HIGH-RESOLUTION SIMULATION ON STRUCTURE FORMATION WITH EXTREMELY LIGHT BOSONIC DARK MATTER

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ABSTRACT

A bosonic dark matter model is examined in detail via high-resolution simulations. These bosons have particle mass of the order of $10^{-22}$ eV and are noninteracting. If they do exist and can account for structure formation, these bosons must be condensed into the Bose–Einstein state and described by a coherent wave function. This matter, also known as fuzzy dark matter, is speculated to be able, first, to eliminate the subgalactic halos to solve the problem of overabundance of dwarf galaxies, and, second, to produce flat halo cores in galaxies suggested by some observations. We investigate this model with simulations up to 1024\(^3\) resolution in a 1 \(h^{-1}\) Mpc box that maintains the background matter density \(\Omega_m = 0.3\) and \(\Omega_\Lambda = 0.7\). Our results show that the extremely light bosonic dark matter can indeed eliminate low-mass halos through the suppression of short-wavelength fluctuations, as predicted by the linear perturbation theory. But in contrast to expectation, our simulations yield singular cores in the collapsed halos, where the halo density profile is similar, but not identical, to the Navarro–Frenk–White profile. Such a profile arises regardless of whether the halo forms through accretion or merger. In addition, the virialized halos exhibit anisotropic turbulence inside a well-defined virial boundary. Much like the velocity dispersion of standard dark matter particles, turbulence is dominated by the random radial flow in most part of the halos and becomes isotropic toward the halo cores. Consequently, the three-dimensional collapsed halo mass distribution can deviate from spherical symmetry, as the cold dark matter halo does.

Key words: dark matter – Galaxy: structure – large-scale structure of universe

Online-only material: color figures

1. INTRODUCTION

Observations of low surface brightness galaxies and dwarf galaxies indicate that the cores of galactic halos have shallow density profiles (Dalcanton et al. 1997; Swaters et al. 2000) instead of central cusps predicted by cold dark matter (CDM; Navarro et al. 1997). In addition, the number density of dwarf galaxies in Local Group turns out to be an order of magnitude fewer than that produced by CDM simulations (Klypin et al. 1999). These two features cast doubt on the validity magnitude fewer than that produced by CDM simulations (Klypin et al. 1999). These two features cast doubt on the validity of standard CDM. There have been at least three different solutions proposed to resolve these problems: (1) warm dark matter, (2) collisional dark matter, and (3) fuzzy dark matter.

Warm dark matter can suppress small-scale structures by free streaming. It seems to be able to both solve the overabundance problem of dwarf galaxies and the singular core problem. In this model, the flat core is embedded within a radius a couple of percent of the virial radius (Jing 2001; Colins et al. 2008), and the core smoothly connects to the Navarro–Frenk–White (NFW) profile (Navarro et al. 1997) outside. However, this modification may generally adversely affect structures of somewhat larger scales (Hu et al. 2000), despite that fine tuning of the thermal velocity of dark matter particles may still be able to have the larger scale structures consistent with observations (Abazajian 2006; Viel et al. 2008).

For collisional dark matter, the halo core can be flattened and dwarf galaxies destroyed, and \(N\)-body simulations confirm this conjecture (Spergel & Steinhardt 2000). But simulations also show that very frequent collisions can yield even more singular cores than the standard collisionless CDM does (Yoshida et al. 2000). This opposite behavior is indicative of the requirement of fine tuning for collisional parameters.

The third solution to the problem is to treat dark matter as an extremely light bosonic dark matter (ELBDM) or fuzzy dark matter (Press & Ryden 1990; Sin 1994; Hu et al. 2000). Axion has been thought to be a candidate of light bosonic dark matter. But for the light dark matter to erase the singular galactic core and suppresses low-mass halos, the particle mass must be far smaller than axion \((m \sim 10^{-22} \text{ eV})\), so low that the uncertainty principle operates on the astronomical length scale. Much like axions, the ELBDM is in a Bose–Einstein condense state produced in the early universe. These extremely light particles share the common ground state and is described by a single coherent wave function. Its de-Broglie wavelength is comparable to or even somewhat smaller than the Jean’s length (Davies & Widrow 1997), where the quantum fluctuation provides effective pressure against self-gravity. Several previous works have pondered on such an idea or its variance (Sin 1994; Hu et al. 2000; Siddhartha & Uréna-López 2003), in which the wave mechanics is described by the Schrödinger–Poisson equation with Newtonian gravity or by the Klein–Gordon equation with gravity. The Schrödinger–Poisson system addresses the scale-free regime of quantum mechanics, where the Jean’s length is a dynamical running parameter. On the other hand, the Klein–Gordon system makes use of the Compton wavelength as a natural length scale to create the flat core in a halo. Widrow and Kaiser conducted simulations for the two-dimensional Schrödinger–Poisson system to approximate the standard collisionless cold dark matter (Widrow & Kaiser 1993). In the two-dimensional case, the \(1/r\) gravitational potential is replaced by \(\log(r)\), and the two-dimensional force law in their simulation becomes of longer range than it actually is in three dimensions. Due to the lack of three-dimensional numerical simulations, some authors resort to spherical symmetry (Sin 1994;
Siddhartha & Uréña-López (2003) or even one-dimensional (Hu et al. 2000) to study this problem. These simplifications may not capture what actually results in a three-dimensional system with realistic initial conditions. In particular, the existence of a flattened core has been derived or inferred from these previous works of one-dimensional system or with spherical symmetry. In this paper, we report high-resolution fully three-dimensional simulations for this problem. Surprisingly, our simulations reveal that the singular cores of bound objects remain to exist even when the core size is much smaller than the Jean’s length.

In Section 2, we provide an explanation for the possible existence of the Bose–Einstein state for the extremely low mass bosons under investigation here. We then discuss two different representations of ELBDM and the evolution of linear perturbations for the two representations. In Section 3, the numerical scheme and initial condition are described. We present the simulation results in Section 4. In Section 5, we look into the physics of collapsed cores with detailed analyses from different perspectives. Finally, the conclusion is given in Section 6. In the Appendix, we present results of one- and two-dimensional simulations and demonstrate that singular cores do not occur in one- and two-dimensional cases.

2. THEORY

2.1. Bose–Einstein Condensate

A Bose–Einstein condensate (BEC) is a state of bosons cooled to a temperature below the critical temperature. BEC happens after a phase transition where a large fraction of particles condense into the ground state, at which point quantum effects, such as interference, become apparent on a macroscopic scale. The critical temperature for a gas consisting of noninteracting relativistic particles is given by (Burakovsky & Horwitz 1996)

\[
T_c \approx \left( \frac{n_{\text{ch}}}{3m} \right)^{1/2},
\]

where the Boltzmann’s constant and speed of light have been set to unity. Given the extremely low particle mass assumed here, \( T_c \) is derived from the relativistic Bose–Einstein particle–antiparticle distribution with the chemical potential set to particle mass \( m \). Here, the “charge” density \( n_{\text{ch}} \equiv n_+ - n_- \), where \( n_+ \) and \( n_- \) are the numbers of densities of particles and antiparticles in excited states. On the other hand, we have \( n_{\text{ch}} \sim (m/T)n_+ \), and it follows that \( T_c \sim \left( \frac{m}{3T} \right)^{1/2} \). Note that \( n_+ \) scales as \( a^{-3} \) and \( T \) as \( a^{-1} \), and it follows \( T_c \) scales as \( a^{-1} \). It means that when \( T \) is below \( T_c \) at some time after a phase transition, the temperature will remain subcritical in any later epoch. As an estimate, if we assume 1% of ELBDM to be in the excited states after its decoupling, the current critical temperature becomes

\[
T_c \approx 3 \times 10^{-14} \left( \frac{m}{eV} \right)^{-1/2} \left( \frac{T}{eV} \right)^{-1/2} \text{eV}.
\]

Substituting \( m \sim 10^{-22} \text{ eV} \) and \( T \sim 10^{-4} \text{ eV} \), the same as the present photon temperature, we find that the current critical temperature \( T_c \approx 0.3 \text{ eV} \gg T \). Hence ELBDM, if exists and accounts for the dark matter, may very well be in the BEC state ever since a phase transition in the early universe. Despite ELBDM particles in the excited state are with a relativistic temperature, almost all particles are in the ground state and described by a single nonrelativistic wave function.

2.2. Basic Analysis

The Lagrangian of nonrelativistic scalar field in the comoving frame is

\[
L = \frac{a^3}{2} \left[ i \hbar \left( \psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) - \frac{\hbar^2}{2a^2m} (\nabla \psi)^2 - 2m V \psi^2 \right],
\]

and the equation of motion for this Lagrangian gives a modified form of Schrödinger’s equation (Siddhartha & Uréña-López 2003):

\[
i \hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2a^2m} \nabla^2 \psi + m V \psi,
\]

where \( \psi \equiv \phi(n_0/a^3)^{-1/2} \) with \( \phi \) being the ordinary wave function, \( n_0 \) the present background number density, and \( V \) is the self-gravitational potential obeying the Poisson equation,

\[
\nabla^2 V = 4\pi G a^2 \delta \rho = \frac{4\pi G}{a} \rho_0 (|\psi|^2 - 1).
\]

The only modification to the conventional Schrödinger–Poisson equation is the appearance \( a^{-1} \) associated with the comoving spatial gradient \( \nabla \), and the probability density \( |\psi|^2 \) to be normalized to the background proper density \( \rho/m \). In the above,

\[
\rho_0 \equiv \frac{3H^2_{\text{0}}}{8\pi G} \Omega_m = m n_0
\]

is the background mass density of the universe.

To explore the nature of the ELBDM, we first adopt the hydrodynamical description to investigate its linear evolution. This approach is not only more intuitive than the wave function description, its advantage will also become apparent later. Let the wave function be

\[
\psi = \sqrt{\frac{n}{n_0}} e^{i \frac{\psi}{\hbar}}
\]

where \( n = \tilde{n} a^3 \), the comoving number density and \( \tilde{n} \) is the proper number density. The quadrature of Schrödinger’s equation can be split into real and imaginary parts, which become the equations of acceleration and density separately,

\[
\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{a^2} \mathbf{v} \cdot \nabla \mathbf{v} + \frac{\nabla V}{m} - \frac{\hbar^2}{2m a^2} \nabla \left( \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right) = 0
\]

\[
\frac{\partial n}{\partial t} + \frac{1}{a^2} \nabla \cdot (n \mathbf{v}) = 0,
\]

where \( \mathbf{v} \equiv \nabla S/m \) is the fluid velocity. There is a new term depending on the third-order spatial derivative of the wave amplitude \( \sqrt{n} \) in the otherwise cold-fluid force equation. This term results from the “quantum stress” that acts against gravity, and it can be cast into a stress tensor in the energy and momentum conservation equation (Chiuhe 1998, 2000). The quantum stress becomes effective only when the spatial gradient of the structure is sufficiently large.

The fluid equations, Equations (5), (8), and (9), are linearized and combined to yield

\[
\frac{\partial}{\partial t} a^2 \frac{\partial}{\partial t} \delta n - \frac{3H^2_{\text{0}} \Omega_m}{2a} \delta n + \frac{\hbar^2}{4m a^2} \nabla^2 \nabla^2 \delta n = 0.
\]

Upon spatially Fourier transforming \( \delta n \), it follows

\[
\frac{d}{dt} a^2 \frac{d}{dt} \delta n_k - \left( \frac{3H^2_{\text{0}} \Omega_m}{2a} \right) n_k + \frac{\hbar^2 k^4}{4m a^2} n_k = 0,
\]
which can be recast into
\[ x^2 \frac{d^2 n_k}{dx^2} + (x^2 - 6)n_k = 0, \]
and the solution to this equation is
\[ n_k = \frac{(3 \cos x - x^2 \cos x + 3x \sin x)}{x^2}. \]
where \( x \equiv \hbar k^2/(m \sqrt{H_0^2 a}) \) and \( a = (t/t_0)^{2/3} \) and \( \Omega_m = 1 \) appropriate for early universe have been assumed. In the small-
\( k \) limit, \( x \) is small and \( n_k \sim x^{-2} \), which grows in time as \( a \); for large \( x \) the solution oscillates. Figure 1 depicts the solution, Equation (12) we can easily identify the oscillating solutions when \( x^2 \geq 6 \), thereby defining the Jeans wave number:
\[ k_j = (6a)^{1/4} \left( \frac{m H_0}{\hbar} \right)^{1/2}. \]

Beyond the Jeans wave number, the perturbation is suppressed by quantum stress. Moreover, the Jeans wave number scales as \( a^{1/2} \) and is proportional to \( m^{1/2} \) (Hu et al. 2000). We shall come back in a later section to examine up to what evolutionary stage the linear solution of \( n_k \) can remain valid.

Next, we linearize the Schrödinger–Poisson equation to derive a governing equation for an alternative wave-function representation. The wave function can be separated into real and imaginary parts,
\[ \psi = 1 + R + i I, \]
with \( R, I \ll 1 \). In the linear regime, we have the real part of linearized Schrödinger’s equation
\[ -\hbar \frac{\partial}{\partial t} R = -\frac{\hbar^2}{2a^2 m} \nabla^2 R + mV. \]
and the imaginary part
\[ \hbar \frac{\partial}{\partial t} I = -\frac{\hbar^2}{2a^2 m} \nabla^2 I. \]
The Poisson equation becomes
\[ \nabla^2 V = \frac{8\pi G}{a} \rho_0 R. \]
The spatial Fourier components of gravitational potential and \( \psi \) satisfy
\[ V_k = -\frac{8\pi G \rho_0}{k^2} R_k, \]
\[ -\frac{\hbar}{dt} I_k = \frac{\hbar^2 k^2}{2a^2 m} R_k + mV_k, \]
and
\[ I_k = \frac{2a^2 m d R_k}{\hbar k^2 dt}. \]
Combing the above, we have, as Equation (11), that
\[ \frac{d}{dt} a^2 \frac{d}{dt} R_k = \left( \frac{3H_0^2 \Omega_m}{2a} \right) R_k + \left( \frac{3a^2}{4m^2 a^2} \right) R_k = 0, \]
and \( R_k \) has, up to a constant factor, the same solution as \( n_k \). Note that since \( dR_k/dt = \dot{a} R_k/a \) for low-\( k \) modes, it follows that \( |I_k| = (2m H_0 a^{1/2}/\hbar k^2)|R_k| = (k_j^2/\sqrt{3}k^2)|R_k| \gg |R_k| \). This feature will serve as one of the indicators for the validity of the linear regime in the wave function representation.

3. NUMERICAL SCHEME AND SIMULATIONS

3.1. Numerical Scheme

We normalize the length to the computational grid size, \( \Delta \), and further define \( \eta = (m \Delta^2 H_0)/\hbar \). The value of \( \eta \) determines the size of Jean’s length relative to the computational grid size. Set \( \nabla = \frac{1}{\Delta} \hat{\nabla} \). The dimensionless Schrödinger–Poisson equations becomes
\[ i \frac{\partial \psi}{\partial \tau} = -\frac{1}{2a^2 \eta} \hat{\nabla}^2 \psi + \frac{3\Omega_m}{2a} U \psi, \]
and
\[ \hat{\nabla}^2 U = (|\psi|^2 - 1), \]
where \( U(x) = V(x)/(32a^2) \), the dimensionless gravitational potential, and \( \tau = H_0 t \).

Given a Hamiltonian \( H \), one can evolve the wave function through Equation (23). It is simply a unitary transformation of the system,
\[ \psi^{i+1} = e^{-iHdt} \psi^i. \]
We use the pseudo-spectral method to solve the Schrödinger equation in the comoving box. Let \( K \) be the kinetic energy operator \( (K = -\frac{1}{2a^2} \hat{\nabla}^2 \rightarrow k^2/2a^2 \) in Fourier space) and \( W \) the potential operator \( (W = \frac{32a^2}{2a^2} U \) in real space). The evolution is then split into
\[ e^{-iHdt} = e^{-i(K+W)dt} = 1 - i(K + W)dt \]
\[ -\frac{1}{2} (K^2 + 2KW + W^2) dt^2 + O(dt^3). \]
On the other hand, we need to consider the noncommutative relation between \( K \) and \( W \), where
\[ e^{-iKdt} e^{-iWdt} = 1 - i(K + W)dt - \frac{1}{2} K^2 dt^2 - \frac{1}{2} W^2 dt^2 - KW dt^2 + O(dt^3), \]
It follows that we obtain, to the second-order accuracy,

\[ e^{-i W dt} e^{-i K dt} = 1 - i(W + K) dt - \frac{1}{2} W^2 dt^2 - \frac{1}{2} K^2 dt^2 - W K dt^2 + O(dt^3). \]  

(28)

It follows that we obtain, to the second-order accuracy,

\[ e^{-i(K+W)dt} \approx \frac{1}{2}[e^{-iKdt} e^{-iWdt} + e^{-iWdt} e^{-iKdt}], \]  

(29)

which will be adopted to advance the time steps.

For each time step, the kinetic energy operator is calculated in the Fourier domain,

\[ \psi_k^{i+1} = e^{-i \frac{\pi}{2} \frac{\eta a^2}{k_{\text{max}}}} \psi_k^i \]  

(30)

and \( \psi \) is advanced in real space with the potential energy operator,

\[ \psi(x)^{i+1} = e^{-i \frac{2\pi \eta a^2}{k_{\text{max}}} dt} \psi(x)^i. \]  

(31)

To ensure numerical stability, we restrict the magnitude of \( dt \) that rotates the phase angle of wave function less than \( \frac{\pi}{2} \) in each time step,

\[ dt \leq \frac{\pi}{2} \frac{\eta a^2}{k_{\text{max}}^2}. \]  

(32)

\[ dt \leq \frac{\pi a}{6 \Omega_m \eta U_{\text{max}}} |. \]  

(33)

In the early stage \( (a \sim 10^{-3}) \), the stability condition is governed by the kinetic energy term, where \( k_{\text{max}}^2 = 3 \pi^2 \) and \( dt \leq (6\pi)^{-1} (\eta a^2) \). At the late time, the gravitational potential becomes ever deeper, and therefore \( dt \) is controlled by the potential energy, where \( U_{\text{max}} \) is the greatest value of potential in the real space.

3.2 Simulation Scale

We prepare the initial conditions with CMBFAST (Seljak & Zaldarriaga 1996) at \( z = 1000 \) with \( \Lambda \)CDM cosmology. Such initial conditions differ from that of Hu et al. (2000), where the Compton length of ELBDM already has imprints on the power spectrum at \( z = 1000 \). We choose this initial condition because only a few low-\( k \) modes can grow for our choice of Jean’s length and the details of initial power spectrum are irrelevant at the late time. The simulations run up to \( 1024^3 \)-grid resolution in a \( 1 \)\( h^{-1} \)Mpc comoving box. For simulations in a much larger box, the background density averaged over a \( 1 \)\( h^{-1} \)Mpc box can often change with time, the so-called environment effects, where galaxies prefer to form in regions of high background density. Here, we ignore the environment effect by fixing \( \Omega_m = 0.3 \). We let the dimensionless parameter \( \eta = 1.22 \times 10^{-2} \) and \( 4.88 \times 10^{-2} \) for the \( 1024^3 \) and \( 512^3 \) simulation boxes, which give a Jeans wavelength 50 kpc at \( z = 0 \). This value of \( \eta \) corresponds to \( m \sim 2.5 \times 10^{-22} \) eV. In the rest of this paper, we shall report only the simulation results of the highest resolution.

4. RESULTS

4.1 Validity of Linear Perturbation Theory

We depict the early evolution of density power spectrum in Figure 2 from \( z = 100 \) to \( z = 10 \). The density power spectrum...
waves function representation predicts that what the linear theory predicts. In particular, the linear theory of before $z$ suppressed by quantum pressure. It is surprising to find that the highest-$k$ modes already become nonlinear. The difference arises $k$ modes even at high $z$, despite that the density power spectrum $|n_k|^2$ vastly deviates from, and is much less than, $4|R_k|^2$ even since early on in the evolution.

To examine this peculiar feature, we construct the imaginary part of wave function $I(x)$ from $I_k$ of a few lowest $k$ modes and depict $|(I^2)_k|^2$ on the same plot as $4|R_k|^2$ at $z = 100$ in Figure 3. It is found that $|(I^2)_k|^2$ coincides with $4|R_k|^2$ at low-$k$. Since $n_k \approx 2R_k + (I^2)_k$, it follows that $(I^2)_k$ has the opposite sign but approximately the same magnitude as $2R_k$ so that the two terms of $n_k$ almost cancel to yield $|n_k|^2 \ll 4|R_k|^2$. Thus, nonlinearity already sets in as early as $z = 100$ in the wave function representation. On the other hand, the fluid representation does not have such a problem. We find that $|n_k|^2$ of low-$k$ modes in the fluid representation agrees with what the linear fluid theory predicts even as late as $z = 1$, despite that the high-$k$ modes already become nonlinear. The difference arises from that $(VS/m)(\equiv \psi)$ in the fluid representation remains small at low-$k$, in contrast to $I = S$ in the wave function representation which has a large amplitude for low-$k$ modes even at high $z$. 

**Figure 5.** Two-dimensional projections of density in real space in a $1h^{-1}$ Mpc comoving box at $z = 3$ (left panel) and $z = 0$ (right panel). Halos A and B are at the top left and bottom left. (A color version of this figure is available in the online journal.)

**Figure 6.** Nonlinear evolution of the power spectrum from $z = 1.5$ to $z = 0$. The highest-$k$ modes acquire their full power after $z = 0.4$, indicative of the creation of singular halo cores. (A color version of this figure is available in the online journal.)

**Figure 7.** Comparison of the halo power spectrum $P_h$ (circle), the background power spectrum $P_b$ (triangle), and the full power spectrum $P$ (square) at $z = 0$. Note that $P_b$ matches $P$ at high $k$ and $P_h$ matches $P$ at low $k$. (A color version of this figure is available in the online journal.)
4.2. Weakly Nonlinearity Regime

Shown in Figure 4 is the evolution of $|n_k|^2 = |2R_k + (R^2 + I^2)_k|^2$ for $1.5 < z < 10$. The initial $n_k$ of high-$k$ mode has been linearly suppressed and is later replaced by the high-$k$ modes that are nonlinearly generated beginning around $z = 5$. The nonlinear coupling arises from the coupling $V \psi$ in the Schrödinger equation. Since $V$ is dominated by low-$k$ modes, the nonlinear coupling transfers modal energy local in $k$ space from one mode to the neighboring mode, and from low $k$ to...
Figure 11. Two-dimensional slice of density for halo A through the core. (A color version of this figure is available in the online journal.)

Figure 12. Same two-dimensional slice of the velocity field for halo A in the comoving frame. (A color version of this figure is available in the online journal.)

high $k$. The gravitational potential $V$ is barely evolving in the weakly nonlinear regime, and hence the dynamics in this regime is for the wave function to settle into almost static potential wells. We note that if $V$ were exactly static, the Schrödinger equation would have been linear and the coupling from low to high $k$ modes would simply be the linear evolution of wave function starting with a non-eigenstate. This argument may explain why
the linear theory of low-$k$ modes works so well even after high-$k$ modes are excited in this regime. Shown in the left panel of Figure 5 is the real-space configuration of $\delta n$ at $z = 3$ in this weakly nonlinear regime, where the wave function settling into individual quasi-static potentials well is underway.

4.3. Strong Nonlinearity Regime

Plotted in Figure 6 is the evolution of density power spectrum after $z = 1.5$. This is the strongly nonlinear regime where the gravitational potential develops deep wells at the collapsed cores. To illustrate the contribution of the few collapsed objects to the final high-$k$ power spectrum, $P(k, 0)$, at $z = 0$, we remove all matter outside the virial radii of these collapsed halos and construct the power spectrum of these artificial objects, $P_b(k, 0)$. The power spectrum of the removed matter, $P_b(k, 0)$, is also constructed for reference. The two power spectra along with the original power spectrum are depicted in Figure 7 for comparison. Clearly the bound objects contribute to almost all the power contained in the original power spectrum, except for the low-$k$ modes that are contributed dominantly by $P_b(k, 0)$. These few lowest-$k$ modes are grown out of the initial noise and remain so in the final configuration. That is, despite that the initial condition possesses many independent degrees of freedom, the final configuration has only a few degrees of freedom, where almost all randomly placed, small collapsed objects seen in standard CDM simulations are entirely suppressed.

During the final collapse phase, the core undergoes large-scale oscillations that send out waves to remove excess angular momentum deposited into the core region, rendering the core to settle into an almost stationary configuration in the physical space. Figure 8 shows the wavy structures of this nature around the collapsed halo A at $z = 1$. Even in the quasi-stationary state of these halos at $z = 0$, we find that this wave phenomenon is still pronounced around the halos, as will be discussed later.

There are two collapsed halos, A and B, of mass $5.7 \times 10^9 M_{\odot}$ and $5 \times 10^9 M_{\odot}$, respectively, at $z = 0$ in our simulation as shown in the right panel of Figure 5. Halo B is subject to major merger around $z = 0.7$. Shown in Figure 9 are halo B before and after the major merger. The final density profiles of halos A and B are plotted in Figure 10. Interestingly, they both develop singular
cores, in spite of the presence of quantum pressure. Both power-law singular cores have a power index $-1.4$, reminiscent of that of the standard CDM (Ghigna et al. 2000). The density profiles of the outskirts also obey power law, with a power index $-2.5$, slightly shallower than that produced by the CDM (Navarro et al. 1997). Similar density profiles arising from different formation processes, i.e., accretion versus merger, suggest that the profile can be universal.

Note that the final collapsed core contains angular momentum through angular dependence in the wave function. The angular dependence manifests itself with large-amplitude, small-scale fluctuations in the wave function. To examine this aspect of the halo, we let the wave function be represented by $\psi = f e^{iS}$. The specific kinetic energy is obtained through the real part of the expression

$$- \frac{\psi^* \nabla^2 \psi}{2\eta^2|\psi|^2} = \frac{1}{\eta^2} \left[ \left( \frac{(\nabla S)^2}{2} - \frac{\nabla^2 f}{2f} \right) - i \left( \frac{\nabla f}{f} \cdot \nabla S + \frac{\nabla^2 S}{2} \right) \right].$$  \hspace{1cm} (34)

Combining the two, the specific internal energy is obtained. Plotted in Figure 11 is the two-dimensional slice of density for halo A, showing large-amplitude, small-scale fluctuations in density. We also plot the same slice for the two-dimensional flow velocity $\nabla S/\eta \ (= -i \nabla (\psi^2/|\psi|^2)/(2\eta \psi^2/|\psi|^2)$ in Figure 12. Figures 13 and 14 are the density and the flow velocity of halo B. The velocity patterns clearly reveal well-defined boundaries of turbulence regions in halos A and B against the infall. The flow becomes randomized inside this sharp boundary. Despite the boundary outlines an accretion shocklike structure, we find in the density slice that there is no obvious jump at the boundary and it is not a shock. Thus, there is no analogy of such a structure.

On the other hand, the specific flow energy can be evaluated through the real part of the following:

$$- \frac{\nabla^2 (\psi^2/|\psi|^2)}{4\eta^2(\psi^2/|\psi|^2)} = \frac{1}{\eta^2} \left[ \left( \frac{\nabla S)^2}{2} - i \left( \frac{\nabla^2 S}{2} \right) \right] \right].$$  \hspace{1cm} (35)

Figure 15. Virial ratios of the kinetic energy integrated up to a radius $r$ (square) to the potential energy integrated up to $r$, the specific flow energy (plus), and specific internal energy (star) for halos A (upper panel) and B (lower panel). This virial ratios are about 0.5 at the average radii of the infall boundaries. The specific internal energy is about twice as large as the specific flow kinetic energy in the interiors of the two halos. (A color version of this figure is available in the online journal.)
We next investigate the virial conditions in the two halos. Plotted in Figures 15(a) and (b) are the ratios of the kinetic energy integrated up to a radius $r$ to the potential energy $\left( \int_0^r 4\pi r^2 (3\Omega_m \eta/4)[n(U - U_{\text{min}})]dr' \right)$ integrated up to $r$ for halos A and B. The virial ratios are 0.5 at the average radii of the infall boundaries, within which turbulence occurs. In addition, we also plot the specific flow energy and the specific internal energy, respectively, in Figure 15. It is found that the specific internal energy is twice as large as the specific flow kinetic energy in the interiors of two halos.

Virialization can be correlated with the flow equipartition. Plotted in Figure 16 are the random tangential specific flow energy and the random radial specific flow (subtracted off the mean radial infall) energy averaged over spherical shells for the two halos. The tangential flow energy is about twice the radial flow energy only well within the halos, thus providing evidence of equipartition at the halo cores. In the outskirts of the halos, the random radial flow energy is larger than the equipartition value. This aspect is reminiscent of the velocity dispersion in a standard CDM halo.

Equipartition is also related to the sphericity of mass distribution. Significant large-scale angular dependence in the wave function can yield aspherical halos. We define the quadrupole-to-monopole ratio as

$$Q \equiv \left( \frac{(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2}{2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)} \right)^{1/2},$$

where $\lambda_1, \lambda_2$, and $\lambda_3$ are the eigenvalues of $\int (rr/\rho^{\beta})nd^3r$, with $\beta = 3.5$ to weigh in favor of the core. The $Q$ value characterizes the low-order angular dependence of wave function, which assumes the extreme value zero or unity when the density profile has spherical symmetry or a one-dimensional shape. It is found that $Q = 0.34$ for halo A and 0.11 for halo B, respectively. Perhaps violent relaxation after a major merger accelerates halo...
B to assume spherical symmetry. We note that the quantum stress is in fact anisotropic: $T_{ij}^Q = (\partial_i \sqrt{n})(\partial_j \sqrt{n})/\eta^2 - \delta_{ij}(\nabla^2 n)/4\eta^2$ (Chiueh 2000). Unlike fluid dark matter (Yoshida et al. 2000), the density asphericity arises from the anisotropic stress of quantum mechanics, similar to that produced by collisionless dark matter. The similarity between the quantum dynamics and the collisionless particle dynamics in fact motivated Widrow & Kaiser (1993) to propose a model that approximates the latter by the former.

5. CONCLUSION

As far as we know, this work presents the first result for the study of BEC under self-gravity via high resolution (1024\(^3\) grids) simulation. Hu et al. (2000) conjectured that if the dark matter is ELBDM, it can solve the long standing problem of far too many low-mass halos present in the standard CDM simulations, and also explains the existence of flat cores in some galaxies. In this work, we confirm that low-mass halos are indeed suppressed by quantum stress even when the small-scale fluctuations are abundant in the initial power spectrum. This result is a consequence of long-time linear suppression of the small-scale modes. We also find, from our simulations of different grid resolutions, that collapsed halos develop singular cores regardless of the halo formation processes. All these runs produce convergent density profiles. Our 1024\(^3\) highest resolution run gives singular density profiles similar to what standard CDM simulations produce.

In retrospect this singular-core result may not be too surprising, as it arises from an almost scale-free Schrödinger–Poisson system. This system is not exactly scale free because there exists a Jeans length for fluctuations that are small in amplitude. However, when the local density much exceeds the background density, the latter becomes locally ill-defined, the Jeans length no longer has any physical significance, and the Schrödinger–Poisson system becomes locally scale free. Being locally scale free, the system develops singularities within a finite time. By contrast, the conservation of phase-space density in classical particle dynamics precludes the space density of standard dark matter particles from developing any singularity (Chiueh & Woo 1997; Dalcanton & Hogan 2001), and explains the existence of a flat core in the warm dark matter model. Note that such a phase-space constraint does not exist for nonlinear wave
dynamics; one example of this nature is a system described by the nonlinear Schrödinger equation with attractive self-interaction (Sulem & Sulem 1999).

Most recent observations of rotation curve in low surface brightness galaxies indicate inconclusive results, as far as the existence of singular halo core is concerned. Some galaxies are claimed not to possess singular halo cores, and some are if noncircular motion is taken into consideration. Among those that do, many possess concentration parameters inconsistent with the constraint given by ΛCDM cosmology (Swaters et al. 2003; Zakurisson et al. 2006; Kuzio de Naray et al. 2008). Given the present status of observations, if galaxies indeed contain singular cores, ELBDM will likely be the only viable candidate for the dark matter that, on one hand, permits the galactic-scale, NFW-like halo cores, and on the other hand, suppresses the subgalactic low-mass halos.

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APPENDIX

COMPARISON WITH ONE- AND TWO-DIMENSIONAL SIMULATIONS

We also examine the one- and two-dimensional simulations with the same numerical scheme used for the three-dimensional simulations. It is hoped that we can verify the scheme by reproducing the results of previous works.

As far as we know, the only work on one-dimensional simulation of the same physical system is the work of Hu et al. (2000). In this work, they simulated the one-dimensional sheet collapse in a three-dimensional expanding universe. Two cases of different Jeans lengths were examined. For the short Jeans length case, they found fast oscillating solutions, in time and space, which correspond to a solution of repeated collapses and rebounds. For the long Jeans length case, they found slow oscillating solution. In either case, the collapsed sheet cores are smooth with no cusp.

The only two-dimensional simulation in the literature is that of Widrow & Kaiser (1993). Unfortunately, they did not examine individual profiles of collapsed filaments and no comparison can be made, as the focus of that work was placed on the equivalence of light bosonic system and the CDM system.

We run one-dimensional simulation with 8192 grids and a small-amplitude initial random fluctuation. The evolution follows the linear growth described in the text. Nonlinearity sets in quite quickly when the density fluctuation grows to a amplitude comparable to the background density. The saturation amplitude of $δρ/ρ$ is around 10. The solution indeed oscillates, corresponding to repeated collapses and rebounds. We show the solutions in Figure A1 for two nearby time steps where $Δt = 0.001$ near $a = 1$. It is apparent that the solution has no singular core. This solution is similar to what was found in the work of Hu et al. (2000).

The two-dimensional case has also been examined with the same scheme as the three-dimensional code with 4096×2 grids. We find that the collapsed filaments at $z = 0$ do not contain singular cores; rather, several dense clumps of $δρ/ρ ∼ 1000$ are scattered over a area of finite radius in the core region, as shown in the zoom-in image in Figure A2.

It is understandable why low-dimensional objects do not develop singularities. This is due primarily to weakening of the focusing power in a three-dimensionally expanding universe. It appears that two-dimensional is the critical dimension, where the singularities are just not to appear. In three-dimensional, the self-focusing power can be so strong as to bind the dense clumps tightly to form singularity.

REFERENCES

Abazajian, K. 2006, Phys. Rev. D, 73, 063513
Burakovsky, L., & Horwitz, L. P. 1996, Phys. Rev. D, 54, 4029
Chiueh, T. H. 1998, Phys. Rev. E, 57, 4150
Chiueh, T. H. 2000, Phys. Rev. E, 61, 3823
Chiueh, T. H., & Woo, T. P. 1997, Phys. Rev. E, 55, 1048
Collins, P., Valenzuela, O., & Avila Reese, V. 2008, ApJ, 673, 203
Dalcanton, J. J., & Hogan, C. J. 2001, ApJ, 561, 35
Dalcanton, J. J., Spergel, D. N., Gunn, J. E., Schmidt, M., & Schneider, D. P. 1997, AJ, 114, 635
Davies, G., & Widrow, L. M. 1997, ApJ, 485, 484
Ghigna, S., Moore, B., Governato, F., Lake, G., Quinn, T., & Stadel, J. 2000, ApJ, 544, 616G
Hu, W., Barkana, R., & Gruzinov, A. 2000, Phys. Rev. Lett., 85, 1158
Jing, Y. P. 2001, Mon. Phys. Lett., A16, 1795
Klypin, A., Kravtsov, A. V., Valenzuela, O., & Prada, F. 1999, ApJ, 522, 82
Kuzio de Naray, R., McGaugh, S. S., & de Blok, W. J. G. 2008, ApJ, 676, 920
Kuzio de Naray, R., McGaugh, S. S., de Blok, W. J. G., & Bosma, A. 2006, ApJS, 165, 461
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Press, W. H., & Ryden, B. S. 1999, Phys. Rev. Lett., 64, 1084
Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437
Siddhartha, F. G., & Uréna-López, A. L. 2003, Phys. Rev. D, 68, 024023
Sin, S. J. 1994, Phys. Rev. D, 50, 3650
Spergel, D. N., & Steinhardt, P. J. 2000, Phys. Rev. Lett., 84, 3760
Sulem, C., & Sulem, P.-L. 1999, The Nonlinear Schrodinger Equation, Self-Focusing and Wave Collapse (New York: Springer-Verlag) ISBN: 978-0-387-98611-1
Swaters, R. A., Madore, B. F., & Trewella, M. 2000, ApJ, 513, L107
Swaters, R. A., Madore, B. F., van der Bosch, F. C., & Balcells, M. 2003, ApJ, 583, 732
Viel, M., Becker, G. D., Bolton, J. S., Haehnelt, M. G., Rauch, M., & Sargent, W. L. W. 2008, Phys. Rev. Lett., 100, 041304
Widrow, L. M., & Kaiser, K. 1993, ApJ, 416, L71
Yoshida, N., Springel, V., White, S. D. M., & Tormen, G. 2000, ApJ, 535, L103
Zakurisson, E., Bergvall, N., Marquart, T., & Ostlin, G. 2006, A&A, 452, 857