Geometry influence on silent chain – sprocket friction

L Jurj¹, R Velicu¹ and M T Lates¹

¹Product Design, Mechatronics and Environment Department, Transilvania University of Braşov, Romania
E-mail: rvelicu@unitbv.ro

Abstract. The silent chains in theory should not present friction at the level of the plate entering in contact with the sprocket, based on the literature. Experimental results have shown that there is a certain amount of friction, which can be inspected even visually. This scientific paper presents a geometric simulation using the exact geometry of the plate and the sprocket, to calculate the amount of mechanical work lost by friction at the entering of the plate in contact with the sprocket teeth. This modelling considers the different deviations of the pitch of the chain, resulted from nominal tolerances, elastic elongation and wear.

1. Introduction
Different studies were performed on the topic of friction reduction in internal combustion engine components. One of the field where we are interested in is the valve timing system. Within this area of the internal combustion engine, different belt and chain drives were studied. Related to chain drives, in [1] different methods were used to reduce the friction in chain transmissions with different chain types, like surface coatings, different material compositions and improved manufacturing. Different mathematical models and testing procedures were elaborated by [2], these methods helped to estimate the losses due to friction between the elements of the chain, and between the chain and the components it is in contact with.

Within the PhD thesis of [3] experimental and theoretic studies were conducted to optimize the guiding system from the point of view of reducing friction losses in chain transmissions. The contact between the chain and the guide is the subject of a model for simulation with results on the type of lubrication between the elements, and optimisation of the system.

In [4], the pitch deviation, friction and centrifugal effects were studied over the load distribution in a bush chain transmission. It is concluded that the load on the driving sprocket teeth is higher than on the driven sprocket teeth in presence of friction. The pitch deviation due to wear determines a supplementary movement of the bush on the surface of the sprocket teeth, inducing friction and wear between the elements.

Studies on bush chains concerning the contact from the joints of the chain are presented in [5]. Contact points and contact angles between bush and sprocket’s tooth were determined for different pitches and the different values of pitch deviation.

The only model for friction losses in silent chains transmissions, that we found studying chain references, is the basic one considering only the pin-plate friction, neglecting possible friction between plates and sprocket’s teeth.

Another aspect is that the silent chains are not standardized. Based on measuring dimensions of existing silent chains, the effects of the friction force on the contact force between the silent chain and
the sprocket are presented in [6]. Contact forces between silent chain and sprocket are inputs for any model of silent chain friction losses.

This paper proposes a model for friction losses based on pitch deviations (pitch of chain is bigger than pitch on the sprocket). The influence of geometrical parameters on chain-sprocket friction losses is presented together with a comparison with the known friction losses due to pin-plate friction.

2. Theoretical model

In theory, the silent chain should get in contact with the sprocket without any friction appearing. On experimentally tested chains with their corresponding sprockets even wear marks can be found at the level of the silent chain plates and the sprocket teeth. This can be explained by sliding between these elements which can be caused by deviations in pitch on the chain. These deviations could come from manufacturing inaccuracies, or nominal deviations; from deviations caused by tensioning of the chain and from elongation of the chain due to wear. Due to wear at the level of the chain’s cylindrical joints the clearances get bigger, up to 0.1…0.3% of chain pitch in case of bush chains. Because of the lack of information about pitch deviations of the silent chain, in this scientific study it will be considered an interval of 0.1…0.6% of chain pitch per link. These pitch deviations of the chain, disturb the plates positioning between the sprocket’s teeth gaps, in a way that the contact point can have a certain travel at the sprocket teeth flank, while the plate rotates with the corresponding pitch angle of the sprocket.

The theoretical model is based on a few simplifying hypotheses, presented on figure 1a:
- It is considered that a single point on the chain plate is in contact with the sprocket’s teeth all the time, while in reality this point is moving along the plate’s outer flank. In this way, the distance travelled with sliding friction will be (with our calculation model) slightly reduced.
- The sprocket’s teeth has a straight profile, while in reality, it has an curved profile, meaning that the distance travelled by the contact point with friction will be, by our calculation model, slightly increased.

\[
p_{\text{real}} = p + a + a_i + a_w = p + a_p
\]  

(1)

where: \( p \) is the pitch of the chain; \( a \) is the nominal deviation of the chain; \( a_i \) is the deviation due to tensioning and \( a_w \) represents the deviation of the pitch due to wear. The total pitch deviation \( a_p \) will be considered in the range:

\[
a_p = (0.1…0.6)\% \text{ of } p.
\]  

(2)

**Figure. 1.** Geometry of the tooth-plate contact point travelling due to chain pitch elongation.

The deviation \( a_r \) resulted from pitch deviation is on a perpendicular direction on the silent chain’s outer flange and can be calculated with the following formula:
The deviation along the line linking the contact point with the centre point of the pin, $a_r$, can be calculated using:

$$a_r = a_f \times \frac{1}{\cos[90 - (\alpha_x + \alpha_\gamma)]}$$

Based on [7], the position of the contact point is defined by the geometrical parameters $d_x$ and $d_y$, shown in Figure 1.

The first step in the theoretical model towards finding the distance over which the contact point lose mechanical work is defining the first the position of the contact point in relation with the pin centre point:

$$d_r = (D_x + D_y)^{1/2}$$

where:

$$D_x = p + d_x \sin \alpha_x - d_y \cos \alpha_x$$

$$D_y = d_x \cos \alpha_x + d_y \sin \alpha_x$$

Using the generalized Sinus theorem the angle between the pitch line, and the line linking the contact point with the centre of the pin can be written as:

$$\alpha_r = a \sin \left[ \frac{(d_x + d \cos \alpha_x) \sin \alpha_x}{d_r} \right]$$

The angle between the tangent line over the silent chain plate’s outer flange through the contact point and the line linking the position of the contact point after the pitch deviation can be written using the generalized Sinus Theorem in the corresponding triangle as follows:

$$\omega = a \sin \left[ \frac{d_x \sin [180 - (\alpha_x + \alpha_\gamma)]}{d_r + a_r} \right]$$

The angle needed to calculate the length of arc described by the contact point travel:

$$\theta_l = \alpha_x + \alpha_\gamma - \omega$$

The length of the contact point displacement on the sprocket can be computed by:

$$l_{f \_sprocket} = \frac{\pi (d_x + a_y) \theta_l}{180}$$

while the length travelled by the contact point calculated in function of $p$ is:

$$l_{f \_sprocket}/p = \frac{l_{f \_sprocket}}{p}$$

and the same length expressed in percentage:

$$l_{f \_sprocket}/p\% = l_{f \_sprocket}/p \times 100.$$
where \( \mu \) is the friction coefficient between plate and sprocket, \( F_{z1} \) represents the force that is coming as a reaction force between the sprocket and the silent chain teeth [5] and \( l_{f_{sprocket}} \) is the distance the contact point slides, during the pitch angle rotation, along the sprocket’s teeth flank.

For comparison, there will be also calculated the friction torque in the cylindrical joint of the pin-plate, for each point of chain entering or exiting a sprocket [8]:

\[
T_{f_{pin}} = \mu d_{pin} F_{1}
\]

where \( \mu \) is the friction coefficient between pin and plate (considered the same with the friction coefficient between plate and sprocket), \( d_{pin} \) - pin diameter; \( F_{1} \) – force in the chain arm.

The friction torque resulted from the friction between the sprocket and the plate, also for each point of chain entering or exiting a sprocket, is calculated with:

\[
T_{f_{sprocket}} = \frac{\mu F_{x} l_{f_{sprocket}}}{\theta}
\]

where \( \theta \) is the pitch angle.

3. Results
For the theoretical model of the contact point displacement due to pitch deviation, and for the work consumed along the path described by the contact point, there were used different parameters of the silent chain LD8 with pitch of 8 mm, and a sprocket with 23 teeth. The input parameters of are shown in Table 1.

| Table 1. Input parameters for the theoretical model. |
|-----------------------------------------------|
| \( p \) | 8 \( \text{mm} \) |
| \( z \) | 23 |
| \( d_{x}/p \) | 0.35 \( \ldots \) 0.6 |
| \( d_{z}/p \) | 0.2 \( \ldots \) 0.5 |
| \( x \) | 50 \( \ldots \) 60 \( \text{deg} \) |
| \( a_{p} \) | \( 0.1 \ldots 0.6 \) \( p \) \( \text{mm} \) |
| \( F_{1} \) | 1000 \( \text{N} \) |
| \( F_{x} \) | 500 \( \text{N} \) |
| \( \mu \) | 0.1 |

Working with the relative values of \( d_{x}/p \) and \( d_{z}/p \) [5] the resulting diagrams can be used to study different situations regardless of the value of the chain pitch.

The position of the contact point is defined by the distance \( d_{z} \) and, for involute sprockets [7], this distance depends on the number of teeth of the sprocket and the chosen profile displacement.

Figure 2 and Figure 3 are showing the effects of the contact point position (\( d_{z}/p \)) over the friction length and the friction torque that appears in the considered silent chain transmission.

Figure 2 presents the dependency of the friction length depending on the pitch of the chain in percentage over the pitch deviation presented in percentage. On Figure 3, values of the friction torques from the chain plate and the sprocket and the friction torque that appears between the pin and the chain plate are presented in function of the pitch deviation.

It is clear that the plate-sprocket friction losses increase with increased deviation of the chain pitch.

Figure 4 highlights the effects of the chain plate dimension \( d_{x} \) on the sliding friction length and Figure 5 shows the effects of the angle \( a_{x} \) on the sliding friction length.
Figure 2. Sliding friction length in function of the corresponding pitch deviation with the effect of the contact point position.

Figure 3. Friction torque at the level of the sprocket and the pin of the chain transmission with the effect of the contact point position.

Figure 4. Friction length in function of the corresponding pitch deviation with the effect of the plate dimension.
Figure 5. Sliding friction length in function of the corresponding pitch deviation with the effect of chain plate flank angle.

4. Conclusions
Based on the theoretical model of the silent chain – sprocket contact point displacement it can be seen that the contact point travel presents different values with the variation of the contact point position, the chain plate dimensions and the chain’s outer flange angle. The theoretical model has shown that the position of the contact point has the greatest effect over the friction length. It also has shown that the friction torque has highest values with the increase of the parameter \( d_z/p \).

Presenting the values of the friction torque in the cylindrical joint, on the same chart, helps the better understanding of the friction losses that can appear in a silent chain transmission. This comparison shows that the friction in the pin-plate joint, can be as much as twice the value of the friction torque from the chain-sprocket contact.

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