NEUTRINOLESS DOUBLE BETA DECAY
CONSTRAINED BY THE EXISTENCE
OF LARGE EXTRA DIMENSIONS

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We present the possible influence on the half-life of neutrinoless double beta decay coming from the existence of n extra spatial dimensions. The half-life in question depends on the mass of the electron neutrino. We base our analysis on the Majorana neutrino mass mechanism in Arkani-Hamed–Dimopoulos–Dvali model.

1. Introduction

Theories and models with additional spatial dimensions have drawn much attention during the last few years (see e.g. [1] and references therein for a complete review). The idea, which dates back to the 20’s of the previous century, has been recently rediscovered due to the development of string theory, which requires for a consistent formulation ten, rather than three, spatial dimensions. Moreover, new multidimensional generalizations of strings, called branes, emerge from this theory in a natural way. Recent models suggest that our observable universe could be embedded in such a brane, which in turn floats in a higher dimensional bulk, possibly interacting with fields that populate the bulk as well as with other branes.

The primary goal of the ADD model ([2-4] and references therein) was to explain the huge difference between the scales of electroweak interactions (∼1 TeV) and gravity (i.e. Planck energy ∼ 10^{16} TeV). It is achieved by assuming that the Standard Model (SM) is localized on a three-dimensional brane which is embedded into a (4 + n)-dimensional space-time. The only boson that feels the additional space (the bulk) is the graviton, and therefore the only interaction which may freely propagate through the bulk is gravity. This mechanism gives a natural suppression of that interaction, coming from the volume of the bulk.

In the present paper we discuss the implications of possible existence of extra dimensions on an exotic nuclear process: the neutrinoless double beta decay.
2. Theoretical Background

Let us assume, that space-time is \((4+n)\)-dimensional. Since we do not observe such a situation in everyday life, the additional \(n\) spatial dimensions must be compactified. For simplicity we will treat them as curled into circles with very small, identical radii \(R\). The whole SM is restricted to a three-dimensional brane, a topological object, which is a higher dimensional generalization of a 1-D string and 2-D membrane.

In the ADD model there exists a second brane, parallel to our SM brane, from which particles carrying non-zero lepton number (call them \(\chi\)) may escape into the bulk. These particles, in turn, interact with SM fields on our brane, which leads to a Majorana neutrino mass term, naturally suppressed by the distance between the branes. We end up with a mass term of the form

\[
m_{\text{Maj}} \sim v^2 R^{\frac{n(n-1)}{n+2}} M_{\text{Pl}}^{\frac{2(n-1)}{n+2}} \Delta_n(R, m_\chi),
\]

where \(v\) is the vacuum expectation value (vev) of the Higgs field \((v = 174\ \text{GeV})\), and \(M_{\text{Pl}}\) is the Planck mass. The \(n\)-dimensional propagator for the messenger particle with mass \(m_\chi\), travelling between branes, is explicitly given by

\[
\begin{align*}
R m_\chi \ll 1 : & \quad \Delta_2(R, m_\chi) \sim -\log(R m_\chi), \\
& \quad \Delta_{n>2}(R, m_\chi) \sim \frac{1}{R^{n-2}}, \\
R m_\chi \gg 1 : & \quad \Delta_2(R, m_\chi) \sim \frac{e^{-R m_\chi}}{\sqrt{R m_\chi}}, \\
& \quad \Delta_{n>2}(R, m_\chi) \sim \frac{e^{-R m_\chi}}{R^{n-2}}.
\end{align*}
\]

A small but non-zero Majorana neutrino mass term gives rise to suppressed lepton number violating processes, like the neutrinoless double beta decay \((0\nu 2\beta)\)

\[A(Z, N) \rightarrow A(Z+2, N-2) + 2e^-\]

This decay requires that two neutrinos emitted in beta decays annihilate with each other. It is readily seen that this process violates lepton number by two units, thus is forbidden in the framework of SM. As a matter of fact, \(0\nu 2\beta\) has not been observed, but restrictions on the amplitude of the decay, which follow from its non-observability, set valuable constraints on the shape of non-standard physics.

One expects that the half-life of \(0\nu 2\beta\) depends on the mass of the electron neutrino, and that is indeed the case. Under the following assumptions about the neutrino mass eigenstates: (i) the contribution coming from \(m_3\) is neglected (this is justified by the CHOOZ results[^6]), (ii) the remaining masses are nearly degenerate \(m_1 \approx m_2\), one obtains for the half-life of the \(0\nu 2\beta\) decay[^9]

\[
T_{1/2}^\text{th} > \kappa \cdot 10^{\frac{2 m_1 - 150}{\pi^2} R^{\frac{2(n-1)}{n+2}} \Delta_n(R)^{-2}} \text{ y}.
\]

[^3]: Here, the uncertainty factor \(\kappa\) satisfies \(0.74 < \kappa < 1.17\). In the derivation of Eq. \((3)\) the experimental values established by the IGEX collaboration \[^{10}\] \(T_{1/2}^{\text{IGEX}} > 1.57 \cdot 10^{25} \text{ y}\) and the effective neutrino mass \((m_\nu) = 0.4 \text{ eV}\), as suggested by the Heidelberg–Moscow project \[^{8}\] have been used.
We proceed with a detailed analysis of Eq. (3), taking into account different cases, according to Eq. (2). One must, however, be aware of the bounds on the mass of the messenger is in general rather weak, except for a narrow region of $2 < R < 10^{-3}$ mm for $n = 2$, $R < 2.6 \times 10^{-9}$ mm for $n = 3$, and $R < 3.4 \times 10^{-10}$ mm for $n = 4$. The same goes to the experimental limit on $T_{1/2}$ which is at present $T_{1/2}^{\text{IGEX}} > 1.57 \times 10^21$ y.

Our results are presented in Figs. 1–3. Notice first, that the dependence on the mass of the messenger is in general rather weak, except for a narrow region of very small $m_\chi$. We see that the closest to experimental lower bound is the case of $n = 2$ with a heavy messenger mass $m_\chi$ of a few MeV. Another possibility is $n = 4$ and a heavy messenger with a pretty arbitrary mass. The remaining cases shown in Figs. 1 and 2 are completely non-verifiable experimentally. There is also an exceptional case $n = 4$ with a light messenger (not illustrated in the figures),
Fig. 3. Lower limits on $T_{1/2}$ in the cases $n = 4$ and $n = 5$ (see text for details).

in which the dependence on $R$ in Eq. (3) is lost. Explicitly we get $T_{1/2} \sim 10^{27}$ years, with the neutrino mass $m_\nu \sim 10^{-6} \text{eV}$ and an arbitrary $R$. This possibility is quite reasonable and is in perfect agreement with all experimental data and most theoretical predictions for the present day. We have found that there is no point in discussing $n > 4$ (see Fig. 3), at least in the context of $0\nu\beta\beta$. If this happens to be the case, such a decay will be practically forbidden.

One has to bear in mind, that all these results have meaning only in the framework of the ADD model. In fact, as for today there is no experimental hint, which supports such ideas. The only theoretical motivation comes from string theory, but the realization of extra dimensions may be of course completely different.

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