Design of 2D Flexible Hinge Stage with Multi-Parameter Feedback Control

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Abstract. Based on modern control principles, a method of multi-parameter feedback control for 2D flexible hinge stage is presented. The structure of the stage is worked out in order to attach the sensors, which is applied to get the feedback signal. The structure is optimized with software of Ansys, the vibration model of the stage is involved.

1. Introduction

With the development of modern science and technology, the microstructure is studied more frequently, and the positioning accuracy is becoming higher and higher. The positioning system with high precise and high resolution is often seen in modern industry and scientific research areas, such as micro-parts machining and assembly, self-adaptive optical system, fiber connection, surgeon operation, biological operation, animal gene engineering and so on. The positioning system is the key of the precise and ultra-precise machining and measuring device [1,3,7]. The splendid prospect has attracted more and more enterprise and scientific institute.

The micro-positioning system consists of three parts: mechanical body, detecting system and actuating control system. The PZT micro-positioning device has been studied deeply in every respect. Byung-Ju Yi et. al. from Korea, has developed a parallel device, which can be controlled in three dimensions of x, y and \( \phi \), and the effect of proper modeling on the positioning accuracy is given [4]. REINHARD LERCH and Klaus Prume et. al. from German has well studied the performance of PZT, and finite element analysis, simulation is completed [5,6].

The positioning accuracy of existing 2D stage is obtained through PZT control. And the micrometer and sub-micrometer measurement and machining can be achieved, but in nanometer practices, this control method cannot meet the demand. For this reason, the different factors, such as the actuator, the X/Y displacement, and X/Y direction coupling, which is concerned with the positioning accuracy and stability, is fed back to control the stage in order to promote the accuracy and stability.

2. Designing the structure of the hinge

To get the multi-parameters feedback signal, selecting the proper sensors with feedback signals inputs from each part should be put in the first place. The resistance strain sensor will be adopted in each part to get micro-position feedback signal due to its small size, light weight, convenience for setting and debugging, good dynamic characteristics. Figure 1 shows the basic structure of the hinge. However, the resistance strain sensor cannot be attached in the distortion sensitive part of the structure.

So we designed three kinds of structures of the hinge which are easy to stick the strain gauges, shown in figure 2 (a,b,c).
3. Optimized analysis of the structures of the hinge

Optimize the three kinds of the structures with the finite element method, which is easy to stick the strain gauges according to the stress situation. Orthogonal design is used in the analysis.

In all parts of the micro-positioning system, the weakest part is the center of the incision of the flexible hinge, shown in figure 3 A-A part. So the pulling stress ($\sigma$) of the weak section of the flexible hinge is [2]

$$\sigma_x = \frac{F}{bt} = \frac{E \delta^2}{4LtV'}$$

(1)

where: $F_x$—Driving force of X direction; $L$—Length of the girder; $t$—Thickness of the hinge;

$$V' = \int \frac{r \sin{\alpha}}{2r + t - 2rc \sin{\alpha}} d\alpha, (r, \alpha \text{ are in polar coordinate}).$$

The formula of the max bending stress is:

$$\sigma_2 = \frac{M}{I_1} \frac{t}{2} = \frac{\delta E}{2\beta'Lt^2}$$

(3)

So:

$$\sigma = \sigma_1 + \sigma_2 = \frac{E \delta^2}{4LtV'} + \frac{\delta E}{2\beta'Lt^2}$$

(4)

Where: $\beta' = \int \frac{r \sin{\alpha}}{(2r + t - 2rc \sin{\alpha})} d\alpha, (r, \alpha \text{ are in polar coordinate}).$

Based on the formula above, it can be found that when the thickness ($t$) of the hinge is smaller, the stress of the weak section is greater. So $t$ must be big enough, to assure the max bending stress $\sigma$ is less than the allowable stress [$\sigma$] of the material, to meet the strength requirement.
## Parameters of the hinge

| Radius(r) | Width (b) | Thickness(t) | Force (N) |
|-----------|-----------|--------------|-----------|
| 0.5       | 18        | 1            | 80        |
| 1         | 20        | 1.5          | 100       |
| 1.5       | 22        | 2            | 120       |

## Property of the material: Al 6061T6

| Elastic modulus (E(GPa)) | Poisson's ratio (δ) | Density (ρ(Kg/m³)) | Bending stress (σmax(GPa)) |
|--------------------------|---------------------|---------------------|-----------------------------|
| 69                       | 0.3269              | 2.7*10³             | 26                          |

With Orthogonal Design, the three kinds of hinges is analyzed through Ansys software, and got the stress distribution, showed in figure 4.

**Figure 4.** The stress analysis of three structure hinges.

### Table 1. The parameters of the three kinds of hinges with the Arrangement of the Orthogonal Design and Finite Element Analysis.

| A | B | C | D | \(\delta_{\text{max}}/\mu\text{m}\) | \(\varepsilon_{\text{max}}/\text{kPa}\) |
|---|---|---|---|--------------------------------|----------------------------------|
| t/mm | b/mm | r/mm | F/N | I | II | III | I | II | III |
| 1 | 0.5 | 18 | 1 | 80 | 1.2107 | 1.6035 | 11.039 | 948.89 | 838.95 | 952.64 |
| 2 | 0.5 | 20 | 1.5 | 100 | 1.4811 | 1.7974 | 13.458 | 1060.9 | 962.04 | 1079.1 |
| 3 | 0.5 | 22 | 2 | 120 | 1.9504 | 2.2224 | 16.001 | 1210.9 | 1127.2 | 1212.3 |
| 4 | 1 | 18 | 1.5 | 120 | 0.41759 | 0.50865 | 2.4405 | 348.69 | 306.67 | 366.23 |
| 5 | 1 | 20 | 2 | 80 | 0.27285 | 0.30761 | 1.5789 | 214.88 | 195.99 | 222.93 |
| 6 | 1 | 22 | 1 | 100 | 0.29658 | 0.40616 | 1.5927 | 257.45 | 204.86 | 266.05 |
| 7 | 1.5 | 18 | 2 | 100 | 0.15363 | 0.17729 | 0.69981 | 136.9 | 121.16 | 139.15 |
| 8 | 1.5 | 20 | 1 | 120 | 0.18316 | 0.25038 | 0.70552 | 159.97 | 112.48 | 165.84 |
| 9 | 1.5 | 22 | 1.5 | 80 | 0.098316 | 0.12216 | 0.43218 | 94.967 | 74.274 | 94.069 |

k₁j 4.6422 5.6233 60.498 1.7819 2.2944 14.1793 1.6904 2.6300 13.2357 1.5819 2.03327 13.05066
k₂j 0.98702 1.22242 5.6086 1.03711 1.235539 15.74242 1.99797 2.42821 16.3306 1.9313 2.38085 15.74701
k₃j 0.4351 0.50863 1.83751 2.3453 2.75072 18.02238 2.3769 2.7073 18.2797 2.5512 2.98143 19.14702
Rj 1.4024 1.691156 12.88683 0.1878 0.13576 1.2810233 0.2288 0.149064 1.44666 0.3231 0.3160533 1.5323133
k₁j 3220.69 2928.19 3044.04 1434.48 1266.78 1485.02 1366.3 1156.29 1344.53 1258.7 1109.214 1269.639
k₂j 821.02 707.52 855.21 1435.75 1270.51 1467.87 1504.5 1342.984 1539.39 1455.2 1288.06 1484.3
k₃j 391.837 370.914 390.059 1563.31 1406.33 1572.49 1562.68 1444.35 1574.38 1719.5 1546.35 1744.37
Rj 942.951 873.4253 948.327 42.946 46.518 38.133 65.457 96.02 63.2833 153.60 145.712 158.24566

\(\delta\) max 6.064326 7.39555 47.94411

\(\varepsilon\) max 4433.547 3943.624 4498.309

\(\delta\) max 5935.55 7.39555 47.94411

\(\varepsilon\) max 4433.547 3943.624 4498.309
The 1<sup>st</sup> kind of the hinge structure:

δ<sub>max</sub>:(1) From the value of range Rj, the order of the importance of each factor is: A>D>C>B;(2) In this case, the smaller the tested parameters is, the better it is; according to the min(k<sub>1j</sub>,k<sub>2j</sub>,k<sub>3j</sub>) the optimized condition can be achieved, that is: A<sub>3</sub>B<sub>1</sub>C<sub>1</sub>D<sub>1</sub>.

σ<sub>max</sub>:(1) From the value of range Rj, the order of the importance of each factor is: A>D>C>B;(2) In this case, the smaller the tested parameters is, the better it is; according to the min(k<sub>1j</sub>,k<sub>2j</sub>,k<sub>3j</sub>) the optimized condition can be achieved, that is: A<sub>3</sub>B<sub>1</sub>C<sub>3</sub>D<sub>1</sub>.

The 2<sup>nd</sup> kind of the hinge structure:

δ<sub>max</sub>:(1) From the value of range Rj, the order of the importance of each factor is: A>D>C>B;(2) In this case, the smaller the tested parameters is, the better it is; according to the min(k<sub>1j</sub>,k<sub>2j</sub>,k<sub>3j</sub>) the optimized condition can be achieved, that is: A<sub>3</sub>B<sub>1</sub>C<sub>1</sub>D<sub>1</sub>.

σ<sub>max</sub>:(1) From the value of range Rj, the order of the importance of each factor is: A>D>C>B;(2) In this case, the smaller the tested parameters is, the better it is; according to the min(k<sub>1j</sub>,k<sub>2j</sub>,k<sub>3j</sub>) the optimized condition can be achieved, that is: A<sub>3</sub>B<sub>1</sub>C<sub>1</sub>D<sub>1</sub>.

The 3<sup>rd</sup> kind of the hinge structure:

δ<sub>max</sub>:(1) From the value of range Rj, the order of the importance of each factor is: A>D>C>B;(2) In this case, the smaller the tested parameters is, the better it is; according to the min(k<sub>1j</sub>,k<sub>2j</sub>,k<sub>3j</sub>) the optimized condition can be achieved, that is: A<sub>3</sub>B<sub>1</sub>C<sub>1</sub>D<sub>1</sub>.

σ<sub>max</sub>:(1) From the value of range Rj, the order of the importance of each factor is: A>D>C>B;(2) In this case, the smaller the tested parameters is, the better it is; according to the min(k<sub>1j</sub>,k<sub>2j</sub>,k<sub>3j</sub>) the optimized condition can be achieved, that is: A<sub>3</sub>B<sub>1</sub>C<sub>1</sub>D<sub>1</sub>.

Comprehensively considering these three kinds of hinges, the 2<sup>nd</sup> kind of the hinge structure is best in the design.

4. Vibration model analysis of the two-dimensional flexible hinge

Setting the model and meshing it in Ansys, the meshed model is shown in Figure 5(a), and stress distribution shown in figure 5(b):

![Meshed model](image1)

![Stress distribution](image2)

**Figure 5.** Meshed model for Ansys and stress distribution.

To further analyze the stage according to the practice, the parameters in Table 2 are involved. Loading the forces of X and Y directions, and making the model analysis, the result of the analysis is shown in table 3.

| Radius(r) | Width (b) | Thickness (t) | Force (F) |
|----------|----------|--------------|-----------|
| 2        | 20       | 2            | 80        |

**Table 2.** The parameter of the hinge stage.

| δ<sub>max</sub>(µm) | σ<sub>max</sub>(kpa) | first-order frequency(hz) | second-order frequency(hz) | Third-order frequency(hz) |
|----------------------|----------------------|----------------------------|-----------------------------|---------------------------|
| 16.164               | 14.471               | 585.79                     | 706.54                      | 1064.6                    |

**Table 3.** The result of model analysis.
These three orders of vibration model analysis are shown in figure 6(a,b,c). The first-order frequency is corresponding to the movement in Y direction, the second-order frequency is corresponding to the movement in X direction, the third-order frequency is corresponding to the movement circumrotated in X-Y plane. The analytical result shows that, the first-order frequency and the second-order frequency are smaller than the third-order frequency, so the first and second orders are primary movement; the third-order frequency affects the dynamic performance of the structure so slightly that it can be ignored.
5. Conclusion and discussion

Considering the installation of the sensors for getting the multi-feedback signal, the structure of the hinge is devised and analyzed with the software of Ansys. The structure of the stage is shown in figure 7. From the result of the finite element analysis, the structure in figure 2(b), to which the strain gauge is attached, has the smaller stress; the first, second and third order vibration model of the stage is also simulated with Ansys. The conclusion that the third-order vibration has little effects on the accuracy and the stability is obtained, which would lay a strong basis for practicing the control system in the future.

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