Probing the dynamics of dark energy with novel parametrizations

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We point out that the CPL parametrization has a problem that the equation of state $w(z)$ diverges in the far future, so that this model can only properly describe the past evolution but cannot depict the future evolution. To overcome such a difficulty, in this Letter we propose two novel parametrizations for dark energy, the logarithm form $w(z) = w_0 + w_1 \left( \frac{\ln(2 + z)}{1 + z} - \ln 2 \right)$ and the oscillating form $w(z) = w_0 + w_1 (\sin(\ln 2) - \sin(1))$, successfully avoiding the future divergency problem in the CPL parametrization, and use them to probe the dynamics of dark energy in the whole evolutionary history. Our divergency-free parametrizations are proven to be very successful in exploring the dynamical evolution of dark energy and have powerful prediction capability for the ultimate fate of the universe. Constraining the CPL model and the new models with the current observational data, we show that the new models are more favored. The features and the predictions for the future evolution in the new models are discussed in detail.

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The properties of dark energy are mainly characterized by the equation of state parameter (EOS), $w$. Extracting the information of EOS of dark energy from observational data is very challenging owing to the accuracy of current data and our ignorance of dark energy. For probing the dynamical evolution of dark energy, under such circumstance, one has to parameterize $w$ empirically, usually using two or more free parameters. Among all the parametrization forms of EOS, the Chevallier-Polarski-Linder (CPL) model\textsuperscript{[1]} is the most widely used one and has been explored extensively. The form of the CPL parametrization is

$$w(z) = w_0 + w_1 \frac{z}{1 + z},$$  \hspace{1cm} (1)

where $z$ is the redshift, $w_0$ is the present-day value of the EOS, and $w_1$ is the derivative of the EOS with respect to the scale factor $a$. The direct motivation of proposing such a parametrization form is to overcome the divergency feature of the linear form $w(z) = w_0 + w_1 z$ at high redshifts. Furthermore, as Linder\textsuperscript{[1]} suggested, the CPL parametrization has several advantages, such as a manageable two-dimensional phase space, well behaved and bounded behavior for high redshifts, high accuracy in reconstructing many scalar field equations of state, simple physical interpretation, etc.

However, we have to point out that there exists a problem in the CPL model. The CPL model only explores the past expansion history properly, but cannot describe the future evolution due to the fact that $|w(z)|$ grows increasingly and finally encounters divergency as $z$ approaches $-1$. Undoubtedly, this is a nonphysical feature. Such a divergency problem prevents the CPL parametrization from genuinely covering the scalar-field models as well as other theoretical models.

To overcome the shortcoming of the CPL model, we are interested in proposing novel parametrization forms of $w(z)$. The new parametrizations will be contrived to exceed the CPL model entirely: inheriting the advantages of the CPL model, avoiding the disastrous divergency in the far future, and being more favored by the observational data. In this Letter, we are devoted to exploring more insightful parametrization forms for dark energy and probing the dynamics of dark energy in light of the novel parametrizations.

The leading proposal we put forth for the EOS of dark energy is of the form:

$$w(z) = w_0 + w_1 \left( \frac{\ln(2 + z)}{1 + z} - \ln 2 \right),$$  \hspace{1cm} (2)

where $w_0$ also denotes the present-day value of $w(z)$, and $w_1$ is another parameter characterizing the evolution of $w(z)$. Note that a minus $\ln 2$ in the bracket is kept for maintaining $w_0$ to be the current value of $w(z)$ and for easy comparison with the CPL model. Obviously, this new parametrization has well behaved, bounded behavior for both high redshifts and negative redshifts. Thanks to the logarithm form in the parametrization, a finite value for $w(z)$ can be ensured, via the application of the L’Hospital’s rule, in both limiting cases, $z \rightarrow \infty$ and $z \rightarrow -1$. This is the reason why we introduce a logarithm form in the new parametrization. For clearness, we list the values of $w(z)$ in the limiting cases:

$$w(z) = \begin{cases} w_0, & \text{for } z = 0, \\ w_0 - w_1 \ln 2, & \text{for } z \rightarrow +\infty, \\ w_0 + w_1 (1 - \ln 2), & \text{for } z \rightarrow -1. \end{cases}$$  \hspace{1cm} (3)

At low redshifts, the new form reduces to the linear one, $w(z) \approx w_0 + w_1 z$, where $w_1 = -(\ln 2)w_1$. Of course, one can also recast it at low redshifts as the CPL form, $w(z) \approx w_0 + \tilde{w}_1 z/(1 + z)$, where $\tilde{w}_1 = \left(1/2 - \ln 2\right)w_1$. Therefore, it is clear to see that the new parametrization exhibits well-behaved feature for the dynamical evolution of dark energy. Without doubt, such a two-parameter form of EOS can genuinely cover many scalar-field models (including quintom models with two scalar fields and/or with one field with high derivatives) as well as other theoretical scenarios.

It is well-known that dark energy drives the cosmic acceleration only at the late times ($z \sim 0.5$), whereas at the early times dark energy can be totally neglected due to the extremely low density compared to the matter or radiation component. Thus, one can only justify that the EOS of dark energy is around $-1$ in the recent epoch, but can tolerate more possibilities for
the early-time EOS of dark energy. For example, the EOS of dark energy might exhibit oscillating feature during the evolution [2]. This is a fascinating possibility, deserving a detailed investigation. Based on this consideration, we further extend the above new parametrization (2) to an oscillating one, by replacing the logarithm function with a sine function:

\[
    w(z) = w_0 + w_1 \left( \frac{\sin(1 + z)}{1 + z} - \sin(1) \right),
\]

(4)

Such a replacement is rather reasonable, lying in the fact that the two parametrizations roughly coincide in the recent epoch (low redshifts), since \( \sin(1) \approx \ln 2 \) and \( \cos(1) \approx 1/2 \) [note that \( \sin(1) \approx 0.841 \), \( \ln 2 \approx 0.693 \), and \( \cos(1) = 0.540 \)]. Hence, the parametrization (4) describes the same behavior as the logarithm form (2) at low redshifts, but exhibits oscillating feature from a long term point of view. Also, we list the values of \( w(z) \) in the following limiting cases:

\[
    w(z) = \begin{cases} 
        w_0, & \text{for } z = 0, \\
        w_0 - w_1 \sin(1), & \text{for } z \to +\infty, \\
        w_0 + w_1(1 - \sin(1)), & \text{for } z \to -1.
    \end{cases}
\]

(5)

We find that the two parametrizations, (2) and (4), also roughly coincide in the limiting cases, \( z \to \infty \) and \( z \to -1 \).

In what follows, we shall explore the dynamical evolution of dark energy via the CPL parametrization and the new parametrizations. For convenience, we call the new parametrizations the logarithm form and oscillating form, respectively, hereafter. Since our aim is to probe the dynamics of dark energy, we should try to avoid other indirect factors weakening the observational limits on the EOS; thus we assume a flat universe, \( \Omega_k = 0 \), consistent with the inflationary cosmology. From the Friedmann equation, the Hubble expansion rate can be written as

\[
    H(z) = H_0 \left[ \Omega_m(1 + z)^3 + \Omega_r(1 + z)^4 + (1 - \Omega_m - \Omega_r) f(z) \right]^{1/2},
\]

(6)

where \( \Omega_r = \Omega_r (1 + 0.2271 N_{\text{eff}}) \), with \( \Omega_r = 2.469 \times 10^{-5} h^{-2} \) for \( T_{\text{cmb}} = 2.725 \) K, \( N_{\text{eff}} \) the effective number of neutrino species (in this Letter we take its standard value, 3.04 [3]), and \( f(z) = \exp[3 \int_0^z d\bar{z}'(1 + w(z'))/(1 + z')] \).

For constraining \( w(z) \), we use the current observational data from the type Ia supernovae (SN), the baryon acoustic oscillations (BAO), and the cosmic microwave background (CMB). Such a combination of data sets is the most widely used one, sufficiently satisfying our aim of testing the new parametrizations and making a comparison. Of course, one can also add other data sets such as gamma-ray bursts, \( H(z) \), and so on, but we feel that this is not necessary for our present aim and leave a more sophisticated analysis to a future work with different goal.

For the SN data, we use the 557 Union2 data compiled in Ref. [4]. The theoretical distance modulus is defined as

\[
    \mu_\text{th}(z_i) = 5 \log_{10} D_L(z_i) + \mu_0,
\]

(7)

where \( \mu_0 = 42.38 - 5 \log_{10} h \) with \( h \) the Hubble constant \( H_0 \) in units of 100 km/s/Mpc, and the Hubble-free luminosity distance

\[
    D_L(z) = (1 + z) \int_0^z \frac{dz'}{E(z', \theta)},
\]

(8)

where \( E \equiv H/H_0 \), and \( \theta \) denotes the model parameters. Correspondingly, the \( \chi^2 \) function for the 557 Union2 SN data is given by

\[
    \chi^2_{SN}(\theta) = \sum_{i=1}^{557} \frac{[\mu_{\text{obs}}(z_i) - \mu_\text{th}(z_i)]^2}{\sigma^2(\mu_i)},
\]

(9)

where \( \sigma \) is the corresponding 1\( \sigma \) error of distance modulus for each supernova. The parameter \( \mu_0 \) is a nuisance parameter but it is independent of the data points. Following Ref. [5], the minimization with respect to \( \mu_0 \) can be made trivially by expanding \( \chi^2 \) of Eq. (7) with respect to \( \mu_0 \) as

\[
    \chi^2_{SN}(\theta) = A - 2\mu_0 B + \mu_0^2 C,
\]

(10)

where

\[
    A(\theta) = \sum_{i=1}^{557} \frac{[\mu_{\text{obs}}(z_i) - \mu_\text{th}(z_i; \mu_0 = 0, \theta)]^2}{\sigma^2(\mu_i)},
\]

\[
    B(\theta) = \sum_{i=1}^{557} \frac{\mu_{\text{obs}}(z_i) - \mu_\text{th}(z_i; \mu_0 = 0, \theta)}{\sigma(\mu_i)},
\]

\[
    C = \sum_{i=1}^{557} \frac{1}{\sigma^2(\mu_i)}.
\]

Evidently, Eq. (10) has a minimum for \( \mu_0 = B/C \) at

\[
    \chi^2_{SN}(\theta) = A(\theta) - \frac{B(\theta)^2}{C}.
\]

(11)

Since \( \chi^2_{SN, \text{min}} = \chi^2_{SN, \text{min}} \) instead minimizing \( \chi^2_{SN} \) we will minimize \( \chi^2_{SN} \) which is independent of the nuisance parameter \( \mu_0 \).

For the BAO measurement, we use the data from SDSS DR7 [6]. The distance ratio (\( d_L \)) at \( z = 0.2 \) and \( z = 0.35 \) are

\[
    d_{0.2} = \frac{r_s(z_{0.2})}{D_V(0.2)}, \quad d_{0.35} = \frac{r_s(z_{0.35})}{D_V(0.35)},
\]

(12)

where \( r_s(z) \) is the comoving sound horizon at the baryon drag epoch [7], and

\[
    D_V(z) = \left[ \left( \int_0^z \frac{dz'}{H(z')} \right)^2 \frac{z}{H(z)} \right]^{1/3}
\]

(13)

encodes the visual distortion of a spherical object due to the non Euclidianity of a FRW spacetime. The inverse covariance matrix of BAO is

\[
    (C^{-1}_{BAO}) = \begin{pmatrix}
        30124 & -17227 \\
        -17227 & 86977
    \end{pmatrix}.
\]

(14)
The $\chi^2$ function of the BAO data is constructed as:

$$X_{BAO}^2 = (d_l^0 - d_l^{\text{obs}})(C_{BAO}^{-1})_{ij}(d_l^j - d_l^{\text{obs}}),$$  \hspace{1cm} (15)

where $d_l = (d_{0.2}, d_{0.35})$ is a vector, and the BAO data we use are $d_{0.2} = 1.905$ and $d_{0.35} = 1.097$.

The CMB is sensitive to the distance to the decoupling epoch via the locations of peaks and troughs of the acoustic oscillations. In this Letter, we employ the “WMAP distance priors” given by the seven-year WMAP observations [3]. This includes the “acoustic scale” $l_A$, the “shift parameter” $R$, and the redshift of the decoupling epoch of photons $z_s$. The acoustic scale $l_A$ describes the distance ratio $D_A(z_s)/r_s(z_s)$, defined as

$$l_A \equiv (1 + z_s)\frac{\pi D_A(z_s)}{r_s(z_s)},$$  \hspace{1cm} (16)

where a factor of $(1 + z_s)$ arises because $D_A(z_s)$ is the proper angular diameter distance, whereas $r_s(z_s)$ is the comoving sound horizon at $z_s$. The fitting formula of $r_s(z)$ is given by

$$r_s(z) = \frac{1}{\sqrt{3}} \int_0^{\sqrt{1/(1+z)}} \frac{da}{a^2H(a)\sqrt{1 + (3\Omega_b/4\Omega_r)a}},$$  \hspace{1cm} (17)

where $\Omega_b$ and $\Omega_r$ are the present-day baryon and photon density parameters, respectively. In this Letter, we fix $\Omega_r = 2.469 \times 10^{-5}$ and $\Omega_b = 0.02246h^{-2}$ given by the seven-year WMAP observations [3]. We use the fitting function of $z_s$ proposed by Hu and Sugiyama [8]:

$$z_s = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{0.2}],$$  \hspace{1cm} (18)

where

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, \quad 82 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}.$$  \hspace{1cm} (19)

The shift parameter $R$ is responsible for the distance ratio $D_A(z_s)/H^{-1}(z_s)$, given by [9]

$$R(z_s) \equiv \sqrt{\Omega_m H_0^2}(1 + z_s)D_A(z_s).$$  \hspace{1cm} (20)

Following Ref. [3], we use the prescription for using the WMAP distance priors. Thus, the $\chi^2$ function for the CMB data is

$$X_{CMB}^2 = (x_i^h - x_i^{\text{obs}})(C_{CMB}^{-1})_{ij}(x_j^h - x_j^{\text{obs}}),$$  \hspace{1cm} (21)

where $x_i = (l_A, R, z_s)$ is a vector, and $(C_{CMB}^{-1})_{ij}$ is the inverse covariance matrix. The seven-year WMAP observations [3] give the maximum likelihood values: $l_A(z_s) = 302.09$, $R(z_s) = 1.725$, and $z_s = 1091.3$. The inverse covariance matrix is also given in Ref. [3]:

$$(C_{CMB}^{-1})_{ij} = \begin{pmatrix}
2.305 & 29.698 & -1.333 \\
29.698 & 6825.27 & -113.180 \\
-1.333 & -113.180 & 3.414
\end{pmatrix}.$$  \hspace{1cm} (22)

Since the SN, BAO and CMB are effectively independent measurements, we can combine the data sets by simply adding together the $\chi^2$ functions. Thus, we have

$$\chi^2 = \chi_{SN}^2 + \chi_{BAO}^2 + \chi_{CMB}^2.$$  \hspace{1cm} (23)

Note that $\chi_{SN}^2$ is free of $h$, while $\chi_{BAO}^2$ and $\chi_{CMB}^2$ are still relevant to $h$.

The three models have the same free model parameters, namely, $\theta = (\Omega_m, w_0, w_1, h)$. According to the joint data analysis, we obtain the best-fit parameters and the corresponding $\chi^2_{\text{min}}$. The best-fit, 1$\sigma$ and 2$\sigma$ values of the parameters with $\chi^2_{\text{min}}$ of the three models are all presented in Table I.

Figure 1 shows the likelihood contours for the three models in the $w_0 - w_1$ and $\Omega_m - h$ planes. For the CPL parametrization, we have $\Omega_m = 0.279$, $w_0 = -1.066$, $w_1 = 0.261$ and $h = 0.699$, with $\chi^2_{\text{min}} = 544.186$. We plot the likelihood contours for the CPL model in the panels (a) and (b) of Fig. 1. For the logarithm parametrization, the fitting results are $\Omega_m = 0.280$, $w_0 = -1.067$, $w_1 = -0.410$ and $h = 0.695$, with $\chi^2_{\text{min}} = 543.986$, which is the smallest of the three. The likelihood contours for this case are shown in the panels (c) and (d) of Fig. 1. For the oscillating parametrization, we obtain the fitting results: $\Omega_m = 0.280$, $w_0 = -1.061$, $w_1 = -0.410$ and $h = 0.695$, with $\chi^2_{\text{min}} = 544.081$, smaller than that of the CPL model. The likelihood contours for this case are shown in the panels (e) and (f) of Fig. 1. According to the $\chi^2_{\text{min}}$, the new parametrizations are indicated to be more favored by the observational data.

Next, we reconstruct the expansion history of the universe in light of the above fitting results for the three models, and then make a comparison for them. Since we focus on the properties of dark energy, we only reconstruct the evolutionary behaviors of the EOS of dark energy, $w(z)$, and the deceleration parameter of the universe, $q(z)$. The reconstructing results are shown in Fig. 2. As has been pointed out in the above, the CPL model has a problem: $w(z)$ diverges when $z$ approaches $-1$. So, the CPL parametrization can only properly describe the past evolution history but cannot genuinely depict the future evolution; it is incomplete in describing the evolutionary history of dark energy. Consequently, the CPL parametrization is not capable of covering other dark-energy theoretical models. Such a problem can be explicitly seen in the reconstructed evolution plots, panels (a) and (b) of Fig. 2. We can see from these two plots, $w(z)$ and $q(z)$, that although the CPL model can do a good job in describing the past evolution of dark energy, it totally loses the prediction capability for the future evolution of dark energy. The novel parametrizations successfully overcome the shortcoming of the CPL model. The reconstructed evolutionary plots, panels (c) and (d) for the logarithm form (2) and panels (e) and (f) for the oscillating form (4), indicate that the both new models can nicely describe the whole evolution history of dark energy.

Comparing the panels (a), (c) and (e) of Fig. 2, we find that all the three models favor a quintom behavior [10] that the EOS crosses $-1$ around the recent epoch. This $w = -1$ crossing feature is only mildly favored by the CPL and the oscillating models, but is explicitly favored by the logarithm model. For the CPL model, since it forfeits the prediction capability for the future evolution, we cannot say anything about the ultimate fate of the universe. For the oscillating model, the fate of the universe is not definitely determined: the big rip may or may not occur. For the logarithm model, the universe will definitely move towards its tragic destiny: the cosmic doomsday
TABLE I: The fitting values for the CPL, logarithm (Log) and oscillating (Sin) models.

| Model | $\Omega_m$ | $w_0$ | $w_1$ | $\sigma_8$ | $\chi^2_{min}$ |
|-------|------------|-------|-------|-------------|----------------|
| CPL   | 0.279^{+0.022}_{-0.020} \ (1\sigma) | -1.066^{+0.267}_{-0.232} \ (1\sigma) | 0.261^{+0.096}_{-0.103} \ (1\sigma) | 0.698^{+0.025}_{-0.036} \ (1\sigma) | 544.186 |
| Log   | 0.280^{+0.028}_{-0.026} \ (1\sigma) | -1.067^{+0.148}_{-0.165} \ (1\sigma) | -1.049^{+0.186}_{-0.168} \ (2\sigma) | 0.695^{+0.024}_{-0.036} \ (1\sigma) | 544.081 |
| Sin   | 0.280^{+0.033}_{-0.029} \ (1\sigma) | -1.066^{+0.172}_{-0.146} \ (1\sigma) | -0.042^{+0.246}_{-0.745} \ (2\sigma) | 0.695^{+0.024}_{-0.036} \ (1\sigma) | 543.986 |

FIG. 1: The probability contours at 1\sigma and 2\sigma confidence levels in the $w_0 - w_1$ and $\Omega_m - h$ planes for the CPL, logarithm (Log) and oscillating (Sin) models.
will happen at about $3\sigma$ level. If we only focus on the past evolution, we find that the CPL model performs better than the oscillating model. The best one among the three is without doubt the logarithm model, not only for the description of the past evolution but also of the future evolution. From the panels (c) and (e) we see clearly that the oscillating model behaves similarly to the logarithm within the range from $z = -1$ to $z = 2$, while the oscillating feature emerges in the oscillating model from a long term perspective. Comparing the panels of (b), (d) and (f) of Fig. 2, we also find that the logarithm model performs the best. From the reconstructed $q(z)$ plots we see that the accelerated expansion of the universe starts at a redshift around $0.5 - 0.7$. According to the new models, for the future evolution, the change rate of the cosmic acceleration, $|dq(z)/dz|$, will first increase and then decrease, with the pivot around $z \approx -0.45$. We find that for the far future, the change rate $|dq(z)/dz|$ for the logarithm model is still rather large but for the oscillating model approaches zero.

In summary, we have proposed two novel parametrizations for the EOS of dark energy, successfully avoiding the future
divergency problem of the CPL parametrization, and used them to probe the dynamics of dark energy not only in the past evolution but also in the future evolution. We pointed out that the CPL parametrization can only properly describe the past evolution history of dark energy but cannot genuinely depict the future evolution of dark energy owing to the divergency of \(w(z)\) as \(z\) approaches \(-1\). Such a divergency feature forces the CPL parametrization to lose its prediction capability for the fate of the universe and to fail in providing a complete evolution history for the dark energy. Consequently, the CPL model cannot genuinely cover scalar-field models as well as other dark energy theoretical models. Our new proposals, the logarithm form \(w(z) = w_0 + w_1 \left( \frac{\ln(1+z)}{1+z} \right) - \ln 2 \) and the oscillating form \(w(z) = w_0 + w_1 \left( \sin(1+z) \right) - \sin(1) \), exhibit well-behaved features for the EOS of dark energy in all the evolution stages of the universe. We constrained the new models by using the current observational data including the 557 Union2 SN data, BAO data from SDSS DR7 and CMB data from 7-yr WMAP. The fitting results show that the oscillating parametrization gains a minimal \(\chi^2_{\text{min}}\) among the three models, seemingly more favored by the data. However, by reconstructing the whole evolutionary histories of \(w(z)\) and \(q(z)\) from past to future via the fitting results, we found that the logarithm parametrization is more tightly constrained by the data. The reconstruction results show that the \(w = -1\) crossing feature is favored. Furthermore, the new models predict that in the future the cosmic acceleration will first speed up and then slow down. In the logarithm model, the cosmic doomsday seems inevitable, which will happen at about \(3\sigma\) level. We believe that the novel parametrizations proposed in the present Letter deserve further investigations.

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