Reexamined radiative Decay $B_q^* \rightarrow B_q \gamma$ in Light Cone QCD

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Abstract

The radiative decay $B_q^* \rightarrow B_q \gamma (q = u, d or s)$ is reexamined with a modified light-cone QCD sum rule method, in which adequate chiral operators are chosen as the interpolating fields in the correlators used for a sum rule estimate of the relevant coupling $g_{B_q^* B_q \gamma}$. The resulting sum rules not only show the physical picture consistent with the underlying physics in $B_q^* \rightarrow B_q \gamma$ but also avoid the pollution by the nonlocal matrix element $\langle \gamma(q) | \bar{q}(x) \gamma(0) \gamma_5 q(0) | 0 \rangle$, which starts with twist-3 and thus may bring a large uncertainty to the sum rule predictions. Also, a comparison is made with the previous results from light-cone QCD sum rules and chiral perturbation theory.

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1. INTRODUCTION

The physics of exclusive $B$ decays is particularly interesting for a detailed test of the standard model (SM) and a careful search for potential signature of new physics. With a large amount of available data from the BaBar and Belle, it is believed that a considerable progress will be made in the area in the near future. A good many theoretical works contribute to this subject. A recent progress deserving mention is the presentation of the QCD factorization formula for $B \to \pi\pi$, $\pi K$ and $\pi D$\cite{1}. Nevertheless, to make a reliable estimate of physical amplitudes demands that we calculate precisely the relevant hadronic matrix elements. It is practically beyond our present ability to do such a calculation from first principle. At present, most of efforts have been devoted to looking for phenomenological approaches to nonperturbative QCD dynamics. Among the most successful models is QCD sum rule method \cite{2}. As it is often case, however, this approach has a restricted application to heavy-to-light transitions, failing to give a viable behavior of the form factors with the heavy quark mass at small momentum transfer. This originates technically from a cut-off of vacuum condensate terms in the short distance expansion where only the operators of lower dimension remain, and physically from neglect of the finite correlations between the quarks or the gluons in the physical vacuum. The case can improve by using the operator product expansion (OPE) around the light cone $x^2 \approx 0$ instead of at the short-distance $x \approx 0$. The calculational framework established in such a way is termed light-cone QCD sum rule \cite{3,4}, which combines the well-developed description of exclusive process in terms of perturbative QCD factorization formula with traditional QCD sum rules. It has found wide applications \cite{3-11} in the literature, especially applying to a study on heavy-to-light transitions, and the obtained predictions are encouraging. However, new nonperturbative inputs, the light-cone wavefunctions of light mesons classified by twists, are involved in light-cone sum rules. A question we must answer is whether these wavefunctions correctly reflect the underlying nonperturbative dynamics, in other word, whether they are phenomenologically better known. To date, only the components of leading twist have undergone a systematic investigation \cite{6} for the distribution amplitudes of the nonsinglet, while the high twist wavefunctions, some of which play a role as important as the twist-2 ones in most cases, require a further examination or refinement for improving the accuracy of numerical results. To prevent the resulting sum rules from suffering the possibly sizable contamination by the twist-3 components, an effective prescription \cite{3,11} has been suggested to estimate heavy-to-light form factors. The trick, to be specific, is to choose an adequate chiral current correlator, as allowed by the sum rules. As a result, not only the resulting new sum rules work effectively but also some of the twist-3 components, which make predominant contributions with respect to the other twist-3 wavefunctions in conventional light-cone QCD sum rule calculations, vanish automatically. From the theoretical viewpoint, this procedure offers a certain modification of the existing light-cone QCD sum rule approach to heavy-to-light transitions, making the nonperturbative effects well controlled.

Also, among the important exclusive processes are the $B$ decays with a photon emission. They
have in fact received a variety of model investigations. It is in [4] that the first application was presented of light-cone QCD sum rule technique to this type of processes, where the radiative decay $\Sigma \to PG$ is discussed in detail. Later on, the same approach was applied to study $B \to \rho \gamma$ [7,8], $B \to l \nu \gamma$ [9] and $B_q^* \to B_q \gamma \gamma$ ($q = u, d$ or $s$) [10]. However, the problems with $B_q^* \to B_q \gamma \gamma$ have been at issue. The yielded decay widths in different approaches disagree in numerical results and, to some extent, also in order of magnitude. For instance, light-cone QCD sum rules [10] predict $\Gamma (B_u^* \to \Sigma_c \gamma) = 0.63 \text{KeV}$ and $\Gamma (B_d^* \to \Sigma_d \gamma) = 0.16 \text{KeV}$ while chiral perturbation theory [12] gives $\Gamma (B_u^* \to \Sigma_c \gamma) = 0.14 \text{KeV}$ and $\Gamma (B_d^* \to \Sigma_d \gamma) = 0.09 \text{KeV}$. A definite conclusion depends strongly on our ability to calculate the relevant $B_q^* B_q \gamma \gamma$ coupling constant $g_{B_q^* B_q \gamma \gamma}$, which is intrinsically a nonperturbative quantity and is defined as

$$\langle \gamma (q, \eta) B_q^* (p, e) \mid B_q (p + q) \rangle = g_{B_q^* B_q \gamma \gamma} \epsilon_{\alpha \beta \rho \sigma} p^\alpha q^\beta \eta^\rho e_\gamma^\sigma.$$  

(1)

where $\eta_\mu (e_\mu), q (p)$ are the polarization vector and the momentum of the photon (the $B^*$ meson) respectively.

In the paper we would reexamine the radiative decay $B_q^* \to B_q \gamma \gamma$ by performing a calculation on the coupling $g_{B_q^* B_q \gamma \gamma}$ with an improved light-cone QCD sum rule approach, in which the appropriate chiral currents operators act as the interpolating fields. Our findings, are intriguing: The resulting sum rules may not only be of a clear physical meaning but also avoid the pollution by the nonlocal matrix element $\langle \gamma (q) \mid \bar{q} (x) \gamma_\mu \gamma_5 q (0) \mid 0 \rangle$ starting with twist-3, which is known very poorly and thus may bring a large uncertainty to the sum rule results.

This presentation is organized as follows. In the following section we propose our improved approach. Then a detailed light cone QCD calculation is preformed of the relevant correlator in Section 3. Section 4 is devoted to a numerical discussion on the sum rule results for $g_{B_q^* B_q \gamma \gamma}$ and then an estimate of $\Gamma (B_q^* \to B_q \gamma \gamma)$. We give a simple summary in the last Section.

2. CHIRAL CURRENT CORRELATOR

At the quark level, the radiative transition $B_q^* \to B_q \gamma \gamma$ is dominated by the magnetic moment interaction between the light quark and electromagnetic field due to small recoil of the decaying meson. This fact reveals, from the viewpoint of QCD, that the process in question belongs in the long distance type. It is well known that the current operators used as interpolating fields are by no means uniquely determined in QCD sum rules, and it is allowed to take different correlators providing that the validity of the resulting sum rule remains. This offers a possibility to explore the radiative decay $B_q^* \to B_q \gamma \gamma$ in a way consistent with the physical picture. In order to obtain such a desired sum rule result for $g_{B_q^* B_q \gamma \gamma}$ it is in order to adopt the following correlator,

$$F_\mu (p^2, (p + q)^2) = i \int d^4x e^{ipx} \langle \gamma (q, \eta) \mid T J_\mu^{V-A} (x) J_{\mu}^{S+P} (0) \rangle 0 \rangle$$

$$= \epsilon_{\mu \nu \alpha \beta} p^\nu q^\alpha \eta^\beta F (p^2, (p + q)^2) + \text{Vector Part},$$

(2)

where the chiral current operators $J_{\mu}^{V-A} = \bar{q} \gamma_\mu (1 - \gamma_5) b$ and $J_{\mu}^{S+P} = \bar{b} i (1 + \gamma_5) q$ are chosen as the interpolating fields. For the invariant function $F (p^2, (p + q)^2)$, remarkably, the resulting
hadronic representation \( F^H \left( p^2, (p + q)^2 \right) \) depends not only on resonances of \( J^P = 0^-, 1^- \) but also on those of \( J^P = 0^+, 1^+ \). However, we can safely isolate the pole contribution of \( B_q \) and \( B_q^* \), due to a sizable mass difference between the lowest negative and positive parity states, and express the high state contribution starting from the threshold parameters \( s_0 \) and \( s'_0 \), which are set around the squared masses of the lowest \( 1^+ \) and \( 0^- \) states respectively, in a form of dispersion integral. The result is

\[
F^H \left( p^2, (p + q)^2 \right) = \frac{m_{B_q}^2 m_{B_q}^*}{m_b + m_q} \frac{f_{B_q} f_{B_q^*} g_{B_q^* B_q}}{(p^2 - m_{B_q^*}^2)^2 (p + q)^2 - m_{B_q}^2} + \int \int ds_1 ds_2 \frac{\rho^H (s_1, s_2)}{(p^2 - s_1)^2 (p + q)^2 - m_{B_q}^2},
\]

with the decay constants defined as

\[
\langle B | \bar{b} i \gamma_5 q | 0 \rangle = \frac{m_{B_q}^2}{m_b + m_q} f_{B_q},
\]

\[
\langle 0 | \bar{\tau} \gamma_\mu b | B_q^* \rangle = m_{B_q} f_{B_q^*} e^{(\lambda)}.
\]

After the Borel improvement \( P^2 \rightarrow M_1^2 \), \( (p + q)^2 \rightarrow M_2^2 \), Eq. (3) is of the following form:

\[
F^H \left( M_1^2, M_2^2 \right) = \frac{1}{M_1^2 M_2^2} \left( \frac{m_{B_q}^2 m_{B_q}^*}{m_b + m_q} f_{B_q} f_{B_q^*} g_{B_q^* B_q} e^{-\frac{m_{B_q}^2}{M_1^2} - \frac{m_{B_q}^2}{M_2^2}} + \int \int ds_1 ds_2 \rho^H (s_1, s_2) e^{-\frac{s_1^2}{M_1^2} - \frac{s_2^2}{M_2^2}} \right).
\]

3. LIGHT CONE OPE

In this section, we will make use of the light cone OPE and give a detailed derivation of the theoretical expression for the correlator in Eq.(2). Schematically, the main contributions in the OPE come from the four Feynman diagrams shown in Fig. 1. All these diagrams can be calculated one by one for large space-like momentum regions \( p^2 \ll 0 \) and \( (p + q)^2 \ll 0 \), where the \( b \) quark is far off shell so that the OPE goes effectively.

(1). SHORT DISTANCE CONTRIBUTION

The short distance contribution, as will be shown, is less important from Fig.1 (a) and (b) but its derivation turns out to be more complicated with respect to the long distance case. We have the UV-convergent loop integrals to deal with:

\[
I = \int d^4k \frac{1}{[(p + k)^2 - m_b^2][(k + p + q)^2 - m_q^2](k^2 - m_q^2)},
\]

\[
I^\mu = \int d^4k \frac{k^\mu}{[(p + k)^2 - m_b^2][(k + p + q)^2 - m_q^2](k^2 - m_q^2)}.
\]
To express it in a form of dispersion integral we use the exponential form for Fig. 1 (b), with

\[ F(a)(p^2, (p + q)^2) = \frac{3Q_b}{2\pi^2} \int_0^1 dx \int_0^1 dy \frac{m_q x \overline{x}}{m_q^2 x + m_q^2 \overline{x} - p^2 x \overline{x} \overline{y} - (p + q)^2 x \overline{y} \overline{y}} \]

for Fig. 1 (a) and

\[ F(b)(p^2, (p + q)^2) = \frac{3Q_q}{2\pi^2} \int_0^1 dx \int_0^1 dy \frac{m_q x^2}{m_q^2 x + m_q^2 x - p^2 x \overline{x} \overline{y} - (p + q)^2 x \overline{y}} \]

for Fig. 1 (b), with \( \overline{x} = 1 - x \) and \( \overline{y} = 1 - y \). The total perturbative contribution can be written as

\[ F(a+b)(p^2, (p + q)^2) = \frac{3m_q}{2\pi^2} \int_0^1 dx \int_0^1 dy \int_0^\infty d\alpha x \left[ Q_q x + Q_q \overline{x} \right] e^{-\alpha (m_q^2 + m_q^2 + Q_q^2 x \overline{x} \overline{y} + Q_q^2 x \overline{y})} \]

To express it in a form of dispersion integral we use the exponential form

\[ F(a+b)(p^2, (p + q)^2) = \frac{3m_q}{2\pi^2} \int_0^1 dx \int_0^1 dy \int_0^\infty d\alpha x \left[ Q_q x + Q_q \overline{x} \right] e^{-\alpha (m_q^2 + m_q^2 + Q_q^2 x \overline{x} \overline{y} + Q_q^2 x \overline{y})} \]

where \( Q_1^2 = -p^2 \) and \( Q_2^2 = -(p + q)^2 \). Following the method proposed in [13], we arrive immediately at the perturbative spectral density \( \rho^{(a+b)}(s_1, s_2) \),

\[ \rho^{(a+b)}(s_1, s_2) = \frac{3m_q}{2\pi^2} \int_{x_1}^{x_2} \frac{dx}{x} \left( m_q x + m_q \overline{x} \right) \delta (s_1 - s_2) \Theta \left( s_1 - (m_b + m_q)^2 \right) \Theta \left( s_2 - (m_b + m_q)^2 \right) \]

with

\[ x_1 = \frac{s_1 + m_b^2 - m_q^2 - \sqrt{s_1^2 - 2 \left( m_b^2 + m_q^2 \right) s_1 + \left( m_b^2 - m_q^2 \right)^2}}{2 s_1} \]

\[ x_2 = \frac{s_1 + m_b^2 - m_q^2 + \sqrt{s_1^2 - 2 \left( m_b^2 + m_q^2 \right) s_1 + \left( m_b^2 - m_q^2 \right)^2}}{2 s_1} \]

Furthermore, completing the integral over \( x \) yields

\[ \rho^{(a+b)}(s_1, s_2) = \frac{3m_q}{2\pi^2} \delta (s_1 - s_2) \Theta \left( s_1 - (m_b + m_q)^2 \right) \Theta \left( s_2 - (m_b + m_q)^2 \right) \]

\[ \times \left[ (Q_b - Q_q) \lambda (1, A, B) + Q_q \ln \frac{1 - A + B + \lambda (1, A, B)}{1 - A + B - \lambda (1, A, B)} \right], \]

with \( A = m_b^2/s_1, ~ B = m_q^2/s_2 \) and \( \lambda (1, A, B) \) defined as

\[ \lambda (1, A, B) = \sqrt{1 + A^2 + B^2 - 2 A - 2 B - 2 A B}. \]

Putting everything together, we have the Borel improved result

\[ F^{(a+b)}(M_1^2, M_2^2) = \frac{3m_q}{2\pi^2 M_1^2 M_2^2} \int ds \Theta \left( s - (m_b + m_q)^2 \right) e^{-\frac{s}{M_2^2}} \]

\[ \times \left[ (Q_b - Q_q) \lambda (1, A, B) + Q_q \ln \frac{1 - A + B + \lambda (1, A, B)}{1 - A + B - \lambda (1, A, B)} \right], \]

with \( M^2 = M_1^2 M_2^2/(M_1^2 + M_1^2) \). It is explicitly shown that the short distance contribution is strongly suppressed from the perturbative hard quarks owing to smallness of the light quark mass \( m_q \), just as one may expect on the basis of a naive analysis.
(2). LONG DISTANCE CONTRIBUTION

Fig.1 (c) and (d) denote the long distance effects induced by the light quark propagating in vacuum and interaction of the light quark in vacuum with the external electromagnetic field, respectively. In the light cone OPE, these soft quarks in vacuum are considered to be in a state of finite correlation, namely they are of a non-vanishing momentum [14], which stems from the high dimension operators neglected in conventional QCD sum rule calculations. We might describe the effect by introducing the concept of nonlocal quark condensate [14].

In the first place, we consider the contribution of Fig.1 (c). In this case the nonlocal quark condensate \( \langle \bar{q}(x)q(0) \rangle \) appears in the light cone OPE. To model its behavior near the light-cone \( x^2 = 0 \), a pragmatic strategy is to assume it to obey a distribution of the Gaussian type [14]

\[
\langle \bar{q}(x)q(0) \rangle = \langle \bar{q}q \rangle e^{-\frac{x^2}{\sigma^2}},
\]

with \( x_E^2 = -x^2 \). If comparing Eq.(18) with the short distance expansion

\[
\langle \bar{q}(x)q(0) \rangle = \langle \bar{q}q \rangle - \frac{m_0^2}{16} \langle \bar{q}q \rangle x_E^2 + \cdots
\]

with \( m_0^2 = 0.8 \text{ GeV}^2 \), the parameter \( \rho \) is found to be \( 5 \text{ GeV}^{-2} \). Introducing the parameters \( \alpha, \beta \) and \( \gamma = \alpha + \beta + \rho \) and then performing the cumbersome loop-integral, we obtain the following result for the contribution of the nonlocal quark condensate,

\[
F^{(c)}(p^2, (p + q)^2) = -2\rho^3Q_b \langle \bar{q}q \rangle \int_0^\infty d\alpha d\beta \frac{1}{\gamma^3} e^{-\frac{\alpha\rho}{\gamma^2} + \frac{\beta}{2\gamma^2}Q_E^2 - (\alpha + \beta)m_0^2},
\]

where \( P_E^2 = -p^2, Q_E^2 = -(p + q)^2 \). Furthermore, it can be converted into the Borel transformed form

\[
F^{(c)}(M_1^2, M_2^2) = -2Q_b \langle \bar{q}q \rangle \frac{1}{M_1^2 M_2^2} e^{-\rho m_0^2 \left( \frac{M_1^2 + M_2^2}{\rho M_1^2 M_2^2 - M_1^2 M_2^2} \right)} \Theta \left( \rho - \frac{1}{M_1^2} - \frac{1}{M_2^2} \right).
\]

The \( \Theta \) function restricts the Borel parameters \( M_1^2 \) and \( M_2^2 \) to the range \( \frac{1}{M_1^2} + \frac{1}{M_2^2} < \rho \). We have already checked that as \( \rho \to \infty \) the above result returns precisely to what it should be in the case in which only the leading term remains in (19).

A subtle approach has been suggested to deal with that type of dynamics in Fig.1 (d). The idea is to introduce so called photonic light-cone wavefunctions to parametrize the long distance dynamics. This can be explained specifically as follows: On contracting the \( b \) quark operators, one has the matrix element \( T \sim iQ_q \eta^{(\lambda)}_p \int dze^{i\nu z} \langle 0 | J^\rho_{em}(z) \mathcal{O}(x, 0) | 0 \rangle \), where \( J^\rho_{em}(z) \) is the electromagnetic current operator of a light quark \( q \) with the charge \( Q_q \) and \( \mathcal{O}(x, 0) \) expresses a certain nonlocal operator. If the underlying light quarks are hard we can perform the standard OPE. This is typically the case of Fig.1 (b). For the soft light quarks we leave the light quark operators without contraction and treat \( J^\rho_{em}(z) \) as the interpolating field of a photon state with the momentum \( q \). In such a way we can define a matrix element of the nonlocal operator \( \mathcal{O}(x, 0) \) between the vacuum and the photon state as

\[
\langle \gamma(q) | \mathcal{O}(x, 0) | 0 \rangle = iQ_q \eta^{(\lambda)}_p \int dze^{i\nu z} \langle 0 | J^\rho_{em}(z) \mathcal{O}(x, 0) | 0 \rangle,
\]

(22)
and further parametrize it via a series of photonic light cone wavefunctions, which is equivalent to a summation over all the relevant condensate terms in the case of the short distance OPE, as in the applications of light cone QCD sum rules to $B \to \pi$ transitions. With such a trick, a straightforward calculation of Fig.1 (d) yields

$$F^{(d)}_{\mu}(p^2, (p+q)^2) = \frac{2i}{(2\pi)^4} \int d^4x \int d^4k e^{i(p-k)\cdot x} \langle \gamma(q) \mid \overline{\gamma}(x) \gamma_\mu \gamma_\nu (1 + \gamma_5) q(0) \mid 0 \rangle \frac{k^\nu}{m_b^2 - k^2}. \quad (23)$$

Using the $\gamma$ algebraic relations

$$\gamma_\mu \gamma_\nu = -i\sigma_{\mu\nu} + g_{\mu\nu}, \quad (24)$$

$$\gamma_\mu \gamma_\nu \gamma_5 = -\frac{1}{2} \epsilon_{\mu \nu \alpha \beta} \sigma^{\alpha \beta} + g_{\mu \nu} \gamma_5, \quad (25)$$

and considering the identities

$$\langle \gamma(q) \mid \overline{\gamma}(x) \gamma_5 q(0) \mid 0 \rangle = 0, \quad (26)$$

$$\langle \gamma(q) \mid \overline{q}(x) q(0) \rangle = 0, \quad (27)$$

we have

$$F^{(d)}_{\mu}(p^2, (p+q)^2) = -\frac{2i}{(2\pi)^4} \int d^4x \int d^4k e^{i(p-k)\cdot x} \frac{k^\nu}{m_b^2 - k^2} \left[ i\langle \gamma(q) \mid \overline{\gamma}(x) \sigma_{\mu\nu} q(0) \mid 0 \rangle \right. \right.$$ \left. + \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} \langle \gamma(q) \mid \overline{\gamma}(x) \sigma^{\alpha \beta} q(0) \rangle \right]. \quad (28)$$

An important observation is, as an appealing feature of our approach, that all the long distance dynamics entering the invariant function $F^{(d)}(p^2, (p+q)^2)$ is encoded in the nonlocal matrix element $\langle \gamma(q) \mid \overline{\gamma}(x) \sigma_{\alpha \beta} q(0) \rangle$, only one surviving after contracting the $b$ quark operators. This answers very well to the actual physical picture in $B^\ast_q \to B_q \gamma$ since such a matrix element stands typically for an effect of the magnetic interaction.

Light-cone expansion of $\langle \gamma(q) \mid \overline{\gamma}(x) \sigma_{\alpha \beta} q(0) \rangle$ involves, to next-to-leading order in $x^2$, the leading twist 2 wavefunction $\phi(u)$ and the two twist 4 distributions $f_1(u)$ and $f_2(u)$. The explicit form is

$$\langle \gamma(q) \mid \overline{\gamma}(x) \sigma_{\alpha \beta} q(0) \rangle = Q_q \langle \overline{q}q \rangle \left( \int_0^1 du \chi \phi(u) F_{\alpha \beta}(ux) + \int_0^1 du x^2 f_1(u) F_{\alpha \beta}(ux) \right.$$ \left.$$+ \int_0^1 du f_2(u) \left[ x_\beta x_\gamma F_{\alpha \gamma}(ux) - x_\alpha x_\gamma F_{\beta \gamma}(ux) - x_\alpha x_\beta F_{\alpha \beta}(ux) \right] \right), \quad (29)$$

where $\langle \overline{q}q \rangle$ denotes the quark condensate density, $\chi$ indicates the magnetic susceptibility of the light quark and $F_{\alpha \beta}$ is the external electromagnetic field tensor. Considering a simple situation $q \to 0$ [4] is helpful to shed light on the physical meaning of Eq.(29). Substituting Eq.(29) into Eq.(28) yields

$$F^{(d)}(p^2, (p+q)^2) = 2Q_q \langle \overline{q}q \rangle \left( \chi \int_0^1 du \frac{\phi(u)}{m_b^2 - (p+uq)^2} \right.$$ \left.$$- \int_0^1 du \left[ f_1(u) - f_2(u) \right] \left[ \frac{4}{[m_b^2 - (p+uq)^2]^2} \right] + \frac{8m_b^2}{[m_b^2 - (p+uq)^2]^3} \right). \quad (30)$$
An emphasis we would put is that the contribution of the high Fock-state \( q \bar{q} g \), which should be taken into account to the present accuracy, has been neglected for it is expected to be negligibly small from most previous calculations. For later continuum subtraction, we have to convert Eq.\( (30) \) into a form of dispersion integral. We do that for the twist-2 term and remain the twist-4 ones unaffected. The relevant spectral density is easy to evaluate by virtue of the technique in \([13]\) once again. The calculation procedure is as follows. First of all, we apply a double Borel operator in \( p^2 \) and \((p + q)^2\) to the twist-2 term, obtaining,

\[
\hat{B} \left( M_1^2, Q_1^2 \right) \hat{B} \left( M_2^2, Q_2^2 \right) \int_0^1 du \frac{\varphi(u)}{m_b^2 - (p + uq)^2} = \frac{1}{M_1^2 + M_2^2} e^{-\frac{m_b^2}{M_1^2 M_2^2}} \varphi \left( \frac{M_1^2}{M_1^2 + M_2^2} \right). \tag{31}
\]

The symmetry of the correlator in Eq.\( (2) \) allows us to set \( M_1^2 = M_2^2 \) so that the wavefunction \( \varphi \left( \frac{M_1^2}{M_1^2 + M_2^2} \right) \) can take its value at the symmetric point \( \mu_0 = 1/2 \). Considering it and making a replacement \( M_1^2 \to \frac{1}{\sigma_1}, M_2^2 \to \frac{1}{\sigma_2} \) in Eq.\( (31) \), we get

\[
\hat{B} \left( \frac{1}{\sigma_1}, Q_1^2 \right) \hat{B} \left( \frac{1}{\sigma_2}, Q_2^2 \right) \int_0^1 du \frac{\varphi(u)}{m_b^2 - (p + uq)^2} = \frac{\sigma_1 \sigma_2}{\sigma_1 + \sigma_2} e^{-m_b^2(\sigma_1 + \sigma_2)} \varphi(1/2) = f \left( \sigma_1, \sigma_2 \right). \tag{32}
\]

Finally, we take the function \( f \left( \sigma_1, \sigma_2 \right) = f \left( \sigma_1, \sigma_2 \right) \) and perform one more Borel transformation in the variables \( \sigma_1 \) and \( \sigma_2 \). The relevant spectral density reads

\[
\rho^{(tw2)} \left( s_1, s_2 \right) = \frac{1}{s_1 s_2} \hat{B} \left( \frac{1}{s_1}, \sigma_1 \right) \hat{B} \left( \frac{1}{s_2}, \sigma_2 \right) \hat{f} \left( \sigma_1, \sigma_2 \right) = \sigma_1 \sigma_2 e^{-m_b^2(\sigma_1 + \sigma_2)} \varphi(1/2) = f \left( \sigma_1, \sigma_2 \right). \tag{33}
\]

At present, it suffices to represent the twist-2 contribution in a form of dispersion relation. We derive the following expression for \( F^{(d)} (p^2, (p + q)^2) \),

\[
F^{(d)} (p^2, (p + q)^2) = 2Q_q \langle \bar{q} q \rangle \left( \chi \int_{m_b^2}^{\infty} ds_1 ds_2 \frac{\delta(s_1 - s_2) \varphi(1/2)}{(p - s_1)(p + q)^2 - s_2} \right.
\]

\[
- \left. \int_0^1 du \left[ f_1 (u) - f_2 (u) \right] \left[ \frac{4}{[m_b^2 - (p + uq)^2]^2} + \frac{8m_b^2}{[m_b^2 - (p + uq)^2]^3} \right] \right). \tag{34}
\]

After the Borel transformation it becomes

\[
\tilde{F}^{(d)} \left( M_1^2, M_2^2 \right) = \frac{1}{M_1^2 M_2^2} \left[ 2Q_q \langle \bar{q} q \rangle \varphi(1/2) \chi \int_{m_b^2}^{\infty} ds e^{-\frac{s}{M^2}} - 8Q_q \langle \bar{q} q \rangle \right.
\]

\[
\times \left. \left[ f_1 \left( \frac{1}{2} \right) - f_2 \left( \frac{1}{2} \right) \right] \left( 1 + \frac{m_b^2}{M_2^2} \right) e^{-\frac{m_b^2}{M_2^2}} \right). \tag{35}
\]

Now we conclude this section with writing down the light-cone OPE result for the invariant function \( F \left( p^2, (p + q)^2 \right) \) in the Borel transformed form,

\[
\tilde{F}^{(OPE)} \left( M_1^2, M_2^2 \right) = \tilde{F}^{(a+b)} \left( M_1^2, M_2^2 \right) + \tilde{F}^{(c)} \left( M_1^2, M_2^2 \right) + \tilde{F}^{(d)} \left( M_1^2, M_2^2 \right). \tag{36}
\]
4. SUM RULES AND NUMERICAL DISCUSSIONS

Matching Eq.(36) with the corresponding hadronic representation (6) and using the quark-hadron duality ansatz, in which the hadronic spectral density $\rho^H(s_1,s_2)$ is usually assumed to coincide with that derived in the light-cone OPE, we arrive at the sum rule for the product $\int_B \int_B' g^2 \langle B \gamma \rangle$:

$$\int_B \int_B' g^2 \langle B \gamma \rangle = \frac{m_b + m_q}{m_B^2 m_{B_q}^2} \left\{ 2Q_q \langle \bar{q} q \rangle \chi \varphi (1/2) \left( e^{-\frac{m_b^2}{M^2}} - e^{-\frac{e_m^2}{M^2}} \right) M^2 - 2Q_b \langle \bar{q} q \rangle e^{-\frac{e_m^2}{M^2}} \right\} \times \left[ (Q_b - Q_q) \lambda (1, m_b^2/s, m_q^2/s) + Q_q \ln \frac{1 - m_b^2/s + m_q^2/s + \lambda (1, m_b^2/s, m_q^2/s)}{1 - m_b^2/s + m_q^2/s - \lambda (1, m_b^2/s, m_q^2/s)} \right]$$

(37)

At this point, we have a minor comment. Nonperturbative QCD dynamics dominates the sum rule; especially the magnetic interaction term makes the leading contribution, while the hard quark contribution modifies merely the sum rule at the level of $m_q$, a negligibly small effect. The nonlocal matrix element $\langle \varphi (q) | q(x) \gamma_\mu q(0) | 0 \rangle$, which starts with twist-3, is effectively eliminated, making the sum rule free of the contamination by its uncertainty. In addition, what we should emphasize is that our subtraction procedure is different from that in Ref.[10], in which continuum subtraction $e^{-\frac{m_b^2}{M^2}} \rightarrow e^{-\frac{m_b^2}{M^2}} - e^{-\frac{e_m^2}{M^2}}$ is made for the twist-3 and -4 parts.

The actual calculation needs a dynamical input. The nonperturbative parameters concerning the light quarks include the light-cone wavefunctions $\varphi (1/2)$, $f_1 (1/2)$ and $f_2 (1/2)$, quark condensate $\langle \bar{q} q \rangle$, magnetic susceptibility $\chi$ and light quark mass $m_q$. Among these, the leading twist-2 wavefunction $\varphi (1/2)$ embodies the underlying dynamics and hence demands a precision determination. Fortunately, it follows from a well-found calculation that $\varphi (u)$ is dependent quite weakly on the scale and deviates little from its asymptotic form [4], with a well-tried universality. Therefore, we can use

$$\varphi (u) = 6u (1 - u),$$

(38)

to high accuracy. According to the analysis given in [7], the set of the twist-4 wavefunctions can be specified as

$$f_1 (u) = -\frac{1}{8} (1 - u) (3 - u),$$

(39)

$$f_2 (u) = -\frac{1}{4} (1 - u)^2.$$  

(40)

Since we work in the case that the QCD radiative correlations are neglected, the appropriate scale, at which the quark condensate $\langle \bar{q} q \rangle$ and magnetic susceptibility $\chi$ take values, should be set by the typical virtuality of the underlying $b$ quark $\mu_b \sim \sqrt{m_{B_q}^2 - m_b^2}$, as in Ref.[8]. We take the results used widely $\langle \bar{u} u \rangle (\mu = 1 GeV) = \langle \bar{d} d \rangle (\mu = 1 GeV) = - (0.24 GeV)^3$,
(\bar{s}s) (\mu = 1 \text{ GeV}) = 0.8 \langle \bar{u}u \rangle (\mu = 1 \text{ GeV}), and \chi (\mu = 1 \text{ GeV}) = -4.4 \text{ GeV}^{-2} \text{ [15]}, from which the use of the Renormalization Group Equation (RGE) gets \langle \bar{u}u \rangle (\mu_b) = \langle \bar{d}d \rangle (\mu_b) = -0.018 \text{ GeV}^3, (\bar{s}s) (\mu_b) = -0.014 \text{ GeV}^3 and \chi (\mu_b) = -3.4 \text{ GeV}^{-2}. Furthermore, the light-quark masses are set as \( m_u = m_d \approx 0, m_s = 0.115 \text{ GeV} \). As far as the \( B \) channel parameters are concerned, we use \( m_b = 4.8 \text{ GeV} \) for the \( b \) quark mass and \( m_{B_{u,d}} = 5.279 \text{ GeV}, m_{B_{u,d}}^* = 5.325 \text{ GeV}, m_{B_s} = 5.369 \text{ GeV} \) and \( m_{B_s}^* = 5.416 \text{ GeV} \) for the set of the \( B \) meson masses. The relevant decay constants, for consistency, require a recalculation in the two point sum rules in which a chiral correlator should be chosen in a proper way. Moreover, we would do that at the leading order in \( \alpha_s (Q^2) \) since in present case the sum rule for \( f_{B_q} f_{B_q^*} g_{B_q B_q^* \gamma} \) works at the same accuracy. In the light of the prescription in [11], the best fits lead to \( f_{B_{u,d}} = 120 \text{ MeV}, f_{B_{u,d}^*} = 141 \text{ MeV} \) with \( s_0 = 32 \text{ GeV}^2 \), and \( f_{B_s} = 147 \text{ MeV}, f_{B_s^*} = 160 \text{ MeV} \) with \( s_0 = 34 \text{ GeV}^2 \). All the above parameters will be used as the central values in the following numerical estimates.

To proceed, one has to find the reasonable ranges of \( M^2 \), from which the desired sum rules can be read off. This is a critical step towards deriving a reliable prediction. For this end, we use the standard procedure where the high state contributions are limited to a level below 30% while the twist-4 ones to an order less than 10%; simultaneously it is required that the resulting sum rules be considerably stable. In the first place, we focus on the derivation of the sum rule for \( g_{B_{u}^* B_{u} \gamma} \). In this case, using the above criterion the fiducial values of \( M^2 \) are found to lie between \( 8 - 14 \text{ GeV}^2 \), with \( s_0 = 32 \text{ GeV}^2 \). In this range, \( M^2 \) dependence of the product \( f_{B_u} f_{B_u^*} g_{B_u^* B_u \gamma} \), as shown in Fig. 2, is quite weak. From the "window" we have the sum rule results
\[
 f_{B_u} f_{B_u^*} g_{B_u^* B_u \gamma} = 10.12 \pm 0.17 \text{ MeV}, \quad g_{B_u^* B_u \gamma} = 0.59 \pm 0.01 \text{ GeV}^{-1},
\]
the quoted errors being due to the change of \( M^2 \). Furthermore, it is necessary to look into the uncertainties arising from the input parameters. Let \( s_0 \) change between 31 – 33 \( \text{ GeV}^2 \) while the other parameters keep fixed, the resulting variation of the product \( f_{B_u} f_{B_u^*} g_{B_u^* B_u \gamma} \) is roughly \( \pm 8\% \). Utilizing this result and incorporating the corresponding changes of the decay constants, the uncertainty in the coupling \( g_{B_u^* B_u \gamma} \) is estimated at the level of \( \pm 5\% \). To look at the impact of the uncertainty in \( m_b \), we concentrate on the sum rule for \( g_{B_u^* B_u \gamma} \) and consider a correlated variation effect, which appears as \( m_b \) changes from 4.6 \( \text{ GeV} \) to 5.0 \( \text{ GeV} \), in such a way in which we substitute the analytical forms of the sum rules for the decay constants into Eq.(37), observe the variation of \( g_{B_u^* B_u \gamma} \) with \( m_b \) by requiring that all the sum rules take only their values from the best fits in both \( s_0 \) and \( M^2 \). A detailed analysis shows that in this case the resulting influence amounts to \( \pm 6\% \). We add also a numerical uncertainty of 30% to all the condensate parameters and see the effects. This results in an error of \( \pm 8\% \) in the sum rule for \( g_{B_u^* B_u \gamma} \). Finally, the resulting total uncertainty in \( g_{B_u^* B_u \gamma} \) can be fixed at about 21%, by adding linearly up all the considered errors. Also, the same procedure is applied to the case of \( B_d^* B_d \gamma \), however, we would like to consider an additional source of uncertainty in the \( B_s^* B_s \gamma \) case, allowing \( m_s \) to change in the region of 115 ± 25 \text{MeV}. The resulting two sum rules turn out to have the Borel intervals and uncertainties different slightly from the corresponding those in the sum rule for \( f_{B_u} f_{B_u^*} g_{B_u^* B_u \gamma} \). We
don’t give details any more for brevity. The final results read:

\[ f_{B_d} f_{B_s^*} B_{d\gamma} = -5.31 \pm 0.15 \text{ MeV}, \quad g_{B_s^* B_{d\gamma}} = -0.31 \pm 0.01 \text{ GeV}^{-1}, \]  

(42)

with \( s_0 = 32 \text{ GeV}^2 \) and \( M^2 = 7 - 14 \text{ GeV}^2 \), and

\[ f_{B_s} f_{B_s^*} B_{s\gamma} = -7.46 \pm 0.23 \text{ MeV}, \quad g_{B_s^* B_{s\gamma}} = -0.32 \pm 0.01 \text{ GeV}^{-1}, \]  

(43)

with \( s_0 = 34 \text{ GeV}^2 \) and \( M^2 = 8 - 15 \text{ GeV}^2 \). The total uncertainties in the two coupling constants are respectively about 20% and 23%. The sum rule stability is illustrated in Fig.2 too, for the products \( f_{B_d} f_{B_s^*} B_{d\gamma} \) and \( f_{B_s} f_{B_s^*} B_{s\gamma} \).

Having the sum rule results for \( g_{B_s^* B_{s\gamma}} \) at hand, we can calculate the decay widths by means of the formula

\[ \Gamma \left( B_q^* \rightarrow B_q \gamma \right) = \frac{g_{B_s^* B_{s\gamma}}^2}{96\pi} \left( \frac{m_{B_s^*}^2 - m_{B_q}^2}{m_{B_s^*}} \right)^3. \]  

(44)

The results are

\[ \Gamma \left( B_u^* \rightarrow B_u \gamma \right) = 0.89 \pm 0.34 \text{ Kev}, \]  

(45)

\[ \Gamma \left( B_d^* \rightarrow B_d \gamma \right) = 0.25 \pm 0.10 \text{ Kev}, \]  

(46)

\[ \Gamma \left( B_s^* \rightarrow B_s \gamma \right) = 0.28 \pm 0.13 \text{ Kev}. \]  

(47)

It is obvious that the sum rule results (45) and (46), within the errors, are in agreement with the estimates from the standard light-cone QCD sum rules [10]: \( \Gamma \left( B_u^* \rightarrow B_u \gamma \right) = 0.63 \text{ Kev} \) and \( \Gamma \left( B_d^* \rightarrow B_d \gamma \right) = 0.16 \text{ Kev} \). If we make a comparison of our sum rule predictions and chiral perturbation theory ones [12], on the other hand, it is demonstrated that there is a considerable numerical difference in the obtained decay widths in the case of \( B_u^* \rightarrow B_u \gamma \); but the obtained predictions are comparable with each other within errors in the case of \( B_d^* \rightarrow B_d \gamma \). Introducing the ratio \( R = \Gamma \left( B_u^* \rightarrow B_u \gamma \right) / \Gamma \left( B_d^* \rightarrow B_d \gamma \right) \) to see \( SU(3) \) breaking effect in the radiative decays, we find \( R = 0.43 - 2.72 \), with a large uncertainty. However, the resulting lower limit of the ratio is close to \( R = 1/3 \approx 0.33 \) obtained in chiral perturbation theory[12].

5. SUMMARY

To sum up, in this paper we present an improved light-cone QCD sum rule approach to the radiative decay \( B_q^* \rightarrow B_q \gamma \) whose model estimates are at issue, an adequate chiral current correlator being used to get a sum rule result consistent with the underlying physics. In this framework, a detailed derivation is given of the sum rules for the relevant coupling constant \( g_{B_s^* B_{s\gamma}} \), which parametrizes all the long distance dynamics. The resulting sum rules are of the two main features: (i). It is the magnetic interaction which makes the leading contribution to the resulting sum rules; on the contrary, the perturbative hard quarks modify the sum rules only at the level of \( m_q \). This typically is the principal dynamical feature the radiative decay \( B_q^* \rightarrow B_q \gamma \) possesses. (ii). The nonlocal matrix element \( \langle \gamma (q) | \bar{\psi}(x) \gamma_\mu \gamma_5 q | 0 \rangle | 0 \rangle \), which is equally important but poorly known
in comparison with the leading matrix element \( \left\langle \gamma (q) \right| \overline{\Psi} (x) \sigma_{\alpha \beta} q (0) | 0 \right\rangle \) [10], disappears in our sum rules, which controls effectively the pollution due to the long distance effects. In addition, we apply the light cone OPE instead of the short distance expansion to the quark condensate contribution, minimizing the uncertainties in the sum rules to the accuracy in consideration and guaranteeing a self-consistent sum rule calculation. It is predicted that \( \Gamma (B_u^* \to B_u \gamma) = 0.89 \pm 0.34 \) KeV, \( \Gamma (B_d^* \to B_d \gamma) = 0.25 \pm 0.10 \) KeV and \( \Gamma (B_s^* \to B_s \gamma) = 0.28 \pm 0.13 \) KeV. All these channels are predicted to be of the decay widths at the level of \( 10^{-7} \) GeV and therefore are accessible at the current running B factories and the future LHC. Also, a comparison is made with other model predictions. There is an approximate agreement between our results and those from the standard light-cone sum rules [10], for \( \Gamma (B_u^* \to B_u \gamma) \) and \( \Gamma (B_d^* \to B_d \gamma) \). We find also that the resulting sum rule predictions deviate more or less from those of chiral perturbation theory [12], depending on different cases. Striving for perfection of phenomenological models and enhancing ones’ ability in controlling nonperturbative dynamics, we believe the procedure presented here to be a valuable attempt in the direction.

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Fig. 1. Diagrams making the predominate contributions to the correlator (2). (a) and (b) depict perturbative contributions, (c) nonlocal quark condensate correction and (d) emission of photon at long distances parametrized by photon wavefunctions. Solid lines denote quarks, crosses vacuum quark condensates and wavy line photon.
$$f_{Bq} f_{Bq}^* g_{Bq} B_q \gamma \ (GeV)$$

Fig.2. The light-cone QCD sum rules for $f_{Bq} f_{Bq}^* g_{Bq} B_q \gamma$, with $s_0 = 32 \ GeV^2$ for $f_{Bq} f_{Bq}^* g_{Bq} B_q \gamma$ (dotted) and $f_{Bq} f_{Bq}^* g_{Bq} B_q \gamma$ (dashed), and $s_0 = 34 \ GeV^2$ for $f_{Bq} f_{Bq}^* g_{Bq} B_q \gamma$ (solid). It should be understood that we have set $f_{Bq} f_{Bq}^* g_{Bq} B_q \gamma$ and $f_{Bq} f_{Bq}^* g_{Bq} B_q \gamma$ positive, for a comparison.