The calibration method of a tandem queueing model with PH service time using NS-3 simulation of a multihop wireless network

A A Larionov¹, A A Mukhtarov¹ and A M Sokolov¹

¹V. A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences
65 Profsoyuznaya street, Moscow 117997, Russia

E-mail: larioandr@gmail.com, mukhtarov.amir.a@gmail.com, aleksandr.sokolov@phystech.edu

Abstract. End-to-end delay is one of the key characteristics of communication network performance. This characteristic determines the possibility of using the network for various delay-critical applications like voice or video transmission. One of the widely used approaches to estimating delays is the use of the queuing theory. According to this approach, a telecommunication network is modeled using a multiphase queuing system. Communication channels are modeled using service devices, and the incoming traffic is modeled with random distributions of the inter-arrival intervals between packets. The accuracy of this network model directly depends on how well the service time distributions are chosen. These distributions must consider the specifics of complex telecommunication protocols, size distributions of the transmitted packets, and, in case of wireless channels, the rate of collisions and retransmissions. The paper presents a study of the accuracy of estimates of end-to-end delays in a multi-hop wireless network using a queuing network with a phase-type (PH) service time distributions. To calibrate the model, PH distributions are found using the moments-matching method based on sample data on the duration of packet transmission in IEEE 802.11 channels. This sample data was obtained using a simulation model written in NS-3, taking into account the features of the IEEE 802.11 protocol and the presence of collisions in the network. To evaluate the accuracy, end-to-end delays are calculated using the queuing network and the wireless network simulation model. It is shown that it is possible to obtain reasonably accurate estimates for small networks, but with an increase in the size of the network, the accuracy decreases. In conclusion, recommendations are given to improve the accuracy of modeling.

1. Introduction
The design of communication networks and the development of new protocols include network performance evaluation, particularly end-to-end delays. These estimates allow us to conclude that the network is suitable for a variety of latency-critical applications and quickly identify performance “bottlenecks.” Researchers often use the queuing theory methods to evaluate network performance characteristics. And the distribution of intervals between packets in the traffic characterizes the arrival flow transmitted by the network. It is necessary to have better distributions to get accurate estimations. However, choosing “good” distributions for channel modeling is a complex problem since telecommunication protocols, especially wireless ones, turn out to be quite complicated. In addition, the transmission time takes into account a distribution of packet sizes and errors occurring. So, it is required to select the distribution depending on the protocol used and traffic transmitted by the network.
In this paper, we consider the particular case of a multi-hop wireless network, in which base stations have the linear topology, and use the same communication channel. Examples of such networks are vehicular ad-hoc networks (VANET) along long highways. We use IEEE 802.11 protocol in our work. Base stations compete with one another to use the shared medium in such a network. We assume that the incoming traffic arrives in the network according to the simplest (Poisson) flow, in which intervals between sequential packets have an exponential distribution. We use phase-type distributions (PH) to model the packet transmission duration. It allows us to describe wired and wireless channels accurately [1].

To calibrate the model and build PH-distributions, we need to have an actual sample of transmission intervals of separate packets in the channel, or at least the first moments of this sample. It is very complicated to use actual wireless equipment for these purposes. A more straightforward method that we apply in our work is implementing a network in the NS-3 modeling system. To construct a sample of time intervals, we also consider that stations located at the beginning and end of the network are less under collisions than intermediate stations. Intermediate stations have to compete with their neighbors for channel access. We use the found PH-distributions in M/PH/1/N → •/PH/1/N →…→ •/PH/1/N queuing model to estimate end-to-end delays. Further, we will compare these estimates with end-to-end delays obtained using the simulated wireless network model implemented in NS-3. As will be shown below, the queuing system allows a fairly accurate estimate of end-to-end delays for networks with a small number of stations. It requires much less calculating time than simulating the wireless network. However, as the network size or arrival traffic rate grow, the accuracy decreases.

2. Related work

The network performance evaluation using queuing methods found a wide application in global literature. Initial research in this area contained simplified assumptions about the nature of information flows in telecommunication networks (L. Kleinrock [2], L. Kleinrock [3], V. Vishnevsky [4,5], etc.). The development of stochastic models has led to widespread use of the phase-type distribution. A tandem queueing system with PH distribution is used in [6-13, et all]. Approximating general distributions by phase-type (PH) distributions is a widespread technique in the stochastic analysis since the Markovian property of PH distributions often allows analytical tractability [14]. Researchers in [1,15-17] carried out the approximation of the time spent in the channel using the PH distribution for IEEE 802.11 wireless networks. Most of the early work used simple simulation models or other mathematical models as a data source, for example, semi-Markov random processes based on the Bianchi model. In contrast to these, in this article, we take the data to construct PH-distributions from the high-precision IEEE 802.11n simulation model in NS-3. Open-Source Project Network Simulator 3 is widely used to simulate wireless networks in many papers [18-23].

The problem of PH-distributions fitting is also presented in many papers [24-31, et all]. In this work, we will use a simple algorithm proposed by Johnson [24] to fit the PH distribution by the first three moments, but instead, you can use more complex techniques, for example, the EM procedure. The methodology of developing a wireless network model proposed in our article does not change from this.

3. Tandem queueing system

Let us consider a multi-hop wireless network with linear topology. Such a network consists of sequentially connected stations, each of which is connected only to its immediate neighbors. We will consider the case when antennas at the stations are omnidirectional, all stations use the same channel and packets enter the network only at the first station, pass all intermediate stations and leave the network at the last one. For the sake of simplicity, we will assume that the stations operate using the IEEE 802.11 DCF protocol without using RTS/CTS. Note that similar transmission protocols are used, for example, in IEEE 802.15.4 (ZigBee) sensor networks.
When modeling a wireless network using the queuing system M/PH/1/N → •/PH/1/N →...→ •/PH/1/N (see Fig. 1), we will model the transmission time in channels using PH-distributions, and buffers at stations, in which packets are awaiting transmission, by queues of finite capacity N. To simplify the model, we will assume that packets have a fixed size B bits, and the intervals between their arrivals have a Poisson distribution with intensity λ. Note that the distribution of packet sizes (in our case, a constant) affects only the service time and is taken into account when constructing PH-distributions.

The PH distribution is a generalization of the exponential distribution and can be used to approximate any positive continuous distribution [24]. Random variable is phase-type distributed if its value can be expressed as the time till absorption in a continuous absorbing Markov chain with generator \( \tilde{S} \). PH-distribution is defined with a subgenerator matrix \( S \in \mathbb{R}^{V \times V} \) and initial probability row vector \( \tau \in \mathbb{R}^V \), such that:

\[
\tilde{S} = \begin{bmatrix} S & -S \mathbf{1} \\ 0 & 0 \end{bmatrix}, \forall i = 1, V: 0 \leq \tau_i \leq 1, \sum_{i=1}^{V} \tau_i = 1.
\]

Since the stations on the network use the same channel, collisions can occur between them. For example, consider the three-hop network shown in Fig. 1. Simultaneous transmissions from station 1 to station 2 and from station 3 to station 4 will lead to a collision at station 2. According to CSMA/CA, on which IEEE 802.11 protocol is built, station 1 will have to retry transmitting its packet at a random time. Notice that the transmission from station 3 may still be successful, since the transmission from station 1 doesn’t interfere with it at station 4. This leads to an increase in losses, especially at stations that are located closer to the data source, and increase in the duration of the transmission of each packet, which will not necessarily be the same at all stations.

In order to take into account this feature of the network operation, we will model service time in each channel with its own PH distribution. To construct these distributions, we will simulate the network shown in Fig. 1 using the NS-3 simulation system. For each channel, we collect samples of transmission durations (from the beginning of the first attempt to transmit a packet until the receipt of an ACK delivery confirmation) for each of the channels t12, t23, and t34. Based on these samples, we calculate the first three moments and, using the method described in Johnson [24], find the PH-distributions for each of the channels. This calibration method is shown in Fig. 2.
PH-distribution fitting and end-to-end delays estimations of the tandem queueing networks were implemented in the tool “pyqumo” implemented in Python 3 language. This is an open-source project available at https://github.com/ipu69/pyqumo.

Before proceeding to the numerical results and analysis of the accuracy of estimates obtained using the queueing network, we will briefly consider the main elements and features of the NS-3 simulation model, which was used in the experiment.

4. Wireless network simulation in NS-3

In this paper we implemented a wireless network simulation model using Network Simulator (NS-3). The model has three parameters: number of stations, simulation time and arrival traffic rate. Settings of the IEEE 802.11n protocol don’t change between experiments, their values are given in the numerical experiments chapter. We used IEEE 802.11n models from NS-3, slightly extended for precise transmission time estimation and random traffic generation.

To simulate network traffic we implemented a traffic generator with exponential distribution of intervals between packets. In terms of NS-3 traffic generator is just an application for scheduling events. Parameters of this application are the mean interval between the packets, the destination station address and the payload size.

Service duration is the time from the moment when the sending station senses the channel and starts DIFS waiting in the first transmission attempt, and till it finishes the reception of ACK frame from the receiving station. Any busy channel waiting intervals, packet transmission attempts and collisions are included in the service duration. If the number of retransmission attempts exceeds the given limit, the packet is considered lost, its service duration is not added to the final statistics. To collect service durations we used the NS-3 events mechanism. To record the timestamp of the first channel sensing we had to slightly modify the model implementation, because the current version doesn’t provide such means. As for ACK delivery time, we used standard event of WiFi MAC level

Model architecture is shown in Fig. 3. Statistic Collector was used to collect service durations, it measured intervals between channel sensing and ACK delivery. To measure mean end-to-end delays we used a standard NS-3 class FlowMonitor. It collects network statistics and determines some network parameters like end-to-end delays, number of lost packets, number of transmitted and received packets, etc.

5. Numerical experiments

In the numerical experiment, simulated stations operated using IEEE 802.11n protocol with bit rate equal to 54 Mbp. Stations were equipped with buffers for 500 packets. Each packet carried 1400 bytes of payload, and in each simulation the traffic generator produced 500’000 packets.

We considered four distinct arrival packet rates \( \lambda = 100, 200, 500 \) and 1000, that corresponded to traffic bitrate of 1.2 Mbp, 2.4 Mbp, 6 Mbp and 12 Mbp, respectively. For each arrival rate we collected service time samples in each channel and computed mean end-to-end delays. Each set of service

![Figure 3. The implementation of the NS-3 tandem model with statistics modules.](image-url)
durations contained about 500,000 samples excluding the packets lost during the transmission. After running the simulation we calculated three moments of service time durations for PH-fitting. Besides non-trivial PH-distributions, we also considered service time approximation with exponential distribution with the mean value equal to the mean service time in the channel, and compared end-to-end estimation accuracy of M/PH/1/N → •/PH/1/N →…→ •/PH/1/N and M/M/1/N → •/M/1/N →…→ •/M/1/N queueing networks.

5.1. End-to-end delays in calibration tandem with four stations
First of all, we studied end-to-end delays in the networks with four stations and three channels like the one shown in Fig. 1. We call this a calibration tandem, since later we use distributions, fitted from samples measured in this model, for delays estimation in networks of arbitrary length. After NS-3 simulation and service time samples collection, we fitted PH and exponential distributions of service time and measured end-to-end delays using M/PH/1/N →•/PH/1/N →…→ •/PH/1/N and M/M/1/N → •/M/1/N →…→ •/M/1/N queueing networks. For each arrival traffic rate $\lambda=100$ (1.2Mbps), $\lambda=200$ (2.4Mbps), $\lambda=500$ (6Mbps) and $\lambda=1000$ (12Mbps) we ran network simulation, service time distribution fitting and end-to-end delays estimations 10 times.

![Normalized moments of packet transmission time in channel](image)

**Figure 4.** Coefficient of variation (CV) and skewness for each channel.

To prove that service time is better fitted with PH than with trivial exponential distribution, Fig. 4 shows the coefficients of variation and coefficients of skewness for each channel in the calibration tandem. It can be seen that most service time distributions have coefficients of variation much higher than 1, especially in cases of low arrival rates. Thus modeling service time with exponential distribution leads to error in variation, which may affect the end-to-end delays estimation precision. However, when the arrival traffic rate is high, the coefficient of variation is closer to 1, and the PH-distribution may be replaced with an exponential distribution with higher precision.

Numerical results of end-to-end delays estimations using NS-3 simulation and queueing models for the network with four nodes are shown in Fig. 5 using box plots. As it can be seen, for low intensity traffic the M/PH/1/N model provides higher accuracy than the M/M/1/N model. For high intensity traffic the M/M/1/N model suddenly provides a slightly better approximation than the M/PH/1/N model. This fact correlates with the observation that the coefficient of variation decreases when the arrival rate grows.
Figure 5. Boxplots for different arrival rates for NS-3 simulation and queueing models for various arrival rates.

5.2. Comparison of end-to-end delay for networks of arbitrary length.
To study the accuracy of $M/PH/1/N \rightarrow •/PH/1/N \rightarrow \ldots \rightarrow •/PH/1/N$ and $M/M/1/N \rightarrow •/M/1/N \rightarrow \ldots \rightarrow •/M/1/N$ models, we investigated the dependency of end-to-end delay on the number of stations in the tandem. Like before, we considered several packet arrival rates using both queueing networks and the NS-3 simulation model. Our key assumption was that all channels can be modeled by distributions from the calibration experiment for four stations. The first and the last channels were modeled using distributions for the first and last channels of the calibration model (1-2 and 3-4, respectively, see Fig. 1), while all the intermediate channels were modeled with the same distribution for the channel 2-3 in the calibration model. This approach allows us to model a tandem network of arbitrary length.

Figure 6. End-to-end delays in networks with different numbers of stations.
Plots in Fig. 6 show the dependency of end-to-end delays on the number of stations for NS-3 simulation and both queueing models. The $M/PH/1/N$ model provides accuracy about 75% in most cases when the arrival traffic rate is low, while the $M/M/1/N$ model has much higher relative error, up to 50%, for large networks. We should note that the accuracy difference between the models for short networks with up to 4 stations isn’t high, and $M/M/1/N$ is fairly suitable for these cases. However, as long as the arrival traffic bandwidth grows, accuracy of both models decrease dramatically.
One of the possible reasons for accuracy degradation under the heavy traffic, is that channel 2-3 is not suitable for modeling intermediate channels in large networks. To study this effect, Fig. 7 shows the mean service times in each channel in the network with 10 stations measured in the NS-3 simulation model. The plots show that for small traffic rates most intermediate channels have very close properties. However, for larger rates the service time mean values in the intermediate channels differ: service time increases till the fifth station, and then it starts decreasing. This can be explained by the fact that most of the collisions and losses occur at the beginning of the transmission, and as the packet moves through the network, the traffic is filtered and the service becomes more deterministic.

6. Conclusion
In this paper we described an approach to multi-hop wireless networks modeling using M/PH/1/N → •/PH/1/N → … → •/PH/1/N tandem queueing systems. We showed that calibrating PH-distributed service times using a simulation network with three hops allows us to estimate end-to-end delays with fairly good precision in large networks with up to ten stations. However, the precision depends on the arrival traffic rate, it degrades as long as the rate grows. For short networks even simple M/M/1/N models provide fairly good accuracy, but using exponential service time approximation leads to errors as long as the network becomes larger. Big networks simulation using NS-3 takes much longer time than estimation of end-to-end delays using queueing networks, so the presented approach can be used when the delays are needed as a property in some larger problem like network topology optimization.

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**Acknowledgments**

The reported study was funded by RFBR, project number 19-29-06043.