Theoretical analysis and evaluation of an optimally controlled full-car vehicle model with a variable-damping semi-active vehicle suspension forced by measured road inputs

Y Ercan
TOBB University of Economics and Technology, Sogutozu, 06560 Ankara, Turkey

E-mail: yercan@etu.edu.tr

Abstract. This study aims to obtain the optimal control algorithm for a full-car model with a variable-damping semi-active suspension, such as a magnetorheological damper, by solving the linear quadratic regulator problem, and then to evaluate the system performance if the control inputs are constrained and delayed, and the vehicle is subjected to measured road inputs. A seven-degree of freedom full-car vehicle model was considered, and the state equations of the system were obtained in bilinear form. An integral performance index involving a weighted combination of the mean squares of average sprung mass acceleration and suspension deflections was defined. Trade-off curves were obtained between the sprung mass acceleration and suspension deflections of the optimally controlled system which is subjected to a measured road profile input. Performance of the optimally controlled system was compared to the performance of the corresponding optimum passive suspension system. For the vehicle parameters and the road input profile considered in this study, a reduction of 6.4% in the average vertical acceleration and 2.8% in the average suspension deflection was achieved by the semi-active suspensions. The response of the system to an initial condition has shown that its transient oscillations are damped out effectively by the semi-active suspension.

1. Introduction
A set of suitable parameters, normally the coefficients of the dampers between the sprung mass and the unsprung masses, are changed in semi-active suspensions to optimize the system performance. Dampers with variable coefficients, such as magnetorheological dampers, are now commercially available, and the use of semi-active suspensions in vehicles has become economically feasible.

Semi-active suspensions were first proposed more than 30 years ago by Karnopp and Crosby [1, 2]. Many studies have been carried out since then, including those employing linear quadratic Gaussian (LQG) control of ¼- and ½-vehicle models, and other control schemes with and without preview [3-15]. Some of the previous studies have been specifically on suspensions with magnetorheological dampers [16]. A review of the major studies may be found in Hrovat [17].

The linear quadratic state regulator optimization yields the optimal linear quadratic Gaussian control if the disturbance input to the system is white noise and the control inputs are not constrained. However, the road profile inputs to a vehicle are not white noise, and the control inputs to a vehicle with a semi-active suspension with variable damping are constrained. Moreover, the control inputs can be applied only with a certain time delay caused by processing of data and the response time of controllers. As a result, performance of a vehicle suspension system using an algorithm based on the
optimal control solution of the LQG problem will be affected by these deviations from the underlying assumptions. The purpose of this study is to obtain the optimal control algorithm for a full-car model with a variable damping semi-active suspension by solving the linear quadratic state regulator problem, that is the LQG problem, and evaluate its performance when the vehicle is subjected to measured road inputs, and the control inputs are constrained.

2. System model and the method of obtaining the optimal control algorithm
A full-car vehicle model with 7 degrees of freedom as shown in figure 1 is considered in this study. Here \( u_1, u_2, u_3, u_4 \) are coefficients of the variable dampers, which may be adjusted to control the system behaviour. \( z_{fl1}, z_{fr1}, z_{rl1}, \) and \( z_{rr1} \) represent the disturbance inputs from the road. The road inputs from the left and right tracks are different from each other. On the other hand, the inputs from the same track at the front and at the back of the vehicle are the same in shape, but there is a time difference between them that is determined by the length \( L \) and speed \( V \) of the vehicle.

![Figure 1. System model.](image)

The state equations of the system, which are not given here to save space, were obtained. The state vector \( x \) has 14 elements. The dynamic suspension clearances \( x_1 = y_{fl2} - y_{fl1}, \ x_4 = y_{fr2} - y_{fr1}, \ x_9 = y_{rr2} - y_{rr1}, \ x_{13} = y_{rl2} - y_{rl1}, \) tire deflections \( x_3 = y_{fl1}, \ x_7 = y_{rl1}, \ x_{11} = y_{fr1}, \) and their rates of change are defined as the state variables. The state equations involve terms in which state and control variables multiply each other. Such systems are called bilinear. It can be shown that the solution of the state regulator problem for bilinear systems may be obtained from the Riccati equation. In the present study, the state regulator problem is solved first. Then, performance of the resulting optimally controlled system is evaluated for constrained control inputs, and in the presence of disturbance inputs which are the measured road elevations.

A quadratic performance index was assumed in this study as follows, involving the average squared heave accelerations of the points on the sprung mass, squares of suspension deflections and tire deflections.

\[
P = \int_0^T \left[ (y_{fl1}^2 + y_{fr1}^2 + y_{rl1}^2 + y_{rr1}^2) + \rho_1 (x_1^2 + x_5^2 + x_9^2 + x_{13}^2) + \rho_2 (x_3^2 + x_7^2 + x_{11}^2 + y_{rr1}^2) \right] dt
\]

(1)

It was shown that the four variable damping coefficients \( u_1, u_2, u_3, \) and \( u_4 \) are not all needed to minimize the selected performance index. Therefore, \( u_4 \) was set to a constant value \( u_{4s} \), and the remaining control variables \( u_{1o}, u_{2o}, \) and \( u_{3o} \) were solved in terms of the state variables by assuming the
final value in the performance $t_f = \infty$, and making use of the corresponding Riccati equation. However, rather than solving the algebraic Riccati equation, differential Riccati equation was integrated in the negative time direction starting from zero final condition until its steady state solution is reached.

3. Performance of a vehicle with a semi-active suspension

Performance of a 7-ton vehicle with a semi-active suspension traveling at a speed of 90 km/h was investigated. However, coefficients of the variable dampers were constrained in applying the optimal control algorithm such that $2000 \text{ N} \cdot \text{s/m} < u_i < 40000 \text{ N} \cdot \text{s/m}$ ($u_i = 1$ to 3), assuming that magnetorheological dampers are used in the system. A randomly selected road profile from The University of Michigan Transportation Research Institute (UMTRI) World Wide Web site was used to disturb the system. Performance of the optimal passive system for the same vehicle was used as a benchmark to evaluate the performance of the semi-active system.

Simulations showed that tire deflections are hardly effected by the semi-active control, and they are within acceptable ranges. Therefore, the weighting coefficient $\rho_2$ was taken zero in the simulations, and the optimal trade-off curve between the average sprung mass vertical acceleration and the average suspension deflections, shown in figure 2, was obtained by changing the values of $\rho_1$.

![Figure 2. Sprung mass acceleration versus suspension deflection trade-off curve.](image)

Minimum sprung mass acceleration was obtained for $\rho_1 = 1.4$, which is 0.2187 m/s² as compared to 0.2336 m/s² for the optimal passive suspension. The corresponding average rms suspension deflections are $1.596 \times 10^{-3}$ m and $1.642 \times 10^{-3}$ m respectively. Hence, a reduction of 6.4% in the rms average vertical acceleration and 2.8% in the average rms suspension deflection is achieved by the semi-active suspensions. The simulations also have shown that either the maximum or the minimum value of the variable damping coefficient is used most of the time; hence, the control is almost like a bang-bang control.

Figure 3 shows the responses of the semi-active system for $\rho_1 = 1.4$, and of the passive system to an initial condition. It is noted that both the average rms vertical acceleration and center of gravity deflection of the sprung mass are reduced effectively by the semi-active suspensions as compared to the passive suspension.
4. Conclusions

A full-car model of a vehicle equipped with variable-damping type semi-active suspensions was developed in this study and bilinear form of state equations was obtained. An analytical study on optimal control of the system has shown that only three variable dampers are sufficient to minimize a performance index consisting of average sprung mass acceleration and suspension deflections. The optimal control algorithm obtained was applied to a vehicle with magnetorheological dampers and moving on a road with a specified profile, and the performance the system was evaluated for constrained control inputs. The results show that rms average sprung mass vertical acceleration is reduced by 6.4% and the average rms suspension deflections by 2.8% by semi-active suspensions as compared to the optimal passive system. The transient oscillations of the system were shown to be effectively damped out by the semi-active suspensions.

5. References

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