Interaction of light and semiconductor can generate quantum states required for solid-state quantum computing: entangled, steered and other nonclassical states

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Abstract
Proposals for solid-state quantum computing are extremely promising as they can be used to build room temperature quantum computers. If such a quantum computer is ever built, it would require built-in sources of nonclassical states required for various quantum information processing tasks. Possibilities of generation of such nonclassical states are investigated here for a physical system composed of a monochromatic light coupled to a two-band semiconductor with direct band gap. The model Hamiltonian includes both photon–exciton and exciton–exciton interactions. Time evolution of the relevant bosonic operators is obtained analytically by using a perturbative technique that provides operator solution for the coupled Heisenberg’s equations of motion corresponding to the system Hamiltonian. The bosonic operators are subsequently used to study the possibilities of observing single- and two-mode squeezing and antibunching after interaction in the relevant modes of light and semiconductor. Further, entanglement between the exciton and photon modes is reported. Finally, the nonclassical effects have been studied numerically for the open quantum system scenario. In this situation, the nonlocal correlations between two modes are shown to violate EPR steering inequality. The observed nonclassical features, induced due to exciton–exciton pair interaction, can be controlled by the phase of input field, and the correlations between two modes are shown to enhance due to nonclassicality in the input field.

Keywords Generation of entanglement · Quantum steering · Solid-state quantum computing · Light–semiconductor interaction
1 Introduction

During the last few decades, a large number of exciting phenomena have been observed in the field of quantum optics. These include the generation of nonclassical states [1,2] of radiation field (e.g., generation of antibunched [3,4], squeezed [5,6], entangled [7] and steered [8] states of light) and the experimental realization of a set of phenomena (e.g., laser without inversion [9], enhancement of refractive index [10], electromagnetically induced transparency [11], absorptionless dispersion [12], coherent population trapping [13]), which were never observed before. The role of squeezed state in the successful detection of gravitational wave [14] and in the noise-free transmissions [6]; the advent of quantum information technology ([15,16] and references therein) as well as quantum state engineering [17–21] have further enhanced the excitement. Specifically, it may be noted that quantum supremacy has been strongly established through the realization of tasks like quantum teleportation [22], unconditionally secure quantum cryptography [23], etc. None of these tasks can be realized in the classical world, and consequently realization of any of these tasks would require one or more quantum states having no classical analogue. Such states are referred to as the nonclassical states and are characterized by the negative values of Glauber–Sudarshan $P$-function [1,2].

As nonclassical states can be used to perform tasks that cannot be done using classical resources, the interest on them is ever increasing. Possibilities of observing nonclassical features have been investigated in various physical systems. Specifically, in the recent past nonclassical properties have been studied in nonlinear optical couplers [24,25], Bose–Einstein condensates [26,27], Raman [28,29] and hyper-Raman [30] processes, optomechanical and optomechanics-like systems [31], etc. However, relatively less attention has been given to the possibilities of generation of nonclassical states in semiconductors, although that may provide quantum resources for solid-state quantum computing [32–35]. Motivated by these facts, in what follows, we study the possibilities of generation of nonclassical states (e.g., squeezed, antibunched, entangled, and steering states) through the interaction of monochromatic light with a two-band semiconductor.

The possibilities of generation of nonclassical states through the light–semiconductor interaction have already been investigated in Refs. [36–47]. These investigations were partially motivated by the easy availability and a few more advantages of the semiconductor over the usual nonlinear crystals. These studies led to a few interesting results. For example, by using the optical Kerr effect, the generation of femtosecond pulsed quadrature squeezed light was reported by Fox et al. in II–VI semiconductor [36]. In addition to this, without going through the cryogenic cooling and cavity enhancement, the generation of strong squeezing in a small interaction length was also observed in II–VI semiconductor [36]. The generation of squeezing and entanglement in nuclear spins were reported in semiconductor quantum dot [37]. Interestingly, phonon-displaced squeezed number state has been extensively investigated in polar semiconductors [38]. In a recent experiment, Zeytinoglu et al. have established that the direct band gap semiconductor can behave as a nonlinear mirror and produce broadband squeezing [39]. Of late, production of stationary squeezed phonon state is reported in a pulsed optical excitation of a semiconductor quantum
well. On the other hand, nonclassical properties of excitons of semiconductor and photons of light propagating through the semiconductor have been studied since last three decades. For example, the existence of squeezed excitons and/or squeezed radiation field in such systems has been reported in the 1990s [40–43]. Almost along the same line, An and Tinh reported that the coupling of a coherent exciton–photon system may lead to quadrature-amplitude and number-phase squeezed excitons [44]; they also reported lower-order [45] and higher-order [46] biexciton squeezing for optical exciton–biexciton conversion; in fact, higher-order squeezing of photon was also reported in Ref. [47].

Keeping in mind the importance of investigating the nonclassical properties of the radiation field coupled to the semiconductor and the results of earlier studies, we aim to study a set of hitherto unstudied nonclassical features which may arise in light–semiconductor interaction. In particular, in this work, we aim to study the possibilities of observing squeezing and antibunching of exciton modes due to interaction between input coherent light and two-band semiconductor. Further, in view of the well-known applications of entanglement and steering in quantum information technology, we would like to investigate the entanglement and steering between the exciton and photon states, too.

The rest of this paper is organized as follows. In Sect. 2, we describe the physical system of our interest and explicitly provide a perturbative operator solution of the Heisenberg’s equations of motion corresponding to the Hamiltonian of the physical system of our interest. Section 3 is dedicated to the nonclassical effects (squeezing, antibunching, and entanglement) that can be observed in the physical system of our interest. Specifically, the existence and evolution of these nonclassical features are reported. Subsequently, in Sect. 4, the evolution of the observed nonclassicality is further investigated in a more general scenario where the field and the semiconductor interact with their surroundings. In this case, stronger correlation, i.e., EPR steering, between exciton and photon modes is also observed. Finally, the paper is concluded in Sect. 5.

2 The model physical system

We consider that a monochromatic light, i.e., a single mode of the electromagnetic field of energy $\omega_2$ (in the units $\hbar = 1$), is propagating through a two-band semiconductor with direct band gap, which is highly excited and allows interband dipole transition. Let the band gap energy of the semiconductor be $E_g$. Then the interaction of this semiconductor with the monochromatic light with $\omega_2 < E_g$ is energetically unable to lift up the electron from the valence band to the conduction band to create free electrons and free holes. Instead, due to the Coulomb attractive interaction, bound states, each is composed of an electron and a hole, can be formed since the energy of such an electron–hole pair lies within the band gap just below the bottom of the conduction band. Such an interacting electron–hole pair is regarded as a kind of quasiparticle referred to in the literature as an exciton. Note that, although both the electron and the hole considered separately are fermions in nature, the exciton itself (i.e., the bound electron–hole pair as a whole) behaves like a boson [41]. Hence, the exciton follows
the bosonic quantization rule, and consequently the Hamiltonian of the semiconductor coupled to the monochromatic electromagnetic field can be expressed in the units $\hbar = 1$ as [43,47]

$$H = \omega_1 a^\dagger a + \omega_2 c^\dagger c - g \left( a^\dagger c + c^\dagger a \right) + \chi a^\dagger^2 a^2, \quad (1)$$

where $a$ ($a^\dagger$) and $c$ ($c^\dagger$) are annihilation (creation) operators for the exciton and photon mode, respectively. The parameter $g$ corresponds to the photon–exciton interaction constant. On the other hand, the parameter $\chi$ is the interaction strength of the exciton–exciton pairs, and hence it is nonlinear in nature. The bosonic quantization of the exciton has energy $\omega_1$ (in the units $\hbar = 1$). Here, we have ignored the possibility of exciton-assisted photon–exciton transition [41]. However, exciton-assisted photon–exciton transition can easily be included in our system by adding a term of the form $\eta(a^\dagger^2 ac + c^\dagger a^\dagger a^2)$ in the Hamiltonian given above.

Some nonclassical features associated with this system have been studied earlier. For example, by using the so-called polariton representation of photon and exciton, the amount of squeezing of light as a function of exciton–exciton interaction was investigated earlier [43] using secular approximation. An approximate closed-form analytical solution of this system for weak nonlinearity (i.e., $\chi \ll 1$) was also reported in [48]. Further, for strong nonlinearity, a numerical investigation was performed and the collapse and revival phenomena in the periodic exchange of energy between atomic oscillator and field were observed. In this numerical investigation, the relevant bosonic operators were replaced by their eigenvalues and hence the $c$-numbered differential equations were formed instead of operator differential equations. This reduced the computational difficulty and led to an exact result at the cost of the phase information which is of great significance in the study of squeezing and entanglement.

In order to investigate the nonclassical properties of the radiation field interacting with two-band semiconductor, we obtain the Heisenberg’s equations of motion involving operators $a$ and $c$ as

$$\dot{a} = -i \left( \omega_1 a - gc + 2\chi a^\dagger a^2 \right),$$

$$\dot{c} = -i \left( \omega_2 c - ga \right). \quad (2)$$

These are coupled nonlinear operator differential equations that are not exactly solvable in closed analytical forms unless $\chi = 0$. Keeping this in mind, in what follows, we report an approximate analytical operator solution of these coupled differential equations (2) using Sen–Mandal approach [24,25,49–51]. This approach is tested in various occasions, and it is established that it provides better solutions than the solutions obtained by the well-known short-time approximation technique [52,53] (for a detail discussion on the advantages of Sen–Mandal approach see [24,25,30,54]). Here, we would use Sen–Mandal perturbative technique, to obtain a second-order operator solution. Specially, the solution would be restricted to the quadratic power of
the interaction constants \( \chi \) and \( g \) considering \( gt < 1 \) and \( \chi t < 1 \). In order to obtain the final solution, we first follow Sen–Mandal approach to write trial solution as follows

\[
a(t) = f_1a(0) + f_2c(0) + f_3a^\dagger(0)a^2(0) + f_4a(0) \\
+ f_5a^\dagger(0)a(0)c(0) + f_6c^\dagger(0)a^2(0) \\
+ f_7a^\dagger(0)a^2(0) + f_8a^\dagger(0)a^2(0),
\]

\[
c(t) = h_1c(0) + h_2a(0) + h_3c(0) + h_4a^\dagger(0)a^2(0),
\]

where \( f_i\)s and \( h_i\)s are unknown parameters. It would be relevant to discuss the physical significance of some of the terms in the above solution (3). For example, the term involving \( f_1 \) is responsible for the free evolution of the exciton mode \( a \). The term \( f_2 \) exhibits a linear coupling nature between exciton and photons. The exciton–exciton nonlinear term is responsible for the appearance of \( f_3 \). In a similar manner, the appearance of the remaining \( f_i\)s and \( h_i\)s can also be explained. Interestingly, the method adopted here does not introduce any restriction on the terms with higher powers of time \( t \) that can be included in the solution. Specifically, proposed solution may contain terms with \( t^3 \) and higher powers of \( t \). This is in sharp contrast with the well-known solutions under short-time approximations [52,53], and this is what leads to better results. In order to obtain the closed-form solution of the evolution of operators \( a \) and \( c \), we have to evaluate the unknown parameters \( f_i\)s and \( h_i\)s. To do so, we substitute Eq. (3) in Eq. (2) and compare the coefficients of same powers of \( a(0), c(0), \) etc. to obtain

\[
\dot{f}_1 = -i\omega_1 f_1,
\]
\[
\dot{f}_2 = -i\omega_1 f_2 + i\hbar h_1,
\]
\[
\dot{f}_3 = -i\omega_1 f_3 + 2\chi |f_1|^2 f_1,
\]
\[
\dot{f}_4 = -i\omega_1 f_4 + i\hbar h_2,
\]
\[
\dot{f}_5 = -i\omega_1 f_5 - 4\chi i|f_1|^2 f_2,
\]
\[
\dot{f}_6 = -i\omega_1 f_6 - 2\chi i f_1^2 f_2^*,
\]
\[
\dot{f}_7 = -i\omega_1 f_7 + i\hbar h_4 - 2\chi i|f_1|^2 f_3,
\]
\[
\dot{f}_8 = -i\omega_1 f_8 - 4\chi i|f_1|^2 f_3 - 2\chi i f_1^2 f_3^*,
\]

and

\[
\dot{h}_1 = -i\omega_2 h_1,
\]
\[
\dot{h}_2 = -i\omega_2 h_2 + i\hbar f_1,
\]
\[
\dot{h}_3 = -i\omega_2 h_3 + i\hbar f_2,
\]
\[
\dot{h}_4 = -i\omega_2 h_4 + i\hbar f_3,
\]

respectively. Finally, we obtain the following analytical expressions for \( f_i\)s and \( h_i\)s, using the initial conditions \( f_1(0) = h_1(0) = 1 \) and \( f_i(0) = h_i(0) = 0 \) for \( i \neq 1 \),
\[ f_1 = e^{-i\omega_1 t}, \]
\[ f_2 = \frac{g e^{-i\omega_1 t}}{\Delta \omega} \left( -1 + e^{i\Delta \omega t} \right), \]
\[ f_3 = -2i \chi t e^{-i\omega_1 t}, \]
\[ f_4 = \frac{g^2 e^{-i\omega_1 t}}{(\Delta \omega)^2} \left( -1 + e^{i\Delta \omega t} - it \Delta \omega \right), \]
\[ f_5 = \frac{4g \chi e^{-i\omega_1 t}}{(\Delta \omega)^2} \left( 1 - e^{i\Delta \omega t} + it \Delta \omega \right), \]
\[ f_6 = \frac{2g \chi e^{-i\omega_1 t}}{(\Delta \omega)^2} \left( -1 + e^{-i\Delta \omega t} + it \Delta \omega \right), \]
\[ f_7 = f_8 = -2\chi^2 t^2 e^{-i\omega_1 t}, \]

and

\[ h_1 = e^{-i\omega_2 t}, \]
\[ h_2 = \frac{g e^{-i\omega_1 t}}{\Delta \omega} \left( -1 + e^{i\Delta \omega t} \right) = f_2, \]
\[ h_3 = \frac{g^2 e^{-i\omega_2 t}}{(\Delta \omega)^2} \left( -1 + e^{-i\Delta \omega t} + i \Delta \omega t \right), \]
\[ h_4 = \frac{2g \chi e^{-i\omega_1 t}}{(\Delta \omega)^2} \left( 1 - e^{i\Delta \omega t} + i \Delta \omega t \right), \]

where \( \Delta \omega = \omega_1 - \omega_2 \) is the difference between the energy of the exciton and photon. Interestingly, the absence of exciton–exciton interaction causes \( f_3 = f_5 = f_6 = f_7 = f_8 = h_4 = 0 \). The consistency of the obtained solution is further checked by confirming that the obtained solution satisfies equal time commutation relation \([a(t), a^\dagger(t)] = [c(t), c^\dagger(t)] = 1 \) and \([a(t), c^\dagger(t)] = [a(t), c(t)] = 0 \). The obtained solution may now be used to calculate the expectation values of various operators that are required for the present investigation on the nonclassical behavior of the exciton and field modes considering that the initial state is a composite coherent state

\[ |\psi(0)\rangle = |\alpha\rangle |\beta\rangle. \]  

This composite coherent state can be viewed as a product of two coherent states \(|\alpha\rangle \) and \(|\beta\rangle \), such that \( a(0) |\alpha\rangle = \alpha |\alpha\rangle \) and \( c(0) |\beta\rangle = \beta |\beta\rangle \). For example, for this initial state, we can obtain the expectation values of the number operators \( \langle N_a(t) \rangle = \langle a^\dagger(t)a(t) \rangle \) and \( \langle N_c(t) \rangle = \langle c^\dagger(t)c(t) \rangle \) as follows

\[ \langle N_a(t) \rangle = |\alpha|^2 + |f_2|^2 \left( |\beta|^2 - |\alpha|^2 \right) \]
\[ + \left\{ f_1^* f_2 \alpha^* \beta - h_1^* h_4 |\alpha|^2 \alpha^* \beta + c.c. \right\} \]  

\[ \text{Springer} \]
and

\[
\langle N_c(t) \rangle = |\beta|^2 + |h_2|^2 \left( |\alpha|^2 - |\beta|^2 \right) + \left\{ (h_1^* h_2^* \beta^* \alpha + h_1^* h_4 |\alpha|^2 \alpha \beta^*) + \text{c.c.} \right\},
\]

where c.c. stands for complex conjugate. In what follows, we consider \( \alpha \) as real, but \( \beta = |\beta|e^{-i\phi} \) as complex, where \( \phi \) is the phase angle of the input coherent state corresponding to the photon mode. Further, \( |\alpha|^2 = \alpha^2 \) would represent the initial number of exciton of the system, whereas \( |\beta|^2 \) is the average photon number before the interaction.

### 3 Nonclassical effects

In the previous section, we have already obtained the analytical solution for the dynamics of the relevant bosonic operators. In the present section, we will use the obtained solution for investigating the nonclassical features present in the physical system of our interest. To begin with, in the following subsection, we investigate the possibility of observing quadrature squeezing.

#### 3.1 Quadrature squeezing

The dimensionless quadrature operators are defined in terms of the usual annihilation and creation operators as follows

\[
X_k = \frac{1}{2} \left[ k(t) + k^\dagger(t) \right],
Y_k = \frac{1}{2i} \left[ k(t) - k^\dagger(t) \right],
\]

where \( k \) (\( k^\dagger \)) is the annihilation (creation) operator of a particular mode. If the variances \( (\Delta X_k)^2 \) or \( (\Delta Y_k)^2 \) goes below \( \frac{1}{4} \), the corresponding quadrature is said to be squeezed. It is clear that the simultaneous squeezing of both the quadrature is prohibited by Heisenberg’s uncertainty principle. Using Eqs. (3), (6)–(8), and (11), we obtain the detailed analytical expressions for the variances involving the exciton mode \( a \) and the photon mode \( c \) as follows

\[
\left[ \frac{(\Delta X_a)^2}{(\Delta Y_a)^2} \right] = \frac{1}{4} \left[ 1 + 2 |f_3|^2 |\alpha|^4 + \left\{ (f_1^* f_5 + 2 f_1 f_6^* ) \alpha^* \beta + \text{c.c.} \right\} + \left\{ (f_1 f_3 + f_1 f_7) \alpha^2 + f_1 f_5 \alpha \beta + 2 f_1 f_6 |\alpha|^2 \alpha^2 + \text{c.c.} \right\} \right],
\]

(12)
\[ \left[ \begin{array}{c} (\Delta X_c)^2 \\ (\Delta Y_c)^2 \end{array} \right] = \frac{1}{4}. \] (13)

It is clear from Eq. (13) that no squeezing is observed in photon mode \( c \). In the past, \([43,47]\), squeezing and antibunching in the photon mode were reported for the physical system of our interest using polariton solution. The perturbative solution obtained here provides us a constant value of quadrature variances in photon mode as \( \frac{1}{4} \), which can be improved by including higher-order terms in the coupling and interaction constants while obtaining the perturbative solution. We have not tried to obtain higher-order perturbative solution as the exact numerical solution for our system failed to reveal nonclassicality in photon mode for the values of physical parameters chosen here. We will further elaborate on this point in the next section. However, it is possible to observe the squeezing effects of the exciton mode \( a \). Interestingly, for \( \chi = 0 \), the squeezing in exciton mode is found to disappear. It substantiates that the effective nonlinearity induced by the exciton–exciton interaction is indeed responsible for the squeezing phenomenon. The analytical expression (12) is now at our disposal to investigate the possibility of observing squeezing of the exciton mode and the evolution of it as a function of the exciton number, photon number and/or the phase of the field mode. However, we restrict ourselves to the study of evolution of \((\Delta X_a)^2\) and \((\Delta Y_a)^2\) with the rescaled time \( gt \) only in Fig. 1a. The figure clearly illustrates the existence of squeezing. In addition to the perturbative analytical expressions for the variance, we have also obtained the exact numerical values for the variance using QuTiP 3.1.0 \([55,56]\) by solving time-dependent Schrödinger equation using the matrix forms of operators and initial state. The results obtained through the numerical analysis are represented by the squares and circles. It is clear from Fig. 1a (and a set of other figures included in this paper, where both numerical and analytical investigations have been conducted) that the numerical results exactly coincide with the corresponding analytical results obtained here.

In Fig. 1 and in the rest of this paper, we have chosen the parameters in analogy with the earlier works of An and others \([40,41,44,45,47]\). The parameters are selected in such a way that they remain consistent with a real physical system. Specifically, we have considered that the semiconductor of our system is CdS and for all the plots, we have used following parameters (which correspond to CdS): \( \omega_1 = 25.277 \, \text{g}, \omega_2 = 24.013 \, \text{g}, \) and \( \chi = 5.304 \, \text{g} \). Here, it may be apt to note that in Ref. [41], these values of the parameters were obtained using the relations \( E_e(k) = E_g + k^2/(2m_e) \) and \( E_h(k) = k^2/(2m_h) \), where \( E_g, E_e(k), \) and \( E_h(k) \) are the band gap energy, energy of the electron, and energy of the hole, respectively. Also, \( m_e(m_h) \) is the effective mass of the electron (hole).

Through Fig. 1a and Eq. (13), we can recognize that squeezing can be observed in exciton mode, but not in the photon mode. Therefore, it will be interesting to
investigate the possibility of intermodal squeezing. In order to do so, we use the following quadrature operators involving the exciton and photon mode [5]:

\[
X_{ac} = \frac{1}{2\sqrt{2}} \left[ a(t) + a^\dagger(t) + c(t) + c^\dagger(t) \right],
\]

\[
Y_{ac} = \frac{1}{2\sqrt{2}i} \left[ a(t) - a^\dagger(t) + c(t) - c^\dagger(t) \right], \tag{14}
\]

Fig. 1 (Color online) Variation of quadrature squeezing with rescaled time \( gt \) using parameters (mentioned in text) that are consistent with the real systems (for CdS) [41], and \( \alpha = 2.0, \beta = 1.0, \) for a exciton mode \( a \) and b the compound mode \( ac \). All the quantities shown here and the rest of the figures are dimensionless.
and the following criterion of intermodal squeezing \((\Delta X_{ac})^2 < \frac{1}{4}\) or \((\Delta Y_{ac})^2 < \frac{1}{4}\).

A bit of calculation using Eqs. (3), (6)–(8), and (14) yields analytical expression for the variance of the coupled mode quadratures as

\[
\left[ \frac{(\Delta X_{ac})^2}{(\Delta Y_{ac})^2} \right] = \frac{1}{4} \left[ 1 + |f_3|^2 |\alpha|^4 + \frac{1}{2} \left\{ f_1^* h_2 + h_1 f_2^* \right. \\
\left. + (f_1^* f_5 + 2 f_1 f_6^*) |\alpha|^2 \right\} \left( 2 f_1 h_4 + h_1 f_5^* \\
+ 2 f_3 h_2^* |\alpha|^2 \right) \right] \left( |f_1|^2 + f_1 f_7 + f_1 h_4 + h_1 f_6 \\
+ h_2 f_3 \right) \alpha^2 + f_1 f_5 \alpha \beta + 3 f_3^2 |\alpha|^2 + \text{c.c.}\right].
\]

The right-hand side of Eq. (15) is plotted in Fig. 1b, which clearly shows the existence of intermodal squeezing in both the quadratures of the coupled mode (of course not simultaneously). The plot also shows that our perturbative analytical results considerably match with the corresponding exact numerical results.

### 3.2 Antibunching

One of the most intensively studied nonclassical phenomena is antibunching of bosons. The photons from an incandescent light source come in a bunch and show bunching. However, there are many physical systems, where antibunching of bosons can be observed [3,4,24,25,30]. The theoretical investigation on the antibunching of bosons is usually performed using the so-called second-order correlation function at zero time delay, which is defined as

\[
g_i^{(2)}(0) = \frac{\langle \hat{i}^\dagger(t) \hat{i}(t) \hat{i}(t) \hat{i}(t) \rangle}{\langle \hat{i}^\dagger(t) \hat{i}(t) \rangle^2} = 1 + \frac{D_i}{\langle N_i(t) \rangle^2},
\]

where

\[
D_i = (\Delta N_i(t))^2 - \langle N_i(t) \rangle,
\]

and \(i \in \{a, c\}\). The condition for the single-mode antibunching in \(i^{th}\) mode is \(g_i^{(2)}(0) < 1\), which is equivalent to \(D_i < 0\), and generally it implies antibunching as well as sub-Poissonian boson (photon/exciton) statistics. In fact, \(g_i^{(2)}(0)\) is more associated with sub-Poissonian boson statistics and ensures the presence of antibunching at least for some time scales ([24] and references therein). Therefore, we have followed this convention here, and have used \(D_i < 0\) as the criterion of antibunching. Using Eqs. (3), (6)–(8), and (17), we can easily obtain an analytical expression for \(D_i\) as follows

\[
D_a = \left\{ 2 \left( f_1^* f_5 + f_1 f_6^* + 2 f_2 f_3^* + f_1^* f_2 f_3 \right) \right. \\
\left. \times |\alpha|^2 \alpha^* \beta + \text{c.c.} \right\}
\]

\[
D_c = \left\{ 2 \left( f_1^* f_5 + f_1 f_6^* + 2 f_2 f_3^* + f_1^* f_2 f_3 \right) \right. \\
\left. \times |\alpha|^2 \alpha^* \beta + \text{c.c.} \right\}
\]
and

\[ D_c = |f_2|^2 |\beta|^2. \]  

(19)

It is clear from the analytical expression of \( D_c \) that the photon mode always remains bunched (at least in the domain of the validity of the perturbative solution) after the interaction, which also establishes that it exhibits super-Poissonian photon statistics. This is so because the right-hand side of Eq. (19) is always positive. However, the exciton mode may be antibunched. In order to study the time evolution of the observed nonclassical property, we plot \( D_a \) as a function of dimensionless interaction time \( gt \) in Fig. 2a. Interestingly, for the chosen set of values of parameters, the antibunching in exciton mode is exhibited. The depth of the antibunching witnessing parameter \( D_a \) is observed to increase with increasing \( gt \). The monotonic nature of \( D_a \) could be attributed to the truncated nature of the solutions for the bosonic operators. Further, the condition of intermodal antibunching can be expressed as

\[ D_{ac} = \left\langle a^\dagger(t)c^\dagger(t)c(t)a(t) \right\rangle - \left\langle a^\dagger(t)a(t) \right\rangle \left\langle c^\dagger(t)c(t) \right\rangle. \]  

(20)

In order to get the flavor of the analytical expression, we plot the right-hand side of Eq. (20) for two values of the phase angle (i.e., for \( \phi = 0 \) and \( \phi = \pi \)) of the input radiation field in the coherent state. The figure establishes that the occurrence of nonclassicality (in this case, intermodal antibunching) can be controlled by the phase of the input radiation field. Specifically, intermodal antibunching is observed for \( \phi = \pi \), while we could not find it for \( \phi = 0 \). Also, the analytical results obtained here match exactly with the numerical results which further establishes the validity of our analytical solution.

### 3.3 Quantum entanglement

With the advent of quantum computing and quantum communication, a large number of applications of entanglement have been reported (see [15,16] for review). We have mentioned about some of them in Sect. 1. Motivated by these applications, and because of the possibility that the exciton–photon entanglement generated in the physical system of our interest can be of use in the solid-state quantum computing, it is of particular importance to investigate the possibility of observing photon–exciton entanglement in the light–semiconductor interaction. In order to obtain the signature of the photon–exciton two-mode entanglement, we use three criteria. Note that we have used here more than one inseparability criterion as each of these criteria is only sufficient witness of entanglement and not necessary. Two criteria used here were proposed by

\[ \left( f_1 f_6^* + f_1 f_3^* h_1 h_2^* \right) |\alpha|^2 a^* b + c.c. \]  

(21)

\[ D_{ac} = \left\langle a^\dagger(t)c^\dagger(t)c(t)a(t) \right\rangle - \left\langle a^\dagger(t)a(t) \right\rangle \left\langle c^\dagger(t)c(t) \right\rangle. \]  

(20)
Fig. 2  (Color online) Variation of antibunching with $gt$ using the same set of values of parameters as in Fig. 1 for a exciton mode $a$ and b the compound mode $ac$ (for two values of the phase angle $\phi = 0$ and $\pi$)

the Hillery and Zubairy [57–59] (from here on, we will refer to these criteria as HZ1 and HZ2, respectively), and the third one was proposed by Duan et al. [60] (which will be referred to as Duan et al.’s criterion).

In our case, HZ1 and HZ2 criteria can be expressed as [57–59]

$$E_{a,c} = \langle N_a(t)N_c(t) \rangle - \left| \langle a(t)c^\dagger(t) \rangle \right|^2 < 0$$  (22)
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The values of parameters are same as used in the previous figures

and

$$E'_{ac} = \langle N_a(t) \rangle \langle N_c(t) \rangle - |\langle a(t)b(t) \rangle|^2 < 0,$$

respectively. Using these two criteria and the solutions obtained above in Eq. (3), we obtain the analytical expressions for the entanglement witnessing parameter $E_{a,c}$ and $E'_{a,c}$ as follows

$$E_{a,c} = |f_3|^2 |\alpha|^4 |\beta|^2 + \left[ (f_1^n f_6 - f_1 f_3^* h_1^* h_2) | \alpha \right| ^2 \alpha \beta^* + c.c.]$$

$$E'_{a,c} = |f_3|^2 |\alpha|^4 |\beta|^2 + \left[ (f_1^n f_6 - f_1 f_3^* h_1^* h_2) | \alpha \right| ^2 \alpha \beta^* + c.c.]$$

Fig. 3 (Color online) Evolution of entanglement witnesses is shown with $gt$ using (a) HZ1 and (b) HZ2 criteria.
and

\[ E'_{a,c} = |f_3|^2 |\alpha|^4 |\beta|^2 - \left( h_1^\dagger h_4 |\alpha|^2 \alpha \beta^* + \text{c.c.} \right), \tag{25} \]

respectively. As these two criteria are only sufficient not necessary, we also investigate the existence of entanglement using Duan et al.’s criterion \cite{60}, which can be expressed as

\[ d_{ij} = \left( (\Delta u_{ij})^2 \right) + \left( (\Delta v_{ij})^2 \right) - 2 < 0, \tag{26} \]

where

\[ u_{ij} = \frac{1}{\sqrt{2}} \left\{ (i + i^\dagger) + (j + j^\dagger) \right\}, \]
\[ v_{ij} = -\frac{i}{\sqrt{2}} \left\{ (i - i^\dagger) + (j - j^\dagger) \right\}. \tag{27} \]

Duan et al.’s criteria and the solution (3) obtained above can now be used together to obtain

\[ d_{ac} = 2 \left[ |f_3|^2 |\alpha|^4 + \frac{1}{2} \left\{ f_1^* h_2 + h_1 f_2^* \right\} \right. \]
\[ + \left( f_1^* f_5 + 2 f_1 f_6^* \right) \alpha^* \beta + \left( 2 f_1 h_4^* + h_1 f_5^* \right) \]
\[ + \left. 2 f_3 h_2^* \right\} |\alpha|^2 + \text{c.c.}. \tag{28} \]

To illustrate the presence of photon–exciton entanglement in our system, we plot the right-hand sides of Eqs. (24), (25), and (28). Clearly, the signature of the intermodal entanglement is seen in the photon–exciton coupled mode using only HZ1 and HZ2 criteria (shown in Fig. 3), but not through Duan et al.’s criterion. Figure 3 further establishes the effect of the phase of the input coherent light on the presence of nonclassicality. Specifically, the presence of entanglement is reflected through HZ1 (HZ2) criterion for the phase angle \( \phi = \pi \) (0), while it failed to show its presence for \( \phi = 0 \) (\( \pi \)). It further establishes that the set of criteria studied here is sufficient only. Further justifying this fact, Duan et al.’s criterion failed to detect entanglement (not shown here). In brief, the use of HZ1 and HZ2 criteria studied here establishes that the photon–exciton mode is entangled for both \( \phi = 0 \) and \( \pi \).

4 Nonclassicality in open quantum system

In this section, we aim to study the nonclassical properties associated with the bosonic modes when the system described above is considered to be under the influence of the environment. Specifically, we now consider a scenario, where both the semiconductor and photon modes are allowed to interact with the ambient environment. In order to
solve the Hamiltonian (1) under this situation, we need to construct a master equation in the Lindblad form [52]. This master equation, under the framework of rotating wave approximation, takes the following form in the interaction picture (in the units of $\hbar = 1$)

$$\frac{d\rho}{dt} = -i [H, \rho] + \sum_{j=1,2,3}\left( L_j \rho L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho - \frac{1}{2} \rho L_j^\dagger L_j \right),$$  

(29)

where $L_1 = \{(n_{\text{th}} + 1) \gamma\}^{1/2} a$, $L_2 = (n_{\text{th}} \gamma)^{1/2} a^\dagger$, and $L_3 = \sqrt{\gamma} c$ are the Lindblad operators, and $\gamma$ denotes the rate of dissipation for both exciton and photon modes. Here, we have assumed same rate of dissipation for both optical and exciton modes. Further, we have considered a vacuum bath ($n_{\text{th}} = 0$) for the optical mode and thermal bath for the exciton mode. The parameter $n_{\text{th}}$ corresponds to the mean number of quanta in the heat bath corresponding to the exciton mode. The numerical simulations are performed by the Quantum toolbox QuTiP 3.1.0 [55,56]. Using the numerical solution with the help of subtool `qutip.mesolve`, we have investigated the nonclassical properties of the aforementioned system.

4.1 Nonclassicality in open quantum system: squeezing and antibunching

We have reported single-mode squeezing and antibunching in the exciton mode and intermodal squeezing and antibunching in exciton–photon mode in Sect. 3. However, we failed to report both single-mode squeezing and antibunching in the photon mode using the perturbative analytic solution. In this section, we have used the criteria of nonclassicality defined in Eqs. (11), (14), (17), and (20) to study time evolution of these witnesses of nonclassicality. The obtained results are illustrated in Fig. 4. Specifically, Fig. 4a shows the variation of quadrature variance $(\Delta X_a)^2$ in the exciton mode in the pure state case (considering dissipation rate zero) and nonzero dissipation rate. It can be clearly observed that with the increase in the value of $\gamma$, the maximum amount of squeezing represented by the least value of the variance decreases which can be seen by decay in the envelope in the corresponding figure. Further, in Fig. 4b, we have shown the evolution of variance of quadrature variable $Y_c$ for photon mode, where nonclassicality can be observed to decay with the increase in dissipation. Further, we can clearly observe that the exact numerical result even in the pure state case does not show squeezing in photon mode for $gt < 0.3$, i.e., in the domain of the validity of the perturbative solution reported in Sect. 2. Thus, the obtained results remain consistent with the analytical results reported previously. However, as the numerical result is valid for much larger timescale, we can now observe the presence of squeezing in photon mode as well. Note that we have not shown the variation of variance in the other quadrature here which were observed to show squeezing.

Thereafter, we report single-mode antibunching in the exciton and photon modes in Fig. 4c, d, respectively. In both closed and open quantum system scenarios, antibunching in the exciton mode over long interaction time $gt$ shows an oscillatory behavior, unlike a monotonically decreasing nature observed in Fig. 2a for short timescale. Further, Fig. 4d fails to show antibunching in the small timescale in the photon mode. This is consistent with the results reported in Sect. 3. However, antibunching can be
Fig. 4 (Color online) Variation of the variance in the measured values of the quadrature variable $a X_a$ for exciton mode and $b Y_b$ for photon mode. The values less than $1/4$ are signature of squeezing. Time evolution of single-mode antibunching in $c$ exciton and $d$ photon modes. Intermodal $e$ squeezing and $f$ antibunching in the exciton–photon mode are also shown here. In all these cases, we have assumed vacuum bath for the exciton mode and the values of the dissipation rate are mentioned in the plot legends.

observed in large timescale for optical mode. Finally, for the sake of completeness of the discussion, we have also reported intermodal squeezing and antibunching in Fig. 4e, f, respectively. The presence of intermodal squeezing in the longer timescale, and its absence in the short timescale, is also consistent with the analytical results reported in
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The effect of thermal excitations \((a, c)\) and dissipation rate \((b, d)\) on two witnesses of entanglement, namely HZ1 \((a, b)\) and HZ2 \((c, d)\). Different values of the parameters are mentioned in the plots. In \((a, c)\), we have chosen \(\gamma = 0.01\), while vacuum bath is considered in \((b, d)\).

Previous section. Finally, all these observed nonclassical features are found to decay with the increase in the dissipation rate assuming a vacuum bath for the exciton mode.

4.2 Entanglement in open quantum system

Here, we will study the effect of change in the average thermal photon number of the thermal bath on the exciton–photon entanglement and compare it with that of the different values of dissipation rate. Using the solution of Eq. (29) with the Hillery–Zubairy criteria of entanglement \((22)–(23)\), we have shown the variation of the entanglement witness in Fig. 5. Specifically, Fig. 5a shows that with the increase in \(n_{th}\) the witness of HZ1 criterion drifts toward the positive side, thus signifying the decay in entanglement and overall quantum to classical transition. Note that, the entanglement may still be present in the timescale we have shown, which was observed to decay eventually for larger values of rescaled time. However, our interest here is to show the effect of different parameter on the observed nonclassicality, so we have shown our results in a small timescale. Further, in case of vacuum bath for exciton mode, the increase in the value of dissipation rate is found to cause a reduction in the value of the witness of nonclassicality (HZ1 entanglement criterion) shown in Fig. 5b. Thus, a collective
The effect of change in the dissipation rate on witness of EPR steering. Different values of the parameters are mentioned in the plots considering interaction of excitons with the vacuum bath. In this case, we have assumed the exciton mode initially in vacuum and photon mode in Fock $|n = 5\rangle$ state. Effect of thermal bath and higher values of dissipation rate ensure nonclassicality to decay more rapidly. Similar studies performed for HZ2 criterion of entanglement and illustrated in Fig. 5c, d establish the same fact. Further, in the short timescale of the analytical solution, we have observed entanglement using HZ1 criterion only for some specific values of the phase angle of the input coherent beam. However, in the larger timescale, entanglement in all the cases is quite evident.

4.3 Steering in open quantum system

Schrödinger introduced the notion of steering in response to EPR paradox [7]. He argued that the choice of measurement on the first subsystem can change the final state of the second subsystem, now known as EPR steering. This correlation is stronger than entanglement and weaker than Bell nonlocality. Incidentally, in the pure state case the presence of entanglement is sufficient to establish stronger correlation, i.e., EPR steering [61]. Therefore, we did not discuss the feasibility of observing steering independently in the previous section.

The EPR steering criteria can be introduced in terms of the HZ1 criterion (22) mentioned in the previous section ([62,63] and references therein) as

$$S_{ac} = E_{ac} + \frac{\langle a^\dagger a \rangle}{2} < 0. \quad (30)$$

Steering is an asymmetric form of correlation as mode $a$ can steer mode $c$ more than the amount by which mode $c$ can steer $a$. The negative values of the witness $S_{ac}$ in Eq. (30) would establish that mode $a$ can steer mode $c$, while the other possibility is obtained by using the number operator of $c$ mode in Eq. (30). In the present case, we observed that the exciton mode can steer the quantum state of the photon mode (shown in Fig. 6).
Clearly, the negative values of the witness, signifying EPR steering, decrease with the dissipation rate when both modes are interacting with the vacuum bath. Specifically, the initial state in this particular case is different from rest of the cases, where both modes were initially coherent. Here, we have assumed the exciton mode is initially in vacuum state and photon mode is in five photon Fock state. In view of the fact that we failed to observe EPR steering with initial coherent states, the presence of this strong correlation in this case manifests the advantage of using nonclassical input states. Particularly, Fock states are known as most nonclassical states and as antibunched state, therefore the photon mode is initially in single-mode nonclassical state while exciton is in vacuum. The interaction leads to strong correlations between two modes showing EPR steering. It is not our purpose here to discuss the advantages of nonclassicality in the initial states, but this shows that the single-mode nonclassicality generated in the system itself has applications in generation of strong bipartite correlations.

5 Conclusion

Nonclassical properties associated with the light-semiconductor interaction are rigorously studied using two physical scenarios. Firstly, the interaction of a coherent light with a two-band semiconductor is investigated, and the existence of several nonclassical features, such as squeezing, intermodal squeezing, antibunching, intermodal antibunching, and entanglement, is established. Interestingly, antibunching and squeezing is observed in the exciton mode using the perturbative solution, but not in photon mode. However, intermodal antibunching, squeezing and entanglement were observed. In some cases (viz., intermodal antibunching and entanglement), the nonclassicality in the output can be controlled by the phase of the input field. This systematic study is performed using analytical technique and is found to coincide exactly with the corresponding numerical results obtained using QuTip 3.1.0. This establishes the validity of the present perturbative solution in the domain in which it is applied here. Finally, in the last section, we have extended the work to a more general scenario, where the system is considered to be under the effect of the ambient environment. This is our second physical scenario, and this particular scenario is investigated using numerical methods only. Interestingly, all the signatures of nonclassicality observed using the perturbative solution are found to be present in this situation, too. On top of that, we have also observed squeezing and antibunching in the photon mode in this case. It is also established that the photon mode can be steered by the exciton mode as correlation between these two modes are stronger than quantum entanglement and show EPR steering. Particularly, we have observed squeezing and antibunching in both exciton and photon modes, intermodal squeezing, antibunching, entanglement, and EPR steering in exciton–photon mode, but what is more interesting and worth mentioning here is that the observed nonclassical features are robust to noise. During this effort we have established the advantage of nonclassical states if used as the input field in the present case as it leads to the EPR steering correlation between two modes. In fact, the origin of the nonclassical features observed here (except EPR steering) can be attributed to the nonlinear interaction between exciton–exciton pairs represented by the interaction constant $\chi$. 

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Here it may be noted that the results summarized above are interesting for various reasons. Firstly, despite the fact that different nonclassical properties of the physical systems related to the present system had been reported earlier, apart from squeezing and photon statistics no other nonclassical features had yet been reported for this particular system (Hamiltonian). Further, entanglement and steering (two of the most useful nonclassical resources for quantum computation and communication) had hardly been reported for any physical system of this kind (see Refs. [36,38–48]). Only in Ref. [64], entanglement was reported for a related but different physical system. Finally, from a set of earlier works on various physical systems, it is already established that Sen–Mandal approach can reveal nonclassicalities, not revealed by solution(s) obtained by the other perturbative methods, specially short-time/length approach (cf. [25,30,49]). However, no effort had yet been made to investigate the interaction of light and semiconductor using Sen–Mandal approach. Thus, the approach as well as the results is new. In addition to this, the results obtained here seem to be interesting for two other reasons, too. Firstly, the system studied here is very general as it can be reduced to various physical systems of particular interest, and thus the obtained results can be used to investigate the possibilities of observing nonclassical features in many other systems. Specifically, the Hamiltonian (1) studied here also corresponds to the single-mode resonant field propagating through a Kerr-like medium [48]. The Hamiltonian (1) also reduces to the interaction between two modes of a beam splitter if $\chi = 0$, which is equivalent to the optical couplers studied in Ref. [65] and references therein. Further, for $g = 0$, it reduces to the Hamiltonian of a system composed of a harmonic oscillator (free field part) and a quartic anharmonic oscillators (which represents light interacting with a Kerr-type medium) that are not interacting with each other. Secondly, it is interesting because of its potential applications in quantum information processing. Specifically, it is expected that the observed entanglement between a semiconductor and the light interacting with it, would be of use to the quantum information community interested in solid-state quantum computing which is extremely exciting at the moment because it leads to a possibility of room temperature quantum computing. Further, the present study has revealed EPR steering between the exciton and photon mode, which has its own several applications in device independence. Moreover, the generated squeezed state may be used to build a solid-state source of squeezed state for use in continuous variable quantum cryptography (see [66] and references therein).

Before, we conclude this paper, it would be apt to note that in order to realize the above-mentioned potential applications of the nonclassical features generated through the interaction of light and semiconductor, it would be useful to consider a more realistic system by generalizing the present system. To be specific, present system (Hamiltonian) may be generalized to include exciton-assisted photon–exciton transition term, and to consider that the signals contain a finite bandwidth. This is doable as the analytic method used here is quite general. Further, the restriction of zero temperature implied here can be easily removed as QuTip allows one to solve Lindblad equations with thermal noise too. Keeping the above in mind, we conclude this work by noting that investigations on the nonclassical properties of such a realistic system will be reported elsewhere in the near future, and thus the applicability of the nonclassical states generated via light-semiconductor interaction will be established in a more general scenario.
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