Level Crossing Analysis of the Stock Markets

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We investigate the average frequency of positive slope $\nu^{+}$, crossing for the returns of market prices. The method is based on stochastic processes which no scaling feature is explicitly required. Using this method we define new quantity to quantify stage of development and activity of stocks exchange. We compare the Tehran and western stock markets and show that some stocks such as Tehran (TEPIX) and New Zealand (NZX) stocks exchange are emerge, and also TEPIX is a non-active market and financially motivated to absorb capital.

I. INTRODUCTION

In recent years, financial markets have been at focus of physicists’s attempts to apply existing knowledge from statistical mechanics to economic problems. Statistical properties of price fluctuations are very important to understand and model financial market dynamics, which has long been a focus of economic research. These markets, though largely varying in details of trading rules and traded goods, are characterized by some generic features of their financial time series. The aim is to characterize the statistical properties of given time series with the hope that a better understanding of the underlying stochastic dynamics could provide useful information to create new models able to reproduce experimental facts. An important aspect concerns the ability to define concepts of activity and degree of development of the markets. Acting on advantageous information moves the price such that the a priori gain is decreased or even destroyed by the feedback of the action on the price. This makes concrete the concept that prices are made random by the intelligent and informed actions of investors, as put forward by Bachelier, Samuelson, and many others. In contrast, without informed traders, the profit opportunity remains, since the buying price is unchanged. Based on recent research for characterizing the stage of development of market, it is well known that the Hurst exponent shows remarkable differences between developed and emerging markets.

Here we introduce "level crossing" to analyze these time series. Which is based on stochastic processes grasps the scale dependency of time series, and no scaling feature is explicitly required. Also this approach has turned out to be a promising tool for other systems with scale dependent complexity (see (35,37) for it application to characterize the roughness of growing surfaces). Some authors have applied this method to study the fluctuations of velocity fields in Burgers turbulence, and in the Kardar-Parisi-Zhang equation in d + 1-dimensions.

The level crossing analysis is very sensitive to correlation when the time series is shuffled and to probability density functions (PDF) with fat tails when the time series is surrogate. The long range correlations are destroyed by the shuffling procedure and in the surrogate method the phase of the discrete Fourier transform coefficients of time series are replaced with a set of pseudo-independent distributed uniform ($-\pi,+\pi$) quantities. The correlations in the surrogate series do not change, but the probability function changes to Gaussian distribution.

Level crossing with detecting correlation is a useful tool to find the stage of development of markets, too. It is known that emerging markets have long-range correlation. This sensitivity of level crossing to the market conditions provides a new and simple way of empirically characterizing the development of financial markets. This means that mature markets the total level crossing are decreased under shuffling effectively, while emerging markets are increased in agreement with the findings of Di Matteo et al (2003) and (2005) which indicate that emerging markets have $H = 0.5$, while mature markets have $H = 0.5$. [H is the Hurst exponent]. The level crossing analysis is more simple calculation than the other methods such as, Detrended Fluctuation Analysis (DFA), Detrended Moving Average (DMA), Wavelet Transform Modulus Maxima (WTMM), Rescaled Range analysis (R/S), Scaled Windowed Variance (SWV), etc. It is well known that, R/S, SWV and other non-detrending methods work well if the records are long and do not involve trends. Also in the detrending method one must make attention that, in some cases, there exist one or more crossover (time) scales separating regimes with different scaling exponents. In this case investigation of the scaling behavior is more complicated and different scaling exponents are required for different parts of the series. Therefore one needs a multitude of scaling exponents (multifractality) for a full description of the scaling behavior. Crossover usually can arise either because of changes in the correlation properties of the signal at different time (space) scales, or can often arise from trends in the data. The level crossing analysis does not require modulus maxima procedure in contrast with WTMM method, and hence does not require lot’s of effort in to write computing code and computing time, than the above methods. So the level crossing analysis is more suitable for short time series.
This paper is organized as follows. In section II we discuss the level crossing in detail. Data description and analysis based on this method for some stocks indices are given in section III. Section IV closes with a discussion of the present results.

II. LEVEL CROSSING ANALYSIS

Let us consider a time series \( \{ p(t) \} \), of price index with length \( n \), and the price returns \( r(t) \) which is defined by \( r(t) = \ln p(t + 1) - \ln p(t) \). Here, we investigate the detrended log returns for different time scales.

Let for time interval \( T \) \( \nu^+_\alpha \) denotes the number of positive difference crossing \( r(t) - \bar{r} = \alpha \) (see fig.1) and also mean value for all the samples be \( N^+_\alpha (L) \) where

\[
N^+_\alpha (T) = E[n^+_\alpha (T)].
\]  
(1)

Since after detrending \( r(t) \), is stationary (i.e. averaged variance saturate to a certain value) if we take second consecutive time interval \( T \) we obtain the same result, and therefore for two intervals together we have

\[
N^+_\alpha (2T) = 2N^+_\alpha (T),
\]  
(2)

from which it follows that, for stationary process, the average number of crossing is proportional to the space interval \( T \). Hence

\[
N^+_\alpha (T) \propto T,
\]  
(3)

or

\[
N^+_\alpha (T) = \nu^+_\alpha T.
\]  
(4)

which \( \nu^+_\alpha \) is the average frequency of positive slope crossing of the level \( r(t) - \bar{r} = \alpha \). We show how the frequency parameter \( \nu^+_\alpha \) can be deduced from the underlying probability distribution function PDF for \( r(t) - \bar{r} \). In time interval \( \Delta t \) the sample can only cross \( r(t) - \bar{r} = \alpha \) with positive difference if it has the property \( r(t) - \bar{r} < \alpha \) at the beginning of this time interval. Furthermore there is a minimum difference at time \( t \) if the level \( r(t) - \bar{r} = \alpha \) is to be crossed in interval \( \Delta t \) depending on the value of \( r(t) - \bar{r} \) at time \( t \). So there will be a positive crossing of \( r(t) - \bar{r} = \alpha \) in the next time interval \( \Delta t \) if, at time \( t \),

\[
\frac{\Delta [r(t) - \bar{r}]}{\Delta t} > \frac{\alpha - [r(t) - \bar{r}]}{\Delta t}.
\]  
(5)

As shown in fig.1 the frequency \( \nu^+_\alpha \) can be written in terms of joint PDF of \( p(\alpha, y') \) as follows

\[
\nu^+_\alpha = \int_0^\infty p(\alpha, y')y'dy'.
\]  
(6)

where \( y' = \frac{r(t+\Delta t) - r(t)}{\Delta t} \). Here we put \( \Delta t = 1 \).

Let us also introduce the quantity \( N_{tot}^+ (q) \) as

\[
N_{tot}^+ (q) = \int_{-\infty}^{+\infty} \nu^+_\alpha |\alpha - \bar{\alpha}|^q d\alpha.
\]  
(7)

FIG. 1: positive slope crossing in a fixed \( \alpha \) level.

FIG. 2: The positive difference crossing of return price for S&P500 and TEPIX market in the same time interval.

where zero moment (with respect to \( \nu^+_\alpha \) \( q = 0 \), shows the total number of crossing for return price with positive slope. The moments \( q < 1 \) will give information about the frequent events while moments \( q > 1 \) are sensitive for the tail of events.

III. APPLICATION ON STOCK MARKET

Investments in the stock market are based on a quite straightforward rule: if you expect the market to go up in the future, you should buy (this is referred to as being long in the market) and hold the stock until you expect the trend to change direction; if you expect the market to go down, you should stay out of it, sell if you can (this is referred to as being short of the market) by borrowing a stock and giving it back later by buying it at a cheaper in the future. It is difficult,
to say the least, to predict future directions of stock market prices even if we are considering time scales of the order of decades, for which one could hope for a negligible influence of noise.

The reason why, in very liquid markets of equities and currency exchanges correlations of returns are extremely small, is because any significant correlation would lead to an arbitrage opportunity that is rapidly exploited and thus washed out. Indeed, the fact that there are almost no correlations between price variations in liquid markets can be understood from simple calculation by $\alpha$.
turns, $r(t)$ is a deviation from the fitting function. According to Eq. 1, the level crossing, $\nu_{\alpha}^+$, is calculated for each index. Figure 2 shows a comparison of $\nu_{\alpha}^+$ for TEPIX and S&P500 as a function of level $\alpha$. It is clear that $\nu_{\alpha}^+$ scales inversely with time, so $\tau(\alpha) = \frac{1}{\nu_{\alpha}^+}$ is a time interval, within this time, the level crossing in average will be observed again. Table I shows the time interval in the high frequency ($\tau(\alpha = 0)$) and the low frequency (tails, $\tau(\alpha = \pm 3\sigma)$) regimes for some indices. The time interval $\tau(\alpha = 0)$ of TEPIX and S&P500 are 7.0 and 4.0 days, respectively. Still, in the tail, it is comparable. Another difference between TEPIX and the other markets (except Amex market) is seen in the time interval of $\tau(\alpha = -3\sigma)$ and right $\tau(\alpha = +3\sigma)$ tails. The time length in left tail is larger than time length in right but also less than other markets and also in TEPIX and NZX the mean $\tau$ is 0.24 and 0.17, respectively. They mean that TEPIX is financially motivated to absorb capital. It is clear that when we apply Eq. 4 for small $q$ regime, high frequency is more significant whereas in the large $q$ regime, low frequency (the tail) is more significant. Figure 3 shows that when $q < 2$, the value of $N_{\alpha}^+$ for TEPIX is smaller than that of S&P500, while for $q > 2$, the value of $N_{\alpha}^+$ for TEPIX gets larger than the other markets. This is because for small $q$ the low frequency events of tails are more significant than the high frequency peak. According to the last section and Eq. 5 the area under the level crossing curve, $N_{\alpha}^+$, shows the total number of crossing. This means that the larger the area, the larger the activity. In essence, by comparing $N_{\alpha}^+(q = 0)$, activity of the index is obtained.

From another point of view, based on recent research of characterizing stage of development of markets, it is shown that the Hurts exponent is sensitive to the degree of development of the market. Emerging markets are associated with high value of Hurts exponent and developed markets are associated with low value of the exponent. In particular, it is found that all emerging markets have Hurts exponents larger than 0.5 (strongly correlated) whereas all the developed markets have Hurts exponents near to or less than 0.5 (white noise or anti-correlated). Here we have shown that the level crossing has ability to characterize degree of development of markets. The sensitivity of the level crossing to the market conditions provides a new and more simple way of empirically characterizing activity and development of financial markets.

Since $N_{\alpha}^+(q = 0)$ is very sensitive to correlation, it increases when the time series is shuffled so that the correlation disappears. Thus, by comparing the change between $N_{\alpha}^+(q = 0)$ and $N_{\alpha}^+(q = 0)$ (shuffled), the stage of development of markets can be determined. Figure 4 shows $\nu_{\alpha}^+$ as a function of $\alpha$ for original and shuffled data in TEPIX and S&P500. The relative changing of $N_{\alpha}^+(q = 0)$ for TEPIX and S&P500 are 0.41 and 0.02 respectively. For the sake of comparison between various stock markets, the two parameters $N_{\alpha}^+$ and $N_{\alpha}^+$, relative variation of $N_{\alpha}^+$, $q = 0$ for original data, its shuffled and Hurst exponent which was obtained by using the detrended fluctuation analysis (DFA) method, are reported in Table II. We notice that TEPIX and NZX belong to the emerging markets category; it is far from an efficient and developed market. These result are comparable with the results reported in and show that Tehran stock exchange belongs to the category of emerging financial markets.

### IV. CONCLUSION

In this paper concept of level crossing analysis has been applied to several stock market indices. It is shown that the level crossing is able to detect activity of markets. This method is based on stochastic processes which should grasp the scale dependency of any time series in a most general way. No scaling feature is explicitly required. Based on the recent research for characterizing the stage of development of markets, it is shown that level crossing is sensitive to degree of development of market, too. This sensitivity of level crossing to market conditions provides a new and simple way of empirically characterizing the activity and development of financial markets. Considering all of the above discussions and results, we notice that Tehran Stock Exchange belongs to the emerging markets category. It is far from an efficient and developed market and also we have found that it is financially motivated to absorb capital. Using this method we classify the activity and stage of development of some markets.

| Market    | $\tau(\alpha = 0)$ | $\tau(\alpha = 3\sigma)$ | $\tau(\alpha = -3\sigma)$ |
|-----------|---------------------|---------------------------|---------------------------|
| S&P500    | 4.0                 | 218.4                     | 115.1                     |
| Djindu    | 4.0                 | 178.3                     | 150.0                     |
| Biogen    | 4.0                 | 179.1                     | 113.8                     |
| 10ytsy    | 4.2                 | 656.3                     | 98.2                      |
| Composit  | 4.3                 | 178.6                     | 140.1                     |
| Amex      | 4.6                 | 115.1                     | 178.4                     |
| NZX       | 5.3                 | 135.3                     | 120.3                     |
| TEPIX     | 7.0                 | 102.8                     | 114.2                     |

| Market    | $N_{\alpha}^+_{sh}$ | $N_{\alpha}^+_{tot}$ | $|N_{\alpha}^+_{sh} - N_{\alpha}^+_{tot}| / N_{\alpha}^+_{tot}$ | $H$ |
|-----------|---------------------|-----------------------|-----------------------------------------------------------------|-----|
| S&P500    | 0.52                | 0.53                  | 0.02                                                            | 0.44±0.01 |
| Djindu    | 0.51                | 0.52                  | 0.02                                                            | 0.46±0.01 |
| 10ytsy    | 0.50                | 0.52                  | 0.04                                                            | 0.47±0.01 |
| Biogen    | 0.48                | 0.51                  | 0.06                                                            | 0.51±0.01 |
| Composit  | 0.50                | 0.52                  | 0.04                                                            | 0.45±0.01 |
| Amex      | 0.45                | 0.50                  | 0.10                                                            | 0.51±0.01 |
| NZX       | 0.40                | 0.52                  | 0.30                                                            | 0.61±0.01 |
| TEPIX     | 0.32                | 0.45                  | 0.41                                                            | 0.74±0.01 |
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