1 Introduction

In a recent paper [1], a new determination of the weak charge of atomic cesium has been reported. The most precise atomic parity violating (APV) experiment compares the mixing among $S$ and $P$ states due to neutral weak interactions to an induced Stark mixing [2]. The 1.2% uncertainty on the previous measurement of the weak charge $Q_W$ was dominated by the theoretical calculations on the amount of Stark mixing and on the electronic parity violating matrix elements. In [1] the Stark mixing was measured and, incorporating new experimental data, the uncertainty in the electronic parity violating matrix elements was reduced. The new result

$$Q_W^{(133\,Cs)} = -72.06 \pm (0.28)_{\text{expt}} \pm (0.34)_{\text{theor}}$$  \hspace{1cm} (1)$$

represents a considerable improvement with respect to the previous determination [2, 3, 4]

$$Q_W^{(133\,Cs)} = -71.04 \pm (1.58)_{\text{expt}} \pm (0.88)_{\text{theor}}$$ \hspace{1cm} (2)$$

On the theoretical side, $Q_W$ can be expressed in terms of the $S$ parameter [5] or the $\epsilon_3$ [6]

$$Q_W = -72.72 \pm 0.13 - 102 \epsilon_3^{\text{rad}} + \delta_N Q_W$$ \hspace{1cm} (3)$$
including hadronic-loop uncertainty. We use here the variables $\epsilon_i$ (i=1,2,3) of ref. \cite{7}, which include the radiative corrections, in place of the set of variables $S$, $T$ and $U$ originally introduced in ref. \cite{8}. In the above definition of $Q_W$ we have explicitly included only the Standard Model (SM) contribution to the radiative corrections. New physics (that is physics beyond the SM) is represented by the term $\delta N Q_W$ including also contributions to $\epsilon_3$. Also, we have neglected a correction proportional to $\epsilon^{\text{rad}_1}$. In fact, as well known \cite{5}, due to the particular values of the number of neutrons ($N = 78$) and of protons ($Z = 55$) in cesium, the dependence on $\epsilon_1$ almost cancels out.

From the theoretical expression we see that $Q_W$ is particularly sensitive to new physics contributing to the $\epsilon_3$ parameter. This kind of new physics is severely constrained by the high energy experiments. From a recent fit \cite{9}, the value of $\epsilon_3$ from the high energy data is $\epsilon_3^{\text{expt}} = (4.19 \pm 1.0) \times 10^{-3}$.

To estimate new physics contributions to this parameter one has to subtract the SM radiative corrections, which, for $m_{\text{top}} = 175 \text{ GeV}$ and for $m_H \text{ (GeV)} = 100, 300$, are given respectively by

$$
m_H = 100 \text{ GeV} \quad \epsilon_3^{\text{rad}} = 5.110 \times 10^{-3}
$$

$$
m_H = 300 \text{ GeV} \quad \epsilon_3^{\text{rad}} = 6.115 \times 10^{-3}
$$

Therefore new physics contributing to $\epsilon_3$ cannot be larger than a few per mill. Since $\epsilon_3$ appears in $Q_W$ multiplied by a factor 102, the kind of new physics which contributes through $\epsilon_3$ cannot contribute to $Q_W$ for more than a few tenth. On the other side the discrepancy between the SM and the experimental data is given by (for a light Higgs)

$$Q_W^{\text{expt}} - Q_W^{\text{SM}} = 1.18 \pm 0.46$$

where we have added in quadrature the uncertainties. This corresponds to 2.6(2.8)-$\sigma$ deviation with respect to the SM for $m_H = 100(300) \text{ GeV}$. The 95%CL limits on $\delta N Q_W$ are

$$0.28 \leq \delta N Q_W \leq 2.08 \quad \text{for} \quad m_H = 100 \text{ GeV}$$

$$0.38 \leq \delta N Q_W \leq 2.18 \quad \text{for} \quad m_H = 300 \text{ GeV}$$

For increasing $m_H$ both bounds increase. One possible contribution to $Q_W$ which was neglected is the difference between neutron and proton spatial distributions in the nucleus. With the increasing APV measurement precision,
this effect has been reconsidered [11] adding an additional $\Delta Q_{W-p}^n = \pm 0.3$ to the theoretical error of eq. (1). Including this additional uncertainty, the theoretical error in eq. (1) becomes $\pm 0.45$ and the deviation with respect to SM for $m_H = 100$ GeV decreases from 2.6-$\sigma$ to 2.2-$\sigma$. Our analysis will be based on the result given in eq. (1). The lower positive bounds from eqs. (1) and (7) exclude the SM at 99%CL and, a fortiori, all the models leading to negative extra contribution to $Q_W$, as for example models with a sequential $Z'$ [11]. This 2.6(2.8)-$\sigma$ deviation with respect to the SM for $m_H = 100(300)$ GeV could be explained by assuming the existence of an extra $Z'$ from $E_6$ or $O(10)$ or from $Z'_{LR}$ of left-right (LR) models [1, 11, 12, 13].

2 Bounds on extra $Z'$ from $Q_W$

Let us now look at models which, at least in principle, could give rise to a sizeable modification of $Q_W$. In ref. [14] it was pointed out that models involving extra neutral vector bosons coupled to ordinary fermions can do the job. The high energy data at the $Z$ resonance strongly bound the $Z - Z'$ mixing [15]. For this reason we will assume zero mixing in the following calculations. In this case $\delta_N Q_W$ is due to the direct exchange of the $Z'$ and is completely fixed by the $Z'$ parameters:

$$\delta_N Q_W = 16 a'_f (2Z + N)v'_u + (Z + 2N)v'_d M_{Z'}^2 M_{Z'}^2$$

(8)

$a'_f, v'_f$ are the couplings $Z'$ to fermions.

We will discuss the following classes of models involving an extra $Z'$: the LR models and the extra-U(1) models. The relevant couplings of the $Z'$ to the electron and to the up and down quarks are given in the Table 1 of [11]. The different extra-U(1) models are parameterized by the angle $\theta_6$. In [11] we used a different definition of the angle. The relation between the angle $\theta_2$ of ref. [11] and $\theta_6$ is given by $\theta_6 = \theta_2 - \arctan \sqrt{5/3}$.

In the case of the LR model considered in [11] the extra contribution to the weak charge is

$$\delta_N Q_W = -\frac{M_{Z'}^2}{M_{Z'}^2} Q_{W}^{SM}$$

(9)

For this model one has a 95%CL lower bound on $M_{Z'_{LR}}$ from Tevatron [16] given by $M_{Z'_{LR}} \geq 630$ GeV. A LR model could then explain the APV data.
Figure 1 - The 95%CL lower and upper bounds for $M_{Z'}$ for the extra-U(1) models versus $\theta_6$. The solid and the dash lines correspond to $m_H = 100$ GeV and $m_H = 300$ GeV respectively. The lower bounds from direct search at Tevatron is about 600 GeV.

allowing for a mass of the $Z'_{LR}$ varying between the intersection from the 95%CL bounds $540 \leq M_{Z'_{LR}}$ (GeV) $\leq 1470$ deriving from eq. (6) and the lower bound of 630 GeV.

In the case of the extra-U(1) models the CDF experimental lower bounds for the masses vary according to the values of the parameter $\theta_6$ which parameterizes different extra-U(1) models, but in general they are about 600 GeV at 95%CL [10]. In particular for the model known in the literature as $\eta$ (or $A$), which corresponds to $\theta_6 = \arctan -\sqrt{5}/3$, the 95%CL lower bound is $M_{Z'_{\eta}} \simeq 620$ GeV, for the model $\psi$ (or $C$), which corresponds to $\theta_6 = \pi/2$, the lower bound is $M_{Z'_{\psi}} \simeq 590$ GeV and for the model $\chi$, which corresponds to $\theta_6 = 0$, the lower bound is $M_{Z'_{\chi}} \simeq 595$ GeV.

By comparing eqs. (3), (4) with eq. (5) we see that the models $\eta$ and $\psi$ are excluded. The bounds on $\delta N Q_W$ at 95%CL can be translated into lower and upper bounds on $M_{Z'}$. The result is given in Fig. 1, where the bounds are plotted versus $\theta_6$. In looking at this figure one should also remember that
Figure 2 - 95%CL lower bounds for $M_{Z'}$ for the extra-$U(1)$ models versus $\theta_6$ from a LC with $\sqrt{s} = 500 \text{ GeV}$, $L = 500 \text{ fb}^{-1}$, $P_{e-} = 0.9$, $P_{e+} = 0.6$ (solid line), $P_{e-} = 0$ (dash line), $\sqrt{s} = 300 \text{ GeV}$, $L = 300 \text{ fb}^{-1}$, $P_{e-} = 0.9$, $P_{e+} = 0.6$ (solid-dot line).

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3 $Z'$ at future colliders

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be upgraded by the future run with $\sqrt{s} = 2 \, TeV$, $L = 1 \, fb^{-1}$ to $M_{Z'} \sim 800 - 900 \, GeV$ and pushed to $\sim 1 \, TeV$ for $L = 10 \, fb^{-1}$. The bounds are based on 10 events in the $e^+e^- + \mu^+\mu^-$ channels and decays to SM final states only is assumed $^{[17]}$. At the LHC with an integrated luminosity of 100 $fb^{-1}$ one can explore a mass range up to $4 - 4.5 \, TeV$ depending on the $\theta_6$ value. Concerning LR models, the 95%CL lower limits from Tevatron run with $\sqrt{s} = 2 \, TeV$, $L = 1(10) \, fb^{-1}$ are $\sim 900(1000) \, GeV$ and extend to $\sim 4.5 \, TeV$ at LHC $^{[17]}$. Therefore if the deviation for $Q_W$ with respect to the SM prediction is not due to a statistical fluctuation but to the presence of new physics like new extra gauge bosons from $E_6$ or LR models, LHC can verify or disprove this possible evidence. However little can be learned on the $Z'$ properties. With $e^+e^-$ colliders the properties of a $Z'$ can be easily investigated if the center-of-mass energy is large enough to produce it. Anyway, from the measurements

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**Figure 3** - Same of Fig. 2 for $M_{Z'_{LR}}$ for the LR models versus $\alpha_{LR}$. 

![Figure 3](image-url)
Figure 4 - 95%CL contours for \((a'_l, v'_l)\) for \(M_{Z'} = 1, 1.5, 3\ \text{TeV}\), and \(\sqrt{s} = 300\ \text{GeV}\) and \(L = 300\ \text{fb}^{-1}\).

of \(e^+e^- \rightarrow \gamma, Z, Z' \rightarrow \bar{f}f\) below threshold one can get information about the nature of the \(Z'\). In ref. \[18\] exclusion limits for \(M_{Z'}\) have been determined from the analysis of all leptonic and hadronic observables. Here we show the upgrading of this analysis for a collider scenario \(\sqrt{s} = 500\ \text{GeV}\), \(L = 500\ \text{fb}^{-1}\) and polarized beams: \(P_{e^-} = 0.9, P_{e^+} = 0.6\) (solid line). The 95%CL lower bounds on \(M_{Z'}\) are given in Fig. 2 for different values of \(\theta_6\) for \(E_6\) models. Also shown is the case of unpolarized positron beam (dash line) and the case of \(\sqrt{s} = 300\ \text{GeV}\), \(L = 300\ \text{fb}^{-1}\) (solid-dot line). The observables which are considered are the total lepton and hadron cross sections, the forward backward asymmetry, the left-right asymmetry (for leptons, hadrons, \(c\bar{c}\) and \(b\bar{b}\) final states), \(R_b\) and \(R_c\). The assumed identification efficiencies are: for leptons \(\epsilon_l = 95\%\), for \(c\bar{c}\) \(\epsilon_c = 40\%\), for \(b\bar{b}\) \(\epsilon_b = 60\%\) and the corresponding systematic errors \(\Delta\epsilon_l/\epsilon_l = 0.5\%\), \(\Delta\epsilon_c/\epsilon_c = 1.5\%\), and \(\Delta\epsilon_b/\epsilon_b = 1\%\). An
uncertainty of 0.5\% on the luminosity and $\Delta P/P = 1\%$ are considered.

The same analysis for the LR models leads to a 95\%CL lower bound for $M_{Z_{LR}}$ shown in Fig. 3. For example the particular LR model considered in [11] corresponds to $\alpha_{LR} = \sqrt{\cot^2 \theta_W - 1}$ and for this model the bound extracted from Fig. 3 is around 7 TeV.

Assuming that the $Z'$ mass is known, one can study the $Z'$ couplings to fermions. The 95\%CL contours on $Z'\ell\bar{\ell}$ couplings for the $\chi$ model for $M_{Z'} = 1, 1.5, 3$ TeV are presented in Fig. 4 for $\sqrt{s} = 300$ GeV and $L = 300$ fb$^{-1}$ and in Fig. 5 for $\sqrt{s} = 500$ GeV and $L = 500$ fb$^{-1}$. In Fig. 6 the 95\%CL contours for $a'_b v'_b$ for $M_{Z'} = 1, 1.5$ TeV, and $\sqrt{s} = 500$ GeV and $L = 500$ fb$^{-1}$ are shown. The dash lines correspond to $P_{e^+} = 0$.

In conclusion, once a $Z'_\chi$ is seen at LHC, already a LC with $\sqrt{s} = 300$ GeV, $L = 300$ fb$^{-1}$ and polarized beams is able to measure the $Z'_\chi$
Figure 6 - 95%CL contours for \((a'_b, v'_b)\) for \(M_{Z'} = 1, 1.5 \, TeV\), and \(\sqrt{s} = 500 \, GeV\) and \(L = 500 \, fb^{-1}\). The dash lines correspond to \(P(e^+e^- = 0\).

couplings to fermions with good precision. For example for a \(Z'_\chi\) with mass of 1 \(TeV\) (which can explain the new APV result) the couplings to leptons can be determined within 10\%, unless a sign ambiguity. By increasing the center of mass energy, the precision improves.

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