Study of the decoherence of a double quantum dot charge qubit via the Redfield equation

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Abstract

By using the Redfield form of the master equation, we investigate the decoherence times of a double quantum dot charge qubit (DQDCQ) in three different cases, namely when it is coupled to (I) the piezoelectric coupling phonon bath (PCPB), (II) the deformation coupling phonon bath (DCPB), and (III) the Ohmic bath. It is found that our results for case (I) and (II) are in the same magnitude with those obtained via the exact path integral methods, while for case (III), the decoherence time is in well agreement with the experimental value.

Keywords: Quantum dot; Decoherence; Redfield equation

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I. INTRODUCTION

The double quantum dot (DQD) charge qubit [1-5] is one of the qubits that are considered to be promising candidates for the realization of the building blocks of quantum information processing. Its two low-energy states are denoted as the local states $|0\rangle$ and $|1\rangle$, which could be controlled via external voltage sources. There exist some effective schemes to prepare the initial states and readout the final states of the qubit [6]. It is also known that this kind of qubit is coupled to its environment inevitably. Therefore, it is believed that the decoherence might be the central impediment for the qubit to be taken as the cell of quantum computer. Hence, finding out the primary origin or dominating mechanism of decoherence for the qubit is a basic task to overcome the difficulty.

The recently experimental realization of the coherent manipulation of electronic states in a double-dot system [7, 8], which was implemented in a GaAs/AlGaAs heterostructure containing a two-dimensional electron gas, stimulated a lot of theoretical interests. Various methods have been tried to study the system. For instance, through density matrix simulation, Fujisawa et al. [9] tried to explain its transportation property. The Born-Markov-type electron-phonon decoherence at large times due to spontaneous phonon emission of the quantum dot charge qubits was investigated by Fedichkin et al. [10]. In 2005, Vorojtsov et al. [11] studied the decoherence of the DQD charge qubit by employing the Born-Markov
approximation. Based on a unitary transformation, Wu et al. [12] investigated the decoherence in terms of a perturbative treatment. Thorwart et al. [13] investigated the decoherence of the DQD charge qubit in a longer time with a numerical exact iterative quasiadiabatic propagator path integral (QUAPI) method [14], while Liang [15] used an iterative tensor multiplication (ITM) method [14] derived from the QUAPI to study decoherence of a DQD charge qubit in both the piezoelectric coupling phonon bath (PCPB) and the deformation coupling phonon bath (DCPB). They found that the decoherence times of the qubit are shorter than the reported experimental ones when the qubit in PCPB and DCPB.

In this paper, we investigate the decoherence times of the (DQD) charge qubit coupled to PCPB, DCPB and the Ohmic bath with another method, through solving the master equations. The model Hamiltonian and the spectral functions for the different baths are introduced in section II. We then develop the Redfield form of the master equation and obtain the decoherence times in section III. Conclusions are given in the last section.

II. QDQ CHARGE QUBIT MODEL

The DQD charge qubit consists of left and right dots connected through an interdot tunneling barrier. Due to the Coulomb blockade, at most one excess electron is allowed to occupy the left and right dot, which defines two basis vectors \( |0\rangle \) and \( |1\rangle \). The energy difference between these two states can be controlled by the source-drain voltage. Neglecting the higher order tunneling between leads and the dots, the effective Hamiltonian in the manipulation process reads [12, 13]

\[
H_{\text{eff}} = H_S + H_B + H_{SB},
\]

where \( H_S \) is the Hamiltonian of the QDQ charge qubit, \( H_B \) is the Hamiltonian for the phonon bath, and \( H_{SB} \) describes the electron-phonon interaction. More explicitly, we have

\[
H_S = \hbar T_c \sigma_x,
\]

\[
H_B = \hbar \sum_q \omega_q b_q^\dagger b_q,
\]

\[
H_{SB} = \hbar \sigma_z \sum_q (M_q b_q^\dagger + M_q^* b_q).
\]
Here, $T_c$ is the interdot tunneling, $\sigma_x$ and $\sigma_z$ are the Pauli matrix, $b_q^\dagger (b_q)$ are the creation (annihilation) operators of phonons, $\hbar \omega_q$ is the energy of the phonons, and $M_q = C_q / \sqrt{2m_q \omega_q \hbar}$, where $C_q$ are the classical coupling constants of the qubit-phonons system. Having written above, we now introduce the spectral density which fully describes the effects of the phonon bath [16, 17]

$$J(\omega) = \sum_q |M_q|^2 \delta(\omega - \omega_q). \quad (2)$$

According to Ref. [12], the spectral density of PCPB is

$$J_{pz}(\omega) = g_{pz}\omega \left[1 - \frac{\omega_d}{\omega} \sin \left(\frac{\omega}{\omega_d}\right)\right] e^{-\omega^2/2\omega_l^2}, \quad (3)$$

where $\omega_d = s/d$ and $\omega_l = s/l$, $d$ denotes the center-to-center distance of two dots, $l$ the dot size, $s$ the sound velocity in the crystal, and

$$g_{pz} = \frac{M}{\pi^2 \rho s^3}\left(\frac{6}{35} + \frac{1}{x}\frac{8}{35}\right). \quad (4)$$

As usual, $M$ is the piezoconstant, $\rho$ is the density of the crystal, and $x$ is the rate of transverse to the longitudinal of sound velocity in the crystal (see for example Refs. [12] and [13]). As in the GaAs crystal $s \approx 5 \times 10^3$ (m/s). With the parameters of GaAs [18], Wu et al. [12] estimated that $g_{pz} \approx 0.035 \text{ (ps)}^{-2}$. The spectral density of DCPB is also obtained. It is

$$J_{df}(\omega) = g_{df}\omega^3 \left[1 - \frac{\omega_d}{\omega} \sin \left(\frac{\omega}{\omega_d}\right)\right] e^{-\omega^2/2\omega_l^2}, \quad (5)$$

with

$$g_{df} = \frac{\Xi^2}{8\pi^2 \rho s^5},$$

where $\Xi$ is the deformation potential. In the same paper, Wu et al. also propose a value $g_{df} \approx 0.029 \text{ (ps)}^{-2}$. With the help of the definite spectral density functions of the baths, one can investigate the dynamics and then the decoherence of the open qubit.

We also consider explicitly the case in which the behavior of the original spectral function $J(\omega)$ has a simple power-law form for $\omega \leq \omega_c$:

$$J(w) = \eta \omega^\delta e^{-\omega/\omega_c}, \quad \eta = \text{const}, \quad (6)$$

with dimensionless damping strength $\eta$ and a cutoff frequency $\omega_c$. On general grounds, the linear low frequency behavior of $J(w)$ is expected in basically all condensed-phase electron transfer (ET) reactions [16], and the frequency $\omega_c$ then corresponds to some dominant bath mode. For its analytic advantages, we use the Ohmic spectral density [17] by setting $s = 1$ in Eq. (6). We also study the case when $\eta = 0.04$, $\omega_c = 0.05 \text{ (ps)}^{-1}$ [19].
III. DECOHERENCE TIMES

In this section, we first review the Redfield form of the master equation. In an open system, the relevant system is characterized by the reduced density-matrix (RDM) $\rho$ which is defined as a trace over all bath variables of the full density matrix $\rho_T$.

$$\rho = tr_B (\rho_T).$$  \hfill (7)

The equation for the time evolution of $\rho$ can be consequently obtained from the Liouville equation for $\rho_T$

$$\frac{\partial \rho(t)}{\partial t} = i\hbar tr_B[\rho_T(t), H].$$  \hfill (8)

The Redfield form of the master equation for the RDM is obtained from Eq. (8) by implementing a series of perturbative approximations [20, 21], namely, the system-bath coupling is treated perturbatively up to the second order. The bath is assumed to remain in equilibrium, and then Markov approximation gets involved. In the eigenstate representation of the system

$$H_s |\mu\rangle = E_s |\mu\rangle,$$

the Redfield equation for the RDM reads [22]

$$\frac{\partial \rho_{\mu\nu}(t)}{\partial t} = -i\omega_{\mu\nu}\rho_{\mu\nu}(t) + \sum_{\kappa\lambda} R_{\mu\nu\kappa\lambda}\rho_{\kappa\lambda}(t),$$  \hfill (10)

where $\omega_{\mu\nu} = (E_\mu - E_\nu)/\hbar$ and $R_{\mu\nu\kappa\lambda}$ is the relaxation or Redfield tensor. The first term on the right-hand side of Eq. (10) describes the isolated system evolution, while the second one represents its interaction with the dissipative environment. The Redfield tensor describing the system relaxation can be expressed as

$$R_{\mu\nu\kappa\lambda} = \Gamma^+_{\lambda\nu\mu\kappa} + \Gamma^-_{\lambda\nu\mu\kappa} - \delta_{\nu\lambda}\sum_\alpha \Gamma^+_{\mu\alpha\alpha\kappa} - \delta_{\mu\kappa}\sum_\alpha \Gamma^-_{\lambda\alpha\alpha\nu},$$  \hfill (11)

with

$$\Gamma^+_{\lambda\nu\mu\kappa} = \frac{1}{\hbar^2} \int_0^\infty dt \langle \lambda | H_{SB}(t) | v\rangle \langle \mu | H_{SB} | \kappa\rangle e^{-i\omega_{\mu\nu}t},$$  \hfill (12)

$$\Gamma^-_{\lambda\nu\mu\kappa} = \frac{1}{\hbar^2} \int_0^\infty dt \langle \lambda | H_{SB} | v\rangle \langle \mu | H_{SB}(t) | \kappa\rangle e^{-i\omega_{\lambda\nu}t},$$  \hfill (13)

$$H_{SB}(t) = e^{iH_Bt/\hbar} H_{SB} e^{-iH_Bt/\hbar},$$  \hfill (14)

where $\langle ... \rangle_B$ denotes the thermal average over the bath.
For the Hamiltonian defined above, the Redfield tensor components by using coherent states can explicitly be expressed as

\[
\Gamma_{\chi_{\mu\kappa}}^+ = \frac{1}{2} \langle \chi | \sigma Z | \mu \rangle \langle \sigma Z | \kappa \rangle J(\omega_{\kappa\mu})(1 + n(\omega_{\kappa\mu})), \quad \text{if } \omega_{\kappa\mu} > 0,
\]

\[
\Gamma_{\chi_{\mu\kappa}}^- = \frac{1}{2} \langle \chi | \sigma Z | \mu \rangle \langle \sigma Z | \kappa \rangle J(\omega_{\kappa\mu}) n(\omega_{\kappa\mu}), \quad \text{if } \omega_{\mu\kappa} > 0,
\]

\[
\Gamma_{\chi_{\mu\kappa}}^{\pm} = \frac{1}{2} \langle \chi | \sigma Z | \mu \rangle \langle \sigma Z | \kappa \rangle J(\omega_{\kappa\mu}) (1 + n(\omega_{\lambda\nu})) \delta_{\lambda\nu}, \quad \text{if } \omega_{\lambda\nu} > 0,
\]

where \(\omega_{\mu} = E_{\mu}/\hbar\) are the system eigenfrequencies, \(n(\omega) = 1/(e^{\frac{\hbar \omega}{kT}} - 1)\) is the distribution function for bosons, \(\kappa\) is the Boltzmann constant, \(T\) is the temperature, and \(J(\omega)\) is the bath spectral function.

To measure effects of the decoherence, one can use the entropy, the first entropy, and many other measures, such as maximal deviation norm, etc. (for example, see Refs. 23, 24, 25). However, the decoherence of an open quantum system is essentially reflected through the decays of the off-diagonal coherent terms of its RDM. In general, the decoherence is produced due to the interaction of the quantum system with other system which has a large number of degrees of freedom, such as the devices of the measurement or environment.

Here, we investigate the decoherence times via directly describing the evolutions of the off-diagonal coherent terms instead of using any measure of decoherence. In the following, we set the initial state of the qubit to \(|\rho(0) = \frac{1}{2}(|0\rangle + |1\rangle)\langle 0| + \langle 1|\rangle\), which is a pure state and it has the maximum coherent terms, and the initial state of the environment is \(\rho_{\text{bath}(0)} = \Pi_k e^{-\beta M_k}/Tr(e^{-\beta M_k})\), where \(M_k = \omega_k b_k^\dagger b_k\), \(\beta = 1/kT\). According to Ref. [12], we set \(\omega_d = 0.02 \, (\text{ps})^{-1}\), \(T_c = 0.1\omega_d\) in the calculations.

We then shall use the Redfield equation to investigate the decoherence time of the DQD charge qubit. The analytical expressions of the elements of the RDM we obtained are

\[
\rho_{11}(t) = \frac{1 + n(\omega_{21})}{1 + 2n(\omega_{21})} - \frac{e^{-2\chi t}}{2(1 + 2n(\omega_{21}))},
\]

\[
\rho_{22}(t) = \frac{n(\omega_{21})}{1 + 2n(\omega_{21})} + \frac{e^{-2\chi t}}{2(1 + 2n(\omega_{21}))},
\]

\[
\rho_{12}(t) = \left(\chi + \sqrt{\chi^2 - \omega_{21}^2}\right) e^{-\chi t + \sqrt{\chi^2 - \omega_{21}^2} t} - \left(\chi - \sqrt{\chi^2 - \omega_{21}^2}\right) e^{\chi t - \sqrt{\chi^2 - \omega_{21}^2} t}
\]

\[
+ \frac{i \omega_{21}}{4 \sqrt{\chi^2 - \omega_{21}^2}} e^{-\chi + \sqrt{\chi^2 - \omega_{21}^2} t} - \frac{e^{\chi - \sqrt{\chi^2 - \omega_{21}^2} t}}{4 \sqrt{\chi^2 - \omega_{21}^2}}.
\]
\[
\rho_{21}(t) = \frac{(\chi + \sqrt{\chi^2 - \omega_{21}^2})e^{(-\chi+\sqrt{\chi^2-\omega_{21}^2})t} - (\chi - \sqrt{\chi^2 - \omega_{21}^2})e^{(-\chi-\sqrt{\chi^2-\omega_{21}^2})t}}{4\sqrt{\chi^2 - \omega_{21}^2}} \\
-\frac{i\omega_{21}e^{(-\chi+\sqrt{\chi^2-\omega_{21}^2})t} - e^{(-\chi-\sqrt{\chi^2-\omega_{21}^2})t}}{4\sqrt{\chi^2 - \omega_{21}^2}},
\]

where \( \chi = \hbar J(\omega_{21})(1 + 2n(\omega_{21}))/2 \). Note that by substituting the parameters \( \omega_{21}, \omega_d, \omega_l, \omega_c, \eta \) and \( T \), in PCPB, DCPB and Ohmic bath into the above expressions, we can readily plot the evolutions of the \( \rho_{12} \) (or \( \rho_{21} \)).

The evolutions of the coherent elements of the RDM of the DQD charge qubit in PCPB and DCPB with different \( \omega_l \) are plotted in Figs. 1 and 2 respectively. And when the environment is modeled with Ohmic bath the evolution of the coherent elements of the RDM is plotted in Fig. 3 as damping strength \( \eta = 0.04, 0.08 \) and \( 0.12 \).

*Fig.1, Fig.2, Fig.3.*

It is shown that the decoherence times of the qubit decreases as the \( \omega_l \) increases in PCPB and DCPB models. And the decoherence time of the qubit decreases as the damping strength \( \eta \) in the Ohmic bath model increases. From Figs. 1-3, we see that the decoherence times of the DQD charge qubit in PCPB and DCPB are much shorter than the experimentally suggested result, which is in agreement with the results of Refs.[13, 15]. However, if we choose the environment as the Ohmic bath, the decoherence time of the qubit agrees with the experimental result when we choose a suitable coupling coefficient of the qubit to the environment. Moreover, we also consider the evolutions of the coherent term at different temperatures when the bath is modeled by PCPB, DCPB and Ohmic bath separately, which are plotted in Figs. 4, 5, and 6, respectively.

*Fig.4, Fig.5, Fig.6.*

It is shown that the decoherence times decreases as the temperature increases no matter which bath is used to model the environment when \( \omega_l \) (in PCPB and DCPB) and \( \eta \) (in Ohmic bath) are fixed, this is physically reasonable.
IV. CONCLUSIONS

In summary, we have investigated the decoherence times of a double quantum dot (DQD) charge qubit when it is coupled to different baths by using the Redfield equation method. Our results show that the qubits have shorter decoherence times than the experimental ones as the environment is modeled by the acoustic phonon baths, which agrees with previous reports. Moreover, when we use Ohmic bath model the environment with the coupling coefficient of the qubit to the environment properly chosen, the decoherence time of the qubit is in well agreement with the experimental result.

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V. FIGURES CAPTIONS

Fig. 1: $\rho_{12}(t)$ vs time $t$. The evolution of the off-diagonal elements of the RDM for the DQD charge qubit in PCPB when $\omega_l = 0.5 \text{ (ps)}^{-1}$ and $\omega_l = 0.7 \text{ (ps)}^{-1}$. Here, $\omega_d = 0.02 \text{ (ps)}^{-1}$, $g_{pz} = 0.035 \text{ (ps)}^{-2}$, and $T = 30 \text{ mK}$. The initial state is described in the text.

Fig. 2: $\rho_{12}(t)$ vs time $t$. The evolution of the off-diagonal elements of the RDM for the DQD charge qubit in DCPB. Here, $\omega_d = 0.02 \text{ (ps)}^{-1}$, $g_{df} = 0.029 \text{ (ps)}^{-2}$. Other parameters are the same as those in Fig. 1.

Fig. 3: $\rho_{12}(t)$ vs time $t$. The evolution of the off-diagonal elements of the RDM for the DQD charge qubit in Ohmic case, with cut-off frequency $\omega_c = 0.05 \text{ (ps)}^{-1}$, $T = 30 \text{ mK}$ and $\eta = 0.04, 0.08, 0.12$, respectively. The initial state is described in the text.

Fig. 4: $\rho_{12}(t)$ vs time $t$. The evolution of the off-diagonal elements of the RDM for the DQD charge qubit in PCPB when $\omega_l = 0.5 \text{ (ps)}^{-1}$, $T = 30 \text{ mK}$, $300 \text{ mK}$, $1 \text{ K}$ respectively. Other parameters are the same as those in Fig. 1.

Fig. 5: $\rho_{12}(t)$ vs time $t$. The evolution of the off-diagonal elements of RDM for the DQD charge qubit in DCPB when $\omega_l = 0.5 \text{ (ps)}^{-1}$, $T = 30 \text{ mK}$, $200 \text{ mK}$, $300 \text{ mK}$, $1 \text{ K}$ respectively.
Other parameters are the same as those in Fig. 2.

Fig. 6: $\rho_{12}(t)$ vs time $t$. The evolution of the off-diagonal elements of the RDM for the DQD charge qubit in Ohmic case when $\eta = 0.04$ and $T = 20$ mK, 30 mK, 40 mK, 50 mK, respectively. Other parameters are the same as those in Fig. 3.
Fig. 2
