Jet-quenching parameter and entropic force for the power-law Maxwell field solution

B. Khanpour and J. Sadeghi

Department of Physics, Faculty of Basic Sciences, University of Mazandaran, P. O. Box 47416-95447, Babolsar, IRAN

Abstract

In this paper, we present general method for the solving the field equation of Einstein - Power law Maxwell-Scalar gravity in $n + 1$ dimension with Liouvil potential. The corresponding solution lead us to achieve some thermodynamics quantity of black hole. On the other hand, in QCD the $Q\bar{Q}$ bound state represents a balance between repulsive kinetic and attractive potential energy. Also, in a hot quark-gluon plasma, the interaction potential experiences medium effects. In that case, the color screening modifies the attractive binding force between the quarks, while the increase of entropy with $Q\bar{Q}$ separation gives rise to a growing repulsion. The jet quenching and entropic force parameters play important role in binding and dissociation length of quark-antiquark. For this reason, we take advantage from $AdS/CFT$ correspondence and obtain the jet quenching and entropic force for the Einstein - Power law Maxwell-Scalar gravity solution. We note that, the effects of $p$ from power law Maxwell field on the jet quenching and entropic force parameters are shown by some figures.

Keywords: $AdS/CFT$ correspondence; Power-law Maxwell field solution; Jet-quenching parameter; Entropic force.

*Electronic address: b.khanpour@stu.umz.ac.ir
†Electronic address: pouriya@ipm.ir
I. INTRODUCTION

In this paper we use AdS/CFT duality and obtain some important parameters in QCD and hydrodynamics. As we know the corresponding duality for the first time is introduced by Maldacena[1]. According to the Maldacena conjecture[1], the strongly coupled gauge theories with conformal symmetries (CFT) in $n$ dimensional are corresponding to theories of gravity in $n+1$ dimensional in anti-de Sitter (AdS) space-time which is known as AdS/CFT correspondence in zero temperature. But, in finite temperature the strongly coupled gauge theories correspond to gravitational theory in AdS black hole [2–8]. So, in that case the gravitational theory of black hole is a thermal system and also it has some thermal properties [9]. So, black hole play important role to understand AdS/CFT correspondence at finite temperature. In this paper, we use the AdS/CFT correspondence in power-law Maxwell field system and calculate jet quenching parameter and entropic force. So, for this reason first we are going to explain generally power-law Maxwell field solution. As we know the present epoch, the universe has a positive acceleration which is described by the standard Friedmann model [10–16]. We note that the Einstein’s gravity with dilaton scalar fields and some low-energy limit of string theory play important role in several aspect of cosmology. On the other words from low-energy limit of string theory, one can reach to Einstein’s gravity along with a dilaton scalar field [17–20]. The most important theory in such direction will be dilaton gravity including power-law Maxwell field term [21]. If we look at to the corresponding theory we will see in the case of scalar field for particular power of the massless Klein-Gordon Lagrangian in arbitrary dimensions [22] and also for electrodynamic Lagrangian in higher dimensions we have conformal invariance. Here we mention that Lagrangian $(F_{\mu\nu}F^{\mu\nu})^{n+1}$ is conformally invariant $(n+1)$-dimensions in Maxwell Lagrangian [23]. But, the Maxwell Lagrangian $F_{\mu\nu}F^{\mu\nu}$ is conformally invariant only in four dimensions. This power-law Maxwell field term in action play important role for the breaking of conformal invariance, so this will be motivation for the employing such theory for the calculating some parameter in QCD. On the other hand, we take advantage from such action and metric background and discuss two phenomena in particle physics as a jet quenching and entropic force. First we are going to give some review to the jet quenching and its properties in QCD. As we know deconfined quark-gluon plasma (QGP’s) are created in ultra relativistic heavy-ion collision BNL, Relativistic Heavy Ion Collider(RHIC) and the CERN Large Hadron Collider(LHC) [24–26]. Two of the most striking
properties of $QGP$'s are the perfect (minimally viscous) fluidity as quantified by their shear viscosity to entropy density $\frac{\eta}{s} \sim 0.1 - 0.2$ \cite{27,31} and the strong quenching of high energy jets quantified by the normalized jet transport coefficient $\hat{q}^T$ \cite{32,34}.

Also from RHIC and LHC experiments shown that the quark gluon plasma ($QGP$) are strongly coupled. Thus, one can not use the perturbation theory in such regime. Fortunately, the AdS/CFT correspondence provides a the suitable method for calculating the quantum chromodynamics ($QCD$) parameter, one of these parameters is jet quenching. In particle physics when heavy ions collide to each other, they are fragmented and produced quark-antiquark will end up back-to-back jets. The quarks have to travel a long way into the $QGP$, in that case they will lose energy in this process. This is jet signal which is received by the detector, such phenomenon is called jet quenching. These produced quark-anti quark as a form of jet give us information about the interaction of the fluid and corresponding particle \cite{36,43}. Also, second step we discuss on the entropic force in quarkonium system in $QCD$. As we know the concept of entropic forces will be result of many-body phenomena and also will be interesting in several branch of physics. Here we note that the effect of such force arises from thermodynamic drive of a many-body system to increase its entropy rather than microscopic system. In that case the more evidence for the describing entropic force coming from Erik Verlinde paper \cite{35}. For the first time he proposed that the gravitational force will be form of entropic force with some isotropic and homogenous background. He extended such theory for the Einstein equation and also obtained all equation in cosmology. On the other hand, we have gauge theory in section of high energy physics such as $QCD$. This theory included abelian and non abelian gauge fields which is bridge between all of interaction of nature. Also Erik Verlinde take entropic force and extend such topic to abelian, non abelian gauge and different matter fields.

The ref \cite{44,47}, discussed quarkonium binding and entropic force. He studied quarkonium binding in terms of potential and shown that the relevant potential is the free energy different $F(T, r)$ between a medium with and one without $Q\bar{Q}$ pair. Also he proved that by increasing of the internal energy $U(T, r)$ with increasing separation distance $r$ and completely correspond to repulsive entropic force. So here we understand that the entropic force also play important role in binding energy in $Q\bar{Q}$ \cite{44,48}. These information give us motivation to take power-law Maxwell field solution and investigate jet quanching parameter and entropic force. Such investigation may
be interesting for the description some phenomena in QCD and also cosmology. The structure of the paper as follows; In section II, we present general method for the solving the field equation of Einstein - Power law Maxwell-Scalar gravity in $n + 1$ dimension. In that case, we consider Liouville potential and obtain black hole solution. Also, we arrange the parameters of the black hole in the corresponding theory. In section III, we take Nambu-Goto action with $\tau, \sigma$ coordinates for the parametrization the world-sheet. So, in order to obtain the jet quenching parameter, we employ holographic description which is related to the Wilson loop joining two light-like lines. Also, we take the corresponding metric background in Nambu-Goto action and use Wilson loop one can achieve the jet quenching parameter. In section IV, we study the entropic force quark-antiquark in distance $L$. In this section, for obtaining the entropic force, one needs to calculate entropy ($S$), temperature ($T$) and quark-antiquark distance ($L$) through AdS/CFT correspondence. Finally in last section we have some conclusion and suggestion.

II. A REVIEW OF POWER-LAW MAXWELL FIELD SOLUTION

Now we are going to investigate power-law Maxwell field action. So, the action for Einstein gravity coupled to a dilaton field with power-law Maxwell field will be following form,

$$S = -\frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \left[ \bar{R} - \frac{4}{n-1} (\nabla \phi)^2 - V(\phi) + \left(-e^{-\frac{4\alpha\phi}{n-1}} F^p\right) \right],$$

(1)

where $\phi$ is dilaton field and $V(\phi)$ is the potential for the dilaton field. $p$ is degree of nonlinearity of Maxwell field and $\alpha$ is strength of coupling of the electromagnetic and scalar field. In order to have solution for the corresponding action, one can consider the dilaton potential with three Liouville-type as [21],

$$V(\phi) = 2\Lambda_1 e^{2\zeta_1\phi} + 2\Lambda_2 e^{2\zeta_2\phi} + 2\Lambda e^{2\zeta_3\phi},$$

(2)

so black hole form solution will be following [21],

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2R(r)^2h_{ij}dx^i dx^j,$$

(3)
\[ f(r) = \frac{k(n-2)(1+\alpha^2)r^{2\gamma}}{(1-\alpha^2)(\alpha^2+n-2)b^{2\gamma}} - \frac{m}{r^{(n-1)(1-\gamma)-1}} - \frac{2\Lambda b^{2\gamma}(1+\alpha^2)r^{2(1-\gamma)}}{(n-1)(n-\alpha^2)} + \frac{2^p p(1+\alpha^2)^2(2p-1)^2b}{\Pi(n+\alpha^2-2p)r^{2\left(\frac{(n-\alpha^2)p-1}{2p-1}\right)}}. \]  

(4)

where \( k \) can be \(-1, 0, 1 \) and \( b \) is a positive constant and also we have following values for \( \gamma, \Pi, \zeta_1, \zeta_2, \zeta_3, \Lambda_1 \) and \( \Lambda_2 \) [21]

\[
\begin{align*}
\gamma &= \frac{\alpha^2}{\alpha^2+1}, \quad \Pi = \alpha^2 + (n-1-\alpha^2)p, \quad \zeta_1 = \frac{2}{(n-1)\alpha}, \\
\zeta_2 &= \frac{2p(n-1+\alpha^2)}{(n-1)(2p-1)\alpha}, \quad \zeta_3 = \frac{2\alpha}{n-1}, \\
\Lambda_1 &= \frac{k(n-1)(n-2)\alpha^2}{2b^2(\alpha^2-1)}, \quad \Lambda_2 = \frac{2^{p-1}(2p-1)(p-1)\alpha^2q^{2p}}{\Pi b^{2p-1}2^{p-1}}.
\end{align*}
\]  

(5)

By using the Einstein equation and field equation, one can arrange \( R(r) \) and \( \phi(r) \) as [21],

\[ R(r) = e^{\frac{2\alpha \phi(r)}{n-1}r}, \]

(6)

and

\[ \phi(r) = \frac{(n-1)\alpha}{2(\alpha^2+1)} \ln\left(\frac{b}{r}\right). \]

(7)

As we know the holography as a gauge/gravity duality is a powerful to study the QCD and hadron physics. On the other hand the dynamics of a moving quark and the motion of a quark-antiquark pair in a strongly coupled plasma in the context of gauge/gravity also be important for the particle physics phenomena. Also, in dual theory, the black hole object play important role for the obtaining some parameter in QCD. Because black hole is thermal object and can be source of some temperature and heat. So, we are going to obtain thermodynamics properties, such as Hawking temperature.

First of all we use the equation \( f(r_+) = 0 \), and obtain \( m \), which is given by [21, 49],

\[
\begin{align*}
m &= \frac{k(n-2)b^{-2\gamma}r_+^{\frac{\alpha^2-n-2}{\alpha^2+1}}}{(2\gamma-1)(\gamma-1)(\alpha^2+n-2)} - \frac{2\Lambda b^{2\gamma}r_+^{\frac{n-\alpha^2}{\alpha^2+1}}}{(n-1)(\gamma-1)^2(n-\alpha^2)} + \frac{2^p p(2p-1)^2b^{-\frac{2(n-2)p\gamma}{2p-1}-\frac{\alpha^2-2p+n}{(2p-1)(\alpha^2+1)}}}{\Pi(\gamma-1)^2(n+\alpha^2-2p)}. \end{align*}
\]  

(8)
In order to determine Hawking temperature, we must calculate the $T = \frac{f'(r_+)}{4\pi}$, so one can obtain following equation,

$$T = \frac{\alpha^2 + 1}{4\pi} (A_1 r_+^{2\gamma - 1} - A_2 r_+^{1 - 2\gamma} - A_3 r_+^{-\eta}),$$

(9)

where

$$A_1 = \frac{k(n - 2)}{b^{2\gamma}(1 - \alpha^2)}, \quad A_2 = \frac{2\Lambda b^{2\gamma}}{n - 1}, \quad A_3 = \frac{2^{\mu} p (2p - 1) b^{-\frac{2(n - 2)\mu}{2p - 1}}}{\Pi q^{-2\mu}},$$

$$\eta = \frac{2p(n - 2)(1 - \gamma) + 1}{2p - 1}, \quad \Lambda = \frac{-[(n - 1)n]}{2l^2},$$

(10)

where here $\Lambda$ is the cosmological constant [35] and we assume the $l = 1$. In the holography point of view the cosmological constant play as a pressure.

The importance of jet quenching and entropic force in $QCD$ lead us to obtain such parameters with use of gauge/gravity duality tools. So we take above information and black holes with power-law Maxwell field and obtain the jet quenching parameter and entropic force.

### III. THE EFFECT OF POWER-LAW MAXWELL FIELD ON JET QUENCHING PARAMETER IN $QCD$

As we know the $AdS/CFT$ correspondence help us to obtain some important parameters in $QCD$. For example in such context we calculate the jet quenching parameter, with use of the some metric background. So, in this section we analyze the behavior of the jet quenching parameter for the power-law metric background (3). Therefore, in order to calculate the jet quenching parameter, we generally consider the light-cone metric. We will take the Nambu-Goto action with $\tau, \sigma$ coordinates for the parametrization the world-sheet. So, in the holographic description, the jet quenching parameter is related to the Wilson loop joining two light-like lines with following equation,

$$< W^A(C) > = exp\left(-\frac{1}{4\sqrt{2}}qL^-L^2\right).$$

(11)

where $W^A(C)$ is adjoint Wilson loop and $C$ is a null-like rectangular Wilson loop formed a dipole with heavy $Q \bar{Q}$ pair. The quark and antiquark are separated by a small length $L$ and travel along the $L^-$ direction. Also, by using relations

$$< W^F(C) >^2 \lesssim < W^A(C) >,$$

(12)
\[
< \mathcal{W}^F(\mathcal{C}) > = e^{-S_I} \tag{13}
\]

one can obtained jet quenching parameter with following formula,
\[
\hat{q} = 8\sqrt{2} \frac{S_I}{L - L'}, \tag{14}
\]

where \(< \mathcal{W}^F(\mathcal{C}) >\) is Wilson loop in the fundamental representation and \(S_I = S - S_0\). Here \(S\) is the total energy of the quark and anti-quark pair and \(S_0\) is the self-energy of the isolated quark and anti-quark. In that case the \(S_I\) is the regularized string world-sheet action.

We are going to start the following metric background and calculate the jet quenching parameter,
\[
ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 R(r)^2 \left[ h_{11} dx^1 dx^1 + h_{22} dx^2 dx^2 + ... \right], \tag{15}
\]

and apply following light-cone coordinates,
\[
x^\pm = \frac{t \pm \sqrt{h_{11}} x^1}{\sqrt{2}}. \tag{16}
\]

so, we have
\[
ds^2 = \left( \frac{r^2 R(r)^2 - f(r)}{2} \right) \left[ (dx^+)^2 + (dx^-)^2 \right] + \left( r^2 R(r)^2 + f(r) \right) dx^+ dx^- + \frac{dr^2}{f(r)}
\]
\[
+ r^2 R(r)^2 \left[ h_{22} (dx^2)^2 + h_{33} (dx^3)^2 + ... \right]. \tag{17}
\]

We pick out the static gauge as \(\tau = x^-\) and \(h_{22} x^2 = \sigma = y\) with \((0 < x^- < L^-)\) and \((-\frac{L}{2} \leq y \leq \frac{L}{2})\).

We consider that the quark and anti-quark pair are located in \(y = \pm \frac{L}{2}\), in that case the string profile is completely obtained by \(r = r(y)\). The only variable on the string world-sheet are \(x^- = \tau\) and \(\sigma = x^2 = y\) and other coordinates such as \(x^+, x^3, ...\) are constant. Therefore, we have string induced metric as,
\[
ds^2 = \left( \frac{r^2 R(r)^2 - f(r)}{2} \right) (dx^-)^2 + \frac{dr^2}{f(r)} + r^2 R(r)^2 dy^2. \tag{18}
\]

In the other hand, we take \(r = r(y), dr^2 = r'^2 dy^2\), where \(r' = \frac{dr}{dy}\). So, one can rewrite \(ds^2\) as,
\[
ds^2 = \left( \frac{r^2 R(r)^2 - f(r)}{2} \right) (dx^-)^2 + \left( r'^2 R(r)^2 + \frac{r'^2}{f(r)} \right) dy^2, \tag{19}
\]
In this case one can arrange the metric as,
\[
g_{\alpha\beta} = \begin{pmatrix} \frac{r^2 R(r) - f(r)}{2} & 0 \\ 0 & r^2 R(r)^2 + \frac{r'^2}{f(r)} \end{pmatrix}.
\] (20)

The Nambu-Goto action will be the following,
\[
S = -\frac{1}{2\pi\alpha'} \int \int_{L^-} d\tau d\sigma \sqrt{-\text{det} g_{\alpha\beta}}
= \frac{L^-}{\pi\alpha'} \int_0^{L/2} dy \sqrt{-\text{det} g_{\alpha\beta}},
\] (21)

and
\[
S = \frac{L^-}{\sqrt{2}\pi\alpha'} \int_0^{L/2} dy \sqrt{\left( f(r) - r^2 R(r)^2 \right) \left( r^2 R(r)^2 + \frac{r'^2}{f(r)} \right)},
\] (22)

where \(\frac{1}{\pi\alpha'}\) is string tension and \(\alpha'\) is related to the 't Hooft coupling constant with \(\frac{1}{\alpha'} = \sqrt{\lambda}\). We consider AdS radius equal one. In the above formula the lagrangian density does not depend to \(y\) explicitly, then corresponding hamiltonian is conserved and one can write following,
\[
\frac{\partial \mathcal{L}}{\partial r'} r' - \mathcal{L} = E,
\] (23)

where \(E\) is the constant energy of motion which is given by,
\[
\mathcal{L} = \sqrt{\left( f(r) - r^2 R(r)^2 \right) \left( r^2 R(r)^2 + \frac{r'^2}{f(r)} \right)}.
\] (24)

One can obtained the equation of motion for \(r\) as,
\[
r' = rR(r) \sqrt{f(r) \left[ \frac{r^2 R(r)^2 \left( f(r) - r^2 R(r)^2 \right)}{E^2} - 1 \right]}. \] (25)

Now, we insert this relation into Nambu-Goto action and we have,
\[
S = \frac{L^-}{\sqrt{2}\pi\alpha'} \int dr \sqrt{\frac{f(r) - r^2 R(r)^2}{f(r)} \left[ 1 - \frac{E^2}{r^2 R(r)^2 (f(r) - r^2 R(r)^2)} \right]^{-1/2}}
\] (26)

Two ends of string has located in turning points \((y = -L/2, \ y = +L/2)\). The corresponding symmetry in string, the boundary condition become as \(r = \pm \frac{L}{2}\) and \(r'(0) = 0\) Equation (25) has two roots, we apply \(r' = 0\) and obtain two solution for the \(f(r)\). First solution is \(f(r) = 0\) and give
us turning point at event horizon \( r = r_h \). The second solution is boundary condition in turning point which is,

\[
f = r^2 R(r)^2 + \frac{E^2}{r^2 R(r)^2},
\]

(27)

where \( r = r_{\text{min}} \) specify turning point near the boundary. We note here in near boundary \( E \) is very small and we have \( f \to r_{\text{min}}^2 R(r)^2 \). Also, we note that in (25) the factor under square root is negative near the black hole horizon \( f(r) \to 0 \) and is positive near the boundary. Of course, we know that \( r' \) is a physical quantity that is always positive. We come back to Nambu-Goto action (26) and in small limit \( E \), we have

\[
S = \frac{L^-}{\sqrt{2\pi \alpha'}} \int_{r_h}^\infty dr \sqrt{g_{--} g_{rr}} \left[ 1 + \frac{E^2}{2r^2 R(r)^2 (f(r) - r^2 R(r)^2)} \right].
\]

(28)

Since action contains self energies of the quark and anti-quark pair, it has divergency. In order to eliminate the divergence it should be subtracted by the self energy. For this reason, we consider the quark and anti-quark as a straight string that stretched from boundary to the event horizon. In this case, in (18), we have \( ds^2 = dy^2 = 0 \)

\[
S_0 = \frac{2L^-}{2\pi \alpha'} \int_{r_h}^\infty dr \sqrt{g_{--} g_{rr}},
\]

(29)

and therefore

\[
S_0 = \frac{L^-}{\sqrt{2\pi \alpha'}} \int_{r_h}^\infty dr \sqrt{\frac{f(r)}{f(r) - r^2 R(r)^2}}.
\]

(30)

For obtaining the \( S_I \), we subtract two equations (30) and (28),

\[
S_I = S - S_0 = \frac{L^-}{2\sqrt{2\pi \alpha'}} \int_{r_h}^\infty dr \sqrt{\frac{f(r) - r^2 R(r)^2}{f(r) - r^2 R(r)^2}} - \frac{E^2}{r^2 R(r)^2 (f(r) - r^2 R(r)^2)},
\]

(31)

and

\[
S_I = \frac{L^-}{2\sqrt{2\pi \alpha'}} \int_{r_h}^\infty dr \sqrt{\frac{E^2}{r^2 R(r)^2 (f(r) - r^2 R(r)^2)}} = \frac{L^- E^2}{2\sqrt{2\pi \alpha'}} I,
\]

(32)

where

\[
I = \int_{r_h}^\infty \frac{dr}{r^2 R(r)^2 \sqrt{f(r) \left( f(r) - r^2 R(r)^2 \right)}}.
\]

(33)
Now, we try to obtain $E$ in terms of separation parameter from quark anti-quark pair as $L$. For this, we have $y = L/2$, then from equation (25), one can write

$$
\frac{dr}{dy} = rR(r) \sqrt{f(r) \left[ \frac{r^2 R(r)^2 (f(r) - r^2 R(r)^2)}{E^2} - 1 \right]}.
$$

(34)

Then one can write $L$ as

$$
\frac{L}{2} = \int_{r_h}^{\infty} \frac{dr}{rR(r) \sqrt{f(r) \left[ r^2 R(r)^2 (f(r) - r^2 R(r)^2) - E^2 \right]}}.
$$

(35)

and

$$
\frac{L}{2E} = \int_{r_h}^{\infty} \frac{dr}{rR(r) \sqrt{f(r) \left[ r^2 R(r)^2 (f(r) - r^2 R(r)^2) \right]}} \left[ 1 - \frac{E^2}{f(r) \left[ r^2 R(r)^2 (f(r) - r^2 R(r)^2) \right]} \right]^{-\frac{1}{2}}.
$$

(36)

In low limit of $E$

$$
\frac{L}{2E} = \int_{r_h}^{\infty} \frac{dr}{rR(r) \sqrt{f(r) \left[ r^2 R(r)^2 (f(r) - r^2 R(r)^2) \right]}} \left[ 1 + \frac{E^2}{2f(r) \left[ r^2 R(r)^2 (f(r) - r^2 R(r)^2) \right]} \right].
$$

(37)

we ignore $E^2$ and then obtain following equation,

$$
\frac{L}{2E} = \int_{r_h}^{\infty} \frac{dr}{r^2 R(r)^2 \sqrt{f(r) \left[ (f(r) - r^2 R(r)^2) \right]}} = I.
$$

(38)

By putting this relation into (32), we obtain

$$
S_I = \frac{L - L^2}{8\sqrt{2}\pi\alpha'I}.
$$

(39)

Finally, by using the relation (14), one can obtain jet quenching parameter as,

$$
\hat{q} = \frac{1}{\pi\alpha'I}.
$$

(40)

Numerically in Fig 1(a), we see the graph of the jet quenching parameter in terms of the black hole temperature. Also in Fig 1(b) we drawn this parameter in terms of radius of event horizon. These figures shown that the jet quenching increase with increasing temperature and radius of event horizon.

In Fig 1(c) and 1(d) for different $p$ we drawn the jet quenching with respect to temperature, in that case we choose $k = 1$ and $\alpha = 0.4$ and $\alpha = 0.6$ respectively. In these figures, we find important result. For example, the jet quenching parameter is increased by increasing of the power-law Maxwell field $p$.

In the next section, we will study the effect of the power-law Maxwell field on the entropic force.
IV. THE EFFECT OF POWER-LAW MAXWELL FIELD ON ENTROPIC FORCE

In this section, we try to calculate the entropic force for the power-law Maxwell field. The entropic force is obtained by the following formula \[35, 50\].
\[ F = T \frac{\partial S}{\partial L}, \]  

where \( L \) is quark-antiquark distance and \( T \) is the temperature of the system. Thus, for obtaining the entropic force, one needs to calculate entropy \( (S) \), temperature \( (T) \) and quark-antiquark distance \( (L) \) through AdS/CFT correspondence.

At first, we assume that QGP is at rest and the frame is moving in one direct. In fact, we assume that quark-antiquark is moving with rapidity of \( \eta \). Therefore, we have to consider different alignments with respect to the plasma wind, parallel\( (\theta = 0) \), transverse\( (\theta = \pi/2) \) and arbitrary direction of the plasma wind. We consider only two first cases. For this reason we boost the frame in the \( x_3 \) direction, so that \( dt = dt' \cosh \eta - dx_3' \sinh \eta \) and \( h_{33} dx_3 = -dt' \sinh \eta + dx_3' \cosh \eta \). Inserting these relations in (3) and dropping the primes, we can write:

\[
\begin{align*}
\text{ds}^2 &= (-f(r) \cosh^2 \eta + r^2 R(r)^2 \sinh^2 \eta) dt^2 + (-f(r) \sinh^2 \eta + r^2 R(r)^2 \cosh^2 \eta) dx_3^2 \\
&\quad + 2 \sinh \eta \cosh \eta (f(r) - r^2 R(r)^2) dt dx_3 + \frac{1}{f(r)} dr^2 + r^2 R(r)^2 h_{11} dx_1^2 + \ldots. 
\end{align*}
\]

Now, we first study \( (\theta = \pi/2) \) and choose the static gauge which is \( t = \tau \) and \( \sqrt{h_{11}} x_1 = \sigma = y \). In this case, quark and antiquark are located at \( x_1 = +L/2 \) and \( x_1 = -L/2 \), so the the induced metric is given by \( dx_2 = dx_3 = \ldots = 0 \):

\[
\begin{align*}
\text{ds}^2 &= (-f(r) \cosh^2 \eta + r^2 R(r)^2 \sinh^2 \eta) dt^2 + r^2 R(r)^2 d\sigma^2 + \frac{1}{f(r)} dr^2, 
\end{align*}
\]

where \( r = r(\sigma) \), therefore \( dr^2 = r^2 d\sigma^2 \) and finally the induced metric will be rewritten by following form,

\[
\begin{align*}
\text{ds}^2 &= (-f(r) \cosh^2 \eta + r^2 R(r)^2 \sinh^2 \eta) dt^2 + \left( \frac{r'^2}{f(r)} + r^2 R(r)^2 \right) d\sigma^2 
\end{align*}
\]

We are going to write the Nambu-Goto action of the \( U \)-shaped string which connected \( QQ \) with together in holographic dimension.

\[
S = -\frac{1}{2\pi \alpha'} \int d\tau d\sigma \sqrt{-g}. 
\]

It means that, one can write,

\[
S = -\frac{1}{2\pi \alpha'} \int d\tau d\sigma \sqrt{g_1(r) + g_2(r)r'^2}, 
\]
Fig. 2: The behavior of LT versus ε for α=0.5, k=1, q=0.5, n=4, b=l=1 with θ = π/2 and p=1 (dash), p=2 (solid)

where $g_1$ and $g_2$ are equals:

$$g_1(r) = r^2 R(r)^2 (f(r) \cosh^2 \eta - r^2 R(r)^2 \sinh^2 \eta)$$
$$g_2(r) = \cosh^2 \eta + \frac{r^2 R(r)^2}{f(r)} \sinh^2 \eta.$$

(47)

Action (46) does not depend on $\sigma$, thus:

$$\frac{\partial \mathcal{L}}{\partial r'} r' - \mathcal{L} = \text{const},$$

(48)

where $\mathcal{L} = \sqrt{g_1(r) + g_2(r)r'^2}$.

By solving the above equation, one can obtain $r'$ as

$$\frac{-g_1(r)}{\sqrt{g_1(r) + g_2(r)r'^2}} = \text{const}.$$  

(49)

The slope of the tangent line for the lowest point in $U$ shape of string is zero, in that case we have $r = r_c$ and $r'_c = 0$. Therefore,

$$\frac{-g_1(r)}{\sqrt{g_1(r) + g_2(r)r'^2}} = g_1(r_c) = g_*.$$  

(50)

From (50), we can obtain $r'$,

$$r' = \sqrt{\frac{g_1(r)^2 - g_1(r)g_*}{g_2(r)g_*}},$$

(51)
Fig. 3, The behavior of LT versus $\varepsilon$ for $\alpha=0.5$, $k=1$, $q=0.5$, $n=4$, $b=l=1$, $\theta=\pi/2$ with $p=2$ and $\eta=0$ (dot), $\eta=0.4$ (dash) and $\eta=0.8$ (solid).

where

$$g_*= g_1(r_c) = r_c^2 R(r_c)^2 (f(r_c)cosh^2\eta - r_c^2 R(r_c)^2 sinh^2\eta),$$ (52)

In this relation the $f(r_c)$ is determined by (4) with condition of $r=r_c$. By integrating of (51), the separation length of the $Q\bar{Q}$ is obtained by,

$$L = 2 \int_{r_c}^{\infty} dr \sqrt{\frac{g_2(r)g_*}{g_1(r)^2 - g_1(r)g_*}}.$$ (53)

Now, we are ready draw $LT$ with respect to $\varepsilon = \frac{r_c}{r_c}$. In fig(2) we drew $LT$ numerically for different power-law Maxwell field for fixed rapidity of $\eta$. If we look at deeply to Fig(2), we will see that by increasing $p$ the maximum point of chart decreases. The maximum $LT = c$ shows the separation boundary of quark-antiquark. In fact, if $LT > c$ the quarks are screened, but if $LT < c$ then the fundamental string is connected.

Also for fixed $p$ and various rapidity of $\eta$, we have plotted $LT$ with respect to $\varepsilon$. As we can see in Fig(3), the increasing of rapidity lead us to have large separation boundary. If we contime our calculation for the case of $\theta = 0$, we take same results as before.

In the following, we are going to calculate the entropic force. The entropic force can be obtained by the following form,

$$F = T \frac{\partial S}{\partial L},$$ (54)
where $T$ is plasma temperature. In order to calculate the entropic force, we need to obtain equation,

$$S = -\frac{\partial F}{\partial T},$$  \hspace{1cm} (55)$$

where $F$ is free energy. For $\theta = \pi/2$ and $LT < c$ the fundamental string connected. In dual theory for calculating free energy one can utilize on-shell action of fundamental string. So, in that case the equations (46) and (51) lead us to have following,
\[ F = \frac{1}{\pi \alpha'} \int_{r_c}^{\infty} dr \sqrt{\frac{g_1(r)g_2(r)}{g_1(r) - g_*}}, \]  

where \( g_* = g_1(r_c) \).

By using (55), numerically one can calculate entropy. By taking \( \theta = \pi/2 \), fixed rapidity and various of \( p \), in fig (4) we plot \( S/\sqrt{\lambda} \) with respect to \( LT \). In this figure one can see that by increasing \( p \), at first time entropy decreases and then it increases. The entropic force, is related to the growth of entropy with distance and is responsible of the dissociating quark-antiquark. The entropic force is increased by large \( p \) and is decreased by large \( LT \). By increasing \( p \) the dissociation length will large and after than it will be small. In figure (5), for \( \theta = \pi/2 \) and fixed \( p \), one can see that increasing of \( \eta \) leads to decreasing of entropic force. In that case, the dissociation length also increasing. Similar results are obtained by \( \theta = 0 \).

V. CONCLUSION

In this paper, we used the black hole solution of Einstein-power law Maxwell-scalar gravity. The AdS/CFT correspondence and black hole solution help us to investigated two important parameter in QCD. One of them is jet quenching parameter and the other one is entropic force. These quantities give us important information about moving quark-antiquark in quark-gluon plasma media and also it’s interaction. The jet quenching shown that how the quark and gluon loses energy in QGP. In that case the entropic force related to dissociation length of quark-antiquark in quark-gluon plasma (QGP). Our main goal this paper is that, we want to show the effect of \( p \) from power law Maxwell field on the jet quenching parameter and entropic force. In order to see such effects, we drew some graph which show us the effect of \( p \) on the corresponding parameters. It may be interesting to consider such solution in case of different \( p \) for the obtaining the imaginary part of potential. Also, one can continue this solution and discuss the relation between the frequency and diffusion constant for the different values of \( p \).
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[1] J. M. Maldacena, Int. J. Theor. Phys. 38, 1113 (1999) [Adv. Theor. Math. Phys. 2, 231 (1998)] [hep-th/9711200].
[2] E. Papantonopoulos (ed.), From Gravity to Thermal Gauge Theories: The AdS/CFT Correspondence. Lecture Notes in Physics, vol. 828 (Springer, Berlin, 2011)
[3] G. Horowitz (ed.), Black Holes in Higher Dimensions Holes in Higher Dimensions (Cambridge University Press, Cambridge, (2012)
[4] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, book:Gauge/String Duality, Hot QCD and Heavy Ion Collisions. Cambridge, UK: Cambridge University Press, 2014 [arXiv:1101.0618 [hep-th]].
[5] S. A. Hartnoll, Class. Quant. Grav. 26, 224002 (2009) [arXiv:0903.3246 [hep-th]].
[6] J. McGreevy, Adv. High Energy Phys. 2010, 723105 (2010) [arXiv:0909.0518 [hep-th]].
[7] N. Iqbal, H. Liu and M. Mezei, arXiv:1110.3814 [hep-th].
[8] M. Natsume, Lect. Notes Phys. 903, pp.1 (2015) [arXiv:1409.3575 [hep-th]].
[9] S. W. Hawking, Commun. Math. Phys. 43, 199 (1975) Erratum: [Commun. Math. Phys. 46, 206 (1976)].
[10] A. G. Riess et al. [Supernova Search Team], Astron. J. 116, 1009 (1998) [astro-ph/9805201].
[11] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [astro-ph/9812133].
[12] J. L. Tonry et al. [Supernova Search Team], Astrophys. J. 594, 1 (2003) [astro-ph/0305008].
[13] A. T. Lee et al., Astrophys. J. 561, L1 (2001) [astro-ph/0104459].
[14] C. B. Netterfield et al. [Boomerang Collaboration], Astrophys. J. 571, 604 (2002) [astro-ph/0104460].
[15] N. W. Halverson et al., Astrophys. J. 568, 38 (2002) [astro-ph/0104489].
[16] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003) [astro-ph/0302209].
[17] M. B. Green, J. H. Schwarz and E. Witten, Superstring Theory, (Cambridge University Press, Cambridge 1987)
[18] B. Zwiebach, A First Course in String Theory, 2nd edn. (Cambridge University Press, Cambridge, 2009)
[19] J. Polchinski, String Theory (Cambridge University Press, Cambridge, 1998)
[20] K. Becker, M. Becker, J.H. Schwarz, String Theory and M theory: A Modern Introduction (Cambridge
21. M. K. Zangeneh, A. Sheykhi and M. H. Dehghani, Phys. Rev. D 91, no. 4, 044035 (2015) [arXiv:1505.01103 [gr-qc]].
22. M. Hassaine, J. Phys. A 40, 5717 (2007) [hep-th/0701285].
23. M. Hassaine and C. Martinez, Phys. Rev. D 75, 027502 (2007) [hep-th/0701058].
24. M. Gyulassy and L. McLerran, Nucl. Phys. A 750, 30 (2005) [nucl-th/0405013].
25. E. V. Shuryak, Nucl. Phys. A 750, 64 (2005) [hep-ph/0405066].
26. B. Muller, J. Schukraft and B. Wyslouch, Ann. Rev. Nucl. Part. Sci. 62, 361 (2012) [arXiv:1202.3233 [hep-ex]].
27. P. Danielewicz and M. Gyulassy, Phys. Rev. D 31, 53 (1985).
28. T. Hirano and M. Gyulassy, Nucl. Phys. A 769, 71 (2006) [nucl-th/0506049].
29. A. Majumder, B. Muller and S. A. Bass, Phys. Rev. Lett. 99, 042301 (2007) [hep-ph/0611135].
30. H. Song and U. W. Heinz, Phys. Rev. C 78, 024902 (2008) [arXiv:0805.1756 [nucl-th]].
31. C. Shen, U. Heinz, P. Huovinen and H. Song, Phys. Rev. C 82, 054904 (2010) [arXiv:1010.1856 [nucl-th]].
32. K. M. Burke et al. [JET Collaboration], Phys. Rev. C 90, no. 1, 014909 (2014) [arXiv:1312.5003 [nucl-th]].
33. R. Baier, D. Schiff and B. G. Zakharov, Ann. Rev. Nucl. Part. Sci. 50, 37 (2000) [hep-ph/0002198].
34. J. Sadeghi, M. R. Setare and B. Pourhassan, J. Phys. G 36, 115005 (2009) [arXiv:0905.1466 [hep-th]].
35. E. P. Verlinde, JHEP 1104, 029 (2011) [arXiv:1001.1753 [hep-th]].
36. H. Liu, K. Rajagopal and U. A. Wiedemann, Phys. Rev. Lett. 97, 182301 (2006) [hep-ph/0605178].
37. K. Bitaghsir Fadafan, B. Pourhassan and J. Sadeghi, Eur. Phys. J. C 71, 1785 (2011) [arXiv:1005.1368 [hep-th]].
38. N. Armesto, J. D. Edelstein and J. Mas, JHEP 0609, 039 (2006) [hep-ph/0606245].
39. S. D. Avramis and K. Sfetsos, JHEP 0701, 065 (2007) [hep-th/0606190].
40. F. L. Lin and T. Matsuo, Phys. Lett. B 641, 45 (2006) [hep-th/0606136].
41. E. Nakano, S. Teraguchi and W. Y. Wen, Phys. Rev. D 75, 085016 (2007) [hep-ph/0608274].
42. E. Caceres and A. Guijosa, JHEP 0612, 068 (2006) [hep-th/0606134].
43. A. Buchel, Phys. Rev. D 74, 046006 (2006) [hep-th/0605178].
44. H. Satz, Eur. Phys. J. C 75, no. 5, 193 (2015) [arXiv:1501.03940 [hep-ph]].
45. S. Tahery and J. Sadeghi, J. Phys. G 44, no. 10, 105001 (2017) [arXiv:1509.01309 [hep-th]].
46. M. R. Pahlavani, J. Sadeghi and R. Morad, J. Phys. G 39, 065004 (2012) [arXiv:1106.2908 [hep-th]].
47. M. R. Pahlavani, J. Sadeghi and R. Morad, J. Phys. G 38, 055002 (2011).
48. K. Bitaghsir Fadafan and S. K. Tabatabaei, Phys. Rev. D 94, no. 2, 026007 (2016) [arXiv:1512.08254 [hep-ph]].
49. J. X. Mo, G. Q. Li and X. B. Xu, Phys. Rev. D 93, no. 8, 084041 (2016) [arXiv:1601.05500 [gr-qc]].
[50] D. E. Kharzeev, Phys. Rev. D 90, no. 7, 074007 (2014) [arXiv:1409.2496 [hep-ph]].