CUTE solutions for two-point correlation functions from large cosmological datasets

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In the advent of new large galaxy surveys, which will produce enormous datasets with hundreds of millions of objects, new computational techniques are necessary in order to extract from them any two-point statistic, the computational time of which grows with the square of the number of objects to be correlated. Fortunately technology now provides multiple means to massively parallelize this problem. Here we present a free-source code specifically designed for this kind of calculations. Two implementations are provided: one for execution on shared-memory machines using OpenMP and one that runs on graphical processing units (GPUs) using CUDA. The code is available at [http://members.ift.uam-csic.es/dmongs/CUTE.html](http://members.ift.uam-csic.es/dmongs/CUTE.html)

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I. INTRODUCTION

The last decades have seen an increasing interest in galaxy surveys as a means of studying the late-time evolution of the Universe. Forthcoming galaxy surveys, such as DES [1], BigBOSS [2], or Euclid [3], will map large regions of the sky (O(10^3−4) sq-deg) to redshifts z > 1 yielding catalogs containing hundreds of millions of objects.

The spatial distribution of these objects on different scales contains invaluable information that could help clarify many open problems in cosmology and astrophysics, such as the nature of dark matter and dark energy or the presence of primordial non-Gaussianities in the density field. One of the simplest observables that can be estimated to quantify the clustering of matter on different scales is the two-point correlation function (2PCF hereon, see section II). Its estimation is based on counting pairs of objects separated by a given distance measure, and therefore its computational time grows with the square of the number of objects in the catalog.

Here we present a CUTE (Correlation Utilities and Two-point Estimation), a free open-source code that estimates different kinds of two-point correlations from discrete cosmological catalogs using various speed-up techniques.

II. THE TWO-POINT CORRELATION FUNCTION(S)

The three-dimensional 2PCF \( \xi(r) \) of a set of discrete points in \( \mathbb{R}^3 \) represents the excess probability of finding two of them inside two small volumes \( dV_1 \) and \( dV_2 \) separated by \( r \) [4]:

\[
\langle dP \rangle = n[1 + \xi(r)] dV_1 dV_2.
\] (1)

When this point distribution comes from a Poisson process based on an underlying random density field \( \delta(x) \), the field’s 2PCF \( \xi_\delta(r) \equiv \langle \delta(x)\delta(x+r) \rangle \) is directly related to that of the point distribution \( \xi \). Note that even though, in principle, the two-point correlation should depend on the positions of both points, \( r_1 \) and \( r_2 \), for homogeneous fields the only dependence is on the separation between them \( r \equiv r_1 - r_2 \).

A. Types of correlation functions

- The 3-D correlation function \( \xi(r,\mu) \) and \( \xi(\sigma,\pi) \). Different observational effects, such as redshift-space distortions or errors in the observed redshifts, transform what would otherwise be an isotropic 2PCF into a function that behaves differently along the line of sight and in the transverse direction. Two coordinate systems are widely used in the literature: the \( \sigma - \pi \) and \( r - \mu \) schemes (see fig. 1), the relation between both being

\[
\pi = r \mu, \quad \sigma = \sqrt{r^2 - \pi^2}.
\] (2)

The \( r - \mu \) scheme has the advantage that the usual multipole expansion is directly written in terms of these variables:

\[
\xi(r,\mu) = \sum_l \xi_l(r) P_l(\mu),
\] (3)

where \( P_l \) are the Legendre polynomials. Note that at the linear level and in the plane-parallel approximation (i.e. the Kaiser formula [5]) only the first three even multipoles \( (l = 0, 2, 4) \) contribute.
used.

FIG. 1: Definition of the different coordinate conventions used.

- **The monopole** $\xi_0(r)$. The first element ($l = 0$) in the expansion above is the angle-averaged correlation function or “monopole”:

$$\xi_0(r) = \frac{1}{2} \int_{-1}^{1} d\mu \xi(r, \mu) \quad (4)$$

This is the only non-zero contribution in the absence of redshift distortions.

- **The radial correlation function** $\xi_r(\bar{z}, \Delta z)$. Correlating only pairs of galaxies aligned with the line of sight, one computes the so-called radial correlation function, which can be made to depend locally only on the redshift difference $\Delta z$ between each pair of galaxies. This quantity is related to the three-dimensional 2PCF through

$$\xi_r(\bar{z}, \Delta z) = \xi(\pi(\bar{z}, \Delta z), \sigma = 0), \quad (5)$$

with

$$\pi(\bar{z}, \Delta z) \simeq \frac{c \Delta z}{H(\bar{z})}, \quad (6)$$

where $\chi(z)$ is the radial comoving distance to redshift $z$.

- **The angular correlation function** $w(\theta)$. The angular correlation function is the 2PCF of the density contrast field projected on the sphere

$$w(\theta) \equiv \langle \delta_s(\hat{n}_1) \delta_s(\hat{n}_2) \rangle, \quad \cos \theta \equiv \hat{n}_1 \cdot \hat{n}_2, \quad (7)$$

$$\delta_s(\hat{n}) \equiv \int dz \phi(z) \delta(r(z) \hat{n}), \quad (8)$$

where $\phi(z)$ is the redshift selection function. The angular correlation function is related to $\xi(r, \mu)$ by

$$w(\theta) = \int dz_1 \phi(z_1) \int dz_2 \phi(z_2) \xi(r(z_1, z_2, \theta), \mu(z_1, z_2, \theta)) \quad (9)$$

$$r(z_1, z_2, \theta) = \sqrt{\chi^2(z_1) + \chi^2(z_2) - 2 \chi(z_1) \chi(z_2) \cos \theta},$$

$$\mu(z_1, z_2, \theta) = \frac{|\chi^2(z_1) - \chi^2(z_2)|}{\sqrt{(\chi^2(z_1) + \chi^2(z_2))^2 - 4 \chi^2(z_1) \chi^2(z_2) \cos^2(\theta)}} \quad (10)$$

- **B. Estimating the 2PCF from discrete data**

As we have said, the two-point correlation function can be understood as the excess probability of finding two objects separated by a given distance with respect to a random distribution, and therefore:

$$1 + \xi = \frac{N_p^d(r) \, dr}{N_p^r(r) \, dr} \quad (11)$$

where $N_p^d \, dr$ is the number of pairs separated by $r \pm dr/2$ in the data, and $N_p^r \, dr$ is the number of pairs that one would expect for a random distribution. The numerator can be easily calculated as

$$N_p^d(r) \, dr = \sum_{i=1}^{N} \sum_{j \neq i} \Theta(r - dr/2 < |x_i - x_j| < r + dr/2), \quad (12)$$

where $N$ is the total number of objects, $\Theta$ is 1 whenever its argument is true and 0 otherwise, and we have explicitly accounted for self-pairs. If the catalog has no boundaries, the number of random pairs could easily be estimated as

$$N_p^r(r) = \frac{N^2}{V} v(r), \quad (13)$$

where $v(r) \approx 4\pi r^2 \, dr$ is the volume of a spherical shell of radius $r$ and thickness $dr$.

As we have said, this is can only be done if the catalog has no boundaries. Effectively this true in the case of an N-body simulation, where a sphere that lies partly outside the simulation box can be “wrapped around” due to the periodic boundary conditions. Thus, in this case a possible estimator is

$$1 + \xi(r) \equiv \frac{V}{N^2 v(r)} \sum_{i,j \neq i} \Theta(r - dr/2 < |x_i - x_j| < r + dr/2), \quad (13)$$

However, when calculating the correlation function from a point distribution with complicated boundaries, as is usually the case in a galaxy survey, several observational difficulties arise: e.g. different parts of the sky may have been mapped to different depths and the radial distribution of objects (selection function) is never uniform. The most usual technique to deal with these
issues is to compare the data catalog with catalogs made of randomly distributed objects that also contain these artificial effects. In this case the 2PCF can be naively estimated as

$$\xi_N = \frac{N_r (N_r - 1) DD}{N_d (N_d - 1) RR} - 1$$

(14)

where $N_d$ and $N_r$ are the number of points in the data and random catalogs respectively and $DD$ and $RR$ are histograms containing the counts of pairs of objects found separated by a given distance in each catalog. It has been shown [7] that the variance of this estimator can be minimized, and its ability to cope with boundary conditions can be enhanced, by making use of the cross-correlation of random and data objects, $DR$. The most widely used estimator is the one proposed by Landy & Szalay [7]:

$$\xi_{LS} = \frac{N_r (N_r - 1) DD - N_d (N_d - 1) DR + RR}{RR}.$$  

(15)

See [8] for a thorough comparison of different estimators.

The most delicate part of the estimation is in fact being able to generate the random catalogs correctly: the background spatial distribution, both in angles and redshift (i.e. the one-point function), of random objects must be exactly the same as in the data. Hence, all observational effects that affect the spatial distribution must be correctly reproduced by the random catalogs. Also, in order to minimize Poisson errors in $DR$ and $RR$, random catalogs should be generated with more particles than the data.

III. CUTE

CUTE (Correlation Utilities and Two-point Estimation) is a free and open-source code for cosmological 2PCF estimation. CUTE is written in C and, in the current public version, comes with two implementations: one parallelized for shared-memory machines using OpenMP and one (CU_CUTE) that performs the correlations in a GPU using NVidia’s CUDA architecture. CUTE calculates 4 different correlation functions ($3$D, monopole, angular and radial) with different binning schemes and speed-up techniques. Here we will explain the parallelization strategies followed by CUTE and some details specific to each type of 2PCF. We refer the reader to the README file accompanying the latest public version of CUTE for the operational options and compilation instructions of the code. In this section we assume some basic knowledge of parallel computing with OpenMP and CUDA by the reader.

It must be noted that there exist two other codes [9][10], recently made public, designed to compute angular correlation functions with GPUs. As in the case of CUTE the speed-up factor (about $10^2$) gained by these codes through the use of graphical devices for parallelization clearly makes it worth the effort of adapting CPU algorithms to run on GPUs.

A. Serial approach

Once the random catalog has been produced, $DD$, $DR$ and $RR$ are computed by autocorrelating or crosscorrelating each pair of catalogs. In a serial code this algorithm is extremely simple, involving one loop over each catalog and performing 3 operations in each iteration: calculating the distance between each pair of objects, determining the bin corresponding to that distance and increasing the histogram count on that bin. The corresponding C-code would be:

```c
int histogram[nbins];
for (i=0; i<npl; i++) {
    for (j=0; j<npl2; j++) {
        //Calculate distance between two objects
double dist=get_dist(x1[i],y1[i],z1[i],x2[j],y2[j],z2[j]);
        //Calculate bin number
        int ibin=bin_dist(dist);
        //Increase histogram count
        histogram[ibin]++;
    }
}
```

As we said before, these two nested loops make this an $N^2$ problem (to be precise, an $n1*n2$ problem), whose computational time will grow very fast as we increase the size of the catalogs. At this point parallelization of, and some kind of fast approximate method is desirable, if not compulsory. In section III B we will describe the parallelization strategies used by CUTE, which complicate this simple algorithm. Other speed-up techniques used by the code are explained in section III C, and some specific details of each type of 2PCF are given in section III D.

B. Parallelization with CUDA and OpenMP

1. Multicore shared-memory machines and OpenMP

OpenMP [22] is an API that gives support for parallel programming in shared-memory platforms. Once a parallel execution block is opened, the programmer can define private (one independent copy per core) or shared (common) variables and easily divide for loops between all available cores. For a thorough review of the different features of OpenMP see [11]. The serial code above takes the following form when parallelized with OpenMP:

```c
#pragma omp parallel default(none) \private(hthread) shared(\ldots) {
    //Initialize private histograms
    for (i=0;i<nbins;i++)
        histo_thread[nbins]=0;
#pragma omp for //Parallelize loop
    for (i=0;i<npl;i++) {
        for (j=0;j<npl2;j++) {
            //Calculate distance between two objects
double dist=get_dist(x1[i],y1[i],z1[i],x2[j],y2[j],z2[j]);
        }
    }
```

```c
```
The strategy in this case is to declare one private histogram per execution thread that will store that thread’s pair counts. The first loop is then divided between all available threads and finally all partial histograms are added, avoiding read/write collisions, into the final shared one. As can be seen, parallelization with OpenMP is effortless, only requiring a few extra lines of code.

2. Graphics cards and CUDA

A GPU (Graphical Processing Unit) is a specialized piece of hardware designed for fast massively parallel manipulation of memory addresses. As their name suggests, GPUs are mainly intended for image building and processing, however their highly parallel structure makes them ideal for intensive numerical computation, providing a relatively cheap flop/s (floating-point operations per second). Hence in the last years GPUs have found their way into different branches of scientific research, computational cosmology being one of them [12, 13]. Initially the main difficulty when trying to use GPUs for scientific computing was the programming of the numerical algorithm, since using the standard APIs meant that data had to be disguised as pixel colors and some mathematical operations had to be encoded as graphics rendering. However lately a few programming models for GPUs have seen the light of day [23, 24] that make general purpose computing on GPUs (GPGPU) a lot easier. Of these we have chosen Nvidia’s CUDA [14] for its syntactic simplicity.

Two main complications arise when one tries to adapt a code to execute on a GPU. First, in a massively parallel environment you must take great care to avoid race conditions due to simultaneous memory read/write processes by different threads. Second, unlike in a multi-CPU machine, the amount of memory “per thread” available in a GPU is very limited, presently of the order of a few GB for hundreds of processors. Besides these, there are other more subtle concerns, such as intra-warp communication or the presence of different types of cached and uncached memory in the GPU, the correct use of which may enhance dramatically the code’s performance. In summary, correctly parallelising a code with CUDA is not as straightforward as it is with OpenMP, and in some cases it may not be worth the effort. For an introduction to CUDA and its many features see [15].

Implementing the serial algorithm above in CUDA would involve executing the following _device_ function in every thread in parallel:

```c
//Calculate bin number
int ibin=bin_dist(dist);

//Increase histogram count
histo_thread[ibin]++;

#pragma omp critical {

  //Add private histograms
  for (i=0;i<nbins;i++)
    histogram[i]+=histo_thread[i]
}

//Add block histograms
atomicAdd(&(histo_thread[threadIdx.x]), histo_thread[threadIdx.x]);
```

As before, we have divided one of the loops (this time the second one) among all the execution threads. The first difference with respect to OpenMP that we can see immediately is that, due to the limited amount of memory of the GPU, we can only declare one partial histogram per block, and not per thread. To do this we declare it as a variable in shared memory, which also has the advantage of having a lower latency than global memory. This introduces a new complication, since now all threads in a block will try to add their pair counts to the same histogram. This has to be done avoiding race conditions by using the CUDA _atomicAdd_ function. This is in fact the bottleneck of any algorithm involving histograms in CUDA (especially if the distribution under study is very degenerate), since many threads may have to remain idle while waiting for other threads to update their histogram entries. The best way to palliate this problem is to use at most as many threads per block as histogram bins (in fact, note that the algorithm above will only work when using as many threads as histogram bins, however it can be easily extended to more general cases). Finally all the partial block histograms are summed up into the global histogram in an ordered manner using again _atomicAdd_. The fact that CUTe uses CUDA atomic functions such as _atomicAdd_ means that it will only run on GPUs that support atomic operations (namely compute capability 2.0 or higher). There exist general algorithms for histograms that work on any CUDA-enabled device [16, 17], however no performance improvement was observed with respect to using _atomicAdd_. Furthermore, the method above reduces the use of shared memory for histograms to a minimum, allowing its use for other purposes.
C. Neighbor searching

Often the maximum scale to which we want to calculate the 2PCF is significantly smaller than the size of our data. In this case, calculating the relative distance between particles that are further away than this maximum scale is useless, and therefore should be avoided. However, how can we determine which pairs to avoid without actually calculating their distances? CUTE makes use of different approaches to minimize the amount of useless pair counts in an efficient way. The main strategy described here is very similar in the three-dimensional case (for the 3-D and monopole 2PCFs) and on the sphere (for the angular correlation), however they differ slightly in the details.

In the three-dimensional case, a box encompassing the whole catalog is first determined and divided into cubical cells. To each cell we associate the positions of all the objects that fall inside it. Assuming that the maximum distance we are interested in is $R_{\text{max}}$ and that the cell size is $a$, we draw a cube of $2 \lceil R_{\text{max}}/a \rceil + 1$ cells per side around each cell $C_i$ (here $\lfloor b \rfloor$ denotes the integer part of $b$). This guarantees that we can draw spheres of radius $R_{\text{max}}$ around any point in $C_i$ and that these spheres will all lie inside the cube (see the top panel in figure 2). Thus we can correlate all the objects inside $C_i$ with the objects in all the other cells inside the cube and safely ignore all other objects. The efficiency of this method depends largely on the number density of the catalog, the range of scales of interest, and the number of cubical cells used.

In the spherical case a similar approach is used. Let us define a spherical cube as a region of the sphere with constant limits in spherical coordinates, i.e. a region with $\phi_0 < \phi < \phi_f$ and $\cos \theta_f < \cos \theta < \cos \theta_0$. It is easy to prove that the spherical cube containing a spherical cap of radius $\theta_{\text{max}}$ centered at the point $(\theta, \phi)$ in spherical coordinates has sides of length:

\[
\Delta(\cos \theta) = \cos(\theta - \theta_{\text{max}}) - \cos(\theta + \theta_{\text{max}}),
\]

\[
\Delta(\phi) = \frac{\sqrt{\cos^2 \theta_{\text{max}} - \cos^2 \theta}}{\sin \theta}.
\] (16)

Now, in the spherical case we can use pixels defined as small spherical cubes instead of the cubical cells of the three-dimensional case. Then the result we have just quoted can be used to define a spherical cube of pixels centered at a given one that safely contains all particles within an angular distance $\theta_{\text{max}}$ of any particle in the central pixel (see the bottom panel in figure 2). Once this is done, the same procedure is followed as in the three-dimensional case.

Another more sophisticated and very popular technique to discard unnecessary correlations is the so-called $k$-Tree method. For a thorough description of this method, see [19].

D. Specifics of the 2PCFs

In the previous section we have described the general strategy followed to parallelize the calculation of any 2PCF with OpenMP and CUDA. However each of the 2PCFs detailed in section [11] requires a different treatment of the data and maybe allows for different, more optimal, approaches. We give the details specific to each of these types here.
FIG. 3: Illustration of the method used to calculate the 3-D correlation function in CUDA. Since the whole 2-dimensional 128×128 bin histogram cannot be fit inside the device’s shared memory, it is split into smaller ones, which are filled separately. Although the catalogs have to be correlated more than once, the bottleneck caused by atomic operations is largely mitigated by the higher number of histogram bins.

1. Radial correlation function

As was said in section II the radial 2PCF is calculated by correlating pairs of aligned objects and binning them according to their relative redshift difference ∆z. Spherical cubes are used by CUTE to quickly find pairs of galaxies subtending an angle smaller than some maximum aperture, which defines aligned pairs. For reasonable apertures (≤ 1°) the number of pairs to correlate is relatively small. Hence, since the computational time in this case is not an issue, there is no need for massive parallelization, and radial correlation functions are only supported by CUTE in its OpenMP version.

2. Angular correlation function

For the calculation of angular 2PCFs CUTE projects all objects in the catalog into the unit sphere and correlates pairs of objects according to their angular separation θ (see figure 1), which is used as a distance measure. Two complementary speed-up techniques can be used by CUTE in this case: if one is not interested in extremely small angular scales one can create a pixel map from the catalog and then correlate the pixels (weighting each of them by the number of objects that fall inside it). This may effectively reduce the number of objects that must be correlated by an order of magnitude and therefore reduce the computational time by a factor of 100. Also, in the calculation of the angular separation, the arc-cosine of the scalar product of two position vectors must be estimated. Calculating the arc-cosine is a very time-consuming operation, and, if one is not interested in very large angular scales, the following approximation can be used,

\[
\arccos(1 - x) \sim \sqrt{2x + \frac{1}{3}x^2 + \frac{4}{45}x^3}
\]

which is precise to 1 part in 10^{-4} for angles below 40° and reduces the computational time by a factor ~ 2. Both these time-saving techniques can be switched on or off in CUTE by the user.

3. 3-D correlation function

The main difference in the calculation of the 3-D correlation function with respect to the other 2PCFs is that pairs are binned in 2-dimensional histograms, according to their (r,μ) or (π,σ) separations. This does not introduce any relevant changes in the OpenMP implementation, as long as the amount of shared memory is large enough to accommodate one private 2-D histogram per thread, however it does matter when adapting the code to CUDA. The reason is that currently the amount of shared memory per block in GPUs is limited to 48 kB, which is too little to allocate, for example, a 128×128 array of long integers. The solution to this problem chosen for CUTE is explained in fig. 3: the catalogs are correlated several times, each time binning only pairs whose separation in one of the two coordinates is within a given range, until the whole histogram is filled. Thus we can declare smaller 2-D histograms in shared memory, and even though the catalogs must be correlated several times, the histogram-filling bottleneck mentioned in section III B 2 is largely alleviated by the higher number of histogram entries, thus conserving a reasonable computational time (see section IV).

4. The monopole in a box

In its current public version, CUTE has a companion program, CUTE_box that calculates the correlation function from data inside a cubical box with periodic boundary conditions. In this case only the calculation of the isotropic 2PCF (the monopole) is supported, using two different types of algorithms:

- **Particle-based algorithms.** In this case CUTE calculates the 2PCF from the pair counts using the estimator in equation [13]. As we have discussed, no random catalog is needed because of the periodic boundary conditions. Two different types of neighbor-searching algorithms are supported: cubical cells and k-Trees.

- **Density grid.** This algorithm is similar to the use of pixels to accelerate the calculation of the angular correlation function. In this case the particle
content of the catalog is first interpolated to a grid and the overdensity field $\delta$ is estimated at every grid point using a TSC algorithm. Then pairs of grid points are correlated, and a weight $\delta_i \delta_j$ is given to each pair. The correlation function is estimated as

$$\hat{\xi}(r) = \langle \delta(x)\delta(x+r) \rangle,$$

where the average is taken over all pairs of grid points separated by a distance $r$. Due to the simplicity of a regular grid, it is trivial to search for the neighboring grid points, and since the relative distances between neighboring points are the same everywhere, these relative distances only have to be calculated once. As a result of this, this method is usually the fastest, however it will only yield reliable results down to the scale of the grid.

IV. PERFORMANCE

We have tested CUTE’s performance in terms of computational time by running it on platforms with different capabilities, listed in table I. The serial version was tested by running CUTE on a single CPU core. We also tested the OpenMP version on a dual-core laptop and on a large shared-memory machine with 80 cores. The CUDA version has been tried on a regular graphics card in a laptop and on a high-end GPU. All the computational times quoted in this section correspond to tests performed without any of the neighbor-searching techniques described in section III C in order to provide a clearer comparison between platforms, and they should therefore be understood as the worst-case scenario. As we have noted, the use of these strategies may improve the computational time significantly with respect to a more naive approach (even by orders of magnitude). However, this improvement depends largely on the number density of the data and the scales of interest.

For this test the monopole 2PCF was calculated for catalogs of different sizes in the range $10^3 - 10^7$. The computational times for one single correlation (i.e. just calculating, for example, $Dr$) in the 5 different platforms are plotted in figure 4. As expected, using GPUs or parallelising the computation on several CPU cores improves the code’s speed by a factor 10 - 100, even using a regular video-game graphics card. The elapsed times were measured using OpenMP and CUDA timing functions, since these give the most accurate estimate of the time spent doing the actual correlation.

For completeness we have also listed in table I the computational times taken by the 5 different devices to calculate different correlation functions. The dataset used for this exercise is a subset of one of the mock catalogs provided by the MICE project [25] [18], with $0^\circ < \text{dec} < 18^\circ$, $0^\circ < \text{R.A.} < 18^\circ$, $0.5 < z < 0.6$, containing $3 \times 10^5$ particles. The 5 different correlation functions are:

- Monopole correlation function: linear binning for $r < 100 \text{Mpc}/h$ and 256 bins.
- Monopole correlation function: logarithmic binning for $r < 100 \text{Mpc}/h$ using 256 bins and 50 bins per decade.
- Angular correlation function: linear binning for $\theta < 30^\circ$ and 256 bins. Calculated by brute-force.
- Angular correlation function: linear binning for $\theta < 30^\circ$ and 256 bins. Calculated using pixels with resolution $\Delta \Omega \equiv 5 \times 10^{-3}$ sq-deg.
- 3-D correlation function: binning in $(\pi, \sigma)$ on a $64 \times 64$-bin histogram.

Figures 5 and 6 show the output produced by CUTE for different kinds of correlation functions.

V. SUMMARY

We have presented CUTE, a parallel code for computing two-point correlation functions from cosmological catalogs. CUTE has been optimized to run on shared-memory machines as well as graphical processing units. It can estimate the 3-D, monopole, radial and angular correlation functions from a set of data using different speed-up techniques and binning schemes. We have shown that great benefits in terms of computational speed can be gained by parallelising the algorithm on GPUs.

The code is publicly available through our website [26]. CUTE is released under the GNU Public License (GPL).
TABLE I: Different devices in which CUTE has been tested: a single CPU core, a dual core, a multi-core shared-memory machine (160 threads), an ordinary graphics card and a high-end GPU.

| Name       | Description                        | #cores |
|------------|------------------------------------|--------|
| CPUs       |                                    |        |
| Sequential | Intel Core i7-2620M                 | 1 core (≡ 1 thread) |
| Laptop-MP  | Intel Core i7-2620M                 | 2 cores (≡ 4 threads) |
| Server-MP  | Intel MP NEHALEM-EX (×8)            | 80 cores (≡ 160 threads) |
| GPUs       |                                    |        |
| Laptop-GPU | NVIDIA NVS 4200M                    | 48 CUDA cores |
| Server-GPU | NVIDIA TESLA C2070 FERMI           | 448 CUDA cores |

TABLE II: Computational times elapsed, for each of the 5 platforms listed in table I during the calculation of 5 different 2PCFs: monopole ($\xi(r)$), monopole with logarithmic binning ($\xi_{\log}(r)$), angular ($w(\theta)$), angular with pixels of resolution $\Delta \Omega \equiv 5 \times 10^{-3}$ sq-deg ($w_{\text{pix}}(\theta)$) and 3-D ($\xi(\sigma, \pi)$). Times are in seconds and correspond to the calculation of the $DR$ histogram (the full calculation of the 2PCF should take 2-3 times longer).

| Platform   | $T(\xi(r))$ | $T(\xi_{\log}(r))$ | $T(w(\theta))$ | $T(w_{\text{pix}}(\theta))$ | $\xi(\sigma, \pi)$ |
|------------|-------------|-------------------|----------------|-----------------------------|-------------------|
| Sequential | 877         | 5230              | 1374           | 21                          | 2238              |
| Laptop-MP  | 389         | 2676              | 628            | 5.3                         | 1064              |
| Laptop-GPU | 113         | 185               | 283            | 6.2                         | 297               |
| Server-MP  | 25          | 52                | 32             | 0.51                        | 50                |
| Server-GPU | 13          | 20                | 22             | 0.46                        | 27                |

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[1] The Dark Energy Survey Collaboration, 2005, arXiv:astro-ph/0510346
[2] Schlegel D. et al., 2011, arXiv:1106.1706 [astro-ph.IM]
[3] Laureijs R. et al., 2011, arXiv:1110.3193
[4] Peebles P. J. E., 1980, The large-scale structure of the universe, Princeton University Press
[5] Martínez V., Saar E., 2001, Statistics of the galaxy distribution, Chapman & Hall/CRC
[6] Kaiser N., 1987, MNRAS, 227, 1
[7] Landy S. D., Szalay A. S., 1993, ApJ, 412, 64
[8] Kerscher M., Szapudi I., Szalay A. S., 2000, ApJ, 535, L13
[9] Ponce R., Cardenas-Montes M., Rodríguez-Vázquez J., Sánchez E., Sevilla I., 2011, Proc. XXI ADASS Conf., Paris, arXiv:1204.6630
[10] Bard D., Bellis M., Allen M. T., Yeremyan H., Krasnochil J. M., arXiv:1208.3658 [astro-ph.IM]
[11] Chapman B., Jost G., Van Der Pas R., 2008, Using OpenMP - Portable Shared Memory Parallel Programming, The MIT Press
[12] Bedorf J., Gaburov E., Zwart S. P., 2012, arXiv:1204.2280
[13] Nakasato N., 2011, arXiv:1112.4539
[14] Nickels et al., 2008, Queue 6, 2, 40-53
[15] Sanders J., Kandrot E., 2011, CUDA by example - An introduction to General-Purpose GPU Programming, Addison-Wesley
[16] Podlozhnyuk V., 2007, NVIDIA, Tech. Rep.
[17] Shams R., Kennedy R. A., 2007, Proc. ISPCS, 418-422
[18] Fosalba P. Gattaia E., Castander F., Manera M. 2008, MNRAS, 391, 435
[19] A. Moore et al., 2000, astro-ph/0012333.
[20] M. Manera, R. Scoccimarro, W. J. Percival, L. Samushia, C. K. McBride, A. Ross, R. Sheth and M. White et al., Mon. Not. Roy. Astron. Soc. 428 (2012) 2, 1036 [arXiv:1203.6609 [astro-ph.CO]].
[21] A. Cabrè and E. Gaztaña, Mon. Not. Roy. Astron. Soc. 393 (2009) 1183 [arXiv:0807.2460 [astro-ph]].
FIG. 5: Monopole correlation function calculated from the PTHalo mock catalogs used in the analysis of the SDSS Ninth Data Release [20]. The solid red line shows the average correlation function, and the shaded area shows the 1-σ region around it, calculated as the r.m.s. over the 600 mocks. The monopole was calculated for the 600 mock catalogs in about 2 hours by running CUTE on the platform Server-MP (see table I).

FIG. 6: 3-D correlation function calculated from a log-normal mock catalog. The catalog contained $\sim 4.3 \times 10^7$ objects in one octant of the sky between redshifts 0.45 and 0.75. The brute-force calculation took $\sim 10$ hours on the platform Server-GPU (see table I). Redshift-space distortions produce a squashing of the correlation function along the transverse direction and create a region of negative correlation along the line of sight. This was first noted in [21].