Hybrid Harvester 3-RPS Robotic Parallel Manipulator

Zhongxing Yang¹ and Dan Zhang¹, *
¹Department of Mechanical Engineering, Lassonde School of Engineering, York University, Toronto, Canada

E-mail addresses: ¹zxyang@yorku.ca; ¹,*dan.zhang@lassonde.yorku.ca

Abstract. This article presents the robotic structure and analysis of a hybrid energy harvester designed in the form of a reconfigurable 3-RPS parallel manipulator, which tracks and collects solar energy as a main function and also harvests energy from wind loads on the large solar panel. The reconfigurable design allows the harvester to lie flat on floor when destructive storms occur and then return to a working configuration after the storms. The workspace of the manipulator is determined by a minimum platform height algorithm that is developed to find all reachable positions that compose the workspace and also bring bending at root to least. The stiffness mapping of the harvester is significant in evaluating its capacity to take wind loads, which a transformation matrix is developed to convert to polar horizontal stiffness at end effector that is crucial to the intelligent control strategy in decision to enter lie-flat configuration.

1. Introduction
A solar tracker or harvester follows the sun to collect energy. Single-axis [1] and dual-axis [2] serial manipulators were designed as solar trackers. Since storms could damage solar tracking fields (North America [3] [4], and Pacific regions [5]), parallel manipulators which have higher stiffness and loading capacity [6] were designed with 2 [7] or 3 [8] DOF (mobility). The design of a general 3-RPS solar tracker [9] could be further improved to enable switch between a lie-flat [10] configuration for safety and a working configuration for enlarged workspace, by reconfiguration [11] methodology.

Loads on solar panels could be another energy source. Piezo-electric materials convert wind loads to electricity, as in applications [12], [13] and [14]. The wind loads on a flat panel could be generally divided to a lift load normal to the panel and a drag load parallel to the panel [15] [16].

This paper presents the design of a solar-piezo hybrid harvester with reconfiguration for dual modes. An algorithm is explained that seeks minimum platform height from a given orientation angle. A transformation matrix is developed that evaluates polar stiffness at end effector on a horizontal plane which is crucial to the intelligent control strategy in decision to enter lie-flat configuration.

2. Mechanical design
This tracking stand is a 3-RPS parallel manipulator with lie-flat and working configuration features.

The orientation angle and height of the solar panel could be adjusted by three actuated legs which are jointed to motor-driven reconfiguration bases. As to meet the dust-proof, load capacity and geometric requirements for out-door working conditions, the linear tables (type: Alpha 15-B-155) [17] and the linear actuators (type: DLA-12-40-311-200-IP65) [18] are selected to function as reconfigurable bases and telescopic legs respectively. The spherical joint with large motion range [19]
and piezo-chips incorporated parallel structures [20] are considered. Figure 1 shows the design in two configurations.

![Figure 1](image1.jpg)

**Figure 1.** The hybrid harvester (a) lie-flat configuration; (b) working configuration.

3. Inverse kinematics

The inverse kinematics solves for the actuation displacements from a given posture of the platform.

In Figure 2. (a), the plane A represents the floor and the plane B represents a parallel plane above floor. The global coordinate system $A_o = X_oY_oZ_o$ is located at $A_o$ underneath $B_o$ by $h_b$.

$$X_o = [1 \ 0 \ 0]^T; \ Y_o = [0 \ 1 \ 0]^T; \ Z_o = [0 \ 0 \ 1]^T \quad (1)$$

$$A_o = [0 \ 0 \ 0]^T; \ B_o = A_o + h_b \cdot Z_o \quad (2)$$

![Figure 2](image2.jpg)

**Figure 2.** Inverse kinematic (a) parallel structure; (b) minimum platform height $z_c$.

Three points $B_1$, $B_2$, and $B_3$ are located on plane B where revolute joints are. One can connect lines $B_oB_i$, and finds angle $\beta_i$ between $X_o$ and $B_oB_i$. The length of $B_oB_i$ is $b_i$ which can be adjusted by reconfiguration. Sub-coordinates $B_i = X_{bi}Y_{bi}Z_o$ are established at $B_1$, $B_2$, and $B_3$. 
\[ X_{bi} = Q_l \cdot X_o ; Y_{bi} = Q_l \cdot Y_o, \text{ where } Q_l = \begin{bmatrix} \cos \beta_i & -\sin \beta_i & 0 \\ \sin \beta_i & \cos \beta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ i = 1,2,3 \] (3)

The platform is a triangle with \( C_o \) at the center, while points \( C_1, C_2, \) and \( C_3 \) are at the three vertices of the triangle where spherical joints are. One can connect lines \( C_oC_i \), the length of \( C_oC_i \) is a fixed value \( r_c \). One can set a sub-coordinate \( C_o - X_cY_cN_c \) at \( C_o \), where \( Y_c \) aligns with vector \( C_oC_1 \), \( X_c \) lies on the triangle plane, and \( N_c \) be a unit vector normal to the platform. The angle \( \beta_i \) is between \( X_c \) and \( C_oC_i \).

A square solar panel is attached above the platform by \( h_p \) in \( N_c \) direction. The four points \( P_1, P_2, P_3 \) and \( P_4 \) are at the four points of the solar panel square. One can connect lines \( P_jP_j \), which forms an angle \( \beta_{pj} \) from \( X_c \), and the length of the lines is a fixed value \( r_p \).

The vectors \( C_oC_i \) and \( P_oP_j \) from the perspective of coordinate \( C_o - X_cY_cN_c \) are given below.

\[ R_{ci} = [r_c \cdot \cos \beta_i, r_c \cdot \sin \beta_i, 0]^T, \ i = 1,2,3 \] (4.a)
\[ R_{pj} = [r_p \cdot \cos \beta_{pj}, r_p \cdot \sin \beta_{pj}, 0]^T, \ j = 1,2,3,4 \] (4.b)

One can connect \( B_iC_i \) along which the prismatic joints are, the length of which is \( q_i \in [q_{min}, q_{max}] \), and a unit vector \( X_qi \) is set along \( B_iC_i \). Vector \( X_qi \) forms an angle \( \beta_{bi} \) above the plane \( B_i \).

In order to make sure that linear actuators operate under the moving platform, an angular constraint \( \gamma_{cm} \) is set which is the minimum angle the vector \( X_qi \) can form to platform plane \( (C_o - C_1C_2C_3) \).

Based on real values of the reconfigurable design, linear actuators and linear tables are selected from manufacturers, so that numerical analysis can be performed. Design parameters are shown in table 1.

Reconfiguration of the \( B_i \) along \( X_{bi} \) introduces an adjustment variable \( b_i - adj \). When \( b_i - adj = 0 \), variable \( b_i \) is determined to allow minimum height of platform at the lie-flat posture.

\[ B_i = b_i \cdot Q_l \cdot X_o + B_o, \text{ where } b_i = q_{min} \cdot \cos \gamma_{cm} + r_c + b_i - adj \] (5)

**Table 1.** Design parameters.

| Parameters   | Units | Values          |
|--------------|-------|-----------------|
| \( h_b \)   | \( m \) | 0.092           |
| \( r_p \)   | \( m \) | 0.5             |
| \( r_c \)   | \( m \) | 0.1             |
| \( h_p \)   | \( m \) | 0.05            |
| \( q_{min}, q_{max} \) | \( m \) | [0.35, 0.55]    |
| \( \beta_{bi} \) \( (i=1,2,3) \) | \( rad \) | \( \frac{\pi}{2}, \frac{\pi}{6}, \frac{7\pi}{6} \) |
| \( \beta_{pj} \) \( (j=1,2,3,4) \) | \( rad \) | \( \frac{\pi}{4}, \frac{\pi}{6}, \frac{3\pi}{4}, \frac{3\pi}{4} \) |
| \( \gamma_{cm} \) | \( rad \) | 0.1745          |

Rotational matrix [21] needs platform rotation variables \( \theta_x \) and \( \theta_y \) around \( X_o \) and \( Y_o \) to convert vectors or matrix in \( C_o - X_cY_cN_c \) to that in \( B_o - X_oY_oZ_o \).

\[ u_x = \frac{\theta_x}{\theta}, \ u_y = \frac{\theta_y}{\theta}, \text{ where } \theta = \sqrt{\theta_x^2 + \theta_y^2} \] (6.a)
One can set \( C_0 = [x_c \ y_c \ z_c]^T \) as its position in global coordinate. As \( B_i \) is a revolute joint, then \( B_0C_i \) is perpendicular to \( Y_{bi} \), which is a relationship to find kinematic solutions for \( x_c \) and \( y_c \).

So that,

\[
C = \begin{bmatrix} Y_{b2} \cdot X_o \\ Y_{b3} \cdot X_o \end{bmatrix}^{-1} \cdot \begin{bmatrix} R \cdot R_{c2} \cdot Y_{b2} \\ R \cdot R_{c3} \cdot Y_{b3} \end{bmatrix}, \text{ which satisfies } (R \cdot R_{cl} + C_0) \cdot Y_{bi} = 0
\]

\[
x_c = C \cdot X_o; \ y_c = C \cdot Y_o
\]  

(7.a)  

(7.b)

The variable \( z_c \) is usually given by the motion planner, however improper selection of this value can result to invalid solution. In this article, an algorithm is explained that seeks the minimum height of platform center \( C_o, z_c \), if an eligible solution exists. This algorithm avoids missing valid solutions while keeping minimum solar panel height to mitigate the wind loads bending effects at structure root.

The minimum height algorithm starts by setting the initial value \( z_{co} \).

\[
C_0^* = [x_c \ y_c \ z_{co}]^T, \text{ where } z_{co} = 0
\]

\[
C_i^* = R \cdot R_{cl} + C_o^*; \quad P_o^* = h_p \cdot R \cdot Z_o + C_o^*; \quad P_j^* = R \cdot R_{pj} + P_o^*
\]  

(8.a)  

(8.b)

In figure 2. (b), possible candidates of \( z_c \) are calculated by determining an offset as a supplement to \( z_{co} \). The offsets are determined considering some criteria. Offset 1: none of \( P_j \) point should stay below plane B; Offset 2: an angular constraint \( \gamma_{cm} \) is met which is the minimum angle the leg can form to platform plane \( (C_o - C_1C_2C_3) \) under this platform; Offset 3: none of \( q_i \) should be less than \( q_{min} \).

Offset 1, \( d_{o1} \):

\[
d_{o1} = h_p - z_{pi-min}, \text{ where } z_{pi-min} = \min (P_i \cdot Z_o)
\]

\[
z_{c-can1} = z_{co} + d_{o1}
\]  

(9.a)  

(9.b)

Offset 2, \( d_{o2} \):

A unit vector \( X_{cl} \) is along the intersection line of planes \( C_o - C_1C_2C_3 \) and \( X_{bi} - B_i - Z_o \).

\[
X_{cl} = \frac{Y_{bi} \times N_c}{|Y_{bi} \times N|}, \text{ where } N_c = R \cdot Z_o
\]  

(10.a)

\( \theta_{ci} \) is an acute angle between planes \( C_o - C_1C_2C_3 \) and \( X_{bi} - B_i - Z_o \).

\[
\theta_{ci} = \cos^{-1}|N_c \cdot Y_{bi}|
\]  

(10.b)

It is imagined that \( X_{cl} \) is rotated at \( C_i \) with an acute angle \( \gamma_{ci} \) about \( Y_{bi} \) to be altered to a new unit vector \( X_{yl} \) which forms an angle \( \gamma_{cm} \) to the platform plane \( C_o - C_1C_2C_3 \) or to its projection on the platform in \( N_c \) direction. The arrowhead of \( X_{yl} \) falls on a plane \( (C_o - C_1C_2C_3)' \) that is parallel to platform plane and \( X_{yl} \) is in plane \( X_{bi} - B_i - Z_o \). The gap between planes \( (C_o - C_1C_2C_3)' \) and \( C_o - C_1C_2C_3 \) is certain, and the intersection lines they make on plane \( X_{bi} - B_i - Z_o \) are parallel and the gap between which can be determined with \( \theta_{ci} \).
\[
X_{y\ell} = Q_{\ell} \begin{bmatrix} \cos \gamma_{c\ell} & 0 & \sin \gamma_{c\ell} \\ 0 & 1 & 0 \\ -\sin \gamma_{c\ell} & 0 & \cos \gamma_{c\ell} \end{bmatrix} \cdot Q_{\ell}^{-1} \cdot X_{c\ell}, \quad \text{where } \gamma_{c\ell} = \sin^{-1}\frac{\sin \gamma_{cm}}{\sin \theta_{c\ell}} (10.c)
\]

If \( \mathbf{C}_l \mathbf{B}_l \) is along \( X_{y\ell} \), the length of \( \mathbf{C}_l \mathbf{B}_l \) for it to reach \( \mathbf{B}_l \) is \( q_{l-\gamma_{cm}} \).

With \( X_{y\ell}, q_{l-\gamma_{cm}} \) and \( \mathbf{C}_l^* \), one can find an imagined point \( \mathbf{B}_{y\ell} \) whose global coordinate in \( Z_o \) is \( z_{bi-c} \).

\[
z_{bi-c} = (q_{l-\gamma_{cm}} \cdot X_{qi} + \mathbf{C}_l^*) \cdot Z_o, \quad \text{where } q_{l-\gamma_{cm}} = \frac{(\mathbf{B}_l - \mathbf{C}_l^*) \cdot x_{bi}}{x_{yi} x_{bi}} (10.d)
\]

Three legs each has its \( \mathbf{B}_{y\ell} \) height, \( z_{bi-c} \), and the minimum among them is selected so that an offset is calculated based on the lowest \( \mathbf{B}_{y\ell} \), thus the other two legs will be satisfied with the offset lift.

\[
d_{o2} = h_b - z_{bi-c-min}, \quad \text{where } z_{bi-c-min} = \min(z_{bi-c}) (10.e)
\]

\[
z_{c-can2} = z_{co} + d_{o2} (10.f)
\]

Offset 3, \( d_{o3} \):

With \( \mathbf{C}_l^* \) and \( q_{min} \), one can find an imagined \( \mathbf{B}_{qi} \) under \( \mathbf{B}_l \) which ensures the minimum length of linear actuators be met in all legs. The global coordinate of \( \mathbf{B}_{qi} \) in \( Z_o \) is \( z_{bi-q} \).

\[
z_{bi-q} = \mathbf{C}_l^* \cdot Z_o - \sqrt{q_{min}^2 - ((\mathbf{B}_l - \mathbf{C}_l^*) \cdot X_{bi})^2} (11.a)
\]

Three legs each has its \( \mathbf{B}_{qi} \) height, \( z_{bi-q} \), and the minimum among them is selected so that an offset is calculated based on the lowest \( \mathbf{B}_{qi} \) and the other two legs will be satisfied with the offset lift.

\[
d_{o3} = h_b - z_{bi-q-min}, \quad \text{where } z_{bi-q-min} = \min(z_{bi-q}) (11.b)
\]

\[
z_{c-can3} = z_{co} + d_{o3} (11.c)
\]

There are three \( z_c \) candidates, each is the minimum height requirement in its offset calculation that means it may have room to be enlarged. The maximum of the candidates is selected so that the other two offset requirements are included under it.

\[
z_c = \max(z_{c-can}), k = 1, 2, 3 (12)
\]

Once \( z_c \) is determined, the posture needs to be calculated again with updated values.

\[
\mathbf{C}_o = [x_c \ y_c \ z_c]^T (13.a)
\]

\[
\mathbf{C}_l = \mathbf{R} \cdot \mathbf{R}_{ci} + \mathbf{C}_o; \mathbf{P}_o = h_p \cdot \mathbf{R} \cdot \mathbf{Z}_o + \mathbf{C}_o; \mathbf{P}_j = \mathbf{R} \cdot \mathbf{P}_{pj} + \mathbf{P}_o (13.b)
\]

The motion of linear actuations are calculated based on the above platform position.

\[
q_{l} = |\mathbf{B}_l - \mathbf{C}_l| \quad \text{and } \theta_{bi} = \tan^{-1}\frac{(\mathbf{C}_l - \mathbf{B}_l) \cdot Z_o}{(\mathbf{B}_l - \mathbf{C}_l) \cdot x_{bi}} (14)
\]

The updated value of \( z_c \) will need to be verified. The posture with updated \( z_c \) need to meet a criteria that none of the linear actuators \( q_c \) extends \( q_{max} \), fully extended size.

Within \( n_r \) pairs of \( (\theta_x, \theta_y) \), program loops calculate \( z_c \) at each \( (\theta_x, \theta_y) \) and then verify. The number of eligible solutions \( n_e \) is an index to evaluate the workspace.
4. Reconfiguration
The original $B_j$ locations allows the lie-flat configuration but provides a limited workspace. Genetic algorithm is implemented to find another configuration that enlarges workspace, known as working configuration [22]. The reconfiguration design enable the switch between the two configurations.

The adjustments $b_{i \rightarrow adj}$ are the variables for the MATLAB optimization algorithm that seeks global minimum of fitness function $f$, while the ultimate aim is to enlarge the workspace index $n_e$. The optimization process and results are displayed in figure 3, table 2 and table 3.

\[
X = [b_{1\rightarrow adj} \ b_{2\rightarrow adj} \ b_{3\rightarrow adj}]
\]

\[
f = -n_e
\]

\[ \text{(15.a)} \]
\[ \text{(15.b)} \]

**Figure 3.** Optimization (a) genetic algorithm for larger workspace; (b) workspace boundary, dash: original; solid: optimized; (c) the $z_c$ position mapping with optimized working configuration.

| Parameters \( X \) | Units \( X \) | Values |
|----------------------|----------|--------|
| lower \( X_{low} \)  | \( m \)  | \([-0.1\ -0.1\ -0.1]\) |
| upper \( X_{up} \)   | \( m \)  | \([0\ 0\ 0]\) |
| search area \( \theta_x \) | \( rad \) | \([-1.5:0.1:1.5]\) |
| search area \( \theta_y \) | \( rad \) | \([-1.5:0.1:1.5]\) |

**Table 2.** Optimization parameters.

| Variable \( X \) | Units | \( f = -n_e \) |
|-------------------|-------|----------------|
| original \( [0\ 0\ 0] \) | \( N.A. \) | \(-126\) |
| optimized \( [-0.098\ -0.098\ -0.097] \) | \( N.A. \) | \(-211\) |
| rounded \( [-0.1\ -0.1\ -0.1] \) | \( N.A. \) | \(-211\) |

**Table 3.** Optimization results.

5. Stiffness Mapping
The 3-RPS manipulator has 3 DOF, that means it has three independent motions, while the other motions in space are compliant to the independent motions. Article [23] has analyzed the inverse kinematics of 3-RRS parallel manipulator, of which the methods could be used to solve for other 3 DOF parallel manipulators.

5.1. Jacobian Matrix
In the global coordinate, the linear and angular velocities system of the end effector, center of the platform $C_o$, is given below.

\[
V_{co} = \begin{bmatrix} \partial x_c \\ \partial y_c \\ \partial z_c \end{bmatrix} \quad \text{and} \quad \omega_{co} = \begin{bmatrix} \partial \theta_{cx} \\ \partial \theta_{cy} \\ \partial \theta_{cz} \end{bmatrix}
\]

\[ \text{(16)} \]
Only three of six space motion variables are independent. The variables have a relationship as below.

\[ Y_{bi} \cdot V_{cl} = 0, \text{ where } V_{cl} = V_{co} + \omega_{co} \times R \cdot R_{cl} \]  
(17.a)

\[ C_v \cdot V_{co} + C_w \cdot \omega_{co} = 0, \text{ where } C_v = [Y_{b1} \quad Y_{b2} \quad Y_{b3}]^T \text{ and } C_w = \begin{bmatrix} R \cdot R_{c1} \times Y_{b1} \\ R \cdot R_{c2} \times Y_{b2} \\ R \cdot R_{c3} \times Y_{b3} \end{bmatrix} \]  
(17.b)

\[ \omega_{co} = -C_w^{-1} \cdot C_v \cdot V_{co} \]  
(17.c)

And the methods in [24] and [25] suggests to eliminate the compliant motion by dot multiplying a vector perpendicular to the compliant motions on both sides of the equation.

\[ V_{qi} + \omega_{qi} \cdot Y_{bi} \times (C_i - B_i) = V_{ci} \]  
(18.a)

\[ (C_i - B_i) \cdot V_{qi} + 0 = (C_i - B_i) \cdot V_{ci} \]  
(18.b)

where,

\[ V_{qi} = \frac{(C_i - B_i)}{|C_i - B_i|} \cdot \partial q_i = X_{qi} \cdot \partial q_i \]  
(18.c)

Rearrange the above, one has the following.

\[ (C_i - B_i) \cdot \frac{(C_i - B_i)}{|C_i - B_i|} \cdot \partial q_i = (C_i - B_i) \cdot (V_{co} + \omega_{co} \times R \cdot R_{ci}) \]  
(19.a)

\[ q_i \cdot X_{qi} \cdot \partial q_i = q_i \cdot X_{qi} \cdot (V_{co} + \omega_{co} \times R \cdot R_{ci}) \]  
(19.b)

\[ X_{qi} = 1 \]  
(19.c)

\[ q_i \cdot \partial q_i = q_i \cdot X_{qi} \cdot (V_{co} - C_w^{-1} \cdot C_v \cdot V_{co} \times R \cdot R_{ci}) \]  
(19.d)

\[ \partial q_i = X_{qi} \cdot (V_{co} - C_w^{-1} \cdot C_v \cdot V_{co} \times R \cdot R_{ci}) \]  
(19.e)

\[ \partial q_i = X_{qi} \cdot V_{co} + X_{qi} \cdot (R \cdot R_{ci}) \times (C_w^{-1} \cdot C_v \cdot V_{co}) \]  
(19.f)

\[ \partial q_i = X_{qi} \cdot V_{co} + X_{qi} \cdot (R \cdot R_{ci}) \times (C_w^{-1} \cdot C_v \cdot V_{co}) \]  
(19.g)

\[ \partial q_i = X_{qi} \cdot V_{co} + X_{qi} \times R \cdot R_{ci} \cdot C_w^{-1} \cdot C_v \cdot V_{co} \]  
(19.h)

\[ \partial q_i = X_{qi}^T \cdot V_{co} + (X_{qi} \times R \cdot R_{ci})^T \cdot (C_w^{-1} \cdot C_v) \cdot V_{co} \]  
(19.i)

\[ \partial q_i = (X_{qi}^T + (X_{qi} \times R \cdot R_{ci})^T) \cdot (C_w^{-1} \cdot C_v) \cdot V_{co} \]  
(19.j)

\[ J_{ri} = (X_{qi}^T + (X_{qi} \times R \cdot R_{ci})^T) \cdot (C_w^{-1} \cdot C_v) \]  
(19.k)

\[ \partial q_i = J_{ri} \cdot V_{co} \]  
(19.l)

The Jacobian matrix is obtained as below.

\[ J = [J_{r1} \quad J_{r2} \quad J_{r3}]^T \]  
(20.a)

This reflects the relationship between actuation velocities and the end effector velocities.

\[ [\partial q_1 \quad \partial q_2 \quad \partial q_3]^T = J \cdot [\partial x_c \quad \partial y_c \quad \partial z_c]^T \]  
(20.b)

5.2. **Stiffness Matrix**

Stiffness of end effector \( K_c \) is expressed as below, where \( K_q \) is the actuation stiffness.
\[ K_c = J^T \cdot K_q \cdot J, \quad \text{where} \quad K_q = \begin{bmatrix} k_{q_1} & 0 & 0 \\ 0 & k_{q_2} & 0 \\ 0 & 0 & k_{q_3} \end{bmatrix} \quad (21) \]

The stiffness along \( X_o, Y_o \) and \( Z_o \) are given.

\[ k_x = (K_c \cdot X_o)^T \cdot X_o; \quad k_y = (K_c \cdot Y_o)^T \cdot Y_o; \quad k_z = (K_c \cdot Z_o)^T \cdot Z_o \quad (22) \]

When \( X_o \) rotates around \( Z_o \) by \( \varphi_z \), it forms a new vector \( X_\varphi \). The vector \( X_o \) expressed in coordinate with \( X_\varphi \) is given below.

\[ (X_o)_\varphi = R_z \cdot X_o, \quad \text{where} \quad R_z = \begin{bmatrix} \cos(-\varphi_z) & -\sin(-\varphi_z) & 0 \\ \sin(-\varphi_z) & \cos(-\varphi_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (23.a) \]

The velocity vector \((V_{co})_\varphi\) represents the end effector velocity in coordinate with \( X_\varphi \). It replaces velocity \( V_{co} \) with transformation matrix \( R_z^{-1} \).

\[ V_{co} = R_z^{-1} \cdot (V_{co})_\varphi, \quad \partial q_i = J_{rl} \cdot R_z^{-1} \cdot (V_{co})_\varphi, \quad \text{where} \quad R_z^{-1} = \begin{bmatrix} \cos \varphi_z & -\sin \varphi_z & 0 \\ \sin \varphi_z & \cos \varphi_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (23.b) \]

This provides a new Jacobian matrix with a rotation angle index of \( \varphi_z \).

\[ J_{\varphi_1} = J_{rl} \cdot R_z^{-1}, \quad \text{so that} \quad J_\varphi = [J_{\varphi_1} \quad J_{\varphi_2} \quad J_{\varphi_3}]^T \quad (23.c) \]

The stiffness \( k_{x\varphi} \) along \( X_\varphi \) could be calculated.

\[ K_\varphi = J_\varphi^T \cdot K_q \cdot J_\varphi \quad (23.d) \]

\[ k_{x\varphi} = (K_\varphi \cdot X_o)^T \cdot X_o \quad (23.e) \]
The leg $B_iC_i$ has the longitudinal stiffness $k_q = 4.9568 \times 10^5$ N/m, which has two components in series, linear actuator stiffness $k_a = 5 \times 10^5$ N/m and eight parallel piezo chips stiffness each with $k_p$. The stiffness of $k_p$ can be calculated with the design sizes [26]. The material Lead Zirconate Titanate (PZT) takes its Young’s module $E_{pzt} = 63$ Gpa from [27].

$$\frac{1}{k_q} = \frac{1}{8k_p} + \frac{1}{k_a}, i = 1,2,3$$ (24)

Stiffness mapping visualize the directional stiffness within the selected range of workspace [28].

Figure 4. (a), (b) and (c) show the stiffness in directions along $X_o, Y_o$ and $Z_o$. However the wind loads could be in any direction. As in figure 4. (d), for a certain configuration and posture of the manipulator, when $\varphi_x$ varies within $0^\circ$ to $360^\circ$, the stiffness $k_{xq}$ with respect to $\varphi_x$ could be visualized with a polar plot, which helps to evaluate the manipulator loading capacity to horizontal winds in specific horizontal directions.

6. Conclusion

The design of a reconfigurable hybrid harvester enables 3-RPS parallel solar tracker to collect solar and wind energy at the same time. This design has a lie-flat feature that protects the manipulator from storms, and an enlarged workspace achieved by reconfiguration. Reconfiguration enables the manipulator to switch between the lie-flat configuration and the working configuration. Minimum
platform height algorithm has been developed and demonstrated to find all eligible orientations of the platform. Stiffness mappings of the manipulator are plotted, after being converted to a polar plot, it assists to evaluate its capability to take wind loads in specific horizontal directions that is significant for lie-flat protection strategy against storms. The design adds favorable features to solar trackers that improves the energy collection efficiency and operational safety of the green industry.

Acknowledgement
The authors would like to thank the financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC). The authors gratefully acknowledge the financial support from Kanef Research Chairs program.

References
[1] T. Tudorache and L. Kreindler, 2010, “Design of a Solar Tracker System for PV Power Plants”, Acta Polytechnica Hungarica, Vol. 7, No. 1, 2010
[2] D. Gnanarathinam,S, Sundaramurthy.S, and A. Wahi, 2015, “Design and Implementation of a Dual Axis Solar Tracking System”, International Journal for Research in Applied Science & Engineering Technology, Vol. 3, Issue X, October 2015
[3] C. Thurston, 2015, “Ensuring Your Solar Array Doesn't Get Caught in the Wind”, Renewable Energy World, Vol. 18, Issue 4, 2015
[4] “Solar Field in Ontario Sees Damage Following Winter Storm”, from http://13wham.com/news/top-stories/solar-field-in-ontario-sees-damage-following-winter-storm
[5] “Tropical Storm Risk (TSR)” from http://www.tropicalstormrisk.com
[6] D. Zhang, Chapter 1 Introduction, Parallel Robotic Machine Tool, Springer 2010
[7] A. Cammarata, 2014, “Optimized Design of A Large-Workspace 2-DOF Parallel Robot for Solar Tracking Systems”, Mechanism and Machine Theory Volume 83, January 2015, Pages 175–186
[8] “Design of Multiaxial Parallel Kinematic Machines with High Dynamic Capabilities” from http://www.ehu.eus/compmech/research-2/projects/design-of-multiaxial-parallel-kinematic-machines-with-high-dynamic-capabilities-dynamech
[9] Ashith Shyam R B and Ghosal A, 2015, “Three-Degree-of-Freedom Parallel Manipulator to Track the Sun for Concentrated Solar Power Systems”, Chinese Journal of Mechanical Engineering, July 2015, Volume 28, pp 793–800, 2015
[10] Z. Yang and D. Zhang, “Reconfigurable 3-PRS Parallel Solar Tracking Stand”, Proceedings of the ASME 2017 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, IDETC2017 August 6-9, 2017, Cleveland, Ohio, USA
[11] D. Zhang, Chapter 7 Reconfigurable Parallel Kinematic Machine Tools, Parallel Robotic Machine Tool, Springer 2010
[12] J. Sirohi and R. Mahadik, 2011, “Piezoelectric Wind Energy Harvester for Low-Power Sensors”, Journal of Intelligent Material Systems and Structures 22(18) 2215–2228, 2011
[13] J.X. Tao, N.V. Viet, A. Carpinteri, and Q. Wang, 2017, “Energy harvesting from Wind by A Piezoelectric Harvester”, Engineering Structures 133 (2017) 74–80
[14] J. Sirohi and R. Mahadik, 2012, “Harvesting Wind Energy Using a Galloping Piezoelectric Beam”, Journal of Vibration and Acoustics Transactions of ASME, Vol. 134, FEBRUARY 2012
[15] L. M. Murphy, 1980, “Wind Loading on Tracking and Field Mounted Solar Collectors”, Solar Energy Research Institute, United States. doi: 10.2172/6889663.
[16] S. Bhadurl and L. M. Murphy, 1985, “Wind Loading on Solar Collectors”, Solar Energy Research Institute, United States, 1985
[17] Linear Table Alpha 15-B-155, HSB alpha Release: 12.12.2016
[18] Linear Actuators: DLA Series and Accessories, 50 mm to 300 mm stroke 1000 N, Transmotec 2018
[19] L. Schreiber and C. Gosselin, 2017, “Passively Driven Redundant Spherical Joint With Very Large Range of Motion”, Journal of Mechanisms and Robotics Transactions of ASME, JUNE 2017, Vol. 9, 2017

[20] G. Yuan and D. H. Wang, 2017, “A Piezoelectric Six-DOF Vibration Energy Harvester Based on Parallel Mechanism: Dynamic Modeling, Simulation, and Experiment”, Smart Materials and Structures, 26 (2017)

[21] “Rotational Matrix” from wikipedia.org

[22] “Genetic Algorithm: Find Global Minima for Highly Nonlinear Problems” from mathworks.com

[23] J. Li, J. Wang, W. Chou, Y. Zhang, T. Wang and Q. Zhang, “Inverse Kinematics and Dynamics of the 3-RRS Parallel Platform”, Proceedings of the 2001 IEEE International Conference on Robotics and Automation, May 21-26 2001, Seoul Korea

[24] Y. Li and Q. Xu, “Kinematics and Stiffness Analysis for a General 3-PRS Spatial Parallel Mechanism”, ROMANSY Conference 2004, June 14-18, 2004, Montreal, Canada

[25] S. R. Babu, V. R. Raju and K. Ramji, 2013, “Design for Optimal Performance of 3-RPS Parallel Manipulator Using evolutionary Algorithms”, Transactions of the Canadian Society for Mechanical Engineering, Vol. 37, No. 2, 2013

[26] R. G. Budynas and J. K. Nisbett, Shigley’s Mechanical Engineering Design, Ninth Edition, McGraw-Hill 2008, pp.1013

[27] “Material: Lead Zirconate Titanate (PZT)” from memsnet.org

[28] C. Gosselin, “Stiffness Mapping for Parallel Manipulators”, IEEE Transactions on Robotics and Automation, Vol. 6, Issue 3, Jun 1990