Optimization of well gas rates for offshore gas-condensate field

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Abstract. Condensate recovery enhancing on offshore gas-condensate fields cannot be realized using the cycling process due to significant financial costs and technical difficulties. Such a problem could be handled by the redistribution of well gas rates. This redistribution assumes the solution the following problem: maximize the total condensate production at a given time interval then gas production requirements are met. The results of solving the problem are functions, each representing the change in the well gas rate over the time. The problem is transformed to the linear optimization model so that the solution algorithms based on the linear programming theory can be applied. Their tests showed, that these algorithms easily find the solution of the optimization problem; and the condensate recovery is greater or equal than the one obtained by the optimization procedures implemented in Eclipse. The search time is an order of magnitude less than the one required by Eclipse.

1. Introduction

A way to enhance the condensate recovery efficiency for an offshore gas-condensate field is the production optimization — the allocation of well gas rates with the aim to maximize the condensate production under the required gas production. This is equivalent to the minimization of condensate loses in the reservoir, hence the loss of reservoir energy. Therefore, the problem is stated as following: to allocate the well gas rates so, that the reservoir energy loss is minimal, and the overall gas production requirements are fulfilled at any time interval.

The optimal rate allocation is influenced by the well interference. To take this effect into account, Charny [1] introduced the interaction coefficients for a homogenous oil reservoir with a far constant pressure external boundary. Meerov et al. [2] formulated the equation describing oil well interaction in the inhomogeneous reservoir, but didn’t propose a method to evaluate the interaction coefficients. Zotov [3] proposed the system of equations for a homogeneous gas-condensate reservoir developing by a well pad and defined explicitly the interaction coefficients.

In contrast, in the current work a new approach is proposed allowing to take into account the well interference in the inhomogeneous gas-condensate reservoir. The interaction coefficients are interpreted as filtration resistance coefficients and expressed explicitly and then are used in the rate allocation algorithms. The solution algorithms are based on two hypotheses: the first one is analogous to Le Chatelier’s principle [4]; the second one uses the relation between reservoir energy and well’s bottom hole pressures. The obtained results confirm the validity of these hypotheses and showed better convergence rate compared with the numerical optimization methods implemented in the standard reservoir simulation software [5].
2. Definition of the filtration resistance coefficients

The key element of the problem statement is the relation between \( P_0 \) – the boundary pressure, \( P_i \) – the bottom-hole pressure (BHP) of the well \( i \) \((i=1, 2, \ldots, n)\) and \( q_1, q_2, \ldots, q_n \) – the well gas rates [3]:

\[
\sum_{j=1}^{n} x_{ij} q_j = P_0^2 - P_i^2, \quad i = 1, n,
\]

where \( n > 1 \), \( q_i > 0 \), \( P_0 \geq P_i > 0 \), and \( x_{ij} \geq 0 \) are parameters that take into account the filtration resistance of gas flows from the drainage area of the \( j \)-th well to the drainage area of the well \( i \).

As follows from equation (1), the increase of \( x_{ij} \) at constant flow rates and the boundary pressure leads to the decrease in BHPs. It means the increase of the reservoir energy required to move the gas-condensate mixture toward the wells. Therefore, the parameters \( x_{ij} \) are called the filtration resistance coefficients (FRC).

The approach to define FRC was presented earlier in [6]. The initial data for estimating the FRC are either the production data or the simulated data obtained by using a hydrodynamic simulator. Specifically, the \( q_i > 0 \) and \( P_i \) are the well gas rate and the BHP of the well \( i \), \( i = 1, 2, \ldots, n \), correspondently, as well as the \( P_0 \) is the boundary pressure at the time \( t \).

According the equation (1), the search for the FRC is reduced to solving the system of linear equations:

\[
\sum_{j=1}^{n} q_j x_{ij} = P_0^2 - P_i^2, \quad i = 1, n,
\]

The system (2) contains \( n \) equations and \( n^2 \) unknown variables – \( x_{ij} \) and, consequently, it has infinite number of solutions. Let’s add to the problem statement the Hypothesis I allowing obtain the unique solution to the system of equations (2): the formation provides maximum resistance to the filtration at any time.

The Hypothesis I is a version of the Le Chatelier principle [4], whereby the system provides the maximum resistance to the action disturbing its equilibrium state. This formulation distinguish from the hypothesis used earlier for FRC evaluation [6] and is all the more valid.

Considering the equation (2) and the Hypothesis I, the search for \( x_{ij} \) is reduced to solving the problem:

\[
\min_{i,j} \{ x_{ij} \} \rightarrow \max_s
\]

\[
\sum_{j=1}^{n} q_j x_{ij} = P_0^2 - P_i^2, \quad i = 1, n,
\]

\[
x_{ij} \geq 0, \quad i = 1, n, \quad j = 1, n.
\]

The problem (3) - (5) has an analytical solution – \( \{ x_{ij}^{(t)} \} \) [6]:

\[
x_{ij}^{(t)} = \Delta_j / Q_j, \quad j \neq i,
\]

\[
x_{ij}^{(t)} = \Delta_j / Q_j + (\Delta_u - \Delta_i) / q_u, \quad j = i,
\]

where

\[
Q_j = \sum_{i=1}^{n} q_i, \quad \Delta_i = \min_{1 \leq i \leq n} \{ \Delta_u \}, \quad \Delta_u = P_0^2 - P_u^2.
\]

3. The mathematical formulation of the optimal distribution of the given overall gas production rate between the wells

After evaluating the FRC, it is possible to form the model of the optimal distribution of the given overall (summarized) gas production rate between the wells \((n > 1)\) in accordance with the criterion of the minimal losses of the reservoir energy at the time \( t \) (or at the moment close to the \( t \), while the
values of FRC can be assumed as constants). In this case, the FRC are input data — \( x^{(t)}_{ij} , i = 1, 2, \ldots, n, j = 1, 2, \ldots, n \). The \( q_i \) and the \( P_i \) — the well gas rates and the BHPs at the time \( t \), respectively — become the solution variables.

Let’s introduce the parameters \( \alpha_{it} \geq 0 \):

\[
\alpha_{it} = \bigg( \frac{P_{it}^2}{P_{max}^2} -\bigg) / q_i,
\]

where \( P_{max} = \max_i \{ P_i \} \).

The last notation shows that \( \alpha_{it} = 0 \) for wells whose BHPs are equal to the \( P_{max} \), i.e. they have the maximum values.

It follows from the equations (6) - (9) that for the time \( t \), the system of equations (1) relating \( q_i \) and \( P_i \) can be reduced to the form:

\[
P_i^2 = P_0^2 - \alpha_{it} q_i - \left ( \Delta_i / Q_i \right ) \sum_{j=1}^{n} q_j, \quad i = 1, n.
\]

In accordance with the statement of the problem, let’s introduce the parameter \( Q \) — the given overall gas production rate (summarized production rate) at some \( t \), as well as the parameters \( a_i \) and \( b_i \) being respectively the minimal and the maximal limits for the gas production rate of the well \( i, i = 1, \ldots, n \). Assume that \( 0 \leq a_i < b_i < Q \).

The basis for the model of optimal distribution of the overall gas production rate between wells is the Hypothesis 2: the minimization of reservoir energy losses is equal to the maximizing well’s bottomhole pressures.

Using our notations, the Hypothesis 2, and the equation (10), the problem takes the form:

\[
\min_{i,j} \left [ P_0^2 - \alpha_{it} q_i - \left ( \Delta_i / Q_i \right ) \sum_{j=1}^{n} q_j \right ]^{1/2} \rightarrow \max q
\]

\[
\sum_{j=1}^{n} q_j \geq Q
\]

\[
a_i \leq q_i \leq b_i, i = 1, n.
\]

Note, that (i) it follows from equation (1) and the criterion (11) that the increase of the BHP of a producing well leads to the decrease of its flow rate, thus, the inequality (12) can be replaced by the equality constraint; (ii) neither \( P_0 \), nor the third summand depend on the index \( i \) in the radicand in the criterion (11); (iii) the radicand increases, its square root increases too; (iv) the product \( (\alpha_{it} q_i) \) in the radicand has the negative sign. The noted particularities of the problem (11)-(13) lead to simplification:

\[
\max_{i,j} \{ \alpha_{it} q_i \} \rightarrow \min q
\]

\[
\sum_{i=1}^{n} q_i = Q
\]

\[
a_i \leq q_i \leq b_i, i = 1, n.
\]
where $I = \{i: \alpha_i > 0\}$.

Let’s consider the case when all BHPs are equal to each other at the time $t$: $P_{it} = P_{\text{max}}$ to all $i = 1, \ldots, n$. In this case, set $\alpha_i = 1$, for $i = 1, \ldots, n$.

4. Optimization of well gas rates at a given time point

Since the original problem (14)-(16) is equivalent to the problem (17)-(20), it suffices to find a solution to the problem (17)-(20). If the set $\{q_i^{ok}, w_{it}\}$ is a permissible solution to the problem (17)-(20), then the set $\{q_i^{ok}\}$ is a feasible solution to the problem (14)-(16). If it is known that $\{q_i^{ok}\}$ is a feasible solution to the problem (14)-(16), then $\{q_i^{ok}, w_{it}\}$ is a permissible solution to the problem (17)-(20), where $w_{it}$ is defined taking into account the criterion (17) and the constraints (19) as:

$$w_{it} = \max \{\alpha_i q_i^{ok}\}.$$  
(21)

Because the problem (17)-(20) is a linear programming model, its solution can be found using, e.g., the simplex method [7]. Nevertheless, in the following an algorithm is proposed that takes into consideration the peculiarities of the problem (17)-(20), and, thus, ensures the effective application even if the original problem (14)-(16) has a high dimension.

The results of the theoretical justification of the algorithm are presented below without the proof, which is caused by the limited volume of the article.

Before proceeding to the algorithm description, let’s consider the necessary and sufficient conditions for the existence of permissible solutions to the problem (14)-(16) and the sufficient condition for their optimality.

The necessary and sufficient condition for the existence of the feasible solutions to the problem (14)-(16) is the satisfaction of the inequality:

$$\sum_{i=1}^{n} a_i \leq Q \leq \sum_{i=1}^{n} b_i.$$  

Let’s introduce $w_i$ – the lower-bound estimate of the objective function (17) for permissible solutions to the problem (17)-(20):

$$w_i = \max_{\{q_i^{ok}\}} \{\alpha_i a_i\}.$$  
(22)

The set of $\{q_i^{ok}\}$ is the optimal solution to the problem (14)-(16), if the following equality is satisfied:

$$w_{it} = w_i,$$  
(23)

where $w_{it}$ is determined by the equation (21).

We now proceed to the investigation of conditions for obtaining an analytical solution to the problem (14)-(16). Let’s introduce the notation: $\{q_i^{opt}\}$ – the optimal solution to the original problem (14)-(16), i.e. $q_i^{opt}$ are optimal well gas rates for all $i = 1, \ldots, n$. Let’s denote $\gamma_i = b_i - a_i$; $I = \{i: \alpha_i > 0\}$, $I_0 = \{1, \ldots, m\}$; $I_0 = \{i: \alpha_i = 0\}$, $I_0 = \{m+1, m+2, \ldots, n\}$; additionally the variables $G$ and $Q_1$ are calculated as

$$G = Q - \sum_{i=1}^{n} a_i; \quad Q_1 = Q - \sum_{i=1}^{n} b_i.$$  
(24)

Let’s consider cases regarding the parameters $G$ and $\gamma_i$ for $i \in I_0$.

1. If the equality

$$G = \sum_{i \in I_0} \gamma_i.$$  
(25)

is satisfied, then the optimal solution to the problem (14)-(16), $\{q_i^{opt}\}$, takes the form:

$$q_i^{opt} = \begin{cases} b_i, & i \in I_0 \\ a_i, & i \in I_i. \end{cases}$$  
(26)

2. If the inequality

$$G > \sum_{i \in I_0} \gamma_i.$$  

is satisfied, then the optimal solution to the problem (14)-(16), $\{q_i^{opt}\}$, takes the form:

$$q_i^{opt} = \begin{cases} b_i, & i \in I_0 \\ a_i, & i \in I_i. \end{cases}$$  
(26)
is satisfied, we introduce $n_0$ – the number of elements in the ensemble $I_0$, i.e. $n_0$ is the number of $\alpha_i$, that are equal to zero.

2.1. Let $\gamma_i \geq G/n_0$ be fulfilled for all $i \in I_0$. Then the optimal solution to the original problem (14)-(16) under the preceding condition (27) takes the form:

$$q_i^{\text{opt}} = \begin{cases} \frac{G}{n_0} + a_i, & i \in I_0 \\ a_i, & i \in I_1. \end{cases}$$

(28)

2.2. Let’s consider the case than there exists $i \in I_0$ for which $\gamma_i < G/n_0$, and the condition (27) is satisfied. Let’s introduce $I_1 = \{i: \gamma_i < G/n_0, i \in I_0\}$, $I_2 = \{i: \gamma_i \geq G/n_0, i \in I_0\}$, $n_1 = |I_1|$, $n_1 \leq n_0$. Note, that $I_0 = I_1 \cup I_2$ and $I_2 \neq \emptyset$, because the inequality (27) is fulfilled.

Let’s denote parameters:

$$H_i \equiv G - \sum_{i \in I} \gamma_i.$$

$$\beta_i \equiv H_i \cdot \gamma_i \cdot \left(\sum_{j \in I} \gamma_j\right)^{-1}, \quad i \in I_2.$$

Then, for this case, the optimal solution to the original problem (14)-(16) takes the form:

$$q_i^{\text{opt}} = \begin{cases} a_i, & i \in I_1 \\ b_i + a_i, & i \in I_2. \end{cases}$$

(29)

3. If the inequality

$$G > \sum_{i \in I_0} \gamma_i,$$

is satisfied, we introduce parameters:

$$\delta \equiv \sum_{i=1}^{\infty} \frac{1}{\alpha_i}; \quad w^* = \frac{Q_i}{\delta}; \quad q_i^* = \frac{w^*}{\alpha_i}, \quad i \in I_i.$$

(31)

If the set $\{q_i^*\}, \quad i \in I_i$ satisfies the constraints (16), then the optimal values of well gas rates in the problem (14)-(16) are determined by the following rule:

$$q_i^{\text{opt}} = \begin{cases} b_i, & i \in I_0 \\ q_i^*, & i \in I_i. \end{cases}$$

(32)

Therefore, the items 1÷3 define the conditions of existence an analytical solution to the original problem (14)-(16). These solutions are expressed by equations (26), (28), (29), and (32). The sufficient optimality condition (23) is fulfilled for all such solutions.

4. Let’s consider the last possible case, when the inequality (30) is satisfied, but the set $\{q_i^*\}, \quad i \in I_i$ does not satisfy the constraints (16).

Firstly, it should be noted, that and in this case, the optimal values of the well gas rates are equal to the maximal permitted values for $i \in I_0$:

$$q_i^{\text{opt}} = b_i, \quad i \in I_0.$$

The search for the optimal values of the problem variables $q_i$ is an iterative procedure for $i \in I_i$. The iterations are aimed to obtain the variables $q_i, \quad i \in I_i$, which should be close to $q_i^*, \quad i \in I_i$, and satisfy the constraints (16) at the same time, whereby the $I_i = \{1,2,\ldots,m\}$ is the same as above.
The least squares criterion is proposed as the measure of proximity to the set \( \{q_i^*\} \). Then the search of \( q_i, i \in I \), is reduced to the solving the quadratic programming problem:

\[
\sum_{i=1}^{m} (q_i - q_i^*)^2 \rightarrow \min_q
\]

\[
\sum_{i=1}^{m} q_i = Q_i
\]

\[
a_i \leq q_i \leq b_i, \quad i = 1, m,
\]

where \( q_i^* \) are determined by equations (31), and \( Q_i \) is determined by the equation (24).

The solution to the problem (33)-(35) is based on the use of Lagrangian relaxation \[8\]. This allows us to solve \( m \) minimization problems with one variable at each iteration instead of the solving the problem (33) - (35) with \( m \) unknowns, i.e. to find \( q_i \) for each \( i \in \{1,2,\ldots,m\} \):

\[
(q_i - q_i^*)^2 - \lambda^{(k)} q_i \rightarrow \min_{q_i}
\]

\[
a_i \leq q_i \leq b_i.
\]

If \( q_i(\lambda^{(k)}) \) is used to denote the solution to the \( i \)-th problem (36), (37), then

\[
q_i(\lambda^{(k)}) = \begin{cases} 
   a_i, & c_i(\lambda^{(k)}) < a_i \\
   c_i, & a_i \leq c_i(\lambda^{(k)}) \leq b_i \\
   b_i, & c_i(\lambda^{(k)}) > b_i,
\end{cases}
\]

\[
c_i(\lambda^{(k)}) = \frac{2q_i^* + \lambda^{(k)}}{2},
\]

where \( i \in \{1,2,\ldots,m\} \).

The set of optimal solutions to the problem (36)-(37) – \( \{q_i(\lambda^{(k)})\} \) is an approximation of the optimal solution to the problem (33)-(35) at iteration \( k \).

Now it is necessary to check whether the constraint (34) is satisfied for the obtained set \( \{q_i(\lambda^{(k)})\} \):

i) if

\[
\sum_{i=1}^{m} q_i(\lambda^{(k)}) = Q_i,
\]

then \( \{q_i(\lambda^{(k)})\} \) is the optimal solution to the problem (33)-(35), which closeness to the solution of the original problem (14)-(16) of can be estimated; otherwise, the following cases should be checked;

ii) if

\[
\sum_{i=1}^{m} q_i(\lambda^{(k)}) < Q_i,
\]

then the next iteration is required, i.e. update \( k \equiv k+1 \), set a new value of the penalty coefficient \( \lambda^{(k)} > \lambda^{(k)} \)

and proceed to the calculation of the new values \( q_i(\lambda^{(k)}) \) using the equations (38), (39);

iii) if

\[
\sum_{i=1}^{m} q_i(\lambda^{(k)}) > Q_i,
\]

then the next iteration is required, i.e. update \( k \equiv k+1 \), set a new value of the penalty coefficient \( \lambda^{(k)} < \lambda^{(k)} \)

and proceed to the calculation of the new values \( q_i(\lambda^{(k)}) \) by the equations (38), (39).

Let’s move on to the choice of penalty coefficients \( \lambda^{(k)} \). It follows from equations (38) and (39) that \( q_i(\lambda^{(k)}) \) are the increasing functions of \( \lambda^{(k)} \), \( i=1,2,\ldots,m \). Therefore, the left-hand side of the equation (40) is also the increasing function of \( \lambda^{(k)} \). The satisfiability of the equation (40) is equivalent to the
optimality condition for solutions to the problem (33)-(35). Therefore, the search for \( \lambda^k \) is reduced to finding the root of the equation:

\[
\sum_{i=1}^{m} q_i (\lambda^k) - Q_i = 0. \tag{41}
\]

Some numerical methods for finding the roots of a nonlinear equation with one variable can be applied to solve the equation (41) with respect to \( \lambda^k \). For instance, the dichotomy method [9] can be used. Iterations continue until the equality (40) will be fulfilled with the required accuracy:

\[
\left| \sum_{i=1}^{m} q_i (\lambda^k) - Q_i \right| / Q_i < \varepsilon, \tag{42}
\]

where \( \varepsilon > 0 \) is the required accuracy.

The set \( \{q_i(\lambda^k), i=1,2,\ldots,m\} \) becomes an approximate solution to the problem (33)-(35), when the inequality (42) is realized. In this case, the set \( \{q_i^{ok}, w_{ok}\} \) is accepted as the final solution of the problem (14)-(16).

Because the problem (17)-(20) and the problem (14)-(16) are equivalent, the parameter \( \theta \) can serve as the measure of the solution accuracy to the original problem (14)-(16).

5. Well rate optimization for the long-time production period

The described in the previous section algorithm may be applied for the prediction of the gas well rate changes in the time. Let’s consider a case, where at the initial time step \( t=0 \) the real well rates, BHPs, and the boundary pressure are known. Let’s assume that at the time \( t \in \{0,1,\ldots,T\} \) the overall production rates are given: \( Q(t), t=0,1,\ldots,T \). One has to find the well gas production rates: \( q_i(t) \), which ensure the minimal loses of the reservoir energy during the period \( [0,T] \) under the fulfillment of the constrains:

\[
\sum_{i=1}^{n} q_i(t) = Q(t), t = 0, T \tag{43}
\]

\[
a_i(t) \leq q_i(t) \leq b_i(t), i = 1,n, t = 0, T \tag{44}
\]

where \( a_i(t) \) and \( b_i(t) \) are the minimal and the maximal limits of the well gas rates for the well \( i, i=1,\ldots,n \), at the time \( t \in \{1,2,\ldots,T\} \), respectively.

The solution of the stated problem is separated into \( T+1 \) stages. At the first stage \( (t=0) \), the parameters \( a_0 \) are estimated on the basis of the real production data, and the problem (14), (43), (44) is solved at \( t=0 \). Using our notations, the set \( \{q_i^{(0)}\} \) is this solution. Then, during the time period \( [0,1] \), the development process is simulated using hydrodynamic modeling with the well production rates \( \{q_i^{(0)}\} \).

The new well production rates and BHPs for all wells will be estimated as result of simulation at the time step \( t=1 \). Well production rates \( q_i^{(0)} \) and the simulated BHPs become analogous real data. The new values of \( a_i \) are found, and, then, the problem (14), (43), (44) is solved at \( t = 1 \), etc. Such calculations are repeated until the time \( t = T \). By this mean, the solutions obtained at the previous stage become the input data for the next one.
Let’s consider the cases where the number of wells is changed during the production period. Let the well \( n \) will be shut at the moment \( t=s \). Then, at the time step \( s \) all calculation procedures are carried out taking into account the well \( n \). Based on these results, \( \alpha_{cr} \), well production rates, and BHPs are evaluated and will be further considered as the real data. Then, the time step \( s \) is recalculated taking into account the “real” data but without modeling the well \( n \).

Let’s consider a case then at the time step \( s \) the well \((n+1)\) comes into operation. Then the production rates and BHPs can be found using the standard group control option including the new well. The obtained data are used as the “real” ones. Taking these data into account the \( \alpha_{cr}, i=1,\ldots,(n+1) \) are estimated and then the problem (14),(43),(44) is solved at \( t=s \).

6. Case study
To evaluate the capabilities of the developed method the results were compared with methods of the overall production rate distribution implemented into the standard reservoir simulation software. Two cases were considered: the results of the implemented optimization algorithm [5] and the rate distribution described above. The final goal of optimization was to maximize the ultimate condensate recovery.

Algorithms were applied to the stochastically created synthetic model of an offshore gas production facility with six production horizontal wells. The wells surround the production unit with equal production sectors of 60°. The length of well’s production interval is 500 m. The offset from the production unit is also 500 m (see Figure 1). Porosity is normally distributed.

Figure 1. Porosity distribution in the synthetic model.

The considered production area is 4×4 km with the total thickness of 100 m. Parameters of the model were generated stochastically: the mean net to gross ratio is 0.5, the average permeability is 250 mD and the average porosity is 0.23. The reservoir was saturated with the gas-condensate mixture and the residual water. The gas saturation is 80%. The initial pressure was 32 MPa. The gas-condensate characteristic is shown in the Figure 2.
Figure 2. Condensate-gas ratio of the reservoir mixture.

The development strategy consisted of two stages. Firstly, the wells come into operation one by one with the same gas production rate of 1.34 mln. st. m$^3$/day. The interval between the well production starts was six months. After 2 years the swinging gas production begins: for 6 months the overall gas production rate was set to 7.6 mln. st. m$^3$/d (simulating the summer period) and for 6 months the overall gas production rate was set at 8.4 mln. st. m$^3$/d (simulating the winter period). The swinging production period continued 5 years. The upper and the lower limits for well’s gas rates were set as $b_i=2.0$ mln. st. m$^3$/d and $a_i=0.5$ mln. st. m$^3$/d, $i=1,...,n$, respectively. Results are shown in the Figure 3. The cumulative condensate recovery for ECLIPSE OPTIMIZE option was 6.09 mln. m$^3$, for the proposed method – 6.12 mln. m$^3$.

Figure 3. Well’s gas rate changing in time: a – OPTIMIZE (Eclipse); b – proposed method.
7. Conclusions

1. The developed method allows to obtain either the optimal or an approximate solution, the error of which can be evaluated. The conditions required to obtain an analytical solution to optimization problem are defined. If these conditions are not met, the method leads to the iterative procedure, whereby the $m$ parameter task is replaced by $m$ one-dimensional optimization problems. This allows to avoid computational difficulties characteristic to the large-dimensional problems. Thus, the method can be recommended for solving problems of optimizing the development of real gas-condensate fields.

2. The given examples of the numerical study of the developed models and methods have confirmed the validity of the proposed Hypotheses and the computational capabilities of these optimization models and algorithms.

3. The case study showed that at the practically the same efficiency with respect to the objective function, the solving time required by the proposed method is significantly less than the one needed by the Eclipse internal optimization features.

4. The main application area of the developed algorithms is the forming the starting point for further search of the optimal field development strategy.

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