QED radiative corrections to parity nonconservation in heavy atoms

M.Yu.Kuchiev * and V.V. Flambaum †
School of Physics, University of New South Wales, Sydney 2052, Australia

Abstract

The self-energy and vertex QED radiative corrections (∼ Zα^2f(Zα)) are shown to give a large negative contribution to the parity nonconserving (PNC) amplitude in heavy atoms. The correction −0.7(2)% found for the 6s − 7s PNC amplitude in ^133Cs brings the experimental result for this transition into agreement with the standard model. The calculations are based on a new identity that expresses the radiative corrections to the PNC matrix element via corrections to the energy shifts induced by the finite nuclear size.

32.80.Ys, 11.30.Er, 31.30.Jv

*kuchiev@newt.phys.unsw.edu.au
†flambaum@newt.phys.unsw.edu.au
It has been discovered recently that there exists a consistent deviation of experimental data on parity nonconservation (PNC) in atoms from predictions of the standard model. This paper demonstrates that this contradiction is removed by the self-energy and vertex QED radiative corrections, which prove to be much larger than anticipated. The corrections are evaluated with the help of a new identity that expresses them via similar radiative corrections to the energy shifts induced by the finite nuclear size.

Experimental investigation of the $6s - 7s$ PNC amplitude in $^{133}$Cs initiated by Bouchiat and Bouchiat [1], was carried further by Gilbert and Wieman [2], and by Wood et al [3] who reduced the error to 0.3%, sparking an interest in the atomic PNC calculations that are crucial for the analysis of the experimental data. Accurate previous calculations of Refs. [4,20] have recently been revisited by Kozlov et al [6] and Dzuba et al [7,8]. Bennett and Wieman [9] analyzed the theoretical data [4,20], comparing it with available experimental data on dipole amplitudes, polarizabilities and hyperfine constants for Cs, and suggested that the theoretical error for the PNC amplitude should be reduced from 1% to 0.4%. It has been recognized recently that several, previously neglected phenomena contribute at the required level of accuracy. Derevianko [10] found that the Breit corrections give $-0.6\%$, the value confirmed in [11,6]. Sushkov [12] pointed out that the radiative corrections may be comparable with the Breit corrections. The calculations of Johnson et al [13] demonstrated that indeed, the QED vacuum polarization gives $0.4\%$, the value confirmed in [14,7,15].

Ref. [9] indicates that there is a $2.3\sigma$ deviation of the weak charge $Q_W$ extracted from the atomic PNC amplitude [3] from predictions of the standard model [16]. More recent works [13,8], in which the Breit corrections ($-0.6\%$) and the QED vacuum polarization ($0.4\%$) were included, give similar deviations $2.2\sigma$ and $2.0\sigma$ respectively. We show that this contradiction is removed by the self-energy and vertex radiative corrections. The corrections of this type were considered previously by Marciano and Sirlin [17] and Lynn and Sandars [18] using the plane wave approximation that resulted in a small value $\sim 0.1\%$. The expectation of Ref. [13] is that the Coulomb field of the atomic nucleus should not produce any drastic effect on these radiative corrections, which should remain small. This assessment is supported by Ref. [14] that mentions in passing preliminary results of calculations indicating that the self-energy and vertex corrections altogether are small. The opposite conclusion of Ref. [7], namely that the self-energy correction may give a substantial contribution, was not decisive, because the model approach pursued in this work was not gauge invariant.

Let us show that there is a precise relation that expresses the QED radiative corrections to the PNC matrix element via similar radiative corrections to the energy shifts of the atomic electron induced by the finite nuclear size (FNS). This relation can be presented as

$$\delta_{\text{PNC,sp}} = \frac{1}{2} (\delta_{\text{FNS,s}} + \delta_{\text{FNS,p}}) ,$$

where $\delta_{\text{PNC,sp}}$ is the relative radiative correction to the PNC matrix element between $s_{1/2}$ and $p_{1/2}$ orbitals. The term relative correction used above indicates that the correction is divided by the matrix element itself

$$\delta_{\text{PNC,sp}} = \frac{\langle \psi_{s,1/2} | H_{\text{PNC}} | \psi_{p,1/2} \rangle_{\text{rad}}}{\langle \psi_{s,1/2} | H_{\text{PNC}} | \psi_{p,1/2} \rangle} ,$$

Similarly $\delta_{\text{FNS,s}}$ and $\delta_{\text{FNS,p}}$ are the relative radiative corrections to the FNS energy shifts $E_{\text{FNS,s}}$, $E_{\text{FNS,p}}$, for the the chosen $s_{1/2}$ and $p_{1/2}$ electron states.
The operator \( H_{\text{PNC}} \) in (2) describes the PNC part of the electron Hamiltonian induced by the Z-boson exchange

\[
H_{\text{PNC}} = (2\sqrt{2})^{-1} G_F Q_W \rho(r) \gamma_5.
\]  

(4)

Here \( G_F \) and \( Q_W \) are the Fermi constant and the nuclear weak charge, and \( \rho(r) \) is the nuclear density. The FNS energy shifts can be presented as matrix elements of the potential \( \delta V_{\text{FNS}}(r) \), which describes the deviation of the nuclear potential from the pure Coulomb one, \( E_{\text{FNS},l} = \langle \psi_{1,1/2}|\delta V_{\text{FNS}}|\psi_{1,1/2}\rangle \), \( l = s, p \). Equality (3) may be established for the sum of all QED radiative corrections (\( \sim Z^2 f(Z\alpha) \)), or specified for any gauge invariant class of them. We concentrate our attention on the self-energy and vertex corrections in the lowest order of the perturbation theory described by the Feynman diagrams in Fig. 1, calling them the e-line corrections, though the vacuum polarization is also briefly discussed below. The vertex in diagrams of Fig. 1(a) originates from the PNC Hamiltonian (4), the left and right external legs describe the wave functions \( \psi_{s,1/2}(r) \) and \( \psi_{p,1/2}(r) \) for the considered atomic states. The sum of these diagrams is gauge invariant, though each one of them is not. We can use this fact for our advantage, choosing a gauge in which the vertex correction Fig. 1(a) is zero. To see that this is possible we can consider, for example, the gauge in which the photon propagator is \( D^{\mu\nu}(k) = (g^{\mu\nu} - \xi f(k)k^\mu k^\nu)/k^2 \), where \( f(k) \) is some nonzero, nonsingular function of \( k \), and tune one parameter \( \xi \) to annihilate the vertex correction for the chosen pair of atomic \( s_{1/2}, p_{1/2} \) levels. In this gauge only the self-energy corrections Fig. 1(b,c) contribute to the PNC transition between this pair of states.

The PNC Hamiltonian (2) is localized inside the nuclear interior \( r \leq R \), where \( R \) is the nuclear radius. In contrast, the radiative corrections take place at separations comparable with the Compton radius \( r \sim m^{-1} \) which is much bigger than the nucleus, \( mR \ll 1 \) (relativistic units \( \hbar = c = 1 \) are used, if not stated otherwise). This difference of the two scales allows us to simplify the problem. Consider the diagram Fig. 1(b) in which the initial wave function (the left leg) is \( \psi_{s,1/2}(r) \). The radiative correction induced by the self-energy operator results in a variation of this wave function \( \delta_{\text{rad}}\psi_{s,1/2}(r) \) which we need to evaluate at the nucleus, where the weak interaction takes place. In this region the shape of the function does not change, because the perturbation caused by the radiative correction is localized far away. Thus, inside the nucleus \( \delta_{\text{rad}}\psi_{s,1/2}(r) = C_{s}^{\text{rad}}\psi_{s,1/2}(r) \), where \( C_{s}^{\text{rad}} \) is an \( r \)-independent factor. Similarly, for the right leg (diagram Fig. 1(c)) \( \delta_{\text{rad}}\psi_{p,1/2}(r) = C_{p}^{\text{rad}}\psi_{p,1/2}(r) \). Using these variations of the wave function we express the relative radiative correction to the PNC matrix element (2) in terms of the factors \( C_{s}^{\text{rad}}, C_{p}^{\text{rad}} \)

\[
\delta_{\text{PNC,sp}} = C_{s}^{\text{rad}} + C_{p}^{\text{rad}}.
\]  

(5)

We took into account here that in the chosen gauge \( \langle \psi_{s,1/2}|H_{\text{PNC}}|\psi_{p,1/2}\rangle_{\text{rad}} = \langle \delta_{\text{rad}}\psi_{s,1/2}|H_{\text{PNC}}|\psi_{p,1/2}\rangle + \langle \psi_{s,1/2}|H_{\text{PNC}}|\delta_{\text{rad}}\psi_{p,1/2}\rangle \).

Let us discuss now the e-line radiative corrections to the FNS energy shift for the \( s_{1/2} \) level that are described by the same Feynman diagrams in Fig. 1 in which the vertex is given by deviation of the potential from the pure Coulomb one \( \delta V_{\text{FNS}}(r) \). We can again choose the gauge in which the vertex correction is zero for the given \( s_{1/2} \) state. Moreover, we can assume that the vertex radiative corrections are zero simultaneously for the FNS
energy shift and for the PNC matrix element (for the chosen pair of \(s_1/2, p_1/2\) states). This is possible because gauge transformations include an infinite number of parameters. The gauge \(D^{\mu \nu}(k) = (g^{\mu \nu} - f(k)k^\mu k^\nu)/k^2\) presents them via an arbitrary function \(f(k)\) that can be chosen to satisfy the conditions formulated above. In this gauge the radiative correction to \(E_{\text{FNS}, s} = \langle \psi_{s,1/2}\rangle \delta V_{\text{FNS}} |\psi_{s,1/2}\rangle\) is expressed via the variation of the wave function \(\delta^{\text{rad}}\psi_{s,1/2}(r)\), which is essential only inside the nucleus where \(\delta V_{\text{FNS}}(r)\) is located. Thus, using arguments similar to the ones that led us to (3), we find the relative correction to the FNS energy shift

\[
\delta_{\text{FNS}, s} = 2C_{\text{rad}}^s,
\]

where the factor 2 accounts for two diagrams (b) and (c) in Fig.4 that give identical contributions. Similarly for the \(p_{1/2}\) level,

\[
\delta_{\text{FNS}, p} = 2C_{\text{rad}}^p.
\]

The factors \(C_{\text{rad}}^s, C_{\text{rad}}^p\) are determined by the radiative corrections, being independent on the nature of a perturbative operator localized on the nucleus. They have, therefore, the same values for the PNC matrix element (5) and FNS energy shifts (6),(7). Combining these equations we immediately derive Eq. (1). The only parameter which governs the accuracy of the presented derivation is the smallness of the nucleus compared with the Compton radius. This makes Eq. (1) a very accurate identity. Similarly we derive this identity for the contribution of the QED vacuum polarization. The result can be compared with Ref. [15] that presents explicit variations of \(s_{1/2}\) and \(p_{1/2}\) wave functions at the origin induced by the vacuum polarization (see Eq.(43) of Ref. [15]). Using these wave functions to calculate corrections to the PNC matrix element and FNS energy shifts, we find perfect agreement with Eq.(1).

Note that we do not consider here radiative corrections of the order \(\sim \alpha/\pi\) which appear in the plane wave approximation. These contributions have been included into the radiative corrections to the weak charge \(Q_W\) (and the renormalization of the charge and electron mass in the case of FNS energy shifts). Correspondingly, we subtract the contribution of the plane waves from Eq.(1), considering only the part of the corrections that depends on the atomic potential \(\sim Z^2\alpha f(Z\alpha)\). For heavy atoms this subtlety is insignificant numerically because the considered \(Z\)-dependent part of the correction is bigger than the omitted \(Z\)-independent one, as we will see below.

Eq. (1) presents the e-line corrections to the PNC matrix element, which are difficult to calculate, in terms of the corrections to the FNS energy shifts that have been well-studied both numerically, by Johnson and Soff [14], Blundell [20], Cheng et al [21] and Lindgren et al [22], and analytically, by Pachucki [23] and Eides and Grotch [24]. Ref. [21] presents the e-line radiative corrections to the FNS energy shifts for \(1s_{1/2}, 2s_{1/2}\) and \(2p_{1/2}\) levels in hydrogenlike ions with atomic charges \(Z = 60, 70, 80, 90\). Eq. (1) contains relative corrections, therefore we need to calculate the FNS energy shifts \(E_{\text{FNS}}\). We did this by solving the Dirac equation with the conventional Fermi-type nuclear distribution \(\rho(r) = \rho_0/\{1 + \exp[(r-a)/c]\}\). Parameters \(a, c\) were taken the same as in [21], namely \(a = 0.523\) fm and \(c\) chosen to satisfy \(R_{\text{rms}} = 0.836A^{1/3} + 0.570\) fm. Using the results of [21] and this calculation we obtained the relative radiative corrections shown in Fig. 4. In order to include the interesting case \(Z = 55\) and to account for all values of \(55 \leq Z \leq 90\) we used interpolation formulae presented in [21]. The relative corrections for the
1s and 2s levels are roughly the same size. This indicates that the radiative processes responsible for the correction take place at separations smaller than the K-shell radius, \( r < Z \alpha m^{-1} \), which is consistent with the assumption \( r \sim m^{-1} \) above. For these separations we can assume that, firstly, the screening of the nuclear Coulomb field in many-electron atoms does not produce any significant effect, and, secondly, the relative corrections does not depend on the atomic energy level because for small separations all atomic \( ns_1/2 \)-wave functions exhibit similar behaviour. These arguments remain valid for the \( p_{1/2} \) states as well, permitting us to presume that the results shown in Fig. 2 for the \( 2s_1/2 \)-levels and \( 2p_{1/2} \)-levels of hydrogenlike ions remain valid for \( s_{1/2} \) and \( p_{1/2} \) states of the valence electron in a many-electron atom. We obtain the e-line radiative corrections for the PNC matrix element using Eq. (1) that expresses them via the found corrections to the FNS energy shifts. The found PNC corrections, presented in Fig. 2 by the dotted line, are negative and large (much larger than the neglected \( Z \)-independent part of the corrections).

Let us discuss the implications for the \( 6s-7s \) PNC amplitude in \(^{133}\)Cs. The standard model value for the nuclear weak charge for Cs \([16]\) is

\[
Q_W(^{133}\text{Cs}) = -73.09 \pm 0.03 \ .
\]  

Ref. \([8]\) refined previous calculations of Ref. \([4]\) extracting from the experimental PNC amplitude of Ref. \([3]\) the weak charge

\[
Q_W(^{133}\text{Cs}) = -72.18 \pm (0.29)_{\text{expt}} \pm (0.36)_{\text{theor}} \ ,
\]  

with the theoretical error 0.5%. It is consistent with \( Q_W(^{133}\text{Cs}) = -72.21 \pm (0.28)_{\text{expt}} \pm (0.34)_{\text{theor}} \) that was adopted in \([13]\) by taking the average of the results of Refs. \([4,20,6]\), and accepting the theoretical error 0.4% proposed in \([4]\). The weak charge in Eq. (9) deviates from the standard model (8) by \( 2.0 \sigma \).

The e-line radiative correction derived from results presented in Fig. 2 is \(-0.73\pm(0.20)\%\), the error reflects the uncertainty of the radiative corrections to the FNS energy shift for the \( 2p_{1/2} \) level in Cs \( E_{\text{FNS},2p_{1/2}} = -0.0001(1) \) eV \([27]\) (more accurate value can, probably, be obtained by extrapolation of much more reliable higher-\( Z \) results shown in Fig. 2). Eq. (9) combined with the e-line correction gives

\[
Q_W(^{133}\text{Cs}) = -72.71 \pm (0.29)_{\text{expt}} \pm (0.39)_{\text{theor}} \ ,
\]  

which brings the PNC experimental amplitude of \([3]\) within the limits of the standard model (8). For heavier atoms the e-line corrections become larger, while the error diminishes. For the Tl atom, which presents another interesting for applications case, we find the e-line correction \(-1.61\%\).

Relations similar to (1) can be derived for any operator which is localized at distances smaller than the Compton radius. One can even try to apply it to the case of the hyperfine interaction (HFI), which has been thoroughly investigated previously, see e.g. \([25,26]\) and references therein, though the HFI has a long-range tail \( \sim 1/r^3 \) that presents an obstacle for our method. However, if convergence of the HFI matrix elements is fast, the relation \( \delta_{\text{FNS},s} \approx \delta'_{\text{HFI},s} \) should hold. Here \( \delta'_{\text{HFI},s} \) is the radiative correction to the HFI for \( s \)-levels, the primed notation indicates that the \( Z \)-independent Schwinger term \( \alpha/(2\pi) \) should be excluded (for heavy atoms this subtlety is not important.) Fig. 3 shows the e-line contribution...
to $\delta_{\text{HFI}, s}'$ that was extracted from [23] using interpolation for all considered values of $Z$. It agrees semi-quantitatively with $\delta_{\text{FNS}, s}$, deviation is less than 33 %. Overall, we observe that $\delta_{\text{FNS}, 1s}, \delta_{\text{FNS}, 2s}, \delta_{\text{FNS}, 2p}$, and $\delta_{\text{HFI}, s}'$ all exhibit similar behaviour, they are negative and large regardless of the perturbative operator considered and quantum numbers of the wave functions involved, which is in line with the main arguments of this paper.

In conclusion, large QED self-energy and vertex corrections to the parity nonconservation amplitude in heavy atoms reconcile the experimental results of Wood et al [3] in Cs with the standard model.

This work was supported by the Australian Research Council. The authors are thankful to K.T. Cheng for his calculations of the self-energy corrections to the FNS energy shifts in Cs Ref. [27].
REFERENCES

[1] M.A.Bouchiat and C.Bouchiat J.Phys. (Paris) 35, 899 (1974); 36, 493 (1974).
[2] S.L.Gilbert and C.E.Wieman, Phys.Rev. A 34, 792 (1986).
[3] C.S.Wood, S.C.Bennett, D.Cho, B.P.Masterson, J.L.Roberts, C.E.Tanner, and C.E.Wieman, Science 275, 1759 (1997).
[4] V.A.Dzuba, V.V.Flambaum, and O.P.Sushkov, Phys. Lett A 141, 147 (1989).
[5] S.A.Blundell, J.Sapirstein, and W.R.Johnson, Phys. Rev. D 45, 1602 (1992).
[6] M.G.Kozlov, S.G.Porsev, and I.I.Tupitsyn, Phys. Rev. Lett. 86, 3260 (2001).
[7] V.A.Dzuba, V.V.Flambaum, and J.S.M. Ginges, hep-ph/0111019 (2001).
[8] V.A.Dzuba, V.V.Flambaum, and J.S.M. Ginges, hep-ph/0204134 (2002).
[9] S.C.Bennett and C.E.Wieman, Phys. Rev. Lett. 82, 2484 (1999); 82, 4153 (1999); 83, 889 (1999).
[10] A.Derevianko, Phys. Rev. Lett. 85, 1618 (2000).
[11] V.A.Dzuba, C.Harabati, W.R.Johnson, and M.S.Safronova, Phys. Rev A 63, 044103 (2001).
[12] O.P.Sushkov, Phys. Rev. A, 63, 042504 (2001).
[13] W.R.Johnson, I.Bednyakov, and G.Soff, Phys. Rev. Lett 87, 233001-1 (2001).
[14] A.I.Milstein and O.P.Sushkov, hep-ph/0109257 (2001).
[15] M.Yu.Kuchiev and V.V.Flambaum, hep-ph/0205012 (2002).
[16] D.E.Groom et al, Eur. Phys. J. C 15, 1 (2000).
[17] W.J.Marciano and A.Sirlin, Phys. Rev. D 27, 552 (1983).
[18] B.W.Lynn and P.G.H.Sandars, J. Phys. B 27, 1469 (1994).
[19] W.R.Johnson and G.Soff, At. Data Nuc. Data Tables 33, 405 (1985).
[20] S.A.Blundell, Phys. Rev. A 46, 3762 (1992).
[21] K.T.Cheng, W.R.Johnson and J.Sapirstein, Phys. Rev A 47, 1817 (1993).
[22] I.Lindgren, H.Persson, S.Salomonson, and A.Ynnerman, Phys. Rev. A 47, 4555 (1993).
[23] K.Pachucki, Phys. Rev. A 48, 120 (1993).
[24] M.I.Eides and H.Grotch, Phys. Rev. A 56, R2507 (1997).
[25] S.A.Blundell, K.T.Cheng, and J.Sapirstein, Phys. Rev. A 55, 1857 (1997).
[26] P.Sunnergren, H.Persson, S.Salomonson, S.M.Sneider, I.Lindgren, and G.Soff, Phys. Rev. A 58, 1055 (1998).
[27] K.T.Cheng. Private communication (2002).
FIGURES

FIG. 1. The QED vertex (a) and self-energy (b), (c) corrections to the PNC matrix element and the FNS energy shifts. For the electron-nucleus PNC interaction the vertex is given in Eq. (4), for the FNS correction the vertex is produced by the short-range potential that describes the spreading of the nuclear charge.

FIG. 2. The relative radiative corrections (in %) induced by the diagramms of Fig.1. Corrections to the FNS energy shifts for $1s_{1/2}, 2s_{1/2},$ and $2p_{1/2}$ levels extracted from [21] as discussed in the text are shown by thick, thin, and dashed lines. Dotted line - predictions of Eq.(1) for the radiative corrections to the PNC matrix element. Dashed-dotted line - relative correction to the HFI taken from [25].
Fig. 1
Fig. 2