Abstract

We perform a general analysis of the R-parity conserving dimension-five operators that can be present beyond the Minimal Supersymmetric Standard Model. Not all these operators are actually independent. We present a method which employs spurion-dependent field redefinitions that removes this “redundancy” and establishes the minimal, irreducible set of these dimension-five operators. Their potential effects on the MSSM Higgs sector are discussed to show that the tree level bound $m_h \leq m_Z$ cannot be easily lifted within the approximations used, and quantum corrections are still needed to satisfy the LEPII bound. An ansatz is provided for the structure of the remaining couplings in the irreducible set of D=5 operators, which avoids phenomenological constraints from flavor changing neutral currents. The minimal set of operators brings new couplings in the effective Lagrangian, notably “wrong”-Higgs Yukawa couplings and contact fermion-fermion-scalar-scalar interactions, whose effects are expected to be larger than those generated in the MSSM at loop or even tree level. This has implications in particular for LHC searches for supersymmetry by direct squark production.
1 Introduction

The Standard Model (SM) and its minimal supersymmetric version (MSSM) are thought to be the low energy limit of a more fundamental theory valid at high scales (string theory, extra dimensions, etc). In the absence of a detailed knowledge of this theory (vacua degeneracy, moduli problem), effective field theories provide a good framework on searches for new physics. In such theories higher dimensional operators are usually present. They can be generated by compactification or, in the case of 4D renormalisable theories, by integrating out massive states of mass $M \gg m_Z$. As a result the low-energy effective Lagrangian below the scale $M$ contains a set of operators of dimension $D>4$. The effective field theory approach resides firstly in organising these operators in a series of powers of $1/M$. In the leading order a smaller number of couplings (parameters) are relevant and this leads to the possibility of making low energy predictions, little dependent on the details of the high scale theory (in many cases unknown anyway). For practical purposes one can consider, in addition to the SM or MSSM Lagrangian, the set of all higher dimensional operators of a given dimension with some unknown coefficients and investigate their implications for electroweak or TeV
scale physics. A second organising principle is that, for a given order in $1/M$, one may use in addition symmetry arguments inspired by phenomenology, to reduce further the number of parameters.

When studying the effects of higher dimensional operators one aspect is often overlooked. This refers to the fact that in an effective field theory not all operators of a given dimension (suppressed by a fixed power of $1/M$) are actually independent. Within a given such set of operators, general field transformations allow one to eliminate those operators which are redundant, and identify the minimal irreducible set of independent operators. The advantage of this result is that it simplifies considerably the study of the models, by removing redundant couplings (parameters) of the theory. The purpose of this work is to show explicitly how one can identify the minimal irreducible set of such operators for a particular example. We consider the MSSM\textsuperscript{1} extended by all dimension-five operators that conserve $R$-parity symmetry \textsuperscript{2} and we identify the minimal irreducible set of these. The method is general and can be applied to other models, too.

Since supersymmetry is broken, the fields’ transformations should take into account effects of supersymmetry breaking associated with the higher dimensional operators. This is done by using spurion-dependent transformations. Some operators are “redundant” in that they can be eliminated completely or they only change/renormalise the standard soft terms and supersymmetric $\mu$-term; such operators can be “gauged away”. In the new fields’ basis the final number of parameters is reduced and calculations and predictions for physical observables can be more easily made. We provide an ansatz for the remaining couplings which allows one to avoid the effects of Flavor Changing Neutral Currents (FCNC), and reduces further the number of these couplings. One consequence is the generation of new effective interactions in the Lagrangian of the type (quark-quark-squark-squark) with potentially large effects in squark production compared to those generated in the MSSM. These are largest for the top/stop quarks. This can be important for LHC supersymmetry searches by direct squark production. Additional “wrong”-Higgs couplings, familiar in the MSSM at the loop level \textsuperscript{3, 4, 5}, are also generated with a numerical coefficient that can be larger than the loop-generated MSSM one. Again, these are largest for the top and also bottom sector at large $\tan \beta$. We discuss some of the associated phenomenological implications.

We show that in the model discussed the Higgs sector is simplified, despite the initial

\textsuperscript{1}For a review see \textsuperscript{4}.
presence of two D=5 operators and their associated spurion dependence. The “redundant”
operator that can be removed by field redefinitions does not change the physics of the Higgs
sector. It also turns out that for the MSSM lightest Higgs the tree level bound $m_h \leq m_Z$ is
not easily lifted by the D=5 operators (with one exception that we discuss). The conclusion
is that in the approximation considered the MSSM Higgs sector is rather stable under the
addition of D=5 operators and quantum corrections are still needed to lift it above LEPII
bound \[6\]. This conclusion changes if the massive states that induce the D=5 operators in
the first instance are sufficiently light not be integrated out but considered together with the
other MSSM states when analysing their implications.

The plan of the paper is as follows. In the next Section we present the general D=5
operators that can be present beyond the MSSM, preserving R-parity. We then identify the
minimal, irreducible set of these operators. Although of dimension-five, they can still induce
too-large, dangerous FCNC effects, for arbitrary coefficients. An ansatz avoiding this problem
is presented, together with its phenomenological implications, in Section \[3\]. In Section \[4\] we
analyse the effects on the Higgs sector that D=5 operators can bring. We show that these
cannot avoid the MSSM tree level upper bound on the lightest Higgs ($m_h \leq m_Z$), with one
exception where a marginal increase above $m_Z$ can be present. We check explicitly that, as
expected, an operator that belongs to the redundant class cannot change the upper bound
on the lightest Higgs and only renormalizes soft masses or the $\mu$ term. In Appendix \[A\] and
Appendix \[B\] we show in detail how the higher dimensional operators of the type discussed
in the text occur at low energies, by integrating out massive supermultiplets (that could be
present beyond MSSM \[7\]), in the absence (Appendix \[A\]) and in the presence (Appendix \[B\])
of gauge interactions. Appendix \[C\] identifies the most general supersymmetry breaking terms
that a particular type of D=5 operator discussed in Section \[2\] can bring. Finally Appendix \[D\]
provides technical details of the calculation of the Higgs spectrum discussed in the text.

2 Higher dimensional operators: a general discussion.

In this section we find the minimal, irreducible set of R-parity conserving dimension-five
operators that can be present beyond the MSSM. Consider

$$\mathcal{L} = \mathcal{L}^{(4)}_{MSSM} + \mathcal{L}^{(5)}$$

(1)
Here $\mathcal{L}^{(4)}_{\text{MSSM}}$ is the standard R-parity conserving MSSM Lagrangian and $\mathcal{L}^{(5)}$ is a Lagrangian of R-parity conserving dimension-five operators, to be introduced shortly. Further

$$
\mathcal{L}^{(4)}_{\text{MSSM}} = \int d^4 \theta \left[ Z_1 H_1^\dagger e^{V_1} H_1 + Z_2 H_2^\dagger e^{V_2} H_2 \right] + \mathcal{L}_K
$$

$$
+ \left\{ \int d^2 \theta \left[ - H_2 Q \lambda_U U^c - Q \lambda_D D^c H_1 - L \lambda_E E^c H_1 + \mu H_1 H_2 \right] + \text{h.c.} \right\} \tag{2}
$$

$\mathcal{L}_K$ accounts for the kinetic terms of the quark and lepton superfields $Q,U^c,D^c,L,E^c$ and for the gauge kinetic part, as well as for their associated soft breaking terms obtained using spurion field formalism. In the MSSM

$$V_1 \equiv g_2 V_W^i \sigma^i - g_1 V_Y, \quad (H_1 \text{ has } Y_{H_1} = -1) \quad \text{and} \quad V_2 \equiv g_2 V_W^i \sigma_i + g_1 V_Y, \quad \text{where } V_Y \text{ and } V_W \text{ are vector superfields of the } U_Y(1)-\text{hypercharge and } SU(2)_L \text{ respectively, and } g_1 \text{ and } g_2 \text{ are the corresponding couplings}^4.$$  

Finally, $\lambda_F, F = U,D,E$ are $3 \times 3$ matrices in the flavor space. Note that

$$Z_i \equiv Z_i(S,S^\dagger), \quad \lambda_F \equiv \lambda_F(S), \quad F : U,D,E, \quad \mu \equiv \mu(S) \tag{3}$$

where $S \equiv M_s \theta^2$ is the spurion parametrising the soft supersymmetry breaking and $M_s$ is the supersymmetry breaking scale. In the following we use the notations

$$Z_1 = 1 + a_1 S + a_1^* S^\dagger + a_2 SS^\dagger, \quad Z_2 = 1 + b_1 S + b_1^* S^\dagger + b_2 SS^\dagger. \tag{4}$$

The complete set of dimension-five operators in MSSM, which preserve R-parity is given by

$$
\mathcal{L}^{(5)} = \frac{1}{M} \left\{ \int d^4 \theta \left[ Q U^c T_Q Q D^c + Q U^c T_L L E^c + \lambda_H(H_1 H_2)^2 \right] + \text{h.c.} \right\}
$$

$$+ \frac{1}{M} \int d^4 \theta \left[ H_1^\dagger e^{V_1} Q Y_U U^c + H_2^\dagger e^{V_2} Q Y_D D^c + H_1^\dagger e^{V_2} L Y_E E^c + \text{h.c.} \right]
$$

$$+ \frac{1}{M} \int d^4 \theta \left[ A(S,S^\dagger) D^\alpha \left( B(S,S^\dagger) H_2 e^{-V_1} \right) D_\alpha \left( \Gamma(S,S^\dagger) e^{V_1} H_1 \right) + \text{h.c.} \right] \tag{5}
$$

$^2 U^c, D^c, E^c$ denote anti-quark/lepton singlet chiral superfields of components $f_{\bar{R}} \equiv (f^c)_L$ and $\bar{f}_R, f = u,d,e,$ while $Q$ and $L$ denote the left-handed quark and lepton superfields doublets.

$^3$We denote a product of two $SU(2)$ doublets (columns) $H_2^\alpha \lambda_{U^c} U^c \equiv H_2^\alpha (i\sigma_2) \lambda_U U^c$ in a matrix notation, which helps us to avoid extra $SU(2)$ indices; also $H_1 H_2 \equiv H_1^\alpha (i\sigma_2) H_2$; similar convention is used below.

$^4$ For a general discussion of D=5 operators with discrete symmetries see [8].
where \( T_{Q,L} \) carry four indices (2 for each up/down sector), and

\[
T_Q \equiv T_Q(S), \quad T_L \equiv T_L(S), \quad \lambda_H \equiv \lambda_H(S), \quad Y_F \equiv Y_F(S,S^\dagger), \quad F : U,D,E
\]

showing the spurion dependence of various couplings. In (5), \( M \) is a mass scale associated with the generation of the dimension-five operators, for example the mass of some heavy particles integrated out. The operator \((H_1 H_2)^2\) is easily generated by integrating out a singlet. The remaining operators in (5) are shown to be generated in Appendix B (see also Appendix A) by integrating out two massive \((SU(2)\) doublets) superfields of mass of order \( M \). Therefore these operators have a natural presence at low energies. The spurion dependence associated to these operators is the most general one can have. Since we assume a spontaneously broken effective Lagrangian, consistency of the integrating out procedure implies the restriction

\[
M_s \ll M .
\]

Also we have in general

\[
A(S,S^\dagger) \equiv \alpha_0 + \alpha_1 S + \alpha_2 S^\dagger + \alpha_3 S S^\dagger
\]
\[
B(S,S^\dagger) \equiv \beta_0 + \beta_1 S + \beta_2 S^\dagger + \beta_3 S S^\dagger
\]
\[
\Gamma(S,S^\dagger) \equiv \gamma_0 + \gamma_1 S + \gamma_2 S^\dagger + \gamma_3 S S^\dagger
\]

The Lagrangian in (1), (2), (5) contains however redundant terms, due to possible field redefinitions which relate various operators as we shall see shortly. Familiar transformations are holomorphic field redefinitions

\[
\Phi_i \rightarrow (1 - k_i S) \Phi_i ,
\]

which are commonly used in MSSM in order to restrict the couplings of the spurion \( S \), and thus, the so-called soft-breaking terms. We shall use this freedom later on. Less familiar are the following (super)field transformations:

\[5\] More exactly, the notation in eq.(5) stands for \( Q^u T_Q Q^d \equiv (Q^u)^\dagger T_Q Q^d \). Similarly, 
\( D^\nu [ B(S,S^\dagger) H_2 e^{-\nu_1} ] D_\alpha [ \Gamma(S,S^\dagger) e^{\nu_1} H_1 ] \equiv D^\nu [ B(S,S^\dagger) H_2^\dagger (i\sigma_2) e^{-\nu_1} ] D_\alpha [ \Gamma(S,S^\dagger) e^{\nu_1} H_1 ] \).

\[6\] From a superpotential \( \mu H_1 H_2 + m \Sigma^2 + \lambda \Sigma H_1 H_2 \) integrating a singlet \( \Sigma \) generates \( \lambda_H(H_1 H_2)^2 \).

\[7\] In Appendix A it is shown how \( H_2 D^2 H_1 \sim D^\alpha H_2 D_\alpha H_1 \) is generated by integrating a massive superfield without gauge interactions. In the presence of gauge interactions one finds the last operator in (5) (Appendix B).

\[8\] To avoid a complicated index notation, the transformations in (10) are written in a matrix notation for the Higgs \( SU(2) \) doublets, thus the presence of \((i\sigma_2)\), although in the superpotential this is not shown explicitly.
\[ H_1 \rightarrow H'_1 = H_1 - \frac{1}{M} D^2 \left[ \Delta_1 H_2^\dagger e^{V_2 (i \sigma_2)} \right]^T + \frac{1}{M} Q \rho_U U^c \]
\[ H_2 \rightarrow H'_2 = H_2 + \frac{1}{M} D^2 \left[ \Delta_2 H_1^\dagger e^{V_1 (i \sigma_2)} \right]^T + \frac{1}{M} Q \rho_D D^c + \frac{1}{M} L \rho_E E^c \]  

(10)

Here

\[ \rho_F = \rho_F(S); \quad F : U, D, E, \quad \Delta_i = \Delta_i(S, S^\dagger) \quad i = 1, 2 \]  

(11)

are arbitrary functions of the spurion, i.e. their coefficients in the Taylor expansion in \( S \) are free parameters, which can be chosen to eliminate redundant dimension-five operators, as we shall see shortly. These coefficients should have values smaller than \( M \) and the same applies to the entries of the \( \rho_F, F = U, D, E \) which are \( 3 \times 3 \) matrices. We take

\[ \Delta_1(S, S^\dagger) = s_0 + s_1 S + s_2 S^\dagger + s_3 S S^\dagger \]
\[ \Delta_2(S, S^\dagger) = s'_0 + s'_1 S + s'_2 S^\dagger + s'_3 S S^\dagger \]  

(12)

Notice that in the R-parity violating MSSM, we would also have the freedom to perform field transformations similar to (10) on quarks and leptons superfields. It is easy to see, however, that all these new transformations, with the exception of (10), violate R-parity and cannot therefore be performed in the R-parity conserving MSSM extension. Notice that field redefinitions (10), in addition of mixing operators from \( L^{(4)}_{MSSM} \) and \( L^{(5)} \), also generate operators of higher-order in \( 1/M \) (dimension-six), of the type

\[ \frac{1}{M^2} \int d^4 \theta D^2 \left[ H_2 e^{-V_1 \Delta_1^\dagger} \right] e^{V_1} D^2 \left[ \Delta_1 e^{-V_1 H_2^\dagger} \right] \]  

(13)

plus a similar one for \( H_1 \). Since the effects of such operators are further suppressed with respect to the dimension-five operators we are considering, we shall neglect them in what follows. One then finds that the original Lagrangian transforms into:

\[ \mathcal{L} = \mathcal{L}_K + \int d^4 \theta \left[ Z_1^\prime H_1^\dagger e^{V_1} H_1 + Z_2^\prime H_2^\dagger e^{V_2} H_2 \right] \]
\[ + \int d^2 \theta \left[ - H_2 Q \lambda_\prime^U U^c - Q \lambda_\prime^D D^c H_1 - L \lambda_\prime^E E^c H_1 + \mu H_1 H_2 \right] + h.c. \]
\[ + \frac{1}{M} \int d^2 \theta \left[ Q U^c T_\lambda^U Q D^c + Q U^c T_\lambda^D L E^c + \lambda_H (H_1 H_2)^2 \right] + h.c. \]
\[ + \frac{1}{M} \int d^4 \theta \left[ H_1^\dagger e^{V_1} Q Y_U^\prime U^c + H_2^\dagger e^{V_2} Q Y_D^\prime D^c + H_2^\dagger e^{V_2} L Y_E^\prime E^c + h.c. \right] + \Delta \mathcal{L} \]  

(14)
\[ \Delta \mathcal{L} = \frac{1}{M} \int d^4 \theta \left[ - \Delta^\dagger_1 H_2 e^{-V_1} D^2 (Z_1 e^{V_1} H_1) - Z_2 H_2 e^{-V_1} D^2 (\Delta^\dagger_2 e^{V_1} H_1) + h.c. \right] \]
\[ + \frac{1}{M} \int d^4 \theta \left[ A(S, S^\dagger) D^\alpha (B(S, S^\dagger) H_2 e^{-V_1}) D_\alpha (\Gamma(S, S^\dagger) e^{V_1} H_1) + h.c. \right] \]

(15)

Above we introduced the notation:

\[ \lambda_F(S) = \lambda_F(S) + \frac{\mu(S)}{M} \rho_F(S) \quad F : U, D, E \]

(16)

and

\[ Y'_U(S, S^\dagger) = Y_U(S, S^\dagger) - 4 \Delta_2(S, S^\dagger) \lambda_U(S) + Z_1(S, S^\dagger) \rho_U(S) \]
\[ Y'_D(S, S^\dagger) = Y_D(S, S^\dagger) - 4 \Delta_1(S, S^\dagger) \lambda_D(S) + Z_2(S, S^\dagger) \rho_D(S) \]
\[ Y'_E(S, S^\dagger) = Y_E(S, S^\dagger) - 4 \Delta_1(S, S^\dagger) \lambda_E(S) + Z_2(S, S^\dagger) \rho_E(S) \]

(17)

and

\[ T'_Q(S) = T_Q(S) + \lambda_U(S) \otimes \rho_D(S) + \rho_U(S) \otimes \lambda_D(S) \]
\[ T'_L(S) = T_L(S) + \lambda_U(S) \otimes \rho_E(S) + \rho_U(S) \otimes \lambda_E(S) \]

(18)

Finally

\[ Z'_1(S, S^\dagger) = Z_1(S, S^\dagger) - \frac{1}{M} \left( 4 \mu(S) \Delta_2(S, S^\dagger) + h.c. \right), \]
\[ Z'_2(S, S^\dagger) = Z_2(S, S^\dagger) - \frac{1}{M} \left( 4 \mu(S) \Delta_1(S, S^\dagger) + h.c. \right) \]

(19)

In eqs. (16), (17), (18) all quantities except \( M \) are functions of the spurion field. Next rescale the Higgs fields for canonical normalisation of their kinetic terms

\[ H_1 \rightarrow \frac{1}{\sqrt{a'_0}} \left[ 1 - k_1 S \right] H_1, \quad H_2 \rightarrow \frac{1}{\sqrt{b'_0}} \left[ 1 - k_2 S \right] H_2, \quad k_1 = \frac{a'_1}{a'_0}, \quad k_2 = \frac{b'_1}{b'_0} \]

(20)

with

\[ a'_0 \equiv Z'_1 \big|_{S,S^\dagger=0}, \quad a'_1 \equiv Z'_1 \big|_{S}, \quad b'_0 \equiv Z'_2 \big|_{S,S^\dagger=0}, \quad b'_1 \equiv Z'_2 \big|_{S} \]

(21)

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Footnote 9: In a matrix notation, in (15) one replaces \( H_2 \rightarrow H_2^T (i \sigma_2) \), and similar for the holomorphic part of (14).

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which can be immediately computed using the definition of \( Z_{1,2}' \), \( Z_{1,2} \) and \( \Delta_{1,2} \) and their spurion dependence given above. After the Higgs fields transformation we obtain

\[
L = L_K + \Delta L + \int d^4 \theta \left[ \left( 1 - \frac{m_2^2}{M_s^2} \right) S S^\dagger H_1^\dagger e^{V_1} H_1 + \left( 1 - \frac{m_2^2}{M_s^2} \right) H_2^\dagger e^{V_2} H_2 \right] \\
+ \int d^2 \theta \left[ - H_2 Q \lambda''_U U^c - Q \lambda''_D D^c H_1 - L \lambda''_E E^c H_1 + \mu' H_1 H_2 \right] + \text{h.c.} \\
+ \frac{1}{M} \int d^2 \theta \left[ Q U^c T_\lambda^c Q D^c + Q U^c T_\lambda^c L E^c + \lambda'_{H}(H_1 H_2)^2 \right] + \text{h.c.} \\
+ \frac{1}{M} \int d^3 \theta \left[ H_1^\dagger e^{V_1} Q Y''_{U} U^c + H_2^\dagger e^{V_2} Q Y''_{D} D^c + H_1^\dagger e^{V_2} L Y''_{E} E^c + \text{h.c.} \right]
\]

(A)bove we introduced the following notation for the spurion dependent quantities:

\[
\lambda''_U(S) = \frac{1}{\sqrt{b_0'}} (1 - k_2 S) \lambda'_U(S) = (1 - b_1 S) \lambda_U(S) + O(1/M), \\
\lambda''_F(S) = \frac{1}{\sqrt{a_0'}} (1 - k_1 S) \lambda'_F(S) = (1 - a_1 S) \lambda_F(S) + O(1/M), \quad F \equiv D, E. \\
\mu'(S) = \frac{1}{\sqrt{a_0' b_0'}} [1 - (k_1 + k_2) S] \mu(S) = (1 - (a_1 + b_1) S) \mu(S) + O(1/M). \quad (23)
\]

Since \( a_0', b_0' \) are \( M \)-dependent, see (19), (21), the couplings \( \lambda''_{U,D,E}(S) \) and also \( \mu'(S) \) have acquired, already at the classical level, a dependence on the scale \( M \) of the higher dimensional operators (threshold correction). This is denoted above by \( O(1/M) \) and can be easily computed using (19), (21). Note that this \( O(1/M) \) correction is relevant for the Lagrangian (22). Similar considerations apply to \( m_{1,2} \) entering in the first line in (22) and their exact expressions (not shown) in terms of initial parameters can be computed in the same way. Further

\[
\lambda''_H(S) = \left( 1 - 2(a_1 + b_1) S \right) \lambda_H(S), \quad Y''_{U}(S, S^\dagger) = \left( 1 - a_1 S^\dagger \right) Y''_{U}(S, S^\dagger) \\
Y''_{D}(S, S^\dagger) = \left( 1 - b_1 S^\dagger \right) Y''_{D}(S, S^\dagger), \quad Y''_{E}(S, S^\dagger) = \left( 1 - b_1 S^\dagger \right) Y''_{E}(S, S^\dagger) \quad (24)
\]

where we ignored terms which bring \( O(1/M^2) \) corrections to (22). Finally, \( \Delta L \) in (22) is that of (15) after applying to it transformation (20). This gives
\[
\Delta \mathcal{L} = -\frac{1}{M} \int d^4 \theta \; t_0 \; H_2 \, e^{-\mathcal{V}_1} \, D^2 \left[ e^{\mathcal{V}_1} \, H_1 \right]
+ \frac{M}{M} \left[ 4 \left( t_1 + t_2 + t_0 (a_1 + b_1) \right) h_2 \, \mathcal{D}_\mu \mathcal{D}^\mu h_1 - 2 \left( t_1 - t_2 + t_0 (b_1 - a_1) \right) h_2 D_1 h_1 \right]
+ 2\sqrt{2} (t_1 + b_1 t_0) h_2 \, \lambda_1 \, \psi_{h_1} - 2\sqrt{2} (t_2 + a_1 t_0) \, \psi_{h_2} \, \lambda_1 \, h_1 - 4 \, t_3 \, F_{h_2} \, F_{h_1} \]
+ \frac{M^2}{M} \left[ -4 \left( t_4 - b_1 \, t_3 \right) h_2 \, F_{h_1} - 4 \left( t_5 - a_1 \, t_3 \right) F_{h_2} \, h_1 + 2 \, t_6 \, \psi_{h_2} \, \psi_{h_1} \right]
+ \frac{M^3}{M} \left[ -4 \left( t_7 - a_1 \, t_4 - b_1 \, t_5 + a_1 \, b_1 \, t_3 \right) h_2 h_1 \right] + h.c. \tag{25}
\]

where the hermitian conjugation h.c. applies to all terms above and where we ignored \(\mathcal{O}(1/M^2)\) corrections. Also \(D_1\) and \(\lambda_1\) are components of the vector superfield \(V_1\) and we also used the component notation \(H_i = (h_i, \psi_{h_i}, F_{h_i})\). In \(\Delta \mathcal{L}\) we replaced \(k_1, (k_2)\) by \(a_1, (b_1)\) respectively, which is correct in the approximation of ignoring \(1/M^2\) terms in the Lagrangian. The coefficients \(t_i\) are given by

\[
\begin{align*}
t_0 &= \alpha_0 \beta_0 \gamma_0 + s_0^* + s_0^{'*}, \\
t_1 &= d_1 - s_2^* - b_1 s_0^{'*}, \\
t_2 &= d_2 - a_1 s_0^* - s_2^{'*}, \\
t_3 &= d_3 - s_1^* - a_1 s_0^* - s_1^{'*} - b_1 s_0^{'*} \tag{26}
\end{align*}
\]

and where \(d_i\) are combinations of input parameters \(\alpha_i, \beta_i, \gamma_i\) of eq.(25)

\[
\begin{align*}
d_1 &\equiv -\beta_1 \alpha_0 \gamma_0 - \alpha_1 \beta_0 \gamma_0/2, \\
d_2 &\equiv -\gamma_1 \beta_0 \alpha_0 - \alpha_1 \beta_0 \gamma_0/2, \\
d_3 &\equiv -\alpha_2 \beta_0 \gamma_0 - \alpha_0 \beta_2 \gamma_0 - \alpha_0 \beta_0 \gamma_2, \\
d_4 &\equiv -\beta_3 \alpha_0 \gamma_0 - \beta_1 \alpha_2 \gamma_0 - \alpha_0 \beta_1 \gamma_2 \\
d_5 &\equiv -\gamma_3 \beta_0 \alpha_0 - \gamma_1 \alpha_2 \beta_0 - \alpha_0 \beta_2 \gamma_1, \\
d_6 &\equiv \alpha_3 \gamma_0 \beta_0 + \alpha_1 \beta_2 \gamma_0 + \alpha_1 \beta_0 \gamma_2 \\
d_7 &\equiv -\gamma_3 \beta_1 \alpha_0 - \gamma_1 \beta_3 \alpha_0 - \gamma_1 \beta_1 \alpha_2. \tag{27}
\end{align*}
\]

A suitable choice of coefficients \(s_0, s_0^{'}, s_2^{'}, s_2\) entering in transformation \(\mathcal{L}\) allows us to set

\[
t_i = 0, \quad i = 0, 1, 2, 3. \tag{28}
\]
This ensures that the non-standard terms in the first, second and third lines of \( \Delta \mathcal{L} \) above are not present. The remaining terms proportional to \( M_s^2 \) and \( M_s^3 \) bring a renormalisation of the soft terms only, which are present anyway in the Lagrangian of (22), thus can be ignored (recall that the auxiliary fields can be replaced onshell by their lowest order (MSSM) values).

Finally, the term \( t_6 \psi_{h_2} \psi_{h_1} \) brings a renormalisation of the supersymmetric \( \mu' \) term \((\mu' H_1 H_2)\) of (22), induced by soft supersymmetry breaking, and is invariant under the general field transformations (11). In principle one could set additional coefficients of the last two lines in \( \Delta \mathcal{L} \) to vanish by a suitable choice of remaining \( s_{1,3}, s'_{1,3}; \) we choose not to do so and instead save these remaining coefficients for additional conditions that can be used to simplify the (couplings or the spurion dependence of our) Lagrangian even further.

We then obtain the minimal set of dimension-five operators beyond the MSSM Lagrangian

\[
\mathcal{L} = \mathcal{L}_K + \int d^4 \theta \left[ \left( 1 - \frac{m_2^2}{M_s^2} S^\dagger S \right) H_1^\dagger e V_1 H_1 + \left( 1 - \frac{m_2^2}{M_s^2} S^\dagger S \right) H_2^\dagger e V_2 H_2 \right]
\]

\[
+ \int d^2 \theta \left[ -H_2 Q \lambda''_U(S) U^c - Q \lambda''_D(S) D^c H_1 - L \lambda''_E(S) E^c H_1 + \mu''(S) H_1 H_2 \right] + h.c.
\]

\[
+ \frac{1}{M} \int d^2 \theta \left[ Q U^c T'_Q(S) Q D^c + Q U^c T'_L(S) L E^c + \lambda'_H(S) (H_1 H_2)^2 \right] + h.c.
\]

\[
+ \frac{1}{M} \int d^4 \theta \left[ H_1^\dagger e V_1 Y''_U(S, S^\dagger) U^c + H_2^\dagger e V_2 Y''_D(S, S^\dagger) D^c + H_2^\dagger e V_2 L Y''_E(S, S^\dagger) E^c \right] + h.c.
\]

(29)

where \( \mathcal{L}_K \) stands for gauge kinetic terms and for kinetic terms of MSSM fields other than \( H_{1,2} \), together with their spurion dependence; \( \mu'' \) now includes the renormalisation due to \( t_6 \) (not shown). This Lagrangian gives the irreducible set of dimension-five R-parity conserving operators that can be present beyond the MSSM and is one of the main results of this work.

As explained above, there is still some remaining freedom of the field redefinitions that will be used in the next section. The couplings entering above are given in eqs. (16), (17), (18), (23), (24) in terms of those in the original Lagrangian. The couplings \( \lambda''_{U,D,E}(S) \) acquired a threshold correction \( \mathcal{O}(1/M) \), which can be obtained from (23). The dimension-five operator that was present in the last line of (13) was completely “gauged away” in the new fields basis, up to effects which renormalised the soft terms (unknown anyway) or the supersymmetric \( \mu \) term. Since the physics should be independent of the fields basis, in this new basis it is
manifest that the last operator in (5) cannot affect the relations among physical masses of the Higgs sector. We discuss this in detail in Section 4.

3 Phenomenology of the new couplings of the MSSM$_5$.

The Lagrangian in (29) has couplings which can have dramatic implications if the scale $M$ is not high enough, in particular due to FCNC effects. Indeed, if $T'_{Q,L}$ and $Y''_F$, $F : U, D, E$, of (29) have arbitrary family dependent couplings, one expects stringent limits from FCNC bounds [9]. It is possible however, under some mild assumptions for the original $\mathcal{L}$ of (1) with (2), (5), that some of the couplings in (29) can be also removed. For example assume that in the original Lagrangian (5) all flavor matrices are proportional to the ordinary Yukawa couplings and similar for $\rho_F$ of (10), (11):

\begin{align*}
T_Q(S) &= c_Q(S) \lambda_U(0) \otimes \lambda_D(0) \\
T_L(S) &= c_L(S) \lambda_U(0) \otimes \lambda_E(0) \\
\rho_F(S) &= c_F(S) \lambda_F(0), \quad F : U, D, E
\end{align*}

and, as usual

\begin{equation}
\lambda_F(S) = \lambda_F(0) (1 + A_F S), \quad F : U, D, E.
\end{equation}

Above $c_{Q,L}(S)$ are some arbitrary input functions of $S$; $\lambda_F(S)$ with $F : U, D, E$ are $3 \times 3$ matrices, while $A_F$ are trilinear couplings. In the following $c_F(S) \equiv c_F^0 + S c_F^1$, $F = U, D, E$ are regarded as free parameters which can be adjusted, together with the remaining $11$ $s_{1,3}$, $s'_{1,3}$, to remove some of the couplings in (29). Indeed, if

\begin{align*}
c_U(S) &= -c_L(S) - c_E(S), \\
c_D(S) &= -c_Q(S) + c_L(S) + c_E(S)
\end{align*}

while $c_E(S)$ remains arbitrary, one obtains

\begin{align*}
T'_Q(S) &= 0, \\
T'_L(S) &= 0
\end{align*}

The eqs in (30) are also motivated by the discussion in Appendix B, eq. (B-7) where a similar structure of $T_{Q,L}$ and $\rho_F$ is generated by integrating out massive $SU(2)$ superfields doublets. $^{11}$ see (10), (12) and (28).
We can therefore remove the associated couplings in \((29)\), the first two terms in the third line of \((29)\). Finally, let us assume that in \((5)\) we also have

\[ Y_F(S,S^\dagger) = f_F(S,S^\dagger)\lambda_F(0), \quad F : U, D, E \]  

where \(f_F\) are spurion-dependent, family-independent functions of arbitrary coefficients:

\[ f_F(S,S^\dagger) = f_{0F}^F + S f_1^F + S^\dagger f_2^F + S S^\dagger f_3^F \]

Using \((24)\), we find that the couplings in \((29)\) are

\[ Y''_F(S,S^\dagger) = \lambda_F(0) \left[ x_0^F + x_1^F S + x_2^F S^\dagger + x_3^F S S^\dagger \right], \quad F = U, D, E \]

One finds

\[ x_0^U = f_0^U - 4 s_0' + c_0^U \]
\[ x_1^U = f_1^U - 4 s_1' + c_1^U + a_1 c_0^U \]
\[ x_2^U = f_2^U - 4 s_2' + a_1 c_0^U - a_1^* x_1^U \]
\[ x_3^U = f_3^U - 4 s_3' + a_1 c_1^U + a_2 c_0^U - a_1^* x_1^U \]

Similar equations exist for \(D\) fields, obtained from those above with replacements \(U \rightarrow D\), \(s_i' \rightarrow s_i\) and \(a_i \rightarrow b_i\). Also for \(E\) fields the replacements are \(U \rightarrow E\), \(s_i' \rightarrow s_i\) and \(a_i \rightarrow b_i\).

Let us examine if the form of \(Y''_F(S,S^\dagger)\) can be simplified using the free parameters that we are left with: these are \(s_{1,3}, s_{1,3}'\) from general transformations \(\Delta_{1,2}\) and \(c_E(S) = c_0^E + S c_1^E\) thus a total of 6 free parameters. We can use \(s_{1,3}'\) \(s_{1,3}\) to eliminate \(S\) and \(S S^\dagger\) parts of \(Y''_U\) \((Y''_D)\), respectively. Using \(c_0^E\) and \(c_1^E\) we can also eliminate the \(S\) and \(SS^\dagger\) of \(Y''_E\). In conclusion we used the remaining 6 free parameters to bring \(Y''_F\) to the form

\[ Y''_F(S^\dagger) \equiv Y''_F(0,S^\dagger) = \lambda_F(0) \left( x_0^F + x_2^F S^\dagger \right), \quad F : U, D, E \]

The coefficients \(x_{0,2}^F\) depend on the arbitrary (input) coefficients \(f_i^F, i = 0, 1, 2, 3, a_i, b_i, c_i\) of the original Lagrangian \((1), (2), (5)\). Other simplifications can occur if we ignore the couplings \(Y\) of the first two families. With these considerations, the Lagrangian in \((29)\) becomes
\[ \mathcal{L} = \mathcal{L}_K + \int d^4 \theta \left[ \left( 1 - \frac{m_1^2}{M_5^2} S^i S \right) H_1^i e^{V_i} H_1 + \left( 1 - \frac{m_2^2}{M_5^2} S^i S \right) H_2^i e^{V_2} H_2 \right] + \int d^2 \theta \left[ -H_2 Q' U(S)^c - Q' D(S) D^c H_1 - L \lambda''_E(S) E^c H_1 + \mu''(S) H_1 H_2 \right] + h.c. \\
+ \frac{1}{M} \int d^4 \theta \left[ H_1^i e^{V_i} Q' U'(S^i) U^c + H_2^i e^{V_2} Q' D'(S^i) D^c + H_1^i e^{V_2} L' E'(S^i) E^c + h.c. \right] \\
+ \frac{1}{M} \int d^2 \theta \lambda''(S) (H_1 H_2)^2 + h.c. \] (39)

with couplings (38), (23)\(^\text{12}\). This defines our MSSM extension with D=5 operators (MSSM\(_5\)).

A detailed analysis of all couplings generated by (39) or by (29) and their phenomenological implications is beyond the scope of this paper. For related studies see also the analysis in [10, 11, 12]. Let us present however all the new couplings generated using component fields and we begin with the couplings proportional to \(M_5\). Part of these are coming from the terms in the second-last line of (39). These include non-analytic Yukawa couplings [4]

\[
\begin{align*}
\frac{M_5}{M} x_2 U (\lambda_0^U)_{ij} (h_1^i q_{L_i}) u_{R_j}^c + h.c. \\
\frac{M_5}{M} x_2 D (\lambda_0^D)_{ij} (h_1^i q_{L_i}) d_{R_j}^c + h.c. \\
\frac{M_5}{M} x_2 E (\lambda_0^E)_{ij} (h_1^i l_{L_i}) e_{R_j}^c + h.c., \quad \lambda_F^E \equiv \lambda_F(0), \quad F : U, D, E. 
\end{align*}
\] (40)

These couplings are not soft in the sense of [13], but “hard” supersymmetry breaking terms (for “non-standard” and “hard” supersymmetry breaking terms see [4, 5]); they are less suppressed than those listed in [4] where they were generated at order \(M_5^2/M^2\). Such couplings can bring about a \(\tan \beta\) enhancement of a prediction for a physical observable, such as the bottom quark mass relative to bottom quark Yukawa coupling [3, 14]. This effect is also present in the electroweak scale effective Lagrangian of the MSSM alone, after integrating out massive squarks at one-loop level, with a result for bottom quark mass [3, 14, 15, 16, 17]

\[
m_b = \frac{v \cos \beta}{\sqrt{2}} \left( \lambda_b + \delta \lambda_b + \Delta \lambda_b \tan \beta \right) \] (41)

\(^\text{12}\) \(\lambda''_F(S)\) acquired a threshold correction in \(M\): \(\lambda''_F(0) = \lambda_U(0) \left[ 1 + 1/M (\mu(0) c_\nu(0) + 2(\mu(0) s_\nu + \mu^*(0) s_\nu^*) \right]\) with similar relations for \(D, E\) obtained by \(s_0 \rightarrow s_0'\) and \(U \rightarrow D, (U \rightarrow E)\). In terms of original parameters, \(s_0 = -[\alpha_0^2 \beta^2 \gamma_0 b_1 - 4 d_3^2 + (f_1^2 + f_2^2 + c_1^2 + c_2^2 + a_1 c_0 + b_1 c_0^*)]/(a_1 - b_1)\) with \(d_3\) as in (27); for the \(D, E\) sectors we use \(s_0' = -\alpha_0^2 \beta^2 \gamma_0 - s_0\). Similar relations exist for non-supersymmetric counterparts, see [23, 24].
where $\lambda_b$ is the ordinary bottom quark Yukawa coupling, $\delta\lambda_b$ its one loop correction and $\Delta\lambda_b$ is a “wrong”-higgs bottom quark Yukawa coupling, generated by integrating out massive squarks. In our case, $\Delta\lambda_b$ receives an additional contribution from the second line in (40). The size of this extra contribution due to higher dimensional operators, can be comparable and even substantially larger than the one generated in the MSSM at one-loop level (for a suitable value for $x^2 M_s/M$ - recall that $x^2$ is not fixed). Such contributions can bring a $\tan\beta$ enhanced correction of the Higgs decay rate to bottom quark pairs. Similar considerations apply to the $U$ and $E$ sectors.

Other similar couplings derived from (39) and proportional to $M_s$ are

$$\frac{M_s}{M} x^U_2 (\lambda_U^0 \lambda_U^0)_{ij} (h_1^\dagger h_2^\dagger) \tilde{u}_{Ri} \tilde{u}_{Rj}^\dagger + h.c.$$ \hspace{1cm} (42)

where we used that $\lambda_{F}^{en}''$ and $\lambda_{F}^{en}$ are equal up to $O(1/M)$ corrections, see (16), (23). The above terms are strongly suppressed due to the square of the Yukawa coupling, in addition to $M_s/M \ll 1$, so their effects are expected to be small, except for the third generation. Their counterparts in the down ($D$) sector are

$$\frac{M_s}{M} x^D_2 (\lambda_D^0 \lambda_D^0)_{ij} (h_2^\dagger h_1^\dagger) \tilde{d}_{Ri} \tilde{d}_{Rj}^\dagger + h.c.$$ \hspace{1cm} (43)

In the lepton sector similar couplings are present, obtained from eq.(43) with $Q \to L$, $D \to E$. All the quartic couplings listed above are renormalisable, but naively they would seem to break supersymmetry in a hard way if inserted into loops with a cutoff larger than $M$. This is of course just an artifact of using a cutoff larger than the energy scale of heavy states that we integrated out.

It is interesting to note that there is no “wrong-Higgs”-gaugino-higgsino coupling generated [4], even though the original Lagrangian in eq.(5) included it, see eq.(25) where

$$\frac{M_s}{M} (\psi_{h_2} \lambda_1 h_1 + h_2 \lambda_1 \psi_{h_1}) + h.c.$$ \hspace{1cm} (44)

was present. Such a coupling can be generated at one loop level, for a discussion see [3]. This coupling was removed in our case by a suitable transformation for the Higgs fields [10]. This
shows that not all “wrong”-higgs couplings are actually independent (this may also apply when such couplings are generated at the loop level).

Note that in the MSSM defined by eq. (39), couplings proportional to $M_s$ involving “wrong”-higgs A-terms are not present, given our ansatz (30), (31) leading to (38). If this ansatz is not imposed on the third generation, then one could have such terms from (29)

$$\frac{M^2}{M} \left[ y_{u,3} h_1^\dagger \tilde{q}_{L,3} \tilde{u}_{R,3}^\dagger + y_{d,3} h_2^\dagger \tilde{q}_{L,3} \tilde{d}_{R,3}^\dagger + y_{e,3} h_2^\dagger \tilde{l}_{L,3} \tilde{e}_{R,3}^\dagger \right]$$ (45)

where $y_{f,3}, f = u, d, e$ are the coefficients of component $S S^\dagger$ of $Y''(S, S^\dagger)$ of third generation.

There are also new, and perhaps most important, supersymmetric couplings generated, that affect the amplitude of processes like quark + quark $\rightarrow$ squark + squark, or involving (s)leptons too. These are

$$\frac{1}{M} x_0^D (\lambda_0^D)_{ij} (\lambda_0^U)_{kl} \tilde{q}_{L,i} \tilde{d}_{R,j} q_{L,k} u_{R,l}^c + h.c.$$ (46)

These couplings can be important particularly for the third generation. The largest effect would be for squarks pair production from a pair of quarks; the process could be comparable to the MSSM tree level contribution to the amplitude of the same process [18]. Indeed, let us focus on the $q \bar{q} \rightarrow \tilde{q} \tilde{q}^*$ in MSSM generated by a tree-level gluon exchange. The MSSM amplitude behaves as

$$A_{q \bar{q} \rightarrow g \rightarrow \tilde{q} \tilde{q}^*} \sim \frac{g_3^4}{\sqrt{s}},$$ (47)

where $s$ is the Mandelstam variable. On the other hand, the operators (46) generate a contact term contributing

$$A_{q \bar{q} \rightarrow \tilde{q} \tilde{q}^*}^{MSSM_5} \sim \frac{\lambda_0^U \lambda_0^D}{M}.$$ (48)

The dimension-five operator for the third generation has therefore a comparable contribution to the MSSM diagrams for energies $E \geq g_3^2 M$, which can be in the TeV range. In MSSM there are other diagrams contributing to this process, in particular Higgs exchange. It can be checked however that at energies above the CP-even Higgs masses, the MSSM amplitude
decreases in energy whereas the contact term coming from the dimension-five operators gives a constant contribution which is sizeable for high energy. Of course, at energies above \( M \) we should replace the contact term by the corresponding tree-level diagram with exchange of massive \( SU(2) \) doublets (or whatever other physics generates this effective operator).

Note that couplings similar to (46) could also be generated by the term \( \int d^2\theta (QU) T_Q (QD) \) of (29). This term is not present in MSSM of (39) due to our FCNC ansatz (30), (33); however, for the third generation this constraint of the ansatz can be relaxed. Therefore the above process of squark production can have an even larger amplitude, from contributions in the third line of (29).

The Lagrangian (39) also contains other (supersymmetric) couplings involving gauge interactions which can be important for phenomenology. They arise from any dimension-five D-term in (39) giving

\[
\mathcal{L} \supset \left( \lambda_U^{ij} \right) \frac{x_U^U}{M} \left[ -h_1^\dagger D_\mu (\tilde{q}_{L_i} u_{R_j}^* ) - \frac{1}{\sqrt{2}} h_1^\dagger \lambda_1 \left( \tilde{q}_{L_i} u_{R_j}^* + q_{L_i} u_{R_j}^* \right) - \frac{1}{\sqrt{2}} \psi_{h_1} \lambda_1 \tilde{q}_{L_i} \tilde{u}_{R_j}^* \right] \\
+ \frac{1}{2} h_1^\dagger D_1 \tilde{q}_{L_i} \tilde{u}_{R_j}^* + i \bar{\psi}_{h_1^i} \sigma^\mu D_\mu \left( \tilde{q}_{L_i} u_{R_j}^* + q_{L_i} u_{R_j}^* \right) \right] \\
+ (U \rightarrow D, H_1 \rightarrow H_2, V_1 \rightarrow V_2) + (Q \rightarrow L, H_1 \rightarrow H_2, V_1 \rightarrow V_2, U \rightarrow E) + h.c. 
\] (49)

where \( D_1, \lambda_1 \) are the auxiliary and gaugino components of \( V_1 \) vector superfield, and

\[
D_1 \equiv -\frac{g_2^2}{2} \left[ h_1^\dagger \sigma h_1 + h_2^\dagger \sigma h_2 + \tilde{q}_{L_i}^\dagger \sigma \tilde{q}_{L_i} + \tilde{l}_{L_i}^\dagger \sigma \tilde{l}_{L_i} \right] \\
+ \frac{g_1^2}{2} \left[ -h_1^\dagger h_1 + h_2^\dagger h_2 + \frac{1}{3} \tilde{q}_{L_i}^\dagger \tilde{q}_{L_i} - \frac{4}{3} \tilde{u}_{R_i} \tilde{u}_{R_i} + \frac{2}{3} \tilde{d}_{R_i} \tilde{d}_{R_i} - \tilde{l}_{L_i} \tilde{l}_{L_i} + 2 \tilde{e}_{R_i} \tilde{e}_{R_i}^* \right] 
\] (50)

Here \( D_\mu \) is the covariant derivative, \( D_\mu = \partial_\mu + i/2 V_{1,\mu} \), where \( V_{1,\mu} \) is the gauge field of the vector superfield \( V_1 \equiv g_2 V_{1i}^\dagger \sigma^i - g_1 V_Y \), introduced in eq. (2). Couplings similar to those above are generated by the substitutions shown in (49). Of the couplings above, phenomenologically relevant could be those involving 2 particles and 2 sparticl es, such as higgs-quark-squark-gaugino, or gauge-quark-higgsino-squark arising from (49). Also notice the presence in this eq of the first term with a “wrong-higgs”-squark-squark derivative coupling.
Yukawa interactions also generate supersymmetric couplings of structure similar to some of those in (49), involving 4 squarks and a higgs or 2 squarks and 3 higgses, or 2 squarks, 2 sleptons plus a higgs. However, these arise at order $\lambda^3$, where $\lambda_F$, $F : U,D,E$ are Yukawa couplings entering (39). Therefore they are suppressed both by the scale $M$ and, relative to the above gauge counterparts, also by an extra Yukawa coupling (this is due to the presence of an extra Yukawa coupling in the third line of (39) relative to ordinary D-terms. The strength of these interactions is also sub-leading to other Yukawa interactions listed so far (which also involved fewer (s)particles).

Finally, supersymmetric couplings with 3 higgses and 2 squarks or 2 sleptons arise from $(H_1 H_2)^2$ of (39), (suppressed by two Yukawa couplings and by the scale $M$); also generated are potentially larger couplings of 2 higgses and 2 higgsinos, being suppressed only by $\lambda_H(0)$ and by the scale $M$. There are also non-supersymmetric couplings with 4 higgs fields, whose effects are discussed in Section 4. This concludes our discussion of all the new couplings generated by dimension-five operators in the MSSM5.

4 The MSSM Higgs sector with dimension-five operators.

In the following we restrict the analysis to the MSSM Higgs sector extended by D=5 operators and analyse their implications. In this sector there are in general two dimension-five operators that can be present and affect the Higgs fields masses, shown in eq.(51) below. According to our previous discussion the last operator in (51) is redundant and can be “gauged away”. However, in this section we choose to keep it, in order to show explicitly that it does not bring new physics of its own.\(^\text{13}\)

The relevant part of MSSM Higgs Lagrangian with D=5 operators is

\[
\mathcal{L}_1 = \int d^4\theta \left[ Z_1(S,S^\dagger) \, H_1^\dagger e^V_1 H_1 + Z_2(S,S^\dagger) \, H_2^\dagger e^V_2 H_2 \right] \\
+ \int d^2\theta \left[ \tilde{\mu} \, (1 + c_1 S) \, H_1 H_2 + \frac{c_3}{M} \, (1 + c_2 S) \, (H_1 H_2)^2 \right] + h.c. \\
+ \frac{1}{M} \int d^4\theta \left\{ A(S,S^\dagger) \, D^{\alpha} \left[ B(S,S^\dagger) \, H_2 e^{-V_1} \right] D_{\alpha} \left[ \Gamma(S,S^\dagger) \, e^{V_1} H_1 \right] + h.c. \right\}
\]

Additional spurion dependence arises from the dimension-five operators considered. For the

\(^{13}\)In the exact susy case, if set onshell this operator brings only wavefunction renormalisation (Appendix B)
definitions of $A(S, S^\dagger)$, $B(S, S^\dagger)$, $\Gamma(S, S^\dagger)$ see eq. [8]. After some calculations, elimination of the auxiliary fields and a re-scaling of the scalar fields, the scalar part of $\mathcal{L}_1$ in (51) becomes:

$$
\mathcal{L}_{1,\text{scalar}} = -\frac{1}{8} (g_1^2 + g_2^2) (|h_1|^2 - |h_2|^2)^2 + \frac{M_\mu}{M} (g_1^2 + g_2^2) (|h_1|^2 - |h_2|^2) (\delta_1 h_1 h_2 + \text{h.c.})
$$

$$
+ \frac{2c_3}{M} (|h_1|^2 + |h_2|^2) (\bar{\mu} h_1 h_2 + \text{h.c.}) - \frac{M_\mu}{M} c_3 (\delta_2 h_1 h_2)^2 + \text{h.c.}
$$

$$
- (|\bar{\mu}|^2 + m_1^2) |h_1|^2 - (|\bar{\mu}|^2 + m_2^2) |h_2|^2 - (h_1 h_2 B_\mu + \text{h.c.}) - h_1^2 D^2 h_1 - h_2^2 D^2 h_2
$$

where

$$
m_1^2 = M_s^2 \left( |a_1|^2 - a_2 \right) + \mathcal{O}(M_s/M)
$$

$$
m_2^2 = M_s^2 \left( |b_1|^2 - b_2 \right) + \mathcal{O}(M_s/M)
$$

$$
B_\mu = \bar{\mu} M_s \left( c_1 - a_1 - b_1 \right) + \mathcal{O}(M_s/M)
$$

The $\mathcal{O}(M_s/M)$ corrections in (53) are not shown explicitly since they only renormalise $m_{1,2}$ and $B_\mu$ which are anyway unknown parameters of the MSSM. In (52) we denoted

$$
\delta_1 = -\beta_1 \alpha_0 \gamma_0 + \gamma_1 \beta_0 \alpha_0 - \alpha_0 \beta_0 \gamma_0 (a_1 - b_1), \quad \delta_2 = c_2 + 2(a_1 + b_1),
$$

From (52) we notice the presence in the scalar potential of three contributions, all introduced by our dimension-five operators. The contributions proportional to $c_3$ in (52) are due to $(H_1 H_2)^2$ in (51) and where discussed in [19] (also [20, 21]; for a review see [22]). The contribution proportional to $\delta_1$ in (52)

$$
(|h_1|^2 - |h_2|^2) (h_1 h_2 + \text{h.c.}),
$$

(55)

was introduced by the dimension-five operator in the last line of (51). This is a new contribution to the scalar potential, and is vanishing if $\alpha_0 = \beta_0 = \gamma_0$. An interesting feature of this new contribution to the MSSM scalar potential is that its one-loop contribution to $h_{1,2}$ self-energy remains soft (no quadratic divergences) despite its higher dimensional origin.\(^{14}\)

\(^{14}\) One can ask what happens to the value of $\delta_1$ after one uses the remaining freedom of rescaling the chiral superfields in (51) as follows: $H_1 \rightarrow (1 - a_1 S) H_1$; $H_2 \rightarrow (1 - b_1 S) H_2$. Under such rescaling $\beta_1 \rightarrow \beta_1 - \beta_0 b_1$, $\gamma_1 \rightarrow \gamma_1 - \gamma_0 a_1$, see [8]. Using the value of $\delta_1$ in (54) (now with $a_1 = b_1 = 0$) and with these new values of $\beta_1, \gamma_1$ one immediately sees that $\delta_1$ is invariant/remains unchanged under this rescaling.
4.1 Higgs mass corrections beyond the MSSM.

Let us consider the implications of (52) for the Higgs masses. The scalar potential is

\[ V = \tilde{m}_1^2 |h_1|^2 + \tilde{m}_2^2 |h_2|^2 + \left( B \mu h_1 h_2 + h.c. \right) + \frac{g^2}{8} \left( |h_1|^2 - |h_2|^2 \right)^2 
\]

\[ + \left( |h_1|^2 - |h_2|^2 \right) \left( \eta_1 h_1 h_2 + h.c. \right) + \left( |h_1|^2 + |h_2|^2 \right) \left( \eta_2 h_1 h_2 + h.c. \right) \]

\[ + \frac{1}{2} \left( \eta_3 (h_1 h_2)^2 + h.c. \right) \]  

(56)

where the definition of \( \eta_{1,2,3} \sim 1/M \) can be read from eq. (52). We take for simplicity \( \eta_i \) real, and therefore \( \eta_3 \geq 0, |\eta_2| \leq \eta_3/4 \). Also

\[ \tilde{m}_1^2 \equiv m_1^2 + |\tilde{\mu}|^2, \quad \tilde{m}_2^2 \equiv m_2^2 + |\tilde{\mu}|^2, \quad g^2 \equiv g_1^2 + g_2^2 \]  

(57)

Consider quantum fluctuations

\[ h_i = \frac{1}{\sqrt{2}} (v_i + \tilde{h}_i + i\tilde{\sigma}_i), \quad i = 1,2 \]  

(58)

where \( v_{1,2} \) are the minimum vev’s of \( V \). Following the details presented in Appendix D and using the minimum conditions for \( V \) one shows that the Goldstone boson has \( m_G = 0 \) and the pseudoscalar Higgs \( A \) has a mass

\[ m_A^2 = -\frac{1 + u^2}{u} B \mu + \frac{u^2 - 1}{2u} \eta_1 v^2 - \frac{1 + u^2}{2u} \eta_2 v^2 - \eta_3 v^2 \]  

(59)

with the notation \( u \equiv \tan \beta \) and \( B \mu < 0 \). Also \( v_1 = v \cos \beta, v_2 = v \sin \beta \) and \( m_Z^2 = g^2/4 \). The masses of the CP even Higgs scalars \( h, H \) are (see also eq. (D-16)):

\[ m_{h,H}^2 = \frac{1}{2} \left[ \frac{m_A^2 + m_Z^2 + \sqrt{w''}}{\sqrt{w''}} \right] \pm \eta_1 v^2 \sin 4\beta \frac{m_A^2}{\sqrt{w''}} \]

\[ + \eta_2 v^2 \sin 2\beta \left[ 1 \pm \frac{m_A^2 + m_Z^2}{\sqrt{w''}} \right] + \frac{\eta_3 v^2}{2} \left[ 1 \mp \frac{(m_A^2 - m_Z^2) \cos 2\beta}{\sqrt{w''}} \right] \]  

(60)

where the upper (lower) signs correspond to \( h \) (\( H \)) respectively and

\[ w'' \equiv (m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos^2 2\beta \]  

(61)
For $\eta_2 = \eta_3 = 0$ one finds from (60)

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2$$

(62)

which is independent of $\eta_1$. Then $\eta_1$ does not affect the relation among physical masses, which is consistent with the result of Section 2 where the last term in (51) responsible for $\eta_1$ term in $V$ could be removed by a suitable field redefinition.

For $\eta_1 = 0$ the result in (60) reproduces that in the first line of eq.(31) in [19] 15. In the limit of large $\tan \beta$ with $m_A$ as a parameter fixed at a value $m_A > m_Z$ one finds:

$$m_h^2 = m_Z^2 + \frac{4 m_A^2 v^2}{m_A^2 - m_Z^2} (\eta_2 - \eta_1) \cot \beta$$

$$- \frac{4 m_A^2 m_Z^2}{m_A^2 - m_Z^2} \left[ 1 - \eta_3 v^2 \frac{m_A^4 + m_Z^4}{2 m_A^2 m_Z^2 (m_A^2 - m_Z^2)} \right] \cot^2 \beta + \mathcal{O}(\cot^3 \beta)$$

(63)

and

$$m_H^2 = m_A^2 + \eta_3 v^2 + \frac{4 (m_A^2 \eta_1 - m_Z^2 \eta_2) v^2}{m_A^2 - m_Z^2} \cot \beta$$

$$+ \frac{4 m_A^2 m_Z^2}{m_A^2 - m_Z^2} \left[ 1 - \eta_3 v^2 \frac{m_A^4 + m_Z^4}{2 m_A^2 m_Z^2 (m_A^2 - m_Z^2)} \right] \cot^2 \beta + \mathcal{O}(\cot^3 \beta)$$

(64)

Therefore

$$\delta m_h^2 = \frac{4 m_A^2 v^2}{m_A^2 - m_Z^2} (\eta_2 - \eta_1) \cot \beta + \mathcal{O}(\cot^2 \beta)$$

$$\delta m_H^2 = \eta_3 v^2 + \frac{4 (m_A^2 \eta_1 - m_Z^2 \eta_2) v^2}{m_A^2 - m_Z^2} \cot \beta + \mathcal{O}(\cot^2 \beta)$$

(65)

in agreement with [19] for $\eta_1 = 0$. The above expansions for large $\tan \beta$ should be regarded with due care, since in fact they are the results of a double series expansion, in $\eta_i$ and $1/\tan \beta$.

Assuming $\eta_3 = 0$ (then $\eta_2 = 0$, too), the term proportional to $\cot \beta$ in (63) is larger than the sub-leading one ($\cot^2 \beta$), giving $m_h^2 - m_Z^2 > 0$ if $|\eta_1/g^2| \geq 1/(4 \tan \beta)$. This bound is however outside the validity of the perturbative expansion in $\eta_1$ as we shall see shortly 16, and then this large $\tan \beta$ expansion is not useful. If $\eta_{1,2} = 0$ and $\eta_3$ non-zero and positive then one could

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15 In the notation of [19], our $\eta_2 = 2 \epsilon_1$, and $\eta_3 = 2 \epsilon_2$, and $v$ has a different normalisation there.

16 See the bounds from (D-18) and discussion below.
obtain \( m_h > m_Z \) if the square bracket in (63) is negative, which is more easily satisfied (for a small \( \eta_3 \)) if \( m_A \) is very close to \( m_Z \), but then the above large \( \tan \beta \) expansion is not reliable.

Let us therefore analyse the validity of the corrections to \( m_{h,H}^2 \) from eqs. (63), (D-16), in the approximation used. For our perturbative expansion in \( \eta_i \) to be accurate we require that the \( \eta_i \)-dependent entries in the mass matrix \( M_{ij} \) (D-2) be much smaller than the corresponding values of these matrix elements in the MSSM case. From this condition one finds\(^{17}\)

\[
\begin{align*}
|3(\eta_1 + \eta_2) v_1^2 + 3(\eta_2 - \eta_1) v_2^2 + 2\eta_3 v_1 v_2| & \ll \frac{1}{2} g^2 v_1 v_2 \\
|6(\eta_2 - \eta_1) v_1 v_2 + \eta_3 v_1^2| & \ll \frac{1}{4} g^2 | v_1^2 - 3 v_2^2 | \\
|6(\eta_2 + \eta_1) v_1 v_2 + \eta_3 v_2^2| & \ll \frac{1}{4} g^2 | 3 v_1^2 - v_2^2 | \\
\end{align*}
\]

(66)

Similar conditions are derived from the pseudoscalar Higgs/Goldstone bosons mass matrix elements \( N_{ij} \) (D-11). From these one can obtain some upper bounds for each \( \eta_i \); lower bounds on \( \eta_i \) can be derived from the condition that the contribution of each \( \eta_i \) or combinations thereof increase \( m_h \) above \( m_Z \) (to avoid the MSSM tree level bound \( m_h \leq m_Z \)). If all these bounds on \( \eta_i \) can be respected simultaneously, then it is possible to obtain \( m_h > m_Z \) in the approximation considered.

Assuming \( \eta_2 = 0 \), then \( m_h > m_Z \) is possible if one or both eqs in (D-18) are respected. One can show that for \( 1 \leq \tan \beta \leq 50 \) and \( m_A/m_Z \geq 1 \) eq. (D-18) has no solution for \( \eta_1 \); therefore \( \eta_1 \) alone cannot change the MSSM bound \( m_h \leq m_Z \) within our approximation. If \( 1 \leq m_A^2/m_Z^2 \leq 2.43 \) there is a somewhat “marginal” solution for \( \eta_3 \) of (D-18), with values of \( m_A/m_Z \) close to unity and with large \( \tan \beta \) preferred, to enforce the “\( \ll \)” inequalities in (66), (D-18). For example, for \( m_A = m_Z \) and \( \tan \beta = 50 \) the lower bound on \( \eta_3/g^2 \) is \( \eta_3/g^2 \geq 0.02 \) while \( \eta_3/g^2 \ll 0.25 \) is also required; in this case, for \( \tan \beta = 50 \) the increase of \( m_h \) relative to \( m_Z \), \( \delta_r = (m_h^2 - m_Z^2)/m_Z^2 \) equals \( \delta_r = -100/2501 + 2 \eta_3/g^2 \). Therefore \( \delta_r = 12\% \) or \( m_h \approx 102 \)

---

\(^{17}\) One may find this condition too restrictive; in principle it may not be necessary to impose the leading \( \eta_i \sim \mathcal{O}(1/M) \) contribution to the mass matrix entries be suppressed relative to the MSSM zeroth order and that one should instead ask that the \( \mathcal{O}(1/M) \) correction dominate over the higher order terms \( \mathcal{O}(1/M^2) \). However, at the quantitative level this leads, for the present case, to results which are similar or even stronger (for example for \( \eta_3 \)) than those derived here from comparing the MSSM zeroth order against the \( \mathcal{O}(1/M) \) terms. (We thank K. Blum, Y. Nir and G.G. Ross for bringing this issue to our attention).

\(^{18}\) Note that a non-zero \( \eta_2 \) requires nonzero \( \eta_3 \) since \( |\eta_2| \leq \eta_3/4 \).
GeV if $\eta_3/g^2 = 0.08$, corresponding to $\eta_3 = 4.4 \times 10^{-2}$. Larger values for $m_h$ should be regarded with care, since would correspond to cases when $\ll$ of (D-18) is not comfortably respected; if $\eta_3/g^2 \approx 0.04$ then $\delta_r \approx 4\%$ or $m_h \approx 95$ GeV. Further, if we now increase $m_A$ even by a small amount relative to $m_Z$, $m_A^2 = 1.5 m_Z^2$ and $\tan \beta = 50$ the lower bound on $\eta_3/g^2$ is 0.118, difficult to comply by a good margin with an upper bound unchanged at $\eta_3/g^2 \ll 0.25$. Even so, then $\delta_r = 2 \times 10^{-3}\%$ only, if $\eta_3/g^2 = 0.118$ ($\eta_3 = 6.48 \times 10^{-2}$), therefore the increase of $m_h$ is negligible. So far we took $\eta_2 = 0$; if we allow a non-zero value for $\eta_2$, which also requires non-zero $\eta_3$, their combined effect on increasing $m_h$ is not larger, and the above results remain valid. Note also that for large $\tan \beta$ regions $1/M^2$-suppressed operators can be important and can affect the results [19].

From this analysis we see that $\eta_1$ alone cannot change the MSSM tree level bound $m_h \leq m_Z$ within the approximation we discuss. This is consistent with Section 2, where it was shown that the operator which induced the $\eta_1$ term could be removed by a general field redefinition of suitable coefficients [19]. However, $\eta_3$ can increase $m_h$ to values $\approx 95 - 100$ GeV if $m_A \approx m_Z$, with the higher values close to the limit of our approximation. Therefore it is the susy breaking term associated to $(H_1 H_2)^2$ that could relax the MSSM tree level bound. This increase brings a small improvement. To conclude, adding the quantum corrections is still needed [19] to bring $m_h$ above the LEP II bound of 114 GeV [6].

These findings show that the MSSM Higgs sector is rather stable under the addition of D=5 operators, in the approximation we considered (expansion in $1/M$) of integrating out a massive singlet or a pair of massive SU(2) doublets which generated the $\eta_{1,2,3}$ contributions. If $M$ is low-enough, the approximation used of integrating out these massive fields becomes unreliable, and one should re-compute the full spectrum with all fields un-integrated out. Then the quartic interactions that the initial massive fields brought can be larger or of similar order (rather than corrections) to their MSSM counterparts, and can change the above conclusions.

5 Conclusions

In this work we considered a natural extension of the MSSM by the addition of R-parity conserving dimension-five operators and analysed some of their implications. As we showed, such operators are a common presence in effective theories, generated by integrating out massive

\footnote{To see this one can also start from (51) and perform a “smaller” version of redefinition (10), with $\rho_F = 0.$}
singlets and $SU(2)$ doublets superfields. As it turns out, not all these higher dimensional operators are independent. We presented a method which employs general, spurion dependent field transformations to identify the minimal, irreducible set of such operators that one has beyond the MSSM. This is done by using field redefinitions suitably chosen to remove some of the “redundant” operators, up to renormalisations of the $\mu$-term and of the soft terms. As a result, the low energy effective theory has the advantage of a smaller number of couplings (i.e. parameters) and its study is simplified. The method can be applied to other, more general models too.

The minimal set of D=5 operators can be reduced further provided that appropriate relations exist between the original couplings of the dimension-five operators and the usual MSSM Yukawa couplings. Such relations are expected to exist in the original Lagrangian to avoid FCNC constraints. In this case, at order $1/M$, one is left with $(H_1 H_2)^2$ and three additional Higgs-dependent D-terms $\langle 39 \rangle$, together with associated, spurion-induced supersymmetry breaking terms of a particular type. The superpotential couplings and their associated soft terms acquire, already at the classical level, nontrivial renormalisations, which depend on the scale $M$ of the higher dimensional operators. If our FCNC ansatz is imposed only for the first two generations, quartic terms in the superpotential $Q_3^c U_3^c D_3^c$ and $Q_3^c U_3^c L_3 E_3^c$ are also irreducible.

The dimension-five Higgs-dependent D-terms leftover affect the couplings of the model MSSM$^5$. In components, these terms contain “wrong”-higgs (susy breaking) Yukawa couplings. These are also known to be generated in the MSSM alone at one-loop level by integrating out massive squarks; our new contributions can be significant if the new physics is not far above LHC energies. The combined effect of the two sources for these couplings brings a tan $\beta$-enhancement of the mass of the bottom quark. Even more interesting are supersymmetric couplings of type quark-quark-squark-squark and also quark-quark-slepton-slepton, that are also generated from the aforementioned D-term operators of dimension five and/or by the quartic superpotential couplings if the FCNC ansatz is made only for the first two generations. These couplings, although suppressed by $1/M$ can contribute significantly, for the case of the third generation, to the process of squark production. This contribution competes with that of the similar process coming from the MSSM at the tree level. This is phenomenologically important since direct squark production can be a first indication of supersymmetry at the LHC and this process is significantly enhanced in the model we discussed.
We also addressed the effects that dimension-five operators have on the Higgs sector. We included all possible contributions of the operators that can be in general present, due to \( \mathcal{O}_1 = A(S,S^\dagger)D^\alpha \left[ B(S,S^\dagger)H_2 e^{-V_1} \right] D_\alpha \left[ \Gamma(S,S^\dagger) e^{V_1} H_1 \right] \) and \( \mathcal{O}_2 = \lambda_H(S)(H_1 H_2)^2 \). The analysis showed that the MSSM tree level bound \( m_h \leq m_Z \) cannot easily be lifted by \( \mathcal{O}_{1,2} \) and their associated susy breaking terms. In the case of \( \mathcal{O}_1 \) this is due to the fact that this is ultimately a “redundant” operator and can be removed by a field redefinition, as showed in Section 2. \( \mathcal{O}_1 \) brings ultimately only a renormalisation of the soft terms and of supersymmetric \( \mu \)-term. Within the approximation used, the non-susy part of \( \mathcal{O}_2 \) can bring (somewhat close to the limit of validity of our approximation), an increase of \( m_h \) to \( m_h \approx 95 - 100 \) GeV, while in that case \( m_A \approx m_Z \). This shows that the MSSM Higgs sector is rather stable under the addition of D=5 operators, in the approximation we considered. This result for the Higgs sector is somewhat expected in an effective theory where additional higher dimensional operators can only bring small corrections to current relations among physical observables of the initial model. Therefore quantum corrections are still needed to increase \( m_h \) above the LEP II bound of 114 GeV.

In conclusion, the natural extension of the MSSM with the minimal, irreducible set of R-parity conserving dimension-five operators that we identified, provides a consistent and very interesting framework for future detailed phenomenological studies. The method presented to identify the minimal set of these operators beyond the MSSM is general and can be applied to sets of operators of higher dimensions and/or of different symmetries.

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Appendix

A Integrating out massive superfields: no gauge interactions present.

In this appendix we examine different methods of integrating out high scale physics and confirm their equivalence, by showing that the same low energy effective Lagrangian is obtained. We ignore gauge interactions, included in Appendix B. We find that integrating out massive states generates in the effective action and in the lowest order in the high scale, a (classical) wavefunction renormalisation while in the next order higher dimensional operators emerge. Operators like $\Phi_2 D^2 \Phi_1$ emerge, which in the presence of gauge interactions becomes $\Phi_2 e^{-V} D^2 e^V \Phi_1$, studied in the text, Section 2. Let us start with a 4D renormalisable model (with $M \gg m$)

$$L_1 = \int d^4\theta \left[ \Phi_1^\dagger \Phi_1 + \chi_1^\dagger \chi_1 \right] + \left\{ \int d^2\theta \left[ \frac{M_2}{2} \chi^2 + m \Phi \chi + \frac{\lambda}{3} \Phi^3 \right] + h.c. \right\}$$  (A-1)

With a transformation $\Phi \equiv (\cos \theta \Phi_1 - \sin \theta \Phi_2)$ and $\chi \equiv (\sin \theta \Phi_1 + \cos \theta \Phi_2)$ one finds

$$L_1 = \int d^4\theta \left[ \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \right] + \left\{ \int d^2\theta \left[ \frac{m_1}{2} \Phi_1^2 + \frac{m_2}{2} \Phi_2^2 + \frac{\lambda}{3} \left( \cos \theta \Phi_1 - \sin \theta \Phi_2 \right)^3 \right] + h.c. \right\}$$  (A-2)

where

$$m_1 = \frac{M_2}{2} \left( 1 - (1 + 4m^2/M^2)^{1/2} \right) = -\frac{m^2}{M} \left( 1 - \frac{m^2}{M^2} \right) + \cdots$$

$$m_2 = \frac{M_2}{2} \left( 1 + (1 + 4m^2/M^2)^{1/2} \right) = M \left( 1 + \frac{m^2}{M^2} + \cdots \right),$$  (A-3)

so $\Phi_2$ is the massive field. We can now integrate out $\Phi_2$ via its equations of motion

$$-\frac{1}{4} D^2 \Phi_2 + m_2 \Phi_2 - \lambda \sin \theta \left( \Phi_1 \cos \theta - \Phi_2 \sin \theta \right)^2 = 0$$  (A-4)

with the solution

$$\Phi_2 = \frac{\lambda}{m_2} \cos^2 \theta \sin \theta \Phi_1^2 - \frac{\lambda^2}{4m_2} \sin^3 2\theta \Phi_1^2 + \frac{\lambda}{4m_2} \cos^2 \theta \sin \theta D^2 \Phi_1^{+2} + O(1/M^3).$$  (A-5)

Keeping the lowest, dimension-five operators of $L_1$, we have

$$L_1 = \int d^4\theta \Phi_1^\dagger \Phi_1 + \left\{ \int d^2\theta \left[ \frac{-m^2}{2M} Z \Phi_1^2 + \frac{\lambda}{3} Z^{3/2} \Phi_1^3 - \frac{m^2\lambda^2}{2M^3} \Phi_1^4 \right] + h.c. \right\} + O(1/M^4),$$  (A-6)
where
\[ Z = 1 - \frac{m^2}{M^2} + O(1/M^4) \] (A-7)

As expected, we find that at low energies (\( \ll M \)) a higher dimensional operator \( \Phi_4^4 \) emerges, suppressed by the scale \( M \) of “new physics” represented by the massive state \( \chi \). Other higher dimensional operators are present beyond that of \( O(1/M^3) \) shown, and these include higher derivative operators involving \( D^2 \Phi_1^1 \). As expected, in the low energy limit, the initial 4D renormalisable theory appears as an effective field theory valid below the scale \( M \).

There is another, equivalent way to analyse the Lagrangian in (A-1) in the low energy limit, which illustrates further the emergence of higher dimensional operators. Start again with eq.(A-1), which gives the following eq of motion for the massive field \( \chi \):
\[ 0 = D^2 \chi - 4 (M \chi + m \Phi) \] (A-8)
with an iterative solution
\[ \chi = \frac{1}{M} \left[ -m \Phi - \frac{m}{4M} D^2 \Phi + \frac{1}{16} \frac{m^2}{M^2} D^2 \Phi - \frac{m}{64M^3} D^2 D^2 D^2 \Phi + \cdots \right] \] (A-9)

Using this solution in original \( \mathcal{L}_1 \) of (A-1), one finds
\[
\mathcal{L}_1 = \int d^4 \theta \left\{ \left[ 1 + \frac{m^2}{M^2} \right] \Phi^\dagger \Phi + \frac{m^2}{8M^2} \left[ \Phi D^2 \Phi + h.c. \right] + \frac{m^2}{16M^4} (D^2 \Phi^\dagger) (D^2 \Phi) \right\} \\
+ \left\{ \int d^2 \theta \left[ -\frac{m^2}{2M} \Phi^2 + \frac{\lambda}{3} \Phi^3 \right] + h.c. \right\} + O(1/M^5) \] (A-10)

After an appropriate re-scaling
\[
\mathcal{L}_1 = \int d^4 \theta \left\{ \Phi^\dagger \Phi + \frac{m^2}{8M^3} \left[ \Phi D^2 \Phi + h.c. \right] + \frac{m^2}{16M^4} (\overline{D^2 \Phi^\dagger}) (D^2 \Phi) \right\} \\
+ \left\{ \int d^2 \theta \left[ -\frac{m^2}{2M} \Phi^2 + \frac{\lambda}{3} \left( Z^{3/2} \overline{\Phi}^3 \right) \right] + h.c. \right\} + O(1/M^5) \] (A-11)

where \( Z = 1/(1 + m^2/M^2) \). After\(^ {20} \) integrating out a massive superfield \( \chi \), higher dimensional derivative operators were generated. These are suppressed by \( M \), below which only an effective

\(^ {20} \) Using \( \overline{D^2 D^2} = -16 \Box \) we find a \(-\Phi^\dagger \Box \Phi \) term; the metric is \((+, -, -, -)\).
theory \((A-11)\) applies. Since the presence of massive states in high scale theories is usually expected, the conclusion is that this type of operators are a generic presence at low energies. There are no ghosts in \(L_1\) of \((A-11)\) as long as one keeps all terms in the series \(^{21}(A-9)\). Once we truncate this series to a given order, such states can be generated, as a signature of the fact that the UV of the theory is unknown. Finally, in order \(1/M^2\) the only effect of the massive state is a wavefunction renormalisation which depends on high scale \(M\).

From this stage there are two approaches one can adopt to continue from eq.\((A-11)\).

I). In the first approach one sets “onshell” the higher dimensional operator, using the equations of motion \(^22\); if one adheres to this procedure, the eq of motion
\[
\mathcal{D}^2 \Phi^+ = -\frac{4m^2}{M} \Phi + 4 \lambda \Phi^2 + \mathcal{O}(1/M^2)
\] (A-12)
can be used back in \((A-11)\); the new Lagrangian so obtained will contain a term \(\Phi \Phi^+ \Phi^2\) which can be removed by a suitable shift
\[
\Phi = \tilde{\Phi} - \frac{\lambda m^2}{2 M^3} \Phi^2
\] (A-13)
to finally find
\[
L_1 = \int d^4 \theta \, \tilde{\Phi}^+ \tilde{\Phi}
\] (A-14)
\[
+ \left\{ \int d^2 \theta \left[ -\frac{m^2}{2M} Z \tilde{\Phi}^2 + \frac{\lambda}{3} \tilde{\Phi}^3 \left( 1 - \frac{3 m^2}{2 M^2} \right) - \frac{\lambda^2 m^2}{2 M^3} \tilde{\Phi}^4 \right] + \text{h.c.} \right\} + \mathcal{O}\left( \frac{1}{M^4} \right)
\]
where \(Z = 1/(1 + m^2/M^2)\). In the approximation \(\mathcal{O}(1/M^4)\) this Lagrangian coincides with that of \((A-6)\), where a different method was used. This confirms that setting the higher derivative operators “onshell” via equations of motion is a correct procedure, within the approximation considered. We again obtained a higher dimensional operator and a scale dependence acquired \textit{classically} by the couplings of the low energy effective theory \(^23\).

II). Finally let us now take the second approach to continue from the Lagrangian in \((A-11)\). This will provide another check that setting onshell the higher derivative operators as done above in I) is indeed a correct procedure. In eq.\((A-11)\) proceed to redefine the fields, to eliminate the \(\Phi \mathcal{D}^2 \Phi\) term. We use a field redefinition
\[
\Phi = \Phi' + c \mathcal{D}^2 \Phi^+
\] (A-15)

\(^{21}\)This is true because the original theory \((A-1)\) had no ghosts; for a detailed discussion see \[23, 24\].

\(^{22}\)For an application see \[29\].

\(^{23}\)To the next order, in \((A-11)\) one has extra D terms \((m^2 \lambda^2/M^4) \tilde{\Phi}^2 \tilde{\Phi}^{12}\) and F terms \((39 m^4/(8M^4)) \Phi^3\).
where the dimensionful coefficient \( c \) is found from the requirement that the coefficient of \( \Phi D^2 \Phi \) vanish in the new Lagrangian. This gives \( c = -m^2/(8M^3) \) and the Lagrangian in (A-11) becomes after some calculations

\[
\mathcal{L}_1 = \int d^4\theta \left[ \Phi^\dagger \Phi + \frac{m^2}{2M^3} \left( \Phi^\dagger \Phi + h.c. \right) \right] + \mathcal{O}(1/M^4) \quad (A-16)
\]

After a shift \( \Phi' = \tilde{\Phi} - \frac{m^2}{2M} \lambda/3 Z^2 / 2Z \), we obtain a low energy Lagrangian identical to that in (A-6), (A-14). This result shows that the three approaches to integrating out the effects of high scale physics (\( \chi \)), using (a) eqs. (A-1) to (A-6), or (b) setting the higher dimensional derivative operators “onshell” eqs. (A-8) to (A-14), and finally (c) using field re-definitions (A-15), are equivalent to the lowest order studied. The approaches gave in all cases the same spectrum and couplings, and checked explicitly that setting onshell the higher derivative operators is correct in the approximation considered. To the lowest order in \( 1/M \) only a wavefunction renormalisation was introduced by integrating out massive states, which classically renormalise low energy couplings. Higher dimensional operators were generated in the next order in \( 1/M \).

**B Integrating out massive superfields: gauge interactions present.**

Here we show how all dimension-five operators of \( \mathcal{L}^{(5)} \) of eq.(5) in Section 2 are generated, and discuss in particular \( \Phi_2 e^{-V} D^2 e^V \Phi_1 \). This appendix also extends the analysis in Appendix A where a similar \( \Phi D^2 \Phi \) was shown to arise, in the absence of gauge interactions. Consider the Lagrangian of a N=1 supersymmetric non-Abelian gauge theory \(^{24}\)

\[
\mathcal{L}_2 = \int d^4\theta \left[ \Phi_1^\dagger e^V \Phi_1 + \Phi_3^\dagger e^V \Phi_3 + \Phi_2 e^{-V} \Phi_2^\dagger + \Phi_4 e^{-V} \Phi_4^\dagger \right]
\]

\[
+ \int d^4\theta \left[ \nu_1 \Phi_1^\dagger e^V \Phi_3 + \nu_2 \Phi_4 \right] + h.c. \quad (B-1)
\]

\(^{24}\)For the link to the MSSM, replace \( V \rightarrow V_1 \equiv g_2 V_w^i \sigma^i - g_1 V_Y \) with \( V_w, (V_Y) \) the \( SU(2), (U(1)_Y) \) gauge fields respectively; also \( \Phi_2 \rightarrow H_2^i, (i\sigma_2), \Phi_1 \rightarrow H_1 \) with \( \Phi_3 (\Phi_4) \) with same quantum numbers to \( \Phi_1 (\Phi_2) \) and \( (i\sigma_2) \) exp\((-\Lambda) = \exp(\Lambda T) (i\sigma_2) \), then \( \Phi_2 e^{-V} \Phi_2^\dagger \rightarrow H_2^i e^{V_2} H_2, \) with \( V_2 \equiv g_2 V_w^i \sigma^i + g_1 V_Y \).
where $M \gg \mu$ and with the notation $V \equiv (V_\mu, \lambda, D/2)$ in the Wess-Zumino gauge. For generality and for phenomenological applications we can allow the presence of another higher dimension term $W' = \int d^2 \theta \xi' (\Phi_1 \Phi_2)^2$, where we assume $\xi' \sim \mathcal{O}(1/M)$; ($W'$ can be generated by integrating out a singlet). The equations of motion for massive $\Phi_{3,4}$ give

$$-rac{\nu_1}{4} \overline{D}^2 \left( e^{V} \Phi_1^\dagger \right) - \frac{1}{4} \overline{D}^2 \left( e^{V} \Phi_3^\dagger \right) + M \Phi_4 = 0$$

$$-rac{\nu_2}{4} \overline{D}^2 \left( e^{-V} \Phi_2^\dagger \right) - \frac{1}{4} \overline{D}^2 \left( e^{-V} \Phi_4^\dagger \right) + M \Phi_3 = 0 \quad (B-2)$$

As in previous section we use these equations to integrate out the massive fields $\Phi_{3,4}$ to find

$$L_2 = \int d^4 \theta \left[ (\Phi_1^\dagger e^V \Phi_1 + \Phi_2 e^{-V} \Phi_2^\dagger + \left( \frac{\nu_1 \nu_2}{4} \xi \Phi_1^\dagger e^V \overline{D}^2 e^{-V} \Phi_2^\dagger + h.c. \right) \right]$$

$$+ \int d^2 \theta \left[ \mu \Phi_1 \Phi_2 + \xi' (\Phi_1 \Phi_2)^2 \right] + h.c. + \mathcal{O}(1/M^2),$$

$$\xi \equiv \frac{1}{M} \quad (B-3)$$

where we ignored higher orders in $1/M$. Again, higher dimensional operators were generated by integrating out massive superfields $\Phi_{3,4}$, as expected in the low energy effective action. Before a detailed analysis of (B-3), let us set on-shell the first dimension-five operator in (B-3) by using the equations of motion for $\Phi_{1,2}$:

$$D^2 \left[ e^{V} \Phi_1 \right] = 4 \mu \Phi_2^\dagger, \quad \overline{D}^2 \left[ e^{-V} \Phi_2^\dagger \right] = 4 \mu \Phi_1 \quad (B-4)$$

We insert these in (B-3), then rescale $\Phi_i \rightarrow \Phi_i^i (1 - \mu \nu_1 \nu_2 \xi/2)$, $i = 1, 2$, to find:

$$L_2 = \int d^4 \theta \left[ \Phi_1^\dagger e^V \Phi_1 + \Phi_2 e^{-V} \Phi_2^\dagger \right]$$

$$+ \int d^2 \theta \left[ \mu (1 - \mu \nu_1 \nu_2 \xi) \Phi_1 \Phi_2 + \xi' (\Phi_1 \Phi_2)^2 \right] + h.c. + \mathcal{O}(1/M^2), \quad (B-5)$$

In conclusion, the supersymmetric higher dimensional operator (generated by integrating out massive superfields), when set on-shell, produced in the leading order (in $1/M$) only wavefunction renormalisation. The D=5 D-term operator in (B-3) was studied in Section 2.

If the superpotential in (B-1) also contains trilinear couplings of the heavy doublets $\Phi_{3,4}$ to the quarks

$$\Delta L_2 = \int d^2 \theta \left[ Q \sigma_u U^c \Phi_4 + Q \sigma_d D^c \Phi_3 + L \sigma_e E^c \Phi_3 \right] + h.c., \quad (B-6)$$
then they change the rhs of (B-2) by extra terms and then new higher dimensional operators are also generated in the low energy effective action in addition to the first one in (B-3). More precisely, the Lagrangian in (B-3) acquires a correction
\[
\Delta \mathcal{L}_2' = -\frac{1}{M} \int d^4 \theta \left[ \nu_1 \Phi^e_1 e^V Q \sigma_u U^c + \nu_2 (Q \sigma_D D^c) e^{-V} \Phi^e_1 + \nu_2 (L \sigma_e E^c) e^{-V} \Phi^e_1 + \text{h.c.} \right] + \frac{1}{M} \int d^2 \theta \left[ (Q \sigma_u U^c)(Q \sigma_D D^c) + (Q \sigma_u U^c)(L \sigma_e E^c) \right] + \text{h.c.} ,
\] (B-7)
where \( \sigma_{u,d,e} \) are 3x3 matrices in the family space. This gives one possible origin of the D=5 operators analysed in Section 2. Eq. (B-7) generates tree-level “wrong-Higgs” couplings and fermion-fermion-sfermion-sfermion couplings, discussed in Section 2 and 3. The structure of the couplings in (B-7) also motivates the ansatz made in Section 3. Eq. (30) would be obtained if \( \sigma_F \propto \lambda_F, F : U,D,E \), which could eventually be enforced by family symmetries.

In the remaining part of this section we present the general offshell form of \( \mathcal{L}_2 \) of (B-3).

Using now this form, we check again that the higher dimensional (derivative) operator in (B-3) brings a wavefunction renormalisation only, in the absence of other interactions coupled to \( \Phi_{1,2} \) (like trilinear terms). After a long calculation, one obtains the offshell form\(^{25}\)
\[
\mathcal{L}_2 = -\phi_1^* D_\mu D^\mu \phi_1 + i \bar{\psi}_1 \sigma^\mu D_\mu \psi_1 - \frac{1}{\sqrt{2}} \left[ \bar{\psi}_1 \lambda \phi_1 + \text{h.c.} \right] + \phi_1^* \frac{D}{2} \phi_1 + |F_1|^2 \\
- \phi_2 D_\mu D^\mu \phi_2 + i \psi_2 \sigma^\mu D_\mu \bar{\psi}_2 + \frac{1}{\sqrt{2}} \left[ \phi_2 \lambda \bar{\psi}_2 + \text{h.c.} \right] - \phi_2^* \frac{D}{2} \phi_2 + |F_2|^2 \\
+ \frac{1}{4} \nu_1^* \nu_2^* \xi \left\{ 4 \left[ F_2 D_\mu D^\nu \phi_1 + \phi_2 D_\mu D^\nu F_1 \right] + 2\sqrt{2} \left[ \psi_2 \sigma^\mu \sigma^\nu D_\mu D_\nu \phi_1 + \phi_2 \lambda \phi_1 \right] \\
+ 2 \left( \phi_2 D F_1 - F_2 D \phi_1 \right) - 2\sqrt{2} \left[ \psi_2 \lambda F_1 - F_2 (\lambda \psi_1) \right] - 2 \phi_2 \bar{\phi}_2 \phi_1 \\
- 4 \psi_2 \sigma^\nu \sigma^\mu D_\nu D_\mu \phi_1 \right\} + \mu \left[ \phi_1 F_2 + F_1 \phi_2 - \psi_1 \psi_2 \right] + \mathcal{W}'|_{g_2} + \text{h.c.} + \mathcal{O}(1/M^2) \quad \text{(B-8)}
\]
where
\[
\mathcal{W}'|_{g_2} = \xi' \left[ - \left( \phi_1 \psi_2 + \psi_1 \phi_2 \right) + 2 \left( \phi_1 \phi_2 \right) \left( \phi_1 F_2 + F_1 \phi_2 - \psi_1 \psi_2 \right) \right] \quad \text{(B-9)}
\]
\(^{25}\)We use \(-4 \psi_2 D_\mu D^\nu \psi_1 = -4 \psi_2 (\sigma^\nu \sigma^\mu - 2 i \sigma^{\nu\mu}) D_\nu D_\mu \psi_1 = -4 \psi_2 \sigma^\nu \sigma^\mu D_\nu D_\mu \psi_1 + 4 \psi_2 \sigma^{\nu\mu} F_{\mu\nu} \psi_1\) and the first term in the rhs is that entering the final expression of \( \mathcal{L}_2 \). Here \( F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + i [V_\mu/2, V_\nu/2] \).
and with

\[ D_\mu = \partial_\mu + i \frac{V_\mu}{2}, \quad \overline{D}_\mu = \overline{\partial}_\mu - i \frac{V_\mu}{2}, \]

The first and second lines in (B-8) are obtained from the first and second terms in (B-3) respectively; the h.c. applies to all terms in the last three lines of (B-8). In the offshell component form of the Lagrangian notice we have an interesting tensor coupling \( \psi_2 \sigma^{\nu} \overline{\sigma}^{\mu} D_\nu D_\mu \psi_1 \) in spite of the minimal gauge coupling in (B-1) and this arises from a coupling \( \psi_2 \sigma^{\mu\nu} F_{\mu\nu} \psi_1 \) coming from the third term in the first line of (B-3), see also the previous footnote \(^{26}\). This coupling could be relevant for tree level calculations of the Feynman diagrams. Next we eliminate the auxiliary fields \( F_{1,2} \) using their equations of motion

\[
F_1^* = -\phi_2 \left( \mu + 2 \xi' (\phi_1 \phi_2) \right) + \frac{1}{4} \nu^2_1 \nu^2_2 \xi \left( -4 \phi_2 \overline{D}_\mu \overline{D}_\mu - 4 \phi_2 \frac{D}{2} + 2 \sqrt{2} \psi_2 \lambda \right)
\]

\[
F_2^* = -\phi_1 \left( \mu + 2 \xi' (\phi_1 \phi_2) \right) + \frac{1}{4} \nu^2_1 \nu^2_2 \xi \left( -4 D_\mu D^\mu \phi_1 + 4 \frac{D}{2} \phi_1 - 2 \sqrt{2} \lambda \psi_1 \right)
\]

(B-11)

In the terms proportional to \( \xi \) in \( \mathcal{L}_2 \) we can replace the derivatives of the fermions by their equations of motion, since the error would be of higher order. We use there

\[
i \overline{\sigma}^{\mu} D_\mu \psi_1 = \mu \overline{\psi}_2 + \frac{1}{\sqrt{2}} \overline{\lambda} \phi_2 + \mathcal{O}(\xi)
\]

\[-i \overline{\psi}_2 \sigma^{\mu} \overline{D}_\mu = \mu \overline{\psi}_1 - \frac{1}{\sqrt{2}} \phi_1 \overline{\lambda} + \mathcal{O}(\xi)\]

(B-12)

We then rescale the scalars and Weyl fermions and after neglecting terms \( \mathcal{O}(\xi \xi') \) we obtain the onshell Lagrangian

\[
\mathcal{L}_2 = -\phi_1^\dagger D^2 \phi_1 + i \overline{\psi}_1 \overline{\sigma}^{\mu} D_\mu \psi_1 - \frac{1}{\sqrt{2}} \left[ \overline{\psi}_1 \overline{\lambda} \phi_1 + \text{h.c.} \right] + \phi_1^\dagger \frac{D}{2} \phi_1
\]

\[ - \phi_2^2 D^2 \phi_2^\dagger + i \overline{\psi}_2 \sigma^{\mu} D_\mu \overline{\psi}_2 + \frac{1}{\sqrt{2}} \left[ \phi_2 \overline{\lambda} \overline{\psi}_2 + \text{h.c.} \right] - \phi_2^\dagger \frac{D}{2} \phi_2^\dagger \]

\[ - \mu^2 \left[ 1 - \mu \nu_1 \nu_2 \xi \right] \left[ \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \right] - \mu \left[ (1 - \mu \nu_1 \nu_2 \xi) \psi_1 \psi_2 + \text{h.c.} \right]
\]

\[ - 2 \xi' \mu \left[ (\phi_1 \phi_2) + \text{h.c.} \right] \left[ \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \right], \quad D^2 = D^\mu D_\mu
\]

(B-13)

\(^{26}\) This coupling is not present in the onshell form of the action, see also \(^{28}\).
This Lagrangian is in agreement with that of (B-5). This shows that on-shell and in the absence of other interactions, only a wavefunction renormalisation effect is present, giving a new \( \mu' = \mu (1 - \mu \nu_1 \nu_2 \xi) \). To conclude, integrating out the massive superfields \( \Phi_{3,4} \) generated a dimension-five operator \( \Phi_2 e^{-V} D^2 e^V \Phi_1 \), which if set on-shell via equations of motion or using the off-shell Lagrangian, brings a (classical) wavefunction renormalisation only, in the absence of additional trilinear interactions. Thus this D=5 operator does not bring new physics of its own, in the absence of additional interactions. One can then ask whether this conclusion remains true \(^{27}\) after supersymmetry is softly broken, and this is answered in the text, Section \( \text{[2]} \) and \( \text{[4]} \). To this purpose the supersymmetry breaking terms associated to this dimension-five operator must firstly be identified, and this is done in Appendix \( \text{[C]} \). Finally, if additional, trilinear interactions were also present, other dimension-five operators of type shown in (B-7) could also generated and these were also analysed in Section \( \text{[2]} \).

### C  Supersymmetry breaking effects and higher dimensional operators.

In this appendix we find all the supersymmetry breaking terms associated with the higher dimensional operator \( \Phi_2 e^{-V} D^2 e^V \Phi_1 \), which were used in Section \( \text{[2]} \) and \( \text{[4]} \). This operator is generated as shown in (A-11) (no gauge interactions) and in (B-3) by integrating out massive superfields. \(^{28}\) To find its associated susy breaking contribution we use the spurion field technique and claim that the most general susy breaking terms coming from this operator are generated by:

\[
\mathcal{L}_{G,S} \equiv \frac{1}{M} \int d^4 \theta \ A(S,S^\dagger) \ D^\alpha \left[ B(S,S^\dagger) \ \Phi_2 e^{-V} \right] D_\alpha \left[ \Gamma(S,S^\dagger) \ e^V \Phi_1 \right] + \text{h.c.} \quad (C-1)
\]

where

\[
A(S,S^\dagger) = \alpha_0 + \alpha_1 S + \alpha_2 S^\dagger + \alpha_3 SS^\dagger
\]
\[
B(S,S^\dagger) = \beta_0 + \beta_1 S + \beta_2 S^\dagger + \beta_3 SS^\dagger
\]
\[
\Gamma(S,S^\dagger) = \gamma_0 + \gamma_1 S + \gamma_2 S^\dagger + \gamma_3 SS^\dagger
\]

\(^{27}\) without setting on-shell this operator.

\(^{28}\) It would be more appropriate to introduce supersymmetry breaking to \( \mathcal{L}_2 \) of (B-1) then integrate again \( \Phi_{3,4} \). It is however easier to start from (B-3) and add to that a general spurion dependence/susy breaking.
\( A, B, \Gamma \) are the most general spurion fields, and \( S = \theta^2 M_s \), where \( M_s \) denotes the scale of supersymmetry breaking. Also \( \alpha_i, \beta_i, \gamma_i \) are arbitrary input parameters of the theory. In (C-1) an overall factor from spurion superfields can always be absorbed into a redefinition of the scale \( M \). This is equivalent to saying that \( \alpha_0, \beta_0, \gamma_0 \) can be set to unity. However, these can also vanish, therefore we kept their presence explicit. After a long calculation one finds

\[
\mathcal{L}_{G,S} = -\frac{\alpha_0 \beta_0 \gamma_0}{M} \int d^4 \theta \, \Phi_2 \, e^{-V} D^2 [e^V \Phi_1]
\]

\[
+ \frac{M_s}{M} \left[ 4(d_1 + d_2) \, \phi_2 D^\mu D_\mu \phi_1 - 2(d_1 - d_2) \, \phi_2 D \phi_1 + 2\sqrt{2} \, d_1 \, \phi_2 \lambda \psi_1 \right]
\]

\[
- 2\sqrt{2} \, d_2 \, \phi_2 \lambda \phi_1 - 4d_3 \, F_2 F_1 \right] + \frac{M_s^2}{M} \left[ -4d_4 \, \phi_2 F_1 - 4d_5 \, F_2 \phi_1 + 2d_6 \, \psi_2 \psi_1 \right]
\]

\[
+ \frac{M_s^3}{M} \left[ -4d_7 \, \phi_2 \phi_1 \right] + h.c. \quad \text{(C-3)}
\]

where the exact susy term can be read from the last three lines of (B-8) proportional to \( \xi \), and \( h.c. \) applies to all terms; the coefficients \( d_i, i = 1, 7 \) are given by:

\[
d_1 = -\beta_1 \alpha_0 \gamma_0 - \frac{1}{2} \alpha_1 \beta_0 \gamma_0, \quad d_2 = -\gamma_1 \beta_0 \alpha_0 - \frac{1}{2} \alpha_1 \beta_0 \gamma_0
\]

\[
d_3 = -\alpha_2 \beta_0 \gamma_0 - \alpha_0 \beta_2 \gamma_0 - \alpha_0 \beta_0 \gamma_2, \quad d_4 = -\beta_3 \alpha_0 \gamma_0 - \beta_1 \alpha_2 \gamma_0 - \alpha_0 \beta_1 \gamma_2 \quad \text{(C-4)}
\]

and

\[
d_5 = -\gamma_3 \beta_0 \alpha_0 - \gamma_1 \alpha_2 \beta_0 - \alpha_0 \beta_2 \gamma_1, \quad d_6 = \alpha_3 \gamma_0 \beta_0 + \alpha_1 \beta_2 \gamma_0 + \alpha_1 \beta_0 \gamma_2
\]

\[
d_7 = -\gamma_3 \beta_1 \alpha_0 - \gamma_1 \beta_3 \alpha_0 - \gamma_1 \beta_1 \alpha_2. \quad \text{(C-5)}
\]

Note the presence of the term \( \phi_2 D \phi_1 \) (assuming \( d_1 - d_2 \neq 0 \)), where \( D \) is the auxiliary gauge field. This term and \( \psi_2 \lambda \phi_1 \) are not present in the MSSM, if we replaced \( \Phi_{1,2} \) by the MSSM Higgs fields \( H_{1,2} \).

D  Mass eigenvalues in the MSSM with higher dimensional operators.

Some details of the calculation in Section 4.1 are given below. From the two minimum conditions for the scalar potential \( V \) of eq.(56) one can express \( \tilde{m}_{1,2} \) there in terms of \( B\mu \),
We introduced the mass eigenvalues $m_1, m_2$ to find:

$$
\tilde{m}_1^2 = -B\mu \frac{v_2}{v_1} - \frac{1}{8} g^2 (v_1^2 - v_2^2) - \frac{\eta_1}{2} \frac{v_2}{v_1} (3v_1^2 - v_2^2) - \frac{\eta_2}{2} \frac{v_1}{v_2} (3v_1^2 + v_2^2) - \frac{\eta_3}{2} v_2^2
$$

$$
\tilde{m}_2^2 = -B\mu \frac{v_1}{v_2} + \frac{1}{8} g^2 (v_1^2 - v_2^2) - \frac{\eta_1}{2} \frac{v_1}{v_2} (3v_1^2 - 3v_2^2) - \frac{\eta_2}{2} \frac{v_2}{v_1} (3v_1^2 + v_2^2) - \frac{\eta_3}{2} v_1^2
$$

which shall be used in the following. The mass matrix is

$$
\mathcal{M}_{ij} = \left. \frac{1}{2} \frac{\partial^2 V}{\partial h_i \partial h_j} \right|_{h_i = v_i, \sigma_i = 0} = X_{ij} + Z_{ij}
$$

where

$$
X_{ij} = \frac{1}{2} \begin{pmatrix}
2\tilde{m}_1^2 + \frac{1}{4} g^2 (3v_1^2 - v_2^2) & 2B\mu - \frac{1}{2} g^2 v_1 v_2 \\
2B\mu - \frac{1}{2} g^2 v_1 v_2 & 2\tilde{m}_2^2 - \frac{1}{4} g^2 (v_1^2 - 3v_2^2)
\end{pmatrix}
$$

and

$$
Z_{ij} = \frac{1}{2} \begin{pmatrix}
6(\eta_1 + \eta_2) v_1 v_2 + \eta_3 v_2^2 & 3(\eta_1 + \eta_2) v_1^2 + 3(\eta_2 - \eta_1) v_2^2 + 2\eta_3 v_1 v_2 \\
3(\eta_1 + \eta_2) v_1^2 + 3(\eta_2 - \eta_1) v_2^2 + 2\eta_3 v_1 v_2 & 6(\eta_2 - \eta_1) v_1 v_2 + \eta_3 v_1^2
\end{pmatrix}
$$

The mass eigenvalues $m^2_{h,H}$ of $\mathcal{M}_{ij}$ are

$$
m^2_{h,H} = M^2_{h,H} \pm \frac{6\eta_1}{\sqrt{w}} \left[ B\mu (v_1^2 - v_2^2) + v_1 v_2 \left( \tilde{m}_1^2 - \tilde{m}_2^2 + \frac{g^2}{4} (v_1^2 - v_2^2) \right) \right]
$$

$$
+ \frac{3\eta_2}{2\sqrt{w}} v_1 v_2 \left( v_1^2 + v_2^2 \right) \left( -4B\mu + g^2 v_1 v_2 \right)
$$

$$
+ \frac{\eta_3}{4} v_1^2 + v_2^2 \left( 2(\tilde{m}_1^2 - \tilde{m}_2^2)(v_1^2 - v_2^2) + g^2 (v_1^2 + v_2^2)^2 - 16B\mu v_1 v_2 \right)
$$

The upper (lower) signs correspond to the lighter $m^2_h$ (heavier $m^2_H$) Higgs field, respectively. We introduced

$$
M^2_{h,H} \equiv \frac{1}{2} \left[ \tilde{m}_1^2 + \tilde{m}_2^2 + \frac{g^2}{4} (v_1^2 + v_2^2) + \frac{1}{2} \sqrt{w} \right]
$$

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where the upper (lower) sign corresponds to $M_h$ ($M_H$) which, if $\eta_{1,2,3} = 0$ reproduce the lighter (heavier) MSSM Higgs field. Above we used the notation

$$w \equiv (4B\mu - g^2v_1v_2)^2 + 4\left(\tilde{m}_1^2 - \tilde{m}_2^2 + \frac{g^2}{2}(v_1^2 - v_2^2)\right)^2$$  \hfill (D-7)

With the values of $\tilde{m}_{1,2}$ expressed in terms of $v_{1,2}$ and $B\mu$ from minimum conditions (D-1), one can re-express $m_{h,H}^2$ of (D-5) as follows

$$m_{h,H}^2 = \frac{m_Z^2}{2} - \frac{B\mu(u^2 + 1)}{2u} \mp \frac{\sqrt{w'}}{2} + v^2 \left[\eta_1 q_1^+ + \eta_2 q_2^+ + \eta_3 q_3^+\right]$$  \hfill (D-8)

with

$$q_{1}^+ = \frac{u^2 - 1}{4u} \mp \frac{(u^2 - 1)}{4u^2(1 + u^2)^2}\sqrt{w} \left[m_Z^2 u(1 - 6u^2 + u^4) + B\mu (1+u^2)(1+18u^2 + u^4)\right]$$

$$q_2^+ = -\frac{1 - 6u^2 + u^4}{4u(1 + u^2)^2} \mp \frac{1}{4u^2 (1 + u^2)^2}\sqrt{w} \left[m_Z^2 u(1 - 14u^2 + u^4) + B\mu (1+u^2)(1+10u^2 + u^4)\right]$$

$$q_3^+ = \mp \frac{2u}{(1 + u^2)^2}\sqrt{w} \left[B\mu(1 + u^2) - m_Z^2 u\right]$$  \hfill (D-9)

where

$$w' \equiv m_Z^4 + \left[B\mu(1 + u^2)^2 + 2m_Z^2 u(1 - 6u^2 + u^4)\right] \frac{B\mu}{u^2(1 + u^2)}$$  \hfill (D-10)

and where we also used $v_1 = v \cos \beta, v_2 = v \sin \beta, u = \tan \beta$ and $m_Z^2 = g^2 v^2/4$. Similar considerations apply for the pseudoscalar Higgs/Goldstone boson sector. The mass matrix is in this case

$$N_{ij} = \left.\frac{\partial^2 V}{\partial \tilde{\sigma}_i \partial \tilde{\sigma}_j}\right|_{h_i = v_i/\sqrt{2}, \tilde{\sigma}_i = 0}$$  \hfill (D-11)

with entries

$$N_{11} = \tilde{m}_1^2 + \frac{g^2}{8}(v_1^2 - v_2^2) + (\eta_1 + \eta_2)v_1v_2 - \frac{\eta_3}{2}v_2^2$$

$$N_{12} = -\frac{\eta_1}{2}(v_1^2 - v_2^2) - \frac{\eta_2}{2}(v_1^2 + v_2^2) - \eta_3 v_1v_2 - \text{Re}(B\mu)$$

$$N_{22} = \tilde{m}_2^2 - \frac{g^2}{8}(v_1^2 - v_2^2) + (\eta_2 - \eta_1)v_1v_2 - \frac{\eta_3}{2}v_1^2$$  \hfill (D-12)
The eigenvalues of $N$ are

$$m^2_{G,A} = \frac{1}{2} (\tilde{m}_1^2 + \tilde{m}_2^2) \mp \frac{1}{8} \sqrt{\kappa}$$

$$\pm \frac{4\eta_1}{\sqrt{\kappa}} \left[ B\mu (v_1^2 - v_2^2) + v_1 v_2 \left( \tilde{m}_1^2 - \tilde{m}_2^2 + \frac{g^2}{4} (v_1^2 - v_2^2) \right) \right] + \eta_2 \left[ v_1 v_2 + \frac{4B\mu}{\sqrt{\kappa}} (v_1^2 + v_2^2) \right]$$

$$\pm \frac{1}{4} \left( - (v_1^2 + v_2^2) \mp \frac{1}{\sqrt{\kappa}} \left[ 8B\mu v_1 v_2 + (v_1^2 - v_2^2) (\tilde{m}_1^2 - \tilde{m}_2^2) + \frac{g^2}{4} (v_1^2 - v_2^2)^2 \right] \right)$$

(D-13)

where

$$\kappa = 16 \left[ 4(B\mu)^2 + \left( \tilde{m}_1^2 - \tilde{m}_2^2 + \frac{g^2}{4} (v_1^2 - v_2^2) \right)^2 \right]$$

(D-14)

where the upper sign corresponds to the Goldstone $m^G$ and the lower sign to $m^2_A$. One can use (D-1) to replace $\tilde{m}_{1,2}$ in terms of $v_{1,2}$ and $m_A$. Using (D-1) one shows that $m^G = 0$ and

$$m^2_A = -\frac{v_1^2 + v_2^2}{2v_1 v_2} \left[ 2B\mu + \eta_1 (v_1^2 - v_2^2) + \eta_2 (v_1^2 + v_2^2) + 2\eta_3 v_1 v_2 \right]$$

$$= -\frac{1 + u^2}{u} B\mu + \frac{u^2 - 1}{2u} \eta_1 v^2 - \frac{1 + u^2}{2u} \eta_2 v^2 - \eta_3 v^2$$

(D-15)

This is the result used in the text, eq.(59). Using eqs.(D-8) and (D-15) to eliminate $B\mu$ between them, one obtains the masses $m_{h,H}$:

$$m^2_{h,H} = \frac{1}{2} \left[ m^2_A + m^2_Z \mp \sqrt{w''} \right] \mp \frac{4m^2_A \eta_1 u (u^2 - 1) u^2}{(1 + u^2)^2 \sqrt{w''}}$$

$$+ \frac{2\eta_2 u v^2}{1 + u^2} \left[ 1 \pm \frac{m^2_A + m^2_Z}{\sqrt{w''}} \right] + \frac{\eta_3 v^2}{2} \left[ 1 \mp \frac{(m^2_A - m^2_Z)(u^2 - 1)^2}{\sqrt{w''} (1 + u^2)^2} \right]$$

(D-16)

where the upper (lower) signs correspond to $h$ ($H$) respectively, and where

$$w'' \equiv m^4_A + m^4_Z - 2m^2_A m^2_Z \frac{1 - 6u^2 + u^4}{(1 + u^2)^2} = (m^2_A + m^2_Z)^2 - 4m^2_A m^2_Z \cos^2 2\beta$$

(D-17)

Replacing $u = \tan \beta$ in $m_{h,H}$ one obtains an equivalent form of $m_{h,H}$ used in the text, eq.(60).

The bounds on $\eta_i$ discussed in Section 4.1 that must be respected in order to increase $m_h > m_Z$ in the approximation considered, are derived from (D-16) with (66) and give
\[
\frac{(\sqrt{\omega} + 1 - \rho) (1 + u^2)^2 \sqrt{\omega}}{32 u (u^2 - 1)} \leq -\frac{\eta_1}{g^2} \ll \min \left\{ \frac{u}{6(u^2 - 1)}, \frac{3u^2 - 1}{24u}, |u^2 - 3| \right\}
\]

\[
\frac{(\sqrt{\omega} + 1 - \rho)(1 + u^2)^2 \sqrt{\omega}}{4 \left(1 + u^2 \right)^2 \sqrt{\omega} - (\rho - 1)(1 - u^2)^2} \leq \frac{\eta_3}{g^2} \ll \min \left\{ \frac{1}{4}, \frac{|u^2 - 3|}{4u^2}, \frac{u^2 - 1}{4u^2}, \frac{u^2 - 1}{4} \right\}
\]

with \( \omega \equiv (\rho - 1)^2 + 16u^2\rho/(1 + u^2)^2 \), \( u \equiv \tan \beta \) and \( \rho \equiv m_A^2/m_Z^2 \). The implications of these eqs are discussed in the text after eq. (66).

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