Adaptive neural consensus tracking control for multi-agent systems with unknown state and input hysteresis

Zhuangbi Lin · Zhi Liu · Yun Zhang · C. L. Philip Chen

Received: 3 March 2021 / Accepted: 20 June 2021 / Published online: 12 July 2021
© The Author(s), under exclusive licence to Springer Nature B.V. 2021

Abstract An indirect adaptive consensus control method is presented for multi-agent systems (MASs) with unknown hysteresis states and input. All system states that can be utilized to design the controller are measured by the sensors subjected to hysteresis, and thus, the system state values are inaccurate. Meanwhile, it is difficult to compensate the input hysteresis for it is coupled with the state hysteresis. The unknown function from agent’s neighbors also increases the difficulty of controller design. To eliminate the influence of unknown input hysteresis, an inverse adaptive compensated method is presented. The problem of state hysteresis is addressed by designing two adaptive laws to approximate the upper and lower bounds of unknown hysteresis coefficient. Neural networks are introduced to handle the unknown dynamics of agent and its neighbors. The proposed control scheme can guarantee that the consensus errors of followers converge to a pre-defined interval of zero asymptotically. In addition, the transient performance of MASs can be further ensured. The simulation examples are included to verify the effectiveness of the presented control approach.

Keywords Adaptive neural control · Input and states hysteresis · Inverse compensation · Consensus control

1 Introduction

Multi-agent systems (MASs) have an extensive application in military, satellite clusters and so on [1–5], and therefore, numerous researchers focus on it. Therein, there are many significant results for linear or low-order nonlinear MASs which have achieved the ideal tracking consensus [6–8]. Moreover, these interesting works were applied in the high-order MASs [9–11]. However, the dynamics of MASs in the important works need to be completely known.

Considering that unknown nonlinearities universally exist in the dynamics of MASs, some novel corresponding methods, such as neural networks (NNs) and fuzzy-logic systems (FLSs), are presented to handle the unknown dynamics. In [12], an innovative consensus tracking control scheme was proposed for MASs with unknown dynamics. In [13], by employing NNs, the consensus control method of MASs with unknown control sign and dynamic uncertainties was presented. In [14], a tracking control scheme for MASs with unknown function existing in each subsystem was pre-
sented. However, as the scale of the MAS expands, the number of the estimated parameters in [14] becomes unbearable, which requires lots of computation. Such a problem was properly solved in [15] for nonlinear system and then in [16,17] for MASs by treating the norm of the ideal weight vector as an unknown constant. This novel scheme is widely adopted in [18–21] for the neural or fuzzy controller design. However, as a result of estimating the norm, control schemes in the aforementioned papers [16–21] only achieve the system stability with unknown tracking error. These schemes introduce some additional terms, and thus, the derivative of Lyapunov function is difficult to be designed negatively. To produce the asymptotic tracking control of nonlinear system, in [22], based on an innovative Lyapunov function design, a performance-oriented fuzzy control approach was proposed. The method in [22] not only ensures the fixed tracking accuracy and transient performance, but also does not increase the computational burden. In [23], a neural control method is presented for uncertain MASs with predefined accuracy. However, there are few results for MASs with fixed tracking performance in the presence of hysteresis.

Adaptive control for nonlinear systems and MASs with input hysteresis has attracted lots of attention, because the hysteresis problem exists in extensive devices and actual systems, such as electromagnetism and mechanical actuators [24–27]. To eliminate the effect of input hysteresis, in [25], by constructing an inverse compensation function, an output-feedback control scheme was proposed. Even so, the perfect compensation requires that the hysteresis model is completely known. To solve the issue of unknown hysteresis, a promising adaptive method was developed in [28]. In [29], an adaptive control method was proposed for stochastic nonlinear systems with input hysteresis and unknown control gain. Moreover, a fuzzy event-triggered-based control scheme was proposed for stochastic MASs with Bouc–Wen hysteresis input in [30].

Nevertheless, the mentioned works are states feedback control methods or output feedback control methods. They require accurate values of the states or the actual system output. In practical application, the system variables are usually measured by sensors with errors, which makes the above methods impractical. It is reported in [31–34] that various sensors suffer from hysteresis and in such case the exact values of system states become unknown. For nonlinear system, the use of inaccurate state value may cause performance degradation and even system instability [35]. There is a fundamental difference between the nonlinearities in actuator and in sensor, because the controller design only can use the inaccurate value. It is a challenging task to relieve the impacts caused by hysteresis existing in sensors. In [24, 36], two adaptive control approaches for linear system with uncertain hysteresis in both actuator and sensor were presented. It is assumed that hysteresis only exists in the sensor measuring the output. To remove this obstacle, Liu et al. [37] discussed an adaptive control for nonlinear systems in the presence of both input and state hysteresis with prescribed accuracy. So far, there are few researches on MASs with state hysteresis and input hysteresis. The control scheme in [37] cannot be applied in MASs because the distributed consensus controller for agent in MAS is designed using information from itself and its neighbor. The states from different agents are subjected to different unknown hysteresis. How to design the controller for MASs with state hysteresis is an interesting work.

Driven by the above discussion, this article explores the predefined precision consensus control of MASs with hysteresis in actuators and measuring sensors. The controllers are fully distributed requiring partial information of MASs. The adaptive compensation scheme is presented to mitigate the effects of state hysteresis. Since the parameters of input hysteresis are unknown, NNs are utilized to estimate the input hysteresis and adaptive laws are given to approximate the weight matrix. The contributions of this article are epitomized as follows.

- Compared with the issue that hysteresis only exists in the actuator [25, 28–30], state hysteresis causes more technical difficulties in back-stepping design. Unlike [24, 36, 37], for the MASs with state hysteresis, the state of agent’s neighbor is subjected to different hysteresis. Hysteresis is related to the states of agent and its neighbors which cannot be obtained in the current step. Compared with [37], the compensation of hysteresis is more difficult because we need to handle several different hysteresis nonlinearities from agent’s neighbors in the same step. To handle the unknown dynamics caused by state hysteresis, adaptive laws are designed at each step to estimate the upper and lower bounds of the time-
2 Problem statement

2.1 Communication graph

The interaction between nodes is described by a directed graph \( G = (\mathcal{V}, \mathcal{E}) \), in which \( \mathcal{V} = \{0, 1, \ldots, K\} \) means a set of agents and \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \) denotes a set of communicated edge. Generally, the node labeled 0 is treated as the leader and \( K \) is the number of followers. The adjacency matrix is defined as \( A = [a_{ik}]_{K \times K}. \) If the \( i \)th node can receive message from the \( k \)th node, \( a_{ik} > 0 \), otherwise \( a_{ik} = 0. \) Moreover, \( \mathcal{L} = \mathcal{D} - A \) denotes Laplacian matrix where \( \mathcal{D} = \text{diag}(d_1, \ldots, d_K) \) and \( d_i = \sum_{j=1}^{K} a_{ik} \). \( b_i \) shows the communication from leader to the \( i \)th follower.

2.2 Radial basis function NNs

It has been proven that radial basis function (RBF) NNs are very influential in dealing with unknown dynamics [38]. This article combines NNs and a novel Lyapunov function to ensure the asymptotic stability of MASs with unknown dynamics. The RBF NNs are employed to approximate the unknown function.

\[
\pi(\bar{x}) = W^* R(\bar{x}) + \Delta(\bar{x})
\]

where \( \bar{x} \) represents the input vector composed of independent variable. \( \Delta(\bar{x}) \) is an unknown but bounded approximation error. \( W^* \) denotes the ideal weight vector defined as

\[
W^* = \arg \min_{W \in \mathbb{R}^r} \{ \sup_{\bar{x} \in \Omega} |\pi(\bar{x}) - W^T R(\bar{x})| \}
\]

\( R = [\nabla_1, \ldots, \nabla_M]^T \) and \( \nabla_p (p = 1, \ldots, M) \) is the Gaussian function defined as

\[
\nabla_p = \exp \left[ -\frac{(\bar{x} - q_p)^T (\bar{x} - q_p)}{\ell_p^2} \right],
\]

where \( \ell_p \) and \( q_p \), respectively, denote the width and the center of basis function.

The following lemma describes a useful property of neural networks which will be helpful in reducing the number of variable inputs.

**Lemma 1** [16] For \( \bar{x}_k = [x_1, \ldots, x_k]^T \) and \( k \leq j \), we have

\[
\| R(\bar{x}_j) \|^2 \leq \| R(\bar{x}_k) \|^2
\]

2.3 Problem formulation

Consider a MAS with hysteresis phenomenon presenting in both actuators and sensors. The dynamics of the followers are modeled as:

\[
\begin{align*}
\dot{x}_{i, p} &= x_{i, p+1} + \pi_{i, p}(\bar{x}_{i, p}) \quad 1 \leq p \leq n_i - 1, \\
\dot{x}_{i, n_i} &= u_i + \pi_{i, n_i}(\bar{x}_{i, n_i}) \quad 1 \leq i \leq N \\
u_i &= H_i(v_i), \quad y_i = x_{i, 1}
\end{align*}
\]

where \( \pi_{i, p}, \pi_{i, n_i} \) and \( \bar{x}_{i, p} = [x_{i, 1}, \ldots, x_{i, p}]^T \) represent unknown smooth function, system output and state vector, respectively. \( v_i \) is the control signal we actually designed and \( u_i \) is the output of actuator acting on the plant. They are not equal when hysteresis exists in actuator. The hysteresis phenomenon can be described by the most popular Bouc–Wen model \( H_i \). While the actuator is not affected by hysteresis, \( H_i(v_i) = v_i \). While hysteresis exists in actuator, it can be described by Bouc–Wen model as

\[
H_i(v_i) = \mu_{i1} v_i + \mu_{i2} \xi_i
\]

where \( \mu_{i1} \) and \( \mu_{i2} \) are unknown constants and sign \( (\mu_{i1}) = \text{sign}(\mu_{i2}). \) \( \xi_i \) is given by

\[
\dot{\xi}_i = \dot{v}_i \bar{h}_i(\xi_i, \dot{v}_i)
\]

and

\[
\bar{h}_i(\xi_i, \dot{v}_i) = 1 - \text{sign}(\dot{v}_i) h_i|\xi_i|^{\mu_{i1}-1} \xi_i - \chi_i |\xi_i|^{\mu_{i2}}
\]

where \( \chi_i \) is a positive constant. \( \dot{v}_i \) is the velocity of input vector \( v_i \).
with $h_i > |\chi_i|$ and $l_i \geq 1$. $h_i$, $\xi_i$ and $l_i$ are the unknown parameters representing the shape, the amplitude and the smoothness, respectively. The Bouc–Wen model is shown in Fig. 1.

The proposed control scheme will discuss these two cases separately.

Remark 1 The important works in [6–11] require that the system dynamics are completely known because their controller design uses all the information of system dynamics. Specially, in [9], the system dynamics are described as $\dot{x}_i = A_i x_i + B_i u_i$, and the controller design uses the matrices $A_i$ and $B_i$. The system model in [11] is similar to the model (4) in this paper. However, the controller design in [10] directly uses the function $\pi_{i,p}$, and therefore, [10] requires that the system dynamics are completely known. By employing RBFNNs, this restriction is removed in this article.

The dynamics of the leader are described as:

$$\dot{x}_0 = f_0(x_0)$$
$$\gamma_0 = x_0$$

Assumption 1 [39] The leader’s output $y_0$ and its derivative $\dot{x}_0$ are bounded. The reference signal of the leader is known to some agents.

In this paper, the exact system states are not available and only the hysteresis states received by sensors $\hat{x}_{i,1}, \hat{x}_{i,2}, \ldots, \hat{x}_{i,n_i}$ can be used in our control design. According to [25, 40], the relationships between the received signals and the genuine system states are given as the Bouc–Wen model:

$$\dot{x}_{i,p} = H_{i,p}(x_{i,p}) = \mu_{i,p1} x_{i,p} + \mu_{i,p2} \xi_{i,p}$$

where $\mu_{i,p1}$ and $\mu_{i,p2}$ are unknown and sign $(\mu_{i,p1}) = \pm \text{sign}(\mu_{i,p2})$. Without loss of generality, the parameters $\mu_{i,p1}$ and $\mu_{i,p2}$ are assumed to be positive. $\xi_{i,p}$ is given by

$$\dot{\xi}_{i,p} = \dot{x}_{i,p} h_{i,p}(\xi_{i,p}, \dot{x}_{i,p})$$
$$h_{i,p}(\xi_{i,p}, \dot{x}_{i,p}) = 1 - \text{sign}(\dot{x}_{i,p}) h_{i,p} |\xi_{i,p}|^{\frac{1}{l_{i,p}} - 1} \xi_{i,p}$$

where $h_{i,p} > |\chi_{i,p}|$ and $l_{i,p} \geq 1$.

Remark 2 New types of sensors and actuators are widely used in industry due to their small size and low energy consumption. However, due to the use of smart materials, nonlinear hysteresis inevitably exists in these sensors and actuators. Some important compensated schemes are proposed to address this issue [32, 36, 41–44]. The model of hysteresis is the key to the controller design. Since the hysteresis phenomenon is caused by smart materials, the hysteresis models of actuator and sensor can be similar. In [42, 43], the Prandtl–Ishlinskii model that is used to describe the hysteresis phenomenon in the actuator is extended to describe the phenomenon in sensors. Moreover, Prandtl–Ishlinskii model can be regarded as a special case of the Bouc–Wen hysteresis model [25]. Therefore, this paper uses Bouc–Wen Model to describe the hysteresis in sensors.

Remark 3 In [25], an output feedback control method is proposed for nonlinear system with unknown input hysteresis. The controller design only requires the function $\pi_{i,p}$, and therefore, [10] requires that the system dynamics are completely known. By employing RBFNNs, this restriction is removed in this article.
information of system output and the unknown hysteresis is perfect compensated by constructing an inverse function. However, the method in [25] still requires the exact value of system output. If the sensor that measures system output is also subjected to hysteresis, the system stability cannot be guaranteed. Moreover, the method in [25] does not take into account information from other agents, such that it cannot handle the hysteresis. RBFNNs are utilized to approximate unknown function at each step. Before our beginning, for the sake of convenience the unknown function \( \pi_{i,p} \) can be expressed as

\[
\pi_{i,p} = k_i^T \psi_{i,p}
\]

where

\[
\psi_{i,p} = \begin{bmatrix}
0 & R_{i,p}^T
\end{bmatrix}^T,
\]

and

\[
k_i = \begin{bmatrix}
W_i^T, \ldots, W_{i,n_i}^T, \Delta_{i,1}, \ldots, \Delta_{i,n_i}
\end{bmatrix}^T
\]

Define

\[
\theta_i = \sup \left\{ \left( k_i^T k_i \right)^{\frac{1}{2}} \right\}
\]

To continue our work, two smooth functions are introduced [45].

\[
s_{g_{i,k}} (z_{i,k}) = \begin{cases}
\frac{\hat{z}_{i,k}}{\xi_{i,k}}, & \hat{z}_{i,k} \geq \epsilon_{i,k} \\
\frac{\hat{z}_{i,k} - \hat{z}_{i,k}^2}{\epsilon_{i,k}^2 - \hat{z}_{i,k}^2} (\hat{z}_{i,k} - \epsilon_{i,k}) + \hat{z}_{i,k}, & \hat{z}_{i,k} < \epsilon_{i,k}
\end{cases}
\]

\[
f_{i,k} (z_{i,k}) = \begin{cases}
1, & \hat{z}_{i,k} \geq \epsilon_{i,k} \\
0, & \hat{z}_{i,k} < \epsilon_{i,k}
\end{cases}
\]

**Remark 5** The function \( s_{g_{i,k}} \) plays the same role in this article as in [45]. As shown in the proof of Theorem 1, by employing the functions \( s_{g_{i,k}} \) and \( f_{i,k} \) to construct Lyapunov function, the tracking accuracy can be designed in advance, that is, \( \lim_{t \to \infty} |z_{i,1}| \leq \epsilon_{i,1} \). However, the method in [45] requires that the function input is the exact tracking error \( z_{i,p} = x_{i,p} - \alpha_{i,p} \), not the \( z_{i,p} \) we define in Eq. (12), such that it cannot be applied in this paper to address the state hysteresis. It is more challenging since the real system states are unavailable in this paper. Moreover, [45] does not consider the communication between agents, so that the unknown terms from agent’s neighbor cannot be handled. In addition, as we mentioned in Remark 8, this...
paper has no need to calculate the derivatives of virtual control laws repeatedly, so that the function \( s_{g_{i,k}} \) is only required to be first-order differentiable. While in [45], it should be \((n_i - p + 1)\)th-order differentiable. In addition, an alternative reduced-order function is provided:

\[
s_{g_{i,k}}(z_{i,k}) = \begin{cases} 
\frac{z_{i,k}}{|z_{i,k}|}, & |z_{i,k}| \geq \epsilon_{i,k} \\
\sin(\frac{z_{i,k}}{\epsilon_{i,k}}), & |z_{i,k}| < \epsilon_{i,k}
\end{cases}
\]  

(20)

The relationship (9) between actual system state and sensor output is rewritten as

\[
x_{i,p} = \frac{\hat{x}_{i,p} - \mu_{i,p} \hat{\xi}_{i,p}}{\mu_{i,p}}
\]  

(21)

Now we can develop the control scheme and the details are presented in the following two cases.

3.1 Case 1: only states hysteresis

In this case, \( H_1(v_1) = v_1 \), that is, the actuator is not affected by hysteresis.

Step 1 From the definition of consensus errors (12) and Eq. (21), \( \hat{z}_{i,1} \) can be derived as

\[
\dot{\hat{z}}_{i,1} = b_1(\mu_{i,11}\hat{x}_{i,1} + \mu_{i,12}\hat{x}_{i,1}) \hat{h}_{i,1}(\hat{\xi}_{i,1}, \hat{x}_{i,1}) - \hat{y}_r
\]

\[
+ \sum_{j=1}^{N} a_{ij}(\mu_{i,11}\hat{x}_{i,1} + \mu_{i,12}\hat{x}_{i,1}) \hat{h}_{i,1}(\hat{\xi}_{i,1}, \hat{x}_{i,1})
\]

\[
- \mu_{j,11}\hat{x}_{j,1} - \mu_{j,12}\hat{x}_{j,1} \hat{h}_{j,1}(\hat{\xi}_{j,1}, \hat{x}_{j,1})
\]

\[
= (d_1 + b_1)(\mu_{i,11} + \mu_{i,12}\hat{h}_{i,1})(\hat{x}_{i,2} + \pi_{i,1}(\hat{z}_{i,1}))
\]

\[- \hat{y}_r
\]

\[
- \sum_{j=1}^{N} a_{ij} \mu_{j,11} + \mu_{j,12}\hat{h}_{j,1}(\hat{x}_{j,2} + \pi_{j,1}(\hat{z}_{j,1}))
\]

\[
= r_{i,1}(\alpha_{i,1} - \mu_{i,22}\hat{\xi}_{2,1} + \hat{z}_{i,2}) + r_{i,2}\mu_{i,21}\hat{\pi}_{i,1} - \hat{y}_r
\]

\[
- \sum_{j=1}^{N} a_{ij} r_{j,1}\hat{x}_{j,2} - \mu_{j,22}\hat{\xi}_{2,1} + \mu_{j,21}\hat{\pi}_{j,1}(\hat{z}_{j,1})
\]

\[
= r_{i,1}(\alpha_{i,1} - \mu_{i,22}\hat{\xi}_{2,1} + \hat{z}_{i,2}) + r_{i,2}\mu_{i,21}\hat{\pi}_{i,1} - \hat{y}_r
\]

\[
- \sum_{j=1}^{N} a_{ij} r_{j,1}\hat{x}_{j,2} - \mu_{j,22}\hat{\xi}_{2,1} + \mu_{j,21}\hat{\pi}_{j,1}(\hat{z}_{j,1})
\]

\[
(22)
\]

where \( r_{i,1} = (d_1 + b_1)(\mu_{i,11} + \mu_{i,12}\hat{h}_{i,1})/\mu_{i,21} \) and \( r_{j,1} = (\mu_{j,11} + \mu_{j,12}\hat{h}_{j,1})/\mu_{j,21} \). From the definition of \( r_{i,1} \), we can directly conclude that \( r_{i,1} \) is positive and bounded. Define \( \eta_{i,p} \) representing the upper bound of \( \frac{1}{\mu_{i,p}} \), \( \hat{\eta}_{i,p} \) is the estimation of \( \eta_{i,p} \) and the error \( \hat{\eta}_{i,p} = \eta_{i,p} - \hat{\eta}_{i,p} \).

Choose a Lyapunov function candidate as

\[
V_{i,1} = \frac{1}{r_{i,1} + 1} \left( |z_{i,1}| - \epsilon_{i,1} \right)^{n_{i,1} + 1} f_{i,1} + \frac{1}{2\gamma_{i}\eta_{i,1}} \hat{\eta}_{i,1}^2
\]

(23)

where \( \epsilon_{i,1} \) is a predefined tracking accuracy and \( \gamma_{i} \) is a positive constant. Subsequently, we have

\[
\dot{V}_{i,1} = \left( |z_{i,1}| - \epsilon_{i,1} \right)^n f_{i,1} s_{g_{i,1}}(z_{i,1}) \hat{x}_{i,1} - \frac{\hat{\eta}_{i,1}}{\gamma_{i}\eta_{i,1}} \hat{\eta}_{i,1}
\]

\[
= \left( |z_{i,1}| - \epsilon_{i,1} \right)^n f_{i,1} s_{g_{i,1}}(z_{i,1}) (r_{i,1}(\alpha_{i,1} + \hat{z}_{i,2})
\]

\[
+ \hat{g}_{i,1})
\]

\[- \frac{\hat{\eta}_{i,1}}{\gamma_{i}\eta_{i,1}} \hat{\eta}_{i,1}
\]

\[
(24)
\]

where the unknown continuous function \( \hat{g}_{i,1}(\hat{Z}_{i,1}) \) satisfies

\[
\hat{g}_{i,1} = [-r_{i,1}\mu_{i,22}\hat{\xi}_{2,1} + r_{i,1}\mu_{i,21}\hat{\pi}_{i,1} - b_1\hat{y}_r
\]

\[
- \sum_{j=1}^{N} a_{ij} r_{j,1}\hat{x}_{j,2} - \mu_{j,22}\hat{\xi}_{2,1} + \mu_{j,21}\hat{\pi}_{j,1}(\hat{z}_{j,1})]
\]

(25)

Since \( \pi_{i,1} \) and \( \pi_{j,1} \) are unknown and contain the exact value of system states, \( \hat{g}_{i,1}(\hat{Z}_{i,1}) \) is unknown. We employ the RBFNN \( \hat{\psi}_{i,1} \) to approximate \( \hat{g}_{i,1}(\hat{Z}_{i,1}) \), and according to Lemma 1, with \( Z_{i,1} = [\hat{x}_{i,1}, \hat{x}_{j,1}]^T \), it can be obtained that

\[
\hat{V}_{i,1} = \left( |z_{i,1}| - \epsilon_{i,1} \right)^n f_{i,1} s_{g_{i,1}}(r_{i,1}(\alpha_{i,1} + \hat{z}_{i,2})
\]

\[
+ \hat{g}_{i,1})
\]

\[
\leq \left( |z_{i,1}| - \epsilon_{i,1} \right)^n f_{i,1} s_{g_{i,1}}(r_{i,1}(\alpha_{i,1} + \hat{z}_{i,2})
\]

\[
+ \hat{g}_{i,1})
\]

\[
+ \hat{y}_r
\]

\[
+ \frac{\hat{\eta}_{i,1}}{\gamma_{i}\eta_{i,1}} \hat{\eta}_{i,1}
\]

\[
(26)
\]

where

\[
\omega_{b_{i,1}} = s_{g_{i,1}} \sqrt{\psi_{i,1}(Z_{i,1})} \psi_{i,1}(Z_{i,1}) + d^2
\]

(27)

with \( d \) representing a design parameter.

Define \( \alpha_{i,p} \) as the upper bound of \( r_{i,p} \), \( \hat{\alpha}_{i,p} \) is an estimate of \( \alpha_{i,p} \) and \( \bar{\alpha}_{i,p} = \alpha_{i,p} - \hat{\alpha}_{i,p} \). To ensure that \( \dot{V}_{i,1} \) is negative, the tuning function, virtual control law and adaptive law are given as follows:

\[
\alpha_{b_{i,1}} = \left( |z_{i,1}| - \epsilon_{i,1} \right)^n \hat{g}_{i,1} \hat{s}_{g_{i,1}}(\omega_{b_{i,1}})
\]

\[
\hat{\eta}_{i,1} = -\gamma_{i} \left( |z_{i,1}| - \epsilon_{i,1} \right)^n f_{i,1} s_{g_{i,1}}(\alpha_{b_{i,1}})
\]

\[
\bar{\alpha}_{i,1} = -s_{g_{i,1}}(z_{i,1}) \left( c_{i,1} + \frac{\hat{\alpha}_{i,1}^2}{4} \right) \left( |z_{i,1}| - \epsilon_{i,1} \right)^n
\]

(28)

(29)
\[ + \tilde{\sigma}_{i,1} \left( \epsilon_{i,2} + 1 \right) \right) - \omega_{\theta_{i,i}} \hat{\theta}_{i} \]
\[ \alpha_{i,1} = \tilde{\eta}_{i,1} \tilde{\alpha}_{i,1} \]  
where \( c_{i,1} \) is a positive design parameter.

From (60), one has \( \dot{\tilde{\theta}}_{i}(0) > 0 \), and therefore, if \( \dot{\tilde{\theta}}_{i}(0) > 0 \) is chosen, \( \dot{\tilde{\theta}}_{i} > 0 \) holds. Similarly, we can prove that \( \tilde{\sigma}_{i,p} > 0 \), and thus, we have
\[ \left( \vert \tilde{z}_{i} \vert - \epsilon_{i,1} \right)^{n_{i}} f_{i,1} s_{g_{i,1}} r_{i,1} \alpha_{i,1} \leq 0 \]  
Hence,
\[ \left( \vert \tilde{z}_{i} \vert - \epsilon_{i,1} \right)^{n_{i}} f_{i,1} s_{g_{i,1}} r_{i,1} \alpha_{i,1} \leq \left( \vert \tilde{z}_{i} \vert - \epsilon_{i,1} \right)^{n_{i}} f_{i,1} s_{g_{i,1}} \frac{1}{\eta_{i,p}} (\eta_{i,p} - \tilde{\eta}_{i,p}) \tilde{\alpha}_{i,1} \]
\[ \leq \left( \vert \tilde{z}_{i} \vert - \epsilon_{i,1} \right)^{n_{i}} f_{i,1} s_{g_{i,1}} \tilde{\eta}_{i,p} \tilde{\alpha}_{i,1} \]
By combining (28)–(33), \( \tilde{V}_{i,1} \) (26) can be calculated as
\[ \tilde{V}_{i,1} \leq - \left( c_{i,1} + \frac{\tilde{\sigma}_{i,1}^{2}}{4} \right) \left( \vert \tilde{z}_{i} \vert - \epsilon_{i,1} \right)^{2n_{i}} f_{i,1} \]
\[ + \left( \vert \tilde{z}_{i} \vert - \epsilon_{i,1} \right)^{n_{i}} f_{i,1} (w_{\theta_{i,1}} |z_{i,2}|) - \tilde{\sigma}_{i,1} \left( \epsilon_{i,2} + 1 \right) + \tau_{\theta_{i,i}} \hat{\theta}_{i} \]

**Remark 6** If the quadratic Lyapunov function \( V = \frac{1}{2} z^{2} \) is chosen and the NN \( W^{T} R + \Delta \) is utilized to approximate the unknown function, we can only achieve that \( \dot{V} \leq cV + \sigma \) where \( c > 0 \) is a constant and \( \sigma \) is unknown [13–21]. This method cannot guarantee that the derivative of Lyapunov function is negative, because the use of Young’s inequality introduces the unknown term into the stability analysis. In this paper, the smooth function \( s_{g} \) is utilized, and thus, the use of Young’s inequality is avoided. With function \( s_{g} \), we can achieve that \( \dot{V} \leq 0 \). The control algorithm for nonlinear system in [22] cannot be directly extended to MASs because of the uncertainties from agent’s neighbor.

**Remark 7** The consensus tracking error is defined in (12) only using the available information. Moreover, due to the existence of state hysteresis, the design of virtual controller \( \alpha_{i,p} \) at step \( p \) may require the states \( x_{i,p+1} \) and \( x_{j,p+1} \). The rate-dependent term \( \dot{\tilde{\eta}}_{i,p} \) introduces the state \( x_{i,p+1} \) into the virtual controller design. However, according to [46, 47], \( x_{i,p+1} \) cannot be obtained in step \( p \). Thus, \( r_{i,p} \) is unavailable. According to Lemma 2, it is easy to prove that \( r_{i,p} \) is bounded and nonnegative. Then, the upper and lower bound of the time-varying term \( r_{i,p} \) are approximated by \( \tilde{\sigma}_{i,p} \) and \( \tilde{\eta}_{i,p} \), respectively. Moreover, Lemma 1 and NNs are introduced to handle the unaccessible state from its neighbor \( x_{j,p+1} \). With these adjustments, we can continue the controller design.

**Step 2** Select the second Lyapunov function as:
\[ V_{i,2} = V_{i,1} + \frac{1}{n_{i}} \left( \vert z_{i,2} \vert - \epsilon_{i,2} \right)^{n_{i}} f_{i,2} \]
\[ + \frac{1}{2 \gamma_{i} \eta_{i,2}} \tilde{\eta}_{i,2}^{2} + \frac{1}{2 \xi_{i}} \tilde{\sigma}_{i,1}^{2} \]  
where \( \zeta_{i} \) is a positive constant. From (12) and (21), one can directly derive that
\[ \dot{z}_{i,2} = \ddot{x}_{i,2} - \dot{\alpha}_{i,1} = r_{i,2} (z_{i,3} + \alpha_{i,2} - \mu_{i,3} \tilde{x}_{i,3}) \]
\[ + r_{i,2} \mu_{i,31} r_{i,2} \]
\[ - \frac{\partial \alpha_{i,1}}{\partial \tilde{x}_{i,1}} (\mu_{i,11} + \mu_{i,12} \tilde{h}_{i,1}) (x_{i,2} + \pi_{i,1} (\tilde{x}_{i,1})) \]
\[ - \sum_{j \in N_{i}} \frac{\partial \alpha_{i,1}}{\partial \tilde{x}_{j,1}} r_{j,1} (\tilde{x}_{j,2} - \mu_{j,22} \tilde{x}_{j,2}) \]
\[ + \mu_{j,21} \pi_{j,1} (\tilde{x}_{j,1}) \]
\[ - \frac{\partial \alpha_{i,1}}{\partial \tilde{\sigma}_{i,1}} \tilde{\sigma}_{i,1} - \frac{\partial \alpha_{i,1}}{\partial \tilde{\eta}_{i,1}} \tilde{\eta}_{i,1} - \frac{\partial \alpha_{i,1}}{\partial \tilde{\theta}_{i}} \tilde{\theta}_{i} - \frac{\partial \alpha_{i,1}}{\partial \gamma_{i}} \tilde{\gamma}_{i} \]
where \( r_{i,2} = (\mu_{i,21} + \mu_{i,22} \tilde{h}_{i,2}) / \mu_{i,31} \). Define \( \eta_{i,2} \) representing the upper bound of \( \frac{1}{r_{i,2}} \tilde{\eta}_{i,2} \) is an estimate of \( \eta_{i,2} \) and \( \tilde{\eta}_{i,2} = \eta_{i,2} - \tilde{\eta}_{i,2} \). There exists an unknown function \( \tilde{g}_{i,2}(\tilde{Z}_{i,2}) \), such that
\[ \tilde{g}_{i,2}(\tilde{Z}_{i,2}) = - r_{i,2} \mu_{i,32} \tilde{x}_{i,3} + r_{i,2} \mu_{i,31} r_{i,2} \]
\[ - \frac{\partial \alpha_{i,1}}{\partial \tilde{x}_{i,1}} (\mu_{i,11} + \mu_{i,12} \tilde{h}_{i,1}) (x_{i,2} + \pi_{i,1} (\tilde{x}_{i,1})) \]
\[ - \sum_{j \in N_{i}} \frac{\partial \alpha_{i,1}}{\partial \tilde{x}_{j,1}} r_{j,1} (\tilde{x}_{j,2} - \mu_{j,22} \tilde{x}_{j,2}) \]
\[ + \mu_{j,21} \pi_{j,1} (\tilde{x}_{j,1}) - \frac{\partial \alpha_{i,1}}{\partial \gamma_{i}} \tilde{\gamma}_{i} \]
\[ \frac{1}{r_{i,2}} \tilde{g}_{i,2}(\tilde{Z}_{i,2}) \]
where \( \tilde{Z}_{i,2} = [\tilde{x}_{i,2}, \tilde{x}_{j,2}, \tilde{x}_{i,1}, \tilde{x}_{j,1}] \)

**Remark 8** The use of backstepping technique requires to calculate the derivatives of virtual control laws repeatedly, which impedes the implementation of our control scheme. To address this problem, the derivatives of virtual control laws are contained in the unknown function which is estimated by a NN. And then, the adaptive law is designed to approximate the ideal norm of the NN. Thus, the calculation of derivative is avoided.
With $Z_{i,2} = [\vec{x}_{i,2}, \vec{x}_{j,2}, \vec{n}_{i,1}, \hat{\hat{n}}_{i,1}, \hat{\sigma}_{i,1}]^T$, according to Lemma 1, the virtual control law is designed as:

$$\tilde{a}_{i,2} = -sg_{i,2} (z_{i,2}) \left[ \left( c_{i,2} + 1 + \frac{\hat{\sigma}_{i,2}^2}{4} \right) \right]$$

$$\left( |z_{i,2}| - \epsilon_{i,2} \right)^{n_{i,1}} - \hat{\sigma}_{i,2} (\epsilon_{i,2} + 1) + d^2 \left( \frac{\partial \alpha_1}{\partial n_{i,1}} \right)^2 + d^2 \left( \frac{\partial \alpha_1}{\partial \sigma_{i,1}} \right)^2$$

$$\left( \frac{\partial \alpha_1}{\partial \theta_{i,2}} \right)^2$$

$$\omega_{i,2} = s g_{i,2} \sqrt{\psi_{i,2} (Z_{i,2})^T} \psi_{i,2} (Z_{i,2}) + d^2$$

$$\alpha_{i,2} = \hat{\hat{n}}_{i,2} a_{i,2}$$

$$\dot{\hat{n}}_{i,2} = -\gamma (|z_{i,2}| - \epsilon_{i,2})^{n_{i,1}-1} f_{i,2} s g_{i,2} a_{i,2}$$

$$\tau_{\theta_{i,2}} = \tau_{\theta_{i,1}} + (|z_{i,2}| - \epsilon_{i,2})^{n_{i,1}-1} f_{i,2} s g_{i,2} \omega_{i,2}$$

$$\hat{\dot{\sigma}}_{i,1} = \zeta (|z_{i,1}| - \epsilon_{i,1})^{n_{i,1}} f_{i,1} |z_{i,2}|$$

where $\gamma$, $\Gamma$ and $c_{i,2}$ are positive constants to be selected.

In the review of (35)–(43), the derivative of $V_{i,2}$ can be obtained

$$V_{i,2} \leq -c_{i,1} (|z_{i,1}| - \epsilon_{i,1})^{2n_{i,1}} f_{i,1}$$

$$- \left( c_{i,2} + \frac{\hat{\sigma}_{i,2}^2}{4} \right) (|z_{i,2}| - \epsilon_{i,2})^{2(n_{i,1}-1)} f_{i,2}$$

$$- (|z_{i,2}| - \epsilon_{i,2})^{n_{i,1}-1} f_{i,2} \left[ \sigma_{i,2} |z_{i,2}| \right]$$

$$- \hat{\sigma}_{i,2} (\epsilon_{i,2} + 1)$$

$$- (|z_{i,2}| - \epsilon_{i,2})^{n_{i,1}-1} f_{i,2} \sqrt{\left( \frac{\partial \alpha_1}{\partial \theta_{i,2}} \right)^2 + 2d^2 (\Gamma_{i} \tau_{\theta_{i,2}} - \omega_{\theta_{i,2}} \hat{\theta}_{i})}$$

$$- \hat{\dot{\theta}}_{i} + M_{i,2} + \hat{\dot{\theta}}_{i} \tau_{\theta_{i,2}}$$

where

$$M_{i,2} = -\frac{\hat{\sigma}_{i,1}^2}{4} (|z_{i,1}| - \epsilon_{i,1})^{2n_{i,1}} f_{i,1}$$

$$+ (|z_{i,1}| - \epsilon_{i,1})^{n_{i,1}} f_{i,1} [\sigma_{i,1} |z_{i,2}|]$$

$$- \hat{\sigma}_{i,1} (\epsilon_{i,2} + 1) - (|z_{i,2}| - \epsilon_{i,2})^{2(n_{i,1}-1)} f_{i,2}$$

Similarly, $\hat{\sigma}_{i,p} > 0$ holds if we choose $\hat{\sigma}_{i,p} (0) > 0$. Then with the work in [45], $M_{i,2} \leq 0$ is always satisfied.

**Remark 9** The proof of $M_{i,2} \leq 0$ needs that $\hat{\sigma}_{i,1} > 0$.

By designing the updated law $\hat{\dot{\sigma}}_{i,1}$ (43) as given in step 2, $\hat{\sigma}_{i,1} > 0$ holds. Accordingly, the updated law $\hat{\dot{\sigma}}_{i,p}$ will be designed in step $p + 1$.

**Step p** Similarly, we choose a Lyapunov function

$$V_{i,p} = V_{i,p-1} + \frac{1}{n_{i,p} - p + 2} (|z_{i,p}| - \epsilon_{i,p})^{n_{i,p} - p + 1} f_{i,p}$$

$$+ \frac{1}{2 \gamma_{i,p} \epsilon_{i,p}} \hat{n}_{i,p}^2$$

$$+ \frac{1}{2 \gamma_{i,p}^{2}} \hat{\sigma}_{i,p-1}^2$$

(46)

Calculating the time derivative of (12), we have

$$\dot{z}_{i,p} = \hat{x}_{i,p} - \hat{\dot{a}}_{i,p-1} = r_{i,p} (z_{i,p+1} + \alpha_{i,p} + \hat{g}_{i,p}$$

$$- \sum_{k=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \sigma_{i,k}} \hat{\sigma}_{i,k} - \sum_{k=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \hat{n}_{i,k}} \hat{n}_{i,k}$$

$$- \frac{\partial \alpha_{i,p-1}}{\partial \hat{\theta}_{i}} \hat{\theta}_{i}$$

(47)

$$\ddot{g}_{i,p} = -r_{i,p} \mu_{i,p+1}) z_{i,p+1} + r_{i,p} \mu_{i,p+1}) \pi_{i,p}$$

$$- \sum_{k=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \hat{x}_{i,k}} \hat{x}_{i,k}$$

$$- \sum_{j=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \hat{y}_{j}} \hat{y}_{j}$$

$$- \sum_{j=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \hat{x}_{i,k}} \hat{x}_{i,k} - \frac{\partial \alpha_{i,p-1}}{\partial \hat{y}_{j}} \hat{y}_{j}$$

(48)

The control law and adaption laws are directly given as

$$\alpha_{i,p} = \hat{n}_{i,p} \hat{a}_{i,p}$$

$$\hat{a}_{i,p} = -sg_{i,p} (z_{i,p}) \left[ \left( c_{i,p} + 1 + \frac{\hat{\sigma}_{i,p}^2}{4} \right) \right]$$

$$\left( |z_{i,p}| - \epsilon_{i,p} \right)^{n_{i,p} - p + 1} + \hat{\sigma}_{i,p} (\epsilon_{i,p} + 1)$$

$$+ \sum_{p=1}^{p-1} \left( \frac{\partial \alpha_{i,p-1}}{\partial \hat{n}_{i,p}} \right)^2 + \sum_{j=1}^{p-1} \left( \frac{\partial \alpha_{i,p-1}}{\partial \hat{\theta}_{i}} \right)^2 + d^2$$

$$+ \left( \frac{\partial \alpha_{i,p-1}}{\partial \hat{\theta}_{i}} \right)^2 + d^2 \cdot \Gamma_{\omega_{\theta_{i,p}}}$$

$$+ \sum_{j=1}^{p-1} \left( \frac{\partial \alpha_{i,p-1}}{\partial \hat{y}_{j}} \right)^2$$

$$\sqrt{\left( \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_{i}} \right)^2 + d^2 \cdot \Gamma_{\omega_{\theta_{j,p}}}$$

$$- \omega_{\theta_{i,p}} \hat{\theta}_{i}$$

(51)

$$\hat{\dot{n}}_{i,p} = -\gamma (|z_{i,p}| - \epsilon_{i,p})^{n_{i,p} - p + 1} f_{i,p} s g_{i,p} \hat{a}_{i,p}$$

$$\tau_{\theta_{i,p}} = \tau_{\theta_{i,p-1}} + (|z_{i,p}| - \epsilon_{i,p})^{n_{i,p} - p + 1} f_{i,p} s g_{i,p} \omega_{\theta_{i,p}}$$

(52)

$$\omega_{\theta_{i,p}} = s g_{i,p} \sqrt{\psi_{i,p} (Z_{i,p})^T} \psi_{i,p} (Z_{i,p}) + d^2$$

(54)

$$\hat{\dot{\sigma}}_{i,p-1} = \zeta (|z_{i,p-1}| - \epsilon_{i,p-1})^{n_{i,p-1} + 1} f_{i,p-1} |z_{i,p}|$$

(55)

© Springer
where $c_{i,p}$ is a positive design parameter.

Then, we can derive the following equation in the same way as in step 2.

$$
\dot{V}_{i,p} \leq \sum_{j=1}^{k} -c_{i,p} \left( |z_{i,j} - \epsilon_{i,j}| \right)^2 f_{i,j} \\
- \frac{\hat{\sigma}^2_{i,p}}{4} \left( |z_{i,1} - \epsilon_{i,1}| \right)^{2(n_{i} - p + 1)} f_{i,p} \\
+ \left( |z_{i,1} - \epsilon_{i,1}| \right)^{n_{i} - p + 1} f_{i,p} \left[ \sigma_{i,p} |z_{i,1} + 1| \\
- \tilde{\sigma}_{i,p} \left( \epsilon_{i,1} + 1 \right) \right] \\
+ \tilde{\theta}_{i} \tau_{\theta,i,p} \\
- \sum_{j=2}^{k} \left( |z_{i,j} - \epsilon_{i,j}| \right)^{n_{i} - j + 1} f_{i,j} \\
\sqrt{\left( \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_{i}} \right)^2 + d^2} \\
\left( \Gamma_{i} \tau_{\theta,i,j} - \hat{\theta}_{i} \right)
$$

(56)

**Step n** In this case, we consider the MASs without input hysteresis. The plant can directly receive control signal, i.e., $H_{i}(v_{i}) = v_{i}$. The distributed consensus controller and the updated laws are given to achieve our control goal:

$$
v_{i} = \tilde{v}_{i,n_{i}} \hat{a}_{i,n_{i}} \tag{57}
$$

$$
\hat{a}_{i,n_{i}} = -sgn(z_{i,n_{i}}) (c_{i,n_{i}} + 1) (z_{i,n_{i}} - \epsilon_{i,n_{i}}) \\
+ \sum_{j=1}^{n_{i}-1} \left( \frac{\partial \alpha_{i,n_{i}-1}}{\partial \hat{\theta}_{i,j}} \right) f_{i,j} \\
+ \sum_{j=1}^{n_{i}-1} \left( \frac{\partial \alpha_{i,n_{i}-1}}{\partial \sigma_{i,j}} \right) f_{i,j} \\
+ d^2 \cdot \Gamma_{i} \tau_{\theta,i,n_{i}} \\
+ \sum_{j=2}^{n_{i}} \left( |z_{i,j} - \epsilon_{i,j}|^{n_{i} - j + 1} f_{i,j} \\
\sqrt{\left( \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_{i}} \right)^2 + d^2} \cdot \Gamma_{i} \tau_{\theta,i,n_{i}} \right) \\
- \omega_{\theta,i,n_{i}} \hat{\theta}_{i} \tag{58}
$$

$$
\hat{\theta}_{i} = \Gamma_{i} \tau_{\theta,i,n_{i}} \tag{59}
$$

$$
\hat{\sigma}_{i,n_{i}-1} = \sigma \left( |z_{i,n_{i}-1} - \epsilon_{i,n_{i}-1}|^{2} f_{i,n_{i}-1} |z_{i,n_{i}} \right) \tag{60}
$$

where $\tau_{\theta,i,n_{i}}$ and $\omega_{\theta,i,n_{i}}$ are defined in (53) and (54). Define the Lyapunov function for this step.

$$
V_{i,n_{i}} = V_{i,n_{i}} - \frac{1}{2} \left( |z_{i,n_{i}} - \epsilon_{i,n_{i}}|^{2} f_{i,n_{i}} + \frac{1}{2\gamma_{i}} \eta_{n_{i}} + \frac{1}{2\tilde{\sigma}^{2}_{i,n_{i}-1} + \frac{1}{2\Gamma_{i}} \hat{\theta}_{i}^{2}} \right) \\
\dot{V}_{i,n_{i}} \leq - \sum_{p=1}^{n_{i}} c_{i,p} \left( |z_{i,p} - \epsilon_{i,p}|^{2(n_{i} - p + 1)} f_{i,k} \right) \tag{63}
$$

With the design of control signal and updated laws, the following theorem can be concluded.

**Theorem 1** Considering the MASs (4) with states hysteresis, if the controller (57) and adaptation laws (59)–(61) are implemented, we can obtain that:

- all variables of the MASs (4) are SGUUB.
- the consensus tracking error asymptotically converges to a predefined bound along with time, i.e., $\lim_{t \to \infty} |z_{i,1} \leq \epsilon_{i,1}$. Furthermore, $\|y - \bar{y}\| \leq \frac{\epsilon_{1}}{\Delta}$, where $\epsilon_{1} = \max(\epsilon_{1})$.

**Proof** Define the Lyapunov function

$$
V = \sum_{i=1}^{N} V_{i,n_{i}} \tag{64}
$$

From (63), one has

$$
\dot{V} \leq - \sum_{i=1}^{N} \sum_{p=1}^{n_{i}} c_{i,p} \left( |z_{i,p} - \epsilon_{i,p}|^{2(n_{i} - p + 1)} f_{i,k} \right) \leq 0 \tag{65}
$$

Thus, $z_{i,p}, \hat{\theta}_{i}, \hat{\sigma}_{i,n_{i}-1}, \hat{\sigma}_{i,n_{i}-1}$, and $\hat{\eta}_{i,p}$ are bounded. Moreover, through the definition of $z_{i,p}$ (12), we can get that $\hat{x}_{i,1}$ is bounded because the output of leader $y_{0}$ is bounded. $\alpha_{i,1}$ is consisting of variables $\hat{\theta}_{i,1}, \sigma_{i,1}, \eta_{1,1}, \hat{x}_{i,1}$ and $\hat{x}_{i,1}$, such that $\alpha_{i,1}$ is also bounded. Subsequently, we can further get that $\hat{x}_{i,2}$ is bounded from (12). In the same way, the boundedness of $\alpha_{i,2}$ and $\hat{x}_{i,2}$ can be proved. Then the control signal $v_{i}$ is also bounded. Noticing Eq. (9), the real system states $x_{i,p}$ are ensured to be bounded. Next it is proved that the consensus errors converge to a predefined neighborhood of zero. For convenience, we define $s_{i} = \left( |z_{i,1} - \epsilon_{i,1}|^{n_{i} - 1} f_{i,1} \right)$. Then one has

$$
\dot{V}_{i,n_{i}} \leq - c_{i,1} s_{i}^{2} \tag{66}
$$

Integrate both sides of the inequality

$$
\int_{0}^{\infty} s_{i}(t)^{2} dt \leq \frac{1}{c_{i,1}} V_{i,n_{i}}(0) \tag{67}
$$

According to Barbalat’s lemma, it is proved that $\lim_{t \to \infty} s_{i} = 0$, i.e., $\lim_{t \to \infty} |z_{i,1} \leq \epsilon_{i,1}$. Along with Lemma 3, $\|y - \bar{y}\| \leq \frac{\epsilon_{1}}{\Delta}$ is achieved. \( \square \)
The \( L_2 \)-norm of the consensus error can be further proved

\[
\|s_i\|_2 = \sqrt{\int_0^\infty s_i(t)^2 dt} \leq \frac{1}{\sqrt{2\epsilon_{i,1}}} \sum_{p=1}^{n_i} \left[ \frac{1}{n_i - p + 2} \right]^{n_i - p + 2} \int_0^\infty f_i, p(z_i, p(0)) + \frac{1}{2\gamma_i} \hat{n}_i, p(0) + \frac{1}{2\epsilon_{i, p-1}(0)} \right]
\]

Remark 10: It is proved that the consensus tracking errors can be designed in advance by using the Lyapunov function with smooth sign function. The \( L_2 \)-norm performance is obviously adjustable. By increasing the design parameters \( \xi_i, \Gamma_i \) and \( \gamma_i \), the transient performance can be improved. The consensus tracking errors can be reduced by selecting a suitable \( \epsilon_{i,1} \).

3.2 Case 2 states hysteresis and input hysteresis

In the just described work, the control method for MASs with states hysteresis was developed. Now the case that actuators affected by hysteresis are further considered. The inverse compensation strategy is given as follows:

\[
v_i = \hat{H}_i (u_{id}) = \frac{1}{\mu_{i1}} u_{id} - \frac{\hat{\mu}_{i2}}{\hat{\mu}_{i1}} \xi_{id}
\]

(69)

\[
\hat{\xi}_{id} = \hat{u}_{id} \hat{h}_i (\xi_{id}, \hat{u}_{id})
\]

(70)

\[
\hat{h}_i (\xi_{id}, \hat{u}_{id}) = 1 - \text{sign}(\hat{u}_{id}) h_i |\xi_{id}|^{l_i id - 1} \xi_{id} - \chi_{id} |\xi_{id}|^{l_i id - 1}
\]

(71)

where \( h_{id}, l_{id}, \chi_{id} \) are positive constants to be chosen. In addition, \( h_{id} \geq |\chi_{id}| \). The error between actual control signal and desired control signal is given by

\[
u_i - u_{id} = \mu_i T \theta_i - \hat{\mu}_i T \theta_i
\]

(72)

\[
\leq \hat{\mu}_i T \theta_i + \Lambda_i
\]

where

\[
\hat{\mu}_i = [\hat{\mu}_{i1}, \hat{\mu}_{i2}] T, \theta_i = [v_i, \xi_i] T, \theta_{id} = [v_i, \xi_{id}] T, \text{ and } \Lambda_i = \mu_i T (\theta - \theta_{id}) = \mu_i (\hat{\xi}_i - \xi_{id})
\]

Since \( \hat{\xi}_i \) and \( \xi_{id} \) are bounded, \( \Lambda_i \) has an unknown upper bound such that \( |\Lambda_i(t)| \leq \Lambda_i \).

Only the actual control law in step \( n_i \) needs to be modified. We have

\[
\hat{\xi}_{i,n} = \hat{\xi}_{i,n-1} - \hat{\xi}_{i,n-1} = r_{i,n} (u_i - u_{id} + u_{id} + \pi_{i,n}) - \hat{\xi}_{i,n-1}
\]

(73)

The RBFNN is employed to approximate the unknown control gain. One has

\[
r_{i,n} (u_i - u_{id}) = W_i T R_i (Z_i) + \Delta_i (Z_i)
\]

(74)

where \( Z_i = [\hat{x}_{i,n}, v_i, \xi_{id}] T \).

\[
r_{i,n} (u_i - u_{id}) = r_{i,n} (\hat{\mu}_i T \theta_i + \Lambda_i)
\]

\[
= \hat{\mu}_i T [W_i T R_i + \Delta_i] + r_{i,n} \Lambda_i
\]

(75)

\[
= \hat{\mu}_i T [W_i T R_i + \hat{W}_i T R_i + \mu_i T \hat{W}_i T R_i]
\]

+ \hat{\mu}_i T \Delta_i + \hat{\mu}_i T \Lambda_i

Then,

\[
\left( |z_{i,n} - \epsilon_{i,n}| f_{i,n} s g_{i,n} r_{i,n} (u_i - u_{id}) \right)
\]

\[
\leq \left( |z_{i,n} - \epsilon_{i,n}| f_{i,n} s g_{i,n} [\hat{\mu}_i T \hat{W}_i T R_i]
\]

\[
- \hat{\mu}_i T \hat{W}_i T R_i + s g_{i,n} \lambda_i
\]

(76)

\[
\hat{\lambda}_i = \|\hat{\mu}_i \| \|\hat{W}_i \| R_i + \|\hat{\mu}_i \| \tilde{\Delta}_i + \sigma_{l,n} \bar{\lambda}_i.
\]

This calculation requires an important property that \( \hat{W}_i \) and \( \hat{\mu} \) always have the upper bound \( W_i \) and \( \mu_i \). The property will be guaranteed by designing the updated laws Eqs. (80) and (81). The control law and updated laws are designed as follows:

\[
u_{id} = \hat{\nu}_i (\bar{\alpha}_{i,n} - s g_{i,n} \hat{\lambda}_i)
\]

(77)

\[
\hat{\nu}_i = -\gamma_i (|z_{i,n} - \epsilon_{i,n}| f_{i,n} s g_{i,n} (\hat{\alpha}_{i,n} - s g_{i,n} \hat{\lambda}_i)
\]

(78)

\[
\hat{\lambda}_i = \beta_i (|z_{i,n} - \epsilon_{i,n}| f_{i,n}
\]

(79)

\[
\hat{\mu}_i = \text{Proj} [\Gamma_{\mu} (|z_{i,n} - \epsilon_{i,n}| f_{i,n} \hat{\nu}_i T R_i]
\]

(80)

\[
\hat{\lambda}_i = \text{Proj} [\Gamma_{\lambda} (|z_{i,n} - \epsilon_{i,n}| f_{i,n} \hat{\nu}_i T R_i]
\]

(81)

where \( \beta_i \) is a parameter to be designed. \( \Gamma_{\mu} \) and \( \Gamma_{\lambda} \) are well-chosen positive definite matrices. Proj(\) represents the projection operator developed in [46].

Remark 11: Unlike [25], the model of input hysteresis is unknown, such that the perfect inverse compensation in [25] cannot be applied in this paper. In this article, we design the updated law Eq. (80) to estimate the model’s parameters instead of parameter identifications, which is easier to be implemented in practice. Compared with [28–30], the control input of MASs is not only affected by the unknown hysteresis in actuators, but also by the unknown control gain caused by states hysteresis, as shown in Eq. (73). The control
coefficient is time-varying such that we cannot directly design an inverse compensation for input hysteresis. A NN is employed to approximate the unknown function.

**Theorem 2** Considering the MASs (4) with states hysteresis and input hysteresis, if the controller (77) and adaptation laws (78)–(81) are implemented, we can conclude that:

- all variables of the MASs (4) are SGUUB.
- the consensus tracking error asymptotically converges to a predefined bound along with time, i.e., \( \lim_{t \to \infty} |z_{i,1}| \leq \epsilon_{i,1} \). Furthermore, \( \|y - \bar{Y}\| \leq \frac{\epsilon_1}{\Delta} \), where \( \epsilon_1 = \max \{\epsilon_{i,1}\} \).

**Proof** Consider the following Lyapunov function

\[
\hat{V}_i,\hat{n}_i = V_i,\hat{n}_i - 1 + \frac{1}{2} \left( |z_{i,n_i}| - \epsilon_{i,n_i} \right)^2 f_{i,n_i}^2 + \frac{1}{2\gamma_i} \eta_i,\hat{n}_i + \frac{1}{2\gamma_i} \hat{\sigma}_{i,n_i}^2 - 1 + \frac{1}{2\gamma_i} \hat{\sigma}_i^2 \tag{82}
\]

and define \( \hat{V} = \sum_{i=1}^{N} \hat{V}_i,\hat{n}_i \). From (82), one has

\[
\dot{\hat{V}} \leq - \sum_{i=1}^{N} \sum_{p=1}^{n_i} c_{i,p} \left( |z_{i,p}| - \epsilon_{i,p} \right)^2 (\hat{n}_i - p + 1) f_{i,k} \leq 0 \tag{83}
\]

Thus, \( z_{i,p}, \hat{\theta}_i, \hat{\sigma}_{i,p-1}, \hat{\eta}_{i,p}, \hat{\lambda}_i, \hat{\mu}_i \) and \( \hat{W}_i \) are bounded. Moreover, through the definition of \( z_{i,p} \) (12), we can get that \( \hat{x}_{i,1} \) is bounded because the output of leader \( y_0 \) is bounded. \( \alpha_{i,1} \) is consisting of variables \( \hat{\theta}_i, \sigma_{i,1}, \eta_{i,1}, \hat{x}_{i,1} \), and \( \hat{x}_{i,2} \), such that \( \alpha_{i,1} \) is also bounded. Subsequently, we can further get that \( \hat{x}_{i,2} \) is bounded from (12). In the same way, the boundedness of \( \alpha_{i,p} \) and \( \hat{x}_{i,p} \) can be proved. Then the control signal \( v_i \) is also bounded. Noticing Eq. (9), the real system states \( x_{i,p} \) are ensured to be bounded.

The property that the consensus errors converge to a predefined neighborhood of zero is also given.

\[
\dot{\hat{V}}_{i,\hat{n}_i} \leq -c_{i,1} \hat{V}_i^2 \tag{84}
\]

According to Barbalat’s lemma, it is proved that \( \lim_{t \to \infty} s_i = 0 \), i.e., \( \lim_{t \to \infty} |z_{i,1}| \leq \epsilon_{i,1} \). Along with Lemma 3, \( \|y - \bar{Y}\| \leq \frac{\epsilon_1}{\Delta} \) is achieved. Since the measured errors are always bounded, the boundedness of the actual system states can be obtained by the boundedness of the measured value.

The \( L_2 \)-norm of the consensus errors can be further proved

\[
\|s_i\|_2 \leq \frac{1}{\sqrt{2c_{i,1}}} \sum_{p=1}^{n_i} \left[ \frac{1}{n_i - p + 2} \left( |z_{i,p}(0)| - \epsilon_{i,p} \right)^{n_i - p + 2} f_{i,p}(z_{i,p}(0)) + \frac{1}{2\gamma_i} \hat{\sigma}_{i,p}^2(0) + \frac{1}{2\gamma_i} \hat{\sigma}_{i,p-1}^2(0) \right] \tag{85}
\]

**Remark 12** If we directly design the updated laws as \( \hat{\mu}_i = \Gamma_i \mu_i \left( |z_{i,n_i}| - \epsilon_{i,n_i} \right) f_{i,n_i} \hat{W}_i^T R_i \) and \( \hat{\lambda}_i = -\Gamma_i \mu_i \left( |z_{i,n_i}| - \epsilon_{i,n_i} \right) f_{i,n_i} R_i \hat{\mu}_i^T \), the term \( \lambda_i \) which includes \( \hat{W}_i \) and \( \hat{\mu}_i \) cannot be ensured to be bounded. Accordingly, the compensation of \( \lambda_i \) cannot be achieved and the system stability cannot be guaranteed. With the projection operator, we can make sure that \( \hat{W}_i \in \Omega_{\hat{W}_i} \) and \( \hat{\mu}_i \in \Omega_{\hat{\mu}_i} \) if \( \hat{W}_i(0) \in \Omega_{\hat{W}_i} \) and \( \hat{\mu}_i(0) \in \Omega_{\hat{\mu}_i} \). With this property, we can continue our work and design the distributed controllers.

**Remark 13** In Eq. (76), we cannot confirm the sign of item

\[
\left( |z_{i,n_i}| - \epsilon_{i,n_i} \right) f_{i,n_i} \gamma_{i,n_i} \gamma_{i,n_i}^T \hat{W}_i^T R_i
\]

in advance, and therefore, we cannot estimate the norm instead of the weight vector to reduce the computational burden. However, the number of estimated parameters depends on the number of NN nodes, so that we can reduce the number of NN node to avoid the computational problem. It does not affect the system stability or increase tracking errors. As shown in Eq. (85), only the transient performance of the system is affected.

**Corollary 1** In both two cases, the MASs (4) still remain stable even if there is bounded disturbances in the system dynamics. Moreover, the conclusion in Theorems 1 and 2 still can be achieved.

**Proof** Let \( \gamma_{i,n_i} \) represent the disturbance. When the disturbances exist in system dynamics, according to
The dynamics of follower agents are given as
\[\dot{x}_{i,2} = u_i - \sin(x_{i,1}x_{i,2}) + 0.1 \sin(t)\]
\[\dot{x}_{i,1} = x_{i,2} + 0.3e^{x_{i,1}^2} - 0.3 \sin(x_{i,1}^2) + 1 \sin(t)\] (88)

where \(\dot{x}_{i,2}\) is the exact system state vector which cannot be used in our controller. The measuring sensors are subject to the hysteresis which is model by
\[\dot{x}_{i,p} = H_i(p(x_{i,p}) = 0.55x_{i,p} + \xi_{i,p}\]
\[\dot{\xi}_{i,p} = \dot{x}_{i,p}\hat{h}_{i,p}(\xi_{i,p}, \dot{x}_{i,p})\] (89)
\[h_{i,p}(\xi_{i,p}, \dot{x}_{i,p}) = 1 - \text{sign}(\dot{x}_{i,p})|\xi_{i,p}|\xi_{i,p} - 0.5|\xi_{i,p}|^2\] (90)

It is more challenging when the input and states are both subjected to hysteresis. The control scheme in Case 2 is implemented for the example. The hysteresis in actuator is modeled by
\[H_i(v_i) = 1.2v_i + 0.5\xi_i\]
\[\dot{\xi}_i = \hat{v}_i \hat{h}_i(\xi_i, \dot{v}_i)\] (92)
\[\hat{h}_i(\xi_i, \dot{v}_i) = 1 - 1.2 \text{sign}(\dot{v}_i)|\xi_i|\dot{v}_i - 0.5|\xi_i|^2\] (93)

The leader’s dynamics are given as follows:
\[\dot{x}_0 = -x_0 + r^3 \exp(-2r) + 1.5 \cos(0.2r)\]
\[y_0 = x_0\] (95)

The design parameters are selected as follows: \(c_{i,1} = 110, c_{i,2} = 3, \zeta_{i,1} = 3, \gamma_i = 0.98, \beta_i = 2, d = 0.78, \Gamma_i = 1.63\). The tracking errors are bounded by the accuracy \(\epsilon_{i,1} = 0.75\) and \(\epsilon_{i,2} = 0.8\). The matrices \(\Gamma_{i,W}\) and \(\Gamma_{i,\mu}\) are set to the identity matrices. \(h_{id}, l_{id}\) and \(\chi_{id}\) in (71) are set as 1.1, 2 and 0.5, respectively. The initial MAS’s states are given as: \(x_1(0) = [0.21, 0]^T, x_2(0) = [0.13, 0]^T, x_3(0) = [0.19, 0]^T, x_4(0) = [0.24, 0]^T, x_5(0) = [0.18, 0]^T\). The leader’s output is initialized as 0.21. The initial values of the adaptive parameters are listed below: \(\dot{\hat{\theta}}(0) = 0.03, \dot{\hat{n}}_{i,1}(0) = \hat{n}_{i,2}(0) = \dot{\tilde{\omega}}_{i,1}(0) = 0.1, \hat{n}_{i,0}(0) = 0.07, \lambda_{i,0}(0) = 0.5, \hat{\mu}_{i,0}(0) = [1, 0.6]^T\) and \(\dot{\hat{W}}_i(0) = \begin{bmatrix} 0.8 & 0.8 \\ 0.8 & 0.8 \end{bmatrix}\). The RBFNNs used in this example contain 3 nodes. Their centers evenly spaced in \([-0.7, 0, 0.7]^T\) and \(\xi_i = 2\). Define the tracking errors \(\epsilon_i = |\hat{x}_{i,1} - y_0|\). The simulation results are shown in Figs. 3, 4, 5, 6, 7 and 8.

We can conclude that adaptive parameters are bounded as shown in Fig. 6. The consensus errors converge to a small domain of zero rapidly as shown in
Fig. 3 The outputs measured by sensors and the leader’s output

Fig. 4 The genuine outputs of agents

Fig. 5 Consensus tracking errors

Therefore, there are still tracking errors in Figs. 3 and 5. Figure 7 shows that the exact system state \(x_{1,2}\) is very different from the measured one \(\hat{x}_{1,2}\), which is the greatest difficulty of the controller design in this paper.

4.2 Application example

Consider the spring–mass–damper MAS that has the same topology as Fig. 2. Each follower is modeled as

\[
\dot{x}_{i,1} = x_{i,2}
\]

\[
\dot{x}_{i,2} = \frac{u_i}{m_i} - \frac{k_{i,1} x_{i,1}}{m_i} - \frac{c_{i,2} x_{i,2}}{m_i}
\]

The leader’s dynamics is given as:

\[
\dot{y}_0 = -y_0 + 4 \exp(-3t)
\]

The system parameters are selected as: \(m_i = 1\), \(k_i = 1 + 0.2i\) and \(c_i = 1.5\). The design parameters are selected as follows: \(c_{i,1} = 6\), \(c_{i,2} = 1\) and \(e_{i,1} e_{i,2} = 0.8\). The initial MAS’s states and the leader’s output are the same as the numerical example. Some initial values of the adaptive parameters have been changed as below: \(\hat{\eta}_{i,1}(0) = \hat{\eta}_{i,2}(0) = 1\), \(\hat{\lambda}_{i}(0) = 0.5\) and \(\hat{\mu}_{i}(0) = [0.2, 0.6]^T\). To illustrate the effectiveness of the proposed method, the control scheme in [23] is considered to make a comparison. We select the same design parameters. Figures 9, 10 and 11 shows the simulation results. It can be obtained from Fig. 11 that the proposed scheme can provide a better system performance. Note that the system performance is adjustable by selecting different design parameters. However, the

Remark 14 As we mentioned in the proof of Theorem 1 and 2, the proposed control method can ensure that the tracking error \(\lim_{t \to \infty} |z_{i,1}| \leq \epsilon_{i,1}\). If we choose \(\epsilon_{i,1} = 0\), the perfect tracking performance can be achieved. However, in this case, the function \(sg_{i,k}\) will be discontinuous which may cause the chattering phenomenon of actuator. Therefore, in practical application we should choose a suitable tracking accuracy according to the actuator. In this simulation example, the tracking accuracy \(\epsilon_{i,1}\) is selected as 0.75, and there-
control scheme in [23] cannot ensure the system stability if the strong hysteresis phenomenon occurs in sensors.

5 Conclusion

The consensus control strategy for MASs with unknown states hysteresis and input hysteresis is researched. Based on backstepping technique, we employ 2 adaptive laws to approximate the upper and lower bounds of the unknown term introduced by state hysteresis. To handle the input hysteresis, NNs are utilized to approximate the unknown control gain which is coupled by input hysteresis and states hysteresis. The proposed scheme guarantees the boundedness of all signal and the consensus errors are ensured to converge to a predefined neighborhood of zero asymptotically. In addition, the $L_2$-norm of the consensus error can be further ensured. Two simulation examples are provided to illustrate the effectiveness of the proposed control approach.

There are still several questions worth exploring in the future:

![Fig. 6 Adaptive parameters](image1)

![Fig. 6 Adaptive parameters](image2)

![Fig. 6 Adaptive parameters](image3)

![Fig. 6 Adaptive parameters](image4)
As mentioned in Remark 14, chattering phenomenon may occur if we choose 0 tracking error. How to achieve perfect tracking performance without chattering is an interesting problem.

The inverse compensation for input hysteresis employs NN, which will bring lots of learning parameters. It is an important work to reduce the computed burden.

Besides, the control for MAS with quantized states and sensor faults is under our consideration.

Acknowledgements This work was supported in part by Guangdong Province Universities and Colleges Pearl River Scholar Funded Scheme, and in part by the National Key...
References

1. Curtain, T.B., Bellingham, J.G., Catipovic, J., Webb, D.: Autonomous oceanographic sampling networks. Oceanography 6(3), 86–94 (1993)
2. Meng, D., Jia, Y.: Robust consensus algorithms for multiscale coordination control of multivehicle systems with disturbances. IEEE Trans. Ind. Electron. 63(2), 1107–1119 (2016)
3. Tomlin, C., Pappas, G., Sastry, S.: Conflict resolution for air traffic management: a study in multiagent hybrid systems. IEEE Trans. Autom. Control 43(4), 509–521 (1998)
4. Zhang, J., Feng, T., Zhang, H., Wang, X.: The decoupling cooperative control with dominant poles assignment. IEEE Trans. Syst. Man Cybern. Syst. PP, 1–9 (2020). https://doi.org/10.1109/TSMC.2020.3011142
5. Zhang, J., Chen, Z., Zhang, H., Feng, T.: Coupling effect and pole assignment in trajectory regulation of multi-agent systems. Automatica 125, 109465 (2021). https://doi.org/10.1016/j.automatica.2020.109465
6. Yu, W., Chen, G., Ming, C.: Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems. Automatica 46(6), 1089–1095 (2010)
7. Su, H., Chen, G., Wang, X., Lin, Z.: Adaptive second-order consensus of networked mobile agents with nonlinear dynamics. Automatica 47(2), 368–375 (2011)
8. Tian, Y.P., Liu, C.L.: Consensus of multi-agent systems with diverse input and communication delays. IEEE Trans. Autom. Control 53(9), 2122–2128 (2008)
9. Zhang, H., Lewis, F.L., Das, A.: Optimal design for synchronization of cooperative systems: state feedback, observer and output feedback. IEEE Trans. Autom. Control 56(8), 1948–1952 (2011)
10. Jiang, F., Wang, L., Jia, Y.: Consensus in leaderless networks of high-order-integrator agents. In: 2009 American Control Conference, pp. 4458–4463 (2009)
11. Hua, C.-C., Li, K., Guan, X.-P.: Leader-following output consensus for high-order nonlinear multiauton systems. IEEE Trans. Autom. Control 64(3), 1156–1161 (2018)
12. Zhang, H., Lewis, F.L.: Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics. Automatica 48(7), 1432–1439 (2012)
13. Shi, P., Shen, Q.: Cooperative control of multi-agent systems with unknown state-dependent controlling effects. IEEE Trans. Autom. Sci. Eng. 12(3), 827–834 (2015)
14. Yoo, S.J.: Distributed consensus tracking for multiple uncertain nonlinear strict-feedback systems under a directed graph. IEEE Trans. Neural Netw. Learn. Syst. 24(4), 666–672 (2013)
15. Chen, B., Liu, X., Liu, K., Lin, C.: Direct adaptive fuzzy control of nonlinear strict-feedback systems. Automatica 45(6), 1530–1535 (2009)
16. Wang, F., Chen, B., Lin, C., Li, X.: Distributed adaptive neural control for stochastic nonlinear multiagent systems. IEEE Trans. Cybern. 47(7), 1795–1803 (2017)
17. Lin, Z., Liu, Z., Zhang, Y., Chen, C Philip: Distributed adaptive cooperative control for uncertain nonlinear multiagent systems with hysteretic quantized input. J. Franklin Inst. 357(8), 4645–4663 (2020)
18. Chen, B., Liu, X.P., Ge, S.S., Lin, C.: Adaptive fuzzy control of a class of nonlinear systems by fuzzy approximation approach. IEEE Trans. Fuzzy Syst. 20(6), 1012–1021 (2012)
19. Liu, Z., Wang, F., Zhang, Y., Chen, X., Chen, C.L.P.: Adaptive fuzzy output-feedback controller design for nonlinear systems via backstepping and small-gain approach. IEEE Trans. Cybern. Auton. Control 44(10), 1714–1725 (2014)
20. Chen, B., Zhang, H., Lin, C.: Observer-based adaptive neural network control for nonlinear systems in nonstrict-feedback form. IEEE Trans. Neural Netw. Learn. Syst. 27(1), 89–98 (2016)
21. Wang, F., Chen, B., Lin, C.: Consensus tracking control for distributed nonlinear multiagent systems via adaptive neural backstepping approach. IEEE Trans. Syst. Man Cybern. Syst. 50(7), 2436–2444 (2020)
22. Lai, G., Zhang, Y., Liu, Z., Chen, C.P.: Indirect adaptive fuzzy control design with guaranteed tracking error performance for uncertain canonical nonlinear systems. IEEE Trans. Fuzzy Syst. 27(6), 1139–1150 (2018)
23. Lu, K., Liu, Z., Lai, G., Chen, C.L.P., Zhang, Y.: Adaptive consensus tracking control of uncertain nonlinear multi-agent systems with predefined accuracy. IEEE Trans. Cybern. 51(1), 405–415 (2021)
24. Chen, X., Ozaki, T.: Adaptive control for plants in the presence of actuator and sensor uncertain hysteresis. IEEE Trans. Autom. Control 56(1), 171–177 (2010)
25. Zhou, J., Wen, C., Li, T.: Adaptive output feedback control of uncertain nonlinear systems with hysteresis nonlinearity. IEEE Trans. Autom. Control 57(10), 2627–2633 (2012)
26. Tao, G., Kokotovic, P.V.: Adaptive control of plants with unknown hystereses. IEEE Trans. Autom. Control 40(2), 200–212 (1995)
27. Su, C.-Y., Wang, Q., Chen, X., Rakheja, S.: Adaptive variable structure control of a class of nonlinear systems with unknown Prandtl–Ishlinskii hysteresis. IEEE Trans. Autom. Control 50(12), 2069–2074 (2005)
28. Zhang, X., Li, Z., Su, C.-Y., Lin, Y., Fu, Y.: Implementable adaptive inverse control of hysteric systems via output feedback with application to piezoelectric positioning stages. IEEE Trans. Ind. Electron. 63(9), 5733–5743 (2016)
29. Yu, Z., Li, S., Yu, Z., Li, F.: Adaptive neural output feedback control for nonstrict-feedback stochastic nonlinear systems with unknown backlash-like hysteresis and unknown control directions. IEEE Trans. Neural Netw. Learn. Syst. 29(4), 1147–1160 (2018)
30. Zhou, Q., Wang, W., Ma, H., Li, H.: Event-triggered fuzzy adaptive containment control for nonlinear multiagent systems with unknown Bouc–Wen hysteresis input. IEEE Trans. Fuzzy Syst. 29(4), 731–741 (2021)
31. Tao, G., Kokotovic, P.V.: Adaptive Control of Systems with Actuator and Sensor Nonlinearities. Wiley & Sons, Inc., Hoboken (1996)

32. Dong, R., Tan, Y.: A model based predictive compensation for ionic polymer metal composite sensors for displacement measurement. Sens. Actuators A Phys. 224, 43–49 (2015)

33. Lei, H., Sharif, M.A., Tan, X.: Dynamics of omnidirectional IPMC sensor: experimental characterization and physical modeling. IEEE/ASME Trans. Mechatron. 21(2), 601–612 (2015)

34. Seco, F., Martín, J.M., Pons, J.L., Jiménez, A.R.: Hysteresis compensation in a magnetostrictive linear position sensor. Sens. Actuators A Phys. 110(1–3), 247–253 (2004)

35. Freeman, R.: Global internal stabilizability does not imply global external stabilizability for small sensor disturbances. IEEE Trans. Autom. Control 40(12), 2119–2122 (1995)

36. Chen, C., Wen, C., Liu, Z., Xie, K., Zhang, Y., Chen, C.P.: Adaptive consensus of nonlinear multi-agent systems with non-identical partially unknown control directions and bounded modelling errors. IEEE Trans. Autom. Control 62(9), 4654–4659 (2016)

37. Liu, Z., Lu, K., Lai, G., Chen, C.L.P., Zhang, Y.: Indirect fuzzy control of nonlinear systems with unknown input and state hysteresis using an alternative adaptive inverse. IEEE Trans. Fuzzy Syst. 29(3), 500–514 (2021)

38. Sanner, R.M., Slotine, J.-J.E.: Gaussian networks for direct adaptive control. In: 1991 American Control Conference, pp. 2153–2159 (1991)

39. Chen, C.P., Wen, G.-X., Liu, Y.-J., Wang, F.-Y.: Adaptive consensus control for a class of nonlinear multiagent time-delay systems using neural networks. IEEE Trans. Neural Netw. Learn. Syst. 25(6), 1217–1226 (2014)

40. Ikhouane, F., MahOsa, V., Rodellar, J.: Adaptive control of a hysteretic structural system. Automatica 41(2), 225–231 (2005)

41. Chen, X., Hisayama, T., Su, C.-Y.: Pseudo-inverse-based adaptive control for uncertain discrete time systems preceded by hysteresis. Automatica 45(2), 469–476 (2009)

42. Parlangeli, G., Corradini, M.L.: Output zeroing of MIMO plants in the presence of actuator and sensor uncertain hysteresis nonlinearities. IEEE Trans. Autom. Control 50(9), 1403–1407 (2005)

43. Zhang, J., Torres, D., Ebel, J.L., Sepúlveda, N., Tan, X.: A composite hysteresis model in self-sensing feedback control of fully integrated v02 microactuators. IEEE/ASME Trans. Mechatron. 21(5), 2405–2417 (2016)

44. Rakotondrabe, M.: Bouc–Wen modeling and inverse multiplicative structure to compensate hysteresis nonlinearity in piezoelectric actuators. IEEE Trans. Autom. Sci. Eng. 8(2), 428–431 (2010)

45. Zhou, J., Wen, C., Zhang, Y.: Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis. IEEE Trans. Autom. Control 49(10), 1751–1759 (2004)

46. Krstic, M., Kokotovic, P.V., Kanellakopoulos, I.: Nonlinear and Adaptive Control Design. Wiley & Sons, Inc., Hoboken (1995)

47. Chen, B., Liu, K., Liu, X., Shi, P., Lin, C., Zhang, H.: Approximation-based adaptive neural control design for a class of nonlinear systems. IEEE Trans. Cybern. 44(5), 610–619 (2013)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.