Entropy Fluctuations in Brane Inflation Models

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We study the development of entropy fluctuations in brane inflation in a warped throat, including the brane-antibrane tachyon as the waterfall field. We find that there is a period at the end of inflation during which the entropy mode associated with the tachyon field increases exponentially. In turn, the induced entropy seeds a contribution to the curvature fluctuation on cosmological scales which grows rapidly and could exceed the primordial curvature perturbation. We identify parameter values for which in the absence of back-reaction the induced curvature fluctuations are larger than the primordial adiabatic ones. In the specific model we study, however, back-reaction limits the growth of the entropy fluctuations. We discuss situations in which back-reaction effects are less constraining. The lesson of our investigation is that the study of the development of entropy fluctuations at the end of the period of inflation can lead to constraints on models of brane inflation and suggests that the curvaton mechanism may contribute significantly to the spectrum of cosmological perturbations.

1. Introduction

In recent years there has been a large effort at inflationary model building in the context of superstring theory. Since string theory contains many scalar field excitations and several of these are massless above the scale of supersymmetry breaking, the hope is that slow-roll inflation could be realized naturally in this context (for recent reviews of inflation in the context of string theory see [1–4]).

A widely studied class of string-inspired inflationary universe models falls under the category of brane inflation [7, 8, 9, 5] (see [9–11] for reviews). Here, the inflaton, the scalar field driving inflation, corresponds to the separation between two branes, or between a brane and an antibraner in a higher dimensional spacetime. When the branes come within a critical distance (given by the string scale) from each other, a
tachyon develops (“tachyon condensation”) and inflation ends. The dynamics of the inflaton in brane inflation is similar to the dynamics of hybrid inflation [12] where inflation ends when a second scalar field, the “waterfall” field, develops a tachyonic mass.

In this paper, we will focus on the dynamics of the initial stages of reheating in brane inflation models. Specifically, we will study the growth of metric fluctuations of entropic type on super-Hubble scales. We find that due to the tachyonic growth in the entropy field, there is an instability to the growth of metric entropy fluctuations on super-Hubble (but sub-horizon) scales. The entropy mode, in turn, induces a growing curvature fluctuation which can dominate over the primordial curvature perturbation for certain values of the model parameters. Demanding that these induced curvature fluctuations do not exceed the observational bounds leads to constraints on the parameter space of brane inflation models. We begin by setting the stage for our study.

In inflation models of chaotic type [13] (also called large-field models), inflation ends when the inflaton field begins to oscillate about the minimum of its potential energy function. As first shown in [14] (see also [15]), these oscillations lead to a parametric resonance instability for fluctuation modes of the inflaton and of fields which couple to the inflaton. This instability rapidly drains the energy from the homogeneous inflaton condensate. This short initial stage of the reheating process is called “preheating” [16]. As shown in [17], the parametric resonance instability persists even if the expansion of the universe is taken into account. For a detailed discussion of preheating see [18].

As was discovered in [19, 20] (see also [21]), the instability to the growth of field fluctuations is qualitatively more efficient in models of hybrid inflation and is called “tachyonic preheating.” Here, the instability is fueled by the tachyonic mode of the background model. In particular, in tachyonic preheating all long wavelength modes are unstable.

Since the metric always couples to the inflaton, it is not far-fetched to expect that oscillations of the inflaton might induce instabilities of metric fluctuations. In fact, it was first suggested in [22] that parametric excitation of super-Hubble metric fluctuations during reheating might be possible. However, in single field inflation models, this effect does not occur [23–25] because the instability is in a gauge mode. However, the effect can be physical in a two field inflation model [26], and concrete models were discussed in [27,28]. In [28], a specific hybrid inflation model was studied as an example. In some of the models studied in [28], the metric fluctuations become nonlinear before back-reaction can stop the instability [29].

In this paper, we will study the excitation of entropy modes of metric fluctuations in brane inflation models of the type considered in [30]. Since the dynamics of preheating is of tachyonic type, we find that there is a homogeneous solution for the entropy mode which increases exponentially for a short time period at the
We show that this growth seeds an exponential instability of long wavelength metric entropy modes, which turns off either once the tachyon field develops a sufficiently large velocity or when back-reaction effects shut off the resonance. We then compute the magnitude of the induced curvature perturbation without taking into account back-reaction and find that for certain parameter values it exceeds the amplitude of the initial curvature fluctuation (one can view this as a particular example of the curvaton scenario [28, 29, 30, 31]). A lesson we thus learn is that the presence of entropy modes needs to be taken into account in brane inflation models. However, in the specific model we study, back-reaction effects may shut off the tachyonic resonance of the fluctuations before the entropy modes reach a sufficient amplitude.

We should stress at the outset that we are only considering one of several entropy modes present in our brane inflation model. We have set all other modes (e.g. modes due to motion in angular directions of the compactification) to zero. An interesting problem for further research would be to perform a systematic study of all of the entropy modes which are present in the setup. In recent work [37–39], there has been progress concerning multiple field perturbations (radial and angular modes) while inflation is still under way.

There has been some previous work on the generation of secondary fluctuations in brane inflation models. Closest to our work is the study of [40] in which the secondary curvature fluctuations due to fluctuations in the tachyon field were considered. However, in that work the tachyonic amplification of the fluctuations after the end of slow-roll inflation was not considered, and as a consequence a much smaller amplitude of secondary perturbations was found. The generation of isocurvature fluctuations at the end of inflationary models of hybrid type due to inhomogeneities in other light fields has been considered by various authors [41, 42], as has “modulated preheating” [43, 44], i.e., the generation of inhomogeneities from variations of coupling constants in the hybrid inflation model. In this case, these variations are due to entropic fluctuations of other light fields which determine the values of the coupling constants [45–48]. Some effects of higher order in perturbation theory have been considered in [49, 50]. Finally, we wish to draw the attention of the reader to the interesting problem of the transfer of energy from the inflaton/tachyon system to matter of the Standard Model, the actual reheating process [51–56].

The outline of this paper is as follows. In the following section, we review the background geometry of warped brane inflation models. Then we discuss the forms of the scalar field potential which describe, respectively, the inflationary phase and the tachyon condensation period. Section 4 contains a discussion of the background dynamics. In section 5, we begin with a review of the formalism of metric entropy

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*In order for the instability to be excited, there needs to be an offset of the tachyon field from its symmetric (and unstable) point when averaged over a volume which corresponds to the comoving Hubble volume at the onset of the period of inflation. A similar assumption was also made in the analysis of [28].
fluctuations, which we proceed to apply to our model. Finally, we study the growth of metric entropy perturbations in our model and confront our conclusions with observational constraints.

2. The background for warped brane inflation

We take the basic setup of a brane moving in a warped throat, as in [30], as our prototypical brane inflation model. The most important point of [30] is that the moduli of the compactification manifold must be stabilized for inflation to occur, so the authors of [30] focus on a class of models in type IIB string theory in which fluxes fix the complex structure moduli [57–61] and a nonperturbative superpotential fixes the Kähler moduli [62]. In these models, the internal geometry is (conformally) Calabi-Yau, which can include singular points. In the most studied case, the singularity considered is the conifold, which may be deformed by a modulus to remove the singularity. In fact, the flux stabilizes the deformation modulus to a finite value, so the conifold becomes nonsingular. In addition, the flux sources a warp factor, which causes the region near the deformed conifold point to become a warped throat. (For a review of these compactifications, see for example [63].)

It is also possible to introduce mobile D3 branes and antibranes into this background. Due to the warp factor, the antibranes sink to the bottom of the throat (near the deformed conifold point) and contribute a positive supersymmetry breaking term to the vacuum energy [62]. To a first approximation, the D3 branes move without constraint, so they experience a Coulomb attraction towards the antibranes, which (hopefully) drives inflation [30]. The (more complicated) details of this situation are discussed below.

In the following, we first review the geometry of the background before and after the flux deforms the conifold singularity. To conclude this section, we then list the values of the key parameters which were assumed in [30].

2.1. Singular Conifold

A compact Calabi-Yau manifold may contain a variety of singularities, one of which is known as the conifold, due to the fact that it is a cone over the Einstein manifold $T^{1,1}$. Focusing on the region around the singularity, the Calabi-Yau metric is approximated by the (noncompact) conifold metric

$$ds_c^2 = dr^2 + r^2 ds_{T^{1,1}}^2 .$$

The base space $T^{1,1}$ has the metric [64]

$$ds_{T^{1,1}}^2 = \frac{1}{9} (g_5)^2 + \frac{1}{6} \sum_{i=1}^4 (g_i)^2 ,$$

where

$$g_5 = \sum_{i=1}^4 g_i ,$$

and $g_i$ are the complex structure moduli.
the $g_i$ denoting a convenient basis of one-forms,

$$g_1 = \frac{e_1 - e_3}{\sqrt{2}}, \quad g_2 = \frac{e_2 - e_4}{\sqrt{2}},$$

(3)

$$g_3 = \frac{e_1 + e_3}{\sqrt{2}}, \quad g_4 = \frac{e_2 + e_4}{\sqrt{2}},$$

(4)

$$g_5 = e_5.$$

(5)

The $e_i$ are a vielbein

$$e_1 = -\sin \theta_1 d\phi_1,$$

(6)

$$e_2 = d\theta_1,$$

(7)

$$e_3 = \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2,$$

(8)

$$e_4 = \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2,$$

(9)

$$e_5 = d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2.$$  

(10)

In type IIB string theory, the full 10D metric allowing for 4D spacetime-filling D3 branes at the tip of the conifold takes the form [65]

$$ds^2 = h^{-1/2}(r)G_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(r)ds_c^2.$$ 

(11)

The external metric $G_{\mu\nu}$ is Minkowski for the known solutions of the 10D field equations, but we will allow it to take FRW form here. Allowing FRW evolution should introduce corrections to this ansatz, but we will assume, as is standard, that they are small.

The warp factor $h(r)$ can be calculated from the 10D equations of motion to read

$$h(r) = 1 + \frac{R^4}{r^4}, \quad R^4 = 4\pi g_s \alpha'^2 \mathcal{N} \frac{N}{v},$$ 

(12)

where $\mathcal{N}$ is the number of background D3 branes and $v$ is the volume ratio $v = \text{Vol} T^{1,1}/\text{Vol} S^5$, i.e. it compares the size of the conifold base to a unit sphere. As is standard, $g_s$ is the string coupling and $\alpha'$ is the squared string length. Very near the tip of the conifold where $r$ is small, $h \approx R^4/r^4$, and the 10D spacetime becomes $AdS_5 \times T^{1,1}$. This region is the so-called conifold throat, which joins on to the bulk Calabi-Yau at large $r$ when $h \approx 1$. Note that the warp factor [12] can be generalized to a harmonic function on the conifold to account for more generally placed D3 branes (or even wrapped D7 branes). Henceforth, however, when we use this form of the warp factor, we will assume that we can neglect the constant term and that we have the simple form $h \approx R^4/r^4$.

### 2.2. Deformed Conifold

The singular conifold is a single point in the moduli space of conifold metrics; it has been shown in [57] that supergravity 3-form flux forces the conifold onto the “deformed conifold” branch of this moduli space. The deformed conifold asymptotes...
to the conifold, but it has a non-shrinking 3-cycle, so it is not a true cone near its core. The deformed conifold metric is [64]
\[
ds_{dc}^2 = \frac{\epsilon^{4/3} K(\tau)}{2} \left\{ \frac{1}{3K^3(\tau)} [d\tau^2 + (g_5)^2] + \cosh^2 \left( \frac{T}{2} \right) [(g_3)^2 + (g_4)^2] + \sinh^2 \left( \frac{T}{2} \right) [(g_1)^2 + (g_2)^2] \right\}.
\]
(13)

These coordinates are dimensionless, so \( \epsilon \) has dimensions of \((\text{length})^{3/2}\). In addition, the function \( K(\tau) \) in (13) is given by
\[
K(\tau) = \frac{[\sinh(2\tau) - 2\tau]^{1/3}}{2^{1/3}\sinh \tau}.
\]
(14)

In particular, the “radial” coordinate of the cone is now denoted by \( \tau \) instead of \( r \), as was the case for the singular conifold (2). To convert between \( \tau \) and \( r \) in the asymptotic conifold region, note that \( K(\tau) \) is asymptotically
\[
K(\tau \to \infty) \approx 2^{1/3} e^{-\tau/3}.
\]
(15)

Therefore, the radial and \( g_5 \) parts of the metric are
\[
g_{\tau\tau} = g_{55} = \frac{\epsilon^{4/3}}{3 \cdot 2^{5/3}} e^{2\tau/3}
\]
at large \( \tau \). Comparing this to the corresponding components of the singular conifold metric (12) in the same limit, we find that \( r \) and \( \tau \) are related by
\[
r^2 = \frac{3}{2^{5/3}} \epsilon^{4/3} e^{2\tau/3},
\]
\[
\tau = \frac{3}{2} \ln \left( \frac{2^{5/3} \epsilon^{2/3}}{3 e^{2\tau/3}} \right) \equiv \ln \left( \frac{r}{r_0} \right).
\]
(17)

Note that while \( r \) has dimension of length, the new variable \( \tau \) is dimensionless. This is a consequence of the new dimensional scale introduced into the theory by the deformation \( \epsilon \). Also, \( r_0 \) is the naively extrapolated value of the radius \( r \) at the bottom of the throat.

In the presence of D3 branes (or D7 branes) or 3-form flux, the deformed conifold develops a warp factor, as in the singular case, so the 10D metric becomes [66]
\[
ds^2 = \hat{h}^{-1/2}(\tau) G_{\mu\nu} dx^\mu dx^\nu + \hat{h}^{1/2}(\tau) ds_{dc}^2.
\]
(18)

(Regarding cosmological metrics, please see the comments following equation (11).)

The warp factor \( \hat{h}(\tau) \) in the deformed case has a more complicated form, which is, for the case of no free D branes [66] (see also [67] for more explicit notation),
\[
\hat{h}(\tau) = (g_5 \mathcal{M}^\alpha)^2 2^{2/3} \epsilon^{-8/3} \times \int_\tau^\infty dx \frac{x \coth x - 1}{\sinh^2 x} [\sinh(2x) - 2x]^{1/3}
\]
(19)
up to an additive integration constant (usually taken to be 1 for a throat attached to a compact Calabi-Yau manifold). Free D3 branes just add a harmonic piece to \( \hat{h} \). Physically, instead of \( \mathcal{N} \) D3 branes, we should now rather think of an effective number of background branes \( \mathcal{N}_{\text{eff}} \): Apart from the \( \mathcal{N} \) D3 branes, the 3-form flux carries D3 brane charge (which is smeared over the deformed conifold, rather than pointlike).

For large \( \tau \), the deformed conifold metric (13) leads back to (12). The integral in (19) cannot be calculated analytically, but at large \( \tau \) it is well approximated by [67, 68]

\[
\hat{h}(\tau) \approx \left( g_s \mathcal{M} \alpha' \right)^2 3 \cdot 2^{1/3} e^{8/3} \tau e^{-4\tau/3} \\
= \frac{81 (g_s \mathcal{M} \alpha')^2}{\epsilon} \frac{1}{r^4} \ln(r/r_0)
\]

(again, up to an additive constant), where (17) has been used to obtain the last expression. As we mentioned above, free D3 branes just add a harmonic term to (20), which looks like \( R^4/r^4 \) if the D3 branes are at the tip of the throat. It is common to push the simplification still further and use the simple warp factor (12) even for the deformed conifold case, though this strictly speaking holds only over short distances.

### 2.3. Warp factor at the bottom of the throat

The presence of supergravity 3-form flux generates a potential for the complex structure moduli, including the conifold deformation modulus \( \epsilon \). In the approximation of a small deformation, \( \epsilon^2 \) is related to the flux by [57]

\[
(\epsilon^2)^{1/3} \approx \sqrt{\alpha'} \exp \left( -\frac{2\pi K}{3Mg_s} \right).
\]

Here, \( \mathcal{M} \) and \( K \) are the quantum numbers of 3-form flux wrapping different cycles. Their product gives the total effective D3 brane charge (modulo free branes) measured at the top of the throat, \( \mathcal{N} = \mathcal{M} \mathcal{K} \). We can integrate (19) numerically to find that

\[
\hat{h}_0 = \hat{h}(\tau = 0) = a (g_s \mathcal{M} \alpha')^2 \epsilon^{-8/3} \\
\approx a (g_s \mathcal{M})^2 \exp \left( \frac{8\pi K}{3Mg_s} \right),
\]

with \( a \approx 1.14 \). Again, we assume that there are no free D3 branes in the throat.

In fact, there is a simple physical argument due to [57] that the integral (19) should be of order unity. The warp factor at a distance \( r \) from \( \mathcal{N} \) D3 branes is \( h(r) \approx g_s \mathcal{N} \alpha'^2 / r^4 \). We have a natural distance scale \( \epsilon^{2/3} \) at the core of the deformed conifold, and (20) tells us that the effective number of D3 antibranes at \( r \lesssim r_0 \) is \( g_s \mathcal{N} \approx (g_s \mathcal{M})^2 \). Therefore, we expect to have \( \hat{h}_0 \approx (g_s \mathcal{M} \alpha')^2 \epsilon^{-8/3} \), which is indeed the case.
Just as we defined a naive value \( r_0 \) for the radius at the bottom of the throat using (17), we can define a similar value for the radius by demanding that the simplest formula for the warp factor (12) (without the constant term) gives the correct value (22) for the deformed conifold. Setting \( \hat{h}_0 \approx R^4/\hat{r}_0^4 \), we find

\[
\hat{r}_0 = \left( \frac{4\pi g_s \alpha'^2 N}{v \hat{h}_0} \right)^{1/4} = \frac{2^{5/6}}{\sqrt{3}} \left( \frac{4\pi K \kappa_{ag} M_v}{g_s M_v} \right)^{1/4} r_0 .
\] (23)

2.4. Example values

In the original KKLMMT paper [30], the authors used the parameters (for the definition of the brane tension see (30))

\[
\frac{T_3}{m_{Pl}^3} \approx 10^{-3} , \quad g_s = 0.1 ,
\] (24)

and hence

\[
\alpha' m_{Pl}^2 \approx 6.4 ,
\] (25)

which is a small hierarchy between the Planck and string scales. In addition, [30] took sample values

\[
K = 8 , \quad M = 20 , \quad \text{therefore} \ N = 160 .
\] (26)

Finally, the volume ratio was taken to be \( v = 1 \), which is equivalent to saying that the base of the conifold is simply the 5-sphere itself. For the Einstein space \( T^{1,1} \), as in (2), we have \( v = 16/27 \approx O(1) \). More generally, however, one can think of \( v \) as a free parameter describing the string background geometry. While (2) is the only explicitly resolved example of a warped throat, it can be viewed as one realization of a a class of backgrounds for which \( v \) can vary over a large range of values. For our purposes, we will take

\[
v = 16/27 .
\] (27)

If we calculate the warp factor \( \hat{h}_0 \) from (22), the above values give

\[
\hat{h}_0 \approx 1.6 \cdot 10^{15} .
\] (28)

We pause here to note that the value of \( 2.6 \cdot 10^{14} \) given in [30] does not precisely correspond to their choice of discrete parameters; however, this value can be achieved with a very small fractional change of \( g_s \).

3. The potential for warped brane inflation

Inflation occurs in the warped throat due to the attraction between D3 branes and antibranes. Due to the warp factor, D3 antibranes sink quickly to the bottom of the throat, but D3 branes are only drawn to the bottom of the throat due to their interaction with the antibranes and to nonperturbative effects related to moduli stabilization. In this section, we briefly review the potential that we use.
3.1. Branes and antibranes

As we mentioned before, the conifold and deformed conifold backgrounds allow for free, mobile D3 branes. At the classical level, these D3 branes and the background are mutually supersymmetric, so there is no force on the brane. D3 antibranes, however, feel a large classical force due to the warp factor, which draws them to the bottom of the throat. For technical reasons, it is standard to assume that there is only a single antibrane at the bottom of the throat. Also at the classical level, the D3 branes and antibranes experience a Coulombic attraction, which draws the D3 brane to the end of the throat.

The situation is more interesting when we consider nonperturbative effects. First of all, the nonperturbative corrections to the potential stabilize any moduli that remain unstabilized by the flux. These terms also generate a potential for D3 brane positions. The effective 4D potential derived from the 10D type IIB action is quite complicated when all the moduli stabilization effects are correctly taken into account [69]; for the potential in several different cases, see [70–73]. In the cases that support slow-roll inflation (which requires some tuning), inflation occurs near an inflection point far from the bottom of the throat. Therefore, by the time the brane reaches the bottom of the throat, it will be rolling quickly. However, since we will find that an increase in the velocity of $\phi$ will only increase the production of entropy modes, we use the naive Coulomb potential between the D3 branes and antibranes to provide a lower limit. In the absence of nonperturbative corrections, this potential allows slow-roll inflation even when the D3 brane approaches the bottom of the throat.

3.2. Inflationary potential

While inflation is under way in the throat (but far away from the bottom $r_0$), the inflaton field $\phi$ is just the radial distance $r$ between the brane and the stack of anti-branes inside the throat, normalized for a canonical kinetic term

$$\phi = \sqrt{T_3} r,$$

where $T_3$ is the D3 brane tension,

$$T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2}.$$ (30)

Note that $\phi$ has dimension of mass.

From [30], we have the Coulomb potential

$$V^{\text{inf}}(\phi) = \frac{M^4}{1 + \left(\frac{\mu}{\phi}\right)^4} \approx M^4 \left[1 - \left(\frac{\mu}{\phi}\right)^4\right],$$ (31)

Technically, the antibranes cannot be introduced into the compactification at the classical level [57], but we ignore that complication as we introduce nonperturbative physics anyway.
for $\phi \gg \mu$. The two parameters $M$ (the overall scale of inflation) and $\mu$ (scale compared to the field value $\phi$ at any given moment) are related to the fundamental string geometry parameters through

$$M^4 = \frac{4\pi^2 v \hat{\phi}_0^4}{N}, \quad \mu^4 = \frac{\hat{\phi}_0^4}{N} = \frac{M^4}{4\pi^2 v},$$

(32)

where $\hat{\phi}_0 = \sqrt{T_3} \hat{r}_0$ is the field value at the bottom of the throat with $\hat{r}_0$ from (23). (We use $\hat{r}_0$ rather than $r_0$ because the Coulomb potential arises from perturbations in the warp factor.) Note in particular that

$$\frac{M}{\mu} = \left(\frac{4\pi^2 v}{\delta \rho}\right)^{1/4},$$

(33)

so we can consider $\mu \approx M$ for $v \approx O(1)$.

The assumptions made in deriving the potential (31) break down when the proper distance between the branes reaches the string scale. We will discuss that situation in the following.

Provided that the linear inflaton fluctuations seed the observed structure in the universe, the mass scale $M$ of inflation can be determined from the COBE constraint [74]

$$\frac{\delta \rho}{\rho} \approx \left(\frac{V^{\text{int}}}{\phi^2}\right)' \approx 10^{-5},$$

(34)

where a prime indicates a derivative with respect to $\phi$. Inserting the potential (31), we obtain

$$\left(\frac{M}{m_{\text{Pl}}}ight)^3 = \frac{\mu^{1/4}}{4\sqrt{3\pi} \rho} \left(\frac{\mu}{\phi_H}ight)^5 
\approx 4.4 \cdot 10^{-8} \left(\frac{\mu}{\phi_H}\right)^5,$$

(35)

where $\phi_H$ is the value of the inflaton when scales of cosmological interest exit the Hubble radius. $\phi_H$ is slightly larger than the value at the waterfall point, which we find in equation (40) below. If we use the waterfall value for the parameter values of [30] (which is $\approx 25.3\mu$), we find

$$\frac{M}{m_{\text{Pl}}} \approx 3.4 \cdot 10^{-5}.$$

(36)

This value is consistent within about a percent with the fact that the scale of the potential is given by the D3 brane tension, $M^4 = 2\hat{h}_0^{-1}T_3$.

\(^c\)Which are that the brane and antibrane interact by closed string modes and that the throat is well approximated by equations (11,12).
3.3. Tachyonic potential

We will now discuss what happens when the D3 brane approaches within a string length of the antibrane and simultaneously reaches the bottom of the throat. Since the brane is close to the bottom of the throat, we have to take into account the deformation of the conifold. In particular, \( r \) ceases to be the appropriate radial coordinate. Near the bottom of the throat, which we will take to be within a string length of the antibrane, the appropriate canonically normalized field is \( \psi \) defined by

\[
\psi = \sqrt{T_3} \frac{\epsilon^{2/3}}{3^{1/6} \cdot 2^{6/5}} \tau \\
= \left( \frac{3}{2} \right)^{5/6} \sqrt{T_3} \frac{\epsilon^{2/3}}{3^{1/6}} \ln \left( \frac{r}{r_0} \right) .
\]

(37)

Due to the presence of the factor \( \epsilon^{2/3} \) in this rescaling, \( \psi \) acquires the correct dimension of mass despite \( \tau \) being dimensionless.

Simultaneously the lightest excitation of the open string stretching from brane to antibrane becomes tachyonic. This tachyon starts rolling down its potential, leading to tachyonic reheating [19, 20]. The total two-field potential after the appearance of the tachyon \( T \) (which replaces (31)) can be modeled as

\[
V_{\text{reh}}(\psi, T) = v^4_0 + \hat{h}_0^{-1/2} \left\{ -\frac{1}{\alpha'} + \frac{\hat{h}_0^{1/2} \psi^2}{T_3 (2\pi \alpha')^2} \right\} T^2
\]

(38)

(see, for example, [7, 75, 77–79]), which is a valid approximation for small tachyon values. Here, \( \hat{h}_0 \) is the warp factor at the bottom of the throat from (22). For simplicity, we have restricted the tachyon to real values, but our argument is not affected by taking \( T \) complex.

The potential (38) is reminiscent of that of hybrid inflation, where the role of the waterfall field being played by \( T \). The “waterfall point” occurs at a value \( \psi = \psi_{\text{strg}} \) which corresponds to a brane/antibrane separation of the string length:

\[
\psi_{\text{strg}} = 2\pi \sqrt{\alpha' T_3 h_0^{-1/4}} = 2\pi a^{-1/4} \sqrt{\frac{T_3}{g_s M}} \epsilon^{2/3} ,
\]

(39)

where we have used (22) in the last equality. We can convert this value to the “long-distance” canonically normalized scalar using (23, 32, 37)

\[
\phi_{\text{strg}} = \phi_0 \exp \left[ \frac{2\pi}{a^{1/4} \sqrt{g_s M}} \left( \frac{2}{3} \right)^{5/6} \right] \\
= \mu \left[ \frac{\sqrt{3}}{2^6/5} \right] \left[ \frac{a g_s M^2 v}{4 \pi} \right]^{1/4} \exp \left[ \frac{2\pi}{a^{1/4} \sqrt{g_s M}} \left( \frac{2}{3} \right)^{5/6} \right] \\
\approx 25.3 \mu ,
\]

(40)
where the last approximation is for the example values from section 2.2.1. For $\psi \lesssim \psi_{\text{strg}}$ (or equivalently $\phi \lesssim \phi_{\text{strg}}$), the tachyon starts rolling down the inverted square potential from $T = 0$.

Setting the potentials (31) and (38) equal at $\phi = \phi_{\text{strg}}$ and $T = 0$ determines the scale $v_0$:

$$v_0^4 \approx M^4 \quad (41)$$

4. Background evolution

Recall that during inflation, i.e. while the potential is given by (31), the inflaton rolls according to

$$\ddot{\phi} + 3H\dot{\phi} + V_{\phi}^\text{inf} = 0 \quad (42)$$

Once preheating and therefore the potential (38) sets in, the equations governing the background fields’ evolution are

$$\ddot{\psi} + 3H\dot{\psi} + V_\psi^\text{reh} = 0 \quad , \quad \ddot{T} + 3H\dot{T} + V_T^\text{reh} = 0 \quad , \quad (43)$$

$$8\pi \frac{\dot{\psi}^2}{3m_{\text{Pl}}^2} + \frac{\dot{T}^2}{2} + V^{\text{reh}}(\psi, T) = H^2 \quad . \quad (44)$$

The astute reader might wonder whether we need to account for the fact that the inflaton and tachyon have noncanonical kinetic terms in the proposed D-brane action with the tachyon (see, e.g., [75–77, 80, 81]). (Reheating with the full tachyon action, minus the inflaton, was studied in [82].) However, as we will see, the velocities and field values we study are all smaller than the local string scale, so we need not concern ourselves with this issue. In this, we agree with [50].

4.1. Tachyon evolution

Let us first consider the evolution of the tachyon during reheating. Later we will find that entropy fluctuations grow when $|\dot{\psi}| > |\dot{T}|$, so we are interested in finding the maximum possible speed of the tachyon in order to set a lower limit on the entropy perturbations. Since we want the greatest tachyon derivative, we can drop the $\psi^2 T^2$ term in the potential (35), which only reduces the tachyonic mass.

Shortly after the tachyon has started rolling, its velocity $\dot{T}$ is still small, but the acceleration $\ddot{T}$ is large. Then the equation of motion (44) can be approximated by

$$\ddot{T} \approx \frac{2}{\alpha'} h_0^{-1/2} T \quad . \quad (46)$$

(in addition, this approximation overestimates the acceleration). Hence, the evolution of the tachyon is described by

$$T(t) \approx T_0 \exp \left[ \frac{C_1}{\sqrt{\alpha'}} t \right] \quad , \quad (47)$$
where \( C_1 = \sqrt{2} \hat{h}_0^{-1/4} \). From this solution, we find that the velocity \( \dot{T} \) is proportional to \( T \) with

\[
\dot{T} \approx \left( \frac{C_1}{\sqrt{\alpha'}} \right) T.
\]  

(48)

We can therefore compare the time scale of the tachyon decay to the time scale of inflation, which from (31) and the Friedmann equation is given by \( H \approx \left( \frac{M^2}{m_{\text{Pl}}} \right) \). For the tachyon decay to be faster than the time scale of inflation, we require

\[
\left( \frac{C_1}{\sqrt{\alpha'}} \right)^{-1} \approx \frac{1}{\sqrt{2}} \hat{h}_0^{1/4} \sqrt{\alpha'} \leq \frac{m_{\text{Pl}}}{M^2}.
\]

(49)

Using the values of section 2.4, i.e., \( \hat{h}_0 \approx 10^{15} \), and the value of \( M/m_{\text{Pl}} \approx 10^{-5} \) given in (36), we find indeed that tachyon decay is faster than the Hubble scale. This justifies, in retrospect, neglecting the Hubble friction term in (44).

### 4.2. Inflaton evolution

We now proceed to determining the inflaton velocity at the beginning of reheating. Since \( \phi \) (or the new field \( \psi \), respectively) does not accelerate further once the potential shifts from (31) to (38) (\( T \) is small initially), a good estimate for the energy in the \( \phi \) (or \( \psi \)) field early on in the reheating phase can be obtained from \( \dot{\phi}_{\text{strg}} \), the inflaton velocity at the waterfall point \( \phi_{\text{strg}} \). To determine \( \dot{\phi}_{\text{strg}} \), let us assume that the slow-roll approximation for \( \phi \) is still valid when the field encounters \( \phi_{\text{strg}} \) moving down the potential (31). We remind the reader again that a large value of \( \dot{\phi} \) (or equivalently \( \dot{\psi} \)) will increase the amount of entropy perturbations, so we are really finding a lower limit.

Simplifying (42) to \( 3H \dot{\phi} \approx -V_{\phi}^{\text{inf}} \), we can estimate the velocity \( \dot{\phi}_{\text{strg}} \) at the waterfall point:

\[
\dot{\phi}_{\text{strg}} = -\sqrt{\frac{2}{3\pi}} \frac{\mu}{\phi_{\text{strg}}} \frac{M^2}{\phi_{\text{strg}}} m_{\text{Pl}}
= -\frac{2}{\sqrt{3}} \hat{\phi}_{\text{strg}}^{1/4} M m_{\text{Pl}} \left( \frac{\mu}{\phi_{\text{strg}}} \right)^{5/4}.
\]

(50)

Note, however, that \( \phi \) is the “old” field used during inflation, and that from the appearance of the tachyon on, we must use \( \psi \) as defined in (37). From this transformation, we see that the velocities \( \psi \) and \( \phi \) are related by

\[
\dot{\psi} = \left( \frac{3}{2} \right)^{5/6} \sqrt{T_3} \epsilon^{2/3} \frac{\dot{\phi}}{\phi},
\]

(51)

so a velocity (50) in \( \phi \) becomes a velocity for \( \psi \):

\[
\dot{\psi}_{\text{strg}} = -\left( \frac{3}{2} \right)^{1/3} \sqrt{T_3} \epsilon^{2/3} \frac{\mu^4 M^2}{\phi_{\text{strg}}^{10}} m_{\text{Pl}}.
\]

(52)
(The astute reader should note that this velocity is, in fact, considerably less than the local string scale.) Therefore, the value of $T$ for which $|\dot{T}| > |\dot{\psi}|$ is

$$\frac{T}{m_{\text{p}1}} > \left(\frac{\sqrt{\alpha}}{C_1}\right) \left(\frac{3}{2}\right)^{1/3} \sqrt{\frac{T_3}{\pi}} \varepsilon^{2/3} \mu^4 \frac{M^2}{\phi_{\text{strg}}} \equiv \frac{T_{\text{eq.},\psi}}{m_{\text{p}1}}.$$  \hspace{1cm} (53)

Simplifying a bit and inserting the values from section 2.4, we find

$$\frac{T_{\text{eq.},\psi}}{m_{\text{p}1}} = \frac{a^{1/4}}{2\pi} \left(\frac{3}{2}\right)^{1/3} \sqrt{v} \mu M \left(\frac{\mu}{\phi_{\text{strg}}}\right)^6 \approx 2.5 \cdot 10^{-9}. \hspace{1cm} (54)$$

This value is in fact close to the Hubble scale and therefore much smaller than the local (warped) string scale at the bottom of the throat, which is given by $M$. Since the value of the tachyon also controls the tachyon velocity, we see that the tachyon velocity is also much smaller than the local string scale.

We now need to compare the value of $T_{\text{eq.},\psi}$ with the initial value of the tachyon once the tachyonic instability sets in. Note from the form of the potential (38) that the tachyonic instability sets in rather suddenly as the inflaton rolls down its potential. At large inflaton values, the tachyon is confined in the steep valley and displaced from $T = 0$ by quantum fluctuations. An upper bound on the amplitude of the quasi-homogeneous tachyon value is obtained by assuming that the mode carries the typical energy density of quantum vacuum fluctuations

$$\rho_{\text{vac}} \approx H^4,$$ \hspace{1cm} (55)

which holds even for very large masses ($m \gg H$) up to numerical coefficients of order $10^{-2}$ [83]. The average displacement $T_0$ of the tachyon can then be estimated by equating the energy density in the tachyon displacement with the above vacuum fluctuation energy density

$$m^2 T_0^2 \approx H^4,$$ \hspace{1cm} (56)

where $m$ is the local tachyon mass which can be read off from (38). Furthermore, by examining the tachyon’s positive frequency modes, it is straightforward to see that the super-Hubble modes account for the entire background. Within a Hubble patch, one may therefore think of $T_0$ as a homogeneous background for the tachyon; while there are fluctuations on shorter wavelengths, they decouple at linear order and affect only the backreaction, as we will discuss later.

However, a refined argument shows that $T_0$ as given in (56) is not truly a homogeneous background for the tachyon. Because the tachyon is massive compared to the Hubble scale during most of inflation, tachyon fluctuations at large wavelength are suppressed by the expansion of the universe. Therefore, if we are interested in studying fluctuations on a particular super-Hubble scale $k$, we need to use the quasi-homogeneous part of the tachyon field, not over a Hubble volume, but a much larger volume of comoving radius $k^{-1}$. Since the tachyon field is massive during inflation, the super-Hubble fluctuations are damped by $\exp(-2N_k)$, where $N_k$ is
the number of e-foldings of inflation since the mode labelled by \( k \) exited the Hubble radius. This leads to an exponential suppression of \( T_0(k) \). To see the exponential suppression, we note that the tachyon fluctuation at a specific wavenumber is

\[
\delta T(k) \sim \sqrt{k/m} a^2 ,
\]

so the total integrated (root-mean-square) background is

\[
T_0(k) \sim (1/m)(k/a)^2 = \exp(-2N_k)H^2/m .
\]

We will return to this point in section 6.5. It is important to remember that \( T_0 \) is not a homogeneous value; rather it is a quasi-homogeneous background and does depend on wave number. However, though the quasi-homogeneous background depends on the wavenumber, it is justified to describe it as a \( k \)-independent, homogeneous background on the scales of interest. In addition, we note that \( T_0 \) is considerably smaller than the Hubble scale, so it is also smaller than \( T_{eq,\psi} \), which allows tachyonic growth.

5. Entropy fluctuations

In our model, there are two scalar fields at play during the phase of reheating, namely the (coordinate-adjusted) inflaton \( \psi \) and the tachyon \( T \). Therefore, as in any multifield inflation model, entropy perturbations should be present. If their growth is fast enough, these entropy perturbations can act as a source for the comoving curvature perturbation \( R \), which is conserved on large scales in single field inflation. The change in \( R \) is described by \cite{84}

\[
\dot{R} = \frac{H}{H} \frac{k^2}{a^2} \Psi - \frac{2H}{\dot{\sigma}^2} V_\sigma \delta s .
\]

In the above, \( \Psi \) is the metric fluctuation in longitudinal gauge, in the gauge in which the metric including linear cosmological perturbations (in the absence of anisotropic stress) takes the form

\[
ds^2 = (1 + 2\Psi)dt^2 - a^2(t)(1 - 2\Psi)d\vec{x}^2
\]

and \( \delta s \) is the entropy perturbation. See \textit{e.g.} \cite{85} for an in-depth survey of the theory of cosmological perturbations, and \cite{86} for a pedagogical overview.

Moreover, \( \dot{\sigma} \) is the “adiabatic” combination of the field derivatives \( \dot{\psi} \) and \( \dot{T} \), defined as

\[
\dot{\sigma} = \sqrt{\dot{T}^2 + \dot{\psi}^2} = \cos(\theta) \dot{\psi} + \sin(\theta) \dot{T}
\]

\(^d\)We thank Jim Clise and Neil Barnaby for emphasizing this point to us.
\(^e\)For the generalization of these expressions to more than two fields, see \cite{87}.
with

\[
\cos(\theta) = \frac{\dot{\psi}}{\sqrt{\dot{\psi}^2 + \dot{T}^2}}, \quad \sin(\theta) = \frac{\dot{T}}{\sqrt{\dot{\psi}^2 + \dot{T}^2}}.
\] (62)

Orthogonal to the adiabatic direction $\sigma$ in field space, the “entropy direction” $s$ is given by

\[
\delta s = \cos(\theta) \delta T - \sin(\theta) \delta \psi.
\] (63)

Note that the background entropy field is constant, $\dot{s} = 0$, and can therefore be set to zero. Finally, $V_s$, the potential’s derivative in the entropy direction, can be expressed as the following combination of the potential derivatives:

\[
V_s = \frac{\dot{\psi}}{\dot{\sigma}} V_{\psi} - \frac{\dot{T}}{\dot{\sigma}} V_T.
\] (64)

Note that we have dropped the index “reh” from the potential. Unless otherwise specified, $V$ from now on always refers to the two-field potential (38).

The evolution equation for the entropy perturbation is

\[
\ddot{\delta s} + 3H \dot{\delta s} + \left(\frac{k^2}{a^2} + V_{ss} + 3\frac{V_s^2}{\dot{\sigma}^2}\right) \delta s
\]

\[
= \frac{\dot{\theta}}{\dot{\sigma}} \frac{k^2}{2\pi G a^2} \Psi,
\] (65)

which contains also the second derivative of the potential with respect to the entropy field, given by

\[
V_{ss} = \frac{\ddot{T}^2}{\ddot{\sigma}^2} V_{\psi\psi} - 2\frac{\dot{\psi}\dot{\psi}T}{\ddot{\sigma}^2} V_{\psi T} + \frac{\dot{\psi}^2}{\ddot{\sigma}^2} V_{TT}.
\] (66)

Evidently, the source term on the right hand side of (65) dies out at large scales, $k/a \to 0$, as does the first term on the right hand side of (59). Therefore, only the term proportional to $\delta s$ remains in (59) as a source for $\dot{R}$ on large scales. To understand its evolution, we need to solve the equation resulting from (65) in the limit $k/a \to 0$,

\[
\ddot{\delta s} + 3H \dot{\delta s} + \left(V_{ss} + 3\frac{V_s^2}{\dot{\sigma}^2}\right) \delta s \approx 0.
\] (67)
First, we calculate explicitly the various derivatives of the potential,

\[ V_\psi = \frac{2\psi}{T_3 \left(2\pi\alpha'\right)^2} = 4\pi g_5 T^2 \psi , \]

\[ V_T = 2\hat{h}_0^{-1/2} T \left( -\frac{1}{\alpha'} + \frac{\hat{h}_0^{1/2} \psi^2}{T_3 \left(2\pi\alpha'\right)^2} \right) , \]

\[ V_{\psi\psi} = \frac{2\psi}{T_3 \left(2\pi\alpha'\right)^2} = 4\pi g_5 T^2 \psi , \]

\[ V_{TT} = 2\hat{h}_0^{-1/2} \left( -\frac{1}{\alpha'} + \frac{\hat{h}_0^{1/2} \psi^2}{T_3 \left(2\pi\alpha'\right)^2} \right) , \]

\[ V_{\psi T} = \frac{2\psi}{T_3 \left(2\pi\alpha'\right)^2} = 8\pi g_5 T \psi , \] (68)

where occasionally (30) has been used to replace \( T_3 \).

The entropy mode \( \delta s \) grows exponentially if the “mass” term in (65) is negative. Of the two terms \( V_{ss} + 3V_s^2/\dot{\sigma}^2 \), only the first one can be negative, so we need

\[ \frac{|V_{ss}|}{3V_s^2/\dot{\sigma}^2} > 1 \Rightarrow V_{ss} < 0 \] (69)

to obtain a tachyonic mass for \( \delta s \). The evolution of the background fields discussed in the previous section now allows us to identify the dominant terms in \( V_{ss} \) and \( V_s^2/\dot{\sigma}^2 \).

6. Growth of the entropy fluctuations

Once the D3 brane has come within a string length of the antibrane, there occurs a short period \( 0 < T < T_{eq,\psi} \), during which \( \dot{T} \) is catching up with \( \dot{\psi} \). After that, for \( T > T_{eq,\psi} \), the fields definitely start to roll down in the \( T \)-direction in field space.

Let us reexamine the condition (69) for tachyonic increase of the entropy mode in the light of this fact. For the moment, we neglect the effects of back-reaction.

6.1. Limit \( |\dot{\psi}| > |\dot{T}| \)

In this limit, we see from (61) that we have \( \dot{\sigma} \approx \dot{\psi} \). Hence, equations (64) and (66) reduce to

\[ V_{ss} = \frac{T^2}{\dot{\psi}^2} V_{\psi\psi} - 2 \frac{T}{\dot{\psi}} V_{\psi T} + V_{TT} \approx V_{TT} < 0 , \] (70)

\[ V_s = V_T - \frac{T}{\dot{\psi}} V_\psi \approx V_T . \] (71)

Using the explicit expressions (68) for the derivatives, we have

\[ \frac{|V_{ss}|}{3V_s^2/\dot{\sigma}^2} \approx \frac{|V_{TT}|\dot{\psi}^2}{3V_s^2} \approx \frac{\dot{\psi}^2 \alpha'}{3\dot{\psi}^2 T^2} . \] (72)
But we see by the definition of $T_{\text{eq,}\psi}$ (assuming $\dot{\psi} = \ddot{\psi}_{\text{strg}}$ constant) that 

$$\left| V_{ss} \right| \approx \frac{T_{\text{eq,}\psi}^2}{3T^2}.$$  

(73)

For small values of $T$, this ratio is much larger than unity, and hence the condition for tachyonic instability of the entropy mode is satisfied. In fact, the tachyonic instability for the entropy mode ends just before $|\dot{T}| = |\dot{\psi}|$. Thus, the tachyonic resonance of the entropy mode continues up to the time when the tachyon velocity starts to exceed the inflation velocity.

6.2. Limit $|\dot{T}| > |\dot{\psi}|$

In this limit, equation (61) gives $\dot{\sigma} \approx \dot{T}$, and therefore from (64) and (66) one finds

$$V_{ss} = V_{\psi\psi} - 2 \frac{\dot{\psi}}{T} V_{\psi T} + \frac{\dot{\psi}^2}{T^2} V_{TT} \approx V_{\psi\psi},$$  

(74)

$$V_s = \frac{\dot{\psi}}{T} V_{\psi T} - V_{\psi} \approx -V_{\psi}.$$  

(75)

Since $V_{\psi\psi}$ is positive, we see that there is no tachyonic resonance in this region. Thus, in order to have any tachyonic resonance of the entropy fluctuations, it is crucial that the initial value $T_0$ be smaller than $T_{\text{eq,}\psi}$.

6.3. Growth of the entropy mode

In the two previous subsections we have seen that, in the absence of back-reaction effects, the tachyonic instability of the entropy mode shuts off once $T > T_{\text{eq,}\psi}$. For the purpose of an order of magnitude estimate for the growth, we can take the Floquet exponent $\mu_F$ to be constant with

$$\mu_F = \left( 2 \frac{\dot{\delta}_0^{-1/2}}{\alpha'} \right)^{1/2} = (16\pi^3 g_s)^{1/4} M.$$  

(76)

Denoting the time when the instability starts with $t = 0$ and the time when it shuts off by $t = t_f$ we have

$$\delta s(t) = e^{\mu_F t} \delta s(0),$$  

(77)

where $\delta s(0)$ is the initial amplitude of the entropy fluctuation. The final value of the entropy mode consequently is

$$\delta s(t_f) = e^{\mu_F t_f} \delta s(0).$$  

(78)

Since in the region of instability the growth rates of the tachyon and that of the entropy mode are the same, see (47), we have

$$T_{\text{eq,}\psi} = e^{\mu_F t_f} T_0,$$  

(79)

and thus

$$e^{\mu_F t_f} = \frac{\delta s(t_f)}{\delta s(0)} = \frac{T_{\text{eq,}\psi}}{T_0}.$$  

(80)
6.4. Induced growth of the curvature fluctuation

Having calculated the resonant growth of the entropy mode, we can now insert the result into the master equation (59) for the induced growth of the comoving curvature perturbation $R$. On large scales, and in the region of the tachyonic resonance where $|\dot{\psi}| > |\dot{T}|$, equation (59) becomes

$$\dot{R} \approx -\frac{2H}{\psi^2} V_T \delta s \approx \frac{2H}{\psi^2} 2\hbar_0^{-1/2} \alpha^{-1} T \delta s .$$

Inserting the solutions (47) and (77) for $T$ and $\delta s$, respectively, we find after integration

$$\delta R \approx \frac{H}{\psi^2} \mu_F T(t_f) \delta s(t_f) .$$

In the absence of back-reaction, $T(t_f) = T_{eq,\psi}$. Since $T_{eq,\psi}$ is just where the tachyon and $\psi$ velocities are equal in magnitude, we obtain

$$\delta R \approx \frac{H}{\mu_F} \delta s(0) \frac{T_3}{T_0} \left( \frac{m_{Pl}^2}{\hbar_0} \right)^{1/4} \frac{\delta s(0)}{T_0} .$$

For the parameters from [30] cited in section 2.4 and used throughout this paper, the resulting amplitude is of the order

$$\delta R(t_f) \approx 10^{-5} \frac{\delta s(0)}{T_0} .$$

We expect that the initial value of the tachyon and the entropy mode are given by the same quantum fluctuation amplitude calculated in (56). More specifically, $\delta s(0)$ is set by $\delta T$ at the wavelength in question, which is given by equation (57). We then normalize $\delta s(0)$ appropriately for a power spectrum by multiplying it by $k^{3/2}$ (since our final goal is calculating the power spectrum of $R$), and we see that $\delta s(0)$ is approximately the same as the quasi-homeogeneous tachyon value $T_0$ obtained in equation (58).

However, if one were to follow the refined argument made above concerning the $k$-dependence of this background $T_0(k)$, the wavenumber $k$ at which one measures $T_0(k)$ should be larger than the one considered for $\delta s(0)$. In this way, we can be sure that the resulting $T_0$ is quasi-homogeneous at the scale in question, and we actually have $T_0 \leq \delta s(0)$, which only enhances the effect. However, since we try to set a lower limit, we will be conservative and treat $T_0$ and $\delta s(0)$ roughly as equal.

Hence, we conclude from (84) that, in the absence of back-reaction effects, the amplitude of the curvature fluctuations induced by the entropy modes is comparable to the amplitude of the primordial linear adiabatic fluctuations. Note that our result
is independent of the specific value of the initial quasi-homogeneous tachyon amplitude $T_0$. The reason for this is that the smaller $T_0$ is, the smaller the initial value of the entropy mode, but the longer the tachyonic instability lasts.

Another useful way to see our result is to consider the ratio of these secondary curvature perturbations to the primary perturbations ($\mathcal{R} = \delta \rho / \rho$):

$$\frac{\delta \mathcal{R}}{\mathcal{R}} = \frac{1}{6\pi} \left( \frac{v}{4\pi g_s} \right)^{1/4} \left( \frac{m_{pl}}{M} \right)^2 \left( \frac{\mu}{\phi_{strg}} \right)^5 \frac{\delta s(0)}{T_0}.$$  \hfill (85)

For the parameter values we use, we find $\delta \mathcal{R}/\mathcal{R} \approx 3.96 \delta s(0)/T_0$, which implies that the secondary anisotropies are actually larger than the primary anisotropies.

Let us now consider a different set of parameter values. Compared to the values from [30] used in the bulk of the paper, we can rescale $g_s \rightarrow xg_s$ and $\mathcal{M} \rightarrow \mathcal{M}/x$ without changing the warp factor (note that this is a discrete choice because flux is quantized and depends on other compactification parameters, as well). Working through the details, it is not hard to find that

$$\left( \frac{\delta \rho}{\rho} \right) \rightarrow \left( \frac{\delta \rho}{\rho} \right) x^{-2}, \quad \left( \frac{\delta \mathcal{R}}{\mathcal{R}} \right) \rightarrow \left( \frac{\delta \mathcal{R}}{\mathcal{R}} \right) x^{3/2}.$$  \hfill (86)

In addition, by changing the compactification volume (again, this would require adjusting other microphysical parameters), we can rescale $(\alpha' m_{Pl}^2) \rightarrow (\alpha' m_{Pl}^2)y$. This is slightly more subtle, because this process also rescales the warp factor $\hat{h}_0$ and the deformation parameter $\epsilon$ [88]. In the end, we find

$$\left( \frac{\delta \rho}{\rho} \right) \rightarrow \left( \frac{\delta \rho}{\rho} \right) y^{-3}, \quad \left( \frac{\delta \mathcal{R}}{\mathcal{R}} \right) \rightarrow \left( \frac{\delta \mathcal{R}}{\mathcal{R}} \right) y^{-11/3}.$$  \hfill (87)

Between the two of these rescalings, it is easy to see that we can maintain the COBE normalization for the primary anisotropies, while the secondary anisotropies would increase in amplitude (assuming for convenience that the primary anisotropies are still calculated using slow-roll physics at $\phi_{strg}$). In other words, we can easily find parameter values where our results present an even sharper problem.

We have thus established our main result, namely that, at least in the absence of back-reaction, there are parameter values in the brane inflation model we have considered for which the secondary fluctuations are larger than the primary ones.

In order to agree with observations, the model parameters will thus have to be normalized to the data using the secondary fluctuations rather than the primary ones. This will lead to different values of the model parameters which are consistent with the data.

### 6.5. Back-reaction effects

Although the quasi-homogeneous value of the tachyon field on the infrared scales relevant to our study is very small, the dispersion of the tachyon field on microphysical scales is large. Using the vacuum values of the small-scale tachyon fluctuations it can easily be shown (see e.g. [20]) that, at the onset of the tachyonic instability,
the small-scale dispersion $\sigma$ of the tachyon field is of the order $m$, where $m$ is the effective mass of the tachyon field before the waterfall point is reached. We denote this initial field dispersion by $\sigma(0)$. Its value is much larger than the value $T_0$ of the quasi-homogeneous tachyon field at the onset of the resonance. Due to the tachyonic resonance, the dispersion $\sigma$ grows exponentially. After a time interval $t_s$ which is given by $m^{-1}$, the dispersion has grown to a field value corresponding to the location $\eta$ of the minimum of the tachyon potential. The time $t_s$ is called the spinodal decomposition time.

Let us model the tachyon potential by the standard potential of a waterfall field in hybrid inflation

$$V(T) = \frac{\lambda}{4} (T^2 - \eta^2)^2$$

(88)

where $\lambda$ is the coupling constant for tachyon field interactions. Fitting $\lambda$ and $\eta$ to our potential (38), we find that $\eta$ is of the order $M^2/m$. Thus, the spinodal decomposition time is of the order of

$$t_s \approx m^{-1} \ln \left( \frac{\eta}{\sigma(0)} \right).$$

(89)

This time must be compared with the time $t_f$ when the tachyonic growth for long wavelength fluctuations stops. This time is given by

$$t_f \approx m^{-1} \ln \left( \frac{T_\text{eq,}\psi}{T_0} \right).$$

(90)

Since $T_0$ is generically much smaller than $m$, the spinodal decomposition time is generically shorter than $t_f$. Thus, the tachyon field becomes nonlinear on microphysical scales before the time $t_f$ is reached. If $T_0$ is given by (56), then the difference between $t_f$ and $t_s$ is only by a logarithmic factor. However, if $T_0$ is exponentially suppressed by $e^{-3N_k/2}$ due to the red-shifting of the long wavelength tachyon fluctuations during the period of slow-roll inflation, $N_k$ being the number of e-foldings of inflation between when the scale $k$ under consideration exits the Hubble radius and the onset of reheating, then the ratio $t_f/t_s$ is of the order $N_k$.

The onset of non-linearity on microphysical scales does not in itself shut off the tachyonic growth on cosmological scales, in the same way that the gravitational collapse of structures on stellar scales in our universe has not shut off the linear growth of perturbations on scales relevant to the cosmic microwave background.

However, once the tachyon dispersion $\sigma$ approaches the minimum of the potential, nonlinear effects in the tachyon field equation become important and generate a positive contribution to the mass term in the tachyon potential. The magnitude of this back-reaction effect depends quite sensitively on the form of the potential.

Working in the context of the above toy model (88), we can make use of the Hartree

\footnotetext{This subsection was added after very useful discussions with Jim Cline and N. Barnaby.}
approximation to estimate the contribution $\delta m_{\text{eff}}^2$ to the effective square mass, and find that it is of the order

$$\delta m_{\text{eff}}^2 \approx \lambda \sigma^2,$$

which dominates over the negative contribution to the square mass as soon as

$$\lambda \sigma^2 > m^2.$$

In the case of the potential \cite{88}, this happens on a time scale $t_s$, and greatly suppresses the efficiency of the tachyonic growth of the entropy fluctuations. In a follow-up study, we plan to study these back-reaction effects in an actual brane inflation model.

### 6.6. Toy model avoiding back-reaction

It is possible that some models of brane inflation allow the tachyon to be light during the last 60 e-foldings of inflation, which means that the entropy mode amplification could continue uninterrupted by back-reaction effects. We will now discuss a toy model in which the brane and antibrane are both located at the tip of the deformed conifold, as recently discussed by \cite{89} (following work by \cite{90}), and point out parameter values in which entropy modes could become important.

As discussed in \cite{89}, nonperturbative corrections to the D-brane action can generate a potential for the angular motion of the brane on the deformed conifold, even in an approximation in which the warp factor is independent of the angular directions. In some cases, \cite{89} found that this potential can support an adequate number of e-foldings of slow-roll inflation; in that case, the potential takes the form

$$V \approx 2\Lambda \left(1 - \frac{1}{16d^4}\psi^4\right),$$

near the top of the potential. We now use $\psi$ to denote the angular position of the brane, which starts near $\psi = 0$, while the antibrane sits at $\psi = \pi/2$, in our model at the antipodal point of the throat’s tip. In this model, we consider the nonperturbative potential to dominate over the Coulomb interaction between the brane and antibrane. Then COBE normalization requires $\Lambda^{1/4} \sim 10^{-3}d$ with $d \lesssim m_{Pl}$.

We can now ask how long the tachyon might be light during this type of slow-roll inflation. If we combine equations (13, 22), we see that the proper radius of the $S^3$ at the tip of the deformed conifold throat is, up to factors of order unity, $\sqrt{g_s\mu\alpha'}$. For the sample values given in section 2.3 (and other commonly taken string parameters), the radius is therefore essentially $\sqrt{\alpha'}$. Thus, we see that it is likely that the brane/antibrane tachyon is no more massive than the warped string scale whenever the brane is on the tip of the deformed conifold. In fact, since the

\[8\text{We use the variable } \psi \text{ to remind the reader that this field is the inflaton in a region of approximately constant warp factor.}\]
brane and antibrane are not at antipodal points of the tip during all of inflation, the tachyon is likely to be substantially lighter than the warped string scale during most of inflation.

Furthermore, as discussed in [56], the supergravity description of the warped throat is reasonable as long as the Hubble parameter $H$ is less than about the warped string scale. Therefore, if $\Lambda$ and hence $H$ are tuned to be large ($H \lesssim \hat{h}_0^{-1/4}/\sqrt{\alpha'}$), it is reasonable that the tachyon is light compared to the Hubble scale during much of slow-roll inflation. In fact, inflation may end by hitting the waterfall point rather than violating the slow-roll conditions. After the waterfall point, we estimate, as before, the evolution of the tachyon by ignoring its coupling to the inflaton. While this overestimates the rate of entropy mode growth, it underestimates the length of time over which the entropy modes can grow. In fact, while the Hubble expansion is still important in the background tachyon evolution (that is, the background tachyon evolution is over-damped), the short wavelength tachyon fluctuations are also over-damped. In essence, the tachyon fails to roll until the inflaton has moved past the waterfall point.

In that case, super-Hubble fluctuations of the tachyon, and therefore $T_0$, will be unsuppressed. This fact means that the long wavelength tachyon fluctuations at the end of inflation will be just as large as the short wavelength modes, so back-reaction from the short wavelength modes will not turn on before the long wavelength entropy modes can amplify the curvature perturbation. In fact, we can immediately estimate the curvature perturbation. If we repeat our analysis, the key results (82) and the first line of (83) are unchanged. In addition, $\mu_F$ is unchanged, still given by the warped string scale. Therefore,

$$\delta R \approx \frac{H}{\mu_F} \frac{\delta s(0)}{T_0}$$

(94)

which can be up to order unity. Therefore, the potential with appropriate parameter values provides an explicit example of a brane inflation where entropy perturbations can affect the normalization of cosmic perturbations to observation.

We should note that this model does not have a parametric separation between the Hubble and warped string scales as we have presented it. Therefore, in order to trust the approximate action we have used for the D-brane tachyon, which is valid for $\dot{\psi}, \dot{T} \lesssim \hat{h}_0^{-1/2}/\alpha'$ and $T \lesssim \hat{h}_0^{-1/4}/\sqrt{\alpha'}$, we need to tune the model a bit more. In particular, if the waterfall point is still within the slow-roll regime, we can satisfy these constraints and $T_0 < T_{eq, \psi}$ when $\sqrt{m_{PL}} H \lesssim \hat{h}_0^{-1/2}/\alpha'$ ($\epsilon$ the usual slow-roll parameter). On the other hand, even when $\dot{\psi}, \dot{T}$ and $T$ are above the warped string scale, it is possible that the entropy mode still grows tachyonically at the beginning of inflation. In [89], it was assumed that the potential should be less than the warped string scale to avoid violating the supergravity approximation. However, since the potential energy is due to the interaction between the D3 brane and a D7 brane elsewhere in the throat, it is not concentrated at the tip of the throat. What is important is that the 10D potential density be less than the 10D string scale, which is possible due to the length of the throat.
of reheating. To determine whether or not that happens, it is necessary to analyze the dynamics of the complete tachyonic action as given, for example, in [76]. While that calculation is beyond the scope of this paper, our results provide considerable motivation for it in future work.

7. Discussion and Conclusions

We have studied the development of entropy fluctuations in brane inflation models of KKLMMT [30] type. We have shown that the tachyonic instability at the end of slow-roll inflation leads to an exponential growth of the entropy mode associated with the tachyon. In turn, the entropy fluctuations lead to an extra contribution to the curvature fluctuations. For the parameter values used in [30], we find that, in the absence of back-reaction, the curvature fluctuations induced by the entropy mode are comparable to the primordial curvature fluctuations. For different parameter values, we find that the secondary fluctuations may be considerably larger than the primary ones.

However, we have also seen that back-reaction may cut off the resonant growth of the entropy modes before these have had a chance to become important. In a simple hybrid inflation model back-reaction effects indeed will truncate the resonance. Whether this will happen in any given brane inflation model will require further study.

Our result shows that the dominant source of curvature fluctuations in brane inflation models of KKLMMT type may be not the primordial inflaton fluctuations, but rather the entropy fluctuations occuring at the end of inflation, see [45]. In this sense, the mechanism is a realization of the “curvaton” [?, ?, ?, ?, ?, 31] scenario. In order for such a model to produce the observed magnitude of density fluctuations, the value of the inflaton mass scale \( M \) must not be fixed by (34), but by demanding that the amplitude from (83) yields the observed value.

Our work is closely related to that of [49, 50] which also discussed entropy modes arising during the early phase of tachyon condensation. The methods used are slightly different. We have introduced an effectively homogeneous background produced by long wavelength fluctuations and reduced the subsequent analysis to a first-order perturbative calculation, whereas [49, 50] assumed that the tachyon averages to zero and studied the generation of entropy fluctuations using techniques of second order perturbation theory.

Finally, we give a few caveats and directions for further research. The most important issue is to clarify the strength of back-reaction effects in the brane inflation model at hand. We should also note that, if the time scale for the tachyon evolution is longer than the Hubble time, there will be no exponential instability for entropy fluctuations because the friction term in (44) cannot be neglected (recall that the en-

\footnote{A similar conclusion was recently found in [91] in the context of another string-inspired inflationary model, namely “Roulette” inflation [92].}
tropy mode and tachyon are essentially identical early in reheating). However, this seems unlikely because the tachyon evolution is dominated by the warped string scale for $\psi \lesssim \psi_{\text{str}}/\sqrt{2}$, which we estimate occurs in less than a Hubble time.

In addition, the Coulombic potential we used is very much a toy model for brane inflation. We should also address how the results we have found might appear in more complete inflection-point inflation models of brane inflation [70, 71] (see also [93–96] for a similar form of inflection-point inflation in the MSSM). There are two main issues that would need to be addressed. First, the inflaton will not follow slow-roll behavior at the onset of tachyon condensation. Instead, the D3 brane will oscillate around the bottom of the throat, perhaps just exiting from a stage of DBI inflation [97]. The second issue to address, which is perhaps more difficult, regards the initial condition for the tachyon at the waterfall point. In our model, we have used the known behavior of a scalar field during inflation; however, at the inflection point, the tachyon would be so massive that it should be integrated out. The tachyon would be deflected from $T = 0$ instead during the early stages of inflaton oscillation or during DBI inflation.

We can make a few comments going beyond our toy model, however. If the tachyon still has a canonical kinetic term (which may be modified somewhat for DBI inflation), the tachyon and entropy mode growth is essentially unchanged from our toy model. Therefore, the key results (82) and the first line of (83) are also unchanged. In addition, $\mu_F$ is unchanged, while $H$ should be no smaller than the value we used. In that respect, we believe that our results give a lower bound on the contribution of entropy modes to curvature perturbations.

In our analysis we have neglected cosmic string production at the end of the phase of tachyon condensation [82, 98]. Once the tachyon approaches the minimum of its potential, string production and interaction dominates the energy transfer [19, 20]. Since the characteristic length scale of string production is much smaller than the Hubble radius, we do not expect this process to effect the long wavelength entropy modes studied here.

We view our work as an initial step at exploring the vast terrain of entropy fluctuation modes in string-inspired inflationary universe models. As shown here, these modes have the potential to rule out large classes of models and to change the parameter values in others. As we comment above, some of our key results may very well apply to more realistic models of inflection point inflation, so it will be important to resolve this issue for those potentials, as well.

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