Koszul-Tate Cohomology
For an Sp(2)-Covariant Quantization
of Gauge Theories With Linearly Dependent Generators

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Abstract

The anti-BRST transformation, in its Sp(2)-symmetric version, for the general case of any stage-reducible gauge theories is implemented in the usual BV [1, 2] approach. This task is accomplished not by duplicating the gauge symmetries but rather by duplicating all fields and antifields of the theory and by imposing the acyclicity of the Koszul-Tate differential. In this way the Sp(2)-covariant quantization can be realised in the standard BV approach and its equivalence with BLT quantization [3, 4] can be proven by a special gauge fixing procedure.
1 Introduction

Without any doubt the most popular and powerful method for the covariant quantization of the gauge systems is the Batalin-Vilkovisky (BV) method \[\text{[1, 2]}\], which uses the nice mathematical structure of Poisson brackets (in fact, antibrackets), canonical transformations, etc., i.e., the attractive features of the hamiltonian approach in the Lagrangian approach, keeping the advantages of a covariant formalism. However, in the standard BV method only the minimal sector of the theory occurs quite natural from the acyclicity of the Koszul-Tate differential \([3]\), and the non-minimal sector, which is crucial for the applications, is difficult to be understood and fixed.

The main purpose of Batalin, Lavrov and Tyutin (BLT) Sp(2)-quantization \([9, 10, 11]\) of the gauge theories is to offer a proper understanding of the non-minimal sector, which becomes a natural part of the minimal sector. Nevertheless, the BLT method, in spite of the fact that it is very similar to BV method, has different structure and a different field content. First of all the symplectic structure of the theory is lost since the usual antibrackets are replaced by the extended antibrackets and the master equation is replaced by two generating equations \([3, 11, 11]\). The field structure of the theory is quite asymmetric since a field \(\phi^A\) is associated with two antifields \(\Phi^{Aa}, a = 1, 2\) and a bar-field \(\Phi_A\), so the symmetry field ↔ antifield is lost in this case. Furthermore, the gauge fixing process is quite tricky and it seems to be difficult to the connection, if any, between BV method and BLT method.

In this paper we shall show that it is possible to reformulate the anti-BRST, Sp(2)-formalism in the usual BV framework just by duplicating all the fields and antifields of the theory and by using homological perturbation theory (HPT) \([4]\). The main ingredient of the HPT is the construction of the Koszul-Tate (KT) differential \(\delta_K\) and its acyclicity. This differential and its properties have been studied details in \([4]\) (see also \([3]\)) in order to prove the existence and the uniqueness for the solution of the master equation. The acyclicity of KT differential determines, in fact, the spectrum of all antifields and therefore of all fields, whether we work within a theory with a symmetric field-antifield structure. In our approach we shall work only with pairs of field-antifield and we shall reduce the BLT quantization for the reducible systems to the usual one.

What is basic in our construction is the remark that if we duplicate the antifields, they form a redundant basis of vectors and in order to identify the algebra of polynomials in the fields and antifields with the algebra of multivectors it is necessary to set \(\Phi_{A1} = \Phi_{A2}\) in the former algebra, where \(\Phi_{Aa}, a = 1, 2\) are all antifields from the theory. The most natural way to accomplish this task is to suppose that the full BRST differential does this job (and not only KT differential). This assumption is quite strong and it implies that all fields (including the ghosts) must occur in pairs and the action must depend only on the sum of these pairs i.e.

\[
S = S(\Phi^{A1} + \Phi^{A2}, \cdots)
\]

where the index \(A\) is common index for all fields and ghosts which occur in our theory. But the action \([14]\) has an additional gauge symmetry \(\Phi^{Aa} \rightarrow \Phi^{Aa} + (-1)^a e^A, a = 1, 2\), which must be taken into account, if we want to quantize properly the theory. Thus during the Sp(2)
quantization, in our version, we have to duplicate the fields and antifields and to consider the new gauge symmetry which we have just mentioned. This has already been done by one of us [21] in the irreducible case. Here we intend to extend our construction for the general case of the reducible systems.

2 The spectrum of antifields

Let us consider a classical set of fields $\Phi^j (j = 1, \cdots, n = n_+ + n_-)$, where $n_+ (n_-)$ is the number of Boson (Fermion) fields. The classical action $S_0(\Phi)$ is supposed to be invariant under the gauge transformation

$$
\Phi^j \rightarrow \Phi'^j = \Phi^j + \delta \Phi^j = \Phi^j + R^j_{\alpha_0}(\Phi) \xi^{\alpha_0}
$$

$$
G_j R^j_{\alpha_0}(\Phi) = S_{0,j}(\Phi) R^j_{\alpha_0}(\Phi) = 0, \quad \alpha_0 = 1, \ldots, m_0, m_0 = m_0 + m_0, \quad (2.1)
$$

where $\xi^{\alpha_0}$ are the parameters of the gauge transformations, with the Grassman parity $\epsilon(\xi^{\alpha_0}) = \epsilon_{\alpha_0}$; $\epsilon(R^j_{\alpha_0}) = \epsilon_j + \epsilon_{\alpha_0}$ and $\epsilon(\Phi^j) = \epsilon_i$, $G_j = S_{0,j}$ the comma here means the derivative of $S_0$ with respect of $\Phi^j$.

For the case of rank($R^j_{\alpha_0}$) = ($r_{0+}, r_{0-}$) with $r_{0\pm} = m_{0\pm}$ when the generators $R^j_{\alpha_0}$ are all independent, the structure and the quantization of the theory have been already discussed in [21].

If $r_{0\pm} < m_{0\pm}$ the generators $R^j_{\alpha_0}$ are linearly dependent and the theory is reducible. In this case the matrix $R^j_{\alpha_0}$ has on mass shell $S_{0,j} = 0$ a number of $m_1 = m_{1+} + m_{1-}$ zero-eigenvalue eigenvectors $Z^{\alpha_0}_{\alpha_1}$ and the numbers $\epsilon_{\alpha_1} = 0, 1$ such that:

$$
R^i_{\alpha_0} Z^{\alpha_0}_{\alpha_1} = S_{0,j} L^i_{\alpha_1}, \quad \alpha_1 = 1, \cdots, m_1; \quad (2.2)
$$

with $\epsilon(Z^{\alpha_0}_{\alpha_1}) = \epsilon_{\alpha_0} + \epsilon_{\alpha_1}$ and the matrices $L^i_{\alpha_1}$ can be chosen to have the properties

$$
L^i_{\alpha_1} = (-1)^{\epsilon_{j\alpha_1}} L^i_{\alpha_1}. \quad (2.3)
$$

If the $\text{rank}Z^{\alpha_0}_{\alpha_1} = (r_{1+}, r_{1-})$ with $r_{1\pm} < m_{1\pm} = m_{0\pm} - r_{0\pm}$ the set $\{Z^{\alpha_0}_{\alpha_1}\}$ is linearly dependent and there is the set of zero-eigenvalue eigenvectors $Z^{\alpha_0}_{\alpha_2}$ and the numbers $\epsilon_{\alpha_2} = 0, 1$ such that

$$
Z^{\alpha_0}_{\alpha_1} Z^{\alpha_0}_{\alpha_2} = S_{0,j} L^{\alpha_0}_{\alpha_2}, \quad \alpha_2 = 1, \cdots, m_2; \quad (2.4)
$$

with $\epsilon(Z^{\alpha_0}_{\alpha_2}) = \epsilon_{\alpha_1} + \epsilon_{\alpha_2}$ and the matrices $L^{\alpha_0}_{\alpha_2}$ can be chosen to have the properties

$$
L^{\alpha_0}_{\alpha_2} = (-1)^{\epsilon_{\alpha_0\alpha_2}} L^{\alpha_0}_{\alpha_2}. \quad (2.5)
$$

In the general case the set $\{Z^{\alpha_0}_{\alpha_2}\}$ could be redundant and we have to continue the process. Thus we have a sequence of reducibility equations for the sets $\{Z^{\alpha_0}_{\alpha_{s-1}}\}, \ (s = 1, \cdots, L)$ of the form:

$$
Z^{\alpha_{s-2}}_{\alpha_{s-1}} Z^{\alpha_{s-1}}_{\alpha_s} = S_{0,j} L^{\alpha_{s-2}}_{\alpha_s}, \quad \alpha_s = 1, \cdots, m_s = m_{s+} + m_{s-}, \quad s = 1, \cdots, L. \quad (2.6)
$$
In order to find the spectrum of fields and ghosts of our theory we shall use a natural way, which differs from the one chosen by Grégoire and Henneaux \[13\] since we will start with the spectrum of the antifields and we define the spectrum of fields just by a simple correspondence antifield $\rightarrow$ field. On the other hand, the spectrum of the antifields is uniquely determined , by the demand that the Koszul-Tate differential $\delta_K$ be acycle. This fact was emphasised by Fisch and Henneaux \[5\] in their attempt to clarify the algebraic structure of the antifield-antibracket formalism for reducible gauge theories in the usual Batalin-Vilkovisky quantization. They showed that the acyclicity of $\delta_K$ forces the antifield spectrum to be just the correct minimal set described by Batalin and Vilkovisky \[1, 2\](see also \[3\]).

We shall use the general scheme developed by Fisch and Henneaux but we are going to introduce a new ingredient in the theory, which will allow us to duplicate all the fields and antifields and to obtain, in one way, the correct set of fields and antifields described by Batalin, Lavrov and Tyutin in their $\text{Sp}(2)$-covariant quantization of gauge theories with liniar dependent generators \[\textbf{11, 12}\]. However, in our approach not only the spectrum of all antifields is correctly obtained but also the form of the quantized action is determined.

At the irreducible level, we start with a doublet of fields $\Phi_{ia}$, where $a = 1, 2$. The action will depend only on the sum of these fields. The Koszul-Tate differential acting on the antifields associated to these fields is given by

$$ \delta_K \Phi^*_{ja} = -S_{0,ja} = -\frac{\delta S_0}{\delta \Phi^*_{ja}} \quad (2.7) $$

where $S_0 = S_0(\Phi^{j1} + \Phi^{j2})$. By subtracting the relations given by the previous equation, we obtain $\delta_K(\Phi^*_j - \Phi^*_j) = 0$, i.e. we have a cycle. But all the cycles have to be trivial, so we introduce a new field $\bar{\Phi}_i$ whose differential is equal with the previous cycle which becomes now a boundary. We thus have

$$ \delta_K \bar{\Phi}_i = \Phi^*_j - \Phi^*_j \quad (2.8) $$

Hitherto, we did not use the symmetries of the theory. We will proceed now to consider them. The Noether theorem tells us that it exist $R^i_{\alpha}$, such that $R^i_{\alpha}S_{0,ia} = 0$, which can be written in another form, as $\delta_K(R^i_{\alpha} \Phi^*_{ja}) = 0$, which introduce another cycle which has to be killed. We kill this cycle by introducing antifields of higher antighost, which will have a supplementary indice, $C^*_{\alpha b|a}$, with

$$ R^i_{\alpha} \Phi^*_{ja} = \delta_K C^*_{\alpha b|a} \quad (2.9) $$

We have again new cycles, given by a specific combination of the antifields just introduced, given by

$$ \delta_K(C^*_{\alpha 1|a} - C^*_{\alpha 2|a}) = 0 \quad (2.10) $$

and

$$ \delta_K(C^*_{\alpha 2|1} - C^*_{\alpha 2|2} - R^j_{\alpha} \bar{\Phi}_j) = 0 \quad (2.11) $$

which again have to be killed. To realize this, one has to introduce new antifields, given by the relations

$$ C^*_{\alpha 1|a} - C^*_{\alpha 2|a} = \delta_K \bar{C}_{\alpha 0|a} \quad (2.12) $$
and
\[ C_{\alpha_0|2}^* - C_{\alpha_0|1}^* = R_{\alpha_0}^j \Phi_j = \delta_K B_{\alpha_0}^* \]  \hspace{1cm} (2.13)
where the fields $\bar{C}_{\alpha_0|a}$ and $\bar{B}_{\alpha_0}$ are introduced in order to kill the nontrivial cycles.

With this we have solved the irreducible theory, i.e. we do not have any nontrivial cycle and we introduced the entire spectrum of antifields. To all the antifields we associate a field, therefore we have besides the previous antifields the fields $\bar{\Phi}$ as:
\[ \bar{\Phi} \leftrightarrow \bar{\Phi}, \bar{C}_{\alpha_0|a}, \bar{C}_{\alpha_0|b}, \bar{B}_{\alpha_0}^a \leftrightarrow B_{\alpha_0}^a, \] where the arrows denote the correspondence between the respective fields and antifields. The total action, which includes all the fields and the antifields is given by
\[ S = \Phi^j \bar{R}^j_{\alpha_0} (C_{\alpha_0|1}^* + C_{\alpha_0|2}^*) \]  \hspace{1cm} (2.14)
\[ + (C_{\alpha_0|2}^* - C_{\alpha_0|1}^* - R_{\alpha_0}^j \bar{\Phi}_j) \bar{R}^j_{\alpha_0} (B_{\alpha_1} + B_{\alpha_2}) + \bar{\Phi}^A (\Phi_{A1} - \Phi_{A2}) \]
where $\Phi^A$ is a collective notation for all the fields.

In this paper we assume that the tensorial character of the ghost $C_{\alpha_0}^1$ and the antighost $C_{\alpha_0}^2$ is the same, the last one belonging to the non-minimal sector in the standard version of BV. This fact is true only for certain types of gauge fixings, so we work by stating this restriction. One example where our approach does not work is the bosonic string in conformal gauge where the ghost and the antighost have different transformations under diffeomorphisms.

We go now to the reducible theory. We call reducible a theory where there exists a relation between the R functions, i.e. there exist functions $Z$, such that $R_{\alpha_0} Z_{\alpha_0} = 0$. In this case the appearance of new cycles which have to be killed by Koszul-Tate differential becomes obvious. They are given by:

1. $Z_{\alpha_1} C_{\alpha_0|b}^*$ which is to be brought to a boundary by introducing new antifields with the relation
\[ \frac{1}{2} Z_{\alpha_1} (C_{\alpha_0|b}^* + C_{\alpha_0|b}) = \delta_K C_{\alpha_1|c|ab} \]  \hspace{1cm} (2.15)
where we imposed the symmetry in a and b indices, a condition required by the Sp(2) symmetry.

2. $C_{\alpha_1|ab}^* - C_{\alpha_2|ab}^*$, which is to be killed by a new bar field introduced with
\[ C_{\alpha_1|ab}^* - C_{\alpha_2|ab}^* = \delta_K (\bar{C}_{\alpha_1|ab}) \]  \hspace{1cm} (2.16)

3. $C_{\alpha_2|1b}^* - C_{\alpha_1|2b}^* + \bar{C}_{\alpha_0|b} Z_{\alpha_0}^a - \frac{1}{2} B_{\alpha_0 b}^* Z_{\alpha_1}^a$, which is killed by introducing a new antifield as:
\[ C_{\alpha_2|1b}^* - C_{\alpha_1|2b}^* + \bar{C}_{\alpha_0|b} Z_{\alpha_0}^a - \frac{1}{2} B_{\alpha_0 b}^* Z_{\alpha_1}^a = \delta_K B_{\alpha_1|c|b} \]  \hspace{1cm} (2.17)

4. $B_{\alpha_1|b}^* - B_{\alpha_2|b}^*$ which is killed by a new bar field given by
\[ B_{\alpha_1|b}^* - B_{\alpha_2|b}^* = \delta_K B_{\alpha_1|b} \]  \hspace{1cm} (2.18)
These are all the antifields which have to be added for a first stage reducible theory. Besides, we have to introduce all the fields which correspond to these antifields with the same correspondence as we used for the irreducible theory.

Using the results for irreducible and first-stage reducible theories, we can now give the formula for general theories, i.e. for the s-stage reducible theories. For these theories, there exist $Z_{\alpha_{i-1}}^{\alpha_i}$, with $i$ from 0 to $s$, such that $Z_{\alpha_{i-1}}^{\alpha_i}Z_{\alpha_i}^{\alpha_{i+1}} = 0$, which are the reducibility conditions.

By induction, we have obtained the following results for the action of the Koszul-Tate differential on the antifields:

$$\delta_K C^*_{\alpha_{i|a_1\cdots a_{s+1}}} = \frac{1}{s+1} S[C^*_{\alpha_{s-a_1\cdots a_s+1}}Z_{\alpha_s}^{\alpha_{s-1}}] \quad (2.19)$$

and

$$\delta_K B^*_{\alpha_{i|a_1\cdots a_s}} = -\epsilon^{ab} C^*_{\alpha_{s-a_1\cdots a_s}} - \frac{1}{s+1} Z_{\alpha_s}^{\alpha_{s-1}} S[B^*_{\alpha_{s-a_1\cdots a_s}}] - \bar{C}_{\alpha_{s-a_1\cdots a_s}} Z_{\alpha_s}^{\alpha_{s-1}} \quad (2.20)$$

where $\epsilon^{ab} = 0$ if $a \neq b$, $1$ if $a = 1$, $b = 2$ and -1 for $a = 2$, $b = 2$ and the symbol $S$ denotes the symmetrisation over the indices which appear after the vertical line in the formulas for $C$ and $B$ because of the Sp(2) symmetry. Moreover we have,

$$\delta_K \bar{C}_{\alpha_{s-a_1\cdots a_s}} = C^*_{\alpha_{s-1}a_1\cdots a_s} - C^*_{\alpha_{s-2}a_1\cdots a_s} \quad (2.21)$$

To all the previously introduced antifields we assign fields using the same conventions as for irreducible and first stage reducible theories.

The minimal action, i.e. the action constructed only with the minimal sector is given by:

$$S = S_0 + \sum_{s=0}^{\infty} \frac{1}{s+1} S[C^*_{\alpha_{s-a_1\cdots a_{s+1}}^{\alpha_{s-1}}}][C^*_{\alpha_{s-1}a_1\cdots a_s} + C^*_{\alpha_{s-2}a_1\cdots a_s}]$$

$$- \epsilon^{ab} C^*_{\alpha_{s-1-a_1\cdots a_s}} + \frac{1}{s+1} Z_{\alpha_s}^{\alpha_{s-1}} S[B^*_{\alpha_{s-a_1\cdots a_s}}] + \bar{C}_{\alpha_{s-a_1\cdots a_s}} Z_{\alpha_s}^{\alpha_{s-1}})[B^*_{\alpha_{s-a_1\cdots a_s}} + B^*_{\alpha_{s-2}a_1\cdots a_s}]$$

$$+ \Phi^A (\Phi^*_{A2} - \Phi^*_{A1}) \quad (2.22)$$

The master equation in our case, as was discussed in [20], is written as

$$(S_T, S_T) = 0 \quad (2.23)$$

where

$$S_T = \Phi^A (\Phi^*_{A2} - \Phi^*_{A1}) + S(\Phi^A, \Phi^*_{A2}, \Phi^*_{A1}) \quad (2.24)$$

where again $\Phi^A$ is the symbol for all the fields i.e. $\Phi^A = \Phi^{A1} + \Phi^{A2}$. 
3 The Sp(2)-BRST transformations for a first stage reducible system

Consider now a first-stage reducible system, which is characterized by the functions $R_{\alpha_0}^j$, $Z_{\alpha_1}^{\alpha_0}$, with the reducibility relations $R^j_{\alpha_0} Z_{\alpha_1}^{\alpha_0} \equiv 0$. This corresponds to an off-shell reducible theory and in the sequel we will discuss only this type of reducible theories, which have the property of being linear in antifields. For an on-shell reducible theory, the solution becomes at least quadratic in antifields which makes it more difficult to solve (in the case of Freedman-Townsend model the solution was obtained by Barnich et al. in [16]).

Moreover, we have the commutativity relation between $R$'s:

$$[R_{\alpha_0}, R_{\beta_0}] = C_{\alpha_0 \beta_0}^{\gamma_0} R_{\gamma_0}^j$$

(3.1)

where $C_{\alpha_0 \beta_0}^{\gamma_0}$ are constants giving the algebra of transformations.

The action, which is a solution of the master equation (2.23) with the boundary conditions (2.22), for the one-reducible theories with a closed algebra exists as a linear functional in the antifields $\Phi^*_A$ and $\bar{\Phi}_A$ and has the general form

$$S_T = S_0(\Phi) + \bar{\Phi}^A (\Phi^1_{A1} - \Phi^1_{A2}) + \Phi^*_A X^{Aa} + \bar{\Phi}_A Y^A$$

(3.2)

where $X^{Aa}$ and $Y^A$ are functions of the fields $\Phi^A = \Phi^{A1} + \Phi^{A2}$. If one defines the following transformations of the fields

$$s^a \Phi^A = X^{Aa}$$

(3.3)

then the master equation (2.23) yields

$$s^a S_0 = 0$$

$$\{s^a, s^b\} \Phi^A = 0$$

$$Y^A = \frac{1}{2} \epsilon_{ab} s^a s^b \Phi^A$$

$$s^a Y^A = 0$$

(3.4)

Starting from these data, we can follow very closely the results of Spiridonov [7], who determined the transformation $s^a \Phi^A$ of fields. Adopting his results to our notations, and using *everywhere* $\Phi^A$ instead of $\Phi^{A1} + \Phi^{A2}$, the results are

$$s^a \Phi^j = R^j_{\alpha_0} C^{\alpha_0 |a},$$

$$s^a C^{\alpha_0 |b} = \epsilon^{ab} B^{\alpha_0} - \frac{1}{2} C^{\alpha_0}_{\beta_0 \gamma_0} C^{\beta_0 |a} C^{\gamma_0 |b} + Z_{\alpha_1}^{\alpha_0} C^{\alpha_1 |ab},$$

(3.5)

$$s^a B^{\alpha_0} = Z_{\alpha_1}^{\alpha_0} B^{\alpha_1 |a} +$$

$$+ \frac{1}{2} C^{\alpha_0}_{\beta_0 \gamma_0} (B^{\beta_0} C^{\gamma_0 |a} + Z_{\alpha_1}^{\beta_0} C^{\alpha_1 |ab} \epsilon^{bc} C^{\gamma_0 |c})$$

$$- \frac{1}{12} (C^{\alpha_0}_{\beta_0 \gamma_0} C^{\beta_0}_{\sigma \rho \sigma 0} + 2 C^{\alpha_0}_{\sigma \rho 0, j} R^j_{\sigma 0}) C^{\alpha_0 |a} C^{\rho_0 |b} \epsilon^{bc} C^{\gamma_0 |c}. \quad (3.6)$$
These relations were constructed only with the variable $C_{\alpha_0\alpha_1\alpha_2}$ and are for the fields which appear in the case of an irreducible theory. For the rest of fields, one needs new variables, which are combinations of $R$, $Z$ and $C$'s.

The corresponding transformations are

$$s^a C^{\alpha_1|bc} = -\epsilon^{ab} B^{\alpha_1|c} - \epsilon^{ac} B^{\alpha_1|b} + A^{\alpha_1}_{\alpha_0\beta_1} C^{\alpha_0|a} C^{\beta_1|bc} - \frac{1}{2} F^{\alpha_1}_{\alpha_0\gamma_0} C^{\alpha_0|a} C^{\beta_0|b} C^{\gamma_0|c} \tag{3.7}$$

and

$$s^a B^{\alpha_1|b} = A^{\alpha_1}_{\alpha_0\beta_1} C^{\alpha_0|a} B^{\beta_1|b} - \frac{1}{2} F^{\alpha_1}_{\alpha_0\beta_0\gamma_0} B^{\alpha_0} C^{\beta_0|a} C^{\gamma_0|b} + \frac{1}{2} A^{\alpha_1}_{\alpha_0\beta_1} Z^{\alpha_0}_{\alpha_1} C^{\alpha_0|a} \epsilon^b c_d - \frac{1}{4} F^{\alpha_1}_{\alpha_0\beta_0\gamma_0} Z^{\alpha_0}_{\beta_1} (3 C^{\beta_0|a} C^{\beta_1|bc} \epsilon^c d C^{\gamma_0|d}) + C^{\beta_0|b} C^{\beta_1|ac} \epsilon^c d C^{\gamma_0|d}) \tag{3.8}$$

It is worthwhile to note that the BRST differential $s$ defined by

$$s F = (F, S_T) \tag{3.9}$$

acts on $\Phi^A = \Phi^{A_1} + \Phi^{A_2}$ as $s \Phi^A = s^1 \Phi^A + s^2 \Phi^A$. Therefore, if one considers only the cohomology of $s$ on the algebra generated by fields $\Phi^A$ and their derivatives then we can define three cohomology groups $H(s)$, $H(s_1)$ and $H(s_2)$. Remembering that any free differential algebra $(A, D)$ can be decomposed into a contractive subalgebra generated by elements of the form $(x, Dx)$ and a minimal algebra $M$ (with the condition $DM \subset M \times M$), the cohomology group of $(A, D)$ is given by $H(M, D)$ because the contractive part does not contribute. Using this observation and the previous transformation of the fields under $s^1, s^2$, it results that the non-minimal of the original BV scheme $C^{\alpha_0|2}, B^{\alpha_0}, C^{\alpha_1|12}, B^{\alpha_1|2}$ do not give any contribution to $H(s_1)$. It also results that $C^{\alpha_0|1}, B^{\alpha_0}, C^{\alpha_1|12}, B^{\alpha_1|1}$ do not give any contribution to the cohomology $H(s_2)$. This proves the compatibility of our approach with the triviality of the non-minimal sector in the usual BV approach (see for example [25]).

Here, the $A$'s and $F$'s which appear in the equations are given by the following relations:

$$Z^{\alpha_0}_{\alpha_1\beta_1} R^{j}_{\beta_0} + C^{\alpha_0}_{\alpha_0\gamma_0} Z^{\gamma_0}_{\alpha_1} + Z^{\alpha_0}_{\beta_0\alpha_1} A^{\beta_1}_{\beta_0\alpha_1} = 0 \tag{3.10}$$

or

$$C^{\alpha_0}_{\beta_0\gamma_0} Z^{\beta_0}_{\alpha_1} Z^{\gamma_0}_{\alpha_1} + A^{\gamma_1}_{\psi_0\beta_1} Z^{\beta_0}_{\alpha_1} Z^{\alpha_0}_{\gamma_1} = 0 \tag{3.11}$$

and

$$A^{\alpha_1}_{\alpha_0\beta_1} A^{\beta_1}_{\beta_0\gamma_1} - A^{\alpha_1}_{\alpha_0\beta_1} A^{\beta_1}_{\beta_0\gamma_1} + A^{\alpha_1}_{\alpha_0\gamma_1} j R^{j}_{\beta_0} - A^{\alpha_1}_{\beta_0\gamma_1} j R^{j}_{\alpha_0} + A^{\alpha_1}_{\alpha_0\beta_1} C^{\gamma_0}_{\alpha_0\beta_0} + 3 F^{\alpha_1}_{\alpha_0\beta_0\gamma_0} Z^{\gamma_0}_{\gamma_1} = 0 \tag{3.12}$$

In the original paper of Spiridonov, the fields $B^\alpha$ appeared as auxiliary fields which were used as Lagrange multipliers for the gauge conditions. In our approach, the B fields are associated
to the antifields introduced with the acyclicity of the Koszul-Tate differential. In the F fields were introduced starting from the equation (3.1) which expresses the closure of the gauge algebra which was assumed in our approach. If we apply the Jacobi identity in the case of field-dependent structure constants for first-stage reducible theory we can introduce the F fields by writing the Jacobi identity in the following form:

\[ C_{\beta_0\rho_0\sigma_0}^\alpha C_{\rho_0\sigma_0}^\beta + R_{\beta_0\rho_0\sigma_0}^\gamma F_{\alpha_1}^\gamma + (\beta_0\rho_0\sigma_0 - \text{cycle}) \] (3.13)

where \( \beta_0\rho_0\sigma_0 \)-cycle means the circular permutation of the indices \( \beta_0, \rho_0, \sigma_0 \).

The A’s coefficients are obtained by taking (3.1), multiplying it by \( Z_{\alpha_1}^{\alpha_0} \) and using:

\[ R_{\alpha_0,k}^{\alpha_1} Z_{\alpha_0}^{\alpha_1} = - R_{\alpha_0}^{\alpha_1} Z_{\alpha_0,k} \] (3.14)

The result of is then introduced into (3.10) which gives the new structure coefficients denoted by A.

In the section 3 we have obtained the action of the BRST operator (which is the Koszul-Tate operator for the case of antifields) on antifields and which is a generalisation of the result of [21] for a theory of any order reducibility. In section 4 we have made the parallel between our approach and Spiridonov approach for the action of the BRST operator on the fields in the case of one-reducible theory.

### 4 Gauge fixing in our approach

In the newest version of the BRST-BV quantization [22, 23, 24] the path integral is proposed to be

\[ Z = \int d\Gamma d\lambda e^{\frac{i}{\hbar}(S+X)} \] (4.1)

where \( \Gamma \) denotes the collection of all fields and antifields of the theory and \( X(\Gamma, \lambda) \) is a hypergauge fixing action, depending on a new set of variables \( \lambda^A \) with \( \epsilon(\lambda^A) = \epsilon_A \), which can be considered as Lagrange multipliers, when \( X \) depends linear in them. In order to assure the gauge independence of the path integral \( Z \) (4.1), \( X \) must satisfy the master equation

\[ (X, X) = 0. \] (4.2)

In the BV quantization the hypergauge action has the form

\[ X = \lambda^A(\Phi_A^* - \frac{\partial \Psi}{\partial \Phi_A}) \] (4.3)

which verifies the master equation (4.2) for any \( \lambda^A \).

In the extended Sp(2) theory we have three types of fields \( \Phi^{A_1}, \Phi^{A_2}, \Phi^A \) and three types of antifields \( \Phi^{*A_1}, \Phi^{*A_2}, \Phi^* \). We want to eliminate the fields \( \Phi^{A_2} \) in the path integral. Therefore we could chose \( X \) to be

\[ X = \mu_A \Phi^{A_2} + \pi^A(\Phi^{*A_1} - \frac{\partial \Psi}{\partial \Phi^{A_1}}) + \lambda^A(\Phi^* - \frac{\partial \Psi}{\partial \Phi^*}) \] (4.4)
where \( \mu_A, \pi^A \) and \( \lambda^A \) are the Lagrange multipliers and \( \Psi = \Psi(\Phi, \bar{\Phi}) \) is the fermion gauge fixing function.

Due to this particular form of the hypergauge action the master equation (4.2) for \( X \) yields

\[
\frac{\partial^2 \Psi}{\partial \Phi^A_1 \partial \bar{\Phi}^B} = \frac{\partial^2 \Psi}{\partial \Phi^B_1 \partial \bar{\Phi}^A}.
\]

These equations have a solution of the form

\[
\Psi = \frac{\partial F}{\partial \Phi^A_1} \bar{\Phi}^A.
\]

Now if we integrate out in \( Z \) the variables \( \mu_A \) and \( \lambda^A \) we get eventually

\[
S_{\text{eff}} = S(\Phi, \Phi_1^*, \Phi_2^*, \bar{\Phi}) - \bar{\Phi}^A \Phi_2^* - \pi^A \Phi_2^* + \pi^A \frac{\partial^2 F}{\partial \Phi^A_1 \partial \Phi^B} \bar{\Phi}^B + (\Phi_A - \frac{\partial F}{\partial \Phi^A}) \lambda^A.
\]

At this point we believe that it is worthwhile to remark that this form of the quantum action coincides with the one proposed by Batalin, Lavrov and Tyutin [9] and by Batalin and Marnelius [24] with the identifications

\[
\pi^A \rightarrow \pi^{A_1}; \quad \bar{\Phi} \rightarrow \pi^{A_2}.
\]

but the roles played by the auxiliary fields \( \pi^{Aa} \) in BLT approach are quite different from the corresponding ones in our approach.

5 Example-topological field theories

As an illustration of the general theory we shall consider the Sp(2)-quantization of the topological Yang-Mills theory. This problem has been solved by Gomis and Roca [26] in the usual BV approach (see however the remark at the end of our discussion) and by Perry and Teo [27] in the usual BRST quantization (see also Baulieu [28]).

In the sequel we shall use the notations employed by Perry and Teo [27]. In these notations the fields \( \Phi^j \) are the \( G \)-valued Yang-Mills 1-form potential \( A \), the ghosts \( \epsilon^{a_0}_{a} \) are the \( G \)-valued ghosts 0-forms \( c \) and \( \bar{c} \) and \( G \)-valued ghost 1-forms \( \psi \) and \( \bar{\psi} \), the ghosts for ghosts \( \epsilon^{a_1}_{ab} \) are the \( G \)-valued ghosts 0-forms \( \Phi \bar{\Phi} \) and the auxiliary \( G \)-valued ghosts 0-forms \( \lambda \). The additional fields \( B^{a_0} \) are the Lagrange multipliers \( b \) and \( k \) and the fields \( B^{a_1}_{a} \) are the odd scalar \( G \)-valued fields \( \eta \) with the ghost number one and its corresponding anti-ghost \( \bar{\eta} \). We have found convenient to change a little this notation and our notation is given in the Table.
The generators of the gauge transformations $R^j_\alpha$ and the coefficients $C^\alpha_0\gamma_0$, $A^\alpha_1\beta_1$ and $F^\alpha_0\beta_0\gamma_0$ can be obtained from the infinitesimal gauge transformation

$$\delta A = \epsilon_1 - D\epsilon$$

with $\epsilon_1$ an infinitesimal $G$-valued 1-form and $\epsilon$ an infinitesimal $G$-valued 0-form and the definitions (3.1), (3.10), (3.11) and (3.12). The gauge generators are off-shell linearly dependent: if we take $\epsilon_1 = D\tau$ and $\epsilon = \tau$, we find that the gauge transformation generated by $\tau$ is 0. Thus we have a first-stage reducible theory, for which we can apply our procedure developed in the previous sections. We will skip the details and give only the forms of the transformations (3.5), (3.6), (3.7) and (3.8) for this model:

$$s_a A = \psi_a - Dc_a$$

$$s_a c_b = \epsilon_{ab}b - [c_a, c_b] + c_{ab}$$

$$s_a \psi_b = \epsilon_{ab}k - [c_a, \psi_b] - Dc_{ab}$$

$$s_a c_{bc} = -\epsilon_{ab}\eta_c - \epsilon_{ac}\eta_b + [c_a, c_{bc}]$$

$$s_a b = -\eta_a + [b, c_a] + \frac{1}{2}[c_{ab}, c_c]\epsilon^{bc} - \frac{1}{12}[c_b, [c_c, c_a]]\epsilon^{bc}$$

$$s_a k = D\eta_a + [k, c_a] - [c_{ab}, \psi_c]\epsilon^{bc}$$

$$s_a \eta_b = (-2)[c_a, \eta_b] + \frac{1}{2}[c_{ac}, c_{bd}]\epsilon^{cd}$$

The quantum action is given by the equation (3.2) adapted for the topological field theory.

Now we believe that it is noteworthy to connect our results with the ones obtained by Gomis and Roca [26]. They have considered an usual, i.e. not an extended Sp(2), BV action $S$ as a solution of the master equation and they tried to find out an additional term $\hat{S}$, which was the generator of the anti-BRST transformations. These two generators can be obtained also in the Sp(2)-BRST quantization in the situation when the solution of the master equation involves only terms linear in the antifields. In this case the solution of the master equation for a general system, irreducible or reducible, in the Sp(2)-BRST
quantization has the form (again we use the notation $\Phi^A = \Phi^{A_1} + \Phi^{A_2}$):

$$S_T = S_0(\Phi) + \Phi^*_A X^{A_1} + \bar{\Phi}^A Y^A + \bar{\Phi}^A (\Phi^{A_1} - \Phi^{A_2}).$$  \hfill (5.9)

In this case, it is easy to verify that the action

$$S = S_0(\Phi) + \Phi^*_A X^{A_1}$$  \hfill (5.10)

as well as the anti-BRST generator

$$\bar{S} = \Phi^*_A X^{A_2}$$  \hfill (5.11)

obtained from (5.9) by identifying $\Phi^*_A = \Phi^*_A = \Phi^*_A$, fulfill the equations

$$(S , S) = (S , \bar{S}) = (\bar{S} , \bar{S}) = 0. \hfill (5.12)$$

On the other hand in the Sp(2)-BRST quantization the ghost numbers of $\Phi^*_A$ and $\Phi^*_A$ are different:

$$gh(\Phi^*_A) = -1 - gh(\Phi^{A_1}) \quad gh(\Phi^*_A) = 1 - gh(\Phi^{A_2}). \hfill (5.13)$$

Thus the identification $\Phi^*_A = \Phi^*_A = \Phi^*_A$ changes the ghost number of $\bar{S}$ which becomes

$$gh(\bar{S}) = +2 \hfill (5.14)$$

and consequently we can not identify $\bar{S}$ with the part of the quantum action but rather with an anti-BRST generator.

Finally we want to remark that our results do not coincide neither with the ones found by Gomis and Roca [26] nor with the one found out by Perry and Teo [27] but they are related by a canonical transformation in the antifield-antibracket formalism. In the standard theory all the anti-ghosts i.e. $c^{\alpha_0|2} = \bar{c}^{\alpha_0}$, $c^{\alpha_1|22} = \bar{c}^{\alpha_1}$, $c^{\alpha_1|12} = \bar{c}^{\alpha_1}$, belong to the contractive part of the BRST free differential algebra. We show now that the action (6.10), (6.11) can be transformed into the standard one via a canonical transformation defined by:

$$\phi^j A = \frac{\partial F_2(\phi, \phi^{*})}{\partial \phi^*_A}; \phi^*_A = \frac{\partial F_2(\phi, \phi^{*})}{\partial \phi^A}; \hfill (5.15)$$

where $\epsilon(F_2) = 1$ and $gh(F_2) = -1$. This canonical transformation preserves the anti-bracket structure (see [29]). Thus in order to define a canonical transformation one should give the function $F_2(\phi, \phi^{*})$.

For the actions (6.10), (6.11) the canonical transformation is generated by:

$$F_2 = \phi^*_j \phi^j + C^{\alpha_0|1|1} C_{\alpha_1|1} + C^{\alpha_0|2} \phi^*_A + B^r_{\alpha_0} s C^{\alpha_0|2} + C^{\alpha_1|11} C_{\alpha_1|11} + C^{\alpha_1|22} C_{\alpha_1|22} C^{\alpha_1|12} C_{\alpha_1|22} + B^r_{\alpha_1} s C^{\alpha_1|12} + B^*_r s C^{\alpha_1|22} \hfill (5.16)$$
The form of the action (6.10) takes a simple form after the canonical transformation:

\[
S = S_0 + \phi_j^* R_{a_0} C^{\alpha_0|1} + C_{\alpha_0|1} \left( Z_{\alpha_1}^{\alpha_0} C^{\alpha_1|1} + \frac{1}{2} C_{\beta_0\gamma_0} C^{\gamma_0|1} C^{\beta_0|1} \right) + \\
+ C_{\alpha_0|2} B_{\alpha_0} + C_{\alpha_1|12} B^{\alpha_1|1} + C_{\alpha_1|22} B^{\alpha_1|2} + \\
C_{\alpha_1|11} \left( A_{\beta_1\alpha_0} C^{\alpha_0|1} + F_{\alpha_0\beta_0\gamma_0} C^{\gamma_0|1} C^{\beta_0|1} C^{\alpha_0|1} \right)
\]

(5.17)

which is the standard form of the action in the BV scheme.

On the other hand, the action \(\bar{S}\) takes a form which is much more complicated. Since it does not play a fundamental role, we will not give it here. The only thing that we want to emphasize is that it coincides with the action \(\bar{S}\) obtained in [26] by using different methods.

6 Conclusions

We started from the results of [21] for irreducible gauge theories and we applied the same ideas for the reducible theories. Our results were similar with those of Batalin-Lavrov-Tyutin for the spectrum of antifields and we obtained the spectrum of fields of Spiridonov directly from cohomology methods and not by introducing auxiliary fields as Lagrangian multiplier. Our method dealt with the cycles by introducing auxiliary variables such that these are to be killed.

We used as an example for our approach the topological field theories and we connected our results with the ones derived in [26], [27].

7 Acknowledgements

Liviu Tătărău would like to thank to W.Kummer from Technische Universität Wien for an extended collaboration.

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