Mathematics for the Imaginary Number $i$

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Abstract—The only objective of this work is to: determine a function that of origin to the value of $\sqrt{-1}$ and, define the segment represent this magnitude of $\sqrt{-1}$. For this it is necessary the rule and compass.

Index Terms—Function for Constant $i$, Leg Theorem, Segment value $\sqrt{-1}$.

I. INTRODUCTION

In this note I do not refer to any article over on the calculation and laws of complex numbers. The sole objective, the solution of the imaginary number $\sqrt{-1}$ is determine their magnitude and point of projection in coordinates axes. If it was already relevant for Physics and Engineering now it will be more so for its calculation accuracy. Let’s remember the presentation pioneer of Rafael Bombelli:[4] $x^2 \rightarrow x^2 = -1$ and the designation of ($i = \sqrt{-1}$) by Leonard Euler in 1777. (Reflection: needless to say, he has the comments that were made in his day of (-1) and where it is located with the passage of time. If it is science with the passage of time, everything has its reason for being).

II. MAIN RESULTS

I start with the $f(y)_{lim(x=b)} \Rightarrow \sqrt{-1}$:

$$y^2 = \frac{b^2 - (x\sqrt{2})^2}{b^2}$$

(1)

I indicate some of the values for $f(y)$ with $b \geq 1$ such as

$x = 0 \Rightarrow y = 1$; $x = \frac{b}{2} \Rightarrow y = \frac{1}{\sqrt{2}}$; $x = \frac{b}{\sqrt{2}} \Rightarrow y = 0$; $x = b \Rightarrow y = \sqrt{-1}$

With ($x = b$) we have

$$\left(\frac{b^2 - (\sqrt{2} * b)^2}{b^2}\right)^{\frac{1}{2}} = \sqrt{-1}$$

In figure 1 the projection of $\sqrt{-1}$ will it’s localize in the quadrant. Let’s leave it like that for now and we start with the ask of the fig 1 ie if we assume that we will can limit the value $\sqrt{-1}$ with a rectangle triangle we resolve the problem $\sqrt{-1}$.

Let’s imagine having a rectangle triangle with sides ($b-1$) and $\sqrt{-1}$; in their at applying Pythagoras theorem we will have

$$\frac{(b-1)^2 + (\sqrt{-1})^2}{b^2} = \frac{b^2 - 2b}{b^2}$$

and

$$\frac{b^2 - 2b - (\sqrt{-1})^2}{b^2} = \frac{b^2 - 2b + 1}{b^2}$$

This leads us to the next question; it will be possible to construct with a ruler and compass a right triangle that has the values of ($b-1$) and $\sqrt{-1}$ by legs

We pass to at development mathematical[1]

A. Proposition 1

We need define a right triangle where their legs is be $\sqrt{(b^2 - 2b)}$, the we will make Fig. 2.

We take a segment of longitude ($b$) we draw on he a semicircle (red) and we mark a segment of value two, then we draw a perpendicular to point (e), this will cut the semicircle at point (p) and from (p) we draw two lines to the ends of (b) and we will have a right triangle

By the Leg Theorem we have

$$(b - 2)^2 = n^2$$

$$b*2 = c^2$$

$$b(b - 2) = m^2$$

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with which the leg (m) has a value of $(b^2 - 2b)$, value need for proposition 1.

B. Proposition 2

Construct the right triangle of the proposition 1 on Fig. 3

To build the right triangle that has by sides the leg (m) and $(\sqrt{-1})$ we will do the following: we draw an arc (blue) of radius $(b - 2 + 1)$ then we extend the segment (c) until inserting in the arc (b-1) from the intersection of both we draw one line to the end of (m) and we will have the right triangle with sides ((m); (x) and $\sqrt{(b^2 - 2b)}$) of the proposition 1

This rectangle triangle thy of base the leg $\sqrt{b^2 - 2b}$ of the triangle of sides (m; c; and b)

$$m^2 = b^2 - 2b$$

and hence we have

$$m^2 = b^2 - 2b$$

and $c^2 = \sqrt{1}$ we have for $\forall b \in \mathbb{Z}$

with which

$$(b^2 - 2b) + c^2 = b^2$$

and for the new rectangle triangle that is be form with the prolonging of the side (c) which in turn forms an angle of 90° at its intersection with the arc of radius (b-1). Being this the magnitude unknown (x) of the triangle that has for hypotenuse (b-1) and for known leg $\sqrt{b^2 - 2b}$ hence we will have by Theorem Pythagoras

$$(b^2 - 2b) + x^2 = (b - 1)^2$$

x admit value of $\sqrt{+1}$ or 1.

However in the fourth dimension (space-time) the Pythagorean Theorem pass to be of the following form

$$(b^2 - 2b) - (\sqrt{-1})^2 = (b - 1)^2$$

thus mathematics is fulfill admitted in the (space-time) as in the Euclidean world.

III. Conclusion

We have defined the right triangle that has by leg the value of $\sqrt{-1}$ for each and every one of the values of (b) in equation (1). The complex numbers are located between the natural numbers (have their projection point and their magnitude of $\sqrt{-1}$ defined and, not admit a different value ) as defined by the function presented here. also is be corroborate mathematically the projections both in Argand plane as every other projections, how is Euler’s formula in the complex plane $(\cos \alpha + isina)$ as well as, every the complex calculation $(a + ib)$ function (1) and Fig. 1 are the only valid ones for this work. check that it is a graph that indicates the point of origin for the projection of $\sqrt{-1}$; It is only possible to construct it with a ruler and a compass if you do the construction of Fig. 3, you will see that the magnitude indicated as $\sqrt{-1}$ is less than unity. Therefore the $\sqrt{-1}$ is a geometric constant and not a numerical constant.

Pythagoras’ theorem in the fourth dimension (space-time).  

\[a^2 - (ix)^2 = \text{hypotenuse squared}\]

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