A Parametric Study of Pile Behavior In Liquefied Soils

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ABSTRACT

Liquefaction in saturated sandy soils under cyclical loads has a significant part in the structural damage cases. Pile foundations, used for soils with bearing capacity problems, might get exposed to various liquefaction-based damages. The finite element program FLAC2D is utilized to understand the pile behavior in liquefied soils under dynamic loads. The 1999 Kocaeli earthquake record was used in the numerical analysis for a single pile element in the layered soil profile with liquefied and non-liquefied soil. The pile-head of the model used in the layered soil sample was left free for rotation. Calculations for a single pile profile where axial load and horizontal load are affected simultaneously were performed by considering both the kinematic and inertial effect. Finite difference analyzes were performed by changing the embedded lengths of pile socket according to the existence and absence of a non-liquefying crust layer on the liquefied soil in the layered soil profile. As the results of the pile-head displacement and maximum moment value output were assessed.

Keywords: Liquefaction, Crust layer, FLAC2D, Embedded length of pile, Inertial-Kinematic effects.

Sıvılaşan Zeminlerde Kazık Davranışına Dair Parametric Bir Çalışma

ÖZ

Tekrarlı yükler altında suya doygun kumlu zeminlerde meydana gelen sıvılaşma olayı yapısal hasarların meydana gelmesinde etkin rol oynamaktır. Taşıma gücü problemleri olan zeminler için kullanılan kazık temeller sıvılaşma olayından dolayı çeşitli hasarlarla maruz kalarlar. Bu çalışmada, dinamik yükler altında sıvılaşan zeminlerdeki kazık davranışını anlamak için numaralı kazık modeli programı FLAC2D kullanılmıştır. Sıvılaşan ve sıvılaşmayan zeminin bulunduğu tabakaların zemin profilindeki tek bir kazık elemanı için yapılan numerik analizde 1999 Kocaeli depremi kaydı kullanılmıştır. Tabakalı zemin örneği içinde kullanılan kazık modelinin kazık başı dönmeye serbest hareket eden biridir. Eksenel yük ve yatay yük aynı anda etki eden tek kazık profili için hem kinematik etki hem de ataletsel etki göz önüne alınarak hesaplanmıştır. Tabakalı zemin profilindeki sıvılaşan zemin üzerine sıvılaşmayan kabuk tabakası bulunması ve bulunmaması durumlarına göre kazık soket boyları değiştirilerek sonuç farkları analiz edilmiştir.

Anahtar kelimeler: Sıvılaşma, Kabuk tabakası, FLAC2D, Kazık soket derinliği, Ataletsel-Kinematik etki.
I. INTRODUCTION

Pore water pressure increases caused by dynamic loads on saturated soils can transform the soils from solid to liquid state hence the liquefaction. Due to liquefaction, effective stress and shear strength decreases. It is known that liquefaction does not occur in all soil layers in the field. Therefore, it is required to assess liquefaction potential. Most critical soils which prone to liquefaction are silty sands, fine clean sands and presence of non plastic fine particles decreases liquefaction resistance. Although liquefaction depth limit is accepted as around 15 to 20 meters, there are liquefaction cases with greater depths. The pile foundations were damaged in the past due to the liquefaction and the lateral spreading events occurred after the earthquakes. Attributing the pile damage under dynamic loads only to the soil behavior is not a comprehensive approach. Understanding pile behavior in liquefied soils is possible through assessing the soil features, the mechanical-physical properties of the pile, embedded length of the pile, and the existence of a non-liquefied crust soil layer on the liquefied soil as a whole. The fact that the pile damages occurred during the recent earthquakes are still observable is due to the liquefied soil behavior and the additional dynamic forces affecting the pile during the earthquake are not completely known at the design stage. Settlements, lateral spreads and bearing capacity losses were observed due to liquefaction during the last earthquakes in the world and in our country. While some of the structures on the liquefied ground collapsed, excessive settlements were also observed.

The pile responses due to only the inertial effects under dynamic loads are analyzed by civil engineers but kinematic effects are generally omitted. However, it is known that the design of pile foundations in liquefied soils under dynamic loads is a complex phenomenon with pile-soil interaction which must be designed for both inertial and kinematic effects [1].

The number of contemporary academic studies conducted to understand the negative effects of liquefied soils on pile foundations have gained momentum. Numerical methods have been applied to determine the permanent displacements caused by liquefaction and pile damages in the past and present [2, 3]. In particular, it was predicted that pile damage usually occurs in the interfaces between the liquefied and non-liquefied soil and the deformations in the interface layer should be higher. Moreover, it was concluded that the most critical design stage for piles is the rotation limitation of the pile-heads, the presence of a liquefied and non-liquefied layered soil conditions. Appropriate analyses considering these effects should be performed earthquake loads [4, 5].

Assessing pile behavior in liquefied soils requires the simultaneous consideration of the effects caused by inertial and kinematic interactions on soil-pile behavior [6-8] Engineering practice generally relies on pseudo-static approaches and neglects the kinematic effect [9]. However, earthquakes experienced in the past and studies show how important the kinematic effect is [10-12].

It is known that large soil displacements caused by kinematic effects are an important criterion for pile control. The kinematic effects increase in the liquefied and non-liquefied soil interface, the piles are also affected [13, 14]. It was observed that the highest bending moments (for fixed-headed piles) occurred at the pile-heads and at the soil interfaces with highest shear wave velocity ratio [15].

Moreover, simple equations and analysis methods were developed to evaluate the kinematic effect at the pile-head and at the interface of the layers [16-22].

It has been observed that inertial effects are effective at a depth of approximately 15 pile diameters from the pile-head, while kinematic effects are effective at greater depths [23]. When the pile-supported structures are exposed to earthquake loads, the inertial loads from the superstructure and the kinematic load generated by the ground motion act together, hence a complex dynamic soil-structure-pile interaction problem occurs. Therefore, ground motion, the state of free field conditions, the
Inertial effects from the superstructure and the soil-pile-structure interaction should be considered to understand the pile behavior in liquefied soils [24]. It is known that the damages occurring near the upper part of the piles after the liquefaction event under earthquake loads are caused by inertial effects. Additionally, the lateral ground movement caused by the liquefaction directs the pile damage towards the middle parts due to kinematic effects [25]. Kinematic and inertial effects on liquefied soils were examined together by some researches [26].

In addition, the liquefied soil thickness and pile socket depth near the ground surface are important factors to determine the pile behavior in liquefied soils. There are numerous sources and abundant research on the subject. Cubrinovski, Kokusho and Ishihara (2006) [27], performed an experimental study through which they changed the pile length and thickness of the liquefied soil layers on single steel and concrete piles. They concluded that flexible piles have the greatest displacement through equal movement with the ground, while rigid piles have less displacement. Dash and Bhattacharya (2007) [28], mentioned that the piles must be inserted into the non-liquefied soil, the shear strength of piles should not exceed the allowable capacity, the axial load on the piles should be designed in a way that impedes buckling and bending failures during liquefaction and settlement of piles should be in allowable values. Dash, Govindaraju and Bhattacharya (2009) [29], examined the pile foundations damaged by liquefaction and lateral spreading in the Kandla port, which was damaged in the 2001 Bhuj earthquake, and asserted that the piles should be inserted into the non-liquefied soil adequately.

It is also important to know the soil properties and the mechanical properties of the pile to assess the pile damage caused by lateral spreading due to liquefaction, as well as whether there is a non-liquefied crust layer on the liquefied soil layer. Several scholars have studied this subject and discussed the effects of the crust layer. Tokida et al. (1993) [30] examined the liquefaction based lateral spreading through shaking table experiments as they found that the length of the spreading soil, the thickness of the liquefied soil layer and the non-liquefied crust layer and the angle of the sloping surface have important effect on this phenomenon.

The presence of the crust layer on the liquefied soil affects the analysis for the pile foundation [31]. Lateral spreading under seismic loads during an interaction between the soil crust layer and the pile-head should be existent and evaluated together [32]. Cases with pile damage in liquefied soils and the presence of non-liquefied crustal layer indicates higher shear force and moment values on piles [33].

Finite difference analysis in the time domain was performed to determine the pile behavior with different diameters in liquefiable soils of different thicknesses and relative densities. In this study finite difference analyses were performed by changing the embedded lengths of pile socket according to the existence and absence of a non-liquefying crust layer on the liquefied soil in the layered soil profile. As the results of these analyses pile-head displacements and maximum moment values were assessed.

II. MATERIALS AND METHODS

A. FINITE DIFFERENCE METHOD

Finite difference method is the process of dividing the problem function into equal intervals and obtaining a result by solving this function. The FLAC2D [34] program utilized in this study is a finite difference program, and the models included in the program and applied during the analyzes are stated below. Since analyses were performed in plane strain condition, behaviour of a single pile with neglecting the effect of pile spacing was assessed.

Mohr-Coulomb model
Mohr-Columb model is a classical model applied to model shear failure in soils and rocks [35]. The Mohr-Coulomb model is visualized by an irregular hexagonal pyramid with the same axis, and this model is shown in Figure 1.

Failure model and yield function are included in the Mohr-Coulomb model, while hardening/softening functions are not [36].

![Figure 1. Mohr-Coulomb and Tresca failure surfaces in stress space [35].](image)

**Finn liquefaction model**

The Finn Model, used in the FLAC finite difference program to simulate liquefied soils, uses the assumed linear elastic-perfect plastic stress-deformation behavior with the Mohr-Coulomb failure criteria [37]. Martin, Finn, and Seed (1975) [38] defined the effect of cyclical loading on pore water pressure as a result of permanent volumetric unit deformations of the soil mass. Hence the spaces between the grains try to decrease and the pore water pressure increases. The increase in volumetric unit deformation ($\Delta \varepsilon_v$) that occurs in any cycle of dynamic loading depends on the shear deformation ($\gamma$) in this cycle and the previously accumulated volumetric deformation ($\varepsilon_v$).

### III. PARAMETRIC ANALYSES MADE WITH FINITE DIFFERENCE METHOD

The finite difference network, boundary conditions and pile element used in these analyzes are presented in Figure 2. As observable in Figure 2, special non-reflective free field conditions were used to model the presence of infinite soil at the side boundaries of the model.
As the first stage effective stress values due to pore water pressures and weight of the soil mass were calculated. After this stage, the pile and the structural element were added to the model, and analysis performed under static loading conditions and the deformations that occurred were nulled to only assess the earthquake-induced deformations. After this stage, dynamic analyses were performed. Piles were modeled with the "pile" element in the FLAC2D analyses by using reinforced concrete section properties. A simple structural element was modeled in the FLAC2D in such a way that the natural vibration of it corresponds to 0.6 seconds (usually the natural vibration period of the viaduct piers is in the range of 0.6-0.8 seconds). Axial loads applied by the structural element to the pile are set as 348 kN for 0.6-meter diameter and 716 kN for 1-meter diameter piles. In a parametric test to understand the pile behavior in liquefied soils, SPT values ($N_{10}$) of liquefiable sand were selected as 5, 10 and 15. The "Kocaeli Earthquake Record", which was recorded during the 1999 Marmara earthquake, was used in the analyses because it creates high spectral accelerations over a wide period of time (Figure 3). Kocaeli earthquake record was filtered according to finite difference grid size and baseline correction was performed to enable the velocity and displacement to equal to zero at the end of the earthquake. Deconvolution process was applied to the Kocaeli earthquake record in DEEPSOIL [39] program to enable the transfer of Kocaeli acceleration recorded on the surface to the base of the FLAC2D model.

**Figure 2.** Boundary conditions and finite difference network used in parametric finite difference analyses.
For the liquefiable soil "Finn Liquefaction Model" was used, the model parameters controlling the pore pressure development (C1, C2) are calculated from corrected SPT values using suggested equations. The layer in which the piles are socketed is assumed to be stiff clay and modeled by “Mohr Columb” as failure criterion with the elastic perfect plastic soil model. The model parameters used are presented in Table 1. Liquefiable soil layer is assumed as clean sand.

Table 1. Soil parameters.

| Non-liquefied soil layer (Clay) | Liquefied soil layer (Sand) |
|---------------------------------|----------------------------|
|                                | (N<sub>1</sub> <sub>60</sub> = 5) | (N<sub>1</sub> <sub>60</sub> = 10) | (N<sub>1</sub> <sub>60</sub> = 15) |
| Undrained Cohesion (kPa)       | Undrained Elasticity Modulus (MPa) | φ° | E (MPa) | C<sub>1</sub> | φ° | E (MPa) | C<sub>1</sub> | φ° | E (MPa) | C<sub>1</sub> |
| 120                             | 40                           | 29° | 25      | 1.164 | 0.344 | 30° | 50      | 0.489 | 0.818 | 32° | 75      | 0.295 | 1.357 |

Pile soil interface elements were used in the analyses, and since these were carried out under plane deformation conditions, the pile section properties were scaled as the pile spacing and diameter ratios were equalized in piles with a diameter of 0.6 and 1 m.

The pile-head is modeled free to rotate. However, as the piles are connected to the superstructure element, the pile-head is not completely free to rotate as in the spring beam (Winkler model) methods. The model is schematically shown in Figure 4.
It was observed that high moments in piles occur in the finite difference analysis at the interface layers and at the top of the pile and it was determined that the moments were in the same phase with the accelerations affecting the superstructure. In other words, the occurrence of maximum accelerations and maximum moments coincide. For example, the graph of the moment in the interface layer and the accelerations at the upper point of the building element is given with solution steps for the pile with 12 meter length and 0.6 meter diameter, socketed to a 8 meter thick liquefiable soil, of 1 meter in the non-liquefiable crust layer, shown in Figure 5. For the accelerations to be apparent in the graph, the accelerations in the unit m/s$^2$ have been magnified 30 times.

This situation indicates that the inertial and kinematic effects must be affected simultaneously in liquefied soils.
For instance, the pore pressure distribution through the analysis obtained from the finite difference analysis for an 8-m long, 0.6-m diameter pile is shown in Figure 6. The deformation vectors obtained for this analysis are provided in Figure 7.

**Figure 6.** Pore pressure distribution at the end of the analysis obtained from finite difference analysis for an 8-m long pile with a 0.6-m diameter.

**Figure 7.** Deformation vectors at the end of the analysis obtained from the finite difference analysis for an 8-m long pile with a 0.6-m diameter.

Piles with a total length of 8 meters of which 7 meters are in the liquefiable soil were modeled in the first group analysis. The results obtained from the analyses are shown in Table 2.

**Table 2.** The results of the finite difference analysis for 8-m piles, with a free rotating pile-head and an embedded length of 1 m.

| Diameter (m) | Pile-head displacement (m) | Maximum moment (kN.m) |
|-------------|---------------------------|-----------------------|
|             | SPT 5                     | SPT 10                | SPT 15                | SPT 5 | SPT 10 | SPT 15 |
| 0.6         | 0.70                      | 0.52                  | 0.32                  | 210   | 190    | 180    |
| 1.0         | 0.68                      | 0.58                  | 0.34                  | 502   | 500    | 467    |

Moment capacities of piles under operating axial loads are 391 kN.m for 0.6-m diameter and 1469 kN.m for 1 m diameter. The moment values obtained through the analysis are below the pile moment capacities. However, pile-head deformations appear to be high. It seems insufficient to socket the piles into 1 meter of non-liquefied ground.
The pile length was changed to 12 m and the embedded length to 4 m, and the analyzes were repeated for liquefiable soils with corrected SPT values of 5, 10 and 15. The results obtained by the analyses are summarized in Table 3.

**Table 3.** The results of the finite difference analyses for 12-m piles, with a free rotating pile-head and an embedded length of 4 m.

| Diameter (m) | Pile-head displacement (m) | Maximum moment (kN.m) |
|--------------|----------------------------|-----------------------|
|              | SPT 5 | SPT 10 | SPT 15 | SPT 5 | SPT 10 | SPT 15 |
| 0.6          | 0.56  | 0.62  | 0.30   | 290   | 270   | 250   |
| 1.0          | 0.37  | 0.37  | 0.16   | 835   | 820   | 790   |

In the simulation with the corrected SPT value of 5, moment values and pile-head displacement values were higher than other other cases. Increasing the embedded length decreased the pile-head deformations and increased the moments. There are two reasons why the moment increases with the pile diameter. First, as the pile diameter increases, higher axial load and hence higher inertial lateral forces are applied because of the safe pile bearing capacity increases. Second, when more rigid elements are exposed to similar deformations, higher cross-section stresses occur.

One meter thick non-liquefiable crust layer was added to the upper part of the soil profile in the consecutive parametric analyses and simulations were repeated for the 12-m long pile. The results are presented in Table 4. The schematic visual of the soil profile with an added crust layer is shown in Figure 8.

**Table 4.** Finite difference analysis results for 12-m long piles with free rotating pile-head, a 4-m embedded length and 1-m non-liquefied crustal layer.

| Diameter (m) | Pile-head displacement (m) | Maximum moment (kN.m) |
|--------------|----------------------------|-----------------------|
|              | SPT 5 | SPT 10 | SPT 15 | SPT 5 | SPT 10 | SPT 15 |
| 0.6          | 0.48  | 0.47  | 0.45   | 940   | 905   | 890   |
| 1.0          | 0.60  | 0.61  | 0.49   | 2505  | 2520  | 2340  |
IV. CONCLUSION

Presence of liquefiable layers impose additional forces on pile foundations. Piles with insufficient rigidity may buckle and sheared. Excessive lateral displacements may occur due to lateral spreading. The results indicate that the crust layer has an impact that immensely increases the moment values. Increasing the SPT values, decreased displacement and moment values observed in the pile-head. For a liquefiable soil with 15 corrected SPT value, the liquefied soil behaved more rigid and supported the pile more, hence less the displacement and moment values observed at the pile head compared to liquefiable soils with 5 and 10 corrected SPT values. In cases where no crust layer exist, the increase in the diameter of the pile caused smaller displacements at the pile head, opposite results were observed when non liquefiable crust layer exist. Moreover, the displacement value at the pile-head decreased as the pile length and socket depth increased in the cases with no crust layer. In the sample with a pile length of 8 m and embedded length of 1 m, the moment value was found to be lower than the sample analysis with the pile length 12 m and embedded length of 4 m as the pile is able to rotate without bending.

Moment capacities are exceeded in both pile simulations with different diameters when a crust layer exists. However, since behaviour of pile cross-section is linear in FLAC2D calculated moments continued to increase.

The pile diameter, length, embedded length, and crust layer properties should be assessed accurately and included in the analysis accordingly for the proper pile design. These results are valid only for the parametric study presented in this study, and it is clear that more analyses and experimental studies such as field measurements and shake-table tests are necessary to reach concrete conclusions.

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