Statistical and Energetic Constraints in Population Synthesis Models

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Abstract

Physical and numerical constraints in building up self-consistent population synthesis models are briefly analysed discussing their application to most of the current synthesis codes widely adopted in Galactic and extragalactic studies.

Major focus is given in particular to the numerical effects of discrete isochrone sampling as well as to the energetic constraints for a proper normalization of the main sequence vs. post-MS luminosity contribution in simple stellar populations.

The case of template models for present-day elliptical galaxies is reviewed in more detail comparing the results from different synthesis codes.

1 Introduction

Stellar population synthesis has extensively been used in the recent years as a main theoretical tool for Galactic and extragalactic studies. In particular, evolutionary synthesis (specifically relying on theoretical tracks for stellar evolution) has quickly become the most popular technique to investigate spectrophotometric properties of external galaxies (cf. e.g. Ellis 1997).

While, in its essence, a synthesis procedure is a quite simple task— we just need to sum up luminosity from stars along isochrone bins to obtain the luminosity of the population as a whole in the different photometric bands— there are however a number of crucial constraints which we must pay attention to in order to achieve a physically (and not only numerically) self-consistent output model.

In this paper we would briefly account for some of these features discussing some reference cases from the main synthesis codes widely adopted in the recent literature.
It is worth stressing that the main aim of our analysis is not to single out any superior model or trying any “ranking” among the available codes, but rather to lead the reader to a more critical appraisal of the current theoretical framework. This, as a little step toward a better use of population synthesis in the interpretative analysis of real stellar systems.

2 Numerical discreteness and statistical sampling

A basic step when assembling a synthetic stellar population concerns the way we choose to sample the theoretical isochrone. Focussing our attention to the general case of a simple stellar population (SSP) intended as a single generation of coeval stars with fixed distinctive parameters and mass distribution, we can envisage two possible procedures in this regard:

i) we might decide to add stars discretely by random sampling according to the isochrone locus and count-density map or

ii) we might attribute each luminosity bin a weight according to its expected relative density normalizing to the total size of the population.

An example of case (i) is shown in Fig. 1 for a 15 Gyr Salpeter SSP with solar metallicity according to Buzzoni’s (1989) code. The advantage of this approach is to provide a more vivid representation of the synthetic c-m diagram of a stellar population, giving a direct hint of the relative distribution of the stars along the different evolutionary phases.

On the other hand, as a main drawback, we might severely be affected by normalization problems and statistical undersampling of all those fast-evolving phases if they predict a few or even less than one star in the whole sample.

For example, according to the Renzini and Buzzoni (1986) fuel consumption theorem (FCT), about 15 stars should be expected along the asymptotic giant branch (AGB) in the SSP of Fig. 1. This means that in general a $1/\sqrt{15} \sim 25\%$ statistical uncertainty in the AGB luminosity contribution could affect a random simulation of this size. While this is a negligible detail when observing in the V band, things might be very different in the infrared where AGB stars provide a much larger fraction of the total luminosity of the population. Following Buzzoni’s (1993) calculations, in the case of the synthetic c-m of Fig. 1 a statistical fluctuation as large as $\pm 0.2$ mag should eventually be expected in the integrated $V - K$ color. Things would get even worst as far as we move to a younger age because of a faster post main sequence (post-MS) evolution.

Note that while statistical uncertainty from sample discreteness could be a real (and often unavoidable) case when observing for instance Galactic globular clusters, the effect could in principle be overcome in theoretical simulations but at cost of increasing the random sample to an exceedingly larger size ($\geq 10^7$ stars).

A synthesis procedure that relies on case (ii) is on the contrary more suitable to achieve an unbiased estimate of the SSP integrated properties. In the latter
Figure 1: Synthetic c-m diagram for a 15 Gyr Salpeter SSP of solar metallicity (from Buzzoni’s 1989 code). The random simulation consists of $10^5$ stars, a size comparable with a typical Galactic globular cluster. Data have been added a Poissonian photometric error such as $\sigma_V = 0.03$ mag at $V = +4$. The main stellar evolutionary phases are labelled in the figure. Note the effect of sample discreteness in the fast-evolving phases (like for instance near the RGB tip or along the AGB phase) that result severely undersampled.

In fact the contribution from each isochrone bin is taken at its face value, and correctly accounted for according to the theoretical luminosity function. Integrated magnitudes and colors will therefore not depend on the size of the population.

As a partial limitation, the latter method somewhat misses a direct picture of star distribution merely restraining the SSP c-m diagram to the locus of the theoretical isochrone. Compare for example the right-hand panel of Fig. 2 displaying the equivalent SSP of Fig. 1 as it appears from the Padova synthesis code (Bertelli et al. 1994).

Preliminary to any fair determination of SSP properties however (either via case (i) or (ii)), a more general problem that deserves our attention for its potential impact on the synthesis results deals with the numerical representation of the star number counts along the theoretical isochrone.
Figure 2: Theoretical luminosity function *(left panel)* and isochrone locus *(right panel)* for a 15 Gyr Salpeter SSP of solar metallicity from the Padova model database (Bertelli et al. 1994). The main evolutionary phases are labelled in the figure. See text for discussion.

In a general approach to isochrone calculation relying on the interpolation within a grid of stellar tracks, the number of stars in each bin eventually derives from the adopted IMF and the estimated width in stellar mass of the “j-th” cell \((\delta M_j)\), namely \(N_j = IMF \times (\delta M)_j\). In spite of any smooth IMF, we should rather expect a “roughness” in the bin number counts such as \(\sigma N \sim \sigma (\delta M)\). A large uncertainty in the values of \((\delta M)_j\) could in facts easily derive from numerical fluctuations in the interpolation algorithm given the extremely thin range of mass variation along post-MS isochrone bins. For example, a red giant branch (RGB) in an old SSP sampled with 50 bins will typically imply \(\delta M \sim 0.0006 M_\odot\) (cf. next section). This means that a 10\(^{-4}\) relative accuracy in the mass interpolation algorithm should be achieved in order to assure a 10% accuracy in the number counts along the theoretical isochrone.

This effect is often neglected in the model analysis but the left-hand panel of Fig. 2 clearly shows how pervasive it can be as far as the SSP luminosity function is taken into account in more detail. Note for example in the figure the two “glitches” about the TO region and the discontinuity about the RGB tip (about bin sequence numbers 50-65) as well as the scatter along the post-AGB evolution (beyond bin number 100). Again, one should carefully consider the differential impact of this effect when accounting for the SSP photometric properties in different wavelength ranges.

The problem of a fair determination of the isochrone luminosity function
eventually calls for another primary question in evolutionary population synthesis dealing with a physical match of MS and post-MS evolution in a SSP, as we will discuss in the next section.

3 Energetic constraints: Post-MS vs. MS contribution

As a general feature, SSP post-MS evolution is recognized to always proceed much faster than stellar MS lifetime at every age. If we get an estimate of the rate of change of the MS stellar Turn Off (TO) mass \( M_{TO} \)， then the expected range \( \Delta M_* \) of actual stellar masses that are experiencing post-MS evolution over a lifetime \( \tau_{PMS} \) at a given SSP age \( T \) is

\[
\Delta M_* = \frac{dM_{TO}}{dt} \tau_{PMS} \sim \frac{M_{TO}}{T} \tau_{PMS}.
\]

This means, in other words, that

\[
\frac{\Delta M_*}{M_{TO}} \sim \frac{\tau_{PMS}}{T} \leq 0.1.
\]

As a consequence, if stars of 1 \( M_\odot \) are crossing the TO point in an old SSP then stars of just 1.1\( M_\odot \) are exhausting their post-MS evolution becoming white dwarfs. It is clear therefore that the SSP post-MS isochrone nearly merges, to a first approximation, with the evolutionary track of a star of fixed mass \( M_* \rightarrow M_{TO} \). In this sense, post-MS lifetime from the stellar track can easily be converted into relative star number counts per isochrone bin:

\[
N_j/N_{PMS} = \tau_j/\tau_{PMS}
\]

where \( \tau_j \) is the lifetime across the j-th bin, so that \( \sum_{PMS} \tau_j = \tau_{PMS} \) and \( \sum_{PMS} N_j = N_{PMS} \). Note in this regard a substantially difference (and a much better performance) of this approach to the definition of the isochrone luminosity function with respect to the classical interpolation algorithm discussed in previous section. With this method in facts we could fully overcome any numerical instability in the bin star counts as \( \tau_j \) is a natural output of the stellar evolutionary track.

As far as the MS evolution is concerned, on the contrary, MS number counts simply derives from the IMF so that for a Salpeter power law we have \( N_j \propto M_j^{-s} \) per bin of fixed width in stellar mass.

The real problem, when assembling the whole synthetic SSP is therefore to properly scale MS and post-MS contribution in order to preserve overall energetic self-consistency in the model.

Clearly, a simple match by grafting contiguous isochrone bins at the MS top and at the Post-MS bottom cannot be a safe solution in this regard as
any uncertainty and numerical noise in the evaluation of both extrema would accordingly reverberate into a magnified scatter in the contribution of each SSP building block.

Rather than relying on such a differential normalization procedure, one could take advantage of an integral approach featuring SSP global properties via the FCT. In this case, following Renzini and Buzzoni (1986, their eq. 14) we have that

\[
\frac{L_{PMS}}{L_{tot}} = B \times \text{Fuel} = (1.76 \pm 0.4) \, m_H. \tag{4}
\]

This simply relates SSP Post-MS luminosity in bolometric to the total amount of fuel spent in post-MS evolution by stars of mass $M_{TO}$.

In the equation, the normalization factor $B$ is the so-called “specific evolutionary flux”; it turns to be about $B = 1.7 \pm 0.4 \times 10^{-11} \, [L_\odot^{-1} \, \text{yr}^{-1}]$ (Buzzoni 1989). The term $m_H$ is the exhausted fuel expressed in Hydrogen-equivalent solar masses. This quantity is a straightforward output of stellar evolution theory.

One can envisage a simple and direct relationship between the SSP post-MS relative contribution from eq. (4) (or its equivalent form in terms of $L_{PMS}/L_{MS}$ ratio) and the $M/L$ ratio of the population. A plot like that in Fig. 3 could
Figure 4: Comparison of template model ellipticals from different theoretical codes for population synthesis. Displayed are the predicted $M/L_V$ ratio and $L_{PMS}/L_{MS}$ $V$-luminosity partition from the models by O’Connell (1976) [OC], Tinsley and Gunn (1976) [TG], Pickles (1985) [PK], Guiderdoni and Rocca-Volmerange (1987) [GR], Bruzual and Charlot (1993) [BC], Worthey (1994) [W], Bressan et al. (1994) [BCF], and Buzzoni (1995) [B]. The Bruzual and Charlot (1996) updated code is also reported by matching both the Padova [(P)] and Geneva [(G)] stellar track database. The effect of artificially doubling post-MS contribution (then relaxing FCT prescriptions) is displayed for Buzzoni’s model by the dotted line.

provide in this sense an effective tool to probe energetic self-consistency in synthesis models. In the figure we took as a reference the standard case of a 15 Gyr Salpeter SSP with solar metallicity displaying both the effect of a change in age and in the IMF slope. Quite interestingly, note that a Salpeter IMF is that providing the highest luminosity per unit stellar mass in a SSP (Buzzoni 1995). In a flatter IMF in facts most of the SSP mass belongs to short-living high-mass stars (so that the $M/L$ ratio is higher than the Salpeter case); for opposite reasons, a steeper IMF will have a large amount of SSP mass locked into low-luminosity dwarf stars (again providing a larger $M/L$ ratio with respect to a Salpeter IMF).

The dotted curve in Fig. 3 tracks the locus of a SSP by relaxing the energetic constraint of the FCT and artificially renormalizing the post-MS contribution by increasing luminosity up to a factor of two. As expected, by overweighting post-MS we will decrease the SSP $M/L$ ratio (because we are mainly enhancing the total luminosity of the population without providing an equivalent extra
mass), and obviously move the reference model toward a higher $L_{PMS}/L_{MS}$ ratio.

An application of this diagnostic plot to the relevant case of template SSPs from different synthesis codes fitting the present-day elliptical galaxies is attempted in Fig. 4 updating Buzzoni’s (1995) original calculations. A striking evidence from the figure is that model ellipticals by Tinsley and Gunn (1976), Pickles (1985), Guiderdoni and Rocca-Volmerange (1987), and Bruzual and Charlot (1993) do not fully comply with the SSP energetic constraint in the sense that all of them appear to be sensibly overestimating post-MS contribution.

This discrepancy appears to be partially alleviated in the updated code of Bruzual and Charlot (1996) (see also Bruzual 1996) once accounting for the Geneva set of stellar evolutionary tracks (Schaller et al. 1992; Charbonnel et al. 1996). It is however at least surprising to note that a similar match with the Padova isochrones (Bertelli et al. 1994) leads to a somewhat different model comparing with Bressan’s et al. (1994) original calculations based on the same theoretical framework. In order to recover this discrepancy one should admit that an important fraction of the (low?) MS luminosity is missing in the Bruzual and Charlot (1996) model.

References

[1] Bertelli, G., Bressan, A., Chiosi, C., Fagotto, F., Nasi, E. 1994, A€ApS, 106, 275
[2] Bressan, A., Chiosi, C., and Fagotto, F. 1994, ApJS, 94, 63
[3] Bruzual, G. 1996 in “From Stars to Galaxies”, ASP Conf. Ser., Vol. 98, eds. C. Leitherer, U. Fritze-v. Alvensliebel and J. Huchra (ASP: San Francisco) p. 14
[4] Bruzual, G., and Charlot, S. 1993, ApJ, 405, 538
[5] Bruzual, G., and Charlot, S. 1996, private communication
[6] Buzzoni, A. 1989, ApJS, 71, 817
[7] Buzzoni, A. 1993, A€Ap, 275, 433
[8] Buzzoni, A. 1995, ApJS, 98, 69
[9] Charbonnel, C., Meynet, G., Maeder, A., Schaerer, D. 1996, A€ApS, 115, 339
[10] Ellis, R.S. 1997, ARAA, 35, 389
[11] Guiderdoni, B., and Rocca-Volmerange, B. 1987, A€A, 186, 1
[12] O’Connell, R.W. 1976, ApJ, 206, 370
[13] Pickles, A.J. 1985, ApJ, 296, 340
[14] Renzini, A., and Buzzoni, A. 1986 in “Spectral Evolution of Galaxies”, eds. C. Chiosi and A. Renzini (Dordrecht: Reidel) p. 195
[15] Schaller, G., Schaerer, D., Meynet, G., Maeder, A. 1992, A€ApS, 96, 269
[16] Tinsley, B.M., and Gunn, J.E. 1976, ApJ, 203, 52
[17] Worthey, G. 1994, ApJS, 95, 107