A New Model of Topcolor-Assisted Technicolor

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Abstract

I present a model of topcolor-assisted technicolor that can have topcolor breaking of the desired pattern, hard masses for all quarks and leptons, mixing among the heavy and light generations, and explicit breaking of all technifermion chiral symmetries except electroweak $SU(2) \otimes U(1)$. These positive features depend on the outcome of vacuum alignment. The main flaw in this model is tau-lepton condensation.
It is not difficult to construct a dynamical model of electroweak symmetry breaking. We have known how at least since 1973 [1]. The difficulties lie in extending this dynamics to flavor: accounting for the masses of all known fermions, including the top quark’s; breaking technifermion chiral symmetries to prevent light technipions with axion-strength couplings to quarks and leptons; and evading the many phenomenological pitfalls—flavor-changing neutral currents to name the most famous and ubiquitous example—that plague any theory of flavor [2]. This paper develops further the topcolor-assisted technicolor approach to accomplishing all this.

Topcolor-assisted technicolor (TC2) is the only scheme known in which there is an explicit dynamical and natural mechanism for breaking electroweak symmetry and generating the fermion masses including \( m_t \simeq 175 \text{ GeV} \). In TC2, there are no elementary scalar fields and no unnatural or excessive fine-tuning of parameters [3]. In Hill’s simplest TC2 model, the third generation of quarks and leptons transforms under strongly-coupled color and hypercharge groups, \( SU(3)_1 \otimes U(1)_1 \), with the usual charges, while the light generations transform under weakly-coupled \( SU(3)_2 \otimes U(1)_2 \). Near 1 TeV, these four groups are broken, somehow, to the diagonal subgroup of ordinary color and hypercharge, \( SU(3)_C \otimes U(1)_Y \). The desired pattern of heavy quark condensation occurs because \( U(1)_1 \) couplings are such that the spontaneously broken \( SU(3)_1 \otimes U(1)_1 \) interactions are supercritical only for the top quark.

Hill did not address the issues of topcolor breaking, generational mixing and chiral symmetry breaking. In addition to these concerns, there are stringent constraints on model-building from the conflict between custodial isospin conservation and the large topcolor \( U(1)_1 \) coupling [4], and from limits on \( B_d - \bar{B}_d \) mixing [5]. These constraints, the cancellation of \( U(1) \) anomalies, and the dynamics of generational mixing and topcolor breaking to \( SU(3)_C \otimes U(1)_Y \) were considered in Refs. [6] and [7]. The main features of the models developed in these studies are:

1. The \( U(1)_1 \) charges of technifermions are custodial-isospin symmetric.
2. Above the electroweak scale, third-generation quarks transform under strongly-coupled \( SU(3)_1 \) while the two light-generation quarks transform under the weaker \( SU(3)_2 \). However, all quarks and leptons transform under the strongly-coupled \( U(1)_1 \).
3. In order that \( Z^0 \) couplings be nearly standard, the breakdown \( U(1)_1 \otimes U(1)_2 \to U(1)_Y \) necessarily occurs at a somewhat higher scale than \( SU(2) \otimes U(1)_Y \to U(1)_{EM} \). This is effected by a higher dimensional technifermion \( \psi \) whose condensate is \( SU(2) \otimes U(1)_Y \).
invariant. The $\psi$-condensate gives rise to a 2–3 TeV $Z'$ boson with much interesting phenomenology \cite{4}, \cite{5}, \cite{8}, \cite{9}, \cite{10}.

(4) The breaking of the color and electroweak symmetries to $SU(3)_C \otimes U(1)_{EM}$ is due to technifermions in the fundamental representation of the TC gauge group, assumed to be $SU(N)$. In particular, the $SU(3)_1 \otimes SU(3)_2$ breaking condensate $\langle \bar{T}^i_1 T^i_R \rangle$, where $T^i$ is a triplet of $SU(3)_i$, is driven by an attractive strong $U(1)_1$ interaction.

(5) Generational mixing is produced by an extended technicolor (ETC) operator which induces the transition $d_L, s_L \leftrightarrow b_R$, but not $d_R, s_R \leftrightarrow b_L$. In this way, the excessive $B_{d \to \bar{b}}$ mixing discussed in Ref. \cite{5} is avoided.

(6) Nontrivial solutions exist to all the $U(1)$ anomaly-cancellation equations. These constraints led to a proliferation of technifermions and a large chiral $SU(N_T)_L \otimes SU(N_T)_R$ symmetry. In the models considered, it was not possible to break explicitly all unwanted chiral symmetries, so that massless or very light technipions occurred.

Explicit chiral symmetry breaking and generational mixing, in the form of quark mass $\bar{q}TTq$ and technipion mass $\bar{TTTT}$ operators, are induced mainly by ETC interactions. Here, $T = (U,D)$ are technifermion isodoublets. Let us define a “complete set” of $SU(2) \otimes U(1)$-invariant 4T operators $\bar{T}^i_L \gamma^\mu T^j_L T^{k}_R \gamma_\mu (a + b \sigma_3) T^l_R$ as one for which no technifermion global symmetry generator commutes with every member of the set. In a complete set, every left-handed and right-handed technifermion field appears in at least one of the operators. (This excludes operators in which the left or right-handed currents involve the same technifermion twice, e.g., operators generated by diagonal ETC or $U(1)_1$ interactions.) Since I have not specified an ETC group and its breaking pattern, it is necessary to assume that the required operators exist, provided they respect all known gauge interactions, including $U(1)_1 \otimes U(1)_2$. For the type of model considered in Ref. \cite{7}, I was unable to find a complete set of 4T operators.

Even a complete set of operators is not sufficient to guarantee that all technipion masses are large. It is also necessary that condensates form so that all 4T operators have nonzero vacuum expectation values, i.e., that they contribute to the vacuum energy

$$E(W) = \langle \Omega | W H' W^{-1} | \Omega \rangle. \quad (1)$$

Here, the hamiltonian $H'$ is the sum over all allowed 4T operators and $W$ is an $SU(N_T)_L \otimes SU(N_T)_R$ transformation. Finding the transformation $W^0$ which minimizes $E(W)$ is known as vacuum alignment \cite{11}. In the correct vacuum, $\langle \bar{T}^i_L T^i_R \rangle \propto W^0_{ij}$, where
$W^0$ is the corresponding $SU(N_T)$ matrix. The models under consideration have a large number of technifermions and 4T operators, and minimization is a complicated numerical task, now under study.

I present here a type of TC2 model which does allow a complete set of 4T operators. For the models of Ref. [7], the difficulty of constructing such a set was due at least in part to the fact that light and heavy quarks transform under different color $SU(3)$ groups. Then their hypercharges were tightly constrained by cancellation of $U(1) [SU(3)]^2$ anomalies and there was no complete set invariant under $U(1)_1 \otimes U(1)_2$. In the model presented here, I adopt the “flavor-universal topcolor” of Chivukula, Cohen and Simmons [12]; also see Ref. [10]. Their model was motivated by the apparent excess of high-$E_T$ events in the CDF jet data [13]. They used two $SU(3)$ groups, but assumed all quarks transform under only the stronger $SU(3)_1$ color group. I find that this allows simpler quark hypercharges than in Ref. [7] and, so, the $U(1)$ constraints for a complete set of 4T operators can be met. A dynamical mechanism for breaking $SU(3)_1 \otimes SU(3)_2 \rightarrow SU(3)_C$ was not provided in Refs. [10] and [12]. I shall use the condensation of technifermions transforming under the two color groups to effect this breaking. The model I present has one obvious bad feature: the tau-lepton has very strong, attractive $U(1)_1$ interactions and, therefore, it has a large condensate and mass.

The fermions in this new model, their color representations and $U(1)$ charges are listed in Table 1. Technifermions $T^i_{L,R}$ transform under $SU(N)$ as fundamentals, while $\psi_{L,R}$ are antisymmetric tensors. As noted above, the condensate $\langle \bar{\psi}_L \psi_R \rangle$ breaks $U(1)_1 \otimes U(1)_2 \rightarrow U(1)_Y$ and $\langle \bar{T}^1_L T^2_R \rangle \neq 0$ breaks $SU(3)_1 \otimes SU(3)_2 \rightarrow SU(3)_C$. The condition on the $U(1)_1$ charges of $T^1$ and $T^2$ required to form this condensate is, in the walking technicolor and large-$N$ limits [7],

$$ (u_1 - v_1)(v_1 - u'_1) > \frac{4\alpha_3}{3\alpha_1}. \quad (2) $$

Here, $\alpha_3$ and $\alpha_1$ are the $SU(3)_1$ and $U(1)_1$ couplings near 1 TeV. Note that this requires and $(u_1 - u'_1)^2 > (u_1 - v_1)(v_1 - u'_1) > 0$. To preserve $U(1)_EM$, $T^1$ and $T^2$ must have equal electric charges, i.e., $u_1 + u_2 = u'_1 + u'_2 = v_1 + v_2$

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1 As I once heard in a similar situation, “the tau-lepton is the bane in mayn haldz” (the bone in my throat). At least, the Goldstone boson from tau-condensation acquires a sizable mass from the ETC contribution to $m_\tau$. 

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To give mass to quarks and leptons, I assume the following ETC operators:

\[ \bar{q}_{iL} \gamma^\mu T^i_L \bar{D}^j_R \gamma_\mu e_{jR} \quad \implies \quad x_1 - x_1' = 0 \]
\[ \bar{q}_{iL} \gamma^\mu T^i_L \bar{T}^j_R \gamma_\mu d_{jR} \quad \implies \quad x_1 - x_1' = 0 \]
\[ \bar{q}_{iL} \gamma^\mu T^i_L \bar{D}^b_R \gamma_\mu T^b_R \quad \implies \quad a - a' = y_1 - y_1' \quad (3) \]
\[ \bar{q}_{iL} \gamma^\mu T^i_L \bar{U}^b_R \gamma_\mu t_{R} \quad \implies \quad b - b' = y_1 - y_1 \]
\[ \bar{q}_{iL} \gamma^\mu T^b_L \bar{D}^b_R \gamma_\mu b_R \quad \implies \quad b - b'' = z_1 - z_1' . \]

To generate \( d_L, s_L \leftrightarrow b_R \), I require the operator \(^2\)

\[ \bar{q}_{iL} \gamma^\mu T^i_L \bar{D}^b_R \gamma_\mu b_R \quad \implies \quad b'' = z_1' - y_1 . \quad (4) \]

Of course, the technifermions in these operators must condense in the correctly aligned ground state. To forbid the transition \( d_R, s_R \leftrightarrow b_L \) and unacceptably large \( B_d - \bar{B}_d \) mixing, ETC interactions must not generate any of the operators \( \bar{q}_{iL} \gamma^\mu T^i_L \bar{D}^b_R \gamma_\mu d_{bR} \) for any \( i, j \). This gives the constraints \( b \neq 0, u_1 - u_1', y_1 - y_1', z_1 - z_1', \) etc.

A complete set of allowed \( SU(2) \otimes U(1) \)-invariant 4T operators is

\[ \bar{T}^1_L \gamma^\mu T^i_L \bar{T}^b_R \gamma_\mu T^1_R \quad \implies \quad u_1 - u_1' = x_1 - z_1' \]
\[ \bar{T}^1_L \gamma^\mu T^b_L \bar{T}^1_R \gamma_\mu T^i_R \quad \implies \quad u_1 - u_1' = z_1 - y_1' \]
\[ \bar{T}^i_L \gamma^\mu T^b_L \bar{T}^b_R \gamma_\mu T^i_R \quad \implies \quad z_1 - z_1' = y_1 - x_1' = 0 \quad (5) \]
\[ \bar{T}^i_L \gamma^\mu T^b_L \bar{T}^i_R \gamma_\mu T^b_R \quad \implies \quad x_1 - x_1' = z_1 - y_1' \]
\[ \bar{T}^2_L \gamma^\mu T^1_L \bar{T}^1_R \gamma_\mu T^2_R \quad \implies \quad x_1 - x_1' = 0 . \]

Note that the equal-charge conditions \( x_1 + x_2 = y_1 + y_2 = z_1 + z_2 \) are implied by these operators. In addition to this set, diagonal 4T operators from broken ETC and \( U(1) \) interactions contribute to the chiral-breaking hamiltonian, \( \mathcal{H}' \).

The requirement that gauge anomalies cancel further constrains \( U(1) \) charge assignments. Taking account of the equal-charge conditions, there are four independent conditions which are linear in the hypercharges (the \( U(1)_{1,2}[SU(3)_2]^2 \) condition is automatically implied by the ETC operators and the anomaly constraints. They are \( \bar{q}_{iL} \gamma^\mu T^i_L \bar{D}^i_R \gamma_\mu b_R \) and \( \bar{q}_{iL} \gamma^\mu T^i_L \bar{D}^i_R \gamma_\mu b_R \). These generation-mixing operators are not simultaneously consistent with the \( U(1) \) symmetries; only one may be assumed to exist.

\(^2\) This choice is not unique. Two other operators are consistent with the hypercharge conditions implied by the ETC operators and the anomaly constraints. They are \( \bar{q}_{iL} \gamma^\mu T^i_L \bar{D}^i_R \gamma_\mu b_R \) and \( \bar{q}_{iL} \gamma^\mu T^i_L \bar{D}^i_R \gamma_\mu b_R \). These generation-mixing operators are not simultaneously consistent with the \( U(1) \) symmetries; only one may be assumed to exist.
satisfied):

\[
U(1)_{1,2}[SU(N)]^2: \quad 3(u_1 - u_1') + y_1 - y_1' + z_1 - z_1' = -\frac{1}{2}(N - 2)(\xi - \xi')
\]

\[
U(1)_{1,2}[SU(3)]^2: \quad 2b - b' - b'' = -2N(u_1 - u_1')
\]

\[
U(1)_{1,2}[SU(2)]^2: \quad a + 3b = -N[3(u_1 + v_1) + x_1 + y_1 + z_1] = N[3(u_2 + v_2) + x_2 + y_2 + z_2].
\]

Taken together with the hypercharge conditions, Eqs. (3) and (7), there follow the relations:

\[
\begin{align*}
  b &= -(u_1 - u_1'), \quad b' = -3(u_1 - u_1'), \quad b'' = z_1' - y_1 = (2N + 1)(u_1 - u_1') \\
  a - a' &= b - b' = y_1 - y_1' = 2(u_1 - u_1') \\
  b - b'' &= z_1 - z_1' = -2(N + 1)(u_1 - u_1') \\
  \xi - \xi' &= 2 \left( \frac{2N - 3}{N - 2} \right) (u_1 - u_1').
\end{align*}
\]

Note that \(bb' > 0\) and \(bb'' < 0\) which favors top, but not bottom, condensation.

There are four anomaly conditions that are cubic in the hypercharges. However, the \(U(1)_Y[SU(2)]^2\) anomaly cancellation guarantees that the \([U(1)_Y]^3\) anomaly also cancels, leaving three independent conditions. They are conveniently given for \([U(1)_1]^3\), \([U(1)_1]^2U(1)_Y\), and \([U(1)_1]^3 + [U(1)_2]^3 - 3[U(1)_1]^2U(1)_Y\):

\[
0 = 2N \left[ 3(u_1^3 - u_1'^3) + y_1^3 - y_1'^3 + z_1^3 - z_1'^3 \right]
+ \frac{1}{2}N(N - 1)(\xi^3 - \xi'^3) + 2a^3 - a'^3 + 3(2b^3 - b'^3 - b''^3)
\]

\[
0 = 2N \left[ 3(u_1 + u_2)(u_1^2 - u_1'^2) + (y_1 + y_2)(y_1^2 - y_1'^2) + (z_1 + z_2)(z_1^2 - z_1'^2) \right]
+ a'^2 - a^2 + b^2 - 2b'^2 + b''^2
\]

\[
0 = 2N \left\{ 3(u_1' - u_1) \left[ (u_1 + u_2)^2 + \frac{1}{4} \right] + (y_1' - y_1) \left[ (y_1 + y_2)^2 + \frac{1}{4} \right] 
+ (z_1' - z_1) \left[ (z_1 + z_2)^2 + \frac{1}{4} \right] \right\} + a - a' + \frac{4}{3}(b - b') + \frac{1}{3}(b - b'').
\]
These conditions have an infinite number of solutions. Following Ref. [7], I found one with $|u_1 - u'_1| = \mathcal{O}(1)$, as required for naturally large couplings in Eq. (2), as follows: I assumed $u_1 = -u'_1$ and $\xi = -\xi'$. Then, for $N = 4$, I chose $z_1 = 8$ and $z_1 + z_2 = 2$. This input has the nontrivial solution

$$a = -15.437, \quad u_1 = -u'_1 = -0.648, \quad u_1 + u_2 = 3.321.$$  \hspace{1cm} (9)$$

The other hypercharges are to be chosen in accord with Eqs.(3)–(7).

The large values $a \simeq a' \simeq -2z_1$ found in the solutions to Eqs. (8) are unavoidable: The $[U(1)_1]^3$ condition has no real solution for $u_1 - u'_1 \neq 0$ and $|a| \lesssim |u_1 - u'_1|$. The large positive value of $aa'$ then suggests that the $U(1)_1$ interactions generate a tau-condensate $\langle \bar{\tau}_L \tau_R \rangle \sim \langle \bar{t}_L t_R \rangle$. Such a hypercharge also raises the question of the triviality of the $U(1)_1$ interaction: does the Landau pole occur at an energy significantly lower than the one at which we can envisage $U(1)_1$ being unified into an asymptotically free ETC group [7]? I know of no choice of chiral symmetry breaking ETC operators and associated hypercharge assignments within the present simple model of flavor-universal topcolor which evades $aa'/(u_1 - u'_1)^2 \gg 1$. It may be possible to find an acceptable model, including a complete set of 4T operators, by enlarging the technifermion sector and/or complicating the light generation hypercharge assignments.

In conclusion, I have constructed a TC2 model with flavor-universal topcolor that seems capable of satisfying all major phenomenological constraints except those involving the tau-lepton. To my mind, the more important task ahead is to show that a nontrivial vacuum-alignment solution exists that results in nonzero masses and mixings for all the fundamental fermions and composite technipions.

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| Particle | $SU(3)_1$ | $SU(3)_2$ | $Y_1$ | $Y_2$ |
|----------|-----------|-----------|-------|-------|
| $\ell^i_L$ | 1 | 1 | 0 | $-\frac{1}{2}$ |
| $e_R, \mu_R$ | 1 | 1 | 0 | $-1$ |
| $q^i_L$ | 3 | 1 | 0 | $\frac{1}{6}$ |
| $u_R, c_R$ | 3 | 1 | 0 | $\frac{2}{3}$ |
| $d_R, s_R$ | 3 | 1 | 0 | $-\frac{1}{3}$ |
| $\ell^h_L$ | 1 | 1 | $a$ | $-\frac{1}{2} - a$ |
| $\tau_R$ | 1 | 1 | $a'$ | $-1 - a'$ |
| $q^h_L$ | 3 | 1 | $b$ | $\frac{1}{6} - b$ |
| $t_R$ | 3 | 1 | $b'$ | $\frac{2}{3} - b'$ |
| $b_R$ | 3 | 1 | $b''$ | $-\frac{1}{3} - b''$ |
| $T^1_L$ | 3 | 1 | $u_1$ | $u_2$ |
| $U^1_R$ | 3 | 1 | $u'_1$ | $u'_2 + \frac{1}{2}$ |
| $D^1_R$ | 3 | 1 | $u'_1$ | $u'_2 - \frac{1}{2}$ |
| $T^2_L$ | 1 | 3 | $v_1$ | $v_2$ |
| $U^2_R$ | 1 | 3 | $v'_1$ | $v'_2 + \frac{1}{2}$ |
| $D^2_R$ | 1 | 3 | $v'_1$ | $v'_2 - \frac{1}{2}$ |
| $T^l_L$ | 1 | 1 | $x_1$ | $x_2$ |
| $U^l_R$ | 1 | 1 | $x'_1$ | $x'_2 + \frac{1}{2}$ |
| $D^l_R$ | 1 | 1 | $x'_1$ | $x'_2 - \frac{1}{2}$ |
| $T^t_L$ | 1 | 1 | $y_1$ | $y_2$ |
| $U^t_R$ | 1 | 1 | $y'_1$ | $y'_2 + \frac{1}{2}$ |
| $D^t_R$ | 1 | 1 | $y'_1$ | $y'_2 - \frac{1}{2}$ |
| $T^b_L$ | 1 | 1 | $z_1$ | $z_2$ |
| $U^b_R$ | 1 | 1 | $z'_1$ | $z'_2 + \frac{1}{2}$ |
| $D^b_R$ | 1 | 1 | $z'_1$ | $z'_2 - \frac{1}{2}$ |
| $\psi_L$ | 1 | 1 | $\xi$ | $-\xi$ |
| $\psi_R$ | 1 | 1 | $\xi'$ | $-\xi'$ |

**TABLE 1**: Lepton, quark and technifermion colors and hypercharges.