Quantum degeneracy and interaction effects in spin-polarized Fermi-Bose mixtures

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Various features of spin-polarized Fermi gases confined in harmonic traps are discussed, taking into account possible perspectives of experimental measurements. The mechanism of the expansion of the gas is explicitly investigated and compared with the one of an interacting Bose gas. The role of interactions on the equilibrium and non equilibrium behaviour of the fermionic component in Fermi-Bose mixtures is discussed. Special emphasis is given to the case of potassium isotopes mixtures.

I. INTRODUCTION

After the achievement of Bose-Einstein condensation (BEC) in dilute vapors of alkali atoms confined in magnetic traps \textsuperscript{13}\textsuperscript{3}C, a challenge for future experiments is the cooling of samples of fermionic atoms, down to the degenerate regime.

This perspective has already motivated several theoretical works (see, for example, \textsuperscript{14}\textsuperscript{33}) devoted to the study of a possible superfluid phase occurring at very low temperature, analogous to the one exhibited by superfluid \textsuperscript{3}He and superconductors. The behaviour of this transition depends crucially on the sign and the size of the scattering length and various predictions have already been made to estimate the value of the corresponding critical temperature. Another peculiar phenomenon exhibited by finite Fermi gases at low temperature is the occurrence of quantum shell effects \textsuperscript{3}, bringing interesting analogies with the physics of atomic nuclei and metal clusters. The observation of such phenomena is seriously limited by the difficulty of reaching the required regime of very low temperatures. In fact both superfluidity and shell effects are expected to occur at temperatures much smaller than the Fermi temperature. For a realistic perspective of experimental measurements in the immediate future it is consequently useful to explore quantum degeneracy effects taking place at higher temperature. These effects have been already the subject of several theoretical works \textsuperscript{4,5}\textsuperscript{4}\textsuperscript{14}\textsuperscript{15}\textsuperscript{16}.

A crucial role in the experimental achievement of the low temperature regime needed to observe quantum degeneracy effects is played by evaporative cooling. This technique requires frequent collisions in order to ensure fast thermalization. Scattering between spin-polarized fermions in \textit{s} wave is inhibited by the Pauli exclusion principle. As a consequence one should look for alternative scattering processes involving different spin states or atoms belonging to different species. Fermi-Bose or Fermi-Fermi mixtures are possible candidates. The resulting scattering processes will eventually provide the relevant thermalization mechanism, allowing for the cooling of the sample (sympathetic cooling \textsuperscript{17}\textsuperscript{18}).

In the following we will mainly discuss the case of Fermi-Bose mixtures where the effects of two-body interactions can be significant also on the equilibrium properties of the fermionic component because of the occurrence of Bose-Einstein condensation which strongly enhances the value of the central density below the critical temperature, thereby emphasizing the effects of the mean field interaction. In this context we will limit the discussion to the case of spin-polarized Fermi gases where the interaction between fermions can be safely neglected. Notice that in the case of unpolarized Fermi gases also the interaction between fermions can produce significant effects on the equilibrium properties if the scattering length is large as happens, for example, in the case of \textsuperscript{6}\textsuperscript{2}Li and \textsuperscript{39}\textsuperscript{41}\textsuperscript{4}K.

The aim of the present paper is to discuss some of the key features exhibited by spin-polarized trapped Fermi gases, with special emphasis to the dynamics of the expansion following the switching off of the trap (Sect. I) and the role of interactions in the case of Fermi-Bose mixtures (Sect. II). We apply the results to the case of \textsuperscript{40}\textsuperscript{41}\textsuperscript{4}K, the fermionic isotope of potassium, which has been recently cooled in a magneto-optical trap \textsuperscript{19}. Potassium is in fact a good candidate for the experiments proposed here because of the presence of two bosonic isotopes, \textsuperscript{39}\textsuperscript{40}K and \textsuperscript{41}K in addition to the fermionic one. The possibility of magnetically trapping these atoms has been already demonstrated \textsuperscript{20}. Collisional cross sections for potassium have been calculated in \textsuperscript{21}\textsuperscript{22}. In the following we assume the values given in \textsuperscript{21}. The analysis presented in the present paper can be easily extended to other fermionic atoms which are presently investigated experimentally as, for instance, \textsuperscript{6}\textsuperscript{4}Li.

II. THE IDEAL FERMI GAS

The thermodynamic behaviour of ideal trapped Fermi gas has already been investigated \textsuperscript{23}\textsuperscript{24}\textsuperscript{25}\textsuperscript{26}. In this section we summarize the main results and we derive explicit formulae for the expansion of the fermionic cloud after
turning off the confining potential. These results could be relevant in view of future measurements on the expanding cloud.

Let us consider \( N \) fermions trapped in an axially symmetric harmonic potential

\[
V(r_{\perp}, z) = \frac{1}{2} m \left( \omega_{\perp}^2 r_{\perp}^2 + \omega_z^2 z^2 \right)
\]

and ignore the effects of the two-body interatomic force. The particle distribution function can be written, in semiclassical approximation, as

\[
n(r, k, \beta) = \frac{1}{(2\pi)^3} \frac{1}{\exp(\beta(\frac{\omega_{\perp}^2 r_{\perp}^2}{2m} + V(r_{\perp}, z) - \mu_f)) + 1},
\]

where \( \beta = (K_B T)^{-1} \) and \( \mu_f \) is the chemical potential, fixed by the normalization condition

\[
N = \int n(r, k, \beta) dr dk = \frac{1}{2(h\omega)^3} \int_0^{\infty} \frac{E^2 dE}{e^{\beta(E - \mu_f)} + 1}.
\]

In eq. (3) \( \omega = (\omega_{\perp}^2 \omega_z)^{1/3} \) is the geometrical average of the trapping frequencies.

If more spin states are occupied, then (2) characterizes the Fermi energy defined by \( E_F = \mu_f(T = 0) \). One finds

\[
E_F = K_B T_F = (6N)^{1/3} h\omega.
\]

The Fermi energy (4) can be used to define typical length and momentum scales (5) characterizing the Fermi distribution in coordinate and momentum space respectively:

\[
E_F = \frac{1}{2} m \omega_{\perp}^2 R_{\perp}^2 = \frac{1}{2} m \omega_z^2 Z^2 = \frac{h^2 K_F^2}{2m}
\]

where \( R_{\perp} \) and \( Z \) are the radial and axial widths of the density distribution at zero temperature:

\[
n(r; T = 0) = \frac{8}{\pi^2} \frac{N}{R_{\perp}^2 Z} \left( 1 - \left( \frac{r_{\perp}}{R_{\perp}} \right)^2 - \left( \frac{z}{Z} \right)^2 \right)^{3/2}
\]

while \( K_F \) is the width of the corresponding momentum distribution

\[
n(k; T = 0) = \frac{8}{\pi^2} \frac{N}{K_F^2} \left( 1 - \frac{k^2}{K_F^2} \right)^{3/2}.
\]

Eqs. (5) and (6) hold for positive values of their arguments. Eq. (6) is the analog of the most familiar momentum distribution \( n(k; T = 0) = 3N/(4\pi K_F^2) \Theta(1 - k^2/K_F^2) \) characterizing uniform Fermi gases.

At finite temperature the total energy \( E \) of the system can be written in the form

\[
E = \frac{1}{2(h\omega)^3} \int_0^{\infty} \frac{E^3 dE}{e^{\beta(E - \mu_f)} + 1}.
\]

While the release energy, given by the energy of the system after switching off the trap is equal to \( E_{rel} = E/2 \), because of the equipartition theorem. At low temperatures one has \( E \approx \frac{3}{4} N \epsilon_F \left( 1 + \frac{2\pi^2}{3} \left( \frac{T}{T_F} \right)^2 \right) \). In Fig. (4) we show the release energy vs \( T \) for a gas of \( N \) fermions confined in a harmonic trap (solid line). In the same figure we also report the behaviour predicted by the classical gas (\( E_{rel} = 3/2NK_BT \), dashed line) as well as the one of an ideal Bose gas with the same number of particles \( N \), in the same confining trap (dot dashed line). For the ideal Bose gas the release energy is given by \( E_{rel} = 1.35N (K_B T)^4 \) for \( T < T_c \) where \( T_c = 0.94 \omega_N N^{1/3}/K_B \) is the critical temperature for Bose-Einstein condensation. Notice that, differently from the Fermi case, Bose gases exhibit a phase transition at \( T = T_c \). This temperature has the same \( N \) dependence as the Fermi temperature (see eq. (8)) and is fixed by the same geometrical average of the trapping frequency. The general relation between the two temperatures is given by

\[
T_F = 1.9T_c \left( \frac{N_f}{N_b} \right)^{1/3} \frac{\omega_f}{\omega_b}
\]

where the suffix \( f \) (\( b \)) refers to fermions (bosons). The figure clearly shows that in order to observe effects of degeneracy in the release energy of a Fermi gas one should go to temperatures considerably smaller than the Fermi temperature. For example in order to obtain a deviation of 20 percent from the classical value one should work at \( T \approx 0.3T_F \). This represents a major difference with respect to the thermodynamic behavior of a Bose gas where the effects of Bose-Einstein condensation in the release energy show up immediately below the critical temperature as clearly shown in the same figure.

Let us now consider the expansion of the cloud after turning off the trapping potential. To obtain the temporal evolution of the density profile one has to solve the Boltzmann transport equation. For cold and dilute Fermi gases one can ignore two body collisions and the distribution function after switching off the trap will consequently follow the ballistic law \( n(r, k, \beta, t) = n_0(r - \mathbf{k}_F t, k, \beta) \) where \( n_0 \) is the distribution function at \( t = 0 \), given by (8). One can then easily calculate the time evolution of the spatial density during the expansion for which we find the simple analytic result

\[
n(r, \beta, t) = \frac{6N_f}{R_{\perp}^2 Z (\pi E_F \beta)^{3/2} (1 + \omega_{\perp}^2 t^2) (1 + \omega_z^2 t^2)^{1/2}} \int_0^{\hat{z}} f_{3/2}(\hat{z}) \]

where \( f_{3/2}(z) = (\Gamma(n))^{-1} \int dy y^{n-1} / (z^{n+1} e^y + 1) \) are the Fermi functions, \( \hat{z} = \exp(\beta(\mu_f - \hat{V}(r, t))) \) and
plays the role of an effective potential fixing, at each instant, the shape of the distribution function. The chemical potential has no time dependence, being fixed by the normalization condition (3). From (1) one can extract the temporal evolution of the mean square radii of the system:

\[
\langle r^2 \rangle = \frac{1}{3N} E_{rel} \frac{2}{m \omega^2} (1 + \omega_t^2 t^2) \tag{11}
\]

\[
\langle z^2 \rangle = \frac{1}{3N} E_{rel} \frac{2}{m \omega_z^2} (1 + \omega_z^2 t^2). \tag{12}
\]

which have been written in terms of the release energy \(E_{rel} = E/2\) of the gas. The structure of these equations is independent of the temperature which enters the problem only through the value of the release energy, fixing the initial value of the widths. Actually the release energy fixes also the asymptotic behavior of the widths, as clearly shown by eqs. (11,12). In Fig. 4 we compare the temporal evolution, at \(T = 0\), of the root mean square radii of the Fermi gas (see eq. [11,12]) with the one of a gas of interacting bosons. The Fermi gas corresponds to \(10^6\) \(^{40}\)K atoms initially confined by a harmonic trap with \(\omega_f = 100\) Hz. The Bose gas instead corresponds to \(10^6\) \(^{39}\)K atoms interacting with a scattering length \(a_{bb} = 80 a_0\) (\(a_0\) is the Bohr radius) and initially confined by a magnetic trap with \(\omega_b = 100\) Hz. The same parameters have been used to calculate the release energy of the interacting Bose gas as a function of temperature (dotted curve in Fig. 1) in the framework of the theory developed in [2]. Fig. 2 clearly shows that the Fermi gas expands more quickly than the Bose gas due to the significantly higher value of the release energy.

Another useful quantity is the aspect ratio of the cloud

\[
R_r(t) = \sqrt{\frac{\langle z^2 \rangle}{\langle x^2 \rangle}} = \frac{1}{\lambda} \sqrt{\frac{1 + \omega_z^2 t^2}{1 + \omega_t^2 t^2}} \tag{13}
\]

Eq. (13) shows that for \(t \to \infty\) the cloud becomes spherical in shape even if it was initially strongly anisotropic. This behaviour is independent of the temperature and is due to the absence of collisions during the expansion. Notice that in a Bose gas the situation is quite different, the asymptotic distribution being anisotropic due to the occurrence of Bose–Einstein condensation, as explicitly pointed out in the first experiments on BEC [1,2].

### III. FERMI-BOSE MIXTURES

It is important to discuss how the scenario presented in the preceding section for the ideal Fermi gas is modified when the system interacts with a Bose gas confined in the same trap. As discussed in the introduction such mixed Fermi-Bose gases might become relevant in future experiments for the achievement of very low temperature regimes via sympathetic cooling.

The study of Fermi-Bose mixtures has been already the subject of theoretical studies at zero [11] as well as finite temperature [2]. Due to the occurrence of Bose-Einstein condensation the bosonic component is characterized by a high density in the central region of the trap where consequently interaction effects play an important role. The presence of the much more dilute Fermi component is not expected to influence the bosonic wave function in a significant way, so that the main effect of interactions between fermions and bosons will result in an additional external field acting on the Fermi component. The Fermi gas can then, in first approximation, be treated again as an ideal gas trapped by the effective potential

\[
V_{eff} = V + g_{bf} n_f(r, T) \tag{14}
\]

where \(V\) is the external potential [1] trapping the fermionic species, and the renormalization arises from the interaction with bosons. The parameter \(g_{bf} = 2 \pi R^2 \alpha_{bf}/m_r\) is the Fermi-Bose interaction coupling constant fixed by the relative scattering length \(a_{bf}\) and by the reduced mass \(m_r = m_b m_f/(m_b + m_f)\). The bosonic density \(n_r\) can be calculated at thermal equilibrium using standard procedures developed to describe Bose condensed gases at finite temperature (see for example [2]).

The new equilibrium properties of the Fermi gas can be simply understood by looking at the shape of the potential [1] as explicitly discussed in [2]. A first important feature is that the interaction term in (14) produces a weaker confinement. This results in an expansion of the Fermi cloud with respect to the non interacting case and a consequent decrease of the average density, as well as of quantum statistical effects. The effects are more pronounced at low temperature where all the bosons are in the condensate and one can use the Thomas-Fermi approximation to calculate the density distribution of the bosonic component. In this case one has \(n_b = 15 N_b (R_b^3 - r^3)/(8 \pi R_b^6)\) where \(R_b = \alpha_{ho}(15 N_b a_{bb}/\alpha_{ho})^{1/5}\) is the classical radius of the Bose gas, \(a_{bb}\) is the boson-boson scattering length and \(\alpha_{ho}\) is \((\hbar/m_b \omega_b)^{1/2}\) is the oscillator length relative to the bosons. One finds that the effective potential felt by the fermions is given, for \(r < R_b\) by

\[
V_{eff} = \frac{m_f}{2 \omega_f^2} \left(1 - \frac{g_{bf}}{g_{bb}} \frac{m_b \omega_b^2}{m_f \omega_f^2}\right) r^2 + \frac{g_{bf}}{g_{bb}} \mu_b \tag{15}
\]

where, for simplicity, we have considered an isotropic trap. For \(r > R_b\) the effective potential instead coincides with the bare potential \(V\). Eq. (14) shows that the oscillator constant is reduced by the interaction with bosons.

As a first example we consider \(10^4\) \(^{40}\)K atoms and \(10^6\) \(^{39}\)K atoms confined in the same harmonic trap (\(\omega_f =...\)
Hence Bose-Fermi mixtures are expected to be available of the two atomic species. For example at the mixture is expected to depend in a crucial way on the equilibrium properties of the Fermi component of cooling. This method can be easily extended to vacuum environmental condition required for evaporative trapped in a double MOT apparatus [18] in the ultra high

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optical trapped from a natural abundance sample. Since in [17] 10

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K potassium atoms could be easily loaded. More recently more than 10

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FIGURE CAPTIONS
Fig. 1 Release energy in units of $NK_BT_F$ as a function of temperature (in units of the Fermi temperature $T_F$). The solid line corresponds to an ideal Fermi gas trapped in a harmonic potential. The dashed line is the linear law of a classical gas, while the dot dashed and the dotted lines are the release energy for a Bose gas with the same number of atoms confined in the same trap in the non-interacting and in the interacting case respectively (see the text).

Fig. 2 Temporal evolution in msec of the root mean square radii (in units of $a_{ho}$) of an ideal Fermi gas of $^{40}K$ (solid line) and of an interacting Bose gas $^{39}K$ (dashed line) at zero temperature after switching off the trap. The two curves refer to a trap with $\omega = 100 \, Hz$ and same number of particles $N = 10^6$. The scattering length for the $^{39}K$ was taken from [20].

Fig. 3 Oscillations at zero temperature of the mean square radius of $10^4 \, ^{40}K$ atoms after the bosons ($10^6 \, ^{39}K$ atoms) are removed. Time and lengths are in units of $\omega/\pi$ and $a_{ho}$ respectively.