Explicit partial and functional differential equations for beables or observables

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Abstract

I provide partial differential equations (PDEs) – in finite cases – and functional differential equations (FDEs) – in field-theoretic cases – which determine observables or beables in the senses of Kuchař and of Dirac. I consider such for a wide range of relational particle mechanics as well as for Electromagnetism, Yang–Mills Theory and General Relativity. I give an underlying reason why pure-configuration Kuchař observables or beables are already well-known: notions of shape, E-fields, B-fields, loops and 3-geometries. I additionally pose the PDEs or FDEs for pure-momentum observables or beables, and for observables or beables which have a mixture of configuration and momentum functional dependence.

1 Introduction

Beables or observables are objects $b_B$ whose classical ‘brackets’ with ‘the constraints’ $c_C$ are ‘equal to’ zero:

$$\|[c_C, b_B]\| \equiv 0 .$$

This requires the use of a set of $c_C$ such that $\|[c_C, c_C]\|$ closes. Beables or observables\(^1\) are more useful than just any configurations $Q^A$’s and momenta $P_A$’s or functionals thereof, through solely containing physical information. At the very least this property is required in phrasing final answers to the physical questions about a theory.

Moreover, there are a number of different possibilities for which constraints, which brackets and even which notion of equality can be involved in the definition. Usually Dirac’s notion of weak equality \(\approx\) is assumed. Dirac [3] and Kuchař [4] notions of beables or observables apply to range of theories considered here. These involve commuting with all first-class constraints and commuting with all first-class linear constraints respectively. Note that Dirac = Kuchař for Particle Physics’ gauge theories, since in this context linear constraints are all. For relational particle mechanics (RPMs) and GR, however, there is a quadratic constraint too, so the two notions are not the same. On the other hand, Supergravity theories require dropping or replacing the latter notion [1, 5, 6], due to their quadratic constraint now being an integrability of their supersymmetric constraint.

I reviewed beables and observables in [1] (see e.g. [7, 4, 8, 9] for other reviews partly or totally on this topic, including the resulting ‘Problem of Beables’ facet of the Problem of Time). The current article lays out intermediate step: explicit partial or functional DEs (PDEs and FDEs) for classical beables and observables. Here the bracket involved is specifically a Poisson bracket (or some generalization thereof, such as the Dirac bracket [2, 10] or the Poisson bracket corresponding to an extended phase space [11, 10]). In the finite case, one has the PDE

$$\sum_A \left\{ \frac{\partial c_C}{\partial Q^A} \frac{\partial b_B}{\partial P_A} - \frac{\partial c_C}{\partial P_A} \frac{\partial b_B}{\partial Q^A} \right\} = 0 .$$

On the other hand, in the field-theoretic case one has the FDE

$$\int d^n z \sum_A \left\{ \frac{\partial(c_C|\partial \zeta^C)}{\partial Q^A(z)} \frac{\partial(b_B|\chi^B)}{\partial P_A(z)} - \frac{\partial(c_C|\partial \zeta^C)}{\partial P_A(z)} \frac{\partial(b_B|\chi^B)}{\partial Q^A(z)} \right\} = 0 .$$

This makes use of smearing, cast in an inner product type form [17]( | ): $(c_W|A^W) := \int d^3 z c_w(z) \langle Q^A(z^i), P_A(z^j) \rangle A^W(z^i)$. Beables equations are, fortunately, quite simple FDEs in some relevant senses; see Appendix A for preliminary results in this direction.

I then present PDE and FDEs for pure configuration Kuchař beables or observables as well as for pure momentum ones. Solutions to the former are already known, whereas solutions to the latter remain largely a mathematical problem posed in the present article, as is the matter of Dirac beables. More specifically, the current paper considers (Sec 2) new examples of Kuchař beables from the new extended set of RPMs presented in [6, 12] (in addition to the earlier RPMs [13, 14] whose beables were already considered in [1] and in more detail for 3 and 4 particles in 2-d in [15, 16]). Sec 3 recollects beables or observables in Electromagnetism and Yang–Mills Theory for useful comparison. Finally, Sec 4 provides the explicit equations for Dirac and Kuchař beables or observables for GR, including posing the pure momentum beables FDEs for this case of Kuchař beables or observables.

\(^1\)I also use an extension from the notion of observables, which eventually carry nontrivial connotations of ‘are observed’, to beables, which just ‘are’. The latter are somewhat more general, so as to cover a number of viable ‘realist’ approaches at the quantum level [1]. Note that this generalization concerns not a change of definition but rather a more inclusive context in which the entities are interpreted.
2 Relational particle model beables or observables

We run down the theories in Fig 1 based upon the groups in Fig 2. The corresponding configuration beables are tabulated in Fig 3, with the corresponding configuration spaces listed in Fig 4.

Figure 1: Summary sketch, including further groups acting upon $\mathbb{R}^d$. $Tr$ are translations, $Rot$ are rotations, and $Dil$ are dilations. I use $P$, $M$ and $D$ for their generators respectively, alongside $K$ for the special conformal transformations', $Sh$ for the shear’s and $Pr$ for the ‘Procrustean stretch’s’. (The last is a top form preserving stretch, for the top form supported by the dimension in question, e.g. area-preserving in 2-d or volume-preserving in 3-d.) Using the abstract Lie group form of brackets, then absences marked X are due to integrability $[[K,P]] \sim D + M$. Absences marked * are due to integrability $[[Sh,Sh]] \sim M$. Finally, absences marked † are due to obstruction $[[K,Sh]]$.

1) The zero total momentum constraint is

$$\mathcal{P} := \sum_{i=1}^{N} q_i = 0 .$$

Then the classical Kuchař beables condition $\{ \mathcal{P}, K_K \} = 0$ gives the PDE

$$\sum_{i=1}^{N} \frac{\partial K_K}{\partial q_i} = 0 ,$$

which is solved by the relative interparticle separation vectors and linear combinations thereof. Indeed the latter are often more convenient, in particular the relative Jacobi coordinates $\rho^A$ [19] whose interpretation is as relative interparticle cluster separation vectors.

2) The zero total angular momentum constraint is

$$\mathcal{L} := \sum_{i=1}^{N} q_i \times p_i = 0$$

(or the 3-component of this in 2-d). The corresponding Kuchař beables condition $\{ \mathcal{L}, K_K \} = 0$ then gives the PDE

$$\sum_{i=1}^{N} \left\{ \frac{\partial K_K}{\partial q_i} \times p_i + q_i \times \frac{\partial K_K}{\partial q_i} \right\} = 0 .$$

This is solved by a number of dot products. In particular the pure-configurational Kuchař beables equation note the symmetry with above

$$\sum_{i=1}^{N} q_i \times \frac{\partial K_K}{\partial q_i} = 0$$

is solved by $q_i \cdot q_i$, and the pure-momentum Kuchař beables equation

$$\sum_{i=1}^{N} \frac{\partial K_K}{\partial p_i} \times p_i = 0$$

is solved by $\frac{\partial K_K}{\partial q_i} \cdot \frac{\partial K_K}{\partial q_i}$.
Figure 2: The corresponding RPM theories’ names. Note that the Möbius case of RPM has so far resisted construction.

is solved by $p_I \cdot p_J$ due to being the $q_I \leftrightarrow q_J$ of the previous equation. The full equation is not solved by $q_I \cdot p_J$ but is solved by $q_I \cdot q_J + p_I \cdot q_J$, which is of course the outcome of applying the product rule to $q_I \cdot q_J$. Note that norms and angles are particular cases among the above, once the Appendix’s Lemma 1 is taken into account.

Next note the composition principle if (5) and (7) both apply, then the solutions are dots of differences, or, often more usefully, dots of relative Jacobi vectors. These are the Kuchař beables [1] for metric shape-and-scale RPM [13].

3) The **zero total dilational momentum constraint** is

$$D := \sum_{I=1}^{N} q_I \cdot p_I = 0 .$$

The corresponding Kuchař beables condition $\{D, K\} \overset{\cdot \cdot}{=} 0$ then gives the PDE

$$\sum_{I=1}^{N} \left\{ \frac{\partial K}{\partial q_I} \cdot p_I + q_I \cdot \frac{\partial K}{\partial q_I} \right\} \overset{\cdot \cdot}{=} 0 .$$

The above can be recognized as an Euler’s homogeneity equation of degree zero, so its solutions are ratios. The pure-configurational Kuchař beables equation is then

$$\sum_{I=1}^{N} q_I \cdot \frac{\partial K}{\partial q_I} \overset{\cdot \cdot}{=} 0 ,$$

and the pure-momentum Kuchař beables equation is (note the $q_I \leftrightarrow p_I$ symmetry again)

$$\sum_{I=1}^{N} \frac{\partial K}{\partial p_I} \cdot p_I \overset{\cdot \cdot}{=} 0 .$$

Then the composition principle gives that if (5) and (13) apply, have ratios of differences, if (7) and (13) apply, have ratios of dots, and if all three apply, have ratios of dots of differences. These are the Kuchař beables [1] for metric shape RPM [13]. See [20, 15, 16] for more on metric pure shapes; [21] contains a summary of the corresponding configuration spaces for these, with comparison to those of GR.

4) One further possibility involves a $\mathbb{R}^d \rightarrow \mathbb{S}^d$ change of underpinning. Here the constraints are $\mathcal{L}$ for $\mathbb{S}^1$, $\mathcal{L}$ in $\mathbb{S}^2$ and the pair $\mathcal{L}, \mathcal{L}'$ in $\mathbb{S}^3$, though also now the unreduced variables are now $\mathbb{S}^p$ angles $\theta^I$ rather than particle positions $q^I$, angles are involved, and there are no $Tr(d)$ or $Dil$. The invariants here are then the spherical metric’s own version of the dot product.

5) A second branch of further possibilities [6] involves extending the zero total angular momentum constraint to the **zero total SL(d, $\mathbb{R}$) momentum constraint**

$$\xi := \sum_{I=1}^{N} q_I \cdot \xi_I \cdot p_I .$$
In 2- and 3-d, it is a volume-preserving constraint, and in general it is a top form preserving constraint for the top form corresponding to the dimension in question. Then the classical Kuchař beables condition \( \{ s, \kappa_k \} \equiv 0 \) gives the PDE

\[
\sum_{I=1}^{N} \left\{ \frac{\partial \kappa_k}{\partial p^I} S_{p_I} - q^I S_{q^I} \frac{\partial \kappa_k}{\partial q^I} \right\} = 0.
\]  

(16)

This is solved in 2-d by areas between pairs vectors, in 3-d by volumes of parallelepipeds formed by triples of vectors, and in arbitrary \( d \) by the top form supported by that dimension formed by \( d \)-tuplets of such vectors. The pure-configuration Kuchař beables equation is

\[
\sum_{I=1}^{N} q^I S_{q^I} \frac{\partial \kappa_k}{\partial q^I} = 0,
\]

(17)

and the pure-momentum Kuchař beables equation is (note that this is no longer symmetric with the previous equation)

\[
\sum_{I=1}^{N} \frac{\partial \kappa_k}{\partial p^I} S_{p_I} = 0.
\]

(18)

The Appendix’s composition principle continues to apply here, so we can have top forms of differences, ratios of top forms and ratios of top forms of differences. The last of these corresponds to affine geometry and the first of these to ‘equi-top-form-al’ geometry (equiareal [22] in 2-d).

6) A third branch involves including instead the special conformal transformations. This follows from the 19th century observation that inversion in sphere also preserves angles, and well-known in 20th century Theoretical Physics through e.g. Conformal Field Theory (CFT) and its subsequent Particle Physics and String Theory applications. Moreover, this is no longer compatible with trivial removal of translations. In the RPM context, including the special conformal transformations leads to the zero total special conformal constraint [6]

\[
\kappa_a := \sum_{I=1}^{N} \{ q^I \delta_a^b - 2q^I q^b \} p_{Ib} = 0.
\]

(19)

Additionally, four subgroups including special conformal transformations are indicated in Fig 1. All require the invariants to be angles (special conformal transformation is a strong condition in this regard). The case with \( Tr(d) \) as well does use the \( \rho^A \) version; the other cases jointly involve the \( q^I \) version.

The classical Kuchař beables equation \( \{ \kappa_a, \kappa_k \} \equiv 0 \) then gives

\[
\sum_{I=1}^{N} \left\{ 2\{ 2p_{Ij}q_{jI} - (q \cdot p)\delta_{Ij} \} \frac{\partial \kappa_k}{\partial p_{Ij}} - \{ q^I q^I - 2q^I q^b \} \frac{\partial \kappa_k}{\partial q^I} \right\} = 0.
\]

(20)

The pure-configuration Kuchař beables equation is

\[
\sum_{I=1}^{N} \{ q^I \delta_{ab} - 2q^I q^b \} \frac{\partial \kappa_k}{\partial q^I} = 0.
\]

(21)

The pure-momentum Kuchař beables equation is (note that this is no longer symmetric with the previous equation)

\[
\sum_{I=1}^{N} \{ 2p_{Ij}q_{jI} - (q \cdot p)\delta_{Ij} \} \frac{\partial \kappa_k}{\partial p_{Ij}} = 0.
\]

(22)

7) In 2-d, the full conformal group becomes infinite-dimensional and thus unsuitable for finite RPMs. One can however consider the \( \text{finite:} \dim(\mathfrak{g}) = 6 \) Möbius group instead (a subgroup of the preceding, corresponding to another elementary piece of 19th century geometry). Then in the RPM context, the zero total Möbius constraint is

\[
\mathcal{M} := \sum_{I=1}^{N} z^I \frac{\partial \kappa_k}{\partial z^I} = 0;
\]

(23)

this comes accompanied by [6] \( \varpi \) and the complex pairing of \( \varphi \) with the 2-d \( \mathcal{L} \) to form \( \sum_{I=1}^{N} z^I \frac{\partial \kappa_k}{\partial z^I} = 0 \). The classical Kuchař beables equation \( \{ \mathcal{M}, \kappa_k \} \equiv 0 \) then gives the PDE

\[
\sum_{I=1}^{N} z^I \left\{ 2p_{Ij} \frac{\partial \kappa_k}{\partial z^I} - z^I \frac{\partial \kappa_k}{\partial z^I} \right\} = 0.
\]

(24)
Figure 3: \( g \)-invariants. \( z_{AB} := z^B - z^A \), with the particular product of two ratios of such forming the cross-ratio invariant.

Figure 4: Corresponding relational configuration spaces [6]. Useful inter-relations are also laid out: a number of configuration spaces are the same as others with one particle less by a Jacobi coordinates taking the same mathematical form as \( N \) point particle coordinates.

The pure-configuration Kuchař beables equation is then

\[
\sum_{I=1}^{N} z_{I}^2 \frac{\partial K}{\partial z_{I}} = 0, \tag{25}
\]

which is indeed solved by the well-known invariants of the Möbius group \( \text{cross-ratios} \). [Since such also occur in real projective geometry, in fact even some of the Ancient Greeks already knew about cross-ratios from that part of Euclidean geometry which eventually developed to become projective geometry.] cross-ratios. On the other hand, the pure-momentum Kuchař beables equation is (note the lack of symmetry once again)

\[
\sum_{I=1}^{N} z_{I}^2 \frac{\partial K}{\partial z_{I}}. \tag{26}
\]

Note 1) \( Conf(d) \) and Möbius\( (2) \) are of value as toy models of more complicated pure momentum beables, the preceding three constraints all being bilinear in its \( P_A \) and \( Q^A \) and the first being free from \( Q^A \).

Note 2) One can also consider the sphere versions of stripping away further layers of geometrical structure, though I leave this matter for a future occasion.

Dirac beables equations for the theories involving whichever subgroup-forming combination of \( \rho, \xi, \mathcal{P} \) and \( \kappa \) the following
extra equation.

\[ 0 \overset{\epsilon}{=} \{\epsilon, \varphi_0\} = \sum_{i=1}^{N} \left\{ \frac{\partial V}{\partial q^i} \frac{\partial \varphi_0}{\partial p_i} - p_i \frac{\partial \varphi_0}{\partial q^i} \right\}. \quad (27) \]

A distinct equation is required in the other cases. E.g., the equiareal and 2-d affine cases each require distinct \( \epsilon \)'s built from cross-products rather than from Euclidean norms.

### 3 Gauge Theory beables or observables

For Electromagnetism, which has the **Gauss constraint**

\[ \varphi := \partial \cdot x, \quad (28) \]

the classical Kuchař beables condition \( \{(\varphi|\xi), (K_K|x^K)\} \overset{\epsilon}{=} 0 \) gives the FDE

\[ \partial \cdot \frac{\delta K_K}{\delta A} = 0. \quad (29) \]

This is solved by \( E_\gamma = \partial \times A \), and thus by a functional \( F[B, E] \) by Lemma 1 in the Appendix. We can also write this in an integrated version in terms of fluxes:

\[ F\left[ \int S E \cdot dS, \int S B \cdot dS \right] = F[W_\gamma, \Phi_S^E] \quad (30) \]

for electric flux \( \Phi_S^E \) and loop variable

\[ W_\gamma := \exp \left( \oint \gamma A \cdot dl \right). \quad (31) \]

This is by use of Stokes’ Theorem with \( \gamma := \partial S \) and then insertion of the exponentiation function subcase of Lemma 1 (this ties the construct to the geometrical notion of holonomy).

All of the above carries over to Yang–Mills theory as well. This has the **Yang–Mills–Gauss constraint**

\[ \varphi_{\gamma} := \partial_a \pi_{\gamma}^a - g f_{IJK} A_a^K \pi^{Ja} = 0. \quad (32) \]

The classical Kuchař beables condition \( \{(\varphi_{\gamma}|\xi^I), (K_K|x^K)\} \approx 0 \) gives

\[ \frac{\partial}{\partial A_{\alpha_J}} \frac{\delta K_K}{\delta A_{\alpha_J}} \approx 0, \quad (33) \]

which is solved by \( E_I \) and \( B_I \), so \( F[E_I, B_I] \) solves also. Once again, this can be rewritten as \( F[W_\gamma, \Phi_S^E] \), now for loop variable

\[ W(\gamma) := \text{Tr} \left( P \exp \left( ig \oint \gamma A_{\alpha} \pi^{\alpha} \right) \right). \quad (34) \]

### 4 GR beables or observables

For GR-as-geometrodynamics [24], the linear **GR momentum constraint** is

\[ \mathcal{M}_i = -2D_i p^i = 0. \quad (35) \]

The corresponding Kuchař beables condition is then \(^4\)

\[ \{(\mathcal{M}_i|\mathcal{L}_i), (K_K|x^K)\} = \left\{ \left( L_{i, h_{ij}} \frac{\delta}{\delta h_{ij}} + L_{i, l_{ij}} \frac{\delta}{\delta l_{ij}} \right) K_K \bigg| x^K \right\} \overset{\epsilon}{=} 0, \quad (36) \]

which corresponds to the unsmeared FDE

\[ 2h_{jk} D_j \frac{\delta K_K}{\delta h_{ij}} + \left\{ D_i p^{ij} - 2 \delta^{ij} \{ D_e p^{le} + p^{le} D_e \} \right\} \frac{\delta K_K}{\delta p^{ij}} \overset{\epsilon}{=} 0. \quad (37) \]

---

\(^2\)Here \( g \) is the coupling constant, \( f_{IJK} \) are structure constants, \( g^{\alpha} \) are group generators, \( D \) the corresponding fibre bundle notion of covariant derivative, and \( P \) is the path-ordering symbol.

\(^3\)\( h_{ij} \) has determinant \( \sqrt{h} \), covariant derivative \( D_i \), Ricci tensor \( R_{ij} \), Ricci scalar \( R \), and conjugate momentum \( p^{ij} \) with trace \( p \). \( \Lambda \) is the cosmological constant.

\(^4\)Here \( L_{\alpha} \) denotes the Lie derivative with respect to \( L_{\alpha} \).
In the weak case, we can furthermore discard the penultimate term. Then the purely configurational solutions of
\[ 2h_{jk}D_i \frac{\delta K K}{\delta h_{ij}} = 0 \] (38)
are 3-geometry quantities \( \mathbf{D}^{(3)} \); by (38) emulating (and moreover logically preceding) the quantum momentum constraint,
\[ 2h_{jk}D_i \frac{\delta \Psi}{\delta h_{ij}} = 0 \] (39)
[26]. Moreover, explicit ‘basis beables’ (see the Appendix) are not known in this case.

On the other hand, the complementary part of this gives a FDE for the associated 3-geometry momenta \( \mathbf{\Pi D}^{(3)} \). These formal entities solve the the GR momentum beables equation
\[ \{ D_i p^{ij} - 2 \delta^i_j \{ D_e p^{el} + p^{el} D_e \} \} \frac{\delta K K}{\delta p^{ij}} = 0. \] (40)
GR also has a quadratic Hamiltonian constraint
\[ \mathcal{H} := N_{ijkl} p^{ij} p^{kl} - \sqrt{h} \{ R - 2\Lambda \} \] (41)
so Dirac beables or observables for geometrodynamics require extra equation \( \{ (\mathcal{H}|J), (dD|\chi^D) \} = 0 \), which gives
\[ \left( \frac{\delta D D}{\delta p} \{ \mathcal{G} - M D^2 \} - \frac{\delta D D}{\delta h} \right) L \{ \chi^D \} \] (42)
for DeWitt vector quantities
\[ \mathcal{G} := \frac{2}{\sqrt{h}} \{ p^{ja} p_a^j - \frac{1}{2} h^{ij} \} - \frac{1}{2\sqrt{h}} \{ p^{ab} p_{ab} - \frac{n^2}{2} \} h^{ij} - \frac{\sqrt{h}}{2} \{ h_{ij} R - 2R^{ij} \} + \sqrt{h} \Lambda h^{ij} \] (43)
familiar from ADM evolution [24] and \( D^2 := D^2 D^j \). In unsmeared form, (42) is the FDE
\[ \{ \mathcal{G} - M D^2 \} \frac{\delta D D}{\delta p} = 2p \frac{\delta D D}{\delta h} . \] (44)
Some examples of Dirac observables in GR for more specialized highly symmetric cases can be found in e.g. [28, 29, 30].

**Acknowledgements** To those close to me gave me the spirit to do this. And with thanks to those who hosted me and paid for the visits: Jeremy Butterfield, John Barrow and the Foundational Questions Institute. Thanks also to Chris Isham and Julian Barbour for a number of useful discussions over the years.

## A Supporting Lemmas

**Lemma 1.** If \( B_B \) are beables, then so are the functionals \( \mathcal{F}[B_B] \).

**Proof.** For \( L \) linear, if \( u \) solves \( L \phi \ = 0 \), then \( f(u) \) and \( F[u] \) also solve it by the chain-rule. And indeed, the PDE or FDE for the beables is an equation of this form.

Note 1) This renders ‘basis beables’ a useful concept: i.e. a sufficient set of Kuchař beables to describe one’s theory by \( K_K \). These are a mutually functionally independent choice of dim(reduced phase space) = \( 2q - c \) reduced phase space quantities (for \( q = \text{dim}(\mathbf{q}) \) and \( c \) the amount of phase space degrees of freedom taken out by the constraints). This is \( 2(q - g) \) in the simplest case of \( g \) first-class constraints alone. See e.g. [15, 16] for RPM examples of ‘basis beables’.

Note 2) In the case of multiple functional dependency restrictions applying, the composite of these applies; see Sec 2 for examples.

**Lemma 2.** The pure-configuration Kuchař beables equation is the same as the equation for what quantities are annihilated by the generators.

Note 3) This refers to the configuration space \( \mathbf{q} \) uplift of the group generators acting upon absolute space \( \mathbf{a} \), by which one passes from \( \mathbf{a} \)-invariants to ones built out of multiple particle positions \( q^I \).

\( ^{5} \)This is under the DeWitt 2-index to 1-index map [27], under which \( h_{ab} \) becomes \( h^A \), \( p^{ab} \) becomes \( p_A \), \( M^{abcd} \) becomes manifestly a metric \( M_{AB} \) and \( N_{abcd} \) becomes \( N^{AB} \). Here also \( M^{abcd} := \sqrt{h}(h^{ac}h^{bd} - h^{ab}h^{cd}) \) is the GR kinetic metric, and \( N_{abcd} := (h_{ab}h_{cd} - h_{ac}h_{bd})/\sqrt{h} \) is its inverse the DeWitt supermetric [27].
Proof. This occurs since the constraints involved are homogeneous linear in $p_{iI}$, so the Poisson bracket removes the $p_{iI}$ factors and replaces them with $\partial/\partial q^{iI}$ factors. [The Poisson bracket term with the other sign is annihilated by the pure-configuration restriction.]

Note 4) The pure-momentum case has no such result as constraints are not confined to be linear in $Q_A$. Those that are double-linear have pure-$P_A$ and pure-$Q_A$ beables close parallel. On the other hand, those which are not diverge more in the forms of these 2 contributions to the ‘basis beables’.

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