On the Ricci dark energy model

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Abstract

We study the Ricci dark energy model (RDE) which was introduced as an alternative to the holographic dark energy model. We point out that an accelerating phase of the RDE is that of a constant dark energy model. This implies that the RDE may not be a new model of explaining the present accelerating universe.
1 Introduction

Holographic principle \[1\] regards black holes as the maximally entropic objects of a given region and implies the Bekenstein entropy bound of \[S_\lambda = (L\lambda)^3 \leq S_{BH} = \pi M_p^2 L^2\] with \[M_p^{-2} = G\], where \(\lambda\) and \(L\) are the UV cutoff and IR cutoff of the given system \[2\]. On the other hand, Cohen et al. \[3\] suggested that the total energy of the system should not exceed the mass of the same-size black hole, \[E_\lambda = \rho_\lambda L^3 \leq E_{BH} = LM_p^2\] with the energy density \(\rho_\lambda = \lambda^4\). This implies the maximum entropy bound of \[S_\lambda \leq S_{3/4}^{BH}\] \[4\]. Based on the energy bound, Li has proposed the holographic dark energy density \[5\]

\[
\rho_L = \frac{3c^2 m_p^2}{L^2}
\]

with a parameter \(c\). Even though we got the holographic energy density, it is not guaranteed that the holographic energy density could describe the present accelerating universe. Hereafter we choose the reduced Planck mass \(m_p^2 = 1/8\pi G = 1\) for simplicity. In order for the holographic energy density to describe the accelerating universe, we have to choose an appropriate IR cutoff \(L\). For this purpose, we may introduce three length scales of the universe: the apparent horizon (=Hubble horizon for flat universe), particle horizon, and future event horizon. The equation of state is defined by

\[
w_L = -1 - \frac{1}{3\rho_L} \frac{d\rho_L}{dx}
\]

with \(x = \ln a\) the scale factor \(a\).

When the cold dark matter (CDM) is present, the Hubble horizon \(L_{HH} = 1/H\) does not describe the accelerating universe because its equation of state \(w_{HH} = 0\) is the same as the CDM does show. Here \(H = \dot{a}/a\) is the Hubble parameter. The first Friedmann equation with \(\rho_{HH} = 3c^2 H^2\) leads to

\[
\left(1 - c^2\right)H^2 = \frac{\rho_m}{3}
\]

with \(\rho_m = \rho_{m0}/a^3 = \rho_{m0}e^{-3x}\). This provides \(\rho_{HH} \propto 1/a^3 = e^{-3x}\), which implies \(w_m = 0 = w_{HH}\) \[6\]. This means that the holographic dark energy model with \(L_{HH}\) does not describe an accelerating phase except the interacting case with the CDM \[7\].
Recently, it was proposed that the energy density which is proportional to Ricci scalar could derive an accelerating universe \([8, 9]\). However, its origin is still enigmatic because it was not proven how an accelerating phase emerges from the Ricci dark energy model. Especially, the role of \(\dot{H}\)-term is not clearly understood to obtain an accelerating phase. On the other hand, the \(\Lambda\)CDM model based on the cosmological constant is a promising candidate to explain the present accelerating universe, even though there exist a lot of dark models.

In this work we show that an accelerating phase of the RDE is that of a constant dark energy model with a constant \(\omega_\Lambda\). This means that one could not know what kind of dark energy derives our accelerating universe in the RDE.

## 2 Ricci dark energy model without dark matter

Let us start with the following holographic dark energy density

\[
\rho_X = 3\alpha \left(2H^2 + \dot{H}\right) \tag{2.1}
\]

where \(\alpha\) is constant to be determined. The first Friedmann equation is

\[
H^2 = \frac{1}{3} \rho_X. \tag{2.2}
\]

We can rewrite the first Friedmann equation as

\[
H^2 = \alpha \left(2H^2 + \frac{1}{2} \frac{dH^2}{dx}\right) \tag{2.3}
\]

Solving the homogenous equation \((2.3)\) for \(H^2\) leads to the conventional Friedmann equation as

\[
H^2 = Ce^{-\left(4 - \frac{2}{\alpha}\right)x} \equiv \tilde{\rho}_X. \tag{2.4}
\]

Substituting \(\tilde{\rho}_\Lambda\) into the energy conservation equation,

\[
\ddot{\rho}_X = -\dot{\rho}_X - \frac{1}{3} \frac{d\tilde{\rho}_X}{dx} \tag{2.5}
\]

we obtain the dark energy pressure

\[
\bar{p}_X = \left(1 - \frac{2}{\alpha}\right) Ce^{-\left(4 - \frac{2}{\alpha}\right)x}. \tag{2.6}
\]
There are two constants $\alpha$ and $C$ to be determined in the expressions (2.4) and (2.6). Also the equation of state $\tilde{\omega}_X$ is defined as

$$\tilde{\omega}_X \equiv \tilde{\rho}_X \tilde{\rho}_X = \frac{1}{3} \left( 1 - \frac{2}{\alpha} \right).$$  \hspace{1cm} (2.7)

For $\alpha < 1$, it describes the dark energy-dominated universe in the future.

### 3 Constant dark energy model

We start with the first and second Friedmann equations

$$H^2 = \frac{1}{3} \rho_\Lambda, \quad \dot{H} = -\frac{\rho_\Lambda + p_\Lambda}{2}$$  \hspace{1cm} (3.1)

where $\rho_\Lambda$ and $p_\Lambda$ are the energy density and pressure in the energy-stress momentum tensor, respectively

$$T^\Lambda_{\mu\nu} = (p_\Lambda + \rho_\Lambda)U_\mu U_\nu - p_\Lambda g_{\mu\nu}.$$  \hspace{1cm} (3.2)

Then the trace of Einstein equation $G_{\mu\nu} = T^\Lambda_{\mu\nu}$ leads to

$$R = -\rho_\Lambda + 3p_\Lambda$$  \hspace{1cm} (3.3)

with the Ricci scalar $R = -6(2H^2 + \dot{H})$  \hspace{1cm} [10]. The conservation law of $\nabla^\mu T^\Lambda_{\mu\nu} = 0$ implies

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0.$$  \hspace{1cm} (3.4)

Introducing a “constant” equation of state parameter $\omega_\Lambda$ like

$$p_\Lambda = \omega_\Lambda \rho_\Lambda,$$  \hspace{1cm} (3.5)

then one solves equation (3.4) to give

$$\rho_\Lambda = \rho_{\Lambda0} a^{-3(1+\omega_\Lambda)} = \rho_{\Lambda0} e^{-3(1+\omega_\Lambda)x}$$  \hspace{1cm} (3.6)

which is a well-known form for a constant $\omega_\Lambda$.

From Eq.(3.3) together with (3.5), we have

$$\rho_\Lambda = \frac{R}{-1 + 3\omega_\Lambda} = \frac{6}{1 - 3\omega_\Lambda} \left( 2H^2 + \dot{H} \right).$$  \hspace{1cm} (3.7)
Now comparing $\rho_\Lambda$ with $\rho_X$ in Eq. (2.1) implies

$$\alpha = \frac{2}{1 - 3\omega_\Lambda}$$

(3.8)

which determines the constant equation of state as function of $\alpha$ as

$$\omega_\Lambda(\alpha) = \frac{1}{3} \left(1 - \frac{2}{\alpha}\right).$$

(3.9)

Plugging this into Eq. (3.6), one finds the energy density and pressure expressed in terms of $\alpha$

$$\rho_\Lambda = \rho_{\Lambda 0} e^{-\left(\frac{4 - \frac{2}{\alpha}}{\alpha}\right)x}, \quad p_\Lambda = \omega_\Lambda(\alpha) \rho_\Lambda.$$ (3.10)

Also we find the correspondence for $\rho_{\Lambda 0} = C$

$$p_\Lambda = \tilde{p}_X.$$ (3.11)

This means that the RDE without dark matter is nothing but a constant dark energy model.

### 4 Ricci dark energy model with dark matter

In this section, we include the dark matter so that the first Friedmann equation takes the form

$$H^2 = \frac{1}{3} \left(\rho_X + \rho_m\right).$$

(4.1)

The above Friedman equation becomes

$$\frac{dH^2}{dx} + \left(4 - \frac{2}{\alpha}\right)H^2 = -\frac{2}{3\alpha} \rho_m.$$ (4.2)

Solving the inhomogeneous equation (4.2) for $H^2$, we obtain a new Friedmann equation

$$H^2 = \frac{\rho_{m 0}}{3} e^{-3x} + \frac{\alpha}{2 - \alpha} \frac{\rho_{m 0}}{3} e^{-3x} + Ce^{-\left(\frac{4 - \frac{2}{\alpha}}{\alpha}\right)x} \equiv \frac{\rho_m + \tilde{\rho}_X}{3}$$

(4.3)

where $C$ is an integration constant. Importantly, $\tilde{\rho}_X$ defined by

$$\tilde{\rho}_X = \frac{\alpha}{2 - \alpha} \rho_{m 0} e^{-3x} + 3Ce^{-\left(\frac{4 - \frac{2}{\alpha}}{\alpha}\right)x}$$

(4.4)
plays the role of a new scaled dark energy density.

Substituting $\tilde{\rho}_X$ into the energy conservation equation,

$$\tilde{\rho}_X = -\tilde{\rho}_X - \frac{1}{3} \frac{d\tilde{\rho}_X}{dx}$$  \hspace{1cm} (4.5)

we obtain the dark energy pressure

$$\tilde{\rho}_X = \left(1 - \frac{2}{\alpha}\right) Ce^{-\left(4 - \frac{2}{\alpha}\right)x}. \hspace{1cm} (4.6)$$

Also the “dynamical” equation of state $\tilde{\omega}_X$ is obtained as

$$\tilde{\omega}_X \equiv \frac{\tilde{p}_X}{\tilde{\rho}_X} = \frac{1}{3} \left(1 - \frac{2}{\alpha}\right) \frac{1}{1 + \frac{\alpha}{3(2 - \alpha)} C e^{(1 - \frac{2}{\alpha})x}}. \hspace{1cm} (4.7)$$

For $\alpha < 2$ and $x \gg 1$, we have an approximately constant equation of state

$$\tilde{\omega}_X \approx \frac{1}{3} \left(1 - \frac{2}{\alpha}\right), \hspace{1cm} (4.8)$$

which describes the dark energy-dominated universe in the future.

## 5 Constant dark energy model with dark matter

In this section, we include the dark matter so that the first Friedmann equation takes the form

$$H^2 = \frac{1}{3} \left(\rho_\Lambda + \rho_m\right). \hspace{1cm} (5.1)$$

The above Friedmann equation leads to

$$\frac{dH^2}{dx} + 3 \left(1 + \omega_\Lambda\right) H^2 = \omega_\Lambda \rho_m. \hspace{1cm} (5.2)$$

Solving the inhomogeneous equation \([5.2]\) for $H^2$, we obtain a new Friedmann equation

$$H^2 = \frac{\rho_m^0}{3} e^{-3x} + \frac{\rho_\Lambda 0 e^{-3(1 + \omega_\Lambda)x}}{3} \equiv \frac{\rho_m + \rho_\Lambda}{3}. \hspace{1cm} (5.3)$$

This equation is compared with Eq.\((4.2)\): two homogeneous equations are the same if one chooses $\omega_\Lambda = \frac{1}{3}(1 - \frac{2}{\alpha})$, while right hand sides are different as $\omega_\Lambda$ and $-2/3\alpha$. Here the “constant” equation of state $\omega_\Lambda$ is obtained as

$$\omega_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda}. \hspace{1cm} (5.4)$$

For $\alpha < 2$ and $x \gg 1$, we have the dark energy-dominated universe $\rho \simeq \rho_\Lambda$ in the future.
6 Discussion

We consider the case without the dark matter. In this case, we note that the relation between energy density and Hubble parameter (its derivative) is found to be

$$\rho_{\Lambda} = 3H^2 = -\frac{R}{1 - 3\omega_{\Lambda}} = \frac{6}{1 - 3\omega_{\Lambda}}(2H^2 + \dot{H}) = \rho_X, \quad (6.1)$$

which shows clearly that the Ricci dark energy model is recovered from a constant dark energy model. The first equality is the first Friedmann equation for constant dark energy, the second one relation indicates the trace of Einstein equation (redundant one), and the last one is the definition of Ricci dark energy density for $\omega_{\Lambda} = \frac{1}{3}(1 - \frac{2}{\alpha})$. This means that the Ricci dark energy model is nothing but a constant dark energy model. Especially, the role of $\dot{H}$-term is clearly understood in the Ricci dark energy model. This appears because the Ricci scalar equation $\Box H$ (trace of Einstein equation) incorporates the first ($H^2$) and second ($\dot{H}$) Friedmann equations in the single, redundant equation. That is, a mysterious term of $\dot{H}$ is nothing but the term in the second Friedmann equation $\Box H$. Finally, $\alpha$ is replaced by a constant $\omega_{\Lambda}$.

In the case with dark matter, two are slightly different because the energy density $\tilde{\rho}_X$ of Eq.(4.4) is composed of two parts: one part evolves like dark matter of $e^{-3x}$ and another part is dark energy term of $e^{-(1-2/\alpha)x}$ with $\alpha < 1$. As a result, we have the “dynamical” equation of state $\tilde{\omega}_X$, while the “constant” equation of state $\omega_{\Lambda}$ is obtained for the constant dark energy model. Even though there exists a slight difference between energy densities $\rho_{\Lambda}$ and $\tilde{\rho}_X$, the Ricci dark energy model describes constant dark energy model for $x \gg 1$.

In conclusion, the origin of Ricci dark energy model is just the constant dark energy model.

Acknowledgments

References

[1] R. Bousso, Rev. Mod. Phys. 74 (2002) 825 [arXiv:hep-th/0203101].
[2] J. D. Bekenstein, Phys. Rev. D 7 (1973) 2333.

[3] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Rev. Lett. 82 (1999) 4971 [arXiv:hep-th/9803132].

[4] G. ’t Hooft, arXiv:gr-qc/9310026.

[5] M. Li, Phys. Lett. B 603 (2004) 1 [arXiv:hep-th/0403127].

[6] S. D. H. Hsu, Phys. Lett. B 594 (2004) 13 [arXiv:hep-th/0403052].

[7] R. Horvat, Phys. Rev. D 70 (2004) 087301 [arXiv:astro-ph/0404204].

[8] C. Gao, X. Chen and Y. G. Shen, arXiv:0712.1394 [astro-ph].

[9] C. J. Feng, arXiv:0806.0673 [hep-th]; C. J. Feng, Phys. Lett. B 670 (2008) 231 [arXiv:0809.2502 [hep-th]]; L. N. Granda and A. Oliveros, Phys. Lett. B 669 (2008) 275 [arXiv:0810.3149 [gr-qc]]; C. J. Feng, arXiv:0810.2594 [hep-th]; L. N. Granda and A. Oliveros, arXiv:0810.3663 [gr-qc]; L. Xu, W. Li and J. Lu, arXiv:0810.4730 [astro-ph]; L. N. Granda, arXiv:0811.4103 [gr-qc]. C. J. Feng, arXiv:0812.2067 [hep-th].

[10] R. Aldrovandi and J. G. Pereira, arXiv:0812.3438 [gr-qc].