Vacuum force on an atom in a magnetodielectric cavity

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(Dated: September 30, 2018)

We demonstrate that, according to a recently suggested Lorentz-force approach to the Casimir effect, the vacuum force on an atom embedded in a material cavity differs substantially from the force on an atom of the cavity medium. The force on an embedded atom is of the familiar (van der Waals and Casimir-Polder) type, however, more strongly modified by the cavity medium than usually considered. The force on an atom of the cavity medium is of the medium-assisted force type with rather unusual properties, as demonstrated very recently [M. S. Tomaš, Phys. Rev. A 71, 060101(R) (2005)]. This implies similar properties of the vacuum force between two atoms in a medium.

PACS numbers: 12.20.Ds, 42.50.Nv, 42.60.Da

It is well known that a neutral atom in the vicinity of a body (mirror) experiences the van der Waals force and, at larger distances, its retarded counterpart, the Casimir-Polder force. This, commonly called the van der Waals force, was considered theoretically numerous times using various methods and for increasingly more complex systems. Apparently, the van der Waals and Casimir-Polder type with rather unusual properties, as demonstrated very recently [M. S. Tomaš, Phys. Rev. A 71, 060101(R) (2005)]. This implies similar properties of the vacuum force between two atoms in a medium.

To account for the force on the cavity medium, which is absent in the traditional approaches to the Casimir effect in material cavities, Raabe and Welsch recently suggested a Lorentz-force approach to the Casimir effect (see also Ref. [22]). As an application of this approach, Raabe and Welsch derived a formula for the force on a magnetodielectric slab in a magnetodielectric planar cavity, as depicted in Fig. 1. Applying their formula to a thin slab, in this work we derive a general expression for the force on an (electrically and magnetically polarizable) atom in a magnetodielectric planar cavity. We demonstrate that, according to this result, the force on an atom is substantially different, depending on whether the atom is embedded in the cavity medium or whether it is a constituent of the cavity medium. The force on the embedded atom behaves in the familiar, although more strongly modified by the cavity medium than found previously, way with the atom-mirror distance and the electric/magnetic properties of the atom and the mirror. Contrary to this, the force on an atom of the cavity medium is a recently introduced medium-assisted force, with very unusual properties. We derive and discuss a number of basic formulas concerning the atom-mirror force in these two cases and establish a connection of these results with their counterparts obtained through a traditional approach. We also address the implications of the obtained results on the properties of the atom-atom force in a medium.

The Raabe and Welsch formula [21] for the force on the slab in the configuration of Fig. 1 can be written as

\[ f(d_1, d_2) = f^{(1)}(d_1, d_2) + f^{(2)}(d_1, d_2), \]  

\[ f^{(1)}(d_1, d_2) = \frac{\hbar}{2\pi^2} \int_0^\infty d\xi \int_0^\infty dk k \times \] 

\[ \sum_{q=p,s} \left( \frac{1}{\varepsilon} \delta_{qp} + \mu \delta_{qs} \right) r_q e^{-2\kappa d_2} - r_1^2 e^{-2\kappa d_1} \] 

\[ \frac{N_q}{N_0}, \]  

\[ f^{(2)}(d_1, d_2) = \frac{\hbar}{8\pi^2 c^2} \int_0^\infty d\xi \xi^2 \mu(n^2 - 1) \int_0^\infty \frac{dk}{k} \times \]
\[
\sum_{q=p,s} [(1 + r^q)^2 - t^q]^2 \Delta_q \frac{r^q e^{-2\kappa d_s} - r^q e^{-2\kappa d_s}}{N^q},
\]
where
\[
N^q = 1 - r^q (r^q e^{-2\kappa d_s} + r^q e^{-2\kappa d_s}) + (r^q - t^q)^2 \frac{r^q e^{-2\kappa (d_s + d_d)}}{N^q}
\]
and \(\Delta_q = \delta_{qp} - \delta_{qs}\). Here
\[
\kappa(\xi, k) = \sqrt{n^2(\frac{\xi^2}{c^2} + k^2)}
\]
is the perpendicular wave vector in the cavity at the imaginary frequency, \(r^q\) and \(t^q\) are Fresnel coefficients for the (whole) slab given by
\[
r^q(\xi, k) = \frac{\mu^q - \gamma^q \kappa_s}{\kappa + \gamma^q \kappa_s}, \quad \gamma^p = \frac{\varepsilon}{\varepsilon_s}, \quad \gamma^s = \frac{\mu}{\mu_s},
\]
are the single-interface medium-slab Fresnel reflection coefficients.

Equation (12) differs from the formula for the Casimir force obtained through the Minkowski tensor calculation \[20\] in the presence of the factors \(1/\varepsilon\) and \(\mu\) which multiply the contributions coming from TM- and TE-polarized waves, respectively. Effectively, these factors diminish the force, so that \(f_{1}^{(1)}\) may be regarded as a medium-screened force. The force \(f_{1}^{(2)}\) owes its appearance to the cavity medium, note that it vanishes when \(n = 1\), and can therefore be regarded as a medium-assisted force. Note that, since the factor \((1 + r^q)^2 - t^q\) is always positive, the sign of each term in Eq. (3) depends only on whether the corresponding mirror \(i\) is dominantly conducting (\(\Delta_q r^q > 0\)) or whether it is dominantly permeable (\(\Delta_q r^q < 0\)).

The force on an atom in a cavity medium can be obtained from the above equations by assuming that the slab consists of a thin, \(d_s \Omega / c \ll 1\), layer \[20\] of the cavity medium with a small number of foreign atoms embedded in it. Then, from Eqs. (6) and (4) we find that to the first order in \(\kappa_s d_s / \Omega d_s / c\)
\[
r^q(\xi, k) \simeq 2\rho^q \kappa_s d_s, \quad [(1 + r^q)^2 - t^q]^2(\xi, k) \simeq 2\kappa d_s / \gamma^q \mu^q / \gamma^q \mu_s.
\]
Also, we have
\[
\varepsilon_s(\xi) = \varepsilon(\xi) + 4\pi N \alpha_e(\xi), \quad \mu_s(\xi) = \mu(\xi) + 4\pi N \alpha_m(\xi),
\]
where \(N\) is the atomic number density and \(\alpha_e(\xi)\) the electric (magnetic) polarizability of an atom. Accordingly, for small \(N\alpha_e(\xi)\)
\[
\kappa_s \simeq \kappa \left[ 1 + 2\pi N (\alpha_e \mu + \alpha_m \varepsilon) \frac{\xi^2}{\kappa^2 c^2} \right]
\]
so that [Eq. (4)]
\[
\rho^q \simeq \pi c^2 \left[ \alpha_e - (\alpha_e \mu + \alpha_m \varepsilon) \frac{\xi^2}{2\kappa^2 c^2} \right]
\]
and \(\rho^s = \rho^p[\varepsilon \leftrightarrow \mu, \alpha_e \leftrightarrow \alpha_m]\).

With the above approximations inserted into Eqs. (6)-(10), the force on the layer can be, to the first order in \(d_s\), written as
\[
f(d_1, d_2) = f_M(d_1, d_2) + N d_s f_a(d_1, d_2),
\]
where
\[
f_M(d_1, d_2) = \frac{\hbar d_s}{4\pi^2 c^2} \int_0^\infty d\xi \xi^2 \mu(n^2 - 1) \int_0^\infty dkk \times
\]
\[
\left\{ \frac{1}{\varepsilon} \left[ \alpha_e - 2\alpha_m \varepsilon \xi^2 \right] - \alpha_m \varepsilon \right\} \mathcal{R}^p(i \xi, k)
\]
\[
+ \mu \left[ \alpha_m - 2\alpha_e \varepsilon \xi^2 \right] - \alpha_e \varepsilon \right\} \mathcal{R}^s(i \xi, k)
\]
\[
+ \mu(n^2 - 1) \left[ \alpha_e \mathcal{R}^p(i \xi, k) - \alpha_m \mathcal{R}^s(i \xi, k) \right]
\]
is the force on the medium (M) layer without the embedded atoms \[22\], and
\[
f_a(d_1, d_2) = \frac{\hbar}{\pi c^2} \int_0^\infty d\xi \xi^2 \int_0^\infty dkk \times
\]
\[
\left\{ \frac{1}{\varepsilon} \left[ \alpha_e - 2\alpha_m \varepsilon \xi^2 \right] - \alpha_m \varepsilon \right\} \mathcal{R}^p(i \xi, k)
\]
\[
+ \mu \left[ \alpha_m - 2\alpha_e \varepsilon \xi^2 \right] - \alpha_e \varepsilon \right\} \mathcal{R}^s(i \xi, k)
\]
is the force on an embedded atom. In these equations,
\[
\mathcal{R}^p(i \xi, k) = \frac{r^q e^{-2\kappa d_s} - r^q e^{-2\kappa d_s}}{1 - r^q e^{-2\kappa (d_s + d_d)}}
\]
Similarly as in Eqs. (11)-(13), the first two terms in Eq. (14) describe a medium-screened force \(f_{a}^{(1)}\) and the last one a medium-assisted force \(f_{a}^{(2)}\) on the atom. Accordingly, the force on an atom in the Minkowski stress-tensor approach, \(f_{a}^{(M)}\), is obtained from the above result for \(f_{a}^{(1)}\) by removing the factors \(1/\varepsilon\) and \(\mu\) from \(p\) and \(s\) contributions to the integrand, respectively. We therefore have
\[
f_{a}^{(M)}(d_1, d_2) = \frac{\hbar}{\pi c^2} \int_0^\infty d\xi \xi^2 \int_0^\infty dkk \times
\]
\[
\left\{ \left[ \alpha_e - 2\alpha_m \varepsilon \xi^2 \right] - \alpha_m \varepsilon \right\} \mathcal{R}^p(i \xi, k)
\]
\[
+ \mu \left[ \alpha_m - 2\alpha_e \varepsilon \xi^2 \right] - \alpha_e \varepsilon \right\} \mathcal{R}^s(i \xi, k)
\]

which coincides with the result obtained by Zhou and Spruch (using the surface mode summation method) \[6\].
but it is generalized by accounting for the magnetic properties of the system. Of course, both Eq. (14) and Eq. (16) give the same result in the case of an empty \((n = 1)\) cavity.

Assuming, for simplicity, a semi-infinite cavity obtained by removing, say, mirror \(1 (s_1^2 = 0)\), we have

\[
R^d(i\xi, k) = R^s(i\xi, k)e^{-2kz}, \tag{17}
\]

where \(R^d \equiv r_d^2\) and \(z \equiv d_2\) is the atom-mirror distance. Then, owing to the above exponential factor, for small, \(z \ll c/\Omega\), atom-mirror distances \(\ll\) the major contribution to the integral in Eq. (16) comes from large \(k\) values. Approximating the integrand with its nonretarded \((k \to \infty)\) counterpart and making the substitution \(u = k/2d\), we find for the leading term of the atom-mirror force

\[
f_a(z) = \frac{\hbar}{8\pi^2} \int_0^\infty \int_0^\infty d\xi \int_0^\infty du u^2 e^{-u} \times \left[ \frac{\alpha e^{-\xi z}}{\varepsilon} R^a_{0m} \left( \frac{u}{2d} \right) + \alpha_m R^a_{m0} \left( \frac{u}{2d} \right) \right] \tag{18}
\]

where \(R^a_{0m}(i\xi, k)\) are reflection coefficients of the mirror in the nonretarded approximation. These coefficients are formally obtained from \(R^s(i\xi, k)\) by letting \(\kappa = k = \kappa_l = k\) for the perpendicular wave vectors in all layers of the mirror. Specially, for a single-medium mirror with the refraction index \(n_m\), \(R^a_{0m}(i\xi, u/2)\) are independent of \(u\) [see Eq. (14)], with \(\{\varepsilon_s, \mu_s\} \to \{\varepsilon_m, \mu_m\}\) performing the elementary integration, in this classical configuration we therefore find

\[
f_a(z) = \frac{3\hbar}{4\pi^2 z^2} \int_0^\infty d\xi \left[ \frac{\alpha e^{-\xi z}}{\varepsilon} + \alpha_m \right] \tag{19}
\]

This generalizes the familiar result for the van der Waals force on an atom close to a medium to magnetodielectric systems. Note the presence of the extra (screening) factors \(1/\varepsilon\) and \(\mu\) in comparison with the corresponding traditionally obtained formula \(\text{[6]}\).

To find \(f_a(z)\) for large \(z\), we make the standard substitution \(\kappa = n\xi p/c\) in Eq. (16) and, since \(\xi \approx c/\varepsilon z\), approximate the frequency-dependent quantities with their static values (denoted by the subscript \(0\)). The integral over \(\xi\) can then be easily performed and we obtain

\[
f_a(z) = \frac{3\hbar}{4\pi^2 z^2} \int_1^\infty dp \frac{dp}{p^3} \times \left[ \frac{1}{\varepsilon} \left[ \alpha e^{-\xi z} + \alpha_m \right] \int_0^\infty \left[ \frac{\alpha e^{-\xi z}}{\varepsilon} + \alpha_m \right] \right] \tag{20}
\]

To illustrate this result, we consider the case of an ideally reflecting \((\varepsilon_m \to \infty \text{ or } \mu_m \to \infty)\) mirror. Letting \(R^d = \pm \Delta_g\) (the minus sign is for an infinitely permeable mirror) in Eq. (21), we obtain

\[
f_a^d(z) = \pm \frac{\hbar}{4\pi^2 z^2 n_0} \left[ \frac{\alpha e^{-\xi z}}{\varepsilon} + \alpha_m \right] \tag{21}
\]

whereas the "traditional" Eq. (16) in this case gives

\[
f_a^M(z) = \frac{3\hbar}{2\pi^2 z^2 n_0} \left( \alpha e^{-\xi z} \alpha_m \right) \tag{22}
\]

Of course, in the case of an empty \((\varepsilon = \mu = 1)\) cavity, we recover from both these equations the Böser generalization \(\text{[7]}\) of the Casimir-Polder formula \(\text{[7]}\).

The force on an atom of the cavity medium \(f_a^M(z)\) can be similarly obtained from Eq. (16). Assuming a dilute medium, \(n^2 \approx 1 + 4\pi M (\alpha_e^2 + \alpha_m^2)\), we have \(f_a^M = f_M/N_M d_s\), where \(N_M\) is the atomic number density in the cavity. This force is, to the first order in \(\alpha_e^2 \text{ and } \alpha_m^2\) and for a single-medium mirror, given by \(\text{[28]}\)

\[
f_a^M(z) = \frac{3\hbar}{4\pi^2 z^2} \int_0^\infty dp \frac{dp}{p^3} \left[ \frac{\alpha e^{-\xi z} \alpha_m}{\varepsilon_M} + \frac{\alpha e^{-\xi z}}{\varepsilon_M} \right] \tag{23}
\]

at small and by

\[
f_a^M(z) = \frac{3\hbar}{4\pi^2 z^2} \left( \alpha e^{-\xi z} \alpha_m^2 \right) \times \int_1^\infty \left[ \frac{\alpha e^{-\xi z} \alpha_m}{\varepsilon_M} + \frac{\alpha}{\varepsilon_M} \right] \tag{24}
\]

at large atom-mirror distances. Here \(s_m = \sqrt{p^2 - 1 + n_s^2}\) as appropriate for a dilute cavity medium. Note that, besides exhibiting very unusual behavior at small atom-mirror distances, the sign of \(f_a^M\) is completely insensitive to the polarizability type of the atom. Since for an ideally reflecting mirror the value of integral in Eq. (20) is \(\pm 2/3\), we see that \(f_a^M\) is comparable with \(f_a\) at large distances [c.f. Eq. (25)].

The above results imply similar properties of the force \(f_a\) between two atoms in a medium. This force can be found in the usual way \(\text{[3]}\) by assuming a single-medium mirror consisting of the cavity medium with a small number of, say, type \(B\) atoms embedded in it, so that \(\varepsilon_m = \varepsilon + 4\pi N_B \alpha_B^2\) and \(\mu_m = \mu + 4\pi N_B \alpha_B^2\). With this inserted in Eqs. (21) and (22), we obtain small and large distance behavior of the force \(f_a^{AB} [\alpha e^{e(m)} \equiv \alpha_{e(m)}^A]\) between two atoms \(A \text{ and } B\) embedded in the medium and from Eqs. (25) and (26) we obtain the corresponding behavior of the force \(f_a^{AB} [\alpha e^{e(m)} \equiv \alpha_{e(m)}^A]\) between an atom of the medium and an embedded atom. Thus, for example, from Eq. (20) we straightforwardly find

\[
f_a^{AB} (r) = \frac{18\hbar}{\pi r^7} \int_0^\infty d\xi \left[ \frac{1}{\varepsilon} \alpha e^{-\xi z} + \frac{1}{\mu} \alpha^A \alpha^B \right], \tag{26}
\]
which predicts stronger medium screening of the van der Waals-London force than found earlier\cite{18}, and from Eq. \ref{20} we obtain

$$f_{aa}^{MB}(r) = \frac{2\hbar}{\pi\varepsilon^2r^5} \int_0^\infty d\xi \xi^2 (\alpha_c^M + \alpha_m^M)(\alpha_c^B - \alpha_m^B),$$

(28)

which implies different properties of the interaction between an atom of the medium and an embedded atom.

In summary, according to the Lorentz force approach to the Casimir effect, the force on an atom embedded in a material cavity differs substantially from the force on an atom on the cavity medium. For embedded atoms, the force consists of a medium-screened and a medium-assisted force. The results for the medium-screened force differ from the corresponding traditionally obtained results in the presence of the extra factors $1/\varepsilon$ and $\mu$ multiplying the contributions of the TM and TE polarized waves, respectively. This, together with the appearance of the medium-assisted force term, predicts a stronger dependence of the atom-mirror force on the medium parameters than usually considered. Accordingly, a number of the classical results for the atom-mirror interaction in various systems are modified with respect to this point. The force on the atoms of the cavity medium is a very recently introduced medium-assisted force\cite{21}, which behaves as the Coulomb force at small and as the Casimir-Polder force at large atom-mirror distances. In addition, contrary to the Casimir-Polder force\cite{3}, its sign is insensitive to the polarizability type (electric or magnetic) of the atom. Clearly, these properties of the atom-mirror force imply similar properties of the atom-atom force in a medium.

Note added. After this work was completed, strong doubts have been raised\cite{26} on the correctness of the stress tensor (brackets denote the average with respect to fluctuations)

$$T_{ij}(r) = \frac{1}{4\pi} \left( D_i E_j + H_i B_j - \frac{1}{2} (\mathbf{D} \cdot \mathbf{E} + \mathbf{H} \cdot \mathbf{B}) \delta_{ij} \right)$$

(29)

employed by Raabe and Welsch\cite{21}. As stressed by Pitaevskii\cite{21}, the first (Minkowski) term here corresponds to the effective stress tensor in a (fluid) medium which is in mechanical equilibrium. If so, the above stress tensor is incomplete since its second term, which gives rise to the force on the medium, is not balanced. We note that such a conclusion is also (implicitly) indicated by rather peculiar properties of the van der Waals and Casimir forces implied by $T_{ij}$ given by Eq. \ref{20}, as we have demonstrated in this work as well as in Ref. \ref{22}.

This work was supported by the Ministry of Science and Technology of the Republic of Croatia under contract No. 0098001.

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\bibitem{24} The integral over $\xi$ in Eq. \ref{20} effectively extends up to a frequency $\Omega$ beyond which either the slab or the corresponding mirror becomes transparent, depending on which of these two quantities is smaller. In Eq. \ref{20}, the corresponding cutoff frequencies are given by the properties of the mirror and the cavity medium. We can therefore take $\Omega$ of the mirror as the cutoff frequency com-
mon to both equations. This introduces the characteristic length $c/\Omega$ by which all other lengths (and thicknesses) in the system may be compared. For a more rigorous consideration of this point, see, e.g., Ref. [8].

[25] The atom-dilute-medium force of the form $f_a(z) = AN/z^\gamma$ implies the atom-atom force of the form $f_{aa}(z) = \gamma(\gamma + 2)A/2\pi r^{\gamma+3}$.

[26] L. P. Pitaevskii, arXiv: cond-mat/0505754.