Tracking Control for a Rehabilitative Training Walker Considering Human-Robot Interaction

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Abstract. A new extended state observer combined with a Backstepping tracking control method were proposed in this paper for the omnidirectional rehabilitative training walker. The aim of this study is to obtain a stable tracking controller, and it considered the interaction forces of the user and walker to ensure that the omnidirectional walker can track accurately specified trajectory. The extended state observer was designed to estimate the unknown interaction forces of user and the walker using output position and speed states. Further, a Backstepping controller based on the observed value was constructed to resist the adverse effects of interaction forces. The asymptotic stability of the trajectory tracking error, the velocity tracking error and the state observation error were guaranteed. Through programming, simulation results show the effectiveness of the proposed design method.

1. Introduction
With the increasing demand for medical rehabilitation training, the rehabilitation robot has attracted the considerable attention by many scholars. There are many kinds of rehabilitation robots, which can effectively help the elderly, the disabled or people with movement disorders to carry out training and restore walking ability, such as lower extremity exoskeleton robot [1, 2], wheeled mobile rehabilitation robot [3, 4], etc. In the research field of rehabilitation robot, control strategy is an important means to realize robot movement, which is a long-term hot topic. Therefore, there are many research achievements in this respect, such as fuzzy sliding mode control [5], robust iterative feedback tuning control [6], and so on. However, in the process of designing the controller through these control strategies, it is ignored that the interaction forces between human and robot can adversely affect the system.

During rehabilitation training, the physical interaction is inevitable due to mutual contact in human and robot. The influence of physical human-robot interaction on mechanical system has attracted great attention in recent years. So far, there have been many studies about physical human-robot interaction, such as interaction control of the human-robot [7, 8], adaptive impedance control for human-robot interactions [9], and so on. However, these studies use sensors to measure human-robot interaction forces, which can lead to the inaccurate results. Since the direction of the action force and the movement direction of the robot determine whether the action force ACTS on the system is resistance or power, the complexity of the time-varying action force is analyzed and considered. Hence, simple measurements do not fully capture interactive information. If the measured interaction forces are put into the control system, the tracking precision will be decreased in the actual application.
In practice, human-robot interaction forces including active motion force, pressure [10, 11], etc., can be regarded as an internal disturbance with uncertainty, which can seriously reduce the tracking performance of the system when the robot tracks a specified path.

In this paper, an omnidirectional rehabilitative training walker (ODW) [12] was taken as the research object, and it is a wheeled rehabilitation and training robot, which can specify the training track to recover the motor function of lower limb injury caused by various reasons. This paper analyzes and proves from the following three aspects:

(1) The effect of the interaction forces is a major challenge for rehabilitative robot control system. Hence, the dynamics model of the ODW was improved. According this model, the extended state observer of estimating the interaction forces was designed by the output position and speed of the ODW.

(2) In the present study, a backstepping controller is designed to compensate the interaction forces. By adjusting the design parameters of the controller and the observer, the trajectory tracking error and velocity tracking error in the experiment are asymptotically stable to reach the expected goal.

(3) As an application, on the basis of the extended state observer, the backstepping tracking control for the ODW was considered. The efficiency of the proposed scheme was demonstrated by simulation results.

2. Interaction Forces Observer Design

2.1. Dynamics Model

The dynamics model is expressed as [13]

\[ M_0 K(\theta) \ddot{X}(t) + M_0 \dddot{K}(\theta, \dot{\theta}) \dot{X}(t) = B(\theta)u_0(t) \]  

In effect, for the mechanical system (1), the input \( u_0(t) \) is a generalized force that should be divided into the control force \( u_1(t) \) and the dissipative force \( u_2(t) \). Specifically, \( u_1(t) \) is input force to be used to realize tracking motion; \( u_2(t) \) is unknown interaction force which will have disadvantageous influences on tracking movement. Therefore, substituting this decomposition into the model (1), we get

\[ M_0 K(\theta) \ddot{X}(t) + M_0 \dddot{K}(\theta, \dot{\theta}) \dot{X}(t) = B(\theta)(u_1(t) + u_2(t)) \]  

where \( B(\theta)u_0(t) \) represents the total interaction forces separated from the generalized control force \( u_0(t) \). we can also use \( F(t) \) to represent \( B(\theta)u_2(t) \), and \( F(t) = [F_\theta(t), F_\phi(t), F_\rho(t)]^T \).

2.2. Extended State Observer of the Interaction Forces

Our objective is to design an extended state observer that can observe the interference caused by man-robot interaction forces in real time during the walking training process.

Denote \( x_1(t) = X(t), x_2(t) = \dot{X}(t) \). Therefore, the dynamic model (2) can be transformed to

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= K^{-1}(\theta)M_0^{-1}B(\theta)u_1(t) + K^{-1}(\theta)M_0^{-1}B(\theta)u_2(t) - K^{-1}(\theta)\dddot{K}(\theta, \dot{\theta}) x_2(t)
\end{align*}
\]  

(3)

Assume that \( x_3(t) = K^{-1}(\theta)M_0^{-1}B(\theta)u_1(t) - K^{-1}(\theta)\dddot{K}(\theta, \dot{\theta}) x_2(t) \), \( x_3(t) \) is an extended state including the interaction forces \( F(t) = B(\theta)u_2(t) \), which is bounded and derivable. Therefore, we can set \( x_3(t) \) to be the following expression.

Therefore, we can set \( x_3(t) \) to be the following expression.

\[ \dot{x}_3(t) = h_0(t) \]  

(4)
So we will transform the model (3) into the extended state
\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= K^{-1}(\theta)M_{\theta}^{-1}u(t) + x_3(t) \\
\dot{x}_3(t) &= h_0(t)
\end{align*}
\]  
(5)
where \(x_i(t) = [X(t), Y(t), \dot{\theta}(t)]^T\), \(x_j(t) = [\dot{X}(t), \dot{Y}(t), \ddot{\theta}(t)]^T\), \(x_3(t) = [X_3(t), Y_3(t), \dot{\theta}_3(t)]^T\).

Let \(\hat{x}_i(t)\) represents the estimate of \(x_i(t)\), and \(\hat{y}(t)\) represents output the estimate \(y(t)\).

The extended state observer is designed as follows
\[
\begin{align*}
\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + \lambda_2(y(t) - \hat{y}(t)) \\
\dot{\hat{x}}_2(t) &= K^{-1}(\theta)M_{\theta}^{-1}B(\theta)u(t) + \hat{x}_3(t) + \lambda_1(y(t) - \hat{y}(t)) \\
\dot{\hat{x}}_3(t) &= (x_3(t) - \hat{x}_3(t)) + \lambda_0(y(t) - X_d(t))
\end{align*}
\]  
(6)

Put the output \(y(t) = x_1(t)\) into the extended state observer equation (6) and get the following expression.
\[
\begin{align*}
\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + \lambda_2(x_1(t) - \hat{x}_1(t)) \\
\dot{\hat{x}}_2(t) &= K^{-1}(\theta)M_{\theta}^{-1}B(\theta)u(t) + \hat{x}_3(t) + \lambda_1(x_1(t) - \hat{x}_2(t)) \\
\dot{\hat{x}}_3(t) &= (x_3(t) - \hat{x}_3(t)) + \lambda_0(x_1(t) - X_d(t))
\end{align*}
\]  
(7)

Set the observation error \(r_i(t)\) as follows
\[
r_i(t) = x_i(t) - \hat{x}_i(t) \quad (i = 1, 2, 3)
\]  
(8)

The observation error (8) is brought into the equation (7). The extended state observer and the velocity observation error are respectively
\[
\begin{align*}
\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + \lambda_2r_1(t) \\
\dot{\hat{x}}_2(t) &= K^{-1}(\theta)M_{\theta}^{-1}B(\theta)u(t) + \hat{x}_3(t) + \lambda_1r_1(t) \\
\dot{\hat{x}}_3(t) &= r_3(t) + \lambda_0(r_1(t) + \hat{x}_3(t) - X_d(t))
\end{align*}
\]  
(9)
\[
\begin{align*}
\dot{\hat{r}}_1(t) &= r_2(t) - \lambda_2r_1(t) \\
\dot{\hat{r}}_2(t) &= r_3(t) - \lambda_1r_1(t) \\
\dot{\hat{r}}_3(t) &= h_0(t) - r_3(t) - \lambda_0(r_1(t) + \hat{x}_3(t) - X_d(t))
\end{align*}
\]  
(10)

3. Backstepping Tracking Controller Design
This part of this paper is mainly to design a controller that can track the predetermined trajectory in the walking training program, so as to suppress the interaction force and ensure the safety and accuracy of the control.

The trajectory tracking error \(e_i(t)\) can express
\[
e_i(t) = \hat{x}_i(t) - X_d(t)
\]  
(11)

The time derivative (11) is given by
\[
\dot{e}_i(t) = \dot{\hat{x}}_i(t) + \lambda_2r_1(t) - \dot{X}_d(t)
\]  
(12)

Set the virtual control variable \(\alpha\) as follows:
\[ \alpha = -c_1e_i(t) + \dot{X}_d(t) - \dot{\lambda}_2r_i(t) \]  
\[ \text{The velocity tracking error } e_z(t) \text{ along the expressions (12) and (13) is obtained by} \]
\[ e_z(t) = c_1e_i(t) + \dot{e}_i(t) \]  
where \( c_1 > 0 \). The time derivative (14) along the equations (10) and (13) is given by
\[ \dot{e}_z(t) = K^{-1}(\theta)M_0^{-1}B(\theta)u_i(t) + \dot{x}_i(t) + \dot{\lambda}_1r_z(t) - \dot{X}_d(t) + \dot{\lambda}_2r_i(t) + c_1\dot{e}_i(t) \]
\[ \text{In order to resist the interaction forces, ideal control input is designed as} \]
\[ u_i(t) = \dot{B}(\theta)M_0K(\theta)(\dot{X}_d(t) - e_i(t) - \dot{\lambda}_2r_z(t) - \dot{\lambda}_1r_z(t) + \dot{\lambda}_2^2r_i(t) - c_1e_i(t) - c_2e_z(t) - \dot{x}_i(t)) \]  
where \( c_2 > 0 \) and \( \dot{B}(\theta) = B^T(\theta)(B(\theta)B^T(\theta))^{-1} \) is the pseudo-inverse matrix of \( B(\theta) \).

The parameters in the extended state observer (10) are designed by the Lyapunov function as follows:
\[ \lambda_2 = 1 + \frac{\kappa^2}{2}, \quad \lambda_1 = 1 + \frac{1}{2\kappa}, \quad \lambda_0 = (h_0 + r_2(t))R(t) \]  
where \( \lambda_2 > 0, \lambda_1 > 0, \lambda_0 \) is a time varying parameter, \( R(t) = (r_1(t) + e_1(t))^T[(r_1(t) + e_1(t))(r_1(t) + e_1(t))^T]^{-1} \) is the pseudo-inverse matrix of \( r_1(t) + e_1(t) \).

By taking the trajectory tracking error \( e_i(t) \) into the equations (10), the velocity observation error is
\[ \begin{cases} 
\dot{r}_i(t) = r_i(t) - \dot{\lambda}_2r_i(t) \\
\dot{r}_z(t) = r_z(t) - \dot{\lambda}_1r_z(t) \\
\dot{r}_0(t) = h_0(t) - r_i(t) - \dot{\lambda}_0(r_i(t) + e_i(t)) 
\end{cases} \]  
\[ \text{Theorem 1. For the model (2), the feedback controller (16) and velocity observation error (18) are designed. The controller parameters } c_1, c_2 \text{ which are given positive constants and the observer parameters } \lambda_0, \lambda_1, \lambda_2 \text{ realize the trajectory tracking error } e_i(t), \text{ the velocity tracking error } e_z(t), \text{ the position observation error } r_i(t), \text{ the velocity observation error } r_z(t) \text{ and the observation error of the extended state } r_0(t) \text{ asymptotically stable.} \]

\textbf{Proof.} the Lyapunov function is defined as follows:
\[ V(t) = \frac{1}{2}e_i^T(t)e_i(t) + \frac{1}{2}e_z^T(t)e_z(t) + \frac{1}{2}r_i^T(t)r_i(t) + \frac{1}{2}r_z^T(t)r_z(t) + \frac{1}{2}r_0^T(t)r_0(t) \]  
The time derivative of \( V(t) \) get with the velocity observation error (18), and the expressions (14), (15) and (19) satisfies the following:
\[ \dot{V}(t) = -c_1e_i^T(t)e_i(t) - c_2e_z^T(t) + \dot{\lambda}_2r_i^T(t)r_i(t) + \dot{\lambda}_1r_z^T(t)r_z(t) - \dot{\lambda}_2r_i^T(t)r_i(t) \\
+ r_i^T(t)r_i(t) - r_z^T(t)r_z(t) + r_0^T(t)h_0(t) - \dot{\lambda}_0r_0^T(t)r_0(t) - \dot{\lambda}_0^2r_i^T(t)e_i(t) \]  
According to the Young’s theorem,
\[ r_i^T(t)r_i(t) \leq \frac{\kappa^2}{2}r_i^T(t)r_i(t) + \frac{1}{2\kappa^2}r_0^T(t)r_0(t) \]  
where \( \kappa > 0 \). Combining (21) with (20), it follows that
\[ V(t) \leq -c_1 e_1^T(t) e_1(t) - c_2 e_2^T(t) e_2(t) - r_1^T(t) r_1(t) - (\lambda_2 - \frac{\epsilon^2}{2}) r_1^T(t) r_1(t) - (\lambda_1 - \frac{1}{2\epsilon^2}) r_2^T(t) r_2(t) + r_3^T(t) (r_3(t) + h_0(t)) \]  

Substituting observational parameter (17) into (22), and \( \|h_0(t)\| \leq h_0 \), we obtain

\[ V(t) \leq -c_1 e_1^T(t) e_1(t) - c_2 e_2^T(t) e_2(t) - r_1^T(t) r_1(t) - r_2^T(t) r_2(t) - r_3^T(t) r_3(t) + r_3^T(t) (h_0(t) - h_0) \]

Thereby, in terms of (23), we can get \( V(t) \leq 0 \), and then we can infer that \( V(t) = 0 \) with \( e_1(t) = 0 \), \( e_2(t) = 0 \), \( r_1(t) = 0 \), \( r_2(t) = 0 \) and \( r_3(t) = 0 \). Therefore, based on Lasalle's principle, the trajectory tracking error, velocity tracking error, position observation error, velocity observation error and extended state observation error are asymptotically stable. And that completes the proof of theorem 1.

4. Simulation Results

This section verifies the proposed tracking control algorithm based on the extended state observer based on observable interactions through ODW simulation. The path \( X_d(t) \) can express as follows:

\[\begin{align*}
    x_d(t) &= 20 \cos(0.1t) - 20 \\
y_d(t) &= 10 \sin(0.1t) \\
    \theta_d(t) &= \frac{\pi}{4}
\end{align*}\]

Some relevant setting parameters are \( M = 58kg \), \( L = 0.4m \), \( I_0 = 27.7 kg \cdot m^2 \), \( r_0 = 0.1m \), and \( \beta = \frac{\pi}{4} rad \). The user load \( m = 80kg \). The controller parameters \( c_1 = 6 \), \( c_2 = 4 \); the observer parameter \( \gamma = 0.01 \). The speed boundary \( h_0 = [230 \ 225 \ 30]^T \). The velocity initial value of the extended state \( h_0(0) = [-4 \ -3 \ -2]^T \). The simulation results are shown in the figures below.

Figure 1 plots the path tracking. The position observation errors, the speed observation errors and the observation errors of the extended state tend to zero in figures 2-4. It was evident that the extended state observer was effective in the backstepping tracking controller (16).

![Figure 1. Path tracking of ellipse.](image1)

![Figure 2. Position observation errors.](image2)
5. The Final Conclusion
In this paper, a backstepping control algorithm based on the extended state observer which has been used to estimate the interaction forces in real time has been proposed. The controller and the observer parameter have been designed to stabilize the walker by using the Lyapunov function. The proposed method focuses on estimating and rejecting the interaction forces of user and the walker. From the simulation results, we can know about the proposed method can effectively solve the training path problem planned by physiotherapists.

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