QCD Factorization and Rare $B$ Meson Decays

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Some recent progress in China in the study of charmless $B$ decays with QCD factorization is reviewed. Chirally enhanced power corrections and infrared divergence problem are stressed.

1. Introduction

$B$ meson weak decays are very important for testing the standard model and probing new physics. The two $B$ factories have accumulated large data sets while Tevatron and LHCb will have even more data. The main task for theorists is to compute all experimental observables in a reliable way. This amounts to compute hadronic matrix elements reliably. There are several methods in the literature. The earliest is the so called naive factorization 1. In this method the matrix element has no renormalization scheme and scale dependence, so cannot cancel the corresponding dependence of the Wilson coefficients. Further more, it cannot account for “nonfactorizable” contributions. The generalized factorization 2 takes into account the one-loop radiative corrections to recover the renormalization scheme and scale dependence for the hadronic matrix elements. But at quark level, to avoid the infrared divergence, it has to assume the external quarks to be off-shell by $-p^2$. This results in gauge dependence. To account for “nonfactorizable” contributions, a phenomenological parameter $N_{c}^{\text{eff}}$ is introduced and it is assumed that $N_{c}^{\text{eff}}$ is universal. But actually $N_{c}^{\text{eff}}$ is process-dependent.

In order to overcome the shortcomings of the above methods, in 1999, Beneke, et al. (BBNS) 3 proposed a new scheme based on QCD. In this talk, I will concentrate on our work 4, 5, 6, 7, 8 about the chirally enhanced power corrections and infrared divergence problem and the application of the QCD factorization approach. My talk is organized as follows: In Section 2, I shall give an simple introduction to QCD factorization. Section 3 is devoted to the chirally enhanced power corrections and the cancellation of the infrared divergence in vertex corrections. In section 4~6,
the application of QCD factorization including chirally enhanced power corrections to the two-body charmless $B$-decays is reviewed. Section 7 is for conclusions.

2. General remarks on QCD factorization

Beneke, et al. (BBNS) proposed a new factorization scheme based on QCD. They pointed out that, in the heavy quark limit ($m_b \to \infty$)

$$\langle M_1 M_2 | Q_i | B \rangle = F_{B \to M_2}^{Q_i} (q^2) \int_0^1 dx T_I^i (x) \Phi_{M_1} (x) + (M_1 \leftrightarrow M_2) + \sum_j \int_0^1 d\xi dx dy T_{ij}^{II} (\xi, x, y) \Phi_B (\xi) \Phi_{M_1} (x) \Phi_{M_2} (y) + \mathcal{O}(\Lambda_{QCD}/m_b) \quad (1)$$

where the short distance hard scattering kernels $T_I^{I,II}$ is calculable order by order in perturbative theory; the long distance quantities, e.g. decay constants, form factors, light-cone distribution amplitudes are inputs from either experimental measurements or other theory; $M_1$ is a light meson or a charmonium state, $M_2$ (contains the spectator in $B$) is any light or heavy meson. If $M_2$ is heavy (for example $D$), the second line in Eq.(1) is $1/m_b$ suppressed.

At the zeroth order of $\alpha_s$, Eq.(1) reduces to naive factorization. At the higher order, the corrections can be computed systematically. The renormalization scheme and scale dependence of $\langle Q_i \rangle$ is restored. In the heavy quark limit, the “nonfactorization” contributions is calculable perturbatively. It does not need to introduce $N_c^{\text{eff}}$. The strong phase is suppressed by either $\alpha_s$ or $1/m_b$. $W$-exchange and $W$-annihilation diagrams are $1/m_b$ suppressed. There is no long distance interactions between $M_1$ and $(BM_2)$. $T_I^i$ includes tree diagram, non-factorizable gluon exchange, e.g. vertex corrections [Fig.1(a)-(d)], penguin corrections [Fig.1(e)-(f)], $\cdots$. $T_{ij}^{II}$ includes hard spectator scattering [Fig.1(g)-(h)], $\cdots$. Later on we shall apply Eq.(1) to $B$ meson rare decays.
3. Chirally Enhanced Power Corrections and Infrared Divergence
Cancellation in Vertex Corrections

Superficially, $\Lambda_{QCD}/m_b \sim 1/15$ is a small number. But in some cases, such power suppression fails numerically. For example

$$\langle Q_6 \rangle_F = -2 \sum_{q} \langle P_1 | (\bar{q}q')_{S-P} | 0 \rangle \langle P_2 | (\bar{q}'b)_{S+P} | B \rangle$$

is always multiplied by a formally power suppressed but chirally enhanced factor $r_\chi = 2\mu_P/m_b$ (where $\mu_P = m_P^2/(m_1 + m_2)$, $m_{1,2}$ are current quark masses, and $P_{1,2}$ denote pseudoscalar mesons).

In the heavy quark limit, $m_b \to \infty$, $r_\chi \to 0$. But for $m_b \sim 5\text{GeV}$, $r_\chi \sim \mathcal{O}(1)$, no $1/m_b$ suppression. So we should consider the chirally enhanced power corrections. For example, in $B \to \pi K$, dominant contribution to the amplitude $\sim a_4 + a_6 r_\chi$, and $a_4 \sim a_6 r_\chi$, so one half comes from the chirally enhanced power corrections.

Possible sources of power corrections are: high twist wave functions, quark transverse momentum $k_\perp$, and annihilation topology diagrams. In this talk, I only discuss the chirally enhanced power corrections from high twist wave functions.

All the chirally enhanced power corrections involve twist-3 light cone distribution amplitudes $\Phi_p(x)$ and $\Phi_s(x)$. If we want to include chirally enhanced power corrections consistently, we must prove that the hard scattering kernels are infrared finite, at least for vertex corrections.

The decay amplitude of $B \to M_1 M_2$ reads

$$A(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_{q=(u,c)} \sum_{i=1} v_q a_i^q(\mu) \langle M_1 M_2 | Q_i | B \rangle_F$$

For the hard kernel $T_i'$ in Eq. (11), there is no contributions from twist-3 wave function insertion for $(V - A) \otimes (V - A)$ and $(S + P) \otimes (S - P)$ operators. But for $(V + A) \otimes (V - A)$, it is subtle! For twist-3 light cone distribution amplitudes, if we use dimensional regularization, the infrared divergences of Fig.1(a)-(d) can not cancel! This is because the twist-3 wave functions are only defined in $4 = 3 + 1$ dimensions. We need to give gluon a small mass and regularize the infrared integrals in 4-dimensions.

After a lengthy calculation, for the operator $Q_5$, the vertex correction terms are

$$\text{Fig.1(a)} \sim - \frac{\alpha_s C_F}{4\pi N_c} \frac{\Phi_\sigma(v)}{v} \left[ \frac{\ln^2 \lambda}{2} + 2\ln(-v)\ln\lambda - 4\ln v \ln\lambda + \ln\lambda + \text{finite terms} \right]$$

$$\text{Fig.1(b)} \sim + \frac{\alpha_s C_F}{4\pi N_c} \frac{\Phi_\sigma(v)}{\bar{v}} \left[ \frac{\ln^2 \lambda}{2} + 2\ln(-\bar{v})\ln\lambda - 4\ln \bar{v} \ln\lambda + \ln\lambda + \text{finite terms} \right]$$

$$\text{Fig.1(c)} \sim + \frac{\alpha_s C_F}{4\pi N_c} \frac{\Phi_\sigma(v)}{v} \left[ \frac{\ln^2 \lambda}{2} + 2\ln(-v)\ln\lambda + 3\ln\lambda + \text{finite terms} \right]$$

$$\text{Fig.1(d)} \sim - \frac{\alpha_s C_F}{4\pi N_c} \frac{\Phi_\sigma(v)}{\bar{v}} \left[ \frac{\ln^2 \lambda}{2} + 2\ln(-\bar{v})\ln\lambda + 3\ln\lambda + \text{finite terms} \right]$$

where $\lambda = m_g^2/m_b^2$. 
In the above, we can see that after summing over Fig.1(a)-(d), the infrared divergences cancel only when \( \Phi_{\sigma}(v) = \Phi_{\sigma}(\bar{v}) \), where \( \bar{v} = 1 - v \). This is not a surprise because we neglected the contribution of three-body twist-3 light cone distribution amplitudes (LCDAs). For the asymptotic form, \( \Phi_{\sigma}(v) = 6v\bar{v} \) which is the same for both \( \pi \) and \( K \) meson.

After our work [4], BBNS [9] also discussed the infrared divergence problem. They implemented the equation of motion

\[
\bar{u}(k_1) \cdot j_1 \cdots v(k_2) = 0
\]

(8)
to show the infrared safety without assuming \( \Phi_{\sigma}(v) \) to be symmetric. However Eq. (8) is justified only when 2-particle twist-3 LCDAs are asymptotic, i.e. \( \Phi_{\rho}(v) = 1 \) and \( \Phi_{\sigma}(v) = 6v\bar{v} \) by neglecting the contribution from the 3-particle twist-3 LCDAs. Our result is based on a more general situation than that BBNS considered. Therefore, BBNS result is consistent with ours.

We can prove at the order of \( O(\alpha_s) \) that the decay amplitude is independent of the renormalization scale. We can also prove the gauge invariance when chirally enhanced power corrections are included. The detailed proof can be found in [4]. We do not discuss it here. Now we come to the application below.

4. \( B \to PP, PV \) Charmless Decays and \( CP \) Violation

Now we give the prediction of the branching ratios and \( CP \) asymmetries. We included the contributions of the hard spectator scattering and annihilation topology. We also include the chirally enhanced power corrections. We compute the branching ratios \( (Br) \) and \( CP \) asymmetries of \( B \to PP : \pi\pi, \pi K, KK, K\eta(\prime), \pi\eta(\prime), \eta(\prime)\eta(\prime), \cdots \); \( B \to PV : \pi\rho, \pi\omega, \pi K^* \rho K, \omega K, K^*\rho(\prime), \rho\eta(\prime), \cdots \). All our numerical results can be found in the tables in Reference [5,6]. From our calculated result we see that:

- Owing to including the chirally enhanced power corrections, the scale \( \mu \) dependence of \( Br \) is smaller; \( Br \) of \( B^\pm \to \pi^0 \pi^- \) (pure tree) and \( B^\pm \to K^0\pi^\pm \) (pure penguin) are in good agreement with data; \( Br \) of \( B^0 \to \pi^+\pi^- \) is larger than data; \( Br \) of \( B \to \pi K \) seem smaller than data; For \( B \to K\eta(\prime) \), if consider di-gluon fusion, we can fit the data; For \( B^0 \to \pi^0\pi^0 \), our prediction is much smaller than data; For \( B^0 \to K^+K^- \), the result is very small, only weak annihilation diagram contributes; There are large uncertainties from CKM elements, form-factors, annihilation parameters. But a global analysis can fit the data very well.

- For \( CP \) asymmetries, the numerical results are not reliable because vertex, penguin, hard spectator scattering and annihilation diagrams can all give imaginary part to decay amplitudes, so strongly affect \( A_{CP} \). Because of space limitation, for more detail, see [5]. For \( B \to PV \), the differences from \( B \to PP \) are: If the emitted meson is vector meson (vertex diagrams and penguins), then the twist-3 wave functions of \( V \) do not contribute (power suppressed, so can be neglected). But for hard spectator scattering, even the emitted meson is vector one, there are still twist-3 contributions. Note, there are large uncertainties on \( CP \) asymmetries. So we cannot
make good predictions on $A_{CP}$ in QCDF scheme. The calculated branching ratios of $B \rightarrow PV$ for $b \rightarrow d$ and $b \rightarrow s$ transitions and $CP$ asymmetries can be found in Reference 7.

5. Global Analysis of $B \rightarrow PP$, $PV$ Charmless Decays

Beneke, et al. have done a global analysis on $B \rightarrow PP$. But their fitted $\gamma \sim 90^\circ$, a bit large compare with the standard CKM fit. Now there are many new data on both $B \rightarrow PP$ and $PV$. It is necessary to do the global fit of $B \rightarrow PP$, $PV$ at the same time. We have done it!

For $B \rightarrow \pi\pi$, $\pi K$, the fit is sensitive to $|V_{ub}|$, $\gamma (\rho, \eta)$, $F_{B \rightarrow \pi}$, $F_{B \rightarrow K}$, $X_A$, $f_B/\lambda_B$, and $m_s$. To include seven $B \rightarrow PV$ decay channels, only $A_0 \rightarrow \rho$, $X_{PV}^\gamma$ are newly involved sensitive parameters. So the inclusion of $B \rightarrow PV$ will lead more stringent test to QCDF. The experimental constrains of $CP$ asymmetries are not implemented, because the QCDF’s predictions on $CP$ asymmetries in $B$ decays are rough! So we do not use $CP$ asymmetries for the global fit. The best fit values of the branching fractions is presented in Table 1.

Table 1. Fit1 and Fit2 mean the best fit values of $Br$ in unit of $10^{-6}$ with and without the contributions of the chirally enhanced hard spectator and annihilation topology, respectively.

| modes                | Exp. | Fit1  | Fit2  | modes                | Exp. | Fit1  | Fit2  |
|----------------------|------|-------|-------|----------------------|------|-------|-------|
| $B^0 \rightarrow \pi^\pm \pi^\mp$ | 4.77±0.54 | 4.82  | 5.68  | $B^+ \rightarrow \pi^+ \pi^0$ | 5.78±0.95 | 5.35  | 3.25  |
| $B^0 \rightarrow K^+\pi^-$          | 18.5±1.0   | 19.0  | 18.8  | $B^+ \rightarrow K^+\pi^0$      | 12.7±1.2   | 11.4  | 12.6  |
| $B^+ \rightarrow K^0\pi^+$          | 18.1±1.7   | 20.1  | 20.2  | $B^0 \rightarrow K^0\pi^0$      | 10.2±1.5   | 8.2   | 7.3   |
| $B^+ \rightarrow \eta\pi^+$          | <5.2        | 2.8   | 1.8   | $B^0 \rightarrow \pi^+\rho^-$    | 25.4±4.3   | 26.7  | 29.5  |
| $B^+ \rightarrow \pi^+\rho^0$       | 8.6±2.0    | 8.9   | 8.5   | $B^0 \rightarrow K^+\rho^-$      | 13.1±4.7   | 12.1  | 5.1   |
| $B^+ \rightarrow \phi K^+$           | 8.9±1.0    | 8.9   | 7.1   | $B^0 \rightarrow \phi K^0$       | 8.6±1.3    | 8.4   | 6.7   |
| $B^+ \rightarrow \eta\rho^+$         | <6.2        | 4.6   | 3.8   | $B^0 \rightarrow \omega K^0$     | 5.9±1.9    | 6.3   | 1.2   |

The main results are:

The best fit of $\gamma \sim 79^\circ$ which is consistent with recent fit results $37^\circ < \gamma < 86^\circ$.

For $B \rightarrow \pi^0\pi^0$, the best fit is around $1 \times 10^{-6}$.

Exp: the BaBar and Belle average is $(1.90±0.49)\times10^{-6}$

For $B^+ \rightarrow \omega K^+$, the best fit is $6.25\times10^{-6}$.

Exp: Belle $(9.2^{+2.6}_{-2.3}\pm1.0)\times10^{-6}$ CLEO $<8\times10^{-6}$.

For $B^+ \rightarrow \omega\pi^+$, the best fit is $6.66\times10^{-6}$.

Exp: BABAR $(6.6^{+2.1}_{-1.8}\pm0.7)\times10^{-6}$ Belle $<8.2\times10^{-6}$

For $B^+ \rightarrow \pi^+K^{*0}$, the best fit $\sim 10^{-6}$.

Exp: LP03 $\sim (10.3\pm1.2^{+1.0}_{-1.2})\times10^{-6}$ ICHEP04 $\sim (9.0\pm1.3)\times10^{-6}$.

For more details, see Reference 11.
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6. $B_s \to PP, PV$ Charmless Decays

We include: i) chirally enhanced power corrections, ii) weak annihilation, iii) hard spectator scattering. We list the computed branching ratios and $CP$ asymmetries of $B_s \to PP, PV$ in Reference 8. Only $B_s \to K^{(*)} K (10^{-6} \sim 10^{-5})$, $K^{(*)} \pi^\mp (10^{-6})$, $\rho^\mp (10^{-5})$, $\eta^{(*)} \eta^{(*)} (10^{-6} \sim 10^{-5})$ have large branching ratios.

7. Conclusions

We have shown that

- chirally enhanced power corrections must be included in QCDF
- twist-3 light cone distribution amplitude $\Phi_p(x), \Phi_\sigma(x)$ must be considered simultaneously
- the infrared divergences in vertex corrections cancel only twist-3 wave function of light pseudoscalar is symmetric, so chirally enhanced power enhanced power corrections can be included consistently
- the calculated branching ratios for $B \to PP, PV$ charmless decays are, principally, in agreement with data
- global analysis of $B \to PP, PV$ can fit the data very well including $B^0 \to \pi^0\pi^0$ and $\gamma \sim 79^\circ$
- $B_s \to PP, PV$ charmless decays are waiting to be tested in Tevatron and LHCb

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