Thermal Relaxation Time in Chemically Non-equilibrated Quark-Gluon Plasma

Xiao-Fei Zhang\textsuperscript{a,b}, Wei-Qin Chao\textsuperscript{a,b}

\textsuperscript{a}CCAST (World Laboratory), P. O. Box 8730, Beijing 100080, People’s Republic of China

\textsuperscript{b}Institute of High Energy Physics, Academia Sinica, Beijing 100039, People’s Republic of China

Abstract

The definition of thermal relaxation time is extended to chemically non-equilibrated quark-gluon plasma and the chemical non-equilibrated thermal relaxation times for partons are calculated using the non-equilibrium Debye mass as the infrared regulator. The dependence of the thermal relaxation time on the fugacity is given and the influence of the chemical non-equilibration is discussed. We find that there are threshold fugacities $\lambda_g^*$ and $\lambda_q^*$ for gluons and quarks. For $\lambda_g < \lambda_g^*(\lambda_q < \lambda_q^*)$, $\tau_{g, q}^{\text{NEQ}} / \tau_{g, q}^{\text{EQ}}$ decreases strongly with increasing fugacity, while for $\lambda_g > \lambda_g^*(\lambda_q > \lambda_q^*)$, the ratios are almost 1. It is shown that there is also the two-stage equilibration in a chemically non-equilibrated plasma. We also discussed the effect of using the non-equilibrium Debye mass as the infrared cutoff.

PACS number(s): 12.38.Mh, 24.85.+p, 52.25.Dg
I. INTRODUCTION

One of the main objectives of the future experimental programs at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC) is the production of Quark-Gluon Plasma (QGP) through heavy ion collisions[1]. The parton gas produced in heavy ion collisions is not immediately in thermal and chemical equilibrium. The parton gas relaxes to the equilibrium state through secondary interactions. Because the preequilibrium phase may influence the QGP signals, it is essential to discuss nonequilibrium properties of QGP and to investigate whether QGP produced in RHIC and LHC could reach equilibrium. Quantum transport theory[2] in principle can be used to describe the properties of nonequilibrium QGP and there are some discussions based on it[3]. However, its applicability to heavy ion collisions is still far from being realistic. The Boltzmann equation in the relaxation time approximation is often used to discuss the transport coefficients and the evolution of the QGP [4-5]. In this approximation method the collision terms are determined by the corresponding thermal relaxation time which measures the time scale of the nonequilibrium system approaching equilibrium and can be estimated from the parton interaction rates[6-8]. The equilibration of QGP considering of parton expansion effect is discussed recently [5] However, the thermal relaxation time has been considered so far mostly for a chemically equilibrated plasma. As discussed by many models[9-11], in the early stage of ultra-relativistic heavy ion collisions, the parton gas is dominated by gluons and far from chemical equilibrium. First, the system is evolving to thermal equilibrium mainly through elastic scattering, which leads to the local isotropy in momentum distribution. The system evolve towards chemical equilibrium later and sometimes the plasma could never reach chemical equilibrium during its life time. In this paper we calculate the thermal relaxation time in a chemically non-equilibrated plasma and discuss how the chemical nonequilibration influences the parton thermal relaxation.

The thermal relaxation time in QGP diverges strongly at small momentum transfer in the naive perturbation theory. This difficulty could be overcome by the Bratten-Yuan method[12] which proposed that the soft contribution $k < k^*$ and the hard part $k > k^*$ could be calculated separately by introducing a separation
scale $k^*$ for the momentum transfer. The soft part is treated using the resummed propagators and vertices proposed by Braaten and Pisarski[13], whereas the bare Green functions are sufficient for the hard one. Assuming $gT < k^* < T$, the final result is independent of the separation $k^*$. There is another much simpler and widely applied approximation method based on the naive perturbation theory, using bare propagators with the Debye mass as infrared regulator[14]. It was demonstrated that this method also worked well for calculating the thermal relaxation time[6]. We will use this approximation approach, and the nonequilibrium Debye mass as the infrared regulator in this paper.

This paper is organized as follows, in Section II we first define the interaction rates in a chemically non-equilibrated plasma. Then, in Section III, the thermal relaxation time for quarks and gluons are calculated using the definition of section II. Finally, in section IV, we give some numerical results and discussions.

II. DEFINITION OF THE INTERACTION RATES IN A CHEMICALLY NON-EQUILIBRATED GAS

In this section we will extend Weldon’s discussion[17] and define the interaction rates for a chemically non-equilibrated gas. There are two kinds of interaction rates. One is the ordinary interaction rate defined as the damping rate for partons, which is usually called interaction rate. The other is the so called transport interaction rate obtained from the ordinary one by introducing a transport weight containing the scattering angle in the center of mass system. Because large angle scattering is the most efficient mechanism for the dissipative momentum transfer in the case of a plasma with long range interaction, hence the mean free path, as well as the thermal relaxation time, may rather be defined by the inverse of the transport interaction rate[15]. Let us first define the ordinary interaction rate for a chemically non-equilibrated plasma from the Bolzmann equation[16],

$$v_1 \cdot \partial_1 f_1(p_1) = -\frac{\nu}{2p_1} \int \frac{d^3p_2}{(2\pi)^32p_2} \int \frac{d^3p_3}{(2\pi)^32p_3} \int \frac{d^3p_4}{(2\pi)^32p_4} \left[ f_1 f_2 (1 \pm f_3)(1 \pm f_4) - (1 \pm f_1)(1 \pm f_2)f_3f_4 \right]$$
\[(2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4) |M_{12\rightarrow34}|^2,\]  

where \(P_i = (p_i, \vec{p}_i), v_i^\mu = p_i^\mu/p_i, \partial_i^\mu = \partial/\partial(x_i)\mu. f_i\) is the distribution function \(f(p_i)\) for partons. \(\pm\) are used for bosons (gluons) and fermions (quarks and anti-quarks). \(|M_{12\rightarrow34}|^2\) is the squared matrix element for corresponding parton scattering process and is summed over final states and averaged over initial states. The spin and color factor \(\nu\) is 16 for gluons and 6 \(N_f\) with \(N_f\) flavors of quarks or anti-quarks.

To obtain the definition of the interaction rates\[17\], we assume that \(f_1\) is away from equilibrium and \(f_i, i = 2, 3, 4,\) are in thermal equilibrium but not in chemical equilibrium, which can be expressed as the following form,

\[f_i^e = \lambda_i \frac{1}{e^{\beta p_i} \mp \lambda_i}, \quad i = 2, 3, 4,\]  

where \(\lambda_i\) is the fugacity of the corresponding partons. It measures how far the system is from the chemical equilibrium. Using the relation for the distribution function Eq.(2),

\[1 \pm f_i^e = \lambda_i^{-1} e^{\beta p_i} f_i^e, \]  

we obtain from the Boltzmann equation,

\[v_1 \cdot \partial_1 f_1(p_1) = -\Gamma(p_1)(f_1 - f_1^e), \]  

\[\Gamma(p_1) = \frac{\nu}{2p_1} \int \frac{d^3p_2}{(2\pi)^32p_2} \int \frac{d^3p_3}{(2\pi)^32p_3} \int \frac{d^3p_4}{(2\pi)^32p_4} \frac{f_2^e(1 \pm f_3^e)(1 \pm f_4^e)}{(2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4)|M_{12\rightarrow34}|^2}.\]  

The above equation is the definition of the interaction rates in a chemically non-equilibrated plasma. By putting \(\lambda_i = 1,\) Eq.(5) is just the interaction rates for a chemically equilibrated plasma\[17\].

We will assume \(f_1^e = f_3^e, f_2^e = f_4^e\) in the following discussion. This simplification holds as long as \(k = |\vec{p}_1 - \vec{p}_3| = |\vec{p}_2 - \vec{p}_4|\) is not too large or \(-t\) is not of the order of \(s,\) with \(s, t, u\) being the usual Mandelstam variables\[8,16\]. Now the transport interaction rates and the thermal relaxation time can be defined from Eq.(5) after introducing a transport weight \(\sin\theta^2/2,\)
\[
\frac{1}{\tau} = \Gamma^{\text{trans}} = \frac{\nu}{2p_1} \int \frac{d^3p_2}{(2\pi)^3} \int \frac{d^3p_3}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3} f_2^e(1 \pm f_4^e) \\
(2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4) |M_{12 \rightarrow 34}|^2 \frac{\sin^2 \theta}{2}.
\]

(6)

**III. RELAXATION TIME FOR QUARKS AND GLUONS**

In this section we use Eq.(6) to calculate the thermal relaxation time for quarks and gluons in chemically non-equilibrated plasma. As thermalization is achieved mainly through momentum changes in elastic scattering to the leading order. Here we only consider the contribution of the elastic scattering. In the lowest order the relevant matrix elements \(M^2\) are [18]

\[
M_{gg \rightarrow gg}^2 = \frac{9}{2} \left[ 3 - \frac{ut}{s^2} - \frac{us}{s^2} - \frac{st}{u^2} \right],
\]

(7)

\[
M_{gq \rightarrow gq}^2 = \frac{-4}{9} \frac{u^2 + s^2}{us} + \frac{u^2 + s^2}{t^2},
\]

(8)

\[
M_{q_1q_2 \rightarrow q_1q_2}^2 = \frac{4}{9} \frac{s^2 + u^2}{t^2}.
\]

(9)

Using the definition of the differential cross section[19]

\[
\frac{d\sigma}{dt} = \frac{g^4}{16\pi s^2} |M|^2
\]

for gluons, we obtain

\[
\frac{1}{\tau_g} = 16 \int \frac{d^3k}{(2\pi)^3} f_g^e(k)[1 + f_g^e(k)] \int dt \frac{d\sigma}{dt}_{gg \rightarrow gg} \frac{2tu}{s^2}
\]

\[
+ 12N_f \int \frac{d^3k}{(2\pi)^3} f_q^e(k)[1 - f_q^e(k)] \int dt \frac{d\sigma}{dt}_{gq \rightarrow gq} \frac{2tu}{s^2}.
\]

(11)

Where \(f_g\) and \(f_q\) are the chemical non-equilibrated distribution function of gluons and quarks respectively.

The interaction rates diverges at small \(k\). As has been discussed, for a chemically equilibrated plasma, it can be regulated by the Debye mass which is the gluon
self energy at zero momentum in the high temperature limit [20]. For a chemically non-equilibrated plasma, we use the non-equilibrium Debye mass as the infrared regulator,

$$m_{DNEQ}^2 = 4\pi(\lambda_g + \frac{N_f}{6}\lambda_q)\alpha_s T^2.$$  \hspace{1cm} (12)

In the following calculation, we assume $f_i$ can be approximated by its factorized form [10],

$$f_i = \lambda_i \frac{1}{e^{\beta p_i} \pm 1}.$$  \hspace{1cm} (13)

Using Eqs. (11-13) and after some direct calculations, we obtain the thermal relaxation time for gluons

$$\frac{1}{\tau_g} = \frac{72T^3}{\pi s_{gg}}\alpha_s^2[2\lambda_g\xi(3) - 2\lambda_g^2\xi(3) + \frac{\lambda_g^2\pi^2}{3}][\ln(s_{gg}/q^{N_{EQ}}) - 19/15]\,$$

$$+ \frac{24N_fT^3}{\pi s_{gg}}\alpha_s^2[\frac{3}{2}\lambda_q^2\xi(3) - \frac{3}{2}\lambda_q^2\xi(3) + \frac{\lambda_q^2\pi^2}{6}][\ln(s_{gg}/q^{N_{EQ}}) - 1.26],$$  \hspace{1cm} (14)

where $\xi(3) = 1.202$, the infrared cutoff $q^{N_{EQ}} = m_{DNEQ}^2$. We assume that $s_{gg}$ and $s_{gq}$ in the above equation can be replaced by its thermal average value, $< s_{gg} >= 2 < p_g > < p_g >, < s_{gq} > = 2 < p_g > < p_q >$ where the expression for $< p_i >$ is

$$< p_i > = \frac{\int d^3 p \alpha_s f(p_i) \beta p_i}{\int d^3 p \beta p_i f(p_i)}.$$  \hspace{1cm} (15)

It can be seen from Eq(15) that $< s >$ is approximately the same for the chemically non-equilibrated and equilibrated gluon gases i.e., $< s_{gg} >= 14.59T^2, < s_{gq} >= 16.96T^2$.

For the thermal relaxation time of quarks we consider the processes $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$ and $qg \rightarrow qg$. The thermal relaxation time for quarks we obtained is

$$\frac{1}{\tau_q} = \frac{32T^3}{\pi s_{qq}}\alpha_s^2[2\lambda_g\xi(3) - 2\lambda_g^2\xi(3) + \frac{\lambda_g^2\pi^2}{3}][\ln(s_{qq}/q^{N_{EQ}}) - 19/15]\,$$

$$+ \frac{32N_fT^3}{3\pi s_{qq}}\alpha_s^2[\frac{3}{2}\lambda_q^2\xi(3) - \frac{3}{2}\lambda_q^2\xi(3) + \frac{\lambda_q^2\pi^2}{6}][\ln(s_{qq}/q^{N_{EQ}}) - 1.26]$$  \hspace{1cm} (16)

Where $s_{qq} = 19.72T^2$ in the above equation.
IV. NUMERICAL RESULTS AND DISCUSSIONS

The thermal relaxation time depends on the fugacity of gluons and quarks. To completely determine it we must know the initial conditions of heavy ion collisions, which only can be obtained from models now. Focusing on the effect of the chemical non-equilibration on the thermal relaxation time in this paper, we give a general discussion here. Fig. 1-2 show the ratio of the thermal relaxation time in chemically nonequilibrium QGP to the one in chemical equilibrium, $\tau_{NEQ}^g/\tau_{EQ}^g$ and $\tau_{NEQ}^q/\tau_{EQ}^q$, v.s. fugacity $\lambda_g, \lambda_q$ respectively. It is found that the results depend weakly on the ratio between the gluon fugacity and quark fugacity. We give the results of two cases, $\lambda_q = \lambda_g$ and $\lambda_q = \frac{1}{2} \lambda_g$. $\tau_{NEQ}^g/\tau_{EQ}^g$ decreases as the fugacity increases in both cases. There are corresponding threshold fugacities $\lambda_g^*$ and $\lambda_q^*$. For $\lambda_g < \lambda_g^*$ ($\lambda_q < \lambda_q^*$), $\tau_{NEQ}^g/\tau_{EQ}^g$ ($\tau_{NEQ}^q/\tau_{EQ}^q$) depends strongly on the fugacity, while for $\lambda_g > \lambda_g^*$ ($\lambda_q > \lambda_q^*$), the ratio is almost 1. These threshold fugacities depend on the QCD coupling constant. From Fig. 1-2, one can also see that the bigger the coupling constant, the smaller the threshold fugacity. The results of Fig.1-2 can be explained as follows: When the system is approaching chemical equilibrium, more and more partons are produced and the fugacity increases continuously. The parton density of chemically non-equilibrated plasma is smaller than the one in the equilibrium state at the same temperature. As a result, the relaxation time in the chemically non-equilibrated plasma is longer than the one in chemical equilibrium.

We also check the influences of using non-equilibrium Debye mass as the infrared cutoff and the result is shown Fig. 3. Using the nonequilibrium Debye mass as the infrared regulator restrains the increase of $\tau^{NEQ}/\tau^{EQ}$ with decreasing fugacity strongly.

Based on parton interaction rates calculated for the chemically equilibrated plasma, Shuryak argued that equilibration of the plasma proceeds via two stages in ’hot gluon scenario’ [21], where gluons equilibrate much faster than quarks. To investigate if the situation is changed for a chemically non-equilibrated plasma we also calculate the ratio $\tau_q/\tau_g$ in a chemically non-equilibrated plasma. The results are shown in Fig.4. We find that the ratios are around 2 and do not change much with the fugacity, although it changes a little more for longer coupling constants.
From our calculation we can conclude that there is also the two-stage equilibration in a chemically non-equilibrated plasma, i.e., gluons also equilibrated much faster than quarks.

As a final remark, we should point out that only the contribution of the elastic scattering to the thermal relaxation is taken into account in this work. The contribution of the inelastic scattering should also be considered in the future study.

ACKNOWLEDGEMENT

This work is supported in part by the National Nature Science Foundation of China. We would like to thank Y. Pang, X. Q. Li, A. Tai and X. X. Yao for useful discussions.
REFERENCES

[1] for a recent review see Quark Matter'95, Nucl. Phys. A 590 (1995) 1.

[2] H. T. Elze and U. Heinz, Phys. Rep. 183, (1989) 81.

[3] X. F. Zhang and J. R. Li, Phys. Rev. C 52, (1995) 864 ; Ann. Phys. 250, (1996) 433.

[4] G. Baym, Phys. Lett. B 128 (1984) 18; S. Gavin, Nucl. Phys. B 435 (1984) 826; A. Hosoya and K. Kajantie, Nucl. Phys. B 250 (1985) 666.

[5] H. Heiselberg and X. N. Wang, Phys. Rev. C 53 (1996) 1892.

[6] M. H. Thoma, Phys. Rev. D 49 (1994) 451.

[7] G. Baym, H. Monien, C. Pethick, and D. G. Ravenhall, Phys. Rev. Lett. 64 (1990) 1867.

[8] F. Reif, Fundamentals of Statistical and Thermal Physics (McGraw-Hill, New York, 1965).

[9] K. Geiger and J. I. Kapusta, Phys. Rev. D 47 (1993) 4905.

[10] T. S. Biró, E. V. Doorn, B. Müller, M. H. Thoma and X. N. Wang Phys. Rev. C 48 (1993) 1275.

[11] E. Shuryak, Phys. Rev. Lett. 68 (1992) 3270.

[12] E. Braaten and T. C. Yuan, Phys. Rev. Lett 66 (1991) 2183.

[13] E. Braaten and R. D. Pisarski, Phys. Rev. D 46, (1992) 1289.

[14] E. Braaten and M. H. Thoma, Phys. Rev. D 44 (1991) 1289.

[15] E. M. Lifshitz and L. P. Pitaevskii, Physical Kinetics (Pergamon, New York, 1981).

[16] S. R. de Groot, W. A. van Lecuwen and Ch. G. van Weert, Relativistic Kinetic Theory (North-Holland, Amsterdam, 1980).

[17] H. A. Weldon, Phys. Rev. D 28 (1983) 2007.
[18] L. Combridge, J. Kripfganz and J. Pethick, Phys. Lett. 70 (1977) 234.

[19] J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill, New York, 1964).

[20] H. A. Weldon, Phys. Rev. D 26 (1982) 1394.

[21] E. Shuryak, Phys. Rev. Lett 22 (1992) 3270.
Figure Capture:

Fig.1. The ratio of the thermal relaxation time of gluons in chemically nonequilibrium plasma to the one in chemical equilibrium, $\frac{\tau_{g}^{NEQ}}{\tau_{g}^{EQ}}$, v.s. fugacity $\lambda_g$.

Fig.2. The ratio of the thermal relaxation time of quarks in chemically nonequilibrium plasma to the one in chemical equilibrium, $\frac{\tau_{q}^{NEQ}}{\tau_{q}^{EQ}}$, v.s. fugacity $\lambda_q$.

Fig.3. The thermal relaxation time obtained using non-equilibrium Debye mass as the infrared regulator comparing with the one using equilibrium Debye mass.

Fig.4. The ratio of the thermal relaxation time of quarks to the thermal relaxation time of gluons in chemically nonequilibrium plasma, $\frac{\tau_{q}^{NEQ}}{\tau_{g}^{NEQ}}$, v.s. fugacity $\lambda_g$. 
Fig. 3
Fig. 1

- $\alpha_s = 0.1$, $\lambda_q = \lambda_g$
- $\alpha_s = 0.1$, $\lambda_q = 0.5\lambda_g$
- $\alpha_s = 0.2$, $\lambda_q = \lambda_g$
- $\alpha_s = 0.2$, $\lambda_q = 0.5\lambda_g$
\[ \frac{\tau_{\text{eq}}}{\tau_g} \]

- \( \alpha_s = 0.1 \lambda_q = \lambda_g \)
- \( \alpha_s = 0.1 \lambda_q = 0.5\lambda_g \)
- \( \alpha_s = 0.2 \lambda_q = \lambda_g \)
- \( \alpha_s = 0.2 \lambda_q = 0.5\lambda_g \)

Fig. 4
Fig. 2