Non-perturbative evidence for non-decoupling of heavy fermions

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Abstract

We investigate, using a $1/N$ expansion, the behavior of a parameter in the scalar–fermion sector of the standard model that shows perturbative non–decoupling as the fermion becomes heavy. This low energy parameter is related to the $S$ parameter defined through the $W_3 - B$ vacuum polarization tensor. We obtain the leading $1/N$ contribution to this parameter that, if expanded perturbatively, collapses to its constant one–loop result; remarkably all the higher–order terms in the series vanish. Non–perturbatively, however, we find that as the mass of the fermion approaches the built–in cutoff scale of the theory — the triviality scale — the parameter is highly dependent on the implementation of the cutoff; it is non–universal, and shows non–decoupling.

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When the mass of a particle is generated by a coupling constant, there are physical effects at low energy that do not vanish as the particle mass becomes very heavy. These so-called non-decoupling effects [1] are crucial in that they provide a window into the physics of higher energies than is currently available. This is evidenced by the current restrictions placed on the top quark and Higgs boson masses in the standard model [2] due to precision measurements [3].

Non-decoupling effects have also raised an important issue regarding the attempts to formulate chiral theories on the lattice [4]. One of the main problems in this program is the inevitability of the existence of the unwanted fermion doublers as required by the Nielsen–Ninomiya theorem [5]. In some of the approaches to this problem, one generates masses of the order of the cutoff scale of the theory for the unwanted fermions using an effectively Yukawa-like coupling. It has been pointed out that at one loop, this procedure leaves behind non-decoupling effects, so it is unlikely to be equivalent to the model without the unwanted fermions in the low energy theory [4]. (Other possible problems have also been pointed out, some previously [4][7].)

To date, non-decoupling effects have been studied within perturbation theory mostly to one and, on few occasions, to two loops. As the mass of the particle becomes heavier, of course the perturbation theory becomes less reliable. It is, therefore, essential to study these issues non-perturbatively and it is necessary to do so when the mass of the particle is of the order of the cutoff scale. Such a study will enable us to determine how these parameters behave outside the perturbative regime and establish the limits of validity of perturbation theory. Also, there can be, and will be, important qualitative effects that do not arise within perturbation theory, as we shall see.

Let us consider a version of the standard model with spontaneous breakdown of a global $SU(2)_L \times U(1)_Y$ symmetry in which gauge couplings have been turned off. The effective lagrangian for the Nambu–Goldstone bosons with heavy fermions integrated out can be written as (we use the spacelike signature ($-+++$) for the metric)

$$-\mathcal{L}_\chi = \frac{1}{2} Z_3 \left( \partial_\mu \chi^3 \right)^2 + Z_+ \left| \partial_\mu \chi^+ \right|^2 + \text{interactions}$$  \hspace{1cm} (1)

In general this lagrangian will be non-local and $Z_3, Z_+$ will be momentum dependent functions (in momentum space) amenable to a non-perturbative calculation in the Yukawa coupling. Let us define the parameter $\tilde{S}$ by

$$\tilde{S} \equiv -2\pi v^2 \left. \frac{d}{dp^2} Z_3(p^2) \right|_{p^2=0}$$  \hspace{1cm} (2)

where $v$ is the corresponding vacuum expectation value that signals the spontaneous breakdown of the symmetry. Obviously, $\tilde{S}$ is “gauge invariant” in the sense that it cannot depend on any gauge-fixing parameters. To one loop we can compute, for instance, the contribution of a doublet of massive fermions to $\tilde{S}$, and this yields $\tilde{S} = 2 \times 1/(12\pi)$. It is mass-independent and in particular, independent of the amount of the mass splitting. The two Yukawa couplings cancel out in the definition of $\tilde{S}$ because the derivative pulls out
two inverse powers of the fermion mass. So $\tilde{S}$ shows perturbative non–decoupling, and as a matter of fact, it counts the number of heavy fermions that have obtained their masses through the mechanism of spontaneous symmetry breaking.

When we turn the gauge couplings on, the lagrangian of Eq. (1) induces an interaction between the gauge bosons when the Nambu–Goldstone bosons are “eaten”. To lowest order in the gauge couplings $g, g'$ (which are known to be small) this interaction can be expressed as

$$-\mathcal{L}_\chi = \frac{1}{2} Z_3 \left( \partial_\mu \chi^+ - \frac{v}{2} (g W_\mu^3 - g' B_\mu) \right)^2 + Z_3 \left| \partial_\mu \chi^+ - \frac{g v}{2} W_\mu^3 \right|^2 + \text{other terms} \quad (3)$$

where $Z_3, Z_+$ may be non–perturbative in the Yukawa coupling. We then discover that $\tilde{S}$ is the contribution to lowest order in $g, g'$ of the longitudinal part of the gauge bosons to the $S$ parameter as defined by Peskin and Takeuchi \[8\]

$$S \equiv -\frac{16\pi}{gg'} \frac{d}{dp^2} \Pi_{W^3B} \bigg|_{p^2=0} \quad (4)$$

which characterizes the amount of $W^3 - B$ mixing and is a measurable quantity. For instance, the one–loop contribution of a heavy degenerate doublet to $S$ and $\tilde{S}$ are identical, namely $S = \tilde{S} = 1/(6\pi)$ \[9\]. However, $S$ and $\tilde{S}$ are not identical in general since, for instance, in the case of a non–degenerate heavy doublet, $\tilde{S}$ remains the same but $S$ receives an extra logarithmic contribution, $S = \frac{1}{16\pi} (1 - Y_L \log \frac{m^2}{m^2_{B^3}})$, where $Y_L$ is the hypercharge of the left handed doublet and $m_{U,D}$ are the masses of the up and down–type fermions in the doublet.

In this paper, we shall study the non–perturbative behavior of $\tilde{S}$ when the Yukawa coupling (or the fermion mass) becomes very large. We choose $\tilde{S}$ because it has the same perturbative characteristics as $S$ as far as non–decoupling is concerned — which is what we are interested in studying — but allows a much simpler $1/N$–type of non–perturbative treatment than $S$. Moreover, it seems quite reasonable to us that $\tilde{S}$, being determined by the dynamics of the symmetry breaking sector, captures the essence of the non–decoupling phenomena found in the $S$ parameter. After all it is because of the spontaneous symmetry breakdown that, at least perturbatively, non–decoupling occurs.

Apart from the phenomenological interest, non–perturbative aspects of non–decoupling effects in renormalizable quantum field theories are of general importance which should, we believe, be studied when possible. In this regard, amongst the presumably trivial theories, chiral Yukawa theories, such as the model we are considering, necessarily contain non–decoupling effects when fermions are massive, making them a natural setting to study these issues. Also, these systems have been studied on the lattice extensively \[6\] \[7\] \[10\] and we hope that our simple calculation might serve as an useful guide for possible future numerical computations.

In a previous study of the $\rho$ parameter\[11\], it was found that the cutoff effects in the non–perturbative regime saturated the perturbative growth with the Yukawa coupling to a constant, making the behavior milder than what is naively expected from perturbation theory. Therefore, an expectation one might harbor in the case of $\tilde{S}$ is that again cutoff
effects diminish the constant value obtained in perturbation theory, making it vanish as the mass approaches the cutoff. If this were to happen, this would be very welcome for the aforementioned problem of decoupling the fermion doublers on the lattice. From a different perspective, $\hat{S}$ is independent of the fermion mass to one loop so that it is somewhat hard to imagine why $\hat{S}$ should be sensitive to whether the fermion mass is close to the cutoff or not. One perhaps expects that while it may not vanish, it may still be rather insensitive to cutoff effects. However, we find within the $1/N_F$ expansion that as the mass approaches the cutoff, the parameter $\hat{S}$ does not vanish and is cutoff dependent, in other words, it is non-universal.

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The version of the standard model we want to study using the $1/N_F$ expansion has the following lagrangian

$$-\mathcal{L}_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi + \lambda \left( \phi^\dagger \phi - v^2 / 2 \right)^2 + \bar{q}_L \phi q_L + \bar{U}_R \Phi U_R + y \left( \bar{q}_L \phi U_R + \bar{U}_R \phi^\dagger q_L \right)$$

where $\phi$ is in an $N_F$ dimensional irreducible representation of $SU(N_F)$. The scalar field develops a vacuum expectation value $\langle \phi \rangle = (v/\sqrt{2} \ 0 \ 0 \ldots)^T$ that breaks the symmetry of the lagrangian from $U(N_F)$ down to $U(N_F - 1)$ and gives mass to the $U$–fermion. We define the $\chi^0$ and $\chi^-$ of Eq. \(1\) as $\phi = ((v + H + i\chi^0)/\sqrt{2}, i\chi^-, \ldots)^T$ where $\phi$ has $N_F$ components. There are one massive real scalar $H$, with tree–level mass $\sqrt{2} \lambda v$, and $2N_F - 1$ Nambu–Goldstone bosons. Within the scalar sector, $q_L$ and $U_R$ are an $N_F$ and a 1 of $SU(N_F)$, respectively. We can think of the $q_L$ field as $q_L = (U_L D^1_L, D^2_L \ldots D^{N_F-1}_L)^T$.

To study the model non–perturbatively, we use the $1/N_F$ expansion by keeping $y^2N_F$, $\lambda N_F$ and $v^2/N_F$ fixed as we take $N_F$ to infinity. In this limit, the leading quantum corrections only contribute to the propagator for the Higgs field, $H$, and the $U$–fermion. The scalar sector and the fermion sector can be solved independently. Except for a trivial shift, $v$ remains unrenormalized so the remaining renormalizations are only those of $\lambda$ and $y$. We refer the reader to \[12\][13][14] for details. Let us only mention that, to leading order in $1/N_F$, the $U$–propagator reads

$$S_U(p) = \left\{ i\not{\! p} \left[ A_{R,bare}(p^2) P_R + P_L \right] + y_{bare} v \right\}^{-1}, \quad A_{R,bare}(p^2) \equiv 1 - \frac{y_{bare}^2 N_F}{2(4\pi)^2} \ln \frac{p^2}{s_{bare}^U} .$$

where $P_L, P_R$ are projection operators onto the left, right–handed fields and $s_{bare}^U$ denotes a regulator dependent quantity. The Yukawa coupling is renormalized according to

$$y^2(s_0) = \frac{y_{bare}^2}{1 - y_{bare}^2 N_F / (32\pi^2) \ln s_0 / s_{bare}^U}$$

with an arbitrary renormalization scale $s_0$. In this renormalization scheme, the renormalized coupling constant has the physical meaning as the effective coupling constant at the momentum–squared scale $s_0$, measured, for instance, through cross sections. This coupling diverges at a scale $s_{triv}^U \equiv s_0 \exp\{32\pi^2/(y^2(s_0)N_F)\}$ which we identify with the physical
cutoff scale in the theory, the triviality scale. This quantity has the generic form of a non–perturbative effect. The mass, \( m_v \), and the width, \( \Gamma_v \), of the fermion are determined from the location of the pole of the full fermion propagator in the complex plane. For convenience, we choose the renormalization scale at the mass scale, \( s_0 = |m_v - i\Gamma_v/2|^2 \), in what follows. In this convention, since the cross sections need to be finite at least at the scale of the mass of the fermion, \( y^2(s_0) \) has to be finite and positive within the physical region. In fig. \( \text{[1]} \) we show the fermion mass and width as well as the triviality scale as a function of the Yukawa coupling \( y^2(s_0) \) evaluated at a scale \( s_0 \). The mass of the fermion is smaller than \( 5.0\sqrt{v^2/N_F} \) when the coupling constant \( y^2(s_0) \) is positive and finite.

This \( 1/N_F \) generalization of the standard model is largely dictated by simplicity. For instance, it would be of interest to study also the case where custodial symmetry is unbroken, perhaps using the large–\( N \) limit of \( \text{[1]} \). However, the model in this case seems substantially more complicated.

The leading order corrections to the two point function of the neutral Nambu–Goldstone boson, \( \Pi_{\chi^3} \), arise from the class of one–particle irreducible graphs in fig. \( \text{[2]} \) and is of the order \( \mathcal{O}(1/N_F) \). The contribution of a fermion multiplet to \( \Pi_{\chi^3} \) may be computed as \( \text{[1]} \)

\[
\Pi_{\chi^3}(p^2) = 2y^2(s_0) \int \frac{d^4k}{(2\pi)^4} \frac{A_R((k+p)^2)k(k+p) + \hat{m}_v^2}{[A_R(k^2)k^2 + \hat{m}_v^2][A_R((k+p)^2)(k+p)^2 + \hat{m}_v^2]} \tag{8}
\]

where \( \hat{m}_v^2 \equiv y^2(s_0)v^2/2 \) and \( A_R(s) \equiv 1 - y^2(s_0)N_F/(32\pi^2)\ln s/s_0 \). The wave function renormalization factor \( Z_3 \) in \( \text{[1]} \) is related to this contribution as

\[
Z_3(p^2) = 1 - \frac{d}{dp^2}\Pi_{\chi^3}(p^2) \tag{9}
\]

Using this relation and the definition of \( \tilde{S} \) in \( \text{[2]} \), we obtain the following expression (in euclidean space) after some algebra:

\[
\tilde{S} = \frac{16\pi\hat{m}_v^4}{3} \int_{k^2 < \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{\mathcal{N}}{k^2(A_R(k^2)k^2 + \hat{m}_v^2)^5} \tag{10}
\]

with

\[
\mathcal{N} \equiv \alpha_y (k^2)^2 \left[A_R^2(k^2) - 3\alpha_y A_R(k^2) + 3\alpha_y^2\right] + \hat{m}_v^2 k^2 \left[3A_R^2(k^2) - 7\alpha_y A_R(k^2) + 6\alpha_y^2\right] + \alpha_y \hat{m}_v^4
\]

(11)

where we used the shorthand \( \alpha_y \equiv y^2(s_0)N_F/(32\pi^2) \). In the above expression, there is a pole in the integrand above the triviality scale so that the integral is ill–defined unless we restrict the integration region. The pole is always larger than the triviality scale so that we cutoff the integral at a scale \( \Lambda^2 \) below \( s_{\text{triv}} \), which is consistent with the existence of the intrinsic cutoff scale \( s_{\text{triv}} \) in the theory. This is how the physical cutoff comes to play the active role that one naturally expects and that is always missed in any perturbative treatment. The integral \( \text{[10]} \) may be computed after some work to be

\[
\tilde{S} = \frac{1}{12\pi} \left[ 1 + \frac{x_\lambda^2\alpha_y(-2A_R(\Lambda^2) + 3\alpha_y) + 4x_\lambda(-A_R(\Lambda^2) + \alpha_y) - 1}{(A_R(\Lambda^2)x_\lambda + 1)^4} \right] \text{ where } x_\lambda \equiv \frac{\Lambda^2}{\hat{m}_v^2}
\]

(12)
If we expand this expression for $\tilde{S}$ in powers of the coupling constant as we would in perturbation theory, the need to restrict the integration region disappears. The truly remarkable fact regarding this parameter in this case is that to all orders in perturbation theory, this parameter $\tilde{S}$ is $1/(12\pi)$ and is independent of the Yukawa coupling, or equivalently the fermion mass, to leading order in the $1/N_F$ expansion; in other words, all the higher order terms in the expansion for $\tilde{S}$ in (10) surprisingly cancel. In fact, it is clear from (12) that $\tilde{S}$ reduces to its constant value in the limit cutoff goes to infinity. The above expressions for $\tilde{S}$ in (10) or (12) include contributions from one–particle irreducible graphs of arbitrary high order (cf. fig. 2) and these contributions are ultimately crucial, so that this is not a trivial fact. The dependence of $\tilde{S}$ on the mass of the fermion, then, comes solely from the necessity of imposing the cutoff in the theory, which makes this parameter an ideal setting for investigating the physical effects of the triviality cutoff.

We may compute the parameter numerically and our results are plotted in fig. 3 and fig. 4 against the renormalized coupling constant $y^2(s_0)$ and the mass of the fermion $m_U$, respectively for a few cutoff values, $\Lambda/\sqrt{s_{\text{triv}}} = 0.1, 0.5$ and 0.8. As the Yukawa coupling grows, the cutoff $\Lambda$ decreases and eventually the physical fermion mass would be larger than the cutoff. We have plotted only the region where the fermion mass is smaller than the corresponding cutoff, $\Lambda$. All calculations agree in the perturbative regime. As the mass approaches the cutoff scale, the results depend on the cutoff scale and deviate from the perturbative result. As the mass increases to $m_U = 3.12v$, the $\tilde{S}$ parameter computed with $\Lambda/\sqrt{s_{\text{triv}}} = 0.8$ differs 1% from the perturbative result, at which point, $\sqrt{s_{\text{triv}}} = 90v$, $\Gamma_U/m_U = 0.51$ and $y^2(s_0) = 23.5$. The maximum mass of the theory in the large–$N_F$ limit is $3.52v$ so that the deviations from the perturbative result are appreciable only when the mass is close to its maximum value. As we can see, the contribution to $\tilde{S}$ does not vanish within the physical region defined by $m_U < \Lambda$, although there is an apparent decreasing trend at large couplings that is stronger for low values of the cutoff. If there is a way to make sense of the region $m_U > \Lambda$ in some framework, whether $\tilde{S}$ can vanish in this region might deserve some further investigation.

In closing, we point out that this contribution to $\tilde{S}$ can be understood as the effect of operators of dimension eight or higher in the effective scalar theory. At dimension eight, there is effectively only one operator, $\mathcal{O} \sim \phi D\phi D\phi \phi \phi$. The first constant term in (12) is generated by an operator like $\mathcal{O}/v^4$ and the cutoff dependent terms are generated by $\mathcal{O}/\Lambda^4$, in both cases, up to higher dimension operators. The former does not fall off with the cutoff and is a perturbatively relevant, however a cutoff independent contribution. The latter is a cutoff dependent but a perturbatively irrelevant contribution. The sole reason this term is not negligible is because the cutoff scale cannot be taken to infinity since it needs to be smaller than the triviality scale.

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Figure Captions

Fig. 1. The plot of the triviality scale $s_{\text{triv}}$, the mass, $m_U$, and the width, $\Gamma_U$, against the renormalized coupling constant $y^2(s_0)$.

Fig. 2. The class of one-particle irreducible graphs contributing to the propagator of the neutral Nambu–Goldstone boson. Dashed and solid lines represent Nambu–Goldstone bosons and fermions, respectively.

Fig. 3. $12\pi\tilde{S}$ plotted against the renormalized coupling constant $y^2(s_0)$ for the values of $\Lambda/\sqrt{s_{\text{triv}}} = 0.1, 0.5$ and $0.8$, which we call "cutoff" in the plot.

Fig. 4. $12\pi\tilde{S}$ plotted against $m_U/v$ for the values of $\Lambda/\sqrt{s_{\text{triv}}} = 0.1, 0.5$ and $0.8$, which we call "cutoff" in the plot.