Beamforming design for MIMO full-duplex SWIPT IoT system under imperfect CSI

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Abstract
This paper proposes a multiple-input multiple-output full-duplex relay Internet of Things system under imperfect channel state information, where the relay uses time switching relaying protocol to harvest energy. Particularly, due to the uncertainty channel circumstance, the self-interference in full-duplex relay cannot be eliminated completely. For such a system, a joint source-relay beamforming optimization problem which maximizes the system achievable rate subject to the transmit power and harvested energy constraints is studied. In light of the intractability of the problem, a singular value decomposition based geometric programming algorithm is investigated. The considered problem is reformulated to a diagonalized form by using singular value decomposition method. Then, the geometric programming algorithm is adopted to obtain the optimal solution for the reformulated problem. Numerical results validate the effectiveness and superiority of the proposed optimization scheme.

1 | INTRODUCTION

The Internet of Things (IoT) has become one of the mainstream emerging communication paradigms that encompasses massive embedded devices which are generally wireless-interconnected to gather and exchange data [1]. It is expected that the number of connected IoT users will reach approximately 50 billion in 2020 [2]. Recently, relay technique has been introduced into IoT networks to assist a long-distance transmission and ensure uninterrupted communication [3, 4]. However, there are two critical issues in implementing relay-aided IoT networks deployment, one of which is the energy supply limitation [5], that is, the relay nodes have limited battery reserves.

Energy harvesting (EH) is a promising and green solution for the energy limitation issue [6, 7]. Particularly, simultaneous wireless information and power transfer (SWIPT)-aided relay technique is considered as an efficient way [8]. It lets energy-constrained relay harvest energy from the radio frequency (RF) signal emitted by the source and then forward the source information to the destination using the harvested energy, which consequently prolong the battery lifetime of relay nodes, provide sustainable energy supplies, and improve the energy efficiency in IoT networks [9–11]. Based on time switching (TS) and power splitting (PS) receiver architectures, two popular relaying protocols, namely, TS relaying (TSR) protocol and PS relaying (PSR) protocol have been proposed in [12–15]. In [14], the reliability performance of a SWIPT-enabled relaying network considering both PS and TS protocols was studied in the finite blocklength (FBL) regime. In [15], an amplify-and-forward (AF) network considering both TSR and PSR protocols was investigated and the achievable throughput performance at the destination was determined by deriving analytical expressions. Compared with the PSR, in the TSR, relay periodically switches between EH and information transmission (IT), which has gained many research interests. In [9], a TS-based decode-and-forward (DF) two-way cooperative cognitive radio network (CCRN) was proposed; herein, a pair of IoT devices (IoDs) were used as TS relays to provide assistance for primary users communication. In [16], the outage optimization problem for two-way network with TSR scheme was investigated.

The other is the spectrum resources limitation. Full-duplex (FD) relay is treated as a promising technique to double the spectrum efficiency and increase the capacity of the IoT networks, since it enables simultaneously transmission and
reception signals over the same frequency band [17–19]. To enable FD relaying transmission, the self-interference (SI) must be suppressed effectively via various SI cancellation techniques [20–22]. Based on such SI cancellation techniques, in [23], a FD AF relay system for low-latency and high-reliability (LLHR) IoT was proposed. Recently, much interest has been turned to introduce FD into TSR networks. In [24], the instantaneous throughput maximization problem of a multiple-input multiple-output (MIMO) FD decode and forward (DF) system with TSR protocol was studied to optimize receive and transmit beamformers at the relay and the TS parameters.

One of the prerequisites considered in all the prior research works is that the perfect channel state information (CSI) can be obtained at the transmitters. However, this perfect CSI assumption is highly unrealistic due to channel delay, channel estimation (CE), and quantization errors [25]. In practice, the acquired CSI at the transmitters is usually an imperfect estimate which may cause a serious affect on the networks performance. Therefore, robust performance optimization under imperfect CSI is a critical concern [26–29]. Thereinto, in [27], the ergodic secrecy rate performance was analyzed for AF three-hop cooperative relay network under imperfect CSI in the IoT. In [28], the source and relay precoders for MIMO AF half-duplex (HD) SWIPT relay system with imperfect CSI were designed to investigate the achievable R-E regions. In [29], the source-relay beamforming for MIMO FD system with PSR protocol under imperfect CSI was designed to minimize the total mean squared error (MSE). However, to the best of our knowledge, joint source-relay beamforming design based on achievable rate maximization in MIMO FD TSR-based IoT system under imperfect CSI has not yet been studied.

Here, a MIMO FD AF IoT system with TSR protocol is studied. For the proposed system, the practical assumption of imperfect CSI is considered at the transmitters. The advantages of the discussed system lie in ensuring uninterrupted communication, extending the longevity and guaranteeing the sustainability of the IoT network, and significantly raising the spectral efficiency. The main contributions of this paper can be concluded as follows. First, contrary to [30], we consider the FD relay which brings higher spectral efficiency than HD one, while the SI generated by the FD relay to itself cannot be fully eliminated with imperfect CSI and the system performance need to be further evaluated under this condition. Second, a joint optimization problem for source and relay beamforming based on the achievable rate maximization is established under the constraints of transmit power and harvested energy. Finally, to address the non-convex optimization problem, a singular value decomposition (SVD) based geometric programming (GP) algorithm is exploited. We cast the primary problem as a scalar form by using SVD method, and then the optimal solution to the reformulated problem can be derived by using GP algorithm. Particularly, we investigate the general expression of the derivation of GP algorithm in the scenario that the number of antennas equipped at relay nodes is no less than that equipped at source nodes. Comparisons between the proposed scheme and two benchmark schemes are presented and the numerical results expose that our scheme can improve the achievable rate.

![FIGURE 1 The system model of MIMO FD AF IoT network with TSR protocol under imperfect CSI](image)

## 2 | SYSTEM MODEL

We consider a three-node MIMO FD AF IoT system with TSR protocol illustrated in Figure 1, which comprises \( M \)-antenna source \( S \), \( 2N \)-antenna relay \( R \), and \( M \)-antenna destination \( D \). \( S \) desires to send signal to \( D \) with the help of \( R \). The channel gains from \( S \) to \( R \) and that from \( R \) to \( D \) are denoted as \( H_1 \in \mathbb{C}^{N \times M} \) and \( H_2 \in \mathbb{C}^{M \times N} \), respectively, and the self-interference channel at \( R \) is represented as \( G_{rr} \in \mathbb{C}^{N \times N} \). In the system, we assume that all channels are reciprocal in incoming and outgoing directions, statistically independent and quasi-static Rayleigh blockfading, and are fixed over a block time \( T \). Furthermore, the direct link between \( S \) and \( D \) is not considered due to the severe path attenuation and shadowing.

Considering the realistic scenario with CE errors in the whole system, all channels \( H_i (i \in \{1, 2\}) \) and \( G_{rr} \) can be modelled as

\[
H_i = \overline{H}_i + \Delta H_i, \quad (1)
\]

and

\[
G_{rr} = \overline{G}_{rr} + \Delta G_{rr}, \quad (2)
\]

where \( \overline{H}_i \) and \( \overline{G}_{rr} \) are the estimate values of \( H_i \) and \( G_{rr} \), respectively. \( \Delta H_i \) and \( \Delta G_{rr} \) are the CE errors, which are considered as independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (ZMCSCG) random matrices with variances \( \sigma_M^2 \mathbf{I}_M \) and \( \sigma_N^2 \mathbf{I}_N \), respectively.

## 3 | AF FD TSR PROTOCOL WITH IMPERFECT CSI

In the AF FD TSR protocol, the block duration \( T \) is divided into two phases, that is, EH phase and IT phase. During the EH phase, \( R \) harvests the energy from RF signals emitted by \( S \). During the IT phase, \( S \) transmits its signals to \( R \) and \( R \) broadcasts the amplified previously received signals to \( D \) simultaneously. The harvested energy during EH phase is stored in a extreme-capacitor and is consumed by \( R \) together with battery energy when forwarding the source information to \( D \).
switches between EH and IT in a time division manner, in such manner, $\alpha T$ is allocated for EH, where $\alpha$ is the ratio of $T$ used for EH with $\alpha \in (0, 1)$, and the remaining $(1 - \alpha) T$ is used for IT. For the simplicity of presentation, we set $T = 1$ throughout the paper.

### 3.1 Energy harvesting

In the EH phase, at time slot $t$, the received signal at $R$ is given as

$$y_n(t) = H_t F_t x(t) + n_n(t),$$

where $x(t) \in \mathbb{C}^{M \times 1}$ is the transmitted signal by $S$ with normalized power, $F_t \in \mathbb{C}^{M \times M}$ is the source beamforming matrix, and $n_n(t) \sim CN(0, \sigma_n^2 I_N)$ denotes the zero-mean additive white Gaussian noise (AWGN) at $R$.

Using (3), the harvested energy $E_{\nu}$ at $R$ in the EH time $\alpha T$ can be written as

$$E_{\nu} = \eta \alpha T \mathbb{E} \left[ |H_t F_t F_t^H| \right],$$

where $\eta \in (0, 1]$ represents the energy conversion efficiency depending on the energy harvesting circuitry [31]. Substituting (1) into (4) and considering that the estimated channels and CE errors are independent, $E_{\nu}$ under imperfect CSI is given by

$$E_{\nu} = \eta \alpha T \mathbb{E} \left[ |H_t F_t F_t^H| \right] + \sigma_n^2 \mathbb{E} \left[ |F_t^H| \right].$$

### 3.2 Information transmission

In the IT phase, at time slot $t$, the FD relay $R$ receives signal $y_n(t)$ and transmits $x_n(t)$ to $D$ concurrently. The received signal $y_r(t)$ at $R$ is combination of the signal from $S$, the SI signal from $R$, and the noise, which can be stated as

$$y_r(t) = H_r F_r x_r(t) + G_n x_r(t) + n_r(t),$$

where $x_r(t)$ denotes the transmitted signal by $R$ with $\mathbb{E} \{ |x_r(t)|^2 \} = 1$, where $\mathbb{E} \{ \}$ means the expectation operation.

Since the relay has full knowledge of its own signal $x_r(t)$, the interference cancellation method proposed in [32] can be applied to mitigate the SI. However, due to the estimate error $\Delta G_n$, the SI cannot be completely cancelled. Then, the post-cancellation signal at $R$ can be represented as

$$\hat{y}_r(t) = H_r F_r x_r(t) + \Delta G_n x_r(t) + n_r(t).$$

With the AF mode, $R$ forwards its received signal after multiplying it by a beamforming matrix $F_r \in \mathbb{C}^{N \times N}$. Then, the transmitted signal $x_r(t)$ at $R$ is given by

$$x_r(t) = F_r (H_r F_r x_r(t - \tau) + \Delta G_n x_r(t - \tau)) + \Delta H_r F_r n_r(t - \tau),$$

where $\tau \in [1, \infty)$ is the symbol processing delay at $R$, $\hat{n}_r(t - \tau) \triangleq n_r(t - \tau) + n_p(t - \tau)$ is the equivalent noise vector, $n_p(t)$ is the processing noise caused by RF-to-baseband conversion [33] with $n_p(t) \sim CN(0, \sigma_p^2 I_M)$. Suppose that $\sigma_p^2 \ll \sigma_n^2$, then $n_n(t - \tau)$ can be ignored. According to [29], (8) can be further expressed as

$$x_r(t) = F_r \sum_j (\Delta G_r F_r) \mathbb{E} \left[ |H_r F_r x_r(t - j\tau - \tau)\right]$$

$$+ \mathbb{E} \left[ |n_r(t - j\tau - \tau)\right],$$

following this, let $J = \Delta G_r F_r$, and the transmit energy $E_{x_r}$ at $R$ can be computed as

$$E_{x_r} = (1 - \alpha) T \mathbb{E} \left[ \sum_j J H_r F_r \mathbb{E} \left[ |F_r^H| \right] + \sigma_n^2 \sum_j (J^H) F_r^H \right].$$

### 3.3 Achievable rate computation

According to Shannon theorem, the achievable rate of the system from source $S$ to destination $D$ in the presence of imperfect CSI per block time can be computed as

$$R(F_r, F_r) = (1 - \alpha) \log_2 \left( 1 + \Phi(F_r, F_r) \right),$$

where $\Phi(F_r, F_r)$ is the expectation of the corresponding end-to-end signal-to-interference-plus-noise ratio (SINR) defined as

$$\Phi(F_r, F_r) = \frac{\mathbb{E} \left[ |H_r F_r|^2 \right]}{\mathbb{E} \left[ |H_r F_r|^2 \right] + \sigma_n^2 \mathbb{E} \left[ |H_r F_r|^2 \right] + \sigma_p^2 \mathbb{E} \left[ |H_r F_r|^2 \right]}$$

$$+ \sigma_n^2 \mathbb{E} \left[ |H_r F_r|^2 \right] + \sigma_p^2 \mathbb{E} \left[ |H_r F_r|^2 \right],$$

where $L = H_r F_r$, $I = L A G_n x_r(t - \tau), M = L H_r F_r$.

In order to simplify the expression of $R(F_r, F_r)$, the term $\mathbb{E} \left[ |H_r F_r|^2 \right]$ in (13) needs to be analyzed first. Substituting (9) into (13) and letting $K = H_r F_r$, we obtain

$$E \left[ |H_r F_r|^2 \right] = \mathbb{E} \left[ \sum_j (J^H) F_r^H \right]$$

$$+ \sigma_n^2 \mathbb{E} \left[ \sum_j (J^H) F_r^H \right].$$
To further simplify (14), we give the following expressions
\[
\Gamma_1 = E[Tr(LJ)^{j+1}(K^H(J^H)^{j+1}L^H)],
\]
and
\[
\Gamma_2 = \sigma_n^2 E[Tr(LJ)^{j+1}(L^H)].
\]

According to [34]
\[
Tr(ABCD) = (vec(C^T))^T(B^T \otimes D) vec(A),
\]
where \(\otimes\) represents the Kronecker product; \(vec(\cdot)\) denotes the vectorization operator; and \(A, B, C, D\) are arbitrary matrices; and the properties of \(Tr(A \otimes B) = Tr(A)Tr(B)\) and \(Tr(A^T) = Tr(A)\), the two expressions can be re-written as
\[
\Gamma_1 = (\sigma_n^2)^{(j+1)}Tr(KK^H(FF^H)J^HTr(LL^H)),
\]
and
\[
\Gamma_2 = \sigma_n^2 (\sigma_n^2)^{(j+1)}Tr(FF^H)Tr(LL^H).
\]

With the error variance model presented in [35], the variances of CE errors \(\sigma_n^2, a \in \{i, r\}\) are small at high SNRs. From this, \((\sigma_n^2)^{(j)}\) will be small enough to ignore when \(j > 1\). Besides, given that \(\sigma_n^2\) is small, \(\sigma_n^2 \times \sigma_n^2\) can approximate to zero. Then, we can respectively obtain
\[
\Gamma_1 = \begin{cases} 
(\sigma_n^2)^{(j+1)}Tr(FK^H(F^H)J^HTr(LL^H)), & \text{if } j = 0 \\
0, & \text{if } j > 0
\end{cases},
\]
and
\[
\Gamma_2 = 0, \quad \forall j \geq 0.
\]

Based on (20) and (21), (14) can be simplified as
\[
E(Tr(I^H)) = \sigma_n^2 Tr(FK^H(F^H)J^HTr(LL^H)).
\]

Putting (22) into (13) and considering \(H, \Phi(FF^H)\) with CE errors, the achievable rate under the constraints of transmit power and harvested energy. The joint optimization problem considering imperfect CSI can be formulated as
\[
\begin{align*}
\max_{F_r, F_s} & \quad R(F_r, F_s) \\
\text{s.t.} & \quad Tr(FF^H) \leq P_r
\end{align*}
\]

where (26b) means that the transmit power at \(S\) should not be greater than the peak power \(P_r\). (26c) ensures that the harvested energy at \(R\) is greater than the minimum energy \(Q_r\) that should be harvested at the node. (26d) guarantees that the transmit power at \(R\) is upper-bounded by the sum of peak power \(P_r\) supplied by \(R\) and the harvested power.

However in (26), variables \(F_r\) and \(F_s\) are coupled, which makes the problem non-convex and difficult to solve. Despite the optimization challenges, we propose an algorithm that can theoretically get the optimal solution to (26). To implement the algorithm, we first need to reformulate (26a) into an equivalent form. Since \(R(F_r, F_s)\) is a monotonically increasing function of \(\Phi(F_r, F_s)\) (see (12)), maximizing \(R(F_r, F_s)\) is equivalent to maximizing \(\Phi(F_r, F_s)\), which becomes equivalent to minimizing
\[
1/\Phi(F_r, F_s).
\]

Accordingly, based on (23), (26) can be re-defined

\[
R(F_r, F_s) = (1 - \alpha)
\]

Furthermore, employing the same procedure above mentioned to deal with \(\sum_j r_j^i\) in (10), \(E_x, \) with imperfect CSI becomes
\[
E_x = (1 - \alpha)Tr(H_sF_s^HTr(R_s) + \sigma_n^2 Tr(R_s)) + \sigma_n^2 Tr(F_rF_s^H) + \sigma_n^2^2,
\]

\section{PROBLEM FORMULATION}

With these definitions in place, we focus on the joint optimization of source and relay beamforming to maximize the achievable rate under the constraints of transmit power and harvested energy.
5 \ PROBLEM OPTIMIZATION

In order to optimize problem (27), a SVD-based GP algorithm is designed. For the algorithm, we first consider channel parallelization technique, that is, employ SVD method to decompose the estimated MIMO channels \( \overline{H}_i \) and \( \overline{G}_{rr} \) into scalar parallel ones so as to cast the primary problem as a scalar form. Then, the function transformation is introduced to convert the reformulated problem into a standard GP. Finally, the optimal solution to the GP can be efficiently obtained using available software packages.

5.1 \ Channel parallelization

In the \( N \geq M \) scenario, exploiting SVD on the channels \( \overline{H}_i \) and \( \overline{G}_{rr} \), we have

\[
\overline{H}_1 = U_1 \Sigma_1 V_1^H, \quad \overline{H}_2 = U_2 \Sigma_2 V_2^H,
\]

and

\[
\overline{G}_{rr} = U_r \Sigma_r V_{rr}^H,
\]

where \( U_1 \in \mathbb{C}^{N \times N}, U_2 \in \mathbb{C}^{M \times M}, V_1 \in \mathbb{C}^{M \times N}, V_2 \in \mathbb{C}^{N \times N}, U_r \in \mathbb{C}^{N \times N}, \) and \( V_{rr} \in \mathbb{C}^{N \times N}, \) are unitary matrices, \( \Sigma_1 = (0_{(N-M)\times M})^T \), \( \Sigma_2 = (\Lambda_{1_2} \ 0_{(N-M)\times M})^T \), and \( \Lambda_{rr} \in \mathbb{C}^{N \times N} \), where \( \Lambda_{1_1}, \Lambda_{1_2}, \) and \( \Lambda_{rr} \) are non-negative diagonal matrices. In order to parallelize the channels in (28), (29), the source and relay beamforming matrices \( F_s \) and \( F_r \) are decomposed into

\[
F_s = V_1 \Lambda_s V_1^H,
\]

and

\[
F_r = V_2 \Lambda_s U_1^H,
\]

where \( \Lambda_s \in \mathbb{C}^{M \times M} \) and \( \Lambda_r \in \mathbb{C}^{N \times N} \) are diagonal matrices with non-negative diagonal elements which are to be optimized. Then, by substituting (28)–(31) into (27), the problem can be expressed as

\[
\min_{\Lambda, \Lambda_s} \frac{\Omega_1(\sigma_s^2 \Omega_1 + \sigma_r^2) + \sigma_r^2}{\Omega_3 + \sigma_1^2 \Omega_2 + \sigma_2^2 \Omega_2 + \Omega_6},
\]
5.2 Optimization with GP

However, problem (33) is a polynomial minimization problem, which is still non-convex and basically intractable. To this end, aroused by [36], we propose a scheme that can optimize problem (33) by solving a series of geometric programs. Specifically, we have the following proposition.

**Proposition 1.** Defining a polynomial \( F(x, y) = \sum_{w=1}^{W} Z_w(x, y) \), where \( Z_w(x, y) \geq 0 \) is the \( i \)-th monomial of \( F(x, y) \), we can obtain the following inequality

\[
\sum_{w=1}^{W} Z_w(x, y) \geq \prod_{w=1}^{W} \left( \frac{Z_w(x, y)}{\varepsilon_w} \right)^{\varepsilon_w},
\]

where \( \varepsilon_w = \frac{Z_w(x, y)}{F(x, y)} \).

**Proof.** Here, we prove the Proposition 1. Based on the definition that \( \varepsilon_w \geq 0 \) and \( \sum_{w=1}^{W} \varepsilon_w = 1 \), we have

\[
\sum_{w=1}^{W} \varepsilon_w \xi^w \geq \prod_{w=1}^{W} \xi^w,
\]

(34)

by using the arithmetic-geometric inequality. Letting \( Z_w = \varepsilon_w \xi^w \), (34) can be turned into

\[
\sum_{w=1}^{W} Z_w(x, y) \geq \prod_{w=1}^{W} \left( \frac{Z_w(x, y)}{\varepsilon_w} \right)^{\varepsilon_w},
\]

(35)

Since \( F(x, y) = \sum_{w=1}^{W} Z_w(x, y) \), we can approximate \( F(x, y) \) by a monomial function

\[
F(x, y) = \sum_{w=1}^{W} Z_w(x, y) \geq \prod_{w=1}^{W} \left( \frac{Z_w(x, y)}{\varepsilon_w} \right)^{\varepsilon_w}.
\]

(36)

Proposition 1 is proved.

Using proposition 1, and considering initial values \( a_{n, n}^{(in)} \) and \( a_{n, n}^{(in)} \), the polynomials in the denominator of (33a) satisfy the following inequalities:

\[
\sum_{m=1}^{\hat{M}} \lambda_{1, m} \geq \prod_{m=1}^{\hat{M}} \left( \frac{\lambda_{1, m}}{\varepsilon_{w1}} \right)^{\varepsilon_{w1}},
\]

(37a)

\[
\sum_{m=1}^{\hat{M}} \left( \sigma_2^2 \lambda_{1, m} + \lambda_{2, m} \right) \geq \sum_{l=1}^{L} \lambda_{2, m} \prod_{m=1}^{\hat{M}} \left( \frac{\sigma_2^2 \lambda_{1, m}}{\varepsilon_{m2}} \right)^{\varepsilon_{m2}},
\]

(37b)

\[
\sum_{m=1}^{\hat{M}} \left( \sigma_2^2 \lambda_{6, m} + \lambda_{2, m} \right) \geq \sum_{l=1}^{L} \lambda_{2, m} \prod_{m=1}^{\hat{M}} \left( \frac{\sigma_2^2 \lambda_{6, m}}{\varepsilon_{m3}} \right)^{\varepsilon_{m3}},
\]

(37c)
and then, they can be replaced by the corresponding lower bound, respectively. Therefore, based on (38), (39a) and (39b), the problem (33) can be reformulated as

\[
\min \{ \varphi_{\omega, \theta} \}, \quad (38),
\]

\[
\text{s.t.} \sum_{i=1}^{M} \lambda_i \leq P, \quad (40a)
\]

\[
\frac{Q_r}{\prod_{i=1}^{M} \left( \frac{(\lambda_i + \sigma^2 \lambda_i) \eta_n}{\varepsilon \omega_k} \right)} \leq 1, \quad (40c)
\]

\[
(1 - \alpha) \sum_{i=1}^{M} \lambda_i \leq 1, \quad (40d)
\]

It is noteworthy that the post-conversion problem (40) is a standard GP problem, and it can be effectively solved by the gplab toolbox [36] through the interior point method in [37]. According to the analysis of the above problems, we can summarize the algorithm shown in Algorithm 5.2.

In Algorithm 1, \( F^{(\text{opt})}_1 = \sqrt{\frac{\sum_{i=1}^{M} I_M}{M}}, F^{(\text{ini})}_1 = \sqrt{\frac{\sum_{i=1}^{M} I_M}{\rho}}, \) where \( O = Tr[I_M \cdot (1 + \sigma^2 \lambda_\omega \hat{\lambda}_i^2 H^{(\text{ini})}_1 H^{(\text{ini})}_1 H^{(\text{ini})}_1 H^{(\text{ini})}_1 + \sigma^2 + \sigma^2 M)] \), \( \delta \) is the error tolerance, ITER is the maximum number of iterations.

**Property 1.** The proposed algorithm 1 is convergent.

**Proof.** According to [38], we let

\[
f_1(a_{\omega, \sigma}^{(\text{opt})}, a_{\sigma, \sigma}^{(\text{opt})}) = \sum_{i=1}^{M} \lambda_i + \sigma^2 \lambda_i \sum_{i=1}^{M} \lambda_i + \sigma^2 \lambda_i \sum_{i=1}^{M} \lambda_i, \quad (a_{\omega, \sigma}^{(\text{opt})}), \quad (40a)
\]

\[
\overline{f}_1(a_{\omega, \sigma}^{(\text{opt})}, a_{\sigma, \sigma}^{(\text{opt})}) = \prod_{i=1}^{M} \left( \frac{(\lambda_i + \sigma^2 \lambda_i) \eta_n}{\varepsilon \omega_k} \right) \leq 1, \quad (40c)
\]

\[
\overline{f}_2(a_{\omega, \sigma}^{(\text{opt})}) = \left( \lambda_i + \sigma^2 \lambda_i \right) \eta_n \lambda_i \leq 1, \quad (40d)
\]

\[
f_3(a_{\omega, \sigma}^{(\text{opt})}) = P_i + \eta_n \sum_{i=1}^{M} \lambda_i + \sigma^2 \lambda_i \lambda_i + \sigma^2 \lambda_i \lambda_i
\]

\[
\overline{f}_3(a_{\omega, \sigma}^{(\text{opt})}) = \left( \lambda_i + \sigma^2 \lambda_i \right) \eta_n \lambda_i \leq 1, \quad (40d)
\]

\[
f_4(a_{\omega, \sigma}^{(\text{opt})}) = \sum_{i=1}^{M} \lambda_i + \sigma^2 \lambda_i \lambda_i + \sigma^2 \lambda_i \lambda_i
\]

\[
\overline{f}_4(a_{\omega, \sigma}^{(\text{opt})}) = \left( \lambda_i + \sigma^2 \lambda_i \right) \eta_n \lambda_i \leq 1, \quad (40d)
\]

\[
f_5(a_{\omega, \sigma}^{(\text{opt})}) = \sum_{i=1}^{M} \lambda_i + \sigma^2 \lambda_i \lambda_i + \sigma^2 \lambda_i \lambda_i
\]

\[
\overline{f}_5(a_{\omega, \sigma}^{(\text{opt})}) = \left( \lambda_i + \sigma^2 \lambda_i \right) \eta_n \lambda_i \leq 1, \quad (40d)
\]

\[
\text{Previous iteration} \quad [39], \quad \text{we obtain} \quad \text{obj}_{i}^{(\text{opt})} \leq \text{obj}_{i}^{(\text{iter} - 1)}.
\]

Consequently, the objective value of approximated problem (40) and the original problem (33) monotonically decreases, which testifies the convergence of our proposed algorithm.

**6 | NUMERICAL RESULTS**

In this section, we present some simulation results to sufficiently evaluate the performance of the proposed algorithm. The simulation settings are set as follows unless specified otherwise. 100 groups of random flat Rayleigh fading channels are generated. The variances of noises are set to be \( \sigma^2 = \sigma^2 = \sigma^2 \), the peak power is set as \( P_i = 10E_r \) and \( P_i = 20E_r \), and the signal to noise ratio (SNR) is computed as \( SNR = 10log_{10}(E_r/\sigma^2) \). \( E_r \) denotes the power of signal, which is a unit power. We assume the relay radius as \( r = 10 \), and the absolute distance of \( S \) to \( D_r \) is 1000, then the relative distance of \( S \) to \( R \) is \( D_r \), it can be computed as \( D_r/r = 100 \). Besides, we set the relative distance of \( R \) to \( D_D = D_r \) to 100. Meanwhile, the energy harvesting requirement \( Q_{\omega, \sigma} = 0.1 \), and we set the data stream \( S = 2 \). For simplicity, we assume \( M = N = 2 \). For comparison, two benchmark schemes are considered: (1) **Unaided scheme.** In such scheme, the beamforming matrices \( F_i \) and \( F_r \) are set as initial
values; (2) SVD-based alternating optimization (AO) scheme. In such scheme, we first scalarize the optimization problem by using SVD technique, then an iterative method is employed to convert the reformulated problem into two subproblems corresponding to variables $a_{rr}$ and $a_{ii}$, and solve them alternately.

Figure 2 shows the influence of SNR on the achievable rate for three schemes under variances of CE errors $\sigma^2_a = 0.05$ and $\sigma^2_a = 0.1$, $a \in \{i, rr\}$. It can be seen that the achievable rate of all schemes increases as SNR increases from 0 to 24, and for the same $\sigma^2_a$, our proposed SVD-based GP scheme always achieves the best performance compared to the Unaided scheme and SVD-based AO scheme in this range of SNR. Furthermore, it can be observed that with the same SNR, decreasing the $\sigma^2_a$ causes an increase in the value of achievable rate for all the schemes. This is because the achievable rate is a decreasing function with the variance of CE error (see (24)); a small value of $\sigma^2_a$ leads to a high achievable rate. Besides, no matter $\sigma^2_a$ is 0.05 or 0.1, our proposed scheme still performs much better than the benchmark schemes for the same SNR.

Figure 3 shows the effect of the source transmit power on the achievable rate under different EH ratios, that is, $\alpha = 0.3$ and $\alpha = 0.7$, for three schemes. From Figure 3, we can see that the achievable rate of all schemes has an obvious increase with the increasing source transmit power under the same $\alpha$, and our proposed SVD-based GP one obtains the best performance compared to other benchmarks. Besides, decreasing $\alpha$ improves the system performance. This is because a lower value of $\alpha$ means more power can be used for information transmission at $R$ (see (10)), and the achievable rate is a decreasing function with $\alpha$ (see (24)); a small $\alpha$ results in a large achievable rate. Besides, the comparison results under the same source transmit power still indicate the superiority of our proposed scheme with respect to other benchmarks.

In order to evaluate the convergence of the proposed algorithm, the behaviours of the achievable rate versus number of iterations are shown in Figure 4. From Figure 4, we can observe that the proposed scheme converges very fast and it exhibits higher performance than the SVD-based AO scheme for different SNRs when the curve converges. We compare the performance of the three schemes with respect to the number of antennas in Figure 5.

From Figure 5, the achievable rate has a noticeable growth as the number of antennas increases for all schemes. That is because when the number of antennas increases, more antennas can be applied to suppress multipath fading with antenna diversity, which can raise the system capacity and improve the performance [40]. It is also observed that the value of achievable rate of the proposed scheme over the others are much more...
FIGURE 5 Achievable rate versus SNR for three schemes with different numbers of antennas. Thereinto, $\sigma^2_a$ are set to be 0.05, $\alpha$ is set to be 0.5.

FIGURE 6 Achievable rate versus SNR for two schemes under various CSI qualities with $N = M = 4$.

better with a larger number of antennas. For example, when $\Delta SNR = 24$ dB and $N = M = 8$, the proposed scheme increases the achievable rate by 25.8% compared with the SVD-based AO scheme counterpart, while when $N = M = 4$, the increment is 24.2% at the same SNR.

Figure 6 shows the influence of SNR to achievable rate for the two schemes under various CSI qualities. It is obvious that, the performance for two schemes increases by increasing SNR value, the performance of the proposed scheme achieved with perfect CSI is the best, and imperfect CSI can cause performance degradation in system achievable rate. Meanwhile for the same SNR, as $\sigma^2_a$ decreases, the gap between the achievable rate achieved with perfect CSI and imperfect CSI becomes smaller. For instance, at $\Delta SNR = 12$ dB, the gap corresponding to $\sigma^2_a = 0.1$ is 2.59, and corresponding to $\sigma^2_a = 0.05$ is 2.21.

7 | CONCLUSION

This study established the joint optimization problem for source and relay beamforming to maximize the achievable rate in MIMO FD SWIPT IoT system with imperfect CSI, where the relay uses TSR protocol to realize energy harvesting. To solve the non-convex problem efficiently, we proposed a SVD-based GP algorithm, and we investigated the general expression of the derivation of GP algorithm in the scenario that $N \geq M$. The numerical results demonstrated that our proposed algorithm outperforms other benchmarks at different SNR, variances of CE errors, source transmit power and EH ratio. In future work, we plan to extend the model for multi-user and multi-relay system, and investigate various network parameters on system performance, such as the number of users and the number of relay nodes. Besides, we will introduce two-way relay technique into MIMO FD SWIPT system to enhance transmission efficiency and improve network throughput.

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