Nonequilibrium dynamics of quantum fields

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Abstract

The nonequilibrium effective equation of motion for a scalar background field in a thermal bath is studied numerically. This equation emerges from a microscopic quantum field theory derivation and it is suitable to a Langevin simulation on the lattice. Results for both the symmetric and broken phases are presented.

Key words: nonequilibrium field dynamics; relativistic heavy-ion collisions; lattice simulations

Relativistic heavy-ion experiments produce highly excited hadronic matter at high temperatures and densities. One important question in the study of the properties of the produced excited matter is the understanding of the dynamics of the quantum fields governing the effective degrees of freedom. In the present work we will discuss the derivation of stochastic Ginzburg-Landau-Langevin (GLL) type of equations with additive and multiplicative noises from quantum corrections to the effective action of a self-interacting \( \lambda \phi^4 \) scalar field theory. We apply the real time Schwinger’s closed time path formalism, as explained for instance in Ref. [1], originally done for the symmetric phase of the model, and we here extend that application to the broken phase of the model. Results of numerical lattice simulations of the derived effective GLL equation in 3 spatial dimensions are then presented for both the symmetric and broken phases. Particular attention is devoted to the question of the the renormalization of the stochastic GLL equation to obtain equilibrium results that are independent of the lattice spacing.

Consider the scalar field model with action

\[
S[\phi] = \int d^4 x \left[ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \right]
\] (1)
where the minus (plus) sign in the quadratic term for the potential is for the symmetric (broken) phase. The effective action in terms of Feynman diagrams, up to $O(\lambda^2)$, is given by

$$
\Gamma[\varphi_c] = S[\phi] + \sum + \sum + \sum + \sum + O(\lambda^3). \quad (2)
$$

Following [1], the real time Schwinger’s closed time path formalism [2] is used for obtaining the effective action of the theory. We will not repeat here the derivation done in [1], that can also be carried out analogously for the broken phase, and only give here the final result for the effective equation of motion of the order parameter. In the derivation there appears imaginary terms in the effective action, that can be associated to functional integrations over Gaussian fluctuation fields $\xi$ by making use of a Hubbard-Stratonovich transformation. The fields $\xi$ act as fluctuation sources for the scalar field configuration (order parameter) $\varphi_c = \langle \phi \rangle$. In the original calculation of [1] there appears two fluctuation (stochastic) fields, $\xi_1$ and $\xi_2$. $\xi_1$ couples to $\varphi_c$, leading to a multiplicative noise term $(\varphi_c \xi_1)$ in the equation of motion for $\varphi_c$, while the field $\xi_2$ gives origin to an additive noise term. A Langevin-like equation for $\varphi_c$ emerges after a series of physically motivated approximations, related to spatial and temporal nonlocalities, are considered. For instance, considering only slowly-varying contributions in space and time (this is a valid assumption for systems near equilibrium, when $\varphi_c$ is not expected to change considerably with time [3]), the final result that we obtain for the effective equation of motion for $\varphi_c$ is of the form

$$
\left[ \Box + m_T^2 \right] \varphi_c(x, t) + \frac{\lambda_T}{3!} \varphi_c^3(x, t) + \eta \varphi_c^2(x, t) \dot{\varphi}_c(x, t) = \varphi_c(x, t) \xi_1(x, t), \quad (3)
$$

where $\lambda_T$ and $m_T$ are renormalized finite temperature coupling constant and the renormalized finite temperature mass. In the high temperature approximation, $T/m \gg 1$, and within perturbative values of $\lambda$, which is the regime we will be exploring in the simulations, $\lambda_T$ and $m_T$ are well approximated by $\lambda_T \simeq \lambda$ and

$$
m_T^2 \simeq \begin{cases} 
m^2 + \frac{\lambda T^2}{4!} & \text{symmetric phase} \\
- m^2 + \frac{\lambda}{4!} T^2 + \frac{\lambda}{2!} \nu^2 & \text{broken phase} \end{cases} \quad (4)
$$

where $\nu$ is the vacuum expectation value for the scalar field in the broken phase [4] and $\eta$ is the dissipation coefficient associated with the multiplicative noise field $\xi_1$ and it is given by
\[
\eta \bigg|_{T \gg m_T} \approx \frac{96}{\pi T} \ln \left( \frac{2T}{m_T} \right)
\]  

Thermal corrections can restore the symmetry, the potential can change from double well to single well for a temperature larger than a critical temperature \( T_c \) given by \( T_c^2 = \frac{m^2}{(\lambda/4!)} \). Note that the additive noise term drops out at \( O(\lambda^2) \) [1].

The noise treatment involves some care due to appearance of Rayleigh-Jeans ultraviolet divergences at long times when simulating the equation on a discrete lattice. These divergences manifest themselves through a lattice-spacing dependence of the equilibrium solutions. We simulate the GLL equations discretizing time in steps of \( \Delta t \) using a leapfrog algorithm and treat the spatial coordinate using Fast-Fourier transform on a cubic lattice of side \( L \) (as in Ref. [5]). In order to illustrate the dependence of the solutions to the lattice spacing \( h = L/N \), where \( N \) is the number of lattice sites in each spatial direction, we show in Fig. 1 results for a simple double-well potential in dimensionless units such that the temperature is \( T = 1 \), the dissipation coefficient is \( \eta = 1 \) and \( \Delta t = 0.001 \).

Equilibrium solutions of the GLL equation insensitive to lattice spacing can be obtained by the introduction of counterterms in the effective potential. Since the classical theory in three spatial dimensions is super-renormalizable, only two terms, corresponding to the first one-loop and the last two-loop diagrams terms shown in (2), are divergent. The divergent parts of these graphs can be isolated using a lattice regularization and then subtracted from the effective potential in the GLL equation. Explicit expressions for these counterterms can be found in [6]. Once these are included, one obtains the results shown in Fig. 1, where one can see on the right panel that equilibrium solutions insensitive to lattice spacing are obtained. Since the counterterms are calculated with the equilibrium partition function, some sensitivity to lattice spacing remains for short times. Our results for the symmetric and broken phases of the effective GLL equations for the \( \lambda \phi^4 \) theory are shown.

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Fig. 1. Illustration of the lattice size dependence of the solutions of the GLL equation for different lattice spacings \( L/N \) (left) and the effect of counterterms (right).
Fig. 2. Results for the volume average of the order parameter in the symmetric (left) and broken (right) phases.

In Fig. 2. Since our results are obtained under the conditions of λ small (in Fig. 2 λ = 0.25) and for $T/m \gg 1$, the double-well in the broken phase is very shallow and therefore the thermal fluctuations are large. The field then quickly moves to a lower vacuum expectation value, but still different from zero, since the temperature is still smaller than the Ginzburg temperature for the transition.

In conclusion, we have studied the nonequilibrium dynamics for a scalar field background configuration. The effective equation of motion describing the dynamics can be obtained entirely from microscopic considerations and it is in a form of a Langevin equation with multiplicative noise and field dependent dissipation terms. It is suitable to the study of the nonequilibrium dynamics of fields using standard Langevin dynamics methods on the lattice. Further details and a more extended discussion will be presented elsewhere [7].

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