Learning to Plan via Neural Exploration-Exploitation Trees

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Abstract

Sampling-based planning algorithms such as RRT and its variants are powerful tools for path planning problems in high-dimensional continuous state and action spaces. While these algorithms perform systematic exploration of the state space, they do not fully exploit past planning experiences from similar environments. In this paper, we design a meta path planning algorithm, called Neural Exploration-Exploitation Trees (NEXT), which can utilize prior experience to drastically reduce the sample requirement for solving new path planning problems. More specifically, NEXT contains a novel neural architecture which can learn from experiences the dependency between task structures and promising path search directions. Then this learned prior is integrated with a UCB-type algorithm to achieve an online balance between exploration and exploitation when solving a new problem. Empirically, we show that NEXT can complete the planning tasks with very small search trees and significantly outperforms previous state-of-the-arts on several benchmark problems.

1 Introduction

Planning paths efficiently in a high-dimensional continuous state and action space is a fundamental yet challenging problem in many real-world applications, such as robot manipulation and autonomous driving. Since the general path planning problem is PSPACE-complete (Reif, 1979), one typically resorts to approximate or heuristic algorithms.

Sampling-based planning algorithms, such as probabilistic roadmaps (PRM) (Kavraki et al., 1996), rapidly-exploring random trees (RRT) (LaValle, 1998), and their variants (Karaman & Frazzoli, 2011), provide principled approximate solutions to a wide spectrum of high-dimensional path planning tasks. However, these generic algorithms typically employ a uniform proposal distribution for sampling which does not make use of the structures of the problem at hand and thus may require lots of samples to obtain an initial feasible solution path for complicated tasks, e.g., a narrow passage in a map. To improve the sample efficiency, researchers designed algorithms to take problem structures into account, such as the Gaussian sampler (Boor et al., 1999), the bridge test (Hsu et al., 2003), the reachability-guided sampler (Shkolnik et al., 2009), the informed RRT* (Gammell et al., 2014) and the batch informed trees (Gammell et al., 2015), to name a few. Despite that, all these improved samplers are designed manually to address specific structural properties, which may or may not be valid for a new task, and thus, may lead to even worse performance compared to the uniform proposal.

Online adaptation in path planning has also been investigated in the literature. For instance, Hsu et al. (2005) exploits online algorithms to dynamically adapts the mixture weights of several manually designed
Can we exploit past path planning experiences, and learn an efficient and generalizable sampling algorithm for future planning tasks? Pioneering works in this direction are limited in one way or the other. Zucker et al. (2008), [Zander et al. (2018)] treat the sampler as a stochastic policy to be learned and apply policy gradient methods to improve the policy. Finney et al. (2007), Ye & Alterovitz (2017); Bowen & Alterovitz (2014); Ichter et al. (2018); Kim et al. (2018); Kuo et al. (2018) apply imitation learning based on the collected demonstrations to introduce bias for better sampler via multiple probabilistic models, e.g., (mixture of) Gaussians, conditional VAE, GAN, HMM and RNN. However, each of these approach either simply rely on special hand-designed local features or assume the tasks are indexed by special parameters. As we will show in the experiments, such representations limit the generalization ability of these algorithms. Although deep learning based approach, such as value iteration networks (VIN) (Tamar et al. 2016) and gated path planning networks (GPPN) (Lee et al. 2018), can learn task representations, they can only handle discrete state and action spaces within low dimensional problems.

Another key issue in learning to plan which has been largely ignored is the online exploration-exploitation trade-off. Rickert et al. (2009) constructed a heuristic to exploit the workspace information but ignored the configuration space information and previous experiences. Paxton et al. (2017) separated the action space into high level discrete options and low level continuous actions, and only considered the trade-off at the discrete option level, ignoring the exploration-exploitation in the fine action space. Arguably the stochasticity in the biased sampler will provide a certain degree of exploration, and the inductive biases in the learned samplers will lead to exploitation. However, existing algorithms do not explicitly balance the exploration and exploitation in a principled online fashion in a new planning task.

In this work, we propose a substantially improved algorithm for learning in path planning, leveraging recent advances in representation learning and infinite-armed bandit problem. Our algorithm, called, Neural Exploration-Exploitation Tree (NEXT), contains a novel neural architecture which can learn from experiences the dependency between task structures and promising search directions. The proposed architecture can extract the latent embedding of the task within the configuration space, on which the planning module can be executed effectively. Then this learned neural prior is integrated with a UCB-type algorithm to achieve an online balance between exploration and exploitation when solving a new problem. These two components are benefit to each other: the UCB-type algorithm will guided by the learned neural network, while the network will be evolved through the successful path generated by the UCB-type algorithm. Empirically, we show that NEXT can exploit past experience to reduce the sample requirement drastically for solving new planning problems, and significantly outperforms previous state-of-the-arts on several benchmark problems.

## 2 Settings for Path Planning Problem

The optimal path planning problem is generally referred to as the problem of finding the shortest path from a given starting position to the goal region if it exists. Let $\mathcal{S} \subset \mathbb{R}^d$ be a high-dimensional continuous planning state space, e.g., the configuration space which contains all the configurations of a robot including the location information in a workspace, and $\mathcal{S}_{obs}$ denotes obstacle region in the state space. Therefore, $\mathcal{S}_\text{free} := \mathcal{S} \setminus \mathcal{S}_{obs}$ denotes the free state space. Since both $\mathcal{S}_\text{free}$ and $\mathcal{S}$ can be highly irregular, explicit discretization is not viable as it may lead to very dense graphs in high dimensions. Let $s_{\text{init}} \in \mathcal{S}_\text{free}$ be the starting position and $\mathcal{S}_{\text{goal}} \subset \mathcal{S}_\text{free}$ be the goal region. Define a path as a continuous function over a time interval $[0, 1]$, $\mathcal{X}_{\text{all}} := \{\xi (\cdot) : [0, 1] \rightarrow \mathcal{S}\}$, then the space of all feasible collision-free paths can be defined as $\mathcal{X} := \{\xi (\cdot) : [0, 1] \rightarrow \mathcal{S}_{\text{free}}\}$. Furthermore, given a cost functional over a path $c (\cdot) : \mathcal{X}_{\text{all}} \rightarrow \mathbb{R}$, the optimal planning problem can be formally defined as finding a feasible path $\xi^*(\cdot)$ which minimizes the cost functional

$$
\xi^* = \arg\min_{\xi \in \mathcal{X}} c (\xi) \\
s.t. \, \xi (0) = s_{\text{init}}, \, \xi (1) \in \mathcal{S}_{\text{goal}}.
$$
Figure 1: Different views of the same planning problem. In (a) we color the obstacles, the starting and the goal position of the robot in deep blue, orange and brown, respectively. The stick robot can move and rotate. The corresponding configuration space is 3d, as visualized in (b), with the extra dimension being the rotation angle w.r.t. the x-axis. The blue region indicates the feasible state space, i.e., the set of collision-free states. The starting and the goal position are denoted with an orange and a brown dot, respectively. Although the workspace looks trivial, the configuration space is irregular, which makes the planning difficult.

Note that a path $\xi$ can also be a sequence of $T$ interconnected line segments, in this case we will represent $\xi$ as $[s_0, s_1], [s_1, s_2], \ldots, [s_T-1, s_T] \subset S$, with $\xi(0) = s_0$ and $\xi(1) = s_T$. Without loss of generality, we assume $c(\xi)$ is additive as $\sum_{i=1}^{n} c([s_i, s_{i+1}])$. The Figure 1(a) illustrates a concrete planning problem for a stick robot in 2d workspace map. With one extra continuous action for rotation, the configuration state is visualized in Figure 1(b), which is highly irregular and unknown to the planner.

In our setting, we consider the general path planning problems where we do not assume the complete knowledge of the configuration space structure:

**Assumption:** We only assume that the map in the physical workspace is known. The dimension of this map is typically 2 or 3, much lower than the original planning state (or configuration) space.

This is reasonable since 2 or 3d workspace maps are generally available or can be obtained with low cost and computation. Since the map is a low dimensional function, it can be discretized into grids where models such as convolutional neural networks (CNN) can be applied for representation learning. We note that, in contrast, the original configuration space cannot be tractably discretized, since it can involve much higher dimensions. For instance, a robot arm with 20 degrees of freedom will correspond to a configuration space with 20 dimensions. It has been demonstrated that in such general planning problems, algorithms designed for a discretized space either achieve suboptimal solutions or require high-resolution discretization which is computationally prohibitive. Thus, current dominant planners for high dimensional planning problems are based on sampling algorithms (Elbanhawi & Simic, 2014).

To emphasize the availability of task maps, we will use the tuple $U := (s_{\text{init}}, S_{\text{goal}}, S_{\text{free}}, \text{map}, c(\cdot))$ to denote a planning task, where the map is function $\text{map}(\cdot) : \mathbb{R}^2$ or $\mathbb{R}^3 \mapsto \{0, 1\}$ containing information about free spaces (0) and obstacles (1). Moreover, different from the traditional continuous path planning setting, we will go beyond a single planning task and address the multitask path planning problem:

**Problem setting:** Can we find a self-improving planner that can generalize and achieve better performance as it experiences more planning tasks $U$ sampled from the same distribution $P_{\text{task}}$?

We emphasize that our problem setting is much more challenging than the one investigated in VIN (Tamar et al., 2016) and GPPN (Lee et al., 2018):
1. We are targeting the continuous state and action planning problem;
2. We only have the map information that is in the workspace, but the complete state space for planning is not explicitly available;
3. We do not have expert routes as supervision information on training tasks.

These significant differences make VIN and GPPN not applicable to our setting. Therefore, a practical and efficient algorithm for our problem setting is much needed.

3 Preliminaries

In this section, we first present a unifying view of many sampling-based planning algorithms, which help us to understand and improve planning algorithms later. Then we introduce the upper confidence bound (UCB) algorithm, which plays a vital role in exploration-exploitation trade-off and the design of our new algorithm.

Tree-based sampling algorithms. Since the path planning problem in (1) needs to be performed in high dimensional continuous space with very complex constraints (due to the obstacles), it is difficult to parameterize a path in a closed form and optimize the parametric form. Thus, nonparametric forms, such as a sequence of interconnected line segments, are typically used to represent a path, and such line segment representation is constructed and refined incrementally with sampling algorithms.

There is a large family of sampling algorithms which maintain a search tree $T$ rooted at the initial point $s_{init}$ and connecting all sampled points $V$ in the configuration space with edge set $E$. Then a path $\xi$ from the initial position to any sampled point can be constructed via the shortest path in the search tree. Furthermore, this tree will be expanded incrementally by incorporating more sampled points until some tree leaf reaches the goal region $S_{goal}$. At this point, a feasible solution for the path planning problem is found, corresponding to the shortest path between the root and this particular leaf. After that, more sampling can be conducted, and the tree can be further expanded to refine the path.

The template of tree-based sampling algorithms is summarized in Algorithm 2 and illustrated in Figure 2. A key component of the algorithm is the tree $Expand$ operator, which can be instantiated differently in...
Algorithm 3: EST :: Expand($T$, $U$)

Data: $T = (V, E), U = (s_{\text{init}}, S_{\text{goal}}, S, S_{\text{free}}, \text{map}, c(\cdot))$

1. $s_{\text{parent}} \sim \phi(s), s \in V$; \hspace{1cm} \triangleright Sample a tree node
2. $s_{\text{new}} \leftarrow \text{Unif}(B(s_{\text{parent}}))$; \hspace{1cm} \triangleright Sample neighborhood
3. return $s_{\text{nearest}}, s_{\text{new}}$;

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Figure 2: Illustration for one iteration of Algorithm 1. The left and right figures illustrate two different cases where the sample returned by the Expand operator is unreachable and reachable from the search tree.

different concrete algorithms (more discussion later). The Expand operator returns an existing node in the tree $s_{\text{parent}} \in V$ and a new state $s_{\text{new}} \in S$ sampled from the neighborhood of $s_{\text{parent}}$. Then the line segment $[s_{\text{parent}}, s_{\text{new}}]$ is passed to function ObstacleFree for collision checking. If the line segment $[s_{\text{parent}}, s_{\text{new}}]$ is collision-free (no obstacle in the middle, or called reachable from $T$), then $s_{\text{new}}$ is added to the tree vertex set $V$, and the line segment is added to the tree edge set $E$. If the newly added node $s_{\text{new}}$ has reached the target $S_{\text{goal}}$, the algorithm will return. Optionally, some concrete algorithms can define a Postprocess operator to refine the search tree. For an example of the Expand operator, as shown in Figure 3 (c), since there is no obstacle on the dotted edge $[s_{\text{parent}}, s_{\text{new}}]$, i.e., $s_{\text{new}}$ is reachable, the new state and edge will be added to the search tree (connected by the solid edges).

Now we will provide two concrete algorithm examples. For instance,

- If we instantiate the Expand operator as Algorithm 2, then we obtain the rapidly-exploring random trees (RRT) algorithm ([LaValle, 1998]), which first samples a state $s$ from the configuration space $S$ and then pulls it toward the neighborhood of current tree $T$ measured by a ball of radius $\eta$:

$$B(s, \eta) = \{ s' \in S \mid \| s' - s \| \leq \eta \}.$$ 

Moreover, if the Postprocess operator is introduced to modify the maintained search tree as in RRT* ([Karaman & Frazzoli, 2011]), the algorithm is provable to obtain the optimal path asymptotically.

- If we instantiate the Expand operator as Algorithm 3, then we obtain the expansive-space trees (EST) algorithm ([Hsu et al., 1997; Phillips et al., 2004]), which samples a state $s$ from the nodes of the existing tree, and then draw a sample from the neighborhood of $s$.

One major limitation of existing algorithms is that they solve each new planning problem from scratch and do not exploit past planning experiences in similar environments. The uniform sampling strategies will result in many useless samples, and waste the computational resources. Intuitively, one can learn from previous experience to improve the sample efficiency. Our proposed algorithm will also follow the template in Algorithm 1 but will explicitly consider the exploration-exploitation trade-off.

UCB-based algorithms. The exploration versus exploitation trade-off in search, i.e., exploring the environment to find better actions while exploiting the empirical best action until now, has been investigated
extensively in online learning and multi-armed bandit settings (Auer et al., 2002; Langford & Zhang, 2008). One straightforward and provable algorithm for K-armed bandit problem is the upper confidence bound algorithm (UCB). More specifically, the algorithm will first play each of the arms once, and then keep track of the average reward \( \bar{r}_i \) and the visitation count \( n_i \) for each arm. After \( T \) rounds of trials, the UCB algorithm will maintain a set of information \( \{(\bar{r}_i, n_i)\}_{i=1}^K \) with \( \sum_{i=1}^K n_i = T \). Then, for the next round, the UCB algorithm will select the next arm based on the one-sided confidence interval estimation provided by the Chernoff-Hoeffding bound,

\[
a^*_T+1 = \arg\max_{i \in \{1, \ldots, K\}} \bar{r}_i + \lambda \sqrt{\frac{\log T}{n_i}},
\]

where \( \lambda \) controls the exploration-exploitation trade-off. It has been shown that the UCB algorithm achieves \( O(\log T) \) regret.

Kocsis & Szepesvári (2006) generalized the UCB policy to searching tree, which leads to the Monte Carlo tree search (MCTS), or also known in the upper confidence trees (UCT). The UCT (MCTS) algorithm has demonstrated its power in balancing exploration and exploitation in the searching with huge number of discrete actions (Silver et al., 2017; Guez et al., 2018). There have been many attempts to generalize the UCB and UCT algorithms to continuous state-action spaces (Chu et al., 2011; Krause & Ong, 2011; Couëtoux et al., 2011; Yee et al., 2016). For instance, contextual bandit algorithms allow continuous arms but involve a non-trivial high dimensional non-convex optimization to select the next arm. In UCT, the progressive widening technique has been designed to deal with continuous actions (Wang et al., 2009). However, since it is designed with extra exploration steps on visited states to handle the potential uncertainty in dynamical environment, which is unnecessary in path planning problems, the UCT will waste huge number of samples in path planning, and thus not efficient. Although these off-the-shelf algorithms are not directly applicable to our path planning setting, their successes show the importance of exploration-exploitation trade-off and will provide the principles for our algorithm for continuous state-action planning problems.

4 Neural Exploration-Exploitation Trees

As we discussed in Section 3, the performance of tree-based sampling algorithms depends crucially on the design of the Expand operator. We will leverage ideas from UCB-based algorithms to explicitly take exploration versus exploitation trade-off into account.

There are two major challenges in directly applying ideas from UCB-based algorithms to path planning problems. First, the state and action of path planning problem live in high dimensional continuous spaces with extreme complex constraints due to obstacles in the map and feasibility of actions. This requires us to design an intelligent progressive expansion algorithm to explore these spaces. Second, we do not have access to an oracle for providing the reward of an action. In fact, obtaining one value of such reward function is as difficult as solving a planning problem.

We address these challenges below, leading to a novel expansion operator in Algorithm 4 which employs a meta self-improving procedure for training (Algorithm 5). Integrating this with the tree-based sampling algorithm template in Algorithm 1, we obtain the Neural Exploration-Exploitation Trees (NEXT).

4.1 Guided Progressive Expansion

In this section, we first introduce the Guided Progressive Expansion (GPE) with the assumption that a value function oracle \( V^*(s|U) \) is provided for any state \( s \) of a problem \( U \). That is,

\[
V^*(s|U) := \min_{\xi \in \Xi} c(\xi) \quad \text{s.t. } \xi(0) = s, \ \xi(1) \in S_{\text{goal}},
\]

the cost of the shortest path from state \( s \) to the goal, is provided. Since we do not have direct access to \( V^* \), we will estimate a value function \( \tilde{V}^* \) instead. We will postpone the learning of such value function from past experience to Section 4.2.
The purpose of the \textbf{Expand} operator is to expand the current search tree $T$ with a new neighboring state $s_{new} \in B(T)$. Inspired by the \textbf{Expand} operator in EST (Algorithm 3), we also consider the expansion as a two-step procedure:

\begin{enumerate}
    \item We select a state $s$ from the existing tree $T$;
    \item We select a state $s_{new}$ in the neighborhood of the selected $s$.
\end{enumerate}

We note that both steps will depend on the search history $T$ and task environment $U$. Next, we will make step i) and ii) more concrete.

\textbf{Selection from $T$}. Recall that in every step, we always have a finite number of nodes in the search tree $T(V,E)$. Then, step (i) shares some similarity with the multi-armed bandit problem by viewing the existing nodes $\{s\} \in V$ as arms and the negative value function estimate $-\hat{V}^*(s|U)$ as the rewards $r(s)$. However, as the algorithm executed, the number of states is increasing. Then, we cannot use the vanilla UCB algorithm. The problem can be solved by smoothing the reward $r(s)$ via parametrization. Then, the upper confidence bounds (UCB) of the reward function, $\phi(s) : V \to \mathbb{R}$ can be computed, and the selection of the next node $s \in V$ to expand will be carried out using $\phi(s)$, i.e.,

$$s^{t+1} = \text{argmax}_{s \in V} \phi(s).$$

We denote the sequence of selected nodes from the current tree as $S_t = \{s_1, s_2, \ldots, s_t\}$. Note that some nodes in the tree may be selected multiple times. We list two examples of constructing the UCB due to different smoothing parametrizations:
which is illustrated in Figure 3(b) and (c). It bypasses the difficulties in naively applying UCB algorithms. We next show how we can obtain the value function estimator $V^* (s|U)$ and learn the guiding policy $\tilde{\pi}^* (s'|s, U)$ in the expansion operator, as illustrated in Figure 3(d) using experiences from previous planning tasks.

1With a slight abuse of notation, we use $\pi (s'|s)$ and $\pi (a|s)$ interchangeably in this paper.
4.2 Neural Architecture

In this section, we introduce the neural architecture and learning scheme for $\tilde{V}^*(s|U)$ and $\tilde{\pi}^*(s'|s, U)$. The design of the neural architecture is inspired by the VIN \cite{Tamar2016} and GPPN \cite{Lee2018}, but with significant differences. The major challenge is that we only have access to low dimensional workspace map (as per our assumption), but we need to predict the value and construct policy for high dimensional states in configuration space. To address this challenge, we will design a novel attention-based configuration space embedding modules, which allows us to perform value iteration in the embedded space.

**Value iteration network.** Value iteration networks \cite{Tamar2016} employ neural networks to embed the value iteration algorithm from planning and then use this embedded algorithm to extract input features and define downstream models such as value functions and policies.

Specifically, VIN mimics the following recursive application of Bellman update operator $G$ to value function $V^*$,

$$V^*(s|U) = (GV^*)(s) := \min_a \sum_{s'} P(s'|s,a) (c([s, s']) + V^*(s'|U)).$$

where $P(s'|s,a)$ is the state transition model. When the state space for $s$ and action space for $a$ are low dimensional, these spaces can be discretized into grids \cite{Tamar2016}. Then, the local cost function $c([s, s'])$ and the value function $V^*(s'|U)$ can be represented as matrices (2d) or tensors (3d) with each entry indexed by grid locations. Furthermore, if the transition model $P(s'|s,a)$ is local, that is $P(s'|s,a) = 0$ for $s' \notin B(s)$, it resembles a set of convolution kernels, each indexed by a discrete action $a$. And the Bellman update operator essentially convolves $P(s'|s,a)$ with $c([s, s'])$ and $V^*(s'|U)$, and then performs a min-pooling operation across the convolution channels.

Inspired by the above computation pattern of the Bellman operator, value iteration networks design the neural architecture as follows,

$$\tilde{V}^{*0} = \min \left( W_1 \oplus \left[ \text{map}, \tilde{R} \right] \right) \quad (7)$$

$$\tilde{V}^{*t} = \min \left( W_1 \oplus \left[ \tilde{V}^{*t-1}, \tilde{R} \right] \right) \quad (8)$$

where $\oplus$ is the convolution operation, both $\text{map}$, $\tilde{V}^{*t}$ and $\tilde{R}$ are $d \times d$ matrices, and the parameter $W_1$ are $k_c$ convolution kernels of size $k \times k$. The min implements the pooling across $k_c$ convolution channels.

Moreover, the optimal policy $\pi^*(s'|s,U)$ can be characterized as

$$\pi^*(s'|s,U) = \arg\min_{s' \in S} c([s, s']) + V^*(s'|U), \quad (9)$$

which implies the optimal policy can be derived from $V^*$. Therefore, the parameterization of $\pi^*(s'|s,U)$ shares the same architecture with $V^*(\cdot|U)$ but different final layer.

**Configuration space embedding.** Since the configuration space is high dimensional and continuous, VIN and GPPN cannot be directly applied. We will design a novel attention-based network to embed the configuration space into a 3d tensor, on which the recursive structure similar to value iteration networks can be applied.

More specifically, we will use $s^w$ to explicitly denote the workspace part of state $s \in S$, and $s^h$ to denote the remaining dimensions of the state, i.e. $s = (s^w, s^h)$. $s^w$ and $s^h$ will be embedded using different neural architectures. For simplicity of notation, we will focus on the 2d workspace case here, where $s^w \subset \mathbb{R}^2$.

However, we emphasize that our method applies to the 3d workspace as well.

\footnote{For simplicity, we assume that each grid point belongs unambiguously to either free space or obstacle space.}
We can obtain the overall architecture of the attention-based embedding module in Figure 4. The result of the embedding of a state $s$ is a $d \times d \times d_a$ tensor attention map $\mu(s)$ with $\mu(s)_{ijl} \geq 0$, and $\sum_{ijl} \mu(s)_{ijl} = 1$. The upper part is inspired by the design of the CoordConv layer [Liu et al., 2018] to learn the spatial attention better.

For $s^w$, the embedding $\mu_{\theta^w}(s^w)$ is a $d \times d$ matrix of the same size as map, and it is computed using $k_w$ convolution layers, i.e.

$$\begin{align*}
\mu_{\theta^w}(s^w) &= \text{softmax}_2d(f_{k_w}^w(s^w)), \\
    f_{i+1}^w(s^w) &= \text{relu}(\theta_{i}^w \circ f_i^w(s^w)),
\end{align*}$$

where each $\theta_{i}^w$ is a convolution kernel of size $1 \times 1$, $f_{i}^w(s^w) \in \mathbb{R}^{d \times d \times k_w}$ for $i > 0$, $k_w$'s are hyper-parameters, $\theta_0^w$ is a convolution kernel of size $1 \times 1 \times 1 \times 4$, and $f_0^w(s^w)$ is a tensor of size $d \times d \times 4$ filled with

$$f_0^w(s^w)_{ijl} = \begin{cases} 
    s^w_{ijl}, & \text{if } l \in \{1, 2\}, \\
    i, & \text{if } l = 3, \\
    j, & \text{otherwise}.
\end{cases}$$

For $s^h$, the higher dimensional configuration embedding $\mu_{\theta^h}(s^h)$ is computed using $k_h$ fully-connected layers, i.e.

$$\begin{align*}
\mu_{\theta^h}(s^h) &= \text{softmax}(f_{k_h}^h(s^h)), \\
    f_{i+1}^h(s^h) &= \text{relu}(\theta_{i}^h \circ f_i^h(s^h) + \theta_{i}^{hb}),
\end{align*}$$

where $\mu_{\theta^h}(s^h) \in \mathbb{R}^{d_a}$ and $f_0^h(s^h) = s^h$.

We can obtain $\mu_{\theta}(s)$ by multiplying $\mu_{\theta^w}(s^w)$ with $\mu_{\theta^h}(s^h)$,

$$\mu_{\theta}(s)_{ijl} = \mu_{\theta^w}^w(s^w)_{ij} \cdot \mu_{\theta^h}^h(s^h)_{jl}. $$

The overall architecture of the attention-based space embedding module is illustrated in Figure 4. The result of the embedding of a state $s$ is a $d \times d \times d_a$ tensor attention map $\mu(s)$ with $\mu(s)_{ijl} \geq 0$, and $\sum_{ijl} \mu(s)_{ijl} = 1$. The bottom part learns the higher dimensional configuration attention via several dense layers from $z$. Finally, the full attention $\mu(s)$ is computed via an element-wise outer-product of the two sub-attentions.
where \( \theta \) denotes the parameters in the embedding network. One can think of \( d_a \) as the level of learned discretization of the configuration space \( s^h \), and the entries in \( \mu \) softly assign the actual state \( s \) to these discretized locations.

**Overall model architecture.** With the attention-based embedding of the configuration spaces, we can apply value iteration networks (planning module) on top of it. First, we will produce the embedding \( \mu_\theta(s_{\text{init}}) \) and \( \mu_\theta(s_{\text{goal}}) \) of the initial state \( s_{\text{init}} \) and the goal state \( s_{\text{goal}} \) respectively. Then we parameterize \( \tilde{V}^* (s|U) \) and \( \tilde{\pi}^* (s'|s, U) \) in the following way.

The input to the planning module is computed by a convolution neural network,

\[
\tilde{V}^{*0}, \tilde{R} = \sigma (W_0 \oplus [\mu_\theta(s_{\text{goal}}) , \text{map}]),
\]

where \([\mu_\theta(s_{\text{goal}}), \text{map}]\) denotes the concatenation of \( \mu_\theta(s_{\text{goal}}) \) and \( \text{map} \) along the 3rd dimension, and \( W_0 \) is a 3d convolution kernel of size \( k \times k \times (d_a + 1) \). After \( T \) iterations of Bellman update

\[
\tilde{V}^{*t} = \min \left(W_1 \oplus \left[ \tilde{V}^{*(t-1)}, \tilde{R} \right] \right),
\]

we have \( \tilde{V}^{*(T)} \). Notice that since the Bellman update is performed in the embedding space, \( \tilde{V}^{*t} \) is a tensor of size \( d \times d \times d_a \), and \( W_1 \) is a 3d convolution kernel of size \( k \times k \times (d_a + 1) \), where \( d_a \) is a multiple of \( d_e \), \( d_e = d_a \times p \).

Then we extract \( \psi(s_{\text{init}}) \in \mathbb{R}^p \) by multiplying \( \tilde{V}^{*(T)} \) (of size \( d \times d \times d_a \times p \)) with attention \( \mu_\theta(s_{\text{init}}) \),

\[
\psi(s_{\text{init}})_e = \sum_{ijl} \tilde{V}^{*(T)}_{ijl} \cdot \mu_\theta(s_{\text{init}})_{ijl},
\]

and finally we obtain

\[
\tilde{V}^{*} (s_{\text{init}}|U), \tilde{\pi}^* (s'|s_{\text{init}}, U) \equiv h_{W_2}(\psi(s_{\text{init}})),
\]

where \( h \) is fully connected dense layers.

We draw the overall model architecture in Figure 5. The parameters \( W = (W_0, W_1, W_2, \theta) \) will be learned together. For the details of the parameterization in our implementation, please refer to the Appendix A.

### 4.3 Meta self-improving learning

The learning of the parameters in \( \tilde{V}^{*} (s|U) \) and \( \tilde{\pi}^* (s'|s, U) \) are carried out simultaneously with planning. We do not have an explicit training and testing phase separation. Particularly, we use a mixture of RRT : Expand
We compared NEXT with RRT* with the goal biasing heuristic $\ell$ where we can reconstruct the successful path $TSA$ learning-enhanced strategy (Gammell et al., 2015) to accelerate the planning process. CVAE-plan and Reject-plan are two BIT* et al., 2018) and Reject-plan (Zhang et al., 2018) in terms of planning time and solution optimality. RRT* is enforced implicitly via imitating the successful samples. The overall learning procedure is summarized in Algorithm 5 and illustrated in Figure 3.

Experiments

We compared NEXT with RRT* (Karaman & Frazzoli, 2011), BIT* (Gammell et al., 2015), CVAE-plan (Ichter et al., 2018) and Reject-plan (Zhang et al., 2018) in terms of planning time and solution optimality. RRT* and BIT* are two widely used effective instances of TSA in Algorithm 4. In our experiments, we equipped RRT* with the goal biasing heuristic $\ell$ to improve its performance. BIT* exploits the batch informed sampling strategy (Gammell et al., 2015) to accelerate the planning process. CVAE-plan and Reject-plan are two learning-enhanced TSA planners proposed recently. CVAE-plan learns the important sampling region with a

Algorithm 5: MSIL: Meta Self-Improving NEXT Learning

1. Initialize dataset $D$;
2. for epoch $i \leftarrow 1$ to $N$ do
   3. Sample a planning instance $U$ from some distribution;
   4. $T \leftarrow TSA(U)$ with $\epsilon \sim Unif[0, 1]$, and $\epsilon \cdot \text{RRT}^* \leftarrow \text{Expand} + (1 - \epsilon) \cdot \text{NEXT}^* \leftarrow \text{Expand};$
   5. Postprocessing with $\text{RRT}^* \leftarrow \text{Postprocess};$
   6. $D_n \leftarrow D_n \cup \{T, U\}$;
   7. for $j \leftarrow 0$ to $L$ do
   8. Sample $\{T, U\}$ from $D_n$;
   9. Reconstruct optimal path $\{s^i\}_{i=1}^m$ and the cost of paths based on $T$;
   10. Update the $W \leftarrow W - \eta \nabla_W \ell(\hat{V}^*, \hat{\pi}^*; T)$;
   11. Anneal $\epsilon = \alpha \epsilon$, $\alpha \in (0, 1)$;
12. return $W$

and NEXT $\leftarrow \text{Expand}$ with probability $\epsilon$ and $1 - \epsilon$, respectively, inside the tree-based planning algorithm TSA in Algorithm 4. The RRT* postprocessing step will be used in the template. The $\epsilon$ is set to be 1 at the initial stage since the $\{\hat{V}^*, \hat{\pi}^*\}$ is not well-trained, and thus, the algorithm behaves like RRT*. As the training proceeds, we anneal $\epsilon$ gradually as the sampler becomes more and more efficient.

The dataset $D_n = \{T_j, U_j\}_{j=1}^m$ for the $n$-th training epoch is collected from the previous planning experiences across multiple random tasks. For a tree $T \in D_n$, let $m$ denote the length of the solution path, we can reconstruct the successful path $\{s^i\}_{i=1}^m$, and the value for each state in the path will be the sum of cost to the end of the path, i.e., $\{y^i := \sum_{i=1}^{m-1} c([s^i, s^{i+1}])\}_{i=1}^m$. We learn $\{\hat{V}^*, \hat{\pi}^*\}$ by optimizing,

$$\min_W \sum_{T \sim D_n} \ell(\hat{V}^*, \hat{\pi}^*; T),$$

where $\ell(\hat{V}^*, \hat{\pi}^*; T) := -\sum_{i=1}^{m-1} \log \hat{\pi}^*(s^{i+1}|s^i) + \sum_{i=1}^m (\hat{V}^*(s^i) - y^i)^2 + \lambda \|W\|^2$. We will apply the stochastic gradient descent to minimize\footnote{With small probability, use the goal state as input instead of sample randomly.} $W$.

On the one hand, the objective\footnote{With small probability, use the goal state as input instead of sample randomly.} is making the policy imitate the successful policy and improving the value function estimation based upon the outcomes from the NEXT algorithm itself on previous tasks. On the other hand, the updated $\{\hat{V}^*, \hat{\pi}^*\}$ will be applied in the next epoch to improve the performance of planning. Therefore, we named the learning algorithm as Meta Self-Improving Learning (MSIL). We want to emphasize that empirically we find it is unnecessary to introduce extra constraints on the policy to guarantee the reachability explicitly. Since all the trajectories we collected in the datasets are feasible, the reachability is enforced implicitly via imitating the successful samples. The overall learning procedure is summarized in Algorithm 5 and illustrated in Figure 3.

5 Experiments

We compared NEXT with RRT* (Karaman & Frazzoli, 2011), BIT* (Gammell et al., 2015), CVAE-plan (Ichter et al., 2018) and Reject-plan (Zhang et al., 2018) in terms of planning time and solution optimality. RRT* and BIT* are two widely used effective instances of TSA in Algorithm 4. In our experiments, we equipped RRT* with the goal biasing heuristic $\ell$ to improve its performance. BIT* exploits the batch informed sampling strategy (Gammell et al., 2015) to accelerate the planning process. CVAE-plan and Reject-plan are two learning-enhanced TSA planners proposed recently. CVAE-plan learns the important sampling region with a
conditional VAE (Sohn et al., 2015). We used the optimal paths produced by RRT* to train the sampler. Reject-plan learns to do rejection sampling with policy gradient methods.

We compared these algorithms in several benchmark tasks. To evaluate their generalization ability, for each experiment, we generated 3000 different tasks from the same distribution such that planners cannot plan on the same map twice. We trained the learning-based baselines CVAE-plan and Reject-plan using the first 2000 tasks, reserved the rest for testing. We let NEXT improve itself using MSIL over the first 2000 tasks. In this period, for every 200 tasks, we updated its parameters and annealed $\epsilon$ once. The value of the annealing $\epsilon$ was set to be the following:

$$
\epsilon = \begin{cases} 
1, & \text{if } i < 1000, \\
0.5 - 0.1 \cdot \lfloor (i - 1000)/200 \rfloor, & \text{if } 1000 \leq i < 2000, \\
0.1, & \text{otherwise,}
\end{cases}
$$

with $i$ denoting the task number.

5.1 Benchmark Environments

We carefully designed three benchmark environments to evaluate the algorithms. Their configuration space are in $\mathbb{R}^2$, $\mathbb{R}^3$ and $\mathbb{R}^5$, respectively. All of their workspace lies in $\mathbb{R}^2$. For each task $U = (s_{\text{init}}, s_{\text{goal}}, S, S_{\text{free}}, \text{map}, c(\cdot))$, the workspace map, starting position $s_{\text{init}}$ and center of the goal region $s_{\text{goal}} \subset S_{\text{goal}}$ are given. For a $d$-dimensional planning task, $\{s_{\text{init}}, s_{\text{goal}}\} \subset S_{\text{free}} \subset \mathbb{R}^d$, and map $\in \{0, 1\}^{d \times d}$ is represented using an occupancy grid. The workspace maps were generated with the recursive backtracker algorithm. Initial and goal positions were sampled uniformly randomly in free space. Three experiments differ in the choice of robots:

- **Workspace planning** The robot is abstracted with a point mass moving in the plane. Without higher dimensions, this problem reduces to planning in the workspace.

- **Rigid body navigation** A rigid body robot, abstracted as a thin rectangle, is used here. The extra rotation dimension is added to the planning problem. This robot can rotate and move freely without any constraints in the free space.

- **3-link snake** The robot is a 5-DOF snake with two joints. Two more angle dimensions are added to the planning task. To prevent links from folding, we restrict the angles to the range of $[-\pi/4, \pi/4]$.

5.2 Results and Analysis

We first illustrate some examples of all three types of tasks in Figure 6 where the NEXT finds high-quality solutions as shown. We also compared the search trees on a workspace planning instance of NEXT and RRT* in Figure 7(a)-(c). Obviously, the proposed NEXT algorithm can achieve better quality solutions with fewer samples. The learned $\tilde{V}^*$ and $\tilde{\pi}^*$ are also shown in Figure 7(d). As we can see, the learned $\tilde{V}^*$ and $\tilde{\pi}^*$ are consistent with our expectation, towards the ultimate target in the map. For more search tree comparison for all three types of tasks, please check Figure 12, 13, and 14 in Appendix B.

To systematically evaluate the algorithms, we recorded the time (measured by the number of collision checks used) needed to find a collision-free path, the success rate within time limits, and the cost of the solution path for each run. The results of the reserved 1000 test tasks of each experiment are shown in the bottom row of Figure 8. To make a fair comparison, we capped the number of samples for all algorithms to be 500, except RRT*-10k and BIT*-10k, which can use 10,000 samples. Both the kernel smoothing (KS-UCB) and the Gaussian process (GP-UCB) version of our algorithms outperform other competitors by a large margin under all three criteria.

We plot the performance improvement curve of our algorithms on the 5D planning tasks in the top row of Figure 8. Due to the space limitation, we put the results on point mass robot and rigid body navigation
Figure 6: The solution path produced by NEXT in a workspace planning task, rigid body navigation task, 3-link snake task from left to right. The orange dot and the brown dot are starting and goal locations, respectively.

(a) RRT* search tree (b) NEXT-KS search tree (c) NEXT-GP search tree (d) learned $\hat{V}^*$ and $\hat{\pi}^*$

Figure 7: Column (a) to (c) are the search trees produced by the RRT, NEXT-KS, and NEXT-GP on the same workspace planning task. The learned $\hat{V}^*$ and $\hat{\pi}^*$ from NEXT-KS are plotted in column (d). In the figures, obstacles are colored in deep blue, the starting and goal locations are denoted by orange and brown dots, respectively. In column (a) to (c), samples are represented with hollow yellow circles, and edges are colored in green. In column (d), the level of redness denotes the value of the cost-to-go estimate $\hat{V}^*$, and the light blue arrows point from a given state $s$ to the center of the proposal distribution $\hat{\pi}^*(s'|s,U)$. We set the maximum number of samples to be 500.

in Figure[11] and the quantitative evaluation in Table[1][2][3] in Appendix[3]. Please refer the details there. For comparison, we also plot the performance of RRT* and BIT*. At the beginning phase of self-improving, our algorithms are comparable to RRT*. They then gradually learn from previous experiences and improve themselves as they see more tasks and better solutions. In the end, NEXT-KS is able to match the performance of RRT*-10k using only one-twentieth of its samples! While the competitors perform consistently without any improvements. Two choices of the variance part in $\phi(s)$, KS and GP were compared in the experiments. Empirically, KS performed slightly better than GP.

The VIN [Tamar et al., 2016] or GPPN [Lee et al., 2018] requires explicit discretization of the configuration space, and thus, they are not directly applicable to planning tasks with higher dimensions, such as the rigid body navigation and the 3-link snake tasks. We only involved the comparison with GPPN on the workspace planning task empirically, which is reported better than VIN [Lee et al., 2018]. We also provide more preferences to GPPN in the experiment with the shortest paths as the supervision information, which is not known to our algorithm. The results are illustrated in Figure[8]. The GPPN generated the whole path without automatic adaption via collision checks, which lead to lower success rate. Moreover, the quality of the found
Figure 8: The first row displays the improvement curves of our algorithms on all 3000 tasks of the 5D experiments. The second row shows the comparison between the NEXT and existing algorithms, in terms of the success rate, the average collision checks, and the average cost of the solution paths. The performance of all algorithms is evaluated with the last 1000 tasks for each experiment. All algorithms are restricted to use only 500 samples, except RRT*-10k and BIT*-10k, which are allowed to use 10,000 samples. The value of collision checks and path costs are normalized w.r.t. the performance of RRT*. In the 2D experiment, the result of GPPN is obtained from running it on the corresponding discretized versions of the original problems. The training and testing data split of GPPN is the same as CVAE-plan and Reject-plan.

paths by GPPN is also worse than that of our algorithm, demonstrating the advantage of our algorithm.

To further demonstrate the power of NEXT::Expand, we replace the NEXT::Expand with breadth-first search (BFS) [Kim et al. 2018], another expanding strategy in the algorithm framework Algorithm 1, while keeps other components the same. Specifically, BFS uses a search queue in the planning. It repeatedly pops a state s out of the search queue, samples k states from π(·|s), and pushes all samples and state s back to the queue, until the goal is reached. For fairness, we use the learned sampling policy π(s′|s, U) by NEXT-KS in BFS. As shown in Figure 8, BFS obtained worse paths with a much larger number of collision checks and far lower success rate, which justifies the importance of the balance between exploration versus exploitation in the proposed NEXT::Expand.

6 Conclusion

In this paper, we propose a self-improving planner, NEXT, which can generalize and achieve better performance with experiences accumulated. The algorithm achieves a delicate balance between exploration versus exploitation via our carefully designed UCB-type expansion operation within the sampling-based planning template. To obtain the generalizable ability across different tasks, we proposed a new parametrization for the value function and policy, which captures the Bellman recursive structure in the high-dimensional continuous state and action space. We demonstrate the power of the proposed algorithm by outperforming previous state-of-the-art planners with significant margins.

Several interesting directions can be explored in the future. One of the limitations of this work is that
the workspace map and the current state of the agent must be known. It is possible that one can relax the
assumption to only seeing the local surrounding obstacles so that the algorithm must gradually estimate the
agent state information and learn the environment as the robot traverses the world. To address this, one may
incorporate the QMDP-net ([Karkus et al. 2017]) and embed the SLAM algorithm ([Cadena et al. 1996]) as
a neural network into this work. One may also try to deploy this algorithm to a more realistic simulation
environment, or even apply it in real-world path planning tasks.

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Appendix

A  Policy and Value Network Architecture

We explain the implementation details of the proposed parametrization for policy and value function. Figure 9 and Figure 10 are neural architectures for the attention module, the policy/value network, and the planning module, respectively.

Figure 9: left: attention module, instantiating the Figure 4; right: policy/value network, instantiating the Figure 5.

In the figures, we use rectangle blocks to denote inputs, intermediate results and outputs, stadium shape blocks to denote operations, and rounded rectangle blocks to denote modules. We use different colors for
different operations. In particular, we use blue for convolutional/LSTM layers, green for dense layers, and orange for anything else. For convolutional layers, "Conv 1×1, 32, relu" denotes a layer with 1×1 kernels, 32 channels, followed by a rectified linear unit; for dense layers, "Dense, 64, relu" denotes a layer of size 64, followed by a rectified linear unit.

The attention module (Figure 9-left) embeds a state to a $d \times d \times d_a$ tensor. See equation (11) and (13) for details for computing $f_w(s)$ and $\mu_\theta(s)$. The planning module (Figure 10) is a one-step LSTM update which takes the result of a convolutional layer as input. Both the input and hidden size of the LSTM cell are $d_e$. All $d \times d$ locations share one set of parameters and are processed by the LSTM in one batch.

The main architecture is illustrated in Figure 9-right. It takes maze map, state and goal as input, and outputs the action and the value. Refer to equation (14) for details for computing $\psi(s)$. In our experiments, we set the values of the hyper-parameters to be $(d, d_e, d_a, p) = (15, 64, 8, 8)$.

### B Experiment Results

#### B.1 Details of Quantitative Evaluation

More detailed results are shown in Table 1, 2, 3 including learning-based and non-learning-based ones, on the last 1000 tasks in each experiment. We normalized the number of collision checks and the cost of paths based on the solution of RRT*. The success rate result is not normalized.

The best planners in each experiment are in **bold**. Our algorithm, i.e., NEXT-KS, achieves competitive or even better results with others using 20 times fewer samples. In all tasks, the proposed algorithms, including NEXT-KS and NEXT-GP outperform the current state-of-the-art planning algorithm with large margins.

![Planning module](image)

**Figure 10**: planning module

| #samples-10k | RRT* | BIT* | BFS | CVAE | Reject |
|--------------|------|------|-----|------|--------|
| 2D           | 0.988| 0.981| 0.967| 0.735| 0.185  |
| 3D           | 0.943| 0.841| -   | 0.490| 0.121  |
| 5D           | 0.883| 0.768| -   | 0.455| 0.030  |

Table 1: Success rate results. The higher the better. The NEXT-KS achieves the best results using only one-twentieth of samples.
Table 2: Average number of collision checks results. The lower the better. The score is normalized based on the solution of RRT∗. The NEXT-KS performs the best.

| NEXT  | KS  | GP  | RRT∗ | BIT∗ | BFS  | CVAE | Reject | #samples-10k |
|-------|-----|-----|------|------|------|------|--------|--------------|
| 2D    | 0.177 | 0.243 | 1.500 | 1.154 | 9.247 | 1.983 | 1.011   | 4.635 | 1.030 | 14.308 | 5.008  |
| 3D    | 0.694 | 1.334 | 1.000 | 1.033 | 7.292 | 2.162 | 0.988   | 5.023 | 1.638 | 57.430 | 5.318  |
| 5D    | 0.888 | 1.520 | 1.000 | 1.004 | 5.758 | 1.188 | 0.997   | 8.525 | 1.888 | 24.373 | 8.329  |

Table 3: Average cost of paths. The lower the better. The score is normalized based on the solution of RRT∗. The NEXT-KS achieves the best solutions.

| NEXT  | KS  | GP  | GPPN | RRT∗ | BIT∗ | BFS  | CVAE | Reject | #samples-10k |
|-------|-----|-----|------|------|------|------|------|--------|--------------|
| 2D    | 0.172 | 0.193 | 0.272 | 1.000 | 1.050 | 2.811 | 1.649 | 1.050   | 0.167 | 0.188 | 0.154 | 0.165  |
| 3D    | 0.116 | 0.315 | -    | 1.000 | 1.886 | 1.720 | 1.734 | 0.984   | 0.129 | 0.480 | 0.680 | 0.123  |
| 5D    | 0.215 | 0.426 | -    | 1.000 | 1.835 | 1.780 | 0.961 | 1.020   | 0.261 | 1.128 | 0.299 | 0.259  |

We demonstrated the performance improvement curves for 2D workspace planning, 3D rigid body navigation Figure [11]. As we can see, similar to the performances on 5D 3-link snake planning tasks in Figure [8] in these tasks, the NEXT-KS and NEXT-GP improve the performances along with more and more experiences collected, justified the self-improvement ability by learning $\hat{V}^*$ and $\hat{\pi}^*$.

B.2 Search Trees Comparison

We illustrate the search trees generated by RRT∗ and the proposed NEXT algorithms with 500 samples in Figure [12], Figure [13], and Figure [14] on several workspace planning, rigid body navigation, and 3-link snake planning tasks, respectively. Comparing to the search trees generated by RRT∗ side by side, we can clearly see the advantages and the efficiency of the proposed NEXT algorithms. In all the tasks, even in 2D workspace planning tasks, the RRT∗ indeed randomly searches without realizing the goals, and thus cannot complete the missions, while the NEXT algorithms search towards the goals with the guidance from $\hat{V}^*$ and $\hat{\pi}^*$, therefore, successfully provides high-quality solutions.
Figure 11: The first and second rows display the improvement curves of our algorithms on all 3000 tasks of the 2D workspace planning and 3D rigid body navigation problems. We compare our algorithms with RRT* and BIT*. Three columns correspond to the success rate, the average collision checks, and the average cost of the solution paths for each algorithm.
Figure 12: Column (a) to (c) are the search trees produced by the RRT, NEXT-KS, and NEXT-GP on the same workspace planning task. The learned $\tilde{V}^*$ and $\tilde{\pi}^*$ from NEXT-KS are plotted in column (d). In the figures, obstacles are colored in deep blue, the starting and goal locations are denoted by orange and brown dots, respectively. In column (a) to (c), samples are represented with hollow yellow circles, and edges are colored in green. In column (d), the level of redness denotes the value of the cost-to-go estimate $\tilde{V}^*$, and the light blue arrows point from a given state $s$ to the center of the proposal distribution $\tilde{\pi}^*(s'|s,U)$. We set the maximum number of samples to be 500.
Figure 13: Each column corresponds to one example from the rigid body navigation problem. The top and the bottom rows are the search trees produced by the RRT and NEXT-KS, respectively. In the figures, obstacles are colored in deep blue, and the rigid bodies are represented with matchsticks. The samples, starting states, and goal states are denoted by yellow, orange, and brown matchsticks, respectively. Edges are colored in green. We set the maximum number of samples to be 500.
Figure 14: Each column corresponds to one example from the 3-link snake problem. The top and the bottom rows are the search trees produced by the RRT and NEXT-KS, respectively. In the figures, obstacles are colored in deep blue, and the rigid bodies are represented with matchsticks. The samples, starting states, and goal states are denoted by yellow, orange, and brown matchsticks, respectively. Edges are colored in green. We set the maximum number of samples to be 500.