Mathematical Model of Gas Dilution in Two Phases Flows of Fluid and Gas

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Abstract. Various models of solubility of gas in two-phase flows of gas and liquid are considered. Such phenomena play an important role in various technical devices especially at modern nuclear power plants. Mathematical models of bubble flows with gas dissolution, gas phase model, and model of liquid phase dissolution were considered and discussed.

1. Introduction

Recently the phenomenons connected with solubility of gases in two-phase flows of gas and liquid play an important role in various technical devices. Special attention to such phenomena arose after the well known crash on Fukushima Daiichi Nuclear Power Plant in Japan. This crash was related with hydrogen explosion previously soluble in liquid. The hydrogen was discharged from the multi-phase mixture and this process was initiated by energy inflow from the earthquake.

In the present work various models of solubility of gas in two-phase currents of gas and liquid are considered. The law of mass balance for bubble flows with gas dissolution, the momentum conservation law for bubble flows with gas dissolution, gas phase model, and liquid phase dissolution model were considered and discussed.

2. Mathematical model of two-phases media

Let’s consider the model of the mathematical model of two-phase flow. Let \( X_k(x, t, \nu) \) is an indicator function for the \( k \) component The examination is limited by the equations of mass and movement. that takes a value of 1 for this implementation of the \( \nu \) current, if phase \( k \) is present at point \( x \) at time \( t \), and the receiving value is 0 otherwise. The volumetric content of the phase \( k \) is defined as the averaging of the indicator function

\[
\alpha_k = \overline{X_k} = \frac{X_k}{\sum_k \overline{X_k}}.
\]

The average values of density, phase velocity and pressure are defined as follows:

\[
p_k = \frac{\overline{X_k\rho}}{\overline{\alpha_k}}, \quad \overline{u_k} = \frac{\overline{X_k\rho u}}{\overline{\alpha_k p_k}}, \quad \overline{p_k} = \frac{\overline{X_k\rho}}{\overline{\alpha_k}}.
\]

Further we shall neglect the difference of pressures between phases ( \( \overline{p_k} = \overline{p}, \forall k \) ) and tangent stresses on the interphase surface. We denote through \( \Gamma_k \). The mass flow through the interphase...
surface $k \sum \Gamma_k = 0$. Then the equation of mass and movement conservation laws can be presented in the following form for phase $k$:

$$\frac{\partial \alpha_k \rho_k}{\partial t} + div(\alpha_k \rho_k \bar{u}_k) = \Gamma_k,$$

(1)

$$\frac{\partial \alpha_k \rho_k \bar{u}_k}{\partial t} + div(\alpha_k \rho_k \bar{u}_k \otimes \bar{u}_k) + \alpha_k \nabla p = div(\alpha_k \bar{T}_k) - \alpha_k \bar{p}_k \bar{g} e_z + M_k,$$

(2)

where $\bar{T}_k$ effective stresses, including the deviator component of the stress tensor and pulsation of stress tensor of phase $k$, $e_z$ - a unit vertical vector, $g$ is a gravity (gravity is directed against the direction of the $z$ axis of Cartesian coordinate system), $M_k$ is a vector of surface forces through the interphase surface ($\sum k M_k = 0$). Let us remind an expression for divergence from the vectors product in the Cartesian coordinate system ($e_x, e_y, e_z$)

$$div(\alpha_k \rho_k \bar{u}_k \otimes \bar{u}_k) = div[(\alpha_k \rho_k \bar{u}_k \cdot e_x) \bar{u}_k e_x] + div[(\alpha_k \rho_k \bar{u}_k \cdot e_y) \bar{u}_k e_y] + div[(\alpha_k \rho_k \bar{u}_k \cdot e_z) \bar{u}_k e_z].$$

Next, let us consider a two-phase system consisting of the liquid phase $k = l$ and the gaseous phase $k = g$. For this system we can evaluate the parameters of the mixture the following equations

$$\rho_m = \alpha_l \rho_l + \alpha_g \rho_g,$$

(3)

$$\rho_m \bar{u}_m = \alpha_l \rho_l \bar{u}_l + \alpha_g \rho_g \bar{u}_g,$$

(4)

$$T_m = \alpha_l T_l + \alpha_g T_g,$$

(5)

$$p_m = p.$$

(6)

Summing up the equations (1-2) by phases with respect to expressions (3-6), we get the following equations for the parameters of the mixture

$$\frac{\partial \rho_m}{\partial t} + div(\rho_m \bar{u}_m) = 0,$$

(7)

$$\frac{\partial \rho_m \bar{u}_m}{\partial t} + div(\rho_m \bar{u}_m \otimes \bar{u}_m) + \nabla p_m = div T_m - \rho_m \bar{g} e_z - div[\sum k \alpha_k \rho_k (\bar{u}_k - \bar{u}_m) \otimes (\bar{u}_k - \bar{u}_m)].$$

(8)

Let us denote the relative velocity between phases through $u_r = \bar{u}_g - \bar{u}_l$, then the last member in equation (8) can be rewritten as follows

$$\sum k \alpha_k \rho_k (\bar{u}_k - \bar{u}_m) \otimes (\bar{u}_k - \bar{u}_m) = \alpha_g \rho_g (1 - \frac{\alpha_g \rho_g}{\rho_m}) u_r \otimes u_r.$$

Assuming that $(\alpha_g \rho_g)/\rho_m \ll 1$, we can ignore by the last member in the equation (8). Then (8) can be overwritten in the following form

$$\frac{\partial \rho_m \bar{u}_m}{\partial t} + div(\rho_m \bar{u}_m \otimes \bar{u}_m) + \nabla p_m = div T_m - \rho_m \bar{g} e_z.$$

(9)

For the density of the mixture $\rho_m$ we have

$$\rho_m = \rho_l + \alpha_g (\rho_g - \rho_l),$$

(10)

and since $\rho_g \ll \rho_l$ it is possible to write

$$\rho_l - \rho_m \approx \alpha_g \rho_l.$$

(11)
The main feature in the consideration of bubble flows with the dissolution of the mixture of non-condensed gases is the equation of state balance. Let’s denote through ⟨⟩ the operator of large-scale averaging on Reynolds. In addition, we denote as \( m_{N_g} \) the mass of the mixture of non-condensed gases in the unit volume of the mixture. Then we have

\[
m_{N_g} = \alpha_g (M_{N_1} C_{N_1} + M_{N_2} C_{N_2}).
\]

(12)

It is assumed that the gas mixture consists of steam and two non-condensing gases \( N_1 \) and \( N_2 \). We suggest that non-condensed gases satisfy the equation of the state of ideal gas. We denote the molar concentrations of steam and non-condensed gases, respectively, through \( C_i \).

We often this effect is neglected for simplicity. In result equation (16) can be rewritten in the following form

\[
\frac{\partial}{\partial t} \langle m_{N_g} \rangle + \text{div}(\langle m_{N_g} \rangle \langle \pi_g \rangle) = \langle \Gamma_{N_g} \rangle - \text{div}(\langle m_{N_g} \pi_g \rangle - \langle m_{N_g} \rangle \langle \pi_g \rangle).
\]

(13)

Let us use the simplest closure for turbulent pulsations in the following form

\[
-\text{div}(\langle m_{N_g} \pi_g \rangle - \langle m_{N_g} \rangle \langle \pi_g \rangle) = \text{div}(D_g \nabla \langle m_{N_g} \rangle).
\]

(14)

where \( D_g \) is a diffusion tensor of gas. In result, formula (13) is transformed into the following equation

\[
\frac{\partial}{\partial t} \langle m_{N_g} \rangle + \text{div}(\langle m_{N_g} \rangle \langle \pi_g \rangle) = \langle \Gamma_{N_g} \rangle + \text{div}(D_g \nabla \langle m_{N_g} \rangle).
\]

(15)

In the present case only non-condensed gases are involved in the process of gas transfer from the bubbles to the solution and back. The change in the mixture of these gases can be described by the relaxation equation proposed in [1]:

\[
\langle \Gamma_{N_g} \rangle = -K [(C_{N_1}^* - C_{dN_1}) + (C_{N_2}^* - C_{dN_2})],
\]

(16)

where \( K \) - the constant of the mass exchange rate, \( D \approx 10^{-9} m^2/s \) - is a coefficient of molecular diffusion of gas in water, \( D \approx 10^{-4} \pm 10^{-5} m \) is an efficient diffusion path (empirical parameter), \( A_b \) is a surface area of the bubbles (surface area of the phase section), \( C_{dNi} \) is a gas \( N_i \) concentration in solution, \( C_{Ni}^* \) is a saturated gas concentration \( N_i \) in solution.

The saturated concentration is defined according to the Henry law [1-2] as follows

\[
C_{Ni}^* = p_{Ni} \cdot \Gamma_{Ni}, \quad i = 1, 2.
\]

where \( p_{Ni} \) is the partial pressure in the gas contacting with the liquid, \( \Gamma_{Ni} \) constant Henry for components \( N_i \) of a mixture of gases. Note that for bubbles with radius \( a \), large than \( 10^{-5} \pm 10^{-6} m \), exceeding the pressure of \( \Delta p = p_g - p = 2\sigma/\alpha \) inside the vial above hydrostatic pressure \( p \) can be neglected (\( \sigma \approx 0.073 \text{ newton/m} \) coefficient of surface tension of water). Therefore, it can be assumed that

\[
C_{Ni}^* = C_{0Ni}^* \frac{p_{Ni}}{p_0}.
\]

where \( C_{0Ni}^* \) is the saturated concentration of gas \( N_i \) at atmospheric pressure \( p_0 \). The value of \( C_{0N_i}^* \) depends on the temperature of the liquid and therefore can be change in the flow. Very often this effect is neglected for simplicity. In result equation (16) can be rewritten in the following form

\[
\langle \Gamma_{N_g} \rangle = -\frac{D}{\delta} S_b [(C_{0N_1}^* \frac{p_{N_1}}{p_0} - C_{dN_1}) + (C_{0N_2}^* \frac{p_{N_2}}{p_0} - C_{dN_2})].
\]

(17)
3. The momentum conservation law for bubble flows with gas dissolution

Within the $k - \varepsilon$ model of turbulence, after the Reynolds averaging, a member, containing $T_m$ equation (9) can be omitted. For the pressure approximation it is possible to take into account only the spherical part Reynolds stresses as follows

$$
\tilde{p}_m = p_m + \frac{2}{3}k.
$$

(18)

Then the equation (9) can be rewritten in the following form (the sign of averaging is omitted here and further for simplicity)

$$
\frac{\partial p_m u_m}{\partial t} + \text{div}(p_m u_m \otimes u_m) + \nabla \tilde{p}_m = \text{div}[\mu_T(\nabla u_m + \nabla^T u_m)] - \rho_m g e_z,
$$

(19)

$$
\mu_T = \rho_m C_{\mu} \frac{k^2}{\varepsilon}, \quad C_{\mu} = 0.09.
$$

The equations for coefficients of $k - \varepsilon$ model have the following form

$$
\frac{\partial (\rho_m k)}{\partial t} + \text{div}(\rho_m u_m k) = \text{div}\left(\frac{\mu_T}{\sigma_k} \nabla k\right) + \Pi_k - \rho_m \varepsilon,
$$

(20)

$$
\frac{\partial (\rho_m \varepsilon)}{\partial t} + \text{div}(\rho_m u_m \varepsilon) = \text{div}\left(\frac{\mu_T}{\sigma_\varepsilon} \nabla \varepsilon\right) + \frac{\varepsilon}{k} \left(c_{\varepsilon 1} \Pi_k - \rho_m c_{\varepsilon 2} \varepsilon\right),
$$

(21)

where

$$
\Pi_k = \frac{\mu_T}{2 \rho_m} \left(\frac{\partial u_m}{\partial x_i} + \frac{\partial u_m}{\partial x_j}\right)^2, \quad (i, j = 1, 2, 3);
$$

(22)

$$
c_{\varepsilon 1} = 1.44, \quad c_{\varepsilon 2} = 1.92, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3.
$$

Let us mention that $k$ is a specific kinetic energy of turbulent pulsations ($k = \frac{1}{2} u_m^2 u_m^2$), and $\varepsilon$ is a dissipation of specific kinetic energy of turbulent pulsations ($\varepsilon = \frac{2 \rho_m \sigma_\varepsilon}{\mu_T s_{ij} s_{ij}}$),

$$
s_{ij} = \frac{1}{2} \left(\frac{\partial u_m}{\partial x_j} + \frac{\partial u_m}{\partial x_i}\right).
$$

4. Gas phase model

Let’s consider the gas phase model proposed [3]. This model does not take into account processes of gas phase bubbles merging and destruction. Then the average number of gas bubbles in the volume unit $N_b$ satisfies to

$$
\frac{\partial N_b}{\partial t} + \text{div}(N_b u_g) = \text{div}(D_g \nabla N_b),
$$

(23)

where

$$
u_g = u_m + w_b(r_b)e_z,
$$

$$
w_b = \left\{
\begin{array}{l}
4474 \text{m/s} \times r_b^{1.357}, \quad \text{if } 0 \leq r_b \leq 7 \times 10^{-4} \text{ m } ,

0.23 \text{ m/s}, \quad \text{if } 7 \times 10^{-4} \text{ m } < r_b \leq 5.1 \times 10^{-3} \text{ m } ,

4.202 \text{ m/s} \times r_b^{0.547}, \quad \text{if } r_b > 5.1 \times 10^{-3} \text{ m } ,
\end{array}\right.
$$

$$
D_g = \frac{\mu_T}{\rho_m Sc_g}.
$$

Here the radius of the bubbles $r_b$ is expressed in meters, the dispersion coefficient of $D_g$ is taken in accordance with the isotropic model proposed in [4], and the Schmidt number $Sc_g$ is taken
equal to 1. The velocity distribution in the formula (24) is chosen by to satisfaction of the experimental data of Haberman and Morton [5].

Let us remind that by definition, the Schmidt number $Sc$ has a ratio of kinematic viscosity to the diffusion coefficient ($Sc = \nu / D$). Note that the right side of equation (23) describes the process of diffusion of gas bubbles carried out by the turbulent pulsations.

Let us mention that alternative approach is based on the consideration of the momentum conservation law taking into account the forces of interphase interaction and lifting forces. However, the difference between two approaches is very small because the dynamic effects in considering phenomenon are poorly influenced on the pressure distribution.

Now let us consider the gas mixture as a mixture of steam and two non-condensed gases $N_1$ and $N_2$. We will assume that non-condensed gases satisfy to equation of the state of ideal gas. Let us denote the molar concentrations of steam and non-condensing gases, respectively, via $C_v$, $C_{N_1}$ and $C_{N_2}$ (molar concentration is the number of moles per unit volume). Then the volumetric content of the gas phase $g$ can be presented in the following form

$$\alpha_g = \alpha_v + \frac{RT_g(C_{N_1} + C_{N_2})}{p_g},$$

$$\rho_g = \frac{C_vM_v + C_{N_1}M_{N_1} + C_{N_2}M_{N_2}}{\alpha_g}.$$  \hfill (24) \hspace{1cm} (25)

Here $\alpha_v$ - volumetric steam content, $M_v$, $M_{N_1}$ and $M_{N_2}$ molecular water vapor weights and non-condensing gases (kg/kmol).

The mass balance equations for non-condensing components of the gas mixture have the following form

$$\frac{\partial C_{N_i}}{\partial t} + \text{div}(C_{N_i} u_g) = S_{N_i} + \text{div}\left(\frac{\mu_T}{\rho_mSc_g} \nabla C_{N_i}\right), \ i = 1, 2;$$ \hfill (26)

where $S_{N_1}$ and $S_{N_2}$ are source parts describing the process of gas dissolving in liquids and reverse process. Let us note, that

$$(\Gamma_{Ng}) = S_{N_1}M_{N_1} + S_{N_2}M_{N_2},$$

$$(m_{Ng}) = C_{N_1}M_{N_1} + C_{N_2}M_{N_2}.$$ \hfill (27) \hspace{1cm} (28)

Knowing $N_b$, $\alpha_v$, $C_{N_1}$ and $C_{N_2}$, by using equation (21) we can find the typical volume of the bubble $v_b$ by the formula

$$v_b = \frac{p_g\alpha_v + RT_g(C_{N_1} + C_{N_2})}{p_gN_b}.$$ \hfill (29)

In [6-9] more complex models are considered, taking into account the processes processes of merging and destruction of gas phase bubbles. In these models instead of the equation (21) uses a somewhat more complex equation in the following form

$$\frac{\partial N_b}{\partial t} + \text{div}(N_b u_g) = \text{div}(D_g\nabla N_b) + K_{break}N_b - K_{coal}N_b^2.$$ \hfill (30)

Here $K_{break}$ and $K_{coal}$ - are coefficients of processes of destruction and merging of bubbles [9].

The process of bubble destruction can be described in terms of the Weber number. This number is the ratio of forces of turbulent pulsations, destroying bubbles, and surface tension in the bubbles, preventing destroying. The number of Weber is calculated by the formula

$$We = \frac{2.8(2\varepsilon r_b)^{1/3}}{\sigma},$$

Here $\varepsilon$ is the volume fraction of steam in the bubble $r_b$ and $\sigma$ is surface tension of steam.
where $\sigma$ - the surface tension ratio.

The process of bubbles destroying begins when the number of Weber exceeds the some critical value, usually equal to 1.2 in turbulent flows. The velocity of the bubbles also depends on the frequency of collisions between the bubbles and the vortex of the same size [6,8]. As a result, in [9] the following expression is proposed for evaluation of $K_{\text{break}}$ ratio

$$\begin{align*}
\text{if} \quad We > W_{e\text{crit}} = 1.2 & \quad \text{then} \quad K_{\text{break}} = C_{\text{break}} \frac{(2\pi r_b)^{1/3}}{2r_b} W_1 \cdot W_2, \\
\text{if} \quad We \leq W_{e\text{crit}} = 1.2 & \quad \text{then} \quad K_{\text{break}} = 0.
\end{align*}$$

Here

$$W_1 = \left(1 - \frac{W_{e\text{crit}}}{We}\right)^{1/2}, \quad W_2 = \exp\left(-\frac{W_{e\text{crit}}}{We}\right).$$

Value $C_{\text{break}}$ according to [8,9] is evaluated by the following formula

$$C_{\text{break}} = 1.4C_{TI} r_b \left(\frac{4\pi N_b}{3\alpha_g}\right)^{1/3}.$$ 

According to experimental data from [10] $C_{TI} = 2$. Then in result

$$\begin{align*}
\text{if} \quad We > W_{e\text{crit}} = 1.2 & \quad \text{then} \quad K_{\text{break}} = 1.4 \left(\frac{4\pi N_b}{3\alpha_g}\right)^{1/3} (2\pi r_b)^{1/3} W_1 \cdot W_2, \\
\text{if} \quad We \leq W_{e\text{crit}} = 1.2 & \quad \text{then} \quad K_{\text{break}} = 0.
\end{align*}$$

The merging of the bubbles, according to [6-8], occurs when a pair of collisions between them. The expression for the merge velocity is output using similar hypotheses to the hypotheses of the kinetic theory of ideal gas. In result we have the following expression for $K_{\text{coal}}$ evaluation

$$K_{\text{coal}} = 1.4(2\pi r_b)^{1/3}(2r_b)^2.$$ 

The more complex model takes into account the decrease of the free path of bubbles with the increase of volumetric content of gas phase $\alpha_g$. In this model according to [9] the following expression for $K_{\text{coal}}$ is valid

$$K_{\text{coal}} = C_{\text{coal}} (2\pi r_b)^{1/3}(2r_b)^2 \frac{1}{1 - \alpha_g^{1/3}}.$$ 

Value $C_{\text{coal}}$ according to [8,9] is evaluated as follows

$$C_{\text{coal}} = \frac{(1.4)^2 C_{RC}}{\pi (2r_b)^2} \left(\frac{36\pi \alpha_g^2}{N_b^2}\right)^{1/3} (2\pi r_b)^{1/3},$$

where $C_{RC} = 0.0565$ - a constant defined in experiments in [8]. Then finally

$$K_{\text{coal}} = \frac{0.11074}{\pi} \left(\frac{36\pi \alpha_g^2}{N_b^2}\right)^{1/3} (2\pi r_b)^{2/3} \frac{1}{1 - \alpha_g^{1/3}}.$$
5. Liquid phase Dissolution model

Now, following to [3], let us consider the dissolution processes in the liquid phase. We designate molar concentrations dissolved in the liquid phase of non-condensing gases as $C_{dN_i}$ and $C_{dN_2}$. The unit of measure $C_{dN_1}$ and $C_{dN_2}$ is $\text{mol} \cdot \text{m}^{-3}$, i.e. mol per cubic meter of liquid phase.

The mass balance equations for non-condensed gases have the form

$$
\frac{\partial C_{dN_i}}{\partial t} + u_l \cdot \nabla C_{dN_i} = S_{dN_i} + \text{div} \left( \frac{\mu_T}{\rho_m S_{Cl}} \nabla C_{dN_i} \right), \quad i = 1, 2; \quad (31)
$$

Where $u_l$ - the velocity of the liquid phase is equal to the velocity of the mixture $u_m$ and the turbulent number Schmidt number for liquid phase $S_{Cl}$ is taken equal to 0.83. Source parts in dissolved gas equations (31) and source parts mass balance equations the gas phase are related by the following expression

$$
S_{dN_i} = -S_{N_i}, \quad i = 1, 2. \quad (32)
$$

The value of the source parts is determined by the surface area between liquid and gas (bubbles) $A_b$ and molar gas flow $F_{N_i}$ and is calculated as their product

$$
S_{N_i} = A_b F_{N_i}, \quad i = 1, 2. \quad (33)
$$

According to [3] the molar gas flow $F_{N_i}$ is evaluated according to the following simple formula

$$
F_{N_i} = h_m (C_{dN_i} - C_{N_i,t}) , \quad i = 1, 2; \quad (34)
$$

where $h_m$ is the mass transfer ratio, $C_{dN_i}$ is the concentration of dissolved gas $N_i$ near the surface of the phase separation, and $C_{N_i,t}$ is the gas concentration in the gas phase on the surface of the phase separation. In the case of local thermodynamic equilibrium on the surface of the phase separation we have

$$
C_{N_i,t} = \Gamma_{N_i} \cdot p_{N_i}, \quad i = 1, 2; \quad (35)
$$

where $\Gamma_{N_i}$ - is Henry constant for $N_i$ gas and $p_{N_i}$ is the partial gas pressure of $N_i$ in bubble. This pressure is related with the pressure of the gas phase $p_g$ as follows (Dalton Law)

$$
p_{N_i} = \frac{p_g C_{N_i}}{C_v + C_{N_1} + C_{N_2}}, \quad i = 1, 2. \quad (36)
$$

Assuming that all the bubbles of the gas are spherical and the same, we have the following estimation for the surface area of the phase separation

$$
A_b = (36\pi)^{1/3} v_b^{2/3} N_b. \quad (37)
$$

According to (29) we have

$$
N_b = \frac{p_g \alpha_v + RT_g (C_{N_1} + C_{N_2})}{p_g v_b}. \quad (36)
$$

Then from (37) we will find

$$
A_b = \left( \frac{36\pi}{v_b} \right)^{1/3} \frac{p_g \alpha_v + RT_g (C_{N_1} + C_{N_2})}{p_g}. \quad (38)
$$

For spherical bubbles

$$
\left( \frac{36\pi}{v_b} \right)^{1/3} = \frac{3}{r_b}. \quad (38)
$$
In result we will obtain the following final expressions for source parts

\[
S_{N_i} = \frac{3\left[p_g\alpha_v + RT_g(C_{N_1} + C_{N_2})\right]h_m}{p_g r_b} \left(C_{dN_i} - \Gamma_{N_i} \frac{p_g C_{N_i}}{C_v + C_{N_1} + C_{N_2}}\right), \quad i = 1, 2. \tag{39}
\]

Here the mass transfer ratio \(h_m\) is assumed to be the same for all non-condensing gases. Henry constant values at 20°C for oxygen equals \(1.3516 \text{ mol} \cdot \text{m}^{-3} \cdot \text{bar}^{-1}\) and for nitrogen is equal to \(0.6788 \text{ mol} \cdot \text{m}^{-3} \cdot \text{bar}^{-1}\).

Detailed study of source parts as a function of the bubble radius is presented in [3].

The disadvantage of the formulas (34) and (39) is that the formula (34) does not describe the process of solution by gas saturation. At the same time, this process describes the formulas (16) and (17) discussed early. Therefore, to determine the molar flow of gas \(F_{N_i}\) instead of formula (34), we will use the following formula together with expressions (16) and (17). Then we have

\[
F_{N_i} = \frac{D A_b}{\delta} (C_{dN_i} - C_{N_i}^*), \quad i = 1, 2. \tag{40}
\]

Further by using formulas (35-38), we will obtain the following final formulas for evaluation of source parts (instead of formula (40))

\[
S_{N_i} = 3D\frac{[p_g\alpha_v + RT_g(C_{N_1} + C_{N_2})]}{\delta p_g r_b} \left(C_{dN_i} - C_{N_i}^*\right), \quad i = 1, 2. \tag{41}
\]

The main advantage of the formula (41) in comparison with the formula (39) is that it describes the processes of fluid saturation by gas. Let us note that this formula can be also presented in the following form

\[
S_{N_i} = \frac{3D[p_g\alpha_v + RT_g(C_{N_1} + C_{N_2})]}{\delta p_g r_b} \left(C_{dN_i} - C_{0N_i}^* \frac{p_{N_i}}{p_0}\right), \quad i = 1, 2. \tag{42}
\]

Here \(C_{0N_i}^*\) the saturated gas concentration of \(N_i\) at atmospheric pressure \(p_0\). Let us note that \(C_{0N_i}^*\) depends on the temperature. It is evident from the formula (42) those shows that the gas phase influence on the gas dissolution process is transmitted through the partial pressure of the \(N_i\) gas in the gas phase of \(p_{N_i}\). For example, the increase of this pressure leads to an increase of the threshold value of \(C_{0N_i}^* \frac{p_{N_i}}{p_0}\) and therefore to the increased amount of gas that can be dissolved in the liquid. Examples of oxygen, hydrogen and nitrogen solubility curves in water from [11] are shown on the following Fig. 1.

Fig.1. Solubility of oxygen, hydrogen and nitrogen in the water
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