Quantum Corrections to a Finite Temperature Blon

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ABSTRACT: In this paper, we will analyze a finite temperature Blon, which is a finite temperature brane-anti-brane wormhole configuration. We will analyze the quantum fluctuations to this Blon solution using the Euclidean quantum gravity. It will be observed that these quantum fluctuations produce logarithmic corrections to the entropy of this finite temperature Blon solution. These corrections to the entropy also correct the internal energy and the specific heat for this finite temperature Blon. We will also analyze the critical points for this finite temperature BIonic system, and analyze the effects of quantum corrections on the stability of this system.

KEYWORDS: Black hole; Thermodynamics; Thermal fluctuations; Massive Gravity.
1 Introduction

In string theory, it is possible to analyze certain physical objects in a region of space-time in terms of very different objects. Thus, it is possible to analyze a system of many coincident strings in terms of D-brane geometry, and this is done in the BIon solution [1, 2]. So, this BIon solution can describe an F-string coming out of the D3-brane or a D3-brane parallel to an anti-D3-brane, such that they are connected by a wormhole with F-string charge. This configuration is called as the brane-antibrane-wormhole configuration. It is also possible to use such a solution to analyze D-branes probing a thermal background [3, 4]. This can be done using the blackfold approach [5–8]. In this method, a large number of coincident D-branes form a brane probe. Furthermore, as this probe is in thermal equilibrium with the background, this method has been used to heat up a BIon. This was done by putting it in a hot background. It is also possible to analyze the thermodynamics of this finite temperature BIon solution [3, 4]. In this paper, we will analyze the effects of thermal fluctuations on the thermodynamics of this system.

The entropy-area law of black holes thermodynamics [9, 10], is expected to get modified near Planck scale due to quantum fluctuations [11]. These quantum fluctuations in the geometry of any black object are expected to produce thermal fluctuations in its associated thermodynamics. It is interesting to note that the thermal fluctuations produce a logarithmic correction term to the thermodynamics of black objects [12–15]. The consequences of such logarithmic correction have been studied for a charged AdS black hole [18], charged hairy black hole [19], a black saturn [20], a Hayward black hole [21] and a small singly spinning Kerr-AdS black hole [22], and a dyonic charged AdS black hole [23]. In non-perturbative quantum general relativity, the density of microstates was associated with the conformal blocks has been used to obtain logarithmic corrections to the entropy [24]. It has also been demonstrated that the Cardy formula can produce logarithmic correction terms for all black objects whose microscopic degrees of freedom are characterized by a conformal field theory [25]. The logarithmic correction has also been studied from the black hole in the presence of matter fields [26] and dilatonic black holes [27]. Leading order quantum corrections to the semi-classical black hole entropy have been obtained [28],
and applied to G"odel black hole \cite{29, 30}. The logarithmic corrections were also used to study different aspects of regular black holes satisfying the weak energy condition \cite{31}, three-dimensional black holes with soft hairy boundary conditions \cite{32}, and certain aspects of Kerr/CFT correspondence \cite{33}. The logarithmic corrected entropy also corrects some hydrodynamical quantities, and so the the field theory dual to such corrected solutions has also been studied \cite{34–38}.

The logarithmic corrections to the entropy of a various black hole have also been obtained using the Euclidean Quantum Gravity \cite{39–41}. In this approach, Euclidean Quantum Gravity \cite{42} is used to obtain the partition function for the black hole, which is then used to obtain the logarithmic corrections to the thermodynamics of that black hole. As the logarithmic corrections occur almost universally in the thermodynamics of black objects, in this paper we will analyze the consequences of such corrections for a thermal BIon. We compute the quantum correction to the black hole entropy, internal energy, specific heat using the Euclidean Quantum Gravity \cite{42}. We find that the logarithmic correction affects the critical points, and the corrections significantly change the stability of this system.

\section{Euclidean Quantum Gravity}

Now we start with the Euclidean Quantum Gravity that is obtained by performing a Wick rotation on the temporal coordinates in the usual path integral. Thus, we obtain gravitational partition function in Euclidean Quantum Gravity \cite{42},

\[ Z = \int [D] e^{-\mathcal{I}_E} = \int_0^\infty \rho(E) e^{-\beta E} dE, \]  

where \( \mathcal{I}_E \) is the Euclidean action for the BIon solution \cite{3, 4}, and \( \beta \propto 1/T \). The density of states \( \rho(E) \) is easily obtained from (2.1) by performing an inverse Laplace transform, so that one obtains

\[ \rho(E) = \frac{1}{2\pi i} \int_{\lambda - i\infty}^{\lambda + i\infty} e^{S(\beta)} d\beta. \]  

Here, \( S \) is the entropy and its exact form is given in terms of the partition function and the total energy as \( S(\beta) = \beta E + \ln Z \). The complex integral (2.2) can be evaluated using the steepest decent method around the saddle point \( \beta_0 \) so that \( \partial S(\beta)/\partial \beta \big|_{\beta=\beta_0} \) vanishes and the equilibrium relation \( E = -[\partial \ln Z(\beta)/\partial \beta]_{\beta=\beta_0} \) is satisfied. Therefore, the equilibrium temperature is given by \( T_0 = 1/\beta_0 \), and we can expand the entropy \( S(\beta) \) around the equilibrium point \( \beta_0 \) as follows

\[ S(\beta) = S_0 + \frac{1}{2} (\beta - \beta_0)^2 \left( \frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta=\beta_0} + \cdots. \]  

Here, the first term \( S_0 := S(\beta_0) \) denotes the entropy at the equilibrium, the second term represents the first order correction over it. If we restrict ourselves to this first order and replace (2.3) into (2.2), we obtain

\[ \rho(E) = \frac{e^{S_0}}{\sqrt{2\pi}} \left( \frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta=\beta_0}^{-\frac{1}{2}}, \]  

\[ -2 - \]
for \([\partial^2 S(\beta)/\partial \beta^2]_{\beta=\beta_0} > 0\), where we choose \(a = \beta_0\) and \(\beta - \beta_0 = ix\) with \(x\) being a real variable. Thus, the expression of the microcanonical entropy \(S\) turns out to be \([12, 13]\)

\[
S = \ln \rho(E) = S_0 - \frac{1}{2} \ln \left( \frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta=\beta_0}.
\]  

(2.5)

Note that the entropy \(S(\beta)\) given in (2.3) is different from \(S\) as given by (2.5), the former \(S(\beta)\) being the entropy at any temperature, whereas the latter one, \(S\) is the corrected microcanonical entropy at equilibrium, which is computed by incorporating small fluctuations around thermal equilibrium. However, the result obtained in (2.5) is completely model independent and, it can be applied to any canonical thermodynamical system including a BIon solution. Thus, the first order correction is solely governed by the term \([\partial^2 S(\beta)/\partial \beta^2]_{\beta=\beta_0}\). This can be simplified to a generic form of the entropy correction given by \(\ln(CT^2)\) \([12, 13]\).

Furthermore, it can be demonstrated that for any black object whose degrees of freedom can be counted using a CFT, we can write \(\ln(\partial^2 S(\beta)/\partial \beta^2)_{\beta=\beta_0} = \gamma \ln(S_0 T^2)\) \([13–15]\), where \(\gamma\) is a free parameter. Now as this holds for any black object whose degrees of freedom can be analyzed using a CFT \([13–15]\), and it has been argued that degrees of freedom of a BIon can also be analyzed using using a CFT \([16, 17]\), we can use a similar equation for BIon. So, we propose that the quantum correction to the entropy of a BIon can be expressed as

\[
S = S_0 - \frac{\gamma}{2} \ln (S_0 T^2) Y \sim S_0 - \frac{\gamma}{2} \ln (S_0 T^2) - \frac{\gamma}{2} Y,
\]  

(2.6)

where \(S_0\) is the original entropy of the BIon solution \([3, 4]\), and \(Y\) is in general a function of other quantities such as the dependence on the D3-brane and string charges. Thus, a full analysis of this system should incorporate such a quantities, but as a toy model, we will analyze the effect of temperature and entropy on the thermodynamics of such a system, and neglect the effect of \(Y\). This can possible be justified by fixing certain quantities in the system, and analysing it as a toy model.

It may be noted that such logarithmic corrections terms are universal, and occur in almost all approaches to quantum gravity. However, the coefficient of such logarithmic correction term is model dependent. As the expression used in this paper involves a free parameter \(\gamma\), it will hold even using different approaches. As any other approaches to this problem can only change the value of this coefficient \(\gamma\), which is not fixed in this paper. Thus, the validity of the (2.6) can be argued on general grounds, and the main aim of the paper is to analyze the effects of such a logarithmic corrections on the thermodynamics of a BIon solution.

So, to obtain quantum corrections to the entropy of a BIon solution, we need to use the original entropy \(S_0\) of the BIon solution \([3, 4]\). Now a Bionic system is a configuration in a flat space of the D-brane which is parallel to anti-D-brane and they are connected by a wormhole which has a F-string charge. Geometrically, it is composed of \(N\) coincident D-branes which are infinitely extended and has \(K\) units of F-string charge, ending in a throat with minimal radius \(\sigma_0\) and at temperature \(T\). To construct a wormhole solution from this, all we have to do, is to attach a mirror solution at the end of the throat.
It is well known that the DBI action can be used to describe the D-brane for probing the zero temperature background. However, it was shown that one can also use DBI action for probing the thermal backgrounds [4], where it is ensured that the brane is not affected by the thermal background, but the degrees of freedom living on the brane are ‘warmed up’ due to the temperature of thermal background. Thus, the thermal background acts as a heat bath to the D-brane probe, and due to this, the probe stays in thermal equilibrium with the thermal background, which is a ten dimensional hot flat space. This is constructed in the blackfold approach, which is a general description for the black holes in the regime where they can be approximated to black brane curved along the sub-manifold of the space-time background. The thermal generalization of BIon solution has also been carried out and the thermodynamic quantities for this configuration are given by [3, 4]

\[
M = \frac{4T_{D3}^2}{\pi T^4} \int_{\sigma_0}^{\infty} d\sigma \frac{\sigma^2 (k \cosh^2 \alpha + 1) F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0) \cosh^4 \alpha}},
\]

(2.7)

\[
S_0 = \frac{4T_{D3}^2}{\pi T^5} \int_{\sigma_0}^{\infty} d\sigma \frac{4\sigma^2 F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0) \cosh^4 \alpha}},
\]

(2.8)

\[
F = \frac{4T_{D4}^2}{\pi T^4} \int_{\sigma_0}^{\infty} d\sigma \sqrt{1 + z'^2(\sigma)} F(\sigma),
\]

(2.9)

which are the total mass, entropy and free energy, respectively. Here, \(T_{D3}\) is the D3-brane tension, \(z\) is a transverse coordinate to the branes and \(F(\sigma) = \sigma^2 (4 \cosh^2 \alpha - 3) / \cosh^4 \alpha\), with \(\sigma\) being the world volume coordinate and \(\sigma_0\) being the minimal sphere radius of the throat or wormhole. Here \(\alpha\) is a function of the temperature. The chemical potentials for the D3-brane and F-string are as follows

\[
\mu_{D3} = 8\pi T_{D3} \int_{\sigma_0}^{\infty} d\sigma \frac{\sigma^2 \tanh \alpha \cos \zeta F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}},
\]

(2.10)

\[
\mu_{F1} = 2T_{F1} \int_{\sigma_0}^{\infty} d\sigma \frac{\tanh \alpha \cos \zeta F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}},
\]

(2.11)

It should be noted that these relations satisfy the first law of thermodynamics \(dM = TdS_0 + \mu_{D3}dN + \mu_{F1}dK\) as well as the Smarr relation, \(4(M - \mu_{D3}N - \mu_{F1}K) - 5TS_0 = 0\).

One can also calculate internal energy and the specific heat of the BIon solution as

\[
U_0 = \frac{4T_{D3}^2}{\pi T^4} \int_{\sigma_0}^{\infty} d\sigma F(\sigma) \left[ \sqrt{1 + z'^2(\sigma)} + \frac{4\sigma^2}{\sqrt{F^2(\sigma) - F^2(\sigma_0) \cosh^4 \alpha}} \right],
\]

(2.12)

\[
C_0 = T \left( \frac{dS_0}{dT} \right) = -\frac{20T_{D3}^2}{\pi T^5} \int_{\sigma_0}^{\infty} d\sigma \frac{4\sigma^2 F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0) \cosh^4 \alpha}},
\]

(2.13)

which indicates that the system has a negative specific heat.

3 Corrected Thermodynamics for the BIon

Let us now look for the thermal corrections to the above equations by considering logarithmic correction to the entropy \(S\) given by the equation (2.6). The entropy (2.8) of \(N\)
Coincident D-branes with a throat solution gets corrected as

$$S = \frac{4 T_{D3}^2}{\pi T^3} \int_{\sigma_0}^{\infty} d\sigma \frac{4\sigma^2 F(\sigma)}{\cosh^4 \alpha \sqrt{F^2(\sigma) - F^2(\sigma_0)}} - \frac{\gamma}{2} \ln \left[ \frac{4 T_{D3}^2}{\pi T^3} \int_{\sigma_0}^{\infty} d\sigma \frac{4\sigma^2 F(\sigma)}{\cosh^4 \alpha \sqrt{F^2(\sigma) - F^2(\sigma_0)}} \right].$$

In order to analyze the expression for the corrected entropy we can assume $\cosh^2 \alpha(\sigma_0) = \frac{3}{4}$, which means that $F(\sigma_0) = 0$ leading to a relatively easy solution. However, we would like to work on the regime where the branch connected to the extremal BIon, which was also utilized in [3]. In this formulation one obtains

$$\text{Figure 1. Behavior of the corrected entropy as a function of $\tilde{T}$ for $K = 1$ and $T_{D3} = 1$.}$$

$$\text{Figure 2. Behavior of the corrected entropy as a function of $\sigma_0$ for $K = 1$, $T_{D3} = 1$ (a) $\tilde{T} = 0.5$ (b) $\tilde{T} = 1$.}$$
demonstrates the behavior of the entropy and we have considered shows that, although the variation of the internal energy is smaller with the set \( \sigma \). In this case, we see that there exists a critical \( \bar{\sigma} \approx \pi T \) by varying the \( \bar{\sigma} \), with \( \bar{\sigma} \approx \pi T \approx 1 \) and \( \bar{\sigma} \approx \pi T \approx 0.99 \), as shown in Fig. 1(c), where we see that corrected entropy is smaller than the uncorrected entropy. Therefore, it suggests that the behavior of the entropy after the logarithmic correction depends on both of the parameters, namely, the temperature and \( \sigma_0 \).

In Fig. 2, we plot the corrected entropy as a function of \( \sigma_0 \) for \( \bar{T} \geq 0.5 \), i.e. \( \sigma_{\text{min}} = 0.25 \). In this case, we see that there exists a critical \( \sigma_c \) for each of the plots, and when the value of \( \sigma_0 \) is less than the value of \( \sigma_c \), the corrected entropy is larger than the uncorrected one. Whereas, when \( \sigma_0 > \sigma_c \), the corrected entropy is less. However, the critical points depend on the temperature, for instance, in the case when \( \bar{T} = 0.5 \), i.e. in Fig. 2(a), we notice that \( \sigma_c \approx 0.7 \), while in Fig. 2(b), i.e. for \( \bar{T} \approx 1 \), \( \sigma_c \approx 1.6 \). We should note that the region compatible with our assumption is \( \sigma_0 > 1 \). Fig. 3 demonstrates the behavior of the entropy with respect to \( \gamma \), which is the coefficient that determines the amount of correction that is imposed into the system. By choosing \( \bar{T} \geq 1 \), we can see that the entropy is a decreasing function of \( \gamma \) for \( \sigma_0 = 2 \), while it is an increasing function of \( \gamma \) for \( \sigma_0 = 1 \). It means that for the small throat (smaller than \( \sigma_c \)), the effect of the thermal fluctuation is to increase the entropy which may yield more stability to the system with maximum value of the entropy. On the other hand, a bigger throat may render the instability to the system.

The logarithmic correction also modifies the internal energy and the specific heat. Let us analyze the effects for the modification of the internal energy first, which we compute as follows

\[
U = \frac{4T^2_{D3}}{\pi T^4} \int_{\sigma_0}^{\infty} d\sigma F(\sigma) \left[ \sqrt{1 + z^2(\sigma)} + \frac{4\sigma^2}{\sqrt{F^2(\sigma) - F^2(\bar{\sigma}) \cosh^4 \alpha}} \right] - \frac{\gamma T}{2} \ln \left[ \frac{4T^2_{D3}}{\pi T^3} \int_{\sigma_0}^{\infty} d\sigma \frac{4\sigma^2 F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\bar{\sigma}) \cosh^4 \alpha}} \right].
\]

(3.3)

We can perform a graphical analysis similar to the entropy to give similar results. However, we focus only on the behavior of the internal energy with the variation of the parameter \( \gamma \). Fig. 4 shows that, although the variation of the internal energy is smaller with the
Figure 3. Behavior of the corrected entropy in terms of $\gamma$. We set $\mathcal{K} = 1$, $\bar{T} = 1$ and $T_{D3} = 1$.

logarithmic corrections, however, the slope of the increasing or decreasing functions depends on the value of the temperature and the radius of the throat, as expected. For larger radius, the entropy is decreased due to the thermal fluctuations, while for the smaller radius the entropy is increased. Let us now see the effects on the specific heat. The exact expression

Figure 4. Behavior of internal energy in terms of $\gamma$, with $\mathcal{K} = 1$ and $T_{D3} = 1$ (a) $\bar{T} = 0.9$ (b) $\bar{T} = 1$. 
of the corrected specific heat is given by

\[ C = T \frac{d}{dT} \left( S - \frac{\gamma}{2} \ln[ST^2] \right), \tag{3.4} \]

which has been analyzed in Fig. 5. Here, we show the effect of the logarithmic correction on the specific heat inside the allowed region \( 1 \leq \sigma_0 \) for Fig. 5(a), and for the whole range

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**Figure 5.** Behavior of the specific heat with respect \( \sigma_0 \) for \( K = 1 \) and \( T_{D3} = 1 \) (a) \( \bar{T} = 0.9 \) (b) \( \bar{T} = 0.7 \).

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**Figure 6.** Behavior of the specific heat with respect to \( \gamma \) for \( K = 1, \bar{T} = 0.9 \) and \( T_{D3} = 1 \).
in Fig. 5(b) in order to explore the general approximate behavior. In the case of $\gamma = 0$, we find that the specific heat is entirely negative, however, in the presence of the thermal fluctuations there is some region where it is positive. It means that in the presence of the logarithmic correction, there is a special radius $\sigma_s$ for which the specific heat is negative, $\sigma_0 > \sigma_s$, while it is positive for $\sigma_0 < \sigma_s$. As always, the value of the $\sigma_s$ depends on the temperature, for instance, in Fig. 5(a) it is obvious that when we choose $\bar{T} = 0.9$, we obtain $\sigma_s \approx 1.03$. On the other hand, in Fig. 5(b), when we increase $\sigma_0$, the difference between the corrected and uncorrected case slowly vanishes. It indicates that the thermal fluctuation becomes relevant for the smaller radius. Also, we can see from the Fig. 5(b) an asymptotic behavior which may be interprets as a phase transition as found in [3], which we show that it is due to the thermal fluctuations. Finally, we demonstrate the variation of the specific heat with the parameter $\gamma$ in Fig. 6. It is obvious that the effect of the logarithmic correction is to increase the specific heat and, for $\gamma > 0.65$ (approximately) the specific heat is completely positive, while for $\gamma = 0$ it is completely negative.

4 Conclusion

Quantum fluctuation is an important phenomenon while dealing with objects of very small length scales (close to the Planck length). It can be neglected while the object is large enough compared to the Planck scale, however, for small objects the quantum fluctuation become important. We analyze the quantum corrections for the BIon systems. We also identify the critical points by including the correction terms over the standard thermodynamical quantities. Moreover, by analyzing the stability conditions, we show how relevant these fluctuation are in the given context. As it turns out that the quantum fluctuation highly affects the critical points and, thus, the stability of the system. The stability is increased under certain relevant conditions, for instance, when the throat is smaller, the inclusion of fluctuation effects increases the stability. Our analysis explicitly shows how the quantum fluctuation terms dominate with the decrease of the radius. As for instance, Fig. 5(b) tells that while we increase the radius $\sigma_0$, the correction term slowly vanishes and the result merges with that of the uncorrected one. Apart from the stability analysis, we have computed the corrections to the internal energy and specific heat due to the quantum fluctuation. We have also demonstrated the change of the behavior of the corrected system with the temperature.

It is possible to construct a BIon solution in M-theory using a system of M2-branes and M5-branes [43]. It would be interesting to analyze the system at a finite temperature. Then the thermodynamics of this system can be studied. It would be possible to study the quantum fluctuations to the geometry of a BIon in M-theory, and these could produce thermal fluctuations in the thermodynamics of this system. It would be interesting to analyze the critical points for such a system, and study the effects of these fluctuations on the stability of this system. It may also be noted that the thermodynamics of AdS black hole been studied in M-theory [44, 45]. It is possible to analyze the quantum corrections to these black holes, and this can also be done in Euclidean Quantum Gravity. It would also be interesting to generalize the work of this paper to such AdS black holes in M-theory.
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