Non singular, bouncing M theory universe

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ABSTRACT

We propose a set of equations as a simple model for non singular evolutions of a 10 + 1 dimensional M theory universe. Our model uses ideas from Loop Quantum Cosmology and offers a solution to the important problem of singularity resolutions. We solve the equations numerically and find that an M theory universe in this model evolves non singularly and with a bounce: going back in time, its density reaches a maximum and decreases thereafter whereas its physical size reaches a non vanishing minimum and increases thereafter. Taking the constituents of the universe to be the most entropic ones (which are four sets of intersecting M branes) leads to an effectively 3 + 1 dimensional spacetime as the M theory universe expands, both in the infinite past and future.
The $9 + 1$ dimensional superstring theory, equivalently the $10 + 1$ dimensional M theory, is a candidate for a quantum theory of gravity and is also expected to describe the matter contents and the quantum evolution of our universe. Consider our universe. It is $3 + 1$ dimensional and, at densities and temperatures small compared to Planckian ones, its cosmological evolution is well described by general relativity equations for a homogeneous isotropic universe. However, general relativity equations lead to a big bang singularity in the past where the densities and temperatures exceed Planckian values and diverge to infinity.

Such singularities are expected to be resolved upon quantising gravity, thus within string/M theory. See [1] – [13] for several string/M theoretic ideas for resolving the big bang singularities. To mention an example: String theory has a T duality symmetry under which the radius $R$ of a compact direction is transformed to its inverse, namely $R \rightarrow l_s^2 R^{-1}$ where $l_s$ is the string length. For an universe whose spatial directions may all be taken as circles, one then expects that the sizes of all the circles will be bounded below by $l_s$ and, hence, that the curvature of the universe will be bounded above by $\simeq l_s^{-2}$. Such a limiting curvature may then resolve the big bang singularities.

Enormous progress has been made in string/M theory towards, for example, understanding the entropy and the Hawking radiation of extremal and near extremal black holes. However, no comparable progress has been made towards understanding the big bang singularities. The stringy mechanisms resolving the big bang singularities are still not understood in full detail. Nor is there any simple model leading to non singular evolution of a string/M theory universe. Also, a successful model should lead to an effectively $3 + 1$ dimensional spacetime as the string/M theory universe expands. See [3], [4], [14] – [19] for several ideas for obtaining a $3 + 1$ dimensional universe in string/M theory.

An alternative candidate for a quantum theory of gravity is the $3 + 1$ dimensional Loop Quantum Gravity (LQG) constructed using Ashtekar variables [20] – [26]. It leads to Loop Quantum Cosmology (LQC) upon restricting to homogeneous variables of cosmology and quantising them. The resulting quantum evolutions of the $3 + 1$ dimensional universe have been extensively studied and found to resolve the big bang singularities [27] – [38]. It has also been found that these non singular quantum evolutions are well described by a set of effective equations which reduce to general relativity equations in the ‘classical limit’.

Recently, one of us have empirically generalised the effective LQC equa-
tions to $d + 1$ dimensional universes and to include arbitrary functions; and then studied analytically their salient features [39] – [42]. In this letter, we will propose these generalised effective equations as a simple model for the evolution of a $10 + 1$ dimensional M theory universe. The model will have one arbitrary function $f(x)$. General relativity equations follow for $f(x) = x$. We will take $f(x) = \sin x$ and solve the effective equations numerically. The resulting evolution of an M theory universe will be non singular and have a bounce. Namely, as one goes back in time, the density of the universe will reach a maximum and then start decreasing thereafter. Correspondingly, the physical size of the universe will reach a non vanishing minimum, bounce back, and then start increasing thereafter. Our model thus uses ideas from LQC, applies them in an M theory context, and offers a solution to the important problem of singularity resolutions.

The line element $ds$ for an M theory universe is taken to be given by

$$ds^2 = -dt^2 + \sum_i e^{2\lambda_i} (dx^i)^2$$

where $i = 1, 2, \cdots, 10$ and the scale factors $e^{\lambda_i}$ are functions of $t$ only. Let $\rho$ and $p_i$ be the total density of the constituents of the universe and their total pressures in the $i^{th}$ direction. Define the quantities $G_{ij}$, $G^{ij}$, $\Lambda$, and $r^i$ by

$$G_{ij} = 1 - \delta_{ij} \quad G^{ij} = \frac{1}{9} - \delta^{ij} \quad \Lambda = \sum_i \lambda_i \quad r^i = \sum_j G^{ij} (\rho - p_j)$$

let $f(x)$ be the function which characterises the model and which is required to $\to x$ as $x \to 0$ but is arbitrary otherwise; let $m^i$, $i = 1, 2, \cdots, 10$ be a new set of variables, to be related to the time derivatives of the scale factors using $f(x)$; and, define the functions $f^i$, $g_i$, and $X_i$ by

$$f^i = f(m^i) \quad g_i = \frac{d f^i}{dm^i} \quad X_i = g_i \sum_j G_{ij} f^j$$

Then we propose that the equations governing the evolution of the scale factors $e^{\lambda_i}$ be given by [43]

$$l_{qm} \lambda_i = \sum_j G^{ij} X_j$$

$$\sum_{ij} G_{ij} f^i f^j = 2 l_{qm}^2 \kappa^2 \rho$$

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\( (m^i)_t + \sum_j \frac{(m^i - m^j)}{9 \, l_{qm}} \, X_j = l_{qm} \, \kappa^2 \left( r^i - \frac{2p}{9} \right) \) \hspace{1cm} (6)

or equivalently

\( (m^i)_t + \Lambda_t \, m^i = l_{qm} \, \kappa^2 \, r^i + \sum_{jk} \frac{G_{jk} \, (m^j \, g_j - f^j)}{9 \, l_{qm}} \, f^k \), \hspace{1cm} (7)

and the standard conservation equation

\( \rho_t + \sum_i (\rho + p_i) \, \lambda^i_t = 0 \). \hspace{1cm} (8)

Equation (8) follows from equations (2) – (6); equivalently, one may take equation (5) to follow from other equations. In the above equations, \( \kappa^2 = 8 \pi G_{11} \), the \( t \)–subscripts denote the time derivatives, and \( l_{qm} = O(1) \, \kappa^{\frac{2}{3}} \) is a length parameter which, in LQC, would characterise the quantum of area. The 10 + 1 dimensional general relativity equations \cite{19} follow in the ‘classical limit’ where \( f(x) = x \), hence \( g_i = 1 \) and \( l_{qm} \lambda^i_t = f^i = m^i \); more generally, any linear function \( f(x) = bx + c \) where \( b \) and \( c \) are constants gives \( l_{qm} \lambda^i_t = bf^i \) and the general relativity equations with \( \kappa^2 \) replaced by \( b^2 \kappa^2 \).

Substituting for \( \rho \) and \( p_i \) the densities and the pressures of the constituents of an M theory universe will then give its evolution in a model specified by a function \( f(x) \).

Consider the constituents of an M theory universe. It is natural to assume that they must be the most entropic ones \cite{16}. As explained lucidly in \cite{12,13}, such constituents are \( N \) sets of brane configurations which intersect according to the Bogomol’nyi – Prasad – Sommerfeld (BPS) rules, with highest possible \( N \) \cite{49} – \cite{54}. According to the BPS rules, two stacks of M5 branes intersect along three common spatial directions; two stacks of M2 branes intersect along zero common spatial directions; a stack of M2 branes intersect a stack of M5 branes along one common spatial direction; and each stack of branes is smeared uniformly along the other brane directions. There can be a wave along the common intersection direction \cite{49,50,51}. High entropies of these configurations arise due to the phenomenon of fractionation of branes \cite{12,13}. Requiring atleast three spatial directions to be not wrapped by any intersecting branes, so that an M theory universe may resemble ours, then restricts \( N \) to be \( \leq 4 \) \cite{52}. Thus \( N = 4 \) for the most entropic configurations which, with no loss of generality, we may take to be
given by four stacks of intersecting M branes which wrap the seven directions, labelled \(1, 2, \cdots, 7\) : namely, two stacks each of \(M^2\) and \(M^5\) branes wrap respectively the directions \(12, 34, 13567,\) and \(24567\).

The densities \(\rho_I\) and the pressures \(p_{iI}\) of the \(I^{\text{th}}\) stack of branes, \(I = 1, 2, \cdots, N\), in such BPS configurations are mutually noninteracting and separately conserved. Thus

\[
\rho = \sum_I \rho_I, \quad p_i = \sum_I p_{iI}, \quad (\rho_I)_t + \sum_i (\rho_I + p_{iI}) \lambda_i^I = 0. \tag{9}
\]

To proceed further, equations of state are needed which determine the pressures \(p_{iI}\) in terms of \(\rho_I\). For black brane configurations, they follow from the M theory action. For cosmology, they were derived from first principles in \cite{12, 13} under certain assumptions. One may also show that the U–duality symmetries of M theory require that the density \(\rho(I)\) of the \(I^{\text{th}}\) stack and its pressures \(p_{\parallel(I)}\) and \(p_{\perp(I)}\) along the parallel and transverse directions must be related as follows \cite{17, 18, 19}:

\[
p_{\parallel(I)} = -\rho(I) + 2 p_{\perp(I)}. \tag{10}
\]

Specifying \(p_{\perp(I)}\) as a function of \(\rho(I)\) will determine the equations of state for \(p_{\parallel(I)}\) and thereby for all the pressures \(p_{i(I)}\). The U–duality symmetries further require this function to be the same for all \(I\). Hence, specifying a single function \(p_{\perp}(\rho)\) determines all \(p_{iI}\) in terms of \(\rho_I\) where \(i = 1, 2, \cdots, 10\) and \(I = 1, 2, \cdots, N\). The result derived in \cite{12, 13} follows as a special case where \(p_{\perp}(\rho) = 0\).

For the \(N = 4\) case, with the four stacks of branes denoted by \(I = 2, 2', 5, 5'\), the pressures \(p_{iI}\) are then given by \cite{17, 18, 19}

\[
\begin{align*}
\{(\rho - p_{i})_{(2)}\} & : (2, 2, 1, 1, 1, 1, 1, 1) u \rho_{(2)} , \\
\{(\rho - p_{i})_{(2')}\} & : (1, 1, 2, 1, 1, 1, 1, 1) u \rho_{(2')} , \\
\{(\rho - p_{i})_{(5)}\} & : (2, 1, 2, 1, 2, 1, 1, 1) u \rho_{(5)} , \\
\{(\rho - p_{i})_{(5')}\} & : (1, 2, 1, 2, 2, 1, 1, 1) u \rho_{(5')} ; \tag{11}
\end{align*}
\]

and the corresponding \(r^i_I = \sum_j G^{ij} (\rho_I - p_{jI})\) by

\[
\{r^i_{(2)}\} : (-2, -2, 1, 1, 1, 1, 1, 1) \frac{\rho_{(2)}}{3},
\]

and
\{r^{(2)}_i\} : (1, 1, -2, -2, 1, 1, 1, 1, 1) \frac{u \rho^{(2)}_i}{3},

\{r^{(5)}_i\} : (-1, 2, -1, 2, -1, -1, 2, 2) \frac{u \rho^{(5)}_i}{3},

\{r^{(5')}_i\} : (2, -1, 2, -1, -1, -1, 1, 1) \frac{u \rho^{(5')}_i}{3}. \quad (12)

Thus, the simple model we propose for the evolution of an M theory universe is given by equations (4) – (9) with \(f(x) = \sin x\) and by the equations of state (11) or equivalently (12).

We obtain the evolution of an M theory universe in our model by numerically solving equations (4) – (9) for \(\lambda_i(t)\), \(m_i(t)\), and \(\rho_I(t)\). We consider the \(N = 4\) case, which is the most entropic one, as well as the \(N < 4\) cases by taking one or more \(\rho_I\) to vanish. In our numerical studies, we set \(l_{qm} = \kappa^2 = 1\) with no loss of generality by measuring the time in units of \(l_{qm}\) and the densities and the pressures in units of \(l_{qm}^{-2} \kappa^{-2}\); set \(\lambda_i = 0\) for all \(i\) at an initial time \(t_0\); and, for the sake of definiteness, take \(u = \frac{2}{3}\) which corresponds to \(p_\perp = \frac{\rho}{3}\). We then obtain the numerical solutions for \(\lambda_i(t)\), \(m_i(t)\), and \(\rho_I(t)\) for various sets of initial values \(m_0^i = m^i(t_0)\) and \(\rho_{I0} = \rho_I(t_0)\); some of the \(\rho_{I0} = 0\) for the \(N < 4\) cases. The results we deduce from our numerical solutions are listed below and they show clearly that the evolutions are non singular and have bounces.

- The densities \(\rho_I(t)\) and the total density \(\rho(t)\) have finite maxima. In the limit \(t \to \pm \infty\), all the non vanishing \(\rho_I\) become equal to each other and \(\to 0\). See Figure 1.

- The total volume factor \(e^{\Lambda(t)}\) has a non vanishing minimum and \(\to \infty\) as \(t \to \pm \infty\). See the plot of \(\Lambda(t)\) in Figure 2.

- For all \(i\), \(m^i(t)\) remain finite, \(\to 0\) as \(t \to \infty\), and \(\to \pi\) as \(t \to -\infty\). See Figure 3. In the limit \(t \to \pm \infty\) then the function \(f(x) = \sin x\) is linear and, hence, the evolutions are as in general relativity.

- For all \(i\), \(\lambda^i(t)\) remain finite. See Figure 4. In the limit \(t \to \pm \infty\), the scale factors \(e^{\lambda^i} \sim |t|^\alpha^i\) and \(t \lambda^i \sim \alpha^i\) where \(\alpha^i\) are constants.

The exponents \(\alpha^i\) can be calculated from general relativity equations when the non vanishing \(\rho_I\) are equal to each other. Depending on
which of the $\rho_I$ are non-vanishing, the exponents $\alpha^i$ for different $i$ may be negative, vanishing, or positive. Then, in the limit $t \to \pm \infty$, the corresponding $\lambda^i$ may $\to -\infty$, $c^i_{\pm}$, or $\infty$ where $c^i_{\pm}$ are constants; the corresponding scale factor $e^{\lambda^i}$ may or may not have a bounce. See the plots of $\lambda^i(t)$ and $t \lambda^i(t)$ in Figures 5 – 7.

We illustrate these features in figures 1 – 7 in a few select cases. In these figures, $u = \frac{2}{3}$, the initial time $t_0 = e^{-10}$, and the initial values are: $\lambda^i(t_0) = 0$; $\{m^i_0\} = (-24, 05, 91, -78, 22, 32, 42, 67, -29) \ast (0.01)$ in figures 1 – 5 and $= (-24, 20, 52, 91, -78, 10, 32, 42, 67, -29) \ast (0.01)$ in figures 6 and 7; and, $\{\rho_{I_0}\} \propto (4.75, 21.8, 183, 373)$ in figures 1 – 5, $\propto (4.75, 0.0, 0.0, 373)$ in figure 6, and $\propto (1.0, 0.0, 0.0, 0.0)$ in figure 7. The proportionality constants in $\{\rho_{I_0}\}$ are fixed by requiring that equation [5] be satisfied at $t_0$. In the figures, we have not labelled the curves with their $i$ or $I$ indices since such labellings are not illuminating for our purposes here.

Figure 1: Plots of total $\rho(t)$ and $\rho_I(t)$ for all $I$ showing their finite maxima. All four $\rho_I$ are non-vanishing.
Figure 2: Plot of $\Lambda(t)$ showing its finite minimum and bounce. All four $\rho_I$ are non vanishing.

Figure 3: Plots of $m^i(t)$ for all $i$ showing their finiteness and their approach to $0$ as $t \to \infty$ and to $\pi$ as $t \to -\infty$. All four $\rho_I$ are non vanishing.
Figure 4: Plots of $\lambda_i^t(t)$ for all $i$ showing their finiteness. All four $\rho_I$ are non vanishing.

The plots of total $\rho$, $\rho_I$, $\Lambda$, $m^i$, and $\lambda_i^t$, shown above in figures 1 – 4, are qualitatively similar in all the cases we studied. The plots of $\lambda_i^t$ and $t\lambda_i^t$ can be different and are shown in figures 5 – 7 when all four $\rho_I$ are non vanishing, only $\rho_{(2)}$ and $\rho_{(3')}$ are non vanishing, and when only $\rho_{(2)}$ is non vanishing. The abscissae in these figures are $ln t$ for the figures on the right hand side and $ln (-t)$ for the figures on the left hand side. In the limit $t \to \pm \infty$, the non vanishing $\rho_I$ become equal to each other, the scale factors $e^{\lambda_i^t} \to |t|^{\alpha_i^t}$, hence $t\lambda_i^t \to \alpha_i^t$, and the evolutions are as in general relativity. The exponents $\{\alpha_i^t\}$ can then be calculated analytically. They are given, after a straightforward calculation, by $(0, 0, 0, 0, 0, 0, 1, 1, 1) \ast \frac{1}{2}$ in figure 5, $(0, -1, 1, 0, 0, 0, 1, 1, 1) \ast \frac{3}{7}$ in figure 6, and by $(-2, -2, 1, 1, 1, 1, 1, 1, 1) \ast \frac{1}{4}$ in figure 7.
Figure 5: Plots of $\lambda^i(t)$ (upper panel) showing their approach to $\infty$ or $c^i_{\pm}$, and of $t\lambda^i(t)$ (lower panel) showing their approach to $\alpha^i$, as $t \to \pm\infty$. Analytically, $\{\alpha^i\} = (0, 0, 0, 0, 0, 1, 1, 1) \ast \frac{1}{2}$. All four $\rho_I$ are non vanishing.
Figure 6: Plots of $\lambda'(t)$ (upper panel) showing their approach to $\pm \infty$ or $c_\pm$, and of $t\lambda_i(t)$ (lower panel) showing their approach to $\alpha^i$, as $t \to \pm \infty$. Analytically, $\{\alpha^i\} = (0, -1, 1, 0, 0, 0, 1, 1, 1) * \frac{3}{7}$. Only $\rho(2)$ and $\rho(5')$ are non vanishing.
Figure 7: Plots of $\lambda^i(t)$ (upper panel) showing their approach to $\pm \infty$, and of $t\lambda^i(t)$ (lower panel) showing their approach to $\alpha^i$, as $t \to \pm \infty$. Analytically, \{\alpha^i\} = (-2, -2, 1, 1, 1, 1, 1, 1, 1) * \frac{1}{4}. Only $\rho(2)$ is non-vanishing.

The behaviour of $\lambda^i$ in figures 5 – 7 in the limit $t \to \pm \infty$ can also be explained physically as follows [19]: In this general relativity limit, $m^i \propto \lambda^i$ and, hence, the terms $r_I^i$ on the right hand side of equation (6) may be thought of as a force due to the $I^{th}$ stack of branes on $\lambda^i$. It follows from equations of state (12) that, when $\rho_I$ are equal to each other, directions parallel to M2 or M5 branes experience a contracting force of strength 2 or 1 in some units, and directions transverse to them experience an expanding force of strength 1 or 2. Therefore, $\lambda^i$ for the directions with net contracting or vanishing or expanding force will $\to -\infty$, or $\to c^i_\pm$ or $\to \infty$ in the limit $t \to \pm \infty$.

Thus, if only $\rho(2)$ is non-vanishing then $\lambda^i \to -\infty$ at the rate of 2 in some units for $i = 1, 2$ and $\to \infty$ at the rate of 1 for the remaining $i$, thus leading to an effectively 8+1 dimensional spacetime in the limit $t \to \pm \infty$. If only $\rho(2)$ and $\rho(5')$ are non-vanishing then $\lambda^i \to -\infty$ at the rate of 3 for $i = 2$, $\to c^i_\pm$ for $i = 1$. 

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for $i = 1, 4, 5, 6, 7$, and $\rightarrow \infty$ at the rate of 3 for $i = 3, 8, 9, 10$, thus leading to an effectively 4 + 1 dimensional spacetime in the limit $t \rightarrow \pm \infty$. If all the four $\rho_I$ are non vanishing, as in the most entropic case, then $\lambda^i \rightarrow c^i_\pm$ for $i = 1, \cdots, 7$ and $\rightarrow \infty$ at the rate of 6 for $i = 8, 9, 10$, thus leading to an effectively 3 + 1 dimensional spacetime in the limit $t \rightarrow \pm \infty$ [18, 19]. These features can be seen clearly in Figures 5 – 7.

In summary, we have presented a simple model leading to non singular evolutions of a 10 + 1 dimensional M theory universe. Our model uses ideas from LQC and offers a solution to the important problem of singularity resolutions. Also, modelling the M theory constituents in the most entropic case as in [12, 13, 18, 19] leads to an effectively 3 + 1 dimensional spacetime as the M theory universe expands.

There are several avenues for further studies. One may study numerically, or analytically where possible, the evolution of higher dimensional universes within our model but for different functions, for example $f(x) = \tanh x$. It seems possible [47, 48] to derive the present model for which $f(x) = \sin x$ using the higher dimensional formulation of LQG given in [44] – [46]. But it is not clear which other functions may be allowed in such an approach, see [35]. One may study higher derivative actions which may lead to the effective equations of the present model, perhaps following the approach of [36]. Such actions may then be used for, among other things, studying inhomogeneous perturbations and their evolutions in non singular universes. One may also analyse systematically whether or not string/M theoretic higher derivative actions lead to non singular evolutions of an universe; and, if yes, study their similarities and differences with the non singular evolutions seen in this letter.

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