Passive Reaction Analysis for Grasp Stability

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Abstract—In this paper we focus on the following problem in multi-fingered robotic grasping: assuming that an external wrench is being applied to a grasped object, will the contact forces between the hand and the object, as well as the hand joints, respond in such a way as to preserve quasi-static equilibrium? In particular, we assume that there is no change in the joint torques being actively exerted by the motors; any change in contact forces and joint torques is due exclusively to passive effects arising in response to the external disturbance. Such passive effects include for example joints that are driven by highly geared motors (a common occurrence in practice) and thus do not back drive in response to external torques. To account for non-linear phenomena encountered in such cases, and which existing methods do not consider, we formulate the problem as a mixed integer program used in the inner loop of an iterative solver. We present evidence showing that this formulation captures important effects for assessing the stability of a grasp employing some of the most commonly used actuation mechanisms.

Note to practitioners: Once a grasp of a given object has been chosen, our method has multiple possible applications. First, it can be used to determine how the choice of a pre-load (i.e., the torques applied to the joints as the grasp is created) affects the stability of the grasp. Second, once a pre-load has been chosen, our method can be used to determine which external disturbances can be resisted solely through passive effects, without further changes to the commands sent to the motors. We believe this approach is particularly relevant for the large family of grasping devices that are not equipped with tactile or proprioceptive sensors, and are thus unable to sense external disturbances or to control joint torques, but are still effective thanks to passive resistance effects.

Index Terms—Grasping, Grasp Stability Analysis, Grasp Force Analysis, Multi-Finger Hands, Underactuated Hands

I. INTRODUCTION

StABILITY analysis is one of the foundational problems for multi-fingered robotic manipulation. Grasp planning, or the problem of determining an appropriate grasp for a given object using a particular hand design, can be posed as a search over the space of possible grasps looking for instances that satisfy a measure of stability. The formulation and characteristics of the stability measure thus play a key role in this search, and, by extension, in any task that begins by planning and executing a grasp.

In turn, multi-fingered grasp stability relies on studying the net resultant wrench imparted by the hand to the grasped object. Ferrari and Canny [1] introduced a very efficient geometric method for determining the total space of possible resultant wrenches as long as each individual contact wrench obeys (linearized) friction constraints. This method answers the simplest form of what we refer to here as the existence problem: given a desired output, are there legal contact wrenches that achieve it, and, if so, how large is their needed magnitude in relation to the output? This approach has been at the foundation of numerous planning algorithms proposed since.

Consider a grasp that scores highly according to the quality metric described above. This means that any desired resultant can be produced by a computable set of contact wrenches (of bounded magnitude). In turn, the contact wrenches can be balanced by a set of joint torques, which can also be computed [2]. However, this approach is based on a string of assumptions:

- First, we have assumed that at every contact we can actively control the forces exerted.
- Second, we have assumed that, at any given moment, the control mechanism knows what resultant wrench is needed on the object.
- Third, we have assumed that the joint torques needed to balance this resultant can be actively commanded by the motor outputs.
- Fourth and finally, we have assumed that the desired motor output torques can be obtained accurately.

In practice, these assumptions do not always hold. First, many robotic hands contain members with limited mobility. That is, the contact forces on any such member cannot be arbitrarily controlled. This is of particular importance when enveloping ‘power’ grasps are considered, where contact between the object is not only made at the fingertips, but also at the proximal links and the palm of the hand. The force at a contact can only directly be affected through the actuators preceding the contact in the kinematic chain. Hence, for a contact on the palm the forces cannot be directly controlled using finger actuators. Instead, the force at such a contact will arise indirectly due to the externally applied wrench and the contact forces exerted at other contacts.

Second, the external wrench in need of balancing is difficult to obtain: to account for gravity and inertia, one needs the exact mass properties and overall trajectory of the object, which are not always available; any additional external disturbance will be completely unknown, unless the hand is equipped with tactile sensors. Third, the kinematics of the hand may not permit explicit control of joint torques. This is of particular importance for the large class of underactuated hands, as by definition, the joint torques may not be independently
controlled. Instead they are determined by the kinematic structure of the hand. Finally, many commonly used robot hands use highly geared motors unable to provide accurate torque sensing or regulation.

A much simpler approach to establishing a stable grasp, applicable to more types of hardware, is to simply select a set of motor commands that generate some level of pre-load on the grasp, and maintain that throughout the task. This method assumes that the chosen motor commands will not only lead to an adequate and stable preload for grasp creation, but also prove suitable for the remainder of the task. A key factor that allows this approach to succeed is the ability of a grasp to absorb forces that would otherwise unbalance the system without requiring active change of the motor commands.

Following the arguments above, we believe it is important to not only consider the wrenches the hand can apply actively by means of its actuators, but also the reactions that arise passively. Thus, in this study we are interested in the distinction between active force generation, directly resulting from forces applied by a motor, and passive force resistance, arising in response to forces external to the contacts or joints. Consider the family of highly geared, non-backdrivable motors: the torque applied at a joint can exceed the value provided by the motor, as long as it arises passively, in response to torques applied by other joints, or to external forces acting on the object. Put another way, joint torques are not always the same as output motor torques, even for direct-drive hands.

The distinction between active and passive force resistance is also an important feature of underactuated hands. Wrench resistance is highly reliant on passive compliance, and contact forces in the nullspace of the transposed grasp Jacobian are common due to the underactuated nature of the hand. These contact forces will have no effect on the actuator but cause a purely passive reaction. The existing grasp stability analysis tools are not equipped to account for the behavior of non-backdrivable actuators or underactuated kinematics.

From a practical standpoint, a positive answer to the existence problem outlined above is not useful as long as the joint torques necessary for equilibrium will not be obtained given a particular set of commands sent to the motors. Here, we focus on stability from an inverse perspective: given a set of commanded torques to be actively applied to the robot’s joints, what is the net effect expected on the grasped object, accounting for passive reactions?

Overall, the main contribution of this paper is a quasi-static grasp stability analysis framework to determine the passive response of the hand-object system to applied joint torques and externally applied forces. This method was designed to account for actuation mechanisms such as non-backdrivable or position-controlled motors without explicit torque regulation. Furthermore, it enables the analysis of the passive behavior of some of the most commonly used underactuated hand mechanisms.

An initial report from this work appeared in the 2016 Workshop on the Algorithmic Foundations of Robotics. This study extends on the previous report in multiple ways. We improve the discussion of the underlying physics of the problem, including the aspects that can not be handled exactly and require approximation in our approach. We extend the formulation to account for tendon-driven underactuated hands (which proves to be a generalization of the per-joint treatment shown previously), and present additional experiments to validate the applicability of our method.

II. RELATED WORK

The problem of force distribution between an actively controlled robotic hand and a rigid object has been considered by a number of authors [3], [4], [5], [6]. A great simplification to grasp analysis is the assumption that any contact force can be applied by commanding the joint motors accordingly. This assumption neglects the deficiency of the kinematics of many commonly used robotic hands in creating arbitrary contact forces. The idea that the analysis of a grasp must include not only the geometry of the grasp but also the kinematics of the hand is central to this paper.

Bicchi [7] showed that for a kinematically deficient hand only a subset of the internal grasping forces is actively controllable. Using a quasi-static model, the subspace of internal forces was decomposed into subspaces of active and passive internal forces. Making use of this decomposition Bicchi proposed a quadratic program formulation for the problem of optimal distribution of contact forces with respect to power consumption and given an externally applied wrench [8]. He proposed a definition of force-closure that makes further use of this decomposition and developed a quantitative grasp quality metric that reflects on how close a grasp is to losing force-closure under this definition [9].

Prattichizzo et al. [10] made use of the previous work by Bicchi to compute the subset of actively controllable internal contact forces and proposed two grasp quality measures that are applicable to kinematically deficient grasps. They analyze how far a set of contact forces are from violating their contact constraints and derive a potential contact robustness (PCR). Any wrench with magnitude less or equal to this parameter can be reacted without violation of these constraints. They furthermore define a potential grasp robustness (PGR), which is similar to the PCR, but allows for contacts either breaking or sliding. Both PCR and PGR are conservative metrics and the calculation of the PGR can be computationally infeasible at higher number of contacts due to its combinatorial nature. Prattichizzo et al. [11] also investigated the controllability of object motion and contact forces in underactuated hands using a quasi-static grasp model. Specifically, they consider compliant hands that exhibit passive mechanical adaptation and make use of “postural synergies”.

There have been rigid body approaches to the analysis of active and passive grasp forces. Yoshikawa [12] studied the concept of active and passive closures and the conditions for these to hold. Melchiorsi [13] decomposed contact forces into four subspaces using a rigid body approach. Burdick and Rimon [14] formally defined four subspaces of contact forces and gave physically meaningful interpretations. They analyzed active forces in terms of the injectivity of the transposed hand Jacobian matrix. They note that the rigid body modeling approach is a limitation, as a compliance model is required to draw conclusions on the stability of a grasp.
An important distinction between our work and that of the above authors lies in the definition of what qualifies as a “passive” contact force. In addition to contact forces that lie in the null space of the transposed hand Jacobian, we consider contact forces arising from joints being loaded passively (due to the non-backdrivability of highly geared motors), and not arising from the commanded joint torque, as passive. Furthermore, we define preload forces as the internal forces that arise from selecting a set of motor commands that achieve a grasp in stable equilibrium, previous to the application of any external wrench.

III. PROBLEM STATEMENT

Consider a grasp establishing multiple contacts between the robot hand and the grasped object. We denote the vector of contact wrenches by \( c \). In equilibrium, the grasp map matrix \( G \) relates contact wrenches to the wrench applied to the object externally \( w \), while the transpose of the grasp Jacobian \( J \) relates contact wrenches to joint torques \( \tau \):

\[
Gc = -w \quad (1) \quad J^Tc = \tau \quad (2)
\]

One way to apply this in practice is to check, for a given wrench \( w \), if contact forces exist that satisfy (1). For a comparison of different methods that can be used to compute contact forces, that satisfy this and other constraints, see the recent work by Cloutier et al.\(^{[15]}\). In order to also consider the hand mechanism, one could then also check if joint torques exist that further satisfy (2). However, as we discuss below, this application method has important shortcomings.

Assume that, for a given wrench \( w \), a given set of contact forces and joint torques have been found to satisfy the equilibrium conditions above; denote these by \( c_{eq} \) and \( \tau_{eq} \) respectively. The most straightforward way to use this would be to command the motors to achieve \( \tau_{eq} \); in other words, if \( \tau_{eq} \) is the command sent to the motors, we simply ask that \( \tau = \tau_{eq} \). However, this approach is subject to the assumptions outlined in the Introduction: it requires that \( w \) be known, that applying \( \tau_{eq} \) at the joints in the presence of \( w \) actually results in the desired contact forces \( c_{eq} \), and finally that we can produce desired joint torques \( \tau_{eq} \). The final point further implies that the hand has the needed control authority over all needed degrees of freedom (which is rare, since most fingers are kinematically deficient with at most 3 joints), and that motors can regulate their torque output as needed (also rare, as most hands used in practice are position- and not force-controlled).

A much more common approach is to command a given \( \tau \), and rely on \( \tau_{eq} \neq \tau \), arising through passive reactions. For the large family of hands powered by geared, non-backdrivable motors, at any joint \( i \) the resulting torque \( \tau_i \) can exceed the commanded value, but only passively; in response to the torques \( \tau_j, j \neq i \) and the wrench \( w \). If this happens as desired for a range of disturbances \( w \), then the job of controlling the hand is greatly simplified: one just needs to always command \( \tau \), and the reaction that stabilizes any particular \( w \) happens passively. However, the field currently lacks a method to accurately analyze this problem.

We state our problem as follows: given commanded torques \( \tau_c \), can the system find quasi-static equilibrium for a disturbance wrench \( w \), assuming passive reaction effects at the joints?

A. The classical approach

In combination with a contact constraint model, solving the relatively simple system of Eqs. (1)&(2) for \( c \) and \( \tau \) when \( w \) is given amounts to solving a force distribution problem. However, for rigid bodies, this problem of computing the exact force distribution across contacts in response to an applied wrench is statically indeterminate.

Previous studies such as those of Bicchi\(^{[7,8,9]}\) make use of a linear compliance matrix that characterizes the elastic elements in a grasp and solves the indeterminacy. For a comprehensive study on how to compute such a compliance matrix see the works by Cutkosky and Kao\(^{[16]}\), and Malvezzi et al. for an extension to underactuated hands\(^{[17]}\). A compliance matrix allows us to consider the force distribution across contacts as the sum of a particular and a homogeneous solution. The contact forces \( c_0 \) create purely internal forces and hold the object in equilibrium. This is the homogeneous solution and, as noted in the Introduction, it can be of great importance to grasp stability. The contact forces \( c_{eq} \) associated with the application of an external wrench \( w \) are considered the particular solution. Bicchi formulates a force distribution problem\(^{[8]}\) given by \( c = c_{eq} + c_0 = G_K^R w + c_0 \) where \( G_K^R \) is the \( K \)-weighted pseudoinverse of the grasp map matrix \( G \). \( K \) is the stiffness matrix of the grasp and is given by the inverse of the grasp compliance matrix.

Given the subspace of controllable internal forces \( I \), the particular solution computed in this way can be used to compute a homogeneous solution such that \( c \) satisfies all contact constraints. Using Eq. (2) the required equilibrium joint torques that satisfy this system \( \tau_{eq} \) can then be calculated.

B. Limitations of this approach

The use of a linear compliance matrix is an important limitation, as it assumes a linear stiffness of the contacts and the joints. However, a contact force may only “push”, it cannot “pull” and hence contact forces behave in a nonlinear fashion. Furthermore, a linear compliance model disregards the nonlinearity of frictional forces obeying Coulombs law of friction. We consider contacts of the point contact with friction type, which means that the contact force must lie within its friction cone. A linear compliance model has no notion of this friction constraint and thus cannot distribute forces accordingly once the frictional component of a contact force reaches its limit.

To illustrate this issue, consider Fig.\(^{[1]}\) Fig.\(^{[1a]}\) shows a homogeneous solution to a force distribution problem. Fig.\(^{[1b]}\) shows the sum of the homogeneous and particular solutions when an external wrench pushing the object towards the palm of the robot is applied. Applying a downward force has caused the contact forces on the distal links to violate the friction
constraint (they lie outside their respective cones), perhaps leading us to believe that we have to increase the internal forces in the grasp in order to resist the applied wrench. In reality, however, the contacts on the distal links will only apply as much frictional force as they may, and more force will be distributed to the contacts on the proximal links. Indeed, experimental results indicate that this grasp withstands arbitrary downward forces applied to the object even in the absence of internal forces.

An attempt to alleviate this issue has been made by Prat-tichizzo et al. in their work on the PGR quality measure [10]. The computation of this metric allows for a contact to slide or break entirely. In order to linearize the problem, however, a sliding contact may not exert any frictional forces at all. Thus, the contact forces obtained through this method may not be physically motivated and the PGR measure tends to be a conservative quality metric.

A further issue with these approaches is that the compliance of the joints in many commonly used robotic hands is also non-linear. A joint powered by a highly geared motor will passively resist very large external torques (up to the mechanical failure of the gears). Thus, if the contact force on a link increases due to the external wrench, the joint torque will passively increase to match. However, even if contact force decreases, the joint torque can not decrease below the level actively applied by the motor. This is a non-linear effect that existing analysis methods can not account for.

IV. GRASP MODEL

Due to the above limitations of linear models, we propose a model that accounts for non-linear effects due to the behavior of contact forces and non-backdrivable actuators. To capture the passive behavior of the system in response to external disturbance, we (as others before [18], [7], [8], [9], [10]) rely on computing virtual object movements in conjunction with virtual springs placed at the contacts between the rigid object and the hand mechanism. Unlike previous work however, we attempt to also capture effects that are non-linear with respect to virtual object movement: joint torques that can not decrease below the commanded levels (but can increase if the joint does not backdrive), as well as contact forces restricted to the inside of the (linearized) friction cone.

A. Friction Model

In order to express (linearized) friction constraints at each contact, contact forces are expressed as linear combinations of the edges that define contact friction pyramids, and restricted to lie inside the pyramid:

\[
D\beta = c \quad (3)
\]
\[
F\beta \leq 0 \quad (4)
\]

Details on the construction of the linear force expression matrix \(D\) and the friction matrix \(F\) can be found in the work of Miller and Christensen [19]. We note that, while the friction model is linear, frictional forces are not linearly related to virtual object movements (in contrast to the linear compliance model discussed in the previous section). In fact, friction forces are not related to virtual object motion at all. Instead, we propose an algorithm that searches for equilibrium contact forces everywhere inside the friction cones. In this study we use the Point Contact With Friction model, approximating Coulomb friction between rigid bodies. However, the formulation is general enough for other linearized models, such as the Soft Finger Contact [20].

B. Compliance Model

Assuming virtual springs placed at the contacts, the normal force at a contact \(i\) is determined by the virtual relative motion between the object and the robot hand at that contact in the direction of the contact normal. This can be expressed in terms of virtual object displacements \(x\) and virtual joint movements.
\( \mathbf{q} \). As it is only the relative motion at the contact in the direction of the contact normal we are interested in, a subscript \( n \) denotes a normal component of a contact force or relative motion. For simplicity, we choose unity stiffness for the virtual contact spring (\( k = 1 \)).

\[
c_{i,n} = k(G^T x - Jq)_{i,n}
\]

(5)

However, a contact may only apply positive force (it may only push, not pull). Hence, if the virtual object and joint movements are such that the virtual spring is extended from its rest position, the contact force must be zero. Thus, the virtual springs operate in two regimes:

1) The object and hand are moving such as to compress the virtual spring at the contact. The contact force is positive (\( c_{i,n} \geq 0 \)) and the equality in (5) holds.

2) The object and hand are moving away from each other at the contact. The contact force is zero (\( c_{i,n} = 0 \)) and [5] no longer holds: \( c_{i,n} - k(G^T x - Jq)_{i,n} \geq 0 \).

We can devise the following set of equations, which capture this behavior.

\[
c_{i,n} \geq 0
\]

(6)

\[
c_{i,n} - k(G^T x - Jq)_{i,n} \geq 0
\]

(7)

\[
c_{i,n} \cdot (c_{i,n} - k(G^T x - Jq)_{i,n}) = 0
\]

(8)

This is a non-convex quadratic constraint and as such not readily solvable. (Note that if re-posed as a Linear Complementarity Problem it produces a non-positive-definite matrix relating the vectors of unknowns). However, the same problem can be posed as a set of linear inequality constraints instead, which can be solved by a mixed-integer programming solver.

\[
c_{i,n} \geq 0
\]

(9)

\[
c_{i,n} \leq k_1 \cdot y_i
\]

(10)

\[
c_{i,n} - k(G^T x - Jq)_{i,n} \geq 0
\]

(11)

\[
c_{i,n} - k(G^T x - Jq)_{i,n} \leq k_2 \cdot (1 - y_i)
\]

(12)

Each contact \( i \) is assigned a binary variable \( y_i \in \{0, 1\} \) determining the regime, in which the virtual spring operates and hence if the normal force at that contact is equal to the force in the virtual spring (for positive spring forces) or zero. Constants \( k_1 \) and \( k_2 \) are virtual limits that have to be carefully chosen such that the magnitude of the expressions on the left-hand side never exceed them. However, they should not be chosen too large or the problem may become numerically ill-conditioned.

\section*{C. Joint Model}

The mechanics of the hand place constraints on the virtual motion of the joints. To clarify this point, consider the equilibrium joint torque \( \tau \), at which the system settles, and which may differ from the commanded joint torque \( \tau_c \). At any joint \( j \) the torque may exceed the commanded value, but only passively. In non-backdrivable hands this means the torque at a joint may only exceed its commanded level if the gearing between the motor and the joint is absorbing the additional torque. In consequence, a joint at which the torque exceeds the commanded torque may not display virtual motion. Similarly to the behavior of the virtual springs at the contacts, the relationship between joint torque and virtual joint motion exhibits two distinct regimes. Therefore - defining joint motion which closes the hand on the object as positive - this constraint can be expressed as another linear complementarity.

\[
q_j \geq 0
\]

(13)

\[
\tau_j - \tau_{c,j} \geq 0
\]

(14)

\[
q_j \cdot (\tau_j - \tau_{c,j}) = 0
\]

(15)

Similarly to the linear complementarity describing normal contact forces this constraint can be posed as a set of linear inequalities.

\[
q_j \geq 0
\]

(16)

\[
q_j \leq k_3 \cdot z_j
\]

(17)

\[
\tau_j - \tau_{c,j} \geq 0
\]

(18)

\[
\tau_j - \tau_{c,j} \leq k_4 \cdot (1 - z_j)
\]

(19)

Each joint is assigned a binary variable \( z_j \) that determines if the joint may move or is being passively loaded and hence stationary. Similarly to \( k_1 \) and \( k_2 \) the constants \( k_3 \) and \( k_4 \) are virtual limits and should be chosen with the same considerations in mind.

\section*{D. Underactuation Model}

The above joint model assumes a direct drive robotic hand kinematic, where an actuator command equates to an individual joint torque command. However, our framework is well suited to model hand kinematics, where the joint torques can be expressed as linear combinations of actuator forces. This includes underactuated designs with fewer actuators than degrees of freedom such as, for example, a tendon driven hand with fewer tendons than joints. This implies, that a tendon - and hence an actuator - can directly apply torques to multiple joints by means of a mechanical transmission. We thus define matrix \( \mathbf{R} \), which maps from forces at the actuators \( \mathbf{f} \) to joint torques \( \mathbf{\tau} \). Note, that its transpose maps from joint motion to the motion of the mechanical force transmission at the actuator.

\[
R \mathbf{f} = \mathbf{\tau}
\]

(20)

Again, we assume the actuators to be non backdrivable and hence at an actuator \( l \) we may see a force \( f_l \) that exceeds the commanded value \( f_{l,c} \) - and again, this can only occur passively. This means, that an actuator force can only exceed the commanded value, if the actuator is being backdriven, and hence the mechanical transmission does not exhibit any virtual motion. Defining transmission motion \( \mathbf{R}^T \mathbf{q} \) that closes the hand around the object as positive, we can again express this constraint was a linear complementarity.

\[
(R^T \mathbf{q})_l \geq 0
\]

(21)

\[
f_l - f_{c,l} \geq 0
\]

(22)

\[
(R^T \mathbf{q})_l \cdot (f_l - f_{c,l}) = 0
\]

(23)
ties this constraint can be posed as a set of linear inequalities.

Related work introduces joint model and will reduce as such if the actuators may move or is being loaded, we assign a binary variable to each actuator. This variable determines if the actuator - and hence the connected transmission - move. The constants $k_5$ and $k_6$ are virtual limits just as constants $k_0$ through $k_4$.

This actuation model is a generalization of the previously introduced joint model and will reduce as such if the actuators control individual joint torques directly.

V. Solution Method

The computational price we pay for considering these non-linear effects is that virtual object movement is not directly determined by the compliance-weighted inverse of the grasp map matrix; rather, it becomes part of the complex mixed-integer problem we are trying to solve. In general, if a solution exists, there is an infinite number of solutions satisfying the constraints. The introduction of an optimization objective leads to a single solution. A physically well-motivated choice of constraints. The introduction of an optimization objective leads to a single solution. A physically well-motivated choice of constraints. The introduction of an optimization objective leads to a single solution. A physically well-motivated choice of constraints. The introduction of an optimization objective leads to a single solution.

As an exact treatment of the underlying physical laws in this formulation is non convex, we have devised an approximate iterative scheme that aims to eliminate unnatural object motion. We constrain the object movement such that motion is only allowed in the direction of the unbalanced wrench acting on the object: $x = sw$, $s \in \mathbb{R}_{\geq 0}$. We remove the equilibrium constraint (1) and replace the objective of the optimization formulation such as to minimize the net resultant wrench $r = w + G^T \beta$ (the net sum of the applied wrench and contact forces) acting on the object. However, under this new constraint on virtual object motion, the solver will generally not be able to completely balance out the wrench and achieve equilibrium in a single step. Even after the optimization, some level of unbalanced wrench may remain. To eliminate it, we call the same optimization procedure in an iterative fashion, where, at each step we allow additional object movement in the direction of the unbalanced wrench $r$ remaining from the previous call. For stability of the numerical scheme, we limit the step size by a parameter $\gamma$.

$$x_{\text{next}} = x + sr, \quad 0 \leq s \leq \gamma \quad (28)$$

After each iteration, we check for convergence by comparing the incremental improvement to a threshold $\epsilon$. If the objective has converged to a sufficiently small net wrench (we chose $10^{-8}$ N), we deem the grasp to be stable; otherwise, if the objective converges to a larger value, we deem the grasp unstable. Thus, we formulate a movement constrained passive response problem as outlined in Algorithm 2 to be solved iteratively as outlined in Algorithm 3.

The computation time of this process is directly related to the number of iterations required until convergence. A single iteration takes of the order of $10^{-2}$ to $10^{-1}$ seconds, depending on the complexity of the problem. Most problems converge within less than 50 iterations. All computations were performed on a commodity computer with a 2.80GHz Intel Core i7 processor.

We use this procedure to answer the question if a grasp, in which the joints are preloaded with a certain commanded torque can resist a given external wrench. In much of the analysis introduced in the next section we are interested in how the maximum external wrench, which a grasp can withstand depends on the direction of application. We approximate the maximum resistible wrench along a single direction using a binary search limited to 20 steps, which requires computation time of the order of tens of seconds. In general, investigating the magnitude of the maximum resistible wrench in every direction involves sampling wrenches in 6 dimensional space. Within our current framework this is prohibitively time consuming and hence we limit ourselves to sampling directions in 2 dimensions and then using the aforementioned binary search to find the maximum resistible wrench along those directions.
Fig. 2. Illustration of the shortcomings of directly solving the PRP problem defined above. A force normal to the closing plane of the fingers (illustrated by the green arrow) is applied to the object at its center of mass. The translational and rotational components of the resulting object movement are shown in violet. The PRP algorithm finds a way to wedge the object between the fingers by rotating the object in a way that would not occur in practice as it violates conservation of energy. This enables the solver to find ways to resist arbitrary wrenches. The iterative approach yields the natural finite resistance.

Algorithm 2

**Input:** $\tau_c$ or $f_c$ - commanded joint torques/ actuator forces, $w$ - applied wrench, $x$ - previous object displacement, $r$ - previous net wrench

**Output:** $c$ - contact forces, $x_{next}$ - next step object displacement, $r_{next}$ - next step net wrench

**procedure** MOVEMENT CONSTRAINED PRP ($\tau_c, w, x, r$)

- **minimize:** $r_{next}^T r_{next}$  

- **subject to:**
  - Eq. (2)  
  - Eqs. (3) & (1)  
  - Eqs. (9) - (12)  
  - Eqs. (16) - (19) or (20) & (24) - (27)  
  - Eq. (28)

**return** $c, x_{next}, r_{next}$

end procedure

Algorithm 3

**Input:** $\tau_c$ or $f_c$ - commanded joint torques/ actuator forces, $w$ - applied wrench

**Output:** $c$ - contact forces, $r$ - net resultant

**procedure** ITERATIVE PASSIVE RESPONSE PROBLEM ($\tau_c, w$)

$x = 0$

$r = w$

**loop**

$(c, x_{next}, r_{next}) = $ Movement Constrained PRP ($\tau_c, w, x, r$)

- if $\text{norm}(r - r_{next}) < \epsilon$ then
  - break
- end if

$x = x_{next}$

$r = r_{next}$

**end loop**

**return** $c, r$

end procedure

VI. Analysis and Results

We illustrate the application of our method on three example grasps using the Barrett hand and an underactuated gripper. We show force data collected by replicating the grasp on a real hand and testing resistance to external disturbances. We model the Barrett hand as having all non-backdrivable joints. Our qualitative experience indicates that the finger flexion joints never backdrive, while the spread angle joint backdrives under high load. For simplicity we also do not use the breakaway feature of the hand; our real instance of the hand also does
not exhibit this feature. We model the joints as rigidly coupled for motion, and assume that all the torque supplied by each finger motor is applied to the proximal joint.

To measure the maximum force that a grasp can resist in a certain direction, we manually apply a load to the grasped object using a Spectra wire in series with a load cell (Futek, FSH00097). In order to apply a pure force, the wire is connected such that the load direction goes through the center of mass of the object. We increase the load until the object starts moving, and take the largest magnitude recorded by the load cell as the largest magnitude of the disturbance the grasp can resist in the given direction.

1) Case 1: We consider first the case illustrated in Fig. 4. This grasp can be treated as a 2D problem, considering only forces in the grasp plane, simple enough to be studied analytically, but still complex enough to give rise to interesting interplay between the joints and contacts. Since our simulation and analysis framework is built for 3D problems, we can also study out-of-plane forces and in-plane moments.

Consider first the problem of resisting an external force applied to the object CoM and oriented along the Y axis. This simple case already illustrates the difference between active and passive resistance. Resistance against a force oriented along positive Y requires active torque applied at the joints in order to load the contacts and generate friction. The force can be resisted only up to the limit provided by the preload, along with the friction coefficient. If the force is applied along negative Y, resistance happens passively, provided through the contacts on the proximal link. Furthermore, this resistant force does not require any kind of preload, and is infinite (up to the breaking limit of the hand mechanism, which does not fall within our scope here).

For an external force applied along the X axis, the problem is symmetric between the positive and negative directions. Again, the grasp can provide passive resistance, through a combination of forces on the proximal and distal links. For the more general case of forces applied in the XY plane, we again see a combination of active and passive resistance effects. Intuitively, any force with a negative Y component will be fully resisted passively. However, forces with a positive Y component and non-zero X component can require both active and passive responses. Fig. 3 shows the forces that can be resisted in the XY plane, both predicted by our framework and observed by experiment. Note that our formulation predicts the distinction between finite and infinite resistance directions, in contrast to the results obtained using the linear compliance model.

For both real and predicted data, we normalize the force values by dividing with the magnitude of the force obtained along the positive direction of the Y axis (note thus that both predicted and experimental lines cross the Y axis at y=1.0). The plots should therefore be used to compare trends rather than absolute values. We use this normalization to account for the fact that the absolute torque levels that the hand can produce, and which are needed by our formulation in order to predict absolute force levels, can only be estimated and no accurate data is available from the manufacturer. The difficulty in obtaining accurate assessments of generated motor torque generally limits the assessments we can make based on absolute force values. However, if one knows the real
TABLE I
PREDICTED AND MEASURED RESISTANCE TO FORCE APPLIED ALONG THE POSITIVE X AXIS IN THE GRASP PROBLEM IN Fig. 6. Each row shows the results obtained if the preload is applied exclusively by finger 1 or finger 2 respectively. Experimental measurements were repeated 5 times for finger 1 (to account for the higher variance) and 3 times for finger 2. Predicted values are non-dimensional, and hence the ratio between the two preload cases is shown.

|       | Measured resistance | Predicted |   |
|-------|---------------------|------------|---|
|       | Values(N)           | Avg.(N)    | St. Dev. | Ratio | Value | Ratio |
| F1 load | 12.2, 10.8, 7.5, 7.9, 9.3 | 9.6 | 1.9 | 2.23 | 1.98 | 2.48 |
| F2 load | 3.7, 4.1, 5.0 | 4.3 | 0.7 |     | 0.80 |      |

Fig. 6. Top and side views for grasp example 2 also indicating finger labels. Note that the spread angle degree of freedom of the Barrett hand changes the angle between finger 1 and finger 2; the thumb is only actuated in the flexion direction.

Moving outside of the grasp plane, Fig. 5 shows predicted and measured resistance to forces in the XZ plane. Again, we notice that some forces can be resisted up to arbitrary magnitudes thanks to passive effects, while others are limited by the actively applied preload.

2) Case 2: One advantage of studying the effect of applied joint torques on grasp stability is that it allows us to observe differences between different ways of preloading the same grasp. For example, in the case of the Barrett hand, choosing at which finger(s) to apply preload torque can change the outcome of the grasp, even though there is no change in the distribution of contacts. We illustrate this approach on the case shown in Fig. 6. Using our framework we can compute regions of resistible wrenches for two different preloads (see Fig. 7).

We compare the ability of the grasp to resist a disturbance applied along the X axis in the positive direction if either finger 1 or finger 2 apply a preload torque to the grasp. Our formulation predicts that by preloading finger 1 the grasp can resist a disturbance that is 2.48 times higher in magnitude than if preloading finger 2. Experimental data (detailed in Table I) indicates a ratio of 2.23. The variance in measurements again illustrates the difficulty of verifying such simulation results with experimental data. Nevertheless, experiments confirmed that preloading finger 1 is significantly better for this case.

This result can be explained by the fact that, somewhat counter-intuitively, preloading finger 1 leads to larger contact forces than preloading finger 2, even if the same torque is applied by each motor. Due to the orientation of finger 1, the contact force on finger 1 has a smaller moment arm around the finger flexion axes than is the case for finger 2. Thus, if the same flexion torque is applied in turn at each finger, the contact forces created by finger 1 will be higher. In turn, due to passive reaction, this will lead to higher contact forces on finger 2, even if finger 1 is the one being actively loaded. Finally, these results hold if the spread degree of freedom is rigid and does not backdrive; in fact, preloading finger 1 leads to a much larger passive (reaction) torque on the spread degree of freedom than when preloading finger 2.

Referring to Fig. 7 we note that actively preloading finger 1 results in greater resistance only in some directions. There is much structure to the prediction made by our framework that could be exploited to make better decisions when preloading a grasp with some knowledge of the expected external wrenches.

3) Case 3: We now apply our framework to a grasp with a two-fingered, tendon-driven underactuated gripper (see Fig. 8). The gripper has four degrees of freedom, but only two actuators driving a proximal and a distal tendon. The
proximal tendon has a moment arm of 5mm around the proximal joints. The distal tendon has moment arms of 1.6mm and 5mm around the proximal and distal joints respectively. The actuators are non backdrivable and hence the tendons not only transmit actuation forces, they also provide kinematic constraints to the motion of the gripper’s links.

As the tendons split and lead into both fingers, we assume that they are connected to the actuator by a differential. This introduces compliance to the grasp: if one proximal joint closes by a certain amount, this will allow the other proximal joint to open by a corresponding amount. This compliance means that the underactuated grasp in Fig. 8 will behave fundamentally different than the very similar grasp in Fig. 4. To see this, consider the region of resistible wrenches in the XY plane (Figs. 3 & 9). In both cases the object is gripped by two opposing fingers, however, while in the case of the Barrett hand the grasp could withstand forces pushing the object directly against a finger, our framework predicts this to be impossible in the case of the underactuated hand due to the compliance.

We used our framework to apply two different preloads and analyzed the resistance of the resulting grasp to an externally applied wrench. As our real underactuated hand does not contain a differential between the left and the right halves of the tendons, we can only compare grasps to the simulation if the applied wrench does not cause any asymmetry in the grasp. Hence, we chose to apply a torque around the X axis. The two preload cases we considered are an active load on the proximal/ distal tendon only, leaving the distal/ proximal tendon to be loaded passively. For equal actuator force, our framework predicts, that a preload created by actively loading the distal tendon leads to almost twice as much resistance to torques applied in the direction of the Y axis than actively loading the proximal tendon only.

Experimental verification of this prediction proved to be difficult, as results had high variance and application of a pure torque to the object along an axis that penetrates the distal links of the gripper was complicated. However, we mounted the gripper such that the grasp plane was in the horizontal and placed weights on the top end of the box object. We found the resistance to these applied wrenches indeed to be much higher when actively loading the distal tendon as opposed to the proximal.

VII. DISCUSSION

1) Limitations: In a subset of cases, the solver reports maximum resistible wrenches with very different magnitude relative to neighboring states. For example, in the grasp Case 2 from the previous section (Fig. 6), when computing resistance to disturbances sampled from the XY plane (Fig. 7), we obtain two outliers for each preload case that do not follow the trend of the surrounding points. These outliers are quite rare and tend to fall within the area deemed to contain resistible wrenches (shaded). These effects will require further investigation.

Our iterative approach allows us to constrain virtual object movement to the successive directions of unbalanced wrenches. However, such an iterative approach is not guaranteed to converge, or to converge to the physically meaningful state of the system. We would like to explore other formulations and iterative schemes to better approximate the non linear and non convex physical laws governing the behavior of the grasp.

Our current real underactuated hand only allows experimental validation of a subset of our analysis results. We are working on designing a hand that we can use to further validate our framework and study the effects of underactuation on grasp stability. For instance, our framework predicts that wrench resistance is highly dependent on the torque ratios at the joints due to the kinematics of the force transmission. We would like to experiment with a variety of underactuated hands, with varying kinematic and actuation models, to investigate these effects.

Furthermore, we would like to analyze the effect of uncertainties (e.g. in exact contact location) on our model. We believe exploring the sensitivity of the model to such uncertainties may yield many valuable insights.
2) Alternative Approaches: As was described in the Problem Statement, a simpler alternative is to disregard non linear effects with respect to virtual object movement, i.e. assume that the joints are fixed (thus joint torque can both increase and decrease passively), and that friction forces also behave in spring-like fashion. The price for this simplicity is, that the results may not be physically sound.

At the other end of the spectrum, our iterative approach allowing successive virtual object movements in the direction of the net resultant wrench shares some of the features of a typical time-stepping dynamics engine. One could therefore forgo the quasi-static nature of our approach, assume that unbalanced wrenches produce object acceleration or impulses, and perform time integration to obtain new object poses. This approach can have additional advantages: even an unstable grasp can eventually transform into a stable one, as the object settles in the hand; a fully dynamic simulation can capture such effects. However, in highly constrained cases, such as grasps at or near equilibrium, any inaccuracy can lead to the violation of interpenetration or joint constraints, in turn requiring corrective penalty terms which add energy to the system. Our quasi-static approach only attempts to determine if an equilibrium can exist in the given state, and thus only reasons about virtual object movements, without dynamic effects.

VIII. CONCLUSIONS

In this paper, we have introduced an algorithm that aims to answer what we believe to be not only a meaningful theoretical question, but also one with important practical applications: once a given joint preload has been achieved, can a grasp resist a given wrench passively, i.e. without any change in commanded joint torques? In the inner loop of a binary search, the same algorithm allows us to determine the largest magnitude that can be resisted for a disturbance along a given direction.

In the examples above we show how the actively set joint preload combines with passive effects to provide resistance to external wrenches; our algorithm captures these effects. Furthermore, we can also compute how preloads set for some of the hand joints can cause the other joints to load as well, and the combined effects can exceed the intended or commanded torque levels. We can also study what subset of the joints is preferable to load with the purpose of resisting specific disturbances. Our grasp model captures well the effect compliance and underactuation have on grasp stability.

Our directional goal is to enable practitioners to choose grasps for a dexterous robotic hand knowing that all disturbances they expect to encounter will be resisted without further changes in the commands sent to the motors. Such a method would have wide applicability, to hands that are not equipped with tactile or proprioceptive sensors (and thus unable to sense external disturbances) and can not accurately control joint torques, but are still effective thanks to passive resistance effects.

In its current form, the algorithm introduced here can answer “point queries”, for specific disturbances or disturbance directions. However, its computational demands do not allow a large number of such queries to be answered if a grasp is to be planned at human-like speeds; furthermore, the high dimensionality of the complete space of possible external wrenches generally prevents sampling approaches. GWS-based approaches efficiently compute a global measure of wrenches that can be resisted assuming perfect information and controllability of contact forces. We believe passive resistance has high practical importance for the types of hands mentioned above, but no method is currently available to efficiently distill passive resistance abilities into a single, global assessment of the grasp. We will continue to explore this problem in future work.

REFERENCES

[1] C. Ferrari and I. Canny, “Planning optimal grasps,” in IEEE International Conference on Robotics and Automation, 1992, pp. 2290–2295.
[2] D. Prattichizzo and J. Trinkle, “Grasping,” Springer Handbook of Robotics, 2008.
[3] J. Salisbury and B. Roth, “Kinematic and force analysis of articulated mechanical hands,” ASME Journal of Mechanisms, Transmissions, and Automation in Design, vol. 105, pp. 35–41, 1983.
[4] M. Aicardi, G. Casalino, and G. Cannata, “Contact force canonical decomposition and the role of internal forces in robust grasp planning problems,” International Journal of Robotics Research, vol. 15, no. 4, pp. 351–364, 1996.
[5] J. Kerr and B. Roth, “Analysis of multifingered hands,” International Journal of Robotics Research, vol. 4, no. 4, pp. 3–17, 1986.
[6] T. Yoshikawa and K. Nagai, “Manipulating and grasping forces in manipulation by multifingered robot hands,” IEEE Transactions on Robotics and Automation, vol. 7, no. 1, pp. 67–77, 1991.
[7] A. Bicchi, “Force distribution in multiple whole-limb manipulation,” in Robotics and Automation, 1993. Proceedings., 1993 IEEE International Conference on. IEEE, 1993, pp. 196–201.
[8] ——, “On the problem of decomposing grasp and manipulation forces in multiple whole-limb manipulation,” Robotics and Autonomous Systems, vol. 13, no. 2, pp. 127–147, 1994.
[9] ——, “On the closure properties of robotic grasping.” The International Journal of Robotics Research, vol. 14, no. 4, pp. 319–334, 1995.
[10] D. Prattichizzo, J. K. Salisbury, and A. Bicchi, contact and grasp robustness measures: Analysis and experiments. Berlin, Heidelberg: Springer Berlin Heidelberg, 1997, pp. 83–90. [Online]. Available: http://dx.doi.org/10.1007/978-3-540-48865-7
[11] D. Prattichizzo, M. Malvezzi, M. Gabiccini, and A. Bicchi, “On motion and force controllability of precision grasps with hands actuated by soft synergies,” IEEE Transactions on Robotics, vol. 29, no. 6, 2013.
[12] T. Yoshikawa, “Passive and active closures by constraining mechanisms,” in IEEE International Conference on Robotics and Automation, vol. 2, 1996, pp. 1477–1484.
[13] C. Melchiorri, “Multiple whole-limb manipulation: An analysis in the force domain,” Robotics and Autonomous Systems, vol. 20, no. 1, pp. 15–38, 1997.
[14] J. Burdick and E. Rimon, “Wrench resistant multi-finger hand mechanisms,” in International Conference on Robotics and Automation, 2016.
[15] A. Cloutier and J. Yang, “Grasping force optimization approaches for anthropomorphic hands,” ASME Journal of Mechanisms and Robotics, vol. 10, 2018.
[16] M. R. Cutkosky and I. Kao, “Computing and controlling the compliance of a robotic hand,” IEEE Transactions on Robotics and Automation, vol. 5, no. 2, 1989.
[17] M. Malvezzi and D. Prattichizzo, “Evaluation of grasp stiffness in underactuated compliant limbs,” in IEEE International Conference on Robotics and Automation, 2013, pp. 2074–2079.
[18] H. Hanafusa and I. Asada, “Stable prehension by a robot hand with elastic fingers,” in Proc. of the 7th ISIR, Tokyo, 1977.
[19] A. Miller and H. Christensen, “Implementation of multi-rigid-body dynamics within a robotic grasping simulator,” in IEEE Intl. Conference on Robotics and Automation, 2003, pp. 2262–2268.
[20] M. Ciocarlie, C. Lackner, and P. Allen, “Soft finger model with adaptive contact geometry for grasping and manipulation tasks,” in Joint Eurohaptics Conference and IEEE Symp. on Haptic Interfaces, 2007, pp. 219–224.
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