LITHIUM DEPLETION IN THE SUN: A STUDY OF MIXING BASED ON HYDRODYNAMICAL SIMULATIONS

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Abstract. Based on radiation hydrodynamics modeling of stellar convection zones, a diffusion scheme has been devised describing the downward penetration of convective motions beyond the Schwarzschild boundary (overshoot) into the radiative interior. This scheme of exponential diffusive overshoot has already been successfully applied to AGB stars. Here we present an application to the Sun in order to determine the time scale and depth extent of this additional mixing, i.e. diffusive overshoot at the base of the convective envelope. We calculated the associated destruction of lithium during the evolution towards and on the main-sequence. We found that the slow-mixing processes induced by the diffusive overshoot may lead to a substantial depletion of lithium during the Sun’s main-sequence evolution.

1. Introduction
Since lithium is destroyed already at temperatures of $2.5 \cdot 10^6$ K by nuclear burning in stellar interiors, its surface abundances can be considerably affected by a sufficiently deep reaching surface convection zone. The solar Li problem is the long-standing conflict between the observed photospheric Li depletion of the Sun by 2.15 dex Anders & Grevesse, 1989 and the predictions of stellar evolution models based on the standard mixing-length prescription. The latter show only moderate Li depletion during the pre main-sequence (PMS) phase (0.3–0.5 dex) whereas the depletion during the main-sequence evolution is negligible. In contrast, observations of open clusters indicate that effective Li depletion takes place on the main-sequence. Consequently, in order to account for the observations, at least one additional mixing mechanism must operate in the radiative regions below the bottom of the surface convection zone.

Suggested solutions include mass loss to expose depleted matter from the interior (e.g. Schramm et al., 1990), microscopic diffusion leading to a leakage of Li out of the surface convection zone (e.g. Michaud, 1986), mixing due to internal gravity waves arising from pressure fluctuations in convective flows (e.g. Press, 1981), rota-
tionally induced mixing by meridional circulation, and rotationally induced mixing due to shear instabilities associated with differential rotation (e.g. Zahn, 1992; or Pinsonneault et al., 1992; Deliyannis & Pinsonneault, 1997 (“Yale” models)).

Among G and K stars in general, the current situation appears still controversial. For example, Stephens et al. (1997) conclude that the combined evidence of Li and Be in G and K stars rules out all but the “Yale” models. On the other hand, Martin & Claret (1996) criticize the angular momentum loss law adopted in these models, and find that rotation inhibits, rather than enforces, depletion.

For more details on the question of Li depletion see, e.g., Pinsonneault (1997) or Chaboyer (1998) and references therein.

2. Our approach to the solar Li problem

Based on radiation hydrodynamics modeling of stellar convection zones, a diffusion scheme has been devised describing the mixing process due to the downward penetration of convective flows into the radiative interior Freytag et al., 1996.

Freytag et al. (1996) have investigated the interface between stellar surface convection zones and the radiative interior in favourable cases where modeling of the entire convection zone and adjacent overshoot region has been possible: white dwarfs and A-type main-sequence stars. Below the classical overshoot layers (characterized by a well defined (anti)correlation between velocity and temperature fluctuations), the numerical models show an extended region where the rms velocity fluctuations decrease exponentially with depth. The existence of this low-amplitude velocity field is in the end simply a consequence of the conservation of mass. Randomly modulated by deep-reaching plumes, the resulting flow gives rise to diffusive mixing without significant temperature perturbations. The nature of the underlying velocity field is fundamentally different from that of propagating gravity waves: while the latter represent oscillating motions with amplitudes increasing with depth, the former are the extension of closed convective flows decaying into the stable layers (for details see Freytag et al. 1996). Extended overshoot leads to slow mixing of a total mass that can exceed that of the convection zone proper by a large factor. It is much more efficient than microscopic diffusion, but otherwise similar. The corresponding depth-dependent diffusion coefficient can be derived from the hydrodynamical models and expressed in terms of an efficiency parameter $f$, the ratio of the scale heights of rms velocity and pressure.

Fig. 1 illustrates the corresponding hydrodynamical simulation of the shallow convection zones of an A-type star. Two convective plumes penetrate deeply into radiative regions: The Schwarzschild border of convective instability is located at an height of $\approx -6000\, \text{km}$ whereas the plumes extend down to $-8000\, \text{km}$, corresponding to about one pressure scale height. Convection carries up to 30% of the total energy flux.

In the case of the Sun we are not yet able to include the entire convective envelope in our simulation box and, thus, do not know $f$. However, we know its...
approximate value for surface convection of A-type stars, $f \approx 0.25$, and main-sequence core convection, $f = 0.02$, respectively (Freytag et al., 1996; Herwig et al., 1997). In the following, we will address the question of whether this slow mixing, with a “reasonable” choice of $f$, can be considered a viable alternative to rotational mixing for explaining the depletion of Li during the main-sequence phase of the Sun.

3. Application to stellar evolution calculations

The scheme of exponential diffusive overhoot has already successfully been applied in stellar evolution calculations to core and deep envelope convection Herwig et al., 1997. Introducing one single efficiency parameter, $f$, it was possible to account for the observed width of the main-sequence as well as for important properties of AGB stars, namely efficient dredge-up processes to produce carbon stars at low luminosities as required by observations, and the formation of $^{13}\text{C}$ within the intershell region during the thermal pulses as the neutron source ($^{13}\text{C}(\alpha,\text{n})^{16}\text{O}$) for the $s$-process. Since these calculations deal with the very deep stellar interior, $f$ was found to be considerably smaller than for the shallow surface convection of A-type stars, namely 0.02, in accordance with the corresponding quasi-adiabatic conditions which allow only small growth rates for convective perturbations. Note, that only with the inclusion of additional mixing processes dredge up was obtained,
and that sufficient amounts of $^{13}$C are formed only due to slow mixing schemes. For more details, see Herwig et al. (1997) or Blöcker (1998).

Abundance changes due to nuclear burning (nuc) and mixing (mix) are calculated according to

$$\frac{dX_i}{dt} = \left(\frac{\partial X_i}{\partial t}\right)_{\text{nuc}} + \frac{\partial}{\partial m_r} \left[ \left(4\pi r^2 \rho\right)^2 D \frac{\partial X_i}{\partial m_r}\right]_{\text{mix}}$$

with $X_i$ being the mass fraction of the respective element, $m_r$ the mass coordinate, and $D$ the diffusion coefficient. Nuclear burning is treated with a detailed nucleosynthesis network. The choice of $D$ depends on the mixing model. Within convectively unstable regions according to the Schwarzschild criterion we follow Langer et al. (1985) and adopt $D_{\text{conv}} = 1/3 v_c l$ with $l$ being the mixing length and $v_c$ the average velocity of the convective elements according to the mixing length theory Böhm-Vitense, 1958. The depth-dependent diffusion coefficient of the extended overshoot regions is given by Freytag et al. (1996):

$$D_{\text{over}} = D_0 \exp \left(-\frac{2z}{H_v}\right), \quad H_v = f \cdot H_p,$$

where $z$ denotes the distance from the edge of the convective zone ($z = |r_{\text{edge}} - r|$ with $r$: radius), and $H_v$ is the velocity scale height of the overshooting convective elements at $r_{\text{edge}}$, given as a fraction of the pressure scale height $H_p$. Consequently, $f$ expresses the efficiency of the mixing process. For $D_0$ we take the value of $D_{\text{conv}}$ near the convective boundary $r_{\text{edge}}$. Note that $D_0$ is well defined because $D_{\text{conv}}$ drops almost discontinuously at $r_{\text{edge}}$.

4. Model Calculations

Solar models were evolved from the birthline in the HRD through the pre-main-sequence (PMS) and main-sequence phase up to an age of 10 Gyr. The calculations are based on the code described by Blöcker (1995) and Herwig et al. (1997). We use the most recent opacities of Iglesias et al. (1992) and Iglesias & Rogers (1996) complemented with the low-temperature tables of Alexander & Ferguson (1994). With an initial composition of $(Y, Z) = (0.277, 0.02)$, we get a mixing length parameter of $\alpha = 1.66$ to fit solar radius and luminosity at $t = 4.6$ Gyr. At the solar age, the depth of the convection zone is $0.282 R_\odot$, a value slightly lower than the currently adopted helioseismic value of $0.287 R_\odot$.

Convection zones in PMS models are deep-reaching and massive. Fig. 2 illustrates that during the PMS evolution the depth of the convection zone changes rapidly in mass (compared to the time scales of the main-sequence evolution). We cannot assume that $f$ is constant during this phase and that it has the same value as on the main-sequence. Thus, an initial ZAMS model was generated by evolving a PMS model with properly adjusted (mixing) parameters to fit the ‘observed’ Li depletion of 0.3 dex in accordance with results from young open clusters (Jones et al., 1997).
The structural and nuclear evolution was calculated for a total of ten main-sequence $1 \, M_\odot$ models, each with a fixed value of $f$ ranging between $f = 0.02$ and 0.31. The dependence of structural properties of the models on $f$ was found to be negligible. Apart from taking into account mixing, these are standard models and do not include any effects of rotation, microscopic diffusion, internal gravity waves, accretion, magnetic fields, or mass loss.

5. Results and discussion

Fig. 2 shows the time evolution of the mass of the convection zone in a $1 \, M_\odot$ model during the pre-main-sequence (PMS) and main-sequence evolution. Each vertical dash-dotted line corresponds to one stellar model (for most of the time the models are so closely spaced that a continuous band appears). The deep-reaching convection of the PMS evolution corresponds to a track in the HRD starting at the Hayashi limit. There is a well-defined transition to the shallow, low-mass surface convection zone of the main-sequence star ($\log t \approx 7.5$).

Fig. 3 shows the predicted depletion of lithium during the main-sequence life of a $1 \, M_\odot$ star (solid lines). The set of solid curves corresponds to different values of the mixing efficiency parameter, $f = 0.02, 0.05, 0.06, 0.07, 0.08, 0.10, 0.15, 0.20, 0.26$ and 0.31. (from top to bottom). The present solar value, -2.15, is represented.
by a diamond, and is close to the $f = 0.07$ curve. The PMS value has been adjusted to -0.3 dex according to observations of young open clusters (Sect. 4).

Fig. 4 illustrates the steady dredge-up of $^3$He, an intermediate product of the p-p chain that has accumulated around $m \approx 0.6 M_\odot$ during the first few Gyr of main-sequence evolution. The different curves have the same meaning as in Fig. 3, but in this case $f$ increases upwards. The ZAMS value of $^3$He is the sum of primordial $^3$He and D, the latter being converted into $^3$He during D burning in the PMS phase.

Bochsler et al. (1990) have used solar-wind data to constrain the enrichment of photospheric $^3$He from the ZAMS to the present. They derive an upper limit of 10 to 20% and emphasize that this isotope is a sensitive tracer for mixing processes. The amount of mixing predicted by our $f = 0.07$ model (which reproduces the observed Li depletion) is less than 1%, compatible with their upper limit. We notice that a 10% increase of $^3$He would imply a mixing efficiency of $f \approx 0.2$ (see Fig. 4) which, according to our model, leads to total destruction of Li.

6. Conclusions

We have shown that slow mixing, a diffusion process related to extended convective overshoot that operates in the almost radiative layers underneath a surface convection zone, may lead to substantial depletion of lithium during the main-sequence evolution of a $1 M_\odot$ star. The mixing efficiency parameter required to reproduce the observed depletion of lithium in the Sun, $f \approx 0.07$, is intermediate between the parameter range of $0.25 \pm 0.05$ inferred by Freytag et al. (1996) from hydrodynamical models of the shallow surface convection of main-sequence A stars and the value of $f = 0.02$ derived empirically by Herwig et al. (1997) from stellar evolution calculations for core and deep envelope convection.

We believe that the existence of an exponentially decaying velocity field below a surface convection zone is a general feature of overshoot. Although the results of the simulations for A-type stars can certainly not be readily applied to the solar case, the basic situation seems not entirely different: as in A-type stars, a substantial fraction of the total flux is in fact carried by radiation in the lower part of the solar convection zone. In this study we have attributed the depletion of Li exclusively to extended overshoot. If other mixing processes should prove to contribute as well, the efficiency of overshoot will be smaller than derived here, i.e. $f < 0.07$. However, in view of the success of the simple mixing model presented here we feel that the potential importance of slow mixing in the context of stellar convection deserves further study.

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