Do varying physical constants provide solution to the lithium problem?

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ABSTRACT

We have used the recently published varying physical constants (VPC) approach to resolve the primordial lithium abundance problem. The value of the ratio of $^7\text{Li}$ to hydrogen $^7\text{Li}/\text{H} = 1.374 \times 10^{-10}$ we have calculated using this approach is about four times lower than that estimated using the standard lambda cold dark matter ($\Lambda$CDM) cosmological model, and is consistent with the most agreed observational value of $1.6 (\pm 0.3) \times 10^{-10}$. In the VPC approach Einstein equations are modified to include the variation of the speed of light $c$, gravitational constant $G$ and cosmological constant $\Lambda$ using the Einstein-Hilbert action. Application of this approach to cosmology naturally leads to the variation of the Plank constant $\hbar$ and the Boltzmann constant $k_B$ as well. They approach fixed values at the scale factor $a \ll 1$: $c = c_0/e$, $G = G_0/e^3$, $\hbar = \hbar_0/e$ and $k_B = k_{B0}/e^{6/4}$, where $e$ is the Euler’s number ($= 2.7183$). Since the VPC cosmology reduces to the same form as the $\Lambda$CDM cosmology at very small scale factors, we could use an existing Big-Bang nucleosynthesis (BBN) code AlterBBN with the above changes to calculate the light element abundances under the VPC cosmology. Among other abundances we have calculated at baryon to photon ratio $\eta = 6.1 \times 10^{-10}$ are: $^1\text{He}/\text{H} = 0.2469$, $\text{D}/\text{H} = 1.564 \times 10^{-5}$, $^3\text{He}/\text{H} = 1.642 \times 10^{-5}$. Also, we determined that the neutron lifetime is increased by $e^{1/2}$ and nuclear reaction rates are enhanced by $e^{1/6}$ at BBN.

Key words. nuclear reactions, nucleosynthesis, abundances – primordial nucleosynthesis – cosmology: theory

1. Introduction

The standard model has been immensely successful in explaining cosmological observations better than any of the alternatives offered from time to time. However, it is not able to resolve the so called ‘lithium problem’ (e.g. Israelian 2012, Howk et al. 2012, Singh et al. 2019, Clara & Martins 2020): a gaping discrepancy exists between the observed primordial lithium abundance and that predicted by the standard model’s Big-Bang nucleosynthesis (BBN). The subject has been extensively discussed in the literature. Here we cite the most recent reviews (Cyburt et al. 2016, Tanabashi et al. 2018) which have exhaustive citations on BBN in general and the lithium problem in particular. Many astronomers consider the problem acute enough to suggest the existence of a new physics (e.g. Jedamzik 2004, Pospelov & Pradler 2010, Howk et al. 2012, Cyburt et al. 2016, Clara & Martins 2020).

Many alternative solutions for the lithium problem have been offered. Mathews et al. (2019) have succinctly reviewed them. The offered solutions include: nuclear – reactions destroying lithium after BBN (Richard et al. 2005, Fu et al. 2015); nuclear – reactions destroying lithium during BBN (Lamia et al. 2019); and cosmological – new physics beyond the standard BBN approach. Their focus was to overview a variety of possible cosmological solutions, and their shortcomings. The possibility of physical process that may modify the velocity distribution of particles from that given by Maxwell-Boltzmann statistics has also been considered (Hou et al. 2017, Kusakabe et al. 2019). The modification may be due to inhomogeneous spatial distribution of domains of primordial magnetic field strength (Luo et al. 2019). Such modified velocity distribution could lead to reduced lithium abundance. Another possibility is that scattering with the mildly relativistic electrons in the background plasma alters the baryon distribution to one resembling a Fermi-Dirac distribution (Sasankan et al. 2018, McDermott & Turner 2018). However, the models that are able to reduce the $^7\text{Li}$ production invariably overproduce $\text{D}$. Hybrid models have been developed to resolve the resulting D/H problem as well as the $^7\text{Li}/\text{H}$. However, the hybrid models result in over producing $^7\text{Li}$. Mathews et al. (2019), therefore, concluded that none of the alternatives are able to satisfactorily resolve the lithium problem.

In this paper our attempt is to resolve the lithium problem using the variable physical constant approach. The concept of varying physical constants has been in existence since 1883 as we know it (Thomson & Tait 1883; Weyl 1919; Eddington 1934). It got traction when Dirac (1937; 1938) suggested the variation of the constant $G$ based on his large number hypothesis. Later Brans and Dicke (1961) developed the $G$ variation theory based on general relativity in which constant $G$ was raised to the status of scalar field potential. Even Einstein, who developed his ground breaking theory of special relativity based on the constancy of the speed of light, considered its possible variation (Einstein 1907). The varying speed of light theories were developed comprehensively by Dicke (1957), Petit (1988) and Moffatt (1993a; 1993b). Albrecht and Magueijo (1999) and Barrow (1999) developed such a theory in which Lorentz invariance is broken as there is a preferred frame in which scalar field is minimally coupled to gravity. Other theories include locally invariant theories (Avelino & Martins 1999; Avelino et al 2000) and vector field theories that cause spontaneous violation of Lorentz invariance (Moffat 2016).

A comprehensive review of the varying fundamental physical constants was undertaken by Uzan (2003) followed by his more recent review (Uzan 2011). An update of the observational and experimental status of the constancy of physical constants was done by Chiba (2011).

Several attempts have been made to resolve the lithium problem by varying some constants. The variation of fine structure constant in BBN was considered by Ichikawa and Kawasaki (2002) who also included the non-standard expansion rate of the universe, and most recently by Clara and Martins (2020) who determined that the fine structure constant was larger at the time of BBN than it is now. Landau et al. (2006) attempted to resolve the problem by assuming that the gauge coupling constants were different during the Big-Bang era than they are now and thus affected the nuclear reaction rates. Dmitriev et al. (2004) tried the variation in the deuterium
binding energy. Coc et al. (2007) considered the variation of
Yukawa couplings and the fine structure constant as well as the
variation of the deuterium binding energy. Berengut et al. (2010)
considered the effect of quark mass variation on big bang
nucleosynthesis.

In most of the proposed theories variable physical constants
(VPC) are introduced at the cost of either not conserving energy-
momentum or violating Bianchi identities. This leads to breaking
the covariance of the theory. Such theories are considered
inconsistent or ad hoc (Ellis & Uzan 2005) or quasi-
phenomenological (Gupta 2019). One requires action principle to
take into account the variation of the fundamental constants that
are being considered for generalization of Einstein equations. This
approach was attempted by Costa et al. (Costa et al. 2019;
Franzmann 2017) by considering the speed of light c, the
gravitational constant G, and the cosmological constant Λ as scalar
fields. They introduced the Einstein-Hilbert action that is
considered consistent with the Einstein equations, and the general
constraint that is compliant with contracted Bianchi identities and
standard local conservation laws. It preserves the invariance of the
general relativity and thus is general covariant. The approach of
Costa et al. has confirmed the findings of our quasi-
phenomenological model (Gupta 2019).

Based on the above, we recently were able to develop a VPC
model and show (Gupta 2020) that the model: (a) fits the
supernovae 1a observational data marginally better than the ΛCDM
model; (b) determines the first peak in the power spectrum of the
cosmic microwave background temperature anisotropies at
multipole value of l = 217.3; (c) calculates the age of the universe
as 14.1 Gyr; and (d) finds the BAO acoustic scale to be 145.2 Mpc.

These numbers are within a couple of percent of the accepted
values. This success of the model encouraged us to consider
applying the model to BBN as well.

We begin with establishing the theoretical background for our
work in Section 2. We will confine ourselves to consider only what
needs to be modified in an existing, well established BBN code,
developed for the standard ΛCDM model, in order to make it
compliant with our VPC model.

Section 3 presents the results that we have obtained by
modifying the well-known AlterBBN code (Arbey et al. 2018) that
we found relatively easy to modify as compared to some other
codes we considered, for example PRIMAT (Pitrou et al. 2018). In
Section 4 we discuss the findings of this paper and in Section 5 we
present our conclusions.

2. Theoretical Background

Following Costa et al. (2019) we may write the Einstein equations
with varying physical constants with respect to time t - speed of
light c = c(t), gravitational constant G = G(t) and cosmological
constant Λ = Λ(t) - applicable to the homogeneous and isotropic
universe, as follows:

\[ G_{\mu\nu} = \left(\frac{8\pi G(t)}{c^4}\right)T_{\mu\nu} - \Lambda(t)g_{\mu\nu}. \]  

(1)

Here $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor with $R_{\mu\nu}$ the Ricci
tensor and $R$ the Ricci scalar, and $T_{\mu\nu}$ is the stress energy tensor.

Applying the contracted Bianchi identities, torsion free continuity
and local conservation laws

\[ \nabla^{\mu}g_{\mu\nu} = 0 \text{ and } \nabla^{\mu}T^{\mu\nu} = 0, \]  

(2)

one gets a general constraint equation for the variation of the
physical constants

\[ \left[ \frac{1}{G} \frac{dG}{dc} - \frac{1}{2} \nabla^{\mu} \left( \frac{8\pi G(t)}{c^4} \right) T^{\mu\nu} - \frac{d\Lambda}{dt} \right] g^{\mu\nu} = 0. \]  

(3)

Now the FLRW (Friedmann–Lemaître–Robertson–Walker)
metric for the geometry of the universe is written as:

\[ ds^2 = -c^2(t)dt^2 + a^2(t) \left[ \frac{1}{1-\sigma} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]  

(4)

with $k = -1, 0, +1$ depending on the spatial geometry of the
universe: $-1$ for negatively curved universe, 0 for flat universe,
and $+1$ for positively curved universe.

The stress-energy tensor, assuming that the universe contents
can be treated as perfect fluid, is written as:

\[ T^{\mu\nu} = \frac{1}{c^2(t)} (\epsilon + p) U^{\mu} U^{\nu} + pg^{\mu\nu}. \]  

(5)

Here $\epsilon$ is the energy density, $p$ is the pressure, $U^{\mu}$ is the 4-velocity
vector with the constraint $g_{\mu\nu}U^{\mu}U^{\nu} = -c^2(t)$. (Unless necessary
to avoid confusion, we will drop showing $t$ variation, e.g. $c(t)$ is
written as $c$.)

Solving the Einstein equation (such as by using Maple 2019)
then yields VPC compliant Friedmann equations:

\[ H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G(t)}{3c^2} \left( \frac{\dot{a}}{a} \right)^2 - \frac{\Lambda}{3c^4}, \Rightarrow \ddot{a} = \frac{\dot{a}}{a} \left( \frac{8\pi G(t)}{3c^2} + \frac{\Lambda}{3c^4} \right), \]  

(6)

\[ \dot{a} = -\frac{8\pi G(t)}{3c^4} (\epsilon + p) + \frac{\dot{a}}{a} \]  

\[ \Rightarrow H = \frac{\dot{a}}{a} \]  

(7)

Here a dot on top of a variable denotes the time derivative of that
variable, e.g. $\dot{c} \equiv dc/dt$. Taking time derivative of Eq. (6), dividing
by $2\dot{a}$ and equating it with Eq. (7), yields the general continuity
equation:

\[ \dot{\epsilon} + \frac{3}{a} (\epsilon + p) = -\left[ \frac{G}{c^4} - 4\frac{\dot{c}}{c} \right] \epsilon + \frac{c^4}{8\pi G(t)} \frac{\dot{\Lambda}}{\Lambda}, \]  

(8)

Eq. (3) for the FLRW metric and perfect fluid stress-energy
tensor reduces to:

\[ \left[ \frac{G}{c^4} - 4\frac{\dot{c}}{c} \right] \frac{8\pi G(t)}{c^4} \epsilon + \frac{\dot{\Lambda}}{\Lambda} = 0, \]  

(9)

therefore,

\[ \dot{\epsilon} + \frac{3}{a} (\epsilon + p) = 0. \]  

(10)

Using the equation of state relation $p = w\epsilon$ with $w = 0$ for matter
and $w = 1/3$ for relativistic particles, the solution for this equation
is $\epsilon = \epsilon_0 a^{-3\omega}$, where $\epsilon_0$ is the current energy density of all the
components of the universe when $a = a_0 = 1$.

Next we need to consider the continuity equation Eq. (9). When
$\Lambda$ is constant, $G/\dot{G} = 4\dot{c}/c$. However, one could choose any
relationship between $G$ and $c$, say $G/\dot{G} = \sigma \dot{c}/c$. Then from Eq. (9),
by defining $\varepsilon_\Lambda = c^4 \Lambda/(3c^4)$, we have

\[ \frac{\dot{\varepsilon}_\Lambda}{c} \dot{c} (4-\sigma) \varepsilon = \Lambda, \Rightarrow \dot{\varepsilon}_\Lambda = \frac{\dot{\varepsilon}_\Lambda}{c} \dot{c} (4-\sigma) \varepsilon = \frac{\dot{\varepsilon}_\Lambda}{c} \dot{c} (4-\sigma) \varepsilon = \frac{\dot{\varepsilon}_\Lambda}{c} \dot{c} (4-\sigma) \varepsilon \]  

(11)
The parameter $\sigma$ may be determined based on the physics or by fitting the observations. We have determined in the past (Gupta 2018) that $\sigma = 3$ analytically, i.e. $G/G = 3c^2/c$ and confirmed it by fitting the supernovae (SNe) 1a data (Gupta 2019, 2020). Thus, we must have $\epsilon_\Lambda = c\Lambda e/(cA)$.

The most common way of defining the variation of the constant is by using the scale factor powerlaw (Barrow & Magueijo 1999; Salzano & Dabrowski 2017) such as $c = c_0 a^{\alpha}$ which results in $c = a \Lambda/a = aH$. The advantage is that it results in very simple Friedmann equations. However, as $a \to 0$ the variable constant tends to zero or infinity depending on the sign of $\alpha$. So, it yields reasonable results when $a = 1/(1+z)$ corresponds to relatively small redshift $z$, but not for large $z$. Therefore we have tried another relation that results in:

$$c = c_0 \exp[(a^\alpha - 1)]; G = G_0 \exp[3(a^\alpha - 1)]; \text{ and } A = \Lambda_0 \exp[(a^\beta - 1)].$$

(12)

Their limitation is that $c$ can decrease in the past at most by a factor of $e = 2.7183$ (Euler number) and $G$ can decrease by a factor of $e^{-3}$ (for positive $\alpha$ within the region of their applicability). Using relations of Eq. (12), we can now write Eq. (11) for $\sigma = 3$,

$$\frac{c^\Lambda}{8\pi G}, \epsilon_\Lambda = \frac{c^\Lambda}{c^\Lambda} = \frac{a}{\beta} a^{\alpha-\beta} \epsilon.$$

(13)

The first Friedmann equation, Eq. (6), becomes

$$H^2 = \frac{8\pi G}{3c^2} (\epsilon + \Lambda^e) - \frac{kc^2}{\sigma^2} = \frac{8\pi G}{3c^2} \left(1 + \frac{a}{\beta} a^{\alpha-\beta}\right) - \frac{kc^2}{\sigma^2}.$$

(14)

Here energy density $\epsilon = \epsilon_m + \epsilon_e = \epsilon_m a^{-3} + \epsilon_e a^{-4}$ with subscript $m$ for matter and $r$ for radiation (relativistic particles, e. g. photons and neutrinos). Dividing by $H_0^2$, we get

$$\frac{\mu^2}{H_0^2} = \frac{8\pi G}{3c^2} \left(\epsilon_m a^{-3} + \epsilon_e a^{-4}\right) \left(1 + \frac{a}{\beta} a^{\alpha-\beta}\right) - \frac{kc^2}{\sigma^2} a^2.$$

(15)

At $t = t_0$ (current time), $a = 1$ and $H = H_0$. Therefore,

$$1 = \frac{8\pi G_0}{3c^2 H_0^2} \left(\epsilon_{m,0} + \epsilon_{e,0}\right) \left(1 + \frac{a}{\beta} a^{\alpha-\beta}\right) - \frac{kc^2}{H_0^2}.$$

(16)

Here we have defined the current critical density as $\epsilon_{c,0} = 3c^2 H_0^2/8\pi G_0$, $\Omega_{m,0} = \epsilon_{m,0}/\epsilon_{c,0}$ and $\Omega_{e,0} = \epsilon_{e,0}/\epsilon_{c,0}$. Thus, by defining $\Omega_0 = \left(\Omega_{m,0} + \Omega_{e,0}\right)(1 + a/\beta)$, we may write Eq. (16).

$$\Omega_0 \equiv -\frac{kc^2}{H_0^2} = 1 - \Omega_0,$$

(17)

$$\frac{\mu^2}{H_0^2} = \exp[(a^\alpha - 1)] \left(\Omega_{m,0} a^{-3} + \Omega_{e,0} a^{-4}\right) \left(1 + \frac{a}{\beta} a^{\alpha-\beta}\right) + \Omega_{k,0} \exp[2(a^\alpha - 1)] a^{-2}.$$

(18)

Now we have determined $a = 1.8 = -\beta$ (Gupta 2020). As $a \to 0$, applicable for the nucleosynthesis epoch, $\exp[(a^\alpha - 1)] \to 1/e$, and Eq. (18) approaches

$$\frac{\mu^2}{H_0^2} = \frac{1}{e} \left(\Omega_{m,0} a^{-3} + \Omega_{e,0} a^{-4}\right).$$

(19)

This equation is the same as for the $\Lambda$CDM model except for the factor $1/e$ that effectively alters the matter and radiation densities.

But since both the densities are reduced by the same factor, the baryon to photon ratio $\eta$ remains the same for the VPC model and the $\Lambda$CDM model.

We can see that for $a < 1, c = c_0/e$ and $G = G_0/e^3$. We had also shown (Gupta 2020) that the reduced Planck constant $\hbar$ evolves as $\hbar_0 \exp[(a^\alpha - 1)]$ and the Boltzmann $k_B$ evolves as $k_{B,0} \exp[(5/4)(a^\alpha - 1)]$, which become $\hbar = \hbar_0/e^3$ and $k_B = k_{B,0}/e^{5/4}$ for $a < 1$. This leads to the conclusion that we can use any proven BBN code provided we meticulously redefine all the equations and parameters that may contain these constants. This is a nontrivial task as all codes use mixed units. Since we will be using the AlterBBN code (Arbey et al. 2018, Jenssen 2016), we refer reader to these references and citations therein for the theory used in the code. Nevertheless, we will consider specificities unique to VPC models.

Since predictions from any BBN model rely strongly on nuclear reactions rates and neutron lifetime measured at the present time, it is important to establish whether or not they have the same value back at the time when BBN took place.

Let us first consider how the nuclear reaction rates depend on VPCs. We may write for the general case of two body reactions between incoming particles $i$ and $j$ resulting in outgoing particle $k$ and $l$ (Jenssen 2016)

$$i + j \leftrightarrow k + l.$$

(20)

The forward reaction rate is defined as

$$\nu_{ij-kl} = \frac{n_in_j}{4 \pi ^2 \delta_{ij}} (\sigma v)_{ij-kl}.$$

(21)

Here $n_i$ and $n_j$ are the number densities of the reacting particles, and $(\sigma v)_{ij-kl}$ is the product of the scattering cross section $\sigma$ and the velocity $v$ averaged over the appropriately normalized velocity distribution. The factor with the Kronecker delta $\delta_{ij}$ covers the possibility of $i$ and $j$ being the same type of particles. For three body and higher multibody reactions the equations are more complex. However, our concern is not to determine the reaction rates, but just how they depend on the VPCs. This dependency is determined directly by knowing the dependency of $(\sigma v)$ on the VPCs.

A textbook expression for $(\sigma v)$ is (Maoz 2016)

$$\langle \sigma v \rangle = \left(\frac{\mu}{\sigma \mu}\right) \left(\frac{S_0}{(kT)^2}\right) \int_0^{\infty} e^{-\frac{E}{kT}} e^{-\frac{E}{\sqrt{kT}}} dE.$$
The integrand in Eq. (22) has a peak at $E_0 = (K_B T / 2)^{2/3} E_G^{1/3}$ with a Gaussian shape that has almost zero value at $E \approx 0$. Therefore the lower limit on the integral in Eq. (22) can be changed to $-\infty$ without influencing the result and can then be analytically integrated. After integrating we may write the equation

$$\langle \sigma \nu \rangle = \frac{2\pi^2}{\sqrt{3} \mu} S_0 \frac{N_e}{(k_B T)^{3/2}} \exp \left[ -3 \left( \frac{E_G}{4k_B T} \right)^{3/2} \right].$$

(24)

Substituting for $E_G$ from Eq. (23), we may write

$$\langle \sigma \nu \rangle = \frac{2\pi^2}{\sqrt{3} \mu} S_0 \frac{N_e}{(k_B T)^{3/2}} \exp \left[ -3 \left( \frac{(\pi a Z_i)^2}{4k_B T} \right)^{3/2} \right].$$

$$= \frac{2\pi^2}{\sqrt{3} \mu} \left( \frac{c_0 \exp(a^2-1)}{k_B T \exp(2(a^2-1))} \right)^{2/3} \exp \left[ -3 \left( \frac{(\pi a Z_i)^2}{4k_B T \exp(2(a^2-1))} \right)^{3/2} \right].$$

$$= \frac{2\pi^2}{\sqrt{3} \mu} \left( \frac{c_0 \exp(a^2-1)}{k_B T \exp(2(a^2-1))} \right)^{2/3} \exp \left[ -3 \left( \frac{(\pi a Z_i)^2}{4k_B T \exp(2(a^2-1))} \right)^{3/2} \right].$$

(25)

where we have taken the limit $a \to 0$, applicable to BBN, in the final expression. An inspection of this equation reveals that the original $\langle \sigma \nu \rangle$ is modified due to VPCs as follows:

(a) a factor $(a^{0.25})^{2/3} = 1.1814$ outside the exp function, and
(b) a factor $(a^{-0.75})^{1/3} = 0.4724$ inside the exp function.

For the case (b) the AlterBBN code automatically looks for the reactions rate corresponding to the lower temperature. We therefore need to explicitly consider the modification only due to (a). Since the abundance is a linear function of the reaction rates (Arbey et al. 2018), we can effectively include (a) by multiplying the abundance calculated by AlterBBN by this factor.

The neutron lifetime plays a critical role in the nucleosynthesis. So we need to consider how it will be affected by the reduced value of the constants in the BBN epoch. The theory of neutron lifetime $\tau_n$ (Wietfeldt 2018) is related to the theory of the beta decay (Nanni 2019). In addition, it appears there is some dependency of the lifetime on the plasma medium of the early universe through Fermi-blocking of decay electrons and neutrinos by plasma which results in the neutrons living longer (Yang et al. 2018). All this makes it not only difficult but also uncertain to reliably determine the dependency on VPCs of all the parameters involved in the neutron lifetime theory, and consequently to properly capture the dependency of $\tau_n$ on the VPCs. Therefore, we postulate the neutron lifetime to evolve as follows:

$$\tau_n = \tau_{n,0} \exp[-\frac{1}{2}(a^{1.8} - 1)] \to \tau_{n,0} a^{1/2} \text{ as } a \to 0,$$

(26)

with $\tau_{n,0}$ the current neutron lifetime. We will verify its correctness from the results we will obtain.

3 Results

We will now test the VPC model against observed BBN abundances. For this we have modified the AlterBBN code (Arbey et al. 2018) developed for the standard $\Lambda$CDM model to make it compliant with the VPC model. The significant input parameters used for the VPC model are $\eta = 6.1 \times 10^{-10}$, number of neutrino species 3.046, neutron lifetime $1451 = 880.2 \times 10^{1/2}$ s, and uncertainty in nuclear lifetime 2 s. All the modifications we have made are presented in the Appendix.

We have calculated abundances of the important light nuclei to see how they vary with the baryon to photon ratio $\eta$. The results are graphically presented in Figure 1. We calculated the abundances for equally spaced $\eta$ values from $1 \times 10^{-10}$ to $1 \times 10^{-9}$, and connected the points with smoothing lines. Each of the abundances has three lines corresponding to error in the neutron lifetime and the reaction rates. The lines are indiscernible except for $^7$Li. The lines in the stacked plot show abundances of $^4$He, $^3$He, and $^7$Li as functions of $\eta$ using the VPC model. The vertical band represents the cosmic microwave background (CMB) determined baryon to photon ratio with its errors and uncertainties (Planck collaboration 2019).

Figure 1. This stacked plot shows BBN abundances of $^4$He ($Y_i$), $^3$He, and $^7$Li as functions of baryon to photon ratio $\eta$. The lines are from the VPC model. Three lines for each element represent error in the neutron lifetime and the reaction rates. The lines are indiscernible except for $^7$Li. The vertical band represents the cosmic microwave background based $\eta$ with its errors and uncertainties. The rectangular patches on the band represent the observed abundances and their variation and errors among various observations. Observed abundances for D and $^4$He overlap and thus are represented with a single rectangle.

The rectangular patches on the band represent the observed abundances and their variation and errors among various observations. Observed abundances for D and $^4$He overlap and thus are represented with a single rectangular patch. The plot in Figure 1 is slightly different from the BBN Schramm plot (e.g. Cyburt et al. 2016), but we believe it is displays the analysis of our findings more explicitly.

Cyburt et al. (2016) have discussed the helium abundance data available from several authors. With errors taken into consideration,
BBN is considered a peephole on the earliest universe through the phenomenon of the nucleosynthesis of primordial elements. Most studies assume that such phenomenon is no different than what we study currently in our laboratories. That requires extrapolation over a huge time scale and energy densities that are involved in BBN. So it is not surprising that BBN findings based on such assumptions are not entirely satisfactory. They rely strongly on the laboratory measured nuclear reaction rates and neutron lifetime. These in turn depend on the assumption that the physical constants are fixed in time. By relaxing this assumption within the constraints of general relativity we have successfully resolved three astrometric anomalies which correspond to $\alpha = 1$ (Gupta 2019), and have shown that the cosmology up to the scale factor of CMB ($\alpha \sim 0.001$) can be comfortably explained with the variable physical constants approach (Gupta 2020). This encouraged us to explore if BBN at $\alpha \sim 10^{-9}$ is also amenable to this approach.

Results reported in the last section indeed confirm that BBN abundances are correctly determined with the varying physical constants approach. It naturally resolves the age old ‘lithium problem’ without compromising any of the achievements of the standard ΛCDM model. It even retains all the significant parameters used in the BBN code for the standard model except the neutron lifetime; the constraint on the neutrino species and baryon to photon ratio remains the same.

Recent observations suggest that the bounds we have considered for D/H may be unrealistic, and the acceptable observed value of D/H is possibly significantly higher than we have calculated. The ground based observation of Cooke et al. (2018) that relates to the O/H absorptions system toward the quasar Q1243+307 yields D/H = (2.527 ± 0.030) × 10^{-5}. However, in an earlier paper from the same group, Pettini and Bowen (2001) determined D/H = (1.65 ± 0.35) × 10^{-5} from Hubble Space Telescope (HST) observation towards QSO 2206-199. In the same paper they mention other QSO related observations with D/H determined by other authors with values up to 4 × 10^{-5}. The observations were further analysed by Pettini et al. (2008).

It is claimed that the absorption system in these observations are near-pristine with extremely low metallicities and thus represent BBN abundances of elements. However taking mean of several such observations may not be prudent if the creation of deuterium in post-BBN universe is not ruled out. Mullen and Linsky (1998) state “Contrary to a widespread assumption, deuterium is not simply destroyed in stars: deuterium is also synthesized in the atmospheres of active stars. This non-primordial synthesis of D arises when protons accelerated in flares interact with the atmosphere, create a flux of free neutrons, and these neutrons then undergo radiative capture on atmospheric protons. Radiative capture does not result in excess production of Li, Be, or B.” Thus, the lowest value D/H = (1.65 ± 0.35) × 10^{-5}, the one determined from the HST observation, may be considered closest to the BBN abundance of deuterium. The model predicted value of D/H = 1.564 ± 0.502 × 10^{-5} is within the bounds of this value. As discussed in Sec. 1, the models that reduce 3Li invariably over produce D (Mathews et al. 2019). It does not happen when we apply the VPC model.

Varying physical constants in the context of BBN have been tried by several researchers. We have mentioned some of them in the introduction (Ichikawa & Kawasaki 2004, Dmirtiev et al. 2004, Landau et al 2006, Coc et al. 2007, Berengut et al. 2010, Clara & Martins 2020). However, in all these studies variation of the constants were determined by fitting the measured light element abundances. The VPC approach is different. In the VPC model the most important parameter $\alpha = 1.8$ is determined analytically.

We immediately notice that the theoretical curves for the abundances of the elements considered here are in good agreement with the observations. Obviously, there is no lithium problem.

The actual values of the abundances we computed using the VPC model at $n = 6.1 \times 10^{-10}$ are as follows: $Y_p = 0.2469 \pm 0.02469$, D/H = 1.564 ± 0.502 × 10^{-5}, $^3$He/H = 1.642 ± 0.629 × 10^{-5}, and $^7$Li/H = 1.374 ± 0.270 × 10^{-10}. In addition, we also computed $^6$Li/H = 6.126 ± 0.372 × 10^{-15} for the $^6$Li abundance and $^7$Be/H = 1.255 ± 0.348 × 10^{-10} for the $^7$Be abundance; the former is 59.6% of the value calculated from the ΛCDM model and the latter only 23.2%. To our knowledge, there is no reliable observational data to compare them with.

Let us now consider how the BBN abundances for the elements depend on the neutron lifetime. This is depicted in Figure 2. The plot shows percent variations in BBN abundances of $^4$He ($Y_p$), D, $^3$He, and $^7$Li as a function of the neutron lifetime for the VPC model. The variations are relative to their values at the neutron lifetime of 1451 s which is $e^{0.5}$ times greater than 880.2 s used in the standard ΛCDM model.

Figure 2. This plot shows percent changes in BBN abundances of $^4$He ($Y_p$), D, $^3$He, and $^7$Li as a function of the neutron lifetime for the VPC model. The zero change point is taken at 1451 s which is $e^{0.5}$ times greater than 880.2 s used in the standard ΛCDM model.

Let us now consider how the BBN abundances for the elements depend on the neutron lifetime. This is depicted in Figure 2. The plot shows percent variations in BBN abundances of $^4$He ($Y_p$), D, $^3$He, and $^7$Li as a function of the neutron lifetime for the VPC model. The variations are relative to their values at the neutron lifetime of 1451 s which is $e^{0.5}$ times greater than 880.2 s for the standard ΛCDM model. The variation is most significant for helium ($Y_p$), about the same for $^7$Li, and least significant for $^3$He. This is the same as for the standard BBN model: $Y_p$ ($^4$He/H) is most sensitive to the neutron lifetime and $^3$He/H the least.
measuring the time of flight $\tau$ of the laser photons. If we ignore the variation of the speed of light in the measurement of the distance $r = c\tau$, then we get the upper limit on $\mathcal{G}/G$ several orders of magnitude lower than other methods. The reason for this is the cancellation of the $\mathcal{G}/G$ by the $3e/c$, which can be easily seen by examining the Kepler’s 3rd law $P^2 = 4\pi^2r^3/(GM)$. Taking logarithmic derivative we get $\frac{\mathcal{G}}{G} = 3t/r - 2\dot{P}/P - M/M$. (Merkovitz 2010). If we substitute $r = ct$, we get $\mathcal{G}/G \sim 3/c$ and not $\mathcal{G}/G$. But in the VPC approach $\mathcal{G}/G \sim 3/c$. This will explain very low value of the upper limit on $\mathcal{G}/G$ determined by LLR (e.g. Hofmann & Muller 2018). Similar situations arise in other methods of determining $\mathcal{G}/G$ when complete or near complete cancellation of the variability of physical constants is not taken into account, such as in asteroseismic method (Bellinger & Christensen-Dalsgaardere 2019), in the BBN method (Alveya et al. 2020), in the white dwarf cooling method (Benvenuto et al. 1999), etc.

5. Conclusions

We have shown that the varying physical constants approach can naturally resolve the primordial $^7$Li abundance problem. The value of $^{\text{Li}}/^{\text{H}} = 1.374 \pm 0.07 \times 10^{-10}$ we have calculated is only about 25% of that estimated using the standard cosmological model, and is consistent with the most agreed observational value of $1.6 (\pm 0.3) \times 10^{-10}$. Since the same approach also resolved three astrometric anomalies and fitted well various cosmological observations, including the first peak in the CMB anisotropy power spectrum and supernovae 1a data, we suggest that the VPC approach be tested on various astrophysical observations. We urge that in order to get true VPC dependent results such studies should consider the variation of all the physical constants involved rather than selected one or few, and also carefully consider how various equations involved are affected due to the time dependency of the physical constants, especially when the scale factor is not very small. One additional finding of this work is that the neutron lifetime evolves as $\tau_n = \tau_{n,0}\exp[-(u^{1.8} - 1)/2]$. As the half-life of isotopes may evolve similarly, it could have significant ramifications on the geological and astrophysical studies that assume the isotopes lifetimes to be constant in time.

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Appendix A
Modification of the AlterBBN Code

The purpose of this appendix is to show explicitly the changes that we have made to the AlterBBN v.2 code so that the code could represent the VPC model. We have copied the lines directly from the code. Data used are those included in the AlterBBN v.2 publicly available code (Arbey et al. 2018).

File: src.include.h

Original for the standard CDM model:

```c
#define g_to_GeV 5.66958884538932e+26 /* conversion factor M(kg) * kg_to_GeV = M(GeV) */
#define sigma_SB 0.1649430668482282 /* Stefan-Boltzmann constant */
#define m_to_GeV 5.067738582705779e+15 /* conversion factor l(m) * m_to_GeV = L(GeV^-1) */
#define cm_to_GeV 5.067738582705779e+13 /* conversion factor l(cm) * cm_to_GeV = L(GeV^-1) */
#define s_to_GeV 1.519267407871377e+24 /* conversion factor t(s) * s_to_GeV = t(GeV^-1) */
#define G 6.70809142443796e-39 /* Newton constant in GeV^-2 */
#define Mplanck 1.2209102930946623e+19 /* mass difference between neutron and proton in GeV */
```

```c
#define Gn 6.67428e-8 /* in cm^3.g^-1.s^-2 */
#define K_to_eV 8.16733063738339e-05 /* conversion factor T(10^9 K) * K_to_eV = T(GeV) or T(K) * K_to_eV = T(eV) */
#define m_e 5.109984616e-6 /* electron mass in GeV */
#define g_to_GeV 5.66958884538932e+23 /* conversion factor M(g) * g_to_GeV = M(GeV) */
```
```
#define zeta 1.6103162253325862 /*
3*k_B/(2*c^2*Mu) in GeV^-1 */
#define k_B 8.617330e-5 /* Boltzmann's constant in GeV/GK */
#define alphaem 0.007297353 /* fine-structure constant */

Modified for the VPC model:

#define Gn 3.32292834666267E-09 /*
VPC (1/e^3) in cm^3.g^-1.s^-2 */
#define K_to_eV 2.46890656372771e-05 /* VPC (1/e^1.25) T(10^9 K) * K_to_eV = T(eV) */
#define m_e 69.15618710e-6 /* VPC (1/e^2) electron mass in GeV */
#define g_to_GeV 0.759175295231707e+23 /* VPC (1/e^2) M(g) * g_to_GeV = M(GeV) */
#define kg_to_GeV 0.75917529523170e+26 /* VPC (1/e^2) M(kg) * kg_to_GeV = M(GeV) */
#define sigma_SB 0.16449340668482282 /*
VPC (unchanged) Stefan-Boltzmann constant = pi^2/60 */
#define m_to_GeV 37.4457455698794e+15 /* VPC (e^2) L(m) * m_to_GeV = L(GeV^-1) */
#define cm_to_GeV 37.4457455698794e+13 /* VPC (e^2) L(cm) * cm_to_GeV = L(GeV^-1) */
#define s_to_GeV 4.1297968738617e+24 /*
VPC (e) t(s) * s_to_GeV = t(GeV^-1) */
#define G 1.34750054524155e-37 /* VPC (e^3) Gn*pow(cm_to_GeV,3.)*pow(g_to_GeV,-1.)*pow(s_to_GeV,-2.) Newton constant in GeV^2 */
#define Mplanck 0.27241750391653e+19 /*
VPC (1/e^1.5) in GeV, more precise definition than before */
#define Dmpn 0.00017504 /*
VPC (1/e^2) mass difference between neutron and proton in GeV */
#define zeta 11.898716926 /*
3*k_B/(2*c^2*Mu) in GeV^-1 */
#define k_B 2.468906e-5 /*
VPC (1/e^1.25) Boltzmann's constant in GeV/GK */
#define alphaem 0.007297353 /* VPC (unchanged) fine-structure constant */
```

Original for standard ΛCDM model:

```c
q9[i]=reacparam[i][9] = reacparam[i][9]/2.117; // VPC: mass to eV
sum_DeltaMdY_dt+=Dm[i]/7389.*dY_dt[i]/(M_u*g_to_GeV);
```

Modified for VPC model:

```c
q9[i]=reacparam[i][9]/2.117; // VPC: mass to eV
sum_DeltaMdY_dt+=Dm[i]/7389.*dY_dt[i]/(M_u*g_to_GeV);
```

Original for standard ΛCDM model:

```c
paramrelic->life_neutron=880.2; // Neutron lifetime (PDG2018) 880.2 s
paramrelic->life_neutron_error=1.; // Neutron lifetime uncertainty (PDG2017) 1 s
double rhorad=pi*pi/30.*geffT*pow(T,4.); double rho_photon_1MeV=pi*pi/15.*1.e-12;
```

Modified for VcGΛ model:

```c
paramrelic->life_neutron=1451.; // Neutron lifetime (PDG2018) 880.2 s VPC (e^0.5)
paramrelic->life_neutron_error=2.; // Neutron lifetime uncertainty (PDG2017) 1 s VPC 2 s
double rhorad=pi*pi/30.*2.71828*geffT*pow(T,4.); // VPC (e)
double rho_photon_1MeV=pi*pi/15.*2.71828*1.e-12; // VPC (e)
```

File: src.cosmodel.c

Original for standard ΛCDM model:

```c
double q = 1.29333217e-3/m_e; // q=(mn-mp)/me;
```

Modified for VcGΛ model:

```c
double q = 1.29333217e-3/7.3890561/m_e; // q=(mn-mp)/me; VPC 1/e^2
```