Distortion-free Golden Precoded Alamouti for MISO Systems

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Abstract—Although the discrete Fourier transform (DFT) precoded Alamouti (DPA) scheme with limited feedback offers numerous benefits in the Rayleigh fading channel, this scheme inherently generates geometric mean distortion due to the error exponential of the DFT precoding (DP). Consequently, the receiver suddenly loses the average received channel power. Instead of the conventional DPA scheme, we propose a Golden precoded Alamouti (GPA) scheme to construct a distortion-free codeword generator that can be used in multiple-input and single-output (MISO) systems. The Golden section with continuous geometric proportion and the Golden precoding (GP) pattern can be used to construct some good space-time block codes (STBCs) with orthogonal properties and simple inverse computation for use in future wireless communications. In this paper, we overcome the Grassmannian subspace packing problem and compute the pairwise error probability (PEP) to verify the bit-error-rate (BER) performance. Our Monte Carlo simulations show that the proposed GPA scheme performs better than the conventional DPA scheme.

Index Terms—MISO systems, Alamouti code, the DPA and GPA scheme, subspace packing problem, BER performance.

I. INTRODUCTION

THE use of orthogonal space-time block codes (OSTBCs) is an important technique in wireless communications to improve the network reliability and offer a high data rate and diversity gain [1]. A well-known space-time block code (STBC) is the Alamouti code, which is a complex orthogonal space-time code specialized for the use with two transmission antennas to achieve full diversity [1]. In addition, OSTBCs provide the benefit of low-complexity symbol-wise decoding [2]. An open loop is generally used in STBCs because they operate without the benefit of channel state information (CSI) at the transmitter [3] and because of the results of studies on closed loops with STBCs to improve the performance of MIMO systems [4]. Closed-loop schemes are based on the type of CSI that is available at the transmitter, such as quantized CSI, although occasionally only statistical CSI may be available on the channel. However, we assume that the channel is known at both the transmitter and the receiver. The CSI is estimated at the receiver and is returned to the transmitter with limited feedback [3], [5]. Thus, in this study, we consider limited feedback with updated CSI, where the precoder index can be given based on the CSI. Instead of full CSI, the corresponding index can be given as feedback to the transmitter.

The discrete Fourier transform precoding (DP) codebook was first proposed in [3], [5], where the authors used the optimal codebook selection criterion to drive a criterion for finding codebooks with low average distortion. Hai et al. [6] showed that the performance of discrete fractional sine transform (DFRST) is equivalent to that of DFT precoding, although they attempted to reduce the hardware complexity. Unfortunately, most existing works have not completely removed the mean distortion from the codeword generator. Thus, in this work, we design a distortion-free Golden precoding codebook (GPC) that constructs an error-free codeword generator.

STBCs with Golden numbers were proposed in [7], [8]. In [7], the authors showed that the Golden code satisfies the properties of full-rank, full-rate, and non-vanishing determinants, which increase the minimum determinants. The Golden and Silver code were compared with diagonal precoding for non-OSTBCs to evaluate the pairwise error probability [8]. Many existing works have introduced a Golden number for STBCs, but Golden precoding has not yet been introduced for OSTBCs. Thus, this paper designs a Hadamard-based GPC scheme to produce a distortion-free codeword generator that measures the minimum chordal distance and computes the pairwise error probability to verify the system bit-error-rate (BER) performance. The rest of this paper is organized as follows. First, we investigate the system model in Section II. Second, we describe the conventional DFT precoded Alamouti (DPA) scheme in Section III. We then propose the Golden precoded Alamouti (GPA) scheme and compute the pairwise error probability (PEP) in Section IV and Section V, respectively. Finally, the Monte Carlo simulations and conclusions are presented in Sections VI and VII, respectively.

II. SYSTEM MODEL

We consider multiple-input and single-output (MISO) systems with a flat Rayleigh fading channel $h \in \mathbb{C}^{1 \times N_T}$ that is employed with multiple transmit antennas $N_T$ and a single receive antenna $N_R = 1$. The receive signal $y \in \mathbb{C}^{1 \times T}$ at a period $T$ can be modeled as

$$y = hX + z,$$

where $X \in \mathbb{C}^{N_T \times T}$ is a transmit codeword structure of the STBCs and $z \in \mathbb{C}^{1 \times T}$ is a circularly symmetric complex Gaussian with zero mean and unit variance.

III. DPA SCHEME

The DPA codeword generator $X$ is given by [3]

$$X = W_D S,$$
where the $M \times T$ complex Alamouti code with a length of $M \leq N_T$ is
\[
S_k = \begin{bmatrix}
    s_{11} & -s_{21} \\
    s_{21} & s_{11}
\end{bmatrix}, \quad k = 1, 2, \ldots, T,
\]
(3)
where $W_D \in \mathbb{C}^{N_T \times M}$ is the $N_T \times M$ DFT precoding matrix given in [3] with $\frac{1}{\sqrt{N_T}}e^{\frac{2\pi k i}{N_T}}$ at entry $(k, l)$, which is chosen from the codebook generator in [3], [5-6]:
\[
F_D = \{\theta_D^{-1}W_D\}, i = 1, 2, \ldots, L - 1.
\]
(4)
The $N_T \times M$ remaining DFT precoding matrices are given by
\[
W_{D,i} = \theta_D^{-1}W_{D,1}, \quad i = 2, \ldots, L,
\]
(5)
where $L$ is the size of the codebook; $W_{D,1}$ is an $N_T \times M$ DFT-based first precoding matrix, the $(k, l)$ of which is given as $\frac{1}{\sqrt{N_T}}e^{\frac{2\pi k i l}{N_T}}$; and $\theta_D$ is the diagonal matrix with the free variable $\{u_i\}_{i=1}^{N_T}$ to be determined in [3].

Problem statement: The given DPA scheme has two main problems. The first problem is the geometric mean distortion of the DPA scheme due to the error exponential of the DP matrix, and the second problem is the minimum chordal distance problem.

A. Geometric mean distortion of the DPA scheme

The DP matrix contains an error exponential that produces a slight distortion at a low-order DPA scheme, although this problem gradually arises at a high-order DPA scheme. Consequently, codeword matrix suffers a spectral leakage problem; for example, the $2 \times 2$ DPA third codeword matrix is given by (2)-(5) as follows:
\[
[X_3] = \frac{\theta_D^2}{\sqrt{2}} \begin{bmatrix}
    1 & 1 \\
    1 & e^{j\pi}
\end{bmatrix} \begin{bmatrix}
    s_{11} & -s_{21} \\
    s_{21} & s_{11}
\end{bmatrix}
= \begin{bmatrix}
    0.0000 + 0.0000i \quad 0.8315 - 0.5556i \\
    0.1379 + 0.6935i \quad 0.1379 + 0.6935i
\end{bmatrix},
\]
(6)
where $i = 3, s_{11} = 0.7071 + 0.7071i$ and $s_{21} = -0.7071 - 0.7071i$ are accounted for by the quadrature amplitude modulation (QAM) constellation, and $(\cdot)^*$ is the complex conjugate operator. Now, we increase the size of the DP matrix in (6). For example, we set $i = 3, M = 2$, and $N_T = 4$ in (6), and we obtain the $4 \times 2$ DPA third codeword matrix in (7):
\[
[X_3] = \frac{\theta_D^2}{\sqrt{4}} \begin{bmatrix}
    1 & 1 \\
    1 & e^{j\pi}
\end{bmatrix} \begin{bmatrix}
    s_{11} & -s_{21} \\
    s_{21} & s_{11}
\end{bmatrix}
= \begin{bmatrix}
    0.0000 + 0.0000i \quad 0.8315 - 0.5556i \\
    0.1379 + 0.6935i \quad 0.1379 + 0.6935i \\
    0.0000 - 1.0000i \quad -0.0000 - 0.0000i \\
    0.7071 - 0.0000i \quad -0.7071 + 0.0000i
\end{bmatrix}.
\]
(7)

Using (6) and (7), we draw Fig. 1, which depicts the mean distortion with the DPA generator. We observe in (6) and Fig. 1 (a) that the DPA generator contains a very small distortion, which has gradually increased in the high-order DP matrix in Fig. 1 (b). Hence, the codeword generator cannot be generated as anti-diagonal blocks in the DPA matrix, and consequently, the decoder requires more frequent CSI feedback to meet the constraint. This problem arises from the error exponential of the DP matrix, which degrades the BER performance by at least $10^{-1}$ steps. However, we solve this problem in Section IV.

B. Minimum chordal distance problem

The minimum chordal distance is the major issue for packing subspaces in the Grassmann manifold. We formulate the minimum chordal distance problem as follows: The optimal DP matrix, $W_{opt}$, is given by
\[
W_{opt} = \arg \max_{W \in \mathbb{C}^{N_T \times M}} \| HW_D \|_F^2,
\]
(8)
where the error matrix $\bar{S} = S_1 - S_j$ and $\bar{S}^H = \alpha I$ with a constant $\alpha$. When $W_{D,i}F$ is not imposed, the above optimum solution $W_{opt}$ is not unique. Thus, the average loss in the received channel power is
\[
E \left\{ \min_{W} \| HW_{opt}\|_F^2 - \| HW \|_F^2 \right\} \leq \left\{ \lambda_1^2(H) \right\} E \left\{ \min_{W} \frac{1}{2} \| \tilde{V}^H - W^H \|_F^2 \right\},
\]
(9)
where the singular value decomposition (SVD) technique of channel $H = US\tilde{S}^H$, the diagonal entry of $\Sigma$, is in descending order; $\tilde{V}$ is unitary; $\lambda_1^2(W_{opt}) = 1$; and $E[\cdot]$ is called the expectation with regard to the random channel $H$. Hence, the optimum codebook is designed to maximize the minimum chordal distance [3], [5-6], [9]:
\[
\delta_{min,D} = \min_{2 \leq i \leq L} d(W_{D,1}, \theta_D^{-1}W_{D,1}).
\]
(10)

IV. PROPOSED GPA SCHEME

In this section, we will design a distortion-free GPA scheme. For this purpose, we first design a Golden Hadamard precoding matrix. Let $N_T = 2^q$ and a $2^q \times M$ precoding matrix, $W(2^q)$, is constructed by $M$ columns of the $2^q \times 2^q$ recursive Golden Hadamard precoding matrices:
\[
W_G(2^q) = \frac{1}{\sqrt{2^q}} \begin{bmatrix}
    \theta_GW_G(2^{q-1}) & \theta_GW_G(2^{q-1}) \\
    \theta_GW_G(2^{q-1}) & -\theta_GW_G(2^{q-1})
\end{bmatrix},
\]
(11)
for $2 \leq q \leq N_T$ in [10], where $W(1) = [1, \xi = n((1 + n)^q - (1 - n)^q)/2^n]$, $n$ indicates the root of the geometric number,
where \( \theta _G \) is the Golden number in [11] that satisfies \( \theta _G \theta _G^{-1} = 1 \), \( (\theta _G^{-1} - \theta _G) = -1 \), and its continuous geometric proportion is \( \theta _G : 1 : 1/\theta _G \), which indicates that the geometric mean between \( \theta _G \) and \( 1/\theta _G \) is unity. Thus, we incorporate (11) in (5) and construct the remaining Hadamard-based Golden precoding (GP) matrices as follows:

\[
W_{G,i}(2^q) = (-1)^{q} W_{G,1}(2^q), i = 2, \ldots, L, \tag{12}
\]

where \( W_{G,1}(2^q) \) is an \( N_T \times M \) Hadamard-based first Golden precoding matrix. Now, we state Theorem 1 as follows.

Theorem 1: The DPA codeword generator \( X \) becomes distortion-free if the precoding matrix \( W_{G}(2^q) \) is a Golden Hadamard.

Proof of Theorem 1:

A. Golden Hadamard Preceding

Case I (Real Golden Number): We consider that \( \theta _G \) is the real-valued root of \( \theta _G^2 = X^2 - X - 1 \); \( \theta _G = \frac{1 + \sqrt{5}}{2} \) is known as the real Golden number [7,11], and its algebraic conjugate is obtained as \( \theta _G = 1 - \theta _G = \frac{1 - \sqrt{5}}{2} \). We incorporate \( i = 3, q = 2, n = \sqrt{5} \), and \( \theta _G \) values in (11-12) to obtain the 4 \times 4 real Golden Hadamard matrices as

\[
W_{G,i,3}(4) = (-1)^2 W_{G,i,2}(4), i = 2, \ldots, L, \tag{13}
\]

where \( (\theta _G^{-1} - \theta _G) = -1 \), \( W_{G,i,2}(4) = (-1) W_{G,i,1}(4) \), and

\[
W_{G,i,1}(4) = \frac{\theta _G}{\sqrt{3}} \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1
\end{bmatrix}. \tag{14}
\]

Case II (Complex Golden Number): Let \( \theta _G \) be the real-valued root of \( \theta _G^2 = X^2 - jX - 1 \); \( \theta _G = \frac{j + \sqrt{3}}{2} \) is known as the complex Golden number, and its algebraic conjugate is obtained as \( \theta _G = \frac{-j + \sqrt{3}}{2} \). Similarly, we incorporate \( i = 3, q = 2, n = \sqrt{3} \), and \( \theta _G \) values in (11-12) to obtain the 4 \times 4 complex Golden Hadamard matrices as

\[
W_{G,i,3}(4) = (-j)^2 W_{G,i,2}(4), i = 2, \ldots, L, \tag{15}
\]

where \( (\theta _G^{-1} - \theta _G) = -j \), \( W_{G,i,2}(4) = (-j) W_{G,i,1}(4) \), and

\[
W_{G,i,1}(4) = \frac{\theta _G}{\sqrt{3}} \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1
\end{bmatrix}. \tag{16}
\]

Thus, we observe that (14) and (16) are the exact real and complex Golden Hadamard matrix, respectively. Finally, our designed GPC matrices are given by

\[
F_G = \begin{cases}
((-1)^{q-1} W_{G,i,2}(2^q)), & \text{for case I}, \\
((-1)^q W_{G,i,2}(2^q)), & \text{for case II},
\end{cases} \tag{17}
\]

where \( i = 1, 2, \ldots, L \).

B. Distortion-free DPA Generator Matrix: Let \( i = 3, q = 2, N_T = 4, M = 2 \), and \( n = \sqrt{5} \). Then, by substituting (14) in (7), we obtain the 4 \times 2 GPA third codeword generator matrix by using case I as

\[
[X_3] = \begin{bmatrix}
0.0000 + 0.0000i & 1.0233 - 1.0233i \\
1.0233 + 1.0233i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & 1.0233 - 1.0233i \\
1.0233 + 1.0233i & 0.0000 + 0.0000i
\end{bmatrix}, \tag{18}
\]

Fig. 2: Distortion-free GPA scheme: (a) case I (18) and (b) case II (19).

Using (18) and (19), we draw Fig. 2 against Fig. 1 and observe that the proposed GPA scheme is completely distortion-free. Thus, Theorem 1 is proved.

V. PAIRWISE ERROR PROBABILITY (PEP)

Let \( S_i \) and \( S_j \) be the transmitted and decoded space-time codewords, respectively, where \( j \neq i \). Thus, the codeword PEP is \( P_r(S_i \rightarrow S_j) \), where \( H \) is the given channel and \( W_{G,i}(2^q) \) is the precoding matrix. An upper bound of the total PEP can be given via the union bound in [12, Sec 5.2], and the average error rate [13, Eq.4.2.2] is upper bounded by

\[
P_{\text{r,tot}} \leq \frac{1}{|\chi|} \sum_{S_i \in \chi} \sum_{S_j \notin \chi, j \neq i} P_r(S_i \rightarrow S_j), \tag{20}
\]

where \( |\chi| \) is the number of elements in the matrix constellation \( \chi \). The PEP depends upon the product distance \( SS^H \), where the error matrix \( \bar{S} = S_j - S_i \) for the STBC scheme and \((\cdot)^H \) represents the Hermitian. The upper bound of the PEP is

\[
P_r(S_i \rightarrow S_j | H) = Q \left( \sqrt{\frac{\gamma_0 \|HW_{G,i}(2^q)S\|^2}{2 \cdot 2^q}} \right) \leq \exp \left( \sqrt{\frac{\gamma_0 \|HW_{G,i}(2^q)S\|^2}{4 \cdot 2^q}} \right), \tag{21}
\]

where \( \gamma_0 \) is the signal-to-noise ratio (SNR) and \( Q(\cdot) \) is the Gaussian Q-function. Thus, the proposed performance measures of the packing minimum chordal distance for both real and complex cases are as follows:

\[
\delta_{\text{min},G_r} = \min_{2 \leq i \leq L} d \{ W_{G,i,1}(2^q), (-1)^{i-1} W_{G,i,1}(2^q) \}, \tag{22}
\]

and

\[
\delta_{\text{min},G_r} = \min_{2 \leq i \leq L} d \{ W_{G,i,1}(2^q), (-j)^{i-1} W_{G,i,1}(2^q) \}. \tag{23}
\]
Table I: Computation of Minimum Chordal Distance

| Minimum chordal distance ($\delta_{\text{min}}$) when $i = 2, q = 2, N_T = 4, M = 2$ | DPC Scheme (10) | Proposed GPC Scheme Case I (22) | Case II (23) |
|---|---|---|---|
| | 0.8392 | 1.0000 | 1.0000 |

Fig. 3: BER performance of GPA scheme against DPA with 16-QAM, when $q = 2, N_T = 4, M = 2, L = 64$, and $\gamma = 20\text{dBs}$.

VI. SIMULATION RESULTS

We compare our designed GPC scheme in (17) with the conventional DPA scheme in (4) for a MISO flat block fading Rayleigh channel. Our designed GPC matrix leads to the minimal chordal distance $\delta_{\text{min,G}} = 1 = \sin(90^0)$; in comparison, the minimal chordal distance of the DPC matrix is $\delta_{\text{min,D}} = 0.8392 = \sin(57.05^0)$ in Table I. Thus, the resulting GPC provides distortion-free performance, as shown in Fig. 3 and Fig. 4. We apply two different codebook schemes, 16-QAM and 64-QAM, at SNR values of 20 and 30 dB, respectively. In both the low- and high-SNR regimes, in Case I (the real case), the GPA scheme performs significantly better than DPA. In contrast, in Case II (the complex case), the performance of the GPA scheme is lower because of the reduction of the Golden ratio. However, the average array gains of both the real (Case I) and complex (Case II) GPA schemes are increased by almost $1.0 \sim 3.0$ dB with $\log_2 L = \log_2 2^6 = 6$-bit feedback.

VII. CONCLUSIONS

We investigated the problem of geometric mean distortion of the DPA scheme due to the error exponential of the DP matrix and proposed the GPC approach, which is achieved with a distortion-free codeword. We addressed the problem of the packing minimum chordal distance and demonstrated the distortion-free performance of our proposed GPA scheme. We may further deal with the presented results in multiuser systems. In future work, we will investigate multi-layer GPC scheme to further improve the sum-rate performance.

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