A non-ergodic spectral acceleration ground motion model for California developed with random vibration theory

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Abstract

A new approach for creating a non-ergodic pseudo-spectral acceleration (PSA) ground-motion model (GMM) is presented, which accounts for the magnitude dependence of the non-ergodic effects. In this approach, the average PSA scaling is controlled by an ergodic PSA GMM, and the non-ergodic effects are captured with non-ergodic PSA factors, which are the adjustment that needs to be applied to an ergodic PSA GMM to incorporate the non-ergodic effects. The non-ergodic PSA factors are based on the effective amplitude spectrum (EAS) non-ergodic effects and are converted to PSA through Random Vibration Theory (RVT). The advantage of this approach is that it better captures the non-ergodic source, path, and site effects through small-magnitude earthquakes. Due to the linear properties of the Fourier Transform, the EAS non-ergodic effects of the small events can be applied directly to the large magnitude events. This is not the case for PSA, as response spectra are controlled by a range of frequencies, making PSA non-ergodic effects dependent on the spectral shape, which in turn is magnitude-dependent. Two PSA non-ergodic GMMs are derived using the ASK14 (Abrahamson et al. in Earthq Spectra 30:1025–1055, 2014) and CY14 (Chiou and Youngs in Earthq Spectra 30:1117–1153, 2014) GMMs as backbone models, respectively. The non-ergodic EAS effects are estimated with the LAK21 (Lavrentiadis et al. in Bull Earthq Eng) GMM. The RVT calculations are performed with the V75 (Vanmarcke in ASCE Mech Eng Mech Division 98:425–446, 1972) peak factor model, the $D_{0.05-0.85}$ estimate of AS96 (Abrahamson and Silva in Appendix A: empirical ground motion models, description and validation of the stochastic ground motion model. Tech. rep., Brookhaven National Laboratory, New York) for the ground-motion duration, and BT15 (Boore and Thompson in Bull Seismol Soc Am 105:1029–1041, 2015) oscillator-duration model. The California subset of the NGAWest2 database (Ancheta et al. in Earthq Spectra 30:989–1005, 2014) is used to fit both models. The total aleatory standard deviation of each of the two non-ergodic PSA GMMs is approximately 25% smaller than the total aleatory standard deviation of the corresponding ergodic PSA GMMs. This reduction has a significant impact on hazard calculations at large return periods. In remote areas, far from stations and past events, the reduction of aleatory variability is accompanied by an increase in epistemic uncertainty.

Keywords Probabilistic seismic hazard analysis · Ground-motion model · Random vibration theory

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1 Introduction

Ground-motion models (GMMs) are used to estimate the distribution of a ground-motion intensity measure (IM) for a given earthquake scenario. The most common IM is pseudo-spectral acceleration (PSA), as it is a good estimator of seismic loading for a wide range of structures. PSA is defined as the absolute maximum response of a single-degree-of-freedom oscillator (SDOF) to an input ground motion. SDOFs are defined by their natural period ($T_0$) or natural frequency ($f_0 = 1/T_0$) and damping ($\zeta$); in GMMs, typically, $T_0$ ranges from 0.01 to 10 s and, $\zeta$ is equal to 5%. The response of the oscillator depends on the frequency content and timing (compactness of energy) of the ground motion. From the entire frequency content of the ground motion, the response of the oscillator mainly depends on the amplitudes of the frequencies near and below $f_0$. Therefore, at small $T_0$ (high $f_0$), the response of the oscillator depends on the entire frequency content of the ground motion (i.e., spectral shape) and not just a narrow frequency bin. This makes the coefficients of a PSA GMM at small $T_0$ magnitude dependent even for linear effects, as the shape of the acceleration response spectrum changes with magnitude. The peak of an acceleration response spectrum will be at 0.1 s for a magnitude ($M$) 3 event and at 0.3 s for a $M$ 7.5 event (Fig. 1); this means that at small magnitudes, the PGA scaling (e.g., $V_{S30}$ coefficient) will be consistent with the scaling of $T_0 = 0.1$ s, while at large magnitudes, the PGA scaling will be consistent with $T = 0.3$ s. This is also observed by Stafford et al. (2017), who showed that linear site amplification factors are magnitude and distance dependent. A detailed discussion of the differences between the scaling of FAS and PSA is given by Bora et al. (2016).

GMMs fall into two main categories: ergodic GMM and non-ergodic GMM. Ergodic GMMs assume that the statistical properties of a ground motion IM do not change in space (Anderson and Brune 1999), and therefore, earthquakes and recordings from all around the world can be merged into a single dataset to estimate the GMM coefficients. Models developed under this assumption tend to have stable median estimates but large aleatory variability. Some models developed with the ergodic approach are: the NGA West GMMs for California, e.g., Abrahamson et al. (2008), and the Douglas et al. (2014) GMM for Europe. Non-ergodic GMMs recognize that source, path, and site effects are systematically different in different parts of the world and account for these differences in model development.

![Fig. 1 Schematic of normalized response spectra for $M$ 3.0 and 7.5 earthquakes computed based on Abrahamson et al. (2014) ergodic ground-motion model](image-url)
Non-ergodic GMMs have smaller aleatory variability than ergodic GMMs, but in areas with sparse data, where the systematic effects are unknown, the reduced aleatory variability is accompanied by an increase in the epistemic uncertainty in the median ground motion. The use of non-ergodic GMMs in Probabilistic Seismic Hazard Analysis (PSHA) is very promising, as it improves the accuracy of the median and reduces the aleatory variability, which together can have a large impact on the seismic hazard at large return periods, in the range of 1000 years return periods, improving the accuracy of the site-specific hazard. A more in-depth discussion of ergodic and non-ergodic GMMs is provided in the accompanying paper Lavrentiadis et al. (2021).

Most PSA GMMs do not explicitly account for the magnitude dependence of the coefficients, such as the $V_{S30}$ scaling or distance scaling; instead, they often use a limited range of magnitudes where the magnitude dependence of the coefficients is not pronounced. For instance, the dataset that was used in the development of the NGA West1 GMMs had a limited set of magnitudes that ranged from $M_{4.5}$ to $M_{8}$ Power et al. (2008). The approach of using a smaller range of magnitudes works when developing an ergodic GMM, as there are enough moderate-to-large magnitude events globally to estimate the coefficients, but it can be problematic when developing a non-ergodic GMM as the available data may be insufficient to discern the regional differences in ground-motion scaling.

For the NGA West2 GMMs, the data set was extended to down to $M_{3}$ with the objective of setting the reference ergodic model that could be used to evaluate regional differences in the site, path, and source terms based on small magnitude data (Ancheta et al. 2014). The NGA West2 GMMs modified the magnitude scaling to capture the average effect of the magnitude dependence of the coefficients, but this does not accurately model the magnitude dependence of the site and path effects.

The estimation of the non-ergodic terms requires a large set of regional data. To achieve that, the datasets used in the development of non-ergodic GMMs need to have a wider range of magnitudes to include the more frequent small-to-moderate earthquakes. It is this expansion of the magnitude range that makes the magnitude dependence of the GMM coefficients a more significant issue in non-ergodic GMMs. One solution to this problem is, first, to develop a non-ergodic GMM for an IM whose scaling does not suffer from the magnitude dependence, as PSA does, and then for a scenario of interest, calculate the non-ergodic PSA based on the estimate of the previous non-ergodic IM.

The effective amplitude spectrum ($EAS$), defined in Goulet et al. (2018) and Kottke et al. (2021), is one such IM: the $EAS$ is a smoothed rotation-independent average power Fourier amplitude spectrum ($FAS$) of the two horizontal components of an acceleration time history. In $EAS$, the amplitude at each frequency is largely independent of the amplitudes of the adjacent frequencies making the coefficients of an $EAS$ GMM magnitude independent. Random vibration theory (RVT) provides a framework to calculate $PSA$ from $EAS$. It relies on extreme-value statistics to estimate the peak response of the oscillator directly in the Fourier domain; it does not require a phase-angle spectrum to first convert the ground motion in the time domain to compute the peak oscillator response. RVT has been used in the past to compute $PSA$ based on $FAS$ from seismological theory (Hanks and McGuire 1981; Boore 1983, 2003) Other studies, such as Boore and Joyner (1984), Liu and Pezeshk (1999) Bora et al. (2015) and, Boore and Thompson (2012), focused on semi-empirical adjustments to the RVT framework to correct for the assumptions not satisfied by ground motions, mainly the fact that acceleration time histories are not stationary signals. More recently, Kottke et al. (2021) provided recommendations for the peak factor selection for the eastern US.
In this study, we developed two non-ergodic PSA GMMs. The average PSA scaling is determined by backbone ergodic PSA GMMs. The non-ergodic effects are defined in terms of non-ergodic PSA factors, which are estimated by combining the Lavrentiadis et al. (2021) non-ergodic EAS GMM with RVT.

2 Ground-motion data

A subset of the NGAWest2 data-set (Ancheta et al. 2014) was used in this study. The selected subset contains the earthquake and stations that are located in California, western Nevada, and northern Mexico. Recordings that were flagged as questionable in Abrahamson et al. (2014) were removed from the regression subset. Figure 2 shows the spatial distribution of earthquakes and stations. Most of the stations are located in Los Angeles, Bay Area, and San Diego metropolitan areas, whereas the spatial density of the stations is lower in less populated areas, such as north-eastern California. The regression dataset contains 7520 records from 185 earthquakes recorded at 1410 stations. Figure 3 shows the magnitude-distance distribution of the data and the number of records per frequency. The magnitude of the earthquakes ranges from 3.1 to 7.3, and the distance of most records ranges from 10 to 200 km. The usable frequency range of the majority of EAS records spans from 0.4 and 20 Hz, while the minimum usable frequency of most PSA records is 0.5 Hz. Both the PSA and EAS minimum usable frequencies are based on the high-pass filter corner frequency used in the processing of the NGAWest2 records. The largest EAS usable frequency is based on the corner frequency of the low-pass filter used during processing. The PSA usable frequency range is not limited by the low-pass filter corner frequency.

Fig. 2 Spatial distribution for earthquakes and stations used in this study
3 Model development

3.1 Model overview

The Model Development Section is organized into three subsections. Subsection: Non-ergodic PSA factors presents the framework of the proposed model. Subsection: Constant Shift and Aleatory Model summarizes the constant adjustments to the backbone ergodic models and the aleatory variability of the proposed non-ergodic PSA models. Subsection: Modeling Procedure lists the steps for implementing the proposed non-ergodic ground motion models. Subsection: Random-Vibration Theory provides an overview of the Random Vibration Theory and presents the validation of the recommended RVT procedure.

3.2 Non-ergodic PSA factors

The non-ergodic effects of the proposed PSA GMM are expressed in terms of a non-ergodic PSA factor ($F_{\text{nerg PSA}}$); that is, the difference of the logs the non-ergodic PSA estimate for a scenario of interest over the ergodic PSA estimate for the same scenario (Eq. 1). The non-ergodic PSA values are calculated with RVT and the Lavrentiadis et al. (2021) non-ergodic EAS GMM (LAK21), while the ergodic PSA values are calculated with RVT and the Bayless and Abrahamson (2019) ergodic EAS GMM (BA18). The scenarios of interest are defined in terms of magnitude ($M$), closest-rupture distance ($R_{\text{rup}}$), the time-average shear-wave velocity at the top 30 m ($V_{S30}$), etc., which are input parameters to both the ergodic and non-ergodic EAS GMMs, but also the earthquake and site coordinates, $t_E$ and $t_S$, which control the source, path and site non-ergodic effects in LAK21. The LAK21 non-ergodic EAS GMM uses BA18 as a backbone model for the average scaling; thus, the ratio of the spectral accelerations computed by these models ($F_{\text{nerg PSA}}$) is only affected by the total non-ergodic effects. In this approach, there are no separate terms for the earthquake, path, and site non-ergodic effects. The spectral dependence of the non-ergodic effects is captured.

Fig. 3 Selected data from the NGAWest2 database. a Magnitude–Distance distribution, b number of PSA and EAS recordings per frequency used in the regression analysis.
by the RVT calculation, which considers the entire frequency content of the underlining EAS.

\[
F_{\text{nerg}}(T_0, M, R_{\text{rup}}, V_{S30}, t_E, t_S, \ldots) = \ln \left( \frac{\text{PSA}_{\text{RVT}}[\text{IR}(T_0) \text{EAS}_{\text{LAK21}}(M, R_{\text{rup}}, V_{S30}, t_E, t_S, \ldots)]}{\text{PSA}_{\text{RVT}}[\text{IR}(T_0) \text{EAS}_{\text{BA18}}(M, R_{\text{rup}}, V_{S30}, \ldots)]} \right)
\]

The proposed non-ergodic PSA GMM is developed by coupling the aforementioned non-ergodic factors with an existing ergodic PSA GMM:

\[
y_{\text{nerg}}(M, R, V_{S30}, t_E, t_S, \ldots) = y_{\text{erg}}(M, R, V_{S30}, \ldots) + F_{\text{nerg}}(M, R, V_{S30}, t_E, t_S, \ldots)
\]

where \( y_{\text{nerg}} \) is the natural log of the non-ergodic PSA median estimate, and \( y_{\text{erg}} \) is the natural log of the ergodic median estimate. The benefit of this approach is that it separates the non-ergodic effects from the average ground-motion scaling. \( F_{\text{nerg}} \) does not affect the average scaling of the non-ergodic PSA GMM, as LAK21 is based on BA18, and thus, their average scaling is canceled out. Furthermore, the small bias of RVT (Eq. 3.5.6) is also canceled out in this approach, as the same RVT procedure is used to compute \( PSA_{\text{erg}} \) and \( PSA_{\text{nerg}} \). For the average scaling of the non-ergodic PSA GMM, \( y_{\text{erg}} \), we chose the Abrahamson et al. (2014) (ASK14) and Chiu and Youngs (2014) (CY14) ergodic PSA GMMs. Hereafter, the non-ergodic GMM that is based on ASK14 is called non-ergodic GMM1, and the non-ergodic GMM that is based on CY14 is called non-ergodic GMM2. The main reasons ASK14 and CY14 are selected to develop the non-ergodic GMM are: i) they were developed with the same dataset as BA18, and ii) they include complex scaling terms, such as hanging-wall effects, which can be passed to the non-ergodic GMMs.

The non-ergodic PSA GMM was not developed directly with RVT and LAK21 because this approach led to an overestimation of the median PSA at medium-to-large periods. Figure 4 compares the four NGAWest2 GMMs: ASK14, BSSA14, CB14, and CY14 (Abrahamson et al. 2014; Boore et al. 2014; Campbell and Bozorgnia 2014; Chiu and Youngs 2014) with the spectral acceleration response spectrum created with RVT and BA18. The NGAWest2 GMMs are in good agreement with the PSA from BA18 for the \( M 5 \) event, but the comparison worsens as the size of the earthquake increases. For periods \( T_0 = 2 – 4 \text{sec} \), for the \( M 8 \) earthquake, the PSA from BA18 is a factor of two higher than the NGAWest2 GMMs, indicating

![Fig. 4](image-url) Comparison of PSA spectra developed with the BA18 EAS GMM and RVT, shown with the black line, and PSA spectra estimated using the NGAWest2 GMMs, shown with the colored lines. (a) \( M 5.0 \), (b) \( M 6.5 \), and (c) \( M 8.0 \) earthquake scenario with \( R_{\text{rup}} = 30 \text{ km} \) and \( V_{S30} = 400 \text{ m/sec} \)
that, in this period range, BA18 has a stronger magnitude scaling than the NGAWest2 GMMs. Since LAK21 is based on BA18, a non-ergodic PSA GMM developed with RVT and LAK21 will also have a stronger magnitude scaling than the NGAWest2 GMMs. Due to the effort involved in the development of the NGAWest2 GMMs, we judge that their magnitude scaling is more likely to be correct, which is why we used the non-ergodic factors approach to develop the non-ergodic GMM; however, future studies should further investigate the cause of this difference in magnitude scaling.

The epistemic uncertainty of the non-ergodic PSA GMM is captured by sampling the non-ergodic terms of LAK21 GMM multiple times and calculating the $F_{\text{nerg PSA}}$ for each sample. As shown in the example in Sect. 4.1, it is important to consider the inter-frequency correlation of the non-ergodic EAS terms as otherwise the epistemic uncertainty is underestimated.

### 3.3 Constant shift and Aleatory model

The constant shift ($\delta c_0$), between-event residuals ($\delta B^0_e$), and within-event within-site residuals ($\delta W^0_{e,s}$) are estimated by fitting a mixed-effects linear model to the total residuals of the non-ergodic models with the NGA West2 data:

\[
\epsilon_{e,s} = \delta c_0 + \delta B^0_e + \delta W^0_{e,s}
\]  

(3)

The magnitude dependence of $\delta B^0_e$ and $\delta W^0_{e,s}$ of the two non-ergodic PSA GMMs for $T_0 = 0.25\text{sec}$ is evaluated in Fig. 5. The mean of $\delta B^0_e$ and $\delta W^0_{e,s}$ shows no trend but their empirical standard deviation decreases with $M$. Similarly, the $R_{rup}$ and $V_{S30}$ dependence of the $\delta W^0_{e,s}$ for $T_0 = 0.25\text{sec}$ is evaluated in Fig. 6 where no significant trends are found in either the mean or the standard deviation.

![Fig. 5](image)

Fig. 5 Between-event and within-event within-site residuals for $T_0 = 0.25\text{sec}$ versus magnitude. a $\delta B^0_e$ of non-ergodic GMM1, b $\delta W^0_{e,s}$ of non-ergodic GMM1, c $\delta B^0_e$ of non-ergodic GMM2, and d $\delta W^0_{e,s}$ of non-ergodic GMM2
Figure 7 shows the estimated and smoothed $\delta c_0$ of the two non-ergodic PSA GMMs. Non-ergodic GMM$_2$, which is based on CY14, is only estimated up to $T_0 = 5$ sec because, at larger periods, $\delta c_0$ deviated significantly from zero. The negative offset of the model intercept is attributed to the left skewness of the log-normal distribution, which is used to model the non-ergodic effects in LAK21. Because of this, the zeroth moment ($m_0$) of the non-ergodic EAS, which includes the random samples of the non-ergodic effects to capture the epistemic uncertainty, is, on average, larger than the $m_0$ of the ergodic EAS resulting in a positive bias which is corrected by the negative $\delta c_0$. This effect is more pronounced at small periods as a small-period oscillator is sensitive to the entire frequency content of the ground motion.

Based on the empirical standard deviation of the non-ergodic residuals (Fig. 5), both $\phi_0$ and $\tau_0$ are modeled as magnitude dependent (Eqs. 4 and 5). Figure 8 shows the period dependence of $\phi_0$ and $\tau_0$ for small and large magnitudes. The magnitude dependence of $\phi_0$ and $\tau_0$ is more significant at small periods. The increase of the within-event aleatory

Fig. 6 Within-event within-site residuals for $T_0 = 0.25$ sec versus $R_{rup}$ and $V_{S30}$. a $\delta WS$$_0^e$ of non-ergodic GMM$_1$ and b $\delta WS$$_0^e$ of non-ergodic GMM$_2$ versus $R_{rup}$, c $\delta WS$$_0^e$ of non-ergodic GMM$_1$ and d $\delta WS$$_0^e$ of non-ergodic GMM$_2$ versus $V_{S30}$

Fig. 7 Estimated and smoothed $\delta c_0$ versus $T_0$. a non-ergodic GMM$_1$, b non-ergodic GMM$_2$
variability at the small periods of small magnitudes may be caused by radiation patterns, which make the amplitude of the ground motion sensitive to the azimuthal angle. For large magnitudes, which can be thought of as many small events, the radiation patterns have less impact on the ground-motion variability, because the individual radiation patterns interfere with each other due to the different azimuthal angles (Kotha et al. 2019). Similarly, the larger between-event aleatory variability at the small periods of small magnitudes is believed to be caused by differences in stress drop, which shifts the ground motions at frequencies above the corner frequency of the earthquake. Due to the larger rupture dimensions of the large events, any variability in the stress drop along the rupture averages out, resulting in reduced between-event variability.

\[
\phi_0 = \begin{cases} 
\phi_{0M_1} + (\phi_{0M_2} - \phi_{0M_2})(M - 5)/(6.5 - 5) & \text{for } M < 5 \\
\phi_{0M_2} & \text{for } 5 < M < 6.5 \\
\phi_{0M_2} & \text{for } M > 6.5 
\end{cases}
\]

(4)

\[
\tau_0 = \begin{cases} 
\tau_{0M_1} + (\tau_{0M_2} - \tau_{0M_2})(M - 5)/(6.5 - 5) & \text{for } M < 5 \\
\tau_{0M_2} & \text{for } 5 < M < 6.5 \\
\tau_{0M_2} & \text{for } M > 6.5 
\end{cases}
\]

(5)

With Fig. 9 proposed relationships for \(\phi_0\) and \(\tau_0\) are compared with the standard deviations of the binned residuals for \(T_0 = 0.25\,\text{sec}\). Overall, the aleatory models are in good agreement with the empirical standard deviations. The discrepancy at large magnitudes

Fig. 8 Period dependence of aleatory model parameters. a period dependence of \(\phi_{0M_1}\) and \(\phi_{0M_2}\) for non-ergodic GMM\(_1\), b period dependence of \(\tau_{0M_1}\) and \(\tau_{0M_2}\) for non-ergodic GMM\(_1\), c period dependence of \(\phi_{0M_1}\) and \(\phi_{0M_2}\) for non-ergodic GMM\(_2\), d period dependence of \(\tau_{0M_1}\) and \(\tau_{0M_2}\) for non-ergodic GMM\(_2\).
is considered acceptable, as the number of large magnitude events is small to estimate the empirical standard deviations reliably.

Figure 10 compares the total standard deviation of the two non-ergodic GMMs with the total standard deviations of ASK14 and CY14, as well as the total standard deviation of the SWUS15 partially non-ergodic GMM (Abrahamson et al. 2015). The standard deviations of the non-ergodic GMMs are approximately 25% smaller than the total standard deviation of the ergodic counterparts, 45% smaller in terms of variance. The standard deviations of non-ergodic GMMs are within the low and high branches of SWUS15 for the entire period range for both small-to-moderate and large events. For small-to-moderate magnitude

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**Fig. 9** Magnitude dependence of $\phi_0$ and $r_0$ for $T_0 = 0.25sec$. Circular markers denote the standard deviations of the binned residuals, and solid lines correspond to the standard deviation models. (a) $\phi_0$ for non-ergodic GMM 1, (b) $r_0$ for non-ergodic GMM 1, (c) $\phi_0$ for non-ergodic GMM 2, and (d) $r_0$ for non-ergodic GMM 2.

**Fig. 10** Comparison of total standard deviation of non-ergodic GMM 1 and GMM 2 with total standard deviation of the ASK14 and CY14 ergodic GMMs and the total standard deviation of the SWUS15 partially non-ergodic GMM. (a) small-to-moderate magnitude comparison, and (b) large magnitude comparison.
events and \( T_0 < 1 \text{sec} \), the total standard deviations of GMM\(_1\) and GMM\(_2\) are larger than the median branch of SWUS15. At large events, the total standard deviations of GMM\(_1\) and GMM\(_2\) are between the central and lower branch of SWUS15. The GMM\(_1\) and GMM\(_2\) \( \sigma_0 \) values are expected to be less than SWUS15 \( \sigma_{55} \) central branch because, in addition to the systematic site effects, GMM\(_1\) and GMM\(_2\) capture the systematic source and path effects; however, the fact that the \( \sigma_0 \) GMM\(_1\) and GMM\(_2\) are larger than the lower branch of SWUS15 indicates that the majority of the systematic effects captured by GMM\(_1\) and GMM\(_2\) are related to the site effects. The LAK14 non-ergodic EAS GMM does not capture the full systematic path effects; it includes the large-distance systematic path effects through the cell-specific anelastic attenuation, but it misses the short-distance systematic path effects that are dominated by a non-ergodic geometrical spreading.

### 3.4 Modeling procedure

This subsection outlines the pseudocode for implementing the proposed non-ergodic PSA GMM framework.

For each ground-motion scenario in the seismic hazard analysis, defined in terms of the source, path, and site parameters (e.g., \( M, R_{rup}, V_{S30} \)) and the source and site locations (\( t_E \) and \( t_S \)):

1. compute the median ergodic EAS estimate using the BA18 GMM \( \text{EAS}_{BA18}(M, R_{rup}, V_{S30}, \ldots) \)
2. draw multiple samples of the non-ergodic EAS using the LAK21 GMM to capture the full range of epistemic uncertainty \( \text{EAS}_{LAK21}(M, R_{rup}, V_{S30}, \ldots, t_E, t_S) \)
3. convert the ergodic EAS\(_{BA18}\) to PSA with RVT
4. convert all non-ergodic samples of EAS\(_{LAK21}\) to PSA with RVT
5. obtain the non-ergodic PSA factors, \( F_{\text{nerg\_PSA}} \), by computing the log-difference of non-ergodic PSA samples from Step 4 from the ergodic PSA from Step 3
6. estimate the ergodic PSA base scaling with either ASK14 or CY14 median GMM and choose the corresponding constant adjustment, \( \delta c_0 \)
7. obtain the mean non-ergodic log PSA samples, representing the epistemic uncertainty, by summing the log of the ergodic PSA and \( \delta c_0 \) from Step 6, with the non-ergodic factors from Step 5
8. evaluate the distribution of the non-ergodic GMM with the samples of the median non-ergodic PSA from Step 7 and aleatory standard deviation from Sect. 3.3

### 3.5 Random-vibration theory

RVT uses Parseval’s theorem and extreme-value statistics (EVS) to estimate the PSA based on the frequency content (i.e., FAS) and the duration of a ground motion. Parseval’s theorem is used to calculate the root-mean-square of the oscillator’s response \( x_{rms} \) to the input ground motion. A peak factor (PF), based on EVS, is used to estimate the absolute peak response of the oscillator, which is the definition of PSA, based on \( x_{rms} \). PFs assume that the ground motion is a stationary stochastic process and that it can be described as a band-limited white Gaussian noise with zero mean. The first assumption means that the amplitudes of the ground motion are identically distributed, and the second assumption means that the phase angles of the ground motion are randomly distributed. Although earthquake ground motions violate both assumptions, numerous studies have shown that RVT provides
PSA estimates that are in agreement with observed ground motions (Hanks and McGuire 1981; Boore 1983, 2003)

### 3.5.1 Oscillator response

The response of an oscillator to a ground motion can be computed by convolving the ground motion with the impulse response ($IR$) of the oscillator. The $IR$ is the response of an oscillator to a very brief acceleration pulse; that is a Dirac delta function. For an SDOF oscillator, the Fourier transform of the impulse response is:

$$IR(f, f_0, \zeta) = \frac{-f_0^2}{f^2 - f_0^2 - 2j \cdot \zeta \cdot f_0 \cdot f}$$

where, $f_0$ is the natural frequency of the oscillator, and $\zeta$ is the damping of the oscillator.

As an example, Fig. 11 shows the PSA impulse response, in time and Fourier domain, for an SDOF oscillator with $f_0 = 2\text{Hz}$ and $\zeta = 5\%$. In the Fourier domain, the convolution is performed by multiplying the ground motion $FAS$ with $IR$; therefore, the response of an SDOF oscillator to a ground motion is:

$$X(f) = FAS(f) \cdot IR_{SD}(f, f_0, \zeta)$$

The $x_{rms}$ of the oscillator’s response is defined as:

$$x_{rms} = \sqrt{\frac{1}{D_{rms}}} \int_{-\infty}^{+\infty} x(t)^2 dt$$

where $D_{rms}$ is a measure of the duration, which is defined in Sect. 3.5.4. Parseval’s theorem states that the amount of energy in the time domain is equal to the amount of energy in

![Fig. 11](image-url) Impulse response of a single degree of the oscillator; a Time domain, b Fourier domain

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the Fourier domain \( \left( \int_{-\infty}^{+\infty} x(t)^2 \, dt = 2 \int_{0}^{+\infty} X(f)^2 \, df \right) \) which allows computing \( x_{\text{rms}} \) directly in Fourier domain:

\[
x_{\text{rms}} = \sqrt{\frac{1}{D_{\text{rms}}} \int_{0}^{+\infty} X(f)^2 \, df} = \sqrt{\frac{m_0}{D_{\text{rms}}}} \tag{9}
\]

with \( m_0 \) being the zeroth moment of FAS. The \( k^{th} \) moment of FAS is defined as:

\[
m_k = 2 \int_{0}^{+\infty} (2\pi f)^k X(f)^2 \, df \tag{10}
\]

### 3.5.2 Peak factor

The peak factor relates the \( x_{\text{rms}} \) with the maximum response of the oscillator \( (x_{\text{max}}) \), which is the definition of the PSA.

\[
PSA = PF \, x_{\text{rms}} \tag{11}
\]

In general, \( PFs \) fall into two main categories: those based on the Cartwright (1956) peak factor, abbreviated as CLH56, and those that are based on the Vanmarcke (1975) peak factor, abbreviated as V75.

In the first group, CLH56 peak factor assumes that the peaks of a time history occur independently according to a Poisson process. In a series of papers, Boore and colleagues (Boore 1983; Boore and Joyner 1984; Boore 2003) developed peak factors (BJ83) based on a reformulated version of CLH56 and removed an integrable singularity. Davenport (1964) proposed the a peak factor model (D64) based on an asymptotic form that approximates CLH56 for long time histories.

The main difference between V75 (Vanmarcke 1975, 1976) and the \( PFs \) of the first group is that V75 removed the Poisson process assumption. Because of this, V75 \( PF \) accounts for the time spent outside the threshold, which is important for a narrow-band process, and considers that the peaks could be clustered in time, which is important for a wide-band process. Der Kiureghian (1980) noted that the D64 peak factor overestimates the number of zero crossings, and developed a new \( PF \) model (DK80) by modifying D65 \( PF \) so that it is asymptotically consistent with V75. V75 and D80 are in general agreement, but they deviate in time histories with a small number of zero crossings.

The V75 \( PF \) is selected for the development of the non-ergodic \( PSA \) GMM. V75 is preferred over the first group of \( PF \) due to the simplified assumptions in CLH56. The complete form of V75 is preferred over the asymptotic forms, as the former is more accurate for the wide range of ground motions considered in this project. This choice is consistent with the \( PF \) used in Kottke et al. (2021).

V75 expressed the probability distribution of the peaks as a first-passage problem. For a Gaussian process, the first-passage probability (i.e., the probability of no crossing) of a \( \pm a \) threshold (type-D barrier) in the time interval \((0, t)\) is equal to:

\[
P(|z| < r) = A \exp \left( -f_z t \exp(-r^2/2) \frac{1 - \exp(-\sqrt{\pi/2} \delta_e r)}{1 - \exp(-r^2/2)} \right) \tag{12}
\]

where \( r \) is the normalized barrier level \((r = a/x_{\text{rms}})\), \( A \) is the probability of starting within the thresholds \((A = 1 - \exp(-r^2/2))\), \( f_z \) is the average rate of zero crossings, and \( \delta_e \) is an
semiempirical measure of bandwidth \( \delta_e = \delta^{1+b} \), in which \( b \) is a non-negative constant equal to 0.2, and \( \delta \) is a measure of bandwidth based on the spectral moments (Vanmarcke 1972) defined as:

\[
\delta = \sqrt{1 - \frac{m_1^2}{m_0 m_2}} \tag{13}
\]

The cumulative distribution function (CDF) of the peak values is obtained by setting \( t \) equal to the ground motion duration \( (D_{gm}) \) in Eq. (12); that is, the probability of the peak of the time history being less than \( r \times x_{rms} \) is equal to the probability that the time history will remain within the thresholds \( \pm r \times x_{rms} \) for the entire ground-motion duration. With that, the CDF of \( PF \) is equal to:

\[
F_{PF}(r) = (1 - \exp(-r^2/2)) \times \exp\left(-f_z D_{gm} \exp(-r^2/2) \frac{1 - \exp(-\sqrt{\pi} / 2 \delta_e r)}{1 - \exp(-r^2/2)}\right) \tag{14}
\]

The expected value of \( PF \) can be computed with the probability density function (PDF) of \( PF \) (Eq. 15), which requires the derivation of the PDF. However, \( PF \) is continuous and defined on the positive side of the real line; thus, the expected value of \( PF \) can be computed directly from the CDF with equation Eq. (16).

\[
E[PF] = \int_0^{+\infty} r f_{PF}(r) \, dr \tag{15}
\]

\[
E[PF] = \int_0^{+\infty} (1 - F_{PF}(r)) \, dr \tag{16}
\]

The mean estimate of the RVT PSA can be computed by substituting the expected value of the V75 PF in Eq. (11).

### 3.5.3 Ground-motion duration

In RVT, a measure of duration is needed in two steps: in the calculation of the peak factor, and in the calculation of \( x_{rms} \). Due to the transient nature of a ground motion, the duration measures used in these two steps are often different. \( D_{gm} \) is the ground-motion duration, which is used in the calculation of \( PF \); \( D_{rms} \) is the duration measure for the calculation of \( x_{rms} \), which is defined in Sect. 3.5.4.

In seismology, the ground-motion duration is most commonly defined either as bracketed or as significant duration. Bracketed duration is the time interval between the first and last time the ground motion exceeds a threshold. Significant duration is the difference in the time the normalized Arias intensity or normalized integral of squared velocity reaches two specific values. The measures of significant duration with the “a” subscript are based on Arias intensity, while the measures of significant duration with the “v” subscript are based on the normalized integral of squared velocity. For instance, the \( D_{a0.05–0.75} \) significant duration is the difference between the time the normalized Arias intensity is at 5%, and the time the normalized Arias intensity is at 75%. 

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In some RVT methods, $D_{gm}$ is set to a measure of significant duration, but in others, $D_{gm}$ is treated as a free parameter with units of time. For instance, Boore (2003) used the $D_{a0.05−0.95}$ significant duration as $D_{gm}$, while Bora et al. (2015) and Bora et al. (2019) treated $D_{gm}$ as a free parameter and developed a duration GMM with the goal to minimize misfit between the observed PSA and the PSA computed with RVT. Kolli and Bora (2021) compared different measures of duration, both those that are based on significant duration and those that are treated as free parameters; they concluded that the significant duration $D_{a0.05−0.75}$ and $D_{a0.05−0.80}$ performed better in predicting the PSA form real data and stochastic simulations respectively, while the $D_{rv}$ duration measure estimated by minimizing the RVT to observed PSA misfit has similar scaling to measures of significant duration.

In this study, $D_{gm}$ is defined as an interval of significant duration. Different intervals of significant duration were tested as $D_{gm}$ candidates to find the one that minimized the misfit between the PSA of the used dataset ($PSA_{NGA}$) and the PSA estimated with RVT ($PSA_{RVT}$); the results of this comparison are shown in the Electronic supplement, Section S1. The $D_{a0.05−0.85}$ significant duration resulted in the best fit of $PSA_{NGA}$ for the entire frequency range, 0.1 to 100 Hz. The Abrahamson and Silva (1996) duration GMM (AS96) was selected for estimating $D_{a0.05−0.85}$ for new scenarios, as to our knowledge, AS96 is the only GMM that provides an estimate for the selected duration interval. Despite the previous results, the $D_{a0.05−0.75}$, $D_{a0.05−0.95}$, and $D_{a0.05−0.95}$, and $D_{a0.05−0.95}$ estimates of the Kempton and Stewart (2006) duration GMM and $D_{a0.05−0.75}$, $D_{a0.05−0.95}$ and $2D_{a0.20−0.80}$ estimates of the Afshari and Stewart (2016) duration GMM were evaluated as candidates for $D_{gm}$, but the $D_{a0.05−0.85}$ of AS96 resulted to a better fit of $PSA_{NGA}$. The results of this comparison can be found in the Electronic supplement, Section S2.

The AS96 functional form for the mean estimate of the $D_{a0.05−0.75}$ duration is:

$$\ln D_{a0.05−0.75} = \begin{cases} \ln \left( \frac{1}{f_c} + c_1 (R_{rup} - R_c) + c_2 S \right) & \text{for } R_{rup} \geq R_c \\ \ln \left( \frac{1}{f_c} + c_2 S \right) & \text{for } R_{rup} < R_c \end{cases}$$

(17)

where $f_c$ is the corner frequency of the earthquake:

$$f_c = 4.9 \times 10^6 \left( \frac{\Delta \sigma}{10^{1.5M+16.05}} \right)$$

(18)

$\beta$ is the shear-wave velocity at the source, and $\Delta \sigma$ is the stress drop. $1/f_c$ is the source duration, $c_1 (R_{rup} - R_c)$ captures the distance dependence, and $c_2 S$ captures the site dependence. The scaling of AS96 has a physical basis because the distance and site dependence terms are additive, instead of multiplicative, to the source duration. The rationale for an additive distance dependence is that small and large magnitude earthquakes are expected to have a similar increase of duration with increasing distance due to the scattering of the seismic waves. Similarly, the duration increase due to the site effects is also expected to be independent of the earthquake size. In AS96, other intervals of significant duration can be calculated with Eq. (19).

$$\ln \left( \frac{D_{a0.05−I}}{D_{a0.05−0.75}} \right) = a_1 + a_2 \ln \left( \frac{I - 0.05}{1 - I} \right) + a_3 \ln \left( \frac{I - 0.05}{1 - I} \right)^2$$

(19)

where $D_{a0.05−0.75}$ is the 5 to 75% of Arias intensity significant duration, $I$ is the upper limit for the significant duration of interest, and $D_{a0.05−I}$ is the significant duration of interest from...
5 to 1% of Arias intensity. The estimated values of the model coefficients are: $a_1 = -0.532$, $a_2 = 0.552$, and $a_3 = -0.0262$.

### 3.5.4 Correction for non-stationarity

One of RVT’s main assumptions that is violated when applied in ground motions is that the signal is stationary. Especially when predicting PSA for large $T_0$, an SDOF oscillator will not abruptly stop at the end of the ground motion; instead, it will have a transient decaying response, which, if not considered, would lead to an overestimation of $x_{rms}$. To solve this problem, Boore and Joyner (1984) (JB84) proposed to include the oscillator duration ($D_o$) in $D_{rms}$ as shown in Eq. (20); $D_o$ is not included in the calculation of the PF because the response of the oscillator follows a steady decay after the end of the excitation. Liu and Pezeshk (1999) (LP99) improved the estimate of $D_o$ by considering the spectral shape of the input time history in the $D_o$ scaling. Boore and Thompson (2012) (BT12), and Boore and Thompson (2015) (BT15) proposed a relationship for $D_{rms}/D_{gm}$; they used a more flexible functional form compared to the previous studies and considered the magnitude and distance scaling of $D_{rms}/D_{gm}$.

$$D_{rms} = D_{gm} + D_o$$

The BT15 oscillator duration model was selected for the subsequent analyses, as in preliminary evaluations, the RVT PSA estimates with BT15 provided a better fit to the recorded PSA than the alternative models. Although BT12 performed equally well in estimating the PSA of medium-to-large earthquakes, it was not selected because its is not applicable to magnitudes less than 4.

### 3.5.5 Extrapolation of EAS

To ensure that entire frequency content of the ground-motion is captured in the RVT calculations and that the zeroth moment is not underestimated, both the ergodic and non-ergodic EAS spectra are extrapolated at low and high frequencies. At low frequencies, EAS is extrapolated to 0.01 Hz with an omega-squared model (Brune 1970):

$$\Omega(f) = \frac{f^2}{1 + f^2/f_c^2}$$

$$EAS(f < f_{min}) = A_{f_{min}} \Omega(f)$$

where $f_c$ is the corner frequency (Equation (18)), and $A_{f_{min}}$ is the amplitude of the omega-squared model at the minimum frequency of the EAS ($f_{min}$). The stress drop for the calculation of $f_c$ for the omega-squared model is estimated with the Atkinson and Boore (2011) empirical relationship. $A_{f_{min}}$ is estimated based on the EAS amplitudes of 1.00$f_{min}$ to 1.05$f_{min}$ frequency bin:

$$A_{f_{min}} = mean\left(\frac{EAS(f)}{\Omega(f)}\right) \text{ for } f \in [1.0f_{min}, 1.05f_{min}]$$

At high frequencies, EAS is extrapolated to 100Hz with a kappa model (Anderson and Hough 1984):
\[ D(f) = \exp(-\pi \kappa f) \]

\[ EAS(f > f_{\text{max}}) = A_{f_{\text{max}}} D(f) \]

(23)

\( \kappa \) defines the rate of decay of the high frequencies, and \( A_{f_{\text{max}}} \) is the amplitude of the kappa model at the largest EAS frequency, \( f_{\text{max}} \). The value of \( \kappa \) is estimated with the Ktenidou et al. (2014) \( \kappa - V_{S30} \) empirical relationship:

\[ \ln(\kappa) = -0.4 \ln\left( \frac{V_{S30}}{760} \right) - 3.5 \]

(24)

\( A_{f_{\text{max}}} \) is estimated based on the EAS amplitudes in the \( 0.95f_{\text{max}} \) to \( 1.00f_{\text{max}} \) frequency bin:

\[ A_{f_{\text{max}}} = \text{mean}\left( \frac{EAS(f)}{D(f)} \right) \text{ for } f \in [0.95f_{\text{max}}, 1.00f_{\text{max}}] \]

(25)

This approach assumes that the rate of amplitude decay of the extrapolated frequencies, imposed by the Ktenidou et al. (2014) \( \kappa - V_{S30} \) model, is the same for the entire region; however, the amplitude of the extrapolated frequencies are controlled by the amplitude of the non-ergodic frequencies, which are dependent on the systematic source, site, and path effects. This is not a critical assumption, as \( m_0 \) is not sensitive to the high-frequency content of the EAS due to their small amplitude. As an example of the extrapolation procedure, the EAS amplitude of the useable frequency range of the RSN 5827 ground motion record from NGA West 2 is extended to high and low frequencies using the omega-squared and kappa models in Fig. 12, which shows that the amplitudes of the extended frequencies are in agreement with the EAS over the usable frequency range.

3.5.6 RVT summary and validation

In summary, all subsequent RVT calculations are performed with: the V75 PF, the median estimate of AS96 for \( D_{a0.05-0.85} \) as \( D_{gm} \), BT15 for \( D_{rms} \), and the extrapolation procedure described in the previous subsection.

**Fig. 12** Extrapolation of EAS to low and high frequencies. EAS from RSN 5827 ground-motion time history from the NGA West2 dataset
As a validation, Fig. 13 shows the residuals between the natural-log of $PSA_{ANGA}$ and the natural-log of $PSA_{ARVT}$ with the recommended RVT procedure. In this approach, the standard deviation represents the precision of the RVT procedure, and the mean misfit represents the accuracy. Overall, $PSA_{ARVT}$ is in good agreement with $PSA_{ANGA}$ for the entire period range ($T_0 = 0.01 - 10 \text{sec}$) with the fit improving for $M > 5$. Figure 14 shows the mean and the standard deviation of the residuals versus $T_0$. The residuals have a positive bias at $T_0 = 1 - 4 \text{sec}$, indicating that the $PSA_{ARVT}$ under-predicts the $PSA_{ANGA}$ in that range. However, this is not propagated in the non-ergodic PSA GMM, as the GMM is developed using non-ergodic factors, defined in the previous subsection (Sect. 3.2). The non-ergodic factors are estimated using the same RVT procedure for the non-ergodic and ergodic GMMs in Eq. (1) canceling out this bias. The standard deviation of the residuals is approximately 0.2 natural-log units for the entire period range.
4 Applications

4.1 Effect of EAS inter-frequency correlation in \( F_{\text{nerg}} \) PSA

In most GMMs, the ground-motion amplitude (i.e., PSA or EAS) at every frequency is estimated independently; however, an actual ground-motion recording has peaks and troughs. That is, the amplitudes of neighboring frequencies are correlated. For instance, if the amplitude of some frequency is above the average, it is likely that the amplitudes of the nearby frequencies will also be above the average. This inter-frequency correlation is important in RVT, as the response of an SDOF oscillator does not only depend on the ground-motion amplitude at \( T_0 \) but also at the frequency content around \( T_0 \). Bayless and Abrahamson (2018) showed that the \( \text{PSA}_{\text{RVT}} \) variability is underestimated if the inter-frequency correlation of FAS is not considered.

To illustrate the effect of the inter-frequency correlation in the calculation of \( F_{\text{nerg}} \) PSA, we applied the proposed RVT methodology without the inter-frequency correlation in EAS. In both cases, the scenario of interest is a \( M_7 \) earthquake on the Hayward Fault 8km away from a site in Berkeley, CA. The ergodic and non-ergodic EAS of the two approaches and the corresponding non-ergodic PSA spectra are shown in Fig. 15. The non-ergodic EAS in Fig. 15a is developed without inter-frequency correlation, whereas the non-ergodic EAS in Fig. 15b is developed using the inter-frequency correlation model in Lavrentiadis et al. (2021).

In EAS space, both approaches resulted in the same median and epistemic uncertainty range, but in PSA space, only the median is the same. The epistemic uncertainty of PSA is larger when the EAS inter-frequency correlation is considered because if EAS is at an extreme at \( T_0 \), it will generally stay at the extreme over the neighboring frequencies; thus, all the frequencies which influence the response of the oscillator will constructively interfere leading to a range of PSA amplitudes that is wider. In contrast, if the EAS amplitudes are uncorrelated, they negate the effect on the response of the oscillator, resulting in a narrower range of PSA. This shows the importance of considering the EAS inter-frequency correlation in the non-ergodic PSA calculations, as otherwise, the epistemic uncertainty of the PSA is underestimated.

4.2 Magnitude dependent \( F_{\text{nerg}} \) PSA

As an application example, Fig. 16 presents the EAS and PSA non-ergodic GMM predictions for \( T_0=0.1 \) s \((f_0 = 10 \text{ Hz})\) for a \( M_3 \) and \( M_8 \) earthquake on the San Andreas fault. The EAS non-ergodic factors are magnitude independent; the median estimate and epistemic uncertainty of \( F_{\text{nergEAS}} \) is the same in both events (Fig. 16a and c). The magnitude independence allows \( F_{\text{nergEAS}} \) to be estimated from the more frequent small magnitude earthquakes and directly applied to the large magnitude events, which typically are of more interest. This is not the case for the \( \text{PSA} \) non-ergodic factors; \( F_{\text{nergPSA}} \) depend on the spectral shape; which is why \( F_{\text{nergPSA}} \) are different in the \( M_3 \) and \( M_8 \) earthquakes (Figs. 16e and g). This illustrates why the non-ergodic \( \text{PSA} \) GMM is developed with non-ergodic factors that are based on EAS. Most of the regional data that are used to estimate the non-ergodic effects are in form of small-magnitude events. But, we can recoup the magnitude dependence when the EAS factors are converted to PSA through RVT, which couldn’t be used if PSA non-ergodic effects were estimated directly.
In addition, Figs. 16b and d, as well as f and d, show the spatial distribution of the epistemic uncertainty for $EAS$ and $PSA$, respectively. In this example, where the location of the earthquake is fixed, the spatial distribution of the epistemic uncertainty depends on the path and site non-ergodic effects. Both the $EAS$ and $PSA$ epistemic uncertainties are small close to stations that have recorded past events, whereas, in remote areas with no available ground-motion data to constrain the non-ergodic terms, the epistemic uncertainties are larger.

The evaluation of the magnitude dependence of the $EAS$ and $PSA$ non-ergodic factors is further examined in Fig. 17. The three scenarios in this comparison are $M 3, 5.5, and 8$ events on the San Andreas Fault, 105 km from the site in San Francisco, CA. As mentioned previously, the non-ergodic $EAS$ factors are the same for all three events (Fig. 17b), while the non-ergodic $PSA$ factors are different, especially at short periods (Fig. 17d), $T_0 < 0.1$ s. This happens because, for $f_0 > 10$Hz ($T_0 < 0.1$ s), there is little ground-motion content in $EAS$ to resonate the SDOF oscillator, making its response, and subsequently $PSA$, dependent on the peak of each spectrum. Similarly, the non-ergodic $PSA$ factors for $T_0 < 0.1$ s depend on the non-ergodic $EAS$ factors at the peak of each spectrum. In this example, the...
Fig. 16  EAS non-ergodic factors ($F_{\text{nerg EAS}}$) and PSA non-ergodic factors ($F_{\text{nerg PSA}}$) for $f_0 = 10$Hz for an earthquake on the San Andreas fault. The star corresponds to the earthquake location, and the dots correspond to the station locations in the used dataset. 

- **a** mean of $F_{\text{nerg EAS}}$ for $M = 3.0$,
- **b** epistemic uncertainty of $F_{\text{nerg EAS}}$ for $M = 3.0$,
- **c** mean of $F_{\text{nerg EAS}}$ for $M = 8.0$,
- **d** epistemic uncertainty of $F_{\text{nerg EAS}}$ for $M = 8.0$,
- **e** mean of $F_{\text{nerg PSA}}$ for $M = 3.0$,
- **f** epistemic uncertainty of $F_{\text{nerg PSA}}$ for $M = 3.0$,
- **g** mean of $F_{\text{nerg PSA}}$ for $M = 8.0$,
- **h** epistemic uncertainty of $F_{\text{nerg PSA}}$ for $M = 8.0$.
$M_3$ event has the largest non-ergodic PSA factors at $T_0 < 0.1$ s, because the non-ergodic EAS factors are predominately positive over its peak ($f = 2$ to 6 Hz). The $M_8$ event has the smallest non-ergodic PSA factors at $T_0 < 0.1$ s because its peak ($f < 0.1$ to 6 Hz) encompasses the dip of the non-ergodic EAS factors that occur from $f = 0.3$ to 2 Hz.

### 4.3 Example hazard calculations

A comparison of the ergodic and non-ergodic PSHA results for $PSA(T_0 = 0.25$ s) for a site in Berkeley, CA, is presented in Fig. 18. The PG&E source model was used in all hazard calculations (Pacific Gas and Electric Company (PG&E) 2015, 2017). The ergodic hazard calculations were performed with the ASK14 and CY14 GMMs, with equal weights, while the non-ergodic hazard calculations were performed with non-ergodic GMM$_1$ and GMM$_2$, with equal weights. The epistemic uncertainty of the non-ergodic GMMs was captured by 100 realizations of $F_{nerg PSA}$. This leads to a logic tree with 200 branches on the
The difference between the two non-ergodic hazard calculations is that, in Fig. 18b, only the regional systematic site effects are constrained, while, in Fig. 18c, recordings from past earthquakes are assumed to be available at the site and thus, both the regional and site-specific site effects are constrained. In LAK21, the regional site effects, captured by $c_{1a,s}$, have a continuous finite correlation-length spatial variability and are modeled as a function of the site location. Due to the finite correlation length, $c_{1a,s}$ can be inferred at new locations based on the $c_{1a,s}$ values from nearby stations. The site-specific site effects, captured by $c_{1b,s}$, have a spatially-independent spatial correlation (i.e., zero correlation length); thus, they can only be estimated from site-specific information, such as past recordings or through a site-specific site response analysis. Further information on the LAK21 components is provided in Lavrentiadis et al. (2021).

For the ergodic hazard calculations, the mean hazard curve is flatter than the non-ergodic hazard curves due to the large aleatory variability of ASK14 and CY14, and the epistemic uncertainty is small as it only encompasses the epistemic uncertainty in the seismic source characterization. Comparing the two non-ergodic calculations, the mean hazard curve is flatter and the epistemic uncertainty is larger in Fig. 18b as $c_{1b,s}$ is free. This example shows the impact of non-ergodic GMM in PSHA where at moderate-to-large return periods, it can lead to a factor of two to four change in the mean ground-motion level.

5 Conclusions

A new approach for developing non-ergodic PSA GMMs is presented in this study which considers the magnitude dependence of the non-ergodic terms. Due to the linear properties of the Fourier Transform, a non-ergodic EAS GMM is used to estimate the non-ergodic effects from the small magnitude events and transfer them to the events of interest (i.e., large magnitudes). RVT is used to compute the non-ergodic PSA effects based on the non-ergodic EAS effects, while the average scaling of the non-ergodic PSA GMM is controlled by an existing ergodic PSA GMM.
Two non-ergodic PSA GMMs are developed in this study. The first one uses the ASK14 GMM as a backbone model for the average scaling and is applicable to periods $T_0 = 0.01–10$ s. The second one uses the CY14 GMM as a backbone model for the average scaling and is applicable to periods $T_0 = 0.01–5$ s. The non-ergodic PSA effects are quantified in terms of non-ergodic PSA factors, that is, the difference between the log of PSA estimated with RVT and the non-ergodic EAS and the log of PSA estimated with RVT and the ergodic EAS. The LAK21 GMM is used for the non-ergodic EAS and the BA18 GMM is used for the ergodic EAS. The RVT calculations are performed with the V75 PF, the median estimate of $D_{a0.05–0.85}$ from AS96 for the ground-motion duration, and the BT15 for the oscillator duration. The RVT components were chosen based on a thorough evaluation of alternative models for the peak factors, ground-motion duration, and oscillator duration. The objective of the evaluation was to minimize the misfit between the observed PSA and the PSA computed with RVT.

The advantages of developing the non-ergodic GMM with an ergodic backbone model and non-ergodic PSA factors, instead of developing it directly with RVT and the LAK21 are: (i) the elimination of the small bias of RVT at $T_0 = 1–4$ s, (ii) the separation of the non-ergodic effects from average scaling, and (iii) the adoption of complex scaling terms present in ergodic PSA GMMs. Compared to the recorded PSA, the PSA estimated with RVT has a small positive bias at $T_0 = 1–4$ s. This bias is not propagated in the non-ergodic PSA factors; it is canceled out, as both the ergodic and non-ergodic RVT PSA estimates are calculated with the same approach.

The aleatory variability of the two non-ergodic PSA GMMs is approximately 25% smaller than the aleatory variability of an ergodic PSA GMM.

Future studies should reevaluate the RVT and EAS models so that when combined, they result in PSA predictions consistent with PSA GMMs. Furthermore, the proposed non-ergodic GMMs were developed with a subset of the NGAWest2 database, which was compiled in 2014. As larger data sets that include more recent and more frequent small-magnitude events become available, the proposed models should be assessed and potentially expanded with additional non-ergodic terms. Similarly, 3D broadband numerical simulations or inferred intensity measurements from historical earthquakes should be used to evaluate the efficacy of the proposed models.

6 Software and resources

The RVT calculations were performed with the pyRVT library (Kottke 2020) in the computer language Python (Van Rossum and Drake 2009). The linear mixed-effects regressions were performed with the lme4 package (Bates et al. 2015) in the statistical environment R (R Core Team 2020). The PSHA calculations were performed with HAZ45.3 (Abrahamson 2021).

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**Code availability** The are python scripts for the non-ergodic regressions are provided at: https://github.com/glavrentiadis/NonErgodicGMM_public.

**Declarations**

**Conflict of interest** The authors declare that they have no conflict of interest.

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