Longitudinal Properties of High Energy Collisions

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Abstract. The rapidity distribution and longitudinal energy distribution are analysed within the thermodynamical formalism with the non-extensive Tsallis statistics. Our approach is based on a model for fireball rapidity distribution recently proposed where the fireball moves longitudinally according to a q-Gaussian function rapidity distribution. We show that the rapidity distribution of charged hadrons are correctly obtained as compared to the experimental data for rapidities up to $\sim 10.5$. The energy density as a function of the rapidity is studied for the first time. We show that most of the energy carried by hadrons are in the high rapidity region. Our results show that the energy density is a good tool for studying the longitudinal aspects of high energy collisions.

1. Introduction

There is an increasing use of power-law distributions to describe high-energy experimental data [1–5], notably for $p_T$-distributions. Among the different formulas the one that can be consistently derived from the non-extensive ideal gas [6] presents some advantages, since its parameters are related to the known physical quantities through the usual equations from Therodynamics. An additional feature is that this non-extensive thermodynamics based on Tsallis statistics [7,8] allows the encompassment of the Hagedorn’s self-consistent principle [9] giving as results a limiting temperature, $T$, which can be understood as a critical temperature for the transition between confined and deconfined regimes of the hadronic matter, a characteristic entropic index, $q$, that is a measure of the deviation from the usual Boltzmann-Gibbs statistics, and a new formula for the hadron-mass spectrum [10].

Several analyses [6, 11–13, 26, 27] of $p_T$-distributions have shown that the non-extensive thermodynamics can correctly reproduce the experimental data in a large range of $p_T$ for different collision energies and particle species [6,11–13] with free parameters $T$ and $q$ resulting to be systematically around the values $T = 68$ MeV and $q = 1.146$, in accordance with the theoretical predictions [10], as shown in Fig. 1. Thus a complete thermodynamical description of the hot and dense system formed after high energy collisions, which we call fireballs, is obtained. Applications
of this non-extensive thermodynamics to neutron-stars [14, 15], LQCD [16] and hydrodynamics have already been developed.

Recently the non-extensive approach was used to describe also rapidity distributions giving for the first time a complete description of the results of high energy in proton-proton(pp) collisions [17]. In this work we address the longitudinal distribution of the energy carried by hadrons of different masses.

2. Non-extensive Thermodynamics of fireballs

The starting point is the non-extensive entropy for an ideal quantum gas defined as [6]

$$ S_q = -g \sum_i [n_i^q \ln_q n_i + (1 - n_i)^q \ln_q (1 - n_i)] . $$

(1)

From here one can derive all thermodynamical functions, particularly the occupation number, which is given by

$$ n(\varepsilon) = \pm \{ [1 + (q - 1)\beta(\varepsilon - \mu)] \mp 1 \}^{-1} $$

(2)

where the + and − signs refer respectively to bosons and fermions distributions.

More interesting for applications in High Energy Physics is the transverse momentum distribution that can be obtained from Eq. 2 resulting [6]

$$ \frac{dN}{dp_T} = \frac{p_T^2 m_T}{T^2} \frac{dN}{dy} \bigg|_{y=0} \frac{(2 - q)(3 - 2q)}{2 - q} \frac{m_0}{2m_0T + 2T^2} \left[ 1 + (q - 1) \frac{m_0}{T} \right]^{\frac{1}{q-1}} \left[ 1 + (q - 1) \frac{m_T}{T} \right]^{\frac{q}{q-1}} $$

(3)

With the equation above it is possible to describe the $p_T$-distribution for any particle with mass $m$ and relate it directly to the central rapidity density, $\frac{dN}{dy}|_{y=0}$ in terms of $T$ and $q$.

For the description of rapidity distributions additional information is needed. Indeed the longitudinal distribution still carries some memory of the initial state which can be included in the theoretical formulations as a longitudinal expansion of the system described by a rapidity distribution of thermodynamically equilibrated fireballs [18, 19]. In Ref. [17] this distribution was

$$ \nu(y_f) = G_q(y_0, \sigma; y_f) + G_q(-y_0, \sigma; y_f), $$

(4)

where

$$ G_q(y_0, \sigma; y_f) = \frac{1}{\sqrt{2\pi} \sigma} e_q \left( -\frac{(y_f - y_0)^2}{2\sigma^2} \right) $$

(5)

and $e_q(x)$ is the q-exponential function defined as

$$ e_q(x) \equiv [1 - (q - 1)x]^{1/(q-1)}. $$

(6)

In Eq. 5 $y_0$ and $\sigma$ are respectively the peak position and the width of the q-Gaussian function used as free parameters to be adjusted in analyses of experimental data.

The double differential yield turns out to be [17]

$$ \frac{1}{N} \frac{d^2N}{dp_T dy} = \frac{1}{A} \int_{-\infty}^{\infty} \nu(y_f) \times \frac{p_T^2 m_T \cosh(y - y_f)}{(2\pi)^2} \left[ 1 + (q - 1) \frac{m_T \cosh(y - y_f)}{T} \right]^{-\frac{q}{q-1}} dy_f $$

(7)
where $N$ is the multiplicity and $A$ a normalization constant. Integrating on $p_T$ one obtains the rapitity distribution. In Fig. 1 we show the good agreement between the experimental data and theoretical calculation.

From the systematic analysis of rapidity distributions performed in Ref. [17] it turns out that the parameters $y_o$ and $\sigma$ in the q-Gaussian function show a simple pattern, as can be seen in Fig. 2. We note that the error bars are large particularly for energies above $\sqrt{s} = 1$ TeV because the available experimental data are limited to a small range of rapidity, however it is possible to observe that for energies above $\sim 500$ GeV both $y_o$ and $\sigma$ are approximately constant. While one would normally expect the peak position of the fireball rapidity distribution present a clear dependence on the beam rapidity, the fact that these parameters are approximately constant within a large range of collision energy could indicate that the expanding system detaches itself from the leading hadrons moving with rapidities close to the beam rapidity. Below $\approx 550$ GeV the scaling observed above disappears, as can be seen in Fig. 2. It is possible that at these collision energies the chemical potential is not null hence more studies on the direction of verifying the role played by the chemical potential at low energies must be carried out, as performed in [20–22].

3. Yield and Energy Distributions

In what follows we assume that $y_o$ and $\sigma$ are constant even for energies above those analysed in the present work. Notice that this assumption is considered for identified hadrons but there are evidences that, at least for lower collision energies, different hadrons present slightly different rapidity
Figure 2. Values for the parameters $\sigma$ and $y_0$ in Eq. 4 as a function of the collision energy. Figure taken from Ref. [17].
distributions [20]. However we will consider that for higher energies those differences can be neglected.

Within the hypothesis given above, the energy density carried by hadrons with mass $m$ is given by

$$\frac{d^2E}{dp_T dy}(m) = \frac{gV}{(2\pi)^2} \rho(m) \varepsilon(p_T, y) \frac{d^2N}{dp_T dy}, \quad (8)$$

where

$$\varepsilon(p_T, y) = m_T \cosh(y), \quad (9)$$

and the total energy carried by baryons is

$$E = \int_0^\infty \rho(m) E(m) dm \quad (10)$$

where $\rho(m)$ is the baryon mass spectrum. The function $\rho(m)$ was determined by the non-extensive self-consistent theory [10] and is given by

$$\rho(m) = \frac{\gamma}{m^{5/2}} \left[ 1 + \frac{(q - 1)m}{T} \right]^{\frac{q-1}{q}} \quad (11)$$

In Ref. [29] it was shown that this formula reproduces correctly the observed hadronic mass spectrum up to a mass of approximately 2 GeV.

For hot fireballs at temperatures close to the limiting temperature the calculation of the total energy with Eq. 10 is rather cumbersome because of the singularity at the critical point. An alternative way is to substitute the integration in that equation by a summation over the known hadrons, as

$$E = \sum_{i=1}^{m_h} E(m_i). \quad (12)$$

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**Figure 3.** Energy density for different hadrons

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| Hadron | Mass (GeV) |
|--------|------------|
| $\Omega$ | 1.672 |
| $\pi$   | 0.140 |
| $p$     | 0.938 |
Since according to our supposition in Eqs. 10 and 12 the parameters $q$, $T$, $y_o$ and $\sigma$ do not depend on the total energy, only the multiplicity $N$ can be a function of the collision energy. Therefore one can write

$$N(E) = KE,$$

where

$$K = \left[ \frac{1}{A} \sum_{i=1}^{n} \int_{-\infty}^{\infty} \int_{0}^{\infty} \varepsilon(p_T, y) \frac{d^2N}{dp_T dy} (m_i) dp_T dy \right]^{-1}. \quad (14)$$

In Ref. [17] considering charged hadrons with masses up to 5 GeV it was found that $K = 0.009\text{GeV}^{-1}$, which falls a factor 2 above the value $K = 0.004\text{GeV}^{-1}$ obtained by fitting the multiplicity for different energies. The difference was attributed to the fact that massive particles that were not taken into account in the calculation of $K$ with Eq. 12 give a significant contribution to the total energy. It will be interesting, therefore, to analyse in more details the individual contributions of different hadrons.

Eq. 8 allows us to calculate the energy carried by any particle with mass $m$ or even its energy density as a function of the rapidity, which is obtained by integrating that equation on $p_T$. In Fig. 3 the show the energy density calculated for a few hadrons where we observe that for all particles most of the carried energy is concentrated in the high rapidity region ($y \geq 5$), while the minimum is located at the central region, $|y| \sim 0$. Also, for any fixed rapidity the energy increases as the particle mass increases. These results are straightforward consequences of the fact that the particle energy is given by

$$\varepsilon = m_T \cosh y \quad (15)$$

where $m_T = \sqrt{p_T^2 + m^2}$.

We also observe a peak, more pronounced for more massive particles, at intermediate rapidities ($y \sim 10$). This peak results from a combination of the decreasing $q$-exponential function from the thermodynamical factor (see Eq. 7) and the increasing hyperbolic cosine from the particle energy (Eq. 15). It is important to notice here that the $q$-exponential factor depends on $y - y_f$, therefore even high rapidity particles can be produced with high probability for sufficiently high fireball rapidity. For $|y| > 10.5$ the energy density increases very fast showing a divergence as the particle rapidity increases. Of course this result evidences limitations in the model used here, namely, the fireball rapidity distribution cannot follow a simple $q$-Gaussian, as given by Eq. 4 function for rapidities close to the beam rapidity, but must decrease faster than the hyperbolic cosine of the rapidity.

The study of the energy density allow us to observe that this quantity is strongly dependent on the particle mass even for relatively low rapidities therefore it represents a good tool for investigation of longitudinal properties of identified particles. Measurements of energy density for hadrons would help us to test different models for rapidity distributions since it is more sensitive to small variations of yield density.

4. Conclusions

In this work we present a comprehensive study of the longitudinal properties of particles generated in high energy proton-proton collisions. The analysis is performed within the non-extensive
thermodynamics approach using Tsallis statistics. The rapidity distribution is obtained with a model proposed recently, where it is supposed that the system formed after the high energy collision constitutes of two fireballs moving in the beam direction with opposite rapidities $y_o$. Each of these fireballs is formed by fireballs with rapidities distributed in the main fireball frame according to a $q$-gaussian function with width $\sigma$. It is assumed, based on a previous systematic study, that $y_o$ and $\sigma$ are independent of the collision energy.

We analyze the particle energy density as a function of rapidity. The analysis evidences that the simple $q$-Gaussian distribution of the fireball rapidity, $y_f$, is not consistent for $y_f$ close to the beam rapidity, although it leads to a good agreement of the particle rapidity distribution in a wide range of rapidities. We argue that for very high rapidities the fireball rapidity distribution must decrease faster than the hyperbolic cosine function.

The results also show that most of the energy is carried by particles with rapidity around $y = 8$. In addition, it is discussed that the energy density is a good tool for investigating longitudinal properties of high energy collisions since it is very sensitive to the particle mass and rapidity.

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