Composite vertices that lead to soft form factors

L. C. Liu

*Theoretical Division, Los Alamos National Laboratory,*

*Los Alamos, NM 87545*

Q. Haider

*Physics Department, Fordham University, Bronx, NY 10458*

J. T. Londergan

*Department of Physics and Nuclear Theory Center, Indiana University,*

*Bloomington, IN 47405*

(February 15, 1995)

Abstract

The momentum-space cut-off parameter $\Lambda$ of hadronic vertex functions is studied in this paper. We use a composite model where we can measure the contributions of intermediate particle propagations to $\Lambda$. We show that in many cases a composite vertex function has a much smaller cut-off than its constituent vertices, particularly when light constituents such as pions are present in the intermediate state. This suggests that composite meson-baryon-baryon vertex functions are rather soft, i.e., they have $\Lambda$ considerably less than 1 GeV. We discuss the origin of this softening of form factors as well as the implications of our findings on the modeling of nuclear reactions.

PACS numbers: 14.20.-c,Gk; 21.30.+y; 25.80.-e
I. INTRODUCTION

Hadronic meson-baryon-baryon (MBB) and three-meson (MMM) vertex functions or form factors are basic inputs to theories of nuclear reactions. They play an important role in many-body nuclear physics as they provide models for making specific off-shell extrapolation of elementary amplitudes to high momenta. In particular, the momentum-space cut-off scale $\Lambda$ of a vertex affects the contribution of a loop diagram in which the vertex appears. The larger the $\Lambda$ of a given vertex is, the smaller the momentum cut-off effects will be, and the greater the magnitude of the loop integration will be for that vertex. It is, therefore, important to understand the quantities that determine the range of a form factor.

Although hadronic form factors can in principle be calculated microscopically, the high complexity of the strong-interaction processes involved in vertex corrections makes such calculations very difficult. As a result, hadronic form factors are typically parameterized by an ad hoc function. This function defines the fall-off of form factors with increasing momentum transfer and is often assumed to depend on a single cut-off parameter, $\Lambda$. Clearly, this parametric function is not unique. One can use, for example, a Gaussian or exponential parameterization of the form factors [1].

By far the most commonly used form factor is the multipole form, of which one specific type is $[\Lambda^2/(\Lambda^2 - q^2)]^n$, with $q^2$ being the square of the four-momentum transfer, and $n = 1(2)$ denoting a monopole (dipole) form factor [2]. The value of $\Lambda$ is then determined from fitting some specific reaction data. The phenomenological nature of this approach creates difficulties in deriving and interpreting the cut-off parameter of the vertex function. First, because the form factor serves as a convenient free input to the theories, inadequacies in the reaction dynamics may be compensated by adjusting $\Lambda$. Second, in electromagnetic interactions, introduction of form factors leads to difficulties in maintaining gauge invariance [3].

The difficulty in determining the cut-off parameter of a form factor is not simply an academic question, as the value of $\Lambda$ for a given form factor can depend strongly on the
types of reaction data that are being fitted, as well as on the theoretical model used to deduce that form factor. We give two examples which illustrate this dependence. The first involves deduction of pion-nucleon coupling constants and cut-off momenta from models which fit nucleon-nucleon scattering data. The second involves inference of $\Lambda_{\pi NN}$ from deep inelastic scattering (DIS) of leptons from nucleons.

Many of the features of phenomenological $NN$ interactions were demonstrated in dispersion relation calculations of correlated and uncorrelated two-pion-exchange by the Paris and Stony Brook groups [4–7]. These calculations showed the role of multipion exchange in the $NN$ interaction. In particular, the dispersion relation treatment highlighted the importance of intermediate $\Delta$ formation in certain spin-isospin channels. These models motivated the development of meson-exchange potentials (MEP) of the nucleon-nucleon interaction, notably the Bonn potential [8,9]. Such potentials were able to incorporate many of the important physical effects of the dispersive calculations, while retaining an ease of computation which allowed them to be used in calculations of few-body systems.

The cutoff parameter for the Bonn potential, $\Lambda_{\pi NN} \sim 1.3$ GeV, has greatly influenced subsequent calculations of the offshell behavior of $NN$ amplitudes. This range was necessary to reproduce the properties of the deuteron and low-energy $NN$ scattering in the Bonn potential. However, it is now known that such hard form factors are not a necessary feature of $NN$ interactions, and that several physical effects argue for a significantly softer form factor. For example, the Jülich group [10] have shown that inclusion of $\pi\pi$ interactions allows one to avoid the need for hard form factors. Janssen et al. [11] further showed that the inclusion of $\pi\rho$ scattering can reduce the value of $\Lambda_{\pi NN}$.

Thomas and Holinde [12] showed that one could obtain a good fit to $NN$ data with a soft $NN\pi$ form factor, if one simultaneously introduced an additional short-ranged, isovector tensor force. Recently, Haidenbauer et al. [13] have produced a model where both one- and two-pion exchanges are included explicitly with soft $NN\pi$ and $\Delta N\pi$ vertices. Smaller cut-offs are also advocated by an earlier theoretical investigation [14] which demonstrates that the boson nature of the pion requires a cut-off parameter much smaller than 1 GeV. Small
Λ_{ρNN} and Λ_{ρNΔ} have also been obtained by Haider and Liu [15] and by Deister et al. [16] when composite models are used for the ρNN and ρNΔ vertices.

Sullivan [17] showed that nonperturbative contributions from coupling to mesons could contribute to DIS even at very high energies. If one assumes such a mechanism, then antiquark contributions can arise from a process where a nucleon emits a virtual pion and a DIS occurs from the antiquark in the meson. The mesonic contribution to this process is very sensitive to the πNN form factor. The nonperturbative contribution from the antiquarks in the meson cannot exceed the total antiquark contribution, and analyses of DIS including pionic contributions through the Sullivan mechanism [18,19] with a monopole form factor give Λ_{πNN} ∼ 0.65 GeV. However, if in addition to pions one includes other mesons (including strange mesons [20]), the inferred monopole cut-off parameter is then somewhat larger (∼ 800 MeV) [21], but is still much less than the 1.3 GeV of the Bonn potential [8].

In order to better understand the Λ of a hadronic vertex, one must go beyond the use of phenomenological parameterizations of the form factor and study the underlying reaction dynamics. As explicit considerations of reaction mechanisms give rise necessarily to composite vertex functions, we carry out in this paper a study of the property of composite vertices. In particular, we investigate a model which demonstrates how the cut-off momentum Λ of a vertex is determined. Our emphasis is on the general dependence of the Λ for a composite vertex function on the masses of the constituent particles and the cut-off momenta of its constituent vertices (or subvertices). The basic analysis is given in Section II. We show that when a light constituent like the pion is present, the composite vertex has a very soft form factor (i.e., small Λ) even if the constituent MBB or MMM form factors are hard (i.e., having very large Λ). We further show that the softening of form factors has a geometrical origin. Section II contains discussion on the implications of our findings on nuclear physics studies. The conclusions are given in Section IV.
II. COMPOSITE VERTEX FUNCTION

Let us consider the lowest-order composite MBB vertex shown in Fig. 1, where we denote the initial baryon, the final baryon, and the meson as \(a\), \(c\), and \(d\), respectively. This vertex exemplifies a composite vertex because the meson \(d\) decays into two mesons (denoted 1 and 2) which subsequently interact with the baryons \(a\), \(b\), and \(c\). In the rest frame of \(c\), the four-momenta of these external particles can be parameterized as \(p_c = (w, 0)\), \(p_a = (a^0, -p)\), and \(p_d = (w - a^0, p)\). If we denote the four-momentum variable of the loop integration as \(q\), then the momenta of the internal particles can be parameterized as \(p_1 = (q^0; q)\), \(p_2 = (w - a^0 - q^0, p - q)\), and \(p_b = (a^0 + q^0, -p + q)\). After projecting out the angular momentum dependence, the form factor of the composite MBB vertex function, \(V\), is a scalar and depends on two independent four-momenta which can be chosen, for example, as \(p_a + p_d\) and \(p_c - p_a\). From these two momenta, one can form three independent scalar variables. Hence, in the most general case, we can write, for example, \(V = V(p_a^2, s, t)\) with \(s = (p_a + p_d)^2 \equiv w^2\) and \(t = (p_c - p_a)^2\). For simplicity, we shall consider a less general case in which the particle \(a\) is on its mass shell, i.e., \(p_a^2 = m_a^2\) or \(a^0 = E_a(p) = \sqrt{p^2 + m_a^2}\). Consequently, \(V\) depends only on two scalar variables which can be chosen as \(s\) and \(t\).

For definiteness, we assume \(s\)-wave interactions for the overall \(a + d \to c\) process and for all subprocesses \(d \to 1 + 2, 1 + a \to b\), and \(2 + b \to c\). An actual example of such case is the vertex function for \(f^0 NN\) coupling via intermediate \(\pi \pi N^* (1535)\). However, in the following, we shall treat the masses \(m_1, m_2,\) and \(m_b\) as variables in order to explore the general feature of a composite vertex function. We further assume that all baryons have spin \(-\frac{1}{2}\). The composite MBB vertex function \(V_{da;c}\) is then given by

\[
V_{da;c}(s, t) = \frac{i}{(2\pi)^4} C_I \sum_{\sigma, \sigma'} \bar{u}^{\sigma'}(p_c) \int d^4 q \frac{g_{d12}(p_d^2)F_{d12}(\Lambda_0^2, t_{12})}{q^2 - m_1^2 + i\varepsilon} \times \frac{g_{1a:b}(p_b^2)F_{1a:b}(\Lambda_1^2, q^2)}{(p_d - q)^2 - m_2^2 + i\varepsilon} \frac{g_{b2c}(w)F_{b2c}(\Lambda_2^2, p_b^2)}{(p_a + q)^2 - m_b^2 + i\varepsilon} \gamma \cdot (p_a + q) + m_b \cdot \frac{\frac{\gamma}{(p_a + q)^2 - m_b^2 + i\varepsilon} u^{\sigma}(p_a) ,}{(p_a + q)^2 - m_b^2 + i\varepsilon} \]

(1)

where \(C_I\) denotes the isospin coefficient, \(u^{\sigma}\) and \(\bar{u}^{\sigma'}\) are Dirac spinors, \(t_{12} \equiv (p_1 - p_2)^2/4\), and \(\gamma\) is the Dirac matrix. In Eq. (1), we have followed the convention adopted in the literature.
by assuming that the energy-dependence of each vertex function is entirely contained in its
coupling constant $g$, while the residual form factor $F$ depends only on the momentum cut-
off and the momentum transfer. The coupling constants of the MBB vertices are denoted
by $g_{1a:b}$ and $g_{2b:c}$. They are dimensionless. The coupling constant of the MMM vertex is
$g_{d;12}$, and it has the dimension of an inverse momentum. For the sake of brevity, we will,
hereafter, denote the coupling constants $g_{d;12}$, $g_{1a:b}$, $g_{2b:c}$ and the associated form factors
by $g_0$, $g_1$, $g_2$ and $F_0$, $F_1$, $F_2$, respectively. In the literature, the $g_i (i = 0, 1, 2)$ are further
fixed at the values calculated with $p_d^2 = m_d^2$, $p_b^2 = m_b^2$, $w^2 = m_c^2$, respectively. Needless to
say, this latter choice introduces implicitly a specific off-shell behavior of the vertices.

It is worth emphasizing that the multipole order $n$ of the subvertex form factors $F$
should be large enough to ensure the convergence of the integration in Eq. (1). For s-wave
interactions considered above, it suffices to use monopole form factors ($n = 1$). When higher
partial wave interactions ($\ell > 0$) are present at the subvertices, there will be a $q^\ell$-dependence
at the corresponding subvertex. Furthermore, propagators of particles having spins higher
than 1/2 will also introduce higher powers of momenta. In these latter cases, higher-order
multipole form factors ($n > 2$) need to be used to ensure the convergence of the integration
[15]. Although this complication introduces more tedious angular momentum algebra, it
will not alter the qualitative conclusion obtained with the present s-wave study (see Section
IV.)

A. Effects of intermediate particle propagations

It is instructive to first examine Eq. (1) in the limit of $\Lambda_i \to \infty (i = 0, 1, 2)$, which
corresponds to having contact interactions at all the subvertices in coordinate space. In this
limit, the form factors $F_0 = F_1 = F_2 = 1$ and

$$
V_{d,0:2}(s, t) = \frac{i}{(2\pi)^4} C_1 g_0 g_1 g_2 \sum_{\sigma, \sigma'} u^{\sigma'}(p_c) \int d^4 q \frac{1}{q^2 - m_\Lambda^2 + i\varepsilon}
\times \left[ \frac{1}{(p_d - q)^2 - m_d^2 + i\varepsilon} \right] \frac{\gamma \cdot (p_a + q) + m_b}{(p_a + q)^2 - m_b^2 + i\varepsilon} u^\sigma(p_a). \tag{2}
$$
To evaluate Eq. (2), we introduce two Feynman parameters ($\alpha$ and $\beta$) and rewrite it as

$$V_{dac}(s, t) = \frac{i}{(2\pi)^4} C_I g_0 g_1 g_2 \sum_{\sigma, \sigma'} \bar{u}^\sigma(p_c) \Gamma(3) \int_0^1 \frac{d\alpha}{d\beta} \int_{-\infty}^{+\infty} d^4q \times$$

$$\left\{ \alpha[q^2 - m_1^2] + \beta[(p_d - q)^2 - m_2^2] + (1 - \alpha - \beta)[(p_a + q)^2 - m_2^2] + i\epsilon \right\}^3 u^{\sigma}(p_a).$$

(3)

Introducing the new integration variable $q' = q + A$ with $A \equiv (1 - \alpha - \beta)p_a - \beta p_d$, we obtain

$$V_{dac}(s, t) = \frac{2iC_I}{(2\pi)^4} g_0 g_1 g_2 \sum_{\sigma, \sigma'} \bar{u}^{\sigma'}(p_c) \int_0^1 \frac{d\alpha}{d\beta} \int_{-\infty}^{+\infty} d^4q' \gamma \cdot (p_a + q' - A) + m_b \frac{(q'^2 - D^2 + i\epsilon)}{(q^2 - D^2 + i\epsilon)} u^{\sigma}(p_a).$$

(4)

where we have defined

$$D^2 = A^2 - [(1 - \alpha - \beta)(p_a^2 - m_b^2) + \beta p_d^2 - \alpha m_1^2 - \beta m_2^2].$$

(5)

Because of the symmetric integration on $q'$, the linear term $\gamma \cdot q'$ in Eq. (4) does not contribute. The $q'$–integration can then be performed with the aid of the relation [22]

$$\int \frac{d^4q'}{(q'^2 - D^2 + i\epsilon)^3} = -\frac{i\pi^2}{2(D^2 - i\epsilon)}. $$

(6)

We have, therefore,

$$V_{dac}(s, t) = \frac{\pi^2}{(2\pi)^4} C_I g_0 g_1 g_2 \sum_{\sigma, \sigma'} \bar{u}^{\sigma'}(p_c) \int_0^1 \frac{d\alpha}{d\beta} \frac{\gamma \cdot (p_a - A) + m_b}{D^2 - i\epsilon} u^{\sigma}(p_a).$$

(7)

For $w < m_b + m_2$, one has $D^2 > 0$. Hence, the $i\epsilon$ can be dropped and Im($V$) = 0.

We introduce the function

$$R(s, p) = \frac{V_{dac}(s, p^2)}{V_{dac}(s, 0)} .$$

(8)

and define the momentum-space cut-off, $\Lambda$, of the composite vertex function as the value of $|p|$ such that

$$R(s, \Lambda) = \frac{V(s, \Lambda^2)}{V(s, 0)} = \frac{1}{2} .$$

(9)

In Fig. 2, we show the $R(s, p)$ calculated at $s = m_N^2$ and $m_a = m_N (=939$ MeV). Three sets of intermediate particle masses were used. Set I (the solid curve) corresponds to $m_1 = m_2 =$
$m_\pi (=139.6 \text{ MeV})$ and $m_b = m_N$. Set II (the dashed curve) is obtained with $m_1 = m_2 = m_\pi$ but with $m_b = m_N^* (=1535 \text{ MeV})$. Finally, Set III (the dot-dashed curve) is the result of using $m_1 = m_2 = 2m_\pi$ and $m_b = m_N$. Figure 2 indicates that even when all the cut-offs associated with the subvertices ($\Lambda_0, \Lambda_1, \Lambda_2$) are infinite, the momentum-space cut-off $\Lambda$ of the composite vertex function is finite. Furthermore, $\Lambda$ increases when the masses of the intermediate particles increase. This finite $\Lambda$ reflects the nonlocality of the $d + a \rightarrow c$ interaction in coordinate space, arising from the propagation of the intermediate particles. The more massive these particles are, the shorter distances they will travel. Consequently, the nonlocality will decrease and $\Lambda$ will increase. We have summarized the results in Table I.

B. Effects of finite momentum-space cut-offs of constituent vertices

To examine quantitatively the effects of finite momentum-space cut-offs of subvertices, one must specify the form factors $F_i (i = 0, 1, 2)$. While it is tempting to employ covariant form factors in their simplest form, e.g., in a monopole form $\Lambda_i^2 / (\Lambda_i^2 - q'^2)$, a word of caution for using such simple parameterization is in order. We recall that the monopole form factor and its multipole variants have their origin in mechanisms based on $t$-channel “pole dominance.” They have been very successful in fitting experimental data in the spacelike region where $-q'^2 > 0$. However, it is also well known that in the timelike region where $-q'^2 < 0$, this simple form no longer holds and dispersion relations have to be used to write down the relevant form factor [23]. Because in a loop integration both these regions can be reached at a subvertex, the use of one single covariant multipole form factor for a given subvertex, therefore, becomes problematic. In particular, it can introduce spurious singularities in $q'^0$. To avoid such difficulties, the “static approximation” has often been invoked to drop the dependence on $q'^0$ and to use accordingly the resulting noncovariant form factors $\left[\Lambda_i^2 / (\Lambda_i^2 + q'^2)\right]^n$, with $n = 1(2)$ for monopole (dipole) [8,11]. The use of noncovariant form factors will necessarily make the calculations frame-dependent and break
the crossing symmetry property of the composite vertex function. We have estimated this covariance breaking by comparing the $V_{a,c}(s,t)$ evaluated separately at the rest frames of the particles $a$ and $c$. We have found that the effect of covariance breaking is about 15%.

In view of the phenomenological nature of the hadronic form factors used in the literature, we consider the static approximation as acceptable. We will, therefore, use the following noncovariant parameterizations:

$$F_0 = \frac{\Lambda_0^2}{\Lambda_0^2 + \left(\frac{p}{2} - q'\right)^2} ,$$

$$F_1 = \frac{\Lambda_1^2}{\Lambda_1^2 + q'^2} ,$$

and

$$F_2 = \frac{\Lambda_2^2}{\Lambda_2^2 + (p - q')^2} .$$

We are thus led to evaluate Eq. (4) in the following form:

$$V_{a,c}(s,t) = \frac{2iC_1}{(2\pi)^4} g_0 g_1 g_2 \sum_{\sigma,\sigma'} \vec{u}'(p_c) \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int_{-\infty}^{+\infty} d^4q' F_0 F_1 F_2 \frac{\gamma \cdot (p_a + q' - A) + m_b}{(q'^2 - D^2 + i\varepsilon)^3} u^\sigma(p_a) ,$$

where the $F_i (i = 0, 1, 2)$ are given by Eqs. (10)–(12). We can first perform the $q'^0$ integration in Eq. (13) either in upper-half or in lower-half complex plane of $q'^0$. Equation (13) then becomes

$$V_{a,c}(s,t) = \frac{12}{(2\pi)^3} \sum_{\sigma,\sigma'} \vec{u}'(p_c) \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int d\mathbf{q}' \frac{\gamma \cdot (p_a + q' - A) + m_b}{(2E(q'))^3} \frac{\Lambda_0^2}{\Lambda_0^2 + \left(\frac{p}{2} - q'\right)^2} \frac{\Lambda_1^2}{\Lambda_1^2 + q'^2} \frac{\Lambda_2^2}{\Lambda_2^2 + (p - q')^2} ,$$

where $q' \equiv (0, \mathbf{q'})$.

We have evaluated Eq. (14) in the kinematic region $s = w^2 = m_N^2$ and $m_2 + m_b > w$ with $m_1 = m_2 = m_\pi$ and $m_b = m_N$. Three sets of subvertex cut-offs were used. They are: $\Lambda_0 = \Lambda_1 = \Lambda_2 = 1.2$ GeV (denoted Set IV); $\Lambda_0 = 0.46$ GeV, $\Lambda_1 = \Lambda_2 = 1.2$ GeV (denoted
Set V); and $\Lambda_0 = 0.46$ GeV, $\Lambda_1 = \Lambda_2 = 0.65$ GeV (denoted Set VI). (See also Table I.) The $R(s,p)$ calculated with these three sets are shown in Fig. 3 as the solid, dashed, and dot-dashed curves, respectively. An inspection of Fig. 3 indicates clearly the trend that the $\Lambda$ of the composite vertex function decreases with the cut-offs of the subvertices. For the purpose of further examining the effect of intermediate particle masses in the presence of finite subvertex cut-offs, we also calculated $R(s,p)$ by keeping $\Lambda_i (i = 0, 1, 2)$ to be the same as those of Set IV but increasing $m_b$ from $m_N$ to $m_N^*$. The result is given in Table I as Set VII.

Upon comparing the $\Lambda$ of Set I (respectively, Set II) of Table I with that of Set IV (respectively, Set VII) of Table I, we find that the presence of finite $\Lambda_i (i = 0, 1, 2)$ reduces significantly the $\Lambda$ of the composite vertex. On the other hand, comparing the $\Lambda$ of Sets IV and VII of Table I shows that it increases with increasing intermediate particle masses as in the case with infinite $\Lambda_i$.

**C. Domain of softening of form factors**

In many cases, the composite vertex is a functional of itself. For example, one of the three subvertices of the composite $\pi NN$ vertex considered in Ref. [11] is the $\pi NN$ vertex itself. The result $\Lambda < \Lambda_i$ for $i \in (0, 1, 2)$ then corresponds to a softening of the form factor of the $i$th vertex. We have explored in detail the dependence of $\Lambda$ on the masses ($m_1, m_2, m_b$) and the subvertex cut-offs ($\Lambda_0, \Lambda_1, \Lambda_2$). Here, we present the results obtained in two of the cases studied. In the first case, we fixed $m_b$ at $m_N$ and considered two variables $m^* \equiv m_1 = m_2$ and $x \equiv \Lambda_0 = \Lambda_1 = \Lambda_2$. We calculated the $\Lambda$ by varying $x$ and using three different $m^*$. We chose $m^* = 140, 550, 770$ MeV in order to simulate the masses of light ($\pi$), medium ($K, \eta$), and heavy ($\rho, \omega$) mesons commonly encountered in intermediate energy physics. In the second case, we further fixed $m_1$ at $m_{\pi}$. (Fixing $m_2$ while varying $m_1$ gave essentially the same results.) In Figs. 4 and 5, we present the dependence of $\Lambda$ on $x$ for these two cases. In these two figures, the region under the diagonal line can be termed the
“domain of softening” because in this region the momentum-space cut-off of the composite vertex is smaller than the cut-offs of the constituent vertices, i.e., \( \Lambda < x \). The region above the diagonal line corresponds to the reversed situation where \( \Lambda > x \). From both figures, we note that the minimal \( x \) (denoted \( x_0 \)) for which \( \Lambda \leq x \) can occur increases with meson masses. However, \( x_0 \) is dramatically reduced when a pion is present in the intermediate state.

We recall that in the composite model of the \( \pi NN \) vertex of Ref. [11], the incoming pion decays virtually into a pion and a rho-meson which subsequently interact with the nucleon. Hence, in our notation, \( m_1 = m_{\rho} = 770 \text{ MeV} \), \( m_2 = m_{\pi} = 140 \text{ MeV} \), and \( m_b = m_N \). Further, the cut-off parameters of the constituent vertices in Ref. [11] correspond to an \( x = 1.3 \text{ GeV} \). From extrapolating the dot-dashed curve in Fig. 5, we see that the parameters used in that work are situated well within the region where a softening of the form factor can take place. A similar analysis applies to the appearance of soft composite \( \rho \)-baryon-baryon vertices in Ref. [15].

**III. DISCUSSION**

We have shown the reason and the conditions under which a composite vertex function can have a momentum-space cut-off much smaller than its constituent vertices. We emphasize that our finding is very general. This is because we have demonstrated that the softening process is closely related to nonlocalities associated with the “elementary” interactions and the particle propagations. In view of this, we assert that softening of the form factor will take place in the presence of different types of meson-baryon interaction so long as the kinematics are favorable. This assertion is clearly supported by various published results [11,15].

When the \( i \)th subvertex is the composite vertex itself, such as in the model of the \( \pi NN \) vertex considered by Janssen et al. [11], the result \( \Lambda < \Lambda_i \) can be termed the “softening” of the form factor of the \( i \)th vertex. However, under this circumstance, it will be necessary
to iterate the calculations so as to obtain a convergent solution of $\Lambda$. Such self-consistent calculations are indeed possible, as has been demonstrated in the study of composite $\rho NN$ and $\rho N\Delta$ vertices by Deister et al. [16] and by Haider and Liu [15], where a set of coupled integral equations of the composite vertex functions is established and iterated to all orders.

The Feynman diagram given in Fig. 1 represents the lowest order dynamics of a composite vertex function. Clearly, there are many higher-order diagrams, such as meson loop corrections to the composite vertex itself [15] and rescattering between mesons [1]. Because higher-order diagrams contain more subprocesses and particle propagations, each of which increases the overall coordinate-space nonlocality of the composite vertex, we can anticipate that inclusion of higher-order diagrams will further decrease the momentum space cut-off $\Lambda$ of the composite vertex. Indeed, it has been noted in Ref. [11] that the inclusion of $\pi\rho$ rescattering softens further the composite $\pi NN$ form factor.

It is of interest to examine the analyticity of the composite vertex function $V(s, t)$. Once the Feynman-diagram representation of the dynamics is given, calculation of the diagrams will determine both the real and imaginary parts of the composite vertex function, $\text{Re}(V)$ and $\text{Im}(V)$. If the Feynman propagators are not approximated, then $\text{Re}(V)$ and $\text{Im}(V)$ will satisfy causality relations. This is the approach used in this study. Furthermore, the analytic structure of the propagators ensures that $\text{Im}(V) = 0$ in the kinematical regions where the $t$-channel $d \to 1 + 2$ process and the $s$-channel $2 + b \to c$ process are energetically forbidden.

An alternative method of relating $\text{Re}(V)$ and $\text{Im}(V)$ is to use dispersion relations. This is the approach used, for example, in Refs. [11] and [7], where $\text{Im}(V)$ is first calculated from the underlying reaction diagrams. The calculated $\text{Im}(V)$ is then inserted into a dispersion relation to generate the $\text{Re}(V)$.

It is useful to recall that the advantage of using dispersion relation resides in the fact that the imaginary part of the scattering amplitude can be directly obtained from experimental cross sections through the optical theorem. In other words, the imaginary part is fixed by experiment and only the real part of the amplitude depends on the parameters of the theory. In this regard, we would like to emphasize that the possibility of using experimental input
to obtain the imaginary part of the composite vertex, $\text{Im}(V)$, is severely limited. This is because, in general, the overall $da \rightarrow c$ and its subprocesses are experimentally inaccessible. This experimental difficulty should not be underestimated, as it will increase considerably the model dependence of dispersion relation calculations.

We have mentioned that the momentum-space cut-off of a vertex function plays an important role in the modeling of nuclear reactions as it affects the magnitude of the contribution by an elementary process to loop diagrams. In most of the published MEP studies, simple multipole MBB form factors were used to fit the nucleon-nucleon phase shifts. These fits gave $\Lambda_{\pi NN}$, $\Lambda_{\pi N\Delta}$, $\Lambda_{\rho NN}$, and $\Lambda_{\rho N\Delta}$ of about 1.3 GeV. However, as mentioned in Section I, such large $\Lambda_{\pi NN}$ and $\Lambda_{\pi N\Delta}$ are in conflict with the DIS results [18–21]. In addition to these difficulties, adverse effects due to the use of large $\Lambda_{\rho NN}$ and $\Lambda_{\rho N\Delta}$ were also reported in the analyses of pion production experiments [24]. While smaller $\Lambda$ have been obtained by means of using composite $\pi NN$, $\rho NN$, and $\rho N\Delta$ vertex functions [11,15], it remains an open question as to the completeness of the physics described by the original MEP models. One can reasonably expect that when composite, instead of phenomenological multipole, MBB form factors are used in MEP models to refit the phase shifts, one or both of the following scenarios could occur. Either the fit would give increased MBB coupling constants, or more meson-baryon interaction diagrams would have to be added in order to compensate numerically the loss of reaction strength, arising from smaller cut-offs of the composite vertices. In either of the above scenarios, our understanding of the meson-exchange dynamics will be significantly improved. In fact, it has already been noted that using a soft $\pi NN$ form factor to fit the $NN$ scattering data requires at least the addition of some more new diagrams, such as the correlated $\pi\pi$ and $\pi\rho$ diagrams [10,12,25].

The use of smaller $\Lambda$ associated with composite hadronic vertices will equally challenge current fits to other reactions. This is because most of these best fits were based on the use of $\Lambda_{MBB} \gtrsim 1.2$ GeV. We would like to mention, in particular, the pion-nucleus double charge exchange reactions. These reactions are still far from being fully understood, and indeed, high sensitivities of these reactions to the $\pi NN$ cut-offs have been noted [26,27]. In the light
of the present study, we believe that this sensitivity might be used to assess objectively the importance of certain reaction mechanisms and, in so doing, to discover new directions for improving the theories.

**IV. CONCLUSIONS**

Mesonic corrections to elementary processes give rise naturally to composite vertex functions. Use of these composite vertices is, therefore, required by microscopic nuclear reaction theories. In most cases, especially when one of the intermediate particles is a light meson such as the pion, the momentum-space cut-off of the composite vertex is much smaller than those of the constituent vertices. When the composite vertex is a functional of itself, this reduction of the momentum-space cut-off can be termed as the softening of the vertex or form factor. However, as has been shown in Refs. [15] and [16], it is necessary to carry out self-consistent calculations in order to ensure the convergence of the iterative calculations. Clearly, the notion of a composite vertex can be extended to the regime of very high momentum transfers to include the exchanges of quarks and gluons among the hadrons.

The softening of form factors has been observed in theoretical calculations of $\rho$-baryon-baryon [15] and $\pi$-baryon-baryon vertices [11]. Although these vertex calculations involve very different types of intermediate particles and meson-baryon interactions, the results obtained all follow the pattern outlined in Section I. In particular, one notes from Ref. [15] that the softening of composite $\rho$-baryon-baryon vertex functions occur in the presence of p- and d-wave interactions at various subvertices. This general agreement with the result given by the present s-wave model strongly supports our findings on the principal role played by the nonlocality in the softening of form factors.

The use of composite vertices and the associated small momentum-space cut-offs will inevitably lead to a reexamination of many existing fits to various reactions, which were obtained with form factors having a monopole cut-off of 1 GeV or greater. On the other hand, this reexamination will aid us to clarify the underlying physics governing meson-
The use of composite vertex functions not only provides a dynamical realization of soft MBB form factors ($\Lambda < 1\text{GeV}$), but also naturally extends it to the domain of complex variables. Indeed, as can be seen from the analytic structure of the Feynman propagators in Eq. (1), for values of $s$ or $t$ above the corresponding inelastic thresholds, $V(s, t)$ is complex-valued. As pointed out in Ref. [15], while it may be a good approximation to employ a real-valued vertex function in analyzing nucleon-nucleon scattering below the pion production threshold, the situation is very different in meson production experiments where $V$ can be complex-valued. The non-vanishing imaginary part of the composite vertex functions can introduce interference effects in nuclear reaction calculations. This new aspect of the nuclear dynamics has so far not been studied in the literature and definitely merits a systematic investigation in the future.

**ACKNOWLEDGEMENTS**

Two of us (Q.H. and L.C.L.) would like to thank the staff of the Nuclear Theory Center of Indiana University, Bloomington, IN, where some of the work reported here was carried out, for their hospitality. This work was done under the auspices of the U.S. Department of Energy and the National Science Foundation.
REFERENCES

[1] D.J. Ernst and M.B. Johnson, Phys. Rev. C 22, 651 (1980).

[2] It is important to note that slightly different versions of multipole form factors can correspond to significantly different “cut-offs.” In comparing different parameterizations, one must be careful to compare similar forms, or to note differences from the “standard” form we present here. We will try to note such differences in this paper.

[3] J.L. Friar and S. Fallieros, Phys. Rev. C 13, 2571 (1976).

[4] M. Lacombe, B. Loiseau, J.-M. Richard, R. Vinh Mau, P. Pirès, and R. de Tourreil, Phys. Rev. D12, 1495 (1975); M. Lacombe, B. Loiseau, J.-M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. C21, 861 (1980).

[5] G.E. Brown and A.D. Jackson, The Nucleon-Nucleon Interaction (North-Holland, Amsterdam, 1976), p. 135. Note that Brown and Jackson use the form \[ \frac{\Lambda^2}{\Lambda^2 - t} \frac{\Lambda^2}{\Lambda^2 - u} \], where \( t \) and \( u \) are the Mandelstam variables.

[6] A.D. Jackson, D.O. Riska, and B. Verwest, Nucl. Phys. A249, 392 (1975).

[7] J.W. Durso, A.D. Jackson, and B.J. Verwest, Nucl. Phys. A282, 404 (1977).

[8] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. 149, 1 (1987).

[9] Note that the Bonn group uses the non-covariant form factor \[ \frac{(\Lambda^2 - m^2)}{(\Lambda^2 - q^2)} \]^n where \( m \) is the mass of the exchanged meson and \( q \) is the three-momentum transfer. One must exercise care in comparing the cut-off of this form factor with ours.

[10] H.-C. Kim, J.W. Durso, and K. Holinde, Phys. Rev. C 49, 2353 (1994); C. Schütz, J.W. Durso, K. Holinde, and J. Speth, ibid. C 49, 2671 (1994).

[11] G. Janssen, J.W. Durso, K. Holinde, B.C. Pearce, and J. Speth, Phys. Rev. Lett. 71, 1978 (1993).
[12] A.W. Thomas and K. Holinde, Phys. Rev. Lett. 63, 2025 (1989); K. Holinde and A.W. Thomas, Phys. Rev. C 42, R1195 (1990).

[13] J. Haidenbauer, K. Holinde, and A.W. Thomas, Phys. Rev. C 49, 2331 (1994).

[14] D. Schütte and A. Tillemans, Phys. Lett. B 206, 1 (1988).

[15] Q. Haider and L.C. Liu, Phys. Lett. B 335, 253 (1994); B 338, 521 (1994).

[16] S. Deister, M.F. Gari, W. Krümpelmann, and M. Mahlke, Few-Body Systems, 10, 1 (1991).

[17] J.D. Sullivan, Phys. Rev. D 5, 1732 (1972).

[18] L.L. Frankfurt, L. Mankiewicz, and M.I. Strikman, Z. Phys. A 334, 343 (1989).

[19] S. Kumano, Phys. Rev. D 43, 59 (1991).

[20] A.W. Thomas, Phys. Lett. B 126, 97 (1983).

[21] W. Melnitchouk and A.W. Thomas, Phys. Rev. D 47, 3794 (1994); A. Szczurek and J. Speth, Nucl. Phys. A 555, 249 (1993).

[22] See for example, Silvan S. Schweber, *Introduction to Relativistic Quantum Theory* (Row and Harper, Inc., New York, 1964). The identity can be readily established by using Wick’s rotation.

[23] T.A. Griffy and L.I. Schiff, *High-Energy Physics* (Academic Press, New York, 1967), Vol. VI.

[24] B.K. Jain and A.B. Santra, Nucl. Phys. A 519, 697 (1990).

[25] G. Janssen, K. Holinde, and J. Speth, Phys. Rev. Lett. 73, 1332 (1994); Phys. Rev. C 49, 2763 (1994).

[26] Q. Haider and L.C. Liu, J. Phys. G 14, 1527 (1988); G 15, 934 (1989).
[27] Q. Haider and L.C. Liu, Z. Phys. A 335, 437 (1990).
TABLES

TABLE I. The cut-offs $\Lambda$ (in GeV) of the composite vertex function calculated in the kinematic domain $s = w^2 = m_N^2$ and $m_2 + m_b > w$.

| Set | $m_b$ | $m_1$ | $m_2$ | $\Lambda_0$ | $\Lambda_1$ | $\Lambda_2$ | $\Lambda$ |
|-----|-------|-------|-------|-------------|-------------|-------------|---------|
| I   | $m_N$ | $m_\pi$ | $m_\pi$ | $\infty$   | $\infty$   | $\infty$   | 0.60    |
| II  | $m_{N^*}$ | $m_\pi$ | $m_\pi$ | $\infty$   | $\infty$   | $\infty$   | 1.14    |
| III | $m_N$ | $2m_\pi$ | $2m_\pi$ | $\infty$   | $\infty$   | $\infty$   | 1.30    |

TABLE II. The cut-offs $\Lambda$ (in GeV) of the composite vertex function calculated in the presence of finite subvertex cut-offs.

| Set | $m_b$ | $m_1$ | $m_2$ | $\Lambda_0$ | $\Lambda_1$ | $\Lambda_2$ | $\Lambda$ |
|-----|-------|-------|-------|-------------|-------------|-------------|---------|
| IV  | $m_N$ | $m_\pi$ | $m_\pi$ | 1.20        | 1.20        | 1.20        | 0.42    |
| V   | $m_N$ | $m_\pi$ | $m_\pi$ | 0.46        | 1.20        | 1.20        | 0.36    |
| VI  | $m_N$ | $m_\pi$ | $m_\pi$ | 0.46        | 0.65        | 0.65        | 0.31    |
| VII | $m_{N^*}$ | $m_\pi$ | $m_\pi$ | 1.20        | 1.20        | 1.20        | 0.55    |
FIGURES

FIG. 1. A leading-order Feynman diagram for a composite $d + a \rightarrow c$ vertex. The solid and dashed lines denote, respectively, the baryons and mesons. The circle symbolizes the corresponding phenomenological vertex which approximates the internal dynamics by means of a multipole form factor.

FIG. 2. $R(s, p)$ calculated at $s = m_N^2$ and $m_a = m_N$ with the parameters given by Set I (solid curve), Set II (dashed curve), and Set III (dot-dashed curve) in Table I.

FIG. 3. $R(s, p)$ calculated at $s = m_N^2$ and $m_a = m_N$ with the parameters given by Set IV (solid curve), Set V (dashed curve), and Set VI (dot-dashed curve) in Table II.

FIG. 4. Relation between $\Lambda$ and $x$ at $s = m_N^2$ and $m_a = m_b = m_N$. The solid, dashed, and dot-dashed curves represent, respectively, the results obtained with $m^* = 140, 550, 770$ MeV.

FIG. 5. Relation between $\Lambda$ and $x$ at $s = m_N^2$, $m_a = m_b = m_N$, and $m_1 = m_\pi$. The solid, dashed, and dot-dashed curves represent, respectively, the results obtained with $m_2 = 140, 550, 770$ MeV.