Hybrid Position/Force Control with Virtual Impedance Model of Robot Manipulators

Guanhua Hu¹, Qingjiu Huang², * Takuya Hanafusa²

¹School of Information and Electronic Engineering, Zhejiang Gongshang University, Hangzhou, Zhejiang Province, 310018
²Control System Laboratory, Graduate School of Engineering, Kogakuin University, Tokyo, 163-8677, Japan

*Corresponding author’s e-mail: huang@cc.kogakuin.ac.jp

Abstract. Industrial robots require force control in applications such as grinding, and impedance control is a representative method of force control. However, the current impedance control has no clear solution to the hybrid control of position and force for robot manipulators. Therefore, this paper distinguished the position control and force control of the top of robot according to the axis, converted the desired force to position compensation based on the kinematics and dynamics of the robot, and proposed a hybrid position/force control with virtual impedance model of robot manipulators. Moreover, this paper established a grinding simulation model of a three-joint robot manipulator on Matlab/Simulink, and verified the effectiveness of the control method proposed in this paper by a numerical analysis of grinding simulation.

1. Introduction

The industrial robots are applied in the automated production, according to whether the robot has an interaction force with the external environment, their work can be divided into contact-work and non-contact-work. Among them, spraying, welding, etc. which do not produce the interaction force with the external environment, are non-contact operations. The contact-work that interacts with the external environment includes polishing, grinding, and assembly. Therefore, in the contact-type work of the robot, it is necessary to introduce force control in order to adjust the output force of the top of the manipulator in real time and improve the flexibility and safety of the external environment and the workpiece.

Regarding representative control methods of robot force control, there are hybrid position/force control and impedance control. The hybrid position/force control implements position control and force control in different axis directions of workpiece according to the operation requirements. Impedance control allows the manipulator to obtain better compliance with the outside world by adjusting the impedance parameters of the target environment.

Impedance control of robot is usually based on force feedback control. The robot controller receives the force command and the contact force of the robot in the environment [1]. Because the high-frequency characteristics of force control are too strong and the stability margin is small, the system tends to diverge [2]. At present, the current impedance control has no clear solution to the hybrid control of position and force for robot manipulators. Therefore, this paper proposes a hybrid position/force control with virtual impedance model of robot manipulators [3]. It distinguishes the position control and force control of the manipulator according to the axis based on the existing impedance control, and converts the change of the desired force at the top of the manipulator into a position compensation in the
impedance control system based on the kinematics and dynamics of the robot, and superimpose it on the target position [4]. At the same time, this paper establishes a three-joint manipulator wall polishing simulation model on Matlab/Simulink, and the effectiveness of the control method proposed in this paper is verified by the numerical analysis of the polishing process.

2. Mechanical structure and dynamic model of three-joint manipulator

2.1. Mechanical structure of three-joint manipulator

In this study, the three-joint manipulator is used as the controlled object to verify the control method. The mechanical structure of the three-joint manipulator is shown in Figure 1. It is a series structure with three joints $\theta_1$, $\theta_2$, $\theta_3$ and three connecting links L1, L2, L3.

2.2. Dynamic model of three-joint manipulator

For the above three-joint manipulator, in work conditions contacting with the external environment, the dynamic equations can be derived according to the Lagrange energy balance formula:

$$D(\theta)\dot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) = \tau + J^TF$$

Where $\theta, \dot{\theta}, \ddot{\theta}$ are the vectors of the joint angle, angular velocity, and angular acceleration in each joint space; $D(\theta)$ is a $3 \times 3$ joint inertia matrix; $C(\theta, \dot{\theta})$ is a $3 \times 3$ centrifugal force and Coriolis force matrix; $g(\theta)$ is a $3 \times 1$ gravity term vector; $\tau$ is a $3 \times 1$ joint torque vector; $J$ is a $3 \times 3$ Jacobian matrix; $F$ is a $3 \times 1$ vector, representing the contact force between the manipulator and the external environment.

3. Hybrid position/force control with virtual impedance model

3.1. Impedance model based on virtual dynamics

In order to realize the impedance control of the manipulator, we virtualize a dynamic model composed of mass, spring and damper between the manipulator and the workpiece. This impedance model based on virtual dynamics embodies the balance between the change in the position of the end of the manipulator and the change in the external environmental force it receives. Taking the x-axis direction of the top of the manipulator as an example, the mathematical expression of the impedance model based on virtual dynamics is as follows.

$$\Delta F = M_i \Delta \ddot{x} + D_i \Delta \dot{x} + K_i \Delta x$$

$$\Delta F = F - F_d \quad \Delta x = x - x_d$$

$\Delta F$ is the change between the external force and the target force. $\Delta x$, $\Delta \dot{x}$, $\Delta \ddot{x}$ respectively are the difference between the position of the top of manipulator and the target position, and its velocity and acceleration. $M_i$, $D_i$, $K_i$ are the mass, damping coefficient and spring coefficient of the virtual dynamic model respectively. In actual work, the received external force can be obtained using a force sensor installed at the top of manipulator. And the target values of force and position are set according to task
needs, and $\Delta F$ and $\Delta x$ can be calculated and renewed by these target values. According to the virtual dynamic model of equation (2), $\Delta F$ can be converted to $\Delta x$ by equations (3) and (4).

$$\Delta \ddot{x} = (\Delta F - K_i \Delta x_0 - D_i \Delta x_0)/M_i$$

$$\Delta x = \int (\int \Delta \ddot{x} dt) dt$$  

$\Delta x_0$, $\Delta \dot{x}_0$ are the $\Delta x$, $\Delta \dot{x}$ of the previous sampling period in the real-time calculation. The $\Delta x$ obtained according to equation (4) is superimposed on the target position $x_d$ of the top of the manipulator as a compensation to the target position, thereby achieving the target force $F_d$. The control flow chart is shown in Figure 2.

$$x_d' = x_d + \Delta x$$  

Through equation (6), the corrected value of target position $x_d'$ at the top of the manipulator is converted into the target angle $\theta_d$ of each joint of the manipulator.

$$\theta_d = \Lambda^{-1}(x_d')$$  

$\Lambda^{-1}$ is the inverse kinematics of the manipulator converting the position and posture to joint angles. Through the position control of joint angles, each joint of the manipulator can track target angles $\theta_d$.

$$\tau = \Gamma(e_\theta) = \Gamma(\theta - \theta_d)$$

$\Gamma$ is the feedback control of the angular positioning of the joint of the manipulator, and a suitable controller can be selected according to the dynamic model of the manipulator and the dynamic characteristics of the system.

In this way, the hybrid position/force control of the manipulator is realized by the impedance model and the feedback control of the joint angle. Where, because the feedback control used in the force control is the position control of the joint angle, the stability margin of the system becomes higher, and it is not easy to diverge, thereby a better force control can be obtained [6].
4. Simulation of wall-contact work with three-joint manipulator

4.1. Simulation model and control flow of wall-contact work

In order to facilitate the establishment of the model and the simulation of the contact work, reflect the characteristics of the control method, the three-joint manipulator as shown in Figure 1 was selected as the modeling object. The basic structural parameters of the manipulator are shown in Table 1. In Matlab/Simulink, a simulation model of the wall-contact work shown in Figure 4 was constructed, and the control system was designed.

First, a wall perpendicular to the ground and with a stiffness $K_w = 1000$ [kg/m] was placed at 1 m in front of the manipulator. The top of the manipulator gradually approaches the wall along the x axis direction from static state, until it contacts the wall, and the force between it and the wall reaches the preset target value. Next, let the top of the manipulator perform translational movement in the y axis and z axis directions while maintaining a certain force against the wall in the x axis direction, and at the same time consider the friction with the wall. The period of the entire simulation process is 8 seconds. Figure 4 shows the initial state of the manipulator; 0-3 seconds means that the manipulator moves forward from the initial state until it contacts the wall. At this time, the state of the manipulator is shown in Figure 5; 3-4 seconds for the manipulator to be static state; 4-6 seconds for the top of manipulator to make translation movement contacting on the wall surface shown in Figure 6; 6-8 seconds for the manipulator to return to a static state. The sampling time of the control system is 0.01 seconds. The contact force between the top of the manipulator and the wall is continuously sampled during the working process, and the position compensation $\Delta x$ is calculated by the hybrid position/force impedance control model. Figure 4 and 5 respectively are schematic diagrams of the initial position of the manipulator and the position after contacting the wall.

Table 1. structural parameters of the three-joint manipulator.

|   | L [m] | Lg [m] | M [kg] | I [kg \cdot m^2] |
|---|-------|--------|--------|-----------------|
| L1 | 0.5   | 0.25   | 3.0    | 0.0272          |
| L2 | 0.5   | 0.25   | 2.5    | 0.0226          |
| L3 | 0.5   | 0.25   | 1.0    | 0.0090          |

Figure 4. The initial state of the manipulator for wall contact.  
Figure 5. Manipulator touches the front wall.
4.2. Analysis of inverse kinematics of manipulator

The lengths of the three-joint manipulator links in this paper are L1, L2, and L3, respectively. The target position of the top of the manipulator corrected by the impedance model is \( x'_d = [x', y', z'] \). According to the geometric relationship between the position of the cartesian coordinate space of the three-joint manipulator and the joint angle in the joint coordinate space, the joint angle \( \theta_d = [\theta_{d1}, \theta_{d2}, \theta_{d3}] \) of the three joints of the manipulator with respect to the target position \( x'_d \) can be calculated as follows.

\[
\theta_{d1} = \arctan(\frac{y'}{x'})
\]

\[
\theta_{d2} = -\arctan\left(\frac{z' - L}{\sqrt{x'^2 + y'^2}}\right) + \arctan(\frac{a}{b})
\]

\[
a = \sqrt{x'^2 + y'^2 + (z' - L)^2 - b^2}
\]

\[
b = \frac{p_x^2 + p_y^2 + (p_z - L)^2}{2L^2}
\]

If \( p_x > L \),

\[
\theta_{d3} = -\arctan(\sqrt{1 - \left(\frac{p_x^2 + p_y^2 + (p_z - L)^2}{2L^2}ight)^2})
\]

If \( p_x < L \),

\[
\theta_{d3} = \arctan(\sqrt{1 - \left(\frac{p_x^2 + p_y^2 + (p_z - L)^2}{2L^2}\right)^2})
\]

4.3. The contact force model between the top of the manipulator and the wall

The contact force in the x axis direction between the top of the manipulator and the wall can be simulated by a wall model composed of a spring and a damper. The formula for simulation is as follows.

\[
F_x = K_w \Delta x_w + D_w \Delta x_w
\]

Among them, \( K_w \) and \( D_w \) are the stiffness coefficient and damping coefficient of the wall, respectively. \( \Delta x_w \) and \( \Delta x_w \) are the deformation of the wall due to the squeeze of the top of the manipulator and the differential of the deformation.

There are the following sliding friction forces between the top of the manipulator and the wall in the y direction and the z direction.

\[
F_y = F_z = \mu F_x
\]

Among them, \( \mu \) is the sliding friction coefficient between the top of the manipulator and the wall. \( F \) between the manipulator and the wall can be defined as follows.

\[
F = [F_x, F_y, F_z]^T
\]

In order to find \( \Delta x_w \) and \( \Delta x_w \) in equation (12), first we can write the homogeneous transformation matrix of the three-joint manipulator as follows.

\[
T = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Then,

\[
x = T(1,1) \quad y = T(2,1) \quad z = T(3,1)
\]
Next, we can find the Jacobian matrix of the manipulator as follows.

\[
J = [J_1 \ J_2 \ J_3]
\]

(17)

The differential of \(x\) can be obtained by the following formula.

\[
\dot{x} = J[\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]^T
\]

(18)

\(\theta_1, \theta_2, \theta_3, \) and \(\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3\) are obtained from the manipulator dynamics described in equation (1).

\[
\dot{x} = x(1,1) \quad \dot{y} = x(2,1) \quad \dot{z} = x(3,1)
\]

(19)

\[
\Delta x_w = x - x_w \quad \Delta \dot{x}_w = \dot{x} - \dot{x}_w = \dot{x}
\]

(20)

4.4. Dynamic analysis of manipulator

The dynamic equation of the manipulator in this simulation is shown in equation (1), and its solution is as follows.

\[
\ddot{\theta} = \left( \tau + J^T F - C(\theta, \dot{\theta}) \dot{\theta} - g(\theta) \right) / D(\theta)
\]

(21)

\[
\theta = \int \dot{\theta} \, dt
\]

(22)

4.5. Simulation numerical analysis

The control block diagram for the simulation of the wall contact work of the manipulator is shown in Figure 7. In the position control of the joint angle of the manipulator, we use the PID control method. The entire simulation process of the works is described in Section 4.1. The numerical analysis results of the simulation are shown in Figure 8-14[7].

The contact force \(F\) between the top of the manipulator and the wall can be obtained by equation (14). Substituting the obtained contact force \(F\) and the target force \(F_d\) into equation (2) (3) (4), the position compensation \(\Delta x\) of the top of the manipulator that achieves the target value \(F_d\). The corrected value \(x'_w\) of the target position at the top of the manipulator can be obtained by equation (5). By using equations (8) (9) (10) (11), the target value of the joint angle \(\theta_d\) corresponding to the corrected value of the target position \(x'_d\) can be obtained. Finally, through the PID controller, the control torque \(\tau\) of the manipulator joint can be obtained, so as to carry out simulation numerical analysis.

It can be seen from Figure 8 that a contact force is quickly generated in the x axis direction after the manipulator touches the wall at 3 seconds. After the contact force is controlled by the impedance of this study, it can basically reach the set target force of 5N, though the manipulator horizontally moves on the wall for 4-6 seconds. After the manipulator stopped moving at 6 seconds, the contact force stabilized at 5N. Although the contact force of the manipulator is fluctuating within ±0.3N during the translation, from the perspective of the allowable range of the breaking strength of the processing tool and the workpiece, this is the error range allowed by the force control.

The target forces of the manipulator in the y axis direction and z axis direction are 0N, so that the manipulator can do free processing movement in the y axis direction and the z axis direction. It can be seen from Figure 9 and Figure 10 that in order to offset the sliding friction force, the manipulator generates resistance force in the y axis direction and the z axis direction in 4-5 seconds and 5-6 seconds, respectively.

Figure 11-13 is the position target value of the manipulator in the three directions during the whole simulation process and the corrected target position value obtained by the impedance control. It can be seen that the corrected value is very close to the target value.

Figure 14 shows the position compensation in the x axis, y axis, and z axis directions. In 3-4 seconds, because the control system gives the manipulator an initial instantaneous increase in the target force of the step signal, the contact force suddenly overshoots. Designing an initially gradually increasing target
force can eliminate the instantaneous overshoot of contact force, which will be the next topic of this research. In 4-6 seconds, it can be seen that in order to maintain the target force in the three directions, the position compensation of no more than ± 0.25mm is generated, which is in line with the formula (2)-(4) of impedance control theory.
Observation of the above simulation results, it can be concluded that the hybrid position/force with virtual impedance model of this paper can not only maintain a certain contact force between the manipulator and the wall, but also eliminate the influence of the friction force between the manipulator and the wall, and can achieve high-precision machining trajectory of the manipulator.

5. Conclusion
Industrial robots require force control in applications such as grinding, and impedance control is a representative method of force control [8]. However, the current impedance control has no clear solution to the hybrid control of position and force for robot manipulators. Based on the existing impedance control, this paper distinguished the position control and force control of the top of robot according to the axis, converted the desired force to position compensation based on the kinematics and dynamics of the robot, and proposed a hybrid position/force control with virtual impedance model of robot manipulators. Moreover, this paper established a grinding simulation model of a three-joint robot manipulator on Matlab/Simulink, and verified the effectiveness of the control method proposed in this paper by a numerical analysis of grinding simulation.

The control system of this paper gave the manipulator an initial instantaneous increase in the target force of the step signal, resulting in instantaneous overshoot of the contact force [9]. Designing an initial gradually increasing target force to eliminate the instantaneous overshoot of contact force will make the control method proposed in this paper more practical, which will be the next topic of this study.

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