The problem of Mass and Mass Generation

Harald Fritzsch

Ludwig–Maximilians–Universität, Sektion Physik,
Theresienstraße 37, D–80333 München

Max–Planck–Institut für Physik, Werner–Heisenberg–Institut,
Föhringer Ring 6, D–80805 München
E–Mail: bm@hep.physik.uni–muenchen.de

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Towards the end of the last century the electron was discovered. In retrospect this discovery marked the beginning of a remarkable development, which eventually led to the emergence of the “Standard Model of Fundamental Particles and Forces” in the 70ies. According to the letter all the visible matter in the universe is composed of fundamental objects of two different categories – leptons (among them the electron) and quarks. The latter do not exist as free particles, but are bound among each other to form the protons and neutrons, the building blocks of the atomic nuclei.

The dynamics of matter in our universe can be traced back to the action of four types of fundamental forces: the strong forces among the quarks, the electromagnetic forces among charged particles, the weak interactions responsible for the phenomenon of radioactivity, and gravity. With the exception of the strong forces all other interactions were known, at least through their indirect effects, at the turn of the last century. The first effects of the strong nuclear force were found in 1919 by Rutherford and his collaborators by studying the interactions of $\alpha$–particles and nitrogen nuclei. Finally in 1932 the neutron was found by Chadwick and soon thereafter identified as a second building block of the atomic nucleus besides the proton.

The “Standard Model” constitutes a consistent theory of the fundamental forces, based on quantum field theory and the concepts of non–Abelian gauge theories. Since about 1980 the quantitative predictions of the “Standard Model” have been subjected to severe experimental tests, with the result that no departures from the theoretical expectations have been found. The “Standard Model” provides us not only with a fairly good description of the fundamental particles and forces, but gives an excellent picture of reality. Not a single
confirmed result from the particle physics experiments are in conflict with it.

With the help of the Large Electron Positron collider (LEP) at CERN one was able in the recent years to test the predictions of the “Standard Model”, especially its electroweak sector, with a high order of precision. Furthermore the study of the collisions at LEP has helped to unify particle physics, astrophysics and cosmology to a coherent picture of the cosmic evolution. The collisions at LEP recreate the conditions which were present in the universe about $10^{-10}$ seconds after the Big Bang.

According to the “Standard Model” there exist two different categories of fundamental particles: the matter particles (quark, leptons) carrying spin 1/2 and the force particles (photons, $W$, $Z$, gluons) carrying spin 1. The latter are gauge bosons, and their interactions with the matter particles and with themselves are dictated by the principles of non–Abelean gauge symmetry. The symmetry group is given by the direct product of three simple groups: $SU(3)_c \times SU(2)_w \times U(1)$ (c: color, w: weak).

The gauge bosons are given in the following table:

| Gauge Bosons (spin 1) | Mass  | Electric Charge | Color |
|-----------------------|-------|-----------------|-------|
| $\gamma$              | 0 GeV | 0               | 0     |
| $W^-$                 | 80.2  | $-1$            | 0     |
| $W^+$                 | 80.2  | $+1$            | 0     |
| $Z$                   | 91.2  | 0               | 0     |
| $g$                   | 0     | 0               | 8     |
The visible matter in the universe is composed of the elements of the first lepton–quark family:

\[
\begin{pmatrix}
\nu_e & u_r & u_g & u_b \\
e^- & d_r & d_g & d_b
\end{pmatrix}
\]

\((r, g, b: \text{color index})\).

The fact that the electric charges of these eight objects add to zero indicates that the two leptons and six quarks are related to each other in a way which cannot be described within the “Standard Model”, but is subject to the yet hypothetical physics beyond the “Standard Model”, perhaps described by a grand unification of all interactions.

All known particles can be described in terms of three lepton–quark–families:

\[
\begin{pmatrix}
\nu_e & u \\
e^-. & d
\end{pmatrix}
\begin{pmatrix}
\nu_\mu & c \\
\mu^- & s
\end{pmatrix}
\begin{pmatrix}
\nu_\tau & t \\
\tau^- & b
\end{pmatrix}
\]

\(I \quad II \quad III\).

It should be stressed that several algebraic properties of the “Standard Model” cannot be deduced from the underlying symmetry, e. g. the charge and spin assignments of the leptons and quarks, or the number of families. It is remarkable that the number “three” plays a three–fold significant role:

a) Quarks come in three colors, hence the nucleons consist of three quarks.

b) There exist three families of leptons and quarks.
c) There are three different gauge interactions, based on the three
different gauge groups (gravity is not considered here).

The question arises whether there are connections between three different
“threenesses” of the Standard Model. For example, the number of
families and colors could be related by an underlying, yet unknown
symmetry principle.

The most unsatisfactory feature of the “Standard Model” is the fairly
large number of free parameters which need to be adjusted. First of
all, the values of the three gauge coupling constants $g_1, g_2, g_3$ have to
be taken from experiment. All other parameters are related to the
masses of the fermions or gauge bosons. They are: the mass of the
$W$–boson, the masses of the three charged leptons $m_e, m_\mu, m_\tau$ (the
neutrinos will be kept massless in our discussion), the masses of the
six quark flavors $m_u, m_d, m_c, m_s, m_t, m_b$ and the four mixing angles of
the weak interaction sector $\Theta_{sd}, \Theta_{sb}, \Theta_{bd}$ and $\delta$ ($\Theta_{sd}$ denotes the angle,
which describes primarily the $s$ – $d$ mixing etc., $\delta$: phase angle). Thus
17 parameters are needed to describe the observed particle physics
phenomena.

We point out that 13 free parameters of the “Standard Model” are
associated with the masses of the matter fermions (nine masses, four
mixing parameters, which arise due to a mismatch between the mass
matrices of the $u$–type quarks and the $d$–type quarks). Thus a deeper
insight into the dynamics of the mass generation for the leptons and
quarks is needed in order to reduce the number of free parameters –
an insight which would definitely carry us beyond the physics of the
“Standard Model”. Certainly the problem of mass and mass generation
are on the top of the priority list of problems in fundamental physics for
the immediate future.
In all physics phenomena the different interactions and mass scales enter in a variety of ways, thus producing the multitude of phenomena we observe. Before discussing the problem of the lepton and quark masses, let me stress that the problem of the nucleon mass (and therefore of the masses of the atomic nuclei, which represent more than 99% of the mass of the visible matter in the universe) has found an interesting solution within the framework of QCD. Due to the renormalization effects the QCD gauge coupling constant $\alpha_s$ depends on the scale at which the interaction is studied. It decreases logarithmically at high energies. At a scale $\mu$ its decrease is given by $\alpha_s(\mu^2) = \text{const.}/\ln(\mu^2/\Lambda^2)$, where $\Lambda$ is a scale parameter, which serves as the fundamental mass scale of the theory, i.e. a mass scale which fixes all other mass scales in strong interaction physics (the effects of the quark masses are neglected). Phenomenologically $\Lambda$ is about 150...200 MeV, i.e. $\Lambda^{-1}$ corresponds to the typical extension of a hadron.

Using powerful computers and sophisticated nonperturbative methods one has been able to calculate the masses of the lowest-lying hadrons (nucleon, $\Delta$-resources, $\rho$-mesons ...) in terms of $\Lambda$, with impressive results. Especially the mass ratios $(m_{\Delta}/m_p), (m_{\rho}/m_p)$ etc. can be calculated with high precision. Thus we can say that the problem of the nuclear masses, especially of the proton mass, has found a solution. The mass of a proton (about 940 MeV) is nothing but the field energy of the quarks and gluons, which are confined within a radius of the order of $10^{-13}$ cm (of order $\Lambda^{-1}$). Therefore, a direct link exists between the nucleon mass and the size of the nucleon. Both have to be of the same order of magnitude – more specifically the nucleon mass is expected to be of the order of $3 \cdot \Lambda$, where 3 denotes the number of the constituent quarks. Furthermore the nucleon mass is a truly
nonperturbative phenomenon, directly related to the confinement aspect of QCD. The QCD gauge interaction itself creates its own mass scale – the nuclear mass is generated dynamically.

This phenomenon of dynamical mass generation is not directly related to the quark substructure of the hadrons, but rather to the gluonic degrees of freedom. This can be seen by studying the mass spectrum of “pure QCD”, i.e. QCD without quarks. This theory of eight interacting gluons displays at low energies a discrete mass spectrum, which starts at the mass of the lowest lying glue meson. Thus unlike “pure QED” the theory displays a mass gap which is generated by nonperturbative effects.

The theory of the nucleon mass described above is remarkable in the sense that the mass of an elementary particle can in principle be calculated. Note that the mass is directly related to the field energy density inside the nucleon. It would be of high interest to know whether the masses of the leptons, the \( W^- \), \( Z^- \)-particles and the quarks are due to a similar mechanism, or are generated by a qualitatively different mechanism.

In the standard electroweak theory these masses are due to a spontaneous breaking of the electroweak gauge symmetry caused by an elementary scalar field \( \phi \). The order parameter of the symmetry breaking is given by the vacuum expectation value \( v \) of the field \( \phi \) which in turn is related to the Fermi constant \( G \) and the \( W^- \)-mass:

\[
\frac{G}{\sqrt{2}} = \frac{g_{w^2}}{8M_{w^2}} = 12v^2
\]

\((g_w: \text{gauge coupling constant})\).
The main consequence of this mechanism is a relation between the $Z$–mass and the $W$–mass in terms of the electroweak mixing angle $\Theta_w$: $M_z = M_w / \cos \Theta_w$. Since the mixing angle $\Theta_w$ can be determined independently by studying the neutral current interaction of leptons and quarks, this mass relation is a nontrivial constraint, in excellent agreement with the experimental data.

The success of the electroweak mass relation does not necessarily imply that the mechanism of the spontaneous symmetry breaking is realized in the real world, but it implies that an alternative mechanism must lead to the same mass relation. This is the case, for example, in technicolor models in which the scalar field $\phi$ is replaced by a field composed of new fermions which are tightly bound by the new technicolor interaction.

If the standard electroweak model is correct, it implies the existence of a particle, the “Higgs” particle, whose couplings are given by the observed particle masses. The mass of this particle is unknown, but it can hardly be larger than about 1000 GeV. The LEP experiments exclude the mass region lower than about 60 GeV.

Certainly the most interesting question is the one about the origin of the lepton– and quark masses. The spectrum extends over five orders of magnitude, starting with the electron (neutrino masses are not considered here), and ending with the $t$–quark with a mass of about 180 GeV. Thirteen free parameters are needed to describe the properties of the lepton and quark mass spectrum: the three lepton masses, the six quark masses, and the four parameters describing the mixing of the quark flavors. Unlike the masses of the leptons, the quark masses cannot be determined directly, but have to be inferred from the properties of the hadronic spectrum. Furthermore they are scale dependent, i. e. they vary...
logarithmically, if the corresponding renormalization point is shifted. In
lowest order this change is given by \( m_q(\mu) = m_q(\mu_0)(1 - \frac{\alpha_s(\mu)}{\pi} \ln \frac{\mu^2}{\mu_0^2}) \),
where \( \alpha_s(\mu) \) is the QCD coupling constant. A suitable renormalization
point for the quark masses is the mass of the \( Z \)-boson, which is known
with a high precision: \( M_z = 91.1884 \pm 0.0022 \text{ GeV} \).

One finds:

\[
\begin{align*}
  m_u(M_z) &= 3.4 \pm 0.6 \text{ MeV}, \\
  m_d(M_z) &= 6.3 \pm 0.9 \text{ MeV} \\
  m_c(M_z) &= 880.0 \pm 48.0 \text{ MeV}, \\
  m_s(M_z) &= 118.0 \pm 17.0 \text{ MeV} \\
  m_t(M_z) &= 17.0 \pm 12.0 \text{ GeV}, \\
  m_b(M_z) &= 3.31 \pm 0.11 \text{ GeV}
\end{align*}
\]

A closer inspection of the mass spectrum tells us:

a) The mass spectra of the three flavor channels (charged leptons, \( u \)-
type quarks, \( d \)-type quarks are almost entirely dominated by the
mass of the member of the third generation.

b) The relative importance of the second generation decreases as we
proceed upwards in the charge (\( \mu \rightarrow s \rightarrow c \)). In the lepton case the
muon contributes about 5.6% to the sum of the masses, while in
the charge \((-1/3)\)- channel the \( s \)-quark contributes only 3.2%, and
in the charge \((+2/3)\)- channel the \( c \)-quark contributes only 0.5%.

c) The relative importance of the masses of the members of the first
generation is essentially zero.

d) The entire mass spectrum of the leptons and quarks is dominated
fairly well by the \( t \)-quark alone. For example, in the case \( m_t = 100 \text{ GeV} \)
the \( t \)-quark contributes 97.5% to the sum of all fermion masses.

All other quarks, mostly the \( b \)-quark, contribute only 2.5%.

The spectrum exhibits clearly a hierarchical pattern: The masses of a
particular generation of leptons or quarks are small compared to the
masses of the following generation, if there is any, and large compared
to the previous one if there is any. Furthermore another hierarchical pattern emerges if we consider the weak interaction mixing parameters. The mixing matrix, if written in terms of quark mass eigenstates, is not far from the diagonal matrix (no mixing). The mixing angles are typically rather small; the Cabibbo angle being the largest of all, is about $13^\circ$, while $\Theta_{sb}$ is about $2.2^\circ$ and $\Theta_{bd}$ about $0.2^\circ$.

What kind of symmetry could one discuss in view of the observed lepton–quark mass spectrum? The observed two different hierarchies suggest that we are very close to a limit, which I like to call the “rank 1”–limit, in which both the $u$–type and $d$–type mass matrix can be diagonalized at the same time and in which they both take the diagonal form $(0, 0, 1)$, multiplied by $m_t$ or $m_b$ respectively. Thus the masses of the first two generations vanish (the mass matrix has rank one), likewise all mixing angles.

Of course, it depends on yet unknown details of the mass generation mechanism whether such a limit can be achieved in a consistent way. We simply assume that this is the case. In this limit there exists a mass gap: The third generation is split from the massless first two generations. Obviously nature is not far away from this limit, and therefore one is invited to speculate about the dynamical origin of such a situation. A mass matrix proportional to the matrix

$$
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

can always be obtained from another matrix, namely the one in which all elements are equal:
by a suitable unitary transformation. Matrices of this type, which might be called “democratic mass matrices” have been considered recently by a number of authors.

They can be used as a starting point to construct the full mass matrices of the quarks, including the weak interaction mixing terms. We note that such a matrix plays an important role in other fields of physics, where mass gap phenomena are observed:

a) In the BCS theory of superconductivity the energy gap is related to a “democratic” matrix in the Hilbert space of the Cooper pairs.

b) The pairing force in nuclear physics which is introduced in order to explain large mass gaps in nuclear energy levels has the property that the associated Hamiltonian in the space of nucleon pairs has equal matrix elements, i.e., it has a structure of the type given above.

c) The mass pattern of the pseudoscalar mesons in QCD in the chiral limit $m_u = m_d = 0$. In this limit the $\pi^0$ and the $\eta$ are massless Goldstone bosons, while the $\eta'$ acquires a mass due to the gluon anomaly.

Once we write the mass matrices of the leptons and quarks in their “democratic” form, it is obvious that there exists a symmetry, namely the symmetry $S_3$ of permutations among the three different flavors. This symmetry suggests that one should consider the eigenstates of the quarks.
and leptons in this basis as the fundamental dynamical entities. Let us denote them as \((l_1, l_2, l_3)\) and \((q_1, q_2, q_3)\) respectively.

The heaviest lepton and quark, i.e. the \(\tau\)-lepton, the \(t\) and \(b\) quarks, would be coherent states of the type:

\[
\tau = \frac{1}{\sqrt{3}} (l_1 + l_2 + l_3)
\]

etc.

In view of the scarce information we have at present about the internal dynamics of the leptons and quarks we do not know, whether this description of the fermions in terms of coherent states is more than a specific mathematical representation. In a composite model, for example, the fermion states \(f_1, f_2, f_3\) would be those states which are “pure” in a dynamical sense, e.g. they have simple unmixed wave functions.

We remind the reader that also in the case of superconductivity and of the nuclear pairing force the mass eigenstates are coherent superpositions of “physical” states which are described by simple wave functions (e.g. the Cooper pairs in superconductivity).

Within our approach we see a solution to a problem, which has plagued many models of the physics beyond the standard model, the problem of the near masslessness of the first and to some extent also of the second generation. In the coherent state basis this is easily understood. For example, the electron state \(e = 1\sqrt{2}(l_1 - l_2)\) is nearly massless, since there is a nearly complete cancellation of the \(l_1\)– and \(l_2\)–mass terms, as a consequence of the rank one structure of the dominant lepton–quark mass term.

Although this is not the place to discuss the details of the mass generation for the first two generations, it is important to note that in various dynamical schemes, in particular in one based on a composite structure
of the leptons and quarks, the introduction of these small masses leads to slight breakings of the flavor conservation, especially in reactions associated with large momentum transfers. In particular the reaction $e + p \rightarrow \tau + X$, a reaction, which might be found at the HERA machine in Hamburg, is of interest. Moreover rare decays like $t \rightarrow Z + c$ can occur.

In this talk I have described a number of ideas which one might consider after looking at the pattern of masses exhibited in the lepton–quark mass spectrum. I have emphasized the role of symmetries in the space of the generations of the quarks in providing relations between the various mass eigenvalues and the mixing angles. An approach to the flavor problem and to the hierarchical mass spectrum of the leptons and quarks, based on the introduction of coherent states, was discussed. It was argued that the mass generation for the third lepton–quark generation is nothing but a gap phenomenon and is rather similar to the mass generation for the pseudoscalar mesons in QCD. Thus the third lepton–quark generation is somewhat distinct from the other ones. The same mechanism which leads to the mass generation causes the appearance of flavor changing effects; only in the absence of the lepton and quark masses of the first and second generation the various quark and lepton flavors are conserved.

If our interpretation of the mass gap seen in the lepton–quark spectrum is correct, it would mean that all mass gap phenomena seen in physics – superconductivity, nuclear pairing forces, QCD mass gap, lepton–quark mas spectrum – are due to an analogous underlying dynamical mechanism. The exploration of further details of this mechanism could lead soon to a deeper understanding of the physics beyond the standard model.