Polarization-sensitive Compton Scattering by Accelerated Electrons

Monika A. Mościbrodzka

Department of Astrophysics/IMAPP, Radboud University, P.O. Box 9010, 6500 GL Nijmegen, The Netherlands; m.moscibrodzka@astro.ru.nl

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Abstract

We describe upgrades to a numerical code that computes synchrotron and inverse-Compton emission from relativistic plasma including full polarization. The introduced upgrades concern a scattering kernel that is now capable of scattering the polarized and unpolarized photons on a nonthermal population of electrons. We describe the scheme to approach this problem and we test the numerical code against a known analytic solution. Finally, using the upgraded code, we predict the polarization of light that is scattered off subrelativistic thermal or relativistic thermal and nonthermal free electrons. The upgraded code enables more realistic simulations of emissions from plasma jets associated with accreting compact objects.

Unified Astronomy Thesaurus concepts: Black holes (162); Polarimetry (1278); Radiative transfer (1335); General relativity (641)

1. Introduction

Accreting black holes in active galactic nuclei, X-ray binaries, or γ-ray bursts often produce relativistic jets. Depending on the system size, jets are usually observed in radio and infrared wavelengths. Interestingly, the radio emission is often correlated with the X-rays (Merloni et al. 2003; Falcke et al. 2004). The latter suggests that some of the X-ray emission observed in accreting black holes may be produced by jets as well. In such an environment, the radio and the X-ray photons are produced by electrons which experience acceleration. New insights into black hole accretion and jet emission may be soon provided by simultaneous spectral-timing polarimetry at keV energies by missions such as NASA’s X-ray polarimetry mission Imaging X-ray Polarity Explorer (IXPE; Soffitta et al. 2021) and the Chinese/European Enhanced X-ray Timing and Polarization mission (eXTP; Zhang et al. 2016) (and a few other similar experiments). The first results from IXPE have been recently reported (Krawczynski et al. 2022). We are therefore motivated to find out what information about electron acceleration in accretion flows or jets can be carried by polarization of light, with a particular focus on the inverse-Compton scattered light.

Polarization of X-ray emission (or more generally, higher-energy emission) produced by plasma in strong gravity depends on whether the high-energy emission is of synchrotron origin (direct emission) or arises in the inverse-Compton process (scattered emission). In the latter case the polarization of scattered light may be due to the transfer of polarization of synchrotron emission in the inverse-Compton process, or it may be due to the scattering process itself (Chandrasekhar 1960). Hence the polarization of scattered emission depends on many factors: on the magnetic field configuration in the plasma (which impacts the polarization of synchrotron radiation), on the energy distribution of synchrotron emitting plasma electrons, on Faraday effects, on the opacity of the plasma for scattering, or whether the scattering in the electron frame occurs in the Thomson (TH) or Klein–Nishina (KN) regimes. In addition, photon emission and propagation depends on spacetime curvature and on the overall geometry and dynamics of the accretion flow. The complexity of the theoretical predictions for polarimetric properties of high-energy radiation is large (for complete overview see Krawczynski 2012).

To enable theoretical studies of polarimetric properties of emission from complex systems, we developed radpol, a covariant Monte Carlo scheme for calculating multiwavelength polarized spectral energy distributions (SEDs) of three-dimensional general relativistic magnetohydrodynamic (3D GRMHD) simulations of black hole accretion (Mościbrodzka 2020). The code samples a large number of polarized synchrotron photons, propagates them in curved spacetime, simulates their inverse-Compton scatterings, and collects information about the outgoing spectrum in a spherically shaped detector at a large distance from the center of the model grid. In our modeling we include synchrotron emission, synchrotron self-absorption in all Stokes parameters and Faraday effects, and the inverse-Compton process, and take into account all effects that are important in relativistic plasma in strong gravitational fields of, e.g., black holes. Our method is unique because it is fully covariant which enables spectra calculations assuming an arbitrary metric tensor.

Our numerical code, until now, assumed that electrons in plasma have a thermal distribution function. In this work we overcome this major oversimplification. Here we present a new scattering kernel for the radpol code to permit emission and polarization from plasma in which electrons are accelerating. Our model for scattering is completely covariant and allows us to build more realistic models of emission from relativistic jets.

The structure of the paper is as follows. In Section 2 we write basic equations that describe inverse-Compton scattering of polarized and unpolarized photons off an electron at rest. We then show how scattering is computed for an ensemble of electrons with four energy distribution functions. We show that our numerical method recovers some well-known theoretical expectations. In Section 3 we present examples of scattering in

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1 The radpol code is an extension of grmonty, which originally assumed unpolarized emission and emission and scattering off a thermal population of electrons (Dolence et al. 2009). Notice that most of the polarization-insensitive algorithms in radpol are inherited from grmonty.

2 To integrate radiative transfer equations radpol uses the numerical scheme of another code, ipeole, a ray-tracing scheme for making polarimetric images of black holes, developed by Mościbrodzka & Gammie (2018).
Minkowski spacetime that can be used to understand results from more complex simulations. Section 4 lists other code developments carried out to calculate polarized nonthermal spectra of complex accretion models. We conclude in Section 5.

2. Inverse-Compton Scattering Model for Accelerated Electrons

2.1. radpol Scattering Kernel Description and Upgrades

We begin with improving the original radpol polarization-sensitive inverse-Compton scattering kernel by converting it from an average-intensity-conserving one (originally implemented in radpol) into a photon-conserving one (Schnittman & Krolik 2013). The latter makes the scheme more robust and enables us to include scattering off accelerated electrons with greater precision.

We first reconsider the inverse-Compton scattering of a polarized photon beam in the rest frame of an electron. The differential cross section for the Compton scattering of polarized photons on free electrons is given by the general KN formula (Berestetskii et al. 1982):

\[
\frac{d\sigma^{KN}}{d\Omega} = \frac{1}{4 \pi} \left( \frac{e'_e}{e_e} \right)^2 \left[ F_{00} + F_{11} \xi_1 \xi'_1 + F_1 (\xi_1 + \xi'_1) \right. \\
+ F_{22}(\xi_2 \xi'_2 + F_{33} \xi_3 \xi'_3)].
\]

(1)

where \( r_e = \frac{e^2}{4 \pi \epsilon_0 m_e c^2} \) is the electron classical radius, \( e_e \) and \( e'_e \) are the incident and scattered energy of photons in units of \( m_e c^2 \), \( \xi_{1,2,3} \) are polarization of scattered photons, which are defined as follows: \( \xi_1 \equiv Q/I \), \( \xi_2 \equiv \ell I \), and \( \xi_3 \equiv V/I \). In Equation (1), Stokes \( Q \) and \( I \) (or their fractions \( \xi_{1,2} \)) are measured with respect to the tetrad defined by \( k \) and the scattering plane, i.e., the plane normal to \( k \times k' \) where \( k \) (\( k' \)) is an incident (scattered) photon four-vector in the rest frame of an electron. The coefficients \( F \) are elements of the following scattering matrix (Fano 1949, 1957):

\[
F = \frac{1}{2} \left( \frac{e'_e}{e_e} \right)^2 F_{00} F_{11} F_{22} F_{33} \left( \begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array} \right) + \frac{1}{2} \left( \frac{e'_e}{e_e} \right)^2 
\left( \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0
\end{array} \right)
\]

\[
\times \left( \begin{array}{cccc}
\frac{e'_e}{e_e} + \frac{e_e}{e'_e} - \sin^2 \theta' \cos^2 \theta' & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array} \right) \cos \theta'
\]

(2)

where \( \theta' \) is the polar scattering angle. In the TH regime \( (e'_e = e_e) \), \( F \) becomes the phase matrix for Rayleigh scattering of Stokes parameters (Chandrasekhar 1960).

Equation (1) summed over all possible polarizations of the scattered photon \( (\xi_{123}') \) gives the scattering cross section as a function of the incident light linear polarization:

\[
\frac{d\sigma^{KN}(\xi_{123})}{d\Omega} = \frac{1}{2} \left( \frac{e'_e}{e_e} \right)^2 \left( \frac{e_e + e'_e}{e_e} - (1 - \xi_1)\sin^2 \theta' \right).
\]

(3)

Since \( \xi_1 \) is defined with respect to scattering plane one can rewrite Equation (3) into:

\[
\frac{d\sigma^{KN}(\xi_{123})}{d\Omega} = \frac{1}{2} \left( \frac{e'_e}{e_e} \right)^2 \left( \frac{e_e + e'_e}{e_e} - \sin^2 \theta' - \delta \sin^2 \theta' \cos(2\phi') \right).
\]

(4)

where \( \delta = Q/I = -\delta \cos(2\phi') \) and where \( \phi' \) is the azimuthal scattering angle. The fractional linear polarization of incident light \( \delta = \sqrt{Q^2 + \ell^2} / I \) is invariant to rotations and the azimuthal scattering angle \( \phi' \) is measured with respect to the \( x \)-axis, which is chosen arbitrarily.

Sampling of the \( \theta' \) scattering angle and \( e'_e \) is carried out using an azimuthal angle integrated differential cross section and kinematic relation for scattering energy and a \( \theta' \) angle \( (\cos \theta' = 1 + 1/e_e - 1/e'_e) \). This step is polarization independent.

For unpolarized light the \( \phi' \in (0, 2\pi) \) angle can be randomly chosen from a uniform distribution function, however, if the incident light is polarized, \( \phi' \) cannot be random. The \( \phi' \) angle is sampled from the conditional probability distribution function (see Zhang et al. 2019):

\[
p(\phi'|e'_e) = \frac{1}{2\pi} - \frac{\delta \sin^2 \theta' \cos 2\phi'}{2\pi (\frac{e_e}{e'_e} + \frac{e'_e}{e_e} - \sin^2 \theta')}.\]

(5)

The \( \phi' \) sampling is carried out via inversion of the cumulative distribution function of the equation above which is:

\[
\text{CDF}(\phi') = \frac{\phi'}{2\pi} - \frac{\delta \sin^2 \theta' \sin 2\phi'}{4\pi (\frac{e_e}{e'_e} + \frac{e'_e}{e_e} - \sin^2 \theta')}.
\]

(6)

In the limit of \( \delta = 0 \) or in the limit of \( \cos (\theta') = \pm 1 \) the formula reduces to sampling the \( \phi' \) angle from the uniform distribution.

Given two scattering angles one can construct \( k' \) and define the scattering plane. The fractional Stokes parameters of the scattered photon, \( \xi'_{123} \), can be finally computed using:

\[
\xi'_1 = F_1 + \xi_1 F_{11} \xi'_1 = \frac{F_{22} \xi_2 \xi'_2 + F_{33} \xi_3 \xi'_3}{F_{00} + \xi_1 F_{11}}, \quad \xi'_2 = \xi_2 F_{22} \xi'_2 = \frac{F_{00} \xi_1 F_{11}}{F_{00} + \xi_1 F_{11}}, \quad \xi'_3 = \frac{F_{00} \xi_3 F_{33}}{F_{00} + \xi_1 F_{11}}.
\]

(7)

where \( \xi_{1,2,3} \) are measured with respect to the scattering plane. The scattering kernel defined this way is photon conserving so Stokes \( I \) does not have to be changed in the scattering event. In the originally published version of radpol, we sampled the \( \phi' \) angle from a uniform distribution function so transformation of polarization included transformation of all Stokes parameters, including Stokes \( I \), using Equation (2). Hence, the original

\footnote{The minus sign appears because of the conventions used in this paper and in our numerical code: for fully polarized light, \( \delta = 1 \), EVA = 0 means \( Q = +1 \) and \( \phi' = 90^\circ \) measured from the \( x \)-axis, EVA = 90° corresponds with \( Q = -1 \) and \( \phi' = 0 \) or 180°.}
scheme was not photon conserving but only averaged-intensity conserving (Schnittman & Krolik 2013).

We have tested the new implementation of the Compton scattering in the electron rest frame. If we reconsider scattering of photons in the electron rest frame, the scattering angle \( \phi' \) depends on the polarization degree and angle of the incident light. For fully polarized light, i.e., \( \delta = 1 \), the scattering of polarized light is favored in the direction perpendicular to the polarization angle. In Figure 1 we show that the outcome of our numerical calculations is consistent with these theoretical expectations (numerical calculations is presented by Özel et al. 2000 and Yuan et al. 2003). The energy density of the thermal electrons is

\[
u_{\text{th}} = n_{\text{th}} \Theta_e a(\Theta_e) m_e c^2\tag{11}\]

where \( a(\Theta_e) \approx (6 + 15\Theta_e)/(4 + 5\Theta_e) \) (Gammie & Popham 1998). While the energy density of the accelerated electrons is

\[
u_{\text{pl}} = n_{\text{pl}} \frac{p - 1}{p - 2} \gamma_{\text{min}} m_e c^2,\tag{12}\]

where the simple form of \( \nu_{\text{pl}} \) is due to normalization of the power-law function. We assume that \( n_{\text{pl}} = \eta n_{\text{th}} \) where \( \eta \) is a fourth free parameter of the acceleration model indicating the fraction of thermal energy transferred to the nonthermal tail. Using Equations (11) and (12) we calculate the resulting number density of accelerated electrons, \( n_{\text{pl}} \):

\[
\eta_{\text{pl}} = \eta \frac{p - 2}{p - 1} \gamma_{\text{min}} m_e c^2 \tag{13}\]

In this model the power-law eDF should smoothly connect with the thermal distribution so we require that:

\[
\nu_{\text{th}}(\gamma_{\text{min}}) = n_{\text{pl}}(\gamma_{\text{min}}). \tag{14}\]

For a set of \( \eta \) and \( \Theta_e \), we solve

\[
\gamma_{\text{min}}^4 \beta_{\text{min}} \exp(-\gamma_{\text{min}}/\Theta_e) = 2(p - 2) \eta a(\Theta_e) \Theta_e^3 \tag{15}\]

to find the \( \gamma_{\text{min}}^4 \).

4. \( \kappa \) eDF (Vasyliunas 1968; Xiao 2006; Pierrard & Lazar 2010) is a more natural eDF inspired by kinetic studies of relativistic plasmas.

\[
\frac{1}{n_{\text{pl}}} \frac{d n_{\text{pl}}}{d \gamma} = \gamma^2 - \frac{1}{1 + \frac{\gamma + 1}{\kappa w}}^{(\kappa + 1)} \tag{16}\]

where \( \kappa \) and \( w \) are parameters. For \( \kappa \to \infty \), the \( \kappa \) distribution function becomes a Maxwell–Jüttner distribution.

2.2. Electron Acceleration Models

Next we consider scattering off a population of electrons. We assume the following electron energy distribution functions (eDFs) that are usually considered for astrophysical applications.

1. Relativistic thermal eDF (Petrosian 1981; Leung et al. 2011):

\[
\frac{1}{n_e} \frac{d n_e}{d \gamma} = \frac{\gamma^2 \beta}{\Theta_e K_2(1/\Theta_e)} \exp(-\gamma/\Theta_e) \tag{8}\]

where \( \beta = \sqrt{1 - 1/\gamma^2} \) and \( \Theta_e = k_\text{B} T_e/m_e c^2 \) is the dimensionless electron temperature.

2. Purely power-law eDF (Rybicki & Lightman 1979):

\[
\frac{1}{n_e} \frac{d n_e}{d \gamma} = \frac{(p - 1)}{(\gamma_{\text{min}}^1 - \gamma_{\text{max}}^1)} \gamma^{-p} \tag{9}\]

where \( p, \eta, \gamma_{\text{min}}^1 \) and \( \gamma_{\text{max}}^1 \) are parameters.

3. Hybrid eDF where we assume that the electrons are accelerated from a thermal eDF. The accelerated electrons' energies are described by a power-law distribution:

\[
\frac{1}{n_{\text{pl}}} \frac{d n_{\text{pl}}}{d \gamma} = \frac{(p - 1)}{(\gamma_{\text{min}}^1 - \gamma_{\text{min}}^2)} \gamma^{-p}, \tag{10}\]

where \( \gamma_{\text{min}}^1, \gamma_{\text{max}}^1, \) and \( p \) are parameters of the acceleration model (we will assume that \( \gamma_{\text{max}}^1 \approx 1 \), in practice we assume \( \gamma_{\text{max}}^1 = 10^5 \). The power-law function is “stitched” to the thermal eDF as follows (the same methodology is presented by Özel et al. 2000 and Yuan et al. 2003).
the electron four-momentum $p^\mu$ are isotropic in the fluid comoving frame. An isotropic eDF model limits the discussion to energy sampling.

To sample the electron Lorentz factor $\gamma_e$ in the thermal distribution function we use the sampling procedure introduced by Canfield et al. (1987) (implemented in the grmonty and radpol codes).

In the case of a pure power-law distribution function the electron Lorentz factor is sampled using an inversion of a cumulative distribution function where the inversion has an analytic form:

$$\gamma_e = (\gamma_{\text{min}} - r) + \gamma_{\text{max}} r^{1/(1-p)}$$

(17)

where $r \in (0, 1)$ is a random number and $\gamma_{\text{min}}$, $\gamma_{\text{max}}$, $p$ are the eDF parameters.

To sample the Lorentz factor from hybrid and $\kappa$ distribution functions we rewrite these two eDFs as a product of two probability functions $p_1$ and $p_2$, where $p_1$ is used for tentative sampling and $p_2$ is used for rejection sampling (the procedure closely follows Canfield et al. 1987 but differs in the details of tentative sampling). For both hybrid and $\kappa$DF:

$$p_1 = \frac{1}{n_e} \frac{dn_e(\gamma_e)}{d\gamma_e}$$

(18)

and

$$p_2 = \beta_e$$

(19)

where $\beta_e = \sqrt{1 - 1/\gamma_e^2}$.

In our model the tentative sampling of $\gamma_e$ from $p_1$ is carried out by inversion of the cumulative distribution function. We found analytic forms of the cumulative distribution function of $p_1$ (hereafter modified cumulative distribution function, MCDF) for hybrid and $\kappa$ eDFs. For the hybrid distribution function it is:

$$\text{MCDF}_{\text{hybrid}}(\gamma_e) = 1 - \frac{\exp\left(-\frac{\gamma_e}{\epsilon_0}\right)}{\exp\left(-\frac{1}{\epsilon_0}\right)} \left(2\Theta_e + 2\Theta_{\gamma_e} + \gamma_e^2\right) (1 - f) +$$

$$\begin{cases} 0 & \text{for } \gamma_e < \gamma_{\text{min}} \\ \frac{(p-1)}{(\gamma_{\text{min}} - \gamma_{\text{max}})}(g_{\text{pl}}(\gamma_{\text{min}}, p) - g_{\text{pl}}(\gamma_e, p)) & \text{for } \gamma_e > \gamma_{\text{min}} \end{cases}$$

(20)

where the third term is added only for $\gamma_e > \gamma_{\text{min}}$, where $f = n_{pl}/n_{th}$ (given by Equation (13)) and where

$$g_{\text{pl}}(\gamma_e) = \begin{cases} \sqrt{\gamma_e - 1} + \frac{1}{\gamma_e} & \text{for } p = 3 \\ \sqrt{\gamma_e - 1} + \frac{1}{2\gamma_e} - \frac{1}{2} \arcsin\left(\frac{1}{\gamma_e}\right) & \text{for } p = 4 \\ \sqrt{\gamma_e - 1} + \frac{2}{3\gamma_e} + \frac{1}{3\gamma_e^2} & \text{for } p = 5 \\ \sqrt{\gamma_e - 1} + \frac{3}{8\gamma_e} + \frac{1}{4\gamma_e^2} - \frac{3}{8} \arcsin\left(\frac{1}{\gamma_e}\right) & \text{for } p = 6. \end{cases}$$

(22)

For $\kappa$ eDF the $p_1$ cumulative distribution function for sampling $\gamma_e$ is:

$$\text{MCDF}_\kappa(\gamma_e) = f_{\kappa,1} (f_{\kappa,1} e^{(\kappa - 1)(\gamma_e + \epsilon + 1)}) + f_{\kappa,2} e^{(\kappa - 1)(\gamma_e + \epsilon + 1)})/(\kappa^2 - 3\kappa + 2)$$

(23)

$$e^{(-\kappa \log(\gamma_e + \epsilon + 1) - (\kappa \log(\epsilon + \gamma)) + 2)$$

(24)

where

$$f_{\kappa,1} = w^{\kappa + 1}(2\kappa^2 + w^2 + (2\kappa^2 + 2 + 4\kappa^{\epsilon + 1})w$$

$$+(\kappa^2 + 2)\kappa^{\epsilon + 1} + 2\kappa^\epsilon),$$

$$f_{\kappa,2} = \kappa^2(w^{\epsilon + 1})^2$$

$$+ w^\epsilon (-2\kappa^{\epsilon + 1} w - 2\kappa^{\epsilon + 1}w)\gamma_e^2 +$$

$$w^\epsilon (-2\kappa^{\epsilon + 1} w^2 - 4\kappa^{\epsilon + 1}w^2 - 2\kappa^{\epsilon + 2}w^2),$$

(25)

and the distribution normalizing factor $f_{\kappa,n}$ is given in Pandya et al. (2016) (see their Equation (19)). For fast and accurate numerical MCDF inversion we use the regula falsi root finder (Ford 1995). Since $\beta$ is close to one for relativistic electrons the rejection sampling is efficient.

2.4. Test of the Numerical Scheme against the Analytic Model

To test the numerical code we consider the single scattering of a beam of monochromatic polarized photons off an ensemble of electrons with four eDFs introduced in the previous subsections. Bonometto et al. (1970) provided a semianalytic solution to this problem as long as electron-frame scattering is in the TH limit ($\epsilon' = \epsilon$). The analytic model has already been briefly described in Appendix A of our previous work (Mościbrodzka 2020) and recently also reproduced in more detail by Xiao-lin et al. (2021).

Our numerical model can be confronted with the theoretical expectation for light intensity and polarization with the predictions of Bonometto et al. (1970). In Figure 2 we show the agreement between the theoretical prediction and our numerical kernel calculations using our new updated scattering kernel using thermal, power-law, hybrid, and $\kappa$ electron distribution functions for a single scattering angle. The Monte Carlo simulations with the radpol scattering kernel converge to the predicted values. Our results are also consistent with results presented in Xiao-lin et al. (2021; see their Figure 25) who carried out the same tests using an independent numerical scheme. In all cases the fractional linear polarization is increasing with frequency. In particular, for an eDF with a power-law component (power-law, hybrid, and $\kappa$ eDFs) the fractional linear polarization converges to a constant value at high energies ($\epsilon' \gg \epsilon$) analogous to the fractional linear polarization of the optically thin synchrotron emission (which can be also thought of as a scattering process) from electrons distributed into a power-law eDF.

3. Scattering of Low- and High-energy Thermal and Nonthermal Electrons

Next, we simulate a single inverse-Compton scattering of a monochromatic beam as a function of the incident light
polarization, eDF, and scattering regime (TH and KN). It is expected that scattering of an unpolarized photon beam of hot relativistic plasma should produce no polarization (e.g., Poutanen & Vilhu 1993); here we can test our code against this expectation. Otherwise, the results presented in this section can be used as a guiding line for analysis of more complex models (e.g., radiation produced in accretion disks and jets in GRMHD simulations), keeping in mind that in realistic accretion flows and jets scattering may be multiple. Notice that here we neglect circular polarization of the incident beam because the circular polarization cannot be generated in the scattering process.

In Figure 3 we show intensity (upper panels) and fractional polarization (lower panels) spectra of scattered light when the scattering occurs in the TH regime (i.e., the energy of the incident beam is low compared to the electron rest-mass...
energy, $\epsilon = 2.5 \times 10^{-11}$). Panels from left to right display results for scattering on subrelativistic (characterized by the dimensionless temperature $\Theta_e = 0.1$) and relativistic electrons distributed into thermal (with $\Theta_e = 100$) and $\kappa = 4.5$ eDF. Initially unpolarized light ($S_{in} = (1, 0, 0, 0)$) scattering off an ensemble of subrelativistic electrons becomes polarized and the degree of polarization depends on the geometry (angle) of scattering and on the scattered photons' frequency. Initially polarized light ($S_{in} = (1, 1, 0, 0)$) scattering off cold electrons will stay polarized only for certain scattering angles. Scattering an unpolarized beam off hot (relativistic) electrons does not produce polarization, as expected. (The residual polarization seen in the high energies in this case is a Monte Carlo noise.) The latter is valid for thermal and nonthermal electron distribution functions. For an initially polarized beam scattering off relativistic electrons, the scattered radiation is partially polarized with fractional polarization increasing with frequency from 0% to 100%. Only for a very specific scattering angle ($\theta' = 90^\circ$, $\phi' = 90^\circ$) does the polarization cancel out to zero.

In Figure 4 we display results of the same numerical tests as shown in Figure 3 but with scattering in the KN regime (i.e., the energy of the incident beam is comparable to the electron rest-mass energy, $\epsilon = 1$). Scattering off cold electrons produces a variety of polarizations, which depend on the scattering direction, similar to results for scattering in the TH regime. For KN scattering off relativistic electrons, the initially unpolarized light will not gain any polarization independently of the eDF, consistent with results in the TH regime. However, for incident polarized light the polarization of the scattered light sharply decreases with frequency regardless of the scattering angle, which is the opposite trend compared to the TH scattering. It is noteworthy, as evident in both Figures 3 and 4, that the total intensity of the scattered light only slightly depends on the incident light polarization.

4. Polarimetric Properties of Scattered Light in Complex Models of Accretion

Our upgraded scattering kernel in the radpol code is now well tested and produces results consistent with theoretical expectations for variety of electron distribution functions. Simulating polarized emission and scattering off nonthermal electrons in complex models of accretion (for example in GRMHD simulations of accreting black holes) requires modifications of the photon sampling routines as well as scattering cross sections. Manufacturing photons in radpol is carried out just like in its unpolarized version grmonty (see the method paper by Dolence et al. 2009) with a difference that now all angle-averaged synchrotron emissivities incorporate thermal and nonthermal eDFs. Once a photon wavevector, $k'$, is build in the fluid frame, the photon polarization is assigned to it using corresponding thermal/nonthermal synchrotron emissivities. Finally, to determine the place of scattering along a ray path in the radpol simulation,
an optical depth for scattering is calculated in each step on the geodesic path. The so-called “hot cross section” is calculated to estimate the cross section for a photon interaction with an ensemble of free electrons. This requires integrating the KN (or TH) cross section over the assumed electron distribution function that can now also be nonthermal. In radpol such integrations are done numerically and tabulated. Full exploration of the polarization of high-energy emission produced in complex models of accretion flows with electron acceleration is beyond the scope of this work and will be presented in a forthcoming publication.

5. Conclusion

In Mościbrodzka (2020) we introduced a Monte Carlo code radpol, which is capable of tracing light polarization of synchrotron emission and polarization-sensitive inverse-Compton scattering processes in full general relativity. In the current work we describe a major extension of the code to compute emission and scattering when electrons are nonthermal. The numerical scheme tests converge to the theoretical expectations. Updated code enables more realistic fully relativistic and covariant models of emission for jets produced by accreting objects of any kind.

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ORCID iDs
Monika A. Mościbrodzka © https://orcid.org/0000-0002-4661-6332

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