Hungaria region as possible source of Trojans and satellites in the inner solar-system.

M. A. Galiazzo$^{1*}$, R. Schwarz$^{1}$

$^{1}$Institute of Astrophysics, University of Vienna, A-1180 Vienna, Türkenschanzstrasse 17, Austria

ABSTRACT

The Hungaria Family (the closest region of the Main Belt to Mars) is an important source of Planet-Crossing-Asteroids and even impactors of terrestrial planets. We present the possibility that asteroids coming from the Hungaria Family get captured into co-orbital motion with the terrestrial planets in the inner solar system. Therefore we carried out long term numerical integrations (up to 100 Myr) to analyze the migrations from their original location - the Hungaria family region- into the inner solar system. During the integration time we observed whether or not the Hungarias get captured into a co-orbital motion with by the terrestrial planets. Our results show that 5.5% of 200 Hungarias, selected as a sample of the whole group, escape from the Hungaria region and the probability from that to become co-orbital objects (Trojans, satellites or horseshoes) turns out to be $\sim 3.3\%$: 1.8% for Mars and 1.5% for the Earth. In addition we distinguished in which classes of co-orbital motion the asteroids get captured and for how long they stay there in stable motion. Most of the escaped Hungarias become Quasi-satellites and the ones captured as Trojans favour the $L_5$ lagrangian point. This work highlights that the Hungaria region is a source of Mars and also Earth co-orbital objects.

Key words: celestial mechanics – minor planets, asteroids – Solar system: general – methods: numerical

1 INTRODUCTION

A co-orbital configuration refers to a celestial object (such as an asteroid) that keeps a quasi-constant distance from its parent object (in this work, planet) and it is on a 1 : 1 mean motion resonance (MMR). In this configuration the asteroid has a rotational period around the Sun similar to the planet which is co-orbiting.

The co-orbital bodies are subdivided in classes of objects which depend on their point of libration. In this study we are interested in the following classes for the Inner solar system: (a) Trojan objects, which librate around one of the two stable Lagrangian equilibrium points, $L_4$ and $L_5$, respectively asteroids leading (libration angle $\lambda \sim +60^\circ$) and heading ($\lambda \sim -60^\circ$) the planets orbit, i.e. 2010 TK$_7$ for the Earth (Connors, Wiegert & Veillet 2011) and (b) Satellites (MOs) and quasi-satellites (QSs) orbits, which librates around 0°, but the libration width $\sigma$ is much larger for the QSs (more details are presented in Section 2). In contrast to MOs, QSs orbits lie outside the planet’s Hill sphere, therefore they are not long-term stable. Over time they tend to evolve to other types of resonant motion, where they no longer remain in the planet’s neighborhood.

Currently one Earth Trojan and 9 Martian Trojan asteroids, 5 horseshoe objects (one Martian and 4 of the Earth) and also 6 quasi satellite close to the Earth, are known. All known co-orbital objects – including candidates – in the inner solar system are presented in Tab. 1. Theoretical studies predict that Trojan asteroids are a byproduct of planet formation and evolution and were later captured from the planets. Chaotic capture of Jovian Trojan asteroids in the early Solar System ($\sim 3.4$ My), were presented in the work of Morais & Namouni (2013). Lykawka et al. (2009) and Lykawka & Horner (2010) investigated the origin and dynamical evolution of Neptune Trojans during the formation and migration of the planets. They found that the captured Trojans display a wide range of inclinations ($0^\circ \lesssim i < 40^\circ$). These results were confirmed by Schwarz & Dvorak (2012), who investigated the capture probability of co-orbital objects for the planets Venus, Earth and Mars.

Early work on the origin of NEAs (e.g. Greenberg & Nolan 1989, 1993), suggested that colli-
sions in the main-belt continuously produce new asteroids by fragmentation of larger bodies. These fragments can be injected into the $\nu_6$ and $3:1$ MMR with Jupiter, which causes a change of their eccentricities and brings them into orbits intersecting the orbits of Mars (Mars crossers) and/or Earth (Earth crossers, e.g. Minton & Malhotra 2011). Gravitationally, the NEAs are transported first to Mars, mainly by MMRs, three-body mean motion resonances (3BMMRs, for a description of this kind of resonances see Nesvorný & Morbidelli, 1998) and secular resonances (SRs), and then to other more interior planets due especially to close encounters with Mars. Also non-gravitational forces can play a role in the transportation as was shown by Bottke et al. (2002, 2006); Greenstreet, Ngo & Gladman (2012) and Cuk, Gladman & Nesvorný (2014), but as a first stage we will take into account only gravitational forces in this work and the Yarkovsky effect will be considered in a future work.

In order to describe the primordial main belt before the LHB, a hypothetical inner extension of the main belt (1), has been suggested and dubbed the “E-belt” (Bottke et al. 2012). One motivation for this inference is to provide a source for basin-forming lunar impacts of the LHB. These E-belt asteroids were supposed to have a semi-major axis ranging from 1.7 to 2.1 au. Prior to the giant planet migration described in the Nice model, these asteroids would have been in a more stable orbit, with the $\nu_6$ secular resonance outside the border of this region (Morbidelli et al. 2010) and with the outer giant planets having a more compact configuration with an almost circular orbit (Gomes et al. 2005). Then, during the migration of the giant planets (Gomes et al. 2011), the $\nu_6$ and other related resonances would have destabilized the E-belt population. Most of them would have moved inward onto terrestrial planets as their eccentricities and inclinations increased making impacts with the planets and so some of these asteroids (0.1-0.4 %) would have acquired orbits similar to the Hungarias. In this sense, the Hungarias are supposed to be a remnant of the E-belt; the survivors of the E-belt dispersion (Bottke et al. 2012). This idea is a development of the NICE model (see in particular Morbidelli et al. 2010) and it should make it more consistent. The NICE model has still some gaps, the most important are: (a) it does not explain the presence of Mercury and (b) the rate of the incoming comets and even an explanation of the large-scale mixing of reddish and bluish material (from the photometric point of view) in the asteroid belt (DeMeo & Carry 2014). For this reason we study in this work only the present Hungary group, which might be an evolution of the ancient E-belt.

The importance of considering Hungarias as source of NEAs (which can originate also possible co-orbital bodies of terrestrial planets), is shown very well in Galiazzo, Bazzo & Dvorak (2013a) and in Cuk, Gladman & Nesvorný (2014), who described the dynamical evolution of these mainly E-type asteroids (Carvano et al. 2001; Assandri & Gil-Hutton 2003; Warner et al. 2009) into the NEAs region.

\[ \text{Table 1. All observed Earth and Mars co-orbital asteroids. * depicts an object which is only a candidate. The different motion types are horseshoe orbits } H \text{ and tad-pole orbits in Lagrangian points } L_1 \text{ and } L_5 \text{ or in both of this last two consecutively, like jumping Trojans } JT. \ T_j \text{ represents the Tisserand parameter in respect of Jupiter and } MT \text{ stands for motion type.} \]

| Name | a [au] | e | i [°] | $T_j$ | MT |
|------|--------|---|------|------|----|
| Mars |
| (121514) 1999 UJ$_7$ | 1.5245 | 0.039 | 16.8 | 4.449 | $L_4$ |
| (5261) Eureka | 1.5355 | 0.065 | 20.3 | 4.428 | $L_5$ |
| (101426) 1998 VF$_31$ | 1.5342 | 0.100 | 31.3 | 4.334 | $L_4$ |
| (311999) 2007 NS$_2$ | 1.5237 | 0.054 | 18.6 | 4.339 | $L_5$ |
| (269719) 1998 QH$_{36}$ | 1.5507 | 0.031 | 32.2 | 4.279 | $L_5$ |
| (385250) 2001 DH$_{47}$ | 1.5238 | 0.035 | 24.4 | 4.400 | $L_5$ |
| 2001 SC$_{191}$ | 1.5238 | 0.044 | 18.7 | 4.439 | $L_5$ |
| (88719) 2011 SL$_{25}$ | 1.5238 | 0.115 | 21.5 | 4.415 | $L_5$ |
| 2011 UN$_{43}$ | 1.5237 | 0.064 | 20.4 | 4.427 | $L_5$ |
| (157204) 1998 SD$_4$ | 1.5149 | 0.125 | 13.7 | 4.475 | H |
| Earth |
| 2010 TK$_7$ | 1.0000 | 0.191 | 20.9 | 6.008 | JT |
| (3753) Cruithne | 0.9977 | 0.515 | 19.8 | 5.922 | QS |
| (164207) 2004 GU$_9$ | 1.0013 | 0.136 | 13.7 | 6.041 | QS |
| (277810) 2006 FV$_{35}$ | 1.0013 | 0.378 | 7.1 | 6.003 | QS |
| 2003 YN$_{107}$ | 0.9987 | 0.014 | 4.3 | 6.132 | QS |
| (54509) YORP | 1.0060 | 0.230 | 6.000 | 6.028 | QS |
| 2001 GO$_2$ | 1.0067 | 0.168 | 6.020 | 6.033 | QS |
| 2013 BS$_{45}$ | 0.9939 | 0.084 | 5.773 | 6.106 | H |
| 2010 SO$_{16}$ | 1.0019 | 0.075 | 14.5 | 6.041 | H |
| 2002 AJ$_{99}$ | 0.9926 | 0.102 | 10.8 | 6.100 | H |
| 2006 JY$_{26}$ | 1.0100 | 0.083 | 1.4 | 6.030 | H |
| (85770) 1998 UP$_1$ | 0.9983 | 0.345 | 33.2 | 5.901 | H |

Here we perform a numerical study on the orbits of the asteroids of the Hungaria Family (see also Galiazzo, Bazzo & Dvorak (2013a)), investigating their capture probability into the 1:1 MMR with the terrestrial planets: Venus, Earth and Mars. Hungarias are relatively far out away from the orbit of the terrestrial planets, in fact the inner part starts with a semi-major axis equal to 1.78 au (Galiazzo, Bazzo & Dvorak 2013b). To study the capture of the Trojan asteroids into the inner Solar System it is necessary to consider the interactions (collisions and mass transport) between the Near-Earth-Asteroids (NEAs) and the main-belt asteroids.

During the integration time we observe whether or not the Hungarias get captured into a co-orbital motion with the planets in the inner Solar-system, from Venus to Mars. In addition we distinguish in which classes of co-orbital motion the asteroids get captured and for how long they stay there in stable motion. Therefore we carry out long term numerical integrations up to 100 Myr to analyze the transfers from their original location - the Hungary family region- towards the terrestrial planets.

The paper is organized as follows: the model and the methods are described in Section 2; the results are shown in Section 3 (subdivided in two subsections, subsection 3.1, where we describe some sample cases of Hungary orbital evolution and transport mechanisms).
with a terrestrial planet, its lifetime and orbit in such configuration). The conclusions are in Section 4.

2 MODEL AND METHODS

We do numerical N-body simulations using the Lie integration method (Hanslmeier & Dvorak 1984, Eggl & Dvorak 2010, Schwarz & Dvorak 2012, Galiazzo, Bazsó & Dvorak 2013). We continue the last work of Galiazzo, Bazsó & Dvorak (2013) considering the calculations of the Hungaria group: we take a sub-sample of 200 bodies, representative of the whole group, as the most evolved ones, selected out of the total sample of 8258 asteroids considering a criterion based on the osculating elements. We choose the following variable

\[ d = \sqrt{\left(\frac{a}{a_{\text{esc}}}\right)^2 - \left(\frac{e}{e_{\text{esc}}}ight)^2 \left(\frac{\sin i}{\sin i_{\text{esc}}}\right)^2} \]

and picked up 200 Hungarias as possible source of co-orbital bodies.

Then, after the first integration, 11 fugitives out of 200 are detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated. For any fugitives, 49 clones were generated: random detected and therefore they are again dynamically investigated.

The model for the solar system is now from Venus to Saturn and the integration time is once more 100 Myr. Finally we search for captures with the terrestrial planets (Venus, Earth and Mars). Whenever we find a capture, we integrate again the orbit of the asteroid from the point when they get captured. We perform another integration (with the same simplified solar system) with a smaller time step (100 d) for 20 kyr and studying the orbit in detail.

The aim of this study is to understand the capture of Hungaria asteroids in the Inner Solar system, in particular for 2 different types of captures: 1) Satellite orbits and 2) Tadpole orbits (L4 and L5). In some cases we find Horseshoes orbits and jumping Trojans too.

The classification was done by the help of the libration width \( \sigma \), which is defined as the difference between the mean longitude of the asteroid and the planet (Venus, Earth or Mars) (\( \lambda - \lambda_P \)). \( \lambda, \lambda_P \) are given by \( \lambda = \varpi + M, \lambda_P = \varpi_P + M_P \) were \( \varpi, \varpi_P \) are the longitudes of the asteroid and of the planet and \( M, M_P \) are the mean anomaly of the asteroid respectively of the planet.

In a next step we compared the distributions of the orbital elements \( a, e, \) and \( i \). We also examine the orbital histories of captured objects to determine type of capture and the orbital evolution of objects before and after a capture event.

3 HUNGARIA CO-ORBITAL OBJECTS (HCOS)

There are 7 candidates (out of 11) among the Hungaria fugitives in Galiazzo, Bazsó & Dvorak (2013) which can be captured in to co-orbital motions with terrestrial planets, the initial condition can be found in Table 2.

Among all the Hungaria fugitives we found co-orbital objects (from now on HCOs), like tadpole orbits (\( L_4 \) and \( L_5 \)), MOs, QSs and some horseshoe orbits, too.

3.1 Sample cases of a Trojan and of a Quasi-satellite

We analyse the orbital evolution of the fugitive clones, observing whether they get captured in to co-orbital motion with terrestrial planets. We have to mention that we never find a case where an asteroid get captured by Mars and then by the Earth together and there is no case for Venus co-orbital motion. We find several cases of QSs, i.e. for a clone of (141096) 2001 XB 4 (see the graphics description in Fig. 1 and 2), but also some jumping Trojans.

3.1.1 Orbital evolution of a typical HCO and transport mechanism

There are different possibilities how the clones get captured into co-orbital motion, an example of a capture into co-

| Asteroid | \( a \) [AU] | \( e \) | \( i \) [deg] |
|----------|-------------|--------|-------------|
| (211279) 2002 RN\(_{137}\) | 1.8584 | 0.1189 | 22.82 |
| (152648) 1997 U1\(_{20}\) | 1.9894 | 0.1841 | 28.88 |
| (141096) 2001 XB\(_{48}\) | 1.9975 | 0.1055 | 12.32 |
| (24883) 1996 VG\(_9\) | 1.8765 | 0.1556 | 22.71 |
| (41577) 2000 SV\(_2\) | 1.8534 | 0.1843 | 24.97 |
| (175851) 1999 UF\(_2\) | 1.9065 | 0.1874 | 19.24 |
| (35661) 1992 QA | 1.8697 | 0.1116 | 26.2 |
| (41898) 2000 WN\(_{124}\)* | 1.9073 | 0.1062 | 17.11 |
| (30935) Davasobel* | 1.9304 | 0.1178 | 27.81 |
| (171621) 2000 CR\(_{58}\)* | 1.9328 | 0.1051 | 17.19 |
| (129450) 1991 JM* | 1.8512 | 0.1263 | 24.50 |

\* means they are not candidate HCOs.

2 The orbital data are taken from the ASTORB database (http://www.lowell.edu/elgb)

3 The Hungaria group is defined in this region of osculating elements: 1.78 < \( a / a_{\text{esc}} \) < 2.03, 12° < \( i < 31 \)° and \( e < 0.19 \). A sub-sample of 200 bodies representative of the Hungaria group are integrated in a simplified Solar System (Sun, Mars, Jupiter, Saturn and the mass-less asteroids), for 100 Myr to identify possible escapers, like Galiazzo, Bazsó & Dvorak (2013).

4 In fact analyzing the orbits of the clones of 3 Hungarias (100 clones per asteroid) next to resonances with the Earth and Venus (i.e. V1:4 and E2:5); including also these 2 planets in the integrations, we found only one important deflection out of 300 bodies.

5 Asteroids which jump from \( L_4 \) to \( L_5 \) or vice versa (Tsiganis, Dvorak & Pilat-Lohinger 2000).

6 several clones of different asteroids goes in co-orbital bodies and several ones of 2001 XB\(_{48}\) become QSs.
orbital motion is the candidate 2002 RN$_{137}$. The description of the orbital evolution of one of its clones can help us to understand the co-orbital evolution of the HCOs. A clone of 2002 RN$_{137}$ becomes a satellite of Mars after 73.237 Myr of integration and it stays like this for 6.5 kyr. We check its orbital evolution:

- The close encounters, which change significantly the orbit of the asteroid and consequently its osculating elements, but in particular the semi-major axis. As shown in Tab. [4] the Hungary fugitives have larger inclinations (in comparison with the initial conditions of the asteroid families in the main belt), which will lead to an escape from that region, because of SRs, and later on to close encounters with the terrestrial planets (e.g. Mars or the Earth). In general this fact increases the possibility that the asteroid get captured into co-orbital motion. This was also shown for different initial conditions by Schwarz & Dvorak (2012). The Hungaria candidate 2002 RN$_{137}$ represents the orbital behavior which we described previously. This decrease of the inclination favors the capture into co-orbital motion with Earth. Fig. 3 let us see multiple close approaches to Mars and after that also to the Earth. In the time-span between about 55 Myr and 65 Myr of integration, many close encounters are found, thus the inclination change dramatically and that leads to the Earth asteroids capture: the orbital elements of the captured asteroid lies in the stability window for that planet as shown by Tabachnik & Evans (2000).

- The resonances: MMRs, 3BMMRs and SRs, which change the eccentricity and again the inclination. An example of the most important resonances for this case are visible in the evolution of one of our fugitives: from about 25 Myr to 30 Myr, $g_5$ is active. The asteroid is inside the region of influence of this secular resonance (upper panel of Fig. [4]), having the inclination between $i \sim 24^\circ$ and $i \sim 32^\circ$ and keeping its semi-major axis between 1.9 au and 2.0 au. Then the asteroid travels into the regions of influence of the SRs $g_3$ and $g_4$ (see also Warner et al. 2009; Milani et al. 2010, where these regions are well described) from about

---

**Figure 1.** From top to bottom: 1) Libration angle ($\sigma$) of the Earth QS (141096) 2001 XB$_{48}$; 2) the semi-major axis of the asteroid librating around the one of the Earth; 3) views of the orbits of Earth (curve with a radius of 1 au) and a clone of the asteroid 2001 XB$_{48}$ (captured by the Earth for about 10 kyr) as seen from above the north ecliptic pole in the geocentric plane, emphasizing the eccentric orbit of this quasi satellite; 4) Views of the orbits of the bodies of point (3) seen on the plane perpendicular to the ecliptic. The inclined orbit of the asteroid to the Earth is clear, allowing excursions of roughly 0.2 and 0.3 au above and below the plane.

**Figure 2.** Comparison between a terrestrial satellite and a Quasi-satellite. In the upper panel the critical angle (upper and bottom curves) versus time is represent and the semi-major axis variation (central curves, apart the horizontal line which represents the semi-major axis of the Earth) versus time again. Lower panel represents eccentricity and inclination versus time of the 2 different types of co-orbital bodies. The configurations represent the satellite-state of 2001 XB$_{48}$ (lighter color) and quasi-satellite state (darker color) in co-orbital motion with the Earth.
Hungarias as possible source of co-orbital bodies.

34 Myr and 38 Myr, where it does not have close encounters (Fig. 3, upper panel and 4, bottom panel).

The strongest MMRs and 3BMMRs which influence the orbit of this Hungaria appear to be: initially S12:1 and J13-S9-2 (where J is for Jupiter, S for Saturn and the last number is for the asteroid), J20-S15-3 and J13-S10-2 (Fig. 3). Then from 20 Myr to about 22 Myr, the first order 3BMMR J13-S10-2 is active on the asteroid. More over J19-S15-3 acts together with the g3 for about 5 Myr from 25 Myr to 29.5 Myr and, from 29.5 Myr to about 30.5 Myr, we have J6-S4-1. From about 40 Myr to 50 Myr, M5:7, then from 51 Myr to 52 Myr, J5-S1-1 and in the end between 59 Myr and 66 Myr, when there are no close encounters, in chronological order M11:13 and J16:13 (for 2 Myr), E2:1 (for 1 Myr) and J17-S14-2 (for about 1.5 Myr). All these resonances change significantly the eccentricity and the inclination facilitating close encounters of the asteroid with the planets and thus contributing to the change of osculating elements, in favor of some possible co-orbital orbit.

3.2 Sources of co-orbital bodies (results)

3.2.1 Population distributions

We find that 3.3% of all the clones of all the fugitives (11) become HCOs: 1.8% for Mars and 1.5% the Earth, see Tab. 8 for the distribution of the different classes and Tab. 4 for the probability of becoming an HCO for each single Hungaria fugitive. This percentage represents the capture probability which we calculate from the total number of clones (a summary for the different co-orbital classes are given in Tab. 5). We obtain more Hungaria co-orbital bodies for Mars compared to those of the Earth, even if the difference is not so significant and many escapers experience different types of co-orbital motions. The QS class turns out to be the most likely to become a Mars HCO, the second one has both possibilities in equal measure (Mars or Earth HCO, Tab. 4). We can conclude that 0.6% Hungaria fugitives get captured in \( L_4 \) and 1.1% in \( L_5 \) for Mars; for the Earth, 0.4% Hungaria fugitives get captured in \( L_4 \) and 0.6% in \( L_5 \).

Also a few cases of Hungaria Jumping-Trojans are found and usually the Hungaria Jumping-Trojans stay in this condition for longer times. The maximum life is for a clone of 2002 RN\(_{137}\), whose life-time is 58 kyr (see also Fig 6). Some fugitives have the probability to become an HCO only for a single planet, i.e. 2001 XB\(_{38}\) and 1996 VG\(_5\) for the Earth or 1997 UL\(_{20}\) and 1999 UF\(_5\) for Mars (Tab. 6).

The Hungarias with the highest probability to become a co-orbital asteroids have a probability of 8% to be so and they are 2002 RN\(_{137}\) and 2000 SV\(_2\). 2002 RN\(_{137}\) is more likely to become a Mars HCO, the second one has both possibilities in equal measure (Mars or Earth HCO, Tab. 3).

Table 5 shows the distribution of asteroid captures (subdivided by inclinations and total number too) found in this work and we compare partly our result with the work of Schwarz & Dvorak (2012), “partly” because the initial conditions are different. The work of Schwarz & Dvorak (2012) considered the region of the NEAs that covers also a small part of the Hungaria region. They called it region C, which considered this range of semi-major axis: 1.54 au < \( a \) < 2.20

\[ \text{Figure 3. Evolution of the orbit till the initial instant of co-orbital motion. Upper panel: semi-major axis and Planetocentric distance versus time for a clone of the asteroid 2002 RN}_{137}. \text{ In vertical points in crosses and dot-quadrate, close encounters with respectively: Mars and the Earth. Bottom panel: eccentricity (light color) and inclination (black).} \]

\[ \text{Figure 4. Dynamical evolution of a clone of the asteroid 2002 RN}_{137} \text{ later captured by Mars. The 2 horizontal lines of the upper panel represent approximately the region of influence of the secular resonance } g_3 \text{ for that value of semi-major axis between 25 Myr and 30 Myr. The 2 horizontal lines represent approximately the region of influence of the secular resonances } g_3 \text{ and } g_4 \text{, for that value of semi-major axis between 34 Myr and 38 Myr. On the y-axis inclination in degree and semi-major axis in astronomical units times 10.} \]
au, but only at certain inclinations and eccentricities (see Schwarz & Dvorak 2012).

However much less HCOs were found in the work of Schwarz & Dvorak (2012), compared to us. This is because only certain peculiar regions in orbital elements can drive asteroids in close approaches with terrestrial planets, and even more peculiar ones give rise to asteroids in co-orbital motions.

3.2.2 Life time and orbits of the HCOs

The HCOs have a short lifetime (or libration period), a mean of ~ 10 kyr (9.6 kyr for the Earth and 9.0 kyr for Mars), with an exception for a jumping Trojan of 2002 RN137, which stays in this condition for more than 50 kyr. However HCOs have lifetimes that usually range between 1 kyr and 20 kyr (Fig. 5). These results are in accordance with the life-time values found for real co-orbital asteroids, e.g. about 6.8 kyr for 2010 TK7, a jumping-Trojan for the Earth, (Connors, Wiegert & Veillet 2011) or 1998 VF31, an L5 Mars Trojan, with a lifetime of 1.4 kyr (de la Fuente Marcos & de la Fuente Marcos 2012). These objects are usually transitional objects (with short dynamical life times) and the most stable HCOs have an inclination between i = 10° and i = 17°, see also Fig. 6.

The Hungarias with smaller escape time from their original cloud have a shorter lifetime as co-orbital objects. In fact Horseshoes have a shorter lifetime (or libration period), a mean of 1.5 kyr (1.5 kyr for the Earth and 1.5 kyr for Mars), with an exception for a jumping Trojan of 2002 RN137, which stays in this condition for more than 50 kyr. However HCOs have lifetimes that usually range between 1 kyr and 20 kyr (Fig. 5). These results are in accordance with the life-time values found for real co-orbital asteroids, e.g. about 6.8 kyr for 2010 TK7, a jumping-Trojan for the Earth, (Connors, Wiegert & Veillet 2011) or 1998 VF31, an L5 Mars Trojan, with a lifetime of 1.4 kyr (de la Fuente Marcos & de la Fuente Marcos 2012). These objects are usually transitional objects (with short dynamical life times) and the most stable HCOs have an inclination between i = 10° and i = 17°, see also Fig. 6.

The Hungarias with smaller escape time from their original cloud have a shorter lifetime as co-orbital objects. In fact

Table 3. Percentage of Hungaria captures in different classes (P. H.) from the total, average life time of the capture (t_l). The * means that the total number of satellites is not equal to the sum of the total number of satellites and quasi satellites, because some clones can become QSs or satellites too, during their evolution.

| Class          | P. H.   | t_l [ky] |
|----------------|---------|----------|
|                 | Earth   | Mars     |
| Satellites      | 0.6     | 0.6      | 7.5±8.5 | 7.9±7.1 |
| Quasi-satellites| 1.3     | 1.3      | 15.0±9.7| 7.0±4.9 |
| Satellites (Subtot*) | 1.9 | 1.9      | 9.6±1.6 | 9.0±4.6 |
| Trojans         | 1.1     | 1.1      | 7.9±3.9 | 10.7±4.4 |
| Horseshoes      | 0.0     | 0.4      | -       | 2.6±1.5 |

Table 4. Relative probability of each fugitive to become HCO (Tot.) and for each planet in percentage.

| Asteroid       | Mars   | Earth | Tot. | H | P. H. |
|----------------|--------|-------|------|---|------|
| (211279) 2002 RN137 | 2       | 6     | 8    | 0.6 | 0.6  |
| (152648) 1997 UL20  | 0       | 6     | 6    | 0.6 | 0.6  |
| (141096) 2001 XB48  | 4       | 0     | 4    | 0.6 | 0.6  |
| (24883) 1996 VG9    | 4       | 0     | 4    | 0.6 | 0.6  |
| (41577) 2000 SV2    | 4       | 4     | 8    | 0.6 | 0.6  |
| (175851) 1999 UF5   | 0       | 2     | 2    | 0.6 | 0.6  |
| (39561) 1992 QA     | 4       | 2     | 6    | 0.6 | 0.6  |

Table 5. Captured asteroids from different region in percentage to the total. Regions are described in the text. C− is for i < 17° and C+ for i > 17°. The numbers in the table are rounded to one digit. C is the region mentioned in Schwarz & Dvorak (2012) and H stands for HCOs at their initial conditions.

| PLANET | C−     | C+     | C_{tot} | H−     | H+     | H_{tot} |
|--------|--------|--------|---------|--------|--------|---------|
| Earth  | 0.0    | 0.0    | 0.0     | 0.0    | 1.5    | 1.5     |
| Mars   | 0.1    | 0.1    | 0.0     | 0.0    | 1.9    | 1.9     |
The inclination angles range from \( \sim 3^\circ \) to very high inclined orbits, \( \sim 40^\circ \). The range for the inclination of the EHCs is in accordance with past studies, in fact stability windows for Earth-Trojans are covered and no cases are found between 24\(^{\circ}\) < \( i < 28^\circ \). The inclination of the EHCs range on average between 15\(^{\circ}\) and 18\(^{\circ}\) and for Trojan \( \sim 16^\circ \) (Table 7). Windows for Earth-Trojans, established by past works until now are: (a) \( i < 16^\circ \), (b) \( 16^\circ < i < 24^\circ \) (Tabachnik & Evans 2001) and (c) \( 28^\circ < i < 40^\circ \) (Dvorak, Lhotka & Zhou 2012).

Mars HCOs (from now on MHCs) have in general high inclined orbits close to the original orbits, in fact they are less perturbed by close encounters compared to the EHCs. A remarkable thing is that the MHC satellites have less inclined orbits than other type of MHCs, see Table 7. The EHCs are usually less inclined than the MHCs, but the eccentricity is larger: \( e_{EHC} \approx e_{MHC}/3 \). The eccentricity of Mars and Earth tad-pole orbits are similar, about \( \sim 0.32 < e < \sim 0.36 \) on the average. The Earth Hungaria MOs have larger eccentricities than tad-pole orbits of both planets and also than MHC satellites (summarized in Table 7).

The Tisserand parameter (Table 7) shows different values for each kind of co-orbital objects, in general the \( T_j \) (3 as Jupiter) of the QSs is usually higher. The typical
Table 6. Dispersion in semi-major axis during co-orbital motion. Sat. = satellites, QSs = Quasi Satellites, Troj. = Trojans, hors. = horseshoe orbits, E = Earth and M = Mars. $\Delta_1,M = maximum dispersion in semi-major axis for Martian HCOs and $\Delta_2,M = minimum dispersion in semi-major axis for Martian HCOs. All measures are in units of $10^{-4}$ au.

| Class | $\Delta_E$ | $\Delta_1,E$ | $\Delta_2,E$ | $\Delta_M$ | $\Delta_1,M$ | $\Delta_2,M$ |
|-------|------------|--------------|--------------|------------|--------------|--------------|
| Sat.  | 17 ± 18    | 42           | 11 ± 7       | 19         | 5            |              |
| QSs   | 26 ± 3     | 34           | 16 ± 3       | 22         | 11           |              |
| Troj. | 21 ± 7     | 29           | 13 ± 3       | 18         | 6            |              |
| hors. | 18 ± 2     | 19           | 17           | -          | -            | -            |

Table 7. Orbital ranges for HCOs. EHCs = Earth Hungaria Co-orbital objects and MHCs = Mars Hungaria Co-orbital objects. $T_p$ and $T_j$ are respectively the Tisserand parameter relative to the planet in case (Mars and the Earth) and to Jupiter. $\bar{\Delta}E$ = average semi-major axis for EHCs, $\bar{\Delta}e$ = average eccentricity for EHCs and $\bar{\Delta}i$ = average inclination for EHCs. Sat. = satellite, QSs = Quasi Satellites and Troj. = Trojans.

| Class | $\bar{\Delta}E$ [au] | $\bar{\Delta}e$ | $\bar{\Delta}i$ |
|-------|---------------------|-----------------|-----------------|
| Sat.  | 0.9999 ± 0.0011     | 0.431 ± 0.010   | 17.5 ± 0.9      |
| QSs   | 5.94 ± 0.007        | 2.687 ± 0.007   |                 |
| Troj. | 1.0000 ± 0.0011     | 0.321 ± 0.004   | 15.8 ± 0.2      |
| EHCs  | 1.0000 ± 0.0013     | 0.507 ± 0.011   | 16.1 ± 1.0      |
| MHCs  | 5.923 ± 0.007       | 2.643 ± 0.005   |                 |

Figure 9. Average (during its co-orbital motion) orbital elements of the Earth HCOs in comparison with the real Earth co-orbital asteroids (circles with a dot inside): eccentricity versus inclination. The diameter of the circle is correspondent to their life time (only for the HCOs). Earth is represented by the largest dot with the least eccentricity and the least inclination.

This result – displayed in Fig. 9 and 10 – seems to assert that co-orbital bodies which have a range in inclinations of $5^\circ < i < 40^\circ$ and large eccentricities $0.22 < e < 0.53$ (even if most of the Mars HCOs are between 0.2 and 0.4) can be of Hungarian origin. The majority of the HCOs lie between $i = 5^\circ$ and $i = 17^\circ$. In this case the most probable former Hungarians are: Cruithne, 2006 FV$_{35}$, (85770) 1998 UP1 and YORP for the Earth and, due to their inclination, 1999 UJ$_7$, 1998 FV$_{31}$ and 2011 SL$_{25}$ for Mars.

Concerning the physical characteristics of the known co-orbital bodies for terrestrial planets, some spectral type of these are known: for the Earth, Cruithne is a Q/S type; YORP is S/V type (from TNEADB); for Mars, 1997 UJ$_{27}$ is an X type, 1998 FV$_{31}$ is an S type. Rivkin et al. (2003) says it is an S(I) type (angrite) and Rivkin et al. (2007) suggest also for a S(VII) type (achondrite similar to the spectrum of 40 Harmonia). So Cruithne and 1998 FV$_{31}$ can again still be considered as possible HCOs, because Hungarians have some S-type asteroids, not the majority, but still 17% (Warner et al. 2009) and especially 1997 UJ$_7$, which is an X-type asteroid like the majority of the Hungarians (Carvano et al. 2001, Warner et al. 2009), even if not specified for the sub-group Xe-type and further spectroscopic analysis would be needed.

Considering the size of the terrestrial planets’ co-orbital bodies, we know the sizes which range from the very small 2013 BS$_{45}$ of about 10-40 m to Cruithne of about 3.3 km in approximate diameter, but usually they are less than 1 km, similarly to the standard range of the Hungarians.

$T_j$ of the EHCs ranges around $T_j = 5.923$ and it is higher than the one of the MHCs, $T_j = 4.362$. Then the Tisserand parameter compared to the relative planet (but also in respect to Jupiter) is higher for the MOs than the QSs.

Some real co-orbital objects of the terrestrial planets could be of Hungarian origin as shown in Tab. 6 and 7. We compare the osculating elements (the average ones during the libration life, Fig. 9 and 10) of the HCOs with the real co-orbital asteroids described in Table 1. The HCOs favor co-orbital bodies with large eccentricity and indicate that it could be possible to find co-orbital bodies for Mars and Earth at large inclinations too, even more than $i = 30^\circ$.  

\[ T_j = 5.923 \]

\[ T_j = 4.362 \]  

\[ T_j = 3.3 \]  

\[ T_j = 1.0 \]  

\[ T_j = 0.0005 \]

9 From The Near-Earth Asteroids Data Base (TNEADB) at earn.dlr.de/nea/table1_new.html
planets and finally captured into co-orbital motion are the following ones:

(i) close encounters with Mars and Earth. Especially for the Earth case, the close encounters decrease the inclination in such a way that the Hungarias enter the window of stability for the Earth co-orbital objects as shown in Tabachnik & Evans (2000).

(ii) resonances: SRs, such as $q_5$ and $q_6$; MMRs such as $M_3:4$, $M_{11}:13$ and $J_{16}:13$ and $3BMMRs$, such as $J13-S10-2$ and $J5-S1-1$.

The average libration period of the HCOs is quite short $\sim 10\,\text{kyr}$ (9.6 kyr for the Earth and 9.0 kyr for Mars), in accordance with the real co-orbital objects of terrestrial planets, (e.g. 6.8 kyr for 2010 TK$_7$; Connors, Wiegert & Veillet 2011). Furthermore our investigations show that the Hungarias with the shortest lifetime are the first ones to escape from the Hungary cloud, i.e. 2000 SV$_2$, Tab. 2.

Mars is the planet which captures more Hungarias, because of the shorter distance, even if the difference between Mars and the Earth capture probability is relatively small. Probably this can explain the smaller number of Earth co-orbital asteroids compared to Mars, because many asteroids will be captured by Mars (close encounters make the bodies achieving too large eccentricities to become Earth co-orbital bodies). However, the evolution of other families of the main belt in this sense should be studied in more detail in the future.

Concerning the HCOs’ orbits, they range from $i \sim 3^\circ$ to $i \sim 40^\circ$. The EHCs average inclination range is $15^\circ < i < 18^\circ$ and for Trojans $i \sim 16^\circ$; this is in agreement with Schwarz & Dvorak (2012). The high inclined HCOs are favourable for MHCs instead of for EHcs and among the MHCs the satellites have the lowest inclined orbits ($14.9^\circ$). The eccentricity of the EHCs is on average 3 times the one of the MHCs, only for the tad pole orbits, it is similar. The EHCs satellites have $e = 0.52$, and the Martians $e = 0.37$.

The typical Tisserand parameter with respect to Jupiters for EHCs is $T_j = 5.923$ and for MHCs is less, $T_j = 4.392$ and for satellites $T_j$ is higher than the QSs.

Some real co-orbital asteroids have orbits which have a high probability to be former Hungarias, like Cruithne, (277810) 2006 FV$_{35}$, (85770) 1998 UP$_1$ and YORP for EHCs and (101429) 1998 VF$_{31}$ and, (88719) 2011 SL$_{25}$ for MHCs. In particular Cruithne and 1998 VF$_{31}$ have both orbital elements and physical characteristics typical of the S-type HCOs. Further investigations have to be done to look for the origin of present co-orbital objects of the terrestrial planets, both dynamical and observational studies. In the next work we will perform a new study for HCOs taking into account also non-gravitational forces.

4 CONCLUSIONS

The capture of Hungarias as Earth or Martian Trojans is not only due to the migration of planets, but also to migration of asteroids from the Main Belt, this is proved physically by Rivkin et al. (2003), who show that two Martian Trojans are collision fragments of a larger body. Numerically, this migration towards the terrestrial planet was described by Galiazzo, Bázsó & Dvorak (2013a), which emphasize the gravitational perturbations. Then, even Cuk, Gladman & Nesvorný (2014) show in particular the delivery of the aubrite meteorites by the Hungarian family, using also non-gravitational forces in the computation of the possible orbits.

The existence of HCOs have a low probability, 3.3% of all Hungary fugitives, but nevertheless the contribution of the Hungary region is important in order to give rise to co-orbital objects for terrestrial planets and so the Hungary region is one of the source-regions of the Main Belt for this kind of bodies. The capture of possible co-orbital Mars objects, is about 1.8%, and for the Earth, it is 1.5% of the total amount of the clones of the Hungary fugitives in 100 Myr of evolution. The Hungarias which have the highest probability (8%) to become co-orbital objects of terrestrial planets are 2002 RN$_{137}$ and 2000 SV$_2$. The first time they become co-orbital objects on average is at $\sim 70$ Myr after their orbital evolution from the original (present) position. The HCOs majority become QSs and concerning the former Hungarias captured into tad pole motions, they will be captured around $L_3$. We found less captures for $L_4$ Trojans for both planets and we did not find any Venus HCOs, in agreement with the present observations. There are some cases of Jumping Trojans and with the longest life time, the maximum detected life time is 58 kyr. Also many HCOs behave like transitional co-orbital objects. Some Hungarias can become co-orbital objects of both planet together, i.e. 2000 SV$_2$ and some exclusively of only one like 1997 UL$_{20}$ and 1999 UF$_3$ of Mars and, 2001 XB$_{48}$ and 1996 VG$_9$ of the Earth.

The mechanism found in this work to transport the asteroids from the Hungary region close to the terrestrial
REFERENCES

Assandri M. C. & Gil-Hutton R., 2008, A&A, 488, 339
Barrabés, E., Mikkola, S., 2005, A&A, 432, 1115
Bottke W. F., Morbidelli, A., Jedicke R., Petit J. M., Levison, H. F., Michel, P. & Metcalfe T. S. 2002, Icarus, 156, 399
Bottke W. F., Vokrouhlický, D., Rubincam, D. P. & Nesvorný D. 2006, Annu. Rev. Earth Planet. Sci., 34, 157
Bottke W. F., Vokrouhlický D., Minton D., Simonson B. & Levison H. F. 2012, Nature, 485, 78
Brasser, R., Innanen, K. A., Connors, M., Veillet, C., Wiegert, P., Mikkola, S., Chodas P. W., 2004b, Icarus, 171, 102
Carvano J. M., Lazzaro D., Moth e-Diniz T., Angeli C., A & Florczak M. 2001, Icarus, 149, 173
Connors, M., Chodas, F., Mikkola, S., Wiegert, P., Veillet, C., Innanen, K., 2002, Meteoritics & Planetary Science, 37, 1435
Connors, M., Wiegert, P., Veillet, C., 2011, Nature, 475, 481
Čuk, M., Gladman, B. J. & Nesvorný, D., 2014, Icarus, 239, 154
DeMeo, F. E. & Carry, B., Nature, 2014, 505, 629
de la Fuente Marcos, C., de la Fuente Marcos, R., 2012, MNRAS, 432, L31
Dermott, S.F., Murray, C.D., 1981, Icarus, 48, 12
Dvorak, R., Pilat-Lohinger, E., Schwarz, R., Freistetter, F., 2004, A&A, 426, L37
Dvorak, R., 2006, Freistetter et al. (eds.): Proceedings of the 4th Austrian Hungarian Workshop on Trojans and related topics, Eötvös University Press Budapest, 63
Dvorak R., Schwarz R., 2005, CeMDA, 92, 19
Dvorak, R., Schwarz, R., Síli, Á. and Kotoulas, T., 2007, MNRAS, 382, 1324
Dvorak, R., Lhotka, Ch., Zhou, L., 2012, Astron. Astrophys., 541, A127
Eggl, S., Dvorak, R., 2010, LNP, 790, 431
Galiazzo M. A., Bazsó, A., Dvorak, R., 2013a, P&SS, 84, 5
Galiazzo M. A., 2013c, PhD thesis
Gomes, R., Levison, H. F., Tsiganis, K., Morbidelli, A., 2005, Nature, 435, 466
Greenberg, R. & Nolan, M., In: Binzel, R.P., Gehrels, T., Matthews, M.S. (Eds.), Asteroids II. The University of Arizona Press, Tucson, p. 778 (1989)
Greenberg, R. & Nolan, M., In: Lewis, J., Matthews, M. S., Guerrieri, M. L. (Eds.), Resources of near-Earth space. The University of Arizona Press, Tucson, 473 (1993)
Greenstreet S., Ngo H. & Gladman B. 2012, Icarus, 217, 355
Hanslmeier, A., Dvorak, R., 1984, A & A, 132, 203
Llibre, J., Ollé, M., 2001, A&A, 378, 1087
Lykawka P. S., Horner J., 2010, MNRAS, 405, 1375
Lykawka P. S., Horner J., Jones B.W., Mukai T., 2009, MNRAS, 398, 1715
Marzari, F., Scholl, H., 1998, A&A, 339, 278
Milani, A., Carpino, M., Hahn, G., Nobili, A. M., 1989, Icarus, 78, 212
Milani, A., Knezević, G., Novaković, B., Cellino, A., 2010, Icarus, 207, 769
Minton, D. A., Malhotra, R., 2011, AJ, 732, 53
Morais, M. H. M., Namouni, F., 2013, MNRAS, 436, L30
Morbidelli, A., Brasser, R., Gomes, R., Levison, H. F. & Tsiganis, K., 2010, Astron. J., 140, 1391
Namouni, F., Murray, C. D., 1999, Icarus, 137, 293
Namouni, F., Murray, C. D., 2000, CeMDA, 76, 131
Nesvorny, D., Morbidelli, A., 1998, AJ, 116, 3029
Rivkin, A. S., Binzel, R. P., Howell, E. S., Bus, S. J. and Grier, J. A., 2003, Icarus, 165, 349
Rivkin, A. S., Trilling, D. E., Thomas, C. A., DeMeo, F., Spahr, T. B. & Binzel, R. P., 2007, Icarus, 192, 434
Robutel, P., Gahagan, F., Tzorbas, A., 2005, CeMDA, 92, 53
Sándor, Zs., Érdi, B., Efthymiopoulous, C., 2000, CeMDA, 78, 113
Schwarz, R., Dvorak, R., Síli, Á., Érdi, B., 2007, A&A, 474, 1023
Schwarz, R., Dvorak, R., 2012, CeMDA, 113, 23
Tabachnik, S. A., Evans, N. W., 2000, MNRAS, 319, 63
Tsiganis, K., Dvorak, R. & Pilat-Lohinger, E., 2000, 1091
Warner B. D., Harris A. W., Vokrouhlický D., Nesvorný D. & Bottke W. F. 2009, Icarus, 204, 172
Wiegert, P. A., Innanen, K. A., Mikkola, S., 1998, Astron. J., 115, 2604
