Precise determination of $V_{ud}$ and $V_{us}$

Summary of the WG1 contributions,
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Abstract

The actual limit of the Unitarity condition of the first row of the CKM matrix $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$ is $\Delta_{CKM} = -0.0001(6)$. In 2010 the same was $\Delta_{CKM,2010} = +0.0001(6)$. Despite the only difference of a sign, and with an absolute change of the value of one third of the accuracy, a substantial amount of work has been done in the last two years to improve the knowledge of all the contributions to this stringent limit to CKM unitarity, and more is expected in the next years. In this paper we present an organized summary of all the important contributions presented during the WG1 sessions, referring as much as possible to the contribution papers prepared by the individual authors.

New bounds on violations of CKM unitarity translate into significant constraints on various new physics scenarios. Such tests may eventually turn up evidence of new physics. If the couplings of the W to quarks and leptons are indeed specified by a single gauge coupling, then for universality to be observed as the equivalence of the Fermi constant $G_F$ as measured in muon and hadron decays, the CKM matrix must be unitary. Currently, the most stringent test of CKM unitarity is obtained from the first-row relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$; here we shortly discuss the ingredients that contribute to its accuracy and its possible new physics implications.

We start with Vincenzo Cirigliano [1] that in his talk emphasized a model-independent EFT approach to $\beta$ decays and Cabibbo universality tests. Given the hierarchy $|V_{ud}|^2 \gg |V_{us}|^2$, let us focus on effects of physics beyond the Standard Model (BSM) to $|V_{ud}|$. Assuming that right-handed neutrinos do not appear as low-energy degrees of freedom, new physics introduces five operators

$$
\epsilon_\Gamma \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_l \cdot \pi \Gamma u \quad (\Gamma = V, A), \quad \epsilon_\Gamma \bar{\ell} (1 - \gamma_5) \nu_l \cdot \pi \Gamma d \quad (\Gamma = S, P, T)
$$

(1)
into the Lagrangian of the $d \to ul\nu$ transitions. The (axial-)vector and scalar couplings $\epsilon_{\{V,A,S\}}$ are constrained at the level of $10^{-3}$ from the superallowed nuclear $\beta$ decays. The current precise knowledge of $|V_{ud}|$ from the nuclear decays together with a future competitive determination from neutron decays could constrain $\epsilon_{\{S,T\}}$ at 0.02% level. The pseudo-scalar coupling $\epsilon_P$ is strongly constrained from the ratio $\Gamma(\pi \to e\nu_e)/\Gamma(\pi \to \mu\nu_\mu)$. The outlook is therefore quite positive: the effective couplings of all the BSM charged-current operators are currently probed or will be soon probed at the level of $10^{-3}$ or better. This corresponds to probing maximal BSM physics scales $\Lambda$ ranging from 7 TeV (for scalar and tensor interactions) to 11 TeV (for vector interactions).

New physics can also modify the Lagrangian of the muon decays. Its effects would be encoded in the Fermi constant $G_F$, which is best determined by the measurement of the positive muon lifetime $\tau_\mu$. Tim Gorringe [2] reported results from the MuLan measurement of the positive muon lifetime, conducted at the Paul Scherrer Institute. The result is characterized by a part-per-million accuracy, largely dominated by the statistical contribution, and rock-solid systematic effects study. The MuLan measurement translates into $G_F = 1.1663787(6) \times 10^{-5}\text{GeV}^{-2}$. The 0.5 ppm error is dominated by the 1.0 ppm uncertainty of the lifetime measurement, with contributions of 0.08 ppm from the muon mass measurement and 0.14 ppm from the theoretical corrections. $\tau_\mu$ and $G_F$ at 0.1 ppm is now the open challenge for the next years.

$|V_{ud}|$ is best determined using $0^+ \to 0^+$ super-allowed nuclear $\beta$ decays. The $|V_{ud}|$ state of the art has been presented by Dan Melconian [3]. Given the limited number of new experimental contributions, no new survey on $|V_{ud}|$ has been done and its world average is unchanged from the 2010 CKM edition [4]. The $0^+ \to 0^+$ transitions benefit from the conservation of the vector current and from small isospin-breaking corrections. The experimental inputs are combined in a quantity that should be nucleus-independent to first order. This is true only applying also the isospin-breaking corrections, $\delta_C$. These are a dominant contribution to the $|V_{ud}|$ accuracy and have been studied in recent years using a variety of theoretical methods. A new approach, proposed by Towner and Hardy [5], allow one to experimentally test the SU(2) correction. They succeeded in measuring the $\delta_C$ for the elements with the largest isospin-breaking correction, $^{32}\text{Ar}$ and $^{32}\text{Cl}$, and found it in agreement with the theoretical calculations. Other groups are beginning to develop complementary models of SU(2) in superallowed decays that will be important checks and may lead to smaller systematic error on this $|V_{ud}|$ determination.

The alternative ways to determine $|V_{ud}|$, from neutron lifetime, from mirror decays, and from pion $\beta$ decay, are still limited by the experimental accuracy. It is nevertheless worth improving their accuracy because the involved processes probe different BSM operators.
As a probe free from nuclear structure corrections, the decay of the free neutron has the potential to provide the most accurate value of $|V_{ud}|$. However, the experimental sensitivity still needs to be further improved to become competitive with superallowed nuclear $\beta$ decays. The current status of the neutron decay studies relevant for the determination of $|V_{ud}|$ has been presented by Oliver Zimmer [6].

In the standard V-A theoretical description of neutron decay both the vector and the axial-vector currents contribute, with $g_V$ and $g_A$ coupling constants respectively, so that two observables are needed to access $|V_{ud}|$: the neutron lifetime $\tau_n$ and the $g_A/g_V$ ratio. For $\tau_n$, the accuracy needed to compete with $0^+ \rightarrow 0^+$ transitions in the $|V_{ud}|$ determination is $\sim 0.3$ s on $\tau_n$ (a factor three from the present accuracy). In this respect, a novelty is the solution of the 5.4 $\sigma$ tension between different determinations, mainly due to a single 2005 measurement. A wide effort in the field and a new measurement in 2010, pushed the authors of the 2005 result to scrutinize their procedures and to recently publish a corrected value. With these changes included, the tension is reduced to 1.4 $\sigma$ even if the the 1.8 scale factor applied by the PDG indicates that systematic uncertainties are not properly taken into account in all experiments. The highest experimental sensitivity on the $g_A/g_V$ ratio has been achieved measuring the $\beta$ asymmetry coefficient. The accuracy goal here is $\sim 0.0003$ on $g_A/g_V$, about one order of magnitude below the present determination. For both $\tau_n$ and $g_A/g_V$ many projects are in the pipeline using ultra-cold neutrons or magnetic trapping. Some are in a very advanced state and have the potential to reach the accuracy goal in the next years.

Nuclear mirror transitions occur between isobaric analogue states within an isospin doublet, where initial and final states have the same spin and parity. The determination of $|V_{ud}|$ from nuclear mirror transitions and the current experimental efforts aimed at improving its precision have been presented by Oscar Naviliat-Cuncic [7]. In addition, the mirror transitions are driven by a mixing of vector and axial-vector interactions, and the present $|V_{ud}|$ accuracy is dominated by the experimental error in the determination of the mixing ratio. As in the neutron lifetime case, two experimental inputs are needed. These are the half-life of the decay, and one of the following: the $\beta - \nu$ angular correlation, the $\beta$ asymmetry, or the $\nu$ asymmetry. Recently a substantial activity has been initiated on the experimental and theoretical sides to improve all the relevant inputs: many experiments are on going, applying different techniques, to measure the $\beta - \nu$ angular correlation, and there are plans to measure also relative $\beta$ asymmetries. These efforts should enable significant improvements in the precision on $|V_{ud}|$ from mirror transitions, which is currently a factor $\sim 8$ less precise than the value extracted from Fermi transitions.

Moving to $|V_{us}|$, its best determination arises from $K_{\ell 3}$ and $K_{\ell 2}$ decays. A very comprehensive review of the $|V_{us}|$ determination from kaons and of its effects on the unitarity test of the first row has been presented by Matthew Moulson [8].
there have been a few significant new measurements and some important theoretical developments. The experimental inputs for the determination of $|V_{us}|$ from $K_{\ell 3}$ decays are the rates and form factors for the decays of both charged and neutral kaons. There have been no new branching ratio measurements since the 2010 review. On the other hand, both the KLOE and KTeV collaborations have new measurements of the $K_S$ lifetime. Finally, the NA48/2 experiment has recently released preliminary results for the form factors for charged kaon decays. This is important because it helps to resolve a controversy: the older measurements of the $K_{\mu 3}$ form factors for $K_L$ decays from NA48 are in such strong disagreement with the other existing measurements that they have been excluded from the FlaviaNet averages. The new NA48/2 measurements, on the other hand, are in good agreement with other measurements. For all of the above efforts, however, the value of and uncertainty on $|V_{us} f_+(0)|$ are essentially unchanged. This is because the new results are nicely consistent with the older averages, and neither the $K_S$ lifetime nor the phase space integrals were significant contributors to the overall experimental uncertainty. The latter is dominated by the lifetime accuracy for the $K_L$ and by the branching ratio for the $K_S$ and $K^\pm$ determinations. In the near future there are no kaon experiments planning new branching ratio or lifetime measurements. For the $K^\pm$, also the uncertainty on the theoretical isospin-breaking correction gives the largest contributions to the $|V_{us} f_+(0)|$ uncertainty.

Besides the latter, advances in algorithmic sophistication and computing power are leading to more and better lattice QCD estimates of the hadronic constants $f_+(0)$ and $f_K/f_\pi$, which enter into the determination of $|V_{us}|$ from $K_{\ell 3}$ and $K_{\mu 2}$ decays, respectively. Due to their non-perturbative nature the only systematically improvable way to compute them are simulations of lattice QCD. Since the ultimate goal is a test of the SM, any model-dependence should be avoided and this is where progress in lattice simulations is currently being made. In addition, two groups working on the classification and averaging of results from lattice QCD have joined their efforts, constituting the newly formed Flavor Lattice Average Group (FLAG-2) \[9\] to provide recommended values of these constants.

The talk by Andreas Jüttner \[10\] reported on the status and ongoing improvements of determinations of $f_+(0)$. A key observation that allows a precise extraction of $f_+(0)$ is the conservation of the vector current at zero momentum transfer in the $SU(3)$ limit: the normalization is fixed in this limit and corrections start at the second order in $SU(3)$ breaking effects. Because of this fortunate situation, recent lattice computations determine $f_+(0)$ at the level of 0.5 – 1.0% and show an excellent agreement among them. The uncertainty of $f_+(0)$ in the state-of-the-art calculations is dominated by the statistical error and the error due to the extrapolation to the physical pion mass. The latter is about to be removed by simulating very close to or at the physical mass.

In his talk Jack Laiho \[11\] presented the lattice progress and the future prospects for $f_K/f_\pi$. This is a key input in the determination of $|V_{us}|/|V_{ud}|$ via $K_{\ell 2}$ decays,
which probe different BSM operators than the $K_{l3}$ decays. While the normalization in the $SU(3)$ limit is fixed, only the axial-vector current contributes to the decay constants and $f_K/f_\pi$ can receives unsuppressed chiral corrections. However, recent simulations on realistic lattices, particularly those at small or even physical pion mass, have calculated this ratio to sub-percent precision leading to the world average with a 0.4% accuracy. A further reduction of the discretization error as well as effects of finite lattice volumes is needed to improve the precision, say, to a level of 0.2%. We expect more simulations with different lattice formulations at the physical pion mass in the future for better understanding and control of these systematic uncertainties.

At the level of precision now reached in the lattice determinations of $f_+(0)$ and $f_K/f_\pi$, the uncertainties of the electromagnetic and isospin corrections are becoming non-negligible. Traditionally, these corrections have been estimated by means of chiral perturbation theory (ChPT), in which an extension to higher orders is generally not easy due to rapidly increasing numbers of relevant diagrams and unknown effective couplings. An interesting possibility is to include these corrections into the lattice determinations in a fully non-perturbative way.

In his talk Nazario Tantalo [12] presented a first principle lattice calculation of the QCD isospin corrections, namely those coming from the small difference of the up and down quark masses $\Delta m_{ud} = (m_u - m_d)/2$ in the absence of the electromagnetic interactions. Isospin is an approximate symmetry of QCD and most of the theoretical predictions on phenomenologically relevant hadronic observables have been derived by assuming the exact validity of isospin symmetry. This is also the case for most of the non-perturbative theoretical predictions on hadronic matrix elements obtained over the years by performing lattice QCD simulations, like the hadronic constants $f_+(0)$ and $f_K/f_\pi$. The RM123 collaboration has recently performed a lattice calculation of the QCD isospin corrections from numerical simulations of isosymmetric QCD combined with a systematic expansion of the partition function with respect to the small parameter $\Delta m_{ud}$. Their new method, therefore, does not need time-consuming simulations of the isospin-broken theory. They obtained encouraging results for the isospin corrections, which are of the same order of magnitude, though higher, of the ChPT estimate. It should be noted that the separation of QED from QCD isospin-breaking effects is prescription-dependent. It is therefore important to calculate the full (QCD+QED) correction on the lattice, which could be an interesting alternative to the conventional ChPT approach.

The hadronic $\tau$ decays provide an alternative way to measure $|V_{us}|$ and to probe the relation of the first row of the CKM matrix. Measurements of $|V_{us}|$ from $\tau$ decays are complementary to those from kaon decays because new physics scenarios that couple primarily to the third generation could cause deviation between measurements of $|V_{us}|$ in the kaon and $\tau$ systems.

In her talk Elvira Gámiz [13] gave an overview of the theoretical issues related to
the $|V_{us}|$ extractions from inclusive and exclusive hadronic $\tau$ decays. Some exclusive decay channels, such as $\tau \to K\nu$ and $\tau \to K\pi\nu$, can be used to extract $|V_{us}|$ in a similar manner to $K_{l2}$ and $K_{l3}$ decays. This method is therefore sensitive to the same lattice QCD uncertainties. The most precise measurement could be offered from the SU(3)-breaking effect in the inclusive rate
\begin{equation}
\delta R = \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2},
\end{equation}
where $R_{\tau,S} = \Gamma(\tau \to X_s\nu_\tau)/\Gamma(\tau \to e\overline{\nu}_e\nu_\tau)$ is the Cabibbo-suppressed hadronic rate into strange particles ($X_s$) and $R_{\tau,V+A} = \Gamma(\tau \to X_{non-s}\nu_\tau)/\Gamma(\tau \to e\overline{\nu}_e\nu_\tau)$ is the Cabibbo-allowed hadronic rate. The SU(3)-breaking effect $\delta R$ can be estimated from finite energy sum rules (FESR). This technique makes the hadronic $\tau$ decays an ideal system to study low-energy QCD under rather clean conditions, allowing the determination of the strong coupling with the same precision achieved by lattice determinations.

To profit from $\delta R$, we need to measure the inclusive strange and non-strange spectral density functions, which are constructed from the sum of invariant mass distributions for each of the strange and non-strange decay modes and normalized to the corresponding branching fractions. Since there are no solid predictions for the branching fractions of hadronic individual $\tau$ decays, all possible modes must be measured or upper bounds have to have placed on them. This technique is completely
Figure 2: $|V_{ud}|$ from $0^+ \rightarrow 0^+ \beta$ decays, $|V_{us}|$ from $K_{l3}$ decays, and $|V_{us}/V_{ud}|$ from $K_{l2}$ decays in the plane of $(|V_{ud}|, |V_{us}|)$ [8]. The yellow ellipse indicates the 1σ confidence interval for the fit to fix $|V_{ud}|$ and $|V_{us}|$. The unitarity constraint is shown by the solid line.

independent of the kaon measurements, and if all of the branching fractions and spectral functions were updated with the whole data sets of BELLE and BABAR experiments, this method would be expected to provide the most precise measurement of $|V_{us}|$.

Ian Nugent [14] presented an overview of the current status of the experiments. Currently, both the exclusive and the inclusive decays determine $|V_{us}|$ with an accuracy of about 1.0%, which is slightly larger than 0.6% achieved in the determinations from $K_{l2}$ and $K_{l3}$ decays. More importantly, the updated experimental results lead to only a slight change from the previous workshop [4] in $|V_{us}|$ from the inclusive decays. This result is about 3σ below the $K_{l2}$ and $K_{l3}$ determinations as shown in Fig. 1.

Since the uncertainty is limited by the experimental precision, further experimental data are needed before drawing any significant conclusion. While the reliability of the FESR analysis has been studied, a more thorough study, for instance about the stability against the choice of the weight, is also welcome.

Presently, the most precise value of $|V_{ud}|$,

$$|V_{ud}| = 0.97425(22),$$

is obtained from $0^+ \rightarrow 0^+$ nuclear decays and is unchanged from the previous CKM workshop. This together with the updated values of $|V_{us}| = 0.2254(13)$ from $K_{l3}$ decays
and $|V_{us}/V_{ud}| = 0.2317(11)$ from $K_{l2}$ can be combined in a single fit to determine the CKM elements and $\Delta_{CKM}$ \cite{8}. As plotted in Fig. 2 the fit does not change the input value of $|V_{ud}|$ and yields

$$|V_{us}| = 0.2256(8), \quad \Delta_{CKM} = +0.0001(6)$$

in perfect agreement with unitarity. The uncertainty of $\Delta_{CKM}$ is equally shared by $|V_{ud}|$ and $|V_{us}|$. The precision of $|V_{ud}|$ is determined from the uncertainty of the transition independent radiative correction \cite{15}, which has been stable over the last several years. The uncertainty of $|V_{us}|$ is currently dominated by uncertainties in the lattice results for $f_+(0)$ and $f_K/f_\pi$, which are at the level of 0.5%. Thus, at the moment, the lattice offers the most certain prospects for further improvement.

Although the accuracy of $\Delta_{CKM}$ is essentially unchanged from the previous workshop, the current precision already allow us to put significant constraints on new physics as summarized in the beginning of this paper. An interesting question is whether the low-energy observables from the nuclear, neutron, kaon and $\tau$ decays have a higher sensitivity than the rest of observables, for instance, from energy-frontier experiments. In his talk Martín González-Alonso \cite{16} answered this question by comparing new physics bounds from the low-energy observables with those obtained by the CMS Collaboration analyzing 5 fb$^{-1}$ of data recorded at $\sqrt{s} = 7$ TeV in the $pp \rightarrow e + MET$ (Missing Transverse Energy) channel. Assuming that the heavy mediators that generate the BSM interactions in Eq. (1) are too massive to be produced at the LHC, we can again employ an EFT approach and put bounds on the $\epsilon_i$ couplings from the LHC search. For the (axial) vector and pseudo-scalar ones low-energy probes are much more powerful. There is an interesting competition between low- and high-energy searches for the scalar and tensor couplings. In addition to Eq. (1), we can also introduce BSM interactions involving right-hand neutrinos, for which the LHC will dominate the search.

Even if the LHC sensitivity will get better as more data is collected, future low-energy experiments, such as with (ultra)cold neutrons, will improve in finding the bounds on new physics. Currently, the precise determination of $|V_{ud}|$ and $|V_{us}|$ provides one of the most stringent tests of the Standard Model and will play an important role, complementary to the collider searches, in probing new physics.

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