The effect of tick size on trading volume share in three competing stock markets

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Abstract. The relationship between tick sizes and trading volume share in two and three competing markets is studied theoretically. By introducing a simple model which is equipped with multiple markets and non-strategic traders, we analytically calculate the share. It is shown that share is shifted from a market with a larger tick size to a market with a smaller tick size, and the size of share-shift is determined by difference between tick sizes not by ratio between tick sizes in both cases of two markets and three markets.

1. Introduction

Multiple stock markets coexist in one country and compete with each other in many cases. For example, 13 stock markets exist in the U.S.[1]. Also in Japan, other than Tokyo Stock Exchange, two Proprietary Trading Systems (PTS), which are opened by online stock trading companies exist. One property different among markets is “tick size”. Tick size is a unit of order price. It is expected that a smaller tick size allows the traders buy and sell at better price and hence leads to the increase in market share[2]. Actually in Japan, by the end of 2013, PTS got about 10% of share in total trading of Nikkei average adoption stocks. And then Tokyo Stock Exchange reduced tick sizes of 100 stock brands in the first section in January and July, 2014. After reducing tick sizes, Tokyo Stock Exchange recovered its market share[3].

Mizuta et al. conducted a simulation study on effect of tick size on trading share by using a market model which is equipped with two markets with different tick sizes and strategic traders. It was found that a market with a larger tick size is deprived of its initial major share by the competitive market, but a market with a larger tick size keeps its share at least to some extent even if a tick size of the competitive market is infinitely small[4]. Nagumo et al. explained the mathematical background of the above result by showing that the size of share-shift is determined by difference between tick sizes, not ratio between tick sizes, using a simple model with two markets and non-strategic traders[5].

In this paper, we will expand this simple model with two markets into a model with three markets. Using the model, we examine whether the mathematical background of share-shift in case of three markets is consistent with that of two markets.

2. Model

There are markets which have different tick sizes. We assume only selling orders. We also assume that these prices are fluctuated randomly in the markets from a “ideal potential price”. This ideal potential price is assumed to be always 0, and the fluctuation of the price is assumed to be determined by standard uniform distribution $U(0, 1)$. The fraction of the prices under tick sizes are rounded up.
Comparing the prices of markets, share of market X is defined according with the following rule. If a price of at least one market is cheaper than the price of market X, market X is not chosen. If the price of market X is the cheapest among the markets, and n other markets also have the same price \((n \geq 0)\), market X is chosen at a probability \(\frac{1}{n+1}\). The probability at which market X is chosen is defined as its share.

Tick sizes of the markets are based on 1 which is the standard deviation of standard uniform distribution. In case of two markets, we define tick sizes of market A and B as \(a\) and \(b\). We suppose that \(a, b\) and 1 satisfy \(0 < b \leq a \leq 1\) and have multiple relations with the others. In the same way, in case of three markets, we define tick sizes of market A, B and C as \(a, b\) and \(c\). We suppose that \(a, b, c\) and 1 satisfy \(0 < c \leq b \leq a \leq 1\) and have multiple relations with the others.

The schematic diagrams of the model are shown in Fig. 1. This pictures show probabilities at which market A is chosen for each case of (I), (II) and (III) in case of three markets.

3. Results
Firstly, we review the result of the case of two markets\([5]\), and next, we argue on the case of three markets.

3.1. Two markets
We define the prices in market A and B as \(p_A\) and \(p_B\). Those size relations are divided into

\[
(i) \ p_A < p_B, \quad (ii) \ p_B < p_A, \quad (iii) \ p_A = p_B.
\]

The probabilities of each case are defined as \(P'_1, P'_2\) and \(P'_3\). They are derived as follows. We define \(p_A\) and \(p_B\) as \(p_A = ma\) and \(p_B = nb\). \(m\) is varied for \(m = 1, 2, \cdots, 1/a\), and each probability is \(a\). \(n\) is varied for \(n = 1, 2, \cdots, 1/b\), and each probability is \(b\). By these definitions, \(P'_1\) and \(P'_3\) represent a probability that \(n > \frac{ma}{b}\) is satisfied and a probability that \(n = \frac{ma}{b}\) is satisfied, respectively. Therefore, \(P'_1, P'_3\) and \(P'_2\) are

\[
P'_1 = \sum_{m=1}^{1/a} a \sum_{n=1}^{1/b} b = \frac{1 - a}{2},
\]

(1)

\[
P'_3 = \sum_{m=1}^{1/a} ab = b,
\]

(2)

\[
P'_2 = 1 - P'_1 - P'_3 = \frac{1 + a - 2b}{2}.
\]

(3)
Market A is chosen at a probability 1 in case of (i) and 1/2 in case of (iii). Likewise, market B is chosen at a probability 1 in case of (ii) and 1/2 in case of (iii). Therefore, share of market A and B should be

\[ S_A^* = 1 \cdot P'_1 + \frac{1}{2} P'_3 = \frac{1}{2} - \frac{1}{2} (a - b), \]  
\[ S_B^* = 1 \cdot P'_2 + \frac{1}{2} P'_3 = \frac{1}{2} - \frac{1}{2} (b - a). \]  

Therefore, it is found that share is shifted from a market with a larger tick size to a market with a smaller tick size. Moreover, the size of share-shift is determined by difference between tick sizes, not ratio between tick sizes.

### 3.2. Three markets
We define the prices in market A, B and C as \( p_A, p_B \) and \( p_C \). Those size relations are divided into

- (i) \( p_A < p_B, p_C \),
- (ii) \( p_B < p_A, p_C \),
- (iii) \( p_C < p_A, p_B \),
- (iv) \( p_B = p_C < p_A \),
- (v)\( p_C = p_A < p_B \),
- (vi) \( p_A = p_B < p_C \),
- (vii) \( p_A = p_B = p_C \).

The probabilities of each case are defined as \( P_1, P_2, \cdots, P_7 \), respectively. They are derived as follows.

We define \( p_A, p_B \) and \( p_C \) as \( p_A = la, p_B = mb \) and \( p_C = nc \). \( l \) is varied for \( l = 1, 2, \cdots, 1/a \), and each probability is \( a \). Likewise, \( m \) and \( n \) are varied for \( m = 1, 2, \cdots, 1/b \) and \( n = 1, 2, \cdots, 1/c \), and each probability is \( b \) and \( c \). By these definitions, for example, \( P_1 \) represents a probability that \( m > la, n > la/c \) is satisfied. Therefore, \( P_1 \) is

\[ P_1 = \sum_{l=1}^{1/a} a \sum_{m=1+la/b}^{1/b} b \sum_{n=1+la/c}^{1/c} c = \frac{1}{6} (1-a)(2-a). \]  

In the same way, \( P_2, \cdots, P_7 \) are derived as

\[ P_2 = \frac{1}{12} (4 + 3a - 9b - a^2 + 3ab), \]  
\[ P_3 = \frac{1}{12} (4 + 3a + 3b - 12c - a^2 + 3ab), \]  
\[ P_4 = \frac{1}{2} (1 + a - 2b), \]  
\[ P_5 = \frac{1}{2} (1 - a), \]  
\[ P_6 = \frac{b}{2} (1 - a), \]  
\[ P_7 = bc. \]  

Market A is chosen at a probability 1 in case of (i), 1/2 in case of (v),(vi), and 1/3 in case of (vii). Likewise, market B is chosen at a probability 1 in case of (ii), 1/2 in case of (iv),(vi), and 1/3 in case of (vii), and market C is chosen at a probability 1 in case of (iii), 1/2 in case of (iv),(v), and 1/3 in case of (vii).
Therefore, share of market A, B and C should be:

\[ S_A^* = P_1 + \frac{1}{2}(P_5 + P_6) + \frac{1}{3}P_7 \]
\[ = \frac{1}{3} - \frac{1}{2} \left( \frac{a - b + c}{2} \right) + \frac{1}{12}(2a^2 - 3ab - 3ac + 4bc), \] (13)

\[ S_B^* = P_2 + \frac{1}{2}(P_4 + P_6) + \frac{1}{3}P_7 \]
\[ = \frac{1}{3} - \frac{1}{2} \left( \frac{b - c + a}{2} \right) + \frac{1}{12}(-a^2 + 3ac - 2bc), \] (14)

\[ S_C^* = P_3 + \frac{1}{2}(P_4 + P_5) + \frac{1}{3}P_7 \]
\[ = \frac{1}{3} - \frac{1}{2} \left( \frac{c - a + b}{2} \right) + \frac{1}{12}(-a^2 + 3ab - 2bc). \] (15)

Using an approximation that we ignore second-order terms of \( a, b \) and \( c \) by the assumption of \( a, b, c \ll 1 \), \( S_A^* \), \( S_B^* \) and \( S_C^* \) should be:

\[ S_A^* = \frac{1}{3} - \frac{1}{2} \left( \frac{a - b + c}{2} \right), \] (16)

\[ S_B^* = \frac{1}{3} - \frac{1}{2} \left( \frac{b - c + a}{2} \right), \] (17)

\[ S_C^* = \frac{1}{3} - \frac{1}{2} \left( \frac{c - a + b}{2} \right). \] (18)

Therefore, it is found that share is shifted from a market with a larger tick size to a market with a smaller tick size. Moreover, the size of share-shift is determined by difference between tick sizes, not ratio between tick sizes. This feature is the same as the case of two markets.

4. Summary and Conclusion
We found that the size of share-shift is determined by difference between tick sizes in both cases of two markets and three markets. Therefore, in both cases of two markets and three markets, markets with any tick sizes keep at least a few amount of share even though tick sizes of the competitive markets is infinitely small.

It is known in case of two markets that even a market with a larger tick size can keep its major initial share against the competitive market with a smaller tick size if traders strongly prefer a market with large share when they choose a market[5]. It is a future task to examine whether this feature is consistent with the case of three markets if such “share-preference” introduced into the model.

References
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