Effects of Finite Top Lifetime at the $t\bar{t}$ Threshold

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In this talk electroweak corrections related to the finite top quark lifetime to the total top pair threshold $e^+e^-$ cross section at NNLL order are discussed. We include the absorptive parts in electroweak matching conditions of the NRQCD operators related to the top decay and use the optical theorem. Gauge invariance is maintained. The corrections lead to ultraviolet phase space divergences and NLL mixing effects. The corrections can amount to several percent and are phenomenologically relevant.

1. Introduction

Because in the Standard Model the top quark width $\Gamma_t \approx 1.5$ GeV is much larger than the typical hadronization energy $\Lambda_{\text{QCD}}$, the total cross section $\sigma(e^+e^- \to t\bar{t})$ in the threshold region $\sqrt{s} \approx 2m_t$ can be computed with perturbative methods to very high precision. The rapid rise of the cross section in the threshold region will allow for a measurement of the top quark mass (in a threshold mass scheme) with experimental and theoretical uncertainties at the level of only 100 MeV. Other parameters such as $\alpha_s$, $\Gamma_t$ or (if the Higgs boson is light) the top Yukawa coupling $g_{tth}$ can also be determined if the normalization and the exact form of the line-shape can be computed with small theoretical errors at the percent level. This is required because the observable cross section is a convolution of the theory prediction with the partly machine-dependent $e^+e^-$ luminosity spectrum.

The theoretical instrument to make first principles predictions for top threshold observables is based on NRQCD, an effective theory (EFT) of QCD for heavy quark pairs with small relative velocity $v \sim \alpha_s \ll 1$. Within “velocity” NRQCD (vNRQCD), which we will use here, it is possible to sum Coulomb singularities and (using renormalization group methods) logarithms of $v$ to all orders of QCD perturbation theory while a systematic and coherent $v$ power counting is maintained. The summation of logarithms avoids large normalization uncertainties that were obtained in earlier fixed-order predictions. At NNLL order (i.e. including corrections of order $v^2$ and the summation of terms $\alpha_s^2(\alpha_s \ln v)^n$) all QCD ingredients for the threshold cross section are presently known except for still missing subleading mixing effects in the running of the heavy quark pair production current. The current normalization QCD uncertainties of the total cross section are estimated to be around 6%.

In this talk we are interested in electroweak effects. At leading order, the three basic electroweak effects are the $t\bar{t}$ production process itself, the finite top lifetime and the luminosity spectrum mentioned above. The latter is accounted for in the experimental simulations, the two former effects are described by NRQCD. Here we want to discuss finite top lifetime effects at the subleading level including QCD interference effects. Previous partial analyses have indicated that the corrections can reach the level of a few percent. In particular, we investigate the role of absorptive parts related to the top quark decay in electroweak loop corrections to the NRQCD matching conditions that contribute to the NNLL total cross section. Interestingly, these corrections lead to UV phase space divergences that would not exist for stable top quarks and that lead to a NLL renormalization of $(e^+e^-)(e^+e^-)$ operators that also contributes to the total cross section. For details of our analysis we refer the interested reader to Ref. [10].

2. Power Counting and Matching Conditions

Let us first recall the power counting to classify the order at which electroweak effects can contribute by considering the matching conditions for the vNRQCD bilinear quark field operators. Electroweak corrections are obtained by
matching 2-point functions in the effective theory to those in QCD and the electroweak theory. The NNLL result including also all QCD contributions has the form

$$\mathcal{L}_{\text{bilinear}}(x) = \sum_p \bar{\psi}_p^i(x) \left\{ i D^\mu - \frac{(p - i D)\gamma^\mu}{2m_t} + \frac{p^4}{8m_t^2} + \frac{i}{2} \Gamma_\ell \left( 1 - \frac{p^2}{2m_t^2} \right) - \delta m_t \right\} \psi_p^i(x) + (\psi_p \to \chi_p),$$

(1)

where the fields $\psi_p$ and $\chi_p$ destroy top and antitop quarks with momentum $p$ and positive energies, $D^\mu = (D^0, -D) = \partial^\mu + igA^\mu$ is the ultrasoft gauge covariant derivative and $\Gamma_\ell$ is the top quark width defined at the (electroweak) top quark pole. The $v$-counting is $D^0 \sim m_t v^2 \sim \Gamma_\ell \sim m_t g^2$ since the top width is of order of the typical top kinetic energy, which defines the ultrasoft scale $m_t v^2$. This leads to the counting $\bar{v} \sim \alpha_s \sim g \sim g'$ for the (SU(2) and U(1)) gauge couplings $g$ and $g'$, where we can treat the weak mixing being of order one. The term $\delta m_t$ is a residual mass term of order $v^2$ that arises within threshold mass schemes. Its electroweak contributions are straightforward to compute and will not be discussed here further. The term $i \Gamma_\ell (1 - p^2/2m_t^2)$ describes the finite lifetime at LL order and the NNLL time dilatation effects. It produces the known replacement rule $E \to E + i \Gamma_\ell$. To account for finite lifetime effects at LL order. Although it renders the effective Lagrangian formally non-hermitian (since hermitian conjugation does not change its sign), the EFT inherits unitarity from the underlying theory, so we can later use the optical theorem to compute the total rates. It is crucial to understand that the effects of the top decay represent hard physics that can be integrated out (i.e. treated by the matching condition $i \Gamma_\ell$) and that, therefore, one can determine the corresponding EFT matching conditions for on-shell top quark amplitudes. This is analogous to the treatment of photons in an absorptive medium in the optical theory. So to account for all NNLL order electroweak effects to Eq. 2 one still has to include the one-loop electroweak and the $O(\alpha_s)$ and $O(\alpha_s^2)$ QCD corrections to the on-shell top width $\Gamma_t$ which will also not be discussed here any further.

Concerning the $t\bar{t}$ interaction at LL order we only need to consider the Coulomb potential, $\mathcal{L}_{\text{pot}} = \sum_{p,p'} \frac{ieC_{\gamma A} v(m_t)}{(p - p')^2} \bar{\psi}_p^i \gamma^\mu \psi_{p'}^j \chi_{-p'}^i \gamma_\mu \chi_{-p'}$, where $v$ is the vNRQCD renormalization scaling parameter. Since effects from the top decay are hard ($\sim m_t$), we can neglect the momentum exchange $p - p'$ in the electroweak loops during the matching computation. From this it is easy to see that all the $g^2$ (vertex and wave function) corrections to the Coulomb potential cancel due to SU(3) gauge invariance. Since all electroweak corrections to the $1/m_t$ suppressed potentials are beyond NNLL order simply by counting powers of $v$ and $g$, we also don’t have to consider any electroweak corrections to these potentials. The same argument applies to electroweak corrections to the interactions of top quarks with the soft gluons ($k \sim m_t v$) or potential four-quark operators caused by electroweak box diagrams; the former can only contribute to $O(\alpha_s)$ corrections to the potentials, and the latter are already $1/m_t^2$ suppressed without even accounting for the powers of $g$. Moreover, the Coulomb interaction does not generate any UV divergences, so electroweak NNLL contributions due to mixing also do not exist.

The dominant operators used to describe $t\bar{t}$ spin-triplet production have the form

$$O_{V,p} = \left[ \bar{e} \gamma_j e \right] O_{p,1}^j, \quad O_{A,p} = \left[ \bar{e} \gamma_j \gamma_5 e \right] O_{p,1}^j,$$

(2)

where $O_{p,1}^j = \left[ \psi_p^i \sigma_j (i\sigma_2) \chi_{-p}^i \right]$. They give the contribution $\Delta \mathcal{L} = \sum_p (C_V O_{V,p} + C_A O_{A,p}) + \text{h.c.}$ to the effective theory Lagrangian where the hermitian conjugation (which gives the corresponding annihilation operators) is referring to the operators, but does not affect their Wilson coefficients. Since we neglect QED radiative corrections, the $e^\pm$ fields act like classic fields, but we need them due to electroweak gauge invariance. The leading order matching conditions to $C_{V/A}$ are obtained from the full theory Born diagrams with photon and Z exchange and are of order $g^2$. We will see in Sec. 3 that $C_{V/A}$ will receive imaginary matching conditions from one-loop electroweak corrections that contribute at NNLL order and play a role similar to the imaginary terms in Eq. 1.

It has been shown in Refs. 12 that up to NLL order for $\sigma_{\text{tot}}$ there are no QCD interference effects from (ultrasoft) gluon radiation off the top/antitop quark or its decay products. (For this consideration only ultrasoft gluons are relevant because they cannot kick a top quark off-shell.) The proof was conducted by explicit computation of diagrams. Using the analysis of matching conditions we can show that this statement even holds at NNLL order. For the time-like ultrasoft $A^0$ gluons, QCD gauge invariance ensures that the dominant electroweak matching corrections to the $A^0$ interaction vertex shown in Eq. 1 vanish because we can again approximate the ultrasoft gluon momentum

0402
Figure 1: (a) Full theory diagrams in Feynman gauge needed to determine the electroweak absorptive parts in the Wilson coefficients $C_{V/A}$ related to the physical $bW^+$ and $\bar{b}W^-$ intermediate states. Only the $bW^+$ cut is drawn explicitly. (b) Full theory diagrams describing the process $e^+e^- \rightarrow bW^+\bar{b}W^-$ with one or two intermediate top or antitop quark propagators. The circle in the first diagram represents the QCD form factors for the $t\bar{t}$ vector/axial-vector currents.

In the electroweak loop as zero. Moreover, the exchange of time-like ultrasoft $A^0$ gluons does itself not contribute at LL because they can be removed from the LL particle-antiparticle sector in Eq. (1) by a redefinition of the top and antitop fields related to static Wilson lines [13]. An anomalous interaction in analogy to the $g-2$ is not covered by this argument but suppressed by a factor $1/m_t^2$. Accounting also for powers of $g$ one finds that interference from $A^0$ gluons does not contribute at NNLL order. For the space-like ultrasoft $A$ gluons the conclusion is the same because they couple to the quarks with the $p.A/m_t$ coupling. The $g^2$-suppression from an additional electroweak loop correction to the interaction vertex then leads to a contribution beyond the NNLL order.

3. Absorptive Matching Conditions

To determine the absorptive parts related to the top decay of the matching conditions of the operators $O_{V,P}$ and $O_{A,P}$ that contribute to the total cross section at NNLL order, we have to consider the $bW^+$ and $\bar{b}W^-$ cuts of the full theory diagrams shown in Fig.1. To obtain the contributions at this order the external (on-shell) top quarks can be taken to be at rest. The full theory amplitude has the form

$$A = i \left[ \bar{u}_{e^+}(k') \gamma^\mu (iC_{V}^{bW,abs} + iC_{A}^{bW,abs} \gamma_5) u_{e^-}(k) \right] \left[ \bar{u}_t(p) \gamma_\mu v_t(p) \right], \quad (3)$$

where $k + k' = 2p = (2m_t, 0)$. The amplitude of the charge conjugated process describing top pair annihilation reads

$$\tilde{A} = i \left[ \bar{u}_{e^-}(k) \gamma^\mu (iC_{V}^{bW,abs} + iC_{A}^{bW,abs} \gamma_5) v_{e^+}(k') \right] \left[ \bar{v}_t(p) \gamma_\mu u_t(p) \right]. \quad (4)$$

We used the cutting equations to obtain expressions for $A_{V,P}^{bW,abs}$ and checked electroweak gauge invariance by carrying out the computation in unitary and Feynman gauge. Analytic formulae are provided in Ref. [13]. The results are consistent with results obtained earlier in Ref. [8]. It is a consequence of the unitarity of the underlying theory that the sign of the imaginary part of the amplitude does not change in the charge conjugated amplitude. It is straightforward to match the amplitudes for the operators $O_{V/A,P}$ and $O_{V/A,P}^{\dagger}$ to the full theory results in Eqs. (3) and (4). The resulting matching conditions for the operators $O_{V/A,P}$ and $O_{V/A,P}^{\dagger}$ read

$$C_{V/A}(\nu = 1) = C_{V/A}^{\text{born}} + i C_{V/A}^{bW,abs}, \quad (5)$$

where we have also indicated the Born level contributions. In a full treatment of electroweak effects the coefficients $C_{V/A}$ also include the real parts of the full set of electroweak one-loop diagrams indicated in Fig.1. A comprehensive examination of these contributions will be provided in subsequent analyses.
4. Time-Ordered Product and Renormalization

Through the optical theorem the NNLL order corrections to the total cross section that come from the absorptive one-loop electroweak matching conditions for the operators $O_{V/A,p}$ and from the time dilatation corrections can be computed from the imaginary part of the $(e^+e^-)(e^+e^-)$ forward scattering amplitude,

$$\sigma_{tot} \sim \frac{1}{s} \text{Im} \left[ \left( C_{V}^{2}(\nu) + C_{A}^{2}(\nu) \right) L^{lk} A_{t}^{lk} \right],$$

where $L^{lk} = \frac{1}{2} (k + k')^2 (\delta^{lk} - \hat{e}^{l}\hat{e}^{k})$ is the spin-averaged lepton tensor and

$$A_{t}^{lk} = i \sum_{p,p'} \int d^{4}x \ e^{-i\hat{q} \cdot x} \left\langle 0 \left| T O_{p,1}^{l} (0) O_{p',1}^{k} (x) \right| 0 \right\rangle = 2 N_{c} \delta^{lk} G_{0}^{0} (a, v, m_{t}, \nu)$$

is the time-ordered product of the $t\bar{t}$ production and annihilation operators $O_{p,1}^{l}$ and $O_{p',1}^{k} \not\equiv \hat{q}$. Here we used $k + k' = (\sqrt{s}, 0)$, $\hat{e} = k/|k|$ and $\hat{q} \equiv (\sqrt{s} - 2m_{t}, 0)$, $\sqrt{s}$ being the c.m. energy. The result reads

$$\Delta \sigma_{tot}^{\Gamma,1} = 2 N_{c} \text{Im} \left\{ 2 i \left[ C_{V}^{\text{born}} C_{V}^{\text{abs}} + C_{A}^{\text{born}} C_{A}^{\text{abs}} \right] G_{0}^{0} (a, v, m_{t}, \nu) + \left[ (C_{V}^{\text{born}})^{2} + (C_{A}^{\text{born}})^{2} \right] \delta G_{T}^{0} (a, v, m_{t}, \nu) \right\},$$

where $a \equiv -V_{c}^{\psi} (\nu)/4\pi = C_{F} \alpha_{s} (m_{t} \nu)$. The term $G_{0}^{0}$ is the zero-distance S-wave Green function of the non-relativistic Schrödinger equation which is obtained from the LL terms in Eqs. [1] and the Coulomb potential. In dimensional regularization it has the form

$$G_{0}^{0} (a, v, m_{t}, \nu) = \frac{m_{t}^{2}}{4\pi} \left\{ a \left[ \ln \left( -\frac{i\nu}{\nu} \right) - \frac{1}{2} + \ln 2 + 2 \gamma_{E} + \psi \left( 1 - \frac{i\nu}{2v} \right) \right] \right\} + \frac{m_{t}^{2} a}{4\pi} \frac{1}{4\epsilon},$$

where $v = ((\sqrt{s} - 2m_{t} + \delta m_{t}) + ii\Gamma_{t})/m_{t}^{1/2}$. The term $\delta G_{T}^{0}$ represents the corrections originating from the time dilatation correction in Eq. [1] and reads $\delta G_{T}^{0} (a, v, m_{t}, \nu) = -i \frac{1}{m_{t}} \left[ 1 + \frac{v}{2m_{t}} + \frac{\nu}{m_{t}} \right] G_{0}^{0} (a, v, m_{t}, \nu)$. Note that the Wilson coefficients $C_{V/A}$ do not have a LL anomalous dimension, so only the matching conditions at $\nu = 1$ appear in Eq. [5].

One can check that the terms proportional to $C_{V/A}^{\text{abs}}$ in Eq. [5] are in agreement with the full theory matrix elements from the interference between the double-resonant amplitudes for the process $e^{+}e^{-} \rightarrow t\bar{t} \rightarrow bW^{+}bW^{-}$ (first diagram in Fig. 1b) and the single-resonant amplitudes describing the processes $e^{+}e^{-} \rightarrow t + bW^{-} \rightarrow bW^{+}bW^{-}$ and $e^{+}e^{-} \rightarrow bW^{+}\bar{t} \rightarrow bW^{+}bW^{-}$ in the $t\bar{t}$ threshold limit for $m_{t} \rightarrow \infty$ (subsequent diagrams in Fig. 1b). To find literal agreement between full and effective theory matrix elements one has to replace the $ic\nu$ terms in the resonant full theory top propagators by the Breit-Wigner term $im_{t}\Gamma_{t}/2$. As discussed above, there are no further QCD corrections in the non-relativistic limit due to the cancellation of the QCD interference effects caused by gluons with ultrasoft momenta.

The corrections given in Eq. [5] exhibit UV $1/\epsilon$-divergences that have interesting features. Physically they arise from a logarithmic high energy behavior of the top-antitop effective theory phase space integration for matrix elements containing a single insertion of the Coulomb potential. Technically they enter the imaginary part of the forward scattering amplitude because the imaginary parts of the matching conditions of Eqs. [5] lead to a dependence on the real part of $G_{0}^{0}$ (see Eq. [9]). In the full theory this logarithmic behavior is regularized by the top quark mass. It is important that the divergences only exist because the top quark is not treated as a stable particle. In particular, the UV divergences from the time dilatation corrections arise from the Breit-Wigner-type high energy behavior of the effective theory top propagator derived from Eq. [11], which differs from the one for a stable particle. Likewise, the interference effects described by the absorptive electroweak matching conditions for the operators $O_{V,p}$ and $O_{A,p}$ would not have to be accounted for if the top quarks were stable particles. UV divergences of the same kind for the NNLL total cross section have been observed and noted before but no resolution of the issue was provided. From the point of view of having an EFT with non-hermitian contributions it is obvious that these UV divergences must be handled in the canonical way using renormalization. Since the divergences are directly related

0402
to operators with non-hermitian Wilson coefficients, the renormalization procedure will naturally involve operators with non-hermitian Wilson coefficients.

The operators that are renormalized by the UV divergences displayed in Eq. (8) are the two \((e^+e^-)(e^+e^-)\) operators

\[
\hat{O}_V = -\left[ \bar{e} \gamma^\mu e \right] \bar{e} \gamma_\mu e, \quad \hat{O}_A = -\left[ \bar{e} \gamma^\mu \gamma_5 e \right] \bar{e} \gamma_\mu \gamma_5 e,
\]

which give the additional contribution \(\Delta L = \hat{C}_V \hat{O}_V + \hat{C}_A \hat{O}_A\) to the effective theory Lagrangian, \(\hat{C}_{V/A}\) being the Wilson coefficients. Because in this work we neglect QED effects, the electron and positron act as classic fields and therefore \(\hat{C}_V\) and \(\hat{C}_A\) run only through mixing due to UV divergences such as in Eq. (8). Since only the imaginary parts of the coefficients \(\hat{C}_{V/A}\) are relevant for the discussion, we neglect the real contributions in the following. Using the standard \(\overline{\text{MS}}\) subtraction procedure to determine the (non-hermitian) counterterms of the renormalized \(\hat{C}_{V/A}\) operators and standard methods to compute and to solve the anomalous dimensions, one obtains the following form of the Wilson coefficients \(\hat{C}_{V/A}\) for scales below \(m_t\) \((\nu < 1)\),

\[
\hat{C}_{V/A}(\nu) = \hat{C}_{V/A}(1) + i \frac{2N_c m_t^2 C_F}{3\beta_0} \left[ \left( C_{V/A}^{\text{born}} \right)^2 + \left( C_{V/A}^{\text{W}} \right)^2 \right] \frac{\Gamma_t}{m_t} + \frac{4 C_{V/A}^{\text{born}} C_{V/A}^{\text{W,abs}}}{\rho(z)} \ln(z) - \frac{4 C_F}{\beta_0} \left( C_{V/A}^{\text{born}} \right)^2 \ln^2(z) + \frac{4(C_A + 2 C_F)}{\beta_0} \left( C_{V/A}^{\text{born}} \right)^2 \rho(z),
\]

where \(z \equiv \alpha_s(m_t\nu)/\alpha_s(m_t)\), \(\rho(z) = \frac{z^2}{2} - \frac{1}{2} \ln^2(z) + \ln \ln(z) - \text{Li}_2 (\frac{1}{z})\) and the \(\hat{C}_{V/A}(1)\) are the hard matching conditions. The operators \(\hat{O}_{V/A}\) lead to an additional contribution to the total cross section of the form

\[
\Delta \sigma_{tot}^{\Gamma_2} = \text{Im} \left[ \hat{C}_V + \hat{C}_A \right].
\]

For details of the computations see Ref. [10]. Parametrically \(\Delta \sigma_{tot}^{\Gamma_2}\) is of order \(g^6\) and thus constitutes a NLL contribution as one can also see from the fact that the corresponding UV divergences were generated in NNLL order effective theory matrix elements. The correction \(\Delta \sigma_{tot}^{\Gamma_2}\) is energy-independent, but it is scale-dependent and compensates the logarithmic scale-dependence in the NNLL contribution \(\Delta \sigma_{tot}^{\Gamma_1}\). The matching conditions \(\hat{C}_{V/A}(\nu = 1)\) are presently unknown and will be analysed in subsequent work. For now we set them to zero in the numerical analysis presented below. We note that it was shown in [10] that the difference between the full theory phase space (which is cut off by the large, but finite \(m_t\)) and the effective theory phase space (which is infinite in the computation of the forward scattering amplitude) contributes to \(\hat{C}_{V/A}(\nu = 1)\) and also represents a NLL effect.

5. Numerical Analysis

In Fig. 2 we have plotted the sum of \(\Delta \sigma_{tot}^{\Gamma_1}\) and \(\Delta \sigma_{tot}^{\Gamma_2}\) in picobarn in the 1S mass scheme [9] for \(M_{1S} = 175\) GeV. the fine structure constant \(\alpha = 1/125.7, s_w^2 = 0.23120, V_{tb} = 1\) and \(M_W = 80.425\) GeV with the renormalization scaling parameter \(\nu = 0.1\) (solid curves), 0.2 (dashed curves) and 0.3 (dotted curves). For the QCD coupling we used \(\alpha_s(M_Z) = 0.118\) as an input and employed 4-loop renormalization group running. Note that in the 1S scheme \(\delta m_t = M_{1S}(V_c^{s}(\nu)/4\pi)^2/8\). For the top quark width we adopted the value \(\Gamma_t = 1.43\) GeV. Note that in a complete analysis of electroweak effects the top quark width depends on the input parameters given above and is not an independent parameter. For the purpose of the numerical analysis in this work, however, our treatment is sufficient. We find that the sum of the corrections is negative and shows a moderate \(\nu\) dependence. We find that the corrections are around \(-10\%\) for energies below the peak, between \(-2\%\) and \(-4\%\) close to the peak and about \(-2\%\) above the peak. Interestingly, they partly compensate the sizeable positive QCD corrections found in [11, 12]. The peculiar energy dependence of the corrections, caused by the dependence on the real part of the Green function \(G^0\), also leads to a slight displacement of the peak position. Relative to the peak position of the LL cross section one obtains a shift of \((30,35,47)\) MeV for \(\nu = (0.1,0.2,0.3)\). This shift is comparable to the expected experimental uncertainties of the top mass measurements from the threshold scan [2].
The sum $\Delta \sigma_{1,1}^{\Gamma_{\text{tot}}} + \Delta \sigma_{2,2}^{\Gamma_{\text{tot}}}$ in pb for $M_{\text{S}} = 175$ GeV, $\alpha = 1/125.7$, $s_{w}^{2} = 0.23120$, $V_{tb} = 1$, $M_{W} = 80.425$ GeV, $\Gamma_{t} = 1.43$ GeV and $\nu = 0.1$ (solid curves), 0.2 (dashed curves) and 0.3 (dotted curves) in the energy range $346$ GeV $< \sqrt{s} < 354$ GeV.

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