Experimental Research on Size Distribution of Suspended Particles in water Based on Mie Scattering Theory

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Abstract. With the rapid development of social economy, it has brought serious environmental pollution problems. This thesis mainly summarizes the measurement methods of particle size and distribution of suspended particles. We have analyzed the relationship between scattering coefficient and concentration of different wavelengths at each scattering angle; the relationship between scattering coefficient and concentration of different scattering angles at each wavelength; and the relationship between scattering coefficient and wavelength of different concentrations at each scattering angle. The result shows that particle size and distribution of suspended particles can be accurately reversed when scattering angle is 15 degree, denoising factor is 200 and wavelength range is 450 nm-500 nm. The analysis can effectively filter out suspended particles in water, which is essential to resolve air pollution and purify water quality.

Keywords: Suspended Particles, Mie Scattering Theory, Experimental Research.

1. Introduction
Suspended particles refer to particles suspended in air or mixed in water with an aerodynamic equivalent diameter is less than or equal to 100 μm. With the rapid development of society and economy, deforestation and burning of forests will cause serious air or water pollution problems. Therefore, the content and distribution of suspended particles are very important indicators to measure air quality evaluation system [1].

The size and chemical composition of suspended particles determine its harm degree to human body. Studies have shown that the most harmful to human body is suspended particles with particle size less than 2.5 μm, called PM 2.5. Suspended particles are dispersed in gas or liquid in different distribution forms, if people are exposed to these polluted gas or water for a long time, the number of human respiratory diseases will increase day and day. So accurately measure the size and distribution of suspended particles and effectively filter them out become particularly important. Therefore, it has great significance for protecting human health and improving environmental issues which improving the measurement accuracy of particle size distribution of suspended particles and finding effective filtering methods [3].
2. Mie Scattering Theory

According to Mie scattering theory [1], Figure 1 shows the dimensionless particle size parameter \( \alpha = \frac{\pi d}{\lambda} \), where \( d \) is diameter of spherical scattering particles, \( m \) is relative refractive index, and \( I_0 \) is intensity of incident light.

\[
\alpha = \frac{\pi d}{\lambda},
\]

\[
is\ is\ diameter\ of\ spherical\ scattering\ particles,\ m\ is\ relative\ refractive\ index,\ I_0\ is\ intensity\ of\ incident\ light.
\]

![Figure 1. Schematic diagram of spherical particle scattering](image)

Considering absorption and scattering, when the light of wavelength \( \lambda \) passes through particle, the attenuation degree of incident light intensity can be expressed as:

\[
I = I_0 e^{-(q + \tau)l}
\]

(1)

Where \( q \) is absorption coefficient of the medium; \( l \) is distance that light passes through the absorbing medium; \( \tau \) is attenuation coefficient.

When a single particle is scattered under the irradiation of a plane wave, the intensity of scattered light generated at a certain point \( P \) is:

\[
I_p = \frac{S^2(\theta, \varphi)}{k^2 r^2} I_0
\]

(2)

Where \( S^2(\theta, \varphi) \) is amplitude function; \( \theta, \varphi \) is azimuth angle of scattered light; \( r \) is distance from particle to point \( P \); \( k \) is wave number.

So intensity of scattered light at a certain point \( P \) is:

\[
I(r, \theta, \varphi) = \frac{\lambda^2 I_0}{4\pi^2 r^2} \left[ i_1(\theta) \sin^2 \varphi + i_2(\theta) \cos^2 \varphi \right]
\]

(3)

With \( i_1(\theta) \) and \( i_2(\theta) \) are scattering intensity functions, and:

\[
i_i(\theta) = \left| S_i(\theta) \right|^2
\]

(4)
According to Mie scattering theory, the expression of the scattering amplitude function is:

$$ S_1(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left\{ a_n \pi_n(\cos \theta) + b_n \tau_n(\cos \theta) \right\} $$

(6)

$$ S_2(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left\{ a_n \pi_n(\cos \theta) + b_n \tau_n(\cos \theta) \right\} $$

(7)

Where $$ a_n, b_n, \pi_n, \tau_n $$ is Mie scattering coefficient, and $$ \pi_n, \tau_n $$ related to scattering angle $$ \theta $$, which is given in Eq. (8).

$$ a_n = \psi_n(\alpha)\psi'_n(m\alpha) - \psi_n(m\alpha)\psi'_n(\alpha) $$

$$ \xi_n(\alpha)\psi'_n(m\alpha) - m\xi_n(\alpha)\psi'_n(\alpha) $$

$$ b_n = m\psi_n(\alpha)\psi'_n(m\alpha) - \psi_n(m\alpha)\psi'_n(\alpha) $$

$$ m\xi_n(\alpha)\psi'_n(m\alpha) - \xi_n(\alpha)\psi'_n(\alpha) $$

$$ \pi_n = \frac{P^{(1)}_n(\cos \theta)}{\sin \theta} = \frac{dP^{(1)}_n(\cos \theta)}{d\cos \theta} $$

$$ \tau_n = \frac{dP^{(1)}_n(\cos \theta)}{d\theta} $$

With $$ \psi_n(z) $$ is semi-integer order Bessel function, $$ \xi_n(z) $$ is second class Hankel function.

$$ \psi_n(z) = \left(\frac{\pi z}{2}\right)^{\frac{1}{2}} J_{n+\frac{1}{2}}(z) $$

$$ \xi_n(z) = \left(\frac{\pi z}{2}\right)^{\frac{1}{2}} H^{(2)}_{n+\frac{1}{2}}(z) $$

(9)

With $$ P_n(\cos \theta) $$ is Legendre function about $$ \cos \theta $$; $$ P^{(1)}_n(\cos \theta) $$ is first-order associated Legendre function about $$ \cos \theta $$.

Therefore, the value of $$ a_n, b_n, \pi_n, \tau_n $$, amplitude function $$ S_1 $$ and $$ S_2 $$, intensity function $$ i_1 $$ and $$ i_2 $$ can be resolved by Mie term number $$ n $$.

In summary, no matter the incident light is natural light or polarized light, extinction coefficient $$ Q_e $$, scattering coefficient $$ Q_s $$ and absorption coefficient $$ Q_a $$ can be calculated.
Mie scattering theory is a classic algorithm for solving analytical solution of the interaction between a spherical scatter and an electromagnetic wave field. It is also the most widely used and most basic algorithm in the field of particle scattering [2].

3. Research Methods
Based on Mie scattering theory [1], Figure 2 shows the optical measurement and analysis process of particle size distribution of suspended particles.

We set up an experimental platform to collect data, measure the scattered light intensity of suspended particles, and use MATLAB tools for data preprocessing. Then we perform statistical analysis on the preprocessed data and use Origin drawing tools to analyze the relationship between scattering coefficient and concentration of different wavelengths at each scattering angle; the relationship between scattering coefficient and concentration of different angles at each scattering wavelength; and the relationship between scattering coefficient and wavelength of different concentrations at each scattering angle. The specific technical research line is shown in Figure 3.

4. Results & Discussion

4.1. Scattering angle and denoising factor
This thesis use a spectrometer to measure original signal plot of scattering coefficient and concentration at different scattering angles. Figure 4 shows scattered light signal graphs at different scattering angles when concentrations is 0.025 g/L or 0.1 g/L.

We can know that when scattering angle is 15°, high-density noise is smaller than low-density noise; when scattering angle is increased to 45°, high-density noise is greater than low-density noise.

The result shows the influence of noise is relatively large. Therefore, removing the influence of noise is very important for entire experiment and the accuracy of experimental results.
This paper uses wavelet transform method to denoise experimental data, set different denoising factors, and compare the denoising results. Figure 5 and 6 show that when scattering angle is 15° and denoising factor is 200, the denoising effect significantly.

**Figure 4.** Scattered light signal plots at different scattering angles with concentration is 0.025 g/L (a) or 0.1 g/L (b)

**Figure 5.** Denoising graph with denoising factor is Lev=1 (a) and Lev=200 (b) at low concentration
4.2. Scattering angle and wavelength

Figure 7 shows that when scattering angle is 15° and scattering wavelength is 450 nm, the relationship between scattering coefficient and wavelength of single particle size, single particle size directly added, and mixture of two suspensions in 1:1.

When concentration is 0.1 g/L, choosing Five-squares fit equation to get two fitting curves, Y1 fitting curve is single particle size directly added; and Y2 fitting curve is mixture of two suspensions in 1:1. By comparing coefficients of order terms in two fitting equations, the following relationship is:

\[
y_1 = -15.52895 + 0.13369x - 4.5681 \times 10^{-4}x^2 + 7.75502 \times 10^{-7}x^3 - 6.54938 \times 10^{-10}x^4 + 2.2045 \times 10^{-13}x^5
\]  
(11)

\[
y_2 = -4.75506 + 0.04186x - 1.46697 \times 10^{-4}x^2 + 2.55981 \times 10^{-7}x^3 - 2.2263 \times 10^{-10}x^4 + 7.7271 \times 10^{-14}x^5
\]  
(12)

Comparing the coefficients of the various sub-terms, the approximate Eq. (13) is obtained:

\[
y_i = 3.27y_2
\]  
(13)

In the same way, the coefficients of fitting equation at other concentrations are resolved, which are 3.11, 2.99, 3.01, 2.98, 2.94. The analysis shows that the fitted curve has a strong linear correlation, and the relationship deviation between coefficients of various sub-terms in fitting equation is small. Therefore, when the relationship between mixed solution scattering coefficient and wavelength or between the sum of particle solution scattering coefficient and wavelength is known, the particle size distribution of suspended particles can be inverted more accurately.
Figure 7. Relationship between scattering coefficient and wavelength of different concentrations at scattering angle is 15°

4.3. Inversion result of particle size ratio
Table 1 shows distribution of particle size ratio under various combinations of different scattering angles and different scattering wavelengths. By comparing with original particle size distribution 1:1, the most suitable scattering angle and the most suitable wavelength range for inversion of particle size of suspended particles is resolved.

Table 1. Particle size distribution ratio under different angles and wavelength combinations

| wavelength                              | angle 15° | 30°  | 45°  |
|-----------------------------------------|-----------|------|------|
| (450 nm, 475 nm, 500 nm)               | 0.92:0.98 | 0.79:0.81 | 0.76:0.71 |
| (450 nm, 550 nm, 650 nm)               | 0.60:0.65 | 0.36:0.32 | 0.31:0.27 |
| (475 nm, 575 nm, 625 nm)               | 1.23:3.46 | 1.02:2.80 | 0.95:2.59 |
| (525 nm, 550 nm, 575 nm)               | 1.36:0.58 | 1.05:0.57 | 1.05:0.56 |

We randomly select several groups of measured values of scattering spectra at different wavelengths in the experiment, such as: (450 nm, 475 nm, 500 nm), (450 nm, 550 nm, 650 nm), (475 nm, 575 nm, 625 nm), (525 nm, 550 nm, 575 nm), and then we compare and analysis the particle size distribution ratio and inverse performance deviation of each group, the following results can be obtained:

(1) when scattering angle is 15°, the inverse particle size distribution is 0.92:0.98, 0.60:0.65, 1.23:3.46, 1.36:0.58, normalization after is 1:1.07, 1:1.08, 1:2.81, 1:0.43;
(2) when scattering angle is 30°, the inverse particle size distribution is 0.79:0.81, 0.36:0.32, 1.02:2.80, 1.05:0.57, normalization after is 1:1.03, 1:0.89, 1:2.75, 1:0.54;
(3) when scattering angle is 45°, the inverse particle size distribution is 0.76:0.71, 0.31:0.27, 0.95:2.59, 1.05:0.56, normalization after is 1:0.93, 1:0.87, 1:0.03, 1:0.53.

Data above shows that when scattering angle is 15° and wavelength range is 450 nm-500 nm, the deviation between particle size distribution calculated by inversion and original particle size ratio is smallest.

5. Conclusions
We have analyzed in detail the relationship between scattering coefficient and concentration of different wavelengths at each scattering angle; the relationship between scattering coefficient and concentration of different scattering angles at each wavelength; and the relationship between scattering coefficient and wavelength of different concentrations at each scattering angle. The research shows that suspended
particulate matter is main component in atmosphere and water. As shown by our results, when scattering angle is 15°, denoising factor is 200, wavelength range is 450 nm-500 nm, the size and distribution of particle size can be reversed with high precision. Particle size analysis can effectively filter suspended particulates in water, which has important theoretical value and research significance for monitoring air pollution index and testing water purity.

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