HARD DIFFRACTIVE SCATTERING *

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Abstract

I discuss hard diffractive scattering in the framework of perturbative QCD and Regge-parametrization.

1. Diffractive deep inelastic scattering

1.1. Kinematics

Measurements of deep inelastic diffractive scattering at HERA provided us with some of the most interesting experimental results which allow one to test new theoretical ideas on diffractive processes in an unexplored regime. The data triggered much theoretical work. The concept of diffractive parton number densities (or differential fracture functions) with standard DGLAP evolution appears to be the key ingredient of the QCD description. It was conjectured that these new types of parton number densities factorize similarly to the parton number densities of non-diffractive

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Deep inelastic scattering and satisfy the same Altarelli-Parisi evolution equation. The Ingelman-Schlein model\textsuperscript{6} emerges subsequently by assuming Regge-behaviour\textsuperscript{7} for the $x_P$ and $t$ dependence.

Diffractive deep inelastic scattering is defined by restricting our studies into final states when the hadrons can be classified into a high mass ($M_X$) and a low mass ($M_Y$) group with large rapidity gap between them. Denoting the four-momenta of virtual photon, proton and electron with $q$, $p$ and $k$ and the four-momenta of the high mass and low mass hadronic system with $p_X$ and $p_Y$, respectively, the measured differential cross-section can be written in terms of the diffractive structure function $F^D_2$ as

$$\frac{d\sigma}{dx dQ^2 dx_P d|t|} = \frac{4\pi\alpha^2}{xQ^4} \left(1 - y + \frac{y^2}{2}\right) \frac{dF^D_2}{dx_P d|t|}(x, Q^2, x_P)$$ (1)

where

\[
Q^2 = -q^2, \quad y = \frac{q.p}{k.p}, \quad x = \frac{Q^2}{2p.q}, \quad x_P = \frac{q.p - q.p_y}{q.p} \approx \frac{Q^2 + M_X^2}{Q^2 + W^2},
\]

\[
\beta = \frac{x}{x_P} = \frac{Q^2}{2q.p - 2q.p_y} \approx \frac{Q^2}{Q^2 + M_X^2}, \quad W^2 = (q + p)^2, \quad t = (p - p_Y)^2
\] (2)

Requiring small $M_Y$ and $x_P$ the rapidity gap is inevitable. For example the H1 collaboration required $M_Y < 1.6$ GeV$^2$ and $x_P < 0.05$. In the H1 measurement, the momentum transfer from the proton to the $Y$-system $t = (p - p_Y)^2$ is not measured but is restricted to the small interval $|t_{\text{min}}| < |t| < 1.0$ GeV$^2$. The ZEUS collaboration collected also data when the $Y$ system was identified with a fast forward proton with a measured value of $t$. The longitudinal cross-section was assumed to be zero.

\subsection*{1.2. Diffractive parton number densities}

The factorization theorem of deep inelastic scattering allows the decomposition

$$F_2(x, Q^2) = \sum_a \int dx' f_{a/p}(x', \mu) \hat{F}_{2a}(x/x', Q^2, \mu^2)$$ (3)

It is natural to ask whether similar decomposition remain valid also for diffractive deep inelastic scattering giving

$$\frac{dF^D_2}{dx_P d|t|}(x, Q^2, x_P, t) = \sum_a \int dx' \frac{dP_{a/p}}{dP_{a|t}|} (x', \mu, x_P, t) \hat{F}_{2a}(x/x', Q^2, \mu^2)$$ (4)

where in both equations $\hat{F}_2$ denotes the finite hard scattering contribution. The dependence on $t$ and $x_P$ is completely absorbed in the diffractive parton number densities. Berera and Soper\textsuperscript{5} pointed out that the operator definition of the parton number densities can be generalized for diffractive processes. The quark parton
number density for example defined as

\[ f_{q/p}(x, \mu) = \frac{1}{4\pi} \sum_{s,X} \int dy e^{i x p \cdot y} \langle p, s | \bar{q}(0, y^-) \rangle |X > \gamma^+ < X | \bar{q}(0, 0, 0) | p, s > \]  

(5)

is modified in the case of diffractive densities as

\[
\frac{df_{q/p}}{dx P d|t|}(\beta, \mu, x_P, t) = \frac{1}{4\pi} \sum_{s,s',X} \int dy e^{i \beta p \cdot y} \langle p, s | \bar{q}(0, y^-, 0) \rangle |p', s' ; X > \gamma^+ < p' , s' ; X | \bar{q}(0, 0, 0) | p, s > \]

(6)

where \(X\) indicates all possible final states with a proton of momentum \(p'\) and polarization \(s'\) and \(\bar{q}\) is the color singlet quark operator given in terms of quark and gluon fields as

\[
\bar{q}(0, y^-, 0) = \mathcal{P} e^{ig} \int_y^\infty dx A_+^{(0,x^-,0)} T_c q(0, 0, 0) \]

(7)

where \(T_c\) is the color matrix and \(\mathcal{P}\) demands path ordering in color space. In fig.1 the factorization in DIS is shown graphically. The crucial observation is that the ultraviolet structure of the bilocal operators defining the non-diffractive and diffractive parton number densities are the same therefore they fulfill the same renormalization group equation. As a result, the diffractive parton number densities at fixed values of \(x_P\) and \(t\) (assuming \(t\) is small) also have to fulfill the DGLAP evolution equation. The apparent difference in the \(Q^2\) behaviour of the DIS and diffractive DIS data must have its origin in the dramatically different initial parton distributions. Factorization theorem with evolution equation has been suggested by Trentadue and Veneziano in the context of deep inelastic scattering with one observed final state hadron allowing also target fragmentation region. In the QCD improved parton model collinear
singularities given by low $p_T$ gluons emitted into the target fragmentation region can not be absorbed into the standard fragmentation and structure functions and a new non–perturbative function, the so called fracture function, had to be introduced. In diffractive deep inelastic scattering the observed forward proton is in the target fragmentation region. Therefore the fracture function formalism applies. Berera and Soper pointed out that if the forward proton transverse momentum remains unintegrated and is required to be small the diffractive parton number densities can be identified with the fracture functions. With explicit calculation Graudenz has shown that the factorization theorem for fracture functions is fulfilled in next-to-leading order accuracy. It is generally believed that it remains valid for diffractive deep inelastic scattering to all orders in perturbation theory.

1.3. Regge factorization and parametrization

As we have seen above both the standard parton number densities and the diffractive parton number densities are defined as matrix element of bilocal operator between proton states and they are given by 'soft physics', they can not be calculated in perturbation theory. The parametrization of the initial distribution at some $Q_0^2$ scale, however, can be motivated by phenomenological models. For diffractive scattering Regge theory is the most successful framework. Assuming the dominance of the Pomeron trajectory one obtains

$$\frac{d f_{a/P}}{d x_P d |t|} (\beta, \mu, x_P, t) = \frac{|\gamma(t)|^2}{8\pi^2} x_P^{-2\alpha_P(t)} f_{a/P} (\beta, t; \mu)$$  \hspace{1cm} (8)

where $\alpha_P(t)$ is the trajectory function and $\gamma(t)$ is its coupling to the proton. From fits to hadronic cross-sections one obtains $\alpha_P \approx 1.08$. The function $f_{a/P} (\beta, t; \mu)$ is called the parton distribution of the Pomeron since formally the cross-section of diffractive deep inelastic scattering is given by folding the hard scattering cross-section with this function. This description was suggested by Ingelman and Schlein using the notion of 'parton constituents of the Pomeron'. The latter concept should be treated with care since the Pomeron can not be interpreted as a particle emitted by the proton before the parton distribution was probed. One of the most significant qualitative consequences of the assumption of the Pomeron parametrization is the factorization of the $x_P$ and $t$ dependence. The HERA diffractive structure function data, however, show a modest amount of factorization breaking. It could, however, be accommodated by invoking a sum over Regge trajectories, each with a different intercept and structure function:

$$F_2^D (\beta, Q^2; x_P, t) = \sum R F_R(t)x_P^{-2\alpha_R(t)+1} F_2^R (\beta, Q^2) ,$$  \hspace{1cm} (9)

which would yield an effective power of the $x_P$ dependence which depends on $\beta$ but is approximately independent of $Q^2$. Integrating over $t$ and small bins of $x_P$ what is measured is a linear combination of $F_2^R$ structure functions or, equivalently, the
parton distributions in an effective colour singlet target:

\[
\int dx_P dt \, F_2^D(\beta, Q^2; x_P, t) = \sum_{R} A_R \, F_2^R(\beta, Q^2) = \beta \sum_q e_q^2 \sum_{R} q_R(\beta, Q^2), \quad (10)
\]

where the coefficients \(A_R\) are independent of \(\beta\) and \(Q^2\). Since the DGLAP equations are linear in the parton distributions, the \(Q^2\) evolution of the integrated structure function \(F_2^D\) should also be calculable perturbatively. The \(x_P\) dependence was fitted by treating \(\alpha_P(0)\) as a free parameter and the H1 and ZEUS collaboration have found that its value is somewhat larger than the soft Pomeron values. The value of a recent fit obtained by H1 is \(\alpha_P \approx 1.20 \pm 0.04\). The values obtained by H1 and ZEUS are consistently between the values of the intercept of the soft and hard (BFKL) Pomeron.

Fig. 2. Starting parton distributions and their evolved values

Various fits for the parton distributions in the ‘Pomeron’ \(f_{q/P}(x, \mu^2)\) have been proposed, ranging from the two extremes of mainly gluons to mainly quarks ( ).
Already the early data favoured the hard gluon parametrization. From the recent H1 analysis of the 1994 data one can conclude that the parametrization which only assumes quarks at $Q^2 = 3 \text{ GeV}^2$ is excluded. Since the diffractive parton number densities do not have momentum sum rule the overall normalization of the quark singlet $\Sigma(x, \mu^2)$ and the gluon $g(x, \mu^2)$ densities have to also be fitted from the data. The normalization, however, will not change with $Q^2$ evolution. It is the product of $f_{q/P}$ and $f_P$ that appears in the expression for the structure function, therefore it is convenient to simply impose a momentum sum rule on the parton distributions and absorb an overall normalization $\mathcal{N}$ into $f_P$. In fig. 2 the initial parton distributions are shown at $\mu^2 = 4 \text{ GeV}^2$ for hard gluon (3) no-gluon (1) and soft gluon (2) fits.

There has been some warning that one can not accept the fitted gluon distribution if it is peaked at $x = 1$ since the higher twist contributions become important in the range $1 - x \mu^2 < 1 \text{ GeV}^2$. This criticism, however, should concern only the measured data points. If the data data points in this range are excluded from the fit the obtained densities can be meaningfully calculated using the DGLAP evolution equation also in this range. One can easily see that the densities in this range can not be directly related to measurable quantities without adding also higher twist terms.

Finally I note that a number of models have been investigated in the literature to calculate the diffractive deep inelastic structure function at some fixed scale. Interesting semi-classical description has been developed by Buchmueller and Hebbeker, consistent with the aligned jet parton picture. The Pomeron is soft and in leading order the intercepts is $\alpha(0) = 1.0$. Models with simple two gluon exchanges have also been considered.

2. Hard diffractive processes in hadron-hadron scattering

Assuming the validity of factorization theorem also for diffractive hard scattering processes using the DIS value of the densities the cross-section values in hadron-hadron collisions can be calculated by using the standard formula

$$\frac{d\Delta\sigma^{SD}}{dx_Pdt} = \sum_{a,b} \int f_{a/P}(x_a, \mu) \frac{df_{a/P}}{dx_Pdt}(x_b, \mu, x_P, t) \Delta\sigma(x_a x_b s, ..) \quad (11)$$

Here the label SD makes reference to single diffractive production. The generalization of this formula for double diffractive scattering is trivial. Data on single diffractive jet production has been obtained first by UA8. Its comparison with the theory was not clear because of high values of $t$. The validity of the factorization theorem, however, is debated. In particular it is expected that it may be strongly violated. The soft exchanges before the hard scattering do not cancel if hadrons are observed in the target fragmentation region because not all the final states containing the hard-scattered object (jet or $W$-boson or Higgs boson etc.) are summed over. Interaction between spectator partons can lead to extra particles to fill the rapidity gap. It has been suggested to correct the prediction of the naive application of the factorization
and the so called survival probability of the rapidity gap. The parton model is predicted to overestimate the cross-sections of hard diffractive scattering by an amount given by such a survival probability. To gain experimental information on this relevant question one may proceed by comparing the naive predictions with the data. The cleanest process to study would appear to be weak boson production at the Tevatron $p\bar{p}$ collider. The naive prediction was first calculated before the HERA data in Ref. neglecting possible effects of $Q^2$ evolution. It has been found that the single diffractive component of the total $W$ cross section could be as large as 20%. It is convenient to study integrated ‘single diffractive’ events by $x_p < 0.1$, integrating over all $t$ and to normalize the result to the total $W$-production cross-section. In practice, of course, the events are defined by rapidity gaps of a certain minimum size, and therefore the observed diffractive cross section must be corrected to the theoretical prediction based on, say, $x_p < 0.1$ using a Monte Carlo simulation. Using the various fits discussed above, it is straightforward to reevaluate the single diffractive $W$ cross section with $Q^2$ evolution and with updated values for the Pomeron flux factors. Two recent analysis have found values in the range

$$\frac{\Delta \sigma_{SD,th}^{W}}{\sigma_{W}} \approx 4\%–6\%.$$  \hfill (12)

The important point to note here is that the predictions coming from different fits are quite similar. The single diffractive $W$ cross section at the Tevatron samples the quarks at $\langle x_q/p \rangle \sim 0.4$. At low $Q^2$ the quark distributions at this $x$ are constrained by the HERA $F_2^D$ data to be roughly the same. As $Q^2$ increases the distributions diverge, reflecting the quantitatively different gluon contributions to the DGLAP evolution. However at $Q^2 \sim 10^4 \text{GeV}^2$, the relevant value for $W$ production, the difference between the quarks in the three models is still not very large, and the predictions for $\sigma_{SD}(W)$ are correspondingly similar since they are directly related to the HERA data. In recent measurements the CDF Collaboration has found a smaller ratio

$$\frac{\Delta \sigma_{W,th}^{SD}}{\sigma_{W}} \approx 1.55 \pm .55$$  \hfill (13)

The surviving probability of the factorized result is about $S \approx 20 – 30\%$. Similar analysis for inclusive jet production gave even smaller value $S \approx 10\%$. These results indicate that the use of the (invalid) factorization theorem very likely strongly overestimates the cross-section values of diffractively produced heavy quarks and Higgs bosons at LHC. It would be important to develop some theoretical model in which the survival probability could be estimated.

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