The minimum orbital period in thermal time-scale mass transfer

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ABSTRACT

We show that the usual picture of supersoft X-ray binary evolution as driven by conservative thermal time-scale mass transfer cannot explain the short orbital periods of RX J0537.7–7034 (3.5 h) and 1E 0035.4–7230 (4.1 h). Non-conservative evolution may produce such periods, but requires very significant mass loss, and is highly constrained.

Key words: binaries: close – stars: evolution – stars: individual: 1E 0035.4–7230 – stars: individual: RX J0537.7–7034 – X-rays: stars.

1 INTRODUCTION

The importance of episodes of mass transfer in a semidetached binary on a thermal time-scale has recently been emphasized in a number of contexts. Such episodes can arise in either of two ways:

(i) the donor star fills its Roche lobe while already undergoing thermal expansion across the Hertzsprung gap; or

(ii) the donor fills its Roche lobe with a mass ratio \( q = M_2/M_1 \) (where \( M_2 \) denotes the donor mass, \( M_1 \) the accretor mass) large enough that the Roche lobe radius \( R_L \) shrinks on mass transfer more rapidly than the thermal equilibrium radius \( R_{th}(M_2) \) (where this exists) appropriate to the donor’s current mass \( M_2 \).

In case (i) the thermal time-scale episode will end once the donor attains a new thermal equilibrium radius, e.g. at some point after reaching the Hayashi line. In case (ii) the donor is continually trying to expand thermally beyond \( R_L \) in order to reach \( R_{th} \); so mass is transferred on a thermal time-scale while this condition holds. Generally, this case involves shrinkage of the orbit to some minimum Roche lobe size, followed by orbital expansion as the mass ratio \( q \) reverses. The thermal time-scale episode ends only when \( R_{th} < R_L \) (cf. Fig. 1; see also van den Heuvel 1992 and references therein). Thereafter normal mass transfer continues, driven either by systemic angular momentum losses (decreasing \( R_L \) ) or nuclear evolution of the donor (increasing \( R_{th} \)).

Case (i) arises in the formation of intermediate-mass X-ray binaries with black hole accretors, such as GRO J1655–40 (Kolb et al. 1997; Kolb 1998). In similar systems with neutron-star accretors, such as Cyg X-2, the condition for case (ii) may hold simultaneously (King & Ritter 1999; Kolb et al. 2000; Podsiadlowski & Rappaport 2000; Tauris, van den Heuvel & Savonije 2000). In this paper we are mainly concerned with case (ii). This has received most attention in connection with the supersoft X-ray binaries (van den Heuvel et al. 1992). Thermal–time-scale mass transfer from a donor initially on or close to the main sequence, on to a white dwarf (WD) accretor, offers a way of driving accretion rates \( \dot{M}_1 \sim 10^{-7} M_\odot \text{yr}^{-1} \) high enough to allow steady nuclear burning. In addition to potentially explaining the observed supersoft systems, this process allows the white dwarf mass \( M_1 \) to grow. If one can arrange that \( M_1 \) reaches the Chandrasekhar mass \( M_C = 1.44 M_\odot \) this suggests a way of making type Ia supernovae. However, the difficulty of computing mass transfer on these time-scales meant that early studies of this process simply used the assumption that mass transfer occurred on a thermal time-scale, and were therefore unable to predict the evolution of the binary parameters (masses, period and mass transfer rate). Detailed calculations of these have only recently begun to emerge (Deutschmann 1998) but are not yet exhaustive.

Discussion of the evolution of supersoft X-ray binaries would be greatly eased if observation provided reliable masses. However, this is very difficult for several reasons. For example, Greiner, Orio & Schwarz (2000) suggest rather low masses \( M_1 = 0.6 M_\odot \), \( M_2 = 0.35 M_\odot \) for the short-period system RX J0537.7–7034 we shall discuss extensively in this paper. However, these and similar estimates use the assumption that the donor is close to the main sequence. By definition this cannot be true in thermal time-scale mass transfer, since the star is not in thermal equilibrium; although it may be quite close to its main-sequence radius, it can also be considerably smaller than this (cf. Deutschmann 1998). Moreover, the mass function of Greiner et al. uses the He II emission line, and thus may not reflect the dynamical motion of the accretor.

Since mass information is so hard to come by, a crucial test for the thermal time-scale model for the supersoft X-ray binaries is provided by the discovery of systems with fairly short orbital periods. In particular, RX J0537.7–7034 (Greiner et al. 2000) has a period of about 3.5 h, and 1E 0035.4–7230 (=SMC 13) (Schmidke et al. 1996) has a period of 4.126 h. Since the initial mass ratio \( q_i \) must be \( \leq 1 \) for this type of evolution (see below), one expects initial donor star masses \( M_{2i} \geq 1 M_\odot \) for a typical white dwarf accretor, and thus an initial orbital period \( P_i \approx 10 \text{ h} \) (see Section 3 and Fig. 3). Evidently, considerable orbital shrinkage would be needed for such systems to reach the periods of RX J0537.7–7034 and 1E 0035.4–7230. This is unlikely if the
mass transfer is conservative, as we shall show. However, it is probable that much of the transferred mass is not accreted by the white dwarf, but blown away from it as a wind (e.g. Li & van den Heuvel 1997), allowing greater orbital shrinkage. Mass loss induced in some way by the mass transfer process is probably the only way of significantly increasing the orbital shrinkage, as other angular momentum loss processes such as magnetic braking or gravitational radiation generally take place on time-scales far longer than the 10^7-yr characteristic of mass transfer in the supersoft X-ray binaries.

In this paper we consider thermal time-scale mass transfer and investigate how much mass loss from the accretor is required if RX J0537.7–7034 and 1E 0035.4–7230 are products of the standard picture of the supersoft binaries. Our method is to compute the minimum orbital period analytically for specified rates of mass and angular momentum loss from the binary.

2 ORBITAL EVOLUTION

We consider the orbital evolution of a semidetached binary in which a fixed fraction 1-\(\eta\) of the mass transferred from the donor (star 2) is lost from the accretor (star 1) with \(\beta\) times the specific angular momentum of the latter. [The quantity 1-\(\eta\) is called \(\alpha\) by King & Kolb (1995) in their general treatment of such ‘consequential angular momentum loss’ [CAML] mechanisms; the use of \(\eta\) allows more compact formulae in what follows.]

Following the general method of van Teeseling & King (1998), setting their quantities \(M_n = J_{\text{sys}} = 0\), \(\beta_i = \beta\), we find

\[
M_{n1} = (1-\eta)M_2,
\]

so that

\[
M_1 = M_2 + M_{n1} = -\eta M_2.
\]

These definitions give

\[
J = (1-\eta)M_2\beta M_2/M_1 M,
\]

with \(M = M_1 + M_2\). Kepler’s third law links angular momentum \(J\) and period \(P\) of the orbit as

\[
J^3 = G^2 M_1^2 M_2^3 P^2 / 2\pi^2.
\]

The evolution of the orbital period then obeys

\[
P = \frac{3 M_2 + 3 M_1}{M_2 M_1} \frac{2(1-\eta)M_1 M_2}{M} + 3(1 - \eta)(\beta - 1) M_2 M_2 / MM_1.
\]

From (2) we have \(M_1 = C - \eta M_2\), \(M = C - (1 - \eta)M_2\), with \(C = M_{10} + \eta M_{20}\), where \(M_{10}\), \(M_{20}\) are the initial values of the two masses. Thus (5) integrates to

\[
P \propto M_2^{-3} M_1^{-3} \beta^{-1} / n M_1^{2 - \beta}
\]

for \(\eta \neq 0, 1\), the corresponding expression for \(\eta = 0\) being given by the limit of this expression as \(\eta \rightarrow 0\), while we obtain the well-known result

\[
P \propto M_2^{-3} M_1^{-3},
\]

for conservative mass transfer, i.e. \(\eta = 1\). In terms of the mass ratio \(q = M_2/M_1\) we can express the relation between \(M_2\) and \(M_1\) as

\[
q M_1 = M_2 + \frac{1}{\eta} (M_{10} - M_1),
\]

leading to

\[
M_{10} = \frac{q M_1}{q - q M_1}.
\]

Thus with \(M = (1 + q)M_1\), etc, (6) gives finally

\[
\frac{P}{P_1} = \left(\frac{q_1}{q}\right)^3 \left(\frac{1 + q_1}{1 + q}\right)^{2(3 - \beta)} \left(\frac{1 + q_1}{1 + q}\right)^{2 + 3(\beta - 1)/n},
\]

where \(P_1\) is the initial period.

As a check we note that if all the transferred mass is blown away from star 1, with the specific angular momentum of that star, we have \(\eta = 0\), \(\beta = 1\); we take the limit of (10) as \(\eta \rightarrow 0\) by noting that \((1 + q_1)^{3/\eta} = \exp(3/\eta \ln(1 + q_1))\) and using l’Hôpital’s rule on the exponent to give the limit as \(\eta \rightarrow 0\) as \(e^{3\beta}/\beta\). We then obtain

\[
\frac{P}{P_1} = \left(\frac{q_1}{q}\right)^3 \left(\frac{1 + q_1}{1 + q}\right)^{2(3 - \beta)}/\beta,
\]

or \(P \propto M_2^{-3} M_2^{-2} \exp[3M_2/M_1]\), as found by, for example, King & Ritter (1999) for this case.

We are interested in the minimum orbital period attained during the binary evolution. Regarding \(P\) in (10) as a function of \(q\), i.e. with \(P_1\), \(q_1\) fixed, we find

\[
1 \frac{\partial P}{\partial q} = \frac{3}{q} - \frac{3 \beta - 1}{q + 1} + \frac{3 \beta + 5 \eta}{q + 1},
\]

so that \(P\) is an extremum at

\[
q = q_m = \left(1 - \eta\right) + \sqrt{(1 - \eta)^2 + 9(\eta + \beta - \eta \beta)} / (3(\eta + \beta - \eta \beta).
\]

From (12) we can show that

\[
1 \frac{\partial^2 P}{\partial q^2} = \frac{3}{q(1 + q)} + \frac{(3 \beta - 1)(1 - \eta)}{(1 + q)^2 (1 + q)}
\]
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![Figure 2. \(P_m/P_i\) as a function of \(q_i\) for \(\eta = 1\), 0 according to (18). Additional constraints for various initial WD masses resulting from the Chandrasekhar limit are indicated by broken lines in the conservative case. They mark the critical values of \(q_i\) where the WD mass has grown to \(M_C\) precisely upon reaching \(P_m\). An upper limit at \(q_i = 3\) caused by delayed dynamical instability (DDI) is also indicated.](https://academic.oup.com/mnras/article-abstract/321/2/327/980449)

at \(q = q_m\), so \(P\) has a minimum there, assuming \(0 \leq \eta \leq 1\). Hence for given \(P_i\), \(q_i\), the smallest orbital period that can be attained in evolution with mass loss is given by (10) with \(q = q_m\). For a given initial period, we still have the freedom to vary the initial mass ratio \(q_i\). (This corresponds to the fact that the period \(P_i\) essentially determines the mean density of the donor, almost independently of the primary mass. So a donor of given initial mass \(M_2\) may initiate mass transfer at various \(P_i\), depending on its evolutionary state, with different \(q_i\), determined by \(M_0\).) For fixed \(P_i\), the minimum value of \(P\) clearly has a maximum, regarded as a function of \(q_i\), at \(q_i = q_m\) (see also Fig. 2 which shows the special case \(\beta = 1\). In other words, \(d[P(q = q_m)]/(d\eta)(p|\text{fixed}) < 0\). Since \(q\) decreases from its initial value \(q_i\) through the evolution this means that given an initial period \(P_i\), the smallest value of the minimum period is given by the largest possible value \(q_m\) of \(q_i\). Hence given an initial period \(P_i\), the minimum possible orbital period \(P_m(P_i)\) is given by

\[
\frac{P_m}{P_i} = \left(\frac{q_m}{q_i}\right)^3 \frac{1 + q_m}{1 + q_i}^{3\theta - 1} \frac{1 + q_m \eta}{1 + q_i \eta}^{5 + 3\theta/\eta},
\]

with \(q_m\) given by (13), and \(q_i\) the largest possible value of the initial mass ratio \(q_i\). Although \(\beta\) is of course irrelevant in the conservative case (\(\eta = 1\)), larger angular momentum losses in the wind allow even smaller ratios \(P_m/P_i\) for the same \(q_i\).

It is likely that thermal time-scale mass transfer is still going on at the minimum orbital period, since by Roche geometry we have

\[
R_L \propto f(q) M_2^{3/2} P^{2/3},
\]

where \(f(q)\) is a slowly increasing function of \(q\) given, for example, by the approximation from Eggleton (1983). Thus

\[
\frac{R_L}{R_i} = \frac{\dot{q} d(f)}{f dq} + \frac{M}{3M} + \frac{2P}{3P},
\]

which tells us immediately that the period reaches its minimum value at larger mass ratios than \(R_L\) as \(q < 0\), \(M \leq 0\). In general, \(R_L\) initially shrinks much more rapidly than \(R_i\). Thus the condition \(R = R_i\) signalling the end of thermal time-scale mass transfer is reached only once \(R_L\) is increasing after passing through its minimum value, i.e. thermal time-scale mass transfer usually stops only after the minimum orbital period is reached. However, it is conceivable that in some cases \(R_i\) shrinks very rapidly on mass loss, and individual cases must be checked. For the purposes of this paper we note the true minimum period for the thermal time-scale could if anything be longer than the values of \(P_m\) we find here.

In what follows we will assume that \(\beta = 1\), i.e. that the mass lost from the accretor has the same specific angular momentum as this star. This includes, for example, any form of mass loss from orbits with circular symmetry about the accretor, as might occur from an accretion disc. In this case (15) simplifies to

\[
\frac{P_m}{P_i} = \left(\frac{q_m}{q_i}\right)^3 \frac{1 + q_m}{1 + q_i}^{3\theta - 1} \frac{1 + q_m \eta}{1 + q_i \eta}^{5 + 3\theta/\eta},
\]

with \(1 \leq q_m \leq 1.387\) now given by

\[
q_m = \frac{(1 - \eta) + \sqrt{(1 - \eta)^2 + 9}}{3}.
\]

and \(q_i\) is the largest possible value of the initial mass ratio \(q_i\). Fig. 2 shows the ratio \(P_m/P_i\) as a function of \(q_i\) for the two extreme cases \(\eta = 1, 0\).

3 Minimum Periods

The work of the last section, especially equation (15), shows that to compute the ratio \(P_m/P_i\) of the minimum to the initial period we need to specify the largest possible value \(q_i\) of the initial mass ratio. For a white-dwarf accretor the short observed periods of RX J0537.7–7034 and 1E 0035.4–7230 then constrain the mass-loss parameter \(\eta\). Clearly, to reach such short periods it is preferable to start from the shortest initial periods \(P_i\), which in turn are given by assuming that the donor is still very close to the zero-age main sequence (ZAMS) when it initiates mass transfer. We can easily show that the initial donor mass \(M_2\) must be \(\geq 1.9 M_\odot\) in order to explain the 3.5-h period of RX J0537.7–7034, by iterating the formula (18), using the fact that we must clearly have \(q_i > q_m\). For the lowest-mass white dwarf we consider (\(M_2 = 0.7 M_\odot\)) this implies \(M_2 > 0.7 M_\odot\). Thus in the conservative case \(\eta = 1\) we have \(q_i = 1, M_2 > 0.7 M_\odot\), and Fig. 3 shows that \(P_i \approx 6 h\) for a ZAMS star of this mass. We can now use Fig. 2 with the restriction \(P_m/P_i < 3.6/6 = 0.58\) to find \(q_i > 2.36\) and thus \(M_2 > 1.66 M_\odot\). Fig. 3 now shows that \(P_i = 10 h\), and we may iterate using Fig. 2 to obtain \(q_i > 3.38\), \(M_2 > 2.37 M_\odot\). Further iteration fails, as Fig. 3 shows that the new higher estimate for \(M_2\)
Figure 3. Typical range of $P_i$ versus $M_2$, for different WD masses, where the curves indicate the main-sequence boundaries. For initial donor star masses in the expected range between 1 and 5 $M_\odot$, the short period edge (=ZAMS) lies at about 10 h, and is fairly constant for $M_2 \approx 1.3 M_\odot$. The additional line marks the terminal main sequence (TMS), more precisely the point of minimum $T_{\text{eff}}$ (where applicable).

does not increase the estimate for $P_i$. In a similar way we find $q_\text{il} > 2.7, M_2 > 1.89 M_\odot$ for the other extreme case $\eta = 0$.

This argument essentially fixes the minimum value of $P_i$ at about 10 h (see Fig. 3), so the orbital shrinkage required to explain the 3.5-h period of RX J0537.7–7034 has $P_m/P_i < 0.35$.

Given an initial white dwarf mass $M_{1i}$ and mass ratio $q_\text{il}$ we can predict the period ratio $P_m/P_i$ for any given $\eta$. The initial mass ratio $q_\text{il}$ must obey two constraints (added as vertical lines in Fig. 2):

(i) the system must avoid the ‘delayed dynamical instability’ (DDI; see Webbink 1977; Hjellming 1989), which occurs when sustained thermal time-scale mass transfer exposes inner layers with a flat entropy gradient; and

(ii) the white dwarf mass cannot exceed $M_C$ before the minimum period $P_m$ is reached.

In practice the first of these constraints requires $q_\text{il} < q_{\text{DDI}} = 3$. Hjellming (1989) found this value for a donor with $M_2 = 3 M_\odot$ near the terminal main sequence (TMS), whereas Kalogera and Webbink (1996) seem to prefer even smaller limiting ratios around 2.5. Kolb et al. (2000) used Mazzitelli’s stellar code to calculate a test sequence with constant primary mass 0.75 $M_\odot$ where mass transfer starts from a 3 $M_\odot$ near-TMS star. A DDI occurred at a donor mass of 2.6 $M_\odot$, in perfect agreement with Hjellming’s prediction. The critical maximum mass ratio $q_{\text{DDI}}$ depends on the stellar structure, and therefore on the stellar input physics. In particular, $q_{\text{DDI}}$ is probably sensitive to the degree of convective core overshooting, as this determines the size of the convective core. This is highlighted by the fact that Kolb et al. (2000) find $q_{\text{DDI}} = 2.9$ for early massive case B mass transfer, while Tauris et al. (2000) find $q_{\text{DDI}} = 3.6$, using an updated Eggleton code.

The second constraint is quite severe for large initial white dwarf masses and $\eta = 1$. In particular, for $M_{1i} = 0.7 M_\odot$ it requires $q_\text{il} < 3.11$, in mild contradiction with the requirement $q_\text{il} > 3.38$ we found above.

Fig. 4 shows $P_m/P_i$ versus $\eta$ for various values of $q_\text{il}$. The stars denote combinations $M_{1i}, q_\text{il}, \eta$ where $M_1$ reaches $M_C$ precisely at $q = q_m$. To the right of these positions we take $P_m$ as the minimum period actually achieved, i.e. the period where the white dwarf reaches $M_C$.

Each panel of this figure shows:

(i) the horizontal line $P_m/P_i = 0.35$, i.e. the upper limit required by RX J0537.7–7034.

Figure 4. $P_m/P_i$ versus $\eta$ for various $q_\text{il}$ as labelled. Only the part of each curve in thick linestyle reaches $P_m$ before the WD grows to the Chandrasekhar mass and a SN Ia occurs, with an asterisk marking the critical $\eta$. Beyond that the current period at the SN event has been given as the smallest possible fraction $P_m/P_i$ during the thermal time-scale mass transfer phase. The horizontal dotted line at $P_m/P_i = 0.35$ marks the fraction at least required to create a system such as RX J0537.7–7034 starting from the ZAMS with $P_i = 10 h$. The numbers in brackets show pairs of masses $[M_1(q_{\text{il}}), M_2(q_{\text{il}})]$ each belonging to a set of parameters $(\eta, q)$ just fulfilling this requirement and marked by a diamond. DDI constraints (that would exclude the two larger initial mass ratios) have not been considered in this graph.

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J0537.7–7034 is predicted to be close to the Chandrasekhar limit unless the DDI limit imposes a more severe restriction than the lowest possible WD mass (as is the case in Fig. 5 for \( q_i \simeq 3, \dot{M}_{M} \geq 0.7 M_\odot \)).

Even given these tight constraints, the evolutions discussed above require the system to have come into contact with the donor still very close to the ZAMS. As orbital angular momentum losses (e.g. via magnetic braking) are probably negligible for the likely initial masses \( M_2 \simeq 2 M_\odot \), this also requires the initial orbital separation to lie in an extremely narrow range. Evidently, to make a system such as RX J0537.7–7034 or 1E0035.4–7230 by the thermal time-scale route is a very rare event. In line with this, Deutschmann (1998) found no orbital periods shorter than about 6h in his detailed calculations with solar metallicity. Furthermore, these requirements have been derived assuming that RX J0537.7–7034 and 1E0035.4–7230 are just at the minimum period \( P_{in} \), so the conditions might be even harder to meet. The probability of observing such a system is larger near the end of the thermally unstable phase because the mass transfer rate decreases (in both Deutschmann’s and our own full computations).

So far we have neglected the effect of tidal interactions on the orbital evolution of the binary. Tauris & Savonije (2000) show that these can be important in low-mass X-ray binaries by translating spin angular momentum losses (via, for example, magnetic stellar wind braking) into orbital losses when tidal synchronization occurs. However, for the binaries we consider here, any effect before the beginning of mass transfer simply allows a shorter \( P_i \) for a wider range of systems. Once synchronism is achieved, the angular momentum of a lobe-filling donor is less than about 10 per cent of the orbital angular momentum (because the gyration radii of typical donor stars are small, \( r_\star^2 \leq 0.10–0.20 \)). Even transferring all of this to the orbit during the course of the evolution would lead to only a marginal shift towards longer minimum periods, leaving our conclusions unchanged.

The work of this paper suggests that while the thermal time-scale mass transfer model for the supersoft X-ray binaries has many desirable features, it may not be possible to use it to describe all of the supersoft binaries, in addition to SNe Ia progenitors. For example, if highly non-conservative mass transfer is as common as seems to be required to explain RX J0537.7–7034 or 1E0035.4–7230, this would make building up the white dwarf mass to produce a type Ia supernova highly problematic. It may therefore be necessary to consider other possibilities (e.g. King & van Teeseling 1998; van Teeseling & King 1998).

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Figure 5. Borderline in the plane of \( \eta \) and \( q_i \) needed to obtain \( P_{in}/P_i = 0.35 \). The allowed range is limited to the small area above the curved line and below the horizontal DDI limit at \( q_i = 3.0 \). Additional constraints are shown for various \( M_1 \) (broken curves). The upper limits to \( q_i(\eta) \) (or \( \eta(q_i) \)) result from the WD growing beyond its Chandrasekhar mass before reaching \( P_{in} \), see Fig. 4 and text for details. Initially massive WDs with \( \geq 1.0 M_\odot \) restrict the allowed range to \( \eta \leq 0.3 \) independently of DDI.

(ii) only those curves that actually manage to cross this line for \( \eta \simeq 0 \). In particular, we plot the curve for the limiting value of \( q_i \) such that the curve just crosses \( P_{in}/P_i = 0.35 \) at \( \eta = 0 \). This gives a lower limit on \( q_i \), which is the same for each mass of the WD;

(iii) at some of the extreme solutions, i.e. wherever the line \( P_{in}/P_i = 0.35 \) is crossed before a supernova occurs, the current values of \( M_1, M_2 \) are given. Note that with growing \( M_1 \) the donor mass has to go up as well for the same \( q_i \). Furthermore, \( M_1 \) at the minimum period comes closer and closer to the Chandrasekhar mass as we increase \( \eta \).

4 DISCUSSION

Fig. 4 already shows that the short orbital period of RX J0537.7–7034 poses very severe constraints if this system results from thermal time-scale mass transfer. In particular,

(i) Conservative evolution (\( \eta = 1 \)) is possible only for initial mass ratios \( q_i > 3 \) which probably make the system vulnerable to delayed dynamical instability. Full evolutionary calculations are required to check whether there are any evolutionary tracks that avoid it. If such tracks exist, the initial white dwarf mass in RX J0537.7–7034 must have been low (\( \leq 0.7 M_\odot \)), but the current system masses must be fairly high (e.g. \( q_i = 4 \) requires \( M_1 = 1.17 M_\odot, M_2 = 2.33 M_\odot \)). These are of course in conflict with the mass estimates of Greiner et al. (2000), but this may not by itself be fatal (see the remarks in the introduction).

(ii) Non-conservative evolution (\( \eta < 1 \)) does allow tracks with \( q_i \leq 3 \) which probably avoid the delayed dynamical instability. However, the ranges of \( \eta \) and \( q_i \) are still very tightly constrained. Fig. 4 shows that \( 2.7 < q_i < 3.0 < \eta < 0.3 \), with \( \eta \) and \( q_i \) correlated as in Fig. 5. For \( M_{M} \simeq 1.0 M_\odot \) \( \eta \) has to be even lower, and generally upper limits on \( q_i \) are given for each \( M_{M} \) in addition to DDI. Note that for the largest values of \( \eta \) the white dwarf in RX

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