Which FLRW comoving 3-manifold is preferred observationally and theoretically?

BOUDEWIJN F. ROUKEMA

Toruń Centre for Astronomy, Nicolaus Copernicus University
ul. Gagarina 11, 87-100 Toruń, Poland

Abstract

The lack of structure greater than $10h^{-1}$ Gpc in Wilkinson Microwave Anisotropy Probe (WMAP) observations of the cosmic microwave background (CMB) favours compact Friedmann-Lemaître-Robertson-Walker (FLRW) models of the Universe. The present best candidates based on observations are the Poincaré dodecahedral space $S^3/I^*$ and the 3-torus $T^3$. The residual gravity effect favours the Poincaré space, while a measure space argument where the density parameter is a derived parameter favours flat spaces almost surely.

1 FLRW models: curvature and topology

Comoving space in the Friedmann-Lemaître-Robertson-Walker (FLRW) models is a 3-manifold $M = \tilde{M}/\Gamma$. This can be thought of (i) embedded in a higher dimensional euclidean space, e.g. $T^2$ as the surface of a doughnut-with-a-hole in $\mathbb{R}^3$; (ii) as a fundamental domain with identified faces, e.g. a square with identified edges for $T^2$ (like in some video games); or (iii) as a tiling of the covering space (apparent space) $\tilde{M}$ by many copies of the fundamental domain, e.g. squares tiling $\mathbb{R}^2$. The group of holonomy transformations $\Gamma$ identifies multiple images of any single physical object in $\tilde{M}$, e.g. $\Gamma = \mathbb{Z}^2$ (linear span of two vectors) for $T^2$.

2 Observations

A first-order argument following immediately from thinking of the fundamental domain of the 3-manifold is that no physical object larger than the fundamental domain can exist. Hence, observational statistics representing structure (density perturbations) in the Universe should show a lack of structure on scales larger than the size of the fundamental domain. In the apparent space, more detailed calculations show that an approximate cutoff in structure in the cosmic microwave background (CMB) should occur. This cutoff was suspected in COBE data and was confirmed at scales above $\sim 10h^{-1}$ Gpc in Wilkinson Microwave Anisotropy Probe (WMAP) maps of the CMB.

Which specific 3-manifold best fits the observations? Well-proportioned spherical spaces, i.e. those with equal sizes in different fundamental directions, are expected to more easily fit the WMAP data than other spaces. Among these, the Poincaré dodecahedral space, $S^3/I^*$, has become a particularly good (though disputed) candidate given the WMAP CMB data. On the other hand, while some identified circle analyses failed to find evidence for the simplest flat spaces (e.g. $T^3$; infinite space is not simple), several other analyses find that the $T^3$ model gener-
ally provides a better fit to the WMAP data than infinite flat models [23, 24, 25, 26].

3 Theoretical arguments

Possible elements of a theory of cosmic topology include quantum gravity work investigating the decay from pure quantum to mixed states [27], smooth topology evolution [28] and some approaches to deciding which 3-manifold should be favoured by quantum cosmology [29, 30].

At a much simpler level, a recent heuristic result concerns a dynamical feedback effect of cosmic topology. In the presence of a density perturbation (massive point particle above a homogeneous background), a residual weak-limit gravitational acceleration effect exists [31]. Well-proportioned spaces [9] are also “well balanced” according to this effect, in the sense that the first-order effect cancels and the remaining effect is only third order in the fractional displacement of a test particle from the massive particle [31, 32]. What is even more surprising is that the space that has raised considerable interest in empirical analyses, the Poincaré dodecahedral space \( S^3/I^* \), is even “better balanced” than the other well-proportioned, well-balanced spaces. The first and third order terms both cancel, leaving an effect dominated by the fifth-order term [32]. Thus, the observational analyses favouring the Poincaré space are matched by this theoretical argument showing that the Poincaré space is an optimal space in terms of the residual gravity effect. Nevertheless, although this is an exciting coincidence, it is still a long way from constituting a physical theory.

In fact, a theoretical argument exists in favour of flat compact models, by assuming that the density parameter \( \Omega \) is a derived rather than fundamental parameter [33]. Suppose that the processes at the exit of the quantum epoch that select a spatial 3-manifold result in a global mass-energy and a Hubble parameter in a way that is independent of curvature and topology. Then, contrary to the usual assumption that \( \Omega \) is a free parameter, any 3-manifold (of negative, zero, or positive constant curvature) allows just one value of \( \Omega \) [Eqs (6), (7), (8), Ref. 33]. If, moreover, the injectivity radius \( r_{\text{inj}} \) is used to to define a probability space over the set \( F \) of compact, comoving, 3-spatial sections of FLRW models, then the natural measure is the Lebesgue measure, and it is normalisable, resulting in a probability space. In this case, flat models should occur with probability one, i.e. almost surely (a.s.), and non-flat models should occur with probability zero, i.e. they will a.s. not occur [33]. This argument is related to the rigidity of curved spaces.

4 Conclusion

Both the Poincaré dodecahedral space \( S^3/I^* \) and the 3-torus \( T^3 \) are observationally viable candidates for the spatial section of the Universe. The residual gravity effect favours the former, while a measure space argument where the density parameter is a derived parameter favours compact flat spaces almost surely.

References

[1] A. Friedmann, Mir kak prostranstvo i vremya (The Universe as Space and Time) (Leningrad: Academia, 1923).
[2] G. Lemaître, MNRAS 91, 490 (March 1931).
[3] H. P. Robertson, ApJ 82, p. 284 (November 1935).
[4] A. A. Starobinsky, JETP Letters 57, 622 (May 1993).
[5] D. Stevens, D. Scott and J. Silk, Phys. Rev. Lett. 71, 20 (July 1993).
[6] D. N. Spergel, et al., Astroph. J. Supp. 148, p. 175 (September 2003), arXiv:astro-ph/0302209.
[7] C. J. Copi, D. Huterer, D. J. Schwarz and G. D. Starkman, Phys. Rev. D 75, 023507 (January 2007), arXiv:astro-ph/0605135.
[8] C. J. Copi, D. Huterer, D. J. Schwarz and G. D. Starkman, MNRAS 399, 295 (October 2009), arXiv:0808.3767.
[9] J. Weeks, J.-P. Luminet, A. Riazuelo and R. Lehoucq, *MNRAS* **352**, p. 258 (July 2004), arXiv:astro-ph/0312312

[10] J. Luminet, J. R. Weeks, A. Riazuelo, R. Lehoucq and J. Uzan, *Nature* **425**, p. 593 (October 2003), arXiv:astro-ph/0310253

[11] B. F. Roukema, B. Lew, M. Cechowska, A. Marecki and S. Bajtlik, *Astron. & Astr.* **423**, p. 821 (September 2004), arXiv:astro-ph/0402608

[12] R. Aurich, S. Lustig and F. Steiner, *Class. Quant. Grav.* **22**, p. 3443 (September 2005), arXiv:astro-ph/0504656

[13] R. Aurich, S. Lustig and F. Steiner, *Class. Quant. Grav.* **22**, p. 2061 (June 2005), arXiv:astro-ph/0412569

[14] J. Gundermann, *ArXiv e-prints* (2005), arXiv:astro-ph/0503014

[15] J. S. Key, N. J. Cornish, D. N. Spergel and G. D. Starkman, *Phys. Rev. D* **75**, 084034 (April 2007), arXiv:astro-ph/0604616

[16] A. Niarchou and A. Jaffe, *Phys. Rev. Lett.* **99**, 081302 (August 2007), arXiv:astro-ph/0702436

[17] S. Caillerie, M. Lachiéze-Rey, J.-P. Luminet, R. Lehoucq, A. Riazuelo and J. Weeks, *Astron. & Astr.* **476**, p. 691 (May 2007), arXiv:0705.0217v2

[18] B. Lew and B. F. Roukema, *Astron. & Astr.* **482**, p. 747 (May 2008), arXiv:0801.1358

[19] B. F. Roukema, Z. Buliński, A. Szaniecka and N. E. Gaudin, *Astron. & Astr.* **486**, p. 55 (June 2008), arXiv:0801.0006

[20] B. F. Roukema, Z. Buliński and N. E. Gaudin, *Astron. & Astr.* **492**, p. 673 (July 2008), arXiv:0807.4260

[21] N. J. Cornish, D. N. Spergel and G. D. Starkman, *ArXiv Gen.Rel. & Quant.Cosm. e-prints* (1996), arXiv:gr-qc/9602039

[22] N. J. Cornish, D. N. Spergel, G. D. Starkman and E. Komatsu, *Phys. Rev. Lett.* **92**, p. 201302 (October 2004), arXiv:astro-ph/0310233

[23] R. Aurich, S. Lustig, F. Steiner and H. Then, *Class. Quant. Grav.* **24**, 1879 (April 2007), arXiv:astro-ph/0612308

[24] R. Aurich, *Class. Quant. Grav.* **25**, p. 225017 (March 2008), arXiv:0803.2130

[25] R. Aurich, H. S. Janzer, S. Lustig and F. Steiner, *Classical and Quantum Gravity* **25**, 125006 (June 2008), arXiv:0708.1420

[26] R. Aurich, S. Lustig and F. Steiner, *ArXiv e-prints* (March 2009), arXiv:0903.3133

[27] S. W. Hawking, *Nuclear Physics B* **244**, 135 (September 1984).

[28] F. Dowker and S. Surya, *Phys. Rev. D* **58**, p. 124019 (December 1998), arXiv:gr-qc/9711070

[29] S. Masafumi, *Phys. Rev. D* **53**, p. 6902 (1996), arXiv:gr-qc/9603002v1

[30] M. Anderson, S. Carlip, J. G. Ratcliffe, S. Surya and S. T. Tschantz, *Class. Quant. Grav.* **21**, p. 729 (January 2004), arXiv:gr-qc/0310002

[31] B. F. Roukema, S. Bajtlik, M. Biesiada, A. Szaniecka and H. Jurkiewicz, *Astron. & Astr.* **463**, 861 (March 2007), arXiv:astro-ph/0602159

[32] B. F. Roukema and P. T. Różański, *Astron. & Astr.* **502**, p. 27 (July 2009), arXiv:0902.3402

[33] B. F. Roukema and V. Blanloeil, *ArXiv e-prints* (December 2009), arXiv:0912.2300