MICROLENSING BY KUIPER, OORT, AND FREE-FLOATING PLANETS

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\textbf{Abstract:} Microlensing is generally thought to probe planetary systems only out to a few Einstein radii. Microlensing events generated by bound planets beyond about 10 Einstein radii generally do not yield any trace of their hosts, and so would be classified as free floating planets (FFPs). I show that it is already possible, using adaptive optics (AO), to constrain the presence of potential hosts to FFP candidates at separations comparable to the Oort Cloud. With next-generation telescopes, planets at Kuiper-Belt separations can be probed. Next generation telescopes will also permit routine vetting for all FFP candidates, simply by obtaining second epochs 4–8 years after the event. At present, the search for such hosts is restricted to within the “confusion limit” of $\theta_{\text{confus}} \sim 0.25''$, but future WFIRST observations will allow one to probe beyond this confusion limit as well.

\textbf{Key words:} astrometry – gravitational microlensing – planets – stars

1. \textbf{Introduction}

\textsuperscript{1}Sumi et al. (2011) have reported evidence of a vast population of free-floating planets (FFP) based on an excess of short timescale ($t_L \sim 1$ day) microlensing events seen in observations toward the Galactic bulge by the Microlensing Observations in Astrophysics (MOA) collaboration. As the authors recognize, these planets are not necessarily unbound: they may simply be so far from their hosts that the host leaves no trace on the microlensing event. \textsuperscript{1}Sumi et al. (2011) searched for two such effects: (1) “bumps” (perhaps of very low amplitude) due to microlensing by the host long before or after the short “FFP” event; (2) distortions in the short event (relative to a point lens event) due to shear from the host. For each FFP candidate, they quantified the limits that could be put on hosts due to the absence of both effects.

Generically, one expects the hosts to be detectable only for planets at projected separations of up to about 5 host Einstein radii. Of course, any particular host might be detected at much larger separation provided that the source trajectory is sufficiently closely aligned with the planet-star separation axis. For example \textsuperscript{2}Poleski et al. (2013) and \textsuperscript{1}Sumi et al. (2016) detected two such examples, which they characterized as “Uranus” and “Neptune” analogs, respectively. However, for a large fraction of such ice-giant analogs, the host would have left no trace. And this would be even more true of more distant planets.

Plants that leave no trace may turn out to be in Kuiper-Belt like orbits at $15 \lesssim a/r_{\text{snow}} \lesssim 150$, or in Oort-Cloud like orbits $150 \lesssim a/r_{\text{snow}} \lesssim 10^4$. Here

$$r_{\text{snow}} \sim 2.7 \text{ AU} \frac{M}{M_\odot}$$

is the snow line, assumed to scale linearly with stellar mass. The snow line can be related to the Einstein radius $r_E = D_L \theta_E$ by

$$\frac{r_{\text{snow}}}{r_E} = 2.7 \frac{\pi_L(M/M_\odot)}{(\kappa M \pi_{\text{rel}})^{1/2}} = 0.95 \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{D_L D_L S/D_S}{\text{kpc}} \right)^{-1/2}$$

Here, $\kappa = 4G/c^2 \text{AU} = 8.14 \text{ mas} M_\odot^{-1}$, $\pi_L = AU/D_L$ is the lens parallax, $\pi_{\text{rel}} = \pi_L - \pi_S$ is the lens-source relative parallax, and $D_LS = D_S - D_L$. Thus, for typical lens masses ($M \sim 0.3 M_\odot$) and distances $D_L D_L S/D_S \sim 1$ kpc, we have $r_{\text{snow}} \sim 0.5 r_E$. Therefore, Kuiper planets are at least 7.5 Einstein radii from their hosts, and thus it is very unlikely that their hosts will induce microlensing signatures on the “FFP” event.

2. \textbf{Confusion Limit of Microlensing}

On very general grounds, the optical depth to microlensing is of order $\tau \sim \mathcal{O}(v^2/c^2)$ where $v$ is the characteristic velocity of the system. For the Milky Way, this predicts $\tau \sim 10^{-6}$ which agrees with observations. That is, the probability that some lens will lie within $n$ Einstein radii of a given location is $p = n^2 \tau$. Or, in other words, we expect of order one such lens to lie within $\theta_{\text{confus}} = \tau^{-1/2} \theta_E \gtrsim 10^4 \theta_E$ of any given point. Since typically $\theta_E \sim 0.25$ mas, this corresponds to $\theta_{\text{confus}} \sim 0.25''$.

In particular, for genuine FFPs, we expect the nearest star (other than the source) to be separated by of order $\Delta \theta \sim \theta_{\text{confus}}$. The only exception would be a companion to the source. Of course, by chance, there could be
unassociated stars that lie closer than this. However, the prior probability for, e.g., an unassociated star at $\Delta \theta = 0.1 \theta_{\text{confus}} \sim 25 \text{ mas}$ would be just 1%. While not much can be concluded from the detection of such a nearby star in any particular case, even a few such detections in a sample of a few dozen “FFPs”, would be evidence for a population of well-separated planets.

Such closely separated stars cannot be resolved even in excellent-seeing ground-based survey data. Therefore, special efforts and/or new space-based surveys are required to detect them.

3. Oort Planets from AO on Existing Telescope

Batista et al. (2015) and Bennett et al. (2015) have detected a lens star that had separated from the source by $\sim 60 \text{ mas}$ due to $\sim 8 \text{ yr}$ of relative proper motion using Keck adaptive optics (AO) and Hubble Space Telescope (HST) observations, respectively. In that case, the lens and source were of comparable brightness. Hence, it is plausible that one could detect very faint hosts of FFPs at $\Delta \theta \sim 100 \text{ mas}$, i.e., roughly $400 \theta_E$ or $800 r_{\text{snow}}$.

If such a star were detected, it would not automatically imply that it was the host to the “FFP”. The star could be a companion to the source, a companion to the true host, which was itself much closer to the planet, or a random interloper. The first of these possibilities is easily tested by taking a second epoch of observations a few years later. If the detected star is a companion to the source, the two will have the same proper motion..

Then, assuming it passes this test, there are only three possibilities: host, companion to the host, or random interloper. Either of the first two possibilities would imply that this is not an FFP. As mentioned above, the last would occur with probability $p \sim 16\% (\Delta \theta / 100 \text{ mas})^2$. Hence, by searching for such hosts of a few dozen FFPs, one could place significant limits on such hosts (or confirm their existence).

However, even if the result of this statistical analysis were that a significant fraction of putative FFPs had distant companions, this still would not distinguish Oort planets from Kuiper planets whose hosts happen to have substantially more distant companions.

4. Kuiper Planets from AO on Next Generation Telescopes

Next generation telescopes such as the Giant Magellan Telescope (GMT), Thirty Meter Telescope (TMT) and Extremely Large Telescope (ELT) will have diameters 2.5 to 4 times larger than Keck and will operate effectively in J band compared to the K band observations of Keck, and so will be able to search for hosts that are closer by a factor 5–8. To be concrete, I adopt a factor 6, i.e., $\Delta \theta \sim 17 \text{ mas}$ or about $70 \theta_E$. This is within the Kuiper range of planetary orbits that I defined above. The chance of random interlopers is reduced by a factor $6^2 = 36$ at this limit, i.e., to of order $p \sim 0.5 \%$. While, this is still not low enough to absolutely rule out this possibility in any individual case, it would enable very strong statistical statements with even three detections out of a few dozen searches. Moreover, the possibility that the detected star is a companion to the host rather than the host itself will be far more restricted for the close-in detections enabled by next-gen telescopes. That is, an FFP candidate would have to be a least a few $\theta_E$ from a putative host to avoid the two detectable effects mentioned above (bump from host and distortion of planetary event due to shear from host).

5. AO Imaging of All FFP Candidates

Indeed, all FFP candidates can be directly tested for distant companions “simply” by obtaining two high resolution images: one at the time of the event and one at a sufficiently later time that the source and lens can be separately resolved. If the planet has a luminous host (i.e., not a brown dwarf, neutron star or black hole), this will appear next to the source at the second epoch.

I have placed “simply” in quotation marks because with present technology, the second epoch must wait about 25 years to achieve 100 mas separation, even assuming typical proper motions of $\mu = 4 \text{ mas yr}^{-1}$. To be relatively certain that the failure to observe the host was not just due to abnormally low proper motion, one should really wait 50 years.

However, with next generation telescopes, these numbers will each come down by a factor 6. Hence, one could obtain a good statistical understanding after just 4 years, and an excellent one after 8 years.

6. Probing Kuiper Planets with WFIRST

WFIRST is a planned NASA mission, currently in Phase A. It will have a substantial microlensing component, probably consisting of six 72-day campaigns centered on quadrature and covering $2.8 \text{ deg}^2$ with a 15 min cadence (Spergel et al., 2015). True FFPs detected by WFIRST should be point-lens events with blended light fractions consistent with zero. In principle, there could be blended light from a companion to the source or from a random interloper. However, as mentioned above the first possibility is easily vetted by checking for common proper motion with the source over several years using.
AO imaging. The prior probability of the second is, as before, \( p = (\Delta \theta / \theta_{\text{confus}})^2 \). For example, if the blended light is \( f_b = 10\% \) of the source light, and the putative FFP event is detected with total \( (S/N) = 100 \), then the offset between the blend and the source (and so lens) can be measured with precision \( \sigma(\Delta \theta) = \theta_{\text{confus}}/f_b(S/N) \rightarrow 10 \text{mas} \). Here \( \theta_{\text{confus}} \) is the characteristic size of the point spread function. Thus, these measurements could both reliably detect offsets and make strong statistical statements against interlopers.

7. Beyond the Confusion Limit

The confusion limit of \( \theta_{\text{confus}} = 0.25'' \) corresponds to about 2000 AU, which is well inside the zone containing the Oort Cloud comets that come by the Sun after being perturbed by random stars. Moreover, it would be difficult to reliably identify a star even a factor few closer than this limit as the host of a putative FFP based on statistical arguments alone. Is it possible to probe out to – or beyond – this confusion limit?

Zhu & Gould (2015) showed that parallaxes of FFPs by simultaneously observing the \textit{WFIRST} fields from a network of ground-based observatories. See also Han et al. (2004) and Yee (2013). Zhu & Gould (2015) showed that particularly for low mass FFPs \((m_p \lesssim M_{\text{Jup}})\), measurements of the full 2-D microlens parallax \( \pi_E \) are possible. Here

\[
\pi_E = \pi_E \mu_{\text{rel}} / \mu_{\text{rel}}, \quad \pi_E = \pi_{\text{rel}} / \theta_E \tag{3}
\]

Gould (1992), where \( \mu_{\text{rel}} \) is the lens-source relative proper motion.

Each candidate host that is observed in AO images can be vetted as follows. First one can check whether the direction of proper motion \( \mu_{\text{rel}} \) is consistent with the direction of the \( \pi_E \) parallax of the FFP. Now, in fact, there is a slight wrinkle here because \( \pi_E \) is measured in the geocentric frame whereas the relative proper motion of the putative host and the source would be measured in the geocentric frame. These differ by

\[
\delta \mu_{\text{rel}} = \mu_{\text{rel}, \text{hel}} - \mu_{\text{rel}, \text{geo}} = \frac{\pi_{\text{rel}}}{\text{AU}} \mathbf{v}_{\odot, \perp} \tag{4}
\]

where \( \mathbf{v}_{\odot, \perp} \) is the velocity of Earth projected on the sky at the time of the event. However, first \( \mathbf{v}_{\odot, \perp} \) is typically small for \textit{WFIRST} because the observations are centered at quadrature. Second, \( \pi_{\text{rel}} \) of the putative host can be estimated photometrically. Hence, \( \delta \mu_{\text{rel}} \) is both small and partly calculable. For example, if \( \mathbf{v}_{\odot, \perp} = 10 \text{km s}^{-1} \) and one estimates \( \pi_{\text{rel}} \) with an error of \( \sigma(\pi_{\text{rel}}) = 0.03 \text{mas} \), then the error in the estimate of this difference is \( \sigma(\delta \mu_{\text{rel}}) \approx 0.06 \text{mas yr}^{-1} \), which is quite small compared to typical proper motions of microlensing events, \( \mu_{\text{rel}} \approx 4 \text{mas yr}^{-1} \).

Second, in some cases, one can estimate the magnitude of the proper motion as well as the direction. That is, \( \mu_{\text{rel}} = \theta_E / \pi_E = \pi_{\text{rel}} E / \pi_E \). For putative hosts in the bulge, \( \pi_{\text{rel}} \) cannot be estimated precisely enough for this equation to be useful. However, for hosts in the disk, \( \pi_{\text{rel}} \) can often be estimated photometrically to much better than a factor 2. Hence, for these, both the magnitude and direction of \( \mu_{\text{rel}} \) can be predicted well enough to eliminate most random interlopers as hosts.

Finally, in a significant minority of cases for which \( \pi_E \) can be measured by combined ground-based and \textit{WFIRST} observations, \( \theta_E \) can also be measured (Zhu & Gould, 2015). For these, the full proper motion of putative hosts can be predicted. Moreover, their photometrically estimated relative parallax must be consistent with that of the FFP, i.e., \( \pi_{\text{rel}} = \pi_E \theta_E \).

It is notable that very low mass FFPs, i.e., in the Mars–Earth range, are the most likely to yield parallax measurements. If the mechanisms that drive objects into Oort-Cloud like orbits are similar to those in the Solar System, then these are most feasibly applied to such low mass objects.

8. Dynamical Stability of Oort Planets

If planets were launched into orbits similar to those of Oort-Cloud comets when a given planetary system formed, would they remain bound until today?

To address this I consider first the deflection due to the closest single encounter with a passing star (to either the planet or the host), which has impact parameter \( b \) satisfying \( 2\pi b^2 m T \approx 1 \), where \( n \) is the number density of ambient stars, \( v \) is their typical velocity relative to the host, and \( T \) is the age of the planetary system. In this closest impact, the host or planet will be deflected in velocity by \( \delta v = 2 G m / b v \) where \( m \) is the mass of the passing star. I then set \( (\delta v)^2 = G m^2 / a \) as the condition to ionize the planet, where \( m^2 \) is the host mass and \( a \) is the semimajor axis. For simplicity, I take \( m^2 = m \) and then find

\[
a_{\text{max}} = \frac{v}{8 \pi G \rho T} \tag{5}
\]

where \( \rho \equiv n m \) is the stellar density. For disk lenses, \( \rho \sim 0.05 M_\odot \), \( v \sim 50 \text{ km s}^{-1} \), and \( T = 5 \text{ Gyr} \), which implies \( a_{\text{max}} \sim 2 \text{ pc} \). This is consistent with the survival of the Oort Cloud in our own Solar System.

For basically self-gravitating stellar systems like the Galactic bulge, one may first note that \( v^2 / 8 \pi G \rho \sim R^2 \) where \( R \) is the size of the system. Hence

\[
a_{\text{max}} \rightarrow \frac{R^2}{v T} \quad \text{(self – gravitating).} \tag{6}
\]

Adopting \( R \sim 1 \text{ kpc} \), \( T = 10 \text{ Gyr} \) and \( v = 150 \text{ km s}^{-1} \) for the bulge, one finds \( a_{\text{max}} \sim 0.7 \text{ pc} \).

A more precise calculation would take account of diffusive processes from multiple sub-ionizing encounters. However, it is clear from this calculation that planets in Oort-like orbits are permitted even in the Galactic bulge.

9. Conclusion

Microlensing is generally thought to probe planetary systems on scales of the Einstein radius, or perhaps a few Einstein radii. I have shown that by combining microlensing observations of seemingly isolated planets
with high resolution imaging, one can probe planetary systems out to the confusion limit $\Delta \theta \sim \theta_{\text{confus}} \sim 0.25''$, and even beyond.

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