RESEARCH ARTICLE

Loading rates dependency of strength anisotropy in coal: Based on the three-dimensional reconstruction modeling technology

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Abstract
We explored the dependency of strength anisotropy on loading rate in coal both experimentally and numerically. An improved numerical modeling method was developed for this investigation to reduce the random error caused by the mechanical differences of coal specimens. After that, specimens with different anisotropy angles (the angle between the coring direction and bedding plane orientation) were processed (0°, 15°, 30°, 45°, 60°, and 90°). These specimens were then scanned by the X-ray Computed Tomography (CT) and tested by unconfined loading frame. Uniaxial compressive simulations were performed on 15 numerical models with different anisotropy angles at various loading rates ($1.0 \times 10^{-6}$, $5.0 \times 10^{-6}$, $10.0 \times 10^{-6}$, and $20.0 \times 10^{-6}$ mm/step). The results indicate that the uniaxial compressive strength (UCS) of coal generally first decreases and then increases as the anisotropy angle increases from 0° to 90° for all loading rates. It is also found that the UCS increases significantly with the increasing loading rate at anisotropic angles of 30° and 45°, while it is less obvious at anisotropic angles of 0° and 90°. This results in a reduced UCS anisotropy at high loading rates. Meanwhile, a negative exponential equation is obtained to describe the variation of UCS with the loading rate at different anisotropic angles. And a unified equation is then developed and verified to define the UCS as a function of the loading rate and anisotropic angle.

KEYWORDS
loading rate, strength anisotropy, three-dimensional reconstruction, X-ray CT
1 | INTRODUCTION

Coal is known to be an inherently anisotropic medium due to the complex distribution of microstructures formed during the sedimentary and geological process, including mineral inclusions, cleats, and random cracks.\(^1\)\(^-\)\(^5\) As a consequence, the strength of coal generally shows anisotropic features as loaded in different directions.\(^6\)\(^-\)\(^8\) Coal mining is a time and structural dependent engineering activity, where loading rates and strength anisotropy correlate with various failures in coal mass.\(^9\)\(^,\)\(^10\) Thus, unraveling the influences of loading rates on the strength of coal is significant for the roadway support,\(^11\) dynamic disaster preventing,\(^12\) and the geo-stress measurement.\(^13\)

The loading rate has a significant effect on the mechanical responses of coal and rock and has been broadly investigated in the past few decades.\(^14\)\(^-\)\(^20\) In general, loading/strain rates variation affects the initiation, propagation, and nucleation of micro-cracks,\(^21\)\(^,\)\(^22\) interferes with the primary fracture formation,\(^23\) and even alters the constitutive model of coal and rock.\(^16\) This is found to be the possible reason for the increment of the uniaxial compressive strength (UCS),\(^24\)\(^,\)\(^25\) tensile strength,\(^26\) fracture toughness,\(^21\) and elasticity modulus\(^27\) with the increasing loading/strain rate. Apart from the mechanical properties mentioned above, the strength anisotropy in coal and rock is also dependent on loading/strain rate. The anisotropy of tensile strength in granitic rocks\(^28\) and fracture toughness\(^29\)\(^,\)\(^30\) in coal are well revealed, which are found to be reduced with the increasing loading/strain rate. However, the loading/strain rate dependency of UCS in coal has only partially reported by Okubo et al.\(^8\) since only three loading directions are tested (perpendicular to the bedding plane, major cleat, and minor cleat). Therefore, the effect of loading rates on the UCS anisotropy in coal remains unanswered and need to be systematically researched.

X-ray computed tomography (CT) is a nondestructive imaging technique which is capable of providing the microstructure characterization without destructing samples.\(^31\)\(^-\)\(^35\) The usage of X-ray CT in geoscience and geo-energy engineering was also well developed in the past decade, such as the characterizations of cleats/fractures,\(^36\)\(^-\)\(^38\) minerals distribution,\(^39\)\(^,\)\(^40\) and gas sorption properties\(^37\)\(^,\)\(^41\) of coal. Three-dimensional (3-D) reconstruction modeling technology is a numerical simulation method developed based on the X-ray CT in conjunction with the numerical modeling technique.\(^7\)\(^,\)\(^31\)\(^,\)\(^42\)\(^,\)\(^43\) It offers opportunities in evaluating the microstructural deformation,\(^44\) gas and fluid flow,\(^45\)\(^-\)\(^47\) and mechanical parameter variation in coal and rock, based on natural specimens. This approach was also introduced and verified by Zhao et al.\(^48\) and considered suitable for investigating the loading-rate-dependent UCS of coal. A significant benefit brought about by this approach is the reduced amount of specimen consumption. However, improvements are still necessary in this research to make the results obtained by the 3-D reconstruction modeling technology more reliable.

Thus, to investigate the effect of loading rate on the anisotropy of UCS in coal, we first improve the previous 3-D reconstruction modeling technology. After that, we process a series of cylinder coal specimens that, respectively, have anisotropic angles of 0°, 15°, 30°, 45°, 60°, which are then scanned by X-ray CT. Numerical models with different anisotropic angles are thus created, and finally, the uniaxial compressive test is simulated under various loading rates.

2 | MODELING METHODOLOGY

In this section, the 3-D reconstruction modeling technology used in forgoing researches for the mechanical property investigation is first introduced, and improvements on it are then proposed.

2.1 | The 3-D reconstruction modeling method

In general, mineral inclusions, coal matrix, and cracks are three basic components in coal.\(^34\)\(^,\)\(^40\)\(^,\)\(^45\) Due to the difference in their densities, the gray scale of individual voxels representing them in the X-ray CT images is also different.\(^49\)\(^,\)\(^50\) Based on this feature, microstructures in coal specimens can be reconstructed and meshed by systematically stacking a series of X-ray CT images in the reconstruction and meshing software,\(^7\)\(^,\)\(^48\) and the numerical models can then be built based on these meshed models. This technique allows the investigation of the mechanical responses of these materials based on the natural specimen structures under various loading conditions.

However, due to the discrepancy in microstructures distribution in different coal and rock specimens, the ISRM (International Soc. for Rock Mechanics) recommended to use the average value obtained from at least three specimens to balance the random errors and make sure the mechanical measurements are representative. As a result, it might not be able to obtain a persuasive result by using only one randomly scanned, reconstructed, and meshed specimen model to investigate the mechanical response under various loading conditions. Thus, improvements are necessary to reduce the random error caused by the previous 3-D reconstruction modeling technique.

2.2 | Improvements

Considering the mechanical differences existing in coal specimens and the limited number of models reconstructed
by the forgoing studies due to the expensive X-ray CT scanning, two improvements are correspondingly proposed to reduce the random error and make the research result more representative.

a. Representative specimen selection

As a nondestructive seismic technique, P-wave velocity test is usually employed to characterize the mechanical properties of rocks, since the P-wave velocity is generally positively correlated with the UCS, point load strength index, and SCH Schmidt hammer rebound number. By taking advantage of this feature, specimens with an axial P-wave velocity close to the average value is chosen for the X-ray CT scanning.

b. Increasing the specimen numbers

The digital feature of two-dimensional X-ray CT images and the development of computer technology permits the volume editing of the reconstructed microstructures. Various models in different part of the scanned specimen can be obtained and meshed by systematically selecting and reconstructing different parts/areas of the X-ray CT images in the software. Thus, more numerical models with different mechanical properties can be created by one scanned specimen, which could reduce the random error encountered in the previous studies using 3-D reconstruction modeling technology.

In this study, the experimental work and numerical modeling are implemented based on these two improvements.

3 | EXPERIMENTAL WORK

Specimens used for uniaxial compressive failure tests are processed with different anisotropic angles. The P-wave velocity testing, X-ray CT scanning, and unconfined failure test are introduced in this part.

3.1 | Specimen preparation

Specimens are processed based on the block coal excavated from Wu Dong coal mine, the No. 45 coal seam, in western China. As indicated by multiple sets of measurements, the moisture content and density of the prepared coal specimens are ~1.8% and ~1.46 g/cm³, respectively. The composition of coal was investigated by the X-ray diffraction (XRD) analyzing, and the result indicates mineral presenting with the minor proportion which was majorly consisted of kaolinite (62.0%) and nacrite (26.5%).

Six anisotropic angles (as presented in Figure 1) are chosen, namely, 0°, 15°, 30°, 45°, 60°, and 90°. Fifteen specimens are cored with a diameter of 50 mm and a height to diameter ratio of 2, and they are divided into five groups with three specimens in each group, based on the recommendation of the ISRM. Before the coring process, anisotropy angles were, respectively, marked in coal blocks, and then the surfaces of coal blocks were trimmed flat to make the angle between coring direction and orientation of bedding plane consistent with the given anisotropy angle. An automatic coring machine (ZS-200, Ebang) was employed during this process, which has a coring diameter range of 25-200 mm and a maximum drilling depth of 400 mm.

3.2 | Primary wave velocity testing

The ultrasonic analyzer used here has a sensor frequency of 50 kHz, a sampling frequency of 10 MHz, and a timing precision of 0.05 μs. The P-wave velocity for each specimen and the average primary wave (P-wave) velocity of every anisotropy angle are summarized in Table 1. Specimens with axial P-wave velocity close to the average value in different groups are selected for the X-ray CT scanning. Five specimens with anisotropic angles of 0°, 30°, 45°, 60°, and 90° are scanned, respectively, with serial numbers of 3-0, 3-30", 3-45, 3-60′, and 3-90′.

**FIGURE 1** Specimens and anisotropy angles: (A) The schematic of the anisotropy angle (φ); (B) Specimens used in this research
3.3 | X-ray CT scanning

A high power micro-CT system, Nano Voxel 4000 (Sanying, China), is used for the X-ray micro-CT scanning. It has optional voltage of 225 kV, 240 kV, and 300 kV with a submicron spatial resolution (≤0.5 μm). The spatial resolution used here is 0.5 μm under the voltage of 225 kV. The X-ray micro-CT scanner and the scanning schematic are shown in Figure 2.

4 | MODELING APPROACH

In this section, finite difference model geometries are created based on the improved 3-D reconstruction modeling technique, and then the estimation of material properties and simulating conditions are described.

4.1 | Modelling process

In this research, the improved 3-D reconstruction modeling technology is illustrated in Figure 3. Three different 3-D cylinder specimens with a diameter of 10 mm and a height to ratio of 2 are, respectively, segmented from the whole reconstructed specimens. Positions of these specimens are marked as A, B, C, as shown in Figure 3B,C.

Then, these specimens are then exported to 3-Matic (Materialise Inc., France) for 3-D model reconstruction, material segmentation, and surface mesh creation, as shown in Figure 3D. During the 3-D modeling process, internal cleats, cracks, and matrix are wrapped with surface meshes. After that, triangular elements are created wrapping the solid model and the boundaries of cleats and cracks, as shown in Figure 3C,D. During the generation of the volume-filling meshes, the quality and amount of the tetrahedral elements are controlled by the maximum mesh size and the mesh quality.

The mesh confirmation and grid property resolution in these 3-D models are performed in ANSYS (ANSYS Inc., Canonsburg, USA), and then, these models are exported to FLAC3D (Itasca Inc., USA) for stress analyses, as shown in Figure 3E. Due to the limited number of models reconstructed in MIMICS, only one model was segmented, reconstructed, and meshed for one time, and three specimens were excavated from this reconstructed model based on the segmentation in its different parts.

In this study, a total of 15 finite difference geometries are, respectively, created from 5 natural specimens with anisotropic angles of 0°, 30°, 45°, 60°, and 90°.

4.2 | Material properties estimation

Mineral inclusions, coal matrix, and cleats and cracks are basic components in these numerical models, as shown in Figure 3A,B. Since cleats and cracks are automatically represented by void space, thus, only the mechanical properties of coal matrix and mineral inclusions need to be experimentally obtained.48 Here, we use the indentation hardness to estimate the mechanical properties of coal matrix and mineral inclusions, because the correlation of indentation hardness and mechanical properties in various kinds of rocks already obtained in forgoing literature57 and its applicability in mechanical properties estimation is also verified.48

In this study, a polished cuboid coal specimen with 10 mm thick is processed for the micro-indentation testing by using an MH-6 micro-indentation tester. Six indentation points are selected and tested in the regions of coal matrix and mineral inclusions under a load controlling condition, and the maximum load is 918 mN (duration at the maximum load is 5 seconds). The indentation hardness of coal matrix and mineral inclusions is recorded in Table 2.

| Anisotropy angle/° | 0   | 30  | 45  | 60  | 90  |
|-------------------|-----|-----|-----|-----|-----|
| Velocity (km/s)   |     |     |     |     |     |
| 1.31              | 1.38| 1.04| 1.17| 1.59|
| 1.69              | 0.98| 1.20| 1.21| 1.36|
| 1.25              | 1.24| 0.94| 1.35| 1.23|
| Average (km/s)    | 1.42| 1.20| 1.06| 1.24| 1.39|

TABLE 1 The axial P-wave velocity in coal specimens with different anisotropy angles

FIGURE 2 The X-ray CT scanner: (A) X-ray CT scanner; (B) the schematic of the X-ray CT scanner used in this research
Meanwhile, the mineral and the coal matrix are assumed as isotropic. The constitutive models are empirically chosen. The mineral inclusion is set as Mohr-Coulomb, while the coal matrix is chosen as strain softening in FLAC3D. Based on the correlation between the mechanical properties and the indentation hardness of the material reported by Guihua, the mechanical properties of coal matrix and mineral inclusions are thus quantitatively estimated and summarized in Table 3.

### 4.3 Boundary condition

In this study, a speed controlling loading condition is simulated. All side nodes of the numerical model are displaced freely under the uniaxial compressive condition with no lateral confinement. While the nodes at the base and top of the numerical model are laterally constrained due to friction. A series of displacement rates of $1.0 \times 10^{-6}$, $5.0 \times 10^{-6}$, $10.0 \times 10^{-6}$, $20.0 \times 10^{-6}$ mm/step are chosen and applied on the top node of the numerical model. The step used here represents the time step in the simulation.

### 5 RESULT AND DISCUSSION

The loading-rate-dependent strength anisotropy in coal is investigated in this section, the application of the numerical modeling methodology is first verified, and the correlation between the loading rate and the UCS in specimens with different anisotropic angles are then researched.

#### 5.1 Feasibility verification

It has been proved by previous studies that the 3-D reconstruction modeling technology is applicable in simulating the uniaxial compressive tests. Thus, this subsection is mainly focused on the feasibility verification on the new proposed numerical modeling approach with the improvements mentioned in Section 2.2, namely, capabilities in eliminating the random errors caused by the mechanical difference in coal specimens.

The UCS at the loading rate of $1.0 \times 10^{-6}$ mm/step is used in this verification and the measured USC with average
values are listed in Table 4. The UCS-anisotropy angle curve obtained by the average value of UCS is U-shaped, as shown in Figure 4.

The capability of the new 3-D reconstruction modeling approach in eliminating random errors is further analyzed based on the same series of the UCS and anisotropy angle data in Table 4. There are totally fifteen data points which are divided into five groups based on five anisotropy angles (0°, 30°, 45°, 60°, and 90°). There are totally 243 different UCS-anisotropy angle curves could be possibly generated if only one data point (UCS) is chosen from each group. One example of these 243 curves is illustrated in Figure 4 by the dashed lines. This randomly generated curve shows a W-shape, which is inconsistent with the one resulting from the average UCS values. Apparently, the improved 3-D reconstruction modeling technology can eliminate the random error caused by the mechanical difference in various coal specimens. Otherwise, a misleading conclusion may be obtained, since a W-shaped feature will be observed.  

Base on the two analyses above, the application of the new reconstruction method provides the opportunity of reducing the random errors and is helpful in obtaining a convincing conclusion.

### 5.2 Loading rate and strength anisotropy

The average UCS of specimens with different anisotropy angles increases with the increasing loading rate, as shown in Table 5. It gains from 18.26 MPa to 19.18 MPa as the loading rate increases from 1.0 × 10⁻⁶ to 20.0 × 10⁻⁶ mm/step. A variation coefficient is defined here as the ratio of standard deviation of UCS in specimens with different anisotropy angles and their average UCS. It is shown that this UCS variation coefficient decreases from 12.50% to 11.12% as the loading rate increases from 1.0 × 10⁻⁶ to 20.0 × 10⁻⁶ mm/step.

The UCS-anisotropic curves for specimens under different loading rates are U-shaped, as shown in Figure 5. However, the effect of loading rate on the UCS varies with the anisotropy angle. As the loading rate gains from 1.0 × 10⁻⁶ to 20.0 × 10⁻⁶ mm/step, the strength increment is significant at the anisotropy angle of 30° and 45°, respectively, with values of 1.10 and 1.14 MPa, while it is less obvious at the anisotropy angle of 0° and 90°, respectively, with an increment of 0.61 and 0.8 MPa. The increment is greater as the anisotropy angle closer to 45°. This results in a reduced anisotropy in UCS as the loading rate increases.

### 5.3 Loading rates dependent coal strength

To evaluate the strength anisotropy variation with the loading rate, the equation developed by Okubo et al. in describing the correlation between the loading rate (10⁻⁶/s-10⁻⁵/s) and the UCS is introduced, namely,

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{\epsilon_1}{\epsilon_2}\right)^{\frac{1}{d+1}}$$  \hspace{1cm} (1)

where \(\sigma_1\) is the peak strength at strain rate of \(\epsilon_1\), \(\sigma_2\) is the peak strength at strain rate of \(\epsilon_2\), \(d\) is a parameter dependent on loading rate and material properties. In order to describe the strength behavior of the coal for a wider strain rates range, Equation (1) was further modified here in the form as

$$\frac{\sigma_1}{\sigma_2} = e^{K\left(\frac{V_1}{V_2}\right)^{\frac{J}{1.5}}}$$  \hspace{1cm} (2)

where \(K\) is a material- and anisotropy-related constant, \(\sigma_1\), \(\sigma_2\), \(V_1\), \(V_2\) represent the UCS and corresponding loading rate, and \(J\) is a constant related to loading rate and material properties. Here, \(J\) was chosen as 1.5 after numerous attempts. Thus, Equation (2) can be rewritten as

$$\frac{\sigma_1}{\sigma_2} = e^{K\left(\frac{V_1}{V_2}\right)^{\frac{J}{3}}}$$  \hspace{1cm} (3)

TABLE 4 The simulated UCS at the loading rate of 1.0 × 10⁻⁶ mm/step

| Anisotropy angle (°) | 0°    | 30°   | 45°   | 60°   | 90°   |
|----------------------|-------|-------|-------|-------|-------|
| UCS (MPa)            | 19.29 | 16.67 | 15.21 | 18.27 | 21.98 |
|                      | 20.23 | 15.58 | 16.17 | 16.06 | 22.84 |
|                      | 19.13 | 16.36 | 16.92 | 17.68 | 21.41 |
| Average (MPa)        | 19.55 | 16.20 | 16.10 | 17.34 | 22.08 |

FIGURE 4 Comparison of the anisotropic angle curves, respectively, obtained by the average UCS values and a random set of UCS data.
After that, the regression analysis is, respectively, performed on data obtained from specimens with different anisotropy angles, as summarized in Table 5. The regression curves in Figure 6 fit well with the data obtained from the numerical simulations, and the correlation coefficients ($R^2$) of all fitting curves are greater than 0.94, as shown in Table 6. This confirms the applicability of Equation (3) in describing the loading-rate-dependent UCS in coal specimens with different anisotropy angles.

The parameter $K$ in Table 6 changes with the anisotropy angle. It increases significantly at the anisotropy angle of 0-30°, while less obviously at the anisotropy angle range of 30-45°, and then it reduces greatly at the anisotropy angle range of 45-90°. This indicates a greater increment of the UCS at the anisotropy angle of 30-45°, which is consistent with the UCS variation we observed in Figure 5.

5.4 Unified equations for strength anisotropy and loading rates

In order to develop constitutive models and improve safety mining designs, it is important to define a unified equation correlating the loading rate with the UCS anisotropy. Thus, the variation of $K$ with the anisotropy angle needs to be investigated.

The variation of $K$ with the anisotropy angle is plotted in Figure 7. Contrary to the trend of the UCS, the $K$-anisotropy angle curve shows an inverted U-shaped feature. After multiple attempts, it is found that the variation of $K$ may be represented as

$$K = A + B\cos^2(\phi - \phi_{min})$$  \hspace{1cm} (4)

where $A$ and $B$ are constants related to the mechanical properties of the coal, $\phi$ is the anisotropy angle, and $\phi_{min}$ is the anisotropy angle with minimal $K$ value, 37.5°.

The regression analysis is also performed based on the $K$ summarized in Table 6. The fitting result plotted in Figure 7 shows a good correlation with the data. The correlation

| Table 5 | The simulated UCS of specimens under different loading rate conditions |
|---|---|---|---|---|---|---|
| Anisotropy angle (°) | 0° | 30° | 45° | 60° | 90° |
| Average UCS (MPa) | | | | | |
| 1.0 × 10^{-6} mm/Step | 19.55 | 16.20 | 16.10 | 17.34 | 22.08 |
| 5.0 × 10^{-6} mm/Step | 19.64 | 16.31 | 16.26 | 17.41 | 22.13 |
| 10.0 × 10^{-6} mm/Step | 19.74 | 16.57 | 16.52 | 17.64 | 22.33 |
| 2.0 × 10^{-5} mm/Step | 20.18 | 17.30 | 17.24 | 18.31 | 22.88 |
| Average (MPa) | | | | | |
| Variation coefficient (%) | | | | | |

| Table 6 | Regression obtained parameters of Equation (3) in specimens with different anisotropy angles |
|---|---|---|---|---|---|
| Anisotropy angle (°) | 0 | 30 | 45 | 60 | 90 |
| $K$ | 3.65 | 7.94 | 8.37 | 6.57 | 4.33 |
| Correlation coefficient ($R^2$) | 0.96 | 0.98 | 0.94 | 0.94 | 0.96 |
coefficient is 0.710, with A and B, respectively, of 4.29 × 10⁻⁴ and 3.56 × 10⁻⁴. Combining Equation (3) and Equation (4) yields
\[
\frac{\sigma_1}{\sigma_2} = e^{(A + B\cos(\varphi - \varphi_{\min}))\left(1 - \frac{V_1}{V_2}\right)^{1.5}}
\]
(5)
where all the parameters are as defined in Equations (1)-(4). The same regression analysis is performed based on Equation (5) and the UCS data summarized in Table 5. A good correlation of Equation (5) and the UCS and anisotropy angle is observed, as shown in Figure 8, and the correlation efficient is 0.97. The A and B here, respectively, are 4.29 × 10⁻⁴ and 3.56 × 10⁻⁴. This verified the applicability of Equation (5) in presenting the UCS variation with anisotropy angle and loading rate.

Equation (5) can be used to estimate the UCS based on the loading rate and anisotropy angle, which facilitates us to have a fundamental understanding of the effects of loading rates and anisotropy on the coal strength. Based on the experimental work implemented on the same series of coal, the UCS obtained at the loading velocity of 1.0 × 10⁻⁶ mm/step in this research is similar to that of 1 mm/step at the experimental condition. In combining with the loading velocity used in this research, the applicable range of this equation should be 10⁻⁴/s-10⁻³/s.

6 | CONCLUSION

We investigated the dependency of strength anisotropy in coal on loading rate in this study. This is performed based on an improved 3-D reconstruction modeling technology, P-wave velocity tests, X-ray CT scanning, and uniaxial compressive tests on a series of coal specimens with anisotropy angles of 0°, 30°, 45°, 60°, and 90°. The conclusions are summarized as below:

a. The improved 3-D modeling method developed in this research is verified to be able to reduce the random error caused by mechanical differences in different coal specimens.
b. As loading rate increases, its effect on the coal strength anisotropy is reduced. The increment of the UCS with the increasing loading rate is significant at the anisotropy angle of 30° and 45°, while it is less obvious at the anisotropy angle of 0° and 90°.
c. A negative exponential equation is obtained to describe the variation of UCS with the loading rate at different anisotropy angles. On this basis, a unified equation is developed and verified to describe the UCS as a function of the loading rate and anisotropy angle.

In addition, this research revealed the influence of loading rate and anisotropy on the UCS, which can provide some insights on the investigation of failure mechanism in coal and rock related to loading rate and anisotropy. However, since the equations were obtained based on the numerical simulations in this study, it might be beneficial to perform more experimental works in the future for further validation and verification.

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