Abstract

The escape panic version of the Social Force Model (SFM) is a suitable model for describing emergency evacuations. In this research, we analyze a real-life video, recorded at the opening of a store during a Black Friday event, which resembles an emergency evacuation (November 2017, South Africa). We measure the flow of pedestrians entering the store and found a higher value ($\langle J \rangle = 6.7 \pm 0.8 \text{ p/s}$) than the usually reported in “laboratory” conditions. We performed numerical simulations to recreate this event. The empirical measurements were compared against simulated evacuation curves corresponding to different sets of parameters currently in use in the literature. The results obtained suggest that the set of parameters corresponding to calibrations from laboratory experiments (involving pedestrians in which the safety of the participants is of major concern) or situations where the physical contact is negligible, produce simulations in which the agents evacuate faster than in the empirical scenario. To conclude the paper, we optimize two parameters of the model: the friction coefficient $k_t$ and the body force coefficient $k_n$. The best fit we found could replicate the qualitative and quantitative behavior of the empirical evacuation curve. We also found that many different combinations in the parameter space can produce similar results in terms of the goodness of fit.

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1. Introduction

In the last years, a growing interest appeared in the research of pedestrian dynamics and emergency evacuations. The increasing number of tragedies in night clubs [1, 2], sports events [3, 4], and mass gatherings in general [5, 6], has concerned many people around the world. The urgency for understanding and preventing these tragedies has called the attention of researchers from different disciplines.

There are three main approaches to study evacuations and pedestrian dynamics in general: the theoretical approach [7, 8, 10], the experimental approach (i.e., performing laboratory experiments) [11, 12, 13], and the empirical approach (i.e., analyzing real-life data) [15, 16].

The emergency evacuation in a panic state is a very complex phenomenon that requires the three approaches to be properly addressed. Each of these approaches has its strengths and limitations. The laboratory experiments (experimental approach) allow the researcher to have control over the analyzed situation while recreating some of the traits that occur in a real-life scenario. However, doing experiments that truly reproduce emergency evacuations is impossible due to safety reasons. The analysis of real-life situations (empirical approach) gives the researcher the possibility of studying the system with a closer degree of realism. However, the limitation of this approach is that there are not many documented reports on real-life emergency evacuations yet.

The theoretical approach is the most common one. This approach does not have to deal with the difficulties of the experimental and empirical approaches. Nevertheless, the theoretical models may produce unrealistic results if they are not validated or adequately calibrated.

Many pedestrian dynamics models were proposed in the last decades (see
There are two main groups of pedestrian models: the discrete models [10, 9] in which space and time are discretized and the continuous models [18] which consider space and time as continuous variables. The Social Force Model [7, 18] belongs to the group of continuous models and it is one of the most developed models in pedestrian dynamics. Since its first version, there have been many revisions, extensions, and improvements to solve multiple problems that go beyond the emergency evacuations [14, 27].

In this paper, we analyze a real-life video (empirical approach) of a Black Friday event where pedestrians rush to enter into a Nike store. The video was specially selected because the crowd behavior resembles an emergency evacuation. We also perform numerical simulations (theoretical approach) to recreate the results of the empirical scenario. We are only focusing on the escape panic version of the social force model [18] because this version is specially oriented to reproduce emergency evacuations.

The paper is organized as follows. In the following section, we describe the forces and the proposed set of parameters that involve the escape panic version of the social force model. In section 2, we mention the research scope and clarify some concepts that will be useful throughout the paper. In section 3, we provide details of the empirical measurements and the numerical simulations that we performed. Section 5 shows the results from both the empirical and numerical analyses. In the last part, we summarize the main conclusions of the work.

2. The escape panic version of the Social Force Model

2.1. The forces of the model

The escape panic version of the social force model came up for the first time in the year 2000 [18]. Unlike other versions [7, 38], this one reduces the force description of the pedestrian evacuations to just a small number of parameters.

The dynamic of the simulation is determined by an equation of motion involving both socio-psychological forces and physical forces. The equation
of motion for any pedestrian $i$ (of mass $m_i$) reads

$$m_i \frac{dv_i}{dt} = f_d^{(i)} + \sum_{j=1}^{N} f_s^{(ij)} + \sum_{j=1}^{N} f_p^{(ij)}$$  \hspace{1cm} (1)$$

where the subscript $j$ corresponds to the neighboring pedestrians or the surrounding walls. The desired force $f_d$ represents the acceleration (or deceleration) of the pedestrian due to his (her) own will. The social force $f_s$, instead, describes the tendency of the pedestrians to stay away from each other. The physical force $f_p$ stands for both the sliding friction and the body force repulsion. These forces are essential for understanding the particular effects of panicking crowds.

The pedestrians’ own will is modeled by the desired force $f_d$. This force stands for the acceleration (deceleration) required to move at the desired velocity $v_d$. This parameter is associated with the anxiety level to reach a specific target. The parameter $\tau$ is a characteristic time scale that reflects the reaction time. Thus, the desired force is modeled as follows

$$f_d^{(i)} = m \frac{v_d^{(i)} \hat{v}_d^{(i)}(t) - v^{(i)}(t)}{\tau}$$  \hspace{1cm} (2)$$

where $\hat{v}(t)$ represents the unit vector pointing to the target position and $v(t)$ stands for the pedestrian velocity at time $t$.

The tendency of any individual to preserve his (her) “private sphere” is accomplished by the social force $f_s$. Therefore, $f_s$ reflects the psychological tendency of two pedestrians to stay away from each other. The mathematical expression to model this kind of “socio-psychological” behavior is as follows

$$f_s^{(i)} = A e^{(R_{ij} - r_{ij})/B} \hat{n}_{ij}$$  \hspace{1cm} (3)$$

where $r_{ij}$ means the distance between the center of mass of the pedestrians $i$ and $j$, and $R_{ij} = R_i + R_j$ is the sum of the pedestrian’s radius. The unit vector $\hat{n}_{ij}$ points from pedestrian $j$ to pedestrian $i$, meaning a repulsive interaction.

The parameter $B$ is a characteristic scale that plays the role of a fall-off length within the social repulsion. Besides, the parameter $A$ represents the
intensity of the social repulsion.

The expression for the physical force (say, the sliding friction plus the body force) has been borrowed from the granular matter field; the mathematical expression reads as follows

\[ f_p^{(ij)} = k_t g(R_{ij} - r_{ij}) (\Delta v^{(ij)} \cdot \hat{t}_{ij}) \hat{t}_{ij} + k_n g(R_{ij} - r_{ij}) \hat{n}_{ij} \]  

(4)

where \( g(R_{ij} - r_{ij}) \) equals \( R_{ij} - r_{ij} \) if \( R_{ij} > r_{ij} \) and vanishes otherwise. \( \Delta v^{(ij)} \cdot \hat{t}_{ij} \) represents the relative tangential velocities of the sliding bodies (or between the individual and the walls). Further details can be found in Ref. [18].

The sliding friction occurs in the tangential direction \( \hat{t}_{ij} \) while the body force occurs in the normal direction \( \hat{n}_{ij} \). Both forces are assumed to be linear with respect to the net distance between contacting pedestrians (overlap). The sliding friction is also linearly related to the difference between the (tangential) velocities. The parameter \( k_t \) is a friction coefficient, while the parameter \( k_n \) is a stiffness coefficient (analogous to the constant factor in the Hooke’s law). For simplicity, we will omit the units of the parameters \( k_n \) and \( k_t \). Remember that the friction coefficient has units \([k_t] = \text{Kg m}^{-1} \text{s}^{-1}\) and the body stiffness coefficient \([k_n] = \text{Kg s}^{-2}\).

2.2. The proposed sets of parameters for the model

The escape panic version of the SFM involves 8 parameters: \( m_i, R_i, \tau, v_d, A, B, k_n \) and \( k_t \). The original version of the model fixes the mass \( m_i = 80 \text{ kg} \), \( R_i \) was uniformly distributed in the interval \([0.25 \text{ m}, 0.35 \text{ m}]\), to represent average soccer fans. The parameters \( A = 2000 \text{ N}, B = 0.08 \text{ m} \) take these values to reproduce the distance kept at normal desired velocities and the measured flows through bottlenecks [59]. \( \tau = 0.5 \text{ s} \) is said to be a reasonable estimate for the acceleration time. Regarding the parameters of the physical forces, \( k_n = 1.2 \times 10^5 \) and \( k_t = 2.4 \times 10^5 \), Ref. [18] does not express reasons for choosing these values, but \( k_n \) may be related to the research in Ref. [28]. Although this set of parameters has been used in many research studies [29, 30, 31, 32, 33, 34, 49], there are other sets of parameters that have also been proposed to fit specific situations.
In Ref. [22], Li et al. calibrate the parameters $A, v_d, k_t, k_n$ from a real-life video analysis. The situation studied was an emergency evacuation of a classroom during the 2013 Ya'an earthquake in China [62]. They use a Differential Evolution algorithm to fit the numerical simulation results to the empirical evacuation curve (cumulative number of evacuees vs. time).

In the research conducted by Haghani et al. [21], the authors perform a sensitivity analysis where they find that the parameters $\tau$ and $k_t$ are the most sensitive (the simulation’s output is strongly dependent on the value of these parameters). They also perform a laboratory experiment where participants are asked to evacuate a place through a narrow door. They measured the evacuation time for 3 rushing levels of the participants and 6 different door widths (varying from 60 cm to 120 cm). Afterward, they successfully calibrated the most sensitive parameters $\tau$ and $k_t$ to fit the experimental data.

In Ref. [23], Lee et al. perform an evacuation laboratory experiment varying the proportion of rush/no-rush participants. They used the Wasserstein distance metric to obtain the parameters $A$ and $B$ that best fitted the experimental results. They found many different sets of values $A$ and $B$ depending on the proportion of rush/no-rush participants. For the purpose of this work, we only selected the parameter values corresponding to 100% rush individuals.

The work of Frank et al. uses a subtle modification of the parameters of the model. They omit the body force ($k_n = 0$) while fixing the other parameters with the same values proposed by Helbing et al. [18]. The subject of study of this research is the emergency evacuations in the presence of obstacles [24].

In the research of Tang et al. [20], the authors perform pedestrian tracking in a Beijing metro station. They use a regression approach based on the least square method to fit the parameters $A$ and $B$. They obtain two sets of values depending on whether pedestrians are in an “approach” situation or a “leave” situation [20]. The first one denotes pedestrians getting close to each other while the latter means pedestrians detaching from each other. We are only interested in the “leave” approach since we only consider repulsive interaction between pedestrians.
| Authors            | $A$ (N) | $B$ (m) | $k_n$ (Kg s$^{-2}$) | $k_t$ (Kg m$^{-1}$ s$^{-1}$) | $\tau$ (s) | Refs. |
|-------------------|--------|--------|---------------------|-----------------------------|----------|-------|
| Helbing et al.    | 2000   | 0.08   | $1.2 \times 10^5$   | $2.4 \times 10^5$           | 0.50     | [18]  |
| Li et al.         | 998    | 0.08   | 819                 | 510                         | 0.50     | [22]  |
| Haghani et al.    | 2000*  | 0.08*  | $1.2 \times 10^5*$  | 5500                        | 0.12     | [21]  |
| Lee et al.        | 2600   | 0.012  | 750                 | 3000                        | 0.50     | [23]  |
| Frank et al.      | 2000   | 0.08   | 0                   | $2.4 \times 10^5$           | 0.50     | [24]  |
| Tang et al.       | 729**  | 0.10   | $1.2 \times 10^5*$  | $2.4 \times 10^5*$          | 0.60     | [20]  |
| Sticco et al.     | 2000   | 0.08   | $1.2 \times 10^5$   | $1.2 \times 10^6$          | 0.50     | [19]  |

Table 1: Sets of parameters selected for this research. Each row corresponds to a different set proposed/used in the literature. The cells with an asterisk * correspond to values that were completed with the Helbing’s proposed values. The cell with ** is actually $9.18 \times m$ with $m = 79.5$. 

In the research of Sticco et al. [19], The authors explore the role of the body force, the friction force, and their dynamical consequences in bottle-necks and corridors. They increased the parameter $k_t$ with respect to the original version of the SFM. The increment was done in order to reproduce the qualitative behavior of the fundamental diagram obtained from the measurements performed in Ref. [60]. In the research from Ref. [60], the authors analyze a video recording of the massive muslim pilgrimage at the entrance of the Jamaraat bridge.

We list in Table 1 the sets of parameters that were selected for this research. Notice that these correspond to the set of parameters proposed/used in the works mentioned above. The list of parameters explicitly excludes the mass, radius, and desired velocity because we will set those parameters according to a different criterion from the ones proposed by the authors. In this work, we set the mass value $m = 79.5$ kg. For the pedestrian radius, we assigned two gaussian distributions: $\mathcal{N}(37.7 \text{ cm}, 0.09 \text{ cm})$ for females and $\mathcal{N}(41.8 \text{ cm}, 0.1 \text{ cm})$ for males. See section 4.2 for the details.

There are many similar works that we did not select because they calibrate other versions of the model (different from the escape panic version) [35, 36, 37, 38, 39, 40, 41, 42, 58]. The one in Ref. [43] studies the interval of parameters $A$ and $B$ that avoid oscillations in the model. This paper was excluded because the author explicitly avoids the bumping (and overlapping) in his theoretical derivation. This could be interpreted as an
improvement of the model in the low-density limit (which is not the case for high-densities reported in emergency evacuations). The novel work of Bode [44] calibrates the escape panic version of the SFM using the experimental data from Ref. [45]. Nevertheless, this paper does not aim to provide a set of parameters but to promote a robust method for model calibration.

3. Scope of this research and clarifications

This investigation has two main goals. The first goal is to present the results of a novel empirical measurement from a real-life video analysis. The second goal is to compare the empirical measurements with numerical simulations. We will mention the strengths and weaknesses of the sets of parameters proposed by different authors across the literature. We do not pretend to provide an exhaustive description of all the phenomena presented here, but to explain the more salient features. We stress that this investigation is a call to set the SFM parameters with caution, rather than offering updated values for these parameters. We realize that more empirical data is required before we can assess definite estimates on this matter.

We based the research on the hypotheses that the video analyzed resembles an emergency evacuation. For simplicity reasons, we will refer to the rush to enter the Nike store as an evacuation process (although in evacuations, pedestrians feel the urge to escape from a particular place rather than being urged to enter into a building).

To simplify the analysis, we set the same parameter values to all the simulated agents (except for the pedestrian’s diameter). We are aware that the reality is more complex, but we decided to leave this kind of analysis to future research.

In section 5.2 we optimize two parameters of the model ($k_n$ and $k_t$). We decided to focus on these two parameters because they are critical in high-density scenarios. The optimization of parameters like $A$ and $B$ (affecting the social repulsion) can be achieved in low-density situations. Whereas the optimization of $k_n$ and $k_t$ requires physical contact among pedestrians (like in the analyzed video). No further questioning will be done on the mutual relation between parameters in the reduced-in-units context of the SFM. This
is out of the scope of the investigation.

4. Empirical measurements and numerical simulations

In this section we describe the process undertaken to accomplish the empirical measurements. We also explain the details of the numerical simulations that we performed.

4.1. Empirical measurements

The core of this work is the analysis obtained from a human rush event video. The video is available on YouTube [48] and shows a crowd of pedestrians rushing to enter a Nike store during a Black Friday event. The event took place at East Point Shopping Centre in the Ekurhuleni municipality (South Africa).

Fig. 1a is a snapshot of the first frame of the video. It shows a crowd of pedestrians waiting for the security personnel to allow them to get in. In the subsequent frames (see the video [48]), it is possible to see one person wearing a red t-shirt and white cap that manages to sneak into the store. Immediately after this, the crowd starts pushing towards the door, the security personnel steps aside, and the crowd begins to enter massively. Only at the end of the video, the pedestrians start to enter in a non-rushing fashion.

The door’s capacity is exceeded by the large number of pedestrians trying to enter the store at the same time. The glass panel placed at the left side of the door is cracked as a result of the high pressures it supports. The pedestrians exhibit high competitive, aggressive, and non-cooperative behavior. We hypothesize that this situation could be similar to an emergency evacuation where the crowd is under panic, and self-interest may prevail over rational and collaborative behavior.

The video is composed by 1848 frames. We excluded the first 219 frames and the last 458. The first part was excluded because the crowd is waiting to enter the store. In a similar way, the last part was excluded because the crowd starts to enter in a much orderly fashion, and no competitive behavior is observed. The analysis was done from the frame 220 to the frame
1419 (including the border values). This is 1200 frames, equivalent to 40 s of recording time (since the video frequency is 30 frames/s).

The selected 1200 frames correspond to the part of the video where the most competitive behavior was observed. Recall that we are only interested in scenarios like this, because the escape panic version of the SFM aims to reproduce emergency evacuations. The selected 1200 frames were divided into “segments” of 60 frames each (equivalent to a sampling period of $\tau_{\text{samp}} = 2$ s). The criteria for choosing this sampling period was to be large enough to count a significant number of pedestrians who enter the store but, to be small enough to achieve a reasonable amount of measurements.

Once we divided the selected 1200 frames into 20 segments of 60 frames each, we proceed to count the number of ingressed pedestrians in each of the 20 segments.

The video’s quality is good enough to distinguish the pedestrians involved in the event. Therefore we did the counting manually frame-by-frame. No pedestrian tracking software was employed to achieve these measurements. In each frame, we draw a straight line that divided the inside of the store from the outside. As the video recording is almost steady, we only moved that dividing line once. Then, we counted how many pedestrians crossed the line in each segment of the video.

Once we counted how many pedestrians entered the store in each segment, we were able to calculate the mean flow (averaging) and the cumulative number of evacuees vs. time (evacuation curve).

4.2. Numerical simulations

Since the escape panic version of the SFM aims to simulate emergency evacuations, we decided to recreate the conditions of the empirical measurements to test the SFM and different sets of parameters proposed in the literature.

The SFM was implemented on the LAMMPS simulation software. Additional modules for LAMMPS were written in C++ to expand the software capabilities. To recreate the Black Friday event conditions, we set a
door size of width \( w = 1.6 \) m. We set this value since it is almost equivalent to twice the size of a South African average door [61] (notice that the entrance of the store is a two-leaf door). Another criteria that made us choose \( w = 1.6 \) m is that the door is almost equivalent to 4 pedestrians’ width (approximately 1.6 m).

The total number of pedestrians was fixed as \( N = 303 \). The number \( N \) is defined as \( N = n + m \). Where \( n = 268 \) is the total number of pedestrians that entered the store, while \( m = 35 \) is the number of pedestrians that appear close to the door in the last analyzed frame. These \( m \) pedestrians get inside the store a few frames after the last analyzed frame.

We set the simulated agents’ diameter following the Ref. [26]. We used the shoulder width (biacromial breadth) of adults corresponding to the ethnic group of “non-Hispanic black”. In the video, it is possible to observe that there is roughly an equal proportion of males and females. Therefore, we set half of the simulated agents as females and the other half as males. For each half, we set a gaussian distribution of shoulder widths. For males we assigned \( \mathcal{N}(41.8 \text{ cm},0.1 \text{ cm}) \) and for females we assigned \( \mathcal{N}(37.7 \text{ cm},0.09 \text{ cm}) \) [26].

We set a mass of \( m = 79.5 \) kg to all the simulated agents. This value is the average weight of men and women reported in Ref. [26]. We tested whether variations in the mass value and diameter value could change the results. We found that the model is robust to mild variations in mass and diameter.

The initial conditions are configurations of agents that are located around the door (to reproduce the first frame of the video). To create these initial configurations, the \( N \) agents were placed in a square enclosure with random positions and speeds. Then they were made to go to the door area but with the door closed (so that they do not leave the enclosure). After 20 s the individuals form a semicircle around the door. That configuration is saved as an initial condition. We repeated this process 50 times, varying the initial positions and speeds of the agents.

In each simulation process, we recorded the number of agents that entered the store every 0.05 s. The simulation process finished when \( n = 268 \) simulated agents entered the store (which is the total number of ingressed
pedestrians in the empirical measurements). We explored the interval of desired velocities $2 \, \text{m/s} \leq v_d \leq 5 \, \text{m/s}$. This interval corresponds to anxious individuals (but not extremely fast runners).

We use different sets of parameters to perform numerical simulations. The sets of parameters that we used are shown in the Table I. For a given set of parameters and a given desired velocity, we run 50 iterations varying the initial conditions. The Eq. (1) was numerically integrated employing the velocity Verlet algorithm, with a timestep value $\Delta t = 10^{-4} \, \text{s}$.

![First frame of the analyzed video. A big crowd of pedestrians is close to the entrance of the store. The crowd is waiting for the security personnel to open the door.](image1.png)

![Snapshot of the performed numerical simulations. The numerical simulations recreate the real-life situation from the video.](image2.png)

5. Results

We present in this section the main results of our investigation. The section is divided into two parts. In the first part (section 5.1), we compare the empirical measurements with numerical simulations by using different sets of parameter available in the literature. In the second part (section 5.2), we present the results of the parameter optimization by means of a genetic algorithm.
5.1. Testing

In this section, we present and discuss our empirical results, and we further compare the numerical simulations with these empirical measurements.

In the previous section, we described the video analysis procedure to obtain the empirical measurements. Using this data, we were able to plot the empirical evacuation curve, which is the number of people entering the store as a function of time (cumulative number of evacuees vs. time).

The empirical evacuation curve is a monotonically increasing function (see the black dotted line in Fig. 2). There are no time intervals in which the empirical evacuation process is completely stopped, at least for the sampling frequency employed in this research ($\tau_{\text{samp}} = 2$ s). We may speculate that a lower $\tau_{\text{samp}}$ would capture the stop-and-go mechanism produced by the blocking clusters close to the door [50].

We used the empirical measurements to calculate the mean flow of pedestrians entering the store $\langle J \rangle = 6.7 \pm 0.8$ p/s. The flow value is higher than other experimental measurements at bottlenecks where individuals are not allowed to harm one another [52, 53, 54, 55, 56, 57, 13, 63, 64].

The specific flow is defined as the flow scaled by the door size (say, $\langle J \rangle / w$). This quantity is used to compare evacuation processes with different door dimensions (to a certain extent). The mean specific flow corresponding to our measurement is $\langle J_s \rangle = 4.1 \pm 0.5$ p/m.s (assuming a door width of 1.6 m). To our knowledge, only the specific flow reported in Refs. [63, 56] is comparable to ours. However, the specific flow from Refs. [63, 56] corresponds to door sizes much narrower ($w \leq 0.75$ m) than the estimated door size of the Nike store ($w = 1.6$ m). It is worth mentioning that some researches have reported that narrower doors produce a moderate increment in the specific flow [53] while other researches concluded the opposite result [63].

Fig. 2 shows the cumulative number of pedestrians that entered the store as a function of the time (for both empirical measurements and numerical simulations). Fig. 2 is a collection of four figures, all of them include the empirical result (shown as a black dotted line) and the results of the numerical simulations (shown as colored curves). Each of these curves is associated
with different sets of parameters (corresponding to the selected papers mentioned in Table 1).

Each figure from Fig. 2 is associated to a different desired velocity in the interval $2 \text{ m/s} \leq v_d \leq 5 \text{ m/s}$. The numerical results of Fig. 2a and Fig. 2b correspond to $v_d = 2 \text{ m/s}$ and $v_d = 3 \text{ m/s}$, while the numerical results of Fig. 2c and Fig. 2d correspond to $v_d = 4 \text{ m/s}$ and $v_d = 5 \text{ m/s}$, respectively. Recall that we assign the same desired velocity to every simulated agent from the beginning of the simulation until the simulated agent manages to enter the store.

The evacuation time $t_e$ is defined as the time for which all the pedestrians evacuated the place (for example, the evacuation time for the empirical measurements is $t_e = 40 \text{ s}$). The Faster-is-Slower is a phenomenon that occurs when increasing the desired velocity $v_d$ reduces the evacuation time. The Faster-is-Faster is the opposite effect; this means that the higher the value of $v_d$, the lower the evacuation time. Within this framework, we can distinguish two categories of evacuation curves: the curves that exhibit Faster-is-Faster (Lee, Li, Haghani) and the curves that exhibit Faster-is-Slower (Sticco, Frank, Tang). The curves corresponding to the parameters proposed by Helbing et al. do not display any of these effects significantly.

Within the three sets of parameters that exhibit Faster-is-Slower, the most noticeable curves are the corresponding to the parameter set proposed by Sticco et al. Recall that this parameter set is similar to Helbing’s but with the friction coefficient increased by a factor of five. This correction was suggested in order to reproduce the qualitative behavior of the fundamental diagram at the entrance of the Jamaraat bridge [19]. The evacuation curve shown in Fig. 2b (for $v_d = 2 \text{ m/s}$), seems to attain a good agreement with the empirical measurements until time $t \approx 20 \text{ s}$. Above this time, the simulated curve tends to increase faster than the empirical curve. For higher desired velocities ($v_d \geq 3 \text{ m/s}$), the parameters proposed by Sticco et al. produce very large evacuation times due to the high friction value.

The parameter sets proposed by Frank et al. and Tang et al. produce a mild Faster-is-Slower effect. They seem to be the parameter sets that produce the best agreement with this empirical data. Notice that the evacuation times (i.e., the time corresponding to the last measurement) surpass the evac-
uation time from Helbing et al. (for any of the desired velocity explored). In
the case of Frank et al., this difference can be explained because the body
force is not considered ($k_n = 0$), this produces a more significant overlap be-
tween pedestrians, which ultimately leads to an increment in the friction [19].

In the case of the parameters proposed by Tang et al., it is more diffic ult
to compare them with Helbing’s because three parameters are modified ($A$, $B$, and $\tau$). Nevertheless, the value of $A$ is reduced and the value of $B$ is
increased. These changes tend to diminish the social force repulsion, which
presumably leads to an effective increment of the overlap (and the friction
force). This is analogous to the $k_n$ reduction stated by Frank et al.

As we mentioned before, the parameters proposed by Haghani, Lee and
Li yield Faster-is-Faster effect. The three of them produce evacuation times
below the empirical measurements (and also below the evacuation time corre-
spending to Helbing). This result holds true for any of the explored desired
velocities.

The set of parameters proposed by Haghani et al. and Lee et al. were
able to reproduce the experimental results in laboratory conditions where
pedestrians are not allowed to have aggressive behavior against each other.
The Black Firday event (empirical condition) reported in this paper involves
highly aggressive and competitive behavior (see the video from Ref. [48]).
This discrepancy between laboratory and empirical conditions seems to be
the reason why the sets of parameters proposed in Refs. [21, 23] do not pro-
duce evacuation curves in agreement with the empirical data analyzed in this
paper.

The set of parameters proposed by Li et al. was calibrated using a real-life
(empirical) video analysis. The situation analyzed is an emergency evacua-
tion of a classroom due to the 2013 Ya'an earthquake in China. In the
video [62], it is possible to observe that students rush to evacuate the class-
room. Despite this, the physical contact between the students involved is
almost negligible. This particular trait of the Ya'an earthquake evacuation
may have underestimated the value of the friction coefficient $k_t$, and conse-
quently, the expected impact that $k_t$ could have in a high-density situation
such as the Black Friday event analyzed in this work.
We may summarize this Section as follows. We reported the evacuation curve of the Black Friday event and measured the flow $\langle J \rangle = 6.7 \pm 0.8 \text{ p/s}$ (and estimated the specific flow $\langle J_s \rangle = 4.1 \pm 0.5 \text{ p/m.s}$). This is a higher value than the flow reported under laboratory conditions throughout the literature. We performed numerical simulations to reproduce the Black Friday event’s conditions using different sets of parameters available in the literature. We divided the results into two categories (parameter sets that yield Faster-is-Slower pattern and parameter sets that produce Faster-is-Faster pattern). The parameter sets that yield Faster-is-Slower seem to have better agreement with the empirical curve (except for the parameter set of Sticco et al. in the interval $v_d \geq 3 \text{ m/s}$). The sets of parameters that yield Faster-is-Faster were calibrated from laboratory and empirical situations. In these situations, the pedestrians involved do exhibit neither aggressive behavior nor significant physical contact. This fact may have underestimated the value of the friction coefficient $k_t$, leading to evacuation curves with evacuation times lower than the empirical case.

5.2. Parameter optimization

In this section, we present the results of the parameter optimization. We perform an optimization of the parameters $k_n$ and $k_t$; these are the parameters associated with the physical forces: the body force and the friction. The optimization was done using the Differential Evolution (DE) algorithm, which is a technique that belongs to the family of genetic algorithms (see Ref. [46] for a detailed explanation of the algorithm).

The goal of DE is to optimize the parameters of the model in order to achieve an evacuation curve consistent with the empirical measurements obtained from the Black Friday video. The optimization process finished when the algorithm reached 100 generations. Since we aim to optimize two parameters, we set $2 \times 10 = 20$ different sets of random $(k_n, k_t)$ values following the recommendations of Storn [46]. The crossover probability and the differential weight were set as $CR = 0.3$ and $F = 0.5$, respectively.

The algorithm requires an objective function (a function to minimize),
Figure 2: (color on-line only) Cumulative number of evacuees vs. time (evacuation curve). The black dotted curve corresponds to the empirical measurements; the rest of the curves correspond to numerical simulation results associated to the parameters proposed/used by different authors (see the legend and Table 1). The simulation consisted of $N = 303$ agents and finished when $n = 268$ evacuated the place (to reproduce the empirical situation). The door’s width was $w = 1.6$ m. The mass ($m = 79.5$ kg) and shoulders width was set according to Ref. [26]. 50 iterations were run for each simulated curve, varying the initial conditions (the position and velocity of each agent). Each of the four figures corresponds to different desired velocities: (a),(b),(c),(d) correspond to $v_d = 2$ m/s, $v_d = 3$ m/s, $v_d = 4$ m/s and $v_d = 5$ m/s, respectively.
we defined this function as

\[ f(k_n, k_t) = \frac{1}{M} \sum_{i=1}^{M} |t^s_i(k_n, k_t) - t^e_i| \]  

where \( t^s_i(k_n, k_t) \) is a time value corresponding to the simulated evacuation curve, \( t^e_i \) is a time value corresponding to the empirical evacuation curve and \( M = 21 \) is the total number of measurements in the evacuation curve. The index \( i \) is associated with a fixed number of evacuees. For instance, \( i = 0 \) corresponds to zero evacuees, while \( i = 21 \) corresponds to 268 evacuees (both in the empirical curve and in the simulated curve). \( f(k_n, k_t) \) is a real number that expresses how much the simulation differs from the empirical measurement. Notice that this estimator avoids the usual over weighting of large deviations encountered in quadratic estimators.

The numerical simulations have the same characteristics as the simulations from the previous section (total number of evacuees, door size, shoulders width, etc.). The only two parameters that we aim to optimize are the friction coefficient \( k_t \) and the body force parameter \( k_n \). The other parameters were fixed to the values proposed by Helbing et al. [18] (except for the pedestrian mass and radius).

Given the values \((k_n, k_t)\), the algorithm calculates the mean evacuation curve over 50 iterations. The mean evacuation is then compared against the empirical evacuation curve (see Eq. (5)) to obtain a value \( f(k_n, k_t) \). Fig. 3 shows the fitness landscape, the horizontal axis stands for the parameter \( k_n \) and the vertical axis stands for the parameter \( k_t \). The color bar represents the value of the objective function \( f(k_n, k_t) \).

Fig. 3a shows the fitness landscape for pedestrians with a desired velocity \( v_d = 2 \text{ m/s} \) while Fig. 3b corresponds to pedestrians with \( v_d = 5 \text{ m/s} \). Both fitness landscapes show that the optimal values of the parameters \( k_n \) and \( k_t \) are correlated. The solid black line is a visual guide to see the correlation in the parameter space. In both plots, it is possible to observe that increasing the value of \( k_n \) requires increasing the parameter \( k_t \) to achieve a similar fitness value \( f(k_n, k_t) \).

The correlation can be explained because the friction force depends on the value of \( k_t \), but it also depends on the overlap between contacting pedes-
trians (see the first term in Eq. (4)). The value of $k_n$ affects the overlap value (which subsequently affects the friction). Many different combinations of the parameters may produce similar results (see Ref. [47] for a further discussion). The black solid line has a different slope depending on the value of $v_d$. Increasing $v_d$ reduces the slope because $v_d$ directly affects the overlap between pedestrians (the higher the desired velocity, the higher the overlap). In other words, if the desired velocity is higher, the friction coefficient $k_t$ needs to be lower in order to achieve the same results.

Figure 3: Fitness landscape for the parameters $k_n$ (horizontal axis) and $k_t$ (vertical axis). The rest of the parameters are the same as Helbing’s proposal in Ref. [18]. The mass ($m = 79.5$ kg) and shoulders width was set according to Ref. [26]. The color scale represents the value of the objective function $f(k_n, k_t)$ (the result of comparing the numerical simulation against the empirical data). The circular dots were produced by the DE algorithm to minimize $f(k_n, k_t)$. The diamond dot represents the parameters of Helbing et al. [18] (a) Corresponds to simulated agents with $v_d = 2$ m/s while (b) corresponds to simulated agents with $v_d = 5$ m/s.

Fig. 4 shows the empirical evacuation curve and the curve corresponding to the best set of parameters (red line) for $v_d = 5$ m/s. The best set of parameters was obtained with the DE algorithm. It corresponds to $f(k_n, k_t) = 0.57$ and parameters: $(k_n, k_t) = (0.036 \times 10^5, 3.05 \times 10^5)$. The friction coefficient is higher than the value corresponding to the Helbing’s original proposal (which is $k_t = 2.4 \times 10^5$). This result is consistent with the friction increment suggested in Ref. [47]. On the other hand, the body force coefficient is lower than the original value ($k_n = 1.2 \times 10^5$). This value has already been questioned in Ref. [66] since empirical data suggests a lower value for the human compres-
sion coefficient. However, we emphasize that there is no unique combination that produces the optimal result since many different parameter combinations can produce similar outcomes.

If the desired velocity was a known value, we could determine the combination of parameters \((k_n, k_t)\) that best fits the empirical data. Although we did not estimate the value of \(v_d\), the fitness landscapes shown in this research can help to narrow down the possible parameters that could fit the empirical evacuation curve.

For future researches, it will be necessary to find a set of parameters that are able to fit situations like the Black Friday event and also situations such as the Jamarat pilgrimage (where \(v_d\) is low, but the density is extremely high). In the latter case, it would be convenient to fit the parameters using the fundamental diagram measurements reported in Ref. [60].

![Figure 4: Cumulative number of evacuees as a function of time (evacuation curve). The black dotted line corresponds to empirical measurements while the red line corresponds to the numerical simulation result that best reproduces the empirical data. The desired velocity is \(v_d = 5\, \text{m/s}\) . The parameters of the best fit are \((k_n, k_t) = (0.0364 \times 10^5, 3.05 \times 10^5)\), the rest of the parameters are the same as Helbing’s proposal in Ref. [18]. The mass \((m = 79.5\, \text{kg})\) and shoulders width was set according to Ref. [26].](image)

We conclude this second part of the results by mentioning that the Differ-
ential Evolution algorithm could achieve a set of parameters for \( v_d = 5 \) m/s \((k_n, k_t) = (0.0364 \times 10^5, 3.05 \times 10^5)\) that fits the empirical data with an agreement value of \( f(k_n, k_t) = 0.57 \). This result suggests that the friction value should be increased respect the original value proposed in Ref. [18]. We also stress that there is no unique combination of optimal parameters but a collection of possible combinations that lead to similar results in terms of the objective function.

6. Conclusions

The escape panic version of the Social Force Model has been widely considered in pedestrian dynamics. The purpose of the model is to simulate emergency evacuations under high emotional stress. We have analyzed a real-life video (Black Friday event) that resembles an emergency evacuation and then turns out to be suitable for testing and optimizing the model.

We measured the Black Friday event flow \((\langle J \rangle = 6.7 \pm 0.8 \) p/s\) and found that it is higher than the flow measurements reported in most experimental studies. We argue that this discrepancy is related to the fact that in controlled experiments, the participants are not extremely competitive (due to safety considerations). On the other hand, in the real-life situation analyzed in this paper, it is possible to observe a high level of competitiveness and aggression, which may have lead to such a distinct result.

We performed numerical simulations that recreate the Black Friday event. We used different sets of parameters that were previously proposed/used in the literature. The parameter sets calibrated from laboratory controlled experiments or situations where the physical contact is negligible produced simulations which display a strong disagreement with our empirical measurements. These sets of parameters produce simulated agents that evacuate too fast. We think that even though those calibrations fit well the situations analyzed in their researches, they are not completely suitable for analyzing high stress situations like the Black Friday event because the friction contribution turns out to be underestimated.

The sets of parameters from Refs. [24, 20] provide better reproduction of the empirical results, although they do not fully reproduce the empirical
evacuation curve of the Black Friday event.

In order to explore the possibility of getting better values of the parameters $k_n$ and $k_t$, we implemented a Differential Evolution algorithm to optimize them. The optimization criteria was to minimize the difference between the simulated evacuation curve and the empirical evacuation curve. The values that produce the best fit are: $(k_n, k_t) = (0.0364 \times 10^5, 3.05 \times 10^5)$, for $v_d = 5$ m/s. These values reproduce the qualitative and quantitative behavior of the empirical measurement. We also found that $k_n$ and $k_t$ are correlated in the fitness landscape. The consequence of this is that there are multiple combinations of $k_n$ and $k_t$ that produce the same fitness values. We did not attempt to identify the source of correlation, but we are currently working on this topic for an upcoming investigation.

We stress that physical forces (friction and body force) are a critical factor in reproducing emergency evacuations where the pedestrians are subject to high-density and high anxiety conditions. Therefore, we suggest calibrating the model parameters using real-life emergency evacuations or similar situations (like the Black Friday stampede analyzed in this paper). The results obtained after performing the above mentioned optimization, indicate that the friction coefficient $k_t$ is generally underestimated.

We believe that in the coming years, there will be an increasing number of videos like the one analyzed in this work. Situations like Black Friday events, music concerts, and human stampedes are more often documented and published on the internet. This data, together with computer vision techniques, will contribute to study these singular events. We hope that future researches based on real-life video analysis will improve the theoretical models aimed at simulating emergency evacuations.

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