Experimental quantum end-to-end learning on a superconducting processor

Xiaoxuan Pan1,6, Xi Cao2,6, Weiting Wang1, Ziyue Hua1, Weizhou Cai1, Xuegang Li1, Haiyan Wang1, Jiaqi Hu7, Yipu Song1, Dong-Ling Deng1,3,4, Chang-Ling Zou1,5, Re-Bing Wu2,4,5 and Luyan Sun1,4,5

Machine learning can be enhanced by a quantum computer via its inherent quantum parallelism. In the pursuit of quantum advantages for machine learning with noisy intermediate-scale quantum devices, it was proposed that the learning model can be designed in an end-to-end fashion, i.e., the quantum ansatz is parameterized by directly manipulable control pulses without circuit design and compilation. Such gate-free models are hardware friendly and can fully exploit limited quantum resources. Here, we report the experimental realization of quantum end-to-end machine learning on a superconducting processor. The trained model can achieve 98% recognition accuracy for two handwritten digits (via two qubits) and 89% for four digits (via three qubits) in the MNIST (Mixed National Institute of Standards and Technology) database. The experimental results exhibit the great potential of quantum end-to-end learning for resolving complex real-world tasks when more qubits are available.

RESULTS

Preliminaries

The basic idea of end-to-end quantum learning is to parameterize the quantum ansatz by physical control pulses that are usually applied to implement abstract quantum gates in variational quantum classifiers. In this way, a feedforward QNN can be constructed by the control-driven evolution of the quantum state.
In the k-th iteration, a randomly selected image of a handwritten digit in the MNIST dataset is converted to a vector \( x^{(k)} \) and then transformed by a matrix \( W^{(k)} \) to the control variables \( \theta^{(k)}_m \) that steer the quantum state to \( |\psi^{(k)}(t_f)\rangle \) of the qubits in the QNN. This process encodes \( x^{(k)} \) to \( |\psi^{(k)}\rangle \). Subsequent inference control pulses \( \theta^{(k)}_m \) are applied to drive \( |\psi^{(k)}(t_j)\rangle \) to \( |\psi^{(k)}(t_{f-j})\rangle \) that is to be measured. The parameters in \( W^{(k)} \) and \( \theta^{(k)}_m \) are updated for the next iteration according to the loss function \( \mathcal{L} \) and its gradient obtained from the measurement. The circled numbers represent the specific points in the data flow and the corresponding learning performances are shown in Fig. 4. The top right is the false-colored optical image of the six-qubit device used in our experiment.

In the experiment, we select a batch of \( b \) samples in each iteration to reduce the fluctuation of \( \mathcal{L} \) for faster convergence of the learning process. The gradient of the loss function \( \mathcal{L} \) with respect to the encoding control \( \theta^{(k)}_m \) and the inference control \( \theta^{(k)}_m \) can be evaluated with the finite difference method by making a small change of the each control parameter \( \theta^{(k)}_m \). The gradient of \( \mathcal{L} \) with respect to \( W^{(k)} \) can be derived from the gradient of \( \mathcal{L} \) with respect to \( \theta^{(k)}_m \) by minimizing \( \mathcal{L} \) on the training dataset \( D \) (see Methods for details of the algorithms). Once the model is well trained, one can use fresh samples from a testing dataset to examine the recognition performance of the handwritten digits.

**Experiments and simulations**

The end-to-end model is demonstrated in a superconducting processor, as shown in Fig. 1. All qubits take the form of the flux-tunable Xmon geometry and are driven with inductively coupled flux bias lines and capacitively coupled RF control lines. Among the six qubits, \( Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 \) are dispersively coupled to a half-wavelength coplanar cavity \( B_3 \) and \( Q_2, Q_3, Q_6 \) are dispersively coupled to another cavity \( B_2 \). Each qubit is dispersively coupled to a quarter-wavelength readout resonator for a high-fidelity single-shot readout and all the resonators are coupled to a common transmission line for multiplexed readouts (see details of the experimental setup in Supplementary Notes I–III). The qubits that are not relevant to the QNN is biased far away and can be ignored from the system Hamiltonian, therefore, the static Hamiltonian of the QNN can be written in the interaction picture as

\[
H_0/h = \sum_{q=p} J_{qp} \left( a_q^p a_p + a_p^q a_q \right) - \sum_{i=1}^M E_{C,q} a_q^p a_q a_q^i,
\]

where \( J_{qp} \) is the coupling strength between the \( p \)-th and \( q \)-th qubits mediated by the bus cavity, \( E_{C,q} \) denotes the qubit anharmonicity, and \( a_q \) is the annihilation operator of the \( q \)-th qubit.

Throughout this work, we set the encoding block with \( E = 2 \) layers followed by an inference block with \( L = 2 \) layers. As shown in Fig. 1, for the \( q \)-th qubit in the \( n \)-th (\( n = 1, 2, 3, 4 \)) layer of the QNN, there are \( c = 2 \) control parameters \( \theta_{m,n-1}(t_i) \) and \( \theta_{m,n}(t_i) \), which are associated with the control Hamiltonians \( H_{2q-1} = (a_q + a_q^+)/2 \) (rotation along the \( x \)-axis of the Bloch sphere).
$H_2 = i(\alpha_2 - \alpha_1^2)/2$ (rotation along the $y$-axis of the Bloch sphere), respectively. The control parameters are the variable amplitudes of the Gaussian envelopes of two resonant microwave sub-pulses, each of which has a fixed width of $4\sigma = 40\ \text{ns}$. All the quantum controls in the same time interval are exerted simultaneously. For an $N$-digit classification task, we take $M = \log_2 N + 1$ qubits for the QNN; the classification results are mapped to the computation bases of the first $\log_2 N$ qubits (label qubits) by the majority vote of the collective measurement performed on label qubits, while one additional qubit is introduced for a better expressibility of the model. Therefore, the QNN in our experiment involves totally $M(E + \hat{n}) = 8M$ control parameters.

We perform the 2-digit (‘0’ and ‘2’) classification task ($N = 2$) with $Q_1$ and $Q_2$ ($M = 2$). The working frequencies are 6.08 GHz and 6.45 GHz, respectively, which are also the flux sweet-spots of the two qubits. The effective coupling strength $J_{2q}/2\pi = 4.11\ \text{MHz}$. We take $Q_3$ as the label qubit and assign the classification result to be ‘0’ or ‘2’ if the respective probability of measuring $|g\rangle$ or $|e\rangle$ state is larger.

The end-to-end model is initialized with $W = W_0$ and $\theta_i = \theta_0$, where all elements of $W_0$ are $10^{-5}$ and each element of $\theta_0$ is tuned to induce a $\pi/4$ rotation of the respective qubit. The parameter update is realized as follows. Firstly, we obtain the loss function $\mathcal{L}$ according to Eq. (3) by measuring $Q_3$. We perturb each control parameter in the control set ($\theta_{i_0}$, $\theta_0$) and obtain the corresponding gradient of $\mathcal{L}$. The $\mathcal{L}$ and its gradient averaged over a batch of two training samples ($b = 2$) are sent to a classical Adam optimizer\cite{Adam} for updating $W$ and $\theta_{i_0}$. All control parameters are linearly scaled to the digital-to-analog converter level of a Tektronix arbitrary waveform generator 70002A, working with a sampling rate of 25 GHz, to generate the resonant RF pulses directly. The control pulses comprised of in-phase and quadrature components are sent to each qubit with the corresponding RF control line. To obtain the classification result, we repeat the procedure and measure the label qubit for 5000 times.

In the 4-digit (‘0’, ‘2’, ‘7’, and ‘9’) classification task ($N = 4$), we take $Q_1$, $Q_5$, and $Q_6$ ($M = 3$) to construct the QNN, whose working frequencies are 6.08 GHz, 6.45 GHz, and 6.19 GHz, respectively. $Q_1$ and $Q_3$ are measured for the classification output. The target digits correspond to the four computational bases spanned by the two label qubits. The training procedure and algorithms are the same as those for the $N = 2$ task.

The typical training process is shown in Fig. 2a, b. For better clarity, the curves are smoothed out by averaging each data point from its neighboring four ones. For the 2-digit (4-digit) classification task, the experimental loss function $\mathcal{L}$ converges to 0.14 (0.22) in 300 (500) iterations. The training loss can potentially be reduced from its neighboring four ones. For the 2-digit (4-digit) classification task, as shown in Fig. 3, the average confidence $1 - \mathcal{L}$ varies little with $\tau$ when $T_1$ or $T_2$ is sufficiently small because the coherent control is overwhelmed by the strong decoherence. For larger $T_1$ or $T_2$ (e.g., $T_1 = 20\ \text{ms}$), the average confidence initially increases with $\tau$, but then decreases after reaching the peak. This trend clearly indicates the trade-off between the gained entanglement and the lost coherence, and thus $\tau$ as well as the number of layers should be optimally chosen for the best balance.

![Fig. 2](image)

**Fig. 2** Results of training the quantum end-to-end model. a, b The typical training process of the end-to-end model. For better clarity, all data points are averaged over the neighboring four points. c–f The classification performance of the trained model. The horizontal labels show the digits to be classified, while the vertical labels show the majority vote of the computational basis measurement results. The hollow bars (nearly invisible) in the experimental results (c, e) correspond to the standard deviation of multiple repeated measurements. c, d Experimental and simulation results for the 2-digit classification task, respectively. The averaged accuracies for the classification are 0.986 ± 0.001 and 0.982 in the experiment and the simulation, respectively. e, f The 4-digit classification of the QNN. The averaged accuracies are 0.894 ± 0.015 and 0.889 in the experiment and the simulation, respectively.

![Fig. 3](image)

**Fig. 3** The error analysis of the average confidence. a The simulated average confidence as a function of the qubit relaxation time $T_1$ and the pulse length $\tau$, while the qubit pure dephasing time $T_2$ is infinite; b The simulated average confidence as a function of $T_2$ and $\tau$, while $T_1$ is infinite.
The end-to-end learning scheme provides a seamless combination of quantum and classical computers through the joint training of the pulse-based QNN and the classical data encoder $W$. To understand their respective roles in the classification, we check how the data distribution varies along the flow $x \rightarrow \bm{\theta}_{\text{ENC}} \rightarrow \psi(t) \rightarrow \psi(t_{i+1}) \rightarrow y$ (see $\Diamond \sim \Diamond$ in Fig. 1) in the 2-digit classification process. To facilitate the analysis, we use the Linear Discriminant Analysis (LDA)\(^\text{28}\) that projects high-dimensional data vectors into two clusters of points distributed on an optimally chosen line (see details in the Supplementary Notes: Linear Discriminant Analysis). The LDA makes it easier to visualize and compare data distributions whose dimensionalities are different.

The projected clusters are plotted in Fig. 4. In each sub-figure, the distance between the centers of the two clusters is normalized, and hence we can quantify the classifiability by their standard deviations (i.e., the narrowness of distribution). As can be seen in Fig. 4a, b, the classical data encoder $W$ effectively reduces the original 784-dimensional vector $x$ to a 8-dimensional vector of control variables $\theta_{\text{ENC}}$, but the standard deviation is increased from 0.1658 for the original dataset to 0.2903 for the transformed control pulses. Then, the control-to-state mapping, which is both non-linear and quantum, sharply reduces the standard deviation to 0.0919 for the encoded quantum state (Fig. 4c), while the following quantum inference block does not make further improvement (Fig. 4d). These results indicate that the classical data encoder is responsible for the compression of the high-dimensional input data, while the classification is mainly accomplished by the QNN.

**DISCUSSION**

To conclude, our proof-of-principle experiment has clearly demonstrated the feasibility of the end-to-end quantum machine learning framework. The pulse-based QNN is experimentally easy to implement and scale up. Through joint training of the classical encoder and the QNN, high-precision classification is achieved for MNIST digits without downsizing the dataset. Our experiment indicates that the limited quantum resources on NISQ devices can be more efficiently exploited than pure gate-based quantum models.

It should be noted that no quantum advantage is claimed here over classical ML algorithms, which is still an ongoing pursuit for all types of variational quantum algorithms. When more qubits are available and the noise level is sufficiently low, quantum advantage may be approached owing to the enhanced expressive power by the exponentially scaled quantum Hilbert space. We expect that, with more elaborately designed training algorithms, the framework of quantum end-to-end learning can be applied to more complicated real-world ML applications (e.g., unsupervised and generative learning).

**METHODS**

**Algorithm for the classical optimizer**

The algorithm for evaluating the loss function and the calculation of the gradient is shown in Algorithm 1. The Adam optimizer that is used in our experiment is shown in Algorithm 2.

**Algorithm 1.** Calculate the loss function and the gradients with respect to the model parameters

| Input $W, \theta_{\text{ENC}}$, Batch of training data $\{x^{(k)}, y^{(k)}\}$ |
|---|
| $P_{\text{out}} g_{\text{out}} g_{\text{in}} \leftarrow 0$ |
| $\delta \in \mathbb{R}^+$ |
| for $\ell = 1 : b$ do |
| $P_0 = P_0 \left( y^{(k)}_x \vert x^{(k)}_x, W, \theta_{\text{ENC}} \right)$ |
| $\theta_{\text{ENC}} = W \ast x^{(k)}$ |
| for $i = 1 : \text{Length}(\theta_{\text{ENC}})$ do |
| $\theta_i = [\theta_{\text{ENC}}(1), \ldots, \theta_{\text{ENC}}(i) + \delta, \ldots]$ |
| $g_i(i) = \left( P(\psi(x^{(k)}_x) \vert x^{(k)}_x, \theta_1, \theta_{\text{ENC}}) - P_0 \right) / \delta$ |
| end for |
| for $i = 1 : \text{Length}(\theta_{\text{ENC}})$ do |
| $\theta_i = [\theta_{\text{ENC}}(1), \ldots, \theta_{\text{ENC}}(i) + \delta, \ldots]$ |
| $g_i(i) = \left( P(\psi(x^{(k)}_x) \vert x^{(k)}_x, W, \theta_2) - P_0 \right) / \delta$ |
| end for |
| $g_W = g_W - g_1 \ast x^{(k)}_w$ |
| $g_{\text{ENC}} = g_{\text{ENC}} - g_2 \ast b$ |
| $P_{\text{out}} = P_{\text{out}} + P_0 / b$ |
| end for |
| return $P_{\text{out}}, g_{\text{out}}, g_{\text{in}}$ |

**Algorithm 2.** Adam optimizer

| Input Training dataset $(x, y)$ |
|---|
| $s_{\text{ENC}}, r_{\text{ENC}}, r_{\text{ ENC}} \leftarrow 0$ |
| $W, \theta_{\text{ENC}} \leftarrow W_0, \theta_{\text{ENC,0}}$ |
| $lr = 10^{-5}, \beta_1 = 0.9, \beta_2 = 0.999, \epsilon = 10^{-8}$ |
| for $k = 1 : N$ do |
| $g_W, g_{\text{ENC}} = \text{Algorithm1} \{ W, \theta_{\text{ENC}}, \{ x^{(k)}, y^{(k)} \} \}$ |
| $s_w = \beta_1 \ast s_w + (1 - \beta_1) \ast g_w$ |
| $r_w = \beta_2 \ast r_w + (1 - \beta_2) \ast g_w \ast g_w$ |
| $s_i = s_i / (1 - \beta^2)$ |
| $r_i = r_w / (1 - \beta^2)$ |
| $W = W - lr \ast s_i / (\sqrt{r_i} + \epsilon)$ |
| $s_{\text{ENC}} = \beta_1 \ast s_{\text{ENC}} + (1 - \beta_1) \ast g_{\text{ENC}}$ |
| $r_{\text{ENC}} = \beta_2 \ast r_{\text{ENC}} + (1 - \beta_2) \ast g_{\text{ENC}} \ast g_{\text{ENC}}$ |
| $s_{\text{ENC}} = s_{\text{ENC}} / (1 - \beta^2)$ |
| $r_{\text{ENC}} = r_{\text{ENC}} / (1 - \beta^2)$ |
| $\theta_{\text{ENC}} = \theta_{\text{ENC}} - lr \ast s_{\text{ENC}} / (\sqrt{r_{\text{ENC}}} + \epsilon)$ |
| end for |
| return $W, \theta_{\text{ENC}}$ |
The gradient of the loss function $\mathcal{L}$ for parameter update in each iteration is obtained by averaging the gradients of the conditional probability $P$ over a batch of randomly selected input samples ($b = 2$). This can reduce the fluctuation of $\mathcal{L}$ for faster convergence in the learning process. For the inference block, the gradient $g_{\theta_n}$ of $\mathcal{L}$ with respect to each parameter in $\theta_n$ is directly obtained by rerunning the experiment with a small change in $\theta_n$ and calculating the difference of $P$. As for the encoding block, the gradient $g_W$ with respect to the elements of the classical matrix $W$ is needed. To reduce the experimental cost, we can equivalently calculate $g_W$ from the outer product between the measured gradient $g_\theta$ with respect to the encoding controls and the input data vector.

Once the gradients with respect to the model parameters are obtained, we take Adaptive Moment Estimation (Adam) \footnote{Adam} update algorithm to update the corresponding parameters. The Adam algorithm is popular for its efficiency and stability in stochastic optimization of learning problems. In the Adam algorithm, $\beta_1, \beta_2, \epsilon$ refer to algorithm configuration parameters and are chosen as default empirical values. The intermediate parameters $s_{b\theta}, s_{\theta W}, p_{\theta W}, s_\theta$ will be passed to the next iteration so as to adaptively control the parameter updating rate.

**DATA AVAILABILITY**

All data that is relevant with the figures of the article can be found in https://github.com/zuo2hu2424/Experimental-Quantum-End-to-End-Learning-on-a-SuperconductingProcessor.

**CODE AVAILABILITY**

The code used for simulations is available from the corresponding authors upon reasonable request.

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**REFERENCES**

1. Nielsen, M. A. & Chuang, I. L. Quantum Computation and Quantum Information (Cambridge Univ. Press, 2000).
2. Biamonte, J. et al. Quantum machine learning. Nature 549, 195–202 (2017).
3. Dunjko, V. & Briegel, H. J. Machine learning & artificial intelligence in the quantum domain: a review of recent progress. Rep. Prog. Phys. 81, 074001 (2018).
4. Sarma, S. D., Deng, D.-L. & Duan, L. Machine learning meets quantum physics. Phys. Today 72, 48 (2019).
5. Lloyd, S., Mohseni, M. & Rebentrost, P. Quantum principal component analysis. Nat. Phys. 10, 631–633 (2014).
6. Rebentrost, P., Mohseni, M. & Lloyd, S. Quantum support vector machine for big data classification. Phys. Rev. Lett. 113, 130503 (2014).
7. Dunjko, V., Taylor, J. M. & Briegel, H. J. Quantum-enhanced machine learning. Phys. Rev. Lett. 117, 130501 (2016).
8. Gao, X., Zhang, Z.-Y. & Duan, L.-M. A quantum machine learning algorithm based on generative models. Sci. Adv. 4, eaat9004 (2018).
9. Liu, Y., Arunachalam, S. & Temme, M. A rigorous and robust quantum speed-up in conditional probability $P$ each iteration is obtained by averaging the gradients of the encoded quantum circuit. Nat. Phys. 7, 195–203 (2011).
10. Havlíček, V. et al. Supervised learning with quantum-enhanced feature spaces. Nature 567, 209–212 (2019).
11. Schuld, M. & Killoran, N. Quantum machine learning in feature hilbert spaces. Phys. Rev. Lett. 122, 040504 (2019).
12. Gao, X., Anschuetz, E. R., Wang, S.-T., Cirac, J. I. & Lukin, M. D. Enhancing generative models via quantum correlations. Phys. Rev. X 12, 021037 (2022).
13. Du, Y., Hsieh, M.-H., Liu, T. & Tao, D. Expressive power of parameterized quantum circuits. Phys. Rev. Res. 2, 033051 (2019).
14. Benedetti, M., Lloyd, E., Sack, S. & Fiorentini, M. Parameterized quantum circuits as machine learning models. Quantum Sci. Technol. 4, 043001 (2019).
15. Schuld, M., Bochvarov, A., Svore, K. M. & Wiebe, N. Circuit-centric quantum classifiers. Phys. Rev. A 101, 032308 (2020).
16. Cong, I., Choi, S. & Lukin, M. D. Quantum convolutional neural networks. Nat. Phys. 15, 1273–1278 (2019).

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AUTHOR CONTRIBUTIONS
X.P. and X.C. are co-first authors. X.P. performed the experiment and analyzed the data with the assistance of X.C. R.B.W. proposed the experiment. R.B.W., C.L.Z., and D.L.D. provided theoretical support. L.S. directed the project. X.P., X.C., and J.H. performed the numerical simulations. W.C. fabricated the J.P.A. W.W. and X.P. designed the devices. X.P. fabricated the devices with the assistance of W.W., H.W., and Y.P.S. Z.H., W.C., and X.L. provided further experimental support. X.P., X.C., R.B.W., and L.S. wrote the manuscript with feedback from all authors.

COMPETING INTERESTS
The authors declare no competing interests.

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Correspondence and requests for materials should be addressed to Re-Bing Wu or Luyan Sun.

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