Numerical nonlinear analysis of a membrane structure subjected to dynamic load

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Abstract. The aim of this paper is the numerical analysis of a thin technical textile in shape of hyperbolic paraboloid. It is the membrane structure, which can function only in tension and it is geometrically nonlinear, therefore nonlinear analysis has to be carried out. In this case, the structure was subjected to the dynamic loads effects. Natural vibration was applied as a fundamental dynamic characteristic. All of this results in natural frequencies and mode shapes and deflections. Acquired results describe the dynamic behaviour of nonlinear system. This part of the analysis could represent a first step in the dynamic analysis. To complete the dynamic analysis of the membrane structure, the forced vibrations as nonlinear time history analysis will have to be performed in a future.

1. Introduction

Membrane structures can be defined as a material of 6. generation along the traditional ones: brick, wood, steel, stone and concrete. Membrane are made of the technical textiles. This type of material directly relates with their positive characteristics. They are very light-weighted and can be used for the wide-span structures without a need of intermediate supports. With this fact coheres a material saving and therefore a lower initial costs. And the economic aspect is completed with the fact that the membrane structures function only in tension, it means that we are trying to use their positive characteristics to extend their life cycle.

But as every other material even the technical textiles have some negative aspects. They are strongly geometrically non-linear structures which makes it more difficult to design. It means that, in difference to the traditional structures, the membranes change their shape in a process of loading, so we need to separate it step by step. A non-linear analysis with an iteration process has to be carried out to obtain the assessment of the structure.

Another specificity is the fact that they need to be pre-stressed to carry out the external load. Technical textile is just piece of a fabric first and initial conditions requires pre-stress of a membrane surface. And because the membrane has to be pre-stressed, we do not know their initial shape in general. For this purpose, form-finding process is used to find the shape. We can say that it presents equilibrium between stress and displacement (shape).

Form-finding is very specific step in design of the membrane structures without which is not possible to continue in a static or dynamic analysis. It can not be described by simple mathematical methods. We have to use physical models or nowadays more appropriate computational methods.

In general, the membrane structures consist of 3 main parts: technical textile itself as a surface load-bearing element, linear load-bearing elements providing pre-stress and therefore stability (e. g. rope), and primary structure allowing transfer of internal forces from these structures to the foundation.

Structures of light-weighted architecture, which include membranes, we can define as which can transmit the same amount of load at a lower self-weight in terms of internal force transfer. Like any
building construction, textile architecture is subject to strict design rules and all tentative requirements for mechanical resistance and stability must be respected, taking into account their specific characteristics listed above.

The usage of the membrane structures is dated thousands of years ago in the form of a temporary dwellings and later used for tents, for example circus tens called chapiteau. But light-weighted structures as we know it today began with the dissertation thesis *Das Hängende Dach* from Otto Frei in 1952. In 1967 he participated in the design of the German pavilion (figure 1) at the World Expo in Montreal. Although the original design of the pavilion was a stretched membrane structure, it was ultimately built as a rope net, using the fabric only as a secondary overlay [1].

![Figure 1. German pavilion at the World Expo [1].](image)

2. The numerical model
For the numerical analysis, the membrane structure in form of the hyperbolic paraboloid was modeled. Computational software Dlubal RFEM was used for this purpose. 3D hypar structure is made of orthotropic material with modulus of elasticity in the warp direction (as principal direction) $E_x=1057$ MPa and modulus of elasticity in the weft direction $E_y=612$ MPa [2]. It simulates the most important material characteristics of technical textile Serge Précontraint 502.

![Figure 2. Membrane after form-finding.](image)

Another parameters which characterise the geometrical size of the membrane are floor plan $2\times2$ m with overall height of $1$ m which represent the rise equal to the overhang $0.5$ m. On the edges of the
surface element are linear elements – ropes with diameter of 8 mm. They are modeled as stainless steel ropes with modulus of elasticity E=130 GPa. They provide the pre-stress of the membrane surface in the form of absolute sag with value of 0.2 m. The pre-stress of the structure is completed with the pre-stress of the surface in both of directions (warp and weft) equal to $n_x=n_y=4 \text{kN/m}$ [3].

The membrane stabilized by the edge ropes is anchored articulated with usage of 2 anchor rods and 2 actuators. Actuators are elements which can change their length and therefore change the stress distribution of membrane. The structure become adaptive with this process. This part of the model will be use in a future research, for now we consider the structure without change of anchorage.

Anchorages are modeled as a solid circular cross-section (D46 and D145). They represent masses on the experimental device in laboratory on which the whole numerical model was created. Verification with the numerical model will be carry out in a next phase of the research on this laboratory device.

Dlubal RFEM calculate any structure based on Finite Element Method (FEM). Therefore finite elements was used: 1D for edge ropes and anchor rods and 2D elements for membrane surface. The mesh size of 2D elements was 50 mm and we used triangular finite shell element. 1D elements are divided with 10 divisions per member (for cable and anchorage also) [3, 4]. The membrane was modeled as 2 triangulars connected in the middle between lower anchors with third vertices in the upper anchors.

Before the dynamic analysis of natural vibrations, process form-finding was performed to find initial shape according to the established pre-stress. Dlubal RFEM uses Update Reference Strategy (URS) for this aim. It basically loads the membrane structure step by step with pre-stress value and trace the displacement of nodes in mesh [5].

In figure 2 we can see surface stresses and internal forces in achores after form-finding. It is necessary to point that form-finding is performed without considering dead load.

### 3. Natural vibration analysis and results

Dlubal RFEM as FEM software has a few add-on modules available. One of them is RF-DYNAM PRO, which serves for dynamic analysis. In this paper, natural vibration analysis was performed to achieve natural frequencies of mode shapes. One mass case was created and defined as permanent. This one represents only self-weight of the structure.

From this mass case one natural vibration case was made of. For the analysis general setting had to be considered. Number of lowest eigenvalues to calculate was established for 10 and scaling of mode shapes was chosen as recommended in manual for $\{u_j\}^T [M] \{u_j\}=1$. This setting is related to non-linear analysis of membrane [6].

For natural vibration case consistent mass matrix and Lanczos method for solving eigenvalue problem were used. It is necessary remark that because of the membrane structure and form-finding process, stiffness modification was considered from load case of form-finding.

In the table 1 you can see all of 10 mode shapes of mass case with only self-weight, with values of eigenvalues $\lambda$, angular frequencies $f$ and natural periods $T$. In the figure 3 you can see all of the mode shapes for this mass case also with deflection.

| Mode | Angular Frequency $v$ [rad/s] | Natural frequency $f$ [Hz] | Natural period $T$ [s] |
|------|-----------------------------|-----------------------------|------------------------|
| 1    | 57.186                      | 9.101                       | 0.110                  |
| 2    | 60.415                      | 9.615                       | 0.104                  |
| 3    | 66.512                      | 10.586                      | 0.094                  |
| 4    | 92.390                      | 14.704                      | 0.068                  |
| 5    | 99.473                      | 15.832                      | 0.063                  |
| 6    | 180.252                     | 28.688                      | 0.035                  |
| 7    | 220.289                     | 35.060                      | 0.029                  |
| 8    | 314.905                     | 50.119                      | 0.020                  |
| 9    | 343.091                     | 54.605                      | 0.018                  |
| 10   | 376.551                     | 59.930                      | 0.017                  |
4. Conclusion
The aim of this contribution was to show modal analysis and natural vibration of the membrane structure. This structure is strongly geometrically non-linear and it requires specific step in the beginning of static or dynamic analysis. This step make membrane structures special in the design stage in compare to the typical materials.

As we can see in the table 1 and figure 3, the lowest natural frequencies begin with value of little bit higher that 9 Hz. We can say that this frequency is lower than 10 Hz which represents dynamic wind load, but it is far more than 5 Hz which can we consider as considerable factor of wind. For this purpose, it is necessary to supplement FEM model also with steel frame, as the experimental device has, so we can suggest that it will have significant influence on these parameters. It will approach real behaviour of the structure and it will make results more precise.

Also in the future research forced vibration will be performed as dynamic load effect and it will extend the results of this paper. We can consider forced vibration caused by harmonic variable or periodic force and every load case can be damped and undamped.

It all can results in comparison and verification of the numerical model with experimental device.

5. References
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