Precision Cosmology

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Abstract

The good agreement between large-scale observations and the predictions of the now-standard ΛCDM theory gives us hope that this will become a lasting foundation for cosmology. After briefly reviewing the current status of the key cosmological parameters, I summarize several of the main areas of possible disagreement between theory and observation: big bang nucleosynthesis, galaxy centers, dark matter substructure, and angular momentum, updating my earlier reviews [1]. The issues in all of these are sufficiently complicated that it is not yet clear how serious they are, but there is at least some reason to think that the problems will be resolved through a deeper understanding of the complicated astrophysics involved in such processes as gas cooling, star formation, and feedback from supernovae and AGN. Meanwhile, searches for dark matter are dramatically improving in sensitivity, and gamma rays from dark matter annihilation at the galactic center may have been detected by H.E.S.S.

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1 Introduction

Modern cosmology – the study of the universe as a whole – is undergoing a scientific revolution. New ground- and space-based telescopes can now observe every bright galaxy in the universe. We can see back in time to the cosmic dark ages before galaxies formed and read the history of the early universe in the ripples of heat radiation still arriving from the Big Bang. We now know that everything that we can see makes up only about half a percent of the cosmic density, and that most of the universe is made of invisible stuff called “dark matter” and “dark energy.” The cold dark matter (CDM) theory based on this (ΛCDM) appears to be able to account for many features of the observable universe, including the heat radiation and the large scale distribution of galaxies, although there are possible problems understanding some details of the structure of galaxies. Modern cosmology is developing humanity’s first
story of the origin and nature of the universe that might actually be true – in the sense that it will still be true in a thousand years. Although this talk is entitled “Precision Cosmology,” I think we should be even more impressed that modern cosmology is true than that it is precise.

Building on the work of Copernicus, Brahe, Kepler, and Galileo, Newton established the basis of what we now call classical physics. Although there have been many scientific revolutions in physics since the Newtonian synthesis, none of them have overthrown Newtonian physics the way that the Copernicano-Newtonian scientific revolution overthrew earlier Aristotelian and Ptolemaic ideas. Ptolemy was never afterward taught as science, only as history, but Newtonian physics will always be taught. The subsequent revolutions in physics – wave optics, field theory, thermodynamics, relativity, and quantum mechanics – encompassed Newtonian physics rather than overthrowing it [2].

Once a well-confirmed basis for further progress is established, such as that provided by Newtonian physics, a scientific field can expand its range of successful applicability without any further overthrowing revolutions, and in this sense it can be said to be progressive. I think it is likely that the modern revolution in cosmology has now established such a basis for progress. Even though there is so much that we still do not know – in particular, the nature of the dark matter and dark energy, and the origin of the initial conditions – what we do know is now so well confirmed by diverse data that it is likely to be true.

Ever since Einstein’s general relativity provided the essential language for cosmology, the field has progressed in the normal scientific style, with predictions followed by confirmations. Friedmann and Lemaitre predicted the expansion of the universe, which was subsequently confirmed by Hubble in 1929. Gamow, Alpher, and Hermann in 1948 predicted the existence of the cosmic background radiation (CBR) which was found by Penzias and Wilson in 1965, with its thermal spectrum confirmed by the FIRAS instrument on the COBE satellite in 1989. The cold dark matter theory [3] predicted the amplitude of the CBR fluctuations, which were discovered by the DMR instrument on COBE in 1992 and found to have the predicted amplitude. By early 1992, it was clear that the only viable simple versions of CDM were $\Lambda$CDM with $\Omega_m \approx 0.3$ and $\Omega_\Lambda \approx 0.7$, and Cold+Hot DM (CHDM) with $\Omega_m = 1$ and $\Omega_\nu = 0.2 - 0.3$. A few years later CHDM and all other $\Omega_m = 1$ cosmologies were ruled out by the discovery of abundant high redshift galaxies and by the discovery of strong evidence for $\Lambda$ using high-redshift supernovae in 1998. The combination of CDM and cosmic inflation predicted the acoustic peak in the CBR angular power spectrum, which was discovered by the BOOMERANG and MAXIMA balloon experiments and the DASI instrument at the South Pole in 2000-2002. Now WMAP has confirmed and extended the CBR observations of ground- and balloon-based instruments, and both the CBR angular power spectrum
and the galaxy power spectrum look exactly like the predictions of ΛCDM [4].

Our modern cosmological synthesis is based on several assumptions of simplicity, in particular the “cosmological principle” (we don’t live at a special place in the universe) and the assumption that the same laws of physics that describe phenomena in our laboratories on earth and nearby are valid at all times and places throughout the universe. These assumptions are being checked against observations. For example, comparison of the details of atomic spectra in the laboratory and from galaxies at various redshifts suggested that there might be variations in the fine structure constant \( \alpha = e^2/(\hbar c) \) of \( \Delta \alpha /\alpha = -0.574 \pm 0.102 \times 10^{-5} \) [5], but the latest results from another group and telescope do not see that effect, finding \( \Delta \alpha /\alpha = 0.06 \pm 0.06 \times 10^{-5} \) [6]. This seems to me good news, since such a variation might be inconsistent with the entire framework of relativistic quantum field theory [7]. But of course it will be necessary to test this new result, and to understand the origin of the earlier apparent variation in \( \alpha \).

2 Cosmological Parameters

In the mid-1990s there was a crisis in cosmology, because the age of the old globular cluster stars in the Milky Way, then estimated to be 16 ± 3 Gyr, was higher than the expansion age of the universe, which for a critical density (\( \Omega_m = 1 \)) universe is 9 ± 2 Gyr (using Hubble parameter \( h = 0.72 \pm 0.08 \)). But when the data from the Hipparcos astrometric satellite became available in 1997, it showed that the distances to the globular clusters had been underestimated, which (combined with improved stellar evolution models) implied that their ages are 12 ± 3 Gyr.

The successful Hubble telescope key project on the extragalactic distance scale determined that the Hubble parameter \( H_0 = 100h \) km s\(^{-1}\) Mpc\(^{-1}\) is \( h = 0.72 \pm 0.08 \) [8]. Several lines of evidence – including CBR, supernovae, and clusters – now show that the universe does not have \( \Omega_m = 1 \), but rather \( \Omega_{tot} = \Omega_m + \Omega_{\Lambda} = 1 \) with \( \Omega_m \approx 0.3 \), which gives an expansion age of about 14 Gyr. The WMAP cosmic background data alone give an expansion age of 13.4 ± 0.3 Gyr, which becomes 13.7 ± 0.2 with the WMAP running power spectrum index model [4].

A new type of age measurement based on radioactive decay of Thorium-232 (half-life 14.1 Gyr) measured in a number of stars gave a completely independent age of 14 ± 3 Gyr. A similar measurement, based on the first detection in a star of Uranium-238 (half-life 4.47 Gyr), gave 12.5 ± 3 Gyr; a second such star gave an age of 14.1 ± 2.5 Gyr [9]. These stellar lifetimes are of course lower limits on the age of the universe.
All the recent measurements of the age of the universe are thus in excellent agreement. It is reassuring that three completely different clocks – stellar evolution, expansion of the universe, and radioactive decay – agree so well.

Ever since the cosmological crisis regarding the age of the universe was thus resolved, all the data has been consistent with the cosmology described above, with the main cosmological parameters now all determined to about 10% or better [4,11] with the sole exception of $\sigma_8$, which measures the amplitude of the (linear) power spectrum on the scale of $8 \, h^{-1} \, \text{Mpc}$. However, $\sigma_8$ is a crucial cosmological parameter which has a big influence over the growth of fluctuations in the early universe. The current analyses lead to values of $\sigma_8$ between about 0.7 and 1.1. But unless $\sigma_8$ is at least 0.85 or so, it is very hard to see how the universe could have formed stars and quasars early enough to have become ionized at $z \sim 17$ [12] as indicated by the WMAP detection of large-angle polarization [13]. The latest analysis of the cosmological parameters, for the first time including the Lyman $\alpha$ forest observed in the SDSS quasar spectra along with the first year WMAP data and the SDSS galaxy clustering data, finds $\sigma_8 = 0.90 \pm 0.03$ and $\Omega_\Lambda = 0.72 \pm 0.02$ [11]. This study finds the primordial spectral index of scalar fluctuations $n_s = 0.98 \pm 0.02$ with no evidence for running of the spectral index, equation of state parameter $w \equiv P/\rho = -0.98^{+0.10}_{-0.12}$ at redshift $z = 0.3$ with no evidence for variation with redshift, and a stringent upper limit on neutrino mass $\sum m_\nu < 0.42 \, \text{eV}$. However, the tiny quoted errors do not include systematic uncertainties in the interpretation of the Lyman $\alpha$ forest data, which require further analysis.

3 Possible Problems

Of course, there are a number of areas in which cosmological theory and observations are not in obvious agreement. The space available does not permit an exhaustive review, so I will concentrate on the following topics which seem to me to be the most important: big bang nucleosynthesis, galaxy centers, dark matter substructure, and angular momentum. Fortunately, some areas which had seemed problematic now seem less so. For example, the low values of the quadrupole and octopole CMB anisotropies reported by WMAP are revised upward in an improved analysis, and are now in good agreement with the $\Lambda$CDM predictions [14]. And the possibility that the first year WMAP data imply that the visible universe might be topologically complex and smaller than the horizon [15] has now been stringently constrained by the absence of pairs of circles in the WMAP data [16]. Another topic that has received more attention in the press than is warranted by the science is the discovery of massive galaxies at high redshifts [17]. Although these observations challenge some overly simplified theories of galaxy formation [18], there are plenty of sufficiently massive halos at the relevant redshifts (with $\sigma_8 = 0.9$) to host the
Fig. 1. Curves show the cumulative number density of dark matter halos more massive than $10^{11}$ to $10^{15} M_\odot$ (from top to bottom, as labeled), as a function of redshift, calculated using an improved Press-Schechter formula [19]. Points show the estimated comoving number densities of several observed populations, as follows. Hexagons: galaxies with stellar masses greater than $10^{11} M_\odot$ (lower point) and $2.5 \times 10^{10} M_\odot$ (higher point), obtained by integrating the stellar mass function from SDSS+2MASS; open square: Extremely Red Objects; open triangle: K20 galaxies; crosses: sub-mm galaxies; six pointed stars: Lyman break galaxies, filled square: quasars. For all populations, $\Lambda$CDM predicts that there are enough dark matter halos massive enough to plausibly host the observed objects. (From Ref. [20], which gives the references for the data points plotted.)

galaxies in question, as Figure 1 shows.

3.1 Big Bang Nucleosynthesis (BBN)

It is reassuring that the same baryon abundance $\Omega_b h^2 = 0.0214 \pm 0.0020$ implied by the deuterium abundance D/H in low-metallicity Lyman limit systems in quasar spectra [21] agrees with that implied by the relative heights of the first two peaks in the WMAP angular power spectrum [4], giving $\Omega_b h^2 = 0.0224 \pm 0.0009$. This is also in agreement with the baryon abundance deduced from clusters [22] and with the lower limit from the opacity of the Lyman alpha forest [23]. However, there are uncertainties in the measured deuterium abundance evolution [24], and potential problems – or perhaps clues to new physics – in discrepancies between the observed helium and lithium abundances and the predictions of BBN.

The abundance by mass of $^4$He measured in low-metallicity ionized regions in
nearby galaxies implies an extrapolated primordial abundance \( Y_p = 0.2421 \pm 0.0021 \) according to the latest published data [25], which corresponds according to standard BBN to a baryon abundance \( \Omega_b h^2 = 0.012^{+0.003}_{-0.002} \), lower by about \( 3\sigma \) than the value just mentioned from D/H and CMB measurements. It remains to be seen whether this could be remedied by improved analyses (for example, based on more realistic models of these low-metallicity galaxies or of their HII regions), or alternatively whether it is perhaps an indication of problems with standard BBN.

The D/H [21] \( \Omega_b \) in standard BBN implies a primordial \( ^7\text{Li}/H \approx 3.3 - 6.0 \times 10^{-10} \), in serious disagreement with the value of \( ^7\text{Li}/H = 1.23^{+0.34}_{-0.16} \times 10^{-10} \) measured in atmospheres of galactic halo metal-poor stars in the “Spite plateau” (i.e. with metallicity [Fe/H] less than about -2) [26]. It disagrees even with the puzzlingly higher value \( ^7\text{Li}/H = 2.19^{+0.30}_{-0.27} \times 10^{-10} \) from a sample of globular cluster stars [27]. It is possible that some of the \( ^7\text{Li} \) in such stars is destroyed by astration, consistent with the small range of \( ^7\text{Li} \) abundance in the Spite plateau stars [28], but this may not resolve the discrepancy.

These disagreements call into question the usual assumptions of standard BBN, for example, the assumption of no significant electron neutrino asymmetry and \( N_\nu = 3 \) light neutrino species [29]. However, an alternative possibility that might neatly account for the \( ^7\text{Li} \) discrepancy is the injection of energetic nucleons around 1000 s after the big bang, for example due to decay of the next-to-lightest supersymmetric partner particle into the lightest one [30].

### 3.2 Galaxy and Cluster Centers

When the first high-resolution simulations of cold dark matter halos became available [31], they had a central density profile approximately \( \rho(r) \propto r^{-1} \), which has come to be known as the central “cusp.” It was soon pointed out [32,33] that this central behavior was inconsistent with the HI observations of dwarf galaxies that were then becoming available, which suggested that the central density is roughly constant, and also that the first cluster lensing observations appeared to be inconsistent with a \( r^{-\alpha} \) central cusp with \( \alpha = 1 \) [32]. Many additional rotation curves of low surface brightness (LSB) galaxies were measured, and they also were claimed to imply that the central density of these galaxies is rather flat. It was subsequently realized that the HI observations of galaxies were affected by finite resolution (“beam smearing”), and that when this was taken into account the disagreement with simulations is alleviated [34]. More recently, higher resolution H\( \alpha \) and CO rotation curves have been obtained for a few nearby dwarf and low surface brightness galaxies [35], and the highest resolution two-dimensional data imply a variety of central density profiles ranging from \( \alpha \approx 0 \) to 1, with evidence for radial motion
especially in the $\alpha \approx 0$ cases [36].

Meanwhile, theorists have done simulations with improving resolution. On the basis of simulations with tens of thousands of particles per dark matter halo, Navarro, Frenk, & White (NFW) [37] showed that halos from galaxy to cluster scales have density profiles that are described fairly well by the fitting function $\rho_{\text{NFW}}(r) \equiv \rho_s (r/r_s)^{-1}(1 + r/r_s)^{-2}$. Subsequently, James Bullock [38] and Risa Wechsler [39] improved our understanding of halo evolution in their dissertation research with me, which included analyzing thousands of dark matter halos in a high-resolution dissipationless cosmological simulation by Anatoly Klypin and Andrey Kravtsov. Defining the (virial) concentration $c_{\text{vir}} \equiv R_{\text{vir}}/r_s$ (where $R_{\text{vir}}$ is the virial radius, Bullock et al. [38] showed that at fixed halo mass $c_{\text{vir}}$ varies with redshift $z$ as $(1 + z)^{-1}$, and developed an approximate mathematical model that explained the dependence on mass and redshift. (An alternative model was proposed in [40], but it appears to be inconsistent with a recent simulation of small-mass halos [41].) Wechsler et al. [39] determined many halo structural merger trees, and showed that the central scale radius $r_s$ is typically set during the early phase of a halo’s evolution when its mass is growing rapidly, while $c_{\text{vir}}$ subsequently grows with $R_{\text{vir}}$ during the later slow mass accretion phase. Higher resolution simulations with roughly a million particles per halo gave central density profiles $\rho(r) \propto r^{-\alpha}$ with $\alpha$ as steep as 1.5 [42], although more recent very high resolution simulations are finding less steep central profiles with $\alpha \approx 1$ or shallower as $r \to 0$ [43]. The disagreement between the theoretical $\alpha \approx 1 - 1.5$ vs. the observed $\alpha \approx 0 - 1$ may just reflect the effects of baryonic matter in the centers of galaxies and the difference between circular velocity and rotation speed likely to arise in gaseous disks embedded within triaxial halos [44].

The mean density $\Delta_{V/2}$ inside the radius $r_{V/2}$ (where the rotation velocity reaches half the maximum observed value) appears to be somewhat smaller than the $\Lambda$CDM prediction with $\sigma_8 = 0.9$, but more consistent with $\Lambda$CDM with $\sigma_8 = 0.7$ [45] or “quintessence” models with equation of state parameter $w < -1$ [46], but as we have mentioned these possibilities appear to be inconsistent with other data (e.g. [11]).

In several clusters of galaxies, after removing the baryonic contribution the central dark matter profile appears to be rather shallow, with $\alpha \approx 0.35$ for cluster MS2137-23 [46]. The apparent disagreement with CDM worsens if adiabatic compression of the dark matter by the infalling baryons is considered [47]. However, dynamical friction of the dense galaxies moving in the smooth background of the cluster dark matter counteracts the effect of adiabatic compression, and can lead to energy transfer from the galaxies to the dark matter which heats up the central cuspy dark matter and softens the cusp. N-body simulations [48] show that the dark matter distribution can become very shallow, with $\alpha \approx 0.3$ for a cluster like MS2137, in agreement with observations.
Taking the triaxial shapes of cluster centers [49] into account may also help to bring theory and observations into agreement [50].

3.3 Substructure

Many fewer small satellite galaxies are seen around the Milky Way and M31 than the number of small dark matter halos seen in semianalytic models [51] and high-resolution simulations [52,53] of such systems. But for ΛCDM the discrepancy arises only for satellites smaller than the LMC and SMC [52], and such small satellites are expected to form stars very inefficiently [54,55]. Semianalytic models taking both reionization of the universe and local feedback from supernovae into account appear to be in good agreement with observations of the relative numbers of faint and bright galaxies in environments ranging from the local group to clusters [55]. This is encouraging, but it is important to check whether such models and high-resolution simulations [56] are capable of accounting for more detailed properties of low-mass galaxies, such as their radial distribution, metallicity and ages [57], and the “fundamental line” of dwarf galaxy properties [58].

“Millilensing” by DM halo substructure may be required to account for anomalous flux ratios in radio lensing of quasars by galaxies producing multiple images [59]. A concern is whether there enough halo substructure in the inner ∼10 kpc of galaxies, where it appears to be needed to account for such lensing, although a recent analysis suggests that substructure along the line of sight to these galaxies can help account for the observations [60]. Spectroscopic observations can determine the mass of the lensing perturber by comparing the magnification of different regions, for example whether the tiny broad line region nearest the AGN is lensed, or also the much larger narrow line region [61]. Lensing of AGN jets can also be a useful diagnostic for substructure [62]. Further observations are needed.

3.4 Angular Momentum

There are two angular momentum problems [63]: (1) overcooling, and (2) the wrong distribution of angular momentum in halos. (1) Overcooling: For many years, realistic spiral galaxies did not form in hydrodynamic simulations [64]. However, it is plausible that unrealistically effective cooling (“overcooling”) in the simulations was responsible for the loss of angular momentum [65]. More realistic disk galaxies have formed in recent, higher-resolution simulations including feedback [66]; an appropriate equation of state for the gas in galactic disks may play a particularly important role [67]. (2) Wrong angular momentum distribution: The standard tidal torque picture of how dark matter halos
and the galaxies that they host get their angular momentum [68] suggests that the dark matter and baryons will have similar angular momentum distributions. The distribution of the specific angular momentum among the dark matter particles can be described by a simple fitting function, but disk galaxies like those seen would not form if the baryons have this same angular momentum distribution [69]. My colleagues and I have developed an alternative picture focussing on the angular momentum growth of the largest progenitor of a given halo, in which the halo’s angular momentum comes mainly from the orbital angular momentum of the accreted halos, and we showed that this model accurately reproduces simulation results [70]. In this model, large angular momentum changes occur following major mergers – but the gas (which shocks) and the dark matter (which does not) would be expected to behave differently in such mergers. Steadily improving hydrodynamic simulations are being done to study these processes in the standard ΛCDM cosmology [71], and new techniques are being developed to compare the outputs to observations [72], but it remains to be seen whether the results will be a good match to the mostly irregular galaxies observed at high redshift turning into the observed Hubble sequence of galaxies at low redshift. Our model of angular momentum growth of dark matter halos implies that the halos that have not had a recent merger have lower spin parameter (dimensionless angular momentum) than average. A perhaps surprising consequence is that the halos that host elliptical galaxies that formed from major mergers are expected to have higher angular momentum than those that host spiral galaxies (since major mergers destroy galactic disks). This is contrary to naive expectations [70,73].

4 Conclusions

On large scales, the agreement between ΛCDM and observations is spectacular. ΛCDM simulations also account remarkably well for smaller scale observations such as galaxy clustering [74], and even for the radial distribution [75] and total mass associated with galaxies [76].

The problems of big bang nucleosynthesis may be a clue to new particle physics or astrophysics. On cusps there has been tremendous progress on observing velocity fields in nearby galaxies, and also real progress in improving simulations. Observed simulations may agree with observed velocities in galaxy centers better than seemed likely a few years ago. But it is something of a scandal that there is still so little theoretical understanding of dark matter halo central behavior, although people are making progress on this problem [77]. It is likely that triaxial halo structure and poorly understood gas-physics will turn out to be relevant. On dark matter halo substructure, it looked last year as if a challenge might be turning into a success for
CDM, if the amount of substructure predicted by ΛCDM is indeed what is required to account for the number of satellites seen and for the flux anomalies observed in radio lensing. The main question is whether the amount of dark matter in subhalos and the predicted radial distribution of such substructure agrees with lensing and observed satellites. Much work remains to be done to test the theory quantitatively. Regarding angular momentum problems, resolving the crucial issues again involves developing better understanding of messy astrophysics. Fortunately, in all these areas wonderful new telescopes are providing crucial data that will help develop and test theory.

At this conference, the updates on direct dark matter detection experiments showed impressive progress, with greatly expanded parameter space now being probed experimentally. Unfortunately, the relevant weakly interacting massive particle (WIMP) elastic scattering cross section is quite uncertain. The annihilation cross section is much better constrained, since this is what determines the WIMP abundance today in most models, but the dark matter density at the center of the Milky Way is uncertain because there are competing processes that can enhance it or diminish it. It is possible that the new H.E.S.S. array of atmospheric Cherenkov telescopes (ACTs) has discovered dark matter annihilation at the galactic center, with a dark matter particle mass of approximately 18 TeV [78], which is unexpectedly high but not impossible for the lightest supersymmetric partner particle. The high energy gamma rays appear to come from the sort of centrally peaked density profiles predicted as a consequence of scattering of WIMPs by the star cluster surrounding the central supermassive black hole Sag A* [79], and the necessary dark matter density is consistent with theoretical expectations [80] if there is baryonic contraction [47,81]. It is unlikely that the gamma rays come from near the black hole event horizon, since there does not appear to be any time variability in the gamma rays although the X-ray and optical radiation from the black hole is quite variable. The main alternative explanation for the high energy gamma rays is that they come from the supernova remnant known as Sag A* East, which covers a region several pc across surrounding the galactic center. These alternatives can perhaps be distinguished via the different angular distributions expected, when data taken with all four H.E.S.S. ACTs are analyzed. Another important discriminant is the energy spectrum of the gamma rays, which must sharply cut off if the gamma rays come from annihilation.

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