
\]

In the asymptotic limit $t \to \infty$, the non-diagonal terms vanish and the system is left in a mass eigenstates with probability $\pi_0 = \frac{1}{2}(1 + \cos 2\theta)$ and $\pi_1 = \pi_{\gamma=\pi/2} = \frac{1}{2}(1 - \cos 2\theta)$. Thus, the mass eigenstates become pointer states. The asymptotic value is independent of the dispersion $s_N$. If $\gamma = \theta$, the initial mixing angle, then
\[
\pi_{\theta} = 1 - \frac{\sin^2 (2\theta)}{2} \left( 1 - \exp \left[ -\frac{(s_Nt)^2}{2} \right] \cos (t) \right). \tag{10}
\]

Similar results are found if three mass eigenstates are considered [7]. If the environment is oriented along an arbitrary direction $\alpha$ in the $(x, z)$ plane the interaction becomes
\[
H^{(\text{coup})} = \frac{1}{2} \sigma_\alpha \otimes \sum_k g_k \sigma^{(k)}_\alpha; \quad \sigma_\alpha = \sigma_z \cos \alpha + \sigma_x \sin \alpha. \tag{11}
\]

We rotate the basis states of the environment, so that $\Psi^{(\alpha)}_n$ are eigenstates of the operator $\sum_k g_k \sigma^{(k)}_\alpha$ with the same eigenvalues $\delta_m \beta_n$. The rotation operator is written
\[
R_n (-\sigma_z + \beta_n \sigma_\alpha) R_n^\dagger = \omega_n \sigma_z; \quad R(\gamma) = \exp(i\gamma \sigma_y/2), \tag{12}
\]
which requires
\[
\cos \gamma = \frac{\beta_n \cos \alpha - 1}{\omega_n}; \quad \sin \gamma = \frac{\beta_n \sin \alpha}{\omega_n}; \quad \omega_n = (\beta_n^2 - 2\beta_n \cos \alpha + 1)^{1/2}. \tag{13}
\]

We replace, as before, the discrete sum by integrals $\zeta_i(\alpha, t), \zeta_x(\alpha, t), \zeta_y(\alpha, t)$, whose explicit expression are found in [7]. The density matrix is written
\[
\rho(\alpha, t) = \frac{1}{2} \left( I + \sum_{i=z,x,y} \zeta_i(\alpha, t) \sigma_i \right) = \pi_+ \rho_+ + \pi_- \rho_-, \tag{14}
\]
where
\[
\pi_+ - \pi_- = \left( \sum_i \zeta_i^2(\alpha, t) \right)^{1/2} \quad \rho_\pm = \frac{1}{2} I \pm \frac{1}{2(\pi_+ - \pi_-)} \sum_i \zeta_i(\alpha, t) \sigma_i.
\]

The probability of detecting a neutrino at time $t$ in the initial state $\rho_\theta$ is
\[
\pi_{\theta} = \frac{1}{2} \left( 1 + \cos 2\theta \zeta_z(\alpha, t) + \sin 2\theta \zeta_x(\alpha, t) \right). \tag{15}
\]

Thus, while $\rho(0)$ represents a pure state, $\rho(t)$ represents a mixed state. In the next Section we shall briefly discussed two cases of interest. The details are presented in Refs.[7, 8].
### 3. Results and Discussions

Case a) Let us first consider the case of neutrinos produced outside the solar system. The length associated with one oscillation of the density matrix is of the order

\[ \lambda = \frac{\nu h c}{\delta m} \approx 2.8 \times 10^6 \text{ km eV/}\delta m. \]

This estimation may be compared with the length predicted for periodic oscillations of neutrinos

\[ l \approx 1.3 \times 10^6 \text{ km eV/}\delta m, \]

which is of the same order of magnitude than \( \lambda \). Assuming an energy of about 20 MeV one gets

\[ l \approx 10^{14} \text{ km}. \]

From these values one gets a solution consisting of \( p = 0.77 \) 1 - \( p = 0.23 \) that is about 23 \% of the neutrinos arriving to the earth from galactic distances are represented by the particular linear combination

\[ \frac{1}{\sqrt{2}}(\Psi_\mu + \Psi_\tau). \]

Since the distance sun-earth is \( 1.5 \times 10^8 \text{ km} \ll l \), the decoherence mechanism will not affect neutrinos produced within the solar system, since it will require a distance six orders of magnitude larger.

Case b) We shall then consider decoherence effects in fluxes of neutrinos coming from a microquasar [8]. The time evolution of the occupation number of neutrinos is governed by the equation of motion

\[
\dot{\rho}_f = \left[ \frac{M^2 c^4}{2E h} + \sqrt{2} G_F \rho, \rho_f \right],
\]

where the squared brackets reads for the commutator, the dot represents the time derivative and \( M^2 c^4 \) is the mass-squared matrix in the flavor basis. The quantity \( \rho_f \) is the density matrix in the flavor basis, and \( \rho \) is a matrix whose diagonal terms are the neutrino number densities, \( E \) stands for the neutrino energy and \( G_F \) is the Fermi constant.

The mass-squared matrix in the flavor basis can be obtained by a transformation of the mass-squared matrix in the mass basis \((m^2 c^4)\) through the unitary mixing matrix \( U \). That is:

\[
M^2 c^4 = U m^2 c^4 U^\dagger = \frac{1}{2} \left( m_1^2 + m_2^2 \right) c^4 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2} \sin 2\theta \left( m_2^2 - m_1^2 \right) c^4 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \frac{1}{2} \cos 2\theta \left( m_2^2 - m_1^2 \right) c^4 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

As explained previously, we can write the mass matrix and the matrix \( \rho_f \) in terms of Pauli matrices. For two-neutrino mass eigenstates they are written

\[
M^2 c^4 = \frac{1}{2} \text{tr} \left( m^2 c^4 \right) I + \frac{1}{2} w \bar{B} \cdot \vec{\sigma},
\]

\[
\rho_f = \frac{1}{2} \text{tr} (\rho_f) I + \frac{1}{2} \bar{P}_f \cdot \vec{\sigma},
\]

where \( \bar{P}_f \) is the polarization vector in the flavor basis, \( w = \frac{\delta m^2}{2 E h} \), and \( \delta m^2 = m_2^2 c^4 - m_1^2 c^4 \) is the mass-squared difference between the mass eigenstates, \( \bar{B} \) is an unitary vector which fixes the orientation of the background.

\[
\bar{B} = \begin{pmatrix} \sin 2\theta \\ 0 \\ -\cos 2\theta \end{pmatrix}.
\]

One can performed a rotation of the coordinate system in the flavor space and choose the direction of \( \bar{B} \) in the z axis. This direction does not necessary correspond to the external physical magnetic field. The equation of motion is then re-written as

\[
\frac{\partial \bar{P}_w}{\partial t} = (w \bar{B} + \mu \bar{P}) \times \bar{P}_w,
\]
where $\mu$ stands for the neutrino-neutrino interaction and

$$ P = \int \bar{P}_w \, dw. \quad (22) $$

The order parameter that measures coherence is defined as the ratio between the modulus of the perpendicular polarization vector at time $t$ and time $t = 0$, that is:

$$ R_\theta(t) = \frac{|\bar{P}_\perp(t)|}{|\bar{P}_\perp(0)|}, \quad (23) $$

where $\bar{P}_\perp(t) = \bar{P} - (\bar{P} \cdot \bar{B}) \bar{B}$. The average in angles can be computed as

$$ R(t) = \frac{\int R_\theta(t) \, d\theta}{\int d\theta}. \quad (24) $$

The time that neutrinos remain inside the jet can be computed as $R_{\text{max}}/c$. This time is of the order of $1.4 \times 10^{-3}$ s and in windy microquasar it is $1.7 \times 10^{-4}$ s. Therefore, inside the microquasar’s jet, the decoherence has no effect in the neutrino distribution. For the active-sterile neutrino mixing, we found that the length of the vector $\bar{P}$ is reduced to zero for the non-interacting case. Meanwhile, for small values of the interaction the length of the vector $\bar{P}$ is depleted. For the case of the mixing between active neutrinos, if one fixes the neutrino boundaries in the region of the order of $10^8$ cm the time they spend there is of the order of $1.6 \times 10^{-5}$ seconds, a time shorter than the time needed to evolve into pointer states. However, if the coupling with sterile neutrinos is activated, the onset of decoherence becomes clear from the results. The same behavior is then expected for the outside region. A more precise determination of the effect requires the comparison between the length of the flavor oscillations and the length of decoherence under the microquasar’s conditions [8].

4. Conclusions

Assuming neutrinos in an initial (pure) flavor state we have obtained the probability of detecting them in a mixed state by taking into account the environment and using a flavor-flavor schematic interactions. Added to the oscillations between flavor-eigenstates we have a rate of decay $\approx \exp(-s_N^2 t^2/2)$, where the parameter $s_N$ measures the width of the residual distribution of eigenvalues in the environment. The amplitude of the oscillations is affected by the exponential decay and it disappears for values of $s_N$ larger than the inverse of the period. We conclude that the probability of detecting a traveling neutrino in a given flavor state is significantly affected by the environment that it may have crossed along its path from the source, an effect which should be considered in addition to flavor oscillations.

5. ACKNOWLEDGMENTS

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