Ratio $B(E2, 4\rightarrow 2)/B(E2, 2\rightarrow 0)$ in Even-Even Nuclei: Apparent Anomalous Behavior of the Chromium Isotopes

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Abstract

We consider the ratio $RE_4 = B(E2, 4\rightarrow 2)/B(E2, 2\rightarrow 0)$ for the lowest $2^+$ and $4^+$ states in even-even nuclei. In the rotational and vibrational models and the shell model calculations here considered, $RE_4$ is greater than one, however empirically, using NNDC adopted half-lives and energies for $^{48}$Cr and $^{50}$Cr, this ratio is less than one.

1 Introduction and Motivation

In shell model calculations of $B(E2)$s it is necessary to use effective charges if one hopes to get reasonable agreement with experiment. A popular choice, but my no means the only one is 1.5 for the proton and 0.5 for the neutron (as compared with the bare effective charges of 1 and 0 respectively). One may also attempt to calculate these effective charges from fundamentals. If for a given nucleus one gets a good fit to $B(E2, 2\rightarrow 0)$, one cannot regard this as an impressive result. One has chosen the effective charge so that the theory fits experiment—one parameter has been adjusted to fit one experimental datum.

In this work we define $RE_J$ as $B(E2, J\rightarrow J-2)/B(E2, 2\rightarrow 0)$. We will focus mainly on $RE_4=B(E2, 4\rightarrow 2)/B(E2, 2\rightarrow 0)$, as there exists significantly more experimental data on $B(E2)$s for $2^+$ and $4^+$ states than there is for higher states. By so doing, to a large extent we take the effective charge out of play. There is considerable empirical data in the National Nuclear Data Center, NNDC, on half-lives of the lowest $2^+$ and $4^+$ states of many even-even nuclei so that we can obtain $RE_4$ empirically.

Before proceeding to the shell model we note that there are simple formulas for this ratio in the rotational model and the vibrational model.

$$RE_{4rotational} = 10/7 \approx 1.429$$  \hspace{1cm} (1)

$$RE_{4vibrational} = 2$$  \hspace{1cm} (2)

This comes from more detailed formulae of Bohr and Mottelson [2]:

$$RE_{Jrotational} = \frac{\langle J\ 0\ 2\ 0|\ (J-2)\ 0 \rangle}{\langle 2\ 0\ 2\ 0|0\ 0 \rangle}$$  \hspace{1cm} (3)

$$RE_{Jvibrational} = J/2$$  \hspace{1cm} (4)
2 Shell Model

The main thrust of this work will be a comparison of the empirical adopted values of $RE_4$ from NNDC \[1\] against calculated values. Besides the simple rotational and vibrational results provided in the previous section we also consider the shell model calculations of Robinson et al. \[3, 4\]. In the $f-p$ shell, the program NuShell \[7\] is used in the FPD6 and GXFP1 interactions for the lighter nuclei. The program ANTOINE \[8\] is used for the JJ45 and JUN45 interactions for the heavier nuclei. In Table 1, the empirical results in the second column should be compared with the shell model results using an FPD6 interaction in the third column. We will discuss the fourth column later.

| Nuclide | Empirical | FPD6 | $T_0$FPD6\(^1\) | GXPF1 |
|---------|-----------|------|-----------------|-------|
| $^{44}$Ti | 2.269 | 1.362 | 1.083 | 1.291 |
| $^{46}$Ti | 1.033 | 1.419 | 1.153 | 1.264 |
| $^{48}$Ti | 1.254 | 1.519 | 1.192 | 1.414 |
| $^{48}$Cr | 0.834 | 1.396 | 1.289 | 1.351 |
| $^{50}$Cr | 0.7558 | 1.450 | 1.278 | 1.422 |
| $^{52}$Fe | 1.839 | 1.460 | 1.182 | 1.304 |

Table 1: The B(E2) ratio $RE_4$ empirical vs. shell model calculations

Table 2: Quadrupole moments of $2^+/4^+$

| Nuclide | Empirical | FPD6 | $T_0$FPD6 | GXPF1 |
|---------|-----------|------|----------|-------|
| $^{44}$Ti | -21.572/-28.918 | -0.945/-7.391 | -5.132/-16.378 |
| $^{46}$Ti | -23.505/-31.02 | -8.777/-17.263 | -12.72/-22.985 |
| $^{48}$Ti | -18.807/-20.724 | -9.402/-8.767 | -13.719/-11.424 |
| $^{48}$Cr | -35.416/-45.472 | -22.825/-33.718 | -30.16/-40.424 |
| $^{50}$Cr | -32.914/-41.867 | -22.317/-31.931 | -28.34/-35.738 |
| $^{52}$Fe | -33.642/-38.486 | -25.686/-23.508 | -30.4509/-37.5636 |

The first thing to notice is that the shell model results all have $RE_4$ bigger than one; this is also the case for the rotational and vibrational models. The shell model results are closer to the rotational value of 10/7 than the large value of 2 from the vibrational model.

The empirical results seem to fluctuate quite a lot from nucleus to nucleus as compared with the steadier FPD6 shell model results. Of particular note is the fact that for $^{48}$Cr and $^{50}$Cr, the $RE_4$ values are less than one—0.862 and 0.756 respectively. We here do not attempt to explain this behavior but present it as a nuclear structure puzzle. Using data from Brandolini, et al. \[3, 4\], instead of data from NNDC, the values of the $RE_4$ of $^{48}$Cr and $^{50}$Cr are 0.997 and 0.921 respectively. Although they are somewhat larger, these values remain less than one and well below the shell model results.

Also worth mentioning is the fact that for $^{44}$Ti we get a very large $RE_4$, even exceeding the vibrational value of 2.

In Table 2 we present the raw NNDC data—the excitation energies of the lowest $2^+$ and $4^+$ states ($E_2$ and $E_4$) as well as the ratio $E_4/E_2$. We also show the half lives of these states $T_{1/2}$. We obtain $RE_4$ from this data by the formula

\[
RE_4 = \frac{(T_{1/2})_{2^+}}{(T_{1/2})_{4^+}} \left(\frac{E_{2^+}}{E_{4^+} - E_{2^+}}\right)^5
\]

\(^1\)T0FPD6 is derived from FPD6 by setting all the T=0 2-body matrix elements of FPD6 to zero but keeping the T=1 elements unchanged.
Table 3: NNDC adopted values of half-lives and energy levels used to obtain RE4s

| Nuclide | Yrast State | Half-life (ps) | Energy (keV) | Empirical RE4 | Energy Ratio E4/E2 |
|---------|-------------|---------------|--------------|---------------|-------------------|
| $^{44}$Ti | $2^+$ | 3.1 | 1083.06 | 2.285 | 2.2643 |
|          | $4^+$ | 0.42 | 2452.33 |  |  |
| $^{46}$Ti | $2^+$ | 5.32 | 889.28 | 1.034 | 2.2601 |
|          | $4^+$ | 1.62 | 2009.84 |  |  |
| $^{48}$Ti | $2^+$ | 4.04 | 983.539 | 1.255 | 2.3341 |
|          | $4^+$ | 0.76 | 2295.65 |  |  |
| $^{48}$Cr | $2^+$ | 7.3 | 752.19 | 0.862 | 2.4707 |
|          | $4^+$ | 1.23 | 1858.47 |  |  |
| $^{50}$Cr | $2^+$ | 9.08 | 783.32 | 0.756 | 2.4017 |
|          | $4^+$ | 2.22 | 1881.31 |  |  |
| $^{52}$Fe | $2^+$ | 7.8 | 849.45 | 1.839 | 2.8072 |
|          | $4^+$ | 0.22 | 2384.55 |  |  |

There are calculations in heavier nuclei by Robinson et al.\cite{4} from which one can extract RE4 but there is as of yet no empirical data for these nuclei. These results are presented in Table 4 for the interactions JUN45 and JJ4B. The nuclei involved are $^{88}$Ru, $^{92}$Pd and $^{96}$Cd. The calculated shapes are quite different, oblate, near zero deformation, and prolate, respectively, however the calculated values of RE4 are similar, all bigger than one and close to the rotational limit.

Table 4: Calculated values of RE4 for heavier nuclei

| Nuclide | JUN45 | T0JUN45 | JJ4B | T0JJ4B |
|---------|-------|--------|------|--------|
| $^{88}$Ru | 1.554 | 1.314 | 1.458 | 1.416 |
| $^{92}$Pd | 1.256 | 1.040 | 1.359 | 2.662\(^2\) |
| $^{96}$Cd | 1.356 | 1.102 | 1.330 | 1.296 |

Table 5: Quadrupole moments $2^+/4^+$ for heavier nuclei

| Nuclide | JUN45 | T0JUN45 | JJ4B | T0JJ4B |
|---------|-------|--------|------|--------|
| $^{88}$Ru | 36.721/43.26 | 7.714/13.59 | 28.9297/36.513 | -5.85/-2.9049 |
| $^{92}$Pd | -3.559/-7.968 | -3.778/4.929 | 4.5599/11.161 | 8.4856/-5.48 |
| $^{96}$Cd | -19.296/-21.486 | -7.328/-12.778 | -16.4119/-15.208 | -15.175/-14.044 |

We last discuss the column T0FPD6 of Table 1. In their 2005 work Robinson et al.\cite{3} wanted to see the effects of the T=0 interaction on energy levels and B(E2)s. They did so by removing and then replacing all the 2-body T=0 matrix elements. In T0FPD6 they kept the T=1 matrix elements of FPD6 unchanged but set all the T=0 ones to zero. Focusing here on RE4 we see that with T0FPD6 the values of RE4 are much closer to one than with the full FPD6 interaction. We can thus say that the effect of the T=0 interaction is to make RE4 larger and close to the rotational limit.

\(^2\)Both values are extremely small, a factor of 200-300 x smaller than the full values
We here add new results on static quadrupole moments of the lowest $2^+$ and $4^+$ states. These are shown in Tables 2 and 4. There are no experiments to compare with. For the most part when the $T$=0 interaction is turned on the magnitudes of the quadrupole moments increase. This is true both for prolate and oblate cases.

A note should be made on the rotational model. As $J$ increases, $R\varepsilon J$ also increases in this model. However, in the shell model, $R\varepsilon J$ ultimately decreases. This is shown not only theoretically\cite{4} but also in the works of Brandolini et al. where they determined several values of $B(E2; J \rightarrow J-2)$ in $^{48}$Cr, $^{50}$Cr\cite{5} and $^{46}$Ti\cite{6}. Given the apparent anomalies for the Chromium nuclei, one can either question the theory or one can question the experiment. At this point we are unable to make a definitive statement in either direction, but we feel it makes the most sense to first encourage more experiments on these nuclei. It is possible that the technology has improved and the relevant values can be measured more precisely.

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References

[1] National Nuclear Data Center, NNDC, online http://www.nndc.bnl.gov
[2] A. Bohr and B.R. Mottelson, Nuclear Structure, Vol. 2, Benjamin, Reading, MA (1975).
[3] S.J.Q. Robinson, A. Escuderos, and L. Zamick Phys. Rev. C 72, 034424 (2005).
[4] S.J.Q. Robinson, T. Hoang, L. Zamick, A. Escuderos, and Y.Y. Sharon, Phys. Rev. C 89, 014316 (2014).
[5] F.Brandolini et al., Nucl. Phys. A 642 (1998) 387.
[6] F. Brandolini et al., Phys. Rev. C 70, 934302 (2004).
[7] B.A. Brown, W.D.M. Rae, E. McDonald, and M. Horoi, NuShellX@MSU.edu.
[8] E. Caurier, shell model code ANTOINE 1989-2004; E. Caurier and F. Nowacki, Acta. Phys. Pol. 30, 705 (1999).