Predicting COVID-19 peaks around the world

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The official data for the time evolution of active cases of COVID-19 pandemics around the world is available online. For all countries, a peak has been either observed (China and South Korea) or is expected in near future. The approximate dates and heights of those peaks imply in important epidemiological issues. Inspired by similar complex behaviour of volumes of transactions of stocks at NYSE and NASDAQ, we propose a q-statistical functional form which appears to describe satisfactorily the available data of all countries. Consistently, predictions become possible of the dates and heights of those peaks in severely affected countries unless efficient treatments or vaccines, or sensible modifications of the adopted epidemiological strategies, emerge.

It is possible to predict the thermostatistical properties of uncountable physical systems at thermal equilibrium through the one-body distribution $p(\epsilon) \propto \omega(\epsilon)e^{-\beta \epsilon}$, where $\omega(\epsilon)$ is the density of states as a function of the energy $\epsilon$, multiplied by the celebrated Boltzmann factor $e^{-\beta \epsilon}$, $\beta$ being the inverse temperature. The function $\omega(\epsilon)$ comes from mechanical considerations (classical, quantum, relativistic) related with the number of degrees of freedom, and does not depend on the temperature; the exponential weight comes instead from standard statistical-mechanical considerations. In many cases it is, either exactly or approximatively, $u(\epsilon) \propto e^{\alpha}$ ($\alpha \in R$). For thermostatistical properties of the stationary- or quasi-stationary-state of wide classes of complex systems, the Boltzmann factor is to be generalised into the q-exponential factor $e_q^{-\beta \epsilon} = [1 - (1 - q) \beta \epsilon]^{1/(1-q)}$ ($q \in R; \epsilon_q^2 = e^\gamma$) [13]. This procedure yielded quite satisfactory results for the high-frequency stock-market in NYSE, NASDAQ and elsewhere [4-6].

Let us focus now on the data available for the COVID-19 pandemics. Briefly after the beginning of the pandemics, several studies analysing the available data, and employing different models and candidate functions, started to consider the q-statistical form (1) does yield satisfactory results for both China and South Korea, the $(\gamma, q)$

\[ N = C(t-t_0)^\alpha e^{-\beta(t-t_0)^\gamma} = \frac{C(t-t_0)^\alpha}{[1+(q-1)\beta(t-t_0)^\gamma]^{1/(q-1)}} \]  

with $C > 0; \alpha > 0; \beta > 0; \gamma > 0, q > 1$ and $t_0 \geq 0$. The constant $t_0$ indicates the first day of appearance of the epidemics in that particular region; it is conventionally chosen to be zero for China; for the other countries it is the number of days elapsed between the appearance of the first case in China and the first case in that country. The normalising constant $C$ reflects the total population of that particular country. For $\alpha = 0$, if $\gamma = 1$ we recover the standard q-exponential expression; if $\gamma = 2$, it is currently referred in the literature as q-Gaussian; for other values of $\gamma$, it is referred to as stretched q-exponential. Through the inspection of the roles played by the four nontrivial parameters, namely $(\alpha, \beta, \gamma, q)$, it became rather transparent that $(\alpha, \beta)$ depend strongly on the epidemiological strategy implemented in that region, in addition to the biological behaviour of the coronavirus in that geographical climate. In contrast, the parameters $(\gamma, q)$ appear to be sort of universal, mainly depending on the coronavirus. Therefore, we investigate several countries, which do not reach their peaks yet, with the basic assumption that these two parameter would not change much from one country to another and we fixed these values at the values that we determine for China since this country has already had nearly the full evolution of the pandemics. This assumption seems to be working. For other countries whose peaks are already reached, we use the same functional form (1) but adjusting all parameters for a better fitting. The results of China and South Korea are given in Fig. 1. It is evident that, although the functional form (1) does yield satisfactory results for both China and South Korea, the $(\gamma, q)$

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parameter values differ somewhat for each of these countries. On the other hand, as can be easily seen from Fig. 2, our assumption is corroborated by several countries that we have numerically analysed. In Table 1 we present the forecasted dates and heights of the peaks, as well as the values of the fitting parameters using the data accumulated until 28 of April, 2020.

![Graph showing the COVID-19 active cases evolution in China and South Korea](https://example.com/graph.png)

**FIG. 1**: Two possible fits for the COVID-19 active cases evolution in (a) China and (b) South Korea: the dots are the available data at 28 April 2020, and the lines are fittings using Eq. (1). For China a strange kink is present in the ascending part of the data curves, which makes that it is not possible to make a single fitting which would satisfactorily account for both increasing and decreasing parts. We present here two log-log representation of different curves which describe either the increasing part or the decreasing part but not both. Inset: linear-linear representation of the same data.

We also test our formula (1) and assumption for the evolution of deaths per day. Again we fixed the \((\gamma, q)\) parameter values at China’s results, which can be seen in Fig. 3, and tried to fit the data of the same 6 countries. It is quite surprising to see that the results given in Fig. 4 seem to suggest that the evolution of death per day is also possible to be fitted without changing these two parameters.

We may summarise as follows. The death curve of South Korea is atypical, in the sense that it sensibly differs from all those that we analysed in the present work. Because of that, we have not included it here. We remind that the values for \((\gamma, q)\) used for the South Korea evolution curve of active cases also differ from those used for all the other countries, which reinforces the fact that some sort of exceptionality exists there, for reasons that are unknown to us. For all countries that do not reach their peak values yet, we have adopted the values of \((\gamma, q)\) obtained from
FIG. 2: Fittings of the active cases data available in 28 April 2020 for various severely affected countries around the world with Eq. (1). The fitting parameters $\gamma$ and $q$ are fixed at the values of China for all countries which do not reach their peak values yet. Notice that, in the case of Brazil, a disruption occurred in the publicly available data in middle April. We do not know the cause of this. Coincidentally, however, precisely in those days the President of Brazil decided to change his Minister for Health.

The inspection of the entire curve of China.

We verify that the present work appears to belong to the realm of complex systems which includes not only, as mentioned above, high-frequency financial transactions (with $\alpha > 0$ [4, 5], but also anthropological issues such as
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| Country | $t_0$ | $C$ | $\alpha$ | $\beta$ | $\gamma$ | $q$ | Prediction interval of peak height | Prediction interval of peak day |
|---------|--------|-----|----------|--------|--------|----|----------------------------------|-----------------------------|
| Brazil  | $1.55 \times 10^{-5}$ | $5.65$ | $8.00 \times 10^{-8}$ | $3.037$ | $1.26$ | $[43400, 58965]$ | $[0.00021, 0.00028]$ | $[109, 114]$ |
| Turkey  | $8.0 \times 10^{-3}$ | $4.625$ | $1.52 \times 10^{-3}$ | $3.025$ | $1.02$ | achieved at $80808$ | achieved on day 93 |
| Italy   | $4.0 \times 10^{-3}$ | $4.68$ | $1.10 \times 10^{-5}$ | $3.083$ | $1.43$ | achieved at $106100$ | achieved on day 96 |
| USA     | $3.4 \times 10^{-9}$ | $8.60$ | $1.65 \times 10^{-5}$ | $3.037$ | $1.26$ | $[934680, 997830]$ | $[0.00028, 0.00030]$ | $[112, 117]$ |
| France  | $1.6 \times 10^{-6}$ | $6.56$ | $9.20 \times 10^{-6}$ | $3.037$ | $1.26$ | $[94551, 98726]$ | $[0.0014, 0.0015]$ | $[12, 17]$ |
| UK      | $1.2 \times 10^{-8}$ | $7.60$ | $9.60 \times 10^{-6}$ | $3.037$ | $1.26$ | $[158700, 186610]$ | $[0.0024, 0.0028]$ | $[16, 20]$ |

TABLE I: The values of the parameters and predictions for the maximum number of active cases and for the day at which the maximum will be achieved. The second line in the column of prediction interval of active cases peak height shows the same data divided by the total population of the country.
FIG. 3: Evolution of deaths per day due to COVID-19 in China. The data are those available at 28 April 2020. The fitting parameters $\gamma$ and $q$ are fixed at one of the two choices of the values for China, namely $(\gamma, q) = (3, 1.26)$. The other fitting parameters, namely $(C_{\text{death}}, \alpha_{\text{death}}, \beta_{\text{death}})$, have been chosen to better fit the available data for China.

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FIG. 4: Fittings of deaths per day data for various strongly affected countries around the world with Eq. (1). The data are those available at 28 April 2020. The fitting parameters $\gamma$ and $q$ are fixed at one of the two choices of the values for China, namely $(\gamma, q) = (3.126)$. The other fitting parameters, namely $(C_{\text{death}}, \alpha_{\text{death}}, \beta_{\text{death}})$, have been chosen to better fit the available data for each country.