Plasma resonance and remaining Josephson coupling in the “decoupled vortex liquid phase” in layered superconductors.

A.E.Koshelev

Material Science Division, Argonne National Laboratory, Argonne, IL 60439, and Institute of Solid State Physics, Chernogolovka, Moscow District, 142432, Russia

(March 23, 2022)

We relate the frequency of the Josephson plasma resonance in layered superconductors with the frequency dependent superconducting response. We demonstrate that the sharp resonance can persist even when the global superconducting coherence is broken provided the resonance frequency is larger than the frequency of interlayer phase slips. In this situation the plasma frequency is determined by the average Josephson energy, which can be calculated using the high temperature expansion. We also find the temperature dependence of the average Josephson energy from the Monte Carlo simulations and determine the applicability region of the high temperature expansion.

Interlayer Josephson coupling between superconducting layers has crucial influence on thermal and pinning properties of layered superconductors in the mixed state. A new, powerful tool to probe the Josephson coupling in a wide range of temperatures and fields has been introduced recently. A sharp microwave absorption resonance has been discovered in the vortex state of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ [3, 5], which was shown to be caused by electric field across the layers [5] and was attributed to the earlier predicted plasmon resonance [3, 5]. It was found that the resonance can be tuned by temperature and magnetic field. The resonance field corresponding to the fixed microwave frequency of 30-90 GHz increases with temperature at low temperatures having a maximum at the irreversibility temperature and afterwards smoothly decreases with temperature. The resonance frequency at a given temperature and field can serve as a measure of the effective Josephson coupling. Unexpectedly, the plasmon resonance has been observed up to high temperatures $\sim 75 \text{ K}$ [3] where the Josephson coupling should be strongly suppressed by thermal fluctuation of pancake vortices. It was theoretically predicted that pancake fluctuations should actually destroy the global superconducting coherence above the decoupling line [2, 3]. Matsuda et al [3] suggested that the decoupling line, if it exists, should lie above the temperature-field line at which the sharp plasmon resonance at a given microwave frequency is observed. However, their line of resonance, $B_0(T)$, clearly lies above the decoupling line observed by $I-V$ measurements [3] and falls into the region where linear resistivity in the c-direction was observed [3, 11]. In this Letter we resolve this controversy. We demonstrate that the sharp plasmon resonance can actually persist even above the decoupling transition provided the resonance frequency is higher than the typical frequency of the interplane phase slips. In this situation the plasma frequency is determined by the average Josephson energy $E_{J}^{\text{eff}} = E_{J} \langle \cos \phi_{nn+1} \rangle$ (or, equivalently, the average Josephson current) as was suggested in Refs. [3, 5]. Here $\phi_{nn+1} = \phi_{n+1} - \phi_n - \frac{2\pi}{\Phi_0} A_z$ is the gauge invariant phase difference between layers and $E_{J}$ is the bare Josephson energy. The decoupling transition manifests itself by a disappearance of the linear superconducting response along the c-axis at zero frequency while the average Josephson energy should not change much at this point and it actually stays finite at any temperature.

The superconducting response, $Q_\omega$, determines the response of the electric current to the external alternating vector-potential, $A_\omega$ [20]:

$$j_\omega = -Q_\omega A_\omega. \tag{1}$$

In superconducting state $Q_{\omega \to 0} > 0$. For the layered superconductor with the Josephson energy $E_{J} = -E_{J} \int \sum_n \cos \phi_{nn+1}$, one can obtain a general thermodynamic expression for the z-component of $Q_\omega$ using the fluctuation-dissipation theorem (FDT) [see, e.g., Ref. [17]]. The external vector-potential, $A_\omega$, creates the energy perturbation, $E_{\Delta} = -\frac{\mu_0}{c} \int \sum_n \sin \phi_{nn+1} A_\omega$, with $j_\omega = \frac{2\pi}{\Phi_0} E_{\Delta}$ being the Josephson current. The average current along the z-direction $j_z = j_\omega \langle \sin \left( \phi_{nn+1} - \frac{2\pi A_z}{\Phi_0} \right) \rangle_\omega$ in linear with respect to $A_\omega$ approximation is given by:

$$j_z = j_\omega \left( -\langle \cos \phi_{nn+1} \rangle_0 \frac{2\pi s A_\omega}{\Phi_0} + \langle \sin \phi_{nn+1} \rangle_\omega \right). \tag{2}$$

Here $\langle ... \rangle_0$ notates the thermodynamic average with undisturbed energy and $\langle ... \rangle_\omega$ notates averaging which takes into account the perturbation $E_{\Delta}$. The second term describes the average Josephson current, which emerges due to the phase perturbations caused by the external field. Using the FDT relation [17] this term can be connected with the correlation function of the Josephson currents and we obtain the following expressions for the real $(Q'_\omega)$ and imaginary $(Q''_\omega)$ parts of the superconducting response along the z-direction.
\[ Q'_\omega = sj_j \left[ \frac{2\pi}{\Phi_0} \langle \cos \phi_{nn+1} \rangle + \frac{j_j}{cT} \int dr \int_0^\infty dt \sum_n \cos(\omega t) \langle \sin \phi_{nn+1}(0,0) \sin \phi_{nn+1}(r,t) \rangle \right] \] (3)

\[ Q''_\omega = -\frac{\omega sj_j^2}{cT} \int dr \int_0^\infty dt \sum_n \cos(\omega t) \langle \sin \phi_{nn+1}(0,0) \sin \phi_{nn+1}(r,t) \rangle \] (4)

Substituting the material relation \( \Phi_0 \) into the Maxwell equations one can see that the plasma resonance exists at the frequency \( \omega_{pl} \) connected with \( Q'_\omega \) by relation \( \omega_{pl}^2 = 4\pi \alpha Q'_\omega / \epsilon \) (5) provided \( Q'_\omega \gg Q''_\omega \). Here \( \epsilon \) is the dielectric constant. Due to the frequency dependence of \( Q'_\omega \) the last expression is actually an equation for \( \omega_{pl} \) rather than its definition. As one can see from Eq. (5) \( Q'_\omega \) is given by the sum of two terms, the first is determined by the average Josephson current, \( sj_j \langle \cos \phi_{nn+1} \rangle \), and the second is determined by the correlation function of the Josephson currents. In the decoupled liquid phase \( Q'_{\omega=0} = 0 \), which means that at \( \omega = 0 \) the second term in square brackets of Eq. (3) exactly compensates the first one. The frequency dependence is totally determined by the second correlation term. This term is strongly suppressed when the frequency exceeds the typical “phase slip” frequency \( \omega_{ps} \), which is determined by the typical decay time of the correlation function \( \langle \sin \phi_{nn+1}(0,0) \sin \phi_{nn+1}(0,t) \rangle \). In the high frequency regime, \( \omega \gg \omega_{ps} \), we have \( Q'_\omega \approx \frac{2\pi}{\Phi_0} s_j j_j \langle \cos \phi_{nn+1} \rangle \), i.e., the plasma resonance probes the average Josephson current. In the mixed state \( \langle \cos \phi_{nn+1} \rangle \) is suppressed by misalignment of pancake vortices induced by pinning and thermal fluctuations. In Ref. [3] this quantity has been connected with the density correlation functions assuming Gaussian distribution for \( \phi_{nn+1} \).

In the decoupled liquid phase \( \langle \cos \phi_{nn+1} \rangle \) can be calculated using the high temperature expansion with respect to the Josephson energy which gives

\[ \langle \cos \phi_{nn+1} \rangle = \frac{E_J}{2T} \int dr \lim_{f \to 0} \left\langle \left[ i \left( \phi(r) - \phi(0) \right) - \frac{2\pi}{\Phi_0} \int_0^r dA \right] \right\rangle \] (6)

where

\[ S_1(r) = \lim_{f \to 0} \left\langle e^{i\left( \phi(r) - \phi(0) \right) - \frac{2\pi}{\Phi_0} \int_0^r dA} \right\rangle \] (7)

is the phase correlation function for a single 2D layer. In the vortex liquid state the decay length of \( S_1(r) \) is given by the average intervortex spacing and Eq. (3) can be rewritten as

\[ \langle \cos \phi_{nn+1} \rangle = \frac{E_J}{2TB} f, \] (8)

where \( f = \frac{1}{\Phi_0} \int dr \left\langle |S_1(r)|^2 \right\rangle \) is the dimensionless function of order unity with weak temperature dependence. We find from simulations that the high-T expansion is quantitatively accurate if \( \langle \cos \phi_{nn+1} \rangle \lesssim 0.5 \). This corresponds to the field and temperature range \( TB > \Phi_0 E_J \). Combining Eqs. (3) and (4) we obtain

\[ \omega_{pl}^2 = \frac{2\pi f sj_j^2 \Phi_0}{cTB}. \] (9)

Taking into account the weak temperature dependence of \( f \), this expression gives an explanation for the experimentally observed dependence \( \omega_{pl} \propto 1/\sqrt{T} \) [3]. Note that such scaling indicates almost decoupled layers rather than a persistence of coupling at high temperatures.

The magnitude and shape of the function \( f(T) \) depend upon the physical realization of the 2D liquid state in the layers. In pin free superconductors this function is universal and depends only on the dimensionless parameter \( T (4\pi\lambda_{ab})^2 / s^2 \Phi_0^2 \). Below we calculate this universal function using Monte Carlo simulations. In real samples random pinning increases disorder in vortex arrangements even in the liquid state, which leads to extra suppression of \( f(T) \) and further smoothes its temperature dependence. On the other hand, the pinning effect becomes progressively weaker at higher temperatures.

Substituting (3) into (4) we obtain the high-T expansion for \( Q'_\omega \)

\[ Q'_\omega = \frac{\omega sj_j^2}{2cT} \int dr \int_0^\infty dt \sin(\omega t) S(r,t), \] (10)

with \( S(r,t) = \langle \exp[i(\phi_{nn+1}(r,t) - \phi_{nn+1}(0,0))] \rangle \) being the dynamic phase correlation function. One can immediately see that, indeed, in this regime \( Q'_{\omega=0} = 0 \), indicating decoupled layers.

The result (5) can be generalized to the case when the magnetic field is tilted with respect to the \( z \)-axis. In presence of the \( y \)-component of the magnetic field, \( B_y \), and in the lowest order with respect to \( E_J \), the phase difference acquires an extra contribution \( \phi_{nn+1} \rightarrow \phi_{nn+1} + \frac{2\pi}{\Phi_0} B_y x \), which gives an extra factor \( \exp(i\frac{2\pi}{\Phi_0} B_y x) \) in the RHS of Eq. (3). Approximating \( S_1(r) \) by a Gaussian function

\[ \langle \cos \phi_{nn+1} \rangle \approx \exp(-\frac{\frac{2\pi}{\Phi_0} B_y x}{2\Phi_0}) \] (11)

The Josephson coupling in tilted fields in presence of coexistent Abrikosov and Josephson lattices was studied in Ref. [5]. As follows from Eq. (11) the typical \( ab \)-component of the field is given by \( \sqrt{B_y} \Phi_0 / \pi \Phi_0 \) and coincides with the one found in Ref. [5] at high fields \( B_y > \Phi_0 / \gamma s^2 \). For \( B_y = 100 \, G \) this typical field is \( \sim 1.7 \, T \) which corresponds to a tilt angle \( \sim 0.3 \, ^\circ \). In contrast to the lattice state, the average Josephson current
does not experience oscillations caused by commensurability of Abrikosov and Josephson lattices but, instead, decreases monotonically with $B_0$.

One can expect that in the liquid state the phase slips are strongly assisted by thermal motion of pancake vortices. In real samples this motion is strongly hindered by pinning as follows from the thermally activated behavior of the in-plane component of resistivity $\rho_{ab}$ in a wide temperature range. An important experimental fact is that the temperature dependencies of $\rho_{ab}$ and $\rho_c$ are characterized by the same value of the activation energy $[16, 18]$. This indicates that the interlayer phase slips are, indeed, induced by motion of pancake vortices. These phase slips are very well described by a simple model which assumes that the dissipation processes are caused by a small concentration $n$ of mobile pancakes with a thermally activated diffusion constant $D$ $[18]$. Within this model the dynamic phase correlation function decays in time and space as

$$S(r, t) = \exp \left( -\frac{\pi B_0^2}{2\Phi_0} \ln \frac{r_{cut}}{r} - \pi n D t \ln \frac{r_{cut}}{D t} \right),$$

(12)

where $r_{cut}$ is the in-plane phase coherence length. Substituting the last equation into Eq. $[11]$ we obtain

$$Q' \approx \frac{s j^2 \Phi_0}{cTB} \frac{\omega^2}{\omega^2 + \omega_{ps}^2},$$

where the typical phase slip frequency, $\omega_{ps}$, is given by $\omega_{ps} = 2\pi n D\ln(n r_{cut}^2)$. Because the same mobile pancakes determine the dissipation along the $ab$-plane, this frequency can be connected with $\rho_{ab}$ $[18]$

$$\omega_{ps} = 2\pi \ln(n r_{cut}^2) \left( \frac{c}{\Phi_0} \right)^2 \frac{T}{s} \rho_{ab},$$

(13)

or $\omega_{ps} \approx 2 \cdot 4 \cdot 10^6 [1/s] \cdot T [K] \cdot \rho_{ab} [\mu\Omega \cdot \text{cm}]$. Taking $T = 45 K$ and $\rho_{ab} = 10^{-3} \mu\Omega \cdot \text{cm}$ corresponding to the resonance field $B \approx 0.1T$ $[3]$ we obtain $\omega_{ps} \approx 1 \cdot 2 \cdot 10^6 [1/s]$, which is indeed much smaller than the microwave frequency $\omega = 2\pi \cdot 4.5 \cdot 10^{10} [1/s]$ used in Ref. $[3]$. As follows from Eqs. $[13]$ $Q'' \approx (\omega_{ps}/\omega)Q'_a$ at $\omega \gg \omega_{ps}$, i.e. the plasma resonance is not suppressed by the dissipation in this regime. Eq. $[13]$ holds only in the “phase slip” regime, i.e., until the quasiparticle contribution to the $c$-axis transport can be neglected.

The analytical calculation of the correlation function $[7]$ in the liquid state is not possible even for a pin-free superconductor. To obtain quantitative information about the Josephson coupling in the liquid state and to check the high-T expansion we perform Monte Carlo simulations of the frustrated three dimensional XY model which was used to investigate phase transitions in the vortex state $[21, 22]$. The energy of the model is given by

$$E = \sum_{n, \alpha} V_0 (\phi(n + \delta) - \phi(n) - a_\alpha(n))$$

(14)

Here $\phi(n)$ are the phases of the order parameter located at the sites of a mesh $1 \leq n_x, n_y \leq N_x, 1 \leq n_z \leq N_z$, with periodic boundary conditions in all directions. The vector potential $a_\alpha(n) (\alpha = x, y, z)$ is taken in the Landau gauge $a_\beta(n_x) = 2\pi b n_x$, where the dimensionless magnetic field $b$ determines the fraction of the grid sites occupied by vortices. We chose $N_x = 72$, $N_y = 4, 10$, and $b = 1/36$ which gives 144 vortex lines. The in-plane phase-phase interaction function, $V_x(\theta) = V_y(\theta)$, determines the in-plane phase stiffness. The choice of the low angle asymptotics, $V_{x,y}(\theta) \approx -1 + \theta^2/2$, fixes the energy and temperature scale as $s\Phi_0^2/(\pi (4\pi \lambda)^2)$.

An unrealistic feature of the model is that vortices are subject to the periodic pinning potential created by the numerical grid. This effect can be reduced significantly by optimizing the shape of $V_x(\theta)$. We chose $V_x(\theta) = -3\pi/4 - (1 - r) \cos(\theta) - (r/4) \cos(2\theta)$, and adjusted the coefficient $r$ to minimize the energy barrier for the vortex jump to the neighbor site, which gives $r = 0.376$. The coupling between planes is given by the standard Josephson interaction, $V_\perp(\theta) = -1/2 \cos(\theta)$, with the amplitude determined by the anisotropy parameter $\Gamma$. The coupling between the two dimensional lattices in the layers is determined by a scaled magnetic field $h = B/B_c$. The dimensionless units $h = Tb$. The parameter $\Gamma$ should not be considered as the anisotropy of a real layered superconductor but rather as a parameter which allows us to vary the scaled magnetic field $h$. We apply the standard Metropolis algorithm to the Hamiltonian $[14]$. The evolution of the phase diagram with changing anisotropy will be reported elsewhere. In this Letter we will focus on the calculation of the quantity $\langle \cos \phi_{nn+1} \rangle$ and verification of the high-T expansion for this parameter.

Fig. $[4]$ shows the numerically obtained temperature dependencies of the low frequency superconducting re-
sponse $Q' = Q'_{\omega=0}$ and the parameter $\langle \cos \phi_{nn+1} \rangle$ for two values of the anisotropy ($\Gamma = 100$ and $225$). As one can see from this plot the temperature dependencies of these parameters are almost identical above the level $\langle \cos \phi_{nn+1} \rangle = 0.6$. Below this level $Q'(T)$ starts to drop fast, indicating the decoupling transition, and $\langle \cos \phi_{nn+1} \rangle$ still decreases smoothly. For high anisotropies $\langle \cos \phi_{nn+1} \rangle$ has a small drop at the decoupling transition. One can see also that for $\Gamma = 100$, $Q'(T)$ shows a strong size effect for $n_z < 10$, which indicates that this parameter is not simply determined by the interaction of two neighbor layers but rather characterizes the superconducting coherence along the whole sample thickness.

To verify the high-$T$ expansion we calculate the phase correlation function $\langle \cos \phi \cos \phi \rangle$ at different temperatures and integrate it to determine the universal function $f(T)$ in Eq. 15. The last equation takes the following form in dimensionless units

$$\langle \cos \phi_{nn+1} \rangle = f(\tilde{T})/(2\Gamma \tilde{b}).$$

We found that the temperature dependence of $f(\tilde{T})$ is, indeed, rather weak and can be approximated by the formula $f(\tilde{T}) = 0.862 - 0.976 \tilde{T}$ for $0.1 < \tilde{T} < 0.6$. For parameters corresponding to BiSSCO, this interval covers the temperature range $24 \text{ K} < T < 66 \text{ K}$. In Fig. 2 we plot the temperature dependencies of the directly calculated $\langle \cos \phi_{nn+1} \rangle$ at different values of $\Gamma$ together with its high-$T$ expression (15). As one can see from this plot, all curves merge with the same universal curve at high enough temperatures which is very well approximated by the high-$T$ expansion. Systems with lower anisotropy merge with this curve at higher temperatures.

![FIG. 2. Temperature dependencies of the parameter $\Gamma \langle \cos \phi_{nn+1} \rangle$ for different $\Gamma$. High-T expansion curve for this parameter given by Eq. 15 is also plotted.](image)

We can use these results for quantitative analysis of the plasma resonance data at high temperatures. Using the relation $J_{\parallel} = 2eB_0/ [s(4\pi \lambda_{ab})^2]$ one can extract the anisotropy ratio $\gamma$ from Eq. 15. Taking the resonance field $B_0 = 0.065 \text{ T}$ at $T = 53.9 \text{ K}$ and $\omega/2\pi = 45 \text{ GHz}$ from Ref. 3 and using the material parameters $\epsilon = 20$ and $\lambda_{ab}(T) = 1800A/\sqrt{1-(T/T_C)^2}$ we calculate from the simulation results $\langle \cos \phi_{nn+1} \rangle = 0.11$ and $\gamma \approx 330$.

This work was initiated by discussions with L. Bulaevskii. Very useful discussions with M. Tachiki, Y. Matsuda, and M. Gaifullin are also appreciated. The work was supported by the National Science Foundation Office of the Science and Technology Center under contract No. DMR-91-20000, and by the US Department of Energy, BES-Materials Sciences, under contract No. W-31-109-ENG-38. The author gratefully acknowledges use of the Argonne High-Performance Computing Research Facility. The HPCRF is funded principally by the U.S. Department of Energy Office of Scientific Computing.

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