Quintessence Reconstruction of Interacting HDE in a Non-Flat Universe

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Abstract In this paper we consider quintessence reconstruction of interacting holographic dark energy in a non-flat background. As system’s IR cutoff we choose the radius of the event horizon measured on the sphere of the horizon, defined as $L = \alpha r(\tau)$. To this end we construct a quintessence model by a real, single scalar field. Evolution of the potential, $V(\phi)$, as well as the dynamics of the scalar field, $\phi$, is obtained according to the respective holographic dark energy. The reconstructed potentials show a cosmological constant behavior for the present time. We constrain the model parameters in a flat universe by using the observational data, and applying the Monte Carlo Markov chain simulation. We obtain the best fit values of the holographic dark energy model and the interacting parameters as $c = 1.0576 \pm 0.3010 \pm 0.3052$ and $\xi = 0.2433 \pm 0.6374 \pm 0.6377$, respectively. From the data fitting results we also find that the model can cross the phantom line in the present universe where the best fit value of the dark energy equation of state is $w_D = -1.2429$.

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1 Introduction

A wide range of observational evidences support the present acceleration of the Universe expansion. The first evidence for the mentioned acceleration is the cosmological observations from Type Ia supernovae (SN Ia).[1] Subsequently such acceleration was repeatedly confirmed by Cosmic Microwave Background (CMB) anisotropies measured by the WMAP satellite.[2] Large Scale Structure,[3] weak lensing,[4] and the integrated Sach–Wolfe effect.[5] Based on the Einstein’s theory of gravity, such an acceleration needs an exotic type of matter with negative pressure, usually called dark energy (DE) in the literatures. This new component consists more than 70% of the present energy content of the universe. The simplest alternative which can explain the phase of acceleration is the so called cosmological constant, which originally was presented by Einstein to build a static solution for the universe in the context of general relativity. Although cosmological constant can explain the acceleration of the universe but it suffers the “fine tuning” and “coincidence” problems. Of interesting models of DE are those which called scalar field models. A typical property of these models is their time varying equation of state parameter ($w = P/\rho$) favored by cosmic observations.[6–8] A plenty of these models have been presented in the literature which an incomplete list is quintessence, tachyon, K-essence, agegraphic, ghost and so on (see Refs.[9–11] and references therein).

Among different candidate to DE, holographic dark energy (HDE) is one which contains interesting features. This model is based on the holographic principle which states that the entropy of area[12] and it should be constrained by an infrared cutoff.[13] Applying such a principle to the DE issue and taking the whole universe into account, then the vacuum energy related to this holographic principle is viewed as DE, usually called HDE.[13–15] According to these statements the holographic energy density can be written as[13]

$$\rho_D = \frac{3c^2M_p^2}{L^2}, \quad (1)$$

where $c^2$ is a numerical constant, $M_p^2 = (8\pi G)^{-1}$, and $L$ is an infrared (IR) cutoff radius. It is worth mentioning that the holographic principle does not determine the IR cutoff and we have still freedom to choose $L$. Different choices for IR cutoff parameter, $L$, have been proposed in the literature, among them are, the particle horizon,[16] the future event horizon,[17] the Hubble horizon,[18–19] and the apparent horizon.[20] Each of these choices solves some features and leads new problems. For instance in the HDE model with Hubble horizon, the fine tuning problem is solved and the coincidence problem is also alleviated, however, the effective equation of state for such vacuum energy is zero and the universe is decelerating[18] unless the interaction is taken into account.[19] For a complete list of papers concerning HDE one can refer to Ref.[21] and references therein.

Nowadays, every model which can explain the accel-
eration of the Universe expansion, and is consistent with observational evidences, could be accepted as a DE candidate. Due to the lack of observational evidences about DE models, many approaches are presented to answer the puzzle of the unexpected acceleration of the Universe. The number and variety of these models are so increasing which we should classify them in any way. One main task in this way is to find equivalent theories presented in different frameworks, however, they seem to have distinct origins. One valuable approach which recently has attracted a lot of attention is to make scalar field dual to DE models partly comes from the fact that scalar fields naturally arise in particle physics including supersymmetric field theories and string/M theory. Beside with clarifying the status of the models in the literature maybe sometimes we can use the corresponding scalar field dual of DE model predicting new features and setting observational constraints on the free parameters.

In this paper our aim is to establish a correspondence between the HDE and quintessence model of DE in a non-flat universe. As system’s IR cutoff we shall choose the radius of the event horizon measured on the sphere of the horizon, defined as $L = ar(t)$. Quintessence assumes a canonical scalar field $\phi$ and a self interacting potential $V(\phi)$ minimally coupled to the other component in the universe. Quintessence is described by the Lagrangian of the form

$$L = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi).$$

(2)

The energy-momentum tensor of quintessence is

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[ \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi) \right].$$

(3)

In quintessence model we choose a convenient potential $V(\phi)$ to obtain desirable result in agreement with observations. Hence, our goal in this paper is to reconstruct the potential $V(\phi)$ corresponds to the HDE and investigate the evolution of different parameters in the model. Our work differs from Ref. [22] in that we consider the interacting HDE model in a non-flat universe, while the author of Ref. [22] studied the non-interacting case in a flat universe. It also differs from Refs. [27–28], in that we take $L = ar(t)$ as system’s IR cutoff not the Hubble radius $L = H^{-1}$ proposed in Ref. [27], nor the Ricci scalar like cutoff, $L^{-2} = \alpha H^2 + \beta \dot{H}$, introduced in Ref. [28].

This paper is organized as follows. In Sec. 2, we reconstruct the non-interacting holographic quintessence model with $L = ar(t)$ as IR cutoff. In Sec. 3, we extend our study to the case where there is an interaction between DE and dark matter. In order to check the viability of the model, in Sec. 4, we constrain the holographic interacting quintessence model by using the cosmological data. We summarize our results in Sec. 5.

2 Quintessence Reconstruction of HDE

Consider the non-flat Friedmann–Robertson–Walker (FRW) universe which is described by the line element

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right),$$

(4)

where $a(t)$ is the scale factor, and $k$ is the curvature parameter with $k = -1, 0, 1$ corresponding to open, flat, and closed universes, respectively. The first Friedmann equation is

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} (\rho_m + \rho_D).$$

(5)

We introduce, as usual, the fractional energy densities such as

$$\Omega_m = \frac{\rho_m}{3M_p^2 H^2}, \quad \Omega_D = \frac{\rho_D}{3M_p^2 H^2}, \quad \Omega_k = \frac{k}{H^2 a^2},$$

(6)

thus, the Friedmann equation can be written as

$$\Omega_m + \Omega_D = 1 + \Omega_k.$$ 

(7)

We shall assume the quintessence scalar field model of DE is the effective underlying theory. The energy density and pressure for the quintessence scalar field are given by

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi),$$

(8)

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi).$$

(9)

Thus the potential and the kinetic energy term can be written as

$$V(\phi) = \frac{1 - w_D}{2} \rho_\phi,$$

(10)

$$\dot{\phi}^2 = (1 + w_D) \rho_\phi.$$ 

(11)

Next we implement the HDE model with quintessence field. The holographic energy density has the form (1), where the radius $L$ in a non-flat universe is chosen as

$$L = ar(t),$$

(12)

and the function $r(t)$ can be obtained from the following relation

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^{\infty} \frac{dt}{a} = \frac{R_h}{a}.$$ 

(13)

It is important to note that in the non-flat universe the characteristic length, which plays the role of the IR-cutoff is the radius $L$ of the event horizon measured on the sphere of the horizon and not the radial size $R_h$ of the horizon. Solving the above equation for general case of the non-flat FRW universe, we have

$$r(t) = \frac{1}{\sqrt{k}} \sin y,$$

(14)

where $y = \sqrt{k} R_h / a$. For latter convenience we rewrite the second Eq. (6) in the form

$$HL = \frac{c}{\sqrt{\Omega_D}}.$$ 

(15)

Taking derivative with respect to the cosmic time $t$ from Eq. (12) and using Eqs. (14) and (15) we obtain

$$\dot{L} = HL + ar(t) = \frac{c}{\sqrt{\Omega_D}} \cos y.$$ 

(16)

Consider the FRW universe filled with DE and dust (dark matter), which evolves according to their conservation laws

$$\dot{\rho}_D + 3H \rho_D (1 + w_D) = 0,$$

(17)

$$\dot{\rho}_m + 3H \rho_m = 0,$$ 

(18)
where \( w_{\text{D}} \) is the equation of state parameter of DE. Taking the derivative of Eq. (1) with respect to time and using Eq. (16) we find
\[
\dot{\rho}_{\text{D}} = -2H\rho_{\text{D}} \left( 1 - \frac{\sqrt{\Omega_{\text{D}}} c \cos y}{3c} \right). \tag{19}
\]
Inserting this equation in conservation law (17), we obtain
\[
w_{\text{D}} = \frac{1}{3} - \frac{2\sqrt{\Omega_{\text{D}}} c \cos y}{3c}. \tag{20}
\]
Differentiating Eq. (15) and using relation \( \dot{\Omega}_{\text{D}} = \Omega_{\text{D}}' H \), we reach
\[
\Omega_{\text{D}}' = \Omega_{\text{D}} \left( -2 \frac{\dot{H}}{H^2} - 2 + \frac{2c}{c} \sqrt{\Omega_{\text{D}}} \cos y \right), \tag{21}
\]
where the dot and the prime denote the derivative with respect to the cosmic time and \( x = \ln a \), respectively. Taking the derivative of both side of the Friedmann equation (5) with respect to the cosmic time, and using Eqs. (7), (15), (17), and (18), it is easy to show that
\[
\frac{2\dot{H}}{H^2} = -3 - \Omega_k + \Omega_{\text{D}} + \frac{2\Omega_{\text{D}}^{3/2}}{c} \cos y. \tag{22}
\]
Substituting this relation into Eq. (21), we obtain the equation of motion of HDE
\[
\Omega_{\text{D}}' = \Omega_{\text{D}} \left[ (1 - \Omega_{\text{D}}) \left( 1 + \frac{2}{c} \sqrt{\Omega_{\text{D}}} \cos y \right) + \Omega_k \right]. \tag{23}
\]
We have plotted in Figs. 1 and 2 the evolutions of the \( w_{\text{D}} \) and \( \Omega_{\text{D}} \) for the HDE with different parameter \( c \). One can see from Fig. 1 that increasing \( c \) leads to a faster evolution of \( w_{\text{D}} \) toward more negative values, while a reverse behavior is seen for \( \Omega_{\text{D}} \) and increasing \( c \) results a slower evolution of \( \Omega_{\text{D}} \).

Now we suggest a correspondence between the HDE and quintessence scalar field namely, we identify \( \rho_{\phi} \) with \( \rho_{\text{D}} \). Using relation
\[
\rho_{\phi} = \rho_{\text{D}} = 3M_p^2H^2\Omega_{\text{D}}
\]
and Eq. (20) we can rewrite the scalar potential and kinetic energy term as
\[
V(\phi) = M_p^2H^2\Omega_{\text{D}} \left( 2 + \frac{\sqrt{\Omega_{\text{D}}} c \cos y}{c} \right), \tag{24}
\]
\[
\dot{\phi} = M_pH \left( 2\Omega_{\text{D}} - \frac{2}{c} \Omega_{\text{D}}^{3/2} \cos y \right)^{1/2}. \tag{25}
\]
Finally we obtain the evolutionary form of the field by...
integrating the above equation

\[ \phi(a) - \phi(a_0) = M_p \int_{a_0}^{a} \frac{da}{a} \sqrt{2 \Omega_D - \frac{3}{c} \Omega_D^{3/2} \cos y}, \]

where \( a_0 \) is the present value of the scale factor, and \( \Omega_D \) is given by Eq. (31). Basically, from Eqs. (31) and (34) one can derive \( \phi = \phi(a) \) and then combining the result with Eq. (32) one finds \( V = V(\phi) \). Unfortunately, the analytical form of the potential in terms of the scalar field cannot be determined due to the complexity of the equations involved. However, we can obtain it numerically. For simplicity we take \( \Omega_k \approx 0.01 \) fixed in the numerical discussion. The reconstructed quintessence potential \( V(\phi) \) and the evolutionary form of the field are plotted in Figs. 2, where we have taken \( \phi(a_0 = 1) = 0 \). A notable point in this figure is that the reconstructed potentials for different values of \( c \) have a nonzero value at the present time, which can be interpreted as a cosmological constant behavior of the model desirable from the perspective of ΛCDM model.

3 Quintessence Reconstruction of Interacting HDE Model

In this section, we consider the interaction between dark matter and DE. In this case the continuity equations take the form

\[
\dot{\rho}_m + 3H \rho_m = Q, \]
\[
\dot{\rho}_D + 3H \rho_D (1 + w_D) = -Q, \]

where \( Q \) denotes the interaction term and can be taken as \( Q = 3 \zeta H \rho_D (1 + u) \) with \( \zeta \) being a coupling constant and \( u = \rho_m / \rho_D \) is the energy density ratio. Inserting Eq. (19) in conservation law (28), we obtain the equation of state parameter

\[
w_D = -\frac{1}{3} - \frac{2 \sqrt{\Omega_D}}{3c} \cos y - \frac{\zeta}{\Omega_D} (1 + \Omega_k). \]

Substituting this relation into Eq. (21), we obtain the equation of motion of HDE

\[
\Omega_D = \Omega_D \left[ (1 - \Omega_D) \left( 1 + \frac{2}{c} \sqrt{\Omega_D} \cos y \right) - 3\zeta (1 + \Omega_k) + \Omega_k \right]. \]

We plot in Fig. 3 the evolutions of \( w_D \) and \( \Omega_D \) for interacting HDE with different parameter \( c \). Now we implement a correspondence between interacting HDE and quintessence scalar field. In this case we find

\[
V(\phi) = M_p^2 H^2 \Omega_D \left( 2 + 3\zeta (1 + \Omega_k) + \frac{\sqrt{\Omega_D}}{c} \cos y \right), \]
\[
\dot{\phi} = M_p H \left( 2 \Omega_D - 3\zeta (1 + \Omega_k) - \frac{2}{c} \Omega_D^{3/2} \cos y \right)^{1/2}. \]

Finally, the evolutionary form of the field can be obtained by integrating the above equation. We obtain

\[
\phi(a) - \phi(a_0) = M_p \int_{a_0}^{a} \frac{da}{a} \sqrt{2 \Omega_D - \frac{3}{c} \Omega_D^{3/2} \cos y - 3\zeta (1 + \Omega_k)}. \]

where \( \Omega_D \) is given by Eq. (31). Again, the analytical form of the potential in terms of the scalar field cannot be determined due to the complexity of the equations involved and we do a numerical discussion. The reconstructed quintessence potential \( V(\phi) \) and the evolutionary form of the field are plotted in Figs. 4 and 5, where we have taken \( \phi(a_0 = 1) = 0 \). For simplicity we take \( \Omega_k \approx 0.01 \) fixed in the numerical discussion. In the interacting case there exist a different manner of evolution for \( w_D \). In the previous section we found that increasing \( c \) leads a faster evolution for \( w_D \) toward more negative values while in the interacting case increasing \( c \) causes \( w_D \) to evolve toward less negative values which can predict a slower rate of expansion for the future HDE dominated universe. Also one can find from Figs. 4 and 5 that the reconstructed potentials evolve toward a nonzero minima at the present as mentioned in the noninteracting case.

![Fig. 3](https://example.com/fig3.png)  
(a) \( \zeta=0.10 \)  
(b) \( \zeta=0.10 \)  

\( \Omega_{D0} = 0.72 \) and \( \Omega_k = 0.01 \).
4 Model Fitting

In this section we will fit the interacting Quintessence HDE model, in a flat universe, by using the cosmological data. To obtain the best fit values of the model parameters, we apply the maximum likelihood method. In this method the total likelihood function \( L_{\text{total}} = e^{-\chi^2_{\text{tot}}}/2 \) can be defined as the product of the separate likelihood functions of uncorrelated observational data with

\[
\chi^2_{\text{tot}} = \chi^2_{\text{SNIa}} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} + \chi^2_{\text{gas}},
\]

where SNIa stands for type Ia supernovae, CMB for cosmic microwave background radiation, BAO for baryon acoustic oscillation and gas stands for X-ray gas mass fraction data. The detail of obtaining each \( \chi \) is discussed in Ref. [29]. Best fit values of parameters are obtained by minimizing \( \chi^2_{\text{tot}} \). In the current paper we will use CMB data from seven-year WMAP,[30] type Ia supernovae data from 557 Union2,[31] baryon acoustic oscillation data from SDSS DR7,[32] and the cluster X-ray gas mass fraction data from the Chandra X-ray observations.[33] We apply a Markov chain Monte Carlo (MCMC) simulation on the parameters of the model by using the publicly available CosmoMC code[34] and considering the parameter vector \( \{\Omega_k h^2, \Omega_{\text{DM}} h^2, \zeta\} \).

### Table 1

The best fit values of the cosmological and model parameters in the interacting Quintessence HDE model in a flat universe with 1σ and 2σ regions. Here CMB, SNIa and BAO and X-ray mass gas fraction data together with the BBN constraints have been used. For comparison, the results for the \( \Lambda \)CDM model from the Planck data are presented as well.[35]

| Parameter | Best fit value | ACMD |
|-----------|----------------|------|
| \( \Omega_k h^2 \) | 0.7021 ± 0.0034 | 0.2214 ± 0.00024 |
| \( \Omega_{\text{DM}} h^2 \) | 0.1099 ± 0.0026 | 0.1187 ± 0.0017 |
| \( \Omega_D \) | 0.7358 ± 0.0034 | 0.692 ± 0.010 |
| \( c \) | 1.0576 ± 0.0063 | – |
| \( \zeta \) | 0.2533 ± 0.0073 | – |
| \( H_0 \) | 70.77 ± 1.09 | 67.8 ± 0.77 |
| \( w_D \) | −1.2492 ± 0.4590 | –1 |
The MCMC simulation results are summarized in Table 1 and the two-dimensional contours are plotted in Fig. 6. From Table 1 it is clear that the main cosmological parameters $\Omega_b h^2$, $\Omega_{DM} h^2$, $\Omega_D$ are compatible with the results of the $\Lambda$CDM model\cite{35} as one can see from the third column in Table 1. We can see that the best fit value for the dark energy equation of state crossed the phantom line where $w_D = -1.249243^{+0.624537}_{-0.66920}$. In addition the best fit value of the HDE parameter $c = 1.0576^{+0.3010}_{-0.3052}$ is compatible with the previous numerical analysis works such as $c = 0.91^{+0.21}_{-0.18}$ in Ref. [36], $c = 0.84^{+0.14}_{-0.12}$ in Ref. [37], and $c = 0.68^{+0.03}_{-0.02}$ in Ref. [38]. Here we obtain a positive best fit value for the interacting parameter in 1$\sigma$ and 2$\sigma$ confidence levels as $\zeta = 0.2433^{+0.0373}_{-0.0251}$ in spite of we took negative values in the prior of the parameter $\zeta$ as well. This positive value suggests only conversion of dark matter to dark energy. Therefore in this model there is no chance for converting of DE to dark matter. The interacting parameter in the HDE model has been constrained by observational data by many authors although with different parametrization of the interaction term $Q$. In Ref. [44] the authors have considered the same parametrization as ours in this paper but they have chosen the prior on parameter $\zeta$ as $\zeta = [0, 0.02]$ and therefore obtained the best fit value of parameter $\zeta$ as $\zeta = 0.0006 \pm 0.0006$.

Fig. 6 2-dimensional constraint of the cosmological and model parameters contours in the flat interacting Quintessence HDE model with 1$\sigma$ and 2$\sigma$ regions. To produce these plots, Union2+CMB+BAO+X-ray gas mass fraction data together with the BBN constraints have been used.

5 Conclusions

Adopting the viewpoint that the quintessence scalar field model of DE is the effective underlying theory of DE, we are able to establish a connection between the quintessence scalar field and interacting HDE scenario. This connection allows us to reconstruct the quintessence scalar field model according to the evolutionary behavior of interacting holographic energy density. We have reconstructed the potential as well as the dynamics of the quintessence scalar field which describe the quintessence cosmology. Unfortunately, the analytical form of the potential in terms of the scalar field cannot be determined.
due to the complexity of the equations involved. However, we have plotted their evolution numerically. A close look at these figures shows several notable points. In the noninteracting case, we found that increasing $c$ leads to a faster evolution for $w_D$ toward more negative values, while in the interacting case, increasing $c$ causes $w_D$ to evolve toward less negative values which can predict a slower rate of expansion for the future HDE dominated universe. Also the evolutionary behavior of the potential, $V(\phi)$, revealed that in both interacting/noninteracting cases the potential evolves a non zero value at the present time implying a cosmological constant behavior of the model in this epoch of its evolution.

By constraining the cosmological parameters of the quintessence HDE model in a flat universe, we found that the best fit values of the main cosmological parameters $\Omega h^2$, $\Omega DM h^2$, $\Omega D$ are in agreement with the $\Lambda$CDM model as one can see from Table 1. The best fit values of the HDE parameter $c$ and interacting parameter $\zeta$ are compatible with the results of the previous constraining works on the HDE in the presence of interaction between DE and dark matter. Moreover, according to our data fitting our model can cross the phantom line in 1$\sigma$ confidence level in the present time of the Universe expansion.

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