Magnetic moments of the doubly charmed and bottomed baryons

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The chiral corrections to the magnetic moments of the spin-1/2 doubly charmed baryons are systematically investigated up to next-to-next-to-leading order with heavy baryon chiral perturbation theory (HBChPT). The numerical results are calculated up to next-to-leading order: \( \mu_{\Xi_{c+}^+} = -0.25 \mu_N \), \( \mu_{\Xi_{b+}^+} = 0.85 \mu_N \), \( \mu_{\Omega_{c+}^+} = 0.78 \mu_N \). We also calculate the magnetic moments of the other doubly heavy baryons, including the doubly bottomed baryons (bbq) and the doubly heavy baryons containing a light quark, a charm quark and a bottom quark ([bc]q and [bc]q): \( \mu_{\Xi_{bb}^0} = -0.84 \mu_N \), \( \mu_{\Xi_{bb}^-} = 0.26 \mu_N \), \( \mu_{\Omega_{bc}^-} = 0.19 \mu_N \), \( \mu_{\Xi_{bc}^0} = -0.54 \mu_N \), \( \mu_{\Xi_{bc}^+} = 0.56 \mu_N \), \( \mu_{\Omega_{bc}^+} = 0.49 \mu_N \), \( \mu_{\Xi_{bc}^-} = 0.69 \mu_N \), \( \mu_{\Omega_{bc}^-} = -0.59 \mu_N \), \( \mu_{\Omega_{bc}^0} = 0.24 \mu_N \).

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I. INTRODUCTION

SELEX Collaboration first reported evidence for the doubly charmed baryon \( \Xi_{c+}^+ \) (3520) in the decay mode \( \Xi_{c+}^+ \to \Lambda_{c+}^- K^- \pi^+ \) with the mass \( M_{\Xi_{c+}^+} = 3519 \pm 1 \text{MeV} \), although other experimental collaborations like FOCUS [2], BABAR [3] and Belle [4] did not find any evidence of the doubly charmed baryons. Recently, LHCb collaboration observed \( \Xi_{c+}^+ \) in the \( \Lambda_{c+}^- K^- \pi^+ \pi^+ \) mass spectrum with the mass \( M_{\Xi_{c+}^+} = 3621.40 \pm 0.72(\text{stat}) \pm 0.27(\text{syst}) \pm 0.14(\Delta M) \text{MeV} \).

In the past decade, there have been many investigations of the doubly charmed baryon masses [6–40]. However, the electromagnetic form factors, especially the magnetic moments play a pivotal role in describing the inner structures of hadrons. In the quark-model, the doubly charmed baryons are just like the light baryons with two light quarks replaced by two charm quarks. The magnetic moments of doubly charmed baryons were first investigated by Lichtenberg in Ref. [41] with nonrelativistic quark model. Since then, more elaborate quark models have been developed to study the magnetic moments of doubly charmed baryons. In Ref. [8], various static properties, including magnetic moments were studied within non-relativistic quark model using the Faddeev formalism. Magnetic moments were also evaluated in the relativistic quark model [12, 13]. In Ref. [44], the radiative decays of double heavy baryons were studied in a relativistic constituent three-quark model including hyperfine mixing.

Besides the quark models, the magnetic moments of the doubly charmed baryons have been studied with other approaches, such as the MIT bag model [13, 14], the Dirac equation formalism [17], the Skyrmin model [18], the hyper central description of the three-body system [19] and lattice QCD [20, 21]. In Refs. [20, 21], the authors studied the electromagnetic properties of baryons in 2+1 flavor lattice QCD. They found that the magnetic moments of the singly charmed baryons are dominantly determined by the light quarks, while the charm quarks play a more important role in the doubly charmed baryons, which is confirmed in this paper.

Unfortunately most of the above models miss the chiral corrections. The Goldstone boson cloud effect can be taken into account through Chiral perturbation theory (ChPT) [52], which organizes the low-energy interactions order by order. Since the baryon mass \( M \) does not vanish in the chiral limit, the convergence of the chiral expansion is destroyed by the large energy scale \( M \). To overcome the above difficulty, heavy baryon chiral perturbation theory (HBChPT) was proposed [53, 54], which has been successfully used in the investigation of baryons. For the doubly charmed baryons, the two charm quarks are so heavy that they can be treated as spectators. Thus, the remaining light quark dominates the chiral dynamics of the doubly charmed baryons.
In this work, we will investigate the magnetic moments of the spin-$\frac{1}{2}$ doubly charmed or bottomed baryons with HBChPT. Right now, there does not exist any experimental measurement of the magnetic moments of the doubly charmed baryons. We use quark model to estimate the corresponding low energy constants (LECs) and calculate the chiral corrections to the magnetic moments order by order. The numerical results are presented up to next-to-leading order while the analytical results are calculated to next-to-next-to-leading order.

Our work is organized as follows. In Section II, we discuss the electromagnetic form factors of the spin-$\frac{1}{2}$ doubly charmed baryons. In Section III, we introduce the effective chiral Lagrangians. We calculate the chiral corrections to the magnetic moments order by order in Section IV and present our numerical results in Section V. A short summary is given in Section VI. We collect the coefficients of the loop corrections in the Appendix A.

II. ELECTROMAGNETIC FORM FACTORS OF SPIN-$\frac{1}{2}$ DOUBLY CHARMED BARYON BARYON

For the spin-$\frac{1}{2}$ doubly charmed baryons, the matrix elements of the electromagnetic current is similar to that of the nucleon,

$$<\Psi(p')|J_{\mu}|\Psi(p)> = e\bar{u}(p')\mathcal{O}_{\mu}(p', p)u(p),$$

with

$$\mathcal{O}_{\mu}(p', p) = \frac{1}{M_H}[P_{\mu}G_E(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2}G_M(q^2)].$$

where $P = \frac{1}{2}(p' + p)$, $q = p' - p$, $M_H$ is the doubly charmed baryon mass.

As the doubly charmed baryons are very heavy compared to the chiral symmetry breaking scale, we adopt the heavy-baryon formulation. In the heavy baryon limit, the spin-$\frac{1}{2}$ doubly charmed baryon field $B$ can be decomposed into the large component $H$ and the small component $L$.

$$B = e^{-iM_H v \cdot x}(H + L),$$

$$H = e^{iM_H v \cdot x}\frac{1 + \frac{1}{2}v \cdot x}{2}B, \quad L = e^{iM_H v \cdot x}\frac{1 - \frac{1}{2}v \cdot x}{2}B,$$

where $v = (1, \vec{0})$ is the velocity of the baryon. Now the doubly charmed baryon matrix elements of the electromagnetic current $J_{\mu}$ read

$$<H(p')|J_{\mu}|H(p)> = e\bar{u}(p')\mathcal{O}_{\mu}(p', p)u(p).$$

The tensor $\mathcal{O}_{\mu}$ can be parameterized in terms of electric and magnetic form factors.

$$\mathcal{O}_{\mu}(p', p) = v_{\mu}G_E(q^2) + \frac{[S^\mu, S^\nu]q^\nu}{M_H}G_M(q^2),$$

where $G_E(q^2)$ is the electric form factor and $G_M(q^2)$ is the magnetic form factor. When $q^2 = 0$, we obtain the charge ($Q$) and magnetic moment ($\mu_H$),

$$Q = G_E(0), \quad \mu_H = \frac{e}{2M_H}G_M(0).$$

III. CHIRAL LAGRANGIANS

A. The strong interaction chiral Lagrangians

To calculate the chiral corrections to the magnetic moment, we construct the relevant chiral Lagrangians. We follow Refs. [56–58] to define the basic chiral effective Lagrangians of the pseudoscalar mesons.

The spin-$\frac{1}{2}$ doubly charmed baryon field reads

$$\Psi = \begin{pmatrix} \Xi^{++}_{cc} \\ \Xi^{+}_{cc} \\ \Omega^{+}_{cc} \end{pmatrix} \Rightarrow \begin{pmatrix} ccu \\ ccd \\ ccs \end{pmatrix}.$$
The leading order pseudoscalar meson and doubly charmed baryon interaction Lagrangians read
\[ \mathcal{L}^{(1)} = \bar{\Psi}(i\not{D} - M_H)\Psi, \]
\[ \mathcal{L}_{\text{int}}^{(1)} = \frac{g_A}{2} \bar{\Psi}\gamma^{\mu\nu}\gamma_5 u_\mu \Psi, \]
where \( M_H \) is doubly charmed baryon mass,
\[ D_\mu \Psi = \partial_\mu \Psi + [\Gamma_\mu, \Psi]. \]

We also need the second order pseudoscalar meson and doubly charmed baryon interaction Lagrangians. Recall that for SU(3) group representations,
\[ 3 \otimes \bar{3} = 1 \oplus 8, \]
\[ 8 \otimes 8 = 1 \oplus 8_1 \oplus 8_2 \oplus 10 \oplus 10 \oplus 27. \]

Both \( u_\mu \) and \( u_\nu \) transform as the adjoint representation. When the product of \( u_\mu \) and \( u_\nu \) belongs to the \( 8_1 \) and \( 8_2 \) flavor representations, we can write down two independent interaction terms of the second order pseudoscalar meson and baryon Lagrangians:
\[ \mathcal{L}_{\text{int}}^{(2)} = \frac{ig_{h1}}{4M_B} \bar{\Psi}\sigma^{\mu\nu}[u_\mu, u_\nu] \Psi + \frac{ig_{h2}}{4M_B} \bar{\Psi}\sigma^{\mu\nu}\{u_\mu, u_\nu\} \Psi, \]
where the superscript denotes the chiral order, \( M_B \) is the nucleon mass and \( g_{h1, h2} \) are the coupling constants. The \( g_{h2} \) term vanishes because of anti-symmetric lorentz structure. Thus, there is only one linearly independent low energy constant (LEC) \( g_{h1} \) which contributes to the present investigations of the doubly charmed baryon magnetic moments up to \( \mathcal{O}(p^4) \).

In the framework of HBChPT, the leading order nonrelativistic pseudoscalar meson and doubly charmed baryon interaction Lagrangians read
\[ \mathcal{L}_{0}^{(1)} = \bar{H}(i\not{D})H, \]
\[ \mathcal{L}_{\text{int}}^{(1)} = g_A \text{Tr} \bar{H} S^\mu u_\mu H, \]
where \( \mathcal{L}_{0}^{(1)} \) and \( \mathcal{L}_{\text{int}}^{(1)} \) are the free and interaction parts respectively. \( S_\mu \) is the covariant spin-operator. We do not consider the mass differences among different doubly charmed baryons. We estimated the \( \phi HH \) coupling \( g_A = 0.5 \) with the help of quark model in Section \( \Box \). For the pseudoscalar meson masses, we use \( m_\pi = 0.140 \) GeV, \( m_K = 0.494 \) GeV, and \( m_\eta = 0.550 \) GeV. We use the nucleon masses \( M_B = 0.938 \) GeV.

The second order pseudoscalar meson and baryon nonrelativistic Lagrangians read
\[ \mathcal{L}_{\text{int}}^{(2)} = \frac{g_{h1}}{2M_B} \bar{H}[S^\mu, S^\nu][u_\mu, u_\nu]H. \]

The above Lagrangians contribute to the doubly charmed baryon magnetic moments in diagram (e) of Fig. 2.

### B. The electromagnetic chiral Lagrangians at \( \mathcal{O}(p^2) \)

The lowest order \( \mathcal{O}(p^2) \) Lagrangian contributes to the magnetic moments of the doubly charmed baryons at the tree level
\[ \mathcal{L}_{\mu \nu}^{(2)} = a_1 \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu]\tilde{F}_{\mu \nu}^+ H + a_2 \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu]H \text{Tr}(F_{\mu \nu}^+), \]
where the coefficients \( a_{1, 2} \) are the LECs. The chirally covariant QED field strength tensor \( F_{\mu \nu}^\pm \) is defined as
\[ F_{\mu \nu}^\pm = u^1 F_{\mu \nu}^R u \pm u F_{\mu \nu}^L u^1, \]
\[ F_{\mu \nu}^R = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu], \]
\[ F_{\mu \nu}^L = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu], \]
where \( r_\mu = l_\mu = -eQ_H a_\mu \) and \( Q_H = \text{diag}(2, 1, 1) \). The operator \( \tilde{F}_{\mu \nu}^+ = F_{\mu \nu}^+ - \frac{i}{3} \text{Tr}(F_{\mu \nu}^+) \) is traceless and transforms as the adjoint representation. Recall that the direct product \( 3 \otimes \bar{3} = 1 \oplus 8 \). Therefore, there are two independent interaction terms in the \( \mathcal{O}(p^2) \) Lagrangians for the magnetic moments of the doubly charmed baryons.
C. The higher order electromagnetic chiral Lagrangians

To calculate the magnetic moments to $O(p^3)$, we also need the $O(p^4)$ electromagnetic chiral Lagrangians at the tree level. Recalling flavor representation in Eqs. (12), (13) and considering that we only need the leading-order terms of (18), (21) contribute to the doubly charmed baryon magnetic moments at

$$L^{(4)}_{\mu H} = d_1 \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu] H \text{Tr}(\chi^+ F_{\mu \nu}^+) \quad + d_2 \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu] \{F_{\mu \nu}^+, \chi^+\} H \quad + d_3 \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu] \chi^+ H \text{Tr}(F_{\mu \nu}^+)$$

(21)

where $\chi^+ = \text{diag}(0,0,1)$ at the leading order and the factor $m_{ab}$ has been absorbed in the LECs $d_{1,2,3}$. There are two more terms which also contribute to the doubly charmed baryon magnetic moments.

$$L^{(4)}_{\mu H} = a_1' \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu] F_{\mu \nu}^+ H \text{Tr}(\chi^+) \quad + a_2' \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu] H \text{Tr}(F_{\mu \nu}^+) \text{Tr}(\chi^+)$$

(22)

However, their contributions can be absorbed through the renormalization of the LECs $a_{1,2}$, i.e.

$$a_1 \rightarrow a_1 + \text{Tr}(\chi^+) a_1', \quad \text{ (23)}$$

$$a_2 \rightarrow a_2 + \text{Tr}(\chi^+) a_2', \quad \text{ (24)}$$

IV. FORMALISM UP TO ONE-LOOP LEVEL

We follow the standard power counting scheme as in Ref. [59]. The chiral order $D_\chi$ is given by [60]

$$D_\chi = 4N_L - 2I_M - I_B + \sum_n nN_n, \quad \text{ (25)}$$

where $N_L$ is the number of loops, $I_M$ is the number of the internal pion lines, $I_B$ is the number of the internal baryon lines and $N_n$ is the number of the vertices from the $n$th order Lagrangian. The chiral order of the magnetic moments $\mu_H$ is $(D_\chi - 1)$ based on Eq. (4).

We assume the exact isospin symmetry with $m_u = m_d$ throughout this work. The tree-level Lagrangians in Eqs. (15, 21) contribute to the doubly charmed baryon magnetic moments at $O(p^1)$ and $O(p^3)$ as shown in Fig. 1. The Clebsch-Gordan coefficients for the various doubly charmed baryons are collected in Table 1. All doubly charmed baryon magnetic moments are given in terms of $a_1, a_2, d_1, d_2$ and $d_3$. There are six Feynman diagrams contribute to the doubly charmed baryon magnetic moments at one-loop level as shown in Fig. 2. All the vertices in these diagrams come from Eqs. (16-18). In diagrams (a), the meson vertex is from the strong interaction terms while the photon vertex is from the meson photon interaction term. In diagram (b), the photon-meson-baryon vertex is from the $O(p^2)$ tree level magnetic moment interaction in Eq. (16). In diagram (c), the two vertices are from the strong interaction and seagull terms respectively. In diagrams (d), the meson vertex is from the strong interaction terms in Eq. (16) while the photon vertex from the $O(p^2)$ tree level magnetic moment interaction in Eq. 18. In diagram (e), the meson-baryon vertex is from the second order pseudoscalar meson and baryon Lagrangian in Eq. (17) while the photon vertex is also from the meson photon interaction term. In diagram (f), the meson vertex is from the strong interaction terms while the photon vertex is from the $O(p^2)$ tree level magnetic moment interaction in Eq. (18).
FIG. 2: The one-loop diagrams where the doubly charmed baryon is denoted by the solid line. The dashed and wiggly lines represent the pseudoscalar meson and photon respectively.

The diagram (a) contributes to the tensor \( eO_\mu \) in Eq. 14 at \( O(p^3) \) while the diagrams (b-f) contribute at \( O(p^4) \). The diagram (c) vanishes in the heavy baryon mass limit. In particular,

\[
J_c \propto \int \frac{d^4l}{(2\pi)^4} \frac{i}{l^2 - m^2 + i\epsilon} (S \cdot l) \frac{i}{v \cdot l + i\epsilon} S^\mu \propto S \cdot v = 0.
\]  

In other words, diagram (c) does not contribute to the magnetic moments in the leading order of the heavy baryon expansion. The diagram (f) indicates the corrections from the wave function renormalization.

Summing all the contributions to the doubly charmed baryon magnetic moments in Fig. 2, the leading and next-to-leading order loop corrections can be expressed as

\[
\mu_{H}^{(2, \text{loop})} = - \sum_{\phi=\pi,K} \frac{\tilde{g}_\phi^2 m_\phi M_N \beta_\phi^2}{64\pi f_\phi^2},
\]

\[
\mu_{H}^{(3, \text{loop})} = \sum_{\phi=\pi,K} \left[ \frac{\beta_\phi^2 m_\phi^2 \ln \frac{m_\phi^2}{\lambda^2}}{128\pi^2 f_\phi^2} + \frac{\beta_\phi^2 m_\phi^2 \ln \frac{m_\phi^2}{\lambda^2}}{16\pi^2 f_\phi^2} \right] + \sum_{\phi=\pi,K,\eta} \left[ \frac{-\beta_\phi^2 \gamma_A m_\phi^2 \ln \frac{m_\phi^2}{\lambda^2}}{512\pi^2 f_\phi^2} (\ln \frac{m_\phi^2}{\lambda^2} - 2) + \frac{-3\beta_\phi^2 \gamma_A m_\phi^2 \ln \frac{m_\phi^2}{\lambda^2}}{256\pi^2 f_\phi^2} \right]
\]  

where \( \lambda = 4\pi f_\pi \) is the renormalization scale. Here, we use the number \( n \) within the parenthesis in the superscript of \( X^{(n,...)} \) to indicate the chiral order of \( X \). \( \beta_\phi^a-f \) arise from the corresponding diagrams in Fig. 2. We collect their explicit expressions in Tables VII and VIII in the Appendix A.

With the low energy counter terms and loop contributions (27, 28), we obtain the magnetic moments,

\[
\mu_H = \{ \mu_H^{(1)} \} + \{ \mu_H^{(2, \text{loop})} \} + \{ \mu_H^{(3, \text{tree})} + \mu_H^{(3, \text{loop})} \}
\]

where \( \mu_H^{(1)} \) and \( \mu_H^{(3, \text{tree})} \) are the tree-level magnetic moments from Eqs. [18, 21].

V. NUMERICAL RESULTS AND DISCUSSIONS

There are not any experimental data on the doubly charmed baryon magnetic moments so far. We do not have any experimental inputs to fit the LECs. In this paper, we use quark model to estimate the leading-order low energy constants. At the leading order \( O(p^1) \), there are two unknown LECs \( a_{1,2} \). The charge matrix \( Q_H \) is not traceless.
which is different from that in the case of the light baryons. Notice that the $a_1$ parts are proportional to the light quark charge within the doubly charmed baryon. The $a_2$ parts are the same for the three doubly charmed baryons and arise solely from the two charm quarks.

At the quark level, the flavor and spin wave function of the $\Xi^{++}_{cc}$ reads:

$$|\Xi^{++}_{cc}; \uparrow\rangle = \frac{1}{3\sqrt{2}} [2c \uparrow c \uparrow u \downarrow c \downarrow u \uparrow -c \downarrow c \uparrow u \uparrow +2c \uparrow c \uparrow -c \downarrow u \uparrow c \uparrow -c \downarrow u \downarrow c \uparrow -c \downarrow u \uparrow c \uparrow -c \downarrow u \downarrow c \uparrow] ,$$

(30)

where the arrows denote the third-components of the spin. Replacing the $u$ quark by the $d$ and $s$ quark, we get the wave functions of the $\Xi^{++}_{cc}$ and $\Omega^{++}_{cc}$ respectively. The magnetic moments of the doubly charmed baryons in the quark model are the matrix elements of the following operator in Eq. (30),

$$\vec{\mu} = \sum_{i} \mu_{i} \vec{\sigma}_{i} ,$$

(31)

where $\mu_{i}$ is the magnetic moment of the quark.

$$\mu_{i} = \frac{e_{i}}{2m_{i}} , \quad i = u, d, s .$$

(32)

We adopt the $m_{u} = m_{d} = 336$ MeV, $m_{s} = 540$ MeV, $m_{c} = 1660$ MeV as the constituent quark masses and give the results in the second column in Table III. The light quark magnetic moments contributes to the LEC $a_1$, which is proportional to the light quark charge. The heavy quark magnetic moments contributes to the LEC $a_2$, which are the same for the three doubly charmed baryons. The magnetic moments of the three doubly charmed baryons are given in the second column in Table III.

Up to $\mathcal{O}(p^2)$, we need include both the leading tree-level magnetic moments and the $\mathcal{O}(p^2)$ loop corrections. At this order, there exists only one new LEC $\tilde{g}_{A}$. We also use the quark model to estimate $\tilde{g}_{A}$. Considering the $\pi^{0}$ coupling at the hadron level,

$$\mathcal{L}_{\pi^{0} \Xi^{++}_{cc} \Xi^{++}_{cc}} = -\frac{1}{2F_{0}} \frac{\tilde{g}_{A}}{2} \bar{\Xi}^{++}_{cc} \gamma^{\mu} \gamma_{5} \partial_{\mu} \pi^{0} \Xi^{++}_{cc} .$$

(33)

At the quark level, the $\pi^{0}$ quark interaction reads

$$\mathcal{L}_{\pi^{0} \text{quark}} = \frac{1}{2} g_{0} \bar{\Psi}_{q} \gamma^{\mu} \gamma_{5} \partial_{\mu} \pi^{0} \Psi_{q} .$$

(34)

With the help of the flavor wave functions of $\Xi^{++}_{cc}$, we obtain the matrix elements at the hadron level

$$\langle \Xi^{++}_{cc} , s = \frac{1}{2} | i \mathcal{L}_{\Xi^{++}_{cc} \Xi^{++}_{cc} \pi^{0}} | \Xi^{++}_{cc} , s = \frac{1}{2} ; \pi^{0} \rangle \sim -\frac{1}{2F_{0}} \frac{\tilde{g}_{A}}{2} q_{3} ,$$

(35)

and at the quark level,

$$\langle \Xi^{++}_{cc} , s = \frac{1}{2} | i \mathcal{L}_{\text{quark}} | \Xi^{++}_{cc} , s = \frac{1}{2} ; \pi^{0} \rangle \sim -\frac{1}{6} g_{0} q_{3} .$$

(36)

After comparison with the axial charge of the nucleon,

$$-\frac{1}{2} \tilde{g}_{A} = \frac{2}{3} g_{0} = 0.5 .$$

(37)

one obtains $\tilde{g}_{A} = \frac{2}{3} g_{A} = 0.5$. Thus, we obtain the numerical results of $\mathcal{O}(p^2)$ chiral loop corrections in the third column in Table III. We list the numerical results of $\mathcal{O}(p^2)$ magnetic moments of the three doubly charmed baryons in the fourth column in Table III. We also compare the numerical results of the magnetic moments when the chiral expansions are truncated at $\mathcal{O}(p^1)$ and $\mathcal{O}(p^2)$ respectively in Table III.

Up to $\mathcal{O}(p^3)$, there are six unknown LECs: $a_{1,2,3}, g_{h1,2}, d_{1,2,3}$. Unfortunately, we are not able to present numerical results since it is impossible to fix all these LECs with the available experimental information. We present our analytical results in Eqs. (27), (28) and Table III. Our analytical results may be useful to the possible chiral extrapolation of the lattice simulations of the doubly charmed baryon electromagnetic properties.
After the similar calculation, one obtains the axial charge of the doubly bottomed baryons to next-to-leading order in Table III. We collect the numerical results of doubly bottomed baryon magnetic moments to next-to-leading order in Table IV and the tree level magnetic moments of the three doubly bottomed baryons in Table V respectively.

We also calculate the magnetic moments of the other doubly heavy baryons. At the quark level, the flavor and spin wave functions of the doubly bottomed baryons are the same as those of the doubly charmed baryons after replacing the c quarks by the b quarks. After the similar calculations of Eqs. (30)-(37), one obtains the axial charge of doubly bottomed baryons \( \tilde{g}_A(\mathbf{bbq}) = \frac{2}{3}g_A \) and the tree level magnetic moments of the three doubly bottomed baryons in the second column in Table III. We collect the numerical results of doubly bottomed baryon magnetic moments to next-to-leading order in Table III.

We also calculate the magnetic moments of the doubly heavy baryons containing a light quark, a charm quark and a bottom quark. We refer to the charm quark and the bottom quark as a diquark. There are two different multiplets of the doubly heavy baryons. The symmetric diquark \( \{bc\} \) has spin 1, while the antisymmetric diquark \( \{bc\} \) has spin 0.

At the quark level, the flavor and spin wave function of the \( \{bc\} \) baryons reads,

\[
|\{bc\}; \uparrow\rangle = \frac{1}{\sqrt{2}} (|bcq\rangle + |bcq\rangle) \otimes \frac{1}{\sqrt{6}} (|\uparrow\downarrow\downarrow\rangle - |\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle),
\]

while the flavor and spin wave function of the \( \{bc\} \) baryons reads,

\[
|\{bc\}; \uparrow\rangle = \frac{1}{\sqrt{2}} (|bcq\rangle - |bcq\rangle) \otimes \frac{1}{\sqrt{2}} (|\uparrow\downarrow\downarrow\rangle - |\downarrow\downarrow\downarrow\rangle).
\]

After the similar calculations, one obtains the axial charge of the \( \{bc\} \) baryons \( \tilde{g}_A(\{bc\}) = \frac{2}{3}g_A \) and the axial charge of the \( \{bc\} \) baryons \( \tilde{g}_A(\{bc\}) = -\frac{2}{3}g_A \). We collect the tree level magnetic moments of the \( \{bc\} \) baryons in the second column in Table IV and the tree level magnetic moments of the three \( \{bc\} \) baryons in the second column in Table V. We collect the numerical results of the \( \{bc\} \) and \( \{bc\} \) baryon magnetic moments to next-to-leading order in the fourth column in Table IV and Table V respectively.
possible chiral extrapolation of the lattice simulations. For future experimental measurement of the magnetic moments. Our analytical results may also be useful to the property of the doubly charmed baryons encodes crucial information of their inner structure. In this work, we have

such an accidental strong cancelation renders the leading order magnetic moment of the $\Xi_{cc}^+$ meson contributes to the chiral correction to $\mu_{\Xi_{cc}^+}$, while only $K^+$ mesons contribute to the chiral corrections to $\mu_{\Omega_{cc}^+}$ at this order. For comparison, the up and charm quark contributions to the $O(p^1)$ magnetic moment of the $\Xi_{cc}^{++}$ are destructive. Such an accidental strong cancelation renders the leading order magnetic moment of the $\Xi_{cc}^{++}$ is much smaller than those of its partner states. In contrast, both the $\pi^+$ and $K^+$ mesons contribute to the chiral corrections to $\mu_{\Xi_{cc}^{++}}$ at $O(p^2)$. In other words, the leading order magnetic moment of the $\Xi_{cc}^{++}$ is reduced while the loop correction is enhanced. As a result, the loop correction is numerically very important and even slightly larger than the leading order term. Such a unique feature can be exposed by future lattice QCD simulation.

In Table VI we compare our results obtained in the HBCChPT with those from other model calculations such as quark model (QM), relativistic three-quark model (RTQM), nonrelativistic quark model in Faddeev approach (NQM) [8], relativistic quark model (RQM) [12], skyrmion description [48], confining logarithmic potential (CLP) [47], MIT bag model [4], nonrelativistic quark model (NQM) [49] and lattice QCD (LQCD). All these approaches lead to roughly consistent results.

As the byproducts, we have also calculated the magnetic moments of the other doubly heavy baryons, including the $bbq$ baryons, the $\{bc\}q$ baryons and the $\{bc\}q$ baryons. Especially, the magnetic moments of $\{bc\}q$ baryons are quite interesting as their magnetic moments totally arise from the light quarks as shown in Table V.

We hope our calculation may be useful for future experimental measurements. As there are several unknown LECs up to next-to-next-to-leading order, we are looking forward to further progresses in both theory and experiment so that we can check the chiral expansion convergence of the three doubly charmed baryons. Our results may be useful for future experimental measurement of the magnetic moments. Our analytical results may also be useful to the possible chiral extrapolation of the lattice simulations.

| Baryons | $O(p^1)$ tree | $O(p^2)$ loop | Total |
|---------|----------------|----------------|-------|
| $\Xi_{bc}^+$ | $\frac{1}{3}(2\mu_b + 2\mu_c - \mu_u) = -0.41$ | -0.13 | -0.54 |
| $\Xi_{bc}^0$ | $\frac{1}{3}(2\mu_b + 2\mu_c - \mu_d) = 0.52$ | 0.04 | 0.56 |
| $\Omega_{bc}^0$ | $\frac{1}{3}(2\mu_b + 2\mu_c - \mu_s) = 0.40$ | 0.09 | 0.49 |

TABLE IV: The magnetic moments of doubly heavy baryons (\{bc\}q) to the next-to-leading order (in unit of $\mu_N$).

| Baryons | $O(p^1)$ tree | $O(p^2)$ loop | Total |
|---------|----------------|----------------|-------|
| $\Xi_{bc}^+$ | $\mu_u = 1.86$ | -1.17 | 0.69 |
| $\Xi_{bc}^0$ | $\mu_d = -0.93$ | 0.34 | -0.59 |
| $\Omega_{bc}^0$ | $\mu_s = -0.58$ | 0.82 | 0.24 |

TABLE V: The magnetic moments of doubly heavy baryons (\{bc\}q) to the next-to-leading order (in unit of $\mu_N$).

VI. CONCLUSIONS

The discovery of the $\Xi_{cc}^{++}$ inspired heated theoretical investigation of the doubly charmed baryons. The doubly charmed baryons are so special that the chiral dynamics is dominated by the single light quark. The electromagnetic property of the doubly charmed baryons encodes crucial information of their inner structure. In this work, we have performed a systematical calculations of the chiral corrections to the magnetic moments of doubly charmed baryons up to the next-to-next-to-leading order in the framework of heavy baryon chiral perturbation theory. We use quark model to estimate the low energy constants and present the numerical results up to next-to-leading order: $\mu_{\Xi_{cc}^{++}} = -0.25\mu_N$, $\mu_{\Xi_{cc}^{++}} = 0.85\mu_N$, $\mu_{\Omega_{cc}^{++}} = 0.78\mu_N$.

From Table III the magnetic moments of the $\Xi_{cc}^+$ and $\Omega_{cc}^+$ are dominated by the leading order term while the chiral corrections are quite small. To be specific, the numerical values of the $O(p^1)$ magnetic moments of the $\Xi_{cc}^+$ and $\Omega_{cc}^+$ are enhanced since the charge of the down and strange quark is $-\frac{1}{3}$ while the charm quark charge is $+\frac{2}{3}$. Only the $\pi^+$ meson contributes to the chiral correction to $\mu_{\Xi_{cc}^+}$ at $O(p^2)$ while only $K^+$ contributes to $\mu_{\Omega_{cc}^+}$ at this order.

As the byproducts, we have also calculated the magnetic moments of the other doubly heavy baryons, including the $bbq$ baryons, the $\{bc\}q$ baryons and the $\{bc\}q$ baryons. Especially, the magnetic moments of $\{bc\}q$ baryons are quite interesting as their magnetic moments totally arise from the light quarks as shown in Table V.
TABLE VI: Comparison of the decuplet to octet baryon transition magnetic moments in literature including quark model (QM) [41], relativistic three-quark model (RTQM) [43], nonrelativistic quark model in Faddeev approach (NRQM) [8], relativistic quark model (RQM) [42], skyrmion description [48], confining logarithmic potential (CLP) [47], MIT bag model [45], nonrelativistic quark model (NQM) [49] and lattice QCD (LQCD) [51] (in unit of $\mu_N$).

| Baryons | $\Xi_{cc}^{++}$ | $\Xi_{cc}^+$ | $\Omega_{cc}^+$ |
|---------|----------------|-------------|--------------|
| QM [41] | -0.124 0.806 0.688 |
| RTQM [43] | 0.13 0.72 0.67 |
| NRQM [8] | -0.206 0.784 0.635 |
| RQM [42] | -0.10 0.86 0.72 |
| Skyrmion [48] | -0.47 0.98 0.59 |
| CLP [47] | -0.154 0.778 0.657 |
| MIT bag model [45] | 0.17 0.86 0.84 |
| NQM [49] | -0.208 0.785 0.635 |
| LQCD [51] | — 0.425 0.413 |
| This work | -0.25 0.85 0.78 |

TABLE VII: The coefficients of the loop corrections to the doubly charmed baryon magnetic moments from Figs. 2(a), 2(b) and 2(d).

| Baryons | $\beta_a^a$ | $\beta_a^K$ | $\beta_b^a$ | $\beta_b^K$ | $\beta_d^a$ | $\beta_d^K$ |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\Xi_{cc}^{++}$ | 2 | 2 | $-4a_1$ | $-2a_2$ | $\frac{1}{6}(a_1 - 12a_2)$ |
| $\Xi_{cc}^+$ | -2 | 0 | $4a_1$ | $2a_1 + 24a_2$ | $\frac{1}{6}(a_1 + 24a_2)$ |
| $\Omega_{cc}^+$ | 0 | -2 | 0 | $4a_1$ | $\frac{1}{6}(a_1 + 24a_2)$ |

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Appendix A: COEFFICIENTS OF THE LOOP CORRECTIONS

In this appendix, we collect the explicit formulae for the chiral expansion of the doubly charmed baryon magnetic moments in Tables VII and VIII.
| Baryons | $\beta^{\pi}_{cc}$ | $\beta_{cc}^{\pi}$ | $\beta_{J}^{\pi}$ | $\beta_{cc}^{\pi}$ | $\beta_{J}^{\pi}$ |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\Xi^{++}_{cc}$ | $g_{h1}$ | $g_{h1}$ | $2(a_1 + 6a_2)$ | $\frac{4}{3}(a_1 + 6a_2)$ | $\frac{2}{3}(a_1 + 6a_2)$ |
| $\Xi^{+}_{cc}$ | $-g_{h1}$ | 0 | $-a_1 + 12a_2$ | $-\frac{4}{3}a_1 + 8a_2$ | $-\frac{1}{3}a_1 + \frac{4}{3}a_2$ |
| $\Omega^{+}_{cc}$ | 0 | $-g_{h1}$ | 0 | $-\frac{4}{3}a_1 + 16a_2$ | $-\frac{5}{3}(a_1 - 12a_2)$ |

TABLE VIII: The coefficients of the loop corrections to the doubly charmed baryon magnetic moments from Figs. 2(e) and 2(f).

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