Experimental study of homogenisation in grid turbulence

J I Cardesa-Dueñas¹, T B Nickels and J R Dawson
Department of Engineering, Cambridge University, Trumpington Street, Cambridge CB2 1PZ, United Kingdom
E-mail: jic27@cam.ac.uk

Abstract. Turbulence statistics have been measured immediately downstream of a regular grid made of round rods with rod spacing \( M \). 2D-2C PIV was used to analyse a measurement area of \( 14M \times 4M \) in the down and cross-stream directions respectively. The relevant Reynolds number span the range \( Re_M = U_\infty M/\nu = 5500 − 16500 \). The Reynolds shear stresses recorded on two parallel measurement planes differently located relative to the grid exhibit significant discrepancies over the first \( 5M \), but have completely homogenised in the cross-stream direction by \( x/M = 7 \). The downstream evolution of the two-point velocity correlation functions shows a progressive loss of coherence and a clear trend towards the expected isotropic behavior. The same conclusions apply to measurements taken in the wake of another regular grid made of square rods. Changes in the vortex shedding pattern from the grid were observed at the lowest Reynolds number, with two of the four rod wakes captured shedding in phase with each other but in anti-phase with a third one. The impact of this early flow coherence on the turbulence statistics did not persist due to the homogenisation process.

1. Introduction

The theory of isotropic turbulence, as first put forward by Taylor (1935) and von Karman & Howarth (1938), led to an evolution equation linking the double and triple two-point velocity correlation functions. This provided a significant simplification of the Navier-Stokes equations in a statistical sense. However, the Karman-Howarth equation still contains enough complexity for problems such as the evolution of turbulent kinetic energy and length scale growth in a freely decaying flow to depend on further assumptions than isotropy alone. In this context, grid turbulence has been the main experimental workhorse of validation of these assumptions and the various results they lead to (Taylor, 1935; Batchelor, 1948; Saffman, 1967; George & Wang, 2009). More than seventy years after the first wind tunnel experiments using grids, the volume of experimental data allows one to find evidence which backs all these different predictions (Taylor, 1935; Comte-Bellot & Corrsin, 1966; Bennett & Corrsin, 1978; Lavoie et al., 2007; Seoud & Vassilicos, 2007; Krogstad & Davidson, 2010). As a result, controversy is still dominant in the study of grid turbulence. There is one thing, nevertheless, which should bring everyone into agreement: if there is a place where the flow is directed towards complying with one set of assumptions or another, it should be at the grid. Yet the bulk of the experimental measurements taken in grid turbulence have focused on the region which is thought to be a close approximation to isotropic turbulence. This region is often identified as that passed \( 50M \)
downstream from the grid (Corrsin, 1963), where $M$ is the grid’s rod spacing. The focus of the present work is on the turbulence properties immediately after the grid. We intend to study the persistence or disappearance of the flow’s coherence, on which any impact of initial conditions on the downstream dynamics hinges.

2. Experimental setup and procedure

Experiments were carried out in an open water channel, with perspex walls and floor for full optical access. The measurement plane was parallel to the channel floor. Three cameras placed side by side covered a field of view of $4M \times 14M$ in the cross- and downstream directions respectively, which started at the grid. The wakes of four grid rods were captured in width, located roughly in the middle of a $13M \times 27M$ grid. The channel dimensions, camera and laser arrangement are all depicted on figure 1. The grid was located at the end of a 3-to-1 contraction. The background turbulence was 0.7% of the mean flow, measured with Laser Doppler Anemometry (LDA). The water was seeded with silver coated hollow glass sphere particles of 10$\mu$m mean diameter and 1.4$g/cm^3$ specific density. The illumination of the particles was provided by one head of an Nd:YLF laser with 25mJ per head pulse. The time delay between two frames used was $\delta t = 2ms$, and the time between two frame pairs was $\Delta t = 4M/U_\infty$.

Two grids were used to generate the turbulence, which were both biplanar and square meshed with $M = 3.2cm$. They differed only in their rod cross-section shape and rod diameter $D$. The defining parameter of such grids is the grid solitry ratio $\sigma = D/M(2 - D/M)$, which was 34% for the square and 44% for the round rods respectively. They will be refered to as $Sq34$ and $Rd44$. These two grid geometries have different drag coefficients, but their different areas result in both of them producing close levels of turbulent kinetic energy in the flow (Corrsin, 1963). The grids were oriented to the flow so that the array of vertical rods is downstream from the horizontal rods. Figure 2 shows the grid geometries along with the locations of the measurement plane. The latter could be positioned at a height aligned with the centreline behind a horizontal rod ($Sq34b$ and $Rd44b$) or midway between two horizontal rods ($Sq34m$ and $Rd44m$). The data sets consist of 12 series of 10600 vector fields each. The twelve series correspond to the experimental settings gathered in table 1, where the short-hand notation used for all 12 settings is also provided. The Reynolds number $Re_M = MU_\infty/\nu$ was varied by changing the freestream velocity $U_\infty$.

3. Data analysis

3.1. PIV analysis settings

The acquired images were stitched and then cross-correlated with the commercial software Davis 7.2 from Lavision GmbH. A 4-pass constant window size algorithm was used, with $32 \times 32$ pixels interrogation windows and 50% overlap. Prior to the image stitching, the average background
intensity was estimated and subtracted separately from each camera. The only further pre-
processing used was particle intensity normalisation of the stitched images. During cross-
correlation, the outlier removal criterion used was based on the $Q$ factor, which is the ratio of
largest to second largest peak in the correlation plane. It was set to 1.5, and missing vectors were
replaced by zero values which were discarded from the averaging routines used for estimating
turbulence statistics. The average $Q$ factor throughout the data series was close to 3, and over
95% valid vectors were found at first pass. A Gaussian filter with $3 \times 3$ kernel was active during
multi-pass processing for smoothing purposes, and a 3-point Gaussian fit was used to estimate
the location of the correlation peaks.

3.2. Turbulence statistics of interest in the present work
In the present work, the time average of a quantity $\theta$ at a given point in space will be denoted
as $\langle \theta \rangle$, and is taken over successive vector fields from times $t_1, t_2, ..., t_N$ at that given point. It
differs from the space average since the spatial inhomogeneity forbids use of ergodic theory to

Figure 1. Sketch of the water channel, the grid and the PIV system along with relevant
dimensions. The coordinate system used has also been included.

Figure 2. Sketch of the two different grid geometries used and of two measurement planes. For
both grids, measurements were taken separately on two different planes: one behind a horizontal
rod and one midway between two horizontal rods. On this figure, the two measurement planes
shown are $Rd44b$ (left) and $Sq34m$ (right).
establish their equivalence (Batchelor, 1953). Thus, the mean velocity \( \langle U \rangle \) subtracted to obtain the turbulent field was a time-averaged mean for each specific location. In this inhomogeneous flow region where the grid blockage forces the flow to be faster in the vicinity of the grid, the time mean will depend on position \( x \). Hence, the Reynolds decomposition takes the form

\[
U(x, t) = \langle U(x) \rangle + u(x, t).
\]

The field \( \langle U(x) \rangle \) in the wake of a single rod is plotted on figure 3 for \( Rd44m_{\text{high}} \), which shows the extent of the backflow region.

The main statistical quantities used to measure homogenisation will be the normalised two-point, one-time autocorrelation functions \( f(x_0, r) \) and \( g(x_0, r) \). They are defined as

\[
f = \frac{\langle v(x_0 + re_y)v(x_0) \rangle}{\langle v^2(x_0) \rangle} \tag{2}
\]

\[
g = \frac{\langle u(x_0 + re_y)u(x_0) \rangle}{\langle u^2(x_0) \rangle} \tag{3}
\]

where \( u \) and \( v \) are the components of \( u(x, t) \) along the \( x \)- and \( y \)-axes respectively (see frame of reference on figure 2) and \( e_y \) is the unit vector in the \( y \)-direction. From now on, they will be referred to simply as correlation functions, or CFs. In inhomogeneous turbulence, the CFs are also a function of the position vector \( x_0 \), as opposed to homogeneous turbulence where they only depend on the separation \( r \) along any direction (Batchelor, 1953). The position vector \( x_0 = xe_x + ye_y \) was located along the centreline of the wake behind the rod closest to \( y = 0 \): the “reference” rod. So keeping \( y_0 \) constant behind the reference rod centreline, \( f \) and \( g \) become functions of \( xe_x \) and \( re_y \) only, where \( r = y - y_o \). In the discussion to come, the “reference point” will refer to that point on \( x_0 \) with which we correlate velocities at a given downstream position.

The Reynolds shear stress per unit volume \( \langle uv \rangle \) that will be dealt with was obtained by averaging as previously described in this section.

3.3. Uncertainty analysis
The present data set is not affected by peak locking, a major source of bias errors in PIV data. The impact of the stitching on the measured flow statistics was found to be confined to the region where the stitching was applied, i.e. \( 4.5 \leq x/M \leq 5 \) and \( 9 \leq x/M \leq 9.5 \). All these conclusions come from a separate study where the uncertainty and error sources in these data sets were analysed (Cardesa-Dueñas et al., 2011). Bias errors aside, the remaining random error can be accounted for by statistical uncertainty analysis. The error bars displayed in the present work represent 99\% confidence intervals and are obtained as outlined by Benedict & Gould (1996). Particular attention was paid by Cardesa-Dueñas et al. (2011) to the suitability of the PIV resolution in the current data sets for estimating several turbulence statistics. It was found that stresses were resolved adequately at this resolution. It remains to be shown that CFs are also free of any dependence on PIV resolution. Figure 4 displays the estimation of \( g \) at \( x/M = 1 \) for \( Rd44m_{\text{high}} \) by using \( 16 \times 16 \) pixels final interrogation windows, keeping all other PIV parameters equal. From the agreement of the two curves on figure 4, it can be concluded that in the following discussion the PIV resolution is not affecting the CFs.

4. Results and discussion
The evolution of the Reynolds shear stresses away from the grid can be seen on figure 5. For both grids, the shear stresses vanish along any cross-stream position at \( x/M \approx 7 \). Before \( x/M = 5 \),
they vary significantly depending on cross-stream location. It was shown by Cardesa-Dueñas et al. (2011) that the normal stresses $\langle u^2 \rangle$ and $\langle v^2 \rangle$ have become independent of cross-stream location by $x/M = 2$ for the present flow conditions. These results suggest that the shear stresses are therefore the lastest ones to homogenise accross the flow. That they vanish completely is a prerequisite of isotropy (von Karman & Howarth, 1938).

Figure 3. Mean velocity field behind a rod in wake of Rd44m_high. Regions of mean flow reversal reach $x/M = 1$. 16 × 16 pixels final interrogation windows were used.

Figure 4. Correlation function $g$ measured at $x/M = 1$ using two different final pass interrogation window sizes: □ 32 × 32 pixels; ■ 16 × 16 pixels. An interrogation window overlap of 50% was applied at both resolutions.

Figure 5. Downstream evolution of shear stresses along four different cross-stream locations: □Sq34b_high and Rd44b_high halfway between 2 vertical rods; ● Sq34m_high and Rd44m_high halfway between 2 vertical rods; ▲ Sq34b_high and Rd44b_high along centreline behind a vertical rod; ▶ Sq34m_high and Rd44m_high along centreline behind a vertical rod. (a) Sq34; (b) Rd44.
Figure 6 shows the evolution of $g$ in the downstream direction. At $x/M = 1$, the shape of $g$ on figure 6(a) reveals a clear influence from the presence of the grid rods. This close to the grid, small peaks in correlation occur in the cross-stream direction at intervals exactly equal to the rod spacing $1M$. At $x/M = 4$ one such peak still subsists, which is now further than $1M$ away from the reference point - see figure 6(b). This larger flow coherence could be due to the fact that, on average, two wakes are merging which result in a larger flow feature. Figure 6(c) shows that at $x/M = 8$ a large zone of cross-stream decorrelation exists, starting roughly $1M$ away from the reference point. But the interesting dynamics do not end here. Some degree of large-scale readjustment must still be taking place at this stage, since between $x/M = 8$ and $x/M = 12$ a marked negative excursion in $g$ appears - see figure 6(d) and figure 7. This can be thought of as a move towards isotropy, where $g$ is observed to be negative (Corrsin, 1963). Figure 7 shows that at the furthermost downstream position, both $f$ and $g$ assume their typical isotropic-like shape: $f$ remains positive and $g$ exhibits its negative region. Yet when estimating $f$ from $g$ by using a standard relation based on incompressibility and isotropy (Hinze, 1975, p. 185, eq. 3-10), disagreement between the measured $f$ and the $f$ predicted by isotropy is still significant at $x/M = 14$ - see figure 7.

A Reynolds number dependence on the wake dynamics of the grid was observed. The peaks in $g$ visible on figure 6(a) hint at a small degree of residual correlation in the vortex shedding of neighbouring rods. The magnitude of these correlation peaks decreases when moving towards more distant rods. This picture, which is valid at $Re_M = 16500$, differs significantly from

![Figure 6](image_url)
that at $Re_M = 5500$ and plotted on figure 8. At the lower Reynolds number, we observe two marked negative dips in correlation which dip to very similar negative values. Two things can be concluded from this curve: the two contiguous rods giving rise to the negative troughs in $g$ are strongly correlated with each other, and they must both result from a vortex shedding process which is (on average) close to anti-phase with that of the reference rod. The interaction between the vortex shedding of rows of bluff bodies close to each other has been studied previously for round or square cylinders - see for example Kumar et al. (2008); Huang et al. (2006) and references therein. It is therefore very likely that complex patterns of flow resonance are occurring between grid rods in both cross-stream directions and in a highly Reynolds number-dependent manner. A natural question that comes to mind is if this flow coherence, so prominent at $x/M = 1$, persists further downstream. Figure 8 suggests that such coherence is very short-lived, since $g$ measured at $x/M = 4$ bears no more signs of it. In fact, it appears very similar to $g$ on figure 6(b) recorded at the same position but for $Re_M = 16500$. This rapid loss of memory is probably a consequence of the vortex shedding from the horizontal rods, which shed their own vortices coming in and out of the measurement plane. These disturbances act towards breaking any initial coherence, and accelerate the homogenisation process.

5. Conclusions

The study of the turbulence statistics in the wake of two different grids reveals that the impact of the grid geometry on the shear stresses is negligible passed $x/M = 7$, where they have become effectively zero irrespective of cross-stream location. Their homogenisation is therefore slower than that of the normal stresses (Cardesa-Dueñas et al., 2011). The impact of the vicinity of the grid on the correlation functions is prominent at $x/M = 1$, where the rod spacing is clearly visible. Rapidly, this footprint of the grid disappears into a larger flow coherence, in what appears to be a strong but gradual move towards the expected isotropic turbulence behavior. At the furthermost downstream measurement location of $x/M = 14$, however, isotropy is still far from being satisfied as evidenced by the relation that should link $f$ and $g$. For the highest Reynolds number case, there is little correlation left between two adjacent rods. This contrasts with the interference patterns in the vortex shedding from contiguous rods visible at the lower Reynolds number regime. The impact of this early flow coherence is found to be short-lived, indicating a loss of memory resulting from the homogenisation that acts in the wake of the present grid geometries.
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