Where do we stand with a 3-D picture of the proton?

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Abstract. We are on the way to obtaining multi-dimensional “pictures” of the proton. The field is bursting with activities, both from the theoretical and experimental side. A brief selection of important achievements of the last years and open challenges for the future is presented. They are already well documented in the various reviews of this Special Issue, but they are gathered here in a more condensed way for convenience, together with some additional remarks. The choice of the items included in this short overview is far from being complete and represents the view of the author.

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1 Why 3-D maps are interesting?

This Special Issue is entirely devoted to the description of the state of the art of the field of 3-D proton “maps.” It gives an impressive overview, both wide and deep, which perfectly conveys the idea of how vibrant this field of research is. Reading the excellent reviews of this Special Issue, it is unavoidable to be surprised by the balanced mixture of theoretical and experimental advances, by the long list of achievements and the even longer list of to-dos. The field can be rightfully considered as expanding.

What are the characteristics that make this field of research flourish in these years?

First of all, this field is focused on one of the “core missions” of Physics: understanding the fundamental constituents of matter.

This question has changed its meaning in the course of the last century. We changed our perspective on the meaning of “fundamental,” “constituent,” and even “matter.”

For what concerns the word “matter,” we discovered the existence of dark matter and possibly even of dark energy. Normal matter around us is just a small fraction of the total matter of the universe. Our curiosity urges us to understand dark matter and dark energy, but this does not diminish our interest in normal matter, which remains of particular relevance for our life and existence.

Concerning the word “constituents,” one century ago we thought that the constituents of matter were electrons, nuclei, photons. Nowadays the list of elementary particles includes three families of leptons, three families of quarks, photons, gluons, weak bosons, the Higgs boson, and all the antiparticles. A good number of these particles is never actually “seen” in nature. We see their indirect effects.

For what concerns the word “fundamental,” nowadays at the fundamental level of our description of Nature we place quantum fields, which have no underlying components and are elementary. To the best of our knowledge, electrons are elementary. Hadrons are not elementary quantum fields, they do not appear in the Lagrangian of the Standard Model, they emerge as bound states of quarks and gluons. Hadrons are in fact the smallest non-elementary structures of the universe: understanding them is the most basic example of going from an underlying layer of “simple” constituents to an emerging layer of “complex” structures.

Hadrons are not elementary, but they are fundamental. We can consider atoms to be the fundamental building blocks in Chemistry, cells are the fundamental building blocks in Biology. In the same way, protons and neutrons can be rightfully counted among the fundamental constituents of matter.

In short, when studying the structure of protons, neutrons, and hadrons in general, we are focusing our attention on a few non-elementary constituents of a small fraction of matter. However, this statement can be turned into a different one: we are focusing our attention on the most relevant fundamental constituents of our world’s matter.

A second crucial characteristic of this field of research is that it is an interface field. Interface between theory and experiment (and computation), between high-energy and nuclear physics, perturbative and nonperturbative, elementary and complex.

For instance, the study of the 1-D structure of the proton, encoded in collinear PDFs, started in the Seventies at facilities sitting at the edge of the energy frontier. The emphasis at that time was understanding the structure of the proton and QCD. In the following decades, predictions...
based on QCD factorization theorems – in particular scaling violations and DGLAP evolution equations – became one of the most important and successful tests of perturbative QCD. In the last decades, PDFs are studied not only because of the information they provide about QCD, but mainly because they are useful to access and understand other aspects of the physics of the Standard Model and beyond. This is the natural destiny of many scientific discoveries: first we understand, then we use. First we understood how to define and extract PDFs, then we used (and still use) them to make predictions and look for new physics. The two steps are however not always clearly separated, nor strictly consequential. For instance, very often we use information without completely understanding it. And using it, we often improve our understanding. To come again to the example of PDFs, we can claim that we understand how to extract PDFs in a reliable way (at least for certain parton flavors and in certain range of $x$). But we have to admit that we do not understand how the details of PDFs arise from QCD, since we are not able to solve QCD in the nonperturbative regime. We are using something that we do not fully understand.

What is the meaning of understanding the structure of the proton? A possible answer could be: being able to encode our knowledge of the proton in some formulas or software that is able to describe what we have measured about the proton and make predictions for something that we have not measured yet. This task is extremely difficult, but not hopeless. We are already able to obtain some excellent results from lattice QCD. They are however limited to certain aspect of proton physics and only indirectly related to 3-D pictures. Furthermore, in order to prove itself to be really useful, lattice QCD should be able to make predictions that can be tested against new measurements. The field of 3-D pictures may become the best arena to develop and test lattice QCD.

Maybe understanding the proton’s structure will turn out to be too an ambitious goal. Nevertheless, the amount of information we are gathering with 3-D proton studies is so large that we will certainly find ways to use it. We will partially follow the footsteps of 1-D studies and use our knowledge to make predictions for processes involving hadrons. For most observables, the knowledge of the 1-D structure of the hadrons is enough. But for some observables, the details of the 3-D structure are necessary.

Stretching our imagination even further, we could wonder if there will ever be some application based on the detailed knowledge of the structure of the proton. This is obviously a very speculative topic. The energies “stored” in nucleons are hundreds of times larger than those stored in nuclei. To tap this source of energy, we would need to be able to transform one hadron to another. It is not something that we can do with natural elements, therefore it is unlikely that we can produce energy using QCD. However, in principle it could be possible to use QCD to store energy. More generally, the closest we get to “using” QCD are applications such as hadron therapy and nuclear fusion. Neither of them make use of the knowledge of the internal structure of the proton. It is therefore at present very difficult to foresee any application of this knowledge. The same, however, can be said of the Higgs boson and of gravitational waves, and yet they remain strong driving forces for progress in science and elsewhere.

In the next sections, a brief selection of important achievements of the last years and open challenges for the future is presented. They are already well documented in the various reviews of this Special Issue, but it is convenient to partially repeat them also here. However, the list of achievements and open challenges is so long that it is impossible to be complete. The choice represents the view of the author. Also, the list of references is necessarily incomplete and the reader is invited to look into the individual reviews for a more extended bibliography.

2 TMDs

From the point of view of TMDs, we have accomplished gigantic steps forward in the last ten years.

TMD factorization, universality, and evolution properties have been placed on a firm theoretical ground, thanks to important contributions from several groups (see, e.g., Refs. [1,2,3,4,5] and the reviews by Rogers and Diehl in this Special Issue [6,7]). We have established that T-odd distribution functions such as the Sivers function can exist [8]. We discovered that TMDs, in particular T-odd ones, get modified when going from a process to another. The origin of these differences rests on the structure of the Wilson lines included in the proper definition of TMDs [9,10]. In practice, this results in a sign difference between semi-inclusive DIS and Drell-Yan [11]. More complicated color prefactors were supposed to appear in other processes, for instance in proton-proton collisions to hadrons [12,13]. However, it was later demonstrated that in these processes it is not possible to disentangle the Wilson line structure and factorization is thus broken [14,15,16]. Fragmentation functions are not affected by universality problems, therefore no differences are expected going, for instance, from semi-inclusive DIS to $e^+e^-$ annihilation [17]. TMD evolution equations were derived in a clear way, and the connection with the literature of transverse-momentum resummation was made manifest. At least for the unpolarized TMD case, the elements involved in the evolution equations are now known up to Next-to-Next Leading Logarithmic accuracy (see, e.g., Ref. [18]).

From the experimental point of view, a vast amount of data coming from semi-inclusive DIS has been collected, thanks to measurements performed mainly at HERMES and COMPASS (see also the reviews by Avakian, Bressan, Contalbrigo and Prokudin, Boglione [19] in this Special Issue). HERMES played a truly pioneering role in these advances, with the first measurements of single transverse-spin asymmetries in DIS [20], closely followed by COMPASS [21], which keeps delivering new data also today. The driving interest initially was to measure transverse-spin dependent azimuthal modulations, with the goal of accessing the transversity distribution function. Longitudinally polarized azimuthal asymmetries have been measured long before, as well as unpolarized observables. Grad-
The driving interest to study transverse spin observables was the goal of studying the transversity distribution through the so-called Collins effect. This milestone has been achieved in a series of papers that carried out the analysis in a constantly improving way (see, e.g., Refs. [46, 53]). The state-of-the-art extractions make full use of TMD evolution equations [59]. All extractions so far are in acceptable agreement with each other and they are also in fair agreement with independent extractions based on collinear factorization in dihadron production (see Refs. [61, 62]) and the review by Pisano, Radici in this Special Issue [64]). In conclusion, we can claim that we are able to extract the transversity distribution, that the up transversity is sizable in the medium-large $x$ region and positive (the absolute sign actually cannot be fixed by experiments), that the down transversity can be large and negative, although its determination is affected by larger errors compared to the up quark. Nucleon tensor charges (i.e., combinations of integrals of transversity distributions) have been estimated, although they are affected by extrapolation uncertainties: the extraction in Ref. [53] is in agreement with most recent lattice QCD estimates for the isovector combination $u - d$ [65-68]. The extraction in Ref. [59] is estimated at a higher scale compared to the lattice calculations. Considering the flavors separately, the extraction of the $u$ tensor charge are typically lower than lattice results. None of the extractions has taken sea quarks into considerations.

The Sivers function is large and positive for up quarks. For down quarks it has an opposite sign, but is, somewhat unexpectedly, of the same size as the up quark [57, 58]. Up and down sea quark contributions are small in the $x$ region explored so far, strange quarks are poorly constrained [50].

Unpolarized TMDs are not yet constrained in a satisfactory way. They are present in all measurements, therefore we would expect them to be better known than anything else. But since they are so ubiquitous, it is not sufficient to describe their qualitative features: some precision is required. In order to make some simple statements, let us consider the position of the peak in the distribution $|k_T f(x, k^2_T)|$ as a measure of the "width" of a TMD $f(x, k^2_T)$. For a TMD with a Gaussian shape, this width is equal to $\sqrt{\langle k^2_T \rangle / 2}$. This particular definition is useful because it can be applied to any function, even if not integrable, which would not be the case for, e.g., the average transverse momentum squared $\langle k^2_T \rangle$. Present extractions (see, e.g., Refs. [51, 52, 55, 60]) indicate that the width of the TMDs at low scales, 1-2 GeV, is around 300-500 MeV. The width increases to more than 1 GeV at the $Z$ mass, due to TMD evolution. Data indicate also that the width is probably increasing as $x$ decreases and there is room for a strong flavor dependence, even though also a flavor-independent scenario is not ruled out [61].

Several model calculations of TMDs, GPDs and Wigner distributions have been presented in the last years (see references in the contribution by Burkardt and Pasquini to this Special Issue [20]). They are in general able to capture the qualitative features of form factors and collinear PDFs, but they are still far from giving a description that

| nucleon pol. | U | L | T |
|--------------|---|---|---|
| U | $f_1$ | $h_1^T$ |  |
| L | $g_1$ | $h_{12}$ |  |
| T | $f_{1T}$ | $g_{1T}$ | $h_1$ $h_{1T}$ |

Table 1. Twist-2 Transverse-Momentum-dependent Distribution functions (TMDs). The U,L,T correspond to unpolarized, longitudinally polarized and transversely polarized nucleons (rows) and quarks (columns). Functions in boldface survive transverse momentum integration.
is satisfactory from the quantitative point of view. However, models have been used especially to illustrate the physics content of TMDs, to predict their qualitative behavior and to give an estimate of the possible size of unknown observables. The most important example of the relevance of model calculations has been the proof that the Sivers function can be nonzero \cite{8}. Models also usually display some nontrivial relations among TMDs and GPDs (see, e.g., Refs. \cite{71,72,73,74,75,76,77,78,79,80}). Some of these relations are broken in perturbative QCD. However, they could still be valid approximations at the threshold between the nonperturbative and perturbative regimes. As such, they could be useful conditions to guide TMD parameterizations.

In spite of these impressive results, the formalism has to be tested at a higher level of precision than currently done. It is not easy to find a clear-cut way to check the validity of the formalism. Even in the case of collinear PDFs, what “proves” that the formalism is valid is our ability to perform global fits that include data from different processes and at different energies. For the case of TMDs this has been done only in a limited way. TMDs have been extracted from Drell-Yan and Z-production data for values of $Q$ between 5 and 100 GeV (see, e.g., Refs. \cite{81,82,83}). In semi-inclusive DIS, data from fixed-target experiments have played a truly pioneering role for TMDs, but by themselves are not sufficient to check the validity of the formalism. First of all, the range of $Q$ values is limited, going from 1 to about 5 GeV. Secondly, there is a strong correlation between the kinematical variables $Q$ and $x$. To study the effects of TMD evolution, it is necessary to keep $x$ fixed and vary $Q^2$. So far, this has been possible only for a few bins of COMPASS measurements. This has been investigated in Ref. \cite{83} and led to the conclusion that the effects of TMD evolution are small in COMPASS data. To make the situation worse, the data themselves are supposedly affected by some normalization error that hamper our conclusion (see Erratum of Ref. \cite{83}).

There is a first clear-cut check that needs to be done: verifying the sign change of the Sivers function in Drell-Yan compared to semi-inclusive DIS. This is one of the rare occasions when a sharp prediction has some profound origin and leads to vast consequences. The prediction is based on factorization theorems and the structure of Wilson lines in the definition of TMDs. If this expectation were falsified, it would mean that there is something very general that we do not understand about TMD factorization. In the less dramatic scenario, it would mean that we misunderstand the nature of the final-state interactions giving rise to T-odd effects, which we presently attribute to soft light-cone gluon exchanges. In a more dramatic scenario, the falsification of the sign-change prediction could demand an extensive check also of what we normally take for granted not only in TMD factorization, but also in collinear factorization.

If the Sivers function sign change is confirmed, this would give us confidence in the validity of the TMD framework. It would however be just the beginning of a long journey into the comprehension of single-spin asymmetries. These asymmetries are present not only in the relatively clean context of semi-inclusive DIS and Drell-Yan twist-2 observables, but also in the historically famous $A_N$ asymmetries in hadron-hadron collisions (see, e.g., Refs. \cite{84,85,86,87,88}). To mention only the most recent ones, and the review by Aschenauer, D’Alesio, Murgia in this Special Issue \cite{89}), in exclusive proton-proton collisions \cite{90,91,92}, in inclusive pion electroproduction \cite{93,94,95,96,97}. It should be possible to explain these phenomena with a single common language, but at the moment we just started scratching the surface of this question. For hadron-hadron collisions, at high transverse momentum we know that the appropriate language is that of collinear twist-3 factorization \cite{98,99,100,101,102,103}. We also know that there are connections between this language and twist-2 TMD factorization. We are far from understanding to which extent these connections are phenomenologically useful and survive QCD corrections. In the meanwhile, $A_N$ asymmetries can be qualitatively described in the context of the so-called Generalized Parton Model \cite{98,99,100}. One should be a connection with the Wandzura-Wilczek approximation for twist-3 DIS observables \cite{101}. However, in this approximation T-odd components are normally discarded, in contrast with the Generalized Parton Model. Therefore, if anything, the latter should be considered as a Wandzura-Wilczek approximation with the addition of twist-2 T-odd terms.

An open problem from the point of view of the TMD formalism is twist-3 factorization. We know that there are unexpected mismatches between the results obtained with the TMD approach at twist 3 and the collinear twist-3 approach \cite{102,103}. Maybe this is a signal of the impossibility to obtain factorization at twist 3, but maybe this is an opportunity to push further our knowledge of QCD and its technology.

The question of TMD factorization breaking in $pp$ collisions to hadrons demands further scrutiny at all levels: formal, experimental, and phenomenological. From the formal point of view, efforts should be made in finding observables that are not affected by color entanglement problems, for instance transverse-momentum-weighted or Bessel-weighted quantities \cite{104}. On the contrary, identifying observables that are clearly related to factorization breaking would be extremely useful \cite{105,106,107,108,109}. From the experimental point of view, the challenge is to either find the signs of factorization breaking, or to constrain their size. In order to do that, probably the only safe avenue is to collect as much data as possible in $pp$ collisions, combine it with data from other processes and look for tensions in phenomenological studies.

From the experimental point of view, we need to extend the measurements in all possible ways. For what concerns semi-inclusive data at fixed target experiments, we are waiting for a new analysis of COMPASS multiplicities, for the final publication of all measured azimuthal asymmetries from HERMES and COMPASS, for further data
from COMPASS, and for the whole mass of results coming from JLab at 12 GeV. All of the measurements should be done with multidimensional binning. Where possible, they should be done for different targets (proton, deuterium, helium) and for different final-state hadrons (pions, kaons, protons). Attention should be paid to twist-3 observables. Beyond this, we look with great expectations to the construction of an Electron Ion Collider (EIC), the ultimate machine for 3-D proton studies (see the dedicated review by R. Ent in this Special Issue).

Even if we extend the range of kinematics with an EIC, semi-inclusive DIS will not be sufficient by itself to disentangle the role of PDFs and FFs. Drell-Yan data will give an invaluable contribution. We look forward to having results from COMPASS, Fermilab, Brookhaven, and even the LHC. COMPASS data will give unprecedented information also about the 3-D structure of the pion, the simplest of all hadrons, and yet only poorly known \[^{110}\]. Brookhaven, with its unique feature in being the first and only polarized proton-proton collider, should provide us with extended measurements of transverse single-spin asymmetries for the production of jets, direct photons, and inclusive hadrons (sensitive to twist-3 functions), for the production of W, Z, and Drell-Yan lepton pairs (sensitive to TMDs), for the production of hadron pairs in the same jet (sensitive to transversity), and finally for the production of two jets or two hadrons in different jets (potentially affected by TMD factorization breaking). Even if all the above facilities can perform experiments with polarization, it should not be forgotten that we need unpolarized data with the right characteristics for 3-D studies, in particular transverse-momentum dependence and multidimensional binning.

To take full advantage of semi-inclusive DIS data, it is also essential to obtain information about fragmentation functions and their 3-D dependence (see the review by Garzia and Giordano in this Special Issue). High-luminosity electron-positron colliders are ideal to push these studies forward. The single most important missing piece of information is at the moment the transverse-momentum dependence of unpolarized fragmentation functions. Priority should be given to measurements with sensitivity to these quantities, with full multidimensional dependence \((z_1, z_2, q_T)\), if possible with different hadron types (all combinations of two pions, pion-kaon, pion-proton, etc.). It is also important to scan different values of center-of-mass energy. Future measurements at BELLE and BESIII can perfectly suit these needs and will be complementary to each other thanks to the different energies. A careful study of experimental acceptance effects will be necessary, possibly calling for the need of Monte Carlo event generators that include spin effects \[^{111,112}\].

From the phenomenological side, extractions of TMD PDFs and FFs are now entering the so-called “Phase 2,” where the proper QCD definition of these objects is taken into account and evolution equations are considered. The questions to answer are many: how do TMDs change as a function of \(x\)? What are the differences between different flavors? What is the role of the nonperturbative part of TMD evolution?

Last but not least, very little is known about gluon TMDs of all kinds, starting from the simplest unpolarized one. Several interesting measurements have been proposed, especially related to the gluon Boer-Mulders distribution, describing linearly polarized gluons in an unpolarized target \[^{113,114,115,116,117,118}\]. These measurements could be carried out also at the LHC, or in the proposed fixed-target experiment at LHC, AFTER \[^{119}\]. Several challenging questions need to be addressed, among which certainly the farthest reaching one is the connection with the language of low-x gluon TMDs (often called in this context unintegrated distribution functions): the formalism at low \(x\) is very different from that of the standard TMDs, but it should be possible to find a common framework that contains both versions of the formalism as limiting cases (see, e.g., Refs. \[^{120,121,122,123}\]).

### 3 GPDs

In the last decade, a tremendous quantity of results related to GPDs was obtained. Compared to TMDs, less activity from the formal side has taken place. This is mainly due to the fact that the framework to analyze data and extract GPDs was already laid out in a rigorous way (see the review by M. Diehl in this Special Issue \[^{7}\]). However, several experimental measurements have been published, by collaborations at DESY and Jefferson Lab. The “golden channel” for GPD studies is Deeply Virtual Compton Scattering (DVCS) (see the review by Kumiricki, Linti and Montarde in this Special Issue \[^{124}\]). JLab Hall A has measured the DVCS unpolarized and beam-polarized cross sections \[^{125}\]. CLAS measured beam spin asymmetries and longitudinally target spin asymmetries \[^{126,127}\] and HERMES measured the complete set of beam charge, beam spin and target spin asymmetries \[^{128,129,130,131,132,133,134,135}\]. The DVCS unpolarized cross section has been measured also by the H1 and ZEUS collaborations \[^{136,137}\] at much higher energy compared to fixed-target experiments.

Complementary information comes also from Deeply Virtual Meson Production (DVMP), where it is possible to probe different combinations and different types of GPDs compared to DVCS (see the review by Favart, Guidal, Horn and Kroll in this Special Issue \[^{138}\]). Moreover, nucleon form factors are related to integrals of GPDs. Therefore, form factor measurements indirectly constrain GPDs and have a sharp impact on 3-D studies. Several parameterizations of GPDs have been presented in the literature in the last years (see, e.g., Refs. \[^{139,140,141,142,143,144}\] and see the detailed review by Guidal, Montarde, and Vanderhaeghen in Ref. \[^{135}\]).

DVCS is and probably will remain the cleanest source of information about GPDs. Even in this process, however, GPDs are not probed directly, but they rather appear in Compton Form Factors, which are weighted integrals of GPDs. In the past years, some efforts went into the extractions not of GPDs, but rather of Compton Form
Factors (see, e.g., Refs. [16,17,13,15,49]). This approach has the advantage that it does not require GPD modeling. However, it can only be considered as a step toward the final goal of GPD extractions.

On top of this, GPDs depend on three variables, e.g., $H(x, \xi, t)$. It is practically impossible to scan their multidimensional dependence. This problem, referred to as the “curse of dimensionality” [24], makes it extremely relevant to build models of GPDs that are at the same time solid and flexible. They should fulfill all fundamental properties of GPDs: this may seem an obvious statement, but its implementation is not so obvious. Moreover, even if all assumptions are correctly incorporated, there will still be too much freedom in the parameterization. Two extreme approaches can be followed: either we accept to constrain only limited regions of GPDs (e.g., the cross-over line, $x = \xi$), or we resort to models to extrapolate our knowledge beyond what is directly accessible. Probably, the right approach sits in between these two extremes: in addition to the formal requirements, some assumptions inspired by model calculations are required to limit the flexibility of the parameterization and constrain the extrapolations. This consideration holds true not only for GPDs, but in general for multi-dimensional studies. The model assumptions should be well-motivated and the use of several different models should help reaching more robust conclusions.

The interpretation of DVMP data is more challenging than DVCS data. This is first of all due to the presence, beside GPDs, of meson Distribution Amplitudes, nonperturbative objects that are not well known, describing the probability amplitude to find a $q \bar{q}$ pair. Secondly, there are some difficulties in explaining the behavior of DVMP observables. The leading-twist handbag diagram formalism predicts a well-defined behavior of the longitudinal and transverse components of the cross section. Data, however, do not confirm these predictions. The transverse cross section is typically larger than expected in the region where data exist. The longitudinal cross section does decrease with $Q^2$ as expected. These mismatches could be due to power and logarithmic corrections, but this still needs to be checked [150,151,152]. To understand the situation, it is important to have a separation of longitudinal and transverse cross sections for all the processes of interest. At the moment, such separation is not available in most of pseudoscalar meson measurements.

It was proposed to explain the large contribution of transverse photons in the light meson channels in terms of transversity GPDs [153,154]. This explanation has extra complications from the theoretical side, since it involves twist-3 pion wave functions and the presence of transverse momentum [155,156], calling for a transverse-momentum-dependent generalization of the established formalism. Phenomenological estimates seem to be able to describe data. It will be useful if these estimates can be put on stronger theoretical foundations.

In any case, the general conclusion is that DVMP observables, at least in experimentally accessible kinematics, are more challenging to explain in terms of GPDs than DVCS. Future data are expected from JLab after the 12 GeV upgrade. They will be useful to clarify some of the open issues and to understand to which extent DVMP can be reliably used to extract GPDs. In the future, the EIC will reach higher $Q^2$ values and make it possible to explore also the contributions from sea quark and gluon GPDs.

Table 2 presents the full list of twist-2 GPDs, with their dependence on polarization. Parametrizations in agreement with data are available for the first two columns (chiral even sector) [139,140,153,156,141,142,143,144]. These parameterization make use of subsets of all available GPD data. Some of them rely only on the knowledge of the collinear PDFs and the nucleon Form Factors, and obtain the full GPD on the basis of some specific choice of functional form.

Different GPD extractions/models are based on different assumptions and functional forms. The first class of fits is based on the so-called Double Distributions (DDs), introduced by Radyushkin [157] and Müller [158]. DDs are convenient ways to represent GPDs and automatically fulfill some fundamental properties of GPDs, such as polynomiality (related to Lorentz invariance). Examples of parameterizations based on DDs are the Vanderhaeghen-Guichon-Guidal VGG model [159], the Goloskokov-Kroll (GK) model [140,153]. The DD models in general reproduce the data fairly well. However underestimate the unpolarized DVCS cross section, they do not describe very well the beam spin asymmetry data at low $t$ and they have problems reproducing some of the azimuthal moments [145]. The GK model is also able to describe some DVMP observables.

Another family of GPD parameterization is based on the use of conformal moments. In the so called “dual parameterization,” GPDs are expanded on a series of Gegenbauer polynomials, which makes it easier to apply evolution equations. Phenomenological studies based on the dual parameterization have been published in Refs. [159,160,161]. Qualitatively, the results are the same as for the DD parameterizations. Another conformal-moment parameterization uses the Mellin–Barnes representation of GPDs, and has been worked out by Kumericki and Müller in Refs. [153,164]. The parameterization in Ref. [143].

| nucleon pol. | quark pol. |
|-------------|------------|
| U | L | T |
| H | $E_T + 2H_T$ | $E_T$ |
| $H$ | $E_T$ | $H_T$ |

**Table 2.** Twist-2 Generalized Parton Distribution functions (GPDs). The U,L,T correspond to unpolarized, longitudinally polarized and transversely polarized nucleons (rows) and quarks (columns). Functions in boldface survive in the forward limit.
is at present the one that can describe most of available DVCS data in a satisfactory way. One of the main differences between this extraction and the others is the presence of a large $\tilde{H}$ GPD.

A fourth type of GPD parameterization is based on model results. The parameterization of Ref. [156] is based on a spectator model for the nucleon, with additional Regge-inspired flexibility. The agreement with data is similar to that of the DD parameterization.

The plots in the recent experimental paper of Ref. [127] give a good overall idea of the ability of the various parameterizations to describe new data: the situation is qualitatively good, especially if we consider that the observables to describe are diverse and most of the parameterization are based on simple concepts with few free parameters.

Apart from the details of fitting techniques, GPDs can be used to reconstruct parton density maps in a two-dimensional transverse position space and a one dimensional longitudinal momentum space. Already from the study of Form Factors and their Fourier transform, we can obtain transverse maps in impact parameter space, integrated over longitudinal momentum, and only for valence quark combinations [139] [142]. From the Fourier transform of the Dirac form factors, it turns out that the distribution of valence up quarks is narrower than the down. The root-mean-square impact parameter is about 0.7 fm. Among other things, this means that a high-energy probe sees a core of positive charge in the center of the proton and a cloud of negative charge around it. In a transversely polarized proton, we know that the densities of up and down quarks are distorted in opposite ways, and the distortion of down quarks is larger than for up quarks. This distortion indirectly suggests that the up quarks have a large orbital angular momentum opposite to the proton spin. Vice-versa for the down quarks.

When considering also the longitudinal momentum dependence, it turns out that at high $x$ the impact-parameter distribution of down quarks seems to be wider than up (root-mean-square impact parameters are equal to 0.8 fm and 0.4 fm respectively) [142]. At low $x$, the width of the distribution becomes wider and can even diverge, in the same way for up and down quarks. These features were obtained from the study of form factors together with PDFs, but assumptions are needed to connect the two limits of the parent GPD. A first attempt to obtain this information directly from Compton Form Factor measurements was illustrated in Ref. [145]. Assuming that the Compton Form Factor can give us direct information about $H(x, x, t)$, the reconstruction of impact-parameter distributions requires an extrapolation to the forward limit $H(x, 0, t)$ (also called “deskewing” correction), which is model dependent, and a Fourier-Bessel transform about $t$. The outcome of this study confirms that the the width of the impact-parameter distribution widens at lower $x$.

Thanks to collider measurements at HERA, it is possible also to extract the GPDs for sea quarks and gluons. The outcome of the fits is that there is no drastic difference between the behavior of sea quarks and gluons. Gluons are slightly narrower than sea quarks. Their root-mean-square impact parameters are 0.7-0.9 fm and 0.6-0.8 fm, respectively. Another observation is that the width in impact parameter space for sea quarks and gluons has no strong dependence on the longitudinal variable $x$ [163] [156].

4 Wigner distributions

An alluring frontier to reach is the possibility of experimentally accessing Wigner distributions [165] [166] [167] or at least their Fourier transform, generalized TMDs (GT-MDs) [168] [169] [170]. There are 16 complex-valued GT-MDs at leading twist. In the forward limit, they reduce to the eight TMDs of Tab. [1] and upon integration over transverse momentum they reduce to the eight GPDs of Tab. [2]. It would already be of extreme interest to identify a process that can be used in principle, even if it turns out to be too difficult to realize in practice. A theoretical QCD analysis of this process should lead to factorization theorems involving Wigner distributions and to their evolution equations, which should eventually match onto the TMD and GPD evolution equations in their respective limits. Very recent work in this direction was presented in Ref. [171] [172].

Even if we are eventually able to extract/reconstruct Wigner distributions, there is still an important clarification to make. Referring to TMDs or GPDs, we speak routinely about 3-D imaging or 3-D mapping. We should remember that we are talking about “probability densities,” not about complete images. Reconstructing a digital image means knowing exactly the color of each pixel, given the color of all others, not just knowing the probability that a pixel in a certain position is red. Information analogous to real images corresponds to multiparton correlation functions, which tell us what is the probability of finding a parton in a certain condition given the conditions of the other partons (ideally, all of them). Multiparton correlation functions have been the subject of intense theoretical and experimental studies in the last years (see, e.g., [173] [174] [175] [176] [177]).

5 Nucleon spin decomposition

A fundamental problem to address is the composition of nucleon’s spin. Without entering too much into the details of the study of parton angular momentum, let us summarize here some general considerations (for more details, see the dedicated review by Lorcé and Liu in this Special Issue [178]). There are various equally valid definitions of partonic total and orbital angular momentum, generally falling into the two classes of kinetic and canonical angular momentum, connected to the two historical approaches of Ji [179] and Jaffe-Manohar [180], respectively. It is in principle possible and interesting to “measure” Orbital Angular Momentum (OAM) in both definitions. The difference between the two can tell us something nontrivial about QCD dynamics. In practice, it would already be an
A full decomposition of the nucleon’s angular momentum means that we should measure spin and total angular momentum, or spin and orbital angular momentum, for all quark flavors and gluons. Even assuming a flavor-blind sea ($u = d = s = \bar{s}$), this means we should measure eight contributions. It will be extremely hard if not impossible to check the validity of the spin sum rule (i.e., that the sum of all contributions adds up to 1/2). We will probably assume the validity of the sum rule and use it to determine those contributions that are harder to access. Therefore, even measuring only some of the contributions remains a valuable achievement.

We should also keep in mind that the various contributions to total spin can occur with different signs. Therefore, if we determine that the sum of certain contributions is small, it does not mean that all the contributions in the sum are small. They can be large and opposite. There has been considerable excitement in the last year concerning the fact that the contribution of the gluon spin is maybe sufficient to saturate the spin sum rule [181,182]. This estimate is affected by large uncertainties, and still leaves much room for a large net contribution of orbital angular momentum. In any case, even if it will remain true that spin contributions add up to something close to 1, we still have to check and justify why orbital angular momenta from different partons add up to zero.

Another important observation is that we would like to know also the angular momentum density, i.e., as a function of $x$ (and $k_T$ and $b_T$). The situation in this case becomes more complex and also frame-dependent. What we have learned for sure up to now is that the integrand of Ji’s relation \( xH(x,0,0) + xE(x,0,0) \) cannot be interpreted as an angular momentum density [183–185,188]. Orbital angular momentum can be defined starting from Wigner distributions, since they contain information about both $k_T$ and $b_T$ [184,188]. Depending on the choice of Wilson line in the definition of the Wigner distribution, it was shown that both definitions of orbital angular momentum can be recovered: the kinetic definition using a straight Wilson line, the canonical definition using a Wilson line of the same shape as in the definition of TMDs [189]. It was pointed out that the difference between the two results can be connected to a “chromomagnetic torque” experienced by the active quark due to the effect of final state interactions [190]. Furthermore, the kinetic definition can be related to twist-3 collinear GPDs [191–192,193,194,195,196]. The connection between the Wigner distribution definition and the collinear definition has been clarified in Ref. [197], which also suggests a connection to the pure twist-3 part of the structure function $g_2$, accessible in experimental measurements.

For the canonical definition, at present only the definition in terms of a Wigner distribution is available [189,188]. However, as for all other Wigner distributions, no way to access it experimentally has been proposed.

At the moment, the only practical way to extract partonic total angular momentum is to use Ji’s relation [179]. As is well known, the relation contains two terms, one of which is well known, being the second Mellin moment of the unpolarized collinear PDFs, i.e., the partonic fractional linear momentum. The second piece is obtained by integrating the forward limit of the GPD $E(x, \xi, t)$. Note, however, that some model calculations showed some inconsistencies, still not completely understood, when total angular momentum is split into various flavors according to Ji’s relation [187,198,199].

In any case, there is evidence from GPD and TMD studies that total angular momentum (in the kinetic definition) for up quarks is large and positive, while for down quarks is small and negative [180,201,50,142]. Roughly speaking, this leaves room for about 50% of the proton spin to be carried by gluons. The latest lattice QCD calculations are in partial agreement with these indications: they also observe a large and positive total angular momentum for up quarks and a small and negative total angular momentum for down quarks. However, there is the unexpected indication that orbital angular momentum for sea quarks is large [202], compensated by the fact that gluons carry only less than 30% of the proton spin. It should be kept in mind that these lattice calculations are done on a quenched lattice and require extrapolations to the physical pion mass. Unquenched calculations typically increase the momentum fraction of gluons and therefore they might also increase the fraction of angular momentum. Extrapolation to the physical pion mass may cause an overestimate of the up quark angular momentum.

6 Further topics

Even if this topical review is devoted to the 3-D structure of the proton, there is still plenty of interesting physics to measure and understand in the 1-D case, or even in the 0-D case. For instance, we know the electric charge of the proton very well (even though we still do not know why it has that value), but other “charges” can be defined: the axial, tensor, and scalar charges.

Charges represent one-dimensional information about the proton. They could be interesting in two ways, as is always the case with hadronic quantities: they could reveal something new about QCD, and they can be used to reveal something new about other fields of research, through processes that involve hadrons. Low-energy precision measurements of such processes can provide unique probes of new physics at much higher scales. For example, there is the possibility that nucleon beta decay or the neutron electric dipole moment receive contributions from interactions not included in the Standard Model. The knowledge of the nucleon charges is necessary to extract the corresponding hypothetical couplings from precision measurements. At present, this information is taken from lattice QCD, but in the future it should be possible to obtain it from experimental measurements (see, e.g., Refs. [203,204,205]).

The axial charge is related to the integral of helicity PDF, the tensor charge to integrals of the transversity PDF, and the scalar charge to the integral of the twist-3
PDF $e(x)$ \cite{200,201,203,204,210}. The latter can be accessed, for instance, in semi-inclusive DIS \cite{211,212,213,214,215}.

In several points of our discussion higher-twist parton distributions were mentioned. This topic by itself can become a focus theme in the next years. It is unavoidably present in all measurements, but it is notoriously difficult to disentangle and interpret. The first step is therefore to separate higher-twist contributions from the rest (see, e.g., Refs. \cite{216,217,35}). To make it possible, it is desirable to have the largest possible $Q^2$ span, keeping all other kinematics as fixed as possible. At the same time, studies of peculiar twist-3 signals (e.g., specific modulations in the cross section that vanish at twist 2) can provide unique information on the physics of the nucleon and challenge our theoretical interpretation. The $e(x)$ twist-3 PDF mentioned above represents a good example, but much more can be studied.

As we discussed at the beginning, understanding the proton means that we know how to compute its characteristics, reproducing what is measured and predicting what it is not. The most successful approach so far has been that of lattice QCD. In the past, lattice QCD was able to study only specific hadronic quantities. In the field of 3-D structure, lattice QCD traditionally provides only calculations of Mellin moments of PDFs and GPDs. This is already an important achievement, and it is already non-trivial to obtain results in agreement with experimental information, as we have already discussed for the case of the tensor charge.

A step forward into lattice studies of 3-D nucleon structure was performed by pioneering investigations on TMDs \cite{218,219,220,221}. This approach makes it possible to calculate transverse moments of TMDs and leads also to a lattice-based understanding of the effect of different Wilson-line choices, including the difference between kinetic and canonical partonic angular momenta.

Recently, studies have appeared that suggest a way to compute on the lattice the full functional dependence of parton distributions, including 3-D ones \cite{222,223,224,225,226}. The calculation is performed in a frame with finite longitudinal momentum (producing the so-called “quasi PDFs”) and the connection to light-front parton distributions is performed using perturbative matching conditions. This approach is potentially extremely powerful and opens up the possibility of calculating the structure of the nucleon instead of extracting it from data. There is however still a very long way ahead: present estimates of 1-D PDFs are very crude and not superior to any basic model calculation \cite{227}. Problems of different nature affect both the high-$x$ and low-$x$ region. Nevertheless, improvements in computational efficiency and in the understanding of quasi PDFs could in the future lead us to a stage comparable to \textit{a priori} calculations of electronic orbitals.

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