Optimal Installation of Overhead Lines (Tegusu) to Reduce Predation from Piscivorous Birds to Plecoglossus altivelis (Ayu) Taking into Account of Its Decaying Protection Effect

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Abstract: This paper develops a simple mathematical model for determining an optimal installation timing of the overhead lines (Tegusu) to reduce predation from piscivorous birds, such as Phalacrocorax carbo (Kawau), to Plecoglossus altivelis (Ayu) taking into account of its decaying protection effect. Temporal dynamics of the total number and the individual body weight of P. altivelis are described with the system of ordinary differential equations for non-renewable population. Then, this paper formulates and numerically solves the governing equation of the optimal installation timing to simultaneously minimize the total predated P. altivelis and the cost to install and maintain the overhead lines. Extensive comparative statics of the optimal installation timing against model parameters is carried out to investigate its dependence on river environment and attitudes of decision makers.

Keywords: Inland fishery; Optimal timing; Population dynamics; Feeding damage; Plecoglossus altivelis; Phalacrocorax carbo

1 Introduction

Plecoglossus altivelis (Ayu) has been economically, culturally, and ecologically important inland fish in Japan (Aino et al., 2015, Tsuibo et al., 2015). Natural P. altivelis is a diadromous fish that has unique life history (Takahashi and Azuma, 2006). Adults of the fish spawn eggs during autumn in downstream reaches of their living river and die soon afterwards. Hatched larvae descend to coastal areas of a downstream water body of the river. In the coming spring, the juvenile fishes ascend toward midstream of the river to mature until the coming autumn.

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Recently, excessive predation pressure from piscivorous birds, such as Phalacrocorax carbo (Kawau) and Ardea cinerea (Aosagi) have caused significant feeding damage to P. altivelis. This problem should be urgently solved since it may devastate Japanese inland fishery and destabilize river ecosystems (Yamamoto, 2008). For overcoming this severe situation, inland fishery cooperatives and local municipalities have conducted various countermeasures for reducing predation from piscivorous birds to the fish (Yamamoto, 2010). One of the most common countermeasures is installing the overhead lines (Tegusu) to reduce the predation (Yamamoto, 2010). The overhead lines are removed from rivers before the opening time of harvesting P. altivelis. The magnitude and timing of installing the overhead lines have been empirically determined so far, and their theoretical investigation have not been performed yet, except for the authors’ previous research (Yaegashi et al., 2016b). They employed the theory of optimal timing (stopping) (Dixit and Pindyck, 1994), which is a germinating approach for the above-mentioned problems. Existence and uniqueness of the optimal timing were mathematically analyzed, and comparative statics of the optimal timing was also numerically performed for comprehending its dependence on model parameters. This model assumes that the protection effect is everlasting once they are installed. However, it has been pointed out that the protection of the overhead lines is not everlasting (Yamamoto, 2010), but it decays as the time elapses, which is not considered in the previous model. Cost-effective installation of the overhead lines should therefore assume that the protection effect decays as the time elapses. This is the main motivation of this paper.

The objective of this paper is to develop and analyze a new and simple mathematical model for cost-effective installation of the overhead lines taking into account of its decaying protection effect. Since this is a germinating research topic, comprehension of characteristics of the model is of importance at this stage. The governing equation of the optimal installation timing is derived and its solution is numerically approximated. A brief mathematical analysis is also carried out to qualify dependence of the optimal installation timing on model parameters.

The rest of this paper is organized as follows. Section 2 formulates the mathematical model. Section 3 numerically performs comparative statics of the optimal installing timing. Section 4 concludes this paper.

2 Mathematical model

2.1 Population dynamics model

Deterministic population dynamics of P. altivelis is
considered (Yaegashi et al., 2016b). This paper considers a river where the population of \( P. \) \( altivelis \) comes from releases by local fishery cooperatives, and the number of \( P. \) \( altivelis \) which ascends naturally from the sea is almost negligible; one example is Hii River, Japan, focused on later. In addition, \( P. \) \( altivelis \) does not reproduce during the period considered in this paper, namely from April to June in a year. Therefore, the river can be theoretically considered as a closed environment. The time is denoted as \( t \) (day). The period to decide the timing of installing the overhead lines is \([0, T]\) with the terminal time \( T > 0 \) (day). At the initial time \( t = 0 \) (day), juveniles of natural \( P. \) \( altivelis \) ascend to mainstream of a river, or cultured fishes are released there. At the time \( t = T \) (day), the overhead lines should be removed from the rivers because harvesting of \( P. \) \( altivelis \) begins, after which piscivorous birds tend to avoid human. Total number of the individuals at the time \( t \) is denoted as \( N_t \) (-) and its representative body weight at the time \( t \) is denoted as \( W_t \) (kg). The body weight is assumed to be homogenous in the population. The governing ordinary differential equations (ODEs) of \( N_t \) and \( W_t \) are set as

\[
\frac{dN_t}{dt} = [R + p \{1 - z(\lambda_{\text{ex}} t)\}]N_t \tag{1}
\]

and

\[
\frac{dW_t}{dt} = rW_t \left(1 - \frac{W_t}{K}\right) \tag{2}
\]

with

\[ z = z \exp(-\mu(t - \tau)) \tag{3} \]

where \( R \) (1/day) is the natural mortality rate of the fish, \( p \) (1/day) is the predation pressure from piscivorous birds, \( \lambda_{\text{ex}} \) is the indicator function for generic set \( S \subset [0,T] \) such that \( \lambda_{\text{ex}}(t) = 1 \) for \( t \in S \) and \( \lambda_{\text{ex}}(t) = 0 \) for \( t \notin S \), \( \tau \in [0,T] \) is the time to install the overhead lines, \( 0 < z_{\text{ex}}(\leq z) \leq 1 \) (-) is effectiveness to reduce the predation pressure with the exponential decay rate \( \mu > 0 \) (1/day) in time, \( z \) (-) is the upper bound of effectiveness, \( r \) (1/day) is the intrinsic growth rate of the fish, and \( K \) (kg) is the upper limit of its body weight. The ODE (1) is modelled based on the pure death process in which the total number of \( P. \) \( altivelis \) monotonically decreases as the time elapses. The ODE (2) represents the growth of individual fish based on the classical Verhulst model (Blowmick et al., 2014), which is the simplest sigmoid type growth curve model. Coupling of ODEs like Eqs. (1) and (2) is often adopted as a deterministic mathematical model for describing fishery resources dynamics (Pascoue et al., 2012). Eq. (3) serves as a minimal model to represent the situation that the piscivorous birds gets accustomed to the overhead lines and its effectiveness decays as the time elapses. The initial conditions of (1) and (2) are \( N_0 \geq 0 \) (-) and \( W_0 > 0 \) (kg); the former is the total number of \( P. \) \( altivelis \) at the initial time \( t = 0 \) in the river, and the latter is the average initial body weight of \( P. \) \( altivelis \). The system of ODEs (1) and (2) are decoupled each other. Using a standard method of quadrature, the ODEs are solved as

\[
N_t = N_0 \exp \left\{ -\frac{(R + p)t}{\mu} + \frac{\lambda_{\text{ex}}(\tau) p}{\mu} \{1 - \exp[-\mu(t - \tau)]\} \right\} \tag{4}
\]

and

\[
W_t = \frac{KW_0 \exp(rt)}{W_0 \exp(rt) + K - W_0}, \tag{5}
\]

respectively.

### 2.2 Optimal timing problem

The objective function \( J_\tau \) to be maximized with an optimal installation timing \( \tau' \in [0, T] \) is established from a bio-economic viewpoint. The objective function represents the net profit of a fishery cooperatives, which includes the profits of cooperative members as a main part. According to the above-mentioned assumption, \( J_\tau \) is set as

\[
J_\tau = \sum_{s=1}^{T} pN_{s}W_{s} \exp(-\delta s) ds - \int_{0}^{T} p(1 - z_{\text{ex}})N_{s}W_{s} \exp(-\delta s) ds - I \exp(-\delta \tau) \tag{6}
\]

where \( I \) (kg) is the cost of installing and maintaining the overhead lines and \( \delta \) (1/day) is the discount rate that represents the situation where the cost of maintaining the installed overhead lines increases as the installation is carried out earlier, and damage in inland fishery sectors increases as predated fish increases earlier. The first (second) term in the right hand side of (6) represents the total predated weight of the fish before (after) installing the overhead lines. The third term represents the cost of installing and maintaining the overhead lines. The maintenance cost consists of monetary costs and labors for partial re-installation of the overhead lines due to damages by winds, birds, and floods. The optimal timing to install the overhead lines is determined so that the sum of the total predated weight of \( P. \) \( altivelis \) and the cost to install and maintain the overhead lines are minimized. In the present model, \( \tau' = T \) means that no installation of the overhead lines is optimal. The installation cost term \( I \exp(-\delta \tau) \) in the objective function is supposed to contain the cost of installing overhead lines at the time \( \tau \) and their maintenance cost after that time. Mathematical modelling with the discount rate \( \delta \), which appears in all the terms of the objective function, is possibly one of the simplest approaches to mathematically formulate the above-mentioned consideration. This kind of approaches have been successfully employed in operations research (Dixit and Pindyck, 1994).

It is shown that \( J_\tau \in C^2(0, T) \cap C((0, T)) \), namely, \( J_\tau \) is continuous over \([0, T]\) and is continuously twice differentiable in \((0, T)\). The present model reduces to the previous counterpart (Yaegashi et al., 2016b) under the limit of \( \mu \to +0 \) where the protection effect of the overhead lines is everlasting. It should be noted that, in the
2.3 Analytical method

Based on the theory of standard static optimization problem of single variable, a maximizer \( \tau = \tau^* \) such that \( \tau^* \in (0, T) \) must satisfy the first- and second-order conditions of the optimality (Rao and Rao, 2009)

\[
\frac{\partial J_r}{\partial \tau} = 0 \quad \text{and} \quad \frac{\partial^2 J_r}{\partial \tau^2} < 0 \, ,
\]

respectively. With the help of the Leibnitz’s rule (Yoshioka and Unami, 2013), the left of (7) is expressed as

\[
\frac{dJ_r}{d\tau} = \int_I \frac{pz}{\mu} \exp\left(-\mu(s - \tau^*) - \delta s\right) N_w \, dz + pN_w \, \exp\left(-\delta \tau^*\right) - \frac{I}{\delta} \exp\left(-\delta \tau^*\right) \, .
\]

Similarly, the right of (7) is expressed as

\[
\frac{d^2 J_r}{d\tau^2} = \frac{1}{\mu} \frac{dJ_r}{d\tau} + \frac{I}{\delta^2} \exp\left(-\delta \tau^*\right) + pN_w \, \exp\left(-\delta \tau^*\right) \left[-R + p \left(1 - z \exp\left(-\mu(t - \tau^*)\right)\right)\right] + \frac{r}{1 - K} - \delta \, .
\]

It follows directly from (8) and (9) that \( \tau^* \) complying with (7) is increasing with respect to \( I \) and \( N_0 \). In addition, assuming the condition \( pz \leq \mu \) and sufficiently large \( I \) or \( N_0 \) shows that no installation of the overhead lines is optimal. This implies that installing the overhead lines with small protection effect should be abandoned.

3 Numerical computation

3.1 Computational conditions

Extensive comparative statics of \( \tau^* \) is numerically carried out to investigate its dependence on the model parameters. The parameters investigated in this paper are \( R \), \( p \), \( \delta \), \( \mu \), \( I \), \( N_0 \), and \( z \). Two of them are chosen in each numerical computation, independently changed, while other model parameters are fixed at the benchmark values. For each couple of the chosen parameters, \( \tau^* \) are found directly from (6) using the classical Simpson rule (Kendall, 1989). The benchmark value and the range of the model parameters used in this paper are shown in Table 1. The benchmark values are specified according to the case of Hii River, where feeding damage from \( P. \) carbo to \( P. \) altivelis has significantly been increasing. The intrinsic growth rate \( r \) is estimated as \( 7.1 \times 10^{-2} \) (1/day) from the growth curve of \( P. \) altivelis in Hii River (Yaegashi et al., 2016c). The natural mortality rate \( R \) is estimated as \( 3.9 \times 10^{-3} \) (1/day) (Miyaji et al., 1963). By considering the number of \( P. \) carbo in the colony at Lake Nakaumi, which is the nearest colony from Hii River, the order of the predation pressure \( p \) is estimated as \( 5.0 \times 10^{-3} \) (1/day). Based on the data presented by Hii River fishery cooperatives, the number of released \( P. \) altivelis \( N_0 \) is specified as \( 1.9 \times 10^5 \), the initial average body weight of \( P. \) altivelis \( W_0 \) as \( 9.4 \times 10^{-3} \) (kg), the upper bound of the body weight \( K \) as \( 5.1 \times 10^{-2} \) (kg), and the opening time of catching \( P. \) altivelis \( T \) as 60 (day). The cost of installing and maintaining the overhead lines \( I \), the discount rate \( \delta \) and the upper bound of effectiveness \( z \) are set so that \( \tau^* \) exists between \( (0, T) \) for most of the computational cases.

The constant \( \mu \) is set as \( 3.3 \times 10^{-3} \) (1/day) following the observation results (Fisheries Agency, 2003), which correspond to the situation where the protection effect significantly decays after one month. Each range of the parameters is set for including each benchmark value. The benchmark parameter values may not be accurate at the present stage, while we consider that the order of each parameter is reasonable. Identifying more accurate values and ranges of the parameters, which is an important future topic, will be specified through the undergoing collection of data in Hii River and interviews to officers of local Fishery cooperatives and local residents.

3.2 Computational results

The computational results of the optimal installation timing \( \tau^* \) are presented and their implications to management of \( P. \) altivelis are discussed. Figure 1 through 6 show computed \( \tau^* \) with a variety of couples of the parameters. The results obtained from these figures can be summarized as follows. Figures 1(a)-1(d) and 2(a)-2(b) show that \( \tau^* \) increases when \( \delta \) increases. Figures 2(a), 2(c), 2(d) and Figures 3(a)-3(c) show that \( \tau^* \) increases when \( I \) increases. In addition, Figures 2(b), 3(b), 3(d), and 4(a)-4(c) indicate that \( \tau^* \) also increases when \( z \) increases. On the other hand, Figures 1(a), 3(c), 4(c), and 5(a)-5(c) show that \( \tau^* \) decreases when \( N_0 \) increases. Furthermore, Figures 1(d), 3(a), 4(b), 5(a), and 6(a)-6(b) indicate that \( \tau^* \) decreases when \( z \) increases. In summary, the above-mentioned computational results indicate that increasing the discount rate \( \delta \), the cost of installing and maintaining the overhead lines \( I \), or the exponential decay rate \( \mu \) delays or abandons the installation. While, increasing the initial number of \( P. \) altivelis \( N_0 \) or the upper bound of effectiveness \( z \) leads to hastens the installation. These results are considered to be in good accordance with our intuition.

Unlike the above mentioned computational results, dependences of \( \tau^* \) on the predation pressure \( p \) and the natural mortality rate \( R \) do not seem to be monotone. According to Figures 1(c), 2(d), 4(a), 5(c), and 6(a), \( \tau^* \) increases when \( R \) increases; however, its dependence on \( R \) seems not to be monotone as demonstrated in Figure 6(c). As the natural mortality rate \( R \) increases, decreasing \( \tau^* \) or abandon of the installation is optimal except for the
cases where the predation pressure $p$ is excessively large. The result inferred that for large $R$, it is no use installing the overhead lines. In such a case, the decision maker should firstly improve the river environment so that $R$ gets smaller, and then consider when and how to install the overhead lines. Moreover, these figures show that the dependence of $r^*$ on $R$ is more sensitive than that on the other parameters investigated, namely, the natural mortality rate $R$ is a critical parameter in this model. This implies that accurate estimation of $R$ is important for real application of the present model.

According to Figures 1(b), 2(c), 5(b), and 6(b)-6(c), for large and small $p$, no installation of the overhead lines is optimal. This result implies that it is no use installing the overhead lines for large predation pressure, while there is no need to install them for small predation pressure. In such a case, the decision maker should decrease predation pressure by taking other countermeasures such as shooting at first for more effective installations of overhead lines.

The tendency of the change of the optimal installation timing $r^*$ is almost the same in both presented and previous models (Yaegashi et al., 2016b); however, the presented model gives larger $r^*$ than that of the previous model where the effect of the overhead lines is everlasting. By setting the model parameters as the benchmark parameters, the reference optimal timing $r^*$ in Hii River can be calculated as $r^* = 37$ (day) with the presented model and $r^* = 9$ (day) with the previous model. The result recommends that, with the specified model parameter values, the overhead lines should be installed after about one month from intensive release events of the juvenile $P. altivelis$. In addition, the computational results suggest that the installation of the overhead should be hastened if the effectiveness of the overhead lines improves, namely $\mu$ decreases or $z$ increases.

### 3.3 Possible future scenarios

This sub-section considers possible future scenarios of river environment and the associated changes of the value of the optimal installation timing $r^*$.

When rock-attached algae (Abe, 2013) as a staple food of $P. altivelis$ does not grow well, or there is an outbreak of cold water disease (Iida and Mizokami, 1996), which is a fatal disease of $P. altivelis$ and occurs when temperature of a river remains between 16°C and 20°C, the natural mortality rate $R$ would dramatically increase. The computational result predicts that the optimal timing $r^*$ increases except for the case with severe predation.

The discount rate $\delta$ represents the attitude of the decision maker: fishery cooperatives. As $\delta$ becomes larger, the decision maker attaches more importance to the number of $P. altivelis$ at early stage, namely, the decision maker avoids decrease of the number of $P. altivelis$ at early stage. Thus, when the decision maker attaches more importance to the number of $P. altivelis$ at early stage, the computational results predict that $r^*$ increases. The results also suggest that installation of the overhead lines should be hastened if fishery cooperatives decreases the number of the juveniles of $P. altivelis$ released at the time $t = 0$ or the natural ones that ascend to the river at the time decreases. This would happen if financial situation of fishery cooperatives get worse due to the decrease of the fish catches and degradation of river environment.

### Table 1: Model parameters

| Parameter | Unit | Benchmark Value | Range |
|-----------|------|-----------------|-------|
| $r$       | 1/day| $7.1 \times 10^{-2}$ | -     |
| $R$       | 1/day| $3.9 \times 10^{-3}$ - $1.0 \times 10^{-3}$ |       |
| $P$       | 1/day| $5.0 \times 10^{-3}$ - $1.0 \times 10^{-3}$ |       |
| $\delta$  | 1/day| $1.0 \times 10^{-5}$ - $1.0 \times 10^{-4}$ |       |
| $\mu$     | 1/day| $3.3 \times 10^{-2}$ - $1.0 \times 10^{-3}$ |       |
| $K$       | kg   | $5.1 \times 10^{-2}$ | -     |
| $I$       | kg   | $1.0 \times 10^{5}$ - $1.0 \times 10^{6}$ |       |
| $W_0$     | kg   | $9.4 \times 10^{-3}$ | -     |
| $N_0$     |      | $1.9 \times 10^{6}$ - $1.0 \times 10^{4}$ |       |
| $z$       |      | 0.50            | 0.0 - 1.0 |
| $T$       | day  | 60             | -     |

### 4 Conclusions

This paper presented a new and simple deterministic model for cost-effective installation of the overhead lines for reducing predation from piscivorous birds to $P. altivelis$ (Ayu) taking into account of its decaying protection effect. In addition, this paper numerically analyzed characteristics of the model in detail. The numerical comparative statics results of the optimal timing to install the overhead lines was extensively conducted, which revealed its dependence on the model parameters. The authors are currently collecting data for verifying the presented model.

The presented model can be extended to a stochastic counterpart where finding the optimal installation criteria is reduced to resolution of a free-boundary problem of partial differential equations (Dixit and Pindyck, 1994). Mathematical analysis of the deterministic and stochastic models will be also carried out to better comprehend dependence of the optimal installation timing on the model parameters and to propose more cost-effective management strategies of inland fishes in the future.

Application of the model to more realistic problems, which is an important research topic, is currently undergoing via collection of data in Hii River. Interviews to officers of local Fishery cooperatives and local residents will also be carried out for verifying the model against their empirical knowledge. Verifying the model will be discussed as a next step of our research.

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Figure 1: $\tau^*$ for different values of (a) $(\delta, N_0)$, (b) $(\delta, p)$, (c) $(\delta, R)$ and (d) $(\delta, z)$

Figure 2: $\tau^*$ for different values of (a) $(I, \delta)$, (b) $(\mu, \delta)$, (c) $(I, p)$ and (d) $(I, R)$

Figure 3: $\tau^*$ for different values of (a) $(I, z)$, (b) $(\mu, I)$, (c) $(\mu, N_0)$ and (d) $(\mu, p)$

Figure 4: $\tau^*$ for different values of (a) $(\mu, R)$, (b) $(\mu, z)$ and (c) $(\mu, N_0)$

Figure 5: $\tau^*$ for different values of (a) $(z, N_0)$, (b) $(p, N_0)$ and (c) $(R, N_0)$

Figure 6: $\tau^*$ for different values of (a) $(R, z)$, (b) $(p, z)$ and (c) $(R, p)$
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