Junctions and the Fate of Branes in External Fields

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Abstract

We discuss the processes of brane bubble nucleation induced by the external branes. The quasiclassical solution for the nucleation by the single external brane has been found in the case when the brane junctions are possible. Exponential factor in the production rate has been calculated. The process induced by the fundamental or D string in the background of two D3 branes is analyzed and its interpretation from the D3 worldvolume theory viewpoint is described.

1 Introduction

During the recent years branes were recognized as an important ingredient in description of the nonperturbative sector of string and field theories. Essentially speaking the branes are multi-dimensional objects having tension and bearing some quantum numbers which can be considered as the higher-dimensional generalizations of the electric and magnetic charges (see

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for instance [1] as a review). In general the branes are very natural objects to study, since they are natural generalizations of the familiar pointlike particles. Including them into the string theory considerations has enriched the string physics as much as Maxwell theory physics is enriched by adding charges and monopoles.

In the context of string theory the branes are mainly considered as background for string dynamics. Some dynamical questions of brane physics have also been studied in the classical approximation. Consideration of the Schwinger-type processes in external fields involves the simplest quasiclassical behavior and the paper is devoted to their analysis.

The spontaneous Schwinger-type processes have previously been considered in [2], [3], [4], [5], [6] solutions to the equations of motion of Born-Infeld (BI) brane action which support the string like configuration, representing electrically or magnetically charged pointlike object on the brane worldvolume, have been discussed in [7], [8], [9]. Recently similar solutions involving string junctions were found in [10].

The basic example of the brane tunneling phenomena was analyzed in [2] where the brane creation in the external RR field has been described. The RR fields couple to the higher dimensional generalization of electric charges and like the point particle Lagrangian includes a term involving the potential one form integrated over the worldline of the particle, the $p$-brane action includes the RR (or NS) $p+1$-form integrated over $p+1$-dimensional worldvolume of the brane; a coefficient in front of this term is the higher dimensional generalization of the electric charge.

We remind that the Schwinger’s pair production in the external electric field is intuitively described as virtual charge anticharge pair being accelerated by the external field in the classically forbidden region until their energy reaches their mass threshold. The acceleration in the classically forbidden region is neatly formulated in terms of imaginary time trajectories, that is, in terms of trajectories in Euclidean space which solve the Euler-Lagrange equations obtained from the original Euler-Lagrange equations by the analytical continuation of time. In the case of uniform external electric field the corresponding trajectories are circles of a fixed radius. Analogously, brane production in the external RR fields is described in terms of the brane world surfaces which minimize (extremize) the Euclidean brane action. In the case of uniform
RR field the corresponding surfaces describing $p$-brane production are $p+1$-dimensional spheres (bubbles) of a given radius. The action computed on such spheres defines the exponential factor in the production rate. The process described can be characterized as a spontaneous brane production in the external RR field.

Apart from the spontaneous brane production one can consider induced brane production, that is production of branes in the presence of some original brane configuration, which is our main concern here. Since in the nonperturbative string theory branes can couple to branes the external branes can couple to the bubbles and hence can deform them changing the brane production rate. The physics arising is a higher dimensional generalization of the one discussed in [11] (see also consideration of the case of nonzero temperature [12] or density [13]), where it was shown that in the particle induced vacuum decay the bubble is deformed in the presence of external particle(s). The key point concerning the induced decay is the existence of the localized mode of the particle on the kink or antikink. Due to zero mode the initial particle can transfer its energy into the energy of tunneling. The same phenomenon takes place in the brane bubble creation thanks to the existence of junction [14] which substitutes the particle-kink vertex in the higher dimensional case. Indeed the initial brane looses almost all its initial energy after the junction point and this energy leads to the increase of the nucleation rate.

Hence the crucial point for our further analysis is the existence of the string junctions when three $(p,q)$ strings join in the vertex which is subject to the charge conservation and zero total tension condition [14]. It was shown [16, 17] that the string junction keeps $1/4$ of the original SUSY and can therefore be treated classically so that the generic string networks can be developed [18]. More recently junctions have been promoted to the M-theory configurations [19] where the vertex is resolved smoothly. Note also that the analogous webs have been elaborated for the 5-branes within string or M-theory approach [13]. In what follows we shall use junctions to find out the explicit form of the Euclidean solution corresponding to the induced tunneling.

When considering strings stretched between external D3 branes interesting interpretation of the induced tunneling emerges. The key point is that the end of the fundamental (F) string on the D3 brane can be considered as the point charge [21], the end of the D string as the monopole [20] and the end of $(p,q)$ string as the dyon. Hence any process involving the strings between
D3 branes amounts to the specific process in the SUSY gauge theory on the D3 worldvolume \cite{23}. For instance the processes with the exchange of the quantum numbers in the bulk have the interpretation of the nonperturbatively mediated phenomena involving the generic dyonic states, such as a well known process of the scattering of the monopoles to dyons \cite{24}. In what follows we shall use junction configuration with the background D3 branes \cite{25} to describe such processes in the external field.

It is perhaps important that the induced brane production can be given a slightly different interpretation - the one of quantization of the external branes in an external RR field. The bubbles then describe contribute to imaginary part in self-energy (“self-tension”) of the external branes. Contribution of such ”brane loops” has been discussed recently for topological strings in the M theory framework \cite{26}. Furthermore, in terms of the field theory on the inducing brane, the external field is a perturbation of the theory. Therefore the induced bubble has something to do with the renormalization group flow.

The rest of the paper is organized as follows. In section 2 we introduce some necessary background. In section 3 we discuss our main example - the induced brane production with one external brane. The production rate in the leading exponential approximation is calculated . Section 4 contains the consideration of the \((p, q)\) strings between D3 branes in the external field and the interpretation of the induced tunneling in terms of the gauge theory on the D3 brane. Discussion on the related issues and open questions can be found in Section 5.

2 Spontaneous brane production

Consider a \(p\)-brane omitting the fermionic degrees of freedom. Its action, essentially, consists of two pieces

\[
S = S_{\text{tension}} + S_{\text{charge}},
\]

where \(S_{\text{tension}}\) is

\[
S_{\text{tension}} = T \int_V \sqrt{\det (G_{\mu\nu} + \mathcal{F}_{\mu\nu})},
\]

the integration is over the \(p + 1\)-dimensional world-volume \(V\) of the brane, \(G_{\mu\nu}\) is the metric induced on the worldvolume via its embedding into the target space and the coefficient \(T\) is
the tension of the brane. Additional contribution \( \mathcal{F}_{\mu\nu} = \mathcal{F}_{\mu\nu} - B_{\mu\nu} \) is due the U(1) YM field and the pullback of the background NS two-form B field. The target space is taken to be the flat Euclidean one in this section, its metric being

\[
ds^2 = \Sigma_i (dx^i)^2. \tag{3}\]

The second term \( S_{\text{charge}} \) is different for the fundamental and D branes. For fundamental brane it has a simple form

\[
S_{\text{charge}} = Q \int_V \hat{C}, \tag{4}\]

where \( \hat{C} \) is NS \( p+1 \) form potential integrated over the worldvolume of the brane. For D brane the additional CS term comes from the RR external fields and has the structure

\[
S_{\text{charge}} = \int \exp \mathcal{F} \wedge \tilde{C}, \tag{5}\]

where \( \tilde{C} \) collects all relevant RR forms. In the most interesting case of D string it includes the excited two-form and axion fields

\[
S_{D-\text{str}} = \int \tilde{C}_2 + \tilde{C}_0 \wedge \mathcal{F}. \tag{6}\]

In external fields the branes are produced in the form of bubbles. Hence if the brane worldvolume forms a closed manifold (the bubble), the CS term can be rewritten as

\[
S_{\text{charge}} = Q \int_U \hat{H}, \tag{7}\]

where \( H = dC \) and \( U \) is the manifold with the boundary \( V \). In the simplifying assumption that \( H \) is a uniform \( D = p+2 \)-form in the flat Euclidean space of dimension \( D \), the last term reads

\[
S_{\text{charge}} = \pm Q \Phi \int_U \sqrt{\det(\hat{G}_{\mu\nu})}, \tag{8}\]

where \( \Phi \) is a flux density of the field \( H \). The two possible signs in Eq. (8) refer to possible orientations of \( V \).

\footnote{The assumption about dimension is not restrictive because the \( D \)-dimensional Euclidean space can be a subspace of the target space supporting the flux of the field \( H \); it is energetically profitable that the bubbles are produced in this subspace and in the quasiclassical consideration the rest of the target space is irrelevant.}
Thus, in view of Eqs. (2, 8), the brane bubble action Eq. (1) becomes a sum of surface and volume terms (the latter having negative coefficient for the relevant bubbles)

\[ S = T \int_V \sqrt{\text{det}(\hat{G}_{\mu\nu})} - Q\Phi \int_U \sqrt{\text{det}(\hat{G}_{\mu\nu})}, \]

where \( Q\Phi \) is positive for the relevant bubbles. Hence there is a competition of the two terms: the surface term suppresses small bubbles while the volume term blows up sufficiently large bubbles. The one which extremize the action Eq. (1) is the critical bubble. In the present case the critical bubble is a \( p + 1 \)-dimensional sphere of radius

\[ R = \frac{(p + 1)T}{Q\Phi}. \]

The critical bubble can be obtained in many ways. We shall now briefly describe two of them because we believe each of them is instructive. Probably the simplest one is to make the spherically symmetric anzatz and to extremize the action Eq. (9) with respect to the radius. This gives an algebraic equation giving the critical radius \( R \) from Eq. (10). The extremal value of the action Eq. (9)

\[ S_c = \frac{1}{p + 1} \Omega_{p+1} TR_c^{p+1} \]

defines the exponential of the production rate

\[ \Gamma \sim e^{-S_c} \]

where \( \Omega_p = \frac{2\pi^{\frac{p+1}{2}}}{\Gamma(\frac{p+1}{2})} \) is volume of unit \( p \)-sphere. This type of argument was given in [29].

Note that under variation of the form of the bubble the surface term in the action Eq. (1) gives the so-called “Laplace pressure” (or, in other words, trace of the external curvature) multiplied by tension \( T \), while the volume term gives \(-Q\Phi\), so in this case one gets the equation

\[ T(\Sigma_i \frac{1}{R_i}) = Q\Phi, \]

where \( \frac{1}{R_i}, i = 1, \ldots, p+1 \) are principal curvatures of the world-volume of the brane. In the case of spherical symmetry all \( R_i \)'s are equal and one comes back to Eq. (10). From this picture it is immediately seen that for \( p = 0 \) brane the bubble in any case can only be glued from arcs of the circle of radius \( R \) Eq. (10) [11].
The other way, followed in [28], consists in choosing among the Euclidean coordinates a "time" coordinate and assuming the spherical symmetry only for the remaining ones. Then Eq.(8) reduces to the effective action

\[ S_{\text{eff}} = T \Omega_p \int dt \rho^p \sqrt{1 + \dot{\rho}^2} - \frac{Q\Phi}{p+1} \Omega_p \int dt \rho^{p+1} \]  

where \( \rho \) is the radial coordinate in the spherical coordinate system:

\[ ds^2 = \Sigma_i (dx^i)^2 = dt^2 + dr^2 + r^2 (d\Omega_p)^2 \]  

and \((d\Omega_p)^2\) is the metric on the unit sphere. Extremization of \( S_{\text{eff}} \) from Eq.(14) gives an equations for \( \rho(t) \) which defines the critical bubble. One can use time translation symmetry of the effective Lagrangian Eq.(14) to reduce the order of the equation for \( \rho(t) \) from second one to the first one. Namely, the first integral reads

\[ E = - \frac{R \rho^p}{\sqrt{1 + \rho^2}} + \rho^{p+1}, \]  

where \( R \) is as in Eq.(10). Since the tunneling proceeds at \( E = 0 \) Eq.(16) becomes an equation of the sphere of the radius \( R \). The section \( t = 0 \) of the critical bubble is the configuration which is born in Minkowski space and then blown up by the external field.

### 3 One brane induced brane production

Essentially, the picture of the induced brane production in the external field looks as follows. There is an infinite brane which do not interact with the external field so asymptotically its worldvolume is flat. If the worldvolume of this brane (we shall call it external) can glue to (or, end on) worldvolume of branes which interact with the external field then the bubble made of the worldvolume(s) of the new brane(s) can arise somewhere in the middle of the world volume of the external brane. The brane action computed on such configuration of branes (minus action computed on the configuration which consists of the external brane alone) defines exponential factor in the brane production rate. A configuration which emerges (and then blows up) in Minkowski space consists of a spacelike slice of the external brane worldvolume with the equatorial slice of the bubble somewhere in the middle of it.
In string theory, the brane worldvolume can glue to worldvolume of a brane of the same
dimension or to a worldvolume of a brane of higher dimension. In the former case, gluing is
possible for the \((P,Q)\)-branes, in the latter case gluing is possible when the higher dimensional
brane bears an excitation of internal degrees of freedom, basically, an excitation of the gauge
field living on the brane. In the present paper we concentrate on the first case.

In IIB string there are \((P,Q)\) strings (1-branes) and \((P,Q)\) 5-branes \[3\]. The \((P,Q)\)-branes
are branes bearing two types of charges - RR and NS ones, correspondingly there are two types
of CS terms in \((P,Q)\)-brane action. As to the tension of the \((P,Q)\)-brane, it depends on the
charges in the following way:
\[
T_{P,Q} = T|P + Q\tau|,
\]
where the complex parameter \(\tau\) encodes axion and dilaton v.e.v. The week string coupling
limit corresponds to \(\tau \rightarrow i\infty\) case. Note that in what follows we shall assume that the RR
axion field is not excited.

From the analysis of the string junctions and \((P,Q)\) webs \[15, 13\], it is known that \((P,Q)\)
branes worldvolumes can glue under two conditions: 1) conservation of the charges; 2) mechanical
equilibrium of tension forces. Thus, e.g., \((1,0)\)-brane couples to \((1,1)\) and \((0,-1)\) branes, and at
\(\tau = i\) the angle between \((1,0)\) and \((1,1)\) ones will be equal to \(3\pi/4\), and the angle between the
\((1,0)\) and \((0,-1)\) branes will be equal to \(\pi/2\).

Obviously, the conditions of gluing are not locally affected by the external field. What is
affected by the external filed is the shape of the worldvolume (as seen from Eq.(13)). However,
for any brane of a given configuration of charges one can choose such a linear combination of
RR and NS external fields that the brane does not feel it in the leading approximation. Below
we assume that the external field is of NS type, while the external brane is a D-brane, that is
with charges of \((L,0)\) type, and hence it does not interact with the external field. \((L,0)\)-brane
can couple to \((P,Q)\) and \((L - P, -Q)\) branes and worldvolumes of the latter are curved by the
external NS field. Locally they are glued according to the same condition 1) and 2) above, that

\[3\] Actually there are also \((P,Q)\) 7 branes but we shall not discuss them later.
is the angle $\alpha_{(L,0)(P,Q)}$ between $(L,0)$ and $(P,Q)$ is defined according to

$$\cos\alpha_{(L,0)(P,Q)} = -\frac{P + Q \text{Re} \tau}{|P + Q \tau|}$$

and analogously the angle $\alpha_{(L,0)(R-P,-Q)}$ is given by

$$\cos\alpha_{(L,0)(R-P,-Q)} = -\frac{L - P - Q \text{Re} \tau}{|P - L + Q \tau|}.$$  

Note that there is an essential difference here from the $(P,Q)$-web case - the internal branes are not flat, their worldvolumes form sort of “caps” (see Fig.1) which are glued to the external brane worldvolume at the angles defined by Eqs.(18),(19), with one of the caps is glued at the angle defined by Eq.(18) from above and the other one - at the angle defined by Eq.(19) from below.

Let us turn to the discussion of the gauge field contribution. Thanks to a point junction Gauss law on the worldsheet D string theory has to be modified since the F string inserts the point source. Due to the discontinuity of the electric field $\Delta E = g$, where $g$ is the IIB string coupling, the $A_0$ component has to be piecewise linear and according to BPS condition has
to be correlated with one of the scalars representing the transverse fluctuation of the string [16, 17]. This condition is fulfilled at the junction and is equivalent to the total zero tension condition.

In the case considered instead of the point like junction point we have junction circle around the bubble. The Gauss law now reads as

\[
div E = g(\delta(x - x_0(t)) - \delta(x + x_0(t))
\]

where \( \rho \) denotes the radius of the circle in the \((x,t)\) plane (the plane of the external string), \( x_0^2 + t^2 = \rho^2 \). Solution to the Gauss law constraint provides the discontinuity of the electric field. Once again BPS condition amounts to the total zero tension along the circle which is consistent with the equations of motion to the gauge field and scalar. Let us emphasize that we use the BI action to describe the D string. Introducing the canonical momentum for the gauge field \( \Pi = \delta L_{BI}/\delta E \) and performing the Legendre transform to the Hamiltonian description one immediately recognizes the action of the string with tension \( T_{P,Q} \) since the canonical momentum along the ”caps” is constant due to the equations of motion.

To find the shape of the caps it is most convenient to turn the second of the two possibilities described at the end of section 2. A coordinate \( z \), orthogonal to the external brane worldvolume (see footnote in section 2), will be considered as “time” in the effective problem. \( ^4 \) With this choice, the effective action reads

\[
S = S_{\text{external}} + S_{(P,Q)} + S_{(L-P,-Q)}
\]

where \( S_{\text{external}} \) includes only tension term for the external brane and \( S_{(P,Q)} \) and \( S_{(L-P,-Q)} \) include both the tension and CS terms (cf. Eq.(14):

\[
S_{(P,Q)} = T_{(P,Q)} \Omega_p \int_0^{\tau_+} dt p^p \sqrt{1 + \rho^2} - \frac{Q \Phi}{p + 1} \Omega_p \int_0^{\tau_+} dt \rho^{p+1}
\]

Perhaps, it is worth to stress that this is unphysical time, which is convenient to describe the shape of the caps, because the caps are obviously spherically symmetric in the rest of coordinates. The physical time - the one in which the brane production should be interpreted - is, in fact, one of the coordinates along the external brane.

\( ^4 \)
and
\[ S_{(L-P, -Q)} = T_{(L-P, -Q)} \Omega_p \int_{t_-}^{0} dt \rho^p \sqrt{1 + \rho^2} - \frac{Q\Phi}{p+1} \Omega_p \int_{t_-}^{0} dt \rho^{p+1}, \tag{23} \]
where tensions \( T_{(P,Q)} \) and \( T_{(L-P, -Q)} \) are defined in Eq.(17), and \( t_+, \rho = 0 \) (\( t_-, \rho = 0 \)) is a top (bottom) point of the upper (lower) cap. Notice that the volume terms in Eq.(22) and Eq.(23) have the same coefficient which is traced back to the charge conservation.

Instead of Eq.(11) of section 2 we now have two first integrals - the one for the upper cap and the other for the lower cap. Moreover it is clear that the first integral constant \( E \) vanishes, so the caps are segments of spheres. The radii of the spheres are those of the spheres in the spontaneous brane production:
\[ R_{(P,Q)} = \frac{(p+1)T|P + Q\tau|}{Q\Phi} \tag{24} \]
for the \((P,Q)\)-cap and
\[ R_{(L-P, -Q)} = \frac{(p+1)T|P - L + Q\tau|}{Q\Phi} \tag{25} \]
for the \((L-P, -Q)\) cap. Given the radii Eqs.(24), (25), the segments are uniquely defined by the angles Eqs.(18), (19). To verify that the caps fit into a bubble one has to check that the \( p \)-spheres on the boundaries of the segments have the same radius which is indeed the case.

To find the exponential of the brane production rate one should take the value of the effective action Eq.(22) on the configuration described and subtract from it action computed on the configuration which consists of the external brane alone.

The contribution from the external string in Eq.(22) is, of course, infinite. However, the decay rate is defined by the difference between the critical value of the action from Eq.(22) and the value of the action when only flat external brane (without any bubble) is present. This difference is finite and defines the following rate of the induced brane production
\[ \Gamma = \exp\left\{ -\frac{1}{(p+1)(p+2)} \Omega_{p+1} \left( \frac{(p+1)|P + Q\tau|T|^{p+2}}{(Q\Phi)^{p+1}} \frac{1}{\pi \arcsin \left| \frac{QIm\tau}{P + Q\tau} \right|} - \frac{1}{(p+1)(p+2)} \Omega_{p+1} \left( \frac{(p+1)|P - L + Q\tau|T|^{p+2}}{(Q\Phi)^{p+1}} \frac{1}{\pi \arcsin \left| \frac{QIm\tau}{P - L + Q\tau} \right|} + \frac{1}{(p+1)(p+2)} \Omega_{p+1} \left( \frac{(p+1)|T|^{p+2}}{(\Phi)^{p+1}} |Im\tau|^{p+1} \right) \right) \right\} \tag{26} \]
\( \Gamma \) can equally be considered as an imaginary part of the \((L,0)\) tension in the external NS field.

The effective contribution from the external string comes from the difference between the action of the external brane without and with the bubble solution and in the string case reads

\[
S_{\text{ext}} = -T_{1,0} \pi \rho^2,
\]

where \( \rho \) is the intersection radius. Let us emphasize that the contribution \( S_{\text{ext}} \) can be treated differently as the contribution due to the Wilson loop along the matching circle. This interpretation is in perfect accord with the Gauss law if we attribute the Wilson loop to the effective "charges". Note once again that the effects due to the classical gauge fields on the caps are taken into account by tensions of the branes.

It is in order here to compare the rates of the spontaneous and induced brane nucleation. Of course, this comparison cannot be taken literally since the final states are different. However it is instructive to identify the enhancement factors for the induced processes in different regimes.

Let us recall the situation for the false vacuum decay induced by a particle of mass \( m \) in \((1+1)\) dimensions. The relevant parameter is the ratio \( \epsilon = m/2\mu \) where \( \mu \) is the kink mass (the tension of the bubble). At small \( m \) the external particle doesn’t disturb classical bubble solution significantly. Therefore the enhancement factor is just exponential of the particle action inside the bubble, \( \exp^{2mR_0} \), where \( R_0 = \mu/\epsilon \) is the radius of the unperturbed bubble. Opposite limit corresponds the so-called “sphaleron” region \( \epsilon \sim 1 \) where the exponential factor disappears since the initial particle can decay into the kink-antikink pair without external field. Note however that in the sphaleron region large quantum corrections make inapplicable the saddle point approximation.

Now turn to the brane induced processes. The relevant parameter analogous to the parameter \( \epsilon \) above is the parameter \( \tau \) defining the ratio of tensions of \( D \) and \( NS \) branes. The region of the slightly disturbed bubble corresponds to the limit \( \text{Im} \tau \to \infty \) and the enhancement factor in this limit is again given by exponential of the external brane action inside the bubble,

\[
\Gamma_{\text{enh}} = \exp\left\{ \frac{1}{(p+1)} \Omega_p L \frac{((p+1)T)^{p+2}}{(\Phi)^{p+1}} |\text{Im} \tau|^{p+1} \right\}
\]

The “sphaleron” region is where the initial brane can pass into a pair of branes without external field. The corresponding “line of marginal stability" (where the tension of the initial
brane equals sum of the tensions of pair of the produced branes) is reached at $\text{Im}\tau = 0$. Discussion of quantum corrections to the “line of marginal stability” is beyond the scope of the paper. Eq. (26) predicts that there is no suppression of the induced brane production at this line. However, in analogy with the particle induced vacuum decay, Eq. (26) cannot be trusted in the sphaleron region because of strong string corrections.

Let us now comment on the choice of the initial D-brane state. The subtlety concerns the possibility to have the initial brane at rest. If we consider the initial D-brane one can wonder about the dependence of the general BI action on the NS B field and therefore the corresponding interaction of D brane. To handle this issue for a string we can suppose that gauge fields are not excited in the initial state and there is no axion $\tilde{C}_0$ field at all. However in the generic case the noncommutative geometry induced by NS B field is involved hence the noncommutative BI Lagrangian actually governs the dynamics.

It is natural to discuss the more general case when two external branes are involved in the induced process. Let us consider two $(1, 0)$ branes as external branes (see fig.2) inducing the tunneling. In this case the bubble describing branes production consists of two caps, one above and one below, and a barrel in the middle. One of the caps has charges $(1, 1)$, the other one - $(1, -1)$, and the barrel - $(0, 1)$. The external branes (“legs”), the caps, and the barrels are glued at the angles defined by Eq. (18). Notice, that in vicinity of the bubble the external branes are not parallel and not flat. The effective action consists of various pieces - those for the legs, for the caps and for the barrel. One can see that the caps are again segments of spheres, while for the barrel the first integral - analogue of Eq. (16) is not zero. Its external curvature has two different eigenvalues. What concerns to the legs, there is no “volume term” in the effective action for them which allows them to spread to infinity, and the first integral is again nonzero which makes them curve. There is no principal difficulties in describing the bubble in this case, but the formulae are not as transparent as in the one brane induced case, and we will not bother the reader by them.

Now, after describing all these induced bubbles, we would like to come back to the spontaneous brane creation. It is now clear, that in addition to the basic case of round bubble made out of one brane, there are other bubbles of branes spontaneously arising in the external
field. For example, one can have a bubble glued from two curved branes bearing \((P,Q)\) and \((-P - L, -Q)\) charges ("caps") and one flat brane in the middle with \((L,0)\) charge. The production rate of such bubbles is described by Eq. (26) with the change of \(L\) to \(-L\).

4 Nucleation of brane bubbles in D3 background

So far we considered the one-brane induced process in the flat space. It is interesting to consider the string induced process in the background of two D3 branes far apart from each other. Namely consider the initial F or D string stretched between two D3 branes along some direction. In this case the ends of the string on D3 branes represent monopoles \([20]\) (D string) or charges \([21]\) (F string) in the 4d SYM theory on the D3 brane worldvolume. If distance between D3 branes corresponding to the vacuum expectation value of the scalar in 4d theory is large we can ignore the metric deformation due to D3 branes and consider the perturbative
Figure 3: Time slice of the Euclidean solution for the induced process in D3 background domain in 4d theory. The configuration under consideration is presented at Fig.3.

In the external B field the process looks as follows. The initial D string creates a bubble in the Euclidean space which then evolves in Minkowski space. Since the simplest decay mode for D string is to F string and (1,-1) string, from the point of view of the 4d observer on D3 brane the process looks like the exponentially suppressed decay of monopole to charge and dyon in the external field during the finite time determined by the bubble action.

The time-evolution for the process looks as follows. The critical (turning point) configuration is the one which arises in the Minkowski region. Then the external field accelerates the (1,1) and (-1,0) strings and at some time size of the bubble becomes comparable with the distance between D3 branes. At this moment in terms of the SU(2) theory on D3 branes (1,1) dyon and (-1,0) are created.

Note that we can believe such consideration if the vacuum expectation value of the scalar is much larger than the critical radius, otherwise the AdS$_5 \times S^5$ structure of the near horizon D3 brane metric

$$ds^2 = H^{-\frac{1}{2}} dx_\parallel^2 + H^\frac{1}{2} (dr^2 + r^2 d\Omega_5^2)$$

$$H = 1 + \frac{4\pi g_\alpha' r^2}{r^4},$$

where $X_\parallel$ are four coordinates along the D3 worldvolume and $d\Omega_5^2$ is the five-sphere metric has to be taken into account.

Let us note that the process looks somewhat unexpected from the point of view of 4d action on D3 brane where two-form NS field enter through the BI term while the RR one through CS one. Since the BI action supports the F and D string excitations we can claim that
these excitations are unstable in the RR or NS two form background respectively. Having this in mind we can interpret the process from the D3 brane observer viewpoint as the "instanton" like monopole decay in the NS B field (for D string) or charge decay in the RR B field (for F string). Both processes are exponentially suppressed due to the dyon production rate by the Euclidean tunneling exponent.

The decay rate can be presented in the form
\[ \Gamma = f_{\text{mink}} \exp(-S_{\text{Eucl}}), \] (31)
where \( S_{\text{Eucl}} \) has been found in the previous section while the \( f_{\text{Mink}} \) stands for the Minkowski part of evolution till the moment when dyonic strings touch D3 branes. Minkowski contribution to the action is purely imaginary
\[ S_{\text{Mink}} = i \int p_{\text{Mink}}(r)dr, \] (32)
where \( p_{\text{Mink}} \) is the canonical momentum conjugate to the radial variable. The subtle point is the integration region. In principle there are two different situations; in the first case the effective potential for the radius has no any additional extremum in the D3 brane background and the contribution of the Minkowski part of the evolution doesn’t play the important role. However one can’t exclude the possibility that there is such extremum and the Minkowski evolution allows the finite motion. In this case the amplitude develops the poles related to the energy levels for the radial variable in the Minkowski region. The possibility of such scenario requires the careful analysis of all gravitational effects and deserves further investigation.

Let us turn to the case of two D strings. Without external field we have two monopoles with the nontrivial moduli space. The brane configuration fits perfectly Nahm description of the moduli space [20]. Nahm equations themselves
\[ D_s T_i = -\epsilon_{ijk} [T_j, T_k], \] (33)
describe the condition of the BPS invariance of the configuration and \( s \) is identified with the direction along the strings. Matrixes \( T_i \) correspond to the positions of D string ends on the D3 branes. Solution to the Nahm equations with the proper boundary conditions amounts to the moduli space of the two-monopole configuration and provides the hyperkahler metric.
External field yields the deformation of the picture. Consider the SU(2) theory on the worldvolume of D strings. The external field enters the Lagrangian through the BI action and hence deforms Nahm equations. The new B field dependent metric can in principle be found in a standard way using the construction of the spectral curve from the solution of the Nahm equation or using the hyperkahler structure [24].

From the generic point of view the state of two monopoles can be represented by the point in the moduli space while their slow relative motion is governed by the geodesic motion in the hyperkahler metric. In the external B field the unstable submanifold in the moduli space is developed. Indeed assuming that the bubble size is smaller than the distance between D3 branes we have to conclude that any point at this submanifold is unstable due to the decay of two static monopoles to the dyons due to the bubble solution. The instability reflects the presence of the negative mode which results in the exponentially suppressed process.

One more interesting comment is in order here. The Nahm equations are the monopole generalization of the ADHM construction for instantons. It is also known that the monopole can be thought of as the chain of instantons. In the brane picture above this can be recognized viewing the D1 string as the bound state of the infinite number of D(-1) branes representing instantons. Since the distance from D3 brane corresponds to the instanton size [27] we can conjecture that the bubble creation can be considered as the nontrivial deformation of the instantons of the sizes related with the radius of the bubble.

The case of the nontrivial momenta of monopoles looks more complicated. Even in the absence of the external fields there are some nonperturbative phenomena in this case. We can mention the scattering at the right angle in the forward collision or the transition to the dyons at the generic kinematics [24]. In the external field the picture is even more rich. The inspection of the matching conditions for the bubble solution with the forward collision of two D string in the B field shows that there is a possibility to have different kinematics at the final state therefore besides the scattering at the right angle there is the exponentially suppressed processes with generic angle kinematics. Moreover there is a plenty of possibilities in the generic kinematics for the nonperturbative phenomena. For instance there is the process of transition of the pair of initial monopoles or charges into the generic state of several dyons through the
more complicated "bubble" type solution.

5 Discussion

In this paper we developed quasiclassical approach to a higher dimensional generalization of the induced tunneling processes. Utilizing the existence of the brane junction configuration the explicit quasiclassical tunneling exponent has been calculated. Our main example involves the stringy induced amplitudes but the 5 brane case is treated along the same routing. The solutions found above can be combined with the string-like solutions to the BI equations of motion to get the generic webs in the external fields. Similar arguments can be applied to more generic processes analogous to the charge and monopole decays mentioned above. Let us also remark that in the usual field theory framework there are slightly different processes similar to the induced false vacuum decay via bubble creation, for instance the processes yielding the baryon number non-conservation. The corresponding nonperturbative solution involves instanton-antiinstanton pair and the collective coordinate relevant for tunneling is the Chern number. It seems that the results of the paper might be useful for the quantitative description of their higher dimensional analogies.

Not much can be said about the quantum corrections to the induced processes. It is not clear if the quantum correction can be reduced to the geometric characteristics of the solution. The only remark available concerns the effect of the quantum fluctuations of the gauge fields to the stringy induced process since the relevant 2d YM action is almost topological and depends only on the area of the manifold which the YM theory is defined on. In our case the relevant manifold has the topology of the sphere therefore the expectation value of the Wilson loop in 2d YM theory is of interest. The Wilson loop expectation can be formulated in term of group characters \cite{30} and even calculated at the large N limit \cite{31}. It appears that the dependence on the area exhibits some phase transition behavior but for large areas it manifests the standard area law and therefore actually yields the tension renormalization.

Let us mention that the proper playground for the theory with the external NS B field is the noncommutative geometry. Having in mind that the rate of the process of the creation of
branes depends nonanalytically on the B field one can expect that the partition function of the theory can manifest some singular structure similar to the singularities at the complex coupling plane in the usual field theory due to instantons. In particular, since the ratio of the process can be considered as imaginary (nonanalytical in $B$) part of the brane tension, one can expect a change of the notion of BPS states in the noncommutative case, as compared to [32]. We shall be back to this subject elsewhere [33].

Let us turn now to the gauge theories on the worldvolume on the emerging branes. The situation is most transparent in the case of 5 brane webs. The particular configuration of 5 branes with some ”external 5 brane legs” amounts to SU(N) 5d gauge theory on the worldvolume of N 5 branes stretched between the external ones [15]. In the external field some brane production is possible which means the process of nonperturbative change of the rank of the gauge group whose rate depends nonanalytically on the external field. Moreover the brane diagram itself can be treated as a kind of Euclidean solution corresponding to the nonperturbative phenomena from the point of view of the gauge theories on the external 5 branes. Finally let us mention that the Minkowski evolution of the 5 brane bubbles acquires the meaning of some RG flow in the theory on their worldvolumes.

It would be interesting to discuss the case when temperature or density play the role of the inducing factor. In the temperature case it is necessary to find the Euclidean solution periodic in the Euclidean time. The temperature fixes the size of the matching p-sphere and therefore gives rise to the action on the solution. The formulae in this case are similar with the substitution of the temperature instead of the tension of the external brane. It is not clear how the density case has to be treated since the chemical potential for the state with branes has to be defined properly. Let us also note that the induced process can be considered in the AdS spaces generalizing the consideration in [3].

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