A self-synchronizing stream cipher based on chaotic coupled maps

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Abstract

A revised self-synchronizing stream cipher based on chaotic coupled maps is proposed. This system adds input and output functions aim to strengthen its security. The system performs basic floating-point analytical computation on real numbers, incorporating auxiliarily with algebraic operations on integer numbers.

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In the recent tens years, since the pioneer work of chaos synchronization by Pecora and Carrol [1], secure communication by utilizing chaos has attracted much attention. Most of the secure communication systems suggested are based on chaos synchronization and they belong to self-synchronizing stream ciphers (SSSC) [2, 3, 4, 6, 7, 8, 9, 10]. For SSSC the signals transmitted in the public channels (i.e., ciphertext) serve as the drivings for the synchronization of the receivers, and this structure can be effectively used by intruders to expose much information of the secret key [11, 12, 13, 14, 15, 16]. In [17, 18, 19, 20] we suggested to use spatiotemporal chaos, i.e., one-way coupled chaotic map lattices, incorporating with some simple conventional algebraic operations, to enhance the security of chaotic SSSC. In this paper we approach a revised system adding input and output functions aim to strengthen the security of the system. The key length is 128 bits, and the key can be expanded to sub-keys to be used. In this cipher all keys, plaintexts, ciphertexts and keystreams are integers of 32 bits defined in \([0, 2^{32})\).

I. ENCRYPTION TRANSFORMATION

Encryption transformation of the transmitter has two parts: the keystream generator and encryption function. The keystream generator utilizes one-way coupled logistic maps to produce the keystream, incorporating with few simple algebraic operations on integer numbers. The keystream generator has three parts: the input function; coupled chaotic maps; and the output function.

The input function:

\[
D_{n+1} = E(FE(FE(c_n \oplus k_9) \oplus k_{10}) \oplus k_{11} \oplus k_{12})
\]

\[
x_{n+1}(0) = \frac{D_{n+1}}{2^{32} - 1}
\]

Where \(c_n\) is the nth ciphertext and \(x_n(0)\) is the state variable for the nth interaction of the 0th map. Symbol \(\oplus\) is bitwise XOR. \(k_9, k_{10}, k_{11}, k_{12}\) are the expanded sub-keys to be defined later. \(E\) and \(F\) are nonlinear functions:

\[
F(A) = A \oplus (A >>> 3) \oplus (A <<< 11)
\]

\[
E(A) = A_1|A_2|A_3|A_4' = s(A_1)|s(A_2)|s(A_3)|s(A_4)
\]

\[A = A_1|A_2|A_3|A_4\]
where the operation $A >>> (<<<)m$ denotes a right (left) cycle shift of $A$ by $m$. Nonlinear function $E(A)$ is performed as follows: first, we split a given 32-bit number $A$ into four bytes $A_1|A_2|A_3|A_4$, according to the order from high to low bit significance. Every byte takes a nonlinear map to $A'_i$, $i = 1, 2, 3, 4$ (i.e., S-box operation) from $A_i$, and then we obtain transformed four bytes $A'_1|A'_2|A'_3|A'_4$. Finally, four new bytes are combined to a new 32-bit integer $A'$. The input function guarantees that any one bit difference in the input ciphertext $c_{n+1}$ can possibly change all 32 bits of the input $D_{n+1}$.

The one-way coupled maps:

$$x_{n+1}(i) = \varepsilon f_i(x_n(i)) + (1 - \varepsilon)f'_i(x_n(i - 1))$$

$$f_i(x) = (3.75 + \frac{a_i}{4})x(1 - x), a_i \in [0, 1]$$

$$f'_i(x) = (3.75 + \frac{b_i}{4})x(1 - x), b_i \in [0, 1]$$

$$a_i = \frac{k_i}{2^{32} - 1}, b_i = \frac{k_{i+4}}{2^{32} - 1}, i = 1, 2, 3, 4 (3)$$

$$y_{n+1} = \text{int}(x_{n+1}(4) \times 2^{52}) \mod 2^{32}$$

where $\varepsilon$ is the coupled parameter, and $\varepsilon = 2^{-16}$. The integer key $(k_1, k_2, k_3, k_4)$ and $(k_5, k_6, k_7, k_8)$ are changed to real number $a_i$ and $b_i$, $i = 1, 2, 3, 4$, for the computations of the logistic maps.

The output function:

$$z_{n+1} = E(E(FE(Fy_{n+1} \oplus k_{13}) \oplus Fy_{n+1} \oplus k_{14}) \oplus k_{15})$$

(4)

where $z_{n+1}$ is the keystream of the system.

The ciphertext $c_{n+1}$ is produced by

$$c_{n+1} = z_{n+1} \oplus I_{n+1}, \mod 2^{32}$$

(5)

where $I_{n+1}$ is the corresponding plaintext.

II. THE DECRYPTION TRANSFORMATION

The decryption transformation of the receiver has also two parts, the keystream generator and decryption function. The keystream generator of the receiver is exactly the same as that of the transmitter and the decryption function is written as
\[ I'_{n+1} = z_{n+1} \oplus c_{n+1}, \mod 2^{32} \]  

With the same key as that of the transmitter, the receiver can reach chaos synchronization with the transmitter, and successfully recover the true plaintext as

\[ z'_{n+1} = z_{n+1}, I'_{n+1} = I_{n+1} \]  

III. KEY EXPANSION

The sub-keys in the system are derived from the main keys. With the 128-bit main key \((k_1, k_2, k_3, k_4)\), the other sub-keys are produced as

\[ k_i = E(F(k_{i-4}) \oplus k_{i-3} \oplus k_{i-2} \oplus k_{i-1}), i = 5, 6, \ldots, 15 \]  

Every sub-key is defined in \([0, 2^{32})\).

IV. S-BOX TRANSFORMATION

S-box is a nonlinear transformation, and usually used in cryptosystems. In our system s-box is defined by a transformation from another 8-bit integer to 8-bit integer. We produce random maps, \(y_i = s(x_i)\), and choose one form them with optimal statistic properties. The input values of this map are 0,1,2,...,255, and the following output values are:

\[
\begin{align*}
181 & 176 64 243 172 14 177 8 90 30 15 133 207 38 130 41 \\
66 & 76 111 2 221 163 115 236 193 211 145 137 35 79 233 155 \\
125 & 18 190 92 187 156 94 58 81 21 225 158 153 52 1 85 \\
69 & 88 140 185 146 254 166 223 151 87 12 143 170 113 231 249 \\
159 & 97 216 164 78 253 227 54 106 229 121 28 13 240 37 16 \\
99 & 210 217 57 77 252 129 114 110 199 220 184 161 232 10 107 \\
101 & 84 178 202 157 165 239 128 228 132 235 183 27 43 5 104 \\
112 & 61 150 116 17 148 25 242 138 192 29 51 230 191 213 26 \\
180 & 70 194 238 105 63 255 251 11 134 123 135 42 173 212 89 \\
247 & 4 141 175 209 83 147 196 53 59 222 82 20 246 23 234 \\
96 & 168 224 56 203 204 86 32 144 218 33 50 241 188 118 80
\end{align*}
\]
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