Global quench dynamics of 3-state quantum Potts spin chain

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Abstract. The global quench dynamics of 3-state quantum Potts spin chain is investigated for different initial states using quantum renormalization group method. The results show that the value of negativity is maximum or minimum at quantum phase transition point and the steady value of negativity is also dependent on the initial states, which means that the dynamic behavior is strongly dependent on the initial states. The range of negativity is decreases as the size of system increases for different steps of renormalization group transformations.

1. Introduction

The quantum quench is an important nonequilibrium process [1, 2]. Due to progress of experiments with ultracold atoms in optical lattices, the quantum quench dynamics become very active area of research, which have triggered intensive theoretical research to understand the properties of nonequilibrium relaxation process of closed quantum systems [3-6]. The spin system is a simple tool to stimulate the nonequilibrium process. The propagation of pairwise entanglement in initial Bell states exhibited that entanglement wave evolving from a Bell state turns out to be localized in one-dimensional spin systems [7].

The dynamics of transverse Ising chain prepared in the “thermal ground state” of the pure Ising model is investigated in [8]. The times at which the entanglement reaches its first maximum is obtained and the derivative of this quantity with respect to the reciprocal field is minimal at the critical field. The evolution of entanglement in a one-dimensional Ising chain also shows that entanglement between remote spins is generated under various initial conditions [9, 10].

The ground state fidelity and Loschmidt echo of three-site interacting XX chain in presence of staggered field show dip only at special point in the entire phase diagram and hence fail to detect any quantum phase transition associated with the present model [11]. The structure of revivals for the cluster-XY model after critical quantum quenches presents a nontrivial behavior depending on the phase of the initial state and critical point [12].

The time evolution of the extended quantum compass model exhibits a universal structure of revivals in transverse field after critical quantum quench which is independent of the initial state and the size of the quench [13]. The quantum criticality is neither a sufficient nor necessary condition of the Loschmidt echo to exhibit an observable revival structure in extended Su-Schrieffer-Heeger model and three-site spin-interacting XY model [14].

The entanglement dynamics of three-qubit system in an initial X state undergoing decoherence exhibited nonstandard behavior of the tripartite negativity entanglement metric [15]. “Light-Cone” dynamics after quantum quenches in spin chains shows that the spreading velocities vary substantially with the initial density matrix and a striking data collapse when the spreading velocity is considered to
be a function of the excess energy [16]. The static and dynamical patterns of entanglement in an anisotropic XY model with an alternating transverse magnetic field showed a nonmonotonic behavior of entanglement with respect to temperature in the antiferromagnetic and paramagnetic phases and entanglement is always ergodic [17].

QRG method is an important method to investigate the static critical property of spin systems [18-23]. Recently it also used to study the quench dynamics of spin system, for example, the transverse Ising chain prepared in the “thermal ground state” of the pure Ising model [8] and the extended quantum compass model [13]. Motivated by the above mentioned works, in this paper, global quench dynamics of 3-state quantum Potts spin chain is numerically investigated using QRG method. The outline of this paper is as follows: in Sec. 2, Hamiltonian of Potts model and the initial states are given. In Sec. 3, using the negativity, the global quench dynamics of Potts model with different initial states are numerically investigated and the results are presented for different cases. In Sec. 4, the results are summarized and discussed.

2. Hamiltonian of Potts model and the initial states

In this paper, we will investigate the dynamic behavior of 3-state quantum Potts model on spin chain using the QRG method. The Hamiltonian of q-state quantum Potts model is [24-27]
\[ H_q = -J \sum_i \left( \sum_p M_{i+1}^{q-p} M_{i+1}^{q-p} - h M_i^z \right), \]  
(1)

where \( M_i^{x,p} = \begin{pmatrix} 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \), \( M_i^{z,p} = \begin{pmatrix} q-1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \), are x and z components of Potts spin operators, which act on ith site. \( 1_{q-1} \) is an \((q-1) \times (q-1)\) identity matrix. We consider coupling coefficient \( J > 0 \), which is ferromagnetic interaction and assume that \( h \) is positive. When \( q = 2 \), it is a transverse-field Ising model. For \( q = 3 \), it is a 3-state quantum Potts model.

The first part of the Hamiltonian (1) shows that the effect of the Potts coupling is to raise one spin and to lower the neighboring spin. However, the Potts spin isn’t real spin. Raising the Potts spin when it is in its highest state \([3]\) brings it to its lowest state \([1]\). The eigenstates of two-site Hamiltonian of pure Potts spin chain (with no magnetic field applied) can be easily obtained

\[ |\psi_1\rangle = |\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle |\uparrow\downarrow\rangle / \sqrt{3}, \]
\[ |\psi_2\rangle = |\downarrow\uparrow\rangle + |\uparrow\down\rangle + |\uparrow\down\rangle |\down\uparrow\rangle / \sqrt{3}, \]
\[ |\psi_3\rangle = |\down\down\rangle + |\up\up\rangle + |\up\up\rangle |\down\down\rangle / \sqrt{3}, \]  
(2)

where \([1]\), \([2]\), \([3]\) are the eigenstates of \( M^z \).

In order to using QRG method, we choose two spins as a block in the Potts spin chain. The block Hamiltonian is \( H_b^\prime = -J \sum_{p=1,2} M_{i+1}^{q,p} M_{i+1}^{q,p} - h M_{i+1}^z \) and the corresponding ferromagnetic ground state is three degenerate states \( |\phi_1\rangle = |\chi\rangle |\down\down\rangle + |\up\up\rangle |\up\up\rangle / \sqrt{\chi^2 + 2}, |\phi_2\rangle = |\chi\rangle |\down\up\rangle + |\up\down\rangle |\down\up\rangle / \sqrt{\chi^2 + 2}, |\phi_3\rangle = |\chi\rangle |\up\up\rangle + |\down\down\rangle |\up\down\rangle / \sqrt{\chi^2 + 2}, \) where \( \chi = [3h - J + \sqrt{3}\delta] / 2J, \delta = [3h^2 - 2hJ + 3J^2]. \) The projection operator of the I block is \( P_i = |\phi_1\rangle \langle \Phi_1| + |\phi_2\rangle \langle \Phi_2| + |\phi_3\rangle \langle \Phi_3| \), where \( |\Phi_1\rangle, |\Phi_2\rangle, |\Phi_3\rangle \) are the effective eigenvectors after QRG transformations. The resulting coarse-grained Hamiltonian is \( P^\dagger HP \) and the QRG functions can be written as [22]
\[ \tilde{J} = J(2\chi^2 + 1) / (\chi^2 + 2), \]
\[ \tilde{h} = h(\chi^2 - 1) / (\chi^2 + 2), \]  
(3)
where, \((J, h)\) and \((\tilde{J}, \tilde{h})\) are the parameters of Potts spin chain before and after QRG transformation.

The effective magnetic field \(g\) and \(\tilde{g}\) are defined as \(g = h / J\), \(\tilde{g} = \tilde{h} / \tilde{J}\). The QRG function also can be written as

\[
\tilde{g} = g(\chi^2 - 1) / (2\chi + 1),
\]

where the quantum transition point is \(g_c = 1.0\) and the correlation length critical exponent is \(\nu^{-1} = \log_2\frac{d\tilde{g}}{dg} \big|_{g = g_c} = 0.8464\) [22-23].

3. Global quench dynamics of Potts spin chain

A quantum quench is a sudden changing parameter in the Hamiltonian of quantum 3-state Potts spin chain. The initial states are the states of two-spin cluster of pure Potts chain, \(|\Psi_i(0)\rangle = |\psi_i\rangle\) and \(|\Psi_2(0)\rangle = (|\psi_1\rangle + |\psi_2\rangle + |\psi_3\rangle) / \sqrt{3}\). The density matrix corresponding to initial state is expressed as \(\rho(0) = |\Psi_i(0)\rangle \langle \Psi_i(0)|\) or \(\rho(2(0)) = |\Psi_2(0)\rangle \langle \Psi_2(0)|\).

At time \(t = 0\), the parameter \(h\) of the two-spin Hamiltonian is suddenly switched on the system. Then the evolution of system is under this Hamiltonian \(H = -J \sum_{p=1,2} M_i^{+p} M_2^{-3-p} - h(M_1^i + M_2^i)\). The time-evolution operator is \(U(t) = \exp[-iHt / \hbar]\). The density matrix at time \(t\) is \(\rho(t) = U(t) \rho(0) U^\dagger(t)\). Due to the second part of Hamiltonian (1) acting on all sites of spin chain, this is a global quantum quench.

The negativity is suitable for quantum entanglement between any two states, where the state may be have many components, so it is widely used in many-body physics, in particular quantum information and condensed matter physics [28-32]. The quantum quench dynamics of the system is investigated using negativity. The negativity \(N(g)\) is

\[
N(g) = \sum_i |\mu_i|,
\]

where \(\mu_i\) is the negative eigenvalues of \(\rho_{12}^x\). \(T_2\) is partial transpose for spin 2 in \(\rho_{12}\). From the time-dependent density matrix \(\rho(t)\), the negativity \(N(g)\) of different size system \(2^i\) can be obtained using ith QRG transforms [8].

In figure 1, the negativity \(N(g)\) versus \(t\) is exhibited in spin chain with different \(g\) for different initial states \(|\Psi_1(0)\rangle\) and \(|\Psi_2(0)\rangle\). Similarly to the Ising model [8], the negativity \(N(g)\) exhibits periodic pattern in figure 1(a). As \(g\) increases from \(g = 0.4\) to \(g = 1.6\), the minimum of negativity increases from \(N(g) = 0.22892\) to \(N(g) = 0.85362\) and the period of negativity \(T\) decreases from \(T \approx 2.0\) to \(T \approx 0.7\). In figure 1(b), the initial state is chosen \(|\Psi_2(0)\rangle\) which is only a superposition state of the pure Potts spin chain. At \(t = 0\), the negativity \(N(t)\) of different \(g\) is zero. When \(t\) is small \((t < 1.0)\), the negativity \(N(g)\) increases as \(t\) increases. After \(t > 1.0\), the negativity \(N(g)\) is oscillating as \(t\) increases, but it doesn't exhibit periodic behavior. The reasons is that after quantum quench the period of the negativity is different for \(|\psi_1\rangle, |\psi_2\rangle\) and \(|\psi_3\rangle\).

In figure 2, the negativity \(N(g)\) versus \(t\) for different QRG steps in spin chain with initial state \(|\Psi_1(0)\rangle\) are exhibited with \(g = 0.8\) and \(g = 1.2\). They show that the change of negativity is periodic and the minimum value of negativity increases at step of QRG increases. This means that the size of system is more lager the range of negativity is more smaller. When the size of system is big enough,
the negativity reaches to 1 and it does not change in figure 2(a). In figure 2(b), the result is same to figure 2(a). Compared to figure 2(a), the period is small, this means that the period of negativity $N(g)$ increases as $g$ increases.

**Figure 1.** The negativity $N(g)$ versus $t$ for different $g$ in spin chain with different initial states $|\Psi_1(0)\rangle, |\Psi_2(0)\rangle$ in (a) and (b).

![Figure 1](image1.png)

**Figure 2.** The negativity $N(g)$ versus $t$ for different QRG steps in spin chain with initial state $|\Psi(0)\rangle$, where $g = 0.8$ in (a) and $g = 1.2$ in (b).

![Figure 2](image2.png)

**Figure 3.** The negativity $N(g)$ versus $g$ for different QRG steps in spin chain with initial state $|\Psi_1(0)\rangle$, where $t = 0.4$ in (a) and $t = 1.5$ in (b).

![Figure 3](image3.png)

**Figure 4.** $T_{min}$ versus $g$ for different QRG steps in (a) and $\ln(dT_{min} / dg_{|_{min}})$ versus $\ln(n)$ in (b) of spin chain with initial state $|\Psi_1(0)\rangle$.

![Figure 4](image4.png)
In figure 3, the negativity $N(g)$ versus $g$ for different QRG steps are presented in spin chain with (a) $t = 0.4$ and (b) $t = 1.5$. The initial state is $|\Psi_1(0)\rangle$ in figure 3(a). The negativity exhibits a sharp dip at the critical point with $t = 0.4$. For $g > 1.0$, the negativity shows oscillating structure and the steady value of negativity is 1. The negativity also exhibits a sharper dip at the critical point with $t = 1.5$ in figure 3(b). For $g < 1.0$, there is a dip at different QRG steps. This is different to the results in figure 3(a). For $g > 1.0$ the amplitude is smaller and the amplitude decreases as $g$ increases. The oscillating structure is more obviously. Similar to figure 3(a), the steady value of negativity is equal to the negativity of $g = 0$.

In figure 4, $T_{\text{min}}$ versus $g$ for different QRG steps and $\ln(dT_{\text{min}}/dg|_{\text{min}})$ versus $\ln(n)$ of spin chain with initial state $|\Psi_1(0)\rangle$ are presented, where $T_{\text{min}}$ is time at the first minimum of negativity and $n$ is the number of spin. In figure 4(a), $T_{\text{min}}$ is 0.547 at $g = 1.0$ for different steps of QRG. For $g < 1.0$, the value of negativity is larger than the value of $g > 1.0$ and the lines of the different QRG steps cross at phase transition point. In figure 4(b), the scaling behavior is given, $dT_{\text{min}}/dg|_{\text{min}} \sim n^0, \theta = 0.894$.[24]

![Figure 5](image1.png)  \hspace{1cm}  ![Figure 6](image2.png)

**Figure 5.** The negativity $N(g)$ versus $g$ for different QRG steps in spin chain with initial state $|\Psi_2(0)\rangle$, where $t = 0.4$ in (a) and $t = 1.5$ in (b).

**Figure 6.** The steady value of negativity $N(g)$ versus $t$ in spin chain with initial state $|\Psi_2(0)\rangle$.

In figure 5, the negativity $N(g)$ versus $g$ for different QRG steps are exhibited in Potts spin chain with $t = 0.4$ and $t = 1.5$. The initial state is $|\Psi_2(0)\rangle$. At the critical point $g = 1.0$, the negativity $N(g)$ sharply increases as the QRG step increases in figure 5(a). The maximum of negativity is 0.4782. For $g > 1.0$ the negativity oscillating decay to the steady value $N(g) = 0.278$. The negativity increases to the maximal value $N(g) = 0.962$ in figure 5(b). At the critical point, there
is a dip, the minimal value is 0.68. For \( g > 1.0 \), the negativity oscillating decreases to 0.90282. Compared to figure (a), the steady value of negativity is bigger.

In figure 6, the steady value of negativity \( N(g) \) versus \( t \) in spin chain with initial state \( |\Psi_2(0)\rangle \). The steady value of negativity \( N(g) \) is periodic oscillation as \( t \) increases. The period is \( t \approx 6.28 \). The range of negativity \( N(g) \) increase from 0 to 1.0 and there are two peaks in every period.

4. Summary
In this paper, we present dynamic of negativity in spin chain near quantum critical point. Our results indicate that the behavior of dynamic is strongly dependent on the initial states. If the initial state is eigenstate \( |\psi_1(0)\rangle = |\psi_1\rangle \) of first part in system Hamiltonian, it shows periodic behavior. If the initial state is superposition state \( |\Psi_2(0)\rangle = |\psi_1\rangle + |\psi_2\rangle + |\psi_3\rangle / \sqrt{3} \), it shows oscillating behavior as time \( t \) increases. As size of system increases, the range of negativity decreases. At the quantum critical point, the negativity shows minimum or maximum for different initial states. Far from the critical point \( g = 1.0 \), the value of negativity will be approach to same value for different steps of QRG. The steady value is dependent on the initial state. The initial state is eigenstate \( |\psi_1(0)\rangle \), it will be approach to negativity of initial state. The initial state \( |\Psi_2(0)\rangle \) is superposition state, it decreases to the steady value and the steady value of negativity \( N(g) \) is periodic oscillation as \( t \) increases.

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