A DYNAMICAL MODEL FOR THE RESONANT MULTIPOLES
AND THE $\Delta$ STRUCTURE

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We show that recent experiment data for the ratios $E_{1+}/M_{1+}$ and $S_{1+}/M_{1+}$ can be explained in a dynamical model for electromagnetic production of pions, together with a simple scaling assumption for the bare $\gamma^* N\Delta$ form factors. Within our model we find that the bare $\Delta$ is almost spherical and the electric $E2$ and Coulomb $C2$ quadrupole excitations of the physical $\Delta$ are nearly saturated by pion cloud contribution in $Q^2 \leq 4.0$ GeV$^2$.

1 Introduction

The study of excitations of the hadrons can shed light on the nonperturbative aspects of QCD. One case which has recently been under intensive study is the electromagnetic excitation of the $\Delta(1232)$ resonance. At low four-momentum transfer squared $Q^2$, the interest is motivated by the possibility of observing a $D$ state in the $\Delta$. The existence of a $D$ state in the $\Delta$ has the consequence that the $\Delta$ is deformed and the photon can excite a nucleon through electric $E2$ and Coulomb $C2$ quadrupole transitions. In a symmetric SU(6) quark model, the electromagnetic excitation of the $\Delta$ could proceed only via magnetic $M1$ transition. In pion electroproduction, $E2$ and $C2$ excitations would give rise to nonvanishing $E_{1+}^{(3/2)}$ and $S_{1+}^{(3/2)}$ multipole amplitudes. Recent experiments give nonvanishing ratio $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)} \sim -0.05$ at $Q^2 = 0$ which has been widely taken as an indication of the $\Delta$ deformation.

At sufficiently large $Q^2$, the perturbative QCD (pQCD) is expected to work. It predicts that only helicity-conserving amplitudes contribute at high $Q^2$, leading to $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)} \rightarrow 1$ and $R_{SM} = S_{1+}^{(3/2)}/M_{1+}^{(3/2)} \rightarrow const$. This behavior in the perturbative domain is very different from that in the nonperturbative one. It is an intriguing question to find the region of $Q^2$ which signals the onset of the pQCD.

In a recent measurement$^b$, the electromagnetic excitation of the $\Delta$ was studied at $Q^2 = 2.8$ and $4.0$ GeV$^2$ via reaction $p(e,e'p)\pi^0$. The extracted ratios $R_{EM}$ and $R_{SM}$ remain small and negative. This disagrees with the previous analysis$^3$ of the earlier DESY data which gave small but positive $R_{EM}$ and $R_{SM}$ at $Q^2 = 3.2$ GeV$^2$, though both analyses indicate that pQCD

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is still not applicable in this region of $Q^2$. In this talk, we want to show that the recent data of Ref. 2 can be understood from the dominance of the pion cloud contribution at low $Q^2$ in both $E_{1+}^{(3/2)}$ and $S_{1+}^{(3/2)}$ multipoles, as predicted by a dynamical model for electromagnetic production of pion, together with a simple scaling assumption for the bare $\gamma^*N\Delta$ form factors.

2 Dynamical Model for $\gamma^*N \rightarrow \pi N$

The main feature of dynamical approach to pion photo- and electro-production is that the unitarity is built in by explicitly including the final state $\pi N$ interaction in the theory, namely, t-matrix is expressed as

$$t_{\gamma\pi}(E) = v_{\gamma\pi} + v_{\gamma\pi}g_0(E)t_{\pi N}(E),$$

where $v_{\gamma\pi}$ is the transition potential operator for $\gamma^*N \rightarrow \pi N$, and $t_{\pi N}$ and $g_0$ denote the $\pi N$ t-matrix and free propagator, respectively, with $E \equiv W$ the total energy in the CM frame.

Multipole decomposition of Eq. (1) gives the physical amplitude in channel $\alpha$

$$t_{\gamma\pi}^{(\alpha)}(q_E, k; E + i\epsilon) = \exp \left( i\delta^{(\alpha)} \right) \cos \delta^{(\alpha)} \times \left[ v_{\gamma\pi}^{(\alpha)}(q_E, k) + P \int_0^\infty dq' R_{\pi N}^{(\alpha)}(q_E, q'; E) v_{\gamma\pi}^{(\alpha)}(q', k) \right],$$

where $\delta^{(\alpha)}$ and $R^{(\alpha)}$ are the $\pi N$ scattering phase shift and reaction matrix in channel $\alpha$, respectively; $q_E$ is the pion on-shell momentum and $k = |k|$ is the photon momentum.

The multipole amplitude in Eq. (2) manifestly satisfies the Watson theorem and shows that $\gamma\pi$ multipoles depend on the half-off-shell behavior of $\pi N$ interaction. We remark that the use of K-matrix unitarization scheme as employed in, e.g., Ref. 7 would amount to approximating Eq. (2) with

$$t_{\gamma\pi}^{(\alpha)}(q_E, k; E + i\epsilon) = \exp \left( i\delta^{(\alpha)} \right) \cos \delta^{(\alpha)} v_{\gamma\pi}^{(\alpha)}(q_E, k).$$

The difference between Eqs. (2) and (3) lies in the fact that only the on-shell rescatterings are included in the K-matrix unitarization scheme.

In the resonant (3,3) channel in which $\Delta(1232)$ plays a dominant role, the transition potential $v_{\gamma\pi}$ consists of two terms

$$v_{\gamma\pi}(E) = v_{\gamma\pi}^B + v_{\gamma\pi}^\Delta(E),$$

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where $v_{B\gamma\pi}^\pi$ is the background transition potential which includes Born terms and vector mesons exchange contributions, as described in Ref. 8. The second term of Eq. (4) corresponds to the contribution of bare $\Delta$.

With Eq. (4), we may decompose Eq. (1) in the following way

$$t_{\gamma\pi}(E) = t_{B\gamma\pi}^\pi(E) + t_{\Delta\gamma\pi}^\pi(E), \tag{5}$$

where

$$t_{B\gamma\pi}^\pi(E) = v_{B\gamma\pi}^\pi + v_{B\gamma\pi}^\pi g_0(E) t_{\pi N}(E), \tag{6}$$

$$t_{\Delta\gamma\pi}^\pi(E) = v_{\Delta\gamma\pi}^\pi + v_{\Delta\gamma\pi}^\pi g_0(E) t_{\pi N}(E). \tag{7}$$

Here $t_{B\gamma\pi}^\pi$ includes contributions from the nonresonant background and renormalization on the vertex $\gamma^*N\Delta$ due to $\pi N$ scattering. The advantage of such a decomposition is that all the processes which start with the electromagnetic excitation of the bare $\Delta$ are summed up in $t_{\Delta\gamma\pi}^\pi$.

Multipole decomposition of Eq. (6) takes the same form as Eq. (2) and is used to calculate the multipole amplitudes $M_{1+}^B(W,Q^2)$, $E_{1+}^B(W,Q^2)$, and $S_{1+}^B(W,Q^2)$ with $R_{\pi N}^{(\gamma)}(q,E;q',E)$ obtained from a meson exchange model for $\pi N$ interaction. Note that to make the principal value integration in Eq. (2) associated with $v_{B\gamma\pi}^\pi$ convergent, we introduce an off-shell dipole form factor with cut-off parameter $\Lambda=440$ MeV. The gauge invariance, violated due to the off-shell rescattering effects, is restored by the substitution

$$J^B_{\mu} \rightarrow J^B_{\mu} - k_{\mu} k \cdot J^B / k^2,$$

where $J^B_{\mu}$ is the electromagnetic current corresponding to the background contribution $v_{B\gamma\pi}^\pi$.

3 $\gamma^*N \leftrightarrow \Delta$ transition form factors

Now let us consider the $\Delta$ resonance contribution $t_{\gamma\pi}^\Delta$ in Eq. (3). In keeping with the standard way of experimental analysis and constituent quark model (CQM) calculations, we describe the resonant multipoles $t_{\Delta\gamma\pi}^\Delta$ with a Breit-Wigner type of energy dependence, as was done in the isobar model of Ref. 8,

$$t_{\Delta\gamma\pi}^\Delta(W,Q^2) = A_{\Delta}(Q^2) - f_{\gamma\Delta} \Gamma_{\Delta} M_{\Delta} f_{\pi\Delta} \frac{M_{\Delta}^2 - W^2 - i M_{\Delta} \Gamma_{\Delta}}{M_{\Delta}^2 - W^2 - i M_{\Delta} \Gamma_{\Delta}} e^{i\phi}, \tag{8}$$

where $f_{\gamma\Delta}(W)$ is the usual Breit-Wigner factor describing the decay of the $\Delta$ resonance with total width $\Gamma_{\Delta}(W)$ and physical mass $M_{\Delta}=1232$ MeV. The expressions for $f_{\gamma\Delta}$, $f_{\pi\Delta}$ and $\Gamma_{\Delta}$ are taken from Ref. 8. The phase $\phi(W)$ in Eq. (8) is to adjust the phase of $t_{\Delta\gamma\pi}^\Delta$ to be equal to the corresponding pion-nucleon scattering phase $\delta^{(33)}$. Note that at the resonance energy $\phi(M_{\Delta}) = 0$. 3
The main parameters in the bare $\gamma^* N\Delta$ vertex are the $\tilde{A}^\Delta(Q^2)$’s in Eq. (8). For the magnetic dipole $\mathcal{M}^\Delta$ and electric quadrupole $\mathcal{E}^\Delta$ transitions, they are related to the conventional electromagnetic helicity amplitudes $A^\Delta_{1/2}$ and $A^\Delta_{3/2}$ by

$$\tilde{M}^\Delta(Q^2) = \frac{1}{2}(A^\Delta_{1/2} + \sqrt{3}A^\Delta_{3/2}), \quad \tilde{E}^\Delta(Q^2) = \frac{1}{2}(-A^\Delta_{1/2} + \frac{1}{\sqrt{3}}A^\Delta_{3/2}).$$

At the photon point $Q^2 = 0$, the bare amplitudes $\tilde{M}^\Delta(0)$ and $\tilde{E}^\Delta(0)$ of Eq. (8) are determined from the best fit to the results of the recent analyses of Mainz\cite{10} (open circles) and VPI group\cite{11} (full circles), as shown in Fig. 1. The dashed curves denote the contribution from $t^B_{\gamma\pi}$ only. The dotted curves correspond to the K-matrix approximation to $t^B_{\gamma\pi}$, namely, without the inclusion of principal value integral term. Solid curves are the full results of our calculation with bare $\Delta$ excitation.

The numerical values for $\tilde{M}^\Delta$ and $\tilde{E}^\Delta$ and the helicity amplitudes, at $Q^2 = 0$, are given in Table 1. Here we also give “dressed” values obtained using K-matrix approximation for $t^B_{\gamma\pi}$. One notices that the bare values determined above for the helicity amplitudes amount to only about 60% of the corresponding dressed values and are close to the predictions of the CQM, as pointed out by Sato and Lee\cite{12}. The large reduction of the helicity amplitudes from the dressed to the bares ones results from the fact that the principal value...
integral part of $t^B_{\gamma\pi}$, which represents the effects of the off-shell pion rescattering, contributes approximately half of the $M_{1+}$ as indicated by the dashed curves in Fig. [1]

Table 1: Comparison of the "bare" and "dressed" values for the amplitudes $\tilde{A}^\Delta$, $A^\Delta_{1/2}$ and $A^\Delta_{3/2}$ (in $10^{-3}$ GeV$^{-1/2}$).

| Amplitudes | "bare" | "dressed" | PDG |
|------------|--------|------------|-----|
| $\tilde{M}^\Delta$ | $158 \pm 2$ | $289 \pm 2$ | $293 \pm 8$ |
| $\bar{E}^\Delta$ | $0.4 \pm 0.3$ | $-7 \pm 0.4$ | $-4.5 \pm 4.2$ |
| $A^\Delta_{1/2}$ | $-80 \pm 2$ | $-134 \pm 2$ | $-140 \pm 5$ |
| $A^\Delta_{3/2}$ | $-136 \pm 3$ | $-256 \pm 2$ | $-258 \pm 6$ |

We now turn to the $Q^2$ evolution of the multipoles $\tilde{M}^\Delta(Q^2)$ and $\bar{E}^\Delta(Q^2)$. In the present work, we parametrize the $Q^2$ dependence of the dominant $\tilde{M}^\Delta$ amplitude by

$$\tilde{M}^\Delta(Q^2) = \tilde{M}(0) \frac{|k|}{k_\Delta} (1 + \beta Q^2) e^{-\gamma Q^2} G_D(Q^2),$$

(10)

where $G_D$ is the nucleon dipole form factor. For the small $\bar{E}^\Delta$ and $S^\Delta$ amplitudes, we follow Ref. 8 and assume that they have the same $Q^2$ dependence as $\tilde{M}^\Delta$ (scaling assumption). This is motivated by the scaling law which has been observed for the nucleon form factors.

We remind the reader that, in contrast to Ref. 8, amplitudes $\tilde{M}_{1+}$ and $\bar{E}_{1+}$ and the corresponding helicity amplitudes in Eq. (3) correspond to the "bare" $\gamma N\Delta$ transition. For the real photon, they are equal to the standard $M1$ and $E2$ amplitudes of the $\Delta \leftrightarrow N\gamma$ transition defined in accordance with the convention of the Particle Data Groups. At the resonance energy, they can be easily expressed in terms of the Dirac-type form factors $g_1$ and $g_2$ used in Ref. 7, or Sachs-type form factors $G_M$ and $G_E$ used in Ref. 12. The relation between $\tilde{M}_{1+}$ amplitude and the bare $G_M$ form factors is as follows

$$\tilde{M}_{1+}(Q^2) = e \frac{|k|}{k_\Delta} \sqrt{\frac{k_\Delta M_\Delta}{m}} G_M(Q^2),$$

(11)

where $k_\Delta = (M_\Delta^2 - m^2)/2M_\Delta$ with $m$ and $M_\Delta$ denoting the nucleon and $\Delta$ mass, respectively. Expression for the electric amplitude is similar, but with opposite sign. Relation between physical $M_{1+}^{3/2}$ multipole and experimentally measured $G_M^*$ form factor is given by Eq. (24) of Ref. 8. Note that we employ the "Ash" definition for the $G_M^*$, which differs from the $\Delta$ form factor used in Ref. 2 by a factor $f = \sqrt{1 + Q^2/(m + M_{\Delta})^2}$, i.e., $G_M^*(\text{our}) = G_M^*(\text{Ref. 2})/f$.  


4 Results and Discussion

Using the $\beta$ and $\gamma$ in Eq. (10) as free parameters, we fit the recent experimental data as well as old one quoted in Ref. 8 on the $Q^2$ dependence of the $M_{1+}^{(3/2)}$ or equivalently, the $G_M^*$ form factor. Our result is shown in Fig. 2. The obtained values for the $\beta$ and $\gamma$ parameters are: $\beta = 0.44 \text{ GeV}^{-2}$ and $\gamma = 0.38 \text{ GeV}^{-2}$. Here the dashed curve corresponds to contribution from the bare $\Delta$, i.e., $t^\Delta_{\gamma\pi}$ of Eq. (7). The results for the ratios $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ and $R_{SM} = S_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ are shown in the right column of Fig. 2. It is seen that they are in good agreement with the results of the model independent analysis of Ref. 2 up to $Q^2$ as high as $4.0 \text{ GeV}^2$. Note that since the bare values for the $E2$ and $C2$ excitations are small, the absolute values and shape of these ratios are determined, to a large extent, by the pion rescattering contribution. The bare $\Delta$ excitation contributes mostly to the $M_{1+}^{(3/2)}$ multipole.

Pion cloud has been found to play an important role in hadron structure in many studies. For example, in the cloudy bag model (CBM), a reasonably good agreement with the measured $R_{EM}$ can be obtained with bag radius of $R = 0.6 - 0.8 \text{ fm}$. In a recent improved CBM calculation, where the relativistic effects and CM motion were better treated, it was also found that the pion cloud gives large contribution to $G_M^*(Q^2)$. It is generally concluded
that the smaller the bag radius, the larger the pion cloud contribution. Similar conclusion is reached with respect to the proton EM form factors within the cloudy bag model. In fact, it has been found that the recent Jlab data on the scaling violation in the electromagnetic form factors can be explained within CBM with \( R = 0.7 \) fm. One might then be tempted to interpret our result as another indication of preferring a smaller bag radius. However, at these small bag radii, the pion field is so strong that the use of the perturbative approach employed in these CBM studies is questionable. In this connection, we should mention that in a nonperturbative calculation within a chiral chromodielectric model and a linear \( \sigma \)-model, Fiolhais, Golli, and Sirca reached a similar conclusion as ours, namely, the large experimental values of \( R_{EM} \) and \( R_{SM} \) could be explained in terms of the pion contribution alone.

In nonrelativistic CQM it is well known that, the values of \( R_{EM} \) and \( R_{SM} \) obtained with a \( D \)-state admixture in the \( \Delta \) generated by one-gluon-exchange hyperfine interaction are in general too small. It has recently been suggested by the Tüibigen group that this problem can be fixed with the inclusion of exchange currents in the calculation. The reason is that two-body exchange currents could flip the spins of the two quarks in the nucleon to convert it into a \( \Delta \). Such a transition would just be a transition between the S-states in the nucleon and \( \Delta \), and accordingly is greatly enhanced. We would like to point out here that the diagram involving a two-body exchange current induced by the exchange of a pion between two quarks in the nucleon can also be interpreted as pion rescattering effects, as considered in the current study.

5 Summary

In summary, we calculate the \( Q^2 \) dependence of the ratios \( E_{1+}/M_{1+} \) and \( S_{1+}/M_{1+} \) in the \( \gamma^* N \rightarrow \Delta \) transition, with the use of a dynamical model for electromagnetic production of pions. We find that both ratios \( E_{1+}/M_{1+} \) and \( S_{1+}/M_{1+} \) remain small and negative for \( Q^2 \leq 4.0 \) GeV\(^2\). Our results agree well with the recent measurement of Frolov et al. but deviate strongly from the predictions of pQCD. Our results indicate that the bare \( \Delta \) is almost spherical and hence very difficult to be directly excited via electric \( E2 \) and Coulomb \( C2 \) quadrupole excitations. The experimentally observed \( E^{(3/2)}_{1+} \) and \( S^{(3/2)}_{1+} \) multipoles are, to a very large extent, saturated by the contribution from pion cloud, i.e., pion rescattering effects. It remains an intriguing question, both experimentally and theoretically, to find the region of \( Q^2 \) which will signal the onset of pQCD.
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