Proton Decay in Intersecting D-brane Models

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Abstract

We analyze proton decay via dimension six operators in certain GUT-like models derived from Type IIA orientifolds with $D6$-branes. The amplitude is parametrically enhanced by a factor of $\alpha_{GUT}^{-1/3}$ relative to the corresponding result in four-dimensional GUT’s. Nonetheless, even assuming a plausible enhancement from the threshold corrections, we find little overall enhancement of the proton decay rate from dimension six operators, so that the predicted lifetime from this mechanism remains close to $10^{36}$ years.

April, 2003

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1 Introduction

Grand Unified Theories in four dimensions have had impressive successes, in accounting for the quantum numbers of fermions [1], in predicting – with the aid of supersymmetry – a value of $\sin^2 \theta_W$ that is in excellent agreement with experiment [2], and in pointing twenty years in advance of decisive measurements to the right order of magnitude of neutrino masses [3]. GUT theories also make the exciting prediction that the proton may decay with a lifetime close to present experimental bounds.

With supersymmetry, even if one assumes an $R$-parity symmetry to avoid catastrophic proton decay at low energies, there actually are two different GUT-based mechanisms for proton decay. The proton may decay due to dimension five operators of the form $\int d^2 \theta Q^3 L$, where $Q$ and $L$ are quark and lepton superfields. Or it may decay by dimension six operators of the form $\int d^4 \theta Q^3 Q^* \tilde{Q}^* \tilde{L}^*$ coming from gauge boson exchange; this effect is similar to proton decay in GUT’s without supersymmetry, but is significantly slower because supersymmetry raises the GUT scale. In the simplest models, proton decay by dimension five operators dominates, and in fact present experimental bounds make life difficult for these models [4, 5, 6] (see, however, [7]). This suggests the possibility that some mechanism suppresses the dimension five operators and that gauge boson exchange may after all be the dominant mechanism. Various methods to suppress the dimension five operators while preserving the successes of GUT’s are known, though they tend to be somewhat technical. For example, one construction based on discrete symmetries [8] is fairly natural in the class of models that we will consider in this paper.

Most GUT-like string-based models of particle physics do not precisely lead to four-dimensional GUT’s, since unification takes place only in higher dimensions (for an early review, see [9], chapters 15 and 16), leading among other things to possibilities for GUT symmetry breaking by discrete Wilson lines [10] and to a higher-dimensional mechanism...
for doublet-triplet splitting [11]. Generally, models such as the heterotic string on a Calabi-Yau manifold lead to qualitatively similar issues concerning proton decay to those in GUT’s, though the details are somewhat different. One important reason that the details are different is that because of the higher-dimensional structure, there is generally in these models no precise answer to the question, “Which color triplet gauge boson is the \( SU(5) \) partner of the standard model gauge bosons?” There is a lightest color triplet gauge boson, whose wave-function in the compact dimensions depends on the details of the model, and its exchange may, depending on the model, give the right order of magnitude though not the correct numerical value of the dimension six part of the proton decay amplitude.

Recently [12], these issues were reconsidered in another class of models – \( M \)-theory on a \( G_2 \) manifold. (Actually, in many cases these models give dual descriptions, useful in a different region of the parameter space, of the same models that can be studied via the heterotic string on a Calabi-Yau manifold.) The GUT threshold correction to proton decay was computed in this class of models and was seen to give a potential enhancement of the proton decay rate. It was also shown that in this type of model, because of the way quarks and leptons are localized, exchange of the lightest color triplet gauge boson does not dominate the proton decay amplitude. On the contrary, a field theoretic attempt to compute a proton decay amplitude by summing over Kaluza-Klein harmonics runs into an ultraviolet divergence, and consequently the correct answer depends on the cutoff provided by \( M \)-theory. Formally, this UV divergence enhances the proton decay amplitude by a factor of \( \alpha^{-1/3}_{GUT} \) compared to what it would be in a four-dimensional model. (This factor must be combined with the threshold factor, of course.) The coefficient of \( \alpha^{-1/3}_{GUT} \) is universal and independent of the details of the model (such as how the GUT symmetry is broken) – in fact, the dominant four-fermi operator is local and invariant under the full GUT symmetry, in contrast to the usual situation in four-dimensional GUT’s. However, present knowledge of \( M \)-theory does not make it possible to compute the numerical coefficient of this operator.

As an alternative, we will in the present paper consider another dual class of models in which the calculation can be done. These models fall in the general class of intersecting brane worlds [13, 14, 15, 16, 17, 18, 19] – Type IIA superstrings with gauge bosons supported on \( D6 \)-branes and chiral matter multiplets at intersections of the \( D6 \)-branes. Many supersymmetric GUT-like models have been constructed along these lines [20, 21, 22], though so far none with the mechanism of symmetry breaking by Wilson lines on the \( D6 \)-brane that we will assume in the present paper to justify borrowing various above-cited results.\(^1\) (In actual model-building, one typically con-

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\(^1\)In the dual context of \( M \)-theory on \( G_2 \) manifolds, some examples of symmetry breaking by Wilson lines were considered in [23].
siders branes in an orbifold of a six-torus, but more generally one may start with any Calabi-Yau manifold: see, for example, [24].) Threshold corrections in intersecting D6-brane GUT-like models were calculated in [25]. These models are dual to M-theory on a manifold of $G_2$ holonomy as $g_s$, the Type IIA string coupling constant, becomes large. The chiral matter fields are localized (at brane intersections) in a similar way to what happens in M-theory on a $G_2$ manifold, and accordingly we will find the same anomalous factor of $\alpha^{-1/3}_{\text{GUT}}$ in the proton decay amplitude. The difference is that in perturbative Type IIA string theory, everything is explicitly calculable, and hence we will be able, at least for $g_s \ll 1$, to be precise about the numerical factors. Regrettably, the numerical factors accompanying $\alpha^{-1/3}_{\text{GUT}}$ are such that even assuming a plausible enhancement from the threshold factors, the proton decay rate is comparable to or only slightly greater than that in standard SUSY-GUT’s. Of course, the precise factors that appear in the corresponding M-theory (or large $g_s$) limit are still unknown.

In this paper we will not be concerned with any specific model, but will rather try to incorporate universal features of the GUT-like intersecting D-brane models relevant to calculation of the proton decay rate. Typically one needs a stack of D6-branes intersecting an orientifold fixed sixplane along $3 + 1$ dimensions. On the covering space we then have a stack of D6-branes intersecting an image set of D6'-branes. If there are 5 D6-branes in the stack, then on the covering space we find gauge group $SU(5) \times SU(5)$, with open strings localized at the intersection transforming in the bifundamental representations $(5, \bar{5}) + (\bar{5}, 5)$. After the orientifold projection we find $SU(5)$ gauge theory with matter in $10 + \bar{10}$. Our goal is to calculate the 4-fermion contact term whose $SU(5)$ structure is $10^2\bar{10}2$. Such an interaction mediates proton decay processes such as $p \rightarrow \pi^0e^+_L$ (and, depending on the assumed flavor structure, other modes with $e^+_L$ replaced by $\mu^+_L$ and/or $\pi^0$ by $K^0$). The calculation is conveniently carried out on the covering space where we need the 4-point function for two $(5, \bar{5})$ states and two $(\bar{5}, 5)$ states. The calculation is sensitive to the local structure of the intersection, and is insensitive to how the D6-branes are wrapped around the compact space, as long as its size is somewhat greater than the string scale so that worldsheet instantons are suppressed.

Four-dimensional GUT’s also have dimension six operators $10\bar{10}5\bar{5}$ which lead to proton decay modes such as $p \rightarrow \pi^0e^+_R$ and $p \rightarrow \pi^+\nu$; as observed in [12] in the analogous $M$-theory case, such interactions do not arise generically in a model of this type. To get such an interaction, we need two stacks of branes that are not mirror images to meet on an orientifold plane; in general, this would be a coincidence. Both four-dimensional GUT’s and the brane worlds have additional $5^2\bar{5}^2$ interactions (which in the brane worlds arise from brane intersections away from the orientifold planes), but these do not violate baryon number.
2 Vertex Operators

Without a loss of generality we may assume that the D6-branes are oriented in the 0123468 directions. The D6'-branes intersect them along the 0123 directions that will have the interpretation of a 3 + 1 dimensional ‘intersecting brane world.’ To specify its orientation in the six transverse directions, we define the complex coordinates

\[ z_1 = x^4 + ix^5, \quad z_2 = x^6 + ix^7, \quad z_3 = x^8 + ix^9. \]  

(1)

In order to preserve \( N = 1 \) supersymmetry in 3 + 1 dimensions, the rotation must act as an \( SU(3) \) matrix on the three complex coordinates [13]. Choosing the matrix to be diagonal, we see that the rotation that turns the D6-branes into the D6'-brane acts as

\[ z_1 \rightarrow \exp(\pi i \theta_1) z_1, \quad z_2 \rightarrow \exp(\pi i \theta_2) z_2, \quad z_3 \rightarrow \exp(\pi i \theta_3) z_3, \]  

(2)

where

\[ \theta_1 + \theta_2 + \theta_3 = 2 \mod 2 \mathbb{Z}. \]  

(3)

We need to construct the vertex operators for the 6 − 6' and 6' − 6 open strings. Such operators, when inserted on the boundary of the disk, create discontinuities in the boundary conditions for the transverse coordinates and their fermionic partners [26]. A standard method for studying correlators on the upper half-plane is the doubling trick where the half-plane is replaced by the entire plane, but with only the holomorphic part of the field on it (for a review, see [27]). If we consider complex fields \( X^i \) corresponding to the transverse coordinates \( z^i \), then their Laurent expansion around the insertion of the vertex operator has mode numbers shifted by \( \theta_i \) [35]. This is analogous to what happens near a twist field introduced in orbifold theories and we can use similar methods for calculating the correlation functions.

Consider a bosonic twist field \( \sigma_+ \) which creates a discontinuity in a complex coordinate \( X \). Then we have the OPE [28, 29, 30]

\[ \partial X(z) \sigma_+ (w) \sim (z - w)^{\theta - 1} \tau_+, \quad \partial \bar{X}(z) \sigma_+ (w) \sim (z - w)^{-\theta} \tau'_+. \]  

(4)

The dimension of \( \sigma_+ \) is \( \Delta_\sigma = \theta (1 - \theta)/2 \), and we will assume that \( \theta \) lies between 0 and 1. Now consider the complex fermion \( \psi \), which is a worldsheet superpartner of \( X \). By worldsheet supersymmetry, its mode numbers are also shifted by \( \theta \). In the Ramond sector, \( \sigma_+ \) must be accompanied by the fermionic twist \( s_+ \) such that

\[ \psi(z) s_+(w) \sim (z - w)^{\theta - 1/2} t_+, \quad \bar{\psi}(z) s_+(w) \sim (z - w)^{1/2 - \theta} t'_+. \]  

(5)

If we bosonize \( \psi \) into a real scalar \( H \), whose Green’s function is

\[ \langle H(z) H(w) \rangle = -\log(z - w), \]  

(6)
\[ \psi = \exp(iH), \quad \bar{\psi} = \exp(-iH), \quad s_+ = \exp[i(\theta - 1/2)H]. \]  

(7)

This means that the dimension of \( s_+ \) is

\[ \Delta_s = \frac{1}{2} \left( \theta - \frac{1}{2} \right)^2. \]

(8)

Therefore,

\[ \Delta_s + \Delta_\sigma = 1/8, \]

(9)

independent of the rotation angle \( \theta \).

The complete vertex operator that creates a massless \( 6 - 6' \) string in the R sector is

\[ V_+ = \lambda^I_\dot{I}_u_u e^{-\dot{\phi}/2} S^{\alpha} e^{ik_\mu X^\mu} \prod_{i=1}^{3} (\sigma^i_\dot{s}^i_+) \]

(10)

where \( \dot{\phi} \) is the bosonized ghost field. If we bosonize the fermions \( \psi^\mu, \mu = 0, 1, 2, 3 \), into scalars \( h_1 \) and \( h_2 \), then the \( 3 + 1 \) dimensional spinor \( S^\alpha \sim \exp[i(s_1 h_1 + s_2 h_2)/2] \), where \( s_1, s_2 = \pm 1 \). The GSO projection for \( 6 - 6' \) strings requires that \( s_1 s_2 = 1 \), which restricts us to a spinor of definite chirality. \( \lambda^i_\dot{I} \) is a Chan-Paton factor with an index \( i \) in the fundamental of the first \( SU(5) \), and an index \( I \) in the antifundamental of the second \( SU(5) \). Since \( h(e^{-\dot{\phi}/2}) = 3/8 \) and \( h(S^\alpha) = 1/4 \), we see that the requirement that the dimension of \( V_+ \) is 1 gives the 4-d massless dispersion relation \( k^2 = 0 \).

The vertex operator \( V_- \) for \( 6' - 6 \) strings is constructed similarly, except we need to replace the \( \sigma_+ \) and \( s_+ \) twists by \( \sigma_- \) and \( s_- \). The latter are defined by sending \( \theta \to 1 - \theta \) in the OPE. The GSO projection now requires that \( s_1 s_2 = -1 \); this corresponds to 4-d spinor chirality opposite to that of the \( 6 - 6' \) string. Thus, we have the vertex operator

\[ V_- = \tilde{\lambda}^I_\dot{I}_\bar{u} e^{-\dot{\phi}/2} \bar{S}^{\dot{\alpha}} e^{ik_\mu X^\mu} \prod_{i=1}^{3} (\sigma^i_\dot{s}^i_-) \]

(11)

3 The 4-fermion Amplitude in String Theory

Our goal is to calculate the 4-fermion amplitude

\[ \int_{1}^{0} dx \langle V_-(0)V_+(x)V_-(1)V_+(\infty) \rangle. \]

(12)

The crucial ingredient in this calculation is the knowledge of the correlator of the bosonic twist fields \( \sigma \). In the orbifold theories, these were calculated in [28, 29, 30]. We will take the square root of their result to account for the fact that we have the boundary twist fields rather than the bulk ones. (This intuitively plausible result has
been derived in [31] and, up to normalization, in [32]; see also [33, 34] for a treatment of cases where the intersecting D-branes have different dimensionalities.) This gives

\[ \langle \sigma_{-}(0)\sigma_{+}(x)\sigma_{-}(1)\sigma_{+}(\infty) \rangle \sim \sqrt{\sin(\pi \theta)}[x(1-x)]^{-2\Delta_{s}}[F(x)F(1-x)]^{-1/2}, \tag{13} \]

where

\[ F(x) \equiv F(\theta, 1-\theta; 1; x) \tag{14} \]

is the hypergeometric function and \( \sim \) represents a numerical constant.

On the other hand, the correlator of the fermionic twists is simply

\[ \langle s_{-}(0)s_{+}(x)s_{-}(1)s_{+}(\infty) \rangle \sim [x(1-x)]^{-2\Delta_{s}}, \tag{15} \]

while the ghost factor contributes

\[ \langle e^{-\phi/2}(0)e^{-\phi/2}(x)e^{-\phi/2}(1)e^{-\phi/2}(\infty) \rangle \sim [x(1-x)]^{-1/4}. \tag{16} \]

The correlator of the 4-d spin fields multiplied by spinor polarizations gives an \( x \)-independent factor

\[ \langle \exp[i(h_{1}-h_{2})/2](0)\exp[i(h_{1}+h_{2})/2](x)\exp[-i(h_{1}-h_{2})/2](1)\exp[-i(h_{1}+h_{2})/2](\infty) \rangle. \tag{17} \]

Putting together all the parts of the vertex operators, defining as usual

\[ s = -(k_{1} + k_{2})^{2}, \quad t = -(k_{2} + k_{3})^{2}, \tag{18} \]

and including both \( s \)- and \( t \)-channel diagrams, we find the amplitude

\[ A_{4}(k_{1}, k_{2}, k_{3}, k_{4}) = iC(2\pi)^{4}\delta^{4}\left(\sum_{i=1}^{4} k_{i}\right)\bar{\pi}_{1}\gamma_{\mu}u_{2}\bar{\pi}_{3}\gamma_{\mu}u_{4} \quad \text{Tr} \left( \tilde{\lambda}_{1}\lambda_{2}\tilde{\lambda}_{3}\lambda_{4} + \tilde{\lambda}_{1}\lambda_{4}\tilde{\lambda}_{3}\lambda_{2} \right) \]

\[ \cdot \int_{0}^{1} dx x^{-1-a'}(1-x)^{-1-a'} \prod_{i=1}^{3} \sqrt{\sin(\pi \theta_{i})}[F(\theta_{i}, 1-\theta_{i}; 1; x)F(\theta_{i}, 1-\theta_{i}; 1; 1-x)]^{-1/2}, \tag{19} \]

where \( C \) is a normalization that we will fix later, while the \( \lambda \)'s and \( \tilde{\lambda} \)'s are Chan-Paton matrices of 6 – 6' and 6' – 6 strings. In four dimensions, with \( \bar{\pi}_{1}, \bar{\pi}_{3} \) having one chirality and \( u_{2}, u_{4} \) the other, we have \( \bar{\pi}_{1}\gamma_{\mu}u_{2}\bar{\pi}_{3}\gamma_{\mu}u_{4} = -\bar{\pi}_{1}\gamma_{\mu}u_{4}\bar{\pi}_{3}\gamma_{\mu}u_{2} \). This insures the antisymmetry of the amplitude under permutation of particles 2 and 4 (or of particles 1 and 3).
We note that the hypergeometric functions are crucial for the convergence of the integral at the endpoints of the integration region for vanishing momenta \( k_i \). As \( x \to 0 \),
\[
F(x) \to 1 \, , \quad F(1 - x) \to \frac{\sin(\pi \theta)}{\pi} \ln(\delta/x) \, ,
\]
where
\[
\ln \delta(\theta) = 2\psi(1) - \psi(\theta) - \psi(1 - \theta) \, .
\]

Therefore, even with \( s = t = 0 \), we find a convergent integral. Its behavior near \( x = 0 \) is
\[
\sim \pi^{3/2} \int_0^1 \frac{dx}{x} [\ln(1/x)]^{-3/2}
\]
The physical reason for the absence of an IR divergence at vanishing momentum is the following. Even though there is a massless intermediate state from the untwisted sector, the special kinematics of this problem gives no IR divergence. For example, if the \( s \)-channel intermediate state is a massless \( 6' - 6' \) string, which is a gauge boson, then it can carry arbitrary momentum \( q \) along the directions of the \( D6' \)-brane orthogonal to the intersection. These three components of momentum have to be integrated over:
\[
\int d^3q \int_0^1 dx x^{\alpha' q^2 - \alpha' s - 1} = \pi^{3/2}(\alpha')^{-3/2} \int_0^1 dx x^{1 - \alpha' s} [\ln(1/x)]^{-3/2} \, ,
\]
which converges near \( x = 0 \) even for \( s = 0 \). Thus, going from the effective field theory to the form of the string integrand near \( x = 0 \), we find the replacement
\[
\int \frac{d^3q}{q^2 - s} \to \pi^{3/2}(\alpha')^{-1/2} \int_0^1 dx x^{-1 - \alpha' s} [\ln(1/x)]^{-3/2} \, .
\]
Performing the integral on the RHS of (25) with the replacement (24), we find
\[
i \text{Tr}(\tilde{\lambda}_1 \lambda_2 \tilde{\lambda}_3 \lambda_4) K^2 g_s(\alpha')(2\pi)\delta^4(\sum_{i=1}^{4} k_i) \pi_1 \gamma^\mu u_2 \pi_3 \gamma_\mu u_4 \pi^{3/2} \int_0^1 x^{-1-\alpha's}[\ln(1/x)]^{-3/2}. \quad (28)
\]
Equating this to the contribution to \( A_4 \), given in eqn. (19), from the \( s \)-channel \( 6' - 6' \) massless intermediate state,
\[
i \text{Tr}(\tilde{\lambda}_1 \lambda_2 \tilde{\lambda}_3 \lambda_4) C (2\pi)^4 \delta^4(\sum_{i=1}^{4} k_i) \pi_1 \gamma^\mu u_2 \pi_3 \gamma_\mu u_4 \pi^{3/2} \int_0^1 x^{-1-\alpha's}[\ln(1/x)]^{-3/2}, \quad (29)
\]
we find that
\[
C = (2\pi)^{-3} K^2 g_s \alpha' = 2\pi g_s \alpha'. \quad (30)
\]
The low-energy limit of the 4-fermion amplitude is hence,
\[
A_4(k_1, k_2, k_3, k_4) = i(2\pi g_s \alpha') I(\theta_1, \theta_2, \theta_3)(2\pi)^4 \delta^4(\sum_{i=1}^{4} k_i) \pi_1 \gamma^\mu u_2 \pi_3 \gamma_\mu u_4 \text{Tr} \left( \tilde{\lambda}_1 \lambda_2 \tilde{\lambda}_3 \lambda_4 + \tilde{\lambda}_1 \lambda_4 \tilde{\lambda}_3 \lambda_2 \right), \quad (31)
\]
where
\[
I(\theta_1, \theta_2, \theta_3) = \int_0^1 \frac{dx}{x(1-x)} \prod_{i=1}^{3} \sin(\pi \theta_i) [F(\theta_i, 1 - \theta_i; 1; x)F(\theta_i, 1 - \theta_i; 1; 1 - x)]^{-1/2}. \quad (32)
\]
This has been derived for intersections of two stacks of branes without any orientifolding. Thus, in an intersection of a stack of \( N D6 \)-branes with a stack of \( M D6' \)-branes, we would have a \( U(N) \times U(M) \) gauge theory with the fermions supported at the intersection points transforming as bifundamentals, and the above four-fermi interactions. To get an \( SU(5) \) theory with fields transforming in the \( 10 \), we should consider the case that \( N = M = 5 \) and the two stacks are exchanged by an orientifolding operation \( \Omega R \) and intersect on the orientifold plane (\( \Omega \) is the worldsheet parity; \( R \) acts by complex conjugating all three complex coordinates: \( z_i \rightarrow \overline{z}_i, i = 1, 2, 3 \)). Moreover, we pick an orientifolding operation that projects onto fermions whose wavefunctions \( \lambda_i \) and \( \tilde{\lambda}_j \) transform as antisymmetric, rather than symmetric, tensors of \( SU(5) \). Tree level amplitudes in the orientifold theory are obtained by computing tree level amplitudes on the covering space for states that are invariant under the orientifolding projection, and then dividing by 2. The \( 10^2 \overline{10}^2 \) interaction in an orientifold theory is thus derived
from
\[ A_4(k_1, k_2, k_3, k_4) = i(\pi g_s \alpha') I(\theta_1, \theta_2, \theta_3) (2\pi)^4 \delta^4 \left( \sum_{i=1}^{4} k_i \right) \mathbf{p}_1 \gamma^\mu u_2 \mathbf{p}_3 \gamma_\mu u_4 \]
\[ \text{Tr} \left( \tilde{\lambda}_1 \lambda_2 \tilde{\lambda}_3 \lambda_4 + \tilde{\lambda}_1 \lambda_4 \tilde{\lambda}_3 \lambda_2 \right) . \]  

(33)

**Evaluation Of I**

Now let us discuss the numerical evaluation of the integral \( I \). After writing
\[ \frac{1}{x(1-x)} = \frac{1}{x} + \frac{1}{1-x} , \]
noting that the two terms contribute equally, and setting \( x = e^{-t} \) to evaluate the contribution of the first term, we can write \( I \) as
\[ I(\theta_1, \theta_2, \theta_3) = 2 \int_{0}^{\infty} dt \prod_{i=1}^{3} \sqrt{\sin(\pi \theta_i)} \left[ F(\theta_i, 1-\theta_i; 1; e^{-t}) F(\theta_i, 1-\theta_i; 1; 1-e^{-t}) \right]^{-1/2} . \]  

(35)

Let us try to evaluate the integral on the RHS numerically for the most symmetric choice of rotation angles: \( \theta_1 = \theta_2 = \theta_3 = 2/3 \). It turns out that for sufficiently large \( t \), it is hard to maintain numerical precision in evaluating the hypergeometric functions.

To deal with this problem, we will break up the integral into the range from 0 to \( T \) and from \( T \) to \( \infty \). In the first region we use Mathematica to evaluate it numerically, while in the second we may use the asymptotics (20) to replace the integral by
\[ 2\pi^{3/2} \int_{T}^{\infty} dt (t + 3 \ln 3)^{-3/2} = 4\pi^{3/2} (T + 3 \ln 3)^{-1/2} , \]
where we used \( \ln \delta(2/3) = 3 \ln 3 \). In practice, the sum of the two is insensitive to \( T \) in a certain range; we have checked this for \( T \) between 15 and 25. We find
\[ I(2/3, 2/3, 2/3) \approx 11.52 . \]  

(37)

It is interesting that a large number 11.52 emerges from an explicit string calculation.

Now consider a generalization to \( \theta_1 = \theta_2 = \theta \) and \( \theta_3 = 2 - 2\theta \). For the purpose of numerical evaluation, we approximate \( I(\theta, \theta, 2-2\theta) \) by
\[ 2 \sin(\pi \theta) \sqrt{-\sin(2\pi \theta)} \int_{0}^{T} dt \frac{\left[ F(2-2\theta, 2\theta - 1; 1; e^{-t}) F(2-2\theta, 2\theta - 1; 1; 1-e^{-t}) \right]^{-1/2}}{F(\theta, 1-\theta; 1; e^{-t}) F(\theta, 1-\theta; 1; 1-e^{-t})} + 2\pi^{3/2} \int_{T}^{\infty} \frac{dt}{(t + \ln \delta(\theta)) \sqrt{t + \ln \delta(2-2\theta)}} . \]  

(38)
It is interesting to ask how $I$ behaves when one of the rotation angles becomes small. This can be determined from (38) by letting $\theta$ approach $1/2$ from above, so that $\theta_3 = 2 - 2\theta$ corresponds to rotation by close to $\pi$. It turns out that $I$ decreases rather slowly from its maximum value at $\theta = 2/3$. For example, $I(0.55, 0.55, 0.9) \approx 9.89$ and $I(0.51, 0.51, 0.98) \approx 6.69$. Even when all three rotation angles become effectively small, $I$ does not fall off very rapidly. Using (38), we find $I(0.9, 0.9, 0.2) \approx 7.205$; $I(0.95, 0.95, 0.1) \approx 5.505$; $I(0.99, 0.99, 0.02) \approx 2.47$. We conclude that for a broad range of angles the quantity $I$ lies in the range $7 - 11.5$.

4 Comparison To Four-Dimensional GUT’s

According to [35], eqns. (12.1.10b) and (13.3.24), the gravitational action for Type IIA superstrings is

$$\frac{1}{(2\pi)^7\alpha'^4} \int d^{10}x \sqrt{-G} e^{-2\Phi} R.$$ (39)

The string coupling constant is $g_s = e^\Phi$. After reduction to four dimensions on a six-manifold $X$ of volume $V_6$, the gravitational action in four dimensions is

$$\frac{V_6}{(2\pi)^7 g_s^2 \alpha'^4} \int \sqrt{-g} R.$$ (40)

As the coefficient of $R$ in four-dimensional General Relativity is conventionally $(16\pi G_N)^{-1}$, with $G_N$ Newton’s constant, we have

$$g_s^2 \alpha'^4 = 8V_6 G_N \frac{(2\pi)^6}{(2\pi)^6}.$$ (41)

The gauge coupling $g_{D6}$ of gauge fields on a $D6$-brane is defined by saying that the effective action for the gauge fields is

$$\frac{1}{4g_{D6}^2} \int d^7x \sqrt{-g_7} \text{Tr} F_{ij} F^{ij},$$ (42)

where $F_{ij}$ is the Yang-Mills field strength and Tr is the trace in the fundamental representation of $U(N)$. If we take the $D6$-brane worldvolume to be $\mathbb{R}^4 \times Q$, where $Q$ is a compact three-manifold of volume $V_Q$, then the action in four dimensions becomes

$$\frac{V_Q}{4g_{D6}^2} \int d^4x \text{Tr} F_{ij} F^{ij}.$$ (43)

However, before comparing to four-dimensional GUT’s, we must recall that it is conventional to expand the gauge fields as $A_i = \sum_a Q_a A_i^a$, where $\text{Tr} Q_a Q_b = \frac{1}{2} \delta_{ab}$. Similarly,
one conventionally expands $F_{ij} = \sum_a F_{ij}^a Q_a$. If this is done, the action becomes

$$\frac{V_Q}{8g_{D6}^2} \int d^4 x \sum_a F_{ij}^a F^{ij}.$$

(44)

The GUT action is conventionally written

$$\frac{1}{4g_{GUT}^2} \int d^4 x \sum_a F_{ij}^a F^{ija},$$

(45)

where $g_{GUT}$ is the GUT coupling. Hence, we have $g_{GUT}^2 = 2g_{D6}^2/V_Q$. Since $g_{D6}^2 = (2\pi)^4 g_s \alpha'^{3/2}$ according to eqn. (13.3.25) of [35], we have

$$g_{GUT}^2 = \frac{2(2\pi)^4 g_s \alpha'^{3/2}}{V_Q}.$$

(46)

The volume $V_Q$ enters in the running of $SU(3) \times SU(2) \times U(1)$ standard model gauge couplings from very high energies down to the energies of accelerators. Roughly speaking, $V_Q^{-1/3}$ plays the role of $M_{GUT}$, the mass scale of unification, in a four-dimensional GUT theory. To find the precise relation between them, one must compute the one-loop threshold corrections to gauge couplings. This was done in [12] for $M$-theory on a manifold of $G_2$ holonomy. The one-loop threshold corrections are the same in Type IIA as in $M$-theory, since they come from Kaluza-Klein harmonics on $Q$ that are the same in the two theories. So we can borrow the result of [12]. According to that result (see eqn. (3.30) of [12]), the precise relation between $M_{GUT}$ understood as the mass at which the low energy coupling constants appear to unify and $V_Q$ is

$$V_Q = \frac{L(Q)}{M_{GUT}^2}.$$

(47)

where $L(Q)$ is a certain topological invariant of $Q$ (together with Wilson lines on $Q$ that break the GUT symmetry to the standard model) that is known as the Reidemeister or Ray-Singer torsion. $L(Q)$ depends on a model, but is readily computable in a given model. For example, in a model considered in [12] in which $Q$ is a lens space $S^3/\mathbb{Z}_q$ for some positive integer $q$ (which must be prime to 5 if we want an $SU(5)$ model), and the Wilson line on $Q$ has eigenvalues $\exp(2\pi i \delta_i/q)$ with $\delta_i = (2w, 2w, 2w, -3w, -3w)$ for some integer $w$ prime to $q$, one has

$$L(Q) = 4q \sin^2(5\pi w/q).$$

(48)

For a slightly more general lens space, also described in [12], whose definition depends on another integer $j$, this is replaced by $L(Q) = 4q |\sin(5\pi w/q)\sin(5\pi j w/q)|$. The
factor $L(Q)$ can be important in determining the proton lifetime, as will become clear. For example, for the minimal choice that leads to the standard model gauge symmetry, which is $q = 2$, $w = 1$, we have $L(Q) = 8$, and the threshold correction will prove to enhance the proton decay rate significantly. It is possible for lens spaces to make $L(Q) < 1$, but only if $q$ is extremely large.

Using (48), we can express (46) in the form

$$
\alpha_{GUT} = (2\pi)^3 L(Q)^{-1} g_s \alpha'^{3/2} M_{GUT}^3,
$$

(49)
where as usual $\alpha_{GUT} = g_{GUT}^2 / 4\pi$. Alternatively,

$$
g_s^2 \alpha'^3 = \frac{\alpha_{GUT}^2 L(Q)^2}{(2\pi)^6 M_{GUT}^6}.
$$

(50)

Ideally, we would like to compute all of the parameters in the string compactification in terms of the quantities $G_N = 1/M_{Pl}^2$, $M_{GUT}$, and $\alpha_{GUT}$, about which we have at least some experimental knowledge. The Planck mass is well-known ($M_{Pl} \simeq 1.2 \times 10^{19}$ GeV), but $M_{GUT}$ and $\alpha_{GUT}$ are somewhat model-dependent. The most commonly cited values based on extrapolation from low energy data are $M_{GUT} \simeq 2 \times 10^{16}$ GeV, $\alpha_{GUT} \simeq .04$. Unfortunately, the string theory really depends on four unknowns $V_6$, $V_Q$, $g_s$, and $\alpha'$.

To parametrize our ignorance, we may introduce the dimensionless quantity

$$
\lambda = \frac{V_6}{V_Q},
$$

(51)
which is of order one if $X$ is fairly isotropic, and solve for the other stringy parameters in terms of $G_N$, $M_{GUT}$, $\alpha_{GUT}$, and $\lambda$. For $V_Q$, this has already been done in (47). Dividing (41) by (50), we get

$$
\alpha' = \frac{8\lambda G_N}{\alpha_{GUT}^2}.
$$

(52)
We can similarly solve for $g_s$,

$$
g_s = \frac{\alpha_{GUT}^4 L(Q)}{2^{9/2}(2\pi)^3 M_{GUT}^3 G_N^{3/2} \lambda^{3/2}}.
$$

(53)

The factor about which we have the least intuition is $\lambda$, which comes from a ratio of volumes. To try to get some intuition about the possible values of $\lambda$, and also about whether the model makes sense, let us examine quantitatively the formula (53) for $g_s$:

$$
g_s = 0.1 \left( \frac{\alpha_{GUT}}{.04} \right)^4 \left( \frac{2 \times 10^{16} \text{ GeV}}{M_{GUT}} \right)^3 \frac{L(Q)}{\lambda^{3/2}}.
$$

(54)
Our calculation does not make much sense if \( g_s \gg 1 \), since then we should do the computation in \( M \)-theory (leading back to the discussion in section 5 of [12]) rather than in Type IIA superstring theory. If \( g_s \ll 1 \), our computation makes sense, but it is implausible for the vacuum to be stabilized after supersymmetry breaking in a way that would enable the world as we see it to exist. So the discussion makes most sense if \( g_s \) is relatively close to 1. We note that happily (54) is compatible with having \( g_s \) near 1 while the GUT parameters have their usual values, \( L(Q) = 8 \), and \( \lambda \) is not too far from 1.

In view of the preceding discussion, it is perhaps more useful to use (54) to solve for \( \lambda \), parametrizing our ignorance via the unknown value of \( g_s \), which we expect to be near 1:

\[
\lambda = \alpha_{GUT}^{8/3} \frac{M_{pl}^2 L(Q)^{2/3}}{M_{GUT}^2 S(2\pi)^2 g_s^{2/3}}. \tag{55}
\]

Now (52) becomes

\[
\alpha' = \frac{\alpha_{GUT}^{2/3} L(Q)^{2/3}}{(2\pi)^2 g_s^{2/3} M_{GUT}^2}. \tag{56}
\]

Finally, the amplitude for proton decay is, from (33),

\[
A_{st} = \pi \alpha' g_s I(\theta_1, \theta_2, \theta_3) = \frac{\alpha_{GUT}^{2/3} L(Q)^{2/3} g_s^{1/3} I(\theta_1, \theta_2, \theta_3)}{4\pi M_{GUT}^2}, \tag{57}
\]

where \( I \) is the integral discussed in the last section. It is interesting to compare this result with the M-theory estimate of [12]:

\[
A_M \sim \frac{\alpha_{GUT}^{2/3} L(Q)^{2/3}}{M_{GUT}^2}. \tag{58}
\]

We find the same scaling with \( \alpha_{GUT} \), \( L(Q) \) and \( M_{GUT} \) as in M-theory. If we keep these parameters and the rotation angles \( \theta_i \) fixed, then we may study the amplitude as a function of \( g_s \). String theory indicates that this function behaves as \( g_s^{1/3} \) for small \( g_s \); M-theory tells us that it approaches a constant for large \( g_s \). Strictly speaking, (57) is reliable for small \( g_s \), but we will take it as a rough approximation for \( g_s \) of order 1.

**Analogous Field Theory Amplitude**

Let us compare this to the analogous four-fermion amplitude in four-dimensional GUT’s. For simplicity, we will assume that (as in \( SU(5) \) models) all superheavy gauge bosons have the same mass \( M_X \). \( M_X \) is comparable to the unification scale \( M_{GUT} \) inferred from the running of the low energy gauge couplings, but differs from it, in
general, by model-dependent factors. Exchange of superheavy gauge bosons of mass $X$ gives an amplitude
\[ \frac{g_{\text{GUT}}^2}{M_X^2} \sum_a \langle A_1|J_{\mu a}|A_2\rangle \langle A_3|J_\mu^a|A_4\rangle, \]  
(59)
where we have labeled initial and final fermion states as $A_1, A_2, A_3$, and $A_4$. The sum runs over the superheavy gauge bosons, but since the standard model gauge bosons contribute baryon-number conserving interactions anyway and taking the sum over all generators of the gauge group will lead to a simpler formula, we will do this. (Standard model gauge boson masses are near zero, not near $M_X$, so (59) is not a good approximation to their contribution.) Note that, ignoring worldsheet instantons, the string theory four-fermion amplitude is $SU(5)$-invariant, since it comes from a local contribution that does not see the symmetry breaking by Wilson lines, while the four-fermion operator in GUT's is of course not $SU(5)$-invariant.

We first consider the case that the fermions transform as $5$'s of $SU(5)$, though the resulting $5^2\overline{5}^2$ amplitude is actually baryon number conserving. To get baryon nonconservation, we need to incorporate $10$'s as well, as we will do presently. Also, to get a better match to the local string theory construction with Chan-Paton factors, we take the gauge group to be $U(5)$ instead of $SU(5)$; the extra $U(1)$ gauge field does not violate baryon number anyway. (In string theory, the local construction at a particular brane intersection point has $U(5)$ gauge symmetry, but globally in realistic models the extra $U(1)$ is Higgsed by absorbing an RR mode.)

Let us work out the group theory part of the matrix element in eqn. (59). For this, we introduce column and row vectors $\alpha_i$ and $\overline{\alpha}_j$ for states transforming in the $5$ or $\overline{5}$; the group theory factor of the current-current matrix element becomes
\[ \sum_a (\overline{\alpha}_1 T_a \alpha_2) (\overline{\alpha}_3 T^a \alpha_4), \]  
(60)
where the $T_a$ are traceless $5 \times 5$ matrices that generate $U(5)$. As we noted earlier, the $T_a$ are conventionally normalized so that $\text{Tr} T_a T_b = \frac{1}{2} \delta_{ab}$. It is straightforward to show that
\[ \sum_a (\overline{\alpha}_1 T_a \alpha_2) (\overline{\alpha}_3 T^a \alpha_4) = \frac{1}{2} (\overline{\alpha}_1 \alpha_4) (\overline{\alpha}_3 \alpha_2). \]  
(61)
The $5^2\overline{5}^2$ interaction in GUT's is hence
\[ \frac{g_{\text{GUT}}^2}{2M_X^2} [ (\overline{\alpha}_1 \alpha_4)(\overline{\alpha}_3 \alpha_2) + (\overline{\alpha}_1 \alpha_2)(\overline{\alpha}_3 \alpha_4) ], \]  
(62)
where we sum over the two channels in which the gauge boson can be exchanged.

Now we move on to the $10^2\overline{10}^2$ interaction. In order to make clear the origin of a certain factor of 2, we will introduce the $10$ by a field theory version of the orientifold
that we used in string theory. The gauge group is $U(5) \times U(5)$, with gauge fields $A$, $A'$ and the fermions consist of fields $\psi, \psi'$ transforming as $(5, 1) + (1, \overline{5})$ plus a field $S$ transforming as $(\overline{5}, 5)$. The Lagrangian is invariant under an “orientifolding” symmetry $\Theta$ that exchanges $A$ with $(A')^T$, where $(A')^T$ is the transpose (or equivalently, as $A'$ is hermitian, the complex conjugate) of $A'$, breaking $U(5) \times U(5)$ to a diagonal $U(5)$. $\Theta$ exchanges $\psi$ and $\psi'$, leaving a 5 of the unbroken $U(5)$, and we take it to act on $S$ with a suitable sign such that the invariant modes in $S$ transform as the 10 of the diagonal $U(5)$, corresponding to an antisymmetric $5 \times 5$ matrix $S^{ij}$.

The kinetic energy is taken to be a general $\Theta$-invariant expression:

$$I = \frac{1}{4g_{\text{GUT}}^2} \text{Tr}(F(A)^2 + F(A')^2) + \overline{\psi} i \gamma \cdot D\psi + \overline{\psi'} i \gamma \cdot D\psi' + \overline{S} i \gamma \cdot DS.$$  

where sums over all indices of $\psi, \psi'$, and $S$ are understood. Let us work out the $S^2\overline{S}^2$ interaction prior to orientifolding. It comes from exchanges of gauge bosons in the two $U(5)$ groups. In working out the contribution from exchange of a gauge boson in either $U(5)$ factor, the indices of $S$ that transform under the other $U(5)$ are spectators. The amplitude due to exchange of either $U(5)$ is thus just like (62), except that when we include the spectator index, the wavefunctions all are $5 \times 5$ matrices $\tilde{\lambda}_i$ and $\lambda_j$ rather than row and column vectors, and the pairing of row and column vectors is replaced by a trace. The amplitude due to exchange of either $U(5)$ is hence

$$\frac{g_{\text{GUT}}^2}{2M_{\text{GUT}}^2} \overline{\pi}_1 \gamma^\mu u_2 \overline{\pi}_3 \gamma_\mu u_4 \text{Tr}(\tilde{\lambda}_1 \lambda_2 \tilde{\lambda}_3 \lambda_4 + \tilde{\lambda}_1 \lambda_4 \tilde{\lambda}_3 \lambda_2).$$

The total amplitude due to exchange of a gauge boson in one or the other $U(5)$ is hence

$$\frac{g_{\text{GUT}}^2}{M_{\text{GUT}}^2} \overline{u}_1 \gamma^\mu u_2 \overline{u}_3 \gamma_\mu u_4 \text{Tr}(\tilde{\lambda}_1 \lambda_2 \tilde{\lambda}_3 \lambda_4 + \tilde{\lambda}_1 \lambda_4 \tilde{\lambda}_3 \lambda_2).$$

Orientifolding is carried out by restricting to $\Theta$-invariant states and dividing the action by 2. As a result, we can drop $A'$ and $\psi'$, but we must divide the $S$ kinetic energy by 2. The resulting kinetic energy is then

$$I' = \frac{1}{4g_{\text{GUT}}^2} F(A)^2 + \overline{\psi} i \gamma \cdot D\psi + \frac{1}{2} \sum_{ij} \overline{S}^{ij} i \gamma \cdot DS^{ij}. $$

The $S$ kinetic energy has been divided by 2, but is still canonically normalized in the following sense. $S$ is an antisymmetric matrix, so for example has a $2 \times 2$ block looking like

$$\begin{pmatrix} 0 & e^+ \\ -e^+ & 0 \end{pmatrix}.$$
The kinetic energy is canonically normalized for fields like $e^+$ appearing above the diagonal in $S$.

As for the $10^2\overline{10}^2$ interaction in the orientifold theory, we can borrow it from (65). We merely have to take the wavefunctions $\lambda_i$ and $\tilde{\lambda}_j$ to be invariant under $\Theta$, which means that they are $5 \times 5$ antisymmetric matrices. Also, we have to divide by 2, just as we divided the $\Theta$-invariant part of the classical action by 2 to get (66). So the $10^2\overline{10}^2$ interaction is finally

$$\frac{g_{GUT}^2}{2M_{GUT}^2} \overline{u}_1 \gamma^\mu u_2 \overline{u}_3 \gamma_\mu u_4 \text{Tr}(\tilde{\lambda}_1 \lambda_2 \tilde{\lambda}_3 \lambda_4 + \tilde{\lambda}_1 \lambda_4 \tilde{\lambda}_3 \lambda_2).$$

(68)

Of course, we could have derived this directly starting with the kinetic energy (66).

\textit{Comparison}

To compare string theory to four-dimensional gauge theory, we therefore need merely compare the string theory amplitude $\pi g_s \alpha'/I$ from (33) to the field theory amplitude $g_{GUT}^2/2M_X^2 = 2\pi \alpha_{GUT}/M_X^2$ from (68). The comparison is thus

$$\pi g_s \alpha'/I \leftrightarrow 2\pi \frac{\alpha_{GUT}}{M_X^2}.$$  

(69)

We call the left hand side the string amplitude $A_{st}$, and the right hand side the corresponding GUT amplitude $A_{GUT}$. Using (57), we find the ratio to be

$$\frac{A_{st}}{A_{GUT}} = \frac{L(Q)^{2/3} I g_s^{1/3}}{8\pi^2} \frac{g_s^{1/3} \alpha_{GUT}^{1/3}}{M_X^2} M_{GUT}^2.$$  

(70)

For other parameters fixed, this ratio scales as $1/\alpha_{GUT}^{1/3}$, which is the same as in the M-theory calculation of [12]. The negative power of $\alpha_{GUT}$ reflects the fact that in this type of model, proton decay is a stringy effect, rather than being dominated by exchange of a single Kaluza-Klein mode. Exploring a possible enhancement due to this factor was the motivation for the present paper, but the factor of $8\pi^2$ in the denominator is clearly unfavorable. We can rewrite (70) as

$$\frac{A_{st}}{A_{GUT}} \approx 0.037 L(Q)^{2/3} I g_s^{1/3} \left(\frac{0.04}{\alpha_{GUT}}\right)^{1/3} \frac{M_X^2}{M_{GUT}^2}.$$  

(71)

Happily, $g_s$, of which we can only say that we expect it to be more or less close to 1, is here raised to a relatively small power, helping to reduce the uncertainty.

Using the most commonly cited values $M_{GUT} \approx 2 \times 10^{16}$ GeV, $\alpha_{GUT} \approx 0.04$, a recent evaluation of the proton lifetime in four-dimensional SUSY $SU(5)$ due to gauge boson exchange gave the value $1.6 \times 10^{36}$ years if $M_X = M_{GUT}$ [5] (we have taken eqn. (17)
of this paper with $M_X = 2 \times 10^{16}$ GeV). If $M_X$ or $\alpha_{\text{GUT}}$ is changed, the proton lifetime scales as:

$$\tau_p = 1.6 \times 10^{36} \text{years} \left(\frac{0.04}{\alpha_{\text{GUT}}}\right)^2 \left(\frac{M_X}{2 \times 10^{16} \text{GeV}}\right)^4.$$  \hfill (72)

Using modified values of $\alpha_{\text{GUT}}$ and $M_{\text{GUT}}$ is natural in the present discussion if doublet-triplet splitting is accomplished with discrete symmetries as in [8], since this necessitates extra matter fields (in complete $SU(5)$ multiplets) surviving to energies below the GUT scale. For example, in [36], in models with such additional multiplets, it was found plausible to have values such as $\alpha_{\text{GUT}} = 0.2$, $M_{\text{GUT}} = 8 \times 10^{16}$ GeV. The net effect of these changes is to roughly double the proton lifetime.

To obtain the proton lifetime $\tau_{p,\text{st}}$ in this class of string theories from (72), we multiply by $(A_{\text{GUT}}/A_{\text{st}})^2$ and replace the prefactor 1.6 by 2 for a reason explained below. Thus:

$$\tau_{p,\text{st}} = 2 \times 10^{36} \text{years} \left(\frac{0.037L(Q)^{2/3}Ig_9^{1/3}}{\alpha_{\text{GUT}}}\right)^{-2} \left(\frac{0.04}{\alpha_{\text{GUT}}}\right)^{4/3} \left(\frac{M_{\text{GUT}}}{2 \times 10^{16} \text{GeV}}\right)^4.$$  \hfill (73)

A proton lifetime of order $10^{36}$ years due to gauge boson exchange is considered unobservably small for the foreseeable future; the present experimental bound on $p \to \pi^0 e^+$ is about $4.4 \times 10^{33}$ years (for example, see [37]), while a next generation detector (described for example in [38]) may reach a limit close to $10^{35}$ years. However, it is clear that a relatively modest enhancement in the proton decay amplitude might save the day. Looking at (71), we see a few large and small factors that tend to cancel. This formula contains the small overall factor 0.037, along with the factor $I$, which we have found to be roughly $7 - 11$ for plausible values of the angles, and the threshold factor $L(Q)^{2/3}$, where $L(Q)$ need not be large but is in fact equal to 8 for the minimal lens space with fundamental group $\mathbb{Z}_2$, and a little larger for many of the lens spaces with other small fundamental groups. Combining these factors, the best we can say is that in a model based on intersecting $D6$-branes, rather as in four-dimensional GUT’s, the proton lifetime due to dimension six operators is likely to be close to $10^{36}$ years.

Note that one major source of uncertainty in GUT’s is absent here: the proton lifetime directly involves the scale $M_{\text{GUT}}$ that can be probed using low energy data, rather than the heavy gauge boson mass $M_X$ whose relation to $M_{\text{GUT}}$ is model-dependent.

What we have evaluated in this paper is a local, stringy contribution to proton decay in a certain class of models based on intersecting Type II $D6$-branes. If the compactification volume is even slightly larger than the string scale, so that worldsheet instanton effects are at least slightly suppressed, what we have computed is likely to be the dominant contribution (from dimension six operators) in this class of models. The result we have obtained has an anomalous power of $\alpha_{\text{GUT}}$ because it is a short distance stringy effect. As in [12], this contribution yields only a $10^2 \mathbf{T}\mathbf{O}^2$ interaction,
which contributes to $p \rightarrow \pi^0 e_L^+$. In four-dimensional GUT’s, there is also a $10 \overline{10} 5 \overline{5}$ contribution, leading to $p \rightarrow \pi^0 e_R^+$. With the assumptions made in [5], the ratio of $p \rightarrow \pi^0 e_R^+$ to the total is $y = 1/(1 + (1 + |V_{ud}|^2)^2)$, where $V_{ud} \cong 1$ is a CKM matrix element. Hence in comparing the stringy proton decay rate to that in field theory, we should include a factor of $1/(1 - y)$ in the proton lifetime in the string model to account for the missing $p \rightarrow \pi^0 e_L^+$. We included this factor in taking the prefactor in (73) to be 2.

The last paragraph has been formulated loosely; with different assumptions about the flavor structure at the GUT scale, the $\pi^0$ could be a $K^0$ and the $e^+$ could be a $\mu^+$ (in which case the lepton polarization in the final state would be measurable). In either the GUT theory or the string model, the proton lifetime could be increased further by unfavorable assumptions about flavor structure (mixing with the third generation, for instance, or mixing with new GUT-scale fermions). At any rate, the assertion of this class of models that proton decay is caused mainly by $10^2 \overline{10}^2$ interactions is testable in principle.

**Acknowledgments**

We thank M. Cvetic and S. Raby for useful discussions. I.R.K. gratefully acknowledges the hospitality of the George P. & Cynthia W. Mitchell Institute for Fundamental Physics at Texas A&M University, where some of this work was carried out. This material is based upon work supported in part by the National Science Foundation Grants No. PHY-9802484 and PHY-0070928. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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