Compilation of Coordinated Choice

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May 5, 2020

Abstract

Recently, we have proposed coordinated choices, which are nondeterministic choices equipped with names. The main characteristic of coordinated choices is that they synchronize nondeterministic decision among choices of the same name.

The motivation of the synchronization mechanism is to solve a theoretical problem. So, as a practical programming language, we still want to use coordinated choices like standard ones. In other words, we want to avoid synchronization. Now, there are two problems: (i) practically, it is a bit complicated work to write a program using coordinated choices in which execution synchronization never happens; and (ii) theoretically, it is unknown whether any programs using standard choices can be written by using only coordinated ones.

In this paper, we define two simply typed lambda calculi called $\lambda^b$ equipped with standard choices and $\lambda^c$ equipped with coordinated choices, and give compilation rules from the former into the latter. The challenge is to show the correctness of the compilation because behavioral correspondence between expressions before and after compiling cannot be defined directly by the compilation rules. For the challenge, we give an effect system for $\lambda^c$ that characterizes expressions in which execution synchronization never happens. Then, we show that all compiled expressions can be typed by the effect system. As a result, we can easily show the correctness because the main concern of the correctness is whether synchronization happens or not.

1 Introduction

Nondeterministic choice, written $(M_1 \parallel M_2)$ in this paper, is a rather well-known construct formalizing nondeterministic computation. Nondeterminism is typically accomplished by the following two operational rules: $(M_1 \parallel M_2) \longrightarrow M_1$ and $(M_1 \parallel M_2) \longrightarrow M_2$. So, for example, $(1 \parallel 2) + (3 \parallel 4)$ is evaluated into one of the three possible values: 4, 5, 6.

Recently, we had proposed coordinated choices \cite{4}, written $(M_1 \parallel^\Phi M_2)$. It is a kind of nondeterministic choice but equipped with a name $\Phi$. The characteristic point of coordinated choices is that nondeterministic evaluation is synchronized among choices of the same name. That means, for instance, $(1 \parallel^b 2) + (3 \parallel^a 4)$ only results 4, 6. The essential purpose of developing coordinated choices is to solve a theoretical problem that happens when we try to introduce nondeterministic choices into a
dependently typed system. In other words, the proposed system does not have nondeterministic choices.

Now, it is a natural and important demand for practically and theoretically, how we use coordinated choices like standard ones. In this paper, we consider how to simulate usual choices with coordinated choices. Concretely, we give a compilation algorithm from a simply typed lambda calculus with nondeterministic choices into one with coordinated choices; and show an evaluation in the former calculus is simulated by the latter calculus, and vice versa.

2 Compilation

In this section, we introduce the compilation rules and discuss the correctness of the given rules. First of all, we recall how a coordinated choice is synchronized. The following is an example of a reduction sequence in which synchronization happens.

\[
\text{add}(1 \parallel 2)(3 \parallel 4) \xrightarrow{\emptyset} \ast \text{add} 1 \parallel \text{add} 2)(3 \parallel 4)
\]

\[
\emptyset \xrightarrow{\ast} \text{add} 1(3 \parallel 4) \parallel \text{add} 2(3 \parallel 4))
\]

\[
\emptyset \xrightarrow{\ast} \text{add} 1(3 \parallel 4) \parallel \text{add} 2(4)
\]

\[
\emptyset \xrightarrow{\ast} 4 \parallel 6
\]

In the previous work, we had made two design decisions to formalize nondeterministic choices. One is that we express the possibility of execution as of the form of a choice rather than branches of reduction sequences as we have seen in Section 1. So, the result of execution above becomes the choice, which expresses the possible results are only 4 and 6. Another is that we adopt call-time semantics [1], which makes nondeterministic choice happening before a function call. The distribution of the first argument for \(\text{add}\) in the first line, for example, accomplishes this semantics.

The devices that enable synchronization are following rules, which are used in the evaluation from the second line to the third line.

\[
M_1 \xrightarrow{\Delta \cup \{\Phi_+\}} M'_1
\]

\[
(M_1 \parallel_\Phi M_2) \xrightarrow{\Delta} (M'_1 \parallel_\Phi M_2)
\]

\[
(M_1 \parallel_\Phi M_2) \xrightarrow{\Delta \cup \{\Phi_+\}} M_1
\]

So, concentrating on one step of an evaluation, we can easily find that a sufficient condition to prevent synchronization is that every name of choices in an expression before the evaluation is distinct. However, this condition is easily broken by evaluation since some reduction rules, e.g., \((\lambda x.M) V \xrightarrow{\Delta} M[x := V]\), duplicate an expression. In other words, it is easy to prevent synchronization in the first step of an evaluation, but it is hard to prevent synchronization in every step after the first step of an evaluation.
The following reduction sequence shows synchronization caused by duplication of an expression. In the first line, recursive call of $\lambda x$.f duplicates the name $\Phi$.

$$(\mu f. \lambda x.(f x A f x)) 0 \not\rightarrow (f' 0 A f' 0)$$

$$(f' 0 A f' 0) A (f' 0 A f' 0) \not\rightarrow (f' 0 A f' 0) \not\rightarrow \ldots$$

where $f' = \mu f. \lambda x.(f x A f x)$.

To avoid the synchronization, the proposed system also has name abstraction mechanism, i.e., name abstractions $\Lambda x.\Phi$; name applications $\Phi \cdot \Phi$; and name concatenations $\Phi \cdot \Phi$. Fortunately, thanks to call-by-value semantics, names that will be duplicated and become a cause of synchronization only occur in the body of lambda abstractions. So we abstract names (giving a name containing variables) occurring in the body of lambda abstractions and instantiate the names as distinct ones where the lambda abstractions are called. The following two evaluation sequences show the effect of this idea.

$$(\mu f. \lambda x.\Lambda a.(f x a A f x a)) 0$$

$$(f' 0 A f' 0 A) \not\rightarrow (f' 0 A f' 0 A) \not\rightarrow (f' 0 A f' 0 A)$$

where $f' = \mu f. \lambda x.\Lambda a.(f x a A f x a)$. 

In the first sequence, the duplicated name $\Phi$ causes synchronization. In the second sequence, on the other hand, the name of a coordinated choice becomes different one at every time that a lambda abstraction is applied (so synchronization does not happen).

Summing up the discussion, concrete compilation rules become the ones in Figure 1. The function $\|\|_\phi^a$ gives the compilation, where $\phi$ and $a$ are additional parameters, called seeds, used for generating distinct names during a compilation. Note that the same seeds are given for the sub-expressions’ compilation of a choice. It does not
cause a problem because the left and right sides of a choice are completely separated, i.e., the sub-expressions never collaborate.

2.1 Correctness of compilation

The main contribution of this paper is that we have formally shown that the compilation is correct. Informally, correctness of a compilation is stated as—a compiled expression behaves the same as the original expression does. This behavioral correspondence between compiled expressions and original expressions is formally defined as a binary relation, called bisimulation, between them. For instance, if we use the compilation rules as the relation, what we need to show is stated as the following two propositions.

**Proposition 2.1.** If $\|e\|_\Phi = M$ and $e \rightarrow e'$, then $M \xrightarrow{\Delta} M'$ and $\|e'\|_\Phi = M'$.

**Proposition 2.2.** If $\|e\|_\Phi = M$ and $M \xrightarrow{\Delta} M'$, then $e \rightarrow e'$ and $\|e'\|_\Phi = M'$.

However, we can easily find a counter-example for the reduction $(\lambda x.x)\lambda x.x \rightarrow \lambda x.x$ as follows.

A seed is alternated by the reduction. Unfortunately, weakening the proposition as follows does not help us, because, if a sub-expression (for example, the argument part of an application) is evaluated, only the seed for the sub-expression part is alternated and the whole expression after the evaluation does not follow the compilation rule.

**Proposition 2.3.** If $\|e\|_\Phi = M$ and $e \rightarrow e'$, then $M \xrightarrow{\Delta} M'$ and $\|e'\|_\Phi = M'$ for some $\Phi'$.

Ultimately, we believe that the relation cannot be obtained directly by a compilation algorithm. That means it is not because of how the compilation rules are defined. To prevent coordination of coordinated choice, we need to give distinct names for each coordinated choice, which is a sub-expression, but as we have seen in the counter-example, a sub-expression could pop-up to the top level, which leads us to an inconsistency.

Summarizing the discussion, we could see that coordination does not happen in the first step of the evaluation for a compiled expression; but we cannot guarantee the property after the first step because an expression can have no relation to the compilation rules. So, we take another indirect strategy. Firstly, we define a relation characterizing expressions that do not cause coordination and is preserved by an evaluation, and then we show the image of the compilation is included in the relation. After accomplishing this, we can easily obtain a bisimulation relation since the compilation just inserts name abstractions and applications—an evaluation after compilation just involves (R-Stigma) reduction in some points.
Formal System

We formalize the idea as $\lambda^\|$, a simply typed lambda calculus with a fix-point operator and non-collapse choices, and $\lambda^{\|\omega}$, a simply typed lambda calculus with a fix-point operator and coordinated choice; and give a compilation rule from the former to the latter.

3.1 Source Language: $\lambda^\|

The syntax, semantics, and type system of $\lambda^{\|\omega}$ are defined as Figure 2, Figure 3, and Figure 4, respectively. Those are standard ones for a simply typed lambda calculus and with straightforward extensions for a fix-point operator $\mu f . e$ and non-collapse choices $(e_1 \parallel e_2)$. "Non-collapse" means that a choice does not really choose an alternative (like $(e_1 \parallel e_2) \longrightarrow e_1$) as coordinated choice does not, but also it never coordinate alternatives.

3.2 Target Language: $\lambda^{\|\omega}$

The syntax and semantics of $\lambda^{\|\omega}$ are shown in Figure 5 and Figure 6, respectively. As far as the semantics, $\lambda^{\|\omega}$ is just a simply typed lambda calculus with coordinated choice, c.f. $\lambda^{H\|\omega}$. An interesting part of $\lambda^{\|\omega}$ is an effect system [3], which is a kind of type system. The effect system of $\lambda^{\|\omega}$ estimates names happening during execution and rejects an expression that will cause synchronization, and so this is the relation that we have mentioned at the last of Section 2.

We range over effects with a meta-variable $\varphi$. Effects of $\lambda^{\|\omega}$ are denoted by regular
\[ \begin{align*}
\emptyset & \vdash \text{ok (SW-EMPTY)} \\
\Gamma, x : T & \vdash \text{ok (SW-Push)} \\
\Gamma & \vdash x : T & (\text{ST-Var}) \\
\Gamma & \vdash e_1 : T_1 \rightarrow T_2 & \Gamma & \vdash e_2 : T_1 & (\text{ST-App}) \\
\Gamma & \vdash e_1 e_2 : T_2 \\
\Gamma, x : T & \vdash e : T_2 & (\text{ST-Ans}) \\
\Gamma & \vdash \lambda x . e : T_1 \rightarrow T_2 \\
\Gamma, f : T_1 \rightarrow T_2 & \vdash \lambda x . e : T_1 \rightarrow T_2 & (\text{ST-Fix}) \\
\Gamma & \vdash e_1 : T & \Gamma & \vdash e_2 : T & (\text{ST-Choice}) \\
\Gamma & \vdash (e_1 \parallel e_2) : T
\end{align*} \]

\[ \text{Figure 4: Typing rules for } \lambda^\parallel \]

\[ \begin{align*}
\Phi & ::= \alpha | e | \emptyset | \bullet | \Phi_1 \Phi_2 \\
\varphi & ::= \emptyset | \Phi | \varphi_1 \varphi_2 | \varphi_1 + \varphi_2 | \varphi^* \\
\tau & ::= \text{nat} \mid \tau_1 \leftrightharpoons \tau_2 \mid \forall \alpha. \varphi \tau \\
M & ::= x \mid M_1 M_2 \mid \lambda x . M \mid \mu f . M \mid (M_1 \parallel M_2) \\
\Xi & ::= \emptyset \mid \Xi, x : \tau \mid \Xi, \alpha \\
\nu & ::= \lambda x . M \mid \Lambda \alpha . M
\end{align*} \]

\[ \text{Figure 5: Syntax of } \lambda^{\parallel_0} \]
\[(\lambda x. M) \xrightarrow{\Delta} M [x := V] \quad \text{(TR-BETA)} \quad (\Lambda \alpha. M) \xrightarrow{\Delta} M[\alpha := \Phi] \quad \text{(TR-SIGMA)}\]
\[
\mu f. \lambda x. M \xrightarrow{\Delta} (\lambda x. M)[f := \mu f. \lambda x. M] \quad \text{(TR-FIX)}
\]
\[
(M_1 \parallel^\Phi M_2) M_3 \xrightarrow{\Delta} (M_1 M_3 \parallel^\Phi M_2 M_3) \quad \text{(TR-DISTAPPL)}
\]
\[
V (M_1 \parallel^\Phi M_2) \xrightarrow{\Delta} (V M_1 \parallel^\Phi V M_2) \quad \text{(TR-DISTAPPR)}
\]
\[
(M_1 \parallel^\Phi M_2) \Phi_2 \xrightarrow{\Delta} (M_1 \Phi_2 \parallel^\Phi M_2 \Phi_2) \quad \text{(TR-DISTSAPP)}
\]
\[
M_1 \xrightarrow{\Delta} M'_1 \\ M_2 \xrightarrow{\Delta} M'_2 \\
\text{(TR-APPL)} \\ M \xrightarrow{\Delta} M' \\ V \xrightarrow{\Delta} V'
\]
\[
M \Phi \xrightarrow{\Delta} M' \Phi \quad \text{(TR-SAPP)} \\
M_1 \xrightarrow{\Delta} M'_1 \\ M_2 \xrightarrow{\Delta} M'_2 \\
\text{(TR-CHOICE)} \\
(M_1 \parallel^\Phi M_2) \xrightarrow{\Delta} (M_1 \parallel^\Phi M_2)
\]
\[
(M_1 \parallel^\Phi M_2) \xrightarrow{\Delta} M_1 \quad \text{(TR-WORLD)} \\ (M_1 \parallel^\Phi M_2) \xrightarrow{\Delta} M_2 \quad \text{(TR-WORLD)}
\]

Figure 6: Operational semantics of $\lambda^{\parallel\Phi}$

\[
\begin{align*}
\text{nat} &\ll\text{nat} \quad \text{(TS-NAT)} \\
\frac{\tau_{21} \ll \tau_{11} \quad \tau_{12} \ll \tau_{22} \quad (L(\varphi_1) \subseteq L(\varphi_2))}{\tau_{11} \xrightarrow{\varphi_1} \tau_{12} \ll \tau_{21} \xrightarrow{\varphi_2} \tau_{22}} \quad \text{(TS-ARROW)} \\
\frac{\forall \alpha. \varphi_1 \ll \tau_1 \quad (L(\varphi_1) \subseteq L(\varphi_2))}{\forall \alpha. \varphi_1 \ll \tau_2 \quad (L(\varphi_1) \subseteq L(\varphi_2))} \quad \text{(TS-FORALL)}
\end{align*}
\]

Figure 7: Effect system of $\lambda^{\parallel\Phi}$ (1): subtyping rules

expressions: the empty set $\varnothing$; literal strings $\Phi$ including the empty string $\epsilon$; concatenations $\varphi_1 \varphi_2$; alternations $\varphi_1 + \varphi_2$; and Kleene star $\varphi^*$. Function types and forall types are annotated by effects, which express the effect happening when a function is used. Forall types $\forall \alpha. \varphi \tau$ bind $\alpha$ in $\varphi$ and $\tau$.

**Definition 3.1.** A closed term is one in which both free variables and free name variables are empty. We denote closed terms with over-barred meta-variables, e.g., $\overline{\Phi}$, $\overline{\varphi}$.

**Definition 3.2.** We denote the language, a set of names, represented by an effect $\varphi$ as $L(\varphi)$.
\[ \emptyset \text{ ok (TW-EMPTY)} \quad \Xi \text{ ok} \quad \Xi \vdash \tau \quad (x \notin \text{dom}(\Xi)) \quad (\text{TW-PUSH}) \]

\[ \Xi \text{ ok} \quad (a \notin \text{ndom}(\Xi)) \quad (\text{TW-PushS}) \]

\[ \Xi \text{ ok} \quad (a \in \Xi) \quad (\text{TW-NVAR}) \quad \Xi \text{ ok} \quad (\text{TW-Eps}) \quad \Xi \text{ ok} \quad (\text{TW-On}) \]

\[ \Xi \vdash \bullet \quad (\text{TW-Off}) \quad \Xi \vdash \Phi_1 \quad \Xi \vdash \Phi_2 \quad (\text{TW-APPEND}) \quad \Xi \text{ ok} \quad (\text{TW-EMPTY}) \]

\[ \Xi \vdash \varphi_1 \quad \Xi \vdash \varphi_2 \quad (\text{TW-DOT}) \quad \Xi \vdash \varphi_1 + \varphi_2 \quad (\text{TW-PLUS}) \]

\[ \Xi \vdash \varphi^\ast \quad (\text{TW-STAR}) \quad \Xi \vdash \tau_1 \quad \Xi \vdash \tau_2 \quad (\text{TW-ARRROW}) \quad \Xi, \alpha \vdash \tau \quad \Xi, \alpha \vdash \varphi \quad (\text{TW-FORALL}) \]

Figure 8: Effect system of \( \lambda^{\varphi_0} \) (2): well-formedness rules

\[ \Xi \vdash M_1 : \Phi\bar{\varphi}_1 \quad \Xi \vdash M_2 : \tau_1 & \Phi\bar{\varphi}_2 \quad \Xi \vdash M_2 : \tau_1 & \Phi\bar{\varphi}_3 \quad (\bigcup_{k \in \{1,2,3\}} L(\bar{\varphi}_k)) \quad (\text{TT-APP}) \]

\[ \Xi, x : \tau_1 \vdash M : \tau_2 & \varphi \quad (\text{TT-APP}) \]

\[ \Xi \vdash \lambda x. M : \tau_1 \rightarrow \tau_2 & A \quad (\text{TT-ABS}) \]

\[ \Xi \vdash \forall \alpha \varphi_1 : \Phi & \Phi' \bar{\varphi}_2 \quad \Xi \vdash \Phi \quad (\ll(\varphi_1[\alpha := \Phi])) \subseteq \ll(\Phi' \bar{\varphi}_2) \quad (\ll(\bar{\varphi}_2) \cup \ll(\bar{\varphi}_3)) \quad (\text{TT-SApp}) \]

\[ \Xi \vdash M : \tau \quad (\text{TT-SAbs}) \]

\[ \Xi \vdash \mu f. \lambda x. M : \tau_1 \rightarrow \tau_2 & A \quad (\text{TT-Fix}) \]

\[ \Xi \vdash M_1 : \tau & \Phi' \bar{\varphi}_1 \quad \Xi \vdash M_2 : \tau & \Phi' \bar{\varphi}_2 \quad \Xi \vdash \Phi \quad (\ll(\Phi) \subseteq \ll(\Phi' \bar{\varphi}_2)) \quad (\ll(\bar{\varphi}_2) \cup \ll(\bar{\varphi}_3)) \quad (\text{TT-Choice}) \]

\[ \Xi \vdash (M_1 \parallel M_2) : \tau & \Phi' (\bar{\varphi}_1 + \bar{\varphi}_2) \quad (\text{TT-SUB}) \]

\[ \Xi \vdash M : \tau_1 & \varphi_1 \quad \Xi \vdash \tau_2 \quad \Xi \vdash \varphi_2 \quad r_1 \ll r_2 \quad (\ll(\varphi_1) \subseteq \ll(\varphi_2)) \quad (\text{TT-SUB}) \]

Figure 9: Effect system of \( \lambda^{\varphi_0} \) (3): typing rules
3.3 Effect System

The effect system for $\lambda^{\|}$ consists of the subtyping relation $\tau_1 \ll \tau_2$; the well-formedness relations $\Xi \text{ok}$, $\Xi \vDash \Phi$, $\Xi \vDash \varphi$, and $\Xi \vDash \tau$; and typing relation $\Xi \vdash M : \tau \& \varphi$.

Well-formedness rules just check a closedness of types by (TW-hyphen.scNV /a.sc/r.sc) and (TW-hyphen.scF/o.sc/r.sc/a.sc/l.sc/l.sc). Typing rules are the heart of the effect system. The judgment $\Xi \vdash M : \tau \& \varphi$ denotes a usual typing relation by the left side of $\&$ and what names will occur during evaluation of $M$ by the right side of $\&$. So, ignoring the right side of $\&$, rules are already familiar. For the effect part, the rules become complicated because of showing meta-properties. (TT-hyphen.scV /a.sc/r.sc), (TT-hyphen.scA/b.sc/s.sc), (TT-hyphen.scSA/b.sc/s.sc), and (TT-hyphen.scF/i.sc/x.sc) are rather easy. Corresponding expressions for those are never evaluated (fix-point operator is evaluated but soon reaches a value); and therefore, those produce no names examined. This is why the effect part of those rules becomes empty. Note that functions’ body is evaluated when an argument is given; so the effect of the body is recorded on types and added at an application point, c.f. (TT-Abs) and (TT-App).

(TT-App), (TT-SApp), and (TT-Choice) have complicated side-conditions. To explain the rules, we will start from the following rather ideal and simple rule for coordinate choices.

$$
\begin{align*}
\Xi \vdash M_1 : \tau \& \varphi_1 & \quad \Xi \vdash M_2 : \tau \& \varphi_2 & \quad \Xi \vDash \Phi \\
\Xi \vdash (M_1 \parallel \Phi M_2) : \tau \& \varphi_1 + \varphi_2 + \Phi
\end{align*}
$$

This rule just estimates the names which occur along an evaluation and impose no restriction. $M_1$ will cause names represented by the regular expression $\varphi_1$, $M_2$ will cause names represented by the regular expression $\varphi_2$, and the choice itself already causes the name $\Phi$. So the whole effect is the alternation of those.

The next step is to introduce a restriction to the rule. The idea of restriction is so simple—coordination never happens if names of sub-expressions do not overlap. So the rule will become the following.

$$
\begin{align*}
\Xi \vdash M_1 : \tau \& \varphi_1 & \quad \Xi \vdash M_2 : \tau \& \varphi_2 & \quad \Xi \vDash \Phi \\
\Xi \vdash (L(\varphi_1) \cup L(\varphi_2) \cup L(\Phi)) \\
\Xi \vdash (M_1 \parallel \Phi M_2) : \tau \& \varphi_1 + \varphi_2 + \Phi
\end{align*}
$$

The last step comes from a technical reason. The naive side-condition breaks subject reduction lemma. We need the following property for the subject reduction lemma.

**Proposition 3.3.** If $\Xi, \alpha \vdash M : \tau \& \varphi$ and $\text{fnv}(\Phi) \subseteq \text{ndom}(\Xi)$, then $\Xi \vdash M[\alpha := \Phi] : \tau[\alpha := \Phi] \& \varphi[\alpha := \Phi]$.

However, the name substitution for the effect breaks the disjunctivity. (For instance, $L(\alpha)$ and $L(\emptyset)$ are disjunctive, but $L(\alpha[\alpha := \emptyset])$ and $L(\emptyset[\alpha := \emptyset])$ are not.)
To amend the problem we adopt more specific side-condition as follows and make the effect in the conclusion as $\Phi'(\bar{\varphi}_1 + \varphi_2 + \bar{\varphi}_3)$.

\[
\begin{align*}
\text{L}(\varphi_1) &\subseteq \text{L}(\Phi') \\
\text{L}(\varphi_2) &\subseteq \text{L}(\Phi') \\
\text{L}(\Phi) &\subseteq \text{L}(\Phi') \\
\text{L}(\bar{\varphi}_1) &\not\subseteq \text{L}(\varphi_2) \cup \text{L}(\bar{\varphi}_3)
\end{align*}
\]

The point is that string variables are gathered into the prefix and name disjunctivity is guaranteed by the closed effects, which are not altered by a substitution. Indeed, this side-condition quite respects the compilation rules, which create a fresh name by appending the constants $\varnothing$, $\bullet$ to the tail of seed and a string variable is pushed to the head of the seed.

The $(\text{TT-Choice})$ is almost obtained. We can drive away the first two subset relation into $(\text{TT-Sub})$. Additionally, it can be seen that the effects from left and right side of a choice, namely $\varphi_1$ and $\varphi_2$, need not be distinct since both sides never collaborate. So, we can take one large effect $\bar{\varphi}$ which subsume $\varphi_1$ and $\varphi_2$ instead of $\bar{\varphi}_1$ and $\bar{\varphi}_2$, i.e., $\text{L}(\varphi_1) \subseteq \text{L}(\Phi')$ and $\text{L}(\varphi_2) \subseteq \text{L}(\Phi')$.

$(\text{TT-App})$ and $(\text{TT-SApp})$ are constructed in a similar manner.

4 Property

In the proof of this section, we implicitly use well-known properties about regular expressions, e.g., $\text{L}(\psi \varphi) = \text{L}(\varphi)$, $\text{L}(\varphi_1 \varphi_2 + \varphi_1 \varphi_3) = \text{L}(\varphi_1 \varphi_2 + \varphi_1 \varphi_3)$, etc. This is one reason we have not fully mechanized the proofs yet.

4.1 Type Soundness of $\lambda^\omega$

We start from investigation for the effect system: the effect system prevents coordination by [Corollary 4.6](#) and the property is preserved by the reduction by [Lemma 4.3](#).

**Lemma 4.1** (Substitution). If $\Xi, x : \tau \vdash M : \tau \& \varphi$ and $\Xi \vdash M' : \tau' \& \text{A}$, then $\Xi \vdash M[x := M'] : \tau \& \varphi$.

**Lemma 4.2** (Name substitution). If $\Xi, \alpha \vdash M : \tau \& \varphi$ and $\text{fnv}(\Phi) \subseteq \text{ndom}(\Xi)$, then $\Xi \vdash M[\alpha := \Phi] : \tau[\alpha := \Phi] \& \varphi[\alpha := \Phi]$.

**Lemma 4.3** (Subject reduction). If $\emptyset \vdash M : \tau \& \varphi$ and $M \xrightarrow{\Delta} M'$, then $\emptyset \vdash M' : \tau \& \varphi$.

**Definition 4.4**. Non-coordinated reduction relation, denoted as $M \rightarrow M'$, is derived from the rules in [Figure 6](#) by replacing $\Rightarrow$ with $\Rightarrow$ and removing (TR-WORLDL) and (TR-WORLDR).

**Lemma 4.5**. If $\emptyset \vdash M : \tau \& \varphi$, $M \xrightarrow{\Delta} M'$, and $\text{L}(\varphi) \not\subseteq \{\omega \mid \omega_1 \in \Delta \lor \omega_2 \in \Delta\}$; then $M \Rightarrow M'$.
Corollary 4.6 (Non-coordination). If $\emptyset \vdash M : \tau \& \varphi$ and $M \xrightarrow{\emptyset} M'$, then $M \xrightarrow{\emptyset^*} M'$.

4.2 Soundness of Compilation

Towards the goal we show a well-typed expression in $\lambda\parallel$ is mapped into a well-typed expression in $\lambda\parallel_{\omega}$ by the compilation.

Definition 4.7 (Compilation for types). Compilation for types is given as follows.

\[
\begin{align*}
\llbracket \text{nat} \rrbracket & = \text{nat} \\
\llbracket T_1 \rightarrow T_2 \rrbracket & = \llbracket T_1 \rrbracket \rightarrow^A \forall\alpha.\varphi(\alpha) \llbracket \llbracket T_2 \rrbracket \rrbracket
\end{align*}
\]

Definition 4.8 (Compilation for typing environment). Compilation for typing environment is given by the following obvious way.

\[
\begin{align*}
\llbracket \emptyset \rrbracket & = \emptyset \\
\llbracket \Gamma, x : T \rrbracket & = \llbracket \Gamma \rrbracket, x : \llbracket T \rrbracket
\end{align*}
\]

Lemma 4.9. $\emptyset \models \llbracket T \rrbracket$.

Lemma 4.10. $\llbracket \Gamma \rrbracket$ ok.

Lemma 4.11. If $\Gamma \vdash e : T$, then $\llbracket \Gamma \rrbracket, \alpha \vdash \llbracket e \rrbracket^\emptyset : \llbracket T \rrbracket$ & $\alpha \Phi(\emptyset) \llbracket T \rrbracket$.

Proof. The proof is by induction on the given derivation. \hfill \Box

4.3 Bisimulation

Next we show a well-typed expression in $\lambda\parallel_{\omega}$ behaves as same as the expression of $\lambda\parallel$ which is obtained by erasing the name related parts.

Definition 4.12 (strong bisimulation between $\lambda\parallel$ and $\lambda\parallel_{\omega}$). A binary relation $R$ between $\lambda\parallel$ and $\lambda\parallel_{\omega}$ expressions is called strong bisimulation iff the following conditions hold.

- If $e \ R \ M$ and $e \xrightarrow{\cdot} e'$, then $M \xrightarrow{\emptyset} M'$ and $e' \ R \ M'$.
- If $e \ R \ M$ and $M \xrightarrow{\emptyset^*} M'$, then $e \xrightarrow{\cdot} e'$ and $e' \ R \ M'$.

Definition 4.13 (weak bisimulation between $\lambda\parallel$ and $\lambda\parallel_{\omega}$). A binary relation $R$ between $\lambda\parallel$ and $\lambda\parallel_{\omega}$ expressions is called weak bisimulation iff the following conditions hold.

- If $e \ R \ M$ and $e \xrightarrow{\cdot} e'$, then $M \xrightarrow{\emptyset^*} M'$ and $e' \ R \ M'$.
- If $e \ R \ M$ and $M \xrightarrow{\emptyset^*} M'$, then $e \xrightarrow{\cdot^*} e'$ and $e' \ R \ M'$. 

11
\[
\begin{align*}
\{x\} &= x \\
\{M_1 M_2\} &= \{M_1\} \{M_2\} \\
\{\lambda x. M\} &= \lambda x.\{M\} \\
\{M \Phi\} &= \{M\} \lambda x. x \\
\{\Lambda a. M\} &= \lambda x.\{M\} \\
\{\mu f. M\} &= \mu f.\{M\} \\
\{(M_1 \parallel^{\Phi} M_2)\} &= (\{M_1\} \parallel \{M_2\})
\end{align*}
\]

Figure 10: Name erasure function

**Definition 4.14** (weak bisimulation for \(\parallel\)). We also call a binary relation \(R\) between \(\parallel\) expressions weak bisimulation iff the following conditions hold.

- If \(e_1 \ R \ e_2\) and \(e_1 \longrightarrow^* e_1'\) and \(e_2 \longrightarrow^* e_2'\) and \(e_2' \ R \ e_2'\),
- If \(e_1 \ R \ e_2\) and \(e_2 \longrightarrow^* e_2'\) and \(e_1 \longrightarrow^* e_1'\) and \(e_1' \ R \ e_2'\).

**Definition 4.15.** We define the name erasure function \(\cdot\) from \(\parallel\) expressions into \(\parallel\) expressions as in Figure 10. The important point of the definition is that name abstractions and applications are replaced by dummy lambda abstractions and applications. If we do not do that, i.e., just erase the name abstractions and applications, it will happens that a value of \(\parallel\) which cannot be evaluated, is evaluated in \(\parallel\) after applying the name erasure function. (Consider \(\Lambda a.(\lambda x. x)\ \lambda x. x\) and name erased expression \((\lambda x. x)\ \lambda x. x\).)

**Lemma 4.16.** \(\{M[x := M']\} = \{M\}[x := \{M'\}]\).

*Proof.* The proof is routine by structural induction on \(M\).

**Lemma 4.17.** \(\{M[a := \Phi]\} = \{M\}\).

*Proof.* The proof is routine by structural induction on \(M\).

**Lemma 4.18.** If \(\emptyset \vdash M : \tau \land \varphi\) and \(\{M\} \longrightarrow e'\), then \(M \longrightarrow^* M'\) and \(\{M'\} = e'\).

*Proof.* The proof is by induction on the given derivation of \(\emptyset \vdash M : \tau \land \varphi\). Note that well-typedness of \(M\) is necessary because function applications and name applications are collapsed by name erasing.

**Lemma 4.19.** If \(M \Rightarrow M'\), then \(\{M\} \longrightarrow [M']\).

*Proof.* The proof is by induction on the given derivation.

**Corollary 4.20.** If \(\emptyset \vdash M : \tau \land \varphi\) and \(M \longrightarrow^* M'\), then \(\{M\} \longrightarrow [M']\).
\[ [x] = x \]
\[ [e_1 e_2] = [e_1] [e_2] \lambda x x \]
\[ [\lambda x.e] = \lambda x.\lambda y.[e] \text{ where } y \text{ is fresh} \]
\[ [\mu f.e] = \mu f.[e] \]
\[ [(e_1 || e_2)] = ([e_1] || [e_2]) \]

Figure 11: Pseudo compilation

**Proof.** This is a corollary of Corollary 4.6 and Lemma 4.19.

**Definition 4.21.** We give the binary relation between source expressions and target expressions, written \( e \sim M \) as follows.

\[ e \sim M \iff \emptyset \vdash M : \tau \land \phi \land [M] = e \]

**Corollary 4.22.** \( \sim \) is a strong bisimulation.

**Proof.** This is a corollary of Lemma 4.3, Lemma 4.18, and Corollary 4.20.

**Corollary 4.23.** If \( \emptyset \vdash e : T \), then \( [e]_\emptyset^\alpha[\alpha := e] \sim [e]_\emptyset^\alpha[\alpha := e] \).

**Proof.** This is a corollary of Lemma 4.2 and Lemma 4.11.

**Definition 4.24.** We define the pseudo compilation from/to \( \lambda \parallel \) expressions as shown in Figure 11. This function is not essential for our discussion, but we use the function for convenience writing in the following.

**Lemma 4.25.** \( [\llbracket e \rrbracket] = [e] \).

**Lemma 4.26.** \( [e[x := e']] = [e][x := [e']] \).

**Lemma 4.27.** If \( e \rightarrow^* e' \), then \( [e] \rightarrow^* e'' \) and \( [e'] = e' \).

**Lemma 4.28.** If \( e \rightarrow^* e' \), then \( [e] \rightarrow^* e'' \) and \( [e'] = e'' \).

**Definition 4.29.** We give the binary relation between \( \lambda \parallel \) expressions, denoted by \( \approx \), as \( e \approx [e] \).

**Corollary 4.30.** The binary relation \( e \approx [e] \) is a weak bisimulation.

**Theorem 4.31.** If \( \emptyset \vdash e : T \), then \( e \approx \ast \sim [e]_\emptyset^\alpha[\alpha := e] \).

The result is a bit blurred since expressions before and after compilation are related by two relations. More directly, the correspondence can be shown as follows.

\[ e \rightarrow^* e' \]
\[ [e]_\emptyset^\alpha[\alpha := e] \rightarrow^* M' \]

and

\[ [e'] = [M'] \]
5 Conclusion

We give a compilation algorithm from $\lambda^\parallel$, a simply typed lambda calculus with nondeterministic choices, into $\lambda^{\parallel\omega}$, a simply typed lambda calculus with coordinated choices; and show the compilation is sound and correct.

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