Spherical collapse model and cluster number counts in power law $f(T)$ gravity

M. Malekjani$^1$ *, S. Basilakos $^2$, N. Heidari $^1$

$^1$ Department of Physics, Bu Ali Sina University, Hamedan 65178, Iran
$^2$ Academy of Athens, Research Center for Astronomy and Applied Mathematics, Soranou Efessiou 4, 11-527 Athens, Greece

Accepted ?, Received ?, in original form December 23, 2016

ABSTRACT

We study the spherical collapse model (SCM) in the framework of spatially flat power law $f(T) \propto (-T)^b$ gravity model. We find that the linear and non-linear growth of spherical overdensities of this particular $f(T)$ model are affected by the power-law parameter $b$. Finally, we compute the predicted number counts of virialized haloes in order to distinguish the current $f(T)$ model from the expectations of the concordance $\Lambda$ cosmology. Specifically, the present analysis suggests that the $f(T)$ gravity model with positive (negative) $b$ predicts more (less) virialized objects with respect to those of $\Lambda$CDM.

Key words: cosmology: methods: analytical - cosmology: theory - dark energy - large scale structure of Universe.

1 INTRODUCTION

The idea of the accelerated expansion of the universe is supported by several independent cosmological experiments including those of supernova type Ia (Riess et al. 1998; Perlmutter et al. 1999; Kowalski et al. 2008), cosmic microwave background (CMB) (Komatsu et al. 2009, 2011; Jarosik et al. 2011; Planck Collaboration XIV 2016), large scale structure and baryonic acoustic oscillation (Percival et al. 2010; Tegmark et al. 2004; Cole et al. 2005; Eisenstein et al. 2005; Reid et al. 2012; Blake et al. 2011a), high redshift galaxies (Alcaniz 2004), high redshift galaxy clusters (Allen et al. 2004; Wang & Steinhardt 1998) and weak gravitational lensing (Benjamin et al. 2007; Amendola et al. 2008; Fu et al. 2008). Cosmic acceleration can well be interpreted in the framework of general relativity (GR) by invoking the dark energy (DE) component in the total energy budget of the universe. Although, the earliest and simplest candidate for DE is the traditional cosmological constant $\Lambda$ with constant equation of state (EoS) parameter $w_{\Lambda} = -1$ (Peebles & Ratra 2003), the well known issues which are associated with the fine-tuning and cosmic coincidence problems, (Weinberg 1989; Sahni & Starobinsky 2000; Carroll 2001; Padmanabhan 2003; Copeland et al. 2006) has led the scientific community to propose a large family of dynamical DE models: quintessence (Caldwell 2002), phantom (Caldwell et al. 2003; Amendola et al. 2007), quintom (Elizalde et al. 2004), Chaplygin gas (Kamenshchik et al. 2001) and generalized Chaplygin gas (Bento et al. 2002) etc. in which $w_{\text{DE}} \neq -1$.

On the other hand, one can consider that cosmic acceleration reflects on the physics of gravity on cosmological scales. Indeed, modifying the Einstein-Hilbert action and using the Friedmann-Robertson-Walker (FRW) spacetime as a background metric one can obtain the modified Friedmann’s equations which can be used in order to understand the accelerated expansion of the universe. As an example, one of the most popular modified gravity models is the $f(R)$ scenario in which we allow the Lagrangian of the modified Einstein-Hilbert action to be a function of the Ricci scalar $R$ (Capozziello & Francaviglia 2008; Nojiri & Odintsov 2011; Sotiriou & Faraoni 2010). Alternatively, among the large group of extended theories of gravity the so-called $f(T)$ gravity plays an important role in this kind of studies. This theory is based on the old definition of the teleparallel equivalent of general relativity (TEGR), first introduced by Einstein (1922) (see also Hayashi & Shirafuji 1979; Maluf 1994). Here, instead of using the curvature defined through the Levi-Civita connection one can assume an alternative approach based on torsion $T$ via the Weitzenböck connection in order to extract the torsion scalar (Hayashi & Shirafuji 1979). Inspired by the methodology of $f(R)$ gravity, a natural extension of TEGR is the theory of $f(T)$ gravity which we assume that the Lagrangian of the modified Einstein-Hilbert action is a function of $T$ (Ferraro & Fiorini 2007; Linder 2010). It is worth noting that in $f(T)$ gravity we have second-order field equations while in $f(R)$ gravity we deal with fourth-order field equations which may lead to pathologies as discussed in the work of Capozziello & Vignolo (2009, 2010). In the literature, there are plenty of papers available that study the cosmological properties of different $f(T)$ models. In particular, the background history and the cosmic acceleration can be found in Refs. (Bengochea & Ferraro 2009; Linder 2010; Myrzakulov 2011; Dent et al. 2011; Zhang et al. 2011; Capozziello et al. 2011; Geng et al. 2011; Bamba et al. 2012). The dynamical aspects and the cosmological constraints of the $f(T)$ models have been investigated in Refs. (Wu & Yu 2010a,
2 M. Malekjani et al.

2011; Dent et al. 2011; Bamba et al. 2011; Capozziello et al. 2011; Geng et al. 2011; Wei 2012; Karami et al. 2013; Bamba et al. 2012) and in Refs. (Wu & Yu 2010b; Nunes et al. 2016; Saez-Gomez et al. 2016; Geng et al. 2012; Wei et al. 2012; Geng et al. 2012; Cardone et al. 2012; Iorio et al. 2015). Also the connection between $f(T)$ and scalar field theory can be found in (Yerzhanov et al. 2010; Chen et al. 2015; Sharif & Rani 2013). Lastly, at the perturbation level we refer the reader the works of Refs. (Chen et al. 2011; Zheng & Huang 2011; Wu & Geng 2012b; Li et al. 2011; Wu & Geng 2012a; Izumi & Ong 2013; Geng & Wu 2013; Basilakos 2016).

It is well known that in order to distinguish modified gravity models from scalar field DE models we need to study the growth of matter perturbations in linear and non-linear regimes. Specifically, the growth index $\gamma$ of linear matter fluctuations (first introduced by Peebles 1993) in $f(T)$ gravity is investigated in Zheng & Huang (2011); Basilakos (2016). Basilakos (2016) found that the asymptotic form of the power-law $f(T)$ model is given by $\gamma \approx 6/11$ which naturally extends that of the $\Lambda$CDM model, $\gamma_{\Lambda} \approx 6/11$.

The spherical collapse model (hereafter SCM), first introduced by Gunn & Gott (1972), is a simple analytical approach to study the evolution of the growth of matter fluctuations in the non-linear regime. Notice, that the scales of SCM are much smaller than the Hubble radius and the velocities are non-relativistic. The central idea of the SCM is based on the fact that due to self-gravity, we expect that the spherical overdensities expand with slower rate than the Hubble expansion. Therefore, at a certain redshift the overdense region completely decouple from the background expansion (reaching to a maximum radius) and it starts to ‘turn around’. This redshift is the so-called turn around redshift, $z_{\text{vir}}$. After $z_{\text{vir}}$, the spherical region collapses due to self gravity and finally it reaches the steady state virial radius at a certain redshift $z_{\text{vir}}$. In the framework of General Relativity (GR), the SCM has been investigated in several independent works (Fillmore & Goldreich 1984; Bertschinger 1985; Hoffman & Shaham 1985; Ryden & Gunn 1987; Avila-Reese et al. 1998; Subramanian et al. 2000; Ascasibar et al. 2004; Williams et al. 2004; Mehraj et al. 2017). Also, the SCM has been extended for various cosmological models, including those of DE (Mota & van de Bruck 2004; Moar & Lahav 2005; Basilakos & Voglis 2007; Basilakos et al. 2009; Li et al. 2009; Pace et al. 2010; Wintergerst & Pettorino 2010; Basse et al. 2011; Pace et al. 2012; Naderi et al. 2013; Abramo et al. 2007, 2009; Malekjani et al. 2015), scalar-tensor and modified gravity (Schaefer & Koyama 2008; Pace et al. 2014a; Nazari-Pooya et al. 2016; Fan et al. 2015). We would like to stress that the general formalism of SCM can be used in the case where Birkhoff’s theorem is valid. As an example, $f(R)$ gravity models which are based on metric formalism can not accommodate Birkhoff’s theorem, while in the case of Palatini formalism this theorem holds (Sotiriou & Faraoni 2010; Capozziello et al. 2007; Faraoni 2010). In $f(T)$ gravity, it has been shown that the Birkhoff’s theorem is valid (Meng & Wang 2011) and thus one can extend the SCM in the context of $f(T)$ models.

In the present article, we attempt to study the non-linear growth of matter overdensities and the corresponding number counts of the power law $f(T)$ model (Linder 2010; Ferraro & Fiorini 2007, 2008) (see also Cai et al. 2016, and references therein). To the best of our knowledge, we are unaware of any previous investigation regarding the SCM in $f(T)$ gravity and thus we believe that the current analysis can be of theoretical interest. Notice, that the growth of matter perturbations in the linear regime has been investigated in (Chen et al. 2011; Basilakos 2016).

We organize our paper as follows. In section 2, we briefly present the basic cosmological properties of $f(T)$ gravity and then we focus on the power-law model. In section 3 we study the growth of matter fluctuations in the linear and non-linear (SCM) regimes respectively. In section 4, we compute the predicted mass function and the number counts of the power-law $f(T)$ model and we discuss the differences from the concordance $\Lambda$ cosmology. Finally, we provide our conclusions in section 5.

2 BACKGROUND HISTORY IN POWER LAW F(T) MODEL

In this section we briefly present the main points of the $f(T)$ gravity (see also Basilakos 2016, and references therein). In particular, the action in the case of $f(T)$ gravity is given by

$$I = \frac{1}{16\pi G_N} \int d^4 x e (T + f(T)) [L_m + L_r] ,$$

where $L_m$ and $L_r$ are the matter and radiation Lagrangians respectively. Notice, that $e = \det(e^{\mu}_\nu) = \sqrt{-g}$ and $e_{\mu}(x^\nu)$ are the vierbein fields. In this context, the gravitational field is expressed in terms of torsion tensor which produces (after the necessary contractions) the torsion scalar $T$ (Hayashi & Shirafuji 1979).

Varying the above action with respect to the vierbeins the modified Einstein’s field equations are

$$\varepsilon^{-1}\partial_{\mu}(e_{\nu}^{\lambda}S_{\rho}^{\mu\nu})(1 + f_T) + e_{\nu}^{\lambda}S_{\rho}^{\mu\nu}\partial_{\nu}(T)f_T = -(1 + f_T)e_{\nu}^{\lambda}T^\nu_{\rho\lambda}S_{\rho}^{\mu\nu} + \frac{1}{4}e_{\nu}^{\lambda}[T + f(T)] = 4\pi G e_{\nu}^{\mu}T_{\nu\rho}^{_{\text{em}}} ,$$

where $f_T = \partial f/\partial T$, $f_{TT} = \partial^2 f/\partial T^2$, and $T_{\nu\rho}^{_{\text{em}}}$ represents the standard energy-momentum tensor. Considering the description of perfect fluids the energy momentum tensor takes the form

$$T_{\mu\nu}^{_{\text{em}}} = P g_{\mu\nu} - (\rho + P)u_{\mu}u_{\nu} ,$$

where $u^{\nu}$ is the fluid four-velocity, $\rho = \rho_m + \rho_r$ is the total pressure and $P = P_m + P_r$ is the total pressure with $(P_m, P_r) = (0, \rho_m/3)$. Of course $\rho_m$ ($\rho_r$) and $(P_m, P_r)$ denotes the energy density and pressure of the non-relativistic matter (radiation) respectively. In the matter dominated era and prior to the present time we can neglect the radiation component from the cosmic expansion. Through out the current work we consider the usual form of the vierbeins $e_{\nu}^{\lambda} = \text{diag}(1, a, a, a)$, which leads to a flat FRW metric

$$ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j ,$$

where $a(t)$ is the scale factor of the universe. Now, inserting the aforementioned vierbeins and the energy momentum tensor into the field equations (2) we can provide the modified Friedmann equations

$$H^2 = \frac{8\pi G_N}{3} (\rho_m + \rho_r) - \frac{f}{6} + \frac{T f_T}{3} ,$$

$$H = -\frac{4\pi G_N (P_m + P_r + \rho_m + \rho_r)}{1 + f_T + 2T f_{TT}} ,$$

where the overdot represents the derivative with respect to cosmic time $t$ and $H \equiv \dot{a}/a$ is the Hubble parameter. The Hubble parameter $H$ in $f(T)$ gravity is given in terms of $T$ via the relation

$$T = -6H^2 .$$
From equation (8), it is easy to prove that the dimensionless Hubble parameter is given by

\[ E^2(a) \equiv \frac{H^2(a)}{H_0^2} = \frac{T(a)}{T_b}, \tag{9} \]

which gives

\[ \frac{d\ln T}{d\ln a} = 2T_b E(a) \frac{d\ln E}{d\ln a}, \tag{10} \]

where \( H_0 \) is the Hubble constant and \( T_b \equiv -6H_0^2 \). From equations (6 & 7) we can obtain the energy density and the pressure of the effective DE component as follows (Linder 2010)

\[ \rho_{de} \equiv \frac{3}{8\pi G_N} \left[ -\frac{f}{6} + \frac{Tf_T}{3} \right], \tag{11} \]

\[ P_{de} \equiv \frac{1}{16\pi G_N} \left[ f - \frac{f_T}{T} + 2T^2 f_T T \right]. \tag{12} \]

The corresponding effective equation of state (EoS) parameter is written as

\[ w_{de} \equiv P_{de} / \rho_{de} = -1 - \frac{d\ln T}{d\ln a} \left( f/T - 2f_T \right). \tag{13} \]

Utilizing equation (6) and the nominal relations \( \rho_m = \rho_m a^{-3} \) and \( \rho_e = \rho_e a^{-4} \) we compute the dimensionless Hubble parameter

\[ E^2(a) = \Omega_{m0} a^{-3} + \Omega_e a^{-4} + \Omega_{F0} X(a), \tag{14} \]

where \( \Omega_{m0} = \frac{8\pi G_N}{9H_0^2} = 0 \), \( \Omega_{F0} = 1 - \Omega_{m0} - \Omega_\Lambda \) and the function \( X(a) \) is given by

\[ X(a) = \frac{1}{T_0\Omega_{F0}} (f - 2Tf_T). \tag{15} \]

Evidently, the Hubble expansion in \( f(T) \) cosmology is affected by the extra term \( \Omega_{F0} X(a) \) which is given in terms of functional form of \( f(T) \), as indicated from equation (15).

For the rest of the paper we focus our analysis on the power law \( f(T) \) pattern (Bengochea & Ferraro 2009) in which the form of \( f(T) \) is given by

\[ f(T) = \alpha (-T)^b, \tag{16} \]

where \( \alpha = (6H_0^2)^{-b} \frac{\Omega_{m0}}{9-b} \). Substituting (16) into equations (13) and (15) we can get

\[ X(a, b) = E^{2b}(a, b), \tag{17} \]

\[ w_{de} = -1 - \frac{2b}{3} \frac{d\ln E}{d\ln a}, \tag{18} \]

and inserting (17) into equation (14) we arrive at

\[ E^2(a, b) = \Omega_{m0} a^{-3} + \Omega_\Lambda a^{-4} + \Omega_{F0} E^{2b}(a, b). \tag{19} \]

As expected, for \( f(T) = \text{const.} \), the above cosmological quantities boil down to those of \( \Lambda \)CDM (\( \Omega_{\Lambda,0} \equiv \Omega_{F0} \)). Theoretically, it has been found that in order to treat the accelerated expansion of the universe the free parameter \( b \) needs to satisfy the condition \( b \ll 1 \) (Linder 2010; Nesseris et al. 2013). Under these circumstances the \( f(T) \) power law model can be viewed as a perturbation around the \( \Lambda \)CDM cosmology (Nesseris et al. 2013; Basilakos 2016). Hence, we can perform a Taylor expansion of \( E^2(a, b) \) around \( b = 0 \) as

\[ E^2(a, b) = E^2(a, 0) + \frac{dE^2(a, b)}{db} \bigg|_{b=0} b + \ldots \]

or

\[ E^2(a, b) = E^2_\Lambda(a) + \Omega_{F0} \frac{dX(a, b)}{db} \bigg|_{b=0} b + \ldots . \tag{20} \]

where for the latter equality we have used Eq.(15). Utilizing equation (17), we can easily provide a useful approximate formula of the dimensionless Hubble parameter (see also Basilakos 2016)

\[ E^2(a, b) \approx E^2_\Lambda(a) + \Omega_{F0} \ln \left[ E^2_\Lambda(a) \right] b, \tag{21} \]

where \( E^2_\Lambda(a) = E^2(a, 0) = \Omega_{m0}/a^3 + \Omega_\Lambda a^4 + \Omega_{F0} \). Obviously, the background evolution of universe depends directly from the free parameters \( b \) and \( \Omega_{m0} \). Notice, that as we have already mentioned above at late enough times we can neglect the radiation component from the Hubble parameter which means that \( \Omega_{F0} \) is determined via \( \Omega_{F0} = 1 - \Omega_{m0} \) for a spatially flat FRW metric.

Recently, using the latest observational data that include SNIa (Suzuki et al. 2012), BAO (Blake et al. 2011b; Percival et al. 2010) and Planck shift parameter (Shafer & Huterer 2014) it has been found that \( \Omega_{m0} = 0.286 \pm 0.012, b = -0.081 \pm 0.117 \) (Basilakos 2016). These results are in agreement (within 1σ uncertainties) with those of Nesseris et al. (2013) who found \( \Omega_{m0} = 0.274 \pm 0.008, b = -0.017 \pm 0.083 \). We observe that the above analysis provide a small and negative value for \( b \) but the 1σ error is quite large. In order to realize the differences of the powerlaw \( f(T) \) model from the \( \Lambda \) cosmology at the expansion level we plot in Fig.(1) the evolution of the EoS parameter \( w_{de}(z) \) (top panel), \( \Delta E = \left[ (E(a, b) - E_\Lambda)/E_\Lambda \right] \times 100 \) (middle panel) and \( \Delta \Omega_{de} = (\Omega_{de}(a, b) - \Omega_\Lambda)/(\Omega_\Lambda) \times 100 \) (bottom panel). Notice that the solid, dashed and dotted-dashed lines correspond to different values of the \( b \) parameter, namely 0, 0.05 and −0.05. Concerning the value of \( \Omega_{m0} \) we have set it to 0.30 which means that \( \Omega_{F0} = 0.70 \). Overall, the evolution of the aforementioned cosmological quantities depends on the model parameter \( b \). We verify that in the case of \( b < 0 \) the effective EoS parameter of the power law \( f(T) \) model remains in the quintessence regime (\( w_{de} > -1 \)), while it goes to phantom (\( w_{de} < -1 \)) for \( b > 0 \). Furthermore, from Fig.(1) (see middle and bottom panels) we observe that in the case of \( b > 0 \) the cosmological quantities \( E(z) \) and \( \Omega_\Lambda(z) \) of the \( f(T) \propto (-T)^b \) model are large with respect to those of the reference \( \Lambda \)CDM model. The opposite holds for negative values of \( b \).

Regarding, the Hubble parameter we find that close to \( z \sim 1 \) the relative deviation \( \Delta H \) lies in the interval \([-0.6\%, 0.6\%]\) for \(-0.05 \leq b \leq 0.05 \), while the relative difference \( \Delta \Omega_{de} \) can reach up to \( \pm 10\% \) at large redshifts \( z \sim 2 \).

3 GROWTH OF OVERDENSITIES IN \( F(T) \propto (-T)^B \) GRAVITY

In this section we explore the growth of matter over-densities in the \( f(T) \propto (-T)^b \) model. First, we focus on the linear perturbation theory and then with the aid of the SCM we study the non-linear matter fluctuations.

3.1 linear growth factor

Let us start with the linear growth of non-relativistic (\( P_m = 0 \)) perturbations. In general at the sub-horizon scales matter perturbations \( \delta_m \) satisfies the following differential equation

\[ \ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{eff}\rho_m\delta_m = 0, \tag{22} \]

where \( G_{eff} \) is the effective Newton’s parameter and in the case of \( f(T) \) gravity models it takes the form (Zheng & Huang 2011)

\[ G_{eff} = \frac{G_N}{1 + f_T}. \tag{23} \]

MNras 000, 1–9 (2015)
where \( G_N \) is the Newton’s constant. Of course for Einstein’s gravity we have \( G_{\text{eff}} = G_N \). Now, combining equation (16) and equation (23) we obtain

\[
G_{\text{eff}} = \frac{G_N}{1 + \frac{M_{\Lambda}a}{(1-2\Omega_m)E_\Lambda^2(1-a)}} \tag{24}
\]

and utilizing a first order Taylor expansion around \( b = 0 \) we find

\[
G_{\text{eff}} \approx G_N \left( 1 - \frac{\Omega_{F0}}{E_\Lambda^2(a)} b \right) \tag{25}
\]

Inserting equation (25) into equation (22) and changing variables from cosmic time to scale factor \((d/dt = aHd/da)\) we find after some calculations

\[
\delta_m'' + \left( \frac{3}{a} + \frac{E'(a)}{E(a)} \right) \delta_m' = \frac{3(1-\Omega_m)\Omega_{F0}b}{2a^2E_\Lambda^2(a)} (1 - \frac{\Omega_{F0}b}{E_\Lambda^2(a)}) \delta_m = 0 , \tag{26}
\]

where \( \delta_m' = d\delta_m/da \), \( \delta_m'' = d^2\delta_m/da^2 \) and \( E(a) \) is given by equation (21). As expected for \( b = 0 \) the above equation reduces to that of \( \Lambda \)CDM presented in (Pace et al. 2010, and references therein).

Now we numerically integrate equation (26) starting from the initial scale factor \( a_i = 10^{-4} \) till the present epoch \( a = 1 \). Regarding the initial conditions we adopt the following case: \( a_i = 10^{-4} \) we use \( \delta_m(a_i) = 1.5 \times 10^{-5} \). Additionally, we also adopt the initial conditions and \( \delta_m(0) = \delta_m/\Omega_m \) which guarantees that matter perturbations grow in the linear regime (see also Batista & Pace 2013; Mehrabi et al. 2015a,b; Malekjani et al. 2017). Once the linear matter overdensity \( \delta_m \) is found we compute the linear growth factor scaled to unity at the present time \( D(a) = \delta_m(a)/\delta_m(a = 1) \). In Fig. (2), we show \( D(a)/a \) as a function of redshift \( z = 1/(a - 1) \). It is well known that for the Einstein de-Sitter (EdS) model \( (\Omega_m = 1) \) the growth factor is proportional to \( a \) which implies that \( D(a)/a \) is always equal to unity. For the concordance \( \Lambda \) cosmology \( (b = 0 \) black solid curve), the growth factor \( D_\Lambda(a)/a \) is higher than the EdS model at high redshifts and progressively it starts fall down at low redshifts. The decrement of the growth factor at late times shows that the cosmological constant \( \Lambda \) dominates the energy budget of the universe and consequently suppresses the growth of matter overdensities. The opposite is true at high redshifts, meaning that the effect of cosmological constant \( \Lambda \) on the growth of perturbations is actually negligible and thus \( D_\Lambda(a)/a \) reaches a plateau. The above general behavior holds also for the power-law \( f(T) \) model with one difference namely, for \( b = 0.05 \) (or -0.05) the amplitude of \( D(a)/a \) is somewhat larger (or lower) than the \( \Lambda \)CDM model at high redshifts. Specifically, for \( z \geq 3 \) we find that the relative difference is \( \sim \pm 1\% \). Qualitatively speaking, these results are in agreement with those of DE models (see Pace et al. 2010; Devi & Sen 2011; Pace et al. 2014a; Nazari-Pooya et al. 2016).

3.2 The spherical collapse model

The spherical collapse model (Gunn & Gott 1972) is a simple but still a useful tool utilized to investigate the growth of bound systems in the universe through gravitational instability (Peebles 1993). It is well known that the main quantities of the SCM, such as the linear overdensity parameter \( \delta \) and the virial overdensity \( \Delta_{\text{vir}} \), are affected by the presence of dark energy ( Lahav et al. 1991; Wang & Steinhardt 1998; Mota & van de Bruck 2004; Horellou & Berge 2005; Wang & Tegmark 2005; Abramo et al. 2007; Basilakos & Voglis 2007; Pace et al. 2010, 2012; Batista & Pace 2013; Pace et al. 2014a; Nazari-Pooya et al. 2016).
et al. 2014a,b; Malekjani et al. 2015; Naderi et al. 2015). Here our aim is to extent SCM within the $f(T)$ cosmological scenario, in order to derive the non-linear structure formation in such models and study the differences with the corresponding predictions of the usual ΛCDM cosmology.

Since Birkhoff’s theorem holds here, we can start from the different equation that describes the growth of matter overdensities in the non-linear regime (see also Pace et al. 2014a)

$$\delta_m + 2H\delta_m - \frac{4}{3}\frac{\delta_m^2}{1 + \delta_m} - 4\pi G eff \rho_m (1 + \delta_m) = 0 . \quad (27)$$

In the linear regime the above equation reduces to equation (22) as it should. Also, in the case of GR the full derivation of equation (22) can be found in Ref. (Abramo et al. 2005). It is interesting to mention that the non-linear matter fluctuations are affected by the law of gravity via the form of $G_{eff}$. In the case of $f(R)$ we refer the reader the work of Schaefer & Koyama (2008).

In order to understand the differences of the $f(T) \propto (-T)^b$ model from the concordance Λ cosmology we plot in Fig. (3) a comparison of the evolution of $G_{eff}/G_N$. Notice, that the solid, dashed and the dotted-dashed curves correspond to $b = 0.00$ (ΛCDM), 0.05 and −0.05. We observe that at high redshifts $f(T) \propto (-T)^b$ tends to GR ($G_{eff} \rightarrow G_N$), but as we approach the present time the ratio $G_{eff}/G_N$ starts to deviate from unity. As an example, at $z = 0$ the relative deviation from GR is close to ±4% for $b = ±0.05$. We also find that a positive value of $b$ implies that $G_{eff} < G_N$, while the opposite holds for $b < 0$.

The obvious connection between $G_{eff}$ and $b$ implies that the free parameter $b$ should leave an imprint in the non-linear matter perturbations via equation (27). Indeed, using equation (25) and changing the variables from $t$ to $a(t)$ we obtain

$$\delta_m'' + \left( \frac{3}{a} - \frac{E'}{E} \right) \delta_m' - \frac{4}{3} \frac{\delta_m^2}{1 + \delta_m} - \frac{8\Omega_{\text{m}_0}^3}{20\pi^2 E^2} \times \left( 1 - \frac{\Omega_{\text{m}_0}^2 b}{E^2} \right) \delta_m (1 + \delta_m) = 0 . \quad (28)$$

Now in order to determine $\delta_c$ and $\Delta_{vir}$ we follow the general approach of (Pace et al. 2010, 2012; Malekjani et al. 2015; Pace et al. 2014b). Specifically, regarding $\delta_c$ we utilize a two-step process. First, we numerically solve equation (26) between the epoch $z_i$ and the collapse redshift $z_c$. As we have already mentioned in the previous section concerning the value of the initial scale factor of the universe $a_i = 1/(1 + z_i)$ we use $10^{-4}$. Our attempt is to calculate the initial values $\delta_{m_i} = \delta_{m}(a_i)$ and $\alpha'_{m_i} = \delta_{m_i}/a_i$, for which the collapse takes place at $a = a_c$ such that $\delta_{m_i}(a_c) \approx 10^7$ (see also Malekjani et al. 2015; Nazari-Pooya et al. 2016). Second, we utilize the values for $\delta_{m_i}$ and $\alpha'_{m_i}$ obtained in the first step as the initial conditions for the linear equation (26) a numerical solution of which provides the critical overdensity threshold above which structures collapse $\delta_c \equiv \delta_c(z_c)$. We remind the reader that in the case of the Einstein-de Sitter model $\delta_c$ is strictly equal to 1.686. In Fig. (4), we show $\delta_c$ as a function of the collapse redshift $z_c$ for the models explored here. We verify that $\delta_c$ converges to the Einstein-de Sitter value at high redshifts, since the matter component dominates the cosmic fluid. The $f(T)$ critical overdensity starts to deviate from that of ΛCDM for $z \leq 1.5$. In this redshift regime we observe that the critical overdensity satisfies: $\delta_c(z_c) > \delta_{m_i}(z_c)$ for $b = 0.05$ and $\delta_c(z_c) < \delta_{m_i}(z_c)$ in the case of $b = -0.05$. This result is compatible with that of DE cosmologies (see Pace et al. 2010; Devi & Sen 2011; Pace et al. 2014a; Nazari-Pooya et al. 2016).

Furthermore we apply the following fitting function (see also

Kitayama & Suto 1996; Weinberg & Kamionkowski 2003) to $\delta_c$ calculated in power law $f(T)$ gravity

$$\delta_c(z) = \frac{3(12\pi)^{2/3}}{20} \left( 1 + \beta \log_{10} \Omega_{\text{m}}(z) \right) , \quad (29)$$

and obtain the constant coefficient $\beta$ in terms of parameter $b$ as

$$\beta = -0.04b + 0.013 \quad (30)$$

Another important quantity is the density contrast at virialization which is defined as $\Delta_{vir} = \xi(x/y)^3$, where $\xi$ is the density contrast at the turnaround point, $x = a_c/a_{vir}$ is the normalized scale factor with respect to the turn around scale factor and $y$ is the ratio between virial radius and turn-around radius, $y = R_{vir}/R_{a}$ (Wang & Steinhardt 1998). It is well known that for the Einstein-de Sitter model we have $(a_c/a_{vir})_{EIS} = (1 + z_c)/(1 + z_0) = 2^{2/3},$ $y = 1/2$, $\xi = (\frac{2}{3\pi})^2 \approx 5.6$ and thus $\Delta_{vir} \approx 18\pi^2 \approx 178$. However, in DE cosmologies the above quantities varies with the collapse redshift (Lahav et al. 1991; Wang & Steinhardt 1998; Mota & van de Bruck 2004; Horellou & Berge 2005; Wang & Tegmark 2005; Abramo et al. 2007; Basilakos & Voglis 2007; Pace et al. 2010, 2012; Batista & Pace 2013; Pace et al. 2014a,b; Malekjani et al. 2015; Naderi et al. 2015).

In the upper panel of Fig. (5) we plot the evolution of the density contrast at turn around. Also, in the lower panel of the same figure we present the relative difference deviation of the turn around density contrast $\xi(z_c)$ for the power law $f(T)$ model with respect to the Λ model $\xi_{\Lambda}(z_c)$. Obviously, the difference from the ΛCDM case is small, namely at $z_c \sim 0$ we find $\sim 1.2\%$ for $b = \pm 0.05$. As expected, at very large redshifts $\xi$ tends to the Einstein-de Sitter value ($\sim 5.6$). Moreover, in the top panel of Fig. (6) we provide $\Delta_{vir}$ as a function of $z_c$ and in the bottom panel of the same figure we show the behavior of $\Delta_{vir}(%) = [\Delta_{vir} - \Delta_{vir}^{\Lambda}] / \Delta_{vir}^{\Lambda} \times 100$. At low redshifts we find $\Delta_{vir}(%) \sim 2\%$ for $b = \pm 0.05$. Therefore, in the case of positive (negative) values of $b$ we expect that the tendency for a large scale overdensity (candidate structure) is to collapse in a more (less) bound system, with respect to the ΛCDM cosmological model.

4 NUMBER OF HALOES

In this section we compute the cluster-size halo number counts within the framework of the cosmological models studied in this article. Using the Press-Schechter formalism the abundance of virialized haloes can be expressed in terms of their mass (Press & Schechter 1974). The conning number densities of virialized

Figure 3. The evolution of $G_{eff}/G_N$ in the case of power-law $f(T)$ model.

SCM in $f(T)$ cosmology
Figure 4. The critical overdensity $\delta_c$ as a function of the collapse redshift $z_c$. The corresponding curves are explained in the caption of Fig. (1).

Figure 5. Upper panel: The evolution of the overdensity $\xi$ at the turn around point. Lower panel: The fractional difference $\Delta\xi$ between the power law $f(T)$ model and the reference $\Lambda$CDM model. The lines correspond to the same styles as in Fig. (1).

Figure 6. Upper panel: The virial overdensity $\Delta_{\text{vir}}$ as a function of the collapse redshift. Lower panel: The fractional difference $\Delta_{\text{vir}}(\%)$ versus $z_c$.

haloes with masses in the range of $M$ and $M + dM$ is given by (Press & Schechter 1974; Bond et al. 1991)

$$\frac{dn(M,z)}{dM} = \frac{\rho_{m0}}{M} \frac{d\sigma^{-1}}{dM} f(\nu),$$  \hspace{1cm} (31)

where $\nu(M,z) = \delta_c/\sigma$, $\rho_{m0} = \Omega_{m0}\rho_{c,0}$ is the background density at the present time and $\rho_{c,0} \simeq 2.775 \times 10^{11} h^2 M_\odot/Mpc^3$ is the corresponding critical density. In the standard Press-Schechter approach the mass function is Gaussian $f(\sigma) = \sqrt{2/\pi\nu(\delta_c/\sigma)} \exp(-\nu^2/2)$. Notice, that $\sigma^2$ is the variance of the linear matter perturbations

$$\sigma^2(R,z) = \frac{D^2(z)}{2\pi^2} \int_0^\infty k^2 P(k) W^2(kR) dk,$$  \hspace{1cm} (32)

where $R = (3M/4\pi\rho_{m0})^{1/3}$ is the radius of the spherical region, $P(k)$ is the linear power spectrum and $W(kR) = 3(\sin(kR) - kR \cos(kR))/(kR)^3$ is the Fourier transform of a spherical top-hat profile. We utilize the cold dark matter (CDM) spectrum $P(k) = A_k n T^2(\Omega_{m0}, k)$, with $T(\Omega_{m0}, k)$ the CDM transfer function according to (Eisenstein & Hu 1998) and $n \simeq 0.96$, following the Planck Collaboration XIII (2015) results. In this framework, the rms matter fluctuations is normalized at redshift $z = 0$ so that for any cosmological model one has (for more detail see Basilakos et al. 2010): $\sigma^2(R,z) = \sigma_8^2(z) \Psi(\Omega_{m0}, R)$ with

$$\Psi(\Omega_{m0}, R) = \int_0^\infty k^{n+2} P(k) W^2(kR) dk$$

and

$$\sigma_8(z) = \sigma_8(0) D(z),$$

where $\sigma_8(0) \equiv \sigma_8$ the rms mass fluctuation on $R_8 = 8h^{-1}$ Mpc scales at redshift $z = 0$. Concerning the value of $\sigma_8$ we have set it to $\simeq 0.815$ based on the Planck 2015 results (Planck Collaboration XIV 2016). It is worth noting that the Gaussian mass-function has a well known caveat, namely it over-predicts/under-predicts the number of low/high mass halos at the present epoch (Sheth & Tormen 1999; Jenkins et al. 2001; Sheth & Tormen 2002; Lima & Marassi 2004). In order to avoid this problem in the present treatment we adopt the Sheth-Torman (ST) mass function (Sheth & Tor-
men 1999, 2002):

\[ f(\nu) = 0.2709 \sqrt{\frac{2}{\pi}} \left( 1 + 1.1096\nu^{0.6} \right) \exp \left( -\frac{0.707\nu^2}{2} \right). \]  

(33)

Now given the determined mass range, say \( M_1 \leq M \leq M_2 \) we can derive the halo number counts, \( N(z) \) via the integration of the expected differential halo mass function as

\[ N(\geq M, z) = \int_M^{M_2} \frac{dn(z)}{dM} dM. \]  

(34)

In Fig.(7), we display the expected ratio \( N(f(T)/N_{\Lambda}) \) as a function of \( M/M_0 \) and above a limiting halo mass, which is \( M_1 \equiv 10^{13} h^{-1} M_\odot \). Concerning the upper mass limit we have set it to \( M_2 \equiv 10^{15} h^{-1} M_\odot \). We remind the reader that \( M_8 = 6 \times 10^{14} \Omega_M h^{-1} M_\odot \) mass inside the radius of \( R_8 = 8 h^{-1} Mpc \) (Abramo et al. 2007). Also the panels in Fig. (7) correspond to different redshifts, namely \( z = 0 \) (top left panel), \( z = 0.5 \) (top right panel), \( z = 1.0 \) (bottom left panel) and \( z = 2.0 \) (bottom right panel). The results indicate that the number variation of the differences between the \( f(T) \) power law model and \( \Lambda \) cosmology model is affected by variations in the value of \( z \). Considering \( b = -0.05 \) (or \( b = 0.05 \)) we find that significant model differences should be expected for \( z \gtrsim 1 \), with the \( f(T) \) model abundance predictions being always less (or more) than those of the corresponding \( \Lambda \) cosmology. In particular, at \( z = 1 \) the \( f(T) \) model with \( b = 0.05 \) (or \( b = -0.05 \)) has roughly 1% (2%) more (less) haloes than the standard \( \Lambda \)CDM model at the low-mass tail \( M/M_8 \leq 0.5 \). Obviously, as we approach the high mass haloes (see for example \( M/M_8 = 5.55 \)) the corresponding differences become more severe. Indeed, we observe that the \( f(T) \) model with \( b = 0.05 \) (or \( b = -0.05 \)) produces \( \sim 15\% \) (\( \sim 12\% \)) more (less) haloes with respect to those of \( \Lambda \)CDM. Furthermore, the deviation between the \( f(T) \) and \( \Lambda \)CDM models becomes even higher at \( z = 2 \). Specifically, for the low-mass tail \( M/M_8 = 0.05 \) we find that the difference between \( f(T) \) and \( \Lambda \)CDM can reach up to \( \pm \sim 5\% \) for \( b = \pm 0.05 \), while for the high mass end (\( M/M_8 = 5.55 \)) we show that the \( f(T) \) gravity with \( b = 0.05 \) (or \( b = -0.05 \)) predicts \( \sim 52\% \) (or \( \sim 36\% \)) more (or less) virtualized haloes. We would like to point that the aforementioned predictions of the power law \( f(T) \) model are similar to those of DE models (quintessence and phantom) which adhere to GR (see Pace et al. 2014b). We have expected such a similarity because in the case of \( b < 0 \) (or \( b > 0 \)) the power law \( f(T) \) model is in the quintessence (or phantom) regime, namely the effective EoS parameter obeys \( w_{de} > -1 \) (or \( w_{de} < -1 \)) [see Fig. (1)].

Although our analysis is self-consistent, in the sense that we compare the expectations of \( f(T) \sim (−T)^b \) model with respect to those of the concordance cosmology using the same mass function, we want to investigate how sensitive are the observational predictions to the different mass functions fitting formulas. For comparison, we use the mass function provided by Reed et al. (2007):

\[ f(\nu) = 0.2709 \sqrt{\frac{2}{\pi}} \left( 1 + 1.1096\nu^{0.6} + 0.2G_1 \right) \exp \left( -\frac{0.763\nu^2}{2} \right), \]  

(35)

where

\[ G_1 = \exp \left( -\frac{\ln \sigma^{−1} - 0.4)^2}{0.72} \right). \]  

(36)

We conclude that the difference between ST mass function and Reed et al. mass function is negligible at low mass tails and low-redshifts respectively. However, as we approach the high mass tail at \( z = 2 \), we find \( 3\% - 6\% \) differences between the two mass functions. Specifically, for \( b = 0.05 \) (or \( b = -0.05 \)) the mass function of Reed et al. (2007) provides \( \sim 6\% \) (\( \sim 3\% \)) more (less) haloes with respect to ST mass function. Overall, we verify that there are observational signatures that can be used to differentiate the power law \( f(T) \) gravity from the \( \Lambda \)CDM and possibly from a large class of DE models (see also Basilakos et al. 2010; Malekjani et al. 2015).

5 CONCLUSION

In this article, we have studied the spherical collapse model (SCM) and the number counts of massive clusters beyond the concordance \( \Lambda \) cosmology by utilizing the power law model for the \( f(T) \sim (−T)^b \) gravity.

First, at the level of the resulting cosmic expansion we have found that the evolution of the main cosmological quantities are affected by the power-law parameter, \( b \). In particular, for \( b < 0 \) we have shown that the effective EoS parameter of the \( f(T) \) gravity is in the quintessence regime \( w_{de} < -1 \), while it goes to phantom \( (w_{de} < -1) \) in the case of \( b > 0 \). Concerning the Hubble parameter, we have found that the \( f(T) \sim (−T)^b \) model is close to that of the \( \Lambda \)CDM model (the relative difference can reach up to \( \sim 0.6\% \)), as long as they are confronted with the quoted set of observations.

Second we have investigated analytically and numerically the linear and non-linear (via SCM) regimes of the matter perturbations in the context of the current \( f(T) \) gravity. In this case we have found that the general behavior of the growth factor is similar to that of the \( \Lambda \)CDM cosmological model, although the relative difference is close to 1% at high redshifts. We have showed that at low redshifts the linear growth of matter perturbations are suppressed due to the modifications of gravity while at high redshifts the effect of modified gravity is less important. Extending the \( f(T) \) model in the non-linear phase of matter perturbations, we have computed the well known SCM parameters, namely the linear overdensity \( \delta_c \), and the virial overdensity \( \Delta_{virc} \). We have showed that \( \delta_c \) and \( \Delta_{virc} \) are affected by the value of \( b \). As expected both quantities tend to those of Einstein-deSitter model at high redshifts. Also, we have found that the predictions of SCM model in the power law \( f(T) \) model are similar with those DE models (quintessence or phantom) which adhere to GR (for comparison see Pace et al. 2014b).

Finally, despite the fact that the \( f(T) \sim (−T)^b \) model closely reproduce the \( \Lambda \)CDM Hubble parameter, we have shown that the \( f(T) \) model can be differentiated from the reference \( \Lambda \) cosmology on the basis of their number counts of cluster-size halos. Indeed, using the Press-Schechter formalism in the framework of Sheth-Tormen (ST) mass function (Sheth & Tormen 1999, 2002), we have found clear signs of difference, especially at \( z \geq 1 \), with respect to the \( \Lambda \)CDM predictions. Therefore, the power-law \( f(T) \) gravity model can be distinguished from the \( \Lambda \)CDM and possibly from a large class of DE models, including those of modified gravity. Also, using the mass function of Reed et al. Reed et al. (2007) we found that the difference between the two mass functions is negligible at low mass tails and low-redshifts respectively. However, as we approach the high mass tail at \( z = 2 \) we found that the relative difference lies in the interval \( 3\% - 6\% \). To this end, in the light of future cluster surveys the methodology of cluster number counts appears to be very competitive towards testing the nature of dark energy on cosmological scales.
Figure 7. The expected ratio $N_f(T)/N_f$ as a function of $M/M_8$. Notice, we provide our results for different redshifts: $z = 0$ (top left), $z = 0.5$ (top right), $z = 1.0$ (bottom left) and $z = 2.0$. The style of curves can be found in the caption of Fig.(1).

References

Abramo L. R., Batista R. C., Liberato L., Rosenfeld R., 2007, JCAP, 11, 12
Abramo L. R., Batista R. C., Liberato L., Rosenfeld R., 2009, Phys. Rev. D, 79, 023516
Alcaniz J. S., 2004, Phys. Rev. D, 69, 083521
Allen S. W., Schmidt R. W., Ebeling H., Fabian A. C., van Speybroeck L., 2004, Mon. Not. Roy. Astron. Soc., 353, 457
Amendola L., Kunz M., Saponi D., 2008, JCAP, 0804, 013
Armendariz-Picon C., Mukhanov V., Steinhardt P. J., 2001, Phys. Rev. D, 63(10), 103510
Ascasibar Y., Yepes G., Gottlöber S., Müller V., 2004, MNRAS, 352, 1109
Avela-Reese V., Firmani C., Hernandez X., 1998, Astrophys. J., 505, 37
Bamba K., Geng C.-Q., Lee C.-C., Luo L.-W., 2011, JCAP, 1101, 021
Bamba K., Myrzakulov R., Nojiri S., Odintsov S. D., 2012, Phys. Rev., D85, 104036
Basilakos S., 2016, Phys. Rev., D93, 083007
Basilakos S., Voglis N., 2007, Mon. Not. Roy. Astron. Soc., 374, 269
Basilakos S., Sanchez J. C. B., Perivolaropoulos L., 2009, Phys. Rev. D, 80, 043530
Basilakos S., Phinios M., Lima J. A. S., 2010, Phys. Rev., D82, 083517
Basse T., Bjelde O. E., Wong Y. Y. Y., 2011, JCAP, 10, 38
Batista R., Pace F., 2013, JCAP, 1306, 044
Bengochea G. R., Ferraro R., 2009, Phys. Rev., D79, 124019
Benjamin J., et al., 2007, Mon. Not. Roy. Astron. Soc., 381, 702
Bento M. C., Bertolami O., Sen A. A., 2002, Phys. Rev., D66, 043507
Bertschinger E., 1985, ApJS, 58, 39
Blake C., Brough S., Colless M., Contreras C., Couch W., et al., 2011a, MNRAS, 415, 2876
Blake C., Kazin E., Beutler F., Davis T., Parkinson D., et al., 2011b, MNRAS, 418, 1707
Bond J. R., Cole S., Efstathiou G., Kaiser N., 1991, ApJ, 379, 440
Cai Y.-F., Capozziello S., De Laurentis M., Saridakis E. N., 2016, Rept. Prog. Phys., 79, 106901
Calderwood R. R., 2002, Phys. Lett. B, 545, 23
Calderwood R. R., Dave R., Steinhardt P. J., 1998, Phys. Rev. Lett., 80, 1582
Capozziello S., Francaviglia M., 2008, Gen. Rel. Grav., 40, 357
Capozziello S., Vignolo S., 2009, Class. Quant. Grav., 26, 175013
Capozziello S., Vignolo S., 2010, Analea Phys., 19, 238
Capozziello S., Stabile A., Troisi A., 2007, Phys. Rev., D76, 104019
Capozziello S., Cardone V. F., Farajollahi H., Ravanpak A., 2011, Phys. Rev., D84, 043527
Cardone V. F., Radicella N., Camera S., 2012, Phys. Rev., D85, 124007
Carroll S. M., 2001, Living Reviews in Relativity, 380, 1
Chen S.-H., Dant J. B., Dutta S., Saridakis E. N., 2011, Phys. Rev., D83, 023508
Chen Z.-C., Wu Y., Wei H., 2015, Nucl. Phys., B894, 422
Cole S., et al., 2005, MNRAS, 362, 505
Copeland E. J., Sami M., Tsujikawa S., 2006, IJMP, D15, 1753
Dent J. B., Dutta S., Saridakis E. N., 2011, JCAP, 1101, 009
Devi N. C., Sen A. A., 2011, Mon. Not. Roy. Astron. Soc., 413, 2371
Einstein A., 1928, Sitz. Preuss. Akad. Wiss., p.217, ibid p. 224
Eisenstein D. J., Hu W., 1998, Astrophys. J., 496, 605
Eisenstein D. J., et al., 2005, ApJ, 633, 560
Elizalde E., Nojiri S., Odintsov S. D., 2004, Phys. Rev., D70, 043539
Erickson J. K., Caldwell R., Steinhardt P. J., Armendariz-Picon C., Mukhanov V. E., 2002, Phys. Rev. Lett., 88, 121301
Fan Y., Wu P., Yu H., 2015, Phys. Rev., D92, 083529
Faraci V., 2010, Phys. Rev., D81, 044002
Ferraro R., Fiorini F., 2007, Phys. Rev., D75, 084031
Ferraro R., Fiorini F., 2008, Phys. Rev., D78, 124019
Fillmore J. A., Goldreich P., 1984, ApJ, 281, 1
Fu L., et al., 2008, Astron. Astrophys., 479, 9
Geng C.-Q., Wu Y.-P., 2013, JCAP, 1304, 033
Geng C.-Q., Lee C.-C., Saridakis E. N., Wu Y.-P., 2011, Phys. Lett., B704, 384
Geng C.-Q., Lee C.-C., Saridakis E. N., 2012, JCAP, 1201, 002
Gunn J. E., Gott J. R., 1972, ApJ, 176, 1
Hayashi K., Shirafuji T., 1979, Phys. Rev., D19, 3524
Hoffman Y., Shaham J., 1985, ApJ, 297, 16
Horellou C., Berge J., 2005, MNRAS, 360, 1393
Iorio L., Radicella N., Ruggiero M. L., 2015, JCAP, 1508, 021
Izumi K., Ong Y. C., 2013, JCAP, 1306, 029

MNRAS 000, 1–9 (2015)
Jarosik N., et al., 2011, ApJS, 192, 14
Jenkins A., et al. 2001, Mon. Not. Roy. Astron. Soc., 321, 372
Kamenshchik A. Yu., Moschella U., Pasquier V., 2001, Phys. Lett., B511, 265
Karami K., Abdolmaleki A., Asadzadeh S., Safari Z., 2013, Eur. Phys. J., C73, 2565
Kitayama T., Suto Y., 1996, Astrophys. J., 469, 480
Komatsu E., Dunkley J., Nolta M. R., et al. 2009, ApJS, 180, 330
Kowalski M., Rubin D., Aldering G., et al. 2008, ApJ, 686, 749
Lahav O., Lilje P. B., Primack J. R., Rees M. J., 1991, MNRAS, 251, 128
Li M., Li X. D., Wang S., Zhang X., 2009, J. Cosmology Astropart. Phys., 6, 036
Li B., Sotiriou T. P., Barrow J. D., 2011, Phys. Rev., D83, 104017
Lima J. A. S., Marassi L., 2004, Int. J. Mod. Phys., D13, 1345
Linder E. V., 2010, Phys. Rev., D81, 127301
Malekjani M., Naderi T., Pace F., 2015, Mon. Not. Roy. Astron. Soc., 453, 4148
Malekjani M., Basilakos S., Davari Z., Mehrabi A., Rezaei M., 2017, Mon. Not. Roy. Astron. Soc., 464, 1192
Maluf J. W., 1994, J. Math. Phys., 35, 335
Mao I., Lahav O., 2005, JCAP, 7, 3
Mehrabi A., Basilakos S., Pace F., 2015a, MNRAS, 452, 2930
Mehrabi A., Basilakos S., Malekjani M., Davari Z., 2015b, Phys. Rev., D92, 123513
Mehrabi A., Pace F., Malekjani M., Del Popolo A., 2017, Mon. Not. Roy. Astron. Soc., 465(3), 2687
Meng X.-h., Wang Y.-b., 2011, Eur. Phys. J., C71, 1755
Mota D. F., van de Bruck C., 2004, A&A, 421, 71
Myrzakulov R., 2011, Eur. Phys. J., C71, 1752
Naderi T., Malekjani M., Pace F., 2015, MNRAS, 447, 1873
Nazeri-Pooya N., Malekjani M., Pace F., Jassur D. M.-Z., 2016, Mon. Not. Roy. Astron. Soc., 458, 3795
Nesseris S., Basilakos S., Saridakis E. N., Perivolaropoulos L., 2013, Phys. Rev., D88, 103010
Nojiri S., Odintsov S. D., 2011, Phys. Rept., 505, 59
Nunes R. C., Pan S., Saridakis E. N., 2016, JCAP, 1608, 011
Pace F., Waizmann J. C., Bartelmann M., 2010, MNRAS, 406, 1865
Pace F., Fedeli C., Moscardini L., Bartelmann M., 2012, MNRAS, 422, 1186
Pace F., Moscardini L., Crittenden R., Bartelmann M., Pettorino V., 2014a, Mon. Not. Roy. Astron. Soc., 437, 547
Pace F., Batista R. C., Del Popolo A., 2014b, MNRAS, 445, 648
Padmanabhan T., 2002, Phys. Rev. D, 66, 021301
Padmanabhan T., 2003, Phys. Rep., 380, 235
Peebles P. J. E., 1993, Principles of physical cosmology. Princeton University Press
Peebles P. J., Ratra B., 2003, Reviews of Modern Physics, 75, 559
Pericw W. J., Reid B. A., Eisenstein D. J., et al. 2010, MNRAS, 401, 2148
Perlmuter S., Aldering G., Goldhaber G., et al., 1999, ApJ, 517, 565
Planck Collaboration XIII 2015, ArXiv e-prints, 1502.01589
Planck Collaboration XIV 2016, Astron. Astrophys., 594, A14
Press W. H., Schechter P., 1974, ApJ, 187, 425
Reed D., Bower R., Frenk C., Jenkins A., Theuns T., 2007, Mon. Not. Roy. Astron. Soc., 374, 2
Reid B. A., Samushia L., White M., Percival W. J., Manera M., et al., 2012, MNRAS, 426, 2719
Riess A. G., Filippenko A. V., Challis P., et al. 1998, AJ, 116, 1009
Ryden B. S., Gunn J. E., 1987, ApJ, 318, 15
Saez-Gomez D., Carvalho C. S., Lobo F. S. N., Tereno I., 2016, Phys. Rev., D94, 024034
Sahni V., Starobinsky A. A., 2000, JIMP, 9, 373
Schafer B. M., Koyama K., 2008, Mon. Not. Roy. Astron. Soc., 385, 411
Shafer D. L., Huterer D., 2014, Phys. Rev., D89, 063510
Sharif M., Rani S., 2013, Astrophys. Space Sci., 345
Sheth R. K., Tornion G., 1999, MNRAS, 308, 119
Sheth R. K., Tormen G., 2002, MNRAS, 329, 61
Sotiriou T. P., Faraoni V., 2010, Rev. Mod. Phys., 82, 451
Subramanian K., Cen R., Ostriker J. P., 2000, ApJ, 538, 528
Suzuki N., Rubin D., Lidman C., Aldering G., et al. 2012, ApJ, 746, 85
Tegmark M., et al., 2004, Phys. Rev. D, 69, 103501
Wang L., Steinhardt P. J., 1998, ApJ, 508, 483
Wang Y., Tegmark M., 2005, Phys. Rev. D, 71, 103513
Weh I., 2012, Phys. Lett., B712, 430
Wei H., Qi T.-Y., Ma X.-P., 2012, Eur. Phys. J., C72, 2117
Weinberg S., 1989, Reviews of Modern Physics, 61, 1
Weinberg N. N., Kamionkowski M., 2003, Mon. Not. Roy. Astron. Soc., 341, 251
Williams L. R., Babul A., Dalcanton J. J., 2004, ApJ, 604, 18
Wintergerst N., Pettorino V., 2010, Phys. Rev. D, 82, 103516
Wu Y.-P., Geng C.-Q., 2012a, JHEP, 11, 142
Wu Y.-P., Geng C.-Q., 2012b, Phys. Rev., D86, 104058
Wu P., Yu H. W., 2010a, Phys. Lett., B692, 176
Wu P., Yu H. W., 2010b, Phys. Lett., B693, 415
Wu P., Yu H. W., 2011, Eur. Phys. J., C71, 1552
Yerzhanov K. K., Myrzakul S. R., Kulnazarov I. L., Myrzakulov R., 2010
Zhang Y., Li H., Gong Y., Zhu Z.-H., 2011, JCAP, 1107, 015
Zheng R., Huang Q.-G., 2011, JCAP, 1103, 002

MNRAS 000, 1–9 (2015)