Safety With Limited Range Sensing Constraints For Fixed Wing Aircraft

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Abstract—In this paper we discuss how to use a barrier function that is subject to kinematic constraints and limited sensing in order to guarantee that fixed wing unmanned aerial vehicles (UAVs) will maintain safe distances from each other at all times despite being subject to limited range sensing constraints. Prior work has shown that a barrier function can be used to guarantee safe system operation when the state can be sensed at all times. However, in this paper we show that this construction does not guarantee safety when the UAVs are subject to limited range sensing. To resolve this issue, we introduce a method for constructing a new barrier function that accommodates limited sensing range from a previously existing barrier function that may not necessarily accommodate limited range sensing. We show that, under appropriate conditions, the newly constructed barrier function ensures system safety even in the presence of limited range sensing. We demonstrate the contribution of this paper in a scenario of 20 fixed wing aircraft, where because of the proposed algorithm, the vehicles are able to maintain safe distances from each other even though the vehicles are subject to limited range sensing.

I. INTRODUCTION

Given the proliferation of fixed-wing unmanned aerial vehicles (UAVs) [1], an important deployment consideration for fixed-wing unmanned aerial vehicles (UAVs) is collision avoidance where vehicles must be able to maintain safe distances [2]. Dynamics constraints imply that vehicles must begin avoidance maneuvers well in advance of a potential collision but if available sensing offers too short of a horizon to begin those maneuvers then safety may not be maintained. In this paper focusing on fixed-wing vehicles, we show how to achieve collision avoidance while simultaneously taking into account dynamics, actuator constraints, and limited range sensing.

There have been a variety of approaches to fixed-wing collision avoidance that have explicitly modeled sensing and kinematic constraints including potential fields [3]–[5], POMDPs [6], [7], model predictive control [8]–[10], first order lookahead approaches [11], [12], and optimal control [13]. Similarly, barrier functions have been used in the context of limited sensing [14]–[17] and allow for safety guarantees so that when the system starts safe it will remain safe for all future time. For instance, in [14], the authors provide a minimum sensing radius in order to ensure a system of double integrator robots maintain safe distances from each other. Further, they reformulate a Quadratic Program (QP) that only requires knowing the relative position to other agents while still ensuring safety for both collaborative and non-collaborative neighbors. In [15], the authors provide a decentralized strategy for collision avoidance that does not require knowing neighbor barrier function parameters. While [16] does not address multi-agent systems, it does consider collision avoidance under limited range sensing for 3D quadrotors. The authors design a sequential Quadratic Program that translates position-based constraints into rotational commands to ensure safety. More generally, sensing limitations can be addressed by adding a disturbance to the system dynamics. For instance, in [17] the authors model road curvature changes as a bounded disturbance in order to apply barrier functions to adaptive cruise control and lane keeping.

The above examples require designing a barrier function that satisfies sensor constraints, where the additional consideration of the sensor can complicate the construction of a barrier function. Thus, in this paper, we decouple the construction of a barrier function from the sensor constraint. We do this by demonstrating how to adjust a barrier function that has been developed without considering sensing limitations into a barrier function that can still be used to guarantee safety when sensing limitations are present. We build on the work of [18] where it was shown how to ensure a system of $k$ UAVs maintain safe distances for all time while taking into account dynamics constraints. However, [18] did not consider limited range sensing. Thus, in this paper we relax this limitation with the following contributions. First, we show that the barrier functions do not necessarily guarantee safety when the UAVs are subject to limited range sensing. Second, we introduce a method for constructing a new barrier function that accommodates limited sensing range from a previously existing barrier function that may not necessarily accommodate limited range sensing. Finally, we conduct an experiment consisting of a scenario of 20 fixed wing aircraft, where because of the proposed algorithm, the vehicles are able to maintain safe distances from each other even though the vehicles are subject to limited range sensing.

This paper is organized as follows: Section II provides the necessary background for barrier functions. Section III discusses issues that can arise when using a barrier function to ensure safety when there is limited range sensing. Section IV proposes a novel approach to derive a barrier function that can be used to make safety guarantees despite

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UAVs’ sensing limitations. Section IV analyzes the set of safe control inputs when using the proposed approach. Section V shows experiments with twenty fixed-wing aircraft to verify the theoretical developments of this paper.

II. BACKGROUND

In this paper, we study the problem of collision avoidance among a team of UAVs with limited sensing range. For ease of exposition, we focus on collision avoidance between two UAVs. Because collision avoidance is a pairwise constraint, we can then apply the method developed in this paper to all pairwise combinations of aircraft to ensure safety of the entire set of UAVs. We model fixed-wing UAVs similarly to [18], where the state of UAV \( i \) \((i \in \{1,2\})\) is given by \( x_i = [p_{i,x} \ p_{i,y} \ \theta_i \ p_{i,z}]^T \) with dynamics

\[
\dot{x}_i = [v_i \cos \theta_i \ v_i \sin \theta_i \ \omega_i \ \zeta_i]^T,
\]

where \( v_i \in [v_{min}, v_{max}] \) is the linear velocity, \( \omega_i \in [-\omega_{max}, \omega_{max}] \) is the turn rate, and \( \zeta_i \in [-\zeta_{max}, \zeta_{max}] \) is the altitude rate. This model is applicable in cases where the roll and pitch of the aircraft are small [19]. Similar models have been used in fixed-wing collision avoidance as in [4], [5], [8], [11], [20], [21]. The dynamics of two aircraft is then

\[
\dot{x} = [\dot{x}_1^T \ \dot{x}_2^T]^T.
\]

The system (1) is a particular instance of a control affine system

\[
\dot{x} = f(x) + g(x)u
\]

where \( f \) and \( g \) are locally Lipschitz, \( x \in \mathbb{R}^n \), \( u \in U \subset \mathbb{R}^m \), and we assume the system has a unique solution defined for all \( t \geq 0 \) given a starting condition \( x(0) \) and input function \( u \).

Assume there is a safe set \( C \) that is the superlevel set of a continuously differentiable function \( h \) so that

\[
C = \{x \in \mathbb{R}^n : h(x) \geq 0\}.
\]

\( C \) is called the safe set and it represents the set of aircraft states where collision avoidance is guaranteed. In the following, \( L_fh(x) = \frac{\partial h(x)}{\partial x}f(x) \) and \( L_g h(x) = \frac{\partial h(x)}{\partial x}g(x) \)
do not Lie derivatives.

Definition 1. [22] Given a set \( C \subset \mathbb{R}^n \) defined in (4) for a continuously differentiable function \( h : \mathbb{R}^n \rightarrow \mathbb{R} \), the function \( h \) is called a zeroing control barrier function (ZCBF) defined on an open set \( D \subset C \subset \mathbb{R}^n \), if there exists a Lipschitz continuous extended class \( K \) function \( \alpha \) such that

\[
\sup_{u \in U} |L_fh(x) + L_g h(x)u + \alpha(h(x))| \geq 0 \ \forall x \in D.
\]

The admissible control space,

\[
K(x) = \{u \in U : L_fh(x) + L_g h(x)u + \alpha(h(x)) \geq 0\},
\]

can be used to find sufficient conditions for the forward invariance of \( C \), meaning that if \( x(0) \in C \) then \( x(t) \in C \) for all \( t \geq 0 \) where \( x(t) \) is the solution to the closed loop system under a fixed controller.

Theorem 1. [22] Given a set \( C \subset \mathbb{R}^n \) defined in (4) for a continuously differentiable function \( h \), if \( h \) is a ZCBF on \( D \), then any Lipschitz continuous controller \( u : D \rightarrow U \) such that \( u(x) \in K(x) \) will render the set \( C \) forward invariant.

In addition to being able to make system safety guarantees, a ZCBF can be used to calculate a safe control input in an online manner through a quadratic program. Given a nominal control input \( \hat{u} \) and a set of actuator constraints that can be expressed as linear inequalities, \( Au \geq b \), we can calculate a safe control input closest in norm to \( \hat{u} \) as follows:

\[
\begin{align*}
\min_{u \in \mathbb{R}^n} & \frac{1}{2} \|u - \hat{u}\|^2 \\
\text{s.t.} & \quad L_fh(x) + L_g h(x)u + \alpha(h(x)) \geq 0 \\
& \quad Au \geq b.
\end{align*}
\]

Importantly, the application of Theorem 1 is conditional upon the definition of a suitable barrier function that captures the safety requirement. This can be difficult when the system is subject to actuator constraints and nonlinear dynamics, as is the case for fixed-wing UAVs. Motivated by this, in [18], it was shown how to systematically construct a barrier function for fixed-wing UAVs. Although the formulation in [18] does not consider sensing limitations, we will show in this paper that the formulation can be adapted to create a barrier function that allows for safety guarantees even when there are sensing restrictions.

In particular, let \( \rho : D \rightarrow \mathbb{R} \) be a safety function that must be nonnegative at all times for the system to be considered safe. Let \( \gamma : D \rightarrow U \) be a nominal evading maneuver. Then a barrier function can be defined as the worst case safety function value when using \( \gamma \) for all future time,

\[
\hat{h}(x;\rho,\gamma) = \inf_{\tau \in [0,\infty)} \rho(\hat{x}(\tau)),
\]

where we define \( \hat{x} \) by

\[
\hat{x}(\tau) = x(t) + \int_0^\tau \hat{\dot{x}}(\eta)d\eta,
\]

where \( \hat{\dot{x}}(\tau) = f(\hat{x}(\tau)) + g(\hat{x}(\tau))\gamma(\hat{x}(\tau)). \)

Theorem 2. [18] Given a dynamical system (3) and a set \( C \subset D \) defined in (5) for a continuously differentiable \( h \) defined in (8) with a safety function \( \rho \) and locally Lipschitz evading maneuver \( \gamma \), \( h \) satisfies (5) for all \( x \in C \). If in addition, \( L_g h(x) \) is non-zero for all \( x \in \partial C \) and \( \gamma \) maps to values in the interior of \( U \), then \( h \) is a ZCBF on an open set \( D \) where \( C \subset D \).

In [18], two cases of \( \gamma \) and \( \rho \) are considered for the calculation of \( h \) in (8) in closed form between vehicles 1 and 2. However, in these two cases sensing limitations were not considered. In the first case

\[
\gamma_{\text{turn}} = \begin{bmatrix} \sigma v & \omega & 0 & v & \omega & 0 \end{bmatrix}^T
\]
is an evasive maneuver encoding a constant rate turn for both vehicles with possibly different forward velocity where $0 < \sigma \leq 1$. The safety function is

$$\rho_{\text{turn}}(x) = \sqrt{d_{1,2}(x) - 2\delta + \delta \sin(\theta_1) - \delta \cos(\theta_1)} - D_s,$$

(12)

where $\delta > 0$ is a scalar, $d_{1,2}(x)$ is the squared distance between vehicles 1 and 2, and $D_s$ is the minimum distance between the vehicles for the system to be considered safe. When $h$ is constructed from $\gamma_{\text{turn}}$ and $\rho_{\text{turn}}$ we denote the resulting $h$ from equation (8) by $h_{\text{turn}}$. The second case where $h$ can be calculated in closed form is given by the evasive maneuver

$$\gamma_{\text{straight}} = \begin{bmatrix} v_1 & 0 & \zeta_1 & v_2 & 0 & \zeta_2 \end{bmatrix}^T,$$

(13)

where $v_1 \neq v_2$, which encodes each vehicle maintaining a straight trajectory. The safety function is

$$\rho_{\text{straight}}(x(t)) = \sqrt{d_{1,2}(x) - D_s}.$$

(14)

When $h$ is constructed from $\gamma_{\text{straight}}$ and $\rho_{\text{straight}}$ we denote the resulting $h$ from equation (8) by $h_{\text{straight}}$. Motivated by the existence of a closed form solution to (8) for $h_{\text{turn}}$ and $h_{\text{straight}}$ we will consider $h_{\text{turn}}$ and $h_{\text{straight}}$ throughout this paper. However, this particular choice is not central to the contribution of this paper.

### III. MOTIVATION

We assume there is a sensor modeled via a set $S \subset \mathcal{D}$ such that, if the system state $x$ is such that $x \in S$, then $x$ is completely known to both vehicles, whereas if $x \notin S$, then all that is known is that $x \notin S$. This is the case for instance when the sensor has limited range, as was considered in [14], [16]. In the case of UAV collision avoidance where each UAV is equipped with an omnidirectional sensor with range $R$, $S = \{x \in \mathcal{D} : d_{1,2}(x) \leq R^2\}$. In this section we present two motivating examples to illustrate two distinct issues that can arise when using barrier functions in the presence of limited range sensing. In both cases, the critical problem is that $K(x)$ cannot be calculated for all $x \in \mathcal{D}$ because $S \subset \mathcal{D}$.

The first issue that limited range sensing introduces is that the safety cannot no longer be guaranteed. In particular, we construct a scenario such that $h(x(0)) \geq 0$ and, because $K(x)$ cannot be calculated, there is a future time for which $h(x(t)) < 0$. In other words, we can have a continuously differentiable barrier function $h$ that satisfies (5) but still not be able to guarantee safety. The second issue we examine is that there can be discontinuities in actuator commands even though $h$ is continuously differentiable. This can cause alarm or discomfort for systems designed to ensure safety of human passengers (e.g., cruise control [22]). To provide concrete examples of these issues, consider two aircraft equipped with omnidirectional sensors (e.g. radar) of radius $R$, with dynamics governed by a nominal controller $\hat{u}(x)$. We consider $h_{\text{turn}}$ constructed from $\gamma_{\text{turn}}$ and $\rho_{\text{turn}}$ as well as $h_{\text{straight}}$ constructed from $\gamma_{\text{straight}}$ and $\rho_{\text{straight}}$, respectively, as $h_{\text{turn}}$ and $h_{\text{straight}}$ were shown to be ZCBFs in [18]. The examples are illustrated in Figures 1a and 1b.

### Example 1. A Barrier Function Without a Safety Guarantee.

Suppose the two vehicles start at $x_1(0) = [r_1 + R/2 \ r_1 - \pi/2 \ 0]^T$, $x_2(0) = [-r_2 - R/2 \ r_2 - \pi/2 \ 0]^T$, respectively, where vehicle 1 has a nominal control input of $\hat{u}_1 = [v_1 \ -\omega]^T$ and vehicle 2 has a nominal control input of $\hat{u}_2 = [v_2 \ \omega]^T$ so that they both follow a circular trajectory with radius $r_1 = v_1/\omega$ and $r_2/\omega$, respectively (see Figure 1a). Then for this initial state, $h_{\text{straight}}(x(0)) = (r_1 + r_2 + R) - D_s \geq 0$ as long as $R \geq \max(0, D_s - r_1 - r_2)$ so the vehicles start safe according to $h_{\text{straight}}$. Further, note that because the vehicles cannot sense each other, $K(x)$ cannot be calculated. This is because to calculate $K(x)$, the values of $L_f(h(x), L_0h(x),$ and $h(x)$ are required. Because $K(x)$ cannot be calculated, there is no means to ensure that the control input applied to the vehicle will be in $K(x)$. In particular, it means that is is unknown whether the nominal control input $\hat{u}$ is in $K(x)$ and a design decision must be employed for what actuation input to apply to the vehicles. If the design decision is, for instance, to apply the nominal controller to the vehicles then the vehicles will reach $[R/2 \ 0 \ \pi \ 0]^T$ and $[-R/2 \ 0 \ 0 \ 0]^T$, respectively. Once the vehicles have reached this state, $h_{\text{straight}}(x) = -D_s$. This means that the vehicles started in a state $x$ at time 0 such that $h(x(0)) \geq 0$ but there exists a later time $t$ such that $h(x(t)) < 0$. This is because $K(x)$ cannot be calculated for $x \notin S$ so the control input applied to the aircraft does not always satisfy (5). In other words, because $K(x)$ cannot be calculated for all $x \in \mathcal{D}$, Theorem 1 cannot be used to guarantee safety.

### Example 2. Loss of Smoothness.

Let there be a barrier function $h_{\text{turn}}$ parameterized by $\rho_{\text{turn}}$ in (12) and $\gamma_{\text{turn}}$ with $v = v_{\text{min}}$ and $\omega = \omega_{\text{max}}$ in (11), respectively. Let the two aircraft have sensor radius $R = (D_s + 2r)\cos(\eta) + 4\delta$ where $r = v_{\text{min}}/\omega_{\text{max}}$ and $\eta = \arcsin(r/(r + D_s/2))$. As in Figure 1b let the vehicles have initial
positions of $[(D_s/2 + r) \cos(\eta) + 2\delta + \epsilon \ 0 \ -\pi \ 0]^T$ and $[(-D_s/2 + r) \cos(\eta) - 2\delta - \epsilon \ 0 \ 0 \ 0]^T$, respectively, where $\epsilon > 0$. Further, let each aircraft have a nominal trajectory that continues toward the origin. Because the aircraft cannot sense each other, $K(x)$ cannot be calculated. This means that there will be no collision avoidance override so the applied actuator command will be equal to the nominal controller command of $\hat{u}_i(x) = [v_{max} \ 0 \ 0 \ 0]^T$ until the vehicles reach states $[(D_s/2 + r) \cos(\eta) + 2\delta \ 0 \ -\pi \ 0]^T$ and $[(-D_s/2 + r) \cos(\eta) - 2\delta \ 0 \ 0 \ 0]^T$, respectively. At this point, the vehicles can sense each other and the constraints (5) in the QP (7) can be calculated. In other words, the sensing limitation causes a discontinuity in the constraint in the QP (7).

IV. CONSTRUCTING A BARRIER FUNCTION FOR SAFETY GUARANTEE DESPITE LIMITED RANGE SENSING RESTRICTIONS

In Section III, we saw that limited range sensing can lead to significant practical issues including the loss of safety guarantees even when there exists a ZCBF, $h$, for the system. In particular, this means that UAVs may collide with each other when there are equipped with limited range sensors. When limited sensing is not taken into account in the design of a ZCBF $h$, the problem is that values of $h$ cannot be evaluated for all $x \in D$, as required by Definition 1 and so $h$ cannot be used to guarantee safety. In this section we provide a solution to this issue.

Definition 2. For a given ZCBF $h$ and a sensor with sensor set $S$, $h$ is sensor compatible if $h$ is a positive constant for all $x \notin S$.

Remark 1. We emphasize that a sensor compatible ZCBF $h$ must be positive outside $S$ since otherwise this would imply for $x \notin S$ that (5) becomes $\alpha(h(x)) < 0$ so (5) does not hold for any $u \in U$.

Remark 2. For the case of UAVs equipped with a limited range sensor, this means the value of $h$ is a positive constant when the vehicles are outside of the sensing range.

Importantly, implementing a safety overriding controller requires an exact calculation of the ZCBF constraint only if $K(x) \neq U$. When $K(x) = U$, there is no need to calculate $h(x)$ or its derivatives because any $u \in U$ is already known to be safe. Because of the additional structure on a sensor compatible ZCBF, we can relax the need to check $u \in K(x)$ for all $x \in D$ in Theorem 1 because it is already known that $u \in K(x)$ for all $u$ when $x \notin S$, as is made precise in the following Corollary.

Corollary 1. Suppose $h$ is a sensor compatible ZCBF. Then any Lipschitz continuous controller $u : D \to U$ such that $u(x) \in K(x)$ for all $x \in S$ will render the set $C$ forward invariant.

Proof. By assumption $u(x) \in K(x)$ for all $x \in S$. Suppose then that $x \notin S$ so that $h(x)$ is a positive constant. Then $K(x) = U$ since $Lf h(x) + Lg h(x) u + \alpha(h(x)) = \alpha(h(x)) > 0$ is satisfied for all $u \in U$. Hence $u(x) \in K(x)$ for all $x \in D$ so the assumptions of Theorem 1 are satisfied.

Remark 3. The difference between Theorem 1 and Corollary 1 is that the condition $u(x) \in K(x)$ only needs to be the case for $x \in S$ rather than $x \in D$ due to the extra structure on a sensor compatible ZCBF. This is an important distinction because when there are sensing limitations, it may not be possible to calculate $K(x)$ for all $x \in D$.

Remark 4. For an arbitrary sensor, neither $h_{straight}$ nor $h_{turn}$ are necessarily sensor compatible. To see this for $h_{straight}$, let $R > 0$ and consider the case where $x_1 = \begin{bmatrix} R + \epsilon & D_s & \pi & 0 \end{bmatrix}^T$ and $x_2 = \begin{bmatrix} -R + \epsilon & -D_s & 0 & 0 \end{bmatrix}^T$. Then $x \notin S$ for $\epsilon > 0$ and in this case $h(x) = 0$. However, if $x_2 = \begin{bmatrix} -R + \epsilon & -D_s & 0 & 0 \end{bmatrix}^T$, then $x \notin S$ but $h(x) = D_s$. Then $h_{straight}$ is not sensor compatible because $h(x)$ is not constant for all $x \notin S$. A similar calculation can be done to show $h_{turn}$ is not sensor compatible. Hence, we cannot always apply Corollary 1 to $h_{straight}$ or $h_{turn}$ when there is limited range sensing.

Consider now some ZCBF $h$ that is not necessarily sensor compatible. In [18], it was shown how to create a barrier function for UAV collision avoidance without considering sensing range limitations. Thus, we now show how to create a new barrier function $\tilde{h}$ from $h$ so that $\tilde{h}$ is sensor compatible even though this is not the case for $h$. This allows us to be able to apply Corollary 1 to $\tilde{h}$ and keep aircraft from colliding even though they have a limited range sensor. However, it is not always possible to create $\tilde{h}$ from $h$ so that $\tilde{h}$ is sensor compatible and we therefore consider two cases. First, although $h_{turn}$ is not necessarily sensor compatible for an arbitrary sensor, we give sufficient conditions to construct a ZCBF to be sensor compatible. Second, we show how to verify when it is impossible to construct a sensor compatible ZCBF using the proposed method. It will be shown this is the case for $h_{straight}$.

To construct $\tilde{h}$ we first introduce an interpolation function to ensure that $\tilde{h}$ is continuously differentiable, as required by the definition of a ZCBF. Let $\xi > 0$, $0 < \beta < 1$, and $\psi$ be a continuously differentiable, non-decreasing, real valued function on an open set including $[\beta \xi, \xi]$ chosen to satisfy

$$\psi(\beta \xi) = \beta \xi$$
$$\psi'(\beta \xi) = 1$$
$$\psi'(\xi) = 0.$$  \hspace{1cm} (15)

Example 3. An example of such a function $\psi$ can be found by fitting a quadratic function. Let $\psi(\eta) = c_1 \eta^2 + c_2 \eta + c_3$. Then (15) can be solved for $c_1 = \frac{-1}{2\xi(1-\beta)}$, $c_2 = -2\xi c_1$, and $c_3 = \beta \xi - c_1(\beta \xi)^2 - 2\beta \xi^2$. The

We now define $\tilde{h}$ as follows

$$\tilde{h}(x) = \begin{cases} h(x) & h(x) \leq \beta \xi \\ \psi(h(x)) & \beta \xi < h(x) < \xi \\ \psi(\xi) & \xi \leq h(x) \end{cases} \hspace{1cm} (16)$$
Given $\tilde{h}$, the system designer’s choice of $\xi$ and $\beta$ defines $\tilde{h}$ and $\mathcal{B}_\xi$. According to Theorem 3 when the system designer can identify a $\xi > 0$ that defines $\mathcal{B}_\xi$ where $\mathcal{B}_\xi \subseteq S$, $\tilde{h}$ is a ZCBF compatible with a sensor $s$.

where we let $\mathcal{B}_\xi = \{x \in \mathcal{D} : h(x) \leq \xi\}$ be a sub-level set of $h$ (see Fig 2). With this setup, $\xi$ denotes the maximum value of $h$ for which the safety constraint affects the value of $\tilde{h}$. $\mathcal{B}_\xi$ represents the set of states where the safety constraint affects the value of $\tilde{h}$. $\beta$ is a mixing term for states where $\beta \xi < h(x) < \xi$ and exists to ensure the differentiability of $\tilde{h}$.

Remark 5. With this setup, we can take the following steps to show when a ZCBF $\tilde{h}$ is sensor compatible. First, the system designer chooses $\xi$ which determines $\mathcal{B}_\xi$. Second, the system designer determines if $\mathcal{B}_\xi \subseteq S$. In other words, $\xi > 0$ must be chosen with the sensing range in mind in order to verify that $\mathcal{B}_\xi \subseteq S$. The second step verifies that a sensor exists so that any value of $h(x)$ such that $h(x) < \xi$ can be calculated. Since $\tilde{h}$ does not require knowledge of $x$ for $h(x) \geq \xi$, $\tilde{h}(x)$ can be calculated for all $x \in \mathcal{D}$. We prove this intuition below. Note that when multiple $\xi$ satisfy the above steps, it may be preferable to select larger $\xi$ because $h(x) = h(x)$ for all $x \in \mathcal{D}$ such that $h(x) \leq \beta \xi$.

Lemma 1. Assume $h$ defined in (5) is a ZCBF where $\gamma$ is locally Lipschitz. Let $\tilde{h}$ be defined as in (16). Then $\tilde{h}$ is a ZCBF on $\mathcal{D}$.

Proof. Note that because of how $\psi$ is defined and because $h$ is a continuously differentiable function, that $\tilde{h}$ is a continuously differentiable function. Also note that for $\beta \xi < h(x) < \xi$, $\psi(h(x)) > 0$ since $\psi$ is a non-decreasing function that is positive at $\beta \xi$.

To show that $\tilde{h}$ satisfies (5), let $x \in \mathcal{D}$. We consider three cases. First, if $h(x) \leq \beta \xi$ then the inequality (5) holds for $\tilde{h}(x)$ because $h(x)$ is a ZCBF. If $h(x) \geq \xi$ then (5) for $\tilde{h}(x)$ becomes $\alpha(\psi(\xi)) \geq 0$ which is true for all $u \in U$ because $\psi(\xi) > 0$ and $\alpha$ is a class $K$ function. Finally, suppose $\beta \xi < h(x) < \xi$ and note that because $\psi$ is non-decreasing that $\frac{\partial \psi(h(x))}{\partial h(x)} \geq 0$. Then

\[
L_f \tilde{h}(x) + L_g \tilde{h}(x)\gamma(x) + \alpha(\tilde{h}(x)) = \frac{\partial \psi(h(x))}{\partial h(x)} (L_f h(x) + L_g h(x)\gamma(x) + \alpha(\psi(h(x)) \geq 0.
\]

The first line uses the chain rule. The second line uses the fact that $\frac{\partial \psi(h(x))}{\partial h(x)} \geq 0$ and that $L_f h(x) + L_g h(x)\gamma(x) \geq 0$, as was established in the proof of Theorem 2 in [18]. Finally, note that $\alpha(\psi(h(x))) \geq 0$ because $\psi(h(x)) > 0$ and $\alpha$ is a class $K$ function. Then $\tilde{h}$ is a ZCBF.

Theorem 3. For a given sensor $S$, assume $h$ defined in (5) is a ZCBF where $\gamma$ is locally Lipschitz and there exists a $\xi > 0$ such that $\mathcal{B}_\xi \subseteq S$. Then $\tilde{h}$ defined in (16) is a sensor compatible ZCBF.

Proof. $\tilde{h}$ is a ZCBF by Lemma 1. Suppose $x \notin S$. Then because $\mathcal{B}_\xi \subseteq S, h(x) > \xi$ so $h(x) = \psi(\xi)$. Then $\tilde{h}$ is a positive constant for all $x \notin S$ and is therefore sensor compatible.

Theorem 3 is the justification of the steps described in Remark 5. Combined with Corollary 1, Theorem 3 states how the forward invariance of a set $C$ can be guaranteed even though there is limited range sensing. However, it is predicated on finding a $\xi > 0$ that defines a sublevel set $\mathcal{B}_\xi$ for which $\mathcal{B}_\xi \subseteq S$. We now give an example of such a case for fixed wing collision avoidance where each aircraft is equipped with an omnidirectional sensor with a given range $R$.

Example 4. It was shown in Remark 4 that $h_{\text{turn}}$ is not necessarily sensor compatible for an arbitrary sensor. We now use Theorem 3 to define sensing requirements so that we can create $h$ from $h_{\text{turn}}$ so that $h$ is sensor compatible. Consider $h_{\text{turn}}$ that is parameterized by $\rho_{\text{turn}}$ in (12) and $\gamma_{\text{turn}}$ in (11). Then the trajectory defined in (9) is a circle for each aircraft with radius $r_1 = \sigma v/\omega$ and $r_2 = v/\omega$, respectively. Let $\Delta x(t) = p_{1x}(t) - p_{2x}(t)$, and $\Delta y(t) = p_{1y}(t) - p_{2y}(t)$, so that the vehicles start at a planar distance of $(\Delta x^2(t) + \Delta y^2(t))^{1/2}$ from each other. Assuming the planar distance between the vehicles, $(\Delta x^2(t) + \Delta y^2(t))^{1/2}$, is greater than $2(r_1 + r_2)$, the closest distance can be along the trajectory (9) is $d_{1,2}(x(0))^{1/2} - 2r_1 - 2r_2$.

Assume each vehicle has an omnidirectional sensor with range $R$ large enough so that

\[
((R - 2r_1 - 2r_2)^2 - 4\delta)^{1/2} - D_s > 0.
\]

Equation (17) implies $S$ for this example. Having defined the sensing limitations for this problem, we now follow the steps in Remark 5 to show that $\tilde{h}$ is a sensor compatible ZCBF. First, we carefully choose $\xi$ so that we can prove $\mathcal{B}_\xi \subseteq S$. In particular, we choose $\xi$ with the sensing constraint in mind such that

\[
((R - 2r_1 - 2r_2)^2 - 4\delta)^{1/2} - D_s = \xi > 0.
\]
Second, we show that \( B_\xi \subseteq S \). Suppose \( x(t) \notin S \) so that \( a_{1,2}(x(t)) > R^2 \). Then because the trajectories of each vehicle is a circle in \( S \),

\[
h(x(t)) = \inf_{\tau \in [0, \infty)} \rho(t + \tau) \\
\geq ((d_{1,2}(x(0)))^{1/2} - 2r_1 - 2r_2)^2 - 4\delta)^{1/2} - D_s \\
> (R - 2r_1 - 2r_2)^2 - 4\delta)^{1/2} - D_s \\
\geq \xi \\
> 0.
\]

Then \( x(t) \notin B_\xi \). Then \( B_\xi \subseteq S \). In other words, given a sensor of radius \( R \), we can choose \( \xi \) according to (18) and use \( h \) defined in (16) to ensure safe operations between two fixed wing aircraft.

While Example 2 showed how to use \( h_{\text{turn}} \) with Theorem 3 in order to ensure UAV collision avoidance in spite of limited range sensing, the same cannot be done for \( h_{\text{straight}} \).

**Corollary 2.** Assume \( h \) defined in (8) is a ZCBF where \( \gamma \) is locally Lipschitz. Suppose there exists an \( x \in \mathcal{D} \) such that \( h(x) < 0 \) and \( x \notin S \). Then for all \( \xi > 0 \), \( B_\xi \subseteq S \).

**Proof.** Note that for the given \( x, x \in B_\xi \) for any \( \xi > 0 \) since \( h(x) < 0 \). Then \( x \in B_\xi \) but \( x \notin S \).

**Remark 6.** We now use Corollary 2 to show \( h_{\text{straight}} \) cannot be used with Theorem 3 to guarantee safety. Let the vehicles have sensing radius \( R \), \( \epsilon > 0 \), \( x_1(0) = [-R/2 - \epsilon \ 0 \ 0 \ 0]^T \) and \( x_2(0) = [R/2 + \epsilon \ 0 \ \pi \ 0]^T \). Then \( h(x) = -D_s \) and \( x \notin S \) since the vehicles are further than \( R \) apart.

V. AN INTERPRETATION OF \( \bar{h} \) AS A MORE PERMISSIVE ZCBF THAN \( h \)

Consider a ZCBF \( h \) that is not necessarily sensor compatible but for which it is nevertheless possible to construct \( \bar{h} \) so that \( \bar{h} \) is a sensor compatible ZCBF. In this section we characterize how \( \bar{h} \) is more permissive than \( h \). For notational convenience we denote \( \nabla_h \psi(h(x)) = \frac{\partial \psi(h(x))}{\partial h(x)} \). We also use the following assumptions.

**Assumption 1.** Assume on \((\beta_\xi, \xi)\), \( \alpha \) is continuously differentiable. Further, assume the derivative of \( \alpha \) is non-increasing on \((\beta_\xi, \xi)\).

**Remark 7.** The assumptions on \( \alpha \) can be satisfied for any \( \alpha \) that is linear on the region \((\beta_\xi, \xi)\).

**Assumption 2.** Assume the domain of \( \psi \) is extended by letting \( \psi \) be the identity function for inputs \( h(x) < \beta_\xi \). Further, assume the first derivative of \( \psi \) is strictly positive but non-increasing on \((\beta_\xi, \xi)\) and the second derivative of \( \psi \) is negative on \((\beta_\xi, \xi)\).

**Remark 8.** Note that because the first derivative of \( \psi \) is strictly positive for \( h(x) < \xi \), \((\nabla_h \psi(h(x)))^{-1} \) is well defined on \( h(x) < \xi \).

**Remark 9.** The \( \psi \) discussed in Example 5 satisfies Assumption 2 by letting \( \psi(h(x)) = h(x) \) for \( h(x) \leq \beta_\xi \).

Under Assumption 2 and noting Remark 8 for \( h(x) < \xi \) we have for \( x \in \mathcal{D} \) that

\[
L_f \bar{h}(x) + L_g \bar{h}(x)u + \alpha(\bar{h}(x)) \\
= \nabla_h \psi(h(x))(L_f h(x) + L_g h(x)u) + \alpha(\psi(h(x))) \\
= \nabla_h \psi(h(x))(L_f h(x) + L_g h(x)u + \alpha(\psi(h(x)))) \\
= \nabla_h \psi(h(x))(L_f h(x) + L_g h(x)u + \alpha(\psi(h(x))))^{-1} \alpha(\psi(h(x))).
\]

Then because \( \nabla_h \psi(h(x)) > 0 \) and letting

\[
\alpha_2(h(x)) = (\nabla_h \psi(h(x)))^{-1} \alpha(\psi(h(x))),
\]

we have

\[
\text{sgn}(L_f \bar{h}(x) + L_g \bar{h}(x)u + \alpha(\bar{h}(x))) \\
= \text{sgn}(L_f h(x) + L_g h(x)u + \alpha_2(h(x)))
\]

where \text{sgn} is the sign of the expression.

**Lemma 2.** Suppose \( h \) is a ZCBF, let \( \bar{h} \) be as defined in (16), let Assumptions 7 and 2 hold, and let \( \alpha_2 \) be as defined in (19). If \( h(x) < \xi \) then \( \alpha_2(h(x)) \geq \alpha(h(x)) \).

**Proof.** Suppose \( h(x) \leq \beta_\xi \). Then because \( \psi(h(x)) = h(x) \), \( \alpha_2(h(x)) = \alpha(h(x)) \). Suppose now that \( \beta_\xi < h(x) < \xi \). We prove \( \alpha_2(h(x)) \geq \alpha(h(x)) \) with the comparison lemma [23]. It has already been shown that \( \alpha_2(h(x)) = \alpha(h(x)) \) at \( h(x) = \beta_\xi \). We now show that \( \nabla_h \alpha_2(h(x)) \geq \nabla_h \alpha(h(x)) \) for \( h(x) \in (\beta_\xi, \xi) \). By the chain rule,

\[
\nabla_h \alpha_2(h(x)) \\
= - \left(\frac{1}{\nabla_h \psi(h(x))}\right)^2 \nabla^2 \psi(h(x)) \alpha(\psi(h(x))) \\
+ \left(\frac{1}{\nabla_h \psi(h(x))}\right) \nabla \alpha(\psi(h(x))) \nabla_h \psi(h(x)) \\
\geq \nabla \alpha(\psi(h(x))).
\]

The inequality holds because the second derivative of \( \psi \) is negative and \( \alpha(\psi(h(x))) \geq 0 \) for \( h(x) \in (\beta_\xi, \xi) \). We must now show that \( \nabla \alpha(\psi(h(x))) \geq \nabla h \alpha(h(x)) \) to conclude that \( \alpha_2(h(x)) \geq \alpha(h(x)) \) for \( h(x) \in (\beta_\xi, \xi) \). Because \( \psi(h(x)) = h(x) \) for \( h(x) = \beta_\xi \), the first derivative of \( \psi \) is 1 at \( h(x) = \beta_\xi \) and the first derivative is non-increasing on \( h(x) \in (\beta_\xi, \xi) \), so \( \psi(h(x)) \leq h(x) \) for \( h(x) \in (\beta_\xi, \xi) \). Then because the derivative of \( \alpha \) is non-increasing for \( h(x) \in (\beta_\xi, \xi) \), \( \nabla \alpha(\psi(h(x))) \geq \nabla h \alpha(h(x)) \).

**Remark 10.** Note that \( \alpha_2 \) is a class \( K \) function. By definition, \( \alpha_2(0) = 0 \) and is strictly increasing on \((0, \beta_\xi)\). To see that \( \alpha_2 \) is strictly increasing on \((\beta_\xi, \xi)\), note that it has already been proven that \( \nabla h \alpha_2(h(x)) \geq \nabla \psi(\psi(h(x))) \geq \nabla h \alpha(h(x)) \). Further \( \nabla h \alpha(h(x)) > 0 \) since \( \alpha \) is a class \( K \) function. Then \( \nabla h \alpha_2(h(x)) > 0 \).
Theorem 4. Suppose $h$ is a ZCBF, assume $\tilde{h}$ as defined in (16) is sensor compatible, and let Assumptions 1 and 2 hold. Then $K(x) \subseteq \bar{K}(x)$ for all $x \in D$.

Proof. Suppose $x$ is such that $h(x) < \xi$ and $u \in K(x)$. Then $L_fh(x) + L_g\hat{h}(x)u + \alpha(h(x)) \geq 0$. Then since $\alpha_2(h(x)) \geq \alpha(h(x))$ from Lemma 2 we have $L_fh(x) + L_f\hat{h}(x)u + \alpha_2(h(x)) \geq 0$. Then from (20), $L_fh(x) + L_g\hat{h}(x)u + \alpha(h(x)) \geq 0$. Then $u \in \bar{K}(x)$.

Suppose $x$ is such that $h(x) \geq \xi$ and $u \in K(x)$. Then because $u \in U$, $u \in \bar{K}(x)$ since $\bar{K}(x) = U$.

Remark 11. In particular, Theorem 4 gives the conditions under which any $u$ satisfying the QP (7) using $h$ will be satisfied in a QP (7) when using $\tilde{h}$.

VI. SIMULATION EXPERIMENTS

In this section we conduct a simulation experiment with SCRIMMAGE [24], a multi-agent simulator designed with a plugin interface that allows users to easily swap out motion models such as unicycle dynamics in (1) and 6-DOF models. We consider two vehicles whose initial states are $[-200 0 0 0]^T$ and $[200 0 \pi 0]^T$ with goal positions of $[-200 0 0]^T$ and $[200 0 0]^T$, respectively.

We conduct the experiment using $h_{\text{turn}}$ parameterized by $\rho_{\text{turn}}$ in (12) and $\gamma_{\text{turn}}$ in (11), respectively. To specify $\gamma$ in (11), we let $v = 0.9v_{\text{min}} + 0.1v_{\text{max}}$ and $\omega = 0.9\omega_{\text{max}}$ where $v_{\text{min}} = 15$ meters/second, $v_{\text{max}} = 25$ meters/second, $\delta = 0.01$ meters$^2$, and $\omega_{\text{max}} = 13$ degrees/second. The choice of $\omega_{\text{max}}$ results from assuming a 30 degree max bank angle while traveling at $v_{\text{max}}$ and applying the constant rate turn formula described for instance in [25].

In this experiment, we examine the effect of sensing range on the resulting closest distance the vehicles experience during the simulation. In Example 4 we showed how to apply the steps described in Remark 5 by choosing $\xi$ so that $B_\xi \subseteq S$ to conclude that that $\tilde{h}$ is sensor compatible. The conclusion required that the sensing range was above a threshold in (17). Using the parameters of this experiment, equation (17) implies $R > 318.4$. Because the inequality is strict, we start the experiment with $R = 319$. As shown in Figure 3, the vehicles are able to maintain safe distances in spite of having a limited range sensor.

In the second experiment we repeat the experiment of [18] where 20 vehicles are positioned around a circle with a nominal controller that cause the vehicles to arrive at the origin at the same time. We note that satisfying multiple constraints simultaneously with barrier certificates has been previously addressed [17], [18], [26], [27] and we use the method discussed in [18] for this experiment. The difference in this experiment from [18] is that we include a limited sensing range of 350 for each vehicle and start the vehicles 1250 feet from the origin so they start the scenario without being able to sense each other. A screenshot of the initial conditions and evasive maneuver are shown in Figure 4. The vehicles are able to maintain safe distances throughout the simulation.

VII. CONCLUSION

In this paper we have discussed practical issues that arise when attempting to use a barrier function to ensure safe operations when there are sensor range restrictions in the context of fixed-wing UAV collision avoidance. The proposed solution derives a new barrier function that can be used to ensure that a system will stay safe for all future times even though the system is still subject to limited range sensing. We verified the results in simulation of twenty fixed wing UAVs that were subject to a limited range sensing constrain.

REFERENCES

[1] FAA Aerospace Forecast, Fiscal Years 2020-2040, Federal Aviation Administration, 2020.
[2] P. Kopardekar, J. Rios, T. Prevot, M. Johnson, J. Jung, and J. Robinson, “Unmanned aircraft system traffic management (utm) concept of operations,” in AIAA Aviation 2016.

[3] S. Mastellone, D. M. Stipanović, C. R. Graunke, K. A. Intlekofer, and M. W. Spong, “Formation control and collision avoidance for multi-agent non-holonomic systems: Theory and experiments,” *The International Journal of Robotics Research*, vol. 27, no. 1, pp. 107–126, 2008.

[4] E. J. Rodriguez-Seda, “Decentralized trajectory tracking with collision avoidance control for teams of unmanned vehicles with constant speed,” in *American Control Conference (ACC)*, 2014. IEEE, 2014, pp. 1216–1223.

[5] P. Panyakeow and M. Mesbahi, “Decentralized deconfliction algorithms for unicycle uavs,” in *American Control Conference (ACC)*, 2010. IEEE, 2010, pp. 794–799.

[6] S. Temizer, M. Kochenderfer, L. Kaelbling, T. Lozano-Pérez, and J. Kuchar, “Collision avoidance for unmanned aircraft using markov decision processes,” in *AIAA guidance, navigation, and control conference*, 2010, p. 8040.

[7] T. B. Wolf and M. J. Kochenderfer, “Aircraft collision avoidance using monte carlo real-time belief space search,” *Journal of Intelligent & Robotic Systems*, vol. 64, no. 2, pp. 277–298, 2011.

[8] B. Di, R. Zhou, and H. Duan, “Potential field based receding horizon motion planning for centricity-aware multiple uav cooperative surveillance,” *Aerospace Science and Technology*, vol. 46, pp. 386–397, 2015.

[9] M. Defoort, A. Kokosy, T. Floquet, W. Perruquet, and J. Palos, “Motion planning for cooperative unicycle-type mobile robots with limited sensing ranges: A distributed receding horizon approach,” *Robotics and autonomous systems*, vol. 57, no. 11, pp. 1094–1106, 2009.

[10] J. Shin and H. J. Kim, “Nonlinear model predictive formation flight,” *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, vol. 39, no. 5, pp. 1116–1125, 2009.

[11] E. Lalish, K. A. Morgansen, and T. Tsukamaki, “Decentralized reactive collision avoidance for multiple unicycle-type vehicles,” in *American Control Conference*, 2008. IEEE, 2008, pp. 5055–5061.

[12] P. Fiorini and Z. Shiller, “Motion planning in dynamic environments using velocity obstacles,” *The International Journal of Robotics Research*, vol. 17, no. 7, pp. 760–772, 1998.

[13] C. Tomlin, G. J. Pappas, and S. Sastry, “Conflict resolution for air traffic management: A study in multiagent hybrid systems,” *IEEE Transactions on automatic control*, vol. 43, no. 4, pp. 509–521, 1998.

[14] U. Borrman, L. Wang, A. D. Ames, and M. Egerstedt, “Control barrier certificates for safe swarm behavior,” *IFAC-PapersOnLine*, vol. 48, no. 27, pp. 68–73, 2015.

[15] L. Wang, A. Ames, and M. Egerstedt, “Safety barrier certificates for heterogeneous multi-robot systems,” in *American Control Conference (ACC)*, 2016. IEEE, 2016, pp. 5213–5218.

[16] G. Wu and K. Sreenath, “Safety-critical control of a 3d quadrotor with range-limited sensing,” in *ASME 2016 Dynamic Systems and Control Conference*. American Society of Mechanical Engineers, 2016, pp. V001T05A006–V001T05A006.

[17] X. Xu, J. W. Grizzle, P. Tabuada, and A. D. Ames, “Correctness guarantees for the composition of lane keeping and adaptive cruise control,” *IEEE Transactions on Automation Science and Engineering*, 2017.

[18] E. Squires, P. Pierpaoli, R. Konda, S. Coogan, and M. Egerstedt, “Composition of safety constraints for fixed-wing collision avoidance amidst limited communications,” *arXiv preprint*, 2020.

[19] S. P. Jackson, “Controlling small fixed wing uavs to optimize image quality from on-board cameras,” Ph.D. dissertation, UC Berkeley, 2011.

[20] L. Pallottino, V. G. Scordio, A. Bicchi, and E. Frazzoli, “Decentralized cooperative policy for conflict resolution in multi-vehicle systems,” *IEEE Transactions on Robotics*, vol. 23, no. 6, pp. 1170–1183, 2007.

[21] A. Krontiris and K. E. Bekris, “Using minimal communication to improve decentralized conflict resolution for non-holonomic vehicles,” in *Intelligent Robots and Systems (IROS)*, 2011 IEEE/RSJ International Conference on. IEEE, 2011, pp. 3235–3240.

[22] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, “Control barrier function based quadratic programs for safety critical systems,” *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 3861–3876, 2017.

[23] K. K. Hassan, *Nonlinear systems*. Prentice Hall, Upper Saddle River, NJ 07458, 2002.

[24] K. DeMarco, E. Squires, M. Day, and C. Pippin, “Simulating collaborative robots in a massive multi-agent game environment (SCRIM-MAGE),” in *Int. Symp. on Distributed Autonomous Robotic Systems*, 2011.

[25] L. J. Clancy, *Aerodynamics*. Halsted Press, 1975.

[26] L. Wang, D. Han, and M. Egerstedt, “Permissive barrier certificates for safe stabilization using sum-of-squares,” in *2018 Annual American Control Conference (ACC)*. IEEE, 2018, pp. 585–590.

[27] X. Xu, “Constrained control of input-output linearizable systems using control sharing barrier functions,” *Automatica*, vol. 87, pp. 195–201, 2018.