Application of the unified method to solve the ion sound and Langmuir waves model

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A B S T R A C T

We present the unified method and use it to integrate the ion sound and Langmuir waves (ISLW) model to retrieve optical soliton solutions. Some new dynamical optical solitons involving the combo of rational, trigonometric, and hyperbolic function solutions are added in this study. The derived optical soliton solutions display various properties such as beat pattern and oscillation with increasing, decreasing, and simultaneously increasing and decreasing amplitudes. Moreover, kink, dark bell, singular kink, single breather, multiple breathers, dark-bright, and dark bright periodic waves are founded. Finally, some dynamical characteristics of the acquired solutions are depicted.

1. Introduction

Optical solitons are one of the rising research areas for the development of the telecommunication industry. We cannot imagine the operating of optical fiber, email, internet, mobile phones and many other communications except the idea of a solitary wave [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]. In recent years, nonlocal integrable NLS [16], [17] and mKdV [18], [19] equations have been systematically analyzed through the Hirota bilinear technique [20] and the Riemann-Hilbert algorithm [21]. With the development of the soliton concept, various nonlinear models like the Dullin-Gottwald-Holm model [22], the first integro-differential KP hierarchy model [23], the geophysical Korteweg-de Vries model [24], the Korteweg-de Vries-Burgers model [25], the Jimbo-Miwa model [26], and others [27], [28], [29], [30], [31] are emerging in the telecommunication industry. A well-known model, named as the ion sound and Langmuir waves models, was first presented in 1972 [32] and has since received numerous accolades. This model was extensively employed in various solitary wave propagation including plasma. This model can describe the dynamical behavior of the ion sound wave, which is caused by a high-frequency field for the impact of the ponderomotive force and the Langmuir wave, which is one kind of nonlinear evolution equation [33], [34]. To express soliton solutions of the ISLW model, various integration algorithms such as inverse scattering [35], trial equation scheme [36], Sine-Gordon expansion scheme [37], modified Kudraysoy scheme [38], extended direct algebraic mapping method [39], and others [40], [41] have been studied. The fractional ISLW model is studied in ref. [42], [43], and the fractional generalized HSC KdV model is studied in ref. [44]. The first goal of this manuscript is to acquire optical soliton solutions that clarify the physical structures of the governing model by the unified technique [45], [46], [47]. This technique is a generalization of two well-known schemes, known as tanh-function, and G'/G-expansion, shown by Akdagli and Aydemir [45]. In 2021, Ullah and others found the singular solution to the LPD equation [46]. This method was also used to solve the Biswas-Arshed nonlinear structure in 2022 [47]. We affirm that the optical soliton solution of the ISLW model in this method has not been studied yet.

2. Governing model

The ISLW model has the following form [33], [34]:

\[ iQ_t + \frac{1}{2}Q_{xx} - RQ = 0, \quad R_{tt} - R_{xx} - 2(\left| Q \right|^2)_x = 0. \]  

In Eq. (1), \( Q \) expresses the normalized electric field of the Longmuir oscillation, \( Q \) signifies the electrostatic field excitations, \( \omega_p \) is the

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plasma frequency, and the parameter $R$ denotes the normalized density perturbation. In engineering and applied sciences, especially in high-frequency cases, the ISLW model is applicable for the impact of ponderomotive force.

3. The ODE structure of the model

Assume the subsequent relation for solving Eq. (1):

$$Q(x,t) = U(y)e^{i\psi}, \quad R(x,t) = V(y), \quad \psi = px + gt, \quad \delta = kx + wt,$$

in which $p, g, k, \text{and} w$ are real parameters need to be measured afterward. Combining Eq. (2) and Eq. (1) yields

$$\frac{i(g + pk)U'}{2} + \frac{p^2U''}{2} + 2wU - UV = 0,$$

Separating imaginary and real parts from the first part of Eq. (3) correspondingly gives

$$(g + pk)U' = 0,$$

$$p^2U'' - (2w + k^2)U - 2UV = 0.$$  \hspace{1cm} (5)

From Eq. (4) we get $g = -pk$. Integrating the 2nd part of Eq. (3) two times with regard to $\psi$, we have

$$V(\psi) = \frac{2}{k^2 - 1} U^2(\psi),$$

where $k \neq \pm 1$. From Eq. (5) and Eq. (6) we have

$$p^2 [k^2 - 1] U'' - (k^2 - 1)(2w + k^2)U - 4U^3 = 0.$$  \hspace{1cm} (7)

4. Summary of the unified method and its application

Suppose the trial solution of Eq. (7) is

$$U(\psi) = \sum_{n=0}^{N} l_n \xi(\psi)^n + \sum_{n=1}^{N} \xi(\psi)^{-n},$$

$$\xi(\psi) = \frac{1}{\psi + \chi}.$$  \hspace{1cm} (9)

Eq. (9) contains nine types of solutions in three cases:

**Case-01:** Hyperbolic function (when $M$ is negative):

$$\xi(\psi) = \left\{ \begin{array}{ll}
\sqrt{-G^2 + H^2} M & G \sinh(2\sqrt{-M}(\psi + \chi)) \\
-G \sinh(2\sqrt{-M}(\psi + \chi)) & G \cosh(2\sqrt{-M}(\psi + \chi)) \\
\sqrt{M} - 1 & G \cosh(2\sqrt{-M}(\psi + \chi)) \\
-G \cosh(2\sqrt{-M}(\psi + \chi)) & G \sinh(2\sqrt{-M}(\psi + \chi)) \\
\sqrt{-G^2 + H^2} M & -G \sinh(2\sqrt{-M}(\psi + \chi)) \\
-G \sinh(2\sqrt{-M}(\psi + \chi)) & -G \cosh(2\sqrt{-M}(\psi + \chi)) \\
\sqrt{M} + 1 & -G \cosh(2\sqrt{-M}(\psi + \chi)) \\
-G \cosh(2\sqrt{-M}(\psi + \chi)) & -G \sinh(2\sqrt{-M}(\psi + \chi)) \end{array} \right.$$  \hspace{1cm} (10)

**Case-02:** Trigonometric function (when $M$ is positive):

$$\xi(\psi) = \left\{ \begin{array}{ll}
\sqrt{G^2 + H^2} M & G \sinh(2\sqrt{M}(\psi + \chi)) \\
-G \sinh(2\sqrt{M}(\psi + \chi)) & G \cosh(2\sqrt{M}(\psi + \chi)) \\
\sqrt{M} + 1 & G \cosh(2\sqrt{M}(\psi + \chi)) \\
-G \cosh(2\sqrt{M}(\psi + \chi)) & -G \sinh(2\sqrt{M}(\psi + \chi)) \end{array} \right.$$  \hspace{1cm} (11)

**Case-03:** Rational function (when $M = 0$)

$$\xi(\psi) = \frac{1}{\psi + \chi}.$$  \hspace{1cm} (12)

when $G \neq 0$, $\chi$, and $H$ are arbitrary parameters.

By balancing principle for $U^3$ and $U''$ we have $N = 1$. Putting $N = 1$ in Eq. (8) gives

$$U(\psi) = l_0 + l_1 \xi(\psi) + l_{-1} \xi(\psi)^{-1}.$$  \hspace{1cm} (13)

Combining Eq. (9), Eq. (13) and Eq. (7) and doing simple calculation gives the following three solution sets

$$l_0 = 0, \quad l_{-1} = 0, \quad l_1 = \frac{\sqrt{2}}{2} \sqrt{2k^2 - 2}, \quad m = p^2 M - \frac{1}{2} k^2.$$  \hspace{1cm} (14)

$$l_0 = 0, \quad l_{-1} = 0, \quad l_1 = \frac{\sqrt{2}}{2} \sqrt{2k^2 - 2}, \quad m = p^2 M - \frac{1}{2} k^2.$$  \hspace{1cm} (15)

$$l_0 = 0, \quad l_{-1} = \frac{\sqrt{2}}{2} \sqrt{2k^2 - 2}, \quad l_1 = \frac{\sqrt{2}}{2} \sqrt{2k^2 - 2}, \quad m = -4p^2 M + k^2.$$  \hspace{1cm} (16)

Using Eq. (2), Eq. (6), Eq. (10)–Eq. (12) and Eq. (14) gives the subsequent eighteen exact solutions of Eq. (1).
Again, using Eq. (2), Eq. (6), Eq. (10)-Eq. (12) and Eq. (15) offer the following sixteen exact solutions of Eq. (1).

\begin{align*}
Q_{21}(x,t) &= l_1 \left( \sqrt{- (G^2 + H^2) M - G \sqrt{- M \cos(2 \sqrt{-M}(\psi + \chi))}} \right)^{-1} e^{i \delta}, \\
Q_{22}(x,t) &= l_1 \left( \sqrt{- (G^2 + H^2) M - G \sqrt{- M \cos(2 \sqrt{-M}(\psi + \chi))}} \right)^{-1} e^{i \delta}, \\
Q_{23}(x,t) &= l_1 \left( \sqrt{- M} \right)^{-1} e^{i \delta}, \\
Q_{24}(x,t) &= l_1 \left( \sqrt{- M} \right)^{-1} e^{i \delta}, \\
Q_{25}(x,t) &= l_1 \left( \sqrt{- (G^2 + H^2) M - G \sqrt{- M \cos(2 \sqrt{-M}(\psi + \chi))}} \right)^{-1} e^{i \delta}, \\
Q_{26}(x,t) &= l_1 \left( \sqrt{- (G^2 + H^2) M - G \sqrt{- M \cos(2 \sqrt{-M}(\psi + \chi))}} \right)^{-1} e^{i \delta}, \\
Q_{27}(x,t) &= l_1 \left( \sqrt{M} \right)^{-1} e^{i \delta}.
\end{align*}

where \( \psi = px + gt \), \( \delta = kx + mt \), \( p = \sqrt{p^2 - \frac{1}{4}} \) and \( l_1 = \pm \sqrt{2k^2 - 2} \).

Using Eq. (2), Eq. (6), Eq. (10)-Eq. (12) and Eq. (16) grant the next eighteen exact solutions of Eq. (1).

\begin{align*}
Q_{31}(x,t) &= l_1 \left( \sqrt{- (G^2 + H^2) M - G \sqrt{- M \cos(2 \sqrt{-M}(\psi + \chi))}} \right)^{-1} \frac{G \sinh(2 \sqrt{-M}(\psi + \chi)) + H}{G \sinh(2 \sqrt{-M}(\psi + \chi)) + H} e^{i \delta}, \\
Q_{32}(x,t) &= l_1 \left( \sqrt{- (G^2 + H^2) M - G \sqrt{- M \cos(2 \sqrt{-M}(\psi + \chi))}} \right)^{-1} \frac{G \sinh(2 \sqrt{-M}(\psi + \chi)) + H}{G \sinh(2 \sqrt{-M}(\psi + \chi)) + H} e^{i \delta}, \\
Q_{33}(x,t) &= l_1 \left( \sqrt{- M} \right)^{-1} e^{i \delta}, \\
Q_{34}(x,t) &= l_1 \left( \sqrt{- M} \right)^{-1} e^{i \delta}, \\
Q_{35}(x,t) &= l_1 \left( \sqrt{- (G^2 + H^2) M - G \sqrt{- M \cos(2 \sqrt{-M}(\psi + \chi))}} \right)^{-1} \frac{G \sinh(2 \sqrt{-M}(\psi + \chi)) + H}{G \sinh(2 \sqrt{-M}(\psi + \chi)) + H} e^{i \delta}, \\
Q_{36}(x,t) &= l_1 \left( \sqrt{- (G^2 + H^2) M - G \sqrt{- M \cos(2 \sqrt{-M}(\psi + \chi))}} \right)^{-1} \frac{G \sinh(2 \sqrt{-M}(\psi + \chi)) + H}{G \sinh(2 \sqrt{-M}(\psi + \chi)) + H} e^{i \delta}, \\
Q_{37}(x,t) &= l_1 \left( \sqrt{M} \right)^{-1} e^{i \delta}.
\end{align*}
\[ Q_{34}(x, t) = \begin{bmatrix} l_1 \left( -i \sqrt{M} - \frac{2G \sqrt{M}}{G + \cos(2\sqrt{M}(\psi + \chi))} \right) + l_{1-1} \left( -i \sqrt{M} + \frac{2G \sqrt{M}}{G + \cos(2\sqrt{M}(\psi + \chi))} \right) \end{bmatrix} e^{it}. \]

\[ Q_{35}(x, t) = -\left[ \frac{l_1}{\psi + \chi} + l_{1-1} \left( \frac{1}{\psi + \chi} \right) \right] e^{it}. \]

\[ R_{36}(x, t) = \begin{bmatrix} l_1 \left( \frac{\sqrt{M} + \frac{2G \sqrt{M}}{G + \cos(2\sqrt{M}(\psi + \chi))}}{G \sinh(2\sqrt{M}(\psi + \chi)) + H} \right)^2 + l_{1-1} \left( \frac{\sqrt{M} - \frac{2G \sqrt{M}}{G + \cos(2\sqrt{M}(\psi + \chi))}}{G \sinh(2\sqrt{M}(\psi + \chi)) + H} \right)^2 \end{bmatrix} \frac{1}{k^2 - 1}. \]

\[ R_{37}(x, t) = \begin{bmatrix} l_1 \left( \frac{-2G \sqrt{M}}{G + \cos(2\sqrt{M}(\psi + \chi))} \right)^2 + l_{1-1} \left( \frac{-2G \sqrt{M}}{G + \cos(2\sqrt{M}(\psi + \chi))} \right)^2 \end{bmatrix} \times \frac{1}{k^2 - 1}. \]

\[ R_{38}(x, t) = \begin{bmatrix} l_1 \left( \frac{1}{\psi + \chi} \right)^2 + l_{1-1} \left( \frac{1}{\psi + \chi} \right)^2 \end{bmatrix} \frac{1}{k^2 - 1}. \]

\[ \text{where } \psi = px + gt, \delta = kx + mt, m = -\frac{\sqrt{2} \sqrt{k^2 - 2}}{2}, \quad l_1 = \pm \frac{\sqrt{2} \sqrt{k^2 - 2}}{2}, \quad l_{-1} = \pm \frac{\sqrt{2} \sqrt{k^2 - 2}}{2}. \]

5. Figure analysis

The solutions \( Q_{11}, Q_{12}, Q_{21}, Q_{22}, Q_{31}, \) and \( Q_{32} \) give similar behavior as depicted in Fig. 1 (a-c) by \( Q_{21} \). The character of this graph changes based on the condition for parameter \( x \). We see that Fig. 1 (a) displays oscillations with rapidly growing amplitudes at \( x = -1 \); Fig. 1 (b) illustrates oscillations with rapidly reducing amplitudes, then drops to 0, and then again oscillations with rapidly growing amplitudes at \( x = -1/7 \); and Fig. 1 (c) exhibits oscillations with rapidly reducing amplitudes at \( x = 1 \). A combination of rational polynomials and periodic solitons produces periodic waves \( Q_{11} \) and \( Q_{32} \), as depicted in Fig. 2 (a-d) by \( Q_{32} \). The combo periodic optical soliton solutions \( Q_{13}, Q_{23}, \) and \( Q_{33} \) exhibit similar behavior and can increase or decrease wave amplitudes. Optical communication systems make use of this property extensively. In particular, we depicted the solution \( Q_{13} \) in Fig. 3 (a-c). We see that for \( k > 0 \), the wave amplitude increases with a constant height after a definite time (see Fig. 3 (a)); for \( k = 0 \) (see Fig. 3 (b)), it remains the same; and for \( k > 0 \), it decreases (see Fig. 3 (c)). The solutions \( Q_{14}, Q_{24}, \) and \( Q_{34} \) represent double periodic optical solutions as depicted in Fig. 4 (a-d) by \( Q_{34} \). This diagram displays oscillations.
Fig. 2. Outlook of wave solution \( Q_{39} \) for \( p = 5, k = 4, M = 0, G = 4, H = 1, \lambda = 1, l_0 = l_{-1} = 0 \): (a, c) cubic wave form, (b, d) planner wave form.

Fig. 3. Plot of wave solution \( Q_{15} \) for \( p = 10, M = -1, G = 1, H = 1, \lambda = 1, l_0 = l_{-1} = 0 \) at \( x = 0 \).

Fig. 4. Profile of solution \( Q_{14} \) for \( p = 10, k = -0.1, M = -0.5, G = H = \chi = 1, l_0 = l_{-1} = 0 \): (a, c) cubic wave form, (b, d) planner wave form.

with rapidly decreasing amplitudes, which diminish to 0 at the origin and then increase rapidly. Moreover, the solutions \( Q_{17}, Q_{18}, Q_{19}, Q_{20}, Q_{21}, Q_{22}, Q_{23}, Q_{24}, Q_{25}, Q_{26}, Q_{27}, Q_{28}, Q_{29}, Q_{30}, Q_{31}, Q_{32}, Q_{33}, Q_{34}, Q_{35}, Q_{36}, Q_{37}, Q_{38}, Q_{39} \) and \( Q_{40} \) also display beat pattern, as shown by the solution \( Q_{25} \) in Fig. 5 (a-f). When \( G = 1 \), the solution represents a singular wave (see Fig. 5 (a-c)). When \( G = 2 \), the solution is not a singular wave (see Fig. 5 (d-f)). The solutions \( R_{13}, R_{23}, R_{14} \) and \( R_{15} \) exhibit kink soliton, as pictured by the solution \( R_{13} \) in Fig. 6.1 (a, b). \( R_{14} \) and \( R_{15} \) exhibit dark bell soliton solution, as pictured by the solution \( R_{14} \) in Fig. 6.2 (a, b). \( R_{21}, R_{22}, R_{31}, R_{32} \) exhibit singular kink soliton, as pictured by the solution \( R_{22} \) in Fig. 6.3 (a, b). The solutions \( R_{11}, R_{12}, R_{19}, R_{24}, R_{25} \) and \( R_{30} \) exhibit a single breather wave solution as shown in Fig. 7.1 (a, b) by the solution \( R_{11} \). The solutions \( R_{17}, R_{18}, R_{22}, R_{27}, R_{37} \) and \( R_{38} \) exhibit multiple breather wave solutions as depicted in Fig. 7.2 (a, b) by \( R_{17} \). The solutions \( R_{15}, R_{16}, R_{25}, R_{26}, R_{35} \) and \( R_{36} \) exhibit the same solution as plotted by the solution \( R_{25} \) in Fig. 8 (a-c). This solution represents a dark periodic wave if \( G^2 < H^2 \) (see Fig. 8 (a)), a dark-bright periodic wave if \( G = \pm H \) (see Fig. 8 (b)), and a bright periodic wave if \( G^2 > H^2 \) (see Fig. 8 (c)). We can say that the results acquired for the ISLW model in the unified approach are unprecedented and newest.

6. Conclusion

This paper has presented the unified method to integrate the ISLW model to retrieve optical solitons that can be used in birefringent fibers. Fig. 2 illustrates periodic waves whose amplitudes increase and decrease. The derived optical soliton solutions exhibit some dynamics as beat pattern, oscillation together increasing, decreasing, and jointly increasing-decreasing amplitudes as shown in Fig. 1, 3–5. Kink, dark bell, and singular kink waves are shown in Fig. 6. Single breather and multiple breather waves are shown in Fig. 7. Moreover, dark, dark-bright, and bright solitons are shown in Fig. 8. In mathematics and engineering, the mentioned technique is an elegant algorithm for finding optical solitons of nonlinear complex models. In addition, applications of fractional differential equations to ISLW models are described in Ref. [42], [43]. In future studies, we need to develop the fractional differential equation of this model by the unified method.
Fig. 5. Shape of $Q_{13}$ for $p = 5$, $k = -1/5$, $M = 2$, $H = \chi = 1$, $l_0 = l_1 = 0$: (a-c) at $G = 1$, (d-f) at $G = 2$ (a, d) 3D view, (b, e) 2D view, and (c, f) absolute value view.

Fig. 6.1. Outlook of $R_{13}$ for $p = 1$, $k = 2$, $M = -0.5$, $G = 4$, $H = \chi = 1$, $l_0 = l_1 = 0$: (a) 3D surface, (b) 2D surface.

Fig. 6.2. Outlook of $R_{14}$ for $p = 1$, $k = -1/10$, $M = -1/2$, $G = H = \chi = 1$, $l_0 = l_1 = 0$: (a) 3D surface, (b) 2D surface.

Fig. 6.3. Outlook of $R_{22}$ for $p = 2$, $k = -5$, $M = -2$, $G = 3$, $H = \chi = 1$, $l_0 = l_1 = 0$: (a) 3D surface, (b) 2D surface.
Fig. 7.1. Graphics of $R_{1}$ for $p = 5$, $k = -1/2$, $M = -1$, $G = 4$, $H = \chi = 1$, $l_{0} = l_{-1} = 0$: (a) 3D view, (b) 2D view.

Fig. 7.2. Graphics of $R_{1}$ for $p = 1$, $k = -3$, $M = 2$, $G = H = \chi = 1$, $l_{0} = l_{-1} = 0$: (a) 3D shape, (b) 2D shape.

Fig. 8. Wave pattern of wave solution $R_{23}$ for $p = 1$, $k = -1/3$, $M = 1/2$, $\chi = l_{1}$, $l_{0} = l_{-1} = 0$: (a) dark periodic wave at $G = 2$, $H = 3$, (b) dark-bright periodic wave at $G = H = 3$, and (c) bright periodic wave at $G = 4$, $H = 3$.

Declarations

Author contribution statement

Dulal Chandra Nandi: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data. Mohammad Safi Ullah: Conceived and designed the experiments; Wrote the paper. Harun-Or-Roshid; M. Zulfikar Ali: Performed the experiments.

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The authors declare no conflict of interest.

Additional information

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References

[1] H. Triki, A. Biswas, S.P. Moshokoa, M. Belic, Optical solitons and conservation laws with quadratic-cubic nonlinearity, Optik 128 (2017) 63-70.
[2] B.Q. Li, Y.L. Ma, Periodic solutions and solitons to two complex short pulse (CSP) equations in optical fiber, Optik 144 (2017) 149-155.
[3] M. Arshad, A.R. Seadawy, D. Lu, Exact bright-dark solitary wave solutions of the higher-order cubic-quintic nonlinear Schrödinger equation and its stability, Optik 138 (2017) 40-49.
[4] B. Kühler, J. Fatome, C. Finot, G. Millot, F. Dias, G. Genty, N. Akhmediev, J.M. Dudley, The Peregrine soliton in nonlinear fiber optics, Nat. Phys. 6 (2010) 790–795.
[5] B.Q. Li, J.Z. Sun, Y.L. Ma, Soliton excitation for a coherently coupled nonlinear Schrödinger system in optical fibers with two orthogonally polarized components, Optik 175 (2018) 275-283.
[6] B.Q. Li, Y.L. Ma, Periodic and N-kink-like optical solitons for a generalized Schrödinger equation with variable coefficients in an inhomogeneous fiber system, Optik 179 (2019) 854-860.
[7] M.S. Ullah, H.O. Roshid, W.K. Ma, M.Z. Ali, Z. Rahman, Interaction phenomena among lump, periodic and kink wave solutions to a $(3+1)$-dimensional Sharma-Tasso-Olver-like equation, Chin. J. Phys. 68 (2020) 699–711.
[8] M.F. Hoque, H.O. Roshid, Optical soliton solutions of the Biswas-Arshed model by the tanh(θ/2) expansion method, Phys. Scr. 95 (2020) 075219.
[9] M.S. Ullah, M.Z. Ali, H.O. Roshid, A.R. Seadawy, D. Baleanu, Collision phenomena among lump, periodic and soliton solutions to a $(2+1)$-dimensional Bogoyavlenski’s breaking soliton model, Phys. Lett. A 397 (2021) 127263.
[10] M.S. Ullah, H.O. Roshid, M.Z. Ali, Z. Rahman, Dynamical structures of multi-soliton solutions to the Bogoyavlenski’s breaking soliton equations, Eur. Phys. J. Plus 135 (3) (2020) 262.
[11] M.S. Ullah, O. Ahmed, M.A. Mahbub, Collision phenomena between lump and kink wave solutions to a $(3+1)$-dimensional Jimbo-Miwa-like model, Partial Differ. Equa. Appl. Math. 5 (2022) 100324.
[12] M.S. Ullah, H.O. Roshid, F.S. Alshammari, M.Z. Ali, Collision phenomena among the solitons, periodic and Jacobi elliptic functions to a $(3+1)$-dimensional Sharma-Tasso-Olver-like model, Results Phys. 36 (2022) 105412.
[13] M.S. Ullah, H.O. Roshid, M.Z. Ali, N.F.M. Noor, Novel dynamics of wave solutions for Cahn-Allen and diffusive predator-prey models using MSE scheme, Partial Differ. Equa. Appl. Math. 3 (2021) 100017.
[14] T. Sulaïman, A. Yusuf, A. Abdellajbah, M. Alquran, Dynamics of lump collision phenomena to the $(3+1)$-dimensional nonlinear evolution equation, J. Geom. Phys. 169 (2021) 104347.
[15] W.X. Ma, B. Fuchsteiner, Explicit and exact solutions to a Kolmogorov-Petrovskii-Piskunov equation, Int. J. Non-Linear Mech. 31 (1996) 329–338.
[16] W.X. Ma, Riemann-Hilbert problems and soliton solutions of nonlocal reverse-time NLS hierarchies, Acta Math. Sci. 42B (1) (2021) 1–14.
[17] W.X. Ma, X.H. Huang, F.D. Wang, Inverse scattering transforms for non-local reverse-space matrix non-linear Schrödinger equations, J. Appl. Math. 33 (2022) 21.
[18] W.X. Ma, Reduced nonlocal integrable mKdV equations of type $(\sigma, J)$ and their exact soliton solutions, Commun. Theor. Phys. 74 (2022) 065002.
[19] W.X. Ma, Nonlocal integrable mKdV equations by two nonlocal reductions and their soliton solutions, J. Geom. Phys. 177 (2022) 105422.
V.E. M.S. A.R. W.X. S.T.R. A.R. M. D.C. Nandi, M.S. Ullah, H.-O. Roshid et al. Heliyon 8 (2022) e10924

Mahony-Burgers
gation
Burgers
using
Random
ferromagnetic
4094–4104.

problems, Partial Differ. Equ. Appl. Math. 4 (2021) 100190.

Dispersion of propagation wave solutions to unidirectional shallow water wave Dullin-Gottwald-Holm system and modulation instability analysis, Math. Methods Appl. Sci. 44 (5) (2021) 4094–4104.

Analytical wave solutions of the (2+1)-dimensional first integro-differential Kadomtsev-Petviashvili hierarchy equation by using modified mathematical methods, Results Phys. 15 (2019) 102775.

Burgers equation as a geophysical Korteweg-de Vries equation, Results Phys. 19 (2020) 103661.

H. Ahmad, A.R. Seadawy, T.A. Khan, Numerical solution of Korteweg-de Vries-Burgers equation by the modified variational iteration algorithm-II arising in shallow water waves, Phys. Scr. 95 (2020) 045210.

A transformed rational function method and exact solutions to the 3+1 dimensional Jimbo-Miwa equation, Chaos Solitons Fractals 42 (2009) 1356–1363.

I. Ali, A.R. Seadawy, S.T.R. Rizvi, M. Younis, K. Ali, Conserved quantities along with Painleve analysis and optical solitons for the nonlinear dynamics of Heisenberg ferromagnetic spin chains model, Int. J. Mod. Phys. B 34 (2020) 2050283.

A.R. Seadawy, M. Arshad, D. Lu, The weakly nonlinear wave propagation of the generalized third-order nonlinear Schrödinger equation and its applications, Waves Random Complex Media 32 (2) (2022) 819–831.

A.R. Seadawy, M. Arshad, D. Lu, The weakly nonlinear wave propagation theory for the Kelvin-Helmholtz instability in magnetohydrodynamics flows, Chaos Solitons Fractals 139 (2020) 110141.

L. Lu, A.R. Seadawy, M. Arshad, Bright-dark optical soliton and dispersive elliptic function solutions of unstable nonlinear Schrödinger equation and its applications, Opt. Quantum Electron. 50 (2018) 23.

M.S. Ullah, M.Z. Ali, H.O. Roshid, M.F. Hoque, Collision phenomena among lump, periodic and stripe soliton solutions to a (2+1)-dimensional Benjamin-Bona-Mahony-Burgers model, Eur. Phys. J. Plus 136 (4) (2021) 370.

V.E. Zakharov, Collapse of Langmuir waves, Zh. Eksp. Teor. Fiz. 62 (1972) 1745–1759.

A. Tripathy, S. Saohee, Exact solutions for the ion sound Langmuir wave model by using two novel analytical methods, Results Phys. 19 (2020) 103494.