R&D subsidies under asymmetric Cournot competition
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With a three-stage game model, this article theoretically assesses the effectiveness of different research and development subsidy strategies under asymmetric duopoly. The findings indicate that subsidising the small firm instead of the large is the optimum for the maximisation of social welfare in general. Meanwhile, if the initial marginal costs of the two firms are close to each other, providing subsidies to the small firm leads to more social R&D investment and higher aggregate production, but lower consumer surplus. Conversely, while the cost gap of the duopoly is large, subsidising the big firm becomes the preferable option for the authority to stimulate both R&D investment and total output of the industry, while sacrificing consumer surplus.

Keywords: Research and development subsidies; asymmetric duopoly; social welfare
JEL classification: D43; L13; L52; 038

1. Introduction
Realising the significant contribution of innovation to economic growth (Chen & Nie, 2014), both developed countries and developing countries formulate a lot of public policies to stimulate corporate research and development (R&D) investment. Among them, R&D subsidy is one of the most commonly used measures, due to the fact that firms may under-invest in innovation activities in the absence of R&D subsidies (Antonelli & Crespi, 2013; Jou & Lee, 2001). Some countries, like Israel, Korea, and Japan have accumulated rich experience and achieved great success in the utilisation of R&D subsidies.

However, not all studies support the idea to provide R&D subsidies. Some researchers argue that offering R&D subsidies to firms may result in unfair competition, since some participants may strengthen their market power by external incentives (Klette, Møen, & Griliches, 2000). Therefore, the following issues arise: Whether it is an effective policy to offer R&D subsidies? Under asymmetric competition, what are the effects of R&D subsidies on firms’ R&D investment and the market equilibrium? Further, what is the optimal choice for the policymaker? To address these issues, we employ a three-stage game model to examine the effectiveness of R&D subsidies under asymmetric competition.

The remainder of this article is organised as follows. Section 2 summarises the related literature. Section 3 describes the model under asymmetric duopoly with a three-stage game. Then, section 4 derives the equilibrium solutions of the model, taking differentiated subsidy patterns into consideration. By mathematic analysis and numerical

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simulation, a comparison and further discussions are presented in section 5. The final section gives concluding remarks.

2. Literature review
Numerous studies focus on the effectiveness of R&D subsidies, but the findings and policy implications are differentiated. Some literature tests the effectiveness of R&D subsidies empirically. Blanes and Busom (2004) studied the case of Spanish manufacturing firms, concluding that even firms in the same industry face different hurdles to participate in R&D subsidy programmes. González and Pazó (2008) also selected Spanish manufacturing firms to examine the effects of R&D subsidies by a matching approach. The results implied that subsidies would not crowd out private R&D expenditure. Alecke, Mitze, Reinkowski, and Untiedt (2011) analysed the effects of R&D subsidies on East German firms. The empirical findings showed that subsidised firms are more active in R&D activities.

Meanwhile, some empirical studies investigate the effects of R&D subsidies on small- and medium-sized enterprises (SMEs). Justman and Zuscovitch (2002) evaluated the effect of subsidies on industrial R&D of Israel. The estimation result indicated that small firms contribute 28% gains with 17% subsidies. Likewise, based on the data of Israeli manufacturing firms, Lach (2002) found that R&D subsidies greatly stimulate the expenditures on innovation of SMEs but reduce the R&D inputs of large firms. Since large firms would undertake R&D projects without subsidy but small firms would not, R&D subsidies are more likely substitutions for the innovation expenditures of large firms but complements for small firms. Kang and Lee and Cin (2010) demonstrated that R&D subsidies could stimulate the innovation performance of SMEs through sharing risk and reducing capital cost.

Besides, some researchers discuss this issue by establishing mathematic models with game theory. Under the assumption of symmetric duopoly, Yang (2014) took government preference into consideration and found that stronger preference to consumer surplus leads to a higher R&D subsidy rate. Gretz, Highfill, and Scott (2012) applied the Monte Carlo simulation to investigate the two R&D subsidy structures, cost sharing and reward for performance. They noticed that government subsidy and firm investment for R&D are complementary in the former structure but substitutionary in the latter structure. Klette et al. (2000) argued that, due to spillover effects and competition, both sponsored firms and those unfunded firms would be affected by R&D subsidies, thus changing the market structure and equilibrium.

Unlike empirical analysis, most theoretical studies discuss the effectiveness of R&D subsidies without considering the differences between competing firms, while only a few of them take heterogeneity into consideration. Lahiri and Ono (1999) investigated a two-stage model under asymmetric duopoly. They demonstrated that the firms with lower marginal cost should be subsidised for more R&D investment, while those without cost advantage should be taxed instead. Chor (2009) studied the effects of subsidies for heterogeneous multinational corporations, highlighting the conclusion that there exists welfare gain induced from selection effects. By classifying firms into two groups with differentiated cost, Ishida, Matsumura, and Matsushima (2011) found that an increase in the number of high-cost firms could benefit low-cost firms with more R&D investments and net profits.

There is no doubt that the effectiveness of R&D subsidy is influenced by many factors, like firm characteristics, industry attributes, R&D evaluation criterion, and firm
location (Herrera & Nieto, 2008; Lee, 2011; Wanzenböck, Scherngell, & Fischer, 2013). Zúñiga-Vicente, Alonso-Borrego, Forcadell, and Galán (2014) reported a literature review about the relationship between R&D subsidies and private R&D spending. They summarised the key issues, including: subsidy history, time lag, components of R&D, financial constraints, subsidy amount and the sources of funding. Absolutely, firm characteristics, such as size and productivity, determine the effectiveness of R&D subsidy partly and basically. However, as mentioned above, most theoretical models just ignore this by assuming the industry structure is symmetric.

Another issue is that most literature only concerns the equilibrium with a given R&D subsidy, which is regarded as an exogenous variable. The policy implications of the models are limited for the reason that it is hard for the government to decide the optimal subsidy rate. Assuming R&D subsidy as an endogenous variable may do help to solve the problem.

By relaxing the assumption of symmetry, this article contributes to the theory concerning the effectiveness of R&D subsidies with a three-stage game model. We search for the optimal subsidy policy for policymakers by comparing the welfare effects under different R&D subsidy strategies. As a result, some interesting conclusions arise and the optimal subsidy rate is captured.

3. The model
In this section we formally establish the model. Consider a market with two asymmetric firms. The two firms can be identified from two aspects, initial marginal cost and R&D efficiency. Generally, the one with lower initial marginal cost and higher R&D efficiency has advantages over its rival. As a result, it occupies a higher market share than its competitor with more outputs. Without loss of generality, we denote the smaller one as firm A and the larger as firm B.

The order of the three-stage game is outlined as follows. In stage 1, the government formulates the R&D subsidy policy. In stage 2, both firm A and B determine their R&D investment in accordance with the subsidies. In stage 3, the two firms choose their production quantities and compete in the product market.

**Consumers.** Write the utility function of the representative consumer as:

\[ U(p, q_A, q_B) = \alpha(q_A + q_B) - \frac{1}{2}(q_A + q_B)^2 - p(q_A + q_B), \]

where \( \alpha \) is a positive constant and \( p \) stands for the market price. Meanwhile, \( q_A \) and \( q_B \) represent the output of firm A and firm B respectively. Hence, the market supply is \( Q = q_A + q_B \). Taking partial derivative of the utility function with respect to \( q_A \) or \( q_B \), we can get the inverse demand function as follows:

\[ p = \alpha - q_A - q_B. \]

**Producers.** Both firm A and B aim to maximise their net profits. Respectively, their profit functions are given by:

\[ \pi_A = (\alpha - q_A - q_B)q_A - (m_A - \frac{1}{2}k_A)q_A - (1 - s_A)k_A^2, \]

\[ \pi_B = (\alpha - q_A - q_B)q_B - (m_B - \frac{1}{4}k_B)q_B - (1 - s_B)k_B^2, \]
where \( k_A \) is the R&D investment of firm A, and \( k_B \) is the R&D investment of firm B. Meanwhile, \( s_A \) and \( s_B \) denote the R&D subsidies for firm A and firm B respectively. Notice that \( s_A, s_B \in [0, 1] \) and we can get the R&D spending expressions for the two firms as \((1-s_A)k_A^2\) and \((1-s_B)k_B^2\). Further, there is no fixed cost at the beginning, and firm A’s marginal cost is \( c_A = m_A - r_Ak_A \), while \( m_A \) represents its initial marginal cost and \( r_A \) stands for its R&D efficiency. Likewise, firm B’s marginal cost is \( c_B = m_B - r_Bk_B \). Since firm B benefits more from economies of scale as a big enterprise, its initial marginal cost is lower than firm A, thus implying \( m_B < m_A \). Meanwhile, due to the fact that the effect of learning curve weakens as the production increases, firm B faces more difficulties to decrease the production cost per unit than firm A. Therefore, with a given R&D investment, the production cost per unit of firm A declines faster than firm B. That is, there exists \( r_A > r_B \), which is also valid in accordance with many empirical studies (see Justman & Zuscovitch, 2002). To simplify the analysis, we set: \( r_A = 1/2 \) and \( r_B = 1/4 \).

**Government.** For the government, to achieve the maximum social welfare is always an appropriate objective. In the process of formulating R&D subsidy policy, the government needs to take several factors into consideration, including consumer surplus, profits of producers and the expenditure of itself. Therefore, the function of social welfare can be defined as:

\[
SW = \int_0^{q_A+q_B} p(Q)d(Q) - p(q_A + q_B) + \pi_1 + \pi_2 - s_Ak_A^2 - s_Bk_B^2. \tag{5}
\]

### 4. Equilibrium solutions

Now we characterise the equilibrium solutions by backward induction approach formally. The government has three alternatives, namely providing no R&D subsidy, subsidising SMEs, and subsidising large firms. Accordingly, we discuss the model under the three possible cases, aiming to make a comparison of them and find out the optimal policy. Notice that we only consider asymmetric subsidies and exclude the case of subsidising both firms at the same time in our model.

#### 4.1. Equilibrium without R&D subsidy

Without R&D subsidy, the model degenerates into a two-stage game with \( s_A = s_B = 0 \). The two firms decide their R&D investments in the first stage and then compete in quantities in the second stage. The equilibrium solutions of R&D investments are given by:

\[
k_A^{*,1} = \frac{2}{373} (23\alpha - 47m_A + 24m_B), \tag{6}
\]

\[
k_B^{*,1} = \frac{4}{373} (5\alpha - 11m_B + 6m_A). \tag{7}
\]

Analogously, the optimal solutions of yield and net profits of the two firms are shown as:

\[
q_A^{*,1} = \frac{6}{373} (23\alpha - 47m_A + 24m_B), \tag{8}
\]
\[ q_{A}^{*1} = \frac{24}{373} (5x - 11m_{B} + 6m_{A}), \]  
\[ \pi_{A}^{*1} = \frac{32}{373^2} (23x - 47m_{A} + 24m_{B})^2, \]  
\[ \pi_{B}^{*1} = \frac{560}{373^2} (5x - 11m_{B} + 6m_{A})^2. \]  

Definitely, the equilibrium solutions of firm A and firm B are mainly determined by three parameters, namely \( \alpha, m_{A} \) and \( m_{B} \). Since the two firms compete in a Cournot fashion, their investment decisions and production decisions are not only depend on themselves, but also partially determined by the rival’s initial marginal cost. The social welfare is outlined as:

\[ SW^{*1} = \frac{2}{373^2} (32,105x^2 + 50185m_{A}^2 + 46696m_{B}^2 - 35,594xm_{A} - 28,616m_{B} - 64776m_{A}m_{B}). \]  

4.2. The case of subsidising firm A

In this case, only firm A receives R&D subsidy from the government, implying \( s_{A} \in (0, 1] \) in equation (3) and \( s_{B}=0 \) in equation (4). That is, the government focuses on stimulating the innovation ability of SMEs instead of large enterprises. In the first phase, the government sets the R&D subsidy rate. Then, firm A and firm B determine their innovation input with the given subsidies. In the last phase, the two firms compete with each other in the market. The equilibrium solutions of R&D subsidy rate and innovation investments can be derived as:

\[ s_{A}^{*2} = \frac{71(3x - 71m_{A} + 68m_{B})}{3(2,123x - 6067m_{A} + 3944m_{B})}, \]  
\[ k_{A}^{*2} = \frac{2}{33,133} (2,123x - 6067m_{A} + 3944m_{B}), \]  
\[ k_{B}^{*2} = \frac{4}{33,133} (443x - 1003m_{B} + 560m_{A}). \]  

Accordingly, the equilibrium production and net profits of the two firms are:

\[ q_{A}^{*2} = \frac{8}{33,133} (1,539x - 3290m_{A} + 1751m_{B}), \]  
\[ q_{B}^{*2} = \frac{24}{33,133} (443x - 1003m_{B} + 560m_{A}), \]  
\[ \pi_{A}^{*2} = \frac{16}{3 \times 33,133^2} (1,539x - 3290m_{A} + 1751m_{B})(16,345x - 33413m_{A} + 17068m_{B}). \]
Therefore, the social welfare is given by:

\[
SW^* = \frac{4}{33,133} (3,823x^2 + 6067m_A^2 + 5644m_B^2 - 4,246zm_A - 3,400zm_B - 7888m_Am_B).
\]  

Compared to the case without any R&D subsidy, the equilibrium is quite different. Even though the government only provides R&D subsidies to firm A, both of the two competitors change their investment and production strategies. Since firm A may invest more in innovation with the financial support, firm B needs to make some adjustments to fight back and keep its market share. Otherwise, it may lose its original advantages in the competition. As a result, social welfare adapts to a new equilibrium.

4.3. The case of subsidising firm B

Besides the two cases above, another possible strategy for the government is to subsidise firm B instead of firm A. Since the innovation ability of firm B is stronger than firm A, providing R&D subsidies to firm B seems a reasonable choice. Meanwhile, as a large enterprise with abundant resources, it is much easier for firm B to obtain subsidies from the government than firm A. What we are concerned with is whether firm B could improve its position in the competition. As assumed, we set \( s_A = 0 \) in equation (3) and \( s_B \in (0, 1] \) in equation (4). We get the optimal solutions of R&D subsidy rate and R&D investments as

\[
s_B^* = \frac{17(71m_A - 68m_B - 3z)}{12(230m_A - 331m_B + 101z)}, \quad (21)
\]

\[
k_A^* = \frac{2}{7,861} (485z - 997m_A + 512m_B), \quad (22)
\]

\[
k_B^* = \frac{4}{7,861} (101z - 331m_B + 230m_A). \quad (23)
\]

Definitely, the optimal R&D subsidy rate for firm B is determined by \( m_A \), \( m_B \) and \( z \). That is to say, while the government chooses firm B as the only one that can get financial support, it also needs to take firm A into consideration. Otherwise, the competition equilibrium cannot reach an appropriate position to maximise social welfare. Respectively, the equilibrium production and net profits are given by:

\[
q_A^* = \frac{6}{7,861} (485z - 997m_A + 512m_B), \quad (24)
\]

\[
q_B^* = \frac{2}{1861} (1,263z - 2816m_B + 1553m_A), \quad (25)
\]

\[
\pi_A^* = \frac{32}{7,861^2} (485z - 997m_A + 512m_B)^2, \quad (26)
\]
\[ x_B^* = \frac{4}{3 \times 7,861^2} (1,263x - 2816m_B + 1553m_A)(3,688x - 8117m_B + 4429m_A). \] (27)

Taking consumer surplus and producer surplus into consideration, the equilibrium solution of social welfare is:

\[ SW^* = \frac{4}{7,861} (907x^2 + 1423m_A^2 + 1324m_B^2 - 1,006xm_A - 808xm_B - 1840m_Am_B). \] (28)

5. **Comparative analysis**

In section 4, we solve the competition equilibrium of the model in section 3. Aiming to obtain the optimal R&D subsidy policy, we make a comparative analysis in this section. The analysis is carried out from the perspectives of stakeholders, including the government, consumers and producers. Besides the mathematical analysis, we conduct a model simulation to get specific results.

5.1. **Mathematical analysis**

**Government.** For the government, to maximise social welfare is an appropriate objective in the process of formulating and implementing public policies. The equilibrium solutions of social welfare under different R&D subsidy patterns have been derived in section 4. By mathematical comparison, we can get the following conclusion.

**Proposition 1.** If \( m_A = \frac{1}{\pi} (3x + 68m_B) \), social welfare sticks to the same level no matter which kind of R&D subsidy policy is implemented. Besides, subsidising firm A can always create higher social welfare than the other two strategies.

In most cases, subsidising the small enterprise, represented as firm A in our model, is the optimal strategy for the government. Only under the condition \( m_A - m_B = \frac{3}{\pi} (x - m_B) \), it makes no difference among the three subsidy patterns from the perspective of social welfare. To explain this, two causes should be highlighted. Firstly, as assumed in the model, subsidising the small enterprise could reduce more production costs per unit than subsidising the large enterprise. Secondly, subsidising the small firm does help to stimulate the competition, thus bringing higher social welfare. Therefore, providing R&D subsidies to SMEs is better than subsidising large firms (Keizer, Dijkstra, & Halman, 2002; Justman & Zuscovitch, 2002; Lach, 2002).

In addition to social welfare, the total R&D investment of the society is also a key consideration to the government. Through aggregating the R&D investments of firm A and firm B, we can obtain the total investments under the three subsidy patterns. For simplicity, we set \( K(0) \), \( K(A) \) and \( K(B) \) as the cases of no subsidy, subsidising only firm A and subsidising only firm B respectively. The results are shown as follows.

\[ K(0) = \frac{2}{373} (33x - 35m_A + 2m_B), \] (29)

\[ K(A) = \frac{6}{1949} (59x - 97m_A + 38m_B), \] (30)
\[ K(B) = \frac{6}{7,861} (229\alpha - 179m_A - 50m_B). \] (31)

Conducting comparison of the above formula, we have:

**Proposition 2.** The cost gap between firm A and firm B has a great impact on the subsidy strategy to stimulate the social R&D investment. While \( m_A \in (m_B, \frac{3\alpha + 68m_B}{7}) \), there exists \( K(A) > K(0) > K(B) \), indicating that subsidising firm A is the optimum. For \( m_A > \frac{1}{7} (3\alpha + 68m_B) \), we have \( K(B) > K(0) > K(A) \). With the condition \( m_A = \frac{1}{7} (3\alpha + 68m_B) \), the effects of all strategies are the same.

The policy implication of proposition 2 is obvious. While the cost gap of the duopoly is limited in a small range, subsidising the small one creates the most social R&D investment. With R&D subsidies, the small firm invests more in innovation activities, while its competitor has to do exactly the same thing under the competition pressure. As a result, giving subsidies to the small firm enhances total R&D investment of the industry. However, if the cost gap is beyond a certain scope, policymakers should subsidise the big one to maximise the total innovation investment. Because of the great cost disadvantage, it becomes inefficient to subsidise the small firm. At the critical point \( m_A = \frac{1}{7} (3\alpha + 68m_B) \), providing subsidies or not are identical. The result is consistent with the findings of Kilponen and Santavirta (2007), while they assessed the effects of R&D subsidies on R&D investment from the perspective of competition.

**Consumers.** Unlike the government, consumers always focus on consumer surplus, which is the major method to measure consumers’ utility and welfare. Likewise, we set \( CS(0) \), \( CS(A) \) and \( CS(B) \) to represent the strategies with no subsidy, subsidising firm A and subsidising firm B respectively. Notice that consumer surplus is \( CS = \int_0^{q_A + q_B} p(Q) d(Q) - p(q_A + q_B) \), and we can get the solutions under different R&D subsidy policies.

\[ CS(0) = \frac{18}{373^2} (43\alpha - 23m_A - 20m_B)^2, \] (32)
\[ CS(A) = \frac{128}{33,133^2} (1,434\alpha - 805m_A - 629m_B)^2, \] (33)
\[ CS(B) = \frac{8}{7,861^2} (1,359\alpha - 719m_A - 640m_B)^2. \] (34)

The above equations outline the equilibrium of consumer surplus under different situations. However, it is a tough work to make direct comparison of them. We conduct numerical simulations for further discussion after mathematic analysis.

**Producers.** To maximise net profits is the objective of producers. Here we only consider the impact of differentiated R&D subsidy policies on the total output and profits of the whole industry, without taking single firm into consideration. Similarly, denote \( Q(0) \), \( Q(A) \) and \( Q(B) \) as the total production under the differentiated subsidy strategies. The equilibrium solutions are:

\[ Q(0) = \frac{6}{373} (43\alpha - 23m_A - 20m_B), \] (35)
\[ Q(A) = \frac{16}{33,133} (1,434\alpha - 805m_A - 629m_B), \] (36)
By comparison, we can derive a unanimous conclusion for the whole industry as proposition 2.

**Proposition 3.** To maximise the total output of the duopoly, the optimal option depends on the cost gap between firm A and firm B. Specifically, for \( m_A \in (m_B, \frac{3a + 68m_B}{2}) \), we have \( Q(A) > Q(0) > Q(B) \). That is, offering subsidies to firm A is better than the other two strategies. If \( m_A = \frac{1}{7}(3a + 68m_B) \), there exists \( Q(A) = Q(0) = Q(B) \), implying no difference among them. With \( m_A > \frac{1}{7}(3a + 68m_B) \), the optimal choice converses with \( Q(B) > Q(0) > Q(A) \).

Similar to the conclusion of proposition 2, while the initial marginal cost of the small firm is not higher than its competitor too much, subsidising the small firm does help to stimulate innovation and competition, thus boosting total production. In comparison, providing subsidies to the large firm under this condition leads to a lower industry output, since the small one may quit production and withdraw from the market, leaving its rival as a monopolist. Conversely, if the cost gap is too large, the effect of subsidising the disadvantaged one is worse than the other two strategies. That is, the misallocation of R&D subsidies can be counterproductive (Catozzella & Vivarelli, 2011).

In addition, the total net profits under the differentiated R&D subsidies, denoted as \( R(0), R(A) \) and \( R(B) \), can be derived from the formula of total outputs. Because of the difficulties in comparison with complicated expressions, we do not report the specific results here. Instead, we apply a model simulation for further analysis, the same as the case in consumer surplus.

### 5.2. Model simulation

We conduct numerical simulations in this part. In the experiment, we set \( \alpha = 20 \), and \( m_B = 4 \). In order to find out what would happen if the cost gap between firm A and firm B keeps growing, we leave \( m_A \) as a dynamic variable as long as \( m_A \geq 5 \). This setting is to ensure that the initial marginal cost of firm A is higher than firm B, which is one of the assumptions of the model. Furthermore, to facilitate the distinctions in figures, green lines are set to represent the situation without any R&D subsidy, while yellow lines and red lines denote the cases under subsidising only firm A and subsidising only firm B respectively. In addition, we apply MATLAB 7.0 for simulations and plotting.

In mathematical analysis, we have captured the policy implication from the perspective of social welfare. Hence, social welfare is not the concern in the experiment. Instead, we focus on the effects of the differentiated subsidies on the other three aspects, including social R&D investment, consumer surplus and total profits of the industry. Figure 1 plots the effects of different R&D subsidy policies on social R&D investment.

Generally speaking, if the initial marginal cost of firm A is close to firm B, both firms need to invest in innovation under the great competition pressure, thus enhancing social R&D investment. With the expansion of the cost gap between firm A and firm B, social R&D investment decreases. If the cost gap is large enough, firm A may quit production and firm B would become the monopolist in the market. In such a situation, social R&D investment would drop down to zero due to the lack of competition. Furthermore, compared with the other two strategies, providing R&D subsidies to firm B is the most effective way to stimulate innovation activities in this specific case. To explain this, we have to highlight the cause that firm B has advantages with economies of scale,
while the effects of subsidising firm A would be cut down by the cost disadvantages. The result implies that R&D subsidy policy can be inefficient if the cost gap between the two competing firms is large enough (Antonelli & Crespi, 2013; Montmartin, 2010). In addition, satisfying the condition $m_A > \frac{1}{7} (3a + 68m_B)$, the findings of this case are fully consistent with proposition 2.

Figure 2 plots the results of consumer surplus under the differentiated subsidies. In the experiment, the preferable R&D subsidy policy for consumers is determined by the initial marginal cost of firm A. If the cost disadvantage of firm A is relatively insignificant, subsidising firm B is the optimum for consumers. Since firm B performs better than firm A in innovation and productivity, subsidising firm B does help to expand market supply and lower product price more faster. Clearly, consumers benefit from both more supply and lower price. However, if the cost disadvantage of firm A is significant, the market environment becomes tough for firm A, since it may lose all market shares in the competition. Offering subsidies to firm A turns to be the best choice for consumers in this situation, while possible monopoly may cause potential loss to consumer surplus. If the government insists to give R&D subsidy to firm B when the cost gap is large and obvious, firm B can easily occupy the whole market with strong market power, while both firm A and consumers have to suffer. That is why consumers’ preference to R&D subsidy patterns would change under different conditions.

To maximise net profits is the objective of producers. The simulation result of the whole industry’s net profits is shown in Figure 3. Existed literature has shown that R&D subsidies improve corporate performance with more outputs and profits (Colombo, Grilli, & Murtinu, 2011; Einio, 2014). Interestingly, the most favourable R&D subsidy policy for the whole industry is different from the government and consumers. For the whole industry, if the initial marginal cost of firm A is not much higher than its competitor, providing subsidies or not matters not that much, since the effects of the
differentiated policies are almost the same. However, if the cost gap of the duopoly is significant, offering subsidies to firm A becomes the optimum. Otherwise, firm B would enlarge its production and get extra market power with stronger cost advantage, while

Figure 2. Consumer surplus.
Source: Numerical simulations on the basis of the settings given in assumption.

Figure 3. Total profits of the industry.
Source: Numerical simulations on the basis of the settings given in assumption.
the market position of firm A tends to be marginalised. As a result, firm A may quit from the competition eventually and firm B may occupy the whole market. Then, the monopolist can adapt the price to get more revenue, thus enhancing net profits of the industry. Under such situation, subsidising firm A benefits not only the firm itself but also the whole industry for more net profits. That is why the government should subsidise firm A to stimulate competition.

6. Conclusion
This article establishes a three-stage game model to investigate the effects of different R&D subsidy strategies under asymmetric competition. The results of mathematical analysis and numerical simulations are in line with the previous studies of Lach (2002) and Justman and Zuscovitch (2002). Specifically, the findings indicate that, in the asymmetric duopoly market, subsidising the small firm instead of the large one does help to maximise social welfare in most cases, and it is conducive to enlarge the profits of the industry. Meanwhile, if the government intends to stimulate social R&D investment and total outputs, the optimal strategy depends on the cost gap of the asymmetric duopoly. Offering R&D subsidies to the large firm becomes the optimal choice for the authority if the cost gap is large enough. As a result, the policy implications vary in accordance with the relative market positions of the competitors.

In addition, there exist some limitations that remain for further extensions. Firstly, we assume that information is complete in the model, meaning all stakeholders have access to adequate information before making decisions. Since incomplete information matches with the reality better, the introduction of incomplete information in further studies is necessary. Secondly, the products are set to be homogenous in this article. For a better understanding of this topic, product substitutability should be considered.

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