Bandgap Narrowing in Quantum Wires

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Abstract

In this paper we consider two different geometry of quasi one-dimensional semiconductors and calculate their exchange-correlation induced bandgap renormalization (BGR) as a function of the electron-hole plasma density and quantum wire width. Based on different fabrication scheme, we define suitable external confinement potential and then leading-order GW dynamical screening approximation is used in the calculation by treating electron-electron Coulomb interaction and electron-optical phonon interaction. Using a numerical scheme, screened Coulomb potential, probability of different states, profile of charge density and the values of the renormalized gap energy are calculated and the effects of variation of confinement potential width and temperature are studied.

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1 Introduction

A highly dense electron-hole plasma can be generated in a wide variety of semiconductors by optical pumping. The band structure and the optical properties of highly excited semiconductors differ from those calculated for non-interacting electron-hole pairs due to many-body exchange-correlation effects arising from the electron-hole plasma \([1-3]\). One of the important many-body effect in high density electron-hole plasma is a density-dependent renormalization of the fundamental band gap of the semiconductor, which causes an increasing absorption in the spectral region below the lowest exciton resonance. The exchange-correlation correction of the fundamental band gap due to the presence of free carriers (electrons in the conduction band and holes in the valence band) in the system is referred to as the band gap renormalization (BGR) effect. Optical nonlinearities, which are strongly influenced by Coulomb interaction in the electron-hole plasma, are typically associated with the bandgap renormalization phenomenon.

The band gap re-normalization has been widely studied in bulk and quasi-two dimensional (quantum well) semiconductors \([4-6]\). In recent years, quasi-one dimensional semiconductor quantum wires (QW) have been fabricated in variety of geometric shape with atomic scale definition, and QW optical properties have been studied for their potential device applications such as semiconductor lasers\([7-9]\). There has, however, been little work on the BGR in QW systems, both experimentally and theoretically. Recently different geometries of quantum wires, such as rectangular, V-shaped and T-shaped quantum wires have been fabricated and studied, and various experimental technics for fabrication and growth of these structures have been developed\([2,3]\). Square quantum well wires have been studied by Hu and Das Sarma\([10]\). They have calculated the value of the band gap re-normalization for this case in GW approximation. Band gap renormalization in photoexcited semiconductor quantum wire in GW approximation has been studied by Hwang and Das Sarma\([11]\). Rinaldi and Cingolani have studied optical properties of 1D quantum structures specially the case of V-shaped quantum wire\([12]\). They have considered confinement potential of the form \(V(y) = -\frac{V_0}{\cosh^2(\alpha y)}\) but they have not calculated the band gap re-normalization with this confinement potential theoretically. Bener and Haug have considered plasma-density dependence of the optical spectra for quasi-one-dimensional quantum well wires\([13]\). Tanatar has studied band gap re-normalization in quasi-one dimensional systems in simple plasmon-pole (quasi-static) approximation\([14]\). On the other hand, T-shaped quantum wires recently have been considered by some authors. For ex-
ample Sedlmaier and his coworkers have studied band gap re-normalization of modulation doped T-shaped quantum wires. They have presented self-consistent electronic structure calculations for this device. These calculations show a band gap re-normalization which, when corrected for excitonic energy and its screening, are largely insensitive to the 1D electronic density[15]. Stopa, using density functional theory, has calculated the electronic structure of a modulation doped and gated T-shaped quantum wire. He also has calculated the band gap re-normalization as a function of the density of conduction band electrons[16]. Lin, Chen and Chuu have found the dependence of the bound states of L-shaped and T-shaped quantum wires to some asymmetric parameter in an inhomogeneous magnetic fields[17]. Sedlmaier and his coworkers have calculated the band-gap re-normalization of modulation doped quantum wires by considering the photoluminescence spectra as a function of the one dimensional density specially in T-shaped quantum wires[18]. Nozari and Madadi, recently have proposed a theoretical framework for calculation of band gap re-normalization in V-shaped quantum wires. They have calculated numerically the value of this re-normalization in various temperatures and carrier density and have found exact solutions for some geometry of this type of quantum wires in a simplified random phase approximation[19]. So far most of calculations have been done in the static screening approximation or in the simple plasmon-pole approximation, which is a simplified version of the random-phase approximation (RPA). The plasmon-pole approximation consists of ignoring the weight in the single particle excitations and assuming that all free carrier contributions to the dynamical dielectric function lies at the effective plasma frequency. The advantages of the plasmon-pole approximation are its mathematical simplicity and simple physical meaning. However, a certain degree of arbitrariness in the choice of the effective plasmon pole parameters leads to considerable difficulties in applying the theory to semiconductors with complex band structures. In this paper, we calculate the BGR of the V-shaped and T-shaped quantum wire structures based on the RPA dynamical screening (GW) scheme by taking into account special mathematical definitions for confinement potentials which differ considerably from existing literature specially [12] and [19]. In doing so, the full frequency dependent dielectric response in the two component one-dimensional electron-hole plasma is considered.

The structure of the paper is as follows: In section 2 we provide formal theory of bandgap renormalization in a general quasi one-dimensional structure using dynamical random phase approximation. Section 3 considers V-shaped confinement potential. First a suitable mathematical definition for this form of confinement potential is given. Then a
numerical scheme for solutions of exact equations is provided and various quantities are calculated numerically. Section 4 considers the same analysis for T-shaped confinement potential. Conclusions and discussion are given in section 5.

2 BGR in Dynamical Random Phase Approximation

The exchange-correlation induced correction of the fundamental band gap due to the presence of free carriers (electrons in the conduction band and holes in the valence band) in the system is referred to as the band gap re-normalization effect. The BGR is given by the sum of the self-energies for electrons and holes at band edges

\[ \Delta = \text{Re} \left( \Sigma_e(0,0) \right) + \text{Re} \left( \Sigma_h(0,0) \right), \]  

(1)

where \( \text{Re} \) stands for real part. The total electronic self-energy within the leading order effective dynamical interaction (GW) in a two-component electron-hole plasma is

\[ \Sigma_e(k, \omega) = i \int \frac{dq}{2\pi} \int \frac{d\omega'}{2\pi} G_0(k - q, \omega - \omega') \frac{V_s(q, \omega')}{\epsilon(q, \omega')}, \]  

(2)

where \( G_0 \) is the Green’s function for noninteracting electron gas, \( V_s = V_c + V_{ph} \) the total effective interaction and \( \epsilon(q, \omega) = 1 - V_s(q, \omega)\Pi_0(q, \omega) \) the effective dynamical dielectric function. \( V_c, V_{ph} \) and \( \Pi_0(q, \omega) \) are direct Coulomb interaction, the longitudinal optical phonon mediated electron-electron interaction, and irreducible polarizability, respectively. The self-energy can be separated into the frequency-independent exchange and correlation parts

\[ \Sigma_i(k, \omega) = \Sigma_i^{\text{Ex}}(k) + \Sigma_i^{\text{Cor}}(k, \omega), \quad i = e, h \]  

(3)

where,

\[ \Sigma_i^{\text{Ex}}(k) = - \int_{-\infty}^{\infty} \frac{dq}{2\pi} n_F(k + q)V_s(q). \]  

(4)

Here, \( n_F(k + q) = \theta(k_F - |k + q|) \) is the Fermi function at \( T = 0 \) and \( \theta \) is Heaviside step function. In GW approximation, the \( \Sigma_i^{\text{Cor}}(k, \omega) \) can be written in the following form,

\[ \Sigma_i^{\text{Cor}}(k, \omega) = \Sigma_i^{\text{line}}(k, \omega) + \Sigma_i^{\text{pole}}(k, \omega), \]  

(5)

where

\[ \Sigma_i^{\text{line}}(k, \omega) = - \int_{-\infty}^{\infty} \frac{dq}{2\pi} V_s(q) \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left( \frac{1}{\xi_{k+q} - \omega - i\omega'} \right) \left[ \frac{1}{\epsilon(q, i\omega')} - 1 \right], \]  

(6)
and
\[ \Sigma_{i}^{\text{pole}}(k, \omega) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left[ \theta(\omega - \xi_{k+q}) - \theta(-\xi_{k+q}) \right] V_s(q) \left[ \frac{1}{\epsilon(q, \xi_{k+q} - \omega)} - 1 \right] \] (7)

where \( \xi(k) \) is defined as
\[ \xi(k) = \frac{\hbar^2 k^2}{2m^*} - \mu, \] (8)

and \( m^* \) is effective electron mass[11].

In general the effective quasi-one dimensional Coulomb interaction in the framework of dynamical random phase approximation is given by
\[ V_s(k) = \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dy' \int_{-\infty}^{+\infty} dx e^{ikx} \frac{e^2}{\epsilon_0 [(x-x')^2 + (y-y')^2]^{3/2}} |\phi(y)|^2 |\phi(y')|^2 \]
\[ = \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dy' \nu(k, y - y') |\phi(y)|^2 |\phi(y')|^2. \] (9)

In this equation \( \phi(y) \) is transverse wave function and \( \nu(k, y - y') = \frac{2\epsilon^2}{\epsilon_0} K_0(||k(y - y')||) \) where \( K_0(x) \) is zeroth order modified Bessel function of second kind. As we will show, in our situation the problem is effectively two dimensional(quantum well wire structure).

### 3 BGR in V-Shaped Quantum Wires

Using formalism of the last section, here we calculate density of states(profile of charges distribution) and band gap re-normalization in the geometry of V-grooved quantum quantum wire. There are many possibilities to define mathematically the geometry of V-grooved confinement potential[3,12]. These possibilities have originated from different fabrication and growing scheme. A possible and in some sense, technically suitable form of V-shaped confinement potential can be written as,

\[ V(x, y) = \begin{cases} 
0 & \text{if } \begin{cases} 
-w_x \leq x \leq w_x \\
\alpha x \leq y \leq \alpha x + w_y \\
-\alpha x \leq y \leq -\alpha x + w_y 
\end{cases} \\
\infty & \text{elsewhere}
\end{cases}, \] (10)
For simplicity in numerical calculations, we define re-scaled quantities \( \tilde{x} = \frac{x}{w_x}, \tilde{y} = \frac{y}{w_y} \) and \( \tilde{\alpha} = \frac{w_x w_y}{w_y} \) for \( x, y \) and \( \alpha \) respectively. So the re-scaled confinement potential becomes,

\[
\tilde{V}(x, y) = \begin{cases} 
0 & \text{if } \begin{cases} -1 \leq \tilde{x} \leq 1 \\
\tilde{\alpha} \tilde{x} \leq \tilde{y} \leq \tilde{\alpha} \tilde{x} + 1 \\
-\tilde{\alpha} \tilde{x} \leq \tilde{y} \leq -\tilde{\alpha} \tilde{x} + 1
\end{cases} \\
\infty & \text{elsewhere}
\end{cases}
\]

(11)

With this form of re-scaled confinement potential, one can compute band gap narrowing in V-grooved quantum wire. To do this end, one should calculate effective two-dimensional transverse wave function. According to above geometry, \( z \)-direction is without confinement, so carriers move in this direction freely. Now the Schrödinger equation for carrier in two direction of confinement is,

\[
(-\frac{\hbar^2}{2m})\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)\phi(x, y) + V(x, y)\phi(x, y) = E\phi(x, y).
\]

(12)

This can be written, using re-scaled quantities, as

\[
(-\frac{\hbar^2}{2m})\left( \frac{1}{w_x^2} \frac{\partial^2}{\partial \tilde{x}^2} + \frac{1}{w_y^2} \frac{\partial^2}{\partial \tilde{y}^2} \right)\phi(x, y) + V(x, y)\phi(x, y) = E\phi(x, y).
\]

(13)

Defining, \( \tilde{E} = \frac{2mEw_y^2}{\hbar^2}, \tilde{V} = \frac{2mVw_y^2}{\hbar^2} \), and \( c = \frac{w_y}{w_x} \), we find finally

\[
(c^2 \frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2})\phi(x, y) - \frac{2mV(x, y)w_y^2}{\hbar^2}\phi(x, y) = \frac{-2mEw_y^2}{\hbar^2}\phi(x, y).
\]

(14)

This equation with potential as (10), can be solved analytically. The screened coulomb potential now is given by,

\[
V_s(k) = \frac{2e^2}{\varepsilon_0} \int dx dy \int dx' dy' K_0|k\Delta r||\varphi(x, y)||\varphi(x', y')|^2,
\]

(15)

or

\[
V_s(k) = \lambda \frac{2e^2}{\varepsilon_0} w_x^2 w_y^2 \int dx' dy' \int dx'' dy'' K_0|k\Delta r||\varphi(x, y)||\varphi(x', y')|^2,
\]

(16)

where \( \lambda \) is re-scaling factor equal to \( 10^{-18}/m^2 \) and \( k\Delta r = k\sqrt{(x - x_0)^2 + (y - y_0)^2} \). Since we want to solve the integral of equation (16), and this integral can not be solved analytically, we need to compute it numerically and therefore the ground state energy and ground state wave function of Schrödinger equation should be calculated numerically. Therefore,
we first provide a suitable numerical scheme for calculations. Basically, we have to solve 2D-Schrödinger equation (13) for potential as given in equation (10), which is not, theoretically, solvable yet. Therefore, we are going to calculate the ground state wave function numerically. We use the usual finite difference algorithm to solve this eigenvalue problem[20]. First, we do define the re-scaled values for $x, y, E$ and $V$ respectively as $\frac{x}{w_x}, \frac{y}{w_y}, \frac{2mEw_x^2}{\hbar^2}$ and $\frac{2mVw_y^2}{\hbar^2}$. Then, we do discretize the $x, y$ axes’s to discretized space $dx = 0.1$ and $dy = 0.1$, therefore the equation (13) will now be changed to form of the matrix equation as:

$$H\phi = E\phi,$$

(17)

where $H$, and $\phi$ are the Hamiltonian matrix and state wave function array respectively and are defined as:

$$\phi = \begin{pmatrix}
\vdots \\
\phi(x_i, y_j) \\
\phi(x_{i+1}, y_j) \\
\vdots \\
\phi(x_i, y_{j+1}) \\
\phi(x_{i+1}, y_{j+1}) \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{pmatrix},$$

(18)
respectively. In all computations in this paper, we have assumed that the ratio of screened potential versus the wave vector for different \( \beta \) is normalized to 0. The error of computation of \( \phi(x, y) \) is in the order of \( O(dx^2) \). We diagonalize the matrix and calculate the ground state wave function numerically. The screened potential is calculated by ground state wave function as a function of \( k \). Figure 1 shows the calculated screened potential versus the wave vector for different \( \alpha \). In this figure, \( V_\alpha(k) \) is normalized by \( \frac{2e^2}{\epsilon_0} \) and the \( k \) is normalized to \( kw_y \). Now, we are able to calculate the self energies of electrons and holes. For this goal, first we define the re-scaled \( \beta_e \) and \( \mu_e^0 \) respectively as \( \frac{\beta e^2}{2m_e^* w_y^2} = \beta \frac{574.5}{w_y^2} \) and \( \mu_e^0 \beta_e \), where \( w_y \) is width of quantum well in \( y \) direction in Nanometer. For holes, we also define the re-scaled \( \beta_h \) and \( \mu_h^0 \) as \( \beta_h = \beta_e \frac{m_e^*}{m_h^*} \) and \( \mu_h^0 = \mu_e^0 \frac{m_e^*}{m_h^*} \) respectively. In all computations in this paper, we have assumed that the ratio \( \frac{m_h^*}{m_e^*} \) is equal to 0.3 and \( m_e^* \simeq 0.067m_e \). By calculating the self energies of holes and electrons, we compute the band gap re-normalized energy for V-grooved confinement potential in dynamical random phase(GW) approximation. We consider the case of \( T = 0 \). The result is shown in figure 2 which shows the values of bandgap renormalization in terms of the width of the V-shaped confinement potential. In all figures, we have assumed that chemical potentials for electron is 1meV and \( E_g \) is normalized by \( 2e^2/\epsilon_0 \).
4 BGR in T-Shaped Quantum Wires

Now we want to do the same calculations for geometry of T-shaped quantum wires. It is evident that one can consider the external T-shaped confinement potential as follows,

\[ V(x, y) = \begin{cases} 
0 & -\infty < x < \infty, \quad -\frac{w_x}{2} \leq y \leq \frac{w_y}{2} \\
0 & \frac{-w_x}{2} < y < \infty, \quad -\frac{w_x}{2} \leq x \leq \frac{w_x}{2} \\
\infty & \text{elsewhere.}
\end{cases} \tag{20} \]

Note that we want to simulate actual situation with mathematical functions. The actual problem is a quantum wire which is a quasi-one dimensional structure but its external confinement potential should be effectively two dimensional to provide two direction of confinement. Therefore, this form of confinement potential is considered effectively two dimensional. To proceed the same algorithm as preceding section, we first re-scale some quantities. First of all the confinement potential can be written as,

\[ \tilde{V} = \begin{cases} 
0 & -\infty < x < \infty, \quad -\frac{1}{2} \leq y \leq \frac{1}{2} \\
0 & \frac{-1}{2} < y < \infty, \quad \frac{-1}{2} \leq x \leq \frac{1}{2} \\
\infty & \text{elsewhere.}
\end{cases} \tag{21} \]

The re-scaled Schrödinger equation becomes

\[-\left( \frac{\partial^2}{\partial x^2} + \alpha^2 \frac{\partial^2}{\partial y^2} \right) \phi(x, y) + \frac{2m\tilde{V}(x, y)}{\hbar^2 w_y^2} \phi(x, y) = \frac{2mE}{\hbar^2 w_y^2} \phi(x, y), \tag{22} \]

where now \( \alpha \) is ratio of well width potential in \( x \) direction to the \( y \) direction, i.e. \( \alpha = \frac{w_x}{w_y} \).

The same numerical scheme has been used to calculate various quantities. By diagonalization of the Hamiltonian matrix, we calculate the ground state wave function numerically. Using ground state wave function, one can calculate the concentration of carriers in quantum wire structure and therefore density of probability for these carriers. This procedure is as previous section. One can show that by decreasing \( \alpha \), the wave function of carriers will be localized in \( x \) directions while by increasing \( \alpha \) the wave function will be more localized in \( y \) directions of the well relative to \( x \) direction. From physical grounds, this is reasonable to say that probability density and charge distribution are function of external confinement potential.

Now we should calculate the screened Coulomb potential. In our situation the screened Coulomb potential should be considered more carefully, since now the confinement potential of quantum well wire is effectively two dimensional. It is important to note that
this means that quantum mechanical problem under consideration is effectively two-dimensional. For these reasons, the screened Coulomb potential can be written as

\[
V_s(k) = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \nu(k, \rho - \rho')|\varphi(x, y)|^2|\varphi(x', y')|^2
\]

(23)

where \( \rho = |\hat{x}i + \hat{y}j| \) is the radius polar coordinate in plane and \( \varphi(x, y) \) is transverse wave function and \( \nu(k, \rho - \rho') = \frac{2e^2}{\varepsilon_0} K_0[|k(\rho - \rho')|] \) where \( K_0(x) \) is zeroth order modified Bessel function of second kind. With this point in mind the screened Coulomb potential (16) is calculated by two dimensional ground state wave function. We have calculated the screened Coulomb potential, as a function of \( k \) in figure (3). In this figure, \( V_s(k) \) is normalized by \( \frac{2e^2}{\varepsilon_0} \) and the \( k \) is normalized to \( 1/W_y \) and the figure shows that screened potential is going to zero at lower wave length, and it is infinite at higher wave length for different \( \alpha \) ratio. The screened potential for a constant wave vector of carriers will increase by increasing \( \alpha \). This indicates that screening effects for carriers are affected by the ratio of width of well potentials.

Now to calculate the self energies of electrons and holes first we define the re-scaled \( \beta_e \) and \( \mu_e^0 \) respectively as before. By calculating the self energies of holes and electrons, we compute the band gap energy renormalization for different temperatures and width of the quantum wells for T-shaped potential. Figure (4) shows the relative difference of band gap energy renormalization for T-shaped potential versus the width of quantum well in \( y \) directions. Increasing temperature leads to more relative renormalization of gap energy and this is in agreement with the experimental results[1,13]. Also it is evident that in our situation the relative width of the well, i.e. \( \alpha \) has significant effect on the value of band gap renormalization. Also our analysis shows a temperature dependence of gap energy renormalization. Generally, under high optical excitation, the band gap for 2D and bulk systems is found to decrease with plasma density due to exchange-correlation effects. The observed band gaps are typically normalized by \( \sim 20meV \) within the range of plasma densities of interest which arise from the conduction band electrons and valence band holes. Since Coulomb screening decreases with reduction of semiconductor dimensions, typical values of band gap renormalization in the case of bulk semiconductor is more than gap renormalization in quantum well structures. In the same manner typical band gap renormalization for quantum wires is less than that of quantum wells. Our numerical results, when are compared with experimental data[13], show the good agreement with experiments at least in general behavior.
5 Results and Discussions

In this paper, a numerical approach has been proposed to calculate band gap renormalization in a V-shaped and T-shaped quantum wires. Quantum field theoretical random phase approximation within the leading-order screening has been used to calculate screened Coulomb potential in terms of carrier density and width of the confinement potentials. The essential question is about the form of confinement potentials. As we have argued the quantum mechanical problem under consideration is effectively two dimensional. In other words since we have two dimension of confinement, simulation of external confinement with mathematical functions, leads to an effectively two dimensional confinement potential which requires solution of two dimensional Schrödinger equation. Based on this viewpoint we have shown that using two dimensional Cartesian and polar coordinates for V-shaped and T-shaped quantum wire respectively, one can use the formalism of Hu and Das Sarma with re-definition of some quantities. Our numerical calculations show that the distribution of probability for carriers concentration is a function of the ratio of the well width in two dimensions. This is natural since probability distributions is a function of confinement potential. In other words, the geometric shape of external confinement itself restricts the shape of wave functions. As the figures show, by decreasing $\alpha$, the wave function of carriers will be localized in $x$ directions while by increasing $\alpha$ the wave function will be more localized in $y$ directions of the well relative to $x$ direction. It is important to note that by re-scaling procedure which we have considered, we have fixed the geometry of the wires but now Schrödinger equation becomes re-scaled via the presence of ratio $\alpha$. Actually one should consider the possibility for changing the geometry also. But our investigation show that the main physical results do not change considerably. Calculation of screened Coulomb interaction shows that the screened potential is going to zero at lower wave length, and it is infinite at higher wave length for different $\alpha$ ratio. The screened potential for a constant wave vector of carriers will increase by increasing $\alpha$. This indicates that screening effects for carriers are affected by the ratio of width of well potentials. Also calculation of band gap renormalization shows that increasing temperature leads to more relative renormalization of gap energy and this is in agreement with the experimental results. Also it is evident that in our situation the relative width of the well, i.e. $\alpha$ has significant effect on the value of band gap renormalization. This is not surprising since the screening effect itself is dependent on the shape of confinement potential. Since Coulomb screening decreases with reduction of semiconductor dimensions, typical values of band
gap renormalization in the case of bulk semiconductor is more than quantum well structures. In the same manner typical band gap renormalization for quantum wires is less than that of quantum wells. The relative values of band gap renormalization in various fabrication pattern (i.e. V-shaped, T-shaped and rectangular) quantum wire structure is dependent to the shape and depth of the potential well. Our numerical results, when are compared with experimental data[13], show the good agreement with experiments at least in general behavior.

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Figure 1: The calculated screened potential versus the wave vector for different $\alpha$, where we have set $\alpha = 0.5$ for lower curve and $\alpha = 1.0$ for upper one. The screened potential is normalized by $\frac{2e^2}{\epsilon_0}$ and the $k$ is normalized to $kw_y$. 
Figure 2: Renormalization of gap energy calculated numerically for V-shaped potential versus the width of quantum well wire $w_y$. In all figures, we have assumed that chemical potentials for electron is $1\text{meV}$. The $\Delta E_g$ is normalized by $2e^2/\epsilon_0$. 
Figure 3: The calculated screened Coulomb potential versus the wave vector. The screened potential is normalized by $\frac{2e^2}{\epsilon_0}$ and the $k$ is normalized to $1/w_y$. 
Figure 4: This figure shows the difference of band gap renormalized energy for T-shaped confinement potential versus the width of quantum well in $y$ directions for different temperatures and different ratio of the well width in two dimensions, $\alpha$. In all figures, we have assumed that the chemical potentials for electron is 1$meV$. The $\Delta E_g$ is normalized by $2e^2/\epsilon_0$. 

$\alpha = 1$

$\Delta E_g(W_y, T) - \Delta E_g(1, T) / \Delta E_g(1, T)$

$W_y$

$\times 10^{-30}$

$4.5$

$4$

$3.5$

$3$

$2.5$

$2$

$1.5$

$1$

$0.5$

$0$

$0$

$2$

$4$

$6$

$8$

$10$

$12$

$14$

$16$

$18$

$20$

$T=4K$

$T=10K$

$T=15K$

$T=20K$