Modeling of unsteady flow of viscous fluid in the channel of complex geometry

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Abstract. The article concerns an issue of exploring the mechanism of wave influence on the process of filtration. To describe a filtration flow, the porous medium is represented as a capillary, the radius of which varies sinusoidally. In this article we are presenting the results of numerical modeling of pulsating liquid flow in a sinusoidally-shaped channel. Numerical research was conducted with the help of the program complex FlowVision. As a result of series of calculations we received fields of velocities and pressure in the axial section of flowing channel. It was found that it is the flow in narrow isthmuses between the pores that contributes to the pressure difference the most. We revealed the signs of steady-state liquid flow in the channel when imposing fluctuations of pressure in the absence of pressure gradient. The conditions of formation of such flow are revealed.

1. Introduction
Effective mastering of oil reserves is a topical and complicated task, the resolution to which requires fundamentally new approaches which would allow drastic reducing unit power expenses and material expenses. Numerous theoretical and experimental researches, which took place in laboratory or in trading, show high efficiency of wave method of increasing oil recovery [1-6]. The essence of wave method lies in the way elastic vibrations influence on productive formation and bottom-hole area of the hole. The most perspective thing to do is to combine the methods of increasing oil recovery with wave influence on productive formation [7-10]. Variety of events that take place under this influence can be summed to two factors which provide increasing the efficiency of oil production. These are acceleration of filtration process and involvement of dead oil into filtration process [11-13]. This article is an attempt to model the filtration process under wave influence on formation numerically which is aimed at studying the mechanism of acceleration of filtration flows in porous mediums when imposing elastic vibrations.

To describe a filtration flow in porous mediums it is quite common to use mathematical models in which the real porous medium is represented as a system of capillaries, cracks, packages with spheres etc. One of the most popular ones is the capillary model which consists of a beam of straight parallel capillaries, radiuses of which are allocated according to some function of allocation.

The subject of our investigation is the flow of fluid in a cylindrical sinusoidally-shaped channel when imposing a wave field. Geometrical sizes of this channel are commensurable with sizes of pores in productive formations of oil and gas fields.
2. Subject of investigation and setting the task
Geometrical model of isotropic porous medium is represented as a narrow channel, the radius of which varies sinusoidally. I.e., pores and narrow isthmuses, which connect adjacent pores, alternate inside of the channel. The sketch of the channel, in which tests were run, is depicted in Figure 1. Characteristic sizes of the channel are the following: diameter of a pore is D=3·10^{-5} m, diameter of a flow section (where it narrows) of the channel is d=0,1·D. The surface of calculation area is a complex of flat polygons that are facets, on which boundary conditions are defined.

![Figure 1. Calculation area. Borders: 1 – input, 2 – wall, 3 – output.](image)

To describe hydrodynamics of the flow we used the model ‘laminar liquid’ as a mathematical model, which describes the flow of viscous liquid with low Mach and Reynolds numbers. The model includes Navier–Stokes equation and continuity equation, the solution to which is performed by the finite volume method with an implicit algorithm of splitting into variables with the program complex FlowVision. Computer modeling was performed for a steady-state flow as well as for the one with imposed pulsations. Pressure pulsations, which were set at the entrance to the calculation area, modeled wave influence on filtration process.

The following boundary conditions were applied to give a task:
1) On the wall as on the border of area the condition of sticking is set:
   \[ \mathbf{V} \bigg|_{w} = 0 \] 
2) At the entrance to the channel pressure-time \( t \) relation is set:
   \[ p(t)=p+p_0\sin(2\pi ft), \] 
where \( p \) is for pressure difference, Pa; \( p_0 \) represents the amplitude of fluctuations, Pa; \( f \) is for frequency of fluctuations, Hz.
3) At the outflow section there is a free exit with zero pressure on the border.

The initial condition was velocity of liquid flow being equal to zero, i.e. liquid is resting and pressure in the channel is equal to the initial one.

The properties of liquid during the calculations are set by the database FlowVision, that is the clean water. Physical characteristics of liquids are the following: density is 998 kg/m^3, viscosity is 0.001 Pa·s. Initial values: temperature is 293 K, pressure is 101 kPa.

3. Results of the numerical research
To investigate influence of different factors (amplitude and frequency of fluctuations) on non-steady-state process of liquid flow in the channel, we conducted series of calculations in an identical calculation area and an identical calculation net. The amplitude of fluctuations varied from 5 to 20
kPa, frequency changed from 200 to 3000 Hz. To make the numerical experiment, a step of time was set for each series and it was equal to $1/(1000f)$.

Figures 2 and 3 depict characteristic allocation of pressure and velocity on a plane of the channel with steady-state flow. From these figures it follows that pressure inside of the ‘pore’ is spread practically evenly. The pressure difference is observed in a narrow part of the channel which connects adjacent ‘pores’. Analysis of velocity allocation allows concluding that maximum velocity is observed in the narrow part of the channel. Meanwhile, the flow is observed in the central part of the channel. Inside of the channel the velocity of liquid decreases to almost zero as we move away from an axis.

![Figure 2. The field of allocation of pressure in the channel.](image)

![Figure 3. The field of allocation of velocity of the liquid in the channel.](image)
For a better understanding of character of pulsating flow of liquid we took into consideration characteristics of velocity by time. The flow is the pulsating motion with a frequency of pressure pulsation at the entrance. However, a deeper analysis leads to a conclusion that pulsations of velocity are imposed on some steady-state flow (average value $\overline{u}$). The numerical value $\overline{u}$ is ten times less than the amplitude of pulsation; nevertheless it is much higher than characteristic value of velocity of filtration.

We studied the average velocity in the narrowing part on different distance from an axis of the channel. We received a non-dimensional profile of relative velocity which is typical for all the series of tests and coincides with the profile of laminar steady-state flow. The dispersion of relative velocities for different frequencies and amplitudes does not exceed 0.1%.

Processing and analyzing the results of numerical experiment allowed defining how average velocity depends on amplitude of fluctuations and frequency. Figure 4 depicts how average velocity on an axis in the narrow part of the channel depends on an amplitude and frequency of fluctuations.

![Figure 4](image_url)

**Figure 4.** Dependence of average velocity of flow in the narrow channel on frequency $f$ and amplitude of fluctuations $p_0$: 1 – 20 kPa; 2 – 15 kPa; 3 – 10 kPa; 4 – 5 kPa.

To determine functional dependence of average velocity of the flow on defining factors, we inserted the following non-dimensional parameters:

$$\frac{\overline{u}}{fd} \quad \text{and} \quad \frac{p_0}{\rho c fd}$$

where $\rho$ represents density of the liquid, kg/m$^3$; $c$ represents speed of sound in the liquid, m/s.

Figure 5 depicts interrelation of these non-dimensional parameters which is approximated by power function.

![Figure 5](image_url)

**Figure 5.** Interrelation of non-dimensional parameters.
From this power function it follows that average velocity is defined the following way:

$$\bar{u} = 1.83 \cdot \left( \frac{p_u}{p_c} \right)^{1.24} \cdot (fd)^{-0.24} \quad (4)$$

The calculated data, averaged by the integer of fluctuation periods, demonstrated that the liquid flow, which is directed from the emission, is formed when imposing pressure fluctuations in the absence of steady-state flow. The velocity allocation through a section of the channel is similar to a steady-state flow during some pressure difference.

4. Conclusion

Analysis of results of the numerical research demonstrated that imposing wave influence on motionless liquid, which is located in a sinusoidally-shaped channel, leads to its pulsating motion. Average velocity is not equal to zero, i.e. steady-state flow arouses towards the allocation of fluctuations; we defined its functional relation. The results open a new direction for researches in the field of influence of pressure fluctuations on the flow of liquids in complex-shaped channels and its practical use for filtration in porous mediums. The results can be used for perfecting methods of wave influence on productive formations.

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