M Theory and Cosmology

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Abstract: This is a series of lectures on M-theory for cosmologists. After summarizing some of the main properties of M-theory and its dualities I show how it can be used to address various fundamental and phenomenological issues in cosmology.

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1. Introduction

This is a series of lectures on superstring/M-theory for cosmologists. It is definitely not a technical introduction to M-theory and almost all technical details will be omitted. A secondary aim of these lectures (or rather the lecture notes – for there will probably not be many professional string theorists at the actual sessions) is to proselytize for a certain point of view about M-theory, which is not the conventional wisdom. A crude statement of this point of view is that many of the key questions of M-theory can be asked only in the cosmological context, in particular the central phenomenological question of vacuum selection. I also believe that some of the fundamental structure of M-theory, and the relation between quantum mechanics and spacetime geometry is obscured when one tries to study only Poincaré invariant vacuum states of the theory, and ignore cosmological questions. The latter ideas are very speculative however, and I will not discuss them here.

The classic justification of string theorists for studying states of M-theory with $d \geq 4$ Poincaré invariance, in a world which is evidently cosmological, is that the universe we observe is locally approaching a Poincaré invariant vacuum. Many of the properties of the world should be well approximated by those of a Poincaré invariant state. It is a philosophy rooted in particle physics, and we shall see that it has been quite successful in M-theory as well. One of the key features of such states is that they can have superselection sectors (a special case of which is the phenomenon of spontaneous breaking of global symmetries). There can be different Poincaré invariant states in the same theory which “do not communicate with each other” in the following sense: Certain finite energy excitations of Poincaré invariant vacua can be classified as asymptotic states of a number of species of particles. States with any finite number of particles differ from the vacuum only in a local vicinity of the particles’ asymptotic trajectories (this is more or less the cluster property). We can construct the scattering matrix for particle excitations of a given vacuum state and it is unitary. No initial multiparticle excitation of a given vacuum ever scatters to produce excitations of another $^1$. In supersymmetric (SUSY) theories it is quite common to have superselection sectors

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$^1$I have taken pains here not to use arguments from local field theory, which can be only approximate in M-theory.
that are not related by a symmetry. A single theory can produce many different kinds of physics, which means that it does not make definite predictions.

In the early days of string theory, when the vast vacuum degeneracy of string perturbation theory was discovered, it was hoped that nonperturbative effects would either lift the degeneracy or show us that many of the apparent classical ground states were inconsistent (as e.g. an $SU(2)$ gauge theory with an odd number of isospin one half fermions is inconsistent). From the earliest times there were arguments that this was unlikely to be true for those highly supersymmetric ground states which least resemble the real world. Recent discoveries in string duality and M-theory make it virtually impossible to believe that these archaic hopes will be realized.

From the very beginning I have argued (mostly in private) that the resolution of this degeneracy would only come from the study of cosmology\(^2\). That is, the physics that determines the correct Poincaré invariant vacuum took place in the very early history of the universe. To understand it one will have to understand initial conditions, and not just stability criteria for possible endpoints of cosmological evolution. Not too much progress has been made along these lines, but there are not many people thinking about the problem (I myself have probably devoted a total of no more than two years since 1984 to this issue.). Nonetheless, I hope to convince you that it is a promising area of study.

Associated with the vacuum degeneracy, there are massless excitations. I do not have an argument for this which does not depend on an effective field theory approximation. In effective field theory, the vacuum degeneracy is parametrized by the zero modes of a collection of scalar fields (which we will call the moduli fields or simply the moduli\(^3\)) with no potential. Fields like this spell trouble for phenomenology. It is difficult to find arguments that they couple significantly more weakly than gravity (see however [19]), and there is no reason for them to couple universally. Thus, they should affect the orbits of the planets and Eotvos-Dicke experiments.

On the other hand, we know that SUSY is broken in the real world, and then there is no reason for scalar fields to remain massless. This however does not eliminate all of the problems and opportunities associated with the moduli. First of all, one can argue that the potential for the moduli vanishes in many different, phenomenologically unac-

\(^2\)The earliest conversation of this type that I remember was with Dan Friedan and took place in 1986 or 1987.

\(^3\)The term moduli space is used by mathematicians to describe multiparameter families of solutions to some mathematical equations or conditions. Thus one speaks of “the moduli space of Riemann surfaces of genus $g$” or “the moduli space of solutions to the X equation”. Physicists have adopted this language to describe spaces of degenerate ground states of certain supersymmetric theories. We will be making a further abuse of the terminology in our discussion of cosmology.
ceptable, extreme regions of moduli space, where supersymmetry is restored. Examples of such regions are weakly coupled SUSY string compactifications and regions where the world has more than four large dimensions. A quite general argument [55] shows that one cannot find a stable minimum of the system by any systematic expansion in the small parameters which characterize those extreme regions. Either one must accept the possibility of different orders in an asymptotic expansion being equally important in a region where the expansion parameter is small, or one is led to expect that the moduli vary with time on cosmological time scales. The latter option typically leads to unacceptable time variation of the constants of nature. Of course, it might also provide interesting models of the fashionable “quintessence” [9], if these difficulties can be overcome.

Even if one finds a stable minimum for the modular potential there are still difficulties. These are a consequence of additional assumptions about the nature of SUSY breaking. It is usually assumed that SUSY has something to do with the solution of the gauge hierarchy problem of the standard model. If so, the masses of superpartners of quarks should not be more than a few TeV and one can show that this implies that the fundamental scale of SUSY breaking cannot be larger than about $10^{11}$ GeV. One then finds that the moduli typically have masses and lifetimes which are such that the universe is matter dominated at the time nucleosynthesis should have been occurring. This is the cosmological moduli problem. There have been several solutions proposed for it, which are discussed below. On the other hand, there has been much recent interest in models with very low scales of SUSY breaking. These include gauge mediated models and models with TeV scale Planck/String mass and large extra dimensions. Here the cosmological moduli problem is more severe, though a recent paper claims that it can be solved by thermal inflation [18].

Another potential problem with moduli was pointed out by [54]. Since the SUSY breaking scale is smaller by orders of magnitude than the natural scale of the vacuum energy during inflation (in most models) one must find an explanation of the discrepancy. A favored one has been that the true vacuum lies fairly deep in an extreme region of moduli space, typically the region of weak string coupling. The universe then begins its history at an energy density many orders of magnitude larger than the barrier which separates the true vacuum from the region of extremely weak coupling where time dependent fundamental parameters and unwanted massless particles destroy any possibility of describing the world we see. Why doesn’t it “overshoot” the true vacuum and end up in the weak coupling regime? We will discuss a cosmology at the end of

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4The latter are often discussed without reference to SUSY, since the hierarchy problem is “solved” by the low Planck scale. However, if they are to be embedded into M-theory they must have SUSY, broken at the TeV scale.
these lectures that resolves this problem.

Not all the news is bad. One of the things I hope to convince you of in these lectures is that M-theory moduli are the most natural candidates in the world for inflaton fields. The suggestion that moduli are inflatons was first made in [24]. The word natural is used here more or less in its technical field theoretic sense; that large dimensionless constants in an effective Lagrangian require some sort of dynamical explanation. For moduli, in order to get $n$ e-foldings of slow roll inflation one needs dimensionless parameters of order $1/n$ in the Lagrangian. Another interesting point is that with moduli as inflatons, the scale of the vacuum energy that is required to explain the amplitude of primordial density fluctuations is the same as the most favored value of the unification scale for couplings and close to the scale determining the dimension five operator that gives rise to neutrino masses. These numbers fit best into an M-theory picture similar in gross detail to that first proposed by Witten [44] in the context of the Hořava-Witten [43] description of strongly coupled heterotic strings. In such scenarios, $10^{16}$ GeV is the fundamental length scale and the fields of the standard model live on a domain wall in an eleven dimensional space with 7 compact dimensions of volume $\sim 10^4$ fundamental units. The four dimensional Planck scale is an artifact of the large volume. The SUSY breaking vacuum energy responsible for inflation must also come from effects confined to a (perhaps different) domain wall.

To summarize, M-theory has a number of features which require cosmological explanations and a number of potentially interesting implications for cosmologists. The two subdisciplines have very different cultures, but they ought to see more of each other. The plan of these lectures is as follows. I will first introduce the elements of string duality and M-theory, starting from the viewpoint of 11D SUGRA, which involves the smallest number of new concepts for cosmologists. The key ideas will be the introduction of the basic half SUSY preserving branes of 11D SUGRA and the demonstration of how various string theories arise as limits of compactified versions of the theory. We will see that the geometry and even the topology of space as seen by low energy observers can change drastically in the course of making smooth changes of parameters. Another key concept is that of the moduli space of vacua which preserve a certain amount of SUSY, and the various kinds of nonrenormalization theorems which allow one to make exact statements about the properties of these spaces.

From this we will turn to a discussion of the fundamentals of quantum cosmology. This discussion will be incomplete since the material is still under development. We will review the problem of time in quantum cosmology and a standard resolution of it based on naive semiclassical quantization of the Wheeler-DeWitt equation. We will argue that M-theory promises to put this argument on a more reliable basis, and in particular that the peculiar Lorentzian metric on the space of fields, which is the basis for the success
of the Wheeler-DeWitt approach to the problem of time, can be derived directly from the duality group of M-theory (at least in those highly supersymmetric situations where the group is known). This leads naturally into a discussion of whether cosmological singularities can be mapped to nonsingular situations via duality transformations (it turns out that some can and some can’t and that the distinction between them defines a natural arrow of time). We also present a weak anthropic argument which attempts to answer the question of why the world we see is not a highly supersymmetric stable vacuum state of M-theory. Finally, as an amusement for aficionados of heterodoxy, we present some suggestions for M-theoretic resolutions of certain cosmological conundra without the aid of inflation.

The remainder of the lectures will be devoted to more or less standard inflationary models based on moduli and will examine in detail the properties of these models adumbrated above. Compared to much of the inflation literature, these sections will be long on general properties and short on specific models which can be compared to the data. M-theory purports to be a fundamental theory of the universe, rather than a phenomenological model. Inflaton potentials are objects to be calculated from first principles rather than postulated in order to fit the data. There is nothing wrong with phenomenological models of inflation, but they are not the real province of M-theory cosmology. Unfortunately, our understanding of the nonperturbative properties of M-theory in the regime where the supersymmetry algebra is sufficiently small to allow for a potential on the moduli space (the alert reader may have already noted that the preceding phrase contains an oxymoron) is too limited to allow us to build reliable models of the potential. Thus, if we are honest, we must content ourselves with general observations and the definition of a set of goals.

2. M-theory, Branes, Moduli and All That

2.1 The story of M

Once upon a time there were six string theories. Well, actually there were five (because one, the Type IA theory, was an ugly duckling without enough Lorentz invariance) and actually there were an infinite number, or rather continuous families . . . . What’s going on here? The basic point is the following: what string theorists called a string theory in the old days was a set of rules for doing perturbation theory. What was perhaps misleading to many people is that these rules were usually given in terms of a Lagrangian, more generally a superconformally invariant $1+1$ dimensional quantum field theory, (with some extra properties). We are used to think of Lagrangians as defining theories. The better way to think of the world sheet Lagrangians of string theory is by imagining
a quantum field theory with many classical vacua. Around each vacuum state we can construct a loop expansion. The quadratic terms in the expansion around a vacuum state define a bunch of differential operators, whose Green’s functions are the building blocks of the perturbation expansion. Using Schwinger’s proper time techniques we can describe these Green’s functions in terms of an auxiliary quantum mechanics, and if we wish we can describe this quantum mechanics in terms of a Feynman path integral with a Lagrangian. The world sheet path integrals of string theory are the analogs of these proper time path integrals. One of the beautiful properties of string theory is that, unlike field theory, the Lagrangian formulation of the propagator completely determines the perturbation expansion. To compute an $n$ particle scattering amplitude in tree level string theory one does the path integral on a Riemann surface with no handles and (for theories whose perturbation expansion contains only closed strings) $n$ boundaries. The boundary conditions on the boundaries are required to be superconformally invariant and carry fixed spacetime momentum. The Lagrangian itself is superconformally invariant, and the allowed boundary conditions are generated by acting on a particular boundary condition which defines the ground state of the single string with a set of vertex operators which represent small perturbations of the action which preserve superconformal invariance. A given vertex operator creates a state of the string which propagates as a particle with given mass and quantum numbers. The higher orders in perturbation theory just correspond to computing the same path integral on Riemann surfaces of higher genus. One sums over all Riemann surfaces, or in some cases only over oriented ones.

The conditions of superconformal invariance have many solutions. Classically (in the sense of two dimensional classical field theory – this should not be confused with tree level string theory which corresponds to summing all orders in the semiclassical expansion of the world sheet field theory, on Riemann surfaces with no handles), for the particular case of Type II string theories, the bosonic terms in the most general superconformal Lagrangian have the form

$$L = (G_{\mu\nu}(x) + iB_{\mu\nu}(x))\partial x^\mu \bar{\partial} x^\nu + h.c. + \Phi(x)\chi$$

where the derivatives are taken with respect to complex coordinates on a Euclidean world sheet. $G_{\mu\nu}$ is symmetric and $B_{\mu\nu}$ is antisymmetric. $\chi$ is the Euler density of the world sheet, a closed two form (for more on forms, closed and otherwise, see below) whose integral is the Euler Character. Quantum mechanically, there are restrictions on the functions, $G, B,$ and $\Phi$. To lowest order in the world sheet loop expansion the

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5Actually the situation is a bit more complicated, and to do it justice one must use the BRST formalism. For our purposes we can ignore this technicality.
condition of superconformal invariance coincides with the equations of motion coming from a spacetime Lagrangian

$$\mathcal{L}_{st} = \sqrt{-g}e^{-2\Phi}[R + 4(\nabla\Phi)^2 + (dB)^2]$$  \hspace{1cm} (2.2)

This fact, combined with the fact that vertex operators are allowed perturbations of these equations, shows us that string theory is a theory of gravity. One cannot choose the background metric arbitrarily; it must satisfy an equation of motion.

In classifying consistent solutions of all these rules, string theorists found a number of discrete choices depending on the number of fermionic generators in the world sheet superconformal group and on the types of Riemann surfaces allowed. This led to the five different types of string theory. Once these discrete choices were made, there still seemed to be a multiparameter infinity of choices. However, it was soon understood that the continuous infinity corresponded to expanding the same basic theory around different solutions of its classical equations of motion (approximately the equations generated by (2.2)). This was a little surprising, because one of the rules for the perturbation expansion was that there be some number of flat Poincaré invariant dimensions (I explained in the introduction why string theorists insisted on this). So each of these solutions was a static classical vacuum state. Why are there so many vacua? The answer is spacetime SUSY. Indeed almost every known perturbatively stable vacuum state of string theory is supersymmetric\(^6\). It is well known that spacetime SUSY often leads to nonrenormalization theorems which prevent the existence of potentials for scalar fields. The strongest theorems of this type come when there is enough SUSY to guarantee that the scalars are in supermultiplets with gauge or gravitational fields, but there are other examples. As noted in the introduction, we will call the space of classical vacua the moduli space. It should be noted that the moduli space is not connected (\textit{e.g.} the branches with different amounts of SUSY are generally disconnected from each other), nor is it a manifold. The reason for the latter property is that often, new massless states can appear on submanifolds of the moduli space. Often these include scalars, which, as long as the original moduli are restricted to the submanifold, have no potential. One can then define a new branch of moduli space on which the original moduli are restricted to the submanifold, but the new massless scalar fields have expectation values. Thus the moduli space has several disconnected components, each of which is a bunch of manifolds of different dimension, glued together along singular submanifolds.

Thus, circa 1994-95 we had five discrete classes of string theory, Type \(II_{A,B}\); Heterotic\(_{A,B}\) (\(A\) refers to the \(E_8 \times E_8\) heterotic string theory and \(B\) to the \(SO(32)\)

\(^6\)There are no known exceptions. Recently however [8] some constructions which appear to be stable at least through two loops have been found. This is the reason for the word almost in the text.
version) and Type $I_B$. The Type $I_B$ theory has the same symmetries in spacetime as the $Het_B$ theory although the world sheet theories are completely different. In Type $I_B$ the gauge quantum numbers are carried on the ends of open strings (like flavor quantum numbers on QCD strings) and nonorientable world sheets appear in the perturbation expansion. The heterotic theory has only closed strings, orientable world sheets and gauge quantum numbers carried by the body of the string. There is also a Type $I_A$ theory which has a similar relation to $Het_A$. This theory does not have ten dimensional Lorentz invariance because it has two $8 + 1$ dimensional domain walls at the ends of a finite or infinite $9 + 1$ dimensional “interval”. It has $SO(16) \times SO(16)$ gauge symmetry carried by the ends of open strings which can only propagate on the domain walls.

The labels A and B refer to theories which are different in 10 (the maximal dimension for perturbative strings) dimensions but are actually equivalent to each other when compactified on a circle. The equivalence is due to a stringy symmetry called T duality. The momentum of a string on a circle is the integral of the time derivative of its coordinate; i.e. the time derivative of the center of mass position.

$$P = \int d\sigma \partial_t \theta. \quad (2.3)$$

Strings on a circle carry another quantum number called winding number, which is defined by

$$w = \int d\sigma \partial_\sigma \theta. \quad (2.4)$$

The Euclidean world sheet Lagrangian for the string coordinate $\theta$ is

$$\mathcal{L}_{ws} = (\partial_\tau \theta)^2 + (\partial_\sigma \theta)^2 \quad (2.5)$$

Instead of $\theta$ we can introduce a new coordinate by the two dimensional analog of an electromagnetic duality transformation

$$\partial_a \theta = \epsilon_{ab} \partial_b \tilde{\theta}. \quad (2.6)$$

It turns out that when one performs this transformation one automatically takes a Type A theory to a Type B theory.

We learn two things from this: First, there are only half as many different string theories as we thought, and second, to see that theories are the same we may have to compactify them. Decompactification loses important degrees of freedom (in this case string winding modes, which go off to infinite energy) which are necessary to see the equivalence. There are no more such equivalences which can be seen in perturbation theory, but we might begin to suspect that there are further equivalences which might
only appear nonperturbatively. How can we hope to realize this possibility in a theory which is formulated only as a perturbation series?

The key to answering this question is the notion of SUSY preserving or BPS states. To explain what these are, let me introduce the SUSY algebra

\[
\{\bar{Q}_a, Q_b\} = \gamma_{ab}^\mu P_\mu
\]  

(2.7)

Actually, this is only the simplest SUSY algebra one can have in a given dimension. We will see more complicated ones in a moment. If we look at particle representations of the SUSY algebra, then \(P_\mu\) is either a timelike or a lightlike vector. In the timelike case, the matrix on the right hand side of \(2.7\) is nondegenerate, while in the lightlike case it is degenerate – half of the states in the representation are annihilated by it. This means that in the lightlike case half of the SUSY generators annihilate every state in the representation. Thus massless supermultiplets are smaller than massive ones. This means, that in general in a supersymmetric theory, small changes in the parameters will not give mass to massless particles. In order to do so one must have a number of massless multiplets which fit together to form a larger massive multiplet (the super Higgs mechanism) in order for states to be lifted. If this is not the case for some values of the parameters, then small changes cannot make it so and the massless particles remain. Of course, we did not really need SUSY to come to this conclusion for massless particles of high enough spin. In that case it is already true that the Lorentz group representations of massless and massive particles are of different multiplicity.

The new feature really comes if we compactify the theory in a way which preserves all the SUSY generators. This can be done by compactifying on a torus with appropriate boundary conditions. The SUSY algebra remains the same, but now some of the components of the momentum are discrete. Also, the Lorentz group is broken to the Lorentz group of the noncompact dimensions, so the spinor representation breaks up into some number of copies of the lower dimensional spinor. The algebra now looks like

\[
\{\bar{Q}_a^i, Q_b^j\} = \gamma_{ab}^\mu P_\mu \delta^{ij} + \delta_{ab} Z^{ij}.
\]  

(2.8)

Spinor and vector indices now run over their lower dimensional values, and \(i, j\) label the different copies of the lower dimensional spinor in the higher dimensional one. The generators \(Z^{ij}\) are scalars under the lower dimension Lorentz group. They are combinations of the toroidal momenta and are examples of what are called central charges.

Now consider a state carrying nonzero values of the central charge in such a way that the higher dimensional momentum is lightlike. It represents a massive Kaluza-Klein mode of the massless particle in the higher dimension. In a nonsupersymmetric theory
the masses of Kaluza Klein modes of higher dimensional massless fields are renormalized by quantum corrections. But in a theory with the extended SUSY algebra (2.8) we can ask whether the representation is annihilated by half the supercharges (other fractions are possible as well). If it is, we get a computation of the particle’s nonzero mass in terms of its central charges. This mass cannot change as parameters of the theory are varied in such a way as to preserve extended SUSY, or rather the formula for its variation with parameters may be read directly from the SUSY algebra. We will see below that it is possible to realize the central charges $Z_{ij}$ of the extended lower dimensional SUSY algebra in other ways. Instead of representing KK momenta in a toroidal compactification, they might arise as winding numbers of extended objects, called branes, around the compact manifold. The italicized conclusion will be valid for these states as well.

The argument for the statement in italics above, is again based on the smaller dimension of the representation. To see it more explicitly, work in the frame where the spatial momentum is zero, and take the expectation value of the anticommutator in states of a single particle with mass $M$ (actually we mean a whole SUSY multiplet of particles). Then (2.8) reads

$$M \delta_{ij} + \gamma^0 Z_{ij} = < [Q_i^a, Q_j^b]_+ > \geq 0.$$  (2.9)

The last inequality follows because we are taking the expectation value of a positive operator. It says that the mass is bounded from below by the square root of the sum of the squares of the eigenvalues of the matrix $Z$, which are also called the central charges.

Equality is achieved only when the expectation value vanishes, which, since the SUSY charges are Hermitian, means that some of the charges annihilate every state in the representation. These special representations of the algebra have smaller dimension and cannot change into a generic representation, which satisfies the strict inequality, as parameters are continuously varied.

Thus, in theories with extended SUSY, certain masses can be calculated exactly from the SUSY algebra. These special states are called Bogolmony Prasad Sommerfield or BPS states, since these authors first encountered this phenomenon in their classical studies of solitons [10]. The connection to SUSY, which makes the classical calculations into exact quantum statements, was noticed by Olive and Witten [12].

Notice that although we motivated this argument in terms of Kaluza-Klein states, it depends mathematically only on the structure of the extended SUSY algebra. Thus if we can obtain this algebra in another way, we will still have BPS states. An alternative origin for central charges and BPS states comes from “wrapped branes” of a higher dimensional theory.
To understand this notion, note that, strictly from the point of view of Lorentz invariance, the SUSY algebra could contain terms like

\[
\{Q_a, Q_b\} = \gamma_{ab}^\mu P_\mu + \gamma_{ab}^{\mu_1...\mu_p} Z_{\mu_1...\mu_p} \tag{2.10}
\]

The multiple indices are antisymmetrized. The famous Haag-Lopusanski-Sohnius [11] generalization of the Coleman Mandula theorem, tells us that this pth rank antisymmetric tensor charge, must vanish on all finite energy particle states. On the other hand, the purely spatial components of it have precisely the right Lorentz properties to count the number of infinite energy \( p \)-branes, or \( p \)-dimensional domain walls, oriented in a given hyperplane. We will have more to say about these brane charges when we talk about branes and gauge theories below.

Now suppose we have compactified \( p \) or more dimensions, and the resulting compact space has a topologically nontrivial \( p \)-dimensional submanifold, or \( p \)-cycle. To see what we mean, consider the two torus

![A Two Torus With Nontrivial Cycles Labelled.](image)

It has two different kinds of nontrivial 1 cycle, labelled a and b in the figure. The whole torus is a nontrivial two cycle. The word nontrivial cycle or just cycle implies that the submanifold cannot be contracted to a point “because it wraps around a hole in the manifold”. If we wrap a \( p \)-brane around the \( p \)-cycle, we get a finite energy particle state. The tensor charge with all indices pointing in the compact directions is a scalar charge in the remaining noncompact directions and is allowed to appear as a central charge in an extended SUSY algebra. Often, the corresponding particles have the BPS property.

With this background, we can get on with the story of M. Practitioners of string duality realized that BPS states gave them a powerful handle on nonperturbative physics.
For example, consider a weakly coupled string theory and a solitonic state, whose mass in string units is proportional to $g_{S}^{-7}$. If it is a BPS state then as the coupling becomes infinitely strong, it becomes infinitely lighter than the string scale (if not for the BPS property, we could not trust the weak coupling formula at strong coupling). In all the cases which have been studied one can, by thinking about the lightest BPS states in the strong coupling limit, realize that they are just the elementary states of another weakly coupled theory. In most cases, this is another string theory, but there is a famous exception.

If one considers Type IIA string theory in ten dimensions, it contains a single $U(1)$ gauge field. None of the perturbative states are charged under this field; they have only magnetic moment couplings. If one considers hypothetical BPS states charged under this $U(1)$, then it is easy to show that their spectrum in the strong coupling limit is precisely that of the supergravity multiplet in 11 dimensions. Thus one is led to conjecture [35] that the strong coupling limit of Type IIA string theory has eleven flat dimensions and a low energy limit described by SUGRA.

None of this was much of a surprise to the SUGRAistas [27]. It had long been known that the low energy limit of IIA string theory was a ten dimensional SUGRA theory which was the dimensional reduction of 11D SUGRA, with the string coupling appearing as the ratio of the three halfs power of the radius of the reducing circle to the eleven dimensional Planck mass. The SUGRAistas even had a correct explanation of where the strings come from. As we will see 11D SUGRA couples naturally to a membrane, the M2 brane. If we wrap one leg of the M2 brane around the circle whose radius is being shrunk to zero we get a string whose tension is going to zero in Planck units.

String theorists get a C for closed mindedness for ignoring the message of the SUGRAistas for so long. Behind their resistance lay the feeling that because both 11D SUGRA and the world volume theory of membranes are nonrenormalizable, one could not trust conclusions drawn on the basis of these theories. It was only with the advent of an unambiguous, string theoretic construction of the KK gravitons of 11D SUGRA as bound states of D0 branes [28] [29] that the last bastions of resistance fell. What one should have realized from the beginning was that conclusions about BPS states, based as they are only on the symmetry structure of the theory, can be extrapolated from effective theories far beyond the limited range of validity these low energy approximations.

The picture as we understand it today\textsuperscript{8} is illustrated by the famous “modular

\textsuperscript{7}In string theory both $r = 1, 2$ are realized. $r = 2$ corresponds to a conventional soliton, arising as a solution of the classical equations of motion. $r = 1$ corresponds to Dirichlet brane or D brane states.

\textsuperscript{8}or rather a cartoon of it, for moduli space is much more complicated than a two dimensional
There is a single theory, which we now call M-theory\(^9\) which has a large moduli space. All of the known perturbative string theories, and 11D SUGRA are limits of this theory in certain extreme regions, or boundaries, of moduli space (the cusps in the picture). There is another class of limits, called F-theory, which are not amenable to complete analysis, but about which many nontrivial statements can be made. An example of an F-theory region is the strongly coupled heterotic string compactified on a two torus.

One of the lessons of duality is that no one of these regions is \textit{a priori better} than any other. Each of them tells a partial story about M-theory, and we learn a lot by trying to patch these stories together. However, the 11D SUGRA limit has a distinct advantage when one is trying to explain M-theory to non-string theorists, particularly if they have a good background in GR. In this limit, most of the arguments are completely geometrical and can be understood on the basis of classical field theory and the classical Lagrangians for various extended objects. For this reason, I will begin in the next section with a discussion of the 11D SUGRA Lagrangian\(^10\).

\(^9\) or at least some of us do. Some people reserve the name M-theory for the region of moduli space where 11D SUGRA is a good approximation. I consider that a waste of a good name since we can call this the 11D SUGRA region.

\(^10\) at least its purely bosonic part. Fermions, like virtue in the world of politics, are entities often talked about, but rarely seen, in discussions of SUSY theories.
3. Eleven Dimensional Supergravity

In eleven dimensions, the graviton has 44 spin states transforming in the symmetric traceless tensor representation of the transverse (transverse to the graviton’s lightlike momentum) $SO(9)$ rotation group. The gravitino is a tensor-spinor of this group, satisfying the constraint $\gamma^i_{ab} \psi^i_b = 0$ which leaves 128 components. The remaining $84 = \frac{9 \times 8 \times 7}{3 \times 2 \times 1}$ bosonic states in the SUGRA multiplet transform as a third rank antisymmetric tensor.

The covariant Lagrangian for the bosonic fields in the multiplet is

$$M_p^9 \sqrt{-g} [ -\frac{1}{2} R + \frac{1}{48} g^{\mu_1 \nu_1} \ldots g^{\mu_4 \nu_4} G_{\mu_1 \ldots \mu_4} G_{\nu_1 \ldots \nu_4} ] - \frac{\sqrt{2}}{3456} \epsilon^{\mu_1 \ldots \mu_{11}} C_{\mu_1 \ldots \mu_3} G_{\mu_4 \ldots \mu_7} G_{\mu_8 \ldots \mu_{11}} ]$$

(3.1)

The supersymmetry transformation of the gravitino is

$$\delta \psi^\mu_\alpha = D^\mu \epsilon_\alpha + \frac{\sqrt{2}}{288} (\Gamma^{\lambda \kappa \sigma}_\mu - \delta^{\nu}_{\mu} \Gamma^{\lambda \kappa \sigma}) G_{\nu \lambda \kappa \sigma} \epsilon_\alpha.$$ 

(3.2)

The existence of a three form gauge potential, suggests that the theory may contain a membrane, which couples to the three form via

$$Q_2 \int C_{\mu \nu \lambda} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \frac{\partial x^\lambda}{\partial \xi^c} d\xi^a d\xi^b d\xi^c$$

(3.3)

where the $\xi^a$ are coordinates on the membrane world volume. The dual of the four form field strength $G_{\mu \nu \lambda \kappa}$ is a seven form $G_7$, whose source, defined by $d \ast G_7 (\equiv \ast dG_4) = J_6^{11}$, is a six form current. This is a current of five dimensional objects, which we will call M5 branes.

The low energy SUGRA approximation to M-theory gives us evidence that both M2 and M5 branes exist, since there are soliton solutions of the SUGRA equations of motion with the requisite properties. This would not be a terribly convincing argument, since the scale of variation of the fields of these objects is (what else?) the eleven dimensional Planck scale, and SUGRA is only an effective field theory. However, these solitons have the BPS property. That is, we can find them by insisting that half of the gravitino SUSY variations vanish. This leads to first order equations, which are much easier to solve than the full second order equations, but give a subclass of solutions of the latter. Since these solutions are constructed so that half of the SUSY variations vanish, their Poisson brackets with half the SUSY generators (in a canonical formulation of 11D SUGRA) vanish. This is the classical approximation to the statement that the

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11The large number of spacetime dimensions lead me to resort to differential form notation even for an audience of cosmologists.
quantum states represented by these soliton solutions are annihilated by half of the SUSY generators.

The solutions are

$$ds^2 = (1 + \frac{k}{r^6})^{-2/3}(-dt^2 + d\sigma^2 + d\rho^2) + (1 + \frac{k}{r^6})^{1/3}dx_8^2$$

(3.4)

$$A_{\mu \nu \lambda} = \epsilon_{\mu \nu \lambda}(1 + \frac{k}{r^6})$$

(3.5)

$$k \equiv \frac{L_p^9 T_2}{3 \Omega_7}$$

(3.6)

$$ds^2 = (1 + \frac{T_5}{r^3})^{-1/3}(-dt^2 + dx_5^2) + (1 + \frac{T_5}{r^3})^{2/3}dy_5^2$$

(3.7)

$$G_{\mu_1 \ldots \mu_4} = 3 T_5 \epsilon_{\mu_1 \ldots \mu_4 \nu} \frac{y^\nu}{r_5}$$

(3.8)

for the two brane and five brane respectively.

In each of these equations, \( r \) denotes the transverse distance from the brane. These solutions contain arbitrary parameters \( T_2 \) and \( T_5 \) which control the strength of the coupling of these objects to the three form gauge potential and to gravity. However, an elementary argument leads to a determination of these parameters. Compactify the theory on a seven torus and wrap the M2 brane around two of the dimensions of this torus and the M5 brane around the other five. The integral of the three form over the two torus on which the membrane is wrapped give us an ordinary Maxwell (1 form) potential. It is easy to see that the wrapped membrane is a charged particle with charge \( Q_2 \) with respect to this Maxwell field (the membrane coupling of (3.3) dimensionally reduces to the standard Maxwell coupling to a charged particle). It is a little harder to see that the wrapped M5 brane is a magnetic monopole for this field. Thus, the Dirac quantization condition implies

$$2\pi Q_2 Q_5 \in \mathbb{Z}$$

(3.9)

This is a quick and dirty proof of the Nepomeche-Teitelboim [30] generalization of the Dirac quantization condition to \( p \)-form gauge fields. This condition determines the tension of the minimally charged M2 and M5 branes to be:

$$T_5 = \frac{1}{2\pi} T_2^2$$

(3.10)

$$T_2 = L_p^{-3}$$

(3.11)

Given the existence of these infinite flat branes, we can also study small fluctuations of them which describe (slightly) curved branes moving in spacetime. The most useful
way to do this is to introduce a world volume field theory which contains the relevant
fluctuations. Among the variables of such a theory should be a set of scalar fields which
describe small fluctuations of the brane in directions transverse to itself. It turns out
that these world volume theories are, in the present case, completely determined by
SUSY.

Let us begin with the M2 brane. The SUSYs that preserve the brane satisfy
\[ \gamma^3 \ldots \gamma^{10} Q = Q \] . There are 16 solutions to this equation, which transform as 8 spinors under the \( SO(2,1) \) Lorentz group of the brane world volume. Each two component
world volume spinor transforms in the eight dimensional spinor representation of the
transverse \( SO(8) \) rotation group. From the point of view of the world volume field
theory, the latter is an internal symmetry. We expect the world volume theory to
contain 8 scalar fields, representing the transverse fluctuations of the membrane. A
SUSY Lagrangian containing these fields is given by
\[ \mathcal{L}_{M2} = \partial_a x^i \partial_a x^i + \bar{\theta}^J \Gamma^a \partial_a \theta^J \] (3.12)
where \( \Gamma^a \) are three world volume Dirac matrices. The SUSY generators are:
\[ Q^I_\alpha = \int d^2 \xi \gamma^i_{JK} [\partial_0 x^i \theta^K_\alpha + \Gamma^a_{o,j} \theta^K_\beta \partial_a x^i] \] (3.13)
Using the canonical commutation relations for the world volume fields it is straightforward to verify that these satisfy the SUSY algebra. Here we have used \( \gamma^i \) to represent
the eight dimensional Dirac matrices, despite the possibility of confusion with the eleven
dimensional matrices of the paragraphs above and below.\(^{13}\)

The world volume theory of the M5 brane is more interesting. The SUSYs pre-
served satisfy \( \gamma^6 \ldots \gamma^{10} Q = Q \) (using the same argument as in the footnote above).
The world volume Lorentz group \( SO(5,1) \) has two different chiralities of spinor repre-
sentation, and (ultimately because the product of all eleven dimensional Dirac matrices
is 1) this condition says that all the SUSY generators have the same chirality. There
are sixteen real solutions of these constraints which can be arranged as two complex 4
representations of the world volume Lorentz group. Under the transverse \( SO(5) \) rota-
tion group, they transform as four copies of the fundamental pseudoreal spinor. This
kind of SUSY is called \((2,0)\) SUSY in six dimensions.

\(^{12}\)To see this, note that the condition specifying which charges annihilate the membrane state must be linear (the sum of two such charges is another) and invariant under both the transverse rotation and world volume Lorentz groups. This is the only such condition since the product of all the 11D Dirac matrices is 1.

\(^{13}\)We have also passed over in silence the two different types of eight dimensional spinor which appear in these equations. Experts will understand and amateurs would only be confused by this detail.
The coordinates of transverse fluctuations are five scalar world volume fields, which transform as the vector of $SO(5)$. There is a unique SUSY representation which includes these fields. Their superpartners are two fermions in the $\bar{4}$ representation of the Lorentz group, and a second rank antisymmetric tensor gauge field, $B_{\mu\nu}$ whose field strength satisfies the self duality condition $H_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\sigma\rho} H_{\sigma\rho}$. Indeed, in a physical light cone gauge, $B_{AB}$ (the A,B indices indicate the four transverse dimensions in the lightcone frame inside the world volume) has 6 components and the self duality cuts this down to 3. Combined with the five scalars this makes eight bosonic degrees of freedom which balance the eight degrees of freedom of a Weyl fermion. The self dual antisymmetric tensor field is chiral and its field equations cannot be derived from a covariant Lagrangian (without a lot of extra complications and gauge symmetries). As we will see, this is the origin of the world sheet chirality of the heterotic string.

4. Forms, Branes and BPS states

4.1 Differential forms and topologically nontrivial cycles

Before proceeding with our discussion of compactification of 11D SUGRA, and its relation to string theory, I want to insert a short remedial course on the mathematics of differential forms. We have already used this above, and will use it extensively in the sequel. Differential $n$ forms or totally antisymmetric covariant tensor fields, were invented by mathematicians as objects which can be integrated over $n$ dimensional submanifolds of a manifold of dimension $d$. The basic idea is that at each point, such a form picks out $n$ linearly independent tangent vectors to the manifold and assigns a volume to the corresponding region on the submanifold. If $t_\mu^i$ are the tangent vectors, then $\omega_{\mu_1\ldots\mu_n} t_1^\mu \ldots t_n^\mu$ is the volume element.

Mathematicians introduced Grassmann variables $dx^\mu$ as placeholders for the $n$ independent tangent vectors. Thus, an $n$ form becomes $\omega_{\mu_1\ldots\mu_n} dx^{\mu_1} \ldots dx^{\mu_n}$, which is a commuting or anticommuting element of the Grassman algebra according to whether $n$ is even or odd. In this way, the set of all forms of rank $0 \to d$ is turned into an algebra. A derivative operator $d$ is defined on this algebra by $d = dx^\mu \frac{\partial}{\partial x^\mu}$. This definition is independent of the metric or affine connection on the manifold. Note that $d^2 = 0$. Forms satisfying $d\omega = 0$ are called closed. Trivial solutions of the form $\omega = d\lambda$ are called exact, and the set of equivalence classes of non exact closed forms (modulo addition of an exact form) is called the cohomology of the manifold.

If a submanifold is parametrized as a mapping $x^\mu(\xi)$ from some $n$ dimensional parameter space into the manifold, then the integral of a form over the submanifold is
given by
\[ \int_M \omega = \int d\xi^{a_1} \ldots d\xi^{a_n} \frac{\partial x^{\mu_1}}{\partial \xi^{a_1}} \ldots \frac{\partial x^{\mu_n}}{\partial \xi^{a_n}} \omega_{\mu_1 \ldots \mu_n}(x(\xi)) \] (4.1)

If \( \omega \) is an \( n-1 \) form and \( S \) an \( n \) dimensional submanifold with boundary \( \partial S \), then the generalized Stokes theorem says that \( \int_S d\omega = \int_{\partial S} \omega \). In particular, the integral of an exact form over a submanifold without boundary, vanishes.

Most \( n \) dimensional submanifolds without boundary, are themselves boundaries of \( n + 1 \) dimensional submanifolds. However, in topologically nontrivial situations there can be exceptions, called nontrivial \( n \)-cycles. You can see this in the example of the \( a \) or \( b \) cycle in Fig. 1. Generally there are many such nontrivial cycles if there are any, but they often differ by trivial cycles (think of two different circles which go around the circumference of the torus). Again, the \( n \) cycle is considered to be the equivalence class of nontrivial submanifolds modulo trivial ones.

One of the most important theorems in mathematics is the de Rham theorem, which states that there is a one to one correspondence between the cohomology of a manifold and the independent nontrivial closed cycles. That is, one can choose a basis \( \omega_i \) in the space of closed modulo exact \( n \) forms such that \( \int_{C_j} \omega_i = \delta_{ij} \). Here \( C_j \) are the independent nontrivial \( n \) cycles.

So much for pure mathematics. The reason that all of this math is interesting in M-theory is that the theory contains dynamical extended objects called \( p \)-branes, and the theory of differential forms allows us to understand the most important low energy dynamical properties of these objects as a beautiful generalization of Maxwell’s electrodynamics. In addition this leads us to a new and deep mechanism for generating nonabelian gauge groups, which is connected to the theory of singularities of smooth manifolds. This in turn allows us to obtain an understanding of certain spacetime singularities in terms of Wilson’s ideas about singularities of the free energy at second order phase transitions. It was long known that the free energy of statistical systems at second order phase transitions had singularities as a function of the temperature and other thermodynamic variables. Wilson realized that these singularities could be understood using the equivalence between statistical mechanics and Euclidean quantum field theory. At values of the parameters corresponding to phase transitions, massless particles appear in the field theory and the singularities of the free energy are attributed to infrared divergences coming from integrating over the fluctuations of these particles.

In classical geometry, singularities of manifolds can be classified by asking which nontrivial cycles shrink to zero as parameters are varied in such a way that a smooth manifold becomes singular. In M-theory there are states described by BPS branes wrapped around these nontrivial cycles, which become massless when the cycles shrink to zero. The singularities in classical geometry are then understood to be a reflection
of the quantum fluctuations of these massless particles. That is, singular quantities in classical geometry can be calculated in terms of Feynman diagrams with loops of the massless states that M-theory predicts at these special points in moduli space (only these states contribute to the infrared divergence). The quantum theory itself is nonsingular at these points, but its description in terms of classical geometry breaks down because there are light degrees of freedom (the wrapped branes) other than the gravitational field. Branes and singularities are at the heart of string duality.

Let us begin the discussion of branes by recalling the Lagrangian for the coupling of the electromagnetic field to a charged particle. It is

$$\int dt A_\mu(x(t)) \frac{dx^\mu}{dt} = \int_C A_1,$$

(4.2)

where in the second equality we have used the notation of forms. If the particle path is closed, this action is invariant under gauge transformations \(A_1 \rightarrow A_1 + d\Lambda_0\). If we add to the action the simplest gauge invariant functional of the \(A_1\) field \(\int dA_1 \star dA_1\), we obtain Maxwell’s theory of the coupling of charged particles to electromagnetism.

There is an obvious generalization of all of this to the coupling of a \(p+1\) form potential to a \(p\)-brane. The interaction is given by

$$\int_{C_{p+1}} A_{p+1},$$

(4.3)

where the integral is over the world volume of the \(p\)-brane. By the generalized Stokes theorem, this enjoys a gauge invariance \(A_{p+1} \rightarrow A_{p+1} + d\Lambda_p\). By virtue of the fundamental equation \(d^2 = 0\), \(dA_{p+1}\) is a gauge invariant object and we can write an immediate generalization of Maxwell’s action,

$$\int d^m x \star F \wedge F.$$

(4.4)

Once we have normalized \(A\) by writing the free Maxwell Lagrangian, we are left with a free coefficient in the coupling of the brane to the gauge field. In the electromagnetic case, we know that this coefficient, the electric charge, is in fact quantized if we introduce magnetic monopoles and quantum mechanics, an observation first made by Dirac.

The analogous observation for general \(p\)-branes was made by Nepomechie and Teitelboim [30]. A \(p\)-brane couples to a rank \(p+2\) field strength. In \(d\) spacetime dimensions we can introduce, given a metric, a dual field strength

$$\ast F_{\mu_1 \ldots \mu_{d-p-2}} = \epsilon_{\mu_1 \ldots \mu_d} F^{\mu_d \ldots \mu_{d-p-1}}$$

(4.5)

[14]Here the \(\ast\) denotes the Hodge dual, but for the purposes of this lecture we can just think of this as a shorthand for Maxwell’s action.
where we have raised indices with the metric tensor. One thus sees that the natural dual object to a $p$-brane is a $d - p - 4$ brane.

The easiest way to see the Dirac-Nepomechie-Teitelboim condition is to compactify the system on a torus of dimension $d - 4$. We wrap the $p$-brane around $p$-cycles of this torus and its dual around the remaining $d - p - 4$. The integral, $\int_{T^p} A_{p+1}$ defines a one form or Maxwell field in the uncompactified four dimensional spacetime, and the $p$-brane is an electrically charged particle. It is easy to convince oneself that the wrapped dual brane is a monopole. Thus we obtain a quantization condition relating the couplings of the two dual branes to the $p + 1$ form gauge potential. Nepomechie and Teitelboim show that there are no further consistency conditions.

4.2 SUSY algebras and BPS states

Now let us recall what we learned in the previous section about BPS states. I will repeat that material briefly here, but readers who feel they have absorbed it adequately can skip the first few paragraphs. We pointed out above that many SUSY theories have classes of massive states whose masses are protected from renormalization in the same way that those of massless particles of spin greater than or equal to one half are. These are called Bogolmony-Prasad-Sommerfield or BPS states. The easiest to understand are the Kaluza Klein states of toroidal compactification, but there is a vast generalization of this idea. The theorem of Haag Lopuzanski and Sohnius (a generalization of the Coleman-Mandula theorem) \[11\] shows that the ordinary SUSY algebra is the most general algebra compatible with an $S$ matrix for particle states. Purely algebraically though, the right hand side of the SUSY algebra could have contained higher rank antisymmetric tensor charges (the general representation appearing in the product of two spinors).

Our discussion of branes and gauge fields provides us with a natural source of such charges, as well as showing us the loophole in the HLS theorem. Indeed, following our analogy with Maxwell electrodynamics, it is easy to see that an infinite, flat, static $p$-brane carries a conserved rank $p$-antisymmetric tensor charge as a consequence of the equations of motion of the $p + 1$ form gauge field it couples to. The fact that these branes are infinite extended objects and carry infinite energy is the loophole in the theorem. It referred only to finite energy particle states. All of the tensor charges vanish on finite energy states.

However, when we compactify a theory, we can imagine wrapping one of these $p$-branes around a nontrivial $p$-cycle in the compact manifold. The resulting state propagates as a particle in the noncompact dimensions. It has finite energy, proportional to the volume of the cycle it was wrapped around. Its tensor charge becomes a scalar in the noncompact dimensions and is called a central charge of an extended
(i.e. larger than the minimal algebra in the noncompact dimensions) SUSY algebra. Thus the central charges in extended SUSY algebras in low dimensions may come from wrapped brane charges as well as KK momenta.

Perhaps the most remarkable fact about this statement is that as the volume shrinks to zero, the mass of the wrapped BPS state does as well. If the volume of the relevant $p$-cycle parametrizes a continuous set of supersymmetric vacuum states, then this conclusion is exact and can be believed in all regimes of coupling even though it was derived by crude semiclassical reasoning. Indeed, even if we don’t know the theory we are trying to construct, we can still believe in the existence of massless wrapped brane states as long as we posit that the SUSY algebra is a symmetry. We will make extensive use of this argument in the sequel.

5. Branes and Compactification

5.1 A tale of two tori

We are now in a position to study many of the important dualities of M-theory, at least at a cursory level. We will not have time to delve here into the many computations and cross checks which have convinced most string theorists that all of these dualities are exact. Many of the duality statements remain conjectures supported by a lot of circumstantial evidence. Obviously, they cannot be proven until a full nonperturbative form of M-theory is discovered. However, an important subclass of the dualities can actually be proven in a Discrete Light Cone Quantization (DLCQ) of M-theory known as Matrix Theory [31]. It applies to Compactifications of M-theory with at least 16 unbroken SUSYs and at least 6 noncompact Minkowski spacetime dimensions. In DLCQ, one gives up Lorentz invariance by compactifying a lightlike direction on a circle. One then gets an exact description of M-theory in terms of an auxiliary quantum field theory living on a fictitious internal space. All of the duality symmetries relevant to this class of compactifications (the only ones we will talk about in these lectures) can be derived as properties of the auxiliary field theory. This includes statements (such as rotation invariance of the Type IIB theory constructed by compactifying M-theory on a two torus) for which there was no other evidence prior to the advent of Matrix Theory.

We begin by compactifying M-theory on a circle of radius $R_{10}$. When $R_{10}$ is much larger than the eleven dimensional Planck length $L_P$ there is a good description of the low energy physics of the system in terms of 11D SUGRA compactified on a circle. The SUGRA Lagrangian incorporates all of the low energy states of the system and gives a good approximation to their low energy scattering amplitudes.
As $R_{10}$ drops below $L_P$, this description breaks down. An 11D low energy physicist might guess that the low energy states of the system are just the zero modes (on the circle) of the SUGRA fields. This gives 10D Type IIA SUGRA, which has the following fields: $g_{\mu\nu}, \phi, B_{\mu\nu}, C^{RR}_{\mu\nu\lambda}, C^{RR}_\mu$. These can be identified as the ten dimensional metric (in string conformal frame), the dilaton field which describes local variations of the radius of the eleventh dimensional circle, a two form gauge potential which is given by the integral of the 11D three form around the circle, the three form itself, and a Kaluza-Klein one form gauge field. The effective Planck mass, $M_{10}^P$, of this ten dimensional theory is given by

$$(M_{10}^P)^8 \sim R_{10}(M_{11}^P)^9$$  \hspace{1cm} (5.1)

Thus, when $R_{10}$ is small, the effective 10D SUGRA description breaks down at a much lower energy scale than the 11D Planck mass.

In fact, the existence of BPS M2 brane states tells us that there is an even lower energy scale in the problem. Consider a configuration of an M2 brane “wrapped on the circle”:

$$x^\mu(t, \sigma, \xi) = x^\mu(t, \sigma); \ \mu = 0 \ldots 9 \hspace{1cm} (5.2)$$

$$x^{10}(t, \sigma, \xi) = R_{10}\xi \hspace{1cm} (5.3)$$

The main part of the action of an M2 brane is the volume of the world surface swept out by the brane, multiplied by the brane tension, which is of order $L_P^{-3}$. For wrapped configurations, this reduces to the ten dimensional area swept out by the string $x^\mu(t, \sigma)$ in units of the string tension $L_S^{-2} \sim R_{10}L_P^{-3}$. This gives an energy scale for string oscillations $m_S \sim \sqrt{R_{10}M_{11}^P}$ which is much smaller than the ten dimensional Planck mass.

Thus, we are led to expect that M-theory on a small circle is dominated at energies below the eleven dimensional Planck scale by low tension string states. At the energy scale set by the string length gravitational couplings are weak. This can be seen by rewriting the dimensionally reduced action in terms of the string length. The coefficient of the Einstein action becomes $(L_P/R_{10})^3L_S^8$, indicating that at the energy scale defined by the string tension, gravitational couplings are determined by a small dimensionless parameter, $g_S^2 = e^{2\phi(\infty)} = (R_{10}/L_P)^3$. In fact, using the technology of Matrix Theory, [32] one can show that in the small $g_S$ limit, M-theory becomes a theory of free strings.

There is in fact a unique consistent ten dimensional theory of free strings with the supersymmetry algebra of 11D SUGRA compactified on a circle (the so called IIA algebra). It is the Type IIA superstring. In fact, one can directly derive the full Green-Schwartz action for the superstring by considering the supermembrane action of [33] restricted to the wrapped M2 brane configurations above. However, this derivation is
entirely classical, while the existence of the string and its action actually follow purely from SUSY and are therefore exact quantum mechanical results.

This, the first of many dualities, exhibits the general strategy of the duality program. Starting from a limiting version of M-theory valid only in a certain domain of moduli space and/or energy scale we exhibit some heavy BPS state whose mass can be extrapolated into regimes where the original version of the theory breaks down, and goes to zero there. We then find that the effective theory of these new light states is another version of the theory. For the most part, we find only weakly coupled string limits and limits where 11D SUGRA is a valid approximation. This is exactly true if we restrict attention to vacuum states with three or more noncompact space dimensions and 32 supercharges. With less SUSY there are limiting regimes (many of which are called by the generic name F-theory) where we do not have a systematic expansion parameter, though many exact results can be derived.

If we try to repeat this exercise on a two torus something really interesting happens. The new regime corresponds to taking the area of the torus to zero with the rest of its geometry fixed. As is well known, up to an overall scale, the geometry of a two torus is determined by a parallelogram in the complex plane with one side going from zero to one along the real axis. This parallelogram describes the periodic boundary conditions which define the torus. It is completely fixed by its other side, which is a complex number \( \tau \) in the upper half plane. \( \tau \) is called the complex structure of the torus. In fact, different \( \tau \)s can describe the same torus. The \( SL(2, \mathbb{Z}) \) group generated by \( \tau \rightarrow \tau + 1 \) and \( \tau \rightarrow -1/\tau \) maps all complex numbers which define the same torus onto each other.

In the zero area limit, we can define a whole set of low tension strings, by choosing a closed path of nontrivial topology on the torus, and studying M2 branes wrapped on this path. The inequivalent nontrivial paths on the torus are characterized by two fundamental cycles, called \( a \) and \( b \) in Figure 1. A general path consists of going \( p \)-times around \( a \) and \( q \) times around \( b \). It turns out that the \((p,q)\) strings with relatively prime integers are stable and can be viewed as bound states of the \((1,0)\) and \((0,1)\) strings. When the integers are not relatively prime the state is not bound. This picture is derived from the BPS formula [34] for the string tension, which follows from a classical calculation in 11D SUGRA and is promoted to an exact theorem by invoking SUSY. The proof that the states with integers having a common divisor are not bound is more complicated [29].

Something even more interesting occurs when we consider M2 branes which wrap the whole torus. A state with an \( m \) times wrapped brane has energy

\[
\sim mAL_P^3 \equiv \frac{m}{R_B} \quad \text{(5.4)}
\]
in the limit that the area goes to zero. It turns out [29] that the \( m \) wrapped states are stable against the energetically allowed decay into \( m \) singly wrapped states. So in the area goes to zero limit we get a new continuum. Any other state in the theory can bind with these wrapped membranes at little cost in energy. The result is that the states are labelled by a new continuous quantum number in addition to their momenta in the eight noncompact dimensions. Even more remarkable (the only extant proof of this requires Matrix Theory [6]) is that the new continuum is related to the old one by an \( SO(9,1) \) Lorentz symmetry [6]. Thus, in M-theory \( 11 - 2 = 10 \).

Since the origin of the new Lorentz group is obscure, we have to resort to Matrix Theory again to find out which kind of ten dimensional SUSY the theory has\(^{15}\). It turns out that both of the ten dimensional Weyl spinors have the same chirality, and we are in the IIB theory.

There is of course a weakly coupled string theory with this SUSY algebra; the Type IIB Green Schwarz superstring. In fact, the SUGRA limit of this theory has an \( SL(2,R) \) symmetry which one can argue is broken to \( SL(2,Z) \) by instanton effects. It acts in the expected way on \( \tau \). Furthermore there are actually two different two form gauge potentials, which form an \( SL(2,Z) \) doublet. Thus we expect to find two different kind of strings, the F(undamental) string and the D(irichlet) string. The latter is a soliton, whose tension goes to infinity in the weak coupling limit. Consulting the eleven dimensional picture we realize that the weak coupling limit should be identified with the \( \text{Im} \tau \to \infty \) limit in which one of the cycles of the torus is much smaller than the other. The F(D) string is then identified with the M2 brane wrapped around the shorter (longer) cycle.

This trick of dimensional reduction by \( 2 - 1 \) dimensions is interesting because it gets around old theorems which stated that Kaluza Klein reduction cannot produce chirality. It can be generalized in the following interesting way. Certain higher dimensional manifolds can be viewed as “elliptic fibrations”. That is, they consist of an \( m \) dimensional base manifold with coordinates \( z \) and a family of two tori \( \tau(z) \) (the area also varies with \( z \)), making altogether an \( m + 2 \) dimensional manifold. Now one varies parameters in such a way that the area of the two tori all shrink to zero. Naively this would give a dimensional reduction by two dimensions. However, given enough SUSY one can again verify that an extra noncompact dimension appears in the limit so that the result is dimensional reduction by one. The name given to this general procedure

\(^{15}\)Actually, a consideration of the field content of the low energy theory is enough. In particular the fact that variations of the complex structure \( \tau \) over the noncompact dimensions should appear as a complex scalar field, is enough to tell us that we are in the IIB theory. The ten dimensional Type IIA theory has only a single real massless scalar. Matrix Theory is only necessary to prove that the statement is consistent at all energies.
is F-theory [13]. It is very useful for describing strong coupling limits of the heterotic string.

If we try to pull the shrinking torus trick in 3 dimensions we run into a disappointment. The new low tension state which appears is a membrane obtained by wrapping the M5 brane around the torus. The effective low energy theory is then M-theory again with a new Planck length defined in terms of the light membrane tension. Indeed it can be shown [35] [5] that for three or more noncompact dimensions the only limiting theories one can obtain without breaking any SUSY are the Type II string theories and 11D SUGRA. We will actually prove this theorem below in our discussion of extreme limits of the moduli space.

For my last example of a duality I will study the moduli space of M-theory compactifications which break half of the 11D SUSY. This is achieved by compactifying on four dimensional spaces called K3 manifolds. We will have to understand a little bit about the geometry of such manifolds, but I promise to keep it simple. The equation for the SUSY variation of the gravitino is

$$\delta\psi_\mu = D_\mu \epsilon$$

This must vanish for certain values of the SUSY parameters $\epsilon$ in order to leave some SUSY unbroken. A consistency condition for this vanishing is

$$R_{\mu\nu}^{ab} \sigma_{ab} \epsilon = 0$$

where $R_{\mu\nu}^{ab}$ is the curvature tensor in an orthonormal frame and $\sigma_{ab}$ are the spin matrices in the Dirac spinor representation. We will always be dealing with strictly Euclidean $n$ dimensional manifolds so these are generators of $O(n)$.

In two and three dimensions, the spinor has only two components and the generators are the Pauli matrices. The only solution of (5.6) is to set the curvature equal to zero, but then we do not break any SUSY. We can do better with four compact dimensions. The group $SO(4) = SU(2) \times SU(2)$ has two different two dimensional spinors (familiar to particle physicists after an analytic continuation as left and right handed Weyl spinors), transforming as $(1,2)$ and $(2,1)$ under the two $SU(2)$ subgroups. Thus, if the curvature lies in one of these two subgroups and we choose $\epsilon$ to be a singlet of that subgroup, then the consistency condition is satisfied.

The stated condition on the curvature tensor is

$$R_{\mu\nu}^{ab} = \epsilon^{abcd} R_{\mu\nu}^{cd}$$

It is easy to see, using one of the standard identities for the Riemann tensor, that this implies that the Ricci tensor, $R_{\mu}= R_{\mu\nu}^{ab} \epsilon_{\nu b}$, vanishes. Thus, insisting that half the
SUSY is preserved implies that the manifold satisfies the vacuum Einstein equations (Euclidean) or is, as we say, Ricci flat.

There is one more immediate consequence of the SUSY equations which I want to note. Just like the spinor representation, the second rank antisymmetric tensor representation of $SO(4)$ breaks up into a direct sum of $(1, 3)$ and $(3, 1)$. Thus, there will be, on a manifold which preserves half the SUSY, three independent covariantly constant (and therefore closed and nowhere vanishing) two forms, $\omega^a_{\mu\nu}$. Modulo some technical questions which we will ignore, this implies that the manifold is hyperkähler. Compact, four dimensional hyperkähler manifolds are called K3 manifolds (this is part of an elaborate joke having to do with the famous Himalayan peak K2).

No one has ever seen a K3 metric, but mathematicians are adept at dealing with objects they can’t write down explicitly. We will only need a tiny bit of the vast mathematical literature on these spaces. In particular, I want to remind you of the famous de Rham theorem, which relates topologically nontrivial submanifolds in a space to the cohomology of differential forms. Remember that a differential form is just a totally antisymmetric tensor multiplied by Grassmann variables that mathematicians call differentials

$$\omega = \omega_{\mu_1...\mu_p} dx^{\mu_1} \ldots dx^{\mu_p} \quad (5.8)$$

The operator

$$d = \frac{\partial}{\partial x^\mu} dx^\mu \quad (5.9)$$

maps $p$-forms into $p + 1$-forms and satisfies $d^2 = 0$. This defines what mathematicians call a cohomology problem. Namely, one wants to characterize all solutions of $d\omega = 0$, modulo trivial solutions of the type $\omega = d\psi$ (such trivial solutions are called exact forms) where $\psi$ is a well defined $p - 1$-form. This is a generalization of finding things with zero curl which cannot be written as gradients. A well known physics example is a constant magnetic field on the surface of a sphere or a torus. The set of closed but not exact $p$-forms is called the cohomology at dimension $p$.

The importance of $p$-forms stems from the fact that their integrals over $p$-dimensional submanifolds are completely defined by the differential topology of the manifold. No metrical concepts are needed to define these integrals.

Another important concept is that of a nontrivial $p$-cycle on a manifold. Basically this is a $p$-dimensional submanifold which cannot be shrunk to a point because of the topology of the manifold. The simplest examples are the $a$ and $b$ 1-cycles on the torus of Figure 1. Actually, it is an equivalence class of submanifolds because any curve which circles around the $a$ cycle and then does any kind of topologically trivial thing on the rest of the torus is equivalent to the $a$ cycle. de Rham’s theorem tells us that there is a one to one correspondence between $p$-cycles and $p$-forms, as we have mentioned above.
After that brief reminder, we can turn to the question of what the cohomology of K3 manifolds is. Since it is a topological question we can answer it by examining an example. Every 4-manifold has cohomology at dimension 0 (the constant function) and dimension 4 (the volume form, \( \epsilon_{abcd} e^a_{\mu_1} e^b_{\mu_2} e^c_{\mu_3} e^d_{\mu_4} \)). The simplest K3 manifold is the “physicists K3”, the singular orbifold \( T^4/\mathbb{Z}_2 \). This is defined by taking a rectilinear torus with axes \( 2\pi R_i \) and identifying points related by \( x^i \rightarrow \pm x^i + 2n_i \pi R_i \). This has 16 fixed points in the fundamental domain: \( x^i = R_i (1 \pm 1) \pi / 4 \). The space is flat except at the fixed points but has curvature singularities there. It can be verified that the holonomies around the fixed points are in a single \( SU(2) \) subgroup of \( O(4) \) so the space is a K3.

It is easy to see that the nontrivial one cycles on the torus all become trivial on the orbifold (the corresponding one forms are odd under the orbifold transformation and are projected out). The torus has six obvious 2 cycles, which are the six different \( T^2 \)s in the \( T^4 \). In addition, when one studies this singular manifold as a limit of smooth K3’s by the methods of algebraic geometry (realizing the manifold as the solution set of polynomial equations) one finds that each of the fixed points is actually a two sphere of zero area. Thus there are twenty two non-trivial two cycles on a K3 manifold. By the de Rham theorem, there are twenty two linearly independent elements of the cohomology at dimension two of K3.

One can introduce a bilinear form on two forms in a four manifold. The product of two two cycles is a four form, which can be integrated over the manifold. Define:

\[
I_{ij} = \int \omega_i \omega_j
\]  (5.10)

Remember that \( \int *\omega \omega \) is the usual Euclidean Maxwell action for a two form field strength is thus positive definite. The form \( I \) is thus negative on antiselfdual tensors and positive on self dual ones. We have already established that there are three independent antiselfdual covariantly constant (and therefore closed but not exact) two forms. It can be shown that the rest of the cohomology consists of self dual two forms, so that \( I \) has signature \((19, 3)\). A basis can be chosen in which it has the form \( I = \sigma_1 \oplus \sigma_1 \oplus \sigma_1 \oplus E_8 \oplus E_8 \), where \( \sigma_1 \) is the familiar Pauli matrix and \( E_8 \) is the Cartan matrix of the Lie Group \( E_8 \) (the matrix of scalar products of simple roots).

This is very suggestive. The heterotic string compactified on a three torus, has nineteen left moving and three right moving currents (the sixteen \( E_8 \times E_8 \) gauge currents and linear combinations of the momentum and winding number currents on the torus). Indeed, Narain [16] introduced the same scalar product, where left movers have opposite signature to right movers, in his study of heterotic compactifications on tori. At this point, readers who are not familiar with the heterotic string will undoubtedly benefit from . . .

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5.2 An heterotic interlude

The bosonic string “lives” in 26 bosonic dimensions, while the superstring lives in 10. This discrepancy has two sources, both of which have to do with the difference between the world sheet gauge groups of the two theories. The bosonic string has only worldsheet diffeomorphism invariance and the 26 is required to cancel the anomaly in this symmetry against a corresponding anomaly coming from Fadeev-Popov ghosts. Type II superstrings have worldsheet supergravity\textsuperscript{16}. On the one hand, this requires the embedding coordinates $X^\mu$ to have superpartners $\psi^\mu$ which also contribute to the anomaly. On the other hand, since the world sheet gauge group is larger, there are more ghosts. The net result of these two effects is to reduce to 10 the maximal number of Minkowski dimensions in which the Type II strings can propagate. Smaller numbers of $X^\mu$ can be achieved by compactification.

In two dimensions, the smallest SUSY algebra is called $(1,0)$ and has a single right moving spinor supercharge. There is a corresponding chiral worldsheet supergravity. Type II strings have the vector like completion of this, $(1,1)$ SUSY, which consists of one left moving and one right moving supercharge. The heterotic string is defined as a perturbative string theory with only $(1,0)$ worldsheet SUSY. Its maximal number of Minkowski dimensions is 10.

In ten Minkowski dimensional target space, the world sheet field theory of any string theory is a collection of free massless fields each of which can be separated into its left and right moving components. The ten dimensional heterotic string has 10 right moving $X^\mu$s and their superpartners, and 26 left moving $X^\mu$s. In order to eliminate an extra continuum from the 16 extra bosonic dimensions, we can compactify them on a torus. This simply means that we eliminate all states which are not periodic functions in these 16 coordinates.

The restriction to toroidal compactification in fact follows from a deeper principle. The construction outlined so far was a consistent gauge fixed quantum theory with infinitesimal $(1,0)$ superdiffeomorphism invariance in two dimensions. We have seen above that perturbative string theory requires us to evaluate the world sheet path integral on Riemann surfaces of arbitrary genus. For genus one and higher, there are disconnected pieces of the diffeomorphism group and we must require invariance under those as well. This is called the constraint of modular invariance. It turns out that this restriction is satisfied iff we choose the sixteen dimensional torus to be the Cartan torus of one of the groups $E_8 \times E_8$ or $SO(32)$. The operators $(\partial_\tau - \partial_\sigma)X^i$, with $i = 1 \ldots 16$, are then the current algebra for the $U(1)^{16}$ Cartan subgroup. The

\textsuperscript{16}Not to be confused with spacetime supergravity, which is another beast entirely.

\textsuperscript{17}$\tau$ and $\sigma$ are worldsheet coordinates and the $X^i$ satisfy $(\partial_\tau + \partial_\sigma)X^i = 0$. 

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currents corresponding to raising and lowering operators of the group have the form of exponentials $e^{ir_i X^i}$ where $r_i$ are the roots of the algebra.

If we further compactify the heterotic string on a $d$-torus, we get $d$ pairs of $U(1)$ currents (which, for generic radii of the torus are not completed to a nonabelian group) from $(\partial_r \pm \partial_s) X^a$. Half of these are purely left moving and the other half purely right moving. The vectorlike combinations are simply the Kaluza Klein symmetries expected when compactifying GR on a torus. The axial combinations couple to the winding number of strings around the $d$ torus. Perturbative string theory always has a two form gauge field $B$, which couples to the string world sheet as $\int_{\text{worldsheet}} B$. When integrated around the $d$ 1-cycles of the torus, it gives rise to $d$ one forms which couple to string winding number. In addition to this gain in the rank of the symmetry group, a generic toroidal compactification will lose the nonabelian parts of the group. This is because we can have Wilson lines around the cycles of the torus. Thus, at a generic point on the moduli space of toroidal compactifications of the heterotic string, the gauge group is $U(1)^{16+d} \times U(1)^d$, where we have separated the contributions coming from left and right moving currents.

Thus, one way of viewing toroidally compactified heterotic string theory is to say that it consists of the modes of $16 + d$ left moving and $d$ right moving scalar fields, where the zero modes of these fields live on independent tori (there are also fermionic partners and fields representing the noncompact dimensions, but we do not need to discuss them here). A given compactification can then be specified by talking about the allowed values of the dimensionless momenta around the torus, a discrete set of numbers $(l^R_m, l^L_n)$. One must insist that the vertex operators with any allowed momenta are all relatively local on the world sheet in order that the expressions for tree level string amplitudes make sense\textsuperscript{18}. Furthermore one must impose a condition called modular invariance to guarantee that one loop amplitudes make sense. These conditions turn out to be equivalent \textsuperscript{16} to the restriction

$$(l^L)^2 - (l^R)^2 \in 2\mathbb{Z} \quad (5.11)$$

combined with the requirement that the lattice of all possible momenta be self dual\textsuperscript{19}. Such lattices turn out to be unique up to an $O(16 + d, d)$ rotation. It can be shown that the parameters of these rotations are equivalent to choices of background Wilson lines.

\textsuperscript{18}Left moving or right moving fields are not local operators. The vertex operators are exponentials of these fields and are generally not local either. But certain discrete subsets of these vertex operators are relatively local.

\textsuperscript{19}The dual of a lattice with a scalar product defined on it is the set of all vectors which have integer scalar product with the vectors of the original lattice.
lines, constant antisymmetric tensor fields on the $d$ torus and the choice of the flat metric on the torus.

Thus, heterotic string theory compactified on a $d$-torus with generic Wilson lines has a natural $O(16+d,d,Z)$ invariant scalar product defined on the space of worldsheet currents. The fact that the same sort of scalar product arises as the intersection matrix of cohomology classes of K3 manifolds is the first hint that the two systems are related.

5.3 Enhanced gauge symmetries

One of the reasons Type II strings did not receive much attention after the discovery of the heterotic string was that they did not appear to have the capability of producing gauge groups and representations like those of the standard model. The same was true of 11D SUGRA. However, the suggestive connection with heterotic strings leads one to suspect that a mechanism for producing nonabelian gauge symmetries has been overlooked. The theory of singularities of K3 manifolds was worked out by Kodaira and others in the 1950’s. It turns out that one can characterize singular K3 manifolds in terms of topologically nontrivial cycles which shrink to zero size. The singularity is determined by the intersection matrix $I$ restricted to the shrinking cycles. It turns out that in almost all cases, the resulting matrix was the Cartan matrix of some nonabelian Lie group. In the purely mathematical study of four manifolds, there is no way to understand where the Lie group is.

However, viewed from the point of view of M-theory compactification on K3, a nonabelian group jumps into view. Indeed, imagine BPS M2 branes wrapped around the shrinking cycles of the singularity. These will be massless particles in the uncompactified seven dimensional spacetime. Since we have 16 SUSYs in the effective seven dimensional theory these must include massless vector fields, since the smallest representation of this SUSY algebra is the vector multiplet. Furthermore, even away from the singularity, we have 22 $U(1)$ vector multiplets. Indeed one can write three form potentials in 11D SUGRA of the form $A_{\mu\nu\lambda}dx^\mu dx^\nu dx^\lambda = a_\mu^i(X) dX^\mu \omega_i$, where $\omega_i$ are the 22 independent harmonic two forms on K3, and $X$ are the seven noncompact coordinates. The $a_\mu^i$ are gauge potentials in a product of $U(1)$ algebras which will be the Cartan subalgebra of the nonabelian group that appears at the singularity. Since membranes are charged under the three form, we see, using the de Rham connection between forms and cycles, that the new massless vector bosons are charged under the Cartan subalgebra, i.e. we have a nonabelian gauge theory.

One further point of general interest. As is obvious from the $T^4/Z_2$ orbifold example, the singularities that give rise to nonabelian gauge groups live on manifolds of finite codimension (or branes) in the compact space. If the volume of the compact space is large, this will lead to a large ratio between the gauge and gravitational couplings in the
noncompact effective field theory. We will discuss the phenomenological implications of this observation in the context of the Hořava Witten scenario below.

The emergence of nonabelian gauge theory from singularities is one of the most beautiful results of the M-theory revolution. It combines Wilson’s observation that singularities in the free energy functional at second order phase transitions could be correlated with the appearance of massless states, with the mysterious occurrence of Lie groups in singularity theory, uniting physics and mathematics in a most satisfying fashion. One can go much further along these lines. When studying singularities of Calabi-Yau manifolds of dimension three or four one encounters cases which cannot be explained in terms of gauge theory, but which do have an explanation in terms of nontrivial fixed points of the renormalization group. The interplay between SUSY, singularity theory, and the theory of the renormalization group in these examples, is a stunning illustration of the power of M-theory [17].

So far, we have seen how enhanced gauge symmetries arise from M-theory on K3 but have not yet delivered on our promise to make a connection with the heterotic string. We have seen in toroidal examples that the key to string duality is the existence of light BPS states when cycles of a manifold shrink to zero. The limit of M-theory on K3 which gives rise to weakly coupled heterotic string theory (on a torus, $T^3$) is the limit where the K3 volume shrinks to zero in Planck units. The M5 brane wrapped around the K3 gives rise to a low tension string in this limit [14]. Recall that the world volume of the fivebrane carries an antisymmetric tensor gauge field with self-dual 3 form field strength, $H = * H$, which satisfies $dH = 0$. For configurations of the fivebrane wrapped on K3 one can study configurations of $H$ of the form $H = j_i \omega_i$, where $j_i$ is a world volume one form which depends only on the two coordinates of the world volume which are not wrapped on K3, and $\omega_i$ is one of the 22 harmonic forms on K3. In order to satisfy $H = * H$, $j_i$ must satisfy $j_i = \epsilon_i * j_i$, where $\omega_i = \epsilon_i * \omega_i$ (recall that 19 of the forms on K3 have $\epsilon_i = 1$, while for the other three it is negative). $dH = 0$ implies $dj_i = 0$. In more familiar notation, the string formed from the M5 brane wrapped on K3 will have 19 left moving ($j^a = \epsilon^{ab}$) and 3 right moving conserved currents. This is precisely the bosonic field content of the (bosonic form of) the heterotic string on a three torus. The evident SUSY of the wrapped brane configuration guarantees the existence of the appropriate world sheet fermions.

The heterotic string was discovered as a solution to the consistency conditions of perturbative string theory. Though it was obviously the perturbative string most closely connected to real physics, no one ever claimed that it was beautiful. The derivation of its properties from the interplay between the K3 manifold and the M5 brane of 11D SUGRA can make such an aesthetic claim. It is another triumph of string duality.

This construction automatically gives rise to the heterotic string compactified on a
three torus. Note that again the geometry of K3 disappears from the ken of low energy observers and is replaced by a space of a different dimension and topology. Following Aspinwall [15] we can try to recover the ten dimensional heterotic string from the K3 picture. The mathematics is somewhat complex but in the end one recovers the picture of Hořava and Witten [43] (if one is careful to keep the full $E_8 \times E_8$ gauge symmetry manifest at all times). That is, one finds 11D SUGRA compactified on an interval, with $E_8$ gauge groups living on each of two 10 dimensional boundaries\textsuperscript{20}. The heterotic string coupling is related to the size of the interval, $L$, by $g_s = (L/L_P)^{3/2}$.

The Hořava-Witten description of the strongly coupled heterotic string in ten dimensions was originally motivated by considerations of anomaly cancellation and matching onto various weakly coupled string limits. It is somewhat more satisfying to realize it as a singular limit of compactification of M-theory on a K3 manifold.

Witten [44] has pointed out that the strong coupling limit of this picture can resolve one of the phenomenological problems of weakly coupled heterotic string theory. Among the few firm predictions of heterotic perturbation theory is the equality between the gauge coupling unification scale $M$ and the four dimensional Planck scale $m_P$. In reality these differ by a factor of 100. Careful consideration of threshold corrections brings this discrepancy to a factor of 20, but one may still find it disturbing. Witten points out that in the picture of 11D SUGRA on an interval it is easy to remove this discrepancy. Indeed, if we compactify the system to four dimensions on a Calabi-Yau 3-fold of volume $V_6$\textsuperscript{21}, then the four dimensional gauge couplings are given approximately by $1/g_0^2 \sim (V_6/L_P^6)$, while the four dimensional Planck mass is given by $m_P^2 \sim (L V_6/L_P^8)$. Tuning $L$ and the volume to the experimental numbers (and taking into account various numerical factors), gives a linear size for the 3-fold of order $2L_P$ and $L \sim 70L_P$. The unification scale $M$ is of order $L_P^{-1}$.

I want to emphasize three features of this proposal. First, the four dimensional Planck mass is not a fundamental scale of the theory. Rather, it is the unification scale, identified with the eleven dimensional Planck scale, which plays this role. Secondly, since we have seen that gauge groups generically arise on branes in M-theory, Witten’s proposal may be only one out of many possibilities for resolving the discrepancy between the unification and Planck scales. A possible advantage of a more flexible scenario might be the elimination of all large dimensionless numbers from fundamental physics, in particular the factor of 70 in Witten’s scenario. If the codimension of the space on

\textsuperscript{20}As one takes the limit corresponding to infinite three torus, one is forced to K3 manifolds with two $E_8$ singularities.

\textsuperscript{21}Actually, due to details which we will not enter into, the Calabi-Yau volume varies along the interval. The parameter $V_6$ is its value at the end of the interval where the standard model gauge couplings live.
which the standard model lives is large, then the factor of order 100 which is attributed
to the volume of this space in Planck units, might just be $2^6$. Finally, let us note that
in this brane scenario, the bulk physics enjoys a larger degree of SUSY (twice as much)
than the branes. This will be useful in our discussion of inflationary cosmology below,
and may also help to solve the SUSY flavor problem [45] [46].

5.4 Conclusions

In this brief summary of M-theory and its duality symmetries, we have seen that
classical geometry can undergo monumental contortions while the theory itself remains
smooth. When there are enough noncompact dimensions and enough supersymmetry,
there are exact moduli spaces of degenerate vacua which interpolate between regions
which have very different classical geometric interpretations by passing through regions
where no geometrical interpretation is possible (for the compact part of the space). The
most striking example is perhaps the K3 compactification, where the 80 geometrical
parameters describing K3 manifolds are interpreted in an appropriate region of the
parameter space as the geometry, and background gauge and antisymmetric tensor
fields, of a three torus with heterotic strings living on it. The clear moral of the story
is that “geometry is in the eye of the (low energy) beholder”, and must actually be a
low energy approximation to some other concept, which we do not as yet understand.

Equally important for our further discussion is that the modular parameters inter-
polate smoothly between different geometrical regions and exist even in regions which
can not be described by geometrical concepts. In different regimes of moduli space,
the moduli can be viewed as zero modes of different low energy fields living on differ-
ent background geometries. But, although their interpretation can change, the moduli
remain intact, and (with enough SUSY), their low energy dynamics is completely de-
termined. In subsequent sections we argue that they are the appropriate variables for
discussing the evolution of the universe.

6. Quantum Cosmology

6.1 Semiclassical cosmology and quantum gravity

In today’s lecture we will leave behind our brief survey of M-theory and duality and
proceed to cosmological questions. We will begin by discussing some “fundamental”
issues in quantum cosmology and proceed to a somewhat more practical application of
M-theory to inflationary models. None of this work will lead to the kind of detailed
model building and comparison with observation that is the bread and butter of most
astroparticle physics. In my opinion, the current theoretical understanding of M-theory
does not warrant the construction of such detailed models. Detailed inflationary model
building requires, among other things, knowledge of the inflaton potential. In an M-
theory context this means that we have to have control over SUSY breaking terms in
the low energy effective action. Even the advances of the last few years have not helped
us to make significant progress in understanding SUSY breaking.

My aim in these lectures will be to address general questions like what the inflaton
is likely to be, the relation between the energy scales of inflation and SUSY breaking,
the connection between various scales and pure numbers encountered in cosmology with
the fundamental parameters, and so on. We will see that a rather amusing picture can
be built up on this basis, which is quite different from most conventional cosmological
models. I will concentrate here primarily on my own work (and that of my collabora-
tors) rather than trying to give a survey of all possible approaches to cosmology within
M-theory. Prof. Veneziano will be giving a detailed exposition of one of the other
major approaches, so between the two of us you will get some idea of what is possible.

The discussion will be divided into two parts, one more “fundamental” and the
other more “practical”. The aim of the first part will be to pose the problem of how
the conventional equations of cosmology may eventually be derived from a fully quan-
tum mechanical system. We will also begin to address the question of why M-theory
does not choose one of its highly supersymmetric vacua for the description of the world
around us. We end this exposition by introducing a heterodox antiinflationary cosmol-
ogy. The “practical” section will concentrate on issues related to moduli and SUSY
breaking. We will see that cosmological considerations suggest a vacuum structure sim-
ilar to that proposed by Hořava and Witten, and put further constraints on the form
of SUSY breaking. One also obtains an explanation of the size of the fluctuations in
the microwave background in terms of the fundamental ratio between the unification
and Planck scales. We will conclude with an inflationary cosmology very different from
most of those in the literature. Among its virtues is the possibility of supporting a QCD
axion with decay constant as large as the fundamental scale. Indeed, the assumption
that such an axion exists gives an explanation of the temperature of matter radiation
equality in terms of the fundamental parameters of the theory.

We will begin our discussion of “fundamental” cosmology by recalling the treatment
of quantum cosmology in GR. One of the more bizarre consequences of an attempt to
marry GR to QM is the infamous Problem of Time. A generally covariant theory
is constructed for the precise purpose of not having a distinguished global notion of
time. In classical mechanics this is very nice, but quantum mechanically it turns the
conventional Hamiltonian framework on its head. The problem can be seen in simple
systems with time reparametrization invariance, that is, actions of the form

\[ \int dt e L(q, \dot{q}/e) \]  

(6.1)

where \( q \) represents a collection of variables which transform as scalars under time reparametrization, and \( e \) is an einbein (i.e. \( edt \) is time reparametrization invariant). We can use the symmetry to set \( e \) equal to a constant (gauge fixing), but the \( e \) equation of motion then says that the canonical Hamiltonian of the \( q \) vanishes.

\[ H = \dot{q} \frac{\partial L}{\partial \dot{q}} - L = 0. \]  

(6.2)

In simple covariant systems like Chern-Simons gauge theory, one can solve this Hamiltonian constraint and quantize the system in the sense that the classical observables are realized as operators in a Hilbert space. However, the notion of time evolution is still somewhat elusive. More generally, in realistic systems where the constraints are not explicitly soluble, one recovers time evolution by finding classical variables. For example, if spacetime has a boundary, with asymptotically flat or asymptotically Anti deSitter boundary conditions, then one can use one of the symmetry generators of the classical geometry at infinity as a time evolution operator.

In cosmology one generally does not have the luxury of a set of variables whose quantum fluctuations are frozen by the boundary conditions. The notion of time evolution is tied to a semiclassical approximation for a particular set of variables. Different cosmological evolutions may not be described by the same semiclassical variables. One of the challenges of this framework is to find a generic justification for the semiclassical approximation. To see how the idea works, one “quantizes” the \( g_{00} \) Einstein equation by writing it in Hamiltonian form and naively turning the canonical momenta into differential operators (at the level of sophistication of this analysis, it does not make sense to worry about ordering ambiguities). This gives the Wheeler-DeWitt equation, a second order PDE which is supposed to pick out the physical states of the system inside a space of functionals of the fields on a fixed time slice. The challenge is to put a positive metric Hilbert inner product on the space of physical states and identify a one parameter group of unitary operators that can be called time evolution.

It is well known that, viewed as a conventional field theory, the conformal factor of the gravitational field has negative kinetic energy. In quantization of GR in perturbation theory around any classical solution of the field equations, the negative modes are seen to be gauge artifacts and a positive definite Hamiltonian is found for gravitons.

In general closed cosmologies, the analogous statement is the following: the Wheeler DeWitt constraint completely eliminates all negative modes from the physical Hilbert
space. It is convenient to think of GR in synchronous gauge, where the $g_{0i}$ components of the metric vanish and $g_{00} = 1$. Such a gauge is built by choosing a spacelike hypersurface and following timelike geodesics orthogonal to this hypersurface to define the evolution into the future. It can then be shown that all of the negative modes represent the freedom to change the choice of the initial hypersurface (the many fingered time of GR). The Wheeler-DeWitt equation is then the constraint which says that physics must be independent of this choice. It is often convenient to solve the constraint in stages. Namely, among all spacelike surfaces in a given spacetime geometry, there are one parameter families related to each other by propagation along orthogonal timelike geodesics. The choice of such a family eliminates all but one of the negative modes, the last one being related to the choice of which surface in the family is called the initial surface. That is, it is related to the time as measured by observers following the timelike geodesics which define the family\textsuperscript{22}. It can be chosen to be any monotonic function along these trajectories, and it is often convenient to choose the volume of the spatial metric.

The upshot of all this, is that once a family of hypersurfaces is chosen, one still has a single component of the Wheeler-DeWitt constraint which has not yet been imposed. Classically this is the familiar Friedmann equation relating the expansion rate to the energy density. A naive quantization of this equation gives a hyperbolic PDE on a space with signature $(1,n)$. A form of this equation sufficiently general for our purposes is

$$[hG^{ab}(m)\partial_a\partial_b + g^{AB}(q,m)\partial_A\partial_B + \frac{1}{h}V(m) + U(q,m)]\Psi = 0 \quad (6.3)$$

We have separated the variables into classical ($m^a$) and quantum ($q^A$) and introduced a formal parameter $h$ to organize the WKB like approximation for the classical variables. The metric $G$ is hyperbolic with one negative direction, while the metric $g$ has Euclidean signature. The analysis we are presenting goes back to [37]. Up to terms of order $h$, it is easy to see that the solution of this equation has the form

$$\Psi(m,q) = e^{iS(m)/h}A(m)\psi(m,q) \quad (6.4)$$

where

$$-G^{ab}\nabla_a S \nabla_b S + V = 0 \quad (6.5)$$

$$G^{ab}(\nabla_a S \nabla_b A + A \nabla_a \nabla_b S = 0) \quad (6.6)$$

$$iG^{ab}\nabla_a S \nabla_b \psi + H \psi = 0 \quad (6.7)$$

\textsuperscript{22}This discussion is purely classical, but mirrors the less intuitive mathematical operations which one carries out in semiclassical quantization of the WD equation.
\[ H \equiv g^{AB} \nabla_A \nabla_B + U \quad (6.8) \]

The first of these equations has a Hamiltonian-Jacobi form. It can be solved by finding classical motions \( \dot{m}^a(t) = G^{ab} \nabla_a S(m) \). \( S \) is then the action of the classical solution, and (6.5) is satisfied if the solution has zero "energy". The existence of real zero energy solutions (and thus real \( S \)) depends on the fact that \( G^{ab} \) has nonpositive signature.

Using the classical solution \( m^a(t) \), we recognize that (6.7) can be written as a conventional Schrödinger equation:

\[ i \frac{\partial \psi}{\partial t} = H \psi \quad (6.9) \]

Positivity of the Hamiltonian (6.8) requires that \( g_{AB} \) have Euclidean signature. Note that since \( H \) depends on the \( m \)'s, the Hamiltonian will in general be time dependent. Furthermore, the quantum fluctuations of the classical variables \( m \) will have a sensible Hamiltonian only if the metric \( G_{ab} \) has only a single negative eigenvalue. Thus, we see that within the naive approach to quantization of Einstein’s equations, the existence of a Hilbert space interpretation of the physical states, with a positive definite scalar product and a unitary time evolution with a sensible Hamiltonian operator, is closely tied to the fact that Einstein’s equations coupled to matter with positive kinetic energy have a hyperbolic metric with exactly one negative eigenvalue (after gauge fixing).

One may wonder whether these observations will survive in a more realistic theory of quantum gravity. We know that Einstein’s action is only a low energy effective description of M-theory. Even those heretics who refuse to admit that M-theory is the unique sensible theory of quantum gravity\(^{23}\) are unlikely to insist that quantization of this famously nonrenormalizable field theory by the crude procedure described above is the final word on the subject of quantum gravity. I would like to present some evidence that in M-theory the \((1,n)\) signature of the metric on the space of classical variables is indeed guaranteed by rather robust properties of the theory.

Before doing so I want to point out how M-theory addresses the question of the existence of semiclassical variables \( m^a \). There are actually two desiderata for the choice of such variables: we want the semiclassical approximation for these variables to be valid during most of cosmic history\(^{24}\). Secondly, given the notion of energy implied by the

\(^{23}\)One hopes that the world has not come to a state in which one has to emphasize that a sentence like this is a joke, but let me record that fact in this footnote just to be on the safe side.

\(^{24}\)As Borges pointed out long ago \cite{7} it is almost impossible to avoid self referential paradoxes when trying to conceptualize a system in which the notion of time is an illusion or an approximation. According to the paragraphs above, cosmic history and its implied notion of time only exist because of the classical nature of the \( m^a \). Rather than attempting the impossible task of being logically and linguistically precise, I will make the common assumption that “any sensible physicist who has followed my discussion understands exactly what I mean by these imprecise phrases”.

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classical solution for the $m^a$, one often wants to be able to make a Born Oppenheimer approximation in which the $m^a$ are slow variables or collective coordinates. Note that the classical nature of the $m^a$ is crucial, while the Born-Oppenheimer approximation is not. Without classical variables we would have no notion of time evolution. The Born-Oppenheimer approximation allows us to discuss the evolution of the classical variables in terms of an effective action in which other degrees of freedom are ignored. This is particularly useful below the Planck energy, since we have no idea how to describe the full set of degrees of freedom of the theory in the Planck regime, but are comfortable with a quantum field theory description below that. Nonetheless, we will argue below that the classical variables might still provide a useful notion of time evolution during the Planck era, as long as the variable which we identify as the spatial volume of classical geometry below the Planck scale, is large. It is important for any such pre-Planckian endeavor that M-theory gives an unambiguous meaning to (at least highly supersymmetric) moduli spaces even in regimes not describable by low energy Einstein equations. In a regime of super-Planckian energy and large volume, one would have to know something about the dynamics and the state of all the degrees of freedom in the system to understand how they effect the evolution of the classical variables.

As suggested in the last paragraph, both classicality and slow evolution can be understood in M-theory if we identify the $m^a$ as moduli, though with a slightly different definition of that word than the usual “parameters describing continuous families of supersymmetric vacua with $d \geq 4$ asymptotically flat dimensions”. In a theory of quantum gravity, SUSY can only be defined nonperturbatively if we insist on studying states with certain a priori boundary conditions. The SUSY charges, just like the Hamiltonian, are defined as generators of certain asymptotic symmetries of the whole class of metrics satisfying the boundary conditions. However, if we restrict attention to the classical SUGRA equations, then we can define what we mean by solutions which preserve a certain amount of supersymmetry. Since the Hamiltonian appears in the SUSY algebra, they will all be static solutions. To find them, we simply require that certain SUSY variations of all the fields vanish at the solution. Typically, we find a moduli space of continuously connected solutions preserving a particular SUSY subalgebra. The parameters $m^a$ are coordinates on this space. In particular, for 11D SUGRA, each solution will be a static, compact ten geometry, and the volume of the compact space, $V$ will be one of the moduli.

Now consider classical motions in which the $m^a$ become functions of time. The effective action derived by plugging such time dependent moduli into the SUGRA action has the form

$$S = G_{ab}(m)\dot{m}^a\dot{m}^b e^{-1}(t)$$

(6.10)
where $e$ is an einbein which imposes time reparametrization invariance. $G$ is a hyperbolic metric with signature $(1, n)$. In fact, it is easy to see that the only modulus with negative kinetic energy is the volume $V$. This is because our choice of parametrization of the spacetime metric has implicitly chosen a family of spacelike hypersurfaces in these spacetimes (those of constant $t$). The constraint equation coming from varying $e$ can be written

$$\left(\frac{\dot{V}}{V}\right)^2 = G_{ab} \dot{\hat{m}}^a \dot{\hat{m}}^b \quad (6.11)$$

and is just the Friedman equation for a Robertson Walker cosmology. The hatted quantities stand for the moduli space of SUSY solutions with volume 1.

It is easy to prove (and well known to those who have studied cosmologies with minimally coupled massless scalar fields) that the field equations of this system give, for the Volume variable, exactly the equations of a Robertson-Walker universe with equation of state $p = \rho$. The “energy density” $\rho$ then scales like $1/V^2$. The $\hat{m}$ variables satisfy the equations of geodesic motion in the metric $\hat{G}$, under the influence of cosmological friction. This is equivalent to free geodesic motion in the reparametrized time $s$ defined by $ds/dt = V^{-\frac{1}{2}}$. The volume is always monotonically decreasing or increasing in these solutions. The derivation of these facts is an enjoyable exercise in classical mechanics which I urge the students to do.

Finally I want to note that under the transformation $V \rightarrow cV$, the action scales as $S \rightarrow cS$. Thus, Planck’s constant $h$ can be absorbed in $V$, and the system is classical at large $V$. I want to emphasize that the actual spacetime geometries described by these evolution equations can be quite complex. That is, the $m^a$ might parametrize a set of Calabi-Yau manifolds. However the simple properties of the evolution on this moduli space described above are unaffected by the complexity of the underlying manifolds.

The point of all of these classical SUGRA manipulations is that, given enough SUSY, there are nonrenormalization theorems which protect this structure in regimes where the classical SUGRA approximation is invalid. For example, if there are 16 or more SUSYs preserved, then one can prove that there is no renormalization of the terms with $\leq 2$ derivatives in the effective Lagrangian for the moduli, to all orders in the expansion around classical SUGRA. Furthermore, these are the cases where SUGRA is dual to Type II (32 SUSYs) or Heterotic (16 SUSYs) string theories, compactified on tori. The weak coupling string expansions are in some sense expanding around the opposite limit from the SUGRA expansion (extremely small volumes, in $L_P$ units, of compact submanifolds rather than extremely large ones). To all orders in the weak coupling string expansions one can establish that the moduli space exists (i.e. that no potential term is generated in the effective Lagrangian for the moduli) and that its
topological and metrical structure is the same as that given by 11D SUGRA\textsuperscript{25}.

There is thus ample evidence that there is some exact sense in which the configuration space of M-theory contains regions which map precisely on to the classical moduli spaces of SUGRA solutions preserving at least 16 SUSYs. For 8 SUSYs, the situation is a bit more complicated. The well understood regions of moduli space here correspond to 11D SUGRA (or Type II strings) compactified on a manifold which is the product of a Calabi-Yau 3-fold and a torus, or heterotic strings compactified on K3 manifolds times a torus. Calabi-Yau 3-folds come in different topological classes, but there is a conjecture that all of these regions are on one continuously connected moduli space once quantum mechanics is taken into account. This statement depends on the fact that the strong nonrenormalization theorems described above are not valid. The metric on moduli space is modified by higher order corrections. However, one can still prove a nonrenormalization theorem for the potential on moduli space (namely that it is identically zero) so that the moduli space still exists as an exact concept.

This is all that is needed to establish that the moduli are good candidates to be the semiclassical, Born-Oppenheimer variables that are necessary for the derivation of a cosmology from a generally covariant quantum system. Indeed, the absence of a potential on moduli space means that the classical moduli can execute arbitrarily slow motions with arbitrarily low energy. Thus, in regimes where the classical motion has energy density small compared to the fundamental scale of M-theory, they are good Born-Oppenheimer variables. Furthermore, the $V$ rescaling symmetry of the action shows that whenever $V$ is large the moduli will behave classically. Indeed this will even be true in regions where the Born-Oppenheimer approximation breaks down, that is regions where the energy density is of the order of or larger than the Planck scale, but the volume is large. In such a regime, a description of the evolution in terms of classical moduli coupled to a stochastic bath of high energy degrees of freedom might be appropriate. The mystery will be to understand the equation of state of the stochastic bath.

The necessity of coupling the moduli to another, stochastic set of degrees of freedom appears also very late in the history of the universe. The modular energy density scales to zero much faster than either matter or radiation. Thus if there is any mechanism which generates matter or radiation, they will quickly dominate the energy density of the universe. In [25] it was shown that when the moduli can be treated as the homogeneous modes of quantum fields, there is an efficient mechanism for converting

\textsuperscript{25}If one is willing to decompactify three of the toroidal directions and view the remaining moduli as zero modes of fields in $3 + 1$ dimensional Minkowski space, then one can prove these statements from SUSY without recourse to any expansion. It is likely that these proofs can be adapted to the completely compactified situation, but this has not yet been done.
modular energy into radiation. Thus, at late times, one must study the motion of the moduli coupled to a stochastic bath of radiation and/or matter.

To summarize, the existence of a set of approximately classical, low energy collective coordinates which take values in a space of hyperbolic signature $(1, n)$ seems to be a very robust property of M-theory. These would seem to be just what we need for a derivation of cosmology from the theory. From the point of view of someone who is deeply attached to “the real world”, the only problem with this analysis is that the universes it describes become highly supersymmetric in the large volume limit. We will defer the discussion of moduli in the context of broken SUSY to section 7.

6.2 Extreme moduli

In this subsection I will present some results about the beginning and end of cosmic evolution in the highly supersymmetric situations I have just described. One motivation for this is to provide a controlled model for more realistic cosmologies. Another is to try to address the question with which we began these lectures, of why the universe as described by M-theory does not end up in a highly supersymmetric state. Finally, we will discover some very interesting results about duality and singularities which are closely related to the hyperbolic structure of moduli space and the question of the arrow of time.

We will discuss only the case of maximally SUSY moduli spaces, which are obtained by compactifying M-theory on a ten torus. The parameters are a flat metric on the torus, and the expectation value of the three form potential, $A_{\mu\nu\lambda}$ on three cycles of the torus. Most of these are compact angle variables. Among the metric variables, only the radii $R_i$ of a rectilinear torus are noncompact, while the three form expectation values are all angle variables because of the Dirac-Nepomechie-Teitelboim quantization condition [30] (their conjugate momenta are quantized). Thus, intuitively, we can restrict our discussion of the possible extreme regions of moduli space to the radii of a rectilinear torus. This argument can be made mathematically precise using the description of the moduli space as a homogeneous space. We will call the restricted rectilinear moduli space, the Kasner moduli space.

The metrics which describe motion on the Kasner moduli space have the form

$$ds^2 = -dt^2 + R_i^2(t)(dx^i)^2$$

(6.12)

where the $x^i$ have period $2\pi$. Inserting this ansatz into the action, we find that the solution of the equations of 11D SUGRA for individual radii are

$$R_i(t) = L_p(t/t_0)^{p_i}$$

(6.13)
where
\[ \sum p_i^2 = \sum p_i = 1 \]  
(6.14)

Note that the equation (6.14) implies that at least one of the \( p_i \) is negative. We have restricted attention to the case where the volume expands as time goes to infinity. We will see below that, although the equations are time reversal invariant, all of these solutions visit two very different regions of moduli space at the two endpoints of their evolution. One of the regions has a simple semiclassical description, while the other does not. This introduces a natural arrow of time into the system – the future is identified as the regime where the semiclassical approximation becomes better and better.

It is well known that all of these solutions are singular at both infinite and zero time. Some of the radii shrink to zero at both ends of the evolution. Note that if we add a matter or radiation energy density to the system then it dominates the system in the infinite volume limit and changes the solutions for the geometry there. However, near the singularity at vanishing volume both matter and radiation become negligible (despite the fact that their densities are becoming infinite) and the solutions retain their Kasner form.

All of this is true in 11D SUGRA. In M-theory we know that many regions of moduli space which are apparently singular in 11D SUGRA can be reinterpreted as living in large spaces described by weakly coupled Type II string theory or a dual version of 11D SUGRA. The vacuum Einstein equations are of course invariant under these U-duality transformations. So one is lead to believe that many apparent singularities of the Kasner universes are perfectly innocuous.

Note however that phenomenological matter and radiation densities which one might add to the equations are not invariant under duality. The energy density truly becomes singular as the volume goes to zero. How then are we to understand the meaning of the duality symmetry? The resolution is as follows. We know that when radii go to zero, the effective field theory description of the universe in 11D SUGRA becomes singular due to the appearance of new low frequency states. We also know that the singularity in the energy densities of matter and radiation implies that scattering cross sections are becoming large. Thus, it seems inevitable that phase space considerations will favor the rapid annihilation of the existing energy densities into the new light degrees of freedom. This would be enhanced for Kaluza-Klein like modes, whose individual energies are becoming large near the singularity.

Thus, near a singularity with a dual interpretation, the contents of the universe will be rapidly converted into new light modes, which have a completely different view of what the geometry of space is. The most effective description of the new situation is in terms of the transformed moduli and the new light degrees of freedom. The latter can
be described in terms of fields in the reinterpreted geometry. We want to emphasize strongly the fact that the moduli do not change in this transformation, but are merely reinterpreted. This squares with our notion that they are exact concepts in M-theory. By contrast, the fields whose zero modes they appear to be in a particular semiclassical regime, do not always make sense. The momentum modes of one interpretation are brane winding modes in another and there is no approximate way in which we can consider both sets of local fields at the same time. Fortunately, there is also no regime in which both kinds of modes are at low energy simultaneously, so in every regime where the time dependence is slow enough to make a low energy approximation, we can use local field theory.

This mechanism for resolving cosmological singularities leads naturally to the question of precisely which noncompact regions of moduli space can be mapped into what we will call the safe domain in which the theory can be interpreted as either 11D SUGRA or Type II string theory with radii large in the appropriate units.

### 6.3 The moduli space of M-Theory on rectangular tori

In this section, we will study the structure of the moduli space of M-theory compactified on various tori $T^k$ with $k \leq 10$. We are especially interested in noncompact regions of this space which might represent either singularities or large universes. As explained above, the three-form potential $A_{MNP}$ will be set to zero and the circumferences of the cycles of the torus will be expressed as the exponentials

$$\frac{R_i}{L_p} = s^{p_i}, \quad i = 1, 2, \ldots, k. \quad (6.15)$$

The remaining coordinates $x^0$ (time) and $x^{k+1} \ldots x^{10}$ are considered to be infinite and we never dualize them. It is important to distinguish the variable $s$ here from the time in the Kasner solution. Here we are just parametrizing possible asymptotic domains in the moduli space, whereas the Kasner solution is to be used as a metric valid for all values of the parameter $t$. We will see that it interpolates between two very different asymptotic domains.

The radii are encoded in the logarithms $p_i$. We will study limits of the moduli space in various directions which correspond to keeping $p_i$ fixed and sending $s \to \infty$ (the change to $s \to 0$ is equivalent to $p_i \to -p_i$ so we do not need to study it separately). In terms of this parametrization of the extreme regions of moduli space, we can see that a Kasner solution with parameters $p_i$ will visit the regime of moduli space characterized by $p_i$ as $t \to \infty$ and the regime $-p_i$ as $t \to 0$.  

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6.4 The 2/5 transformation

M-theory has dualities which allow us to identify the vacua with different $p_i$’s. A subgroup of this duality group is the $S_k$ which permutes the $p_i$’s. Without loss of generality, we can assume that $p_1 \leq p_2 \leq \ldots \leq p_{10}$. We will assume this in most of the text. The full group that leaves invariant rectilinear tori with vanishing three form is the Weyl group of the noncompact $E_k$ group of SUGRA. We will denote it by $G_k$. We will give an elementary derivation of the properties of this group for the convenience of the reader. $G_k$ is generated by the permutations of the cycles on the torus, and one other transformation which acts as follows:

$$(p_1, p_2, \ldots, p_k) \mapsto (p_1 - \frac{2s}{3}, p_2 - \frac{2s}{3}, p_3 - \frac{2s}{3}, p_4 + \frac{s}{3}, \ldots, p_k + \frac{s}{3}).$$

(6.16)

where $s = (p_1 + p_2 + p_3)$. Before explaining why this transformation is a symmetry of M-theory, let us point out several of its properties (6.16).

- The total sum $S = \sum_{i=1}^k p_i$ changes to $S \mapsto S + (k - 9)s/3$. So for $k < 9$, the sum increases if $s < 0$, for $k = 9$ the total sum is an invariant and for $k > 9$ the sum decreases for $s < 0$.

- If we consider all $p_i$’s to be integers which are equal modulo 3, this property will hold also after the 2/5 transformation. The reason is that, due to the assumptions, $s$ is a multiple of three and the coefficients $-2/3$ and $+1/3$ differ by an integer.

- As a result, from any initial integer $p_i$’s we get $p_i$’s which are multiples of 1/3 which means that all the matrix elements of matrices in the 2/5 transformation are integer multiples of 1/3.

- The order of $p_1, p_2, p_3$ is not changed (the difference $p_1 - p_2$ remains constant, for instance). Similarly, the order of $p_4, p_5, \ldots, p_k$ is unchanged. However the ordering between $p_{1..3}$ and $p_{4..k}$ can change in general. By convention, we will follow each 2/5 transformation by a permutation which places the $p_i$’s in ascending order.

- The bilinear quantity $I = (9 - k) \sum (p_i^2) + (\sum p_i)^2 = (10 - k) \sum (p_i^2) + 2 \sum_{i<j} p_i p_j$ is left invariant by $G_k$.

The fact that 2/5 transformation is a symmetry of M-theory can be proved as follows. Let us interpret $L_1$ as the M-theoretical circle of a type IIA string theory. Then the simplest duality which gives us a theory of the same kind (IIA) is the double T-duality. Let us perform it on the circles $L_2$ and $L_3$. The claim is that if we combine
this double T-duality with a permutation of \( L_2 \) and \( L_3 \) and interpret the new \( L_1 \) as the M-theoretical circle again, we get precisely (6.16).

Another illuminating way to view the transformation 2/5 transformation is to compactify M-theory on a three torus. The original M2-brane and the M5-brane wrapped on the three torus are both BPS membranes in eight dimensions. The tension of the original M2-brane is of order \( L^{-3}_p \), while that of the membrane which comes from the wrapped M5 is \( VL_p^{-6} \) where \( V \) is the volume of the three torus. When the three torus is large and the 11D SUGRA approximation is valid, the wrapped M5-brane is much heavier than the M2-brane, while in the small volume limit, the opposite is true. We have seen previously that in limits where classical geometrical descriptions are breaking down, one can find a new classical description by following those BPS states which become lightest in the limit. This suggests that we try to define \( \bar{l}_p^{-3} = VL_p^{-6} \) and \( \bar{V} = L_p^{-3} \bar{p} \) and try to imagine a duality transformation in M-theory which takes a compactification on a small three torus to a compactification on a large one, with corresponding redefinition of the Planck scale. Aharony [38] has given arguments that such a duality transformation exists, and it can be demonstrated rigorously in Matrix Theory. In the limit in which one of the cycles of the \( T^3 \) is small, so that a type II string description becomes appropriate, it is just the double T-duality of the previous paragraph. The fact that this transformation plus permutations generates \( G_k \) was proven by the authors of [39] for \( k \leq 9 \). I leave it to the reader to verify that the effect of this transformation on the variables \( p_i \) is precisely that described above.

In the following subsection we will use this group of duality transformations to prove that extreme regions of the moduli space fall into a number of distinct categories. One is such that some kind of semiclassical description of the physics is valid, and breaks up into regions that are described by 11D SUGRA or weakly coupled Type IIA or IIB string theory. The other is completely mysterious and has no known semiclassical description. Each Kasner solution visits both of these regions at the extreme ends of its trajectory. It is thus reasonable to identify the past with the unknown region and the future with the semiclassical regime.

The derivations below are based primarily on elementary algebra and the definition of the duality transformations given above. However, many cosmologists may want to skip the technical details.

### 6.5 The boundaries of moduli space

There are three types of boundaries of the toroidal moduli space which are amenable to detailed analysis. The first is the limit in which eleven-dimensional supergravity becomes valid. We will denote this limit as 11D. The other two limits are weakly coupled type IIA and type IIB theories in 10 dimensions. We will call the domain
of asymptotic moduli space which can be mapped into one of these limits, the safe domain.

- For the limit 11D, all the radii must be greater than $L_p$. Note that for $t \to \infty$ it means that all the radii are much greater than $L_p$. In terms of the $p_i$’s, this is the inequality $p_i > 0$.

- For type IIA, the dimensionless coupling constant $g_{s}^{IIA}$ must be smaller than 1 (much smaller for $t \to \infty$) and all the remaining radii must be greater than $L_s$ (much greater for $t \to \infty$).

- For type IIB, the dimensionless coupling constant $g_{s}^{IIB}$ must be smaller than 1 (much smaller for $t \to \infty$) and all the remaining radii must be greater than $L_s$ (much greater for $t \to \infty$), including the extra radius whose momentum arises as the number of wrapped M2-branes on the small $T^2$ in the dual 11D SUGRA picture.

If we assume the canonical ordering of the radii, i.e. $p_1 \leq p_2 \leq p_3 \leq \ldots \leq p_k$, we can simplify these requirements as follows:

- 11D: $0 < p_1$
- IIA: $p_1 < 0 < p_1 + 2p_2$
- IIB: $p_1 + 2p_2 < 0 < p_1 + 2p_3$

To derive this, we have used the familiar relations:

$$\frac{L_1}{L_p} = (g_{s}^{IIA})^{2/3} = \left(\frac{L_p}{L_s}\right)^2 = \left(\frac{L_1}{L_s}\right)^{2/3} \quad (6.17)$$

for the 11D/IIA duality ($L_1$ is the M-theoretical circle) and similar relations for the 11D/IIB case ($L_1 < L_2$ are the parameters of the $T^2$ and $L_{IIB}$ is the circumference of the extra circle):

$$\frac{L_1}{L_2} = g_{s}^{IIB}, \quad 1 = \frac{L_1L_s^2}{L_3^2} = \frac{g_{s}^{IIB}L_2L_s^2}{L_3^2} = \frac{L_{IIB}L_1L_2}{L_3^2}, \quad (6.18)$$

$$\frac{1}{g_{s}^{IIB}} \left(\frac{L_p}{L_s}\right)^4 = \frac{L_1L_2}{L_p^2} = \frac{L_p}{L_{IIB}} = \left(\frac{L_s}{L_{IIB}}\right)^{4/3} \quad (6.19)$$

Note that the regions defined by the inequalities above cannot overlap, since the regions are defined by $M, M^c \cap A, A^c \cap B$ where $A^c$ means the complement of a set.
Furthermore, assuming \( p_i < p_{i+1} \) it is easy to show that \( p_1 + 2p_3 < 0 \) implies \( p_1 + 2p_2 < 0 \) and \( p_1 + 2p_2 < 0 \) implies \( 3p_1 < 0 \) or \( p_1 < 0 \).

This means that (neglecting the boundaries where the inequalities are saturated) the region outside \( 11D \cup \Pi A \cup \Pi B \) is defined simply by \( p_1 + 2p_3 < 0 \). The latter characterization of the safe domain of moduli space will simplify our discussion considerably.

The invariance of the bilinear form defined above gives an important constraint on the action of \( G_k \) on the moduli space. For \( k = 10 \) it is easy to see that, considering the \( p_i \) to be the coordinates of a ten vector, it defines a Lorentzian metric on this ten dimensional space. Thus the group \( G_{10} \) is a discrete subgroup of \( O(1,9) \). The direction in this space corresponding to the sum of the \( p_i \) is timelike, while the hyperplane on which this sum vanishes is spacelike. We can obtain the group \( G_9 \) from the group \( G_{10} \) by taking \( p_{10} \) to infinity and considering only transformations which leave it invariant. Obviously then, \( G_9 \) is a discrete subgroup of the transverse Galilean group of the infinite momentum frame. For \( k \leq 8 \) on the other hand, the bilinear form is positive definite and \( G_k \) is contained in \( O(k) \). Since the latter group is compact, and there is a basis in which the \( G_k \) matrices are all integers divided by 3, we conclude that in these cases \( G_k \) is a finite group. In a moment we will show that \( G_9 \) and \( a \text{ fortiori} \ G_{10} \) are infinite. Finally we note that the \( 2/5 \) transformation is a spatial reflection in \( O(1,9) \). Indeed it squares to 1 so its determinant is \( \pm 1 \). On the other hand, if we take all but three coordinates very large, then the \( 2/5 \) transformation of those coordinates is very close to the spatial reflection through the plane \( p_1 + p_2 + p_3 = 0 \), so it is a reflection of a single spatial coordinate.

\[
\begin{array}{c}
\text{IIA strings} \\
\text{M-theory in 11 dim.} \\
\text{IIB strings} \\
\end{array}
\]

\[ p_1 \]

\[ p_1 > p_2 \]

\[ \text{permuted region} \]

\[ p_1 \]

**Figure 3:** The structure of the moduli space for \( T^2 \).
We now prove that \( G_9 \) is infinite. Start with the first vector of \( p_i \)'s given below and iterate (6.16) on the three smallest radii (a strategy which we will use all the time) and sort \( p_i \)'s after each step, so that their index reflects their order on the real line. We get

\[
(-1, -1, -1, -1, -1, -1, -1, -1, -1) \\
(-2, -2, -2, -2, -2, -2, +1, +1, +1) \\
(-4, -4, -4, -1, -1, -1, +2, +2, +2) \\
(-5, -5, -5, -2, -2, -2, +4, +4, +4)
\] (6.20)

so the entries grow (linearly) to infinity.

### 6.6 Covering the moduli space

We will show that there is a useful strategy which can be used to transform any point \( \{p_i\} \) into the safe domain in the case of \( T^k, k < 9 \). The strategy is to perform iteratively 2/5 transformations on the three smallest radii.

Assuming that \( \{p_i\} \) is outside the safe domain, i.e. \( p_1 + 2p_3 < 0 \) (\( p_i \)'s are sorted so that \( p_i \leq p_{i+1} \)), it is easy to see that \( p_1 + p_2 + p_3 < 0 \) (because \( p_2 \leq p_3 \)). As we said below the equation (6.16), the 2/5 transformation on \( p_1, p_2, p_3 \) always increases the total sum \( \sum p_i \) for \( p_1 + p_2 + p_3 < 0 \). But this sum cannot increase indefinitely because the group \( G_k \) is finite for \( k < 9 \). Therefore the iteration process must terminate at some point. The only way this can happen is that the assumption \( p_1 + 2p_3 < 0 \) no longer holds, which means that we are in the safe domain. This completes the proof for \( k < 9 \).

For \( k = 9 \) the proof is more difficult. The group \( G_9 \) is infinite and furthermore, the sum of all \( p_i \)'s does not change. In fact the conservation of \( \sum p_i \) is the reason that only points with \( \sum p_i > 0 \) can be dualized to the safe domain. The reason is that if \( p_1 + 2p_3 \geq 0 \), also \( 3p_1 + 6p_3 \geq 0 \) and consequently

\[
p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9 \geq p_1 + p_1 + p_1 + p_3 + p_3 + p_3 + p_3 + p_3 + p_3 \geq 0. \quad (6.21)
\]

This inequality is saturated only if all \( p_i \)'s are equal to each other. If their sum vanishes, each \( p_i \) must then vanish. But we cannot obtain a zero vector from a nonzero vector by 2/5 transformations because they are nonsingular. If the sum \( \sum p_i \) is negative, it is also clear that we cannot reach the safe domain.

However, if \( \sum_{i=1}^9 p_i > 0 \), then we can map the region of moduli space with \( t \to \infty \) to the safe domain. We will prove it for rational \( p_i \)'s only. This assumption compensates
for the fact that the order of $G_9$ is infinite. Assuming $p_i$’s rational is however sufficient because we will see that a finite product of 2/5 transformations brings us to the safe domain. But a composition of a finite number of 2/5 transformations is a continuous map from $\mathbb{R}^9$ to $\mathbb{R}^9$ so there must be at least a “ray” part of a neighborhood which can be also dualized to the safe domain. Because $\mathbb{Q}^9$ is dense in $\mathbb{R}^9$, our argument proves the result for general values of $p_i$.

From now on we assume that the $p_i$’s are rational numbers. Everything is scale invariant so we may multiply them by a common denominator to make integers. In fact, we choose them to be integer multiples of three since in that case we will have integer $p_i$’s even after 2/5 transformations. The numbers $p_i$ are now integers equal modulo 3 and their sum is positive. We will define a critical quantity

$$C = \sum_{1 \leq i < j} (p_i - p_j)^2. \quad (6.22)$$

This is a priori an integer greater than or equal to zero which is invariant under permutations. What happens to $C$ if we make a 2/5 transformation on the radii $p_1, p_2, p_3$? The differences $p_1 - p_2, p_1 - p_3, p_2 - p_3$ do not change and this holds for $p_4 - p_5, \ldots p_8 - p_9$, too. The only contributions to (6.22) which are changed are from $3 \cdot 6 = 18$ “mixed” terms like $(p_1 - p_4)^2$. Using (6.16),

$$(p_1 - p_4) \mapsto (p_1 - \frac{2s}{3}) - (p_4 + \frac{s}{3}) = (p_1 - p_4) - s \quad (6.23)$$

so its square

$$(p_1 - p_4)^2 \mapsto [(p_1 - p_4) - s]^2 = (p_1 - p_4)^2 - 2s(p_1 - p_4) + s^2 \quad (6.24)$$

changes by $-2s(p_1 - p_4) + s^2$. Summing over all 18 terms we get $(s = p_1 + p_2 + p_3)$

$$\Delta C = -2s[6(p_1 + p_2 + p_3) - 3(p_4 + \ldots + p_9)] + 18s^2 = 6s^2 + 6\left(\sum_{i=1}^{9} p_i - s\right) = 6s\sum_{i=1}^{9} p_i. \quad (6.25)$$

But this quantity is strictly negative because $\sum p_i$ is positive and $s < 0$ (we define the safe domain with boundaries, $p_1 + 2p_3 \geq 0$).

This means that $C$ defined in (6.22) decreases after each 2/5 transformation on the three smallest radii. Since it is a non-negative integer, it cannot decrease indefinitely. Thus the assumption $p_1 + 2p_3 < 0$ becomes invalid after a finite number of steps and we reach the safe domain.

Now let us turn to the fully compactified case. As we pointed out, the bilinear form $I \equiv 2 \sum_{i<j} p_ip_j$ defines a Lorentzian signature metric on the vector space whose

50
components are the $p_i$. The 2/5 transformation is a spatial reflection and therefore the group $G_{10}$ consists of orthochronous Lorentz transformations. Now consider a vector in the safe domain. We can write it as

$$(-2, -2 + a_1, 1 + a_2, \ldots, 1 + a_9)S, \quad S \in \mathbb{R}^+$$

(6.26)

where the $a_i$ are positive. It is easy to see that $I$ is positive on this configuration. This means that only the inside of the light cone can be mapped into the safe domain. Furthermore, since $\sum p_i$ is positive in the safe domain and the transformations are orthochronous, only the interior of the future light cone in moduli space can be mapped into the safe domain.

We would now like to show that the entire interior of the forward light cone can be so mapped. We use the same strategy of rational coordinates dense in $\mathbb{R}^{10}$. If we start outside the safe domain, the sum of the first three $p_i$ is negative. We again pursue the strategy of doing a 2/5 transformation on the first three coordinates and then reordering and iterating. For the case of $G_9$ the sum of the coordinates was an invariant, but here it decreases under the 2/5 transformation of the three smallest coordinates, if their sum is negative. But $\sum p_i$ is (starting from rational values and rescaling to get integers congruent modulo three as before) a positive integer and must remain so after $G_{10}$ operations. Thus, after a finite number of iterations, the assumption that the sum of the three smallest coordinates is negative must fail, and we are in the safe domain. In fact, we generically enter the safe domain before this point. The complement of the safe domain always has negative sum of the first three coordinates, but there are elements in the safe domain where this sum is negative.

It is quite remarkable that the bilinear form $I$ is proportional to the Wheeler-De Witt Hamiltonian for the Kasner solutions:

$$\frac{I}{t^2} = \left( \sum_i \frac{dL_i}{dt} \frac{1}{L_i} \right)^2 - \sum_i \left( \frac{dL_i}{dt} \frac{1}{L_i} \right)^2 = \frac{2}{t^2} \sum_{i<j} p_i p_j.$$ 

(6.27)

The solutions themselves thus lie precisely on the future light cone in moduli space. Each solution has two asymptotic regions ($t \to 0, \infty$ in (6.12)), one of which is in the past light cone and the other in the future light cone of moduli space. The structure of the modular group thus suggests a natural arrow of time for cosmological evolution. The future may be defined as the direction in which the solution approaches the safe domain of moduli space. All of the Kasner solutions then, have a true singularity in their past, which cannot be removed by duality transformations.

Actually, since the Kasner solutions are on the light cone, which is the boundary of the safe domain, we must add a small homogeneous energy density to the system
in order to make this statement correct. The condition that we can map into the safe domain is then the statement that this additional energy density is positive. Note that in the safe domain, and if the equation of state of this matter satisfies (but does not saturate) the holographic bound of \cite{36}, this energy density dominates the late time evolution of the universe, while near the singularity, it becomes negligible compared to the Kasner degrees of freedom. The assumption of a homogeneous negative energy density is manifestly incompatible with Einstein’s equations in a compact flat universe so we see that the spacelike domain of moduli space corresponds to a physical situation which cannot occur in the safe domain.

The backward lightcone of the asymptotic moduli space is, as we have said, visited by all of the classical solutions of the theory.

To summarize: the U-duality group \( G_{10} \) divides the asymptotic domains of moduli space into three regions, corresponding to the spacelike and future and past timelike regimes of a Lorentzian manifold. Only the future lightcone can be understood in terms of weakly coupled SUGRA or string theory. The group theory provides an exact M-theoretic meaning for the Wheeler-De Witt Hamiltonian for moduli. Classical solutions of the low energy effective equations of motion with positive energy density for matter distributions lie in the timelike region of moduli space and interpolate between the past and future light cones. We find it remarkable that the purely group theoretical considerations of this section seem to capture so much of the physics of toroidal cosmologies.

6.7 Moduli spaces with less SUSY

We would like to generalize the above considerations to situations which preserve less SUSY. This enterprise immediately raises some questions, the first of which is what we mean by SUSY. Cosmologies with compact spatial sections have no global symmetries in the standard sense since there is no asymptotic region in which one can define the generators. We will define a cosmology with a certain amount of SUSY by first looking for Euclidean ten manifolds and three form field configurations which are solutions of the equations of 11D SUGRA and have a certain number of Killing spinors. The first approximation to cosmology will be to study motion on a moduli space of such solutions. The motivation for this is that at least in the semiclassical approximation we are guaranteed to find arbitrarily slow motions of the moduli. In fact, in many cases, SUSY nonrenormalization theorems guarantee that the semiclassical approximation becomes valid for slow motions because the low energy effective Lagrangian of the moduli is to a large extent determined by SUSY. There are however a number of pitfalls inherent in our approach. We know that for some SUSY algebras, the moduli space of compactifications to four or six dimensions is not a manifold. New moduli can appear
at singular points in moduli space and a new branch of the space, attached to the old one at the singular point, must be added. There may be cosmologies which traverse from one branch to the other in the course of their evolution. If that occurs, there will be a point at which the moduli space approximation breaks down. Furthermore, there are many examples of SUSY vacua of M-theory which have not yet been continuously connected on to the 11D limit, even through a series of “conifold” transitions such as those described above [41]. In particular, it has been suggested that there might be a completely isolated vacuum state of M-theory [42]. Thus it might not be possible to imagine that all cosmological solutions which preserve a given amount of SUSY are continuously connected to the 11D SUGRA regime.

Despite these potential problems, we think it is worthwhile to begin a study of compact, SUSY preserving, ten manifolds. Here we will only study examples where the three form field vanishes. The well known local condition for a Killing spinor, \( D_\mu \epsilon = 0 \), has as a condition for local integrability the vanishing curvature condition

\[
R_{\mu\nu}^{ab} \gamma_{ab} \epsilon = 0
\]  

(6.28)

Thus, locally the curvature must lie in a subalgebra of the Lie algebra of \( Spin(10) \) which annihilates a spinor. The global condition is that the holonomy around any closed path must lie in a subgroup which preserves a spinor. Since we are dealing with 11D SUGRA, we always have both the 16 and \( \bar{16} \) representations of \( Spin(10) \) so SUSYs come in pairs.

For maximal SUSY the curvature must vanish identically and the space must be a torus. The next possibility is to preserve half the spinors and this is achieved by manifolds of the form \( K3 \times T^7 \) or orbifolds of them by freely acting discrete symmetries.

We now jump to the case of 4 SUSYs. To find examples, it is convenient to consider the decompositions \( Spin(10) \supseteq Spin(k) \times Spin(10 - k) \).

The 16 is then a tensor product of two lower dimensional spinors. For \( k = 2 \), the holonomy must be contained in \( SU(4) \subseteq Spin(8) \) in order to preserve a spinor, and it then preserves two (four once the complex conjugate representation is taken into account). The corresponding manifolds are products of Calabi-Yau fourfolds with two tori, perhaps identified by the action of a freely acting discrete group. This moduli space is closely related to that of F-theory compactifications to four dimensions with minimal four dimensional SUSY. The three spatial dimensions are then compactified on a torus. For \( k = 3 \) the holonomy must be in \( G_2 \subseteq Spin(7) \). The manifolds are, up to discrete identifications, products of Joyce manifolds and three tori. For \( k = 4 \) the holonomy is in \( SU(2) \times SU(3) \). The manifolds are free orbifolds of products of Calabi-Yau threefolds and K3 manifolds. This moduli space is that of the heterotic
string compactified on a three torus and Calabi-Yau three-fold. The case $k = 5$ does not lead to any more examples with precisely 4 SUSYs.

It is possible that M-theory contains U-duality transformations which map us between these classes. For example, there are at least some examples of F-theory compactifications to four dimensional Minkowski space which are dual to heterotic compactifications on threefolds. After further compactification on three tori we expect to find a map between the $k = 2$ and $k = 4$ moduli spaces.

It is clear that the metric on the full moduli space still has Lorentzian signature in the SUGRA approximation. In some of these cases of lower SUSY, we expect the metric to be corrected in the quantum theory. However, we do not expect these corrections to alter the signature of the metric. To see this note that each of the cases we have described has a two torus factor. If we decompactify the two torus, we expect a low energy field theoretic description as three dimensional gravity coupled to scalar fields and we can perform a Weyl transformation so that the coefficient of the Einstein action is constant. The scalar fields must have positive kinetic energy and the Einstein term must have its conventional sign if the theory is to be unitary. Thus, the decompactified moduli space has a positive metric. In further compactifying on the two torus, the only new moduli are those contained in gravity, and the metric on the full moduli space has Lorentzian signature.

Note that as in the case of maximal SUSY, the region of the moduli space with large ten volume and all other moduli held fixed, is in the future light cone of any finite point in the moduli space. Thus we suspect that much of the general structure that we uncovered in the toroidal moduli space, will survive in these less supersymmetric settings.

The most serious obstacle to this generalization appears in the case of 4 (or fewer) supercharges. In that case, general arguments do not forbid the appearance of a potential in the Lagrangian for the moduli. Furthermore, at generic points in the moduli space one would expect the energy density associated with that potential to be of order the fundamental scales in the theory. In such a situation, it is difficult to justify the Born-Oppenheimer separation between moduli and high energy degrees of freedom. Typical motions of the moduli on their potential have frequencies of the same order as those of the ultraviolet degrees of freedom. In section 7 we will try to present a solution to this conundrum.

6.8 Chaotically avoiding SUSY

The considerations of this section also allow us to achieve some insight into the problem of why M-theory has not chosen to set in one of its stable highly supersymmetric vacua in the world we observe. The discussion which follows is completely rigorous on the
branches of moduli space with 16 or more SUSYs. It is probably valid for 8 SUSYs as well, for in that case the moduli space exists although its topology and metric are not determined by classical considerations. Nonetheless, all known extreme regions of the moduli space have the properties we will use below.

The key point is that our analysis of extreme regions of moduli space showed a monotonic flow from the unsafe to the safe regions. We have neglected extreme regimes corresponding to partial decompactification, and also the motion of the other moduli, and of the non modular degrees of freedom which surely dominate the energy density in regimes where the universe has expanded a lot. In fact, inclusion of these other degrees of freedom reinforces the conclusion that the universe will always end up in the safe domain.

Horne and Moore [63] have shown that motion on the full moduli space (as opposed to its Kasner subspace) is chaotic. Furthermore, the Euclidean metric on the subspace of moduli with unit spatial volume has finite volume in the metric on moduli space, which means that the extreme regions of this space (which correspond to partial decompactifications) have vanishingly small measure. The chaotic nature of the motion, as well as the fact that the moduli are, at least at late times, coupled to a stochastic radiation bath, imply that the generic cosmological solution will in fact sample regions of the moduli space in proportion to the measure defined by the kinetic energy of the moduli. In particular, partial decompactifications, which are of measure zero on the moduli space, will not be generic final states of the cosmological evolution.

We conclude that the generic cosmological solution in these supersymmetric regions of the moduli space will asymptote to a ten or eleven dimensional universe filled with radiation. All of low energy physics is weakly coupled, there are no finite energy scales apart from the Planck or string scales, and there are no apparent candidates for long lived nonrelativistic particles. It seems safe to conclude that none of these model universes could ever contain galaxies. Thus, if we are willing to entertain the very weak form of the anthropic principle which claims that galaxies are necessary for intelligent life, we can find an explanation of why we do not live in a universe with 8 or more SUSYs.

I do not claim to find this a completely satisfactory resolution of the question. On the one hand, I maintain that this sort of use of anthropic reasoning is scientifically valid. That is, we appear, in M-theory, to be faced with a model of physics which predicts the possibility of alternate universes which do not resemble what we observe. I have tried to give an honest account of what happens to a generic universe of this sort (within the class with maximal SUSY) and found that it lacks what would appear to

\[\text{26 I do not see any source for a population of large and therefore long lived black holes.}\]
be a very weak requirement for the existence of life. I did not have to speculate about unknown results in extra universal biology to come to this conclusion. On the other hand, one might wish for a sharper distinction between our own universe and these unobservable ones. Wouldn’t it be nicer if they all suffered some sort of satisfyingly final cosmic catastrophe and sank back into the ultraviolet muck of creation? Or perhaps one could, with a more comprehensive knowledge of M-theory, argue that generic cosmological solutions of the whole theory do not end up in the maximally SUSY regions.

One direction in which to search for such an argument has to do with inflation. I have purposely avoided mentioning that cosmologies which remain on the moduli spaces with 8 or more SUSYs cannot inflate. The obvious retort to such a remark is that inflation could have occurred somewhere else in configuration space, and the system could then have rolled down to the moduli space. One cannot investigate the probability of such a motion without a much more thorough understanding of M-theory than we now possess. So the galactothropic explanation of the absence of SUSY ground states is the best we can do at the moment. Perhaps it will be the best we can ever do.

6.9 Against inflation

To an audience of astroparticle physicists the suggestion that inflation might not be a necessary feature of our explanation of the universe is akin to heresy. I therefore thought it would be amusing to insert some speculations here about alternative ways to solve the cosmological conundra which led to the invention of inflation. Those of my readers who actually attended these lectures will not that I did not actually present this material. Let me assure you that it was only for lack of time, and not because I was afraid of being mauled by an angry crowd of true believers.

To begin our trek down the path of heterodoxy let me attack the common wisdom about the horizon problem. This is the observation that in conventional Big Bang cosmology, the horizon at early times is much smaller than the backward extrapolation of our current horizon. Thus, regions of the universe that we can observe today were out of causal contact. How one then asks can their contents be in thermal equilibrium at a uniform temperature? I would like to contend that the M theorist’s answer might be “very easily”. Local field theory is only an approximation to M-theory. At sufficiently high energies it is clear that locality breaks down in some way. The typical high energy state in perturbative string theory is an extremely long single string. Beyond the perturbative approximation, large branes of other dimensions may be relevant.

\footnote{We will see something of the sort happening to another class of undesirable universes in the next section.}
Although brane interactions are local on the brane (e.g. strings split and join at points in spacetime) this does not seem to be an argument which forces one to conclude that the correct state of the string is unlikely to be a typical member of the ensemble of strings with given energy (as one argues for a quantum field theory in a Big Bang cosmology when one says that the fields in causally disconnected regions have not had a chance to thermalize). If the system is in thermal equilibrium at very high energy, and if the expansion is slow enough, then it will remain at equilibrium at lower energies.

Another argument against naive locality at the fundamental scale (which might be much lower than $10^{19}$ GeV) has to do with black holes. Once the typical energy and impact parameter in particle collisions are such that black hole formation is common, the spacetime geometry is distorted in a way which modifies the naive causality arguments. If we believe that black hole evaporation is a unitary process, then standard causality arguments are only valid outside black hole horizons (I am assuming that if the universe is closed, then its radius is much larger than the relevant black hole horizons). All states associated with a given black hole are in thermal equilibrium with each other, and black holes will tend to coalesce, bringing more and more of the system into equilibrium.

The claim then is that the horizon problem is not a problem (I am being deliberately provocative here – I don’t know whether I believe these arguments). Rather, the principle of thermodynamic equilibrium i.e. that systems tend to be in typical states consistent with their energy content is more fundamental than the causality principle applied to a simple averaged classical geometry and a model of its matter content as localized particles interacting via local field theory.

Similar remarks apply to the monopole problem, at least in those regions of moduli space where there is no grand unified group below the fundamental scale. Monopoles then belong to the high energy theory, and the conventional field theoretic estimates (again based on causality) of their abundance are incorrect.

One can make an even more convincing attack on the arguments for the flatness problem. This puzzle is based on the model of a homogeneous isotropic universe. This should properly be regarded as a phenomenological model rather than a fundamental starting point for cosmology. Indeed although descriptions of inflationary cosmology usually start from standard Robertson-Walker ideology, they in fact reject that ideology. Homogeneity and isotropy arise as late time fixed point behavior. However, if one is going to start from more general initial conditions, one can get rid of the flatness problem in a simpler way. Indeed, I have argued above that a more fundamentally motivated approach to cosmology might start from geometries (and configurations of other fields) on a moduli space of static classical solutions of the SUGRA equations. It is a generic feature of such models, that unless the energy density is allowed to
be negative, the universe evolves monotonically toward large volume. Thus spatial curvatures (and many of the Calabi-Yau manifolds on these moduli spaces are curved) are generally evolving towards zero without any fine tuning. The general cosmological solution for motion on a moduli space of geometries, coupled to positive energy density matter evolves toward zero spatial curvature if we are only willing to wait long enough. The only real issue left among the conventional cosmological puzzles is the Entropy Problem.

To explain this in more detail let us consider a simple example of the kind of model we are discussing. Consider the moduli space of solutions of weakly coupled heterotic string theory compactified on a three torus large compared to the string scale, times a Calabi-Yau threefold. Let us agree to ignore the phenomenological problems with the dilaton which make this regime problematic as a model of the real world. The Friedmann equation for this model has the form

\[ m_p^2 \left( \frac{\dot{a}}{a} \right)^2 = m_p^4 \left[ b/a^6 + d/a^4 + e/m_P a^3 + \Lambda \right]. \tag{6.29} \]

\( a \) is the scale factor of the three torus, and \( b, d, e \) and \( \Lambda \) represent the contributions to the energy density of the moduli, radiation, nonrelativistic matter, and a cosmological constant, all measured in Planck units. We choose conventions such that \( a = 1 \) is the present scale factor. The volume of the torus is \( a^3 V_0 \), where \( V_0 \) is the volume today. Observation tells us that the periods of the three torus are of the same order as, or larger than our horizon volume, whose size is \( 10^{60} \) Planck units. We neglect processes which convert one form of energy density into another and do not attempt to explain why all of the constants \( d, e \) and \( \Lambda \) are within an order of magnitude or so of each other.

The moduli of the torus are the ratios of its periods, the angles between the different toroidal directions, Wilson lines for the heterotic gauge fields and “Wilson two surfaces” for the antisymmetric tensor potential of heterotic string theory. These evolve as a nonlinear sigma model of Goldstone type. The analysis of \([25]\) implies that motion on this space stops early in the history of the universe, its kinetic energy being converted into a gas of momentum modes of the corresponding fields, which contributes to the constant \( d \). The torus then expands indefinitely with fixed shape. Thus, if we wait long enough, all remnant of the finiteness of space is wiped out, without fine tuning of initial conditions. In cases where the moduli space in question is a family of curved Calabi-Yau spaces, the same analysis applies and the spatial curvature is erased without any fine tuning.

The real difficulty for this solution of the flatness problem is simply that if we wait long enough for spatial finiteness and curvature to be stretched away, there may not be enough matter and radiation in our model to account for the universe we observe. This is what is commonly referred to as the Entropy problem in the literature of
inflationary cosmology. The models we are discussing show that it is logically separate from the flatness problem, which is rather specific to homogeneous isotropic models, where the spatial geometry at each instant is not a static solution of the Einstein equations. In these models, generic initial values for curvature would have long ago led to a curvature dominated regime of expansion and substantially modified much of cosmic history. In models based on moduli, generic initial conditions would not have changed the expansion rate very much at late times and would probably not show up in local physics. Their discrepancy with observation would simply come from the absence of evidence for global structure or anisotropy in the background geometry.

Another way to phrase the Entropy Problem is the discrepancy between the universe’s energy content and its size at the “moment of the Big Bang”. If one follows the conventional Robertson-Walker cosmology back to the Planck energy density, then the linear size of our horizon volume at that time is $10^{27}$ Planck units. The size of any closed universe would have to be larger than this. I do not have any explanation of this large pure number in the present context. In inflationary cosmology it is solved by creating the matter and radiation after a period of inflationary expansion.

At the level of the semiclassical analysis we have done, there does not seem to be any strong objection to such initial conditions. We have emphasized that the semiclassical treatment of the moduli requires only that the volume of the universe be large. At energies above the Planck scale, there will be new terms in the equations of motion of the moduli representing their interaction with the full set of high energy degrees of freedom of M-theory. But in principle one could imagine following the evolution back to a Planck size for the whole universe before the semiclassical approximation breaks down. The statement of the Entropy Problem at that time would be that the energy density was many orders of magnitude higher than the Planck scale. Is there some principle which prevents this?

It would be nice to find one, because one would like to have a clean reason for rejecting alternatives to inflation. Alternatively, it would be interesting to find an explanation of this large number, and to take the anti-inflationary cosmology more seriously. In the latter event one would be required to come up with an explanation for the fluctuations in the cosmic microwave background at least as convincing as that provided by inflationary models.

What should the serious cosmologist take away from this discussion? I hardly hope or wish to convince anyone to abandon the inflationary paradigm. However, I think it is salutary to recognize that many of the theoretical arguments which one thinks of as the basic raison d’être of inflationary cosmology, are on rather shaky ground in the light of current theory. The clear cut triumphs of inflation are reduced to two: the explanations of the entropy of the current universe and of the fluctuations in the
microwave background.

In the next section, we will abandon this heresy and pursue a more orthodox path.

6.10 Conclusions

We argued that the supersymmetric moduli of M-theory were the natural semiclassical variables which provide the clock for cosmology. Our argument was based on the naive Wheeler-Dewitt quantization of gravity but we presented some evidence that the general structures assumed in that quantization were more robust than their derivation from a low energy effective theory would have led us to believe. We showed that duality transformations resolve some but not all cosmological singularities, and provided a first draft of an argument for the absence of highly supersymmetric vacuum states of M-theory in the list of Natural Phenomena in the Real World. We also briefly explored a heterodox, noninflationary, approach to cosmology which resolves some but not all of the problems that inflation was invented to solve.

7. Moduli and Inflation

7.1 Introduction

In this lecture we will finally start to discuss more realistic sectors of M theoretic cosmology. As I have warned you several times, this area is still under development and there is no justification for trying to build detailed models which can be compared to observation. Indeed, towards the end of my presentation I will describe my own favorite scenario for cosmology in M-theory. It turns out that its viability depends heavily on numerical factors of order one which cannot be reliably calculated at present. Such factors in fundamental quantities have a tendency to get raised to high powers in a cosmological context (e.g. the widths of unstable states depend on the cube of their masses and the square of their couplings. These in turn might be estimated by formulae which depend on high powers of some fundamental scale. Mistakes of order one can thus be amplified.). Also, experience with weakly coupled string theory shows that order of magnitude estimates can miss factors like $16\pi^2$. Our fundamental contention about M-theory is that neither the true vacuum state nor the point where inflation takes place are likely to sit in one of the weakly coupled or large radius regimes where systematic calculations can be done. Thus, we are unlikely to be able to extract detailed numbers from M-theory until we learn a lot more about the nonperturbative formulation of the theory. In this situation it seems wisest to try to investigate very general problems, and that is what we will try to do. I will deviate from this formula only towards the end of my lectures, in order to present the amusing scenario that I favor.
7.2 Moduli as inflatons?

In view of our discussion in the previous section, one might have thought that the appropriate title for this section was “Cosmology on the Moduli space with 4 SUSYs”. At first sight, the phrase in quotes does not appear to make any sense. M-theory has no global internal symmetries – all of its symmetries are residual gauge symmetries which leave some class of configurations invariant\(^{28}\). With only 4 SUSYs, supersymmetry alone permits a superpotential on the space of chiral superfields. The full effective potential is the sum of a term coming from the so-called D-terms of continuous gauge groups, and a term coming from the superpotential\(^{29}\). The D-terms are positive, and the moduli space of fields on which they vanish can be parametrized in terms of gauge invariant composite fields. The superpotential can be viewed as a function on this space. The only symmetries which act on the composites are discrete gauge symmetries\(^{30}\). In most cases, a discrete symmetry cannot imply the vanishing of a function on an entire submanifold (we will explore the exception below).

The apparent implication of this is that the phrase “moduli space of M-theory compactifications with 4 SUSYs” has no apparent meaning. There is no moduli space in the true sense of the word (with the exception noted in the last parenthesis). Nonetheless, the authors of [24] proposed and [25] and others explored the idea, that moduli of such compactifications were the natural inflaton candidates in string/M-theory. Note that inflatons, by their nature, must have a potential so the idea of moduli as inflatons is truly oxymoronic.

However, I hope to demonstrate for you that this idea is not at all idiotic, and that it has many attractive features. The original proposals were based on string perturbation theory. Here the idea of a moduli space of \textit{quadrisusic}\(^{31}\) compactifications makes perfect sense.

\(^{28}\)As usual, there are two arguments for this, one based on SUGRA, the other on perturbative string theory. Their agreement is taken as evidence that the statement is exact. The SUGRA argument is simply that all symmetries of SUGRA are diffeomorphisms, thus gauge symmetries. Global symmetries arise only as diffeomorphisms which leave invariant the asymptotic behavior of the noncompact portion of space. All other symmetries are gauged. In perturbative string theory an internal symmetry would arise as a symmetry of the superconformal field theory describing the internal space. One can show, \cite{[40]}, that a continuous global symmetry implies the existence of a Kac-Moody current algebra in the superconformal field theory (basically just Noether’s theorem plus conformal invariance – up to technicalities). The Kac-Moody currents can be used to construct vertex operators for massless gauge bosons.

\(^{29}\)See Keith Olive’s lectures at this school for a concise introduction to four dimensional SUSY, chiral superfields, superpotentials, D terms, \textit{etc}..

\(^{30}\)The only difference between gauged and nongauged discrete symmetries from a practical point of view is the absence of stable domain walls for gauged discrete symmetries.

\(^{31}\)A recently rediscovered ancient Latin word meaning: having four supersymmetries.
mathematical sense. At string tree level, a vacuum state is characterized as a conformal field theory with certain extra properties. There is an exact theorem which guarantees the existence of continuous families of solutions to this constraint. The most famous among them are those which correspond to compactification of the heterotic string on a CY 3-fold with the standard embedding of the spin connection of the manifold in the gauge group. Here the theorem follows from the fact that the same conformal field theories can be used to compactify Type II string theories to four dimensions, preserving 8 spacetime SUSYs. The extra spacetime SUSY guarantees the existence of moduli. The heterotic and Type II theories compactified on these backgrounds differ at the one loop level and beyond, and the heterotic theory has only 4 SUSYs. Nonetheless, to all orders in the loop expansion, no superpotential is generated on the tree level moduli space in the heterotic theory. Indeed, the heterotic coupling, like a generic gauge coupling, can be viewed as the real part of a chiral superfield $S = \frac{8\pi}{g_s^2} + i\theta$, whose imaginary part is an axionlike field called the model independent string axion. This field arises by a duality transformation on a second rank antisymmetric tensor gauge field. As a consequence, to all orders in perturbation theory there is a continuous shift symmetry $S \rightarrow S + ia$. This symmetry, combined with holomorphy, forbids any perturbative correction to the superpotential.

The idea behind most previous work on the subject was that the real world corresponds to a point in moduli space where the perturbative estimates of the superpotential were correct. The string coupling was supposed to correspond more or less to the perturbative gauge couplings we see in nature, or to be related to them by simple group theoretical factors. The superpotential on the perturbative moduli space was then much smaller than the fundamental scales of the theory, and it made sense to think about an approximate moduli space.

This set of ideas had a number of related difficulties. The first was the Dine Seiberg problem [55]. These authors made the simple observation that for most functions, the leading asymptotic formula in some extreme region (here the weak coupling region) is monotonic and does not have minima. There have been two mechanisms proposed for stabilizing M-theory in the weak string coupling regime, which go under the names of Kähler stabilization [48] and racetrack models [4]. Both imply that, although the couplings are weak, many quantities cannot be calculated in a systematic expansion.

A related cosmological problem with the weak coupling regime was pointed out by Brustein and Steinhardt [54]. There is a distinct possibility that the universe would

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32Exceptions to this are somewhat pathological. The leading asymptotic behavior could contain a factor $\sin(1/g^2)$ which has an infinite number of more and more closely spaced minima as one approaches the weak coupling regime.
“overshoot” a weak coupling minimum and evolve into the regime of extreme weak coupling where M-theory is in violent disagreement with observation.

When combined with Witten’s analysis [44] of the possible resolution of the discrepancy in the weak coupling prediction of the ratio between the unification and Planck scales, these observations compel one to consider the possibility that weakly coupled string theory is not a good description of nature. A somewhat better starting point is the 11D SUGRA analysis begun in [43]. The analyses of [44] and [45] indicate that

- In the regime of moduli determined by the fit to the unified coupling strength and the four dimensional Planck mass, the volume of the Calabi-Yau manifold on the brane where the standard model lives is not really in the regime where the SUGRA expansion can be trusted. However, the small size of the four dimensional effective coupling, combined with holomorphy, is enough to guarantee the usual tree level unification relations between standard model couplings. This gives rise to a situation similar to that hypothesized in the Kähler stabilization mechanism, where holomorphic quantities can be calculated reliably but the Kähler potentials of chiral fields are unknown.

- Witten’s hypothesis that the coupling of the gauge fields on the second brane is strong, and gives rise to a gaugino condensate whose magnitude is of order the unification scale (which is also the fundamental 11D Planck scale) induces too high a scale of SUSY breaking on the standard model brane. We will discuss a resolution of this problem below.

- In the analysis of [45] the SUSY breaking F term comes from the modulus which parametrizes the radius of the single large dimension transverse to the Hořava-Witten ninebranes. To leading order in the SUGRA expansion this leads to no-scale SUSY breaking with vanishing cosmological constant, and also gives rise to degenerate squarks.\(^{33}\)

- The radial mode is not stabilized in this approximation and we have a sort of Dine-Seiberg problem within the SUGRA approximation. It is unclear how many of the good features of the model will survive the resolution of this problem. It is clear that the vanishing cosmological constant will not.

\(^{33}\)The degeneracy in mass of the squarks is a desirable phenomenological feature. To the extent that it is valid it eliminates unwanted flavor changing neutral currents which threaten the viability of generic SUSY models. This success of the scenario is mitigated by the failure to stabilize the radial mode. The terms necessary to stabilize the radius come from corrections to its Kähler potential. Similar corrections could ruin the degeneracy of squarks.
In short, this scenario is better than perturbative string theory, but not without its own flaws. On the other hand, the observation that gauge theories arise on branes of finite codimension is generic in M-theory and leads one to expect that Witten’s explanation of the Planck and unification scales is a correct one.

At first sight, the above conclusions would seem to rule out the idea of modular inflation. If we are in the strong coupling regime and there is no reason for the superpotential to be small then what is our excuse for separating the moduli out from all the other variables of M-theory? What does the word moduli mean in the strong coupling regime with only four SUSYs? Worse, one of the points of [25] was that within the context of modular inflation, the energy scale during inflation is predicted to be near the unification scale. In Witten’s scenario, this scale is identified with the fundamental scale of quantum gravity and it seems unreasonable to use any sort of effective field theory description to describe this situation.

In fact, I claim that the Hořava-Witten scenario and Witten’s use of it to explain the ratio between \( m_P \) (the four dimensional Planck scale \( \sim 2 \times 10^{18} \text{ GeV} \)) and \( M \) (the unification scale \( \sim 2 \times 10^{16} \text{ GeV} \)) may resolve all of these problems. The key is that the higher dimensional theory has more SUSY than the effective theory below the KK scale. The higher dimensional SUSY is broken by the branes, but if the bulk volume is large then this breaking can be ignored for some purposes. In particular, we can identify the moduli space as that of the higher dimensional theory. Thus, in such scenarios, a clearcut notion of approximate moduli survives at all energy scales, as long as we remain in a regime where the compact volume is large. We will call these approximate moduli the \textit{inflamoduli} to distinguish them from certain fields we will discuss below, which get their potential only from lower energy physics.

Note that this is all compatible with the existence of a superpotential of order \( M^3 \) for the inflamoduli, and indeed this order of magnitude is reasonable for fields which parametrize properties of the bulk higher dimensional theory \textit{only if there is enhanced SUSY in the bulk}. Otherwise we would have expected the effective superpotential of the moduli to contain a factor of the volume of the internal space. On the other hand, if the superpotential comes only from the vicinity of the branes, it has, by dimensional analysis, the form

\[
W = M^3 w(\theta_a)
\]  

(7.1)

where \( \theta_a \) are dimensionless parameters characterizing the internal geometry. On the other hand, the kinetic term for these zero modes, just like the Einstein term for the zero modes of the gravitational field, is proportional to the volume \( V_7 \) of the internal manifold, and has the form

\[
M^9 V_7 \sqrt{-g} G_{ab}(\theta) \nabla \theta_a \nabla \theta_b.
\]  

(7.2)
Note that $M^9 V_7 = m_P^2 = \frac{1}{8\pi G_N}$ is, as the notation indicates, the same coefficient which multiplies the Einstein action. Furthermore, although the volume $V_7$ is itself a modulus, when we pass to the Einstein conformal frame in which $V_7$ is replaced by its vacuum value, the kinetic term of the moduli is rescaled in precisely the same manner as the gravitational action. It is then natural to define canonical scalar fields by $\phi_a = m_P \theta_a$. Their action has the form

$$\int \sqrt{-g} \left[ G_{ab}(\phi/m_P) \nabla \phi^a \nabla \phi^b - \frac{M^6}{m_P^2} v(\phi/m_P) \right]. \quad (7.3)$$

Now let us examine the implications of a Lagrangian of this form for inflationary cosmology. The slow roll equations of motion derived from this action are

$$3H d\phi^a/dt = -\frac{M^6}{m_P^2} G^{ab} \partial_v G_{ab} \partial_b v. \quad (7.4)$$

and lead to the equation

$$dv/dt = \frac{M^3}{3m_P^2 \sqrt{v}} \partial_a v G^{ab} \partial_b v. \quad (7.5)$$

where $\partial_a$ refers to the derivative with respect to the dimensionless variable $\theta^a$. We have also used the slow roll expression for $H$ in terms of the potential. From (7.5) we immediately derive an expression for the number of $e$-foldings

$$N_e = 3 \int \frac{v}{\partial_a v G^{ab} \partial_b v} \partial_c v d\theta^c. \quad (7.6)$$

where the integral is over the trajectory in moduli space that the system follows during the time interval when the slow roll approximation is valid. We see that in order to obtain a large number of $e$-foldings we need a potential which is flat in the sense that $|\partial v|/v \sim 1/N_e$. The phenomenologically necessary $N_e \sim 60$ can be achieved with only a mild fine tuning of dimensionless coefficients. Correspondingly, the conditions on the potential which ensure the validity of the slow roll approximation are order one conditions on the derivatives of the potential and do not contain any exponentially small dimensionless numbers.

An additional feature of modular dynamics, which provides extra frictional damping of the motion of the moduli, was discovered in [25]. If we completely ignore the potential on moduli space, it is still an interacting nonlinear system. In [25] the equations for small fluctuations of the modular field theory around a solution of the equations of motion (without potential) for the zero modes, was studied, and an unstable mode was found. This was interpreted as an efficient mechanism for converting kinetic energy of the zero modes into energy of a gas of nonzero modes. It was estimated that the zero
modes were effectively brought to a halt by this mechanism in less than a Lyapunoff
time of the chaotic motion on moduli space. In the inflationary context, this mecha-
nism will act as a source of friction which should make inflation much more probable.
In particular, it is an avenue in which the large dimension of the moduli space (which
can be a number of order $10^2$) could effect inflation, by providing a large number of
degrees of freedom for efficient frictional damping of the zero mode motion. This is a
topic which has not been investigated and deserves much more thorough study.

The fact that actions of the form (7.3) give rise to inflation with minimal fine
tuning, and that such actions naturally arise for moduli in string theory was pointed
out in [25]. The general point that moduli might provide the flat potentialled, weakly
coupled fields necessary to inflation was first made in [24]. Here we note that in brane
scenarios, it is the bulk inflamoduli which play this role. There may also be moduli
associated with branes, but they will have a natural scale $M$ and have a quite different
role to play.

Another pleasant surprise awaits us when we plug the potential from (7.3) into
the standard formula for the amplitude of the primordial energy density fluctuations
generated by inflation. Up to numbers of order one we find

$$\frac{\delta \rho}{\rho} \sim N_\lambda (M/m_p)^3 \sim 10^{-5}$$

(7.7)

where the numerical value comes from the measured cosmic microwave background
fluctuations, and $N_\lambda \sim 50$. This gives $M \sim (2/10)^{1/3} \times 2 \times 10^{16}$ GeV, which, given the
crudeness of the calculation, is the unification scale. To put this in the most dramatic
manner possible, we can say that a brane scenario of the Horava-Witten type, given
the unification scale as input, predicts the correct amplitude for inflationary density
fluctuations. Furthermore, the whole scenario only makes sense because of the same
large volume factor that underlies Witten’s explanation of the ratio between the Planck
and unification scales. This is necessary at a conceptual level to understand why it is
sensible to think about a modulus with a super potential of order the fundamental
scale, and at a phenomenological level to understand the magnitude of the density
fluctuations.

A detailed calculation of the fluctuation spectrum as opposed to its absolute normal-
ization requires more knowledge of the potential $v$ than we possess. A crucial question
(posed during my lecture by Andre Linde) is how natural the phenomenologically nec-
essary flat spectrum is in this context. I leave it as an exercise for the enterprising
student.

Although it has no connection with our discussion here I cannot resist pointing
out the other piece of evidence for a scale of the same order as $M$. Any theory of
the type we are discussing would be expected to contain corrections to the standard model Lagrangian of the form (in superfield notation) \( \frac{1}{M} LLH^2 \), which gives rise to neutrino masses. It is a matter of public record [58] now that such masses exist, with an estimated value for \( M \) between \( .6 \) and \( 1.8 \times 10^{15} \) GeV. Although this is an order of magnitude shy of the unification scale I believe the uncertainties in coefficients of order one in dimensional analysis could easily make up the difference. If not, we will have the interesting problem of explaining the existence of two close but not identical energy scales in fundamental physics. [61].

We also want to note that this scenario for inflation does not suffer from the runaway problem pointed out by Brustein and Steinhardt [54]. These authors noted that the inflationary vacuum energy is much larger than the SUSY breaking scale. Furthermore, the minimum of the effective potential was assumed close to the region of weak string coupling. There was then a distinct possibility that the inflaton field would overshoot the small barrier separating it from the extreme weak coupling regime where string theory is incompatible with experiment. In the present scenario, the coupling is not assumed to be weak (nor the volume extremely large). Furthermore the inflationary potential has nothing to do with SUSY breaking. There is no runaway problem at all.

The authors of the papers in [25] agonized over the discrepancy between the unification scale and the scale of SUSY breaking. In fact, they discussed and discarded what I now believe is the obvious solution of this problem, because of problems specific to weakly coupled string theory\(^{34}\). The obvious way to avoid SUSY breaking at the scale \( M \), is to insist that the superpotential (7.1) has a SUSY minimum. In fact, the existence of such minima is generic, requiring only the solution of \( n \) complex equations for \( n \) unknowns. However, in general, the superpotential will not vanish at such a minimum but instead will give rise to a negative cosmological constant.

It was pointed out in [26] that in postinflationary cosmology, the universe’s attempt to access such a SUSY minimum of the effective potential leads to a very welcome cosmological disaster. The key point is that inflation has completely eliminated the spatial curvature terms from the cosmological equations, so that the Friedmann equation reads

\[
m_p^2 (\dot{a}/a)^2 = G_{AB} \dot{m}^A \dot{m}^B + V
\]  

(7.8)

This does not have static solutions with \( m^A \) resting at a minimum of \( V \) with negative value. What happens instead is that a generic solution of the cosmological equations\(^ {35} \)

\(^{34}\)Namely the fact that superpotentials are exponentials of exponentials of the canonically normalized dilaton field.

\(^{35}\)There are very special solutions in which the universe is static and the scalar fields oscillate in the potential with exactly zero energy, and I once thought that these were relevant to the cosmological constant problem. However, they are unstable to small perturbations.
reaches a point where $\dot{a} = 0$ and then begins to recollapse to infinite energy density. This happens on a microscopic time scale. Thus inflationary cosmology eliminates generic SUSY preserving minima of the effective potential from the list of late time attractors of the cosmological equations.

The stable postinflationary attractors of a supersymmetric cosmology are points in inflamoduli space with vanishing superpotential and SUSY order parameters. These can be characterized in terms of a symmetry. Namely, any complex R symmetry forces the superpotential to vanish, and if there are no fields of R charge 2 then the SUSY order parameter vanishes as well. The R symmetry must of course be discrete, since we are discussing M-theory. If in addition, there do exist fields of R charge 0, then there will be an entire submanifold on which the superpotential vanishes and SUSY is preserved. Our future considerations will concentrate on this submanifold, which from now on we call the true moduli space, since it is the oft advertised exception to our statement that quadrisusic backgrounds had no moduli space. It is the locus of restoration of a discrete R symmetry with the above properties. We should expect the true moduli space to have more than one connected component, each characterized by a different R symmetry.

7.3 Radius stabilization

Every silver lining has its cloud. The discussion above treated the four dimensional Planck scale as a fixed parameter. In fact, in the Hořava-Witten scenario, it is determined by the radius of the fifth dimension, which is one of the moduli. In fact it is one of the bulk moduli and might be expected to vary during inflation.

At first glance, the situation appears to be much worse than that. In the limit of large $R$, the Lagrangian of the field $R$ is highly constrained by extended SUSY. In this limit the Kähler potential of the superfield $T$ which contains $R$ is fixed to be $-3m_R^2 \ln(T + T^*)$. In the analysis of [44] [45], the superpotential was supposed to be generated only by gaugino condensation on the hidden brane, separated by a distance $R$ from the brane where the standard model lives. This is a function only of a particular linear combination $S$, where $S$ is the superfield which controls the coupling of the hidden sector gauge group. The superpotential can also depend on the other moduli, e.g. the complex structure moduli of the Calabi-Yau threefold, as well as the vector bundle moduli in the hidden gauge group. Although this superpotential is not explicitly calculable, it will generically have a supersymmetric point with $S$ fixed to be small (the hidden gauge theory is strongly coupled) and the complex structure and hidden sector gauge bundle moduli fixed. Unless there are points of enhanced discrete

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36This is an example of the nonexistence of continuous global symmetries.
R symmetry, as described above, the superpotential will be nonvanishing at the SUSY point and of order $M^3$.

The fact that the superpotential is of order $M^3$ means that it cannot really be considered to have originated in some “low energy effective theory”, but comes from physics at the fundamental M-theory scale. The possibility of superpotentials generated at short distance was not appreciated in [44] and [45], nor as far as I can tell in any of the papers on M-theory phenomenology which have appeared since that time. I do not see any good argument for omitting such terms in the low energy Lagrangian. However, there is a symmetry argument that such a superpotential will be of the form $\sum_{n>0} w_n(S,C)e^{-nkmpT/M^2}$, where $k$ is a number of order one. The factors in the exponent will be explained below. Here $C$ is a collection of superfields representing the complex structure moduli, as well as vector bundle moduli for the gauge configurations on each wall. The imaginary part of $T$ comes from a pure gauge mode of the bulk graviphoton, which is chosen to vanish on the hidden sector wall. The gauge symmetry becomes a shift symmetry for Im$T$. One may expect this symmetry to be broken by effects involving membrane instantons stretched between the walls, and by fivebranes (which, in the walls, are gauge theory instantons). As a consequence, a discrete remnant of the shift symmetry remains, and this is what constrains the superpotential in the manner described above.

Thus, in the large $R$ limit, one expects the Kähler potential of the field $T$ to be given by its asymptotic form, and the superpotential to be independent of $T$. As a consequence, even if we assume the inflamoduli are slowly rolling at some point away from the minimum of their potential, the dynamics of the universe will be strongly influenced by the motion of $T$. It is easy to see that the real part of $T$ is, in Einstein frame, related to a canonically normalized scalar field with an exponential potential. The slope in the exponent is outside the range in which (power law) inflationary solutions of the equations of motion exist. Other sources of friction for $T$ must be found if inflation is to take place.

There are several obvious sources for such extra friction. The first is the imaginary part of $T$, which, in the large $R$ approximation, behaves like a Goldstone field. Unfortunately, this means that the energy density associated with this field, and the extra friction associated with it, scales away like $1/a^6$. While I have not done a proper numerical study of this system it seems unlikely that it will have long periods of inflation for generic initial conditions.\footnote{Here and henceforth we restrict attention to CY threefolds with only a single Kähler modulus and disregard the possibility of inserting M5 branes in the bulk between the two walls.}

\footnote{Remember that unlike the case of the other moduli fields, there are no unknown parameters in the asymptotic Lagrangian for $T$.}
Two other sources of extra friction are the excitation of nonconstant modes of the $T$ field, and Kaluza-Klein particle production. In [25] it was argued that the first of these mechanisms is very efficient at stopping the chaotic motion on moduli space with no potential. As noted above, there is an instability which converts modular zero mode kinetic energy into a gas of nonzero modes within less than a Lyapunov time of the chaotic motion on moduli space. It seems quite plausible that in the presence of an exponential potential one would then have inflationary solutions. Kaluza-Klein particle production is also to be expected in the presence of a rapidly moving $T$ field, because the real part of $T$ directly influences the masses of these particles.

Obviously, more work is needed to see whether these mechanisms can really salvage the inflationary scenario of the previous section. Even if they do, one mystery still remains. Although some combination of these effects can explain why $T$ is slowly varying during inflation, there is no explanation of why it is close to its vacuum value. Since the four dimensional Planck mass (and through it our successful prediction of the magnitude of energy density fluctuations) depends exponentially on the canonically normalized field constructed from the real part of $T$, it is extremely important to explain this coincidence.

Another possibility for rescuing inflation comes from the recognition that the radial modulus has a Dine Seiberg instability. That is to say, although we would like to be doing a systematic asymptotic expansion in $R$, we know that we will never find a stable minimum for $T$ in this approximation. Thus we should admit that near the vacuum value for $T$, the large radius expansion for (at least) the effective potential of this field has broken down. Let us recall that we defined $T$ in terms of the deviation of the radius from its vacuum value [45]. Thus, $RM \sim (m_P T/M^2)$. On physical grounds, we expect corrections to the asymptotic form of the Lagrangian to be functions of $RM$. In the case of the $T$ dependence of the superpotential discussed above, this guess can be verified by analytic continuation from the region of weakly coupled string theory [45].

The potential for $T$ during inflation has two terms. The first, coming from the F terms of the other moduli was discussed above, and is all that exists in the extreme asymptotic limit. In that limit, it gives an exponential potential with slope of order $1/m_P$ for the canonically normalized field $\sim m_P \ln \text{Re} (T/m_P)$. The second term has the form:

$$V \sim e^{K/m_P} [K^{TTT} K_T/m_P]^2 - 3||W/m_P||^2,$$

where $K$ is the Kähler potential of $T$. The implication of the previous paragraph is that there is a region of $T/m_P$ of order one, where $K$ is very different from its asymptotic form, and varying rather rapidly as a function of this variable. Now consider initial conditions where $RM$ starts out close to one and growing. The $T$ field will then have
to cross a regime in which the rapidly varying piece of the potential is significant before it can access the asymptotic regime. If the other moduli are slowly rolling, it is clear that it will instead be rapidly driven very close to the minimum of its potential. Unfortunately, I have no argument that this is the same as its VEV.

Indeed, we will see in the next section that the minimum of the low energy potential for $T$ is the same as that of (7.9). There is no obvious reason to expect the first term of the potential (proportional to the F terms of other chiral fields) to be negligible compared to (7.9). Thus, although this mechanism saves inflation, it is not clear that it preserves our explanation of the size of primordial fluctuations.

Our discussion of the end of inflation is also modified. Once the contribution of $T$ is taken into account, the cosmological constant (at the end of inflation, but neglecting low energy gauge dynamics) is given by the value of (7.9) at its minimum, with the other moduli set at SUSY preserving values. Points with nonvanishing superpotential will now have SUSY spontaneously broken by the F term of the $T$ field. If we insist that the low energy cosmological constant vanishes exactly (in the scenario with discrete R symmetry broken by low energy dynamics), then these points will also have vanishing cosmological constant and will be attractors of the postinflationary cosmological equations. This is unfortunate, because these points have gravitino masses of order $M^3/m_P^2$, and are ruled out by phenomenology. It would have been pleasant to find that they were also disfavored by cosmological evolution. In the next section we will see that we can still recover acceptable phenomenology at points of enhanced R symmetry (broken only by low energy dynamics).

There is a (weakly anthropic) way of understanding why points in moduli space with R symmetry broken at high energies could be ruled out by cosmology as well as phenomenology, if we accept that there is a nonvanishing cosmological constant in the world we observe. Then the ratio of cosmological constants between the R asymmetric worlds and our own is $\sim (M/\mu)^6$, where $\mu$ is the scale of low energy R symmetry breaking. If one insists on a low energy SUSY breaking scale of order a TeV $\mu$ is fixed at about $10^{13}$ GeV (see below). This gives the R asymmetric worlds a De Sitter horizon size of about a light year. There is certainly no galaxy formation in such a universe, and it does not take a degree in exobiology to conclude that no life is possible there. There is no plausible initial (post primary inflation) matter distribution which leads to any appreciable late time matter inside a horizon volume, unless it is collapsed into black holes.

Finally, one should note that at small values of $T$ (values of $RM$ of order 1) there might be a SUSY minimum of the potential for $T$. This regime is hard to discuss because effective field theory does not apply to it and the notion of effective potentials, approximate moduli, and classical spacetime are all suspect. However, even if one
assumed that such a minimum existed, one would find that one could not access it after inflation and it would be irrelevant to macroscopic physics.

I have divided the discussion of inflation on moduli space into two parts, initially ignoring the problem of the radial modulus, because I suspect that it may be possible to find other scenarios in which this problem is completely absent. I will make a similar division of the discussion of SUSY breaking below.

### 7.4 SUSY breaking

Before proceeding to the discussion of SUSY breaking on the true moduli space, we should introduce the final characters in our story, the boundary or brane moduli. In Calabi-Yau compactification of weakly coupled string theory, there are moduli which correspond to the parameters of the $E_8 \times E_8$ gauge field configuration on the manifold (these are called vector bundle moduli in the string compactification literature). In a brane scenario these moduli should be thought of as living on the branes where the gauge fields live. In the strong coupling regime, these fields will have a superpotential of the form $M^3 W(b/M)$ and it is not clear that they should be called moduli at all. Some of them may be invariant under the discrete complex $R$ symmetry, and thus belong to the true moduli space. In perturbative string theory, some vector bundle moduli have components $\theta_b$ which couple to gauge fields like axions: $\theta_b F \tilde{F}$. The decay constants of these axions are of order $M$ because, since they live on the brane, no other scale can enter their kinetic terms.

In our later considerations, we will have need of a field with a decay constant of order $M$ and a very small potential energy. The vector bundle moduli on the standard model wall have the first of these properties. In perturbative string theory these fields have Peccei-Quinn symmetries which are broken only by world sheet instantons. It is then plausible that in the Hořava-Witten regime the dominant breaking of these symmetries comes from nonperturbative QCD. The potential energy of one of the gauge bundle axions would be much smaller than any fundamental scale, and would have the form $\Lambda_{QCD}^4 u(a/M)$. We will consider the possibility that there are other moduli of this type, with a variety of scales replacing $\Lambda_{QCD}$.

In addition to these moduli fields, any brane scenario will contain a variety of gauge fields and matter fields in nontrivial representations of the gauge group. The moduli will interact with these fields via the moduli dependence of bare gauge and yukawa coupling parameters in the effective theory as well as thru a variety of irrelevant operators. If the gauge couplings are asymptotically free and do not run to infrared fixed points at low energy, this description of the physics only makes sense if the bare gauge couplings are sufficiently small that the scale at which the effective coupling becomes large is substantially below the scale $M$. Otherwise it is not consistent to include the gauge
degrees of freedom in the low energy effective theory. The weakness of bare couplings in these scenarios is not evident a priori, as it would be in a purely perturbative approach. The underlying physics is assumed to be strongly coupled. Witten [44] has shown how the small unified coupling of the standard model can be explained in terms of a product of a large number of factors of order one in a geometry of large dimensions. We will assume that similar numerical factors explain the strength of the gauge interactions that lead to SUSY breaking.

The main role of the gauge interactions is not to break SUSY, but rather the discrete R symmetry. If we fix the moduli and treat the gauge theory as a flat space quantum field theory, then SUSY remains unbroken even though a nonperturbative superpotential is generated. The scale of this superpotential is determined via a standard renormalization group analysis in terms of the bare gauge coupling function \( f(\phi/m_P, \chi/M) \), where we have indicated dependence on both bulk and boundary moduli. For simplicity we assume that \( f \) is a large constant \( f_0 \) plus a smaller, moduli dependent, term. The conclusions are not affected by this assumption. The scale \( \mu \) of the nonperturbative superpotential is then determined by \( f_0 \). It takes the form

\[
W_1 = \mu^3 w_1(\phi/m_P, \chi/M)
\]

We have eliminated all (composite) superfields related to the gauge interactions from this expression by solving their F and D flatness conditions for fixed values of the moduli. The possibility of doing this is equivalent to the statement that the gauge theory does not itself break SUSY. We assume that \( W_1 \) does not vanish at any minimum of the effective potential. This is the statement of spontaneous R symmetry breaking. As a consequence, SUSY minima of the potential have negative cosmological constant constant of order at least \( \mu^6/m_P^2 \) and are not attractors of the cosmological equations. Thus, cosmologically, R symmetry breaking forces the moduli to choose a minimum with spontaneously broken SUSY.\(^{39}\)

Phenomenology puts an upper bound on the value of \( \mu \) because it contributes directly to squark masses. The nonvanishing F terms are of order \( \mu^3/m_P^3 \). A standard argument shows that squark masses will be of order \( \mu^3/m_P \), about the same as the gravitino. Assuming this is about a TeV we find \( \mu \sim 10^{13} \) GeV. An attractive feature of this scenario is that the positive and negative terms in the SUGRA potential are naturally of the same order of magnitude. Although we have no real understanding of why the cosmological constant is so small, this fact of nature is an indication of a relation between the scales of R symmetry breaking and of SUSY breaking. In models in which

\(^{39}\)The tunneling amplitudes of such nonsupersymmetric vacua into supersymmetric AdS vacua are incredibly tiny and might be identically zero, as discussed in [26].
the SUSY breaking F term originates as a bulk modulus the correct order of magnitude relation between these scales arises automatically.

As we now recall, a deficiency of this scenario for SUSY breaking is that it leads to the cosmological moduli problem. The scalar fields in the bulk moduli multiplets acquire masses from the SUSY violating potential of order $m_M \sim \mu^3/m_P^2$ which is the same order of magnitude as the gravitino and squark masses, i.e. a TeV. They have only nonrenormalizable couplings to ordinary matter, scaled by $m_P$. Thus, their nominal reheat temperature, $\sqrt{m_M^3/m_P}$ is of order $\sim 3 \times 10^{-2}$ MeV, and the universe is matter dominated at the time that nucleosynthesis is supposed to be taking place. The thermal inflation scenario [56] can solve this problem, and we will now review another solution [49].

Suppose that the coefficient in the order of magnitude relation between the moduli mass and the fundamental parameters is $m_M = 5 \times \mu^3/m_P^2$, while the squark mass is actually $m_{\tilde{q}} = \mu^3/4m_P^2 = 1$ TeV. Then the reheat temperature for the bulk moduli is multiplied by a factor of $20^{3/2} \sim 10^2$ and is just above 1 MeV. Thus, an innocent looking insertion of factors of order one can cause the moduli to decay just in time to light the furnace in which the primordial elements are forged.

One still has to account for baryogenesis. Adopting a mechanism suggested long ago by Holman, Ramond and Ross [59] we aver that this can come from the decay of the moduli themselves. All of their interactions are of order the fundamental scale of M-theory, so there is no reason for them to preserve accidental symmetries like baryon and lepton number. It is quite reasonable that they also violate CP, though the status of CP in M-theory is somewhat more obscure. The decay itself is an out of equilibrium process, so all of the Sakharov criteria for baryogenesis are fulfilled. However, we must also take note of the theorem of Weinberg [60], according to which baryon number violating terms in the Hamiltonian must act twice in order to generate an asymmetry. In the decay of moduli, the first action of the Hamiltonian comes at no cost in amplitude, because the modulus must decay somehow and there is no reason for its baryon number violating decays to be significantly smaller than those which conserve baryon number. However the second baryon number violating interaction should not be highly suppressed if we want to generate a reasonable baryon asymmetry. Indeed, a 10 TeV, gravitationally coupled, particle which produces a baryon asymmetry of order one in its decay, also produces of order $(10$ TeV$/3$ MeV) or $\sim 3 \times 10^6$ photons. Thus a large suppression of the average baryon number per decay would give too small a baryon asymmetry. A way out of this difficulty is to admit renormalizable baryon number violating operators in the supersymmetric standard model. Discrete symmetries such as a $Z_2$ lepton parity [57] can adequately suppress all unobserved baryon and lepton number violating processes in the laboratory, while allowing such operators with coefficients as large as $5 \times 10^{-3}$.
This might be large enough to produce the observed baryon asymmetry.

An unfortunate casualty of this mechanism is the lightest SUSY particle. The LSP is no longer stable in the scenario described above and we have to look elsewhere for a dark matter candidate. However, there are natural candidates for dark matter. Imagine a boundary modulus whose potential energy is substantially smaller than the estimate \( \mu^3/M^2 \) coming from (7.10). We will call this the dark modulus, because it will be our dark matter candidate. It has a potential of the form \( U = \Lambda^4 u(D/M) \). (In [49], where this scenario was first proposed, the candidate was a QCD axion field. This model works, but the mechanism is much more general and does not require energy densities as small as those of the axion.)

Now, briefly review cosmic history. First we have inflation generated by bulk moduli fields which are not on the true moduli space (which we have called inflamoduli). This period ends after of order 100 e-foldings, and the universe is heated by inflamoduli decay to a temperature of order \( 10^9 \) GeV. The primordial plasma quickly redshifts away. Furthermore, as soon as the inflamoduli potential energy density falls to \( \mu^6/m_P^2 \), the universe becomes dominated by the coherent oscillations of the true bulk moduli. The dark modulus remains frozen at some generic point on its potential until the Hubble parameter falls to the mass scale of this field. At this point the energy density of the universe is of order \( \rho \sim m_P^2 \Lambda^4/M^2 \) which is of order \( (m_P/M)^2 \sim 10^4 \) times larger than the energy density of the dark modulus. The important point now is that this ratio is preserved by further cosmic evolution until the true bulk moduli decay. After that time, the dark energy density grows linearly with the inverse temperature relative to radiation, and matter radiation equality occurs at \( 10^{-4} \) MeV. This is close enough to the true value for the observable universe that the factors of order one which we have neglected throughout might account for the difference. \( \Lambda \) must satisfy two constraints in order for this scenario to work: the dark moduli must remain frozen until the true bulk moduli begin to oscillate, and the dark modulus must have a lifetime at least as long as the age of the universe. The second constraint is by far the stronger, and leads to \( \Lambda < 3 \times 10^6 \) GeV. Axions satisfy this constraint by a large margin. Note that this scenario completely removes the conventional cosmological constraint on the axion decay constant. Axions will be very weakly coupled and will escape all of the usual schemes for detecting them.

Another possible mechanism for baryogenesis in this scenario is that of Affleck and Dine [2]. Indeed the authors of [3] have investigated a scenario with a 10 TeV modulus and Affleck Dine baryogenesis and found that it can account for all cosmological data.

\[\text{\textsuperscript{40}This was suggested to me by a student at the school. I thank M.Dine for detailed discussions of it and for pointing out the reference below.}\]
In this scenario the dark matter can either be an LSP, or if we have strong R parity violating interactions, the dark modulus (or a combination).

All in all, this seems to be the simplest solution of the cosmological moduli problem, and has the added virtue of allowing an invisible axion solution of the strong CP problem. I am also fond of the way in which the version of this scenario with a dark modulus predicts the correct (within an order of magnitude) temperature for matter radiation equality in terms of fundamental parameters.

For completeness, we should also discuss the possibility that SUSY breaking itself is caused by gauge interactions which are weakly coupled at the fundamental scale. This is required if we assume, with Dine [47] [42] [41], that moduli are fixed at some enhanced symmetry point. Scenarios of this sort are attractive because they allow us to use the idea of gauge mediation [50] to solve the SUSY flavor problem. Gauge interactions generate superpotentials of the form $\mu_1^3 w_{g_1}(C_1/m_1) + \mu_2^3 w_{g_2}(C_2/m_2)$, where the $C_i$'s are composite superfields and the $m_i$ the nonperturbative low energy scales generated by asymptotic freedom. Here, in order to cancel the cosmological constant, we must introduce an R breaking gauge theory with scale $(m_1)$, which preserves SUSY and a SUSY breaking gauge theory, with scale related by $m_1^6 = m_2^2 m_4^2$. This is the price one must pay for giving up the idea that true bulk moduli are the instigators of SUSY breaking. The ratio of scales between SUSY and R breaking no longer comes out naturally, but must be put in by hand. In compensation there is no cosmological moduli problem in this picture, since all moduli are assumed to be frozen by the initial superpotential.

7.5 The effects of a dynamical radius

We now have to include the dynamics of the radial modulus $T$. The R symmetry violating superpotential has an expansion\footnote{It is important that, as a consequence of our assumption of an R symmetry under which $T$ is neutral, all terms in this expansion are proportional to the R breaking scale $\mu^3$. This means that we cannot invoke mechanisms like that of [1] to explain the stabilization of the radius.}

$$W = \sum_{n=0}^{\infty} \mu^3 W_n(m/m_P)e^{-nT/m_P}$$

At large radius the exponential terms are negligible. We then have no scale SUSY breaking even if all other bulk moduli have SUSY minima\footnote{However, once we take into account the SUSY violating potential coming from the $F$ term of $T$, there is no reason to assume that the other fields sit at their SUSY minima. The minimum of the potential might be achieved with $F$ terms for all the fields.}. One can then hope, as in [45], that the radius is stabilized by higher order terms in the Kähler potential. This
would give a SUSY breaking scale close to $\mu^2/m_P^2$. The resulting scenario is similar to that of the previous section.

There is a much more substantial difference in the case where (what we previously called) the true moduli space is a point. $T$ still plays the role of a true modulus, and we again get no-scale SUSY breaking when the low energy theory violates R symmetry without breaking SUSY. We can, if we wish, also add a low energy SUSY breaking sector, but to leading order in $R$ this leads to a large positive cosmological constant. This is true no matter what we choose for the relative scales of low energy SUSY breaking and R symmetry breaking (as long as we try to be consistent with the lower bound on superpartner masses). Thus, once the radius is allowed to be dynamical there do not seem to be consistent scenarios with gauge mediated SUSY breaking.

### 7.6 Generalizing Hořava-Witten

As we have noted, the moduli space of 11 dimensional SUGRA compactifications which preserve $\mathcal{N} = 1$ SUSY in four Minkowski dimensions splits into three components. These are Joyce sevenfolds, F theory limits of compactification on Calabi-Yau fourfolds, and Heterotic limits of compactification on $K3 \times CY_3$. These may be continuously connected when short distance physics is properly taken into account. In addition, there may be many branches of moduli space which join onto these through generalized extremal transitions. The moduli space is thus highly complex.

The cosmological arguments of these lectures indicate that the phenomenologically relevant compactifications may belong to a highly constrained submanifold of this complicated space. Namely, they should preserve eight supercharges in the bulk. The breaking to $\mathcal{N} = 1$ should occur only on branes. SUGRA compactifications preserving eight SUSYs are much more constrained. The holonomy must be contained in $SU(3)$ which implies that the manifold is the product of a Calabi-Yau threefold times a torus, modded out by a discrete group $\Gamma$. In order to obtain a smooth manifold with eight SUSYs, $\Gamma$ should act freely and the holonomy around the new cycles created by $\Gamma$ identification should be in $SU(3)$. Clearly, a way to obtain Hořava-Witten like scenarios is to allow fixed manifolds of $\Gamma$, on which an additional SUSY is broken. The original scenario of Hořava and Witten was a $CY_3 \times S^1$ compactification in which $\Gamma$ is a $Z_2$ reflection on the $S^1$. The fixed planes carry $E_8$ gauge groups, and one must also choose

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43 It should be noted that a large positive cosmological constant is not a disaster only for our ability to “fit the data”. The size of the event horizon for a De Sitter space with energy density of scale 1 MeV is about a light second in linear size, and for the scale of SUSY breaking it is smaller by a factor of $10^{12}$. In a theory with multiple late time attractors it is not hard to explain why we are not there to observe such a universe.
an appropriate gauge bundle. A further generalization allows five branes wrapped on
two cycles of $CY_3$ to live between the planes.

It seems likely that more complicated choices of $\Gamma$ might lead to a wider class of
scenarios. The problem of classifying scenarios of this type seems quite manageable\textsuperscript{44}. The moduli space of compactifications of M-theory on $CY_3$ times a torus has a rea-
sonably complicated structure, replete with extremal transitions. Nonetheless, it is
considerably simpler than the fourfold or Joyce manifold problem, and we know much
more about its structure. Thus, if cosmology really points us in the direction of gener-
alized Hořava-Witten compactifications, we have made real progress in the search for
the true vacuum of M-theory.

\section{7.7 Conclusions}

Witten’s explanation of the discrepancy between the Planck and unification scales in
the context of Hořava-Witten compactifications, poses a challenge for inflationary cos-
mology and particularly for the notion that moduli are inflatons. In fact, the enhanced
bulk SUSY of these compactifications gives us a clean definition of modular inflatons.
The scenario then makes an order of magnitude prediction of the amplitude of primor-
dial density fluctuations in terms of the unification scale. The major problem with
this inflationary scenario comes from stabilization of the radius of the Hořava-Witten
orbifold. In leading order in the large radius approximation, radial dynamics appears
to destroy inflation. We pointed out several sources of friction for the radius field,
which could restore inflationary solutions, but there is more work to be done here and
a mystery remains. Assuming the radion is slowly rolling during inflation, why is it
near its vacuum value?

An alternative, which seems more compelling, is to recognize that the Dine Seiberg
problem for the radial field probably requires us to contemplate the breakdown of the
large radius expansion for its Kähler potential near the true VEV of this field. We
argued that this meant that the Kähler potential was rapidly varying (as a function
of $T/m_P$) near the low energy VEV and that this implied that the radius would not
be an inflaton but instead would rapidly be driven to the minimum of its potential
during inflation. It is not clear whether the inflationary minimum is close enough to
the VEV to salvage our explanation of the size of density fluctuations. This depends
on properties of the Kähler potential which are, at the moment, incalculable.

In the context of this large class of inflationary scenaria, arguments first discussed
in [26] then focus attention on the true moduli space of M-theory, a locus of enhanced
discrete R symmetry. Such a space almost certainly exists [52]. It is the attractor of

\textsuperscript{44} Preliminary results on the classification problem have been obtained by L.Motl.
postinflationary cosmological evolution. The further evolution of the universe then depends on whether this space contains bulk moduli. In the attractive scenario in which it does, the initial Hot Big Bang generated by inflation, is soon dominated by the energy density stored in coherent oscillations of true bulk moduli. By making optimistic but plausible assumptions about coefficients of order one in order of magnitude estimates, one obtains a reheat temperature above that required by nucleosynthesis. The decay of true bulk moduli, rather than that of the inflaton, generates the Hot Big Bang of classical cosmology. The baryon asymmetry might also be generated in these decays, and this is possible if the SUSY standard model contains renormalizable baryon number violating interactions (compatible with laboratory tests of baryon and lepton number conservation). As a consequence of this, there is no LSP dark matter candidate. Instead, boundary moduli with a suppressed potential energy act as a natural source of dark matter. Indeed, the ratio between the Planck and unification scales appears again in this scenario, this time in explaining the temperature at which matter and radiation make equal contributions to the energy density of the Universe. This estimate comes out an order of magnitude too high, but given the crudity of the calculation it seems quite plausible that this mechanism could be compatible with observation. The “dark modulus” which appears in this scenario could be a QCD axion with decay constant of order the unification scale. Our unconventional origin for the Hot Big Bang completely removes the cosmological upper bound on this decay constant. Such a particle would be undetectable in presently proposed axion searches.

An alternative is to postulate the Affleck-Dine mechanism as the source of the baryon asymmetry in this late decaying modulus scheme. Dark matter could then be an LSP, a unification scale QCD axion, or some combination of the two.

If a cosmology like that outlined here turns out to be correct, one might be tempted to revise Einstein’s famous estimate of the moral qualities of a hypothetical Creator. The current standard model of cosmology was constructed in the sixties. Since then there has been much speculation about cosmology at times earlier than that at which the primordial elements were synthesized. Most of it has been based on an eminently reasonable extrapolation of the Hot Big Bang to energy densities orders of magnitude higher. If the present scenario is correct, no such extrapolation is possible, and the conditions in the Universe in the first fraction of the First Three Minutes were considerably different from those at any subsequent time. There was a prior Big Bang after inflation, whose remnants may be forever hidden from us. The dark matter which dominates our universe is so weakly coupled to ordinary matter that its detection is far beyond the reach of currently planned experiments. The QCD and electroweak phase transitions never occurred.

The only dramatic prediction of this scenario for currently planned experiments
is the occurrence of renormalizable baryon number violation in the low energy SUSY world\textsuperscript{45}. The details of the baryogenesis scenario envisaged here should be worked out more carefully, and combined with laboratory constraints, to nail down precisely which kind of operators are allowed. The scenario is thus easily falsifiable, but even the discovery of renormalizable baryon number violating interactions among SUSY particles will not be a confirmation of our cosmology. Similarly, any evidence for the existence of more or less conventional WIMP dark matter will be a strong indication that the present speculations are incorrect, but the failure to discover WIMPS will not prove that they are correct.

Instead one will have to rely on the slow accumulation of evidence against alternatives: ruling out vanishing up quark mass and spontaneous CP violation as solutions to the strong CP problem, the failure of conventional axion and WIMP searches, the discovery of renormalizable B violation. These will be steps on the road to proving that this cosmology is correct, but the end of that road is not in sight.

We have travelled a long road, from the exotic reaches of M-theory to what I hope have been glimpses of more practical applications of modular physics to cosmology. I hope I have convinced you that the moduli of M-theory are likely to play a crucial role in any inflationary cosmological model and that many of the phenomenological and fundamental problems of M-theory are likely to be resolved in a cosmological context. Perhaps the somewhat unorthodox cosmological scenarios presented here will also prove to be more than just a theorist’s toys, and will play some role in the future of cosmology.

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