VECTOR MESON AT NON-ZERO BARYON DENSITY AND
ZERO SOUND

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We present simulation results of the $(2 + 1)d$ four-fermion model with a baryon chemical potential $\mu$. We examine temporal correlation functions of the vector meson, and find evidence of phonon-like behavior characterised by a linear dispersion relation in the long wavelength limit. We also discuss the consistency of our numerical results with analytical solutions to the Boltzmann equation corresponding to zero sound. We argue that our results provide the first evidence for a collective excitation in a lattice simulation.

We study the $(2 + 1)d$ Gross-Neveu (GN) model with a $Z_2$ chiral symmetry describing $N_f$ flavors of self-interacting fermions, with continuum lagrangian

$$\mathcal{L} = \bar{\psi}(\not\partial + \mu \gamma_0)\psi - \frac{g^2}{2N_f}(\bar{\psi}\psi)^2. \quad (1)$$

As $\mu$ is increased, the model exhibits a sharp first-order transition at a critical $\mu_c$ from a phase where a fermion mass $m_f$ is dynamically generated but baryon density $n_B = \langle \bar{\psi} \gamma_0 \psi \rangle$ vanishes to one where $m_f = 0$ and $n_B \propto \mu^2$ [1]. The main new feature emerging for $\mu > \mu_c$ is the existence of a new scale, the Fermi momentum $k_F$. Since the lowest energy excitations of the ground state have $|\vec{k}| \approx k_F$, measurements of Euclidean timeslice correlators with $\vec{k} \neq 0$ are mandatory.

If certain conditions are fulfilled, such as an effective interaction between quasiparticles which is both short-ranged and repulsive, then there is a massless bosonic excitation in the spectrum as $T \rightarrow 0$. Carrying zero baryon number, it is a phonon, ie. a quantum of a collective excitation.
called zero sound [2]. Sound propagation occurs in any elastic medium; zero sound happens when the elasticity originates not from collisions between individual particles, but from the force on a single particle due to its coherent interaction with all others present in the medium. It can be pictured as a propagating distortion in the local shape of the Fermi surface, and its speed of propagation exceeds that of conventional “first” sound.

In [3] we solved the self-consistent equation for zero sound in the framework of Fermi liquid theory in the case of the 3d GN model to leading non-trivial order in $1/N_f$. We found solutions for sound propagation with speed $\beta_0 > \beta_F$, the Fermi velocity. Here we discuss our numerical results and compare them with the analytical solutions presented in detail in [3].

The lattice regularized action of our model uses staggered fermions. Details of the formulation and simulation algorithm are given in [1,4]. The simulation parameters are $N_f = 4$ and $a/g^2 = 0.75$, corresponding to a physical fermion mass at $\mu = 0$ of $m_f \simeq 0.17a^{-1}$. All results are taken in the chirally-restored phase, i.e. with $\mu > \mu_c \simeq 0.16a^{-1}$.

By analysing the decay of the appropriate correlation function with Euclidean time, we calculated the dispersion relations $E(k)$ for the spin-$\frac{1}{2}$ quasiparticle, which carries a baryon charge, and $\omega(k)$ for various meson states of the form $\bar{\psi}\Gamma\psi$ which are best thought of as excitations formed from a particle-hole pair. We simulated $L_s^2 \times 48$ systems with $L_s = 32$ and 48, with $\mu$ ranging from 0.2 to 0.6, and measured $E(k), \omega(k)$ for $\vec{k} = (k, 0)$ with $k = 0, 2\pi/L_s, \ldots, \pi/2$. The increased momentum resolution offered by $L_s = 48$ has proved to be important. Note that the staggered fermion action is only invariant under translations of two lattice spacings, which restricts the space of accessible momenta. In [3,5] we measured the quasiparticle propagator and showed that the transition between particle and hole behavior is rather sudden, characteristic of a well-defined Fermi surface. We also presented [5] the theoretical predictions of Fermi liquid properties to leading non-trivial order in $1/N_f$ for $\mu \gg \mu_c$. The results for the Fermi velocity $\beta_F$ and the Fermi momentum $k_F$ are given by:

$$\beta_F = 1 - \frac{1}{16N_f} \simeq 0.984 ; \quad \frac{k_F}{\mu \beta_F} = 1 - \frac{1}{8N_f} \simeq 0.969,$$

(2)

As discussed in [3,5] the agreement between the analytical predictions and the numerical results on $48^3$ lattices for $\mu = 0.2, \ldots, 0.6$ and with discretization effects taken into account is at best qualitative. It should be noted that analytic predictions of $\beta_F$ and $k_F$ to $O(1/N_f)$ for finite $\mu$ strictly requires a knowledge of the gap equation to two loops, which is not yet available.
For the remaining analysis we will therefore assume the free field values $k_F = \mu, \beta_F = 1$.

Next we consider the meson sector, ie. correlators of the form

$$C_\Gamma(\vec{k}, t) \equiv \sum_\vec{x} \langle \bar{\psi}_\Gamma(0) \psi_\Gamma(x, t) \rangle e^{i\vec{k}.\vec{x}},$$

(3)

which carry zero baryon number. In [5] we showed that the generic large-distance behaviour in any given channel is dominated by zero-energy particle-hole pairs, and as a result the decay is algebraic, ie. $C_\Gamma(t) \propto t^{-\lambda(k)}$, with the exponent $\lambda$ determined by the geometry of the overlap between two Fermi disks with relative displacement $\vec{k}$ between their centres. A particularly favourable configuration occurs for $|\vec{k}| \approx 2\mu$, in which case the Fermi surfaces just kiss, and $\lambda = \frac{3}{2}$. In this paper we focus on point-split meson operators corresponding to spatial components of the conserved vector current, ie. of the form

$$j_i(x) = \eta_i(x) \frac{1}{2} [\bar{\chi}(x) \chi(x + \hat{i}) + \bar{\chi}(x + \hat{i}) \chi(x)],$$

(4)

where $\chi, \bar{\chi}$ are staggered fermion fields, $\eta_1(x) = (-1)^i$, and $\eta_2(x) = (-1)^{i+x_1}$. We set $\vec{k} = (k, 0)$, and define $C_\parallel$ in terms of the correlator $\langle j_1(0)j_1(x) \rangle$ and $C_\perp \sim \langle j_2(0)j_2(x) \rangle$. In Fig. 1 we show $|C_\parallel, \perp|(k, t)$ data taken on a $48^3$ lattice at $\mu = 0.5$, for both $k = \frac{\pi}{3}$ and $k = \pi - \frac{\pi}{3}$ (the modulus is taken because we use a log-linear scale, and $C$ fluctuates in sign). Whereas $C_\perp$ and $C_\parallel(k = \frac{\pi}{3})$ all show behaviour consistent with algebraic decay, the decay in the $C_\parallel(k = \pi - \frac{\pi}{3})$ channel is much faster, and resembles the exponential decay expected of an isolated pole. We have fitted it with the form

$$C_\parallel(\pi - k, t) = A \exp(-\Omega(k)t) + (-1)^iB \exp(-\omega(k)t),$$

(5)

in most cases employing datapoints with $t \in [10, 38]$. For small $k$, the coefficient $B \gg A$, suggesting that the correlator is dominated by a pole in the alternating channel. In Fig. 1 we plot the resulting dispersion relation $\omega(k)$ for $\mu = 0.2, \ldots, 0.6$. The behaviour $\omega \propto k$ as $k \to 0$ suggests the presence of a massless pole similar to that of a phonon. Support for this interpretation comes from recasting the meson bilinear in terms of continuum-like fields $q^\alpha_a, \bar{q}^{\alpha}_a$ having spinor index $\alpha = 1, \ldots, 4$ and ‘color’ index $a = 1, 2$ [6]. We obtain

$$(-1)^{x_1}(-1)^i \bar{\chi}(x) \chi(x + \hat{i}) \sim i\bar{q}(\gamma_0 \otimes \tau_a^\ast)q,$$

(6)

demonstrating that the excitation can be viewed as an oscillation of local baryon density.
Figure 1. Left: Meson propagators $|C_\parallel|$ and $|C_\perp|$ for both $k = \frac{\pi}{3}$ and $\pi - \frac{\pi}{3}$ for $\mu = 0.5$. The dashed line shows a fit of the form (5). Right: Dispersion relation $\omega(k)$ for various $\mu$ for the channel defined by the correlator $C_\parallel(\pi - k)$. Data are from 48$^3$.

Although we are examining the $k \to 0$ limit, we are trying to model a Fermi surface phenomenon on the lattice; the appropriate scale with which to compare $\beta_0$ is the “bare” Fermi velocity (ie. with no discretisation correction) which for free fields with $E(k) = -\mu + \sinh^{-1}(\sin k)$ is given by

$$\beta_0^{\text{bare}} = \left. \frac{\partial E}{\partial k} \right|_{E=0} = \sqrt{\frac{1 - \sinh^2 \mu}{1 + \sinh^2 \mu}}.$$  

The general trend in our data is that the speed ratio $s = \beta_0/\beta_F^{\text{bare}}$ increases towards unity as $\mu$ increases (for $\mu = 0.2$ $s = 0.896(8)$ and for $\mu = 0.6$ $s = 0.959(2)$). In [3] we used Fermi liquid interaction to leading nontrivial order in $1/N_f$ to find analytical solutions to the Boltzmann equation corresponding to zero sound. The excitation is spin and isospin symmetric, and has speed ratio $s > 1$ for almost all $\mu > \mu_c$. To what extent can we be sure that our numerical results describe the same physical phenomenon? Solutions with $s < 1$ allow the possibility of emission of a phonon with momentum $\vec{q}$ and energy $\beta_0|\vec{q}|$ from a quasiparticle with momentum $\vec{k}$ and energy $\mu + \beta_F(|\vec{k}| - k_F)$. This is allowed kinematically for $s < 1$, the angle of emission $\phi$ satisfying

$$\cos \phi = s + \frac{|\vec{q}|}{2|\vec{k}|}(1 - s^2) > s.$$  

All radiation is emitted within a cone of half-angle $\cos^{-1} s$ centred on the quasiparticle trajectory. This well-known phenomenon is variously known as Landau damping, Čerenkov radiation, or most appropriately in the current context, as a sonic boom.
First let us argue how our numerical results can support a state resembling a simple pole but with $s < 1$, in apparent contradiction to the above. On a spacetime lattice, the radiation process is constrained because there is a natural lower bound for the angle of emission, $\phi_{\text{min}} \sim 2\pi/L_s|\vec{q}|$. Landau damping is thus kinematically forbidden for

$$s > \cos \phi_{\text{min}} \simeq 1 - \frac{2\pi}{L_s^2|\vec{q}|^2}.$$  

With a conservatively high $|\vec{q}| \sim \mu/2$, then on a $48^3$ lattice we have no damping for $s > 0.73$ at $\mu = 0.2$, rising to $s > 0.97$ at $\mu = 0.6$. It is plausible therefore that Landau damping is suppressed in our lattice data and that the phonon is described by an isolated pole even if $s < 1$.

Simulations on volumes considerably larger than those used here, however, will be needed to disentangle the various possible systematic effects due to finite $L_s$, finite $\mu/T$, and non-zero lattice spacing in order to determine the sign of $s - 1$ for finite $N_f$. We think the most probable explanation for $s < 1$ is an effect of non-zero $T \equiv (aL_t)^{-1}$; the physical temperature of the $L_t = 48$ lattice decreases by a factor of roughly two going from $g^{-2} = 0.75$ to $g^{-2} = 0.6$, and according to our lattice data $|s - 1|$ decreases by roughly the same factor. It is known empirically that the speed of sound in liquid $^3$He increases as $T \to 0$ and the dominant mode of propagation changes from first sound to zero sound [7]. In principle, since our extracted value of $s \lesssim 1$ has relied on a rescaling of $\beta_F$ to take account of discretisation artifacts, we also need to go to considerably finer lattices, i.e. with $\beta_F^{\text{bare}} \approx 1$, in order to demonstrate a clear distinction between our signal and first sound with expected propagation speed $\beta_1 \simeq 0.7$. If, however, the theoretical arguments about zero sound dominating as $T \to 0$ can be taken seriously, then for the first time we have succeeded in identifying a collective oscillation in a lattice simulation.

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