Computation of Schenberg response function by using finite element modelling

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Abstract. Schenberg is a detector of gravitational waves resonant mass type, with a central frequency of operation of 3200 Hz. Transducers located on the surface of the resonating sphere, according to a distribution half-dodecahedron, are used to monitor a strain amplitude. The development of mechanical impedance matchers that act by increasing the coupling of the transducers with the sphere is a major challenge because of the high frequency and small in size. The objective of this work is to study the Schenberg response function obtained by finite element modeling (FEM). Finally, the result is compared with the result of the simplified model for mass spring type system modeling verifying if that is suitable for the determination of sensitivity detector, as the conclusion the both modeling give the same results.

1. Introduction
SCHENBERG is a spherical resonant-mass Gravitational Wave (GW) detector [1, 2] being built in the Department of Materials and Mechanics of the University of Sao Paulo, Brazil, can be seen on figure 1. The sphere with 65 cm in diameter and weighting 1.15 ton, is made of a copper-aluminum alloy [3] with 94% Cu and 6% Al. The detector will have six electromechanical transducers (motion sensors that monitor the sphere surface movement, transforming the mechanical oscillation in electrical signal [4, 5]), arranged on the spheres surface in a half-dodecahedron distribution. The sensors will be located as if in the center of the six connected pentagons in a dodecahedron surface, following the studies by Merkowitz and Johnson [6, 7] confirmed by Magalhaes and collaborators [8].

By analyzing the signal of such sensors, the amplitudes and the direction of the incoming gravitation wave can be determined [9, 10]. The Schenberg group has decided to use microwave parametric transducers as motion sensors like the one used in the Australian GW detector NIOBÉ [11]. In this kind of transducers, a superconducting cavity is pumped with monochromatic resonant microwaves and when the size of the cavity changes due to vibration (one of the cavity wall is connected to the sphere by the mechanical impedance matcher), two side bands are created in the microwave signal that leaves the cavity. The amplitude of the side band is proportional to the amplitude of the sphere vibration. Such transducer was tested in SCHENBERG [12]. The history of the decisions process that has determined Schenberg characteristics can be seen in [13].
External seismic noise can degrade the detector sensitivity, to decrease such noise the detector will have a suspension (Figure 2) that has the responsibility to decrease this noise below the thermal noise of the sphere, in such a way, Schenberg is a vibrational system as the output being measured is the vibration of the sphere surface.

The simulation was done using SolidWorks Simulation 2010-2011 version.

![Figure 1. Schenberg schematics.](image)

In this work, the system is simulated connecting one impedance matcher on the sphere surface then a white force is applied in the opposite side of the sphere and the frequency response is obtained. Then the frequency response graphic is compared with the results expected from a simplified theoretical model that compares the detector with a system of spring and masses, usually used in the analyses of such detectors. Using of the experience gained with the work with the simulations of the mechanical part of the transducers connected to the sphere surface using finite element modelling.

2. Material used
The materials used in the FEM simulation are summarized in the table 1.

3. The sphere by itself
First of all the sphere was simulated by itself. The values of the frequencies found were (in Hz) 3157.87; 3157.95; 3158.06; 3158.21 and 3158.32 Hz. These values have a mean of 3158.08 Hz with a standard deviation of about 0.1%. The snapshots sides of the ball movement can be in
Figure 2. Schenberg suspension connected to the sphere [14].

Table 1. Material properties.

| Properties       | Symbol | Value 4 |
|------------------|--------|---------|
| Material         | -      | 94%Cu-6%Al |
| Specific mass    | $\rho$ | 8077.5 kg/m$^3$ |
| Young modulus    | $E$    | $1.303 \times 10^{11}$ Pa |
| Poisson ratio    | $\nu$  | 0.364 |
| Damping ratio    | -      | 0.0005 |

figure 3. The sphere oscillates from (a) to (b) then from (b) to (c) and then from (c) to (a). The loss of the five-fold quadrupole degeneracy is due to the finite element modeling approximation. If the sphere have a central hole for the suspension the frequencies are: 3172.50; 3183.00; 3213.60; 3222.90 and 3240.00 Hz. In the second case the central hole breaks the spherical symmetry.

4. The mechanical impedance matcher

The shape chosen for the mechanical impedance matcher was one that is the easiest possible to have its frequency changed into the finite element program: mushroom shape. This shape can be seen in the figure 4 together with its normam modes frequencies calculated by the FEM on the sphere surface. While this does not address the actual format to be used in the detector, its engagement with the antenna is physically similar, which can be simulated by masses and springs.

For simplicity, only one transducer was used in this simulation set of simulations. To correct tune the correct frequency of the impedance matcher to that of the sphere, their dimensions were changed until the resulting scattered modes be as close as possible. The original estimate was based on the frequency described in the book of natural shapes and frequencies by Blevins.
The dimensions of the impedance matcher are: 50 mm of external diameter, 10 mm of base diameter and 1.615 mm of thickness. It is made of the same material of the sphere.

The five modes showed in figure 4 are located in a bandwidth of 30.1 Hertz. The model to compute this bandwidth can be seen in [15]. In order to compute this it is necessary the effective mass of the sphere [16], that gives 313.4 Kg and the effective mass of the impedance matcher [17] what gives 20 g. The calculated bandwidth is of the order of 50 Hz.
5. The FEM noise spectral density
The frequency response due to a white force applied to the surface of the sphere in the opposite side is shown in Figure 5. The upper curve is the deformation in the impedance matcher edge (which has the biggest amplitude) and the lower curve is the deformation in the region where the white force of 1 N^2/Hz is applied, in the region close to 3960 Hz, a peak due to tangential quadrupole mode can be seen and in the region close to 3160 Hz the radial quadrupole mode can be seen. Figure 6 is the same curve closer to the central frequency, it is much easier to estimate the bandwidth in the amplified figure. The bandwidth is around 30 Hz and the amplification is around 10,000 times (length squared).

![Figure 5](image.png)

**Figure 5.** The frequency response on the edge of the impedance matcher due to a white force applied to the white of the sphere.

6. Conclusions
The amplification of the vibration, that can be seen as the difference of the curve in figure 5 in the peak around frequency of 3166 Hz is 100 times (considering the length, the figure shows the length squared over Hertz), in the peak around 3136 Hz the amplification, in length, is higher around 300 times. It should be, by the mathematical model equal to the square root of the ratio between the effective masses, which is of the order of 125.

For the the range of the two which is at the order of 30 Hz in figure 5, which is around 105 times smaller than the mean frequency. The bandwidth computed to be 50 Hz, an interesting feature is that the frequency range in figure 5 in the valley between the two peaks is almost 50 Hz. These differences and the one of the amplification factor are compatibles, it could be improved if the steps in the frequencies were smaller, giving a better resolution for the frequency response curve, the next step of this research is to minimize these steps and improve the output.

Another interesting factor is that the amplification and the ratio between mean frequency and the bandwidth are compatible, as for theory in [15], both should be the square root of the effective masses which gives 125. So the FEM simulation presents compatible results of the one predicted by the modelling using springs and masses, confirming both approaches.
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