Comparison of Various PID Control Algorithms on Coupled-Tank Liquid Level Control System

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Abstract. On account of the unsatisfactory control effects of traditional PID control theory on dealing with complicated or nonlinear control system, the paper focused on multiple intelligent control theories, including fuzzy self-adaptation modulation PID control, BP neural network modulation PID control and RBF neural network modulation PID control. By setting double-Tank model as an example, and adopting MATLAB software programming, the research is aimed to realize tank level control via four PID control algorithms. By comparing the simulation curves of the four control algorithms, the superiority of the REF neural network in arbitrarily accurate approximating any continuous function is proved.

1. Introduction
PID control is to realize the controlling relationship between control signal and the controlled object by transmitting deviation signal to proportion (P), integration (I) and differential (D) links subjecting to linear combination[1]. Thanks to the characteristics such as simple algorithm, high reliability and robustness, it has been applied in a variety of control fields and has achieved excellent effects[2]. Due to the existing of the actual production objects, traditional PID control is not applicable to closed-loop optimization control or complex nonlinear system for the complicated parameter setting and other factors. Under such a background, intelligent PID parameter setting emerged, including input variables fuzzy control and neural networks control artificial neural network involved in. The paper summarized PID control theories of traditional PID, fuzzy PID, BP neural network PID and RBF neural network PID. By setting coupled-tank as an example, it simulated water tank level under the four PID controllers by adopting MATLAB software. Through comparing the simulated curves, it can be seen that REF neural network shows the optimal liquid level control effect for coupled-tank.

2. Mathematical Model for Coupled-tank:
Mathematical model for coupled-tank is selected as shown in Fig.1. It is composed of the upper tank, lower tank, water storage tank, electric control valve, pipeline and solenoid valve etc. \( h_1 \) and \( h_2 \) refer the liquid level, \( q_1 \) and \( q_2 \) refer the inflow volume, \( A_1 \) and \( A_2 \) refer the base area, \( R_1 \) and \( R_2 \) refer the flow resistance of the upper and lower tank respectively.
The differential equation of coupled-tank is derived from tank material balance equation:

\[
T_1 T_2 \frac{d^2 \Delta h}{dt^2} + (T_1 + T_2) \frac{d \Delta h}{dt} + \Delta h = K \Delta x
\]  

(1)

In formula (1), \(T_1\) refers to the time constant of the upper tank, \(T_1 = R_1 A_1 = R_1 C_1\); \(T_2\) refers to the time constant of the lower tank, \(T_2 = R_2 A_2 = R_2 C_2\); and \(K\) is the amplification coefficient of the object \(K = k R_2\).

Specific transfer function equation of coupled-tank adopted in the paper is:

\[
G(s) = \frac{5}{18s^2 + 8.5s + 1}
\]  

(2)

The paper selected the liquid level of the lower tank is selected as the controlled object to study the liquid level changes as being affected by inlet valve, outlet valve opening and other factors.

### 3. Basic Principle and Results of Three PID Control Methods

#### 3.1 Control Principle and Results of Traditional PID Control

Traditional PID controller utilizes deviation signal subjecting to proportion, integration and differential links as the controlled object. The deviation \(e(t)\), namely the difference value between liquid level output and preset value, is the input signal of PID controller. The control principle of traditional PID control can be expressed as:

\[
u(t) = k_p e(t) + \frac{1}{t_i} \int_0^t e(t') dt + t_d \frac{d}{dt} e(t)
\]

(3)

In formula (3), \(u(t)\) is the output signal of PID controller, \(k_p\) is the proportion constant, \(k_i\) is the integral time, \(t_d\) is the differential time, and \(e(t)\) is the input signal of PID controller. On the basis of traditional PID theory, the paper adopted incremental PID control algorithm, namely adopting analog signal discretization and control all the data acquired through deviation calculation. The formula is as shown below:

\[
u(k) = k_p [e(k) + \frac{t}{t_i} \sum_{i=0}^{k-1} e(i)] + \frac{k_d}{t_d} e(k - 1)]
\]

(4)

\[
u(k-1) = k_p [e(k-1) + \frac{t}{t_i} \sum_{i=0}^{k-2} e(i)] + \frac{k_d}{t_d} e(k - 2)]
\]

(5)

The operational formula of incremental PID control algorithm is obtained based on the difference between the control amount at the time \(k\) and that at the time \(k-1\):

\[
\Delta u(k) = u(k) - u(k-1) = k_p [e(k) - e(k-1)] + k_i e(k) + k_d [e(k) - 2e(k-1) + e(k-2)]
\]

(6)
In formula (6), proportionality coefficient is \( k_p = \frac{1}{\delta} \); integration coefficient is \( k_i = k_p \frac{t}{t_i} \); differential coefficient is \( k_d = k_p \frac{t}{t_d} \).

Incremental PID double-capacity water tank level control simulation curve realized via formula programming as mentioned above:

![Figure 1](image)

**Figure 1**. Results of incremental PID

It can be seen from Fig. 2 that traditional incremental PID algorithm requires more oscillations. The adjusting time is 22s and stabilization time is 25s. It takes long time.

3.2 Fuzzy Self-adaptation Setting PID Control and Results

During industrial production process, the controlled objects are subject to changes on characteristics, parameters, and results as being affected by the load and interference factors [3]. The adaptive control in modern control theory has the advantages on real-time changing the control strategy and improving the control quality. However, the control effect is closely related to the precision of identification model. Therefore, for complicated and large scale system, if the control model precision is unsatisfactory, the control effect will be poor. The fundamental principle of fuzzy self-adaptation PID controller is to take error \( e \) and error change \( e_c \) as controller input thus to satisfy PID parameter self-setting requirements via \( e \) and \( e_c \) at any time. A fuzzy control rule table is established to modify PID parameters [4].

It is an algorithmic program by running MATLAB software, preparing fuzzy control rules table of KP, KI and KD, and fuzzy self-adaptation PID double-capacity water tank control. Liquid level control simulation results and running results of fuzzy self-adaptation PID double-capacity water tank are as shown in Fig. 3 below. Fig. 3 (a) is the simulation output curve of fuzzy PID double-capacity water tank level. And Fig. 3 (b) is the error curve.
Figure 3(a). Results of Fuzzy adaptive PID

It can be seen from Fig. 3 that fuzzy adaptive PID setting requires less oscillation comparing to traditional PID control. However, it still requires long stabilization time 19s.

3.3 BP Neural Network PID Control and Results

BP neural network is to deal with the data automatically at complicated network environment and to analyze the normal running status of the system based on the defined learning rate, learning step size, and performance index [5]. By adopting BP network, a self-learning PID controller can be established for $K_P$, $K_I$ and $K_D$. And it can realize PID control optimization through self-learning of neural network, weighted coefficient adjusting for neural network output corresponding to certain control rule.

Figure 4 shows BP neural network PID controller structure. The control algorithm steps are summarized as following:

1. To determine the structure of input layer, implication layer and output layer including each layer neuron number and activation function of BP neural network. The activated function is as shown in the following formula. An original value is assigned for each layer of weight number. Choose appropriate learning rate and operation step [6]:

$$g(x) = \frac{1}{2(1+\tanh(x))} = \frac{e^x}{e^x + e^{-x}}$$

(7)

2. To obtain the sum of $r_in(k)$ and $y_out(k)$ through sampling and calculate the error of initial time $e(k)$;

3. To perform neuron data treatment, input and output through the defined parameters for each layer of neural network [7];

4. To calculate the output of PID controller of formula (6) for $u(k)$;
(5) To perform self-adaption learning thus to realize automatic adjustment under certain rules, input and output calculation and automatic setting of the final output of KP, KI and KD control parameters [8-9];

(6) To set \( k = k + 1 \), and return to (1).

Liquid level control simulation results and running results of BP neural network PID double-capacity water tank are as shown in Fig. 5 below. Fig. 5 (a) is the simulation output curve of BP neural network PID double-capacity water tank. And Fig. 5 (b) is the error curve.

![Figure 5(a). Results of BP neural network PID](image1)

![Figure 5(b). Error curve](image2)

### 3.4 RBF Neural Network Setting PID Control

Radial primary function RBF neural network has three layers of feed forward network in single hidden layer structure. It is a local approximation network. It has been approved that it can get close to any continuous function at an arbitrary precision[10-12].

The principle of RBF neural network PID setting is as shown below. The physical quantity formula of control error \( e(k) \) is as shown in formula (8):

\[
e(k) = rin(k) - yout(k)
\]  

(8)

The three inputs of PID controller are as shown in formula (9):

\[
x_1(1) = e(k) - e(k - 1) \\
x_2(2) = e(k) \\
x_3(3) = e(k) - 2e(k - 1) + e(k - 2)
\]  

(9)

Incremental control algorithm is as shown in Formula (6): Performance index function is as shown in formula (10):

\[
E(k) = 1/2e(k)^2
\]  

(10)

Proportion and integration. The dynamic adjustment approach of differential coefficient \( K_p \), \( K_i \) and \( K_D \) is the same with that of BP neural network. For negative gradient descent, the computational process is as shown in formula (11):

\[
\Delta k_p = -\eta \frac{\partial E(k)}{\partial y_{out}} \frac{\partial y_{out}}{\partial u} \frac{\partial u}{\partial k_p} = \eta e(k) \frac{\partial y_{out}}{\partial u} x(1)
\]  

(11)

\[
\Delta k_i = -\eta \frac{\partial E(k)}{\partial y_{out}} \frac{\partial y_{out}}{\partial u} \frac{\partial u}{\partial k_i} = \eta e(k) \frac{\partial y_{out}}{\partial u} x(2)
\]  

\[
\Delta k_D = -\eta \frac{\partial E(k)}{\partial y_{out}} \frac{\partial y_{out}}{\partial u} \frac{\partial u}{\partial k_D} = \eta e(k) \frac{\partial y_{out}}{\partial u} x(3)
\]  

The paper adopted 3 neurons of RBF neural network input layer, 6 neurons of implication layer and 1 neuron of output layer which form the network structure as shown in 3-6-1. Liquid level control
simulation results and running results of RBF neural network PID double-capacity water tank are as shown in Fig. 6 below. Fig. 6 (a) is the output curve of RBF neural network PID double-capacity water tank. And Fig. 6 (b) is the error curve.

![Image](example.png)

**Figure 6(a).** Results of RBF neural network PID

**Figure 6(b).** Error curve

It can be seen from Fig. 6 that the time required by RBF control algorithm has been decreased significantly. However, the algorithm result is idealistic since the transfer function is quite simple.

4. Comparison of the Results of Four PID Control Algorithms

The simulation curves of traditional PID, fuzzy PID, BP neural network PID and RBF neural network PID on double-capacity water tank level control system are obtained as shown above. Through comparing the simulation curves, the parameters of the four control modes are as shown in Table 1 and 2.

| Control algorithm | Latency time (s) | Rising time (s) | Peak time (s) | Adjusting time (s) | Overshoot (s) |
|-------------------|-----------------|-----------------|--------------|--------------------|---------------|
| Traditional PID   | 2               | 2               | 3.5          | 22                 | 58            |
| Fuzzy PID         | 3               | 3.5             | 5            | 18                 | 30            |
| BP PID            | 4.8             | 5               | 9            | 12                 | 5             |
| RB FPID           | 0.5             | 1.8             | 2            | 2                  | 0             |

| Control algorithm | Time to reach stabilization/s | $k_p$ | $k_i$ | $k_d$ |
|-------------------|-------------------------------|------|------|------|
| Traditional PID   | 30                            | 12   | 8    | 9    |
| Fuzzy PID (algorithm) | 19                        | 3    | 0.0028 | 7.365 |
| BP PID            | 12                            | 0.406| 0.125| 0.51 |
| RBF PID           | 2                             | 1000.05| 0.4   | 0.13 |

From the control algorithm results as shown in Table 1 and Table 2, as well as combining with the curves of the four control methods, it can be seen that traditional PID algorithm requires more oscillations, longer adjusting time and higher overshoot; the difference on adjusting time between fuzzy PID algorithm and BP neural network PID algorithm is not significant. However, the overshoot of BP neural network PID is much lower than that of fuzzy PID. Comparing with former three kinds of algorithms, RBF neural network has no overshoot and can achieve stability in the shortest time. We get the final conclusion that RBF neural network algorithm is the optimal method for double-capacity water tank level control.
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