Phenomenology of Dirac NeutrinoNeogenesis
in Split Supersymmetry

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November 16, 2005

Abstract

In Split Supersymmetry scenarios the possibility of having a very heavy gravitino opens the door to alleviate or completely solve the worrisome “gravitino problem” in the context of supersymmetric baryogenesis models. Here we assume that the gravitino may indeed be heavy and that Majorana masses for neutrinos are forbidden as well as direct Higgs Yukawa couplings between left and right handed neutrinos. We investigate the viability of the mechanism known as Dirac leptogenesis (or neutrino-genesis), both in solving the baryogenesis puzzle and explaining the observed neutrino sector phenomenology. To successfully address these issues, the scenario requires the introduction of at least two new heavy fields. If a hierarchy among these new fields is introduced, and some reasonable stipulations are made on the couplings that appear in the superpotential, it becomes a generic feature to obtain the observed large lepton mixing angles. We show that in this case, it is possible simultaneously to obtain both the correct neutrino phenomenology and enough baryon number, making thermal Dirac neutrinoogenesis viable. However, due to cosmological constraints, its ability to satisfy these constraints depends nontrivially on model parameters of the overall theory, particularly the gravitino mass. Split supersymmetry with $10^5 \text{ GeV} \lesssim m_3/2 \lesssim 10^{10} \text{ GeV}$ emerges as a “natural habitat” for thermal Dirac neutrinoogenesis.

1 Introduction

The phenomenology of split supersymmetry models \cite{susy1,susy2} in which a hierarchy is generated between the gaugino masses and the masses of the scalar sparticles has received a great deal of attention in recent times. The main advantage of such scenarios is that they circumvent a wide variety of data pressures on theories with supersymmetry breaking on a lower scale that arise from potentially dangerous radiative corrections involving TeV-scale scalar sparticles for the Higgs Mass, flavor-violating effects, etc. This is done by making the scalar masses heavy, while keeping the gaugino masses at the TeV scale or below to constitute the dark matter and to preserve gauge coupling unification.

In one particularly simple scenario, which we refer to as loop-split supersymmetry (after the PeV-scale supersymmetry of \cite{loop-susy}), supersymmetry is broken at an intermediate scale, around $\sim 10^5 - 10^7 \text{ GeV}$, and all scalars in the theory, (with the exception of one light Higgs
particle) are given masses around the PeV scale while the gauginos acquire masses at the TeV scale [4]. The preferred method of achieving this hierarchy is by invoking anomaly mediation in the gaugino sector (but not the scalar sector). This can be arranged by charging the chiral supermultiplet $X$ responsible for the transmission of supersymmetry effects under some symmetry, so that the term that normally provides the dominant contribution to the gaugino mass is forbidden by gauge invariance. The dominant contribution to the gaugino masses now arises only at the one-loop level [4] and is given by

$$M_\lambda = \frac{\beta g_\lambda}{g_\lambda} \left( \frac{\langle F^\dagger_X F_X \rangle}{M_P^2} \right)^{1/2},$$

where the index $\lambda$ labels the gauge groups in the theory. Charging $X$ will not affect the scalar masses, which are still manifestly gauge-invariant whether $X$ is charged or not, and we obtain the desired hierarchy. The advantage of this scenario is its simplicity: no additional symmetries are required to keep the gaugino masses from being evolved up to the SUSY-breaking scale. The theory also imposes severe restrictions on both the identity (which dictates ought to be predominantly either Wino or Higgsino for strict one-loop AMSB) and mass of the lightest supersymmetric particle (LSP), and it has been shown that dark matter composed primarily of such particles could have observable consequences at the next generation of $\gamma$-ray telescopes [5, 6, 7]. Also, in loop-split SUSY, characteristic signatures of gluino decays may be observable at colliders [8, 9].

As mentioned above, one of the reasons why split supersymmetry is such an attractive phenomenological model is that it provides a convenient way of circumventing data pressures on weak-scale superpartner masses. However, for a theory to describe the universe we live in, it is not enough that it evade all present experimental bounds: the theory must be cosmologically viable as well, in the sense that it does not disrupt big bang nucleosynthesis (BBN), that it is compatible with cosmic inflation, that it permits some mechanism by which baryogenesis may occur, etc. In this paper, we examine the cosmological ramifications of split supersymmetry, and in particular show that a viable model of baryogenesis can be realized via Dirac neutrino genesis (i.e. Dirac leptogenesis) [10, 11]. This is not a trivial problem: it turns out that constraints arising from gravitino cosmology fix the reheating temperature associated with cosmic inflation to $\sim 10^{10}$ GeV or lower for substantial regions of $m_{3/2} - m_{\text{LSP}}$ parameter space. This heightens the tensions already inherent among the model parameters (mass scales, couplings, etc.) of the theory. Data from neutrino oscillation experiments impose an additional battery of constraints any phenomenologically viable theory must satisfy. As a result, getting Dirac neutrino genesis to work in split supersymmetry requires careful consideration of both cosmology and neutrino physics.

Our aim in this paper is threefold. First, we examine the astrophysical and cosmological constraints on thermal leptogenesis models, especially those associated with baryogenesis and gravitino cosmology. Second, we investigate the neutrino spectrum constraints imposed on Dirac neutrino genesis models by neutrino oscillation experiments. Third, we solve the full system of Boltzmann equations for Dirac neutrino genesis and assess its viability—i.e. its ability to satisfy all aforementioned constraints.
2 Dirac Neutrinoogenesis

The universe we live in is manifestly asymmetric between baryons and antibaryons. This statement can be quantified by introducing a parameter $\eta$, defined as $\eta = n_B/n_\gamma$. Here $n_B = n_b - n_{\bar{b}}$, where $n_b$ and $n_{\bar{b}}$ are the baryon density and antibaryon density of our universe, respectively, and $n_\gamma$ is the present number density of photons. The value of $\eta$ has recently been measured with great precision by WMAP [14] to be within the range

$$\eta = (6.1 \pm 0.3) \times 10^{-10}.$$  

In the standard cosmology, which is symmetric with respect to baryons and antibaryons, one would expect to find $\eta = 0$. As this is not to be the case, we must find a method of baryogenesis – the generic term for a process through which a baryon asymmetry might evolve in the early universe – to account for this. A set of generic criteria required for any successful baryogenesis scenario were first established by Sakharov [15]: first, there must be baryon number $B$ violation; second, there must be $C$ and $CP$ violation; and third, there must be some departure from thermal equilibrium. If any one of these conditions is not met, baryogenesis fails\(^1\).

A variety of viable baryogenesis models exist, including electroweak baryogenesis, in which the CP-violation occurs at a bubble wall, or phase boundary, and Affleck-Dine baryogenesis [18], in which the baryon asymmetry is generated by moduli fields charged under $B - L$. In this paper, we will focus on models which achieve baryogenesis through a framework known as leptogenesis [19, 20]. In this scenario, decays of heavy particles in the early universe which violate both $CP$ and lepton number $L$ produce an initial lepton asymmetry, which is then converted to a nonzero baryon asymmetry by sphaleron processes associated with the SU(2) electroweak anomaly [17]. Leptogenesis is a particularly attractive model because in addition to its ability to yield a realistic value for $\eta$ [21], it can also explain why the standard model neutrinos have small but nonzero masses. In its most common form, which we will call Majorana leptogenesis, the heavy particle whose decays violate $L$ is taken to be the right handed neutrino, which being a gauge singlet, may be given a large Majorana mass $M_{\nu_R}$. Since the most general realizable superpotential

$$W \ni yLNH_u + M_{\nu_R}NN$$  

will also contain a term which gives rise to a Dirac mass $m_D = yv \sin \beta$, where $v$ is the standard model Higgs VEV, the neutrino mass matrix will contain small off-diagonal terms mixing $\nu_L$ and $\nu_R$. When this matrix is diagonalized, the resulting mass spectrum contains three light neutrinos with masses

$$m_\nu = m_D \frac{1}{M_{\nu_R}} m_D^1,$$  

which are identified with the standard model neutrinos, as well as three heavy neutrinos with masses $\sim M_{\nu_R}$. This see-saw mechanism [22] is an example of one of the strongest assets

\(^1\)Actually the third condition is only required in theories where the Hamiltonian preserves $CPT$. For a theory that is not $CPT$-invariant, such as the spontaneous baryogenesis of [16], departure from thermal equilibrium is not necessary for baryogenesis. In the scenarios we are considering, we will assume that $CPT$ is a good symmetry of the Hamiltonian and that some departure from thermal equilibrium is required.
of leptogenesis scenarios: they all provide some explanation for the lightness of neutrino masses, in addition to explaining the origin of the observed baryon asymmetry.

Recently, another promising leptogenesis model, which is often called Dirac leptogenesis or Dirac neutrino genesis [10, 11], has received some attention. In this scenario, an additional symmetry is introduced, and charges are assigned under this new symmetry in a manner which forbids both the Majorana and Dirac mass terms appearing in (3) and additional heavy, vector-like fields are introduced whose decays will violate $CP$. These decays build up equal and opposite lepton asymmetries $L_\ell$ and $L_{\nu_R}$ in the left-handed lepton and right-handed neutrino sectors, while maintaining an overall lepton number for the universe $L_{tot} = L_\ell + L_{\nu_R} = 0$.² In the fermion sector these stores do not equilibrate due to the smallness of the neutrino Dirac mass term, which only appears in the low-energy effective theory suppressed by powers of the mass scales associated with the heavy vector-like fields. The electroweak sphaleron processes which convert $L_\ell$ into a baryon asymmetry $B$ effectively shut off before $L_\ell$ and $L_{\nu_R}$ have a chance to equilibrate. The result, depicted in fig. 1, is that the universe ends up with a net positive lepton number as well as baryon number.

We will consider a model with the same field content as the one presented in [11]. In addition to the usual quark and lepton supermultiplets (of which the left-handed lepton multiplet $L$ and the Higgs multiplets $H_u$ and $H_d$ will be pertinent to leptogenesis), we introduce a right-handed neutrino superfield $N$, an exotic chiral multiplet $\chi$, and a number $N_\Phi$ of vector-like pairs of chiral multiplets $\Phi_i$ and $\overline{\Phi}_i$ (the precise value of $N_\Phi$ is unspecified: any choice of $N_\Phi \geq 2$ is allowed from a baryogenesis standpoint). The charge assignments under the additional symmetry, whatever it may be, are to be arranged such that the most

²Unlike in Majorana leptogenesis, no explicit lepton-number-violating terms are present here.
general superpotential that can be written is
\[ W \ni \lambda_{i\alpha} N_\alpha \Phi_i H_u + h_{i\alpha} L_\alpha \Phi_i + M_{\Phi_i} \Phi_i \Phi_i + \mu H_u H_d, \quad (5) \]
where \( \alpha \) is a family index, \( \lambda_{i\alpha} \) and \( h_{i\alpha} \) are Yukawa couplings, \( M_{\Phi_i} \) are (supersymmetry-respecting) mass terms for the \( \Phi_i \) fields, and \( \mu \) is the usual Higgs mass parameter. The particular symmetry introduced and charge configuration employed in arriving at this superpotential is not of particular relevance to us, nor is it the aim of this paper to explore the model-building possibilities afforded by different such configurations, but an example (taken from [11]) of one that works is provided in table 1. \( M_{\Phi_i}, \lambda_{i\alpha}, \) and \( h_{i\alpha} \) may in general be complex. Both the scalar and fermionic components of the \( \Phi \) and \( \Phi \) multiplets, which we denote by \( \phi, \bar{\phi}, \psi_{\Phi}, \) and \( \psi_{\Phi} \) will play the role that the \( \nu_R \) play in Majorana leptogenesis. As for neutrino masses, they will appear in the low-energy effective superpotential obtained by integrating out \( \Phi_i \) and \( \Phi_i \):
\[ W_{\text{eff}} \ni \frac{\lambda_{i\alpha} h_{i\beta}}{M_{\Phi_i}} \chi L H_u N + \mu H_u H_d. \quad (6) \]
If we arrange for the scalar component of \( \chi \) to acquire a VEV, then the first term in \( W_{\text{eff}} \) translates into a neutrino Dirac mass
\[ m_{\nu\alpha\beta} = \langle \chi \rangle \nu \sin \beta \sum_i \frac{\lambda_{i\alpha} h_{i\beta}}{M_{\Phi_i}}. \quad (7) \]
This can be attained via an O’Raifeartaigh model of the type employed in [13], in which the F-term of \( \chi \) acquires a large VEV \( \langle F \rangle \sim m_3/2M_P \) and supergravity effects give rise to a nonzero VEV \( \langle \chi \rangle \sim 16\pi m_3/2\kappa^{-3} \) for the scalar component of \( \chi \), where \( \kappa \) is an undetermined dimensionless coupling constant. Requiring that \( \langle \chi \rangle \ll M_{\Phi_1} \), where \( M_{\Phi_1} \) denotes the lightest of the \( M_{\Phi_i} \), and that the \( \lambda_{i\alpha} \) and \( h_{i\alpha} \) are \( \mathcal{O}(1) \) or smaller, this setup yields small yet nonzero neutrino masses without the aid of the see-saw mechanism. Furthermore, because of the simple structure of the neutrino mass matrix given in (7), Dirac neutrino genesis can, under certain conditions, yield interesting predictions about the mass hierarchy among the standard model neutrinos.

Just like in Majorana Leptogenesis, some intermediate scale is required to set the mass scale of the heavy fields responsible for the small neutrino masses and for the lepton number generation. In our case, we are less constrained since the neutrino masses are set not only by the masses \( M_{\Phi_i} \) of the heavy fields, but also by the VEV \( \langle \chi \rangle \) of the new field \( \chi \). This leaves us more freedom to choose the mass scale of the heavy fields, but can also be thought of as a drawback since we have lost some predictiveness. One suggestive idea would be to make use of the \( \chi \) field and the additional symmetry to generate the supersymmetric \( \mu \) term. The effective superpotential could then look like
\[ W_{\text{eff}} \ni \frac{\lambda_{i\alpha} h_{i\beta}}{M_{\Phi_1}} \chi L H_u N + Y_\chi \chi H_u H_d. \quad (8) \]
Table 1: One possible set of charge assignments, taken from [11], that leads to the Dirac
neutrinoogenesis superpotential given in (5). Here the additional symmetry employed is a
$U(1)$ (which may in principle be either global or local). Only the charges of the fields
relevant to leptogenesis, which include the Higgs doublets $H_u$ and $H_d$, the left-handed
lepton superfield $L$, the right-handed neutrino superfield $N$, the heavy fields $\Phi$ and $\Phi^\dagger$, and the additional field $\chi$, have been included. Here, $U(1)_L$, $SU(2)$, and $U(1)_Y$ respectively
denote lepton number, $SU(2)$, and $U(1)$ hypercharge quantum numbers, and $U(1)_N$ denotes
the charge under the additional $U(1)$.

where $Y_\chi$ is an $O(1)$ coupling constant, and where the Higgses should now be charged under
the new hidden sector symmetry (and therefore enlarging the list of potential problems of
the model, like cancelation of anomalies, etc.). In this situation, we have $\mu = Y_\chi \langle \chi \rangle$
and any observation or limits on Higgsino dark matter would directly constrain the VEV $\langle \chi \rangle$.
(Split) supersymmetry might also give us some ideas as to how to relate the heavy fields $\Phi$
to the SUSY breaking scale. We will come back to some of these issues when we deal in
detail with neutrino phenomenology in section 4.

If $y_{i\alpha}$ and $h_{i\alpha}$ contain nontrivial, $CP$-violating phases, a net $CP$ asymmetry will be
generated as the component fields of $\Phi_1$ and $\Phi^\dagger_1$ decay, the leading contribution to which
results from the interference of tree-level and one-loop-level diagrams. Those relevant to $\phi$
and $\phi^\dagger$ decay are shown in fig. 2 These contributing to $CP$
fermion fields $\psi_{\Phi_1}$ and $\psi_{\Phi^\dagger_1}$ undergo similar decays,
and stores of lepton number are built up in the lepton fields $\nu_R$ and $\ell$, and in the associated
slepton fields $\tilde{\nu}_R$, $\tilde{\ell}$. The $CP$ amplitudes (and resultant $CP$-asymmetries) generated from
the decays of the charged fields in the fields charged under SU(2) will be the same as those
generated from the decays of the neutral fields, since SU(2) is unbroken. Additionally,
in the approximation of unbroken supersymmetry, the amplitudes (and resultant $CP$-
asymmetries) in the fermion case will be the same as those for the scalar case. We will
assume that any lepton number stored in the slepton sector rapidly equilibrates through
the large $\langle F_\chi \rangle$ term, and thus we need only pay attention to the lepton number stored in
the lepton sector.

We will begin by examining a toy model in which there are only two sets of $\Phi$ and $\Phi^\dagger$, the minimum number required for $CP$-violation to take place, and make the further
simplifying assumption that the masses $M_{\Phi_i}$ are all much larger than $\mu$. In this case [23],
one may parameterize the associated lepton number violation by defining a single decay
asymmetry $\epsilon$, which represents the amount of lepton number generated in any particular

\begin{table}
\begin{center}
\begin{tabular}{|c|c|c|c|c|}
\hline
Field & $U(1)_L$ & $U(1)_N$ & $SU(2)$ & $U(1)_Y$ \\
\hline
$N$ & -1 & +1 & 1 & 0 \\
$L$ & +1 & 0 & 2 & $-\frac{1}{2}$ \\
$H_u$ & 0 & 0 & 2 & $\frac{1}{2}$ \\
$H_d$ & 0 & 0 & 2 & $-\frac{1}{2}$ \\
$\phi$ & +1 & -1 & 2 & $-\frac{1}{2}$ \\
$\phi^\dagger$ & -1 & +1 & 2 & $\frac{1}{2}$ \\
$\chi$ & 0 & -1 & 1 & 0 \\
\hline
\end{tabular}
\end{center}
\end{table}
Figure 2: Diagrams that give the leading contribution to the $CP$ asymmetry from decays of the scalar fields $\phi_1$ and $\phi_1^\dagger$. Similar $CP$ asymmetries are generated during the decay of the fermionic fields $\psi\Phi_1$ and $\psi\Phi_1^\dagger$.

lepton-number-carrying species by the decay of a single heavy particle. This implies the relations

$$\Gamma(\Phi_1 \rightarrow N_\alpha^c H_u^\alpha) - \Gamma(\Phi_1^c \rightarrow N_\alpha H_u) \equiv \epsilon \Gamma_D$$

(9)

$$\Gamma(\Phi_1 \rightarrow L_\alpha \chi) - \Gamma(\Phi_1 \rightarrow L^c_\alpha \chi^c) \equiv -\epsilon \Gamma_D$$

(10)

$$\Gamma(\Phi_1 \rightarrow L^c_\alpha \chi^c) - \Gamma(\Phi_1 \rightarrow L_\alpha \chi) \equiv \epsilon \Gamma_D$$

(11)

$$\Gamma(\Phi_1 \rightarrow N_\alpha H_u) - \Gamma(\Phi_1^c \rightarrow N^c_\alpha H_u^\alpha) \equiv -\epsilon \Gamma_D,$$

(12)

where $\Gamma_D$ is the total decay width of any of the heavy fields in the $\Phi_1$ or $\Phi_1^\dagger$ supermultiplets, and we have used the superfield notation for $\Phi_1$, $N$, etc. to indicate that the decay asymmetries are the same for all supersymmetrizations of the diagrams in fig. 2. Explicit calculation of $\Gamma_D$ and $\epsilon$ yields

$$\Gamma_D = \frac{1}{16\pi} M_{\Phi_1} \sum_\alpha \left( |\lambda_{1\alpha}|^2 + |h_{1\alpha}|^2 \right).$$

(13)

and

$$\epsilon = \frac{\text{Im}(\lambda^*_{1\alpha} \lambda_{2\alpha} h_{1\beta} h_{2\beta} M_{\Phi_1} M_{\Phi_2})}{4\pi(|M_{\Phi_2}|^2 - |M_{\Phi_1}|^2)(|\lambda_{1\gamma}|^2 + |h_{1\gamma}|^2)},$$

(14)

where in both equations, a sum over the repeated indices $\alpha$, $\beta$ and $\gamma$ is assumed. Defining $\delta \equiv |M_{\Phi_1}|/|M_{\Phi_2}|$, $\epsilon$ can be expressed in the more revealing form

$$\epsilon = \frac{\text{Im}(\lambda^*_{1\alpha} \lambda_{2\alpha} h_{1\beta} h_{2\beta} e^{i\psi})}{4\pi(|\lambda_{1\gamma}|^2 + |h_{1\gamma}|^2)} \left( \frac{\delta}{1 - \delta^2} \right),$$

(15)

where $\psi$ is the relative phase between $M_{\Phi_1}$ and $M_{\Phi_2}$. This tells us that for small values of $\delta$, the final baryon-to-photon ratio will be approximately proportional to $\delta$. Since $\ell_\alpha$ and $n_{R\alpha}$ have equal and opposite charges under the global $U(1)_L$ symmetry, the individual $L_\ell$ and $L_{\nu_R}$ lepton numbers respectively stored in left-handed leptons and right-handed neutrinos will likewise be equal and opposite. Thus no net lepton number is produced by the decays of $\Phi$ and $\Phi^\dagger$; the generation of both $L_{\text{tot}} \neq 0$ and $B \neq 0$ occurs via electroweak sphaleron processes, which conserve $B - L_\ell$ but violate $B + L_\ell$. As these are associated with the
SU(2) × U(1)y electroweak anomaly, they will only affect \( L_\ell \) and not \( L_\nu_R \), and thus create both a nonzero value for \( B \) and a disparity between the two stores of lepton number (see fig. 1).

In order for these two stores of lepton number not to be equilibrated away through the effective Higgs coupling, the equilibration rate must not become significant compared to the expansion rate of the universe until after the electroweak phase transition \( T_c \), at which point sphaleron interactions have effectively turned off. This equilibration rate may be estimated on dimensional grounds to be

\[
\Gamma_{\text{eq}} \sim \frac{|\lambda|^2|\bar{h}|^2\langle\chi\rangle^2}{M_{\Phi_1}^2}g^2T, \tag{16}
\]

where \( g \) is an \( \mathcal{O}(1) \) gauge or top Yukawa coupling and \( T \) is temperature. The expansion rate may be expressed by the Hubble parameter \( H = \frac{1.66g_*^{1/2}T^2}{M_P} \), where \( g_* \) is the number of interacting degrees of freedom (in the minimal supersymmetric model (MSSM) during the baryogenesis epoch, \( g_* \approx 205 \)). Requiring that \( \Gamma_{\text{eq}} < H \) for \( T > T_c \) leads to the condition

\[
\frac{|\lambda||\bar{h}|\langle\chi\rangle}{M_{\Phi_1}} \leq 10^{-8}, \tag{17}
\]

which can be translated into a limit on the neutrino mass using equation (7):

\[
m_\nu \leq 1.74\sin \beta \text{ keV}. \tag{18}
\]

This condition has to be met in any case because of cosmological constraints on the absolute neutrino mass, and therefore left-right equilibration would happen well after sphalerons have shut off, and Dirac neutrino genesis can then yield an appropriate value for \( \eta \).

As we mentioned earlier, an important advantage of Dirac leptogenesis is its versatility: there is nothing in the theory that fixes any of the parameters \( M_{\Phi_1}, M_{\Phi_2}, \lambda_{\alpha}, h_{\alpha}, \) and \( \langle\chi\rangle \) to any physical scale. There are, however numerous constraints on these parameters. First, the neutrino masses must be appropriately small. Second, we must satisfy the Sakharov criterion that the abundances of \( \phi_1 \) and \( \bar{\phi}_1 \) depart from their equilibrium values. This occurs when \( \Gamma_D \) is slower than the rate of the expansion of the universe at the temperature \( T \sim M_{\Phi_1} \) when \( \phi_1 \) can no longer be treated as effectively massless and its abundance begins to fall off, or in other words

\[
\frac{\Gamma_D}{H(M_{\Phi_1})} = 9.97 \cdot 10^{-2}\sqrt{g_*} \frac{M_P}{M_{\Phi_1}} \sum_{\alpha}(|\lambda_{1\alpha}|^2 + |h_{1\alpha}|^2) \lesssim 1. \tag{19}
\]

This constraint also favors small couplings and large \( M_{\Phi_1} \). However, in addition to these two requirements, we must ensure that the present value of \( \eta \) satisfy the bounds in equation (2).

To derive the precise limits this requirement places on the model parameters, one must solve the full system of Boltzmann equations, which we do in section 5. For present purposes, a rough estimate can be made using the “drift and decay” approximation, in which we assume that the \( \phi \) decays occur well out of equilibrium and that the effects of inverse decays and \( 2 \leftrightarrow 2 \) processes where \( \Delta L_\ell \neq 0 \) are negligible. Including contributions from both scalar and fermion decays, this gives the result

\[
L_\ell = \frac{2\epsilon n_{\phi_1}^{\text{init}}}{\pi s} = \frac{90\epsilon}{\pi^2 g_*} K_2(1) = 7.32 \times 10^{-3} \epsilon, \tag{20}
\]
where \( n_{\phi_1}^{\text{init}} \) is the initial number density of \( \phi \) and \( K_2(x) \) denotes the modified Bessel function of the second kind, evaluated at \( x \). Since the final baryon-to-entropy ratio \( B \) (related to \( \eta \) by \( B = \eta/7.04 \)) generated by sphaleron processes will be on the same order (\( B \approx 0.35 L_\epsilon \)), this can serve as a rough estimate for \( B \). Thus even if equation (19) is satisfied, the final baryon-to-entropy ratio of the universe will be proportional to \( \epsilon \); and from equation (14), we see that for \( \epsilon \) to be large, either the couplings must be large or the splitting between \( M_{\Phi_1} \) and \( M_{\Phi_2} \) must be small. Thus there are tensions among the model parameters in Dirac leptogenesis, but they can be reconciled without too much difficulty.

The real tensions among \( M_{\Phi_1}, M_{\Phi_2}, \lambda_{i\alpha}, h_{i\alpha}, \) and \( \langle \chi \rangle \) are not those inherent in the Dirac neutrino-genesis framework, however, but those that arise when we demand that the model respect the full battery of additional constraints from cosmology and from neutrino physics. We now turn to address these constraints and their implications for Dirac neutrino-genesis.

### 3 Astrophysical Constraints

Because the particle spectrum of models with high-scale supersymmetry breaking may differ significantly from that of models with weak-scale supersymmetry breaking, it is necessary to investigate whether any such alterations will affect the astrophysical constraints (from BBN, inflation, etc.) the theory must satisfy. In addition, Dirac neutrino-genesis introduces three additional light, sterile neutrino fields \( \nu_{R\alpha} \), which could affect BBN. We must make certain that neither of these considerations presents an insurmountable problem for our model.

We will first address the issue of the additional neutrino species. It has been shown [11] that the presence of the three additional light neutrino fields \( \nu_{R\alpha} \) at the time of BBN does not violate the bound [25] on the number of additional light neutrino species \( N_\nu \leq 0.3 \), since their contribution to \( N_\nu \) is suppressed by a large entropy factor. Still, there are other potential cosmological problems that might arise for split supersymmetry: in particular, constraints on the reheating temperature \( T_R \) associated with cosmic inflation become constraints on \( M_{\Phi_1} \) in thermal leptogenesis models. We must ensure that such constraints do not make Dirac neutrino-genesis unworkable.

The most severe constraints on \( T_R \) are rooted in gravitino cosmology. There are two distinct varieties of gravitino problem that must be addressed: first, late gravitino decays can disrupt BBN by releasing energy in the form of photons and other energetic particles into the system; second, as the gravitino decays predominately into the LSP and its standard model superpartner, the potentially large, non-thermal population of stable particles produced in this manner could overclose the universe—or if the right amount is produced, could make up the majority of cold dark matter (CDM). Both of these issues are contingent on the gravitino lifetime \( \tau_{3/2} \), which for cases where \( m_{3/2} \gg m_{LSP} \), is approximated by [26]

\[
\tau_{3/2} = 4.0 \times 10^8 \left( \frac{m_{3/2}}{100\text{GeV}} \right)^{-3} \text{s.}
\]

Careful analysis of the BBN constraints [27] reveals that unless \( m_{3/2} \gtrsim 10^5 \text{ GeV} \), \( T_R \) cannot be greater than around \( 10^8 \text{ GeV} \), and for \( m_{3/2} \lesssim 5 \times 10^3 \text{ GeV} \), cannot exceed \( 10^6 \text{ GeV} \). In models where \( m_{3/2} \) is at the PeV scale, however, \( \tau_{3/2} \approx 10^{-4} \text{ s} \), which implies that gravitinos

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produced in the thermal bath decay long before the BBN epoch (at $t_{\text{universe}} \sim 1$ s), and thus there is no gravitino problem of the former type for models with $m_{3/2}$ at or above these scales. However, since a weakly interacting LSP decouples on a timescale $t_f \sim 10^{-11}$ s, it will have long frozen out by the time gravitino decay occurs unless $m_{3/2}$ is larger than around $10^8$ GeV; hence in theories with smaller gravitino masses (including simple PeV-scale supersymmetry with anomaly-mediated gaugino masses), LSPs produced by gravitino decay will be unable to thermalize and the latter type of gravitino issue cannot be ignored.

In order to avoid any complications from late gravitino decay, we require not only that the LSP not overclose the universe, but that its surviving relic density $\Omega_{LSP}$ must be less than (or ideally, if the LSP is to constitute the majority of cold dark matter, equal to) the relic density of CDM as measured by WMAP \cite{28},

$$ \Omega_{\text{CDM}} h^2 = 0.11 \pm 0.01 \text{ (WMAP 68\% C.L.)}. \quad (22) $$

In general, $\Omega_{LSP}$ will have both a thermal and a non-thermal component, so that $\Omega_{LSP} = \Omega_{LSP}^{T} + \Omega_{LSP}^{NT}$. The thermal component $\Omega_{LSP}^{T}$ may be ascertained by solving the relevant set of Boltzmann equations for the LSP abundance at freeze-out. The results, for the case where the LSP is essentially either a pure Wino or Higgsino, are \cite{2}

$$ \Omega_{LSP}^{T} h^2 = 0.02 \left( \frac{|M_2|}{\text{TeV}} \right) \text{ for Wino LSP} \quad (23) $$

$$ \Omega_{LSP}^{T} h^2 = 0.09 \left( \frac{\mu}{\text{TeV}} \right) \text{ for Higgsino LSP}. \quad (24) $$

Now we turn to evaluating $\Omega_{LSP}^{NT}$. We begin by addressing the regime in which there is no significant reduction in $\Omega_{LSP}^{NT}$ from LSP annihilations. Assuming for the moment that the dominant contribution to the non-thermal relic abundance comes from late gravitino decays and that all the LSPs produced from such decays survive until present day, $\Omega_{LSP}^{NT}$ is given by

$$ \Omega_{LSP}^{NT} = \frac{m_{LSP} (3) T_0}{\pi^2 \rho_{\text{crit}}} Y_{3/2}(T_{3/2}), \quad (25) $$

where $\rho_{\text{crit}}$ is the critical density of the universe, $T_0$ is the present temperature of the universe, and $Y_{3/2}(T_{3/2})$ is the number of gravitinos per co-moving volume at the characteristic temperature $T_{3/2}$ at which the gravitino decays, which is given by \cite{29}

$$ Y_{3/2}(T_{3/2}) = 0.856 \times 10^{-11} \left( \frac{T_R}{10^{10}\text{GeV}} \right) \left( 1 - 0.0232 \ln \left( \frac{T_R}{10^{10}\text{GeV}} \right) \right). \quad (26) $$

Substituting this into equation (25) yields

$$ \Omega_{LSP}^{NT} h^2 = 2.96 \times 10^{-4} \frac{m_{LSP}}{\text{GeV}} \left( \frac{T_R}{10^{10}\text{GeV}} \right) \left( 1 - 0.0232 \ln \left( \frac{T_R}{10^{10}\text{GeV}} \right) \right). \quad (27) $$

In fig. 3, we plot the contours corresponding to the WMAP upper and lower bounds from equation (22) on the total LSP relic abundance, as well as the simple overclosure bound $\Omega_{LSP} h^2 = 1$, taking into account both thermal and non-thermal contributions, as a function of $m_{LSP}$ and $T_R$. The region above and to the right of the WMAP upper bound is excluded. In order for the LSP to make up a significant fraction of the dark matter, $\Omega_{LSP}$
Figure 3: Contours of $\Omega_{\text{LSP}} h^2$ corresponding to the upper and lower bounds from WMAP \cite{22}, as well as a simple overclosure bound, as a function of the LSP mass $m_{\text{LSP}}$ and the reheating temperature $T_R$ associated with cosmic inflation. The requirement that $\Omega_{\text{LSP}}$ not overclose the universe severely constrains $T_R$, and hence the temperature scale of thermal leptogenesis, for a theory like PeV-scale loop-split supersymmetry, in which the LSP is particularly heavy.

should fall within the narrow strip between the upper and lower bounds on $\Omega_{\text{CDM}}$ given in \cite{22}, but if some other particle makes up the majority of CDM, then the entire region below and to the left of the WMAP upper bound is phenomenologically allowed. In order to achieve an appropriate LSP relic density by thermal means alone, it can be seen that $T_R$ must be quite low–around $10^9$ GeV; if the majority of CDM is generated non-thermally, $T_R$ may be raised a bit, but is still constrained to be below $\sim 5 \times 10^{10}$ GeV. As we shall see in section 5, $T_R \sim 10^9$ GeV turns out to be problematic for Dirac neutrinoogenesis (in terms of the final baryon-to-photon ratio generated), largely due to the out-of equilibrium condition in \cite{19}, this implies that if we want to raise $T_R$ above $10^9$ GeV and still have the LSP relic density dominate $\Omega_{\text{CDM}}$, the dark matter must be essentially non-thermal in origin.

We now turn to address the regime where LSP annihilations do play a role in reducing $\Omega_{\text{LSP}}^{NT}$, and thus the upper bound on $T_R$ may be raised. This effect becomes important when $m_{3/2} \gg m_{\text{LSP}}$. When it is taken into account \cite{31}, the non-thermal LSP relic density is modified to

$$\Omega_{\text{LSP}}^{NT} = \min \left(\Omega_{\text{LSP}}^{NT(0)}, \Omega_{\text{LSP}}^{NT(\text{ann})}\right),$$

where $\Omega_{\text{LSP}}^{NT(0)}$ is the relic density given in equation \cite{22}, and $\Omega_{\text{LSP}}^{NT(\text{ann})}$ is the relic density obtained by solving the full system of Boltzmann equations for the LSP. For a Wino LSP, $\Omega_{\text{LSP}}^{NT(\text{ann})}$ is given by

$$\Omega_{\text{LSP}}^{NT(\text{ann})} = 2.41 \times 10^{-2} \frac{(2 - x_W)^2}{(1 + x_W)^{3/2}} \left(\frac{m_{\text{LSP}}}{100 \text{GeV}}\right)^3 \left(\frac{m_{3/2}}{100 \text{TeV}}\right)^{-3/2} \times \left(1 - \left(\frac{m_{\text{LSP}}}{m_{3/2}}\right)\right)^3 \left(1 + \frac{1}{3} \left(\frac{m_{\text{LSP}}}{m_{3/2}}\right)\right),$$

(29)
where $x_W \equiv m_W/m_{LSP}$; for a Higgsino LSP, which annihilates far less efficiently, $\Omega_{LSP}^{NT(ann)}$ will be even higher.

In fig. 4 we show the relationship between $\Omega_{LSP}^{NT(ann)}$ and $m_{LSP}$ for several values of $m_{3/2}$. To be of any benefit in reducing $\Omega_{LSP}^{NT}$, $\Omega_{LSP}^{NT(ann)}$ itself must not exceed the WMAP upper bound, which is included for reference. From this plot it is evident that annihilations are only effective in reducing the LSP relic abundance below this bound when $m_{3/2}$ is much larger than $m_{LSP}$. This is a problem for loop-split supersymmetry with $m_{3/2}$ around the PeV scale, where the ratio of $m_{LSP}$ to $m_{3/2}$ is explicitly determined by the anomaly-mediation relation (1). It is not, however, a problem for more general split SUSY scenarios, in which the splitting between $m_{LSP}$ and $m_{3/2}$ can be much larger [29, 45]. Furthermore, when $m_{3/2}$ is increased beyond around $10^6$, $\tau_{3/2}$ becomes short enough that gravitino decay occurs before LSP freeze-out, and $\Omega_{LSP}^{NT}$ drops to zero regardless of what the ratio of $m_{LSP}$ to $m_{3/2}$ is.

Figure 4: The variation of $\Omega_{LSP}^{NT(ann)}$ (the LSP relic density function when LSP annihilations are taken into account) with LSP mass for several different values of $m_{3/2}$. When $m_{3/2} = 10^6$ GeV, only for $m_{LSP} \leq 500$ GeV will $\Omega_{LSP}^{NT(ann)}$ not generate an LSP relic density that exceeds the WMAP upper bound on the CDM relic density, and thus only for such an LSP mass can the reheating temperature $T_R$ be increased beyond the naive upper bound from fig. 3. For larger gravitino masses $m_{3/2} \geq 10^7$ GeV, annihilations effectively reduce $\Omega_{CDM}^{NT}$ and allow for $T_R$ to be increased above the naive upper bound.

Since thermal leptogenesis requires that a thermal population of $\Phi_1$ and $\Phi_1$ be generated during reheating after inflation, we require that $M_{\Phi_1} \lesssim T_R$; hence an upper bound on $T_R$ translates into an upper bound on $M_{\Phi_1}$. This means that there is substantial model tension when the $T_R$ is constrained to be at or below $\sim 5 \times 10^{10}$ GeV. This is not in any way peculiar to Dirac leptogenesis either: in a Majorana leptogenesis model, the constraints still apply with the lightest right-handed neutrino mass $M_{\nu_R}$ in place of $M_{\Phi_1}$. Here, the out-of-equilibrium condition [13] demands that $\lambda_{1a}$ and $h_{1a}$ be at most $O(10^{-4})$ for $M_{\Phi_1} \sim 10^{10}$ GeV; on the other hand, [14] and [20] imply that unless there is a large hierarchy among the Yukawa couplings to different sets of $\Phi$ and $\Phi$ (so that $\lambda_{1a} \ll \lambda_{2a}$ for all $a$), or unless
$M_{\Phi_1}$ and $M_{\Phi_2}$ are essentially degenerate, making the $\lambda_{i\alpha}$ and $h_{i\alpha}$ any smaller than $O(10^{-4})$ will yield insufficient baryon number—and we haven’t yet taken into account the effects of $2 \leftrightarrow 2$ processes and inverse decays. In short, leptogenesis becomes difficult when there are upper bounds on $T_R$, which occurs when $m_{3/2} \lesssim 10^8$ GeV and LSP annihilations are ineffective.

There are two methods for increasing the baryon density in models where $M_{\Phi_1}$ is tightly constrained by bounds on the reheating temperature, as indicated in equation (14). One possibility, to which we have already alluded, is somehow to arrange a large hierarchy among the Yukawa couplings to different sets of $\Phi$ and $\Phi$ (i.e. to adjust the matrixes $\lambda$ and $h$). Since the couplings to $\Phi_2$ only influence the physics via their CP-violating phases in $\epsilon$ (14) and via their contribution to the mass matrix (7), increasing them will cause no phenomenological problems as long as a realistic neutrino mass spectrum is obtained. The second possibility is to make $M_{\Phi_1}$ and $M_{\Phi_2}$ very close together in order to achieve a resonance condition in $\epsilon$ (i.e. to adjust $\delta$, the ratio of these two quantities). Since the perturbation theory we have used in calculating equation (14) is good as long as the separation of $M_{\Phi_1}$ and $M_{\Phi_2}$ is substantially greater than the value of the off-diagonal elements $M_{ij}$ in the $\Phi - \Phi$ mass mixing matrix induced at the one-loop level:

$$M_{ij} \sim g^i g^j (M_{\Phi_1} + M_{\Phi_2}) I_{ij},$$

(30)

where $g_i$ and $g'_j$ represent the appropriate $\lambda$ and $h$, summed over the fermion family index, and $I_{ij}$ is a numerical factor on the order of $1/16\pi^2$ from the loop integral. Thus for small $\lambda$ and $h$, $\delta$ can be set very close to one and $M_{\phi_1}$ and $M_{\phi_2}$ may be very nearly degenerate. This allows for the possibility of resonant leptogenesis, as has been done in the Majorana leptogenesis case, which can be invoked to generate a large baryon number in cases where the bounds on $T_R$ are more severe. As we shall see in the next section, scenarios with small $\delta$ can naturally produce a realistic neutrino spectrum; thus we will restrict our attention to the case where $\delta$ is small.

While the problems that can arise for small gravitino masses have now been thoroughly addressed, the caveats associated with extremely large $m_{3/2}$ should also be mentioned. As has been shown in 33, split supersymmetry models with a large hierarchy between the gravitino and gaugino masses can suffer from phenomenological problems associated with the overproduction of gluinos. While a precise ceiling for $m_{3/2}$ must wait until gluino decay is better understood, it is known that this ceiling falls somewhere in the $m_{3/2} \simeq 10^{10} - 10^{12}$ GeV range. There are also caveats associated with the gravitino-producing decays of scalar sparticles in models where one or more scalars has a mass larger than $m_{3/2}$ 32.

The message here is that cosmological considerations can make thermal Dirac neutrinogenesis substantially more difficult when $m_{3/2} \lesssim 10^8$ GeV, as substantial tensions arise among the out-of-equilibrium decay criterion, overclosure bounds related to the reheating temperature $T_R$, the equation that determines the decay asymmetry $\epsilon$, etc. For light gravitinos ($m_{3/2} \lesssim 10^5$ GeV), BBN limits severely constrain $T_R$, and hence $M_{\Phi_1}$. For slightly heavier gravitinos ($10^5$ GeV $\lesssim m_{3/2} \lesssim 10^8$ GeV), there are still constraints on $T_R$ from nonthermal decays which are only alleviated (via LSP annihilations) when $m_{LSP} \ll m_{3/2}$. Since this is not the case in loop-split supersymmetry, where the ratio of $m_{LSP}$ to $m_{3/2}$ is specified, making Dirac neutrinoogenesis succeed is quite difficult in this framework. Still, if we allow for non-thermal generation of $\Omega_{CDM}$ (or for the possibility that the LSP is not a significant contributor to the dark matter density), Dirac neutrinoogenesis may still be viable
in loop-split SUSY (while there are apparent tensions in this case, they can be resolved by invoking some mechanism, such as resonant leptogenesis, for amplifying the lepton number produced). In more general split supersymmetry scenarios, however, $T_R$ and $M_{\Phi_1}$ may be increased to $10^{12}$ GeV or more, leptogenesis should proceed without too much trouble, and the possibility for thermal dark matter still exists. Whatever the case, it will be necessary to solve the full system of Boltzmann equations to ascertain whether or not there is any region of parameter space in which Dirac neutrinoogenesis can be successful, which we do in section 5.

4 Neutrino Physics

In addition to respecting constraints arising from cosmological considerations, in order to be phenomenologically viable, a given Dirac neutrinoogenesis model must yield a neutrino spectrum that accords with current experimental constraints. The most stringent such constraints come from solar and atmospheric neutrino oscillation experiments, and place limits both on the mass splittings

$$\Delta m_{ab}^2 = m_{\nu_a}^2 - m_{\nu_b}^2,$$  \hspace{1cm} (31)

where the indices $a$ and $b$ label the different neutrino mass eigenstates, and on the mixing angles $\theta_{ab}$ between these eigenstates. The primary connection between the latter and observable physics occurs through the leptonic mixing matrix

$$U_{\text{MNS}} = U^{(\nu)} U^{(e)},$$  \hspace{1cm} (32)

where $U^{(\nu)}$ is the neutrino mixing matrix and $U^{(e)}$ is the charged lepton mixing matrix. In the basis where the charged lepton mass matrix is diagonal, $U_{\text{MNS}}$ is the neutrino mixing matrix and may be expressed in terms of the neutrino mixing angles $\theta_{ab}$ as

$$U_{\text{MNS}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i \delta_{CP}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta_{CP}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{-i \delta_{CP}} & -s_{13} e^{-i \delta_{CP}} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{-i \delta_{CP}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{-i \delta_{CP}} & c_{23} c_{13} \end{pmatrix},$$  \hspace{1cm} (33)

where $c_{ab} = \cos \theta_{ab}$, $s_{ab} = \sin \theta_{ab}$, and $\delta_{CP}$ is a $CP$-violating phase. The present limits\(^5\) on the $\Delta m_{ab}^2$ and $\theta_{ab}$ are \[34\]

$$\sin^2 \theta_{12} = 0.30^{+0.04}_{-0.05}, \hspace{1cm} \sin^2 \theta_{23} = 0.50^{+0.14}_{-0.12}, \hspace{1cm} \sin^2 \theta_{13} \leq 0.031,$$

$$\Delta m_{21}^2 = (7.9^{+0.6}_{-0.6}) \times 10^{-5} \text{eV}^2, \hspace{1cm} |\Delta m_{31}^2| = (2.2^{+0.7}_{-0.5}) \times 10^{-3} \text{eV}^2.$$

The smaller of the two mass splittings, $\Delta m_{21}^2$, is to be identified with the $\Delta m_{\odot}^2$ obtained from solar neutrino data (the MSW-LMA solution); the larger, $\Delta m_{21}^2$, with the $\Delta m_{A}^2$ from atmospheric neutrino data. The constraints in \[34\] imply a $U_{\text{MNS}}$ matrix with bounds

$$|U_{\text{MNS}}| = \begin{pmatrix} .79 - .86 & .49 - .58 & 0 - .18 \\ .30 - .58 & .40 - .68 & .61 - .80 \\ .19 - .46 & .50 - .77 & .59 - .79 \end{pmatrix},$$  \hspace{1cm} (35)

\(^5\)We do not take into account the LSND result which would require an extra neutrino mass eigenstate. In the case that forthcoming data from experiments such as MiniBooNE corroborate the LSND signal, it will be necessary to extend the neutrino content of our model.
but they leave the sign of the largest mass squared difference $\Delta m^2_{31}$ undefined. This implies that the physical neutrino $\nu_3$ can be either the heaviest of the three mass eigenstates, i.e. $m_1 < m_2 \ll m_3$ (normal hierarchy) or the lightest, i.e. $m_3 \ll m_1 < m_2$ (inverted hierarchy).

The unitary matrix $U_{\text{MNS}}$, in the charged lepton basis, is responsible for diagonalizing the squared neutrino mass matrix, i.e.

$$ (m^2_{\nu})^{\text{diag}} = U_{\text{MNS}}^\dagger m_{\nu} m_{\nu}^\dagger U_{\text{MNS}}. $$

We can therefore estimate the generic form of the neutrino mass matrix squared, since it has to be diagonalized by the $U_{\text{MNS}}$ matrix.

We should have for a Normal Hierarchy

$$ (m^2_{\nu})^\text{Norm} \sim \Delta m^2_{31} \begin{pmatrix} \xi & \xi & \xi \\ . & 1 & 1 \\ . & . & 1 \end{pmatrix}, $$

and for an Inverted Hierarchy

$$ (m^2_{\nu})^\text{Inv} \sim \Delta m^2_{31} \begin{pmatrix} 1 & \xi & \xi \\ . & 1 & 1 \\ . & . & 1 \end{pmatrix}, $$

where the $\xi$ are small entries (not necessarily equal) compared to the $O(1)$ entries. In both cases, the order of magnitude of $\xi$ is constrained by the ratio $\rho_{32} \equiv \Delta m^2_{31}/\Delta m^2_{21}$, which, according to (34), must respect the bounds

$$ 20.0 < \rho_{32} < 39.7. $$

We are interested in finding a simple connection between the heavy $\Phi$ fields and the neutrino sector, which does generically reproduce the observed neutrino spectrum and is also compatible with successful baryon number generation.

Let us first consider the spectrum of a model with only one set of $\phi$ and $\bar{\phi}$, or alternatively, a theory in which $M_{\phi_i} \ll M_{\bar{\phi}_i}$ for all $i > 1$. In such a case, the mass matrix is proportional to an outer product of the family-space vectors $\lambda_{1\alpha}$ and thus its eigenvalues are

$$ m_{\nu_1} = 0 \hspace{1em} m_{\nu_2} = 0 \hspace{1em} m_{\nu_3} = \sum_\alpha \lambda_{1\alpha} h_{1\alpha}. $$

For each additional set of $\phi$ and $\bar{\phi}$ with a mass similar to $M_{\phi_1}$, an additional neutrino acquires a nonzero mass. Thus in “short-suited” models in which there are effectively only two sets of $\phi$ and $\bar{\phi}$ involved in determining the neutrino spectrum, one neutrino mass eigenstate is massless, and the other two neutrino masses are given by $m^2_{\nu_2} \approx \Delta m^2_{21}$ and $m^2_{\nu_3} \approx \Delta m^2_{31}$. Furthermore, leptogenesis in such models is well-approximated by the toy model considered in section 2 in which there were only two sets of $\Phi$ and $\bar{\Phi}$.

From (7), the neutrino mass-squared matrix is given by

$$ |m_{\nu}|^2_{\alpha\beta} = \left( v \langle \chi \rangle \sin \beta \right)^2 \sum_{i,j=1}^{2 \text{ (or 3)}} \sum_{\gamma=1}^{3} \lambda_{i\gamma}^* \lambda_{j\gamma} h_{i\alpha}^* h_{j\beta} \frac{1}{M_{\Phi_i} M_{\Phi_j}}. $$

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As long as $\lambda$ and $h$ are completely generic and there are at least three sets of $\phi$ and $\bar{\phi}$, it is apparent that a matrix of this form can yield an arbitrary neutrino mass spectrum. On the one hand this is good, for it means that the theory is perfectly viable; on the other hand, this arbitrariness comes at the price of introducing many additional free parameters, whose relative values must be determined by some additional underlying physics.

Guided by the idea of linking the neutrino data to the $\Phi$ fields sector, we find two simple conditions that generically reproduce the form of the normal hierarchy neutrino mass squared matrix ($\Phi$).

- **Normal Hierarchical ansatz A**

A quick inspection of (40) shows that if the spectrum of heavy fields is hierarchical, i.e. if $M_{\Phi_1} \ll M_{\Phi_2} (\ll M_{\Phi_3})^6$ then we can very simply write the neutrino mass squared matrix to zero order in $\delta = M_{\Phi_1}/M_{\Phi_2}$ as

$$|m_\nu|^2 = \left( v \langle \chi \rangle \sin \beta \right)^2 \frac{|h_{11}|^2}{M_{\Phi_1}^2} \begin{pmatrix} |h_{11}|^2 & h_{11}h_{12} & h_{11}h_{13} \\ |h_{12}|^2 & |h_{12}| & |h_{12}h_{13}| \\ |h_{13}|^2 & |h_{13}| & |h_{13}| \end{pmatrix} + O(\delta),$$

(41)

where $\Lambda_1 = \sum_\gamma |\lambda_{1\gamma}|^2$.

Another hierarchical assumption, namely the requirement that the off-diagonal elements of the coupling matrix $h$ be much greater than the diagonal elements ($h_{12} \sim h_{13} \equiv \tilde{h} \gg h_{11}$), allows us to rewrite the previous equation as

$$|m_\nu|^2 = \left( v \langle \chi \rangle \sin \beta \right)^2 \frac{|\tilde{h}|^2\Lambda_1}{M_{\Phi_1}^2} \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} + O(\delta),$$

(42)

where $\varepsilon = h_{11}/\tilde{h}$.

This reproduces the structure required for a normal hierarchy neutrino sector. It is interesting that the two simple hierarchical requirements, $M_{\Phi_1} \ll M_{\Phi_2,3}$ and $h_{11} \ll h_{12} \sim h_{13}$ (with $h_{11} \neq 0$), generically give rise to the correct neutrino phenomenology.

- **Normal Hierarchical ansatz B**

The previous ansatz suggests that we look for some setup that would ensure that the diagonal element $h_{11}$ is small compared to the off diagonal terms $h_{12}$ and $h_{13}$. An antisymmetric condition on the matrix $h$ can do the job, but from eq. (41) it becomes clear that we need to include a higher order term in the small parameter $\delta$.

When we assume that all the diagonal elements of $h$ are zero (or extremely small), and the rest of elements in $h$ or $\lambda$ are of order 1, we obtain the structure

$$|m_\nu|^2 \propto \begin{pmatrix} \delta^2 & \delta & \delta \\ \delta & 1 & 1 \\ \delta & 1 & 1 \end{pmatrix}.$$ 

(43)

Again we find the generic structure required for a normal hierarchy, but now the value of $\delta$ fixes the ratio $\rho_{23}$.

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6Of course $M_{\Phi_2}$ cannot be too large since then, as remarked earlier, we effectively generate the mass of just one physical neutrino leaving the other two massless.
Clearly, either ansatz A or B (or a combination of both) can explain the neutrino data and large mixing angles. Ideally, we would like to have a theoretical framework that would provide both the form of the superpotential required for Dirac neutrino mass generation [5] and the necessary flavor structure, like the ones presented in the previous examples. A careful construction of such a model is out of the scope of this paper, but we nevertheless would like to point at least in one interesting direction in which simple assumptions enable us to reproduce the needed conditions, while at the same time reducing the number of new free parameters.

In many grand unified theories and models with non-Abelian flavor symmetries, the Yukawa matrices (and in general the flavor interactions) can be symmetric, antisymmetric, or both (see for example [35, 36, 37] and references therein). Operating in this paradigm, let us assume that the SM left-handed leptons have the same flavor charge as the heavy fields $\Phi$, and the SM right-handed charged leptons have same flavor charge as the fields $\Phi$. Upon breaking of the flavor symmetry, let us assume that the charged lepton Yukawa matrix is symmetric due to a symmetric flavon VEV configuration $\langle S_{\alpha\beta} \rangle$. The corresponding effective superpotential is

$$ W \supset Y_L \frac{\langle S_{\alpha\beta} \rangle}{M_F} H_d L_{\alpha} e_{\beta} + Y_\Phi \langle S_{\alpha\beta} \rangle \Phi_\alpha \Phi_\beta + \text{h.c.} \quad (44) $$

where $Y_L$ and $Y_\Phi$ are dimensionless couplings, and $\alpha$ and $\beta$ are flavor indices. The charged lepton Yukawa matrix $(y_l)_{\alpha\beta}$ and the mass matrix $(M_\Phi)_{\alpha\beta}$ of the heavy fields $\Phi$ would be both symmetric and proportional

$$ (M_\Phi)_{\alpha\beta} = M_F \frac{Y_\Phi}{Y_L} (y_l)_{\alpha\beta} \quad (45) $$

This specific structure predicts exactly the mass spectrum for the $\phi$ fields in terms of the flavor scale $M_F$, which is at the origin of the intermediate scale required for successful thermal Leptogenesis.

$$ M_{\Phi_1} = m_e \frac{M_F}{v}, \quad M_{\Phi_2} = m_\mu \frac{M_F}{v} \quad \text{and} \quad M_{\Phi_3} = m_\tau \frac{M_F}{v} \quad (46) $$

If the flavor scale is of the order of some GUT scale $M_F \sim 10^{16} \text{ GeV}$, we would then expect $M_{\Phi_1} \sim 10^{11} \text{ GeV}$, with $\delta = M_{\Phi_1}/M_{\Phi_2} = m_e/m_\mu \sim 5 \times 10^{-3}$ being a small parameter. If one worries about the dependance on the Flavor symmetry scale it is also possible to link the intermediate scale to the large SUSY breaking scale $M_{\text{SUSY}}$. We can imagine that if all fields in the theory are charged, the spurion field $X$ of supersymmetry breaking might as well be charged, and perhaps it is charged under the same symmetry $U(1)_N$ responsible for obtaining the superpotential [5]. Before any kind of breaking we can imagine the superpotential as

$$ W \supset \frac{S_{\alpha\beta}}{M_F} (Y_L H_d L_{\alpha} e_{\beta} + Y_\Phi X \Phi_\alpha \Phi_\beta) + \text{h.c.} \quad (47) $$

where again $Y_L$ and $Y_\Phi$ are dimensionless couplings. Supersymmetry breaking effects can provide the field $X$ with a VEV $\langle A_X \rangle \sim (m_{3/2}M_{\text{Pl}})^{1/2}$, and upon flavor symmetry breaking we could get the effective superpotential

$$ W_{\text{eff}} \supset (y_l)_{\alpha\beta} \left( H_d L_{\alpha} e_{\beta} + \sqrt{m_{3/2}M_{\text{Pl}}} \Phi_\alpha \Phi_\beta \right) + \text{h.c.} \quad (48) $$

where we have assumed that the original constants $Y_L \sim Y_\Phi$. In this situation, for example with $m_{3/2} \sim 10^{10} \text{ GeV}$, the mass of the lightest $\Phi$ field would be $M_{\Phi_1} \sim 10^{11} \text{ GeV}$. 

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Now, let us also assume that the flavon VEV $\langle S_{\alpha\beta} \rangle$ is symmetric and that the coupling $\bar{\Phi}_L \chi$ becomes antisymmetric upon flavor breaking, i.e. the superpotential can be written as

$$W \supset (y_l)^{\text{sym}}_{\alpha\beta} H_d L_\alpha e_\beta + (M_\Phi)^{\text{sym}}_{\alpha\beta} \bar{\Phi}_\alpha \Phi_\beta + \lambda_{\alpha\beta} N_\alpha \Phi_\beta H_u + h^{\text{antisym}}_{\alpha\beta} \bar{\Phi}_\alpha L_\beta \chi$$

(49)

where we have $\lambda_{\alpha\beta} \equiv \langle A^N_{\alpha\beta} \rangle$ and $h^{\text{anti}}_{\alpha\beta} \equiv \langle A^x_{\alpha\beta} \rangle$, with the flavon $A^N$ acquiring an antisymmetric VEV configuration and with the VEV configuration of $A^x$ being arbitrary in flavor space (it is not important to our purposes its flavor structure although probably in a specific model of flavor it would end up being some linear combination of symmetric and antisymmetric VEV configurations). Let us see what the implications of these ingredients are for our model.

In general, the charged lepton Yukawa matrix $(y_l)_{\alpha\beta}$ can be diagonalized by a biunitary transformation of the type

$$U^\dagger y_l V = y_l^{\text{diag}},$$

(50)

but when $y_l$ is symmetric, this biunitary transformation takes the simpler form

$$U^T y_l U = y_l^{\text{diag}}.$$  

(51)

If $(M_\Phi) \propto (y_l)$, as in the setup described above (see equation (15)), then $(M_\Phi)$ will be diagonalized by the same transformation. When the mass matrices for the charged leptons and the $\phi - \bar{\phi}$ system are simultaneously diagonalized (that is, when we go to the charged lepton basis), $\lambda$ and $h$ transform as

$$\lambda' = U^T \lambda U \quad h^{\text{anti}} = U^T h^{\text{anti}} U.$$  

(52)

Transformations of this type preserve the antisymmetry of $h$ (and $\lambda$ if also antisymmetric); thus in the charged lepton basis the matrix $h$ remains antisymmetric and $(M_\Phi)$ is real and diagonal; this reproduces the Hierarchical ansatz B described earlier, in which the diagonal elements of $h$ are zero and it is the smallness of $\delta$ the responsible for the large mixing angles observed in the lepton mixing matrix $U_{\text{MNS}}$.

In the setup discussed above, $(M_\Phi)$ and $y_l$ could be simultaneously diagonalized because $(M_\Phi) \propto y_l$, which fixes $\delta = M_{\Phi_1}/M_{\Phi_2} = m_e/m_\mu$. Such proportionality is not requires, however: any matrix of the form

$$(M_\Phi) = Ay_l + BI_{3\times3},$$

(53)

where $A$ and $B$ are arbitrary constants and $I_{3\times3}$ it the $3 \times 3$ identity matrix, can be diagonalized along with $y_l$. Thus when $B \neq 0$, $\delta$ can in principle take any value (as long as that value is consistent with the observational bounds on the neutrino spectrum).

- **Constrained Hierarchical Dirac Leptogenesis**

Motivated by the preceding remarks, we define Constrained Hierarchical Dirac Leptogenesis (CHDL) as the phenomenological setup in which $(M_\Phi)$ is real and diagonal, $\lambda$ and $h$ are antisymmetric, and the smallness of $\delta = M_{\Phi_1}/M_{\Phi_2}$ explains the large mixing angles in the neutrino sector. We may parameterize $\lambda$ and $h$ as
\[
\lambda = f \begin{pmatrix} 0 & 1 & a_2 \\ -1 & 0 & a_3 \\ -a_2 & -a_3 & 0 \end{pmatrix}, \quad h = f \begin{pmatrix} 0 & b_1 & b_2 \\ -b_1 & 0 & b_3 \\ -b_2 & -b_3 & 0 \end{pmatrix},
\]

which is convenient when the \(a\) and \(b\) are all roughly \(O(1)\). Since the assumption of a hierarchy among the \(M_{\Phi_i}\), leads to the neutrino mass-squared matrix of the form \([13]\), we expect that \(a_3\) and \(b_3\), the effects of which show up only at the \(O(\delta)\) level, will be less tightly constrained than the rest of the \(a_i\) and \(b_i\), which contribute to the leading term. This is in fact the case: if \(a_1\), \(a_2\), \(b_1\), or \(b_2\) deviates significantly from one, the neutrino spectrum cannot satisfy the constraints in \([33]\). It is therefore appropriate, since the value of \(f\) is unimportant as far as this set of constrains are concerned (any rescaling of \(f\) can be compensated for by a similar rescaling of \(\langle \chi \rangle\)), to analyze constrained hierarchical models as functions of \(a_3\) and \(b_3\) alone.

In fig. 5 we show the region of viability in \(a_3 - b_3\) space for two different values of \(\delta\): in the left-hand panel, we set \(\delta = m_e/m_\mu\) (and \(M_{\Phi_2}/M_{\Phi_3} = m_\mu/m_\tau\)) as required in the minimal version of CHDL discussed above; in the right-hand panel, we set \(\delta = 10^{-1}\). We consider a given combination of \(a_3\) and \(b_3\) to be phenomenologically viable if there is any combination of the remaining \(a_i\) and \(b_i\) for which the combination simultaneously obeys all the neutrino oscillation constraints in \([34]\). This plot demonstrates two important features of the Yukawa matrices in CHDL: first, it is indeed possible to satisfy the neutrino oscillation constraints for \(\delta = m_e/m_\mu\); second, while \(b_3\), like most of the other \(a_i\) and \(b_i\), is constrained to lie fairly close to 1, \(a_3\) can be quite large when \(\delta\) is small.

In the same figure, we also show contours for the value of \(\sin \theta_{13}\), the value of which will be measured or constrained in future neutrino experiments. It is seen that the value of \(\sin \theta_{13}\) increases with increased \(a_3\) until reaching its maximum experimental bound.

Let us now take a moment to address how these results affect leptogenesis. Since we are assuming that \(M_{\Phi_3} \gg M_{\Phi_1}, M_{\Phi_2}\), the formula \([15]\) for the decay asymmetry \(\epsilon\) tells us that \(\text{Im}(\lambda_{1\alpha}^* \lambda_{2\beta} h_{13}^* h_{23} M_{\Phi_1} M_{\Phi_2}^*)\) will vanish (when it involves diagonal elements of \(\lambda\) or \(h\)) unless \(\beta = \alpha = 3\). This means that

\[
\epsilon \propto \text{Im}(a_2^* a_3 b_2^* b_3) \frac{\delta}{1 - \delta},
\]

in CHDL; the two panels in fig. 5 then show that for a given \(\delta\), the largest amount of left-handed lepton number \(L_L\) (and therefore the largest baryon asymmetry) will be obtained when \(a_3 = a_3^{\text{max}}(\delta)\), where \(a_3^{\text{max}}(\delta)\) is the maximum possible value of \(a_3\) for a given \(\delta\) consistent with neutrino masses and mixings. It is interesting to note that since the maximum experimental value of \(\sin \theta_{13}\) sets the value of \(a_3^{\text{max}}\), then the maximum baryon asymmetry will be obtained when \(\sin \theta_{13}\) acquires its maximal experimental value, that we take to be \(\sin^2 \theta_{13} = 0.031\). Because \(\delta = m_e/m_\mu \simeq 4.83 \times 10^{-2}\) is fixed to be quite small and since \(\epsilon\) is approximately proportional to \(\delta\) for small \(\delta\), one might worry that such a small \(\delta\) might doom baryogenesis. However, as indicated in the left panel of fig. 5, \(a_3^{\text{max}}(m_e/m_\mu) \approx 95\), so that a hierarchy between couplings to different sets of \(\Phi\) and \(\overline{\Phi}\) is permitted, and the result is that \(\epsilon\) is only suppressed by an \(O(1)\) numerical factor. The
Figure 5: Here the regions of $a_3-b_3$ space (see equation (54) for a description of the Yukawa-matrix parametrization used) for which all constraints on neutrino masses and mixings (34) are simultaneously satisfied for some combination of the remaining $a_i$ and $b_i$ are shown for two different values of $\delta$. Additionally, contours depicting the ranges for $s_{13} = \sin \theta_{13}$ (which depends primarily on $a_3$, but varies slightly with the remaining $a_i$ and $b_i$) are shown. In the left-hand panel, $\delta \equiv M_{\Phi_1}/M_{\Phi_2} = m_e/m_\mu$; in the right-hand panel, $\delta = 10^{-1}$. The plots reveal that while $b_3$ is constrained, along with most of the other $a_i$ and $b_i$, to lie reasonably near 1, $a_{3 \text{max}}$ is on the order of $10^2$, which, when $a_3 \approx a_{3 \text{max}}$, results in an increased $\epsilon$ and—at least in the drift-and-decay limit—the baryon-to-photon ratio $\eta$, making it easier to achieve a realistic value for $\eta$. For smaller $\delta$, $a_{3 \text{max}}(\delta)$ is much lower, as will be $\eta$. In each panel, the configuration that yields the greatest decay asymmetry $\epsilon$ is marked with an arrow.

right panel of fig. 5 shows that $a_{3 \text{max}}(\delta)$ drops sharply as $\delta$ increases and $O(\delta)$ corrections become important in $m_2^\nu$, since $a_{3 \text{max}}(1/10) \approx 4.5$.

We are also assuming that we have maximal CP violation in the decays of the fields $\Phi$ and $\overline{\Phi}$ (i.e. the overall phase in the product $a_2^* a_3 b_2^* b_3$ must be $\pi/2$). In this case one can obtain a value for the effective CP violation in the lepton sector. This can be defined in a phase invariant way in terms of the quantity $J = \text{Im}(U_{12} U_{22}^* U_{23} U_{13}^*)$ \cite{40,41}, where $U_{ij} \equiv (U_{\text{MNS}})_{ij}$. Taking for example the two points that will generate the most baryon number in the left and right handed panels of fig. 5 (marked with an arrow) we find $J \approx 0.034$ for the point in the left panel and $J \approx 0.030$ for the point in the right panel (since the maximal value for $J$ is $|U_{12} U_{22}^* U_{23} U_{13}^*|$, which for these points yields a value of $J \approx 0.038$, the CP-violation here is close to maximal). It could be interesting to do a more detailed study of the issue of linking more generally the effective CP violation in the lepton sector to the CP violation in the interactions of the heavy fields $\Phi$ and $\overline{\Phi}$. Nevertheless, we will not pursue this issue further in the present work.

To summarize this section, we have shown that it is possible to construct a variety of Dirac neutrinosynthesis scenarios (including the simple, theoretically motivated CHDL) that
are capable of satisfying current constraints on the light neutrino spectrum. In CHDL, which will become our primary focus from this point forward, the set of parameters relevant to neutrino physics and to leptogenesis is quite small, given the antisymmetry condition on the two coupling matrices $\lambda$ and $h$. The other important parameter is the ratio of masses $\delta = M_{\Phi_1}/M_{\Phi_2}$. It remains to be seen whether CHDL—or indeed any Dirac neutrino-ogenesis—model can also simultaneously satisfy the constraints from gravitino physics and yield a realistic baryon number for the universe. As we shall see in the next section, thermal Dirac neutrino-ogenesis will indeed turn out to be workable in a variety of split-supersymmetry models.

5 Boltzmann Equations

Let us now turn to the numerical calculation of $\eta$ and the solution of the full Boltzmann equations. The heavy fields aside, in Dirac neutrino-ogenesis there are six particle species charged under lepton number ($\nu_R$, $\tilde{\nu}_R$, $\ell$, $\tilde{\ell}$, and the right-handed charged lepton and slepton fields $e_R$ and $\tilde{e}_R$), and thus six individual stores of lepton number to keep track of: $L_\ell$, $L_{\nu_R}$, $L_{\tilde{\ell}}$, $L_{\tilde{\nu}_R}$, $L_{e_R}$, and $L_{\tilde{e}_R}$. However, since $\ell$, $\tilde{\ell}$, $e_R$, and $\tilde{e}_R$ participate in $SU(2)$ and/or $U(1)_Y$ gauge interactions, which we assume to be sufficiently rapid (compared to other processes relevant to leptogenesis) that these species will always be in chemical equilibrium with one another, any lepton number stored in any one of them will be rapidly distributed among $L_\ell$, $L_{\tilde{\ell}}$, $L_{e_R}$, and $L_{\tilde{e}_R}$ in proportion to the relative number of degrees of freedom of each respective field. Furthermore, since we are assuming that $\tilde{\nu}_R$ and $\nu_R$ equilibrate rapidly through a large $\langle F_\chi \rangle$ term, $\tilde{\nu}_R$ will also be in equilibrium with these fields. Thus $\tilde{\nu}_R$, $\ell$, $\tilde{\ell}$, $e_R$, and $\tilde{e}_R$ can be viewed as one lepton sector with an aggregate lepton number $L_{agg}$. This leaves only the right-handed lepton field $\nu_R$, which does not participate in any of the aforementioned rapid interactions and thus retains its own distinct lepton number $L_{\nu_R}$.

In the limit of rapid equilibration within the $L_{agg}$ sector, the set of Boltzmann equations governing the evolution of lepton and baryon number simplifies considerably. In addition to the relations that describe the dynamics of the heavy fields, only three equations are required: one for $L_{agg}$, one for $L_{\nu_R}$, and one for $B$. However, before we state these equations (a derivation of which is provided in Appendix A), it will be useful to digress for a moment and discuss the rates for the various processes involved. We then return to a discussion of the Boltzmann equations themselves, followed by a presentation of the results from our numerical computations.

While there are a large number of $2 \leftrightarrow 2$ processes which transfer lepton number between different particle species, we will only need to calculate the rates for those which transfer it between $L_{agg}$ and $L_{\nu_R}$. As for the rest, which collectively serve to equilibrate lepton number among the fields in the $L_{agg}$ sector, it only matters that they are rapid compared to other rates in the problem. The diagrams for the $s$-channel $2 \leftrightarrow 2$ processes which shuffle lepton number between $L_{agg}$ and $L_{\nu_R}$ are pictured in fig. 6. In addition to these, there are contributions from the $t$-channel transforms (two per diagram) of these diagrams. In order
to evaluate diagrams containing virtual heavy fermions, we define a Dirac spinor

$$\Psi_{Di} = \left( \begin{array}{c} (\psi_\Phi) \alpha \\ (\bar{\psi}_\Phi) \dot{\alpha} \end{array} \right),$$

where $$(\psi_\Phi) \alpha$$ and $$(\bar{\psi}_\Phi) \dot{\alpha}$$ are the Weyl spinor components of the $\Phi_i$ and $\bar{\Phi}_i$ superfields.

Numerical calculation of the thermally averaged cross-sections for the diagram pictured on the left in the top row of fig. 6, which involves two Yukawa-type couplings of a scalar to two fermions, yields

$$\langle \sigma v \rangle_{ia\beta} \approx \sigma_i^{(2Y)} |\lambda_{ia}|^2 |\lambda_{i\beta}|^2,$$

where

$$\sigma_i^{(2Y)} \equiv 10^{-2} \frac{T^2}{(M_{\Phi_i}^2 + T^2)^2}. \quad (57)$$

For the diagram on the right in the first row of fig. 6, which include one Yukawa-type coupling and one trilinear scalar coupling proportional to $M_{\Phi_i}$, the result is very nearly temperature independent and well approximated by

$$\langle \sigma v \rangle_{ia\beta} \approx \sigma_i^{(1Y1S)} |\lambda_{ia}|^2 |h_{i\beta}|^2,$$

where

$$\sigma_i^{(1Y1S)} \equiv 0.5 \times \frac{1}{M_{\Phi_i}^2}. \quad (58)$$

The two diagrams in the second row of fig. 6, which involve two Yukawa couplings and a mass insertion from the heavy fermions, yield this same contribution. These interactions dominate among $2 \leftrightarrow 2$ processes. The diagram in the bottom row of fig. 6, which involves a trilinear scalar coupling to the down-type Higgs, may be approximated by

$$\langle \sigma v \rangle_{ia\beta} \approx \sigma_i^{(Hd)} |\lambda_{ia}|^2 |\lambda_{i\beta}|^2,$$

where

$$\sigma_i^{(Hd)} \propto \frac{\mu}{M_{\Phi_i}^3}. \quad (59)$$

where the constant of proportionality is $O(1)$, and is thus suppressed relative to the rate given in (58) by $\mu/M_{\Phi_i}$. Here, we will assume that $\mu$ is several orders of magnitude smaller than all the $M_{\Phi_i}$, and therefore the effect of these processes can be neglected.

Taking into account contributions involving virtual fields in the $\Phi_2$ and $\bar{\Phi}_2$ supermultiplets, as well as those in $\Phi_1$ and $\bar{\Phi}_1$, the total interconversion rate between $L_{agg}$ and $L_{\nu_R}$ is

$$\Gamma_{2\leftrightarrow 2} \approx 3n_\gamma \sum_\alpha \sum_\beta \left( \sigma_i^{(2Y)} |\lambda_{1\alpha}|^2 |\lambda_{1\beta}|^2 + \sigma_i^{(2Y)} |\lambda_{2\alpha}|^2 |\lambda_{2\beta}|^2 \right) + 9n_\gamma \sum_\alpha \sum_\beta \left( \sigma_i^{(1Y1S)} |\lambda_{1\alpha}|^2 |h_{1\beta}|^2 + \sigma_i^{(1Y1S)} |\lambda_{2\alpha}|^2 |h_{2\beta}|^2 \right) + 3n_\gamma \sum_\alpha \sum_\beta \left( \sigma_i^{(Hd)} |\lambda_{1\alpha}|^2 |\lambda_{1\beta}|^2 + \sigma_i^{(Hd)} |\lambda_{2\alpha}|^2 |\lambda_{2\beta}|^2 \right) \quad (60)$$

where we have assumed an equilibrium number density for all non-leptonic light species (e.g. $H_u$, $\chi$) involved.
Figure 6: Diagrams for $2 \leftrightarrow 2$ s-channel processes which transfer lepton number between $L_{agg}$ (the aggregate lepton number in the sector comprising the fields $\ell$, $\bar{\ell}$, $\nu_R$, $e_R$, and $\bar{e}_R$), which are assumed to be in chemical equilibrium with one another due to rapid gauge and $\langle F_\chi \rangle$-term equilibration interactions); and $L_{\nu_R}$ (the lepton number stored in right handed neutrinos). The two t-channel interactions associated with each diagram appearing above must also be included in calculating the full thermally averaged cross-section.

In addition to the $2 \leftrightarrow 2$ processes acting, we must also discuss the rate associated with sphaleron processes, which represent saddle-point transitions between distinct electroweak vacua possessing different values of $B$ and $L_{agg}$, for these will be responsible for the conversion of lepton number to baryon number. As these processes are mediated by the $SU(2)$ electroweak anomaly, they only affect $L_\ell$ (and hence $L_{agg}$) and not $L_{\nu_R}$. We will primarily be concerned with the high temperature sphaleron interaction rate, by which we mean the interaction rate at temperatures $T \gg T_c$, where $T_c$ is the temperature at the weak scale. This rate takes to form

$$\Gamma_{Sph} = c T \alpha_2^4,$$

(61)

where $\alpha_2 \equiv g_2^2/4\pi \approx 1/30$ and $c$ is an $O(1)$ constant, whose value calculations \cite{42, 43, 44} place very close to 1.

We are now ready to write the full set of Boltzmann equations governing the evolution of baryon number $B$ in the early universe. We will need to include equations for $L_{agg}$, $L_{\nu_R}$, $B$, as well as ones for the lepton numbers stored in the heavy fields $\phi_1$ and $\bar{\phi}_1$, which we respectively dub $L_{\phi_1}$ and $L_{\bar{\phi}_1}$, and the number-density-to-entropy ratio of one of these heavy fields or their conjugates (we choose $\phi^c$), which we call $Y_{\phi^c}$. In terms of the variable $z \equiv M_{\phi_1}/T$, they are
\[ \frac{dB}{dz} = \frac{z}{H(M_{\Phi_1})} \left[ -\langle \Gamma_{sph} \rangle (B + \frac{8}{15} L_{agg}) \right] \]  
(62)

\[ \frac{dL_{agg}}{dz} = \frac{z}{H(M_{\Phi_1})} \left[ -2\epsilon \langle \Gamma_D \rangle (Y_{\phi_\beta}^c - Y_{\phi_\beta}^{eq}) + \langle \Gamma_L \rangle (L_{\phi_\beta} + L_{\phi_\bar{\beta}}) + \langle \Gamma_R \rangle L_{\phi_\bar{\beta}} - 2L_{agg} \left( \langle \Gamma_D \rangle_{ID} + \langle \Gamma_L \rangle_{ID} \right) + (L_{\nu_R} - \frac{1}{7} L_{agg}) \langle \Gamma_{2\leftrightarrow2} \rangle - \langle \Gamma_{sph} \rangle (B + \frac{8}{15} L_{agg}) \right] \]  
(63)

\[ \frac{dL_{\nu_R}}{dz} = \frac{z}{H(M_{\Phi_1})} \left[ 2\epsilon \langle \Gamma_D \rangle (Y_{\phi_\beta}^c - Y_{\phi_\beta}^{eq}) + L_{\phi_\beta} \langle \Gamma_R \rangle - 2L_{\nu_R} \langle \Gamma_R \rangle_{ID} - (L_{\nu_R} - \frac{1}{7} L_{agg}) \langle \Gamma_{2\leftrightarrow2} \rangle \right] \]  
(64)

\[ \frac{dY_{\phi_\beta}}{dz} = \frac{z}{H(M_{\Phi_1})} \left[ -\langle \Gamma_D \rangle (Y_{\phi_\beta}^c - Y_{\phi_\beta}^{eq}) + \frac{1}{2} L_{agg} \langle \Gamma_L \rangle_{ID} + \frac{1}{2} L_{\nu_R} \langle \Gamma_R \rangle_{ID} \right] \]  
(65)

\[ \frac{dL_{\phi_\beta}}{dz} = \frac{z}{H(M_{\Phi_1})} \left[ -\langle \Gamma_D \rangle L_{\phi_\beta} + 2L_{agg} \langle \Gamma_L \rangle_{ID} + 2L_{\nu_R} \langle \Gamma_R \rangle_{ID} \right] \]  
(66)

\[ \frac{dL_{\phi_\bar{\beta}}}{dz} = \frac{z}{H(M_{\Phi_1})} \left[ -\langle \Gamma_D \rangle L_{\phi_\bar{\beta}} + 2L_{agg} \langle \Gamma_D \rangle_{ID} \right] \]  
(67)

Here, the inverse decay rates \( \langle \Gamma_D \rangle_{ID}, \langle \Gamma_L \rangle_{ID}, \)  and \( \langle \Gamma_R \rangle_{ID} \) are defined by

\[ \langle \Gamma_D \rangle_{ID} = \frac{1}{\ell n_{\gamma}} \left( \frac{K_1(z)}{K_2(z)} \right) \Gamma_D, \]  
(68)

\[ \langle \Gamma_L \rangle_{ID} = \frac{1}{\ell n_{\gamma}} \left( \frac{K_1(z)}{K_2(z)} \right) \Gamma_L, \quad \text{and} \]  
(69)

\[ \langle \Gamma_R \rangle_{ID} = \frac{n_{\phi_\beta}^{eq}}{n_{\gamma}} \left( \frac{K_1(z)}{K_2(z)} \right) \Gamma_R, \]  
(70)

where \( \Gamma_D \) is the total decay width of \( \phi_1 \) given in equation (68), \( n_{\phi_\beta}^{eq} \) is the equilibrium number density of \( \phi_1 \) (and \( \phi_\bar{1}, \text{etc.} \)), and the quantities \( \Gamma_L \) and \( \Gamma_R \) represent the partial decay widths for \( \phi \rightarrow \nu_R \bar{H} \) (or \( \phi \rightarrow \bar{\nu}_R H \)) and \( \phi \rightarrow \ell \bar{\chi} \) (or \( \phi \rightarrow \bar{\ell} \chi \)). The ratio of modified Bessel functions \( K_1(z) \) and \( K_2(z) \) appearing in these rates is a result of averaging over time-dilation factors (see \( 63 \) in Appendix A). Explicit definitions for these quantities, along with a derivation of the Boltzmann equations themselves, are provided in Appendix A.

We solve this system of equations numerically as a function of the input parameters \( \delta \) and the reparameterized coupling strength \( f \), defined in eq. (54), and present our results in fig. 7. Here, the regions of \( f - \delta \) parameter space in which the final value of \( \eta \) generated falls within the WMAP-allowed range given in (2), appear as thin ‘ribbons’, each corresponding to a different value of \( M_{\phi_1} \). In fig. 7 the effects of the processes detailed above are apparent, and certainly nontrivial. Physically, these effects can be interpreted as follows: increasing the strength of the neutrino-sector couplings (here parameterized by \( f \)) increases \( \Gamma_D \), which in turn increases the initial value of \( B \); however, from equation (60), increasing \( f \) also increases the rates for the \( 2 \leftrightarrow 2 \) processes which shuffle lepton number back and forth between \( L_{agg} \) and \( L_R \). Furthermore, it increases the rate for inverse decays. This allows
two possibilities for generating a realistic final value for $B$. In the first case, where $f$ is small, the initial baryon number produced by $\phi$ and $\bar{\phi}$ decays is approximately within the range allowed by WMAP, and $2 \leftrightarrow 2$ and inverse decay processes are so slow as to be negligible; this is the “drift-and-decay limit” of equation (20). In the second case, where $f$ is large, a surfeit of baryon number will initially be produced, but these processes, which occur more rapidly for larger $f$, subsequently reduce $B$ to a phenomenologically acceptable level; this we refer to as the “strong-washout regime”. These two possibilities are shown in the two panels of fig. 8 in which the dynamical evolution of $B$, $L_{agg}$ and other relevant quantities has been plotted, for $\delta = 4.83 \times 10^{-3}$ and $M_\Phi = 10^{12}$ GeV. In fig. 8 the two regimes are represented respectively by the lower and upper portions of each ribbon.

Having discussed the general effects of the washout processes described above as a group, it is also important to address their characteristics relative to one another. Inverse decays dominate over $2 \leftrightarrow 2$ processes only for a brief period, where $1 \lesssim z \lesssim 50$, but during this period they are extremely effective in reducing lepton number, and in fact are the primary factor in determining the final value of $\eta$. For larger $z$, until they freeze out, the $2 \leftrightarrow 2$ interactions dominate and further reduce $L_{agg}$ and $L_{\nu R}$ (and consequently $B$). It should be noted, however, that the total $B - L_{tot}$ number of the universe is manifestly conserved by the Boltzmann equations (62) - (67) (the sum of the rates for the various lepton numbers involved is zero), and since we began with $B - L_{tot} = 0$, we end up with $B - L_{tot} = 0$ in any case.

For a given $M_{\Phi_1}$ and $\delta$, then, there are at most two points of parameter space at which all salient constraints from neutrino physics, cosmology, and baryogenesis are satisfied (represented in fig. 7 by the points at which a given ribbon intersects the grey, vertical line of constant $\delta$). At each of these points, the value of $\langle \chi \rangle$ is obtained by requiring that the neutrino masses given by equation (7) satisfy the experimental bounds in equation (34). For $M_{\Phi_1} = 10^{12}$ GeV and $\delta = m_\nu/m_\mu$, the strong washout case (with $f = 8.3 \times 10^{-2}$) corresponds to $\langle \chi \rangle \sim 5$ GeV; in the drift-and-decay case (where $f = 1.5 \times 10^{-3}$), $\langle \chi \rangle \sim 50$ TeV. In general, for the strong washout regime, the value of $\langle \chi \rangle$ increases with decreasing $M_{\Phi_1}$; for the drift and decay case, it decreases with decreasing $M_{\Phi_1}$. These trends continue until the two points converge into one at $M_{\Phi_1} \simeq 10^{10}$ GeV, for which $\langle \chi \rangle \sim 1$ TeV.

In CHDL, the flavor structure of neutrinos rises from a hierarchy among the $M_{\Phi_i}$ in which $\delta$ is small. As is evident from fig. 7 it does not seem possible simultaneously to obtain a realistic neutrino spectrum and obtain the correct baryon number when $M_{\Phi_1} < 10^{10}$ GeV. Since for higher $M_{\Phi_1}$ we require $T_R \gtrsim 10^{10}$ GeV, the reheating temperature constraints discussed in section 3 are of genuine concern. In particular, the constraints from BBN and nonthermal LSP production make things very difficult for CHDL when $m_{3/2} \lesssim 10^5$ GeV and BBN constraints come into play. While it is possible to get Dirac neutrino genesis to work when $10^5$ GeV $\gtrsim m_{3/2} \lesssim 10^8$ GeV, this requires one of two things: either $m_{LSP}$ is light enough that the naive reheating temperature bound permits a reheating temperature above $10^{10}$ GeV (see fig. 8), or else the ratio $m_{3/2}/m_{LSP}$ must be large enough that LSP annihilations are effective (fig. 9). While it is possible to get leptogenesis to work in the former case, it is far from optimal in the sense that nonthermal (or non-LSP) dark matter is required. In the latter, thermal CDM is still possible, but the ratio $m_{3/2}/m_{LSP}$ has to be be far larger than the $\beta/g_\lambda$ that results from the one-loop splitting given in equation (11). As discussed earlier, this means there will be great difficulty getting CHDL to work in loop-split SUSY when $m_{3/2}$ is on the PeV scale. There is, however, a window where $M_{\Phi_1} \sim$
Figure 7: Bands in $f - \delta$ parameter space for which the final baryon number $B_f$ falls within the range permitted by WMAP, for different choices of $M_{\Phi_1}$. Here $f$ parameterizes the couplings of the $\Phi_i$ fields (see eq. (54)). The configuration of the left (right) panel is the one marked with a dot in the left (right) panel of fig. 5, in which $a_3 = 95$ ($a_3 = 4.5$). The shaded vertical lines show the constraint on the value of $\delta$ coming from neutrino mixings and masses ($\delta = m_e/m_\mu \approx 4.83 \times 10^{-3}$ in the left and $\delta = 10^{-1}$ in the right). Note that there are two points that yield a realistic value of $B_f$ for a given $\delta$ (the points at which the grey vertical line intersects a given ribbon). When $f$ is small enough, the initial baryon number generated is just enough to be consistent with WMAP, while the washout effect of inverse decays and $2 \leftrightarrow 2$ processes is negligible. In this situation the baryon number generated is independent of $M_\Phi$ and its the final value is proportional to the CP violating parameter $\epsilon$ (see eq. (20)). The dark dotted curve shows the band of consistent baryon number calculated in this “drift and decay” limit. Each “X” marks the point in which $\Gamma_D/H = 1$ for each different $M_{\Phi_1}$. At these points, the “strong washout” regime starts. The now active washout processes reduce an initial surfeit of baryon number (due to a larger $f$) down to an acceptable level (see fig. 8).

$10^{10}$ GeV and $m_{LSP} \lesssim 300$ GeV in which CHDL can be made to work in such scenarios. For $m_{3/2} \gtrsim 10^8$ GeV (where the gravitino decays rapidly and the LSP relic abundance is entirely thermal), reheating temperatures greater than $10^{10}$ GeV are permitted, and CHDL succeeds in providing an explanation for the origin of the observed baryon asymmetry and neutrino mixings. Since constraints from gluino cosmology become relevant for $m_{3/2} \gtrsim 10^{10}$ GeV, it appears that split supersymmetry models where $10^8$ GeV $\lesssim m_{3/2} \lesssim 10^{10}$ GeV are the “natural habitat”, so to speak, for thermal Dirac neutrinoogenesis with thermal CDM.
Figure 8: These two plots show the evolution of baryon number $B$ for $M_{\Phi_1} = 10^{12}$ GeV and $\delta = m_e/m_{\mu} = 4.83 \times 10^{-3}$ (the CHDL value), in the two different regimes that produce a realistic value for the final baryon number of the universe, $B_F$. For $f = \sqrt{\lambda_{23} h_{23}} = 1.5 \times 10^{-3}$, as shown in the left panel, the effects of $2 \leftrightarrow 2$ lepton-number-changing processes are negligible and the final baryon-to-entropy ratio is the same as that initially produced by $\phi$ and $\bar{\phi}$ decays. For stronger coupling $f = 3.8 \times 10^{-2}$, as shown in the right panel, baryon number is initially overproduced, but $2 \leftrightarrow 2$ processes, which are stronger for stronger coupling, reduce $B$ to an acceptable level by the time they freeze out. The left and right panels correspond respectively to the lower and upper parts of the ‘ribbon’ in fig. 4.

6 Conclusion

Dirac neutrinogenesis is an interesting alternative to the standard Majorana leptogenesis, and one which is equally capable of explaining both the baryon asymmetry of the universe and the smallness of neutrino masses (without the seesaw mechanism). We have shown that thermal Dirac neutrinogenesis is not only an interesting theory, but a genuinely viable phenomenological model: it is simultaneously capable of producing a baryon-to-photon ratio $\eta$ for the universe that matches that observed by WMAP, yielding a neutrino spectrum that agrees with the predictions of solar and atmospheric neutrino data, and avoiding reheating temperature constraints and other potential cosmological and astrophysical problems. Furthermore, it is capable of doing all this in the context of a simple setup in which the observed neutrino masses and mixings are explained by imposing (anti)symmetry conditions on couplings in the superpotential and a hierarchy among the masses $M_{\Phi_i}$ of the additional heavy fields in the theory.

We have also shown that Dirac neutrinogenesis is naturally accommodated in split supersymmetry regimes involving either a heavy gravitino (with a mass above around $10^8$ GeV) or a very large hierarchy between the gravitino mass and the mass of the LSP. In models with a lighter gravitino and only a small splitting between $m_{3/2}$ and $m_{LSP}$, LSP annihilations are generally ineffective, the reheat temperature $T_R$ associated with inflation...
cannot be raised above $\sim 10^{10}$ GeV, and constraints on $M_{\Phi_1}$ become quite severe. As a result, leptogenesis becomes extremely difficult (for ease of reference, we classify the different gravitino-mass regimes with regard to leptogenesis considerations in table 2). It is interesting to note that Dirac neutrino genesis, at least in its constrained hierarchical form (CHDL), can be realized in simple, one-loop AMSB in a small region of parameter space in which $M_{\Phi_1} \sim 10^{10}$ GeV and $m_{\text{LSP}} \lesssim 300$ GeV (which sets $m_{3/2} \sim 10^5$ GeV). In more general split SUSY models, cosmological constraints are less severe. In models where $m_{3/2} \gtrsim 10^8$ GeV, the constraints altogether disappear and Dirac neutrino genesis can be made to work without sacrificing the possibility for thermal LSP cold dark matter. Thus Dirac neutrino genesis, at least in a regime with heavy gravitinos, provides an interesting and viable alternative to Majorana leptogenesis.

| Gravitino Mass Range (GeV) | Maximum $M_{\Phi_1}$ (GeV) | Workability of CHDL? | Comments |
|---------------------------|----------------------------|----------------------|----------|
| $m_{3/2} \lesssim 10^5$   | $10^6 - 10^8$              | Very Low             | $\tilde{G}$ decay during or after BBN. Insufficient $\eta$ generated. |
| $10^5 \lesssim m_{3/2} \lesssim 10^8$ | $10^9 - 10^{10}$ or higher | Depends on $m_{\text{LSP}}$ and the ratio $m_{\text{LSP}}/m_{3/2}$ | LSP annihilations ineffective unless $m_{\text{LSP}}/m_{3/2}$ is small. $T_R$ constrained by nonthermal LSP abundance from $\tilde{G}$ decay. Loop-split SUSY works only for $m_{3/2} \sim 10^5$ GeV. More general split SUSY theories can be successful. |
| $10^8 \lesssim m_{3/2} \lesssim 10^{10}$ | None                      | Excellent             | $\Omega_{\text{LSP}}$ is thermal, since $\tilde{G}$ decays before LSP freeze-out. $M_{\Phi_1} > 10^{11}$ GeV allowed. $\nu$ spectrum requirements compatible with CHDL. |
| $10^{10} \lesssim m_{3/2} \lesssim 10^{12}$ | None                      | Questionable (depends on gluino properties) | $\nu$ sector and baryogenesis okay, but model may have a cosmological gluino problem. |
| $10^{12} \lesssim m_{3/2}$ | None                      | Very Low              | Potential gluino problem becomes a serious concern. |

Table 2: The various gravitino mass regimes for split supersymmetry models and the viability of thermal Constrained Hierarchical Dirac Leptogenesis (CHDL) in each case.

Another asset of the theory is that there are experimental checks on its viability. The major prediction of Dirac neutrino genesis is that neutrinoless double-beta decay will not be observed to any degree, for this process relies on the existence of a Majorana mass term for right-handed neutrinos. The discovery of such a process experimentally would rule the theory out. In addition, forthcoming results from MiniBooNE should either confirm or deny the LSND result, which will reveal whether or not the neutrino spectrum produced by Dirac neutrino genesis is in fact the one present in nature. Finally, thermal Dirac neutrino genesis yields some definite predictions about the neutrino mixing parameter $\sin \theta_{13}$ (see fig. 5), the value of which will be measured in future experiments.
Acknowledgments

We would like to thank James Wells for his useful comments and discussions and for carefully reading the manuscript. We also thank Jason Kumar, David Morrissey, Aaron Pierce, Kazuhiro Tobe, Liliana Velasco-Sevilla and Ting Wang for useful comments or related discussions. B.T. and M.T. are supported by D.O.E. and the Michigan Center for Theoretical Physics (MCTP).

A Derivation of the Boltzmann Equations

In this appendix, we derive the Boltzmann Equations that appear in equations (63) – (62), following the methods of [46]. The Boltzmann equations for any particle species \( a \) in the early universe can be written in terms of \( Y_a \equiv n_a/s \), where \( n_a \) is the number density of \( a \) and \( s \) is the entropy density, as [24]

\[
\frac{dY_a}{dt} = \frac{1}{s} \int \frac{d^3p_a}{(2\pi)^3} \frac{d^3p_i}{(2\pi)^3} \frac{d^3p_j}{(2\pi)^3} \frac{d^3p_k}{(2\pi)^3} (2\pi)^4 \delta(\sum_{n=a,i,j,...k} p_n) \left[ \sum_{\text{int.}} |\mathcal{M}(a...i \rightarrow j...k)|^2 (f_a...f_i) - \sum_{\text{int.}} |\mathcal{M}(j...k \rightarrow a...i)|^2 (f_j...f_k) \right],
\]

(71)

where \( f_i \) is the phase-space distribution function of particle \( i \), \( |\mathcal{M}(a...i \rightarrow j...k)|^2 \) are the squared matrix elements for particle-number-changing interactions involving \( a \), and the sums are over all interaction processes which create or destroy \( a \). Since for the scalar fields \( \phi \) and \( \phi^c \), the leading processes include only decays and inverse decays, the Boltzmann equations for the abundances \( Y_\phi \) and \( Y_{\phi^c} \) are given by

\[
\frac{dY_\phi}{dt} = -\frac{1}{s} \Lambda_{12}^\phi \left[ f_\phi \left| \mathcal{M}(\phi \rightarrow \bar{\ell} \bar{\ell}) \right|^2 + f_\phi^c \left| \mathcal{M}(\phi \rightarrow \nu_R \bar{H}_u^c) \right|^2 \right.
\]

\[
- f_\phi \bar{f}_\ell \left| \mathcal{M}(\bar{\ell} \phi \rightarrow \phi) \right|^2 - f_{\nu_R} f_{\bar{H}_u^c} \left| \mathcal{M}(\nu_R \bar{H}_u^c \rightarrow \phi) \right|^2 - f_{\nu_R} f_{\bar{H}_u^c} \left| \mathcal{M}(\nu_R \bar{H}_u^c \rightarrow \phi) \right|^2 \right]
\]

(72)

and

\[
\frac{dY_{\phi^c}}{dt} = -\frac{1}{s} \Lambda_{12}^{\phi^c} \left[ f_{\phi^c} \left| \mathcal{M}(\phi^c \rightarrow \bar{\ell} \bar{\ell}) \right|^2 + f_{\phi^c} \left| \mathcal{M}(\phi^c \rightarrow \nu_R \bar{H}_u^c) \right|^2 \right.
\]

\[
- f_{\phi^c} \bar{f}_\ell \left| \mathcal{M}(\bar{\ell} \phi^c \rightarrow \phi) \right|^2 - f_{\nu_R} f_{\bar{H}_u} \left| \mathcal{M}(\nu_R \bar{H}_u \rightarrow \phi^c) \right|^2 - f_{\nu_R} f_{\bar{H}_u} \left| \mathcal{M}(\nu_R \bar{H}_u \rightarrow \phi^c) \right|^2 \right],
\]

(73)

where \( \Lambda_{12}^{a...i} \) has been used as a shorthand to denote the appropriate phase-space integral and we use the notation \( l \) and \( H_u \) to denote the usual Lepton and Higgs-up doublets of \( SU(2) \).

Let us also define the particle asymmetry \( A \equiv (Y_a - Y_a^c) \), where \( Y_a \) and \( Y_a^c \) are the entropy normalized abundances of particle \( a \) and its conjugate \( a^c \). For a relativistic species \( a \) (i.e. \( T \gg m_a \)) and as long as both \( A \) and \( \mu_a \) are small, we have

\[
A \simeq \frac{n_a}{s} g_a (e^{2\mu_a/T} - 1),
\]

29
which leads to $e^{\mu_a/T} \simeq 1 + \frac{s}{n_\gamma} \frac{A}{g_a}$ where $g_a$ is the number of degrees of freedom of particle $a$, $s$ is the entropy density, and $n_\gamma$ is the photon density. If we apply the energy conservation relation to the inverse decay processes of the form $(a+ i \rightarrow b)$ appearing in equations (72) and (73), in which particle $i$ is in chemical equilibrium with other particles in the thermal bath (i.e. $\mu_i = 0$), the previous relation allows us to write

$$ f_a f_i \simeq f_b^{eq}(1 + \frac{s}{n_\gamma} \frac{A}{g_a}), \quad (74) $$

where $f_b^{eq}$ is the equilibrium distribution of $b$, and the negative sign occurs when $a$ is a conjugate particle.

It will also be useful to relate the particle asymmetries of Leptons to the net Lepton number abundance carried by each Lepton species. We define the Lepton abundances as

$$ L_\ell = Y_\ell - Y_{\ell^c} \quad (75) $$

$$ L_{\nu_R} = -(Y_{\nu_R} - Y_{\bar{\nu}_R}) \quad (76) $$

$$ L_{\tilde{\ell}} = Y_{\tilde{\ell}} - Y_{\tilde{\ell}^c} \quad (77) $$

$$ L_{\bar{\nu}_R} = -(Y_{\bar{\nu}_R} - Y_{\bar{\nu}_R}). \quad (78) $$

Relation (74), along with the CPT conservation relation

$$ |\mathcal{M}(a \rightarrow ij)|^2 = |\mathcal{M}(\bar{i} \bar{f}^c \rightarrow a^c)|^2, \quad (79) $$

allows us to simplify the Boltzmann equations for $\phi$ and $\bar{\phi}^c$ significantly. The results may be stated in terms of $L_\ell$, $L_{\nu_R}$, $L_{\bar{\nu}_R}$ and $L_{\tilde{\ell}}$ as

$$ \frac{dY_{\phi}}{dt} = -\frac{1}{s} \int \frac{d^3p_\phi}{(2\pi)^3} \left[ (f_{\phi} - f_{\phi}^{eq}) \Gamma_D - \frac{s}{2n_\gamma} f_{\phi}^{eq} \left( \frac{L_{\tilde{\ell}}}{2} \Gamma_{\ell} + L_{\nu_R} \Gamma_{R} \right) \right] $$

$$ \frac{dY_{\phi^c}}{dt} = -\frac{1}{s} \int \frac{d^3p_{\phi^c}}{(2\pi)^3} \left[ (f_{\phi^c} - f_{\phi^c}^{eq}) \Gamma_D + \frac{s}{2n_\gamma} f_{\phi^c}^{eq} \left( \frac{L_{\tilde{\ell}}}{2} \Gamma_L + L_{\nu_R} \Gamma_R \right) \right], $$

where the interaction rates $\Gamma_L = \Gamma(\phi \rightarrow \tilde{\ell} + \chi)$ and $\Gamma_R = \Gamma(\phi \rightarrow \nu_R + \tilde{\nu}_R)$ are defined by the relations

$$ \Gamma_L = \int \frac{d^3p_\ell}{(2\pi)^3} \left| \mathcal{M}(\phi \rightarrow \ell + \chi) \right|^2 $$

$$ \Gamma_R = \int \frac{d^3p_{\nu_R}}{(2\pi)^3} \left| \mathcal{M}(\phi \rightarrow \nu_R + \tilde{\nu}_R) \right|^2, $$

with $\Gamma_L, R$ being the rates of the conjugate processes and $\Gamma_D = \Gamma_L + \Gamma_R$ is the total decay rate for $\phi$, $\bar{\phi}$, etc. given in (13). Because of supersymmetry, the total decay rate of the fermion components of the heavy supermultiplets $\Phi$ and $\overline{\Phi}$ will also be $\Gamma_D$, with the same $\Gamma_L$ and $\Gamma_R$ as their partial rates.

We also define the Lepton asymmetry $L_{\phi} \equiv Y_{\phi} - Y_{\phi^c}$ associated to the new heavy leptons. After integrating over the incoming momentum $\vec{p}_\phi$ we obtain for $L_{\phi}$:

$$ \frac{dL_{\phi}}{dz} = -\langle \Gamma_D \rangle \left( \frac{L_{\phi}}{2} - L_{\nu_R} \right) + Y_{\phi}^{eq} s \frac{L_{\tilde{\ell}}}{2} \left( \langle \Gamma_L \rangle + L_{\nu_R} \langle \Gamma_R \rangle \right) $$

$$ + \frac{2}{2n_\gamma} Y_{\phi}^{eq} \frac{L_{\bar{\nu}_R}}{2} \left( \langle \Gamma_L \rangle + L_{\nu_R} \langle \Gamma_R \rangle \right)$$

$$ + \frac{2}{2n_\gamma} Y_{\phi}^{eq} \frac{L_{\bar{\nu}_R}}{2} \left( \langle \Gamma_L \rangle + L_{\nu_R} \langle \Gamma_R \rangle \right) \quad (84) $$

30
Here, \( \langle \Gamma_A \rangle \) is the respective decay rate averaged over time-dilation factors [46]:

\[
\langle \Gamma_A \rangle = \frac{K_1(M_\Phi/T)}{K_2(M_\Phi/T)} \Gamma_A
\]  

(85)

where \( K_1(x) \) and \( K_2(x) \) are modified Bessel functions. The Boltzmann equations for the scalar component of the \( \Phi \) superfield (and its conjugate), as well as the ones for the fermion components of \( \Phi \) and \( \bar{\Phi} \), are obtained in a similar manner.

We now turn to address the evolution of the Lepton number abundance of the particle species \( \ell, \nu_R, \bar{\ell}, \) and \( \bar{\nu}_R \), in which we must take into account the effect of \( 2 \leftrightarrow 2 \) processes which transfer lepton number between \( L_\ell, L_{\nu_R}, L_{\bar{\ell}}, \) and \( L_{\bar{\nu}_R} \). We will make one important distinction among them: the rates for interactions which shuffle lepton number between \( \ell, \bar{\ell}, \nu_R, \) and \( \bar{\nu}_R \), and the right-handed charged lepton and sleptons fields \( e_R \) and \( \tilde{e}_R \) will be much larger than those for the interactions which shuffle lepton number between \( \nu_R \) and any of these other fields. This is due to the fact that \( \nu_R \) only interacts via the processes pictured in figure [46] which involve a virtual \( \phi_\Phi \), \( \phi_{\bar{\Phi}} \), etc. while all of the other fields either take part in \( SU(2) \) and/or \( U(1)_Y \) gauge interactions, or—in the case of \( \bar{\nu}_R \)—left-right slepton equilibration through an assumed large \( \langle F_\chi \rangle \) term. We will represent the effects of these rapid equilibration processes by including terms \( \Sigma_A \) (where \( A \) is the relevant particle asymmetry) to represent them in the Boltzmann equations. The slower \( 2 \leftrightarrow 2 \) processes through which right handed neutrinos \( \nu_R \) interact with lepton doublets \( \ell \) and \( \bar{\ell} \) and with right handed sneutrino \( \bar{\nu}_R \) will be included separately as \( C_{\nu_R \leftrightarrow \ell}, C_{\nu_R \leftrightarrow \bar{\ell}} \) and \( C_{\nu_R \leftrightarrow \bar{\nu}_R} \). To simplify further the notation we will define the terms \( F_A \) to account for the collective contribution from decays (and inverse decays) of the fermionic components of \( \Phi \) and \( \bar{\Phi} \) (which we will not write explicitly, being of similar form to the contribution from the scalar components).

We obtain for the Lepton number abundance \( L_\ell \):

\[
\frac{dL_\ell}{dt} = -\epsilon \langle \Gamma_D \rangle (Y_{\phi_\Phi}^e + Y_{\phi_\bar{\Phi}}^{eq}) + L_{\phi_\bar{\Phi}} (\langle \Gamma_L \rangle - Y_{\phi_\Phi}^{eq} s_{\nu_R}) \frac{1}{2} L_\ell (\langle \Gamma_L \rangle - \frac{1}{2} \epsilon \langle \Gamma_D \rangle)
\]

\[
+ F_\ell + \Sigma_\ell + C_{\nu_R \leftrightarrow \ell}
\]  

(86)

Similarly, the equations for \( L_{\nu_R}, L_{\bar{\ell}}, \) and \( L_{\bar{\nu}_R} \) are given by

\[
\frac{dL_{\nu_R}}{dt} = \epsilon \langle \Gamma_D \rangle (Y_{\phi_\Phi}^e + Y_{\phi_\bar{\Phi}}^{eq}) - L_{\phi_\Phi} (\langle \Gamma_R \rangle + Y_{\phi_\Phi}^{eq} s_{\nu_R}) \frac{1}{2} L_{\nu_R} (\langle \Gamma_R \rangle - \frac{1}{2} \epsilon \langle \Gamma_D \rangle)
\]

\[
+ F_{\nu_R} - C_{\nu_R \leftrightarrow \bar{\nu}_R} - C_{\nu_R \leftrightarrow \bar{\ell}} - C_{\nu_R \leftrightarrow \ell}
\]  

(87)

\[
\frac{dL_{\bar{\ell}}}{dt} = -\epsilon \langle \Gamma_D \rangle (Y_{\phi_\Phi}^e + Y_{\phi_\bar{\Phi}}^{eq}) + L_{\phi_\bar{\Phi}} (\langle \Gamma_L \rangle - Y_{\phi_\Phi}^{eq} s_{\nu_R}) \frac{1}{2} L_{\bar{\ell}} (\langle \Gamma_L \rangle + \frac{1}{2} \epsilon \langle \Gamma_D \rangle)
\]

\[
+ F_{\bar{\ell}} + \Sigma_{\bar{\ell}} + C_{\nu_R \leftrightarrow \bar{\ell}}
\]  

(88)

\[
\frac{dL_{\bar{\nu}_R}}{dt} = \epsilon \langle \Gamma_D \rangle (Y_{\phi_\Phi}^{eq} + Y_{\phi_\bar{\Phi}}^e) - L_{\phi_\bar{\Phi}} (\langle \Gamma_R \rangle - Y_{\phi_\Phi}^{eq} s_{\nu_R}) \frac{1}{2} L_{\bar{\nu}_R} (\langle \Gamma_R \rangle + \frac{1}{2} \epsilon \langle \Gamma_D \rangle)
\]

\[
+ F_{\bar{\nu}_R} + \Sigma_{\bar{\nu}_R} + C_{\nu_R \leftrightarrow \bar{\nu}_R}
\]  

(89)

Before going any further we still need to compute the terms \( C_{\nu_R \leftrightarrow \bar{\ell}}, C_{\nu_R \leftrightarrow \bar{\nu}_R} \) and \( C_{\nu_R \leftrightarrow \ell} \) corresponding to the \( 2 \leftrightarrow 2 \) processes mediated by heavy fields.
We begin by calculating $C_{\nu R \to \ell}$, which is given by

$$
C_{\nu R \to \ell} = \frac{2}{n_\gamma} \Lambda_{12}^{-1} e^{-(E_1+E_2)/T} \left[ \left| \mathcal{M}' (\bar{H}_u^c \nu_R \to \bar{\ell} \chi) \right|^2 - \left| \mathcal{M}' (\bar{\ell} \chi \to \nu_R^c \bar{H}_u^c) \right|^2 \right]
+ \frac{(L_{\nu R} - \frac{1}{2} L_\ell)}{2} \left[ \left| \mathcal{M}' (\bar{H}_u^c \nu_R \to \bar{\ell} \chi) \right|^2 + \left| \mathcal{M}' (\bar{\ell} \chi \to \nu_R^c \bar{H}_u^c) \right|^2 \right]
$$

(90)

$\mathcal{M}'$ here refers to the amplitude for the specified $2 \leftrightarrow 2$ process to which we have subtracted the contribution from the resonant intermediate state (RIS) in which a real field $\phi_k$ is produced and then decayed into the 2 particle final state. The RIS contribution must be subtracted since we have already counted contributions from decays of real $\phi_k$ fields.

The leading term in the difference between a $2 \leftrightarrow 2$ process of the form $ab \to ij$ involving a heavy intermediary $k$ and its conjugate process depends on the contribution of the on-shell (resonant) intermediate state:

$$
|\mathcal{M}'(ab \to ij)|^2 - |\mathcal{M}'(ij \to ab)|^2 = |\mathcal{M}_{\text{RIS}}(ij \to ab)|^2 - |\mathcal{M}_{\text{RIS}}(ab \to ij)|^2
$$

(91)

with

$$
|\mathcal{M}_{\text{RIS}}(ab \to ij)|^2 \approx \frac{\pi}{m_\phi \Gamma_D} \delta(s - m_\phi^2)|\mathcal{M}(ab \to k)|^2 \times |\mathcal{M}(k \to ij)|^2
$$

(92)

where $k$ represents the intermediate-state particle, and $s$ is the usual kinematic variable $s = (p_1^{|ab|} + p_2^{|ij|})^2$. In our case, making use of the equality $\Gamma_L \Gamma_R - \Gamma_L \Gamma_R = \epsilon \Gamma_D$ we find

$$
|\mathcal{M}_{\text{RIS}}(\bar{\ell} \chi \to \nu_R^c \bar{H}_u^c)|^2 - |\mathcal{M}_{\text{RIS}}(\bar{H}_u^c \nu_R^c \to \bar{\ell} \chi)|^2 \approx \epsilon \frac{\pi}{m_\phi \Gamma_D} \delta(s - m_\phi^2) |\mathcal{M}_\text{tot}|^4
$$

(93)

and substituting this result in eq. (90) we finally obtain

$$
C_{\nu R \to \ell} = 2 e Y_{\phi_k} \langle \Gamma_D \rangle + \left( L_{\nu R} - \frac{1}{2} L_\ell \right) n_{\nu_{\nu R \to \ell}} (v_{\nu_{\nu R \to \ell}} + v_{\nu_{\nu R \to \nu R}}).
$$

(94)

For the other sets of $2 \leftrightarrow 2$ processes, $C_{\nu R \to \bar{\nu}_R}$ and $C_{\nu R \to \ell}$, the procedure is essentially the same. The rates $\langle \Gamma_{\nu R \to \ell} \rangle \equiv n_{\nu} (v_{\nu_{\nu R \to \ell}} + v_{\nu_{\bar{\nu}_R \to \nu R}})$ and $\langle \Gamma_{\nu R \to \nu R} \rangle \equiv n_{\nu} (v_{\nu_{\nu R \to \ell}} + v_{\nu_{\nu R \to \nu R}})$, associated with these interactions are calculated in section 5. We will denote the total contribution from these processes as $\langle \Gamma_{2 \leftrightarrow 2} \rangle$.

At this point, the full Boltzmann system comprises fourteen individual differential equations: four for $\phi_k$, $\phi_{\bar{k}}$, and their conjugates (or alternatively, the abundances $Y_{\phi_k}$, $L_{\phi_k}$, etc.); four for the the Lepton asymmetries $L_\ell$, $L_{\nu_R}$, $L_{\bar{\nu}_R}$, and $L_{\bar{\ell}}$; two for the Lepton asymmetries $L_{e_R} \equiv e_R - e_R$ and $L_{\bar{e}_R} \equiv \bar{e}_R - \bar{e}_R$ of the right-handed charged lepton and slepton fields; and an additional four for the fermionic superpartners in the $\Phi$ and $\bar{\Phi}$ supermultiplets.

Let us make the assumption that the rates for the processes denoted by $\Sigma_A$ are sufficiently rapid that chemical equilibrium is achieved among the particle species that interact via these processes. In this case, any lepton number stored in $\ell$, $\bar{\ell}$, $\nu_R$, $e_R$, or $\bar{e}_R$ should be rapidly distributed among all of these particles in proportion to the relative number of degrees of freedom for each field. The lepton number stored in each field then becomes

$$
L_\ell, L_{\bar{\ell}} \to L_{\text{eq}},
$$

(95)

$$
L_{\nu_R}, L_{e_R}, L_{\bar{e}_R} \to L_{\text{eq}}/2,
$$

(96)
where we have defined an equilibrium lepton number $L_{eq}$ which represents the effects of rapid equilibration through $\Sigma_A$. Because of this rapid equilibration, it is no longer necessary to keep track of the individual lepton numbers $L_\ell$, $L_{\bar{\ell}}$, $L_{e_R}$, and $L_{\bar{e}_R}$; we need only one differential equation representing the total lepton number in this equilibrated, aggregate sector $L_{agg} = L_\ell + L_{\bar{\ell}} + L_e + L_{\bar{e}} = \frac{1}{2} L_{eq}$. The equation is

$$\frac{dL_{agg}}{dt} = -\epsilon \langle \Gamma_D \rangle (Y^c_{\phi_\Phi} - Y^{eq}_{\phi_\Phi}) + \langle \Gamma_L \rangle (L_{\phi_\Phi} + L_{\phi_\overline{\Phi}}) + \langle \Gamma_R \rangle L_{\phi_\overline{\overline{\Phi}}} - \frac{s}{2n_\gamma} \frac{1}{2} L_{agg} (\langle \Gamma_D \rangle + \langle \Gamma_L \rangle) + \langle L_{\nu_R} - \frac{1}{7} L_{agg} \rangle \langle \Gamma_{2\leftrightarrow 2} \rangle + F_{net},$$

(97)

where $F_{net}$ represents the overall contribution to $L_{agg}$ from the fermionic components of $\Phi$ and $\overline{\Phi}$, and small terms proportional to $\epsilon$ times $L_\ell$, $L_{\nu_R}$, $L_{\bar{\ell}}$ or $L_{\bar{\nu}_R}$ have been dropped. Here, we have used the fact that the Boltzmann equations for $L_{e_R}$ and $L_{\bar{e}_R}$ are trivial, consisting of only $\Sigma_A$ terms, which all cancel after taking the sum of all Lepton abundances.

The one task that remains is to deal with the fermionic components of $\Phi$ and $\overline{\Phi}$. However, it can be shown that in the approximation that the $\Sigma_A$ interactions are rapid, the decay and inverse decay contributions to $L_{agg}$ and $L_{\nu_R}$ from these fermions are the same as those from the scalars, and thus no additional Boltzmann equations for $\psi_\Phi$, $\psi_\overline{\Phi}$, etc. are needed. We therefore require only five differential equations to describe the evolution of lepton number in the universe. These are

$$\frac{dL_{agg}}{dt} = -2\epsilon \langle \Gamma_D \rangle (Y^c_{\phi_\Phi} - Y^{eq}_{\phi_\Phi}) + \langle \Gamma_L \rangle (L_{\phi_\Phi} + L_{\phi_\overline{\Phi}}) + \langle \Gamma_R \rangle L_{\phi_\overline{\overline{\Phi}}} - 2L_{agg} \langle \langle \Gamma_D \rangle_{1D} + \langle \Gamma_L \rangle_{1D} \rangle + \frac{1}{7} L_{agg} \langle \Gamma_{2\leftrightarrow 2} \rangle + (L_{\nu_R} - \frac{1}{7} L_{agg}) \langle \Gamma_{2\leftrightarrow 2} \rangle$$

(98)

$$\frac{dL_{\nu_R}}{dt} = 2\epsilon \langle \Gamma_D \rangle (Y^c_{\phi_\Phi} - Y^{eq}_{\phi_\Phi}) + L_{\phi_\Phi} \langle \Gamma_R \rangle - 2L_{\nu_R} \langle \Gamma_R \rangle_{1D} - \langle \Gamma_D \rangle_{1D} + \frac{1}{2} L_{agg} \langle \Gamma_L \rangle_{1D} + \frac{1}{2} L_{\nu_R} \langle \Gamma_R \rangle_{1D}$$

(99)

$$\frac{dY^c_{\phi_\Phi}}{dt} = -\langle \Gamma_D \rangle (Y^c_{\phi_\Phi} - Y^{eq}_{\phi_\Phi}) + \frac{1}{2} L_{agg} \langle \Gamma_L \rangle_{1D} + \frac{1}{2} L_{\nu_R} \langle \Gamma_R \rangle_{1D}$$

(100)

$$\frac{dL_{\phi_\Phi}}{dt} = -\langle \Gamma_D \rangle L_{\phi_\Phi} + 2L_{agg} \langle \Gamma_L \rangle_{1D} + 2L_{\nu_R} \langle \Gamma_R \rangle_{1D}$$

(101)

$$\frac{dL_{\phi_\overline{\Phi}}}{dt} = -\langle \Gamma_D \rangle L_{\phi_\overline{\Phi}} + 2L_{agg} \langle \Gamma_D \rangle_{1D},$$

(102)

where $\langle \Gamma_D \rangle_{1D} = \frac{1}{i} \frac{n^{eq}_{\phi_\Phi}}{n_\gamma} \langle \Gamma_D \rangle$, $\langle \Gamma_L \rangle_{1D} = \frac{1}{i} \frac{n^{eq}_{\phi_\Phi}}{n_\gamma} \langle \Gamma_L \rangle$ and $\langle \Gamma_R \rangle_{1D} = \frac{n^{eq}_{\phi_\Phi}}{n_\gamma} \langle \Gamma_D \rangle$. Once again, small terms proportional to $\epsilon L_{agg}$ or $\epsilon L_{\nu_R}$ have been neglected. Of course we have not yet included the coupling of baryon number $B$ to $L_{agg}$, which is accomplished by introducing a differential equation for $B$ (making a total of six coupled equations) and the addition of a sphaleron interaction term which mixes $B$ and $L_{agg}$. With this addition one obtains equations (62) – (67).

Before closing this appendix we can easily solve these equations in the “drift and decay” limit, in which we assume that all the rates $\langle \Gamma_A \rangle$ are much smaller than the rate of expansion
of the universe $H$. In that limit the equations simplify greatly:

$$\frac{dL_{agg}}{dt} = -2\epsilon (\Gamma_D) (Y^c_{\phi} - Y^{eq}_{\phi})$$

(103)

$$\frac{dL_{\nu_R}}{dt} = 2\epsilon (\Gamma_D) (Y^c_{\phi} - Y^{eq}_{\phi})$$

(104)

$$\frac{dY^c_{\phi}}{dt} = -(\Gamma_D) (Y^c_{\phi} - Y^{eq}_{\phi})$$

(105)

from which we can write

$$\frac{dL_{\nu_R}}{dt} = -\frac{dL_{agg}}{dt} = -2\epsilon \frac{dY^c_{\phi}}{dt}.$$  

(106)

Since the initial values of $L_{\nu_R}$ and $L_{agg}$ vanish, the final abundances after a long time will be

$$L_{\nu_R}^{final} = -L_{agg}^{final} = 2\epsilon Y^{eq}_{\phi} (t=0)$$

(107)

where we have assumed that $Y^{final}_{\phi} = 0$ and $Y^{initial}_{\phi} = Y^{eq}_{\phi} (t=0)$ with $t = 0$ defined as the time in which $T = M$. 

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