Sideband cooling beyond the quantum backaction limit with squeezed light

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Quantum fluctuations of the electromagnetic vacuum produce measurable physical effects such as Casimir forces and the Lamb shift. They also impose an observable limit—known as the quantum backaction limit—on the lowest temperatures that can be reached using conventional laser cooling techniques. As laser cooling experiments continue to bring massive mechanical systems to unprecedentedly low temperatures, this seemingly fundamental limit is increasingly important in the laboratory. Fortunately, vacuum fluctuations are not immutable and can be ‘squeezed’, reducing amplitude fluctuations at the expense of phase fluctuations. Here we propose and experimentally demonstrate that squeezed light can be used to cool the motion of a macroscopic mechanical object below the quantum backaction limit. We first cool a microwave cavity optomechanical system using a coherent state of light to within 15% of this limit. We then cool the system to more than two decibels below the quantum backaction limit using a squeezed microwave field generated by a Josephson parametric amplifier. From heterodyne spectroscopy of the mechanical mode, we measure a minimum thermal occupancy of 0.19 ± 0.01 phonons. With our technique, even low-frequency mechanical oscillators can in principle be cooled arbitrarily close to the motional ground state, enabling the exploration of quantum physics in larger, more massive systems.

Progress in the control and measurement of massive mechanical oscillators has enabled tests of fundamental physics, and applications in sensing and quantum information processing. However, the performance of these experiments is often limited by thermal motion of the mechanical mode. Although the most sophisticated refrigeration technologies can be sufficient for cooling high-frequency mechanical structures to the ground state, observing quantum behaviour in lower-frequency mechanical systems requires other cooling methods. Recent efforts using active quantum feedback have been successful in preparing motional states with low entropies. So far, however, only laser cooling techniques similar to those that revolutionized the coherent control of atomic systems have yielded thermal uncertainties below one quantum. Nevertheless, vacuum fluctuations impose a lowest possible temperature that can be achieved using these techniques. This limit is now being encountered in state-of-the-art experiments involving macroscopic oscillators.

The concept of sideband cooling relies on the removal of mechanical energy by scattering incident drive photons to higher frequencies. In general, however, this photon up-conversion (anti-Stokes) process competes with a down-conversion (Stokes) process that adds energy to the mechanical system. In cavity optomechanics, a light–matter interaction arises owing to a parametric modulation of the resonance frequency of an optical cavity with the position of a mechanical oscillator. When the cavity is driven at detuning below its resonance frequency, the difference in the density of states of the cavity at the mechanical sideband frequencies leads to a dominant anti-Stokes scattering rate (see Fig. 1). The disparity between the Stokes and anti-Stokes scattering rates grows as the linewidth of the cavity decreases relative to the mechanical resonance frequency of the oscillator. Accordingly, optomechanical systems in the ‘resolved sideband’ limit can be cooled to low temperatures with coherent states of light. A full quantum analysis, however, shows that vacuum fluctuations always stimulate some degree of Stokes scattering, which prevents true ground-state cooling.

Although the sideband cooling limit rests on an intuitive set of assumptions, it is not a fundamental limit. Proposals to cool below this limit include pulsed cooling schemes, dissipative coupling, optomechanically induced transparency and nonlinear interactions. For atomic laser cooling, it has been proposed that squeezed light can yield an advantage over laser cooling with coherent states. Here we propose and implement an analogous quantum-enhanced cooling scheme for cavity optomechanical systems. Using the Heisenberg–Langevin equations, we show that driving an optomechanical cavity with a pure squeezed state of light can coherently null the Stokes scattering process, theoretically eliminating the quantum backaction limit from sideband cooling. We then implement this squeezing-enhanced cooling method in a microwave cavity optomechanical system to achieve cooling below the traditional quantum backaction limit.

The Hamiltonian that governs the interaction between cavity mode and mechanical mode is $\hat{H}_m = -\hbar g (\hat{a}^\dagger \hat{a}) (\hat{b}^\dagger + \hat{b})$, where $\hat{a}$ (annihilates) a photon and $\hat{b}$ (creates) a phonon. Here, $g$ is the parametrically enhanced optomechanical coupling rate, which is proportional to the amplitude of the drive field, $\hat{b}_0$, and $\hbar$ is the reduced Planck constant. When driving the system with a pure coherent state, the Stokes and anti-Stokes scattering rates are proportional to $\Gamma_+ (n_{m} + 1)$ and to $\Gamma_-, n_{m}$, respectively, in the weak coupling limit ($g \ll \kappa$). Here, $n_{m}$ is the equilibrium phonon occupancy of the mechanical mode and

$$\Gamma_\pm = \frac{4g^2 \kappa}{\kappa^2 + 4(\Omega \pm \Delta)^2} \tag{1}$$

As $g$ increases, the products $\Gamma_+ (n_{m} + 1)$ and $\Gamma_- n_{m}$ tend towards equilibrium, which removes the cooling power of the scattering processes. Accordingly, the minimum possible phonon occupancy $n^0_m$ should satisfy the condition $\Gamma_+ n^0_m = \Gamma_- (n^0_m + 1)$, or

$$n^0_m = \frac{\Gamma_-}{\Gamma_+ - \Gamma_-} = \frac{\kappa^2 + 4(\Delta + \Omega)^2}{16 \Delta^2} \tag{2}$$

The expression for $n^0_m$ is minimized at the optimal drive detuning $\Delta_0 = -\frac{1}{2} \sqrt{\kappa^2 + 4 \Omega^2}$. Even at this ideal detuning, however, $n^0_m$ does not vanish.

It is convenient to describe the cooling process by modelling the mechanical mode as being coupled to two thermal reservoirs held at different temperatures. One of these reservoirs is provided by the surrounding thermal environment, which results in an initial thermal phonon occupancy $n^t_m$. The coupling rate to this environment is given

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The critical squeezing strength depends solely on the ratio $\kappa/\Omega$ and is deeply related to the `ponderomotive squeezing' produced by off-resonantly driving the cavity with a coherent state (see Methods). This critical behaviour for the cooling is in stark contrast to that observed when driving the cavity at $\Delta = 0$, where full suppression of radiation pressure shot noise can only be asymptotically approached with infinite levels of squeezing.23,24

An optomechanical cavity driven with a pure, critically squeezed state at the traditional optimal detuning $\Delta_0$ yields $n_m^{\text{bath}} = 0$. Note that both $\Delta_0$ and $r_c$ are completely determined by $\kappa$ and $\Omega$. For more strongly squeezed states ($r > r_c$), there are two optimal detunings $\Delta_0^{(r)} = -\Omega \cosh(2r) \pm \frac{1}{2} \sqrt{4\kappa^2 \sinh^2(2r) - \kappa^2}$. At both of these detunings, $n_m^{\text{bath}} = 0$. Figure 2 illustrates this behaviour by mapping $n_m^{\text{bath}}$ as a function of $\kappa/\Omega$ and $\Delta/\Omega$ for several values of the squeezing. The plots show how the use of squeezed light pushes colder mechanical bath temperatures into the `bad cavity' limit ($\kappa \gg \Omega$), in which traditional sideband cooling with coherent states is less effective.

Realistic laboratory conditions place practical bounds on the strength and the purity of the squeezing that can be generated to achieve a cooling enhancement. To address these limitations, we exploit the freedom to model any impure Gaussian state as a thermal state subject to an ideal entropy-preserving squeezing operation.25 In this case, the product of the major and minor axes of the noise ellipse of the light field obeys $\langle (\Delta X)^2 (\Delta Y)^2 \rangle = (1 + 2n)^2/16$. The purity of the squeezing is parameterized by $n_0$, which denotes the effective thermal occupancy of the light field before the squeezing operation. We emphasize that any type of impurity introduced to the light field (for example, by loss or parasitic nonlinear processes) will be treated using this model (see Methods).

In the presence of impure squeezing, equation (4) generalizes to

$$n_m^{\text{bath,eff}} = n_m^{\text{bath}} + n_1 (I_2 + I_2^*) \cosh(2r) - 2 \sqrt{I_1 I_2^*} \sinh(2r) \Gamma_{opt}$$

An examination of equation (6) reveals that the critical squeezing strength $r_c$ and the optimal drive detuning(s) $\Delta_0^{(r)}$ remain unaltered when $n_1 > 0$. However, the impurity of the squeezing limits the lowest achievable mechanical bath temperature to $n_m^{\text{bath,eff}} \geq n_1$. This inequality saturates to $n_m^{\text{bath,eff}} = n_1$ provided that `strongly' squeezed light fields ($r \geq r_c$) drive the system at the optimal detunings. Thus, even impure squeezed states of light can be used to remove the quantum backaction limit from the cooling physics entirely.

To experimentally demonstrate a cooling enhancement, we drive a microwave cavity optomechanical system with a squeezed state generated by a Josephson parametric amplifier (JPA).24 The microwave cavity consists of a vacuum-gap parallel plate capacitor26 shunted by a 15-nH spiral inductor, yielding a cavity resonance frequency of $\omega_c = 2\pi \times 6.4$ GHz. Our experiments are performed in a dilution cryostat held at a temperature of $T = 37$ mK, resulting in an initial mechanical occupancy of approximately 75 phonons. For all experiments, the cavity is driven at the traditional optimal detuning $\Delta_0$. Furthermore, we operate in the limit of strong damping, in which $\Gamma_{opt} = 2\pi \times 36$ kHz exceeds the thermal decoherence rate $\Gamma_m^{\text{bath}}$ by a factor of approximately 30. As such, the mechanical temperature is primarily determined by the quantum state of the drive field and not by the thermal environment. The linewidth of the cavity ($\kappa = 2\pi \times 27$ MHz) is comparable to twice the resonance frequency of the primary flexural mode of the aluminium membrane ($\Omega = 2\pi \times 10.1$ MHz). However, the internal loss rate of the cavity ($\kappa_0 < 2\pi \times 100$ kHz) is small enough relative to $\kappa$ that the effects of internal cavity loss can be neglected (see Methods).

Figure 3a displays measured power spectral densities of the two mechanical sidebands when the system is driven at various squeezing...
The upper-left map represents the theory for traditional sideband cooling with a pure coherent state (0 dB). The other maps show the bath temperatures established by −2.5 dB, −4.8 dB and −6 dB of pure squeezing. The squeezing phase is optimized to \( \theta = \theta_0 \) at all points in phase space to consider the lowest possible bath temperatures. Taken together, the maps illustrate how the squeezing required to achieve \( n_{\text{bath}}^\text{th} = 0 \) increases with \( \kappa/\Omega \).

Figure 3 | Experimentally measured power spectral densities of the upper and lower mechanical sidebands. The power spectral densities are normalized to the shot-noise limit. The spectra were measured using various squeezed states to drive the optomechanical cavity at detuning \( \Delta = \Delta_0 \). The shaded regions show the theoretically predicted behaviour.

As the squeezing phase passes through \( \theta_0 \) (or \( \theta_0 - \pi \)), however, the sidebands exhibit Lorentzian line shapes. These two cases yield extrema in the mechanical mode temperature, which are of primary interest. Correctly inferring the mechanical temperature from the heterodyne spectra can be subtle. Although the optimum squeezing phase yields Lorentzian mechanical sidebands, an interference effect persists that distorts the results of standard sideband thermometry. For the case of uncorrelated drive noise, this effect has been well-characterized experimentally. The effects of correlated noise are more complicated, but can be handled by introducing effective phonon occupancies for the upper sideband \( n_{\text{eff}}^\text{up} \) and for the lower sideband \( n_{\text{eff}}^\text{lo} \). Relative to the noise floor, the Stokes scattering rate scales as \( \Gamma_0 (n_{\text{eff}}^\text{up} + 1) \), whereas

\[ \Delta \Omega \]

\[ n_{\text{bath}}^\text{th} = 0 \]

\[ r/c = 0.55 \]
we then increase the squeezing parameter within 15% of the quantum limit, bands to measure the thermal occupancy of the mechanical mode. In a man-

squeezed state. Figure 3b illustrates this effect as mechanical sidebands dip below the heterodyne noise floor. In a man-

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In other words, a critically squeezed state nulls both effective phonon occupancies reduce to

\[ n_{\text{eff}} = n_m \mp n_l \cos(2\theta) \pm \sinh(\theta) \mp \frac{1}{2} \sin(2\theta) \]  

(7)

Therefore, extracting \( n_m \) from the detected sidebands amounts to subtracting these interference terms from the experimentally measured values \( n_{\text{eff}} \).

Equation (7) contains an important signature of the onset of critical squeezing in the strong damping limit (\( F_{\text{opt}} \gg F_{\text{th}} \)). For any \( n_l \geq 0 \), the effective phonon occupancies reduce to \( n_{\text{eff}} = 0 \) and \( n_{\text{eff}} = -1 \) when \( r = r_c \) and \( \Delta = \Delta_0 \). In other words, a critically squeezed state nulls both mechanical sidebands to the noise floor, regardless of the purity of the squeezed state. Figure 3b illustrates this effect as \( r \) is increased through \( r_c = 0.55 \). When \( r > r_c \) (and the detuning is held to \( \Delta = \Delta_0 \)), the mechanical sidebands dip below the heterodyne noise floor. In a manner analogous to active feedback cooling, these dips signal the onset of mechanical heating.

In Fig. 4, we use heterodyne spectroscopy of both mechanical sidebands to measure the thermal occupancy of the mechanical mode. Starting first with a coherent state, we cool the mechanical mode to within 15% of the quantum limit, \( n_{\text{eff}} = 0.33 \). Without changing \( F_{\text{opt}} \), we then increase the squeezing parameter \( r \) by increasing the pump power to the JPA (while holding \( \theta = \theta_0 \)). As shown in Fig. 4b, the heights of the measured sidebands remain nearly identical for any value of \( r \). Using equation (7), we extract \( n_m \) from the sidebands as a function of the squeezing strength. These results are then compared against theory, which is represented by the grey zone in Fig. 4a. As expected, our results show that the cooling enhancement improves with the strength of the correlations for small values of the squeezing parameter\( (r < 0.25) \). As \( r \) is increased, however, the limited microwave transmit-

\[ n_m \sim 0.19 \pm 0.01 \text{ at } r = 0.3 \]  

Our data demonstrate that squeezed light can be used to cool a cavity optomechanical system below the quantum backaction limit. Analytical expressions for the mechanical mode temperature indicate that the primary cooling bottleneck is set by the purity of the applied squeezing. A natural extension of this work would be to parametrically swap an itinerant squeezed light field into the mechanical state, which could result in mechanically squeezed states of unprecedented strength. Looking forward, the generic technique that we have demonstrated could immediately improve the cooling of any cavity optomechanical system. State-of-the-art optical squeezing of approximately 15 dB (ref. 30) is sufficiently pure to cool a cavity optomechanical system with \( r/\Omega > 60 \) to less than one quantum. Likewise, this technique will allow systems that are already in the resolved sideband regime to achieve laser cooling to unprecedented temperatures.

Online Content Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

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Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to J.D.T. (john.teufel@nist.gov).

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METHODS

System parameters and measurement set-up. Extended Data Table 1 summarizes the experimentally determined parameters of our device. Extended Data Fig. 1 depicts the thermalization stages of the dilution cryostat used to perform the experiments. The room-temperature microwave set-up (not pictured) was identical to that described in ref. 24, with only the heterodyne detection pathways selected. We note an increase in the cold-stage attenuation of the microwave drive lines to the Josephson parametric amplifier (JPA) and to the optomechanical circuit. This change was intended to minimize any observable thermal noise on the microwave field driving the optomechanical circuit, which we estimate to be less than 0.05 thermal photons. The initial thermal occupancy of the mechanical mode (approximately 75 phonons) agrees with the measured base temperature ($T = 37$ mK) of the dilution cryostat.

Measurement of $\rho_m$. Here we describe the procedure used to retrieve the equilibrium phonon occupancy $\rho_m$ from the measured mechanical sidebands. To do so, it can be helpful to draw a direct comparison to the more familiar case of driving an optomechanical cavity with a pure coherent state of light. For the case of a coherent-state drive, the spectral densities of the upper mechanical sideband (USB) and lower mechanical sideband (LSB) take the form (normalized to the shot-noise floor of the light field)

$$S_{\text{USB}}(\omega) = 1 + \Gamma_s \Gamma_t \omega n_m$$

$$S_{\text{LSB}}(\omega) = 1 + \Gamma_s \Gamma_t \omega (n_m + 1)$$

where $\Gamma_s$ and $\Gamma_t$ are defined in Equation (1) and $\Gamma_s = \Gamma + \Gamma_s - \Gamma_t$. The mechanical susceptibility $\chi_m(\omega)$ gives the spectrum its Lorentzian line shape:

$$|\chi_m(\omega)|^2 = \frac{4}{\Gamma_s^2 + 4(\omega - \Omega)^2}$$

Here, $\Omega$ denotes the mechanical resonance frequency in the presence of the 'optical spring' ($a < 1\%$ correction in our experiments). From equations (8) and (9), it is clear that the areas of the sidebands scale linearly with the final phonon occupancy of the mechanical mode $n_m^\text{eff}$. When the cavity is driven with squeezed light, the power spectral densities of the sidebands generally display 'Fano' line shapes (see Fig. 3a), with the exact profiles depending on the phase of the applied squeezing $\theta$. However, at two important squeezing phases, $\theta = \theta_0$ and $\theta = \theta_0 - \pi$, these sidebands once again exhibit Lorentzian line shapes. Again, $\theta_0$ denotes the squeezing phase that yields optimal sideband cooling:

$$\theta_0 = \tan^{-1}\left[\frac{-4\Delta\kappa}{\kappa^2 + 4(\Omega^2 - 4\Gamma^2)}\right]$$

In contrast, the squeezing phase $\theta = \theta_0 - \pi$ leads to maximal heating of the mechanical mode. At these critical squeezing phases, the areas of the Lorentzians no longer depend on $n_m$ as in equations (8) and (9).

To see how the mechanical occupancy is extracted at these squeezing phases, we first consider the simplest case of driving a strongly overcoupled cavity with a pure squeezed state of light. When $\theta = \theta_0$, for example, equation (8) becomes

$$S_{\text{USB}}(\omega) = 1 + \sinh^2(\theta) + \Gamma_s \Gamma_t |\chi_m(\omega)|^2 \times [n_m - \sinh^2(\theta) + \frac{\sinh^2(\theta)}{\sinh^2(\theta) + \frac{(\omega - \Omega)^2}{\Gamma_s^2 + 4(\omega - \Omega)^2}}]$$

$$S_{\text{LSB}}(\omega) = 1 + \sinh^2(\theta) + \Gamma_s \Gamma_t |\chi_m(\omega)|^2 [n_m - \sinh^2(\theta) + \frac{\sinh^2(\theta)}{\sinh^2(\theta) + \frac{(\omega - \Omega)^2}{\Gamma_s^2 + 4(\omega - \Omega)^2}}]$$

A similar expression follows for the lower sideband (see below). Equation (13) reveals that $n_m^\text{eff}$ no longer directly weights the heights of the Lorentzians in equations (8) and (9). Instead, the heights of the Lorentzian mechanical sidebands can be expressed as

$$S_{\text{USB}}(\omega) = 1 + \sinh^2(\theta) + \Gamma_s \Gamma_t |\chi_m(\omega)|^2 n_{\text{eff}}$$

$$S_{\text{LSB}}(\omega) = 1 + \sinh^2(\theta) + \Gamma_s \Gamma_t |\chi_m(\omega)|^2 (n_{\text{eff}} + 1)$$

where $n_{\text{eff}}^\text{op}$ represent the effective thermal occupancies of the mechanical mode retrieved from the upper (+) and lower (−) sidebands. Because $n_{\text{eff}}^\text{op} = n_{\text{eff}}$ and $n_{\text{eff}}^\text{op} = n_m$, standard coherent-state sideband thermometry techniques are not appropriate.

In the presence of an arbitrarily impure state of squeezed light, $n_{\text{eff}}^\text{op}$ are related to the actual mechanical occupancy $n_m$ according to

$$n_{\text{eff}}^\text{op} = n_m - n_e \cosh(2\theta) - \sinh^2(\theta) + \sinh(2\theta) + \frac{\Gamma_s \Gamma_t |\chi_m(\omega)|^2}{\Gamma_s^2 + 4(\omega - \Omega)^2}$$

$$n_{\text{eff}}^\text{op} = n_m + n_e \cosh(2\theta) + \sinh^2(\theta) + \sinh(2\theta) + \frac{\Gamma_s \Gamma_t |\chi_m(\omega)|^2}{\Gamma_s^2 + 4(\omega - \Omega)^2}$$

where the top (bottom) sign corresponds to the case where $\theta = \theta_0 (\theta = \theta_0 - \pi)$. Here, $r$ denotes the squeezing parameter for an ideal, unitary squeezing process applied to an initial thermal state of occupancy $n_\text{th} = \text{tr}(\rho^\text{th} |\alpha| |\alpha|^\dagger)$. As emphasized in the main text, we use these parameters to describe any impure Gaussian state of the light field driving the system. However, equations (16) and (17) assume a strongly overcoupled cavity as is appropriate for our experimental parameters (the effects of cavity loss are considered in Methods).

To use equations (16) and (17) to extract $n_m$, we first retrieve system parameters $g$, $\kappa$, and $\Delta$ by interrogating the optomechanical system with an unsqueezed microwave field using standard techniques. With these parameters determined, we vary the state of the squeezing by changing the amplitude and phase of the pump driving the JPA (see Extended Data Fig. 1). Equations (16) and (17) make it clear that $n_{\text{eff}}^\text{op}$ vary as a function of $\kappa$, $\Omega$, $\Delta$, and $\theta$, which are known. They also vary as a function of $r$ and $n_\text{th}$, which we treat as unknown quantities.

The unknown parameters $r$ and $n_\text{th}$ are uniquely determined by two measurements of $n_{\text{eff}}^\text{op}$ (or of $n_{\text{eff}}^\text{op}$) at the two squeezing phases of interest, $\theta = \theta_0$ and $\theta = \theta_0 - \pi$. Because $r$ and $n_\text{th}$ also specify $n_m$ (in conjunction with the known parameters $g$, $\kappa$, $\Omega$, $\Delta$, and $\theta$), we are able to retrieve the mechanical temperature without presupposing the state of the squeezing. In other words, the measurements of each sideband reveal both the state of the squeezing driving the mechanical mode and the phonon occupancy of the mechanical mode. These measurements were repeated for the upper and lower sidebands at each value of $r$ (Fig. 4), yielding two independently retrieved values of $n_m$ (one per sideband). Independent measurements of the squeezing using homodyne tomography were used to compute the grey (theory) zone in Fig. 4.

Theory of sideband cooling with squeezed light. In this section we augment important theoretical ideas that were not discussed at length in the main text. In particular, we develop an intuition for the existence of a critical squeezing strength required for ground state cooling. Additionally, we discuss several effects that limit the cooling enhancement provided by the squeezing. These results can be straightforwardly derived using the steady-state solutions of the Heisenberg–Langevin equations. We therefore begin by specifying the assumptions used to solve those equations.

The Hamiltonian that describes the coupling of the cavity mode $\hat{a}$ to the mechanical mode $\hat{b}$ is given by

$$\hat{H}_{\text{int}} = - \hbar g (\hat{b}^\dagger \hat{a} + \hat{a} \hat{b}^\dagger)$$

where $g_\text{op}$ represents the vacuum optomechanical coupling rate. As expressed in the main text, however, we take the common approach of working in a linearized regime in which a coherent build-up of cavity photons $(\hat{a}^\dagger \hat{a}) = \kappa^2 \theta$ (that is, the 'stick' in the 'ball and stick' picture of a Gaussian state) enhances the coupling rate according to $\theta = g_\text{op} \kappa$. The cavity and mechanical field quadrature fluctuations then couple together according to

$$\hat{H}_{\text{int}} = - \hbar g (\hat{b}^\dagger \hat{a} + \hat{a} \hat{b}^\dagger)$$

Under the linearized coupling assumption, the evolution of the fields is described by coupled Heisenberg–Langevin equations:

$$\dot{\hat{a}} = - \left(\frac{1}{2} - i \Delta \right) \hat{a} + ig_\text{th} (\hat{b}^\dagger + \hat{b}) + \sqrt{\kappa_\text{ext}^\text{th}} \sin {\theta} + \sqrt{\kappa_\text{int}^\text{th}} \cos {\theta}$$

$$\dot{\hat{b}} = - \left(\frac{1}{2} + i \Delta \right) (\hat{b}^\dagger + \hat{b}) + ig_\text{th} (\hat{a}^\dagger + \hat{a}) + \sqrt{\kappa_\text{ext}^\text{th}} \cos {\theta} + \sqrt{\kappa_\text{int}^\text{th}} \sin {\theta}$$

where $\kappa_\text{ext}^\text{th}$ and $\kappa_\text{int}^\text{th}$ represent environmental noise operators for modes $\hat{a}$ and $\hat{b}$. These noise operators couple to the cavity and mechanical fields at rates $\kappa_\text{ext}^\text{th}$ and $\kappa_\text{int}^\text{th}$, respectively. $\kappa_\text{ext}^\text{th}$ denotes the noise environment set by the squeezing. $\kappa_\text{int}^\text{th}$ parameterizes the coupling rate to the feed line carrying the squeezed field. The internal dissipation of the cavity is modeled by introducing an 'internal' noise operator $\xi_\text{ext}$, which is assumed to be at zero temperature. The mechanical bath operators...
\[ \hat{a} \text{ are assumed to satisfy the characteristics of a thermal state with average thermal occupancy } n_0 = \text{tr}(\rho_0 \hat{b} \hat{b}^\dagger). \] 
All baths are assumed to be Markovian (that is, memoryless). Finally, the input–output relation

\[
\hat{a}_{\text{ext-out}} = -\hat{a}_{\text{ext}} + \sqrt{\Gamma_{\text{ext}} \delta}
\]

is used to evaluate the heterodyne spectrum of the reflected drive field \( \hat{a}_{\text{ext-out}} \).

From the solutions to these coupled equations, we derive the expressions for the final mechanical occupancy and power spectral densities presented in the text. Several additional assumptions were imposed to reach the exact form of these expressions. For example, we assume operation in the weak coupling limit, in which \( g \ll \kappa \). To see how the weak coupling assumption affects these expressions, we describe the optomechanical interaction in terms of cavity and mechanical susceptibilities, \( \chi_c \) and \( \chi_m \):

\[
\chi_c^{-1}(\omega) = \kappa / 2 - i(\omega + \Delta) \\
\chi_m^{-1}(\omega) = \Gamma / 2 - i(\omega - \Omega)
\]

In the weak coupling limit, the following approximation can be made:\(^{31}\):

\[
\chi_m(\kappa) = 1 + g^2 (\chi_c(\omega) - \chi_c(-\omega)) \chi_m(\omega) \approx \left[ \frac{\Gamma_{\text{ext}}}{2} - i(\omega - \Omega_{\text{det}}) \right]^{-1}
\]

Under this approximation, the right-hand side of equation (25) represents the ‘dressed’ mechanical susceptibility^*, which includes optical damping and optical spring effects. For our experimental parameters, the optical spring amounted to a <1% correction to the bare mechanical resonance frequency \( \Omega \). Nonetheless, all references to the mechanical resonance frequency \( \Omega \) strictly refer to \( \Omega_{\text{det}} \), where

\[
\Omega_{\text{det}} = \Omega + g^2 \left[ \frac{\Delta - \Omega}{\kappa/4 + (\Delta - \Omega)^2} + \frac{\Delta + \Omega}{\kappa/4 + (\Delta + \Omega)^2} \right]
\]

Owing to the comparatively high quality factor of the mechanical mode \( (Q_m \approx 6.7 \times 10^5) \), we imposed two additional assumptions to arrive at the exact form of the equations presented here. First, we assumed that the mechanical mode was narrow enough (even when being optically damped) that \( \chi_c(\omega) \) could be treated as being constant over a bandwidth given by \( \Gamma_m \). Additionally, we imposed a mechanical rotating-wave approximation, which amounts to assuming that the mechanical quality factor \( Q_m \) obeys

\[
Q_m \gg 1/16
\]

However, a cavity rotating-wave approximation, which assumes that

\[
\left( \frac{4\Omega^2}{\kappa} \right) \gg 1
\]

was not imposed (for our experimental parameters, \( 4(\Omega/\kappa)^2 \approx 2 \)). Indeed, the resolved sideband cooling limit itself arises from considering the effect of the counter-rotating cavity terms.

**Connection with ponderomotive squeezing.** One of the most counterintuitive aspects of the cooling enhancement is the requirement of a minimum level of applied squeezing to achieve ‘true’ ground state cooling (that is, a complete suppression of the Stokes scattering rate). In this section, we show that this critical level of squeezing is closely related to the minimum level of ‘ponderomotive’ squeezing\(^{32-36}\) that can be generated by driving the optomechanical cavity with a coherent state in the strong damping limit \( (\Gamma_m \gg \Gamma_{\text{ext}}) \). To sketch this picture, we derive the limit of the ponderomotive squeezing that can be achieved for off-resonant coherent state drives. The critical squeezing parameter \( \kappa_r \) and two detuning solutions \( \Delta_0 \) can be derived from this limit. The discussion will conclude with a calculation of the remaining ponderomotive squeezing after the optomechanical system is seeded with a squeezed state of arbitrary strength \( r \).

The interaction between an applied coherent state and an optomechanical cavity squeezes the reflected light field near the mechanical resonance frequency \( \Omega \) (which we assume includes ‘optical springing’ effects). Typically, such ponderomotive squeezing is generated using a resonant coherent drive tone (\( \Delta = 0 \)) because, in this case, the resultant squeezing scales with the applied drive power. Nevertheless, ponderomotive squeezing persists even in the case where the drive field is tuned below cavity resonance (that is, when \( \Delta < 0 \)). In this case, however, the squeezing does not scale arbitrarily high with pump power. Instead, at any red drive detuning \( \Delta < 0 \), the squeezing parameter of the state \( \rho_{\text{OM}} \) saturates to

\[
\sinh(r_{\text{OM}}) = 2 \left( \frac{\Gamma_{\text{OM}}}{\Gamma_m} \right)^{\frac{1}{2}}
\]

where \( \Gamma_m \) are defined in equation (1). Equation (29) assumes negligible thermal mechanical noise, such that \( \Gamma_m \gg \Gamma_{\text{out}} \). At zero temperature, equation (29) also holds under the weaker condition that \( \Gamma_m \gg \Gamma \).

The expression for \( r_{\text{OM}} \) in equation (29) confirms that the squeezing parameter does not depend on the applied pump strength (which is parameterized by the linearized coupling rate \( g \)). Indeed, equation (29) can also be expressed in terms of the coupling-independent resolved sideband cooling limit \( n_{\text{res}}^2 \):

\[
\sinh(r_{\text{OM}}) = 2 \sqrt{n_{\text{res}}^2 (n_{\text{res}}^2 + 1)}
\]

Equation (30) shows that the ponderomotive squeezing increases under conditions where resolved sideband cooling is less effective. This observation explains the need for stronger levels of injected squeezing for ground state cooling of systems in the ‘bad cavity limit’ (\( \kappa/\Omega \gg 1 \)). It also explains why more strongly squeezed states are needed for drive detunings away from the optimal coherent-state drive detuning

\[
\Delta_0 = -\frac{1}{2} \sqrt{\kappa^2 + 4\Omega^2}
\]

To gain further intuition for equations (29) and (30), it is useful to define the normalized drive detuning

\[
\tilde{\Delta} = \Delta / \Delta_0
\]

In terms of \( \tilde{\Delta}, \rho_{\text{OM}} \) is described by

\[
\sinh(r_{\text{OM}}) = \frac{1}{2} \sqrt{\left( \frac{\tilde{\Delta}}{1 - \frac{\tilde{\Delta}}{\Delta_0}} \right) \left( \frac{\kappa}{2\delta} \right)^2 + \left( 1 - \frac{\tilde{\Delta}}{\Delta_0} \right)^2}
\]

Equation (33) reveals several results:

\[
r_{\text{OM}}(\tilde{\Delta} = 0) = 2\kappa_c
\]

\[
r_{\text{OM}}(\tilde{\Delta} = 0) \geq 2\kappa_c
\]

\[
r_{\text{OM}}(\Delta) = \text{tanh}(\Delta^{-1})
\]

Equations (34) and (35) show that the critical squeezing parameter \( r_c \) emerges directly from considering the ponderomotive squeezing without any consideration of ground state cooling. The symmetry of the squeezing with respect to \( \Delta \) (equation (36)) accounts for the behaviour displayed in Extended Data Fig. 2. The next section will help to elucidate why this symmetry also accounts for the symmetric splitting of the optimal detunings predicted in Fig. 2a.

As discussed in the main text, squeezing the lower (Stokes) mechanical sideband opens up the possibility of true ground state cooling. To show when this condition is met, we calculate the expected power spectral density of an ideal homodyne detection of the squeezed drive field after its interaction with a cavity optomechanical system. The calculation confirms that the power spectral density at the mechanical resonance frequency flattens when the system is driven with a critically squeezed state.

When driving an optomechanical system with a squeezed state at the optimum squeezing phase \( \theta = \theta_0 \) (equation (11)), the quadrature amplification of the incident light field driving the optomechanical cavity is completely out of phase with the amplification provided by the JPA; Extended Data Fig. 3 illustrates this behaviour. For example, a weakly squeezed \( (r < r_c) \) light field driving the optomechanical system is expected to yield a net reduction in the level of ponderomotive squeezing near \( \kappa = \Omega \). By increasing the level of injected squeezing (that is, squeezing from the JPA) to the point at which \( r = 2\kappa_c \), the ponderomotive squeezing at the mechanical sideband is re-amplified by the optomechanical system back to the shot-noise limit. Stronger injected fields \( (r > 2\kappa_c) \) become further amplified above the shot-noise limit near \( \kappa = \Omega \).

In addition to reversing the deamplification of the optomechanical system, the injection of squeezed light also suppresses the noise floor away from the mechanical sideband frequency. When \( r = r_{\text{OM}} \Omega = r_c \), this suppression of the noise floor exactly meets the rising noise level at \( \kappa = \Omega \), which flattens the power spectral density of the reflected drive field. At this critical point, the squeezed field scatters from the cavity as if it was free of any optomechanical interaction (including a frequency-dependent rotation of the squeezing phase).
When an optomechanical cavity produces strong ponderomotive squeezing, it is necessary to inject a correspondingly strong level of externally generated squeezing (in our case, from the JPA) to suppress the mechanical sidebands. It follows from equation (30) that sideband cooling optomechanical devices in the bad cavity limit (in which $\kappa_n^0$ is large) also require stronger levels of injected squeezing. This observation accounts for the behaviour exhibited in Fig. 2, which shows that squeezing pushes colder temperatures into the bad cavity limit. Similarly, drive detunings $\Delta = \Delta_0$ that produce stronger levels of ponderomotive squeezing require more strongly squeezed fields ($r > \chi$) to cool the mechanical mode to the ground state. This explains the bifurcation behaviour of the optimal drive detunings (see Fig. 2).

Indeed, solving equation (33) for the detuning roots reproduces the detuning solutions $\Delta^2 = -\Omega (cosh (2 \alpha) \pm 1) (4 \Omega^2 sinh^2(2 \alpha) - \chi^2)$, provided that the identification $r = \pi \alpha / 2$ is made.

**Limits of sideband cooling with squeezed light.** In this section we consider three important effects that limit the cooling enhancement provided by the squeezing. They are presented in order of practical importance with respect to our experimental parameters.

In our experiments, the impurity of the squeezed state driving the optomechanical system primarily limited the cooling enhancement. Using conventional homodyne tomography and methods of optomechanical quantum nondemolition measurements of the light\(^4\), we experimentally verify that the impurity can be largely explained by microwave loss in the critical path between the JPA and the optomechanical circuit (Extended Data Fig. 4a). Modelling the variance of the amplitude quadrature of the squeezed field $\theta$ in in $\hat{X} = \frac{1}{2} (\alpha + \alpha^*)$ as a pure squeezed state subject to such loss yields

$$\langle (\Delta X)^2 \rangle = \frac{1}{4} \left( 1 - \eta_m + \eta_m^4 (cosh(2 \eta_m) - cos(\theta) sinh(2 \eta_m)) \right)$$

(37)

Here, $\theta$ denotes the squeezing phase, $\eta_m$ is the squeezing parameter for the pure squeezed state being injected into the critical path and $\eta_m$ is the critical path microwave transmittance ($\eta_m \approx 57\% \pm 2\%$). Extended Data Fig. 4b plots the variance as the squeezing phase $\theta$ is rotated for several different values of $\eta_m$.

Modelling the squeezing in this way can be convenient because only a single parameter $\eta_m$ is needed to specify a squeezed state subject to a given level of loss. However, for other reasons it can be more convenient to use the model described in the main text, where the state is specified by an effective thermal photon occupancy $n_b$ and an ideal squeezing parameter $r$. This approach greatly simplifies the expression for the full bath temperature $\eta_m^0$, in the weak coupling limit (equation (6)). Additionally, the loss model empirically shows poor agreement with our experimental data at higher inferred values of $\eta_m$. In this regime, the squeezed state entering the critical path can no longer be assumed to be pure. Therefore, at high squeezing strengths ($>8$ dB), two parameters are once again required to completely specify the quantum state of the drive field in our experiments.

After a thermal state of occupancy $n_b$ is subject to an ideal squeezing operation, the amplitude quadrature variance obeys

$$\langle (\Delta X)^2 \rangle = \frac{1}{4} \left( 1 - \eta_m + \eta_m^4 (cosh(2 \eta_m) - cos(\theta) sinh(2 \eta_m)) \right)$$

(38)

Equation (38) confirms that the squeezing process is entropy-preserving, because the uncertainty product of the maximally squeezed ($\theta = 0$) and antisqueezed ($\theta = \pi$) quadratures is maintained for any value of $r$. A Gaussian state that has been specified by $\eta_m$ and $\eta_m$ can be equivalently specified by $n_b$ and $r$:

$$r = \ln \left( \frac{1 - \eta_m + \eta_m e^{2 \eta_m}}{1 - \eta_m - \eta_m e^{-2 \eta_m}} \right)^{1/4}$$

(39)

$$n_b = \frac{1}{2} \left( \frac{1 - \eta_m + \eta_m e^{-2 \eta_m}}{1 - \eta_m + \eta_m e^{2 \eta_m}} \right)$$

(40)

The behaviour of $n_b$ and $r$ for given values of $\eta_m$ and $\eta_m$ is plotted in Extended Data Fig. 4d, e. In our experiments, we found that the measured squeezing and the resulting cooling enhancement agrees well with the $(n_b, n_m)$ pairs computed by equations (39) and (40) (assuming $\eta_m = 57\%$, provided that $r < 0.5$ (which corresponds to approximately $8$ dB of injected squeezing). As $r$ is increased further, the inferred values of $n_b$ grow appreciably faster than the model predicts, suggesting the introduction of uncorrelated noise from the JPA.

The second theoretical limitation to achieving complete ground state cooling is the onset of effects of strong coupling. When the applied drive power is increased to the point at which the linearized coupling rate $g$ exceeds the mechanical and cavity dissipation rates ($\Gamma$ and $\kappa$, respectively), the interaction enters the strong coupling regime. In this regime, the approximation in equation (25) is no longer valid. The correction to the resolved sideband cooling limit imposed by strong coupling effects (for example, normal mode splitting) has been studied for the case of coherent-state drive fields.\(^3,37–39\). For a coherent state of the drive field, the final phonon occupancy $n_m$ can be expanded in terms of the normalized parameters $\kappa / \Omega$ and $g / \Omega$ at zero temperature to yield\(^3,37\)

$$n_m \approx \frac{\kappa}{\Omega} \! \left( 1 + \frac{1}{2} \frac{g}{\Omega} \right)^2$$

(41)

The first term in equation (41) corresponds to $n_m^0$ (equation (2)) in the resolved sideband limit ($\kappa / \Omega \ll 1$) when $\Delta = \Delta_0$. The $g / \Omega$ dependence can be viewed as a strong coupling effect because it becomes comparable to $|\kappa / (4\Omega^2)|^2$ when $g \approx \kappa$.

Extended Data Fig. 5 generalizes this correction to the full mechanical bath occupancy $n_m^\text{bath,eff}$ resulting from using squeezed drive fields. $n_m^\text{bath,eff}$ is numerically computed as a function of the normalized coupling strength $g / \kappa$, assuming pure squeezed drive fields at the optimal squeezing phase ($\theta_0$) and detuning ($\Delta_0$). A deeply sideband-resolved optomechanical system ($\kappa / \Omega = 0.1$) has been assumed. Extended Data Fig. 5 shows how ($g / \Omega$)\(^2\) continues to bottleneck the cooling. However, for the highest coupling rates in our experiments, this contribution to $n_m^\text{bath,eff}$ falls two orders of magnitude below the heating effect arising from the impurity of the squeezing (see previous section). Even when accounting for the ambient thermal environment in our experiments ($n_m^0 \approx 75$), the ($g / \Omega$)\(^2\) contribution would in principle limit our experiments to $n_m \approx 10^{-2}$ with pure squeezed states. Therefore, it does not meaningfully limit our experimental results.

The third mechanism that will limit the final mechanical occupancy acquired with squeezing-enhanced cooling is internal cavity loss. In our experiments, the optomechanical circuit was strongly overcoupled to the feed line carrying the squeezed microwave field. This ‘external’ coupling rate $\kappa_{\text{ext}} \approx 2 \pi \times 27$ MHz dominated the ‘internal’ cavity loss rate $\kappa_b < 2 \pi \times 100$ kHz by at least two orders of magnitude. Accordingly, microwave loss inside of the cavity did not play an important part in limiting the achievable cooling enhancement. Nevertheless, it is worth considering the effects of internal cavity loss because they become most important when operating in the desirable resolved sideband cooling limit. Moreover, the effect of cavity loss is subtle; it cannot be correctly modelled as a loss channel outside of the cavity.

We now briefly describe how internal cavity loss limits sideband cooling for any Gaussian state of the drive field. To simplify this discussion, we neglect the strong coupling effects that were discussed in the previous section. Additionally, to consider only fundamental limits, we assume access to arbitrarily strong (pure) squeezed states at the input port of the cavity. We also assume that the cavity is held at low enough temperatures that the internal loss process can be treated as coupling to microwave vacuum. Finally, we focus the discussion on the behaviour of the effective mechanical bath temperature in the presence of cavity loss, $\eta_m^\text{bath,eff}$ (as opposed to the final phonon occupancy $n_m$ in equation (3)).

In describing how $n_m^\text{bath,eff}$ is affected by $\kappa_{\text{ext}}$, it is useful to define $\eta = \kappa_{\text{ext}} / \kappa_b$. For any $0 < \eta < 1$,

$$\eta_m^\text{bath,eff}(\kappa) = \eta_m^\text{bath}(\kappa) + (1 - \eta) \eta_m^\text{bath}(\kappa)$$

(42)

where $\kappa = \kappa_{\text{ext}} + \kappa_b$. In equation (4), $\eta_m^\text{bath}(\kappa)$ denotes the resolved sideband cooling limit for coherent states (equation (2)) and $\eta_m^\text{bath}(\kappa)$ represents the mechanical bath temperature in the presence of pure squeezing (equation (4)). It is can be interpreted as the result of coupling the mechanical mode to $n_m^\text{bath}(\kappa)$ and to $\eta_m^\text{bath}(\kappa)$ via a beamsplitter of transmittance $\eta$.

By assuming access to pure and arbitrarily strong squeezing, it is always possible to choose a squeezing parameter $r$ such that $n_m^\text{bath}(\kappa) = 0$. Furthermore, by eliminating the first term in equation (42), it becomes clear that the drive detuning that minimizes $n_m^\text{bath,eff}(\kappa)$ is the same detuning $\Delta_0$ that minimizes $n_m^\text{bath}(\kappa)$. We conclude that

$$n_m^\text{bath,eff}(\kappa) \geq \eta_m^\text{bath}(\kappa) \! \left( 1 + \frac{\kappa_{\text{ext}} + \kappa_b}{2 \Omega} \right)^2 - 1$$

(43)

where the term in brackets gives the $\Delta = \Delta_0$ expression for $n_m^\text{bath}(\kappa)$. Because $n_m^\text{bath,eff}(\kappa)$ decreases as $\kappa_{\text{ext}} \to 0$ (given some internal loss rate $\kappa_b$ being held constant), we conclude that

$$n_m^\text{bath,eff}(\kappa) \geq \eta_m^\text{bath}(\kappa)$$

(44)

Equation (44) conveys that the cooling limit for any Gaussian state is set by the internal loss rate of the cavity.

Despite the limit imposed by $\kappa_{\text{ext}}$, Extended Data Fig. 6 illustrates how pure squeezed light always provides a colder mechanical bath temperature than...
unsqueezed light given $\kappa_{\text{ext}} > 0$. The lower temperature bound given by equation (44) is asymptotically approached in the weakly undercoupled limit in which $\kappa_{\text{ext}} \ll \kappa_0$. On the other hand, in the strongly overcoupled limit ($\kappa_{\text{ext}} \gg \kappa_0$), equation (43) yields a distinctly different behaviour from that of $n_m^b$: 

$$n_m^{\text{bath, eff}}(\kappa) \approx \frac{\kappa_{\text{ext}}}{4\Omega}$$

$$n_m^b(\kappa) \approx \frac{\kappa_{\text{ext}}}{4\Omega}$$

The coherent-state limit $n_m(\kappa)$ diverges as $\kappa_{\text{ext}} \to \infty$, whereas $n_m^{\text{bath, eff}}(\kappa)$ asymptotes to a value given by the internal loss rate. Therefore, given a strongly overcoupled cavity, the cooling advantage conferred by a pure state of squeezed light increases as $\kappa_{\text{ext}}/\kappa_0$ grows. Moreover, in the 'intrinsic' bad cavity limit in which $\kappa_0/(2\Omega) \gg 1$, $n_m^{\text{bath, eff}}(\kappa)$ becomes largely insensitive to changes in $\kappa_{\text{ext}}$.

**Data availability.** The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Extended Data Figure 1 | Cryogenic measurement set-up. LPF, low-pass filter; JPA, Josephson parametric amplifier; OM, optomechanical; HEMT, high-electron-mobility transistor amplifier; μ-wave, microwave. The components shaded in grey signify broadband microwave attenuators. The rectangular components shaded red indicate directional couplers.
Extended Data Figure 2 | Calculated power spectral density of the ponderomotive squeezing and antisqueezing that would be detected by an ideal homodyne receiver. The solid lines indicate ponderomotive squeezing and the dashed lines antisqueezing. SNL denotes the shot-noise limit (vacuum fluctuations). The indicated mechanical resonance frequency $\Omega$ includes optical springing effects. The colour of each curve denotes the resultant power spectral density produced by either of two coherent-state pump detunings satisfying $(\Delta/\Delta_0)^\pm \pm 1 = \text{constant}$. Here, $\Delta_0$ denotes the traditional optimal drive detuning when sideband cooling with coherent states (equation (31)). All curves assume a mechanical quality factor $Q_m = 5 \times 10^5$, a constant (and strong) optical damping rate $\Gamma_{\text{opt}} = \Gamma_+ + \Gamma_- = 103 \Gamma$, a lossless optomechanical cavity and zero temperature. Additionally, a normalized cavity linewidth $\kappa/\Omega \approx 2.85$ has been assumed, which yields $-10 \text{ dB}$ of ponderomotive squeezing when $\Delta = \Delta_0$ (see equation (30)).
Extended Data Figure 3 | Calculated power spectral density (PSD) of the reflected drive field as would be detected by an ideal homodyne receiver. The field quadrature being plotted is that which is maximally squeezed (ponderomotively). SNL denotes the shot-noise limit (vacuum fluctuations). All panels assume the same conditions as for Fig. 2, with the exception of a constant drive detuning ($\Delta = \Delta_0$) and a constant drive cooperativity $C = 4g^2/(\kappa \Gamma) = 5,000$. Each panel assumes a different level of injected squeezing, although the optimal squeezing phase $\theta = \theta_0$ (equation (11)) is assumed throughout. The strength of the injected squeezing is parameterized by $r$, expressed in units of the critical squeezing parameter $r_c$. The critical squeezing for this cavity is $-5$ dB.
Extended Data Figure 4 | Modelling the impurity of the squeezing.

**a.** A pure squeezed state (with squeezing strength parameterized by $r_{in}$) is subject to loss via a beamsplitter of transmittance $\eta_{in}$. Although the loss is to vacuum, the resulting squeezed state is impure and does not obey the minimum uncertainty relation: $\langle (\Delta X)^2 (\Delta Y)^2 \rangle = 1/16$.

**b.** Amplitude variance $\langle (\Delta X)^2 \rangle$ of a pure $-10$ dB squeezed state subject to loss as the squeezing phase $\theta$ is rotated (see inset).

**c.** The Gaussian state in the right-hand panel of a can be equivalently represented with $n_1$ and $r$. The impurity is captured by the uncertainty product $\langle (\Delta X)^2 (\Delta Y)^2 \rangle = (1 + 2n_1)^2 / 16$, which is maintained in the presence of an ideal entropy-preserving squeezing operation (parameterized by $r$). The dashed circles denote the standard vacuum fluctuations for reference.

**d, e.** Computed values of $r$ (d) and $n_1$ (e) given various values of input squeezing $r_{in}$ and $\eta_{in}$ (see equations (39) and (40)).
Extended Data Figure 5 | Numerically computed effective mechanical bath occupancy plotted against the normalized coupling rate \( g/\kappa \) for various pure squeezed states. By numerically computing the full bath occupancy \( n_{\text{bath, eff}} \), we account for the role of ‘strong coupling’ physics discussed elsewhere. Various squeezing parameters \( r \) were considered (expressed in units of the critical squeezing parameter \( r_c \) in equation (5)). All curves assume drive detuning \( \Delta = \Delta_0 \) (equation (31)) and squeezing phase \( \theta = \theta_0 \) (equation (11)). The optomechanical system is assumed to be deep in the resolved sideband cooling limit \( \kappa/\Omega = 0.1 \). The grey dashed line shows the strong coupling limit to the squeezing-enhanced cooling, which scales as \( (g/\Omega)^2 \).
Extended Data Figure 6 | Comparison of the mechanical bath temperature with squeezed and unsqueezed light in the presence of internal cavity loss. The solid and dashed lines correspond to the mechanical bath temperature with squeezed ($n_{\text{bath,eff}}^\text{bath,eff}$; equation (42)) and unsqueezed ($n_m^0$; equation (2)) light, respectively. The bath occupancies are plotted as a function of the external coupling rate $\kappa_{\text{ext}}$ normalized by the internal loss rate of the cavity $\kappa_0$. Each colour corresponds to a different value of $\kappa_0$. All curves assume a drive detuning $\Delta_0$ (equation (31)) from cavity resonance and the corresponding optimal squeezing phase $\theta_0$ (equation (11)). $n_{\text{bath,eff}}^\text{bath,eff}$ and $n_m^0$ approach the same limit (equation (44)) as $\kappa_{\text{ext}}/\kappa_0 \to 0$, indicating that internal cavity loss limits the coldest temperatures that can be reached when sideband cooling with any Gaussian state. For $\kappa_{\text{ext}} \gg \kappa_0$, $n_m^0$ diverges (equation (46)) whereas $n_{\text{bath,eff}}^\text{bath,eff}$ asymptotically approaches the ratio $\kappa_0/(4\Omega)$ (equation (45)).
## Extended Data Table 1 | Summary of notation

| Parameter                                                      | Symbol | Value                  |
|---------------------------------------------------------------|--------|------------------------|
| Mechanical resonance frequency                                | \( \Omega \) | \( 2\pi \times 10.1 \text{ MHz} \) |
| Total cavity linewidth                                        | \( \kappa \) | \( 2\pi \times 27 \text{ MHz} \) |
| Resolved sideband cooling limit (phonon occupancy)            | \( n_m^0 \) | 0.33                   |
| Critical squeezing parameter                                  | \( r_c \) | 0.55                   |
| Cavity resonance frequency                                    | \( \omega_c \) | \( 2\pi \times 6.4 \text{ GHz} \) |
| Cavity internal loss rate                                     | \( \kappa_0 \) | \( < 2\pi \times 100 \text{ kHz} \) |
| Intrinsic mechanical linewidth                                | \( \Gamma \) | \( 2\pi \times 15 \text{ Hz} \) |
| Vacuum optomechanical coupling rate                           | \( g_0 \) | \( 2\pi \times 260 \text{ Hz} \) |
| Microwave critical path transmittance (between JPA and OM)    | \( \eta_{\text{tn}} \) | 57 \( \pm \) 2%      |
| Input squeezing parameter (before critical path)              | \( r_{\text{in}} \) | -                      |
| Effective thermal occupancy of the squeezing                  | \( n_1 \) | -                      |
| Effective entropy-preserving squeezing parameter               | \( r \) | -                      |
| Detection chain noise temperature                             | \( T_N \) | 4.6 K                  |
| Effective heterodyne detection efficiency                     | \( \eta_{\text{det}} \) | 6.5%                   |
| Base temperature of dilution cryostat                         | \( T \) | 37 mK                  |

The corresponding values of any system constants are also given.