I. Introduction

The hyperloop is a concept for a high-speed mass transportation system that uses an enclosed low-pressure environment and small autonomous vehicles ("pods") to enable an unparalleled combination of short travel times, low energy consumption, and ultra-high throughput. From an engineering perspective, hyperloop system design is a highly-coupled optimization problem with many recursive design relationships. It is also a clean-sheet design problem – there is no operational precedent to use as a baseline – so there is no "initial guess" from which to begin. It is ripe for the application of rigorous system optimization.

Hyperloop system designers must answer questions such as: How fast should the pod travel? How quickly should the pod accelerate? How many passengers should each pod be able to carry? How heavy does a pod need to be to achieve these things? How big should the tube be? How low should the air pressure inside the tube be? How big does each portal need to be? How big does a pod fleet need to be? The answers to these questions and many more are not only important to enable detailed design but they are also inextricably intertwined.

To further complicate matters, the goal is to design a superlative mode of transportation; one that offers the shortest journey time, highest departure frequency, lowest energy consumption, cheapest ticket price, highest passenger throughput, and highest levels of safety and comfort. We therefore need a disciplined way of trading between these often-competing objectives and making engineering decisions based on their impact on an appropriately chosen objective function.

Fortunately, advances in powerful mathematical solvers and user-friendly optimization modeling languages have made it possible to formulate, modify, and solve large and complex optimization problems quickly and in a way that is approachable and useful in a practical engineering context.

We have leveraged such advances to create a hyperloop system optimization tool that has been used extensively in the development of the Virgin Hyperloop system. The purpose of this paper is to describe both the hyperloop system optimization problem in general, and the tool we have developed.

The structure of the paper is as follows. First, to provide context, we give a brief overview of the defining characteristics of a hyperloop system.

Next, we extend this to explain what makes the design of such a system not only an interesting optimization problem but also an extremely tightly coupled one that is uniquely well suited to formal and rigorous system optimization. We then explain how this motivated the development of HOPS, a hyperloop system optimization tool that allows engineers to answer the questions above (and many more) using convex optimization techniques. This work presents a system optimization tool, HOPS, that has been adopted as a central component of the Virgin Hyperloop design process. We discuss the choice of objective function, the use of a convex optimization technique called geometric programming, and the level of modeling fidelity that has allowed us to capture the system's many intertwined, and often recursive, design relationships. We also highlight the ways in which the tool has been used. Because organizational confidence in a model is as vital as its technical merit, we close with discussion of the measures taken to build stakeholder trust in HOPS.

II. Hyperloop

Although Elon Musk coined the name and repopularized [1–3] the concept with the Hyperloop Alpha white paper [4], the notion of what constitutes a hyperloop has evolved significantly since its publication in 2013, with different companies pursuing different system architectures. Time will tell which architecture(s), if any, will be commercially successful; however, all of the most promising architectures share certain key features.

By virtue of these features, we claim that hyperloop system design presents a singular opportunity for the application of multidisciplinary design optimization (MDO), agnostic of the choice of architecture. However, we also claim that using an MDO tool that considers the full system-wide impacts of each design choice is crucial to choosing the best system architecture.

A. The Key Features of a Hyperloop System

To achieve the high-level objectives of low travel time, low wait time, low energy consumption and high throughput, a hyperloop system must have the following key features:

1. Small\(^b\) pods to allow highly demand-responsive direct-to-destination service.

2. An enclosed low pressure environment to enable low energy consumption despite high speeds and small pods.

3. “Pod-side switching”, i.e. no moving parts on the wayside, to allow ultra-high throughputs\(^c\) despite small pods and direct-to-destination service.

There are four major elements of a hyperloop system: the pod fleet, the linear infrastructure (“hyperstructure”), the pressure management system, and the stations (“portals”). The pod fleet carries passengers to their destinations. The hyperstructure encloses the low pressure environment and supports any necessary track elements for propulsion, levitation, guidance, and emergency braking. The enclosed nature of the system also allows it to be more resilient to weather and safety hazards than other transportation systems. The pressure management system establishes and maintains a low-pressure environment inside the tube. The portals provide an interface between the hyperloop system and the outside world; they are where passengers board and disembark pods and where pods’ resources are replenished. Figure 1 shows a rendering of a pod inside a tube to help visualize these key elements.

\(^b\) How small (i.e. how many passengers) is obviously one of the most important variables to optimize, but intuitively we can say that they should carry more people than a car and fewer people than a regional jet.

\(^c\) Ultra-high throughput is not only important for designing a system that is equipped to handle the demands of future population growth, but it is also a key part of reducing the total cost per passenger by enabling much higher utilization than a conventional rail or maglev system.
B. A Singular Opportunity for Multidisciplinary Design Optimization

Aircraft design is often cited as the exemplar of an interesting and challenging multidisciplinary optimization problem because of its conflicting design pressures from different technical domains (e.g. aerostuctural optimization of a wing), its recursive design relationships (e.g. the size of a wing depends on the weight of the aircraft which depends on the weight of the wing), and its strong coupling between subsystems (e.g. for a multi-engine aircraft the size of the vertical tail might depend on the thrust of the engines) [5, 6]. As a result, a vast amount of research has gone into developing tools for multidisciplinary design optimization of aircraft [7–9].

We claim that hyperloop system design presents a singular opportunity for multidisciplinary design optimization because of four properties: it requires modeling from many engineering disciplines, it is tightly coupled and highly recursive, it is a clean-sheet design problem, and the behavior of the system is expected to be highly deterministic.

1. Multidisciplinary

A hyperloop system invokes many engineering disciplines including structures, electronics, aerodynamics, electromagnetics, thermodynamics, controls, manufacturing, and civil engineering. Because it is a mass transportation system and involves large infrastructure projects, there are also important economic models to capture.

2. Coupled and Recursive

Hyperloop system design is an even more coupled problem than aircraft design. The most obvious reason for this is that the environment in which pods travel and the structure containing this environment are also key parts of the design and they affect, and are affected by, the design of the pod and the pressure management system. For example, the tube pressure and blockage ratio (pod-to-tube-area ratio) are key drivers of aerodynamic drag and therefore pod energy consumption, while the size depends on the size of the vertical tube, which in turn depends on the power of the engine.

Whilst airports and railway stations can impact the design of their respective vehicles, hyperloop portals are a more tightly coupled form of passenger terminal, because their size and cost are more closely linked to the site of the pods and the pod turnaround time. For battery-powered pods, turnaround time is often gated by charging rather than passenger boarding/disembarking, which introduces a coupling between portal size and pod performance that is unique for a mass transportation system.

Pods do not operate in isolation but rather as part of a fleet, and this adds another source of coupling, with both favorable and adverse effects. For example, more pods operating close together can reduce average aerodynamic drag, but having more pods in the system also means more sources of air leak into the tube, which can increase the optimal tube pressure.

Finally, not only do all of the major subsystems have recursive design relationships (e.g. the propulsion system mass depends on the pod mass which depends on the propulsion system mass) but they are also even more coupled to each other than subsystems on vehicles that operate in atmospheric pressure. Because there is very little convective cooling in a near-vacuum environment, the thermal management system plays a significant role in system design by introducing a strong coupling between efficiency and mass; even small decreases in efficiency can cause a significant "mass spiral" effect under certain conditions.

Figure 2 captures the coupling between system elements and pod subsystems. Every line represents the interdependence of two models, with variables flowing to and/or from the connected models.

It should be noted that not every potential hyperloop system architecture has the same degree of coupling, and while more coupling can make design more difficult, it is often a byproduct of architecture choices that unlock other key advantages.

3. Deterministic

Another factor that makes hyperloop design well-suited to rigorous system optimization is that, as an autonomous, track-based system operated in a controlled environment, it has a well-defined nominal operation for any given route; the environmental conditions can be well characterized and the energy consumption and thermal profiles are therefore easy to model with relative confidence. In other words, operations are highly deterministic. While there are other sources of uncertainty and some subsystems will necessarily be sized by off-nominal scenarios, the pod design is not subject to extreme, rare, and/or difficult-to-analyze weather phenomena and can largely be optimized around well-defined nominal operations.

4. Clean-Sheet Design

The other rare optimization opportunity afforded to hyperloop design at this time is the lack of any legacy infrastructure or standards that need to be accommodated. Hyperloop system design is a clean-sheet problem, governed by physics, economics, and human psychology. As a nascent industry, commercial success is also not (yet) dominated by questions of supply chain management, as is the case in the automotive industry. This means that an MDO tool can have more impactful than in mature industries; its outputs can be used directly for setting design specifications and requirements. Projected cost savings attained through design optimization can be realized because their is no industry inertia to overcome; they don’t require expensive re-design of tooling, infrastructure (e.g. airports), or legislation (e.g. Federal Aviation Regulations) [10].

III. Geometric Programming: A Goldilocks Optimization Technique

Due to the complex and intricately coupled nature of the hyperloop design space, a powerful, disciplined, and effective optimization tool is needed. The rapidly iterative nature of engineering design, and our need to be able to "explore the design space," require that such a tool also be fast.

In optimization there is, broadly speaking, a tradeoff between model generality – how accurately an arbitrarily complicated design problem can be represented – and solver efficiency. At one extreme, linear programs are a well-known class of convex optimization problem that can be solved extremely quickly but are only valid for problems that can be described with linear constraints. At the other extreme, many general nonlinear local and global optimization techniques have been developed for solving arbitrarily complex design problems, but these techniques are often computationally intensive and provide no guarantees about the quality of their solutions.

One goldilocks technique is called geometric programming [11]. Geometric programs (GPs) are able to model relatively complex relationships and can be transformed into a convex optimization problem which can be solved efficiently using off-the-shelf software that guarantees global optimality (if a solution exists). Crucially, these solvers also do not require an initial guess. Being able to find optimal solutions without an initial guess makes the technique particularly useful for the conceptual design of a transportation system with no precedent.

These impressive properties are possible because GPs represent a restricted subset of nonlinear optimization problems. In particular,
the objective and constraints can only be composed of monomial and posynomial functions.

A monomial is a function of the form

\[ m(x) = c \prod_{j=1}^{n} x_j^{a_j}, \]

where \( a_j \in \mathbb{R}, \ c \in \mathbb{R}^+, \ \text{and} \ x_j \in \mathbb{R}^+. \) For instance, the familiar expression for lift, \( \frac{1}{2} \rho V^2 C_L S, \) is a monomial with \( x = (\rho, V, C_L, S), \ c = 1/2, \ \text{and} \ a = (1, 2, 1, 1). \)

A posynomial is a function of the form

\[ p(x) = \sum_{k=1}^{K} c_k \prod_{j=1}^{n} x_j^{a_{jk}}, \]

where \( a_k \in \mathbb{R}^n, c_k \in \mathbb{R}^+, \ \text{and} \ x_j \in \mathbb{R}^+. \) Thus, a posynomial is a sum of monomial terms, and all monomials are posynomials of just one term.

A GP minimizes a posynomial objective function, subject to monomial equality constraints and posynomial inequality constraints. The standard form of a geometric program is:

\[
\begin{align*}
\text{minimize} & \quad p_0(x) \\
\text{subject to} & \quad p_j(x) \leq 1, \ j = 1, \ldots, n_p,
\end{align*}
\]

where the \( p_j \) are posynomial (or monomial) functions, the \( m_k \) are monomial functions, and \( x \in \mathbb{R}^n_+ \) are the decision variables.

While this may seem restrictive, a perhaps-surprisingly broad range of physical and economic relationships can be described using these sorts of relationships, either exactly, through some algebraic manipulation or changes of variable; by use of suitable approximations; or by fitting of GP-compatible surrogate functions [12, 13].

One simple and frequently used modeling technique that is central to the GP design paradigm, known as the “epigraph method” [14], is the practice of relaxing a posynomial equality constraint into an inequality to make it GP-compatible. When the constraint is active, this relaxation is tight, meaning that equality will hold at the optimum.

It turns out that it is possible to model a full hyperloop system to an appropriate level of fidelity using geometric programming for every relationship except for one which requires a signomial constraint.

A signomial is a generalization of a posynomial that allows subtraction of monomial terms, i.e. \( c_k \in \mathbb{R}. \) Similarly, a signomial program is a generalization of a geometric program that allows signomial constraints in addition to monomial and posynomial constraints. A signomial program can be solved by taking a local approximation of any signomial constraints, solving the resulting geometric program, and repeating this process until convergence is attained [11].

Signomial programs do not offer the same guarantee of global optimality as geometric programs. However, from a practical engineering
perspective, when only one constraint is signomial, and when the range of possible values for such a variable is well understood, the quality of solutions can be verified by plotting a sweep over that variable.

Geometric and signomial programming have been used previously in a variety of engineering applications, including the design of aircraft [15, 16], communication systems [17], and digital circuits [18].

A number of domain-specific languages have been written to empower engineers to express design problems as convex optimization problems without needing to be experts in the details of optimization algorithms [12, 19]. GPKit is one such modeling toolkit specifically designed for formulating, manipulating, and solving geometric and signomial programs [20]. It includes a number of powerful features such as automatic unit conversion, model debugging, variable vectorization, solution "diffs", and a solution post-processor that warns users if constraints are unexpectedly "tight" or "loose". GPKit supports several backend solvers including MOSEK [21] and CVXOPT [22].

IV. HOPS: The Hyperloop System Optimization Tool

The remainder of this paper describes HOPS, the hyperloop system optimization tool developed at Virgin Hyperloop\(^4\). From an engineering perspective, HOPS is a collection of coupled subsystem models and a tool for quickly performing system-level trade studies and sensitivity analyses. It encompasses everything from the propulsion and levitation subsystems, to the pod aerodynamics, to the pressure management pumps, to the size of the tube and the number of podbays needed in each portal.

From a mathematical perspective, HOPS is a signomial program that is solved as a sequence of geometric programs. The default HOPS case at the time of writing has 4512 free variables and 6140 constraints, all of which are strictly GP-compatible (monomial or posynomial) except for one important but simple signomial constraint.

From a software perspective, HOPS is a Python package that uses GPKit as its modeling toolkit and MOSEK as its solver. At its most distilled, HOPS encodes the optimization problem in three objects: an objective function, a list of constraints, and a dictionary of input values for any constants or variables whose values are fixed. It takes a 2016 MacBook Pro with a sixth-generation Intel i5 processor about 4 seconds to return an optimal solution for the default case.

The list of constraints is structured as a collection of models. A description of the subsystem (and sub-subsystem) models follows, with explanations of how the model was made GP-compatible, where appropriate. A discussion of the relationship that needs a signomial constraint follows as well. But first, we address the most important question of an optimization problem: what is the objective function?

A. Objective function

At a high-level, hyperloop system optimization can be thought of as a trade-off between the cost to build a system, the cost to operate the system, and the performance of the system, so it is appropriate to choose an objective function that captures these three metrics. The function we minimize is called the total cost per passenger-km.

\[
\begin{align*}
\min & \quad C_{\text{total, per pax-km}} \\
\text{subject to} & \quad \text{Constraints}
\end{align*}
\]

It is defined as the route-length-normalized sum of capital expenditure (CapEx) per passenger, operating expenditure (OpEx) per passenger, and a virtual time cost per passenger\(^7\).

\[
C_{\text{total, per pax-km}} = C_{\text{total, per pax}} / \text{route length}
\]

\[
C_{\text{total, per pax}} = C_{\text{capex, per pax}} + C_{\text{opex, per pax}} + C_{\text{time, per pax}}
\]

Whilst the route-length normalization is not strictly necessary for single-route optimization, it has two advantages. Firstly, it allows for easy comparison of solutions for different route lengths as well as comparison against existing and competing modes of transportation, for which the equivalent value can be calculated. Secondly, it enables weighted optimization over multiple routes of different lengths.

CapEx includes the cost to build the linear infrastructure, the portals and the pressure management system, as well as the cost to manufacture the pod fleet and is measured as an absolute cost (e.g. $). OpEx is the cost to operate and maintain the system, including both energy and non-energy costs, and is measured as a cost-per-unit-time (e.g. $/year). The time cost is a measure of the lost utility of a passenger's time—the opportunity cost of the time spent traveling—and is measured as a cost-per-passenger (e.g. $/pax). Estimating the cost of passenger time is a standard government practice for evaluating the social benefit of transportation infrastructure projects [24].

To combine these three concepts into a single value, they must all be normalized to units of cost-per-passenger ($/pax). To do this, both CapEx and Opex must be divided by the number of passengers (i.e. trips) per year, \(N_{\text{paxyear}}\). CapEx must also be multiplied by a parameter known as the capital recovery factor, \(f_{CR}\), which can be thought of as the inverse of a present value (PV) factor, and is a function of the effective interest rate and project payback period.

\[
C_{\text{capex, per pax}} = f_{CR}(\text{CapEX})_{\text{total, per paxyear}}
\]

\[
C_{\text{opex, per pax}} = (\text{OpEX})_{\text{total, per paxyear}}
\]

In its most simple form, the time cost per passenger is calculated by multiplying travel time by a value of passenger time, \(V_{\text{time}}[\$/hr]\), that depends on how a population values productivity and leisure [25]. The Department of Transportation (DOT) periodically publishes an updated value of passenger time for the United States [26]. The cost of passenger time calculation can be enhanced by also considering the waiting time and using a wait-time multiplier, \(M_{\text{waittime}}\), to quantify the psychological phenomenon that makes time spent waiting less bearable than time spent in motion [27].

\[
C_{\text{time, per pax}} = V_{\text{time}}(t_{\text{travel}} + M_{\text{waittime}}t_{\text{wait, average}})
\]

Figure 3 shows an icicle plot of the total cost breakdown for an example solution. The objective for HOPS is to shrink the total height of this chart as much as possible, by minimizing and trading between the constituent cost buckets.

Although HOPS solves a single-objective optimization problem, it could be thought of as a multi-objective optimization problem where the key weighting parameters are the capital recovery factor, the cost of energy, and the value of passenger time.

B. Constraints

Rather than being thought of as design-space limits, the constraints in HOPS encode the relationships between all the variables. They are organized into a hierarchy of models, where each model comprises three elements: a declaration of variables (along with their respective units), a list of constraints, and a dictionary of input values for variables that are fixed, where applicable. These models are constantly evolving, as constraints are added, removed, simplified and replaced, and input values are updated, in the continuous pursuit of representing the latest understanding of system and subsystem characteristics and performance.

This pursuit is made more challenging by the tension between making models accurate to the latest design point and capturing the sensitivities and coupling of the broader design space. The fidelity of models has generally increased over time but certain models have also been simplified, for example when off-the-shelf hardware choices have been made or when scaling laws have been deemed more appropriate than first-principles physics models. In addition, another key feature of HOPS is that different system and subsystem architectures have been modeled, allowing discrete trade studies to be performed. As such, this section should be thought of as a glimpse at a snapshot of the model at the time of writing, with the current default architecture.

\(^{\text{4}}\)Virgin Hyperloop is the latest name for a company that has previously also been known as Hyperloop Technologies, Hyperloop One, and Virgin Hyperloop One.

\(^{\text{7}}\)We call the sum of CapEx per passenger-km and Opex per passenger-km the total hard cost per passenger-km or the Levelized Cost of Transportation (LCOT). This is analogous to a concept in power systems engineering called the Levelized Cost of Energy (LCOE) [23].

\(^{\text{6}}\)These inputs are referred to as "substitutions" in GPKit verbiage to reflect the notion that any free variable can be substituted with a fixed value.
Figure 3. An icicle plot shows the breakdown of total cost for an example solution. By expressing all costs on a per-passenger-km basis, an apples-to-apples comparison can be made of capital cost, operating cost, and time cost.

Figure 4 shows the hierarchy of HOPS models. There are models for each major system element (Pod, Linear Infrastructure, Portal, and Pressure Management System) as well as models for the Route, Network, and System Cost. Most of these models have many levels of hierarchy of their own sub-models.

Although all models share a common structure and most have numerical inputs, the Route and Network models can be thought of as the high-level input models. Similarly, the System Cost model can be thought of as the output model: all other models feed into it and it encodes the cost rollups that build up to the objective function. That said, it is important to re-emphasize that HOPS is not a procedural design tool; it is fundamentally a list of constraints, not a sequence of calculations. The added benefit of formulating the design as a convex optimization problem is that it also offers a cleaner way to represent the recursive relationships of a hyperloop system.

Note the separation of the Pod model and the Pod Performance model (which sits inside the Pod Fleet model). While it may seem redundant, this structure of modeling allows the separation of variables (and constraints) that are route-invariant (e.g. pod mass) and those that depend on the route(s) over which a pod is being optimized (e.g. battery cell depth of discharge). This in turn allows HOPS to be used in several different ways:

1. Single-point optimization (nominal): Optimizing the system design and performance for a given route
2. Fixed-design optimization: Optimizing the system for a sizing case, then freezing the design and optimizing only the performance for a different route
3. Multi-point optimization: Optimizing a single system design over multiple routes each with an optimal performance

Thanks to the design of GPkit, the separation of sizing constraints (Pod) and performance constraints (Pod Performance) allows implementation of these variants to be no more than a couple lines of code. Perhaps counterintuitively, the Pod Fleet model is also a performance model because the size of a fleet can vary significantly for different routes and business cases, and is necessarily a key optimization variable.

The remainder of this section provides an overview of each top-level model, along with more detailed descriptions of a few key sub-models, where appropriate. The purpose is to demonstrate the breadth and depth of models, and in doing so, give the reader a sense for HOPS’ level of fidelity, which ranges from fairly crude for the Route model to quite detailed for key subsystems of the Pod model.

1. Route

The Route model encodes the high-level inputs for an origin-destination pair. This includes a crude representation of the alignment (i.e. the path of the linear infrastructure) and the expected passenger traffic. Key inputs include the route length, the net elevation change, the maximum grade, the fraction of the alignment that is below grade (tunneled), a parameter that describes the straightness of a route (discussed further in paragraph i), and the peak and average hourly passenger throughput. In the current model, throughput is taken as fixed and does not depend on travel time; however it can easily be adapted to include a demand response to performance.

2. Network

A key part of the hyperloop value proposition is the ability to offer direct-to-destination service, a feature that requires on- and off-ramps, much like a highway. While certain networks will allow local reduced speed in the vicinity of portals, the only general solution that does not adversely impact throughput capability is for the mainline to have a constant cruise velocity. Therefore, ramps need to provide enough distance to accelerate to full cruise velocity. The length of these ramps is determined by the acceleration and deceleration capabilities of pods, and in turn directly impact the total length, and thus cost, of linear infrastructure that needs to be built. The total network length is captured as a simple-but-critical posynomial constraint:

$$l_{\text{network}} \geq l_{\text{longestroute}} + n_{\text{ramps}}l_{\text{ramp}}$$  \hspace{1cm} (10)

For the purpose of this work, only single-origin-destination-pair networks are considered. Because it does not need ramps, optimizing around such a network would yield a system with low acceleration capability and thus limited long-term value proposition, a parameter can be used to represent the number of “extra” ramps, as a surrogate for a more sophisticated implementation with multiple explicitly-defined origin-destination pairs. Discussion of this more sophisticated implementation is left for future work.
3. Pod and Pod Fleet

The pod is the most complex and multidisciplinary system element and the most ripe for optimization. As such, the models that describe its design and performance are the most detailed, and its system-level specifications are amongst the most important products of HOPS. The high-level sizing variables include the pod mass, the pod capacity (i.e., number of passengers), the pod cost, and the variables that describe its geometry. The key performance variables for a given route are those that describe the velocity profile (e.g., launch acceleration and cruise velocity), travel time, energy consumption, and thermal losses.

The Pod model comprises sizing models for each pod subsystem. Each sizing model captures the mass and cost of that subsystem, using a combination of roll-up constraints (e.g., $m \geq m_1 + m_2 + \ldots$), scaling law constraints, and input values. Some subsystems also have models that capture their geometry (e.g., volume, frontal area, length, height, and width) and these variables tie into constraints that capture considerations such as aerodynamic drag, pod structure sizing, tube sizing, and portal sizing. Key active subsystems have corresponding performance models (under Pod Performance) that capture maximum power and total energy consumption. Subsystems with significant losses also include performance models for (instantaneous) thermal power, (aggregated) thermal energy, and drag, where applicable. There is also a simple model to account for the maintenance costs of each subsystem.

As Figure 4 shows, there are 16 major pod subsystem models, however only 7 of these are modeled to a level of fidelity where they have significant direct coupling to other subsystems. These are the High-Voltage Battery, Power Electronics, Propulsion, Levitation, Guidance, Secondary Brake, and Thermal Management Systems. All of these, with the exception of the Secondary Brake, also have corresponding performance models. Two examples of these subsystem models are highlighted below to show the level of fidelity considered and to demonstrate how modeling tricks are able to make a relatively complex power loss model GP-compatible.

There is also a performance model to capture the aerodynamic drag experienced by the pod throughout its trajectory. We highlight it below to show how GP-compatible fits can be used to capture complex non-analytical design spaces. We also describe the model vectorization that allows us to account for the different aerodynamic drag experienced by different pods in a convoy.

Pods do not operate in isolation; they are part of a fleet and they are capable of traveling in convoys. The constraints that model the fleet are in the top-level Pod Fleet model. The number of pods in the fleet for a given route is a key free variable because the cost of the fleet represents a significant proportion of total CapEx and therefore total cost per passenger-km. The size of the fleet depends on the peak throughput for the route, the pod capacity, the travel time and the in-portal turnaround time. Because the turnaround time is typically constrained by the battery charging time (with current battery technology) for all but the shortest routes, the fleet size is therefore coupled not only to the travel time but also to the energy efficiency of the pods.

Not all pods in a convoy have the same energy consumption, but they must all follow the same velocity profile. As such, the critically important constraints that model travel time and the velocity profile, including the only non-GP constraint in HOPS, are part of the Convoy Performance model.

i. Convoy Performance The velocity profile model underpins all of the other pod performance models and is therefore critical to system optimization. The trajectory modeled in HOPS has a launch (acceleration) phase, a constant-speed cruise phase, and a braking (deceleration) phase. The launch and braking phases are discretized into a user-specified number of steps (default value: 25) to improve the accuracy of the model, for example by allowing the acceleration rate to vary which is particularly important for capturing the behavior of the system when it is power-limited. This means that all launch and braking constraints are “vectorized” (i.e., duplicated for every step) so every launch or braking constraint actually represents 25 constraints. Thanks to a GPkit feature called Vector Variables, each constraint can just be written once and can mix Vector Variables with (scalar) Variables. For example, the constraint derived from Newton’s second law

$$\mathbf{F}_{\text{launch}} \geq m_{\text{pod}} a_{\text{launch}} + D_{\text{launch}}$$

actually represents

$$\mathbf{F}_{\text{launch},0} \geq m_{\text{pod}} a_{\text{launch},0} + D_{\text{launch},0}$$

$$\mathbf{F}_{\text{launch},1} \geq m_{\text{pod}} a_{\text{launch},1} + D_{\text{launch},1}$$

$$\ldots$$

$$\mathbf{F}_{\text{launch},24} \geq m_{\text{pod}} a_{\text{launch},24} + D_{\text{launch},24}$$

To ensure a GP-compatible formulation, this discretization is performed using uniform velocity steps (instead of uniform time steps) as illustrated in Figure 6.
The travel time model is important not only because of the time cost component of the objective function, but also because travel time directly impacts the size of the fleet needed for a given route.

The velocity-based discretization allows us to model travel time using the following posynomial constraint, where $t_{route}$ is the total route length, $v_c$ is the cruise velocity, $n_i$ is the number of discretizations of the launch phase, $a_i$ is the acceleration rate during each launch segment, $n_f$ is the number of discretizations of the braking phase, and $d_f$ is the deceleration rate during each braking segment.

$$t_{route} \geq \frac{v_c}{a_1} + \sum_{j=2}^{n_f} \left( \frac{2n_j - 2i + 1}{n_j^2} \right) + \sum_{j=2}^{n_f} \left( \frac{2j - 1}{n_j^2} \right)$$  \hspace{1cm} (15)

Although at first glance, this may not appear GP-compatible due to the negative terms, these terms do not include any model variables and simplify to give a posynomial expression.

As mentioned previously, there is one signomial constraint in HOPS. This is the constraint that governs cruise length, which must have a lower bound constraint because other models apply downward pressure on both cruise length and cruise time:

$$f_{cruise} \geq 1 - 2l_{ramp}/l_{route}$$  \hspace{1cm} (16)

where $f_{cruise}$ is the fraction of the trajectory in cruise and $l_{ramp}$ is the ramp length. We know this constraint is not GP-compatible by inspection because its right-hand side is an example of the “one-minus” atom, which is log-log concave [19]. There are many possible formulations of this constraint that would achieve the same intent but they all generally have the same structure. It is also possible that a clever GP-compatible formulation exists but has not yet been derived. Note also that if the value of ramp length is fixed or a sweep of ramp length is conducted, then this constraint is no longer a signomial and HOPS can be solved with a single GP rather than a sequence of GPs.

Unfortunately, not all of the alignments proposed for prospective routes are able to support a constant-cruise-velocity profile at the velocities a hyperloop system is capable of, due to curvature and passenger comfort constraints. HOPS is not able to optimize over arbitrarily complex velocity profiles but optimizing over an ideal trajectory can significantly underestimate the electrical, thermal, and travel time impacts of more complex routes and limiting the cruise velocity throughout the trajectory would be unrealistically conservative. It is therefore necessary to have a way to account for these effects.

We use a parameter known as the “number of full slowdowns” for this purpose, where a full slowdown is defined as a deceleration to rest followed by a re-acceleration to cruise velocity. An appropriate value for this parameter is provided by an in-house simulation tool that can determine the equivalent number of full slowdowns for an arbitrary velocity profile. Figure 7 shows an example velocity profile with two full slowdowns, though it should be noted that non-integer values can be used.

ii. Battery

The size of the battery is one of the most important design decisions for a podside-powered hyperloop system. Not only does the battery significantly impact key performance metrics like acceleration capability and nonstop range, it is also the heaviest subsystem, the largest subsystem inside the bogie structure, and the biggest contributor of thermal energy, so it has a significant impact on both the design of other major subsystems (e.g. levitation, propulsion, thermal management) and the overall size of the pod structure (and in turn, the tube structure).

As with other major subsystems, the battery is also subject to critical recursive design relationships. If it gets heavier, the whole pod gets heavier, which then requires the battery and other key subsystems to be more powerful (i.e. heavier) to achieve the same performance. Such mass spirals can be caused by any change to the major subsystems.

**DESIGN CONSIDERATIONS**

The battery must be able to supply the power demanded of it by other subsystems in all phases of a trajectory and it must store enough energy to complete the journey in both nominal and off-nominal scenarios. Because the system uses regenerative braking, this energy constraint applies to the lowest depth-of-discharge point of a route, which occurs at the end of cruise for a nominal trajectory, as shown in Figure 8.

These requirements must be met whilst also respecting important design constraints. First, the battery must operate within the current and voltage capabilities of the cells, during all phases. These limits can come from cell manufacturer datasheets as well as test results or other design considerations. Second, heat generated by the battery must be absorbed, or otherwise dissipated, by the thermal management system (TMS). This can be thought of as a bilateral constraint on both the battery and the TMS (i.e. should the battery generate less heat or should the TMS get bigger?). Third, cells need to be replaced when their performance deteriorates to a certain point, but cell replacement costs can be a significant portion of system OpEx. The battery replacement rate is therefore another key operating decision, trading off replacement cost with the capacity, efficiency and thermal load impacts of using cells for longer. Finally, battery charging time is often (though not always) the gating factor for pod turnaround time, which directly impacts the size (and therefore cost) of portals.
Modeling Fidelity. The battery is modeled using basic cell physics, including the heat generated by, and voltage drop due to, ohmic losses. These constraints are duplicated for every phase of a trajectory, including each discretization step of the launch and braking phases:

\[ P_{\text{cell,cruise}} + P_{\text{loss,cell,cruise}} \leq V_{\text{opencircuit,cell}} I_{\text{cell,cruise}} \]  
\[ P_{\text{cell,cruise}} = V_{\text{terminal,cell,cruise}} I_{\text{cell,cruise}} \]  
\[ P_{\text{loss,cell,cruise}} \geq I_{\text{cell,cruise}} (R_{\text{internal,cell,cruise}} + R_{\text{contact,cell}}) \]

The model also includes constraints that capture the effect of cell aging on internal resistance and capacity, using GP-compatible fits of empirical data. An example of a fit of the internal resistance of a cell as a function of discharge current is shown in Figure 9.

Cell properties such as beginning-of-life capacity and maximum charge and discharge current limits are inputs to the model and different sets of inputs can be selected to represent different cell options and perform trade studies. Input sets can also be used to study the impact of potential future improvements in cell technology.

iii. Propulsion System. The proprietary propulsion system [28] consists of on-board linear motors that interact electromagnetically with a passive-wayside track and provide bi-directional longitudinal forces, enabling the pod to accelerate, overcome drag, and climb grades, while also providing nominal regenerative braking.

The capability of the motors directly impacts the pod’s acceleration and deceleration performance, and therefore the length of ramps. As with most motors, under full load they operate in two regimes: constant force and constant power. The speed at which the transition from constant force to constant power occurs is called the base speed.

Design Considerations. The propulsion system must be capable of delivering (or absorbing) the force and power required in every phase of a trajectory. In the launch and cruise phases, this force includes the drag, any acceleration, and any grade climbing. The total drag is the sum of the aerodynamic drag, the drag due to the levitation and guidance systems, and, perhaps confusingly, the drag due to the motors themselves because of losses in the track. The losses generated by the motors are another major design consideration, both from the perspective of sizing the power electronics and the battery, and sizing the thermal management system. To improve the overall efficiency of the propulsion drive train (including the power electronics and battery) in cruise, a parameter allows use of a subset of the motors during low-power operation, so that the power electronics can be operated more efficiently.

As with every other subsystem, we must also consider the mass, cost, and geometry of the motors.

Another significant design consideration is the cost of the propulsion track, which depends on the cross-sectional area of the track and therefore on the size of the motors.

Modeling Fidelity. The motor is modeled using a combination of basic physics models and geometric scaling laws. For example, motor force scales with the active area, using an input for force-per-unit-area taken from detailed electromagnetic simulation work, and validated through testing. The basic geometry of every key component (e.g. coils, cold plates) in the motor is modeled and the motor active width is the most significant sizing variable. Some design decisions like coil pitch are made using more detailed electromagnetic analysis and taken as inputs to HOPS.

The various sources of loss both on-board and in the track are modeled in relative detail. The losses in the coils are modeled as functions of current and resistance, where the resistances of the coils are modeled as functions of geometry. Other on-board losses are modeled as a function of flux and motor geometry. Track-side losses are modeled as a function of motor geometry, pod velocity, and flux. Accurately capturing the loss models is made more complicated because different relationships apply to the constant-force and constant-power phases of acceleration. To keep relationships GP-compatible, the maximum power loss and the energy loss (i.e. the integral of power loss) must therefore be modeled separately. Several GP-compatible fits are necessary to make this model work, including some fits of the integral of certain variables.

For example, the total loss in one of the coils during launch is calculated as the integral of power over the time reach cruise:

\[ E_{\text{loss,coil,launch}} = \int_0^{t_c} P_{\text{loss,coil}} dt = 3R_{\text{coil}} \int_0^{I_{\text{rated}}^2} \frac{I^2}{v} dt \]

But \( I_{\text{rated}} \) is a piecewise function of velocity with different behavior either side of the base speed \( v_b \), i.e. in the constant-force and constant-power phases:

\[ I_{\text{rated}}^2 = \begin{cases} I_{\text{rated}}^2 \left( \frac{v}{v_b} \right)^2 + F_{\text{peak}} \left( 1 - \frac{v}{v_b} \right)^2, & v \leq v_b \\ I_{\text{rated}}^2 \left( \frac{v}{v_b} \right)^2 + F_{\text{peak}} \left( 1 - \frac{v}{v_b} \right)^2, & v > v_b \end{cases} \]

so it needs to be integrated piecewise:

\[ \int_0^{I_{\text{rated}}^2} \frac{I^2}{v} dt = \int_0^{I_{\text{rated}}^2} F_{\text{rated}}^2 \frac{1}{v} dt + \int_0^{\frac{1}{v}} F_{\text{peak}}^2 \left( \frac{v}{v_b} \right)^2 + F_{\text{peak}} \left( 1 - \frac{v}{v_b} \right)^2 \frac{1}{v} dt \]

\[ = I_{\text{rated}}^2 \frac{v_b}{d_0} + \frac{m_{\text{peak}}}{P_{\text{peak}}} \left( F_{\text{peak}}^2 + F_{\text{rated}}^2 \right) v_b \log \left( \frac{v}{v_b} \right) \]

The cumbersome expression in square brackets is not GP-compatible, but we can approximate it well using a posynomial fit with 2% RMS error as shown in Figure 10.

![Graph showing GP-compatible fit of test data for cell internal resistance as a function of discharge current](image1)

![Graph showing a GP-compatible fit of the term in square brackets as a function of cruise velocity and base speed over a likely range of values](image2)
We can now express the loss with the following constraint, where $\psi$ is the posynomial function plotted in Figure 10.

$$E_{\text{loss,coil,launch}} \geq 3R_{\text{coil}} \left( \frac{I_{\text{peak}}}{a_0} \frac{v_0}{m_{\text{pod}}} \frac{P_{\text{peak}}}{P_{\text{peak}}} \psi \right)$$

(24)

$$\psi^{0.151} \geq 2.34v_{\text{c}}^{0.395} - 0.0037 \frac{A_{\text{tube}}}{A_{\text{pod,frontal}}} (25)$$

iv. Aerodynamic Drag Although a defining feature of a hyperloop is a low pressure environment, there is still enough air in the tube to cause aerodynamic drag that, while small, cannot be neglected.

The drag coefficient depends primarily on three factors: the velocity of a pod, the tube pressure, and the blockage ratio (the ratio of pod frontal area to tube cross-sectional area). In certain high-traffic scenarios, the aerodynamic drag can also depend on the convoy-to-convoy spacing. We use a GP-compatible fit of CFD data to build surrogate models that take some or all of these dimensions into account. Figure 11 shows slices of one such model (RMS error: 4%).

A key feature of hyperloop operations is the ability for pods to travel in convoys, much like a train but without any physical coupling between pods. This enables a passenger throughput capacity that eclipses those of the best high-speed rail systems in the world, while still supporting direct-to-destination service. However, convoy operations also offer another benefit: reduced aerodynamic drag when averaged across all pods thus reducing overall system energy consumption.

The average part is important; different pods in a convoy experience different levels of aerodynamic drag. CFD analysis has shown that, perhaps counterintuitively, the last pod in a convoy feels the highest level of drag, approximately double the average drag for a 4-pod convoy. This difference is significant enough to justify modeling the two cases separately and simultaneously. While special "caboose" pods could be designed (with slightly larger batteries and thermal management systems) this would be challenging from an operational flexibility perspective, particularly when pods need to be able to peel off from convoys to reach different destinations. A single pod design therefore needs to be able to complete a given journey regardless of its position in a convoy. The pod should be optimized for the weighted-average performance but capable of performing the most challenging case. This means the last pod in a convoy is the "sizing" case.

GPkit allows this functionality to be implemented with a single command to "Vectorize" the Pod Performance model. This duplicates all of the constraints and each Pod Performance variable now has a two-element solution, one for the lead pods in a convoy, which all experience similar levels of drag, and one for the last pod. Because the Pod Performance models represent a significant proportion of the constraints in HOPS, this duplication significantly increases the size of the problem, however thanks to the efficiency of the solvers used, the increased computational cost is low in practical terms; the difference between 2 and 4 seconds for a typical point-solution.

The difference in the solution is significant enough to be worthwhile. For typical solutions with a 4-pod convoy, there is a 10-20% difference between the average energy consumed and the energy consumed by the last pod, which translates to a non-negligible difference in the system design, compared to what it would be if only optimized for the last pod performance.

Importantly, the model can also still be run in "lone-pod" mode, with a single Pod Performance model, for scenarios where convoys are less relevant (e.g. low traffic routes). This model vectorization could also be extended to other domains of the model where it is helpful to optimize over a distribution of cases within a fleet, such as different battery ages and different payload mass.

4. Linear Infrastructure

The linear infrastructure, or hyperstructure, provides an enclosed environment through which pods travel, while supporting the track elements needed for propulsion, levitation, guidance, and emergency braking. Most routes will likely have an alignment with both above-grade (i.e. elevated) and below-grade (i.e. tunnelled) portions, which requires a different hyperstructure configuration. The above-grade hyperstructure comprises a substructure (i.e. columns) and a superstructure (i.e. tube); the below-grade hyperstructure comprises a tube that can be constructed as part of the tunnel boring process. HOPS does not perform alignment optimization, so the fraction of a route that is below-grade is taken as an input.

The predominant consideration for the hyperstructure is cost and the hyperstructure CapEx is often the largest single contributor to the total cost per passenger-km. As discussed previously, for a network with multiple ramps, the total length of infrastructure is a significant driver of total infrastructure cost. However, the cost of each unit-length of hyperstructure is also important and depends on the cost of the civil infrastructure (e.g. columns, tunnel boring etc.), the hyperloop-specific infrastructure (tube, tunnel liner etc.), the track elements, and any side communication equipment.

The cost of the tube and the track elements are directly coupled to the design of the pod. There are two main considerations for tube size: geometric constraints and aerodynamic blockage. The tube must be large enough to fit the pod with appropriate range-of-motion envelopes and to prevent significant aerodynamic drag, as discussed in paragraph iv. The blockage ratio, $r_{\text{blockage}}$, also must consider the cross-sectional area of track elements inside the tube.

$$r_{\text{blockage}} \geq \frac{A_{\text{pod,frontal}}}{A_{\text{tube}}}$$

(26)

$$A_{\text{ube,unobstructed}} + A_{\text{ube,obstructed}} \leq A_{\text{tube}}$$

(27)

The cost of each track element depends on its size, which is directly coupled to the geometry of the engine with which it interacts, and therefore the pod mass and performance. Track cost also depends on whether the track is laminated or solid, which also affects the performance of the engines.

An important caveat for modeling the hyperstructure is that a large portion of the cost is very insensitive to, or even independent of, the design. This is in part due to the relatively (and necessarily) crude nature of infrastructure design and construction, and in part because a lot of infrastructure cost has nothing to do with engineering at all [29]. These large fixed costs, which can also be heavily project-dependent, must be accounted for, not only in pursuit of accuracy for total cost, but also to be able to communicate total cost sensitivities. Understanding how the optimal system design looks under a variety of infrastructure cost scenarios is important to producing a valid and defensible design.

5. Pressure Management System

The Pressure Management System (PMS) has two main functions: maintaining the operating tube pressure and pumping the system down.
to operating pressure after maintenance or any other required re-purification of the tube.

Maintaining operating pressure involves pumping out any air that leaks into the system during the course of normal operations. Air leaks into the tube both from outside the tube and from inside the pods, so the capacity of the PMS is coupled not only to the length of linear infrastructure, but also to the number of pods in the system.

For pumpdown, the PMS must be able to return the system to nominal operating pressure in a suitably short period of time to allow overnight maintenance operations. Pressure management facilities are collocated with gate valves so that sections of the tube can be isolated for re-pressurization. Therefore pumpdown requirements constrain not only the number of pumps but also the spacing of pump facilities.

The key design variable for the PMS is the tube pressure. Other important design variables include the total number of pumps and the spacing of the pump facilities. In addition to the capital cost associated with the pumps and the supporting equipment needed at every pressure management facility, the PMS operating costs must be accounted for. These include the energy consumption for both pressure maintenance and pumpdown operations, as well as pump maintenance costs.

The performance of the pumps is captured using GP-compatible fits of pump manufacturer data for flow rate and power consumption as a function of pressure. A fit for flow rate is shown in Figure 12. Different pumps are modeled to allow comparisons of different technologies.

![Figure 12. GP-compatible fit of pump curve data: flow rate as a function of pressure.](image)

6. **Portals**

A portal can be decomposed into technology-agnostic infrastructure (i.e., a building or underground structure), hyperloop-specific infrastructure (e.g., tubes), hyperloop-specific hardware (e.g., battery chargers, coolant supply equipment, and airlocks), and other equipment. Each portal has a number of podbays where passengers board and disembark while pods are charged and replenished with coolant. Larger portals may have multiple branches of podbays to help with traffic flow.

The vast majority of system energy consumption occurs in the portal, because that is where pod batteries are charged and coolant is produced. Passengers board pods through airlocks, gateways that provide a pressurized connection between the pod interior and the platform. Each time a pod departs, the air must be removed from the airlock, which also has an associated energy consumption.

The size of each portal depends on the size and number of podbays needed to serve the pod traffic during peak hour. The size of podbays depends on the size of pods, and the number of podbays depends on how long it takes for a pod to turn around, which is constrained by both the time it takes to charge the battery and the time it takes for passenger loading and unloading. The time needed to charge the battery depends on how much energy the pod consumed during its inbound journey.

![Figure 13. Pod mass against time for the first design cycle.](image)
to these, it can be useful and illustrative to explore the broader design space with sweeps of inputs to see, for example, when certain constraints become active. Because HOPS solves a single design point in just a few seconds, performing even large and multi-dimensional sweeps is relatively quick. This allows system designers to understand the sensitivity to any input, from low-level variables like motor loss model parameters or the leak rate of the tube, to high-level variables like the route length or value of passenger time.

For example, Figure 14 shows the effect of a sweep of the maximum cell charge current on six key variables. As can be seen, what may feel like a subsystem-level input can have relatively dramatic system-wide impacts. Increasing the allowable charge current reduces charging time which in turn reduces the number of podbays needed in each portal. However, increasing the charge current beyond 22 amps does not reduce turnaround time further because turnaround time is also constrained by passenger loading/unloading time. Increasing the charge current also allows the battery to reduce in size quite substantially, which in turn allows the pod to get lighter. Meanwhile the optimal cruise velocity and tube pressure also increase. The sweep’s effect on ramp length is particularly interesting. Initially the optimal ramp length decreases as charge current increases, because charge current constrains the pod’s regenerative braking capability. Above 16 amps, this is no longer the active constraint for ramp length but a constraint on maximum instantaneous cell power loss becomes active, which causes the optimal ramp length to increase again as the optimal cruise velocity also continues to increase. Above 20 amps, a constraint on charge voltage becomes active and the optimal ramp length stabilizes: further increases in charge current do not afford meaningful cost reduction. We can identify these constraints becoming active by plotting the sensitivities to each input as shown in Figure 15. In practice, a sweep like this tells us over which ranges we should care about the cell charge current limit (from a cost perspective) and why (from a design perspective).

![Figure 14. The effect of sweeping maximum charge current on six key variables, shown as percentage changes from the first point.](image)

Because all design relationships are expressed as constraints rather than a sequence of calculations, sweeps can be performed on variables that would usually be considered free variables, i.e. outputs. For example, a designer can sweep over a range of pod mass values to understand what the optimal design and performance would be at each point. This can be helpful for developing intuition and identifying opportunities where the model is not adequately capturing true design considerations.

Two-dimensional sweeps (which rely even more on HOPS’ speed) can illustrate relative sensitivities, answering questions such as “how much more should you be willing to pay for track to improve the efficiency of your propulsion system?”

![Figure 15. Changes in sensitivity help us to identify when constraints become active in the sweep shown in Figure 14. The vast majority of inputs have a flat or shallow slope. A sensitivity of <1 means that decreasing that value by 1% will decrease (improve) the objective function by 1%](image)

D. Trade studies

HOPS has been used to perform trade studies between different architectures for the propulsion, levitation, power electronics, and thermal management systems. Optimal solutions for two or more discrete architectures can then be compared to show differences in both high-level cost and subsystem designs. For example, the choice of a power electronics technology may significantly affect the frequency of battery replacement, while the thermal management system architecture may significantly change the optimal battery size and pressure management system design.

What HOPS provides is a common platform from which to debate quantitative merits and drawbacks of each architecture. HOPS can only provide a “dry” cost optimization perspective though, and has nothing to say on questions of development risk, upgradability, and safety.

VI. Implementation, Transparency, and Trust

HOPS has been adopted as a central component of the Virgin Hyperloop system design process. Given its potential to therefore influence a broad range of significant decisions, and given that formal system optimization (let alone geometric programming) is a relatively niche and nascent field in most industries, it has been important to take extensive measures to develop confidence in the tool.

This need to build confidence has significantly influenced HOPS’ implementation from a cultural perspective. The core tenets of this implementation have been transparency, testing, and distribution of ownership and responsibility.

Finding the right level of involvement has been a challenge, but the most natural and practical balance has been to have design engineers, manufacturing engineers, engineering analysts, and business analysts provide models for their respective domains in whatever format feels most natural to them (usually an Excel spreadsheet or a MATLAB script) which is then transposed to a geometric programming formulation and HOPS syntax by a HOPS developer.

A. Documentation

A key piece of transparency comes from having comprehensive and up-to-date model documentation. Given the size of the model and the frequency with which changes are made, manually updating such documentation would be impractical. Instead, a custom “autodocumentation” tool parses all model code to generate a LaTeX document that includes all of the constraints and numerical inputs. This allows stakeholders to review and check the model without needing to navigate the code repository or be familiar with Python and GPKit syntax. HOPS’ syntax has been designed to ensure uniformity and thus enable full documentation coverage with as little maintenance and customization as possible. This uniformity has the added benefit of making the code easier to read and navigate. Because the documentation is automatically generated with every model change, it is guaranteed to be up-to-date.
B. Testing

One challenge of maintaining a large, monolithic, all-at-once optimization tool is the implementation of effective model testing. Because of the many inter-model coupling variables, unit testing is difficult to implement and would itself require a lot of maintenance. In lieu of unit testing, a suite of full-model test cases are run with each model change to ensure that all valid model configurations remain functional.

A “diff” comparing each new solution to its predecessor is generated and saved with the test solutions. These diffs are then manually inspected to ensure that the effect on each test case is accurate and understood. Solution visualizations are also used extensively to identify unexpected or erroneous results.

Finally, HOPS models are cross-validated against other in-house design tools. For example, system-level pod performance results have been compared with the outputs of a pod simulation tool and subsystem solution values have been compared against the original models from which they were generated.

C. Interactivity

An interactive browser-based tool called iHOPS allows users across the company to create and solve cases and review solutions without needing to install HOPS (and all its dependencies) or be familiar with Python and GPkit. iHOPS empowers the engineering and business teams to perform their own point optimizations, sensitivity studies (“sweeps”), or comparisons (“diffs”). This helps build confidence in HOPS, while also helping to identify any surprising model behavior or inputs worthy of further investigation. iHOPS also provides access to key “reference” solutions and therefore serves as a central repository for design values and assumptions.

D. Check-ins

We encourage widespread buy-in to HOPS results by holding regular (e.g. monthly) “check-in” meetings with all stakeholders to review models, inputs, and solution values, and to discuss potential improvements and areas of concern. These check-ins foster a sense of shared ownership of, and responsibility for, the development of HOPS and the quality of its solutions.

HOPS is imperfect in many ways. Despite rigorous efforts to quality-control the models, the possibility of errors cannot be ruled out. There are also plenty of high-level inputs that can be described as best guesses or judgement calls. Despite this, HOPS serves an important purpose as a unified repository of information, a tool that aligns members of every engineering team to a single design point, regardless of whether it is truly optimal [32]. This is only possible because HOPS describes the full hyperloop system: every subsystem designer’s interests (and concerns) are represented. The purpose of check-ins is to make clear to stakeholders that they have the power to change that representation at any time.

VII. Conclusion

The design of a hyperloop system presents an interesting and highly-coupled optimization problem due to the often-competitive objectives of minimizing CapEx, OpEx, and travel time, and the many recursive design relationships inherent to the system architecture. Combining this with the fact it is a clean-sheet design problem makes hyperloop a unique opportunity to impactfully apply rigorous MDO techniques.

A system optimization tool, HOPS, has been developed that is capable of providing a fast and disciplined way of solving such an optimization problem, by formulating it as a sequence of geometric programs that can be solved using commercially available software. HOPS minimizes total cost per passenger-km and models everything from the diameter of the tube down to the current in the motor coils. The default case has 4512 free variables and solves in 4 seconds on a laptop.

HOPS has played a central role in the Virgin Hyperloop system design process, and has been used to optimize the design, set system requirements, perform sensitivity analysis, and inform trade studies.

References

[1] Goddard, E. C., “Vacuum tube transportation system,” June 20 1950, US Patent 2,511,979.
[2] Oster, D., Kumada, M., and Zhang, Y., “Evacuated tube transport technologies (ETT) tm: a maximum value global transportation network for passengers and cargo,” Journal of Modern Transportation, Vol. 19, No. 1, 2011, pp. 42–50.
[3] Bierfrai, M., Axhausen, K., and Abay, G., “The acceptance of modal innovation: The case of Swissmetro,” Swiss Transport Research Conference, 2001.
[4] SpaceX, “Hyperloop Alpha,” 2013, https://www.tesla.com/sites/default/files/blog_images/hyperloop-alpha.pdf.
[5] Henderson, R. P., Martins, J. R., and Perez, R. E., “Aircraft conceptual design for optimal environmental performance,” The Aeronautical Journal, Vol. 116, No. 1175, 2012, pp. 1–22.
[6] Kroo, I., Altus, S., Braun, R., Gage, P., and Sobieski, I., “Multidisciplinary optimization methods for aircraft preliminary design,” 5th symposium on multidisciplinary analysis and optimization, 1994, p. 4325.
[7] Martins, J. R. and Lambe, A. B., “Multidisciplinary design optimization: a survey of architectures,” AIAA journal, Vol. 51, No. 9, 2013, pp. 2049–2075.
[8] Martins, J. R., Alonso, J. J., and Reuther, J. J., “High-fidelity aerostructural design optimization of a supersonic business jet,” Journal of Aircraft, Vol. 41, No. 3, 2004, pp. 523–530.
[9] Sobieszczanski-Sobieski, J. and Hafikta, R. T., “Multidisciplinary aerospace design optimization: survey of recent developments,” Structural optimization, Vol. 14, No. 1, 1997, pp. 1–23.
[10] Lee, E., “Don’t look for commercial BWB airplane any time soon, says Boeing’s future airplanes head,” April 2018, https://leehamnews.com/2018/04/03/dont-look-for-commercial-bwb-airplane-any-time-soon-says-boeings-future-airplanes-head/.
[11] Boyd, S., Kim, S.-J., Vandenberghe, L., and Hassibi, A., “A tutorial on geometric programming,” Optimization and Engineering, Vol. 8, No. 1, 2007, pp. 67–127.
[12] Burnell, E., A Worker-Centered Approach to Convex Optimization in Engineering Design, Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, Sept. 2020.
[13] Hoburg, W., Kirschen, P., and Abbeel, P., “Data fitting with geometric-programming-compatible softmax functions,” Optimization and Engineering, Vol. 17, No. 4, 2016, pp. 897–918.
[14] Boyd, S. and Vandenberghe, L., Convex Optimization, Cambridge University Press, New York, 2004.
[15] Hoburg, W. and Abbeel, P., “Geometric programming for aircraft design optimization,” AIAA Journal, Vol. 52, No. 11, 2014, pp. 2414–2426.
[16] Kirschen, P. G., York, M. A., Ozturk, B., and Hoburg, W. W., “Application of signomial programming to aircraft design,” Journal of Aircraft, Vol. 55, No. 3, 2018, pp. 965–987.
[17] Chiang, M., Geometric programming for communication systems, Now Publishers Inc, 2005.
[18] Boyd, S. P., Kim, S.-J., Patil, D. D., and Horowitz, M. A., “Digital circuit optimization via geometric programming,” Operations research, Vol. 53, No. 6, 2005, pp. 899–932.
[19] Agrawal, A., Diamond, S., and Boyd, S., “Disciplined geometric programming,” Optimization Letters, Vol. 13, No. 5, 2019, pp. 961–976.
[20] Burnell, E., Dumen, N. B., and Hoburg, W., “GPkit: A Human-Centered Approach to Convex Optimization in Engineering Design,” Conference on Human Factors in Computing Systems (CHI), Association for Computing Machinery, Honolulu, 2020, p. 12.
[21] MOSEK, “MOSEK Optimizer API for C. Version 8.1,” 2019, https://docs.mosek.com/8.1/capi/index.html.
[22] Andersen, M. S., Dahl, J., and Vandenberghe, L., “CVXOPT: A Python package for convex optimization,” abel. ee. ucla. edu/cvxopt, Vol. 88, 2013.
[23] Ashuri, T., Zaaijer, M. B., Martins, J. R., Van Bussel, G. J., and Van Kuik, G. A., “Multidisciplinary design optimization of offshore wind turbines for minimum levelized cost of energy,” Renewable energy, Vol. 68, 2014, pp. 893–905.

[24] Lee Jr, D., “Methods for evaluation of transportation projects in the USA,” Transport policy, Vol. 7, No. 1, 2000, pp. 41–50.

[25] Small, K. A., “Valuation of travel time,” Economics of transportation, Vol. 1, No. 1-2, 2012, pp. 2–14.

[26] White, V., “Revised Departmental Guidance on Valuation of Travel Time in Economic Analysis,” Office of the Secretary of Transportation, US Department of Transportation, Available at: https://www.transportation.gov/sites/dot.gov/files/docs/2016%20Revised%20Value%20of%20Travel%20Time%20Guidance.pdf, 2016.

[27] Wardman, M., “Public transport values of time,” Transport policy, Vol. 11, No. 4, 2004, pp. 363–377.

[28] Jedinger, A., “Homopolar Linear Synchronous Machine,” March 2020, U.S. Patent 20200091807A1.

[29] Levy, A., “Low- and Medium-Cost Countries,” November 2020, https://pedestrianobservations.com/2020/09/25/low-and-medium-cost-countries.

[30] Pillai, P. P., Burnell, E., Wang, X., and Yang, M. C., “Early-Stage Uncertainty: Effects of Robust Convex Optimization on Design Exploration,” Journal of Mechanical Design, 2020, pp. 11.

[31] Meluso, J., Austin-Breneman, J., and Uribe, J., “Estimate Uncertainty: Miscommunication About Definitions of Engineering Terminology,” Journal of Mechanical Design, Vol. 142, No. 7, July 2020, pp. 13.

[32] Burnell, E., Pillai, P. P., and Yang, M. C., “Maps, Mirrors, and Participants: Design Lenses for Sociomateriality in Engineering Organizations,” arXiv, Vol. 2008.06616, 2020, pp. 9.