BAM: Bias Assignment Method to generate mock catalogs

A. Balaguera-Antolínez⋆1,2, Francisco-Shu Kitaura†1,2, Marcos Pellejero-Ibáñez1,2, Cheng Zhao3 and Tom Abel4

1 Instituto de Astrofísica de Canarias, s/n, E-38205, La Laguna, Tenerife, Spain
2 Departamento de Astrofísica, Universidad de La Laguna, E-38206, La Laguna, Tenerife, Spain
3 National Astronomy Observatories, Chinese Academy of Science, Beijing, 100012, P.R. China
4 Kavli Institute for Particle Astrophysics and Cosmology, Stanford University, SLAC National Accelerator Laboratory, Menlo Park, California 94025, USA

18 June 2018

ABSTRACT
We present BAM: a novel Bias Assignment Method envisaged to generate mock catalogs by linking the continuous cosmic dark matter field to a discrete population of tracers, such as dark matter halos or galaxies. Using a reference high resolution cosmological N-body simulation to extract a biasing scheme we can generate halo catalogues starting from much coarser density fields calculated from downsampled initial conditions using efficient structure formation solvers. We characterize the halo-bias relation as a function of a number of properties (e.g. local density, cosmic web type) to the dark matter density field defined on a mesh of a $3 h^{-1} \text{Mpc}$ cell side resolution, derived from the fast structure formation solvers. In this way our bias description automatically includes stochastic, deterministic, local and non-local components directly extracted from full N-body simulations. We sample the halo density field according to the observed halo bias, such that the two-point statistics of the mock halo catalog follows the same statistics as the reference. By construction, our approach reaches percentage accuracy, $\sim 1\%$ in the majority of the $k$-range up to the Nyquist frequency without systematic deviations for power spectra (about $k \sim 1 h^{-1} \text{Mpc}$) using either particle mesh or Lagrangian perturbation theory based solvers. When using phase-space mapping to compensate the low resolution of the approximate gravity solvers, our method is able to reproduce the bispectra of the reference within $10\%$ precision studying configurations tracing the quasi-nonlinear regime. Therefore BAM promises to become a standard technique to produce mock halo and galaxy catalogs for future galaxy surveys and cosmological studies being highly accurate, efficient and parameter free.

Key words: cosmology: – theory - large-scale structure of Universe

1 INTRODUCTION
The analysis of cosmological large-scale structure experiments such as eBOSS (Dawson et al. 2016), DES (The Dark Energy Survey Collaboration 2005), Euclid (Amendola et al. 2016) and DESI (Levi et al. 2013) demands exact models of galaxy clustering and precise estimates of covariance matrices (e.g. Dodelson & Schneider 2013; Taylor et al. 2013). In the lack of analytically accurate models capturing the highly complex nonlinear gravitational evolution underlying a galaxy distribution, the physical processes such as galaxy bias, baryon effects (e.g. Eisenstein & Hu 1998; Rudd et al. 2008), redshift space distortions (e.g. Kaiser 1987) and systematic effects (e.g. survey geometry), the construction of large sets of mock catalogs based on N-body simulations has been adopted as the standard approach to assess robust error estimates on cosmological observables. This imposes some practical restrictions, given the considerably high time and/or memory requirements that a state-of-the-art N-body simulation requires to generate hundreds to thousands of realizations and light-cones. In order to speed-up the construction mock catalogs, a number of alternatives have been designed. There are some pioneering works such as PINOCCHIO (Monaco et al. 2002) and the PEAK-PATCH method (Bond & Myers 1996). These approaches generate mock galaxy (or halo) catalogues in a predictive way by using approximate gravity solvers based on analytical approaches, such as Lagrangian perturbation theory and prescriptions to compute the formation of halos. The problem of these methods is that such approximations do not accurately describe structure formation towards small scales, deviating beyond 5%...
accuracy from the true power spectrum (at \( \Delta k \sim 0.2 \, h\text{Mpc}^{-1} \)) already on scales relevant to baryon acoustic oscillations, redshift space distortions and non-linear evolution. There are also some recent advances in fast gravity solvers such as ICE-COLA (Tassev et al. 2013; Kodama et al. 2016; Izard et al. 2016) and FastPM (Feng et al. 2016), which do not suffer so severely from these inaccuracies. All these methods however, require resolving the halos and thus demand approximately the same level of memory as a full N-body calculation, being just moderately faster in the computational process (e.g. Blot et al., in preparation). These strategies also limit the methods to generate mock galaxy catalogues, as they work best to resolve distinct halos, which can be then augmented with, e.g. an HOD approach (e.g. Berlind et al. 2003; Zheng et al. 2015). However, techniques such as Halo Abundance Matching (e.g. Behroozi et al. 2010; Trujillo-Gomez et al. 2011) require resolving also the substructures, which can only be obtained with very accurate gravity solvers, and thus remain unreachable for those approaches. Special mention should be made to a different pioneering work, PThalos (Scoccimarro & Sheth 2002), which changed the strategy in later works paving a new way of analysing galaxy surveys with large numbers of mocks (e.g. Manera et al. 2013, 2015). The original path of PThalos is in fact technically more interesting, and was further explored in later works with PATCHY (Kitaura et al. 2014), QPM (White et al. 2014), and EZMOKCS (Chuang et al. 2015a) and HALOGEN (Avila et al. 2015). The idea in these approaches is to rely only on the smooth large-scale dark matter field obtained from approximate gravity solvers, and populate it with halos (or galaxies) following some bias prescriptions.

The problem of the original PThalos approach is that the bias prescription is not trivial at all. Many studies have demonstrated that this quantity can depend on several properties of dark matter halos, such as their formation time (e.g. Gao & White 2007) their local density and mass (e.g. Pillepich et al. 2010; Tinker et al. 2010; Valeanuas 2011), scale-dependency and non linear evolution (e.g. Fry & Gaztanaga 1993; Kravtsov & Klypin 1999; Smith et al. 2007), stochasticity (e.g Mo & White 1996; Dekel & Lahav 1999; Sigad et al. 2000; Somerville et al. 2001; Kitaura et al. 2014), non-locality (e.g. Matsubara 1999; McDonald & Roy 2009; Chan et al. 2012; Pollack et al. 2012; Sheth et al. 2013), the cosmic web type (Yang et al. 2017; Fisher & Faltenbacher 2018), among others. The halo bias, from a practical implementation, also depends on the arbitrarily chosen resolution of the dark matter mesh, approximations of the gravity solver, and mass assignment scheme (MAS hereafter). In fact this remains hitherto as the main problem for all these bias mapping approaches, requiring complex calibration procedures, which are only valid for a particular cosmology and numerical setting. The degree of complexity this field has achieved can be witnessed in Vakili et al. (2017), where Markov Chains are used to break complex degeneracies in the bias parameters using higher order statistics (see also Kitaura et al. 2015). Despite of the calibration difficulties, these bias mapping methods have shown to reach high level of accuracy circumventing the limitations of the approximate gravity solvers (see Chuang et al. 2015b) being able to make large amount of precise mock galaxy catalogues (e.g. Kitaura et al. 2016) with very low memory and computational requirements (e.g. Blot et al., Colavincenzo et al., and Lippich et al., in preparation).

Future galaxy surveys will trace the cosmic web further towards the low density regime, being able to map in more detail the filamentary network with bright, blue, red, and emission-line galaxies (e.g. Comparat et al. 2016; Merson et al. 2018; Merson, Wang, Benson, Faisst, Masters, Kiessling & Rhodes 2014; Benitez et al. 2014). In the light of this situation, and with a calibration process which is becoming too complex and subject to propagation of systematic errors due to an approximate bias modelling, we propose here to radically simplify the calibration problem trying to capture the full complexity, by directly mapping the bias relation from detailed N-body simulations. To this aim, in this letter we present a new method to generate mock catalogs: the Bias Assignment Method (BAM), which applies the idea of halo bias mapping in a free-parameter fashion. To describe the nonlinear dark matter field accurately with low number of particles we resort on the phase-space mapping technique (PSM hereafter) (Abel et al. 2012; Hahn et al. 2013). In order to develop our method we adopted the MINERVA simulations (Grieb et al. 2016) with output at redshift \( z = 1 \). The simulation has a comoving volume of 1500\(^3\) (Mpc/h\(^3\)) with a mass resolution of \( M_{\text{min}} = 2.6 \times 10^{12} M_{\odot} \) and 1000\(^3\) dark matter (DM hereafter) particles. Dark matter halos (DMH hereafter) are identified with a friends-of-friends algorithm (FoF hereafter). The outline of this letter is as follows. We describe the BAM assumptions and method and study its performance. Then we end-up with conclusions and discussion.

2 The BAM Method

BAM exploits the idea of mapping the halo distribution onto a target dark matter density field (TDMF hereafter) with a biasing scheme directly extracted from a reference N-body simulation. Such TDMF is obtained by evolving approximate gravity solvers using the same initial conditions of the reference simulation, downgraded to a lower resolution. In this way one can extract the required mapping operations required to obtain the reference halo catalog with respect to low resolution approximate gravity solvers, which can then be applied to any configuration of initial conditions retaining the same cosmological parameters and numerical setting. This builds on the rank ordering method proposed by Weinberg (1992), extending it to a multivariate bias relation dependent on local and non-local quantities, and relating a continuous field to a discrete realization of tracers (see also an object by object halo mass reconstruction technique Zhao et al. 2015). Moreover, our approach includes an iterative sampling procedure, which is crucial to achieve high accuracy in the resulting power and bispectra.

Our method characterizes the halo bias in the spirit of Dekel & Lahav (1999), measuring the probability distribution of halo number densities conditional to a set of properties of the underlying DM density field such as the local density, the cosmic web-type and environmental density. Note that we encode all possible dependencies of halo bias on the properties of DM density field, instead of using halo properties such as mass, spin, assembling history or concentration (e.g. Paranjape et al. 2018, and references therein). The properties of DMH are expected to be tightly correlated to the DM density field (e.g. Bardeen et al. 1986). Therefore our method, being based on an statistical map-
Figure 1. Halo bias for different cosmic web classifications, in the plane $x = \log(\rho_{dm})$ and $y = \log(\rho_H)$, where $\rho = 1 + \delta$, is the DM and H density on the grid, computed for illustrative purposes using the CIC assignment scheme. The contours in each panel denote the region containing 68%, 95% and 99% of the total number of classified cells. The filled curve denotes the mean relation $\langle y \rangle(x)$ and its rms, while the insets display one element of the full $P(y|x)dx$, where the bin $(x, x + dx)$ is represented by the vertical line.

Panel (a) shows the conditional distribution $P(y|x)$, namely, mean and $\pm$ rms. By comparing the shaded curve and the contours, it is evident that there is more information on halo bias beyond the first and the second moments of the conditional probability distribution. All this information is what approaches like PATCHY (Kitaura et al. 2014) aim to account for, by specifying not only the shape of the mean bias relation, but also the scatter around that mean. The vertical dashed line in the same panel represents a bin of DM density, in which the distribution of the DMH densities behaves as shown in the inset plots.

In order to characterize the halo bias as a function of the cosmic web type (denoted by $t$), DMH are classified as knots, filaments, sheets, or voids according to the behaviour of the eigenvalues of the tidal field (see e.g. Hahn et al. 2007), evaluated at their grid positions. Top-right and bottom panels of Fig. 1 represent the conditional probability distribution $P(y, x, \Delta V)_t$ and its first two moments, for different cosmic-web types. As for the first panel, we show in the inset an example of the conditional distribution $P(y|x, \Delta V)_t$, in order to evidence the difference in the halo bias in each case. Finally, we account for the halo environment (see e.g. Shi & Sheth 2018, and references therein), defined here as large-scale collapsing regions identified as percolation of cells classified as knots. These regions are identified using a FoF algorithm (Zhao et al. 2015). We compute the mass $M_K$ of such regions, labeling each cell therein contained with that value. Our method can be extended to other properties of the DM density field.

The main inputs requested by BAM to generate a mock catalog are i) the reference halo number counts in cells, obtained from the halo catalog constructed from the DM particle distribution, and ii) the TDMF, obtained from an approximated gravity solver using the same initial conditions of the reference simulation. In order to verify the accuracy of our procedure, we first use the DM distribution of the reference as the TDMF. We also validate our method using approximate gravity solvers to show its practical use. With these inputs, the steps followed by BAM are summarized as:

- Classification of the cosmic web based on the TDMF.
- Identification of the large-scale collapsing regions ($M_K$).
- Measurement of the halo bias, i.e., the conditional probability distribution $P(N_H|x, M_K, \Delta V)_t dx dM_K$, accounting for the number of DMH from the reference ($N_H$) in a cell of volume $\Delta V$ with local DM density in the range $(x, x + dx)$, cosmic web type $t$ and embedded in a large-scale collapsing region with mass in the range $(M_K, M_K + dM_K)$, $N_H$.
- Sampling of the TDMF by assigning the number of halos to cells as $N_H \sim P(N_H|x, M_K, \Delta V)_t$. The procedure is performed such that the distribution of number counts replicates that of the reference halo catalog.

In panel (a) of Fig. 2 we show the power spectrum of the mock catalog obtained after these steps. In order to highlight the dependencies described above, we show three cases, namely, i) halo-bias depends solely on the DM density, ii) extending that dependency with the cosmic web type, and iii) including the environment. Panel (b) of the same figure shows the ratio of the power spectrum obtained in each case to the reference halo power spectrum. Assumming that the halo bias only depends on the local DM density, the mock catalog generated by BAM presents a ~ 40% more clustering than the reference. This fraction is reduced to ~ 30% when the information of the cosmic web is included, while accounting for the environment lowers it to ~ 20%. This clustering excess is due in part to other dependencies in the halo bias, not accounted for in our analysis, together with the impact of the mass assignment scheme. To verify this, we have performed some tests on our method: given the DM density field of the reference, interpolated into the mesh with a particular MAS, we can construct a fake reference halo density field using a certain bias prescription (e.g. $N_H \sim \text{Poisson} (\rho_{dm})$). If
Figure 3. The first and second rows represent a slice of width ~ 20 Mpc of the dark matter halo density fields, respectively. We show the reference simulation (Minerva, first column), the density fields after one (second column) and several (third column) iterations with BAM, using the Minerva DM as TDMF. Fourth and fifth columns show the results of using BAM with two approximated gravity solvers, namely, ALPT and FastPM. The last column shows ALPT combined with PSM. The color scale is the same for all dark matter (or halo) density fields, and represents log$_{10}$(σ/δ). The third row shows the corresponding halo power spectrum compared to that obtained in each case (dotted line). The last row shows the reduced bispectrum $Q(k_1,k_2,k_3)$, for a configuration $k_1 = 0.1 h$ Mpc$^{-1}$ and $k_2 = 2k_1$.

BAM uses the same MAS for the TDMF and include all relevant properties assumed by the fake halo bias, then our method is able to create a mock halo density field with the power spectrum of the reference with a < 5% precision. In practice we are marginalizing over properties relevant for halo bias and we do not know the effective mathematical description of the halo finder with which the reference DMH’s have populated the underlying DM field. In order to account for such unknown dependencies, BAM introduces an iterative process summarized as follows:

(i) Obtain the first mock halo catalog as previously described. Define an isotropic kernel in Fourier space, initialized to $K_j \equiv K(k_j) = 1$, where $k_j$ denotes the $j$-th spherical shell in Fourier space.

(ii) Define a bias transfer function $T_{ij} \equiv P_{gal}(P_{ij}$ and update the kernel $K_j$ with the values of $T_{ij}$ as $K_j \equiv T_{ij} \times \ldots \times T_{i-1,j} \times T_{ij}$. At each iteration, the values of $T_{ij}$ are selected according to a Metropolis-Hasting algorithm.

(iii) Convolve the TDMF with the updated kernel and measure a new halo-bias relation.

(iv) Generate a mock halo density field and estimate the mock power spectrum. Return to step (i).

The black solid line in panel (a) of Fig. 2 shows the mock halo power spectrum obtained after this process, with its ratio to the reference shown as a black solid line in panel (b). By construction, our method can recover the reference power spectrum with ~ 1% in the majority of the k-range up to the Nyquist frequency without systematic deviations. The main advantage of BAM can be recognized when applied to an approximated gravity solver, capable to generate dark matter density fields with the correct large-scale structure and with low computational costs. We have explored two examples, the Augmented Lagrangian perturbation theory (ALPT, Kitaura & Hess 2013) and FastPM (Feng et al. 2016), both run using the initial conditions of the reference simulation. In Fig. 3 we summarize the results of these applications. The first (upper) and second row shows the DM and DMH density fields in different stages of BAM. We also show the power spectra of each case and the bispectrum $Q(k_1,k_2,k_3)$ for configurations of $k_1 = 0.1 h$ Mpc$^{-1}$ and $k_2 = 2k_1$, which trace the quasi-nonlinear regime. We highlight that the 3-point statistics is not calibrated in our method. However, when applied on approximated gravity solvers, BAM can recover the shape for this statistics to a good degree, although with a systematic deviation of 10–30% with respect to the reference. This discrepancy is alleviated using the PSM technique. We have applied it to ALPT, considerably improving the agreement with respect to the bispectrum of the reference catalog, as shown in the fifth column of Fig. 3. We highlight that the 3-point statistics is not calibrated in our method, and still, BAM is able to reproduce the reference bispectrum within a ~ 10% difference. Whether such deviation is systematic or not will be studied with the large set of Minerva simulations. Further investigation needs to be done to study whether including more information in the halo-bias can reduce these discrepancies.

3 CONCLUSIONS AND DISCUSSION

In this letter we have described BAM, a novel Bias Assignment method, which promises to become a standard technique to produce mock halo and galaxy catalogs for future galaxy surveys and cosmological studies. In particular we have demonstrated that BAM reproduces the distribution of halos from an N-body simulation (the Minerva simulation,
described in Sect. 1) within 1% precision in the power spectrum up to Nyquist frequencies of \( k \sim 1 \ h \ Mpc^{-1} \) and within 10% in the bispectrum (using only 500 particles to describe the dark matter field and counts in cell in the 2- and 3-point statistics computations). We tested a particle mesh code (FastPM) and a Lagrangian perturbation theory solver (ALPT), performing equally well in the power spectrum. High accuracy in the bispectrum for configurations down to the quasi-nonlinear regime require PSM phase-space mapping, although configurations on very large scales seem to be reproduced equally well without that particular technique. In order to study this more rigorously we plan to use the large set of Minerva simulations. The BAM method is not limited to a particular halo mass to achieve percentage accuracy in the power spectrum, as it does so by construction. But its performance in the bispectrum going to lower masses remains to be investigated in future work using reference catalogs from higher resolution simulations. It is in any case remarkable how our bias assignment sampling scheme and phase-space mapping enables an efficient Lagrangian perturbation theory solver to reach such high accuracies. Our studies with the BAM method have been restricted to halo populations, to real-space and the computation of the different statistics to number counts per cell, so far. We note, that BAM replaces the parametrized deterministic and stochastic bias prescriptions of the PATCHY method. We can then assign dark matter particle positions and velocities to halos within each cell (Kitaura et al. 2015) and generalize it to mock galaxy catalogs by taking the proper reference catalogs (Kitaura et al. 2016). An improved sub-grid model describing the distribution of objects on small scales still needs to be implemented, able to properly account for small scale effects, such as fiber collision (Hahn et al. 2017).

In summary, the BAM approach represents a big step forward towards the efficient production of accurate mock catalogs for the analysis of future galaxy surveys.

Acknowledgements ABA acknowledges financial support from the Spanish Ministry of Economy and Competitiveness (MINECO) under the Severo Ochoa program SEV-2015-0548. FSK thanks support from the grants RYC2015-18693, SEV-2015-0548 and AYA2017-89091-P. MPI acknowledgements from MINECO under the Severo Ochoa program SEV-2015-0548 and AYA2017-89891-P. MPI acknowledges support from MINECO under the grant AYA2012-39702-C02-01. We acknowledge Ariel Sánchez and Claudio Dalla Vecchia for providing us with a realization of the Minerva simulations.

REFERENCES

Abel T., Hahn O., Kaehler R., 2012, MNRAS, 427, 61
Amendola L., et al., 2016, preprint, (arXiv:1606.00180)
Avila S., Murray S. G., Knebe A., Power C., Robotham A. S. G., Garcia-Bellido J., 2015, MNRAS, 450, 1856
Bardeen J. M., Bond J. R., Kaiser N., Szalay A. S., 1986, ApJ, 304, 15
Behroozi P. S., Conroy C., Wechsler R. H., 2010, ApJ, 717, 379
Benitez N., et al., 2014, preprint, (arXiv:1403.5237)
Beutler F., et al., 2013, MNRAS, 436, L78
Bond J. R., Myers S. T., 1996, ApJS, 103, 1
Chuang C.-H., Kitaura F.-S., Hess S., 2013, MNRAS, 439, L21
Chuang C.-H., et al., 2015b, MNRAS, 452, 686
Chuang C.-H., et al., 2016, A&A, 592, A121
Dawson K. S., et al., 2016, AJ, 151, 44
Deek A., Lahav O., 1999, ApJ, 520, 24
Dodelson S., Schneider M. D., 2013, Phys. Rev. D, 88, 063537
Eisenstein D. J., Hu W., 1998, ApJ, 496, 605
Feng Y., Chiu M.-Y., Seljak U., McDonald P., 2016, MNRAS, 463, 2275
Fisher J. D., Faltenbacher A., 2018, MNRAS, 473, 3941
Fry J. N., Gaztanaga E., 1993, ApJ, 413, 447
Gao L., White S. D. M., 2007, MNRAS, 377, L5
Grieb J. N., Sánchez A. G., Salazar-Albornoz S., Dalla Vecchia C., 2016, MNRAS, 457, 1577
Hahn O., Porciani C., Carollo C. M., Dekel A., 2007, MNRAS, 375, 489
Hahn O., Abel T., Kaehler R., 2013, MNRAS, 434, 1171
Hahn C., Scoccimarro R., Blanton M. R., Tinker J. L., Rodriguez-Torres S. A., 2017, MNRAS, 467, 1940
Izard A., Crocce M., Fosalba P., 2016, MNRAS, 459, 2327
Kaiser N., 1987, MNRAS, 227, 1
Kitaura F.-S., Hess S., 2013, MNRAS, 435, L78
Kitaura F.-S., Yepes G., Prada F., 2014, MNRAS, 439, L21
Kitaura F.-S., Gil-Marin H., Scolloca C. G., Chuang C.-H., Müller V., Yepes G., Prada F., 2015, MNRAS, 450, 1836
Kitaura F.-S., et al., 2016, MNRAS, 456, 4156
Koda J., Blake C., Butterf L., Kazin E., Marin F., 2016, MNRAS, 459, 2118
Krvatskov A. V., Klypin A. A., 1999, ApJ, 520, 437
Levi M., et al., 2013, preprint, (arXiv:1308.0847)
Manera M., et al., 2013, MNRAS, 428, 1036
Manera M., et al., 2015, MNRAS, 447, 437
Matsubara T., 1999, ApJ, 525, 543
McDonald P., Roy A., 2009, JCAP, 8, 020
Merson A., Wang Y., Benson A., Faisst A., Masters D., Kiessling A., Rhodes J., 2018, MNRAS, 474, 177
Mo H. J., White S. D. M., 1996, MNRAS, 282, 347
Monaco P., Theuns T., Taffoni G., 2002, MNRAS, 331, 587
Paranjape A., Hahn O., Sheth R. K., 2018, MNRAS, 476, 3631
Pillepich A., Porciani C., Hahn O., 2010, MNRAS, 402, 191
Pollack J. E., Smith R. E., Porciani C., 2012, MNRAS, 420, 3469
Rudd D. H., Zentner A. R., Kravtsov A. V., 2008, ApJ, 672, 19
Scoccimarro R., Sheth R. K., 2002, MNRAS, 329, 629
Sheth R. K., Chan K. C., Scoccimarro R., 2013, Phys. Rev. D, 87, 083002
Shi J., Sheth R. K., 2018, MNRAS, 473, 2486
Sigad Y., Branchini E., Deek A., 2000, ApJ, 540, 62
Smith R. E., Scoccimarro R., Sheth R. K., 2007, Phys. Rev. D, 75, 063512
Somerville R. S., Lemson G., Sigad Y., Deek A., Kauffmann G., White S. D. M., 2001, MNRAS, 320, 289
Tassev S., Zaldarriaga M., Eisenstein D. J., 2013, JCAP, 6, 036
Taylor A., Joachimi B., Kitching T., 2013, MNRAS, 432, 1928
The Dark Energy Survey Collaboration 2005, ArXiv Astrophysics e-prints,
Tinker J. L., Robertson B. E., Kravtsov A. V., Klypin A., Warren M. S., Yepes G., Gottlöber S., 2010, ApJ, 724, 878
Trujillo-Gomez S., Klypin A., Primack J., Romanowsky A. J., 2011, ApJ, 742, 16
Vakili M., Kitaura F.-S., Feng Y., Yepes G., Zhao C., Chuang C.-H., Hahn C., 2017, MNRAS, 472, 4144
Valageas P., 2011, A&A, 525, A98
Weinberg D. H., 1992, MNRAS, 254, 877
White M., Tinker J. L., McBride C. K., 2014, MNRAS, 437, 2594
Yang X., et al., 2017, preprint, (arXiv:1704.02451)
Zehavi I., et al., 2005, ApJ, 630, 1
Zhao C., Kitaura F.-S., Chuang C.-H., Prada F., Yepes G., Tao C., 2015, MNRAS, 451, 4266

MNRAS 000, 1–77 (0000)