Underdetermined 2D-DOD and 2D-DOA Estimation for Bistatic Coprime EMVS-MIMO Radar: From the Difference Coarray Perspective

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Abstract

In this paper, the underdetermined 2D-DOD and 2D-DOA estimation for bistatic coprime EMVS-MIMO radar is considered. Firstly, a 5-D tensor model was constructed by using the multi-dimensional space-time characteristics of the received data. Then, an 8-D tensor has been obtained by using the auto-correlation calculation. To obtain the difference coarrays of transmit and receive EMVS, the decoupling process between the spatial response of EMVS and the steering vector is inevitable. Thus, a new 6-D tensor can be constructed via the tensor permutation and the generalized tensorization of the canonical polyadic decomposition. According to the theory of the Tensor-Matrix Product operation, the duplicated elements in the difference coarrays can be removed by the utilization of two designed selection matrices. Due to the centrosymmetric geometry of the difference coarrays, two DFT beamspace matrices were subsequently designed to convert the complex steering matrices into the real-valued ones, whose advantage is to improve the estimation accuracy of the 2D-DODs and 2D-DOAs. Afterwards, a third-order tensor with the third-way fixed at 36 was constructed and the Parallel Factor algorithm was deployed, which can yield the closed-form automatically paired 2D-DOD and 2D-DOA estimation. The
simulation results show that the proposed algorithm can exhibit superior estimation performance for the underdetermined 2D-DOD and 2D-DOA estimation.

**Index Terms**

Bistatic coprime EMVS-MIMO radar, 2D-DOD and 2D-DOA estimation, difference coarrays, high-order tensor model, PARAFAC algorithm

**I. INTRODUCTION**

Studies about the electromagnetic vector sensors (EMVS) have attracted extensive attention due to their excellent measurement capabilities of angle parameters and polarization parameters [1]-[5]. Compared with the scalar array, EMVS with three orthogonal electric dipoles and three orthogonal magnetic dipoles can obtain the electromagnetic field vector information of the targets. Recently, in order to measure the polarization information of targets in bistatic MIMO radars, the EMVS has been used as the transmit and receive array in the bistatic MIMO radar system.

For the achievement of multi-dimensional parameter estimation in bistatic EMVS-MIMO radar, the ESPRIT algorithm was firstly proposed in [6]. But, this algorithm has high computational complexity due to the singular value decomposition on the covariance matrix of the array received data. In [7], a computationally friendly PM estimator was proposed to construct the virtual direction matrix for bistatic EMVS-MIMO radar. In [8], a tensor subspace based algorithm has been developed to improve the estimated performance by exploiting the inherent multi-dimensional structure of the received data. Additional optimization process in [6]-[8] is inevitable because of the pair matching between 2D-DOD and 2D-DOA. Thus, an improved PM algorithm in [9] has been raised to realize the automatic pair of 2D-DOD and 2D-DOA, and this method can further decrease the computational complexity. The disadvantage of the PM algorithms in [7] and [9] is the relatively poor estimation performance. In [10], the PARAFAC algorithm was utilized to realize the automatically paired 2D-DOD and 2D-DOA estimation. In [11], the bistatic coprime EMVS-MIMO radar was designed to enhance the 2D-DOD and 2D-DOA estimation performance.

It is apparent that the above-mentioned algorithms do not utilize the inherent difference coarray of transmit and receive EMVS, when conducting multi-dimensional parameter estimation in bistatic EMVS MIMO radar. According to [12]-[17], the difference coarray of the original array can offer larger array apertures, which means that the underdetermined DOA estimation is obtain-
able. For bistatic EMVS-MIMO radar, the challenge for constructing the difference coarray of the transmit and receive EMVS is the coupling in the spatial response alongside the steering vector. In this paper, the de-coupling process has been completed with the help of tensor operation. To estimate more targets than elements, the difference coarray of transmit and receive EMVS has been attained by the high order tensor permutation along with the generalized tensoriation of the canonical polyadic decomposition. In addition, two designed selection matrices have been used to remove the repeated elements in the difference coarray via the Tensor-Matrix product rule. The geometry of the difference coarray for both transmit and receive EMVS exhibit centrosymmetry. As discussed in [22]-[26], the DFT beamspace matrix can be exploited to convert the complex array manifold matrix with centro-symmetric structure into the real-value one. This process can enhance the 2D-DOD and 2D-DOA estimation performance in the bistatic EMVS-MIMO radar. Besides, a third-order tensor has been constructed with the dimension of the third-way fixed at 36. Finally, The automatically paired transmit/receive elevation angle, transmit/receive azimuth angle, transmit/receive polarization angle and transmit/receive polarization phase difference can be accurately derived from the estimated transmit and receive factor matrices acquired by the PARAFAC algorithm. Simulation results have verified its effectiveness.

The paper is organized as follows. Section II describes the nomenclature employed in this publication. Section III formulates the signal model of the bistatic coprime EMVS-MIMO radar. Section IV reports the detailed derivation process of the proposed algorithm. Section V presents simulations to verify the effectiveness of the proposed algorithm. Section VI provides a summary.

II. NOMENCLATURE

- $A^*$: The complex conjugate of $A$
- $A^T$: The transpose of $A$
- $A^H$: The Hermitian transpose of $A$
- $A^{-1}$: The inverse of $A$
- $A^\dagger$: The pseudo-inverse of $A$
- $a \otimes b$: The Kronecker product of $a$ and $b$
- $A \odot B$: The Khatri-Rao product of $A$ and $B$
- $A \oplus B$: The Hadamard product of $A$ and $B$
- $a \circ b$: The vector outer product of $a$ and $b$
- $a \times b$: The vector-cross-product of $a$ and $b$
• $(\Pi)_{A}^{\perp}$: The projection matrix of the matrix $A$
• $\sin(\theta)$: The sine of $\theta$
• $\cos(\theta)$: The cosine of $\theta$
• $\tan(\theta)$: The tangent of $\theta$
• $\arcsin(\theta)$: The inverse sine of $\theta$
• $\arccos(\theta)$: The inverse cosine of $\theta$
• $\arctan(\theta)$: The inverse tangent of $\theta$
• $\angle(a)$: The phase angles of $a$
• $\text{real}(A)$: The real part of $A$
• $\min[a,b]$: smaller value taken from $a$ or $b$.
• $D_i(A)$: The diagonal matrix with the $i$-th column elements from $A$.
• $I_N$: The $N \times N$ identity matrix
• $0_N$: The $N \times N$ matrix of zeros.
• $1_N$: The $N \times N$ matrix of ones.
• $\kappa_A$: The $\kappa$-rank of $A$.

Some Definitions of Coprime Array in [12]-[13]

Definition 1 ($D$, Difference Coarray). For a sparse coprime array specified by an integer set $S$, its difference coarray $D$ is defined as

$$D = \{n_1 - n_2 | n_1, n_2 \in S\}$$  (1)

Definition 2 ($U$, maximum central contiguous ULA). Let $S$ denote a coprime array and $D$ be its difference coarray. The maximum central contiguous ULA segment in $D$ is

$$U = \{m, -m, \ldots, -1, 0, 1, \ldots, |m|\}$$  (2)

Definition 3 ($W$, Weight Functions). The weight function $w(m)$ of an array $S$ is defined as the number of sensor pairs that lead to coarray index $m$, i.e., $|\{(n_1, n_2 \in S) | n_1 - n_2 = m\}|$

Some Definitions of Tensor in [18]-[21]

Definition 4 (Mode-$n$ Tensor-Matrix Product). The mode-$n$ product of an $N$-order tensor
set $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$ and a matrix $A \in \mathbb{C}^{J_n \times I_n}$ is defined as

$$\mathcal{Y} = \mathcal{X} \times_n A \in \mathbb{C}^{I_1 \times \cdots \times I_{n-1} \times J_n \times I_{n+1} \times \cdots \times I_N}$$

(3)

**Definition 5** (Outer product of two tensors). The outer product of two tensors $A \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$ and $B \in \mathbb{C}^{J_1 \times J_2 \times \cdots \times J_M}$ is defined as

$$\mathcal{Y} = A \circ B \in \mathbb{C}^{I_1 \times \cdots \times I_N \times J_1 \times J_2 \times \cdots \times J_M}$$

(4)

whose elements are $y_{i_1,i_2,\ldots,i_N,j_1,j_2,\ldots,j_M} = a_{i_1,i_2,\ldots,i_N} \cdot b_{j_1,j_2,\ldots,j_M}$

**Definition 6** (PARAFAC decomposition). For an $N$th-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$, it can be expressed as a linear combination of rank-1 tensors as follows:

$$\mathcal{X} = \sum_{k=1}^{K} a_{k1} \circ a_{k2} \circ \cdots \circ a_{kN}$$

(5)

where $a_{kn} \in \mathbb{C}^{I_n \times 1}$, $k = 1, 2, \ldots, K$ denotes the rank-1 tensor.

**Definition 7** (Generalized Tensorization of a PARAFAC model). For an $N$th-order PARAFAC model $A = \sum_{k=1}^{K} a_{1k} \circ a_{2k} \circ \cdots \circ a_{Nk} \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$ with $a_{nk} \in \mathbb{C}^{I_n}$ ($n = 1, \ldots, N$), let the ordered sets $A_j = [r_{j,1}, r_{j,2}, \ldots, r_{j,L_j}]$ for $j = 1, 2, \ldots, J$ be a partitioning of the dimensions $A = [1, 2, \ldots, N]$, the generalized tensorization of $A$ is defined as

$$A_{h_1,h_2,\ldots,h_J} = \sum_{k=1}^{K} b_{1k} \circ b_{2k} \circ \cdots \circ b_{Jk}$$

$$\in \mathbb{C}^{I_{R_1} \times I_{R_2} \times \cdots \times I_{R_J}}$$

(6)

where $I_{R_j} = \prod_{l=1}^{L_j} I_{r_{j,l}}$ and $b_{jk} = a_{r_{j,L_j}} \otimes a_{r_{j,L_j-1}} \otimes \cdots \otimes a_{r_{j,1}}$.

### III. DATA MODEL

A bistatic coprime EMVS-MIMO radar system with $M$ transmit EMVS and $N$ receive EMVS is demonstrated in Fig. 1. Let the array positions of transmit EMVS and receive EMVS are $L_t$ and $L_r$, respectively

$$L_t = \{M_1 m_2 | 0 \leq m_2 \leq M_2 - 1\} \cup \{M_2 m_1 | 0 \leq m_1 \leq 2M_1 - 1\}$$

(7)
Fig. 1. Bistatic coprime EMVS-MIMO radar system.

$$L_r = \{N_1 n_2 | 0 \leq n_2 \leq N_2 - 1 \} \cup \{N_2 n_1 | 0 \leq n_1 \leq 2N_1 - 1 \} \quad (8)$$

Assuming the number of targets is $K$, the transmit steering vector and receive steering vector of the bistatic coprime EMVS-MIMO radar can be expressed as

$$c_{tk} = a_{tk} \otimes q_{tk} (\theta_{tk}, \phi_{tk}, \gamma_{tk}, \eta_{tk})$$

$$c_{rk} = a_{rk} \otimes q_{rk} (\theta_{rk}, \phi_{rk}, \gamma_{rk}, \eta_{rk})$$

where $\theta_{tk}, \theta_{rk} \in [0, \pi)$, $\phi_{tk}, \phi_{rk} \in [0, 2\pi)$, $\gamma_{tk}, \gamma_{rk} \in [0, \pi/2)$ and $\eta_{tk}, \eta_{rk} \in [-\pi, \pi)$ denote the transmit/receive elevation angle, transmit/receive azimuth angle, transmit/receive polarization angle and transmit/receive polarization phase difference, respectively. $q_{tk} (\theta_{tk}, \phi_{tk}, \gamma_{tk}, \eta_{tk})$ and $q_{rk} (\theta_{rk}, \phi_{rk}, \gamma_{rk}, \eta_{rk})$ represent the spatial response of the transmit EMVS and the receive EMVS of the $k$-th target, respectively. $a_{tk} = [e^{-j2\pi d_{t0} \sin(\theta_{tk})}, \ldots, e^{-j2\pi d_{tM} \sin(\theta_{tk})}]^T$, $a_{rk} = [e^{-j2\pi d_{r0} \sin(\theta_{rk})}, \ldots, e^{-j2\pi d_{rN} \sin(\theta_{rk})}]^T$. For an EMVS, the spatial response $q$ can be expressed as

$$q(\theta, \phi, \gamma, \eta) =$$

$$\begin{bmatrix}
\cos(\phi) \cos(\theta) & -\sin(\phi) \\
\sin(\phi) \cos(\theta) & \cos(\phi) \\
-\sin(\theta) & 0 \\
-\sin(\phi) & -\cos(\phi) \cos(\theta) \\
\cos(\phi) & -\sin(\phi) \cos(\theta) \\
0 & \sin(\theta)
\end{bmatrix} \begin{bmatrix}
\sin(\gamma) e^{j\eta} \\
\cos(\gamma) \\
\end{bmatrix} = F(\theta, \phi) g(\gamma, \eta) \quad (11)$$
where $F(\theta, \phi) \in \mathbb{C}^{6 \times 2}$ denotes the spatial angular location matrix, $g(\gamma, \eta) \in \mathbb{C}^{2 \times 1}$ is representative of the polarization states vector.

According to [1], the elevation angle $\theta$ and azimuth angle $\phi$ can be extracted from the normalized Poynting vector of $q(\theta, \phi, \gamma, \eta)$$$egin{bmatrix} u \\ v \\ w \end{bmatrix} \triangleq \frac{q[1,2,3]}{\|q[1,2,3]\|} \times \frac{q^*[4,5,6]}{\|q[4,5,6]\|} = \begin{bmatrix} \sin(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) \\ \cos(\theta) \end{bmatrix}$$ (12)

By using the orthogonality of the bistatic coprime EMVS-MIMO radar transmit and receive waveforms, the signal model after matched filtering at the receive front can be denoted as

$$y(t) = ((A_t \odot Q_t) \odot (A_r \odot Q_r)) s(t) + n(t)$$ (13)

where $A_t = [a_{t1}, a_{t2}, \ldots, a_{tK}]$, $A_r = [a_{r1}, a_{r2}, \ldots, a_{rK}]$, $Q_t = [q_{t1}, q_{t2}, \ldots, q_{tK}]$ and $Q_r = [q_{r1}, q_{r2}, \ldots, q_{rK}]$ denote the transmit steering matrix, the receive steering matrix, the transmit spatial response matrix and the receive spatial response matrix, respectively. For the total $L$ snapshots, the sample data can be further derived as

$$Y = ((A_t \odot Q_t) \odot (A_r \odot Q_r)) S + N$$ (14)

For the bistatic coprime EMVS-MIMO radar with $M$-element transmit EMVS and $N$-element receive EMVS, the matrix dimension of $(C_t \odot C_r)$ is $\mathbb{C}^{36MN \times K}$. In order to achieve the underdetermined 2D-DOD and 2D-DOA, the difference coarray of the transmit EMVS and receive EMVS will be deployed to construct a new third-order tensor model. Moreover, from (14), it can be seen that the received data $Y$ satisfies the space-time multi-dimensional tensor structure.

Here, some assumptions are made in order to effectively estimate the transmit 4-D parameter $(\theta_t, \phi_t, \gamma_t, \eta_t)$ and receive 4-D parameter $(\theta_r, \phi_r, \gamma_r, \eta_r)$,

- The value of $(\theta_t, \phi_t, \gamma_t, \eta_t)$ and $(\theta_r, \phi_r, \gamma_r, \eta_r)$ should satisfy $\theta_t, \theta_r \in [0, \pi/2)$, $\phi_t, \phi_r \in [0, \pi/2)$, $\gamma_t, \gamma_r \in [0, \pi/2)$, $\eta_t, \eta_r \in [0, \pi/2)$.

- The signals are stationary, non-Gaussian random process and statistically independent of each other.

- The number of signals $K$ is known or corrected estimated by the existing algorithm.

- The $N$ is zero-mean, complex Gaussian white noise.

- The noise is statistically independent of the signals.
IV. THE PROPOSED DIFFERENCE COARRAY STRATEGY

A. Construction of the Difference Coarray

For the estimation of more targets than elements, the difference coarray of transmit and receive EMVS are constructed. From (9) and (10), it can be found that \( \mathbf{a}_{tk} \) and \( \mathbf{q}_{tk} \), \( \mathbf{a}_{rk} \) and \( \mathbf{q}_{rk} \) are copuling, respectively. Therefore, in order to obtain the difference coarrays, the de-coupling is invetiable.

To reserve the multiple space-time structure in \( \mathbf{Y} \), a 5-D tensor \( \mathcal{Y} \) can be constructed as

\[
\mathcal{Y} = \sum_{k=1}^{K} \mathbf{a}_{tk} \odot \mathbf{q}_{tk} \odot \mathbf{a}_{rk} \odot \mathbf{q}_{rk} \odot \mathbf{s}_k + \mathcal{N}_l \tag{15}
\]

Then, a 8-D tensor \( \mathcal{R} \) can be obtained via the following correlation operation between \( \mathcal{Y} \) and \( \mathcal{Y}^* \)

\[
\mathcal{R} = E[\mathcal{Y} \odot \mathcal{Y}^*] = \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}_{tk}^\circ \mathbf{a}_{rk}^* \odot \mathbf{q}_{tk} \odot \mathbf{q}_{rk}^* \odot \mathbf{a}_{rk} \odot \mathbf{q}_{rk}^* + \mathcal{R}_N \tag{16}
\]

where \( \sigma_k^2, k = 1, 2, \ldots, K \) denotes the power of the \( k \)-th source. And, according to Definition 1, the corresponding difference coarray of transmit EMVS and receive EMVS can be obtained by combining the factor vectors \( \mathbf{a}_{tk} \) and \( \mathbf{a}_{tk}^* \), \( \mathbf{a}_{rk} \) and \( \mathbf{a}_{rk}^* \), respectively. Thus, the factor vectors can be rearranged as \( [1, 2, 3, 4, 5, 6, 7, 8] \mapsto [1, 5, 2, 3, 7, 4, 6, 8] \) based on the tensor permutation rule, and a new 8-D \( \mathcal{R}_1 \) can be expressed as

\[
\mathcal{R}_1 = \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}_{tk}^\circ \mathbf{a}_{rk}^* \odot \mathbf{q}_{tk} \odot \mathbf{q}_{rk}^* \odot \mathbf{a}_{rk} \odot \mathbf{q}_{rk}^* + \mathcal{R}_{N1} \tag{17}
\]

Then, the indeces 1 and 2, 4 and 5, 7 and 8 need to be combined by the use of the generalized tensorization of the canonical polyadic decomposition in Definition 7. As a result, a new 5-D tensor \( \mathcal{R}_2 = \mathcal{R}_{1[1,2][4,5][7,8]} \) can be achieved as

\[
\mathcal{R}_2 = \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}_{tk}^\circ \mathbf{q}_{tk} \odot \mathbf{a}_{rk} \odot \mathbf{q}_{rk} + \mathcal{R}_{N2} \tag{18}
\]

where \( \mathbf{a}_{tk} = \mathbf{a}_{tk} \otimes \mathbf{a}_{tk}^*, \mathbf{a}_{rk} = \mathbf{a}_{rk} \otimes \mathbf{a}_{rk}^* \), \( \mathbf{q}_k = \mathbf{q}_k \otimes \mathbf{q}_k^* \).

According to Definition 1, for the \( M \) transmit coprime EMVS with the number of the first subarray is \( M1 \) and the second subarray is \( M2 \), the \( N \) transmit coprime EMVS with the number
of the first subarray is \( N_1 \) and the second subarray is \( N_2 \), the number of continue virtual array elements in the corresponding difference coarrays can be deonted as

\[
\tilde{M} = 2M_1M_2 + 2M_1 + 1
\]

\[
\tilde{N} = 2N_1N_2 + 2N_1 + 1
\]

However, the difference coarrays of the coprime transmit array and the coprime receive array contain many duplicated elements. To attain the maximum central contiguous ULA in the difference coarrays, it is necessary to remove the repeated elements, which can be realized by the application of tensor matrix product rules. Two selection matrices \( J_1 \) and \( J_2 \) are designed on the basis of Algorithm 1. So, a new 6-D tensor can be expressd as

\[
\mathcal{R}_3 = \mathcal{R}_2 \times_1 J_1 \times_3 J_2 
\]

\[
= \sum_{k=1}^{K} \sigma_k^2 \tilde{a}_{tk} \odot q_{tk} \odot \tilde{a}_{rk} \odot q_{rk} \odot q_k + \mathcal{R}_{N2}
\]

where

\[
\tilde{a}_{tk} = \left[ e^{-j\pi \frac{\tilde{M}-1}{2} \sin(\theta_{tk})} \ldots e^{j\pi \frac{\tilde{M}-1}{2} \sin(\theta_{tk})} \right]^T
\]

\[
\tilde{a}_{rk} = \left[ e^{-j\pi \frac{\tilde{N}-1}{2} \sin(\theta_{tr})} \ldots e^{j\pi \frac{\tilde{N}-1}{2} \sin(\theta_{rk})} \right]^T
\]

It can be found that both the transmit and the receive difference coarray can be achievable via the above-mentioned above process. Compared with (15), it is obvious that the proposed process has significantly improved the degree of freedoms.

\section*{B. Formulation of the new third tensor}

It can be seen that \( \tilde{a}_{tk} \) and \( \tilde{a}_{rk} \) in (22) and (23) satisfy the centro-symmetric geometry, so, the DFT beamspace matrices are used to transform the complex value of \( \tilde{a}_{tk} \) and \( \tilde{a}_{rk} \) into the corresponding real-value ones. This operation can improve the estimation performance of the angle parameters and help to reconstruct the spatial response matrices \( Q_t \) and \( Q_r \) easily. Firstly, two DFT beamspace matrices are designed as

\[
W_t = \begin{bmatrix}
    w_{t1}^H \\
    w_{t2}^H \\
    \vdots \\
    w_{tM}^H
\end{bmatrix}
\]
Algorithm 1 pseudocode for coarray selection matrix

**Input:** sparse array $S$, difference coarray $D$, weight function $W$

**Output:** coarray selection matrix $J$

1: construct rectangular grid $S_1$ and $S_2$ via $[S_1, S_2] = \text{ndgrid}(S)$
2: $S_3 = S_1 - S_2$
3: $S_4 = \text{vec}(S_3)$
4: $J = \text{zeros}(|D|, |S_3|)$
5: for $i = 1 : |D|$ 
6: $[\text{row, col}] = \text{find}(S_4 == D(i))$
7: $J(i, \text{row}) = 1/W(i)$
8: return $J$

\[
W_r = \begin{bmatrix}
  w^H_{r1} \\
  w^H_{r2} \\
  \vdots \\
  w^H_{rN}
\end{bmatrix}
\]  
(25)

where

\[
w_{tm} = \begin{bmatrix}
  e^{-j\pi(M-1)u_t(m)/M} \\
  \vdots \\
  e^{j\pi(M-1)u_t(m)/M}
\end{bmatrix}^T
\]

$m = 1, \cdots, \tilde{M}$  
(26)

\[
w_{tn} = \begin{bmatrix}
  e^{-j\pi(N-1)u_r(n)/N} \\
  \vdots \\
  e^{j\pi(N-1)u_r(n)/N}
\end{bmatrix}^T
\]

$n = 1, \cdots, \tilde{N}$  
(27)

where $u_t = [0, 1, \cdots, \tilde{M} - 1]$, $u_r = [0, 1, \cdots, \tilde{N} - 1]$. Then, a 5-D tensor with real-value steering matrices can be obtained by exploiting the tensor-matrix product rules

\[
\mathcal{R}_4 = \mathcal{R}_3 \times_1 W_t \times_3 W_r \\
= \sum_{k=1}^{K} \sigma_k^2 \hat{a}_{tk} \circ q_{tk} \circ \hat{a}_{rk} \circ q_{rk} \circ \hat{q}_k + \mathcal{R}_{N4}
\]  
(28)

where $\hat{A}_t = W_t \bar{A}_t = [\hat{a}_{t1}, \hat{a}_{t2}, \cdots, \hat{a}_{tK}]$ and $\hat{A}_r = W_r \bar{A}_r = [\hat{a}_{r1}, \hat{a}_{r2}, \cdots, \hat{a}_{rK}]$ denote the real-value steering vector matrices of the transmit array and the receive array, respectively, and their detailed forms are as (29) and (30).
The three-way tensor \( \mathcal{R}_5 \) can be attained via combining the indices 1 and 2, 3 and 4

\[
\mathcal{R}_5 = \mathcal{R}_{4[1,2][3,4][5]} = \sum_{k=1}^{K} \mathcal{C}_{tk} \odot \mathcal{C}_{rk} \odot \mathcal{q}_k + \mathcal{R}_{N5} \quad (31)
\]

Afterwards, a new three-way tensor \( \mathcal{R}_5 \) can be attained via combining the indices 1 and 2, 3 and 4

As shown in Fig. 2, the dimension of the newly constructed tensor \( \mathcal{R}_5 \) is \( C^{6\tilde{M} \times 6\tilde{N} \times 36} \), whose third way is fixed at 36. On its basis, the PARAFAC algorithm can be utilized to estimate the
factor matrices $\hat{C}_t$, $\hat{C}_r$, and $\hat{Q}$, respectively. According to Definition 6, the different slices of the third-order tensor $R_5$ can be further obtained as

$$R_{5[i,:,:]}^T = \hat{C}_r D_i(\hat{Q}) \hat{C}_t^T, \quad i = 1, 2, \ldots, 36$$

$$R_{5[,:,:]}^T = \hat{Q} D_j(\hat{C}_t) \hat{C}_r^T, \quad j = 1, 2, \ldots, 6\tilde{M}$$

$$R_{5[,:,:]}^T = \hat{C}_t D_k(\hat{C}_r) \hat{Q}^T, \quad k = 1, 2, \ldots, 6\tilde{N}$$

(32)

where $D_i$, $D_j$, $D_k$ represent the diagonal matrix operation, and the elements on the diagonal matrices are the $k$th-rows of the factor matrices $\hat{C}_t$, $\hat{C}_r$, and $\hat{Q}$, respectively.

In order to realize the estimation of transmit steering matrix $\hat{C}_t$, the receive steering matrix $\hat{C}_r$ and the signal matrix $\hat{Q}$, the trilinear alternating least squares algorithm is deployed as follows

$$\min_{\hat{C}_t^T} = \| [R_{5}^T]_{(1)} - (\hat{C}_r \odot \hat{Q}) \hat{C}_t^T \|^2_F$$

$$\min_{\hat{C}_r^T} = \| [R_{5}^T]_{(2)} - (\hat{Q} \odot \hat{C}_t) \hat{C}_r^T \|^2_F$$

$$\min_{\hat{Q}^T} = \| [R_{5}^T]_{(3)} - (\hat{C}_t \odot \hat{C}_r) \hat{Q}^T \|^2_F$$

(33)

Then, let $\hat{Q}$, $\hat{C}_t$ and $\hat{C}_r$ denote the estimated factor matrices, respectively

$$\hat{C}_t^T = (\hat{C}_r \odot \hat{Q})^T [R_{5}^T]_{(1)}$$

$$\hat{C}_r^T = (\hat{Q} \odot \hat{C}_t)^T [R_{5}^T]_{(2)}$$

$$\hat{Q}^T = (\hat{C}_t \odot \hat{C}_r)^T [R_{5}^T]_{(3)}$$

(34)

For the factor matrices $\hat{Q}$, $\hat{C}_t$ and $\hat{C}_r$ obtained by multiple iterations of PARAFAC-TALS, the transmit steering matrices $\hat{C}_t$ and the receive steering matrices $\hat{C}_r$ are in one-to-one correspondence. This means that the 2D-DOD and 2D-DOA in $\hat{C}_t$ and $\hat{C}_r$ are automatically paired. Therefore, the proposed algorithm need not the construction of an additional pairing optimization function compared to the algorithms in [6]-[8].

C. Estimation of the angle and polarization parameters

For the estimated $\hat{C}_t$ and $\hat{C}_r$, the corresponding transmit elevation angle and receive elevation angle can be obtained. Firstly, structural characteristics of the steering vectors $\hat{a}_{tk}$ in $\hat{A}_t$ and $\hat{a}_{rk}$
in $\hat{A}_r$ are analyzed. The $m$-th and $(m+1)$-th elements in $\hat{a}_{tk}$ and $n$-th and $(n+1)$-th elements in $\hat{a}_{rk}$ can be expressed as

$$
\begin{align*}
[\hat{a}_{tk}]_m &= \sin\left(\frac{\tilde{N}(\sin\theta_{tk} - 2m\pi M)}{2}\right) \\
[\hat{a}_{rk}]_n &= \sin\left(\frac{\tilde{N}(\sin\theta_{rk} - 2n\pi N)}{2}\right)
\end{align*}
\tag{35}
$$

It can be seen that the adjacent elements in $\hat{a}_{tk}$ and $\hat{a}_{rk}$ satisfy the following relationships

$$
\begin{align*}
tan\left(\frac{\pi\sin(\theta_{tk})}{2}\right)\{\cos\left(\frac{m\pi}{M}\right)\}[\hat{a}_{tk}]_m + \cos\left(\frac{(m+1)\pi}{M}\right)\}[\hat{a}_{tk}]_{m+1} &= \sin\left(\frac{m\pi}{M}\right)\}[\hat{a}_{tk}]_m + \sin\left(\frac{(m+1)\pi}{M}\right)\}[\hat{a}_{tk}]_{m+1} \\
	an\left(\frac{\pi\sin(\theta_{rk})}{2}\right)\{\cos\left(\frac{n\pi}{N}\right)\}[\hat{a}_{rk}]_n + \cos\left(\frac{(n+1)\pi}{N}\right)\}[\hat{a}_{rk}]_{n+1} &= \sin\left(\frac{n\pi}{N}\right)\}[\hat{a}_{rk}]_n + \sin\left(\frac{(n+1)\pi}{N}\right)\}[\hat{a}_{rk}]_{n+1} \\
\end{align*}
\tag{36}
$$

On the basis of (37) and (38), the following rotation-invariant relationship for $\hat{A}_t$ and $\hat{A}_r$ can be constructed

$$
\Gamma_{t1}\hat{A}_t\Phi(\theta_t) = \Gamma_{t2}\hat{A}_t
\tag{39}
$$

$$
\Gamma_{r1}\hat{A}_r\Phi(\theta_r) = \Gamma_{r2}\hat{A}_r
\tag{40}
$$

where

$$
\Phi(\theta_t) = \begin{bmatrix} 
\tan\left(\frac{\pi\sin(\theta_{t1})}{2}\right) \\
\vdots \\
\tan\left(\frac{\pi\sin(\theta_{tK})}{2}\right) 
\end{bmatrix}
\tag{41}
$$

$$
\Phi(\theta_r) = \begin{bmatrix} 
\tan\left(\frac{\pi\sin(\theta_{r1})}{2}\right) \\
\vdots \\
\tan\left(\frac{\pi\sin(\theta_{rK})}{2}\right) 
\end{bmatrix}
\tag{42}
$$
The detailed forms of the selection matrices $\Gamma_{t1}$, $\Gamma_{t2}$, $\Gamma_{r1}$, and $\Gamma_{r2}$ are

$$\Gamma_{t1} = \begin{bmatrix} c_{t1} & c_{t2} & 0 & \cdots & 0 \\ 0 & c_{t2} & c_{t3} & \cdots & 0 \\ 0 & 0 & c_{t3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_{t(\tilde{M}-1)} & \end{bmatrix}$$

(43)

$$\Gamma_{t2} = \begin{bmatrix} s_{t1} & s_{t2} & 0 & \cdots & 0 \\ 0 & s_{t2} & s_{t3} & \cdots & 0 \\ 0 & 0 & s_{t3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_{t(\tilde{M}-1)} & \end{bmatrix}$$

(44)

$$\Gamma_{r1} = \begin{bmatrix} c_{r1} & c_{r2} & 0 & \cdots & 0 \\ 0 & c_{r2} & c_{r3} & \cdots & 0 \\ 0 & 0 & c_{r3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_{r(\tilde{N}-1)} & \end{bmatrix}$$

(45)

$$\Gamma_{r2} = \begin{bmatrix} s_{r1} & s_{r2} & 0 & \cdots & 0 \\ 0 & s_{r2} & s_{r3} & \cdots & 0 \\ 0 & 0 & s_{r3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_{r(\tilde{N}-1)} & \end{bmatrix}$$

(46)

where $c_{tm} = \cos\left(\frac{m\pi}{\tilde{M}}\right)$, $m = 0, 1, \cdots, \tilde{M}$, $c_{rn} = \cos\left(\frac{n\pi}{\tilde{N}}\right)$, $n = 0, 1, \cdots, \tilde{N}$, $s_{tm} = \sin\left(\frac{m\pi}{\tilde{M}}\right)$, $m = 0, 1, \cdots, \tilde{M}$, $s_{rm} = \sin\left(\frac{n\pi}{\tilde{N}}\right)$, $n = 0, 1, \cdots, \tilde{N}$. Therefore, in order to estimate the transmit elevation angle $\theta_{t_k}, k = 1, 2, \cdots, K$ and the receive elevation angle $\theta_{r_k}, k = 1, 2, \cdots, K$ in the bistatic EMVS-MIMO radar system, the following rotation-invariant relationship is further constructed

$$(\Gamma_{t2} \otimes I_6)\hat{C}_t = (\Gamma_{t1} \otimes I_6)\hat{C}_t \Phi(\theta_t)$$

(47)

$$(\Gamma_{r2} \otimes I_6)\hat{C}_r = (\Gamma_{r1} \otimes I_6)\hat{C}_r \Phi(\theta_r)$$

(48)
Then, the estimation of $\Phi(\theta_t)$ and $\Phi(\theta_r)$ can be achieved by using the least squares

$$\hat{\Phi}(\theta_t) = ((\Gamma_{t1} \otimes I_6)\hat{C}_t)\dagger((\Gamma_{t2} \otimes I_6)\hat{C}_t)$$

(49)

$$\hat{\Phi}(\theta_r) = ((\Gamma_{r1} \otimes I_6)\hat{C}_r)\dagger((\Gamma_{r2} \otimes I_6)\hat{C}_r)$$

(50)

Furthermore, by performing the singular value decomposition on $\hat{\Phi}(\theta_t)$ and $\hat{\Phi}(\theta_r)$, the corresponding eigenvalues $\lambda_{t1}, \lambda_{t2}, \cdots, \lambda_{tK}$ and $\lambda_{r1}, \lambda_{r2}, \cdots, \lambda_{rK}$ can be attained. Thus, the transmit elevation angle $\theta_{tk}, k = 1, 2, \cdots, K$ and receive elevation angle $\theta_{rk}, k = 1, 2, \cdots, K$ can be estimated as

$$\hat{\theta}_{tk} = \arcsin\left(\frac{2\arctan(\lambda_{tk})}{\pi}\right), \quad k = 1, 2, \cdots, K$$

(51)

$$\hat{\theta}_{rk} = \arcsin\left(\frac{2\arctan(\lambda_{rk})}{\pi}\right), \quad k = 1, 2, \cdots, K$$

(52)

For the estimated $\hat{\theta}_{tk}, k = 1, 2, \cdots, K$ and $\hat{\theta}_{rk}, k = 1, 2, \cdots, K$, the corresponding transmit/receive azimuth angle, transmit/receive polarization angle and transmit/receive polarization phase difference can be gained by the following process. Since the estimation process for $(\phi_{tk}, \gamma_{tk}, \eta_{tk}), k = 1, 2, \cdots, K$ and $(\phi_{rk}, \gamma_{rk}, \eta_{rk}), k = 1, 2, \cdots, K$ are similar, this paper only provides the comprehensive derivation process for $(\phi_{tk}, \gamma_{tk}, \eta_{tk}), k = 1, 2, \cdots, K$.

By exploiting the property of Khatri-Rao product, the transmit spatial response $Q_t$ can be reconstructed as

$$[q_{t1}, \cdots, q_{t1}] = \frac{1}{M} \sum_{m=1}^{M} (\hat{C}_t(6m - 5 : 6m,:))$$

(53)

where $\hat{C}_t(6m - 5 : 6m,:)$ denote $(6m - 5)$-th row to $6m$-th row of $\hat{C}_t$. Due to the real-value transform in (28), there is no need to multiply an additional diagonal matrix when estimating the $Q_t$, which differs from the reconstruction of $Q_t$ in [6]-[11].

For the reconstructed transmit spatial response $Q_t$, the normalized Poynting vector corresponding to the transmit EMVS array is

$$\begin{bmatrix}
\bar{u}_{tk} \\
\bar{v}_{tk} \\
\bar{w}_{tk}
\end{bmatrix} \triangleq \frac{e_{tk}}{||e_{tk}||} \times \frac{h_{tk}^*}{||h_{tk}||} = \begin{bmatrix}
\sin(\theta_{tk}) \cos(\phi_{tk}) \\
\sin(\theta_{tk}) \sin(\phi_{tk}) \\
\cos(\theta_{tk})
\end{bmatrix}$$

(54)

Therefore, the estimated transmit azimuth angle $\phi_{tk}, k = 1, 2, \cdots, K$ can be expressed as

$$\hat{\phi}_{tk} = \arctan\left(\frac{\bar{v}_{tk}}{\bar{u}_{tk}}\right), \quad k = 1, 2, \cdots, K$$

(55)
After obtaining \((\theta_{tk}, \phi_{tk}), k = 1, 2, \cdots, K\), the corresponding transmit polarization state vector \(g_{tk}(\gamma_{tk}, \eta_{tk})\) can be achieved

\[
g_{tk}(\gamma_{tk}, \eta_{tk}) = \begin{bmatrix} g_{1tk} \\ g_{2tk} \end{bmatrix} = \left[ F(\tilde{\theta}_{tk}, \tilde{\phi}_{tk}) \right]^\dagger Q_t, \tag{56}
\]

\(k = 1, 2, \cdots, K\)

Consequently, the polarization parameters \((\gamma_{tk}, \eta_{tk}), k = 1, 2, \cdots, K\) of the transmit EMVS can be obtained as

\[
\tilde{\gamma}_{tk} = \arctan\left( \frac{g_{1tk}}{g_{2tk}} \right), \quad k = 1, 2, \cdots, K \tag{57}
\]
\[
\tilde{\eta}_{tk} = \angle g_{1tk}, \quad k = 1, 2, \cdots, K
\]

Finally, the automatically paired transmit 4-D parameters \((\theta_t, \phi_t, \gamma_t, \eta_t), k = 1, 2, \cdots, K\) can be gained by using the mentioned above process, and the receive 4-D parameters \((\theta_r, \phi_r, \gamma_r, \eta_r), k = 1, 2, \cdots, K\) can be estimated in the same way. **Algorithm 2** offers the framework of the proposed algorithm.

**D. Deterministic Cramer-Rao bound**

According to [10], for a bistatic EMVS-MIMO radar with \(M\) transmit EMVS and \(N\) receive EMVS, the Cramer-Rao bound for the transmit 4-D parameters \((\theta_t, \phi_t, \gamma_t, \eta_t)\) and the receive 4-D parameters \((\theta_r, \phi_r, \gamma_r, \eta_r)\) is

\[
CRB = \frac{\sigma^2}{2L} \left[ \text{real}\left( (D)^H (\Pi)^\dagger_{C_{tr}} (D) \oplus (R_{ss}^T \otimes 1_{8 \times 8}) \right) \right]^{-1} \tag{58}
\]

where \(C_{tr} = (C_t \odot C_r)\) is the joint transmit-receive matrix of the bistatic EMVS-MIMO radar, \((\Pi)^\dagger_{C_{tr}} = I_{36MN} - C_{tr} C_{tr}^\dagger\) denotes the projection matrix of \(C_{tr}\), \(\oplus\) denotes the Hadamard product, \(R_{ss}\) represents the signal covariance matrix, \(1_{8 \times 8}\) is representative of the all-one matrix of dimension \(8 \times 8\), \(D = \begin{bmatrix} \partial C_{tr} \partial \theta_t & \partial A \partial \phi_t & \partial C_{tr} \partial \gamma_t & \partial C_{tr} \partial \eta_t \\
\partial C_{tr} \partial \phi_t & \partial A \partial \theta_t & \partial C_{tr} \partial \eta_t & \partial C_{tr} \partial \gamma_t \\
\partial C_{tr} \partial \gamma_t & \partial C_{tr} \partial \eta_t & \partial A \partial \phi_r & \partial C_{tr} \partial \theta_r \\
\partial C_{tr} \partial \eta_t & \partial C_{tr} \partial \gamma_t & \partial C_{tr} \partial \theta_r & \partial A \partial \phi_r \end{bmatrix}\) is the joint derivative matrix of the \(C_{tr}\) for \((\theta_t, \phi_t, \gamma_t, \eta_t)\) and \((\theta_r, \phi_r, \gamma_r, \eta_r)\).

**E. Identified targets**

Assume the maximum number of identified targets is \(K\) by using the proposed algorithm in this paper. The value of \(K\) is dependent on the PARAFAC algorithm, owing to the characteristic of PARAFAC decomposition, whose uniqueness, as reported in [18], can be guaranteed via the following Kruskal’s theorem.
Algorithm 2 Framework of the proposed algorithm

Input: array received data matrix $\mathbf{Y}$

Output: the transmit 4-D parameters $(\theta_t, \phi_t, \gamma_t, \eta_t)$ and the receive 4-D parameters $(\theta_r, \phi_r, \gamma_r, \eta_r)$

1: construct the 5-D tensor $\mathbf{Y}$ according to (15)
2: compute the covariance tensor $\mathbf{R}$ according to (16)
3: calculate the new 5-D tensor $\mathbf{R}_2$ by using the tensor permutation and the generalized tensorization of the canonical polyadic decomposition according to (17) and (18)
4: remove the repeated elements in the difference coarrays in $\mathbf{R}_2$ according to (21)
5: construct the three-way tensor $\mathbf{R}_5$ according to (24)-(31)
6: perform the PARAFAC algorithm on $\mathbf{R}_5$ to obtain the estimated transmit factor matrix $\hat{\mathbf{C}}_t$ and the receive factor matrix $\hat{\mathbf{C}}_r$ according to (32)-(34)
7: construct the select matrices $\Gamma_{t1}, \Gamma_{t2}, \Gamma_{r1}$ and $\Gamma_{r2}$ to compute $\tilde{\Phi}(\theta_t)$ and $\tilde{\Phi}(\theta_r)$, respectively. And, by using SVD to obtain the estimated $\hat{\theta}_{tk}, k = 1, 2, \cdots, K$ and $\hat{\theta}_{rk}, k = 1, 2, \cdots, K$ according to (43)-(50)
8: estimate $(\phi_t, \gamma_t, \eta_t)$ by using the reconstructed matrix $\mathbf{Q}_t$ according to (53)-(57)
9: estimate the receive 3-D parameters $(\phi_r, \gamma_r, \eta_r)$ by using the similar way according to step 8
10: return $(\theta_t, \phi_t, \gamma_t, \eta_t)$ and $(\theta_r, \phi_r, \gamma_r, \eta_r)$

Theorem 1 (Kruskal’s theorem) For a three-way tensor model $\mathbf{R}_5 \in \mathbb{C}^{\tilde{M} \times \tilde{N} \times 36}$, the PARAFAC decomposition is unique, if

$$\kappa_{\hat{\mathbf{C}}_t} + \kappa_{\hat{\mathbf{C}}_r} + \kappa_{\hat{\mathbf{Q}}} \geq 2K + 2$$

(59)

where $\kappa_{\hat{\mathbf{C}}_t}, \kappa_{\hat{\mathbf{C}}_r}$ and $\kappa_{\hat{\mathbf{Q}}}$ denote the $\kappa$-rank of $\hat{\mathbf{C}}_t$, $\hat{\mathbf{C}}_r$ and $\hat{\mathbf{Q}}$, respectively. The max $\kappa$-rank of $\hat{\mathbf{C}}_t$, $\hat{\mathbf{C}}_r$ and $\hat{\mathbf{Q}}$ are $6\tilde{M}$, $6\tilde{N}$ and 36, respectively. Thus, the value of $K$ can be obtained as

$$K = \frac{6\tilde{M} + 6\tilde{N} + 34}{2}$$

(60)

However, the value of $K$ also relies on the rotation invariant relationship in (47) and (48). The maximum value of $K$ in (47) and (48) should satisfy the following constraint

$$\begin{cases} K \leq 6(\tilde{M} - 1) & \text{in } (47) \\ K \leq 6(\tilde{N} - 1) & \text{in } (48) \end{cases}$$

(61)
According to (60) and (61), if \( \tilde{M} > \tilde{N} \), then \( K = 6(\tilde{N} - 1) \). If \( \tilde{N} > \tilde{M} \), then \( K = 6(\tilde{M} - 1) \). Thus, the maximum number of identified targets \( K \) is

\[
K = \min[6(\tilde{M} - 1), 6(\tilde{N} - 1)]
\]

Resultantly, the maximum number of identified targets is larger than the number of transmit EMVS and receive EMVS. The constructed difference coarrays in this paper are effective for the underdetermined 2D-DODs and 2D-DOAs estimation in the bistatic EMVS-MIMO radar system.

**F. Computational complexity**

Table I compares the main computational complexity of the proposed algorithm, the ESPRIT algorithm in [6], PM algorithm in [7], Tensor subspace-based algorithm in [8] and PARAFAC algorithm in [10]. Here, the CM, SVD and HOSVD denote the covariance matrix calculation, the singular value decomposition and the higher-order singular value decomposition, respectively. For a bistatic EMVS-MIMO radar with \( M \) transmit array and \( N \) receive array, the computational complexity of the proposed algorithm is higher than that of the PM algorithm and the PARAFAC algorithm, while lower than that of the ESPRIT algorithm and the HOSVD algorithm.

**TABLE I**

| Algorithm   | the complexity of major steps | total complexity |
|-------------|------------------------------|-----------------|
|             | CM              | SVD           | HOSVD       | multiple iterations |                      |
| ESPRIT      | \((36MN)^2L\)  | \((36MN)^3\) | \(\times\) | \(\times\) | \(\mathcal{O}((36MN)^2L + (36MN)^3)\) |
| PM          | \((36MN)^2L\)  | \(\times\)   | \(\times\) | \(\times\) | \(\mathcal{O}((36MN)^2L)\) |
| Tensor      | \((36MN)^2L\)  | \(4(36MN)^3\) | \(\times\) |                   | \(\mathcal{O}((36MN)^2L + 4(36MN)^3)\) |
| PARAFAC     | \(\times\)   | \(\times\)   | \(\times\) | \(\kappa(3K^3 + 108MNKL+)\) | \(\mathcal{O}(\kappa(3K^3 + 108MNKL+))\) |
|             |                 |               |             | \(3K^2(36MN + 6NL + 6ML)\) | \(3K^2(36MN + 6NL + 6ML)\) |
| Proposed    | \((36MN)^2L\)  | \(\times\)   | \(\times\) | \(\kappa(3K^3 + 3888\tilde{M}\tilde{N}K+)\) | \(\mathcal{O}(\kappa(3K^3 + 3888\tilde{M}\tilde{N}K+))\) |
|             |                 |               |             | \(3K^2(36\tilde{M}\tilde{N} + 216\tilde{N} + 216\tilde{M})\) | \(3K^2(36\tilde{M}\tilde{N} + 216\tilde{N} + 216\tilde{M})\) |

**V. SIMULATION RESULTS**

In order to evaluate the estimation performance of angle parameters and polarization parameters in bistatic coprime EMVS-MIMO radar, the following simulation experiments are
Firstly, the underdetermined 2D-DODs and 2D-DOAs estimation performance is evaluated by using the proposed algorithm. Assume the number of transmit EMVS $M$ is 9 with $(M_1, M_2) = (3, 4)$, the number of receive EMVS $N$ is 10 with $(N_1, N_2) = (3, 5)$. The corresponding transmit 4-D parameters and receive 4-D parameters of the impinging targets are listed in Table II. Additionally, the SNR is set to $10dB$, the snapshot is 200. The noise is assumed to be independent zero-mean additive Gaussian white noise, and the signal and noise are independent of each other. The scatterplot in Fig. 3 is obtained via 100 trails. Fig. 3(a) and Fig. 3(b) show that the underdetermined 2D-DODs and 2D-DOAs are accurately estimated. In the meantime, Fig. 3(c)-Fig. 3(f) demonstrate that the corresponding transmit 4-D parameters $(\theta_t, \phi_t, \gamma_t, \eta_t)$ and receive 4-D parameters $(\theta_r, \phi_r, \gamma_r, \eta_r)$ are automatically paired via the PARAFAC algorithm. Therefore, the effectiveness of proposed algorithm for the underdetermined 2D-DODs and 2D-DOAs estimation in the bistatic coprime EMVS-MIMO radar has been verified.

Secondly, the RMSEs performance of different algorithms versus SNR are compared. The average root mean square error (RMSE) is calculated as

$$RMSE = \sqrt{\frac{1}{KI} \sum_{i=1}^{I} ||\hat{\alpha} - \alpha||^2} \quad (63)$$

where $I = 200$ denotes the Monte Carlo trails, $\alpha$ and $\hat{\alpha}$ represent the true angle parameters and estimated angle parameters, respectively. Assume that there are $K = 3$ far-field targets with $\theta_t = [40^\circ, 20^\circ, 30^\circ], \phi_t = [15^\circ, 25^\circ, 35^\circ], \gamma_t = [10^\circ, 22^\circ, 35^\circ], \eta_t = [38^\circ, 48^\circ, 56^\circ], \theta_r = [24^\circ, 38^\circ, 16^\circ], \phi_r = [21^\circ, 32^\circ, 55^\circ], \gamma_r = [42^\circ, 33^\circ, 60^\circ], \eta_r = [17^\circ, 27^\circ, 39^\circ]$, respectively. The number of SNR is increased from 0dB to 20dB and the snapshot is set to 200 for different SNRs. The suffixes ’-d’ and ’-p’ in the legend refer to angle parameter and polarization parameter, respectively. Fig. 4 shows that the proposed algorithm exhibits a better angle and polarization parameters estimation

| targets $\theta_t/\theta_r$ | $\phi_t/\phi_r$ | $\gamma_t/\gamma_r$ | $\eta_t/\eta_r$ |
|-----------------------------|-----------------|---------------------|-----------------|
| $K = 13$                    | [10:5:70]       | [5:5:65]            | [5:3.75:50]     |
|                             | [3:5:63]        |                     |                 |
|                             | [15:5:75]       | [20:3.75:65]        | [5:5:65]        |
|                             |                 |                     | [10:5:70]       |
Fig. 3. Scatterplot by using the proposed method.
performance compared with the cutting-edge algorithms on the conditions of the different SNR.

Thirdly, the RMSEs performance of different algorithms versus the number of snapshots are assessed. The snapshots are set to vary from 100 to 1000 with a step of 100 and the SNR is fixed at 10dB. Other simulation parameters are the same as those in the Secondly experiment. As evident from Fig. 5, the RMSE versus snapshots of the proposed algorithm has the smallest estimation error both for the angle parameters and polarization parameters. As a consequence, the difference coarray of the coprime EMVS used in this paper promote the angle parameters and polarization parameters estimation performance.

Fourthly, the RMSEs performance of different algorithms versus the number of impinging sources $K$ has been compared. The number of sources vary from 2 to 8 and the corresponding transmit 4-D parameter $(\theta_t, \phi_t, \gamma_t, \eta_t)$ and receive 4-D parameters $(\theta_r, \phi_r, \gamma_r, \eta_r)$ are chosen from the Table II. The SNR and snapshots are set to 10dB and 200, respectively. It can be seen from Fig. 6 that the proposed method is able to maintain a reasonable estimation error for different number of sources, while the RMSEs for the other three algorithms become large with the increase of $K$. This phenomenon indicates that the proposed algorithm is robust to the different number of sources.

Finally, the estimation performance of the two closely-located targets has been evaluated by using the proposed method. Here, the biases between the true angle/polarization parameters and the mean of estimated angle/polarization parameters versus SNR and snapshots are examined.
Fig. 5. RMSE versus snapshot. (a) angle parameters. (b) polarization parameters.

Fig. 6. RMSE versus $K$. (a) angle parameters. (b) polarization parameters.

The transmit 4-D parameters and receive 4-D parameters of the two closely located targets are set to $\theta_t = [22^\circ, 23^\circ]$, $\phi_t = [26^\circ, 28^\circ]$, $\gamma_t = [45^\circ, 55^\circ]$, $\eta_t = [53^\circ, 63^\circ]$, $\theta_r = [33^\circ, 35^\circ]$, $\phi_r = [34^\circ, 36^\circ]$, $\gamma_r = [20^\circ, 65^\circ]$, $\eta_r = [28^\circ, 47^\circ]$, respectively. As illustrated in Fig. 7, the estimated bias by the proposed method is very small with the increase of SNR/snapshots.

VI. CONCLUSION

A tensor-based algorithm has been investigated to deal with the underdetermined 2D-DOD and 2D-DOA estimation in the bistatic EMVS-MIMO radar system, and its effectiveness has been well verified via simulations. This algorithm is advantageous to the realization of de-coupling.
between the spatial response and steering matrix, and it also contributes to the difference coarrays construction of transmit and receive EMVS, which plays an important role to estimate more targets than elements. More meaningfully, a variety of newly-designed sparse arrays in recent research can also be used as the transmit/receive EMVS in the bistatic MIMO radar system and this method can pave a path to obtain the corresponding difference coarrays to enhance the estimation performance.

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