Models for Pedestrian Behavior

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1 Abstract

The behavior of pedestrians shows certain regularities, which can be described by quantitative (partly stochastic) models. The models are based on the behavior of individual pedestrians, which depends on the pedestrian intentions on one hand, and on the aspects of movement on the other hand.

The pedestrian intentions concerning a sequence of destinations are influenced first by the demand for certain kinds of commodities, second by the location of stores selling these kinds of commodities, third by the expenditures (prices, ways, etc.) to get the required commodities.

The actual pedestrian movement starts and ends at special city entry points like bus stops, parking lots or metro stations. It is guided by the pedestrian intentions, but is subject to deceleration processes and avoidance maneuvers due to obstacles or pedestrians who are in the way. As a consequence, the pedestrians have to speed up in the course of time in order to reach the next destination well-timed. In addition, the pedestrian behavior is influenced by unexpected attractions (e.g. by shop windows or entertainment).

The model for the behavior of individual pedestrians is an ideal starting point for computer simulations of pedestrian crowds. Such simulations take into account the limited capacity of pedestrian ways and places, and allow to de-
termine an optimal design of pedestrian areas and an optimal arrangement of store locations. Therefore, they can be applied for town- and traffic-planning. The model for the behavior of individual pedestrians also allows the derivation of mathematical equations for pedestrian crowds and for pedestrian groups. Pedestrian crowds can be described by a stochastic formulation, by a gaskinetic formulation or by a fluiddynamic formulation. The gaskinetic formulation (mezoscopic level) can be derived from the stochastic formulation (microscopic level), and the fluiddynamic formulation (macroscopic level) from the gaskinetic formulation (mezoscopic level).

2 Introduction

Human behavior is based on individual decisions. In building a mathematical model for the movement of pedestrians, one has to assume that these decisions show certain regularities (e.g. follow stochastic laws). This assumption is justified, because decisions and the behavior of pedestrians are usually determined by utility maximization: For example, a pedestrian takes an optimal path to a chosen destination, and tries to minimize delays when having to avoid obstacles or other pedestrians. The optimal behavior for a given situation can be derived by plausibility considerations, and will be used as a model for pedestrian movement. Of course this optimal behavior is normally not thought about, but by trial and error an individual has automatically learned to use the most successful behavioral strategy, when being confronted with a standard situation.

3 Individual behavior

The behavior of individual pedestrians is the (microscopic) basis for developing models that describe pedestrian groups or pedestrian crowds. A model for the individual behavior has to take into account the pedestrian intentions and the aspects of movement. In the following, the basic ideas of a model of this kind will be described. The mathematical formulation of the model will be presented in a forthcoming paper.
3.1 Pedestrian intentions

Let us consider the case of pedestrians who walk in a shopping area. (This is the most relevant case for town- and traffic-planning.) The pedestrians’ behavior will be determined mainly by their demand, then. This demand will be varying according to a certain distribution, which may depend on the pedestrians’ consumer type. Given a certain demand, a pedestrian’s destinations will be stores, where the required kinds of commodities are offered. The probability to decide for a certain store as next destination will be the greater, the more of the required commodities are offered there, the greater the assortment is, the lower the price level is, and the shorter the way to the store is.

In general, there are several ways to the chosen destination. The probability to decide for a certain way will decrease with the corresponding distance, but the readiness for taking detours is growing with the available time.

When arriving at the chosen destination (store), the pedestrian will buy a commodity of a certain kind. The probability for buying a commodity of a certain kind during a given time interval will be increasing with the assortment, and with the number of commodities which are required of this kind. It will be the lower the higher the price level is. These probabilities determine the time which is necessary for buying one of the required commodities.

The purchase of a required commodity changes the remaining demand and calls for a decision about the next destination. Since it depends on the distance which destination is prefered, the same store will usually be chosen again as long as there is a demand for other commodities that are offered there (if the prices for these commodities are not too high, and if the assortment with respect to the remaining demand is not too low).

A pedestrian will leave the shopping area when the demand is satisfied, i.e. when he or she has bought all of the required commodities.

A detailed model for the route choice behavior of pedestrians and its dependence on their demand has been developed, simulated and empirically tested by Borgers and Timmermans [2, 3].
3.2 Pedestrian movement

The motion of pedestrians starts at special city entry points like bus stops, parking lots or metro stations. The choice of a certain entry points depends on a pedestrian’s demand.

The pedestrian’s movement is, then, guided by his or her desired velocity. Whereas the direction of the desired velocity is given by the way to the chosen destination, the desired speed of pedestrians is distributed Gaussian \[11, 12, 14\]. The desired speed of pedestrians may be varying with time. For example, it is increased in the case of delays in order to reach a certain destination well-timed.

Since unexpected obstacles and other pedestrians have to be avoided, the actual velocity of a pedestrian will normally differ from the desired velocity. Interactions with other pedestrians are characterized by avoiding maneuvers and stopping processes. They determine the capacity of a pedestrian area. During the interaction free time pedestrians are accelerating, and trying to approach their desired velocity again.

Deviations from the originally chosen way also result from unexpected attractions like shop windows or entertainment along the pedestrian area. Such attractions may lead to spontaneous stops (“impulse stops”).

A detailed model for the movement behavior of pedestrians is given in \[6\].

4 Computer simulations

An ideal method of testing the model described in section \[3\] is a Monte Carlo simulation of pedestrian dynamics with a computer. The results of these simulations can be compared with empirical data (see \[2, 3\]) or with films of pedestrian flow.

Computer simulations can be used as a powerful tool for town- and traffic-planning: They allow to determine an optimal design of pedestrian areas and an optimal arrangement of store locations, since they take into account the pedestrian demand, the city entry points, the location of the stores and the capacity of the pedestrian areas. The capacity depends on the pedestrian
density and the pedestrian flows (see sect. 6.3). It is, therefore, a function of the size and the geometry of a pedestrian area.

5 Pedestrian groups

From the behavior of individual pedestrians some results concerning pedestrian groups can be derived. Interesting examples are the formation of freely-forming groups and the behavior in queues.

5.1 Formation of freely-forming groups

Pedestrians who know each other and meet in a pedestrian area by chance may form a group, and stay together for a talk. However, a pedestrian will join another pedestrian only, if the motivation (the attraction) to do so is greater than the motivation to get ahead. The pedestrian will leave the moment at which the motivation to join the group becomes less than the increasing motivation to get ahead with the desired velocity (which is growing according to the delay resulting from the stay). If, right from the beginning, the motivation to get ahead is greater than the motivation to join a certain person or group the pedestrian will normally not stop for a talk.

As a consequence of this joining and leaving behavior, a truncated Poisson distribution results for the group size (see fig. 1) \[6\]. This has been already derived and empirically tested by Coleman \[4, 5\].

5.2 Behavior in a queue

If the front of a queue has come to rest, the following phenomenon can often be observed: After a while, one of the waiting individuals begins to move forward a little, causing the successors to do the same. This process propagates in wave-like manner to the end of the queue, and the distance to move forward increases (see fig. 2).

Why do individuals behave in such a paradoxical way?—They do not get forward any faster but only cause the queue to become more crowded! The reason is that an individual in a queue keeps a distance, which corresponds to
Figure 1: Group size distribution of freely-forming groups: The frequency of groups consisting of $k$ members is given by the truncated POISSON distribution $p_k = \frac{1}{\lambda^{k-1} k!} \lambda^k (k = 1, 2, \ldots)$ with a situation specific constant $\lambda$. 
Figure 2: Behavior in a queue: If an individual moves forward a little due to the growing pressure of time, the successors are motivated to do the same. The distance for the $n$th successor to move forward is usually greater than the one of the $(n-1)$th successor. The old position of a moving individual is indicated by an empty circle.
an equilibrium between the motivation to get ahead (the pressure of time) and
the motivation to keep a certain distance to the predecessor (the readiness to
respect the territory of the individual in the front). However, during waiting
in the queue, the pressure of time increases, whereas the territorial effect is
time independent. As a consequence, the individual moves forward a little
after a while.

6 Pedestrian Crowds

6.1 Stochastic formulation

The mathematical formulation of the model described in section [lead to a
stochastic equation, namely the master equation. This equation is extremely
complicated: it is impossible to be solved analytically and hard to be solved
with a computer. However, from the master equation approximate mean
value equations can be derived, which are similar to Boltzmann equations [8].
These equations can be interpreted as a gaskinetic formulation of pedestrian
movement.

6.2 Gaskinetic formulation

Before gaskinetic equations have been developed for pedestrian crowds, they
have been already used for the description of traffic flow [1, 15, 16, 17]. The
gaskinetic formulation of pedestrian behavior with Boltzmann-like equa-
tions has some analogies with the description of ordinary gases, but it takes
into account the effect of pedestrian intentions and interactions. Some sim-
ilarities and differences between pedestrian crowds and ordinary gases shall
be illustrated by two examples: The behavior an a dance floor on one hand,
and the separation of opposite directions of motion on the other hand.

6.2.1 Behavior on a dance floor

On a dance floor like that of a discotheque, two types of motion can be found:
One type represents individuals, who want to dance, i.e. intend to move with
a high velocity variance $\theta_d$ (“high temperature”). The second type represents individuals, who look on the dancers and do not want to move, i.e. intend to have a low velocity variance $\theta_s$ (“low temperature”). Dancers and spectators are in equilibrium only, if the mutually exerted pressure $P = \rho \theta$ of both groups agrees ($P_d = P_s$) [7]. As a consequence, the dancers are expected to show a lower density $\rho$ than the spectators ($\rho_d < \rho_s$) (see fig. 3). This phenomenon can actually be observed.

Figure 3: Behavior on a dance floor: Dancing individuals (filled circles) show a lower density than the individuals standing around (empty circles), since they intend to move with a greater velocity variance.

6.2.2 Separation of opposite directions of motion

In a street, footway or pedestrian area normally (at least) two opposite directions of motion (two opposite flows) can be found. The interaction rate (i.e. the rate of avoiding maneuvers and stopping processes) becomes minimal, if pedestrians with opposite desired directions of motion use separate lanes (see fig. 4a). Actually, a separation of opposite directions of motion arises, if the pedestrian density exceeds a certain value. The width of the lanes can again be evaluated with the equilibrium condition for the mutual pressure of both flows [7].
Figure 4: (a) Opposite directions of motion normally use separate lanes. Avoiding maneuvers are indicated by arrows. (b) For pedestrians with an opposite direction of motion it is advantageous, if both prefer either the right hand side or the left hand side when trying to pass each other. Otherwise, they would have to stop in order to avoid a collision. The probability $p$ for choosing the right hand side is usually greater than the probability $(1 - p)$ for choosing the left hand side.
A separation of opposite flows and a reduction of stopping processes only results, if pedestrians with an opposite direction of motion both prefer either the right hand side or the left hand side when trying to pass each other (see fig. 4b). In most cases, the right hand side is being preferred, in spite of the fact that a preference of the left hand side would have the same effect. This break of symmetry can be understood as phase transition [6, 9]: With both sides being originally equivalent, one side is being preferred by a growing majority, once it has been preferred at random.

6.3 Fluid dynamic equations

Often, one is only interested in quantities like the density, mean velocity and velocity variance of pedestrians, but not in the detailed velocity distribution. In this case, fluid dynamic equations are sufficient. They can be derived from the gaskinetic equations as mean value equations for the quantities of interest. This has been explicitly shown in [7], but investigations on fluid dynamic properties of pedestrian crowds have been already made by Henderson [10, 11, 12, 13]. The similarity of the motion of pedestrian crowds with the motion of ordinary fluids can be best seen by comparison of quick-motion pictures of pedestrians with streamlines of fluids. Nevertheless, the fluid dynamic equations for pedestrians contain some additional terms, which take into account the intentions and interactions of pedestrians.

From the fluid dynamic equations the following conclusions for the optimization of pedestrian flows can be drawn:

- Crossings of different directions of motion should be avoided (if necessary, by bridges, traffic lights or round-about traffic).
- Opposite directions of motion should use separate lanes. At a narrow passage, pedestrians should walk by turns.
- Great velocity variances should be avoided. This can be done by walking in formation.
- If obstacles or narrow passages are unavoidable, they should be given an aerodynamic design.
7 Conclusions

Models for pedestrian movement can be developed on several levels. On the microscopic level one has to describe the individual behavior, and a stochastic formulation results. However, the microscopic level of pedestrian movement can be treated easier by Monte Carlo simulations with computers.

From the microscopic formulation a gaskinetic formulation can be derived, which describes the mesoscopic level. On the macroscopic level, one is confronted with fluiddynamic equations, which can be derived from the gaskinetic equations.

All levels of description take into account pedestrian intentions and interactions. Models for pedestrian movement can be used as a powerful tool for town- and traffic-planning.

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