Squeezing arbitrary cavity-field states through their interaction with a single driven atom

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Abstract

We propose an implementation of the parametric amplification of an arbitrary radiation-field state previously prepared in a high-Q cavity. This nonlinear process is accomplished through the dispersive interactions of a single three-level atom (fundamental $|g⟩$, intermediate $|i⟩$, and excited $|e⟩$ levels) simultaneously with i) a classical driving field and ii) a previously prepared cavity mode whose state we wish to squeeze. We show that, in the adiabatic approximantion, the preparation of the initial atomic state in the intermediate level $|i⟩$ becomes crucial for obtaining the degenerated parametric amplification process.

PACS: 42.50.Ct, 42.50.Dv

Journal-ref: Phys. Rev. A 68, 061801(R) (2003)
The parametric amplification process represents a central issue in quantum optics since its applications range from fundamental physics to technology. As one of its by-products, the squeezed states of the radiation field have been researched in order to deepen our understanding of the properties of radiation [1] and its interaction with matter [2]. An unequivocal signature of the quantum nature of light has been provided by the antibunching process emerging from squeezed light, and apart from fundamental questions, squeezed states have provoked some striking technological challenges. The improvement of the signal to noise ratio in optical communication [3] and, even more attractive, the possibility of measuring gravitational waves through squeezed fields [4], are just some of the potential applications of squeezed states. Such proposals rely on reducing the quantum fluctuation in one (signal) quadrature component of the field at the expense of amplifying the fluctuation in another (unobservable) component, as ruled by the Heisenberg uncertainty relation [5].

As squeezed light is mainly supplied by nonlinear optical media as running waves (through backward [6] or forward [7] four-wave mixing and parametric down-conversion [8]), standing squeezed fields in high-\(Q\) cavities or ion traps can be generated through atom-field interaction [9]. Although considerable space has been devoted in the literature to the squeezing process in the Jaynes-Cummings model, the issue of squeezing any desired prepared cavity-field state \(|\Psi\rangle\), i.e., the accomplishment of the operation \(S(\zeta)|\Psi\rangle\) in cavity QED (\(\zeta\) standing for a set of group parameters) has not been addressed. Engineering such an operation is the subject of the present letter; it is achieved through the dispersive interactions of a three-level atom simultaneously with a classical driving field and a cavity mode whose prepared state we wish to squeeze. In short, the dispersive interaction of the cavity mode with a driven atom produces the desired operation \(S(\zeta)|\Psi\rangle\).

Selective atomic measurements in cavity QED have been employed to enhance squeezing in the Jaynes-Cummings model (JCM) [10]. Whereas cavity-field squeezing in the JCM is rather modest, about 20% for low average photon number, squeezing of up to 75% can be obtained through selective atomic measurements [10]. However, the squeezed states resulting from selective atomic measurements (and the proposed schemes employing atom-field interactions [11]) do not come from the unitary evolution \(S(\zeta)|\Psi\rangle\). In the present proposal of dispersive interaction of the cavity mode – whose state \(|\Psi\rangle\) is to be squeezed – with a driven atom, we obtain squeezing around 88%. This higher squeezing is crucial to the building of truly mesoscopic superpositions with a large average photon number and also a large...
“distance” in phase space between the centers of the quasi-probability distribution of the individual states composing the prepared superposition \[12\].

As depicted in Fig. 1, the three-level atom is in a ladder configuration where an intermediate atomic level (\(|i\rangle\)) lies between the ground (\(|g\rangle\)) and the excited (\(|e\rangle\)) states. The quantized cavity mode of frequency \(\omega\) couples dispersively both transitions \(|g\rangle \leftrightarrow |i\rangle\) and \(|e\rangle \leftrightarrow |i\rangle\) with coupling constants \(\lambda_g\) and \(\lambda_e\), respectively, and detuning \(\delta = |\omega - \omega_{\ell i}|\) (\(\ell = g, e\)). A classical field of frequency \(\omega_0 = 2\omega + \Delta\) drives dispersively the atomic transition \(|g\rangle \leftrightarrow |e\rangle\) with coupling constant \(\Omega\). (We assume that the transition \(|g\rangle \leftrightarrow |e\rangle\) may be induced by applying a sufficiently strong electric field) While the quantum field promotes a two-photon interchange process, the classical driving field constitutes the source of the parametric amplification. This system has been considered in many theoretical \[13\] and experimental works \[14\].

The Hamiltonian of our model, within the rotating wave approximation, is given by

\[
H = H_0 + V,
\]

where

\[
H_0 = \hbar \omega a^\dagger a - \hbar \omega |g\rangle \langle g| + \hbar \delta |i\rangle \langle i| + \hbar \omega |e\rangle \langle e|,
\]

\[
V = \hbar (\lambda_g a |i\rangle \langle g| + H.c.) + \hbar (\lambda_e a |e\rangle \langle i| + H.c)
+ \hbar (\Omega |e\rangle \langle g| e^{-i\omega_0 t} + H.c.),
\]

(1b)

with \(a^\dagger\) (\(a\)) standing for the creation (annihilation) operator of the quantized cavity mode. Writing \(H\) in the interaction picture (through the unitary transformation \(U_0 = \exp (-iH_0 t/\hbar)\)) and then applying the transformation \(U = \exp [-i\delta t (|g\rangle \langle g| + |e\rangle \langle e|)]\), we obtain the Hamiltonian \(H = U_0^\dagger U HUU_0 - H_0 - \hbar \delta (|g\rangle \langle g| + |e\rangle \langle e|)\) given by

\[
H = \hbar (\lambda_g a \sigma_{ig} + \lambda_e a \sigma_{ei} + \Omega e^{-i\Delta t} \sigma_{eg} + H.c)
- \hbar \delta (\sigma_{gg} + \sigma_{ee}),
\]

(2)

where we have defined the atomic transition operator \(\sigma_{kl} = |k\rangle \langle l|\), \(k, l = g, i, e\). Next, we compare the time scales of the transitions induced by the cavity field, considering the Heisenberg equations of motion for the transition operators \(\sigma_{ig}\) and \(\sigma_{ei}\),

\[
\frac{d}{dt} \sigma_{ig} = \lambda^*_g a^\dagger (\sigma_{ii} - \sigma_{gg}) - \lambda_e a \sigma_{eg} + \Omega^* e^{i\Delta t} \sigma_{ie} - \delta \sigma_{ig},
\]

(3a)

\[
\frac{d}{dt} \sigma_{ei} = \lambda_g a \sigma_{eg} + \lambda^*_e a^\dagger (\sigma_{ee} - \sigma_{ii}) - \Omega^* e^{i\Delta t} \sigma_{gi} + \delta \sigma_{ei}.
\]

(3b)
If the dispersive transitions are sufficiently detuned, i.e., \( \delta \gg |\lambda_g|, |\lambda_e|, |\Omega|, |\Delta| \), we obtain the adiabatic solutions for the transition operators \( \sigma_{ig} \) and \( \sigma_{ei} \), by setting \( d\sigma_{ig}/dt = d\sigma_{ei}/dt = 0 \) [15]:

\[
\sigma_{ig} = \frac{1}{\delta} \left[ \lambda_g^* a^\dagger (\sigma_{ii} - \sigma_{gg}) - \lambda_e a \sigma_{eg} + \Omega^* e^{i\Delta t} \sigma_{i\ell} \right], \quad \text{(4a)}
\]

\[
\sigma_{ei} = \frac{1}{\delta} \left[ \lambda_e^* a^\dagger (\sigma_{ii} - \sigma_{ee}) - \lambda_g a \sigma_{eg} + \Omega e^{i\Delta t} \sigma_{i\ell} \right]. \quad \text{(4b)}
\]

Solving the system (4a, 4b) and inserting these adiabatic solutions for the transition operators \( \sigma_{ig} \) and \( \sigma_{ei} \) into Eq. (2), the Hamiltonian becomes

\[
\mathbf{H} = -\hbar \delta (\sigma_{gg} + \sigma_{ee}) + \hbar \left( \Omega e^{-i\Delta t} \sigma_{eg} + \text{H.c.} \right) - \frac{\hbar}{\delta} \left\{ \left( 2a^\dagger a + 1 \right) \right. \\
\times \left[ |\lambda_g|^2 \sigma_{gg} - (|\lambda_g|^2 + |\lambda_e|^2) \sigma_{ii} + |\lambda_e|^2 \sigma_{ee} + \frac{|\lambda_g|^2 + |\lambda_e|^2}{2\delta} \left( \Omega e^{-i\Delta t} \sigma_{eg} + \text{H.c.} \right) \right] \\
+ 2 \left( \lambda_g \lambda_e a^2 \sigma_{eg} + \text{H.c.} \right) + \frac{1}{\delta} \left( \lambda_g \lambda_e \Omega^* e^{i\Delta t} a^2 + \text{H.c.} \right) \left( \sigma_{gg} + \sigma_{ee} - 2\sigma_{ii} \right) \right\} \quad \text{(5)}
\]

We note that the solution of the system (4a, 4b) must be inserted into a symmetrized Hamiltonian (2), where the operator structure \( a\sigma_{kl} \) must be substituted by \( (a\sigma_{kl} + \sigma_{lk}a)/2 \). Otherwise, the resulting Hamiltonian (5) would depend on the order of the operators, \( a\sigma_{kl} \) or \( \sigma_{lk}a \). The state vector associated with Hamiltonian (5), in the Schrödinger picture, can be written using

\[
|\Psi (t)\rangle = |g\rangle |\Phi_g (t)\rangle + |i\rangle |\Phi_i (t)\rangle + |e\rangle |\Phi_e (t)\rangle, \quad \text{(6)}
\]

where \( |\Phi_\ell (t)\rangle = \int \frac{d\alpha}{\pi} \mathcal{A}_\ell (\alpha, t) |\alpha\rangle \), \( \ell = g, i, e \), the complex quantity \( \alpha \) standing for the eigenvalues of \( a \), and \( \mathcal{A}_\ell (\alpha, t) = \langle \alpha, \ell | \Psi (t) \rangle \) is the expansion coefficients for \( |\Phi_\ell (t)\rangle \) in the basis of coherent states, \( \{|\alpha\rangle\} \). Using the orthogonality of the atomic states and Eqs. (5) and (6) we obtain the uncoupled time-dependent (TD) Schrödinger equations for the atomic subspace \(|i\rangle \) (in the Schrödinger picture):

\[
i\hbar \frac{d}{dt} |\Phi_i (t)\rangle = \mathcal{H}_i |\Phi_i (t)\rangle, \quad \text{(7)}
\]

\[
\mathcal{H}_i = \hbar \omega a^\dagger a + \hbar \left( \xi e^{-i\nu t} a^\dagger a + \xi^* e^{i\nu t} a^2 \right) \quad \text{(8)}
\]

where \( \omega = \omega + \chi \) \( \chi = 2 \left( |\lambda_g|^2 + |\lambda_e|^2 \right)/\delta \) stands for the effective frequency of the cavity mode, while \( \xi = 2\Omega \lambda_g \lambda_e^*/\delta^2 = |\xi| e^{-i\Theta} \) and \( \nu = 2\omega + \Delta \) are the effective amplitude and frequency of the parametric amplification field. For subspace \(|g\rangle, |e\rangle\) there is a TD
Schrödinger equation which couples the fundamental and the excited atomic states. Therefore, when we initially prepare the atom in the intermediate level $|i⟩$, the dynamics of the atom-field dispersive interactions, governed by the effective Hamiltonian (8), results in a cavity mode with shifted frequency submitted to a parametric amplification process.

In the resonant regime the classical driving field has the same frequency as the cavity mode, so that $\nu = 2\omega$ (i.e. $\Delta = 2\chi$). The evolution of the cavity field state, in the interaction picture, is governed by a squeeze operator such as $|\Phi_i(t)⟩ = S(\xi, t)|\Phi_i(0)⟩$, where

$$S(\xi, t) = \exp \left[ -i \left( \xi a^{\dagger 2} + \xi^* a^2 \right) t \right].$$

(9)

The degree of squeezing in the on-resonant regime is determined by the factor $r_{on}(t) = 2 |\xi| t$, while the squeeze angle is given by $\varphi_{on} = \pi/2 - \Theta$. For a specific cavity mode and atomic configuration, the parameter $r_{on}(t)$ can be adjusted in accordance with the coupling strength $\Omega$ and the interaction time $t$. Assuming typical values for the parameters involved, arising from Rydberg states where the intermediate state $|i⟩$ is nearly halfway between $|g⟩$ and $|e⟩$, we get $|\lambda_g| \sim |\lambda_e| \sim 3 \times 10^5 s^{-1}$ \[16\], \[17\]. With such values and assuming the detuning $|\delta| \sim 15 \times |\lambda_g|$ and the coupling strength also $\Omega \sim 3 \times 10^5 s^{-1}$, we obtain $|\xi| \sim 3 \times 10^3 s^{-1}$.

For an atom-field interaction time about $t \sim 2 \times 10^{-4}$s, we get the squeezing factor $r_{on}(t) \sim 1.07$ such that, for the resonant regime, the variance in the squeezed quadrature turns out to be $\langle (\Delta X)^2 \rangle = e^{-2r_{on}(t)}/4 \sim 3 \times 10^{-2}$, representing a squeezing around 88% (for an initial coherent state prepared in the cavity) with the passage of just one atom. Of course, the injection of more atoms through the cavity leads to a squeezing even greater than this remarkable rate. Note that the interaction time considered here is two (one) order of magnitude smaller than the field decay time in closed \[16\] (open \[17\]) microwave cavities used in these experiments. We note that closed cavities do not allow the use of circular Rydberg atoms and, consequently, the atomic decay time becomes a concern \[18\].

Even that the adjustment of the detuning $\Delta$ between the driving field and the atomic transition (such that $\Delta = 2\chi$) is not the main difficulty of implementing the method here proposed we also analyzed the off-resonant regime ($\nu \neq 2\omega$). To solve the Schrödinger Eq. (7) we employed the TD invariants of Lewis and Riesenfeld \[19\] as demonstrated in detail in \[12\]. It is possible to show \[12\], \[20\] that in the off-resonant regime we find three different solutions depending on parameter $\mathfrak{P} = 4 |\xi|/(2\chi - \Delta)$ which is an effective macroscopic coupling: the strong ($|\mathfrak{P}| > 1$), weak ($|\mathfrak{P}| < 1$), and critical ($|\mathfrak{P}| = 1$) coupling parameter.
There is a well-known threshold in the behavior of the TD squeeze factor \( r_{\text{off}}(t) \), arising from the quadratic TD Hamiltonian (8) \([12, 20]\): \( r_{\text{off}}(t) \) increases monotonically for \( |\mathcal{P}| \geq 1 \), while for \( |\mathcal{P}| < 1 \) it oscillates periodically. For this reason, in the present letter we are interested in the strong coupling regime, where we obtain the highest TD squeeze parameters, given by

\[
\cosh (2r_{\text{off}}(t)) = \frac{1}{\mathcal{P}^2 - 1} \left[ \frac{e^{h(t)}}{4} + \mathcal{P}^2 \left( \mathcal{C}^2 + \mathcal{P}^2 - 1 \right) e^{-h(t)} - \mathcal{C} \right].
\] (10a)

\[
\cos \left[ \varphi_{\text{off}}(t) + \nu t - \Theta \right] = \frac{\mathcal{C} - \cosh (2r_{\text{off}}(t))}{\mathcal{P} \sinh (2r_{\text{off}}(t))},
\] (10b)

where the constant \( \mathcal{C} \) and function \( h(t) \) are given, respectively, by

\[
\mathcal{C} = \cosh [2r_{\text{off}}(0)] + \mathcal{P} \cos [\varphi_{\text{off}}(0) - \Theta] \sinh [2r_{\text{off}}(0)]
\] (11a)

\[
h(t) = \mp \sqrt{\mathcal{P}^2 - 1} |\mathcal{P}| 4\xi t + \ln \left[ 2 |\mathcal{P}| \left( \sqrt{(\mathcal{P}^2 - 1)(\mathcal{C}^2 - 1)} + \mathcal{C} |\mathcal{P}| \right) \right],
\] (11b)

the sign being chosen so that \( r_{\text{off}}(t) \geq 0 \). We note that in the limit as \( |\mathcal{P}| \to \infty \), i.e, \( \Delta \to 2\chi \), we obtain the on-resonant interaction from the dispersive strong coupling regime. Therefore, the highest squeezing factor, resulting from the highest intensity of the effective coupling \( \mathcal{P} \), is computed from the on-resonant regime, as observed in Fig. 2, where the ratio \( r_{\text{off}}(t)/r_{\text{on}}(t) \) is depicted as a function of the detuning \( \Delta \).

It is worth stressing that for weak damped systems, as fields trapped into realistic high-\( Q \) cavities, the lifetime of the squeezing is of order of the relaxation time of the cavity \([21]\). Therefore, the dissipative mechanism of the cavity plays a much milder role in the lifetime of the squeezing than in decoherence phenomena \([22]\). Regarding atomic decay, we note that for circular Rydberg levels the spontaneous emission hardly affects the squeezing process for typical interaction time scales. In this connection, next we estimate the on-resonant squeezing factor considering the finite lifetime of the atomic levels as well as the cavity damping rate which are introduced phenomenologically into the equation of motion

\[
\frac{d}{dt} \mathcal{O} = -\frac{i}{\hbar} [\mathbf{H}, \mathcal{O}] - \frac{\Gamma}{2} \mathcal{O},
\] (12)

where \( \Gamma \) stands for the decay rate of the system corresponding to operator \( \mathcal{O} \) and Hamiltonian \( \mathbf{H} \) is given by Eq. \([2]\) \([23]\). To estimate the squeezing factor we compute the variance of the field quadrature \( X = (a e^{-i\varphi_{\text{on}}} + a^\dagger e^{i\varphi_{\text{on}}})/2 \) from the solution of equation

\[
\frac{d}{dt} a = i \left( \lambda_g \sigma_{ig} + \lambda_c \sigma_{ei} \right) - \frac{\Gamma_c}{2} a,
\] (13)
where $\Gamma_c$ indicates the cavity damping rate. Proceeding to the adiabatic solutions of equations

\[
\frac{d}{dt} \sigma_{ig} = \lambda_c^* a^\dagger (\sigma_{ii} - \sigma_{gg}) - \lambda_c a \sigma_{eg} + \Omega^* e^{i \Delta t} \sigma_{ie} - \delta \sigma_{ig} - \frac{\Gamma_i}{2} \sigma_{ig},
\]

\[
\frac{d}{dt} \sigma_{ei} = \lambda_g a \sigma_{eg} + \lambda_e^* a^\dagger (\sigma_{ee} - \sigma_{ii}) - \Omega^* e^{i \Delta t} \sigma_{gi} + \delta \sigma_{ei} - \frac{\Gamma_e}{2} \sigma_{ei},
\]

assuming now that the dispersive transitions are sufficiently detuned such that $\delta \gg |\lambda_g|, |\lambda_e|, |\Omega|, |\Delta|, \Gamma_{ig}, \Gamma_{ei}$, we obtain the solutions in Eqs. (14a) and (14b) except for changing $\delta$ by $\delta - i \Gamma_i$ and $\delta - i \Gamma_e$, respectively.

In what follows we consider three approximations in order to simplify our calculations. First, $i)$ we assume the same lifetime for both atomic levels $|g\rangle$ and $|i\rangle$ to define the atomic decay rate $\Gamma_a = \Gamma_i = \Gamma_e$. Secondly, $ii)$ we assume that the atomic decay will hardly populate level $|g\rangle$ and, consequently, level $|e\rangle$ (which is coupled to $|g\rangle$ through the classical field). (In fact, even in the ideal situation where dissipation is dismissed, the experiment must be restarted when the atom is not detected in the state $|i\rangle$ after interacting with the cavity field.) With this assumption, which considerably simplify the problem, we obtain the commutation $[\sigma_{ii}, H] \propto \sigma_{gg}, \sigma_{ee}, \sigma_{ge} \approx 0$ such that $\sigma_{ii}(t) = e^{-\Gamma_at} \sigma_{ii}(0)$. Substituting the solutions for $\sigma_{ei}$ and $\sigma_{ig}$ (resulting from these two approximations besides the adiabatic one) into Eq. (13) we finally obtain the coupled equations

\[
\frac{d}{dt} \tilde{a} = -i \chi (1 - e^{-\Gamma_at/2}) \tilde{a} + i \xi e^{-\Gamma_at/2} \tilde{a}^\dagger,
\]

\[
\frac{d}{dt} \tilde{a}^\dagger = i \chi (1 - e^{-\Gamma_at/2}) \tilde{a}^\dagger - i \xi e^{-\Gamma_at/2} \tilde{a},
\]

where $\tilde{a} = e^{(\Gamma_c + i \chi)t/2} a$. Next, we proceed to the third approximation $iii)$ noting that for the time interval of the atom-field interaction, about $10^{-4}$s, and for the spontaneous-emission decay times of circular Rydberg states $\Gamma_a \sim 10^2$ s$^{-1}$, the second terms on the right hand side of Eqs. (15a) and (15b) can be dismissed. Therefore, within the above approximations we finally obtain the solution

\[
a = e^{-(\Gamma_c + i \chi)t/2} \left( a_0 \cosh \tilde{r}_{on} + e^{-i \varphi_{on}} a_0^\dagger \sinh \tilde{r}_{on} \right),
\]

from which we obtain the variance in the squeezed quadrature ($\varphi_{on} = \pi/2$)

\[
\langle \Delta X \rangle^2 = \frac{1}{4} \left[ 1 - (1 - e^{-2\tilde{r}_{on}}) e^{-\Gamma_c t} \right]
\]
where the squeezing factor under the atomic decay is \( \tilde{r}_{on} = 4 \mid \xi \mid (1 - e^{-\Gamma_a t/2})/\Gamma_a \). As noted after Eqs. (15a) and (15b), for the time interval of the atom-field interaction \( e^{-\Gamma_a t/2} \approx 1 - \Gamma_a t/2 \), such that the squeezing factor under atomic decay \( \tilde{r}_{on} \) is approximately that of the ideal case \( r_{on} \). However, the damping of the cavity mode, expressed by the time-dependent exponential decay in Eq. (17), contributes substantially to increase the variance of the squeezed quadrature and, consequently, to decrease the squeezing rate. Assuming the decay time of circular Rydberg states \( \Gamma_a \sim 10^2 \) s\(^{-1} \) (when \( n \approx 50 \)) and the typical values considered above for the parameters \( |\lambda_g|, |\lambda_e|, |\Omega|, \delta, \) and \( t \), we obtain \( \tilde{r}_{on} \sim 1.06 \).

Therefore, for the typical decay factor for open high-\( Q \) cavities, \( \Gamma_c \sim 10^3 \) s\(^{-1} \) [17], we obtain the variance in the squeezed quadrature \( \langle \Delta X \rangle^2 \sim 7 \times 10^{-2} \), representing a squeezing around 72%. For closed high-\( Q \) cavities, where \( \Gamma_c \sim 10^{3} \) s\(^{-1} \) and noncircular Rydberg levels with \( n \sim 60 \) are employed, such that, \( \Gamma_a \sim 5 \times 10^3 \) s\(^{-1} \) [16], we obtain \( \langle \Delta X \rangle^2 \sim 4.7 \times 10^{-2} \) and a squeezing around 81%. We note that in closed cavities an external amplification field directly coupled to a second normal mode of the cavity could be used [24].

There are others sensitive points in the experimental implementation of the present scheme. Apart from the atomic detection efficiency and the spread of the atomic velocity not taken into account in the present analysis, the Gaussian profile \( f(x) \) of the cavity field in the transverse direction must also be computed. Due to this Gaussian profile the atom-field couplings \( \lambda_g \) and \( \lambda_e \) becomes time-dependent parameters as well as the effective amplitude of the parametric amplification field which turns to be (without considering dissipation) \( \xi = 2\Omega \lambda_a \lambda_b [f(x)]^2 / \delta^2 \) where \( f(x) = \exp(-x^2 / w^2) \) (\( x \) being the time-dependent atom position from the center of the cavity, and \( w \sim 0.6 \) cm [17] is the waist of the Gaussian). The effect of the field profile can be evaluated by using the analytical results for a time-dependent parametric amplification process, demonstrated in [12], leading to the on-resonant squeezing factor \( r'_{on} = 2 \int_0^\tau \xi(t) dt = (4\Omega \lambda_g \lambda_e / \delta^2) \int_0^\tau [f(x)]^2 dt \). Considering the atom-field interaction time about \( \tau \sim 2 \times 10^{-4} \) s, we get the squeezing factor \( r'_{on} \sim 0.4 \) representing \( \langle \Delta X_1 \rangle^2 \sim 1.1 \times 10^{-1} \) and a squeezing around 55%. To obtain the value \( r'_{on} \sim 1 \) of the ideal case, we must increase the interaction time to \( \tau \sim 5 \times 10^{-4} \) s. However, with this value of the atom-field interaction time the dissipative process becomes more pronounced and a coast-benefit estimative must be computed. A detailed analysis involving both error sources, the dissipative process and the Gaussian profile of the cavity field will be considered elsewhere [25].
In conclusion, we have shown theoretically that the dispersive interaction of a cavity mode prepared in the state $|\Psi\rangle$ with a driven atom would produce the squeezing operation $S(\zeta)|\Psi\rangle$. In the ideal case we would obtain squeezing around 88% of a prepared coherent field state, in the on-resonant regime, with the passage of a single three-level atom through the cavity. We finally stress that the squeezing of previously prepared states is crucial to build truly mesoscopic superpositions with a large average photon number produced by the parametric amplification process we have engineered [12].

Acknowledgments

We wish to express thanks for the support from FAPESP (under contracts #99/11617-0, #00/15084-5, and #02/02633-6) and CNPq (Instituto do Milênio de Informação Quântica), Brazilian agencies.

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**Figure Captions**

Fig. 1. Energy-levels diagram of the three-level atom for the parametric amplification scheme.

Fig. 2. Ratio of the squeezing factors in the off-resonant and on-resonant regimes.
FIG. 1
\[ r(t) = r_{\text{off}}(t) + (t - 1) \Delta (s^{-1}) \]

**FIG. 2**