Local regions with expanding extra dimensions

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Abstract

We study possible spatial domains containing expanding extra dimensions. We show that they are predicted in the framework of $f(R)$ gravity and could appear due to quantum fluctuations during inflation. Their interior is characterized by the multidimensional curvature ultimately tending to zero and a slowly growing size of the extra dimensions.

1 Introduction

It is usually assumed that the nucleation of our Universe is related to quantum processes at the Planck energy scale [1–3]. After nucleation, various manifolds evolve classically, forming a set of manifolds, one of which is our Universe. It is a serious problem to fix the Lagrangian parameters to satisfy the observations. The complexity of this problem is aggravated by inclusion of extra dimensions. The latter are of particular interest because the idea of extra space is widely used in modern research dealing with such problems as grand unification [4,5], the cosmological constant problem [6–8] and so on. The assumption of extra space compactness immediately leads to the question: why is a specific number of dimensions stable or slowly expanding [9–11]. Stabilizing factors could be, for example, scalar fields [9,12] or gauge fields [13]. A static solution can be obtained using the Casimir effect [14,15] or form fields [16]. A contradiction with observations can be avoided if the extra-dimensional scale factor $b(t)$ varies sufficiently slowly [17,18].

One of the ways to stabilize the extra dimensions is based on gravity with higher derivatives, which is widely used in modern research. One of the most promising models of inflation is the Starobinsky model using a purely gravitational action [19]. Attempts to avoid the Ostrogradsky instabilities are made [20], and extensions of the Einstein-Hilbert action attract much attention. Our model contains $f(R)$ gravity with addition of the Kretchmann invariant and the Ricci tensor squared. Its action can be considered as a basis for an effective field theory [21,22]

The recent paper [23] studied the evolution of manifolds after their creation on the basis of a pure gravitational Lagrangian with higher derivatives. The final metrics may differ in different spatial regions if the model admits several stationary states. It is precisely the case for the model discussed in [24] where one of the stationary states was studied. The Lagrangian parameters at low energies are chosen in such a way as to supply an (almost) Minkowski space for the present Universe, the stationarity of the extra-space metric, and to reproduce an inflationary stage of the expansion with the Hubble parameter of the order of $H \sim 10^{13}$ GeV. Here we will discuss another final state admitted by our model, with a comparatively small curvature of the extra dimensions which can still be compatible with observations. Some 3D spatial regions in the Universe could be characterized by such a metric, and it is of interest to study this possibility.
2 Outlook

Self-stabilization of the extra dimensions is one of necessary elements of models based on compact extra spaces. It has been shown in [33, 34] that the models with higher derivatives could lead to stationary solutions. The analysis was based on the model where the initial action was taken in the form [31,32]
\[
S = \frac{1}{2} m^D m^{D-2} \int \sqrt{g} \, d^D x \left[ f(R) + c_1 R^{AB} R_{AB} + c_2 R^{ABCD} R_{ABCD} + L_m \right],
\]
where capital Latin indices cover all \( D \) coordinates, \( g = |\text{det}(g_{MN})| \), \( f(R) \) is a smooth function of the \( D \)-dimensional scalar curvature \( R \), \( c_1, c_2 \) are constants, \( L_m \) is a matter Lagrangian, and \( m^D = 1/r_0 \) is the \( D \)-dimensional Planck mass, so that \( r_0 \) is a fundamental length in this theory. As \( L_m \), we can consider the Casimir energy density in space-time with the metric
\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu - r_0^2 e^{2\beta(x^n)} d\Omega^2_n
\]
where \( x^\mu \) are the observable four space-time coordinates, and \( d\Omega^2_n \) is the metric on a unit sphere \( S^n \). The space-time is a direct product \( M_4 \times M_n \). The function \( f(R) \) is taken in a general quadratic form,
\[
f(R) = a_2 R^2 + R - 2\Lambda_D.
\]
Assuming \( L_m = 0 \), we have obtained an effective scalar-tensor theory with the potential presented in Fig.1. Here, the internal Ricci scalar \( R_n \) actually plays the role of a scalar field \( \phi \). Details can be found in [24,33]

According to the experimental data, the scale of extra dimensions cannot be larger than \( 10^{-17} - 10^{-18} \) cm. For example, the Ricci scalar \( R_n \sim 0.01 M^2_{Pl} \) from Fig.1 is very large, so that the inequality
\[
R_n \gg R_4 \quad \text{(4)}
\]
looks natural. Here \( R_n \) and \( R_4 \) are the Ricci scalars of the extra dimensions and of our 4D space-time, respectively. The inequality [4] was used in the derivation of the effective potential in Fig.1.

The scalar field could tend to any minimum, depending on the initial conditions. Hence, different spatial regions could be filled with different states of \( \phi \), that with \( R_n = \phi_m \) or with a small Ricci scalar \( R \to 0 \) due to the metric fluctuations at high energies.

The metric evolution around the solution \( R_n = \phi_m \) could reproduce the inflationary scenario. One can adjust the model parameters to describe the observational properties of the inflationary

\[\text{Figure 1: The effective potential for a viable version of the model [1]. The minimum of the potential is at the point } \phi_m \approx 0.083, m_D = 1, m_D \sim 0.1 M_{Pl} \]
stage. As usual, strong fine tuning is necessary to produce a small, almost zero 4D cosmological constant.

In this paper, we intend to analyze the space-time metric that can emerge near the minimum \( R \to 0 \). Naively, an effective 4D cosmological constant should tend to zero automatically in such regions of space. Is it applicable for life formation? The question is important even if the answer is negative. Even if we live at the standard minimum where \( R_n = \phi_m \), some neighboring regions could be filled with another minimum where \( R \to 0 \).

Notice that we are interested in those regions where the Ricci scalar tends to zero. In this case, the inequality (4) may look suspicious, but it should evidently hold if the extra dimensions are spherical and small enough to be invisible by modern instruments. But more than that, close to the limit \( R \to 0 \) one more inequality,

\[
a_2 R^2 \ll R,
\]

must hold, which strongly facilitates the analysis.

3 Field equations

Under the assumption \([5]\) we can simply put \( F(R) = R - 2\Lambda_D \) and neglect all the curvature-nonlinear terms in \([1]\).

The action \([1]\) is then reduced to 4 dimensions, in which it takes the form inherent to a scalar-tensor theory specified in 4D space-time with the metric \( g_{\mu\nu} \):

\[
S = \frac{1}{2} V(n) m^2 \int \sqrt{|g|} d^4 x e^{n\beta} \left[ R_4 + \frac{n(n-1)}{r_0^2} e^{-2\beta} + 2n \Box \beta + (n+1)(\partial \beta)^2 - 2\Lambda_D \right],
\]

where \((\partial \beta)^2 = g^{\mu\nu} \beta_{,\mu} \beta_{,\nu}, \Box = \nabla_{\mu} \nabla^{\mu}, \text{and } V(n) = 2\pi^{(n+1)/2}/\Gamma(\frac{1}{2}(n+1))\) is the volume of a unit sphere \( S^n \).

The standard transition to the Einstein frame with the 4D metric

\[
g_{\mu\nu} = e^{n\beta} g_{\mu\nu}
\]

brings the action to the form (up to a full divergence)

\[
S = \frac{1}{2} V(n) m^2 \int \sqrt{|\bar{g}|} d^4 x \left[ \bar{R}_4 + \frac{1}{2} n(n+2)(\bar{\partial} \beta)^2 + \frac{n(n-1)}{r_0^2} e^{-(n+2)\beta} - 2 e^{-n\beta} \Lambda_D \right] + S_m,
\]

where overbars mark quantities obtained from or with \( g_{\mu\nu} \).

Now, if we use \([8]\) to describe cosmological models with the Einstein-frame metric

\[
d s_E^2 = dt^2 - a^2(t) d\bar{x}^2,
\]

and the effective scalar field \( \beta(t) \), we have the Einstein-scalar equations (only two of them are independent, and \( V_\beta = dV/d\beta \)):

\[
\ddot{\beta} + \frac{a}{a^2} \dot{\beta} + \frac{1}{n(n+2)} V_\beta = 0,
\]

\[
3 \frac{a^2}{a^2} = \frac{1}{2} n(n+2) \dot{\beta}^2 + V(\beta),
\]

\[
6 \frac{a^2}{a^2} \left( \dot{a}^2 + a \ddot{a} \right) = -n(n+2) \dot{\beta}^2 - 4 V(\beta),
\]

with the potential \( V(\beta) \) given by

\[
\begin{align*}
r_0^2 V(\beta) &= \lambda e^{-n\beta} - k_1 e^{-(n+2)\beta} + k_C e^{-(2n+4)\beta}, \\
\lambda &= r_0^2 \Lambda_D, \quad k_1 = \frac{1}{2} n(n-1).
\end{align*}
\]

The last term is related to the Casimir effect, with \( k_C \) of the order \( 10^{-3} - 10^{-4} \), see \([31,32]\) and references therein.

As is shown in the next section, the first term in the potential \([13]\) dominates.
4 Can we live in a region with $R \to 0$? Some estimates

Consider the dimensionless version $W(x)$ of the potential $V(\beta)$, with $x = e^{-\beta}$,

$$W(x) = \lambda x^n - k_1 x^{n+2} + k_0 x^{2n+4}. \quad (14)$$

Let us determine the order of magnitude of the variable $x$ such that the condition [3] will be satisfied. It can be estimated by analogy with the well-known 4D models that employ $f(R)$ and, in particular, quadratic gravity. In such models, e.g., [19,35,36], it is supposed that quadratic corrections to GR become important at energy scales of Grand Unification theories, which approximately corresponds to $a_2 \sim 10^{10} r_0^2$ in [37], where $r_0$ is of the order of the Planck length. The same assumption was used in some models predicting a semiclassical bounce instead of a Schwarzschild singularity inside a black hole [37,38]. Now, assuming that the curvature $R$ is of the order $R \sim e^{-2\beta}/r_0^4 = x^2/r_0^4$ and substituting this into the condition [5], we obtain

$$x^2 \ll r_0^2/a_2 = 10^{-10} \quad \Rightarrow \quad x \ll 10^{-5}. \quad (15)$$

With such values of $x$ and the above-mentioned values of $k_C$, it is clear that the Casimir contribution to the potential (14) is also quite negligible, and we can restrict $W(x)$ to the first two terms.

Other restrictions on admissible values of $x$ follow from two evident conditions: (i) A classical space-time description requires that the size of the extra dimensions, $r = r_0 e^\beta = r_0/x$ should be much larger than the fundamental length $r_0 = 1/m_D$, hence $x \ll 1$; (ii) This size should be small enough for the extra dimensions to be invisible for modern instruments, that is, $r = r_0/x \lesssim 10^{-17}$ cm, approximately corresponding to the TeV energy scale. If we assume that $m_D \sim m_4 \sim 10^{-5}$ g $\sim 10^{33}$ cm$^{-1}$, it follows $x \gtrsim 10^{-16}$. Admitting that $m_D$ may be some orders of magnitude smaller than $m_4$ and recalling (15), we can more or less safely assume

$$10^{-13} \lesssim x \ll 10^{-5}. \quad (16)$$

For a further study and further estimates, it is necessary to make an assumption on a conformal frame in which we interpret the observations. This choice ultimately depends on how fermions are included in a (so far unknown) underlying unification theory of all physical interactions. We will consider two options: the Jordan frame with the action (6), directly derived from the original D-dimensional theory, and the Einstein frame with the action (8).

The Einstein frame

Assuming that our observed space-time corresponds to the Einstein frame, we have Eqs. (11) and (12) for the scale factors $a(t)$ and $e^{\beta(t)}$. The Hubble parameter of the present Universe, $H(t) = \dot{a}/a$, is of the order $10^{-17}$ s$^{-1} \approx 10^{-61} r_0^{-2}$ if $r_0 = 1/m_D$; evidently, $\beta$ should be at most of the same order, therefore the same is required from $V(\beta)$. Then it follows from Eq. (11)

$$\lambda x^n - \frac{1}{2} n(n-1) x^{n+2} \lesssim 10^{-122}. \quad (17)$$

Assuming $x = e^{-\beta} \sim 10^{-10}$ (well in the range (16)), and using Eq. (17), we can consider the following two options:

(i) $\lambda = 0, \quad n(n-1) \cdot 10^{-10n-2} \lesssim 10^{-122} \quad \Rightarrow \quad n \geq 12. $

(ii) $\lambda \neq 0$, then we can rewrite (17) as

$$\lambda = \frac{1}{2} n(n-1) \cdot 10^{-20} + O(10^{-122+10n}),$$

thus the estimate of $\lambda$ depends on $n$ which can now be smaller than 12, but a considerable fine tuning is necessary, for example,

$$n = 10 \quad \Rightarrow \quad \lambda = 45 \times 10^{-20} \pm O(10^{-22}),$$

$$n = 5 \quad \Rightarrow \quad \lambda = 10^{-19} \pm O(10^{-72}).$$

In all cases, Eqs. (11) and (12) should be solved numerically.
The Jordan frame

If we assume that the observed space-time metric is \( ds^2 = e^{-n\beta} ds^2_E \), we can write down the cosmological metric as

\[
ds_J^2 = e^{-n\beta} [dt^2 - a_J(t)^2 d\vec{x}^2] = d\tau^2 - a_J^2(\tau) d\vec{x}^2,
\]
where \( \tau \) is cosmological time such that \( d\tau = e^{-n\beta/2} dt \), and \( a_J(\tau) = e^{-n\beta/2} a(t) \) is the Jordan-frame scale factor.

Now, the Hubble parameter is defined as

\[
H = \frac{1}{a_J} \frac{da_J}{d\tau} = \left(\frac{\dot{a}}{a} - \frac{1}{2} n\dot{\beta}\right) e^{-n\beta/2},
\]
which leads to the estimate

\[
\frac{\dot{a}}{a} - \frac{1}{2} n\dot{\beta} \sim H x^{n/2}.
\]

On the other hand, \( \dot{\beta} \) is subject to the observational constraint on possible variations of the effective gravitational constant \( G \) (it is a true constant in the Einstein frame but varies in Jordan’s proportionally to \( e^{-n\beta} \), see (6)): according to [39, 40], we must have \((1/G)|dG/d\tau| \lesssim 10^{-3} H\), whence it follows

\[
|\dot{\beta}| \lesssim 10^{-3} H x^{n/2}.
\]
It means that we can neglect the second term in (20) and simply take \( \dot{a}/a \sim H x^{n/2} \). Applying this to Eq. (11) in the same manner as in the Einstein frame, we arrive at the relation

\[
\lambda - \frac{1}{2} n(n-1)x^2 \sim 10^{-122},
\]
which means, for any reasonable choice of \( x \), an unnatural fine tuning of the value of \( \lambda \). Recalling that \( x = e^{-\beta} \) varies with time while \( \lambda = \text{const} \), we come to the conclusion that the Jordan frame does not lead to a plausible cosmology in the present statement of the problem.

5 The metric of a region with \( R \to 0 \). Numerical simulations

The main result of the previous discussion is the following: if an observer is inside such a region, then an unnaturally strong fine tuning of the model parameter \( \lambda \) is necessary (except for \( n \geq 12 \) in the Einstein frame). Also, the first and second-order derivatives of the potential are not defined at the point \( \phi \equiv R_n \to 0 \), so that there are no oscillations around such a minimum. But such oscillations are necessary for a successful reheating just after inflation. Therefore, our Universe is hardly described by the metric discussed above. Nevertheless, such regions could exist somewhere in the Universe, probably not too close to our Milky Way galaxy, otherwise it could disturb too strongly the well-known observable picture. It is a separate problem how we, being external observers for such regions, could detect signals coming from there. Let us note that a similar problem was discussed some years ago concerning possible large-scale antimatter regions [25, 26].

It looks worthwhile to analyze the possible metric inside such regions. The first field equation for \( \beta(t) \) (excluding \( a(t) \) with (11)) reads

\[
\ddot{\beta} + \dot{\beta} \sqrt{3 \left[ \frac{1}{2} n(n+2)\dot{\beta}^2 + V(\beta) \right] + \frac{1}{n(n+2)} V_\beta} = 0
\]

The second equation (11) is necessary for obtaining the 4D scale factor.

We are not restricted by the observational arguments if a region in question is much smaller than the whole Universe, but it should be assumed to be large and homogeneous enough, so that the cosmological metric (9) could be applicable. On the other hand, no such severe fine
tuning is needed there for the model parameters since it is not necessary to require an extreme smallness of the extra dimensions.

The field motion near a nonzero minimum of \( V(\phi) \) in Fig. 1 corresponds to the observable inflation for specific parameter values \[24\]. We need only one of them, \( \lambda = \Lambda_D = 0.0125 \). Also, we put \( V(\beta) = \Lambda_D e^{-n\beta} \), see \[13\]. A numerical solution to these equations is presented in Fig. 2. One can see that the extra dimensions expand very slowly — we show the dynamics during the most interesting time interval from the sub-Planckian time scale to the post-inflationary period \( 10^9 \sim 10^{-34} \) s in our units). The behavior of the curves remains roughly the same up to the present time. It is of interest that the expansion rate strongly depends on the number \( n \) of extra dimensions: it is inversely proportional to \( n \).

It was the Einstein frame. A transition to the Jordan frame, written explicitly using Eq. \[18\], gives no new results simply because of a very slow variation of the extra-space metric.

6 Conclusion

The space domains containing the expanding extra dimensions like those described above could exist in our Universe. It would be of interest to study light propagation from there. Also, ordinary matter could penetrate inside a domain possessing unusual properties, or they could be connected with our space through wormholes \[27, 41, 42\]. It is also of interest to study the stability of such regions from the point of view of an external observer: do they expand or shrink?

Another possibility arises if we recall that the potential maximum separates two minima as in Fig. 1. In this case, a closed wall is formed \[43\], which could expand or shrink, sweeping out the internal domain. As was discussed in \[28, 29, 34\], shrinking walls could cause multiple formation of black holes in the Universe. It could solve the problem of primordial black hole formation \[25, 30\].

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