Fractals in construction material science

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Abstract. The application of the theory of fractal geometry in construction materials production allows revealing a new possibility to model the properties of materials and processes. The fractal theory can be used in the research of the processes of cement paste hardening as the formation of a cluster structure; the structure and properties of dispersed media; structural characteristics of the porous material; cracking caused by various factors, using data received at predicting the collapse of critical structures. The investigation of surface properties of aggregate, using a fractal approach, allowed one to reveal the direct dependence between the physical and mechanical characteristics of the concrete and the value of the fractal dimension of such aggregate. Thus, application of the method based on the determination of the fractal dimension of the aggregate, allows predicting the properties of concrete as well as to regulate its physical and mechanical properties by selecting or modifying the composition of the aggregate grain size, using the estimation of its fractal dimension.

1. Introduction
Recently, the term "fractal" is used in various fields of science and technology - physics, electronics, biology, medicine, economics, sociology, metallurgy, etc. It didn’t keep away construction materials science. This term is derived from the Latin word fractus translated as fractional, brake. [1]. We accustom to the classical concepts of geometry – point, line, surface, volume, which are associated with one-, two- and three-dimensional objects and the dimensions of the final value. Fractals are geometric objects which have a jagged form and called self-similar pattern, because they have a similar structure on the scale of different levels.

According to one of the plurality of classifications, fractals can be regular and irregular. First ones like the Koch snowflake, Peano curve or Sierpinski carpet are the fruit of the imagination, the result of a mathematical abstraction and they have a geometrically regular, structure (Figure 1, a). The second ones are a product of nature or the result of human activity (Figure 1, b).

2. Relevance
Application of the fractal geometry theory in building materials science allows to see in a new light the possibility of modeling the properties of materials and processes.

The theory of fractals can be used in studying of cement curing process, structure and properties of dispersed media [2]; characteristics of the porous materials structure; cracking caused by various factors, for forecasting of structural failure.
3. Problem statement

The studies of the aggregate surface properties by using of fractal approach enable to determine a direct relationship between the physical and mechanical properties of concrete and the aggregate value of the fractal dimension.

The fractal dimension is a numerical characteristic of fractal. Y.G. Puzanchenko, in his works dedicated to the fractal landscapes, writes: "...the fractal dimension is a good measure of complexity and provides direct information about the general form of relief and the spatial structure of any component". [3]

4. Theoretical part

There are many ways to found the fractal dimension [4,5]. All of these are based on the counting of volume or area of fractal shape and its changes in the scale, when the volume and the shape is increased.

1) The arbitrarily small unit "a" so called "line" is set. Then the length of the curve is measured in via replacing it by a broken line composed of segments with equal length "a". If the line is used N times, the overall length is equal N*a. Further, in accordance with the definition of the Mandelbrot "fractal dimension" of broken line is:

$$D = \lim_{n \to 0} \frac{\log N}{\log \frac{1}{a}}$$  \hspace{1cm} (1)

D will be a "bending degree"of curve [6]. In some cases, the broken number (1) has a constant value at each step.

Then,

$$D = \frac{\log N}{\log \frac{1}{a}} \quad \text{or} \quad N = \left(\frac{1}{a}\right)^D$$  \hspace{1cm} (2)

The greater the value of D, the more jagged the surface is. The smaller the scale used the smaller surface features will be taking into account and will contribute to the measured length. Conversely, when the scale is increased, the length of surface is reduced and the surface is "rectified"[1].

2) The surfaces may be rough, and even fractal up to molecular level. The area of such surfaces can be measured by depositing molecules with acquainted sizes on the test surface. The surface area is
typically determined by measuring the adsorption isotherms. The number of gram-moles of molecules (n) is measured under different values of pressure (p) and define temperature (T): \( n = f_T(P) \). The one method of n determining consists in measuring pressure before and after the interaction with the absorbent surface. Another method consists in direct weighing. In works [7-9], the authors examine surface area determined by the adsorption isotherms. The results show surfaces of many materials are fractal with fractal dimension ranging from 2 to 3. The specific surface area value of the sample depends on the size of the molecules used.

3) The use of optical analysis to determine the fractality [10,11].

Tested particles are photographed under a microscope with determined increase. Fractal dimension of the projection perimeter on the plane is determined. The dependence of the length elements number, covering the perimeter \( N_L \) on the scale is a power function

\[ N_L = \delta^{-D_L} \]  

where \( D_L \) – fractal dimension, \( \delta \) - the scale.

Calculation of fractal dimension of surface particle’s area is produced by the method of counting the number of squares, covering the projection particle area at different scales of the sides of the squares. [2].

4) For the calculation of the fractal dimension offer of M.J. Yablokov can be used [12]:

Square cell grid with size \( \delta \) is superimposed on the black-and-white image. The dependence of the number of cells occupied by black (white) pixels on the size of the cell is determined. Reticulated fractal dimension \( D_r \) is determined in accordance with the slope of line obtained from dependency diagrams:

\[ D_r = \frac{\ln N}{\ln \frac{1}{\delta}} \]  

where \( N \) – number of cells; \( \frac{1}{\delta} \) - cell size.

In the formulas (2) and (4) the value of \( a \) and \( \delta \) shows how many times the whole bigger the part. In accordance with the rules of B. Mandelbrot [13,14] for irregular fractals, fractal dimension greater than dimension calculated by the method of grid on 1:

\[ D = D_r + 1 \]  

5. Results of experimental studies

Comparison of fractal dimension of river sand and crushing screenings once again confirms that the artificial material has a distinct fractality as its micro-relief is much higher than natural material’s one. That is true both for the material as a whole, and for each fraction separately. The data presented in the table 1 shows that the average fractal dimension of screenings of crushing is 16.6 % higher than fractal dimension of river sand.

Table 1. The fractal dimension of the aggregate by "grid" method.

| Sort of sand          | Fractal dimension (D) for fraction, mm | \( D_{avg} \) |
|-----------------------|----------------------------------------|--------------|
|                       | 5-2.5  | 2.5-1.6 | 1.6-0.63 | 0.63-0.315 |               |
| River sand            | 2.559  | 2.246   | 2.133   | 2.253   | 2.298         |
| Crushing screening    | 2.876  | 2.421   | 2.820   | 2.602   | 2.679         |

Studies have shown that the selection of the grain aggregate (fractionation) composition allows you to change the fractal dimension (Table 2).
Table 2. The fractal dimension of the fractionated crushing screening

| Mix | Fraction, mm | Content of fraction, % | Fractal dimension D |
|-----|--------------|------------------------|--------------------|
| 1   | (5-2.5]      | 38                     | 2.718              |
|     | (2.5-0.315]  | 43                     |                    |
|     | (0.315-0.16] | 19                     |                    |
| 2   | (5-1.25]     | 52                     | 2.660              |
|     | (1.25-0.315] | 22                     |                    |
|     | (0.315-0.16] | 26                     |                    |
| 3   | (5-1.25]     | 41                     | 2.694              |
|     | (1.25-0.315] | 35                     |                    |
|     | (0.315-0.16] | 24                     |                    |

Analysis of experimental data shows that composition 2 has the maximum fractal dimension. The value of the fractal dimension also can be compared with a form of grain, which is on par with micro-relief is an important characteristic of the aggregate. Theoretical foundations of Euclidean geometry suggests that the line has dimension 1, smooth curved surfaces and a plane have dimension 2 and a bodies of various shapes are three dimensional. Therefore, in estimating the fractal dimension of the used aggregates it can be argued that the surface of the river sand is smoother while the screenings crushing shape is similar in isometric shape with a developed surface.

The concrete mix on the basis of river sand, unfractonated crushing screenings and fractionated were crushing screenings were designed. Physical and mechanical properties of the concretes are shown on the figure 2.

Figure 2. The strength characteristics of concrete based on various aggregates.

6. Conclusion
The data confirm the feasibility of fractionation. Obviously, it is showed that the bending and compressive strength of concrete on the base of fractionated crushing screenings is higher than those based on natural river sand and unfractonated crushing screenings. An average excess of flexural and compressive strength is equal to 44 and 27%, respectively. Increased strength is due to the fact that crushing screening has more contacts leading to improved adhesion to the cement paste and, consequently, the physical and mechanical characteristics of the concrete are improved.
Thus, the physical and mechanical characteristics of fine grained concrete directly depend on the aggregate fractal dimension, which in turn will depend on the structure of the material, its physical-mechanical characteristics and external process parameters.

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