On Entanglement with Vacuum

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The so-called entanglement with vacuum is not a property of the Fock space, but of some rather pathological representations of CCR/CAR algebras. In some other Fock space representations the notion simply does not exist. We have checked all the main Gedanken experiments where the notion of entanglement with vacuum was used, and found that all the calculations could be performed at a representation-independent level. In particular any such experiment can be formulated in a Fock-space representation where the notion of entanglement with vacuum is meaningless. So, for the moment there is no single experiment where the notion is needed, and probably it is simply unphysical.

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I. INTRODUCTION

The word “entanglement” has been introduced by Schroedinger in 1935 [1], and the phenomenon has been an object of study ever since (and even before - take the famous “EPR paper” [2], published a few months earlier, for example). More recently the concept of “entanglement with vacuum” had been introduced to study non-localities of single-particle states [3, 4]. Though it had been strongly criticized in [5] and later in [6], this concept has been exploited to develop Quantum Key Distribution protocols [7, 8] and further study of single-particle non-locality [9, 10]. Since quantum cryptography is nowadays an object of study ever since (and even before - take the famous “EPR paper” [2], published a few months earlier, for example). More recently the concept of “entanglement with vacuum” had been introduced to study non-localities of single-particle states [3, 4]. Though it had been strongly criticized in [5] and later in [6], this concept has been exploited to develop Quantum Key Distribution protocols [7, 8] and further study of single-particle non-locality [9, 10]. Since quantum cryptography is nowadays an object of study ever since (and even before - take the famous “EPR paper” [2], published a few months earlier, for example).

II. REPRESENTATIONS OF CCR/CAR ALGEBRAS

In [3] authors point out that some states that seem to be entangled in Fock space are merely single-particle ones in configuration space and vice versa. On the other hand, authors of [10] state that: “since the Fock basis is a complete basis, it is just as good as any other to express and calculate quantum physics”. The Fock space mentioned in both papers is the first thing that needs clarification.

The one particle-vacuum entangled state in that space is of the form:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B)$$

Indices A and B denote here modes occupied by the particle. If we would like to be more precise, we should rather write it as:

$$|\Psi\rangle = \ldots |0\rangle_{i_1} |0\rangle_{i_2} |0\rangle_{i_3} |\psi\rangle |0\rangle_{j_1} |0\rangle_{j_2} |0\rangle_{j_3} \ldots$$

Other vacua represent here other modes that are not taken into account during while discussing the experiment. A “total vacuum”, that is the state with no particles in any of the modes, can be put down as follows:

$$|0\rangle = \ldots |0\rangle |0\rangle |0\rangle |0\rangle |0\rangle |0\rangle \ldots$$

State $|\Psi\rangle$ can also be expressed in second quantization formalism as:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (a_A^\dagger + a_B^\dagger) |0\rangle$$

In this, more general, approach entanglement does clearly not exist. It appears only when a specific representation of the CAR or CCR algebra is chosen. If we choose the representation discussed above (we will call it MVR for Multiple Vacua Representation, because the state $|0\rangle$ consists of infinite, and even uncountable, number of empty modes — vacua) the entanglement with vacuum appears. But we can choose another representation of those algebras, like the one proposed by Berezin [11]. Obviously [11] does not change, but the explicit form of creation operators and vacuum does. If we use that representation then the “total vacuum” is a vector:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \end{pmatrix}$$

and this is the only vacuum there, so there can be no entanglement with it (entanglement with a single vector is trivial). Here $|\Psi\rangle$ is also a vector from the Fock space but in this space entanglement with vacuum is not visible.

In [3] authors noticed that the entanglement with vacuum does not appear in second quantization formalism. This statement is imprecise. What we have just shown is that it appears in the Fock space only if a specific representation of CCR or CAR algebra is chosen. It seems that rather than being a physical phenomenon the entanglement with vacuum is only a peculiarity of one of the infinite number of possible representations. In the following section we will try to judge if that is the case.
III. THE CASE STUDY

First of all it is worth noticing that MVR and Berezin’s representation are inequivalent. The fact that the Fock space corresponding to the second one is separable and to the first one is not is enough to prove that. Since entanglement with vacuum appears only in one of them, there are four possibilities:

a) There is a preferred representation of the universe. It is MVR (or similar) and entanglement with vacuum exists and can be exploited.

b) There is a preferred representation of the universe, but it is Berezin’s (or similar) and entanglement with vacuum is a meaningless concept.

c) Both representations are correct but they describe different entities.

d) All irreducible representations are physically equivalent and there is no experiment that can be conducted to decide whether entanglement with vacuum has physical meaning or whether it is just a convenient notion for expressing more complex ideas.

Natural method for finding which of these is true is to repeat calculations done in MVR but this time using another representation or representation independent formalism and compare results. In papers [3, 7 - 10] single photon’s presence is being felt at two spatially separated phase sensitive detectors. By phase sensitive detector we mean here the detection unit which consists of a pair of detectors, beam splitter and light source, such as the homodyne detectors in [4]. The description of the beam splitter which is the only component of experiments described explicitly in those papers, slightly differs but this does not play a significant role. The unitary transformation performed by the beam splitter is described by $B_1$ in [4] and $B_2$ in [5, 9, 10]:

$$B_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad B_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

The symmetric beam splitter $B_1$ might be realized by two back-to-back prisms with an air gap between, while the antisymmetric one $B_2$ by silvering a glass plate on one side [12]. Evolution operators corresponding to those matrices are, respectively:

$$S_1 = e^{i\pi(a_1^\dagger a_2 + a_2^\dagger a_1)} \quad S_2 = e^{i\pi(a_1^\dagger a_2 - a_2^\dagger a_1)}$$

satisfying:

$$S_i^\dagger \bar{a}^{\text{out}} S_i = B_i \bar{a}^{\text{in}}$$

In [3] particles in experiment are fermions, but it does not make much difference. To make calculations easier it is worth noticing that any state transformation described by 2x2 unitary matrix leads to the evolution operator of the form:

$$S = e^{iK} \quad K = \sum_{i,j} c_{i,j} a_i^\dagger a_j$$

When considering the action of $S$ upon any state it is only commutator $[K, a_j^\dagger]$ that matters. It is straightforward to check that

$$[K, a_j^\dagger] = \sum_i c_{i,j} a_i^\dagger$$

is true regardless whether annihilation/creation operators correspond to bosons and follow CCR or fermions and follow CAR.

Having the $S$ operators it is elementary to find the outcome of any of the experiments described in papers [3, 4 - 10] using only CCR and the fact that $a(0) = 0$ (operators corresponding to detectors are, of course, given by $N = a^\dagger a$).

In [3] the matrix operator for the electron-positron annihilation point can not be given since in the number basis this operation is nonlinear. But we can introduce a $S$ operator for this point of the form:

$$S_a = e^{i\theta(a^\dagger b d + a b^\dagger d^\dagger)}$$

where $\sin \theta$ plays the role of probability amplitude of electron-positron annihilation, $a$ corresponds to the state of photon field after annihilation, $b$ and $d$ to electron and positron respectively. If we choose $\theta = \frac{\pi}{2}$ which corresponds to the assumption made by Hardy that if positron and electron meet the annihilation is certain, we can once again get all the results by using only CAR.

As an example of operator nonlinear in the number basis for bosons we can give the one corresponding to the Kerr medium as given in [14]. It is easy to check that the CNOT gate for the dual-rail representation of the qubit that employs Kerr medium, works as it should if positron and electron meet the annihilation is certain, we can once again get all the results by using only CAR.

$$S_B = e^{i\pi(a_1^\dagger a_4(t) - a_3(t) a_4(t))}$$

Kerr medium:

$$S_K = e^{i\alpha_1(t) a_2(t) a_3(t)}$$

and phase shifters:

$$S_\pi = e^{i\pi a_1^\dagger(t) a_4(t)}$$

Fig. 1. CNOT gate for dual-rail representation of the qubit.
Furthermore, since Kerr media, beam splitters and phase shifters are sufficient for quantum computation [14], calculation of any quantum circuit will give the same results regardless of the representation chosen. And this means that the same physics follows from all the Fock representations — including entanglement with vacuum or not — and thus the presence of this questionable concept is only a question of taste.

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