Direct coupling-induced pseudoparity nonconservation scattering: bipolar spin diode and unipolar spin entanglement pairing

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Abstract

The zigzag graphene nanoribbon (ZR) is characterized by the distinct pseudoparity combined with valley-selection rule, which could feature exotic transport phenomena, especially in ZR-based superconducting spintronic devices. However, the ZR with superconductivity induced by proximity of a bulk superconductor (SC) on it still keeps original band properties. Herein, we present a superconducting heterostructure with an SC directly coupling to two ZRs, which is characteristic of pseudoparity-mixing, resulting in pseudoparity nonconservation elastic cotunneling (EC) and crossed Andreev reflection (CAR) processes. It is shown that the mixing leads to the switch effect of the EC and CAR processes manipulated by the SC length, particularly the full spin polarization. In the context of only one magnetized ZR lead, a novel bipolar spin diode behavior on a scale of small SC length and unipolar spin entanglement pairing at some large SC lengths, are both exhibited on a large scale of forward and/or reverse bias voltages. More importantly, the spin-diode can be combined with the quantum spin Hall (QSH) insulator to provide smoking gun evidence for the helical spin texture of the (QSH) insulator, which is still lacking.

1. Introduction

Due to the expectation on the utilization of entanglement effects in quantum communication and computation [1, 2], quantum entanglement as a kind of nonlocal quantum correlation [3], has been an extremely active research area. One of the most key issues for us to confront with is searching for the proper methods of creating entangled particles. A spin entangled electron pair is a better candidate in experiments, which is usually much less sensitive to decoherence than an orbitally entangled one [4]. A superconductor (SC) is deemed as a natural source for generating nonlocal electron pairs [3, 5, 6]. A Cooper pair can be spatially deformed by the inverse nonlocal or crossed Andreev reflection (CAR), a process which emits two spin entangled electrons in two conventional metallic leads, where they can be probed separately [7–11]. Therefore, a lot of Cooper pair splitters have been theoretically put forward and experimentally implemented in part, where an SC is coupled with quantum dots [12, 13], carbon nanotubes [14, 15], Luttinger liquid wires [16], graphenes [17, 18], spin quantum Hall insulators [19, 20], quantum anomalous Hall insulators [6], etc. However, besides the CAR, there also exists local Andreev reflection (LAR) [21], particularly, the CAR is often completely masked by another nonlocal process known as electron cotunneling (EC) [6, 22, 23], which does not involve Cooper pairs and is therefore a parasitic process. To obtain a perfect CAR with its coefficient being 1, one should propose a system, where the LAR, EC, and
normal reflection are all completely suppressed and thus all incident electrons are converted into holes in the other terminal.

Recently, acting as a fundamental and necessary block in modern spin-logic devices, such as spin transistors and magnetic tunneling junctions, spin diode has attracted growing attention [24, 25]. As a spin version of a charge diode, it sustains a fully-spin-polarized current with the property of forward conducting and backwards blocking [26, 27]. Generally, there exist two types of spin diodes. One is a bipolar spin diode with one spin forward conducting but backward blocking and the other spin being just opposite [28, 29]. The other is a unipolar spin diode with one spin forward conducting but backward blocking and the other spin being always nonconducting [30, 31]. Only in a few specific magnetic systems, can both the bipolar spin diode and the unipolar spin diode be realized [24, 25, 27], in which a strong contrast of on and off states in spin filtering is required.

Furthermore, in recent years, the emergence of new materials, graphene and typically graphene-like monolayers, including silicene, germanene, and stanene, has brought a new perspective to spintronics, specifically owing to the presence of two coupled degrees of freedom, spin and valley, which were intensively studied [32–42]. Besides the spin and charge degrees of freedom, as is known, the electron in these two-dimensional materials with the honeycomb lattice structure has linear low energy dispersion around two valleys at the K and K′ points of the Brillouin zone, being degenerate. The two valleys, characterized by a pseudo-spin degree of freedom, are related to each other by time-reversal symmetry like the spin degree of freedom, so a superconducting Cooper pair should be composed of electrons from the two opposite valley bands, which is also the crucial mechanism in valley-based superconducting spintronics [10, 17, 43–46].

Advanced nanoscale technologies have stimulated the fabrication of graphene-based devices, which may exert a significant impact on the electronics and computer industries of the future. Hence, it becomes extremely important to study the physics and modeling of the graphene-based devices. Here, we are only focused on the zigzag graphene nanoribbon (ZR) case [47, 48], since it is relatively easy to produce the ZRs experimentally and their edges are in favor of forming bonds with other atoms or compounds. In addition, the ZRs have a band with zero-energy modes as shown in figures 2(a) and (b), which is absent in the armchair situation. Particularly, for a ZR with even-number chain, the electron wave function having odd or even pseudoparity requires that electrons are only allowed to transport between the same pseudoparity bands, i.e., the valley-selection rule [49–51]. Therefore, in a ZR-based superconducting heterostructure, tuning the gate voltages and magnetizing the corresponding lead would yield a variety of plentiful features in such as EC and CAR processes. However, hitherto in such previous structures, the graphene with superconductivity induced almost by the proximity remains the original band properties, where the superconducting layer is placed on the graphene layer, i.e., so-called home-SC model. Thus, many important superconducting spintronic applications, like spin diode and quantum entanglement in the following parts, are severely hindered by no pseudoparity-mixing thanks to an SC directly coupling to two ZRs [17, 52], which is referred to as bulk-SC model. Thus, it is desirable to maximize the potential for transport phenomena induced by pseudoparity-mixing in superconducting spintronic applications based on ZRs.

Therefore, we study a ZR/SC/ZR heterostructure with the two ZRs directly coupling to the SC layer, as shown figure 1(a), in which there exists pseudoparity-mixing due to the electrons having no valley degree in the bulk SC. Using a Green’s function method, we investigate the corresponding scattering processes. The pseudoparity-mixing, particularly combined with spin freedom in the magnetized ZR (MZR) lead, results in the switch effect of the CAR and EC processes tuned by the SC length. It is found that under the case of only one MZR lead, on a scale of small SC length, there always exists only EC process for one spin direction at the forward bias voltages while for the opposite spin direction at reverse bias voltages. However, at some large SC lengths, only CAR process either for one spin direction at the forward bias voltages or for the opposite spin direction at reverse bias voltages, is exhibited. Therefore, our results pave the way toward the design of novel bipolar spin diodes and unipolar electron entanglement creators. More interestingly, the novel bipolar spin-diode behavior is the same as that in quantum spin Hall (QSH) insulator characterized by helical states with the spin polarization of electron being locked to its direction of motion, and thus can be combined with the QSH insulator [53], so as to provide smoking gun evidence for the helical spin texture being still lacking [54–61].

2. Direct coupling of the SC to two ZRs

The MZR/SC/ZR heterostructure under consideration shown in figure 1(a) can be regarded as a three-terminal device, being infinite along x-axis while finite along y-axis, where the SC lead is grounded, its length is $La$ with $a$ the lattice constant, the MZR and ZR leads are, respectively, biased $V_+$ and $V_R$, and
where the width of the ribbon is denoted by the zigzag chain number $N$. The structure is described by the Hamiltonian

$$H = \sum_{k\sigma} U_{k} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{\langle lm \rangle} t (c_{k\sigma}^\dagger c_{k\sigma} + \text{c.c.}) + \sum_{k\sigma} (\varepsilon_k - \mu) b_{k\sigma}^\dagger b_{k\sigma} + \sum_{k} (\Delta b_{k\uparrow}^\dagger b_{k\downarrow} + \text{c.c.}) + \sum_{\langle\beta\sigma\rangle} (t_{\beta} c_{\beta\sigma}^\dagger b_{\sigma} + \text{c.c.}),$$

(1)

where the first two terms describe the two ZRs with $c_{k\sigma}^\dagger$ ($c_{k\sigma}$) the creation (annihilation) operator at site $l$ for spin $\sigma$ ($\sigma = \pm \uparrow \downarrow$), $\langle lm \rangle$ denoting the summation over the nearest neighbor sites, $t = 2.8 \text{ eV}$ the hopping integral, and $U_{k}$ the lattice site energies of the left, middle, and right regions; the third and fourth terms depict the SC metal lead with $\mu$ the chemical potential and $\Delta$ the superconducting pair potential; the last term stands for the coupling between the SC lead and the ribbon with $t_{\beta}$ the hoping energy and $b_{\sigma}$ the lattice version of the operator $b_{\sigma}$ in the SC lead. In the calculation, the ZR is considered to be composed of slices, where per slice is treated as a big cell and thus the ZR is reduced to a one-dimensional system.

The current flowing in the left MZR lead can be obtained by the standard Keldysh formalism

$$I_{L} = \frac{e}{h} \int dE \text{Tr} [H_{i\uparrow+1} G_{i\uparrow+1}^\text{R}(E) - G_{i\uparrow+1}^\text{L}(E) H_{i\uparrow+1}]_{\sigma},$$

(2)

where $H_{i\uparrow+1}$ the hopping matrix between two neighboring slices of the ZR with $i$ a unit slice index and $G^\text{C} < \text{L} \text{R}$ the lesser Green’s function, are $8N \times 8N$ matrices by considering both the spin and Nambu spaces. In the light of the Keldysh formula $G = G^\Sigma G^\dagger$ with $G^{\Sigma\dagger}$ the retarded (advanced) Green’s function and $\Sigma^\dagger$ the lesser self-energy, the current is reduced to

$$I_{L} = \frac{e}{h} \int dE (T_{\text{EC}} + T_{\text{AC}} + T_{\text{CAR}} + T_{\text{QT}})$$

(3)

with the four terms $T_{\text{EC}} = \text{Tr} \{G^\text{L} G^\text{R} (f_{\text{L}} - f_{\text{R}}) \},$ $T_{\text{AR}} = \text{Tr} \{G^\text{L} G^\text{R} (f_{\text{L}} - f_{\text{R}}) \},$ $T_{\text{CAR}} = \text{Tr} \{G^\text{L} G^\text{R} (f_{\text{L}} - f_{\text{R}}) \},$ $T_{\text{QT}} = \text{Tr} \{G^\text{L} G^\text{R} (f_{\text{L}} - f_{\text{R}}) \}$ representing the EC process, local AR process, and quasiparticle’s tunneling process occurring mainly out of the superconducting energy gap, respectively. Here, $f_{j}$ ($j = \text{L, R, S}$) denotes the Fermi–Dirac distribution function in the $j$th lead with $f_{\text{L(R)}} = f(E + eV_{\text{L(R)}})$ and $f_{\text{L(R)}} = f(E - eV_{\text{L(R)}})$. $\Gamma_{j} = i(\Sigma^+ - \Sigma^\dagger)$ is the linewidth function of the $j$th lead, $\Sigma^\dagger = [\Sigma^+]^\dagger$ refers to the self-energy, and the Green’s function of the SC lead is determined by $G = [E - H_{i} - \Sigma^\dagger(\Sigma^+)]^{-1}$ with $H_{i}$ the Hamiltonian of the SC lead in the Nambu space and $I$ a unit matrix. In addition, the left and right self-energies $\Sigma^\text{L,R}$ can be evaluated from the semi-infinite ideal ZR lead, while that of the SC lead can be analytically obtained as $\Sigma^\text{C} = i \sigma_{0} (E_{i} + \Delta \sigma_{z})/2 \Omega$ with

$$\Omega = \sqrt{E_{i}^{2} - \Delta^{2}} \text{ at } |E_{i}| > \Delta \text{ and } \Omega = i \sqrt{\Delta^{2} - E_{i}^{2}} \text{ at } |E_{i}| < \Delta.$$ In calculations, the zero temperature $T = 0 \text{ K}$ is considered, the pair potential is set as $\Delta = 1 \text{ meV}$, $\Gamma_{\text{L,R}}$ can be calculated directly from a semi-infinite.
Figure 2. (a) Electron and hole energy dispersions of the edge states in the nonmagnetic even ZR with the left (the gate voltages \( U_L(0) \)) and right (\( U_L(0) > 0 \)) panels. The hole band ‘h’ is obtained as a mirror image of the electron band ‘e’ over the Fermi level (horizontal dotted line). '+' and '-' denote, respectively, the even and odd pseudoparities of the edge states. (b) The corresponding dispersions of the edge states in a magnetized even ZR, where the spin-up (-down) electron band overlaps with spin-down (-up) hole band of opposite pseudoparities, and the spin exchange energy causes a shift of the Dirac point from the Fermi level. (c) Corresponds to the bulk SC with the left (the gate voltage \( U_S > 0 \)) and right (\( U_S < 0 \)) panels.

ZR, the linewidth constant of the SC lead is taken to be \( g_S = 2\Delta \), and the zigzag chain number is taken as even number 14.

We focus the dependence of \( T_{\text{CAR}} \) and \( T_{\text{EC}} \) on the SC length \( L \) and bias voltages \( eV_L \) of the left ZR in the ZR/bulk-SC/ZR and MZR/bulk-SC/ZR structures. Since the electron-like and hole-like zero-energy states have opposite pseudoparities for an even ZR as shown in the left panel of figure 2(a), the LAR at the interface of the undoped ZR/SC junction should be prohibited due to the valley-selection rule. However, at a positive gate voltage applied on the ZR (or \( U_L > 0 \)), the energy-dispersion shifts downward as illustrated in the right panel of figure 2(a), only the electron-like zero-energy state is involved in transport, so the LAR is allowed since the electron (e) and hole (h) have the same pseudoparities. In the following parts, we set the gate voltages of the left and right ZRs to be zero, namely \( U_L = U_R = 0 \), so that the LARs are prohibited and there only exist the CAR and EC processes.

3. Switch effect induced by pseudoparity mixing scattering on CAR and EC

We first briefly analyze the pseudoparity-mixing in ZR (MZR)/bulk-SC/ZR structures. The incident electrons with ‘+’ and ‘-’ pseudoparities in the left ZR (MZR) both can propagate into the electron (hole)-like bands of the SC region due to the pseudoparity-mixing in the bulk SC with no valley degree (see figure 2(c)), and then enter into the electron (hole)-like bands of the right ZR region with reverse pseudoparities. This indicates novel opened channels, hence the pseudoparity nonconservation EC (CAR) processes are yielded.

Figure 3 shows \( T_{\text{CAR}} \) and \( T_{\text{EC}} \) as a function of the SC length \( L \) at positive and negative \( eV_L \). It is found that \( T_{\text{CAR}} \) and \( T_{\text{EC}} \) oscillate with \( L \) and can emerge for not only \( eV_L = 0.5\Delta < 0 \) but also \( eV_L = -0.5\Delta < 0 \). The explanation can be given by that at \( eV_L < 0 \), the incident electron of ‘+’ pseudoparity in the left ZR propagates into the electron- and hole-like bands in the SC region due to the pseudoparity-mixing (see the left panel of figures 2(a) and (c)), which allows for the CAR and EC processes \( (T_{\text{CAR}} \neq 0 \text{ and } T_{\text{EC}} \neq 0) \), similarly, at \( eV_L > 0 \), the pseudoparity-mixing also opens new channels for both the CAR and EC processes. However, in the ZR/home-SC/ZR structure [17], due to the valley-selection rule, for \( eV_L > 0 \), only electron-like zero-energy mode is involved in the transport, thus the EC is allowed \( (T_{\text{EC}} \neq 0) \) but the CAR is thoroughly suppressed \( (T_{\text{CAR}} = 0) \), and for \( eV_L < 0 \), \( T_{\text{CAR}} \) and \( T_{\text{EC}} \) are both vanishing due to the system resembling a pnp junction.

In figure 3(a) with \( eV_L > 0 \), for small length \( L \), \( T_{\text{CAR}} \) exhibits a fairly weak oscillation with \( T_{\text{CAR}} \simeq 0 \) or \( = 0 \) at most values of \( L \), while the case is just contrary for large length \( L \), and \( T_{\text{EC}} \) has a fairly strong oscillation by comparison. In figure 3(b) with \( eV_L < 0 \), compared with those for \( eV_L > 0 \), the oscillations of \( T_{\text{CAR}} \) and \( T_{\text{EC}} \) are slightly weaker and slower on the whole, particularly, for the small \( L \), \( T_{\text{CAR}} \) is almost equal to zero. Furthermore, the pseudoparity nonconservation scattering naturally requires a finite time or distance, especially for CAR processes, which can be reflected by the fact that \( T_{\text{CAR}} \) owns a weaker
Figure 3. The CAR and EC probabilities versus the SC length $L$. Here, $eV_L = 0$, $U_L = U_R = 0$, $U_S = -10\Delta$, $eV_L = 0.5\Delta$ (the left column) and $-0.5\Delta$ (the right column).

oscillation at both $eV_L > 0$ and $eV_L < 0$, compared with $T_{\text{EC}}$. This also means that the coherence length exerts a significant influence on the CAR process. The pseudoparity-mixing obviously induces the novel channels with the pseudoparity nonconservation (inversion), which are thoroughly different from the pseudoparity conservation channels as a consequence of the valley-selection rule [17], can bring about peculiar features.

In what follows, we investigate the pseudoparity-mixing combined with the spin freedom and thus introduce an MZR to replace the left ZR lead with the magnetization effect, where the site energy $U_L$ in equation (1) is replaced by the spin exchange term $h \cdot \sigma$ in calculations. The site energy $U_S$ in the SC region ($|U_S| > h > \Delta$), whose variation can adjust the Fermi energy of the SC, must prohibit one spin-species band of the left MZR from contributing to the current. In figure 4, $T_{\text{CAR}}$ and $T_{\text{EC}}$ are plotted as a function of $L$ for the spin-resolved scattering CAR and EC processes. For $eV_L > 0$, irrespective of $L$, $T_{\uparrow \text{CAR}} \approx 0$ and $T_{\uparrow \text{EC}} \approx 0$, while $T_{\uparrow \text{CAR}} \approx 0$ at any small $L$ and most large $L$ except for around several oscillation peaks, and the oscillation of $T_{\uparrow \text{CAR}}$ is on the whole shown to strengthen at large $L$. The oscillation of $T_{\downarrow \text{EC}}$ is generally slightly strengthened with $L$ but much stronger and faster than that of $T_{\uparrow \text{CAR}}$. It follows that, at $eV_L > 0$ and any small $L$ or certain large $L$, there only exists spin-down EC process. Interestingly, the situation for $eV_L < 0$ is just contrary and the same features exhibit, more specifically, there exists only the spin-up EC process at small $L$, while not only the spin-up EC process but also spin-down CAR process at large $L$. These imply that the features are not changed compared with those for the structure with the left ZR, i.e., the corresponding EC and CRA processes still remain. However, in the present structure, they are fully spin-polarized and opposite in spin directions each other, and introducing spin-polarization gives rise to separation of opposite spin directions by applying bias voltages of opposite direction. The oscillation of corresponding processes with $L$ is related to the formation of resonant transmission levels inside the SC region due to the presence of two interfaces in the junction. Due to much different dispersions of the edge states between the spin-up and -down in an MZR, as shown in figure 2(b), the number of novel opened channels contributing to the current for the spin-down EC induced by the pseudoparity-mixing is much larger than that for the spin-up EC. And thus, the dependence of spin-down EC probabilities on $L$ shows the more obviously periodic oscillation. Here, the properties at $U_S = 10\Delta$ in the structure with the left ZR or MZR lead are found to be the same as those at $U_S = -10\Delta$, and thus we do not give the figures for simplicity.
Figure 4. The spin-resolved CAR and EC probabilities versus $L$ with the same parameters as in figure 3 except that $h = 5\Delta$.

The full-polarization of $T_{\uparrow}\sigma$ and $T_{\downarrow}\sigma$ in the present structure originates from the pseudoparity-mixing, being much different from that in the MZR/home-SC/ZR structure owing to the valley selection rule [17]. For the case of small $L$, due to $L$ being much smaller than the coherence length, there is no enough time (or distance) for CAR to take place, thus we consider general case with large $L$. At $eV_L < 0$, an incident spin-up electron of ‘+’ pseudoparity of the left MZR can be scattered into the electron-like band of the SC, and then into the electron-like band of ‘−’ pseudoparity of the right ZR due to pseudoparity-mixing (see the left panels of figures 2(b) and (c)), indicating pseudoparity nonconservation scattering, therefore the spin-up EC emerges. The existence of spin-down scattering CAR process can be similarly explained, in which no pseudoparity-mixing is needed, meaning the pseudoparity conservation. However, the system at $eV_L < 0$ for an incident spin-down electron resembles a polarized pnp junction, and the current flow is blocked due to $|U_S| > h > \Delta$. At $eV_L > 0$, the emergencies of the spin-down EC and spin-up CAR processes can be explained by the same way, where the former is pseudoparity nonconservative and the latter is just contrary. Therefore, the pseudoparity-mixing gives rise to novel opened channels with definite pseudoparity and spin, and thus more spin polarized scattering processes take up.

4. Bipolar spin-diode

To further get insight into the influence of $eV_L$ on the CAR and EC processes, we illustrate the spin-resolved scattering probabilities $T_{\uparrow}\sigma$ and $T_{\downarrow}\sigma$ varying with $eV_L$ in the MZR/bulk-SC/ZR structure at different $U_S$, as
shown in figure 5. One can find that at small $L$ such as 40, no matter whether $U_S$ is positive or negative, $T^\uparrow_{\text{CAR}}$ is approximately regarded as zero for any $eV_L$ regardless of spin orientation. For $T^\uparrow_{\text{EC}}$, however, there only exists the spin-up EC process for any negative $eV_L$, while only the spin-down EC one for any positive $eV_L$ except that at $U_S = 10\Delta$, all processes disappear when $eV_L > \Delta$. These features are quite different from those of the MZR/home-SC/ZR structures [17]. Accordingly, these also indicate that the features in figure 3 are not changed by $eV_L$, originating from that $eV_L$ only adjusts the Fermi energy of the left MZR. Specifically, with $eV_L < 0 (> 0)$ increased, the dispersion curve entirely shifts downward (upward) relative to the Fermi surface and the dispersion of the right ZR remains unchanged. Therefore, only by proper adjustment of the polarity of $eV_L$, can one obtain the spin-filtering current with flowing direction locked to spin orientation, which could be used to manufacture a novel bipolar spin-diode.

5. Unipolar electron entanglement creator

Now we consider the case with the large $L$ and only the left ZR magnetized, in which $T^\downarrow_{\text{EC}} \approx 0$ at most values of $L$ as above-mentioned. At $L = 124$ and 178, just corresponding to $T^\downarrow_{\text{EC}}$ equal to zero in figures 4(a) and (b), respectively, $T^\downarrow_{\text{EC}}$ and $T^\downarrow_{\text{CAR}}$ versus the negative or positive $eV_L$ are shown in figure 6. At $L = 124$, $T^\uparrow_{\text{EC}}$, $T^\downarrow_{\text{EC}}$, and $T^\downarrow_{\text{CAR}}$ are still roughly or just zero and only the spin-down CAR process covers a wide range of negative $eV_L$, $-0.6\Delta < eV_L < 0$, as can be seen from figure 6(a). The feature naturally emerges for other $L$ corresponding to $T^\downarrow_{\text{EC}}$ of zero in figure 4(a), and thus can be used to produce the unipolar electron entanglement. However, at $L = 178$ (see figure 6(b)), there only exists the spin-up CAR process in a wide range of positive $eV_L$, $0.2 < \Delta eV_L < 0.8$. Similarly, the characteristic also certainly occurs for other $L$ corresponding to $T^\downarrow_{\text{EC}}$ of zero in figure 4(b), and hence can be also utilized to yield the unipolar electron entanglement. It then follows that the spin-species of unipolar electron entanglement are greatly relevant to a given $L$.

6. Testification of the helical spin texture in a QSH

The QSH insulator, a topological nontrivial phase of electronic matter, is characterized by the helical edge states. Whereas the spin-up electrons propagate clockwise along the sample edge, the spin-down do counterclockwise, and thus the helical states may be of great importance in the spintronic device and quantum information processing. However, the direct evidence of the helical spin texture of edge state is still lacking despite the fact that the edge-state conduction has been observed [54–61]. Hereinafter, we apply the bipolar spin diode with spin-locking motion direction in this work to helical spin texture of edge state. The proposed setup is shown in figures 1(a) and (b). Due to the spin-momentum locking in the QSH, the
electrons moving to the right along the upper and lower edge channels must have opposite spin directions. If the spin-up electrons starting from terminal 1 go forward along the upper edge channels, they may reach terminals 2, 3, or 4. By magnetizing the left ZR of the bipolar spin diode as shown in figure 1(a), at the applied forward $eV_L$, only spin-up current follows in both the QSH and bipolar spin diode, while at the reverse $eV_L$, the case is just contrary, i.e., only spin-down current follows. However, by magnetizing the left of the bipolar spin diode along the opposite direction, neither spin-up nor spin-down current follows whether $eV_L$ is forward or reverse. Thus a unambiguous evidence of the helical edge states is indicated.

7. Experimental possibility

Finally, we comment on the experimental possibility concisely. To make the proposed spin-diode device come true, a prerequisite, the perfect even ZR in a nanoscale, is needed, more specifically, the first energy-level difference $\delta E \sim t_\pi/\mathcal{N}$ of the ZR is large enough, so that there exist only the zero-energy edge states involved in transport. For a 50 nm ZR, $\delta E = 50$ meV is a lot larger than the correlative quantities $U_i$, $\Delta$, and $\hbar$ used in calculations. Particularly, the nanosize ZR has been successfully fabricated experimentally.

8. Discussion on the electron–phonon interaction

Next, we evaluate the effect of the electron–phonon interaction on the device performance of the present structure. Two fundamental conditions of intrinsic graphene define an overall very weak electron–phonon coupling (EPC), rendering itself not to be an SC [62]. One is the pointlike Fermi surface (Dirac point) with vanishing density of states, the other is the weak electron–phonon pairing potential, arising from the graphene dimensionality [63]. For the Fermi energy close to the Dirac point, the weak electron–phonon interaction in graphene can induce the transport scattering at low temperatures only with the acoustic modes [64], which gives rise to a suppression of the forward scattering. However, for the Fermi energy far away from the Dirac point by proper doping, a conventional superconducting pairing can be induced if the electron–phonon pairing potential is greatly enhanced by the environment [62, 65]. Most favorable is a singlet pairing between different valleys, which is not very sensitive to disorder. As a result, the superconductivity can be produced in the two MZR regions, or the proximity induced by the bulk SC is enhanced. Furthermore, the EPC is one of the main bottlenecks for efficient electronic cooling in graphene [63], as it represents a thermal leakage channel to the phonon bath. Yet electronic cooling in graphene-based superconducting hybrid structures is promising. Under the situation of Fermi energy close
to the Dirac point, due to the weak EPC, the heat is pumped into the superconducting leads, thus avoiding the overheating and preserving the cooling, which is also detrimental for the superconducting state.

Experimentally, during the preparation process of graphene, there exist some structural defects on a substrate [66], which does not change the overall band structure of the samples, still exhibiting the key properties as expected for Dirac fermions in graphene [67]. However, the metallicty of the substrate can be used to control EPC and hence superconductivity [68]. For example, the EPC for semiconducting Ge substrates in graphene is higher than that for Au substrates. Moreover, large mechanical strain of the substrate will result in the distinct deformation of lattice structure and the biaxial tensile strain can greatly enhance the EPC of graphene [62], leading to the superconductivity. A characteristic phonon mode dominates the EPC, approximated by the Einstein model. The superconductivity is ‘intrinsic’, arising solely from the intrinsic graphene properties modified by strain. Similarly, the tensile strain hardly modifies the electronic structure and the electrons scattered by phonon still consist of $\pi$ electrons of graphene.

All the above-mentioned, particularly, the electron–phonon interaction, could have some influence on the device performance, such as the switch effect in the present system. The number of channels for both the CAR and EC processes can be varied, leading to a different manipulation of the two processes by the SC length. However, in our work, we focus on the situation for low energy with Fermi energy close to the Dirac point, the key properties will not be significantly varied and the switch effect still remains.

9. Conclusions

In summary, we have proposed a hybrid junction with an SC sandwiched between either two ZRs or one MZR and one ZR, which is characterized by pseudoparity-mixing, leading to pseudoparity nonconservation EC (CAR) processes. The two structures are shown to reveal the controllable switch effect of the EC and CAR processes by the SC length. It is also demonstrated that the latter setup has two features on a large scale of forward and/or reverse $eV_L$: (1) a novel bipolar spin diode behavior in the range of small SC length and (2) unipolar spin entanglement pairing at some large SC lengths. The devices are likely to be achieved experimentally, in particular, the bipolar spin-diode can be combined with the QSH insulator to provide an evidence of the helical spin texture of the edge states in the QSH insulator, which is still lacking.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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