Full counting statistics as a probe of quantum coherence in a side-coupled double quantum dot system

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Abstract

We study theoretically the full counting statistics of electron transport through side-coupled double quantum dot (QD) based on an efficient particle-number-resolved master equation. It is demonstrated that the high-order cumulants of transport current are more sensitive to the quantum coherence than the average current, which can be used to probe the quantum coherence of the considered double QD system. Especially, the quantum coherence plays a crucial role in determining whether the super-Poissonian noise occurs in the weak inter-dot hopping coupling regime depending on the corresponding dot-lead coupling, and the corresponding values of super-Poissonian noise can be relatively enhanced when considering the spins of conduction electrons. Moreover, this super-Poissonian noise bias range depends on the singly-occupied eigenstates of the system, which thus suggests a tunable super-Poissonian noise device. The occurrence-mechanism of super-Poissonian noise can be understood in terms of the interplay of quantum coherence and effective competition between fast-and-slow transport channels.

Key words: Full counting statistics; Quantum coherence; Super-Poissonian noise; Quantum dots

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1. Introduction

Non-equilibrium electronic full counting statistics (FCS) is a powerful diagnostic tool for probing the nature of electron transport mechanisms inaccessible by the average current measurements\cite{1, 2}. Recently, probing the quantum coherence of coupled quantum-dot (QD) systems by means of FCS, i.e., the transport current high-order cumulants, have attracted considerable attention due to the quantum coherence plays a crucial role for their application in the field of solid-state quantum computing, and some interesting current noise characteristics have been observed or predicted\cite{3, 4, 5, 6, 7, 8, 9, 10}. For example, the interplay of quantum coherence and strong Coulomb blockade/charging energy can induce the super-Poisson behavior of transport current in the parallel\cite{3, 4} and series\cite{5} double-QD (DQD) systems, but for coupled three-QD system\cite{6}, a crossover from sub-Poissonian to super-Poissonian statistics was observed with increasing ratio of tunnel and decoherence rates. In particular, the high-order cumulants, e.g., the shot noise, the skewness, are more sensitive to the quantum coherence effect than the average current in the different types of QD systems\cite{3, 4, 5, 6, 7, 8, 9, 10}, namely, the series DQD\cite{7}, the parallel DQD\cite{8}, the Aharonov-Bohm interferometer with a quantum dot embedded in one of its two current paths\cite{9}, a semiopen Kondo-correlated quantum dot\cite{10}. Moreover, theoretical studies have also shown that the high-order cumulants can be used to detect the positions of the zeros of the generating function\cite{11}, reveal the intrinsic multistability\cite{12}, and extract the fractional charge of charge transfer through an impurity in a chiral Luttinger liquid\cite{13}. However, the shot noise or higher-order cumulants of the current do not provide any more information on decoherence additional than the average current that also was observed in two types of mesoscopic structures with a varying number of propagating channels, e.g., mesoscopic cavities and Aharonov-Bohm rings\cite{14}. Therefore, extracting quantum coherence information from the current high-order cumulants is still an open issue. On the other hand, the FCS of electron transport through a semiconductor quantum dot (QD) have been experimentally, especially the
fifteen-order cumulants\textsuperscript{15, 16} and finite-frequency current statistics\textsuperscript{17} can be extracted from the high-quality real-time single-electron measurements. This provides the opportunity to investigate the relationship between the quantum coherence and the FCS of electron transport through coupled QD systems.

The goal of this paper is thus to study the influence of the quantum coherence on the high-order cumulants of electron transport through a relatively highly coherent quantum system, and analyze the feasibility of extracting quantum coherence information from the current high-order cumulants. Here, we consider a side-coupled DQD system with high quantum coherent for a weak inter-dot hopping coupling relative to the dot-lead coupling. Although super-Poissonian shot noise was studied in this QD system\textsuperscript{18, 19}, the effects of the quantum coherence between singly-occupied eigenstates of the system, i.e. the off-diagonal elements of the reduced density matrix, and the electron spin on super-Poissonian noise, especially the feasibility of probing the quantum coherence of this QD system by means of FCS, are not revealed in previous investigations\textsuperscript{19}. Here, we examine the effects of conduction electron spin and quantum coherence on the FCS in such QDs system. We found that the current high-order cumulants are more sensitive to the quantum coherence than the average current, which can be used to probe the quantum coherence of this side-coupled DQD system. Especially, the quantum coherence of this DQD system plays a crucial role in determining whether the super-Poissonian noise occurs in the weak inter-dot hopping coupling regime, and the corresponding values of super-Poissonian noise can be relatively enhanced when considering conduction electron spin. Moreover, this super-Poissonian noise characteristic, which originates from the quantum coherence between the singly-occupied eigenstates, can be used to design a tunable super-Poissonian noise device because of singly-occupied eigenstates can be tuned by a gate voltage. The paper is organized as follows. In Sec. II, we introduce the considered side-coupled DQD system and outline the procedure to obtain the FCS formalism based on an effective particle-number-resolved quantum master equation approach. The numerical results are discussed in Sec. III, where we discuss the effects of the conduction electron spin and quantum coher-
ence on super-Poissonian noise and analyze the mechanism of their formation. Finally, in Sec. IV we summarize the work.

2. MODEL AND FORMALISM

We consider a side-coupled DQD system weakly coupled to two metallic electrodes [see the Fig. 1]. The system is described by the Hamiltonian $H_{\text{total}} = H_{\text{dot}} + H_{\text{leads}} + H_{T}$. The double-QD Hamiltonian is given by

$$H_{\text{dot}} = \sum_{i,\sigma} \epsilon_{i} d_{i\sigma}^{\dagger} d_{i\sigma} + U_{12} \sum_{\sigma, \sigma'} \hat{n}_{1\sigma} \hat{n}_{2\sigma'} - J \sum_{\sigma} \left( d_{1\sigma}^{\dagger} d_{2\sigma} + d_{2\sigma}^{\dagger} d_{1\sigma} \right),$$

(1)

where $d_{i\sigma}^{\dagger}$ ($d_{i\sigma}$) creates (annihilates) an electron with spin $\sigma$ and energy $\epsilon_{i}$ (which can be tuned by a gate voltage $V_{g}$) in $i$th QD. $U_{12}$ is the interdot Coulomb repulsion between two electrons in the DQD system, where we consider the intradot Coulomb interaction $U \to \infty$, so that the double-electron occupation in the same QD is prohibited but in different QDs is permitted. The last term of $H_{\text{dot}}$ describes the hopping coupling between the two dots with $J$ being the hopping parameter.

The relaxation in the electrodes is assumed to be sufficiently fast so that their electron distributions can be described by equilibrium Fermi functions. The electrodes are modeled as non-interacting Fermi gases and the corresponding Hamiltonian

$$H_{\text{Leads}} = \sum_{\alpha\kappa \sigma} \varepsilon_{\alpha\kappa} a_{\alpha\kappa \sigma}^{\dagger} a_{\alpha\kappa \sigma},$$

(2)

where $a_{\alpha\kappa}^{\dagger}$ ($a_{\alpha\kappa}$) creates (annihilates) an electron with energy $\varepsilon_{\alpha\kappa}$ and momentum $\kappa$ in $\alpha$ ($\alpha = L, R$) electrode. The tunneling between the QD-1 and the electrodes is described by

$$H_{T} = \sum_{\alpha\kappa \sigma} \left( t_{\alpha\kappa} a_{\alpha\kappa \sigma}^{\dagger} d_{1\sigma} + t_{\alpha\kappa}^{*} d_{1\sigma}^{\dagger} a_{\alpha\kappa \sigma} \right).$$

(3)

The QD-electrode coupling is assumed to be sufficiently weak, such that the sequential tunneling is dominant. The transitions are well described by quantum master equation of a reduced density matrix spanned by the eigenstates of the
side-coupled DQD system. The detailed derivation of the FCS formalism based on the particle-number-resolved quantum master equation can be found in Refs. [4, 20, 21], and here we only give the main results. Under the second order Born approximation and Markov approximation, the particle-number-resolved quantum master equation for the reduced density matrix is given by

$$
\dot{\rho}^{(n)}(t) = -iL \rho^{(n)}(t) - \frac{1}{2} \mathcal{R} \rho^{(n)}(t),
$$

with

$$
\mathcal{R} \rho^{(n)}(t) = \sum_{\sigma=\uparrow, \downarrow} \left[ d_{1\sigma}^{\dagger} A_{L,\sigma}^{(-)} \rho^{(n)}(t) + d_{1\sigma}^{\dagger} A_{R,\sigma}^{(-)} \rho^{(n)}(t) + \rho^{(n)}(t) A_{L,\sigma}^{(+)} d_{1\sigma}^{\dagger} + \rho^{(n)}(t) A_{R,\sigma}^{(+)} d_{1\sigma}^{\dagger} - A_{L,\sigma}^{(-)}(t) d_{\sigma}^{\dagger} - A_{R,\sigma}^{(-)}(t) d_{\sigma}^{\dagger} - A_{L,\sigma}^{(+)}(t) d_{\sigma}^{\dagger} - A_{R,\sigma}^{(+)}(t) d_{\sigma}^{\dagger} + \ldots \right] + \text{H.c.},
$$

where $A_{\alpha,\sigma}^{(\pm)} = \Gamma_{\alpha} n_{\alpha}^{(\pm)} (-\mathcal{L}) d_{\sigma}, n_{\alpha}^{\pm} = f_{\alpha}, n_{\alpha}^{-} = 1 - f_{\alpha}$ ($f_{\alpha}$ is the Fermi function of the electrode $\alpha$), and $\Gamma_{\alpha=L,R} = 2\pi g_{\alpha=L,R} |t_{\alpha=L,R}|$. Liouvillian superoperator $\mathcal{L}$ is defined as $\mathcal{L}(\cdots) = [H_{\text{dot}}, (\cdots)]$, and $g_{\alpha=L,R}$ denotes the density of states of the metallic electrodes. $\rho^{(n)}(t)$ is the reduced density matrix of the side-coupled DQD system conditioned by the electron numbers arriving at the right electrode up to time $t$. In order to calculate the FCS, one can define $S(\chi, t) = \sum_{n} \rho^{(n)}(t) e^{in\chi}$. According to the definition of the cumulant generating function, $e^{-F(\chi))} = \sum_{n} \text{Tr}[\rho^{(n)}(t)] e^{in\chi} = \sum_{n} P(n, t) e^{in\chi}$, we evidently have $e^{-F(\chi))} = \text{Tr}[S(\chi, t)]$, where the trace is over the eigenstates of the considered DQD system and $\chi$ is the counting field. Since Eq. (4) has the following form

$$
\dot{\rho}^{(n)} = A \rho^{(n)} + C \rho^{(n+1)} + D \rho^{(n-1)},
$$

$S(\chi, t)$ satisfies

$$
\dot{S} = AS + e^{-i\chi} CS + e^{i\chi} DS \equiv \mathcal{L}_{\chi} S,
$$

where $S$ is a column matrix, and $A$, $C$ and $D$ are three square matrices. In the low frequency limit, the counting time (i.e., the time of measurement) is much
longer than the time of electron tunneling through the considered DQD system. In this case, $F(\chi)$ is given by
\[ F(\chi) = -\lambda_1(\chi)t, \tag{7} \]
where $\lambda_1(\chi)$ is the eigenvalue of $L_\chi$ which goes to zero for $\chi \rightarrow 0$. According to the definition of the cumulants one can express $\lambda_1(\chi)$ as
\[ \lambda_1(\chi) = \frac{1}{t} \sum_{k=1}^{\infty} C_k \frac{(i\chi)^k}{k!}. \tag{8} \]
Here, the first four cumulants $C_k$ are directly related to the transport characteristics. For example, the first-order cumulant (the peak position of the distribution of transferred-electron number) $C_1 = \bar{n}$ gives the average current $\langle I \rangle = eC_1/t$. The zero-frequency shot noise is related to the second-order cumulant (the peak-width of the distribution) $S = 2e^2C_2/t = 2e^2(\bar{n}^2 - \bar{n}^2)/t$. The third-order $C_3 = (\bar{n} - \bar{n})^3$ and fourth-order cumulants $C_4 = (n - \bar{n})^4$, respectively, characterize the skewness and kurtosis of the distribution. Here, $\langle \cdots \rangle = \sum_n \langle \cdots \rangle P(n,t)$. In general, the shot noise, skewness and kurtosis are represented by the Fano factor $F_2 = C_2/C_1$, $F_3 = C_3/C_1$ and $F_4 = C_4/C_1$, respectively. Moreover, the specific form of $L_\chi$ can be obtained by performing a discrete Fourier transformation to the matrix elements of Eq. (4). Inserting Eq. (8) into $|L_\chi - \lambda_1(\chi)I| = 0$ and expanding this determinant in series of $(i\chi)^k$, one can calculate $C_k/t$ by setting the coefficient of $(i\chi)^k$ equal to zero.

3. NUMERICAL RESULTS AND DISCUSSION

We now study the effects of conduction-electron spin and quantum coherence on the FCS of electronic transport through the side-coupled DQD system weakly coupled to two metallic electrodes. We assume the bias voltage ($V_b = \mu_L - \mu_R$) is symmetrically entirely dropped at the QD-electrode tunnel junctions, which implies that the levels of the QDs are independent of the applied bias voltage even if the couplings are not symmetric, and choose meV as the unit of energy which corresponds to a typical experimental situation\textsuperscript{28}. Here, the eigenstates of $H_{dot}$
are chosen to describe the electronic states of this side-coupled double-QD system, which can be obtained by diagonalizing the QDs Hamiltonian $H_{\text{dot}}$ in the basis represented by the electron spins in the QD-1 and QD-2 denoted respectively by $|\sigma_1, \sigma_2\rangle$, i.e., \{0,0⟩, |↑,0⟩, |↓,0⟩, |0,↑⟩, |0,↓⟩, |↑,↑⟩, |↑,↓⟩, |↓,↑⟩, |↓,↓⟩\}. In the present work, we only study the transport above the sequential tunneling threshold, i.e., $V_b > 2\epsilon_{sc}$, where $\epsilon_{sc}$ is the energy difference between the ground state with charge $N$ and the first excited states $N-1$. In this regime, the inelastic sequential tunneling process is dominant. It should be noted that, however, the normalized second-, third- and fourth-order cumulants will deviate from the results obtained by considering only sequential tunneling when taking into account cotunneling, since in the Coulomb blockade regime the current is exponentially suppressed and the electron transport is dominated by cotunneling. In the following numerical calculations, the parameters of the QDs are chosen as $\epsilon_1 = \epsilon_2 = \epsilon$, $U_{12} = 5$, $J = 0.001$ and $k_B T = 0.1$.

We firstly consider the influence of the conduction-electron spin on the FCS of electronic transport. The spinless Hamiltonian of the considered double-QD system is given by

$$H_{\text{spinless}} = \sum_i \epsilon_i d_i^\dagger d_i + U_{12} \hat{n}_1 \hat{n}_2 - J \left(d_1^\dagger d_2 + d_2^\dagger d_1\right).$$

(9)

Figure 2 shows the average current, shot noise, skewness and kurtosis as a function of the bias voltage for the two cases of considering and without considering the conduction-electron spin. Here, the off-diagonal elements of the reduced density matrix, i.e., quantum coherence, are considered in the numerical calculations. For the symmetric coupling of the QD-1 with two metallic electrodes, i.e., $\Gamma_{L\uparrow} = \Gamma_{L\downarrow} = \Gamma_{R\uparrow} = \Gamma_{R\downarrow} = \Gamma$, the magnitude of average current is relatively large for considering the conduction-electron spin case, see the Figs. 2(a) and 2(e). This feature can be explained in terms of the variation of the probability of the corresponding eigenstates of the considered DQD system. For considering and without considering the conduction-electron spin cases, the transport
channel currents can be expressed by \[23, 24, 25, 26, 27\]

\[
\begin{align*}
I_{\psi_{i\sigma}^{\pm}} \rightarrow \psi_{0,0} & = \frac{1}{2} \Gamma R_{\sigma} n_{R} \left( \epsilon_{1,\sigma}^{\pm} - \epsilon_{0,0} \right) P_{\psi_{i\sigma}^{\pm}} \\
I_{\psi_{1,1}^{\pm}} \rightarrow \psi_{1,1} & = \frac{1}{2} \Gamma R_{1} n_{R} \left( \epsilon_{1,1}^{\pm} - \epsilon_{1,1}^{\pm} \right) P_{\psi_{1,1}^{\pm}}, \\
I_{\psi_{1,1}^{\pm}} \rightarrow \psi_{1,0} & = \frac{1}{2} \Gamma R_{1} n_{R} \left( \epsilon_{1,0}^{\pm} - \epsilon_{1,1}^{\pm} \right) P_{\psi_{1,0}^{\pm}}, \\
I_{\psi_{1,1}^{\pm}} \rightarrow \psi_{1,0} & = \frac{1}{2} \Gamma R_{1} n_{R} \left( \epsilon_{1,1}^{\pm} - \epsilon_{1,0}^{\pm} \right) P_{\psi_{1,1}^{\pm}}, \\
I_{\psi_{1,1}^{\pm}} \rightarrow \psi_{1,0} & = \frac{1}{2} \Gamma R_{1} n_{R} \left( \epsilon_{0,0}^{\pm} - \epsilon_{1,1}^{\pm} \right) P_{\psi_{1,0}^{\pm}}, \\
I_{\psi_{1,1}^{\pm}} \rightarrow \psi_{1,0} & = \frac{1}{2} \Gamma R_{1} n_{R} \left( \epsilon_{1,0}^{\pm} - \epsilon_{1,1}^{\pm} \right) P_{\psi_{1,0}^{\pm}},
\end{align*}
\]

and

\[
\begin{align*}
I_{\psi_{i\sigma}^{\pm}} \rightarrow \psi_{0,0} & = \frac{1}{2} \Gamma n_{R} \left( \epsilon_{1,\sigma}^{\pm} - \epsilon_{0,0} \right) P_{\psi_{i\sigma}^{\pm}} \\
I_{\psi_{k\sigma}^{\pm}} \rightarrow \psi_{1,0} & = \frac{1}{2} \Gamma n_{R} \left( \epsilon_{1,0}^{\pm} - \epsilon_{1,1}^{\pm} \right) P_{\psi_{k\sigma}^{\pm}},
\end{align*}
\]

respectively. Here, \(\epsilon_{i}\) is the eigenvalue of the system eigenstate \(|i\rangle\), and \(P_{|i\rangle}\) is the probability of electron occupying state \(|i\rangle\). For the considered system parameters \((\epsilon_{1} = \epsilon_{2} = 1)\), the first current-step induced by the transition processes between the singly-occupied and empty eigenstates, and the Fermi function of the right electrode \(n_{R}^{+} (\epsilon_{1,\sigma}^{\pm} - \epsilon_{0,0}) = 0\), namely, electrons reverse tunneling from the right electrode to the considered DQD system is prohibited. This leads to \(n_{R}^{-} (\epsilon_{1,\sigma}^{\pm} - \epsilon_{0,0}) = 0\), so that the probability of each eigenstate is given by \(1/5\) since there are four singly-occupied and one empty eigenstates in this bias voltage window, one can check from Eq. \(10\) that the total current \(I_{1}/\Gamma = 2/5\).

The second current-step induced by the two kinds of transition processes: (i) between the singly-occupied and empty eigenstates, (ii) between doubly-occupied and the singly-occupied eigenstates, and the corresponding Fermi function of the right electrode \(n_{R}^{+} = 0\), i.e., \(n_{R}^{-} = 0\). In this case, the probability of each eigenstate is \(1/9\) due to there are nine eigenstates in this bias voltage window, and the total current \(I_{2}/\Gamma = 2/3\), see the Fig. 2(c). As for without considering the conduction-electron spin case, based on the same analysis the magnitudes of the first current-step and the second current-step are given by \(I_{1,\text{spinless}}/\Gamma = 1/3\) and \(I_{2,\text{spinless}}/\Gamma = 1/2\), respectively, see the Fig. 2(a). Therefore, the current for considering the conduction-electron spin case is relatively large. In particular, the corresponding high-order cumulants of transport current have a significantly strengthened (the shot noise and skewness) or weakened (the kurto-
sis) especially for the $\Gamma \gg J$ case, see the dotted and short-dashed lines in Figs. 2(b) and 2(f), 2(c) and 2(g), 2(d) and 2(h). These characteristics have demonstrated that the conduction-electron spin cannot be neglected in the systems with a relatively strong quantum coherent effect.

Now, we discuss the effect of quantum coherence on the high-order cumulants of transport current, and choose $\epsilon_1 = \epsilon_2 = 1$ here. For the relatively weak hopping between the two QDs with respect to the couplings of QD-1 with two metallic electrodes, i.e., $J < \Gamma$, Figures 2 and 3 show the first four cumulants of zero-frequency current fluctuation as a function of bias voltage for the cases of considering and without considering quantum coherence at different values of $\Gamma$, respectively. Considering only the diagonal elements of the reduced density matrix, the super-Poissonian noise cannot be observed; but the corresponding Fano factors, which do not depend on the $\Gamma$ value, have the same values, see the Fig. 3. This characteristic can be understood in terms of the specific form of $L_\chi$. For the considered case of $\Gamma_L = \Gamma_R = \Gamma$, the matrix $L_\chi$ can be rewritten as $\Gamma L_\chi'$, and the corresponding $k$-order cumulant is given by $C_k/\Gamma$, so that the Fano factor of $k$-order cumulant, which are defined as $C_k/C_1$, does not depend on the $\Gamma$ value, see the Fig. 3. As a result, the off-diagonal elements of the reduced density matrix should be considered in the numerical calculation. In particular, the quantum coherence, which originates from the two kinds of coherent singly-occupied eigenstates $|\Psi_{1,\sigma}^+\rangle$ and $|\Psi_{1,\sigma}^-\rangle$ determined by the hopping parameter $J$, plays a crucial role in determining whether the super-Poissonian noise occurs for the $\Gamma > J$ case, where this ratio of $\Gamma$ to $J$ should be greater than a certain value (about 3), see the Figs 2(f), 2(g) and 2(h). This characteristic of super-Poissonian noise can be understood in terms of the interplay of quantum coherence and effective competition between correlated electron transport channels, which is a new occurrence-mechanism of super-Poissonian noise and not revealed in the previous studies [4, 19, 23, 24, 25, 26, 29]. For this super-Poissonian noise bias voltage range, electrons can tunnel from the left metallic electrode into and then tunnel out of the DQD system to the right electrode via the transition between the singly-occupied and empty eigenstates. In the case
of $\Gamma \gg J$, when the conduction-electron from the left electrode tunnels from the QD-1 to the QD-2, this electron in the QD-2 will remain for a relatively long time, then again tunnels into the QD-1 and out of the double-QD system to the right metallic electrode. These indirect electron tunneling processes will lead to electron tunneling is blocked since this DQD system can occupy only one electron and form the slow electron transport channels, whereas the direct tunneling processes through QD-1 form the corresponding fast transport channels. The interplay of quantum coherence and effective competition between the fast and slow correlated electron transport channels eventually results in the formation of super-Poissonian noise, and the super-Poissonian behavior will be more obvious for the relatively large ratio of $\Gamma$ to $J$, see the Figs. 2(f), 2(g) and 2(h).

However, the super-Poissonian noise value will be decreased to a sub-Poissonian value as expected for the relatively strong hopping, i.e., $J > \Gamma$. This originates from the fact that with increasing the ratio of $J$ to $\Gamma$ especially for the $J \gg \Gamma$ case the conduction-electron can tunnel back and forth between the two QDs that leads to the indirect electron tunneling processes are almost equivalent to direct electron tunneling processes, so that the effective fast and slow correlated electron transport channels cannot be formed, and the quantum coherence of the two kinds of coherent singly-occupied eigenstates will be weakened, namely, the diagonal elements of the reduced density matrix play a major role in the electron tunneling processes, see the Fig. 4. Consequently, the corresponding Fano factors decrease to the sub-Poissonian values. Another important finding is that the high-order cumulants, e.g., the skewness and kurtosis, are more sensitive to the quantum coherence of this considered QDs system, which is independent of the relative magnitudes of $J$ and $\Gamma$, see the Figs. 2(g) and 2(h), and Fig. 4. This characteristic can be used to detect the quantum coherence of quantum systems, which is contrary to the conclusion of the Ref. [14].

Finally, we study the effect of the ground state (which can be tuned by a gate voltage $V_g$) of the present QDs system on the super-Poissonian behaviors, here the corresponding parameters of $J$ and $\Gamma$ are chosen as $\Gamma = 0.005$ and
\[ J = 0.001. \] The empty, singly-occupied and doubly-occupied eigenstates are given by

\[
\begin{align*}
\epsilon_{0,0} &= 0 \\
\epsilon_{\pm,\sigma} &= \epsilon \pm J \\
\epsilon_{\sigma,\sigma'} &= 2\epsilon + U_{12}
\end{align*}
\]

where \( \epsilon_1 = \epsilon_2 = \epsilon \). Therefore, for the \( \epsilon > 0 \) case the ground state is the electronic unoccupied eigenstate; but for \( \epsilon < 0 \) the ground states are the singly-occupied eigenstates \(|\Psi^{\pm}_{1,\sigma}\rangle\) or doubly-occupied eigenstates \(|\Psi_{\sigma,\sigma'}\rangle\) since \( \epsilon \gg J \).

In the two cases of \( \epsilon > 0 \), and \( \epsilon < 0 \) and \( 2\epsilon + U_{12} > 0 \), the first current-step induced by the transition processes between the singly-occupied and empty eigenstates, the super-Poissonian noise in this bias voltage, which originates from the same mechanism as mentioned above, can occur and the threshold value of the super-Poissonian noise bias voltage depends on \( \epsilon \). This characteristic can be used to tune the super-Poissonian noise bias voltage range by adjusting the two independent gate electrodes for each dot, and suggests a tunable super-Poissonian noise device, see the Fig. 5. As for the \( \epsilon < 0 \) and \( 2\epsilon + U_{12} > 0 \) case, the first current-step induced by the transition processes between the doubly-occupied and singly-occupied eigenstates. For the considered \( \Gamma/J = 5 \) case, when one of the two conduction electrons tunnels out the DQD system to the right electrode, the singly-occupied eigenstates will relax to the state of \(|\sigma\rangle_{1} |0\rangle_{2}\) or \(|0\rangle_{1} |\sigma\rangle_{2}\) because the system needs to be formed a new doubly-occupied eigenstate. If the other conduction electron in the QD-1, i.e., \(|\sigma\rangle_{1} |0\rangle_{2}\), that will remain for a relatively long time until this electron tunnels from the QD-1 to the QD-2, then electron from the left electrode can tunnel into the QD-1. This process will result in electron tunneling is blocked and form the slow transport channels; whereas for the case of the other conduction electron in the QD-2, electron from the left electrode can direct tunnel into the QD-1 and form a new doubly-occupied eigenstate, so that electron tunneling processes can continues until the singly-occupied eigenstates relax again to the \(|\sigma\rangle_{1} |0\rangle_{2}\) state, which will form the corresponding fast correlated transport channels. The interplay of quantum coherence and effective competition between these fast
and slow correlated transport channels is responsible for the super-Poissonian noise behavior, see the short-dashed lines in the Figs. 5(b), 5(c) and 5(d). Furthermore, the skewness and kurtosis for the $\epsilon < 0$ and $2\epsilon + U_{12} > 0$ case, respectively, show a very large negative value relative to the two cases of $\epsilon > 0$, and $\epsilon < 0$ and $2\epsilon + U_{12} > 0$, see the Figs. 5(c) and 5(d). This characteristic can be used to identify whether the ground state is doubly-occupied eigenstate.

4. Conclusions

We have studied theoretically the full counting statistics of electron transport through the side-coupled DQD system above the sequential tunneling threshold. The high-order cumulants of transport current are found to be more sensitive to the quantum coherence than the average current. This characteristic can be used to probe the quantum coherence of the considered side-coupled DQD system. Especially, the quantum coherence of this DQD system plays a crucial role in determining whether the super-Poissonian noise occurs in the weak inter-dot hopping coupling regime, e.g., super-Poissonian noise is observed when the ratio of the inter-dot hopping coupling to dot-lead coupling is smaller than a certain value, and the corresponding values of super-Poissonian noise can be relatively enhanced when considering conduction electron spin. Moreover, this super-Poissonian noise bias range depends on the singly-occupied eigenstates of the system, which thus suggests a tunable super-Poissonian noise device. The super-Poissonian noise characteristics can be qualitatively attributed to the interplay of quantum coherence and effective competition between fast and slow transport channels.

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Figure Caption:

Fig. 1 Schematic representation of the considered side-coupled DQD system coupled to two metallic electrodes.

Fig. 2 (Color online) The first four cumulants of zero-frequency current fluctuation versus bias voltage for different coupling of the side-coupled DQD system with two metallic electrodes. Here, the off-diagonal elements of the reduced density matrix are considered. (a), (b), (c) (d) for the spinless Hamiltonian; and (e), (f), (g), (h) for the spin Hamiltonian. The system parameters: $\epsilon_1 = \epsilon_2 = 1$, $U_{12} = 5$, $J = 0.001$ and $k_B T = 0.1$.

Fig. 3 (Color online) The first four cumulants of zero-frequency current fluctuation versus bias voltage for different coupling of the side-coupled DQD system with two metallic electrodes. Here, the diagonal elements of the reduced density matrix are only considered. (a), (b), (c) (d) for the spinless Hamiltonian; and (e), (f), (g), (h) for the spin Hamiltonian.. The other system parameters are the same as in Fig. 2.

Fig. 4 (Color online) The first four cumulants of zero-frequency current fluctuation versus bias voltage for the two cases for considering off-diagonal elements and only considering diagonal elements at fixed coupling of the side-coupled DQD system with two metallic electrodes, respectively. (a), (b), (c) (d) for $J = 0.005$ and $\Gamma_\uparrow = \Gamma_\downarrow = 0.001$; and (e), (f), (g), (h) for $J = 0.01$ and $\Gamma_\uparrow = \Gamma_\downarrow = 0.001$. The other system parameters are the same as in Fig. 2.

Fig. 5 (Color online) The first four cumulants of zero-frequency current fluctuation versus bias voltage for different values of $\epsilon$ ($\epsilon_1 = \epsilon_2 = \epsilon$), where $J = 0.001$ and $\Gamma_{L\uparrow} = \Gamma_{L\downarrow} = \Gamma_{R\uparrow} = \Gamma_{R\downarrow} = 0.005$. The other parameters are the same as in Fig. 2.
