$J/\psi$-Nucleon Scattering Length and In-medium Mass Shift of $J/\psi$ in QCD Sum Rule Analysis

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Abstract:

We calculate the spin-averaged $J/\psi$-nucleon scattering length $a_{J/\psi}$ by directly applying QCD sum rule to $J/\psi$-N forward scattering amplitude. Our result $a_{J/\psi} = -0.10 \pm 0.02$ fm, predicts the possibility of bound states with nuclei, though the force is weaker than that of the light vector mesons ($\rho, \omega, \phi$)-N cases. Up to dimension-4 gluonic operators, we evaluate the scattering length with twist-2 contribution, which increases the absolute value of the scattering length about 30%. If we apply $a_{J/\psi}$ to the effective mass of $J/\psi$ in nuclear matter on the basis of the linear density approximation, it shows very slight decrease ($4 \sim 7$ MeV) at normal matter density.

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1 Introduction

Theoretical analysis on the in-medium properties of hadrons is increasingly required by various on-going and forthcoming heavy-ion experiments (such as SPS, LHC (CERN) and AGS, RHIC (BNL)) [1]. In particular, experimentally it is important to observe vector mesons, because they decay into lepton pairs and carry the information inside the matter without disturbance of the strong interaction. The properties of light vector mesons in nuclear matter have been extensively studied in various theoretical approaches such as effective hadronic models [2] and QCD sum rules (QSR’s) [3, 4, 5, 6]. Vacuum properties of the vector mesons have been successfully studied by the QSR’s [7, 8]. The method enables us to express physical quantities such as mass and decay width in terms of the parameters in QCD Lagrangian and vacuum condensates. Extending the vacuum QSR to finite density, we can consistently incorporate the effects of nuclear matter into the form of in-medium condensates. There are two methodologically different ways for in-medium QSR. Firstly, T. Hatsuda and S.H. Lee developed the in-medium QSR formalism for light vector mesons [3]. They found a $10 \sim 20\%$ decrease of the $\rho$ and $\omega$ mesons at normal matter density. Secondly, for light vector mesons we formulated in-medium QSR [4, 6] based on the relation between a scattering length and a mass shift [9]. In this approach with the Fermi gas model, in-medium correlation function is divided into vacuum part and one nucleon part. This one nucleon part corresponds to the forward vector meson-nucleon scattering amplitude. The QSR analysis on the forward scattering amplitude enables us to obtain the information for vector meson-nucleon interaction. Moreover, from the information we can estimate the change of spectra for vector mesons in nuclear matter. The difference between these two approaches has been discussed in [10, 6]. Eventually we derived in [6] that both of them are based on almost the same idea and can lead the consistent results with those of the effective models.

In this paper we apply the QSR analysis established in [6] to a heavy quark system with equal mass for quark and antiquark. As a concrete system we focus on $J/\psi$, which is a low-lying charmonium state ($^3S_1$). To study the medium modification of $J/\psi$ has the following reasons:

1. We have the detailed experimental information for the charmonium. In particular, the spectrum of $J/\psi$ is extremely narrow for the leptonic decay ($\Gamma_{\ell+\ell-} \simeq 5$ keV). So it would be a good tool to observe the change of the spectra (e.g., mass shift) in nuclear matter.

2. Since charmonium and nucleons consist of quarks in different kinds of flavors, $J/\psi$-N interaction is purely gluonic without quark exchange to first order in elastic scattering. This simplification reduces our practical calculation.

3. Theoretical studies for $J/\psi$ in QSR have succeeded only in the description of the free state [8, 11].
4. To utilize $J/\psi$ suppression \[12\] as a direct signal of the quark-gluon plasma (QGP) phase, we need to estimate the effect of nuclear absorption theoretically \[13\]. It seems prompt to conclude that the present experimental data \[14\] can be explained only by nuclear absorption, until we investigate the $J/\psi$-nucleon interaction in detail. For that purpose, it is reasonable as the first step to study the $J/\psi$-$N$ elastic scattering at low energy.

Motivated by these, we calculate $J/\psi$-$N$ scattering length and the mass shift of $J/\psi$ in nuclear matter. That is, the first aim is to estimate the essential features of the interaction between $J/\psi$ and $N$ through the scattering length. In practice, by applying QSR to $J/\psi$-$N$ forward scattering amplitude we calculate the scattering length. The scattering length is a physically very important quantity in free space, because it is the unique observable in $J/\psi$-$N$ elastic scattering at low energy. If it is negative, then we could predict attractive nuclear force capable of binding $J/\psi$ to nucleus, so that $J/\psi$ could lead to a bound state with nucleus. Moreover, due to absence of Pauli blocking unlike the case of light vector mesons-$N$ system, the effective $J/\psi$-$N$ interaction will not have a short-range repulsion. The prediction of such exotic state will give exciting new directions in nuclear physics. As is well known, since the nuclear force is repulsive for isovector mesons, $\pi$ meson forms $\pi$-nucleus bound state by Coulomb attractive force. On the other hand $J/\psi$ is expected to be bounded only by attractive nuclear force from the isoscalar property. As was pointed out in \[15\], this interaction should be sufficiently attractive to allow a bound state. The probability of such exotic states has recently discussed for $\eta$, $\omega$ and $D$ cases \[16\].

The second aim is how the superposition of elementary $J/\psi$-$N$ scattering at low energy affects the effective mass of $J/\psi$ in nuclear matter. When we work in the dilute nucleon gas, we find that the mass shift is linearly dependent on the density (linear density approximation).

This paper is organized as follows. In section 2 we summarize the relation between the scattering length and the mass shift in the linear density approximation \[6\]. In the actual calculation we adopt moment sum rule method to the forward scattering amplitude. In section 3 the Wilson coefficients in the OPE side is explicitly given for twist-2 operator. In section 4 in order to obtain unknown hadronic parameters for the forward scattering amplitude we apply moment sum rule to vacuum correlation function. In section 5 the numerical results of the scattering length and the mass shift of $J/\psi$ is shown. Finally concluding remarks are given.

2 The relation between scattering length and mass shift

Let us first review the relation between the scattering length and the mass shift on the basis of QSR method \[4\] \[8\]. The starting point of this approach is the following vector current
correlation function in the ground state of nuclear matter with nucleon density \( \rho_N \).

\[
\Pi_{\mu\nu}^{NM}(q) = i \int d^4xe^{iqx} \langle T J_\mu(x)J_\nu^\dagger(0) \rangle_{NM(\rho_N)},
\]

(2.1)

where \( q^\mu = (\omega, \mathbf{q}) \) is the four-momentum carried by the \( J/\psi \) vector meson current \( J_\mu(x) = \bar{c} \gamma_\mu c(x) \) with the quantum numbers. Following the QSR method, when we apply an operator product expansion (OPE) to this correlator at deep Euclidean region \( (Q^2 = -q^2 > 0) \), it is supposed that the \( \rho_N \)-dependence of this correlator is entirely contained into the \( \rho_N \)-dependence of various condensates. Moreover we assume the Fermi gas model taking into account the Pauli principle among uncorrelated nucleons for the nuclear matter. In this approximation, in-medium correlation function reads

\[
\Pi_{\mu\nu}^{NM}(q) = \Pi_{\mu\nu}^0(q) + \sum_{\text{spin, isospin}} \int_{PF} d^3p \frac{d^3p}{(2\pi)^32p_0} T_{\mu\nu}(q),
\]

(2.2)

where \( \Pi_{\mu\nu}^0(q) \) is in-vacuum correlation function and \( \sum_{\text{spin, isospin}} \) denotes the sum of spin and isospin states for nucleons in nuclear matter. \( T_{\mu\nu}(q) \) is the vector current-nucleon forward scattering amplitude defined as

\[
T_{\mu\nu}(\omega, \mathbf{q}) = i \int d^4xe^{iqx} \langle N(\mathbf{p})|T J_\mu(x)J_\nu^\dagger(0)|N(\mathbf{ps}) \rangle.
\]

(2.3)

Here \( |N(\mathbf{ps}) \rangle \) denotes the nucleon state with four momentum \( p = (p_0, \mathbf{p}) \) and spin \( s \) normalized covariantly as \( \langle N(\mathbf{p})|N(\mathbf{p'}) \rangle = (2\pi)^32p_0^0\delta(\mathbf{p} - \mathbf{p'}) \). \( \Pi_{\mu\nu}^0(q) \) gives the main contribution for \( \Pi_{\mu\nu}^{NM}(q) \) due to the perturbative contribution. On the other hand \( T_{\mu\nu}(q) \) leads small contribution for \( \Pi_{\mu\nu}^{NM}(q) \), but the effect is vital contribution.

If we consider sufficiently low nucleon density such as normal matter density \( (\rho_N \sim 0.17 \text{ fm}^{-3}) \), the integral of the last term in Eq. (2.2) can be approximated up to the first order of nucleon density \( \rho_N \) reasonably well. The linear density term corresponds to the matter with static nucleons \( (\mathbf{p} = 0) \) and higher order correction terms correspond to the velocity-dependent terms involving the effect of Fermi motion \( (\mathbf{p} \neq 0) \) and the complex interaction among nucleons. The linear expression can be calculated model-independently. On the other hand the higher order corrections depend on the model calculation, but in a few effective theory \([7]\) it is known that the effect for the linear result is fairly small \( (\sim 10\%) \) at nuclear matter saturation density. Hatsuda et al. also insist that the Fermi momentum correction is fairly small \( (\sim 10\%) \) up to twist-4 operators in \([10]\). Thus we can safely neglect the effect at the saturation density. Therefore we can set \( p = (M_N, 0) \) for \( T_{\mu\nu}(p, q) \), so that we proceed to discussions based on the assumption that all nucleons are at rest in nuclear matter.

In Eq. (2.2) the second term means the slight deviation from the properties in free state determined by \( \Pi_{\mu\nu}^0 \). By applying QSR method to \( T_{\mu\nu} \) directly, we relate the scattering length extracted from the QSR for \( T_{\mu\nu} \) with the mass shift as one of deviation from the free state in the framework of QSR. Near the pole position of the \( J/\psi \), \( T_{\mu\nu} \) can be associated with the \( T \) matrix for the forward \( J/\psi-N \) helicity amplitude \( T_{hH,h'H'}(\omega, \mathbf{q}) \), where \( h(h') \) and \( H(H') \) are
the helicity of the initial(final) $J/\psi$ and the initial(final) nucleon, respectively. The relation between $T_{\mu\nu}$ and $T_{hH,N'H'}$ is given by the relation

$$\epsilon^{\nu*}_{(h)}(q)T_{\mu\nu}(\omega, q)\epsilon^{\mu}_{(h)}(q) \simeq \frac{-f_{J/\psi}^2 m_{J/\psi}^4}{(q^2 - m_{J/\psi}^2 + i\varepsilon)^2} T_{hH,N'H'}(\omega, q). \quad (2.4)$$

Here we introduce the coupling $f_{J/\psi}$ and the $J/\psi$ mass $m_{J/\psi}$ by the relation $\langle 0|J_{\mu}|J/\psi(h)\rangle = f_{J/\psi} m_{J/\psi}^2 \epsilon^{(h)}(q)$ with the polarization vector $\epsilon^{(h)}$ normalized as $\sum_h \epsilon^{(h)*}(q)\epsilon^{(h)}(q) = -g^{\mu\nu} + q^{\mu}q^{\nu}/q^2$. Taking the spin average on both sides of Eq. (2.4), $T_{\mu\nu}(\omega, q)$ is projected onto $T(\omega, q) = T_{\mu\nu}^\mu/(-3)$ and $T_{hH,N'H'}(\omega, q)$ is projected onto the spin averaged $J/\psi$-$N$ T-matrix, $T(\omega, q)$. At low energy, $q = (m_{J/\psi}, 0)$ and $p = (M_N, 0)$. $T$ is reduced to the spin averaged $J/\psi$-$N$ scattering length $a_{J/\psi} = 1/3(2a_{3/2} + a_{1/2})$ ($a_{1/2}$ and $a_{3/2}$ are the scattering lengths in the spin-1/2 and spin-3/2 channels, respectively) as $T(m_{J/\psi}, q = 0) = 8\pi(M_N + m_{J/\psi})a_{J/\psi}$. We note that the negative $a_{J/\psi}$ corresponds to attraction in our convention.

We relate the parameters of the QCD Lagrangian with the hadronic mass and coupling using the dispersion relation. If one utilizes the retarded correlation function as a useful quantity for dispersion analysis, we obtain the following dispersion relation for $T(\omega, q)$;

$$T(\omega, 0) = \frac{1}{\pi} \int_{-\infty}^{\infty} du \frac{\rho(u, 0)}{u - \omega - i\varepsilon} = \frac{1}{\pi} \int_{0}^{\infty} du' \frac{\rho(u, 0)}{u'^2 - \omega^2}. \quad (2.5)$$

Here the spectral function $\rho(u, q = 0)$ is given with three unknown phenomenological parameters $a, b, c$ in terms of the spin-averaged $J/\psi$-$N$ forward T-matrix $T$ such as

$$\rho(u, q = 0) = \frac{1}{\pi} \text{Im} \left[ \frac{-f_{J/\psi}^2 m_{J/\psi}^4}{(u^2 - m_{J/\psi}^2 + i\varepsilon)^2} T(u, 0) \right] + \cdots \quad (2.6)$$

$$= a \delta'(u^2 - m_{J/\psi}^2) + b \delta(u^2 - m_{J/\psi}^2) + c \delta(u^2 - s_0). \quad (2.7)$$

$\cdots$ term in Eq. (2.6) represents the continuum contribution and $\delta'$ in Eq. (2.7) is the first derivative of $\delta$ function with respect to $u^2$. The first $a$-term is the double pole term corresponding to the on-shell effect of $T$ matrix and the coefficient is associated with the scattering length $a_{J/\psi}$ as $a = 8\pi f_{J/\psi}^2 m_{J/\psi}^4 (M_N + m_{J/\psi})a_{J/\psi}$. The second $b$-term is the simple pole term corresponding to the off-shell effect of $T$ matrix. The third $c$-term is the continuum term corresponding to other remaining effects, where $s_0$ is regarded as the continuum threshold in vacuum. Now the contribution from the inelastic channels is not included in the ansatz of Eq. (2.7). In this system the OZI rule restricts the inelastic channels of $J/\psi$-$N$ interactions to those containing charmed quarks, for example, $J/\psi + N \rightarrow D + \bar{D} + N$ and $J/\psi + N \rightarrow \Lambda_c + \bar{D}$. But all these processes are forbidden at the threshold. So fortunately this system is immune from such inelastic contributions.

The parameters $a, b$ and $c$ in Eq. (2.7) are not completely independent. That is, among these parameters we introduce a constraint relation, which is imposed by low energy theorem for the $J/\psi$ current-nucleon forward scattering amplitude. In the low energy limit $\omega \rightarrow 0$,
\( T(\omega, 0) \) become equivalent to Born term \( T^{\text{Born}}(\omega, 0) \), which is zero in \( J/\psi-N \) system. Now we get the following constraint relation from the low energy theorem,

\[
\frac{a}{m^4_{J/\psi}} + \frac{b}{m^2_{J/\psi}} + \frac{c}{s_0} = 0. \tag{2.8}
\]

Therefore the spectral function is parametrized with two unknown phenomenological parameters \( a \) and \( b \) by removing \( c \) from Eq. (2.8). The phenomenological (PH) side for \( \Pi^{NM}_{\mu\nu} \) can be expressed as the combination of pole position for \( \Pi^0_{\mu\nu} \) and \( T_{\mu\nu} \) such as

\[
\Pi^{NM}_{\mu\nu} = \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \left[ \frac{F}{m^2_{J/\psi} - q^2} + \frac{\rho_N}{2M_N} \left\{ \frac{a}{(m^2_{J/\psi} - q^2)^2} + \frac{b}{m^2_{J/\psi} - q^2} \right\} + \cdots \right]
\]

\[
\propto \frac{F + \Delta F}{(m^2_{J/\psi} + \Delta m^2_{J/\psi} - q^2)} + \cdots, \tag{2.9}
\]

where the pole residue \( F \) in \( \Pi^0_{\mu\nu} \) is equivalent to \( f^2_{J/\psi}m^4_{J/\psi} \) and the deviation \( \Delta F \) is \( \rho_N b/2M_N \). The quantity expressed as the shift of the squared \( J/\psi \) mass in nuclear matter,

\[
\Delta m^2_{J/\psi} = 2m_{J/\psi}\delta m_{J/\psi} = \frac{\rho_N}{2M_N} \frac{a}{f^2_{J/\psi}m^4_{J/\psi}} = \frac{\rho_N}{2M_N} 8\pi(M_N + m_{J/\psi})a_{J/\psi} \tag{2.10}
\]

is proportional to the scattering length \( a_{J/\psi} \) through the double pole term in \( T_{\mu\nu} \). Thus we can calculate the mass shift \( \delta m_{J/\psi} \) in Eq. (2.10) from \( a_{J/\psi} \) obtained by QSR for \( T_{\mu\nu} \).

We explicitly write down PH side with the unknown parameters \( a \) and \( b \) for \( T(q^2) \) using Eq. (2.3), (2.8) and (2.9). We take the \( n \)-th derivative with respect to \( q^2 \) after dividing \( T^{\text{ph}} \) by \( q^2 \) as follows and define it as \( \hat{T}^{(n)} \).

\[
\frac{1}{n!} \left( \frac{d}{dq^2} \right)^n \frac{T^{\text{ph}}(q^2)}{q^2} \equiv \hat{T}^{(n)}(q^2; a, b)
\]

\[
= \frac{a}{m^2_{J/\psi}} \left[ \frac{(n + 1)m^2_{J/\psi}}{(m^2_{J/\psi} - q^2)^{n+1}} + \frac{1}{(m^2_{J/\psi} - q^2)^{n+1}} - \frac{1}{(s_0 - q^2)^{n+1}} \right]
\]

\[
+ \frac{b}{m^2_{J/\psi}} \left[ \frac{1}{(m^2_{J/\psi} - q^2)^{n+1}} - \frac{1}{(s_0 - q^2)^{n+1}} \right]. \tag{2.11}
\]

In order to construct the QSR we calculate the \( n \)-th derivative of OPE side similarly in the next section.

### 3 The calculation of the Wilson coefficients for \( T_{\mu\nu} \)

Now we give the OPE expression for \( T_{\mu\nu} \). The main task in OPE side is to calculate the Wilson coefficients based on perturbative QCD. In the case of \( J/\psi \) the charmed quark mass
is so heavy that the calculation of the Wilson coefficients must be carried out explicitly with
the effect of heavy quark mass. We now expand local operators up to dimension-4 in the
OPE side. Then pure gluonic contributions must be only taken into account for the local
operators. Up to this order of the OPE the nucleon matrix elements of two-gluon operators
\( (GG) \) are most dominant. We note that contrary to vacuum QCD sum rule a new feature in
\( T_{\mu\nu} \) is that the nucleon matrix elements survive not only Lorentz scalar o perators but also
nonscalar operators. That is, we must consider new contributions from twist-2 operators
with two spins for the Wilson coefficients. In order to calculate the coefficient function, we
adopt the valid and well-known method for massive quarks propagating through the couple
to soft gluons working as the external field, namely fixed-point gage method [18]. This gauge
condition is expressed as \( x^\mu A^a_\mu(x) = 0 \). The nucleon matrix element of two-gluon operators
can be decomposed into the scalar part and the twist-2 part with an additional four-vector
\( u_\mu \) \( (u^2 = 1) \), through the simple tensor analysis [19].

\[
\left\langle G^a_{\alpha\beta} G^b_{\gamma\delta} \right\rangle_N = \frac{\delta^{ab}}{96} \left\{ \left\langle G^2 \right\rangle_N \left( g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma} \right) - 4 \left( (u \cdot G)^2 - \frac{1}{4} G^2 \right) \right\}_N \left\{ g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma} \right\},
\]

where we define \( (u \cdot G)^2 \equiv G^a_{\kappa\lambda} G_\rho^a \cdot u^\kappa u^\rho \). By introducing the \( u_\mu \), one can imagine uniformly
moving uncorrelated nucleons \( (p_\mu = M_N u_\mu) \) in nuclear matter, but in this case we set
\( u = (1,0) \). The OPE expression for \( T_{\mu\nu} \) is written as follows by the combination of Eq.(3.1)
and the Wilson coefficients corresponding to each matrix element.

\[
\frac{1}{n!} \left( \frac{d}{dq^2} \right)^n \frac{T_{\text{OPE}}(q^2)}{q^2} \equiv \hat{T}^{(n)}_{\text{OPE}}(q^2)
= \frac{1}{3} \left[ C_G^{(n)}(\xi) \right. \left\{ \frac{\alpha_s}{\pi} \left\langle G^2 \right\rangle_N - 4 \frac{\alpha_s}{\pi} ST(G^a_{0\alpha} G^a_{0\beta}) \right\}_N \]

\[ + \left\{ D_1^{(n)}(\xi) - D_2^{(n)}(\xi) - D_3^{(n)}(\xi) \right\} \left\langle \frac{\alpha_s}{\pi} ST(G^a_{0\alpha} G^a_{0\beta}) \right\rangle_N \].

Here we define the dimensionless parameter as \( \xi = -q^2/4m_c^2 \) \( (m_c; \text{ charmed quark mass}) \)
and \( \rho = \xi/(1 + \xi) \). \( ST \) means making the twist-2 operators symmetric and traceless in its
Lorentz indices. In Eq.(3.2) each coefficient function is given using Gauss hypergeometric
functions \( _2F_1 \) for arbitrary \( q^2 \) as follows:

\[
C_G^{(n)}(\xi) = -\frac{2^n(n + 1)(n + 3)!}{(2n + 5)!!} (4m_c^2)^{-n+2} (1 + \xi)^{-(n+2)} \left( n + 2, -\frac{1}{2}, n + \frac{7}{2}; \rho \right)
\]

\[
D_1^{(n)}(\xi) = \frac{2^{n+3}(n + 1)(n + 1)!}{3(2n + 3)!!} (4m_c^2)^{-(n+2)} (1 + \xi)^{-(n+2)}
\]

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\[ D_{2}^{(n)}(\xi) = -\frac{2^{n+5}(n+1)(n+2)!}{3(2n+5)!}(4m_{c}^{2})^{-(n+2)}(1 + \xi)^{-(n+2)} \times \left[ 2F_{1} \left( n + 2, \frac{1}{2}, n + \frac{7}{2}; \rho \right) - \frac{n+2}{2(1 + \xi)} 2F_{1} \left( n + 3, \frac{1}{2}, n + \frac{7}{2}; \rho \right) \right] \] (3.5)

\[ D_{3}^{(n)}(\xi) = \frac{2^{n+3}(n+1)(n+2)!}{3(2n+5)!}(4m_{c}^{2})^{-(n+2)}(1 + \xi)^{-(n+2)} \times \left[ (n+2) 2F_{1} \left( n + 2, \frac{1}{2}, n + \frac{7}{2}; \rho \right) + 4(2n+5) 2F_{1} \left( n + 2, \frac{1}{2}, n + \frac{5}{2}; \rho \right) \right], \] (3.6)

The Wilson coefficient of Eq.(3.3) for scalar operator have already given in [11] and Eq.(3.4), (3.5) and (3.6) are new contributions for twist-2 operator. Eventually by equating Eq.(2.11) and Eq.(3.2) we obtain the moment sum rule expressed as the form of the n-th derivative with respect to \( q^{2} \),

\[
\hat{T}^{(n)}_{PH}(\xi ; a, b) = \hat{T}^{(n)}_{OPE}(\xi).
\] (3.7)

The manipulation of the derivative ensures the enhancement of low energy part not to depend on the details of high energy part. The vacuum sum rules have been utilized for investigation into the free state of charmonium by Reinders et al. [11] in moment sum rule and by Bertlmann [20] in Borel sum rule. Furnstahl et al. [21] have studied the spectra of \( J/\psi \) at finite temperature using both QCD sum rules. We summarize the well-known behavior of the moment sum rule for the variations of \( n \) and \( q^{2} \).

- The convergence of OPE side is worse with \( n \) larger but better with \( q^{2} \) larger.
- In contradiction to this behavior of OPE side, the unwelcome contributions from the continuum in PH side grow with \( q^{2} \) larger but decrease with \( n \) larger.

We must choose the reliable stability region of moment sum rule for the change of the both \( n \) and \( q^{2} \). The moment sum rule for \( T_{\mu\nu} \) is the method to investigate the deviation from the properties in vacuum obtained from \( \Pi_{0} \). So we should adopt the same regions of \( n \) and \( q^{2} \) as \( \Pi_{0} \), which can reproduce the nature of \( J/\psi \) in the moment sum rule reasonably well.
4 Moment sum rule for $\Pi_0$

In this section we calculate the window of $n$ for various values of $\xi$ by applying the moment sum rule to vacuum correlation function $\Pi_0$ \([11]\).

In OPE side $n$-th derivative for $\Pi_0^{(n)}$ is expressed as

$$
\frac{1}{n!} \left( \frac{d}{dq^2} \right)^n \Pi_0^{\text{OPE}}(q^2) = \frac{\Pi_0^{(n)} \text{OPE}(\xi)}{q^2} = \frac{1}{3} \left[ C_0^{(n)}(\xi) \left\{ 1 + c_1^{(n)}(\xi) \alpha_s(\xi) \right\} + C_0^{(n)}(\xi) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 \right], \tag{4.1}
$$

where $C_0^{(n)}(\xi), c_1^{(n)}(\xi)$ are given by

$$
C_0^{(n)}(\xi) = \frac{9}{4\pi^2} \frac{2^n(n+1)(n-1)!}{(2n+3)!!} (4m_c^2)^{-n}(1+\xi)^{-n} F_1 \left( n, \frac{1}{2}, n + \frac{5}{2}; \rho \right), \tag{4.2}
$$

$$
c_1^{(n)}(\xi) = \frac{(2n+1)!!}{3 \cdot 2^{n-1} n!} \frac{2n+3}{2(n+1)} \frac{1}{2F_1 \left( n, \frac{1}{2}, n + \frac{5}{2}; \rho \right)} \times \left[ \pi - \left\{ \frac{\pi}{3} + \frac{1}{2} \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) \right\} \frac{1}{n+1} 2F_1 \left( n, 1, n+2; \rho \right) + \frac{1}{3(n+1)(n+2)} \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) 2F_1 \left( n, 2, n+3; \rho \right) \right] - \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) - 2n \frac{\ln(2+\xi)}{\pi} \frac{2 + \xi}{(1+\xi)^2} \frac{2F_1 \left( n+1, \frac{1}{2}, n + \frac{5}{2}; \rho \right)}{2F_1 \left( n, \frac{1}{2}, n + \frac{5}{2}; \rho \right)}. \tag{4.3}
$$

In Eq.\(\text{[4.1]}\) the OPE side of moment sum rule in vacuum $\Pi_0^{(n)}$ is the combination of the first term from a bare loop contribution and the second term replaced by the expectation values of the nucleon into it of the vacuum. On the other hand the relation between the $J/\psi$ mass of lowest resonance and $\Pi_0^{(n)}$ in PH side is given as

$$
\Pi_0^{(n)}(\xi) = \frac{9m_{J/\psi}^2}{4g_{J/\psi}^2} \frac{1}{(m_{J/\psi}^2 - \omega^2)^{n+1}} [1 + \delta_n], \tag{4.4}
$$

where $m_{J/\psi}$ and $g_{J/\psi}$ are the parameters of the lowest-lying resonance and $\delta_n$ represents the contributions from higher resonances. Moreover in $\Pi_0^{(n)}$ we add continuum contribution proportional to $1/4\pi \times (1 + \alpha_s/\pi)$, which we for simplicity assume to be constant value without the dependence of charmed quark mass. From Eq.\(\text{[4.4]}\) we eliminate the coupling parameter $g_{J/\psi}$ by taking ratios of the $n$-th moments and the $(n-1)$-th moments. Finally
The bare mass of $J/\psi$ is derived from the following relation,

$$m_{J/\psi} = \left[ \omega^2 + \frac{\Pi_0^{(n-1)}}{\Pi_0^{(n)}} \right]^{1/2}. \quad (4.5)$$

First we fix $\xi$ to be from 0.0 to 3.0 at 0.5 intervals. These values are equivalent to the magnitude of from 0 to 4 [GeV] in $\sqrt{-q^2}$. In Fig.1 we show the $J/\psi$ bare mass determined from Eq.(4.3) for the change of $\xi$. Here we used $s_0 = 3.6^2$ [GeV$^2$] adopted in [11]. We must read off the range of $n$ for each $\xi$ from the Figure 1. The window of $n$ corresponds to find the region stabilizing the $J/\psi$ bare mass for the change of $n$. Then we obtain the windows of $n$ for each $\xi$ as follows: $n_1 = 2, 3, 4$ for $\xi = 0.0$ ($-q^2 = 0.00$ [GeV$^2$]), $n_2 = 3, 4, 5$ for $\xi = 0.5$ ($-q^2 = 3.18$ [GeV$^2$]), $n_3 = 4, 5, 6, 7$ for $\xi = 1.0$ ($-q^2 = 6.25$ [GeV$^2$]), $n_4 = 5, 6, 7, 8$ for $\xi = 1.5$ ($-q^2 = 9.23$ [GeV$^2$]), $n_5 = 6, 7, 8, 9$ for $\xi = 2.0$ ($-q^2 = 12.1$ [GeV$^2$]), $n_6 = 7, 8, 9, 10$ for $\xi = 2.5$ ($-q^2 = 14.9$ [GeV$^2$]), $n_7 = 8, 9, 10, 11$ for $\xi = 3.0$ ($-q^2 = 17.6$ [GeV$^2$]). These points seem to reproduce the bare mass reasonably well for the experimental value $m_{J/\psi} = 3.096$ [GeV].

### 5 Numerical results

By inserting the sets of $\xi$ and $n$ obtained in section 4 into Eq.(4.7), we can determine unknown parameters $a$ and $b$ simultaneously by fitting the left-hand side to the right-hand side. Concretely the order of calculation is as follows: At first we arbitrarily choose two points in the window of $n$ for the fixed $\xi$ and make a simultaneous equation for $a$ and $b$ by inserting two $n$ chosen. We consider all such combinations for each $\xi$ and take the average of $a$ solved for each combination. Eventually the scattering length is easily obtained from $a$.

To calculate the scattering length we use the next values for other various parameters. First of all, in PH side we adopt $m_{J/\psi} = 3.1$ [GeV], $M_N = 0.94$ [GeV] and $s_0 = 3.6^2$ [GeV$^2$] adopted in [11]. The coupling is determined from the experimental value of $\Gamma_{J/\psi}^{e^+e^-}$ as follows:

$$f_{J/\psi}^2 = \frac{3\Gamma_{J/\psi}^{e^+e^-}}{4\pi e_q^2 \alpha^2 m_{J/\psi}} = 1.7 \times 10^{-2}, \quad (5.1)$$

where $e_q$ is electric charge of quark ($e_c = 2/3$ for charm quark) and $\alpha$ is the fine structure constant ($= 1/137$). Indeed this values of the coupling can be also determined by substituting the experimental values of $J/\psi$ bare mass into Eq.(4.3) oppositely. Then QSR for $\Pi^0$ reproduces well the experimental values of the coupling [11]. For QCD Lagrangian parameters we use the following functions dependent on $\xi$ given in [11]:

$$\alpha_s(\xi) = \frac{\alpha_s(4m_c^2)}{1 + \frac{25}{12\pi} \alpha_s(4m_c^2) \ln(1 + \xi)} , \quad \alpha_s(4m_c^2) \simeq 0.3 \quad (5.2)$$

$$m_c(\xi) = 1.28 \times \left[ 1 - \frac{\alpha_s(\xi)}{\pi} \left\{ \frac{2 + \xi}{1 + \xi} \ln(2 + \xi) - 2 \ln 2 \right\} \right] \text{[GeV]} \quad (5.3)$$
| $\xi$ | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
|------|-----|-----|-----|-----|-----|-----|-----|
| $-a_V^{\text{scalar}}$ [fm] | 0.091 | 0.068 | 0.070 | 0.063 | 0.059 | 0.057 | 0.055 |
| $\delta m_V^{\text{scalar}}$ [MeV] | 5.3 | 3.9 | 4.0 | 3.6 | 3.4 | 3.3 | 3.2 |
| $-a_V^{\text{twist-2}}$ [fm] | 0.120 | 0.090 | 0.092 | 0.083 | 0.078 | 0.075 | 0.073 |
| $\delta m_V^{\text{twist-2}}$ [MeV] | 6.9 | 5.2 | 5.3 | 4.8 | 4.5 | 4.3 | 4.2 |

Table 1: $J/\psi$-$N$ scattering length and the mass shift of $J/\psi$ in the case of the scalar operator only and the scalar + twist-2 operator at normal matter density $\rho_N = 0.17$ [fm$^{-3}$].

In OPE side we determine the nucleon matrix elements as follows:

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_N = -(1.222 \pm 0.282) \text{ [GeV}^2 \text{]} \quad (5.4)$$

$$\left\langle \frac{\alpha_s}{\pi} ST(G_0^a G_{0\sigma}) \right\rangle_N = -(0.094 \pm 0.010) \text{ [GeV}^2 \text{]} \quad (5.5)$$

The scalar part is evaluated from the $\pi$-$N$ sigma term [19, 22]. The twist-2 part is determined from the gluon distribution function of a nucleon, which is obtained by leading order parametrization to the experimental data of deep inelastic scattering [19, 23].

We list the results in the case of scalar operator only and the case involving twist-2 operator in Table 1.

### 6 Concluding remarks

The direct application of moment sum rule to the forward $J/\psi$-$N$ scattering amplitude supplies us the fascinating result for the $J/\psi$-$N$ interaction. That is, the $J/\psi$-$N$ scattering length $a_{J/\psi}$ indicates negative value (about $-0.1$ fm). This result suggests that the attractive $J/\psi$-$N$ interaction is not sufficient to form a bound state with one nucleon, but it could make a bound state with nuclei. The absolute value is certainly smaller than the typical hadronic size 1 fm and the scattering length of light vector meson-$N$ systems ($a_\rho \simeq -0.47$, $a_\omega \simeq -0.41$, $a_\phi \simeq -0.15$) [4], but the experimental creation of $J/\psi$ at the threshold would lead to formation of a bound state inside a heavy nucleus. Our result is smaller than those obtained recently by S.J.Brodsky et al. [24] and G.F.de Téramond et al. [25] in the QCD sum rule approach [26]. Their method is based on the on-shell calculation on charmed quark mass ($q^2 = 0$).

In this study we newly calculated the Wilson coefficients for twist-2 gluon operators (dimension-4) in the form with quark mass explicitly. The nucleon matrix elements of twist-2 gluon operators is about 1/10 times as large as that of the scalar part, but the total contribution with the Wilson coefficient makes the absolute value of scalar part larger about 30%.
From $a_{J/\psi}$, we can estimate a total cross section ($\sigma_{J/\psi} = 4\pi a_{J/\psi}^2$). The result is about 1.26 mb at the threshold. The contribution from the elastic channel corresponds to 20% of the nuclear absorption cross section derived from the experimental data, $\sigma_{nab} = 7.3$ mb.

Next in the linear density approximation we can calculate $J/\psi$ mass shift from $a_{J/\psi}$. The result gives very small decrease of mass (about $-4$ to $-7$ MeV), about 0.1 to 0.2% at normal matter density. Since the slight mass shift is of the order of MeV, the change is sufficiently larger than the leptonic decay width of the order keV. So we consider $J/\psi$ is a good probe for the observation of medium effect.

Recently just before I submit this paper, Klingl et al. reported the mass shift of about $-20$ MeV (0.6%) for $J/\psi$ in nuclear matter [27]. The pattern of derivation is similar to the case for light vector meson [3, 4].

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References

[1] Quark Matter ’97, Proceeding of the 13th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions, Tsukuba, Japan, 1-5 Dec. 1997, Nucl. Phys. A638 (1998) 1c–610c.

[2] G.E. Brown and M. Rho, Phys. Rev. Lett. C66 (1991) 2720;
K.-C. Jean, J. Piekarewicz and A.G. Williams, Phys. Rev. C49 (1994) 1981;
H. Siomi and T. Hatsuda, Phys. Lett. B334 (1994) 281;
M. Herrmann, B.L. Friman, and W. Nörenberg, Nucl. Phys. A560 (1993) 411;
M. Asakawa, C.M. Ko, P. Lévai and X.J. Qiu, Phys. Rev. C46 (1992) R1159.

[3] T. Hatsuda and S.H. Lee, Phys. Rev. C46 (1992) R34.

[4] Y. Koike, Phys. Rev. C51 (1995) 1488.

[5] X. Jin and D.B. Leinweber, Phys. Rev. C52 (1995) 3344.

[6] Y. Koike and A. Hayashigaki, Prog. Theor. Phys. 98 (1997) 631.

[7] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385.

[8] L.J. Reinders, H.R. Rubinstein and S. Yazaki, Phys. Rep. 127 (1985) 1.
[9] Y. Kondo and O. Morimatsu, Phys. Rev. Lett. 71 (1993) 2855.
[10] T. Hatsuda, S.H. Lee and H. Shiomi, Phys. Rev. C52 (1995) 3364.
[11] L.J. Reinders, H.R. Rubinstein and S. Yazaki, Nucl. Phys. B186 (1981) 109.
[12] T. Matsui and H. Satz, Phys. Rev. Lett. B178 (1986) 416.
[13] D. Kharzeev, nucl-th/9601023.
[14] NA38 Collaboration, C. Baglin et al., Phys. Lett. B220 (1989) 471;
NA50 Collaboration, M.C. Abreu et al., Phys. Lett. B410 (1997) 327,337.
[15] S.J. Brodsky, I. Schmidt and G.F. de Téramond, Phys. Rev. Lett. 64 (1990) 1011;
D.A. Wasson, Phys. Rev. Lett. 67 (1991) 2237;
M. Luke, A.V. Manohar and M.J. Savage, Phys. Lett. B288 (1982) 355;
A.B. Kaidalov and P.E. Volkovitsky, Phys. Rev. Lett. 69 (1992) 3155.
[16] R.S. Hayano, S. Hirenzaki and A. Gillitzer, nucl-th/9806012;
K. Tsushima, D.H. Lu and A.W. Thomas, nucl-th/9806043;
K. Klingl, T. Waas and W. Weise, hep-ph/9810312;
K. Tsushima, D.H. Lu, A.W. Thomas, K. Saito and R.H. Landau, nucl-th/9810016;
K. Tsushima, nucl-th/9811063.
[17] T.D. Cohen, R.J. Furnstahl, D.K. Griegel, Phys. Rev. C45 (1992) 1881.
[18] V.A. Fock, Sov. Phys. 12 (1937) 404;
J. Schwinger, Particle Sources and Fields, Vol.I (Addison-Wesley, 1970);
C. Cronström, Phys. Lett. 90B (1980) 267;
V.A. Novikov, M.A. Shifman, A.I. Vainstein and Zakharov, Fortschr. Phys. 32 (1984) 585.
[19] X. Jin, T.D. Cohen, R.J. Furnstahl and D.K. Griegel, Phys. Rev. C47 (1993) 2882.
[20] R.A. Bertlmann, Nucl. Phys. B204 (1982) 387.
[21] R.J. Furnstahl, T. Hatsuda and S.H. Lee, Phys. Rev. D42 (1990) 1744.
[22] T. Hatsuda and T. Kunihiro, Nucl. Phys. B387 (1992) 715.
[23] X. Ji, Phys. Lett. 74 (1995) 1071.
[24] S.J. Brodsky and G.A. Miller, Phys. Lett. B412 (1997) 125.
[25] G.F. de Téramond, R. Espinoza and M. Ortega-Rodríguez, hep-ph/9708202.
[26] M.E. Peskin, Nucl. Phys. B156 (1979) 365;
G. Bhanot and M.E. Peskin, Nucl. Phys. B156 (1979) 391.
[27] F. Klingl, S. Kim, S.H. Lee, P. Morath nad W. Weise, nucl-th/9811070.
Figure Captions

**Fig. 1** Stability region in \( n \) for various values of \( \xi \). For comparison the experimental mass values have also been indicated.

**Fig. 2** \( J/\psi \)-\( N \) scattering length determined for various values of \( \xi \) and the stability region of \( n \) obtained from Fig.1.
