Abstract. We propose that the one-fold selfintersecting center-vortex loop, being the stable excitation in the confining phase of SU(2) Yang-Mills thermodynamics of scale $\Lambda \sim 0.5$ MeV, after an electric-magnetically dual interpretation of this theory represents the electron/positron. Our argument invokes recent results on the physics of a strongly and spherically perturbed 't Hooft-Polyakov monopole, the role of the central spatial region in a Harrington-Shepard (HS) (anti)caloron, the latter’s deformation towards maximally non-trivial holonomy and subsequent dissociation into a pair of screened BPS monopole and antimonopole, the energy-density of the deconfining thermal ground state, and the critical temperature $T_c$ for the deconfining-preconfining transition. We estimate the typical spatial extent of the selfintersection region and the monopole core size, implying that the electron/positron, judged by its Compton wave length, is anything but a point particle. Our results support and elucidate the ideas of Louis de Broglie on the thermodynamics of an isolated particle.

INTRODUCTION

In the course of proposing the wave nature of the electron in terms of “phase harmony” of an internal vibration and a wave-like propagation, Louis de Broglie stumbled over an apparent contradiction, arising from the distinct Lorentz-boost transformation properties of an internal-clock frequency $\omega = \sqrt{1-\beta^2} \omega_0$, transforming in the same way as an internal heat $Q = \sqrt{1-\beta^2} Q_0$ or the associated temperature $T = \sqrt{1-\beta^2} T_0$, and the total quantum energy $E = h \frac{\omega_0}{\sqrt{1-\beta^2}} = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$, being the zero component of four-momentum. Here $m_0$ denotes the proper mass, $\beta = \frac{v}{c}$ with $v$ being the (linear) velocity modulus of the boost and $c$ the speed of light in vacuum, $h$ is Planck’s quantum of action, $\omega_0$ represents the circular frequency associated with an internal vibration, and $Q_0 = m_0 c^2$ is the internal heat content of the particle in its rest frame. This puzzle was addressed by de Broglie in decomposing $E$ as [1, 2]

$$E = h \frac{\omega_0}{\sqrt{1-\beta^2}} = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = Q + v p = Q + \mathcal{F} = m_0 c^2 \sqrt{1-\beta^2} + \frac{m_0 v^2}{\sqrt{1-\beta^2}},$$

where the relativistic spatial momentum modulus $p$ is given as $p = \frac{m_0 v}{\sqrt{1-\beta^2}}$, and the quantity $\mathcal{F} = v p$ is interpreted as a “pseudo-vis viva” (translational energy) of the particle. In the limit $v \rightarrow 0$ the particle $E$ reduces to the rest energy $m_0 c^2$ which de Broglie considered the temporal mean of a fluctuating energy being “the result of the continual energy exchanges between the particle and the hidden thermostat” [1].

The purpose of this brief communication is to propose that the region of selfintersection of an SU(2) Yang-Mills center-vortex loop (figure-eight shaped soliton, stable excitation in the confining phase, see FIG. 1) actually satisfies de Broglie’s notion of the electron’s rest mass $m_0$ being the result of quantum induced interactions between a thermal environment, approximated by the energy-density and pressure dominating deconfining thermal ground state close to the deconfining-preconfining phase boundary at temperature $T_0 \sim T_c = \frac{11.87}{\Lambda}$, $\Lambda$ denoting the Yang-Mills scale [3], and a topologically stabilised BPS monopole. Such a monopole is enabled by the dissociation of
FIGURE 1. Schematics of a one-fold self-intersecting center-vortex loop, immersed into the confining phase of SU(2) Yang-Mills theory. SU(2) Yang-Mills theory is to be interpreted in an electric-magnetically dual way, see [3] and SEC. 3. Thus, the (vibrating) core of the perturbed magnetic ’t Hooft-Polyakov monopole, located within the (fuzzy) self-intersection region of radius \( R_0 \) representing deconfining thermal-ground-state energy density \( \rho_{gs} \), actually represents an electric charge. Also, the magnetic center flux, residing in the two wings of the vortex-loop, actually is an electric one, giving rise to a magnetic moment twice that of a single vortex loop. The concentric circles indicate the dilution of the confining phase by expanding high-frequency shells carried by massive off-Cartan modes of the monopole which also balance the negative pressure of the deconfining phase at the fuzzy phase boundary. These spherical shells are the consequence of (anti)caloron-center induced quantum perturbations of the monopole, and they should enable the penetration of the monopole’s electric Coulomb field into the confining phase, compare with SEC. 4.

This paper is organized as follows. In SEC. 2 we review some results of [11] on the physics of a strongly excited ’t Hooft-Polyakov monopole which are relevant for the emergence of the intersection region and the leak-out of a Coulomb-like electric field into exterior space (\( r > R_0 \)). SEC. 3 discusses some aspects of SU(2) Yang-Mills thermodynamics which, together with the work of [11], imply our results of SEC. 4 on the extent of the intersection region and the monopole core. In SEC. 4 we provide a short summary and outlook.

SEC. 2: A STRONGLY AND SPHERICALLY PERTURBED ’t HOOFT-POLYAKOV MONOPOLE

If not explicitly stated otherwise, we work in natural units (\( \hbar = k_B = c = 1 \)) from now on. The normal-mode spectrum of a ’t Hooft-Polyakov, considering small field fluctuations about the static BPS monopole only, was investigated in [12]. In [11] a spherically symmetric, strong perturbation of the ’t Hooft-Polyakov monopole in SU(2) Yang-Mills-Higgs theory was analysed dynamically by virtue of a hyperboloidal conformal transformation of the original field equations for the profiles \( H(r, t) = h(r, t)/r + H_\infty \) and \( w(r, t) \) of the adjoint Higgs and off-Cartan fields, respectively. Their results can be summarised as follows. Considering a spherically symmetric, localised initial pulse as a strong perturbation of the static BPS monopole, the typical dynamical response did not depend on the parameter values of this pulse within a wide range. Namely, there are high-frequency oscillations in \( w \) which form expanding shells decaying in time as \( t^{-1/2} \) (the further away from the monopole core the shell the higher the frequencies that build it). On the other hand, a localised breathing state appears in association with the energy density of the monopole core region whose frequency \( \omega_0 \) approaches the mass \( m_w = eH_\infty \) (\( e \) denoting the gauge coupling) of the two off-Cartan modes in a power-like way in time (natural units):

\[
\omega_0 = eH_\infty - C_w t^{-2/3} \Rightarrow \lim_{t \to \infty} \omega_0 = eH_\infty ,
\]

where \( C_w \) is a positive constant. The amplitude of the oscillation in energy density decays like \( C_a t^{-5/6} \), \( C_a \) again denoting a positive constant. Disregarding for now the question what the physics of the exciting initial condition is,
it thus appears that an internal clock of (circular) frequency $\omega_0 \sim m_w$ is run within the core region of the perturbed monopole.

**SEC. 3: SOME ASPECTS OF THE DECONFINING, PRECONFINING, AND CONFINING PHASES IN SU(2) YANG-MILLS THERMODYNAMICS**

To interpret the location of selfintersection within an SU(2) Yang-Mills center-vortex loop as a quantum dynamically stabilized region of deconfining thermal ground state, interacting with a 't Hooft-Polyakov monopole and immersed in an environment essentially made up of confining phase, we need to rely on a few non-perturbative results on SU(2) Yang-Mills thermodynamics [3].

**Deconfining phase**

The deconfining phase of SU(2) Yang-Mills thermodynamics comprises a thermal ground state estimate due to HS (anti)calorons. This ground state exhibits a linear-in-$T$ energy density

$$\rho^{gs} = 4\pi \Lambda^3 T$$

and is composed of the densely packed HS-(anti)caloron centers, responsible for quantum excitations, and overlapping HS-(anti)caloron peripheries, giving rise to (anti)selfdual dipole densities, the associated permittivity and permeability and thus the associated propagation speed of a wave-like excitation being $T$-independent [3]. There are off-Cartan massive vector modes, which fluctuate quantum-thermodynamically, and a massless “photon” mode which propagates in terms of classical electromagnetic waves at low frequencies and/or intensities and fluctuates quantum-thermodynamically otherwise. A certain class of effective radiative corrections collectively describes an ensemble of screened and unresolved magnetic monopole-antimonopole pairs, generated by (anti)caloron dissociation upon holonomy shift [6, 8, 9, 7, 4]. The maximum non-trivial (anti)caloron holonomy is determined by $A_4(\tau, |\vec{x}| \to \infty) = \pi T t_3$, agreeing on a trace normalization of the SU(2) generators $t^a$ ($a = 1, 2, 3$). For a constituent BPS monopole this implies $H_\infty = \pi T_c$. After screening and considering temperature $T$ to be only mildly above $T_c$, the associated monopole mass $m_m = \frac{8\pi}{e^2} H_\infty$ approximately is given as [13]

$$m_m \sim \frac{8\pi^2}{e^2} H_\infty \sim H_\infty \sim \pi T_c,$$

where the plateau value $e = \sqrt{8\pi}$ of the SU(2) coupling constant, expressing one-loop thermodynamical selfconsistency of deconfining SU(2) Yang-Mills thermodynamics [3], was used. For our purposes this is justified because the singularity of $e$ at

$$T_c = \frac{13.87}{2\pi} \Lambda \sim \frac{H_\infty}{\pi}$$

is logarithmically thin only [3]. Close to $T_c$ deconfining SU(2) Yang-Mills thermodynamics is ground-state dominated. Note that the radius $R_c$ of the monopole core is about

$$R_c \sim H_\infty^{-1}.$$

**Confining and preconfining phase**

The confining phase of an SU(2) Yang-Mills theory exhibits vanishing ground-state pressure and energy density at vanishing temperature. As the temperature variable $T$ approaches the Yang-Mills scale $\Lambda$ the spectrum of excitations is characterized by an over-exponentially growing density of states ($n$-fold selfintersecting center-vortex loops) such that spatially homogeneous thermal equilibrium increasingly is invalidated [3]. At $T \sim \Lambda$ a Hagedorn transition occurs, and the intersection regions of center-vortex loops percolate into a condensate of massless magnetic monopoles/antimonopoles, forming the new thermal ground state of the preconfining phase [3]. This ground state dominates the entire thermodynamics due to the Meissner massiveness of the gauge mode which is massless in the deconfining phase. The latter sets in at $T_c$ [3].
Electric-magnetically dual interpretation of charges in SU(2) Yang-Mills theory

In units were \( c = e_0 = \mu_0 = 1 \) (\( e_0 \) and \( \mu_0 \) denoting the electric permittivity and magnetic permeability of the classical electromagnetic vacuum in SI units) the QED fine structure constant \( \alpha \), which is unitless in any system of units, reads

\[
\alpha = \frac{Q^2}{4\pi\hbar}.
\]  

(7)

On the other hand, the Yang-Mills coupling \( e \) was argued to be \( e = \frac{\sqrt{g_5}}{\sqrt{\alpha}} \) almost everywhere in the deconfining phase [14], based on the observation in [15] that only a small range of (anti)caloron radii, centered about the spatial coarse-graining cutoff \( |\phi|^{-1} \), contributes to the emergent thermal ground state. This value of \( e \) implies the (anti)caloron action to be \( \hbar \). Now, for \( a \) to be unitless it follows that \( Q \propto 1/e \), meaning that an electric charge in the real world is represented by a magnetic charge w.r.t. to the Cartan subalgebra in the SU(2) Yang-Mills theory. In particular, a magnetic monopole in the deconfining phase of an SU(2) Yang-Mills theory thus is to be interpreted as an electric charge in our world.

SEC. 4: EXTENDED ELECTRIC CHARGE AND MAGNETIC MOMENT

Let us now exploit the facts introduced in the previous sections to make contact with de Broglie’s hidden thermodynamics of the isolated electron and to infer typical length scales governing the selfintersection region of the associated center-vortex loop in SU(2) Yang-Mills theory.

First, according to Eq. (3) the energy density of the deconfining ground state, which dominates SU(2) Yang-Mills thermodynamics for \( T \) close to \( T_c \), is linear in \( T \) (\( T = \sqrt{1 - \beta^2 T_0} \)), supporting the interpretation that the internal heat \( Q \) is due to the “thermostat” interacting with the particle: \( Q = \sqrt{1 - \beta^2} Q_0 \).

Second, according to Eq. (2) one may express the rest energy/mass \( m_0 \), associated with the intersection region, as

\[
m_0 \sim e H_\infty \sim \sqrt{8\pi} H_\infty.
\]

(8)

On the other hand, due to energy conservation exhibited by the quantum interaction of the deconfining thermal ground state with the isolated monopole – induced by a sequence of perturbations issued indeterministically by (anti)caloron centers [16] –, we may write (compare with Eqs. (3),(4),(5))

\[
m_0 = m_0 + E_0 \sim H_\infty + \frac{4}{3}\pi R_0^3 \mu_0^2 \sim H_\infty \left(1 + \frac{128\pi}{3} \left(\frac{R_0}{13.87}\right)^3 H_\infty^3\right),
\]

(9)

where \( R_0 \) denotes the spatial radius of a ball-like (yet fuzzy) region of deconfining phase, see Fig. 1. Equating the right-hand sides of Eqs. (8), (9) and solving for \( R_0 \), we arrive at

\[
R_0 \sim 13.87 \left(\frac{3}{128\pi} (\sqrt{8\pi} - 1)\right)^{1/3} H_\infty^{-1} \sim 5.4 H_\infty^{-1}.
\]

(10)

Comparing this to the Compton wave length \( \lambda_C = m_0^{-1} \), we have

\[
R_0 \sim 46.9 \lambda_C \sim 1.1 \times 10^{-10} \text{m}.
\]

(11)

Thus, \( R_0 \) is about twice as large as the Bohr radius \( a_0 \) which, in turn, is about 2.7 (= 5.4/2) times larger than the radius of the monopole core \( R_c \). Therefore, it would be utterly incorrect to consider the electron, if realized in the manner as proposed above, to be a point particle. The reason why scattering experiments do not reveal an inner structure is that quantum thermodynamics, being the state of maximum entropy taking place within radius \( R_0 \), does not represent any discernible structure. As discussed in SEC. 2, based on the important work [11], high-frequency oscillations in the profile function \( \omega \) of the off-Cartan gauge fields, belonging to the perturbed monopole, develop expanding shells. If these shells are considered to penetrate the confining phase \((r > R_0)\), such that the static Coulomb field of the monopole, arising from a temporal average over many cycles of the monopole core-vibration, can actually permeate this phase, then we may estimate the Coulomb-field correction \( \Delta m_0 \) to the “thermodynamical” mass \( m_0 \) as

\[
\Delta m_0 = \int_{R_0}^\infty \frac{r^2}{r^3} = R_0^{-1} - \frac{1}{5.4} H_\infty < \sqrt{8\pi} H_\infty = m_0 \quad \text{or} \quad m_0 \sim 48 \Delta m_0.
\]

(12)
This is a ∼2% correction only, much in contrast to the classical electron radius defining the entire rest mass $m_0$ in terms of Coulomb energy.

Finally, we would like to mention that, in assigning half a Bohr magneton to the magnetic moment carried by a single center-vortex loop, a $g$-factor of two naturally arises by the electron being composed of two such center-vortex loops by virtue of the region of selfintersection, see Fig. 1.

**SUMMARY**

In this contribution we have, based on a recent numerical investigation of an isotropically perturbed ’t Hooft-Polyakov monopole [11], the constituents / structure of the deconfining thermal ground state, and the nature of excitations in the confining phase of SU(2) Yang-Mills thermodynamics [3], elucidated the assertion made by L. de Broglie [2, 1] that, based on distinct Lorentz boost transformation properties of frequency $\omega_0$ and energy $E$, both associated with the rest mass $m_0$, the electron’s charge and mass represent a spatial region of active quantum thermodynamics in SU(2) Yang-Mills theory. Our main result is that the radial extent $R_0$ of this region is comparable to typical atomic radii with the radial extent $R_e$ of the electric charge being about a factor of three smaller: $R_0 / R_e \sim 1/3$, $\alpha_0$, $\alpha_0$ being the Bohr radius. The discussion of the present paper is restricted to a uniformly moving electron / positron. How such a system responds to acceleration by external forces is, as of yet, unclear because this would necessitate to solve the field equations under essentially relaxed symmetry assumptions.

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