Aeroelastic flutter analysis of functionally graded spinning cylindrical shells reinforced with graphene nanoplatelets in supersonic flow

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Abstract
Aeroelastic analysis of functionally graded spinning cylindrical shells reinforced with graphene nanoplatelets in supersonic flow is studied. Multilayer functionally graded graphene platelets reinforced composite cylindrical shell based on the first-order shear deformation theory are examined. The supersonic flow is modeled through the use of first order piston theory. The effective Young’s modulus, mass density and Poisson’s ratio of nanocomposites are calculated based on the modified Halpin-Tsai model and rule of mixture. The coupled governing equations of motion and associated boundary conditions are developed by applying extended Hamilton principle. Galerkin technique is utilized to convert the coupled equations of motion to a general eigenvalue problem. In this investigation, four graphene platelets distribution patterns through the thickness of shell, i.e., UD, FG- L, FG- X and FG- O are considered. The effects of weight fraction, distribution patterns, number of layers, aspect ratio and spinning velocity on the flutter boundary are expressed. The results point out that the larger surface area related to more distributing graphene platelets near the inner and outer surfaces of the cylindrical shell predicts the most effective reinforcing effect. Furthermore, to improve significantly the stiffness of cylindrical shell, a small amount of extra graphene nanoplatelets as reinforcing nanofillers is an efficient way.

Nomenclature

- $a_{GPL}$: Average length of GPL
- $b_{GPL}$: Average width of GPL
- $C_{\infty}$: Speed of sound
- $C$: Damping matrix
- $E^{(k)}$: Effective Young’s modulus of kth layer
- $E_m$: Young’s moduli of the polymer matrix
- $E_{GPL}$: Young’s modulus of GPL
- $h_k$: Thickness of each layer
- $h$: Total thickness of composite plate
- $I_i$: Rotary inertial coefficients
- $I$: Unitary matrix
- $K$: Stiffness matrix
- $k$: The kth layer
- $L$: Length of circular cylindrical shell
- $M_{\infty}$: Mach number
- $M_{ij}$: Moments resultant about axis

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1. Introduction

To enhance mechanical characteristics and material properties of traditional structures, nano-fiber reinforced composite materials such as carbon nanotubes (CNTs) and graphene nanoplatelets (GPLs) are an important procedure to prepare a further strong and lightweight structure. Functionally graded (FG) structures reinforced with GPLs have widely used in aerospace structures, sports equipment and wind turbines [1, 2]. Due to outstanding mechanical and physical properties, they have a great deal of attention among science and engineering. Yang et al [3] considered the buckling and post-buckling properties of FG beams reinforced with GPLs. Nonlinear bending and free vibration of the FG multilayer beams reinforced with GPLs were inspected by
Feng et al [4, 5]. Free vibration behaviors and buckling analysis of FG porous beams reinforced by GPLs investigated by Kitipornchai et al [6]. Wu et al [7] performed the dynamic instability responses of FG multilayer nanocomposite beams reinforced with GPLs under thermal and periodic axial loading. Song et al [8] examined the effects of GPL distribution pattern, weight fraction, geometry and size as well as the total number of layers on the forced and free vibration of FG plates reinforced by GPLs. The mechanical characteristics of epoxy nanocomposites reinforced by GPLs, single and multi-walled CNTs were studied by Rafiee et al [9]. The influences of GPL distribution pattern, the total number of layers, weight fraction, geometry and size on the buckling and vibration of functionally graded multilayer graphene nanoplatelet composite plates were done by Song et al [8, 10]. Buckling and static bending analyses as well as the free vibration behavior of multilayer functionally graded GPLs reinforced composite plates were inspected by Chai et al [11]. Dynamic responses of functionally graded doubly-curved composite panels with arbitrary geometries and boundary conditions was investigated by Rafiee-Anamagh and Bediz [12]. Bending and vibration analyses of functionally graded trapezoidal plates reinforced with GPLs were examined by Zhao et al [13]. Using the generalized differential quadrature method Tornabene et al [14] presented the vibrations analysis of laminated nanocomposite plates and shells reinforced with CNT and fiber. Pashmforoush [15] proposed a statistical study on the free vibration behavior of functionally graded GPLs reinforced composite plates. The effect of the porosity coefficient, weight fraction of GPLs, various porosity distributions and GPL dispersion patterns on the vibration analysis of plates with piezoelectric layers were studied by Nguyen et al [16]. In the frame work of three-dimensional elasticity theory including state-space formulation, The buckling analysis and free vibration of FG cylindrical shell reinforced with GPLs were investigated by Liu et al [17]. Free vibration and buckling responses of the FG porous spinning cylindrical shell reinforced by GPLs were deliberated by Dong et al [18, 19]. Barati and Zenkour [20] examined free vibrational behavior of porous nanocomposite shells reinforced with uniformly and non-uniformly distributed GPLs. Free vibration and static bending analysis of FG composite doubly-curved shallow shells reinforced with GPL are analyzed by Wang et al [21]. Recently, researchers announced the concept of non-uniformly dispersed graphene reinforced polymer nanocomposites in the epoxy matrix and studied the aeroelasticity [22–25], nonlinear free vibration [26], nonlinear bending, buckling and vibration, dynamic stability [27–31] of functionally graded structures reinforced with CNTs/GPLs.

Recently, the aeroelastic behavior of plates and shells subjected to supersonic flow become an important part of researches which attracted lots of considerations by many researchers and engineers in the field of fluid and solid interaction. Therefore, the stability behavior of structures under supersonic flow is a vital and great topic to study. A comprehensive study about nonlinear vibration and stability of plates and shells due to supersonic flow is published by Amabili [32]. In another study Amabili and Pellicano [33, 34] analyzed both linear and nonlinear aeroelastic stability of circular cylindrical shells under supersonic airflow utilizing Donnell’s nonlinear shell theory and Galerkin’s solution procedure. Akhavan and Ribeiro [35] investigated the effects of boundary conditions, fiber angles and airflow on the flutter and divergence instabilities of the composite plate with curvilinear fiber subjected to airflow. Aeroelastic flutter response of a composite laminated circular cylindrical shells combined mechanical and aerodynamic loading was studied by Chen and Li [36]. Donadon and Faria [37] investigated aeroelastic flutter of hybrid composite shells made of shape memory alloy in supersonic flow. The aeroelastic buckling and flutter instability of FG composite circular cylindrical shell reinforced with CNTs in supersonic flow were performed by Asadi and Wang [38]. The supersonic panel flutter of FG circular cylindrical imperfect shells subjected to thermal and aerodynamic loading is investigated by Mahmoudkhani [39]. The flutter and divergence instabilities of truncated conical shells reinforced with CNTs in flowing fluid were scrutinized by Mohammad et al [40]. Mokhtari et al [41] studied the flutter instability of sandwich cylindrical shells under aerodynamic pressure based on the both Donnell-Mushtari theory and first-order shear deformation theory for facing layers and viscoelastic layer, respectively. The aeroelastic instability of magnetorheological fluid sandwich panels under supersonic airflow was discussed in [42].

Determining flutter boundary of FG multilayer nanocomposite spinning circular cylindrical shell reinforced with GPLs subjected to supersonic flow is the main purpose of current study. Using the first-order shear deformation theory, the governing equations of motion and boundary conditions are derived based on the extended Hamilton’s principle theory. Besides, the first order piston theory is applied to simulate the supersonic aerodynamic pressure exerted on the outer surface of circular cylindrical shell. Modified Halpin-Tsai micromechanics model and rule of mixture are used to achieve the effective material properties of FG multilayer nanocomposite circular cylindrical shell reinforced with GPLs. Using Galerkin method, the discretization of governing equations of motion is performed. The obtained results are prepared to consider the effects of GPL distribution pattern, GPL weight fraction, number of layers, geometric parameters, spinning velocity and Mach number on the flutter aerodynamic pressure of the FG multilayer circular cylindrical shells reinforced with GPLs in supersonic flow.
2. Mathematical formulation

A configuration model of a circular cylindrical shell is illustrated in figure 1 which made of metal foams reinforced by GPL, here an orthogonal coordinate system (x, θ, z) is set at the center point of surface. The geometric parameters of shell are as: L is the length of circular cylindrical shell, radius is R and h stands for thickness. $u_0$, $v_0$ and $w_0$ denote the displacements of the middle surface of circular cylindrical shell along the x, θ and z axes, respectively. It is considered that the circular cylindrical shell spins about its longitudinal direction with a constant angular speed $\Omega$.

The displacement fields of circular cylindrical shell based on the first order shear deformation theory are given as below, which defined through five generalized time-dependent displacement variables ($u_0$, $v_0$, $w_0$, $\phi_x$ and $\phi_\theta$) [43]

$$u_1(x, \theta, z; t) = u_0(x, \theta; t) + z\phi_x(x, \theta; t)$$
$$u_2(x, \theta, z; t) = v_0(x, \theta; t) + z\phi_\theta(x, \theta; t)$$
$$u_3(x, \theta, z; t) = w_0(x, \theta; t)$$  \hspace{0.5cm} (1)
Here the displacement components of the any point of circular cylindrical shell are denoted by \( u_i (i = 1, 2, 3) \) in \( x, \theta \) and \( z \) directions, respectively; then, transverse normal rotations about the \( x \) and \( \theta \) directions are shown by \( \phi_x \) and \( \phi_\theta \).

On the basis of first order shear deformation shell theory, strain field, \( \varepsilon^t = [\varepsilon_{xx}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \gamma_{xz}, \gamma_{xy}] \), of the circular cylindrical shell in terms of the middle surface displacement components \( u_0, v_0, w_0 \) and transverse normal rotations \( \phi_x \) and \( \phi_\theta \) can be stated as follow

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{\theta\theta} \\
\varepsilon_{zz} \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\varepsilon^0_{xx} \\
\varepsilon^0_{\theta\theta} \\
\varepsilon^0_{zz} \\
\phi_x + w_0,x \\
\phi_\theta + \frac{1}{R} w_0,\theta
\end{bmatrix} + \frac{P}{E} \begin{bmatrix}
k_{xx} \\
k_{\theta\theta} \\
k_{zz} \\
q_{xz} \\
q_{xy}
\end{bmatrix}
\]

(2a)

Where \( \varepsilon^0_{xx}, \varepsilon^0_{\theta\theta} \) and \( \varepsilon^0_{zz} \) show the normal and shear strains of the middle surface in the \( x - \theta \) plane, and \( k_{xx}, k_{\theta\theta} \) and \( k_{zz} \) stand for the curvatures. The following terms belong to them

\[
\begin{align*}
\varepsilon^0_{xx} &= u_{0,xx}, \quad \varepsilon^0_{\theta\theta} = \frac{1}{R}(v_{0,\theta} + w_0), \quad \varepsilon^0_{zz} = \frac{1}{R}(u_{0,\theta} + v_{0,x}) \\
k_{xx} &= \phi_{x,x}, \quad k_{\theta\theta} = \frac{1}{R}\phi_{x,\theta}, \quad k_{zz} = \frac{1}{R}\phi_{x,x} + \phi_{0,x}
\end{align*}
\]

(2b)

Differentiation with respect to the indicated variables is denoted as commas.

Based on the first order shear deformation shell theory, the relations between strain and stress that are related to the \( k \)th layer of multilayer composite circular cylindrical shell can be given as

\[
\begin{bmatrix}
\sigma_{xx}^{(k)} \\
\sigma_{yy}^{(k)} \\
\sigma_{xy}^{(k)} \\
\sigma_{xz}^{(k)} \\
\sigma_{yz}^{(k)}
\end{bmatrix} = \begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{12} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{66} & 0 & 0 \\
0 & 0 & 0 & Q_{55} & 0 \\
0 & 0 & 0 & 0 & Q_{44}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx}^{(k)} \\
\varepsilon_{yy}^{(k)} \\
\gamma_{xy}^{(k)} \\
\gamma_{xz}^{(k)} \\
\gamma_{yz}^{(k)}
\end{bmatrix}
\]

(3)

Where

\[
\begin{align*}
Q_{11}^{(k)} &= Q_{22}^{(k)} = \frac{E(z)}{1 - \nu^2} \\
Q_{12}^{(k)} &= Q_{21}^{(k)} = \frac{\nu E(z)}{1 - \nu^2} \\
Q_{44}^{(k)} &= Q_{55}^{(k)} = Q_{66}^{(k)} = \frac{1}{2}(Q_{11}^{(k)} - Q_{12}^{(k)}) = \frac{E(z)}{2(1 + \nu)}
\end{align*}
\]

(4)

Where, \( \nu \) and \( E \) are the effective Poisson’s ratio and Effective Young’s modulus of composite plate reinforced with GPLs, respectively. The FG multilayer circular cylindrical shell reinforced by graphene nanoplatelets is made of \( N_L \) layers with equal thickness \( h_L = h/N_L \). The relations of four considered GPL distribution patterns as shown in figure 1(b) in terms of the volume fractions of GPLs of the \( k \)th layer, can be expressed as

\[
\begin{align*}
UD: \quad & V^{(k)}_{\text{GPL}} = V^*_{\text{GPL}}, \\
FG - A: \quad & V^{(k)}_{\text{GPL}} = 2k V^*_{\text{GPL}}/(1 + N_L), \quad k = 1, 2, \ldots, N_L; \\
FG - X: \quad & V^{(k)}_{\text{GPL}} = 4V^*_{\text{GPL}} \left( \frac{1}{2} + \left| k - \frac{N_L + 1}{2} \right| \right)/(1 + N_L), \\
FG - O: \quad & V^{(k)}_{\text{GPL}} = 4V^*_{\text{GPL}} \left( \frac{N_L + 1}{2} + \left| k - \frac{N_L + 1}{2} \right| \right)/(1 + N_L)
\end{align*}
\]

(5)

Where

\[
V^*_{\text{GPL}} = \frac{w_{\text{GPL}}}{w_{\text{GPL}} + (\rho_{\text{GPL}}/\rho_m)(1 - w_{\text{GPL}})},
\]

(6)

In equation (5), the mass density of GPL, mass density of matrix and GPL weight fraction are represented by \( \rho_{\text{GPL}}, \rho_m \) and \( w_{\text{GPL}} \), respectively. The subscripts ‘\( m \)’ and ‘\( \text{GPL} \)’ are shown the matrix and GPL, respectively.

The effective Young’s modulus of composite circular cylindrical shells reinforced with GPLs can be written as [3]
\[
E^{(k)} = 3\left(1 + \xi_L \eta_L V_{\text{GPL}}^{(k)}\right) E_m + 5\left(1 + \xi_T \eta_T V_{\text{GPL}}^{(k)}\right) E_m \\
\eta_L = \frac{(E_{\text{GPL}} / E_m) - 1}{(E_{\text{GPL}} / E_m) + \xi_L}, \quad \eta_T = \frac{(E_{\text{GPL}} / E_m) - 1}{(E_{\text{GPL}} / E_m) + \xi_T}, \\
\xi_L = 2\frac{a_{\text{GPL}}}{t_{\text{GPL}}}, \quad \xi_T = 2\frac{b_{\text{GPL}}}{t_{\text{GPL}}}. 
\]

Where \( E_m \) and \( E_{\text{GPL}} \) stand for Young’s moduli of polymer matrix and GPL, respectively. The average length, width and thickness of GPL are represented by \( a_{\text{GPL}}, b_{\text{GPL}} \) and \( t_{\text{GPL}} \) respectively. The effective mass density \( \rho^{(k)} \) and effective Poisson’s ratio \( \nu^{(k)} \) are obtained as follow via the rule of mixture:

\[
\rho^{(k)} = V_{\text{GPL}}^{(k)} \rho_{\text{GPL}} + (1 - V_{\text{GPL}}^{(k)}) \rho_m, \\
\nu^{(k)} = V_{\text{GPL}}^{(k)} \nu_{\text{GPL}} + (1 - V_{\text{GPL}}^{(k)}) \nu_m, 
\]

Where \( \nu_{\text{GPL}} \) and \( \nu_m \) are the Poisson’s ratio of GPL and polymer matrix, respectively.

### 3. Derivation of equations of motion

Hamilton’s principle is used here to achieve the governing equations of the circular cylindrical shell with spinning motion about axis in supersonic flow as the following form:

\[
\int_{t_0}^{\tau} (\delta U_h + \delta U_e - \delta T + \delta W) dt = 0
\]

Where \( W \) is the work caused by external loads, and there exists in the present circular cylindrical shell model as an aerodynamic pressure due to supersonic flow; \( U_h \) represents the strain energy produced by initial hoop tension and the strain energy of the elastic shell is indicated by \( U_e \). The kinetic energy of circular cylindrical shell in which spinning speed \( \Omega \) is considered by it, named as \( T \). The mentioned terms can be formulated as:

\[
\delta U_h = \int_{0}^{2\pi} \int_{0}^{L} \frac{a}{2} \left[ - \Omega^2 (I_0 u_{,\theta} + I_1 \phi_{,\theta}) \delta u - \Omega^2 (I_0 v_{,\theta} + I_1 \phi_{,\theta}) \delta v \\
+ \Omega^2 (I_0 v_{,\theta} + I_1 \phi_{,\theta}) \delta w - \Omega^2 (I_1 u_{,\theta} + I_2 \phi_{,\theta}) \delta \phi_s \\
- \Omega^2 (I_1 u_{,\theta} + I_2 \phi_{,\theta}) \delta \phi_x \right] dz dx d\theta
\]

\[
\delta U_e = \int_{0}^{2\pi} \int_{0}^{L} \frac{b}{2} \left[ - \left( N_{\theta x} + \frac{M_{\theta \theta}}{R} \right) \delta u - \left( N_{\theta x} + \frac{M_{\theta \theta}}{R} \right) \delta v \\
+ \left( M_{\theta x} - M_{\theta x} - \frac{M_{\theta \theta}}{R} \right) \delta \phi_s + \left( M_{\theta x} - M_{\theta x} - \frac{M_{\theta \theta}}{R} \right) \delta \phi_x \right] dz dx d\theta
\]

\[
\delta T = \int_{0}^{2\pi} \int_{0}^{L} \frac{b}{2} \left[ (I_0 \ddot{u} + I_1 \ddot{\phi}_s) \delta u - (I_0 \ddot{u} + I_1 \ddot{\phi}_s) \delta v + (I_0 \ddot{v} + I_1 \ddot{\phi}_s) \delta w - (I_1 \ddot{v} + I_2 \ddot{\phi}_s) \delta \phi_s \\
+(I_1 \ddot{v} + I_2 \ddot{\phi}_s) \delta \phi_x \right] dz dx d\theta
\]

\[
\delta W = \int_{0}^{2\pi} \int_{0}^{L} \frac{b}{2} \Delta P \delta w dx dz d\theta
\]

Here, the centrifugal force generated the initial hoop tension. Also, the differentiation with respect to time \( t \) corresponds to the conventional differential symbol is shown by the dot-superscript. And \( \Delta P \) represents as the aerodynamic pressure of supersonic flow. Then, the stress resultants and moments defined as below:

\[
[N_{ij}, M_{ij}] = \int_{-h/2}^{h/2} \sigma_g [1, z] dz \quad (i, j = x, \theta, z)
\]

Furthermore, the rotary inertial coefficients written as:

\[
[I_{0x}, I_{0\theta}, I_{1z}] = \int_{-h/2}^{h/2} \rho(z) [1, z, z^2] dz
\]
Moreover, the linear aerodynamic pressure can be expressed as

\[
\Delta P = \frac{\gamma p_\infty M^2_\infty}{\sqrt{M^2_\infty - 1}} \left( w_{x\infty} + \frac{M^2_\infty - 2}{M^2_\infty - 1} \frac{1}{M_\infty C_\infty} w - \frac{1}{2R\sqrt{M^2_\infty - 1}} w \right)
\]  

(17)

Which, \(p_\infty\), \(M_\infty\), \(C_\infty\) and \(\gamma\) represent the free stream static pressure, Mach number, speed of sound and adiabatic exponent of air, respectively. In this simulation the effects of boundary layer is neglected and assumed that the fluid is inviscid [33]. Also, the first, second and third terms of this equation indicate the relative displacement, aerodynamic damping and curvature effect, respectively.

The governing equations of circular cylindrical shell consist of stress resultants, moments and the components of displacement are determined as the following forms through the substituting equations (11)–(14) into (16):

\[
N_{xx,x} + \frac{1}{R} N_{x0,0} + \Omega^2 (I_0 u_{x,0} + I_1 \phi_{x,00}) = I_0 \ddot{u} + I_1 \ddot{\phi}_x
\]  

(18a)

\[
\frac{1}{R} N_{00,0} + N_{x0,x} + \frac{1}{R} Q_{zz} + \Omega^2 (I_0 v_{x,0} + I_1 \phi_{x,00} + 2I_0 \dot{w}) + 2\Omega \dot{I}_0 \dot{w} = I_0 \ddot{v} + I_1 \ddot{\phi}_y
\]  

(18b)

\[
Q_{xx,x} + \frac{1}{R} Q_{x0,0} - \frac{1}{R} N_{00,0} - \Omega^2 [I_0 w_{x,0} - 2(I_0 v_{x,0} + I_1 \phi_{x,00})] - 2\Omega (I_0 \ddot{v} + I_1 \ddot{\phi}_y)
\]  

(18c)

\[
M_{xx,x} + \frac{1}{R} M_{x0,0} - Q_{zz} + \Omega^2 (I_1 u_{x,0} + I_2 \phi_{x,00}) + 2\Omega I_1 \dot{w} = I_1 \ddot{u} + I_2 \ddot{\phi}_y
\]  

(18d)

\[
M_{00,0} + \frac{1}{R} M_{x0,x} - Q_{00} + \Omega^2 (I_1 v_{x,0} + I_2 \phi_{x,00} + 2I_1 \dot{w}) + 2\Omega I_1 \dot{w}
\]  

(18e)

The mechanical edge boundary conditions are considered as

(i) Simply supported

\[
N_{xx} = 0, \quad v = 0, \quad \phi_x = 0, \quad M_{xx} = 0, \quad w = 0,
\]

(19a)

(ii) Clamped

\[
u = 0, \quad \phi_x = 0, \quad \phi_{\theta} = 0, \quad w = 0,
\]

(19b)

4. Solution method

There are a lot of paper which implemented the solid-fluid interaction issue through the use of GDQ and Galerkin’s method [22, 44–46, 47, 48]. Here the Galerkin solution procedure is utilized. In order to solve equation (18) the Galerkin’s solution approach is performed. In this method the displacement field is expressed as a series as below

\[
\begin{bmatrix}
u \\
\dot{v} \\
\dot{w} \\
\phi_x \\
\phi_{\theta}
\end{bmatrix}(t, x, \theta) = \sum_{m=1}^{M} \sum_{n=1}^{N} \begin{bmatrix}
\Psi_\nu(x) \cos(n\theta) q_\nu(t) \\
\Psi_v(x) \sin(n\theta) q_v(t) \\
\Psi_w(x) \cos(n\theta) q_w(t) \\
\Psi_{\phi_x}(x) \cos(n\theta) q_{\phi_x}(t) \\
\Psi_{\phi_{\theta}}(x) \sin(n\theta) q_{\phi_{\theta}}(t)
\end{bmatrix}
\]

(20)

where \(n\) denotes the numbers of circumferential waves, \(\Psi\) and \(q\) (\(i = u, v, w, x, \theta\)) indicate shape mode function and generalized coordinate, respectively. The shape function for three different boundary conditions are expressed as
For simply support:
\[
\begin{pmatrix}
\Psi_0(x) \\
\Psi_1(x) \\
\Psi_2(x) \\
\Psi_3(x) \\
\Psi_4(x)
\end{pmatrix} =
\begin{pmatrix}
\cos(m\pi x) \\
\sin(m\pi x) \\
\sin(m\pi x) \\
\cos(m\pi x) \\
\sin(m\pi x)
\end{pmatrix}
\tag{21}
\]

And for clamped edge boundary condition the following shape mode function can be expressed:
\[
\begin{pmatrix}
\Psi_0(x) \\
\Psi_1(x) \\
\Psi_2(x) \\
\Psi_3(x) \\
\Psi_4(x)
\end{pmatrix} =
\begin{pmatrix}
\partial / \partial x \Psi_0(x) \\
\Psi_0(x) \\
\Psi_1(x) \\
\partial / \partial x \Psi_2(x) \\
\Psi_3(x)
\end{pmatrix}
\tag{22}
\]

Which, the mode shape function \(\Psi_i\) written as
\[
\Psi_i(x) = \cosh(\lambda_i x) - \cos(\lambda_i x) - \zeta_i(\sinh(\lambda_i x) - \sin(\lambda_i x))
\tag{23}
\]

In equation (23), \(\lambda_i\) is a real number associated to number of axial wave and \(\zeta_i\) can be evaluated as:

| Boundary conditions | \(\lambda_i\) |
|--------------------|----------------|
| C-C                | 4.7300, 7.8532, 10.9956, 14.1372, ... |
| C-S                | 3.9266, 7.0686, 10.2102, 13.3518, ... |

Substituting modal expansions (20) into to the governing equation (18) and employing the Galerkin procedure the following standard eigenvalue problem as a matrix form can be achieved,
\[
([D] - \alpha [I])Z(\tau) = 0
\tag{24}
\]

Where
\[
[D] = \begin{bmatrix}
0 & [I] \\
-[M]^{-1}[K] & -[M]^{-1}[C]
\end{bmatrix}
\tag{25}
\]

while \([I]\) is the unitary matrix and \([M], [C]\) and \([K]\) are non-dimensional mass, damping and stiffness matrices of system. To have a nontrivial solution for the equation (24), the determinant of the coefficient matrix vanishes, namely
\[
det([D] - \alpha [I]) = 0
\tag{26}
\]

It is clear from the results that eigenvalues are generally complex quantities, i.e., \(\alpha = Re(\omega) + ilm(\omega)\) because of the system is non-conservative. Also, the imaginary part of the eigenvalue is related to the oscillation frequency, while the real part is the system decaying rate [49–51].

5. Numerical results

In the current section, numerical results are available to find out the dynamic response of FG-GPL reinforced spinning circular cylindrical shell in supersonic flow. The effects of spinning velocity, different distribution patterns of GPL, Mach number and different boundary conditions on the natural frequency and free stream...
The static pressure of FG circular cylindrical shell are numerically studied. First, convergence and comparison studies are undertaken by comparing different types of solutions and they presented methodology in literature. Then, parametric investigation is proposed to study the effects of various mentioned parameters.

To express the primary validation, the first five natural frequencies of isotropic circular cylindrical shell characterized by C-C boundary condition are compared in Table 1 with the results specified by Tornabene et al [52] who employed GDQ method. As illustrated, by comparing the provided data with [52], an excellent convergence related to the current solutions appears when the total mode number is \( N = 14 \).

It is notable that the results of present procedure are very close to those obtained by the mentioned results in the literature. In order to exhibit the reliability and correctness of the proposed model in another way, Table 2 is prepared to show the natural frequency of simply supported circular cylindrical shell that different numbers of modes are assumed. The present results are compared with the results of Dong et al [53] using Galerkin approach to obtain the response of dynamic behavior of isotropic circular cylindrical shell and also, they compared with the experimental results of [54]. As can be observed, there is a very well agreement between the current results and the obtained data by experiments [54]. Similarly, this agreement exists between present results and the results of Dong et al [53] with a small amount of error due to the nonlinear response. Next validation is presented in Table 3 to confirm the applicability and correctness of the proposed formulation and solution method of FGM circular cylindrical shell. First and second fundamental modes of natural frequency of FGM circular cylindrical shell are calculated and compared in Table 3 with those of finite element method (FEM) and DQ technique via [43] in the literature. Different boundary conditions and radius to thickness ratio are assumed, such a way that a close argument is seen for each type. Eventually, to evaluate the accuracy of proposed model, Table 4 is tabulated to

| Table 2. Comparison of the natural frequencies of an isotropic simply supported circular cylindrical shell for different modes number. |
| m | n | Dong et al [53] | Experiment [54] | Present solution |
|---|---|-----------------|-----------------|-----------------|
| 1 | 7 | 488.424 | 484.6 | 484.65 |
| 8 | 494.495 | 489.6 | 489.65 |
| 9 | 551.750 | 546.2 | 546.26 |
| 6 | 555.876 | 553.3 | 553.43 |
| 10 | 642.650 | 636.8 | 636.83 |
| 5 | 723.524 | 722.1 | 722.2 |
| 11 | 756.633 | 750.7 | 750.64 |
| 12 | 888.251 | 882.2 | 882.15 |
| 2 | 10 | 972.446 | 968.1 | 968.27 |
| 11 | 988.407 | 983.4 | 983.49 |

| Table 3. Comparison of the natural frequencies of FG simply supported circular cylindrical shell for different boundary conditions. |
| Boundary conditions | Natural frequency of FGM Cylindrical shell (Hz) |
|---------------------|--------------------------------------------------|
| First fundamental mode | Second fundamental mode |
| Boundary conditions | \( \frac{\rho}{\gamma} \) | FEM [43] | DQ [43] | Present solution | FEM [43] | DQ [43] | Present solution |
| C-C | 40 | 587.30 | 589.60 | 591.65 | 590.80 | 594.40 | 595.31 |
| | 100 | 390.72 | 389.34 | 389.66 | 392.25 | 391.37 | 392.24 |
| C-S | 40 | 528.40 | 530.50 | 529.22 | 535.40 | 539.90 | 536.72 |
| | 100 | 346.93 | 346.98 | 346.51 | 351.82 | 351.99 | 350.33 |
| S-S | 40 | 473.50 | 475.40 | 475.35 | 485.50 | 490.30 | 490.43 |
| | 100 | 302.57 | 302.57 | 303.50 | 316.08 | 317.52 | 317.50 |

| Table 4. Comparison of the natural frequencies of FG circular cylindrical shell for different number of modes (\( V_{\text{CNT}}^0 = 0.12, \frac{k}{\gamma} = 30, L = \sqrt{100R^2}, h = 2 \text{ mm}, M = 3 \)). |
| Number of modes | CC | SS |
|----------------|----|----|
| 5 | 6.306 7408 | 6.864 9991 |
| 7 | 6.341 6002 | 7.064 3797 |
| 9 | 6.291 9123 | 7.074 6908 |
| 12 | 6.284 7296 | 6.969 9427 |
| Asadi and Wang [38] | 6.303 9623 | 6.989 4645 |
determine the critical aerodynamic pressure of simply supported and clamped-clamped FG circular cylindrical shell. The natural frequency of FG circular cylindrical shell is compared with results belong to Asadi and Wang [38] that their results are on the basis of harmonic differential quadrature method (HDQM).

Tabulated results in table 5 investigate the effects of spinning velocity on the free stream static pressure of FG-GPL reinforced circular cylindrical shell. In this table, \( V_{GPL} = 0.2\% \), and \( N_L = 16 \) are considered. The results are listed for pure epoxy circular cylindrical shell and four types of GPL distribution patterns. It can be shown that the free stream static pressure of circular cylindrical shell increases with an increase in spinning velocity, but, no significant change is observed in free stream static pressure with the increase in spinning velocity. As a result, circular cylindrical shell with the higher spinning velocity and FG-X distribution of GPL has more stability. Because, reinforcing effect on the inner and outer surfaces of composite cylindrical shell is more effect on the flexural bending rigidity. Therefore, the stability of FG-X composite cylindrical shell is increased.

The effects of Mach number on the free stream static pressure of FG-GPL reinforced circular cylindrical shell are shown in table 6 for various spinning velocity. The FG-X pattern and \( R = 30 \text{ h}, L = \sqrt[100]{100Rh}, h = 2 \text{ mm} \) are considered. It is observed that the increase of Mach number leads to lower values of free stream static pressure in all values of spinning velocity. Thus, the identical trends of free stream static pressure are observed for different spinning velocity. As can be observed, there is a significant difference between calculated data related to the lowest and highest values of free stream static pressure that state lower value of Mach number can be one of the sufficient factors to have an ideal structure.

The values of free stream static pressure of FG-GPL reinforced circular cylindrical shell with different total number of layers \( N_L \) are tabulated in table 7 in order to show pressure independent of the number of layers. As can be seen, there is no change in free stream static pressure value for the different number of layers related to each GPL distribution patterns. Consequently, there is no ideal approximate for the number of layers to the facility of manufacturing and the construction cost.

**Table 5.** Effect of rotating velocity on the free stream static pressure of FG-GPL reinforced circular cylindrical shell for \( M = 3, R = 30 \text{ h}, L = \sqrt[100]{100Rh}, h = 2 \text{ mm}, N_L = 16 \) and \( V_{GPL} = 0.2\% \).

| \( \Omega \) (Hz) | Pure epoxy | UD | FG-O | FG-A | FG-X |
|-----------------|-----------|----|------|------|------|
| 0               | 1.896 925 | 4.121 846 | 3.778 913 | 4.022 989 | 4.493 093 |
| 20              | 1.897 946 | 4.122 092 | 3.779 103 | 4.023 178 | 4.493 284 |
| 40              | 1.898 388 | 4.122 623 | 3.779 672 | 4.023 746 | 4.493 839 |
| 60              | 1.899 124 | 4.123 582 | 3.780 627 | 4.024 692 | 4.494 817 |
| 80              | 1.899 296 | 4.124 925 | 3.781 969 | 4.026 017 | 4.496 161 |
| 100             | 1.900 629 | 4.126 563 | 3.783 694 | 4.027 721 | 4.497 891 |
| 120             | 1.902 238 | 4.128 765 | 3.785 803 | 4.029 805 | 4.500 005 |
| 140             | 1.904 185 | 4.131 199 | 3.788 295 | 4.032 267 | 4.502 504 |
| 160             | 1.906 405 | 4.134 069 | 3.791 171 | 4.035 109 | 4.505 389 |
| 180             | 1.908 922 | 4.137 323 | 3.794 431 | 4.038 330 | 4.508 660 |
| 200             | 1.911 736 | 4.140 963 | 3.798 075 | 4.041 956 | 4.512 322 |

**Table 6.** Effect of number of Mach number on the free stream static pressure of FG-X circular cylindrical shell for different rotating velocity \( R = 30 \text{ h}, L = \sqrt[100]{100Rh}, h = 2 \text{ mm}, N_L = 16 \) and \( V_{GPL} = 0.2\% \).

| Mach | \( \Omega = 0 \text{ Hz} \) | \( \Omega = 50 \) | \( \Omega = 100 \) | \( \Omega = 200 \) |
|------|-----------------|-----------------|-----------------|-----------------|
| 2    | 7.933 499       | 7.934 168       | 7.936 688       | 7.944 851       |
| 2.5  | 6.364 210       | 6.366 016       | 6.371 436       | 6.393 130       |
| 3    | 4.493 093       | 4.494 305       | 4.497 891       | 4.512 322       |
| 3.5  | 3.363 457       | 3.364 347       | 3.367 020       | 3.377 738       |
| 4    | 2.670 521       | 2.671 234       | 2.673 375       | 2.681 942       |
| 4.5  | 2.216 953       | 2.217 532       | 2.219 271       | 2.226 220       |
| 5    | 1.899 772       | 1.900 268       | 1.901 771       | 1.907 774       |
The effects of GPL distribution patterns on the free stream static pressure of FG circular cylindrical shell are illustrated in figure 2 as a function of GPL volume fraction. Herein, the parameter $R_{30} h$, $L = \sqrt{100Rh}$, $h = 2$ mm and $M = 3$. As can be seen, FG-X pattern is marked to be the best dispersion way, due to the fact that FG-X pattern is capable enough of carrying the highest pressure among all GPL distribution patterns and FG-O pattern predicts the lowest free stream static pressure through the other GPL distribution patterns. Consequently, FG-X circular cylindrical shell has the highest stiffness because of more GPLs distributed closely to surface layers. Obtained results demonstrate that by increasing the volume fraction of GPLs, the static pressure of FG circular cylindrical shell increases drastically. So, higher GPL volume fraction can be one of the best suggestions of the structural performance of FG circular cylindrical shell. It is remarked that pure epoxy circular cylindrical shell is plotted here to compare with other GPL distribution patterns and to show significant difference in system stability that increases as GPL content grows.

To inspect the effect of the Mach number on the free stream static pressure of FG circular cylindrical shell, figure 3 is presented for different GPL distribution pattern and $V_{GPL} = 0.2\%$, $R = 30$ h, $L = \sqrt{100Rh}$, $h = 2$ mm. It can be taken that rising the Mach number causes a visible decrease in the free stream static pressure of the FG circular cylindrical shell. For illustration, the increment of Mach number from 2 to 5 leads to a reduction from 8 to 2.9 MPa in the free stream static pressure parameter. As expected, FG-X pattern of circular cylindrical shell predicts the highest free stream static pressure, moreover FG-X pattern along with less Mach number is an appropriate choice to have more stable system.

Variations of free stream static pressure versus the radius-to-thickness ratio of GPLs are demonstrated in figure 4 to pronounce the effect of Mach number on the free stream static pressure of FG-GPL reinforced circular cylindrical shell. Herein, $R = 30$ h, $L = \sqrt{100Rh}$, $h = 2$ mm and $V_{GPL} = 0.1\%$ are set. It is marked from the figure 4 that the Mach number is a very important factor to calculate the free stream static pressure of FG-GPL reinforced circular cylindrical shell. It can be concluded increasing the Mach number from 2 to 5 cause a substantial reduction in the free stream static pressure of FG-GPL reinforced circular cylindrical shell. One point to note is that, increasing radius-to-thickness ratio plays an essential role to decrease the free stream static pressure of FG-GPL reinforced circular cylindrical shell as far as $R/h = 70$. Therefore, the free stream static pressure of FG-GPL reinforced circular cylindrical shell will be much lower.

| $N_L$ | UD   | FG-O | FG-A | FG-X |
|------|------|------|------|------|
| 2    | 4.121 845 093 | 4.121 901 2626 | 4.093 174 922 | 4.121 901 262 |
| 6    | 4.121 845 093 | 3.857 652 9542 | 4.043 378 319 | 4.042 382 789 |
| 12   | 4.121 845 093 | 3.794 525 2563 | 4.027 194 223 | 4.474 949 703 |
| 16   | 4.121 845 093 | 3.778 918 0904 | 4.022 921 505 | 4.493 101 169 |
| 20   | 4.121 845 093 | 3.769 592 5068 | 4.020 314 865 | 4.504 046 726 |
| 25   | 4.121 845 093 | 3.764 936 3334 | 4.018 204 453 | 4.509 573 110 |

The effects of GPL distribution patterns on the free stream static pressure of FG circular cylindrical shell are illustrated in figure 2 as a function of GPL volume fraction. Herein, the parameter $R = 30$ h, $L = \sqrt{100Rh}$, $h = 2$ mm and $M = 3$ are fixed. As can be seen, FG-X pattern is marked to be the best dispersion way, due to the fact that FG-X pattern is capable enough of carrying the highest pressure among all GPL distribution patterns and FG-O pattern predicts the lowest free stream static pressure through the other GPL distribution patterns. Consequently, FG-X circular cylindrical shell has the highest stiffness because of more GPLs distributed closely to surface layers. Obtained results demonstrate that by increasing the volume fraction of GPLs, the static pressure of FG circular cylindrical shell increases drastically. So, higher GPL volume fraction can be one of the best suggestions of the structural performance of FG circular cylindrical shell. It is remarked that pure epoxy circular cylindrical shell is plotted here to compare with other GPL distribution patterns and to show significant difference in system stability that increases as GPL content grows.
pressure is depending on the radius-to-thickness ratio up to a value \( R/h \) is around 70) and after this value, rising the radius-to-thickness ratio predicts almost a constant value for the free stream static pressure of FG-GPL reinforced circular cylindrical shell.

**6. Conclusion**

In this study, Aeroelastic analysis of FG spinning circular cylindrical shells reinforced with GPLs in supersonic flow was investigated. Based on the first-order shear deformation shell theory and first order piston theory, the governing equations of spinning circular cylindrical shell reinforced with GPLs in supersonic flow were derived. Applying modified Halpin-Tsai model and rule of mixture, the effective Young’s modulus, mass density and Poisson’s ratio were obtained. Galerkin technique is utilized to discretize the coupled equations of motion. The influences of weight fraction, distribution patterns, number of layers, aspect ratio and spinning velocity on the free stream static pressure were studied. It can be observed that the static pressure of FG circular cylindrical shell increases by increasing the volume fraction of GPLs. Also, circular cylindrical shell with FG-X pattern predicts the highest free stream static pressure. Besides, it can be seen that the free stream static pressure of circular cylindrical shell decrease with an increase in the Mach number.

**Data availability statement**

No new data were created or analysed in this study.
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