Reduced Order Estimation of the Speckle Electric Field History for Space-based Coronagraphs

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Received 2019 March 21; revised 2019 June 28; accepted 2019 July 1; published 2019 August 21

Abstract

In high-contrast space-based coronagraphs, one of the main limiting factors for imaging the dimmest exoplanets is the time-varying nature of the residual starlight (speckles). Modern methods try to differentiate between the intensities of starlight and other sources, but none incorporate models of space-based systems that can take into account actuations of the deformable mirrors (DMs). Instead, we propose formulating the estimation problem in terms of the electric field while allowing for dithering of the DMs. Our reduced-order approach is similar to intensity-based PCA (e.g., KLIP) although, under certain assumptions, it requires a considerably lower number of modes of the electric field. We illustrate this by a FALCO simulation of the Wide-field Infra-red Survey Telescope hybrid Lyot coronagraph (the data and the comparison code are available at https://github.com/leonidprinceton/EFOR).

Key words: methods: data analysis – methods: statistical – techniques: high angular resolution – techniques: image processing

1. Introduction

Direct imaging of exoplanets and circumstellar disks requires an observatory that creates a high contrast between the astrophysical source and the residual starlight halo (speckles). The contrast needed to detect and characterize Earth-like planets is on the order of $10^{10}$ and can be achieved only by going to space to avoid the fundamental limitations imposed by atmospheric turbulence (Guyon 2005; Cavarroc et al. 2006). The Coronagraph Instrument (CGI) on NASA’s upcoming Wide-field Infra-red Survey Telescope (WFIRST) is designed to demonstrate the viability of such missions (Douglas et al. 2018). It will employ passive and active starlight suppression systems (coronagraphs, deformable mirrors (DMs), etc.) to block most of the starlight and control the diffraction pattern in a region of the image where the planet is located (Demers et al. 2015; Demers 2018). Yet, the speckle background is expected to remain brighter than the planet and to vary over the lengthy observation time (Shaklan et al. 2011; Ygouf et al. 2016). It is therefore necessary to post-process the images to distinguish between the speckles and the signals incoherent with the star (which include the planet). We will refer to the ratio between the intensity of the speckle floor and the dimmest detectable planet as the post-processing factor.

Post-processing methods for ground-based telescopes exploit various sources of diversity between images to extract the faint signal from the varying background (for a review, see Jovanovic et al. 2018; Pueyo 2018). Some utilize physical properties such as the rotation of the telescope (Marois et al. 2006), phase apodization (Codona et al. 2006; Kenworthy et al. 2007), the spectrum of the planet and the star (Marois et al. 2000; Sparks & Ford 2002; Martinache 2010), and their polarization (Baba & Murakami 2003). Others rely on mathematical manipulations of multiple images, either only from the target star (Lafreniere et al. 2007), or from additional observations of reference stars (Mawet et al. 2012), and even artificial speckles (Jovanovic et al. 2015). These images are often used to construct a low-dimensional representation of the speckle intensity by means of principal component analysis (PCA) (Amara & Quanz 2012; Soummer et al. 2012; Fergus et al. 2014), robust PCA (Gonzalez et al. 2016), or non-negative matrix factorization (Ren et al. 2018).

However, in order to fully capture the contribution of the speckles in each image, one has to take into account the evolution of the speckles and how they are affected by the DMs. For ground-based telescopes this can be done by monitoring the speckles in the coronagraph plane after being partially suppressed by the adaptive optics (phase sorting interferometry, Codona & Kenworthy 2013). However, space-based telescopes lack an adaptive optics system and operate with much lower photon counts ($\sim 10^3$ photons per exposure compared to $\sim 10^5$ in Codona & Kenworthy 2013). The phase diversity therefore has to come from dithering the DMs beyond what is necessary to suppress the speckles (as was shown by Pogorelyuk & Kasdin 2019).

Although DM dithering may lead to a higher post-processing factor, it requires accounting for the DM control history, which can be achieved by formulating the estimation problem in terms of the electric field of the speckles rather than their intensity. In this paper, we extend our previous work by introducing a reduced-order representation of the speckle field, similar to the low-rank treatment of speckle intensities by PCA-based methods.

We also propose that in the context of space-based coronography, formulating the estimation problem in terms of the electric field is more natural than in terms of the intensity. Although our method requires introducing phase diversity via DM dithering, it is able to distinguish between the incoherent signal and the residual starlight in the absence of reference libraries, telescope rotations, spectrographs, or other means to achieve diversity that require multiple images.

In Section 2, we describe an electric field order reduction (EFOR) approach to the estimation of speckle history and incoherent intensity and propose an explanation for its better performance (compared to intensity-based estimation). The suggested algorithm incorporates the DM commands, which may come from those used for high-contrast maintenance.
during the observation phase (i.e., the closed-loop approach, see Miller et al. 2017; Pogorelyuk & Kasdin 2019) or from dithering the DMs to increase the phase diversity necessary for estimating the electric fields (an open-loop approach) or both together. In Section 3, we use a simulated WFIRST observation scenario to illustrate the new algorithm’s performance and compare it to an intensity-PCA-based estimation.1

2. Electric Field Order Reduction

In this section, we formulate an algorithm that computes an estimate of the incoherent intensity (of light sources other than the star) given measurements of photon counts across multiple pixels and time frames and the history of DM controls. It is similar to PCA-based methods such as Karhunen-Loève Image Projection (KLIP, Soummer et al. 2012) and PynPoint (Amara & Quanz 2012) because it considers a low-dimensional basis for the speckles, but is also similar to the a posteriori estimation method suggested in Pogorelyuk & Kasdin (2019) because it formulates an optimization problem in terms of the electric field.

PCA-based methods assume that the speckles lie in an empirically determined subspace(s) of intensities. Besides ignoring the history of control inputs and allowing for negative intensities, these methods might also increase the dimension of the subspace beyond what is necessary for accurately approximating the speckles in a space-based telescope. Indeed, let us assume that the speckles drift due to slowly varying wavefront errors (WFEs), which can be well approximated by a small number (~20) of 2D polynomials (e.g., Zernikes). Then, due to the linear nature of Fourier optics, the variations in the electric field of the speckles are also well approximated by the same small number of electric field modes. However, as we show next, the number of intensity modes necessary to capture the same variations is significantly larger (~200). Under these assumptions, a low-dimensional representation of the electric field reduces the number of model parameters by an order of magnitude (while also allowing the incorporation of the history of DM dither and controls).

Throughout the paper we consider a set of $N$ low-bandwidth photon detectors located in a region of high contrast—the dark hole. Broadband imaging is therefore modeled by multiple detectors of discrete wavelengths and their exact number and location does not play a role in the analysis below (alternatively, one can think of taking multiple images with low-bandwidth filters of varying wavelength, see, for example, Give’on et al. 2007a). We define the electric field vector over the real numbers (to make the discussion of the numerical method and the comparison to intensity PCA easier later in the text), as

$$\mathbf{E}(t) = \begin{bmatrix} \text{Re} E_1(t) \\ \text{Im} E_1(t) \\ \text{Re} E_2(t) \\ \text{Im} E_2(t) \\ \vdots \end{bmatrix} \in \mathbb{R}^{2N}, \quad (1)$$

where $E_i(t)$ is the electric field of the residual start light (speckles) at pixel $i$ and time frame $t$. Here $t = 0$ corresponds to the first exposure after the dark hole has been created (for example, via Electric Field Conjugation—EFC (Give’on et al. 2007b). We take the nominal DM setting used to create the dark hole to be, without loss of generality, $u(0) = 0$. By exploiting the approximately linear effects of DM actuators, we further split the electric field into

$$\mathbf{E}(t) \approx \mathbf{E}^{OL}(t) + G^t u(t), \quad (2)$$

where $\mathbf{E}^{OL}(t)$ is the hypothetical speckle field in the absence of DM actuations (open-loop), $u(t)$ is the deviation of the DM command from its nominal value (due to both dither and possible closed-loop control), and $G^t$ is the control influence matrix (the Jacobian).

The intensity of the speckles is given by

$$\mathbf{I}(t) = B \cdot (\mathbf{E}(t) \circ \mathbf{E}(t)) \in \mathbb{R}^N,$$  \quad (3)

where $\circ$ stands for the Hadamard (element-wise) product and

$$B = \begin{bmatrix} 1 & 1 & 1 & \cdots \\ \vdots & \ddots & \ddots & \vdots \end{bmatrix} = I_{N \times N} \otimes [1 \ 1]. \quad (4)$$

We define the total intensity in the image (from all sources) as

$$\mathbf{I}(t) = \mathbf{I}^S(t) + \mathbf{I}^D + \mathbf{I}^P + \mathbf{I}^L,$$ \quad (5)

where the sources incoherent with the speckles are $\mathbf{I}^D$ —the intensity of the signal (planet), $\mathbf{I}^P$ —the intensity of the zodi, and $\mathbf{I}^L$ —the “effective” intensity of the dark current (thermal electrons detected as photons). For a space telescope pointing in a fixed direction, $\mathbf{I}^P$ and $\mathbf{I}^L$ are assumed to be constant and $\mathbf{I}^D$ is assumed to be constant, known, and uniform. In our analysis, $\mathbf{I}^P$ and nonuniformities in $\mathbf{I}^D$ will be indistinguishable from $\mathbf{I}^L$ and are therefore sources of systematic errors (which could potentially be addressed given a model of the actual optical system and observation scenes).

The measured numbers of photons during frame $t$

$$\mathbb{R}^N \ni \mathbf{y}(t) \sim \text{Poisson} (\mathbf{I}(t)),$$ \quad (6)

is a vector of realizations of independent Poisson distributed variables whose parameters are given by the elements of $\mathbf{I}(t)$. Our goal is, given measurements $\{\mathbf{y}(t)\}_{t=1}^T$ and some model for $\mathbf{E}(t)$, to estimate $\mathbf{I}^D + \mathbf{I}^L$ (and we will assume $\mathbf{I}^P = 0$ for the rest of the paper). In Equation (6), we ignored read noise because it is expected to be negligible when WFIRST is operating in “photon-counting” mode (Harding et al. 2015).

We note that errors in the Jacobian, $G^t$, would translate into errors in estimates of the field via Equation (2). This issue can be addressed by keeping the commands, $u(t)$, approximately zero-mean and small (this requires periodically “recalibrating” $u(0)$, see Pogorelyuk & Kasdin 2019), and by obtaining more accurate estimates of the Jacobian during the dark hole creation phase (Sun et al. 2018).

2.1. Reduced-order Modeling of the Speckle Field and Its Drift

The main idea behind our method is to assume that the electric field, $\mathbf{E}^{OL}(t)$, lies in a low-dimensional subspace,

$$\mathbf{E}^{OL}(t) \in \mathcal{S}_E \subset \mathbb{R}^{2N}. \quad (7)$$

Here, $\dim \mathcal{S}_E = r$ and $r - 1$ is the number of Zernike polynomials (or any other basis) required to get an accurate model of the low-order, slowly varying WFEs in the pupil

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1 The data and the comparison code are available at https://github.com/leonidsprinceton/EFOR.
plane. It is implied that those errors propagate linearly, hence the increments of the hypothetical open-loop field, \( E^{OL}(t) - E^{OL}(t-1) \), lie in an \( r \) - one-dimensional subspace (the extra dimension is for \( E^{OL}(0) \)). Similarly, contemporary methods based on PCA of the intensity assume that the intensity lies in a low-dimensional subspace

\[
I^{E^{OL}}(t) \in S_I \subset \mathbb{R}^N, \tag{8}
\]

where \( I^{E^{OL}} = B \cdot (E^{OL} \circ E^{OL}) \). Here, \( S_I \) and \( S_E \) encompass all of the pixels, but both formulations are applicable to smaller regions of pixels in the dark hole.

The two approaches are consistent in the sense that if the field increments satisfy Equation (7) then the intensity increments satisfy Equation (8). However, in light of Equation (3) and the properties of the Hadamard product

\[
\dim S_I = \left( \frac{r+1}{2} \right) = \frac{r(r+1)}{2}. \tag{9}
\]

That is, the dimension of the intensity’s subspace, \( S_I \), may be up to \((r+1)/2\) times larger than the dimension of the electric field’s subspace, \( S_E \).

We note that the dimension of \( S_I \) can be as small as just four times larger than the dimension of the unknown electric field’s subspace, \( S_E \). In practice, the WFEs do not exactly lie in a low-dimensional subspace; it therefore only makes sense to consider the “effective” dim \( S_I \), which is somewhere between 4 and \((r+1)/2\) times larger than the “effective” dim \( S_E \), depending on how well the WFEs are approximated by a Fourier basis. In any case, we claim that low-order modeling of speckles is more naturally done in terms of the electric field rather than intensity.

To this end, we rewrite Equation (7) as

\[
E^{OL}(t) = \text{colsp}\{G^V(t)\}, \tag{10}
\]

where \( \text{colsp}\{G^V\} \) and \( \text{colsp}\{G^E\} \) are replaced with the column space of some generally unknown matrix \( G^V \in \mathbb{R}^{2N \times r} \). This gives a low-order parameterization of the open-loop electric field

\[
E^{OL}(t) = G^V(t), \tag{11}
\]

with \( v(t) \in \mathbb{R}^r \). The goal of the remainder of this section is to estimate the relatively small number of parameters \( G^V \), \{\( v(t) \}_{t=1}^T \} \) given measurements and control history. This, in turn, will allow us to estimate the planet signal, \( I^P \).

### 2.2. Estimation Based on EFOR

We wish to estimate the intensity of the planet, \( I^P \), given measurements \{\( y(t) \)\}_{t=1}^T \) that depend on the history of the unknown electric field, \( \{E^{OL}(t)\}_{t=1}^T \), and known DM commands, \{\( u(t) \)\}_{t=1}^T \). The proposed method has a single free parameter—the dimension of the electric field subspace, \( r \) (its implementation can be found at https://github.com/leonidprinceton/EFOR).

First, we approximate the incoherent intensity, \( I^I = I^P + I^D \) (assuming \( I^D = 0 \)), by its conditional mean estimate, \( \hat{I}^I \), given by (see Equations (2), (3), and (5)),

\[
\hat{I}^I([E^{OL}(t)]_{t=1}^T) = \text{ramp}_D \left\{ \frac{1}{T} \sum_{t=1}^T \left[ \sum_{t=1}^T (y(t) - B \cdot (E^{OL}(t) + G^U u(t))) \right] \right\}, \tag{12}
\]

where \( \text{ramp}_D \) is the shifted ramp function,

\[
\text{ramp}_D ([y(t)]_{t=1}^N) = \text{max} \{ [y(t), I^P] \}_{t=1}^N. \tag{13}
\]

This ensures that the intensity estimate is above the dark current (which is known).

Second, we express the probability of the measurements given the electric fields and incoherent intensities

\[
p([y(t)]_{t=1}^T | [E^{OL}(t)]_{t=1}^T, I^I) = \prod_{t=1}^T \prod_{i=1}^N \frac{1}{y_i(t)!} \left( \frac{1}{y_i(t)} \right)^{y_i(t)} e^{-\frac{1}{y_i(t)} y_i(t) (y_i(t) + [E^{OL}(t) + G^U u(t)])}, \tag{14}
\]

This expression stems from the fact that \( y_i(t) \) is Poisson distributed with parameter \( I_i(t) = I^P + [E^{OL}(t) + G^U u(t)] \).

Substituting \( \hat{I}^I \) the intensity estimate from Equation (12) in place of \( I^I \), this distribution can be approximated by a function of the field alone

\[
p([y(t)]_{t=1}^T | [E^{OL}(t)]_{t=1}^T) \approx \prod_{t=1}^T \prod_{i=1}^N \frac{1}{y_i(t)!} \left( \frac{1}{y_i(t)} \right)^{y_i(t)} e^{-\frac{1}{y_i(t)} y_i(t) (y_i(t) + \hat{I}^I(t))}, \tag{15}
\]

where \( \hat{I}^I \) is also a function of \{\( E^{OL}(t) \)\}_{t=1}^T.

Lastly, we replace \{\( E^{OL}(t) \)\}_{t=1}^T by its reduced-order parameterization, Equation (11), and propose the maximum likelihood estimator

\[
\hat{G}^V, \{\hat{v}(t)\}_{t=1}^T = \arg \max_{G^V, \{v(t)\}_{t=1}^T} p([y(t)]_{t=1}^T | [G^V v(t)]_{t=1}^T), \tag{16}
\]

We note that the reduced rank of \( v(t) \) compared to \( E(t) \) can be seen as a regularization of the likelihood implied by Equation (15). In Pogorelyuk & Kasdin (2019), the authors regularized the likelihood using the prior distribution of \( E^{OL}(t) \) (with respect to time), which resulted in a maximum a posteriori estimate.

The task of finding the estimate in Equation (16) can be expressed as an optimization problem. We define the cost function

\[
J(G^V, \{v(t)\}_{t=1}^T) = -\log p([y(t)]_{t=1}^T | [G^V v(t)]_{t=1}^T), \tag{17}
\]

where \( p \) is defined via Equation (15). The minima of \( J \) are not unique or isolated, because there are many different choices of basis vectors (columns of \( G^V \)) that span a given subspace. More precisely, for any invertible \( W \in \mathbb{R}^{r \times r} \), we have

\[
J(G^V, \{v(t)\}_{t=1}^T) = J(G^V W, \{W^{-1}v(t)\}_{t=1}^T), \tag{18}
\]

so for any minimizer \( G^V \) there is a family of minimizers (parameterized by \( W \)) with the same cost \( J \). In Section 3, we
find a local minimum of $J$ using a gradient descent algorithm (specifically the Adam optimizer, Kingma & Ba 2014), in which we initialize $G^V$ as a random orthogonal matrix. This method does not appear to have any numerical issues, even though the minima are not isolated. Alternatively, one could add a soft constraint term, $\| \text{transpose}(G^V) G^V - I \|$, to the cost function (see, for example, Brock et al. 2016), or constrain $\{E^\text{OL}(t)\}_{m=1}^R$ to a Grassmann manifold (Edelman et al. 1998; Townsend et al. 2016).

2.3. Incorporating Reference Images

So far we have only considered measurements, $\{y(t)\}_{t=1}^T$, taken during the observation phase. However, existing reduced-order methods incorporate prior observations by keeping a library of speckle images, which they use to compute the intensity subspace. An analogous “library” in our case would be a set of reference images, $\{y^{\text{ref}}(m)\}_{m=1}^R$, and controls, $\{u^{\text{ref}}(m)\}_{m=1}^R$, applied to the DMs when taking those images. Below, we show how to incorporate such data to improve the incoherent intensity estimate.

To compile a library of reference PSFs to be used with KLIP, WFIRST is proposing to periodically chop from the target star to a brighter reference star to obtain new reference PSFs and to occasionally reset the DMs. Although the reference star is usually significantly brighter than the target star, there is no conceptual difference between the measurements. Therefore, we treat the two types of data identically and compute an estimate of the speckle drift subspace, $\text{colsp}(G^\Gamma)$, based on both. This is contrary to PCA approaches, which do not employ DMs to allow “detecting” incoherent sources in the library of reference images (and therefore target images cannot be used as part of that library).

For simplicity, we assume that in addition to the measurements of the target star, $\{y(t)\}_{t=1}^T$, we have a set of images corresponding to a single reference star, $\{y^{\text{ref}}(m)\}_{m=1}^R$. The control’s Jacobian, $G^\Gamma^{\text{ref}}$, depends on the brightness and spectrum of the reference star and therefore differs from the Jacobian of the target star, $G^\Gamma$. The parameterizations of the electric field in Equation (11) may be shifted by some known $E^\text{OL}_{\text{ref}}$, if the nominal controls differ between the two observations. The speckles subspace, on the other hand, is a property of the optical system; it is thus shared between the observations ($G^{\Gamma^{\text{ref}}}_m = G^V$).

A reduced-order parameterization of the open-loop field of the reference star is (see Equation (11)),

$$E^\text{OL}_{\text{ref}}(m) = E^\text{OL}_{\text{ref}}^0 + G^V y^{\text{ref}}(m).$$

(19)

Likewise, the conditional incoherent intensity estimate in Equation (12) becomes

$$I^{\Gamma^{\text{ref}}}(\{E^\text{OL}_{\text{ref}}(m)\}_{m=1}^R)$$

$$\text{ramp}_{\mathcal{P}} \left\{ \sum_{m=1}^R (y^{\text{ref}}(m) - \mathcal{B}) \cdot (E^\text{OL}_{\text{ref}}(m) + G^\Gamma^{\text{ref}} u^{\text{ref}}(m))^{p^2} \right\}.$$ 

(20)

and the cost function for the reference star (defined via Equation (15) and $I^{\Gamma^{\text{ref}}}$) is

$$J^{\text{ref}}(G^V, \{y^{\text{ref}}(m)\}_{m=1}^R) = -\log p(\{y^{\text{ref}}(m)\}_{m=1}^R | \{E^\text{OL}_{\text{ref}}^0 + G^V y^{\text{ref}}(m)\}_{m=1}^R).$$

(21)

We may now find the incoherent intensity estimate of the target star, $I^\Gamma$, by optimizing

$$J^\text{tot}(G^V, \{y^{\text{ref}}(m)\}_{m=1}^R, \{y(t)\}_{t=1}^T) = J^{\text{ref}} + J^{\text{tot}},$$

(22)

and then using Equation (16). Note that measurements corresponding to higher intensity (higher $y^{\text{ref}}$ or $y^{\text{ref}}$), have proportionally larger “weights” in $J^\text{tot}$ due to the power term in Equation (15). This implies that the data gathered from the brighter reference star plays a larger role in determining the speckle drift modes, $G^V$, because they carry more information (per image).

Since Equation (22) treats the measurements from various sources identically, it can be extended to incorporate data from multiple targets (and possibly reference stars),

$$J^\text{tot} = J^{\text{ref}_1} + J^{\text{ref}_2} + \cdots + (J^{\text{ref}_1} + J^{\text{ref}_1} + \cdots).$$

(23)

This will arguably give better characterization of the drift, $G^V$, and consequently result in better estimates of the incoherent intensities for the target stars, $\{I^\Gamma^{\text{ref}_1}, I^\Gamma^{\text{ref}_2}, \cdots \}$. Given enough measurements, one might even be able to improve accuracy of the Jacobian estimate by adding appropriate terms to Equation (22) (similarly to the real-time approach for combined estimation of the electric field and the Jacobian proposed in Sun et al. 2018).

3. Numerical Simulation

This section illustrates the EFOR method for estimating the incoherent intensity given a history of photon counts (images), control inputs, and, possibly, a set of reference images (the data and code for generating the results below are available at https://github.com/leonidprinceton/EFOR). The data was simulated with the Fast Linearized Coronagraph Optimizer (FALCO: Ruane et al. 2018) model of the WFIRST hybrid Lyot coronograph.

3.1. Simulation Details

The hybrid Lyot coronograph starts at a relatively low contrast due to model/system imperfections. Therefore, as a precursor to the observation stage, the EFC (Give’on et al. 2007b) algorithm was used to create a dark hole in a ring between 3 and $9 \lambda/D$ with a nominal contrast of $4.2 \cdot 10^{-10}$ at $\lambda = 546$ nm (to simplify the analysis, we discarded all electric fields of longer wavelengths that were simulated by FALCO). At this point, we introduced wavefront drift through independent random increments of the first 21 Zernike polynomials

$$\Delta z_p^{p-2j}(t) \sim \mathcal{N}(0, 0.1 \text{ nm}, \frac{0.1 \text{ nm}}{p^2}, \lambda), \ 0 < j < p,$$

(24)

where $p$ is the order of the polynomial, $j$ is its azimuthal degree, and $\Delta z_p^{p-2j}$ is its increment over one 100 s frame. To make the comparison with PCA-based methods (Amara & Quanz 2012; Soummer et al. 2012) easier, we chose a Markov drift model in which the Zernike coefficients $z_p^{p-2j}(t)$ fluctuate but remain
1.0 photons
5
observations, the Zenike coefficients where advanced 1000
time steps via Equation (25). Another potential source of high-
order WFE is the time-dependent drift of the DM actuators
themselves (J. T. Trauger 2019, personal communication).
Although not addressed here, its influence on the electric field
would be linear and manifest itself through a low-rank
approximation of the control Jacobian ($G^U$ instead of $G^V$
in Equation (11)). Contrary to what one might expect, the
“effective” rank of the Jacobian is far lower than the number
of actuators, the latter usually being large to prevent actuator
oversaturation.

The simulated data consisted of 500 frames of the target star
(with an exoplanet) and 1500 frames of the reference star,
which is more than sufficient for PCA purposes. The
measurements were “taken” once with fixed DMs and once
with DM dithering, while keeping the drift history, $z_{p}^{-2}(t)$, the
same. The average intensity of the residual light was 1.2 photons
frame in a perfect dark hole, 4.8 photons frame with simulated WFEs (time
average), and 5.7 photons frame with both WFEs and DM dithering.
When pointing at the reference star, all speckle intensities
where 16 times higher. The maximum intensity of the planet
was 1.0 photons frame (see the comparison in Figure 1(b)) and the
effective intensity of the dark current was 0.25 photons frame.

The total number of pixels in the dark hole (for the single
wavelength considered) was $N = 2608$. The history of the
average dark hole intensity when observing the target star is
shown in Figure 1(a), with and without DM dithering (the
dithering for each actuator was sampled from a normal
distribution with 0.01V standard deviation around their
nominal values, which had an average magnitude of 10V). At
each time frame, the number of photons detected at each pixel
was sampled from a Poisson distribution based on the intensity
of that pixel (Equation (6)). We note that instead of just
dithering the DMs, one could also apply an EFC control law to
mitigate the effects of time-varying speckles (Pogorelyuk &
Kasdin 2019). This closed-loop approach would have resulted
in higher contrast, lower shot noise, and a better post-
processing factor; we plan to analyze the combined performance
of EFOR and dark hole maintenance in a future paper.

3.2. Results and Discussion

Figure 2 shows the best estimates of the incoherent intensity
obtained by PCA and the EFOR method introduced in
Section 2. The PCA estimates were computed based on
measurements taken with fixed DMs. The reference images
were used to construct a basis for speckle intensities (via
singular value decomposition) across all pixels (although
splitting the domain into smaller regions could reduce the
optimal number of modes, see Soummer et al. 2012; Fergus
et al. 2014). The two EFOR estimates were obtained from
measurements taken while dithering the DMs: one corresponds
to fitting just the observations of the target star, and the other to
fitting the observations to both the target and the reference stars
(Equation (17) versus (22)). The latter estimate is slightly more
accurate, illustrating the advantage of incorporating multiple
observations. However, once one has a good estimate of the
speckle basis, colsp($G^V$), additional reference images provide
diminishing improvements in accuracy (similarly, PCA
becomes insensitive to the number of reference images beyond
a certain threshold).

To make the comparison between the methods quantitative,
consider the average error in the incoherent intensity estimate,
$I_{\text{rep}}$, in a region where the planet’s intensity is above its half-

Figure 1. (a) Average intensity of the speckles in the dark hole (with and without dithering of the deformable mirrors) as it varies in time due to WFE drift. The maximum intensity of the planet is an order of magnitude below the speckle floor. (b) Side by side comparison of the speckles and the planet on a logarithmic scale. The data was obtained using a model of the WFIRST hybrid Lyot coronagraph implemented in FALCO (Ruane et al. 2018).
In Figure 3(a), the errors in EFOR estimates are plotted as a function of the number of modes, \( r \), while the errors in the PCA estimates are plotted as a function of \( \frac{r(r+1)}{2} \) (in light of Equation (9)). We indeed see that the errors of EFOR with reference data flatten around \( r_{\text{true}} \approx 22 \) modes, while PCA reaches its peak accuracy in the region between 66 and 300 modes (corresponding to roughly 4\( r_{\text{true}} \) and \( \frac{r_{\text{true}}(r_{\text{true}} + 1)}{2} \), see the discussion after Equation (9)). EFOR without reference data, on the other hand, reaches its peak accuracy at around \( r = 15 \) modes after which it overfits the data.

The lowest error, as defined by Equation (26), is achieved by EFOR with reference data. The minimum error of EFOR with data from just the target start is 1.4 times larger, while the lowest PCA error is 2.5 times larger. These errors can be further reduced by incorporating more observations or employing a closed-loop dark hole maintenance scheme (Pogorelyuk & Kasdin 2019) to reduce the shot noise.

Finally, Figure 3(b) shows the cost function \( J \) of the EFOR methods and the truncated singular value of the PCA method as a function of \( r \) and \( \frac{r(r+1)}{2} \) respectively. The steep decrease in the EFOR’s cost function (with reference data) stops at \( r = 15 \), suggesting that this is the number of “important” WFE modes. One would therefore expect the steep decrease in the singular values to stop somewhere between 4\( r = 60 \) and \( \frac{r(r+1)}{2} = 120 \) PCA modes, and it indeed stops at around \( \frac{12 \cdot 13}{2} = 78 \) modes. In the case of a single observation sequence, the cost function of

\[
\text{err}(I^P) = \text{avg} \{|I_i^P - I_i| \ i \leq N, I_i^P > 0.5 \|I^P\|_{\infty}\}.
\]

(26)
EFOR decreases very slowly (red dashed line on Figure 3(b)) and does not provide an indication of the “correct” number of modes, \( r \). However, instead of using reference images, one could incorporate data from multiple targets to get a better estimate of the speckle field basis, \( G^T \) (see the discussion around Equation (23)). Alternatively, one could choose the \( r \) that maximizes some detection criterion based on the PSF of the planet/halo (e.g., a matched filter, Kasdin & Braems 2006, or a Hotelling observer, Caucci et al. 2007).

4. Conclusions

In this work we assumed that the slow evolution of WFEs in space-based coronagraphs can be well approximated by a small number of modes of the electric field in the pupil plane and that these errors are linearly propagated to the image plane. We then showed that a low-dimensional approximation of the speckles requires an order of magnitude fewer electric field modes than intensity modes. Consequently, we introduced a reduced-order algorithm for estimating the history of the electric field and the incoherent intensity (which includes the planet signal). Unlike existing methods (e.g., KLIP), our algorithm relies on small actuations (dithering) of the DMs during the observation phase as well as an accurate estimate of the controls influence matrix (the Jacobian). This introduces the phase diversity required for estimating the electric field, rather than the intensities of the speckles.

We illustrated that because the newly proposed method employs significantly fewer parameters, it has the potential to make substantially more accurate estimates than intensity-based PCA, even without a library of reference images. However, our method does allow for seamlessly incorporating reference data as well as other observations. The resulting scheme searches for an “optimal” basis for speckle field modes across all measurements taken by a given instrument.

More importantly, the proposed algorithm uniquely incorporates the history of control inputs employed during the observation phase. We speculate that combining it with a closed-loop dark hole maintenance scheme will decrease the shot noise and increase the post-processing factor even further. This direction, together with an analysis of the dithering strategy, are subjects of future research.

This material is based upon work supported by the Army Research Office, award number W911NF-17-1-0512. We would like to thank Neil T. Zimmerman for providing technical details for the proposed observation sequence of the WFIRST telescope. We would also like to thank the reviewer of the manuscript for pointing out the lower bound on the number of modes necessary to describe a set of intensity fields in Section 2.1.

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