Composition and decomposition of indestructible Blaschke products

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Abstract. We prove that the composition of two indestructible Blaschke products is again an indestructible Blaschke product. We also show that if an indestructible Blaschke product is the composition of two bounded analytic functions, then both functions are indestructible Blaschke products.

1 Introduction and results

Let $H^\infty$ denote the Banach space of all functions analytic and bounded in the unit disk $\mathbb{D} := \{ z \in \mathbb{C} : |z| < 1 \}$ equipped with the supremum norm $\| \cdot \|_\infty$. The set $\mathcal{B} := \{ f \in H^\infty \text{ nonconstant} : \| f \|_\infty \leq 1 \}$ of nonconstant functions in the unit ball of $H^\infty$ is clearly closed under composition, that is,

$$f \circ g \in \mathcal{B} \quad \text{for all} \quad f, g \in \mathcal{B}.$$ 

There are three well–known subsemigroups inside the composition semigroup $\mathcal{B}$:

(a) the set

$$\text{Aut}(\mathbb{D}) = \left\{ \eta \frac{z - a}{1 - \overline{a} z} : |a| < 1, |\eta| = 1 \right\}$$

of all conformal automorphisms of $\mathbb{D}$. Note that $\text{Aut}(\mathbb{D})$ is actually a group w.r.t. composition.

(b) the set of all finite Blaschke products

$$\eta \prod_{j=1}^{N} \frac{z - a_j}{1 - \overline{a}_j z}, \quad a_1, \ldots, a_N \in \mathbb{D}, \ |\eta| = 1,$$

and

(c) the set of all inner functions, i.e., those functions $F \in H^\infty$ for which the radial limit function

$$F^*(\zeta) := \lim_{r \to 1-} F(\zeta r),$$

satisfies $|F^*(\zeta)| = 1$ for a.e. $\zeta \in \partial \mathbb{D}$. We refer to [17] for a proof of the fact that the set of inner functions is a semigroup and to [18] for more about the structure of this semigroup.

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On the other hand, the set of all Blaschke products\(^1\)

\[
B(z) = \eta \prod_{j=1}^{\infty} \frac{-\overline{a}_j z - a_j}{|a_j| (1 - \overline{a}_j z)}, \quad |\eta| = 1, \quad (a_j) \subseteq \mathbb{D} \text{ s.t. } \sum_{j=1}^{\infty} (1 - |a_j|) < \infty,
\]

is *not* closed under composition. In fact, a celebrated result of Frostman [1, Theorem 6.4] says that every inner function can be written as the composition \(T \circ B\) of a unit disk automorphism \(T\) and a Blaschke product \(B\). In view of Frostman’s result it is natural to consider those Blaschke products \(B\) such that \(T \circ B\) is a Blaschke product for every \(T \in \text{Aut}(\mathbb{D})\). Such Blaschke products are called *indestructible*, see [8, 16]. Clearly, every finite Blaschke product is indestructible, but there are also numerous examples of infinite indestructible Blaschke products (see [16, Chapter 5] and Remark 1.4 below). Indestructible Blaschke products do have a number of intriguing properties, some of which are described e.g. in [8, 16].

Our first result shows that the set of all indestructible Blaschke products forms a composition semigroup.

**Theorem 1.1 (Composition of indestructible Blaschke products)**

The composition of two indestructible Blaschke products is an indestructible Blaschke product.

Knowing that \(B \circ C\) is an indestructible Blaschke product if both \(B\) and \(C\) are indestructible, the second result of this paper deals with the “inverse” problem.

**Theorem 1.2 (Decomposition of indestructible Blaschke products)**

Let \(B, C \in H^\infty\) be nonconstant functions such that \(B \circ C\) is an indestructible Blaschke product. Then \(B\) and \(C\) are both indestructible Blaschke products.

The proofs of Theorem 1.1 and Theorem 1.2 are given in the next two sections.

**Remark 1.3 (Prime indestructible Blaschke products)**

In view of Theorem 1.1 and Theorem 1.2, it is natural to ask for *prime* indestructible Blaschke products, i.e. indestructible Blaschke products \(A\) such that if \(A = B \circ C\) for \(B, C \in H^\infty\), then \(B\) or \(C\) is a unit disk automorphism. It would be interesting to find examples of infinite prime indestructible Blaschke products. We note that the concept of primality in the context of \(H^\infty\)-functions has been considered for a long time for finite Blaschke products (see [14, 15] for recent developments). For inner functions the concept is due to Stephenson [13] (see also [11, 12]) and has been further explored e.g. by Gorkin, Laroco, Mortini and Rupp [18].

**Remark 1.4 (Maximal Blaschke products)**

A result intimately connected to Theorem 1.1 has recently been proved in [6], see also [5]. There, so-called maximal Blaschke products have been studied. Maximal Blaschke products are characterized by an extremal property and constitute an appropriate generalization of the class of finite Blaschke products. They are defined

\(^{1}\)We use the convention \(\frac{1}{0} = 1\) if \(a = 0\).
as follows. Let $F \in H^\infty$ be a nonconstant function, let $C := (z_1, z_2, \ldots)$ denote the critical points of $F$ counting multiplicity, and let $N$ denote the number of times that 0 appears in $C$. Consider the extremal problem
\[
\max \left\{ \Re f^{(N+1)}(0) : f \in H^\infty, f'(z) = 0 \text{ for any } z \in C \right\}.
\]
It is not difficult to see that this extremal problem has a unique solution $F_C \in H^\infty$ and that $F'_C$ vanishes precisely on the sequence $C$. It turns out, see [6, Theorem 1.1], that the extremal function $F_C$ is an indestructible Blaschke product. This generalizes Nehari’s well-known extension [10] of the Ahlfors–Schwarz lemma from 1947. Nehari’s result covers finite sequences $C$, in which cases the extremal functions are precisely the finite Blaschke products. Now, every Blaschke product of the form $T \circ F_C$ with $T \in \text{Aut}(\mathbb{D})$ and $F_C$ the extremal function for the critical set $C$ of some nonconstant $H^\infty$-function is called a maximal Blaschke product. In particular, every finite Blaschke product is maximal and every maximal Blaschke product is indestructible, so maximal Blaschke products provide a large collection of examples of indestructible Blaschke products. Maximal Blaschke products do have a number of striking properties reminiscent of finite Blaschke products and Bergman space inner functions and they are intimately connected to hyperbolic geometry, see [4, 5, 6].

Remark 1.5
Summarizing the results and remarks above, we have the following “zoo” of subsemigroups of $B \subset H^\infty$ involving Blaschke products (BPs):

\[\text{Aut}(\mathbb{D}) \subset \{\text{finite BPs}\} \subset \{\text{max. BPs}\} \subset \{\text{indestr. BPs}\} \subset \{\text{inner functions}\} \subset B.\]

All but possibly one of these inclusions are strict: We do not know whether there exists an indestructible Blaschke product which is not a maximal Blaschke product.

Pioneering work on indestructible Blaschke products can be found in the papers by Heins [2, 3], McLaughlin [8] and Morse [9]. We also refer to the excellent survey on indestructible Blaschke products by Ross [16] and to the Fields Institute Proceedings [7] for a collection of surveys and research articles on Blaschke products and inner functions in general.

2 Proof of Theorem 1.1

The proof of Theorem 1.1 relies on the following characterization of indestructible Blaschke products due to McLaughlin [8, Theorem 1]; see also Ross [16, Theorem 4.1]. We use the following notation. If $F \in H^\infty$ is not constant, then for fixed $a \in \mathbb{D}$ let $\xi_1(F; a), \xi_2(F; a), \ldots \in \mathbb{D}$ denote the solutions to $F(z) = a$ counting multiplicities.
We denote by \( z_1(F), z_2(F), \ldots \in \mathbb{D}\setminus\{0\} \) the non-zero solutions to \( F(z) = F(0) \) again counting multiplicities.

**Lemma 2.1**

Let \( F \in H^\infty \) be an inner function such that

\[
F(z) = F(0) + b_n z^n + b_{n+1} z^{n+1} + \ldots \quad (b_n \neq 0).
\]

Then \( F \) is an indestructible Blaschke product if and only if the following two conditions hold:

\[
\left| \frac{F(0) - a}{1 - \overline{a} F(0)} \right| = \prod_{j=1}^{\infty} \left| \frac{\xi_j(F; a)}{|\xi_j(F; a)|} \right| \quad \text{for any } a \in \mathbb{D}\setminus\{F(0)\} \quad (2.1)
\]

\[
\frac{|b_n|}{1 - |F(0)|^2} = \prod_{j=1}^{\infty} |z_j(F)|. \quad (2.2)
\]

**Remark 2.2**

The statement of Lemma 2.1 in [8] and [16] is slightly weaker, since both sources assume that \( F \) is a Blaschke product. In fact, the proofs in [8, 16] reveal that it is sufficient to assume that \( F \) is an inner function. This simple observation will be crucial in the proof of Theorem 1.1 given below.

**Proof of Theorem 1.1.** Let \( B \) and \( C \) be two indestructible Blaschke products. We can assume that both \( B \) and \( C \) are not constant. Since \( B \) and \( C \) are inner functions, we see that \( A := B \circ C \) is an inner function. Fix \( a \in \mathbb{D} \). Since \( B \) is indestructible, the function

\[
w \mapsto \frac{B(w) - a}{1 - \overline{a} B(w)}
\]

is a Blaschke product. Recalling the convention \( \frac{\xi}{|\xi|} = 1 \) if \( \xi = 0 \), this implies

\[
\frac{B(w) - a}{1 - \overline{a} B(w)} = \eta \prod_{j=1}^{\infty} \frac{-\xi_j(B; a)}{|\xi_j(B; a)|} \frac{w - \xi_j(B; a) w}{1 - \xi_j(B; a) w}, \quad w \in \mathbb{D}, \quad (2.3)
\]

for some \( \eta \in \partial \mathbb{D} \). We now distinguish the two cases

- Case I: \( a \neq A(0) \)
- Case II: \( a = A(0) \).

We start with Case I, so \( a \in \mathbb{D}\setminus\{A(0)\} \). For \( w = C(0) \) equation (2.3) gives

\[
\frac{A(0) - a}{1 - \overline{a} A(0)} = \eta \prod_{j=1}^{\infty} \frac{-\xi_j(B; a)}{|\xi_j(B; a)|} \frac{C(0) - \xi_j(B; a) C(0)}{1 - \xi_j(B; a) C(0)} \quad (2.4)
\]

As \( A(0) \neq a \), this implies in particular,

\[
C(0) \notin \{\xi_1(B; a), \xi_2(B; a), \ldots\}.
\]
Since $C$ is indestructible, Lemma 2.1 therefore shows that
\[
\prod_{k=1}^{\infty} |\xi_k(C; \xi_j(B; a))| = \left| \frac{C(0) - \xi_j(B; a)}{1 - \xi_j(B; a) C(0)} \right|, \quad j = 1, 2, \ldots,
\]
so (2.4) takes the form
\[
\left| \frac{A(0) - a}{1 - \overline{a} A(0)} \right| = \prod_{j,k=1}^{\infty} |\xi_k(C; \xi_j(B; a))|. \quad (2.5)
\]

We now consider the equation $A(z) = a$. Let $z \in \mathbb{D}$ such that $A(z) = a$. This is equivalent to $B(C(z)) = a$, which is the same as
\[
C(z) \in \{\xi_1(B; a), \xi_2(B; a), \ldots\}.
\]

Hence
\[
\{\xi_l(A; a) : l = 1, 2, \ldots\} = \{\xi_k(C; \xi_j(B; a)) : k, j = 1, 2, \ldots\},
\]
so
\[
\prod_{l=1}^{\infty} |\xi_l(A; a)| = \prod_{j,k=1}^{\infty} |\xi_k(C; \xi_j(B; a))|. \quad (2.5)
\]

Inserting this expression into (2.5) leads to
\[
\left| \frac{A(0) - a}{1 - \overline{a} A(0)} \right| = \prod_{l=1}^{\infty} |\xi_l(A; a)|.
\]

Hence we have verified that condition (2.1) holds for $F = A$.

Let us turn to Case II, so assume $a = A(0)$. Now, we need to distinguish two subcases

**Case IIa:** $C(0) \neq 0$

**Case IIb:** $C(0) = 0$.

Let first $C(0) \neq 0$ and suppose
\[
C(z) = C(0) + c_N z^N + c_{N+1} z^{N+1} + \ldots, \quad c_N \neq 0,
\]
\[
B(z) = B(0) + b_M z^M + b_{M+1} z^{M+1} + \ldots, \quad b_M \neq 0.
\]

Then $A(z) = B(C(z))$ has an expansion of the form
\[
A(z) = A(0) + a_N z^N + a_{N+1} z^{N+1} + \ldots
\]
about $z = 0$ with
\[
a_N = N c_N b_M C(0)^{N-1} \neq 0.
\]

Since $a = A(0) = B(C(0))$, we have
\[
C(0) \in \{\xi_j(B; a) : j = 1, 2, \ldots\},
\]

so we may assume that $C(0) = \xi_1(B; a)$. Using (2.3) for $w = A(z)$ with $z \in \mathbb{D} \setminus \{0\}$ and dividing by $z^N$, we get

$$
\frac{1}{z^N 1 - \sigma A(z)} = \eta_a \prod_{j=2}^{\infty} \frac{-\xi_j(B; a)}{|\xi_j(B; a)|} \frac{C(z) - \xi_j(B; a)}{1 - \xi_j(B; a)C(z)} \frac{-C(0)}{1 - C(0)C(z)} \frac{1}{z^N}.
$$

Letting $z \to 0$ on both sides, we obtain

$$
\frac{a_N}{1 - |a|^2} = \eta_a \prod_{j=2}^{\infty} \frac{-\xi_j(B; a)}{|\xi_j(B; a)|} \frac{C(0) - \xi_j(B; a)}{1 - \xi_j(B; a)C(0)} \frac{c_N}{1 - |C(0)|^2}. \quad (2.6)
$$

In particular, since $a_N \neq 0$ and $c_N \neq 0$, we have

$$
C(0) \notin \{\xi_j(B; a) : j = 2, 3, \ldots\}. \quad (2.7)
$$

We now use the assumption that $C$ is indestructible. Lemma 2.1 shows that

$$
\left| \frac{C(0) - \xi_j(B; a)}{1 - \xi_j(B; a)C(0)} \right| = \prod_{k=1}^{\infty} |\xi_k(C; \xi_j(B; a))|, \quad j = 2, 3, \ldots,
$$

and

$$
\frac{|c_N|}{1 - |C(0)|^2} = \prod_{j=1}^{\infty} |z_j(C)|.
$$

Inserting these last two expressions into (2.6), we arrive at

$$
\frac{|a_N|}{1 - |a|^2} = \prod_{j=2}^{\infty} \prod_{k=1}^{\infty} |\xi_k(C; \xi_j(B; a))| \cdot \prod_{j=1}^{\infty} |z_j(C)|. \quad (2.8)
$$

Now, let us find the non-zero solutions to $A(z) = a$, i.e., the points $z_j(A)$, $j = 1, 2, \ldots$. If $z \in \mathbb{D} \setminus \{0\}$ with $A(z) = a$, then $B(C(z)) = a$. If $C(z) = C(0)$, then $z = z_j(C)$ for some $j = 1, 2, \ldots$ and any $z_j(C)$ is a solution to $A(z) = a$. If $C(z) \neq C(0)$, then (2.7) implies $z = \xi_k(C; \xi_j(B; a))$ for some $k = 1, 2, \ldots$ and some $j = 2, 3, \ldots$. Conversely, each such $\xi_k(C; \xi_j(B; a))$ is a solution to $A(z) = a$. Hence, we have shown that

$$
\{z_j(A) : j = 1, 2, \ldots\} = \{z_j(C) : j = 1, 2, \ldots\} \cup \{\xi_k(C; \xi_j(B; a)) : k = 1, 2, \ldots, j = 2, 3, \ldots\}.
$$

This enables us to rewrite (2.8) as

$$
\frac{|a_N|}{1 - |a|^2} = \prod_{j=1}^{\infty} |z_j(A)|.
$$

Hence condition (2.2) holds for $F = A$ in the case $C(0) \neq 0$.

In a final step, we now proceed to establish condition (2.2) for $F = A$ in the remaining case $C(0) = 0$. Let

$$
B(z) = B(0) + b_N z^N + b_{N+1} z^{N+1} + \cdots, \quad b_N \neq 0, N \geq 1,
$$

$$
C(z) = c_M z^M + c_{M+1} z^{M+1} + \cdots, \quad c_M \neq 0, M \geq 1.
$$
Then \( A(0) = B(C(0)) = B(0) = a \) and
\[
A(z) = a + a_{NM}z^{NM} + \ldots, \quad a_{NM} = b_{N}c_{M} \not= 0.
\]

Since \( B \) is indestructible and \( B(z) - a \) has a zero of order \( N \) at \( z = 0 \), we get
\[
\frac{B(w) - a}{1 - \overline{a}B(w)} = \eta_{a}w^{N} \prod_{j=1}^{\infty} \frac{-z_{j}(B)}{|z_{j}(B)|} w - z_{j}(B) \\
1 - z_{j}(B)w, \quad w \in \mathbb{D}.
\]

For \( w = C(z) \) and \( z \in \mathbb{D}\setminus\{0\} \), this leads to
\[
\frac{1}{z^{MN}} \frac{A(z) - a}{1 - \overline{a}A(z)} = \eta_{a} \left( \frac{C(z)}{z^{M}} \right)^{N} \prod_{j=1}^{\infty} \frac{-z_{j}(B)}{|z_{j}(B)|} \frac{C(z) - z_{j}(B)}{1 - z_{j}(B)C(z)}.
\]

Letting \( z \to 0 \), we deduce
\[
\frac{|a_{NM}|}{1 - |a|^{2}} = |c_{M}|^{N} \prod_{j=1}^{\infty} |z_{j}(B)|. \quad (2.9)
\]

Since \( C \) is indestructible, Lemma 2.1 implies
\[
|c_{M}| = \frac{|c_{M}|}{1 - |C(0)|^{2}} = \prod_{j=1}^{\infty} |z_{j}(C)|.
\]
as well as
\[
|z_{j}(B)| = \left| \frac{C(0) - z_{j}(B)}{1 - z_{j}(B)C(0)} \right| = \prod_{k=1}^{\infty} |\xi_{k}(C; z_{j}(B))|, \quad j = 1, 2, \ldots.
\]

Inserting the last two expressions into (2.9), we get
\[
\frac{|a_{NM}|}{1 - |a|^{2}} = \prod_{j=1}^{\infty} \prod_{k=1}^{\infty} |\xi_{k}(C; z_{j}(B))| \cdot \prod_{j=1}^{\infty} |z_{j}(C)|^{N}. \quad (2.10)
\]

Consider the equation \( A(z) = a \) and its non-zero solutions. Let \( z \in \mathbb{D}\setminus\{0\} \) with \( B(C(z)) = A(z) = a \). If \( C(z) = 0 \), then \( z = z_{j}(C) \) is a zero of \( A(z) - a \) of order \( N \). If \( C(z) \not= 0 \), then \( z = \xi_{k}(C; z_{j}(B)) \) for \( j, k = 1, 2, \ldots \). Hence we can write (2.10) as
\[
\frac{|a_{NM}|}{1 - |a|^{2}} = \prod_{j=1}^{\infty} |z_{j}(A)|.
\]

This proves (2.2) for \( F = A \) also in the case \( C(0) = 0 \).

In summary, we have shown that conditions (2.1) and (2.2) are satisfied for \( F = A \). Lemma 2.1 therefore guarantees that \( A \) is an indestructible Blaschke product. The proof of Theorem 1.1 is complete.
3 Proof of Theorem 1.2

We need the following well–known characterization of Blaschke products, see [1, Theorem 2.4].

Lemma 3.1
Let \( f \in H^\infty \), \( \|f\|_\infty \leq 1 \). Then the following are equivalent.

(a) \( f \) is a Blaschke product.

(b) \( \lim_{r \to 1} \int_0^{2\pi} \log |f(re^{it})| \, dt = 0 \).

(c) The least harmonic majorant of \( \log |f| \) is 0.

Proof of Theorem 1.2. We first prove that \( \bar{C} \) is an indestructible Blaschke product. Let \( T \) be a unit disk automorphism. We need to show that \( \bar{C} := T \circ C \) is a Blaschke product. Since \( B \) is not constant, we can choose another unit disk automorphism \( S \) such that \( \bar{B} := S \circ B \circ T^{-1} \) maps 0 to 0. Hence the Schwarz lemma implies

\[ |\bar{B}(z)| \leq |z|, \quad z \in \mathbb{D}. \tag{3.1} \]

Since \( \bar{A} := B \circ C \) is indestructible, the function \( \bar{A} := S \circ A \) is a Blaschke product and \( \bar{A} = \bar{B} \circ \bar{C} \). Using Lemma 3.1 (a) \( \Rightarrow \) (b), for \( f = \bar{A} \), we obtain

\[ 0 = \lim_{r \to 1} \int_0^{2\pi} \log |\bar{A}(re^{it})| \, dt = \lim_{r \to 1} \int_0^{2\pi} \log |\bar{B}(\bar{C}(re^{it}))| \, dt \leq \lim_{r \to 1} \int_0^{2\pi} \log |\bar{C}(re^{it})| \, dt \leq 0. \]

Hence

\[ \lim_{r \to 1} \int_0^{2\pi} \log |\bar{C}(re^{it})| \, dt = 0, \]

so by Lemma 3.1 (b) \( \Rightarrow \) (a), we conclude that \( \bar{C} \) is a Blaschke product.

Now, let us show that \( B \) is an indestructible Blaschke product. Let \( A := B \circ C \), let \( T \) be a unit disk automorphism and consider \( T \circ B \). Denote by \( h : \mathbb{D} \to \mathbb{R} \) the least harmonic majorant of the subharmonic function \( \log |T \circ B| \). Note \( h \leq 0 \). Then \( h \circ C \) is a harmonic majorant of \( \log |T \circ A| \) and \( h \circ C \leq 0 \). Since \( A \) is indestructible, \( T \circ A \) is a Blaschke product, so that by Lemma 3.1 (a) \( \Rightarrow \) (c), the least harmonic majorant of \( \log |T \circ A| \) is 0. It follows that \( h \circ C = 0 \). Since \( C \) is not constant, the function \( h \), the least harmonic majorant of \( \log |T \circ B| \), is 0. Hence Lemma 3.1 (c) \( \Rightarrow \) (a), implies that \( T \circ B \) is a Blaschke product. This shows that \( B \) is an indestructible Blaschke product.
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