Gaussian expansion approach to Coulomb breakup

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An accurate treatment of Coulomb breakup reactions is presented by using both the Gaussian expansion method and the method of continuum discretized coupled channels. As $L^2$-type basis functions for describing bound- and continuum-states of a projectile, we take complex-range Gaussian functions, which form in good approximation a complete set in a large configuration space being important for Coulomb-breakup processes. Accuracy of the method is tested quantitatively for $^8$B+$^{58}$Ni scattering at 25.8 MeV.

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Determination of the neutrino oscillation parameters is one of the central issues in the neutrino physics. The astrophysical factor $S_{17}$, which is essentially equal to the cross section of the $p$-capture reaction $^7$Be($p, \gamma$)$^8$B, plays an important role in the parameter-search procedure, since the prediction value for the flux of atmospheric neutrinos is about 5% in errors. Due to difficulties of direct measurements of $^7$Be($p, \gamma$)$^8$B at very low energies, alternative indirect measurements were proposed; $^8$B Coulomb breakup [3, 4, 5, 6, 7] is a typical example of them.

The method of continuum discretized coupled channels [8, 9] (CDCC), which was shown to describe various projectile-breakup processes [10, 11, 12, 13, 14, 15, 16, 17, 18], has also been applied to $^8$B Coulomb breakup with high degrees of success [19, 20, 21, 22]. In the all analyses above, CDCC describes the reaction system with a three-body model, i.e., $p^+^7$Be+$^8$Ni as shown in Fig. 1.

From $^8$B Coulomb breakup measured at intermediate energies, $S_{17}$ was determined using the first-order perturbation theory. It was shown in Refs. [21, 22] that $S_{17}$ can alternatively be determined by CDCC analysis, combined with the asymptotic normalization coefficient method [23], of $^8$B breakup even at low energies, such as the Notre Dame (ND) experiment [24]. It was found, however, that the ND data contain an irrelevant component to $S_{17}$, corresponding to the $1/2^-$ $^7$Be excited state in the final channel, which of order 10% presents in the $^8$B ground state [25]; effects of this component on $S_{17}$ can be expected around 10%. Therefore, the evaluated value of $S_{17}$ in Refs. [21, 22] should be assumed as the upper limit of $S_{17}$. Even though the irrelevant component can be experimentally removed by detecting the emitted $^7$Be in the ground state, the $1/2^-$ $^7$Be excited state may play a dynamical role during the breakup process of $^8$B. Thus, description of $^8$B breakup beyond the three-body model, i.e., $p^+^7$He+$^4$He+$^{58}$Ni four-body model, is highly expected in order to determine $S_{17}$ with very high accuracy.

Very recently, a new treatment of breakup continuum in CDCC was proposed [26], which uses pseudo-state (PS) wave functions [27, 28] obtained by diagonalizing internal Hamiltonian of the projectile with Gaussian basis functions. For nuclear breakup processes, CDCC with the PS method (PS-CDCC) was found to perfectly reproduce the “exact” breakup $S$-matrix elements $S(k)$ calculated by the standard CDCC, i.e., with the Average (Av) method [8, 29] for discretization of breakup continuum. The most important feature of PS-CDCC is that it generates $S(k)$ as a smooth function of $k$ in its entire range, without assuming any form a priori for $k$-distributions, in contrast to CDCC with the Av method (Av-CDCC).

Additionally, as discussed in Refs. [26, 29], PS-CDCC with Gaussian basis functions makes it possible the four-body CDCC analysis of projectile breakup, a preliminary result of which for $^6$He elastic scattering, including effects of breakup channels of $^6$He, has already been obtained [30, 31]. Before making four-body PS-CDCC analysis of $^8$B Coulomb breakup, however, one must see the applicability of PS-CDCC to breakup processes due to long-ranged Coulomb coupling potentials. This is quite nontrivial because the $r$-space of the projectile needed to describe Coulomb breakup is much larger than that in the case of nuclear breakup; for example, the effective space is $r \lesssim 20$ fm for nuclear breakup of $^6$Li [26], but $r \lesssim 100$ fm for Coulomb breakup of $^8$B [21, 22].

The aim of this brief report is, as the first step towards the four-body CDCC analysis of $^8$B Coulomb breakup, to show that three-body PS-CDCC based on the Gaussian basis functions can well describe the corresponding result of three-body Av-CDCC for $^8$B breakup. As for the test case we take $^8$B breakup from $^{58}$Ni at 25.8 MeV.
Below we briefly recapitulate the formulation of three-body CDCC.  We consider the three-body system of Fig. 1.  The model Hamiltonian of the system is

\[ H = K_r + V_{he}(r) + K_R + U_{bA}(r_{bA}) + U_{cA}(r_{cA}) \]

Coordinates are defined in Fig. 1.  Operators \( K_r \) and \( K_R \) are kinetic energies associated with \( r \) and \( R \), respectively, and \( V_{he}(r) \) is the interaction between \( b \) and \( c \).  The interaction \( U_{bA} \) (\( U_{cA} \)) between \( b \) (\( c \)) and \( A \) is taken to be the optical potential for \( b+A \) (\( c+A \)) scattering; Coulomb breakup is induced by the Coulomb components of the potentials.  For simplicity, in this study we neglect the intrinsic spins of individual constituents of the system.

In CDCC, the states of the projectile are classified by the linear and the angular momenta, \( k \) and \( \ell \), of relative motion between \( b \) and \( c \), which are truncated by \( k \leq k_{\text{max}} \) and \( \ell \leq \ell_{\text{max}} \).  The truncation is the most basic assumption in CDCC, and it is confirmed that calculated \( S \)-matrix elements converge for sufficiently large \( k_{\text{max}} \) and \( \ell_{\text{max}} \).  The converged CDCC solution is the unperturbed solution of the distorted Faddeev equations, and corrections to the solution are negligible within the region of space in which the reaction takes place.  The \( k \)-continuum of the \( b+c \) system are then discretized into a finite number of states, each of which corresponds to a “discretized-continuum state” with a certain positive eigenenergy labeled by \( i \).  The resulting orthonormalized wave functions, \( \{ \hat{\Phi}_{it}(r) \} \), are assumed to form an approximate complete set in the \( r \)- and \( k \)-space being important for the breakup reaction.

The three-body wave function \( \Psi \) of the system is expanded by \( \{ \hat{\Phi}_{it}(r) \} \) and then inserted into the approximate three-body Schrödinger equation \( (H - E)\Psi = 0 \).  One can then obtain a set of coupled differential equations, called CDCC equations, that provide the discrete \( S \)-matrix element, \( \hat{S}_\gamma \), for the transition from the initial channel to a discretized-continuum one \( \gamma = (i,\ell) \).  In order to calculate a breakup cross section of the projectile with a certain range of breakup energies, or a coincidence-cross-section, a smoothing procedure, which constructs continuous \( S_{\ell}(k) \) from the discrete \( \hat{S}_\gamma \), is necessary.  Actually, this is the case in the analysis of \( {^{8}\text{B}} \) Coulomb-breakup experiments.

Discretization of the breakup continuum in the PS method is done by diagonalizing the internal Hamiltonian \( H_{\text{be}} = K_r + V_{he}(r) \) in a space spanned by a finite number of \( L^2 \)-type basis-functions, for which we here take the following pairs of functions:

\[ \phi_{i\ell}^{a}(r) = r^{\ell} \exp\left[-(r/a_j)^2\right] \cos\left[b(r/a_j)^2\right], \]
\[ \phi_{i\ell}^{a}(r) = r^{\ell} \exp\left[-(r/a_j)^2\right] \sin\left[b(r/a_j)^2\right], \]

where \( \{a_j\} \) are assumed to increase in a geometric progression and \( b = \pi/2 \).  We refer to the basis as the complex-range Gaussian basis, since the basis functions can be expressed by Gaussian functions with a complex-range parameter, \( r^{\ell} \exp\left[-(1+ib)(r/a_j)^2\right] \), and its complex conjugate.  The complex-range Gaussian basis functions are oscillating with \( r \), so they can simulate the oscillating pattern of the continuous breakup-state wave-functions as shown in Ref. 23, which is very important for the description of Coulomb breakup by PS-CDCC.  An accurate transformation from \( \hat{S}_\gamma \) to \( S_{\ell}(k) \):

\[ S_{\ell}(k) \approx \sum_i f_{it}(k) \hat{S}_\gamma, \]

with \( f_{it}(k) = \langle \hat{\Phi}_{it}(r) | \hat{\Phi}_{it}(r) \rangle \), is possible when the basis functions form an approximate complete set in the finite configuration space being important for the breakup process.

In the Av method, on the other hand, the \( k \)-continuum \( [0, k_{\text{max}}] \), for each \( \ell \), is divided into a finite number of bins, each with a width \( \Delta_{it} = k_i - k_{i-1} \), and the continuum breakup-states in the \( i \) th bin are averaged with a weight \( w_{it}(k) \).  In this study we take \( w_{it}(k) = 1 \) for simplicity.  The resulting orthonormal state is then described as

\[ \hat{\Phi}_{it}(r) = \frac{1}{\sqrt{\Delta_{it}}} \int_{k_{i-1}}^{k_i} \Phi_{it}(r) dk \]

Inserting Eq. 9 into Eq. 1 leads to \( S_{\ell}^{\text{Av}}(k) = \hat{S}_\gamma \sqrt{\Delta_{it}} \) for \( k_{i-1} < k \leq k_i \), as shown in Ref. 19.

It should be noted that \( S_{\ell}^{\text{Av}}(k) \) is smooth only within each bin, while \( S_{\ell}(k) \) with the PS method, i.e., \( S_{\ell}^{\text{PS}}(k) \), is a smooth function of \( k \) in its entire range.  Details of the discretization and smoothing procedures both for the Av and PS methods can be found in Ref. 26.

In order to see the applicability of PS-CDCC for Coulomb breakup, we calculated the breakup cross section of \( {^{8}\text{B}} + {^{58}\text{Ni}} \) scattering at 25.8 MeV.  In this process both nuclear and Coulomb interactions, with interference, play important roles.  Moreover, first order perturbation theory cannot reproduce the experimental data 24 at all, since higher-order processes cannot be neglected at such a low incident energy.  Thus, it is a good test for PS-CDCC to compare the results with those calculated by Av-CDCC that reproduce the experimental data very well.

For both the Av and PS methods, we took the \( {^{8}\text{B}} \) single-particle wave-function of Esbensen and Bertsch 35 and the same potential as for the ground state was used to generate \( \Phi_{it}(r) \).  In its entire range, details of the discretization and smoothing procedures both for the Av and PS methods can be found in Ref. 26.

In the Av method, the modelspace with \( k_{\text{max}} = 0.66 \text{ fm}^{-1} \) and \( \Delta_{it} = 0.66/16 \text{ fm}^{-1} \) for \( \ell = 1 \) gives convergence of the resulting total breakup cross section.  The maximum internal coordinate \( r_{\text{max}} \) was taken to be 100 fm.  In order to obtain the correct asymptotic form of the Coulombcoupling potentials, we made the following approximation for
where \( j = \text{s-state} \) or \( \text{p-state} \). The horizontal dotted line represents the cut-off momentum \( k_{\text{max}} \) taken to be 0.66 fm\(^{-1}\).

The monopole components:

\[
\psi_{\ell, \ell', L, \lambda, 0}^{c0}(R) = \int_{0}^{r_{\text{max}}} \Phi_{\ell'}(r) \frac{Z_{j}Z_{58\text{Ni}}e^{2}}{r} \Phi_{\ell}(r) r^{2} dr \approx \frac{Z_{j}Z_{58\text{Ni}}e^{2}}{r} \delta_{\ell, \ell'},
\]

where \( j = \text{p or } 7\text{Be} \). In the practical calculation we took account for Coulomb radii for \( p-58\text{Ni} \) and \( 7\text{Be-58Ni} \) and Eq. \( (4) \) has a slightly complicated form \( [21] \). It should be noted that Eq. \( (4) \) is exact when \( r_{\text{max}} \to \infty \); the numerical results shown below were found to converge at \( r_{\text{max}} = 100 \) fm. The result thus obtained, i.e., with Av-CDCC, is assumed to be the "exact" solution.

In the PS method, we used the complex-range Gaussian basis with \( (a_{1} = 1.0, a_{n} = 35.0, 2n = 60, b = \pi/2) \) that gives good convergence. The number of channels included in the CDCC calculation, i.e., \( k \leq k_{\text{max}} = 0.66 \) fm\(^{-1}\), was 18 for both the s- and p-states. The resulting wave functions with positive eigenenergies turned out to oscillate up to about 100 fm. In order to obtain orthonormality of the \( \Phi_{\ell}(r) \), we put \( r_{\text{max}} = 130 \) fm in the calculation of coupling potentials. We show the level sequences of the resulting discrete eigenstates in Fig. 2. One sees each interval of the levels is almost even, which is just the same as in Ref. \([26]\).

We show in Fig. 3 the calculated angular distribution of \( ^{8}\text{B} \) total breakup cross section for \( ^{58}\text{Ni}(^{8}\text{B}, ^{8}\text{B}) \) at 25.8 MeV. The solid and dashed lines correspond to the Av and PS methods, respectively. One sees both results coincide with very high accuracy, which implies that a forthcoming four-body PS-CDCC analysis of \( ^{8}\text{B} \) Coulomb breakup has enough accuracy to determine a reliable value of \( S_{17} \), i.e., within 5% of errors.

In order to see the validity of PS-CDCC for the \( ^{8}\text{B} \) Coulomb breakup more strictly, below we compare \( S_{\ell}^{\text{PS}}(k) \) with \( S_{\ell}^{\text{Av}}(k) \). In the calculation of the latter, i.e., the "exact" \( S_{\ell}(k) \), we put \( \Delta_{\ell} \) for the p-state to be 0.66/32 fm\(^{-1}\). This makes \( k \)-dependence of \( S_{\ell}^{\text{Av}}(k) \) clearer, since \( S_{\ell}^{\text{Av}}(k) \) is constant in \( k \)-region corresponding to each bin with the width of \( \Delta_{\ell} \), as shown in Ref. \([26]\). It should be noted that this refinement of the modelspace for the Av method makes no changes in the physical quantities such as the elastic and total

![FIG. 2: Discretized momenta for \(^{8}\text{B}\); the left (right) side corresponds to the s-state (p-state). The horizontal dotted line represents the cut-off momentum \( k_{\text{max}} \) taken to be 0.66 fm\(^{-1}\).](image)

![FIG. 3: Angular distribution of the total breakup cross section for \(^{58}\text{Ni}(^{8}\text{B}, ^{8}\text{B}) \) at 25.8 MeV. The solid and dashed lines represent the results with the Av and PS methods, respectively.](image)

![FIG. 4: The squared moduli of breakup \( S \)-matrix elements, as a function of \( k \), at \( J = 150 \) for \(^{8}\text{B}+^{58}\text{Ni} \) scattering at 25.8 MeV. The panel (a), (b), (c) and (d) correspond to \( (\ell, L, L_{0}) = (1, 150, 150), (1, 149, 151), (1, 151, 151) \) and \( (0, 150, 151) \), respectively. In each panel, the solid line represents the result of PS-CDCC, while the step line is the result of Av-CDCC assumed as the "exact" \( S \)-matrix elements.](image)
breakup cross sections.

Figure 4 shows the result of the comparison of $|S_1(k)|^2$ at $J = 150$, which corresponds to the scattering angle of $10^\circ$ assuming the classical path. It was found that CDCC calculation with only Coulomb coupling potentials gives a peak at $10^\circ$ in the total breakup cross section. Thus, it can be assumed that Fig. 4 corresponds to the most-Coulomb-like breakup process; in any case, features of the result were found to be almost independent of $J$. In each panel of Fig. 4 the result with $(\ell, L, L_0)$, where $L$ ($L_0$) is the orbital angular momentum between $^{8}\text{B}$ and $^{56}\text{Ni}$ in the final (initial) channel, is shown. The panel (a), (b), (c) and (d) correspond to $(\ell, L, L_0) = (1, 150, 150), (1, 149, 151), (1, 151, 151)$ and $(0, 150, 151)$, respectively; all other components are negligibly small and not shown in the figure. One sees that the result of PS-CDCC (solid line) very well reproduces the "exact" solution (step line) for all $k$ being significant for the $^{8}\text{B}"\text{Coulomb" breakup."

In summary, PS-CDCC proposed in Ref. [26] is shown to describe Coulomb breakup processes very well. Due to the long-ranged Coulomb coupling-potentials, the modelspace required for CDCC is very large. Particularly, one must prepare the internal wave functions of the projectile, both in bound and continuum states, for a wide range of the internal coordinate, say, $0$–$100$ fm, which is in general difficult for PS methods. We found that this can easily be achieved by using the complex-range Gaussian basis, in the case of two-body projectile. According to Ref. [29], the basis is also applicable to the reaction processes with three- and four-body projectiles, since energies and wave functions of the pseudo-states of the projectiles are given easily. Moreover, all coupled-channel potentials in four- and five-body PS-CDCC can analytically be given by expanding each nuclear-optical-potential concerned in terms of Gaussian functions. Then, PS-CDCC based on the complex-range Gaussian basis functions is expected to become an effective method of practical use even for four-body Coulomb breakup of $^8\text{B}$, in which $^8\text{B}$ is assumed to be a bound state of the $p^4\text{He}^4\text{He}$ system. Such a four-body analysis of $^8\text{B}$ dissociation is desirable in order to determine $S_{17}$ with high accuracy and then approach to the solar neutrino problem from the nuclear physics side.

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