Neutron electric dipole moment due to Higgs exchange in Left-Right symmetric models

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In this paper we study the neutron electric dipole moment (EDM) due to Higgs boson exchange in Left-Right symmetric models. In pseudo-manifest Left-Right symmetric models, the neutral Higgs contribution is smaller than that from the charged Higgs. The charged Higgs contribution at the two loop level can be as large as the experimental upper bound. In non (pseudo) manifest Left-Right symmetric models, the neutral Higgs exchange contribution can reach the experimental upper bound. The Higgs exchange contributions can be more important than the ones from W-boson exchange due to $W_L - W_R$ mixing.

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One of the outstanding problems of particle physics today is the origin of CP violation. CP violation has only been observed in the neutral kaon system, and many models have been proposed to explain it \[1\]. In order to determine the source (or sources) responsible for CP violation, it is important to find other processes which also violate CP. The measurement of the neutron EDM, \(D_n\), is a very promising area of investigation. A very stringent upper bound on the neutron EDM \(\langle D_n \rangle\) has been obtained \[2\], \(|D_n| < 1.2 \times 10^{-25} \text{ ecm}\), whereas the standard model \[3\] predicts a very small \(D_n (< 10^{-31} \text{ ecm})\). There are similarly stringent bounds on the electron \[4\] and atomic \[5\] EDMs. Assuming that the strong CP \(\theta\) parameter is negligible, if a neutron EDM within five orders of magnitude of the experimental upper bound should be detected, it signals physics beyond the standard model. In extensions of the standard model it is indeed possible to have a large neutron EDM \[6,7\]. CP violation due to Higgs exchange is an example of such models. Recently, several authors have exploited some new classes of two loop diagrams which induce a large neutron EDM \[8–12\]. In this paper we study these new contributions due to Higgs exchange in Left-Right symmetric models and compare them with the contributions from W-boson exchange due to \(W_L - W_R\) mixing \[11,13–15\]. The neutron EDM due to Higgs exchange at the one loop level in Left-Right symmetric models has been considered before \[16\]. Here we will discuss both the one loop and two loop contributions.

The gauge group of the Left-Right symmetric models is \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \[17\]. Under this group the left and right handed fermions transform as

\[
Q_L = (3, 2, 1, 1/3), \quad Q_R = (3, 1, 2, 1/3),
\]

\[
L_L = (1, 2, 1, -1), \quad L_R = (1, 1, 2, -1), \quad \tag{1}
\]

where \(Q\) and \(L\) are quarks and leptons respectively. In order to give fermion masses through the tree level Higgs-fermion couplings, at least one bi-doublet representation of Higgs boson, transforming as \(\phi = (1, 2, 2, 0)\), is needed. It can be written as

\[
\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \quad \tag{2}
\]
and its vacuum expectation value (VEV) is

\[
\langle \phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\delta} \end{pmatrix}.
\]  

(3)

In this notation, \( \phi \) transforms as \( U_L \phi U_R^\dagger \) under \( SU(2)_L \times SU(2)_R \). In order to break \( SU(2)_R \) at a higher scale, additional Higgs representations are needed. There are two traditional ways of introducing these Higgs representations,

\begin{align*}
& a) \quad H_L = (1, 2, 1, 1), \quad H_R = (1, 1, 2, 1); \\
& b) \quad \Delta_L = (1, 3, 1, 2), \quad \Delta_R = (1, 1, 3, 2).
\end{align*}

(4)

In case a) neutrinos can have only Dirac masses. In case b) neutrinos can have both Dirac and Majorana masses and the lighter neutrinos have naturally small masses due to the see-saw mechanism. For this reason, the case b) is usually favored in the literature. However, for our purposes, the two cases result in similar phenomenology. If the VEV of \( \langle H_R \rangle (\langle \Delta_R \rangle) = v_R \) is larger than \( v_1, v_2 \) and the VEV of \( \langle H_L \rangle (\langle \Delta_L \rangle) = v_L \), the symmetry breaking scales for \( SU(2)_L \) and \( SU(2)_R \) are well separated. If \( v_1 v_2 \neq 0 \), there is a mixing between \( W_L \) and \( W_R \) with a mixing angle \( \zeta \approx v_1 v_2/v_R^2 \) for a) and \( 2 v_1 v_2/v_R^2 \) for b). In the following, we shall adopt case a) for illustrative purpose whenever we need to. For simplicity we will assume \( v_L = 0 \).

In order to make this assumption consistently, it is necessary to impose additional discrete symmetries to eliminate the terms linear in \( H_L \) in the Higgs potential \([18]\).

The Higgs-quark couplings are given by

\[
L_Y = \bar{Q}_L f \phi Q_R + \bar{Q}_L h \tau_2 \phi^* \tau_2 Q_R + H.C.,
\]  

(5)

where \( f \) and \( h \) are \( 3 \times 3 \) matrices. We obtain the mass matrices for quarks

\[
M'_u = f v_1 + h v_2 e^{-i\delta}, \quad M'_d = f v_2 e^{i\delta} + h v_1,
\]  

(6)

which can be diagonalized by the following transformation:

\[
M'_u = V_L^u M_u V_R^u, \quad M'_d = V_L^d M_d V_R^d,
\]  

(7)
where $M_{u,d}$ are the diagonalized mass matrices for up and down quarks respectively. The mixing matrices for the charged currents are

$$V_L = V_L^u V_L^{d\dagger}, \quad V_R = V_R^u V_R^{d\dagger}. \quad (8)$$

In general $V_L$ and $V_R$ are independent. One can always parametrize $V_L$ in the conventional way in which there is only one CP violating phase for three generations of quarks. Then, in general, $V_R$ will have six CP violating phases. In special cases, the number of CP violating phases is reduced. For simplicity, we shall impose the following Left-Right exchange symmetry, $S$:

$$Q_L \leftrightarrow Q_R, \quad \phi \leftrightarrow \phi^\dagger, \quad (9)$$
on the Lagrangian. It implies $f = f^\dagger$ and $h = h^\dagger$. In the following we shall consider three cases.

1. CP is broken explicitly, however $\delta = 0$. In this case the mass matrices are hermitian and can be diagonalized by unitary transformations. Therefore we have

$$V_L = V_R. \quad (10)$$

We shall refer to this case as the manifest Left-Right (MLR) symmetric case. Since the phases in $V_L$ and $V_R$ can be simultaneously removed, we can assume that both are transformed into Kobayashi-Maskawa (KM) form.

2. CP is assumed to be spontaneously broken. In this case, $f$ and $h$ are real and symmetric but $\delta \neq 0$. To diagonalize a symmetric matrix it is possible to use $V_L^* = V_R$ in the bi-unitary transformation. Therefore in arbitrary basis one would have

$$V_R = J_u V_L^* J_d^*. \quad (11)$$

with

$$J_u = \text{diag}(e^{-i\alpha_u}, e^{-i\alpha_c}, e^{-i\alpha_t}), \quad J_d = \text{diag}(e^{-i\alpha_d}, e^{-i\alpha_s}, e^{-i\alpha_b}). \quad (12)$$
We shall refer to this case as the pseudo-manifest Left-Right (PMLR) symmetric case. We shall take the basis in which $V_L$ is in KM form.

3. CP is explicitly broken and $\delta$ is also nonzero. In this case there is no simply relation between $V_L$ and $V_R$. If one also does not insist on the $S$ symmetry of Eq.(9), $V_L$ and $V_R$ are completely independent. We refer to this case as the non-manifest Left-Right (NMLR) symmetric case. An interesting special case of this which produces interesting phenomenological consequences is one in which $V_R$ can be written as

$$V_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & V_{Rcs} & V_{Rcb} \\ 0 & V_{Rts} & V_{Rtb} \end{pmatrix}$$

(13)

This form maximizes the effect of the flavor changing neutral Higgs as we shall show later.

In order to study Higgs contributions to the neutron EDM, we need to find out the physical Higgs couplings to quarks. For simplicity we will choose case a) of Eq.(4) and assume that CP is broken spontaneously in case 2) or explicitly in case 3) from now on. In that case, there is one charged Higgs eigenstate $\chi^+$ which couples directly to quarks

$$\chi^+ = \frac{1}{T} \left[ (v_1^2 - v_2^2)H^+_R + v_R(v_1\phi_1^R + v_2e^{i\delta}\phi_2^R) \right].$$

(14)

where $T^2 = v^2v_R^2 + (v_1^2 - v_2^2)^2$, $v^2 = v_1^2 + v_2^2$. The charged Higgs boson associated with $H_L$ does not mix with the others because of the discrete symmetry and does not couple to fermions at all. There are three physical neutral Higgs bosons which couple to quarks. We analyze in a convenient basis, $\phi_1^0$ and $\phi_2^0$, which are linear combinations of $\phi_1^{0*}$ and $\phi_2^0$ such that $\langle \phi_1^0 \rangle \neq 0$ and $\langle \phi_2^0 \rangle = 0$. The physical neutral Higgs bosons are then expressed as linear combinations of $H_1$, $H_2$ and $H_3$. Here $H_1$ is the the real part of $\phi_1^0$ while $H_2$, $H_3$ are real and imaginary parts of $\phi_2^0$. They can be written explicitly as

$$H_1 = \cos \theta \phi_{1R} + \sin \theta \cos \delta \phi_{2R} + \sin \theta \sin \delta \phi_{2I},$$

$$H_2 = \sin \theta \cos \delta \phi_{1R} - \cos \theta \phi_{2R},$$

$$H_3 = \sin \theta \sin \delta \phi_{1R} - \cos \theta \phi_{2I},$$

(16)
\[ H_2 = -\sin \theta \phi_{1R} + \cos \theta \cos \delta \phi_{2R} + \cos \theta \sin \delta \phi_{2I}, \]  
\[ H_3 = \sin \theta \phi_{1I} - \cos \theta \sin \delta \phi_{2R} + \cos \theta \cos \delta \phi_{2I}. \]  

where \( \cos \theta = v_1/v \) and \( \sin \theta = v_2/v \), and \( \phi_{iR,L} \) denote the real and imaginary parts of \( \phi_i^0 \) respectively.

For case b) of Eq.(4), the situation is more complicated because it is harder to eliminate the term linear in \( \Delta L \) [22]. If these terms remain then \( \langle \Delta L \rangle \neq 0 \) and the singly charged Higgs boson in \( \Delta L \) will also mix with \( \phi_i^+ \) just as \( \Delta R \) does. The neutral components of \( \Delta L,R \) will also mix with \( H_i \) defined in Eq.(15) [23]. However, if \( v_R \gg v_i \) these mixings will be small. The dominant components which couple to quarks are still \( \chi^+ \approx \cos \theta \phi_{1R}^+ + \sin \theta e^{i\delta} \phi_{2R}^+ \) and \( H_i \) just as in the case a).

The neutral Higgs bosons defined in Eq.(15) are in general not mass eigenstates. However in order to simplify the discussion, we will take these particles to be mass eigenstates in the following for PMLR and NMLR models. In these two cases, the mixings in Eq.(15) already reflect the full complexity of the problem as far as the CP violating phenomenology is concern. If CP is explicitly broken in the Higgs self couplings, as is required in the case of MLR models (since \( \delta = 0 \)), the mixings between these neutral Higgs bosons are more complicated and important. We will comment on this later. The Yukawa interactions of these Higgs bosons to the quark sector are

\[
L_{\text{Yukawa}} = \left( \frac{\sqrt{2} G_F}{\cos 2\theta} \right)^{1/2} \left\{ \sqrt{2} \left[ U_L (M_u V_R - V_L M_d \sin 2\theta e^{-i\delta}) D_R 
- \bar{U}_R (V_R M_d - M_u V_L \sin 2\theta e^{-i\delta}) D_L \right] \chi^+ 
+ \bar{U}_L M_u (\cos 2\theta H_1 - \sin 2\theta H_2 + i \sin 2\theta H_3) U_R 
+ \bar{D}_L M_d (\cos 2\theta H_1 - \sin 2\theta H_2 - i \sin 2\theta H_3) D_R 
+ \bar{U}_L (V_L M_d V_R^\dagger) e^{-i\delta} (H_2 - i H_3) U_R 
+ \bar{D}_L (V_L M_u V_R^\dagger) e^{i\delta} (H_2 + i H_3) D_R \right\} + H.c.
\]  

One should note that in contrary to the multi-doublet extensions of Standard Model frequently discussed in the literature [3] the charged Higgs boson \( \chi^+ \) has right-handed couplings.
\(M_u V_R\) that are proportional to up-type quark masses, in addition to the usual left-handed ones. In particular, these new couplings depend on the \(V_R\) mixing matrix which is not severely constrained experimentally. Therefore the \(d_R\) quark can in principle have a large mixing with \(t_L\) through charged Higgs boson. This fact has been observed before \([16]\) but has not been emphasized. Similarly, in the last term the the neutral Higgs couplings is also proportional \(M_u V_R\). They are partly responsible for the large CP violating effects that we shall discuss later.

We will use the standard KM convention \([24]\) for \(V_L\) with \(\text{Im} \ V_{Ltb} = 0\). We also set \(|V_{us}|_{L,R} \approx |V_{us}|_{L,R} = 0.22, |V_{td}|_{L,R} = 0.006\), in PMLR models. In NMLR models, \(V_{Rij}\) can be different from \(V_{Lij}\). We shall assume it is of the form in Eq.(13) to maximize the effect of CP violation. For the quark masses we will use: \(m_u(1 \text{ GeV}) = 4.2 \text{MeV}, m_d(1 \text{ GeV}) = 7.5 \text{MeV}, m_s(1 \text{ GeV}) = 150 \text{MeV}, m_c(m_c) = 1.4 \text{GeV}, m_b(m_b) = 5 \text{GeV}\) and \(m_t(m_t) = 150 \text{GeV}\). Since \(m_t \gg m_b\), a natural value for \(\theta\) is \(\sin 2\theta \approx 2 \frac{m_b}{m_t}\).

The \(H_1\) boson behaves like the Higgs boson of the Standard Model. Its coupling does not mediate flavour changing neutral currents (FCNC) and does not violate CP at the tree level. But \(H_2\) and \(H_3\) do both. Note that in the usual multi-doublet extensions of the Standard Model, such FCNC-mediating Higgs bosons can be avoided by introducing a discrete symmetry \([25]\). However they are essential parts of the usual Left-Right Symmetric Models \([14]\). Therefore, in this case, instead of trying to avoid them, we shall investigate under what circumstances their effect can be large and detectable. Because \(H_2\) induces FCNC at the tree level, its mass must be sufficiently large in order not to yield a too large mass difference between \(K_L\) and \(K_S\). This consideration constrains the mass of \(H_2\) to be larger than \(8 \text{ TeV}\) \([26]\) in the MLR and PMLR models. In PMLR models with spontaneous CP violation, it was difficult to get \(\delta \neq 0\) if one used only minimal Higgs multiplets \([27]\). However it was also observed \([27]\) that such solution can indeed be obtained if one is willing to make a slight extension of Higgs sector. The lower bound on the mass derived from the absence of FCNC only applies to neutral Higgs bosons. In our estimates, for PMLR models we will use \(10 \text{ TeV}\) for neutral Higgs mass. The charged Higgs \(\chi^+\) can have a smaller mass.
When $V_L$ and $V_R$ are independent from each other, if one takes the special form of $V_R$ in Eq.(13), the experimental lower bound for the $H_2$ mass can be smaller.

We are now ready to estimate the Higgs contributions to the neutron EDM. We shall consider the following three interactions which can give important contributions to the neutron EDM,

\[
\text{the quark edm,} \quad O^\gamma = - \frac{d_q}{2} i \bar{q} \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} q , \\
\text{the quark color edm,} \quad O^C_q = - \frac{f_q}{2} i g_s \bar{q} \sigma_{\mu\nu} \gamma_5 G^{\mu\nu} q , \\
\text{the gluon color edm,} \quad O^C_g = - \frac{1}{6} C f_{abc} G^a_{\mu\nu} G^b_{\mu\alpha} \tilde{G}^c_{\nu\alpha} ,
\]

where $F^{\mu\nu}$ is the photon field strength, $G^{\mu\nu}$ is the gluon field strength and $\tilde{G}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$.

There are many ways to estimate the contributions of these operators to the neutron electric dipole moment, $D_n$. Using SU(6) relations we have [6]

\[
D_n(d_q) = \frac{1}{3} (4d_d - d_u), \quad D_n(f_q) = \frac{1}{3} \left( \frac{4}{3} f_d + \frac{2}{3} f_u \right) e . 
\]

The estimate for $O^C_q$ is more uncertain than that of $O^\gamma$. Various other estimates [10] and calculations using sum rule techniques [28] give a range between 0.05 and 1 for the ratio $D_n(f_q)/ef_q$. A recent reevaluation [29] confirms in fact the result of Eq.(18). For the contribution from $O^C_g$, we use the naive dimensional analysis (NDA) to estimate the neutron EDM [8]

\[
D_n \approx \frac{eM}{4\pi} C ,
\]

where $M = 4\pi f_\pi = 1190$MeV is the scale of chiral symmetry breaking. An alternative estimate using QCD sum-rules [30] gives a value smaller by about a factor of 30. The sum-rule result involves additional assumptions such as $\eta$ dominance and its reliability is hard to assess. However, the NDA estimate is also plagued by uncertainties, in this case an arbitrary assumption about the normalization. The comparison of these two estimates may be used as an estimate of the uncertainty in the calculation of hadronic matrix elements.

A non zero–value for $f_s$ will also generate a neutron EDM. It was estimated to give [12]
\[ D_n(f_s) \approx 0.03f_s e \ . \] (20)

As we will show later, in some scenarios, \( f_s \) can give rise to the dominant contribution.

In models of CP violation, the quark edm, \( d_q \), and the quark color edm, \( f_q \), can be generated at the one and two loop levels. The gluon edm, \( O_g^C \), are typically generated at the two loop level. The one loop contribution to \( d_d \) and \( f_d \) from the neutral Higgs boson, as shown in Fig. 1, is given by [16]

\[ d_d \approx \left( -\frac{1}{3} e \right) \frac{m_t G_F}{8\sqrt{2\pi^2}} \frac{m_t^2}{\cos^22\theta m_H^2} \ln \left( \frac{m_H^2}{m_0^2} \right) \eta_d \text{Im} \left( V_{Ld}^* V_{Rd} V_{Lt}^* V_{Rt} e^{2i\delta} \right) , \] (21)

\[ e f_d \approx -3 \frac{\eta_f}{\eta_d} d_d , \]

Note that it is assumed that the neutral Higgs couplings is dominated by the flavor changing neutral current, the last term in Eq.(16). In PMLR models, using Eqs.(11,12),

\[ \text{Im} \left( V_{Ld}^* V_{Rd} V_{Lt}^* V_{Rt} e^{2i\delta} \right) \approx |V_{td}|^2 \sin(\alpha_d + \alpha_b + 2\delta - 2\alpha_t) . \] (22)

For the charged Higgs contribution in Fig. 2, we obtain [10]

\[ d_d \approx \left( \frac{2}{3} e \right) \frac{m_t G_F}{4\sqrt{2\pi^2}} \sin2\theta \frac{m_t^2}{\cos^22\theta m_t^2} \ln \left( \frac{m_t^2}{m_f^2} \right) \eta_d \text{Im} \left( V_{Ld}^* V_{Rd} e^{-i\delta} \right) , \] (23)

\[ e f_d \approx \frac{3}{2} \frac{\eta_f}{\eta_d} d_d . \]

In PMLR models,

\[ \text{Im}(V_{Ld} V_{Rd}^*) = |V_{td}|^2 \sin(\alpha_t - \alpha_d - \delta) . \] (24)

In Eqs.(21,23), \( \eta_d = \left( \frac{\alpha_s(m_t)}{\alpha_s(\mu)} \right)^{16/23} \) and \( \eta_f = \left( \frac{\alpha_s(m_t)}{\alpha_s(\mu)} \right)^{14/23} \) are the QCD correction factors [10].

Note that \( d_d \) is more suppressed by the QCD correction than \( f_d \). Following Ref. [8], we will use \( \alpha_s(\mu) = \frac{\alpha_s(m_t)}{6} \) and \( \alpha_s(m_t) = 0.1 \). As we commented before, Eq.(23) is characterized by its \( m_t^3 \) dependence, a feature which distinguishes it from the the usual multi-doublet models.

Using the numerical values quoted before for the parameters, we find the contribution to \( D_n \) from neutral Higgs exchange to be less than \( 10^{-28} \text{ecm} \) with \( m_H = 10 \text{ TeV} \). Using the same parameters for the charged Higgs boson contribution in PMLR models, we have
\[ D_n(d_d) = \begin{cases} 
3 \times 10^{-28} \sin(\alpha_t - \alpha_d - \delta) \text{ ecm}, & m_\chi = 10 \text{ TeV}, \\
1.3 \times 10^{-26} \sin(\alpha_t - \alpha_d - \delta) \text{ ecm}, & m_\chi = 1 \text{ TeV},
\end{cases} \] (25)

where we have set \( \sin 2\theta \approx \frac{2m_t}{m_t^2} \approx 0.04 \). We see that the one loop level Higgs contributions to the neutron EDM are small. Of course if the mass of the charged Higgs is much lower than 1 TeV, it is possible to have a larger neutron EDM. A similar contribution also comes from \( f_d \) (about 60% of \( d_d \) contribution). The contributions from \( d_u \) and \( f_u \) are smaller because the couplings are smaller.

The contribution to the neutron EDM from \( f_s \) due to the neutral Higgs boson is given by

\[
D_n(f_s) \approx 0.03 f_s e 
\approx 0.03 e \frac{m_b G_F}{8 \sqrt{2} \pi^2 \cos^2 \theta m_H^2} \ln \left( \frac{m_H^2}{m_b^2} \right) \eta_f \text{Im} \left( V_{Lts}^* V_{Rts} V_{Rtb} V_{Ltb} e^{2i\delta} \right) 
= 2 \times 10^{-26} \text{ ecm} \times \frac{1}{(0.04)^2} \text{Im} \left( V_{Lts}^* V_{Rts} V_{Rtb} V_{Ltb} e^{2i\delta} \right), \quad m_H = 1 \text{ TeV}.
\] (26)

There is also a similar contribution from the charged Higgs boson. We have

\[
D_n(f_s) \approx 0.03 e \frac{m_t G_F}{4 \sqrt{2} \pi^2} \sin 2\theta \frac{m_t^2}{m_\chi^2} \ln \left( \frac{m_\chi^2}{m_t^2} \right) \eta_f \text{Im} \left( V_{Lts}^* V_{Rts} e^{-i\delta} \right) 
= 2.7 \times 10^{-26} \text{ ecm} \times \frac{1}{(0.04)^2} \text{Im} \left( V_{Lts}^* V_{Rts} e^{-i\delta} \right), \quad m_\chi = 1 \text{ TeV}.
\] (27)

In the special case of Eq.(13), \( |V_{Rts}| \) can be larger than \( |V_{Lts}| \sim 0.04 \), and therefore these contributions can be near the experimental upper bound. In PMLR models, \( |V_{Rts}| = |V_{Lts}| \) and the neutral Higgs masses are around 10 TeV. Then, only Eq.(27) contributes significantly, with values near those in Eq.(25).

We now turn to the two loop contributions. Once again in this case one can take advantage of the fact that CP violating neutral Higgs couplings can all be proportional to \( m_t \) instead of having at least one of them proportional to \( m_b \) as in the case of the multi-doublet extensions of Standard Model. At this level, the neutral Higgs exchange in Fig. 3 will generate a quark color edm \( f_q \) which is given by [10]
\[
\begin{align*}
    f_q &= \frac{G_F}{16\sqrt{2}\pi^3} m_q \alpha_s(\mu) \left( \frac{\alpha_s(m_t)}{\alpha_s(\mu)} \right)^{37/23} G\left( \frac{m_t^2}{m_H^2}, q \right), \\
    G(z, u) &= f(z) \text{Im}Z_{tu} + g(z) \text{Im}Z_{at}, \\
    G(z, d) &= f(z) \text{Im}Z_{td} + g(z) \text{Im}Z_{dt}.
\end{align*}
\]  

For \( z \ll 1 \),

\[
f(z) \approx g(z) \approx \frac{1}{2} \frac{z}{(\ln z)^2},
\]

where \( \text{Im}Z_{ij} \) are defined through

\[
    \text{Im}Z_{ij} = 2\gamma_i \beta_j.
\]

with

\[
    L_{\text{int}} = (2\sqrt{2}G_F)^{1/2} (m_t \gamma_t \bar{t}t + \text{Im}Z_{td}) + m_d \gamma_d \bar{d}d
    + \text{Im}Z_{td} = -\frac{\sin 2\theta}{\cos^2 2\theta} \text{Im} \left( (V^*_{Lcd} V^*_{Rcd} + \frac{m_t}{m_s} V^*_{Lct} V^*_{Rct} + \frac{m_t}{m_d} V^*_{Lct} V^*_{Rct}) e^{i\delta} \right),
\]

In PMLR models the largest contribution to \( f_d \) is from the term proportional to \( \text{Im}Z_{td} \), we have

\[
    \text{Im}Z_{td} \approx -\frac{m_c}{m_d \cos^2 2\theta} \frac{|V_{cd}|^2}{\cos^2 2\theta} \sin 2\theta \sin(\alpha_d - \alpha_c + \delta).
\]

The contribution to \( D_n \) is again small, \( D_n < 4 \times 10^{-29} \text{ ecm} \) for \( m_H = 1\text{TeV} \). The \( f_u \) contribution is even smaller.

In the special case of Eq.(13), the contribution from \( f_s \) again dominates over other contributions. Changing the subscript \( d \) to \( s \) in equations (28) and (32), we obtain \( f_s \). The resulting value of the neutron EDM is given by

\[
    D_n(f_s) \approx 0.03 f_s e
    \approx 2 \times 10^{-27} \text{ ecm} \text{ Im} \left( V^*_{Lcs} V^*_{Rcs} e^{i\delta} + \frac{m_t}{m_c} V^*_{Lts} V^*_{Rts} e^{i\delta} \right), m_H = 1\text{TeV}.
\]
This contribution is small.

The operator $O_g^C$ will also be generated at the two loop level. We find the neutral Higgs contribution to $D_n$ through this mechanism to be

$$D_n \approx e \xi M \frac{\sqrt{2} G_F}{(4\pi)^2} \text{Im} Z_{tt} h \left( \frac{m_t^2}{m_H^2} \right),$$

$$\xi = \left( \frac{g(\mu)}{4\pi} \right)^3 \left( \frac{\alpha_s(m_b)}{\alpha_s(m_t)} \right)^{-54/23} \left( \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{-54/25} \left( \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right)^{-54/27} \approx 6 \times 10^{-5}. \quad (35)$$

For $z \ll 1$,

$$h(z) \approx \frac{1}{2} z \ln z. \quad (36)$$

We have

$$\text{Im} Z_{tt} = - \frac{m_b \sin 2\theta}{m_t \cos^2 2\theta} \text{Im}(V_{Ltb} V_{Rtb}^* e^{-i\delta}). \quad (37)$$

This effect is extremely small $D_n < 10^{-30}$e.cm. In the special case of Eq.(13), this contribution can be larger ($\sim 10^{-28}$e.cm) because the neutral Higgs mass is less constrained.

The charged Higgs contribution in Fig. 4 to the neutron EDM via the operator $O_g^C$ is given by

$$D_n \approx e \xi' M \frac{\sqrt{2} G_F}{(4\pi)^2} \text{Im} Z' h' \left( \frac{m_t^2}{m_H^2} \right),$$

$$\xi' = \left( \frac{g_s(\mu)}{4\pi} \right)^3 \left( \frac{\alpha_s(m_b)}{\alpha_s(m_t)} \right)^{-14/23} \left( \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{-54/25} \left( \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right)^{-54/27} \approx 3 \times 10^{-4}. \quad (38)$$

For $z \ll 1$,

$$h'(z) \approx \frac{1}{2} z \ln z. \quad (39)$$

$\text{Im} Z'$ is defined by

$$L_{int} = (2\sqrt{2} G_F)^{1/2} (a m_b \bar{t}_L b_R + b m_t \bar{t}_R b_L) \chi^+ \quad (40)$$

$$\text{Im} Z' = 2 \text{Im}(ab^*).$$

We have
\[ \text{Im}Z' = 2 \frac{m_t}{m_b \cos^2 \theta} \text{Im} \left( V_{Rtb} V_{Ltb}^* e^{i \delta} \right), \]  
(41)

and in PMLR models,
\[ \text{Im}Z' = 2 \frac{m_t}{m_b \cos^2 \theta} \sin(\delta + \alpha_b - \alpha_t). \]  
(42)

The neutron EDM from this contribution is
\[ D_n = \begin{cases} 
2.5 \times 10^{-27} \sin(\delta + \alpha_b - \alpha_t) \text{ecm}, & m_\chi = 10\text{TeV}, \\
10^{-25} \sin(\delta + \alpha_b - \alpha_t) \text{ecm}, & m_\chi = 1\text{TeV}. 
\end{cases} \]  
(43)

This result is also valid for the special case of Eq.(13).

Several comments about our results are in order:

1. It is clear from our discussion that the neutral Higgs contributions to the neutron EDM in PMLR models are small, while the charged Higgs contributions can be as large as the experimental upper bound. The one loop contribution from the charged Higgs is smaller than the two loop contribution. However QCD sum rule calculations show that the dimensional analysis estimate for the $O^C_g$ contribution may be overestimated \[30] and the contribution from $f_\eta$ may be larger than the $SU(6)$ prediction \[28]. In this case, the contribution from the charged Higgs at the one loop level may be as important as the two loop contribution.

If $V_L$ and $V_R$ are independent from each other, the neutral Higgs masses can be smaller. The contribution to the neutron EDM can then be close to the experimental upper bound.

In MLR models, because $V_L = V_R$ and $\delta = 0$ all the contributions discussed above are equal to zero if there is no CP violating couplings in the Higgs potential. We have mentioned before that in general such couplings exist. In this case even $\delta = 0$ exchange of Higgs particle will violate CP. The calculations are similar to those discussed before.

One only needs to change the CP violating phases in the previous equations to the CP violating mixing parameters in this case. The Higgs contributions to the neutron EDM are similar to those in PMLR models.
2. Many calculations for the neutron EDM in Left-Right symmetric models have concentrated on the contributions from $W_L - W_R$ mixing. All these contributions are proportional to the mixing angle $\zeta$. A large contribution can be obtained from a four quark operator generated by exchange of the light $W$ boson at the tree level. This was estimated in ref. [31] to be

$$D_n \approx 2 \times 10^{-19} \zeta \text{Im}(V_{Lud}V_{Rud}^*) .$$

It is interesting to note that unless there are fortuitous cancellations, $\zeta$ is bounded from experimental data on $\epsilon'/\epsilon$ to be less than $10^{-5}$ if the CP violating phase involved is close to one [13]. In that case this contribution will be smaller than the charged Higgs contribution if the charged Higgs mass is less than 1 TeV and the phases of $V_{Rtb}V_{Ltb}^* e^{i\delta}$ and $V_{Lud}V_{Rud}$ are the same order of magnitude.

3. Exchange of Higgs particles in Left-Right symmetric models will also generate CP violating electron-nucleon and nucleon-nucleon interactions which will induce a non-zero atomic EDM. The electron-nucleon interactions will be generated by exchange of neutral Higgs at the tree level. We find that these interactions are small [32] ($c_s, c_P < 10^{-10}$). The contribution to CP violating nucleon-nucleon interactions due to the operator $O^C_g$ from the charged Higgs are the largest contributions due to Higgs bosons. However it is also very small [33] ($\eta < 10^{-4}$).

To summarise, we have studied the neutron EDM due to Higgs exchange in Left-Right symmetric models. We find that in PMLR models the most important effect is from the charged Higgs at the two loop level. In NMLR models, the neutral and charged Higgs contributions at the one loop level can reach the experimental upper bound. These contributions can be more important than the contributions from $W_L - W_R$ mixing.
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REFERENCES

[1] L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964); T. D. Lee, Phys. Rev. D8, 1226 (1973); Phys. Reports, 9C, 143 (1974); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973); S. Weinberg, Phys. Rev. Lett. 37, 657 (1976).

[2] K. Smith et al., Phys. Lett. B234, 234 (1990). I. S. Altarev et al., Phys. Lett. B 276, 242 (1992).

[3] E. P. Shabalin, Sov. Phys. Usp. 26, 297 (1983); B. H. J. McKellar, S. R. Choudhury, X.-G. He and S. Pakvasa, Phys. Lett. B197, 556 (1987).

[4] W. Bernreuther and M. Suzuki, Rev. Mod. Phys. 63, 313 (1991).

[5] T. G. Vold et al., Phys. Rev. Lett. 52, 2229 (1984); S. K. Lamoreaux et al., Phys. Rev. Lett. 59, 2275 (1987); S. A. Murthy, D. Krause, Z. L. Li and L. R. Hunter, Phys. Rev. Lett. 63, 965 (1989); P. Cho, K. Sangster and E. A. Hinds, Phys. Rev. Lett. 63, 2559 (1989); K. Abdullah et al., Phys. Rev. Lett. 65, 2347 (1990).

[6] X.-G. He, B. H. J. McKellar and S. Pakvasa, Int. J. Mod. Phys. A4, 5011 (1989); Erratum — ibid A6, 1063 (1991); D. Chang, in The Standard Model and Beyond-Proceedings of Ninth Symposium on Theoretical Physics, Edited by J. E. Kim (World Scientific Pub. Singapore 1991);

[7] S. M. Barr and W. Marciano, in CP violation, Edited by C. Jarlskog(World Scientific Pub. Singapore 1989); S. M. Barr and A. Zee, Phys. Rev. Lett. 65, 21 (1990); Erratum — ibid, 65, 2920 (1990); D. Chang, W.-Y. Keung and T.-C. Yuan, Phys. Rev. D43, R14 (1991).

[8] S. Weinberg, Phys. Rev. Lett. 63, 2333 (1989); Phys. Rev. D42, 860 (1990); E. Braaten, C.-S. Li and T.-C. Yuan, Phys. Rev. Lett. 64, 1709 (1990); Phys. Rev. D42, 276 (1990); J. Dai and H. Dykstra, Phys. Lett. B237, 256 (1990) and Erratum; A. Yu. Morozov, Sov. J. Nucl. Phys. 40, 505 (1984).
[9] D. A. Dicus, Phys. Rev. D41, 999 (1990); D. Chang, W.-Y. Keung, C. S. Li and T.-C. Yuan, Phys. Lett. B241, 589 (1990); J. Dai, H. Dykstra, R. G. Leigh and S. Paban, Phys. Lett. B237, 216 (1990); Erratum-ibid, B242, 547 (1990); M. Dine and W. Fischler, Phys. Lett. B242, 239 (1990); A. DeRujula, M. B. Gavela, O. Pene and F. J. Vegas, Phys. Lett. B 245, 640 (1990); D. Chang, K. Choi and W.-Y. Keung, Phys. Rev. D44, 2196 (1991); R. Arnowitt, M. J. Duff and K. S. Stelle, Phys. Rev. D 43, 3085 (1991); D. Chang, T. W. Kephart, W.-Y. Keung and T.-C. Yuan, Phys. Rev. Lett. 68, 439 (1992); Northwestern Preprint NUHEP-TH-91-17, (Nucl. Phys. B in press). D. Chang, W.-Y. Keung, I. Phillips and T.-C. Yuan, Northwestern Preprint NUHEP-TH-92-8, (Phys. Rev. D in press).

[10] J. Gunion and D. Wyler, Phys. Lett. B248, 170 (1990); D. Chang, W.-Y. Keung and T.-C. Yuan, Phys. Lett. B251, 608 (1990).

[11] D. Chang, C.-S. Li and T.-C. Yuan, Phys. Rev. D42, 867 (1990); D. Atwood, et al., Phys. Lett. B256, 471 (1991); D. Chang, W.-Y. Keung and J. Liu, Nucl. Phys. B355, 295 (1991).

[12] X.-G. He, B. H. J. McKellar and S. Pakvasa, Phys. Lett. B254, 231 (1991).

[13] G. Beall and A. Soni, Phys. Rev. Lett. 47, 552 (1981); D. Chang, Nucl. Phys. B214, 435 (1983); G. Ecker and W. Grimus, Nucl. Phys. B258, 328 (1985); J. F. Nieves, D. Chang and P. B. Pal, Phys. Rev. D33, 3324 (1986); X.-G. He, B. H. J. McKellar and S. Pakvasa, Phys. Rev. Lett. 61, 1267 (1988); J. Liu, C.-Q. Geng and J. Ng, Phys. Rev. D39, 3474 (1989); J. M. Frere et al., Phys. Rev. D45, 259 (1992).

[14] J. Basecq, D. Chang, L. F. Li and P. B. Pal, Phys. Rev. D30, 1601 (1984).

[15] V. M. Khatsymovsky, I. B. Kriplovich and A. S. Yelkhovsky, Ann. Phys. 186, 1 (1988); X.-G. He and B. H. J. McKellar, Preprint UM-P-92/01, OZ-92/01.

[16] M. Wakano, Prog. Theor. Phys. 72, 180 (1984); Phys. Lett. B173, 163 (1986); G.
Ecker, W. Grimus and H. Neufeld, Nucl. Phys. B229, 421 (1983); Phys. Lett. B153, 279 (1985); D. Cocolicchio and G. L. Fogli, Phys. Rev. D35, 3462 (1987).

[17] J. Pati and A. Salam, Phys. Rev. D10, 275 (1974); R. Mohapatra and J. Pati, Phys. Rev. D11, 566 (1975); G. Senjanovic and R. Mohapatra, Phys. Rev. D12, 1502 (1975); R. Mohapatra and D. Sidhu, Phys. Rev. Lett. 38, 667 (1977); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[18] For example, to get rid of the term $H_L^\dagger \phi H_R$ one can impose two discrete symmetries: $S_L$, which changes only $H_L$ by a minus sign, and $S_R$, which changes only $H_R$ by a minus sign. $S_L$ and $S_R$ form a nonabelian discrete group together with the left-right discrete symmetry $S$.

[19] R. N. Mohapatra, F. E. Paige and D. P. Sidhu, Phys. Rev. D17, 2462 (1977).

[20] F. I. Olness and M. E. Ebel, Phys. Rev. D30, 1034 (1984); D. London and D. Wyler, Phys. Lett. B232, 503 (1989); P. Langacker and S. Uma Sankar, Phys. Rev. D40, 1596 (1989).

[21] D. Cocolicchio and G. L. Fogli, Phys. Rev. D32, 3020 (1985).

[22] D. Chang and R. Mohapatra, Phys. Rev. D32, 1248 (1985).

[23] N. G. Deshpande, J. F. Gunion, B. Kayser and F. Olness, Phys. Rev. D44, 837 (1991).

[24] Particle Data Book, Phys. Rev. D45, Volume I–II, III.65 (1992).

[25] S. Weinberg and S. Glashow, Phys. Rev. D15, 1958 (1977).

[26] F. J. Gilman and M. H. Reno, Phys. Rev. D29, 937 (1984).

[27] J. Liu and L. Wolfenstein, Nucl. Phys. B272, 145 (1986).

[28] V.M. Khatsymovsky, I.B. Khriplovich, and A.S. Yelkhovsky, Ann. Phys. 186, 1 (1988); I. I. Kogan and D. Wyler, Phys. Lett. B274, 100 (1992).
[29] V.M. Khatsymovsky, I.B. Khriplovich, Phys. Lett. B, to be published.

[30] I. I. Bigi and N. G. Uraltsev, Nucl.Phys. B353, 321 (1991); M. Chemtob, Phys. Rev. D45, 1649 (1992).

[31] X.-G. He, B. H. J. McKellar, Preprint, UM-P-92/01 (1992).

[32] S. M. Barr, Phys. Rev. Lett. 68, 1822 (1992); Phys. Rev. D45, 4148 (1992); X.-G. He, B. H. J. McKellar and S. Pakvasa, Phys. Lett. B 283, 348 (1992).

[33] X.-G. He and B. H. J. McKellar, Phys. Rev. D 46, 2131 (1992).
FIGURES

FIG. 1. One loop contribution to $d_{d,s}(f_{d,s})$ due to the neutral Higgs bosons $H_{2,3}$. The $m_t^2 m_b$ dependence comes from the couplings in Eq.(16) and the mass $m_b$ insertion in the internal $b$ quark line.

FIG. 2. One loop contribution to $d_{d,s}(f_{d,s})$ due to the charged Higgs boson $\chi^+$. The $m_t^3$ dependence comes from the couplings in Eq.(16) and the mass $m_t$ insertion in the internal $t$ quark line.

FIG. 3. Leading two loop contribution to the quark color edm due to the neutral Higgs exchange and the virtual top quark loop effect.

FIG. 4. Leading two loop contribution to the gluon color edm due to the charged Higgs boson exchange.