We present a general scenario for high-temperature superconducting cuprates, based on the presence of dynamical charge density waves (CDWs) and to the occurrence of a CDW quantum critical point, which occurs, e.g., at doping $p \approx 0.16$ in YBa$_2$Cu$_3$O$_{6.8}$ (YBCO). In this framework, even the pseudogap temperature $T^*$ is interpreted in terms of a reduction of the density of states due to incipient CDW and, at lower temperature to the possible formation of incoherent superconducting pairs. The dynamically fluctuating character of CDW accounts for the different temperatures at which the pseudogap ($T^*$), the CDW onset revealed by X-ray scattering ($T_{\text{on,k}}(p)$), and the static three-dimensional CDW ordering appear. We also investigate the anisotropic character of the CDW-mediated scattering. We find that this is strongly anisotropic only close to the CDW quantum critical point (QCP) at low temperature and very low energy. It rapidly becomes nearly isotropic and marginal-Fermi-liquid-like away from the CDW QCP and at finite (even rather small) energies. This may reconcile the interpretation of Hall measurements in terms of anisotropic CDW scattering [1] with recent photoemission experiments [2].

**INTRODUCTION**

There are essentially two opposite points of view on the basic physical nature of the electronic state in high-temperature superconducting cuprates. Since the early times the idea was put forward (mostly by P. W. Anderson, [3]) that these systems are strongly correlated doped Mott insulators, where the large electron-electron repulsion and the consequent short-range antiferromagnetic (AF) correlations, inside the low-dimensional layered structure of cuprates render these systems intrinsically different from standard metals ruled by the Landau Fermi-liquid (FL) paradigm. The occurrence of this non-FL phase may imply a drastic rearrangement of the fermionic states: while far from the Mott state a FL is present with a large Fermi surface containing $n_h = 1 + p$ holes ($p$ is the doping) per unit cell in the CuO$_2$ planes, approaching the Mott state the metallic character is given by just $p$ carriers residing in four hole pockets in the so-called nodal regions $(\pi/2a)(\pm1, \pm1)$ of the Brillouin zone [4]. This transition is marked by a gradual loss of spectral weight occurring below a $T^*(p)$ temperature, which vanishes at a critical doping $p^* \approx 0.19$. This is the famous pseudogap (PG) temperature, which in this context is clearly related to the “Mottness” of the metallic state.

The opposite point of view is that in two dimensions strong correlations and the proximity to a doped Mott insulator are not enough to spoil the Landau FL and the anomalous behavior of metallic cuprates should be ascribed to the proximity to some form of instability ending at zero temperature into a QCP. In this case, the incipient order, which at low or zero temperature has an intrinsic quantum (and therefore dynamic) character produces strong long-ranged and long-lived fluctuations. In turn these mixed quantum-thermal fluctuations mediate strong scattering between the quasiparticles, spoiling the FL character of (some of) the quasiparticles and possibly mediating a strong superconducting pairing. In this “quantum criticality” scenario a crucial role is obviously played by the type of order that the system would like to realize, were it not for low dimensionality and disorder, which favor the competing presence of superconducting long-range order. Although many proposal have been put forward, the old evidences of CDWs [8] have been strongly revived by the recent ubiquitous observations of CDW with NMR [11], STM [12], [13], sound velocity [14], transport [15], [17], and, most clearly, resonant X-ray scattering [15], [24]. In XRS experiments CDW clearly emerge with an ordering vector $q_c$ along the Cu-O bonds, i.e., along the $(1,0)$ or $(0,1)$ directions of the CuO$_2$ square lattice.

In this CDW framework, the CDW $q_c$ precisely joins the portions of the Fermi surface where the PG first occurs. This naturally leads to the idea that CDW themselves can be responsible for the loss of the fermion spectral weight in the so-called antinodal regions of the Brillouin zone. In particular, it was recently shown [14] that the CDW quantum critical theory provides a coherent scenario rationalizing several issues, like the mechanism for CDW formation and its relation to the normal and superconducting state of cuprates, the mechanism fixing the direction of the CDW modulating wavevector $q_c$ along the Cu-O bonds, the relation among the different CDW onset curves and corresponding QCPs, the relation between CDWs and PG, the role of CDWs in determining the rapid change of the Hall number seen in experiments, the mechanisms leading to a dome-shaped CDW critical...
line, delimited by two QCPs, at \( p_c' \approx 0.08 \) and \( p_c \approx 0.16 \).

Starting from that analysis, whose main results are briefly recalled, we address here yet another relevant issue, namely: How isotropic or anisotropic is the nearly singular scattering mediated by CDW fluctuations and how this (an)isotropy mirrors in transport and spectroscopic experiments?

**DYNAMICAL CDW**

**Fermi-liquid theory and the direction of the CDW wavevector**

Recent theories find that the CDW instability may be driven by retarded nearly critical spin fluctuations. However, in these approaches the CDW instability occurs along the (1,1) direction \( q = (1,1) \), in contrast with experiments, or requires a preliminary nematic deformation of the Fermi surface \( q = (1,1) \). A recent functional renormalization group approach keeping into account the more detailed three orbital structure of the CuO\(_2\) unit cell finds instead the right instability direction \( q = (1,1) \) as it also happens when hole pockets are assumed as a prerequisite from a nearly ordered AF state \( q = (1,1) \). While all these approaches strongly rely on retarded collective spin fluctuations, a CDW mechanism was proposed long ago, which is based instead on the rather common tendency of strongly correlated systems to become unstable under phase separation when even mild attractive forces mediated by short-range AF coupling and/or phonons are present \( q = (1,1) \). Of course, this tendency is hindered by the long-range Coulomb repulsion between the charged quasiparticles and the system then chooses a compromise with short-range charge inhomogeneity. In this case CDW naturally arise \( q = (1,1) \). More specifically, within a standard Random Phase Approximation closely mimicking the strong-correlation approaches (slave-boson or Gutzwiller), one can see that the instability occurs when the denominator of the density-density response function vanishes, as fixed by the condition \( \Pi(q, \omega_n = 0) \approx 0 \), at zero Matsubara frequency, \( \omega_n = 0 \), and \( q = q_c \). Here, the residual interaction among the quasiparticles \( V(q) = U(q) - \lambda + \nu(q) \) arises from three distinct contributions \( q = q_c \): \( U(q) \) is a short-range residual repulsion resulting from the bare large repulsion of a one-band Hubbard model, \( \lambda \) is a weakly momentum dependent short-range attraction promoting charge segregation (it may be due to a local phonon \( \Delta \)), to the instantaneous magnetic interaction present in doped antiferromagnets, or to both mechanisms), and \( \nu(q) \) is the long-range part of the Coulomb interaction. The screening processes are customarily described by the Lindhard polarization bubble \( \Pi(q, \omega_n) \) for quasiparticles having a renormalized band structure fitting the dispersion obtained from angle-resolved photoemission spectroscopy (ARPES) experiments. Two major points should be appreciated at this stage. First of all, the driving force for the CDW formation is the natural tendency to phase separation, which is triggered by non critical interactions, phonons and/or short-range AF. Therefore the proximity to an AF QCP is immaterial here and the Mottness only acts to weaken the metallic character of the FL quasiparticles, making them more easily prone to phase separation. Secondly, the short-range residual interaction between the FL quasiparticles has a momentum structure, which is reported in Fig.\( \text{Fig.1} \) for a typical case.

**Pseudogap, CDW three-dimensional static order, and CDW onset temperature**

Expanding \( V(q) \) and \( \Pi(q, \omega_n) \) around \( q = q_c \) and \( \omega_n = 0 \) one obtains the standard quantum-critical charge-fluctuation propagator

\[
D(q, \omega_n) = \frac{1}{m_0 + \nu(q) + |\omega_n| + \frac{\omega_n^2}{\Omega}}, \tag{1}
\]
where $m_0 \propto 1 + V(q_\perp)\Pi(q_\perp,0)$ is the mean-field mass of the fluctuations, and is proportional to the inverse of the square correlation length of CDW fluctuations, $\nu(q) \approx \bar{\nu}|q-q_c|^2$ is the dispersion law of Landau-damped CDW fluctuations, $\bar{\nu}$ is an electronic energy scale (we work with dimensionless momenta, measured in inverse lattice spacings $1/\alpha$), and $\Pi$ is a frequency cutoff, setting the frequency above which CDW fluctuations become more propagating. The mean-field instability line $T_{CDW}^0(p)$ is characterized by a vanishing $m_0$, and corresponds to the line in the temperature vs. doping diagram where $1 + V(q_\perp)\Pi(q_\perp,0) = 0$. According to the scenario presented in Ref. [1], $T_{CDW}^0(p)$ tracks the PG onset line $T^*(p)$. Therefore, in this scheme, $T^*$ is not related to any exotic realization of non-FL states, but simply occurs because CDW fluctuations start to deplete the states around the antinodal region of the Fermi surface. This explains the identification of our theoretical mean-field line $T_{CDW}^0(p)$ with the experimental PG line $T^*(p)$, ending into a missed QCP at $p^* \approx 0.19$.

The fluctuation suppression of the mean-field critical line $T_{CDW}^0(p)$ is obtained by the self-consistent evaluation of the correction to the mean-field mass $m_0$, due to the fluctuator Eq. [1]. In this way, a renormalized mass $m(T,p) = m_0(T,p) + \delta m(T,p)$ is obtained, which vanishes at the true CDW ordering temperature $T_{CDW}(p)$, which ends into a QCP at $p_c \approx 0.16$. It is worth noticing that in a strictly two-dimensional system, the renormalized ordering temperature for incommensurate CDW would vanish. However, in real layered systems, as cuprates, the planes are weakly coupled and, introducing a small energy scale $\nu_\perp$ related to the interplane coupling, a finite $T_{CDW}(p)$ can be obtained. This temperature, however, is so strongly reduced with respect to the mean-field line $T_{CDW}^0(p)$ that it occurs below the superconducting dome. Superconductivity therefore appears as the stabilizing phase against CDW long-range order. This explains why the experimental data corresponding to long-range CDWs are only detected for magnetic fields large enough to weaken the superconducting phase [14].

While at doping $p \gtrsim p_c$, the complex CDW order parameter fluctuates in modulus and phase, a different situation is met at low doping, where the physics is dominated by fluctuations of the phase of the CDW complex order parameter only [1]. Thus the critical line $T_{CDW}(p)$, that would be monotonically increasing with decreasing doping, in the case ruled by modulus and phase fluctuations, becomes dome-shaped due to the prominent role of phase fluctuations at low doping, and ends into yet another QCP at $p'_c \approx 0.08$, in agreement with the experiments.

It must be noted that in experiments CDW appear in two ways: when fast probes like X-rays are used a dome-like onset temperature $T_{on}(p) \approx 100$–150 K is identified. On the other hand, static CDW are found both with fast and slow probes at the lower $T_{CDW}(p)$. This raises the intriguing issue is why different probes identify different CDW onset temperatures. The key point is the dynamical character of the CDW fluctuations. A probe with very long characteristic timescale $\tau_{pr}$ (like, e.g., NMR or NQR) will only detect static order, otherwise the fluctuating CDWs average to zero during the probing time. This is why these probes identify a true phase-transition line $m(T_{CDW},p) = 0$ at high magnetic field (of course, if in real systems pinning intervenes to create locally a static order, this can be detected by static probes even at larger temperatures and low magnetic fields [11]). On the other hand, a fast probe with a short probing time $\tau_{pr}$ takes a fast snapshot of the fluctuating system and finds a higher “transition” temperature when the CDW order is still dynamical, as long as the CDW characteristic timescale $\tau_{CDW} \propto \xi^2 \propto m^{-1}$ is longer than $\tau_{pr}$. We thus identify the dynamical onset line as the line where $m$ reaches its minimal dynamical value $m \approx \omega_{pr} = \tau_{pr}^{-1}$ [1].

On the one hand, this qualitatively solves the experimental puzzle accounting for different CDW onset lines. However, the value $\omega_{pr} \sim 50$–100 K needed to fit the CDW onset line detected with RIXS [1] corresponds to timescales of order of 0.1 ps that are still too long in comparison with the fast (of order of a few fs) characteristic times of RIXS. We notice that this discrepancy could be reconciled if the onset temperature found by these fast probes were coinciding with $T_{CDW}^0(p)$ (i.e. all CDW fluctuations are slower than the RIXS snapshot) which is 50–100 K higher. We therefore suggest that a higher sensitivity of these experiments could shift at substantially higher temperatures the detection of the CDW peak, much closer to the $T^*$ line.

**ANISOTROPY OF THE QUASIPARTICLE SCATTERING**

The proximity to a CDW QCP provides an anisotropic scattering mechanism between the FL quasiparticles quite similar to the one due to spin fluctuations near an AF QCP [39]. In Ref. [1] it was shown that this anisotropic scattering may account for the rapid changes observed in the Hall constant $R_H$ of YBCO at low temperature, when the doping is increased from $p \gtrsim 0.16$ to $p = 0.19$. The rapid growth of $R_H$ with increasing doping is naturally interpreted in terms of a large Fermi surface corresponding to a hole density $1 + p$ reconstructing to hole pockets enclosing $p$ states only. However, any reconstruction of a Fermi surface must occur via a change in the electronic state. While the possibility can be considered that the pockets are formed in an exotic Mottness-driven non-FL state, it is also natural to associate this new state to the CDW QCP occurring at $p = 0.16$, when the ordered CDW state takes place under strong mag-
netic field. In this latter framework, increasingly slower and more extended CDW correlations are present when the doping is reduced from $p = 0.19$ to $p = 0.16$. This implies that at least in the initial steps of this underdoping the reduction of $R_H$ should be interpreted as due to the increasingly stronger and anisotropic CDW mediated interactions. This allowed for the successful fitting of $R_H$ in terms of such increasingly anisotropic interactions. Of course, other effects neglected in our analysis, like the interplay with CDW-mediated pairing correlations, could gradually add their contribution in this crossover region, possibly promoting the formation of Fermi arcs. However, the experiments cannot exclude that the Fermi surface stays large at zero temperature.

On the other hand, ARPES experiments were recently carried out on Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi2212) samples, finding indications of an isotropic electronic self-energy compatible with a marginal-FL state [40]. At first sight this result seems more naturally explained by a microscopic model based on a $q_c = 0$ instability like the one due to time-reversal symmetry breaking loop-current fluctuations [41] and seems at odds with the anisotropic singular scattering mediated by CDW fluctuations. This section is precisely devoted to a possible solution of this puzzle.

First of all, to get rid of “trivial” anisotropy due to the band structure, we purposely consider a simple isotropic parabolic electronic band $\epsilon_k = -E_0 + tk^2$, where $E_0 = 1$ eV and $t = 0.25$ eV (again $k$ is dimensionless). This gives rise to a large circular Fermi surface of quasiparticles with mass $m_{qp} = 1/2t$. Then, taking the typical parameters of the CDW fluctuator [11] the lowest-order self-energy correction is calculated to be compared with that obtained from ARPES

$$\text{Im}\Sigma(k, \omega) = \frac{g^2}{(2\pi)^2} \int \frac{d\mathbf{q}}{2\pi} \frac{\epsilon_k - \epsilon_{\mathbf{k}-\mathbf{q}}}{\nu(\mathbf{q})} \frac{(\omega - \epsilon_{\mathbf{k}-\mathbf{q}})(b(\epsilon_{\mathbf{k}-\mathbf{q}}) + f(\epsilon_{\mathbf{k}-\mathbf{q}} - \omega))}{(\nu(\mathbf{q}) - (\omega - \epsilon_{\mathbf{k}-\mathbf{q}})^2/\Omega)^2 + (\omega - \epsilon_{\mathbf{k}-\mathbf{q}})^2}$$

where $g$ is the quasiparticle-CDW coupling. Here, $q_c$ is arbitrarily chosen to connect different parts of the perfectly isotropic Fermi surface, in order to introduce the effects of the anisotropic CDW-mediated interaction [see Fig. 2(e)]. To emphasize this effect, we select the mass of the CDW corresponding to $p = 0.16$, as determined from the analysis of Ref. [11, which vanishes at $T = 0$. Fig. 2 reports $\text{Im}\Sigma = \Sigma''$ as a function of energy for $|k| = k_F$ and various angles $\phi$ along the Fermi surface. One can see that at the angles $\phi_{hot}$ corresponding to the
hot spots (where points of the Fermi surface are connected by $q_c$) $\Sigma''(\phi,\omega) \sim \sqrt{\omega}$, according to previous perturbative analyses \cite{39,12}. This is clearly visible in the panels (c) (green and light-blue curves), where the mass in the CDW fluctuator is small because at low temperature ($T = 50$ K) the system is close to the CDW QCP. On the other hand, as soon as one moves away from the hot spots, the behavior of $\Sigma''$ rapidly recovers the quadratic shape typical of a FL. When the temperature grows to $T = 200$ K, the mass of the CDW fluctuations increases (i.e., the correlation length $\xi_{CDW} \sim 1/\sqrt{m}$ decreases) and $\Sigma''(\phi,\omega)$ becomes much more isotropic. This can be quantified by calculating $\sigma(\omega)$, defined as the variance of $\Sigma''(\omega)$ when this is averaged over $\phi$ around the Fermi surface. Fig. 3 shows that $\sigma(\omega)$ is usually small (of order 20-30 percent) both at low and high temperature.

![Variance along $\phi$](image)

**FIG. 3.** Variance $\sigma(\omega)$ of $\Sigma''(\phi,\omega)$ averaged over $\phi$ around the Fermi surface at low $T = 50$ K (purple curve) and intermediate $T = 200$ K (green curve).

The only large variations in $\phi$ occur at low energies (smaller than 50 meV) and low temperature, because in this case a strong anisotropy is present due to the nearly singular scattering at the hot spots. These results clearly solve the (an)isotropy puzzle because they show that the CDW scattering is only quasi-singular and produces non-FL behavior in narrow regions around the hot spots and at low energies. This accounts for the strong effects observed in transport experiments, like in the Hall effect \cite{17}. On the other hand, at finite frequencies the scattering is rather strong but nearly isotropic over the whole Fermi surface. We also notice that $\Sigma''$ becomes rather isotropically linear in frequency, mimicking well a marginal FL behavior. Of course in the isotropic band we purposely adopted, without an upper limit to the energy, no ultraviolet cutoff intervenes to stop this marginal-FL behavior at positive frequencies.

**CONCLUSIONS**

Our work shows that an internally coherent scenario is possible in which $T^*$ marks the initial appearance of CDW fluctuations. Moreover, as it was shown long ago \cite{25}, CDW may mediate d-wave pairing and therefore an additional mechanism of PG formation due to pairing may intervene when the fluctuating CDW glue becomes strong enough. Of course, according to the Mottness supporters, the possibility is still open, that CDW are just an “epiphenomenon” occurring on top of the more fundamental physics ruled by strong correlations spoiling the FL and that the origin of $T^*$ has nothing to do with CDW. The point of view of our work is instead fully within the “quantum criticality” scheme showing that the PG occurring below $T^*$ and all the CDW phenomenology (like, e.g., the Fermi surface reconstructions revealed by transport in strong magnetic field) may all stem from the occurrence of CDW around optimal doping $p \approx 0.16$ (in YBCO) and are not tightly related to the Mott physics and to the disappearance of long-range antiferromagnetism occurring at much lower doping ($p = 0 - 0.02$).

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