Estimates dark halo parameters in S-galaxies

M Butenko¹*, N Kuzmin¹ and I Nichiporov¹

¹Volgograd State University, Volgograd, 400062, Russia
E-mail: *Corresponding author e-mail: maria_butenko@volsu.ru

Abstract. The dark mass problem is the most important for cosmology and galaxies physics. Various observational data of galaxies and galaxies clusters kinematics and dynamics require presence of massive dark matter. We estimate the dark mass within the galaxies optical radius using several different approaches, such as the maximum disk model, the marginal condition of the stellar disk gravitational stability and the vertical equilibrium condition in the galactic disk. The distribution functions of the relative dark mass within the optical radius $R_{\text{opt}}$ for a set of 50 galaxies are constructed. Most of the objects from our set have a relative mass within $\mu = 2$. This agrees well with the results of numerical simulations.

1. Introduction
The dark mass nature is far from any complete understanding, but many observations indicate its existence [1, 2, 3]. We mention only some facts. A spiral galaxies photometry and kinematics joint analysis shows that visible matter is not enough to provide the observed rotation curves [4, 5]. The effects of gravitational lensing indicate a dark matter predominance both in galaxies clusters, and in individual massive galaxies [6, 7]. Cosmological simulations show that the galaxy stellar disk may be formed inside halo resulting angular momentum exchange between the dark and baryon matter and efficient radiative cooling [8, 9].

In this paper, we estimate the dark mass value of spiral galaxies by different methods using various kinematic and photometric data.

2. Restrictions on a dark halo mass from observations
2.1. Galactic components models
Spiral galaxies consist of spheroidal and disk subsystems. The spheroidal subsystem includes the dark halo and stellar bulge, the rotation disk consist of stars and gas. We use the cylindrical coordinate system $(r, \varphi, z)$, where $r$ is radial, $\varphi$ is azimuthal and $z$ is vertical coordinates, correspondly.

Dark halo model can be described by the quasi-isothermal halo model [10]

$$\rho_h(r) = \rho_{h0} \cdot \left(1 + r^2/a^2\right)^{-1},$$

(1)

where $a$ is a spatial scale, $\rho_{h0}$ is a characteristic density.

The stellar bulge model is characterized by the central density $\rho_{b0}$ and the characteristic spatial scale $b$ for the radial profile de Vaucouleurs [11] or King [12].
The stellar disk volume density can be defined as [10]
\[ \rho_d(r, z) = \frac{\sigma_0 \exp(-r/r_d)}{2z_0 \cosh^2(z/z_0)}, \]
(2)
where \( \sigma_0 \) is a central surface density, \( r_d \) is a radial exponential scale, \( z_0 \) is a vertical scale (disk thickness).

The gaseous disk, which is cooler than a stellar disk, can be an important component. Therefore, it is considered that the total circular velocity is proportional to the gas rotation velocity: \( V_c(r) \approx V_g(r) \). For the transition from the stellar disk rotation curve \( V_\star(r) \) to \( V_c(r) \), it is necessary to know the stars velocities dispersion along the radial, azimuthal and vertical coordinates: \( c_r(r), c_\phi(r), c_z(r) \).

We calculated the masses of the stellar disk \( M_\star \), the gaseous disk \( M_g \), the halo \( M_h \), and the bulge \( M_b \) within a fixed radius \( R_{\text{max}} \), using the density components profiles. We also introduce the following notation of the disk subsystem total mass: \( M_d = M_\star + M_g \). As \( R_{\text{max}} \) we chose the optical radius \( R_{\text{opt}} \). The relation \( R_{\text{opt}} \sim 4r_d \) is usually satisfied [10].

2.2. Maximum disk model

The total circular velocity \( V_c(r) \) is determined by the expression
\[ V_c^2(r)/r = \partial \Phi/\partial r, \]
(3)
where \( \Phi = \Phi_d + \Phi_b + \Phi_h \) is the total gravitational potential, and \( \Phi_d \), \( \Phi_b \) and \( \Phi_h \) are the gravitational potentials of the disk subsystem, the bulge and halo, correspondingly. Therefore, the total circular velocity \( V_c(r) \) can be expressed through the circular velocities of the disk subsystem \( V_d(r) \), the bulge \( V_b(r) \) and the halo \( V_h(r) \):
\[ V_c^2(r) = V_d^2(r) + V_b^2(r) + V_h^2(r). \]
(4)

The maximum disk model (MDM) is the simplest way to estimate the lower limit of the dark mass in the halo. Within this approach, the profile \( V_c(r) \) that is closest to the observed rotation curve is constructed. Moreover, for \( r \leq 2r_d \) in the expression (4), the disk component contribution \( V_d(r) \) should be as large as possible, and the dark halo contribution \( V_h(r) \) should be minimal [2, 13]. Usually in the maximum disk model the circular disk velocity \( V_d \) is 75–95 % of the total circular velocity \( V_c \). Using the profile \( V_h(r) \) obtained for this model, we reconstructed the potential \( \Phi_h(r) \) and estimated the halo mass \( M_h \).

Figure 1a, b shows the results of the rotation curve decomposition for four-component galaxy models in the MDM.

2.3. Condition of gravitational stability

The gravitational stability of a stellar disk is characterized by the Toomre parameter [14]
\[ QT \equiv c_r/c_T, \quad c_T = 3.36G\sigma(r)/\kappa(r), \]
(5)
where \( G \) is the gravitational constant, \( \kappa \) is the epicyclic frequency. Considering that on flat part of rotation curves (plateau) \( \kappa \approx \sqrt{2}\Omega \), and the rotation frequency \( \Omega = V_c/r \), from (5) we get the following expression for the surface density of the stellar disk:
\[ \sigma_\star(r) \approx \frac{\sqrt{2}c_r(r)V_c(r)}{3.36rGQT}. \]
(6)
Figure 1. Rotation curve decomposition in the maximal disk model for NGC 300 (a) and NGC 628 (b); decomposition for NGC 628 with the condition of vertical disk equilibrium (c). The crosses show the observed rotation curve. Different lines show component velocities: line 1 is the total rotational velocity; lines 2 and 6 corresponds to the stellar disk; lines 3, 4 and 5 corresponds to the halo, the gaseous disk and the bulge, respectively.

We assume that $M_g \ll M_\ast$ ($M_d \simeq M_\ast$). Then the circular velocity of the disk component $V_d(r)$ was reconstructed from the distribution $\sigma_\ast(r)$. Substituting $V_d(r)$ into (4) and varying $V_h(r)$, we got the profile $V_c(r)$ that is closest to the observed rotation curve. Using the profile $V_h(r)$ found this way, we reconstructed the potential $\Phi_h(r)$ and estimated the halo mass $M_h$ [3].

For the marginal condition of the stellar disk gravitational stability, we used the following approximation for $Q_T(r)$ [15]:

$$Q_T(r) = 1.25 - 0.19r/r_d + 0.134(r/r_d)^2.$$  (7)

Note that the presence of a massive gaseous disk with $M_g \sim (0.1 - 0.25)M_\ast$ can significantly change the gravitational stability condition. Cold gas destabilizes the self-consistent stellar-gaseous disk. Therefore, the stability condition of a multi-component disk depends on the surface gas density radial profiles $\sigma_g(r)$ and the sound speed $c_s(r)$. In this case, the generalized stability criterion is used [16].

2.4. The vertical equilibrium

One of the most direct methods of determining the ratio between the halo and disk masses is to use a vertical velocity dispersion $c_z(r)$ in the stellar disk to determine its surface density [17, 18].

$$\sigma_\ast(r) = \frac{c_z^2(r)}{2\pi G z_0},$$  (8)

where $z_0$ is the exponential disk vertical scale. Assuming that $M_g \ll M_\ast$ ($M_d \simeq M_\ast$), the circular velocity of the disk component $V_d(r)$ was reconstructed from the distribution $\sigma_\ast(r)$. Then, substituting it into (4) and changing $V_h(r)$, we get profile $V_c(r)$ closest to the observed rotation curve. Further we reconstructed the potential $\Phi_h(r)$ and estimated the halo mass $M_h$ from $V_h(r)$.

3. The dark halo relative mass

We applied the methods described in the previous section to estimate the dark mass for a set of 50 galaxies. Rotation curves decomposition was performed using the software [19], which allows
to obtain the total rotation curve taking into account the main galaxies component, to calculate their density profiles and report on the different components masses.

For all objects in our set, the observed rotation curves $V_c(r)$ and the estimates of the photometric scales for the components of the galaxy are known. Approximately half of the objects have stars velocity dispersion values $c_{\text{obs}}(r)$ for $r \simeq 2r_d$. This allows us to estimate the mass of the galaxy components from the condition of gravitational stability. At the specified distance, the disk contribution to the rotation curve is maximum. For $r > 2r_d$ dispersion estimates are usually determined with large errors and the effect of external factors dynamically heating the disk can be more significant. At small $r$ it is often difficult to separate the bulge and disk stars contributions in the observed values $c_{\text{obs}}$, as well as to take into account the bar presence, if it exists in the galaxy.

Our set included 6 LSB and 4 HSB galaxies from [20], in which the observed velocity dispersions at sufficiently large $r$ are shown. We also reviewed 8 S0 galaxies from [21]. For 24 galaxies from [22], only the observed rotation curves are known, so for them we obtained the dark mass estimates only in the MDM for these galaxies. For the NGC 628 and NGC 300 galaxies, we took into account the presence of gaseous disks, whose surface density profiles were obtained from [23, 24]. Figure 1a, b shows the decomposition of these galaxies rotation curves in the MDM. Figure 1c shows the rotation curve decomposition taking into account the vertical disk equilibrium condition according to the observation of dispersion $c_z$.

To estimate the halo mass by vertical equilibrium, it is necessary to know the vertical scale of the stellar disk $z_0$ and the vertical velocity dispersion $c_z$. In our set, these parameters are known only for one galaxy — NGC 628. It is located almost “on face”, so its $c_z$ is approximately equal to the observed velocity dispersion $c_{\text{obs}}$. Note that it is difficult to estimate $z_0$ for galaxies located “on face”. In the calculations, we used the smaller inclination angle $i = 6^\circ$ than specified in [23]. This led to a slight increase of circular velocity. In the MDM, the halo mass within $R_{\text{opt}}$ is $4.2 \cdot 10^{10} M_\odot$, and the approach based on vertical equilibrium gives double value — $9.97 \cdot 10^{10} M_\odot$.

During the decomposition of all objects, we used the quasi-isothermal halo model and the King model for the bulge. As a result of the rotation curve decomposition into components, we obtain the model parameters estimates of spheroidal components (halo and bulge), and the mass estimation for each of the galaxy components. To analyze the obtained data, we introduced the following dimensionless relative mass parameters: $\mu_{\text{DM}} = M_h/M_d$ — the dark mass in mass units of the disk component, $\mu_{\text{spheric}} = (M_h + M_b)/M_d$ — the ratio of the spheroidal subsystem mass to the disk mass, which affect the system dynamics in different ways, $\mu_{\text{baryon}} = M_h/(M_d + M_b)$ — the ratio of dark mass to baryon matter.

Note that the mass estimates obtained using MDM $\mu_{\text{MDM}}$ give us the lower mass limit for the dark halo. This is clearly seen from the figure 2, that shows the galaxies distribution in our set by the values of the relative mass parameters $\mu$. We present the obtained statistical values for the relative halo mass for our set: $\min(\mu) = 0.05 \div 0.18$; $\max(\mu) = 3.59 \div 5.02$; $\langle \mu \rangle = 0.81 \div 1.14$ and the median value $\mu = 0.66 \div 0.9$. The minimum value is $\mu_{\text{baryon}}$, the maximum is $\mu_{\text{spheric}}$ due to the presence of a massive bulge in some galaxies.

The halo relative masses were compared with the rotation velocity $V_c$ and the morphological type of the galaxy $T$ (figure 3). The diagram shows that taking into account the disk stability criterion leads to an increase of dark halo mass for some objects more than 3 times.

SB-galaxies is half amount of our set according to the HYPERLEDA database. From the distributions in figure 3a you can see that for galaxies with a bar, the maximum distribution of the relative mass $\mu_{\text{DM}}$ is shifted toward larger values. This does not contradict the numerical N-body simulations results which unable to obtain a long-lived bar in model with high-mass dark halos. [25]. This result is also visible in figure 3b, c. The coloring of the beans in figure 3a corresponds to figure 2.
Figure 2. The galaxies distributions from our set by the values of the dark halo relative mass: (a) $\mu_{DM}$; (b) $\mu_{spheric}$; (c) $\mu_{baryon}$. The filled rectangles show the mass estimates obtained from the maximum disk model; the shaded rectangles show the mass estimates obtained by the marginal criterion.

4. Conclusion
The three methods of determining the dark halo relative mass we considered are based on three different physical assumptions. The maximum disk method gives an dark halo estimate from below based only on the radial forces balance (assuming that the contribution of dark matter is minimal). Other methods generally lead to large values of the halo relative mass $\mu$. The criterion of the stellar disk gravitational stability increases the dark halo mass estimate by 2–3 times. In this paper, we do not take into account the effect of the gaseous component, but the stellar and gaseous disks stability condition requires large relative values of the halo mass as a cold gaseous disk is a destabilizing factor.

The method based on vertical disk equilibrium seems to be the most accurate. It uses data on the vertical stars velocity dispersion, for which the best orientation of the disk is “on face”

Figure 3. (a) The galaxies distributions from our set by $\mu_{DM}$ values for galaxies with a bar (top) and no bar (bottom). (b) Comparison of $\mu_{DM}$ with a rotation velocity $V_c$. (c) Comparison of $\mu_{DM}$ with morphological type $T$. The slanting crosses in the diagrams correspond to the values of $\mu_{DM}$ from MDM for galaxies with a bar, the straight crosses are — bar galaxies; filled circles — the values of $\mu_{DM}$ for the marginal stability criterion for galaxies with a bar, empty circles — galaxies without a bar.
with $i \simeq 0^\circ$. However, in this case, the rotation curve determination is extremely inaccurate. As the angle $i$ increases, the rotation curve quality increases, but the dispersion accuracy of $c_z$ deteriorates.

Note that our results will be slightly different for a larger set.

Acknowledgments

This work was performed as a part of state assignment of the Ministry of Education and Science (project No. 2.852.2017/4.6). M. A. Butenko thanks for the support of the Russian Federal Property Fund and the Administration of the Volgograd region (grant 18-42-343005). The authors are grateful to A. V. Khoperskov for valuable comments and helpful discussions.

References

[1] Lukash V N and Mikheeva E V 2007 Phys. Usp. 50 971–6
[2] Zasov A V, Saburova A S, Khoperskov A V and Khoperskov S A 2017 Phys. Usp. 60 3–39
[3] Zasov A V, Khoperskov A V and Saburova A S 2011 Astronomy Letters 37 374–84
[4] Martinsson T P K, Verheijen M A W, Westfall K B, Bershady M A, Andersen D R and Swaters R A 2013 Astron. and Astrophys. 557 A131
[5] Kauffmann G, Huang M L, Moran S and Heckman T M 2015 Monthly Notices Royal Astron. Soc. 451 878–87
[6] Schneider P, Kochanek C S and Wambsganss J 2006 Gravitational Lensing: Strong, Weak and Micro ed G Meylan and P Jetzer et al (Berlin: Springer)
[7] Mandelbaum R, Wang W, Zu Y, White S, Henriques B and More S 2016 Monthly Notices Royal Astron. Soc. 457 3200–18
[8] Abadi M G, Navarro J F, Fardal M, Babul A and Steinmetz M 2010 Monthly Notices Royal Astron. Soc. 407 436–46
[9] Khoperskov S A, Shustov B M and Khoperskov A V 2012 Astronomy Reports 56 664–71
[10] Fridman A M and Khoperskov A V 2013 Physics of Galactic Disks (Cambridge International Science Publishing)
[11] de Vaucouleurs G 1953 Monthly Notices Royal Astron. Soc. 113 134–61
[12] King I 1962 Astron. J. 67 471–84
[13] Binney J and Tremaine S 2008 Galactic Dynamics: Second Edition (Princeton University Press)
[14] Toomre A 1964 Astrophys. J. 139 1217–38
[15] Khoperskov A V, Zasov A V and Tyurina N V 2003 Astronomy Reports 47 357–76
[16] Romeo A B and Falstad N 2013 Monthly Notices Royal Astron. Soc. 433 1389–97
[17] van der Kruit P C and Freeman K C 1984 Astrophys. J. 278 81–8
[18] Bershady M A, Verheijen M A W, Westfall K B, Andersen D R, Swaters R A and Martinsson T 2010 Astrophys. J. 716 234–68
[19] Mukhatov D S and Zasov A V 2013 Science journal of Volgograd State University. Mathematics. Physics 18 84–92
[20] Pizzella A, Corsini E M, Sarzi M, Magorrian J, Méndez-Abreu J, Coccato L, Morelli L and Bertola F 2008 Monthly Notices Royal Astron. Soc. 387 1099–116
[21] Simien F and Prugnuel P 2000 Astron. and Astrophys. Suppl. 145 263–7
[22] Spano M, Marcelin M, Amram P, Carignan C, Epinat B and Hernandez O 2008 Monthly Notices Royal Astron. Soc. 383 297–316
[23] Aniyan S, Freeman K C, Arnaboldi M, Gerhard O E, Coccato L, Fabricius M, Kuijken K, Merrifield M and Ponomareva A A 2018 Monthly Notices Royal Astron. Soc. 476 1909–30
[24] Westmeier T, Braun R and Koribalski B S 2011 Monthly Notices Royal Astron. Soc. 410 2217–36
[25] Morozov A G 1981 Sov. Astron. 25 421–6