I. THE NATURE OF THE PROBLEM

Probably one of the most interesting problems in modern physics, since it appears to be one of the most fundamental and, at the same time, one of the simplest, is the problem of interference. The measurements of Tonomura et al. [1], displayed in Fig. 1, reveal the impact of single electrons on a detector screen, only gradually and without regularity evolving into the familiar interference pattern of a two-slit interferometer. In Feynman’s view this experiment ”has been designed to contain all of the mystery of quantum mechanics, to put you up against the paradoxes and mysteries and peculiarities of nature one hundred percent” [2]. His judgement has gained, with these pictures, a particularly striking illustration. The effect is commonly associated with the notion of ”wave-particle duality” [3] and is seen as one of the main examples, where our classical concepts of reality break down, since, after all, how can an electron passing through one slit ”know” whether the second aperture is opened, a knowledge, it seems to possess, because its impact on the screen depends on the setup. These measurements can formally be described by the quantum theory of motion [4], but although the description is highly suggestive, it does not solve the problem, what the ”quantum-mechanical” potential, substantial in this model, is actually supposed to mean. In this sense the claim that the problem is solved [5] seems to be unjustified.

We propose, in this paper, a solution which is a step by step procedure, where every step in the mathematical formalism can be justified by precise and conceptually consistent physical arguments. The solution will be based on our recent analysis of measurement processes [6], which in turn is founded on the notion of extended particles [7]. It will be seen that the solution given by the quantum theory of motion is formally correct, although it requires a full understanding of the ensemble structure of quantum theory to be physically sound.

The paper is organized as follows: first we restate the classical solution to a single slit experiment and show that the actual interference effect cannot be localized. Based on a conjecture about the interaction with the detector screen we propose a measurement to detect an eventual incompleteness of the standard model. Then the ensemble structure of quantum theory is briefly discussed, and it will be shown that the probability interpretation of ψ makes quantum theory (QT) inherently non-local. Based on the concept of quantum ensembles a new interpretation will be given to the quantum potential Q [8], and Q will be found, as Bohm suggested [9], related to information, since it describes the change of ensembles due to the physical environment. Finally we shall analyze the solution for the two-slit interferometer in the quantum theory of motion (QTM), in this case the analysis leads to the result that the simplest way to account for varying amplitudes and volumes of the extended and wave-like objects underneath the fundamental statements of QT is to describe ensembles by varying densities of trajectories of point-like objects. In this sense the ”particle” in QTM has a double meaning: it is a single and well defined physical object but, due to the quantum potential, also a member of the full quantum ensemble. It will be advocated that this is the actual meaning of the term ”guiding-wave”, figuring prominently in de Broglie’s original concept [10].

II. SINGLE-SLIT DIFFRACTION

To analyze the main conceptual difficulties and the inherent non-locality in the classical treatment of interference problems, it is sufficient to consider, in the scalar approximation, a single slit experiment. We assume that the electrons incident in our interferometer can be described as solutions of the Helmholtz equation of their wave-function ψ(r), the wave shall cover a sufficient region for the relation to make sense. With this definition we wish to avoid the discussion of boundaries and coherence of the wave, since it will be seen that the fundamental feature, which is the inherent non-locality, arises as soon as we admit a continuous wave-field as a suitable description. Helmholtz’s equation in the vacuum states:
\[(\Delta + k^2) \psi(\vec{r}) = 0\]  

With Green’s theorem and using the vacuum Green’s function \(\psi(\vec{r})\) is transformed into an integral over the boundary \(R(V)\):

\[
\psi(\vec{r}) = \oint_{R(V)} d^2\vec{f} \left( \nabla' \psi(\vec{r}') + \frac{2\pi i \psi(\vec{r}')}{\lambda} \left| \vec{r}' - \vec{r} \right| \left( 1 + \frac{i}{k|\vec{r}' - \vec{r}|} \right) \right) 
\]

where \(-\vec{R}\) is the source and \(\vec{c}_p\) the velocity of the ”particle”. If we neglect the derivatives of \(\delta\) which, after all, shall only signify the existence of single entities, i.e. the fact that a single electron has limited extension and is small compared to the system, then we get at the moment \(t = 0\):

\[
\psi_{t=0}(\vec{r}) = e^{ik|\vec{R} - \vec{c}_p t|} \Gamma(k, \vec{r}) 
\]

With a Kirchhoff approximation the calculation yields the familiar results of classical electrodynamics: the amplitude \(\Gamma(k, \vec{r})\) depends on the setup geometry of the interferometer and the wavelength \(\lambda\). But it provides the result already at the moment, when the ”particle” passes the slit (\(t = 0\)). Clearly, therefore, it does not describe the causal propagation of single particles, but the probabilities of their impact on the screen, even if this probability is contained in the intensity of the wave. A different way of stating the same result is saying that the effect cannot be localized.

Now let the interferometer be operating in the Fraunhofer limit of diffraction (distance between screen and slit sufficiently high), then the variations of the wavefunction are [13]:

\[
\psi(k, \theta) \propto \frac{\sin(k\theta)}{k\theta} \tag{5} 
\]

where \(k\) is the wavevector and \(\theta\) the azimuthal angle. A phase \(\alpha\) of the wave at its origin \(-\vec{R}\) only adds a phase-factor, which has no effect on the intensity in the conventional model. In the realistic model [9] it does have an effect, though, if the energy effecting the measurement is either only the kinetic component or only the field component of particle energy. Since the field component can be described as the real part of the wave \(\psi\), the intensity from this energy component alone is given by:

\[
dI(k, \theta, \alpha) = \left| \frac{\sin(k\theta)}{k\theta} \right|^2 \cos^2 \alpha d\theta \tag{6} 
\]

The intensity as a function of the azimuthal angle \(\theta\) and the phase \(\alpha\) is shown in Fig. 2. Therefore, a measurement of the interference fringes by using waves of a defined phase may add to our information about the interaction process. Either the result is independent of \(\alpha\), in this case the interaction energy can only be the total energy and the conventional model is sufficient, or the result depends, like in Fig. 2, on the phase \(\alpha\), in this case the interaction energy depends not on total energy but on
an unbalanced composition of field energy and kinetic energy. In either case the result yields an increase of information about the process: this applies to photons as well as electrons.

![Intensity of interference fringes](image)

**FIG. 2.** Intensity of interference fringes due to the field component of energy of the particle. The initial phase determines the visibility of the pattern on the screen in the far-field limit (Fraunhofer limit) of the interferometer.

Summing up the result of this short classical analysis it can be said that in classical field theory the origin of the interference effect cannot be localized, its result is obtained essentially by a summation of all components over the whole system: it is therefore, in a sense, non-local, because the setup can be chosen in such a way that the distance between the detector screen and the slit environment is space-like for a given problem. And since the amplitude is given already at the moment, when the photon passes the slit, its trajectory contains information about the whole system in a non-local manner.

### III. ENSEMBLES IN QUANTUM THEORY

In non-relativistic QT a system is generally described by a suitable Schrödinger equation:

\begin{equation}
\hat{H}\psi = E\psi
\end{equation}

which we assume to be given also in case of a two-slit interferometer. It shall consist of a Hamiltonian with potentials suitable to describe the slit environment. The main conceptual problem here is that the single impacts, e.g. measured by Tonomura et al. [16], are not described by the wavefunction \( \psi(\vec{r}) \), where \( \vec{r} \) shall be a location on the detector screen, since \( \psi(\vec{r}) \) only yields the overall intensity on the screen but not the single hit. One does not need Einstein’s fundamental criticism of QT [1] to be left unsatisfied with a situation, where one observes events, which cannot be explained and even, if one follows the orthodox [13], are unexplainable in principle. Bohr, for example, considered any real event on the atomic level, which is, what these impacts amount to, as beyond the means of scientific analysis, in his view "such an analysis is in principle excluded" [7].

We shall, in the following, interpret the single impacts as a result of the ensemble structure of QT, which lies underneath its fundamental equations, and which recently has been consistently analyzed for the first time [8]. The origin of the quantum ensemble is the unknown phase of \( \psi \), together with an intrinsic and field-like energy component of particle propagation [1], not accounted for in the conventional framework of QT. In particular it was shown that the uncertainty relations can be referred to this intrinsic energy. Considering electrons of defined energy where

\[ E_T = \hbar \omega = mu^2 \]  

is the total energy including the intrinsic components, an external potential \( V(\vec{r}) \) leads to an ensemble wavefunction, described by an integral over k-space:

\[ \psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int_0^{k_1} d^3k \chi_0(\vec{k}) e^{i\vec{k}\cdot\vec{r}} \]

where
\[ k_1 = k_1(\vec{r}) = \frac{m}{h} \sqrt{E_T - V(\vec{r})} \]  

The integral limit \( k_1 \) describes the error margin due to the undefined intrinsic energy components in quantum theory. Since the amplitude \( \chi_0(\vec{k}) \) is undefined, the condition that the density of a single plane-wave component integrated over real space equals the mass of one electron:

\[ \int_{-\infty}^{+\infty} d^3r \left| \chi(\vec{r}, \vec{k}) \right|^2 = \chi_0^2(\vec{k}) =: m_e \]  

leads, together with the probability interpretation of \( \psi \) to the following integral:

\[ \int_{-\infty}^{+\infty} d^3r \left| \psi(\vec{r}, \vec{k}) \right|^2 = \frac{m_e}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3r \times \int_0^{k_1} d^3k d^3k' e^{i\vec{r}\cdot\vec{k}'} \tag{11} \]

For a given potential in the system, \( U = const. \), the integral reduces to an integration over a sphere in k-space:

\[ \int_{-\infty}^{+\infty} d^3r \left| \psi(\vec{r}, \vec{k}) \right|^2 = \frac{4\pi m_e}{3} k_1^3 \quad k_1 = \frac{m}{h^2} \sqrt{E_T - U} \]  

In general the probability interpretation of \( \psi \) thus has the following effects: (i) The amplitude \( \chi_0(\vec{k}) \) has to be renormalized according to the integration of \( \psi \) over the whole system under the condition that [8]:

\[ \int_{-\infty}^{+\infty} d^3r \left| \psi(\vec{r}, \vec{k}) \right|^2 = 1 \]  

\[ \tag{13} \]
And (ii) the ensemble wavefunction at a given location within the system then depends on the potentials and amplitudes in all the other parts of the system. A physical process, described via the wavefunction $\psi$ therefore cannot be localized at a specific point $\vec{r}$ of the system.

IV. ENSEMBLES IN THE QUANTUM THEORY OF MOTION

If we assume, as in QT, that the two-slit interference problem can be solved by solving the Schrödinger equation:

$$\hat{H}\psi = E_{in}\psi$$

(14)

where $\hat{H}$ shall be a suitable Hamiltonian and $\psi$ the wavefunction of the problem, while $E_{in}$ denotes the kinetic energy of the incident electrons, then the potentials of the problem should be known. Considering now, that $\psi$ is the wavefunction of the ensemble, it carries two different informations: (i) The information about the behavior of single electrons in the potential environment, the physical side of the problem, and (ii) the information about the change of the quantum ensemble due to the changes of the environment, the statistical side of the problem.

Not considering the magnetic properties of electrons an analysis of Tonomura’s experiments may be limited to the scalar problem. Which means, that the polarizations of intrinsic fields may be disregarded. From the viewpoint of single wavelets the unknown properties are therefore the amplitude and/or the volume of single "particles" [8,9]. For practical reasons we could assume the volume of a single electron to be constant and, in the simplest possible model, even point-like compared to all relevant distances within our system. Then a single point-particle has to comply with two different constraints: (i) The potentials in our system, the physical constraints, and (ii) the ensemble structure of solutions of Schrödinger’s equation due to the variation of the ensemble with the external potentials. While the first is a strictly classical term, in this formalization of the problem described by classical mechanics, the latter is essentially non-classical and originates from the probability interpretation of the wavefunction. Comparing with the standard expressions of QTM, where the change of $S$, the exponent of the ensemble wavefunction is described by:

$$-\frac{\partial S}{\partial t} = V + \frac{(\nabla S)^2}{2m_e} - \frac{\hbar^2}{2m_e} \frac{\nabla^2 R}{R} \bigg|_{l=Q}$$

(15)

it is obvious that (I) describes the classical energy components of particle motion, while (II), where $R$ is the amplitude of the ensemble wavefunction:

$$\psi = R \cdot e^{iS/\hbar}$$

(16)

is a non-classical term. The main difference here is our interpretation of $R$ and $Q$. In QTM it is assumed that "matter ... [is endowed] ... with a field aspect, mathematically described by \( R^2 \) and \( S \) ... which enables ... [QTM] ... to avoid the paradox of an individual’s properties apparently depending on an ensemble." [13]. In our view $Q$, the "quantum mechanical" potential is not due to some non-classical and still physical field, but describes the change of the ensemble due to the change of the "physical" potential $V(\vec{r})$. In this sense it is a basically statistical term, even if it appears as an energy, and it is, as Bohm suspected [11], related to the information about the ensemble rather than any physical quality of the single "particle". The classical limit of motion then is the case, where $\psi$ does not influence the trajectory $\vec{x}(t)$ of any single member.

FIG. 3. Quantum potential for two Gaussian slits of the interferometer, the potential is viewed from the detector screen, its origin is the change of the quantum ensemble due to the change of the physical environment (from Philippidis et al. [4]).

That this interpretation of $Q$ is better suited to describe its real meaning than the current one can also be seen from the fact that the "particle equation of motion is a deduction from the Schrödinger equation" [20], but not vice versa, and, most importantly, that although the particle " responds to the local value of the field in its vicinity (via $Q$) ... there is no reciprocal action of the particle on the wave" [20]. If the quantum potential were of physical origin, the last statement would be equivalent to a violation of Newton’s third law ($\text{actio} = \text{reactio}$). Therefore it cannot be of physical origin.

From a formal point of view the realistic interpretation and QTM consider the Schrödinger equation valid, and it has been held against Bohm’s concept, that it does
not provide additional information \[21\]. In this case it must be conceded that QTM is logically equivalent with standard QT. Which equally applies to the realistic interpretation. Then the result that the Schrödinger equation is not an exact equation (a result of the realistic interpretation \[3\]) requires the existence of additional terms describing the evolution of a system, and which are not related to the evolution of any single - and well defined - member. And in this case one arrives at the same conclusion: the additional term, showing up in QTM as the quantum potential, can only be related to this evolution of the system, described by the Schrödinger equation. In this sense it is also correct to say that QT does not know any well defined object, because of its fundamental equation.

FIG. 4. Trajectories of single particles in a two-slit interferometer from \[4\]. The unusual curvature of the trajectories is due to the quantum potential Q.

It is interesting to note that, as Peter Holland pointed out, most orthodox physicists attack the quantum theory of motion for two, mutually exclusive, reasons: for being too classical (in its conception of particles, trajectories etc.) and for being not classical enough (in its non-locality, the concept of the quantum potential etc.) \[22\]. Zeh recently gave a similar example in an article with the programmatic title: "Why Bohm’s Quantum Theory?" \[21\], when he criticized the theory for being logically equivalent to QT ("successful only because it keeps Schrödinger’s (exact) wave mechanics") and going beyond QT (although "the rest of it is observationally meaningless", the trajectories it describes are thought to result from "unobservable causes for stochastic events"). Now QTM is, of course, logically equivalent to the Schrödinger equation. And once it is accepted, that it is a simplified account of events - the point particle is the simplest possible model for the single wavelets in QT -, then it teaches us something new, since it combines the notion of an individual with the notion of an ensemble in a consistent and instructive manner - something QT has not been able to achieve in seventy years of arguing. The GRW model \[23\] mentioned can not be considered on an equal footing, since it depends on a non-linear addition to the Hamiltonian: where this term should actually come from, remains a mystery, even if Zeh refers it to "fundamentally irreversible spontaneous localization".

In his treatment of the two-slit problem Philippidis begins by calculating the wavefunction $\psi$ after the slits, assumed Gaussian for convenience, and which is then employed to derive the quantum potential Q from Eq. \[15\]. The numerical calculation used characteristic experimental data of a 1961 electron interference measurement by Jönsson \[24\], it was calculated for the whole region between the slits and the detector screen. It is essentially due to the local constraints of the wavefunction at the two-slits, contrary to the previous chapter, where our main emphasis was on constraints due to external potentials. Given the symmetry of QT (and also QTM) concerning real space and momentum space, the difference should not change the picture: a change of the ensemble gives rise, in QT, to a quantum potential Q. Fig. \[3\] reproduces the result of the calculation, the two parabolic peaks in the back coincide with the slit positions. The particle trajectories in QTM are calculated by integrating the equation:

$$\vec{p} = \nabla S$$

where $\vec{p}$ is the momentum of the particle and $S$ the exponent of the wavefunction. Initially, the trajectories from the two slits fan out like for a single slit interference measurement, it is only where Q becomes appreciable, that distinct kinks appear, which are due to a rapid acceleration at the troughs of the quantum potential. The trajectories of single particles are displayed in Fig. \[4\]. The single hits on the screen reproduce the overall pattern of the intensity calculated in the conventional manner, but the single impacts are at distinct locations: in that sense QTM reproduces the full extent of the experimental findings (in contrast to QT, where only the intensity is given). And while in QT the electron is a particle and a wave simultaneously, it is in QTM a particle, guided by its quantum potential Q, which represents the full quantum ensemble of a given environment.

**V. NON-LOCALITY, OR WHAT?**

The question, whether or not an individual electron "knows" of the setup, most remain open, since, as the analysis reveals, none of our current theories gives a local description of the problem. In classical field theories the fact has been known for quite some time, this is, what we mean, when we speak about "waves", but in QT, due to the essentially abstract framework, the same feature is less obvious.

Non-locality, it has been shown, enters the framework essentially via the normalization of the wavefunction, because this cannot be done without considering the ensem-
ble over the whole system. But while in a classical context the non-locality could be argued on the basis of the rather high field extension, this seems no longer a possibility considering current experimental techniques. With femto-lasers and, most strikingly, atom interference measurements \[2\] the typical extension of a single wavelet is well below the separation of the slits of an interferometer. Conceptually, one either has to concede that the formal non-locality is also an experimental fact, because it is hard to support the notion that interactions of an atom with one slit-environment depend on the arrangement of atoms some \(10^4\) atom diameters away (distance of the second slit about 8 \(\mu m\) \[25\]). Or, along a completely different line of reasoning, one argues that the coherence of the beam is the ultimate origin of non-locality, since it guarantees that a single atom fits into the overall pattern (e.g. via its phase). A possibility, which we shall explore in future publications.

The main improvement, compared to the current state of affairs in QTM, is conceptual. While, for example, the calculation of the quantum potential (a physical cause for the motion of a single particle) from the wavefunction (according to current belief \[18\] a statistical measure of events and locations) must remain logically inconsistent, the procedure becomes perfectly sound, once the wavefunction gains a double meaning (a fundamental result of the realistic interpretation \[8\]) and if we concede that the point-particle is only a very crude approximation. In this sense Bohm’s quantum theory of motion appears to be the simplest mathematical form the realistic interpretation can take. Which means also that it’s extension to all fields, e.g. to the atomic domain, might not make too much sense. Because in fundamental processes, where the physics of a system are as well known as in hydrogen \[26\], the simplifications of the quantum theory of motion may well lead to a completely distorted picture.

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