Solution Domain Analysis of Earth-Moon Quasi-Symmetric Free-Return Orbits

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The solution domain of the Earth-Moon quasi-symmetric free-return orbits (EMQSFRO) is analyzed using a novel strategy proposed in this paper applying the Jet Propulsion Laboratory (JPL) ephemeris dynamic model. EMQSFRO is constrained by altitude at the time of trans-lunar injection (TLI), lunar swing-by altitude and Earth atmosphere re-entry angle. A vehicle on such an orbit can return to Earth without need of additional impulse after TLI. The present research on EMQSFRO and its technical applications are first summarized. Then a novel direct design strategy for EMQSFRO is proposed using a sequential quadratic programming algorithm, which applies the orbital parameters in the Moon perilune inertial coordinate system as design variables, and computes the objective orbital parameters in the TLI and re-entry time using the forward and backward numerical integral method. A simulation example indicates that the method has excellent convergence performance and precision. According to further simulation results, the solution domain cross-profile characteristics of four kinds of the EMQSFRO are discovered, which can give a deeper insight into the dynamic principle of EMQSFRO generation and supply references to the orbit design of aerospace missions.

Key Words: Earth-Moon Transfer Orbit, Quasi-Symmetric Orbit, Free-Return Orbit, Solution Domain Analysis

Nomenclature

μ: gravitational constant of centric celestial body
r: relative position vectors among the lunar spacecraft between the centric celestial body
v: relative velocity vectors among the lunar spacecraft between the centric celestial body
ρ: relative position vectors among the Sun, the Moon and between the Earth
R: Earth or Moon equator average radius
J2: Earth J2 perturbation constant
M: coordinate rotation matrix

Subscripts
E: Earth
M: Moon
S: Sun
EJ2: J2000.0 Earth coordinate system
MJ2: J2000.0 Moon coordinate system
MOI: Moon orbital inertial coordinate system
PMI: Moon perilune coordinate system
prl: perilune
prg: perigee
opt: optimization

Superscripts
\( t_{prl} \): time of perilune
TLI: trans-lunar injection
reen: re-entry

1. Introduction

The Moon is the nearest celestial body to Earth and the first stop for human’s exploration of the universe. The research on lunar probe orbit design is an energetic factor that extends non-linear dynamics and control problems like the circular restricted 3-body problem (CR3BP), which is also an experiment table that explores the orbital dynamic principles of the Earth and space beyond, as well as a number of particular orbit classes. The Earth-Moon quasi-symmetric free-return orbit (EMQSFRO) is a type of orbit which propagates after trans-lunar injection (TLI) from low-Earth-orbit (LEO) to the Earth atmospheric layer via one lunar swing-by at a low perilune height. The whole journey needs no extra orbit control, and the trajectory of EMQSFRO is quasi-symmetric about the Earth-Moon line in inertial space.

CR3BP is one of the dynamic models dealing with the orbits in the Earth-Moon space. Early in 1960, Miele1–3 expanded image trajectories in the Earth-Moon space based on CR3BP. Afterwards, Schwaniger4 and Weber et al.5 specialized circumlunar free return orbit, aiming for high safety of astronauts’ lives during manned lunar landing missions. Egorov6 discovered one class of EMQSFRO with a lunar swing-by during a return journey after TLI. It is named after the Egorov-class orbit by Farquhar in 1980, a concept for exploring the Earth’s geomagnetic tail based on a Sun-synchronous Egorov orbit.7 Thereafter, CR3BP has been used for designing libration point missions and related orbit maneuver problems.8,9

The double 2-body patched-conic model (D2BPCM) is another simplified dynamic model to manage Earth-Moon space orbits. Penzo10 and Gibson11 worked out circumlunar Earth-Moon free-return orbits. Apollo-8, 10, 11 and 12...
adopted Earth-Moon free-return orbits for manned lunar landing missions. When it comes to Apollo-13, a mechanical fault occurred at about 205,000 m into the trans-lunar phase of the LEO, then the spacecraft was taken into a circumsolar free-return orbit, narrowly returned astronauts to the Earth safely. Indisputably, the circumsolar free-return orbit has a unique advantage for manned lunar landing missions. Due to the analytic properties of D2BPCM, research on Earth-Moon spacecraft orbits based on D2BPCM are frequently developed. Luo et al. discussed a double lunar swing-by trajectory. Peng et al. proposed a serial strategy using particle swarm optimization (PSO) and D2BPCM to get an initial value, then using of sequential quadratic programming (SQP) and high-precision dynamic modeling to further optimize the circumsolar free-return parameter. Luo et al. devised a method based on the pseudo-state theory and extended differential-correction method. Li et al. put forward a multi-segment free-return orbit idea that allows a wide range of lunar approach inclinations, but additional velocity impulses are needed in the trans-lunar phase. Jesick and Ocampo produced an automatic four-class symmetric lunar-free-return orbit generation method based on CR3BP and linear perturbation theory. He et al. discovered the influence ranges and sizes of the Sun, Moon and Earth, Earth’s J perturbation assignable forces, when studying the trans-lunar orbit deviation propagation mechanism based on the Jet Propulsion Laboratory (JPL) ephemeris dynamic model. Jing and Gao analyzed the direct and indirect influences of the perturbation effect of the Sun, Moon eccentricity and the orbital plane change on the Earth-Moon space orbit. In summary, the complexity and variability of the gravitational force and perturbation in Earth-Moon space have been realized by scholars, which led to the development of a more precision dynamics model for Earth-Moon space orbits.

This paper reports the skillful establishment of the selenocentric perilune coordinate system by applying EMQSFRO’s quasi-symmetric natural property about the Earth-Moon line, and derives the state parameters conversions between it and the J2000.0 Earth-centric coordinate system (EJ2), which may avoid the decoupled orbital parameters in EJ2. Then, EMQSFRO is classified based on the perilune state parameters. The orbital solution domain characteristics are analyzed based on the JPL ephemeris high-precision dynamic model integral attempting to reveal the mechanism of EMQSFRO generation, which can also provide reference for Moon or deep-space exploration missions.

2. Dynamic Model

It can be seen that dynamic models are serious deficiencies when not considering Sun, Moon and Earth’s J perturbation in Earth-Moon space. Considering with the perturbation forces mentioned above, a dynamic model is set up for the EJ2 as follows.

\[ r = -\mu_E \frac{r}{r^3} - \mu_M \left( \frac{r_M}{r_M^3} + \frac{\rho_M}{\rho_M^3} \right) - \mu_S \left( \frac{r_S}{r_S^3} + \frac{\rho_S}{\rho_S^3} \right) \]

\[ - \left( \frac{3\mu_E J_2 R_S^2}{2r^5} - \frac{15\mu_E J_2 R_S^2 z^2}{2r^7} \right) r - \frac{3\mu_E J_2 R_S^2}{r^5} \left( \begin{array}{c} 0 \\ z \end{array} \right) \]  

Wherein \( \mu_E, \mu_M, \mu_S \) are the Earth, Moon and Sun gravitational constants, respectively; and \( r, r_S, r_M \) and \( \rho_M, \rho_S \) are relative position vectors among the lunar spacecraft, the Sun, the Earth and the Moon, as shown in Fig. 1. \( J_2 \) is the Earth’s J2 perturbation constant, \( \mu_E \) is the Earth equator average radius. The astronomical environment constants in Earth-Moon space are shown in Table 1.

Considering with the quasi-symmetry property of EMQSFRO, which means the orbital perilune is approximately on the line along the Earth and the Moon, the parameters describing EMQSFRO are established in the perilune coordinate system.

1) Transform the orbital parameters in EJ2 into the orbital parameters in the J2000.0 Moon-centric coordinate system (MJ2):

\[
\begin{bmatrix}
R_{\text{per}}^{\text{MJ2}} \\
V_{\text{per}}^{\text{MJ2}}
\end{bmatrix} =
\begin{bmatrix}
R_{\text{per}}^{\text{EJ2}} \\
V_{\text{per}}^{\text{EJ2}}
\end{bmatrix} -
\begin{bmatrix}
R_{\text{M-E}}^{\text{MJ2}} \\
V_{\text{M-E}}^{\text{MJ2}}
\end{bmatrix}
\]  

Here, the superscript \( \text{per} \) indicates the time of perilune, \( R_{\text{per}}^{\text{MJ2}} \) and \( V_{\text{per}}^{\text{MJ2}} \) are perilune position and velocity in MJ2, \( R_{\text{per}}^{\text{EJ2}} \) and \( V_{\text{per}}^{\text{EJ2}} \) are perilune position and velocity in EJ2, and \( R_{\text{M-E}}^{\text{MJ2}} \) and \( V_{\text{M-E}}^{\text{MJ2}} \) are Moon position and velocity in EJ2 at the time of perilune.

2) Translate orbital parameters in MJ2 into the perilune time selenocentric orbital inertial coordinate system (SOI) \( O_{\text{MOI}}-x_{\text{MOI}}y_{\text{MOI}}z_{\text{MOI}} \) (i.e., Moon orbital coordination system), as shown in Fig. 2.

The coordinate system rotation matrix \( M_{\text{MOI-MO}} \) is shown as:
Here, $\Omega_M$, $i_M$, and $\mu_M$ are the Moon’s right of ascension of ascending node (RAAN), inclination and argument of latitude in EJ2 at the time of perilune. $M_i(\theta)(i = x, y, z)$ signifies the coordinate system rotation matrix of $\theta$ (rad) around the $i$ (i.e., $x$, $y$, or $z$) axis, and the specific expressions are:

$$M_i(\theta) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{bmatrix}$$

(4)

$$M_j(\theta) = \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}$$

(5)

$$M_k(\theta) = \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}$$

(6)

Hence, the position and velocity vectors in SOI are:

$$\begin{align*}
\mathbf{r}_{\text{SOI}}^{\text{prl}} &= \mathbf{M}_{\text{M2-SOI}} \mathbf{r}_{\text{M2}}^{\text{prl}} \\
\mathbf{v}_{\text{SOI}}^{\text{prl}} &= \mathbf{M}_{\text{M2-SOI}} \mathbf{v}_{\text{M2}}^{\text{prl}}
\end{align*}$$

(7)

3) Define a new coordinate system named “selenocentric perilune inertial coordinate system (SPI),” $O_M-x_P y_P z_P$. $O_M$ is the Moon’s center and $O_M-x_p$ is the vector points from the Moon’s center to the spacecraft at perilune. $O_M-z_P$ points to the perilune moment orbital plane normal vector, and the Moon’s north pole direction is positive. $O_M-y_P$ makes up a cartesian coordinate system with the other two axes.

Then, the position and velocity in SPI can be solved using Eq. (8):

$$\begin{align*}
\mathbf{r}_{\text{SPI}}^{\text{prl}} &= \mathbf{M}_{\text{SOI-SPI}} \mathbf{r}_{\text{SOI}}^{\text{prl}} \\
\mathbf{v}_{\text{SPI}}^{\text{prl}} &= \mathbf{M}_{\text{SOI-SPI}} \mathbf{v}_{\text{SOI}}^{\text{prl}}
\end{align*}$$

(8)

Here, $\mathbf{M}_{\text{SOI-SPI}}$ is the rotation matrix from SOI to SPI, as shown in Eq. (9). The geometric meanings of parameters $\alpha$ and $\beta$ are illustrated in Fig. 3

$$\mathbf{M}_{\text{SOI-SPI}} = \mathbf{M}_2(\beta)\mathbf{M}_3(\alpha)$$

(9)

The position vector in SPI at the time of perilune

$$\mathbf{r}_{\text{SPI}}^{\text{prl}} = \begin{bmatrix} r_{\text{prl}} \ 0 \ 0 \end{bmatrix}^T$$

(10)

Where, $r_{\text{prl}}$ is the selenocentric distance at the time of perilune.

As shown in Fig. 4, the velocity vector $\mathbf{v}_{\text{SPI}}^{\text{prl}}$ at the time of perilune is perpendicular to position vector $\mathbf{r}_{\text{MPI}}^{\text{prl}}$, and the velocity vector can be presented as Eq. (11):

$$\mathbf{v}_{\text{SPI}}^{\text{prl}} = \begin{bmatrix} 0 & v_{\text{prl}} \cos i_{\text{prl}} & v_{\text{prl}} \sin i_{\text{prl}} \end{bmatrix}^T$$

(11)

If five orbital parameters (i.e., distance from Moon center $r_{\text{prl}}$, position vector parameters $\alpha$ and $\beta$, velocity magnitude $v_{\text{prl}}$ and angle $i_{\text{prl}}$ at the time of perilune) are known, one trajectory of EMQSFR can be obtained in EJ2 by inversely referencing the coordination transformation process mentioned above.

3. Orbit Classification and Design

3.1. Classification of EMQSFR

As shown in Table 2, Jesick and Ocampo20) classified free-return trajectories considering the lunar passage and Earth departure forms based on CR3BP.

EMQSFR has no strict symmetry based on the JPL ephemeris high-precision integral dynamic model, but it can still be divided into four classes with reference to the above factors. Among them, classes a) and b) are of orbital parameters $\alpha \equiv 0$ and $i_{\text{prl}} \equiv \pi$ in SPI, while classes c) and d) are of orbital parameters $\alpha \equiv \pi$ and $i_{\text{prl}} \equiv 0$. The trajectories in EJ2 are as shown in Fig. 5. Type Ai, of EMQSFRs is used to explore the Moon for manned lunar landing missions, and Type Bi can be employed for exploring the Earth’s geomagnetic tail. The other two kinds of EMQSFR lack efficiency due the additional launch cost against the Earth’s rotation.

3.2. Forward/backward integral orbit design strategy

When using the trajectory optimization design problem, based on the JPL ephemeris high-precision dynamic model, it is very difficult to adopt the Pontryagin maximum principle
and other indirect optimization methods. As a result, the direct method is generally used.

There are no path constraints or control impulses (except for mid-course minor correction) in the trans-lunar phase or trans-Earth phase, but three time constraints (i.e., TLI time, perilune time and re-entry time) exist. The TLI impulse increment is usually treated as optimization target parameters, without strict constraints on transfer time from the Earth to the Moon (about three days) for manned lunar landing free-return orbits. On the contrary, the Sun-synchronous Egorov orbit demands a rigorous transfer time to follow the Sun’s track, resulting in little optimization of fuel consumption for the maneuver.

Based on orbital parameters in SPI, a forward/backward integral orbit design strategy process is proposed, as shown in Fig. 6.

As a numerical example, a Sun-synchronous Egorov orbit with 15-day trans-lunar time and 15-day trans-Earth time is designed as follows. Based on the above strategy, the iteration will end up at the same time when the TLI parameters and re-entry parameters constraints are met. Otherwise, the perilune design parameters will be adjusted.

The orbit design problem can be attributed to the non-linear optimization problem with orbital parameters in SPI as the optimization variables.

\[
x_{\text{opt}} = \begin{bmatrix} r_{\text{prl}} & \alpha & \beta & v_{\text{prl}} & t_{\text{prl}} \end{bmatrix}
\]

The optimization objective function is defined as the target absolute value sum of two different parts. One part is TLI time perigee radius \(r_{\text{TLI}}\) and the other is the re-entry Earth nearby perigee radius \(r_{\text{prl}}\).

Table 2. Earth-Moon free-return orbit classifications.

| Characteristic   | Value | Definition  |
|------------------|-------|-------------|
| Lunar passage    | A     | circumlunar |
| B                | cislunar |
| Earth departure  | i     | posigrade   |
| ii               | retrograde |

Table 3. Sun-synchronous Egorov orbit parameters.

| \(r_{\text{prl}}\) (km) | \(\alpha\) (rad) | \(\beta\) (rad) | \(v_{\text{prl}}\) (km/s) | \(t_{\text{prl}}\) (rad) |
|--------------------------|------------------|----------------|--------------------------|--------------------------|
| 1938                     | 3.114            | -0.005         | 2.501                    | 0.000                    |

Fig. 5. Coplanar orbital schematics in EJ2.

Fig. 6. Schematic diagram of the forward/backward integral orbit design strategy.

Fig. 7. Trajectory of Sun-synchronous Egorov orbit.
At the same time, position and velocity at the time of perilune are restrained by orbital kinematics constraints.

\[ r_{prl} = R_M + h_{prl} \]
\[ v_{prl} = \sqrt{2\mu_M (R_M + h_{prl})} \]

Here, \( R_M \) and \( \mu_M \) are Moon equator radius and gravitational constants, as shown in Table 1, \( h_{prl} \) is the perilune altitude, and a higher altitude is set as 200 km. This may be safer than 111 km reported in Berry.\(^{14} \)

For example, set 19 Jan 2025 04:00:00.000 UTCG as the time of perilune, using a global optimization algorithm–Multi-start\(^{24} \) (based on SQP), the Sun-synchronous Egorov orbit parameter in SPI is obtained as shown in Table 3, and the trajectories are as shown in Fig. 7.

4. Analysis of Solution Domain

4.1. Circumlunar class

EMQSFRO solution domain exploration contributes to a deeper understanding of the EMQSFRO mechanism in Earth-Moon gravitational space. As mentioned in Section 2, five parameters in Eq. (12) and perilune time \( t_{prl} \) constitute six-dimension variables to determine one EMQSFRO trajectory. Based on the orbit design strategy in Section 2.2, a large number of shooting simulation results show that the two classes of EMQSFRO in Fig. 5 a) and b) yield perilune parameters \( |\alpha| \leq 0.2 \text{ rad} \) and \( |\beta| \leq 0.2 \text{ rad} \) with fixed perilune time \( t_{prl} \) and perilune radius \( r_{prl} \); while the other two classes in Fig. 5 c) and d) yield perilune parameters \( |\alpha - \pi| \leq 0.2 \text{ rad} \) and \( |\beta| \leq 0.2 \text{ rad} \) with fixed perilune time \( t_{prl} \) and perilune radius \( r_{prl} \).

Set 19 Jan 2025 04:00:00.000 UTCG as perilune time \( t_{prl} \) and \( h_{prl} = 200 \text{ km} \). Solution domain section at \( \alpha = 0 \) and \( \beta = 0 \) are shown in Fig. 8. Here, a), b), c) and d) are contour maps of the trans-lunar orbital TLI perigee radius, TLI inclination, re-entry perigee radius and re-entry inclination, respectively.

It is observed that, for the solution domain of circumlunar-class EMQSFRO, perilune velocity \( v_{prl} \) and inclination \( i_{prl} \) are quasi-symmetric the TLI and re-entry perigee radius present looped distributions as shown in Fig. 8 a) and c). Compared with orbital inclination contour maps b) and d), solution domain sections of the Ai and Aii-type orbits (i.e., divided by the TLI inclination 90 deg line) are connected. The velocity of the former is a little smaller than that of
the latter. TLI inclination and the re-entry inclination quasi-symmetric axis are related to the angle between the Moon’s path plane and equatorial plane at the time of perilune. When perilune velocity angle is equal to $\pi$, the perilune velocity relative to Ai-type orbits is smaller and the velocity relative to Aii-type orbits is larger.

As an example, one singularity orbit between Ai and Aii type trajectories is drawn for EJ2, as shown in Fig. 9. Its perigee altitude is negative, which implies a hazardous crash to a probe on this orbit, returning to Earth like a meteorite.

4.2. Cislunar class

Set 19 Jan 2025 04:00:00.000 UTCG as the time of perilune and $h_{prl} = 200$ km. The solution domain section at $\alpha = \pi$ and $\beta = 0$ is shown in Fig. 10. Here, a), b), c) and d) are contour maps of trans-lunar orbital TLI perigee radius, TLI inclination, re-entry perigee radius and re-entry inclination, respectively.

It is observed that, for the solution domain of cislunar class of EMQSFRO, the perilune velocity $v_{prl}$ and inclination $i_{prl}$ are quasi-symmetric. TLI and re-entry perigee radius present looped distributions, as shown in Fig. 10 a) and c). Compared with the orbital inclination contour maps b) and d), the solution domain sections of Bi and Bii-type orbits (i.e., divided by the TLI inclination 90 deg line) are connected. The velocity of the former is a little slower than the latter. The TLI inclination and re-entry inclination quasi-symmetric axis are related to the angle between the Moon’s path plane and equatorial plane at the time of perilune. When perilune velocity angle is equal to 0, perilune velocity relative to Bi-type orbits is smaller and the velocity relative to Bii-type orbits is larger.

As an example, one singularity orbit between Bi and Bii-type trajectories is drawn for EJ2, as shown in Fig. 11. Its perigee altitude is negative, which implies a hazardous crash to a probe on this orbit and returning to Earth like a meteorite.

5. Conclusions

In the present study, a selenocentric perilune inertial coordinate system is established, and a forward and backward integral orbit design strategy is devised based on the coordinate system and ephemeris high-precision dynamic model for EMQSFRO. Then four classes of EMQSFRO solution domain are analyzed. Some conclusions and the solution domain of the EMQSFRO generation cannot only help us to determine the real Earth-Moon gravitational space produced according to the EMQSFRO mechanism more clearly, but can also be used for manned lunar landing missions or Moon and the beyond space exploration task orbit design.
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