Hinge Spin Polarization in Magnetic Topological Insulators Revealed by Resistance Switch

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We report on the possibility to detect hinge spin polarization in magnetic topological insulators by resistance measurements. By implementing a three-dimensional model of magnetic topological insulators into a multi-terminal device with ferromagnetic contacts near the top surface, local spin features of the chiral edge modes are unveiled. We find local spin polarization at the hinges that inverts sign between top and bottom surfaces. At the opposite edge, the topological state with inverted spin polarization propagates in the reverse direction. Large resistance switch between forward and backward propagating states is obtained, driven by the matching between the spin polarized hinges and the ferromagnetic contacts. This feature is general to the ferromagnetic, antiferromagnetic and canted-antiferromagnetic phases, and enables the design of spin-sensitive devices, with the possibility of reversing the hinge spin polarization of the currents.

Introduction. — The recent discovery of intrinsic magnetic topological insulator (TI) multilayered MnBi$_2$Te$_4$ [1, 2] has boosted the expectations for more resilient quantum anomalous Hall effect [3–6] and observability of axion insulator states [7, 8]. The material platforms to realize the quantum anomalous Hall (QAH) phase can be classified in two- and three-dimensional systems. The former includes monolayer materials with spin-orbit coupling and magnetic exchange [9, 10]. The latter is the case of three-dimensional magnetic TIs, including magnetically doped TIs [11, 12], proximitized surfaces of a TI with a magnetic insulator [13, 14], and the Chern insulator phase of MnBi$_2$Te$_4$ [1, 8]. The distinction that arises in three-dimensional magnetic TIs is that the topological nature comes from contributions from two Dirac-like surfaces that, upon the introduction of a magnetization field throughout the material, become massive with opposite effective masses [15, 16]. Despite the three-dimensional nature of magnetic TIs, they are often analyzed near the surface, as effective two-dimensional systems.

However, compared to their two-dimensional counterparts, three-dimensional magnetic TIs present a higher level of complexity that reflects in layer-to-layer magnetic exchange and termination-dependent surface states, which ultimately dictate the nature and properties of surface magnetism and of topological edge states [17, 18]. The spin texture of topological edge states in both the quantum spin Hall and quantum anomalous Hall (QAH) regimes is usually perpendicular to the material’s surface, limiting the possibility for magnetic-sensitive detection or further spin manipulation protocols. The effective two-dimensional models of these materials are often highly symmetric and may overlook the sublattice and spin degree of freedom. By reducing the symmetry constrains, new spin textures can develop, such as hidden spin polarization of magnetic layers exhibit high-order topological phases and cleavage-dependent hinge modes [25, 27]. Thus, a detailed study of the spin features on a spinful three-dimensional model of the QAHE realized in magnetic TIs multilayers is missing.

In this Letter, we use the generic Fu-Kane-Mele (FKM) model for three-dimensional topological insulators [28] and introduce exchange terms to describe both ferromagnetic (FM) and antiferromagnetic (AFM) multilayered TIs. Contrary to ordinary spin-$z$ polarization of edge states in the QAH regime, the model exhibits an in-plane hinge spin polarization (HSP) which becomes apparent (and observable) in a specific device setup. Indeed, the topological states are characterized by an in-plane HSP perpendicular to both the current flow and the sample magnetization direction. The in-plane polarization reverses sign along the vertical direction, between the top and bottom surfaces. By using efficient quantum transport simulation methods [29] implemented into a three-dimensional multi-terminal device, such peculiar local spin polarization is shown to give rise to a giant resistance switching (or spin valve) triggered upon either inverting the magnetization of the sample, varying the polarization of the magnetic detectors, or reversing the current direction. Such fingerprints of HSP in the QAH regime are rooted in the chiral-like [27, 39] symmetries of the lattice, and are highly robust to Anderson-type of energetic disorder, and to structural edge disorder.

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Hamiltonian of the three-dimensional magnetic TI. —

The magnetic TI is described by a three-dimensional (diamond cubic lattice) FKM Hamiltonian, with magnetic layers modeled by an exchange coupling term that well captures the effect of magnetic impurities or magnetic layers. To simulate a multilayer FM or AFM magnetic TI we tune the orientation of the magnetic moments per layer. The FKM lattice vectors are $a_1 = (1/2, −√1/2, 0)$, $a_2 = (0, √1/2, 0)$, and $a_3 = (1/2, 0, √1/2)$; each unit cell has two sublattices: A with offset $d_1 = (0, 0, 0)$, and B with offset $d_4 = (1/2, 0, 0)$. The other first neighbors of A sites are at relative positions $d_q = d_q - a_q$ for $q = 1, 2, 3$. The full Hamiltonian reads

$$
H_0 = \sum_{ij} c^\dagger_{i,\alpha} t_{ij} c_{j,\alpha}, \quad H_Z = \sum_{i,\alpha} c^\dagger_{i,\alpha} [m_i \cdot s]_{\alpha,\beta} c_{i,\beta}
$$

$$
H_{SO} = \frac{1}{a^2} \sum_{ij,\alpha,\beta} c^\dagger_{i,\alpha} [s \cdot (d^\dagger_{ij} \times d^\dagger_{ij})]_{\alpha,\beta} c_{j,\beta}
$$

$$
H = H_0 + H_{SO} + H_Z.
$$

The Latin indices go over the lattice sites, and the Greek indices over the spin indices in the $s_z$ basis. The Zeeman magnetization vector $m_i$ may depend on the layer of the orbital $i$, and $s$ is a vector of Pauli matrices acting on the spin degree of freedom. The parameter $\lambda_{SO}$ denotes the spin-orbit coupling strength, while $t_{ij}$ describes the first nearest neighbors coupling between sites $i$ and $j$, and takes four different values labeled $t_q$ with $q = 1...4$ according to the relative position of the pair of sites $r_j - r_i = d_q$. As described in Fu et al., the isotropic case where $t_q = t$ defines a multicritical point, but adding anisotropy $t_q = t$ for $q = 1, 2, 3$ and $t_4 > t$, sets the phase to a strong TI, characterized by a non-trivial $Z_2$ invariant. We tune the parameters in units of the in-plane hopping $t$ to the strong TI phase with $t_4 = 1.4t$ and $\lambda_{SO} = 0.1t$. The FKM model can be interpreted as a stack of coupled Rashba layers, with alternating Rashba field. In absence of Zeeman field the strong TI phase is the three-dimensional realization of the Shockley model, hosting surface states that are sublattice polarized. The magnetic moments per layer describe the AFM (alternating magnetization between layers $m_i = \pm m$) or FM (constant magnetization $m_i = m$) coupling between layers. In a slab geometry perpendicular to the $z$ axis, a Zeeman exchange coupling field $m = 0.05t \hat{z}$ normal to the slab leads to a gap opening of the surface states, and sets the QAH phase described by a non-trivial Chern number.

We present the main electronic and spin characteristics of the magnetic topological insulator model in Figure 1. The details of the edge modes vary with the geometric design. For a heterostructure infinite along the $y$-direction but finite in both other directions, we obtain the usual linear energy dispersion of topological edge states seen in Fig 1(a). These states cover the whole side surface of the stack (wall states) with a very large electronic density at the hinges. Interestingly, the projected local spin density of states of the edge state at $k_y = -0.1\pi/a$ ($k_y = 0.1\pi/a$). The edge state covers the side wall of the slab and propagates to the right (left). b) Local density of states of a finite square slab. The edge state circulates around the sample, covers the side surfaces perpendicular to $\hat{x}$, and propagates along the top or bottom hinges of the side surfaces perpendicular to $\hat{y}$. c) Side view of transport setup geometry: metallic leads connect to the whole walls at both ends of the slab (golden color), and ferromagnetic leads connect to the lateral walls only near top hinge (red color). d) Top view and reference numbering of the leads on the transport setup.

FIG. 1. Magnetic TI in the FM phase, $m = 0.05t \hat{z}$. a) Dispersion relation of a slab geometry infinite in the $y$ direction. The left (right) inset depicts the local spin density of states ($s_z$) of the edge state at $k_y = -0.1\pi/a$ ($k_y = 0.1\pi/a$). The edge state covers the side wall of the slab and propagates to the right (left). b) Local density of states of a finite square slab. The edge state circulates around the sample, covers the side surfaces perpendicular to $\hat{x}$, and propagates along the top or bottom hinges of the side surfaces perpendicular to $\hat{y}$. c) Side view of transport setup geometry: metallic leads connect to the whole walls at both ends of the slab (golden color), and ferromagnetic leads connect to the lateral walls only near top hinge (red color). d) Top view and reference numbering of the leads on the transport setup.

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and appears in all phases, that is: FM, AFM, and canted-AFM, irrespective of the canting angle, as long as there is a z-component of the net magnetization. We next explore the possible fingerprints of such anomalous spin features on quantum transport in the QAHE regime.

**Multi-terminal spin transport simulations.** — To analyse the spin transport in the QAHE regime, we use the Kwant software package \[^{29}\] to build the three-dimensional model, and implement a multi-terminal device configuration, shown in Figs. 1c), and d). We perform charge transport simulations of a central scattering region connected with metallic and ferromagnetic leads. The interplay between the states available for transport in the leads and in the scattering region has a central role. The leads \(L_1\) and \(L_3\) are the metallic leads (golden color). They are fully contacting the left and right sides of the slab (all spin projections). The ferromagnetic leads \(L_0\) and \(L_2\) (red color) located on the sides only contact the upper part of the device near the top hinge. They carry electrons with only one spin polarization: \((s_x, \downarrow)\).

In this way, these contacts couple with the edge state in the region of maximal local spin polarization.

The expected resistance measurements for the QAHE are shown on the inset of Fig. 2. We use the notation \(R_{ij,kl}\), for the resistance measured from passing current between terminals \(i\) and \(j\), and measuring the voltage drop between terminals \(k\) and \(l\). The two-terminals (2T) resistance \(R_{kl,kl}\) is noted \(R_{2T,kl}\). The typical values of Hall resistance \(R_{xy} = R_{13,20}\) and the longitudinal resistance \(R_{xx} = R_{01,23}\) of a QAHE insulator \[^{37, 38}\] take the quantized values of \(R_{xy} = 1/4 \times h/e^2\), where \(C\) is the Chern number, and vanishing \(R_{xx}\) inside the gap. The two-terminal resistance \(R_{2T} = h/e^2\) is also quantized in the case of perfect tunneling between the leads and the scattering region. Such is the case of the matching ferromagnetic lead. The matching or mismatching between the spin current carried by the leads and the spin polarization of the edge states gives rise to a remarkable resistance switch, as seen in Fig. 2. The 2T resistance in the matching case is quantized inside the topological gap, while in the mismatching case the resistance increases by more than one order of magnitude.

To test the robustness of the 2T resistance switching effect, we introduce different types of disorder sources. First, we consider the impact of structural disorder described by a random distribution of a certain density of vacancies near the side walls of the slab that makes the scattering region. The impact of this disorder is detrimental to the formation of well-defined HSP, which only occur for wall-states at crystalline edges. Nevertheless, it is relevant for predictions on experiments, since the side walls of material samples have edge disorder. We find that the HSP effect survives to structural disorder, up to 5\% vacancies \[^{39}\]. Next, we simulate Anderson disorder by adding an onsite energy \(d\chi\), where \(\chi\) is a random variable with normal distribution on \([-0.5, 0.5]\). We find robustness of the HSP up to \(d\) much larger than the magnetization strength. In Fig. 2 we use \(d = 2|m| = 0.04t\), and average the resistance curve over 10 disorder realizations. We see that spin transport measurements can still distinguish the peculiar spin texture of the edge states.

The fact that Anderson disorder and structural disorder show the resistance switch is crucial in establishing the robustness of our results. There is a limit case where the edge state is fully polarized in a large region near the top hinge, and the ferromagnetic leads with opposite spins are completely decoupled from the edge state, and thus, from the transport setup. In this case, the voltage probes may have zero transmission probability to any other leads, leaving a floating probe with an arbitrary value of the chemical potential and the voltage \[^{40}\]. However, in our case the ferromagnetic leads are not fully disconnected when the spins do not match, rather they are weakly connected. Even though the value of the 2T resistance is sensitive to the details of the weak coupling, seen on the large standard error in Fig. 2 the trend is clear. In a QAHE thin-film contacted on its lateral sides with ferromagnetic leads, we can selectively get, either full transmission, or blocking of the edge state transport. Such phenomenon is sensitive to the direction of the magnetization of the ferromagnetic leads, the direction of the current, and the net magnetization of the sample.

Another experimentally relevant analysis is to explore the resistance switch for different directions of the mag-
netization $m$ of the slab in the FM phase. Figure 3 shows two measures of 2T resistance, as in Fig. 2 at the charge neutrality point, for different directions of the Zeeman exchange field ($\cos \theta \cos \phi \hat{x} + \sin \theta \cos \phi \hat{y} + \sin \phi \hat{z}$) (see right inset). At low $\phi$ angles ($m$ pointing mostly towards $+\hat{z}$) the configuration $R_{2T,1.2}$ in a) shows large resistance, while the values of $R_{2T,0.1}$ in b) are close to the quantized value $\hbar/e^2$. When sweeping the magnetization to the inverse direction (towards $-\hat{z}$) at $\phi = 180^\circ$, the roles of a) and b) reverse, giving a clear signature of the highly spin-polarized hinges and of the spin-dependent matching and mismatching with the ferromagnetic leads. In the middle of both extremes where $\phi = 90^\circ$, the magnetization lies in the plane of the slab and does not open a gap on the top and bottom surfaces. At intermediate angles, we note that the resistance switch is more robust for $\theta = 90^\circ$, where $m$ tilts towards $\hat{y}$, the transport direction and edge direction that the FM leads contact.

The HSP of the edge states is a good proxy to predict the switch in resistances that is measured in the device shown in Fig. 3. We obtain the spin projection of the forward propagating edge states on the top half of an infinite slab in the $\hat{y}$ direction, and finite in the $xz$ plane, see insets of Fig. 4. At momentum $k = -0.02\pi/a$ we select the positive eigenvalue inside the topological gap, similar to the states shown in the insets of Fig. 1(a). Panels a) and b) of Fig. 4 show finite length arrows that indicate the spin density and components in the $(s_x, s_z)$ plane of the forward propagating state, while the color represents the mostly null $(s_y)$ component. A vanishing arrow length (a point in the plot) indicates that there is no net spin density at that region enclosing that hinge [41]. When the system is in the topological phase, that is, $\phi$ away from $\sim 90^\circ$, there is electronic density in one edge or the other, and spin density near the hinge (a finite arrow). Accordingly, we see that panel b) complements perfectly panel a). In both cases the HSP direction changes with the magnetization angle, giving a notch to control the matching or mismatching cases in a transport setup.

**Conclusions.** — We have demonstrated that the edge states in thin-film ferromagnetic and antiferromagnetic TIs host HSP, spin polarized states at the hinges, which leads to a large resistance switch. The HSP of the edge states is in-plane, but the sign depends on the propagation direction and the magnetization of the sample. For a crystalline edge direction, the local spin polarization reverses across the vertical direction. Thus, the HSP inverts across the vertical direction, and switches sign for the opposite current direction. Carefully engineering ferromagnetic contact leads in a transport setup, allows us to obtain a giant resistance (spin valve effect) upon reversing the current direction or, conversely, tuning the total magnetization of the sample. The $(s_x)$ component of the spin direction in Fig. 4(a) and b) can be directly translated to the resistance values found in Fig. 3(a) and b). This highlights that the resistance switching mechanism, once established, can be used to gain insight about the magnetization of the sample.

We finally observe that the fact that FM and AFM topological insulators are able to host locally spin polarized currents along crystalline edges, with maximum spin
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Note that we remove sites from a slice with 3 sites depth near the wall. Since we remove each site with the same probability, for example \( \%5 \), then the probability at any point on the side wall that at least one of the sites is removed is much larger, around \( \%14 \).

In the Ladauer-Büttiker formalism, after setting one reference probe \( L_0 \) with an arbitrary voltage value \( V_0 \), we eliminate the rows and columns of \( L_0 \) from the transmission matrix to make it non-singular. However, if there is a disconnected voltage probe \( L_d \), there is one extra row and column with zero values corresponding to \( L_d \), and this matrix is again singular. The value that the voltage can take in this probe \( V_d \) is arbitrary, and we must eliminate the corresponding column and row to solve the system.

The norm of the spin components is not conserved, since it can be averaged out within the region where its computed. Therefore, the arrows may vanish without a significant \( \langle s_y \rangle \) component.