Neutrinos: past, present, future

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Abstract

A general overview of neutrino physics is given. The history of the neutrino starting from Pauli and Fermi is briefly presented. Two-component neutrinos, the phenomenological $V - A$ theory and the standard model are discussed. The problem of neutrino masses and mixing is reviewed.

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I. INTRODUCTION

Neutrinos play a special role in particle physics and astrophysics. This is determined by the fact that neutrinos have only weak interactions. Let me list some of the most important discoveries related to neutrinos.

1. In the 1950's the electron neutrino was discovered in the experiments of Reines and Cowan.

2. In 1956 the parity non-conservation in $\beta$-decay was discovered (Wu et al.).

3. In 1957 it was proved that the neutrino is a left-handed particle (Goldhaber et al.).

4. In the 1962 Brookhaven experiment of Lederman, Steinberger, Schwarz et al., the muon neutrino was discovered.

5. In 1973 in a neutrino experiment by the Gargamelle collaboration at CERN a new class of weak interactions (neutral currents) was discovered.

6. In the 1980's in experiments on neutrino beams at CERN and at Fermilab, the quark structure of nucleon was established and investigated.

7. In 1983 at CERN, the intermediate $W$ and $Z$ bosons were discovered (UA1 and UA2 collaborations).

8. In the 70's solar neutrinos were detected in pioneering experiment by R. Davis.

9. In 1987 neutrinos from Supernova 1987A were detected (Kamiokande, IMB, Baksan).

10. In the 1990's in LEP experiments it was found that only three types of light flavour neutrinos exist in nature.

This is an impressive list of discoveries, the importance of which for elementary particle physics and astrophysics is difficult to overestimate.

II. THE PAULI HYPOTHESIS

I will start with the history of the neutrino. The neutrino hypothesis was put forward by Pauli in 1930. At that time protons and electrons were considered as elementary particles, and nuclei were considered as bound states of protons and electrons. Such picture confronted with two fundamental problems:

I. The problem of $\beta$-spectrum.

Continuous $\beta$-spectra cannot be explained if $\beta$-decay is a transition of one nucleus into a two-body final state with another nucleus and electron.
II. The problem of the spin of some nuclei.

The classical example is the nucleus $^7\text{N}_{14}$. If nuclei are bound state of protons and electrons, this nucleus must be a bound state of 14 protons and 7 electrons and must have a half-integer spin. From the experimental data it followed that $^7\text{N}_{14}$ nuclei are particles with integral spin.

To solve these problems Pauli assumed that in addition to $p$ and $e$ there exist a new elementary particle (which he called the neutron) with a spin $1/2$, equal to zero electric charge, mass less than the electron mass and an interaction much weaker than photon interaction. He assumed that "neutrons" are constituents of nuclei (thus solving the problem of the spin of $^7\text{N}_{14}$ and other nuclei) and that $\beta$-decay is a three-body decay with a nucleus, an electron and a "neutron", (which is not detected in the experiment) in final state.

In 1932 the neutron was discovered by Chadwick and all nuclear data (including the spin of $^7\text{N}_{14}$ and other nuclei) can be naturally explained under the assumption that nuclei are bound states of protons and neutrons. The problem of $\beta$-decay remained unsolved.

III. FERMI THEORY OF $\beta$-DECAY

The first theory of $\beta$-decay was proposed by Fermi in 1934. This theory was based on the assumption that nuclei are bound states of protons and neutrons. Fermi assumed that the light Pauli particle (which he named the neutrino) exists and is produced in $\beta$-decay together with an electron in the process

$$n \rightarrow p + e^- + \nu$$

(3.1)

The first Hamiltonian of $\beta$-decay was build by Fermi in analogy with the Hamiltonian of electromagnetic interaction

$$\mathcal{H}_\text{em}^\alpha = e\bar{p}\gamma_\alpha p A^\alpha$$

(3.2)

that describes the transition

$$p \rightarrow p + \gamma$$

(3.3)

Fermi assumed that the Hamiltonian of the process (1) has a similar to (2) vector form:

$$\mathcal{H}^\beta = G_F \bar{p}\gamma_\alpha n \bar{e}\gamma^\alpha \nu + \text{h.c.}$$

(3.4)

where $G_F$ is the interaction constant.

Let us stress an important difference between the Hamiltonians (2) and (4). The interaction constant $G_F$ has dimension $M^{-2}$ ( units $\hbar = c = 1$, $M$ is a mass), while an electric charge $e$ is a dimensionless quantity. This is connected to the fact that the Fermi interaction (4) is a four-fermion interaction and the electromagnetic interaction (2) is the interaction of the pair of fermions with a boson.

The experiments on the investigation of $\beta$-decay, that were done after the Fermi theory appeared, showed that the Fermi interaction is not enough to explain the data. In 1936 Gamow and Teller proposed the following most general four-fermion Hamiltonian of $\beta$-decay:
\[ \mathcal{H}_I^\beta = \sum_i G_i \bar{p} O_i n \bar{e} O^i \nu + \text{h.c.} \]  

(3.5)

Here \( O_i \rightarrow 1 \) (scalar), \( \gamma_\alpha \) (vector), \( \sigma_{\alpha \beta} \) (tensor), \( \gamma_\alpha \gamma_5 \) (axial), \( \gamma_5 \) (pseudoscalar). In the Hamiltonian (5) five arbitrary constants enter and there was general belief that the number of fundamental constants in the \( \beta \)-decay Hamiltonian must be much less. During many years the strategy of \( \beta \)-decay experiments was to find dominant variants of \( \beta \)-decay. The situation remained uncertain up to 1956.

**IV. NON-CONSERVATION OF PARITY**

In 1956 it was discovered that parity is not conserved in \( \beta \)-decay (Wu et al.). This discovery completely changed our understanding of \( \beta \)-decay and neutrino.

In the experiment of Wu et al. the dependence of the probability of \( \beta \)-decay of polarized \( ^{60}\text{Co} \) on the angle between the directions of the vector of polarization and the electron momentum was measured. In the case of non-conservation of parity the probability of \( \beta \)-decay of polarized nuclei is given by the following general expression:

\[ dW_{\vec{P}}(\vec{k}) = dW_0 (1 + \alpha \vec{P} \vec{k}), \]

(4.1)

where \( \vec{P} \) is the polarization vector and \( \vec{k} = \frac{\vec{k}}{p} \) (\( p \) is electron momentum). If parity is conserved, in this case

\[ dW_{\vec{P}}(\vec{k}) = dW_{\vec{P}}(-\vec{k}) \]

(4.2)

and the asymmetry parameter \( \alpha \) is equal to zero (the pseudoscalar \( \vec{P} \vec{k} \) cannot enter in the expression for the probability of the decay in the case of parity conservation). In the experiment of Wu et al. it was found that \( \alpha \approx -0.7 \).

The idea of non-conservation of parity in a weak interaction was put forward by Lee and Yang before the Wu et al. experiment. The Hamiltonian of \( \beta \)-decay that they proposed was a direct generalization of the four-fermion Fermi-Gamow-Teller Hamiltonian:

\[ \mathcal{H}_I^\beta = \sum_i \bar{p} O_i n \bar{e} O^i (G_i + G'_i \gamma_5) \nu + \text{h.c.} \]

(4.3)

This Hamiltonian is the most general four-fermion Hamiltonian in the case of non-conservation of parity. It is characterized by 10 (!) arbitrary constants \( G_i \) and \( G'_i \) (\( i = S, V, T, A, P \)).

**V. TWO-COMPONENT NEUTRINO**

In 1957-58 two fundamental stages completely changed the field and brought us to the correct effective theory of \( \beta \)-decay and other weak processes. The first step was done by Landau, Lee and Yang, and Salam. They proposed the two-component neutrino theory.

Any fermion field \( \psi \) can be presented as the sum of left-handed \( \psi_L \) and right-handed \( \psi_R \) components
\[ \psi = \psi_L + \psi_R, \]  
\[ \psi_{L,R} = \frac{1 \pm \gamma_5}{2} \psi \]  
In the case of massless neutrinos, \( \nu_L (\nu_R) \) is the field of neutrino with left (right) helicity and antineutrino with right (left) helicity.

According to the two-component neutrino theory, the neutrino is a massless left-handed (or right-handed) particle, \( i.e. \) in the Hamiltonian of weak interaction only left-handed \( \nu_L \) (or right-handed \( \nu_R \)) neutrino field enters. Thus, in the case of a two-component neutrino theory \( G'_i = -G_i \ (G'_i = G_i) \) and non-conservation of parity in \( \beta \)-decay is maximal.

In 1957 the two-component theory was confirmed by the experiment of Goldhaber et al. In this experiment by the measurement of the polarization of \( \gamma \)-quanta in the process

\[ e^- + Eu \rightarrow \nu + Sm^* \]

the helicity of neutrinos was determined. It was found that the neutrino is a left-handed particle.

\section*{VI. V - A CURRENT \times CURRENT THEORY}

The next decisive stage in the construction of an effective theory of weak interaction was done by Feynman and Gell-Mann, Marshak and Soudarshan. These authors assumed that in the Hamiltonian of the weak interaction only left-handed components of fields enter. Taking into account that

\[ \bar{e}_L \left( 1 \ ; \ \sigma_{\alpha \beta} \ ; \ \gamma_5 \right) \nu_L = 0 \]  
and that

\[ \bar{e}_L \gamma_\alpha \gamma_5 \nu_L = -\bar{e}_L \gamma_\alpha \nu_L, \]  
for the Hamiltonian of \( \beta \)-decay from (8) they obtained the following expression

\[ \mathcal{H}_I^\beta = \frac{G_F}{\sqrt{2}} 4 \bar{p}_L \gamma_\alpha n_L \bar{e}_L \gamma^\alpha \nu_L \ + \ h.c. \]  
This interaction is characterized by one fundamental constant \( G_F \) and differs from the Fermi interaction (4) only in the change of all fields by left-handed fields. From all available experimental data it follows that the Hamiltonian (14) is the correct effective Hamiltonian of \( \beta \)-decay.

The other process in which a neutrino is produced is \( \mu \)-capture

\[ \mu^- + p \rightarrow \nu + n \]
B. Pontecorvo was the first to notice in the 1950’s that the constant that characterize this process is the Fermi constant. He assumed that the weak interaction is a universal interaction which includes the pairs \((e, \nu)\) and \((\mu, \nu)\). The theory of Feynman and Gell-Mann is a universal theory of the weak interaction. Their Hamiltonian describes not only \(\beta\)-decay and \(\mu\)-capture, but also \(\mu\)-decay, the process in which two neutrinos are produced

\[
\mu^+ \to e^+ + \nu + \bar{\nu}.
\]  

(6.5)

Feynman and Gell-Mann introduced a \(V-A\) weak current

\[
\begin{align*}
  j_\alpha &= 2[\bar{\nu}_e \gamma_\alpha e_L + \bar{\nu}_\mu \gamma_\alpha \mu_L + \bar{\mu} \gamma_\alpha n_L] \\
  &\text{and assumed that the Hamiltonian of the weak interaction has the current \(\times\) current form}
\end{align*}
\]

\[
\mathcal{H}_I = \frac{G_F}{\sqrt{2}} j_\alpha \bar{j}^{\alpha}.
\]  

(6.7)

The non-diagonal terms of (18) are Hamiltonians of \(\beta\)-decay, \(\mu\)-capture, \(\mu\)-decay and other connected processes (like \(\nu_e n \to e^- p, \ldots\)). There are also in (18) diagonal terms as

\[
\mathcal{H}^{d}_I = \frac{G_F}{\sqrt{2}} 4 \bar{\nu}_e \gamma_\alpha \nu_e \bar{e}_L \gamma_\alpha e_L.
\]  

(6.8)

Thus, the current \(\times\) current theory predicted new weak processes such as

\[
\bar{\nu}_e + e \to \bar{\nu}_e + e.
\]  

(6.9)

Let us notice that this process with reactor antineutrinos was observed many years later by Reines et al.. The measured cross section was in agreement with the standard model, that includes the interaction (19) as well as the additional (neutral current) interaction.

VII. \(\nu_\mu\) AND \(\nu_e\) ARE DIFFERENT PARTICLES

In (17) we denoted the field of the neutrino that enter in the Hamiltonian together with the field of electron (muon) as \(\nu_e (\nu_\mu)\). Are electron and muon neutrinos different or are they the same particles? In 1959 B. Pontecorvo proposed an experiment that could allow to answer this question. The idea of the experiment was the following. According to the \(V-A\) theory, the decay of a charged pion into an electron and a neutrino is strongly suppressed. This consequence of the theory was beautifully confirmed by the CERN experiment of Fidecaro et al.. Thus, charged pions decay mainly into a muons and a muon neutrinos. If we produce the beam of pions and give pions the possibility to decay, a practically pure beam of muon neutrinos can be produced. In a neutrino detector only muons would be observed if \(\nu_\mu\) and \(\nu_e\) are different particles. If \(\nu_\mu\) and \(\nu_e\) are identical particles due to \(\mu - e\) universality an equal number of muons and electrons will be produced.

The experiment proposed by Pontecorvo was done at Brookhaven by Lederman, Steinberger, Schwarz et al. in 1962. The Brookhaven experiment was the first experiment with accelerator neutrinos. It was proven that \(\nu_\mu\) and \(\nu_e\) are different particles.
What are the quantum numbers that distinguish muon and electron neutrinos? Let us introduce the electron $L_e$ and muon $L_\mu$ lepton numbers in such a way that $L_e = 1$, $L_\mu = 0$ for $\nu_e$, $e^-$ and $L_e = 0$, $L_\mu = 1$ for $\nu_\mu$, $\mu^-$. The data of the Brookhaven experiment was in agreement with the assumption that the total electron lepton number and the total muon lepton number are conserved

$$\sum L_e = \text{const}, \quad \sum L_\mu = \text{const}. \quad (7.1)$$

**VIII. CABIBBO AND GIM CURRENTS**

Up to now we have not discussed decays of strange particles. Strange particles were included in the current $\times$ current scheme by N. Cabibbo in 1962.

After many years of investigation of semileptonic decays of strange particles

$$K^+ \rightarrow \pi^0 \mu^+ \nu_\mu, \quad \Lambda \rightarrow p e^- \bar{\nu}_e, \quad \Sigma^- \rightarrow n e^- \bar{\nu}_e, \ldots \quad (8.1)$$

three phenomenological rules were formulated:

I. In semileptonic decays the strangeness is changed by one: $|\Delta S| = 1$.

II. The rule $\Delta Q = \Delta S$ is satisfied, where $\Delta Q = Q_f - Q_i$, $\Delta S = S_f - S_i$ and $Q_f$ ($Q_i$) and $S_f$ ($S_i$) are the initial (final) electric charge and strangeness of hadrons.

III. The decays of strange particles are suppressed with respect to decays of non-strange particles.

In order to include strange particles in the $V-A$ scheme, Cabibbo assumed that the weak current is the combination of the components of an SU(3) current. We can easily construct the Cabibbo current if we assume that fields of $u(Q = 2/3, S = 0)$, $d(Q = -1/3, S = 0)$ and $s(Q = -1/3, S = -1)$ quarks enter in the weak current. Let us stress that this is a new point of view: this assumption means that the weak interaction is the interaction of leptons and quarks. Let us accept the Feynman-Gell-Mann conjecture and assume that only the left-handed components of fields enter in the current. There are only the following two quark terms that, like the lepton terms in (17), change the electric charge by one:

$$\bar{u}_L \gamma_\alpha d_L \quad \text{and} \quad \bar{u}_L \gamma_\alpha s_L \quad (8.2)$$

The first term does not change the strangeness. The second term changes the strangeness by one. It is obvious that in the framework of the current $\times$ current scheme this term provides the fulfillment of the $|\Delta S| = 1$ and $\Delta Q = \Delta S$ rules for semileptonic decays of strange particles. In order that rule III be satisfied, Cabibbo introduced an angle $\theta_C$ and assumed that the quark current has the form

$$j^C_\alpha = 2[\cos \theta_C \bar{u}_L \gamma_\alpha d_L + \sin \theta_C \bar{u}_L \gamma_\alpha s_L]. \quad (8.3)$$

He showed that with the help of (24) it is possible to describe the data. For the parameter $\sin \theta_C$ he found the value $\sin \theta_C \simeq 0.2$. 

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Now the weak current can be written in the form
\[ j_\alpha = 2[\bar{\nu}_eL\gamma_\alpha e_L + \bar{\nu}_\mu L\gamma_\alpha \mu_L + \bar{u}_L\gamma_\alpha d^c_L], \quad (8.4) \]
where
\[ d^c_L = \cos \theta_C d_L + \sin \theta_C s_L. \quad (8.5) \]
As seen from (25) the lepton and quark terms have similar structure. However, there is an asymmetry between the lepton and quark parts of the current: there are two lepton terms and one quark term.

In 1970 strong arguments appeared in favour of an additional quark term in the weak current (Glashow, Iliopoulos, Maiani). It was necessary to introduce an additional quark term in order to suppress decays like \( K^+ \rightarrow \pi^+ \nu\bar{\nu} \), in which the strangeness of hadrons is changed and the charge is not changed (such decays were not observed in the experiments).

In order to introduce an additional term in the current it was necessary to assume that a new quark with the charge \( 2/3 \) exist. This quark was called charmed (c). Glashow, Iliopoulos and Maiani assumed that the additional term in the current has the form
\[ j^{\text{GIM}}_\alpha = 2\bar{c}_L\gamma_\alpha s^c_L, \quad (8.6) \]
where
\[ s^c_L = -\sin \theta_C d_L + \cos \theta_C s_L. \quad (8.7) \]
is the combination of the fields of s and d quarks orthogonal to the Cabibbo combination (26). The weak current that includes the Cabibbo and GIM currents has the form
\[ j_\alpha = 2 \left[ \sum_{l=e,\mu} \bar{\nu}_l L\gamma_\alpha l_L + \bar{u}_L\gamma_\alpha d^c_L + \bar{c}_L\gamma_\alpha s^c_L \right]. \quad (8.8) \]
The charmed particles were discovered in 1975. The investigation of the decays of charmed particles and neutrino processes fully confirmed the GIM hypothesis.

**IX. THE STANDARD THEORY OF ELECTROWEAK INTERACTIONS**

Up to now we have discussed only a four-fermion weak interaction. In 1938 O. Klein assumed that there exist a charged heavy intermediate vector boson \( W \) and that fundamental weak interaction is the interaction of two fermions and a \( W \)-boson (like the electromagnetic interaction). From this point of view, the weak processes at small \( Q^2 \) (momentum transfer squared) such as \( \beta \)-decay, are second order processes with a virtual \( W \)-boson.

In the framework of the \( V-A \) theory two alternative theories were considered:

I. The theory of the four-fermion weak interaction.

II. The theory with the intermediate vector boson.
The Hamiltonian of the four-fermion interaction is given by (18). The Lagrangian of the interaction of fermions with vector boson is given by

\[ L_I = -\frac{g}{\sqrt{2}} j_\alpha W^\alpha + h.c. \] (9.1)

where \( g \) is a dimensionless coupling constant.

From the point of the view of the intermediate vector theory the current-current Hamiltonian (18) is an effective Hamiltonian that describes second order weak processes at \( Q^2 << M^2_W \) (\( M_W \) is the mass of the \( W \)-boson). The Fermi constant \( G_F \) is connected with the constant \( g \) and the mass \( M_W \) by the relation

\[ \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M^2_W} \] (9.2)

and naturally has dimension \( M^{-2} \).

The four-fermion theory, as well as the theory with the intermediate vector boson in the lowest order of the perturbation theory describe numerous experimental data. However both theories were unrenormalizable theories.

Over the years there were many attempts to build a renormalizable theory of weak interactions. Success was achieved by way of the unification of weak and electromagnetic interactions in a unified electroweak interaction (Glashow 1961, Weinberg 1968, Salam 1968).

In order to unify weak and electromagnetic interactions it is necessary to assume that a vector intermediate \( W \)-boson exists. The unification is based upon local Yang-Mills gauge invariance. In the case of the simplest SU(2) local gauge invariance with the doublets of the left-handed fermion fields \( \psi_aL \), vector gauge fields \( A^i_\alpha (i = 1, 2, 3) \) are fields of charged and neutral particles. The interaction Lagrangian is given by

\[ L_I = -g \sum_i j^i_\alpha A^{i\alpha} \] (9.3)

where \( j^i_\alpha = \sum_a \bar{\psi}_aL \gamma_\alpha \frac{1}{2} \tau_i \psi_aL \) is an isovector current. The Lagrangian (32) can be written in the form

\[ L_I = -\frac{g}{2\sqrt{2}} j_\alpha W^\alpha + h.c. - gj^3_\alpha A^{2\alpha} \] (9.4)

where \( j_\alpha = 2(j^1_\alpha + ij^2_\alpha) \) is the charged current, \( W_\alpha = \frac{1}{\sqrt{2}} (A^1_\alpha - iA^2_\alpha) \) is the field of charged vector particles and \( A^3_\alpha \) is the field of neutral vector particles. If the fermion doublets are chosen as

\[ \psi_{1L} = \left( \begin{array}{c} \nu'_L \\ \bar{d}'_L \end{array} \right), \quad \psi_{2L} = \left( \begin{array}{c} u'_L \\ \bar{s}'_L \end{array} \right), \quad \psi_{3L} = \left( \begin{array}{c} c'_L \\ \bar{t}'_L \end{array} \right), \quad \psi_{1L} = \left( \begin{array}{c} \nu'_L \\ \bar{d}'_L \end{array} \right), \quad \psi_{2L} = \left( \begin{array}{c} u'_L \\ \bar{s}'_L \end{array} \right), \quad \psi_{3L} = \left( \begin{array}{c} c'_L \\ \bar{t}'_L \end{array} \right), \quad \psi_{1L} = \left( \begin{array}{c} \nu'_L \\ \bar{d}'_L \end{array} \right), \quad \psi_{2L} = \left( \begin{array}{c} u'_L \\ \bar{s}'_L \end{array} \right), \quad \psi_{3L} = \left( \begin{array}{c} c'_L \\ \bar{t}'_L \end{array} \right) \] (9.5)

for the charged current we have the expression

\[ j_\alpha = 2 \left[ \sum_l \bar{\nu}_L \gamma_\alpha \nu'_L + \bar{d}_L \gamma_\alpha d'_L + \bar{s}_L \gamma_\alpha s'_L + \bar{t}_L \gamma_\alpha t'_L \right], \] (9.6)
that is similar to the expression (29) for the phenomenological charged current (in accordance
with the existing data, additional lepton and quark terms are taken into account here).

The last term of the Lagrangian (33) describes the interaction of fermions with neutral
vector bosons. This term includes the neutrino fields and does not conserve parity; it is not
the Lagrangian of the electromagnetic interaction.

In order to unify weak and electromagnetic interactions we must enlarge the symmetry
group and introduce an additional gauge interaction with a neutral vector field. The inter-
action with the charged vector field must be retained. The minimal enlargement is a local
gauge SU(2) × U(1) group. In the case of this group the gauge interaction has the form

\[ L_I = -g j_\alpha A_\alpha - g' j_\alpha^y B^\alpha, \] (9.7)

where \( B^\alpha \) is a U(1) gauge field and \( g' \) is the dimensionless coupling constant.

If the arbitrary U(1) constants are chosen in such a way that the Gell-Mann–Nishijima
rule \( Q = I_3 + \frac{1}{2} y \) is satisfied, in this case current \( \frac{1}{2} j_\alpha^y \) is given by

\[ \frac{1}{2} j_\alpha^y = j_\alpha^{\text{em}} - j_\alpha^3 \] (9.8)

where \( j_\alpha^{\text{em}} \) is electromagnetic current.

The Lagrangian of the interaction of quarks and leptons with neutral vector fields is
equal to

\[ L_0^I = -g j_\alpha A_\alpha^3 - g' (j_\alpha^{\text{em}} - j_\alpha^3) B^\alpha = -c j_\alpha^{\text{em}} A_\alpha - \frac{g}{2 \cos \theta_W} j_\alpha^0 Z^\alpha \] (9.9)

Here \( g' = g \tan \theta_W \), \( g \sin \theta_W = e \), the fields \( Z^\alpha \) and \( A^\alpha \) are connected with \( A_\alpha^3 \) and \( B^\alpha \) by
the relations

\[ Z_\alpha = \cos \theta_W A_\alpha^3 - \sin \theta_W B_\alpha \]
\[ A^\alpha = \sin \theta_W A_\alpha^3 + \cos \theta_W B_\alpha \] (9.10)

and

\[ j_\alpha^0 = 2 j_\alpha^3 - 2 \sin^2 \theta_W j_\alpha^{\text{em}} \] (9.11)

The first term of the expression (38) is the Lagrangian of the electromagnetic interaction,
the second term is the new neutral current interaction of fermions and vector bosons.

Thus, if weak and electromagnetic interactions are unified on the basis of local gauge
symmetry, in this case:

1. Not only the charged vector \( W^\pm \)-bosons but also the neutral vector \( Z \)-boson must
exist.

2. The Lagrangian of the interaction of \( Z \)-bosons and fermions has the form of the product
of the \( Z \)-field and the neutral current \( j_\alpha^0 \), which is a combination of the third component
of isovector current \( j_\alpha^3 \) and electromagnetic current \( j_\alpha^{\text{em}} \); the only parameter which
enters in the neutral current is \( \sin^2 \theta_W \).
3. The parameters of the theory are connected by the relation \( g \sin \theta_W = e \).

A local gauge invariance is an exact symmetry only if the masses of all particles are equal to zero. Thus, in the real world, this symmetry must be broken.

The standard electroweak theory (standard model) is based on the Higgs mechanism of spontaneous violation of the symmetry which requires the existence of a neutral scalar Higgs boson. As a result of the violation of the symmetry:

i. All particles (perhaps with the exception of neutrinos) acquire masses.

ii. The primed fermion fields that enter in the doublets (34) are connected with fermion fields with definite masses by unitary transformation. For the quark fields we have

\[
\begin{pmatrix}
  d'_L \\ s'_L \\ b'_L
\end{pmatrix} = V_L \begin{pmatrix}
  d_L \\ s_L \\ b_L
\end{pmatrix}, \quad \begin{pmatrix}
  u'_L \\ c'_L \\ t'_L
\end{pmatrix} = U_L \begin{pmatrix}
  u_L \\ c_L \\ t_L
\end{pmatrix}
\]

(9.12)

where \( u, d, \ldots \) are fields of physical quarks.

As a result, we come to the following expression for the charged quark current

\[
j_\alpha = 2 \left[ \bar{u}_L \gamma_\alpha d^c_L + \bar{c}_L \gamma_\alpha s^c_L + \bar{t}_L \gamma_\alpha b^c_L \right]
\]

(9.13)

which, for the case of two generations, coincides with the phenomenological current (29). Here

\[
d^c_L = \sum_{q=d,s,b} V_{uq} q^c_L, \quad s^c_L = \sum_{q=d,s,b} V_{cq} q^c_L, \quad b^c_L = \sum_{q=d,s,b} V_{tq} q^c_L,
\]

(9.14)

where \( V = U^*_L \) is the unitary Cabibbo-Kobayashi-Maskawa mixing matrix.

Neutral currents were discovered in the neutrino experiments at CERN in 1973. Their detailed investigation showed impressive agreement of the standard model with experiment.

The relations (43) mean that the fields of quarks enter in the charged current in mixed form. From numerous experimental data that are available today, we have rather detailed information about the elements of the CKM mixing matrix \( V \). What about neutrinos? Are neutrino masses different from zero and the fields of massive neutrinos enter into charged current also in the mixed form? The major aim of present (and future) neutrino experiments is to answer these fundamental questions.

**X. MASSIVE AND MIXED NEUTRINOS**

We will finish this lecture with a short discussion of the problem of neutrino mixing. Let us stress first of all that there are more possibilities for the mixing of neutrinos than for the mixing of quarks. This is connected with the fact that quarks are charged Dirac particles, whereas for massive neutrinos there are two possibilities:
I. Neutrinos can be Dirac particles. In this case the total lepton number \( L = L_e + L_\mu + L_\tau \) is conserved and neutrinos and antineutrinos have opposite lepton numbers. For the neutrino mixing in this case we have

\[
\nu_{lL} = \sum_{k=1}^{3} U_{lk} \nu_{kL} \quad l = e, \mu, \tau
\]  

(10.1)

where \( U^\dagger U = 1 \) and \( \nu_k \) is the field of the neutrino with mass \( m_k \).

II. Neutrinos can be truly neutral Majorana particles. In this case there are no conserved lepton numbers, and for neutrino mixing we have

\[
\nu_{lL} = \sum_{k=1}^{n} \bar{U}_{lk} \chi_{kL}
\]  

(10.2)

where \( \chi_k = \chi_k^c = C\bar{\chi}_k^T \) is the field of the Majorana neutrino with mass \( m_k \) (\( C \) is the charge conjugation matrix).

The number of massive neutrinos in (45) depends upon the model. If only left-handed components \( \nu_{lL} \) enter in the neutrino mass term, in this case \( n = 3 \). If left-handed \( \nu_{lL} \) and right-handed \( \nu_{lR} \) fields enter in the neutrino mass term, in this case \( n = 6 \).

Note that Dirac neutrino masses can be generated by the standard Higgs mechanism. The Majorana neutrino masses can be generated only in the framework of models beyond the standard model.

The existing models cannot allow to predict the values of the neutrino masses. There exist, however, a rather general mechanism of neutrino mass generation that can explain the smallness of neutrino masses with respect to the masses of all the other fundamental fermions. This is the so-called see-saw mechanism. If we assume that, due to the presence of a right-handed Majorana mass term, lepton numbers are violated at a scale \( M \) that is much larger than the fermion masses, in this case for the mass of the light neutrino in each generation we have the see-saw formula

\[
m_k \simeq \frac{(m_F^k)^2}{M_k}, \quad k = 1, 2, 3.
\]  

(10.3)

Here \( m_F^k \) is the mass of the up-quark or charged lepton and \( M_k \gg m_F^k \). Let us stress that if the neutrino masses are generated by see-saw mechanism, in this case:

1. Massive neutrinos are Majorana particles.
2. There is a hierarchy of neutrino masses: \( m_1 \ll m_2 \ll m_3 \).

At the moment the problem of neutrino masses and mixing is investigated in different experiments. There are three experimental methods that allow to reveal the effects of neutrino masses and mixing.
A. Precise measurement of high energy part of beta-spectrum

The classical decay in which neutrino mass is measured is the $\beta$-decay of $^3H$

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e$$  \hspace{1cm} (10.4)

This is the superallowed transition and the electron spectrum is determined by the phase factor

$$\frac{dN}{dT} = C p E (Q - T) \sqrt{(Q - T)^2 - m_\nu^2} F(E),$$  \hspace{1cm} (10.5)

where $E$ and $p$ are the electron energy and momentum, $T = E - m_e$, $Q \simeq 18.6$ keV is the energy release, $F(E)$ is the Fermi function that describes the Coulomb interaction of the final particles, and $m_\nu$ is the neutrino mass. In the real spectrum it is necessary to take into account molecular effects, spectrometer resolution, background, and so on.

From the measured spectrum the Kurie function

$$K(T) = \sqrt{(Q - T) \sqrt{(Q - T)^2 - m_\nu^2}}$$  \hspace{1cm} (10.6)

can be obtained. If $m_\nu = 0$, in this case $T_{\text{max}} = Q$ and $K(T) = Q - T$. If $m_\nu \neq 0$, in this case $T_{\text{max}} = Q - m_\nu$ and a deviation of $K(T)$ from a straight line will be observed near the end point of the spectrum.

No indications in favour of non-zero neutrino masses were found in tritium experiments. From the data of recent experiments the following upper bounds were found:

$$m_\nu < 3.9 \text{ eV} \quad (\text{Troizk}), \quad m_\nu < 5.6 \text{ eV} \quad (\text{Mainz}).$$  \hspace{1cm} (10.7)

B. Search for neutrinoless double-$\beta$ decay

There are many experiments in which neutrinoless double-beta decay

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$$  \hspace{1cm} (10.8)

do different even-even nuclei is searched for. In process (51) the total lepton number is not conserved. Thus neutrinoless double-beta decay is possible only if neutrinos are massive Majorana particles.

Process (51) is of the second order in weak interaction with virtual neutrinos. The matrix element of the process is proportional to

$$<m> = \sum U_{ek}^2 m_k$$  \hspace{1cm} (10.9)

($U_{ek}^2$ is due to two vertices and $m_k$ is due to the the propagator of left-handed neutrino fields). No indications in favour of neutrinoless double-beta decay were found in experiments up to present time. In the $^{76}\text{Ge}$ experiment of the Heidelberg-Moscow collaboration it was found that

$$T^{1/2}_\frac{1}{2} \geq 1.1 \cdot 10^{25} \text{y}$$  \hspace{1cm} (10.10)

From this data it follows that $| <m> | \leq (0.5 - 1.1)eV$. Let us notice that in the nearest years sensitivity of $| <m> | \simeq 10^{-1}eV$ will be reached (NEMO, Heidelberg-Moscow and other experiments).
C. Neutrino oscillations

Neutrino oscillations were first considered by B. Pontecorvo in 1958. From the point of view of quantum mechanics, neutrino oscillations are similar to the famous $K^0 \leftrightarrow \bar{K}^0$ oscillations. If there is neutrino mixing, the state vector $|\nu_l\rangle$ of the flavour neutrino $\nu_l$ with momentum $p$ is the coherent superposition of the states $|k\rangle$ of massive neutrinos with momentum $p$ and energy $E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p}$ (for $p \gg m_k$):

$$|\nu_l\rangle = \sum_k U_{lk}^* |k\rangle.$$  \hspace{1cm} (10.11)

This basic relation is valid if the neutrino mass differences are so small that, due to an uncertainty relation, it is not possible in weak interaction to distinguish one massive neutrino from the other one. If at $t=0$ in weak decays neutrinos $\nu_l$ with momentum $p$ are produced, at time $t$ the neutrino state vector is given by

$$|\nu_l\rangle_t = \sum_k U_{lk}^* e^{-iE_k t} |k\rangle = \sum_{l'} |\nu_{l'}\rangle \sum_k U_{l'k} e^{-iE_k t} U_{lk}^*$$  \hspace{1cm} (10.12)

Thus, if there is neutrino mixing, the beam of neutrinos at some macroscopic distance from the source will be a superposition of states of different flavour neutrinos. For the probability of the transition $\nu_l \rightarrow \nu_{l'}$, we have

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_k U_{l'k} e^{-i\frac{\Delta m^2_{k1} L}{2p}} U_{lk}^* \right|^2.$$  \hspace{1cm} (10.13)

Here $L$ is the source-detector distance and $\Delta m^2_{k1} = m_k^2 - m_{l1}^2$ (we made the usual assumption that $m_1 < m_2 < \ldots$). Taking into account the unitarity of the mixing matrix, for the simplest case of oscillations between two flavour neutrinos from (54) we have

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \delta_{l1} + U_{l2} U_{l2}^* e^{-i\frac{\Delta m^2_{k1} L}{2p}} - 1 \right|^2,$$  \hspace{1cm} (10.14)

where $\Delta m^2 = m_2^2 - m_1^2$. From this expression it is clear that for neutrino oscillations to be observed, it is necessary that

$$\Delta m^2 \gtrsim \frac{E}{L}.$$  \hspace{1cm} (10.15)

Here $L$ is the distance in meters, $E$ is neutrino energy in MeV and $\Delta m^2$ is the difference of neutrino masses squared in eV$^2$. From (58) it follows that different neutrino facilities (accelerators, reactors, atmospheric neutrinos, sun) allow us to study neutrino oscillations in a wide range of $\Delta m^2$, from $\Delta m^2 \simeq 10 eV^2$ till $\Delta m^2 \simeq 10^{-10} eV^2$. For two oscillating neutrinos the mixing matrix is given by

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & -\cos \theta \end{pmatrix},$$
where $\theta$ is the mixing angle. From (57) for the transition probabilities we obtain the following standard expressions

$$P(\nu_l \to \nu_{l'}) = \frac{1}{2} \sin^2 2\theta (1 - \cos \frac{\Delta m^2 L}{2p})$$

$$P(\nu_l \to \nu_l) = 1 - P(\nu_l \to \nu_{l'}), \quad l' \neq l$$

In many experiments with neutrinos from reactors and accelerators no indications in favour of neutrino oscillations were found. There are, however, three experimental indications that neutrinos are massive and mixed. They were found in solar neutrino experiments, in atmospheric neutrino experiments, and in the Los Alamos neutrino experiment.

Let us first discuss solar neutrinos. The energy of the sun is generated in the reactions of the thermonuclear $pp$ and CNO cycles. From the thermodynamical point of view, energy is generated in the transition of four protons into $^4He$:

$$4p \to ^4He + 2e^+ + 2\nu_e.$$  \hspace{1cm} (10.17)

Thus, the generation of the energy of the sun is accompanied by the emission of electron neutrinos. The total flux of neutrinos is connected to the Luminosity of the sun $L_\odot$ by the relation

$$Q \sum_i \left(1 - \frac{\bar{E}_i}{Q}\right) \Phi_i = \frac{L_\odot}{2\pi R^2},$$

where $Q = 26.7$ MeV is the energy release in the transition (60), $R$ is the sun-earth distance, $\Phi_i$ is the total flux of neutrinos from the source $i$, and $\bar{E}_i$ is the average energy. The most important sources of solar neutrinos are the following reactions:

$$p + p \to d + e^+ + \nu_e$$
$$e^- + ^7Be \to \nu_e + ^7Li$$
$$^8Be \to ^8Be + e^+ + \nu_e$$

The first reaction is the main source of solar neutrinos. In this reaction neutrinos with energy less than 0.42 MeV are produced. The second reaction is a source of monochromatic neutrinos with energy 0.86 MeV. This reaction contributes about 10% to the total flux of solar neutrinos. The third reaction contributes only about $10^{-4}$ to the total flux. However, this reaction is the main source of high energy solar neutrinos (up to 15 MeV).

At present the data of the Homestake, GALLEX, SAGE, Kamiokande and Super-Kamiokande experiments are available. These experiments, due to different detection thresholds, allow us to detect neutrinos from different sources: the GALLEX and SAGE experiments allow us to detect neutrinos from all sources, the Homestake experiment allows us to detect mainly $^8B$ neutrinos (about 90%) and $^7Be$ neutrinos; and in the experiment Kamiokande and Super-Kamiokande experiments only $^8B$ neutrinos are detected.

In all experiments the observed event rate is significantly smaller than the rate predicted by the standard solar model. For example, in the Super-Kamiokande experiment the detected flux is equal to
The expected flux of $^8B$ neutrinos is equal to

$$
\Phi_{\text{exp}} = 6.62 \pm 1 \ 10^6 \ cm^{-2} s^{-1}.
$$

All existing data can be explained if, due to an enhancement of the neutrino mixing in matter (MSW effect), there are transitions of initial solar $\nu_e$ into neutrinos of other types that are not detected by existing experiments. For the oscillation parameters $\Delta m^2$ and $\sin^2 \theta$ two possible values were obtained

$$
\Delta m^2 \simeq 5 \cdot 10^{-6} eV^2 \quad \sin^2 \theta \simeq 7 \cdot 10^{-3}
$$

or

$$
\Delta m^2 \simeq 2 \cdot 10^{-5} eV^2 \quad \sin^2 2\theta \simeq 0.8
$$

Let us notice that existing data can be also explained by the vacuum oscillations.

The present analysis of solar neutrino data is based on the standard solar model. Let us stress that the Super-Kamiokande experiment and the future SNO experiment (in which solar neutrinos will be detected by the observation of CC and NC reactions) will allow us to obtain model-independent information about neutrino masses and mixing.

The second indication in favour of neutrino mixing was obtained in atmospheric neutrino experiments. Atmospheric neutrinos are produced in decays of pions and kaons ($\pi (K) \to \mu \bar{\nu}_\mu, \mu \to e \nu_e \nu_\mu$) that are produced in the interaction of cosmic rays with the atmosphere. The ratio of the muon and electron events can be predicted with accuracy about 5% (at relatively small energies this ration is close to 2). In the Kamiokande, IMB and Soudan experiments it was found that the ratio $R$ of the ratio of observed muon and electron events to the predicted ratio is significantly less than one. This atmospheric neutrino anomaly was confirmed recently by the Super-Kamiokande experiment. For the ratio of ratios $R$ in this experiment it was found

$$
R = 0.635 \pm 0.033 \pm 0.053.
$$

The result obtained can be explained by $\nu_\mu \to \nu_e$ oscillations. For the oscillation parameters it was found

$$
\Delta m^2 \simeq 5 \cdot 10^{-3} eV^2, \quad \sin^2 2\theta \simeq 1.
$$

Several long-baseline neutrino oscillation experiments (KEK–Super-Kamiokande, Fermilab–Soudan, CERN–Gran-Sasso, CHOOZ and Palo Verde) that will allow us to investigate the "atmospheric neutrino range" of $\Delta m^2$ in different oscillation channels are now under preparation. The first reactor long-baseline experiment CHOOZ has started recently.

The third indication in favour of neutrino mixing was found in the Los Alamos accelerator experiment. Neutrinos in this experiment are produced in decays at rest of $\pi^+$ and $\mu^+$:

$$
\pi^+ \to \mu^+ \nu_\mu, \quad \mu^+ \to e^+ \nu_e \bar{\nu}_\mu.
$$

The LSND detector at the distance about 30 m from beam stop is searching for electron antineutrinos (by the observation of the process $\bar{\nu}_e p \to e^+ n$). It was found 22 such events.
The expected background is $4.6 \pm 0.6$ events. If negative results of other experiments are taken into account from the results of the LSND experiment the following allowed range of oscillation parameters was obtained

$$0.3 < \Delta m^2 \leq 2 \ eV^2, \ 10^{-3} \leq \sin^2 2\theta \leq 4 \times 10^{-2}$$

(10.26)

Indications in favour of relatively large values of $\Delta m^2$ in $\nu_\mu \to \nu_e$ channel were obtained only in LSND experiment. These data need confirmation from other experiments. It is planned that another experiment KARMEN will reach sensitivity of LSND experiment in about 2 years.

**XI. CONCLUSION**

After Pauli and Fermi neutrino physics have done tremendous progress (see, for example, the books [1–6]). We know that three types of flavour left-handed neutrinos exist in nature and we know interaction of neutrinos with other particles. However, the problem of neutrino properties remains unsolved. The key problem is the problem of the neutrino mass (see, for example, the reviews [7,8]). Today we have different indications that neutrinos are massive (see, for example, the Proceedings [9,10]). All of them require further checks and confirmation. We do not know what is the nature of massive neutrinos (Dirac or Majorana) and how many massive neutrinos exist in nature. We need to know the neutrino mixing matrix. The solution of these problems could bring us to new physics beyond the standard model. The solution of the neutrino mass problem will be very important for astrophysics and in particular for the understanding of the dark matter problem.

The investigation of neutrino properties is the present and future of neutrino physics.

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