A cosmology with variable c

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A new varying-c cosmological model constructed using two additional assumptions, which was introduced in our previous work, is briefly reviewed and the dynamic equation of the model is derived distinctly from a semi-Newtonian approach. The results of this model, using a Λ term and an extra energy-momentum tensor, are considered separately. It is shown that the Universe began from a hot Big Bang and expands forever with a constant deceleration parameter regardless of its curvature. Finally, the age, the radius, and the energy content of the Universe are estimated and some discussion about the type of the geometry of the Universe is provided.

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1. INTRODUCTION

During the last decade, varying speed-of-light (VSL) theories have arisen as alternatives to the inflationary scenario of the Universe [1, 2, 3, 4, 5, 6, 7, 8, 9]. These theories are based on various dynamics but in all of them, the speed of light needs be much more larger than at present to solve the cosmological problems. For instance, in a model provided by Albrecht and Magueijo [2], the speed of light suddenly falls off to its current value during a phase transition. Some authors introduced bimetric gravity theories [4, 7]. Barrow and Magueijo constructed a model in which the speed of light decreases by a smooth power-law function of the cosmological time [3].

There are some criticisms about the varying-c models. For instance, the speed of light is not a dimensionless quantity, hence, going to a new frame, one may cancel its probable variations. Or, if c and, consequently, the coupling constant of the Einstein equation would vary, then observers in different frames would see the evolution of the Universe governed by different rules. However, these arguments are also applicable to any varying-constant model, in which some physical constants are made to vary, for example, any theory with varying gravitational constant G, such as the Brans-Dicke theory of gravitation. Moreover, if a dimensionless parameter seems to vary, for example, the fine-structure constant α, then its included quantities, which are dimensional parameters, are to vary as well. In this case, if one tries to fix the value of these latter quantities, one simply lets the units be rescaled.

Some authors worry about the Lorentz invariance breaking in varying speed-of-light models. In contrast, the authors of ref. [10] claim that they have provided the Fock-Lorentz [11, 12] and Magueijo-Smolin [13, 14] transformations with a varying-c, as redescriptions of special relativity. Also, according to ref. [15], assuming only the first principle of special relativity, namely, the relativity principle, together with Euclidianity and isotropy leads to either Galilean or Lorentz transformations. In the latter transformation, there is an upper local limit for the speed of particles. In addition, the semi-strong principle of equivalence permits the possibility of different numerical contents, namely, the values of the fundamental constants, at different parts of space-time in the Universe. The global laws, whose local approximations are considered in the various local space-time regions, may probably involve the derivatives of these constants [15].

Other objections are due to the fairly well-predicted nucleosynthesis and cosmic microwave background (CMB). Till now, no one has claimed that the varying-constant models cannot predict these phenomena.

At last, some people prefer the frame-independent arguments. It is a plausible opinion in every field of physics except cosmology, for when one considers the Universe as a whole, one cannot ignore the preferred frame of cosmology, or introduce a more useful frame than it; that is, it is of worth for a law to be independent of the observers so as to be identical everywhere, but the cosmological preferred frame is the most global frame that is obtainable everywhere by its homogeneity and isotropy. Moreover, in a Machian view, this frame is an undetachable property of the Universe.

In our previous work [16], we showed that one can construct a simple varying-c model of the Universe using two key assumptions, which we will review in Sect. 2. In addition, this model is also based on the cosmological principle, the Weyl postulate, and the extended Einstein equation, where, by the latter, we mean that the coefficient of the Einstein equation is a function of cosmological time in the preferred cosmological frame. This model provides solutions to

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some of the Standard Big Bang (SBB) problems. It should be mentioned that the consequent results were obtained from two parts: the first part was directly derived from our assumptions that were discussed as the basic concepts. Section 2 gives a brief review of this part. The second part provides the dynamics of the Universe, i.e., the Friedmann equations.

In this work, we are going to consider a different approach to deriving those Friedmann equations, namely, a semi-Newtonian way. Also, a comparison of the results of this model with the Einstein equation when it is equipped with the cosmological “constant”, Λ, is provided in Sect. 4. Section 5 includes a similar way of deriving the Friedmann equations, where Λ is replaced by an extra energy-momentum tensor related to the varying-\(c\). Finally, in Sect. 6, the results of this model are discussed, some ideas about the age and the radius of the Universe are provided and the energy content of it is estimated. In addition, by means of the derived values, the type of the geometry of the Universe is discussed.

2. BRIEF REVIEW

The very plausible assumptions of homogeneity and isotropy of the Universe, which are in agreement with observations on the cosmic scale, lead one to use the Robertson-Walker metric as the preferred frame for cosmology. In general, one can write this metric as

\[
ds^2 = c(t)^2 dt^2 - a(t)^2 \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right) \tag{1}
\]

which is conformal with its usual form, i.e.,

\[
ds^2 = \left( \frac{c(t)}{c_0} \right)^2 \left[ c_0^2 dt^2 - a'(t)^2 \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right) \right] \tag{2}
\]

From now on, we hold all arguments in the preferred frame of cosmology and write them according to the metric (1). This is the preferred metric for the whole Universe and not its constituents.

To construct the model, the first assumption is that the total energy of the Universe, as observed in the preferred frame of the cosmology, is a constant, i.e.,

\[M c^2 = \text{const.}\] \tag{3}

where \(M\) is the total mass of the Universe. This is what has really been accepted in the SBB. However, if one ignores the constancy of the speed of light, then the situation differs completely from the SBB.

The second assumption asserts that the total energy of a particle, including its inertial energy and its gravitational potential energy, measured in the preferred frame of the cosmology, is zero. Mathematically, that is

\[- \frac{G M m}{R} + mc^2 = 0 \] \tag{4}

where \(R\) is the radius of the Universe and \(G\), as we assume, is a constant. In other words, the inertial energy of a particle is due to the gravitational potential energy of the mass content of the Universe upon it. Simplifying the above relation, it gives

\[
\frac{GM}{Rc^2} = 1 \] \tag{5}

This dimensionless relation is a legitimate one that contains the most essential cosmological quantities such as \(M\), \(R\) and \(c\). Inspired from this, one may feel better if one substitutes the second assumption as if one defines \(G\) as a proportionality coefficient with which the relation (5) holds. That is, the Universe is such that the fraction of \(\frac{Rc^2}{M}\) is a constant.

Combining the relations (3) and (5), to omit \(M\), and then differentiating the result, one gets

\[
\frac{dc}{c} = - \frac{1}{4} \frac{dR}{R} \] \tag{6}

Now, using the ordinary dimensionless scale factor, \(a\), as

\[
a = \frac{R}{R_0} \] \tag{7}
one can gain from the relation (6) that
\[ c = c_0 \left( \frac{a}{a_0} \right)^{-\frac{1}{2}} \]  
(8)

where \( c_0 \) and \( a_0 \) are the current values of these quantities. Following the same procedure in deriving the above equation, one can easily get
\[ M = M_0 \left( \frac{a}{a_0} \right)^{\frac{1}{2}} \]  
(9)

The relations (7) and (9) imply that:
\[ \rho \propto \frac{M}{R^3} \propto \frac{1}{a^3} \quad \text{or} \quad \rho = \rho_0 \left( \frac{a}{a_0} \right)^{-\frac{3}{2}} \]  
(10)

where \( \rho \) is the total matter density of the Universe, regardless of its content. Combining the relations (8) and (10), one gets
\[ \varepsilon = \rho c^2 = \rho_0 c_0^2 \left( \frac{a}{a_0} \right)^{-3} \]  
(11)

which is a more familiar relation.

We also found in ref. [16] that the wavelength and the frequency of the light vary as
\[ \lambda = \lambda_0 \frac{a}{a_0} \quad \text{and} \quad f = f_0 \left( \frac{a}{a_0} \right)^{-\frac{5}{2}} \]  
(12)

In addition, the relation between the emitted frequency, \( f_e \), the received frequency, \( f_r \), and the redshift, \( z \), is
\[ \frac{f_e}{f_r} = \left( \frac{a_0}{a_z} \right)^{\frac{5}{2}} = (1 + z)^{\frac{5}{2}} \]  
(13)

where \( a_z \) is the value of the scale factor at the time of emission. Also, we found that the temperature of the Universe varies as
\[ T = T_0 \left( \frac{a}{a_0} \right)^{-\frac{5}{2}} \]  
(14)

### 3. SEMI-NEWTONIAN APPROACH

A semi-Newtonian approach to the dynamics of the Universe, often used in standard text books [17, 18], is mainly based on Newtonian gravitation (except the existence of any absolute space or time) instead of the general relativity.

Using the assumptions of homogeneity and isotropy, one can write the energy equation for a test particle \( m \) at a distance \( r \) away from any center, which can be selected arbitrarily, as
\[ U = \frac{1}{2} m r^2 - \frac{Gm}{r} \left( \frac{4\pi r^3 \rho}{3} \right) \]  
(15)

Rearranging the equation, using the relation \( \vec{r} = a(t) \vec{x} \), where \( \vec{r} \) and \( \vec{x} \) are related as physical and co-moving coordinates, respectively, one gets
\[ \frac{2U}{mx^2} = a^2 - \frac{8\pi}{3} Ga^2 \rho \]  
(16)

The right-hand side of the above relation does not depend on spatial coordinates and consequently neither does the left-hand side, so in fact, one learns that homogeneity requires that \( U \) does indeed vary with \( x^2 \). In addition, inspired by special relativity, one has \( \frac{L}{m} \propto c^2 \). These lead to
\[ kc^2 \equiv \frac{2U}{mx^2} = -a^2 + \frac{8\pi}{3} Ga^2 \rho \]  
(17)
where \( k \) is supposed to be a constant and for further convenience, it should be rescaled to unity. Also, it is worth noting that the second term on the right-hand side of relation (15), regarding the relations (8) and (10), is proportional to the square of the speed of light as well. Therefore, one can write

\[
\left( \frac{\dot{a}}{a} \right)^2 + \frac{k c^2}{a^2} = \frac{8\pi G}{3} \rho
\]  

which is exactly the standard form of the first Friedmann equation. By differentiating this relation with respect to \( t \), one gets

\[
2 \left[ \frac{\ddot{a}}{a} - \left( \frac{\dot{a}}{a} \right)^2 \right] \frac{\dot{a}}{a} + 2 \frac{k c^2}{a^2} \left( \frac{\dot{c}}{c} - \frac{\dot{a}}{a} \right) = \frac{8\pi G}{3} \dot{\rho}
\]

Using the relations (8) and (10), one gets

\[
\frac{\ddot{a}}{a} = -\frac{1}{4} \left( \frac{\dot{a}}{a} \right)^2
\]

which is the analog of the second Friedmann equation in this model. The other equations can be derived from this point on, namely, an ever-expanding Universe, i.e.,

\[
a = a_0 \left( \frac{t}{t_0} \right)^{\frac{4}{5}}
\]

regardless of the type of the geometry of the Universe, with

\[
t_0 = \frac{4}{5} \frac{1}{H_0}
\]

where \( H_0 \) is the value of the Hubble constant, i.e., \( H = \frac{\dot{a}}{a} \) at \( t_0 \). Consequently, one has

\[
c(t) = c_0 \left( \frac{t}{t_0} \right)^{-\frac{2}{5}}, \quad M(t) = M_0 \left( \frac{t}{t_0} \right)^{\frac{2}{5}}, \quad \rho(t) = \rho_0 \left( \frac{t}{t_0} \right)^{-2}, \quad T = T_0 \left( \frac{t}{t_0} \right)^{-1}
\]

However, one should notice that the assumption (17) in the Newtonian view is actually a priori demand for obtaining the relation (21). Also, the time dependency of \( \frac{t}{m} \) cannot be seen in the classical way. Hence, because of these points, it is a semi-Newtonian approach.

4. THE FIELD EQUATION WITH \( \Lambda \)

Although it is not necessary to introduce a \( \Lambda \) term in the Einstein equation when treating this model, for similarity to the known models, in this section, we treat the model as if it has two sources, the ordinary energy-momentum tensor plus a \( \Lambda \) term, namely,

\[
G^{ab} = \frac{8\pi G}{c^4} T^{ab}_m + \Lambda g^{ab}
\]

The index \( m \) in \( T^{ab}_m \) has been introduced in order to distinguish it from the energy-momentum tensor, \( T^{ab} \), introduced in ref. [16], where \( \nabla_a T^{ab} \neq 0 \). Now, by \( T^{ab}_m \), we mean the energy-momentum tensor of a part of the matter for which the conservation still holds, i.e. we assume

\[
\nabla_a T^{ab}_m = 0
\]

In the above, we regard the \( \Lambda \) term as a source of matter than geometry. Also, as the properties of the Robertson-Walker metric are based on the symmetries of space-time, we use its adapted frame as the preferred coordinate, hence, the components of this metric are

\[
g_{00} = 1, \quad g_{11} = -a^2(t) \frac{1}{1 - kr^2}, \quad g_{22} = -a^2(t)r^2, \quad g_{33} = -a^2(t)r^2 \sin^2 \theta
\]
As is obvious, the metric components are as before, and this allows us to use the well-known Friedmann equations with $x^0$, and then one replaces $dx^0$ by $c dt$, see ref. [16] for details.

The inner property of the geometry, i.e. the reduced Bianchi identity, $\nabla_a G^{ab} = 0$, implies that

$$\nabla_a \left( \frac{8\pi G}{c^4} T_m^{ab} + \Lambda g^{ab} \right) = 0 \quad (27)$$

If further, one assumes $T_m^{ab}$ to be a perfect fluid with the equation of state as

$$p_m = (\gamma_m - 1) \rho_m c^2 \quad (28)$$

then, one can simply solve the relation (25) to get

$$\rho_m \propto \frac{1}{a^{3\gamma_m-1/2}} \quad \text{and hence} \quad \rho_m c^2 \propto \frac{1}{a^{3\gamma_m}} \quad (29)$$

or

$$\rho_m = \rho_{m0} \left( \frac{a}{a_0} \right)^{\frac{1}{2} - 3\gamma_m} \quad (30)$$

where $\rho_{m0}$ is the matter density related to $T_m^{ab}$ at $a_0$. Using the relation (25) in the relation (27) yields

$$\partial_a \left( \frac{8\pi G}{c^4} \right) T_m^{ab} + (\partial_a \Lambda) g^{ab} = 0 \quad (31)$$

Hence, as $c$ is not a constant, $\Lambda$ cannot be a constant anymore, and it must be a function of space-time in general. However, in this case, from the Lagrangian point of view [19], for any Lagrangian density that is a function of the metric and its first and second derivatives, where its functionality on the second derivatives of the metric is linear, a varying $\Lambda$ should be related to the curvature scalar. However, we are not going to proceed any further with this view in this work.

By the homogeneity and in the preferred frame of the cosmology, which is being used here, $\Lambda$ is just a function of cosmological time only. Hence, the zero-component of the relation (31) gives

$$8\pi G \partial_0 \left( \frac{1}{c^4} \right) T_m^{00} + \partial_0 \Lambda g^{00} = 0 \quad (32)$$

Using relation (8) yields

$$\Lambda = \Lambda_0 \left( \frac{a}{a_0} \right)^{1-3\gamma_m} \quad (33)$$

where

$$\Lambda_0 = \frac{8\pi G \rho_{m0}}{(3\gamma_m - 1)c_0^2} \quad (34)$$

This reminds us the quintessence instead of the cosmological “constant”.

To obtain $\gamma_m$, one should use the zero-component of (24) to get the first Friedmann equation, i.e.,

$$\left( \frac{\dot{a}}{a} \right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3} \rho_m + \frac{\Lambda c^2}{3} \quad (35)$$

From the relations (8), (30) and (33), it is clear that the two terms on the right-hand side of the above equation have the same order of the scale factor. So, if one compares (18) with the above relation, and then uses relation (10), one gets

$$\frac{8\pi G}{3} \rho_m + \frac{\Lambda c^2}{3} \propto a^{-\frac{2}{3}} \quad (36)$$

which directly implies

$$\gamma_m = 1 \quad (37)$$
The above result corresponds to dust, i.e., \( p_m = 0 \). This simplifies the relations \((33)\) and \((34)\) as

\[
\Lambda = \Lambda_0 \left( \frac{a}{a_0} \right)^{-2}
\]

where

\[
\Lambda_0 = \frac{4\pi G}{c^4} \rho_{m0}
\]

and hence

\[
\Lambda = \Lambda_0 \left( \frac{t}{t_0} \right)^{-\frac{8}{5}}
\]

It is worth noting that the splitting of the total mass of the Universe into \( T_{ab}^m \), the covariant divergence of which vanishes, and a \( \Lambda \) term that varies with the cosmological time, is not unique. That is, \( \rho_m \) is not fixed, and strictly speaking, it depends on how one splits the mass and energy content of the Universe between \( \rho_m \) and \( \Lambda \).

### 5. THE FIELD EQUATION WITH AN ADDITIONAL ENERGY-MOMENTUM TENSOR

A plausible argument for considering the previous section is that the term \( \Lambda \) has been recently used in these types of contexts. However, a better way to consider that section may be to introduce a new energy-momentum tensor, \( T_{ab}^\phi \). In other words, one can split the total energy-momentum in the Einstein equation, \( G_{ab} = \frac{8\pi G}{c^4} T_{ab} \), as

\[
T_{ab} = T_{ab}^m + T_{ab}^\phi
\]

That is, to consider

\[
G_{ab} = \frac{8\pi G}{c^4} (T_{ab}^m + T_{ab}^\phi)
\]

where again, by \( T_{ab}^m \) we mean

\[
\nabla_a T_{ab}^m = 0
\]

Using the Robertson-Walker metric and \( T_{ab}^m \) as a perfect fluid, relation \((30)\) is still valid. In addition, for simplicity, we assume \( T_{ab}^\phi \) to be a perfect fluid too, with the usual equation of state

\[
p_\phi = (\gamma_\phi - 1)\rho_\phi c^2
\]

Applying the covariant divergence to both sides of \((42)\) leads to

\[
\partial_a \left( \frac{1}{c^4} \right) (T_{ab}^m + T_{ab}^\phi) + \frac{1}{c^4} \nabla_a T_{ab}^\phi = 0
\]

Simplification of the zero-component of the above relation, using the Robertson-Walker metric, gives

\[
\dot{\rho}_\phi + \left( 3\gamma_\phi \frac{\dot{a}}{a} - 2\frac{\dot{c}}{c} \right) \rho_\phi - 4\frac{\dot{c}}{c} \rho_m = 0
\]

Using the relation \((8)\) and assuming a plausible assumption that \( \rho_\phi \propto a^\ell \) results in

\[
\rho_\phi = \frac{1}{3\gamma_\phi + \frac{3}{2} + \ell} \rho_m
\]

i.e., \( \rho_m \) and \( \rho_\phi \) are in the same order of \( a \), and consequently from \((30)\), one gets

\[
\ell = \frac{1}{2} - 3\gamma_m
\]
The relation (42) gives
\[
\left( \frac{\dot{a}}{a} \right)^2 + \frac{k c^2}{a^2} = \frac{8 \pi G}{3} (\rho_m + \rho_\phi) \tag{49}
\]
and
\[
\frac{\ddot{a}}{a} + \frac{1}{4} \left( \frac{\dot{a}}{a} \right)^2 = -\frac{4 \pi G}{3} \left[ (3 \gamma_m - 2) \rho_m + (3 \gamma_\phi - 2) \rho_\phi \right] \tag{50}
\]
The total density, \( \rho \), calculated in relation (10) is the sum of the densities \( \rho_m \) and \( \rho_\phi \). Hence, regarding relation (47), it implies that
\[
\rho_\phi \propto \rho_m \propto a^{-\frac{5}{2}} \tag{51}
\]
Therefore, we get exactly
\[
\rho_\phi = \frac{1}{2 - 3 \gamma_\phi} \rho_m \tag{52}
\]
where \( \gamma_\phi \neq \frac{2}{3} \), and
\[
\gamma_m = 1 \tag{53}
\]
which, as in the previous section, shows that \( T^\mu_\nu \) corresponds to dust. Besides, using
\[
p = p_\phi + p_m = p_\phi = (\gamma_\phi - 1) \rho_\phi c^2 \tag{54}
\]
and
\[
p = (\gamma - 1) \rho c^2 = (\gamma - 1)(\rho_m + \rho_\phi)c^2 \tag{55}
\]
and relation (52) gives \( \gamma = \frac{2}{3} \) as expected and as derived in ref. [16]. Moreover, this is also consistent with why \( \gamma_\phi \neq \frac{2}{3} \).

Substituting relations (52) and (53) in relation (50) makes its right-hand side vanish automatically and does not give a new relation. As is discussed at the end of the Sect. 4, \( \rho_m \) is not unique. In addition, relation (52) shows a freedom to choose \( \rho_\phi \) and \( \gamma_\phi \), even when \( \rho_m \) is completely definite. That is, the value of \( \gamma_\phi \) cannot be fixed by these formulae and it should perhaps be determined from its equation of state.

In a special case, when one sets
\[
\rho_\phi = \frac{\Lambda c^2}{8 \pi G} = \rho_\Lambda \tag{56}
\]
the relations (8), (38), (39), (51) and (52) give
\[
\gamma_\phi = 0 \tag{57}
\]
which corresponds to a negative pressure
\[
p_\phi = -\rho_\phi c^2 = -\frac{1}{2} \rho_m c^2 \tag{58}
\]
or
\[
p_\Lambda = -\rho_\Lambda c^2 \tag{59}
\]
of the previous section.
6. SUMMARY AND DISCUSSION

We summarize the dynamics of the Universe and other results of this model. The relations (21) and (23) imply that the Universe began from a hot Big Bang and expands forever, even if \( k = +1 \). In addition, one has from (20) that

\[
\frac{\ddot{a}}{a} = -\frac{1}{4} \left( \frac{\dot{a}}{a} \right)^2 = -\frac{1}{4} H^2
\]

This relation asserts that the Universe always decelerates with a constant deceleration parameter \( q \equiv -\frac{\ddot{a}}{\dot{a}^2} = \frac{1}{k} \).

From measurements, the Hubble constant is given as

\[
H_0 = 100h \frac{km}{s} \frac{1}{Mpc}
\]

where \( 0.55 \leq h \leq 0.8 \) (61)

Hence, by relation (22), the age of the Universe should be

\[
9.78 \times 10^9 \text{ yr} < t_0 < 14.22 \times 10^9 \text{ yr}
\]

in this model. Generally, from relation (21), one gets

\[
t = \frac{4}{5} \frac{1}{H}
\]

One can also calculate the particle horizon as

\[
\int_{r_0}^{r} \frac{dr}{\sqrt{1 - kr^2}} = \int_{0}^{t_0} \frac{c(t)dt}{a(t)} = \int_{0}^{t_0} \frac{c_0 t_0 dt}{a_0} = \frac{c_0 t_0}{a_0} \ln \left( \frac{t}{t_0} \right)
\]

which is obviously infinite. Therefore, the observed radius of the Universe is actually the current radius of the whole Universe. This result means that all parts of the Universe are causally connected and leads to an idea to estimate the current radius of the Universe. From relations (7), (21) and (23), one can get

\[
dR = \frac{4}{5} R_0 t_0^{-\frac{4}{5}} t^{-\frac{1}{5}} dt
\]

and

\[
cdt = c_0 t_0 \frac{1}{5} t^{-\frac{4}{5}} dt
\]

As can easily be seen, both the above relations have the same time dependency, \( t^{-\frac{1}{5}} \), in agreement with the above assertion. To hold the causality, one can equate the relations (65) and (66) and find

\[
R_0 = \frac{5}{4} c_0 t_0
\]

Substituting relation (22) in the above relation, it yields

\[
c_0 = H_0 R_0
\]

or, using the relations (7), (21), (22), (23) and (63), one obtains the general relation

\[
c(t) = H(t) R(t)
\]

This can be regarded as a characteristic equation of the Universe in this model.

The mean value of (62) is \( t_0 \approx 12 \) Gyr, from the relation (67), one gets

\[
R_0 \approx 15 \text{ Gly}
\]

where the light-year unit is based on \( c_0 \). However, there is no contradiction, for \( c \) is time dependent and its value at previous epochs was greater than now.
If one tries to construct a VSL-type theory, endowed with the Robertson-Walker metric, in which the Universe is causally connected, then, if \( R \propto a \propto t^\lambda \) and \( c \propto t^\nu \), one must have

\[
\lambda = \nu + 1 \quad (71)
\]

Assuming that \( \rho c^2 \propto a^{-3} \) and the Universe, at least for large \( t \), is not curvature-dominated, from the first Friedmann equation \([18]\), one can easily find that \( \lambda = \frac{4}{5} \) and \( \nu = -\frac{1}{5} \). This is exactly what has been derived in this model.

The total current mass of the Universe can be calculated from relation \([5]\) as

\[
M_0 = \frac{R_0 c_0^2}{G} \sim 10^{53}\text{kg} \quad (72)
\]

where the traditional value of the gravitational constant has been used. Consequently, one obtains the energy content of the Universe as

\[
E_0 = M_0 c_0^2 \sim 10^{70}\text{J} \quad (73)
\]

which is a constant all over the life of the Universe by our first assumption.

To evaluate the current density of the Universe, one can obviously suppose that the volume of it, when its radius is \( R_0 \), is

\[
V_0 = \frac{4}{3} \pi \beta R_0^3 \quad (74)
\]

where \( \beta \) is introduced to present the geometry of the Universe. In other words, \( 0 < \beta < 1 \), \( \beta = 1 \), and \( \beta > 1 \) correspond to \( k = +1 \), \( k = 0 \), and \( k = -1 \), respectively. Now, by relations \([72]\) and \([74]\), one has

\[
\rho_0 = \frac{3c_0^2}{4\pi \beta G R_0^2} \quad (75)
\]

On the other hand, one can rewrite the first Friedmann equation \([18]\) as

\[
\Omega_0 - 1 = \frac{kR_0^2}{a_0^2 H_0^2} \quad (76)
\]

where \( \Omega_0 \equiv \frac{\rho_0}{\rho_c} \) and \( \rho_c = \frac{3H_0^2}{8\pi G} \). Substituting relation \([75]\) in the above relation and using relation \([68]\), it yields

\[
\frac{2}{\beta} - 1 = k \left( \frac{R_0}{a_0} \right)^2 \quad (77)
\]

Hence, the only possible values of \( \beta \), in agreement with the sign of \( k \), are

\[
\begin{cases} 
0 < \beta < 1 & \text{where } k = +1 \quad \text{& } \Omega_0 > 2 \\
\beta > 2 & \text{where } k = -1 \quad \text{& } \Omega_0 < 1 
\end{cases} \quad (78)
\]

Note that, no exact flat geometry is permitted in this model. In addition, considering the current cosmological data, the second possibility, i.e., the negative curvature, seems more probable. Introducing \( \Omega_k \equiv -k \left( \frac{R_0}{a_0} \right)^2 \), one can also write \([77]\) as

\[
\Omega_0 + \Omega_k = 1 \quad (79)
\]

where, both \( \Omega_0 \) and \( \Omega_k \) are positive constants according to the above discussion.

A more complete model, in which some other fundamental physical constants are supposed to vary, will be introduced later \([20]\).

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