Quantum scissors: teleportation of single-mode optical states by means of a nonlocal single photon

S. A. Babichev, J. Ries, A. I. Lvovsky
Fachbereich Physik, Universität Konstanz, D-78457 Konstanz, Germany
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We employ the quantum state of a single photon entangled with the vacuum \( |1\rangle_A |0\rangle_B - |0\rangle_A |1\rangle_B \)
generated by a photon incident upon a symmetric beam splitter, to teleport single-mode quantum states of light by means of the Bennett protocol. Teleportation of coherent states results in truncation of their Fock expansion to the first two terms. We analyze the teleported ensembles by means of homodyne tomography and obtain fidelities of up to 99 per cent for low source state amplitudes. This work is an experimental realization of the quantum scissors device proposed by Pegg, Phillips and Barnett (Phys. Rev. Lett. 81, 1604 (1998))

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a. Introduction

Quantum teleportation (QT) is the transport of an unknown quantum state \( |\phi\rangle \) over arbitrary distances by means of dual classical and Einstein-Podolsky-Rosen (EPR) channels. To perform teleportation, the sender, Alice, and the receiver, Bob, prearrange the sharing of an EPR-correlated pair of particles. Alice makes a joint measurement on her EPR particle and the source state and sends Bob the classical result of this measurement. Knowing this, Bob can convert the state of his EPR particle into an exact copy of the source state. In this way neither Alice nor Bob obtain any information of his EPR particle into an exact copy of the source state.

This is known as the teleportation phenomenon first described theoretically in 1993 by Bennett, et al. After its proposal in 1993 by Bennett et al. \cite{Bennett}, QT has been implemented experimentally on discrete- \cite{discrete} and continuous-variable optical states \cite{continuous} as well as on molecular spins \cite{molecular}. All these schemes used an EPR pair maximally entangled in a Hilbert space which is isomorphic to the Hilbert space of the source state.

This identity allows, in principle, exact replication of the source state by Bob.

An interesting extension of the Bennett protocol arises, however, if the source state lives in the Hilbert space of higher dimension than the EPR pair. In this case all terms of the source state associated with the dimensions beyond that of the EPR pair will be “cut off” from the teleported state. This is known as the quantum scissors (QS) effect first described theoretically in 1998 by Pegg, Phillips, and Barnett \cite{Pegg} and implemented experimentally in the present work.

In the heart of our teleportation experiment there is an EPR pair implemented by a nonlocal single photon state \( |\Psi^-\rangle = \frac{1}{\sqrt{2}} (|1\rangle |0\rangle - |0\rangle |1\rangle) \) which is generated when a single photon \( |1\rangle \), incident upon a symmetric beam splitter, entangles itself with the vacuum \( |0\rangle \). It is remarkable that our EPR ensemble is formed by just one particle; yet this is a maximally entangled state in the two-dimensional Hilbert space defined by basis vectors \( |0\rangle \) and \( |1\rangle \).

We apply the state \( |\Psi^-\rangle \) to teleport arbitrary single-mode quantum states of the electromagnetic field that belong to the Hilbert space of infinite dimension. If a random quantum state is given by \( |\phi\rangle = \sum a_n |n\rangle \) in the number (Fock) state basis, the scissors effect will truncate the above series, leading to the output of a form \( |\phi_{out}\rangle = a_0 |0\rangle + a_1 |1\rangle \). In simpler words, the higher number terms cannot reach Bob because there is never more than one photon in the original EPR state \( |\Psi^-\rangle \).

In our actual experiment, the role of the source ensemble was played by a coherent state \( |\alpha\rangle \). Since this state has an infinite number of terms in its Fock expansion, it is well suitable for demonstrating quantum scissors; on the other hand it is readily available from the source laser. Alice’s Bell-state measurement is performed by overlapping the source state and her share of the EPR state on a beam splitter and measuring the number of photons in each output (Fig. 1). Events in which the detector \( D_1 \) registers exactly one photon while \( D_2 \) registers zero photons correspond to the two-mode state \( |\Psi^-\rangle_{12} \) entering Alice’s apparatus in modes 1 and 2. If this is the case, Bob’s share of the EPR state (mode 3) is in the state \( |\phi_{out}\rangle \) so no additional manipulations are required from Bob to complete the QT protocol. Restricting to these events, we perform a homodyne measurement on the teleported ensembles in order to characterize them and determine the teleportation fidelity.

Full implementation of the scissors protocol requires, in particular, highly efficient single-photon detectors capable of determining the number of incident photons. Al-
though such detectors are currently being developed [8],
they are not widely available. Fortunately, the protocol
exhibits surprisingly good fidelity even with regular,
non-discriminating single-photon detectors as long as the
amplitude of the source state is sufficiently small [10,11].

The efficacy of the protocol demonstrates that single
photons can be used to achieve quantum information proc-
essing in a cost-effective and practical manner. The
present scheme is highly efficient and employs only
passive optics, which makes it a promising candidate
for future quantum communication experiments.

Preparation of the single photon state is imperfect:
dark counts of our trigger detector (D_T) and optical
losses result in a statistical mixture of one photon and
no photon instead of a pure |1⟩ state. The ensemble
entering the first beam splitter is therefore

\[ \hat{\rho}_{11} = \eta_{11} |1⟩⟨1| + (1 - \eta_{11}) |0⟩⟨0|, \]

\( \eta_{11} \) being the preparation efficiency.

The density matrix of the EPR pair used for teleportation
\( \hat{\rho}^{\text{epr}} \) can be found by applying the beam splitter
transformation operator

\[ \hat{B} |m, n⟩ = \sum_{j, k=0}^{m, n} \sqrt{(j+k)!(m+n-j-k)!} \left( \begin{array}{c} m \\ j \end{array} \right) \left( \begin{array}{c} n \\ k \end{array} \right) \times (-1)^{h^2-(n+m)/2} |j + k, m + n - j - k⟩ \]

(2)
to the incident combination of \( \hat{\rho}_{11} \) and the vacuum:

\[ \hat{\rho}^{\text{epr}} = \hat{B} (|0⟩⟨0| \otimes \hat{\rho}_{11}) \hat{B}^\dagger. \]

(3)

The source coherent state and one of the EPR “parti-
cles” enter Alice’s apparatus where they undergo fur-
ther transformation via another beamsplitter. After this
transformation, the density matrix of the 3-mode
ensemble can be written as

\[ \hat{\rho}_{123} = \hat{B}_{12} (|α⟩⟨α| \otimes \hat{ρ}^{\text{epr}}_{23}) \hat{B}_{12}^\dagger, \]

(4)

where the subscripts refer to the optical modes according
to Fig. 1.

The first two modes of \( \hat{\rho}_{123} \) are subjected to measure-
ments via single-photon detectors. A non-discriminat-
ing detector of quantum efficiency \( \eta_{\text{SPD}} \) is described by the
following positive operator-valued measure (POVM):

\[ \hat{\Pi}_{\text{no-click}} = \sum_{n=0}^{∞} (1 - \eta_{\text{SPD}})^n |n⟩⟨n| \]

\[ \hat{\Pi}_{\text{click}} = \hat{1} - \hat{\Pi}_{\text{no-click}}. \]

(5)

This measurement leads to a collapse of \( \hat{\rho}_{123} \) projecting it
in the event of a “click” in detector \( D_1 \) and “no click” in
detector \( D_2 \) upon the following non-normalized ensemble
in Bob’s channel:

\[ \hat{\rho}_{\text{out}} = \text{Tr}_{123} (\hat{\rho}_{123} \hat{\Pi}_{\text{click}} \hat{\Pi}_{\text{no-click}}). \]

(6)

The probability of a teleportation event is given by

\[ p_{\text{tel}} = \text{Tr}(\hat{\rho}_{\text{out}}). \]

Imperfect spatial, spectral or temporal mode matching
between Alice’s share of the nonlocal single photon
and the source state \( |α⟩ \) leads to partial distinguishability
and more classical-like behavior, reducing the teleporta-
tion fidelity. In case of a complete mode mismatch, the

FIG. 1. Conceptual scheme of the experiment. BS, beam
splitters; \( D_i \), single photon detectors.
behavior of the system is fully described by a semiclassical model in which photons act like particles with no wave properties. Each beam splitter distributes the incident photons randomly into the output channels. The correlated photon number distribution in the three modes can be calculated according to the laws of classical statistics. From this distribution we infer the probability \( p_{\text{out}}^{\text{sc}} \) of the positive Bell measurement outcome as well as the conditional probability \( p_{\text{out}}^{\text{sc}} \) that Bob’s mode contains a photon. The ensemble received by Bob can then be expressed as a density matrix

\[
\hat{\rho}_{\text{out}}^{\text{sc}} = \begin{pmatrix}
1 - p_{\text{out}}^{\text{sc}} & 0 \\
0 & p_{\text{out}}^{\text{sc}}
\end{pmatrix}
\].

In the actual case of partial mode matching, the output ensemble is a mixture of those calculated via classical and semiclassical models

\[
\hat{\rho}_{\text{out}}^{\text{Bob}} = M \cdot p_{\text{out}} N[\hat{\rho}_{\text{out}}^{\text{exp}}] + (1 - M) p_{\text{out}}^{\text{sc}} N[\hat{\rho}_{\text{out}}^{\text{sc}}],
\]

where \( M \) is the mode matching factor \([13] \) and \( N[\hat{\rho}] \) denotes normalization.

Imperfections in the homodyne detection of the teleported state such as poor mode matching between the local oscillator and the signal, linear losses, inefficient photodiodes or imperfect balance can all be modeled by a single beam splitter with one empty input in the signal beam with a reflectivity \( \eta_{\text{HD}} \) (generalized Bernoulli transformation) \([13][21]\).

c. Experimental apparatus  The setup for preparing the single-photon Fock state was the same as in our previous experiments \([12][13]\). A 82-MHz repetition rate train of 1.6-ps pulses generated by a Spectra-Physics Ti:Sapphire laser at 790 nm was frequency doubled and directed into a beta-barium borate crystal for down-conversion. The latter occurred in a type-one frequency-degenerate, but spatially non-degenerate configuration. The single-photon detector \( D_{\text{T}} \), placed into the idler channel of the down-converter, detected photon-pair creation events and triggered all further measurements.

Pulses containing conditionally-prepared photons entered the optical arrangement shown in Fig. 1, which had to be maintained interferometrically stable throughout the experimental run. The coherent source state \( |\alpha\rangle \) and the local oscillator for homodyne detection were provided by the master Ti:Sapphire laser. These two modes had to be matched, spatially and temporally, to the respective modes of the EPR pair. To this end, we modeled the single photon by a classical wave as described in \([13][14]\). The mode matching was then optimized by maximizing the visibility of the interference fringes observed in the beam splitter outputs. The visibility value provided a basis for a ballpark estimation of the mode matching factor \( M \).

Further knowledge of the experimental parameters was gained through an auxiliary tomography measurement in which the ensemble arriving to Bob was characterized without conditioning on Alice’s results. This ensemble is a statistical mixture of states \( |0\rangle \) and \( |1\rangle \) with the single-photon fraction equal to \( \eta_{\text{SPD}} \approx 0.5 \). The homodyne measurement of the teleported state was conditioned upon detectors \( D_{\text{T}} \) firing and the detector \( D_{\text{I}} \) not firing. The digital logic employed featured rigorous synchronization control of the photon count events with respect to each other and to the master laser pulses. This helped us reduce the dark count contribution to a negligible level.

The time-domain homodyne detector used for characterizing the teleported state was described in \([22]\). For each value of \( \alpha \) approximately 20000 events were collected. The phase of the local oscillator was varied with a piezoelectric transducer. The acquired data was used to calculate the density matrix \( \hat{\rho}_{\text{out}}^{\text{exp}} \) of the teleported ensemble by means of the quantum state sampling method \([21]\). The teleportation fidelity was then evaluated as

\[
F = \langle \alpha | \hat{\rho}_{\text{out}}^{\text{exp}} | \alpha \rangle.
\]

d. Results and discussion  For conceptual verification of the teleportation protocol we performed a measurement run in which we varied the phase of the source state instead of the local oscillator. From the classical point of view, this action should not affect the optical field observed by Bob and therefore its quadrature statistics should remain constant. Yet we observed the optical phase of the teleported ensemble vary in accordance with that of the source (Fig. 2). This result is readily explained by quantum mechanics: by changing the source state phase Alice changes the conditions of the measurement performed on one of the members of the EPR pair. This has a nonlocal effect on the other member which is observed by Bob in his homodyne measurement.
Fig. 3 shows the teleportation fidelity determined experimentally along with the theoretical fit calculated according to Eqs. (1)–(4). There were three fitting parameters: quantum efficiencies $\eta_{1}$ and $\eta_{\text{HD}}$ of the single photon preparation and the homodyne detection, respectively, and the mode matching factor $M$. By fitting these parameters with fixed $\eta_{\text{HD}} \cdot \eta_{1}$ we found $\eta_{\text{HD}} = 0.54$, $\eta_{1} = 0.9$, $M = 0.56$. Note that the value of $\eta_{\text{HD}}$ includes not only the homodyne detector efficiency per se, but also the mode matching of Bob’s ensemble with the local oscillator.

Along with the data pertinent to the actual experiment, Fig. 3 also shows the behavior of the fidelity factor for the idealized quantum-mechanical model with number discriminating detectors and the semiclassical particle model discussed above. All three models exhibit similar qualitative behavior. If the source state is vacuum, a photon detected by Alice must originate from the EPR pair, so Bob receives no photons. The ensemble arriving at Bob’s station is in the vacuum state, and the teleportation fidelity is perfect. For high $\alpha$, the input state has almost vanishing vacuum and single-photon terms, the only components of the truncated teleported ensemble. The teleported ensemble is then practically orthogonal to the source state, and the fidelity is low.

Our experimentally measured fidelity is always higher than that predicted semiclassically, showing the importance of quantum nonlocal effects. A remarkable feature is that for low values of $\alpha$ the value of $F$ is very high, up to 99 per cent. To our knowledge, this is the highest fidelity ever achieved in experimental QT.

**Conclusion** We reported an experimental realization of quantum scissors, i.e. teleportation of single-mode optical ensembles using the nonlocal single photon state as the EPR pair. The teleported state was examined by homodyne measurement and the fidelity was found to be well above the classical limit. Since we did not postselect the teleportation events according to Bob’s results, this experiment is of *a priori* nature. To our knowledge, this is the first QT experiment in which the Bell measurement was done in a discrete, and the characterization of the teleported state in a continuous basis.

In perspective we plan to improve our teleportation fidelity by using number discriminating photon detectors [9]. Another possibility would be to extend the QS protocol to synthesize arbitrary truncated superpositions of Fock states $a_{0}[0] + \ldots + a_{n}[n]$ [4–13]. The nonlocal single photon $|0,1\rangle + |1,0\rangle$ is worth further investigation from the point of view of quantum nonlocality [24].

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*Email: Alex.Lvovsky@uni-konstanz.de*

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