Localization of gravitational energy and its potential to evaluation of hydrogen atom properties

Jozef Šima and Miroslav Súkeník
Slovak Technical University, Radlinského 9, 812 37 Bratislava, Slovakia

e-mail: sima@chtf.stuba.sk

Abstract. Vaidya metric as an integral part of the Expansive Nondecelerative Universe (ENU) model enables to localize the energy of gravitational field and, subsequently, to find a deep interrelationship between quantum mechanics and the general theory of relativity. In the present paper, stemming from the ENU model, ionisation energy and energy of hyperfine splitting of the hydrogen atom, energy of the elementary quantum of action, as well as the proton and electron mass are independently expressed through the mass of the planckton, Z and W bosons and fundamental constants.

INTRODUCTION

The time when science will be able to offer a definite solution of relationship between quantum mechanics and the general theory of relativity (GTR) is still far-away. In our previous works [1, 2] it has been documented that results and consequences stemming from the GTR can be applied, apart from the macroworld phenomena of cosmology and astrophysics, to microworld phenomena too. In addition to differences between the principles governing quantum mechanics and gravity, there are also common features of these fields of physics. In unification of these fields, two main obstacles can be identified, namely their different nature (the uncertainty principle as a leading feature of quantum mechanics contrary precise results emerging from the GTR) and inability of up-to-date used physical theories to localize gravitational energy.

As for the localization of gravitational energy, three main streams of opinions can be observed [3]: 1) gravitational field energy is localizable but
There is no corresponding "magic" formula for its density to be found; 2) it is non-localizable in principle; 3) it does not exist at all since the gravity is a pure geometric phenomenon. It seems that solution of this enigma lies in the metrics applied. It has been evidenced by the ENU model that a metric involving changes in matter due to its permanent creation must be used. Utilization of Vaidya metric [4] allowed to offer answers to several open questions, explain some known facts in a new independent light, correct some opinions or demonstrate their limitations and unveiled deep mutual interrelationships between natural phenomena [1, 2]. This paper is aimed to provide further evidence of capability of the ENU model and the GTR in solving the issues of microworld by offering a new approach to a deeper understanding the ionisation energy of the hydrogen atom, the energy of hyperfine splitting, and the energy of elementary quantum of action. Moreover, it is evidenced that the proton and electron mass being themselves fundamental constants, can be expressed through the mass of the planckton, Z and W bosons and other fundamental constants.

THEORETICAL BACKGROUND

The Planck energy $E_{pc}$

$$E_{pc} = m_{pc}.c^2 = \sqrt{\frac{\hbar.c^5}{G}} \approx 10^{19} GeV$$  \hspace{1cm} (1)

where $m_{pc} (2.176711 \times 10^{-8} kg)$ is the planckton mass and $G (6.67259 \times 10^{-11} kg^{-1} m^3 s^{-2})$ is the gravitational constant, is of fundamental importance for space structure and existence of the Universe [5]. This energy plays an important role in unifying the fundamental physical interactions. Planck quantities - energy, length ($1.616051 \times 10^{-35} m$) and time ($5.390563 \times 10^{-44} s$) - represent limits describing the initial phase of the Universe expansion.

In our previous work [1] the density of gravitational energy has been expressed within the first approximation using Tolman equation as

$$\varepsilon_g = - \frac{R.c^4}{8\pi.G} \approx - \frac{3m.c^2}{4\pi.a.r^2}$$  \hspace{1cm} (2)

where $\varepsilon_g$ is the density of gravitational energy emitted by a body with the mass $m$ at the distance $r$, $R$ denotes the scalar curvature (contrary to a
more frequently used Schwarzschild metric, in the Vaidya metric $R \neq 0$ also outside the body) and $a$ is the gauge factor. In ENU model, the gravitational energy is both localizable and quantifiable which enables to bridge quantum mechanics and the GTR and to provide answers to some problems which have been unsolvable up-to-now. Some of the potentials of ENU model are demonstrated further.

We are postulating that the gravitational energy of planckton in Compton volume $V_C$ (determined by integration of the gravitational field density of planckton over Compton volume for any “elementary” particle) is equal to the rest energy $E_0$ of that particle. In other words, the particle energy is created by the planckton gravitational energy. This hypothesis can be expressed by relation

$$E_0 = \int |\varepsilon_{\theta(P_C)}| dV_C \cong m_{PC} \cdot c^2 \frac{\lambda_C}{a(T)}$$

in which $a(T)$ is the gauge factor and $\lambda_C$ is the Compton wavelength. In the above relation (3), the Compton wavelength $\lambda_C$ relates to the mass $m$ of a given particle [6]

$$\lambda_C = \frac{\hbar}{m \cdot c}$$

and the gauge factor $a(T)$ relates to the specific time when it held

$$k \cdot T \cong m \cdot c^2$$

where $T$ is the temperature of the Universe. In the period starting at the beginning of the Universe expansion and finishing at the end of radiation era it had to hold [7]

$$E \approx T \approx a^{-1/2}$$

Taking (4) and (5) into account, relation (6) is obtained

$$a(T) = \frac{m^2_{PC} \cdot l_{PC}}{m^2}$$

where Planck length $l_{PC}$ is defined as

$$l_{PC} = \left(\frac{G \cdot \hbar}{c^3}\right)^{1/2}$$
A substitution of (4) - (8) into (3) leads to identity that is important for understanding a mutual relation between the gravitational and inertial masses and, moreover, proves internal consistency of the used procedure.

RESULTS AND DISCUSSION

This paper is aimed at verification of a more general validity of (3). In case of an effort to find the properties of a particle, Compton volume must be replaced by another volume characteristic for this particle. To verify the justification of a broader usability of (3), the hydrogen atom was chosen. Its “volume” given by the Bohr radius (bearing in mind consequences of the uncertainty principle) and composition are known. In addition, its energy parameters (ionization energy, fine structure constant and hyperfine splitting) have been experimentally measured, their values (belonging to the most precisely determined values in the whole physics) are known and this allows to confront the results obtained within our approach with the reality.

To solve the problems associated with verification of (3), it was necessary to determine the gauge factor corresponding to influence of both gravitational and electromagnetic forces. Further, solutions for some different cases are offered.

1. Ionization energy of the hydrogen atom

In unification of electromagnetic and weak interactions, Z and W bosons play the crucial role. Gravitational influence of Z and W bosons on their surroundings initiated manifests itself just when their Compton wavelength became equal to their effective gravitational radius [1, 2]. At that time, gauge factor (denoted here as $a_{I(H)}$) reached the value

$$a_{I(H)} = \frac{\hbar^2}{G m_{ZW}^3}$$  \hspace{1cm} (9)

where $m_{ZW}$ is the mean mass of Z and W bosons (the actual masses being $1.434 \times 10^{-25}$kg and $1.6262 \times 10^{-25}$ kg, respectively). Introducing (8) to (3) and substituting Compton wavelength in (3) for the Bohr radius $r_H$ (52.917706 pm) [8] we obtain

$$I_H \approx \frac{m_{Pe,c^2} r_H G m_{ZW}^3}{\hbar^2}$$  \hspace{1cm} (10)
The left side of \((10)\) represents the ionisation energy \(I_H\) of the hydrogen atom. Calculation using numerical values of the right-side members leads to the value of 14.0 eV that is very closed to the experimental value 13.6 eV.

2. The mass of the electron

The mass of the electron belongs to fundamental constants without explaining its value. In this part it is shown of how this inertial mass depends on other parameters and fundamental constants. The electron mass is a parameter present in expression of both the ionization energy \((11)\) and Bohr radius \((12)\) of the hydrogen atom [8, 9]

\[
I_H = \frac{m_e e^4}{32\pi^2\varepsilon_o^2\hbar^2} = \frac{\alpha_e^2 m_e c^2}{2} \tag{11}
\]

\[
r_H = \frac{4\pi\varepsilon_o\hbar^2}{m_e e^2} = \frac{\hbar}{\alpha_e m_e c} \tag{12}
\]

In \((11)\) and \((12)\), \(\alpha_e\) \((7.29735 \times 10^{-3})\) is the dimensionless fine structure constant (related to the spin-orbit coupling of the electron), \(\varepsilon_o\) \((8.854187816 \times 10^{-12} \text{ kg}^{-1}\text{m}^{-3}\text{s}^4\text{A}^2)\) is the vacuum permittivity, \(m_e\) \((9.109534 \times 10^{-31} \text{ kg})\) and \(e\) \((1.60217733 \times 10^{-19} \text{ A.s})\) are the electron mass and charge, respectively. (To be exact, the reduced electron mass should be taken into account, the difference of the masses is, however, customarily omitted) Stemming from \((10)\), \((11)\) and \((12)\), relation \((13)\) correctly expressing the electron mass is obtained

\[
m_e = \sqrt{\frac{2m_{ZW}^3}{\alpha_e^2 m_{PC}}} \tag{13}
\]

Calculation leads to the electron mass of \(9.2029 \times 10^{-31}\) kg that is very closed to the actual value. Expressing \(m_{ZW}\) as \([2]\)

\[
m_{ZW} \approx \sqrt{\frac{\hbar^3}{g_F c}} \tag{14}
\]

\((g_F\) is the Fermi constant reaching \(1.41 \times 10^{-62} \text{J.m}^3\), and its subsequent substitution into \((13)\) allows to express the electron mass through fundamental physical constants only as
Equation (15) reveals a deep interrelationship between the electron mass (being itself a fundamental constant) and other fundamental constants, and unveils the reason of its value.

3. Energy of hyperfine splitting in the hydrogen atom

A further gauge factor (denoted as $a_{hf}$) is related to the time of initial gravitational influence of proton on its environment. Similarly to (9) it holds ($m_p$ being the proton mass)

$$a_{hf} = \frac{\hbar^2}{G.m_p^3}$$

Substituting (16) to (3) we obtain

$$E_{hf} \approx \frac{m_p.c^2.r_H.G.m_p^3}{\hbar^2}$$

(17)

It must be connected to the energy of hyperfine splitting since this quantity depends on magnetic moments and thus, in turn, on the inertial masses of proton and electron. The electron mass is included in $r_H$ and calculation based on (17) leads to

$$E_{hf} \approx 2.9 \times 10^{-24} J$$

(18)

The above value is approximately 3 times higher than the experimentally determined energy of hyperfine splitting (a consequence of the interaction of magnetic momentum of the proton and electron in the hydrogen atom [10] being 1420.4057518 MHz or in wavelength units approximately 21 cm). It should be pointed out, however, that the calculation was performed only on the level of first approximation. Moreover, the value in (18) is still much more precise than that obtained by means of usually used classical formula (19)

$$E_{hf} \approx \alpha_e^2 \cdot I_H \cdot \frac{m_e}{m_p}$$

(19)
4. The mass of the proton

Using (17) and (19), a simplified relation determining the mass of the proton emerges

\[ m_p \approx \sqrt[4]{\frac{\alpha_e^5 \cdot m_{Pc} \cdot m_e^3}{2}} \]  

(20)

In analogy with the electron mass, substituting \( m_e \) in (20) for (15) an expression for the interrelationship of the proton mass and fundamental constants emerges exhibiting the reason for its value as well as for the ratio of the electron mass and proton mass. It is worth mentioning that to obtain precise (as closed to experimental as possible) values, instead of simplified expressions their exact (and more complex) forms must be used and, in addition, the level of accord in calculated and measured values depends also on the accuracy of values of all members in the relevant relations.

5. Energy of elementary quantum of action

Due to the fact that the energy of gravitational field exerted by electron to its surroundings is lower than the critical gravitational density, the electron does not exert any gravitational effect on its surroundings at the time being [2]. Gravitational influence of electron will be observable when the gauge factor (denoted here as \( a_{g(e)} \)) reaches the value

\[ a_{g(e)} = \frac{\hbar^2}{G \cdot m_e^3} \]

(21)

Substituting (21) into (3) relation describing the energy of elementary quantum of action

\[ E_{eq} \approx \frac{m_{Pc} \cdot c^2 \cdot r_H \cdot G \cdot m_e^3}{\hbar^2} \approx 4.8 \times 10^{-34} J \]

(22)

is obtained. The above value corresponds (and is closed at a unit frequency) to the Planck constant \( \hbar \).

Conclusions
Utilization of the background of the ENU model, as a model enabling to localize the energy of gravitational field, helped to unveil mutual relationships between some fundamental physical constants associated with the hydrogen atom, and to provide correct values of both its energy parameters and inertial mass of its constituents. The results presented in this contribution clearly documented the applicability of the ENU model in acting as a bridge connecting the macroworld phenomena described by the GTR and the quantum mechanical realm of particles. Verification of the postulate stating that the energy of any particle is created by the planckton gravitational energy using the hydrogen atom as an example, and excellent agreement of calculated and experimental values of its parameters should be taken as a starting point for further investigation and seeking of interrelationships and common features of the macroworld and microworld.

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