Inelastic Neutrino Reactions with Light Nuclei and Standing Accretion Shock Instability in Core-Collapse Supernovae

S Furusawa$^{1,2}$, H Nagakura$^3$, K Sumiyoshi$^1$, S Yamada$^1$

$^1$ Waseda University, 3-4-1, Okubo, Shinjuku, Tokyo 169-8555, Japan
$^2$ Center for Computational Astrophysics, National Astronomical Observatory of Japan, Mitaka, Tokyo, 181-8588, Japan
$^3$ Yukawa Institute for Theoretical Physics, Kyoto University, Oiwake-cho, Kitashirakawa, Sakyo-ku, Kyoto, 606-8502, Japan
$^4$ Numazu College of Technology, Ooka 3600, Numazu, Shizuoka 410-8501, Japan

E-mail: furusawa@heap.phys.waseda.ac.jp

Abstract. We perform numerical experiments to investigate the influence of inelastic neutrino reactions with light nuclei on the standing accretion shock instability. The time evolutions of shock waves are calculated with a simple light-bulb approximation for the neutrino transport and a multi-nuclei equation of state. The neutrino absorptions and inelastic interactions with deuterons, tritons, helions and alpha particles are taken into account in the hydrodynamical simulations in addition to the ordinary charged-current interactions with nucleons. Axial symmetry is assumed but no equatorial symmetry is imposed. We show that the heating rates of deuterons reach as high as $\sim 10\%$ of those of nucleons around the bottom of the gain region. On the other hand, alpha particles heat the matter near the shock wave, which is important when the shock wave expands and density and temperature of matter become low. It is also found that the models with heating by light nuclei have different evolutions from those without it in non-linear evolution phase. The matter in the gain region has various densities and temperatures and there appear regions that are locally rich in deuterons and alpha particles. These results indicate that the inelastic reactions of light nuclei, especially deuterons, should be incorporated in the simulations of core-collapse supernovae.

1. Introduction

The mechanism of core-collapse supernovae is not clearly understood at present because of its intricacy. The neutrino driven mechanism is considered to be the most promising scenario for the core-collapse supernova and it is considered the so-called standing accretion shock instability (SASI) and the convection in multi-dimension are helpful to increase the efficiency of neutrino heating. In addition to these hydrodynamical effects, there are some nuclear physical ingredients that are also supposed to be important for the core-collapse supernovae. The inelastic neutrino interactions and equation of states (EOS) of nuclear matter are also important.

The inelastic interactions between neutrinos and nuclei have been neglected in most hydrodynamical simulations of the neutrino heating phase after the bounce and shock stall. Ohnishi et al. showed that the inelastic neutrino interactions with alpha particles are helpful to revive the shock in 2D simulations if the neutrino luminosity is close to the critical value, which
is the threshold for a shock revival [1]. However, the shocked matter is certainly composed not only of nucleons and alpha particles but also of deuterons, tritons and helions [2]. The energy transfer cross sections of deuterons are comparable to those of nucleons and ten times greater than those alpha particles [3].

EOS is another important input physics and its influences on the dynamics of core-collapse supernovae were investigated. There are currently the two EOS’s widely used for the simulations of core-collapse supernovae, in which only a single representative nucleus and alpha particles are incorporated to approximate the ensemble of heavy and light nuclei, respectively. We have developed an EOS including a large number of nuclei based on the liquid drop model for heavy nuclei with shell effects and nuclear pasta phases the quantum approach for light nuclei and relativistic mean field theory for free nucleons [4, 5]. This EOS can provide the mass fractions of each light nuclei more reliably.

The aim of this article is to investigate effects of these light nuclei in the new EOS on an aspect of supernova dynamics. We explore the impacts of the inelastic neutrino reactions with light nuclei on the SASI. We perform several hydrodynamical simulations of the post-bounce phase in 1D and 2D. In addition to cooling and heating by nucleons, we incorporate the neutrino heating reactions with light nuclei. This article is organized as follows. In section 2, we describe the basic set-up of dynamical simulations and the importance of light nuclei in the shock heating. The paper is wrapped up with a summary and in section 4.

2. Models

The basic set-up of dynamical simulations is the same as that of Ohnishi et al. and Nagakura et al. except for the heating reactions of light nuclei [1, 6, 7]. We perform 2D simulations assuming axial symmetry. Spherical coordinates are used and no equatorial symmetry is assumed. We utilize 300 radial mesh points to cover the whole meridian section. The mass of a central object is assumed to have the same value: \( e = 500 \text{ km} \), where \( r_{in} \) is the inner boundary and is set to be the radius of the neutrino sphere of \( \nu_e \) in the initial state. The 60 angular mesh points are adopted to cover the whole meridian section. The mass of a central object is assumed to be constant and set to be \( 1.4 M_\odot \). Interactions between neutrinos and nucleons are taken into account in the energy equation and the evolution of electron fraction. The calculations of the contributions of nucleons to the energy and electron fraction are done just in the same way as in Ohnishi et al. [6]. The heating rates of deuterons, tritons, helions and alpha particles are also taken into account in the energy equation and they are the new elements in this work.

The heating rates of light nuclei are calculated from the analytic formula [8]. The energy transfer cross sections of deuterons, tritons, tritons alpha particles are available from some references [3, 8, 9, 10, 11]. The cooling reactions involving light nuclei are ignored since the reaction rates are not available at the moment and we focus only the influences of the heating reactions of light nuclei in this paper. The neutrino transport is solved by the simple light bulb approximation. We assume that the temperatures of \( \nu_e, \bar{\nu}_e \) and \( \nu_\mu \) emitted from the neutrino spheres are constant and set to be \((T_{\nu_e}, T_{\bar{\nu}_e}, T_{\nu_\mu}) = (4, 5, 10) \text{ MeV} \). The luminosities of \( \nu_e \) and \( \bar{\nu}_e \) are assumed to have the same value: \( L_{\nu_e} = L_{\bar{\nu}_e} = L \). The luminosity of \( \nu_\mu \) is set to be \( L_{\nu_\mu} = 0.5 \times L \). We employ the multi-nuclei EOS, which gives not only thermodynamical quantities but also the abundance of all nuclei [4, 5].

For the first step of the calculations, we prepare the initial conditions of the stars, which are spherical symmetric steady accretion flows [6, 7]. The neutrino inelastic interactions with light nuclei are included in these calculations for the steady states. We start the dynamical simulations, adding the perturbations of 1 %, which are proportional to \( \cos \theta \), to the initial radial velocities. We investigate the influences of light nuclei on the dynamics under different circumstances, varying the luminosity \( L \) and mass accretions rate \( M \). We define the normalized...
neutrino luminosity $L_{52} \equiv L/(10^{52}\text{erg/s}^{-1})$ and mass accretion rate $\dot{M}_{\text{sun}} \equiv -\dot{M}/(M_{\odot}\text{s}^{-1})$. The detail of the model is described in Furusawa et al. [12].

3. Results
3.1. 1D simulation
To obtain the basic features of the heating by light nuclei in the dynamical settings, we perform a spherically symmetric 1D simulation. Although we do not add any perturbation initially, numerical noises induce small radial oscillations that grow gradually. In Fig. 1, we compare the time evolution of the integrated heating rate of each nuclear species along with the shock and gain radii for the $L_{52} = 5.4$ and $\dot{M}_{\text{sun}} = 1.0$. The shock radius is defined as the iso-entropic surface of $s = 5.0$ $k_B$ where $k_B$ is the Boltzmann constant. The heating rates are integrated over the gain regions as $\int_{\text{gain}} Q_i \text{d}r$ and given in normalized unit of $10^{52}\text{erg s}^{-1}$. We can see that the heating rate of alpha particles changes roughly in step with shock radius, since the larger shock has more alpha particles. It is also found that the heating by alpha particles is more important than that of deuterons after the shock wave revives and goes outward ($t > 400\text{ms}$). On the other hand, the heating rates of deuterons reach a local maximum when the gain and shock radii are small since the matter have high densities and favor deuterons. Furthermore since the deuterons are located closer to the neutrino sphere, the deuterons gain the heating rates as high as which reach $1-10\%$ of those of nucleons. These results indicate that the alpha particles and deuterons heat the matter in different phases in the oscillation of the shock wave. Although the tritons and helions are similar to deuterons, they are quite minor.

![Figure 1. The time evolutions of the shock and gain radii and integrated heating rates of nuclear species. Black dashed and dotted lines denote the shock and gain radii, respectively. Magenta, red, green and blue lines represent the heating rates of $A_i=1$ (nucleons), $A_i=2$ (deuterons), $A_i=3$ (tritons and helions) and $A_i=4$ (alpha particles).](image)

3.2. 2D simulation
Figure 2 displays the time evolutions of average shock radii for four modes, in which all light nuclei, only deuterons, only alpha particles and no light nuclei are taken into account in the heating sources, respectively. The models without deuteron heating do not succeed in the shock revival for $L_{52} = 5.1$, whereas the other two models do though it takes long time. For $L_{52} = 5.2$, on the other hand, all modes produce shock revival. We can see that the heating by deuterons and alpha particles both reduce the time to shock revival. Note that this may be too naive, since the time to shock revival is known to be sensitive to various ingredients such as the initial perturbations when the neutrino luminosity is set to be close to the critical luminosity. We,
however, stress that the heating by light nuclei, especially deuterons, brings the clear changes to the evolutions of shock waves in most cases.

Figure 2. The time evolutions of shock radius of the models with heating by all light nuclei (cyan solid lines), only deuterons (red dashed lines), only alpha particles (blue dashed-dotted lines) and no light nuclei (black dotted lines) for $L_{52} = 5.1$ and 5.2 with $M_{\odot \text{sun}}=1.0$.

We now focus on the model including the heating by all light nuclei with $L_{52} = 5.2$ and $M_{\odot \text{sun}}=1.0$ to explore the role of light nuclei in the evolution of the shock wave. The shock oscillation grows linearly by $t \sim 150 \text{ ms}$ as shown in Fig. 2. The distributions of nucleons and light nuclei are almost spherically symmetric at $t=100 \text{ ms}$. The heating rates of light nuclei are large in the narrow region near the quasi-steady shock wave until $t=100 \text{ ms}$. At $t=200$ and 300 ms, however, the shock waves are deformed and have reached the non-linear regime of SASI. In some regions, the light nuclei are abundant indeed and their heating is efficient accordingly. Figure 3 plots the pairs of $(\rho, T)$ obtained along 5 different radial rays. Although the initial distributions in $(\rho, T)$ plane (the black symbols) are not located in the regions that are rich in light nuclei, the turbulence in the non-linear SASI broadens the distributions. Figure 4 shows the mass fractions and the heating rates of nuclear species along the radial ray with $\theta = 180^\circ$ at $t = 200 \text{ ms}$ and the one with $\theta = 0^\circ$ at $t = 300 \text{ ms}$. The heating rate of deuterons becomes as high as $\sim 10\%$ of that of nucleons at $t=200 \text{ ms}$ around the bottom of the gain region. The shock wave at $t = 200 \text{ ms}$ moves northwards ($\theta = 0^\circ$) and the matter in the southern ($\theta = 180^\circ$) with low entropies goes down deep into the central regions. The orange in the top panel of Fig. 3 also indicate that the matter in the southern side ($\theta = 180^\circ$) has low entropies at the inner part of the gain regions, resulting in more deuterons than the matter in other parts. At $t=300 \text{ ms}$, the shock wave reach at $\sim 400 \text{ km}$ and the heating by alpha particle is dominant. We can see in the bottom panel of Fig.8 that the matter along the radial ray with $\theta = 0^\circ$ has also lower entropies and as a consequence deuterons and alpha particles are abundant at the regions of high and low densities, respectively. Both at $t = 200$ and 300 ms, the deuterons have the heating rates comparable to those of nucleons near the bottom of the gain regions. The heating rates of alpha particles are $\sim 10 \%$ of those of nucleons around the shock wave.

4. Discussion
We have investigated the influences of the inelastic interactions of neutrinos with light nuclei on the dynamics in the post-bounce phase of core-collapse supernovae. We have done numerical simulations of SASI with the assumption of axial symmetry for some representative combinations of the luminosity and mass accretion rate. We have not solved the dynamics of the central part of
the core and replaced it with the suitable boundary conditions and have started the simulations from spherically symmetric steady state, adding some perturbations to the radial velocity. The neutrino transport has been handled by the simple light-bulb approximation. In addition to the ordinary heating and cooling reactions with nucleons, we have taken into account the heating reactions with four light nuclei for the first time. The abundance of light nuclei is provided by the multi-nuclei EOS together with other thermodynamical quantities.

We have found that the evolutions of shock waves in 2D are influenced by the light nuclei heating and the deuterons and alpha particles have different roles in dynamical simulations in 1D and 2D. From the results of 1D dynamical simulations, we have found that the integrated heating rates of deuterons and alpha particles become high at different phases in the oscillations of shock radius: the heating rates of deuterons are the highest when the shock radius hits the minimum and the matter compression is the greatest. Whereas the heating by deuterons is constantly helpful for matter to gain the internal energies, the heating by alpha particles has an impact on the shock revival particularly when the shock wave has large radii and the matter becomes low entropies. The dynamics in 2D are more sensitive to the inclusion of the light nuclei heating because SASI in the non-linear regime make the gain regions more inhomogeneous and there appear the regions that have densities and temperatures favorable for the existence of light nuclei. The heating rates of light nuclei reaches about 10% of that of nucleons locally. As a consequence, the dynamics of the shock revival is influenced by the heating via light nuclei, especially deuterons. Although they are never dominant heating sources, these heating reactions of light nuclei should be included if one were to estimate the critical luminosity and/or explosion energy quantitatively accurately.

The numerical simulations in this paper are admittedly of experimental nature and the numbers we have obtained may be subject to change in more realistic simulations. There is also a room for improvement in our experimental computations. We need more systematic investigations, varying not only the neutrino luminosity and mass accretion rate but also the neutrino temperature, mass of a central object and initial perturbation. The cooling reactions of light nuclei should be incorporated in the calculations. These issues are currently undertaken and will be reported elsewhere.
Figure 4. Mass fractions (left panels) and heating rates per baryon (right panels) for protons and neutrons (black), deuterons (red), tritons and helions (green), and alpha particles (blue) for $t = 200$ ms (top panels) and 300 ms (bottom panels). Dashed lines indicate the cooling regions where the cooling reaction of nucleons is dominant.

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