Summary: The aim of this paper is to introduce and to investigate the basic properties of $q$ convex, $q$-affine and $q$-concave sequences and to establish their surprising connection to Chebyshev polynomials of the first and of the second kind. One of the main results shows that $q$ concave sequences are the pointwise minima of $q$-affine sequences. As an application, we consider a nonlinear selfmap of then-dimensional space and prove that it has a unique fixed point. For the proof of this result, we introduce a new norm on the space in terms of a $q$-concave sequence and show that the nonlinear operator becomes a contraction with respect to this norm, and hence, the Banach Fixed Point theorem can be applied.

MSC:
26A51 Convexity of real functions in one variable, generalizations
39B62 Functional inequalities, including subadditivity, convexity, etc.

Keywords:
$q$-convex sequence; $q$-concave sequence; $q$-affine sequence; Chebyshev polynomials of first and second kind; contraction

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References:
[1] G. H. HARDY, J. E. LITTLEWOOD, AND G. PÓLYA, Inequalities, Cambridge University Press, Cambridge, 1934, (first edition), 1952 (second edition).
[2] X. Z. KRASNIQI, On $\alpha$-convex sequences of higher order, J. Numer. Anal. Approx. Theory 45 (2016), no. 2, 177-182. - Zbl 1399.26021
[3] M. KUCZMA, An Introduction to the Theory of Functional Equations and Inequalities, Prace Naukowe Uniwersytetu Śląskiego, vol. 489, Państwowe Wydawnictwo Naukowe - Uniwersytet, 1970. In cooperation with P. M. Vasić.
[4] D. S. MITRINOVIC’, Analytic inequalities, Die Grundlehren der mathematischen Wissenschaften, Band 165, Springer-Verlag, New York-Berlin, 1970. In cooperation with P. M. Vasić.
[5] D. S. MITRINOVIC’, J. E. PECAHI ‘C’, ANDA. M. FINK, Inequalities Involving Functions and Their Integrals and Derivatives, Mathematics and its Applications (East European Series), vol. 53, Kluwer Academic Publishers Group, Dordrecht, 1991.
[6] D. S. MITRINOVIC’, J. E. PECAHI ‘C’, ANDA. M. FINK, Classical and New Inequalities in Analysis, Mathematics and its Applications (East European Series), vol. 61, Kluwer Academic Publishers Group, Dordrecht, 1993.
[7] C. P. NICULESCU AND E. PERRISON, Convex Functions and Their Applications, CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, 23, Springer-Verlag, New York, 2006. A contemporary approach.
[8] M. NIEZGODA, Remarks on convex functions and separable sequences, II, Discrete Math. 311 (2011), no. 2-3, 178-185. - Zbl 1205.26017
[9] M. NIEZGODA, Inequalities for convex sequences and nondecreasing convex functions, Aequationes Math. 91 (2017), no. 1, 1-20. - Zbl 1364.39022
[10] M. NIEZGODA, Sherman, Hermite-Hadamard and Fejér like inequalities for convex sequences and nondecreasing convex functions, Filomat 31 (2017), no. 8, 2321-2335. - Zbl 1374.26022
[11] T. POPOVICIU, Les fonctions convexes, Hermann et Cie, Paris, 1944.
[12] A. W. ROBERTS ANDD. E. VARBERG, Convex Functions, Pure and Applied Mathematics, vol. 57, Academic Press, New York-London, 1973.
[13] D. F. SOFONEA, T. T, ANDC. ANCA, M. ACU, Convex sequences of higher order, Filomat 32 (2018), no. 13, 4655-4663. - Zbl 1399.26021
[14] JÁ. TÁBOR, J’O. TÁBOR, AND D. M. ZOLDAK, Strongly convex sequences, Inequalities and applications 2010, Internat. Ser. Numer. Math., vol. 161, Birkhäuser, Springer, Basel, 2012, p. 183-188.
[15] SH. WU ANDL. DEBNATH, Inequalities for convex sequences and their applications, Comput. Math. Appl. 54 (2007), no. 4, 525-534. - Zbl 1144.26016
[16] S. YILDIZ, A general matrix application of convex sequences to Fourier series, Filomat 32 (2018), no. 7, 2443-2449. - Zbl 1399.26021

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