Learning to Personalize Treatments When Agents Are Strategic

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Abstract

There is increasing interest in allocating treatments based on observed individual data: examples include targeted marketing, individualized credit offers, and heterogeneous pricing. Treatment personalization introduces incentives for individuals to modify their behavior to obtain a better treatment. This shifts the distribution of covariates, requiring a new definition for the Conditional Average Treatment Effect (CATE) that makes explicit its dependence on how treatments are allocated. We provide necessary conditions that treatment rules under strategic behavior must meet. The optimal rule without strategic behavior allocates treatments only to those with a positive CATE. With strategic behavior, we show that the optimal rule can involve randomization, allocating treatments with less than 100% probability even to those with a positive CATE induced by that rule. We propose a dynamic experiment based on Bayesian Optimization that converges to the optimal treatment allocation function without parametric assumptions on individual strategic behavior.

Keywords: Stackelberg Games, Robustness, Treatment Rules

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1 Introduction

The growing collection of individual-level data has increased the feasibility of personalizing treatments in a wide variety of settings. Treating individuals heterogeneously can improve outcomes compared to a uniform policy. Rossi et al. (1996) estimate a demand model to show that targeting consumers with different coupons depending on their purchase history can improve revenue compared to allocating the same coupon to everyone. The allocation of food inspectors or fire inspectors can be improved by assigning them to establishments that improve safety the most in response to inspections (Athey, 2017; Glaeser et al., 2016). Online lenders allocate credit heterogeneously based on conventional and unconventional data like phone usage (Björkegren and Grissen, 2019).

When treatments have value to individuals, personalization of treatments introduces incentives for individuals to change their behavior used for targeting and receive a better treatment. In this paper, we study how to optimally allocate a binary treatment conditional on observed covariates, when agents report covariates strategically in response to the treatment rule. In the coupon example, a profit-maximizing seller would like to allocate a coupon only to reluctant buyers, who will buy the product only if they receive a coupon. They would like to avoid giving the coupon to customers who would buy the product even without a coupon. The seller, however, cannot observe the buyer’s type directly, so instead relies on proxies like browser activity or past purchases for targeting. An allocation rule that uses online behavior to target thrifty customers with a discount will incentivize other customers to change their behavior to mimic a reluctant buyer and save on their purchase. As a result, the distribution of observed covariates can shift depending on the structure of the allocation rule, which impacts how treatments are optimally allocated.

We model the strategic behavior as a Stackelberg game, where the planner announces a mapping from covariates to treatment probabilities, $n$ individuals report covariates strategically in response, treatments are allocated, and finally the corresponding potential outcome is realized for each individual. The planner’s goal is to design a treatment allocation rule that maximizes expected outcomes in this framework.

There is a large related literature on estimating prediction rules when agents manipulate observed characteristics, which is known as the strategic classification problem, including Hardt et al. (2016); Dong et al. (2018); Perdomo et al. (2020); Björkegren et al. (2020); Miller et al. (2021). Optimal prediction rules when agents are strategic have a distinct structure compared to prediction functions in non-strategic settings; for example, in the linear setting, manipulable characteristics are underweighted (Frankel and Kartik, 2020; Ball,
However, the insights gained in the prediction setting do not apply directly to the causal setting. As described in Athey (2017), the problems of prediction and causal inference, while closely related, are distinct, which prevents us constructing optimal decision rules from an optimal prediction rule in general (Ascarza, 2018; Bertsimas and Kallus, 2020). In non-strategic settings, the literature on policy learning shows that in a variety of frameworks, the optimal allocation rule estimates heterogeneous responses of each individual to the treatment conditional on their observed characteristics, and allocates treatments to those with the largest estimated responsiveness (Manski, 2004; Bhattacharya and Dupas, 2012; Kitagawa and Tetenov, 2018; Hirano and Porter, 2009; Kallus and Zhou, 2020; Athey and Wager, 2020). What is missing from the literature, which this paper seeks to address, is how the structure of the optimal allocation rule is affected by strategic behavior incentivized by treating individuals heterogeneously.

The first result of the paper is to characterize necessary conditions that the planner’s optimal allocation rule must meet; these conditions depend on a combination of behavioral elasticities that result from strategic behavior and provide some insight on the structure of the optimal rule. In the absence of strategic behavior, the Conditional Average Treatment Effect, which is the expected difference between treated and control outcomes for individuals with certain observed covariates, is exogenous to the treatment rule. Then, the optimal rule, as shown in Van der Weele et al. (2019), takes the form of a cutoff rule that assigns treatment with probability one to individuals with a positive CATE. An empirical version of this rule is known as the Conditional Empirical Success Rule (Manski, 2004). When there is strategic behavior, then the distribution of observed covariates depends on how treatments are allocated. The average treatment effect of individuals who report a certain value for \( x \) can change depending on how heterogeneously individuals are treated. As a result, we define a new version of the CATE that is conditional both on reported covariates and on how treatments are allocated.

In the setting with strategic behavior, a cutoff rule is one that allocates treatments with probability 1 to those who have a positive CATE induced by that rule, and probability 0 otherwise. We show that under this new definition, even when a cutoff rule exists, it is not always the optimal allocation rule. Instead, there exist natural settings where the optimal rule allocates the treatment to those with a positive CATE induced by the rule with probability less than one. Those with negative CATEs induced by the rule can receive the treatment with probability greater than zero. For certain forms of strategic behavior, adding some randomization to the treatment rule reduces incentives to engage in strategic behavior and thus can sometimes lead to conditional distributions of treatment effects more amenable
to effective targeting.

In a general model with many multi-valued covariates, strategic behavior can be very complex. To provide some additional insight on what kinds of strategic behavior lead to optimal allocation rules that have a cutoff compared to a randomization structure, we examine more closely the setting where agents report a single binary covariate used for targeting. We provide a simple condition on the sign of a Local Average Treatment Effect (Imbens and Angrist, 1994), the average treatment effect of individuals who are strategic in the region of an existing cutoff rule, that determines whether or not that rule is optimal. We provide two examples to further illustrate this theoretical result.

In the first example, inspired by Rossi et al. (1996), coupons are allocated based on a consumers’ online behavior, and the goal is to allocate coupons only to those with a low willingness to pay, who have a positive individual treatment effect. In this model, in response to discrimination on the basis of the observed online behavior, individuals with a negative individual treatment effect are strategic in a way that increases their probability of treatment. This leads to an allocation rule that does not take a cutoff form, where those with a positive CATE induced by the optimal rule are treated with probability less than 1. Our second example is a model of allocating a product upgrade offer based on a measure of a customers’ expertise. In this model, individuals with a positive individual treatment effect are increasingly strategic in response to discrimination on the basis of the observed covariate. This leads to an optimal allocation rule that takes a cutoff form even in the presence of strategic behavior.

In the absence of strategic behavior, it is straightforward to estimate the optimal rule using an A/B test to estimate CATEs, and assigning individuals with a positive estimated CATE to treatment, as described in Manski (2004). With strategic behavior, however, a traditional A/B test that randomizes treatment does not provide sufficient information to identify the optimal rule, or even to infer whether or not the optimal allocation rule has a cutoff or randomized structure. The other contribution of the paper is to design a sequential experiment that allows the planner to learn the optimal treatment rule over time without any parametric assumptions on agent strategic behavior. We show that the problem of estimating the treatment rule can be cast as a zero-th order stochastic optimization problem. Then, a variety of techniques from the stochastic optimization literature can be used to estimate the optimal rule. With strategic behavior, it is not straightforward to verify that the planner’s objective is convex in the allocation rule. As a result, we rely on a global optimization procedure based on Bayesian Optimization to estimate the optimal treatment allocation rule using sequential noisy function evaluations, which imposes minimal assumptions apart from
smoothness on the objective function.

In the final section of the paper, we present an MTurk experiment that demonstrates that targeting a valuable treatment based on observed behavior induces a shift in Conditional Average Treatment Effects. We use the data from this experiment to run a semi-synthetic simulation that demonstrates our proposed estimation procedure has average regret that decreases rapidly after a low number of noisy evaluations of the objective.

Related Work  This paper introduces a Stackelberg model in a non-parametric causal setting. The Stackelberg model for strategic behavior in the prediction setting was introduced by Hardt et al. (2016) and sparked the growing literature of strategic classification. One strand of this literature examines algorithms that converge to the optimal prediction rule. Dong et al. (2018) uses derivative-free convex optimization that converges to the optimum when agent strategies are costly manipulations of a continuous covariate. In economics, Björkegren et al. (2020) uses a randomized experiment that varies the coefficients of a prediction function to estimate a parametric model of manipulation and compute the prediction rule that is optimal under this model. Subsequent to the first version of this paper, Miller et al. (2021) and Izzo et al. (2021) also compute the optimal prediction rule through variants of derivative free convex optimization by imposing some light structural assumptions. The other strand of the strategic classification literature studies the convergence and optimality of algorithms that approximate the optimal strategy, such as repeated risk minimization (Perdomo et al., 2020; Brown et al., 2020). Our paper is more related to the first strand of literature, in that we introduce a sequential experiment for estimating the optimal treatment allocation rule directly. Prediction can be considered as a type of intervention, as described in Miller et al. (2020). However, as described in the introduction, optimally allocating a binary intervention cannot be reduced to a standard classification problem, so requires new analysis. A second distinction is that the regret properties of our estimator require neither parametric assumptions on agent behavior nor convexity of the objective function in the parameters of the allocation rule; these assumptions, made frequently in the classification literature, are difficult to verify in the more general causal framework.

In the economic theory literature, both Frankel and Kartik (2020) and Ball (2020) show that the optimal linear prediction rule underweights manipulable characteristics. In this paper, we derive complementary results for treatment allocation. We show that there is a form of underweighting in treatment allocation with strategic agents, where the optimal rule can allocate treatments to those with a positive CATE induced by the rule with less than 100% probability.
The literature on policy learning, which assumes agents are non-strategic and the distribution of observed covariates are fixed, has examined optimal treatment allocation rules in a growing variety of settings. The literature takes into account data that is from a randomized experiment (Manski, 2004) or observational (Athey and Wager, 2020; Kallus and Zhou, 2020), and examines the optimal rule defined by both finite sample and asymptotic frameworks (Hirano and Porter, 2009) under budget (Bhattacharya and Dupas, 2012) and other constraints on the allocation rule (Kitagawa and Tetenov, 2018). The theory part of our paper examines the structure of the unconstrained optimal rule when agents are strategic in a setting more comparable with the initial papers in this literature, where treatments are binary, covariates are discrete, and the objective is maximizing expected outcomes.

There is a growing literature that shows that a traditional A/B test is not sufficient to estimate causal quantities of interest under spillover or equilibrium effects, and designs new forms of experiments for these more complex settings (Vazquez-Bare, 2017; Viviano, 2020; Munro et al., 2021). This paper shows that under strategic effects, new experiment designs are also needed to learn how to target treatments effectively.

2 Models of Treatment Allocation

2.1 Treatment Allocation with Exogenous Covariates

We start by discussing the classical setting of Manski (2004). Each of $i \in 1, \ldots, n$ individuals have exogenously characteristics and potential outcomes jointly drawn from some unknown distribution: $X_i, \{Y_i(1), Y_i(0)\} \sim G$. In our formal results, we assume for simplicity that $X_i \in \mathcal{X}$ is discrete, with $|\mathcal{X}| = d$. The treatment allocation proceeds as follows:

1. The planner specifies $\delta(x) = Pr(W_i = 1 | X_i = x)$ for each $x \in \mathcal{X}$.

2. A binary treatment $W_i$ is sampled from Bernoulli($\delta_i$), where $\delta_i = \delta(X_i)$.

3. The observed outcome is $Y_i = Y_i(W_i)$.

Since $X_i$ is discrete, we can represent the function $\delta : \mathcal{X} \rightarrow [0, 1]^d$ as the $d$-length vector $\delta$. The planner would like to choose $\delta \in [0, 1]^d$ to maximize expected outcomes, $E[Y_i(W_i)]$. Let $\tau(x) = E[Y_i(1) - Y_i(0) | X_i = x]$ be the CATE, the average treatment effect among individuals who have covariate value $x$. We can expand the objective to show that $\delta(x)$ enters the objective linearly, and as a result we can use the CATE to construct a simple optimal treatment rule that takes a cutoff form. Using Bayes' rule,
\[ \mathbb{E}[Y_i(W_i)] = \sum_{x \in \mathcal{X}} \delta(x) \mathbb{E}[Y_i(1)|X_i = x] + (1 - \delta(x)) \mathbb{E}[Y_i(0)|X_i = x]. \] (1)

**Proposition 1.** Assume that the density \( f(x) > 0 \) for all \( x \in \mathcal{X} \). The policy that maximizes expected outcomes is defined by \( \delta^0(x) = 1(\tau(x) > 0) \) for \( x \in \mathcal{X} \).

The proof of this proposition is straightforward and appears in different forms in the literature; see for example Setting 3 of Van der Weele et al. (2019). In order to estimate this rule based on a finite sample of data, we require only an estimate of \( \tau(x) \), which can be constructed using data from a Bernoulli randomized experiment,

\[
\hat{\tau}(x) = \frac{\sum_{i=1}^{n} 1(X_i = x, W_i = 1)Y_i}{\sum_{i=1}^{n} 1(X_i = x, W_i = 1)} - \frac{\sum_{i=1}^{n} 1(X_i = x, W_i = 0)Y_i}{\sum_{i=1}^{n} 1(X_i = x, W_i = 0)}.
\]

The choice \( \delta^0(x) = 1(\hat{\tau}(x) > 0) \) for \( x \in \mathcal{X} \) is the Conditional Empirical Success Rule of Manski (2004). In the next section, we will show how under strategic behavior then \( \delta(x) \) no longer enters the planner’s objective linearly, leading to a more complex structure for the optimal rule.

### 2.2 Treatment Allocation with Strategic Agents

We now move to our main model of interest. We are still in a setting with \( i \in \{1, \ldots, n\} \) individuals and \( X_i \in \mathcal{X} \) discrete, with \( |\mathcal{X}| = d \). The covariate for individual \( i \), \( X_i \), is no longer exogenous to the treatment allocation rule, however. We have \( X_i = X_i(\delta) \), where the function \( X_i : [0,1]^d \to \mathcal{X} \) determines how agents are strategic in response to different treatment rules.

Both the potential covariate function and potential outcomes\(^1\) are jointly drawn from some unknown distribution \( X_i(\cdot), \{Y_i(1), Y_i(0)\} \sim G \). The treatment allocation procedure can now be described as a Stackelberg game:

1. The planner specifies \( \delta(x) = Pr(W_i = 1|X_i = x) \) for each \( x \in \mathcal{X} \).

2. For \( i \in [n] \), agent \( i \) reports covariates \( X_i(\delta) \in \mathcal{X} \). In many settings, we can interpret the potential covariates as the result of utility maximization of randomly drawn \( U_i(\cdot) \):

\[
X_i(\delta) = \arg \max_x \delta(x)U_i(x, 1) + (1 - \delta(x))U_i(x, 0)
\]

\(^1\)In this section, where we characterize the structure of the optimal rule, we assume that potential outcomes do not depend on the treatment allocation rule. However, the estimation procedure given in Section 3 is robust to settings where potential outcomes also depend on \( \delta \).
3. For $i \in [n]$, $W_i$ is sampled from $\delta(X_i)$.

4. The outcome $Y_i = Y_i(W_i)$ is observed.

The key difference in this model compared to the previous section is Step 2, which captures how if $\delta(x)$ varies for different values of $x$, then incentives are introduced for individuals to change their default behavior. In many settings, individuals have some potentially costly methods of changing their behavior $X_i$. Furthermore, they may value receiving a treatment more or less than not receiving a treatment. We can capture how individual’s well-being depends on their observed behavior and the treatment through a heterogenous utility function that, when maximized, determines how an individual responds to a certain treatment allocation rule.

In this more complex environment, the objective of maximizing expected outcomes remains the same. $\Pi(\delta) = E[Y_i(W_i)]$, and an optimal rule is defined as

$$\delta^* \in \arg \max_{\delta \in [0,1]^d} E[Y_i(W_i)].$$

With strategic behavior, the CATE is endogenous to the treatment rule. The correlation between the observed data $X_i$ and the individual treatment effect $Y_i(1) - Y_i(0)$ changes depending on the agent’s strategy for reporting $X_i(\delta)$. Thus, we need to define a separate CATE for each possible treatment rule:

$$\tau(x, \delta) = E[Y_i(1) - Y_i(0)|X_i(\delta) = x].$$

A natural first conjecture for a good targeting rule is an extension of the cutoff rule from Proposition 1. The cutoff rule with strategic agents is one that meets the condition

$$\delta^c(x) = 1(\tau(x, \delta^c) > 0).$$

This rule allocates treatments only to individual who have a positive CATE, where the CATE is defined based on the distribution of $X_i$ induced by $\delta^c$. A cutoff rule meeting this fixed point condition may not exist, depending on the form of strategic behavior. In settings where it does exist, we will show that the optimal allocation rule sometimes has this form, but in other cases does not.

With strategic behavior, the dependence of the objective on $\delta$ is much more complex than in the exogenous setting of Equation 1. Let $f(x, \delta) = Pr(X_i(\delta) = x)$. We can use
Bayes rule to expand $\Pi(\delta)$ as

$$\Pi(\delta) = \sum_{x \in X} f(x, \delta)(\delta(x)\mu(1, x, \delta) + (1 - \delta(x))\mu(0, x, \delta)),$$

where $\mu(w, x, \delta) = \mathbb{E}[Y_i(w) | X_i(\delta) = x]$. The treatment rule now enters into the objective in a non-linear way, and it is no longer as straightforward to derive the form of the optimal rule. To make some progress, we first impose some regularity conditions. Strategic behavior is such that the marginal distribution of covariates and the average potential outcomes for individuals reporting a certain covariate value varies smoothly with changes in the allocation rule.

**Assumption 1.** The conditional outcome functions $\mu(1, x, \delta)$ and $\mu(0, x, \delta)$ are differentiable in $\delta$ and bounded for every $x \in X$. $f(x, \delta)$ is also differentiable in $\delta$ for every $x \in X$.

Under assumption 1, the function $\Pi(\delta)$ is also a continuous and bounded function on its domain, and so attains a maximum on $[0, 1]^d$, which we denote

$$v^* = \max_{\delta \in [0, 1]^d} \Pi(\delta).$$

The set of maximizers, $\{\delta : \Pi(\delta) = v^*\}$, any element of which we denote $\delta^*$, is guaranteed to be non-empty. We next provide a necessary condition that any optimal rule must satisfy.

**Theorem 1.** Let $\bar{\mu}(x, \delta) = \delta(x)\mu(1, x, \delta) + (1 - \delta(x))\mu(0, x, \delta)$. Under Assumption 1, the following is a necessary condition that any optimal rule $\delta^*$ must meet: There exists $d$-length vectors $\lambda^1 \geq 0$ and $\lambda^0 \geq 0$ such that for each $x \in X$,

$$f(x, \delta^*)[\tau(x, \delta^*)] + \sum_{z \in X} \left[ \frac{\partial f(z, \delta^*)}{\partial \delta^*(x)} \bar{\mu}(z, \delta^*) + f(z, \delta) \left( \delta^*(z) \frac{\partial \mu(1, z, \delta^*)}{\partial \delta^*(x)} + (1 - \delta^*(z)) \frac{\partial \mu(0, z, \delta^*)}{\partial \delta^*(x)} \right) \right] - \lambda^1_x + \lambda^0_x = 0 \quad (3)$$

$$\begin{align*}
(\delta^*(x) - 1)\lambda^1_x &= 0, \\
\delta^*(x)\lambda^0_x &= 0, \\
0 &\leq \delta^* \leq 1.
\end{align*}$$

If $\Pi(\delta)$ is concave, then this necessary condition is also sufficient to ensure that the rule is a global maximizer of $\Pi(\delta)$. 
We can define the strategic component of the condition in Equation 3 as

\[
s(x, \delta) = \sum_{z \in \mathcal{X}} \left[ \frac{\partial f(z, \delta)}{\partial \delta(x)} \bar{\mu}(z, \delta) \right] \text{Marginal distribution shift} + f(z, \delta) \left( \delta(z) \frac{\partial \mu(1, z, \delta)}{\partial \delta(x)} + (1 - \delta(z)) \frac{\partial \mu(0, z, \delta)}{\partial \delta(x)} \right) \text{Conditional distribution shift}
\]

(4)

The derivative of the objective with respect to \( \delta(x) \) is \( f(x, \delta) \tau(x, \delta) + s(x, \delta) \). If there is no strategic behavior and \( x \) is exogenous to the allocation rule, then \( s(x, \delta) = 0 \) and we are back in the setting of Proposition 1. If the CATE is positive, then, the derivative of the objective with respect to \( \delta(x) \) is always positive, so the optimal rule has \( \delta^*(x) = 1 \). But with strategic behavior, the derivative of the objective with respect to a parameter of the allocation rule depends not only on the CATE induced by the rule, \( \tau(x, \delta) \), but also the effect of \( \delta \) on the marginal distribution of covariates and the conditional distribution of outcomes given covariates. Changes in the treatment rule lead to changes in the distribution of \( X_i \). If the distribution shift in response to a small increase in \( \delta(x) \) results in:

1. More individuals reporting covariates that are linked to higher realized outcomes
2. Conditional potential outcomes \( \mu(1, z, \delta) \) that are more positive for covariates with \( \delta(z) \) closer to 1, and
3. Conditional potential outcomes \( \mu(0, z, \delta) \) that are more positive for covariates with \( \delta(z) \) closer to zero,

then the strategic component of the derivative of the objective function is positive. The strategic component captures how changing the treatment rule can lead to distributions of \( X_i \) which are more or less correlated with the individual treatment effect \( \tau_i = Y_i(1) - Y_i(0) \). This affects how easy it is for a planner to distinguish between individuals who have a positive treatment effect and those with a negative treatment effect and impacts the structure of the optimal allocation rule.

**Corollary 1.** Assume that a cutoff rule of the form

\[
\delta^c(x) = 1(\tau(x, \delta^c) > 0)
\]

exists. If there is any \( \tilde{x} \in \mathcal{X} \) such that \( \text{sgn}(s(\tilde{x}, \delta^c)) \neq \text{sgn}(\tau(\tilde{x}, \delta^c)) \) and \( |s(\tilde{x}, \delta^c)| > |f(x, \delta^c)\tau(x, \delta^c)| \), then \( \delta^* \neq \delta^c \) and the optimal rule does not have a cutoff form. If no such \( \tilde{x} \) exists, and \( \Pi(\delta) \) is concave, then the cutoff rule is optimal even in the presence of strategic behavior.
From the optimality conditions, we can no longer guarantee that a cutoff rule is optimal if the strategic effect \( s(x, \delta^c) \) is large enough and of opposite sign to the CATE \( \tau(x, \delta^c) \). Under the conditions identified in Corollary 1, the optimal rule is an interior solution where for certain values of \( x \in X \), we induce some randomization, with \( 0 < \delta^*(x) < 1 \). When \( X \) can take many possible values, interpreting \( s(x, \delta^c) \) in general is more complex.

If we assume binary \( X \in \{L, H\} \) and a boundary crossing model for gaming, then the form of \( s(x, \delta) \) simplifies and we can provide some intuitive sufficient conditions under which \( s(x, \delta) \) will have the same sign as \( \tau(x, \delta) \) so that the cutoff rule is optimal. We next introduce two corresponding assumptions. In the first assumption, we ensure a cutoff rule exists that discriminates between individuals and assigns treatments only to individuals who report \( H \).

**Assumption 2.** \( X \in \{L, H\} \) is binary. There exists a \( \delta^c : \{L, H\} \rightarrow [0, 1]^2 \) for which \( \delta^c(L) = 0 \) with \( \tau(L, \delta^c) < 0 \), and \( \delta^c(H) = 1 \) with \( \tau(H, \delta^c) > 0 \).

We also restrict strategic behavior to take the form of selection from \( L \) into \( H \) (Heckman and Vytlacil, 2005).

**Assumption 3.** Agents have heterogeneous value for the treatment, \( V_i \). Some proportion \( \rho \) of agents always report \( H \). The remaining agents can report \( L \) for free and have a heterogeneous cost for reporting \( H, C_i \). For the agents whose reporting rule is not constant, we can write their utility function as follows:

\[
U_i(x) = V_i \delta(x) - C_i \mathbb{1}(x = H)
\]

An implication of Assumption 3 is that the strategic reporting function can take one of two possible forms:

\[
X_i(\delta) \in \{H, L + (H - L)\mathbb{1}(\delta(H) - \delta(L) \geq R_i)\},
\]

where \( R_i = \frac{C_i}{V_i} \). Under these assumptions, then Corollary 2 provides a condition that restricts the sign of a Local Average Treatment Effect (LATE). Under this condition, the cutoff rule defined in Assumption 2 meets the necessary conditions of Theorem 1.

**Corollary 2.** Under Assumption 2 and 3, if individuals that are strategic in the local region of the cutoff rule have a positive average treatment effect, so that

\[
\mathbb{E}[Y_i(1) - Y_i(0)|R_i = 1] \geq 0,
\]
then $\delta^c$ meets the necessary KKT conditions for optimality from Theorem 1. If $\Pi(\delta)$ is also concave, $\delta^c$ is a globally optimal rule $\delta^*$.

This corollary allows us to evaluate what kind of strategic behavior can result in a treatment rule that is distinct from the cutoff rules that are always optimal in the exogenous setting. If only individuals with positive ITEs are strategic in the sense of Assumption 3, then the LATE condition will be satisfied and a cutoff rule is optimal. In this case, incentives between the planner and individuals are aligned; individuals who have a positive response to the treatment from the perspective of the planner are also those who prefer the treatment enough to change their behavior from $L$ to $H$ when the gap between $\delta(H)$ and $\delta(L)$ is large enough.

But as will be discussed in the next section, in many natural settings incentives are not aligned. Individuals who have a negative response to the treatment from the perspective of the planner may be those who prefer the treatment enough to change their behavior from $L$ to $H$. If individuals with a negative ITE are strategic, then the LATE condition of Corollary 2 is not satisfied, and an interior solution can be optimal instead of a cutoff rule.

We illustrate this point through two simple models that follow Assumptions 2 and 3. The first is a model of coupon allocation and product demand where the optimal rule involves randomization in the allocation. The second is a model of driving behavior and insurance discounts where where the optimal rule takes the form of a cutoff rule despite the presence of strategic behavior.

### 2.3 Examples with Binary Covariates

In each of these models, we compute the treatment allocation, the CATEs, and the objective function for three different allocation policies:

1. A uniform allocation policy with $\delta(H) = \delta(L) = 0.5$

2. A cutoff rule with $\delta(H) = 1$ and $\delta(L) = 0$

3. The optimal rule that maximizes expected outcomes

**Example 1. Price Discrimination through Coupons**

In the first example, a firm is offering a coupon to certain customers. For each customer $i \in [n]$, there is an unobserved type $\theta_i \sim \text{Bernoulli}(0.5)$ which determines potential outcomes $\{Y_i(1), Y_i(0)\}$ and the reporting behavior $X_i(\delta)$. Customers with $\theta_i = 0$ are always buyers, who will buy the product with or without the coupon. Customers with $\theta_i = 1$ are reluctant buyers, who will not purchase the product without the coupon but will purchase it with
75% probability if they receive a coupon. A product purchase leads to $10 profit for the firm without the coupon and $5 with it. This leads to potential outcomes

\[ Y_i(W_i) = 5 \cdot (0.75\theta_i + (1 - \theta_i))W_i + 10 \cdot (1 - \theta_i)(1 - W_i). \]

Always buyers have a negative ITE and reluctant buyers have a positive ITE. However, the store observes \( X_i \in \{L, H\} \) rather than \( \theta_i \), where \( X_i = H \) indicates the customer has left a product in the cart for more than a few minutes. In the absence of incentives otherwise, always buyers will purchase the item immediately, so they will have \( X_i = L \). No matter the allocation policy, reluctant buyers will always leave the product in their cart and report \( X_i = H \). However, when coupons are allocated on the basis of \( X_i \), this introduces incentives for always-buyers to mimic reluctant buyers by leaving the product in their cart to obtain a coupon and purchase the product for $5 less. This strategic behavior is influenced by an individual specific cost of behavior change \( C_i \in \text{Uniform}(0, 10) \), and is described by the following utility function:

\[ U_i(x) = 5 \cdot \delta(x; \beta) - C_i(1 - \theta_i)\mathbb{1}(x = H). \]

This leads to a behavior function that follows Assumption 3 of:

\[ X_i(\delta) = \theta_iH + (1 - \theta_i)\left( L + \mathbb{1}(5(\delta(H) - \delta(L)) \geq C_i)(H - L) \right) \]

The optimal policy is the coupon allocation procedure that maximizes profit when treatments are allocated on the basis of \( \delta(H) \) and \( \delta(L) \) and the distribution of \( X_i \) is determined by \( X_i(\delta) \).

\[ \delta^* = \arg\max_{\delta} E[Y_i(W_i)] \]

Table 1 describes the performance of three different allocation rules in this model of coupon allocation. Under a uniform treatment assignment policy, \( \tau(H, \delta^0) = 3.75 \), \( \tau(L, \delta^0) = -5 \) and the objective value is $4.688. We can implement a cutoff rule \( \delta^c(H) = 1 \) and \( \delta^c(L) = 0 \) which raises the expected outcomes to $5.626. But, since \( \delta^c(H) \neq \delta^c(L) \) strategic behavior is now induced from those with \( \theta_i = 0 \).

In Appendix B.1, we show that the distribution of \( X_i \) now depends on \( \delta(H) - \delta(L) \).

\[ Pr(X_i(\beta) = H|\theta_i = 0) = \frac{1}{2}(\delta(H) - \delta(L))\mathbb{1}\{\delta(H) > \delta(L)\}. \]

As a result, the CATEs change in response to the cutoff rule compared to the uniform
rule. By using the conditions on Theorem 1, we can derive the optimal rule of $\delta^*(H) = 0.75$ and $\delta^*(L) = 0$, which leads to higher expected outcomes than the cutoff rule. In this model, individuals with a negative individual treatment effect are strategic, and the incentives of the planner and agents are not aligned. As a result of this, the optimal targeting rule involves some randomization in allocating; this reduces the amount of strategic behavior that occurs and ensures enough information about an individuals treatment effect is retained in the distribution of $X_i$.

Example 2. Allocating Product Upgrades

In this second example, a firm is offering a product upgrade for purchase. Again, there are $n$ customers and for each customer $i \in [n]$ an unobserved type $\theta_i \sim \text{Bernoulli}(0.5)$ which determines potential outcomes and strategic behavior. $\theta_i = 0$ indicates naive customers and $\theta_i = 1$ indicates sophisticated customers. $W_i = 1$ indicates that customer $i$ receives an offer of a product upgrade. The firm receives a profit of $5$ from each customer who is not treated. The upgrade offer annoys naive customers who have no use for the upgrade, so they reduce their usage upon receiving treatment, leading to a profit of $1$ per naive customer who is treated. For sophisticated customers, they benefit from the product upgrade and purchase it, leading to a profit of $10$ per sophisticated customer who is treated.

$$Y_i(W_i) = 5(1 - W_i) + (1 - W_i)(10\theta_i - (1 - \theta_i)).$$

The ITE for naive customers is negative and the ITE for sophisticated customers is positive. The firm does not observe customer sophistication directly, but they do observe whether or not they have completed a certification course $X_i \in \{L, H\}$. Naive customers never complete the certification course ($X_i = L$). In the absence of a product upgrade offer, sophisticated customers complete the certification course ($X_i = H$) based on whether some random value $V_i \sim \text{Uniform}(-10, 10)$ is greater than equal to zero. Sophisticated customers receive a value of $5$ from receiving and purchasing the product upgrade, so when the treatment is targeted based certification course completion, this incentivizes additional sophisticated customers to
complete the course.

For sophisticated customers, their utility function is

$$U_i(x) = V_i 1(x = H) + 5 \delta(x).$$

This leads to a behavior function that follows Assumption 3 of

$$X_i(\delta) = (1 - \theta_i)L + \theta_i\left(L + (H - L)1(V_i + 5(\delta(H) - \delta(L)) \geq 0) \right).$$

The optimal policy is the coupon allocation procedure that maximizes profit when treatments are allocated on the basis of $\delta(H)$ and $\delta(L)$ and the distribution of $X_i$ is determined by $X_i(\delta)$.

$$\delta^* = \arg \max_\delta \mathbb{E}[Y_i(W_i)].$$

In this example, those with a positive treatment effect are strategic. As the planner targets the offer more precisely to those who complete the certification course, they incentivize those who would benefit the most from the offer to complete the certification course. As a result, as we might expect from Corollary 2, the optimal rule in this model remains a cutoff rule in the presence of beneficial strategic behavior, see Table 2 for a summary and Appendix B.2 for a derivation of the optimal rule.

### 2.4 Coupon Example with Continuous Covariates

In the previous two examples, we illustrated our theoretical results in a simple models where there is a single binary covariate. A natural question is how these results extend to a setting with continuous covariates, with $X_i \in \mathbb{R}$. With continuous covariates, so that the planner’s optimization problem remains finite dimensional, we restrict the treatment allocation rule to be a parametric function. For this section, we choose a logit function, so that

$$\delta(X_i; \beta) = \frac{1}{1 + e^{-(\beta_0 + X_i \beta_1)}}.$$
With continuous covariates, the optimal allocation rule is \( \delta^* = \delta(X_i; \beta^*) \), where we can define the planner’s objective function in terms of \( \beta \) rather than \( \delta \) directly.

\[
\beta^* = \arg \max_{\beta \in \mathbb{R}^2} E_{\beta}[Y_i(W_i)]
\]

For this section, we construct a modified version of the structural model of Example 1. As in the discrete setting, we have unobserved discrete type \( \theta_i \in \{0, 1\} \), where \( \theta_i = 0 \) represents always-buyers with a negative ITE and \( \theta_i = 1 \) represents reluctant buyers with a positive ITE. We introduce a new variable \( Z_i \in \mathbb{R} \) which can be observed as the agent’s inherent or costless behavior, where \( Z_i \sim \text{Normal}(-1, 2) \) if \( \theta_i = 0 \) and \( Z_i \sim \text{Normal}(1, 2) \) if \( \theta_i = 1 \). Always-buyers, who are potentially strategic, have a utility function

\[
U_i(x, \delta) = 5 \cdot \delta(x; \beta) - C_i(x - Z_i)^2,
\]

while reluctant buyers do not report \( X_i \) strategically. The utility is made up of a value for receiving the treatment and a cost of reporting covariate \( x \). Following the literature on prediction with strategic behavior, we have assumed that the individual has some inherent behavior \( Z_i \) and that the cost for reporting a different covariate \( x \) is quadratic in the distance from \( Z_i \) (Frankel and Kartik, 2020).

This utility specification leads to a reporting rule

\[
X_i(\delta) = \theta_i Z_i + (1 - \theta_i) \arg \max_{x \in \mathbb{R}} U_i(x, \delta).
\]

As in the discrete setting, \( C_i \in \text{Uniform}(0, 10) \) and potential outcomes are

\[
Y_i(W_i) = 5 \cdot (0.75 \theta_i + (1 - \theta_i))W_i + 10 \cdot (1 - \theta_i)(1 - W_i).
\]

Figure 1 plots the allocation rule and the resulting response of individuals for three different settings. Underneath the graph of the treatment rule, the location of 200 individuals who respond to this treatment rule are plotted at a jitter, colored by their ITE. For Figure 1a, we simulate a version of the coupon model with continuous covariates without strategic behavior, so that \( X_i = Z_i \) and those with a lower \( X_i \) are more likely to have a negative ITE. We plot the resulting optimal rule, and as expected, the optimal logit function is a step function, and the step occurs where the CATE transitions from negative to positive. The average profit per agent is $5.72. If this cutoff function is implemented in a population who report \( X_i \) strategically based on the model in this section, as in Figure 1b, then the
distribution of $X_i$ shifts. Those with $\theta_i = 0$ but with inherent behavior $Z_i$ that is close to the cutoff will shift their behavior to report $X_i > 0$ and receive the valuable coupon. As a result, the profit drops to $5.49$ per agent. Taking into account the strategic behavior in Figure 1c, the optimal logit function is no longer a cutoff rule. Instead, there is a fuzzy region where treatments are assigned with below one probability to those with positive $X_i$. There is still some strategic behavior induced (some agents with a negative ITE report $X_i > Z_i$), but the average profit of $5.55$ per agent is improved.

![Figure 1: These figures plot the probability of being assigned a coupon conditional on reporting $X_i$ under different scenarios. The reported $X_i$ for each individual, colored by their ITE, is plotted using a jitter underneath the allocation rules.](image)

Just as in the binary case, with continuous covariates, a rule that assigns treatment with 100% probability to groups with a positive CATE may induced a distribution where targeting is far less effective than one that commits to treating individuals in a more uniform way. Knowing that the optimal rule no longer always takes the form of a cutoff rule in both continuous and discrete settings, our next goal is coming up with estimation methods for $\delta^*$. In the next section, we show how estimating the optimal rule is a type of sequential optimization problem with noisy function evaluations, and design a procedure that recovers
the optimal rule without relying on strong assumptions on the structure of strategic behavior.

3 Estimating the Optimal Rule

Theorem 1 gives a set of conditions that the optimal rule must solve in the setting with discrete covariates. These conditions, which include behavioral changes that occur in response to changes in the allocation rule, indicate that without further assumptions on strategic behavior we cannot estimate the optimal rule without some variation in the treatment rule.

To estimate the rule in finite samples without a model for strategic behavior, we assume that the planner is in a setting of sequential experimentation.

**Assumption 4. Optimization Environment:** At each time \( t = 1, \ldots, T \), a batch of \( n \) agents arrive and the Stackelberg game from Section 2.2 is played for each \( i \in [n] \):

1. Agents respond to treatment rule \( \delta^t \) and report behavior \( X_i(\delta^t) \)
2. Treatments are allocated, with \( \Pr(W^t_i = 1) = \delta^t(X_i(\delta^t)) \)
3. Outcomes are observed, with \( Y^t_i = Y^t_i(W^t_i) \).

At each time \( t = 1, \ldots, T \), a batch of \( n \) agents arrive and are treated by the planner. In the coupon example, we can think of the batch of agents that arrive at each time \( t \) as the customers who arrive to the seller’s website within a fixed time period. We assume that the agent’s decision problem is static, so we can ignore dynamic considerations if an agent arrives repeatedly over time. In this environment, the planner cannot observe \( \Pi(\delta) \) directly with only a finite sample of agents at each time step. However, they can observe a noisy version of \( \Pi(\delta) \),

\[
\Pi_n(\delta^t) = \frac{1}{n} \sum_{i=1}^{n} Y^t_i(W^t_i)
\]

As \( n \) grows large, we have from the Central Limit Theorem that

\[
\sqrt{n}(\Pi_n(\delta^t) - \Pi(\delta^t)) \to_D N(0, \sigma^2).
\]

At time-step \( t \), the planner observes \( \Pi_n(\delta^t) = \Pi(\delta^t) + \epsilon \) where \( \epsilon \) is approximately normally distributed. The goal is a procedure for setting \( \delta^t \) that makes cumulative regret small, defined as

\[
R_T = \sum_{t=1}^{T} \Pi(\delta^*) - \Pi(\delta^t),
\]
where $\delta^* \in \text{arg max}_{\delta \in [0,1]^d} \Pi(\delta)$. This is a standard setting of sequential optimization where only noisy function evaluations are available. Without any continuity assumptions on $\Pi(\delta)$, this optimization problem is NP-Hard. With a strong-convexity assumption on the objective, we can use an approximate gradient approach based on function evaluations, as in Flaxman et al. (2004).

Our choice of an optimization method for $\Pi(\delta)$ is guided by a few factors. First, it is challenging to verify convexity assumptions on the objective $\Pi(\delta)$, since it depends on the unknown structure of strategic behavior. As a result, we make smoothness assumptions but do not make convexity assumptions, so do not use a gradient approximation-based approach. Second, function evaluation is costly. Although we do not model this explicitly, each time a platform changes its targeting rule requires engineering effort and time for customers to adjust to the new rule. As a result, we would prefer a method that finds an optimal rule with a limited number of time steps. In modern machine learning, Bayesian Optimization has been used successfully to find hyperparameters that optimize unknown non-convex functions that are costly to evaluate, see Snoek et al. (2012). Our preferred approach for optimizing targeting rules when agents are strategic is also based on Bayesian Optimization, and is briefly described in the next section.

### 3.1 Gaussian Process Optimization

In Gaussian Process Optimization, we assume that our unknown objective function $\Pi(\delta)$, where $\delta \in [0,1]^d$, is drawn from a Gaussian Process (Williams and Rasmussen, 2006), defined by a mean function $\mu : [0,1]^d \to \mathbb{R}$ and kernel function $k : [0,1]^d \times [0,1]^d \to \mathbb{R}$. The choice of kernel function enforces certain restrictions on $\Pi$, for example if the kernel is the squared exponential kernel, then the objective function is assumed to be in class $C^\infty$. If a function $f$ evaluated with independent normal noise with variance $\sigma^2$ has a prior distribution given by $GP(\mu_0, k)$, then the prior distribution of a vector of function values $Z = f(Q)$ is multivariate normal, so that $Z|Q \sim N(\mu_0(Q), k(Q, Q) + \sigma^2 I)$. Given a set of noisy function evaluations, then the posterior distribution for the unknown objective function is a Gaussian Process with mean function $\mu_s$ and kernel function $k_s$.

Let $\pi^s = [\Pi_n(\delta_t)]_{t=1}^s$ be a vector of $s$ sequential noisy function evaluations, where $\pi^s_t = \Pi_n(\delta_t)$, $D^s = [\delta_t]_{t=1}^s$ is the matrix defining $s$ allocation functions, where $D^s_{tk} = Pr(W^s_t = 1|X^s_t = k)$. Given a set of noisy function evaluations, the posterior distribution for the unknown objective function is a Gaussian Process with mean function $\mu_s$ and kernel function $k_s$.

$$
\mu_s(\delta) = \mu_0(\delta) + k(\delta, D^s)(\Sigma + \sigma^2 I)(\pi^s - \mu(D^s)),
$$
\[ k_s(\delta, \delta') = k(\delta, \delta') - k(\delta, D^s)(\Sigma^s + \sigma^2 I)k(D^s, \delta), \]

where \( \Sigma^s = k(D^s, D^s) \) is the \( s \times s \) kernel matrix.

In this framework, we can choose our evaluation point \( \delta^t \) by following the Gaussian Process Upper Confidence Bound algorithm (GP-UCB), described in Algorithm 1 (Auer, 2002; Dani et al., 2008; Srinivas et al., 2009).

**Algorithm 1: GP-UCB**

**Input:** Prior \( GP(\mu_0, k) \)

**Output:** Estimate of treatment rule \( \delta_T \)

for \( t \in \{1, \ldots, T\} \) do

Choose \( \delta_t = \arg \max_{\delta \in [0, 1]^d} \mu_{t-1}(\delta) + \sqrt{\alpha_t}k_{t-1}(\delta, \delta) \);

Receive feedback \( \pi_t = \Pi_{n_t}(\delta_t) \);

Compute \( \mu_t \) and \( k_t \) based on Bayesian update;

end

This algorithm favors points that have a high posterior mean and posterior variance, so there is substantial uncertainty about the function value at that point. \( \alpha_t \) is a tuning parameter which determines the exploration-exploitation tradeoff. Under a selection procedure for \( \alpha_t \), then Srinivas et al. (2009) provides bounds on the cumulative regret of the GP-UCB algorithm for a class of kernel functions. In the next section, we use the squared exponential kernel in our prior; if it is the case that the objective \( \Pi(\delta) \) is drawn from this prior, then the results of Srinivas et al. (2009) indicate that regret grows at a sublinear rate, with \( R_T = O\left(\sqrt{T(\log T)^{d+1}}\right)\).

### 4 MTurk Experiment

We start by introducing a third example, upon which we build a semi-synthetic simulation that we use to evaluate Algorithm 1.

**Example 3. Preference-Based Targeting** Individuals have a binary preference, \( \theta_i \in \{0, 1\} \) that is not directly observed. The preference is whether or not the individual likes mathematics. A planner would like to allocated a free calculator \( W_i \in \{0, 1\} \) optimally. The outcome is a measure of net benefit taking into account how much the recipient appreciates the calculator as well as the cost of providing it to them. Those who like math have a positive ITE, \( \mathbb{E}[Y_i(1) - Y_i(0) | \theta_i = 1] > 0 \). Those who do not like math have a negative ITE, \( \mathbb{E}[Y_i(1) - Y_i(0) | \theta_i = 0] < 0 \). The planner cannot observe \( \theta_i \) directly, and instead must allocate the equipment based on some observed behavior \( X_i \in \mathcal{X} \) using the function \( \delta : \mathcal{X} \rightarrow [0, 1] \). All individuals have some value for the calculator, since even if it is not used it can be sold,
so some individuals will behave strategically to increase their chance of receiving a calculator, 
\[ X_i = \arg \max_x U_i(x, \delta(x)). \]

We run a simple experiment on MTurk that captures the key features of Example 3 and consists of a survey with two questions. The first asks individuals how much they like math, on a scale of 1-5. This measures \( \theta_i \), where \( \theta_i = 1 \) if the individual reports 4 or 5 and \( \theta_i = 0 \) if they report 1 to 3. The second question measures \( X_i \). This question is labelled as optional, and asks the respondent to report either factors of the function \( 2x^2 - 5x = 3 \). The correct answer is either -0.5 or 3. \( X_i = H \) if the individual responds correctly and \( X_i = L \) otherwise. We allocate a bonus of $1 based on individual’s response to \( X_i \), where the allocation rule changes across two different waves of the survey with \( n = 316 \) observations in each wave.

We map this survey design into Example 3 as follows:

- Rather than allocating a real calculator to respondents, we proxy the value of the treatment by a bonus of $1 which is allocated based on \( X_i \).
- We measure \( \theta_i \) but imagine that it is not observed by the planner. It is used to impute \( Y_i \) as
  \[ Y_i = 5 \cdot \theta_i W_i - 6 \cdot (1 - \theta_i) W_i, \]
  which measures that individuals who like math have a positive ITE and individuals who do not like math have a negative ITE

In survey Wave 1, nobody is allocated a bonus, and individuals are paid a fixed rate to complete the survey. In survey Wave 2, individuals are paid a bonus of $1 only if \( X_i = H \).

Our first result, in Table 3, shows how the distribution of \( X_i \) shifts in response to changes in the treatment allocation rule between Wave 1 and Wave 2. In Wave 1, those who like math respond correctly at a much higher rate than those who do not like math. From Wave 1 to Wave 2, the proportion of all individuals reporting \( X_i = H \) increases 18 percentage points. For those who do not like math, the proportion increases from 29% to 48% of individuals. This affects the usefulness of \( X_i \) for targeting. Based on the ITEs that we assumed for \( \theta_i = 0 \) and \( \theta_i = 1 \), when treatments are allocated uniformly in Wave 1, then the CATE for \( X_i = L \) is negative and for \( X_i = H \) is positive. However, enough individuals who do not like math are induced to respond to the factoring question in Wave 2 that the resulting CATE for \( L \) rises and for \( H \) drops.

The next step is to introduce a a simple structural model of manipulation that follows Assumption 3, and use the data generated from the MTurk experiment to estimate the
Table 3: Summary Statistics of MTurk Experiment

|                               | Wave 1 (Uniform Rule) | Wave 2 (Cutoff Rule) |
|-------------------------------|-----------------------|----------------------|
| $\delta(L)$                   | 0.0                   | 0.0                  |
| $\delta(H)$                   | 0.0                   | 1.0                  |
| $Pr(X_i = H)$                 | 0.41                  | 0.59                 |
| $Pr(Z_i = 1)$                 | 0.43                  | 0.49                 |
| $Pr(X_i = H|Z_i = 0)$          | 0.29                  | 0.48                 |
| $Pr(X_i = H|Z_i = 1)$          | 0.56                  | 0.71                 |
| $\tau(L, \delta)$            | -2.47                 | -2.22                |
| $\tau(H, \delta)$            | 0.566                 | 0.495                |

Table 4: Parameter Estimates for Semi-Synthetic Simulation

| Estimate   | $\lambda$ | $\hat{\rho}_0$ | $\hat{\rho}_1$ | $b_0$ | $b_1$ |
|------------|------------|-----------------|-----------------|-------|-------|
|            | 0.46       | 0.71            | 0.44            | 3.737 | 2.933 |

Close to half of the agents like math. Liking math impacts whether an individual responds correctly to the factoring question. Those with $\theta_i = 1$ are much more likely to respond $H$. Solving for the optimal treatment allocation rule in this structural model, we find that the optimal rule in this setting has $\delta^*(L) = 0$ and $\delta^*(H) = 0.644$. We can now simulate how Algorithm 1 would perform. We assume that the algorithm is run by a planner who does not have any knowledge of the underlying structural model for strategic behavior, but can observe outcomes from a sample of $n = 2000$ agents who respond
according to this estimated model repeatedly over time. In Figure 2, we plot the average regret $\frac{R_T}{T}$ over 500 periods of Algorithm 1, averaged over 50 repetitions of the simulation.

![Figure 2: Average Regret $R_T/T$ of Algorithm 1](image)

We see that Algorithm 1 quickly finds a treatment allocation rule that is close to the optimum and average regret decreases rapidly towards zero.

5 Conclusion

In this paper, we have shown that strategic behavior impacts how a binary treatment is optimally allocated based on observed covariates. These results are complementary to those in the strategic classification literature which show that strategic behavior impacts optimal prediction rules and require new estimation methods. We proposed a sequential experiment based on Bayesian optimization that converges to the optimal treatment assignment function, without making parametric assumptions on the structure of individuals’ strategic behavior.

There are a variety of avenues for future work, including extending the results to more complex settings with multi-valued or continuous treatments, exploring estimation and approximation methods based on observational data, and exploring how strategic behavior impacts the analysis of causal settings with interference between units.
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A Proofs

A.1 Proof of Theorem 1

When we have discrete covariates, then we can write the optimal treatment allocation rule as the solution to a constrained maximization problem

\[
\max_{\delta} \sum_{x \in \mathcal{X}} f(x, \delta)(\delta(x)\mu(1, x, \delta) + (1 - \delta(x))\mu(0, x, \delta)) \\
\text{s.t. } 0 \leq \delta \leq 1
\]

We can write the Lagrangian for this problem as:

\[
L(\delta, \lambda^1, \lambda^0) = \sum_{x \in \mathcal{X}} f(x, \delta)(\delta(x)\mu(1, x, \delta) + (1 - \delta(x))\mu(0, x, \delta)) - \lambda^1(\delta - 1) + \lambda^0(\delta)
\]

Since we assume differentiability of \(f(x, \delta)\) and \(\mu(w, x, \delta)\), in \(\delta\), both the objective and constraints are differentiable functions of \(\delta\), and we can write the KKT conditions corresponding to the constrained optimization problem as, for each \(x \in \mathcal{X}\):

\[
f(x, \delta)[\tau(x, \delta)] \\
+ \sum_{z \in \mathcal{X}} \left[ \frac{\partial f(z, \delta)}{\partial \delta(x)} \mu(z, \delta) + f(z, \delta) \left( \delta(z) \frac{\partial \mu(1, z, \delta)}{\partial \delta(x)} + (1 - \delta(z)) \frac{\partial \mu(0, z, \delta)}{\partial \delta(x)} \right) \right] \\
- \lambda^1_x + \lambda^0_x = 0,
\]

and

\[
(\delta(x) - 1)\lambda^1_x = 0, \\
\delta(x)\lambda^0_x = 0, \\
0 \leq \delta(x) \leq 1, \\
\lambda^1_x, \lambda^0_x \geq 0.
\]

Since we have that constraints are linear, then the KKT conditions are a necessary condition that must hold for any optimum \(\delta^*\), which leads to the theorem statement.

A.2 Proof of Corollary 1

For every \(x \in \mathcal{X}\), we have that \(\delta^c(x) = 1\) or \(\delta^c(x) = 0\). Start with choosing an \(x\) such that \(\delta^c(x) = 1\), so the constraint on \(\delta^c(x)\) holds. In this setting, we have that \(\tau(x, \delta^c) > 0\), by the
definition of $\delta^c$. Furthermore, it must be that $\lambda^0_x = 0$ so that both constraint qualification conditions hold. The remaining conditions to check are $\lambda^1_x \geq 0$, and Equation 3.

We enumerate a few cases:

1. If $s(x, \delta^c) \geq 0$, then there $\lambda^1_x = s(x, \delta^c) + f(x, \delta)\tau(x, \delta) \geq 0$ satisfies Equation 3. The necessary conditions for optimality are not violated.

2. If $s(x, \delta^c) \leq 0$, and $-s(x, \delta^c) < \tau(x, \delta^c)f(x, \delta)$, then $\lambda^1_x = s(x, \delta^c) + f(x, \delta)\tau(x, \delta) \geq 0$ satisfies Equation 3. The necessary conditions for optimality are not violated.

3. If $s(x, \delta^c) \leq 0$, and $-s(x, \delta^c) > \tau(x, \delta^c)f(x, \delta)$, then there is no $\lambda^1_x \geq 0$ that satisfies Equation 3 and $\delta^c$ cannot meet the necessary conditions of optimality.

We can make a similar argument with $\delta^c(x) = 0$. For such an $x$, then we have that $\tau(x, \delta^c) < 0$ by the definition of $\delta^c$. We must have $\lambda^1_x = 0$ so that both constraint qualification conditions hold. The remaining conditions to check are $\lambda^0_x \geq 0$, and Equation 3.

1. If $s(x, \delta^c) \leq 0$, then there $\lambda^0_x = -s(x, \delta^c) - f(x, \delta)\tau(x, \delta) \geq 0$ satisfies Equation 3. The necessary conditions for optimality are not violated.

2. If $s(x, \delta^c) \geq 0$, and $-s(x, \delta^c) > \tau(x, \delta^c)f(x, \delta)$, then $\lambda^0_x = -s(x, \delta^c) - f(x, \delta)\tau(x, \delta) \geq 0$ satisfies Equation 3. The necessary conditions for optimality are not violated.

3. If $s(x, \delta^c) \geq 0$, and $-s(x, \delta^c) < \tau(x, \delta^c)f(x, \delta)$, then there is no $\lambda^0_x \geq 0$ that satisfies Equation 3 and $\delta^c$ cannot meet the necessary conditions of optimality.

For $x$ with a positive or negative CATE, we can summarize Case 3 by the condition in the Corollary, under which $\delta^c$ cannot be optimal. If Case 3 never holds for any $x \in X$, then we have that $\delta^c$ meets the necessary conditions for optimality. These conditions are sufficient if $\Pi(\delta)$ is also concave.

A.3 Proof of Corollary 2

In the binary case, we can write $f(\delta) = f(H, \delta)$ and $f(L, \delta) = 1 - f(\delta)$. At $\delta^c$, we have that $\delta^c(H) = 1$ and $\delta^c(L) = 0$. Using the formula in Equation 4, we can write

$$s(H, \delta^c) = \frac{\partial}{\partial \delta(H)} \left[ f(\delta^c) \mu(1, H, \delta^c) + (1 - f(\delta^c))\mu(0, L, \delta^c) \right].$$  \hspace{1cm} (6)

$$s(L, \delta^c) = \frac{\partial}{\partial \delta(L)} \left[ f(\delta^c) \mu(1, H, \delta^c) + (1 - f(\delta^c))\mu(0, L, \delta^c) \right].$$  \hspace{1cm} (7)
Let $\theta$ represent the unobserved type, so that $Y_i(\cdot) = Y(\cdot, \theta_i)$, $X_i(\delta) = X(\delta, \theta_i)$ and $R_i = R(\theta_i)$. Using Bayes’ Rule, we have that $Pr(X_i(\delta^c) = H, \theta) = Pr(X_i(\delta^c) = H|\theta)p(\theta)$. Furthermore, from Assumption 3, we have that $Pr(X_i(\delta^c) = H|\theta) = \mathbb{1}(\delta^c(H) - \delta^c(L) \geq R(\theta))$, where for those who always report $H$, we can write $R(\theta) = -\infty$. We can then write

$$
\mu(1, H, \delta^c) = \mathbb{E}[Y(1, \theta)|X_i(\delta^c) = H] = \int Pr(X_i(\delta^c) = H, \theta)Y(1, \theta)d(\theta) Pr(X_i(\delta^c) = H) = \int p(\theta)\mathbb{1}(\delta^c(H) - \delta^c(L) \geq R(\theta))Y(1, \theta)d(\theta) f(\delta^c)
$$

Similarly, we can derive

$$
\mu(1, L, \delta^c) = \int p(\theta)[1 - \mathbb{1}(\delta^c(H) - \delta^c(L) \geq R(\theta))]Y(0, \theta)d(\theta) 1 - f(\delta^c)
$$

We also derive the derivatives of the indicator function with respect to $\delta(H)$ and $\delta(L)$ and evaluate at the cutoff rule.

$$
\frac{\partial \mathbb{1}(\delta^c(H) - \delta^c(L) \geq R(\theta))}{\delta(H)} = \mathbb{1}(R(\theta) = \delta^c(H) - \delta^c(L)) = \mathbb{1}(R(\theta) = 1)
$$

$$
\frac{\partial \mathbb{1}(\delta^c(H) - \delta^c(L) \geq R(\theta))}{\delta(L)} = -1(R(\theta) = \delta^c(H) - \delta^c(L)) = -1(R(\theta) = 1)
$$

Plugging these expressions into Equation 6, we have that

$$
s(H, \delta^c) = \frac{\partial}{\partial \delta(H)} \int p(\theta)[\mathbb{1}(\delta^c(H) - \delta^c(L) \geq R(\theta))Y(1, \theta) + (1 - \mathbb{1}(\delta^c(H) - \delta^c(L) \geq R(\theta)))Y(0, \theta)]d(\theta)
$$

$$
= \int p(\theta)\mathbb{1}(R(\theta) = 1)[Y(1, \theta) - Y(0, \theta)]d\theta
$$

$$
= \mathbb{E}[Y_i(1) - Y_i(0)|R_i]
$$

$$
\geq 0
$$

where we use the dominated convergence theorem to pass the derivative through the integral and the inequality follows from the condition on the treatment effect of those who are strategic around the cutoff rule in Corollary 2.

Similarly, we can write

$$
s(L, \delta^c) = -\int p(\theta)\mathbb{1}(R(\theta) = 1)[Y(1, \theta) - Y(0, \theta)]d\theta \leq 0
$$

Note that to meet the condition in Corollary 1 then we must have $s(H, \delta^c) \geq 0$ and
s(L, δc) ≤ 0, since we have τ(H, δc) > 0 and τ(L, δc) < 0. We have shown exactly this and the proof is complete.

B Solving Example Models

B.1 Solving Example 1

We can write the objective function as

\[
\Pi(\delta) = \delta(H)[Pr(X_i = H|\theta_i = 1)Pr(\theta_i = 1)Y(1, \theta_i = 1) + Pr(X_i = H|\theta_i = 0)Pr(\theta_i = 0)Y(1, \theta_i = 0) \\
+ (1 - \delta(H))[Pr(X_i = H|\theta_i = 1)Pr(\theta_i = 1)Y(0, \theta_i = 1) + Pr(X_i = H|\theta_i = 0)Pr(\theta_i = 0)Y(0, \theta_i = 0)] \\
+ \delta(L)[Pr(X_i = L|\theta_i = 1)Pr(\theta_i = 1)Y(1, \theta_i = 1) + Pr(X_i = L|\theta_i = 0)Pr(\theta_i = 0)Y(1, \theta_i = 0)] \\
+ (1 - \delta(L))[Pr(X_i = L|\theta_i = 1)Pr(\theta_i = 1)Y(0, \theta_i = 1) + Pr(X_i = L|\theta_i = 0)Pr(\theta_i = 0)Y(0, \theta_i = 0)]
\]

Y(1, \theta_i = 1) = 3.75, Y(1, \theta_i = 0) = 5, Y(0, \theta_i = 1) = 0, and Y(1, \theta_i = 0) = 10. Pr(X_i = H|\theta_i = 1) = 1, so we just have to derive Pr(X_i = H|\theta_i = 0).

\[
Pr(X_i = H|\theta_i = 0) = Pr(C_i \leq 5(\delta(H) - \delta(L))) \\
= \frac{1}{2}(\delta(H) - \delta(L))
\]

We can then plug into the objective,

\[
\Pi(\delta) = \delta(H)[1.875 + \frac{5}{4}(\delta(H) - \delta(L)) + (1 - \delta(H))\frac{10}{4}(\delta(H) - \delta(L)) \\
+ \delta(L)\frac{5}{4}(2 - \delta(H) + \delta(L)) + (1 - \delta(L))\frac{10}{4}(2 - \delta(H) + \delta(L))
\]

which can be simplified to

\[
\Pi(\delta) = -0.625(2\delta(L)^2 - 4\delta(L)\delta(H) + 4\delta(L) + 2\delta(H)^2 - 3\delta(H) - 8)
\]

Note that this objective is concave. We can find the global optimum using the KKT conditions from Theorem 1, and also noting that based on the problem intuition the optimum has \(\delta(H) > 0\) and \(\delta(L) < 1\), so that \(\lambda^1_L = \lambda^0_H = 0\).

For \(\delta(L)\), we have:

\[
-\frac{5}{2}(\delta(L) - \delta(H) + 1) + \lambda^0_L = 0
\]
For $\delta(H)$, we have:

$$2.5(\delta(L) - \delta(H) + 0.75) - \lambda H = 0$$

We can’t have both $\delta(L)$ and $\delta(H)$ be interior solutions. There is also no solution satisfying the KKT conditions when $\delta(H) = 1$. The solution to the KKT conditions has $\delta^*(L) = 0$ and $\delta^*(H) = 0.75$.

For the CATEs, note that in the uniform allocation we have those with $\theta_i = 0$ (who all have an ITE of -5) reporting $X_i = L$ and those with $\theta_i = 1$ reporting $X_i = L$ (who all have an ITE of 3.75).

The $\tau(L, \delta)$ remains the same for different allocation rules. $\tau(H, \delta)$ is computed as follows for the cutoff and optimal rule.

$$\tau(H, \delta^*) = \frac{0.5}{0.5 + 0.1875} \cdot 3.75 + \frac{0.25}{0.75} \cdot (-5)$$

$$\tau(H, \delta^*) = \frac{0.5}{0.5 + 0.1875} \cdot 3.75 + \frac{0.1875}{0.5 + 0.1875} \cdot (-5)$$

### B.2 Solving Example 2

As in the previous example, we can write the objective function as

$$\Pi(\delta) = \delta(H)[Pr(X_i = H|\theta_i = 1)Pr(\theta_i = 1)Y(1,\theta_i = 1) + Pr(X_i = H|\theta_i = 0)Pr(\theta_i = 0)Y(1,\theta_i = 0) + (1 - \delta(H))[Pr(X_i = H|\theta_i = 1)Pr(\theta_i = 1)Y(0,\theta_i = 1) + Pr(X_i = H|\theta_i = 0)Pr(\theta_i = 0)Y(0,\theta_i = 0)]$$

$$+ \delta(L)[Pr(X_i = L|\theta_i = 1)Pr(\theta_i = 1)Y(1,\theta_i = 1) + Pr(X_i = L|\theta_i = 0)Pr(\theta_i = 0)Y(1,\theta_i = 0)]$$

$$+ (1 - \delta(L))[Pr(X_i = L|\theta_i = 1)Pr(\theta_i = 1)Y(0,\theta_i = 1) + Pr(X_i = L|\theta_i = 0)Pr(\theta_i = 0)Y(0,\theta_i = 0)]$$

$$Y(1,\theta_i = 1) = 10, Y(1,\theta_i = 0) = 1, Y(0,\theta_i = 1) = 5, \text{and} \ Y(0,\theta_i = 0) = 5. \ Pr(X_i = L|\theta_i = 0) = 1, \text{so we just have to derive} \ Pr(X_i = H|\theta_i = 1).$$

$$Pr(X_i = H|\theta_i = 1) = Pr(-V_i \leq 5(\delta(H) - \delta(L)))$$

$$= \frac{1}{4}(\delta(H) - \delta(L)) + \frac{1}{2}$$

We can then plug into the objective,

$$\Pi(\delta) = \delta(H)\left[\frac{5}{4}(\delta(H) - \delta(L)) + \frac{5}{2}\right] + (1 - \delta(H))\left[\frac{5}{8}(\delta(H) - \delta(L)) + \frac{5}{4}\right]$$

$$+ \delta(L)(3 - \frac{5}{4}(\delta(H) - \delta(L)) + (1 - \delta(L))\left[\frac{15}{4} - \frac{5}{8}(\delta(H) - \delta(L))\right]$$
which can be simplified to

$$\Pi(\delta) = \frac{1}{8}[5\delta(L)^2 - 10\delta(L)\delta(H) - 6\delta(L) + 5\delta(H)^2 + 10\delta(H) + 40]$$

Note that this objective is concave and we can find the global optimum using the KKT conditions from Theorem 1:

For $\delta(L)$, we have:

$$\frac{1}{4}(5\delta(L) - 5\delta(H) - 3) + \lambda^0_L = 0$$

For $\delta(H)$, we have:

$$\frac{-5}{4}(\delta(L) - \delta(H) - 1) - \lambda^1_H = 0$$

The KKT conditions are satisfied with $\delta^*(L) = 0$ and $\delta^*(H) = 1$ at which the objective value is 6.875. To compute the CATEs:

$$\tau(H, \delta^0) = \frac{1}{1 + 0.5}(-4) + \frac{0.5}{1 + 0.5}$$

$$\tau(L, \delta^c) = \frac{1}{1 + 0.25}(-4) + \frac{0.25}{1 + 0.25}$$