Comparison of Standard $k$-$\varepsilon$ Model and RSM on Three Dimensional Turbulent Flow in the SEN of Slab Continuous Caster Controlled by Slide Gate

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The fluid flow in the SEN and its outflow characteristic at nozzle port, after installation of slide gate system to control molten steel flow rate, have been investigated by mathematical simulation using standard $k$-$\varepsilon$ model, RSM and experimental measurement by Ultrasonic Doppler Velocimeter (UDV) respectively, and the adaptability of turbulence models has been discussed. The research results indicate that, due to the throttle action of slide gate, a secondary flow and a separated flow are generated under the slide gate, and a swirl flow appears at nozzle port, with the swirl direction from the clogging side via nozzle bottom to the opening side of the slide gate. The swirl direction of outflow at nozzle port calculated by standard $k$-$\varepsilon$ model is contrary to the measurement result by UDV and, the calculated result obtained by RSM is close to the UDV data. Based on kinetic theory of molecular, theoretical deficiencies of the Boussinesq hypothesis has been discussed.

KEY WORDS: slide gate; SEN; standard $k$-$\varepsilon$ model; RSM; Boussinesq hypothesis.

1. Introduction
Slide gate control system, which is installed on the outside of tundish, with the advantage of accurate control precision of molten steel flow rate, consequently, it is widely employed in continuous casting of steel. After installation of slide gate, molten steel is throttled at slide gate region, thus, some complex flow phenomena are emerged, such as sudden expansion flow, separated flow, re-circulated flow, and so on. Therefore, the selection of turbulence models becomes very important. Up to now, the standard $k$-$\varepsilon$ model,1–3 with two additional transport equations of turbulent kinetic energy ($k$) and turbulent kinetic energy dissipation rate ($\varepsilon$), is applied by many researchers around the world. This model is designed for the full developed turbulence, with the advantages of simplicity and low computational complexity, so that it is widely used in engineer fields. However, this model is based on the eddy viscosity hypothesis, and the scalar quantity of eddy viscosity is applied to calculate the Reynolds stress tensor, it is found that there are problems when it is used for strong swirl flow,4) effects of streamline curvature,5,6) sudden changes in strain rate, buoyant flow, separated flow, secondary flow, round jet, etc.7) While, in the Reynolds stress model (RSM), eddy viscosity hypothesis is not used, and Reynolds stress tensor is directly solved, so that, the anisotropy of turbulence can be considered, and the history of turbulence can also be simulated. But there are six partial differential equations (PDEs) to be solved additionally; hence, this model consumes more computer resource.

For the complex flow in the SEN after slide gate adopted, which turbulence model can describe rules of turbulence flow accurately in continuous casting system? Therefore, the standard $k$-$\varepsilon$ model and Reynolds stress model (RSM) are used to calculate flow characteristics in the SEN and the outflow at nozzle port in present study. Meanwhile, a verification experiment is conducted by Ultrasonic Doppler Velocimeter (UDV) using water model, and the adaptability of turbulence models has been discussed.

2. Experimental
The flow characteristics at nozzle port have been investigated by water model, and the schematic of 0.7-scale water model constructed from transparent plastic plates is shown in Fig. 1(a), and its detail geometry parameters are given in Table 1. The flow rate is controlled by a slide gate, with an opening ratio of 30 pct, which moves perpendicular to the mold wide wall. In this experiment, the velocity at SEN inlet is 1.2 m/s, and the corresponding casting speed is 0.94 m/min.

The measurements have been carried out using DOP2000 velocimeter (model 2125, Signal Processing SA, Lausanne, Switzerland) equipped with 1 MHz transducers. Because of the space constraint, it is difficult to measure velocity data in the SEN, the velocity component at the SEN port, which is in the mold cavity, is measured along mold thickness and height direction. The measurement region and distribution of measuring points are shown in Fig. 1(b). There are 8 mea-
suring points in the height direction of the SEN port, the first measuring point locates at port upper edge, the last one is located at port lower edge, and the interval of every two measuring points is 8 mm. In the mold thickness direction, there are 9 measuring points, the 5th measuring point is in the wide-center plane, and there are 4 measuring points on the each side of the wide-center plane, the distance of every two measuring points is 5 mm. The measuring time is 120 s for each measuring point in the experiment, the mean value of measuring data in this time interval is treated as the time-average velocity component at corresponding position.

3. Mathematical Models

3.1. Governing Equations

The fluid flow in the continuous caster is governed by the equations of mass conservation and momentum conservation. The incompressible time average Navier-Stokes equations in Cartesian coordinate system are written in following form:

\[ \frac{\partial \rho u_j}{\partial x_j} = 0 \] .......................... (1)

\[ \rho_j \frac{\partial (\rho u_j)}{\partial x_j} = - \frac{\partial p}{\partial x_j} + g_j + \frac{\partial}{\partial x_j} \left( \tau_{ij} - \rho u_i' u_j' \right) \] .......................... (2)

Where \( \rho \) and \( \rho_j \) denote the time-average velocity in \( i \) and \( j \) direction, \( u_i \) and \( u_j \) stand for the fluctuating velocity, \( p \) is the time-average pressure, \( \rho \) is the density of the fluid, \( \tau_{ij} \) is the mean viscous stress tensor, defined as

\[ \tau_{ij} = 2 \mu \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \] .......................... (3)

and \( -\rho u_i' u_j' \) is Reynolds stress tensor, and it is unknown.

3.2. Standard k-\( \varepsilon \) Model

The basis for standard k-\( \varepsilon \) model is the Boussinesq hypothesis, which postulates that the Reynolds stress is related linearly to mean rate of strain, directly analogous to the relation for the viscous stress in a Newtonian fluid, and the proportionality factor is eddy viscosity. Therefore, the Reynolds stress tensor can be written as:

\[ \tau_{ij} = \rho \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \] .......................... (4)

\[ \mu \frac{\partial \left( \rho \frac{\partial u_i}{\partial x_j} \right)}{\partial x_j} = \rho \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - 2 \mu \frac{\partial u_i}{\partial x_j} \] .......................... (5)

where \( k \) denotes turbulent kinetic energy \((k = \frac{1}{2} u_i' u_j')\), and \( \mu \) stands for the eddy viscosity, defined as \( \mu = \rho C_k k^2 / \varepsilon \), with \( \varepsilon = \frac{\mu_j}{\rho} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \) being the turbulent kinetic energy dissipation rate.

Thus, the eddy viscosity contains two unknown variables \( k \) and \( \varepsilon \). It is necessary to provide the transport equations for \( k \) and \( \varepsilon \) derived from the momentum equation, follow as:

\[ \rho \frac{\partial \left( \rho u_i \frac{\partial u_i}{\partial x_j} \right)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\mu \frac{\partial u_i}{\partial x_j} + \rho \frac{\partial u_i}{\partial x_j}}{\partial x_j} \right) + G - \rho \varepsilon \] .......................... (6)

\[ \rho \frac{\partial (\rho \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\mu \frac{\partial u_i}{\partial x_j} + \rho \frac{\partial u_i}{\partial x_j}}{\partial x_j} \right) + C_1 \frac{\varepsilon}{k} G - C_2 \frac{\varepsilon^2}{k} \rho \] .......................... (7)

where \( G = \rho \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \right) \), and the constants in this model are \( C_1 = 1.44, C_2 = 1.92, C_3 = 0.09, \sigma_k = 1.0 \) and \( \sigma_\varepsilon = 1.3 \) respectively.

3.3. Reynolds Stress Model (RSM)

In RSM, the eddy viscosity approach is not used, but a partial differential equation (transport equation) for the stress tensor is derived from the Navier-Stokes equation, and can be written as following form:

\[ \frac{\partial}{\partial x_j} \left( \rho u_i u_j + \frac{\partial u_i}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left( \rho \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \right) \]

\[ + \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \right) \]

\[ - \rho \left( u_i u_j + g_i' u_j' \right) \]

\[ + \mu \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_j} \]

\[ - 2 \mu \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_j} \]

\[ - 2 \rho \frac{\partial u_j}{\partial x_j} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_j} \]

.......................... (6)
here, $C_k$ is convection term, $D_{t k}$ is turbulent diffusion term, $D_{k k}$ is molecular diffusion term, $P_k$ is stress production term, $G_k$ is buoyancy production term, $\phi_k$ is pressure strain term, $\varepsilon_k$ is dissipation term and $F_k$ is production by system rotation.

The above Reynolds stress transport equation contains two new variables of turbulent energy ($k$) and turbulent dissipation rate ($\varepsilon$), so their transport equations are as follows:

$$
\rho \frac{\partial (\mu k)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \frac{1}{2} \left( P_k + G_k \right) - \rho \varepsilon \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 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seen from +x axis) can be formed at nozzle bottom. Meanwhile, another counter-clockwise recirculation is found under the slide gate. Such flow pattern is reported in previous study.\(^1\) But the calculated result (Fig. 2(b)) from the RSM indicates that, the fluid expands strongly after throttled by slide gate, and then the flow fields become more complex. The most important is that a clockwise recirculation zone appears at nozzle bottom; its swirl direction disagrees with that calculated by standard k-ε model obviously. Therefore, it is necessary to carry out a water model experiment to verify the adaptability of the turbulence models.

Figure 3 illustrates the velocity vectors and streamline in the SEN under slide gate. According to the pictures of Fig. 3(a) and Fig. 3(b), the fluid stream (region a in the Fig. 3(c)) expands weakly after throttled by slide gate, and then forms a recirculation zone, with a length about 200 mm in the vertical direction, is generated under the slide gate (streamline 1 in the Fig. 3(b)). Meanwhile, most fluid, as seen in the b region in the Fig. 3(c), flows downward to nozzle bottom (streamline 2 in the Fig. 3(b)). However, in the pictures of Fig. 3(d) and Fig. 3(e), the fluid in the region a (as seen in Fig. 3(f)), expands rapidly and strongly, and dashes to the inner wall in the clogging side of slide gate, after a half circumference of SEN inner wall flow distance, the fluid flows back to the slide gate opening side (streamline 1 in the Fig. 3(e)), therefore, a clockwise swirl flow appears under the slide gate in horizontal plane at half SEN section (x>0 mm), and an counter-clockwise swirl flow appears at another half SEN section (x<0 mm) also. And then, the above steam, together with the fluid in the region c (seen in Fig. 3(f)), flows along a spiral trace on the inner wall surface to the place just below the slide gate, and encounters with another fluid stream flowing along the opposite half circular arc, and then splits into two steams: the upward steam and the downward steam. The upward steam and the fluid in the region b (as seen in Fig. 3(f)) flow along the inner wall to generate a recirculation zone in vertical direction. The encounter point of the vertical recirculation zone and the horizontal recirculation zone is the lowest dash point of the fluid (streamline 3 in Fig. 3(e)) in the narrow center plane to the inner wall of the SEN. The downward steam together with the stream in region c (as seen in Fig. 3(e)) flow along the inner surface of the SEN, and the fluid in region d (as seen in Fig. 3(e)) flow downward to nozzle bottom.

Figure 4 indicates the time average velocity distributions at nozzle port using standard k-ε model, RSM and UDV respectively. It is very obvious that the calculated result using RSM is close to experimental result measured by UDV, the fluid flow presents clockwise rotation, but the rotation direction calculated by standard k-ε model is contrary to the UDV measurement result.

5. Discussion

To compare the rotation direction of outflow at nozzle port obtained by standard k-ε model, RSM and UDV, it is find that the flow characteristic calculated by standard k-ε model disagrees with the UDV measured result, it may be a result of self-reasons of the turbulence models.

5.1. Kinetics Theory Analysis

An analogy method is adopted in the Boussinesq hypothesis. In the eddy viscosity model, turbulent fluctuation is analogous to the molecular motion, mean velocity of the turbulence is analogous to the mean molecular velocity, and the Reynolds stress produced by turbulent fluctuation is analogous to the viscous shear stress of molecular motion. Hence, the expression of Reynolds stress generated by turbulence should be a similar form to the viscous shear stress generated by molecular motion. Due to the deviatoric stress tensor constitutive equation of Newtonian fluid, the expression of Reynolds stress tensor of incompressible fluid can be obtained as Eq. (3). It is obvious that to introduce the positive scalar of eddy viscosity is just a modification of the dynamic viscosity of fluid, the Reynolds stress generated by the eddy irregular motion of turbulence is summarized as the increase of the effective viscosity.

The concept of eddy viscosity is based on the analogy of turbulent transport of momentum and molecular transport. According to the kinetic theory of ideal gases, the viscosity of ideal gas is given by \( \mu = \frac{1}{3} \rho \bar{c} \lambda \), where \( \bar{c} \) is the mean molecular speed, and \( \lambda \) is the mean free path. Correspondingly, eddy viscosity may be proportional to the product of turbulent velocity scale and turbulent length scale (\( u' \) and \( \ell' \) respectively) \( v_t \propto u' \ell' \), with the turbulent velocity scale playing the role of mean molecular speed and the turbulent length scale playing the role of the mean-free-path. In standard k-ε model, \( \ell' \) is expressed as the function of turbulent kinetic energy (\( k \)) and turbulent kinetic energy dissipation (\( \varepsilon \)), and specified as \( \ell' = k^{1/2} / \varepsilon \). \( u' \) can be written as \( u' = k^{1/2} \), the eddy viscosity can be written as \( v_t = C_s k^2 / \varepsilon \) correspondingly.

However, in simple laminar flow (with shear rate
\( \frac{\partial u}{\partial x_2} = S = U / L \), the ratio of the molecular timescale \( \lambda / C \) and the shear timescale \( S^{-1} \) is \( \frac{\lambda}{C} = \frac{k U}{L C} \approx Kn Ma \), which is very small, about \( 10^{-10} \). From the view of the statistics, the effect of molecular collision disappears rapidly, and the fluid rapidly adjusts to the imposed straining state. Consequently, the shear viscous stress can be represented by the local velocity gradient, and the motion history is not considered. For the turbulence, the ratio of the turbulence timescale \( \tau = k / \varepsilon \) to and the mean shear timescale \( S^{-1} \) is not small, usually bigger than 1.0; the relaxation time of turbulent vortex is very long, which can reach several minutes.\(^{80}\) the turbulence does not recover to imposed mean straining rapidly, and so there is no general basis for a local relationship between the stress and the rate of strain. Additionally, in this period, the turbulent eddies can be carried to a long distance through convection and/or diffusion, the mean values of some turbulent characteristics of these turbulent eddies are affected by the flow history strongly, so the history process of the turbulence flow must be considered.\(^{80}\)

On the other hand, the turbulence fluctuation, with the properties of randomness and irregularity, usually represents anisotropy, and it is strong in the main flow direction, and weak in the other directions. The scalar quantity of eddy viscosity implies that the ratio of Reynolds stress and mean velocity gradient is equal in all directions. In fact, the eddy viscosity should be a second-order tensor, so there is a distinct difference between the Boussinesq hypothesis and real turbulence.

5.2. Isotropy Hypothesis

The Reynolds stress is proportional to the local mean strain-rate, and the ratio between Reynolds stress and mean rate of deformation is the same in all directions. However, this hypothesis is in contradiction with a lot of actual flows, even for some simple flows, the eddy viscosity model encounters great difficulties. Figure 5 is a sketch of homogeneous turbulent shear flow, which is used to analyze the prediction accuracy of the Reynolds stresses using eddy viscosity model. When the homogeneous turbulent shear flow reaches a steady state, the mean velocity gradients \( \partial \bar{u} / \partial x \) and \( \partial \bar{v} / \partial y \) are equal to zero, according to Boussinesq hypothesis; the Reynolds stress in \(-\)-direction can be written as

\[
\tau_{xx} = -\rho u' \bar{u}' = -\rho u'^{\prime 2} = 2 \mu \left( \frac{\partial \bar{u}}{\partial x} \right) - \frac{2}{3} \rho k = -\frac{2}{3} \rho k \ldots \ldots \ldots (10)
\]

\[
\tau_{yy} = -\rho v' \bar{v}' = -\rho v'^{\prime 2} = 2 \mu \left( \frac{\partial \bar{v}}{\partial y} \right) - \frac{2}{3} \rho k = -\frac{2}{3} \rho k \ldots \ldots \ldots (11)
\]

According to Eq. (10) and Eq. (11), the following expressions \( \tau_{xx} = \tau_{yy} \) and \( u' = v' \) can be easily obtained, it means that the Reynolds stress (or fluctuating velocity) in \(-\)-direction is equal to that in \(+\)-direction. However, for this homogeneous shear flow, the Reynolds stress has been measured by Tavoularis\(^{81}\) using experiment and calculated by Moin\(^{80}\) with DNS respectively, the research results are shown in Table 2. Table 2 indicates that the Reynolds stress in the main flow direction (\(+\)-direction) is much bigger than that in the other directions, it is about 2 times and 3 times of that in \(-\) and \(-\)-direction, there is a great disparity with the analysis result (\( \tau_{xx} = \tau_{yy} \)) using eddy viscosity model. Therefore, eddy viscosity models have significant deficiencies; some consequences obtained by eddy viscosity assumption (Eq. (3)) are not being valid. In three-dimensional flows, the Reynolds stress and the strain rate may not be related in such a simple way. This means that the eddy viscosity may no longer be a scalar; indeed, both measurements and simulations show that it becomes a tensor quantity.\(^{11}\) The introduction of eddy viscosity may lead to the equality of normal Reynolds stress in some flow conditions, but in some complex flows, such as, separated flow, secondary flow in long non-circular ducts, strong swirl flow, streamline curvature, which are induced by the normal Reynolds stress, the eddy viscosity model loses simulating capacity of Reynolds stress and predicts the inaccuracy result.

5.3. History Effects

In standard \( k-\varepsilon \) model, eddy viscosity is written to be a function of turbulent kinetic energy and its dissipation rate, it can reflect a certain history effects, but the real turbulence characteristics are not reflected completely. Figure 6 is a sketch of axi-symmetric contraction turbulence flow generated by turbulence-generating grid. According to the eddy viscosity model, the mean strain rate at the axi-symmetric contraction section in \(+\)-direction can be written as
in some simple flows, such as turbulent shear flow, axisymmetric contract turbulent flow, etc., which can further influence the solution of the Navier-Stokes equation. So that for some complex flows, the predicting results from eddy viscosity model are not convinced completely.

The standard $k$-$\varepsilon$ model, based on Boussinesq hypothesis, is the most widely used turbulence model in practical engineer fields. This model is a high Reynolds number turbulence model; it is only used under the circumstances of the equilibrium of production and dissipation of turbulence fluctuation. However, in the suddenly change turbulent flows, the production and dissipation of turbulence fluctuation cannot reach to equilibrium state immediately, it will lead to inaccurate flow predictions. In order to handle complex flow features, the Reynolds stress model (RSM) is designed.

In the RSM, the eddy viscosity is abandoned, and Reynolds stress transport equations are solved directly, so this model can correct some of the Boussinesq approximation’s shortcomings. First, equation (6) automatically accounts for the convection and diffusion of Reynolds stress, so the effects of flow history will be included. The dissipation and turbulent-transport terms in the equation (6) indicates that time scales unrelated to mean-flow time scales are considered, so history effects should be more realistically represented than with standard $k$-$\varepsilon$ model. Second, equation (6) contains convection, production and body-force terms that respond automatically to effects such as streamline curvature, system rotation and stratification. Third, equation (6) gives no a priori reason for the normal stresses to be equal even when the mean strain rate vanishes. Rather, their values will depend upon initial conditions and other flow processes, so that the model should behave properly for flows with sudden changes in strain rate. Consequently, the RSM has an accurate predicting ability for the Reynolds stress in some complex turbulent flow, and the performance of turbulence models has been improved greatly also.

5.5. Discussion of Results

The outflow characteristic at nozzle port is very important, because it governs the flow in the mold. But in some previous works,\textsuperscript{12,13} flow rate control devices (slide gate or stopper rod) are not considered, so these research results can not reveal the fluid laws in the SEN and the mold completely.

Figure 8 compares flow characteristics at nozzle port between previous work\textsuperscript{14} (including calculated result by $k$-$\varepsilon$ model in present study) and UDV measurement result and/or calculated result by RSM in present study. In the reference\textsuperscript{1}, the Argon was injected ($f_a=5.8\%$), and the slide gate opening mode was as the same as in the present study, but its opening ratio was $52\%$, the schematic of outflow at nozzle port is shown in Fig. 8(a). Compared with present result (Fig. 8(b)), it is very obvious that the rotation direction is contrary to each other, but in present study, Argon is not injected, and the slide gate opening ration (30%) is smaller than previous work. Thus, Argon injection and the opening ratio may lead to the different results.

In present study, after introduction of slide gate into the continuous caster, fluid will be throttled at the slide gate region, the strain rate will be change suddenly, and the streamline will be deformed, so that a secondary flow and a
separated flow can be formed under the slide gate; meanwhile, a swirl flow appears at nozzle bottom, these flow phenomena have the anisotropic feature, so that the isotropic standard $k$-$\varepsilon$ model is unable to obtain the accurate results, and it is proved by the UDV measurement data. It is concluded that standard $k$-$\varepsilon$ model is not accommodated in the continuous caster with slide gate. However, RSM accounts for the effects of streamline curvature, swirl, rotation, and rapid changes in strain rate in a more rigorous manner than two-equation models (standard $k$-$\varepsilon$ model), it has greater potential to give accurate predictions for complex flows in SEN controlled by slide gate, and it is verification by water model.

6. Conclusions

The standard $k$-$\varepsilon$ model, based on the isotropic Boussinesq hypothesis, can not accurately predict the sudden expand flow, secondary flow and separated flow caused by slide gate. However, the RSM through directly solving the Reynolds stress transport equations can reflect the flow in the SEN and outflow characteristic at nozzle port, so it is shown that the RSM has strong adaptability in the simulation of complex flows. From the simulation results of the RSM and measurement data by UDV, the following conclusions can be drawn.

1. The flow becomes more complex after introduction slide gate, a secondary flow and a separated flow formed under the slide gate.

2. A swirl flow appears at nozzle port, with the swirl direction from the clogging side via nozzle bottom to the opening side of the slide gate.

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REFERENCES

1) H. Bai and B. G. Thomas: Metall. Mater. Trans. B, 32B (2001), 253.
2) N. Kubo, J. Kubota and T. Ishii: ISIJ Int., 41 (2001), 1221.
3) L. Zhang, Y. Wang and X. Zuo: Metall. Mater. Trans. B, 39B (2008), 534.
4) R. Weber, B. M. Visser and F. Boysan: Int. J. Heat Fluid Flow, 11 (1990), 225.
5) V. C. Patel and F. Sotiropoulos: Proc. Aerospace Sci., 33 (1997), 1.
6) J. L. Xu: Ph. D. Thesis, Graduate School of the Chinese Academy of Sciences, (2008).
7) J. Blazek: Computational fluid dynamics: Principles and applications, Elsevier Science Ltd, Kidlington, UK, (2001), 233.
8) S. T. Cai: Turbulence Theory, Shanghai Jiao Tong University Press, Shanghai, (1993), 143.
9) S. Tavoularis and S. Corrsin: J. Fluid Mech., 104 (1981), 311.
10) M. M. Roger and P. Moin: J. Fluid Mech., 176 (1987), 33.
11) J. H. Ferziger and M. Peric: Computational Methods for Fluid Dynamics, Springer, Berlin, Germany, (2001), 305.
12) H. J. Tucker: Report, McGill University, Mechanical Engineering Department, (1970).
13) M. J. Lee and W. C. Reynolds: Report, Stanford University, (1985).
14) Z. Warhaft: J. Fluid Mech., 99 (1980), 345.
15) D. E. Hershey, B. G. Thomas and F. M. Najjar: Int. J. Numer. Methods Fluids, 17 (1993), 23.
16) D. Gupta, S. Subramaniam and A. K. Lahiri: Steel Res., 62 (1991), 496.
17) C. Real, R. Miranda, C. Vilchis, M. Barron, L. Hoyos and J. Gonzalez: ISIJ Int., 46 (2006), 1183.