Unitarity in $WW \rightarrow WW$ elastic scattering without a Higgs boson

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(Dated: January 30, 2013)

The amplitude for elastic scattering of longitudinally polarized massive $W$ bosons diverges at tree level, thus violating unitarity, in the absence of any additional particle. This is considered one of the strongest arguments for the existence of the Higgs particle. We show that a triplet of antisymmetric tensor bosons interacting with the gauge bosons via a $B \wedge F$ coupling can also cancel the divergence. No Higgs-like excitation is needed.

The Higgs boson [1–3] provides a mechanism for spontaneous symmetry breaking as well as a mass for gauge bosons. As the LHC gathers its first fb$^{-1}$ of data, without as yet observing a signal for an electroweak Higgs boson, we ask if it is possible to have a meaningful electroweak theory without a Higgs boson.

Unitarity provides the basis for the strongest argument for the existence of the electroweak Higgs boson – the amplitude for elastic scattering of longitudinally polarized massive $W$ bosons diverges at tree level, thus violating unitarity, if there is no other particle mediating the interaction [4–7]. Thus, the argument goes, quantum mechanics requires the existence of a Higgs boson, or a similar composite particle, and something fundamental would need to be changed in physics if a Higgs boson does not exist.

Let us review this argument briefly. It is sufficient to consider the processes for an SU(2) gauge group, although extending it to the electroweak SU(2)$\times$U(1) gauge group is a straightforward exercise. Let us also assume for convenience that all of the bosons have the same mass $m$. The observed $W$-$Z$ mass difference will not be relevant to our calculations since we are interested only in the high-energy limit $E \gg m$. We will write the gauge bosons as $W^+, W^-$ and $W^3$.

The tree-level scattering processes involving only the $W$ bosons are shown in Fig. 1.

![Diagram](image)

FIG. 1: (a) s-channel, (b) t-channel, (c) direct quartic.

Writing the magnitude of the 3-momentum in the center of mass frame as $P$, and the cosine of the angle between the initial and final $W^+$ as $c$, we find that in the high energy limit $P \gg m$, the Feynman amplitudes
of these diagrams go as

\[ \mathcal{M}_{1s} = \frac{g^2 P^4}{m^4} (1 - c)(3 + c) + \frac{g^2 P^2}{2m^2} (9 + 7c - 4c^2) + O(P^0) \]  
(1)

\[ \mathcal{M}_{1t} = -4\frac{g^2 P^4}{m^4} c - 9\frac{g^2 P^2}{m^2} c + O(P^0) \]  
(2)

\[ \mathcal{M}_{1q} = -\frac{g^2 P^4}{m^4} (3 - 6c - c^2) - 2\frac{g^2 P^2}{m^2} (2 - 3c - c^2) + O(P^0) \]  
(3)

\[ \sum \mathcal{M}_1 = \frac{g^2 P^2}{2m^2} (1 + c) + O(P^0). \]  
(4)

Clearly, if these are the only diagrams for the \( W^+ W^- \) elastic scattering process, the amplitude diverges as \( P \to \infty \) and unitarity is violated. We note that this is also the sum of these amplitudes for the SU(2) \( \times \) U(1) gauge theory, when the mediating gauge boson can be either a Z or a photon.

In the Standard Model of electroweak interactions, the masses of the gauge bosons come from the Higgs mechanism, i.e. from a complex scalar doublet. Three of the four modes are converted into the longitudinal modes of the three gauge bosons, and the fourth remains as the Higgs particle \( H \), coupling to \( W^\pm \) via a term \( m_H W^\mu_\mu W^{\nu -} \). This leads to two additional diagrams with the Higgs particle as the mediator, shown in Fig. 2, corresponding to amplitudes

\[ \mathcal{M}_{2s} = \frac{g^2 P^2}{2m^2} (1 - c) + O(P^0) \]  
(5)

\[ \mathcal{M}_{2t} = -\frac{g^2 P^2}{m^2} + O(P^0), \]  
(6)

so that the total amplitude from the Higgs mediated diagrams cancel the divergence of the previous diagrams, the \( W^+_L W^-_L \) elastic scattering amplitude remains finite, and unitarity is not violated as \( P \to \infty \). A crucial ingredient here was the 3-point coupling between the Higgs and the \( W \) particles. In the absence of a Higgs particle or a Higgs-like excitation with a 3-point interaction with \( W^+ W^- \), it might appear that there is no way of preventing the death of unitarity in this scattering process.

Here we explore another possible interaction which could contribute to this process even in the absence of a Higgs particle, either fundamental or composite. This comes from a coupling between an SU(2) triplet antisymmetric tensor potential \( B \) and the gauge field strength \( F \) via a term \( \frac{m}{4} \text{Tr} B \wedge F = \frac{m}{4} \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu}^a F_{\rho\lambda}^a \). This leads to a 2-point derivative coupling as well as 3-point couplings shown in Fig. 3, with vertex rules

\[ iV_{\mu,\nu,\lambda}^{ab} = -m \epsilon_{\mu\nu\lambda} k^b g^{ab} \]  
(7)

\[ iV_{\mu,\nu,\lambda}^{abc} = -igm f^{bca} \epsilon_{\mu\nu\lambda}, \]  
(8)
FIG. 3: Vertices from the $B \wedge F$ term

where the momenta are all directed inwards, i.e. towards the vertex, and $\delta^{ab} = 2 \text{Tr}(t^a t^b)$. In order to use these vertices in a diagram, we need propagators for the fields, which come from kinetic terms, $-\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu}$ for the $W$ bosons, and $\frac{1}{12} H_{\mu \nu \lambda}^a H^{a \mu \nu \lambda}$ for the $B$ field, where $H_{\mu \nu \lambda}^a = \partial_{[\mu} B_{\nu \lambda]}^a - g f^{bca} A_{\mu}^b B_{\nu \lambda}^c + \text{cyclic permutations}$. In the Feynman-'t Hooft gauge, and ignoring the 2-point vertex for the moment, the propagators are

$$i \Delta_{\mu \nu}^{ab} = \frac{-ig_{\mu \nu} \delta^{ab}}{k^2 + i\varepsilon},$$

$$i \Delta_{\mu \nu, \mu' \nu'}^{ab} = \frac{i g_\mu [\mu' \nu'] [\nu \mu]}{4} \frac{g_\nu [\lambda \rho] [\sigma \tau]}{k^4 + i\varepsilon} + \ldots,$$

where the square brackets indicate antisymmetrization. The effective tree-level propagator for the $W$ boson is the sum over all possible insertions of the $B$-field as in Fig. 4,

$$i D_{\mu \nu} = i \Delta_{\mu \nu} + i \Delta_{\mu \nu} i V_{\sigma \rho, \mu'} i \Delta_{\sigma \rho, \mu' \nu'} i V_{\nu', \mu'} i \Delta_{\nu', \nu} + \ldots = \frac{-ig_{\mu \nu}}{k^2 + i\varepsilon} \left( 1 + \frac{m^2}{k^2} + \frac{m^4}{k^4} + \ldots \right) = \frac{-ig_{\mu \nu}}{k^2 - m^2 + i\varepsilon},$$

which is the propagator of a massive vector boson of mass $m$. We have suppressed the gauge indices here.

FIG. 4: Massive $W$ propagator by summing over $B$ insertions

The particle interpretation of quantum fields come from the quadratic part of the Lagrangian. We have made the vector bosons massive by diagonalization of the quadratic terms, with no leftover field degree. The $B$ field has only one degree of freedom per gauge index, which provides the longitudinal mode of the massive gauge boson. Thus the $B$ triplet acts similarly to the Goldstone modes of the complex Higgs field, but the SU(2) symmetry is unbroken. All the three vector bosons have the same mass, and nothing analogous to the Higgs particle remains.

However, the kinetic term for the $B$ field contains new interactions between the $B$ and the $W$ fields, leading to the vertices in Fig. 5,

$$i V_{\mu, \lambda, \rho, \sigma}^{abc} = gf^{abc} [(p - q)_\mu g_{\lambda [\sigma g_{\rho] \rho]} + p_{[\sigma g_{\rho]} [\lambda g_{\rho] \rho]} - q_{[\lambda g_{\rho]} [\sigma g_{\rho] \rho]}]$$

$$i V_{\mu, \nu, \lambda, \rho, \sigma}^{abcd} = ig^2 [f^{ace} f^{bde} (g_{\mu \nu} g_{\lambda [\sigma g_{\rho] \rho]} + g_{\mu [\sigma g_{\rho]} [\lambda g_{\rho] \rho]} + f^{ace} f^{bde} (g_{\mu \nu} g_{\lambda [\sigma g_{\rho] \rho]} + g_{\mu [\lambda g_{\rho]} [\sigma g_{\rho] \rho]}].$$

Instead of the $H$-mediated diagrams of Fig. 2, we now find several new diagrams at the tree level. We may ignore those which, by power counting, behave as $P^0$ or less. We have grouped the remaining diagrams into Fig. 6, Fig. 7 and Fig. 8, according to the number of internal $B$ propagators. The amplitudes for all of these diagrams go as $P^2$. In Fig. 6 diagrams (a) and (b) appear only once, but diagrams (c) and (d) have twins,
obtained by exchanging the internal $B$ and $W$ lines. Similarly, the $B$ line can be on any of the external legs in each of diagrams (e) and (f), leading to a multiplicity of 4.

We have calculated the amplitudes corresponding to these diagrams using the vertex rules and propagators given above. For the internal $W$ propagators we have used the resummed propagator of Eq. (11). For internal $B$ propagators we can do a similar resummation, leading again to $k^2 - m^2$ in the denominator. For a $B$ propagator on an external leg, we have used the propagator in Eq. (10). The amplitudes for the diagrams in Fig. 6, including their multiplicities, are

$$M_6 + M_7 = -\frac{3g^2 P^2}{2m^2} (1 + c) + O(P^0)$$  \hspace{1cm} \text{(14)}$$

$$2(M_6 + M_7) = \frac{3g^2 P^2}{m^2} (1 + c) + O(P^0)$$  \hspace{1cm} \text{(15)}$$

$$4(M_6 + M_7) = -\frac{2g^2 P^2}{m^2} (1 + c) + O(P^0).$$  \hspace{1cm} \text{(16)}$$

We note that the diagrams in the last row of Fig. 7 have different amplitudes, even though the diagrams themselves appear to be related by exchanges of $B$ and $W$ lines. Including multiplicities, the amplitudes of Fig. 7 are

$$2(M_7 + M_8 + M_9 + M_10) = \frac{2g^2 P^2}{m^2} (1 + c) + O(P^0)$$  \hspace{1cm} \text{(17)}$$

$$4(M_7 + M_8 + M_9 + M_10) = \frac{4g^2 P^2}{m^2} (1 + c) + O(P^0)$$  \hspace{1cm} \text{(18)}$$

$$4(M_7 + M_8 + M_9 + M_10) = -\frac{4g^2 P^2}{m^2} (1 + c) + O(P^0)$$  \hspace{1cm} \text{(19)}$$

$$2(M_7 + M_8 + M_9 + M_10) = \frac{2g^2 P^2}{m^2} (1 + 3c + 2c^2) + O(P^0).$$  \hspace{1cm} \text{(20)}$$

The remaining diagrams which go as $P^2$ are shown in Fig. 8. There are two of each diagram, corresponding to exchanging the $B$ line between the incoming lines and simultaneously between the outgoing lines. The amplitudes for these are

$$2(M_{11} + M_{12} + M_{13} + M_{14}) = -\frac{4g^2 P^2}{m^2} (1 + 2c + c^2) + O(P^0).$$  \hspace{1cm} \text{(21)}$$

Adding the amplitudes of the diagrams in Fig. 6, Fig. 7 and Fig. 8 we get

$$M_6 + M_7 + M_8 = -\frac{g^2 P^2}{2m^2} (1 + c) + O(P^0),$$  \hspace{1cm} \text{(22)}$$

which, when added to the amplitudes of the purely $W$-mediated diagrams of Fig. 1 cancels the $P^2$ divergence exactly. The $W^+_L W^-_L$ elastic scattering amplitude remains finite as $P \to \infty$, and unitarity is not violated. We
FIG. 6: Scattering diagrams with $P^2$ behavior: I

note that there are other diagrams in this model for the $W^+_L W^-_L$ elastic scattering process, but all those are of the order $P^0$, so do not affect our argument.

This demonstration of unitarity should not come as a great surprise. The action can be written [9] in a form invariant under the Becchi-Rouet-Stora-Tyutin (BRST)-symmetry by using an auxiliary field in a Stueckelberg mechanism [10] for the $B$-field, which shows that the theory is unitary. A detailed calculation shows that the auxiliary field does not contribute at tree-level to the process considered here. It can also be shown [11] that perturbative corrections, contribute no new terms to the action, but only to a rescaling, or renormalization, of the couplings and masses.

The problem of unitarity in $W^+_L W^-_L$ elastic scattering does not depend on how $SU(2) \times U(1)$ is broken. But the mass mechanism using the $B$ field forces the vector boson masses to be all the same. So the symmetry will have to be broken in order for the $W$ and the $Z$ to have different masses. We will show elsewhere that one way
of accounting for the $W - Z$ mass difference in this model is by breaking the $SU(2) \times U(1)$ symmetry is broken via a soft term, and that the $W^+_L W^-_L$ elastic scattering still remains finite as $P \to \infty$.

We conclude that the non-observation of the Higgs boson should not signal the breakdown of quantum field theory: we have shown the preservation of unitarity using ordinary perturbative quantum field theory, without anything exotic, such as extra dimensions or nonlocality, and also without using a Higgs boson.
FIG. 8: Scattering diagrams with $P^2$ behavior: III

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