A Computational Review of The Solar Wind in Oblate Spheroidal Coordinates Perspective

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Abstract. The solar as theoretical is known not perfectly round. Many types of research have been trying to approach the states and properties of the solar. Complexities of these issues are very interesting to study and observe. In this paper presented the review of a solar wind that blows from its surface based on Parker’s model of solar wind that approached using continuity and momentum equations in the fluid. But in this case, the solar wind also approached in oblate spheroidal coordinates. This paper didn’t describe the effect of the magnetic field and only calculated in the radial direction. The solar wind equation in oblate spheroidal coordinates has been observed in around the pole and equator. These approached equations have been solved computationally using parameters of \( G \), \( M^* \), \( a \) and \( c_s \).

1. Introduction
It has been shown theoretically that the shape of the sun isn’t perfectly round, but suffered flattening caused by the rotation. The symptom that attracted the most attention in the review of a stellar is the wind that blows from the surface of the stellar. This is due to several reasons, namely wind interactions with the environment to produce shock, which creates an impression amazing [1]. The presence of a stellar wind is a major supplier of mechanical energy source in an interstellar medium (ISM). The stellar wind is an important medium for the repatriation of stellar nucleosynthesis results into the ISM. A nucleosynthesis is a process of making a novel atomic from nucleons.

The stellar wind is a case of reversal of the process of the stellar accretion. A both of them involve identical equations i.e. the continuity equation, momentum equation, \( M^* \) is the value of the missing mass, and \( u \) is the velocity flow. When it shows acceleration, the forces involved in generating additional variables in the momentum equation. In practice, the stellar wind flows as time-dependent hydrodynamics problems [2]. Symptoms of the stellar wind aren’t detected directly. Most of the stellar are bigger, hotter, denser, spins faster, and has a magnetic field that is stronger, older or younger than the solar [3]. The best study of the stellar wind is the solar wind.

At the time of the gravitational field of the solar isn’t able to hold the shocks of the hot gas, the corona radiates out continuously. The corona with a high temperature, all the elements are ionized and since the chances of recombination are low, the gas that comes out of the solar intangible ion up to a distance of the astronomical unit. The steady flow of ionized gas particles emitted from the solar is
highly conductive plasma giving rise to a magnetic field or also known as the solar wind [4]. The solar wind consists of a hot gas of charged particles emitted from the sun flowing radially outward through the solar system and into interstellar space. It takes several days for its journey to the earth [5]. The photoionization has an important role in the charge state composition of the solar wind [6]. The differences in ion temperature properties are affected by their charge and mass [7].

Aims of this paper are approaching the solar wind in oblate spheroidal coordinates computationally. In this paper also defines the mechanism of the solar wind model and its behavior in the oblate spheroidal. This paper concludes the effect of the magnetic field in solving of the solar wind within the oblate spheroidal and the solar wind calculated is only in the radial direction. This paper also defines fluid basic equation namely continuity and momentum equations in the oblate spheroidal coordinate system which is implemented in the fluid equation to obtain solar wind equation in the oblate spheroidal coordinate.

2. Related works

Contents of the universe are dominated by gas matter. Each gram unit of it consists of particles ordered $10^{24}$ such as atoms, ions, protons, electrons, etc. The state of the gas system is very dynamic, each constituent particle moves freely assumed not to collide and no interaction of force on each particle. It is seen that the universe is a complex system and not easy to describe the patterns of events on it. In many cases, the complexity of the system in the universe is able to be approached with another concept [8]. In this paper to approach this phenomenon is using a fluid concept [9].

The depiction of the dynamics of complex systems isn’t easy as a look and complicated. Some cases due to the complex system could be approached a matter using matter approaching as a fluid. In the implementation of the fluid study involves many macroscopic physics concepts. The model of dynamic fluid analysis is able to perform a hassle of microphysics contained into a single parameter i.e. a state equation. The state equation obtained could be resolved easily with including to dynamic fluid equations.

2.1. Fluids

The term of fluid relates to gasses and liquids. Gasses and liquids have more relation than with solids. Since both of them have molecules that are free to move around and not isolated in a place as in a solid. The hotter the fluid, the faster its molecules move on and the more area the fluid will flow. Fluid constituent atomic particles differ from atoms structuring the solid matter. Inter fluid constituent particle drifted to one another while the solid constituent particles bound to the crystal lattice. Practically the fluid is always composed of microscopic particles, however, the equations of the hydrodynamics have imposed the fluid as a continuous medium with macroscopic properties such as well define and having pressure ($p$), density ($\rho$), and velocity ($u$) given at each point. A medium is described as a fluid at any point defined physical quantities such as density $\rho(r, t)$, velocity $u(r, t)$, where $r$ is the vector position of the time $t$. The magnitude of the fluid dynamic is a well defined on the range of such areas.

2.2. Dynamic Fluid

There is a special perspective when such a system enacted as a fluid, namely number of macroscopic representing averages defined locally in such element fluid within region range. A fluid dynamic equation is a form of a statement of conservation law depicting fluid element motion based on Newton’s law of motion i.e. conservation equation of mass, momentum, and energy. Presentation of the quantities of fluid that can change over time is able to be viewed in two ways namely Lagrangian and Euler. According to Euler, a partial differential $dp/dt$ is implemented to measure changing of $\rho$ at $t$ on position $r$. Meanwhile, according to Lagrangian, the density changing value measured belongs to element fluid move together with fluid. From this case, the changing value is known as Lagrangian
differential and symbolized as a total differential. The basic equations to depict stellar wind are Euler equation and momentum equation.

2.2.1. A mass conservation. In the fluid, consider that there is a volume $V$ limited by surface $S$ so the fluid mass included in the volume given by equation 1.

$$ m = \int_V \rho dV $$

(1)

The mass is in volume $V$ could be changed only through fluid flow penetrating surface $S$, thus the mass conservation written as equation 2.

$$ \frac{d}{dt} \int_V \rho dV = -\int_S \rho u dS $$

(2)

where $dS$ is an area element on the surface $S$. That’s way the volume is fixed, so the left side could be derived and with implementing divergent theorem on the right side it would be obtained equation 3.

$$ \int_V \left( \frac{d\rho}{dt} + \text{div}(\rho u) \right) dV = 0 $$

(3)

Volume $V$ exists in any value but not zero, so the integrant could be neglected in this case it would form as equation 4.

$$ \frac{d\rho}{dt} + \text{div}(\rho u) = 0 $$

(4)

This case could be simplified as equation 5.

$$ \frac{d\rho}{dt} + \frac{d}{dx_j}(\rho u_j) = 0 $$

(5)

2.2.2. A momentum conservation. The momentum equation is obtained with the same way using an average calculation of total momentum changing in volume $V$ as given in equation 6.

$$ \frac{d}{dt} \int_V \rho u dV $$

(6)

In this case, the momentum flux penetrating the surface $S$ couldn’t be neglected with calculating of force at each unit of the volume known as force flux $f_i$ at the fluid and surface strain, in this case given as a strain tensor $T_{ij}$, Thus obtained momentum equation as given in equation 7.

$$ \frac{d}{dt}(\rho u_i) + \frac{d}{dx_j}(\rho u_i u_j) = f_i + \frac{d}{dx_j}(T_{ij}) $$

(7)

The supporting forces that are often considered in this case. One of it is a gravitation force as shown in equation 8.
The gravitation potential equation due to the density presented by Poisson equation (see equation 9)
\[ \nabla^2 \Phi = 4\pi G \rho \]  
(9)
where \( G \) is a gravitation constant. The other force is the magnetic force that is shown in equation 10.
\[ f_i = \{j \wedge B\} \]  
(10)
where \( j \) is a current and \( B \) is a magnetic field while for electrical force written as equation 11.
\[ f_i = \rho_q E_i \]  
(11)
where \( \rho_q \) is an electrical current density and \( E_i \) is a magnetic field.

A strain tensor defined as infinitesimal surface vector elements \( dS \) at fluid. A vector unit is an extent of surface element and vector direction as a normalization on the surface element. So the surface force \( F_i \) is given according to equation 12.
\[ F_i = T_{ij} dS_j \]  
(12)
where \( dS \) and \( F \) are vectors so \( T_{ij} \) is a second ordered tensor. In this case, the strain tensor donated by pressure \( p \) as shown in equation 13.
\[ T_{ij} = -\rho \delta_{ij} \]  
(13)
where \( \delta_{ij} \) is a Kronecker delta. The magnetic force as strain tensors written in equation 14.
\[ m_j = B_i B_j - \frac{1}{2} \delta_{ij} B_i B_k \]  
(14)
Although it seems not to calculate the viscosity effect, but in this case, the strain donated by the viscosity effect is still showing from \( \frac{du}{dx} \). The strain tensor is the average including information in the element fluid relative flow. Physically, this shows a microscopic motion especially the temperature in the gas particles. These particles will take a momentum as long as the average free path. By implementing equation 4 and to replace a partial differential \( \frac{d\rho}{dt} \) obtained a momentum equation or a fluid motion equation as shown in equation 15.
\[ \frac{du}{dt} + u_j \frac{du_i}{dx_j} = -\frac{1}{\rho} \frac{d\rho}{dx_i} - \frac{d\Phi}{dx_i} + \frac{dm_j}{dx_j} \]  
(15)

3. Tensor Approaching
Coordinates and differential operators against time in the fluid equation are contravariant vectors, whereas differentials in Cartesian coordinates replaced with covariant differentials using metric tensor $g_{ij}$ with a line element $dS$ that is given in equation 16 [10].

$$ds^2 = g_{ij} dx^i dx^j$$  \hspace{1cm} (16)

The continuity equation could be written in the tensor notation as written in equation 17.

$$\frac{dp}{dt} + (\rho \nu^i) = 0$$  \hspace{1cm} (17)

with coordinate axis $x^i$, namely a sum of all space components with $i=1,2,3$ and the contravariant velocity vector $\nu^i$ is a space coordinate differential against time $\nu^i=dx^i/dt$. The symbol of semicolon shows a covariant differential so the contravariant vector random $T^i$ is able to enact the equation 18.

$$T^i_{\;j} = \frac{dT^i}{dx^j} + \Gamma^i_{\;j} T^j$$  \hspace{1cm} (18)

with $\Gamma^i_{\;j}$ is the second form of Christoffel symbol and for a scalar number $\phi$, its covariant differential could be written as equation 19.

$$\psi_{,ij} = \frac{d\psi}{dx^i}$$  \hspace{1cm} (19)

So the momentum equation will be as equation 20.

$$\frac{dv^i}{dt} + \nu^i v^j = -g^{ij} \left( \frac{1}{\rho} p_{,j} + \psi_{,j} \right)$$  \hspace{1cm} (20)

where $p$ is a pressure and $\psi$ is a gravitation potential.

4. Fluid equation in oblate spheroidal coordinates

Oblate spheroidal coordinates are coordinate systems of a confocal ellipsoid or known as planetary ellipsoid because the earth shape is close to this. The confocal ellipsoid coordinate system is often used to solve this problem in mechanics, electrodynamic potential, and hydrodynamic. General coordinates $(\xi, \upsilon, \phi)$ are chosen in oblate spheroidal coordinates and the Cartesian coordinate transformation to oblate spheroidal coordinate is shown in equation 21.

$$x = a \cosh \xi \sin \upsilon \cos \phi, \hspace{1cm}$$
$$y = a \cosh \xi \sin \upsilon \sin \phi, \hspace{1cm}$$
$$z = a \sinh \xi \cos \upsilon$$  \hspace{1cm} (21)

with $0<\xi<\epsilon$, $0<\upsilon<\pi$, $x=r \cos \phi$, $y=r \sin \phi$, $0<\phi<2\pi$. The coordinate axis $z$ as a rotation axis and curves focus on the plane spot $x=0$, the centre is zero $(0)$, and radius $a$ for $\xi$ is a fixed value. When the value of $\xi$ is changed, it will be obtained a confocal oblate spheroidal. The scale factor is valued $h_{,i} = h_{,i} = a(\sinh^2 \xi \cos^2 \upsilon)^{1/2}$, $h_i = a \cosh \xi \sin \upsilon$. Oblate spheroidal coordinates are shown in Figure 1.
So the metric space for oblate spheroidal coordinates \((\zeta, \nu, \phi)\) is shown in equation 22.

\[
ds^2 = a^2 \left( \sinh^2 \zeta \cos \nu \left( d\xi^2 + d\nu^2 \right) + a^2 \cosh^2 \zeta \sin^2 \nu d\phi^2 \right) \tag{22}
\]

A diagonal metric of \(g_{ij}\) for \(i = j = k\) and there is no re-added index could be calculated using Christoffel \(\Gamma^i_{\mu\nu}\) using equation 23.

\[
\Gamma^i_{\mu\nu} = 0, 1^i_{\mu} = -\frac{1}{2g_{\mu\nu}} \frac{dg_{\mu\nu}}{dx^i}, \quad 1^i_{\nu} = -\frac{1}{2g_{\mu\nu}} \frac{dg_{\mu\nu}}{dx^i}, \quad 1^i_{\phi} = -\frac{1}{2g_{\mu\nu}} \frac{dg_{\mu\nu}}{dx^i} \tag{23}
\]

It could be obtained the Christoffel symbol existed as below,

\[
\begin{align*}
\Gamma^1_{11} &= \frac{\sinh \zeta \cosh \zeta}{\left( \sinh^2 \zeta + \cos^2 \nu \right)} \\
\Gamma^1_{12} &= \Gamma^1_{21} = \frac{1}{2} \frac{\sin 2\nu}{\left( \sinh^2 \zeta + \cos^2 \nu \right)} \\
\Gamma^1_{22} &= \frac{\sin \zeta \cosh \zeta}{\left( \sinh^2 \zeta + \cos^2 \nu \right)} \\
\Gamma^1_{33} &= -\frac{\sinh \zeta \cosh \zeta \sin^2 \nu}{\left( \sinh^2 \zeta + \cos^2 \nu \right)} \\
\Gamma^2_{11} &= 1 \sin 2\nu \\
\Gamma^2_{12} &= \frac{1}{2} \frac{\cosh^2 \zeta \sin 2\nu}{\left( \sinh^2 \zeta + \cos^2 \nu \right)} \\
\Gamma^2_{22} &= \frac{1}{2} \frac{\cosh^2 \zeta \sin^2 \nu}{\left( \sinh^2 \zeta + \cos^2 \nu \right)} \\
\Gamma^2_{33} &= \tanh \zeta \\
\Gamma^3_{11} &= -1 \csc \nu \\
\Gamma^3_{23} &= -\cot \nu
\end{align*}
\]

From the continuity equation in the oblate spheroidal coordinate system could be written as equation 24.
\[ \frac{d\rho}{dt} + \frac{d\rho u^\zeta}{d\zeta} + \frac{d\rho u^\nu}{d\nu} + \rho u^\nu \left( \frac{2 \sinh \xi \cosh \xi}{\sinh^2 \xi + \cos^2 \nu} \right) + \rho \frac{u^\nu}{\cot \nu \left( \frac{\sin 2\nu}{\sinh^2 \xi + \cos^2 \nu} \right)} = 0 \tag{24} \]

and from the other fluid equation resulted in momentum equation as shown in equation 25, 26, and 27.

\[ \frac{du^\zeta}{dt} + u^\zeta \left( \frac{dt}{d\zeta} \frac{du^\zeta}{d\zeta} + \frac{d\rho u^\zeta}{d\zeta} \frac{\sinh \xi \cosh \xi}{\sinh^2 \xi + \cos^2 \nu} u^\zeta - \frac{1}{2} \frac{\sin 2\nu}{\sinh^2 \xi + \cos^2 \nu} u^\nu \right) + \frac{u^\nu}{\sinh \xi \cosh \xi} \left( \frac{2 \sinh \xi \cosh \xi}{\sinh^2 \xi + \cos^2 \nu} \right) u^\nu - \frac{\sinh \xi \cosh \xi}{\sinh^2 \xi + \cos^2 \nu} u^\nu = 0 \tag{25} \]

\[ \frac{dt}{d\phi} \left( \frac{du^\nu}{d\phi} \frac{\sinh \xi \cosh \xi \sin 2\nu}{\sinh^2 \xi + \cos^2 \nu} u^\nu = \frac{1}{\alpha^2 \sinh^2 \xi + \cos^2 \nu} \left( \frac{1}{\rho \frac{d\zeta}{d\xi}} + \frac{d\phi}{d\xi} \right) \right) + \frac{d\rho}{dt} = 0 \tag{26} \]

\[ \frac{du^\phi}{dt} + \frac{u^\zeta}{\cot \xi u^\phi} + \frac{u^\nu}{\cot \xi u^\phi} \left( \frac{dt}{d\phi} \right) + \frac{d\phi}{d\xi} + \frac{d\theta}{d\phi} + \frac{d\phi}{d\phi} \tag{27} \]

Assumed that the wind flow directs radially (\( \xi \)), \( u^\xi = u^\phi = 0 \), so every rate \( d/d\xi \) and \( d/d\phi \) are neglected or equal to zero (0). So it will obtain continuity and momentum equations more simple as shown in equation 28 and 29 respectively.

\[ \frac{d\rho}{dt} + \frac{d\rho u^\zeta}{d\zeta} + \rho u^\nu \left( \frac{2 \sinh \xi \cosh \xi}{\sinh^2 \xi + \cos^2 \nu} \right) + \frac{d\rho}{dt} + \frac{d\rho u^\nu}{d\nu} = 0 \tag{28} \]

\[ \frac{du^\zeta}{dt} + u^\zeta \left( \frac{dt}{d\zeta} \frac{du^\zeta}{d\zeta} + \frac{d\rho u^\zeta}{d\zeta} \frac{\sinh \xi \cosh \xi}{\sinh^2 \xi + \cos^2 \nu} u^\zeta - \frac{1}{\alpha^2 \sinh^2 \xi + \cos^2 \nu} \left( \frac{1}{\rho \frac{d\zeta}{d\xi}} + \frac{d\phi}{d\xi} \right) \right) = 0 \tag{29} \]

5. Solar wind in oblate spheroidal coordinates
In around the equator, the colatitude degree is about 90° and written as $\nu \approx \pi/2$ so the value of $\cos \nu$ is zero (0) and $\sin \nu$ is one (1). In this case, this approaching is used to implement in around the equator and pole.

### 5.1. Solar wind in around the equator

Based on the momentum equation if enacted in around the equator with $\nu \approx \pi/2$ so it will be obtained as equation 30.

\[
\left( \frac{1}{\rho} \frac{dp}{d\nu} + \frac{d\psi}{d\nu} \right) = 0
\]  
(30)

In this case, the pressure ($p$) and potential ($\psi$) aren’t dependent on $\nu$. The equation for a stellar radius presented in the oblate spheroidal coordinate system as shown in equation 31.

\[
r = a^2 \cosh^2 \xi \sin^2 \nu \cos^2 \varphi + a^2 \cosh^2 \xi \sin^2 \nu \sin^2 \varphi + a^2 \sinh^2 \xi \cos^2 \nu
\]

\[
= a^2 \cosh^2 \xi \sin^2 \nu + a^2 \sinh^2 \xi \cos^2 \nu
\]  
(31)

So the stellar radius from the stellar centre until around the equator, $r_e = a \cosh \xi$, could be found $\psi$ as equation 32.

\[
\psi = -\frac{GM_*}{a \cosh \xi}
\]  
(32)

The momentum equation thus substituted $c_s^2 = \frac{p}{\rho}$ and $\frac{d\psi}{d\xi} = \frac{GM_* \sinh \xi}{a \cosh^2 \xi}$ become equation 33.

\[
u^* \frac{d\nu^*}{d\xi} + \left( \frac{\nu^*}{\cosh \xi} \right)^2 \frac{\cos \varphi}{\sinh \xi} = -\frac{1}{a^2 \sinh^2 \xi} \left( \frac{c_s^2 \frac{dp}{d\xi}}{\rho} + \frac{GM_* \sinh \xi}{a \cosh^2 \xi} \right)
\]  
(33)

The equation 33 is a momentum equation in the first order partial differential equation with independent variable $\rho$ and $\nu^*$ so that the continuity equation could be shown in equation 34.

\[
\frac{d\rho}{d\xi} = -\rho \left( \frac{1}{\nu^*} \frac{d\nu^*}{d\xi} + \frac{2 \cosh \xi}{\sinh \xi} + \tanh \xi \right)
\]  
(34)

The equation 34 then substituted into the equation 33 obtained the momentum equation in the differential equation with independent variable $\nu^*$ as shown in equation 35.

\[
\left( \frac{\nu^* - \frac{c_s^2}{a^2 \sinh^2 \xi} \nu^*}{\cosh \xi} \right) \frac{d\nu^*}{d\xi} + \left( \frac{\nu^*}{\cosh \xi} \right)^2 \frac{\cos \varphi}{\sinh \xi} = -\frac{c_s^2}{a^2 \sinh^2 \xi} \left( \frac{2 \cosh \xi}{a \sinh^2 \xi} + \tanh \xi \right) - \frac{GM_* \sinh \xi}{a \sinh^2 \xi \cosh \xi}
\]  
(35)

The stellar wind equation in around the equator is shown in Figure 2 and 3.
Figure 2. The stellar wind in around the equator with $G=2$, $M_*=1$, $a=1$, and $c_s=1$.

Figure 3. The stellar wind in around the equator with $G=2$, $M_*=1$, $a=2$, and $c_s=1$.

5.2. Solar wind in around the pole

In around the pole, the colatitude degree is assumed 0 and written as $\nu = 0$ so the value of $\cos \nu$ is one (1) and $\sin \nu$ is zero (0). Because the wind velocity is dependent only on the direction $\xi$ so the continuity equation is written as shown in equation 36.

$$\frac{d\rho}{dt} + \frac{d\rho u^\xi}{d\xi} + \rho u^\xi (3 \tanh \xi) = 0$$

Thus the momentum equation is shown in equation 37.

$$\frac{du^\xi}{dt} + u^\xi \frac{du^\xi}{d\xi} + (u^\xi)^2 \tanh \xi = -\frac{1}{a^2 \cosh^2 \xi} \left( \frac{1}{\rho} \frac{d\rho}{d\xi} + \frac{d\psi}{d\xi} \right)$$

Approaching for state around a pole is obtained a stellar radius $r_p = a \sinh \xi$ so assumed the stellar potential as shown in equation 38.

$$\psi = -\frac{GM_*}{a \sinh \xi}$$

So $\frac{d\psi}{d\xi} = \frac{GM_* \cosh \xi}{a \sinh^2 \xi}$ and the steady wind flow is $\frac{d\rho}{dt} = 0$. The continuity is shown in equation 39.

$$\frac{d\rho}{d\xi} = -\rho \left( \frac{1}{u^\xi} \frac{du^\xi}{d\xi} + 3 \tanh \xi \right)$$

and the momentum equation is shown in equation 40.

$$u^\xi \frac{du^\xi}{d\xi} + (u^\xi)^2 \tanh \xi = -\frac{1}{a^2 \cosh^2 \xi} \left( \frac{c^2}{\rho} \frac{d\rho}{d\xi} + \frac{GM_* \cosh \xi}{a \sinh^2 \xi} \right)$$

with substituting the equation 39 into the equation 40, it will result as shown in equation 41.
The stellar wind equation in around the pole is shown in Figure 4 and 5.

\[ \left( u' - \frac{c_s^2}{a^2 \cosh^2 \xi} \frac{1}{u'} \right) \frac{du'}{d\xi} \left( u'^2 \right) \tanh \xi = \frac{3c_s^2 \sinh \xi}{a^2 \cosh^2 \xi} \frac{GM_\ast \sinh \xi}{acosh \xi \sinh^3 \xi} \tag{41} \]

The stellar wind equation in around the pole is shown in Figure 4 and 5.

**Figure 4.** The stellar wind in around pole with \( G=2, M_\ast=1, a=1, \) and \( c_s=1 \).

**Figure 5.** The stellar wind in around pole with \( G=2, M_\ast=1, a=2, \) and \( c_s=1 \).

6. Conclusion
The computational approach to solving differential equations in this paper has been done. The stellar wind equation in oblate spheroidal coordinates has been evaluated in around the pole and the equator. Both of them are same type namely one ordered differential equation as a velocity function at the direction \( \xi \) to the distance \( \xi(U(\xi)) \). These equations have been solved by using a fixed value for \( G, M_\ast, a \) and \( c_s \). Future work of our paper needs more completion research on other colatitude angle and it will also underlie the study of solar wind dynamics process more complex within a magnetic field, viscosity, etc. So it will have an enhanced interpretation.

7. References

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