Reexamining RHDE models in FRW Universe with two IR cutoff with redshift parametrization

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Abstract: In this paper, we have investigated that the gravitational field equations are not compatible with conservation equation in Dixit et al. (Euro Phys J Plus 135:831, 2020). Therefore, the expression for an equation of state parameter along with the dynamics of \( x_T - x_0 T \) plane does not reflect the actual behaviors of RHDE models in \( f(R, T) \) gravity and thus the method and technique given in Dixit et al. (2020) represents a fractured way for analyzing RHDE models in \( f(R, T) \) theory of gravity. We also investigate that the derived Universe is in decelerating phase of expansion for \( 0 < \beta \leq 1.5 \) which is contrary to the result obtained in Dixit et al. (2020).

Keywords: RHDE model; IR cutoff; Cosmology

1. Introduction

In 2004, Li [1] has proposed holographic dark energy principle to search the dark energy scenario and the authors of Refs. [2–4] have investigated that the concept of holographic dark energy can be utilized in quantum gravity. In Wang and Wang [5], a holographic dark energy model inspired by Bekenstein–Hawking entropy has been investigated. We also note that by using the concept of horizon entropy of a black hole which is also known as Tsallis entropy [6], Tavayef et al. [7] have investigated Tsallis holographic dark energy (THDE) model in general relativity. However, the Hubble horizon as an IR cutoff in modified theories of gravity is not a suitable candidate to explain the late time acceleration in the Universe [8, 9]. Some important applications of Tsallis entropy are given in Ref. [10]. Recently, Moradpour et al. [11] have proposed a new holographic model for dark energy model inspired by the concept of Rényi entropy [12] in the framework of general relativity. This new model of dark energy is known as Rényi holographic dark energy (RHDE) model. In the recent past, Tsallis and Rényi entropies [13–17] have been used to study some gravitational events in the framework of general relativity [18–23]. Also, we note that a general approach to THDE and even its generalization is done in Refs. [24, 25].

In the recent time, the modified theories of gravity have been constantly used to solve dark energy problems or issues associated with the standard \( \Lambda \mathrm{CDM} \) model [26–28]. Some alternative theories of gravity have a pretty agreement with astrophysical observations [30–32]. The \( f(R, T) \) theory of gravity [29] presents in its field equations extra contributions from both geometry, through a general dependence on \( R \), and matter, through a general dependence on \( T \), the trace of the energy–momentum tensor. Moreover, the \( T \)—dependence of the geometrical action in \( f(R, T) \) gravity may be due to the existence of some imperfect fluids and intrinsically may have some quantum effects like particle production [33]. Some important applications of \( f(R, T) \) theory of gravity in various physical contexts are given in Refs. [44, 45]. We also note that the particular functional form

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and its trace $T = g^{\mu\nu}T_{\mu\nu}$.

In Dixit et al. [67], the authors have considered the stress–energy tensor as
\begin{equation}
T_{\mu\nu} = -p g_{\mu\nu} + (\rho + p) u_\mu u_\nu.
\end{equation}

With the choice of $\mathcal{L}_m = -p$, with $p$ being the pressure of matter. We have assumed units such that $G = 1$.

Therefore, the gravitational field equation for $f(R, T) = R + 2\xi T$ is obtained as
\begin{equation}
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} - 2f'(T)T_{\mu\nu} - 2f'(T)\Theta_{\mu\nu} + f(T)g_{\mu\nu},
\end{equation}

where $\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu}$ and the prime denotes a derivative with respect to the argument.

For dust filled Universe, $p = 0$, Eq. (4) is reduced to
\begin{equation}
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} + 2f'(T)T_{\mu\nu} + f(T)g_{\mu\nu}.
\end{equation}

Note that Eq. (5) of this manuscript is exactly the same as Eq. (9) of Dixit et al [67]. Further, in Ref. [67], the authors have selected $f(T) = \xi T$ with $\xi$ as a constant.

The spatially flat FRW Universe is represented by the following space-time
\begin{equation}
ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2),
\end{equation}

where $a(t)$ denotes the scale factor.

Therefore, the general field equations for $f(R, T) = R + 2\xi T$ and the metric (6) are obtained as
\begin{align}
3\frac{a^2}{a^2} &= (8\pi + 3\xi)\rho_T, \quad (7) \\
\frac{a^2}{a^2} + \frac{\dot{a}}{a} &= \xi\rho_T. \quad (8)
\end{align}

We note that Eqs. (7) and (8) are exactly the same as Eqs. (11) and (12) of Dixit et al. [67]. With red choice of $\Lambda_{\text{eff}} \propto H^2$, where $H = \frac{\dot{a}}{a}$ is a Hubble function. The general solution the field equations are obtained in Ref. [67] as
\begin{equation}
H(t) = \frac{2(8\pi + 3\xi)}{3(8\pi + 2\xi)} = \frac{2\beta}{3t},
\end{equation}

where $\beta = \frac{8\pi + 3\xi}{8\pi + 2\xi}$. Eq. (9) of this article and Eq. (14) of Dixit et al. [67] are the same. It is worthwhile to note the solution (9) is not new. It has been already given in Harko et al. [29]. Together with some other issues, the main issue in Dixit et al. [67] is that one can not obtain the energy conservation equation $\frac{\delta}{\delta t} + 3H(\rho_T + p_T) = 0$ from the gravitational field equations (7) and (8). Therefore, the expression for equation of state parameter together with the dynamics of $\omega_T - \omega_p$ plane does not show the actual behaviors of RHDE models in $f(R, T)$ theory of gravity.
3. Non-existence of energy conservation law for RHDE in \( f(R, T) \) gravity

In Ref. [67], the conservation equation is

\[
\frac{\partial \rho_T}{\partial t} + 3H(\rho_T + p_T) = 0. \tag{10}
\]

where \( \rho_T \) is the Renyi holographic energy density. \( p_T \) is not defined in Ref. [67] but we understand that it is a pressure of considered fluid in connection to define equation of state parameter \( \omega_T \). It is interesting to note that for obtaining the field equations (7) and (8) in \( f(R, T) \) gravity, which is Eqs. (11) and (12) in Dixit et al. [67]. Moreover, the authors of Ref. [67] have assumed \( p = 0 \) but in the conservation equation, they conveniently admit pressure without a concrete physical reason behind it. However, it is well known that the conservation equation does not hold in \( f(R, T) \) theory of gravity. To make it clear firstly we discuss the conservation equation in the framework of general relativity.

The Einstein field equation with cosmological constant \( (\Lambda) \) is read as

\[
R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = 8\pi T_{ij}. \tag{11}
\]

The field equations for metric (6) are written as

\[
3 \frac{\dot{a}^2}{a^2} = 8\pi \rho + \Lambda = \rho_{\text{eff}}, \tag{12}
\]

\[
2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi p + \Lambda = -p_{\text{eff}}. \tag{13}
\]

where \( \rho_{\text{eff}} = 8\pi \rho + \Lambda \) and \( p_{\text{eff}} = 8\pi p - \Lambda \). Therefore, the effective equation of state parameter is read as

\[
\omega_{\text{eff}} = \frac{\rho_{\text{eff}}}{p_{\text{eff}}} = \frac{8\pi \rho - \Lambda}{8\pi \rho + \Lambda}. \tag{14}
\]

In the absence of matter, \( \rho = 0 \) and \( p = 0 \), the effective equation of state parameter is equal to \( \omega_{\text{eff}} = -1 \). This represents the standard \( \Lambda CDM \) model of the Universe.

Differentiating Eq. (12) with respect to \( t \) and combining the resulting equation with Eq. (13), we obtain the energy conservation equation as follows

\[
\rho_{\text{eff}}' + 3(\rho_{\text{eff}} + p_{\text{eff}}) \dot{H} = 0. \tag{15}
\]

which implies \( d(\rho_{\text{eff}} V) = -p_{\text{eff}} dV \). Note that \( V = a^3 \), the volume of the Universe and the quantity \( \rho_{\text{eff}} V \) represent the total energy of the Universe. As the Universe expands, the amount of dark energy in an expanding Universe increases in proportion to the volume. While in \( f(R, T) \) theory of gravity, one may get a different picture because in \( f(R, T) = R + 2\xi T \) gravity, \( \nabla' T_{ij} \neq 0 \). [29]

\[
\nabla' T_{ij} = \frac{f_T(R, T)}{8\pi - f_T(R, T)} [(T_{ij} + \Theta_{ij}) \nabla' \ln f_T(R, T) + \nabla' \Theta_{ij}] . \tag{16}
\]

where \( f_T(R, T) = \frac{\dot{f}(R, T)}{\dot{f}} \) and \( \Theta_{ij} = g^{ij} \frac{\partial f}{\partial T} \).

In case of \( f(R, T) = R + 2\xi T \), Eq. (16) reduces to

\[
\nabla' T_{ij} = \frac{2\xi}{8\pi - 2\xi} [(T_{ij} + \Theta_{ij}) \nabla' \ln f_T(R, T) + \nabla' \Theta_{ij}] . \tag{17}
\]

Furthermore, we note that for \( \xi = 0 \), we obtain \( \nabla' T_{ij} = 0 \), but in \( f(R, T) = R + 2\xi T \), one can not choose \( \xi = 0 \) because \( \xi = 0 \) reduces \( f(R, T) = R + 2\xi T \) theory into general relativity case. However, for \( \xi \neq 0 \), the conservation equation does not hold. Some sensible researches for violation of conservation law in \( f(R, T) \) theory are given in Refs. [44, 45]. Furthermore, in Refs. [29, 66], it has been explained that the problem of perfect fluids, described by energy density \( \rho \), pressure \( p \), and four-velocity \( u' \) is more complicated and there is no unique definition of matter Lagrangian. Therefore, one may assume that the stress-energy tensor of the matter is given in Eq. (3). Contrary to the above, in Ref. [67], the authors have used Eq. (15), i.e.,

\[
\frac{\partial \rho_T}{\partial t} + 3H(\rho_T + p_T) = 0 \text{ which can neither be obtain from Eqs. (11) and (12) nor it is true for } f(R, T) \text{ theory of gravity. It is worthwhile to note that the physics behind the existence of the conservation equation (15) as well as choosing the negative values of } \beta = \frac{8\pi + 3\xi}{8\pi + 2\xi} \text{ in Dixit et al. [67] are not elaborated. This raises a serious question on the method and approaches used for constructing RHDE models in } f(R, T) \text{ gravity. The authors of Ref. [67] have analyzed the cosmological features of RHDE models through a conservation equation which is not compatible with the basic field equations of the model. Also, we notice that, in Ref. [69], the similar approach has been used for analyzing the k-essence and dilation field models inspired by THDE in } f(R, T) \text{ gravity which also need some moderation to establish THDE models in modified theories of gravity. However, it is worthwhile to note that the correct conservation law in the model under consideration may obtain as follows.}

Equations (7) and (8) are recast as

\[
3H^2 = (8\pi + 3\xi) \rho_T, \tag{18}
\]

\[
2\dot{H} + 3H^2 = \xi \rho_T. \tag{19}
\]

Differentiating Eq. (18) with respect to time, one may express the resulting answer as a linear combination of Eqs. (18) and (19).

Therefore,

\[
\dot{\rho}_T + 3H \left( \frac{8\pi + 2\xi}{8\pi + 3\xi} \right) \rho_T = 0. \tag{20}
\]
while in Dixit et al. [67], the authors have used $\rho_T + 3H(\rho_T + p_T) = 0$, the energy conservation equation of general relativity.

4. Conclusions

In this paper, we have analyzed that the gravitational field equations are not consistent with the energy conservation equation in the framework of $f(R, T) = R + 2\xi T$ gravity. Therefore, the method and approach given in Ref. [67] represent a fractured way for analyzing RHDE models in $f(R, T)$. However, one can use this method to analyze some features of RHDE models in general theory of relativity. Further, we investigate that the dynamics of the deceleration parameter does not favor the result displayed in Dixit et al. [67] for the particular range of $\beta$.

The deceleration parameter is obtained as

$$ q = -\frac{\dot{a}a}{a^2} = -1 - \frac{\dot{H}}{H^2}. $$

Equations (9) and (25) lead to

$$ q = -1 + \frac{3}{2\beta}. $$

Since we have chosen $\beta = \frac{8\pi + 3\xi}{8\pi + 2\xi}$ with $\xi$ as a constant. The negative value of $\beta$ is possible only when $8\pi + 3\xi < 0$. But from Eq. (7), we obtain that $8\pi + 3\xi < 0$ leads to the negative energy density $\rho_T$ and hence a non-physical case. Thus we have omitted the negative values of $\beta$ for describing the behavior of the Universe in the derived model. Figure 1 depicts the plot of $q$ for various numerical values of $\beta$ in the range $[0, 10]$. From Eq. (22) and Fig. 1, we observe that for $\beta = 0$, $q = \infty$. Furthermore, we observe that for $0 < \beta \leq 1.5$, the Universe in the derived model is in decelerating mode of expansion while in Dixit et al. [67], it has been classified as quintessence model of accelerating Universe for the same range of $\beta$ (see particular II of table 1 and 2 of Ref. [67]).

It is customary to note that neither we avoid the possible existence of RHDE cosmological models in $f(R, T)$ theory of gravity nor we decline the viability of $f(R, T)$ gravity with special type dark energy density. But the method and technique for obtaining the cosmological solution of RHDE models in $f(R, T)$ gravity, as given in Dixit et al [67] is not correct. However, the cosmological implications of $f(R, T)$ gravity to describe the late time acceleration of the Universe may be investigated in contrast with topics, like, spherical relativistic vacuum core models in a $\Lambda$-dominated era, Stellar filaments with Minkowskian core in the Einstein-$\Lambda$ gravity. One may define cosmological constant $\Lambda$ as a function of trace $T$, i.e., $\Lambda = \Lambda(T) = \xi(T + 2p)$ [70, 71]. In Shabani and Zaiae [72], the authors have given a clue that the non-conservation of energy from the thermodynamic point of view implies an irreversible matter creation process and such matter/particle creation corresponds to energy flow from the gravitational field to the created matter particles. Also, Shabani and Zaiae [73] have investigated late time solution of the cosmological model of type $f(R, T) = R + \Lambda(T)$ with $\Lambda(T)$ as trace dependent cosmological constant. The present work could be also extended to find the energy of the particles in the modified gravitational field coming from Einstein-$\Lambda$ gravity. The reconstruction of RHDE models in $f(R, T)$ gravity is in progress and further results on this topic will be reported in forthcoming researches. Furthermore, it is worthwhile to mention that the RHDE case under discussion is just a particular version of the generalized holographic dark energy case (see Refs. [2, 74, 75]).

Appendix

The Einstein field equation with RHDE fluid is read as

$$ R_{ij} - \frac{1}{2} g_{ij} R = 8\pi \left( T^m_{ij} + T^R_{ij} \right). $$

where $T^m_{ij}$ and $T^R_{ij}$ are energy momentum tensor of matter and RHDE fluid, respectively.

Thus, the field equations for metric (6) are read as

$$ 3H^2 = 8\pi (\rho + \rho_T), $$

$$ 2\dot{H} + 3H^2 = -8\pi (p + p_T). $$

where $\rho$, $p$, $\rho_T$, and $p_T$ denote energy density of matter, the pressure of the matter, the energy density of RHDE fluid and pressure of RHDE fluid, respectively.

![Fig. 1 The plot of q for numerical values of \beta](image)
Differentiating Eq. (25) with respect to time and combining the resulting equation with Eq. (22), we obtain the following energy conservation equation

$$\dot{\rho} + 3(\rho + p)H + \dot{\rho}_T + 3(\rho_T + \rho_T)H = 0. \quad (26)$$

Moreover, the field equations for metric (6) in the framework of $f(R, T) = R + 2f(T)$ theory of gravity are read as

$$3H^2 = (8\pi + 2f_T)\rho + 2pf_T + f(T), \quad (27)$$

$$2H + 3H^2 = -8\pi p + f(T). \quad (28)$$

From Eqs. (27) and (28), we can pick out a dark energy component due to $f(T)$, described by

$$\rho_T = 2f_T \rho + 2pf_T + f(T), \quad (29)$$

$$\rho_T = -f(T). \quad (30)$$

It is worthwhile to note that one can reconstruct $f(R, T)$ gravity from RHDE model by defining RHDE density in IR cutoff as $\rho_T = \frac{3c_1^2}{8\pi L^2(1+n_0 H)}$ with $c_1$ being a numerical constant. The parameters $\delta$ and $L$ denote a non-additive parameter and length of horizon, respectively.

Differentiating Eq. (27) with respect to time and combining the resulting equation with Eq. (28), we obtain

$$(8\pi + 2f_T)\dot{\rho} + 3[(8\pi + 2f_T)\rho + 8\pi p]H + 2(\rho + p)f_T + (2\dot{\rho} + 6pH + 1)f_T = 0. \quad (31)$$

Since $f(T) = \hat{\zeta}T$ which leads $f_T = \frac{2\hat{\zeta}}{\hat{\zeta}T} = \hat{\zeta}$ and $\dot{f}_T = 0$, therefore, Eq. (31) leads to

$$(8\pi + 2\hat{\zeta})\dot{\rho} + 3[(8\pi + 2\hat{\zeta})\rho + 8\pi p]H + (2\dot{\rho} + 6pH + 1)\hat{\zeta} = 0. \quad (32)$$

From Eq. (32), it is clear that for $\hat{\zeta} = 0$, Eq. (32) converts into the conservation equation of general relativity case as we have discussed in Sect. 3.

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