Stochastic metric perturbations (radial) in gravitationally collapsing spherically symmetric relativistic star

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Abstract

Stochastic perturbations (radial) of a spherically symmetric relativistic star, modeled by a perfect fluid in comoving coordinates for the collapse scenario are worked out using the classical Einstein-Langevin equation, which has been proposed recently. The solutions are in terms of perturbed metric potentials and their two point correlation. For the case worked out here, it is interesting to note that the two perturbed metric potentials have same magnitude, while the potentials themselves are in general independent of each other. Such a treatment is useful for building up basic theory of non-equilibrium and near equilibrium statistical physics for collapsing stars, which should be of interest towards the end states of collapse. Here we discuss the first simple model, that of non-rotating spherically symmetric dynamically collapsing relativistic star. This paves way to further research on rotating collapse models of isolated as well as binary configurations on similar lines. Both the radial and non-radial perturbations with stochastic effects would be of interest to asteroseismology, which encompassed the future plan of study.

1 Introduction

Perturbations of relativistic stars are of interest to studies in asteroseismology [1, 2] and has recently gained more importance in the context of gravitational waves, emitted by collapsing stars or binary mergers [3]. Nevertheless, perturbations of such configurations have a wider range of relevance [4, 5, 6]. The oscillations of stars, both radial and non-radial have been studied and mathematical formulations of the theory has a long standing history [7, 8, 9]. These oscillations enables one to study the interior of the stars, which for the more relativistic cases, is an active area of research. The equations of state of the matter content of a compact star, towards the end state of collapse is open to investigations. Also the dynamics of the interior of such a collapsing star is of interest, and ways to determine this still awaits theoretical as well as observ-
tional developments. The metric perturbations to the spacetime geometry are often treated deterministically, and modes of oscillations addressed for a given compact system under investigation. This is done via the perturbed Einstein’s equations and their solution.

In this article we attempt to show how stochastic effects in the matter content of relativistic star give rise to metric perturbations with induced stochasticity. The aim of such an approach is to set up a theoretical framework for doing non-equilibrium statistical analysis of a relativistic star, which should be of interest and more relevant in observational aspects, towards the end states of dynamical collapse. Here we treat a very simple non-rotating geometry, to show that an analytical method can be developed for the same, though this article is restricted to radial perturbations only. Such a case for static stellar configuration has been recently worked out [10], where we have proposed a classical Einstein Langevin equation in the context. This article takes forward the same theme, for a collapsing star in comoving coordinates and is the next step in the development. On the lines of this dynamical collapse, we will in future attempt to consider rotating geometry and non-radial perturbations which will takes us ahead on the program for more realistic cases.

An interesting feature in using this formalism is that, the relativistic Cowling approximation is ignored [2, 11], hence it is exact solution to Einstein’s equation, along with added stochastic effects which are seen to arise as a result of statistical properties of matter content in the interior of the star. As is apparent from the results obtained form our preliminary calculations, it is not the fluctuations of the pressure and density at a given spacetime point that show up playing a direct role in the metric perturbations, but the covariances of pressure and density inside the star at separated points, which induce these. Hence, one can see that the cowling approximation does not find a way directly in this framework, and we get results from taking the perturbed equations in their complete form. In this analysis the averages of the metric perturbations \( \langle h_{ab}(x) \rangle \), are vanishing, which does in fact fall in line with cowling approximation. Thus one can view it as cowling approximation been hidden in the stochasticity, while we see the real effects of randomness in terms of two point correlations of the metric, and covariances of fluid elements. This will become clearer, when we deal with non-radial perturbations in which case the Cowling approximation is usually applied.

In the specific example that we take up in this article the noise in the system or the source of stochasticity are the pressure and density covariances in the perfect fluid composing a compact object. Thus the randomness in the stress tensor induces the statistical effects in the perturbed geometry as we see below.

This is the most simplified model, more involved cases being those of dissipative and anisotropic fluids [12, 13, 14] in a collapsing scenario which form realistic matter fields towards end states of collapse in highly compact configurations. This can be dealt in future work, the common feature carried forward being the fluctuations of the stress tensor for anisotropic and dissipative fluids, would be defined in terms of, say, fluctuations of heat flux in the interior of the star as it collapses. However, it may be difficult to get analytical solutions for
those cases and, one may have to succumb to numerical solutions. The below formalism sets stage for such cases, where the physical results would encompass dependence of the metric perturbations on statistical properties of dissipative fluids which would show up explicitly and probably a fluctuation-dissipation relation in the system addressed.

2 The collapse model with stochastic effects

For a non-rotating spherically symmetric relativistic star in comoving coordinates given by

$$ ds^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t,r)d\Omega^2 $$

(1)

with a perfect fluid stress tensor

$$ T^t_t = -\rho(t,r), T^r_r = T^\theta_\theta = T^\phi_\phi = p(t,r) $$

(2)

the Einstein’s tensors read

$$ G^t_t = -\frac{F'}{R^2 R'} + \frac{2\dot{R}E^{-2\nu}}{R} (\ddot{R}' - \dot{R}' - \ddot{\psi}R') $$

(3)

$$ G^r_r = -\frac{\dot{F}}{R^2 R'} - \frac{2\dot{R}'}{R} e^{-2\psi}(\ddot{R}' - \dot{R}' - \ddot{\psi}R') $$

(4)

$$ G^\phi_\phi = e^{2(\psi - \nu)}G^r_r = \frac{2e^{-2\nu}}{R} (\ddot{R}' - \dot{R}' - \ddot{\psi}R') $$

(5)

where $F$ is defined by the equation

$$ e^{2\psi} = (1 + e^{-2\nu} \dot{R}'^2 - \frac{F}{R})^{-1} R'^2 $$

(7)

The perfect fluid stress tensor can be treated as random variable, while the randomness can be introduced in the system by internal or external influences. The fluid particles thus undergo random collisions, which may not be of thermal origin, since for compact objects like neutron stars we deal with non-thermal effects, that of quantum origin where the pressure in the star is due to neutron degeneracy and the like. The fluctuations of the macroscopic fluid variables may thus partially capture the quantum effects in the dynamical system. The external influences that can give rise to randomness, may be mechanical in origin like the accretion of matter or implosion effects of the core of the star. Though here we deal with a non-rotating system, these effects are very much known to induce perturbations in the system. Usually these perturbation are treated deterministically. We introduce the randomness in a fashion, such that the system can be modeled by an Einstein Langevin equation [10]

$$ G^{ab}[g + h](x) = T^{ab}[g + h](x) + \xi^{ab}[g](x) $$

(8)
where $\xi^{ab}(x) = (T^{ab}(x) - < T^{ab}(x) >)$ is the Langevin noise defined by $< \xi^{ab}(x) >= 0$, satisfying $\nabla_a \xi^{ab}(x) = 0$. The two point correlation $< \xi^{ab}(x) \xi^{cd}(y) > = N^{abcd}(x, y)$, as discussed later, defines the noise. It is important to note that the noise term $\xi^{ab}$ is defined on the background spacetime $g^{ab}(x)$ and not the perturbed one. In fact it is this noise that is supposed to induce the perturbations in the metric over the background.

The above equation can be written as

$$\delta G^{ab}(x) = \delta T^{ab}(x) + \xi^{ab}(x)$$

(9)
as the Einstein’s equation balances the unperturbed part $G^{ab}[g] = T^{ab}[g]$ over the background geometry.

The perturbed Einstein tensors take the following form, where we assume the model with the perturbation in the area radius $\delta \rho(t, r) = 0$, for mathematical simplicity.

$$\delta G^t_t = 2\delta \nu e^{-2\nu} \frac{\dot{R}}{R} \left( \frac{\dot{R}}{R} + 2\psi \right) - 2\delta \psi e^{-2\psi} \left( \frac{R^2}{R^2} - 2\nu \right)$$

$$-2\Psi \frac{\dot{R}}{R} \frac{\dot{R}}{R} - 2\delta \psi \frac{\dot{R}}{R} e^{-2\psi} - 2\delta \psi \frac{\dot{R}}{R} e^{-2\psi}$$

(10)

$$\delta G^r_r = 2\delta \nu e^{-2\nu} \left( \frac{\dot{R}}{R} + 2\nu \right) - 2\delta \psi e^{-2\psi} \left( \frac{R^2}{R^2} + 2\overline{R} \nu' \right)$$

$$-2\delta \psi \frac{\dot{R}}{R} e^{-2\psi} + 2\delta \psi \frac{\dot{R}}{R} e^{-2\psi}$$

(11)

$$\delta G^\theta_\theta = -\ddot{\rho} \delta \nu' - \dot{\psi} R'$$

(12)

and similar expression for $\delta G^\phi_\phi$ and $\delta G^\phi_\phi$, which we can use in the Einstein Langevin equation (9) for the geometry sector. The perturbed stress tensor has non-zero components, $\delta T^r_t = -\delta \rho, \delta T^r_r = \delta T^\theta_\theta = \delta T^\phi_\phi = \delta \rho$ . For the stochastic term $\xi^{ab}$, we defined noise in the following section.

### 2.1 Noise in the system

The noise term characterized by $\xi^{ab}$ in the Einstein Langevin equation is responsible for the induced stochasticity in the perturbed metric potentials, as we would see further in this article. This is the Langevin noise which for the perfect fluid stress tensor as defined above is given by $\xi^{ab} = 0$ , while the two point correlation gives a non-zero contribution and defines randomness in the system as shown below. The most general definition as proposed in [10] is

$$N^{abcd}(x, x') = < (T^{ab}(x) - < T^{ab}(x) >) (T^{cd}(x') - < T^{cd}(x') >) >$$

$$= < T^{ab}(x) T^{cd}(x') > - < T^{ab}(x) > < T^{cd}(x') >$$

$$= Cov[T^{ab}(x), T^{cd}(x')]$$

(13)

which in a slightly different way with the indices can be written as

$$N^a_{\ b\ c\ d}(x, x') = < \xi^a_{\ b}(x) \xi^c_{\ d}(x') > = Cov[T^a_{\ b}(x), T^c_{\ d}(x')]$$

(14)
For the stress tensor given by (2), the nonzero noise components can be directly read off from the above definition as

\[ N_{t}^{t}(t,r;t',r') = Cov[\rho(t,r),\rho(t',r')] \]
\[ N_{r}^{t}(t,r;t',r') = N_{t}^{r}(t,r;t',r') = -Cov[\rho(t,r),p(t,r)] \]

\[ N_{t}^{t}(t,r;t',r') = N_{t}^{t} \delta(t',t)N_{t}^{t} \delta(r,r') \]
\[ N_{t}^{r} \delta(t',t)N_{t}^{r} \delta(r,r') \]
\[ N_{t}^{\theta} \delta(t',t)N_{t}^{\theta} \delta(r,r') = -Cov[p(t,r),\rho(t,r)] \]
\[ N_{r}^{t} \delta(t',t)N_{r}^{t} \delta(r,r') \]
\[ N_{r}^{\theta} \delta(t',t)N_{r}^{\theta} \delta(r,r') \]
\[ N_{\phi}^{t} \delta(t',t)N_{\phi}^{t} \delta(r,r') \]
\[ N_{\phi}^{\theta} \delta(t',t)N_{\phi}^{\theta} \delta(r,r') \]
\[ = Cov[p(t,r),p(t',r')] \] (15)

2.2 Solution of Einstein Langenvin equation

It is interesting to note here that for our model, in which we assume \( \delta R(t,r) = 0 \), equation (12) gives (as \( \delta T_{t} = 0 \) similarly \( \xi_{t} = 0 \))

\[ \dot{\delta \psi} = -\delta \nu \] (16)

Hence it follows that \( \delta \psi = -\delta \nu \). Thus the two perturbed potential are same, except for the difference in sign, even though there is no such restriction on the potentials themselves. This is an interesting result in itself, which enables one to simplify the complexity of Einsteins’ perturbed equations as well as the Einstein Langevin equation drastically, as we see further in this article.

In view of the above relation between the \( \delta \psi \) and \( \delta \nu \), equations (10) and (11), take the form

\[ \delta G_{t} = 2\delta \nu e^{-2\nu}\dot{R}_{t}(\dot{R} + 5\psi) + 2\delta \nu e^{-2\psi}(\frac{R'^{2}}{R^{2}} - 2\nu' \frac{R'}{R} + 2 \frac{R''}{R}) \]
\[ + 2\delta \nu' (\frac{R}{RR' e^{-2\nu}} + \frac{R'}{R} e^{-2\psi}) \] (17)
\[ \delta G_{r} = 2\delta \nu \left(e^{-2\nu}\frac{R^{2}}{R'^{2}} + 2\frac{R}{R'} \dot{R} + 2\psi \frac{R'}{R'} + e^{-2\psi}(\frac{R'^{2}}{R^{2}} + 2\nu' \frac{R'}{R}) \right) \]
\[ - 2\delta \nu' \left(\frac{R^{2}}{RR'} e^{-2\nu} + e^{-2\psi} \frac{R'}{R} \right) \] (18)

The above two relations can be put in the E-L equation,

\[ \delta G^{t} = \delta T^{t} + \xi^{t} \]
\[ \delta G^{r} = \delta T^{r} + \xi^{r} \] (19)

Using an equation of state \( p = w \rho \), and perturbing it to give \( \delta p = w \delta \rho \), the above two relations can be put together, to get expressions for the metric potential \( \delta \nu \) (and \( \delta \psi \)). Thus

\[ \delta G^{r} = -w \delta G^{t} + w \xi^{t} + \xi^{r} \] (21)

Further using (17) and (18) we obtain

\[ \nu' f_{1}(t,r) + f_{2}(t,r) \delta \nu = -w \xi^{t} - \xi^{r} \] (22)
Its is clear then, that $\langle \delta \nu \rangle = 0$ as expected for the Langevin formalism. The two point correlations are given by
\[
f_1(t, r) = 2(1 + w)(\frac{\dot{R}^2}{RR'}e^{-2\nu} + \frac{R'}{R}e^{-2\psi})
\]
and
\[
f_2(t, r) = -2[e^{-2\psi}\frac{\dot{R}^2}{R^2}(1 + w) + 2\frac{\dot{R}}{R} + \frac{\dot{R}'}{R} + 2w\dot{\psi}\frac{\dot{R}}{R}]
\]
\[
e^{-2\psi}\left[\frac{R'^2}{R^2}(1 + w) + 2\frac{R'}{R}(\nu' - w\psi') + 2w\frac{R''}{R}\right]
\]
Solving for $\delta \nu$,
\[
\delta \nu(t, r) = e^{-\int \frac{f_2(t, r)}{f_1(t, r)} dr} \int e^{-\int \frac{f_1(t', r')}{f_1(t, r)} dr'} (-w\xi_t - \xi_r) dr''
\] (23)

Putting in the noise from sec. 241
\[
< \delta \nu(t, r) \delta \nu(t', r') > = e^{-\int \frac{f_2(t, r)}{f_1(t, r)} dr + \int \frac{f_1(t', r')}{f_1(t, r)} dr'} \int \int e^{-\int \frac{f_2(t', r')}{f_1(t, r)} dr' + \int \frac{f_1(t, r)}{f_1(t', r')} dr} \{w^2 Cov[p(t, r_1'), p(t', r_2')] + Cov[p(t', r_1'), p(t, r_2')]
\]
\[
-w(Cov[p(t', r_1'), p(t, r_2')] + Cov[p(t', r_1'), p(t, r_2')])\}
\]
\[
dr_1' \cdot dr_2'
\] (24)

Another solution arises from taking the other possible set of Einstein’s tensors $\delta G_t$ and $\delta G_r$ with the ‘ derivatives, replaced by ‘ derivatives using the relation 10.

\[
\delta G_t = 2\nu[e^{-2\psi}\frac{\dot{R}}{R}(\frac{\dot{R}}{R} + 2\psi) + e^{-2\psi}(\frac{R'^2}{R^2} + 2w\dot{\psi}\frac{\dot{R}}{R} + 2w\frac{R''}{R}e^{-2\psi})]
\] (26)
\[
\delta G_r = 2\nu[e^{-2\psi}\left(\frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}}{R} + 2w\dot{\psi}\frac{\dot{R}}{R} + \nu e^{-2\psi}\left(\frac{R'^2}{R^2} + 2\frac{R'}{R}e^{-2\psi}\right)\right)]
\] (27)
From the Einstein Langevin equations as done for the previous case, we obtain

\[
2\delta\nu (1+w)\left[\frac{\dot{R}}{R}e^{-2\nu} + e^{-2\psi}\frac{R^2}{RR'}\right] - 2\delta\nu e^{-2\nu}\frac{\dot{R}^2}{RR'}(1+w)
\]

\[
+ 2\frac{\dot{R}}{R}(\psi w + \nu)\right] + e^{-2\psi}\left(\frac{R^2}{2R}(1+w + 2\frac{R'}{R}(\nu' - w\nu') + 2\frac{R''}{R})\right]
\]

\[
= -w\xi^t_t - \xi^r_r
\]  

(28)

This can be written as

\[
\delta\nu f_3(t, r) + \delta\nu f_2(t, r) = -w\xi^t_t - \xi^r_r
\]  

(29)

where,

\[
f_3(t, r) = 2(1+w)\left[\frac{\dot{R}}{R}e^{-2\nu} + e^{-2\psi}\frac{R^2}{RR'}\right]
\]

(30)

we get a similar solution in this case, as the previous one, but in terms of temporal integrals.

\[
\delta\nu(t, r) = e^f \int_{\frac{2\xi^t_t(r)}{f(t, r)}} dt \int e^{-f} \int_{\frac{2\xi^r_r(r)}{f(t, r)}} dt'(-w\xi^t_t - \xi^r_r)dt''
\]  

(31)

Again, as expected \(<\nu(t, r) >= 0\), while the two point correlation is given by

\[
\langle \delta\nu(t, r)\delta\nu(t', r') \rangle = e^f \int_{\frac{2\xi^t_t(r)}{f(t, r)}} dt \int_{\frac{2\xi^r_r(r)}{f(t, r)}} dt' \int e^{-f} \int_{\frac{2\xi^t_t(r)}{f(t, r)}} dt_1 \int_{\frac{2\xi^r_r(r)}{f(t, r)}} dt_2 [w^2 \langle \xi^t_t(t_1, r)\xi^t_t(t_2, r') \rangle + w\langle \xi^t_t(t_1, r)\xi^r_r(t_2, r') \rangle + \langle \xi^r_r(t_1, r)\xi^t_t(t_2, r') \rangle + \langle \xi^r_r(t_1, r)\xi^r_r(t_2, r') \rangle]
\]  

(32)

Putting in the noise from sec. 2.1

\[
\langle \delta\nu(t, r)\delta\nu(t', r') \rangle = e^f \int_{\frac{2\xi^t_t(r)}{f(t, r)}} dt \int_{\frac{2\xi^r_r(r)}{f(t, r)}} dt' \int e^{-f} \int_{\frac{2\xi^t_t(r)}{f(t, r)}} dt_1 \int_{\frac{2\xi^r_r(r)}{f(t, r)}} dt_2 [w^2 Cov[p(t_1, r), p(t_2, r')] - w(Cov[p(t_1, r), p(t_2, r')] + Cov[p(t_1, r), p(t_2, r')] + Cov[p(t_1, r), p(t_2, r')])]
\]  

(33)

Equation 25 and 33 represent two equivalent expressions for the two point correlation of perturbed metric potential \(\delta\nu(t, r)\), both of which show that these depend on the covariances of pressure and density inside the star.

One could also obtain easily from equation 25 the case \(t = t'\), for coincident comoving time the two point correlator with dependencies on covariances of density and pressure between two different radial coordinates, while from 33 the case \(r = r'\), for the coincident radial coordinate, with dependencies on pressure and density at two different comoving times can be obtained.

We further describe the results 25 and 33 giving a clearer picture that emerges from the given expressions. An obvious question to raise here is over
the observable effects of the matter fluctuations or stochasticity in general. As mentioned earlier in the article these stochastic effects could be due to external sources, or internal dynamics of the gravitating system, but more importantly as seen from the expressions, they show up in the form of covariances of pressure and density at two different points inside the collapsing cloud. The pressures or density inside a collapsing star which are treated as random variables can vary differently by significant amounts, with respect to one another at a few points in space and time during the collapse. Specially in the context of collapsing models, this could be significant near the critical phases of collapse, where the two radial points are placed such that one lies inside or at the shell which is imploding while the other which is about to explode or outgoing, and lies at the outer region of the stellar configuration (similar for the temporal coordinates as well). One can think of stages of collapse just before a Supernova burst, when one of the points lies where the core collapse would occur and the other in the outer region which would undergo explosion. It is for a short period in the collapsing stages or non-equilibrium stages of the collapse that this could have observable effects, such that covariances of the pressure and density at two such different points become large enough. In other words, the varying of pressure or density in one of the regions and that in the other region (or time) near or at a critical phase of collapse may show up very different behavior in the statistical sense which would be captured by the terms like $\text{Cov}[p(r,t), p(r', t)]$ and have significant impact on the metric perturbations. It is important to realize that here we are not trying to observe fluctuations of the matter content at a given particular point inside a star which is negligible and would not have observable consequences, but the covariance at two different points in two very different regions, where the matter can have significantly different properties, which show up in how the pressure and density vary with respect to each other at two different points. This effect could be large enough to perturb the metric (while also showing up correlations in the metric perturbation at two points) during the critical phases. As an analogy, one can compare this with complete gravitational collapse of black holes, where interesting behaviour of quantum fluctuation is seen to emerge at Cauchy horizon, that of a divergence of a regularized stress tensor as well as its two point fluctuations [16]. Of course here we are dealing with a classical system, which is a different picture, however in the collapsing scenario, these effects will be more significant in observational sense than a static configuration as is proposed in [10]. This is because the dynamical collapse does give different regions of the massive star, significantly different behavior, and thus a non-equilibrium statistical treatment to the analysis can be more relevant here.

Thus the implications of this results lies in the fact that the two point correlations of the induced metric fluctuations act as a probe to the interior statistical properties the perfect fluid or the matter content during critical phases of dynamical collapse. Moreover, one can see a drastic change in these perturbations from being negligible during smooth collapse phases to becoming important near or at the critical points of collapse.

As a stellar configuration can undergo many critical phases during the col-
lapse, the above analysis can probe into the statistical properties at each stage of criticality and compare and contrast the behavior from the beginning till the final stages of collapse, when the core settles down in an equilibrium configuration and these covariances become insignificant again. By doing such an analysis of a collapsing stellar configuration, one could study detailed interior dynamics statistically. This may provide information about the matter content and its properties and evolution of the interiors through different stages of the collapse which can be of Astrophysical interest. Hence such a theoretical development as discussed in this article is more important than for a static configuration of a relativistic star.

Further significance of the analysis done in this article is discussed below.

3 Discussion and further directions

The statistical correlations of the metric perturbations shown in the analysis here are the basic building blocks in an effort to characterize the dynamical statistical properties, which are induced by those of the matter content of the interior of the star. Thus the correlations in the metric perturbations show up the induced stochastic behavior. The model treated here for the collapsing case is a toy model, with very basic structure, we intend to carry forward this formalism to more realistic situations of collapse in further work, which would treat different stress tensors and matter components of the interior for the star as well as different collapsing geometries like the rotating configurations for collapse and deviation from spherical symmetry. However the method of solution of such systems, is not as simple as is treated in the present case, but will involve developments on the lines of theory of perturbed system for non-radial collapse using different gauges. The most important challenge is to work out formalism on the same lines without using the Cowling approximation, as we need to see the effects of the matter field stochasticity on the spacetime. Though it may seem initially, that these effects would be minor and not of much Astrophysical interest, we expect to see relevant results, in cases, where one is interested in critical stages of collapse in compact relativistic objects or configurations, for both radial as well as non-radial perturbations. The non-equilibrium statistical physics developed for such applications would be of interest to dynamical collapse of gravitating bodies and capture and characterize the interiors in the language of spacetime perturbations. It is likely that near the end states of collapse, this may enable one to study details of the behavior of geometry and affect results and parameters at horizons of interest, viz the event horizon in case of formation of black holes or affecting the bounce models of collapse and relevant parameters therein. It would be of interest to examine if boundary conditions for the collapsing cloud are affected or modified by including the stress tensor fluctuations for different cases of imperfect, dissipative fluid with heat flux and so on, specially when the matter fluctuations in the crust of a relativistic star are significant. Non equilibrium effects may become very interesting for such cases, which is an important research direction that can follow from the work
presented in this article.

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