Discrete dark matter mechanism

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Abstract. We present the Discrete Dark Matter mechanism. In this scenario the stability of the dark matter arises from a spontaneous breaking of a non-Abelian group (flavor symmetry) into one of its Abelian subgroups, this in the framework of a renormalizable extension of the Standard Model. Here we present the inclusion of the quarks on this scheme promoting the quarks to transform in a non-trivial way under the flavor symmetry.

1. Introduction
Non-baryonic Dark Matter (DM) is one of the most compelling problems of modern cosmology. Despite the fact that its existence is well established by cosmological and astrophysical probes, its nature remains elusive. Still, observations can constrain the properties of dark matter and give some hints about its identity. One of the fundamental requirements for a viable dark matter candidate is the stability over cosmological times.

This stability could be due to the existence of a symmetry protecting or suppresing its decay. It has been shown recently that such symmetry can be related to the flavor structure of the Standard Model [1]. The model proposed in [1] is based on an $A_4$ symmetry with four $SU(2)$ Higgs doublets. After the electroweak symmetry breaking, the $A_4$ (even permutation of four objects) group is spontaneously broken into a $Z_2$ subgroup which is responsible for the DM stability. The leptonic sector is also extended. It consist in four right handed neutrinos and the light neutrino masses are generated through the type-I seesaw mechanism and obey an inverted hierarchy mass spectrum with $m_{\nu_3} = 0$ and vanishing reactor angle $\theta_{13} = 0$.

We have also considered models with a different matter content for the right handed neutrinos with the same DM stability mechanism but with different neutrino phenomenology [2] or a model based on the dihedral group $D_4$, where some flavour changing neutral currents are present and constrain the DM sector [3]. There are some models based on flavor symmetries but with decaying DM that have also been considered, see for instance [4, 5]. For a model with non-abelian flavor symmetries with stable DM but where the DM couples to some right handed neutrinos in a similar way to our model see [6].

2. Discrete Dark Matter Mechanism
The discrete dark matter (DDM) mechanism consists on the stability of the dark matter by means a residual $Z_N$ symmetry of a spontaneously broken discrete flavor symmetry, for instance

$^1$ In this model the DM stability is due to an extra $Z_2$ where the DM and some right handed neutrinos are charged under this symmetry.
in the original model the group of even permutations of four objects was used, $A_4$, and it was spontaneously broken by the electroweak symmetry breaking mechanism into a $Z_2$ sub group, $A_4 \rightarrow Z_2$.

The $A_4$ group has two generators: $S$, and $T$, which obey the properties $S^2 = T^3 = (ST)^3 = 1$. $S$ is a $Z_2$ generator while $T$ is a $Z_3$ generator. $A_4$ has four irreducible representations, three singlets 1, 1', and 1'' and one triplet. The generators of $A_4$ in the $S$-diagonal basis are

$$
\begin{align*}
1 & S = 1 & T = 1 \\
1' & S = 1 & T = \omega^2 \\
1'' & S = 1 & T = \omega
\end{align*}
$$

where $\omega^3 = 1$.

The model consist of an extended SM where instead of having one $SU(2)$ doublet of scalar fields, we have three of them transforming as a triplet under $A_4$, and four right handed neutrinos three of them transforming as a triplet and the other one as a singlet of the flavor group. In table 1 we present the relevant quantum numbers for the matter fields. The Yukawa Lagrangian of the model is given by

$$
\mathcal{L} = y_e L_e e H + y_\mu L_\mu\nu H + y_\tau L_\tau \nu H + \frac{y_\nu}{\mu}(N_T\eta_1) + \frac{y_\nu}{\mu}(N_T\eta_1)\nu + \frac{y_\nu}{\mu}(N_T\eta_1)\nu + \frac{y_\nu}{\mu}(N_T\eta_1)\nu + N_1 N_T N_T + M_2 N_3 N_4 + h.c. \tag{2}
$$

This way $H$ is responsible for quark and charged lepton masses, the latter automatically diagonal\(^2\). Neutrino masses arise from $H$ and $\eta$. The vacuum alignment for the scalar fields [1] is

$$
\langle H^0 \rangle = v_h \neq 0, \quad \langle \eta_1^0 \rangle = v_\eta \neq 0 \quad \langle \eta_{2,3}^0 \rangle = 0, \tag{3}
$$

which means the vev alignment for the $A_4$ triplet of the form $\langle \eta \rangle \sim (1,0,0)$. This alignment is invariant under the $S$ generator\(^3\), see eq. (1), therefore the minimum of the potential breaks spontaneously $A_4$ into a $Z_2$ subgroup generated by $S$. All the fields in the model singlets under $A_4$ are even under the residual $Z_2$, the triplets transform as:

$$
\begin{align*}
N_1 & \rightarrow +N_1, \quad \eta_1 \rightarrow +\eta_1 \\
N_2 & \rightarrow -N_2, \quad \eta_2 \rightarrow -\eta_2 \\
N_3 & \rightarrow -N_3, \quad \eta_3 \rightarrow -\eta_3
\end{align*} \tag{4}
$$

The DM candidate is the lightest particle charged under $Z_2$ i. e. the lightest combination of the scalars $\eta_2$ and $\eta_3$, which we will denote generically by $\eta_{DM}$. We list below all interactions of $\eta_{DM}$:

(i) Yukawa interactions

$$
\eta_{DM} \bar{\nu}_i N_{2,3}, \tag{5}
$$

where $i = e, \mu, \tau$.

\(^2\) For quark mixing angles generated through higher dimension operators see reference [7].

\(^3\) $H_s$ is in the 1 representation of $A_4$ and its vev also respects the generator $S$. 

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**Table 1.** Summary of relevant model quantum numbers

| $SU(2)$ | $L_e$ | $L_\mu$ | $L_\tau$ | $l^c_e$ | $l^c_\mu$ | $l^c_\tau$ | $N_T$ | $N_4$ | $H$ | $\eta$ |
|--------|-------|--------|--------|--------|--------|--------|-------|-------|-----|-----|
| $A_4$  | 1     | 1'     | 1''    | 1'     | 1''    | 1''    | 3     | 1     | 1   | 3   |

---
(ii) Higgs-Vector boson couplings

\[ \eta_{DM} \eta_{DM} ZZ, \quad \eta_{DM} \eta_{DM} W W, \quad \eta_{DM} \eta_{2,3} W^\pm Z, \quad \eta_{DM} \eta_{2,3} W^\pm, \eta_{DM} A_{2,3} Z. \]  \hspace{1cm} (6)

(iii) Scalar interactions from the Higgs potential:

\[ \eta_{DM} A_1 A_2 h, \quad \eta_{DM} A_1 A_3 h_1, \quad \eta_{DM} A_1 A_2 h_1, \quad \eta_{DM} A_1 A_3 h_1, \quad \eta_{DM} A_2 A_3 h_3, \quad \eta_{DM} h_1 h_3 h, \quad \eta_{DM} \eta_{DM} h_1 h_1. \]  \hspace{1cm} (7)

After electroweak symmetry breaking, the Higgs fields acquire vacuum (vevs) expectation values, \( v_h \) and \( v_\eta \) for the singlet and the first component of the triplet respectively, additional terms are obtained from those in Eq. (7) by replacing \( h \rightarrow v_h \) and \( h_1 \rightarrow v_\eta \). The dark matter phenomenology of this specific model has been studied in detail in [8].

3. A Discrete Dark Matter model for quarks and leptons

We were looking for a group \( G \) that contains at least two irreducible representations of dimension larger than one, namely \( r_a \) and \( r_b \). We also require that all the components of the irreducible representation \( r_a \) transform trivially under an abelian subgroup of \( G \supset Z_N \) (with \( N = 2, 3 \)) while at least one component of the irreducible representation \( r_b \) is charged with under \( Z_N \). The stability of the lightest component of the matter fields transforming as \( r_b \) is guaranteed by \( Z_N \) giving a potential 4 DM candidate.

The smallest group with this property we found is \( \Delta(54) \), isomorphic to \( (Z_3 \times Z_3) \rtimes S_3 \). This group contains four triplet irreducible representations, \( 3_{1,2,3,4} \), in addition, \( \Delta(54) \) contains four different doublets \( 2_{1,2,3,4} \) and two singlet irreducible representations, \( 1_\pm \). The product rules for the doublets are as follows:

- The product of two equal doublets

\[ 2_k \times 2_k = 1_+ + 1_- + 2_k \]  \hspace{1cm} (8)

- The product of two different doublets give us the other two doublets, for instance:

\[ 2_1 \times 2_2 = 2_3 + 2_4 \]  \hspace{1cm} (9)

Of the four doublets \( 2_1 \) is invariant under the \( P \equiv (Z_3 \times Z_3) \) subgroup of \( \Delta(54) \), while the others transform nontrivially, for example \( 2_3 \sim (\chi_1, \chi_2) \), which transforms as \( \chi_1 (\omega^2, \omega) \) and \( \chi_2 (\omega, \omega^2) \) respectively, where \( \omega^3 = 1 \). We can see that by taking \( r_a = 2_1 \) and \( r_b = 2_3 \) that \( \Delta(54) \) is a perfect choice for our purpose.

| \( L_e \) | \( L_D \) | \( e_R \) | \( l_D \) | \( H \) | \( \chi \) | \( \eta \) | \( \Delta \) |
|--------|--------|------|------|----|----|----|----|
| \( SU(2) \) | 2 | 2 | 1 | 1 | 2 | 2 | 2 | 3 |
| \( \Delta(54) \) | 1_+ | 2_1 | 1_+ | 2_1 | 1_+ | 2_1 | 2_3 | 2_1 |

Table 2. Lepton and higgs boson assignments of the model.

In this way we can choose the scalars in the model to belong to the \( 2_3 \) for instance, while the active scalars to belong to the \( 2_1 \) [9]. The relevant quantum numbers of the model are in

\footnote{Other requirements must fullfilled in order to have a viable DM candidate, such as neutrality, correct relic abundance, and consistency with constraints from DM search experiments.}
In Table 2, we fix the intrinsic neutrino CP–signs [17] as seen in Fig. 1. For neutrinoless double beta decay [12], as in Table 3.

\[
\begin{array}{cccccccc}
Q_{1,2} & Q_3 & (u_R, c_R) & t_R & d_R & s_R & b_R \\
SU(2) & 2 & 2 & 1 & 1 & 1 & 1 \\
\Delta(54) & 2_1 & 1^+ & 2_1 & 1^+ & 1^- & 1^+ \\
\end{array}
\]

Table 3. Quark gauge and flavour representation assignments.

Table 2 and 3. In Table 2, \( L_D \equiv (L_\mu, L_\tau) \) and \( l_D \equiv (\mu_R, \tau_R) \). There are five \( SU(2)_L \) doublets of Higgs scalars: the \( H \) is a singlet of \( \Delta(54) \), while \( \eta = (\eta_1, \eta_2) \sim 2_3 \) and \( \chi = (\chi_1, \chi_2) \sim 2_1 \) are doublets. In order to preserve a remnant \( P \) symmetry, the doublet \( \eta \) is not allowed to take vacuum expectation value (vev).

Let’s start discussing the quark sector. In Ref. [1, 3, 8, 2] quarks were considered blind under the flavor symmetry to guarantee the stability of the DM. Consequently the generation of quark mixing angles was difficult, see [7].

A nice feature of our current model is that with \( \Delta(54) \) we can assign quarks to the singlet and doublet representations as shown in Table 2, in such a way that we can fit the CKM mixing parameters.

The resulting up- and down-type quark mass matrices in our model are given by

\[
M_d = \begin{pmatrix} r a_d & r b_d & r d_d \\ -a_d & b_d & d_d \\ 0 & c_d & e_d \end{pmatrix}, \quad M_u = \begin{pmatrix} r a_u & b_u & d_u \\ a_u & b_u & d_u \\ c_u & r c_u & e_u \end{pmatrix}.
\]

where \( r = \langle \chi_2 \rangle / \langle \chi_1 \rangle \). Note that the Higgs fields \( H \) and \( \chi \) are common to the lepton and the quark sectors and in particular the parameter \( r \). In order to fit of all quark masses and mixings provided \( r \) lies in the range of about \( 0.1 < r < 0.2 \) [9]. We turn to the leptonic sector, where the charged leptons are basically equal to the up quarks under the flavor symmetry, so the charged lepton mass matrix is of the same form as the up quark mass matrix in Eq. (10).

The neutrino masses are generated through the type-II see-saw mechanism [10]. For that we include in the model an \( SU_L(2) \) Higgs triplet scalar field \( \Delta \sim 2_1 \). Regarding dark matter, note that the lightest \( P \)-charged particle in \( \eta_{1,2} \) can play the role of “inert” DM [11], as it has no direct couplings to matter. The conceptual link between dark matter and neutrino phenomenology arises from the fact that the DM stabilizing symmetry is a remnant of the underlying flavor symmetry which accounts for the observed pattern of oscillations. Choosing the vev alignment \( \langle \Delta \rangle \sim (1,1) \) and \( \langle \chi_1 \rangle \neq \langle \chi_2 \rangle \), consistent with the minimization of the scalar potential one finds that

\[
M_\nu \propto \begin{pmatrix} 0 & \delta & \delta \\ \delta & \alpha & 0 \\ \delta & 0 & \alpha \end{pmatrix},
\]

where \( \delta = y_a \langle \Delta \rangle, \alpha = y_b \langle \Delta \rangle \) [9]. Note that this matrix has two free parameters and gives us a neutrino mass sum rule of the form \( m_\nu^2 + m_\nu^2 = m_\nu^2 \) in the complex plane, which has implications for neutrinoless double beta decay [12], as seen in Fig. 1.

We now turn to the second prediction. For simplicity, we consider in what follows only real parameters and we fix the intrinsic neutrino CP–signs [17] as \( \eta = \text{diag}(-,+,+) \), where \( \eta \) is defined so that the CP conservation condition in the charged current weak interaction reads \( U^* = U \eta, U \) being the lepton mixing matrix. The correlations we have are presented in Fig. 2.

4. Conclusions

We have extended DDM to include the quarks in the game. We have described how spontaneous breaking of a \( \Delta(54) \) flavor symmetry can stabilize the dark matter by means of a residual
Figure 1. Effective neutrinoless double beta decay parameter $m_{ee}$ versus the lightest neutrino mass. The thick upper and lower branches correspond to the “flavor-generic” inverse (yellow) and normal (gray) hierarchy neutrino spectra, respectively. The model predictions are indicated by the green and red (darker-shaded) regions, respectively. They were obtained by taking the $3\sigma$ band on the mass squared differences. Only these sub-bands are allowed by the ∆(54) model. For comparison we give the current limit and future sensitivities on $m_{ee}$ [13, 14] and $m_\nu$ [15, 16], respectively.

Figure 2. The left figure (right figure) is the correlation for normal hierarchy (inverse hierarchy). The shaded (yellow) curved band gives the predicted correlation between solar and reactor angles when the solar and atmospheric squared mass splittings are varied within $2\sigma$ for the normal hierarchy spectrum. The solid (black) line gives the global best fit values for $\theta_{12}$ and $\theta_{13}$, along with the corresponding two-sigma bands, from Ref. [18]. The dashed lines correspond to the central values of the recent published reactor measurements [19, 20]. Note that $\theta_{23}$ is also within $2\sigma$

unbroken symmetry. In our scheme left-handed leptons as well as quarks transform nontrivially under the flavor group, with neutrino masses arising from a type-II seesaw mechanism. We have found lower bounds for neutrinoless double beta decay, even in the case of normal hierarchy, as seen in Fig. 1. In addition, we have correlations between solar and reactor angles consistent with the recent Daya-Bay and RENO reactor measurements, see Fig. 2, interesting in their own right. Direct and indirect detection prospects are similar to a generic WIMP dark matter, as provided by multi-Higgs extensions of the SM, see, for example, Ref. [8].

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