Research Article

Effects of Variations of SUSY Breaking Scale on Neutrino Parameters at Low Energy Scale under Radiative Corrections

Kh. Helensana Devi,1 K. Sashikanta Singh,1 and N. Nimai Singh1,2

1Department of Physics, Manipur University, Imphal 795003, India
2Research Institute of Science and Technology, Imphal 795003, India

Correspondence should be addressed to Kh. Helensana Devi; helensanakhumanthem2@manipuruniv.ac.in

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The paper addresses the effects of the variations of the SUSY breaking scale $m_s$ in the range (2-14) TeV on the three neutrino masses and mixings, in running the renormalization group equations (RGEs) for different input values of high energy seesaw scale $M_R$, and in both normal and inverted hierarchical neutrino mass models. The present investigation is a continuation of the earlier works based on the variation of $m_s$ scale. Two approaches are adopted one after another—bottom-up approach for running gauge and Yukawa couplings from low to high energy scale, followed by the top-down approach from high to low energy scale for running neutrino parameters defined at high energy scale, along with gauge and Yukawa couplings. A self-complementarity relation among three mixing angles is also employed in the analysis and it is found to be stable under radiative correction. Significant effect due to radiative corrections on neutrino parameters with the variation of SUSY breaking scale $m_s$ is observed. For comparison of the results, variation of $\tan \beta$ for different $M_R$ is also considered.

1. Introduction

Neutrino physics has registered significant progress in recent years with the measurements of nonzero $\theta_{13}$ [1–3] and the Dirac CP phase [4, 5], thus indicating a possibility for a sizable CP violation in neutrino sector. The T2K team [5] has concluded with $3\sigma$ confidence level that the Dirac Phase $\delta_{CP}$ lies somewhere between $-3.41$ and $-0.03$ for normal hierarchy (NH) and between $-2.54$ and $-0.32$ for inverted hierarchy (IH). The interval includes the CP-conserving value of $-\pi = -22/7 = -3.14$ in case of NH, so that the CP conservation is disfavored only at the modest $2\sigma$ confidence level.

Next generation of neutrino detectors, such as Hyper-Kamiokande in Japan, DUNE in USA, and JUNO in China, may be able to get $5\sigma$ confidence level for confirmation of CP violation in neutrino sector. Neutrino oscillations [6–8] have been well studied with the precise measurements of neutrino oscillation mass parameters and mixing angles. But till date, there are still some unsettled questions in neutrino physics such as the correct mass hierarchical order whether normal or inverted, absolute neutrino mass scale, nature of neutrinos whether Dirac or Majorana type, the exact high energy scale of seesaw mechanism, and the supersymmetric breaking scale if all it exists, to mention a few.

The information related to the absolute neutrino masses has been continuously updating with the recent Planck data on cosmological upper bound [9, 10] on the sum of three absolute neutrino masses $\Sigma m_i < 0.12$ eV, neutrinoless double beta decay [11, 12] results with the upper limit on the effective Majorana neutrino mass $m_{\beta\beta} < (0.036 - 0.156)$ eV from the KamLAND-Zen experiment [13] in Japan, and KATRIN [14] result on direct kinematic measurement with the upper bound $m_{\nu_e} < 0.8$ eV. Neutrino mass model, if any, is bound to be consistent with these upper bounds on absolute neutrino masses. On theoretical front, the presence of supersymmetry (SUSY) [15–17] enables us to ensure the stability of hierarchy between the weak and GUT scales with the possible cancellation of quadratic term in radiative corrections to the Higgs boson mass. It is needed to have a precise unification point of three gauge couplings at high
GUT scale around $2 \times 10^{16}$ GeV [18–20]. It can also provide a natural mechanism for understanding the electroweak symmetry breaking (EWSB) [21, 22] and Higgs physics. Minimal Supersymmetric Standard Model (MSSM) [23] is thus a straightforward extension of the Standard Model (SM) with minimum number of new parameters. All the particles in the same supersymmetric multiplet would have the same mass if the supersymmetry is an exact symmetry. So far, there is no clear evidence for the presence of supersymmetric particles in the ongoing Large Hadron Collider (LHC), and LHC has almost reached its maximum energy of about $14$ TeV [24, 25]. Third run of LHC reaches $13.6$ TeV slightly higher than that of $13$ TeV of the second run [26]. While the existence of supersymmetric particles has been continuously ruling out in LHC, the supersymmetric breaking scale ($m_s$) still remains as an unknown parameter. There are speculations that SUSY particles may have a wide spectrum and are not confined. For simplicity, one can assume a single scale [27, 28] for all supersymmetric particles and this kind of assumption is valid as long as the $m_t$ or $m_s < m_Z$ [29, 30]. We assume that the $m_s$ scale may lie somewhere in between $2$ TeV and $14$ TeV within the scope of LHC. The effects of the variations of SUSY breaking scale on the unification of gauge couplings and also Yukawa couplings in MSSM and SUSY GUT models have already been addressed using the two-loop RGEs for gauge and Yukawa couplings without SUSY in two different steps using the low energy observational input values, bottom-up running from low to high energy scale for gauge and Yukawa couplings, and top-down running from high to low energy scale for neutrino mass parameters and mixing angles, along with gauge and Yukawa couplings which are already evaluated at high energy scale $M_R$.

The present work is confined to the question of stability of neutrino mass models for both normal and inverted hierarchy with the variation of $m_s$ scale and other input parameters $\tan \beta$ and $M_R$ scale. Another important applications of RGE analysis such as low energy magnification of neutrino mixings in quark-lepton unification hypothesis at high energy scale, radiative correction to validate the tribimaximal mixings and golden ratio mixing at high scale, and radiative origin of reactor angle and solar neutrino mass-squared parameter at low energy scale are not addressed in the present investigations.

The paper is organised as follows. In Section 2, we give a brief discussion of gauge and Yukawa coupling RGEs mainly on bottom-up and top-down runnings. In Section 3, we present the numerical analysis and results. In Section 4, we study the effects of variations on neutrino parameters for different values of $\tan \beta$. Summary and Discussion are presented in Section 5. We give relevant RGEs for gauge, Yukawa, and quartic Higgs couplings in two-loops for both the SM and MSSM in Appendix A and RGEs of neutrino parameters in Appendix B.

### 2. Renormalization Group Equations (RGEs)

We study the radiative corrections to neutrino oscillation parameters using the Renormalisation Group Equations (RGEs) [18, 31, 39] with and without SUSY in two different steps using the low energy observational input values, bottom-up running from low to high energy scale for gauge and Yukawa couplings, and top-down running from high to low energy scale for neutrino mass parameters and mixing angles, along with gauge and Yukawa couplings which are already evaluated at high energy scale $M_R$.

#### 2.1. Bottom-Up Running

In the bottom-up running of the RGEs, we divide it into three regions, $m_Z < \mu < m_t$, $m_t < \mu < m_s$, and $m_s < \mu < M_R$. We use the recent experimental data [8, 40] as initial input values at low energy scale, given in Table 1.

The values of gauge couplings, $\alpha_2$ for SU(2)$_L$ and $\alpha_1$ for U(1)$_Y$, are calculated by using $\sin^2 \theta_W(m_Z) = \alpha_{em}(m_Z)/\alpha_2(m_Z)$ and matching condition,

$$\frac{1}{\alpha_{em}(m_Z)} = \frac{5}{3} \frac{1}{\alpha_1(m_Z)} + \frac{1}{\alpha_2(m_Z)}.$$  \hspace{1cm} (1)

| Mass (GeV) | Coupling constants | Weinberg mixing angle |
|------------|-------------------|----------------------|
| $m_t(m_Z) = 91.1876$ | $\alpha_{em}(m_Z) = 127.952 \pm 0.009$ | $\sin^2 \theta_W(m_Z) = 0.23121 \pm 0.00017$ |
| $m_t(m_t) = 172.76$ | $\alpha_1(m_Z) = 0.1179 \pm 0.009$ | |
| $m_t(m_s) = 4.18$ | | |
| $m_t(m_t) = 1.77$ | | |
We can also express the gauge couplings $\alpha_i$'s [18] in terms of normalized couplings $g_i$'s as $g_i = \frac{1}{\pi} \frac{\alpha_i}{m_{\pi}}$, where $i = 1, 2, 3$ denote electromagnetic, weak, and strong couplings, respectively. RGEs at one-loop level [41] is used for evolution of the three gauge coupling constants from $m_Z$ scale to $m_t$ scale, as given below

$$\frac{1}{a_i(\mu)} = \frac{1}{a_i(m_Z)} - \frac{b_i}{2\pi} \ln \frac{\mu}{m_Z},$$

where $m_{\mu} \leq \mu \leq m_t$ and $b_i = (5.30, -0.50, -4.00)$ for non-SUSY case. For fermion masses to define at $m_t$ scale, we use QED-QCD rescaling factor $\eta$ [42], $m_\nu(m_\nu) = m_\nu(m_t)/\eta_\nu$, and $m_\tau(m_t) = m_\tau(m_t)/\eta_\tau$, where $\eta_\tau = 1.53$ and $\eta_\nu = 1.015$. We then convert them to Yukawa couplings, $h_i(m_t) = m_\nu(m_t)/v_0$, $h_\nu(m_t) = m_\nu(m_t)/v_0\eta_\nu$, and $h_\tau(m_t) = m_\tau(m_t)/v_0\eta_\tau$, where $v_0 = 174$ GeV is the vacuum expectation value (VEV) of SM Higgs field. The calculated numerical values for fermion masses, Yukawa, and gauge couplings at $m_t$ scale are given in Table 2.

We study the effect of variation of SUSY breaking scale ($m_\mu$) on gauge and Yukawa couplings for running from $m_t$ to the $M_\nu$ scale using RGEs, which are given in Appendix A. At $m_t$ scale, the following matching conditions are applied at the transition point from SM ($m_t < \mu < m_t$) to MSSM ($m_t < \mu < M_\nu$) as

$$g_i(\text{SUSY}) = g_i(\text{SM})$$

$$h_i(\text{SUSY}) = \frac{h_i(\text{SM})}{\sin \beta} = h_i(\text{SM}) \times \sqrt{1 + \tan^2 \beta} / \tan \beta,$$

$$h_\nu(\text{SUSY}) = \frac{h_\nu(\text{SM})}{\cos \beta} = h_\nu(\text{SM}) \times \sqrt{1 + \tan^2 \beta},$$

$$h_\tau(\text{SUSY}) = \frac{h_\tau(\text{SM})}{\cos \beta} = h_\tau(\text{SM}) \times \sqrt{1 + \tan^2 \beta}. \tag{3}$$

Third generation Yukawa coupling constants are highly affected by input value of tan $\beta$ as shown in Equation (3). A detailed numerical analysis shows that high scale value of $h_\tau$ always decreases with input value on SUSY breaking scale $m_{\mu}$ in the range (2-14) TeV for low and high tan $\beta = 25, 35, 40, 45$, and 55. However, high scale values of $h_\nu$ and $h_\nu$ behave in different patterns. In fact $h_\nu$ at high scale $M_\nu$ increases with the increase of $m_\mu$ scale for low value of tan $\beta = 25$, but it decreases with $m_\mu$ scale for higher values of tan $\beta = 40-55$. For $h_\tau$, it has a similar trend with $h_\nu$ but with a little difference in the range of tan $\beta$. In fact $h_\tau$ at high scale increases with the increase of $m_\mu$ scale for both low and moderate values of tan $\beta = 25, 40$, but it again decreases with $m_\mu$ for higher value of tan $\beta = 55$. For specific case used in the present calculation at input value of tan $\beta = 40$, both $h_\tau$ and $h_\nu$ decrease with the increase of $m_\mu$ scale, but $h_\tau$ increases with the increase of $m_\tau$ scale. This analysis is reflected in Tables 3–6.

The output for Yukawa and gauge couplings at $M_\nu$ scale are given in Table 3 for $M_\nu = 10^{13}$ GeV, Table 4 for $M_\nu = 10^{14}$ GeV, Table 5 for $M_\nu = 10^{15}$ GeV, and Table 6 for $M_\nu = 10^{16}$ GeV, respectively, for common value of tan $\beta = 40$. These values are needed for the next top-down running as input values at high energy scale.

2.2. Top-Down Running. In this running, we use the values of Yukawa and gauge couplings which are found at $M_\nu$ scale as initial inputs given in Tables 3–6. In this work, the high energy seesaw scale is the starting point for running the RGEs and it ends at electroweak scale. We give the sum of three neutrino masses $\Sigma = |m_\nu|$ in the range, 0.114 eV -0.121 eV for NH case and 0.1056 eV -0.1072 eV for IH case. The input values of neutrino masses and three mixing angles are indeed arbitrary in order to study the stability of neutrino mass model under RGEs running at low scale, with variations of other free parameters such as tan $\beta$, SUSY breaking scale $m_{\mu}$, and high energy scale $M_\nu$. We select our input values in Tables 7–8 with the aim to produce low energy values of neutrino oscillation parameters consistent with observational data. In our work, we focus on the high energy seesaw scale at $M_\nu = 10^{14}$ GeV; but for comparison, it has been supplemented by other values of high energy scale.
Table 5: Values of Yukawa and gauge couplings $f_\beta = \ln (10^{15} \text{ GeV}) = 34.53$ for $\tan \beta = 40$, for different choices of $m_i$ scale.

| $m_i$ (TeV) | $h_1$ | $h_2$ | $g_1$ | $g_2$ | $g_3$ |
|------------|--------|--------|--------|--------|--------|
| 2          | 0.6084 | 0.2829 | 0.3579 | 0.6574 | 0.7026 | 0.7378 |
| 4          | 0.5891 | 0.2798 | 0.3601 | 0.6521 | 0.6971 | 0.7341 |
| 6          | 0.5809 | 0.2784 | 0.3613 | 0.6494 | 0.6944 | 0.7323 |
| 8          | 0.5735 | 0.2771 | 0.3624 | 0.6468 | 0.6917 | 0.7305 |
| 10         | 0.5701 | 0.2766 | 0.3629 | 0.6455 | 0.6903 | 0.7296 |
| 12         | 0.5668 | 0.2760 | 0.3635 | 0.6443 | 0.6890 | 0.7287 |
| 14         | 0.5637 | 0.2754 | 0.3640 | 0.6430 | 0.6877 | 0.7278 |

Table 6: Values of Yukawa and gauge couplings evaluated at $f_\beta = \ln (10^{16} \text{ GeV}) = 36.84$ for $\tan \beta = 40$, for different choices of $m_i$ scale.

| $m_i$ (TeV) | $h_1$ | $h_2$ | $g_1$ | $g_2$ | $g_3$ |
|------------|--------|--------|--------|--------|--------|
| 2          | 0.5854 | 0.2676 | 0.3494 | 0.6893 | 0.7089 | 0.7200 |
| 4          | 0.5661 | 0.2647 | 0.3518 | 0.6831 | 0.7032 | 0.7166 |
| 6          | 0.5580 | 0.2634 | 0.3529 | 0.6801 | 0.7004 | 0.7149 |
| 8          | 0.5680 | 0.2687 | 0.3622 | 0.6770 | 0.6664 | 0.7131 |
| 10         | 0.5473 | 0.2618 | 0.3547 | 0.6757 | 0.6963 | 0.7124 |
| 12         | 0.5441 | 0.2613 | 0.3553 | 0.6742 | 0.6949 | 0.7116 |
| 14         | 0.5410 | 0.2608 | 0.3559 | 0.6727 | 0.6936 | 0.7107 |

Table 7: Input set of neutrino parameters at high energy scale $M_R$ for NH case. $\theta_{23}$ is used from SC relation, $\theta_{13} = q \times (\theta_{13} + \theta_{12})$ with $q = 1.1$. This is common for all cases of $m_i$ scale.

| Input parameters | Seesaw scale ($\tan \beta = 40$) |
|------------------|----------------------------------|
|                  | $10^{16} \text{ GeV}$ | $10^{15} \text{ GeV}$ | $10^{14} \text{ GeV}$ | $10^{13} \text{ GeV}$ |
| $m_1$ (eV)       | 0.0262 | 0.0258 | 0.0271 | 0.0274 |
| $m_2$ (eV)       | -0.0263 | -0.0259 | -0.0272 | -0.0275 |
| $m_3$ (eV)       | -0.0615 | -0.0645 | -0.0643 | -0.0664 |
| $\Sigma | m_i |$ | 0.114 | 0.116 | 0.118 | 0.121 |
| $\theta_{23}^{\rho}$ | 32.46 | 33.95 | 33.01 | 32.61 |
| $\theta_{13}^{\rho}$ | 7.39 | 7.56 | 7.64 | 7.70 |
| $\psi_1^{\rho} = \psi_2^{\rho} = \psi_3^{\rho}$ | 180 | 180 | 180 | 180 |
| $\delta_{CP}^{\rho}$ | 240 | 240 | 240 | 240 |

Table 8: Input set of neutrino parameters at high energy scale $M_R$ for IH case ($m_3 \neq 0$). $\theta_{23}$ is used from SC relation, $\theta_{13} = q \times (\theta_{13} + \theta_{12})$ with $q = 1.1$. This is common for all cases of $m_i$ scale.

| Input parameters | Seesaw scale ($\tan \beta = 40$) |
|------------------|----------------------------------|
|                  | $10^{16} \text{ GeV}$ | $10^{15} \text{ GeV}$ | $10^{14} \text{ GeV}$ | $10^{13} \text{ GeV}$ |
| $m_1$ (eV)       | 0.0515 | 0.0501 | 0.0511 | 0.0523 |
| $m_2$ (eV)       | -0.0516 | -0.0502 | -0.0512 | -0.0524 |
| $m_3$ (eV)       | -0.0025 | -0.0021 | -0.0022 | -0.0025 |
| $\Sigma | m_i |$ | 0.1056 | 0.1024 | 0.1045 | 0.1072 |
| $\theta_{12}^{\rho}$ | 31.94 | 32.39 | 31.99 | 32.17 |
| $\theta_{13}^{\rho}$ | 8.53 | 8.29 | 8.35 | 8.40 |
| $\psi_1^{\rho} = \psi_2^{\rho} = \psi_3^{\rho}$ | 180 | 180 | 180 | 180 |
| $\delta_{CP}^{\rho}$ | 240 | 240 | 240 | 240 |

The sum of the three neutrino masses should satisfy the latest Planck cosmological data $\Sigma |m_i| < 0.12 \text{ eV}$, (i) The neutrino mass model should be nearly quasi-degenerate at least in $m_1$ and $m_2$ in order to get high value of $\theta_{13}$, and $m_3$ should be nonzero.

One can also express the neutrino mass eigenvalues after absorbing the Majorana CP phases as diag. $(m_1^{\text{eV}}, m_2^{\text{eV}}, m_3^{\text{eV}})$. As discussed in ref. [33], from the presence of a term $|m_1^{\text{eV}} + m_3^{\text{eV}}|/|\Delta m_{31}^2|$ in the evolution of $\theta_{13}$, a nonzero value of the difference $|\psi_1 - \psi_2|$ of the Majorana phases damps the RG equation. The damping becomes maximal if this difference equals $\pi$, which corresponds to an opposite CP parity of the two nearly degenerate mass eigenstates $m_1$ and $m_2$. A similar term $|m_2^{\text{eV}} + m_3^{\text{eV}}|/|\Delta m_{32}^2|$ is also present in the evolution equation of $\theta_{13}$, and this implies opposite CP parity between $m_1$ and $m_3$, though they are not so degenerate. Under this consideration, we make our input choice of CP parity as diag. $(m_1 - m_2 - m_3)$ in the present work. The RGEs for evolution of $\theta_{23}$ is also directly proportional to a term $A_{23} = (m_2 - m_1)/(m_3 - m_1)$ which is highly sensitive for the nearly degenerate masses between $m_1$ and $m_2$ [22, 33]. Any possible singularity in the running of RGEs may be avoided with the choice of opposite CP parity between $m_1$ and $m_2$ for nearly degenerate case. In this work, we take two Majorana phases $\psi_1$ and $\psi_2$ at $180^0$, which are constrained to be equal for simplicity ($\psi_1 = \psi_3 \neq \psi_2$), and the Dirac CP phase $\delta_{CP}$ angle at $240^0$. Our main aim is to study the neutrino oscillation parameters against varying $m_i$ for different $M_R$ scale.

Using all the necessary mathematical frameworks, we analyze the radiative nature of neutrino parameters like neutrino masses, mixings, and CP phases in the top-down approach with the variations of $m_i$ scale at different $M_R$ scale. The respective RGEs which are given in Appendix B. The input sets are given in Tables 7 and 8.

3. Numerical Analysis and Results

The effects of the variation of $m_i$ on the outputs of neutrino mass parameters and mixing angles are given in
Tables 9–12, along with the graphical representations in Figure 1 for normal hierarchical (NH) model, and in Tables 13–16 and Figure 2 for inverted hierarchical (IH) case. In each case, we also present the results for variation of high energy seesaw scale ($10^{13}$ - $10^{16}$ GeV). Similar patterns with the variations of seesaw scale are observed in all the Figures 1 and 2.

The neutrino oscillation parameters are found to be almost stable with the variation of $m_t$ at low energy scale except $\Delta m_{21}^2$ which is found to be very sensitive with the
change of \( m_s \) scale. One difference between NH and IH model is that in NH case, all the low energy parameters \( \Delta m^2_{31}, \Delta m^2_{21}, \theta_{23}, \theta_{12}, \theta_{13}, \) and \( \delta_{CP} \) are found to increase with the increase of \( m_s \) scale, but in IH case, \( \Delta m^2_{21}, \theta_{23}, \) and \( \theta_{12} \) increase whereas \( |\Delta m^2_{31}| \) and \( \delta_{CP} \) decrease with \( m_s \). The low energy values of \( \Sigma |m_j| \) are below the latest Planck data \( \Sigma |m_j| < 0.12 \) eV, where the values for NH are smaller than those of IH case.

4. Effects of the Simultaneous Variation of \( m_s \) and \( \tan \beta \) Values

In this section, we again study how the estimated low energy values of neutrino oscillation parameters behave with the simultaneous variations of \( m_s \) and \( \tan \beta \) over a wide range. As a representative case, we consider only one seesaw scale.
Table 13: Effects on the output of neutrino parameters at low energy scale, on varying \(m_j\) for IH case (\(\tan \beta = 40, M_R = 10^{13}\) GeV).

| \(m_j\) scale (TeV) | \(|\Delta m_{31}^2| (10^{-3}\text{ eV}^2)\) | \(\Delta m_{21}^2 (10^{-5}\text{ eV}^2)\) | \(\theta_{21}(^\circ)\) | \(\theta_{12}(^\circ)\) | \(\theta_{13}(^\circ)\) | \(\delta_{CP}(^\circ)\) | \(\Sigma \left|m_1\right| (\text{eV})\) |
|---------------------|------------------------------------------|---------------------------------|-----------------|-----------------|-----------------|-----------------|------------------|
| 2                   | 2.58                                     | 5.96                            | 44.70           | 32.21           | 8.41            | 239.94          | 0.119            |
| 4                   | 2.55                                     | 6.64                            | 44.71           | 32.22           | 8.41            | 239.93          | 0.117            |
| 6                   | 2.53                                     | 7.10                            | 44.72           | 32.22           | 8.41            | 239.93          | 0.116            |
| 8                   | 2.52                                     | 7.32                            | 44.73           | 32.23           | 8.41            | 239.92          | 0.115            |
| 10                  | 2.52                                     | 7.52                            | 44.73           | 32.23           | 8.41            | 239.92          | 0.114            |
| 12                  | 2.51                                     | 7.73                            | 44.74           | 32.23           | 8.41            | 239.92          | 0.114            |
| 14                  | 2.50                                     | 7.86                            | 44.74           | 32.23           | 8.41            | 239.92          | 0.113            |

Table 14: Effects on the output of neutrino parameters at low energy scale, on varying \(m_j\) for IH case (\(\tan \beta = 40, M_R = 10^{14}\) GeV).

| \(m_j\) scale (TeV) | \(|\Delta m_{31}^2| (10^{-3}\text{ eV}^2)\) | \(\Delta m_{21}^2 (10^{-5}\text{ eV}^2)\) | \(\theta_{21}(^\circ)\) | \(\theta_{12}(^\circ)\) | \(\theta_{13}(^\circ)\) | \(\delta_{CP}(^\circ)\) | \(\Sigma \left|m_1\right| (\text{eV})\) |
|---------------------|------------------------------------------|---------------------------------|-----------------|-----------------|-----------------|-----------------|------------------|
| 2                   | 2.46                                     | 5.68                            | 44.44           | 32.04           | 8.35            | 239.95          | 0.114            |
| 4                   | 2.44                                     | 6.30                            | 44.46           | 32.05           | 8.35            | 239.94          | 0.112            |
| 6                   | 2.42                                     | 6.75                            | 44.47           | 32.05           | 8.35            | 239.93          | 0.110            |
| 8                   | 2.41                                     | 7.05                            | 44.48           | 32.05           | 8.35            | 239.93          | 0.106            |
| 10                  | 2.40                                     | 7.25                            | 44.48           | 32.06           | 8.35            | 239.93          | 0.109            |
| 12                  | 2.40                                     | 7.37                            | 44.49           | 32.06           | 8.35            | 239.93          | 0.108            |
| 14                  | 2.39                                     | 7.49                            | 44.49           | 32.06           | 8.35            | 239.92          | 0.108            |

Table 15: Effects on the output of neutrino parameters at low energy scale, on varying \(m_j\) for IH case (\(\tan \beta = 40, M_R = 10^{15}\) GeV).

| \(m_j\) scale (TeV) | \(|\Delta m_{31}^2| (10^{-3}\text{ eV}^2)\) | \(\Delta m_{21}^2 (10^{-5}\text{ eV}^2)\) | \(\theta_{21}(^\circ)\) | \(\theta_{12}(^\circ)\) | \(\theta_{13}(^\circ)\) | \(\delta_{CP}(^\circ)\) | \(\Sigma \left|m_1\right| (\text{eV})\) |
|---------------------|------------------------------------------|---------------------------------|-----------------|-----------------|-----------------|-----------------|------------------|
| 2                   | 2.37                                     | 5.56                            | 44.82           | 32.44           | 8.29            | 239.95          | 0.111            |
| 4                   | 2.34                                     | 6.11                            | 44.84           | 32.45           | 8.29            | 239.94          | 0.109            |
| 6                   | 2.33                                     | 6.55                            | 44.85           | 32.45           | 8.29            | 239.94          | 0.107            |
| 8                   | 2.32                                     | 6.73                            | 44.85           | 32.45           | 8.29            | 239.93          | 0.107            |
| 10                  | 2.31                                     | 6.91                            | 44.86           | 32.46           | 8.29            | 239.93          | 0.106            |
| 12                  | 2.30                                     | 7.15                            | 44.87           | 32.46           | 8.29            | 239.93          | 0.105            |
| 14                  | 2.30                                     | 7.23                            | 44.87           | 32.46           | 8.29            | 239.93          | 0.105            |

Table 16: Effects on the output of neutrino parameters at low energy scale, on varying \(m_j\) for IH case (\(\tan \beta = 40, M_R = 10^{16}\) GeV). Four different choices of \(M_R\) scale are presented.

| \(m_j\) scale (TeV) | \(|\Delta m_{31}^2| (10^{-3}\text{ eV}^2)\) | \(\Delta m_{21}^2 (10^{-5}\text{ eV}^2)\) | \(\theta_{21}(^\circ)\) | \(\theta_{12}(^\circ)\) | \(\theta_{13}(^\circ)\) | \(\delta_{CP}(^\circ)\) | \(\Sigma \left|m_1\right| (\text{eV})\) |
|---------------------|------------------------------------------|---------------------------------|-----------------|-----------------|-----------------|-----------------|------------------|
| 2                   | 2.51                                     | 5.22                            | 44.59           | 31.98           | 8.54            | 239.94          | 0.118            |
| 4                   | 2.49                                     | 5.73                            | 44.60           | 31.98           | 8.54            | 239.93          | 0.116            |
| 6                   | 2.47                                     | 6.14                            | 44.61           | 31.99           | 8.54            | 239.93          | 0.115            |
| 8                   | 2.47                                     | 6.26                            | 44.62           | 31.99           | 8.54            | 239.92          | 0.114            |
| 10                  | 2.46                                     | 6.46                            | 44.62           | 31.99           | 8.54            | 239.92          | 0.114            |
| 12                  | 2.45                                     | 6.68                            | 44.63           | 32.00           | 8.54            | 239.92          | 0.113            |
| 14                  | 2.45                                     | 6.74                            | 44.63           | 32.00           | 8.54            | 239.92          | 0.112            |

at \(M_R = 10^{14}\) GeV for both NH and IH, using the same high scale input parameters given in Tables 7 and 8, respectively. We consider the range of \(\tan \beta = 25 - 55\) along with the range of \(m_j = (2 - 14)\) TeV. Significant effects of the variation of \(\tan \beta\) for a given \(m_j\) value have been observed as shown in Tables 17 and 18 and Figures 3 and 4. The following observations on the low energy neutrino oscillation parameters can be drawn.

(i) All the three mixing angles are observed to increase with increasing values of \(\tan \beta\) and \(m_j\) for both NH and IH cases.

For NH case, both \(\Delta m_{21}^2\) and \(|\Delta m_{31}^2|\) decrease with increasing value of \(\tan \beta\), but increase with increasing \(m_j\). For IH case, \(\Delta m_{21}^2\) increases but \(|\Delta m_{31}^2|\) decreases with the increase of both \(\tan \beta\) and \(m_j\).
For NH case, $\delta_{CP}$ decreases with increasing $\tan \beta$ but increases with increasing $m_s$, whereas for IH case, $\delta_{CP}$ decreases with the increase of both $\tan \beta$ and $m_s$.

5. Summary and Discussion

To summarize, the present work is a continuation of the earlier investigations [18, 32] on the effect of the variations of SUSY breaking scale $m_s$ in the running of RGEs for neutrino masses and mixing parameters from high to low energy scale. Among many other applications of RGEs on neutrino physics such as magnification of neutrino mixings at low energy scale in quark-lepton unification hypothesis at high energy scale, generation of suitable radiative corrections to validate the tribimaximal or golden ratio neutrino mixings at high scale, and radiative origin of reactor mixing angle...
and solar neutrino mass-squared parameter at low energy; the present work focuses only on the question of the stability of neutrino mass models for both NH and IH, under RGEs analysis with the variations of SUSY breaking scale $m_s$ and input value of $\tan \beta$.

The numerical analysis in the present investigation is confined to the effects on the variations of three important free parameters in the ranges—high energy seesaw scale $M_R = (10^{13} - 10^{16})$ GeV, the SUSY breaking scale $m_s = (2 - 14)$ TeV, and $\tan \beta = (25 - 55)$. As a special

| $m_t$ (TeV) | $\tan \beta = 55$ | $\tan \beta = 45$ | $\tan \beta = 40$ | $\tan \beta = 35$ | $\tan \beta = 25$ |
|-------------|------------------|------------------|------------------|------------------|------------------|
| 2           | 46.63            | 45.85            | 45.75            | 44.25            | 43.96            |
| 4           | 46.64            | 45.88            | 45.79            | 44.29            | 44.00            |
| 6           | 46.64            | 45.91            | 45.82            | 44.31            | 44.02            |
| 8           | 46.64            | 45.92            | 45.84            | 44.32            | 44.04            |
| 10          | 46.65            | 45.93            | 45.86            | 44.35            | 44.06            |
| 12          | 46.65            | 45.94            | 45.88            | 44.36            | 44.07            |
| 14          | 46.66            | 45.95            | 45.89            | 44.36            | 44.07            |

Table 17: Effects on the neutrino oscillation parameters at low energy scale, on varying $m_t$ and $\tan \beta$ for NH case ($M_R = 10^{14}$ GeV).
representative case, we choose $M_R = 10^{14}$ GeV for $\tan \beta = 40$ in both NH and IH models. For simplicity of comparison, the results for other choices of $M_R$ are also presented. To study the stability criteria of neutrino mass model, we start with arbitrary high energy scale input values of neutrino masses and mixings which satisfy certain conditions including Planck cosmological bound. The input value for the Dirac CP phase angle is taken at 240°.
for all cases and two Majorana phase angles at 180° for simplicity. In order to avoid any possible singularity in running RGEs for nearly quasidegenerate case, we considered Majorana CP conserving parity in the mass eigenvalues \((m_1, -m_2, -m_3)\). Majorana phases are more insensitive as compared to \(\delta_{CP}\) against RGEs analysis and we omit to report these results. We conclude with the following important points of our results.

\[\tan \beta = 55\]
\[\tan \beta = 45\]
\[\tan \beta = 25\]

**Figure 3:** Effects on the low energy output results in \(\theta_{ij}, |\Delta m^2_{ij}|\), and \(\delta_{CP}\) with variation of \(m_s\) for NH case at \(M_R = 10^{14}\) GeV. Five different choices of \(\tan \beta\) are presented.
The input value of tan β sharply affects the evolution pattern of the third generation Yukawa coupling constants ($h_t$, $h_b$, and $h_\tau$) with energy scale. It has been observed that $h_t$ at $M_R$ scale always decreases with the increase of SUSY breaking scale $m_s$ for both low and large tan β values. However, the high energy scale values of $h_b$ and $h_\tau$ are observed...
to increase with the increase in $m_s$ scale for low tan $\beta$ values, but decrease with $m_s$ for larger tan $\beta$ values. For moderate tan $\beta$ values, $h_b$ decreases with $m_t$ again but $h_\tau$ increases with $m_t$.

(i) The effect of the variation of $m_t$ scale on neutrino oscillation parameters at low energy scale is very mild except $\Delta m_{32}^2$ which is very sensitive with $m_t$ and tan $\beta$ values. Both low energy values of $|\Delta m_{32}^2|$ and $\Delta m_{21}^2$ increase with the increase in $m_t$ scale for NH case. However, $\Delta m_{21}^2$ increases with the increase of $m_t$, but $\Delta m_{32}^2$ decreases with $m_t$ for IH case. The low energy scale values of three mixing angles ($\theta_{12}$, $\theta_{13}$, and $\theta_{23}$) have mild increasing trend with the increase of $m_t$ scale for both NH and IH cases. The Dirac CP phase $\delta_{CP}$ at low energy scale increases with the increase in $m_t$ for NH but it decreases with $m_t$ for IH case.

The simultaneous variations of $m_t$ and tan $\beta$ on low scale neutrino oscillation parameters have significant effects. It is observed that all the low energy values of mixing angles ($\theta_{12}$, $\theta_{13}$, and $\theta_{23}$) increase with the increase in $m_t$ and tan $\beta$ values for both NH and IH. Both low energy values of $\Delta m_{32}^2$ and $|\Delta m_{32}^2|$ decrease with the increase of tan $\beta$, but they increase with $m_t$ scale for NH. For IH case, $\Delta m_{21}^2$ increases and $|\Delta m_{32}^2|$ decreases with the increase of $m_t$ and tan $\beta$. The Dirac CP phase $\delta_{CP}$ decreases with the increase of tan $\beta$ but increases with the increase of $m_t$ for NH case. For IH case, the low energy value of $\delta_{CP}$ decreases with the increase of both tan $\beta$ and $m_t$.

The complementarity relation (SC) is found to satisfy at high and low energy scale under RGs with the variations of both tan $\beta$ and $m_t$ scale.

The numerical analysis in the present work shows the stability of both NH and IH neutrino mass models with the variation of SUSY breaking scale $m_t$, and also the other two input parameters $M_R$ scale tan $\beta$ for a wide range of input values. The present analysis can be applied to check the validity at low energy scale of certain mixing patterns such as tribimaximal [51–54] and golden ratio mixing patterns defined at high energy scale [34, 55–58].

Appendix

A. RGEs for Gauge Couplings [39]

The two-loop RGs for gauge couplings are given by

$$\frac{dg_i}{dt} = \frac{b_i}{16\pi^2} g_i^3 + \frac{1}{(16\pi^2)^2} \left[ \sum_{j=1}^{N} b_{ij} g_j^2 g_i^2 - \sum_{j=1, h_t} a_{ij} g_j^2 h_j^2 \right],$$

(A.1)

where $t = \ln \mu$, and $b_i, b_{ij}, a_{ij}$ are $\beta$ function coefficients in MSSM,

$$b_i = (6.6, 1.0, -3.0), b_{ij} = \begin{pmatrix} 7.96 & 5.40 & 17.60 \\ 1.80 & 25.00 & 24.00 \\ 2.20 & 9.00 & 14.00 \end{pmatrix},$$

and, for non-supersymmetric case, we have

$$b_i = (4.100, -3.167, -7.00), b_{ij} = \begin{pmatrix} 3.98 & 2.70 & 8.80 \\ 0.90 & 5.83 & 12.00 \\ 1.10 & 4.50 & -26.00 \end{pmatrix},$$

(A.2)

where

$$a_{ij} = \begin{pmatrix} 5.2 & 2.8 & 3.6 \\ 6.0 & 6.0 & 2.0 \\ 4.0 & 4.0 & 0.0 \end{pmatrix},$$

(A.2)

$$a_{ij} = \begin{pmatrix} 0.85 & 0.5 & 0.5 \\ 1.50 & 1.5 & 0.5 \\ 2.00 & 2.0 & 0.0 \end{pmatrix}.$$
\[ \frac{dh_t}{dt} = \frac{h_t}{16\pi^2} \left( 4h_t^2 + 3h_b^2 - \sum_{i=1}^{3} c'_i g_i^2 \right) \\
+ \frac{h_t}{(16\pi^2)^2} \left[ \sum_{i=1}^{3} \left( c'_i b_i + \frac{c''_i}{2} \right) g_i^4 + \frac{9}{5} g_1^2 g_2^2 \right] \\
+ \left( \frac{6}{5} g_1^2 + 6g_2^2 \right) h_t^2 + \left( \frac{2}{5} g_1^2 + 16g_2^2 \right) h_b^2 \\
+ 9g_1^4 - 10h_t^4 - 3h_b^2 h_t^2 - 9h_b^4 h_t^2 \right] \right), \\
(A.4) \]

where

\[ c_i = \begin{pmatrix} \frac{13}{15}, 3, \frac{16}{13} \end{pmatrix}, c'_i = \begin{pmatrix} 7, 3, \frac{16}{3} \end{pmatrix}, \\
(A.5) \]

For non-supersymmetric case,

\[ \frac{dh_t}{dt} = \frac{h_t}{16\pi^2} \left( \frac{3}{2} h_t^2 - \frac{3}{2} h_b^2 + Y_2(S) - \sum_{i=1}^{3} c_i g_i^2 \right) \]

\[ + \frac{h_t}{(16\pi^2)^2} \left[ \begin{pmatrix} 1187 \\ 600 \end{pmatrix} g_1^4 - \frac{23}{4} g_1^4 - 108g_3^4 \right] \\
- \frac{9}{20} g_1^2 g_2^2 + \frac{19}{15} g_1^4 g_3^2 + 9g_3^4 g_2^2 \\
+ \left( \frac{223}{80} g_1^2 + \frac{135}{16} g_2^2 + 16g_3^2 \right) h_t^2 \\
- \left( \frac{43}{80} g_1^2 - \frac{9}{16} g_2^2 + 16g_3^2 \right) h_b^2 + \frac{5}{2} Y_4(S) \\
- 2\lambda(3h_t^4 + h_b^4) + \frac{3}{2} h_t^4 - \frac{5}{4} h_b^4 + \frac{11}{4} h_b^4 \\
+ Y_2(S) \left[ \frac{5}{4} h_b^2 - \frac{9}{4} h_t^2 \right] - \eta_4(S) + \frac{3}{2} \lambda^2 \right], \]

\[ \frac{dh_b}{dt} = \frac{h_b}{16\pi^2} \left( \frac{3}{2} h_b^2 - \frac{3}{2} h_t^2 + Y_2(S) - \sum_{i=1}^{3} c_i g_i^2 \right) \]

\[ + \frac{h_b}{(16\pi^2)^2} \left[ \begin{pmatrix} 127 \\ 600 \end{pmatrix} g_1^4 - \frac{23}{4} g_1^4 - 108g_3^4 \right] \\
- \frac{27}{20} g_1^2 g_2^2 + \frac{31}{15} g_1^4 g_3^2 + 9g_3^4 g_2^2 \\
- \left( \frac{79}{80} g_1^2 + \frac{9}{16} g_2^2 + 16g_3^2 \right) h_t^2 \\
+ \left( \frac{187}{80} g_1^2 + \frac{135}{16} g_2^2 + 16g_3^2 \right) h_b^2 + \frac{5}{2} Y_4(S) \\
- 2\lambda(3h_t^4 + 3h_b^4) + \frac{3}{2} h_t^4 - \frac{5}{4} h_b^4 + \frac{11}{4} h_b^4 \\
+ Y_2(S) \left[ \frac{5}{4} h_b^2 - \frac{9}{4} h_t^2 \right] - \eta_4(S) + \frac{3}{2} \lambda^2 \right], \]

\[ \frac{d\lambda}{dt} = \frac{1}{16\pi^2} \left[ \begin{pmatrix} 9 \\ 4 \end{pmatrix} \left( \frac{3}{25} g_1^4 + \frac{3}{5} g_1^2 g_2^2 + g_2^4 \right) \right] \\
\left( \begin{pmatrix} 73 \\ -8 \end{pmatrix} \lambda^2 + \begin{pmatrix} \frac{7}{8} g_1^2 + \frac{117}{20} g_1^2 g_2^2 + \frac{1887}{200} g_1^4 \end{pmatrix} \lambda \right) \\
- \frac{341}{1000} g_1^4 - 64g_1^2 h_t^2 + h_b^4 \\
- \frac{8}{5} g_1^2 (2h_t^4 - h_b^4 + 3h_t^4) - \frac{3}{2} g_1^2 Y_2(S) \\
+ 10Y_4(S) + \frac{3}{5} g_1^2 \left( \frac{57}{10} g_1^2 - 21g_2^2 \right) h_t^2 \\
+ \left( \frac{3}{2} g_1^2 + 9g_2^2 \right) h_b^2 + \left( \frac{15}{2} g_1^2 + 11g_3^2 \right) h_b^2 \\
- 24\lambda Y_2(S) - \lambda H(S) + 6h_t h_b^2 \\
+ 20 \left( 3h_t^4 + 3h_b^4 + h_b^6 \right) \ldots 12 h_t h_b^4 + h_b^6 \right] \right], \]

\[ (A.6) \]

where

\[ Y_2(S) = 3h_t^2 + 3h_b^2 + h_t, \]

\[ Y_4(S) = \frac{1}{3} \left( 3\Sigma c_i g_i^2 h_t^2 + 3\Sigma c'_i g_i^4 h_t^2 + 3\Sigma c''_i g_i^2 h_t^2 \right), \]

\[ H(S) = 3h_1^2 + 3h_2^2 + h_3^2, \]

\[ \eta_4(S) = \frac{9}{4} \left( 3h_1^4 + 3h_2^4 + h_3^4 - \frac{3}{2} h_2^4 h_3 \right), \]

\[ (A.7) \]

\[ \lambda = m^2_1/m^2_2 \] is the Higgs self-coupling, \( m_2 = 125.78 \pm 0.26 \text{ GeV} \) is the Higgs mass [59], and \( v_0 = 174 \text{ GeV} \) is the vacuum expectation value.

The beta function coefficients for non-SUSY case are given below:

\[ c_i = (0.85, 2.25, 8.00), c'_i = (0.25, 2.25, 8.00), \text{ and } c''_i = (2.25, 2.25, 0.00). \]
B. RGES for Three Neutrino Mixing Angles and Phases [33]: (Neglecting Higher Order of $\theta_1$)

$$
\dot{\theta}_{12} = -\frac{C_Y^2}{32\pi^2} \sin 2\theta_{12} \sin^2 2\psi_{12} \frac{m_1e^{i\psi_1} + m_2e^{i\psi_2}}{\Delta m^2_{12}},
$$

$$
\dot{\theta}_{13} = \frac{C_Y^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m^2_{13}(1 + \xi)} \times [m_1 \cos (\psi_1 - \delta) - (1 + \xi)m_2 \cos (\psi_2 - \delta) - \xi m_3 \sin \delta],
$$

$$
\dot{\theta}_{23} = -\frac{C_Y^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m^2_{23}} \left[ c^2_{12} |m_2e^{i\psi_2} + m_3|^2 + s^2_{12} |m_1e^{i\psi_1} + m_3|^2 \right],
$$

where $\Delta m^2_{12} = m^2_2 - m^2_1$, $\Delta m^2_{13} = m^2_3 - m^2_1$, and $\xi = \Delta m^2_{12}/\Delta m^2_{31}$.

B.1 RGES for the Three Phases [33]. For Dirac phase $\delta$,

$$
\dot{\delta} = \frac{C_Y^2}{32\pi^2} \delta^{(-1)} + \frac{C_Y^2}{8\pi^2} \delta^{(0)},
$$

where

$$
\delta^{(-1)} = \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m^2_{13}(1 + \xi)} \times [m_1 \sin (\psi_1 - \delta) - (1 + \xi)m_2 \sin (\psi_2 - \delta) + \xi m_3 \sin \delta],
$$

$$
\delta^{(0)} = \frac{m_1m_2}{\Delta m^2_{13}} \frac{s_{12}^2}{\Delta m^2_{13}(1 + \xi)} \sin (\psi_1 - \psi_2),
$$

$$
\dot{\delta} = \frac{m_1m_2}{\Delta m^2_{13}} \frac{s_{12}^2}{\Delta m^2_{13}(1 + \xi)} \sin (\psi_1 - \psi_2),
$$

For Majorana phase $\psi_2$ [33],

$$
\dot{\psi}_2 = \frac{C_Y^2}{8\pi^2} \left[ m_3 \cos 2\theta_{23} \frac{m_1s_{12}^2}{\Delta m^2_{13}(1 + \xi)} \sin \psi_1 + (1 + \xi)m_2c_{12}^2 \sin \psi_2 \right] + \frac{C_Y^2}{8\pi^2} \left[ m_1m_2s_{12}^2c_{23} \sin (\psi_1 - \psi_2) \right].
$$

B.2 RGES for Neutrino Mass Eigenvalues [33].

$$
\dot{m}_1 = \frac{1}{16\pi^2} \left[ \alpha + C_Y^2 (2s_{12}^2s_{23} + F_1) \right] m_1,
$$

$$
\dot{m}_2 = \frac{1}{16\pi^2} \left[ \alpha + C_Y^2 (2c_{12}^2s_{23} + F_2) \right] m_2,
$$

$$
\dot{m}_3 = \frac{1}{16\pi^2} \left[ \alpha + 2C_Y^2 c_{13}c_{23} \right] m_3,
$$

where

$$
F_1 = -s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta + 2s_{13}^2 c_{12}^2 c_{23},
$$

$$
F_2 = s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta + 2s_{13}^2 c_{12}^2 c_{23}.
$$

For MSSM case,

$$
\alpha = -\frac{6}{5} g^2_1 - 6g^2_2 + 6y^2_3,
$$

$$
C = 1.
$$

For SM case,

$$
\alpha = -3g^2_1 + 2g^2_2 + 6y^2_3 + 6y^2_3 + \lambda,
$$

$$
C = -\frac{3}{2}.
$$

and $\lambda$ is the Higgs self-coupling in the SM.

Data Availability

Data related to this work can be accessed through my zenodo doi:10.5281/zenodo.6585877

Disclosure

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.
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