The D1-D5 brane system in six dimensions

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Abstract

We consider scattering of minimal coupled scalars from a six-dimensional black string carrying one and five brane charges but no Kaluza-Klein momentum. The leading correction to the absorption cross section is found by improved matching of inner and outer solutions to the wave equation. The world sheet interpretation of this correction follows from the breaking of conformal invariance by irrelevant Born-Infeld corrections. We note that discrepancies in normalisation are caused by there being two effective length scales in the black string geometry but only one in the effective string model and comment on the implications of our results for the effective string model.

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I. INTRODUCTION

There has been a great deal of progress over the last year in understanding the strong coupling limit of large $N$ gauge theory [1], [2], [3], [4], [5]. Of particular importance is Maldacena’s conjecture [6] that the world volume theories of certain coincident branes are related to string theory or M theory on backgrounds consisting of anti-de Sitter spaces times spheres. Subsequent work in [7] and [8] elaborated on the precise nature of this correspondence and many more papers on the subject have followed. Much of the interest has been focussed on the D3-brane system and on the D1-D5 brane system with momentum along the string direction. In this paper we will be interested in the latter system, although to simplify the calculations from both the world sheet and the supergravity points of view we shall mostly consider the zero momentum extremal system, which corresponds to the zero temperature limit of the effective string model.

The relationship between correlators in the world sheet theory and low energy absorption in the entire black string or black hole metric has been extensively discussed in the literature. The first work in this direction appeared in [9] and was followed by papers demonstrating the precise agreement between the semi-classical and world sheet calculations of the minimal scalar absorption cross section [10], [11] for extreme black holes. In [12] the calculation was extended to near extreme black holes within the so-called dilute gas regime, and many further papers have explored scattering in other regions of the moduli space, including [13], [14], [15] and [16]. Recent discussion of the five dimensional black hole system in the context of the the relationship between the conformal field theory and anti-de Sitter supergravity can be found in [17].

As Maldacena and others have pointed out, if one takes the throat limit of the black string solution one can identify the part of the metric which determines the correlation functions in the conformal limit. However the string geometry is only anti-de Sitter out to a characteristic radius $R$ which is related to the number of D1 and D5 branes. When one specifies the world sheet theory with the DBI action powers of the string scale $\alpha'$ suppress the non renormalisable interactions. Even at strong coupling, when the string geometry is smooth, the corrections are detectable in the energy dependence of the absorption cross section. Such departures from world volume conformal invariance were discussed in detail for the D3-brane in [18] and our discussion parallels theirs in many respects. Related discussions on the absorption of two form perturbations by three-branes are to be found in [19] and absorption by extremal three branes is discussed generally in [20].

From the supergravity point of view we look for corrections to the absorption cross section of minimal scalars by matching the solutions to the wave equation more carefully between inner and outer regions. Just as in [18] we find that the wave equation has a self-dual point, at precisely the radius of the effective anti-de Sitter space. Improved matching leads us to find a leading order logarithmic correction to the absorption cross section. One would expect a correction of the same type for scattering within the related five dimensional black hole carrying three charges.

The logarithmic term encodes the leading order departure from the conformal limit. This breakdown can be interpreted in terms of the effective world sheet action for the string: non renormalisable interactions enter the action at subleading order. We look at the effect of
subleading couplings of minimally coupled scalars to operators of conformal dimension four and higher on the string world sheet. We find that such couplings allow us to reproduce the logarithmic form of the leading corrections to the absorption probability.

However the normalisations of the corrections predicted by the effective string model do not agree with those found semi-classically. The corrections will not agree unless we extend the effective world sheet (or worldvolume) theory to take account of both the five brane and one brane charges. Put another way, there are two length scales in the black string geometry, related to the two distinct charges, and any effective worldvolume model would have to take account of both.

Similarly, although one can predict the leading order cross section for a massless scalar using the duality of the near horizon geometry to a boundary conformal field theory, one cannot successfully predict corrections. The near horizon geometry depends only on one parameter, which appears in the dual conformal field theory, whereas the supergravity calculation depends on two.

The plan of the paper is as follows. In §II we discuss the supergravity background describing a black string carrying two charges in six dimensions and in §III we consider the supergravity analysis of scattering from the string. In §IV we consider the effective string analysis whilst in §V we consider the dual conformal field theory. We present our conclusions in §VI.

II. THE BLACK STRING SPACETIME

The low energy effective action for ten dimensional type IIB string theory contains the terms

\[ \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[ e^{-2\Phi} \left( R + 4(\partial\Phi)^2 \right) - \frac{1}{12} H^2 \right], \]

where \( \Phi \) is the dilaton and \( H \) is the RR three form field strength. The ten-dimensional solution in which we are interested is

\[ ds^2 = (H_1 H_5)^{-\frac{1}{2}} \left[ -dt^2 + dx^2 + H_1 \sum_{i=6}^9 dx_i^2 \right] + (H_1 H_5)^{\frac{1}{2}} [dr^2 + r^2 d\Omega_3^2], \]

where we give the metric in the string frame and the harmonic functions are given by

\[ H_1 = 1 + \frac{r_1^2}{r^2}, \quad H_5 = 1 + \frac{r_5^2}{r^2}, \]

The ten dimensional dilaton is

\[ e^{-2\Phi} = \frac{H_5}{H_1}, \]

which implies that when one wraps the five brane on a four torus the six dimensional dilaton \( \Phi_6 = \Phi - \frac{1}{4} \ln G_{int} \), with \( G_{int} \) the determinant of the metric on the torus, is constant. We will not need the explicit form of the three form in what follows. The effective six dimensional action in the Einstein frame is
$S_0 = \frac{1}{2\kappa_6^2} \int d^6x \sqrt{-g} \left[ R - (\partial \Phi_6)^2 + \ldots \right].$  \hspace{1cm} (5)

The solution for the six-dimensional black string in the Einstein frame is then

$$ds^2 = (H_1 H_5)^{-\frac{1}{2}}(-dt^2 + dx^2) + (H_1 H_5)^{\frac{1}{2}}(dr^2 + r^2 d\Omega_3^2),$$  \hspace{1cm} (6)

If $g$ is the ten-dimensional coupling strength then

$$\kappa_{10}^2 = 64\pi^7 g^2 \alpha'^4.$$  \hspace{1cm} (7)

Dimensionally reducing on a four torus of volume $V$ the associated six-dimensional variables are

$$v = \frac{V}{(2\pi)^4 \alpha'}, \quad g_6 = \frac{g}{\sqrt{v}}, \quad \kappa_6^2 = 4\pi^3 g_6^2 \alpha'^2.$$  \hspace{1cm} (8)

The charges of the black string are given by

$$r_1^2 = \frac{g\alpha' n_1}{v}; \quad r_5^2 = g\alpha' n_5,$$  \hspace{1cm} (9)

where $n_1$ and $n_5$ are the number of units of D1-brane and D5-brane charge respectively. In the decoupling limit, we can neglect the constant terms in the harmonic functions and the metric becomes that of $AdS_3 \times S^3$:

$$ds^2 = \frac{R^2}{z^2} \left[-dt^2 + dx^2 + dz^2\right] + R^2 d\Omega_3^2,$$  \hspace{1cm} (10)

with $z = R^2/r$ and the radius of the effective anti-de Sitter space being defined by

$$R^2 = r_1 r_5 = g_6 \alpha' \sqrt{n_1 n_5}.$$  \hspace{1cm} (11)

For this system, Maldacena’s conjecture is that the $(1 + 1)$-dimensional conformal field theory describing the Higgs branch of the D1-D5 brane system on the torus is dual to type IIB theory on $(AdS_3 \times S^3)_R \times T^4_{(r_1^2/r_5^2)}$, where the subscripts indicate the effective “radii” of the manifolds.

**III. SCATTERING OF MINIMAL COUPLED SCALARS: SEMI-CLASSICAL CALCULATION**

We firstly consider scattering of a minimally coupled scalar in the black string metric. Examples of such scalars include the six dimensional dilaton, which is constant in the background, and transverse graviton components. The equation of motion for a mode of frequency $\omega$ of a minimally coupled scalar $\phi$ in the black string metric is

$$\left[ \frac{1}{r_3} \partial_r (r_3^2 \partial_r) + \omega^2 H_1 H_5 \right] \phi = 0.$$  \hspace{1cm} (12)

The parameter $R^2 = r_1 r_5$ which describes the scale of the anti-de Sitter space plays an important role in determining the form of the solutions. We divide the spacetime into inner
and outer regions defined by $r \ll R$ and $r \gg R$; we will consider low energy scattering and so assume that $\omega R \ll 1$.

In the region $r > R$ we look for a solution of the form $\phi(r) = \psi(\omega r)/r$ where $\psi$ satisfies

$$[\rho^2 \psi'' + \rho \psi' - (1 - \omega^2 Q^2) \psi + \rho^2 \psi] = -\frac{(\omega^4 R^4)}{\rho^2} \psi,$$

with $\rho = \omega r$ and $Q^2 = r_1^2 + r_5^2$. For small $\omega R$ the leading order solution of this equation is, as first discussed in [10],

$$\psi(\rho) = \alpha J_\nu(\rho) + \beta J_{-\nu}(\rho),$$

where

$$\nu = (1 - \omega^2 Q^2)^{1/2}.$$  

(14)

We can regard the right hand side of (13) as a small perturbation in the outer region $r \gg R$.

In the region $r < R$ there is a natural choice of reciprocal variable $y = \omega R^2/r$ in terms of which the wave function $\phi(y) = y f(y)$ satisfies

$$[y^2 f'' + y f' - (1 - \omega^2 Q^2) f + y^2 f] = -\frac{(\omega^4 R^4)}{y^2} f.$$  

(16)

For $r \ll R$ the term on the right hand side is negligible and the leading order solution is thence

$$f(y) = H^{(2)}_\nu(y),$$  

(17)

where we have chosen the solution to be pure infalling at the horizon. Just as for the D3-brane and M-branes, the equation of motion for the minimal scalar has a self-dual point defined by the radius $R$ of the effective anti-de Sitter space.

Matching the amplitude of $\phi$ to leading order at $r = R$, assuming that $\nu \approx 1$ one finds that

$$\alpha = \frac{4i}{\pi \omega},$$  

(18)

with $\beta = 0$. Note that although such a naive matching scheme seems invalid since neither solution holds at $r = R$ it does in fact work. We can find the solution for the scalar field at the self-dual point as an expansion in $\omega R$, and then match the leading order term to obtain this result.

Now the asymptotic form of the infalling wave function is

$$\phi(y) = y H^{(2)}_\nu(y) \approx \sqrt{\frac{2y}{\pi}} \exp\{i (y - \frac{1}{2} \nu \pi - \frac{1}{4} \pi)\},$$  

(19)

which implies that the ingoing flux defined by

$$F_{r=0} = \frac{1}{2i} \{ \phi^* r^3 (\partial_r \phi) - \phi r^3 (\partial_r \phi^*) \}|_{r=0}$$  

(20)
is given by

\[ F_{r=0} = \frac{2\omega^2 R^4}{\pi}. \] (21)

Since the ingoing part of the wavefunction at infinity is given by

\[ \phi(r) \approx \alpha \sqrt{\frac{1}{2\pi \omega r^3}} \exp\{i(\omega r - \frac{1}{2} \nu \pi - \frac{1}{4} \pi)\}, \] (22)

the ingoing flux at infinity is given by

\[ F_\infty = \frac{|\alpha|^2}{2\pi} = \frac{8}{\pi^3 \omega^2}. \] (23)

The s-wave absorption probability is given by the ratio of the flux across the horizon to the ingoing flux at infinity and hence

\[ \sigma_{abs}^s = \frac{1}{4} \pi^3 \omega^4 R^4. \] (24)

Multiplying by \(4\pi/\omega^3\) to obtain the absorption cross-section we find that

\[ \sigma_{abs} = \pi^3 \omega R^4. \] (25)

As we would expect, the absorption cross-section vanishes at zero frequency. The absorption cross section for a minimally coupled scalar in the associated five dimensional black hole under the assumption that \(r_K \ll r_1, r_5\) is [12]

\[ \sigma_{abs} = \pi^3 \omega R^4 \frac{e^{\omega T_H} - 1}{(e^{\omega T_L} - 1)(e^{\omega T_R} - 1)}, \] (26)

where \(T_L\) and \(T_R\) are the temperatures of the left and right moving excitations respectively. In the limit that \(r_K \to 0, T_L, T_R \to 0\), with the Hawking temperature defined as

\[ \frac{1}{T_H} = \frac{1}{2} \left( \frac{1}{T_L} + \frac{1}{T_R} \right), \] (27)

we recover (25) as required. As is by now well-known we can also reproduce this result from the scattering cross sections of BTZ black holes [21]: the cross section for a BTZ black hole is [22]

\[ \sigma = \pi^2 \omega R^2 \frac{e^{\omega R} - 1}{(e^{\omega T_L} - 1)(e^{\omega T_R} - 1)}, \] (28)

where \(R\) defines the asymptotic radius of curvature. One then takes the same limit of the temperatures and multiplies by the volume of the three sphere \(2\pi^3 R^3\) and divides by the the length of the circle direction \(2\pi\) to obtain the string cross section. This limit corresponds to taking the zero mass, zero angular momentum BTZ black hole.
In the near extremal limit of the string, which we obtain by replacing \( g_{tt} \to hg_{tt} \) and \( g_{rr} \to h^{-1}g_{rr} \) where

\[
h = (1 - \frac{r_0^2}{r^2}),
\]  

with \( r_0 \) the extremality parameter, the temperatures of the left and right moving excitations are given by

\[
T_L = T_R = T_H = \frac{r_0}{2\pi R^2}.
\]  

This indicates that the near extremal cross section is

\[
\sigma = \pi^3 \omega R^4 \coth(\frac{\omega}{4T_L}),
\]

which is finite in the limit of zero frequency only for a black string far from extremality.

From a supergravity point of view the dominant corrections to the absorption cross section arise from the matching about the self dual point \( r = R \). Following the approach of [18] we look for scalar field solutions as power series in \( \omega \): note that there does not seem to be an exact solution to the wave equation as was found for scattering within a 3-brane background in [23]. In contrast to the D3-brane and M-brane calculations, we have not one but two dimensionless parameters controlling the corrections, following from the presence of two scales in the semi-classical geometry. This of course follows from the fact that the effective string preserves only one quarter of the supersymmetry. The two dimensionless parameters are \( \omega Q \) and \( \omega R \), where \( Q \) and \( R \) are defined above. In the limiting case \( r_1 = r_5 \), which has been distinguished as a special case several times in the literature, most notably in fixed scalar calculations [24], [25], [26], there is only one scale in the black string geometry.

Since \( Q \) is necessarily greater than \( R \), in the region \( r \ll R \) the right hand side of (13) acts as a small correction to the leading order solution. However, as we approach the self-dual point \( r = R \) the two terms in the equation involving \( \omega Q \) and \( \omega R \) act as corrections to the leading order solution of the same order of magnitude. That is, in the near horizon region \( r < R \), the field equation for \( f(y) = \phi(y)/y \) should be written as

\[
\left[ y^2 f'' + yf' + (y^2 - 1)f \right] = -\omega^2 Q^2 f - \frac{(\omega R)^4}{y^2} f.
\]  

For small \( \omega Q \) and \( \omega R \) the terms on the right hand side act as small corrections even at the self-dual point. We look for a perturbative solution \( f(y) = f_0(y) + f_1(y) \) where the leading order solution is

\[
f_0(y) = H^{(2)}_1(y),
\]

and \( f_1 \) satisfies the inhomogeneous equation

\[
\left[ y^2 f_1'' + yf_1' + (y^2 - 1)f_1 \right] = -\omega^2 Q^2 f_0 - \frac{(\omega R)^4}{y^2} f_0.
\]

Since this is a second order equation we can simply write down the solution for \( f_1 \) as
\begin{equation}
\phi(y) = \frac{2i}{\pi} \left(1 - \frac{1}{2} (\omega Q)^2 \ln(y) \right).
\end{equation}

There are also subleading correction terms of the form \((\omega Q)^2\) multiplied by powers of \(y\); dominant corrections arise as we would expect from the first term in \((35)\) only. If we rewrite this solution in terms of the variable \(u = r/R\) we find that

\begin{equation}
\phi(u) = \frac{2i}{\pi} \left(1 - \frac{1}{2} (\omega Q)^2 \ln(\omega R) + \frac{1}{2} (\omega Q)^2 \ln(u) \right).
\end{equation}

We can repeat the same procedure in the region \(r > R\) to find that the leading order solution in the transition region is given by

\begin{equation}
\phi(\rho) = \phi((\omega R) u) = \omega \frac{\alpha}{2} \left(1 + \frac{1}{2} (\omega Q)^2 \ln(\omega R) + \frac{1}{2} (\omega Q)^2 \ln(u) \right).
\end{equation}

We can compare these solutions at the self-dual point \(u = 1\); since \((\omega R)\) is much smaller than one, both \(\rho\) and \(y\) are small in the matching region, and our perturbative expansions are valid. The mismatch between these solutions requires that one take

\begin{equation}
\alpha = \frac{4i}{\pi\omega} \left[1 - (\omega Q)^2 \ln(\omega R) \right],
\end{equation}

which implies that the absorption cross section behaves as

\begin{equation}
\sigma = \pi^3 \omega R^4 \left[1 + 2(\omega Q)^2 \ln(\omega R) \right].
\end{equation}

It is straightforward to show that higher order corrections to the cross section are of the form one would expect

\begin{equation}
\sigma = \pi^3 \omega R^4 \left[1 + a_1 (\omega Q)^2 + a_2 (\omega Q)^2 (\omega R)^2 + a_3 (\omega R)^4 + \ldots \right.
\end{equation}

\begin{equation}
+ \ln(\omega R) \left(2b_1 (\omega Q)^2 + b_2 (\omega Q)^2 (\omega R)^2 + b_3 (\omega R)^4 + \ldots \right)
\end{equation}

\begin{equation}
+ (\ln(\omega R))^2 \left(c_1 (\omega Q)^2 (\omega R)^2 + c_2 (\omega R)^4 + \ldots \right) + \ldots,\end{equation}
where the ellipses indicate higher powers of \( \omega Q \) and \( \omega R \).

Note that this calculation implies that for scattering from the black string carrying momentum in the string direction one should get corrections to the cross section of the same type. That is, we expect the low energy cross section of the five dimensional black hole to behave as

\[
\sigma = A_h \left( 1 + O((\omega Q)^2 \ln(\omega R)) \right),
\]

where \( A_h \) is the area of the horizon. Evidently for such an expression to hold we need to assume that \( r_K \ll Q \); in the region \( r_K \ll r \leq R \) the metric will then be of the AdS form and this matching scheme will hold. If \( r_K \) is of the same order as \( r_1 \) and \( r_5 \) the matching of the scalar field wave function between regions is more subtle [16].

### IV. THE EFFECTIVE STRING MODEL

We now consider the world sheet origins of the logarithmic corrections to the cross section. Unlike the D3-brane case, we do not have a good description of the action for the system at small \( g \). The heuristic model introduced in [3] and developed in [10], [11], [24] cannot produce the correct results for fixed scalars, although this is not the case for minimal scalars. The refined model discussed in [27] relies on the moduli space approximation, and the higher order couplings in which we are interested lie beyond the scope of this approximation. With these problems in mind, we will use the model of [24] and then investigate what input the semi-classical results have on this model.

So let us assume that the low energy excitations of the system are described by the standard D-string action

\[
S_D = -T_{\text{eff}} \int d^2\sigma e^{-\Phi} \sqrt{\det(\gamma_{ij})} + ..., \tag{43}
\]

with \( \Phi \) the ten-dimensional dilaton and \( \gamma \) the induced string frame metric on the world sheet defined by

\[
\gamma_{ij} = G_{MN} \partial_i X^M \partial_j X^N, \tag{44}
\]

with \( G \) the ten-dimensional string frame metric. As is usual we set the world sheet gauge field to zero and we will choose the static gauge \( X^0 = x^0, X^9 = x^1 \). We are interested in the coupling of a minimally coupled scalar to the string world sheet and will choose this scalar to be the six-dimensional dilaton \( \Phi_6 \). Expanding out the action the relevant terms describing the coupling of the dilaton to the world sheet are [24]

\[
S_D = -T_{\text{eff}} \int d^2x \left[ 1 + \frac{1}{2} \partial_+ X^m \partial_- X_m - \frac{1}{2} \Phi_6 (\partial_+ X^m \partial_- X_m) + \frac{3}{16} \Phi_6 (\partial_+ X)^2 (\partial_- X)^2 + \frac{1}{16} \Phi_6 (\partial_+ X^m) (\partial_- X_m) \left( (\partial_+ X)^2 + (\partial_- X)^2 \right) + ... \right], \tag{45}
\]

where as usual \( \partial_+ = \partial_0 + \partial_1 \) and \( \partial_- = \partial_0 - \partial_1 \). In principle the index \( m \) runs over 1...8 although we expect that only fluctuations within the 5-brane are significant [24], and hence we should sum only over \( m = 5..8 \). There is a subtlety in the subleading terms: to take
account of two bosonised fermion fields $\varphi$ it seems that one should add these fields as follows
\[ (\partial_+ X)^2 \to (\partial_+ X)^2 + (\partial_+ \varphi)^2, \] (46)
and similarly for the left derivatives. Such a correction is required to obtain the correct normalisation for the fixed scalar cross section in the case $r_1 = r_5$. Introducing canonically normalised scalar fields $\tilde{X}^m$ such that
\[ \tilde{X}^m = \sqrt{T_{\text{eff}}} X^m, \] (47)
and rotating to Euclidean signature $x^0 \to ix^0$ we find the action becomes
\[
S_{\text{int}} = -\int d^2\sigma[T_{\text{eff}} \left( -\frac{1}{2} (\partial \tilde{X})^2 + \frac{1}{2} \Phi_6 (\partial \tilde{X})^2 + \frac{3}{16 T_{\text{eff}}} \Phi_6 [(\partial \tilde{X})^2]^2 \\
+ \frac{1}{8 T_{\text{eff}}} \Phi_6 (\partial \tilde{X})^2 \left( (\partial_0 \tilde{X})^2 - (\partial_1 \tilde{X})^2 \right) + \ldots \right)],
\] (48)
where
\[ (\partial \tilde{X})^2 = \sum_m [(\partial_0 \tilde{X}^m)^2 + (\partial_1 \tilde{X}^m)^2]. \] (49)
Hence we find that at linear order the dilaton couples to the world volume through an interaction of the form
\[ S_{\text{int}} = -\int d^2x (\phi \mathcal{O}) = -\int d^2x \phi \left[ \mathcal{O}_2 + \frac{1}{T_{\text{eff}}} \mathcal{O}_4 + \ldots \right], \] (50)
where the subscripts to the operators indicate their conformal dimensions. The effective tension $T_{\text{eff}}$ has length dimension of minus two and as one expects one picks up factors of $1/T_{\text{eff}}$ as one increases the operator dimension. We are interested in calculating the two point function of the operator $\mathcal{O}$: to leading order we can calculate using the infrared limit. However, subleading corrections arise from the effect of irrelevant perturbations which take the theory away from the superconformal limit. Following the same type of analysis as in [18] one finds that the two point function for the operator $\mathcal{O}$ is given by
\[
\langle \mathcal{O}(x)\mathcal{O}(0) \rangle = \int \mathcal{D}X e^{-\int d^2y \left[ \mathcal{O}_2 + \frac{1}{T_{\text{eff}}} \mathcal{O}_4 \right]} \mathcal{O}(x)\mathcal{O}(0) ; \\
= \int \mathcal{D}X e^{-\int d^2y \mathcal{O}_2 \mathcal{O}(x)\mathcal{O}(0) \left( 1 - \frac{1}{T_{\text{eff}}} \int d^2z \mathcal{O}_4(z) \right) ; \\
= \left\langle \mathcal{O}(x)\mathcal{O}(0) \left( 1 - \frac{1}{T_{\text{eff}}} \int d^2z \mathcal{O}_4(z) \right) \right\rangle ; \\
= \left[ \langle \mathcal{O}_2(x)\mathcal{O}_2(0) \rangle - \frac{1}{T_{\text{eff}}} \int d^2z \langle \mathcal{O}_2(x)\mathcal{O}_4(z)\mathcal{O}_2(0) \rangle \right]. \] (51)
The correlators on the right hand side are evaluated in the free gauge theory which is conformal, and all subsequent correlators are implicitly evaluated in the conformal theory.
Note that we have implicitly gone to Euclidean signature which will be used to simplify the correlator calculations. Terms like $\langle O_2(x)O_4(0) \rangle$ vanish since only operators of the same conformal dimension can have a non-vanishing two point function.

To calculate the absorption cross section which follows from the leading order interaction term we could use the methods of [10] taking the zero temperature limit. Since we also wish to calculate the subleading corrections it is instructive to use instead the methods used in [3] and in [18]. At zero temperature the analysis in fact becomes a great deal easier: absorption cross-sections corresponds up to a simple overall factor to discontinuities of two point functions of certain operators on the D-brane world-volume [3]. Here we are considering here minimally coupled massless particles normally incident on the string. If the coupling of the particles to the string is given by

$$ S_{\text{int}} = \int d^2x \phi(x, 0) \mathcal{O}(x), \quad (52) $$

where $\phi(x, 0)$ is a canonically normalised field evaluated on the brane, and $\mathcal{O}$ is a local operator on the brane, then the precise correspondence is

$$ \sigma = \frac{1}{2i\omega} \text{Disc } \Pi(k)|_{k^0 = \omega; k=0}, \quad (53) $$

with $\omega$ the energy of the particle and

$$ \Pi(k) = \int d^2xe^{ik \cdot x} \langle \mathcal{O}(x)\mathcal{O}(0) \rangle. \quad (54) $$

$\text{Disc } \Pi(k)$ is the difference of $\Pi(k)$ evaluated for $k^2 = \omega^2 + i\epsilon$ and $k^2 = \omega^2 - i\epsilon$. The validity of this expression depends on $\phi$ being a canonically normalised field.

In Euclidean space the propagator for a scalar field $\tilde{X}$ is

$$ \langle \tilde{X}(x)\tilde{X}(0) \rangle = \frac{1}{2\pi} \ln(x), \quad (55) $$

where $x^2 = (x_0^2 + x_1^2)$. Note that we are assuming that the string direction is infinite rather than compact. Then,

$$ \langle \partial_i \tilde{X}(x) \partial_I \tilde{X}(0) \rangle = -\frac{1}{2\pi x^2} \left[ \delta_{iI} - \frac{2}{x^2} x_i x_I \right]. \quad (56) $$

From this we can deduce that the term giving the leading order contribution to the absorption cross section is

$$ \Pi(x) = \left\langle \frac{1}{2} (\partial \tilde{X})^2(x) : \frac{1}{2} (\partial \tilde{X})^2(0) : \right\rangle = \frac{1}{8\pi^2 x^4}, \quad (57) $$

and Fourier transforming we find that

$$ \Pi(k) = \int d^2x \Pi(x)e^{ik \cdot x} = -\frac{k^2}{16\pi} \ln(k^2/\Lambda^2), \quad (58) $$
where $\Lambda$ is an ultraviolet cutoff. The leading order absorption cross section is hence given by

$$\sigma = 4 \times \frac{\kappa_6^2}{2i\omega} \text{ Disc } \Pi(s) = \frac{1}{4} \omega \kappa_6^2, \quad (59)$$

where the factor of four originates from the four scalars on the world volume to which the scalar couples and the factor of $\kappa_6^2$ originates from the fact that the dilaton is not canonically normalised in (\ref{eq:11}).

Comparing the expressions for $R$ and $\kappa_6^2$ in (\ref{eq:11}) and (\ref{eq:8}) we see that there is a discrepancy of $n_1n_5$. However one expects that the effective $\alpha'$ on the string world sheet is $\sqrt{n_1n_5}\alpha'$ because of the fractionisation of the open string excitations [28]. Hence the effective $\kappa_6^2$ on the world sheet is indeed equal to $4\pi^3R^4$ and the string cross section agrees with the semi-classical calculation to leading order.

The leading order correction to the cross section will be given by

$$\delta \Pi(x) = - \int d^2z \left\{ \frac{1}{2}(\partial \tilde{X})^2(x) \frac{1}{T_{eff}} \mathcal{O}_4(z) \frac{1}{2}(\partial \tilde{X})^2(0) \right\} = \frac{3}{128\pi^4T_{eff}} \int d^2z \frac{1}{z^4(x-z)^4}, \quad (60)$$

where the form of $\mathcal{O}_4(z)$ follows from (\ref{eq:48}). In momentum space,

$$\delta \Pi(k) = \int d^2x \Pi_1(x)e^{ik\cdot x} = \frac{3}{512\pi^2T_{eff}}k^4(\ln(k^2/\Lambda^2))^2. \quad (61)$$

Then the correction to the absorption cross section is given by

$$\delta \sigma = 16 \times \frac{\kappa_6^2}{2i\omega} \text{ Disc } \delta \Pi(k), \quad (62)$$

where the factor of 16 arises from the fact that four scalars contribute. We expect there to be two bosonised fermions contributing to the subleading interaction term in the action (\ref{eq:11}) as well as the four bosons: when we calculate cross sections for the fixed scalars we need to include them to obtain agreement for the cross section in the case $r_1=r_5$ [24]. However, the terms arising from the fermions do not contribute to the subleading term in the dilaton cross section. We find that

$$\delta \sigma = \pi^3\omega R^4 \left( \frac{3\omega^2}{2\pi T_{eff}} \ln(\omega/\Lambda) \right). \quad (63)$$

To compare this with the supergravity calculation we should take the cutoff to be at $\Lambda = 1/R$. As in the D3-brane calculations, the natural cutoff on the world sheet is $1/\sqrt{\alpha'}$ but the difference between the cutoffs gives a contribution to the cross section of the form

$$\delta \sigma \propto \frac{\omega^3 R^4}{T_{eff}} \ln(R/\sqrt{\alpha'}). \quad (64)$$
Hence the difference between these cutoffs contributes only to the first non-logarithmic correction term in the cross section which we have not calculated.

One then has to decide what value one should use for the effective string tension. Analysis of the entropy and temperature of near-extremal five branes leads to an effective string of tension $1/2\pi r_5^2$ [28]. Extending these methods to the case $r_1 \sim r_5$ implies that

$$T_{\text{eff}} = \frac{1}{2\pi Q^2}. \quad (65)$$

However, most of the scattering calculations do not depend on the effective tension or require $r_1 = r_5$, and so there is an ambiguity in the value one should take for the effective tension. In fact it was shown in [29], [30] that one should choose the value

$$t_{\text{eff}} = \frac{1}{2\pi R^2} \quad (66)$$

to obtain the correct scaling properties of cross sections of higher partial waves of minimal scalars from the effective string model. This is also the value found in the analysis of [27]. This ambiguity illustrates a problem of the effective string model: in this zero temperature limit, there are two scales in the geometry but only one of these scales appears in the effective string action. For the general near extremal five dimensional black hole, there are four scales in the semi-classical geometry $r_0, r_K, Q$ and $R$, where $r_0$ is the extremality parameter and $r_K$ is related to the Kaluza-Klein charge. In the effective string model, however, there are only three length scales, given by $T_{\text{eff}}, T_L$ and $T_R$.

It is interesting to note that if one chooses the first value for the tension then our string correction is a factor of $3/2$ greater than the semi-classical result. One should not be worried by such a numerical discrepancy, since the semiclassical solution is valid in the regime $g_1, g_5 \gg 1$, whereas the perturbative effective string calculation is valid in the region $g_1, g_5 \ll 1$. We can only reliably compare the cross-sections in the decoupling limit for which we cannot calculate perturbatively on the string world sheet.

Even though the first correction agrees in form, if not coefficient, with that calculated semi-classically the effective string model cannot then reproduce the next order correction of the form

$$\omega R^4 \left[ \ln(\omega R)^2 (\omega Q)^2 (\omega R)^2 \right], \quad (67)$$

since the scale set by $R$ does not appear in the effective string model, except as an ultraviolet cutoff in the logarithmic terms.

For finite temperature, we will have to use the finite temperature Green’s functions to calculate the correction to the cross section. One can follow an approach similar to that given in [31] to calculate this correction. The leading order term follows straightforwardly from [31] and reproduces the $r_K \to 0$ limit of the result of [12]. One can also show that the functional dependence of the leading order correction behaves as

$$\frac{\omega^3 R^4}{T_{\text{eff}}} \coth \left( \frac{\omega}{4T_L} \right), \quad (68)$$

for $r_0 \ll R$ as one would expect from (31).
The appearance of two scales in the semi-classical geometry but only one scale in the effective string model is also related to the discrepancy in calculations of fixed scalar cross sections for \( r_1 \neq r_5 \). In the limit of \( T_L = T_R \) the discrepancy in functional dependence of the leading order cross sections disappears: that is, one obtains the same functional form for the cross section from \((1, 3), (2, 2)\) and \((3, 1)\) operators. Given the conformal dimension of these operators, it is easy to see that the leading order cross section for both fixed scalars behaves as

\[
\sigma \sim \frac{1}{T_{\text{eff}}^2} \omega^5 R^4. \tag{69}
\]

However, the precise results calculated semi-classically are \([25], [26]\)

\[
\sigma = \frac{9\pi^3\omega^5 R^{12}}{64(Q^2 \pm \sqrt{Q^4 - 3R^4})^2}, \tag{70}
\]

where the sign depends on which of the two fixed scalars we are considering.

V. THE CFT CORRESPONDENCE

Having considered the world sheet interpretation of subleading effects in the \( g \to 0 \) limit, it is interesting to consider the calculation of the absorption cross-section from the AdS-CFT correspondence in the limit of large \( g n_{1,5} \). In the region \( r \ll R \) the geometry of the black string is that of \([10]\) and we will assume in all that follows that the string direction is not compact.

The AdS-CFT correspondence implies that a massless minimally coupled scalar couples at leading order to an operator of conformal dimension two. The two point function of the operator \( O \) behaves as

\[
\langle O(k)O(q) \rangle \propto \frac{R^4}{\kappa_6^2} k^2 \ln(kR)^2 \delta^2(k + q), \tag{71}
\]

where \( \kappa_6 \) is the six-dimensional gravitational constant. One could obtain this form directly from Fourier transforming the two point functions of \([8]\) and \([32]\), but to fix the normalisation it is convenient to follow an analysis similar to that of \([7]\). Starting with an action for the scalar field of the form

\[
S = \frac{1}{2\kappa_6^2} \int d^6x \sqrt{-g} \left( \frac{1}{2} (\partial \phi)^2 \right), \tag{72}
\]

and substituting for the metric \([10]\) we find that

\[
S = \frac{\pi^2 R^4}{2\kappa_6^2} \int d^2x \left[ \phi \frac{1}{z} \partial_z \phi \right]_R^\infty, \tag{73}
\]

\(^1\)Since the field is massless we do not need to worry about the correction factor discussed in \([32]\).
where we have introduced a cutoff at radius $r = R$ and $x = (t, x)$. The equation of motion for the field $\phi$ is

$$\left[ z \partial_z \frac{1}{z} \partial_z + \eta^{ij} \partial_i \partial_j \right] \phi = 0,$$

(74)

where $i, j = 1, 2$. Finiteness of the action requires that $\phi$ must vanish in the limit that $z \to \infty$ and the appropriate form for $\phi$ is then

$$\phi(x, z) = \frac{1}{(2\pi)^2} \int d^2 k \lambda_k e^{i k \cdot x} \left( \frac{z K_1(kz)}{RK_1(kR)} \right),$$

(75)

where $K_1$ is the modified first Bessel function. Note that we have normalised the scalar field so that it takes a value of one on the boundary $r = R$. Substituting into the action we find to leading order

$$S = -\frac{\pi^2 R^4}{4\kappa_6^2} \int d^2 k d^2 q \lambda_k \lambda_q \left( \frac{1}{(2\pi)^2} \delta^2(k + q) \right) k^2 \ln(kR)^2,$$

(76)

from which it is apparent that the two point function is

$$\langle O(k) O(q) \rangle = -\frac{\pi^2 R^4}{2\kappa_6^2} k^2 \ln(kR)^2 \left( \frac{1}{(2\pi)^2} \delta^2(k + q) \right).$$

(77)

Defining $s = -k^2$ and letting

$$\Pi(s) = \frac{\pi^2 R^4}{2\kappa_6^2} \left[ s \ln(-R^2 s) \right],$$

(78)

then the absorption cross section can be inferred from the discontinuity as one crosses the positive real axis in the $s$-plane of $\Pi(s)$. That is, the cross section behaves as

$$\sigma = \frac{\kappa_6^2}{i\omega} \text{Disc}(\Pi(s))|_{k^0 = \omega, k = 0},$$

(79)

where the factor of $\kappa_6^2$ arises from the fact that $\phi$ is not canonically normalised in (72). Hence we find that

$$\sigma = \pi^3 \omega R^4,$$

(80)

which is the same as the semi-classical result (23) and the leading order string result as expected.

To look for subleading corrections to this cross section we need to postulate what the effective action for the conformal field theory is in the large $gn_1$, $gn_5$ limit. Looking at the form of the semi-classical corrections one sees that to reproduce this result one must have a correction to the two point function coming from a term of the form

$$\int d^2 z \langle O_2(x) O_4(z) O_2(0) \rangle \sim \frac{1}{T} \int d^2 z \frac{1}{z^4(x - z)^4},$$

(81)
where $T$ has length dimension of minus two. Then one infers that the correction to the cross section is

$$\delta \sigma \propto \kappa_0^2 \omega \left( \omega^2 T^{-2} \ln(\omega/\Lambda) \right). \tag{82}$$

Since the only length scale in the CFT is $R$ we should take $T \propto 1/R^2$ which gives the correction to the cross-section as

$$\delta \sigma \propto \omega^3 R^6 \ln(\omega R), \tag{83}$$

where we have also taken the cutoff at $1/R$. The conformal field theory correction does not then agree in normalisation with that of the semi-classical calculation. Such a discrepancy is unsurprising given that the length scale $Q$ does not appear in the boundary conformal theory.

The fixed scalars couple to operators of dimension four, and one can hence find that the absorption cross section for both of them is given by

$$\sigma = \frac{1}{64} \pi^3 \omega^5 R^8, \tag{84}$$

which again does not agree with the semi-classical calculation. The fixed scalars are scattered in the asymptotically flat part of the geometry which is not described by the boundary theory. It is interesting to note that in the CFT approach there is an ambiguity in the operators to which the fixed scalars couple which affects the finite temperature result. We know that the operator has dimension four, but cannot fix whether it is $(3,1)$, $(2,2)$ or $(1,3)$ without further analysis. It is of course natural to assume that the operator is of the $(2,2)$ type, as was done in [33], since this reproduces the functional form of the semi-classical results. The same ambiguity will appear in finite temperature calculations of scattering from D3-branes and M-branes. To fix the ambiguities one needs to know the origins of the terms in the boundary conformal field theories.

**VI. CONCLUSIONS**

Our results demonstrate the limited applicability of the conjectured AdS-CFT correspondence in the calculation of scattering in asymptotically flat systems using CFT methods. Although we can calculate the leading order absorption rates using the properties of the asymptotic near horizon geometry, subleading corrections depend not just on the boundary theory but also on the bulk theory. That is, the asymptotically flat part of the geometry will determine the leading order corrections in the semi-classical absorption rate. The boundary CFT does not have information about the full geometry and hence cannot give the correct answer for scattering in the asymptotically flat geometry.

If one has a non-minimally coupled particle in the semi-classical geometry, then in many important cases the functional dependence of the scattering rate seems to be determined solely by the behaviour in the near horizon region. Scattering in the asymptotically flat region simply corrects the rate by factors depending on the parameters of the near horizon geometry. This will mean that one can predict the functional dependence of the scattering rate from the coupling to the near horizon geometry, and hence from knowing the conformal
dimension of the operator in the CFT to which the particle couples. The normalisation of the cross section could not however be predicted.

Since the Maldacena conjecture relates type IIB theory in the anti-de Sitter background to a dual conformal field theory, there is of course no reason why this conformal field theory should reproduce results in an asymptotically flat spacetime in which only the near horizon geometry is of the anti-de Sitter form. It would hence be interesting to investigate subleading effects in the scattering of massive and massless scalars in the six-dimensional geometry consisting of the BTZ black hole times a three sphere: one would expect that these could be reproduced by the dual conformal field theory. The supergravity analysis of subleading corrections is in this case more subtle because of the timelike boundary at infinity.

Our analysis also illustrates various problems in trying to reproduce the semi-classical results using an effective string model. If the effective string model is to reproduce subleading effects in the semi-classical geometry, one needs to incorporate into the model the four length scales that generally determine the black hole geometry. Of course there is no reason why the subleading effects calculated at $g_{n_1}, g_{n_5} \gg 1$ should be reproduced by an effective string model valid only for small $g$. 
REFERENCES

[1] S. Gubser, I. Klebanov and A. Peet, Phys. Rev. D 54, 3915 (1996); hep-th/9602135.
[2] I. Klebanov, Nucl. Phys. B 496, 231 (1997); hep-th/9702076.
[3] S. Gubser and I. Klebanov, Phys. Lett. B 413, 41 (1997); hep-th/9708005.
[4] S. Gubser, I. Klebanov and A. Tseytlin, Nucl. Phys. B 499, 217 (1997); hep-th/9703040.
[5] A. Polyakov, hep-th/9711002.
[6] J. Maldacena, hep-th/9711200.
[7] S. Gubser, I. Klebanov and A. Polyakov, hep-th/9802109.
[8] E. Witten, hep-th/9802150; hep-th/9803131.
[9] C. Callan and J. Maldacena, Nucl. Phys. B 475, 645 (1996); hep-th/9602043.
[10] S. Das and S. Mathur, Nucl. Phys. B 478, 561 (1996); hep-th/9606185.
[11] S. Das and S. Mathur, Nucl. Phys. B 482, 153 (1996); hep-th/9607149.
[12] J. Maldacena and A. Strominger, Phys. Rev. D 55, 861 (1997); hep-th/9609026.
[13] I. Klebanov and S. Mathur, Nucl. Phys. B 500, 115 (1997); hep-th/9701187.
[14] S. Das, Phys. Rev. D 56, 3582 (1997); hep-th/9702109.
[15] S. de Alwis, hep-th/9804013.
[16] S. Gubser, A. Hashimoto, I. Klebanov and M. Krasnitz, hep-th/9803023.
[17] A. Rajaraman, hep-th/9803082.
[18] S. Mathur and A. Matusis, hep-th/9805064.
[19] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. 69, 1849 (1992); hep-th/9204099.
[20] S. Hyun, G. Lee and J. Yee, Phys. Lett. B 322, 182 (1994); S. Hyun, G. Lee and J. Yee, Phys. Rev. D 51, 1787 (1995), hep-th/9409017; D. Birmingham, I. Sachs and S. Sen, Phys. Lett. B 413, 281 (1997), hep-th/9707188.
[21] S. Gubser and A. Hashimoto, hep-th/9805140.
[22] C. Callan, S. Gubser, I. Klebanov and A. Tseytlin, Nucl. Phys. B 489, 65 (1997); hep-th/9610172.
[23] I. Klebanov and M. Krasnitz, Phys. Rev. D 55, 3250 (1997), hep-th/9612051; Phys. Rev. D 56, 2173 (1997), hep-th/9703120.
[24] S. Hassan and S. Wadia, hep-th/9712213.
[25] D. Freedman, S. Mathur, A. Matsuis and L. Rastelli, hep-th/9804058.
[26] E. Teo, hep-th/9805014.