Triaxial projected shell model study of $\gamma$-vibrational bands in even-even Er isotopes

J. A. Sheikh$^1$, G. H. Bhat$^1$, Y. Sun$^{2,3}$, G. B. Vakil$^1$, and R. Palit$^4$

$^1$Department of Physics, University of Kashmir, Srinagar, 190 006, India
$^2$Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China
$^3$Joint Institute for Nuclear Astrophysics, University of Notre Dame, Notre Dame, Indiana 46556, USA
$^4$Tata Institute of Fundamental Research, Colaba, Mumbai, 400 005, India

We expand the triaxial projected shell model basis to include triaxially-deformed multi-quasiparticle states. This allows us to study the yrast and $\gamma$-vibrational bands up to high spins for both $\gamma$-soft and well-deformed nuclei. As the first application, a systematic study of the high-spin states in Er-isotopes is performed. The calculated yrast and $\gamma$-bands are compared with the known experimental data, and it is shown that the agreement between theory and experiment is quite satisfactory. The calculation leads to predictions for bands based on one- and two-$\gamma$ phonon where current data are still sparse. It is observed that $\gamma$-bands for neutron-deficient isotopes of $^{156}$Er and $^{158}$Er are close to the yrast band, and further these bands are predicted to be nearly degenerate for high-spin states.

PACS numbers: 21.60.Cs, 21.10.Hw, 21.10.Ky, 27.50.+e

I. INTRODUCTION

Recent experimental advances in nuclear spectroscopic techniques following Coulomb excitations, in-elastic neutron scattering, and thermal neutron capture have made it possible to carry out a detailed investigation of $\gamma$-vibrational bands in atomic nuclei $^{1,2,3}$. These bands are observed in both spherical and as well as in deformed nuclei. In spherical nuclei, the vibrational modes are well described using the harmonic phonon model $^{4,5}$. Although exact harmonic motion has never been observed, there are numerous examples of nuclei exhibiting near harmonic vibrational motion. As a matter of fact, one- and two-phonon excitations have been reported in a large class of spherical nuclei. In deformed nuclei, vibrational motion is possible around the equilibrium of deformed shape configuration. The deformed intrinsic shape is parameterized in terms of $\beta$ and $\gamma$ deformation variables. These parameters are related to the axial and non-axial shapes of a deformed nucleus. The one-phonon vibrational mode in deformed nuclei with no component of angular momentum along the symmetry axis ($K = 0$) is called $\beta$-vibration and the vibrational mode with component of angular momentum along the symmetry axis ($K = 2$) is referred to as $\gamma$-vibration. The rotational bands based on the $\gamma$-vibrational state are known as $\gamma$-bands $^{3,5,6}$. One-phonon $\gamma$-bands have been observed in numerous deformed nuclei in most of the regions of the periodic table. There has also been reports on observation of two-phonon $\gamma$-bands $^{7,10}$.

Several theoretical models have been proposed to study $\gamma$-bands with varying degree of success. The quasiparticle phonon nuclear model (QPNM) $^{11,12}$, which restricts the basis to, at the most, two phonon states, has led to the conclusion that two-phonon collective vibrational excitations cannot exist in deformed nuclei due to the Pauli blocking of important quasiparticle components. On the other hand, the multi-phonon method (MPM) $^{13,14}$ embodies an entirely different truncation scheme. It employs only a few collective phonons and restricts the basis to all the corresponding multi-phonon states up to eight phonons. This approach predicts that, for strongly collective vibrations, two phonon $K^\pi = 4^+$ excitations should appear at an energy of about 2.6 times the energy of the one-phonon $K^\pi = 2^+$ state $^{3,15}$. On the other hand, the dynamic deformation model (DDM) $^{16}$, which is quite different from the models mentioned above, constructs collective potential from a set of deformed single-particle basis states accommodating eight major oscillator shells. This model predicts a collective $K^\pi = 4^+$ at almost 2 MeV.

All the above mentioned models (QPNM, MPM, and DDM) do not have their wave functions as eigen-states of angular momentum. Strictly speaking, these methods do not calculate the states of angular momentum, but the $K$-states ($K$ is the projection of angular momentum on the intrinsic symmetry axis). To apply these models, one has to assume that $I \approx K$. However, since an intrinsic $K$-state can generally have its components spread over the space of angular momenta of $I \geq K$, the reliability of these approaches depends critically on actual situation. As pointed out by Soloviev $^{12}$, it is quite desirable to recover the good angular-momentum in the wave functions.

Some algebraic models including the extended version of the interacting boson (sdg-IBM) $^{17,18}$ and pseudo-symplectic models $^{19}$ have also been employed to study the $\gamma$-excitation modes and predict high collectivity for the double $\gamma$-vibration $^{20}$.

Recently, the triaxial projected shell model (TPSM) has been employed to describe $\gamma$-bands $^{21,22}$. This model uses shell model diagonalization approach and in this sense, it is similar to the conventional shell model approach except that the basis states in the TPSM are triaxially deformed rather than spherical. In the present version of the model, the intrinsic deformed basis is constructed from the triaxial Nilsson potential. The good angular momentum states are then obtained.
through exact three-dimensional angular momentum projection technique. In the final stage, the configuration mixing is performed by pairing plus quadrupole-quadrupole Hamiltonian in the projected basis \[23, 24\]. The advantage of the TPSM is that it describes the deformed single-particle states microscopically as in QPNM, MPM, and DDM, but its total many-body states are exact eigen states of angular momentum operator. Correlations beyond the mean-field are introduced by mixing the projected configurations.

It is to be noted that an intrinsic triaxial state in the TPSM is a rich superposition of different \(K\)-states. For instance, the triaxial deformed vacuum state is composed of \(K = 0, 2, 4, \cdots\) configurations. The projected bands from these \(K = 0, 2, 4\) and 4 intrinsic states are the dominant components of the ground-, \(\gamma\), and \(2\gamma\)-bands, respectively \[22\].

In the earlier TPSM analysis for even-even nuclei, the shell model space was very restrictive, including only 0-qp states \[21, 22, 23, 26, 27, 28\]. This strongly limited the application of the TPSM to the low-spin and low-excitation region only. It was not possible to study high-spin states because multi-qp configurations will usually become important for states with \(I > 10\) in the normally deformed rare-earth nuclei. In the present work, the qp-space is enlarged to incorporate the two-neutron-qp, two-proton-qp and four-qp configuration consisting of two protons plus two neutrons. This large qp space is adequate to describe the bands up to second bandcrossing \[24\]. The purpose of the present work is, as a first application of the extended model, to perform a detailed investigation of the high-spin band structures, in particular \(\gamma\)-bands, of Erbium isotopes ranging from mass number \(A = 156\) to 170. In a parallel work \[24\], the TPSM analysis for odd-odd nuclei in a multi-qp space has been performed.

The manuscript is organized in the following manner: in the next section, a brief description of the TPSM method is presented. The results of the TPSM study are presented and discussed in section III. Finally, the work is summarized in section IV.

II. TRIAXIAL PROJECTED SHELL MODEL APPROACH

In the present work, the TPSM qp basis is extended, which consists of projected 0-qp vacuum, 2-proton (2p), 2-neutron (2n), and 4-qp states, i.e.,

\[
\hat{P}_{MK}^I | \Phi >, \\
\hat{P}_{MK}^I a_{p_1}^\dagger a_{p_2}^\dagger | \Phi >, \\
\hat{P}_{MK}^I a_{n_1}^\dagger a_{n_2}^\dagger | \Phi >, \\
\hat{P}_{MK}^I a_{p_1}^\dagger a_{p_2}^\dagger a_{n_1}^\dagger a_{n_2}^\dagger | \Phi >.
\]

In Eq. (1), the three-dimensional angular-momentum operator is \[30\]

\[
\hat{P}_{MK}^I = \frac{2I + 1}{8\pi^2} \int d\Omega D_{MK}^I(\Omega) \hat{R}(\Omega),
\]

with the rotational operator

\[
\hat{R}(\Omega) = e^{-i\alpha J_x} e^{-i\beta J_y} e^{-i\gamma J_z},
\]

and \(|\Phi >\) represents the triaxial qp vacuum state. The qp basis chosen above are adequate to describe the high-spin states up to, say \(I \sim 24\), and in the present analysis we shall restrict to this spin regime. The triaxially deformed qp states are generated by the Nilsson Hamiltonian

\[
\hat{H}_N = \hat{H}_0 - \frac{2}{3} \hbar \omega \left\{ \epsilon \hat{Q}_0 + \epsilon' \hat{Q}_2 + \frac{\hat{Q}_4 - 2}{\sqrt{2}} \right\}
\]

Here \(\hat{H}_0\) is the spherical single-particle Hamiltonian, which contains a proper spin-orbit force \[31\]. The parameters \(\epsilon\) and \(\epsilon'\) describe axial quadrupole and triaxial deformations, respectively. It should be noted that for the case of axial-symmetry, the qp vacuum state has \(K = 0\), whereas in the present case of triaxial deformation, the vacuum state \(|\Phi >\) is a superposition of all the possible \(K\)-values. The allowed values of the \(K\)-quantum number for a given intrinsic state are obtained through the following symmetry consideration. For the symmetry operator, \(S = e^{-i\pi J_x}\), we have

\[
\hat{P}_{MK}^I | \Phi > = \hat{P}_{MK}^I \hat{S} \hat{S}^\dagger | \Phi > = e^{i\pi(K-\kappa)} \hat{P}_{MK}^I | \Phi >,
\]

where \(\hat{S} | \Phi > = e^{-i\pi \kappa} | \Phi >\), and \(\kappa\) characterizes the intrinsic states in Eq. (1). For the self-conjugate vacuum or 0-qp state, \(\kappa = 0\) and, therefore, it follows from the above equation that only \(K = even\) values are permitted for this state. For 2-qp states, the possible values for \(K\)-quantum number are both even and odd depending on the structure of the qp state. For the 2-qp state formed from the combination of the normal and the time-reversed states, \(\kappa = 0\) and, therefore, only \(K = even\) values are permitted. For the combination of the two non-parity states, \(\kappa = 1\) and only \(K = odd\) states are permitted.

As in the earlier projected shell model (PSM) calculations, we use the pairing plus quadrupole-quadrupole Hamiltonian \[23\]

\[
\hat{H} = \hat{H}_0 - \frac{1}{2} \chi \sum_{\mu} \hat{Q}_\mu^\dagger \hat{Q}_\mu - G_M \hat{P}^\dagger \hat{P} - G_Q \sum_{\mu} \hat{P}_{\mu}^\dagger \hat{P}_{\mu}
\]

The interaction strengths are taken as follows: The \(QQ\)-force strength \(\chi\) is adjusted such that the physical quadrupole deformation \(\epsilon\) is obtained as a result of the self-consistent mean-field HFB calculation \[23\]. The monopole pairing strength \(G_M\) is of the standard form

\[
G_M = [21.24 \mp 13.86(N - Z)/A] / A,
\]
with “−” for neutrons and “+” for protons, which approximately reproduces the observed odd–even mass differences in the rare-earth mass region. This choice of $G_M$ is appropriate for the single-particle space employed in the PSM, where three major shells are used for each type of nucleons ($N = 4, 5, 6$ for neutrons and $N = 3, 4, 5$ for protons). The quadrupole pairing strength $G_Q$ is assumed to be proportional to $G_M$, and the proportionality constant being fixed as 0.18. These interaction strengths are consistent with those used earlier for the same mass region [21, 22, 23].

III. RESULTS AND DISCUSSIONS

The triaxial projected shell model calculations have been performed for Er-isotopes ranging from $A = 156$ to 170. The deformation parameters ($\epsilon, \epsilon'$) used in the present work are same as those employed in Ref. [22]. It has already been mentioned in section II that in the present work the mean-field potential is constructed with given input deformation values of $\epsilon$ and $\epsilon'$. In a more realistic calculation, these deformation values for a given system are obtained through the variational HFB calculations. The chosen values of $\epsilon$ for the present calculation are those from the measured quadrupole deformations of the nuclei as is done in the previous projected shell model analysis. The $\epsilon'$ values used in the present work are realistic, which correctly reproduce, for example, excitations of the $\gamma$ band relative to the ground-state [22].

A. Band Diagrams

Band diagrams can bring valuable information regarding the underlying physics [23]. These band diagrams for the studied Er-isotopes are presented in Figs. 1 to 4 and depict the results of the projected energies for each intrinsic configuration. In the diagrams, the projected energies are shown for 0, 2n, 2p and 2p+2n quasiparticle configurations. The qp energies for these configurations are given in the legend of each figure. As already mentioned in the last section that with the triaxial basis, the intrinsic states do not have a well-defined $K$-quantum number. Each triaxial configuration in Eq. (1) is a composition of several $K$-values and bands in Figs. 1 to 4 are obtained by assigning a given $K$-value in the angular-momentum projection operator. To make the discussion easy, we denote a $K$-state of an i-configuration as $(K,i)$, with $i = 0, 2n, 2p$ and 4. For example, the $K = 0$ state of 0-qp configuration is marked as $(0,0)$ and $K = 1$ of 2n-qp configuration as $(1,2n)$

In Figs. 1 to 4, the projected bands associated with the 0-qp configuration are shown for $K = 0, 2$, and 4, namely the $(0,0), (2,0)$, and $(4,0)$ bands. In the literature, these $K = 0, 2$, and 4 bands are referred to as ground-state, $\gamma$-, and $2\gamma$-bands. The ground-state band has $\kappa = 0$ and is, therefore, comprised of only even-$K$ values. We use the same names in the following discussion to be consistent with the literature, but stress that in our final results obtained after diagonalisation, $K$ is not a strictly conserved quantum number due to configuration mixing.

It is evident from Fig. 1 that the $(2,0)$ bands for $^{156}\text{Er}$ and $^{158}\text{Er}$ lie very close to the $(0,0)$ bands. This means that $\gamma$-vibration has low excitation energy in these two nuclei. For high-spin states, it is further noted that the $(0,0)$ and $(2,0)$ band energies become almost degenerate, and as a matter fact for $I = 16$ and above, the energy of even-spin states in the $(2,0)$ band is slightly lower than the $(0,0)$ band. It is a well-known fact that $\gamma$-bands become lower in energy with increasing triaxiality and what is also evident from Fig. 1 that they become favored with increasing angular-momentum. As can be seen from Fig. 1, the $(2,0)$ bands in $^{156}\text{Er}$ and $^{158}\text{Er}$ also depict pronounced signature splitting with the splitting amplitude increasing with spin. The $(4,0)$ band is close to the $(2,0)$ band for $^{156}\text{Er}$ and lies at a slightly higher excitation energy for $^{158}\text{Er}$. The $(4,0)$ bands in these two isotopes are also noted to have signature splitting for higher angular momenta, and the splitting amplitude is nearly the same for the $(2,0)$ and $(4,0)$ bands.

In Fig. 1, several representative multi-qp bands, namely projected 2- and 4-qp configurations, are also plotted. Although the $K = 1$ 2-qp neutron $(1,2n)$ and 2-qp proton $(1,2p)$ bands are close in energy for low spins, but with increasing spin the 2n-qp bands are lower in energy than 2p-qp bands due to larger rotational alignment. It is noted that neutrons are occupying $1h_{13/2}$ and protons are occupying $1h_{11/2}$ intruder sub-shells. For each of the $(1,2n)$ and $(1,2p)$ bands, the projected energies are also shown for the corresponding $\gamma$-bands with configurations $(3,2n)$ and $(3,2p)$. The $(1,2n)$ band is noted to cross the $(2,0)$ and the $(0,0)$ bands at $I = 12$. It is also seen that the $(3,2n)$ band crosses the $(0,0)$ band at a slightly higher spin value of $I = 14$. It is interesting to note that after the band crossing, the lowest even-spin states originate from the $(1,2n)$ band, whereas the odd-spin members are the projected states from the $(3,2n)$ configuration. Finally, the 4-qp $(4,4)$ configuration lies at high excitation energies and does not become yrast, at-least, up to the spin values shown in the figure.

The band diagrams for $^{160}\text{Er}$ and $^{162}\text{Er}$ are presented in Fig. 2. The energy separation between the $(0,0)$ and $(2,0)$ bands is larger as compared to the two lighter isotopes in Fig. 1. In the case of $^{160}\text{Er}$, the $(2,0)$ band energies do come close to the $(0,0)$ energies for spins $I > 12$. The $(1,2n)$ band again crosses the $(0,0)$ band at $I = 12$ for $^{160}\text{Er}$ and at $I = 14$ for $^{162}\text{Er}$. The band diagrams for $^{164}\text{Er}$ and $^{166}\text{Er}$ shown in Fig. 3 depict larger energy gaps among various bands. The signature splitting of the $(2,0)$ band has considerably reduced. It is further noted that 2n-band-crossing is shifted to higher spin values. For the case of $^{164}\text{Er}$, the band crossing is observed to occur at $I = 16$ and for $^{166}\text{Er}$ it occurs at $I = 18$. The band diagrams for $^{168}\text{Er}$ and $^{170}\text{Er}$ shown in Fig. 4 indicate that the $(2,0)$ bands are quite high in excitation energy.
The band crossing for these cases is further shifted to higher spin values.

B. Results after Configuration Mixing

In the second stage of the calculation, the projected states obtained above are employed to diagonalize the shell model Hamiltonian of Eq. (4). It is to be mentioned that for the discussion purpose, only the lowest three bands from the 0-qp configuration and lowest two bands for each other configuration have been shown in band diagram, Figs. 1 to 4. However, in the diagonalisation of the Hamiltonian, the basis states employed are much more, which includes, for example, those $K = 1, 3, 5$ and 7 with $\kappa = 1$ and $K = 0, 2, 4, 6$ and 8 with $\kappa = 0$.

The lowest three bands after the configuration mixing are shown in Figs. 5 and 6 and are compared with the experimental energies wherever available. Although they are of mixed configurations in our model, we still call them yrast, $\gamma$- and $2\gamma$-bands to be consistent with the literature. It is observed from these two figures that the agreement between the calculated and the experimental energies for the yrast and $\gamma$-bands is quite satisfactory. For $^{156-164}$Er, the theoretical yrast line depicts two slopes and these correspond to the slopes of two crossing bands shown in Figs. 1 and 4. This also indicates that the interaction between the two crossing bands is small with the result that these nuclei shall depict a back-bending effect [24]. It is also encouraging to note from Figs. 5 and 6 that the agreement for the $\gamma$-bands is quite good, except that for $^{164}$Er and $^{170}$Er, the signature splitting at the top of the bands is not reproduced properly. For the $2\gamma$-bands, our calculations agree well with the only available data in $^{166}$Er [9] and $^{168}$Er [10].

There is another notable effect about anharmonicity in $\gamma$ vibrations. If we regard the $\gamma$-bandhead as one $\gamma$-phonon vibration and the $2\gamma$-bandhead as two $\gamma$-phonon vibration, it can be easily seen from Figs. 5 and 6 that the vibration is not perfectly harmonic. In fact, in the two lightest isotopes, the $\gamma$-soft $^{156}$Er and $^{158}$Er, the vibration is almost harmonic. As the neutron number increases, a clear anharmonicity is predicted from our calculation and the degree of anharmonicity increases with increasing neutron number.

C. Analysis of Wavefunction

In order to probe further the structure of the bands presented in Figs. 5 and 6, the wavefunction decomposition of the yrast, $\gamma$- and $2\gamma$-bands are shown in Figs. 7, 8 and 9 for $^{156}$Er, $^{164}$Er and $^{170}$Er. For other nuclei, the wavefunction have similar structure and are not presented. It is seen from Fig. 7 that the yrast band for $^{156}$Er is predominantly composed of the (0, 0) configuration up to $I = 10$. The (0, 0) contribution suddenly drops at $I = 10$, and (1, 2n) configuration becomes dominant from $I = 12$ to 16. For $I = 18$ and onwards, there are many configurations with finite values contributing to the yrast states. The band diagram of $^{156}$Er in Fig. 1 suggests that the $\gamma$-band should have the (2, 0) configuration as the dominant component. This is evident from Fig. 7 and it is also noted that (0, 0) is significant for the even spin states up to $I = 8$. The $I = 10$ state is mostly composed of (1, 2n) and for higher spin states the (3, 2n) and (3, 2p) configurations are the dominant components of the $\gamma$ band. The $2\gamma$ band in Fig. 7 is composed of (4, 0) band for the low spin states. $I = 8$ of this band is predominantly composed of the (1, 2n) configuration, but the high spin states are found to have quite a complex structure.

The yrast wavefunction decomposition of $^{164}$Er, shown in the top panel of Fig. 8, indicates that this lowest band is predominantly composed of the (0, 0) configuration up to $I = 12$ and there appears to be very small admixtures of $K = 2$ and other configurations. After the bandcrossing at $I = 16$, the yrast states are dominated by the $(1, 2n)$ configuration. There is also a significant contribution of the $(3, 2n)$ configuration after the bandcrossing. The $\gamma$-band in Fig. 8 is primarily composed of the (2, 0) configuration up to $I = 11$ and above this spin the states are a mixture of different configurations. There is a clear distinction in the composition of the even- and odd-spin states above $I = 11$. The odd-spin states are composed of the $(3, 2n)$ and $(2, 0)$ configurations, and the even-spin states are dominated by the $(1, 2n)$ and $(0, 0)$ structures. The $2\gamma$-band up to $I = 7$ is primarily the $(4, 0)$ configuration. For $I = 8$ and above, this band is a mixture of $(1, 2n)$ and $(3, 2n)$ configurations.

The wavefunction analysis of $^{170}$Er shown in Fig. 9 indicates that the yrast state, as expected for a well deformed nuclei, is mainly comprised of the (0, 0) configuration. This contribution drops smoothly and, on the other hand, the (1, 2n) component increases steadily. For $I = 20$, it is noted that the (0, 0) and (1, 2n) contributions are almost identical and above this spin value, it is expected that the (1, 2n) configuration shall dominate the yrast states. The $\gamma$-band is also noted to have a well defined structure of (2, 0) and only for high spin states, it is observed that the (1, 2n) and (3, 2n) of the 2n-aligned configuration become important. The $2\gamma$-band is dominated mostly by the aligning configurations above $I = 7$. As is evident from the band diagram of this nucleus, presented in Fig. 4, that 2n-aligned band is lower than the $(4, 0)$ band for most of the spin values.

IV. SUMMARY AND CONCLUSIONS

In the present work, the triaxial projected shell model approach with extended basis has been employed to study the high-spin band structures of the Er-isotopes from $A = 156$ to 170. In this model, the Hamiltonian employed consists of pairing plus quadrupole-quadrupole interaction. It is known that Nilsson deformed potential
is the mean-field of the quadrupole-quadrupole interaction and this potential is directly used as the Hartree-Fock field rather than performing the variational calculations. It is, as a matter of fact, quite appropriate to use the Nilsson states as a starting basis because the parameters of this potential have been fitted to large body of experimental data. The parameters of the model are the deformation parameters of $\epsilon$ and $\epsilon'$. The axial deformation parameter $\epsilon$ has been fixed from the observed quadrupole deformation of the system as is done in most of the projected shell model analysis. The non-axial parameter $\epsilon'$ was chosen to reproduce the bandhead of the $\gamma$ band. The pairing strength parameters have been determined to reproduce the odd-even mass differences. The monopole pairing interaction has been solved in the BCS approximation and the qp states generated. In the present work, the qp states considered are: 0-qp, 2-qp neutron, 2-qp proton, and the 4-qp state of 2-neutron plus 2-proton.

In the second stage of the calculations, the three-dimensional angular-momentum projection is performed to project out the good angular-momentum states from these qp states. These projected states are then used as the basis to diagonalise the shell model Hamiltonian in the third and the final stage. The salient features of results obtained in the present work are:

1. $\gamma$-bands are quite close to the yrast line for the neutron-deficient Er-isotopes, in particular, for $^{156}$Er and $^{158}$Er. It is further evident from the present results that these $\gamma$-states become further lower in energies for high-spin states. As a matter of fact, for $^{156}$Er and $^{158}$Er, they become lower than the ground-state band for $I > 14$. We propose that this is a feature of $\gamma$-soft nuclei.

2. $\gamma$-bands are pushed up in energy with increasing neutron number, and further the degree of anharmonicity of $\gamma$ vibration also increases.

3. The wavefunction decomposition of the bands demonstrates that for neutron deficient Er-isotopes, there is a significant mixture of the $\gamma$ configuration in the ground-state band and vice-versa. The neutron rich $^{170}$Er nucleus, on the other hand, has the intrinsic structures as expected for a well deformed nucleus with the ground-state band composed of nearly pure $K = 0$ configuration.

Y.S. is supported by the Chinese Major State Basic Research Development Program through grant 2007CB815005, and by the U. S. National Science Foundation through grant PHY-0216783.

---

[1] C. Fahlander, A. Baklin, L. Hasselgren, A. Kavka, V. Mittal, L. E. Svensson, B. Varnejsting, D. Cline, B. Kotlinski, H. Grein, E. Grosse, R. Kulessaa, C. Michel, W. Spreng, H. j. Wollersheim, and J. Stachel, Nucl. Phys. A 485, 327 (1988).
[2] T. Belgya, G. Molnar, and S. W. Yates, Nucl. Phys. A 607, 43 (1996).
[3] H. G. Borner and J. Joli, J. Phys. G 19, 217, (1993).
[4] A. Bohr and B. R. Mottelson, Nuclear Structure, Vol. 2 (Benjamin Inc., New York, 1975).
[5] R. F. Casten, Nuclear Structure from a Simple Perspective, Second Edition (Oxford University Press, 2000).
[6] A. Guessous, N. Schulz, W. R. Phillips, I. Ahmad, M. Bentaleb, J. L. Durell, M. A. Jones, M. Leddy, E. Lukkiewicz, L. R. Morss, R. Piepenbring, G. Smith, W. Urban, and B. J. Varley, Phys. Rev. Lett. 75, 2280 (1995).
[7] X. Wu, A. Aprahamian, S. M. Fischer, W. Reviol, G. Liu, and J. X. Saladin, Phys. Rev. C 49, 1837 (1993).
[8] H. G. Borner, J. Jolie, S. J. Robinson, B. Krusche, R. Piepenbring, R. F. Lasten, A. Aprahamian, and J. P. Draayer, Phys. Rev. Lett. 66, 691 (1991).
[9] C. Fahlander, A. Axelsson, M. Heinebrodt, T. Härlein, and D. Schwalm, Phys. Lett. B 388, 475 (1996).
[10] T. Härlein, M. Heinebrodt, D. Schwalm, C. Fahlander, Eur. Phys. J. A 2, 253 (1998).
[11] V. G. Soloviev and N. Yu. Shirikova, Z. Phys. A 301, 263 (1981).
[12] V. G. Soloviev, Theory of Atomic Nuclei: Quasiparticles and Phonons (IOP, London, 1992).
[13] J. Leandri and R. Piepenbring, Phys. Rev. C 37, 2779 (1988).
[14] M. K. Jannmari and R. Piepenbring, Nucl. Phys. A 487, 77 (1988).
[15] D. G. Burke and P. C. Sood, Phys. Rev. C 51, 3525 (1994).
[16] K. Kumar, Nuclear Models and Search for unity in Nuclear Physics (Universitetforlaget, Bergen, Norway, 1984).
[17] A. Arima and F. Iachello, Phys. Rev. Lett. 35 1069 (1975).
[18] N. Yoshinaga, Y. Akiyama, and A. Arima, Phys. Rev. 56, 1116 (1986).
[19] O. Castanos, J. P. Draayer, and Y. Leschber, Ann. Phys. (New York) 180, 290 (1987).
[20] P. E. Garrett, M. Kadi, Min Li, C. A. McGrath, V. Sorokin, M. Yehand, and S. W. Yates, Phys. Rev. Lett. 78, 4545 (1996).
[21] J. A. Sheikh and K. Hara, Phys. Rev. Lett. 82, 3968 (1999).
[22] Y. Sun, K. Hara, J. A. Sheikh, J. G. Hirsch, V. Velazquez, and M. Guidry, Phys. Rev. C 61, 064323 (2000).
[23] K. Hara and Y. Sun, Int. J. Mod. Phys. E 4, 637 (1995)
[24] K. Hara and Y. Sun, Nucl. Phys. A 529, 445 (1991)
[25] J. A. Sheikh, Y. Sun, and R. Palit, Phys. Lett. B 507, 115 (2001).
[26] Y. Sun, J. A. Sheikh, and G.-L. Long, Phys. Lett. B 533, 253 (2002).
[27] P. Boutachkov, A. Aprahamian, Y. Sun, J. A. Sheikh, S. Frauendorf, Eur. Phys. J. A 15, 455 (2002).
[28] Y. Sun, G.-L. Long, F. Al-Khudair, and J. A. Sheikh, submitted.
[29] Z.-C. Gao, Y.-S. Chen, and Y. Sun, Phys. Lett. B 634, 195 (2006).
[30] P. Ring and P. Schuck, The Nuclear Many Body Problem (Springer, New York, 1980).
[31] S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wycech, C. Gustafson, I. Lamm, P. Moller, and B. Nilsson, Nucl. Phys. A 131, 1 (1969).
[32] Y. Sun, P. Ring, and R. S. Nikam, Z. Phys. A 339, 51 (1991).
FIG. 1: Band diagrams for $^{156-158}$Er isotopes. The labels (0,0), (2,0), (4,0), (1,2n), (3,2n), (1,2p), (3,2p), (2,4) and (4,4) correspond to ground, $\gamma$, 2$\gamma$, two neutron-aligned, $\gamma$-band on this two neutron-aligned state, two proton-aligned, $\gamma$-band on two this proton-aligned state, two-neutron plus two-proton aligned band and $\gamma$ band built on this four-quasiparticle state.
FIG. 2: Band diagrams for $^{160-162}$Er isotopes. The labels indicate the bands mentioned in the caption of Fig. 1.
FIG. 3: Band diagrams for $^{164-166}$Er isotopes. The labels indicate the bands mentioned in the caption of Fig. 1.
FIG. 4: Band diagrams for $^{168−170}$Er isotopes. The labels indicate the bands mentioned in the caption of Fig. 1.
FIG. 5: Comparison of experimental and the calculated band energies for $^{156-162}$Er.
FIG. 6: Comparison of experimental and the calculated band energies for $^{164-170}\text{Er}$. 
FIG. 7: Wavefunction decomposition for $^{156}$Er. $a_K$ denotes the amplitude of the wavefunction in terms of the projected basis states.
FIG. 8: Wavefunction decomposition for $^{164}$Er. $a_K$ denotes the amplitude of the wavefunction in terms of the projected basis states.
FIG. 9: Wavefunction decomposition for $^{170}$Er. $a_K$ denotes the amplitude of the wavefunction in terms of the projected basis states.