Nonlinear radiation oscillator theory for symmetric and anti-symmetric damping graphene coupled metamaterials

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Abstract
We develop a novel nonlinear radiation oscillator theory, which can describe nonlinear optical responses in symmetric and anti-symmetric damping graphene coupled metamaterials. An ultra-high quality factor of induced transparency can be realized in the proposed model. The results show that the spectral response and quality factor can be effectively tuned by the resonant detuning, the phase difference, the damping factor as well as the field intensity of the pump light. Moreover, the quality factor of induced transparency is increased by more than 60 times when the field intensity of the pump light and the damping factor are changed. Especially, the single induced transparency window splits into double induced transparency windows as the damping factors of dark mode and bright mode show anti-symmetric damping case. These results may pave the way for designing the high performance of nano-optical devices in terahertz.

1. Introduction

Electromagnetically induced transparency (EIT) is a quantum interference effect between electromagnetic fields and the energy level in atomic systems [1]. However, the realization of EIT in atomic systems needs demanding experimental conditions [2]. Plasmon induced transparency (PIT) is an analogue of EIT observed in many plasmonic systems, such as metal-dielectric-metal waveguides [3–5], metasurface [6, 7], metamaterials [8], nano-particles [9] as well as graphene coupled systems [10–13]. Although PIT is easy to realize and also has strong dispersion, the quality factor of its spectra is enslaved to the large loss of plasmonic materials. Thus, it is necessary to enhance the quality factor of induced transparency. Gain materials are recognized as one of the effective ways to increase the quality factor of the spectra in plasmonic systems, so the high quality factor of PIT as well as other interesting phenomenon has been discussed by researchers in recent years [14, 15].

Theoretical analytical models are of great value in the study of optical coupling properties. A perfect theoretical analytical model can prove and predict interesting optical phenomenon in different optical structures. Coupled mode theory [16], transmission line theory [17], radiation oscillator theory [4, 8, 18], scattering matrix theory [15, 19] are commonly used to describe the optical coupling and spectra in optical systems. Cao et al developed a uniform coupled mode theory for investigating the direct coupling and indirect coupling in a metal-dielectric-metal coupled waveguide [16]. Xu et al investigated PIT in coupled graphene by use of coupled mode theory [10]. Liu et al studied multiple PIT in graphene coupled metasurface through coupled mode theory and numerical simulations [13]. Zhong et al used transmission line theory to study sharp and asymmetric transmission response in metal-dielectric-metal plasmonic waveguides [17]. Huang et al investigated the switching of the direction for reflectionless light propagation at exceptional points in non-PT-symmetric structures using phase-change materials by use of the scattering matrix theory [19]. Tassin et al proposed a radiation field model to demonstrate plasmonic analogue of EIT in metamaterial structures, and they also clarified the relationship between macroscopic variables and microscopic variables clearly [18]. However, the nonlinear property and the gain effect have not been considered in previous radiation oscillator theories. In addition, the radiation oscillator theory is rarely used.
for discussing optical responses in graphene systems in terahertz ranges, and the quality factor of PIT in current graphene systems are relatively little.

In this letter, we develop a nonlinear radiation oscillator theory by consideration of the nonlinear property and the gain effect for studying optical responses in symmetric and anti-symmetric damping graphene coupled metamaterials. We find that spectral responses and its quality factors can be effectively tuned by the resonant detuning, the phase difference, the gain or loss coefficient of dark mode as well as the field intensity of the pump light. Moreover, the quality factor of the induced transparency window can reach up to 1001 through the regulation of the field intensity of the pump light and the damping factor of dark mode. What is more, the single induced transparency turns into double induced transparency when damping factors of bright mode and dark mode are additive inverse.

2. Radiation oscillator theory

Figure 1 shows the schematic diagram of the proposed nonlinear radiation oscillator theory model. \(a_1, a_2, a_3, a_4\) represents the amplitude of the light wave traveling to the right side, and \(b_1, b_2, b_3, b_4\) are the amplitude of the light wave reflecting to the left side. The orange parts can be directly excited by the input light, so they can be seen as bright modes for the first and second units, respectively. However, the blue parts are seen as dark modes for the first and second units, respectively. \(\omega_{b1}\) and \(\omega_{b2}\) are resonant angular frequencies of bright modes, \(\omega_{d1}\) and \(\omega_{d2}\) are resonant angular frequencies of dark modes, \(\gamma_{b1}\) and \(\gamma_{b2}\) stand for damping factors of bright modes, \(\gamma_{d1}\) and \(\gamma_{d2}\) stand for damping factors of dark modes for the first and second units, respectively. \(\kappa_1\) and \(\kappa_2\) represent the coupling strength between the bright mode and dark mode in the first and second units, respectively.

Here, we introduce a set of two coupled harmonic oscillators to describe optical properties in the \(i\)th unit \((i = 1, 2)\) as shown in the following equations [18]

\[
\omega_{b1}^{-2} b_i''(t) + \gamma_{b1}\omega_{b1}^{-1} b_i'(t) + b_i(t) = f_i(t) - \kappa_i d_i(t) \tag{1}
\]

\[
\omega_{d1}^{-2} d_i''(t) + \gamma_{d1}\omega_{d1}^{-1} d_i'(t) + d_i(t) = -\kappa_i b_i(t) \tag{2}
\]

where \(b_i(t) = \tilde{b}_i(\omega)e^{-j\omega t}, d_i(t) = \tilde{d}_i(\omega)e^{-j\omega t}\), and \(f_i(t) = f_i(\omega)e^{-j\omega t}\) are assumed in our work. Thus, equations (1) and (2) can be solved in the frequency domain

\[
\tilde{b}_i(\omega) = \frac{D_{bi}(\omega) \cdot f_i(\omega)}{D_{bi}(\omega) \cdot D_{d1}(\omega) - \kappa_i^2} \tag{3}
\]

\[
\tilde{d}_i(\omega) = \frac{\kappa_i \cdot f_i(\omega)}{D_{d1}(\omega) \cdot D_{bi}(\omega) - \kappa_i^2} \tag{4}
\]

where \(D_{bi}(\omega) = 1 - (\omega/\omega_{bi})^2 - j\gamma_{bi}(\omega/\omega_{bi})\) and \(D_{d1}(\omega) = 1 - (\omega/\omega_{d1})^2 - j\gamma_{d1}(\omega/\omega_{d1})\). Based on \(f_i(\omega) \propto E_i(\omega), E_i(\omega)\) is the averaged electric field on the bright mode, and the average polarization current \(\bar{I}(\omega) = -j\omega n \tilde{b}_i(\omega)\), \(n\) is the number of atoms per unit of surface area in the bright mode [18]. In addition, the average dipole moment for the bright mode has the connection with \(E_i(\omega)\) expressed as [18]

\[
\bar{n}\tilde{b}_i(\omega) = \beta E_i(\omega), \text{with } \beta = \varepsilon_0 \sum_N \chi^{(N)} E_i(\omega)^{N-1} \tag{5}
\]

where \(\chi^{(N)}\) is the \(N\)th-order surface susceptibility. Thus, the developed radiation oscillator theory can describe nonlinear optical responses in coupled systems. Through the static limit, \(\beta E_i(\omega) \approx n f_i(\omega)\) [18].
Based on the Ohm’s law \( I(\omega) = \sigma_i E_i(\omega) \), the surface conductivity can be calculated as

\[
\sigma_i \approx \frac{-j\omega \beta D_{\sigma i}(\omega)}{D_{\text{bi}}(\omega)D_{\text{bi}}(\omega) - \kappa_i^2} \tag{6}
\]

The effective transmission, reflection, and absorption responses can be described by an electric current sheet with surface conductivity \( \sigma_i \). The amplitude of the light wave in the bus waveguide can be expressed through scattering matrix as:

\[
\begin{pmatrix}
a_2 \\
b_2
\end{pmatrix} = s_i \begin{pmatrix}
a_i \\
b_i
\end{pmatrix} = \begin{pmatrix}
t_i & r_{i+} \\
r_{i+} & t_i
\end{pmatrix} \begin{pmatrix}
a_i \\
b_i
\end{pmatrix} = \frac{2 + \zeta \sigma_i}{2 + \zeta \sigma_i} \begin{pmatrix}
t_i & r_{i+} \\
r_{i+} & t_i
\end{pmatrix} \begin{pmatrix}
a_i \\
b_i
\end{pmatrix} \tag{7}
\]

where \( \zeta \) is the wave impedance of the external waves and \( i = 1 \). According to the relationship between the scattering matrix and transmission matrix, the transmission matrix can be calculated as

\[
\begin{pmatrix}
a_2 \\
b_2
\end{pmatrix} = T_i \begin{pmatrix}
a_1 \\
b_1
\end{pmatrix} = \begin{pmatrix}
t_i & r_{i+} \\
r_{i+} & t_i
\end{pmatrix} \begin{pmatrix}
a_1 \\
b_1
\end{pmatrix} \tag{8}
\]

where \( i = 1 \). In addition, the phase transformation matrix from \( i \) unit to \( i + 1 \) unit can be expressed as

\[
P_i = \begin{bmatrix} e^{-j\phi} & 0 \\ 0 & e^{j\phi} \end{bmatrix} \tag{9}
\]

Thus, the total transmission matrix when there are two units in our proposed model can be expressed as

\[
T_{\text{total}} = T_2 \cdot P_1 \cdot T_1 = \begin{bmatrix}
\alpha_1e^{\phi} + e^{-j\phi}r_{2-}a & e^{j\phi} \zeta \sigma_i \alpha_2 t_2 \\
-e^{j\phi}r_{1+} & e^{-j\phi} \zeta \sigma_i \alpha_2 t_2 + e^{j\phi} \zeta \sigma_i r_{2-}a \\
t_1 a & t_1 a + e^{-j\phi} \zeta \sigma_i r_{2-}a \\
t_2 b & t_2 b + e^{j\phi} \zeta \sigma_i r_{2-}a \\
\end{bmatrix} \tag{10}
\]

where \( \alpha_i = t_i - r_i - r_i^* / t_i, (i = 1, 2) \). Based on the configuration of the proposed model in figure 1, the relationship between input and output of the proposed model can be described by the following transfer matrix and scattering matrix as follows

\[
\begin{pmatrix}
a_4 \\
b_4
\end{pmatrix} = T_{\text{total}} \begin{pmatrix}
a_1 \\
b_1
\end{pmatrix} = s_{\text{total}} \begin{pmatrix}
a_1 \\
b_1
\end{pmatrix} \tag{11}
\]

Therefore, the total scattering matrix of the proposed waveguide model can be calculated as

\[
s_{\text{total}} = \begin{bmatrix} t_\text{total} & r_{-\text{total}} \\ r_{+\text{total}} & t_\text{total} \end{bmatrix} = \begin{bmatrix}
\frac{e^{j\phi} r_{1+} / t_1 a - e^{-j\phi} r_{2-} / t_1 b}{e^{j\phi} r_{1+} / t_1 b + e^{-j\phi} \alpha_1 t_2 / t_2} & 1 \\
e^{-j\phi} \zeta \sigma_i r_{2-} b / t_1 b - e^{-j\phi} r_{2-} a / t_1 a & e^{j\phi} r_{1+} / t_1 a - e^{-j\phi} \alpha_1 t_2 / t_2 \\
e^{j\phi} r_{1+} / t_1 b + e^{-j\phi} \alpha_1 t_2 / t_2 & e^{-j\phi} \zeta \sigma_i r_{2-} b / t_1 b - e^{-j\phi} r_{2-} a / t_1 a \\
e^{j\phi} r_{1+} / t_1 b - e^{-j\phi} \alpha_1 t_2 / t_2 & e^{-j\phi} \zeta \sigma_i r_{2-} b / t_1 b - e^{-j\phi} r_{2-} a / t_1 a \\
\end{bmatrix} \tag{12}
\]

where \( t_{\text{total}}, r_{+\text{total}}, \) and \( r_{-\text{total}} \) are the transmission coefficient, forward and backward reflection coefficients in the proposed model.

3. Structural model and discussion

In order to prove the correctness of the radiation oscillator theory, we propose a graphene coupled metamaterial as shown in figure 2(a). The metamaterial consists of the upper and lower layers of graphene and the middle dielectric structure. The gray part is chosen to be glass, the permittivity of the glass is taken from the reference book by Palik [20]. For the sake of the clarity of presentation, the Kubo formula has governed the surface conductivity of graphene including the intraband and interband transition contributions. The environment for this work is set at room temperature (\( T = 300 \text{ K} \)) in the THz region, so the Fermi energy \( E_F \gg \hbar \omega, k_B T \), where \( \hbar \) is the reduced Planck constant and \( k_B \) is the Boltzmann constant. Thus the complex conductivity of graphene \( \sigma_g \) can be reduced to a Drude-like expression as follows [10]

\[
\sigma_g = \frac{j e^2 E_F}{\pi \hbar^2 (\omega + i\gamma)} \tag{13}
\]
where $e$ is the elementary charge. $\tau = \mu E_f / (e v_F^2)$ is the carrier relaxation time with $\mu = 1 \text{ m}^2 \text{s}^{-1}$ and $v_F = 10^6 \text{ m s}^{-1}$. The structural parameters are set as $h = 230 \text{ nm}$, $l = 2000 \text{ nm}$, $w = 700 \text{ nm}$, and $p = 150 \text{ nm}$, the upper and lower layers of graphene have the same structural parameters. A TM-polarized wave is injected along the negative direction of z-axis. The spectral responses can be simulated by using the finite-difference time-domain (FDTD) method. In the simulation, the effective area is divided into uniform Yee cells with $\Delta x = \Delta y = \Delta z = 5 \text{ nm}$ and $\Delta t = \Delta x / 2c$ ($c$ is the velocity of light in vacuum) [21, 22]. The perfectly matched layer can be set for the boundary conditions along the direction of z, and the periodic boundary condition is set at the direction of x and y [23]. Figure 2(b) shows the transmission spectra of the proposed graphene coupled metamaterial. The red line shows the FDTD simulation result, and blue circle shows the theoretical result through the developed radiation oscillator theory, and the fitting parameters of the radiation oscillator theory for our proposed graphene coupled metamaterial are shown as:

\[
\begin{align*}
\omega_{b1} &= \omega_{b2} = \omega_{d1} = \omega_{d2} = 3.35 \times 10^{13} \text{ rad s}^{-1}, \\
\beta &= 8.9 \times 10^{-12}, \\
\gamma &= 1.48 \times 10^{-3}, \\
\gamma_{b1} &= \gamma_{b2} = \gamma_{d1} = \gamma_{d2} = 0.05, \\
\varphi &= 0.35\pi, \text{ and } \kappa_1 = \kappa_2 = 0.2.
\end{align*}
\]

Observing from figure 2(b), we can find that the numerical simulation result is well agreement with the radiation oscillator theory result.

Based on the above research result, we further discuss the induced transparency as functions of the phase difference between the two units, the resonant detuning between the bright and dark modes as well as the field intensity of the pump light. Figure 3(a) shows the transmission spectra versus the phase difference between the first and second units in the symmetric damping case ($\gamma_{b1} = \gamma_{b2} = \gamma_{d1} = \gamma_{d2} = 0.05$) when $\zeta = 1.48 \times 10^{-3}$, and $\kappa_1 = \kappa_2 = 0.2$. We can see that the obvious induced transparency shows periodic changing as the phase difference increases from 0 to $2\pi$ when there is no resonant detuning between the bright mode and the dark mode as shown in figure 3(a). Then we investigate transmission spectra as a function of the phase difference in the case of $\Delta = \omega_{b1} - \omega_{d1} = \omega_{b2} - \omega_{d2} = 5 \times 10^{12} \text{ rad s}^{-1}$ as depicted in figure 3(c). Comparing with figure 3(a), we can see that quality factor of the induced transparency decreases, and the asymmetry of the spectral line is enhanced.

Then we discuss the optical responses versus the field intensity $E$ of the pump light. Here the third-order nonlinear effects of the graphene are considered. The linear susceptibility $\chi^{(1)}$ and third-order nonlinear susceptibility $\chi^{(3)}$ can be expressed as [24, 25]

\[
\begin{align*}
\chi^{(1)} &= \frac{\sigma_s^{(1)}}{2\pi\omega d}, \quad \text{where } \sigma_s^{(1)} = -\frac{j e^2 E}{\pi \hbar^2 \omega}, \\
\chi^{(3)} &= \frac{\sigma_s^{(3)}}{2\pi\omega d}, \quad \text{where } \sigma_s^{(3)} = -\frac{9 e^4 v_F^2}{\pi 8 \hbar^2 E \omega^3}.
\end{align*}
\]

where $\sigma_s^{(1)}$ and $\sigma_s^{(3)}$ are linear and third-order nonlinear surface conductivity of the graphene, and $d$ is the thickness of the monolayer graphene. Thus, the variable $\beta$ in equation (3) can be simplified as $\beta = \varepsilon_0 \chi^{(1)} + \varepsilon_0 \chi^{(3)} |E|^2$ as follows

\[
\beta = -j \varepsilon_0 \frac{e^2 E}{2\pi \hbar^2 \omega^2 2d} - j \varepsilon_0 \frac{9 e^4 v_F^2}{2\pi 8 \hbar^2 E \omega^4 d} |E|^2.
\]

The transmission spectra as a function of the field intensity $E$ of the pump light in the case of $\Delta = 0$ and $\Delta = 5 \times 10^{12} \text{ rad s}^{-1}$ are shown in figures 3(b) and 3(d), respectively. From figure 3(b), we can see that the quality factor of the induced transmission window increases, but the transmittance at the transmission window decreases as the field intensity $E$ increases. Interestingly, the induced transparency disappears when the intensity $E$ is larger than $5 \times 10^{12} \text{ V m}^{-1}$. When the resonant detuning $\Delta = 5 \times 10^{12} \text{ rad s}^{-1}$, the trend is similar to the case without detuning as shown in figure 3(d), but the induced transmission window shows
asymmetric spectra comparing with figure 3(b). These results show a variety of modulation methods for induced transparency.

The field intensity $E$ of the pump light and the damping factor of the dark mode are significant factors for the induced transparency. Thus, we study transmission spectra and its quality factor as a function of the intensity $E$ with $\gamma_{b1} = \gamma_{b2} = 0.05$, $\zeta = 1.48 \times 10^{-3}$, $\varphi = 0.35\pi$, $\Delta = 0$, and $\kappa_1 = \kappa_2 = 0.2$ when $\gamma_{d1} = \gamma_{d2} = 0.001$ and $\gamma_{d1} = \gamma_{d2} = 0.005$ as shown in figure 4. From figure 4(a), we can see that the quality factor of the induced transparency sharply increases as the intensity $E$ of the pump light increases when $\gamma_{d1} = \gamma_{d2} = 0.001$. Interestingly, when $\gamma_{d1} = \gamma_{d2} = 0.005$, the induced transparency disappears when
Table 1. The transmittance, central angular frequency, FWHM, and $Q$ versus $E$ when $\gamma_{d1} = \gamma_{d2} = 0.05, 0.005,$ and 0.001, respectively.

| $E(\times 10^{12} \text{ V m}^{-1})$ | $T$ | $\omega_b/\omega_{b(2)}$ | FWHM | $Q$ |
|-----------------------------------|-----|------------------------|-------|-----|
| 0                                | 0.196 | 0.993                 | 0.064 | 15.5 |
| 5                                | 0.088 | 0.993                 | 0.052 | 19.1 |
| 0                                | 0.825 | 0.996                 | 0.060 | 16.6 |
| 5                                | 0.538 | 1.001                 | 0.023 | 43.5 |
| 0                                | 0.167 | 0.999                 | 0.007 | 142.7 |
| 10                               | 0.036 | 1.000                 | 0.005 | 200.0 |
| 0                                | 0.961 | 1.000                 | 0.060 | 16.7 |
| 5                                | 0.876 | 1.001                 | 0.022 | 45.5 |
| 10                               | 0.673 | 1.001                 | 0.007 | 143.9 |
| 15                               | 0.444 | 1.000                 | 0.003 | 333.3 |
| 20                               | 0.250 | 1.000                 | 0.002 | 500.0 |

$E > 15 \times 10^{12} \text{ V m}^{-1}$ as shown in figure 4(b). Comparing with figures 3(b), 4(a) and (b), we can find that more obvious induced transparency and larger quality factor can be realized in the case of stronger field intensity of the pump light and smaller damping factor in the dark mode.

In order to quantitatively describe the transmittance and damping factor versus the field intensity $E$ and the damping factor of dark mode in figure 4, we will show the full width half maximum (FWHM) and quality factor ($Q = \omega_b/(\omega_{b(2)} \times \text{FWHM})$) in table 1. From table 1, we can see that the transmittance and FWHM decrease when the field intensity of the pump light $E$ increases as $\gamma_{d1}$ and $\gamma_{d2}$ keep unchanged. Especially, we can see that the quality factor $Q$ of induced transparency can reach up to 500 when $\gamma_{d1} = \gamma_{d2} = 0.001$ and $E = 20 \times 10^{12} \text{ V m}^{-1}$, which is over 30 times than that in the linear case. So the low loss for the dark mode and nonlinear optical effect can effectively enhance the quality factor of induced transparency.

As is well known, gain material can effectively enhance the quality factor $Q$ for the optical system. Moreover, when the gain and loss are balanced in an optical system will emerge a variety of particular phenomena [26]. Thus, we will study the optical responses when there is gain materials filled on the dark mode graphene with $\zeta = 1.48 \times 10^{-3}$, $\gamma_{b1} = \gamma_{b2} = 0.05$, $\varphi = 0.35\pi$, $\Delta = 0$, and $\kappa_1 = \kappa_2 = 0.2$ as shown in figure 5. The case of over gain represents the damping factor for the dark mode $\gamma_{d1} = \gamma_{d2} < 0$. When $\gamma_{d1} = \gamma_{d2} = -0.001$, we can see that the transmittance and quality factor of induced transparency show obvious increase as the field intensity of the pump light $E$ increases, and the maximum of $Q$ is of $\sim 249.8$. Figure 5(b) shows the transmission spectra as a function of the field intensity of the pump light $E$ when $\gamma_{d1} = \gamma_{d2} = -0.005$. We can find that both the transmittance and $Q$ first increase and then decrease. Interestingly, the single induced transparency window can split into double induced transparency windows when the field intensity of the pump light $E > 5 \times 10^{12} \text{ V m}^{-1}$. Moreover, the maximum of $Q$ for peak 1 and peak 2 can reach up to 498.5 and 1001.2, respectively. At last, we study optical responses in the balanced...
case for the bright mode and dark mode ($\gamma_{b1} = \gamma_{b2} = 0.05, \gamma_{d1} = \gamma_{d2} = -0.05$) as shown in figure 5(c). We can see obvious double induced transparency when the intensity of the pump light $E < 2.5 \times 10^{12} \text{ V m}^{-1}$. In addition, the Q and transmittance decrease when the intensity of the pump light $E$ increases.

4. Conclusion

In summary, a novel nonlinear radiation oscillator theory has been established for demonstrating the nonlinear optical response in symmetric and anti-symmetric damping graphene coupled metamaterials. Induced transparency with the maximum of quality factor $\sim 1001.2$ have been realized in our proposed model in terahertz band. In addition, we have also found that transmission spectra and its quality factor of the induced transparency can be effectively tuned by the resonant detuning, the phase difference, the gain or loss coefficient of dark mode as well as the field intensity of the pump light. Especially, the strong induced transparency can be effectively tuned by the resonant detuning, the phase difference, the gain or loss coefficient of dark mode as well as the field intensity of the pump light. At last, we have also found that the single induced transparency turns into double induced transparency when $\gamma_{b1} = \gamma_{b2} = -\gamma_{d1} = -\gamma_{d2}$. The proposed nonlinear radiation oscillator theory may have important applications for predicting interesting optical phenomena and designing the high performance nano-optical devices.

Disclosures

The authors declare no conflicts of interest.

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References

[1] Boller K-J, Imamoglu A and Harris S E 1991 Observation of electromagnetically induced transparency Phys. Rev. Lett. 66 2593–6
[2] Fleischhauer M, Imamoglu A and Marangos J P 2005 Electromagnetically induced transparency: optics in coherent media Rev. Mod. Phys. 77 633
[3] He Z, Li H, Li B, Chen Z, Xu H and Zheng M 2016 Theoretical analysis of ultrahigh figure of merit sensing in plasmonic waveguides with a multimode stub Opt. Lett. 41 5206
[4] He Z, Li H, Zhan S, Cao G and Li H 2014 Combined theoretical analysis for plasmon-induced transparency in waveguide systems Opt. Lett. 39 5543
[5] Zhan S, Li H, He Z, Li B, Chen Z and Xu H 2015 Sensing based on plasmon-induced transparency in nanocavity-coupled waveguide Opt. Express 23 20313
[6] He Z et al 2020 Tunable Fano resonance and enhanced sensing based on plasmon-induced transparency in nanocavity-coupled waveguide Opt. Express 28 79–86
[7] Xia S-X, Zhai X, Huang Y, Liu J-Q, Wang L-L and Wen S-C 2017 Graphene surface plasmons with dielectric metasurfaces J. Lightwave Technol. 35 4553–8
[8] Liu N, Langguth L, Weiss T, Kästel J, Fleischhauer M, Pfau T and Giessen H 2009 Plasmonic analogue of electromagnetically induced transparency at the Drude damping limit Nat. Mater. 8 758–62
[9] Chen Z, Zhang S, Chen Y, Liu Y, Li P, Wang Z, Zhu X, Bi K and Duan H 2020 Double Fano resonances in hybrid disk/rod artificial plasmonic molecules based on dipole–quadrupole coupling Nanoscale 12 9776–85
[10] Xu H, Li H, He Z, Chen Z, Zheng M and Zhao M 2017 Dual tunable plasmon-induced transparency based on silicon-air grating coupled graphene structure in terahertz metamaterial Opt. Express 25 20780
[11] Enduo G, Zhimin L, Hongjian L, Hui X and Fengqi Z 2019 Dynamically tunable dual plasmon-induced transparency and absorption based on a single-layer patterned graphene metamaterial Opt. Express 27 13884
[12] Xia S X, Zhai X, Wang L L and Wen S C 2018 Plasmonically induced transparency in double-layered graphene nanoribbons Photon. Res. 6 31–41
[13] Liu Z, Gao E, Zhang X, Li H, Xu H, Zhang Z, Luo X and Zhou F 2020 Terahertz electro-optical multi-functional modulator and its coupling mechanisms based on upper-layer double graphene ribbons and lower-layer a graphene strip New J. Phys. 22 053039
[14] He Z, Zhao J and Lu H 2020 Tunable nonreciprocal reflection and its stability in a non-PT-symmetric plasmonic resonators coupled waveguide systems Appl. Phys. Express 13 012009
[15] Huang Y, Veronis G and Min C 2015 Unidirectional reflectionless propagation in plasmonic waveguide-cavity systems at exceptional points Opt. Express 23 39882–95
[16] Cao G, Li H, Zhan S, He Z, Guo Z, Xu X and Yang H 2014 Uniform theoretical description of plasmon-induced transparency in plasmonic stub waveguide Opt. Lett. 39 216
[17] Zhong Z-J, Xu Y, Lan S, Dai Q-F and Wu L-J 2010 Sharp and asymmetric transmission response in metal-dielectric-metal plasmonic waveguides containing Kerr nonlinear media Opt. Express 18 79–86
[18] Tassin P, Zhang L, Zhao R, Jain A, Koschny T and Soukoulis C M 2012 Electromagnetically induced transparency and absorption in metamaterials: the radiating two-oscillator model and its experimental confirmation Phys. Rev. Lett. 109 187401
[19] Huang Y, Shen Y, Min C and Veronis G 2017 Switching of the direction of reflectionless light propagation at exceptional points in non-PT-symmetric structures using phase-change materials Opt. Express 25 27283
[20] Palik E D 1998 Handbook of Optical Constants of Solids (New York: Academic)
[21] He Z, Li Z, Li C, Xue W and Cui W 2020 Ultra-high sensitivity sensing based on ultraviolet plasmonic enhancements in semiconductor triangular prism meta-antenna systems Opt. Express 28 17595–610
[22] Yu P, Yang H, Chen X, Yi Z, Yao W, Chen J, Yi Y and Wu P 2020 Ultra-wideband solar absorber based on refractory titanium metal Renew. Energy 158 227–35
[23] Cao G, Li H, Deng Y, Zhan S, He Z and Li B 2014 Plasmon-induced transparency in a single multimode stub resonator Opt. Express 22 25215
[24] Peres N, Bludov Y V, Santos J E, Jauho A-P and Vasilevskiy M 2014 Optical bistability of graphene in the terahertz range Phys. Rev. B 90 125425
[25] Qasymeh M 2017 Phase-matched coupling and frequency conversion of terahertz waves in a nonlinear graphene waveguide J. Lightwave Technol. 35 1654–62
[26] Lawrence M, Xu N, Zhang X, Cong L, Han J, Zhang W and Zhang S 2014 Manifestation of PT symmetry breaking in polarization space with terahertz metasurfaces Phys. Rev. Lett. 113 093901