Symmetry energy from nuclear masses and neutron-star observations using generalised Skyrme functionals

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Abstract. We study the constraints imposed by nuclear mass measurements and neutron-star observations on the symmetry energy. For this purpose, we use a family of unified equations of state of neutron-star interiors, based on generalised Skyrme functionals that were fitted to essentially all the experimental nuclear mass data while ensuring a realistic neutron-matter equation of state.

1. Introduction

Born from gravitational core-collapse supernova, neutron stars (NSs) are among the most compact objects in the Universe, with a central density which can reach several times nuclear matter density \( n_0 \simeq 0.16 \text{ fm}^{-3} \) (see, e.g., Ref. [1]). The interior of a NS is expected to contain at least four different regions: (i) an “outer crust”, at densities above \( \sim 10^4 \text{ g cm}^{-3} \), composed of fully ionised atoms, arranged in a Coulomb lattice and neutralised by a degenerate electron gas, (ii) an “inner crust”, at densities above \( \sim 4 \times 10^{11} \text{ g cm}^{-3} \), where neutron-proton clusters and electrons coexist with unbound neutrons, (iii) an outer core, at densities above \( \sim 10^{14} \text{ g cm}^{-3} \), consisting of a neutron-rich liquid with a small admixture of protons and leptons, and (iv) an inner core, whose composition is still a matter of debate (see, e.g., Ref. [1]). Throughout this paper, we will consider nucleons and leptons only. The composition of dense matter, which can impact various NS phenomena like cooling, is mainly determined by the symmetry energy. The slope of the symmetry energy is related to the pressure of neutron-matter, and is therefore related to the NS structure. More generally, the equation of state (EoS) of dense matter, i.e., the relation between the average baryon number density \( n \), the pressure \( P \) and the mass-energy density \( \rho \), is a fundamental ingredient for modeling NSs.

In this paper, we consider three different EoSs of dense matter based on generalised Skyrme functionals, which provide a unified description of all regions of a NS. Although all functionals yield the same value for the symmetry energy at saturation, each of them leads to a different prediction for the high-density behaviour of the symmetry energy. We use these EoSs to compute the mass and radius of NSs and we test our predictions against astrophysical observations. In this way, we can obtain a constraint on the symmetry energy. This constraint is compared to other estimates coming from nuclear physics.
2. Unified dense matter equations of state

Although many EoSs of dense matter have been already constructed, most of them are only applicable to specific parts of a NS. On the other hand, a thermodynamically consistent description of all regions of the star is of utmost importance for dynamical simulations since the ad hoc matching of different EoSs may trigger spurious instabilities. For this reason, a family of three different unified EoSs, referred to as BSk19, BSk20, and BSk21 respectively, and reflecting the current lack of knowledge of high-density matter, has been recently developed [2, 3, 4, 5]. These EoSs are based on the nuclear energy density functional (EDF) theory derived from generalised Skyrme effective interactions [2, 6], supplemented with a microscopic contact pairing interaction [7]. These EDFs, employed in the Hartree-Fock-Bogoliubov (HFB) framework with phenomenological collective corrections, fit the 2149 measured masses of nuclei from the 2003 Atomic Mass Evaluation [8] with a root-mean square deviation as low as 0.58 MeV for all three models. These EDFs are also constrained to reproduce three different realistic neutron matter EoSs: BSk19 (BSk21) was adjusted to a soft (stiff) EoS, while BSk20 was fitted to an EoS with an intermediate stiffness (see Ref. [2] for details). Therefore, these EDFs are very well suited for a unified treatment of NS interiors. The corresponding EoSs of the crust were obtained in Refs. [3, 4, 5]. The EoS of the core can be readily obtained from the EDF [2].

3. Symmetry energy constraints

Figure 1. Mass-radius relation of neutron stars for models BSk19, BSk20 and BSk21. The dark (light) shaded regions correspond to the 1-σ (2-σ) probability distributions implied by observations, as shown in the upper right panel of Fig. 9 of Ref. [11].

Using these EoSs, we studied the global properties of NSs. In particular, we found that the maximum masses of non-rotating NSs obtained with our EoSs BSk19, BSk20, and BSk21 are respectively 1.86 $M_\odot$, 2.15 $M_\odot$, and 2.28 $M_\odot$ [9]. The softest EoS BSk19 is incompatible with the recently measured mass of PSR J0348+0432, namely $M = 2.01 \pm 0.04 M_\odot$ [10]. This conclusion remains unchanged if rotation is taken into account [9], even for the most rapidly spinning pulsar known. The maximum NS mass mainly probes the high-density part of the dense-matter EoS. Constraints on the low-density part of the EoS can be obtained from the measurements of the NS mass and radius. In particular, our EoSs are compared with the constraint obtained from observations of three type I X-ray bursters with photospheric radius
expansion, and three transient low-mass X-ray binaries in globular clusters [11]. Once again, as shown in Fig. 1, the EoS BSk19 seems to be too soft whereas our stiffest EoS BSk21 is marginally consistent. On the other hand, the EDF underlying the soft EoS BSk19 seems to be favoured by the analysis of pion production in heavy-ion collisions. If this analysis is confirmed, it might suggest that the dense inner core of such massive NS may undergo a phase transition to non-nucleonic matter with a stiff EoS [12]. Further constraints on the nuclear-matter EoS at densities around normal density can be obtained from nuclear physics, both from theoretical calculations and from experiments. The energy per nucleon of at density \( n \) and charge asymmetry \( \eta = (n_n - n_p)/n \) can be approximately written as \( e(n, \eta) \approx e(n, \eta = 0) + e_{\text{sym}}(n)\eta^2 \). In this approximation, the symmetry energy \( e_{\text{sym}} \) is thus simply given by the difference between the energy per nucleon of neutron matter and that of symmetric nuclear matter. However, because of higher order terms in \( \eta \) this identification is generally not exact, and significant deviations can be found with increasing density. Various studies have been carried out on the symmetry energy, because of its importance in understanding both nuclear and astrophysical phenomena (see, e.g., Refs. [13, 14] and Refs. therein). The behaviour of the symmetry energy around normal density can be characterized by the coefficients \( J \equiv e_{\text{sym}}(n_0) \) and \( L \equiv 3n_0 \left. \frac{\partial e_{\text{sym}}}{\partial n} \right|_{n=n_0} \). While the values of \( J \) obtained from theoretical calculations and experiments lie in a relatively narrow range, the predicted values for \( L \) may differ widely. This coefficient is related to the pressure \( P_0 \) of neutron matter around the normal density, as \( L = 3P_0/n_0 \). Therefore, the uncertainties in \( L \) reflect the uncertainties in the neutron-matter stiffness around normal density. For our three EDFs, \( J \) was set to 30 MeV, a value which was subsequently shown to be optimum for fitting both nuclear masses and realistic neutron-matter EoSs [15]. The values of \( L \) corresponding to the EDFs BSk19, BSk20, and BSk21 are respectively 31.9 MeV, 37.4 MeV and 46.6 MeV [2]. Comparing our EoSs with astrophysical observations thus suggests that the slope \( L \) of the symmetry energy is most likely to have values around \( L \sim 37 \) MeV. This constraint should be considered as indicative given our assumptions about NS matter, as well as the uncertainties in the analysis of observations.

Various other constraints on \( J \) and \( L \) obtained from the analysis of different experiments [13, 14] have been summarised in Fig. 2. It should be pointed out that this required the use of some models so that the uncertainties may be actually larger than those shown. In addition, the constraint may change if high-order terms in the expansion of \( e(n, \eta) \) are considered. The figure shows the constraint deduced by Tsang et al. [16] from heavy-ion collisions (HIC), that derived by Chen et al. [17] from measurement of the neutron skin thickness in tin isotopes, and that obtained from the analysis of giant dipole resonance (GDR) [18, 14]. For comparison, the constraint obtained from our HFB nuclear mass models BSk9-26 as well as unpublished ones are also shown. Values of \( J \) and \( L \) have also been extracted from pygmy dipole resonances [19] and isobaric analogue states [2]. However, we have not included them in the figure because of large experimental and theoretical uncertainties (see, e.g., Refs. [21, 22]). The comprehensive analysis of symmetry energy constraints from Tsang et al. [13] includes data from isobaric analogue states and pygmy dipole resonances. Possibly for this reason, their most preferred values for \( J \) and \( L \) are higher than those found in Ref. [14], namely \( J \sim 32.5 \) MeV and \( L \sim 70 \) MeV.

4. Conclusions
We have found that fitting both nuclear masses and realistic neutron-matter EoSs using generalized Skyrme functionals leads to an optimum value of the symmetry energy coefficient around \( J = 30 \) MeV. Using unified EoSs of dense matter based on such functionals, we have computed the structure of NSs. Comparing our predictions with astrophysical observations, we have found that the most favoured value for the slope of the symmetry energy is \( L \sim 37 \) MeV, consistent with other estimates from nuclear physics.
Figure 2. Experimental constraints on the symmetry energy parameters (see text for details), taken from [14]. The dotted line represents the constraint obtained from fitting experimental nuclear masses using the Brussels-Montreal HFB models with a root-mean-square deviation below 0.84 MeV (best fits are for \( J = 30 \) MeV).

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5. References
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