Estimating gravity acceleration from an atomic gravimeter by Kalman filtering

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Abstract – We present the construction of a two-state model of the atomic gravimeter and the associated Kalman recursion to estimate gravity acceleration from an atomic gravimeter. It is found that the Kalman estimator greatly improves the estimation precision in the short term by removing the white phase noise. The residual noise of the estimates follows \(0.13 \mu \text{Gal}/\sqrt{\text{sf or m or e}}\) than 100s and highlights a precision of 0.34 \(\mu \text{Gal}\) at the measuring time of a single sample, even with no seismometer correction.

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Introduction. – Atom interferometry realizes a versatile tool that offers precise and accurate measurement for inertial sensing [1,2], which greatly complements the state-of-the-art classical instruments [3–12]. Among these inertial sensors, atomic gravimeters [3–7,13] are of great interest for a wide range of essential applications, from geophysics to fundamental physics [14].

When monitoring the waveform\(^1\) of gravity acceleration, the atomic gravimeter reading fluctuates due to various measurement noises including vibration noise, phase noise of Raman lasers, detection noise, and ultimately the quantum projection noise [15]. Experimentally, after implementing a combination of vibration compensation [15–17], phase locking [18,19], and efficient detection scheme [20], the residual measurement noise of atomic gravimetry is dominated by the white phase noise in the short term and demonstrates the corresponding \(\tau^{-1/2}\) relations over the measuring time \(\tau\) [3–5,21–23]. For static atomic gravimetry, the white phase noise is a challenging obstacle for improving the short-term sensitivity [15].

A statistical problem in estimating gravity acceleration from a static atomic gravimeter is waveform estimation [24]. Currently, the solution most commonly used by the community is the average method, which applies long-term integration to achieve high precision at a long measuring time, but the short-term sensitivity is not improved [3,4,21–23]. However, since the average method depends on no statistical model of the atomic gravimeter\(^2\) and is non-real-time, if the period of the phenomena is shorter than the measuring time, some useful information of gravity variation would be distorted or filtered out. Therefore, a quasi-real-time estimation method rooted in the physics of the atomic gravimeter can benefit short-term performance and save static gravimetry from the trade-off between precision and information loss. Such a method significantly filters out the measurement noise, and keeps the useful information of gravity variation in the short term.

For the waveform estimation problem of the system driven by white Gaussian processes and observed with white Gaussian noise, the Kalman recursion is an optimal estimator with the minimum mean square error. It provides quasi-real-time and causal estimation [25,26], and its application [27–29] to atom interferometry benefits inertial sensing applications [30–32]. Despite the promising

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\(^{1}\)“Waveform” stands for the time-varying state of a system, e.g., the time-varying gravity acceleration to be estimated from the gravimeter readings.

\(^{2}\)Note that, in some cases, depending on no statistical model is also an advantage of the average method, since no model means no bias.
expectations, a satisfactory implementation for static atomic gravimetry has not been found, because the statistical model of the static atomic gravimeter is less studied.

Thus, this work presents the construction of a two-state statistical model of the static atomic gravimeter and the associated Kalman recursion, and demonstrates a precise estimation (which is also intrinsically quasi-real-time) of the gravity acceleration. It is found that the Kalman estimator greatly improves the precision of estimates in the short term by removing the white phase noise. The residual acceleration can be written as

$$ y(t) = w_1(t) + \xi_2(t), $$

(2)

where $w_1(t) \sim \mathcal{N}(0, Q_1)$ denotes a white Gaussian noise, and $\xi_2(t)$ denotes an accumulated phase error for the random walk nature of the residual acceleration,

$$ \xi_2(t) = \int_0^t w_2(t')dt', $$

(3)

with the random process $w_2(t) \sim \mathcal{N}(0, Q_2)$ also being Gaussian.

The two-state model of the atomic gravimeter.

First, the apparatus is briefly described. The atomic gravimeter is similar to the one described in ref. [33], except that no seismometer correction is used. The test mass is a free-falling cloud of rubidium 87 atoms. The atoms are initially trapped and cooled by a three-dimensional magneto-optical trap to 3.7 $\mu$K. After state preparation and velocity selection, $N \sim 10^6$ atoms are selected to participate in the interferometry that is composed of three Raman pulses separated by two equal time intervals $T = 82$ ms. After the interferometer sequence, we acquire transition probabilities from both the top (0) and each side ($\pm \frac{1}{2}$) of the central interference fringe (3 shots for each phase modulation), and the measured acceleration is determined by the mid-fringe protocol. The direction of the momentum transfer $k_{\text{off}}$ of Raman transitions is reversed every 9 shots to reject direction-independent systematic errors. For every sampling time of $T_s = 5.7$ s, the interferometer cycles 18 shots, and one gravity acceleration $g(t)$ is read out. Compared to the typical systems required for an atomic gravimeter [2], this apparatus is distinguished by implementing no vibration compensation system. As shown below, the two-state statistical model is constructed to precisely estimate the gravity acceleration from noisy gravimeter readings.

The gravimeter reading fluctuates due to various measurement noises including vibration noise, phase noise of Raman lasers, detection noise, and ultimately the quantum projection noise [15]. The fundamental quantity used to characterize the noise on the gravimeter is the residual acceleration, which is defined as the difference between the gravimeter reading and the true waveform of the gravity acceleration,

$$ y(t) = g(t) - g_0(t), $$

(1)

where $y(t)$ is referred to as gravimeter noise, due to its stochastic nature.

Though describing the true state of the system in the waveform estimation problem, the state model does not resolve the state with unlimited precision due to some physical limits. Here, based on the physics of atom interferometry, the noise contribution to the state stems from the quantum projection noise. The quantum projection noise disturbs the state at each time $t$ by $w_1(t)$, and generates phase error $w_2(t)$ between states at adjacent times. Therefore, the state model is constructed based on the noise characteristics originated in the quantum projection noise, and the residual acceleration can be written as

$$ y(t) = w_1(t) + \xi_2(t), $$

(2)

where $w_1(t) \sim \mathcal{N}(0, Q_1)$ denotes a white Gaussian noise, and $\xi_2(t)$ denotes an accumulated phase error for the random walk nature of the residual acceleration,

$$ \xi_2(t) = \int_0^t w_2(t')dt', $$

(3)

with the random process $w_2(t) \sim \mathcal{N}(0, Q_2)$ also being Gaussian.

The two-state model of the atomic gravimeter. –

The state of the gravimeter is modeled with a two-dimensional state vector

$$ X(t) = \begin{bmatrix} X_{1,1}(t) \\ X_{2,1}(t) \end{bmatrix}, $$

(4)

where the auxiliary quantity $X_{1,1}(t)$ is introduced as an integration of the gravimeter reading

$$ X_{1,1}(t) = \int_0^t g(t')dt', $$

(5)

and $X_{2,1}(t) = \xi_2(t)$. With $X(t)$, the two-state gravimeter model can be written as

$$ \frac{dX(t)}{dt} = FX(t) + Bu(t) + W(t), $$

(6)

where

$$ F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, $$

$$ u(t) = g_0(t), \quad W(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}. $$

$F$ is the state transition matrix, $B$ is the control matrix, $u(t)$ is the control input, and $W(t)$ represents the random process. Though the gravity acceleration is time-varying due to gravimetric Earth tides, atmospheric and polar motion effects, and other phenomena, the periods of the waveform are much longer than the time interval between two readout [34–36]. Thus, $g_0(t)$ can be reasonably treated as cyclostationary.

Now, eq. (6) is discretized based on an integral approximation method, and a discrete-time version of the two-state model along the time axis $t(n) = nT_s + t(0)$ is derived, which can be used to perform Kalman recursion,

$$ X(n) = F_d(T_s)X(n-1) + B_d u_d(n) + W_d(n), $$

(7)

where

$$ F_d(T_s) = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}, \quad B_d = \begin{bmatrix} T_s \\ 0 \end{bmatrix}, \quad u_d(n) = g_0(n), $$

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and
\[ W_d(n) = \int_{(n-1)}^{(n)} F_d[t(n) - t'] W(t')dt', \]
with the associated covariance matrix \( Q \) given by
\[ Q = E[W_d(n)W_d(n)^T] = \begin{bmatrix} Q_1 T_s + Q_2 T_s^3 & Q_2 T_s^2 \\ Q_2 T_s^2 & Q_2 T_s \end{bmatrix}. \]

The subscript “d” indicates the discrete-time version of the variable.

When observations are taken, the readout of the state is limited by the measurement noise. This gives an observation vector \( Z(n) \) and a discrete-time observation equation of the form
\[ Z(n) = H X(n) + V(n), \]
where the measurement matrix \( H \) gives the connection between the measurement and the state vector,
\[ H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \]
and \( V(n) \) describes the noise contribution to the measurement of \( X_{1,1}(n) = T_s \sum_{m=0}^{n} g(m) \). Each gravimeter reading to be summed \( g(m) \) makes an independent measurement, and the noise contribution to \( g(m) \) is Gaussian with a zero mean and a variance of \( R \). Thus, \( V(n) \) also represents a Gaussian noise with a zero mean and the variance defined by summing \( R \) of \( n + 1 \) independent \( g(m) \), i.e., \( V(n) \sim \mathcal{N}(0, (n+1)RT_s^2) \).

In the framework of the two-state model, \( X(n) \) is estimated by Kalman recursion, and the estimate of gravity acceleration is then derived from the auxiliary quantity \( X_{1,1} \) through the following equation:
\[ \hat{g}_0(n) = \frac{\hat{X}_{1,1}(n) - \hat{X}_{1,1}(n-1)}{T_s}. \]

Recently, the authors became aware of the works [37–39] on the two-state model of atomic clocks and its application to Kalman filtering. These works could be generalized to atomic gravimeters after several significant modifications: First, for the state to be estimated, the noises considered and their contributions to the state are determined by the physical limit for resolving the state. Therefore, the model needs to be constructed based on the physics of atom interferometry, and have the ability to theoretically predict the precision limit of the estimation. Secondly, the model needs to be modified to properly describe the new subject. This is because in atomic gravimetry, the state to be estimated is time-varying and related to the absolute phase, but not to phase fluctuations or frequency fluctuations.

The Kalman recursion. – As shown in fig. 1, the gravity acceleration is estimated from the gravimeter readings in a recursive model, where the estimates \( \hat{X}(n) \), \( \hat{g}_0(n) \) and the associated error covariance matrix \( P(n) =
\[ E\{[X(n) - \hat{X}(n)][X(n) - \hat{X}(n)]^T]\} \] are constructed in two steps: the propagation step and the update step.

In the propagation step, \( \hat{X}^-(n) \), and \( P^-(n) \) are predicted conditioned on the previous a posteriori estimates \( \hat{X}(n-1) \) and its covariance \( P(n-1) \) as follows:
\[ \hat{X}^-(n) = F_d(T_s)\hat{X}(n-1) + B_d\hat{g}_0(n), \]
\[ P^-(n) = F_d(T_s)P(n-1)F_d(T_s)^T + Q, \]

where the superscript “minus” indicates the a priori estimate deduced according to the two-state model of the atomic gravimeter. The a priori estimate \( \hat{g}_0^-(n) \) is theoretically generated through the following equation:
\[ \hat{g}_0^-(n) = \hat{g}[t \rightarrow t(0)^{-}]-\hat{g}_{\text{tide}}(0) + \hat{g}_{\text{tide}}(n), \]

where \( \hat{g}[t \rightarrow t(0)^{-}] = \lim_{t' \rightarrow t(0)^{-}} \frac{1}{T_s} \int_{t'}^{t} g(t')dt' \) is an a priori measurement of the local gravity before \( t(0)^{-} \), and \( \hat{g}_{\text{tide}}(n) \) is the gravimetric Earth tides theoretically predicted with an inelastic non-hydrostatic Earth model [40].

Then, in the update step, the a priori estimates \( \hat{X}^-(n) \) and \( P^-(n) \) are improved after each observation \( Z(n) \), and the a posteriori estimates \( \hat{X}(n) \), \( \hat{g}_0(n) \) and the covariance \( P(n) \) are deduced according to
\[ \hat{X}(n) = \hat{X}^-(n) + K(n)[Z(n) - H\hat{X}^-(n)], \]
\[ P(n) = [I - K(n)H]P^-(n), \]
\[ \hat{g}_0(n) = \frac{\hat{X}_{1,1}(n) - \hat{X}_{1,1}(n-1)}{T_s}, \]

where the Kalman gain \( K(n) \) that minimizes the mean square error is calculated as
\[ K(n) = P^-(n)H^T[H P^-(n)H^T + (n+1)RT_s^2]^{-1}, \]

and to avoid growing memory per stack of data, the observation \( Z(n) \) is constructed by summing the gravimeter readings \( g(n) \) recursively:
\[ Z(n) = Z(n-1) + g(n)T_s, \]
where \( Z(n) \) is experimentally obtained, but not through theoretical prediction \( H\hat{X}^{\rightarrow}(n) \).

The parameters \( Q_{1,2} \) (governs the covariance matrix \( Q \)) and \( R \) of the two-state model affect the precision of the estimation. Here, \( Q_{1,2} \) is determined based on the physics of atom interferometry, where the quantum projection noise is the physical limit to determine the state of interferometry phase shift [41]. For the white phase noise \( y(n) \sim w_1(n) \), \( w_1 \) describes the contribution of the quantum projection noise to the state, and its variance can be written as

\[
Q_1 = \left( \frac{1}{k_{\text{eff}} T^2 \sqrt{N}} \right)^2,
\]

and for the random walk phase noise \( g(n) \sim Q_2(n) = \int_0^{t(n)} w_2(t')dt' \), the integrand \( w_2 \) is also Gaussian and stems from phase errors between adjacent readings \( g(n) - g(n - 1) \sim w_2(n)T_s \), the variance of which is

\[
Q_2 = \left( \frac{1}{k_{\text{eff}} T^2 \sqrt{N}} \right)^2 \frac{1}{T_s^2},
\]

under the quantum projection limit. \( R \) describes the measurement noise on gravimeter readings, and it is experimentally determined from the variance of a sample of \( g(n) \).

Based on the parameters determined above, the Allan deviation is theoretically calculated as

\[
\sqrt{\frac{Q_1}{\tau} + \frac{Q_2}{3}},
\]

where the crossover between the white phase noise and the random walk phase noise occurs at \( \tau = \sqrt{\frac{Q_2}{Q_1}} = 9.8 \) s. After \( \tau \), the estimator is dominated by the random walk phase noise \( \sqrt{Q_2} = 0.09 \mu\text{Gal}/\sqrt{s} \) in the short term, which is the precision limit of the estimator. The precision limit could be considered as a systematic error that might bias the gravimetry in the short term. However, for a measuring time of 100 s, this systematic error is kept under 1 \( \mu\text{Gal} \), which is a very acceptable level of accuracy [42], even for a state-of-the-art atomic gravimeter.

The Kalman recursion is initialized according to our \textit{a priori} knowledge about the atomic gravimeter and the local gravity, with

\[
\hat{X}^{-}(0) = \left[ \begin{array}{c} g(0) t \rightarrow t(0) \rightarrow T_s \\ \sqrt{Q_2} T_s \end{array} \right],
\]

\[
P^{-}(0) = Q,
\]

\[
Z(0) = g(0) T_s.
\]

As illustrated in the algorithm above, only one datum, but not the whole data set of the gravimeter readings \( g(n) \) is taken as the input at each iteration of the Kalman recursion. Though being applied to the post-measurement estimation in this work, this intrinsic characteristic of the algorithm makes it easy to be generalized to the quasi-real-time implementation [43].

The data. – The experimental observation was conducted at a seismic station dedicated to seismic studies in Zhaotong city, Yunnan Province, China. The vibration noise of the observation site was estimated to be 89.8 \( \mu\text{Gal}/\text{shot} \), and it dominated the measurement noise. The atomic gravimeter was located at a gravity pillar without any seismometer correction, and a long-term measurement of the local gravity was performed for 200 hours. These data were chosen because they consisted of a fairly long record with sufficient quality. To test the robustness of the Kalman estimator, no preprocessing was needed to remove the outliers and gaps in the time series. The outliers are mainly brought by the vibration noise, because no vibration compensation system was utilized. The gaps were due to temporary failures of the power supply. These failures did not damage the gravimeter, and a remote restart was possible once electrical power was restored.

Performance. – As shown in fig. 2, both the gravimeter readings (blue) and the estimates (red) basically agree well with the theoretical prediction of the gravimetric Earth tides (black). The difference between the estimates and the theoretical prediction mainly reflects the effects of some non-solid-Earth-tide phenomena, such as pressure or underground water changes. Meanwhile, the corresponding residual acceleration is obtained by subtracting the theoretical prediction of the gravimetric Earth tides from the gravity signal. The residual acceleration clearly shows that though the outliers and gaps exist in the gravimeter readings, their effects on the Kalman estimator are not significant. To evaluate the reliability of the Kalman estimator, the estimates are compared directly to the low-pass-filtered gravimeter readings. Note that the true wave-form of the gravity acceleration is more complicated than the theoretical prediction of the gravimetric Earth tides, and it is usually monitored by superconducting gravimeters during the comparison [44]. Here, without superconducting gravimeters, the low-pass-filtered data are taken for the true gravity acceleration, and the cut-off frequency of the low-pass filter is \( \frac{1}{1450s} \). The tracking error of the Kalman estimator \( \epsilon_{KF} \) is obtained by subtracting the low-pass-filtered data \( g_{LP} \) from the estimates \( \mathring{g}_0 \),

\[
\epsilon_{KF} = \mathring{g}_0 - g_{LP}.
\]

After about 15 iterations, the tracking error of the Kalman estimator stabilizes. The rms value of the tracking error is \( \sim 2.5 \mu\text{Gal} \), which is in good agreement with the Allan deviation of the gravimeter readings with \( \tau = 1450 \) s. The result indicates that the tracking error is dominated by the noise of the low-pass-filtered data, but not the residual noise of the estimates. Thus, the Kalman estimator is more precise than the low-pass filter or the integration method used in previous works [3,4,21–23], at least in the short term. Besides, the reliability of the Kalman estimator is confirmed by the numerical simulations in the Supplementary Material Supplementalmaterial.pdf (SM).
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Fig. 2: Long-term measurement of the local gravity for 200 hours and the corresponding residual acceleration by subtracting the gravimetric Earth tides from the gravimeter readings (blue) or the Kalman estimates (red). The tracking error (green) is obtained by comparing the estimates to the low-pass-filtered gravimeter output. And the black line is the theoretical prediction of the gravimetric Earth tides. The inset shows the tracking errors in the first 100 iterations.

Then, the Allan deviation of the residual acceleration is calculated to characterize the short-term sensitivity and the long-term stability of the gravimeter readings and the estimates. As illustrated in fig. 3, the sensitivity of the atomic gravimeter (blue) follows $67 \mu\text{Gal}/\sqrt{\text{Hz}}$ for up to 1000s. The measurement noise is dominated by white phase noise in this regime, and it demonstrates a reduction characteristic of $\tau^{-1/2}$ via integrating in time. Beyond 1000s, the stability is degraded due to instrumental or geophysical effects, such as fluctuations of systematic effects, imperfect tide model, or ocean loading, which leads to the random walk phase noise of $0.06 \mu\text{Gal}/\sqrt{s}$. The Kalman estimator significantly reshapes the residual noise by removing the white phase noise in the short term while leaving the long-term stability unchanged. Then, the residual noise of the
estimates (red) shows $\tau^{1/2}$ integrating, which corresponds to random walk phase noise and follows $0.13\,\mu\text{Gal}/\sqrt{s}$ for more than 100 s. The Allan deviation observed is consistent with the theoretical prediction of the total noise in the short term $\sqrt{0.09^2 + 0.06^2} = 0.11\,\mu\text{Gal}/\sqrt{s}$, which is root-sum-squares of the precision limit of the estimator $0.09\,\mu\text{Gal}/\sqrt{s}$ and the long-term stability observed $0.06\,\mu\text{Gal}/\sqrt{s}$. Though not affecting the long-term stability, the Kalman estimator greatly improves the precision of estimates in the short term and highlights a precision of $0.34\,\mu\text{Gal}$ at the measuring time of a single sample ($T_s = 5.7$s). By contrast, in previous works [3,4,21–23], the precision at this level is obtained at the cost of a sophisticated vibration compensation system and a measuring time of hundreds of samples.

Conclusion. — In conclusion, we shed light on the two-state statistical model that is rooted in the physics of the atomic gravimeter. In the model, the precision limit to estimation is theoretically predicted based on the quantum projection noise, which avoids the estimation from risking the accuracy of gravimetry. The model we proposed makes the implementation of the Kalman recursion in static atomic gravimetry possible, demonstrating a quasi-real-time and precise estimation. Meanwhile, by comparing the estimates to the gravimeter readings and those of the estimates coincide after about 1000 seconds.

The estimation method demonstrated could provide a significant advantage in short measurements, such as comparison or gravity survey with portable gravimeters, where the Kalman estimator helps to reduce the time spent on each system error evaluated or each measurement site. Additionally, as the short-term sensitivity is greatly improved, the estimation method could be applied to detect fast gravity variations in a long-time static measurement of gravity (see the SM). Moreover, the estimation method provided an opportunity to simplify the portable atomic gravimeter and perform precise gravity measurements in relatively quiet locations without a vibration compensation system. This demonstration would be of great interest for those applications involving static measurements of gravity, such as metrology or geophysics [14].

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