A new parton model for the soft interactions at high energies: two channel approximation.

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The primary goal of this paper is to describe the diffraction production using the model that takes into account the Pomeron interaction, and satisfies both $t$ and $s$ channel unitarity. We hope that these features will allow us to describe the diffraction production in a more convenient way than in CGC motivated models, that do not satisfy these unitarity constraints. Unfortunately, we show that both approaches are only able to describe half of the cross section for the single diffraction production, leaving the second half to be estimates of the large mass production in the Pomeron approach. The impact parameter dependance of the scattering amplitudes show that soft interactions at high energies measured at the LHC, have a much richer structure than presumed. We discuss the $t$-dependence of the elastic cross section in wide range of $|t| = 0 \div 1 \text{GeV}^2$. We show that in the kinematic region of the minimum, we cannot use approximate formulae to calculate the real part of the amplitude. The exact calculation in our model, shows that the real part is rather small, and it is necessary to include the Odderon contribution in order to describe the experimental data.

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I. INTRODUCTION

In our recent paper [1] we proposed a new parton model for high energy soft interactions, which is based on Pomeron calculus in $1+1$ space-time dimensions, suggested in Ref. [2], and on simple assumptions of hadron structure, related to the impact parameter dependence of the scattering amplitude. This parton model stems from QCD, assuming that the unknown non-perturbative corrections lead to determining the size of the interacting dipoles. The advantage of this
approach is that it satisfies both the $t$-channel and $s$-channel unitarity, and can be used for summing all diagrams of
the Pomeron interaction including Pomeron loops. Hence, we can use this approach for all possible reactions: dilute-dilute
(hadron-hadron), dilute-dense (hadron-nucleus) and dense-dense (nucleus-nucleus) parton system scattering.

In other words, in this model the dimensional scale, that determines the interaction at high energy,
arises from the non-perturbative QCD approach, which fixes the size of dipoles. Such an approach is quite different
from the Colour Glass Condensate (CGC) one, where this scale originates from the interaction of dipoles at short
distances, and turns out to be large and increases with energy[3]. In spite of the fact, that the model, based on CGC
approach[4–11], describes all available data on soft interactions at high energy as well as the deep inelastic processes,
it has an intrinsic problem: the CGC approach, in it’s present form, does not provide a scattering amplitude, that
satisfies both the $t$ and $s$ channel unitarity[2].

We have shown that the new parton model is able to describe high energy data on the total and elastic cross sections
for proton-proton scattering, but the simple version of Ref.[1] leads to vanishing of diffractive production. In this
paper we propose a two channel model which generates diffraction production in the region of small masses. As it
is well known from the experiences (see for example Refs.[7, 12] ) that diffraction production is the process, which is
difficult to describe and, specifically, this process provides a check of our approach to long distances physics, which is
part of non-perturbative QCD.

We demonstrate that a two channel model is able to describe four experimental observables: $\sigma_{tot}, \sigma_{el}, B_{el}$
and the single diffraction cross sections. We show that this model leads to a rich structure for the impact parameter
dependence of the scattering amplitude. In particular, we study the dependence of the elastic cross section as function
of $|t| = 0 \div 1 GeV^2$. We show that we are able to describe the experimental data on $d\sigma_{el}/dt$ in this $t$-region: the
position of the minimum with $|t|_{min} = 0.52 GeV^2$ at $W = 7 TeV$ and the value and $t$ behaviour for larger $t$.

II. THE NEW PARTON MODEL

A. General approach.

As we have discussed in Ref.[1, 2] the new parton model is based on three ingredients:

1. The Colour Glass Condensate (GCC) approach (see Ref.[3] for a review), which can be re-written in the equivalent
form as the interaction of BFKL Pomeron[13] in a limited range of rapidities ($Y \leq Y_{max}$):

$$ Y \leq \frac{2}{\Delta_{BFKL}} \ln \left( \frac{1}{\Delta_{BFKL}} \right) $$

$\Delta_{BFKL}$ denotes the intercept of the BFKL Pomeron[14]. In our model $\Delta_{BFKL} \approx 0.2 – 0.25$ leading to $Y_{max} = 20 – 30$,
which covers all collider energies.

2. The following Hamiltonian:

$$ H_{NPM} = -\frac{1}{\gamma} \tilde{P} P $$

where NPM stands for “new parton model”. $P$ and $\tilde{P}$ are the BFKL Pomeron fields. The fact that it is self dual is
evident. This Hamiltonian in the limit of small $\tilde{P}$ reproduces the Balitsky-Kovchegov Hamiltonian $H_{BK}$ ( see Ref.[2]
for details). This condition is the most important one for determining the form of $H_{NPM}$. $\gamma$ in Eq. (2) denotes the
dipole-dipole scattering amplitude, which in QCD is proportional to $\tilde{\alpha}_S^2$.

3. The new commutation relations:

$$ \left( 1 - P \right) \left( 1 - \tilde{P} \right) = (1 - \gamma) \left( 1 - \tilde{P} \right) \left( 1 - P \right) $$

For small $\gamma$ and in the regime where $P$ and $\tilde{P}$ are also small, we obtain

$$ [P, \tilde{P}] = -\gamma + ... $$

consistent with the standard BFKL Pomeron calculus (see Ref.[2] for details).

In Ref.[2], it was proved that the scattering matrix for the model is given by

$$ S_{min}^{NPM}(Y) = e^{i \int_0^Y d\gamma \ln(1 - p) \frac{\partial}{\partial \gamma} \ln(1 - \tilde{p} + \tilde{p})} [1 - p(Y)]^m [1 - \tilde{p}(0)]^n |_{p(0) = 1 - e^{-\gamma^m}} \tilde{p}(Y) = 1 - e^{-\gamma^m} $$

$$ = [1 - p(Y)]^m e^{i \int_0^Y d\gamma \ln(1 - \tilde{p} + \tilde{p})} $$

(5)
where \( p(\eta) \) and \( \bar{p}(\eta) \) are solutions of the classical equations of motion and have the form:

\[
P(\eta) = \frac{\alpha + \beta e^{(1-\alpha)\eta}}{1 + \beta e^{(1-\alpha)\eta}}; \quad \bar{P}(\eta) = \frac{\alpha(1 + \beta e^{(1-\alpha)\eta})}{\alpha + \beta e^{(1-\alpha)\eta}}; \tag{6}
\]

where the parameters \( \beta \) and \( \alpha \) should be determined from the boundary conditions:

\[
P(\eta = 0) = p_0; \quad \bar{P}(\eta = Y) = \frac{\alpha}{\bar{P}(\eta = Y)} = \bar{p}_0 \tag{7}
\]

It is interesting to compare the scattering amplitude given by this expression to that obtained from the BK equation, which describes deep inelastic scattering on nuclei in QCD. For which we have

\[
S_{m\bar{n}}^{\text{BK}}(Y) = \int dP(\eta)d\bar{P}(\eta)e^{\frac{1}{\beta} \int_0^\infty d\bar{p} \ln(1-P) \ln(1-\bar{P})} (1 - P(Y))^{-\alpha m}(1 - \bar{P}(0))^{-\bar{p}_0} \tag{8}
\]

In the classical approximation

\[
S_{m\bar{n}}^{\text{BK}}(Y) = e^{\frac{1}{\beta} \int_0^\infty d\bar{p} \ln(1-p) \ln(1-\bar{p})} [1 - p(Y)]^{-\alpha m} \bar{p}(0)^{-\bar{p}_0} \tag{9}
\]

Note, that the solution for \( \bar{P} \), is not relevant for the BK amplitude, which is determined entirely by \( P(Y) \). On the other hand, the scattering amplitude in the NPM depends on \( \bar{P} \). Nevertheless, the two models should be similar in the regime where the BK evolution is valid. The results of the estimates in Ref.\,[2] shows that in the region close to saturation, the differences between BK and NPM are quite significant.

### B. Interrelation with QCD.

As has been mentioned, in the limited range of energies, given by Eq. (1), both QCD and our model describe the interaction of the BFKL Pomeron\,[13]. For weak fields \( P \) and \( \bar{P} \), the model reproduces the BK limit of the CGC approach, assuming that the non-perturbative corrections result in determining the size of the interacting dipoles, and hence, the successful description of the soft data at high energies in CGC approach\,[11] supports the idea that this effective size is rather small. The model leads to the descriptions that satisfy both \( t \)-unitarity and \( s \)-channel unitarity, while, as it was shown in Ref.\,[2], the BFKL Pomeron calculus in the BK limit, as well as the Braun Hamiltonian\,[14] for dense-dense system scattering violates \( s \)-channel unitarity. Unfortunately, we are still far from being able to solve this problem in the effective QCD theory at high energy (i.e. in the CGC /saturation approach).

### C. Two channel approximation

Our model includes three essential ingredients: (i) the new parton model for the dipole-dipole scattering amplitude that has been discussed above; (ii) the simplified two channel model that enables us to take into account diffractive production in the low mass region, and (iii) the assumptions for impact parameter dependence of the initial conditions.

In the two channel approximation we replace the rich structure of the diffractively produced states, by a single state with the wave function \( \psi_D \). The observed physical hadronic and diffractive states are written in the form

\[
\psi_h = \alpha \Psi_1 + \beta \Psi_2; \quad \psi_D = -\beta \Psi_1 + \alpha \Psi_2; \quad \text{where} \quad \alpha^2 + \beta^2 = 1; \tag{10}
\]

Functions \( \psi_1 \) and \( \psi_2 \) form a complete set of orthogonal functions \( \{\psi_i\} \) which diagonalize the interaction matrix \( T \)

\[
A_{i,k}^{k'} = \langle \psi_i \psi_k | T | \psi_i' \psi_k' \rangle = A_{i,k} \delta_{i,i'} \delta_{k,k'}. \tag{11}
\]

The unitarity constraints take the form

\[
2 \text{Im} A_{i,k}(s,b) = |A_{i,k}(s,b)|^2 + G_{i,k}^{\text{in}}(s,b), \tag{12}
\]

where \( G_{i,k}^{\text{in}} \) denotes the contribution of all non diffractive inelastic processes, i.e. it is the summed probability for these final states to be produced in the scattering of a state \( i \) off a state \( k \). In Eq. (12) \( \sqrt{s} = W \) denotes the energy of the colliding hadrons and \( b \) denotes the impact parameter. In our approach we used the solution to Eq. (12) given by Eq. (5) and

\[
A_{i,k} = 1 - S_{i,k}^{\text{NPM}}(Y) \tag{13}
\]
D. The general formulae.

Initial conditions: Following Ref.[1] we chose the initial conditions in the form:

\[ p_i(b') = p_{0i} S(b', m_i) \quad \text{with} \quad S(b, m_i) = m_i b K_1(m_i b); \quad \bar{p}_i(b - b') = p_{0i} S(b - b', m_i) \quad z_m = e^{\Delta(1-p_{0i})Y} \quad (14) \]

Both \( p_{0i} \) and masses \( m_i \), as well as the Pomeron intercept \( \Delta \), are parameters of the model, which are determined by fitting to the relevant data. Note, that \( S(b, m_i) \sim b \gg 1 \quad \exp(-m_i b) \) in accord with the Froissart theorem[16].

From Eq. (14) we find that

\[
\begin{align*}
a_{i,k}(b, b') &\equiv a_{i,k} (p_i, \bar{p}_k, z_m) = \frac{1}{2} \left( p_i + \bar{p}_k \right) + \frac{1}{2} z_m ((1 - p_i)(1 - \bar{p}_k) - D_{i,k}); \\
b_{i,k}(b, b') &\equiv b_{i,k} (p_i, \bar{p}_k, z_m) = \frac{1}{2} \frac{p_i - \bar{p}_k}{1 - p_i} - \frac{1}{2z_m(1 - p_i)} ((1 - p_i)(1 - \bar{p}_k) - D_{i,k}); \\
D_{i,k} &= \sqrt{4p_i(1 - p_i)(1 - \bar{p}_k)z_m - ((1 - p_i)(1 - \bar{p}_k) - (p_i - \bar{p}_k)z_m)^2};
\end{align*}
\]

These equations are the explicit solutions to Eq. (6) and Eq. (7).

Amplitudes: In the following equations \( p_i \equiv p_i(b') \) and \( \bar{p}_k \equiv \bar{p}_k(b - b') \).

\[
z = e^{\Delta(1-p_{0i})Y}
\]

\[
S_{i,k}(a_{i,k}, b_{i,k}, z) \equiv S(a_{i,k}(b, b'), b_{i,k}(b, b'), z_m), \quad X_{i,k}(a, b, z) \equiv X(a_{i,k}(b, b'), b_{i,k}(b, b'), z_m)
\]

\[
X(a_{i,k}, b_{i,k}, z) = \frac{a_{i,k} + b_{i,k}z}{1 + b_{i,k}z}
\]

\[
SS_{i,k}(a_{i,k}, b_{i,k}, z) = -(a_{i,k} - 1)Li_2(-b_{i,k}z) + a_{i,k}Li_2\left(-\frac{b_{i,k}z}{a_{i,k}}\right) + (a_{i,k} - 1)Li_2\left(\frac{a_{i,k} + b_{i,k}z}{a_{i,k} - 1}\right) + \frac{1}{2}a_{i,k} \log^2((1 - a_{i,k})b_{i,k}z)
\]

\[
- (a_{i,k} - 1) \log(b_{i,k}z + 1) \log((1 - a_{i,k})b_{i,k}z) - \left( a_{i,k} \log(z) - (a_{i,k} - 1) \log\left(-\frac{b_{i,k}z + 1}{a_{i,k} - 1}\right) \right) \log(a_{i,k} + b_{i,k}z)
\]

\[
+ a_{i,k} \log(z) \log\left(\frac{b_{i,k}z}{a_{i,k}} + 1\right)
\]

\[
S_{i,k}(a_{i,k}, b_{i,k}, z) = SS_{i,k}(a_{i,k}, b_{i,k}, z) - SS_{i,k}(a_{i,k}, b_{i,k}, z = 1)
\]

The amplitude is given by

\[
A_{i,k}(s, b) = 1 - \exp \left( \frac{1}{p_{01}} \int \frac{m_i^2 d^2 b'}{4\pi} \left( S_{i,k}(a_{i,k}, b_{i,k}, z_m) + a_{i,k}(b, b') \Delta(1-p_{0})Y \right) - \int \frac{m_i^2 d^2 b'}{4\pi} \bar{p}_k(b - b', m_k) X(a_{i,k}, b_{i,k}, z_m) \right)
\]
E. Physical observables.

The physical observables in this model can be written as follows

\[
\text{elastic amplitude: } a_{el}(s, b) = i \left( \alpha^4 A_{1,1} + 2\alpha^2 \beta^2 A_{1,2} + \beta^4 A_{2,2} \right); \\
\text{elastic cross section: } \sigma_{tot} = 2 \int d^2 b a_{el}(s, b); \quad \sigma_{el} = \int d^2 b |a_{el}(s, b)|^2; \\
\text{elastic slope: } B_{el} = \frac{1}{2} \int d^2 b |\text{Im} A_{el}(Y, b)|; \\
\text{optical theorem: } 2 \text{Im} A_{el}(s, t = 0) = 2 \int d^2 b |\text{Im} a_{el}(s, b)| = \sigma_{el} + \sigma_{in} = \sigma_{tot}; \\
\text{elastic cross section: } \frac{d\sigma_{el}}{dt} = \pi |f(s, t)|^2; \quad a_{el}(s, b) = \frac{1}{2\pi} \int d^2 q e^{-iq b} f(s, t) \text{ where } t = -q^2; \\
\text{single diffraction: } \sigma_{sd}^G = 2 \int d^2 b \left( \alpha \beta \left( -\alpha^2 A_{1,1} + (\alpha^2 - \beta^2) A_{1,2} + \beta^2 A_{2,2} \right) \right)^2; \\
\text{double diffraction: } \sigma_{dd}^G = \int d^2 b \alpha^4 \beta^4 \left( A_{1,1} - 2 A_{1,2} + A_{2,2} \right)^2. 
\]

It should be noted, that factor 2 in Eq. (26) takes into account the single diffractive dissociation of the two protons.

III. COMPARISON WITH EXPERIMENTAL DATA FOR PROTON-PROTON SCATTERING

A. The results of the fit in two channel model

As we have seen in the previous section, we introduce three dimensionless parameters: \( \Delta \) - the intercept of the BFKL Pomeron, and \( p_{01} \) (\( p_{02} \)) - the amplitudes of the dipole-dipole scattering at low energies, and \( \beta \) which is related to the contribution of the diffractive production. For \( b \)-dependence we suggested a specific form (see Eq. (14)) which is characterized by the dimensional parameters: \( m_i \). These parameters are determined by fitting to the experimental data. We choose to describe five observables: total and elastic cross sections, the elastic slope and single and double diffractions at low masses (see Eq. (22) - Eq. (27)).

The situation with the experimental data on the single and double diffraction production in proton-proton scattering at high energies, is far from clear. It was well summarized in Ref. [12], to which we refer the reader. We assume that the two channel model is able to describe proton-proton diffraction production in the entire kinematic region of produced mass. As is shown in Ref. [21] for \( \Delta > 0 \) the integral over the produced mass in diffraction is convergent, and the Good-Walker mechanism [20] is able to describe the diffraction production both of small and large masses. However, the simple two channel model is a simplification, but we hope to learn something by attempting to fit all available data using this simple model.

From Fig. 1 one can see that we obtain quite a good description of the data for \( \sigma_{tot} \), \( \sigma_{el} \) and for the slope \( B_{el} \) for \( W \geq 0.5 \text{TeV} \). Comparing with the one channel model of Ref. [11] we start fitting from lower value of \( W = 0.5 \text{TeV} \) instead of \( W = 1 \text{TeV} \). We present The fitting parameters in Table I. One can see that both sets have the same qualitative features: the large value of the amplitude \( A_{1,1} \) and small values of other amplitudes. Note, the values of parameters which describe this large amplitude turns out to be quite different in one and two channels fits. Especially, this difference is seen in the value of \( \Delta \) and masses (\( m_1 \) and \( m_2 \)). The quality of the description of this model and the one channel model of Ref. [11] for \( W = 0.5 \div 13 \text{TeV} \) are more or less the same. However for \( W > 1 \text{TeV} \) the one channel model gives a better description.

However, from Fig. 5a and Fig. 6a it is clear that we failed to describe the data on the single and double diffraction production: roughly speaking we are able to describe only half of the values for single diffraction cross section. Therefore, the simple two channel model is not enough to describe the experimental data on the single diffraction production, in spite of the three new parameters that we have introduced. Actually, we had the same situation in our CGC motivated model of Ref. [7]. Hence, we can conclude that the fact that our model satisfies the unitarity constraints both in \( t \) and \( s \) channel unitarity is not sufficient, and we need to search for a more complicated model for the hadron structure.

The values of parameters which led to the best agreement with the experimental data of are shown in Table I. The two sets of parameters are quite different, but qualitatively they describe the data with large \( A_{1,1} \) and small \( A_{1,2} \) and
Comparing these parameters with the resulting curves in Fig. 1, we see that shadowing corrections play an essential role. First, we note that the value of $\Delta_{\text{dressed}} = \Delta (1 - p_{01})$ is rather large (about 0.5) in all variants. Recall, that means that $\Delta \approx 1$. Factor $(1 - p_{01})$ in $\Delta_{\text{dressed}}$, stems from the enhanced diagrams that contribute to the Green function of the Pomeron. The resulting $\sigma_{\text{tot}} \propto s^{\Delta_{\text{eff}}}$ with $\Delta_{\text{eff}} \approx 0.07$. The reduction from $\Delta_{\text{dressed}}$ to $\Delta_{\text{eff}}$ occurs due to strong shadowing corrections.

From Fig. 2a one can see that we failed to describe the double diffraction production. This reflects the situation which we had in our previous attempts to describe this process [7]. The same problem occurs with other groups (see, for example, Ref. [12] and reference therein). The small size of the double diffraction cross section in our model occurs due to strong shadowing corrections.

To study the role of shadowing in more detail we consider two variants of the fit $\Delta_{\text{dressed}} = \Delta (1 - p_{01})$. We have the same expression as in Eq. (28) for the diagram of Fig. 2-c but we need to replace the bare Pomeron Green’s function by the resulting (‘dressed’) Pomeron Green’s function ($G_{\gamma P}(y,b) \rightarrow G_{\gamma P}^{\text{dressed}}(y,b)$). In our model it turns out that easier to find not the resulting Green’s function but the product $p_i G_{\gamma P}^{\text{dressed}}(y,b)$, which we will denote $A_{i, \gamma P}(y,b)$. We can find this amplitude from the general formulae of section II applying new initial conditions

\begin{eqnarray}
\sigma_{\text{el}}^\text{LM} = 2 \int_0^Y dy' \int d^2b' \; p_i \; G_{\gamma P} \; (Y - y', b - b') \Gamma_{3\gamma P} \; G_{\gamma P}^2 \; (y', b') \; p_k^2
\end{eqnarray}
FIG. 2: The single diffraction production of large masses. Fig. 2-a and Fig. 2-b present the first diagrams for the diffraction production. The blog represents the triple Pomeron vertex. Fig. 2-c corresponds to the contributions of the ‘dressed ’ Pomerons in our model. The wavy lines denote the Pomeron while the double wavy lines describe the resulting Green’s function of the Pomeron in our model.

Instead of Eq. (14): viz.

\[ p_i(b) = p_{0i}S(b,m_i) \quad \text{for} \quad i = 1, 2; \quad p_{P}(b) = p_{0P}S(b,m_P) \]  

From our model it follows that \( p_{0P} = p_{01} \), but the value of the mass \( m_P \) should include the impact parameter dependence of the triple Pomeron vertex. It is known that the radius of the triple Pomeron vertex is much smaller that the size of the proton. We choose the typical mass \( m_P = 3 \text{GeV} \), which means that this radius is in about three times smaller than the radius of the Pomeron-proton vertex. We checked that the numerical estimates are not sensitive to the value of this mass.

FIG. 3: The single diffraction production of large masses including the survival probability. The wavy lines denote the Pomeron, while the double wavy lines describe the resulting Green’s function of the Pomeron in our model. The black circles denote the transition \( \langle \psi_h | \psi_i \rangle = \alpha (i = 1) \) or \( \beta (i = 2) \).

However, we need to multiply Eq. (28) by the survival probability (see Refs. \[24, 25\] for a review). Indeed, together with the processes shown in Fig. 2, a number of parton showers can be produced and gluons (quarks) from these showers will produce additional hadrons which, in particular, can fill the rapidity gap (\( y' \) in Fig. 2). The survival probability factor \( \langle S^2 \rangle \) gives the fraction of the processes in which the production of the parton showers are suppressed. Finally, the contribution of the single diffraction is given by the following expression (see Fig. 3):

\[
\sigma_{\text{sd}}^{LM} = 2 \int_0^Y dy' \int d^2b' \tilde{A}_{i,IP} (Y - y', b - b') \tilde{A}_{k,IP}^2 (y', b') \left( 1 - A_{i,k} (Y,b) \right)^2 \]  

The main contribution to \( \sigma_{\text{sd}}^{LM} \) for set I, stems from \( A_{1,1} \), in spite of small values of \( \langle S^2 \rangle \), since all other amplitude are small. For set II \( A_{1,2} \) leads to the largest cross section due to large \( \langle S^2 \rangle \approx 0.8 \).

For double diffraction in the region of large masses we can write the following formula which follows directly from Fig. 4:

\[
\sigma_{\text{dd}} = 4 \int_0^Y dy'' \int_0^y dy' \int d^2b'' d^2b' \tilde{A}_{i,P} (Y - y', b - b') \tilde{A}_{k,P}^2 (y'' - y', b' - b') \tilde{A}_{k,P} (y'', b - b') \left( 1 - A_{i,k} (Y,b) \right)^2 \]
FIG. 4: The double diffraction production of large masses including the survival probability. The double wavy lines describe the resulting Green’s function of the Pomeron in our model. The black circles denote the transition \( \langle \psi_h|\psi_i \rangle = \alpha(i = 1) \) or \( \beta(i = 2) \).

C. Diffraction production: comparison with the experimental data

Fig. 5-a shows a comparison of our results compared to the single diffraction production data, taken from Ref. [22] and which are shown in Fig. 5-b. One can see that the description is not very good at \( W \approx 0.5 \text{TeV} \). The reason for this is that, the integration over \( y' \) in Eq. (30) leads to the amplitude \( A_{i,h} \) and \( A_{k,b} \) enter at energies smaller than \( W = 0.5 \text{TeV} \). We cannot describe these energies in our model. For larger energies the phase space that corresponds to the unknown region of energies gives much smaller contributions.

The TOTEM value of the single diffraction cross section is \( 9.1 \pm 2.9 \text{(mb)} \) (see Ref.[22]) , while our estimates lead to \( \sigma_{sd} = 12 - 13 \text{mb} \). As can be seen from Fig. 5-a and Fig. 5-b, our model leads to values of the single diffraction cross section, which are close to our predictions from the CGC motivated model of Ref.[7] (the curve GLM in Fig. 5-b). We refer the reader to Ref.[12] in which the situation with tensions between different experimental groups on the single diffraction cross section, has been discussed.

The description of the double diffraction is very poor. The large mass diffraction leads to a large double diffraction cross section at high energies, and we cannot reproduce the values of \( \sigma_{dd} \) at lower energies.

Concluding this section we can claim that the large mass diffraction leads to a considerable contribution, which in this model increases rapidly with energy. This is a direct consequence of the large value of \( \Delta_{dressed} \) in our model (see Table I). For \( W > 0.5 \text{TeV} \), this increase is damped by large shadowing corrections, but in the formulae for diffraction production of large masses, includes energies which are less than \( W = 0.5 \), where the strength of shadowing corrections is not sufficient to lead to a reasonable effective \( \Delta \).
IV. DEPENDENCE ON IMPACT PARAMETERS

In Fig. 7 we plot the scattering amplitudes as a function of the impact parameter $b$. One can see that the two channel model generates a very interesting and unexpected structure. One amplitude $A_{11}(b)$ has reached the unitary limit $A_{11}(b = 0) = 1$ at $W = 0.5\, TeV$ and shows the increasing of the radius of the interaction, with energy. The two other amplitudes are far from the unitarity limit even at ultra high energy $W = 100\, TeV$. They increase as $W^{\Delta_{\text{eff}}}$ with $\Delta_{\text{eff}} \sim 0.1$. The behaviour as a function of $b$ is also unexpected. Both $A_{11}$ and $A_{22}$ decrease monotonically at large $b$, while $A_{12}$ has a maximum which moves to larger values of $b$. The value of the amplitude for this maximum increases as $W^{\Delta_{\text{eff}}}$. On the other hand, $A_{12}(b = 0)$ is almost independent of $W$.

Such dependence of the amplitudes generate the elastic amplitude which is smaller than the unitarity limit even at very high energies (see Fig. 7a). This conclusion is in accord with the recent paper of Ref. [26] in which it is demonstrated that in the Miettinen-Pumplin [27] approach the elastic amplitude $A_{el}(b = 0) \approx 0.92 < 1$ at $W = 57\, TeV$. Note, that this approach is ideologically close to ours and second, that in Ref. [26] the entire set of soft interaction data has been described successfully.

In Fig. 5a we present the comparison between the elastic amplitude in our 2 channel model and in one channel model of Ref. [1]. One can see that these two amplitudes have a different behaviour both as a function of energy and impact parameter. We believe that this figure demonstrates that the modeling of the non-perturbative structure of the hadron is very important in understanding high energy scattering. Fig. 8b shows the behaviour of $d\sigma_{sd}/db^2$ (see Eq. (26))

$$
\frac{d\sigma_{sd}}{db^2} = \left(\alpha\beta(-\alpha^2A_{1,1} + (\alpha^2 - \beta^2)A_{1,2} + \beta^2A_{2,2})\right)^2
$$

One can see that this observable decreases very slowly with energy, and does not show a maximum at large $b$. Such behaviour is quite different from what we obtain in CGC motivated model (see Ref. [7] Fig.7) and from the estimates of Ref. [26].

We believe that the impact parameter and energy behaviours shown in Fig. 7 and in Fig. 5 illustrate the fact that the soft interaction at high energies could have a much richer structure than we previously assumed.

V. DEPENDENCE OF THE ELASTIC CROSS SECTIONS ON $t$

We attempt to describe the elastic cross section for $|t| = 0 \div 1\, GeV^2$ to check the rich structure present in the impact parameter dependence, this stems from our model, which predicts the existence of a minimum in the elastic cross sections, however its position occurs at $|t| \approx 0.3\, GeV^2$, which is much smaller than was observed experimentally by TOTEM collaboration [28].

Assuming that this discrepancy is due to the simplified form of $b$ dependence of our amplitude which is given by
Fig. 7: The scattering amplitudes versus impact parameter $b$ for different energies: Fig. 7-a $A_{el}$; Fig. 7-b $A_{11}$, Fig. 7-c $A_{22}$, Fig. 7-d $A_{12}$.

Fig. 8: The scattering amplitudes versus impact parameter $b$ for different energies: Fig. 8-a: the elastic amplitudes for the one channel model of Ref. [1] (dashed line) and for two channel model of this paper (solid line). For estimates in our model we used set I of parameters in Table I; Fig. 8-b: $d\sigma_{sd}/db^2$ of Eq. (32) for the variant two (solid line) and variant one (dashed line) set of parameters.

As we have seen in Fig. 1 our two channel model does not give a good description of the elastic cross section. Bearing this in mind we made a fit using the one channel model in which $p_{01} \neq 0$ but $p_{02} = 0$. In the Table II we present the parameters that we found for the fit. Fig. 9 shows the comparison with TOTEM data of Ref. [28]. One can see that we obtain good agreement with the experimental data for $|t| < |t|_{min}$ and for $|t| > |t|_{min}$. However, for $|t| \approx |t|_{min}$ the real part of the scattering amplitude turns out to be small, and we obtain a value of the $d\sigma_{el}/dt$ approximately an order of magnitude smaller than the experimental one. It should be stressed that we do not use any of the simplified approaches to estimate the real part of the amplitude, but using our general expression of Eq. (21) for $A_{ik}(s,t)$, we consider the sum $A_{ik}(s,+i\epsilon t) + A_{ik}(u-i\epsilon,t)$, which corresponds to positive signature, and calculated the real part of this sum.

In Fig. 9 we estimate the contribution of the $\omega$-reggeon, using the description taken from the paper of Ref. [29] (note the difference between green dashed line and the blue solid curve). This contribution is small, and can be neglected.

To evaluate the real part of the amplitude we use the relation: However,
\[ \sqrt{s} = 7 \text{ TeV} \]

- **TOTEM**
  - \( \text{Im}^2 + \text{Re}^2, \text{Odderon} \)
  - \( \text{Im}^2 + \text{Re}^2 \)
  - \( \text{Im}^2 \)
  - \( \text{Re}^2 \)

**FIG. 9:** \( d\sigma/dt \) versus \( t \). The black green line describes the result of our fit. The dashed line corresponds to the contribution of the imaginary part of the scattering amplitude to the elastic cross section. The dotted line relates to the real part of our amplitude. The red solid line takes into account the contribution of the odderon to the real part of the pp amplitude, as is shown in Eq. (36). The data, shown in grey, include systematic errors. They are taken from Ref. [28].

\[
\text{Re}A_{11}(s,t) = \frac{1}{2} \pi \frac{\partial}{\partial \ln (s/s_0)} \text{Im}A_{11}(s,t) \bigg|_{\text{Eq. (21)}} \tag{34}
\]

Eq. (34) correctly describes the real part of the amplitude only for small \( \rho = \text{Re}A/\text{Im}A \). In Fig. 10 we plot the \( d\sigma/dt \) with such estimates for the real part. The real part from Eq. (34) turns out to be almost twice larger than the experimental data in the vicinity of \( t_{\text{min}} \). Therefore, at the minimum, where \( \text{Im}A \ll \text{Re}A \), Eq. (34) cannot be used for the real part. However, replacing Eq. (34) by

\[
\text{Re}A_{11}(s,t) = \tan(\rho) \text{Im}A_{11}(s,t) \bigg|_{\text{Eq. (21)}} \tag{35}
\]

we obtain the same result, that the real part of the amplitude turns out to be too large. Actually, Eq. (35) assumes that the scattering amplitude depends on energy as a power \( A(s,t) \propto s^{2\rho/\pi} \). Our amplitude is a rather complex function of energy, and depends on \( \ln(s) \).

| Variant of the fit       | \( \Delta_{\text{dressed}} \) | \( p_{01} \) | \( m_1 \) (GeV) | \( \mu_1 \) (GeV) | \( \nu_1 \) | \( \nu_2 \) | \( \kappa_1 \) |
|-------------------------|-------------------------------|-------------|---------------|---------------|-----------|-----------|-----------|
| one channel model       | 0.48 ± 0.01                  | 0.8 ± 0.05  | 0.860         | 7.6344        | 0.9       | 0.1       | 0.48      |

**TABLE II:** Fitted parameters for \( d\sigma/dt \) dependence. \( \Delta_{\text{dressed}} = \Delta (1 - p_{01}) \).

Concluding, we see that to describe the TOTEM experimental data in the framework of our model, the contribution to the real part of the amplitude from the exchange of the odderon [30] is needed. Hence, our estimates confirm the conclusions of Ref. [31]. In Fig. 9 we plot the description of the elastic cross section in which we have added the
FIG. 10: $d\sigma/dt$ versus $t$. The solid line describes the result of our fit. The dotted line corresponds to the contribution of the real part of the scattering amplitude to the elastic cross section, which is calculated using Eq. (34), with added contribution of the exchange of the $\omega$-reggeon, which is taken from Ref. [29]. We do not show the contribution of the real part without the $\omega$-reggeon since it coincides with the dotted line. The dashed line is the contribution of the imaginary part of the amplitude. The data are taken from Ref. [28].

The primary goal of this paper was to investigate whether the new parton model, which has been developed in Ref. [1], is able to describe the diffraction production. The model is based on Pomeron calculus in $1+1$ space-time, suggested in Ref. [13], and on the simple assumptions on the hadron structure, related to the impact parameter dependence of the scattering amplitude. This parton model stems from QCD, assuming that the unknown non-perturbative corrections lead to fixing the size of the interacting dipoles. The advantage of this approach is that it satisfies both $t$-channel and $s$-channel unitarity, and it can be used for summing all diagrams of the Pomeron interaction including the Pomeron

VI. CONCLUSIONS

The primary goal of this paper was to investigate whether the new parton model, which has been developed in Ref. [1], is able to describe the diffraction production. The model is based on Pomeron calculus in $1+1$ space-time, suggested in Ref. [13], and on the simple assumptions on the hadron structure, related to the impact parameter dependence of the scattering amplitude. This parton model stems from QCD, assuming that the unknown non-perturbative corrections lead to fixing the size of the interacting dipoles. The advantage of this approach is that it satisfies both $t$-channel and $s$-channel unitarity, and it can be used for summing all diagrams of the Pomeron interaction including the Pomeron
the elastic cross sections for $pp$-scattering with the odderon contribution (see Eq. (36)), while the dashed line shows the elastic cross section for $\bar{p}p$-scattering using Eq. (36). The data are taken from Ref. [28].

Unfortunately, we did not find any advantages of our new model, and we have to describe half of the single diffraction cross section by the diffraction production of large masses, in striking similarity with the CGC based models. Certainly, it is not a very encouraging result, especially since the CGC models describe the large mass diffraction production better than this model. Mostly this is due to the fact that $\Delta_{\text{dressed}}$ in this model, turns out to be larger than in CGC one.

The impact parameter dependence of the scattering amplitudes (see Fig. 7) shows that the soft interaction at high energies measured at the LHC have a much richer structure that we presumed in the past. We believe that we have demonstrated that the character of high energy scattering is closely related to the structure of hadron, which presently is described by a simple two channel model.

Our attempt to describe the $t$-dependence of the elastic cross section shows that we can reproduce the main features of the $t$-dependence that are measured experimentally: the slope of the elastic cross section at small $t$, the existence of the minima in $t$-dependence which is located at $|t|_{\text{min}} = 0.52 \text{GeV}^2$ at $W=7 \text{ TeV}$; and the behaviour of the cross section at $|t| > |t|_{\text{min}}$. It should be stressed that our model allows us to find the real part of the scattering amplitude using our general expression of Eq. (21) for $A_{ik}(s,t)$. We consider the sum $A_{ik}(s, +ict) + A_{ik}(u - ict)$, which corresponds to positive signature, and calculated the real part of this sum. It should be stressed that we do not use any of the simplified approaches to estimate the real part of the amplitude which we show (in our model ) which do not reproduce correctly the real part of the amplitude at large $t$. In our model the real part turns out to be much smaller than the experimental one. Consequently, to achieve a description of the data, it is necessary to add an odderon contribution. Hence, our model corroborates the conclusion of Ref. [31].

A topic for future study, is whether the characteristic behaviour of the $A_{ik}(b)$ amplitudes as a function of $b$ stems from the theory of interacting Pomerons, which satisfies both $s$ and $t$ channel unitarity, or is an artifact of the simple two channel approach with the phenomenological input, on the impact parameter dependence.

We are aware that our model is very naive in describing the hadron structure, but hope that further progress in accumulating data on diffraction production, as well as the unsolved problem of treating the processes of the loops. Our hope was that this model will be superior to the model which we developed based on CGC approach [4, 5], and which does not satisfy both $t$ and $s$ channel unitarity.
multiparticle generation in the framework of our approach, will generate a self consistent picture for high energy scattering at long distances.

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