Generalized Random-Phase Approximation Theory of Quasiparticle Spectral Functions: Application to Bilayer Quantum Hall Ferromagnets

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We present a microscopic theory of ground-state spectral function of bilayer quantum Hall systems that includes interactions between Hartree-Fock quasiparticles and quantum fluctuations of the order parameter field. The collective modes in these systems are properly described only when fluctuations in direct and exchange particle-hole channels are taken into account. Using an auxiliary field functional integral approach, we present a generalization of the random phase approximation for quasiparticle self-energy which captures fluctuations in both channels. We discuss its relationship to diagrammatic perturbation theory and an adiabatic approximation. We present simple analytical results for the quasiparticle self-energy and the renormalized order parameter that follow from this theory.

I. INTRODUCTION

The physics of a two-dimensional (2D) electron gas in a perpendicular magnetic field is unique in many respects. Bilayer electron systems consist of a pair of 2D electron gases separated by a distance \( d \) (\( d \approx 100 \) \( \text{Å} \)) which is comparable to the typical distance between electrons within one layer. In the presence of a strong magnetic field perpendicular to the layers, the kinetic energy of electrons is quenched and the physical properties of the system are determined by electron-electron interactions alone. Bilayer quantum Hall systems at Landau level filling factor \( \nu = 1 \) exhibit a rich variety of broken symmetry states and have been extensively studied over the past decade. For filling factor \( \nu = 1 \), the ground state is fully spin polarized and the spin excitations are gapped because of Zeeman coupling of electron spins to the external magnetic field. The only remaining dynamical degrees of freedom are the discrete layer index and the intra-Landau level orbitals. We model the layer index using a pseudospin label where “up” denotes the symmetric bilayer state (S) and “down” denotes the antisymmetric bilayer state (AS). Note that this choice of quantization axis is different from the usual convention where “up” denotes a state localized in the top layer. At total filling factor \( \nu = 1 \), because of strong interlayer correlations, the ground state of the system exhibits spontaneous interlayer phase coherence (easy-plane ferromagnetism in the pseudospin language) and is incompressible. At a critical layer separation \( d_{cr} \), the system undergoes a phase transition from this incompressible quantum Hall state with pseudospin ferromagnetism to a disordered compressible state, possibly with other more exotic intervening states.

In this paper we present a theory which describes effect of quantum fluctuations on the Hartree-Fock quasiparticles of the \( \nu = 1 \) bilayer. Some of the results presented here were published earlier. Our theory is based on an approximate expression for the quasiparticle self-energy which includes the effect of quantum fluctuations and generalizes the random phase approximation (RPA) to cases when both direct and exchange fluctuations are important. This self-energy modifies the spectral function of the mean-field quasiparticles, which in turn renormalizes the physical properties of the system. The most important macroscopic parameter characterizing the phase-coherent state is the pseudospin polarization. We use pseudospin polarization normalized to its mean-field value as a dimensionless order parameter \( m_z \). This order parameter is related to quasiparticle spectral weights integrated up to the Fermi energy, \( m_z = (n_S - n_{AS}) \), where \( n_S \) denote occupation numbers for the symmetric and antisymmetric states. Therefore the order parameter is sensitive to changes in the quasiparticle spectral function. In the mean-field approximation the quasiparticles have sharply defined energies and the spectral functions are \( \delta \)-functions at these energies. In the mean-field ground state, all electrons occupy the symmetric state in the lowest Landau level, which corresponds to all pseudospins pointing along the positive \( z \)-axis, \( m_z = 1 \). We will find that when we include the effect of collective fluctuations, the spectral functions develop a continuum piece and the spectral weight at the \( \delta \)-function peak is reduced. We will calculate the renormalized order parameter by evaluating the fluctuation correction to the mean-field spectral function.

The generalized random phase approximation for quasiparticle self-energy presented in this paper can be usefully adopted to study the effect of quantum fluctuations on mean-field quasiparticles whenever the collective-mode behavior...
is determined by fluctuations in more than one particle-hole channels; for example, to analyze the effect of fluctuations on Wigner crystal mean-field quasiparticles at low filling factors. Traditionally random phase approximations have been used either to take into account the screening of Coulomb interaction where only direct-channel interactions determine the physics, or the effect of spin-waves on a ferromagnetic mean-field state where the low-energy physics is solely determined by exchange interactions. The generalization which we present here systematically takes into account fluctuations in both channels. We study the bilayer quantum Hall system as a test case because its translation invariance combined with the absence of dispersion in the Landau level bands permits many elements of this calculation to be performed analytically, and because the effect of quantum fluctuations on the mean-field ground state can be tuned over a large range simply by changing the layer separation. In the \( d = 0 \) limit, since the interaction Hamiltonian is pseudospin invariant, the mean-field ground state is exact. At finite \( d \), because of the difference between the intralayer Coulomb interaction \( V_A(q) = 2\pi e^2/q \) and the interlayer Coulomb interaction \( V_E(q) = V_A(q)e^{-qd} \), the symmetry of the interaction Hamiltonian is reduced from SU(2) to U(1), and the order parameter \( m_z \) does not commute with the interaction. Therefore the pseudospin-polarized mean-field state is not an eigenstate of the Hamiltonian and in particular, it is not the ground state. The exact ground state of this system incorporates quantum fluctuations in the quantum Hall ferromagnet’s order parameter field.

The paper is organized as follows. In section II we describe the functional integral approach to this problem. It is known that the collective modes in these systems are properly described only when both electrostatic and exchange fluctuations are taken into account. No standard Hubbard Stratonovich (HS) transformation can treat both effects simultaneously. Section II also summarizes the generalized Hubbard-Stratonovich which we employ and establishes some of the notation which we will use. In this section we present the generalized random phase approximation (GRPA) for the quasiparticle self-energy, which takes into account fluctuations in both channels and is based on an approach developed previously by Kerman et al. Section III summarizes the relationship of this GRPA with diagrammatic perturbation theory. In section IV we apply our formalism to a bilayer system at filling factor \( \nu = 1 \). We evaluate the effect of fluctuations on the quasiparticle spectral functions and renormalization of the order parameter due to these fluctuations. Section V summarizes an adiabatic approximation for the quasiparticle self-energy in a quantum Hall ferromagnet and its close relationship with the two more general approaches we discuss. We conclude the paper with a brief discussion in section VI.

II. AUXILIARY FIELD FUNCTIONAL INTEGRAL APPROACH

In bilayer systems, the anisotropy of the Coulomb interaction in pseudospin space makes it necessary to treat fluctuations in the Hartree and exchange channels on an equal footing. A general way to treat fluctuations around an ordered mean-field is the Hubbard Stratonovich transformation. In this approach, by introducing an integral over a Bose field \( \phi \), we convert the two-body fermionic interaction term into a one-body term coupled to this field. Since the effective Hamiltonian obtained is quadratic in fermion operators, the trace over these degrees of freedom can be performed exactly and we obtain an effective action for the auxiliary Bose field. Stationary phase approximations to the bosonic action give various mean-field theories. The nature of the mean-field state depends upon the way the Hubbard Stratonovich transformation is implemented and the predictions for collective excitations depend qualitatively on the mean-field around which Gaussian fluctuations in the Bose field are considered. For example, fluctuations in the exchange mean-field produce spin-wave excitations in ferromagnets, whereas fluctuations in the Hartree mean-field lead to plasmons in an electron gas. For a bilayer system, we need a method which treats both Hartree and exchange fluctuations on an equal footing. It is known that the standard HS transformation does not yield a Hartree-Fock mean-field and cannot capture both Hartree and the exchange fluctuations.

Kerman, Levit and Troudet have presented a generalization of the HS transformation which overcomes these limitations. To establish our notation (which differs from that of Kerman et al.) we briefly review the previous work which describes fluctuation corrections to mean-field approximations for the grand potential. Our approach, on the other hand, enables us to systematically improve upon mean-field approximations for correlation functions.

Consider a many fermion Hamiltonian in second quantized form

\[
\hat{H} = \sum_{\alpha} K_{\alpha\beta} c^\dagger_{\alpha} c_{\beta} + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta\gamma\delta | V | \gamma\delta \rangle c^\dagger_{\alpha} c^\dagger_{\beta} c_{\gamma} c_{\delta},
\]  

(2.1)

where \( K \) is a one-body (kinetic) term and \( V \) is a two-body interaction. Here \( \alpha \) denotes (a set of) single-particle quantum numbers. For example, in the case of a bilayer system \( \alpha = (n, k, \sigma) \) where \( n \) is the Landau level index, \( k \) is the intra-Landau level index and \( \sigma \) is the pseudospin label. Introducing pair-labels \( a = (\alpha') \) and the density matrix \( \hat{\rho}_a = c^\dagger_{\alpha'} c_{\alpha} \), the kinetic term becomes \( K_{\alpha} \hat{\rho}_a \) where summation over repeated index is assumed. Similarly the interaction
term can be written as \( V_{ab} \hat{\Omega}_{ab} \) where \( \hat{\Omega}_{ab} = : \hat{\rho}_a \hat{\rho}_b : \) is the normal ordered two-body term and \( V_{ab} = \langle \alpha' \beta' | V | \alpha \beta \rangle \). Then in an obvious matrix notation, we write (2.1) as \( \hat{H} = K \hat{\rho} + V \hat{\Omega}/2 \). The partition function in the grand canonical ensemble is given by

\[
Z = \text{Tr} \ e^{\alpha N - \beta \hat{H}} = \lim_{\epsilon \to 0} \text{Tr} \ e^{\alpha N} T \prod_{j=1}^{M} \left[ 1 - \epsilon K \hat{\rho}_j - \frac{\epsilon}{2} V \hat{\Omega}_j \right],
\]

(2.2)

where \( T \) denotes the imaginary-time ordered product, \( N \) is the number operator, \( \epsilon = \beta/M \) and in the thermodynamic limit \( \alpha/\beta \to \mu \), the chemical potential. Since the number operator commutes with the Hamiltonian, we can expand only the \( e^{-\beta \hat{H}} \) term. The central idea of this generalized Hubbard transformation is to expand around an arbitrary two-body interaction \( U \), thus treating \( K \hat{\rho} + U \hat{\Omega}/2 \) as the dominant term and \( (V - U) \hat{\Omega}/2 \) as a perturbation. Using the Gaussian identity

\[
1 = \frac{1}{\sqrt{\det U}} \int \prod_{\gamma \delta} \frac{d\phi_{\gamma \delta}(j)}{\sqrt{2\pi/\epsilon}} \exp \left[ -\frac{\epsilon}{2} \sum_{\alpha \beta \alpha' \beta'} \phi_{\alpha \beta}(j) U^{-1}_{\alpha \beta \alpha' \beta'} \phi_{\alpha' \beta'}(j) \right]
\]

(2.3)

at each time index \( j \), a typical term in Eq.(2.2) becomes

\[
\left[ 1 - \epsilon K \hat{\rho}_j - \frac{\epsilon}{2} V \hat{\Omega}_j \right] = \int \mathcal{D}\phi_j e^{-\epsilon \phi_j U^{-1} \phi_j /2} \left[ 1 - \epsilon K \hat{\rho}_j + \epsilon \phi_j \hat{\rho}_j - \frac{\epsilon^2}{2} V \hat{\Omega}_j \frac{\phi_j U^{-1} \phi_j}{N^2} \right].
\]

(2.4)

Here \( N^2 = \mathcal{N} \times \mathcal{N} \) stands for a repeat sum over the \( \mathcal{N} \) single-particle state labels in the Hilbert space and \( \mathcal{D}\phi_j \) implies both a product over single-particle labels and the relevant normalization factors. Introducing these fields at each time-step \( j \) we get

\[
Z = \lim_{\epsilon \to 0} \int \prod_{j=1}^{M} \mathcal{D}\phi_j \exp (-S[\phi]),
\]

(2.5)

\[
S[\phi] = \frac{\epsilon}{2} \sum_{j=1}^{M} \phi_j U^{-1} \phi_j - \ln \text{Tr} \ e^{\alpha N} T \prod_{j=1}^{M} \left[ 1 - \epsilon K \hat{\rho}_j + \epsilon \phi_j \hat{\rho}_j - \frac{\epsilon^2}{2} V \hat{\Omega}_j \frac{\phi_j U^{-1} \phi_j}{N^2} \right].
\]

(2.6)

We stress that since \( \langle \phi^2 \rangle \approx 1/\epsilon \), the limit \( \epsilon \to 0 \) in Eq.(2.6) must be taken after all \( \phi \) integrals are done.

Systematic approximations to the grand potential are obtained by expanding the bosonic action (2.6) around its minimum. In the limit as \( \epsilon \to 0 \), the configuration of fields \( \phi^0 \) which minimizes the action \((\partial S/\partial \phi|_{\phi^0} = 0)\) is given by

\[
\phi^0_{\alpha \beta}(j) = \langle \phi_{\alpha \beta}(j) \rangle_{\phi^0}, \quad j = 1, \ldots, M;
\]

(2.7)

where \( \langle \rangle_{\phi^0} \) indicates thermal average with the mean-field Hamiltonian \( \hat{h}_1 = (K - \phi^0) \hat{\rho} \). We emphasize that Eq.(2.4) is an exact identity; since the variance of the auxiliary field is proportional to \( 1/\epsilon \), the interaction term with prefactor \( \epsilon^2 \) is of the same order as the kinetic term with the prefactor \( \epsilon \) after all the \( \phi \)-integrals are done. The stationary phase approximation, however, corresponds to replacing the measure \( \mathcal{D}\phi_j \) by \( \mathcal{D}\phi_j \delta(\phi_j - \phi^0_j) \). Therefore, in the stationary phase approximation, interaction term with the prefactor \( \epsilon^2 \) can be neglected and does not contribute to the mean-field solution. It is clear from Eqs.(2.7) that the mean-field Hamiltonian is solely determined by the trial interaction \( U \) and is independent of the actual two-body interaction \( V \). We get the Hartree (exchange) mean-field by choosing \( U = -V \) (\( U = +V \epsilon \)), whereas \( -U = (V - V \epsilon) = V A \) (the antisymmetrized interaction) gives the Hartree-Fock mean-field.

We improve upon the stationary phase approximation by considering quadratic fluctuations around the mean field \( \phi^0 \). These fluctuations, of course, depend upon the true microscopic interaction \( V \). Expanding the action (2.6) to second order in the fluctuating fields \( \phi_j = \phi^0 + \delta \phi_j \) gives

\[
\frac{\partial^2 S}{\partial \phi_j \partial \phi_{j'}} = \epsilon M(j, j') = \epsilon \left[ \delta_{jj'} U^{-1} - \epsilon (1 - \delta_{jj'}) \right] D^{kk'} + \epsilon \delta_{jj'} S,
\]

(2.8)

where only time indices are explicitly shown. The matrices \( D \) and \( S \) are defined by

\[
D^{jj'}_{\alpha \beta \gamma \delta} = \langle \rho_{\alpha \beta}(j) \rho_{\gamma \delta}(j') \rangle_{\phi^0} - \langle \rho_{\alpha \beta}(j) \rangle_{\phi^0} \langle \rho_{\gamma \delta}(j') \rangle_{\phi^0},
\]

(2.9)
\[ S_{\alpha\beta\gamma\delta} = U_{\alpha\beta\gamma\delta}^{-1} \frac{\langle V \delta \phi \rangle}{N^2} + \langle \rho_{\alpha\beta}(j) \rangle_{\phi^0} \langle \rho_{\gamma\delta}(j') \rangle_{\phi^0}, \]  

(2.10)

where \( \alpha, \beta \) are the single-particle labels and \( j, j' \) stand for the time indices. The particle-hole response function \( D \) is the same as the Hartree-Fock mean-field susceptibility and the fluctuation matrix \( M^{-1} \) is essentially the propagator for collective modes around the mean-field. It is clear from Eqs.(2.8), (2.9) and (2.10) that properties of the collective modes are determined by the mean-field \( \phi^0 \) and the true microscopic interaction \( V \). We note that among all trial potentials, only the antisymmetrized interaction optimizes the quasiparticle grand potential, \( \Omega_{\beta}(U) = S[\phi^0] + trS/2 \), obtained from the action (2.4). Therefore, we will concentrate on the Hartree-Fock mean field which corresponds to \( U = -V^A \).

We now extend this approach to the evaluation of the one-particle Green’s function \( G_{ab}(\tau_a, 0) = +\langle Tc_a(\tau_a)c_b^\dagger(0) \rangle \). Starting with this definition and introducing auxiliary fields at each time step, we get

\[ G_{ab}(\tau_a) = \lim_{\epsilon \to 0} \frac{1}{Z} \text{Tr} \left[ \prod_{j=1}^{M} \left( 1 - \epsilon \hat{H}_j + \frac{\alpha}{\beta} \hat{N}_j \right) c_a(\tau_a)c_b^\dagger(0) \right], \]  

(2.11)

\[ = \lim_{\epsilon \to 0} \frac{1}{Z} \int \mathcal{D}\phi e^{-S[\phi] + \langle \langle c_a(\tau_a)c_b^\dagger(0) \rangle \rangle_{\phi}}, \]  

(2.12)

where \( S \) is the bosonic action (2.4) and \( \langle \langle c_a(\tau_a)c_b^\dagger(0) \rangle \rangle_{\phi} \) represents the finite-\( \epsilon \) expression for thermal average with the auxiliary Hamiltonian \( \hat{h}_j = (K - \phi^0)\hat{\beta} \).

Systematic approximations to the exact Green’s function are obtained by expanding the action \( S \) and the \( \phi \)-dependent thermal average around the mean-field value \( \phi^0 \). At the mean-field level, we obtain the Green’s function of non-interacting quasiparticles in the self-consistent field \( \phi^0 \)

\[ G_{0ab}(i\omega_n) = (-i\omega_n + K - \phi^0 - \mu)^{-1}, \]  

(2.13)

where \( \omega_n = \pi(2n + 1)/\beta \) is a fermionic Matsubara frequency. To consider the effect of fluctuations, we expand the action (2.4) to second order in fluctuating fields \( \phi_j = \phi^0 + \delta\phi_j \), replace the mean-field Green’s function by a \( \phi \)-dependent Green’s function

\[ G_{ab}(i\omega_n) = (-i\omega_n + K - \phi^0 - \delta\phi - \mu)^{-1} = \left( G_0^{-1} - \delta\phi \right)^{-1}, \]  

(2.14)

and expand (2.14) in powers of \( \delta\phi \) to all orders. The quadratic expansion of the action (2.4) gives non-interacting collective modes around the mean-field \( \phi^0 \). Expanding the \( \phi \)-dependent Green’s function gives an effective interaction between the mean-field quasiparticles and the collective modes. The resulting integrals over the fluctuating fields are performed using Wick’s theorem. In this approximation the Green’s function has a self-energy that is the sum of all 1-particle irreducible diagrams obtained from mean-field Green’s function \( G^0 \) and the collective-mode propagator \( M^{-1} \).

We approximate this self-energy by the first irreducible diagram, which describes a single scattering of a collective mode and a mean-field quasiparticle (Figure 3). The contribution from such a diagram is given by

\[ \Sigma_{ab}(i\omega_n) = -\frac{1}{\beta} \sum_{i\Omega_n} \sum_{\alpha'\beta'} G_{0,\alpha'\beta'}^0 (i\omega_n - i\Omega_n) M^{-1}_{\alpha'\beta',\rho\gamma}(i\Omega_n). \]  

(2.15)

where \( \Omega_n = 2\pi n/\beta \) is a bosonic Matsubara frequency. Eq.(2.15) is a general expression for self-energy of a quasiparticle because of its interactions with the collective modes around any mean-field. We emphasize once again that this calculation is to be done with a finite value of \( \epsilon \) and that the limit \( \epsilon \to 0 \) is taken only after all the \( \phi \)-integrals are evaluated. Since the dimension of the matrix \( M \) is of order \( 1/\epsilon \) and since the matrix elements of \( M \) are dependent on \( \epsilon \), we have to be careful about elements of the inverse matrix. As we are interested in fluctuations around the Hartree-Fock mean field, we use \( U = -V^A \) and Eq.(2.8) for the collective-mode propagator, and arrive at the following expression for the quasiparticle self-energy

\[ \Sigma_{ab}(i\omega_n) = \frac{1}{\beta} \sum_{i\Omega_n} \sum_{\alpha'\beta'} G_{0,\alpha'\beta'}^0 (i\omega_n - i\Omega_n) \left[ (1 + V^A D)^{-1} V^A - V^A \right]_{\alpha'\beta',\rho\gamma}(i\Omega_n). \]  

(2.16)

Here we have used the identity \( A_{ij}^{-1} = \partial \ln \det A/\partial A_{ij} \), and the fact that \( \ln \det M = \ln \det |1 + V^A D| - \text{tr}(V^A D) \). We will call (2.16) as the generalized random phase approximation (GRPA) for quasiparticle self-energy. It is clear
from Eq. (3.16) that the GRPA self-energy includes diagrams with \( n \geq 2 \) interaction terms, and in particular, it does not contain the mean-field self-energy contribution. This approximation for the self-energy arises naturally in our auxiliary field functional integral approach. In the next section we discuss the corresponding approximation in diagrammatic perturbation theory.

### III. GENERALIZED RANDOM PHASE APPROXIMATION

In this section we describe the generalized random phase approximation for the quasiparticle self-energy in diagrammatic language and relate it to expression (3.16) derived in the preceding section. In the case of particle-hole response functions, or the grand potential, appropriate conserving generalizations of random phase approximation which take into account fluctuations in both, direct and exchange, particle-hole channels have been discussed in the literature.

Similar generalizations for the quasiparticle self-energy are less than transparent. Several approaches and approximations have been used in the past. Some start from the “bubble” RPA which describes the screening and use vertex-corrected bubbles, while others start from the “ladders” RPA which captures the spin-wave dynamics and include the Hartree corrections by using screened interaction. We are not aware of a systematic generalization for the self-energy.

The appropriate generalization of the RPA self-energy which we propose is summarized diagrammatically in Fig. 2. The physical content of this self-energy is determined solely by the nature of fluctuations encoded in the four-point vertex \( \Gamma^{(4)}(\vec{q}, i\Omega_n) \). We use the antisymmetrized interaction as the bare four-point vertex, \( \Gamma_0^{(4)} = V^A \). The self-energy obtained from the bare four-point vertex by contracting the incoming and outgoing labels on the top is the same as the the Hartree-Fock mean-field approximation for the self-energy. If instead of the antisymmetrized interaction, we use the direct (exchange) interaction as the bare vertex, we will get the Hartree (exchange) mean-field self-energy.

We encode the fluctuations in direct and exchange channels into the four-point vertex by summing all the particle-hole diagrams in this set, when formally summed, give bare vertex. As expected, it represents only exchange fluctuations, which determine the spin-wave dynamics. The diagrams in this set, when formally summed, give

\[
\Sigma_{ab}(i\omega_n) = \frac{1}{\beta} \sum_{i\Omega_n} \sum_{a'\beta'} G_{a'\beta'}^0(i\omega_n - i\Omega_n) \left[ (1 + V^A D)^{-1} V_A - V_A \right]_{a\alpha',b\beta'}(i\Omega_n).
\]

This is the diagrammatic equivalent of the self-energy approximation (2.15) discussed in the previous section.

The diagrammatic content of this approximation is shown in Fig. 3. The first set of diagrams on the left, (a), has been traditionally used to include the effect of screening in RPA self-energy. This set of diagrams is generated by using the direct interaction as the bare four-point vertex in Eq. (3.1). The diagrams in this set can be formally summed and their contribution to the quasiparticle self-energy is given by

\[
\Sigma_{ab}(i\omega_n) = \frac{1}{\beta} \sum_{i\Omega_n} \sum_{a'\beta'} G_{a'\beta'}^0(i\omega_n - i\Omega_n) \left[ V^{sc} - V \right]_{a\alpha',b\beta'}(i\Omega_n).
\]

where we identify \( V^{sc} = (1 + V D)^{-1} V \) as the screened Coulomb interaction. The second set, (b), has been used to include the effect of spin-waves in a ferromagnet. This set is generated by using the exchange interaction as the bare vertex. As expected, it represents only exchange fluctuations, which determine the spin-wave dynamics. The diagrams in this set, when formally summed, give

\[
\Sigma_{ab}(i\omega_n) = -\frac{1}{\beta} \sum_{i\Omega_n} \sum_{a'\beta'} G_{a'\beta'}^0(i\omega_n - i\Omega_n) \left[ (1 - V^{ex} D)^{-1} V^{ex} - V^{ex} \right]_{a\alpha',b\beta'}(i\Omega_n).
\]
Eq. (3.3) reproduces the results for electron self-energy because of spin-waves in a single-layer quantum Hall system at \( \nu = 1 \), where the physics of spin-waves is determined solely by exchange fluctuations. The generalized random phase approximation introduced here includes another class of diagrams, (c), which represents combinations of (competing) direct and exchange fluctuations. We note that the set of diagrams in Fig. 3 arises naturally in the functional integral approach as do the subsets (a) and (b), when considering fluctuations around the Hartree or the exchange mean-field respectively. We refer to the sum of diagrams in all three sets as the GRPA self-energy. This approximation for the self-energy can also be derived by differentiating the grand potential with respect to the mean-field Green’s function when the appropriate generalized random phase approximation for the grand potential is used.

IV. APPLICATION TO DOUBLE LAYER SYSTEMS

In this section, we apply the formalism developed in preceding sections to a double-layer quantum Hall system at total filling factor \( \nu = 1 \). We consider a completely spin-polarized ground state so that the layer index and the intra-Landau level index are the only dynamical degrees of freedom. The properties of the Landau level wavefunctions allow us to associate a two-dimensional momentum with a pair of orbit-center labels. The existence of this unitary transformation between the 2D wavevectors and pairs of orbit-center labels permits the significant progress that can be achieved analytically in the following calculations.

The Hamiltonian for a bilayer system with interlayer tunneling amplitude \( \Delta_t \) is

\[
\hat{H}_0 - \mu N = \sum_{\mathbf{k}, \sigma} \varepsilon_\sigma \hat{c}_{\mathbf{k}\sigma}^\dagger \hat{c}_{\mathbf{k}\sigma},
\]

\[
\hat{V} = \hat{V}_0 + \hat{V}_x,
\]

\[
\hat{V}_0 = \frac{1}{2} \sum_{\mathbf{k}_1, \sigma_1} \langle \mathbf{k}_1 \mathbf{k}_2 \rangle \langle \mathbf{V}_0 \mathbf{V}_0 \rangle \hat{c}_{\mathbf{k}_1 \sigma_1}^\dagger \hat{c}_{\mathbf{k}_2 \sigma_2} \hat{c}_{\mathbf{k}_3 \sigma_3} \hat{c}_{\mathbf{k}_4 \sigma_4},
\]

\[
\hat{V}_x = \frac{1}{2} \sum_{\mathbf{k}_1, \sigma_1} \langle \mathbf{k}_1 \mathbf{k}_2 \rangle \langle \mathbf{V}_x \mathbf{V}_x \rangle \hat{c}_{\mathbf{k}_1 \sigma_1}^\dagger \hat{c}_{\mathbf{k}_2 \sigma_2} \hat{c}_{\mathbf{k}_3 \sigma_3} \hat{c}_{\mathbf{k}_4 \sigma_4},
\]

Here, \( \sigma = \uparrow = -\bar{\sigma} \) denotes a symmetric state (S) while \( \sigma = \downarrow \) denotes the antisymmetric state (AS), and \( \varepsilon_\sigma = -\sigma \Delta / 2 \) are the bare single-particle energies measured from the chemical potential. The \( \mathbf{k}_i \) are the angular momenta which are good quantum numbers of the single-particle Hamiltonian in the symmetric gauge \( \mathbf{A} = (-B y / 2, B x / 2, 0) \); \( \mathbf{V}_0 = (\mathbf{V}_{A} + \mathbf{V}_{E}) / 2 \) and \( \mathbf{V}_x = (\mathbf{V}_{A} - \mathbf{V}_{E}) / 2 \) are sums and differences of interlayer and intralayer Coulomb interactions. In the present choice of quantization axis in pseudospin-space, the interaction \( \mathbf{V}_x \) reverses pseudospins of the scattering particles and therefore it does not commute with the pseudospin polarization operator.

We start with results for the Hartree-Fock mean-field approximation. The Matsubara Green’s function for the one-body Hamiltonian \( \hat{H}_0 \) is

\[
\hat{G}_{\mathbf{k}\sigma}(i\omega_n) = \frac{-1}{i\omega_n - \varepsilon_\sigma}.
\]

To obtain the Hartree-Fock mean-field Green’s function, we solve the Dyson equation with Hartree-Fock self-energy

\[
\Sigma_{\sigma}^{HF} = \varepsilon_\sigma^0 - \varepsilon_\sigma = -\left[\Gamma_0(0)n_F(\varepsilon_\sigma^0) + \Gamma_x(0)n_F(\varepsilon_\sigma^0)\right],
\]

where \( (\lambda = 0, x, A, E) \)

\[
\Gamma_\lambda(\mathbf{q}) = \frac{1}{A} \sum_{\mathbf{p}} \mathbf{V}_\lambda(\mathbf{p}) e^{-i\mathbf{q}\cdot\mathbf{p}} e^{-i\mathbf{q}\cdot(\mathbf{q}\times\mathbf{p})/2} e^{i\mathbf{q}^2 / 2m / 2},
\]

and \( n_F(x) \) is the Fermi occupation number. The \( \Gamma_\lambda(\mathbf{q}) \) are interactions between an electron and an exchange-hole, whose center is separated by a distance \( q l^2 \). The Hartree-Fock self-energy is diagonal in the orbit center and pseudospin labels, and the Hartree-Fock mean-field Green’s function is given by

\[
\hat{G}_{\mathbf{k}\sigma}^0(i\omega_n) = \frac{-1}{i\omega_n - \varepsilon_\sigma^0}.
\]

At low temperatures the stable solution of Eq. (4.4) is given by \( \varepsilon_\sigma^0 = -[\Delta_t + m_z \Gamma_E(0)] / 2 \equiv -\Delta_{SAS} / 2 \). Particle-hole symmetry at \( \nu = 1 \) implies that \( \varepsilon_{AS}^0 = -\varepsilon_S^0 \). The exchange enhancement of the symmetric-antisymmetric energy
splitting, $\Delta_m = m_\gamma \Gamma_E(0)$, is determined by the interlayer interaction and survives in the limit of vanishing interlayer tunneling, giving rise to spontaneous phase coherence. The spectral function for the Hartree-Fock Green’s function is given by $A_{\gamma}(\omega) = 2\pi \delta(\omega - \epsilon^0_{\gamma})$. In a mean-field approximation, the ground state is fully pseudospin polarized along the positive $z$-direction. The low-lying collective modes around this mean-field involve slow variations in the pseudospin field i.e., the pseudospin waves. Fluctuations out of the $y-z$ plane in pseudospin-space correspond to the transfer of charge from one layer to another whereas pseudospin fluctuations in the $y-z$ plane correspond to variations in the relative phase between states localized in the top and the bottom layer.

We now evaluate the contribution of these pseudospin waves to the quasiparticle self-energy and its effect on the mean-field spectral function. The calculation of four point vertex $\Gamma^{(4)}$ or the fluctuation matrix $M$ is particularly simple in the present case. Since moment and frequency are good quantum numbers, the fluctuation matrix is diagonal in these indices and effectively has only the pseudospin labels. In other words, the matrix equation for the four-point vertex having orbital and pseudospin labels, Eq.(2), reduces to an algebraic equation with matrices having only pseudospin labels. Furthermore, because of the constraints which ensure that the Hartree-Fock susceptibility $D$ is nonzero, only a $2 \times 2$ submatrix of the entire $4 \times 4$ matrix $V^A D$ is nonzero. This non-vanishing $2 \times 2$ submatrix is given by

$$\langle \sigma_1 \sigma_2 | V^A(q) D(i\Omega_n) | \sigma_3 \sigma_4 \rangle = \begin{pmatrix} (v_x - \Gamma_0)_{\text{min}} + \Delta_{\text{PAS}} & (v_x - \Gamma_0)_{\text{min}} - \Delta_{\text{PAS}} \\ (v_x - \Gamma_x)_{\text{min}} + \Delta_{\text{PAS}} & (v_x - \Gamma_x)_{\text{min}} - \Delta_{\text{PAS}} \end{pmatrix}. \tag{4.7}$$

Here $2\pi \Gamma v(\epsilon_\gamma) = e^{-q^2 \epsilon_\gamma^2/2} V_{\gamma}(\epsilon_\gamma)$ represents the effect of electrostatic fluctuations whereas $\Gamma(\epsilon_\gamma)$ represent the effect of exchange fluctuations. The entire $4 \times 4$ fluctuation matrix is a trivial extension of Eq.(4.7). The roots of the equation $\text{det} \ [1 + V^A(q) D(i\Omega)] = 0$ give the bilayer pseudospin-wave dispersion $\Omega = E_{\text{sw}}(q)$ where

$$E_{\text{sw}}^2(q) = a(q) \cdot b(q) = [\Delta_{\text{PAS}} - \Gamma_A(q) + 2v_x(q)] \cdot [\Delta_{\text{PAS}} - \Gamma_E(q)]. \tag{4.8}$$

Thus, the functional integral approach reproduces results obtained earlier by diagrammatic and by the single-mode approximation. Figure 4 shows typical pseudospin-wave dispersions. For a finite interlayer tunneling, $\Delta_t \neq 0$, the collective mode is gapped at zero wavevector because the U(1) symmetry in the $y-z$ plane in pseudospin-space is explicitly broken. As $d$ increases, the minimum in the pseudospin-wave spectrum near $q l \approx 1$ reaches zero at a critical layer separation $d_{cr}$. In Hartree-Fock theory, at this point, the ground state changes from a uniformly pseudospin-polarized state to a pseudospin-density wave state. This softening of pseudospin-wave at a finite wavevector has been associated with the phase transition from a phase-coherent quantum Hall state to a compressible state.

Using the mean-field Green’s function (4.6) and the $4 \times 4$ fluctuation matrix we arrive at the following analytical expression for zero-temperature symmetric state self-energy

$$\Sigma_S(i\omega_n) = \frac{2\pi l^2}{A} \sum_\beta \frac{(E_{\text{sw}}(\vec{p}) + \Delta_{\text{PAS}})^2}{2E_{\text{sw}}(\vec{p})} \left[ \frac{\epsilon(\vec{p}) - E_{\text{sw}}(\vec{p})}{i\omega_n - E_{\text{sw}}(\vec{p}) - \epsilon^0_{\text{AS}}} \right] = -\Sigma_{\text{AS}}(-i\omega_n), \tag{4.9}$$

where $\epsilon(\vec{p}) = [a(\vec{p}) + b(\vec{p})] / 2$. This remarkably simple expression is the first principal result of this work. The Dyson equation relating the full Green’s function to the mean-field Green’s function and the fluctuation self-energy is

$$\left[ G_\sigma(i\omega_n) \right]^{-1} = \left[ G_\sigma^0(i\omega_n) \right]^{-1} + \Sigma_\sigma(i\omega_n). \tag{4.10}$$

We obtain the quasiparticle spectral function from the retarded GRPA self-energy by analytically continuing (4.9) to real frequencies, $i\omega_n \to \omega + i\eta$,

$$A_S(\omega) = -2 \text{ Im } G_S(\omega + i\eta) = -2 \text{ Im } \frac{1}{\omega + i\eta - \epsilon^0_{S} - \Sigma_S(\omega + i\eta)} = A_{\text{AS}}(-\omega). \tag{4.11}$$

This spectral function has a $\delta$-function contribution at frequency $\omega^*$ which satisfies the Dyson equation $\omega^* - \epsilon^0_{S} = \Sigma_S(\omega^* + i\eta)$. The continuum piece of $A_S(\omega)$ is nonzero in the region where the retarded self-energy $\Sigma_S(\omega + i\eta)$ has a branch cut, i.e., at frequencies in the interval $I_S \equiv [\omega_<, \omega>] = [\epsilon^0_{\text{AS}} + E_{\text{sw}}, \epsilon^0_{\text{AS}} + \Delta_{\text{PAS}}]$. Here $E_{\text{sw}}$ denotes the minimum of the pseudospin-wave energy near $q l \approx 1$ and $\Delta_{\text{PAS}} = E_{\text{sw}}(q \to \infty)$ is the mean-field splitting. Explicitly, we can write the spectral function as

$$A_S(\omega) = 2\pi z_S \delta(\omega - \omega^*) + \theta(\omega - \omega_<)\theta(\omega_\gamma - \omega) \frac{-2 \text{ Im } \Sigma_S(\omega)}{[\omega - \epsilon^0_{S} - \text{Re} \Sigma_S(\omega)]^2 + [\text{Im } \Sigma_S(\omega)]^2}. \tag{4.12}$$
where \( z_S = (1 - \partial \Sigma_S / \partial \omega)_{\omega=\mu}^{-1} < 1 \) is the quasiparticle renormalization factor. The mean-field spectral function \( A_S(\omega) = 2\pi \delta(\omega - E^0_S) \) is recovered by neglecting the fluctuation self-energy, which implies \( \omega^* = E^0_S \) and \( z_S = 1 = z_{AS} \).

When we include the fluctuation self-energy, the spectral weight at the \( \delta \)-function peak is reduced to \( z_S < 1 \) and is distributed into a continuum piece at positive energies, \( \omega > \mu \). Figure 3 shows the evolution of continuum part of the symmetric-state spectral function with increasing layer separation. We see that the spectral weight at positive energies, an indicator of the strength of quantum fluctuations in the pseudospin polarized mean-field ground state, increases with increasing layer separation and as required by the sum-rule, the quasiparticle renormalization factor \( z_S \) decreases.

The presence of the spectral weight at positive energies has been detected at low temperatures by Manfra et al. in optical absorption experiments. The redistribution of the spectral weight indicates the mixing of many-body states having antisymmetric quasiparticles into the completely pseudospin-polarized ground state. One measure of this mixing is the suppression of the order parameter

\[
m_z = (n_S - n_{AS}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} n_F(\omega) [A_S(\omega) - A_{AS}(\omega)]
\]

from its mean-field value \( m_z = 1 \). Using the particle-hole symmetry at \( \nu = 1 \) we get \( A_S(\omega) = A_{AS}(\omega) \). This leads to the following simple expression for the zero-temperature renormalized order parameter

\[
m_z(T = 0) = 2z_S - 1,
\]

\[
z_S = \left[ 1 + \frac{2\pi l^2}{A} \sum_{\vec{p}} \frac{(E_{sw}(\vec{p}) + \Delta_{SAS})^2}{2E_{sw}(\vec{p})} \frac{\epsilon(\vec{p}) - E_{sw}(\vec{p})}{(\omega^* - E_{sw}(\vec{p}) - E^0_{AS})^2} \right]^{-1}. \tag{4.15}
\]

Eq. (4.13) is the second principal result of this work. Figure 4 shows contours of renormalized pseudospin polarization in the parameter space \((d, \Delta_0)\). In the mean-field approximation, the pseudospin polarization is not susceptible to the softening of the collective mode at a finite wavevector as \( d \to d_{cr} \). When we include the effect of collective modes, the order parameter is strongly suppressed as the layer separation approaches the critical layer separation. We plot the typical dependence of renormalized polarization \( m_z \) on interlayer spacing \( d \) in Fig. 5. We interpret the strong suppression of the order parameter because of the quantum fluctuations as a precursor of the phase-transition to a compressible state.

V. ADIABATIC APPROXIMATION

In the preceding sections, we presented the generalized random phase approximation for the quasiparticle self-energy using a functional integral approach and discussed its equivalent approximation in diagrammatic perturbation theory. In this section, we present an adiabatic approximation for the fluctuation self-energy. This approximation is intuitively appealing and is applicable to a large class of problems, including metallic ferromagnets, where the effects of dispersive band structure are important. We will show that for bilayer systems, the fluctuation self-energy calculated using the adiabatic approximation is identical to the GRPA self-energy.

The starting point for this approximation is the Hartree-Fock mean-field-theory Hamiltonian where we assume that the ground state is pseudospin polarized. The mean-field Hamiltonian couples the pseudospin polarization \( m_z \) to the self-consistent field \( \phi^0 \) generated by other electrons. In the adiabatic approximation, we treat this field \( \phi \) as a dynamical variable and consider small fluctuations of this field from its self-consistent value \( \phi^0 \). We expand the Hamiltonian, considered as a function of quasiparticles and the field, around the self-consistent value of the field.

The leading-order term in this expansion gives an effective interaction between the mean-field quasiparticles and the collective modes around that mean-field. We approximate the fluctuation self-energy by a single collective-mode exchange diagram similar to the approximation commonly used to treat phonon-exchange in metals.

It is particularly easy to evaluate this self-energy for a bilayer quantum Hall system because the Landau level bands have no dispersion and because particle-hole pair-momentum is a good quantum number. We start with the Hartree-Fock Hamiltonian in the pseudospin polarized state

\[
\hat{H}_{HF} = -\frac{\Delta_F}{2} \sum_{k,\sigma} \sigma c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,\sigma k'\sigma'} \langle k'\sigma' | U_{HF} | k\sigma \rangle c_{k'\sigma'}^\dagger c_{k\sigma}, \tag{5.1}
\]

where the mean-field potential \( \hat{U}_{HF}(\vec{q}) \) is given by
\[ \hat{U}_{HF}(q) = \left[ \tau^0 \cdot \left( v_{\vec{q}} - \frac{\Gamma^q_A}{2} \right) \langle m_{-\vec{q}}^0 \rangle + \tau^x \cdot \left( v_{\vec{q}_x} - \frac{\Gamma^q_A}{2} \right) \langle m_{-\vec{q}}^z \rangle - \tau^y \cdot \frac{\Gamma^q_E}{2} \langle m_{-\vec{q}}^y \rangle - \tau^z \cdot \frac{\Gamma^q_E}{2} \langle m_{-\vec{q}}^z \rangle \right]. \]  

(5.2)

Here \( \tau^\mu \) are the Pauli matrices acting in the pseudospin-space, \( v_{\vec{q}}^\mu \) and \( \Gamma^q_A \) are the direct and exchange interactions, and

\[ m_{q}^\mu = \sum_{k',k} (\langle k'|e^{-i\vec{q}\cdot\vec{r}}|k \rangle) c_{k',\sigma}^\dagger \tau^\mu_{\sigma'\sigma} c_{k\sigma} \]

(5.3)

are the charge and pseudospin densities. To obtain the dynamics of the pseudospin waves and their effective interaction with the mean-field quasiparticles, we linearize Eq. (5.2) around the pseudospin polarized state given by \( \langle m_{\vec{q}}^z \rangle = A \delta_{\vec{q},0}/2\pi l^2 \). The transverse fluctuations \( m_{q}^y \) correspond to charge-imbalance modulations with wavevector \( \vec{q} \) whereas the fluctuations in \( m_{q}^x \) represent the variation of relative phase between states localized in the top and the bottom layer. These fluctuations are canonically conjugate, \( [m^x, m^y] = 2im^z \approx 2iA/2\pi l^2 \), and therefore we quantize them using bosonic creation and annihilation operators

\[ m_{\vec{q}}^x = \sqrt{\frac{A}{2\pi l^2}} (a_{\vec{q}} + a_{\vec{q}}^\dagger) , \quad m_{\vec{q}}^y = i\sqrt{\frac{A}{2\pi l^2}} (a_{\vec{q}} - a_{\vec{q}}^\dagger) . \]

(5.4)

The Hamiltonian for the bosonic modes introduced in (5.4) is obtained from the Hartree-Fock energy functional

\[ E_{HF}[\vec{m}] = \frac{\Delta_t}{2} m_{\vec{q}=0}^2 + \frac{2\pi l^2}{A} \sum_{\vec{p}} \left[ \left( \frac{\Gamma^q_E}{2} \right) m_{\vec{p}}^y m_{-\vec{p}}^y - \frac{\Gamma^q_E}{2} \left( m_{\vec{p}}^y m_{-\vec{p}}^y + m_{\vec{p}}^z m_{-\vec{p}}^z \right) \right]. \]

(5.5)

We expand energy functional (5.5) around the mean-field state \( \langle m_{\vec{q}}^z \rangle = A \delta_{\vec{q},0}/2\pi l^2 \) and quantize the fluctuations using Eq. (5.4). A simple calculation gives the following Hamiltonian for the pseudospin-waves

\[ \hat{H}^{sw} = \sum_{\vec{p}} \left[ \epsilon_{\vec{p}} a_{\vec{p}}^\dagger a_{\vec{p}} + \frac{\lambda_{\vec{p}}}{2} \left( a_{\vec{p}}^\dagger a_{-\vec{p}} + a_{\vec{p}} a_{-\vec{p}} \right) \right], \]

(5.6)

where \( \epsilon(\vec{p}) = [\Delta_{SAS} + v_x - \Gamma_0] = [a(\vec{p}) + b(\vec{p})]/2 \) and \( \lambda(\vec{p}) = [v_x - \Gamma_x] = [a(\vec{p}) - b(\vec{p})]/2 \). The anisotropy of Coulomb interaction in the pseudospin-space is reflected in the fact that the number operator \( \hat{n}_{\vec{p}} = a_{\vec{p}}^\dagger a_{\vec{p}} \) does not commute with the Hamiltonian (5.6). The Hamiltonian is diagonalized by Bogoliubov transformation and we get non-interacting pseudospin-waves with the dispersion \( E_{sw}(\vec{q}) = \epsilon^2(\vec{q}) - \lambda^2(\vec{q}) = a(\vec{q}) \cdot b(\vec{q}) \) obtained in preceding sections. The effective interaction between pseudospin waves and the mean-field quasiparticles is obtained by expanding the Hartree-Fock Hamiltonian (5.4) in terms of pseudospin-wave operators. At leading order we get

\[ \hat{H}^{e-sw}_{k'k} = \sum_{\vec{p}} \frac{1}{2\pi l^2 A} \left( c_{k'k}^{\dagger} c_{kAS} \cdot a_{\vec{p}}^\dagger - c_{k'k}^{\dagger} c_{kAS} c_{kS} \cdot a_{\vec{p}} \right) M_{k'k}(\vec{p}) \]

\[ - \sum_{\vec{p}} \frac{1}{2\pi l^2 A} \left( c_{k'k}^{\dagger} c_{kAS} \cdot a_{\vec{p}}^\dagger - c_{k'k}^{\dagger} c_{kAS} c_{kS} \cdot a_{\vec{p}} \right) N_{k'k}(\vec{p}), \]

(5.7)

where the interaction matrix elements are given by

\[ M_{k'k}(\vec{q}) = i(2\pi l^2) \langle k|e^{-i\vec{q}\cdot\vec{r}}|k' \rangle [v_x(\vec{q}) - \Gamma_0(\vec{q})] , \]

(5.8)

\[ N_{k'k}(\vec{q}) = i(2\pi l^2) \langle k|e^{-i\vec{q}\cdot\vec{r}}|k' \rangle [v_x(\vec{q}) - \Gamma_x(\vec{q})] . \]

(5.9)

A straightforward albeit lengthy calculation that follows line similar to the earlier work gives fluctuation self-energy expression (4.9) identical to the one obtained using the functional integral approach or the diagrammatic approximation.

VI. SUMMARY AND DISCUSSION

This paper discusses an approximation for the self-energy of interacting electrons that goes beyond the venerable random phase approximation (RPA). Depending on context the RPA describes in an approximate way the interactions
of electrons with fluctuations in either direct or exchange particle-hole channels. There are many circumstances, however, where both channels must be treated on an equal footing if the qualitative physics is to be captured. The objective of the approximation scheme discussed here is to fill this need. We have developed our approximation in the language of a path-integral approach developed by Kerman et al., that, unlike standard auxiliary-field path integral approaches, enables expansions to be made around Hartree-Fock mean-field states in which the quasiparticles experience both direct and exchange fluctuating potentials. The same approximation for the self-energy can be derived by differentiating the approximation for the grand potential that follows from this generalized RPA scheme, with respect to the mean-field Green’s function. We have identified the infinite subset of many-body perturbation theory Feynman diagrams that are included in this self-energy approximation. As far as we are aware, this self-energy approximation has not been considered previously.

We have applied our formalism to the ground state of bilayer quantum Hall systems at filling factor $\nu = 1$. This ground state has a broken symmetry in which phase-coherence is spontaneously established between the two layers in order to take advantage of interlayer exchange and improve interlayer correlations. Its collective excitations involve both fluctuations in the interlayer phase and fluctuations in the difference between charge densities in each layer. In order to describe the energetics of these fluctuations, it is necessary to include both direct and exchange particle-hole interactions. Bilayer quantum Hall systems therefore provide an example of a situation where the standard RPA will fail quite badly. Our approximate self-energy properly describes the interactions between quasiparticles in this system and the collective excitations of its order parameter field. Bilayer quantum Hall systems provide an interesting example for our approach because the importance of fluctuations for the ground state properties can be tuned between zero and large values by varying the layer separation. It is now well established by both theory and experiment that bilayer quantum Hall systems have a quantum phase transition at a finite value of $d$ where the phase-coherence is lost. Our approximate self-energy calculation correctly obtains an order parameter that vanishes beyond a critical value of $d$, however, we do not expect it to be reliable very close to the transition point, whether it is first order or continuous. The bilayer quantum Hall case is also favorable because particle-hole excitations of the ground state have momentum as a good quantum number and there is only one excitation at each momentum. In effect, there is only one excited state at each momentum, the collective excitation. Much as in the case of a one-dimensional electron system, there is no particle-hole continuum in the excitation spectrum. This greatly simplifies the calculation and allows us to derive a remarkably simple analytic expression for the self-energy.

We have confirmed the physical content of our theory by demonstrating its equivalence, for the case of bilayer quantum Hall systems, to an adiabatic approximation in which the Hartree-Fock single-particle Hamiltonian is expressed in terms of the order-parameter field, via the density matrix. The possibility of specifying the density matrix that appears in the Hartree-Fock single-particle Hamiltonian in terms of the order parameter field is unique to quantum Hall systems. Therefore this simple relationship cannot always be established. Nevertheless, this example demonstrates the ability of the approximation we discuss to accurately capture the effect of important fluctuations on quasiparticle properties.

We believe that our approximate self-energy will be useful for other situations as well. A potentially interesting example is provided by the case of superconductors for which coupled fluctuations in the phase of the order parameter and of the charge density form elementary excitations and both pairing and electrostatic energies are important. In the case of superconductors it is particle-particle channel fluctuations are important for phase-fluctuations, unlike the case of bilayer quantum Hall systems. The problems can be made quite similar, however, by performing a particle-hole transformation for, say, the down-spins of the superconductor, turning the superconducting order into easy-plane ferromagnetism. In fact, going in the opposite direction, the order of a bilayer quantum Hall ferromagnet can be regarded as an electron-hole pair condensate by making a particle-hole transformation in one of the layers. In the case of a superconductor it is well-known that the long-range of the electron-electron interaction turns the collective modes into gapped plasmons, usually leading to high accuracy for the mean-field treatment of a superconductor. This can change, however, if the collective excitations become soft at large wavevectors as they do in bilayer quantum Hall systems. Such softness could be associated, for example, with partial Fermi surface nesting.

In this work, we have concentrated on quantum fluctuation effects at zero temperature. It is easy to obtain corresponding expressions for finite temperature self-energy, though they are somewhat cumbersome. In the limit of vanishing layer separation, $d \to 0$, the finite-temperature expressions reproduce, with the appropriate identification of pseudospin with spin, the spin-wave contribution to the electron self-energy in a single-layer system. We have neglected the form-factors which encode the effects of finite well-widths since their inclusion does not change the results qualitatively. They can be incorporated without difficulty when modeling a specific system.
VII. ACKNOWLEDGEMENTS

This work was supported in part by the Robert A. Welch Foundation, by the Indiana 21st Century Fund, and by the NSF under grant DMR0115947.

\[ \sum (i\omega) \]

\[ 1\Omega \]

\[ 1\omega - i\Omega \]

FIG. 1. Schematic representation of self-energy approximation (2.14). The directed solid line represents the mean-field Green’s function \( G^{0}(i\omega - i\Omega) \) and the wavy line represents the collective-mode propagator \( M^{-1}(i\Omega) \). The matrix elements at the vertex do no depend upon the energy transfer \( \Omega \).

\[ \sum (i\omega) = \]

\[ 1\Omega \]

\[ 1\omega - i\Omega \]

FIG. 2. Diagrammatic summary of the GRPA for self-energy. We use the antisymmetrized interaction as the bare scattering vertex, \( \Gamma_{0}^{(4)} = V^{A} \), to take into account fluctuations in the direct and the exchange channels. The self-energy is obtained from the scattering vertex \( \Gamma^{4} \) by contracting the incoming and outgoing labels on the top.
FIG. 3. Diagrammatic content of the GRPA self-energy (2.14). The first set of diagrams (a) has been traditionally used to include the effect of screening whereas the second set (b) has been used as the RPA self-energy in itinerant ferromagnets to include the effect of spin-waves. Eq.(2.14) includes another class of diagrams, shown in (c), which captures the contribution of competing Hartree and exchange fluctuations. We refer to the sum of diagrams in all three sets as the GRPA self-energy.
FIG. 4. Typical collective-mode dispersion for a pseudospin polarized state. For a finite interlayer tunneling, $\Delta_t \neq 0$, the collective mode is gapped since the $U(1)$ symmetry in the $y-z$ plane is explicitly broken. The minimum in the pseudospin-wave energy near $q_l \approx 1$ is because of competing Hartree and exchange fluctuations present in the term $a(q)$.

FIG. 5. Continuum part of the symmetric-state spectral function: The zero of energy is at the chemical potential. As the interlayer spacing increases, the spectral weight at positive frequencies increases and the spectral weight $z_S$ in the $\delta$-function peak at the symmetric-state quasiparticle pole decreases. This presence of spectral weight at positive energies has been detected by Manfra et al.\textsuperscript{26}
FIG. 6. Renormalized polarization contours. The rapid suppression of the order parameter only close to the phase-boundary indicates that interactions between the collective modes can be ignored for most part of the parameter space in the phase-coherent regime.

FIG. 7. Typical renormalized polarization at $T = 0$. The mean-field polarization is independent of the changes in the nature of collective mode spectrum which occur as $d$ approaches the critical layer separation, $d_{cr}/l = 2.15$ for $\Delta t = .15(e^2/\ell_l)$. The renormalized polarization vanishes rapidly close to the phase-boundary because of increasing quantum fluctuations.
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