A future linear $e^+e^-$ collider with a clean environment, tunable collision energy, high luminosity, polarized incoming beams, and additional $e^-e^-$, $e\gamma$ and $\gamma\gamma$ modes, will offer precision tools to explore new physics. Here we summarize three papers submitted to the ICHEP04 conference in which polarized $e^+e^-$ and $\gamma\gamma$ beams are exploited to search for CP violation, and universal extra dimensions (UED).

1 Transverse beam polarization at a linear $e^+e^-$ collider and new physics$^{1,2,3}$

A 1 TeV linear $e^+e^-$ collider with high luminosity and polarized beams is now a distinct possibility. If spin rotators can produce transversely polarized beams (TPB), providing two more vectors, they would make it possible to observe CP violation by observing a single final-state particle without measuring its spin. Two specific processes have been considered:

a) $e^+e^- \rightarrow t\bar{t}$

The four-Fermi Lagrangian of beyond the SM contact $e\bar{e}t\bar{t}$ interactions (CI), after Fierz transformation, takes the form

$$\mathcal{L}^{4F} = \sum_{ij} [S_{ij}(\bar{e}e)_i(\bar{t}t)_j + V_{ij}(\bar{e}\gamma_{\mu}e)_i(\bar{t}\gamma^{\mu}t)_j + T_{ij}(\bar{e}\sigma_{\mu\nu}e)_i(\bar{t}\sigma^{\mu\nu}t)_j]$$ (1)

where $(\bar{f}O_f) = \bar{f}O_f \gamma_5$, $S_{RR} = S_{LL}^\text{L}$, $V_{ij} = V_{ij}^\text{L}$, $T_{RR} = T_{LL}^\text{L}$ and non-diagonal $S$ and $T$ vanish.

With TPB the chirality-violating $S$ and $T$ terms interfere with the SM contributions and can be studied, in contrast to the cases of no or longitudinal beam polarization, where they appear only at second order. Assuming $P_{L}$ along the $x$-axis and anti-parallel $P_{R}$, the CP-odd azimuthal asymmetry, defined as

$$A_1(\theta_0) = \frac{N(\sin \phi > 0) - N(\sin \phi < 0)}{N(\sin \phi > 0) + N(\sin \phi < 0)}$$ (2)

is sensitive to

$$\text{Im} S \equiv \text{Im}(S_{RR} + \frac{2c_A^2 c_V}{c_V^2} T_{RR})$$ (3)

Here $N$ is the number of events with a given azimuthal angle $\phi$ and polar angle of the top quark within the $\theta_0$ cut as $\theta_0 < \theta < \pi - \theta_0$. Fig.1 (left) shows the 90% C.L. limit on $\text{Im} S$ for an integrated luminosity of $\int Ldt = 500\text{ fb}^{-1}$ and $\sqrt{s} = 500\text{ GeV}$ and 100% TPB. The limit of $1.6 \cdot 10^{-8} \text{ TeV}^{-2}$ translates to a scale $\Lambda$ of CI of order 8 TeV (for $\sqrt{s}$ of 800 GeV the sensitivity increases to $\sim 9.5$ TeV). For realistic TPB of $P_{L} = 0.8$ and $P_{R} = 0.6$, the limit on $\Lambda$ goes down to about $6.7$ TeV.

A specific example of chirality-violating couplings at tree level has been considered in$^2$ with an $SU(2)_L$ doublet of scalar leptoquarks coupled only to first-generation leptons and third-generation quarks. The azimuthal asymmetry $A_1$ turns to be proportional to $\text{Im}(g_R g_L^*)$ while an asymmetry $A_2$, defined as in Eq.(2) but with $\phi \rightarrow \phi + \pi/2$ and with parallel $P_{L}$ and $\vec{P}_{R}$, is proportional to $\text{Re}(g_R g_L^*)$: $g_i$ is the leptoquark coupling to the chiral current $(ie)_{ij}$. Fig.1 (cen-
ter) shows the 90% C.L. limits on $\text{Re}(g g L)$ and $\text{Im}(g g L^*)$ for leptoquark mass of 1 TeV with realistic TPB. The limit on $\text{Re}(g g L)$ is competitive with $|\text{Re}(g g L^*)| < 0.1$ derived from $\theta_c^w - 2$, while the limit $|\text{Im}(g g L^*)| < 10^{-6}$ from the electron EDM is much stronger.

b) $e^+ e^- \to \gamma Z^3$

Here the final-state particles are both self-conjugate. With TPB, a T-odd, CP-even azimuthal asymmetry can be combined with the T-even, CP-odd forward-backward (FB) asymmetry to give an asymmetry which is both CP- and T-odd. A CP-violating contribution can arise if anomalous CP-violating $\gamma Z$ and $\gamma ZZ$ couplings are present.

The effective CP-violating Lagrangian for $\gamma \gamma Z$ and $\gamma ZZ$ interactions, up to dimension 6 terms, can be written as

$$\mathcal{L} = \frac{e \lambda_1}{2m_Z^2} F_{\mu\nu} \left( \partial^\mu Z^\lambda \partial^\nu Z^\lambda - \partial^\nu Z^\lambda \partial^\mu Z^\lambda \right) + \frac{e \lambda_2}{16c_W s_W m_Z^2} F_{\mu\nu} F^{\nu\lambda} \left( \partial^\mu Z^\lambda + \partial^\nu Z^\lambda \right) \quad (4)$$

To isolate appropriate anomalous couplings three different CP-odd asymmetries, which combine a FB asymmetry with an appropriate asymmetry in $\phi$, have been identified in$^3$. The derived 90% C.L. limits on real and imaginary parts of the $\lambda_1$ and $\lambda_2$ couplings, plotted as functions of the cut-off $\theta_c$, are shown in the right panel of Fig.1.

2 Resonant H/A mixing in the CP-noninvariant SUSY$^4$

The tree level Higgs potential of the MSSM is CP-conserving implying two $h, H$ of the three neutral states to be CP-even, while the third $A$ is CP-odd. With non-vanishing CP phases in the soft SUSY-breaking terms, however, radiative corrections induce the three neutral bosons to mix forming a triplet $(H_1, H_2, H_3)$ with even and odd components in the wavefunctions under CP transformations.

As expected from quantum mechanical rules, the mixing can become very large if the states are nearly mass-degenerate. This situation is naturally realized in the decoupling limit in which two of the neutral states, $H$ and $A$, are heavy. The lightest Higgs $H_1$ then becomes the CP-even SM-like Higgs, and does not mix with the $H/A$ system. In this limit the off-diagonal mixing term of the $2 \times 2$ mass matrix $M^2$ in the $H, A$ basis reads

$$M^2_{HA} = v^2 \left[ \frac{1}{2} \lambda^I_1 c_{2\beta} - \frac{1}{2} (\lambda^I_0 - \lambda^I_2) s_{2\beta} \right] \quad (5)$$

where $\lambda^I_0$ are imaginary parts of loop-induced quartic Higgs couplings$^4$.

For small mass differences, the mixing is strongly affected by the widths of the states and the complex, symmetric Weisskopf–Wigner mass matrix $M_c^2 = M^2 - iM\Gamma$ must be considered in total, not only the real part. Recently a coupled-channel method has been employed$^5$ for the Higgs formation and decay.

Figure 1. The 90% CL limits on: $\text{Im} S$ (left), leptoquark (center), and $\gamma \gamma Z$ and $\gamma ZZ$ couplings (right), as functions of the cut-off angle $\theta_c$. [$\sqrt{s}=500$ GeV, $\int L dt=500$ fb$^{-1}$]
processes at the LHC.

In Ref.\(^4\) an alternative approach has been followed where the full mass matrix \(M^2\) is diagonalized

\[
M^2_{\text{diag}} = C M^2 C^{-1}
\]

For the \(H/A\) system, the complex 2×2 rotation matrix is expressed in terms of a complex mixing angle \(\theta\), which is given by

\[
X = \frac{1}{2} \tan 2\theta = \frac{M^2_{HA} - i M_{HA} \Gamma_{HA}}{M^2_{HH} - M^2_{AA}}
\]

where \(\Gamma_{HA} (M^2_{HH}, M^2_{AA})\) is the off-diagonal (diagonal) entry of the decay (complex mass \(M^2\) matrix.

In Fig.2 the complex \(H/A\) mixing in the MSSM is shown for a typical set of parameters: \(M_S=0.5\) TeV, \(|A_t|=1\) TeV, \(\mu=1\) TeV, \(\tan\beta=5\), while varying the phase \(\phi_A\) of the trilinear parameter \(A_t\).

A future photon collider would be an ideal tool to study resonant CP-violation in the Higgs sector. Two promising signatures have been considered in Ref.\(^4\):

a) \(\gamma\gamma \rightarrow H_i\) formation with polarized beams

For linearly polarized photons, the CP-even component of the \(H_i\) wave-functions is projected out if the polarization vectors are parallel, and the CP-odd component if they are perpendicular. This can be observed in the CP-even asymmetry \(A_{lin}\),

\[
A_{lin} = \frac{\sigma_{||} - \sigma_{\perp}}{\sigma_{||} + \sigma_{\perp}}, \quad A_{hel} = \frac{\sigma_{++} - \sigma_{--}}{\sigma_{++} + \sigma_{--}}
\]

since \(|A_{lin}|<1\) requires both scalar and pseudoscalar \(\gamma\gamma H_i\) non-zero couplings. Moreover, CP-violation due to \(H/A\) mixing can directly be probed via the CP-odd asymmetry \(A_{hel}\) constructed with circular photon polarization, as defined in Eq.(8).

b) polarization of top quarks in \(H_i\) decays

CP-induced correlations between the transverse \(t\) and \(\bar{t}\) polarization vectors \(s_{\perp}, \bar{s}_{\perp}\) in the decay process \(H_i \rightarrow t\bar{t}\),

\[
C_\parallel = \langle s_{\perp} \cdot \bar{s}_{\perp} \rangle, \quad C_\perp = \langle \hat{p}_t \cdot (s_{\perp} \times \bar{s}_{\perp}) \rangle
\]

lead to a non–trivial CP-even and a CP-odd azimuthal correlation, respectively, between the decay planes of \(t \rightarrow bW^+\) and \(\bar{t} \rightarrow bW^-\).

The left panel of Fig.3 shows the asymmetries \(A_{lin}\) (blue solid line) and \(A_{hel}\) (red dashed line) in the \(\gamma\gamma\) collider as the \(\gamma\gamma\) energy is scanned from below \(M_{H_2}\) to above \(M_{H_2}\). The right panel shows the \(E_{1\parallel}\) dependence of the correlators \(C_\parallel\) (blue solid line) and \(C_\perp\) (red dashed line). Both figures are for \(\phi_A = 3\pi/4\), a phase value close to resonant CP-mixing. Detailed experimental simulations would be needed to estimate the accuracy with which they can be measured. However, the large magnitude and the rapid, significant variation of the CP-even and CP-odd asymmetries through the resonance region would be a very interesting effect to observe in any case.
3 Testing the UED at a linear $e^+e^-$ collider$^6$

In the simplest UED scenario considered in$^6$ one extra dimension is accessed by all SM particles and low-energy data constrain the compactification radius $R^{-1} \gtrsim$ a few hundred GeV. The extra dimension $y$ is compactified on a $S^1/Z_2$ orbifold, rendering all matter and gauge fields, viewed from 4d, dependent on $y$ either as $\cos(ny/R)$ or $\sin(ny/R)$, where $n$ is the corresponding KK number. The mass of the $n$th KK state is given by $M_n^2 = M_0^2 + n^2/R^2$, where $M_0$ is the zero mode mass of that field. A remnant $Z_2$ symmetry dictates that the KK parity defined as $(-1)^n$ is conserved, implying (i) the lightest Kaluza-Klein particle (LKP) is stable, and (ii) a single KK state cannot be produced – a reminiscent of supersymmetry with conserved R-parity.

A tree-level degeneracy of the KK modes of light SM particles is lifted by radiative corrections: finite bulk corrections $\Delta M_n^2 \propto \beta/16\pi^4R^2$, and logarithmically divergent orbifold corrections $\Delta M_n = M_n(\beta/16\pi^2) \ln(\Lambda^2/\mu^2)$; $\beta$ represents the $\beta$-functions of the gauge and matter KK fields in the loop, and $\Lambda$ is the UV cut-off scale.

The first KK modes of electrons, $e_1^\pm$, should be copiously produced in $e^+e^- \rightarrow e_1^+e_1^-$ at a future linear collider. The process proceeds through the $s$-channel $\gamma$ and $Z$, and the $t$-channel $\gamma_1$ and $Z_1$ exchanges. The splitting between $e_1$ and $\gamma_1$ turns to be sufficient for the decay $e_1^- \rightarrow e^- + \gamma_1$ with $\gamma_1$ likely to be the LKP and escaping detection. So the final state is $e^+e^-$ and $2\gamma_1$’s carrying away missing energy. The $W_1^\pm$ pair production with $W_1^+ \rightarrow e+\nu_{e_1}$, $e_1+\nu_e$ is numerically insignificant. The main SM background from $W^+W^-$ pair production can be eliminated by requiring that the final electrons are sufficiently soft.

Fig.4 shows the cross section for $e^+e^- \rightarrow e^+e^- + E_{\text{miss}}$ with $e^\pm$ energies between 0.5 and 20 GeV and polar angle away from the beam pipe by 15$^\circ$. Events coming from excited $W_1$ have been neglected. Not much is gained from the beam polarization. The cross section enhances as $\Delta R$ is increased from 2 to 20 due to the change in the $n = 1$ Weinberg angle. Clearly the signal events are quite prominent, and the SM background after cuts is under control.

4 Conclusions

The examples discussed above add an additional weight to the physics case of a linear collider, strengthening its ability to explore physics beyond the standard model.

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