Free vibration and Flutter Stability of Interconnected Double Graded Micro Pipes System Conveying Fluid

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Abstract— Functionally gradient materials and small-scale pipes have a great important in industry because of its wide applications in many engineering fields such as, fluid transport in fluidic devices. The aim of this work is to study the dynamic stability of double FGM micro pipes conveying fluid depending on a modified couple stress theory. The two micro pipes are connected together continuously through elastic spring. The vibration equations with boundary conditions are acquired based on Hamilton’s principle and subsequently, solved by Galerkin’s method. The results of this research were compared with results reported in the literature. A reasonable agreement was found. Also, the influences of a gradient index of the material, a parameter of a length scale, the outer diameter of micro-pipe on the critical flow velocity and resonant frequencies are discussed. The results displayed that the critical velocities and natural frequencies are increased hastily with an increase in a gradient index n

Keywords— Critical flow velocity, graded double micro Pipe conveying fluid, Galerkin’s method, Natural frequency.

I. INTRODUCTION

Pipes conveying fluid have many engineering applications, for examples in air conditioning, power plants, heat exchangers, hydraulic systems, oil pipelines and microfluidic devices. One of a most significant and oldest studies in the field of fluid-structure interaction was conducted by [1] who studied the vibrational conduct for the pipes conveying fluid. In addition, they found that the pipes with a supported end might squander the stability by divergence, whilst the cantilevered pipes encounter the dynamic instability (flutter), at high flow velocities. After that, many investigations are performed to study the different sides of the conduct of a pipe conveying fluid, such as post-buckling by [2], stability analysis , wave propagation [3] and biomechanical properties by [4]. [5] analyzed the problem of free vibration in pipes conveying fluid for several boundary conditions. The critical fluid flow velocities and fundamental natural frequencies are acquired by using (DT) method and compared the results with (DQ) method. [6] discussed the dynamics of fluid conveying that occurs in the Timoshenko pipes. Usually structures conveying fluid lose the stability once the fluid exceeds the critical velocity. The type of the instability depended on the kind of the supports. If the structures (beam or pipe) are cantilevered, flutter instability is noted once the critical velocity is overriding. When he studying a flutter instability for a cantilevered pipe by a linear theory, it has been proved
numerical integration that value of this critical velocity is only for small values of a mass ratio (approximately $\beta < 0.1$).

Micro and Nano scale structures have become a subject of interest in the modern science and technology after their invention. They have significant importance in thermal, a mechanical and electrical performance that is higher than traditional structural materials. Such as micromechanics gyroscope, sensors, fluid storage, actuators, fluid transport and one of the important application of two micro pipes system is for the single pressure transducer. For measuring pressure based on the differential vibration frequency alternating of two micro pipes by [7]. It is totally known that the size effect cannot take into account a mechanical conducts on micro-Nano scale structures in a classical continuum theory (CCT). However, a classical continuum models demand to be extended to look at the small-scale size effects and this can be done by utilizing the nonlocal continuum theory by [8], modified couple stress (MCS) theory [9] the theory of modified strain gradient [10], [11] and [12] studied the free vibration conduct for carbon nanotubes (CNTs) conveying fluid depend on a nonlocal elasticity of an Euler-Bernoulli beam theory. [13] examined the instability analysis for cantilever micro pipes conveying fluid by utilizing a modified strain gradient (MSG) theory. A mathematical derivation was widened in terms of three parameters of length scale in synchronism with the Bernoulli-Euler beam paradigm and solved by Galerkin’s method. The MSGT compared with classical theory and theory of modified couple. They examined the effect of a parameter of length scale, aspect ratio and outside diameter on frequencies and critical speeds (flutter). Results emphasized that (MSG) Theory foretells greater natural frequencies and critical speeds (flutter) are more than that predicted by CT and MCST. In addition, many developments at a dynamics of structures and a size-dependent static in Nano- and micro scales can be pursued in the researches of [9], [14–16], and [17].

On the other hand, due to the modern technological evolutions in materials and science, advanced inhomogeneous composites are discovered recently, these materials are Functionally graded materials, where the mechanical properties alteration from the surface to another continuously. Synthesis is different continuously with a change in a volume fraction of components. In addition, it related to those advanced inhomogeneous composites in which the material properties vary over the volume. Using the metal and ceramic layers at two sides of FG material can be designed to utilize a best mechanical properties of both materials and makes it a suitable candidate for many applications [18]. In this regard, many examine and studies have been conducted on these materials in recent years,[19] studied the instability problem and vibration analysis for a spinning thin-walled (STW) made from FGMs. A thermo mechanical stability for a thin-walled conveying fluid of a cantilevered end condition pipe made from functionally graded and subjected to compressive axial force was investigated by [20]. The pipe was formulated depend on Rayleigh's theory and the extended Galerkin's method was used to solve the vibration equation. They investigated the effects of gradient index, axial compressive force, fluid mass ratio, fluid flow speed, and temperature Variable on a stability of thin-walled FGM pipe. [21] analyzed the size-dependent FG Materials pipe conveying fluid by using a modified strain gradient elasticity (MSGE) theory. They investigated the effects of a parameter of length scale and the FG power-law exponent on a critical fluid flow velocities and natural frequencies of a micro pipe.

With respect to development in mechanical conduct analysis of micro/Nano-beams, it should be noted that some researches polished are about the coupled double Nano-beam and tube system. For example, [22], analyzed the forced vibration of double- carbon nanotube (CNT) system by a moving Nano-particle on the basis of a nonlocal elasticity theory. [23], investigated the axial instability and free vibration of the double Nano-beam system. [24] studied the vibration of the double bonded carbon Nano-tubes conveying flow with nonlocal elasticity theory. [25], investigated vibration and instability responses for the double carbon nanotubes with interior flowing fluid founded on nonlocal elasticity theory. [26], analyzed the forced vibrations for double piezoelectric (FGM) micro pipes which fluid-conveying founded on the flex electricity theory and modified couple stress theory. The non-classical governing equations were solved by Rung–Kutta, and differential quadrature (DQ) methods for various boundary conditions. The influence of the parameter of length scale, material gradient exponent, visco-Pasternak foundation, and fluid velocity was discussed.

In the above review, we note that most of the researchers focused their studies on micro scale structures are related to an analysis of single micro-pipes and did not address the instability and free vibration of double FG material micro-pipe systems. In this article, an effect of Interconnected spring on the stability of double micro pipe conveying fluid made by using functionally graded materials are investigated. Based on the theory of modified coupled stress and Hamilton’s principle, the vibration equation for primary and secondary micro pipe are derived.
these equations are solved by Galerkin's method for cantilever end condition. The influence of the parameter of length scale, volume fraction n, fluid velocity, and the outer diameter on instability for the system is investigated.

II. MODEL DESCRIPTION AND GOVERNING EQUATIONS:

2.1. Material Properties of double FGM micro pipe:

In the present investigation, material properties of coupled FG micro-pipes with a Length \( L \), \( d \) refer to the outer diameter of micro-pipe, fluid flow velocity \( u_f \), and \( K \) refers to the stiffness of spring. The transverse and axial displacements on mid-axis are \( u \) and \( w \), respectively. Material properties are presumed to change continuously during the thickness direction (\( h \)) as shown in Fig. 1. In addition, Table 1 represents the mechanical constituent materials properties the same as that used by [27]:

| Materials | \( E(GPa) \) | \( \rho_p(kg/m^3) \) | \( v \) |
|-----------|-------------|----------------|---|
| Alumina   | 380         | 3800           | 0.23 |
| Aluminum  | 70          | 2700           | 0.23 |

A volume fraction of an outer surface constituent of the FG micro-pipes can be appeared as:

\[
v_m = \left( \frac{2z + h}{2h} \right)^n \quad \text{Where} \ (0 \leq n \leq \infty)
\]

\[
v_c = 1 - v_m
\]

Where \( n \) is the power gradient index, which characterizes the profile of volume fraction during the thickness and is a very positive number; also, subscripts of \( m \) and \( c \) indicate the metal and ceramic layers, respectively.

![Fig. 1 Elastically connected fluid-conveying double-FG Micro-pipes system.](image)

It is clear that the material properties vary continuously from ceramic-rich at the inner surface for a micro-pipes to aluminum-rich at the outer surface. A change of volume fraction \( v_m \) with thickness direction for various values of exponents of volume fraction \( n \) is depicted by Figure 2, it can be noted that when the exponent \( n \) is supposed to be zero, FGM micro-pipe reduces to homogeneous micro pipe [27].

\[
\rho(z) = v_m \rho_m + v_c \rho_c
\]

\[
E(z) = v_m E_m + v_c E_c
\]

where \( \rho \) and \( E \) stand for the density, and Young’s modulus, respectively. Substituting equations (1) and (2) into equations (3) and (4) yields:
\[ E(z) = E_c (E_R - 1) \left( \frac{2z + h}{2h} \right)^n + E_c \]  
(5)

\[ \rho(z) = \rho_c (\rho_R - 1) \left( \frac{2z + h}{2h} \right)^n + \rho_c \]  
(6)

where \( E_R = E_m/E_c \) and \( \rho_R = \rho_m/\rho_c \) represent the ratio of Young’s modulus and density, respectively.

Fig. 2 Difference of volume fraction \( \nu_m \) along thickness direction, for various values of gradient index \( n \).

**2.2. Mathematical formulation:**

Based on the Euler–Bernoulli beam theory, the offset field for a random point along the x and z-axes can be written as:

\[ \bar{u}(x, z, t) = u(x, t) - Z \frac{\partial w(x, t)}{\partial x} \]  
(7)

\[ \bar{w}(x, z, t) = w(z, t) \]  
(8)

where \( Z \) is the coordinate measured from the plane of the neutral fibers. Assuming that micro-pipe is elastic, the stress–strain relation is given by

\[ \sigma_{xx} = E \varepsilon_{xx} \]  
(9)

\[ \varepsilon_{xx} = - \frac{\partial w(x, t)}{\partial x} \]  
(10)

**2.3. Modified couple stress theory:**

The expression of strain energy \( U_s \) indicates to a linear elastic material-taking zone \( \Omega_i \) with very small deformation is given by:

\[ U_s = \sum_i \frac{1}{2} \int_{\Omega_i} (\sigma_{ij} \varepsilon_{ij} + \tau_{ij} \gamma_{ij}) d\Omega_i \]  
(11)

\[ U_s = \frac{1}{2} \int_0^L \left( E(z) z^2 + G(z) f^2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \right) dx + h \int_0^L \left( E(z) z^2 + G(z) f^2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \right) dx \]  
(12)
\[ U_s = \frac{1}{2} \int_0^l \left( EI_{eq} + GA_{eq} l^2 \right) \left( \frac{\partial^2 W_i}{\partial x^2} \right)^2 dx + \frac{1}{2} \int_0^l \left( EI_{eq} + GA_{eq} l^2 \right) \left( \frac{\partial^2 W_i}{\partial x^2} \right)^2 dx \]  
(13)

where:

\[ EI_{eq} = \int_0^{2a_{eq} h} \int_{-h/2}^{h/2} E(z) z^2 dz dr \]  
(14)

\[ GA_{eq} = \int_0^{2a_{eq} h} \int_{-h/2}^{h/2} \frac{E(z)}{2(1+v)} dz dr \]  
(15)

The mass of pipe and fluid per unit length, respectively \( m_p \) and \( m_f \), their expressions can be offered as:

\[ m_p = \int_0^{2a_{eq} h} \int_{-h/2}^{h/2} \rho(z) dz dr, \quad m_f = \rho_f A_f \]  
(16)

Where \( \rho_f \) is the density of the fluid in FGM micro-pipe and \( A_f \) is the cross-sectional area of flow.

### 2.4. Governing equations:

The kinetic energy of FG Materials micro-pipe and fluid is known as follows [21]:

\[ T_p = \frac{1}{2} \int_0^l \rho A_{eq} \left[ \left( \frac{\partial W_i}{\partial t} \right)^2 + \left( \frac{\partial W_i}{\partial t} \right)^2 \right] dx \]  
(17)

The effects of the elastic spring between two FGM micro pipes can be expressed as:

\[ \delta W_b = \frac{1}{2} \int_0^l q_1 \delta W_1 dx + \frac{1}{2} \int_0^l q_2 \delta W_2 dx \]  
(18)

Where \( \delta W_b \) is virtual work of an elastic spring on the top and bottom micro-pipes [26]. Therefore, \( q_1 \) and \( q_2 \) are given as:

\[ q_1 = K_w (w_1 - w_2), \quad q_2 = K_w (w_2 - w_1) \]  
(19)

Then, the vibration equations for single FGM micro-pipes which fluid-conveying are derived by extended Hamilton’s principles (Hosseini et al 2017).

\[ \delta \int_{t_1}^{t_2} (T_p + T_f + W_b - U_s) dt = 0 \]  
(20)

After substitute (13), (17), (18) and (19) into equation (21), the vibration equations for primary and secondary FGM micro pipe conveying fluid are as follows:

\[ (EI_{eq} + GA_{eq} l^2) \frac{\partial^4 W_i}{\partial x^4} + \alpha m_f u_f^2 \frac{\partial^2 W_i}{\partial x^2} + 2m_f u_f \frac{\partial^3 W_i}{\partial x^3 \partial t} + (m_f + m_p) \frac{\partial^2 W_i}{\partial x^2} + K_w (w_1 - w_2) = 0 \]  
(21)
\[(EI_{eq} + GA_{eq}l^2) \frac{\partial^4 w_2}{\partial x^4} + \alpha m_j u_j^2 \frac{\partial^2 w_2}{\partial x^2} + 2m_j u_j \frac{\partial^2 w_2}{\partial x^2 \partial t} + (m_f + m_p) \frac{\partial^2 w_2}{\partial x^2 \partial t} + K_u (w_2 - w_1) = 0 \] (22)

Where \( \alpha \) is a factor referred to the effect of micro-flow velocity [9], the non-dimensional form for the equation of motion for FGM micro pipe conveying fluid is:

\[(\gamma + \mu) \frac{\partial^4 \eta_1}{\partial \xi^4} + (\alpha U^2) \frac{\partial^2 \eta_1}{\partial \xi^2} + 2U\beta^{1.5} \frac{\partial^2 \eta_1}{\partial \xi^2 \partial t} + K_u (\eta_1 - \eta_2) = 0 \] (23)

\[(\gamma + \mu) \frac{\partial^4 \eta_2}{\partial \xi^4} + (\alpha U^2) \frac{\partial^2 \eta_2}{\partial \xi^2} + 2U\beta^{1.5} \frac{\partial^2 \eta_2}{\partial \xi^2 \partial t} + K_u (\eta_2 - \eta_1) = 0 \] (24)

where:

\[
\xi = \frac{x}{L}, \quad \eta_1 = \frac{w_1}{L}, \quad \eta_2 = \frac{w_2}{L}, \quad \beta = \frac{m_f}{m_f + m_p}, \quad k_u = \frac{K_u L^4}{E_i I_o}
\] (25)

\[
\gamma = \frac{EI_{eq}}{E_i I_o}, \quad U = \sqrt{\frac{m_f}{E_i I_o} u_L}, \quad \mu = \frac{GA_{eq} l^2}{E_i I_o}, \quad \tau = \sqrt{\frac{E_i I_o}{(m_f + m_p) L^2}}
\]

The boundary condition for clamped- free end conditions is:

At \( \xi = 0 \rightarrow \eta(\xi) = 0, \frac{\partial \eta(\xi)}{\partial \xi} = 0 \) (26)

At \( \xi = 1 \rightarrow \frac{\partial^2 \eta(\xi)}{\partial \xi^2} = 0, \frac{\partial^3 \eta(\xi)}{\partial \xi^3} = 0 \) (27)

III. SOLUTION METHOD:

It has recognized that partial differential equations (PDEs) are generally hard to obtained analytical solutions for the continuous systems. In this situation, although the execution accuracy is sometimes are not good sufficient, the truncation method is still overwhelmingly used to obtain an approximate solution by converting partial differential equations (PDEs) to ordinary differential ones.

The solution accuracy actually counts on the number of truncation terms and the supposed trial functions [28]. In this portion, the Galerkin's style will be utilized to truncate partial differential equations derived, so let

![Fig. 3 Cantilever double FGM micro pipes](image-url)
\[ \eta_1(\xi, \tau) = \sum_{r=1}^{\infty} \phi_r(\xi) q_{\eta_1}(\tau) \]

\[ \eta_2(\xi, \tau) = \sum_{r=1}^{\infty} \phi_r(\xi) q_{\eta_2}(\tau) \]  

(26)

Substituting Eq. (28) into Eq. (24) and (25) and utilizing the technique of standard Galerkin's, we can gain ordinary differential equations such as:

\[ \gamma + \mu \sum_{r=1}^{\infty} \phi''(\xi) q_{\eta_1}(\tau) + (\alpha U^2) \sum_{r=1}^{\infty} \phi'(\xi) q_{\eta_1}(\tau) + 2U \beta^{0.5} \sum_{r=1}^{\infty} \phi(\xi) q_{\eta_1}(\tau) + \sum_{r=1}^{\infty} \phi(\xi) q_{\eta_1}(\tau) \]

\[ + k_w \left( \sum_{r=1}^{\infty} \phi(\xi) q_{\eta_1}(\tau) - \sum_{r=1}^{\infty} \phi(\xi) q_{\eta_1}(\tau) \right) = 0 \]  

(27)

\[ (\gamma + \mu) \sum_{r=1}^{\infty} \phi''(\xi) q_{\eta_2}(\tau) + (\alpha U^2) \phi'(\xi) q_{\eta_2}(\tau) + 2U \beta^{0.5} \phi(\xi) q_{\eta_2}(\tau) + \phi(\xi) q_{\eta_2}(\tau) \]

\[ + k_w \left( \sum_{r=1}^{\infty} \phi(\xi) q_{\eta_2}(\tau) - \sum_{r=1}^{\infty} \phi(\xi) q_{\eta_1}(\tau) \right) = 0 \]  

(28)

Then, simplifying the equations (27) and (28) yield:

\[ \sum_{r=1}^{\infty} \phi''(\xi) (\gamma + \mu) q_{\eta_1}(\tau) + (\alpha U^2) \phi'(\xi) q_{\eta_1}(\tau) + 2U \beta^{0.5} \phi(\xi) q_{\eta_1}(\tau) + \phi(\xi) q_{\eta_1}(\tau) \]

\[ + k_w \left( \phi(\xi) q_{\eta_1}(\tau) - \phi(\xi) q_{\eta_1}(\tau) \right) = 0 \]  

(29)

\[ \sum_{r=1}^{\infty} \phi''(\xi) (\gamma + \mu) q_{\eta_2}(\tau) + (\alpha U^2) \phi'(\xi) q_{\eta_2}(\tau) + 2U \beta^{0.5} \phi(\xi) q_{\eta_2}(\tau) + \phi(\xi) q_{\eta_2}(\tau) \]

\[ + k_w \left( \phi(\xi) q_{\eta_2}(\tau) - \phi(\xi) q_{\eta_1}(\tau) \right) = 0 \]  

(30)

Where \( \int_0^1 \phi_r \phi_s(\xi) d\xi = \delta_{sr}, \delta_{sr} \) being Kronecker’s delta and \( \phi_r'''(\xi) = \lambda^3 \phi_r [1] \):

\[ \sum_{r=1}^{\infty} \left[ \int_0^1 \phi_r \phi''(\xi) (\gamma + \mu) q_{\eta_1}(\tau) d\xi + \int_0^1 \phi_r (\alpha U^2) \phi'(\xi) q_{\eta_1}(\tau) d\xi + \int_0^1 \phi_r 2U \beta^{0.5} \phi(\xi) q_{\eta_1}(\tau) d\xi \right. \]

\[ + \int_0^1 \phi_r \phi(\xi) q_{\eta_1}(\tau) d\xi + k_w \left( \int_0^1 \phi_r \phi(\xi) q_{\eta_1}(\tau) d\xi - \int_0^1 \phi_r \phi(\xi) q_{\eta_2}(\tau) d\xi \right) \]  

(31)

\[ \sum_{r=1}^{\infty} \left[ \int_0^1 \phi_r \phi''(\xi) (\gamma + \mu) q_{\eta_2}(\tau) d\xi + \int_0^1 \phi_r (\alpha U^2) \phi'(\xi) q_{\eta_2}(\tau) d\xi + \int_0^1 \phi_r 2U \beta^{0.5} \phi(\xi) q_{\eta_2}(\tau) d\xi \right. \]

\[ + \int_0^1 \phi_r \phi(\xi) q_{\eta_2}(\tau) d\xi + k_w \left( \int_0^1 \phi_r \phi(\xi) q_{\eta_2}(\tau) d\xi - \int_0^1 \phi_r \phi(\xi) q_{\eta_1}(\tau) d\xi \right) \]  

(32)

The definite integrals may be estimated, defining the following set of constants

\[ \int_0^1 \phi_r \phi'(\xi) d\xi = \delta_{sr}, \int_0^1 \phi_r \phi''(\xi) d\xi = b_{sr}, \int_0^1 \phi_r \phi'''(\xi) d\xi = c_{sr} \]  

(33)
The equation of the primary micro pipe is:

\[ Iq_{\theta_1} + 2U\beta^{0.5} Bq_{\theta_1} + ((\gamma + \mu)\lambda + \alpha U^2 C + k_u l)q_{\theta_1} - k_u Iq_{\theta_2} = 0 \]  \hspace{1cm} (33)

and, the equation of the secondary micro pipe is:

\[ Iq_{\theta_2} + 2U\beta^{0.5} Bq_{\theta_2} + ((\gamma + \mu)\lambda + \alpha U^2 C + k_u l)q_{\theta_2} - k_u Iq_{\theta_1} = 0 \]  \hspace{1cm} (34)

In general the standard Galerkin’s is:

\[ M\ddot{Q} + G\dot{Q} + KQ = 0 \]  \hspace{1cm} (35)

Where M, G, and K refers to the mass, gyroscopic and stiffness matrices, respectively [28], as:

\[ Q = \begin{bmatrix} q_{\theta_1} \\ q_{\theta_2} \end{bmatrix}, \quad M = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, \quad G = \begin{bmatrix} 2U\beta^{0.5} B & 0 \\ 0 & 2U\beta^{0.5} B \end{bmatrix} \]  \hspace{1cm} (36)

\[ K = \begin{bmatrix} (\gamma + \mu)\lambda + \alpha U^2 C + k_u l & -k_u l \\ -k_u l & (\gamma + \mu)\lambda + \alpha U^2 C + k_u l \end{bmatrix} \]

1 is N×N identity matrix

\[ B_{ij} = \begin{cases} \frac{4}{(2i-1)^2} - 1, & i + j \text{ (odd)} \\ \frac{4}{(2j-1)^2} + 1, & i + j \text{ (even)} \\ 2, & i = j \end{cases} \]  \hspace{1cm} (37)

\[ C_{ij} = \begin{cases} \frac{2(2j-1)\pi_j - (2i-1)\pi_i}{-1 - \left(\frac{2i-1}{2j-1}\right)^2}, & i \neq j \text{ (odd)} \\ \frac{2(2j-1)\pi_j - (2i-1)\pi_i}{-1 - \left(\frac{2i-1}{2j-1}\right)^2}, & i \neq j \text{ (even)} \\ \frac{2j-1}{2}, & i = j \end{cases} \]  \hspace{1cm} (38)

\[ \lambda = \begin{cases} \frac{2i + 1}{2}, & i = j \\ 0, & i \neq j \end{cases} \]  \hspace{1cm} (39)

where \( B_{ij} \), \( C_{ij} \), and \( \lambda \) indicates the matrices elements B, C, and \( \lambda \) with \( i \)th refer to row and \( j \)th refer to the column, respectively. The periodic solution of the Eq. (22), can be assumed in the following complex form [28]:

\[ q_{x_1} = S_1 e^{i\omega t}, \quad q_{x_2} = S_2 e^{i\omega t}, \quad \ldots, \quad q_{x_{q}} = S_{q} e^{i\omega t} \]

\[ q_{x_1} = S_{N+1} e^{i\omega t}, \quad q_{x_2} = S_{N+2} e^{i\omega t}, \quad \ldots \]
\[ q_{2N} = S_2N e^{i\omega t} \]  
(40)

where the imaginary unit is \( i \), \( \omega \) is the vibration frequency and \( S_1 - S_{2N} \) is the amplitudes. Substituting Eq. (40) into Eq. (35) leads to the following equation:

\[ (-w^2 M + iwG + K)S = 0 \]  
(41)

where \( S \) refers to \( [S_1, S_2, \ldots, S_{2N}]^T \). A solution of Eq. (41) requires a determinant of a coefficient matrix equivalent to zero, the characteristic system equation is:

\[ \det(-w^2 M + iwG + K) = 0 \]  
(42)

The stability of pipe conveying fluid depends on the critical velocity and the type of end conditions (conservative or non-conservative system). For the non-conservative system (cantilever micro-pipe) loss stability when fluid velocity exceeds the critical velocities.

Once (42) is evaluated, an equation of the form:

\[ f(w) = a_0 w^n + a_1 w^{n-1} + \ldots + a_{m-1} w + a_m \]  
(43)

where the coefficients, \( a_i \)'s, are functions of \( w \) and \( u \). Once (43) is established, a Hurwitz determinant is formed with its elements being the coefficients in (43), for \( N = 2 (m = 4) \).

\[
T_4 = \begin{bmatrix}
  a_1 & a_0 & 0 & 0 \\
  a_3 & a_2 & a_1 & 0 \\
  0 & a_4 & a_3 & a_2 \\
  0 & 0 & 0 & a_4
\end{bmatrix}
\]  
(44)

And, for \( m > 4 \)

\[
T_m = \begin{bmatrix}
  a_1 & a_5 & a_7 & \cdots & a_{2m-1} \\
  a_0 & a_2 & a_4 & \cdots & a_{2m-2} \\
  0 & a_1 & a_3 & \cdots & a_{2m-3} \\
  0 & a_0 & a_2 & \cdots & a_{2m-4} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & 0 & 0 & a_m
\end{bmatrix}
\]  
(45)

where \( N \) is the number of the basic functions (terms) taken. (See Appendix for details of the ROUTH-HURWITZ method)

**IV. RESULT AND DISCUSSIONS:**

The numerical results for coupled-cantilever micropipes conveying fluid based on MSGT are offered in this section. The geometrical parameters for coupled FGM micro-pipe are used as those utilized by [27] as \( \text{Di/Do} = 0.9, \text{Do}=20 \mu m, \text{length} l \) of FGM micro-pipe is assumed \( 17.6 \mu m \), the spring stiffness parameter \( k=100, \) the density of fluid is \( \rho_f = 1000 \text{kg/m}^3 \) and the mechanical properties for a constituent materials are shown in Table (1). MATLAB packages are developed to investigate an effect of different parameters on the instability and free vibration of coupled FGM micro-pipes.
As the first step, the precision of the method used for the homogenous pipe is verified by comparing with those reported by [1], and we obtained a reasonable agreement as shown in Fig.4.

![Figure 4](image)

**Fig. 4** The validation of a flutter instability with mass ratio for a fully metal cantilever FGM pipe.

Figure 5 shows the influence of the parameter of length scale on the variation of the real component of the dimensionless natural frequency with dimensionless fluid flow velocity for the first mode. It is noted that the critical velocities and the real component of natural frequency decrease with the increase of the parameter of length scale.

It must be noted that the influence of material length scale on a free vibration of coupled FGM micro-pipe conveying fluid is very significant, and it makes a micro pipe more balanced and stable, especially when a diameter of micro-pipe is comparable to a parameter of length scale ($Do/L$).

This is because of the parameter of length scale has an effect on increasing an equivalent bending hardness [$\left(\frac{E I_{eq}}{\nu} + (G A_{eq} L^2)\right)$].

![Figure 5](image)

**Fig. 5** The natural frequency with fluid flow velocity for different values of D/L

Figures 6a, b, and c depict the real and imaginary components of vibration frequencies with dimensionless fluid flow velocity for first, second and third modes when $k=100$ and $D/L=10$ for diverse values of the exponent of volume fraction $n$. 
Fig. 6a illustrates the divergence of first and second modes when $n=0$, we noted that the divergence of the first mode happens at $u=1.88$ and coupled $2^{nd}$ and $3^{rd}$ modes flutter at $u=6.6$ and for the second mode at $u=5.02$. In addition, when $n=1$ the divergence of first modes occurs at $u=3.54$ and coupled $1^{st}$ and $2^{nd}$ modes occurs at $u=8.32$, as shown in Fig. 6b. While for $n=3$, the first mode divergence happens at $u=4.01$, in addition, the $1^{st}$ and $2^{nd}$ mode combined at $u=9.45$, as shown in Fig. 6c. Through this, we can deduce that critical velocities will increase with an increase in the exponent of volume fraction $n$.
Fig. 6 First three natural frequencies of coupled FGM micropipes conveying fluid with dimensionless fluid velocity: (a) $n=0$, (b) $n=1$ and (c) $n=3$.

Fig. 7 explains the influences of gradient index $n$ on the critical velocities and natural frequencies of coupled FGM micro pipes conveying fluid when $D_o = 20 \mu m$ and $D_i/D_o=0.9$. From this figure, it is noted that at $n=0$, the critical velocities and natural frequency are lower as compared with $n=1$, and 10. Therefore, it can deduce that real and imaginary components of the natural frequencies and critical velocities are increased significantly with an increase in the exponent of volume fraction.

![Fig. 7](image1)

Fig. 7: Real and imaginary components of natural frequency with fluid flow velocity for different value of gradient index ($k=100$).

Figs. 8a and b illustrated the stability scheme of the homogenous and FG micro-pipe respectively, with three number of modes (N). It is observed that the FG micro-pipe has the greatest stable region in comparison with homogenous for $N=4$. It should be noted that the velocities below the flutter velocity, the vibrational motion would eventually decay. At velocities above the flutter velocity, any initial dynamic structural disturbance will grow. Thus, the fluid conveying pipe will lose its stability at lower velocity of fluid compared with FG micro-pipe.

![Fig. 8](image2)

Fig. 8 The critical velocity with mass ratio ($M_r$) for a) homogenous micro pipe and b) single FGM micro pipe ($n=0$).

Figure 9 presents the variations of the critical flow velocity for the flutter phenomenon of cantilever (micro-pipe, and FG micro-pipe) versus fluid mass ratio. The results are presented for $N=4$, and $k=10$. The flutter boundaries have several S-shaped segments that are associated to the instability-destabilization-instability sequences. These S-shaped segments indicate that the thin-walled pipe conveying fluid loses its stability in different modes.
It is observed that the FG micro-pipe has the greatest stable region in comparison with other cases. It should be noted that the velocities below the flutter velocity, the vibrational motion would eventually decay. At velocities above the flutter velocity, any initial dynamic structural disturbance will grow. Thus, the fluid conveying pipe will lose its stability at lower velocity of fluid compared with other cases.

![Fig. 9 The critical velocity with mass ratio (Mr) for, homogenous micro-pipe and FGM micro-pipe.](image)

The critical velocity for cantilever FGM micro-pipe conveying fluid with mass ratio at the various value of volume fraction exponent ($n=0.1, 1, \text{ and } 5$) are presented in Figures. 10. It is found that as the volume fraction exponent increases, the flutter velocity and stability reign increase. The flutter boundaries have several S-shaped segments that are associated to the instability-restabilization-instability sequences. These S-shaped segments indicate that the FGM micro-pipe conveying fluid loses its stability in different modes.

![Fig. 10 The critical velocity with mass ratio for double FGM micro pipe for various values of gradient index.](image)

Figure 11 depicts the flutter critical velocity with the mass ratio for different values of length scale parameters ($l = 15e - 6m, l = 20e - 6m, \text{ and } l = 25e - 6m$), it is noted that the critical velocities are increased with increase in length scale $l$ and gradient index $n$. Thus, the stability of coupled FGM micro-pipe are increased.
Figure 12 explains the influence of mass ratio of coupled FGM micro-pipes on the flutter dimensionless critical velocities and critical frequencies predicted from a various value of outer diameter ($D_o = 20e-6, 25e-6, and 30e-6$). It is found that the critical velocities and frequencies are increased with increase the length sale parameter $l$ and decrease with increase the outer diameter.

V. CONCLUSIONS:

The instability and free vibration of cantilever functionally graded FG material coupled micropipes conveying fluid are investigated in this research. The theory of modified coupled stress and Galerkin’s method are used with the extended Hamilton’s principle, the vibration equation of motion and associated boundary conditions are solved to find the natural frequencies and the critical velocities. The influence of the parameter of length scale, gradient index, the outer diameter of the FG micro-pipe the velocity of flow on the mechanical conducts of coupled FGM micro-pipes are investigated. Some main conclusions obtained from the results above as follows:
1. The real component of natural frequency decrease with an increase in the parameter of length scale, when the outer diameter comparable into the parameter of length scale a size effect is very significant for cantilever coupled FGM micro-pipes and it makes FGM micro-pipes more stable.

2. The flutter critical velocities and natural frequencies increase with the increase in gradient index \( n \). therefore the stability of cantilever coupled FG micro-pipes increases with increase in an exponent of volume fraction \( n \) and can be amended by the distribution of natural frequencies easily by designing of an exponent of the volume fraction \( n \).

3. The flutter critical velocities increase with the increase of mass ratio for FGM micro-pipe, homogenous micro-pipe, and pipe. In addition, the flutter velocity for FGM micro-pipe is greater than for homogenous micro pipe and homogenous pipe.

4. The critical velocity and critical frequency are increased with an increase in the length scale \( L \), and decrease with increase in outer diameter \( D \).

**LIST OF SYMBOLS:**

- \( E \) Young’s modulus
- \( \rho_p \) The density of FGM micro pipe
- \( \rho_f \) The density of fluid
- \( \nu \) Poison ratio
- \( R_i \) The inner radius of FGM micro pipe
- \( R_o \) The outer radii of FGM micro pipe
- \( h \) The thickness direction
- \( L \) Length of micro pipe
- \( u \) Fluid velocity
- \( n \) Gradient index
- \( G \) The modulus of rigidity
- \( z \) The distance to the mid-plane of the FG micro pipe
- \( \text{Vi} \) Volume fraction inner
- \( \text{Vo} \) Volume fraction outer
- \( m_f \) The mass of the fluid
- \( m_p \) The mass of the micro pipe
- \( A_f \) Flow cross sectional area
- \( T \) Kinetic energy
- \( \alpha \) The effect of size for micro-flow
- \( U \) Strain energy
- \( Mr \) The mass ratio

**APPENDIX:**

Routh-Hurwitz Stability Criteria Developed and modified by Routh and Hurwitz in (1895), the Routh- Hurwitz stability criteria locate the stability of the polynomial equation without calculating the roots. Assume the \( m^{th} \) degree polynomial is of the style (Petrus 2006)

\[
f(s) = a_0 s^m + a_1 s^{m-1} + a_2 s^{m-2} \]  

(A1)

The Hurwitz array is of form
\[
\begin{pmatrix}
\begin{array}{cccccc}
s^m & a_0 & a_2 & a_4 & \cdots & a_m \\
s^{m-1} & a_1 & a_3 & a_5 & \cdots & a_{m-1} \\
s^{m-2} & b_1 & b_2 & b_3 & \cdots & \ \\
s^{m-3} & c_1 & c_2 & c_3 & \cdots & \ \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\end{array}
\end{pmatrix}
\]

(A2)

Where the \(a_i\)'s are the polynomial coefficients and

\[
\begin{align*}
b_1 &= -\frac{1}{a_1} \begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix}, & b_2 &= -\frac{1}{a_1} \begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix} = -\frac{1}{a_1} \begin{vmatrix} a_0 & a_6 \\ a_1 & a_7 \end{vmatrix} \\
c_1 &= -\frac{1}{b_1} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, & c_2 &= -\frac{1}{b_2} \begin{vmatrix} a_1 & a_5 \\ b_2 & b_4 \end{vmatrix} 
\end{align*}
\]

(A3)

The first five Hurwitz determinants are:

\[
T_0 = a_0, \quad T_1 = a_1, \quad T_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}, \quad T_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_4 \\ 0 & a_4 & a_5 \end{vmatrix}, \quad T_4 = \begin{vmatrix} a_1 & a_0 & 0 & 0 \\ a_3 & a_2 & a_4 & a_6 \\ 0 & a_4 & a_5 & a_7 \\ 0 & 0 & 0 & a_8 \end{vmatrix}
\]

(A4)

And, for \(m \geq 4\)

\[
T_m = \begin{vmatrix}
\begin{array}{cccccccc}
a_1 & a_3 & a_5 & a_7 & \cdots & a_{2m-1} \\
a_0 & a_2 & a_4 & a_6 & \cdots & a_{2m-2} \\
0 & a_1 & a_4 & a_5 & \cdots & a_{2m-3} \\
0 & a_0 & a_2 & a_4 & \cdots & a_{2m-4} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a_m \\
\end{array}
\end{vmatrix}
\]

(A5)

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