Near Maximum-Likelihood Detector and Channel Estimator for Uplink Multiuser Massive MIMO Systems with One-Bit ADCs

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Abstract—In massive multiple-input multiple-output (MIMO) systems that operate with high bandwidths, it may not be power efficient to have a high-resolution analog-to-digital converter (ADC) for each antenna element. In this paper, a near maximum likelihood (nML) detector for uplink multiuser massive MIMO systems is proposed where each antenna is connected to a pair of one-bit ADCs, i.e., one for each real and imaginary component of the baseband signal. To achieve low complexity in the proposed nML detector, a strict constraint on the possible transmitted symbols in the original maximum likelihood (ML) detection problem is relaxed to formulate an ML estimation problem. Then, the ML estimation problem is converted into a convex optimization problem which can be efficiently solved. After obtaining the ML estimate by solving the convex problem, the base station can perform simple symbol-by-symbol detection for the transmitted signals from multiple users. The minimum required number of receive antennas for detectors using one-bit ADCs is also discussed. Numerical results show that the proposed nML detector is efficient enough to simultaneously support multiple uplink users adopting higher-order constellations, e.g., 16 quadrature amplitude modulation. Since our detector makes use of the channel as part of the decoding, an ML channel estimation technique with one-bit ADCs that shares the same structure with our proposed nML detector is also developed. The proposed detector and channel estimator provide a complete low power solution for the uplink of a massive MIMO system.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is a transmission technique for cellular systems that leverages a large number of base station antennas to support many single antenna users [1]. With enough antennas, massive MIMO can eliminate inter-user interference completely using matched beamforming for downlink and matched combining for uplink if the base station has full channel state information (CSI). Because of its simple beamforming and combining structures, massive MIMO will be beneficial not only for cellular systems but also for other wireless communication systems, e.g., vehicular-to-everything (V2X) communications using many antennas [2].

There are several practical constraints that are encountered when implementing massive MIMO systems. Because of the large number of antennas, it may not be possible to deploy expensive and powerful hardware with small noise and distortion at the base station. Prior work studied the impact of several hardware impairments including phase-shifts due to non-ideal oscillators and distortion noise caused by analog-to-digital convertors (ADCs) for massive MIMO systems. It was shown in [3]–[5] that having a large number of antennas helps to mitigate these hardware impairments, which confirms the benefit of massive MIMO.

In this paper, we focus on uplink multiuser massive MIMO systems using extremely low-resolution of one-bit ADCs at the base station. Because the power consumption by ADCs grows exponentially with their resolution level [6], [7], using one-bit ADCs may be a practical way of implementing cost-efficient and green massive MIMO systems. It is expected that using one-bit ADCs is particularly beneficial for wideband communication systems that require high sampling frequency, which will become common in millimeter wave (mmWave) communication systems [8]. Adopting one-bit ADCs for uplink massive MIMO, however, is challenging because of the severe threshold applied to the received signal (after it has passed through the channel and noise was added).

Using low-resolution ADCs for wireless communications has been investigated under various assumptions. It was shown in [9] for one-bit ADCs and in [10] for low-resolution (one to three bits) ADCs that the capacity maximizing transmit signals for single-input single-output channel are discrete regardless of assumptions on CSI, which is different from the unquantized output case. The mutual information of the MIMO channel with quantized output was studied without optimizing input distribution in [11] for one-bit ADCs and in [12], [13] for low-resolution ADCs. The input distributions were optimized to achieve the capacity of the quantized MIMO channel using one-bit ADCs in [14], [15]. Interestingly, the impact of having low-resolution ADCs is not that severe. For example, [13] showed that the rate loss due to using one-bit ADCs is 1.5793 bits/s for 4×4 MIMO with a quadrature phase shift keying (QPSK) constellation. It was proven in [14] that the mutual information of MIMO with one-bit ADCs is only 2/\pi times smaller compared to that of the unquantized MIMO case in the low signal-to-noise ratio (SNR) regime.

Most prior work on quantized MIMO [11]–[15] was restricted to point-to-point communications and focused on studying capacity or designing optimal transmit signals for quantized channel outputs using low-resolution ADCs without proposing specific signal processing algorithms for detecting the received signals. For point-to-point communications with
low, but more than one-bit resolution (e.g., 2-3 bits) ADCs, an iterative decision feedback equalizer was developed in [16], and digital and analog combiners were compared in terms of the achievable rate for mmWave systems in [17]. It is not straightforward to extend the techniques in [16], [17] to multiuser scenarios though.

In this paper, we propose a practical near maximum likelihood (nML) detector for uplink multiuser massive MIMO systems with one-bit ADCs. The exponential growth of the maximum likelihood (ML) detector complexity makes it difficult to use the ML detector in practice. The proposed nML detector, however, can be implemented with standard convex optimization techniques with marginal performance degradation when the number of receive antennas at the base station is large. Our nML detector also can support higher order constellations in a unified way.

The most relevant work with this paper is [18]. In [18], a message-passing algorithm-based multiuser detector was proposed for uplink massive MIMO systems with one-bit ADCs. Although the proposed detector in [18] was shown to perform better than the naïve minimum mean square error (MMSE) detector, the detector relied on the assumption of independence between the real and imaginary components of the transmitted signals. Therefore, it is difficult to use the detector in [18] for arbitrary constellations, which is different from the proposed nML detector. Uplink multiuser massive MIMO was also considered in [19] but the detector was based on several bit ADCs and developed for the spatial modulation scheme in [19] while we consider more general data symbol transmissions with one-bit ADCs in this paper.

We also propose an ML channel estimation technique using one-bit ADCs that is in line with our proposed nML detector. There has been related work on channel estimation with low-resolution ADCs [21]–[25]. Prior work using one-bit ADCs [22], [24], [25] though was not able to estimate the norm of the channel due to the simple zero-threshold setting for one-bit quantization. The work [26] used generalized approximate message passing (GAMP) algorithm combined with asymmetric one-bit quantizers to deliver the the norm information of the target vector but the proposed GAMP algorithm heavily relied on the sparsity of the target vector. Our ML channel estimator is shown to estimate not only the channel direction but also the channel norm and does not make an assumption about sparsity in the channel.

Our contributions are summarized as follows.

• We propose an nML detector for uplink multiuser massive MIMO systems. The proposed nML detector is based on the ML detector developed for distributed reception in [27]. We show that the two problems, i.e., distributed reception and multiuser detection, are essentially the same problem, which makes it possible to exploit the detectors from [27] for our multiuser detection problem. The complexity of the ML detector in [27], however, grows exponentially with the number of uplink users, which prevents its use in massive MIMO with many users. We reformulate the ML detector in [27] to derive an ML estimator and convert the ML estimation problem into a convex problem. Therefore, we can rely on efficient convex optimization techniques to obtain an ML estimate and perform symbol-by-symbol detection based on the ML estimate.

• By using the classical combinatorial geometry results [28]–[30], we derive the minimum required number of receive antennas at the base station, which is a necessary condition, for detectors using one-bit ADCs to perform well. Note that the classical combinatorial geometry results are also exploited in [15] to bound the high SNR capacity.

• We propose an ML channel estimator that has the same structure with the proposed nML detector. The proposed estimator can estimate the direction and norm of the channel more accurately then other channel estimators using low-resolution ADCs. Because of the similar structure, it is possible to implement both the nML detector and the ML channel estimator using a unified framework.

The paper is organized as fellows. We describe our system model using one-bit ADCs in Section II. In Section III, we briefly discuss the detectors which were originally developed for distributed reception in [27]. Then, we propose our nML detector and analyze the minimum required number of antennas in Section IV. We propose a ML channel estimator in Section V. In Section VI we evaluate the proposed techniques by simulations, and the conclusion follows in Section VII.

Notation: Lower and upper boldface letters represent column vectors and matrices, respectively, $||a||_2$ is used to denote the two-norm of a vector $a$, and $A^T$, $A^*$, $A^\dagger$ denote the transpose, Hermitian transpose, and pseudo inverse of the complex matrix $A$. $\Re(b)$ and $\Im(b)$ represent the real and complex part of a complex vector $b$, respectively. $0_m$ is used for the $m \times 1$ all zero vector, and $I_m$ denotes the $m \times m$ identity matrix. $\mathbb{C}^{m \times n}$ and $\mathbb{R}^{m \times n}$ represent the set of all $m \times n$ complex and real matrices, respectively.

II. System Model

We explain our system model and several assumptions that are relevant to our detector design. We also define expressions to for one-bit ADCs in this section.

A. Massive MIMO Received Signal Model

We consider a uplink multiuser cellular system with $N_c$ cells. Each cell consists of a base station with $N_t$ received antennas and $K$ users equipped with a single transmit antenna. All $K N_c$ users transmit independent data symbols simultaneously to their serving base stations. Assuming all users transmit data with power $P$, the received signal at the $i$-th base station $y_i = [y_{1,i}, y_{2,i}, \ldots, y_{N_c,i}]^T$ is

$$y_i = \sqrt{P} \sum_{k=1}^{K} h_{i,ik} x_{ik} + \sqrt{P} \sum_{m=1}^{N_c} \sum_{k=1 \atop m \neq i}^{K} h_{i,mk} x_{mk} + n_i$$

(1)

where $h_{i,ik} \in \mathbb{C}^{N_r \times 1}$ is the channel vector between the $i$-th base station and the $k$-th user associated with the $m$-th base station, $x_{mk}$ is the data symbol, which satisfies $\mathbb{E}[x_{mk}] = 0$ and $\mathbb{E}[|x_{mk}|^2] = 1$, transmitted from the $k$-th user supported
by the $m$-th base station, and $n_i \sim \mathcal{CN}(0_{N_r}, \sigma^2 I_{N_r})$ is the complex additive white Gaussian noise (AWGN) at the $i$-th base station.

To make the paper concise and focus on detection techniques, we make the following assumptions.

**Assumption 1:** The data symbols $x_{mk}$ are from an $M$-ary constellation $\mathcal{S} = \{s_1, \ldots, s_M\}$ for all $m$ and $k$ and satisfy

$$||x_i||^2 = K$$

where $x_i = [x_{i1} \ x_{i2} \ \cdots \ x_{iK}]^T$ for all $i$. If $\mathcal{S}$ is a phase shift keying (PSK) constellation satisfying $|s_m|^2 = 1$ for all $m$, then the norm constraint is trivially satisfied. For a quadrature amplitude modulation (QAM) constellation, the law of large numbers (LLN) gives

$$\sum_{k=1}^K |x_{ik}|^2 \approx K$$

as $K$ becomes large if all users select their data symbols independently and with equal probability within $\mathcal{S}$ (with proper normalization). The approximation (3) holds even when some users adopt PSK constellations while other users use QAM constellations as long as there are many users adopting each constellation so that the LLN applies. Therefore, we assume the norm constraint (2) holds throughout the paper.

**Assumption 2:** We neglect pilot contamination during the channel estimation procedure. Pilot contamination is only a significant impairment with a very large number of antennas [31], [32]. We further assume that each base station has perfect local CSI with which to implement its detector. After implementing the detectors, we relax this assumption in Section V where we consider channel estimation techniques for one-bit ADCs.

In addition to these assumptions, we first focus on a single cell scenario because the detectors considered in this paper do not exploit any kind of inter-cell cooperation. In Section IV-C, we explain how the proposed detector can be adapted to a multicell setting.

For the single cell scenario, the received signal $y$ in (1) becomes

$$y = \sqrt{P} \sum_{k=1}^K h_k x_k + n = \sqrt{P} H x + n,$$

and the SNR is

$$\rho = \frac{P}{\sigma^2}.$$ (5)

We assume the base station knows $\rho$ perfectly while we numerically study the impact of antenna-dependent $\rho$ mismatch in Section V.

**B. Received Signal Representation with One-Bit ADCs**

We focus on a massive MIMO system that uses one-bit ADCs for the real and imaginary parts of the each element of $y$. The conceptual figure of our system is depicted in Fig. 1.

The output of the $n$-th receive antenna after the one-bit ADCs is given as

$$\hat{y}_n = \text{sgn}(\text{Re}(y_n)) + j \text{sgn}(\text{Im}(y_n))$$ (6)

where $\text{sgn}(\cdot)$ is the sign function which is defined as

$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}.$$ (7)

Therefore, we have

$$\hat{y}_n \in \{1 + j, 1 - j, -1 - j, -1 + j\}$$ (8)

for $1 \leq n \leq N_r$. The collection of $\hat{y}_n$ is given as

$$\hat{y} = [\hat{y}_1 \ \hat{y}_2 \ \cdots \ \hat{y}_{N_r}]^T.$$ (9)

**III. POSSIBLE DETECTORS USING ONE-BIT ADCS**

In [27], distributed reception techniques for spatially multiplexed data symbols were studied. Assuming one-bit quantization for the real and imaginary parts of the received signal at each receive node, ML and zero-forcing (ZF)-type detectors at the fusion center were proposed assuming an error-free link between the fusion center and each receive node.

If we map each receive node in [27] into a receive antenna in our system model, the detectors proposed in [27] can be applied to our problem directly. In this section, we reformulate the detectors for distributed reception proposed in [27] into the uplink multiuser massive MIMO setting. We also discuss their characteristics and limitations. This discussion is useful for developing our nML detector in Section V.

**A. ML Detector Reformulation**

Let $g_m^T \in \mathbb{C}^{1 \times K}$ be the $n$-th row of the channel matrix $H$, i.e.,

$$H = [g_1 \ g_2 \ \cdots \ g_{N_r}]^T.$$ (10)

Note that $g_m$ is the channel between the $K$ users and the $m$-th receive antenna. Assuming the real and imaginary components of the Gaussian noise are IID, it is useful when approaching one-bit ADC problems to rewrite the signal model in the real
vector form instead of complex form as

\[
\begin{align*}
G_{R,n} &= \begin{bmatrix} \text{Re}(g_n) & \text{Im}(g_n) \end{bmatrix}^T = [g_{R,n,1}^T g_{R,n,2}^T] \in \mathbb{R}^{2 \times 2K}, \\
n_{R,n} &= \begin{bmatrix} \text{Re}(n_n) \\ \text{Im}(n_n) \end{bmatrix} = [n_{R,n,1}^T n_{R,n,2}^T] \in \mathbb{R}^{2 \times 1}, \\
x_R &= \begin{bmatrix} \text{Re}(x) \\ \text{Im}(x) \end{bmatrix} \in \mathbb{R}^{2K \times 1}
\end{align*}
\]

where

\[
\begin{align*}
g_{R,n,1} &= \begin{bmatrix} \text{Re}(g_n) \\ -\text{Im}(g_n) \end{bmatrix}, \\
g_{R,n,2} &= \begin{bmatrix} \text{Im}(g_n) \\ \text{Re}(g_n) \end{bmatrix}
\end{align*}
\]

and

\[
n_{R,n,i} \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)
\]

for all \(n\) and \(i\). The received signal at the \(n\)-th receive antenna can be also rewritten as

\[
\begin{align*}
y_{R,n} &= \begin{bmatrix} \text{Re}(y_n) \\ \text{Im}(y_n) \end{bmatrix} = \sqrt{P}G_{R,n}x_R + n_{R,n},
\end{align*}
\]

and the vectorized version of the quantized \(\hat{y}_k\) in the real domain is given as

\[
\hat{y}_{R,n} = \begin{bmatrix} \text{sgn}(\text{Re}(y_n)) \\ \text{sgn}(\text{Im}(y_n)) \end{bmatrix} = \begin{bmatrix} y_{R,n,1} \\ y_{R,n,2} \end{bmatrix}.
\]

Based on \(\hat{y}_{R,n}\), the base station generates the \textit{sign-refined} channel matrix for the \(n\)-th receive antenna as

\[
\bar{G}_{R,n} = \begin{bmatrix} g_{R,n,1}^T \\ g_{R,n,2}^T \end{bmatrix}
\]

where \(\bar{g}_{R,n,i}\) is defined as

\[
\bar{g}_{R,n,i} = \hat{y}_{R,n,i}g_{R,n,i}.
\]

Note that \(\hat{y}_{R,n,i} = \pm 1\) depending on the sign of \(y_{R,n,i}\). Define \(S_R\) to be

\[
S_R = \left\{ \begin{bmatrix} \text{Re}(s_1) \\ \text{Im}(s_1) \end{bmatrix}, \ldots, \begin{bmatrix} \text{Re}(s_M) \\ \text{Im}(s_M) \end{bmatrix} \right\}
\]

where \(M\) is the size of the data symbol constellation \(S\).

With these definitions and using similar logic from [27], the ML detector can be defined as

\[
x_{R,ML} = \arg\max_{\bar{x}_R} \prod_{i=1}^{2N_r} \prod_{n=1}^{N_t} \Phi \left( \sqrt{2\rho^2 g_{R,n,i}^T\bar{x}_R} \right)
\]

where \(\Phi(t) = \int_{-\infty}^{t - \frac{1}{\sqrt{2\pi}}} e^{-\frac{u^2}{2}} du\) and \(S_R^K\) is the \(K\)-ary Cartesian product set of \(S_R\), which is ordered with the real parts of the constellations first and the imaginary parts later. The \(\sqrt{2}\) term in (21) comes from the distribution of \(n_{R,n,i}\) given in (15).

### B. ZF-Type Detector Reformulation

By brute-force search, the complexity of the ML detector in (21) is \(M^K\) which grows exponentially with the number of users \(K\). To support a large number of \(K\), the ZF-type detector is proposed in [27]. The base station first obtains the ZF estimate as

\[
\hat{x}_{ZF} = H^\dagger \hat{y}.
\]

Because the norm square of \(x_{ZF}\) may not equal to \(K\), the base station normalizes \(x_{ZF}\) as

\[
\hat{x}_{ZF} = \sqrt{K} \frac{\hat{x}_{ZF}}{||\hat{x}_{ZF}||}
\]

and performs symbol-by-symbol detection using \(\hat{x}_{ZF}\) as

\[
\hat{x}_{ZF,i} = \arg\min_{x \in S} |x_{ZF,i} - \hat{x}_{ZF,i}|^2
\]

where \(\hat{x}_{ZF,i}\) is the \(i\)-th element of normalized \(\hat{x}_{ZF}\). The normalization is not an issue for PSK constellations; however, it is crucial for QAM constellations.

It is shown numerically in [27] that the ZF-type detector approaches the error rate floor much faster than the ML detector in the high SNR regime; the error rate floor in both cases is inevitable due to the one-bit ADCs. Therefore, we propose the nML detector that outperforms the ZF-type detector and requires much less complexity than the ML detector in the next section.

### IV. NEAR ML DETECTOR IMPLEMENTATION

We derive our nML detector by converting the original ML detection problem as a convex optimization problem. This can be done by relaxing constraints on the transmitted vector. We also derive the minimum required number of receive antennas for all detectors using one-bit ADCs to perform better than random guessing.

#### A. Convex Optimization Formulation of ML Detector

Because of the norm constraint (5), we define the ML estimator by relaxing the constraint \(x_R \in S_R^K\) in (21) as

\[
x_{R,ML}^{(1)} = \arg\max_{x_R} \prod_{i=1}^{2N_r} \prod_{n=1}^{N_t} \Phi \left( \sqrt{2\rho^2 g_{R,n,i}^T\bar{x}_R} \right)
\]

where \(\hat{y}_{R,n,i} = \pm 1\) depending on the sign of \(y_{R,n,i}\). Define \(S_R\) to be

\[
S_R = \left\{ \begin{bmatrix} \text{Re}(s_1) \\ \text{Im}(s_1) \end{bmatrix}, \ldots, \begin{bmatrix} \text{Re}(s_M) \\ \text{Im}(s_M) \end{bmatrix} \right\}
\]

where \(M\) is the size of the data symbol constellation \(S\).

With these definitions and using similar logic from [27], the ML detector can be defined as

\[
x_{R,ML} = \arg\max_{\bar{x}_R} \prod_{i=1}^{2N_r} \prod_{n=1}^{N_t} \Phi \left( \sqrt{2\rho^2 g_{R,n,i}^T\bar{x}_R} \right)
\]

It was shown in [27] that \(x_{R,ML}^{(1)}\) converges to the true transmitted vector \(x_R\) in probability as \(N_r \to \infty\) for arbitrary \(\rho > 0\). Therefore, if (26) can be solved, then the detector is guaranteed to achieve good performance when \(N_r\) is large.

The problem is that (26) is not easy to solve in general. The function \(\log \Phi(\cdot)\) is log-concave but the optimization problem in (26) is not convex due to the norm constraint \(||\bar{x}_R||^2 = K\).
To sidestep this challenge, we relax the norm constraint as \( \| \hat{x}_R \|_2 \leq K \) and reformulate the problem as

\[
\hat{x}_{R,\text{ML}}^{(2)} = \arg\max_{ \hat{x}_R \in \mathbb{R}^{2K \times 1} } \sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi \left( \sqrt{2\rho}g_{R,n,i}^T \hat{x}_R \right) \tag{27}
\]

which is a convex optimization problem that can be efficiently solved \(^{[33]}\).

Note that the estimates \( \hat{x}_{R,\text{ML}}^{(1)} \) and \( \hat{x}_{R,\text{ML}}^{(2)} \) may not be the same in general. Because \( \log \Phi(\cdot) \) is an increasing function, the estimator \((27)\) tries to give \( \hat{x}_{R,\text{ML}}^{(2)} \) that makes the arguments of \( \log \Phi(\cdot) \) functions as positive and large as possible. If \( \hat{x}_{R,\text{ML}}^{(2)} \) satisfies

\[
\sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi \left( \sqrt{2\rho}g_{R,n,i}^T \hat{x}_{R,\text{ML}}^{(2)} \right) \geq \sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi \left( \sqrt{2\rho}g_{R,n,i}^T \alpha \hat{x}_{R,\text{ML}}^{(2)} \right) \tag{30}
\]

with arbitrary \( 0 < \alpha < 1 \) because \( \log \Phi(\cdot) \) is an increasing function. Therefore, the norm square of \( \hat{x}_{R,\text{ML}}^{(2)} \) always becomes \( K \) due to the norm constraint, and

\[
\hat{x}_{R,\text{ML}}^{(2)} \in \mathcal{X}^{(1)} \tag{31}
\]

where \( \mathcal{X}^{(1)} \) denotes a set of all possible solutions of \((26)\) that depends on \( N_t \).

Because \( \hat{x}_{R,\text{ML}}^{(2)} \) is likely to satisfy \((28)\) for as many \( n \) and \( i \) as possible, \( \hat{x}_{R,\text{ML}}^{(2)} \) may be a good estimate if \( g_{R,n,i}^T \hat{x}_R \geq 0 \) for all \( n \) and \( i \) where \( x_R \) is the true transmitted vector. Note that the sign refinement in \((19)\) tries to make \( g_{R,n,i}^T \hat{x}_R \geq 0 \) for as many \( n \) and \( i \) as possible. The following lemma shows that the condition holds when \( \rho \) goes to infinity.

**Lemma 1.** When \( \rho \to \infty \), \( g_{R,n,i}^T \hat{x}_R \geq 0 \) in probability for all \( n \) and \( i \).

**Proof:** Please see Appendix \( \Box \)

There is also an intuitive explanation for Lemma \( \Box \). When \( \rho \to \infty \), the transmit power \( P \) becomes much larger than the variance of the noise \( n_{R,n,i} \). Therefore, the effect of \( n_{R,n,i} \) on \( y_{R,n,i} \) becomes negligible, and the one-bit output \( y_{R,n,i} \hat{x}_R \) would be solely determined by the sign of \( g_{R,n,i}^T \hat{x}_R \), which results in

\[
g_{R,n,i}^T \hat{x}_R = \hat{g}_{R,n,i}^T x_R \geq 0 \tag{32}
\]

After obtaining the estimate \( \hat{x}_{R,\text{ML}}^{(2)} \), the base station needs to perform normalization followed by symbol-by-symbol detection similar to the ZF-type detector in \((24)\). If we let \( \bar{x}_{R,\text{ML},i} \) be the \( i \)-th element of normalized \( \hat{x}_{R,\text{ML}} \), the symbol-by-symbol detection is

\[
\hat{x}_{\text{ML},i} = \arg\min_{\hat{x} \in \mathcal{S}} \left| \left( \bar{x}_{R,\text{ML},i} + j \bar{x}_{R,\text{ML},K+i} \right) - \hat{x} \right|^2 \tag{33}
\]

considering the fact that \( \hat{x}_{R,\text{ML}} \) is a \( 2K \times 1 \) real vector.

**Remark 1:** Note that either nML detector based on \((26)\) or \((27)\) is suboptimal compared to the original ML detector \((21)\). The proposed nML detector is based on the ML estimation \((22)\) over the \( 2K \)-dimensional real space with the norm constraint. It can be the case that

\[
\sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi \left( \sqrt{2\rho}g_{R,n,i}^T \hat{x}_{R,\text{ML}}^{(2)} \right) \geq \sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi \left( \sqrt{2\rho}g_{R,n,i}^T x_R \right) \tag{34}
\]

\[
> \sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi \left( \sqrt{2\rho}g_{R,n,i}^T \hat{x}_R \right) \tag{35}
\]

\[
> \sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi \left( \sqrt{2\rho}g_{R,n,i}^T \hat{x}_{R,\text{ML}}^{(2)} \right) \tag{36}
\]

with

\[
\left\| x_R - \hat{x}_{R,\text{ML}}^{(2)} \right\|^2 > \left\| \hat{x}_R - \hat{x}_{R,\text{ML}}^{(2)} \right\|^2 \tag{37}
\]

where \( x_R \) is the true transmitted vector and \( \hat{x}_{R,\text{ML}} \in \mathcal{S}_K \setminus \{ x_R \} \).

Numerical studies in Section \( \text{VI} \) show that the suboptimality of the proposed nML detector comes into play especially when \( N_t \) is small.

**Remark 2:** All detectors discussed in this paper are based on the assumption that \( \hat{g}_{R,n,i}^T x_R \geq 0 \) for all \( n \) and \( i \). When \( \rho \) is not large enough, it is possible that \( \hat{g}_{R,n,i}^T x_R < 0 \) for some \( n \) and \( i \), which degrades the performance of the all detectors. The numerical results in Section \( \text{VI} \) show that \( \rho \) can be not-so-large for detectors to achieve good performance.

### B. Analysis of Required Number of Receive Antennas

In the high SNR regime, the estimator \((27)\) finds \( \hat{x}_{R,\text{ML}}^{(2)} \) that satisfies

\[
g_{R,n,i}^T \hat{x}_{R,\text{ML}}^{(2)} > 0 \tag{38}
\]

for all \( 1 \leq n \leq N_t \) and \( 1 \leq i \leq 2 \). We do not consider the negligible case of \( g_{R,n,i}^T \hat{x}_{R,\text{ML}}^{(2)} = 0 \). This problem has close relation with classical combinatorial geometry \([28]–[30]\). The relation has been used to bound the high SNR capacity of MIMO channels using one-bit ADC in \([15]\). In this subsection, we derive the minimum required number of receive antennas for the nML detector to properly decode the signals from \( K \) users. First, we restate the lemma in \([15]\) that is useful for our analysis.

**Lemma 2.** \( N \) hyperplanes in general position passing through the origin of a \( d \)-dimensional space divide the space into

\[
2 \sum_{k=0}^{d-1} \binom{N-1}{k} \tag{39}
\]

regions.

General position is a strengthened rank condition, and we refer to \([15]\) for the exact definition of general position. Applying Lemma \(2\) to our problem,

\[
d = 2K, \quad N = 2N_t \tag{40}
\]

assuming all channel vectors (in the real domain) are in general position. The number of distinguishable regions in \( 2K \)-dimensional space becomes

\[
2 \sum_{k=0}^{2K-1} \binom{2N_t-1}{k} \tag{41}
\]
Note that the total number of possible transmit vectors \( \mathbf{x} \) is \( M^K \). Therefore, the minimum number of required receive antennas \( N_{r,\text{min}} \) should satisfy

\[
M^K \leq 2 \sum_{k=0}^{2K-1} \binom{2N_{r,\text{min}} - 1}{k}
\]  
(42)

to detect the transmitted vector \( \mathbf{x} \) correctly. We can easily determine \( N_{r,\text{min}} \) numerically from this inequality.

Having \( N_{r,\text{min}} \) receive antennas is just a necessary condition not only for our nML detector but also for all detectors using one-bit ADCs to perform well and does not guarantee to decode \( \mathbf{x} \) perfectly even when \( \rho \to \infty \). The transmitted vector \( \mathbf{x} \) can be detected without error when \( \mathbf{x} \) is the only vector located in a specific region. Considering the fact that the channel vectors are random in general, there is a probability that the true transmitted vector \( \mathbf{x} \) and other candidate vectors \( \hat{\mathbf{x}} \) can be located in the same region.

This is unlikely to be a problem in massive MIMO systems, however, because the number of distinct regions in (41) would become very large even with not-so-large \( N_r \), e.g., \( N_r = 32 \) with \( K = 5 \) gives around \( 5.6 \times 10^{10} \) regions. If all five users adopt an 8PSK constellation for their data symbols, the total number of possible transmit vectors becomes 32768 which is much less than the number of regions. Therefore, it seems reasonable that the probability of having two or multiple possible transmit vectors in a single region would become very low even with not-so-large \( N_r \). For a special case, we can explicitly derive the probability of having two possible transmit vectors in the same region.

**Special case:** Consider two transmit vectors \( \mathbf{x}_1 = [x_1, x_2, \ldots, x_K]^\top \) and \( \mathbf{x}_2 = [-x_1, x_2, \ldots, x_K]^\top \) where \( x_k \) is selected from a standard \( M \)-ary constellation \( S \) with equal probability, is the transmit symbol of the \( k \)-th user. Assume each entry of \( \mathbf{H} \) follows IID Rayleigh fading, and the high SNR regime which gives two received signals \( y_1 = \mathbf{Hx}_1 \) and \( y_2 = \mathbf{Hx}_2 \). Define \( \hat{y}_1 \) and \( \hat{y}_2 \) as the quantized outputs of \( y_1 \) and \( y_2 \) using one-bit ADCs.

**Proposition 1.** For the special case,

\[
\Pr(\hat{y}_1 = \hat{y}_2) = \left(\frac{2}{\pi} \arctan \sqrt{K-1}\right)^{2N_r}.
\]  
(43)

**Proof:** Please see Appendix [B].

The following corollary is a direct consequence of Proposition [I].

**Corollary 1.** As \( N_r \to \infty \), \( \Pr(\hat{y}_1 = \hat{y}_2) \to 0 \).

Corollary [I] shows that the probability of having \( \hat{y}_1 \) and \( \hat{y}_2 \) (which is the same as having \( \mathbf{x}_1 \) and \( \mathbf{x}_2 \)) in the same region becomes zero as \( N_r \) goes to infinity in the high SNR regime. Although it is difficult to generalize Lemma [I] and Corollary [I] for an arbitrary pair of transmit vectors and channel models, we expect that the same conclusion would hold in general.

### C. Extension to Multicell Setting

For the multicell scenario, the inter-cell interference should be taken into account for the proposed nML detector. It is reasonable to assume that the base station can accurately estimate the long-term statistic of the inter-cell interference. Therefore, we assume the \( i \)-th base station knows the average inter-cell interference power at the \( n \)-th receive antenna

\[
\eta_{i,n}^2 = P \sum_{m=1}^{N_e} \sum_{k=1}^{K} E [||h_{i,m,k,n}||^2] \]  
(44)

where \( h_{i,m,k,n} \) is the \( n \)-th entry of \( h_{i,m,k} \) in (1). Note that large-scale fading, e.g., path loss, is reflected in \( h_{i,m,k} \). We further assume that

\[
\eta_{i,1}^2 = \eta_{i,2}^2 = \cdots = \eta_{i,N_r}^2 = \eta_i^2
\]  
(45)

considering the fact that all receive antennas at the base station are closely deployed.

Without having any CSI from out-of-cell users, the base station will consider the inter-cell interference as additional AWGN noise, i.e.,

\[
\sqrt{P} \sum_{m=1}^{N_e} \sum_{k=1}^{K} h_{i,m,k} x_{m,k} \sim \mathcal{CN}(0_{N_r}, P \eta_i^2 I_{N_r}).
\]  
(46)

Then the effective signal-to-interference-noise ratio (SINR) at the \( i \)-th base station is

\[
\rho_{i,MC} = \frac{P}{P \eta_i^2 + \sigma^2},
\]  
(47)

and the proposed nML detector can be adapted to the multicell scenario by substituting \( \rho \) in (27) with \( \rho_{i,MC} \). Note that the large-scale fading of the desired signals at the \( i \)-th base station is still captured in the channel \( h_{i,ik} \) and not reflected in \( \rho_{i,MC} \) while that of the inter-cell interference is abstracted in \( \eta_i^2 \).

### V. CHANNEL ESTIMATION WITH ONE-BIT ADCS

Note that the detectors discussed in the previous sections assume perfect CSI at the base station. For a coherent detector, however, the CSI is normally obtained through an estimate of the channel. In this section, we develop efficient channel estimation techniques assuming one-bit ADCs at the base station. We focus on estimating \( \mathbf{g}_n \), i.e., the channel between the receive antenna \( n \) and \( K \) users, instead of \( \mathbf{h}_k \).

We consider a block fading channel to develop channel estimation techniques. We assume the channel is static for \( L \) channel uses in a given fading block and changes independently from block-to-block. The received signal at the \( n \)-th antenna for the \( \ell \)-th channel use in the \( m \)-th fading block is given as

\[
y_{n,m}[\ell] = \sqrt{\rho} g_{n,m}^* x_{m}[\ell] + w_{n,m}[\ell].
\]  
(48)

Let the first \( T < L \) channel uses be devoted for a training phase and the remaining \( L - T \) channel uses be dedicated to a data communication phase. Put the first \( T \) received signals during the training phase into a vector form as

\[
y_{n,m,\text{train}} = \sqrt{\rho} \mathbf{x}_{m,\text{train}}^* g_{n,m} + w_{n,m,\text{train}}
\]  
(49)
where

\[
Y_{n,m,\text{train}} = \begin{bmatrix} y_{n,m}[0] & \cdots & y_{n,m}[T-1] \end{bmatrix}^* \in \mathbb{C}^{T \times 1},
\]
(50)

\[
X_{\text{train}} = \begin{bmatrix} x_{m}[0] & \cdots & x_{m}[T-1] \end{bmatrix} \in \mathbb{C}^{K \times T},
\]
(51)

\[
w_{n,m,\text{train}} = \begin{bmatrix} w_{n,m}[0] & \cdots & w_{n,m}[T-1] \end{bmatrix}^* \in \mathbb{C}^{T \times 1}.
\]
(52)

In the training phase, \( X_{\text{train}} \) is known to the base station but \( g_{n,m} \) must be estimated. While arbitrary training matrices are possible, for simulation purpose in Section VI, we focus on unitary training where \( X_{\text{train}} \) satisfies

\[
X_{\text{train}}^* X_{\text{train}} = I_T \quad \text{if} \quad K \geq T,
\]
(53)

\[
X_{\text{train}}^* X_{\text{train}} = T I_K \quad \text{if} \quad K < T.
\]
(54)

The normalization term \( T \) in the case of \( K < T \) ensures the average transmit power equals to \( P \) in each channel use.

Similar to the previous sections, we reformulate all expressions into the real domain as

\[
Y_{R,n,m,\text{train}} = \sqrt{\rho} X_{R,n,m,\text{train}} g_{n,m} + w_{R,n,m,\text{train}}
\]
(55)

where

\[
Y_{R,n,m,\text{train}} = \begin{bmatrix} \text{Re}(Y_{n,m,\text{train}}) & \text{Im}(Y_{n,m,\text{train}}) \end{bmatrix} \in \mathbb{R}^{2T \times 1},
\]
(56)

\[
X_{R,n,m,\text{train}} = \begin{bmatrix} \text{Re}(X_{n,m,\text{train}}) & -\text{Im}(X_{n,m,\text{train}}) \end{bmatrix} \in \mathbb{R}^{2K \times 2T},
\]
(57)

\[
g_{R,n,m} = \begin{bmatrix} \text{Re}(g_{n,m}) & \text{Im}(g_{n,m}) \end{bmatrix} \in \mathbb{R}^{2K \times 1},
\]
(58)

\[
w_{R,n,m,\text{train}} = \begin{bmatrix} \text{Re}(w_{n,m,\text{train}}) & \text{Im}(w_{n,m,\text{train}}) \end{bmatrix} \in \mathbb{R}^{2T \times 1}.
\]
(59)

It is important to point out that (55) has the same form as (16) while the roles of the channel and the transmitted signal are reversed. Therefore, using the same techniques that we exploited for the detectors, we can develop channel estimators based on the one-bit ADC outputs and \( X_{R,n,m,\text{train}} \).

We define the \( i \)-th column of \( X_{R,n,m,\text{train}} \) as \( x_{R,n,m,\text{train},i} \) and the \( i \)-th output of the one-bit ADC as

\[
\tilde{y}_{R,n,m,\text{train},i} = \text{sgn}(y_{R,n,m,\text{train},i})
\]
(60)

where \( y_{R,n,m,\text{train},i} \) is the \( i \)-th element of \( Y_{R,n,m,\text{train}} \). Note that there are 2T one-bit ADC outputs in total for the \( n \)-th receive antenna, i.e., \( T \) outputs for each of the real and imaginary parts of the received signal. Based on \( \tilde{y}_{R,n,m,\text{train},i} \), the base station performs the sign-refinement as

\[
\tilde{x}_{R,n,m,\text{train},i} = \tilde{y}_{R,n,m,\text{train},i} x_{R,n,m,\text{train},i},
\]
(61)

and the ML channel estimator is given as

\[
\hat{g}_{R,n,m,\text{ML}} = \arg\max_{\hat{g}_R \in \mathbb{R}^{2K \times 1}} \frac{1}{T} \sum_{i=1}^{2T} \Phi\left( \sqrt{2\rho} \tilde{x}_{R,n,m,\text{train},i} \hat{g}_R \right).
\]
(62)

Due to the fact that \( \log \Phi(\cdot) \) is an increasing function where \( g_{R,n,m} \) is the true channel vector.

To overcome this problem, we impose the norm constraint on \( g_{R,n,m} \) and convert (63) to

\[
\hat{g}_{R,n,m,\text{ML}} = \arg\max_{\hat{g}_R \in \mathbb{R}^{2K \times 1}} \frac{1}{T} \sum_{i=1}^{2T} \log \left( \Phi\left( \sqrt{2\rho} \tilde{x}_{R,n,m,\text{train},i} \hat{g}_R \right) \right)
\]
(65)

which is forced to satisfy \( \| \hat{g}_{R,n,m,\text{ML}} \|^2 = K \) for most of channel models. The norm constraint can be further optimized if the base station knows the long-term statistic of the channel norm.

For comparison purpose, we also define a simple ZF-type channel estimator as

\[
\hat{g}_{R,n,m,\text{ZF}} = \sqrt{K} \left( X_{R,n,m,\text{train}} \right)^\dagger \tilde{y}_{R,n,m,\text{train}}
\]
(66)

which we numerically compare our ML channel estimator and the ZF-type channel estimator in Section VI.

VI. SIMULATION RESULTS

We perform Monte-Carlo simulations to evaluate the proposed techniques. We first consider the single cell scenario to compare the performance of detectors. Then we take the multicell scenario into account to consider more practical settings.

A. Single Cell Scenario

We assume IID Rayleigh fading channels, i.e., all elements of \( H \) are distributed as \( \mathcal{C}\mathcal{N}(0,1) \), although the distribution of the channel was not explicitly incorporated into any of the analysis.

We first evaluate the ML and ZF-type channel estimators discussed in Section V. We focus on estimating \( g_{n,m} \), i.e., the channel between the \( n \)-th receive antenna and \( K \) users. We define the mean squared error (MSE) of a channel estimator \( x \) as

\[
\text{MSE}_x = \mathbb{E} \left[ \| \hat{g}_{n,m} - \hat{g}_{n,m,z} \|^2 \right],
\]
(67)

and the normalized MSE (NMSE) as

\[
\text{NMSE}_x = \mathbb{E} \left[ \left( \frac{\| \hat{g}_{n,m} \|}{\| g_{n,m} \|} - \frac{\| \hat{g}_{n,m,z} \|}{\| g_{n,m,z} \|} \right)^2 \right],
\]
(68)

which are used as performance metrics. The expectations are taken over \( g_{n,m} \). In Fig. 2 we compare the proposed ML and the ZF-type channel estimators to the iterative method from [22] with different training lengths \( T \). We set \( \rho = 20 \text{dB} \) and \( K = 6 \). As shown in the figure, the iterative method fails to estimate the norm of channels, and the iterative method with the norm fixed to \( K \) gives the same MSE.
Fig. 2: MSE and normalized MSE of different channel estimators with $\rho = 20\,\text{dB}$, $K = 6$, and different values of $T$. “Iterative” is from [22], and “Iterative, fixed norm” is same as “Iterative” except the norm square fixed to $K$.

Fig. 4: SER vs. SNR (in dB) of three detectors with $N_r = 10$, $K = 3$, and $M = 4$ (QPSK). The original ML, the proposed nML, and the ZF-type detectors are compared.

Fig. 5: SER vs. SNR in dB scale with $M = 8$ (8PSK), $K = 8$, and different values of $N_r$. The proposed nML and ZF-type detectors are compared.

**Fig. 3:** SER vs. $N_r$ of the proposed ML detector with $K = 4$, $M = 8$ (8PSK), and $\rho = 20\,\text{dB}$. The proposed nML detector is compared with random guessing.

Note that implementing the proposed nML detector based on the normalized channel estimates $\hat{g}_{n,m,\text{ML}}/\|\hat{g}_{n,m,\text{ML}}\|$ would have negligible performance degradation in high SNR regime. Therefore, the detectors studied in Sections III and IV can be implemented based on the accurate channel direction estimates of the proposed ML channel estimator.

From now on, we assume the base station has perfect CSI and evaluate the detectors. We use the average symbol error rate (SER) which is defined as

$$\text{SER} = \frac{1}{K} \sum_{n=1}^{K} \mathbb{E} \left[ \Pr \left( \hat{x}_n \neq x_n \mid x_{\text{sent}}, H, n, \rho, K, N_r, S \right) \right]$$

for the performance metric where the expectation is taken over $x$, $H$, and $n$.

We first verify the analysis on the required number of antennas in (42). In Fig. 3, we plot SER of the proposed ML detector with different numbers of receive antennas. We fix $\rho = 20\,\text{dB}$ to mimic the high SNR regime and set $K = 4$ and $M = 8$ (8PSK). With these parameters, the minimum required number of receive antennas should be $N_{r,\text{min}} \geq 7$. The figure clearly shows that the proposed ML detector is even worse than the random guessing of the transmitted signals when $N_{r,\text{min}} < 7$. The detector starts to work at least for some extent when $N_{r,\text{min}} = 7$, and the performance becomes better.
as the base station is equipped with more receive antennas.

Now, we compare the three detectors: 1) the original ML detector (21) that is based on exhaustive search over all possible transmitted vectors, 2) the proposed nML detector (27) that is based on convex optimization, and 3) the ZF-type detector. Due to the computational complexity of the original ML detector, we set $K = 3$, $M = 4$ (QPSK) for all users, and $N_r = 10$ which may not be considered as massive MIMO. We plot SERs of the three detectors in Fig. 4. The figure shows that the proposed nML detector is clearly suboptimal compared to the original ML detector as explained in Remark 1 of Section IV-A. Still, the proposed nML detector outperforms the ZF-type detector for all SNR regimes.

In Fig. 5 we plot SERs of the proposed nML detector and the ZF-type detector according to SNR with $K = 8$ and $M = 8$ (8PSK). We compare $N_r = 64$ and 128 for both detectors. Note that $N_r = 64$ is a very practical antenna number that is considered in practice [35]. We do not consider the original ML detector in this scenario because of its excessive complexity, i.e., the detector needs to compare $8^8 = 2^{24}$ possible transmit vectors. The figure shows that the proposed nML detector outperforms the ZF-type detector with the same number of $N_r$. The ZF-type detector suffers from the error rate floor while the proposed nML detector does not have such floor until $10^{-5}$ SER with $N_r = 128$.

We adopt the same system setup with Fig. 5 except a $M = 16$ (16QAM) constellation for data symbol in Fig. 6. The proposed nML detector still outperforms the ZF-type detector; however, the nML detector also suffers from the error rate floor even with $N_r = 128$ because of the excessive number of possible transmit vectors, i.e., there are $16^8 = 2^{32}$ possible candidates. We expect that the error rate floor for the nML detector would vanish for practical SNR regimes once the base station is deployed with more antennas, e.g., $N_r = 256$.

Figs. 5 and 6 both show that the proposed nML detector gives similar SER performance with the ZF-type detector using only half of the receive antennas. Therefore, if the base station has sufficient computation power, it is always beneficial to use the proposed nML detector than the ZF-type detector. Note that the computation power at the base station is related to the digital baseband processing in Fig. 1 and different from having one-bit ADCs.

In Fig. 7 we evaluate the impact of having inaccurate SNR information at the base station for the proposed nML detector. The base station assumes the SNR (in dB scale) at the $i$-th receive antenna as

$$\rho_i = \rho + \triangle \rho_i \quad (70)$$

where $\rho$ is the true SNR and $\triangle \rho_i$, which is independently and uniformly distributed between $-\rho_{\text{mis}}$ and $\rho_{\text{mis}}$, denotes the SNR mismatch. Note that $\rho_i$ is different for all $i$. We consider two different SNR values of $\rho = 0$dB and 10dB with $K = 8$, $M = 8$, and $N_r = 64$. The SNR mismatch parameter $\rho_{\text{mis}}$ increases from 0 to 10dB for both $\rho$ values. The figure shows that the inaccurate SNR information has negligible impact on nML detector until $\rho_{\text{mis}} = 4$ for both SNR values. Therefore, we expect the SNR mismatch would not be a problem for the proposed nML detector in practice.

To compare the detectors in a practical setting, we combine the detectors with a low-density-parity-check (LDPC) code. We assume the base station has perfect SNR information for this study. We adopt the rate 1/2 LDPC code with the block length of 672 bits from the IEEE 802.11ad standard [36]. After hard detection by the detectors, the estimated symbols (or bits) are decoded using the bit-flipping decoding algorithm [37]. The coded bit error rates (BERs) of the nML and ZF-type detectors according to SNR with $K = 4$, $N = 64$, and $M = 8$ are shown in Fig. 8. The figure clearly shows that the proposed nML detector outperforms the ZF-type detector even for this practical setting. Further improvements could be expected if further work is put into deriving an appropriate soft decision decoding metric.
Fig. 8: Coded BER vs. SNR in dB scale with $M = 8$ (8PSK), $K = 4$, and $N_r = 64$. The proposed nML and ZF-type detectors are compared using the rate 1/2 LDPC code adopted in the IEEE 802.11ad standard.

### TABLE I: Multicell Simulation Parameters

| Parameter                  | Assumption                                                                 |
|----------------------------|----------------------------------------------------------------------------|
| Cell layout                | 57 hexagonal cells                                                         |
| Cell radius                | 500m                                                                       |
| # of RX antennas per BS ($N_r$) | 64                           |
| # of TX antennas per user  | 1                                                                          |
| # of users per cell ($K$)  | 4                                                                          |
| Min. dist. btw. BS and user| 100m                                                                      |
| User transmit power         | 23dBm                                                                     |
| Path loss per km           | $131.1+42.8\log_{10}(\text{dist.})$ dB                                    |
| System bandwidth           | 5MHz                                                                      |
| Noise spectral density     | -174dBm/Hz                                                                |
| Noise figure               | 5dB                                                                 |
| Constellation              | 8PSK                                                                      |
| Channel coding             | Rate 1/2 LDPC from 802.11ad                                               |

B. Multicell Scenario

For the last numerical study, we consider the multicell scenario where the detailed simulation parameters are listed in Table I. We consider two different user dropping scenarios. 1) All users except a typical user in the center cell are randomly dropped within corresponding cells. The typical user is manually dropped with the distance $d$ from the base station in the center cell. 2) All users except the users in the center cell are randomly dropped within corresponding cells. The users in the center cell are randomly dropped within the range $(d - 20m, d + 20m)$ from the center cell base station. The second scenario can be considered as the case with user scheduling that selects users with similar received signal power while the first scenario corresponds to uncoordinated uplink transmission. We consider coded BER of the typical user for the first scenario while BERs of all $K$ users in the center cell are averaged for the second scenario. The proposed nML detector is performed as explained in Section IV-C.

The BER results for these two scenarios according the distance $d$ are plotted in Fig. 9. We can see that the proposed nML detector outperforms the ZF-type detector for both user dropping scenarios. As $d$ increases, the BER performance of both detectors becomes worse because of the reduced received signal power. Note that the BER performance of the second scenario is much better than that of the first scenario when $d$ is large. For large $d$, the received signal power of the typical user in the first scenario is overwhelmed by other users’ received signals (the near-far effect), resulting in poor BER performance. If all users experience similar SINR as in the second scenario, however, the BER performance is quite good even with large $d$. This shows that the proposed nML detector will perform well with proper user scheduling or uplink power control, which are already common in current cellular systems. Note that the ZF-type detector suffers from the notable error rate floor for the second scenario when $d$ is small (that corresponds to the high SNR regime for the single cell scenario) while the proposed nML detector does not have such floor until $10^{-5}$ BER. In the first scenario, there is no error rate floor even for the ZF-type detector because the received signal of the typical user overwhelms other users’ received signals.

VII. CONCLUSION

We proposed a unified framework of detection and channel estimation techniques for uplink multiuser massive MIMO systems using a pair of one-bit ADCs at each antenna. The proposed techniques are based on off-the-shelf convex optimization methods, which makes it easy to implement in practice. The proposed nML detector gives far better performance than the ZF-type detector for all range of SNR regimes and number of antennas. Numerical studies showed that the proposed nML detector is able to perform well even with not-so-large number of antennas, robust to inaccurate SNR estimation, and outperforms the ZF-type detector for practical channel coded and multicell settings. The proposed ML channel estimator can effectively estimate not only the direction but also the norm of the channel even with one-bit ADCs. Because of the unified structure of the ML detector and
channel estimator, same hardware can be used for both tasks, which make the proposed hardware more attractive for uplink massive MIMO systems using one-bit ADCs.

**APPENDIX A**

**PROOF OF LEMMA**

We prove the lemma using two disjoint cases:

Case 1: When $y_{R,n,i} \geq 0$.

Because $y_{R,n,i}$ is the sign of $y_{R,n,i}$, if $y_{R,n,i} \geq 0$, we have $\hat{y}_{R,n,i} = 1$ and

$$y_{R,n,i} = \sqrt{P_{R,n,i}}^{T}x_{R} + n_{R,n,i}$$

(71)

$$= \sqrt{P_{R,n,i}}^{T}x_{R} + n_{R,n,i}$$

(72)

$$\geq 0$$

(73)

where the second equality is from the definition of $\tilde{g}_{R,n,i}$ in [12]. Then, we have the inequality

$$n_{R,n,i} \geq -\sqrt{2P_{R,n,i}}^{T}x_{R}$$

(74)

and the probability for this inequality is given as

$$Pr \left( n_{R,n,i} \geq -\sqrt{P_{R,n,i}}^{T}x_{R} \right) = 1 - \Phi \left( -\sqrt{2\rho P_{R,n,i}}x_{R} \right)$$

(75)

Therefore, we need to have $\tilde{g}_{R,n,i}^{T}x_{R} \geq 0$. Otherwise,

$$Pr \left( n_{R,n,i} \geq -\sqrt{2P_{R,n,i}}^{T}x_{R} \right) \rightarrow 0$$

(76)

as $\rho \rightarrow \infty$, and it is impossible to have $y_{R,n,i} \geq 0$, which contradicts the assumption.

Case 2: When $y_{R,n,i} < 0$.

In this case, $\hat{y}_{R,n,i} = -1$, and we have

$$\tilde{y}_{R,n,i}y_{R,n,i} = \sqrt{P_{R,n,i}}^{T}y_{R,n,i}\tilde{g}_{R,n,i}^{T}x_{R} + \tilde{y}_{R,n,i}n_{R,n,i}$$

(77)

$$= \sqrt{P_{R,n,i}}^{T}x_{R} - n_{R,n,i}$$

(78)

$$> 0$$

(79)

The probability of this inequality is given as

$$Pr \left( n_{R,n,i} < \sqrt{2P_{R,n,i}}^{T}x_{R} \right) = \Phi \left( \sqrt{2\rho P_{R,n,i}}x_{R} \right)$$

(80)

If $\tilde{g}_{R,n,i}^{T}x_{R} < 0$, the probability converges to zero as $\rho \rightarrow \infty$, which contradicts the assumption. Therefore, we need to have $\tilde{g}_{R,n,i}^{T}x_{R} \geq 0$.

**APPENDIX B**

**PROOF OF PROPOSITION**

Note that

$$Hx_{1} = h_{1}x_{1} + H'x', \quad Hx_{2} = -h_{1}x_{1} + H'x'$$

(81)

where $H' = [h_{2} \ h_{3} \cdots h_{K}]^{T}$ and $x' = [x_{2} \ x_{3} \cdots x_{K}]^{T}$. Because of the assumptions on $H$ and $x_{1}$, we have

$$h_{1}x_{1} \sim CN(0_{N_c}, I_{N_c}), \quad H'x' \sim CN(0_{N_c}, (K-1)I_{N_c})$$

(82)

Therefore, the requirement of $\hat{y}_{1} = \hat{y}_{2}$ can be decomposed into $2N_{r}$ equations $sgn(x - y) = sgn(x + y)$ in the lemma of Appendix C. Because the $2N_{r}$ equations are independent, we have

$$Pr (\hat{y}_{1} = \hat{y}_{2}) = \left( \frac{2}{\pi} \arctan \sqrt{K-1} \right)^{2N_{r}}$$

(83)

which finishes the proof.

**APPENDIX C**

**Lemma.** If $x \sim N(0, (N - 1)/2)$ and $y \sim N(0, 1/2)$ for a given $N \geq 1$, we have

$$Pr (sgn(x - y) = sgn(x + y)) = \frac{2}{\pi} \arctan \sqrt{N - 1}$$

(84)

**Proof:** Since $x \sim N(0, (N - 1)/2)$ and $y \sim N(0, 1/2)$,

$$Pr (sgn(x - y) = sgn(x + y)) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi (N - 1)}} e^{-x^{2}} dx$$

(85)

$$= \pi^{-\frac{1}{2}} \int_{-\infty}^{+\infty} e^{-\frac{x^{2}}{N-1}} dx$$

(87)

$$= \frac{4}{\pi N^{-\frac{1}{2}}} \int_{0}^{+\infty} e^{-z^{2}} e^{-\frac{z^{2}}{N-1}} dz$$

(88)

$$= \frac{2}{\pi} \arctan \sqrt{N - 1}$$

(89)

where (a) follows by letting $z = \frac{x}{\sqrt{N-1}}$. 

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