Intra Prediction-Based Measurement Coding Algorithm for Block-Based Compressive Sensing Images

JIRAYU PEETAKUL\textsuperscript{1}, YIBO FAN\textsuperscript{2}, AND JINJIA ZHOU\textsuperscript{1,3}, (Member, IEEE)

\textsuperscript{1}Graduate School of Science and Engineering, Hosei University, Tokyo 102-8160, Japan
\textsuperscript{2}State Key Laboratory of ASIC and System, Fudan University, Shanghai 201203, China
\textsuperscript{3}Japan Science and Technology Agency (JST) PRESTO, Tokyo 102-0076, Japan

Corresponding author: Jinjia Zhou (jinjia.zhou.35@hosei.ac.jp)

This work was supported by the JST, PRESTO, Japan, under Grant JPMJPR1757.

ABSTRACT Block-based compressive image sensor (BCIS) captures light and represents them as compressed data called measurement. It has potential to revolutionize conventional image and video acquisition system that builds upon high complexity and redundant process. However, by comparing the compression performance between these two systems, BCIS cannot reduce bitrate to similar factor as the compressed media by pixel-based compression algorithms. It still requires enormous amounts of bit to store and transmit data. In this work, we introduce intra prediction based measurement coding algorithm for giving an extra compression performance to measurement. Moreover, importantly, there is a requirement that sensing matrix for BCIS must not be derived from non-uniform distribution in order to control prediction accuracy and quality. Therefore, we use structural sensing matrix made of sequency-ordered Walsh-Hadamard. Furthermore, it allows boundary pixels of adjacent blocks to be accessible through measurement, which helps intra prediction to generate its candidates accurately. The algorithm encodes prediction error between target measurement and multiple prediction candidates, resulting in smaller data size. This work can significantly reduce bpp by 10.90% and simultaneously increase 3.95 dB in PSNR compared to the state-of-the-art works. Moreover, we implemented the proposal on FPGA. It gave 10 times higher throughput than software. The core power consumption is at 50 mW and working at 88 MHz when processing 3840 × 2160 pixels with the sampling rate of 1/4.

INDEX TERMS Compressive sensing, measurement coding algorithm, intra prediction.

I. INTRODUCTION

Over the past few years, block-based compressive image sensor (BCIS) has gained significant interest in imaging technology. It can solve analog-to-digital converter (ADC) problems in conventional image sensor such as slow pixel readout time and power consumption.

A single-pixel camera was successfully developed by using digital micromirror device (DMD) array [1]. It is useful for microscopy and microanalysis applications [2]. Nevertheless, it required long sensing time when the resolution is relatively high. Robucci \textit{et al.} proposed separable-transform BCIS [3]. It could capture image faster than single-pixel camera. Nevertheless, it had limited in frame-per-second (fps).

The associate editor coordinating the review of this manuscript and approving it for publication was Li Minn Ang\textsuperscript{12}.

Later, Oike and El Gamal proposed programmable BCIS with per-column Sigma-Delta ADC to reduce sensing time and increase frame-rate up to 1920 fps [4], which overcame the problems of [1] and [3].

In [5] and [6], they gave an opinion that BCIS can revolutionize image and video capturing and compressing scheme, where bitrate could be varied depending on preferred quality. However, at this stage, it is not suitable for consumer devices because they do not have enough computational resources for decoding the measurement. In the meantime, there is a suitable application for BCIS such as wireless surveillance system because the measurement can be decoded at monitoring sites with unlimited resources [7], [8].

To transmit measurement wirelessly, it must be compressed into more compact format, resulting in lower transmission costs [9], [10]. Therefore, in this work, we propose four
modes intra prediction based measurement coding (IPMC) algorithm including upper, left, average, and no prediction. However, there is a requirement that sensing matrix for BCIS must not be derived from non-uniform distribution in order to control prediction accuracy and quality. Therefore, we use structural sensing matrix (SSM) made of sequency-ordered Walsh-Hadamard (SoWH). It allows boundary pixels of adjacent blocks to be accessible through measurement, which helps intra prediction to generate its candidates accurately. The algorithm encodes prediction error between target measurement and multiple prediction candidates, resulting in smaller data size.

To demonstrate the applicability and versatility, we evaluate the proposal using numerous video datasets in 4K resolution using peak-to-noise-ratio (PSNR), structural similarity index measure (SSIM), and bits-per-pixel (bpp). Moreover, we increase compression throughput by extending the proposal from software to hardware using FPGA.

We establish notation and provide a brief background of compressive sensing theory in subsection A, and related works on measurement coding in subsection B. Section II provides the proposed IPMC algorithm for BCIS and hardware architecture. Section III provides extensive simulation results and compares them with state-of-the-art works. Further, we also report hardware implementation summary. Section IV provides conclusions.

A. COMPRESSIVE SENSING THEORY
Compressive sensing (CS) is built upon two major fundamental conditions consisted of sparsity and incoherent [11], [12]. There are essential conditions in order to apply CS. First, a signal characteristic of \( x \) must be sparse when expressing in a specific orthonormal transform basis (See Appendix A-A). Next, sensing matrix \( \Phi \in \mathbb{R}^{m \times n} \), it can be made of random distribution called random sensing matrix (RSM), where the number of dimensional \( m \) must be less than \( n \), known as sampling rate (SR).

The measurement \( y \in \mathbb{R}^{m \times 1} \) can be obtained by projecting \( \Phi \) to \( x \). Nevertheless, traditional CS is not suitable for a large scale problem because it requires long sensing time. In [13] proposed partitioning approach to traditional CS by dividing an entire frame into multiple non-overlapping blocks. Instead that \( n \) equal to frame size, now \( n \) will be equal to \( b \times b \), where \( b \) is block size. Hence, \( x \) will be sampled with smaller \( \Phi \), resulting in faster projection. It can be expressed mathematically as:

\[
y_i = \Phi x_i
\]

where \( y_i \in \{y_1, y_2, y_3, \ldots, y_m\} \) is measurement of compressible signal of \( x_i \in \{x_1, x_2, x_3, \ldots, x_n\} \) and \( i \) is the block order through raster scanning as shown in Figure 1.

To guarantee a good reconstructed image, the sensing matrix should satisfy restricted isometry property (See Appendix A-B). In this work, we recover \( y \) by using a classical method via convex optimization called \( \ell_1 \)-minimization (See Appendix A-C).

B. RELATED WORKS ON MEASUREMENT CODING
Up to the present time, most of the CS literature has been devoted to study the recovery of sparse signals from a small number of measurement, but less in measurement coding algorithm. By referring to the legacy vector compression algorithms, introduced for lossless coding [24], and its extension for lossy coding [25]. Although, it is possible to use these legacy approaches to encode measurement. Nevertheless, it requires a precise design for each system specifically, which is not convenient.

Scalar quantization (SQ) provided a straightforward approach to compress measurement. By comparing to vector compression algorithms, SQ gave higher performance and versatility than vector compression. Nevertheless, it has been established that SQ is highly inefficient in terms of information-theoretic rate-distortion (RD) performance [26]–[30]. Additionally, it require an iterative recovery algorithm to predict corrupted quantized measurement such as quantized iterative hard thresholding (QIHT) [31], quantized compressed sampling matching pursuit (QCoSAMP), and adaptive outlier pursuit for quantized iterative hard thresholding (AOP-QIHT) [32].

Next, differential pulse-code modulation (DPCM) was introduced to reduce bitrate [33]. DPCM used a single prediction candidate to predict target measurement. However, the disadvantage is that the single prediction candidate may contain irrelevant information to target measurement, resulting in unstable bitrate reduction.

Afterward, spatially directional predictive coding (SDPC) was introduced in [34]. This work was implemented based on DPCM. It gave higher compression performance than SQ and DPCM, while improved image quality. However, this work used RSM as sensing matrix, resulting in unstable quality and unstable bitrate when coding the same image. Importantly, they could not embed this kind of sensing matrix into hardware device, where it can only handle binary signal sources.

Later, intra prediction based measurement coding with modification of RSM was introduced in [35]. This work was inspired by intra prediction concept from conventional pixel-based compression algorithms that uses boundary pixels of adjacent blocks to predict target block. By imitating the conventional approach, they modified sensing patterns of RSM corresponding to obtain boundary pixels of adjacent blocks called hybrid sensing matrix (HSM). They used that
boundary pixels information to generate intra prediction candidates. This work significantly reduced bitrate lower than SQ, DPCM, and SDPC. Nevertheless, it produced sampling artifact to the image due to the modification of sensing matrix.

In our previous work [36], we adopted SSM made of Natural ordered Hadamard (NoH) to obtain boundary information. We proposed four modes intra prediction included upper, left, average, and no prediction. This work significantly reduced bitrate and improved image quality compared to other works in this literature.

II. PROPOSED INTRA PREDICTION BASED MEASUREMENT CODING FOR BCIS

In this work, we improve coding performance and image quality based on the previous work in [36]. The overall architecture including BCIS and IPMC algorithm can be seen in Figure 2. There are three primary signals control BCIS including column selector, row selector, and pixel selector where each pixel will be selected according to the sensing matrix.

Subsequently, we adopt SoWH as SSM. It can gather information more efficiently than RSM, HSM, and SSM made of NoH due to higher orderliness, resulting in better image quality. Furthermore, it allows pixels boundary information of neighboring blocks to be accessible through measurement without modifying the sensing matrix.

After obtained the measurement, it will be transferred to IPMC algorithm to compress. We use boundary pixels of adjacent blocks to deliver four modes intra prediction. We encode target measurement by finding minimum distortion with multiple prediction candidates, resulting in smaller data size.

Next, we apply quantization to reduce the probability symbol for Huffman coding. In addition, we include inverse quantization inside the transmitter to estimate prediction candidates loss by quantization at the receiver. Subsequently, we use that estimated prediction candidates to predict the next target measurement. Otherwise, the decoder at receiver will act as error accumulator, in which a single corrupted measurement can initiate recovery error to the whole image.

Moreover, to increase coding performance, throughput, and to realize IPMC algorithm in real-world applications, we implement the proposal in hardware level and evaluate it on FPGA.

A. SEQUENCE-ORDERED WALSH-HADAMARD SENSING MATRIX

To use BCIS to capture the light, there is an implementation constraint that sensing matrix $\Phi$ must be $[0, 1]$ because the pixel selector can handle only digital signal (i.e., low (0) and high (1)). Let $\Phi_{NoH}$ can be obtained by order $n$, it can be said to be $\Phi_{NoH}$ if the transpose of the matrix $\Phi_{NoH}^T$ is closely related to its inverse. It can be expressed as given below:

$$\Phi_{NoH} \Phi_{NoH}^T = nI_{n \times n}$$  \hspace{1cm} (2)

where $nI_{n \times n}$ is the identity matrix and $\Phi_{NoH}^T$ is the transpose of matrix. By applying Sylvester’s construction to $\Phi_{NoH}$, resulting in Walsh-Hadamard matrix denoted by $\Phi_{WH}$ as the following:

$$\Phi_{WH} \left( 2^k \right) = \begin{bmatrix} \Phi_{NoH} \left( 2^{k-1} \right) & \Phi_{NoH} \left( 2^{k-1} \right) \\ \Phi_{NoH} \left( 2^{k-1} \right) & -\Phi_{NoH} \left( 2^{k-1} \right) \end{bmatrix}$$

$$= \Phi_{NoH} \left( 2^1 \right) \otimes \Phi_{NoH} \left( 2^{k-1} \right)$$  \hspace{1cm} (3)

for $2 \le k \le n$, where $\otimes$ denotes the Kronecker product. Subsequently, we applying bit-reversal and gray-code permutation, resulting in sequency order of $\Phi_{WH}$ denoted by $\Phi_{SoWH}$ this sensing matrix satisfies the RIP with a probability.
of at least $1 - 5/n - e^{-\beta}$ providing $m \geq c(1+\beta)\kappa \log n$ where $\beta$ is a positive constant and $\kappa$ is sparsity level. In general, Hadamard transform will return matrix in $\{-1, 1\}$. Therefore, we binarize them from $\{-1, 1\}$ to $\{0, 1\}$ to possibly embed the sensing matrix into BCIS.

**B. INTRA PREDICTION BASED MEASUREMENT CODING ALGORITHM**

In general, boundary pixels of adjacent blocks have information that closely related to target block. Hence, we use that boundary information to deliver four modes intra prediction including upper, left, average, and no prediction.

Firstly, prediction parameter preparation, different sensing patterns can refer to each order of $\Phi_{\text{SoWH}}$, which use to obtain each element of $\gamma$. It offers several features that allow boundary information to be accessible from measurement. Hence, we can trace back to which pixels in the block had read. In this work, there are three significant sensing patterns as shown in Figure 3.

where white and black squares indicate the pixel that is being read and skip, respectively.

For instance, by multiplying $x$ with $\Phi_{\text{SoWH}_1}$, the first element denoted by $y_1$ is the summation of $4 \times 4$ pixels; the second element $y_2$ can be obtained by multiplying $x$ with $\Phi_{\text{SoWH}_2}$, which is the summation of upper-half $2 \times 4$ pixels; and the third element $y_3$ can be obtained by multiplying $x$ with $\Phi_{\text{SoWH}_3}$, which is the summation of half-left $4 \times 2$ pixels.

However, the parameters that necessary for generating prediction candidates are located in black squares, which are opposite-side of $\Phi_{\text{SoWH}_2}$ and $\Phi_{\text{SoWH}_3}$. To retrieve them, since $y_1$ is a summation of all pixels in the block. Therefore, the data in black squares can be obtained by subtracting $y_1$ with $y_2$ and $y_1$ with $y_3$, resulting in sum of bottom-half $2 \times 4$ pixels and sum of the right-half $4 \times 2$ pixels, respectively.

To understand the concept clearer, we present the subtraction process by referring to patterns subtraction as shown in Figure 4. We note that this method delivers the same results as modifying the sensing matrix to obtain boundary pixels of adjacent blocks. Besides, the image quality will not be disturbed as the work in [35]. Further, we demonstrate the group of pixels after subtraction over image as shown in Figure 5.

At this stage, the parameters are the representation of multiple pixels. It is necessary to average them by dividing by the number of active pixels (i.e., in the case of $2 \times 4$ pixels and $4 \times 2$ pixels, the number of active pixels equals 8).

Afterward, we multiply the averaged parameters with $\Phi_{\text{SoWH}}$ to generate vector known as intra prediction candidate. To sum up, the candidate generation procedure of each mode can be explained by the following equations:

**Up mode:**

$$y_u = \frac{(y'_1 - y'_2)}{\sum_{j=1}^{n}(\Phi_{\text{SoWH}_1,j} - \Phi_{\text{SoWH}_2,j})} \times \Phi_{\text{SoWH}}$$

**Left mode:**

$$y_l = \frac{(y'\_1 - y'\_2)}{\sum_{j=1}^{n}(\Phi_{\text{SoWH}_1,j} - \Phi_{\text{SoWH}_3,j})} \times \Phi_{\text{SoWH}}$$

**Average mode**

$$y_{\text{avg}} = \frac{y_u + y_l}{2}$$

The final prediction candidate $y_p$ can be estimated by finding minimum error between target measurement $y$ with prediction candidates $y_c \in \{y_u, y_l, y_{\text{avg}}\}$. It can be expressed as the following:

$$y_p = \arg\min_{y_c \in \{y_u, y_l, y_{\text{avg}}\}} \|y_l - y_c\|_1$$

In addition, in case there is no prediction candidate selected from $y_c$, $y_p$ will be equal to zero, which means no prediction. The residual measurement $y_r$ can be calculated by subtracting $y$ with $y_p$. It can be expressed by

$$y_r = y - y_p$$
J. Peetakul et al.: IPMC Algorithm for Block-Based Compressive Sensing Images

FIGURE 6. Hardware architecture of the proposed IPMC algorithm.

TABLE 1. Overall performance comparison with the state-of-the-art on various datasets, where $b = 16$, $SR = 1/4$, and $Q_b = 4$.

| Methods              | Beauty dataset | ReadySetGo dataset | Rosphorus dataset | HoneyBee dataset |
|----------------------|----------------|--------------------|-------------------|------------------|
|                      | PSNR (dB)/SSIM | bpp                | PSNR (dB)/SSIM    | bpp              |
| Bernoulli + SQ* [26] | 34.45 / 0.86   | 2.71               | 30.83 / 0.67      | 2.72             |
| NoH + SQ* [27]       | 32.72 / 0.77   | 0.39               | 29.69 / 0.48      | 0.51             |
| Gaussian + SQ* [28]  | 33.27 / 0.75   | 2.62               | 30.79 / 0.50      | 2.64             |
| DPCM + SQ [33]       | 36.15 / 0.88   | 2.71               | 31.23 / 0.72      | 2.71             |
| SDPC + SQ [34]       | 36.27 / 0.88   | 2.71               | 33.31 / 0.73      | 2.70             |
| Intra Pred. + HSM + SQ [35] | 35.57 / 0.87 | 2.30               | 31.04 / 0.70      | 2.18             |
| This work            | 38.90 / 0.92   | 2.21               | 34.36 / 0.87      | 2.05             |
|                      |                |                    | 36.47 / 0.94      | 1.92             |

C. SCALAR QUANTIZATION

We further reduce bitrate and probability symbols of $y_r$ using SQ. It maps residual measurement $y_r$ into a finite sequence of codewords with quantization step equal to $\Delta$. It can be expressed by:

$$\Delta = \left\lfloor \frac{\max(y_r) - \min(y_r)}{2Q_b} \right\rfloor,$$

$$y_q = \left\lfloor \frac{y_r}{\Delta} \right\rfloor,$$

where $Q_b$ is quantization bit and quantized measurement denoted by $y_q$. Subsequently, inverse quantization maps $y_q$ into $y_{iq}$ that is an approximation of $y_r$. It can be expressed by:

$$y_{iq} = \Delta \cdot y_q.$$

In this work, we fixed $Q_b$ at 4 bits, which is sufficient to reduce bitrate and probability symbols. Furthermore, we include inverse quantization inside the transmitter to estimate prediction candidates loss by quantization at the receiver. Subsequently, we use that estimated prediction candidates to predict the next target measurement. If both sides do not have the same prediction candidates information, the decoder will act as error accumulator, in which a single corrupted measurement can ruin the whole image. We note that $y_q$ needs to transmit along with 2 bits side information of prediction mode and $\Delta$ of each block to the receiver.

D. HARDWARE IMPLEMENTATION OF PROPOSED IPMC ALGORITHM FOR BCIS

In this section, we extend IPMC algorithm from software to hardware for increasing throughput. The hardware architecture can be seen in Figure 6. It can be placed next to BCIS.

FIGURE 7. A diagram of vector summation for $m = 4$, where (a) tree-like vector pipeline which the summation can be done within $\log_2(m)$ clock cycles and (b) non-pipeline which the summation can be done within $m - 1$ clock cycles.
Subsequently, we reported RD-curve performance in various setting of $Q_b$ as shown in Figure 10.

Step 6: Subtract $y$ with $y_p$, resulting in $y_r$.

Step 7: Apply quantization to $y_r$, resulting in $y_q$.

Nevertheless, the data structure of $y$ is vector. Without optimization, it requires at least $m - 1$ clock cycles to encode $y$. Therefore, we optimize vector summation module using a tree-like pipeline technique as shown in Figure 7a, and non-pipeline in Figure 7b, in which clock cycle can be shorter from $m - 1$ to $\log_2(m)$. Consequently, it requires slightly higher resources than non-pipeline.

Subsequently, it is necessary to prepare the prediction parameters for the next target measurement. The procedure of prediction parameters preparation can be described as the following step:

Step 1. Apply inverse quantization to $y_q$, resulting in $y_{iq}$.
Step 2. Decode $y_{iq}$ by adding $y_p$, resulting in $\hat{y}$.
Step 3. Extract $\hat{y}$ using vector splitter to obtain prediction parameters (i.e., $\hat{y}_1$, $\hat{y}_2$, and $\hat{y}_{32}$).
Step 4. Subtract $\hat{y}_1$ with $\hat{y}_2$ and $\hat{y}_1$ with $\hat{y}_{32}$.
Step 5. Store the results in registers for the next prediction.

### III. EXPERIMENTAL RESULTS

In this section, we reported the performance of IPMC algorithm using PSNR, SSIM, and bpp. The simulation results delivered by MATLAB using $l_1$-minimization via primal-dual interior-point method [18]. We used multiple 4K datasets [37] consisted of Beauty, ReadySetGo, Bosphorus, and HoneyBee. Lastly, we reported hardware implementation results in terms of device specification and throughput.

1) OVERALL QUALITY COMPARISON OF VARIOUS SENSING MATRICES

As the results shown in Figure 8, we compared SoWH with various existing sensing matrices such as Bernoulli [26], NoH [27], Gaussian [28], and HSM [35] using ReadySetGo and HoneyBee datasets. It can be seen that SoWH gave the best quality in terms of SSIM than other sensing matrices at the
same SR, reflecting a higher ability to gather compressible signals.

2) OVERALL PERFORMANCE COMPARISON WITH STATE-OF-THE-ART WORKS

As the results shown in Table 1, firstly, we compared our proposal with the works that used SQ to code measurement such as Bernoulli + SQ [26], NoH + SQ [27], and Gaussian + SQ [28]. Our proposal overcame them in terms of higher PSNR and SSIM, and lower bpp. Nevertheless, NoH + SQ gave incredible results in bpp reduction because the data structure of measurement has highly uncorrelated. By using the equation (9), it returns a large parameter of $\Delta$. Thus, SQ will give a huge image degradation as can be noticed by artifacts. Based on the uncontrollable performance of SQ, where the performance will be varied depending on sensing matrix. We assume that SQ is an inefficient coding method, which correspond to the opinion stated in the most recent literature. Next, we compared our proposal with state-of-the-art works that utilized measurement coding and SQ such as DPCM + SQ [33], SDPC + SQ [34], and Intra Pred. + HSM [35]. This work significantly outperformed by reduced 10.90% of bpp, increased in PSNR and SSIM by 3.95 dB and 10.17%, respectively.

These results emphasized our opinion that compression performance can be increased by designing a good pair of measurement coding algorithm and sensing matrix. The measurement sampled by SSM has higher data structure consistency, which enabled coding algorithm to perform better, resulting in higher compression performance. Hence, the most important element in measurement will be encoded and will not be ruined by quantization, resulting in an improvement of PSNR and SSIM.

Lastly, we provided visual quality comparison of reconstructed images in Figure 9. This work provided better image quality than state-of-the-art works without compression artifacts at the edge of object.

It can be seen that this work gave a remarkable coding performance, where the data were encoded and not ruined by quantization even at shallow $Q_b$. 

![RD-curves of ReadySetGo dataset](image1)

![RD-curves of HoneyBee dataset](image2)
3) RESULTS OF HARDWARE IMPLEMENTATION

we reported hardware specification of IPMC algorithm in Table 2. The full block diagram and schematic in the Altera Quartus tools is located in Figure 11. The IPMC algorithm consumed total logic utilization by 5,948/41,910 logic elements and total registers by 2,138. The throughput of this algorithm is 5 Gpixels/s and operated at 88 MHz. This architecture cost the power of 50 mW for encoding 3840 × 2160 pixels, where the SR is fixed at 1/4. Lastly, we provided top-level timing diagram of BCIS and IPMC algorithm in Figure 12.

IV. CONCLUSION

BCIS is an innovative approach, in which turned conventional image and video system upside down. In this work, we closed the gap of compression performance between these BCIS and conventional systems. Our proposal capitalizes on a good pair of sensing matrix and the IPMC algorithm, which gave an extra compression performance to the traditional CS paradigm. Further, our proposal gave the highest compression performance compared to state-of-the-art works, which gradually closing the possibility gap to replace image and video acquisition system with BCIS and novel coding algorithm. Moreover, we implemented the proposal on FPGA, which IPMC algorithm gave simpler and less complexity than conventional algorithms.

APPENDIX A

COMPRESSED SENSING THEORY

A. SPARSE SIGNAL CHARACTERISTIC

This characteristic implies that only a few coefficients would contain the majority of the signal information. It can be expressed by:

\[ x = \Psi \theta \]  

where \( x \in \mathbb{R}^{n \times 1} \) is the vectorized signal and \( \theta \in \mathbb{R}^{n \times 1} \) is the sparse vector that contains the projection of \( x \) in the transform basis \( \Psi \in \mathbb{R}^{m \times n} \).

B. RESTRICTED ISOMETRY PROPERTY

To guarantee a good reconstructed image, we should follow a hinge on a characterization of sensing matrix called restricted isometry property [14]. We can determine the lower bound dimensional of \( m \) for non-uniform distributed sensing matrix using the following equation:

\[ (1 - \delta_s) \| x \|_2^2 \leq \| 8 \|_2^2 \leq (1 + \delta_s) \| x \|_2^2 \]  

(13)

where \( \delta_s \in \{1, 0\} \).

C. SPARSE SIGNAL RECOVERY VIA CONVEX OPTIMIZATION

By solving ill-posed linear inverse problems via convex optimization to recover the signal, CS states that if the signal \( x \) is compressible by sparse transform \( \Phi \) and \( \Psi \) is highly incoherent to \( \Phi \). The signal can accurately recover from dimensional \( m \) of incomplete measurement in the coefficient domain as:

\[ \hat{x} = \Phi \Psi^{-1} \Psi x \]  

(14)

where \( \Psi^{-1} \) is inverse transform [15]–[17]. In this work, we recover \( y \) by using a classical method via convex optimization called \( \ell_1 \)-minimization [18] as follows:

\[ \hat{x} = \arg \min \frac{1}{2} \| \Phi x - y \|_2^2 + \lambda \| x \|_1 \]  

(15)

Further, there are greedy-based recovery algorithms included orthogonal matching pursuit (OMP) [19] and its
extension stagewise OMP [20], A*OMP [21], CoSaMP [22], and TwIST2 [23] have been proposed for CS. However, by comparing to $\ell_1$-minimization, greedy-based methods are generally faster because they take advantage of sparsity structure via minimizing a sequence of subspace problem. However, it requires higher prior knowledge and a decent amount of measurements to make a good reconstruction image.

REFERENCES

[1] M. F. Duarte, M. A. Davenport, D. Takhar, J. N. Laska, T. Sun, K. F. Kelly, and R. G. Baraniuk, “Single-pixel imaging via compressive sampling,” IEEE Signal Process. Mag., vol. 25, no. 2, pp. 83–91, Mar. 2008, doi: 10.1109/MSP.2007.914730.

[2] M. Pascucci, S. Ganesan, A. Tripathi, O. Katz, V. Emilianii, and M. Guillon, “Compressive three-dimensional super-resolution microscopy with speckle-saturated fluorescence excitation,” Nature Commun., vol. 10, no. 1, p. 1327, Dec. 2019, doi: 10.1038/s41467-019-09929-7.

[3] R. Robucci, J. D. Gray, L. K. Chiu, J. Romberg, and P. Hasler, “Compressive sensing on a CMOS separable-transform image sensor,” Proc. IEEE, vol. 98, no. 6, pp. 1089–1101, Jun. 2010.

[4] J. Peetakul, J. Zhou, and K. Wada, “A measurement coding system for compressive sensing with quantized side information at the decoder,” IEEE Trans. Inf. Theory, vol. 62, no. 1, pp. 1–22, Jan. 2016.

[5] J. A. Tropp, “The sparsity gap: Uncertainty principles proportional to dimension,” in Proc. 44th Annu. Conf. Inf. Sci. Syst. (CISS), Princeton, NJ, USA, Mar. 2010, pp. 1–6, doi: 10.1109/CISS.2010.5464824.

[6] B. Candes and T. Tao, “Near-optimal signal recovery from random projections: Universal encoding strategies?” IEEE Trans. Inf. Theory, vol. 52, no. 12, pp. 4880–4895, Dec. 2006.

[7] E. J. Candès, J. Romberg, and T. Tao, “Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information,” IEEE Trans. Inf. Theory, vol. 52, no. 2, pp. 489–509, Feb. 2006.

[8] J. A. Tropp, “Bayesian recovery of sparse signals via convex programming,” Tech. Rep., Oct. 2005.

[9] J. A. Tropp and A. C. Gilbert, “Signal recovery from random measurements via orthogonal matching pursuit,” IEEE Trans. Inf. Theory, vol. 53, no. 12, pp. 4655–4666, Dec. 2007, doi: 10.1109/TIT.2007.909108.

[10] D. L. Donoho, Y. Tsaig, I. Drori, and J.-L. Starck, “Sparse solution of underdetermined systems of linear equations by stagewise orthogonal matching pursuit,” IEEE Trans. Inf. Theory, vol. 58, no. 2, pp. 1094–1121, Feb. 2012, doi: 10.1109/TIT.2011.2173241.
YIBO FAN received the B.E. degree in electronics and engineering from Zhejiang University, Hangzhou, China, in 2003, the M.S. degree in microelectronics from Fudan University, Shanghai, China, in 2006, and the Ph.D. degree in engineering from Waseda University, Tokyo, Japan, in 2009. He was an Assistant Professor with Shanghai Jiao Tong University and Fudan University, from 2009 to 2014, and an Associate Professor with Fudan University, from 2014 to 2019. He is currently a Full Professor with the College of Microelectronics, Fudan University. His research interests include image processing, video coding, machine learning, and associated VLSI architecture.

JINJIA ZHOU (Member, IEEE) received the B.E. degree from Shanghai Jiao Tong University, China, in 2007, and the M.E. and Ph.D. degrees from Waseda University, Fukuoka, Japan, in 2010 and 2013, respectively. From 2013 to 2016, she was a Junior Researcher with Waseda University. She is currently an Associate Professor and the Co-Director of the English-based graduate program with Hosei University. She is also a Senior Visiting Scholar with the State Key Laboratory of ASIC and System, Fudan University, China. Since 2020, she has been a specially appointed Associate Professor with Osaka University. Her research interests include algorithms and VLSI architectures for multimedia signal processing. She was selected as JST PRESTO Researcher from 2017 to 2021. She received the Research Fellowship of the Japan Society for the Promotion of Science for the term 2010–2013. She was a recipient of the Hibikino Best Thesis Award in 2011 and the Chinese Government Award for Outstanding Students Abroad of 2012. She was a co-recipient of the Best Student Paper Award of VLSI Circuits Symposium 2010, the Design Contest Award of ACM ISLPED 2010, and the ISSCC 2016 Takuo Sugano Award for Outstanding Far-East Paper. She participated in the design of the world first 8K UHDTV video decoder chip, which was granted the 2012 Semiconductor of the Year Award of Japan. She also works as an Associate Editor for IEEE Access and a Reviewer of journals, including IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS FOR VIDEO TECHNOLOGY, IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I, IEEE TRANSACTIONS ON VERY LARGE SCALE INTEGRATION (VLSI) SYSTEMS, and IEEE TRANSACTIONS ON MULTIMEDIA.

* * *