Dynamic analysis of a rotating stepped shaft with and without defects

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ABSTRACT. Rotating machines have many applications, in several mechanical systems. They typically contain a rotary shaft as an influence conversion unit, that is subject to multiple loads in operation. One of the most widely-used rotary machines is the pump, and pump shafts are exposed to many forces due to fluids, unbalancing due to slight bending in the shaft or errors of design or bearings, bearings-induced forces, etc. These forces can reach an unacceptable level once the rotor is functioning close to its natural frequencies. Excessive vibration levels within the device cause fatigue and thus tiny cracks may grow to a serious extent and ultimately cause failure. In this study, dynamic analysis of the pump shaft was made, with and without cracking. Fundamental bending natural frequencies and torsional natural frequencies, response shaft, the equivalent stresses and total deformations in each dynamic and static cases were evaluated. The dynamic load factor (DLF) was calculated in the presence of cracks of various depths (4mm, 6mm, 8mm, 10mm, 12mm) set at diverse locations (x=80mm, x=166mm, x=210mm) measured from the point of overhanging. The finite element method by ANSYS package was used to conduct the numerical analysis for this study, and a specific experimental test rig was built to verify the experimental results. Results showed that increasing depth of the cracks will lead to reducing in the natural frequency and, as a result, increase instability in the shaft. When the location of the crack is close to the highest bend point in the shaft, the natural frequencies will increase. In addition, the equivalent stress depends on cracking location and it is increased with increased depth of cracking.

1. Introduction

The dynamic rotor is an important element in the dynamic behavior of rotary machines, from large systems such as power plant rotors and turbo generators to small systems such as tooth drills and a variety of rotors such as pumps, air compressors, etc. The history of the evolution of rotor dynamics is important because it illustrates the fundamental nature of the various types of problem encountered by simply rolling bearings and developing them for many applications. Dynamic studies of technological applications started in the 19th century, as industrialization drove the need to include rotation in dynamic behavior analysis. Since 1954, several researchers have created studies that agitated cracks in shafts. Several of those results have been extended to incorporate real rotors and this offers knowledge that is crucial to designers.
In general, most rotors work under cyclic pressure so they will be exposed to some operating problems such as fatigue cracks, where the natural frequencies and critical speeds of these rotors increase with the decrease in the length of the shaft and increase the area of the cross-section [1]. For the purpose of establishing a control system that gives a signal to diagnose operating errors as well as cracks at an early stage of growth and to avoid sudden accidents, it is necessary to identify the vibration parameters of the cracked shafts. Many researchers have studied the effect of cracks on the efficiency of rotating shafts. Some have studied these problems in analytical analysis, while some have dealt with these problems in an approximate way so the important factor to reduce the unfavorable vibrations is to control the geometric unbalance of the rotor [2]. Through calculation and knowledge of the mass of unbalance during rotation, the vibration response can be measured for any system [3]. This response will increase with increase in the depth of the cracks, so that the critical rotation speed of the system will decrease, as will the natural frequency [4]. This will produce local flexibility in the rotor column's metal structure. This flexibility is a function of crack depth [5]. The equivalent stiffness of the shafts is affected by the locations and depths of the cracks [6].

The number of cracks affects the response of the system, and the number of cracks in the shaft may be anticipated by measuring the number of natural frequencies; the number of natural frequencies depends on the number of cracks. The first five natural frequencies must be measured for each crack [7]. The torsional vibrations have a strong dynamic response and this response occurs at a frequency that is twice the value of the speed rotation [8]. The torsional natural frequency has a high sensitivity to the depth of the cracks, especially when the ratio of the speed to the critical speed is between 0.8-1.2 [9]. This sensitivity to the cracking may reduce the ability to add extra forces to the system, because it will accelerate the growth of cracking and consequently cause an increase in system defects [10].

Transverse cracks reduce the cross-section area of rotary shafts and cause significant damage to rotors. These walls are the most dangerous and most common, as shown in Figure 1, and have been extensively studied by researchers [11].

In general, the implications of cracking for engineering structures has been investigated with various techniques by multiple researchers through the years. In 2012, M. J. Jweeg et. Al [12], investigated the effect of crack position and size on the vibration characterizations of the plate structure. Then, in 2017, both M. Al-Waily, [13], and M. J. Jweeg et. al, [14] studied the effect of crack angle, position and size on the vibration properties of pipe-induced vibration. After this, in 2018, M. A. Al-Shammarri [15], investigated the effects of crack size on the vibration characterizations of the sandwich plate structure. Finally, in 2019, H. J. Abbas et. al [16], studied the effect of prediction for the crack defect of fiber optical cable, using a vibration technique.

In this study, a model of the rotor of a particular centrifugal pump shaft was used to analyze the effect of cracking on the natural frequency of the rotor. The importance of this study is that the changes in natural frequencies of any rotor may represent a serious indication of the progress of cracks sufficiently deep to generate fracture.
2. Experimental Part

To simulate a stepped shaft, resembling those employed in the centrifugal pump of some petroleum industries, a carbon steel shaft was manufactured with a density of 7850 kg/m$^3$ and Young's modulus of 210x10$^9$ N/m$^2$. The mass of the shaft was determined as 2.19 kg. The dimensions of the manufactured shaft are in Figure 2. A carbon steel disk of 2.5 kg was connected to the free end (A) of the shaft. Experiments were carried out for the uncracked stepped shaft and the stepped shaft having a transverse crack. A suitable cutting tool was used to produce transverse cracks in the stepped shaft, with depths of 4mm, 6mm, 8mm, 10mm and 12mm. Diverse locations were selected for cracks, to examine the effects of the position, as well as the depth, of cracks on the system response.

Figure 2 shows that the selected locations of cracks were x=80mm, x=166mm, and x=210mm measured from the free end (A) of the shaft. Dynamic analyses were conducted for different rotational speeds (500, 1000, 1500, 2000, 2500, 3000 rpm).

![Figure 2. Crack positions on the shaft model.](image-url)
Figure 3, shows the test rig used in this study. It was manufactured to simulate the operation of actual rotors in devices such as centrifugal pumps and steam turbines. Natural frequencies and forced excitation response to external shaking load or unbalance forces, in addition to critical speeds, can be examined and calculated through this test rig. In this device, the impact hammer is used for the purpose of excitation external force and generating vibration in the model.

![Test Rig](image1)

**Figure 3.** Experimental test rig for the tested stepped shafts.

An accelerometer of a maximum frequency of 200MHz, ADS 1202cl model was used in this study with a suitable amplifier and oscilloscope [17-18]. The accelerometer was installed at six different locations on the test rig, one on the disk and the other on the bearing and the rest in different positions on the shaft, as shown in Figure 4.

![Test Positions](image2)

**Figure 4.** The test positions on the stepped shafts.

The response signal is saved in the oscilloscope as excel data and images. SIGVIEW program was used to analyze and draw the estimated signals that coming from the oscilloscope. FFT spectrum analysis was used in the SIGVIEW program, as shown in Figure 5 [19-21].
Figure 5. FFT analysis of the response wave (SIGVIEW) program.

3. Numerical Part

The finite element method (FEM) was used to evaluate fundamental natural frequency, deflection and stresses for the tested isotropic stepped shaft [22-24]. The FEM technique is well-known for analyzing engineering issues in several fields and was used in this study with the help of the ANSYS workbench 16.0 package [25-28].

Beam models were fastened from two supports by two parallel ball bearings.

All of the elements have 6 DOF (degrees of freedom) at every node: translations within the x,y,z nodal directions and rotations regarding the x,y,z nodal axes. The beam boundary conditions are noted in Figure 6, node boundary conditions are illustrated in Figure 7. The tetrahedron is of the meshing kind, and used mesh component size 1mm with 76417 elements and 129600 nodes for the intact shaft, and 76694 elements and 130001 nodes for the cracked shaft.
4. Results and discussions

4.1. Results for Uncracked stepped shaft

The numerical analyses were conducted for equivalent depths and locations of cracks as within the experimental section. The stepped shaft was analyzed numerically in the static and dynamic conditions. Statically, the maximum equivalent von Mises stress and the maximum total deformation were about 2.08Mpa and 6.3µm, respectively, when applying a torque of 4.77 N.m on the disc, as shown in Figure 8.

Using the same boundary conditions as in the static analysis, the dynamic analysis was performed under the effect of sinusoidal torque $T_0 \sin(\omega t)$ where $T_0=4.77$ N.m. The period used in this analysis was one second, with variable rotation speed (500 - 3000 rpm). Table 1 shows the dynamically numerical results for the tested stepped shaft.
Table 1. Equivalent von Mises stress and total deformation at different spin speeds (N) for the stepped shaft.

| Spin speed(N) rpm | Angular velocity(ω) Rad/Sec | Maximum equivalent stress (MPa) | Maximum total deformation (mm) |
|-------------------|-------------------------------|---------------------------------|-------------------------------|
| 500               | 52.360                        | 3.3868                          | 0.0063648                     |
| 1000              | 104.720                       | 3.3882                          | 0.0063674                     |
| 1500              | 157.080                       | 3.3869                          | 0.0063648                     |
| 2000              | 209.440                       | 3.3865                          | 0.0063642                     |
| 2500              | 261.800                       | 3.3856                          | 0.0063623                     |
| 3000              | 314.160                       | 3.3951                          | 0.0063806                     |

The first mode natural frequency was found numerically to be equal to 174.65 Hz, while the fundamental torsional natural frequency was about 495.98 Hz. The experimental results show good agreement with the numerical results. The experimental fundamental bending natural frequency was measured as an average of six different positions and was about 166.65 Hz. Figure 9 shows the result of experimental for one of those tested positions. The fundamental torsional frequency (465.76Hz) was calculated, as can be seen in Figure 10.

![Figure 9. First tested position natural frequency.](image)

![Figure 10. Torsional natural frequency for uncracked tested stepped shaft.](image)
4.2. Results for cracked stepped shaft

4.2.1. Static and dynamic representations

Statically, static analysis for a cracked stepped shaft with the connected disk under the effect of torque 4.77 N.m. Figure 11 shows the effect of the depth of the cracks and locations on the equivalent von Mises stress of the tested shaft, while Figure 12 represents their effects on total deformation of the shaft. It was very clear that increasing the depth of the crack would lead to an increase in the equivalent von Mises stress and deformations, because of its effect on reducing the second moments of inertia in the shaft. In addition, increasing the distance between the crack and the disc would have the same effects as the depth of the crack. That is because the crack’s position will affect the value of moments where the stresses are at their maximum values.

![Figure 11. Von Mises stress of cracked shaft.](image1)

![Figure 12. Total deformation of cracked shaft.](image2)

Dynamically, the equivalent stresses and deformations will take a sinusoidal shape because the bending moment will change as the angular position of the test point changes at a time (1 sec), as shown in Figure 13. This fact is very clear through Figures 14 and 15, which represent the time dependency of the stress and deformation, respectively, for a stepped shaft having a crack of 4mm depth located at 80 mm from the tip of the stepped shaft when it is running at 3000 rpm.

![Figure 13. Moments time curve at the coupling position (B).](image3)
The dynamic load factor (DLF), the magnitude relationship of dynamic stress to static stress, was calculated. It was 1.64, since the maximum dynamic stress for the tested stepped shaft was about 3.41 MPa while the maximum static stress was about 2.08 MPa.

4.2.2. Natural frequencies for cracked stepped shafts

The results of the numerical and experimental work on bending natural frequencies are given in Table 2. The experimental results for each case represent an average of four readings. The standard deviation between the numerical and experimental results was calculated to be 11.3%. Torsional natural frequencies for the same cracked shafts were also calculated numerically and experimentally, and are listed in Table 3.

The experimental and numerical results show good compatibility, but with some differences. The differences in the results are due to, for example, the laboratory conditions that were inhospitable to the test, such as noise, temperature, etc. All of these conditions affect the vibrations, causing different results.

The effect of the cracks and depths on the natural frequencies of the tested shafts can be observed in Figure 16. Increasing crack depth will lead to a large decrease in bending natural frequencies because of their negative effect on the stiffness of the shaft. Changing the cracks’ positions also affects the lateral natural frequency of the shaft, since the equivalent stress is a function of the crack position.

![Figure 14. Dynamic stress of cracked shaft.](image1.png)

![Figure 15. Dynamic deflection of cracked shaft.](image2.png)
Figure 16. Fundamental bending natural frequencies corresponding to cracks depths and positions.

Table 2. Fundamental bending natural frequencies for cracked stepped shafts.

| No. | Crack depth (mm) | Experimental (Hz) | Numerical (Hz) | Discrepancy percentage (%) |
|-----|------------------|-------------------|----------------|-----------------------------|
| 1   | 4                | 163.98            | 175.36         | 6.93                        |
| 2   | 6                | 161.21            | 175.00         | 8.55                        |
| 3   | 8                | 160.76            | 174.04         | 8.26                        |
| 4   | 10               | 156.81            | 173.14         | 10.41                       |
| 5   | 12               | 154.03            | 171.54         | 11.36                       |

| No. | Crack depth (mm) | Experimental (Hz) | Numerical (Hz) | Discrepancy percentage (%) |
|-----|------------------|-------------------|----------------|-----------------------------|
| 1   | 4                | 159.42            | 172.02         | 7.9                         |
| 2   | 6                | 157.39            | 168.55         | 7.09                        |
| 3   | 8                | 154.75            | 163.45         | 5.62                        |
| 4   | 10               | 143.93            | 156.51         | 8.74                        |
| 5   | 12               | 136.76            | 147.44         | 7.8                         |

| No. | Crack depth (mm) | Experimental (Hz) | Numerical (Hz) | Discrepancy percentage (%) |
|-----|------------------|-------------------|----------------|-----------------------------|
| 1   | 4                | 155.35            | 171.74         | 10.55                       |
| 2   | 6                | 152.76            | 167.71         | 9.78                        |
| 3   | 8                | 148.01            | 161.90         | 9.38                        |
| 4   | 10               | 140.94            | 154.80         | 9.83                        |
| 5   | 12               | 131.86            | 145.64         | 10.45                       |
Table 3. Fundamental torsional natural frequencies for cracked shafts.

**d- At crack position (x=80 mm).**

| No. | Crack depth (mm) | Experimental (Hz) | Numerical (Hz) | Discrepancy percentage (%) |
|-----|------------------|-------------------|----------------|-----------------------------|
| 1   | 4                | 461.52            | 495.8          | 7.42                        |
| 2   | 6                | 454.84            | 493.59         | 8.51                        |
| 3   | 8                | 447.42            | 489.4          | 9.38                        |
| 4   | 10               | 441.49            | 484.07         | 9.64                        |
| 5   | 12               | 435.03            | 476.46         | 9.53                        |

**e- At crack position (x=166 mm).**

| No. | Crack depth (mm) | Experimental (Hz) | Numerical (Hz) | Discrepancy percentage (%) |
|-----|------------------|-------------------|----------------|-----------------------------|
| 1   | 4                | 454.47            | 494.14         | 8.72                        |
| 2   | 6                | 447.51            | 491.69         | 9.87                        |
| 3   | 8                | 439.92            | 487.94         | 10.91                       |
| 4   | 10               | 434.05            | 482.59         | 11.18                       |
| 5   | 12               | 431.78            | 475.17         | 10.04                       |

**f- At crack position (x=210 mm).**

| No. | Crack depth (mm) | Experimental (Hz) | Numerical (Hz) | Discrepancy percentage (%) |
|-----|------------------|-------------------|----------------|-----------------------------|
| 1   | 4                | 449.15            | 495.8          | 10.38                       |
| 2   | 6                | 447.72            | 494.52         | 10.45                       |
| 3   | 8                | 445.62            | 492.49         | 10.51                       |
| 4   | 10               | 445.59            | 489.98         | 9.96                        |
| 5   | 12               | 442.52            | 486.6          | 9.96                        |

5. Conclusions
Cracks affect the fundamental natural frequency; in our study, the fundamental natural frequency has decreased as the depths of cracks increased. Fundamental natural frequencies are affected by the locations of the cracks. It was seen that the natural frequency increases as the location of cracks
approach the location of the mass (disc) near the free end of the shaft. The amplitude of vibration is affected dramatically by the presence of cracks, and the nonlinear vibrations are induced at high speed because of shaft rotation. The equivalent stresses and deformations increase with increasing crack depths. It was observed also that the stress and deformation increased as the location of cracks approached the location of the impeller mass, near the free end of the shaft.

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