Two-phonon structure of the neutron-rich nuclei

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Abstract. Starting from the Skyrme energy-density functional, the microscopic model of the spin-isospin excitations is developed for the $\beta$-delayed $\gamma$-spectroscopy. We take into account the coupling between one- and two-phonon terms in the wave functions of $1^+$ states of daughter nuclei. An extension of the two-phonon space allowing for the additional charge-exchange phonons substantially enriches the $1^+$ spectrum. As an example, the $\beta$-decay rates of neutron-rich nuclei $^{126,128}$Cd are discussed.

A study of the $\beta$-decay properties is an interesting problem not only from the nuclear structure point of view but it is very important for the nuclear astrophysics applications [1]. The $\beta$-decay properties of $r$-process “waiting-point nuclei” $^{129}$Ag, $^{130}$Cd, and $^{131}$In have attracted a lot of experimental efforts recently [2, 3, 4, 5, 6]. It is desirable to have theoretical models which can describe the data and predict the properties related to spin-isospin modes in the nuclei with extreme high $N/Z$ ratio to allow for experimental studies. One of the successful tools for nuclear structure studies is the quasiparticle random phase approximation (QRPA) with the self-consistent mean-field derived from the energy density functional (EDF) [7]. The framework allows to relate the properties of the ground states and excited states through the same EDF. The theoretical analysis has been done within the microscopic-macroscopic finite-range droplet model [8, 9], the continuum QRPA approach with the Fayans EDF [10, 11, 12]. Recently, the proton-neutron relativistic QRPA [13] and the finite-amplitude method [14] calculations with the Skyrme EDF have appeared. Importantly, all the cited papers have used the one-phonon approximation. On the other hand, it would be desirable to overcome the discrepancies between the theoretical predictions low-energy $1^+$ spectrum using the one-phonon QRPA wave functions of the daughter nucleus and the measurements, see e.g., [15].

Our tool is the QRPA with Skyrme EDF with tensor components included. By means of the standard procedure [16] the familiar equations of the QRPA in the one-phonon configuration space are obtained. We find the eigenvalues of the QRPA equations as the roots of a relatively simple secular equation within the finite rank separable approximation [17, 18, 19] which allows one to perform the calculations in large configurational spaces. We have generalized the approach to the coupling between one- and two-phonon terms in the $1^+$ wave functions and the tensor force effects on the $\beta$-decay rates of neutron-rich nuclei [20]. We applied the influence of the phonon-phonon coupling (PPC) on the multi-neutron emission probabilities [22]. The new calculation [23] is extended by enlarging the variational space for the $1^+$ states with the inclusion of the two-phonon configurations. The $1^+$ spectrum of $^{130}$In populated in the $\beta$-decay of $^{130}$Cd was studied. The dominant contribution to the additional $1^+$ states comes from the...
Figure 1. $\beta$-decay windows $Q_\beta$ of the parent nuclei (squares), one-neutron separation energies (circles) for the daughter nuclei. The panels: (a), (b), (c) and (d) correspond to the isotopes: Sn, In, Cd, Ag, respectively. Results of the HF-BCS calculations with the EDF T43 are denoted by the open symbols. Experimental data (filled symbols) are taken from Ref. [21].

$[3^+ \otimes 2^+]_{QRPA}$ two-phonon configurations constructed from the charge-exchange $3^+$ phonons. The coupling with the charge-exchange $3^+$ phonons enriches the low-energy $1^+$ spectrum of the daughter nucleus [23]. In the present report we illustrate this effect by comparing with the $^{126,128}$Cd decay rates and $E2$ transition probabilities of $^{126,128}$In.

The Skyrme EDF T43 is used for all calculations in connection with the surface-peaked zero-range pairing force. The T43 set [24] is one of 36 parametrizations, covering a wide range of the parameter space of the isoscalar and isovector tensor term added with refitting the parameters of the central interaction, where a fit protocol is very similar to that of the successful SLy parametrizations. This choice of the Skyrme EDF has been selected to reproduce the experimental the $Q_\beta$ values for the parent nuclei and the neutron separation energies ($S_n$) for the daughter nuclei in the vicinity of “waiting-point nuclei” $^{129}$Ag, $^{130}$Cd and $^{131}$In (see Fig. 1). Also, the set T43 predicts enough positive value of the spin-isospin Landau parameter $G_0' = 0.14$ at saturation density and it gives a reasonable description of properties of the Gamow-Teller (GT) and charge-exchange spin-dipole resonances [25]. It is worth mentioning that the correct description of the $Q_\beta$ values for the parent nuclei and the $S_n$ values for the daughter nuclei is the important ingredient for the reliable prediction of the outset of the $\beta$-delayed neutron emission of the isotope chain, see e.g., the case of Cd isotopes [22].

To take into account the PPC effects we follow the basic ideas of the quasiparticle phonon model (QPM) [26, 27]. The Hamiltonian can be diagonalized in a space spanned by states composed of one and two QRPA phonons [20],

$$\Psi_\nu(JM) = \left( \sum_i R_i(J\nu)Q_{JMi}^+ \right)$$
weighted with the integrated Fermi function $f$ in the GT transitions, described by the operator $\hat{O}$ \([23]\). The wave functions (1) allow us to determine the ground state, and it corresponds to the ground-state energy of the daughter nuclei $^{126,128}\text{In}$. This means that the two-phonon configurational space consists of the phonon compositions $[\lambda^\pi \otimes \lambda'^\pi]_{\text{QRPA}}$, i.e., $[1^+ \otimes 2^+]_{\text{QRPA}}$, $[3^+ \otimes 2^+]_{\text{QRPA}}$, $[3^+ \otimes 4^+]_{\text{QRPA}}$, $[2^- \otimes 3^-]_{\text{QRPA}}$, $[2^- \otimes 1^-]_{\text{QRPA}}$ and $[1^- \otimes 1^-]_{\text{QRPA}}$. The diagonalization of the Hamiltonian in the space of the one- and two-phonon configurations produces eigenvalues of $1^+_k$ states ($E_k$) \([20]\). The excitation energies are obtained by the following ansatz:

$$E_k(1^+_k) \approx E_i - \Omega.$$  

The QRPA analysis within the one-phonon approximation results in the spin-parity of the ground state, $J^\pi = 3^+$. The $\Omega$ value is the $[3^+_k]_{\text{QRPA}}$ eigenvalue of the QRPA equations, and it corresponds to the ground-state energy of the daughter nuclei $^{126,128}\text{In}$. In the present work, the calculation details are identical to Ref. [23]. The wave functions (1) allow us to determine the GT transitions, described by the operator $\hat{O} = \sum_{i,m} t_- (i) \sigma_{m(i)}$, as

$$B(\text{GT})_k = \left| \langle N - 1, Z + 1; 1^+_k | \hat{O}^- | N, Z; 0^+_D \rangle \right|^2.$$  

Since the correlation effects, produced by the tensor interaction, are taken into account within the $1p-1h$ and $2p-2h$ configurational spaces, any quenching factors are redundant \([28]\).

The $\beta$-decay rates of the parent (even-even) nucleus ($N,Z$) are expressed by summing up the probabilities (in units of $G_A^2/4\pi$) of the energetically allowed GT transitions ($E_x(1^+_k) < Q_\beta$) weighted with the integrated Fermi function $f_0$,

$$T_{1/2}^{-1} = \sum_k \lambda^k_{1f} = D^{-1} \left( \frac{G_A}{G_V} \right)^2 \times \sum_k f_0(Z + 1, A, Q_\beta - E_x(1^+_k)) B(\text{GT})_k,$$  

where $\lambda^k_{1f}$ is the partial $\beta$-decay rate, $G_A/G_V = 1.25$ is the ratio of the weak axial-vector and vector coupling constants and $D=6147$ s \([29]\). The results of the half-life calculation, $0.14$ s of $^{126}\text{Cd}$ and $0.12$ s of $^{128}\text{Cd}$, are in a satisfactory description of the experimental data \([30]\). The partial $\beta$-decay rate has a strong energy dependence which approximately scales like $(Q_\beta - E_x)^5$. As expected, the largest contribution in the calculated half-life comes from the $[1^+_k]_{\text{QRPA}}$ state. The two-phonon space results in the substantial GT strength
Figure 2. β-transition rates in $^{126}$Cd (the upper panels) and $^{128}$Cd (the lower panels). Panels (a) and (c): the β-transition rates are calculated within the QRPA. Panels (b) and (d): the calculation is taken into account the two-phonon configurations.

fragmentation [20, 22]. To illustrate it, the β-decay rates of $^{126,128}$Cd are shown in Fig. 2. The β-decay rates within the QRPA have a rather simple two-peak structure. The main contribution of the GT transition to the $[1^+_1]_{QRPA}$ state is built on the $\{1g_9=2, 1g_7=2\}$ matrix element. Inclusion of the PPC effects leads to the appearance of the weak fragmented satellites.

For both In isotopes, the $1^+_1$ states calculated with PPC effects have a prevailing two-phonon configuration of the $2^+_1$ vibrational phonon and the $3^+_1$ charge-exchange phonon. We find a smaller fragmentation effect, and the wave function normalizations of the $1^+_1$ states particularly mix 3% of the $[1^+_2]_{QRPA}$ configuration and about 97% of the $[3^+_1 \otimes 2^+_1]_{QRPA}$ configuration. This small change in structure has large effect on the partial β-decay rate, i.e. the small one-phonon contribution in the $1^+_1$ state. The calculated values: $E_x = 0.7$ MeV, $\log f t = 4.1$ of $^{126}$In and $E_x = 1.0$ MeV, $\log f t = 4.3$ of $^{128}$In are in a nice agreement for the $1^+_1$ states experimentally identified at 688 keV of $^{126}$In and at 1173 keV of $^{128}$In [30]. Also the two-phonon structure of the $1^+_1$ states is reflected in the $B(E2;1^+_1 \rightarrow 3^+_2)$ values, as is shown in Table I.

The dominant contribution in the wave function of the $1^+_2$ states of both In isotopes come from the $[1^+_1]_{QRPA}$ configuration, but the two-phonon contributions are appreciable. This means that these states has predominantly two-quasiparticle nature $\{1g_9/2, \nu 1g_7/2\}$. As a result, we find the largest β-decay rate in the case of the $1^+_2$ states, see Fig. 2. The main two-phonon components of the $1^+_2$ wave function are the $[3^+_1 \otimes 4^+_1]_{QRPA}$, $[1^+_1 \otimes 2^+_1]_{QRPA}$ and $[3^+_2 \otimes 2^+_1]_{QRPA}$ configurations such contributions leads to the small values of the $B(E2;1^+_2 \rightarrow 3^+_2)$, see Table I.
Table 1. The calculated energies and log $ft$ values of the low-lying $1^+$ states, $E2$ $1^+_k \rightarrow 3^+_g$ transition probabilities of $^{126,128}$In. $B(E2)$ values are given in Weisskopf units ($1 \text{ W.u.} = 5.94 \times 10^{-2} A^{4/3} e^2 \text{fm}^4$).

| $1^+_k$ | Energy (MeV) | $log ft$ | $B(E2)$ (MeV) | $log ft$ | $B(E2)$ (W.u.) |
|--------|--------------|---------|---------------|---------|---------------|
| $1^+_1$ | 0.7          | 4.1     | 17.7          | 1.0     | 4.3           |
| $1^+_2$ | 0.9          | 3.2     | 0.1           | 1.4     | 3.3           |
| $1^+_3$ | 1.5          | 4.9     | 0.1           | 1.5     | 3.5           |
| $1^+_4$ | 2.7          | 3.5     | 0.5           | 2.8     | 3.8           |
| $1^+_5$ | 2.8          | 4.7     | 0.1           | 3.0     | 4.8           |

For both In isotopes, the $1^+_1$ state is mainly characterized by the two-phonon component the \([3^+_1 \otimes 2^+_1]_{QRPA}\) and the \([1^+_2]_{QRPA}\) configuration. They play a key role in explaining the noticeable size of the partial $\beta$-decay rates and the $B(E2; 1^+_i \rightarrow 3^+_g)$ values.

We predict the two-phonon structures of the $1^+$ states experimentally identified at 688 keV of $^{126}$In and at 1173 keV of $^{128}$In. Also a correlation is found between the low-lying $E2$ transition strengths of the parent and daughter isobaric companions [23]. It is shown that two phonon structures have strong influence on electric transition probabilities. For even-even heavy vibrational nuclei, the lowest known $1^-$ state comes from the two-phonon structure composed of the quadrupole and octupole vibrational phonons. At the same time first one-phonon $1^-$ state in the calculations within the QRPA appears above 5 MeV [31, 32]. As shown in Ref. [33], there is an empirical correlation between $B(E1; 1^-_1 \rightarrow 0^+_g)$ and $B(E1; 3^-_1 \rightarrow 2^+_g)$ values. This low-energy $E1$ transition forbidden in the ideal boson picture has been calculated in the QPM [26] taken into account the internal fermion structure of phonons [34].

In summary, by starting from the Skyrme mean-field calculations, we have studied the effects of the phonon-phonon coupling on the $\beta$-decay rates and the electric transitions from the low-energy $1^+$ states to the ground state. As an example, we considered the $^{126,128}$Cd decays. It is shown that an extension of the configurational space allowing for the two-phonon excitations substantially enriches the $1^+$ spectra of $^{126,128}$In. Also the dominant two-phonon configuration $[3^+_1 \otimes 2^+_1]_{QRPA}$ of the $1^+_1$ wave functions leads to the noticeable $B(E2; 1^+_1 \rightarrow 3^+_g)$ values of $^{126,128}$In.

The existence of two-phonon $1^+$ states should be a generic feature of odd-odd nuclei in the vicinity of doubly magic nucleus $^{132}$Sn and further experimental investigation in this region to check this prediction are probably necessary.

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