TEXTURE ZEROS AND CP-VIOLATING PHASES
IN THE NEUTRINO MASS MATRIX

ZHI-ZHONG XING
Institute of High Energy Physics, Chinese Academy of Sciences
P.O. Box 918 (4), Beijing 100039, China
E-mail: xingzz@mail.ihep.ac.cn

We stress that specific texture zeros of lepton mass matrices, which might dynamically arise from a new kind of flavor symmetry, can help us to establish simple and testable relations between the lepton flavor mixing parameters and lepton mass ratios. We present a brief review of one-zero, two-zero and three-zero textures of the neutrino mass matrix. Their phenomenological consequences on neutrino mixing and CP violation are also discussed.

1. Introduction

Impressively robust evidence in favor of neutrino oscillations has been accumulated from the solar\(^1\), atmospheric\(^2\), reactor\(^3\) and accelerator\(^4\) neutrino experiments in the past few years. We are now convinced that neutrinos are massive and lepton flavors are mixed. In spite of such exciting progress in neutrino physics, our quantitative knowledge about neutrino masses and lepton flavor mixing remain rather poor – for example, the absolute scale of three neutrino masses, the smallest lepton mixing angle and three CP-violating phases are still unknown.

In the lack of a convincing flavor theory, five approaches have been tried towards a deeper understanding of fermion mass generation\(^5\): (a) radiative mechanisms\(^6\); (b) texture zeros\(^7\); (c) flavor symmetries\(^8\); (d) seesaw mechanisms\(^9\); and (e) extra dimensions\(^10\). Some of them can be correlated with one another. For instance, lepton mass matrices may have a few texture zeros as a natural consequence of a new kind of flavor symmetry\(^11\), and those texture zeros may guarantee some calculability and allow us to predict the neutrino mass spectrum and lepton flavor mixing parameters via the seesaw mechanism. Note that texture zeros of a fermion mass matrix dynamically mean that the corresponding matrix elements are sufficiently suppressed in comparison with their neighboring counterparts. A very good
lesson from the quark sector is remarkable: reasonable zeros of quark mass matrices allow us to establish some simple and testable relations between the flavor mixing angles and quark mass ratios\(^\text{12}\) – if such relations are more or less unique and experimentally favored, they may have a good chance to be close to the truth – namely, the same or similar relations should be predicted by the underlying (true) theory with much fewer fundamental parameters. Hence a phenomenological study of possible texture zeros of fermion mass matrices does make some sense to get useful hints about flavor dynamics responsible for the generation of fermion masses and the origin of CP violation.

The phenomenology of lepton masses and flavor mixing at low energies can be formulated in terms of the charged lepton mass matrix \(M_l\) and the (effective) neutrino mass matrix \(M_\nu\). The lepton flavor mixing matrix \(V\) arises from the mismatch between diagonalizations of \(M_l\) and \(M_\nu\). There are totally twelve physical parameters: three charged lepton masses \((m_e, m_\mu, m_\tau)\), three neutrino masses \((m_1, m_2, m_3)\), three flavor mixing angles \((\theta_{12}, \theta_{23}, \theta_{13})\), and three CP-violating phases \((\delta, \rho, \sigma)\). Besides \(m_e, m_\mu\) and \(m_\tau\), preliminary values of \(|\Delta m^2_{21}|, |\Delta m^2_{32}|, \theta_{12}\) and \(\theta_{23}\) have essentially been extracted from solar and atmospheric neutrino oscillations. How small \(\theta_{13}\) is remains an open question\(^\text{13}\). The sign of \(\Delta m^2_{32}\) is unknown and the magnitudes of \(\delta, \rho\) and \(\sigma\) are entirely unrestricted. It seems impossible to fully determine \(M_l\) and \(M_\nu\) from the feasible experiments at present or in the near future. In this situation, we hope that possible texture zeros of \(M_l\) and \(M_\nu\) may help us out.

We remark that texture zeros of lepton mass matrices can lead to some simple and testable relations between unknown and known parameters of neutrino oscillations. Of course, such zeros may not be preserved to all orders or at any energy scales in the unspecified interactions from which lepton masses are generated. At the one-loop level and in the flavor basis where \(M_l\) is diagonal and positive, however, the renormalization-group evolution of \(M_\nu\) from the seesaw scale (i.e., the mass scale of the lightest right-handed Majorana neutrino) to the electroweak scale does allow its texture zeros to preserve\(^\text{12}\). Once the approach of texture zeros is combined with the seesaw mechanism, it is possible to simultaneously account for the cosmological baryon number asymmetry via leptogenesis\(^\text{14}\) and current neutrino oscillation data.

This talk is subject to a phenomenological analysis of texture zeros of lepton mass matrices at low energy scales. For simplicity, we restrict ourselves to symmetric \(M_l\) and \(M_\nu\). A symmetric lepton mass matrix totally
has six independent entries. If $n$ of them is/are taken to be vanishing, we will arrive at

$$\binom{6}{n} = \frac{6!}{n!(6-n)!}$$

patterns, which are structurally different from one another. It is obvious that a pattern of $M_l$ or $M_\nu$ with more than three texture zeros (i.e., $n \geq 4$) has no chance to be compatible with the experimental data of lepton masses and flavor mixing angles. Hence we shall pay our attention to 20 three-zero textures, 15 two-zero textures and 6 one-zero textures of $M_\nu$ in the following.

2. Three-zero textures of $M_\nu$

There are twenty three-zero patterns of $M_l$ or $M_\nu$, which can be classified into four categories:

(a) Three diagonal matrix elements are all vanishing (type 0):

$$M_0 = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix},$$

where those non-vanishing entries are simply symbolized by $\times$'s.

(b) Two diagonal matrix elements are vanishing (type I):

$$M_{11} = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \quad M_{13} = \begin{pmatrix} 0 & \times \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix},$$

$$M_{14} = \begin{pmatrix} 0 & \times \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad M_{15} = \begin{pmatrix} 0 & 0 & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix},$$

which are of rank three; and

$$M_{16} = \begin{pmatrix} 0 & 0 & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix},$$

which are of rank two.
(c) One diagonal matrix element is vanishing (type II):

\[
M_{\text{II}_1} = \begin{pmatrix}
\times & \times & 0 \\
\times & 0 & 0 \\
0 & 0 & \times
\end{pmatrix}, \quad
M_{\text{II}_2} = \begin{pmatrix}
\times & 0 & \times \\
0 & 0 & 0 \\
\times & 0 & \times
\end{pmatrix}, \quad
M_{\text{II}_3} = \begin{pmatrix}
0 & \times & 0 \\
0 & 0 & \times \\
\times & 0 & \times
\end{pmatrix},
\]

which are of rank three; and

\[
M_{\text{II}_7} = \begin{pmatrix}
\times & \times & 0 \\
\times & 0 & \times \\
0 & 0 & 0
\end{pmatrix}, \quad
M_{\text{II}_8} = \begin{pmatrix}
\times & 0 & \times \\
0 & 0 & \times \\
\times & 0 & \times
\end{pmatrix}, \quad
M_{\text{II}_9} = \begin{pmatrix}
0 & \times & 0 \\
0 & 0 & \times \\
\times & 0 & \times
\end{pmatrix},
\]

which are of rank two.

(d) Three diagonal matrix elements are all non-vanishing (type III):

\[
M_{\text{III}} = \begin{pmatrix}
\times & 0 & 0 \\
0 & 0 & \times \\
0 & \times & 0
\end{pmatrix}.
\]

This pattern itself does not give rise to flavor mixing.

In the flavor basis where \(M_l\) is diagonal and positive (i.e., of the pattern \(M_{\text{III}}\)) and \(M_\nu\) takes one of the above 20 patterns, we find that none of the 20 combinations of \(M_l\) and \(M_\nu\) can be compatible with current neutrino oscillation data. When \(M_l\) is allowed to have off-diagonal non-vanishing entries, however, the situation will change. A careful analysis\(^\text{15}\) shows that there are totally 24 combinations of \(M_l\) and \(M_\nu\) with six texture zeros, which are compatible with current experimental data at the 3\(\sigma\) level. These 24 patterns can be classified into a few distinct categories:
The lepton mass matrices in each category are isomeric – namely, they have different structures but their phenomenological consequences are exactly the same\(^\dagger\). Note that \(m_1 = 0\) holds for the last four categories of lepton mass matrices, in which \(M_\nu\) is of rank two. Both the neutrino mass spectrum (\(m_1, m_2\) and \(m_3\)) and the Majorana phases of CP violation (\(\rho\) and \(\sigma\)) can well be determined or constrained for these 24 patterns.

3. Two-zero textures of \(M_\nu\)

In the past two years, some particular attention has been paid to two-zero textures of the neutrino mass matrix in the flavor basis where the charged lepton mass matrix is diagonal and positive\(^\ddagger\),\(^\S\). There are totally fifteen possible patterns of \(M_\nu\) with two independent vanishing entries, as illustrated below.

| Pattern A\(_1\) | Pattern A\(_2\) | Pattern B\(_1\) |
|----------------|----------------|----------------|
| \((0 \times \times)\) | \((0 \times 0)\) | \((\times \times 0)\) |
| \((0 \times \times)\) | \((\times \times 0)\) | \((\times 0 \times)\) |
| \((\times \times 0)\) | \((\times 0 \times)\) | \((\times 0 x\times)\) |

| Pattern B\(_2\) | Pattern B\(_3\) | Pattern B\(_4\) |
|----------------|----------------|----------------|
| \((\times 0 \times)\) | \((\times 0 \times)\) | \((\times \times 0)\) |
| \((0 \times \times)\) | \((0 \times 0)\) | \((\times \times 0)\) |
| \((\times \times 0)\) | \((\times 0 \times)\) | \((\times 0 \times)\) |

| Pattern C | Pattern D\(_1\) | Pattern D\(_2\) |
|-----------|----------------|----------------|
| \((\times \times \times)\) | \((\times \times \times)\) | \((\times \times \times)\) |
| \((\times 0 \times)\) | \((\times \times 0)\) | \((\times \times 0)\) |
| \((\times \times 0)\) | \((\times 0 \times)\) | \((\times 0 \times)\) |

| Pattern E\(_1\) | Pattern E\(_2\) | Pattern E\(_3\) |
|----------------|----------------|----------------|
| \((\times \times \times)\) | \((\times \times \times)\) | \((\times \times 0)\) |
| \((\times 0 \times)\) | \((\times \times \times)\) | \((\times \times 0)\) |
| \((\times \times \times)\) | \((\times \times 0)\) | \((\times \times 0)\) |

| Pattern F\(_1\) | Pattern F\(_2\) | Pattern F\(_3\) |
|----------------|----------------|----------------|
| \((\times 0 \times)\) | \((\times 0 \times)\) | \((\times \times 0)\) |
| \((\times \times \times)\) | \((\times \times 0)\) | \((\times \times 0)\) |
| \((\times \times \times)\) | \((\times \times 0)\) | \((\times \times 0)\) |

Among these patterns, seven of them (\(A_{1,2}, B_{1,2,3,4}\) and \(C\)) are found to be compatible with current neutrino oscillation data; and two of them (\(D_{1,2}\)) are only marginally allowed by today’s experimental data. The left six
patterns (E_{1,2,3} and F_{1,2,3}) have been ruled out in phenomenology.

One may reproduce those phenomenologically-favored two-zero textures of $M_{\nu}$ with the help of the seesaw mechanism and calculate the cosmological baryon number asymmetry via leptogenesis\textsuperscript{18}.

4. One-zero textures of $M_{\nu}$

There are totally six different textures of the neutrino mass matrix with one independent vanishing entry, as shown below.

| Pattern A | Pattern B | Pattern C |
|-----------|-----------|-----------|
| (0 × ×)   | (0 × ×)   | (× × 0)   |
| (× × ×)   | (0 × ×)   | (× × ×)   |
| (× × ×)   | (× × ×)   | (0 × ×)   |

| Pattern D | Pattern E | Pattern F |
|-----------|-----------|-----------|
| (× × ×)   | (× × 0)   | (× × ×)   |
| (× 0 ×)   | (× × 0)   | (× × ×)   |
| (× × ×)   | (× 0 ×)   | (× × 0)   |

In the flavor basis where $M_{l}$ is diagonal and positive, one may confront these one-zero textures of $M_{\nu}$ with current neutrino oscillation data. However, it is impossible to fully determine the neutrino mass spectrum and Majorana phases of CP violation in this case.

Note that pattern A is of particular interest, because it predicts $\langle m_{ee} \rangle = 0$ (namely, the effective mass of the neutrinoless double beta decay vanishes). While $\langle m_{ee} \rangle \neq 0$ must imply that neutrinos are Majorana particles, $\langle m_{ee} \rangle = 0$ does not necessarily imply that neutrinos are Dirac particles. The reason is simply that the vanishing or suppression of $\langle m_{ee} \rangle$ may be due to large cancellation induced by the Majorana CP-violating phases. Current neutrino oscillation data together with the condition $\langle m_{ee} \rangle = 0$ can well constrain the neutrino mass ratios and two Majorana phases\textsuperscript{19}.

Now we consider one-zero textures of $M_{\nu}$ with one vanishing eigenvalue ($m_{1} = 0$ or $m_{3} = 0$). Such phenomenological scenarios are interesting, because they may naturally appear in the minimal seesaw model with two heavy right-handed neutrinos\textsuperscript{20}. A careful analysis\textsuperscript{21} shows that patterns A, B and C with $m_{1} = 0$ are compatible with current neutrino data, so are patterns B, C, D and F with $m_{3} = 0$. In particular, the CP-violating phases are calculable in the minimal seesaw model with the Frampton-Glashow-Yanagida-like ansätze\textsuperscript{22}. Radiative corrections to neutrino masses and lepton flavor mixing parameters have also been computed in this framework\textsuperscript{23}. 


5. Concluding remarks

We have briefly described one-zero, two-zero and three-zero textures of the neutrino mass matrix and confronted them with today’s neutrino oscillation data. Some more remarks and comments are in order.

(a) Vanishing entries of simplified $M_l$ and $M_\nu$ (or the flavor mixing matrix $V$) may serve as a symmetry limit of small entries of realistic $M_l$ and $M_\nu$ (or $V$). The latter may come from explicit perturbations or radiative corrections. Model building may start from the possible symmetry limit, in which CP might be conserving. For example, the bi-large mixing pattern of $V$ with small $|V_{e3}|$ at low energies might result from the bi-maximal mixing pattern of $V$ with vanishing $|V_{e3}|$ at GUT scales. In this case, the seesaw threshold effects play a primary role\textsuperscript{13,24}.

(b) Lepton and quark mass matrices could have the same texture zeros, arising from a universal flavor symmetry at a certain energy scale. For instance, a typical four-zero texture of the form

$$M \sim \begin{pmatrix} 0 \times 0 \\ \times \times \times \\ 0 \times \times \end{pmatrix}$$

is favored for quark mass matrices and is seesaw-invariant\textsuperscript{12}. It can be incorporated into a SO(10)-inspired neutrino model\textsuperscript{25}.

(c) It will be very useful to explore the full parameter space of $M_\nu$ by using more accurate experimental data in no assumption of texture zeros\textsuperscript{26}. Such an analysis might more or less favor certain of texture zeros in $M_\nu$, nevertheless.

Our conclusion is that the mass hierarchy of charged leptons and quarks (and perhaps neutrinos) seem to imply certain textures of fermion mass matrices, in which zeros may (approximately) be present. The study of texture zeros could help establish the true bridge between experimental measurables and fundamental parameters of the underlying flavor dynamics.

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