CAN THE GEOMETRIC TEST PROBE THE COSMIC EQUATION OF STATE?

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ABSTRACT

The feasibility of the geometric test as a probe of the cosmic equation of state of the dark energy is discussed assuming the future Two Degree Field QSO sample. We examine the sensitivity of the QSO two-point correlation functions, which are theoretically computed incorporating the light cone effect and the redshift distortions, as well as the nonlinear effect, to a bias model whose evolution is phenomenologically parameterized. It is shown that the correlation functions are sensitive to a mean amplitude of the bias and not to the speed of the redshift evolution. We will also demonstrate that an optimistic geometric test could suffer from the possibility that a signal from the cosmological model can be confused with that from a stochastic character of the bias.

Subject headings: cosmology: theory — dark matter — large-scale structure of universe — quasars: general

1. INTRODUCTION

Recent observations of the cosmic microwave background anisotropies and the distant Type Ia supernovae favor a spatially flat universe whose expansion is currently accelerating (de Bernardis et al. 2000; Lange et al. 2000; Perlmutter et al. 1999b; Riess et al. 1998). Motivated by this fact, variants of the cold dark matter (CDM) model with the cosmological constant (ACDM) have been widely studied. In particular, quintessence is proposed as a plausible model (e.g., Caldwell, Dave, & Steinhardt 1998; Zlatev, Wang, & Steinhardt 1998). An attractive feature of quintessence is that certain quintessence models (tracker models) naturally explain the “coincidence problem,” the near coincidence of the density of matter and the dark energy component at present (Zlatev et al. 1998; Steinhardt, Wang, & Zlatev 1999, and references therein). Observational constraints on the quintessential cold dark matter (QCDM) model have been investigated (e.g., Efstathiou 1999; Perlmutter, Turner, & White 1999a; Wang et al. 2000; Newman & Davis 2000), including attempts to reconstruct the cosmic equation of state or the quintessential potential (Starobinsky 1998; Saini et al. 2000; Nakamura & Chiba 1999; Chiba & Nakamura 2000).

In the meanwhile, recent progress on the Two Degree Field (2dF) QSO redshift (2QZ) survey has been reported by Shanks et al. (2000). An interesting scientific aim of the 2QZ survey is to obtain new constraints on the cosmological constant from the geometric test, which was originally proposed by Alcock & Paczyński (1979), with the QSO clustering statistics. Several authors have proposed possible cosmological tests using the clustering of high-redshift objects (Ryden 1995; Ballinger, Peacock, & Heavens 1996; Matsubara & Suto 1996; Popowski et al. 1998; Nair 1999). In a series of works (Suto, Magira, & Yamamoto 2000; Yamamoto, Nishioka, & Taruya 2001, hereafter Paper I, and references therein), a useful theoretical formula has been developed for predicting the correlation functions of high-z objects, incorporating the light cone effect and the redshift-space distortions, as well as the geometric distortion, simultaneously. The theoretical formula is useful because it is expressed in a semianalytic form, which enables us to compute the two-point correlation functions corresponding to a survey sample based on specific cosmological models without huge numerical simulations. Therefore, it will be worthwhile to examine the feasibility of the geometric test as a probe of the cosmic equation of state of the dark energy with assuming a realistic QSO clustering statistics.

Unfortunately, however, the QSO clustering bias is not well understood at present. This ambiguity of the clustering bias should be crucial for the geometric test because the evolution of bias affects the predicted correlation functions. In Paper I, behavior of the QSO correlation functions is partially examined. On the other hand, the stochastic bias has been discussed as a possible characteristic of the galaxy biasing by several authors (e.g., Dekel & Lahav 1999; Tegmark & Peebles 1998; Taruya, Koyama, & Soda 1998). If the QSO clustering bias possesses the stochastic characteristic, it affects the redshift-space correlation functions (Pen 1998). In the present Letter we consider the feasibility of the geometric test as a probe of the cosmic equation of state of the dark energy component, including the possible stochasticity of the QSO clustering bias.

This Letter is organized as follows. In § 2, we briefly review the theoretical formula for the two-point statistics. In § 3, sensitivity of the correlation functions to the evolution of a bias model is investigated. Section 4 is devoted to the summary and conclusions. Throughout this Letter we use units such that the light velocity $c$ equals 1.

2. CONTENTS FOR THEORETICAL PREDICTION

We restrict ourselves to a spatially flat FRW universe and follow the quintessential cosmological model consisting of a scalar field slowly rolling down its effective potential. The effective equation of state of the dark energy, $w_0 = p_0/\rho_0$, can be a function of redshift in the general case; however, we will consider the QCDM model with a constant equation of state for simplicity. In this case the dark energy density evolves $\rho_0(z) \propto a(z)^{-3(1+w_0)}$, where $a(z)$ is the scale factor. Then the relation between the comoving distance and the redshift is

$$ r(z) = \frac{1}{H_0} \int_0^z \frac{dz'}{\Omega_m (1 + z') \Omega_m (1 + z')^{w_0(z) + 1}} , \quad (1) $$

where $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$ is the Hubble constant and $\Omega_m$ and $\Omega_{\Lambda} (= 1 - \Omega_m)$ denote the density parameters of the matter component and the dark energy, respectively, at present.

Wang & Steinhardt (1998) have given a useful approximate formula for the linear growth index, which we adopt for computation of the $f$-factor defined by $f(z) \equiv d \ln D_r(z)/d \ln a(z)$, where $D_r(z)$ is the linear growth rate. Simple fitting formulae for the linear transfer function and the nonlinear mass pertur-
bution power spectrum are presented by Ma et al. (1999) for the QCDM model, which we adopt for modeling the CDM mass density perturbations. The formulae are applicable for the QCDM model with the constant equation of state \(-1 \leq w_0 \leq -\frac{1}{3}\) and up to \(z \sim 4\) with errors \(\leq 10\%\). In addition, we adopt the normalization by cluster abundance (Wang & Steinhardt 1998; Wang et al. 2000). Throughout this Letter we assume the Harrison-Zeldovich spectrum.

In predicting clustering statistics of high-redshift objects in a redshift survey, one must incorporate several observational effects for careful comparison between theoretical predictions and observational results. A useful theoretical formula for the two-point statistics has been developed incorporating the redshift distortions due to peculiar motion of sources and the light cone effect simultaneously, as well as the geometric distortion (Suto et al. 2000; Paper I). According to the theoretical formula, the two-point correlation function is given by a Fourier transform of the power spectrum on a light cone \(P^{LC}_{1}(k)\):

\[
\xi_{l}(R) = \frac{1}{2\pi^2 l} \int_{0}^{\infty} dk \ k^2 \ j_l(kR) P^{LC}_{l}(k),
\]

where \(P^{LC}_{l}(k)\) is obtained by averaging the local power spectrum \(P^{ss}_{l}(k, z)\) over the redshift

\[
P^{LC}_{l}(k) = \frac{\int dz \ W(z) P^{ss}_{l}(k, z)}{\int dz \ W(z)}
\]

with the weight factor

\[
W(z) = \left( \frac{dN}{dz} \right)^2 \left( \frac{\sigma_z^2}{1} \right)^{-1},\]

where \(dN/dz\) denotes the number count of the objects per unit redshift and per unit solid angle and \(s = s(z)\) denotes the distance-redshift relation of the radial coordinate that we chose to plot a map of sources. In the present Letter we adopt the distance-redshift relation of the Einstein–de Sitter universe, i.e., \(s(z) = 2 \ H_0^{-1} \left[ 1 - (1 + z)^{-1/2} \right]\). In equation (3), \(z\)-integration arises from the light cone effect within the small-angle approximation. The power spectrum \(P^{ss}_{l}(k, z)\) in equation (3) is given by

\[
P^{ss}_{l}(k, z) = \frac{2l + 1}{c_s^2 c_l^4} \int_{0}^{1} d\mu \ L_s(\mu) \\
\times P_{\text{QSO}}(q_s | q_s |, |q_s| \rightarrow k \frac{1 - \mu^2}{c_s^2}, \ z),
\]

where \(P_{\text{QSO}}(q_s, |q_s|, z)\) is the QSO power spectrum, \(q_s (|q_s|)\) is the wavenumber component parallel (perpendicular) to the line-of-sight direction in the real space, and \(L_s(\mu)\) is the Legendre polynomial. In equation (5), we denoted \(k = |k|, c_s = r(z)s(z), \) and \(c_l = dr(z)/dzs(z)\) with the comoving distance in the real space \(r(z)\), equation (1).

We model the power spectrum of QSO distribution by introducing the bias factor \(b(z)\),

\[
P_{\text{QSO}}(q_s | q_s |, z) = \left[ b(z)^2 + 2b(z)f(z)R(z) \right] P_{\text{max}}(z, q_s)D[q_s, \sigma(z)].
\]

where \(q = (q_s^2 + |q_s|^2)^{1/2}\) and \(P_{\text{max}}(z, q)\) is the CDM mass power spectrum. The terms in proportion to \(f(z)\) in equation (6) are traced back to the linear distortion (Kaiser) effect. To describe a cross-correlation between the QSO distribution and the CDM mass distribution, we here introduced the cross-correlation coefficient \(R(z)\). In the case of the deterministic bias, the cross-correlation coefficient can be set \(R(z) = 1\); however, in the case of the stochastic bias, the cross-correlation coefficient is allowed to deviate from unity (e.g., Pen 1998).

In equation (6), \(D[q_s, \sigma(z)]\) is the damping factor due to the finger-of-God effect, for which we adopt the exponential model for the distribution of the pairwise velocity dispersion \(\sigma_p\). For \(\sigma_p\) we adopt an approximate formula whose validity has been investigated by several authors (see Magira, Jing, & Suto 2000; Mo, Jing, & Bornier 1997). The nonlinear effect is not a dominant effect on large length scales; however, it cannot be neglected. Therefore, we here take the nonlinear effects into account for definiteness.

Recently, Boyle et al. (2000) have reported on the evolution of the QSO optical luminosity function using their preliminary result of the 2QZ survey. The evolution of the luminosity function is compared with analytic fitting formulae. For theoretical predictions that correspond to the ongoing 2QZ survey, we adopt their qaoz luminosity function with best-fit parameters of a power-law polynomial evolution of luminosity under the assumption of an Einstein–de Sitter universe. Then the number count \(dn/dz\) brighter than the limiting magnitude \(B \leq 20.85\) can be obtained by integrating the luminosity function.

3. Sensitivity

The QSO clustering bias is a challenging problem, and a few authors have proposed theoretical models (Fang & Jing 1998; Martini & Weinberg 2000; Haiman & Hui 2000). However, these models seem to be prototype models; hence, we here adopt a model whose evolution is phenomenologically parameterized, for simplicity, as (cf. Matarrese et al. 1997)

\[
b(z) = \alpha + [b(z_s) - \alpha] \left[ \frac{1 + z}{1 + z_s} \right]^{\beta},
\]

where \(\alpha, \beta, \) and \(b(z_s)\) are free parameters, \(\beta\) specifies the speed of redshift evolution, \(b(z_s)\) is the amplitude at a mean redshift \(z_s\), and \(\alpha\) corresponds to the amplitude of bias at \(z = 0\) in the limit of \(z_s \gg 1\). Throughout the present Letter we fix \(\alpha = 0.5; \) however, this does not change the results for \(0 \leq \alpha \leq 1\).

\(^1\) For the \(K\)-correction we assume the quasar energy spectrum \(L_s \propto \nu^{-

We define the mean redshift by

$$z_* = \frac{\int_{\text{max}}^{\text{min}} dz zW(z)}{\int_{\text{max}}^{\text{min}} dz W(z)},$$

(8)

with the weight factor (4). Here we consider the sample in the range $0.3 \leq z \leq 2.2$; in this case we have $z_* = 1.2$. We note that $b(z_*)$ is almost same as the mean amplitude of the bias defined in a similar way to equation (8).

For the geometric test, the ratio of the correlation functions $\xi_z(R)/\xi_0(R)$ on the $\beta$-$b(z_*)$ plane will be an important quantity to characterize the geometric distortion effect. Figure 1 shows contours of $\xi_z/\xi_0$ on the $\beta$-$b(z_*)$ plane with the separation $R$ fixed as 20 $h^{-1}$ Mpc for various cosmological models (solid lines). Figures 1a and 1b show the cases of the ΛCDM model ($\Omega_m = 0.3, \omega = -1$) and the QCDM model ($\Omega_m = 0.3, \omega = -\frac{1}{2}$), respectively. In both panels we show the case of the deterministic bias. This figure shows that the value of $\xi_z/\xi_0$ is sensitive only to the parameter $b(z_*)$ and not to the speed of evolution $\beta$.

Recently, it was reported that the QSO correlation function is consistent with being $\xi = (r_0 R_0^{-1})^{-1}$ with $r_0 = 4 h^{-1}$ Mpc from the preliminary result of the 2QZ survey (Shanks et al. 2000). The previous result of the QSO surveys similar to the 2QZ survey reported $r_0 = 6 h^{-1}$ Mpc (Croom & Shanks 1996). The dashed lines on the panels show the contours satisfying $R_\ast = 4 h^{-1}$ Mpc and $R_\ast = 6 h^{-1}$ Mpc, where the characteristic correlation length $R_\ast$ is defined by $\xi_0(R_\ast) = 1$. The observational constraint is not strict at present because of statistical errors. Thus, Figure 1 should be regarded as a demonstration. However, it is instructive. Taking the constraint $4 h^{-1}$ Mpc $\leq R_\ast \leq 6 h^{-1}$ Mpc into account, the ratio of the correlation function is $\xi_z/\xi_0 \approx -2.0 \sim -1.4$ for the ΛCDM model and $\xi_z/\xi_0 \approx -1.4 \sim -1.0$ for the QCDM model. We fixed $R = 20 h^{-1}$ Mpc in Figure 1; however, similar behavior appears at other values of $R$. For example, in the case $R = 30 h^{-1}$ Mpc, $\xi_z/\xi_0 \approx -3.5 \sim -2.3$ for the ΛCDM model and $\xi_z/\xi_0 \approx -2.3 \sim -1.8$ for the QCDM model. The reason why $\xi_z/\xi_0$ depends on the cosmological model can be explained as follows. One reason is the difference of the linear growth rate $D_\ast(z)$, which affects the Kaiser factor. The other effect is the scaling effect due to the geometric distortion, which is described by the factors $c_i$ and $c_\ast$. The latter effect is the most important. It is clear from Figure 2 that the less negative value of $\omega$ ($\omega > 0$) decreases the coefficients $c_i(z)$ and $c_\ast(z)$.

![Figure 1](image1.png)

**Fig. 1.**—Contours of $\xi_z(R)/\xi_0(R)$ at $R = 20 h^{-1}$ Mpc on the $\beta$-$b(z_*)$ plane for various cosmological models (solid lines): (a) ΛCDM model ($\Omega_m = 0.3, \omega = -1$); (b) QCDM model ($\Omega_m = 0.3, \omega = -\frac{1}{2}$); (c) same as (a) but with the stochastic bias case with $R(z) = 0.6$; (d) QCDM model ($\Omega_m = 0.35, \omega = -\frac{1}{2}$). The contour levels of the solid lines are indicated on the figure, and the same contour levels are adopted for each panel. In each panel we adopted $h = 0.7, \Omega_c h^2 = 0.015, \alpha = 0.5$ for the bias model. Except for (c), the deterministic bias is considered. The left (right) dashed line shows the contour that satisfies the condition of the characteristic correlation length $R_\ast = 4 h^{-1}$ Mpc ($R_\ast = 6 h^{-1}$ Mpc).

![Figure 2](image2.png)

**Fig. 2.**—Evolution of $c_i(z)$ and $c_\ast(z)$. The lines show the cases $\Omega_m = 0.3$ and $\omega = -1, -\frac{1}{2}, -\frac{1}{4}, -\frac{5}{4}$, from top to bottom, for both panels.
1b, we might find that the ambiguity of the density parameter $\Omega_m$ is not negligible too.

To solve the degeneracy due to the stochastic character of the bias, $\xi_b(R)$ might be useful. Figure 3 plots contours of $\xi_z/\xi_0$ for the various cosmological models, whose parameters are same as those in Figure 1. Similar features to Figure 1 can be seen in Figure 3. However, Figure 3 shows that $\xi_z/\xi_0$ is rather insensitive to the stochasticity $R$ and the density parameter $\Omega_m$, as is expected from the investigation of the linear stochastic biasing in redshift space (Pen 1998). Therefore, $\xi_z/\xi_0$ might be useful to break the degeneracy, if it could be measured precisely. However, the amplitude of the signal is rather small, on the order of 10% compared with $\xi_z/\xi_0$.

4. DISCUSSION

In the present Letter, we have examined the sensitivity of correlation functions in a QCDM cosmological model to the bias evolution, incorporating the various observational effects, i.e., the light cone effect, the linear distortion effect, and the nonlinear and the finger-of-God effects. Then the feasibility of the geometric test is discussed as a probe of the cosmic equation of state assuming the future 2dF QSO sample. The amplitude of the correlation functions is sensitive to the mean amplitude of the bias and is rather insensitive to the speed of evolution due to the light cone effect. We have found that the systematic difference appears in the ratio of the correlation functions depending on the effective cosmic equation of state, because of the geometric distortion effect. We have also shown that, if the QSO bias has a stochastic character, the signal from the cosmological model can be confused with that from the stochastic one. Hence, the simple geometric test with only $\xi_z/\xi_0$ suffers from the degeneracy between the cosmological parameter and the bias parameter unless the stochasticity character is clarified.

For example, the ACDM model with $R(z) = 0.6$ and the QCDM model with $w_Q = -0.9$ and $R(z) = 1$ predict almost the same value of $\xi_z/\xi_0$ at $R = 20 h^{-1}$ Mpc. Therefore, other cosmological information is required, e.g., the higher order multipole moment of the correlation function $\xi_z/\xi_0$. However, the signal from $\xi_z$ seems to become noisy; more detailed investigations will be needed to determine the viability.

Finally, it will be worthwhile to discuss the robustness of our results for several assumptions adopted in the present Letter. Another choice of the cosmological redshift space $s(z)$ alters the shape of the correlation function (Paper I); hence, the predicted values of $\xi_z/\xi_0$ will be altered. However, the sensitivity to the cosmic equation of state will not be significantly altered; neither will the feasibility of the geometric test. The bias model (eq. [7]) seems to express general evolution of the bias; then our result is not sensitive to the bias model unless the scale dependence of the bias is significant. Concerning the QCDM cosmological model, our investigation is restricted to the case where $w_Q$ is a constant. In addition, we used the fitting formula by Ma et al. (1999), which is applicable to that case. In most cases, the quintessence equation of state changes slowly with time; however, we believe that predictions are well approximated by treating $w_Q$ as an averaged constant value (e.g., Wang et al. 2000).

An open CDM model shows a similar result to that of the QCDM model in $\xi_z/\xi_0$ (Paper I); then our conclusion is based on the assumption of a spatially flat universe.

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