Cavity field ensembles from nonselective measurements

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Abstract
We continue our investigations of cavity QED with time dependent parameters. In this paper we discuss the situation where the state of the atoms leaving the cavity is reduced but the outcome is not recorded. In this case our knowledge is limited to an ensemble description of the results only. By applying the Demkov-Kunike level-crossing model, we show that even in this case, the filtering action of the interaction allows us to prepare a preassigned Fock state with good accuracy. The possibilities and limitations of the method are discussed and some relations to earlier work are presented.

1 Introduction
Cavity QED has become a standard tool of Quantum Optics. It allows the experimentalists to control both atomic and field variables to a high degree, and consequently the system can be utilized to explore many basic features of matter-field interactions. In the limit when the radiation-induced time scales are fast enough, we can neglect the various relaxation rates affecting the system, and then it can be used as a laboratory realization of the well known Jaynes-Cummings model.

In cavity QED physics, the model has usually been introduced with time independent parameters. Its utility is, however, preserved if we allow them to vary at a suitable rate. They should be fast compared with the relaxation processes of the system but slow enough not to destroy the mode structure of the cavity. In our works [1] and [2], we have considered the new possibilities offered by such a model.

Because the Jaynes-Cummings model separates into decoupled two-level systems, we advocated in reference [1] that it may be used to design a filter mechanism to shape the photon distribution in the cavity into a desired form. There are earlier investigations into the effect on the cavity field caused by recording of the atomic state, see for example [3] and [4], but we explore the possibilities deriving from time dependence of the coefficients. In reference [2], we consider the possibility that the desired effects may be derived simply from the shape of the eigenmodes of the cavity field.
In our earlier works, we have assumed that we record the state of every atom exiting the cavity after an interaction. In practical situations, however, the environment rapidly destroys the coherence between the atomic states after leaving the cavity, even when no measurement is performed. In this case the state of the field is reduced to one of its possible configurations without us knowing which one has occurred. We have a situation of nonselective measurements. After the passage of \( m \) atoms through the cavity, our uncertainty about the actual state of the cavity becomes large. This, however, is just the situation to be described by a statistical ensembles of fields, each member occurring with a probability determined from the sequence of atomic reduction events which brought it about. In this paper we consider the evolution of the field in such a situation.

The approach may be useful even if we, for some reason, do not want to record the results of the individual interaction sequence. In particular, we address the question of creating a pure Fock state of the field in the cavity. This problem has been discussed before [5], but all methods tend to become difficult if one aims at large quantum numbers. Our results may be considered to be a contribution to the discussion aiming at the creation of Fock states.

In section 2, we set up the problem and introduce the basic concepts. In section 3 we specialize the Hamiltonian to the solvable Demkov-Kunike model and discuss its properties in its adiabatic and non-adiabatic limits. In the former case we show that it is, ideally, possible to reach arbitrary \( n \)-states of the field. Special cases of the latter situation have been discussed earlier [4] and [6], and hence we only make a numerical investigation of the properties of the result specifically addressing the question of obtaining a Fock-state. Here the concept of trapping states [7] becomes central, and we suggest a way to improve the method by velocity selection of the atoms sent into the cavity. In section 4 we conclude the presentation with a discussion.

2 Adiabatic Jaynes-Cummings model

We consider a system described by the Jaynes-Cummings model [8], which is the simplest formulation of the interaction between matter and a quantized field. The model assumes that the time of interaction is much shorter than other time-scales in the system, such as atomic and cavity field relaxation times. This means that atomic and field losses can be neglected during the interaction. The Hamiltonian for the full system is (\( \hbar = 1 \))

\[
H_{JC} = \Omega a^\dagger a + \frac{\omega}{2} \sigma_3 + g \left( a^\dagger \sigma^- + a \sigma^+ \right),
\]

(1)

where the \( \sigma \)'s are the ordinary Pauli matrices and \( \{a, a^\dagger\} \) are the Boson operators of the cavity mode. Within this rotating wave approximation, the number of excitations is conserved; the operator \( N = a^\dagger a + \frac{1}{2} \sigma_3 \) is a constant of motion. We therefore define the interaction Hamiltonian \( H_I = H_{JC} - \Omega N \), which separates the dynamics into two-level manifolds within the bare states.

The state of the whole atom-field system can be expressed as

\[
|\Psi\rangle = c_0 a^\dagger(0)|0, -\rangle + \sum_{n=1}^\infty c_n \left[ a^\dagger(n)|n-1, +\rangle + a(n)|n, -\rangle \right]
\]

(2)
and after the passage of the atom, the state of the system is determined from $a_\pm^\infty(n)$. The coefficients $a_\pm(n)$ are given by the Schrödinger equation

$$\frac{d}{dt} \begin{bmatrix} a_+(n) \\ a_-(n) \end{bmatrix} = \begin{bmatrix} \frac{\Delta \omega}{2} & g \sqrt{n} \\ g \sqrt{n} & -\frac{\Delta \omega}{2} \end{bmatrix} \begin{bmatrix} a_+(n) \\ a_-(n) \end{bmatrix},$$  

(3)

Here the detuning $\Delta \omega = \omega - \Omega$ and the coupling $g$ are allowed to depend on time. Initially, the atom is taken to be either in its upper or lower level

Case (a) $|a_0^0| = 1$, $a_0^0 = 0$

Case (b) $a_0^0 = 0$, $|a_0^\pm| = 1$

and the initial field distribution is determined from $c_n$.

After the interaction, the atomic state is measured, and from equation (2) it follows that the initial state of the field is modified by $a_\pm^\infty(n)$ or $a_\pm^\infty(n)$, depending on whether the atom is found in its upper $|+\rangle$ or lower $|−\rangle$ state. After $m$ atoms have passed through the cavity and been measured, the field is multiplied by a series of $a_\pm^\infty(n)$s. It is important to note, that the number of atoms detected in their upper and lower levels is not enough to tell what the state of the field is. One also needs the order of upper and lower detection events. This follows from the fact that, if the initial and final atomic states are different, then not only will the field be modified by $a_\pm^\infty(n)$, but the photon distribution will also be shifted by one unit. For example, if an initial lower level atom is found in its upper level after the interaction, the distribution of the field is

$$P_{m,0}(n) = |a_\infty^+(n + 1)|^2 |a_\infty^+(n + 2)|^2 |a_\infty^+(n + 2)|^2 P_0(n + 2).$$  

(5)

If $\nu = \sum j k_j$, then the photon distribution after $m$ atoms can be written, in a compact form, as

$$P_{m,k}(n) = \frac{1}{N_k} A_{m,k}(n) P_0(n + \nu),$$  

(6)

where $A_{m,k}(n)$ is the appropriate sequence of "filter functions" $|a_\pm^\infty(n)|^2$ and $N_k$ is the normalization constant. If the field was initially in a Fock state, then, whatever the filter functions are, the final field will also be in a Fock state, shifted by an amount $−\nu$. 

3
If the initial conditions are as Case (a), the probability for measuring the atomic states $|\pm\rangle$, given a normalized photon distribution $P = P(n)$, is
\begin{align}
P(+|P) &= \sum_n |\alpha^\infty(n+1)|^2 P(n) \\
P(-|P) &= 1 - P(+|P).
\end{align}
(7)

By repeatedly applying the equation above, we generate the probability $P(k|P_0)$ of detecting the sequence $k$, if the initial photon distribution is $P_0(n)$. Note that this probability equals the normalization constant in equation (4)
\begin{equation}
P(k|P_0) = N_k.
\end{equation}
(8)

Now suppose that we perform a nonselective measurement of the atomic state, i.e. that is we do not record the final state of the atom. Then the field is described by a density operator, obtained by tracing over the atomic degrees of freedom
\begin{equation}
\rho^{field} = \text{Tr}_{\text{atom}}(\rho) = \sum_{n,n'} c_n c_n^* \left[ a^\infty(n) a^{\infty*}(n') |n-1\rangle\langle n' - 1| + a^\infty(n) a^{\infty*}(n') |n\rangle\langle n' | \right].
\end{equation}
(9)

Note that, if we only consider the diagonal elements, $\tilde{P}(n) = \rho_{nn}^{field}$, we do not need the phases of the $\alpha^\pm(n)$'s. This $\tilde{P}(n)$ is the ensemble photon distribution as will be explained below. For Case (a) in equation (4), the initial photon distribution is $P_0(n) = |c_{n+1}|^2$ and, after $m$ nonselective measurements, the ensemble photon distribution is obtained from the recurrence relation
\begin{equation}
\tilde{P}_m(n) = |a^\infty(n+1)|^2 \tilde{P}_{m-1}(n) + |a^\infty(n)|^2 \tilde{P}_{m-1}(n-1).
\end{equation}
(10)

A special case of this equation has been studied before, see [4] and [5]. It is of essential importance to point out, that in general $\tilde{P}_m(n) \neq \tilde{P}_{m,k}(n)$, for an arbitrary $k$. The distribution given in (10) is the ensemble average over all possible outcomes $P_{m,k}(n)$
\begin{equation}
\tilde{P}_m(n) = \sum_{\text{All } k} P(k|P_0(n)) P_{m,k}(n) = \sum_{\text{All } k} A_{m,k}(n) P_0(n + \nu) = \overline{P_{m,k}(n)},
\end{equation}
(11)
where we have used equations (4) and (5) in the second step and $\overline{P_{m,k}(n)}$ defines the ensemble average over all possible $k$'s. If we consider cases with a fixed series of outcomes defined by a given $k$, we have
\begin{equation}
P_{m,k'}(n) = P_{m,k}(n), \quad \forall k, k' \Rightarrow \tilde{P}_m(n) = P_{m,k}(n).
\end{equation}
(12)

This happens if for example $P(k|P_0) = 1$ for any $k$; then this outcome has unit probability and is the only possible one, all other outcomes have zero probability. The only possibility that $P_m(n)$ will describe a Fock state exactly, is if all $P_{m,k}(n)$ represent the same Fock state. In other words, if $\tilde{P}(n)$ describes a Fock state, this agrees with the actual state of the field.

With the above formalism, expectation values $\langle \ldots \rangle_m$ with respect to the distribution $\tilde{P}_m(n)$ are related to expectation values $\langle \ldots \rangle_{m,k}$ for the distribution $P_{m,k}(n)$ according to
\begin{equation}
\langle \ldots \rangle_m = \overline{\langle \ldots \rangle_{m,k}}.
\end{equation}
(13)

For example, the variance $\Delta n^2_m$ of the distribution $\tilde{P}_m(n)$ is given by
\begin{equation}
\Delta n^2_m = \overline{(n^2)_{m,k}} - \overline{(n)_{m,k}}^2.
\end{equation}
(14)
With the parameters introduced in (17) the filter functions may be written as

\[ \frac{\Delta \omega(t)}{2} = \bar{E} + E_0 \tanh \left( \frac{t}{\tau} \right) \]  

\[ g(t) = g_0 \text{sech} \left( \frac{t}{\tau} \right). \]

If we consider Case (a) of equation (14) and integrate over time \((-\infty, +\infty)\), the filter functions become

\[ |a_{\infty}^+(n)|^2 = \begin{cases} 
1 - \frac{\sinh \pi T (E_0 + \sqrt{E_0^2 - g_0^2 n}) \sinh \pi T (E_0 - \sqrt{E_0^2 - g_0^2 n})}{\cosh \pi T (E + E_0) \cosh \pi T (E - E_0)} & E > E_0 \\
\cosh \pi T (E + \sqrt{E_0^2 - g_0^2 n}) \cosh \pi T (E - \sqrt{E_0^2 - g_0^2 n}) & E < E_0 
\end{cases} \]

\[ |a_{\infty}^-(n)|^2 = 1 - |a_{\infty}^+(n)|^2. \]  

These expressions contain the three dimensionless parameters

\[ \Lambda_1 = \bar{E} T, \quad \Lambda_2 = E_0 T \quad \text{and} \quad \eta = g_0 T. \]  

Normally the adiabatic limit arises from large \( T \), or equivalently that, at least one of \( \bar{E} \) or \( E_0 \) becomes large. In the non-adiabatic limit \( \bar{E} \approx E_0 \approx 0 \). In the following we consider these two regimes separately.

### 3.1 The adiabatic limit

With the parameters introduced in (17) the filter functions may be written as

\[ |a_{\infty}^+(n)|^2 = \begin{cases} 
1 - \frac{\cosh \pi 2 \Lambda_2}{2 \cosh \pi (\Lambda_1 + \Lambda_2) \cosh \pi (\Lambda_1 - \Lambda_2)} & \Lambda_1 > \Lambda_2 \\
\cosh \pi 2 \Lambda_1 & \Lambda_1 < \Lambda_2 
\end{cases} \]

\[ \Lambda_2 = \frac{E_0}{\eta}. \]  

For no photons, \( n = 0 \), we have \( |a_{\infty}^+(0)|^2 = 1. \) For

\[ n > \left( \frac{\Lambda_2}{\eta} \right)^2, \]  

the last term is an oscillating cosine-term in \( n \). In the adiabatic regime, either \( \Lambda_1 \) or \( \Lambda_2 \) becomes large, and since the denominator grows exponentially in these parameters, the oscillating cosine-term can be neglected. This means that, for photon numbers \( n > (\Lambda_2/\eta)^2 \), the filter functions will become constant, determined by the first terms/term in (18). Then, if the parameters are chosen such that \( \Lambda_2/\eta \approx 1 \) and at least one of \( \Lambda_1 \) or \( \Lambda_2 \) is large, we have to a good approximation

\[ |a_{\infty}^+(n)|^2 = \begin{cases} 
1; & n = 0 \\
\kappa; & n \neq 0 
\end{cases} \]  

5
where $0 \leq \kappa \leq 1$. The number $\kappa$ depends on the parameters $\Lambda_1$ and $\Lambda_2$. For example, if $\Lambda_1 = \pm \Lambda_2$ then $\kappa = 0.5$ or if $\Lambda_1 < \Lambda_2$ (the levels cross) we have $\kappa = 0$, while $\Lambda_2 < \Lambda_1$ (no level crossing) gives $\kappa = 1$. If there are non-adiabatic contributions, it is possible to have a $\kappa$ that differs from the three cases given above. An important point is that, as long as the process is adiabatic, the $\kappa = 0, 1$ cases are not sensitive to the parameters of the problem. On the other hand, the $\kappa = 0.5$ case is sensitive to the condition $|\Lambda_1| = |\Lambda_2|$. The filter function (20) with $\kappa = 0.5$ can also be achieved by using a constant detuning $\Delta \omega$ and an asymmetric field coupling $g(t)$ in the Hamiltonian (3), see [11].

With the result (20), the equation (10) becomes

$$\tilde{P}_m(n) = \kappa \tilde{P}_{m-1}(n) + (1 - \kappa) \tilde{P}_{m-1}(n-1). \quad (21)$$

This equation can be solved by introducing a generating function

$$G_m(z) = \sum_{n=0}^{\infty} z^n \tilde{P}_m(n). \quad (22)$$

The equation (21) then becomes

$$G_m(z) = [\kappa + (1 - \kappa)z]G_{m-1}(z), \quad (23)$$

having the solution

$$G_m(z) = [\kappa + (1 - \kappa)z]^m G_0(z). \quad (24)$$

With the field initially in a vacuum, $P_0(n) = \delta_{n,0}$, we find after $m$ atoms the state

$$G_m(z) = [\kappa + (1 - \kappa)z]^m = \sum_{n=0}^{\infty} \binom{m}{n} \kappa^{m-n}(1 - \kappa)^n z^n. \quad (25)$$

As the coefficient of $z^n$ is the photon probability distribution, we find

$$\tilde{P}_m(n) = \binom{m}{n} \kappa^{m-n}(1 - \kappa)^n. \quad (26)$$

$\tilde{P}_m(n)$ is the form of a Binomial distribution with a variance

$$\Delta n_m^2 = m \kappa (1 - \kappa). \quad (27)$$

If $\kappa = 0$ we note that we have $\tilde{P}_m(n) = \delta_{n,m}$. From the discussion above, we know that if the ensemble state of the field is in a Fock state then the real state of the cavity field is in the same Fock state. In other words, after $m$ atoms has passed through the cavity, the state of the field, initially in vacuum, will be $|m\rangle$. A similar idea for producing a Fock state has been presented in [12].

Physically, the process can be understood from the fact that as each atom traverses the cavity, it is tuned across the field-mode frequency in an adiabatic manner such that it exits the cavity in its lower state. Then, after the passage, the mode will contain one extra photon. On the other hand, if we consider the initial condition according to Case (b) in equation (4) instead, the mode will be left with one photon less. This holds for all initial distributions of the field

$$P_m(n) = P_0(n - m) \quad \text{for Case (a)} \quad \text{(28)}$$

$$P_m(n) = P_0(n + m) \quad \text{for Case (b).} \quad \text{(28)}$$
Obviously, Case (a) will heat up the cavity mode, while Case (b) cools it down. If lower level atoms are sent through the cavity, the field mode will eventually end up in the vacuum. The process can then be used as a "field eraser" [13]. Note that the result in equation (28) agrees with the one in (6).

### 3.2 The non-adiabatic limit

If the atomic and field-mode frequencies are on resonance ($\bar{E} = E_0 = 0$), the filter functions are given by

$$|a_\infty^+ (n)|^2 = \cos^2 (\pi \eta \sqrt{n})$$

$$|a_\infty^- (n)|^2 = \sin^2 (\pi \eta \sqrt{n}),$$

(29)

where the initial condition has been chosen according to Case (a). The ensemble distribution [10], with the filter functions as in equation (29), has been investigated in [14] and [15], so we will not study it in great detail here. However, we specifically investigate its prospects to speed up the production of a Fock state. Figure 1 shows how the distribution $\tilde{P}_m(n)$, initially in a coherent state with $\bar{n} = 47$, evolve as the number of atoms $m$ increases. The filter functions are given by (29) with $\eta = 1$. We see how the distribution builds up around photon numbers $n' = 35, 48$ and 63. These $n'$'s fulfill the trapping condition [7]

$$\sqrt{(n' + 1)\eta} = q, \quad q = 1, 2, 3, \ldots$$

(30)

The corresponding states $\{|n'\rangle\}$, fulfilling the trapping condition (30), are called trapping states and are labelled by ($n', q$). If the field is in a trapping state, then, since $|a_\infty^+ (n' + 1)|^2 = 1$, all atoms will exit the cavity in their upper level and the state of the field is just multiplied by a phase-factor. The integer $q$ is the number of the corresponding maximum of $|a_\infty^+ (n')|^2$, in other words the index of the trapping state. For a given trapping state ($n', q$), there are $q - 1$ other trapping states with photon numbers smaller than $n'$. $q$ also gives the number of Rabi cycles during the interaction [14]. When the atomic velocity can be controlled and consequently the effective interaction time $T \propto \eta$, we may change the number of Rabi cycles $q$ in the trapping state ($n', q$), since $T \sim q$.

Since $|a_\infty^+ (n' + 1)|^2 = 0$ for the trapping states, it is clear from (10) that the states $|n\rangle$ are not coupled to the states $|n' + 1\rangle$. This results in a separation of the space $\{|n\rangle\}$ into disconnected blocks; population within one block will not leak into another. If $q$ is small, the distance between two consecutive trapping states is large, and may exceed the width of the initial photon distribution. Then, if this distribution lies in one such block between two maxima of $|a_\infty^+ (n')|^2$, it will eventually end up in the trapping state with the largest photon number $n'$. In this way, the situation allows us to create any Fock state. However, using this method, the preparation of Fock states with large photon numbers is difficult for two reasons:
1. When the photon distribution "approaches" the trapping state, more and more atoms will exit the cavity in their upper state and leave the field approximately unchanged. This follows from the small curvature of the of the filter functions around one maximum with a large $n'$ and a small $q$. The larger the photon number $n'$ the smaller the curvature, and the number of atoms needed for creating the Fock state is growing rapidly. This means that relaxation of the field may not be negligible.

2. The error in the atomic velocity, or in the interaction time $T$, propagates to $n'$. When this error is of the order of unity, the trapping condition is clearly destroyed. This undesired effect manifests itself for large photon numbers, since the uncertainty in $n'$ grows linearly with the photon number.

The small curvature of the filter functions slows down the process, so if the curvature could be increased as the distribution approaches the trapping state, the number of atoms needed would be reduced. This can be achieved by changing the atomic velocity, and consequently the trapping state, in such a way that $(n', q) \to (n', q')$, where $q' > q$.

In figure 2 we have plotted $\tilde{P}_m(10)$, when the initial state was a coherent one with $\tilde{n} = 4$. The solid line (a) gives the probability when all atoms have the same velocity, such that $n' = 10$ fulfills the trapping condition with $q = 1$. For the solid line (b), the velocities of consecutive atoms are chosen such that $q$ is increased by one unit for each atom. Starting with $q = 1$ it means that the first atom has a velocity in which the state $(n', q) = (10, 1)$ fulfills the trapping condition, and the velocity of the second atom is changed such that $(10, 2)$ instead satisfies the condition. The trapping states thus occur according to

$$(10, 1) \to (10, 2) \to (10, 3) \to \ldots$$

for each atom. This sequence may not be the most efficient one, but it clearly shows how the process is speeded up. The other two lines (dot-dash) in figure 2 describe the same processes as the ones above, but with a 2% Gaussian random error in the effective interaction time $T$. The error in $T$ clearly "washes out" the trapping condition as mentioned above.

4 Conclusion

In this paper we have continued our investigation into the possibilities offered by letting the parameters of an atom-cavity system depend on time. It is essential that this variation is slow enough to retain the identity of the cavity modes and atomic states; adiabatic changes are assumed. On the other hand, the interaction times considered have to be short enough that relaxation processes can be neglected.

In our earlier publications we have investigated how such time dependence combined with observation of the state of the outgoing atoms may be used as a photon state filter. Choosing a suitable time variation, desired photon states may be achieved. In this paper we continue the investigation by assuming
reduction of the atomic state but no recording of the result. This may take place spontaneously or by special design. The result is, however, that knowledge about the cavity state is assembled into a density matrix, representing the ensemble of events compatible with the events having occurred. In general this conveys less information about the state than a selective measurement does. However, if the method can be used to prepare a pure Fock state, the result becomes uniquely defined.

The analysis utilizes a Jaynes-Cummings model with time dependent coefficients. This splits the problem into blocks that can be analysed separately. Using the well known level crossing model presented by Demkov and Kunike, we may look separately at the adiabatic and non-adiabatic limits of this model. Our numerical investigations suggests some scaling of the process:

In the adiabatic limit it takes \( n \) atoms to prepare the \( n \)th Fock state. In the non-adiabatic limit and with the trapping state fixed, the number of atoms grows with \( n^2 \) which may conflict with the requirement of no relaxations occurring. If, however, the consecutive atoms utilize consecutive trapping states, i.e. their velocities are fine tuned before entering the cavity, the number of atoms may be reduced to \( n \) again.

We have not been able to solve the general problem: given that we want to reach an arbitrarily chosen final photon state, is there a time dependence which guarantees this? We have only illustrated the utility of the method for the preparation of a selected Fock state.
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Figure 1: The probability distribution $\tilde{P}_m(n)$ is plotted versus the number of atoms $m$. Here the dimensionless parameter $\eta = 1$. The distribution is built up around the photon numbers $n = 35$, 48 and 63. These photon numbers all fulfill the trapping condition $\text{(30)}$.

Figure 2: This figure shows how the probability $\tilde{P}_m(10)$, i.e. having $n = 10$, evolves with the number of atoms $m$. For the solid line (a), the velocities of the atoms are all the same and chosen such that $n = 10$ fulfills the trapping condition $\text{(30)}$ with $q = 1$. For line (b), the velocities are changed for each atom in such a way, that the trapping states $(n, q)$ occur in the sequence $(10, 1) \rightarrow (10, 2) \rightarrow (10, 3) \ldots$, for consecutive atoms. The two dot-dash lines are the same as the solid lines, but with a $2\%$ Gaussian random error in the atomic velocity and consequently the interaction time. The spread in velocities clearly destroys the trapping effect.
