Wave propagation in a non-homogeneous magneto-thermo-electro-elastic disc of polygonal cross sections immersed in an in viscid fluid

R Selvamani ¹ and G Infant Sujitha²
¹Department of Mathematics, Karunya University, Coimbatore-641114
²Department of Science and Humanities, Sri Krishna College of Engineering and Technology, Coimbatore-641008
E. Mail: selvamani@karunya.edu

Abstract: The study of non-homogeneous wave propagation in a magneto-thermo-electro-elastic polygonal disc immersed in fluid is carried out in this paper using linear theory of elasticity along with Fourier expansion collocation method. The considered material is non-homogeneous transversely isotropic material and the wave equation is applied under the assumption of disc immersed in an in viscid fluid. The analytical results have been obtained for triangle, square, pentagon and hexagonal cross-sections of disc. The stress, strain, mechanical displacement, electrical displacement and temperature distribution have been represented graphically with varying non-homogeneous parameter.

1. Introduction
The thermo electro elastic disc is used in generator which is extracting the waste heat and converting into useable electricity. The application of this is wide in many areas such as intelligent sensors, damage detectors and the machines that coupling the mechanical loads. The non-homogenous wave propagation has a vital role in the generator. The disc guides to design and optimize the new material in the production of composite materials. The uses of this type of discs plays vital role in the production of piezoelectric and piezomagnetic materials. In recent days, the research on the wave propagation of the magneto-thermo-electro-elastic disc is more devoted. The vibration analysis has been carried out for homogeneous transversely isotropic thermo elastic cylindrical panels and cylinders [1-4]. The [5-14] give the discussion on the functionally graded materials and the materials in contact with fluid. The [15-24] analysis the wave propagation on the materials with the rotation and the magnetic thermo- electro- elastic materials. In this paper, we have formulated the problem of the non-homogeneous wave propagation in a magneto-thermo-electro-elastic disc of polygonal cross sections immersed in fluid. Using the double Fourier series and the collocation method the boundary conditions and frequency equations are derived. The analytical results have been obtained for triangle, square, pentagon and hexagonal cross-sections of disc. The stress, strain, mechanical displacement, electrical displacement and temperature distribution have been represented graphically with varying non-homogeneous parameter.
2. Formulation of the model

The considered non homogeneous magneto-thermo-electro-elastic polygonal disc is having the stress and displacement and it is defined by the cylindrical coordinates in the absence of body force for a linearly elastic medium from Sharma and Sharma [1] as follows

\[ \sigma_{r,r} + r^{-1}\sigma_{r,\theta} + r^{-1}(\sigma_{r,\theta} - \sigma_{\theta,\theta}) = \rho u_{r,r} \tag{1} \]
\[ \sigma_{r,\theta} + r^{-1}\sigma_{\theta,\theta} + 2r^{-1}\sigma_{\theta,\theta} = \rho u_{\theta,\theta} \tag{2} \]

where \( \sigma_{rr}, \sigma_{r\theta}, \sigma_{\theta\theta} \) are the stress components and \( \rho \) is the mass density of the material. The above system is defined by the polar coordinates \( r \) and \( \theta \). \( u_{r}, u_{\theta} \) are the displacements in the direction of \( r \) and \( \theta \) respectively. The equation of thermal conductivity is as follows

\[ k(T_{r,r} + r^{-1}T_{r,\theta} + r^{-2}T_{\theta,\theta}) - \rho c_{v}(T + t_{0}) = T_{0}\left( \frac{\partial}{\partial t} + \delta_{i,j}t_{0}\frac{\partial^{2}}{\partial t^{2}} \right) \left[ \beta(u_{r,r} + r^{-1}(u_{\theta,\theta} + u_{r})) \right] \tag{3} \]

The electric conduction equation is given by,

\[ D_{r,r} + r^{-1}D_{r,\theta} + r^{-1}D_{\theta,\theta} = 0 \tag{4} \]

The magnetic conduction equation is given by,

\[ B_{r,r} + r^{-1}B_{r,\theta} + r^{-1}B_{\theta,\theta} = 0 \tag{5} \]

where \( K \) is the thermal conductivity, \( T \) is the temperature change, \( \beta \) is the thermal capacity factor which couples the heat conduction elastic field equations, \( t_{0}, t_{1} \) are the two thermal relaxation times, \( t \) is the time, \( \delta_{ij} \) is the Kronecker delta, \( D_{r}, D_{\theta} \) are electric displacements and the magnetic components are \( H_{r}, H_{\theta} \).

The stress-strain relation is given by

\[ \sigma_{rr} = \lambda(e_{rr} + e_{\theta\theta}) + 2\mu e_{rr} - \beta(T + t_{0}) \tag{6} \]
\[ \sigma_{r\theta} = \lambda(e_{rr} + e_{\theta\theta}) + 2\mu e_{\theta\theta} - \beta(T + t_{0})(\delta_{r\theta}) \tag{7} \]
\[ \sigma_{\theta\theta} = 2\mu e_{\theta\theta} \tag{8} \]
\[ D_{r} = e_{11}E_{r} + m_{11}H_{r} \tag{9} \]
\[ D_{\theta} = e_{11}E_{\theta} + m_{11}H_{\theta} \tag{10} \]

and

\[ B_{r} = m_{11}E_{r} + \mu_{11}H_{r} \tag{11} \]
\[ B_{\theta} = m_{11}E_{\theta} + \mu_{11}H_{\theta} \tag{12} \]

where \( \lambda, \mu \) are Lame’s constants, \( e_{11} \) is the dielectric constant and \( m_{11}, \mu_{11} \) are the electro-magneto material coefficients and magnetic displacements respectively. \( E_{r}, E_{\theta} \) are the electric potentials, \( H_{r}, H_{\theta} \) are the magnetic potentials. The strain \( e_{ij} \) can be represented by,

\[ e_{rr} = u_{rr} \tag{13} \]
\[ e_{\theta\theta} = r^{-1}(u_{r} + u_{\theta,\theta}) \tag{14} \]
\[ e_{r\theta} = u_{r,\theta} - r^{-1}(u_{\theta} - u_{r,\theta}) \tag{15} \]

Using the Lord-Shulman thermo-elastic model by taking \( k=1 \) in the thermal equation and then the stress equation of motion is given by,

\[ \sigma_{rr} = \lambda(e_{rr} + e_{\theta\theta}) + 2\mu e_{rr} - \beta T \tag{16} \]
\[ \sigma_{r\theta} = \lambda(e_{rr} + e_{\theta\theta}) + 2\mu e_{\theta\theta} - \beta T \tag{17} \]
\[ \sigma_{\theta\theta} = 2\mu e_{\theta\theta} \tag{18} \]
also here the magnetic and electric components are given by,
\[ E_r = -E_r, \quad E_\theta = r^{-1} E_\theta \]  (19)
\[ H_r = -H_r, \quad H_\theta = r^{-1} H_\theta \]  (20)
We obtain the stress displacement equations for non-homogeneous medium by considering
\[ \rho = \lambda, r^{2n}, \mu = \mu, r^{2m}, m_{11} = m_{11}, r^{2m}, e_{11} = e_{11}, r^{2m}, \rho = \rho, r^{2m} \]
And also substituting Equations (13)-(15) and (19)-(20) in equations (16)-(18) and (9)-(12) respectively.
\[ \left( \lambda + 2 \mu \right) \left[ u_{r,rr} + r^{-1} u_{r,r} - r^{-2} u_r \right] + \mu r^{-2} u_{r,\theta \theta} + r^{-1} \left( \lambda + \mu \right) u_{\theta,\theta} \\
+ r^{-2} \left( \lambda + 3 \mu \right) u_{\theta,\theta} + 2mr^{-1} \left( \lambda + 2 \mu \right) u_r + \lambda \left( r^{-1} u_{r,\theta} + r^{-1} u_\theta \right) \right) - \beta T_r = \rho u_{r,r} \]  (21)
\[ \mu \left( u_{r,rr} + r^{-1} u_{r,r} - r^{-2} u_r \right) + r^{-2} \left( \lambda + 2 \mu \right) u_{\theta,\theta} + \lambda \left( r^{-1} u_{r,\theta} + r^{-1} u_\theta \right) \right) - \beta T_\theta = \rho u_{\theta,\theta} \]  (22)
\[ k(T_{rr} + r^{-1} T_r + r^{-2} T_{r,00} - \rho \varepsilon_c (T_{r} + t_{r} \dot{T}_{r}) = T_0 \left( \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} \right) \left[ \beta \left( u_{r,r} + r^{-1} u_{\theta,\theta} + u_r \right) \right] \]  (23)
\[ e_{11} \left( E_{r,rr} + r^{-1} E_r + r^{-2} E_{r,00} \right) + m_{11} \left( H_{r,rr} + r^{-1} H_r + r^{-2} H_{r,00} \right) \]  (24)
\[ + 2mr^{-1} \left( m_{11} E_r + m_{11} H_r \right) = 0 \]  (25)
3. Solution of disc problem with fluid
The equations (21)-(25) are coupled partial differential equations with the displacement and magnetic, electric and thermal components. We are seeking the solution in the form by Mirsky [2] to uncouple the solution
\[ u_r \left( r, \theta, t \right) = \sum_{n=0}^{\infty} e_n \left[ \phi_n, r + r^{-1} \Sigma_n, \theta \right] + \left( \phi_n, r + r^{-1} \Sigma_n, \theta \right) \]  (26)
\[ u_\theta \left( r, \theta, t \right) = \sum_{n=0}^{\infty} e_n \left[ r^{-1}, \phi_n, \theta - \phi_n, \theta \right] + \left( r^{-1}, \phi_n, \theta - \phi_n, \theta \right) \]  (27)
\[ T \left( r, \theta, t \right) = \left( \lambda + 2 \mu / \rho \alpha^2 \right) \sum_{n=0}^{\infty} e_n \left( T_n + \bar{T}_n \right) e^{i\omega t} \]  (28)
\[ E \left( r, \theta, t \right) = \sum_{n=0}^{\infty} e_n \left( E_n + \bar{E}_n \right) \]  (29)
\[ H \left( r, \theta, t \right) = \sum_{n=0}^{\infty} e_n \left( H_n + \bar{H}_n \right) \]  (30)
where \( e_n = 1/2 \) for \( n = 0 \) and \( e_n = 1 \) for \( n \geq 1 \). To attain the desired solution following dimensionless quantities are introduced and get the solution in the following form
\[ x = \frac{r}{a}, \tau_0 = (1 + t_c \omega), \bar{\Omega} = K \sqrt{\rho / \mu / \tau_0 \beta^2 a T_0}, T_a = t \sqrt{\mu / \rho / a}, \Omega^2 = \omega^2 a^2 / c_1 \]
\[ c_i^2 = (\lambda + 2 \mu) / \rho, c_i^2 \] is the phase velocity
\[
(\lambda + 2\mu)\nabla^2 \phi_n + mr^{-1}(2\lambda_1 + \mu_1)\phi_{n,r} - 2mr^2\lambda_1\phi_{n} - \rho_t \phi_{n,z} - \beta T_{n} = 0
\]

\[
e_{11} \nabla^2 E_n + m_{11} \nabla^2 H_n + 2mr^{-1}(e_{11} E_{n,r} + m_{11} H_{n,r}) = 0
\]

\[
m_{11} \nabla^2 E_n + \mu_1 r^{-2m} \nabla^2 H_n + 2mr^{-1}(m_{11} E_{n,r} + \mu_1 r^{-2m} H_{n,r}) = 0
\]

\[
\mu_1 \nabla^2 \Psi_n + 2\mu_1 m(r^{-1} \Psi_{n,r} - r^{-1} \Psi_n) = 0
\]

\[
\nabla^2 \phi_n + \left(K \nabla^2 + d\right) \Psi_n = 0
\]

The solution of the problem is considered as follows,

\[
\phi_n(r, \theta, t) = r^{-m} \phi_0(n) \cos n\theta
\]

\[
E_n(r, \theta, t) = r^{-m} E_0(n) \cos n\theta
\]

\[
H_n(r, \theta, t) = r^{-m} H_0(n) \cos n\theta
\]

\[
\Psi_n(r, \theta, t) = r^{-m} \Psi_0(n) \cos n\theta
\]

\[
T_n(r, \theta, t) = r^{-m} T_0(n) \cos n\theta
\]

Substituting (36) in (31)-(35) we get the set of solution

\[
\phi_0(n) = A_{\theta} J_1(a \theta x) + B_{\theta} Y_1(a \theta x)
\]

\[
E_0(n) = \left[ A_{\theta} r^{-p} + B_{\theta} r^{-p} \right] \cos n\theta
\]

\[
H_0(n) = \left[ A_{\theta} r^{-p} + B_{\theta} r^{-p} \right] \cos n\theta
\]

\[
\Psi_0(n) = \left[ A_{\theta} J_1(a \theta x) + B_{\theta} Y_1(a \theta x) \right] \cos n\theta
\]

\[
T_0(n) = \left[ A_{\theta} J_1(a \theta x) + B_{\theta} Y_1(a \theta x) \right] \cos n\theta
\]

The above solution has been obtained for the symmetric mode and attaining the solution for anti-symmetric mode is just replacing the cosine function by the sine function. Since the disc is immersed in the fluid the acoustic pressure affects the material and the radial displacement equation of motion for this effect are of the form Achenbach [20],

\[
\rho \Delta = -B' \left[ a' + r^{-1} (a' + b' \phi) \right]
\]

And the acoustic phase velocity of the fluid is,

\[
c^2 a' = \Delta
\]

Where \( \left(a', b'\right)\) is the displacement vector, the acoustic phase velocity \( c \) is given by \( c = \sqrt{B' / \rho_0} \).

\( B' \) is the adiabatic bulk modulus and \( \rho_0 \) is the fluid density and

\[
\Delta = \left(a' + r^{-1} (a' + b' \phi) \right)
\]

Using the assumption that \( a' = \phi_{/r} \) and \( b' = \phi_{/r} r \)

And driving towards the solution of (39) in the following form

\[
\phi' = \sum_{n=0}^{\infty} E_n \left[ \phi_n \cos n\theta + \phi_n \sin n\theta \right] \sin t
\]

Where,
The solution for the inner fluid consists
\[ f_{\alpha}(\alpha, \theta) = \frac{\rho_0}{\rho} \] in which \( \rho_0 = \rho \) and \( f_{\alpha} \) is the Bessel function of the first kind and \( \frac{\rho_0}{\rho} \) is same kind of \( \frac{\rho_0}{\rho} \). In the case of negative value \((\alpha, a)^2\), the modified Bessel function of second kind is used instead of Bessel function of first kind. The acoustic pressure of the disc will get the following form by substituting \( f_{\alpha} \) in (38)
\[ p' = \sum_{n=0}^{\infty} \varepsilon_n \left( \frac{\xi}{\alpha} \right)^n J_n(\alpha, ax) \cos n \theta e^{i \omega t} \] (43)

4. Boundary conditions and frequency equation

The considered problem is about the vibration of a magneto thermo-electro-elastic disc immersed in an inviscid fluid. Since the boundary is irregular it is difficult to satisfy the boundary conditions directly. Hence, from the idea of Nagaya [4], the boundary conditions are satisfied by the application of Fourier expansion collocation method. Thus the boundary conditions are obtained as,
\[ (\sigma_{xx} + p')_q = (\sigma_{yy})_q = (u - u')_q = (D_r)_q = (B_r)_q = (T_r)_q = 0 \] (44)

Where \( x \) is the coordinate normal to the boundary and \( y \) is the tangential to the boundary \( \sigma_{xx} \) and \( \sigma_{yy} \) are the normal and shearing stress \( T \) is the thermal field and \( ( ) \) is the value at the \( q \)th segment of the cross section. We obtain the stress displacement as follows by transforming the vibration displacements into the Cartesian coordinates \( x, y \), the relation between the displacements for the \( i \)-th segment,
\[ \sigma_{xx} = \lambda \left( u_{xx} + r^{-1} \left( u_r + u_{\theta,\theta} \right) \right) + 2\mu \left[ u_{xx} \cos^2(\theta - \gamma) + r^{-1} \left( u_r + u_{\theta,\theta} \right) \sin^2(\theta - \gamma) \right] - \beta T \]
\[ \sigma_{yy} = \mu \left[ u_{yy} - r^{-1} \left( u_r + u_{\theta,\theta} \right) \sin 2(\theta - \gamma) + \left( r^{-1} \left( u_{\theta,\theta} - u_{\theta,\theta} \right) + u_{\theta,\theta} \right) \sin 2(\theta - \gamma) \right] \]

In the case of non-homogeneity,
\[ \sigma_{xx} = \left( \lambda_1 + \mu_1 \right) \cos^2(\theta - \gamma) + \lambda_1 \sin^2(\theta - \gamma) \left( u_{xx} + r^{-1} \left( \lambda_1 + \mu_1 \right) \sin^2(\theta - \gamma) + \lambda_1 \cos^2(\theta - \gamma) \right) \]
\[ \sigma_{yy} = \mu_1 \left[ u_{yy} - r^{-1} u_{\theta,\theta} \right] \sin 2(\theta - \gamma) + \left( r^{-1} u_{\theta,\theta} + u_{\theta,\theta} - r^{-1} u_{\theta,\theta} \right) \cos 2(\theta - \gamma) = 0 \]
\[ D_r = -E_{xx} r_{xx} - m_{xx} H_{xx} = 0 \]
\[ B_r = -m_{xx} E_{xx} - \mu_{xx} H_{xx} = 0 \] (45)

Applying the solution in the boundary condition (44) we arrive at the transformed form,
\[ \left[ \left( S_{xx} \right)_q + \left( S_{yy} \right)_q \right] \xi e^{i \omega t} = 0 \]
\[ \left[ \left( S_{yy} \right)_q + \left( S_{yy} \right)_q \right] \xi e^{i \omega t} = 0 \]
\[ \left[ \left( S_{yy} \right)_q + \left( S_{yy} \right)_q \right] \xi e^{i \omega t} = 0 \]
\[ \left[ \left( E_{xx} \right)_q + \left( E_{xx} \right)_q \right] \xi e^{i \omega t} = 0 \]
\[ \left[ \left( H_{xx} \right)_q + \left( H_{xx} \right)_q \right] \xi e^{i \omega t} = 0 \] (46)

Where,
\[
S_{sz} = 0.5\left( A_{1s}e_1^1 + A_{2s}e_0^2 \right) + \sum_{n=1}^{\infty} \left( A_{1n}e_n^1 + A_{2n}e_n^2 + A_{3n}e_n^3 \right)
\]
\[
S_{s\theta} = 0.5\left( A_{1s}f_0^1 + A_{2s}f_0^2 \right) + \sum_{n=1}^{\infty} \left( A_{1n}f_n^1 + A_{2n}f_n^2 + A_{3n}f_n^3 \right)
\]
\[
S_r = 0.5\left( A_{1r}g_0^1 + A_{2r}g_0^2 \right) + \sum_{n=1}^{\infty} \left( A_{1n}g_n^1 + A_{2n}g_n^2 + A_{3n}g_n^3 \right)
\]
\[
E_x = 0.5\left( A_{1x}h_0^1 + A_{2x}h_0^2 \right) + \sum_{n=1}^{\infty} \left( A_{1n}h_n^1 + A_{2n}h_n^2 + A_{3n}h_n^3 \right)
\]
\[
H_x = 0.5\left( A_{1x}i_0^1 + A_{2x}i_0^2 \right) + \sum_{n=1}^{\infty} \left( A_{1n}i_n^1 + A_{2n}i_n^2 + A_{3n}i_n^3 \right)
\]

(47)

For anti-symmetric mode,
\[
\overline{S}_{sz} = 0.5\left( A_{3s}e_1^1 + \sum_{n=1}^{\infty} \left( A_{1n}e_n^1 + A_{2n}e_n^2 + A_{3n}e_n^3 \right) \right)
\]
\[
\overline{S}_{s\theta} = 0.5\left( A_{3s}f_0^1 + \sum_{n=1}^{\infty} \left( A_{1n}f_n^1 + A_{2n}f_n^2 + A_{3n}f_n^3 \right) \right)
\]
\[
\overline{S}_r = 0.5\left( A_{3r}g_0^1 + \sum_{n=1}^{\infty} \left( A_{1n}g_n^1 + A_{2n}g_n^2 + A_{3n}g_n^3 \right) \right)
\]
\[
\overline{E}_x = 0.5\left( A_{3x}h_0^1 + \sum_{n=1}^{\infty} \left( A_{1n}h_n^1 + A_{2n}h_n^2 + A_{3n}h_n^3 \right) \right)
\]
\[
\overline{H}_x = 0.5\left( A_{3x}i_0^1 + \sum_{n=1}^{\infty} \left( A_{1n}i_n^1 + A_{2n}i_n^2 + A_{3n}i_n^3 \right) \right)
\]

(48)

Using the Fourier series expansion to the boundary equation (44) along the boundary surfaces are expanded in the form of double Fourier series .The boundary conditions for symmetric mode are obtained as,
\[
\sum_{m=0}^{\infty} e_m \left[ E_{m0}^1 A_{10} + E_{m0}^2 A_{20} + \sum_{n=1}^{\infty} \left( E_{mn}^1 A_{1n} + E_{mn}^2 A_{2n} + E_{mn}^3 A_{3n} \right) \right] \cos m\theta = 0
\]
\[
\sum_{m=0}^{\infty} f_m \left[ F_{m0}^1 A_{10} + F_{m0}^2 A_{20} + \sum_{n=1}^{\infty} \left( F_{mn}^1 A_{1n} + F_{mn}^2 A_{2n} + F_{mn}^3 A_{3n} \right) \right] \sin m\theta = 0
\]
\[
\sum_{m=0}^{\infty} g_m \left[ G_{m0}^1 A_{10} + G_{m0}^2 A_{20} + \sum_{n=1}^{\infty} \left( G_{mn}^1 A_{1n} + G_{mn}^2 A_{2n} + G_{mn}^3 A_{3n} \right) \right] \cos m\theta = 0
\]
\[
\sum_{m=0}^{\infty} h_m \left[ H_{m0}^1 A_{10} + H_{m0}^2 A_{20} + \sum_{n=1}^{\infty} \left( H_{mn}^1 A_{1n} + H_{mn}^2 A_{2n} + H_{mn}^3 A_{3n} \right) \right] \cos m\theta = 0
\]
\[
\sum_{m=0}^{\infty} i_m \left[ I_{m0}^1 A_{10} + I_{m0}^2 A_{20} + \sum_{n=1}^{\infty} \left( I_{mn}^1 A_{1n} + I_{mn}^2 A_{2n} + I_{mn}^3 A_{3n} \right) \right] \cos m\theta = 0
\]

(49)

where, for symmetric mode,
\[ E_{mn}^j = \frac{2\varepsilon_n}{\pi} \sum_{q=1}^{\infty} \int_{\theta_{1,q}}^{\theta_{2,q}} (R_q, \theta) \cos m \theta d\theta \] 
\[ F_{mn}^j = \frac{2\varepsilon_n}{\pi} \sum_{q=1}^{\infty} \int_{\theta_{1,q}}^{\theta_{2,q}} (R_q, \theta) \sin m \theta d\theta \] 
\[ G_{mn}^j = \frac{2\varepsilon_n}{\pi} \sum_{q=1}^{\infty} \int_{\theta_{1,q}}^{\theta_{2,q}} (R_q, \theta) \cos m \theta d\theta \] 
\[ H_{mn}^j = \frac{2\varepsilon_n}{\pi} \sum_{q=1}^{\infty} \int_{\theta_{1,q}}^{\theta_{2,q}} (R_q, \theta) \sin m \theta d\theta \] 
\[ I_{mn}^j = \frac{2\varepsilon_n}{\pi} \sum_{q=1}^{\infty} \int_{\theta_{1,q}}^{\theta_{2,q}} (R_q, \theta) \cos m \theta d\theta \] 
\[ J_{mn}^j = \frac{2\varepsilon_n}{\pi} \sum_{q=1}^{\infty} \int_{\theta_{1,q}}^{\theta_{2,q}} (R_q, \theta) \sin m \theta d\theta \] 

(50)

For anti-symmetric mode, the cosine function will be replaced by the sine function and the sine function will be replaced by cosine function.

The coefficients \( e_n^j \sim \tilde{e}_n^j \) are listed in the appendix.

5. Numerical results and discussions

The dispersion equations are derived from equations (50) and are analyzed numerically for rings of triangular, square, pentagonal and hexagonal cross sections. The material properties used for the computation are as follows: For the solid the Poisson ratio \( \nu = 0.3 \), density \( \rho = 7849 \text{ kg/m}^3 \) and the Young’s modulus \( E = 2.139 \times 10^7 \text{ N/m}^2 \) and for the fluid: the density \( \rho_f = 1000 \text{ kg/m}^3 \) and the phase velocity \( c = 1500 \text{ m/s} \). The polygonal cross sectional ring in the range \( \theta = 0 \) and \( \theta = \pi \) is divided into many segments for convergence of frequency in such a way that we can estimate the distance between any two segments is ignorable. Gauss five point formula has been used to integrate each segment numerically.

5.1. Triangular and pentagonal geometry

The vibrational displacements are symmetrical about the \( x \) axis for the longitudinal mode and anti-symmetrical about the \( y \) axis for the flexural mode in the triangular and pentagonal cross sectional discs since the cross section is symmetrical about only one axis. Therefore \( n \) and \( m \) are chosen as 0,1,2,3,... in (49) for longitudinal mode and \( n,m = 1,2,3,... \) in Equations (50) for the flexural mode.

5.2. Square and hexagonal geometry

In longitudinal vibration of square and hexagonal cross sectional disc, the displacements are symmetrical about both major and minor axes since both the cross sections are symmetrical about both the axes. Therefore the frequency equation is obtained by choosing both terms of \( n \) and \( m \) are chosen as 0,2,4,6,... in equations (49) and (50). The displacements are anti-symmetrical about the major axis and symmetrical about the minor axis for the flexural mode. Hence the frequency equation is obtained by choosing \( n,m = 1,3,5,... \) in Equations (49)-(50).

![Figure 1. Triangular Disc](image1)

![Figure 2. Square Disc](image2)
5.3. Dispersion curves

The variation of radial stress is calculated with the non-homogeneous parameter \( m \) of triangle, square, pentagon and hexagonal cross sectional discs with and without fluid interactions in Figures 3 and 4. Figs. 5 and 6 reveal that the variation of radial displacement of triangle, square, pentagon and hexagonal cross sectional discs with the non-homogeneous parameter \( m \) for the disc with and without fluid. From the Figs. 5 and 6, it is observed that, both in space and fluid the modes of the poly rings are merges for \( 0 < m < 0.4 \) and begin starts decreases monotonically. The merging of curves between the vibrational modes shows that, there is an energy transportation between the modes of vibrations by the effect of fluid interaction. The dispersion curves are drawn for electrical displacement versus the non-homogeneous parameter \( m \) of disc with polygonal cross section in the presence and absence of fluid in Figures 7 and 8. From the Figures 7 and 8, it is noticed that the electric displacement attains maximum at the lower range of non-homogeneous parameter \( m \) and starts to vary linearly in the remaining range of parameter \( m \). The effect of electric displacement is little dispersive in fluid interaction. Figures 9 and 10 shows that the variation of temperature with the parameter \( m \) for the case of disc with fluid and without fluid of various polygonal shapes. The temperature distribution is increases as the non-homogeneous parameter \( m \) increases both in polygonal disc in fluid and in space. The temperature distribution profiles are dispersive in trend for the case of disc in fluid than in space and experience convergence in the non-homogeneous parameter \( 0 \leq |\zeta| \leq 0.2 \). The energy transformation between the vibrational modes has been represented by the cross over points appearing between the vibration modes.
Figure 6. Variation of radial stress and non-homogeneous parameter $m$ of disc in fluid.

![Graph showing variation of radial stress and non-homogeneous parameter $m$ of disc in fluid.]

Figure 7. Variation of radial displacement and non-homogeneous parameter $m$ of disc in space.

![Graph showing variation of radial displacement and non-homogeneous parameter $m$ of disc in space.]

Figure 8. Variation of radial displacement and non-homogeneous parameter $m$ of disc in fluid.

![Graph showing variation of radial displacement and non-homogeneous parameter $m$ of disc in fluid.]

Figure 9. Variation of electrical displacement and non-homogeneous parameter $m$ of disc in space.

![Graph showing variation of electrical displacement and non-homogeneous parameter $m$ of disc in space.]
Figure 10. Variation of electrical displacement and non-homogeneous parameter m of disc in fluid.

Figure 11. Variation of temperature and non-homogeneous parameter m of disc in space.

Figure 12. Variation of temperature and non-homogeneous parameter m of disc in fluid.

6. Conclusion:
The non-homogeneous wave propagation of magneto thermo-electro-elastic disc immersed in the fluid is discussed. The linear theory of elasticity has been applied for the analytical formulation. The
dispersion equation of polygonal cross sectional disc such as square, triangle, pentagon and hexagon is
developed using the Fourier expansion collocation transformation along the irregular boundaries. The
stress, strain, mechanical displacement, electrical displacement and temperature distribution have been
represented graphically with varying non-homogeneous parameter. The impact of fluid on the disc is
also discussed for the above cross sections. The non-homogeneity and the surrounding fluid have
significant effect in the variation of physical parameter. This type of investigation is important in the
construction of structures in diverse engineering field with non-circular cross sections.

Appendix A:

\[ e_n^1 = 2((n^2 - n)J_n(\alpha,ax) + (\alpha,ax)J_{n+1}(\alpha,ax))\cos(2(\theta - \gamma))\cos n\theta \]
\[ - x^2 ((\alpha,ax)^2(\bar{\lambda} + (1 + \cos 2(\theta - \gamma)))J_n(\alpha,ax)\cos n\theta \]

\[ e_n^2 = 2((n^2 - n)J_n(\alpha,ax) + (\alpha,ax)J_{n+1}(\alpha,ax)\sin 2(\theta - \gamma)\sin n\theta \]
\[ - x^2 ((\alpha,ax)^2(\bar{\lambda} + (1 + \cos 2(\theta - \gamma)))J_n(\alpha,ax)\sin n\theta \]

\[ e_n^3 = \bar{x}^2\beta J_n^1(\alpha,ax)\cos n\theta \]
\[ e_n^4 = \bar{x}^2\beta J_n^1(\alpha,ax)\sin n\theta \]

\[ f_n^1 = 2((n^2 - n) - (\alpha,ax)^2)J_n(\alpha,ax) + (\alpha,ax)J_{n+1}(\alpha,ax))\sin 2(\theta - \gamma)\sin n\theta \]
\[ + 2n((\alpha,ax)J_{n+1}(\alpha,ax) - (n-1)J_n(\alpha,ax))\cos 2(\theta - \gamma)\sin n\theta \]

\[ f_n^2 = 2((n^2 - n) - (\alpha,ax)^2)J_n(\alpha,ax) + (\alpha,ax)J_{n+1}(\alpha,ax)\sin 2(\theta - \gamma)\sin n\theta \]
\[ - 2n((\alpha,ax)J_{n+1}(\alpha,ax) - (n-1)J_n(\alpha,ax))\cos 2(\theta - \gamma)\sin n\theta \]

\[ f_n^3 = 2((n^2 - n) - (\alpha,ax)^2)J_n(\alpha,ax) + (\alpha,ax)J_{n+1}(\alpha,ax)\sin 2(\theta - \gamma)\sin n\theta \]
\[ + 2n((\alpha,ax)J_{n+1}(\alpha,ax) - (n-1) - (\alpha,ax)^2)J_n(\alpha,ax)\cos 2(\theta - \gamma)\sin n\theta \]

\[ f_n^4 = 2((n^2 - n) - (\alpha,ax)^2)J_n(\alpha,ax) + (\alpha,ax)J_{n+1}(\alpha,ax)\sin 2(\theta - \gamma)\sin n\theta \]
\[ - 2n((\alpha,ax)J_{n+1}(\alpha,ax) - (n-1) - (\alpha,ax)^2)J_n(\alpha,ax)\cos 2(\theta - \gamma)\sin n\theta \]

\[ g_n^1 = (nJ_n(\alpha,ax) - (\alpha,ax)J_{n+1}(\alpha,ax))\cos n\theta \]
\[ g_n^1 = (nJ_n(\alpha,ax) - (\alpha,ax)J_{n+1}(\alpha,ax))\sin n\theta \]
\[ g_n^2 = nJ_n(\alpha,ax)\cos n\theta \]
\[ g_n^2 = nJ_n(\alpha,ax)\sin n\theta \]
\[ g_n^3 = -(nJ_n^1(\alpha,ax) - (\alpha,ax)J_{n+1}^1(\alpha,ax))\cos n\theta \]
\[ g_n^3 = -(nJ_n^1(\alpha,ax) - (\alpha,ax)J_{n+1}^1(\alpha,ax))\sin n\theta \]
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