Performances of Symmetric Loss for Private Data from Exponential Mechanism

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Abstract—This study explores the robustness of learning by symmetric loss on private data. Specifically, we leverage exponential mechanism (EM) on private labels. First, we theoretically re-discussed properties of EM when it is used for private learning with symmetric loss. Then, we propose numerical guidance of privacy budgets corresponding to different data scales and utility guarantees. Further, we conducted experiments on the CIFAR-10 dataset to present the traits of symmetric loss. Since EM is a more generic differential privacy (DP) technique, it being robust has the potential for it to be generalized, and to make other DP techniques more robust.

Index Terms—Machine learning, Symmetric loss, Differential privacy, Label differential privacy, Exponential mechanism

I. INTRODUCTION

Individual information privacy has become a severe issue with the development of data analysis techniques. Risk of exposing sensitive data would discourage respondents to cooperate honestly with our caring investigations. Even worse, it can also discourage the participation of respondents. Therefore, many researchers propose to corrupt a part of data before publication. Data corruption and publication techniques that reveal the maximum possible amount of knowledge, with minimum possible risk of exposure of sensitive data, are imperative for data analysis.

There are several notions proposed for private data publications such as $k$-anonymity [1] or $\ell$-diversity [2]. One of the most commonly-known notions is DP [3]. Many researchers are interested in DP because it can precisely quantify information leakage, usually called privacy budget, in each publication.

Several researches in DP aim to find the most precise publication under a given privacy budget. Many, such as [4], aim to publish a precise machine learning (ML) output under DP. However, none of them consider relationship between the precision and loss function, which is one of the most important components in several ML approaches.

A. Our Contributions

In this study, we consider a type of loss function called symmetric loss. Symmetric loss, and, in particular, barrier hinge loss, are proven to be robust against corrupted data [5].

As we corrupt data for privacy in DP, we believe that the loss is suitable for DP. Indeed, the corrupted model considered in [5] is identical to the corruption in randomized response (RR) [6], [7]—which is one of the publication techniques under DP. We can immediately apply the results in [5] to RR.

We question if symmetric losses can also be robust for other publication techniques. In this work, we consider publications under EM [8], [9]. While RR requires that all data records must be independently sampled, we do not need that assumption in EM. Exponential mechanism is then more applicable than RR, and thus, is worth considering. Our contributions can be summarized as follows:

1) Theoretical Results: We presented and proved a series of properties between data scale and the expected utility range while implementing EM. Further, we provide numerical guidance of privacy budgets for EM. In particular, we provide theoretical evidence that an accurate learning output can be obtained with almost certain probability using EM and symmetric loss.

2) Experimental Results: We investigated the robustness of barrier hinge loss on an EM processed CIFAR-10 dataset. The experiment results have been presented. Even with a small privacy budget $\epsilon = 0.5$, the output accuracy could reach a level of 80%. The accuracy is larger for larger $\epsilon$. In this sense, we are allowed to use highly protected data.

3) We corrected the mistake in the formula for implementing EM in [9].

B. Related Works

Current DP research includes two major directions. The first direction is concerned with how to quantify privacy distances. The original definition of DP is sometimes too trivial for different privacy demands. Therefore, providing a wide range of definitions, such as relaxed $(\epsilon, \delta)$-DP [10], Rényi-DP [11], and so forth—provides higher dimensions for researchers to demonstrate inspirations for new mechanisms.

The second direction is to provide new mechanisms, several of which are for ML applications. As ML and DP are both data based, applying ML approaches in DP mechanisms has become popular. Specifically, as label-DP regime [4] naturally fits the ML methods, rolling-up ML approaches for better utility DP frameworks is simple, such as in [4], [12]. In [4], many aggregated ML-DP frameworks are presented.

Our discussion is different from the above two directions as follows. Firstly, this work is still based on the traditional DP definition. Therefore, this research can easily be extended

Unfortunately, the robustness proof in [5] assumes that the records are independently sampled. We are considering a robustness proof that does not require the independence.
to other privacy quantifications. Further, we do not focus on
aggregating multiple methods to make new DP frameworks.
We only present performances of loss functions on EM
processed data. Unlike the examples in [4] that purely
pursue better performance with unsupervised or ensemble
learning pre-processing, our experiments still demonstrate the
insufficiency of only depending on loss functions—especially
while the privacy budget is small. However, as loss functions
are fundamental elements in ML, testing them can provide
a reference of modifications for a series of ML-DP hybrid
frameworks.

C. Paper Organization

The rest of the paper is organized as follows. In section
II, we present an introduction to symmetric loss with its
properties, and EM with its implementation. In section
III, we show some theoretical properties of EM. In section
IV, we present the robustness of barrier hinge loss on EM.

According to PDAA regular paper format, proofs of
properties in section III are moved to Appendix, which is not
included in the publication. One can find the full version on
the ArXiv.

II. PRELIMINARIES

A. Symmetric Loss in Machine Learning

Any given data can be represented by a series of real values,
a \((d+k)\)-tuple \((x \in \mathbb{R}^d, y \in \mathbb{R}^k)\). In ML, x are called features,
and y are called labels. Our dataset \(D\) is a collection of those
tuples; i.e., \(D := \{(x_i, y_i)\}_{i=1}^n\) is a dataset with size \(n\). It is
not necessary to have all meaningful values for all entries in a
tuple in a dataset \(D\). For example, we may have 0’s or wrong
values for some entries. Normally, features are treated as fully
and correctly known elements; otherwise, we can move the
unclear entry or entries to the label side.

Machine learning is meant to find the internal relationship
between features and labels in a set, and make predictions for
possible or potential data in the same problem, we normally
restrict the expectations to certain sizes, which arc named
empirical risks, in the form of \(\frac{1}{|D|} \sum_{(x,y) \in D} l(f(x), y)\).

Trivial empirical risk unbiasedly weighs the losses from
evory data point, which outputs functions that suffer from
disparate data treats. Manipulating it is necessary and,
and different predictors can clearly be derived. In binary
classification problems, to fairly evaluate the errors from
positive and negative labeled data, one would leverage the
receiver operating characteristic curve (AUC) [13], or the
balanced error rate (BER) [14], [15]. AUC and BER are
defined in the following definitions.

**Definition 1.** Empirical AUC-risk is defined as:

\[
R_{AUC} = E_{D_P}[E_{D_N}[l(f(x_P) - f(x_N))]]
\]

**Definition 2.** Empirical BER-risk is defined as:

\[
R_{BER} = \frac{1}{2} \left[ E_{D_P}[l(yf(x))] + E_{D_N}[l(yf(x))] \right]
\]

The restrictions to certain datasets still carry in distances
between empirical minimizers and Bayes minimizers. Let \(f^*\)
be the optimal function that describes the relation of the
problem, and \(\mathcal{F}\) is a collection of functions restricted to
\(D\). Such distances are described in a Bayes sense \(R(f^*) -
\inf_{f \in \mathcal{F}} R(f)\), named excess risks [16]. Classical excess risks are
defined on a 0-1 loss function, which is the function
\(l_{01}\) such that \(l_{01}(x) = 1\) for \(y \leq 0\), and \(l_{01}(x) = 0\)
otherwise. However, because of the non-differentiability and
non-convexity of a 0-1 loss at the crucial point 0, which
harms the learning fluency and cost, we normally adopt other
(surrogate) losses instead—like a logic sigmoid function, or
hinge functions, and so forth.

The unavoidable issue is how to guarantee the minimizer
of surrogate excess risk to coincide with the minimizer of 0-1
excess risk. This fundamental property is called classification-
calibration, and has been proven to be satisfied for most
commonly used surrogate losses (e.g., exponential, quadratic,
hinge, and sigmoid [16]). [5] has extended this property to
also be satisfied by symmetric functions, which are function
\(f^*\) such that there is \(C\) where \(f(x) + f(-x) = C\) for all \(x\).
(Therefore, the function is symmetric about the intersection
point with y-axis, not symmetric about y-axis.) Despite this
merit, normal symmetric functions cannot simultaneously
satisfy non-negative values, and convexity [17], [18] (except
for the horizontal lines). To cope with the issue, [5] proposes
the use of a restricted symmetric loss, called a barrier hinge
function, which can be defined as follows:

**Definition 3** ([5]). A barrier hinge function with parameter
\(b, r\) can be defined as follows:

\[
f_{b,r}(x) = \max(-b(r + x) + r, \max(b(x - r), r - x)).
\]
The barrier hinge function is symmetric on \( x \in [-r, r] \), when \( b > 1 \) and \( r > 0 \). Therefore, it satisfies classification-calibration for the sensitive interval \([-r, r]\), and satisfies the non-negativity and the convexity conditions at the same time.

In our experiments, we leverage both AUC and BER risks as complements to each other to evaluate the overall performances of several surrogate losses. Further, we also show the superiority of the barrier hinge loss over other losses on mild privacy cost.

B. Exponential Mechanism

Differential privacy [3] aims to protect sensitive information of each individual with minimum harm to the utilities (learning results). The strategy of adding noise is to obfuscate every possible dataset \( D \) with its most likely set of datasets \( \{D' | D' \sim D\} \) with a distribution corresponding to the similarity (e.g., Hamming distance as in the following). Formally, we say a method or a randomized algorithm \( \mathcal{A} \) is \( \epsilon \)-differential privacy preserving [3] if, for all possible measurements \( \mathcal{M} \),

\[
\frac{\Pr[\mathcal{A}(D) \in \mathcal{M}]}{\Pr[\mathcal{A}(D') \in \mathcal{M}]} \in [\exp(-\epsilon), \exp(\epsilon)],
\]

for any given privacy budget \( \epsilon \).

Label DP [4] is a relaxation version of DP, which constrains the private to only lie on a subset of columns in an information table. Since the result of this relaxation fits the ML regime naturally, without any extra work, ML models could be interspersed. [4] demonstrated their frames of leveraging unsupervised and semi-supervised learning models in improving utilities of private data.

One of the most classical and widely used DP methods is RR, introduced in [6]. By simply randomly flipping the data with a fixed probability, we are able to protect individual privacy, and infer the classification ratio of the sensitive information. However, this classical method requires the plausibly deniable data to be mutually independent. Otherwise, flipping a part of the dependent data would not be sufficient. Unfortunately, several existing datasets are not entirely independent—thus, RR would not be efficient because of this. Instead, researches prefer to adopt EM, an upgraded version from Laplace mechanism [10].

Definition 4 (Exponential Mechanism (EM) [8]). Given any real valued function \( q : \mathcal{D} \rightarrow \mathbb{R} \), with \( \Delta q := \max_{D,D'} |q(D) - q(D')| \). The EM can pose any possible dataset \( D'' \in \mathcal{D} \) with the probability \( P(D'') \propto \exp\left(\frac{q(D'')^2}{2\Delta q}\right) \).

Proposition 1. Exponential mechanism is \( \epsilon \)-differentially private.

Defining the real valued function \( q \) is the essential step [8], which provides the original power of this mechanism. The global sensitivity \( \Delta q \) of the maximum distance of adjacent datasets is used for binding the range of the distribution.

One common approach in defining \( q \) is based on Hamming distance [19]: Let \( \delta(D) = (x_i, y_i)_{i=1}^n, D' = (x_i, y'_i)_{i=1}^n := \{|i| y_i \neq y'_i\} \). We define the Hamming distance \( q \) as \( q(D, D') := |D_p \cap D'_p| + |D_p \cup D'_p| = |D'| - \delta(D, D') \) [9]. The implementation of EM contains the calculations of the probability distribution over \( \mathcal{D} := \{D' = (x_i, y'_i)_{i=1}^n : (y_j)_{i=1}^n \in \{-1, +1\}^n\} \). A straightforward implementation would cost \( O(n^2) \) time and space. Redundant exponential function calculations carry an extra burden. Therefore, we apply the two-step approach [9]. Firstly, we compute the distribution for each possible score by logarithmic recursion. Please note that there is a misprint in [9] where they drop the term \( \epsilon/2 \) there.

\[
\log \Pr(q = i) = \begin{cases} -|D'| \log(1 + \exp(\frac{\epsilon}{2\sqrt{\epsilon}})) & \text{when } i = 0 \\ \log(|D'| - i + 1) - \log i + \frac{\epsilon}{2} + \log \Pr(q = i - 1) & \text{when } i > 0 \end{cases}
\]

Then, we uniform-randomly select one table within the same score group, i.e., from the selected value of \( q \), and select \( D \) from the set \( \{D' \in \mathcal{D} : q(D', D') = q\} \) with uniform probability. This two-step method cuts down the time and space complexity to a linear size. One noticeable point is, by this definition of \( q \), EM will degrade to RR when the data size goes to infinity. Despite this odd fact, this degradation would not happen in real situations, but is still worth our study. Therefore, it is an open problem to theoretically prove the relation between EM and RR.

III. Properties in Exponential Mechanism

There is however one issue of the EM for binary classification using semi-supervised learning. The table, which we use for the classification, might be heavily flipped such that the learning algorithms cannot correctly learn the function. When we use symmetric loss, it is known that a learning algorithm can still provide a good result if the flipping rate is no more than half of the dataset—i.e., \( q \geq |D|/2 \) [5]. In this section, we consider the probability that \( q \geq |D|/2 \) for each dataset size \( n = |D| \), and each privacy budget \( \epsilon \). Specifically, we consider \( \sum_{i \geq |D|/2} \Pr(q = i) \). We conduct theoretical analyses, then give numerical guidance for real implementations.

In the following subsections, we only give out the short and straight proofs for corollaries. The important proofs of properties are demonstrated in the appendices.

A. Success probability and the dataset size

In this subsection, we study the relationship between \( \sum_{i \geq |D|/2} \Pr(q = i) \), and the size of dataset \( n \). We begin our illustration by proving generic properties of a binomial distribution.

Definition 5. A truncated binomial distribution \( S \) with \( n \) trials is the partial sum of the binomial distribution restricted to some certain number of events. For example, between \( j \) and \( k \):

\[
S(n,j,k) = \Pr[j \leq I_{n,p} \leq k] = \sum_{i=j}^{k} \binom{n}{i} p^i (1 - p)^{n-i},
\]
where \( p \) is the probability of the binomial distribution.

**Definition 6.** The upper truncated binomial distribution by

\[
S(n, j) = \Pr[I_{n,p} \leq j] = \sum_{i=0}^{j} \binom{n}{i} p^i (1-p)^{n-i}.
\]

In the lower truncated case, we may just use \( 1 - S(n, j) \).

**Property 1.** Truncated binomial distribution \( S(n, j) \) is monotonically decreasing w.r.t. \( n \), while \( j \in [0, n] \) is fixed.

**Property 2.** Truncated binomial distribution \( S(n, n-k) \) is monotonically increasing w.r.t. \( n \), while \( k \in [0, n_0] \) and \( n_0 \) are fixed.

These two properties seem to be trivial, however, the proofs are not intuitive, see Appendix. Moreover, above properties are simple enough to not involve probability \( p \). With the analysis getting deeper, we are able to deduce the following property considering \( p \) with the help of above properties.

**Property 3.** Truncated binomial distribution \( S(n, \lfloor \frac{n}{2} \rfloor + k) \) is almost monotonic w.r.t. \( n \), except for when \( n \) changes between even and odd—this is while \( k \in [-\lfloor \frac{n}{2} \rfloor, \lfloor \frac{n}{2} \rfloor] \), and \( n_0 \) is fixed. Moreover, if \( p \leq \frac{n-\lfloor \frac{n}{2} \rfloor-k}{n+k} \), \( S \) is almost monotonically increasing; and is decreasing if \( p \geq \frac{n-\lfloor \frac{n}{2} \rfloor-k}{n+k} \), respectively.

The relationship between \( n \) and \( S(n, j) \) is illustrated as discussed in Property 3 in Fig. 1. This observation actually counters the normal intuition, we demonstrate it out for appliers not falling into intuition traps. We then use the property in the following corollary.

**Corollary 1.** The probability that the score \( i \) taken in EM, \( \sum_{i \geq |D|/2} \Pr[q = i] \), is no less than half of the maximum possible is almost monotonically increasing w.r.t. the data size \( n \).

**Proof.** Firstly, such a probability is given by:

\[
\sum_{i \geq |D|/2} \Pr[q = i] = \sum_{i=0}^{n} \binom{n}{i} \exp(\frac{\epsilon^2}{2n}) \frac{1}{1 + \exp(\frac{\epsilon}{2n})}^n = \sum_{i=0}^{n} \binom{n}{i} \left( \frac{1}{1 + \exp(\frac{\epsilon}{2n})} \right)^{n-i} \left( \exp(\frac{\epsilon}{2n}) \right)^i = S(n, [\frac{n}{2}]) = S(n, [\frac{n}{2}] - 1),
\]

which is exactly a truncated binomial distribution with the probability \( p = \frac{1}{1 + \exp(\frac{\epsilon}{2n})} \). As \( \epsilon \geq 0 \) and \( \Delta > 0 \), then

\[
p \leq \frac{1}{2} \leq \frac{n - [\frac{n}{2}] + 1}{n + 1} = \frac{n - [\frac{n}{2}]}{n + 1}.
\]

Then, by Property 3, our probability \( \sum_{i \geq |D|/2} \Pr[q = i] \) is almost monotonically increasing w.r.t. \( n \). \( \square \)

**B. Success probability and privacy budget**

Now, let us consider the relation between the probability \( \sum_{i \geq |D|/2} \Pr[q = i] \) and the privacy budget \( \epsilon \).

**Property 4.** The truncated binomial distribution \( \Pr[I_{n,p} \leq j] \) is monotonically decreasing w.r.t. probability \( p \). Moreover, if \( p = 0 \), then \( \Pr[I_{n,p} \leq j] = 1 \) constantly; if \( p = 1 \), then \( \Pr[I_{n,p} \leq j] = 0 \).

**Corollary 2.** The probability that the score taken in EM is greater than or equal to half of the possible maximum, \( \sum_{i \geq |D|/2} \Pr[q = i] \), is monotonically increasing w.r.t. the privacy budget \( \epsilon \). Further, \( \lim_{\epsilon \to \infty} \sum_{i \geq |D|/2} \Pr[q = i] = 1 \).

**Proof.** Since the mean of the flipping probability \( p = \frac{1}{1 + \exp(\frac{\epsilon}{2n})} \) is negatively related to \( \epsilon \), then it is trivial. \( \square \)

**C. Recommendations on privacy budget decisions**

By the results in the previous two subsections, we can recommend the appropriate value of \( \epsilon \) on each \( n \). First, we recall that the results we have so far are as follows:

**Theorem 1.** The probability that the score taken in EM is greater than or equal to half of the possible maximum, \( \sum_{i \geq |D|/2} \Pr[q = i] \), is (almost) monotonic w.r.t. \( n \) and \( \epsilon \) when the limits tend to 1.

Above results are clean from involving truncated point \( j \), because our proofs did not involve the concentration inequalities [20]. To apply those inequalities, we need to discuss the sign of the subtraction \( j - E(\sum_{i=1}^{n} X_i) = j - np \). Therefore, we are forced to discuss the relation between \( p \) with the truncation parameter \( j \), which would extend to unnecessarily complicated proof procedures for above clean results. Adding more factors to our analysis, we will use central limit theorem and concentration inequalities for the
following proposition, see Appendix. And as one can see, \( j \) is also involved in the relations.

### TABLE I

REPLIED TO PRIVACY BUDGETS FOR 99.9% CONFIDENCE (EMPTY CELLS ARE NOT APPLICABLE)

| \( n \) | 0% | 50% | 45% | 40% | 35% | 30% | 25% | 20% | 15% | 10% | 5% |
|-------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| 1      | 5.360 | 1.562 | 2.349 | 2.000 | 2.355 | 4.098 | 5.560 | 8.487 | 7.272 | 5.360 | 3.499 |
| 10000  | 0.472 | 0.884 | 1.316 | 1.779 | 2.292 | 2.884 | 4.160 | 5.497 | 7.272 | 5.360 | 3.499 |
| 100000 | 0.149 | 0.552 | 0.967 | 1.404 | 1.875 | 2.401 | 3.777 | 4.847 | 6.857 | 5.458 | 4.529 |
| 1000000| 0.041 | 0.449 | 0.860 | 1.290 | 1.751 | 2.260 | 3.647 | 4.529 | 6.613 | 5.605 | 4.636 |

### TABLE II

Refer to Privacy Budgets for 95% Confidence (Empty Cells are Not Applicable)

| \( n \) | 0% | 50% | 45% | 40% | 35% | 30% | 25% | 20% | 15% | 10% | 5% |
|-------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| 1      | 5.360 | 1.562 | 2.349 | 2.000 | 2.355 | 4.098 | 5.560 | 8.487 | 7.272 | 5.360 | 3.499 |
| 10000  | 0.472 | 0.884 | 1.316 | 1.779 | 2.292 | 2.884 | 4.160 | 5.497 | 7.272 | 5.360 | 3.499 |
| 100000 | 0.149 | 0.552 | 0.967 | 1.404 | 1.875 | 2.401 | 3.777 | 4.847 | 6.857 | 5.458 | 4.529 |
| 1000000| 0.041 | 0.449 | 0.860 | 1.290 | 1.751 | 2.260 | 3.647 | 4.529 | 6.613 | 5.605 | 4.636 |

**Proposition 2.** For a given dataset of size \( n \), to leverage EM with minimum probability \( P \), the score should be no less than \( n-j \), i.e., \( \sum_{i=j}^{n-j} \Pr[q = i] \geq P \), and we need a privacy budget to be no less than \( 2\Delta \log \frac{n-j + \sqrt{-n \log (1-P)/2}}{j - \sqrt{-n \log (1-P)/2}} \).

**Proof.** Set \( p = \frac{1}{1+\exp(\frac{-j}{n})} < \frac{1}{2} \). We use \( X_i \) to denote the event for each trial. We then have \( E[\sum_{i=1}^{n} X_i] = np \), and we apply Hoeffding’s inequality \( [20] \) of \( \sum_{i=n-j}^{n-j} \) for a given dataset of size \( n \), if we then have

\[
\Pr \left[ \left| \sum_{i=1}^{n} X_i - np \right| > np \right] \leq 2 \Delta \log \frac{n-j + \sqrt{-n \log (1-P)/2}}{j - \sqrt{-n \log (1-P)/2}}.
\]

By substituting back \( \epsilon \), we obtain

\[
\epsilon \geq 2 \Delta \log \frac{n-j + \sqrt{-n \log (1-P)/2}}{j - \sqrt{-n \log (1-P)/2}}.
\]

**Corollary 3.** To ensure a 99.9% probability of having a flipping rate that is no greater than \( \epsilon \), we need a privacy budget to be no less than \( 2\Delta \log \frac{1+2\sqrt{(1.5 \log 10)/n}}{1-2\sqrt{(1.5 \log 10)/n}} \).

The tables on the left side provide the suggestions for privacy budgets of commonly referred to scenarios with sensitivity 1.

**Remark:** The cases for the speeds of the increments of truncated points to be \( \frac{1}{2} \) (flipping rate no more than \( \frac{1}{2} \)) of \( n \) have been proven. We may further derive the results for different increment speeds following similar steps. However, since other speed factors are not our major concerns in our experiments of EM, we will not spend space to discuss here.

### D. Exponential mechanism for large dataset

Our analyses on the success probability has been completed in the previous subsection. In this section, we use the results obtained to show a relationship between the EM and the randomized response in large dataset.

After the trend of truncated binomial distribution has been learned, we may further investigate the behaviors in extreme situations—for example, \( n \rightarrow \infty \). According to De Moivre–Laplace theorem (Central Limit Theorem for binary distributions [21]), a binomial distribution will converge to a normal distribution \( \mathcal{N}(np, np(1-p)) \). Moreover, the probability mass will converge to the probability density:

\[
\int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi np(1-p)}} \exp \left( -\frac{(i-np)^2}{2np(1-p)} \right) dx.
\]

Hence, we may use an integral to approximate our truncated sum:

\[
\int_{j}^{k} \frac{1}{\sqrt{2\pi np(1-p)}} \exp \left( -\frac{(i-np)^2}{2np(1-p)} \right) di.
\]

Now, let us omit factor \( k \) for context simplicity.

**Property 5.** The truncated binomial distribution \( S(n, \lfloor \frac{n}{2} \rfloor) \) approaches 1 if \( p < \frac{1}{2} \), 0 if \( p > \frac{1}{2} \), and \( \frac{1}{2} \) if \( p = \frac{1}{2} \), as \( n \rightarrow \infty \).

**Corollary 4.** For any \( p < \frac{1}{2} \) (or \( p > \frac{1}{2} \), respectively), there always exists a positive integer \( R \) such that \( S(n+r, \lfloor \frac{n+r}{2} \rfloor + k) \geq S(n, \lfloor \frac{n}{2} \rfloor + k) \) when for all \( r > R \) (or \( S(n+r, \lfloor \frac{n+r}{2} \rfloor + k) \leq S(n, \lfloor \frac{n}{2} \rfloor + k) \), respectively).

**Proof.** By Property 3, we know \( S(n, \lfloor \frac{n}{2} \rfloor + k) \) is monotonic w.r.t. parities. However, by Property 2

\[
S(2m+1, \lfloor \frac{2m+1}{2} \rfloor + k) \leq S(2m, \lfloor \frac{2m}{2} \rfloor + k).
\]

Moreover, it is not necessary that for a fixed odd number \( R_0 \) such that

\[
S(2m+R_0, \lfloor \frac{2m+R_0}{2} \rfloor + k) \geq S(2m, \lfloor \frac{2m}{2} \rfloor + k).
\]
And, Property 5 tells us that
\[
\lim_{\text{odd } R_0 \to \infty} S(2m + R_0, \left\lfloor \frac{2m + R_0}{2} \right\rfloor + k) = 1 \geq S(2m, \left\lfloor \frac{2m}{2} \right\rfloor + k).
\]
Thus, using the definition of the limit, some interchange point \( R \) must exist.

**Property 6.** For any \( \delta \in (0, 1) \), the truncated binomial distribution \( S(n, n(p - \delta), n(p + \delta)) \) approaches 1 as \( n \to \infty \).

**Corollary 5.** EM degrades to RR as \( n \to \infty \).

**Proof.** Property 6 states that the magnitude of the distribution would stack fully on one point if \( n \to \infty \), which means the flip rate of labels would become fixed to \( p = \frac{1}{1 + \exp(-\Delta)} \). Therefore, the two-step EM actually becomes a one-step RR.

The illustration showing how EM degrades to RR is shown in Fig. 2.

**IV. Experiment**

Experiments were conducted on the CIFAR-10 dataset with a convolutional neural network (CNN). The AUC score and balanced accuracy (BER) were used for the evaluation. Data sizes were majorly issued to full scale (10000 for training, 2000 for testing), and 10\% scale (1000 for training, 200 for testing), see Table III, Appendix. Privacy budgets were chosen in a common range. Labels were binary-categorized, and were processed by EM beforehand. The experiment was conducted 10 times, each consisted of 50 epochs. Other commonly used losses are comparing groups. We set the barrier hinge loss parameters as \( b = 200 \) and \( r = 50 \).

Overall, barrier hinge loss presents higher accuracies on medium small privacy parameters \( \epsilon \in [0.1, 3] \), which results from the robustness of symmetry on the noise condition [5]. It also presents acceptable accuracies for moderate privacy \( \epsilon \geq 0.5 \), which could outstrip other losses for real applications. One plausible reason for the slightly lower performances of extremely clean data could be the value of symmetric range \( R \). One may tune \( b \) and \( r \) to fit specific needs. Leveraging EM to process the data also results in higher standard deviations of accuracies than RR [5], which is reasoned from the flexibility of the flipping rate as shown in Fig. 2. Further, as \( n \) gets larger, standard deviation decreases Fig. 3. Training with small \( n \) may result in underfitting, which could be explained by the convergence speed of each loss in [5]. We also observe that the standard deviation of barrier hinge loss converges faster than other loss w.r.t. the increasing of \( \epsilon \). While sigmoid function presents better convergence rate w.r.t. \( n \), sigmoid function also presents its second optimum among all functions in our experiments, which re-confirms its success in recent years. Although in our analysis, convergence rates (e.g., of standard deviations) are not studied, it could be interesting for a further research, because the strength of oscillating of data accuracy is one elementary control corresponding to purposes.

**V. Conclusion**

We explored the performance of barrier hinge loss on EM processed private data. The experimental results show the robustness of this loss over other commonly used loss functions. Although our results were only derived from AUC and BER evaluations, the experiments can be evaluated with other standards easily. Moreover, we analyzed the properties of EM and provided numerical guidance of setting private parameters. Further, it was found that with the common definition of utility, EM would degrade to RR when the data size increases to infinity. This phenomenon is not fully understood, and is worth further investigation.

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TABLE III
MEAN AND STANDARD DEVIATION OF AUC/BER FOR ‘AIRPLANE VS CAT’ WITH DIFFERENT $\epsilon$, WHERE $n = 1000$, $6000$, $10000$

| $\epsilon$ | Loss | 0.1 | 0.5 | 1.0 | 5.0 | 10.0 | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AUC | BER | AUC | BER | AUC | BER | AUC | BER | AUC | BER |
| | | | | | | | | | | | | |
| Barrier | 58.2(3.7) | 80.0(2.2) | 88.9(0.9) | 68.0(1.4) | 88.6(0.6) | 90.9(1.0) | 50.5(2.3) | 50.0(2.1) | 50.7(2.5) | 50.2(2.6) | 51.4(2.7) | 51.6(3.0) |
| Sigmoid | 58.2(3.7) | 80.0(2.2) | 88.9(0.9) | 68.0(1.4) | 88.6(0.6) | 90.9(1.0) | 50.5(2.3) | 50.0(2.1) | 50.7(2.5) | 50.2(2.6) | 51.4(2.7) | 51.6(3.0) |
| Unhinged | 58.2(3.7) | 80.0(2.2) | 88.9(0.9) | 68.0(1.4) | 88.6(0.6) | 90.9(1.0) | 50.5(2.3) | 50.0(2.1) | 50.7(2.5) | 50.2(2.6) | 51.4(2.7) | 51.6(3.0) |
| Savage | 58.2(3.7) | 80.0(2.2) | 88.9(0.9) | 68.0(1.4) | 88.6(0.6) | 90.9(1.0) | 50.5(2.3) | 50.0(2.1) | 50.7(2.5) | 50.2(2.6) | 51.4(2.7) | 51.6(3.0) |
| Logistic | 58.2(3.7) | 80.0(2.2) | 88.9(0.9) | 68.0(1.4) | 88.6(0.6) | 90.9(1.0) | 50.5(2.3) | 50.0(2.1) | 50.7(2.5) | 50.2(2.6) | 51.4(2.7) | 51.6(3.0) |
| Squared | 58.2(3.7) | 80.0(2.2) | 88.9(0.9) | 68.0(1.4) | 88.6(0.6) | 90.9(1.0) | 50.5(2.3) | 50.0(2.1) | 50.7(2.5) | 50.2(2.6) | 51.4(2.7) | 51.6(3.0) |
| Hinge | 58.2(3.7) | 80.0(2.2) | 88.9(0.9) | 68.0(1.4) | 88.6(0.6) | 90.9(1.0) | 50.5(2.3) | 50.0(2.1) | 50.7(2.5) | 50.2(2.6) | 51.4(2.7) | 51.6(3.0) |

Fig. 3. AUC score and balanced accuracy (BER) correspond to binary classification with varying privacy budgets and data sizes. Experiments were conducted 10 times. Means and standard deviations are displayed.

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