Feasibility of a spin light polarimeter at JLab

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Abstract. The future 12 GeV program at JLab includes several high precision experiments that aim to use parity violation in electroweak interactions to search for interactions beyond the Standard Model. These experiments require precision electron polarimetry with an uncertainty of $\sim 0.4\%$. Compton and Møller polarimeters are typically the polarimeters of choice for these experiments. However, a complimentary polarimetry technique based on the spin dependence of synchrotron radiation (SR), referred to as “spin-light,” is often overlooked. In this article we examine the feasibility of a “spin-light” polarimeter at Jefferson Lab (JLab) for 12 GeV experiments.

1. Introduction
The determination of the longitudinal polarization of the electron beam is one of the dominant systematic uncertainties in all the parity violating experiments that have been approved for the 12 GeV JLab. In order to achieve the desired high precision, the polarization of the electron beam must be monitored continuously with an uncertainty of $\sim 0.4\%$. In addition to being precise, the polarimeters must be non-invasive and must achieve the desired statistical precision in the shortest time possible. These ambitious goals can be achieved if multiple independent and high precision polarimeters are used simultaneously. We propose to develop a novel continuous polarimeter based on the spin dependence of synchrotron radiation (SR), referred to as “spin-light”. The proposed spin-light polarimeter can achieve statistical precision of 0.4% in measurement cycles of less than 10 minutes, which would significantly reduce the systematic uncertainty. In this paper we discuss the spin-light polarimeter in detail. The design is based on a 1993 proposal by Karabekov and Rossmanith [1].

2. Spin Light
The exact expression for SR intensity including quantum corrections was calculated by Sokolov, Ternov and Klepikov, based on the solution to the Dirac equation in the framework of quantum electrodynamics [2]. Sokolov and Ternov also developed the mathematics required to describe the spin of relativistic electrons moving in an external electromagnetic field [3, 4], which allowed them to calculate the electron spin related properties of SR. The spin dependence of the SR was verified at the VEPP-4 storage ring in Novosibirsk [5] for 5 GeV transversely polarized electrons. Ordinarily one would expect the quantum effects to become important if the acceleration of the electron is comparable to the acceleration at which a single SR photon would carry away all of the electron’s energy (called the critical condition). The critical magnetic field strong enough to provide this acceleration is found to be $B_c = \frac{m_e^2 c^3}{e \hbar} \sim 4 \times 10^9$ T and the critical energy
$E_e = m_e c^2 \sqrt{\frac{mc^2 R}{\hbar}} \sim 10^6$ GeV [3, 6], where $e$, $m_e$, and $R$ are the charge, mass and orbital radius of the electron. Since the critical energy and field are extremely large compared to those accessible at present day terrestrial accelerators, the quantum corrections to the SR intensity in a magnetic field $B$ and a Lorentz boost of $\gamma$, can be expanded in terms of the critical parameter $\xi = \frac{\mu_B}{m_e c^2 R} \gamma << 1$, and are relatively small. However, it turns out that several quantum effects, such as the spin dependence of SR are important even at considerably lower electron energies and fields.

The total SR power radiated by longitudinally polarized electrons, ignoring spin flip terms and other terms of order $\xi^2$, is given by [3]:

$$P_\gamma(\text{long}) = \frac{9n_e}{16\pi^3} \frac{c^2}{R^2} \frac{\gamma^5}{\gamma_C} \int_0^\infty \frac{y^2 dy}{(1 + \xi y)^4} \int d\Omega (1 + \alpha^2)^2 \times$$

$$\left[ K_{2/3}(z) + \frac{\alpha^2}{1 + \alpha^2} K_{1/3}(z) + j\xi y \frac{\alpha}{\sqrt{1 + \alpha^2}} K_{1/3}(z) K_{2/3}(z) \right],$$

where $n_e$ is the number of electrons, $j$ is the electron spin polarization, $\omega$ is the angular frequency of the SR photon, $\omega_C$ is the critical angular frequency, and $y = \frac{\omega}{\omega_C}$, $K_n(z)$ are the modified Bessel functions, $z = \frac{\omega}{\omega_C} (1 + \alpha^2)^{3/2}$, and $\alpha = \gamma \psi$, with $\psi$ being the vertical angle in the frame of the moving electron. The spin dependent term in equation 1 is an odd function of the vertical angle therefore when integrated over all angle the total SR power for longitudinally polarized electrons is spin independent. However, the power radiated into the space above ($0 < \psi < \pi/2$) and below ($-\pi/2 < \psi < 0$) the orbital plane of the electron are different and the difference between them is spin dependent [3, 4]. This offers a new possibility for direct observation of the polarization characteristics of an electron beam by determining the difference in the SR power above and below the electron beam direction.

To examine the size and characteristics of this spin dependence we have numerically integrated equation 1 for $I_e = 100$ $\mu$ A, $E_e = 11$ GeV, longitudinally polarized electron with 100% polarization, in a 4 T magnetic field. We have integrated over a horizontal angular acceptance of $\Delta \theta = 10$ mrad, and a vertical acceptance of $\alpha = \pm 1$. The total power radiated by longitudinally polarized electrons, $P_\gamma(\text{long})$, per MeV and the spin dependent difference in power radiated above and below the orbital plane of the electron (spin-light), $\Delta P(\text{long})$, per MeV are shown in figure 1(a). The number of SR photons, $N_\gamma(\text{long})$, per MeV and the number of spin-light photons $\Delta N_\gamma(\text{long})$, per MeV, as function of photon energy are

![Figure 1](image-url)
shown in figure 1(b). The asymmetry defined as \( A = \frac{\Delta N_{\gamma}(\text{long})}{N_{\gamma}(\text{long})} \) as a function of photon energy is shown in figure 1(c). This indicates that one should measure the hard tail of the SR spectrum \( (E_{\gamma} > 500 \text{ keV}) \) and avoid the soft part of the spectrum where the asymmetry is low and changes rapidly with energy. Although the asymmetry is small \( \sim 10^{-4} \) the photon flux is high, even at the hard tail of the spectrum, allowing a rapid determination of the asymmetry, with 1% statistical uncertainty \( (\frac{\delta A}{A} = \frac{1}{\sqrt{2N}}) \) within a few tens of seconds, as shown in figure 1(d). The energy dependence of the asymmetry for \( E_{e} = 4 - 11 \text{ GeV} \) and the magnetic field dependence of the asymmetry for \( B_{\text{wriggler}} = 2 - 5 \text{ T} \) are shown in figure 2(a) and figure 2(b) respectively. These figures demonstrate that spin-light based polarimetry is a very promising technique at high energies and can be used to monitor the relative polarization of 11 GeV electrons in very rapid measurement cycles, with high statistical precision, which would help reduce the polarimetry related systematic uncertainties in the PV electron scattering experiments.

2.1. A Conceptual Design

The two basic components of a spin-light based polarimeter are the source of SR and the X-ray detector which can measure the spatial asymmetry.

2.1.1. The SR Source - Wriggler

A three pole wriggler magnet with a magnetic field that has uniform magnitude but reversed direction at each pole and a short-long-short pole arrangement is well suited as a source of SR. The three poles must be symmetric about the center such that the line integral of the magnetic field in the direction of the motion of the electron, \( z \), must be zero (i.e. \( \int B(z)dz = 0 \)), ensuring that it does not effect the electron beam transport and its spin direction (beyond the wriggler). The magnitude of the field being constant at the three poles, flips the sign of the spin dependent spatial asymmetry from any two adjust poles and hence when measured simultaneously it can help reduce systematic uncertainties arising from the vertical motion of the beam.

The intensity and the asymmetry both increase with increasing field strength, while the pole length decreases with increasing field strength. Therefore a field strength of 4 T is a judicious choice for the wriggler field. A 10 mrad bend can be achieved with a pole length of 10 cm. Thus the total magnet length is 40 cm, and the spacing between the poles is optimized for ease of extraction and detection of the SR beam. A separation of 1 m between the poles allows for collimators to be placed that can separate the SR beams spots from the different poles. The small pole length ensures that the effect of spin-flip inducing SR and the fluctuation of the SR power are negligible (< 0.1%).

The number of photons emitted by a particular electron per radian bend will be distributed according to Poisson statistics about a mean value given by \( n = \frac{5}{2\sqrt{3}} \frac{2\gamma}{137} = 20.62E \), where \( n \) is the mean number of photons per radian bend, \( E \) is the beam energy in GeV. The average...
energy of the photons emitted is \( E_c = h\omega_c = \frac{3}{2} h\nu_c \). Therefore the mean energy fluctuation is given by \( \Delta E = \sqrt{nE_c} \). It is interesting to note that the energy fluctuation depends only on the electron beam energy and the bend radius of the wriggler. A beam of 11 GeV electrons in a 4 T wriggler with a 10 m bend radius and a bend angle of 10 mrad, gives \( n \sim 2 \) and \( E_c = 199 \text{ KeV} \). Therefore \( \Delta E/E \sim 2.5 \times 10^{-5} \), which is comparable to the fluctuations due to the recirculating arcs of the JLab accelerator [8].

The SR power spectrum usually peaks at angles of \( \pm 1/\gamma \) with respect to the electron direction. The r.m.s. kick from the emission of \( n \) photons is given by \( \sqrt{n\Delta \theta_c} \). Thus for a 11 GeV beam bend by 10 mrad the r.m.s. kick is \( \sim 1.5 \times 10^{-8} \text{ rad} \), which is negligible. Thus the wriggler magnet would have negligible influence on the electron beam and a spin-light polarimeter can be used for non-invasive monitoring of the relative beam polarization.

2.1.2. The X-ray Detector - Ionization Chamber

The detector used to measure the spatial asymmetry must be sensitive to \( \sim 500 \text{ keV} \) to \( 2.5 \text{ MeV} \) X-rays and must be able to pick out a small asymmetry from a large spin independent background, it must be radiation hard, have low noise and be able to withstand high rates of \( \sim 10^{12} \text{ photons/sec} \). Ionization chambers (IC) are well known for their high rate capability (when operated in current mode), low electronic noise and radiation hardness. Argon/Xenon is an attractive candidate for use as an ionization medium, its high atomic number (18/54) and density (when compressed) gives it a high stopping power for hard X-rays and low energy gamma [10].

Another recent development, is the split collector ionization chamber that have turned the IC into a position sensitive device. Position sensitive ionization chambers are designed to have the collector plate split into two sections in a zig-zag/backgammon pattern such that each half operates as an independent ionization chamber. A prototype of such a chamber has been shown in figure 3. These chambers were developed at the Advanced Photon Source at the Argonne National Lab and at the SPring-8 light source in Japan. They are used to measure the vertical position of X-ray beams and have been shown to have a resolution of \( 5 \mu m \) [9]. These chambers also have very low dark currents in the \( ~pA \) range and have been operated at photon flux of \( 5.0 \times 10^{12} \text{ photons/sec} \). They work by measuring the difference in counts between the two halves of the chamber, i.e. they are differential ionization chambers. A position sensitive differential ionization chamber operated in current mode can be used to measure the spatial asymmetry of the SR generated by longitudinally polarized electrons. A dual, 1 atm. Ar/Xe differential ionization chamber would be ideal for a relative polarimeter. The chamber would consist of Ti or stainless steel windows thick enough to cut down the low energy X-rays (\( < 50 \text{ keV} \)). A split central collector/ground plate would be placed between the anode and the cathode. The collector plate would be split in a backgammon pattern. The current measured on each half of the collector plate is amplified with a current amplifier. The SR beam from two adjacent poles will be incident on the two chambers This ensures that the spin-light spatial asymmetry (above and below the
orbital plane) has opposite sign in the left/right chambers since the magnetic field direction of adjacent poles are opposite. On the other hand any spatial asymmetry due to vertical motion of the beam will have the same sign in the left/right chambers. A schematic of the complete polarimeter is shown in figure 4.

![Schematic of the polarimeter](image)

**Figure 4.** A schematic for a differential spin-light polarimeter (not to scale).

The major sources of systematic uncertainties for a spin-light polarimeter are background asymmetries from processes such as Bremsstrahlung and false asymmetry due to vertical beam motion, differences in chamber efficiency and magnetic field non-uniformity between adjacent poles of the wriggler. The main advantage of operating the ionization chambers as differential detectors is that the false asymmetries will cancel to first order. The 3-pole design ensures that the vertical beam motion related false asymmetry also cancels to first order. The background asymmetries can be monitored using the difference in the signal from the chambers with the wriggler magnets turned on and off. We estimate the systematic uncertainties of a relative polarimeter to be < 0.5 %.

### 2.2. Summary
Spin light based polarimetry was demonstrated over 30 years ago, but has been ignored since then. The figure of merit for such a polarimeter increases with electron beam energy and the strength of magnetic field used. The 11 GeV beam at JLab is well suited for spin light polarimetry and such a polarimeter would help achieve the < 0.5 % polarimetry desired by experiments approved for the 12 GeV era. A 3-pole wriggler with a field strength of 4 T and a pole length of 10 cm would be adequate for such a polarimeter. A dual position sensitive ionization chambers with split anode plates is ideally suited as the X-ray detector for such a polarimeter. The differential detector design would help reduce systematic uncertainties. Locating a reasonable piece of beam-line real estate is however very challenging.

### References
[1] Karabekov I P and Rossmannith R 1993 *Proc. of the 1993 PAC, Washington* I 457; Karabekov I P and Karabekian S I 1996 *Proceedings of 5th European Particle Accelerator Conference (EPAC 96)*, Sitges, Spain 1743.
[2] Sokolov A A, Klepikov N P and Ternov I M 1952 *JETF* 23 632.
[3] Sokolov A A and Ternov I M 1968 *Synchrotron Radiation* Pergamon Press, New York; Sokolov A A and Ternov I M 1986 *Radiation from Relativistic Electrons* A.I.P. Translation Series, New York.
[4] Ternov I M 1978 *Physics - Uspekhi* 38 409.
[5] Belomestnykh S A et al. 1984 *Nucl. Inst. and Meth.* 227 173.
[6] Ternov I M 1978 *Nucl. Instr. and Meth.* 152 213.
[7] Sands M 1970 *SLAC Technical note SLAC-121*. 
[8] Norum B 1985 CEBAF Technical note TN-0019.
[9] Sato K 2001 J. of Synchrotron Rad. 8, 378; Gog T, Casa D M and Kuzmenko I 2000 CMC-CAT technical report.
[10] Bolotnikov A E and Ramsey B 1997 Nucl. Inst. and Meth. A396 360.
[11] Nakamura E et al. 1996 J. of Elec. Spec. and Rel. Phen. 80 421; Baynham D E, Clee P T M, and Thompson D J 1978 Nucl. Instr. and Meth. 152 31.