New directions for gravitational wave physics via “Millikan oil drops”*

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Editors’ note: The following chapter has been the subject of considerable controversy during the review process. Although one reviewer found the claim that gravitational waves could be detected or generated by the method proposed in this paper to be reasonable, the other reviewers found this claim to be highly questionable. In particular, the reviewers and editors believe that some statements in the paper may be inconsistent with the current theory of superfluids. However, that theory could be wrong, and Dr. Chiao’s innovative work proposes an experiment based on an alternative view. We hope that its publication here will stimulate the sort of discussion that leads to scientific progress.

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And God said, “Let there be light,” and there was light. (Gen. 1:3)

1 Introduction

In this book in honor of my beloved teacher, colleague, and friend for over four decades, Professor Charles Hard Townes, I would like to take a fresh look at an old problem we had discussed on many occasions, going back to the days when I was his graduate student at MIT. After a visit to Joseph Weber’s laboratory at the University of Maryland in the 1960s, I can still remember his critical remarks concerning the experiments then being conducted in Weber’s lab using large, massive aluminum bars. He expressed concerns that the numbers that he calculated indicated that it would be extremely difficult to see any observable
effects, and he was therefore worried that Weber would not be able to see any genuine signal. Later, he expressed to me his similar worries about LIGO, especially in light of its large scale and expense.

Here I would like to revisit the problem of generating gravitational radiation, which has many similarities to that of generating electromagnetic radiation. The famous work of Gordon, Zeiger, and Townes on the maser opened up entirely new directions in coherent electromagnetic wave research by generating coherent microwaves by means of the quantum mechanical principle of the stimulated emission of radiation.

Are there new ideas that might stimulate similar developments that would open up new directions in gravitational wave research? I would like to explore here situations in which the principle of reciprocity (i.e., time-reversal symmetry) demands the existence of nonnegligible quantum back-actions of a measuring device on the gravitational radiation fields that are being measured in a quantum mechanical context. I believe that such quantum back-actions may allow the generation of gravitational waves.

The quantum approach taken here is in stark contrast to the classical, test-particle approaches being taken in contemporary, large-scale gravitational wave experiments, which are based solely on classical physics. The back-actions of classical measuring devices such as Weber bars and large laser interferometers on the incident gravitational fields that are being measured are completely negligible. Hence, they can only passively detect gravitational waves from powerful astronomical sources such as supernovae [1], but they certainly cannot generate these waves.

Specifically, I would like to explore here the quantum physics of Planck-mass-scale “Millikan oil drops” consisting of electron-coated superfluid helium drops at millikelvin-scale temperatures in the presence of tesla-scale magnetic fields, as a means to test whether or not some of the large quantum back-action effects predicted here exist.

Recently, our ideas have shifted from the use of superfluid helium drops to the more practical use of magnetically levitated, electrically charged superconducting spheres, whose scattering cross section for an incoming gravitational wave is predicted to be enormously enhanced over that for normal, classical matter by 42 orders of magnitude [2, 3].

This enormous enhancement factor arises from the ratio of the electrostatic force to the gravitational force between two electrons and is a necessary consequence of the uncertainty principle. When this basic quantum principle is applied to the motion of Cooper pairs in a superconductor in the presence of a gravitational wave, supercurrents will result because the uncertainty principle trumps the equivalence principle whenever decoherence is prevented from occurring due to the Bardeen, Cooper, and Schrieffer (BCS) energy gap [4]. These currents lead to an enormous back-action on the wave, which arises from the
Coulomb force between the ensuing separated charges that strongly oppose the gravitational tidal force of the incoming wave. Quantum back-actions are thus predicted to lead in this case to a mirror-like reflection of the wave.

I am in the process of performing some of these experiments with my colleagues at the new tenth campus of the University of California at Merced in order to test some of these ideas. These quantum experiments have become practical to perform because of important advances in ultra–low-temperature dilution refrigerator technology. I will describe some of these experiments below.

### 2 Forces of gravity and of electricity between two electrons

Let us first consider, using only classical, Newtonian concepts (which are valid in the correspondence principle limit and at large distances asymptotically, as seen by a distant observer), the forces experienced by two electrons separated by a distance $r$ in the vacuum. Both the gravitational force and the electrical force obey long-range, inverse-square laws. Newton’s law of gravitation states that

$$|F_G| = \frac{Gm_e^2}{r^2}$$  \hspace{1cm} (1)

where $G$ is Newton’s constant and $m_e$ is the mass of the electron, and Coulomb’s law states that

$$|F_e| = \frac{e^2}{4\pi\varepsilon_0 r^2}$$  \hspace{1cm} (2)

where $e$ is the charge of the electron and $\varepsilon_0$ is the permittivity of free space (I shall use SI units throughout this paper except in Appendix B). The electrical force between two electrons is repulsive, but the gravitational force is attractive.

Taking the ratio of these two forces, one obtains the dimensionless ratio of fundamental coupling constants

$$\frac{|F_G|}{|F_e|} = \frac{4\pi\varepsilon_0 Gm_e^2}{e^2} \approx 2.4 \times 10^{-43}$$  \hspace{1cm} (3)

The gravitational force is extremely small compared to the electrical force and is therefore usually ignored in all treatments of quantum physics. However, it turns out that this force cannot be ignored in the case of a superconductor interacting with an incident gravitational wave [2, 3].

### 3 Gravitational and electromagnetic radiation powers emitted by two electrons

The above ratio of the fundamental coupling constants $4\pi\varepsilon_0 Gm_e^2/e^2$ is also the ratio of the powers of gravitational (GR) to electromagnetic (EM) radiation emitted by two electrons separated by a distance $r$ in the vacuum, when they
undergo an acceleration \( a \) and are moving with a speed \( v \) relative to each other, as seen by a distant observer.

From the equivalence principle, it follows that dipolar gravitational radiation does not exist \([1]\). Rather, the lowest order of symmetry of radiation permitted by this principle is quadrupolar. General relativity predicts that the power \( P_{\text{GR}}^{(\text{quad})} \) radiated by a time-varying mass quadrupole tensor \( D_{ij} \) of a periodic system is given by \([1]\) [3] [6]

\[
P_{\text{GR}}^{(\text{quad})} = \frac{G}{45c^5} \left\langle \frac{\dddot{D}_{ij}^2}{\omega^6} \right\rangle = \omega^6 \frac{G}{45c^5} \left\langle D_{ij}^2 \right\rangle
\]  

(4)

where the triple dots over \( \dddot{D}_{ij} \) denote the third derivative with respect to time of the mass quadrupole moment tensor \( D_{ij} \) of the system (the Einstein summation convention over the spatial indices \((i, j)\) for the term \( D_{ij}^2 \) is being used here), \( \omega \) is the angular frequency of the periodic motion of the system, and the angular brackets denote time averaging over one period of the motion.

Applying this formula to the periodic orbital motion of two point masses with equal mass \( m \) moving with a relative instantaneous acceleration whose magnitude is given by \( |a| = \omega^2 |D| \), where \( |D| \) is the magnitude of the relative displacement of these objects, and where the relative instantaneous speed of the two masses is given by \( |v| = \omega |D| \) (where \( v \ll c \)), with all these quantities being measured by a distant observer, one finds that Equation (4) can be rewritten as follows:

\[
P_{\text{GR}}^{(\text{quad})} = \kappa \frac{2}{3} \frac{Gm^2}{c^3} a^2 \quad \text{where} \quad \kappa = 2 \frac{v^2}{15c^2}
\]  

(5)

The frequency dependence of the radiated power predicted by Equation (5) scales as \( v^2 a^2 \sim \omega^6 \), in agreement with triple dot term \( \dddot{D}_{ij}^2 \) in Equation (4). It should be stressed that the values of the quantities \( a \) and \( v \) are those being measured by an observer at infinity. The validity of Equations (4) and (5) has been verified by observations of the orbital decay of the binary pulsar PSR 1913+16 \([7]\).

Now consider the radiation emitted by two electrons undergoing an acceleration \( a \) relative to each other with a relative speed \( v \), as observed by an observer at infinity. For example, these two electrons could be attached to the two ends of a massless, rigid rod rotating around the center of mass of the system like a dumbbell. The power in gravitational radiation that they will emit is given by

\[
P_{\text{GR}}^{(\text{quad})} = \kappa \frac{2}{3} \frac{Gm^2}{c^3} a^2
\]  

(6)

where the factor \( \kappa \) is given above in Equation (5). Due to their bilateral symmetry, these two identical electrons will also radiate quadrupolar, but not dipolar, electromagnetic radiation with a power given by

\[
P_{\text{EM}}^{(\text{quad})} = \kappa \frac{2}{3} \frac{e^2}{4\pi\varepsilon_0 c^3} a^2
\]  

(7)
with the same factor of $\kappa$. The reason that this is true is that any given electron carries with it mass as well as charge as it moves, since its charge and mass must co-move rigidly together. Therefore, two electrons undergoing an acceleration $a$ relative to each other with a relative speed $v$ will emit simultaneously both electromagnetic and gravitational radiation, and the quadrupolar electromagnetic radiation that it emits will be completely homologous to the quadrupolar gravitational radiation that it also emits.

It follows that the ratio of gravitational to electromagnetic radiation powers emitted by the two-electron system is given by the same ratio of fundamental coupling constants as that for the force of gravity relative to the force of electricity, viz.,

$$\frac{P_{\text{GR}}^{(\text{quad})}}{P_{\text{EM}}^{(\text{quad})}} = \frac{4\pi\varepsilon_0 G m_e^2}{e^2} \approx 2.4 \times 10^{-43} \quad (8)$$

Thus, it would seem at first sight hopeless to try to use any electron system as a practical means for coupling between electromagnetic and gravitational radiation.

Nevertheless, it should be emphasized here that although this dimensionless ratio of fundamental coupling constants is extremely small, the gravitational radiation emitted from the two-electron system must in principle exist, or else there must be something fundamentally wrong with the experimentally well-tested inverse-square laws given by Equations (1) and (2).
4 The Planck mass scale

However, the ratio of the forces of gravity and electricity of two “Millikan oil drops” (see Fig. 3) need not be so hopelessly small [8].

For the purposes of an order-of-magnitude estimate, suppose that each “Millikan oil drop” has a single electron attached firmly to it and contains a Planck-mass amount of superfluid helium, viz.,

$$m_{\text{Planck}} = \sqrt{\frac{\hbar c}{G}} \approx 22 \mu g \quad (9)$$

where $\hbar$ is Planck’s constant/2π, $c$ is the speed of light, and $G$ is Newton’s constant. Planck’s mass sets the characteristic scale at which quantum mechanics ($\hbar$) impacts relativistic gravity ($c$, $G$). Note that the extreme smallness of $\hbar$ compensates for the extreme largeness of $c$ and for the extreme smallness of $G$, so that the order of magnitude of this mass scale is *mesoscopic*, and not astronomical, in size. This suggests that it may be possible to perform some novel *nonastronomical*, tabletop experiments at the interface of quantum mechanics and general relativity, which are accessible in any laboratory. Such experiments will be considered here.

The forces of gravity and electricity between the two “Millikan oil drops” are exerted on the centers of mass and the centers of charge of the drops, respectively. Both of these centers coincide with the geometrical centers of the spherical drops, assuming that the charge of the electrons on the drops is uniformly distributed around the outside surface of the drops in a spherically symmetric manner (like in an $S$ state). Therefore, the ratio of the forces of gravity and electricity between the two “Millikan oil drops” becomes

$$\frac{|F_G|}{|F_e|} = \frac{4\pi \varepsilon_0 G m_{\text{Planck}}^2}{e^2} = \frac{4\pi \varepsilon_0 G (\hbar c/G)}{e^2} = \frac{4\pi \varepsilon_0 \hbar c}{e^2} \approx 137 \quad (10)$$

Now the force of gravity is approximately 137 times stronger than the force of electricity, so that instead of a mutual repulsion between these two charged, massive objects, there is now a mutual attraction between them. The sign change from mutual repulsion to mutual attraction between these two “Millikan oil drops” occurs at a critical mass $m_{\text{crit}}$ given by

$$m_{\text{crit}} = \sqrt{\frac{e^2}{4\pi \varepsilon_0 \hbar c}} m_{\text{Planck}} \approx 1.9 \mu g \quad (11)$$

whereupon $|F_G| = |F_e|$ and the forces of gravity and electricity balance each other in equilibrium. The radius of a drop with this critical mass of superfluid helium, which has a density of $\rho = 0.145 \text{ g/cm}^3$, is

$$R = \left( \frac{3m_{\text{crit}}}{4\pi \rho} \right)^{1/3} = 146 \mu m \quad (12)$$
This is a strong hint that mesoscopic-scale quantum effects can lead to nonnegligible couplings between gravity and electromagnetism that can be observed in the laboratory.

Now let us scale up the mass so that there can still occur a comparable amount of generation of gravitational and electromagnetic radiation power on scattering of radiation from a larger pair of “Millikan oil drops,” each with a larger mass $M$ and with a larger charge $Q$, so that

$$
\frac{P_{\text{quad}}^{\text{GR}}}{P_{\text{quad}}^{\text{EM}}} = \frac{4\pi \varepsilon_0 G M^2}{Q^2} = 1
$$

We shall call this the “criticality” condition. At “criticality,” equal amounts of quadrupolar gravitational and quadrupolar electrical radiation powers will be scattered from the two objects. The factors of $\kappa$ in Equations (6) and (7) still cancel out, if the center of mass of each object co-moves rigidly together with its center of charge. This will happen if the objects remain rigidly in their quantum ground state. Then the scattered power from these two larger objects in the gravitational wave channel will remain equal to that in the electromagnetic wave channel. Of course, it will be necessary for the scattering cross sections of gravitational waves from these objects to be nonnegligible, in order for the scattered power to be experimentally interesting. This turns out to be the case not only for “Millikan oil drops,” but also for the more practical case of a pair of centimeter-scale, charged superconducting spheres [2, 3], which are also levitated in the configuration shown in Figure 3.

Note that any pair of objects whose masses have been increased beyond the critical mass $m_{\text{crit}}$ can still satisfy the “criticality” condition, Equation (13), provided that the number of electrons on these objects is also increased proportionately, so that their charge-to-mass ratio remains fixed, and provided that these objects remain in their quantum mechanical ground states during the passage of an incident GR wave, so that their motion is rigid. Therefore, we can replace a pair of drops of superfluid helium by a pair of spheres of superconductors, provided that these spheres are charged so that their charge-to-mass ratio is maintained at the “criticality” value

$$
\left( \frac{Q}{M} \right)_{\text{criticality}} = \sqrt{4\pi \varepsilon_0 G} = 8.6 \times 10^{-11} \text{ C/kg}
$$

This “criticality” charge-to-mass ratio can be easily achieved experimentally. For the superconducting spheres, it will be necessary for them to remain in the BCS ground state during the passage of a GR wave. This can be achieved by cooling them to ultra low temperatures.

### 5 Maxwell-like equations

To understand the calculation of the scattering cross section of the “Millikan oil drops” to be given below, let us start from a useful Maxwell-like representation of the linearized Einstein equations of general relativity due to Wald (see
Ref. [9] and Appendix A), which describes weak gravitational fields coupled to nonrelativistic matter in the asymptotically flat coordinate system of a distant inertial observer:

\[ \nabla \cdot \mathbf{E}_G = -\frac{\rho_G}{\varepsilon_G} \]  
\[ \nabla \times \mathbf{E}_G = -\frac{\partial \mathbf{B}_G}{\partial t} \]  
\[ \nabla \cdot \mathbf{B}_G = 0 \]  
\[ \nabla \times \mathbf{B}_G = \mu_G \left( -\mathbf{J}_G + \varepsilon_G \frac{\partial \mathbf{E}_G}{\partial t} \right) \]

where the gravitational analog of the electric permittivity of free space \( \varepsilon_G \) is given by

\[ \varepsilon_G = \frac{1}{4\pi G} = 1.19 \times 10^9 \text{ SI units} \]  
and where the gravitational analog of the magnetic permeability of free space \( \mu_G \) is given by

\[ \mu_G = \frac{4\pi G}{c^2} = 9.31 \times 10^{-27} \text{ SI units} \]  

Taking the curl of the gravitational analog of Faraday’s law, Equation (16), and substituting into its right-hand side the gravitational analog of Ampere’s law, Equation (18), one obtains a wave equation, which implies that the speed of gravitational radiation is given by

\[ c = \frac{1}{\sqrt{\varepsilon_G \mu_G}} = 3.00 \times 10^8 \text{ m/s} \]

which exactly equals the vacuum speed of light. In these Maxwell-like equations, the field \( \mathbf{E}_G \), which is the gravitoelectric field, is to be identified with the local acceleration \( \mathbf{g} \) of a test particle produced by the mass density \( \rho_G \), and the field \( \mathbf{B}_G \), which is the gravitomagnetic field produced by the mass current density \( \mathbf{J}_G \) and by the gravitational analog of the Maxwell displacement current density \( \varepsilon_G \frac{\partial \mathbf{E}_G}{\partial t} \), is to be identified with a time-dependent generalization of the Lense-Thirring field of general relativity.

In addition to the speed \( c \) of gravitational waves, there is another important physical property that these waves possess, which can be formed from the gravitomagnetic permeability of free space \( \mu_G \) and from the gravitoelectric permittivity \( \varepsilon_G \) of free space, namely, the gravitational characteristic impedance of free space \( Z_G \), which is given by

\[ Z_G = \sqrt{\frac{\mu_G}{\varepsilon_G}} = \frac{4\pi G}{c} = 2.79 \times 10^{-18} \text{ SI units} \]

As in electromagnetism, the characteristic impedance of free space \( Z_G \) plays a central role in all radiation problems, such as in a comparison of the radiation resistance of gravitational wave antennas to the value of this impedance in order
to estimate the coupling efficiency of these antennas to free space. The numerical value of this impedance is extremely small, but the impedance of all material objects must be much lower than this extremely small quantity before significant power from an incident GR wave can be appreciably scattered or reflected by these objects.

However, all classical material objects, such as Weber bars, have such a high dissipation and such a high radiation resistance that they are extremely poorly “impedance matched” to free space. They can therefore neither absorb norscatter gravitational wave energy efficiently [6, 11, 12, 13]. Hence, it is a common belief that all materials, whether classical or quantum, are essentially completely transparent to gravitational radiation.

Macroscopically coherent quantum matter (e.g., a quantum Hall fluid) can be an exception to this general rule, however, since it can be quantized so as to have a strictly zero dissipation. In the quantum Hall effect, this “quantum dissipationlessness” arises from the large size of the energy gap $E_{\text{gap}} = \hbar \omega_{\text{cycl}}$, where $\omega_{\text{cycl}}$ is the electron cyclotron frequency, when $E_{\text{gap}}$ is compared with the small size of the thermal fluctuations due to $k_B T$ at very low temperatures. The energy gap $E_{\text{gap}}$ is like the BCS gap of superconductors [14]. As in superconductors, because of the absence of excitations with energies within the energy gap, the scattering of the electrons in the quantum Hall fluid by phonons, impurities, etc., in the material is exponentially suppressed, and the quantum many-body system thus becomes dissipationless. For example, persistent currents in annular rings of superconductors have been observed to have lifetimes longer than the age of the universe.

Instead of discussing superconductors here, however, I focus instead on quantum Hall fluids. (For the more practical case of superconductors, see our work in Refs. [2, 3].)

6 Specular reflection of gravitational waves by a quantum Hall fluid

A quantum Hall fluid consists of a two-dimensional electron gas that forms at very low temperatures in the presence of a very strong magnetic field. In solid-state physics, a quantum Hall fluid forms due to the electrons trapped at the interface between two semiconductors, such as gallium arsenide and gallium-aluminum arsenide, when the sample is cooled down to millikelvin-scale temperatures in the presence of tesla-scale magnetic fields. Experimental evidence that the quantum Hall fluid is dissipationless comes from the fact that their quantum Hall plateaus are extremely flat. For example, in the “integer” effect, the transverse Hall resistance is quantized in exact integer multiples of $\hbar/e^2$, but the longitudinal Hall resistance, which is responsible for dissipation, is quantized to become exactly zero [15].

However, I consider here the quantum Hall fluid that forms on the surface of a superfluid helium drop. Impurity, phonon, roton, ripplon, etc., scattering
of the electrons moving on the surface of the drop is exponentially suppressed because of the essentially perfect superfluidity of liquid helium at millikelvin-scale temperatures. Thus, the electrons can slide frictionlessly along the surface of a “Millikan oil drop.” Since the electrons reside in a thin layer at a very small distance of approximately 80 Å away from the surface, which is much smaller than the typical centimeter-scale size of the drops to be used in the proposed experiments, locally the electronic motion is planar and can be well approximated by the two-dimensional motion of an electron gas on a frictionless dielectric plane (see Appendix B).

One important consequence of the zero-resistance property of a quantum Hall fluid is that a mirror-like reflection of electromagnetic waves can occur at a planar interface between the vacuum and the fluid. This reflection is similar to that which occurs when an incident electromagnetic wave propagates down a transmission line with a characteristic impedance $Z$, which is then terminated by means of a resistor $R$ whose value is close to zero. The reflection coefficient $R$ of the wave from such a termination is given by

$$ R = \frac{Z - R}{Z + R} \rightarrow 100\% \text{ when } R \rightarrow 0 \quad (23) $$

which approaches arbitrarily close to 100% when the resistance vanishes. When the resistance $R = 0$, low-frequency electromagnetic radiation fields are “shorted out” by the resistor $R$, and specular reflection occurs.

From the Maxwell-like Equations (15) through (18) and the boundary conditions that follow from them it follows that an analogous reflection of a gravitational plane wave from a planar interface of the vacuum with the quantum Hall fluid should exist, whose reflection coefficient $R_G$ is given by

$$ R_G = \frac{|Z_G - R_G|^2}{|Z_G + R_G|^2} \rightarrow 100\% \text{ when } R_G \rightarrow 0 \quad (24) $$

2Recall the boundary conditions that follow from Maxwell’s equations for electromagnetism. Consider for simplicity a planar boundary. The local normal component of the magnetic field must be continuous across the boundary (this comes from the Maxwell equation $\nabla \cdot B = 0$ applied to a small pillbox that straddles the boundary), and the local tangential component of the magnetic field must have a discontinuous jump across the boundary due to surface currents flowing at the boundary (this comes from the Maxwell equation $\nabla \times B = \mu_0 J$, where $J$ is the electric current density, applied to a small rectangular loop that straddles the boundary). For a quantum Hall fluid moving frictionlessly on the surface of superfluid helium, the surface resistance of the electrons on the surface is strictly zero. This, in conjunction with the Lorentz force law, leads to specular reflection of EM waves from the boundary for one circular polarization, as is shown in Appendix B. But each electron carries mass as well as charge with it when it moves. Therefore, a strictly zero surface resistance in the electrical sector implies a strictly zero surface resistance in the gravitational sector. The gravitational Maxwell-like equations lead to the same local normal and tangential boundary conditions for the gravitomagnetic field in the gravitational sector as the ones for the electromagnetic sector. Thus, specular reflection of GR waves at microwave frequencies should also occur below the cyclotron frequency. While it is true that most of the mass is in the interior of a “Millikan oil drop,” for the validity of the specular boundary conditions, it is the linear response of the electrons on the surface of the drop to the gravitational radiation fields that is crucial.
This counterintuitive result arises from the fact that the quantum Hall fluid can, under certain circumstances, possess a strictly zero dissipation, and therefore an equivalent mass-current resistance $R_G$ that can also be strictly zero, as compared to the characteristic impedance of free space $Z_G = 2.79 \times 10^{-18}$ SI units given by Equation (22). Although the gravitational impedance of free space $Z_G$ is an extremely small quantity, it is still a finite quantity. However, the dissipative resistance of a quantum Hall fluid is quantized and can therefore be exactly zero. When the resistance $R_G = 0$, low-frequency incident gravitational radiation fields are “shorted out” by $R_G$, and specular reflection occurs.

It may be objected that in Equation (24) it is unclear exactly how the thickness of the quantum Hall fluid compares in size relative to any relevant “penetration-depth” length scales, and also that this equation fails to take into account the frequency-dependent complex impedance of the quantum Hall fluid. When properly taken into account, it could have turned out that these effects would have made the reflectivity $R_G$ negligibly small. However, when they are properly taken into account (see Appendix C), the result is that although the reflectivity $R_G$ is not strictly unity, it can nevertheless be nonnegligible. The reflectivity $R_G$ for gravitational waves need only be of the order of unity, and not strictly unity, to be experimentally interesting.

Hence, it follows that under certain circumstances to be spelled out below, specular reflection of gravitational waves can occur from a quantum Hall fluid, just as from superconductors [2, 3]. Therefore, mirrors for gravitational radiation in principle can exist. Curved mirrors can focus this radiation, and Newtonian telescopes for gravitational waves can therefore in principle be constructed. In the case of scattering of gravitational waves from the “Millikan oil drops,” the above specular reflection condition implies hard-wall boundary conditions at the surfaces of these spheres, so that the scattering cross section of these waves from a pair of large spheres can be geometric (i.e., hard sphere) in size.

However, one cannot tell whether these statements about specular reflection of gravitational radiation from quantum Hall fluids are true experimentally without the existence of a source and a detector for such radiation. The quantum transducers based on “Millikan oil drops” to be discussed in more detail below may provide the needed source and detector.

Although we have been focusing in the above discussion on the case of the quantum Hall fluid that forms on “Millikan oil drops,” we should remark that specular reflection of gravitational waves should also occur from a vacuum-superconductor interface. In addition to our recent theoretical work [2, 3], this conclusion may possibly follow from the recent potentially very important experimental discovery [16, 17, 18] (which of course needs independent confirmation) that in an angularly accelerating superconductor, such as a niobium ring rotating with a steadily increasing angular velocity, there seems to be an enormous enhancement of the gravitomagnetic field $B_G$. As a result of the angular acceleration of the niobium ring, a steadily increasing gravitational analog of the London moment in the form of a very large $B_G$ field inside the ring seems to arise, which is increasing linearly in time. The gravitational analog of Faraday’s
law, Equation (16), then implies the generation of loops of the gravitoelectric field \( E_G \) inside the hole of the ring, which can be detected by sensitive accelerometers. The gravitomagnetic field \( B_G \) is thus inferred to be many orders of magnitude greater than what one would expect classically as a result of the mass current associated with the rigid rotation of the ionic lattice of the ring. These observations may have recently been confirmed by replacing the electromechanical accelerometers with laser gyroscopes [19].

A tentative theoretical interpretation of these recent experiments is that the coupling constant \( \mu_G \), which couples the mass currents of the superconductor to the gravitomagnetic field \( B_G \), is somehow greatly enhanced as a result of the presence of the macroscopically coherent quantum matter in niobium. This enhancement can be understood phenomenologically in terms of a ferromagnetic-like enhancement factor \( \kappa_G^{(magn)} \), which enhances the gravitomagnetic coupling constant inside the medium as follows:

\[
\mu'_G = \kappa_G^{(magn)} \mu_G
\]

where \( \kappa_G^{(magn)} \) is a positive number much larger than unity. This ferromagnetic-like enhancement factor \( \kappa_G^{(magn)} \) is the gravitational analog of the magnetic permeability constant \( \kappa_m \) of ferromagnetic materials in the standard theory of electromagnetism.

The basic assumption of this phenomenological theory is that of a linear response of the material medium to weak applied gravitomagnetic fields that is to say, whatever the fundamental explanation is of the large observed positive values of \( \kappa_G^{(magn)} \), the medium produces an enhanced gravitomagnetic field \( B_G \) that is directly proportional to the mass current density \( J_G \) of the ionic lattice. For weak fields, this is a reasonable assumption. However, it should be noted that this phenomenological explanation based on Equation (25) is different from the theoretical explanation based on Proca-like equations for gravitational fields with a finite graviton rest mass, which was proposed by the discoverers of the effect in Refs. [16, 17, 18].

Nevertheless, it is natural to consider introducing the phenomenological Equation (25) to explain the observations, since a large enhancement factor \( \kappa_G^{(magn)} \) due to the material medium is very similar to its analog in magnetism, which explains, for example, the large ferromagnetic enhancement of the inductance of a solenoid by a magnetically soft, permeable iron core with permeability \( \kappa_m \gg 1 \) that arises from the alignment of electron spins inside the iron. This spin-alignment effect leads to the large observed values of the magnetic susceptibility of iron, like those utilized in mu metal shields. Just as in the case of the iron core inserted inside a solenoid, where the large enhancement of the

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3The response of the medium must be not only linear in the amplitude of the weak applied gravitational radiation fields, but also causal. Hence, the real and imaginary parts of the linear response function \( \kappa_G^{(magn)}(\omega) \), as a function of frequency \( \omega \) of the gravitational wave, must obey Kramers-Kronig relations similar to those given by Equations (4) and (5) of Ref. [11].
A solenoid’s inductance disappears above the Curie temperature of iron, it was observed in these recent experiments that the large gravitomagnetic enhancement effect disappears above the superconducting transition temperature of niobium.

If the tentative phenomenological interpretation given by Equation (25) of these experiments turns out to be correct, one important consequence of the large resulting values of $\kappa^{(magn)}_G$ is that a mirror-like reflection should occur at a planar vacuum-superconductor interface, where the refractive index of the superconductor has an abrupt jump from unity to a value given by

$$n_G = \left(\kappa^{(magn)}_G\right)^{1/2}. \tag{26}$$

However, it should be immediately emphasized here that only positive masses are observed to exist in nature, and not negative ones. Hence, gravitational analogs of permanent electric dipole moments do not exist. It follows that the gravitational analog $\kappa^{(elec)}_G$ of the usual dielectric constant $\kappa_e$ for all kinds of matter, whether classical or quantum, in the Earth’s gravitoelectric field $E_G = g$

$$\kappa^{(elec)}_G \equiv \frac{\varepsilon'_G}{\varepsilon_G} = 1. \tag{27}$$

exactly. Hence, one cannot screen out, even partially, the gravitoelectric DC gravitational fields like the Earth’s gravitational field using superconducting Faraday cages, in an “antigravity” effect. In particular, the local value of the acceleration $g$ due to Earth’s gravity is not at all affected by the presence of nearby matter with large $\kappa^{(magn)}_G$.

The gravitational analog of Ampere’s law combined with Wald’s gravitational analog of the Lorentz force law (see Ref. [9], section 4.4)

$$F_G = m\left(E_G + 4v \times B_G\right) \tag{28}$$

where $F_G$ is the force on a test particle with mass $m$ and velocity $v$ (all quantities as seen by the distant inertial observer), leads to the fact that a repulsive component of force exists between two parallel mass currents traveling in the same direction, whereas two parallel electrical currents traveling in the same direction attract each other. A repulsive gravitomagnetic gravitational force follows from the negative sign in front of the mass current density $J_G$ in Equation (18), which is necessitated by the conservation of mass, since upon taking the divergence of Equation (18) and combining it with Equation (15) (whose negative sign in front of the mass density $\rho_G$ is fixed by Newton’s law of gravitation, where all masses attract each other), one must obtain the continuity equation for mass—that is,

$$\nabla \cdot J_G + \frac{\partial \rho_G}{\partial t} = 0 \tag{29}$$

where $J_G$ is the mass current density and $\rho_G$ is the mass density. Moreover, the negative sign in front of the mass current density $J_G$ in the gravitational analog of Ampere’s law, Equation (18), implies an anti-Meissner effect, in which the
lines of the $B_G$ field, instead of being expelled from the superconductor as in the usual Meissner effect, are pulled tightly into the interior of the body of the superconductor, whenever $\kappa_G^{(\text{magn})}$ is a large, positive number.

However, it should again be stressed that what is being tentatively proposed here in this phenomenological scenario does not at all imply an “antigravity” effect, in which the Earth’s gravitational field is somehow partially screened out by the so-called Podkletnov effect, where it was claimed that rotating superconductors reduce by a few percent the gravitoelectric field $E_G = g$, i.e., the local acceleration of all objects due to Earth’s gravity, in their vicinity. Experiments attempting to reproduce this effect have failed to do so [16, 17, 18]. The nonexistence of the Podkletnov effect would be consistent with the above phenomenological theory, since longitudinal gravitoelectric fields cannot be screened under any circumstances; however, transverse radiative gravitational fields can be reflected by supercurrents in coherent quantum matter.

Very large values of $\kappa_G^{(\text{magn})}$ for superconductors would imply that the index of refraction for gravitational plane waves in these media would be considerably larger than unity—that is,

$$n_G = \left(\kappa_G^{(\text{magn})}\right)^{1/2} \gtrsim 1$$

(30)

The Fresnel reflection coefficient $R_G$ of gravitational waves normally incident on the vacuum-superconductor interface would therefore become

$$R_G = \left|\frac{n_G - 1}{n_G + 1}\right|^2 \simeq \text{order of unity}$$

(31)

and could thus be large enough to be experimentally interesting. Again, but for different reasons from those given in [2, 3], Equation (31) would imply mirror-like reflection of these waves from superconducting surfaces (see Appendix C). It should be noted that large values of the ferromagnetic-like enhancement factor $\kappa_G^{(\text{magn})}$, of the index of refraction $n_G$, and of the reflectivity $R_G$ are not forbidden by the principle of equivalence, which has been checked experimentally with extremely high accuracy, but only within the gravitoelectric sector of gravitation.

However, although interesting and possibly very important, the above discussion concerning superconductors as mirrors for gravitational waves is only secondary to the primary purpose of this paper, which is to present the case for the possibility of efficient quantum transducers via “Millikan oil drops.” Nevertheless, superconducting transducers based on the same principles as those of the “Millikan oil drops,” to be described below, should also exist.

7 “Millikan oil drops” described in more detail

Let the oil of the classic Millikan oil drops be replaced with superfluid helium ($^4\text{He}$) with a gravitational mass of approximately the Planck mass scale, and let these drops be levitated in the presence of strong, tesla-scale magnetic fields.
The helium atom is diamagnetic, and liquid helium drops have successfully been magnetically levitated in an anti-Helmholtz magnetic trapping configuration \cite{20, 21}. As a result of its surface tension, the surface of a freely suspended, isolated, ultracold superfluid drop is ideally smooth, i.e., atomically perfect, in the sense that there are no defects (such as dislocations on the surface of an imperfect crystal) that can trap and thereby localize the electron. The absence of any scattering centers for the electrons on the surface of the superfluid helium of a “Millikan oil drop” implies that the electrons can move frictionlessly, and hence dissipationlessly, over its surface.

When an electron approaches a drop, the formation of an image charge inside the dielectric sphere of the drop causes the electron to be attracted by the Coulomb force to its own image. As a result, it is experimentally observed that the electron is bound to the outside surface of the drop in a hydrogenic ground state. The binding energy of the electron to the surface of liquid helium has been measured using millimeter-wave spectroscopy to be 8 K\(^4\) which is quite large compared to the millikelvin-scale temperatures for the proposed experiments. Hence, the electron is tightly bound to the outside surface of the drop so that the radial component of its motion is frozen, but when the drop becomes a superfluid, the electron is free to move frictionlessly tangentially on the surface, and thus free to become delocalized over the entire surface.

Such a “Millikan oil drop” is a macroscopically phase-coherent quantum object. In its ground state, which possesses a single, coherent quantum mechanical phase throughout the interior of the superfluid\(^5\) the drop possesses a zero circulation quantum number (i.e., contains no quantum vortices), with one unit (or an integer multiple) of the charge quantum number. As a result of the drop being at ultra low temperatures, all degrees of freedom other than the center-of-mass degrees of freedom are frozen out, so that a zero-phonon Mössbauer-like effect results, in which the entire mass of the drop moves rigidly as a single unit in response to radiation fields (see below). Therefore, the center of mass of the drop will co-move with the center of charge. In addition, since it remains adiabatically in the ground state during perturbations as a result of these weak radiation fields, the “Millikan oil drop” possesses properties of “quantum rigidity” and “quantum dissipationlessness,” which are the two most important quantum properties for achieving a high coupling efficiency for

\(^4\)See Refs. \cite{22, 23}. In the ground state of the system, the electron resides on the outside surface of a superfluid helium drop, and not within the inside volume of the drop. When the electron is forced to be within the interior of the drop, it will form a bubble with a radius of approximately 1 nm, as a result of the balancing of an outward Pauli pressure with the surface tension of the superfluid (see Ref. \cite{24}). The bubble will then rise to the surface, driven by the Coulomb force of attraction to its own image charge induced in the surface. It will then burst through the surface to uniformly coat the drop with one electron charge on its outside surface. The electron will be in an S-state to minimize the energy of the system. This then is the ground state of the system.

\(^5\)Note that the quantum mechanical ground-state wave function (or complex order parameter) must remain single valued (according to a distant inertial observer) globally at all times everywhere inside the interior of the system during the passage of a gravitational wave. This is another aspect of the “quantum rigidity” of a quantum fluid in its response to the gravitational wave.
Figure 2: “Charged quantum fluid” is a quantum transducer consisting of a pair of “Millikan oil drops” in a strong magnetic field, which converts a gravitational (GR) wave into an electromagnetic (EM) wave. A pair of charged superconducting spheres can also be used as such a transducer.

Note that two spatially separated “Millikan oil drops” with the same mass and charge have the correct bilateral symmetry in order to couple to quadrupolar gravitational radiation, as well as to quadrupolar electromagnetic radiation in the TEM$_{11}$ mode. The coupling of the drops to the electromagnetic TEM$_{00}$ mode, however, vanishes as a result of symmetry. When they are separated by a distance on the order of a wavelength, they should become an efficient quadrupolar antenna capable of generating, as well as detecting, gravitational radiation.

8 A pair of “Millikan oil drops” as a transducer

Now imagine placing a pair of levitated “Millikan oil drops” separated by approximately a microwave wavelength inside a black box, which represents a quantum transducer that can convert gravitational (GR) waves into electromagnetic (EM) waves (see Fig. 7). This kind of transducer action is similar to that of the tidal force of a gravitational wave passing over a pair of charged, freely falling objects orbiting the Earth, which can in principle convert a GR wave into an EM wave [8]. Such transducers are linear, reciprocal devices.

By time-reversal symmetry the reciprocal process, in which another pair

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Gravitational wave antennas [11, 12, 13].

6 Time-reversal symmetry under the _global_ operation of time reversal includes here the reversal of the direction of the applied DC magnetic fields.
of “Millikan oil drops” converts an EM wave back into a GR wave, must occur with the same efficiency as the forward process, in which a GR wave is converted into an EM wave by the first pair of “Millikan oil drops.” The time-reversed process is important because it allows the generation of gravitational radiation and therefore can become a practical source of such radiation. The radiation reaction or back-action by the EM fields on the GR fields via these coherent quantum drops leads necessarily to a nonnegligible reciprocal process of the generation of these fields. These actions must be mutual ones between these two kinds of radiation fields.

This raises the possibility of performing a Hertz-like experiment, in which the time-reversed quantum transducer process becomes the source, and its reciprocal quantum transducer process becomes the receiver of GR waves (see Fig. 8). Faraday cages consisting of nonsuperconducting metals prevent the transmission of EM waves, so that only GR waves, which can easily pass through all classical matter such as the normal (i.e., dissipative) metals of which standard, room-temperature Faraday cages are composed, are transmitted between the two halves of the apparatus that serve as the source and the receiver, respectively. Such an experiment would be practical to perform using standard microwave sources and receivers, provided that the scattering cross sections and the transducer conversion efficiencies of the two “Millikan oil drops” turn out not to be too small.

9 Mössbauer-like response of “Millikan oil drops” in strong magnetic fields to radiation fields

Let a pair of levitated “Millikan oil drops” be placed in strong, tesla-scale magnetic fields, and let the drops be separated by a distance on the order of a microwave wavelength, which is chosen so as to satisfy the impedance-matching condition for a good quadrupolar microwave antenna.

Now let a beam of electromagnetic waves in the Hermite-Gaussian TEM$_{11}$ mode [25], which has a quadrupolar transverse field pattern homologous to that of a gravitational plane wave, impinge at a 45° angle with respect to the line joining these two charged objects. Such a mode has been successfully generated using a “T”-shape microwave antenna [11, 12, 13]. As a result of being thus irradiated, the pair of “Millikan oil drops” will be driven into relative motion in an antiphased manner, so that the distance between them will oscillate sinusoidally with time, according to an observer at infinity. Thus, the simple harmonic motion of the two drops relative to one another (as seen by this observer) produces a time-varying mass quadrupole moment at the same frequency as that of the driving electromagnetic wave. This oscillatory motion will in turn scatter (in a linear scattering process) the incident electromagnetic wave into gravitational and electromagnetic scattering channels with comparable powers, provided that the ratio of quadrupolar radiation powers is that given by the “criticality” condition, Equation (13)—that is, this ratio is of the order of unity, which will be
Figure 3: A Hertz-like experiment, in which EM waves are converted by the lower-left quantum transducer (“Charged quantum fluid”) into GR waves at the source, and the GR waves thus generated are back-converted back into EM waves by the upper-right quantum transducer at the receiver. Communication by EM waves is prevented by the normal (i.e., nonsuperconducting) Faraday cages.
the case if the charge-to-mass ratio of the drops is given by the “criticality” ratio, Equation (14). The reciprocal scattering process will also have a power ratio of the order of unity.

The Mössbauer-like response of “Millikan oil drops” will now be discussed in more detail. Imagine what would happen if one were to replace an electron in the vacuum with a single electron that is firmly attached to the outside surface of a drop of superfluid helium in the presence of a strong magnetic field and at ultra low temperatures, so that the system of the electron and the superfluid, considered as a single quantum entity like that of a “gigantic atom,” would form a single, macroscopic quantum ground state. Such a quantum system can possess a sizable gravitational mass. For the case of many electrons attached to a large, massive drop, where a quantum Hall fluid forms on the outside surface of the drop in the presence of a strong magnetic field, a Laughlin-like ground state results, which is the many-body state of an incompressible quantum fluid [26]. The property of quantum incompressibility of such a fluid is equivalent to the property of “quantum rigidity,” which is one necessary requirement for achieving high efficiency in gravitational radiation antennas, as was pointed out in Refs. [11, 12, 13]. Like superfluids and superconductors, this fluid is also frictionless (i.e., dissipationless). This fulfills the condition of “quantum dissipationlessness,” which is another necessary requirement for the successful construction of efficient gravitational wave antennas [11, 12, 13].

In the presence of strong, tesla-scale magnetic fields, an electron is prevented from moving at right angles to the local magnetic field line around which it is executing tight cyclotron orbits. The result is that the surface of the drop, to which the electron is tightly bound, cannot undergo low-frequency liquid-drop deformations, such as the oscillations between the prolate and oblate spheroidal configurations of the drop that would occur at low frequencies in the absence of the magnetic field. After the drop has been placed into tesla-scale magnetic fields at millikelvin-scale operating temperatures, both the single- and many-electron drop systems will be effectively frozen into the ground state, since the characteristic energy scale for electron cyclotron motion in tesla-scale fields is on the order of kelvins. As a result of the tight coupling of the electron(s) to the outside surface of the drop, also on the scale of kelvins, this would effectively freeze out all low-frequency shape deformations of the superfluid drop.

Since all internal degrees of freedom of the drop, such as its microwave phonon excitations, will also be frozen out at sufficiently low temperatures, the charge and the entire mass of the “Millikan oil drop” will co-move rigidly together as a single unit, in a zero-phonon, Mössbauer-like response to applied radiation fields with frequencies below the cyclotron frequency. This is a result

This single quantum entity can be viewed as if it were a gigantic atom in which the usual atomic nucleus is replaced by the superfluid helium drop, and the usual electronic cloud surrounding the atomic nucleus is replaced by the electrons on the surface surrounding the drop. The large energy gap (Eq. (33)) arising from the large applied magnetic field is what makes this gigantic atom extremely rigid and dissipationless at low temperatures. A pair of such gigantic atoms forms a gigantic diatomic molecule. If the charges and masses of the two drops are slightly different from each other, such a gigantic diatomic molecule will form an entangled state of charge and mass in its ground state at sufficiently low temperatures.
of the elimination of all internal degrees of freedom by the Boltzmann factor at sufficiently low temperatures, so that the system stays in its ground state, and only the external degrees of freedom of the drop, consisting only of its center-of-mass motions, remain.

The criterion for this zero-phonon, or Mössbauer-like, mode of response of the electron-drop system is that the temperature of the system is sufficiently low, so that the probability for the entire system to remain in its ground state without even a single quantum of excitation of any of its internal degrees of freedom being excited is very high—that is,

\[
\text{Prob. of zero internal excitation} \approx 1 - \exp \left( \frac{-E_{\text{gap}}}{k_B T} \right) \rightarrow 1 \quad \text{as} \quad \frac{k_B T}{E_{\text{gap}}} \rightarrow 0 \quad (32)
\]

where \(E_{\text{gap}}\) is the energy gap separating the ground state from the lowest permissible excited states, \(k_B\) is Boltzmann’s constant, and \(T\) is the temperature of the system. Then the quantum adiabatic theorem ensures that the system will stay adiabatically in the ground state of this quantum many-body system during adiabatic perturbations, such as those due to weak, externally applied radiation fields with frequencies below the cyclotron frequency. By momentum conservation, because there are no internal excitations to take up the radiative momentum transfer, the center of mass of the entire system must undergo recoil in the emission and absorption of radiation. Thus, the mass involved in the response to radiation fields is the entire mass of the whole system.

For the case of a single electron (or many electrons in the case of the quantum Hall fluid) in a strong magnetic field, the typical energy gap is given by

\[
E_{\text{gap}} = \hbar \omega_{\text{cycl}} = \frac{\hbar eB}{m} \gg k_B T \quad (33)
\]

where \(\omega_{\text{cycl}} = eB/m\) is the electron cyclotron frequency. This is satisfied by the tesla-scale fields and millikelvin-scale temperatures in the proposed experiments.

10 Estimate of the scattering cross section

Let \(d\sigma_{a \rightarrow \beta}\) be the differential cross section for the scattering of a mode \(a\) of radiation of an incident gravitational wave to a mode \(\beta\) of a scattered electromagnetic wave by a pair of “Millikan oil drops” (Latin subscripts denote GR waves, and Greek subscripts denote EM waves). Then, by time-reversal symmetry\(^{3}\)

\[
d\sigma_{a \rightarrow \beta} = d\sigma_{\beta \rightarrow a} \quad (34)
\]

Since electromagnetic and weak gravitational fields both formally obey Maxwell’s equations (apart from a difference in the signs of the source density and the source current density; see Eqs. (15)–(18)), and since these fields obey the same

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\(^{3}\)As previously noted, time-reversal symmetry under the global operation of time reversal includes here the reversal of the direction of the applied DC magnetic fields.
boundary conditions (see Appendix C and the footnote on page 10), the solutions for the modes for the two kinds of scattered radiation fields must also have the same mathematical form. Let \( a \) and \( \alpha \) be a pair of corresponding solutions and \( b \) and \( \beta \) be a different pair of corresponding solutions to Maxwell’s equations for GR and EM modes, respectively. For example, \( a \) and \( \alpha \) could represent incoming plane waves that copropagate in the same direction, and \( b \) and \( \beta \) could represent scattered, outgoing plane waves that copropagate together in a different direction. Then for a pair of drops with the “criticality” charge-to-mass ratio given by Equation (14), there is an equal conversion into the two types of scattered radiation fields in accordance with Equation (13), and therefore at “criticality”

\[
d\sigma_{a \rightarrow b} = d\sigma_{a \rightarrow \beta}
\]  

(35)

where \( b \) and \( \beta \) are corresponding modes of the two kinds of scattered radiations.

By the same line of reasoning, for this pair of drops

\[
d\sigma_{b \rightarrow a} = d\sigma_{\beta \rightarrow a} = d\sigma_{\beta \rightarrow \alpha}
\]  

(36)

It therefore follows from the principle of reciprocity (i.e., detailed balance or time-reversal symmetry) that

\[
d\sigma_{a \rightarrow b} = d\sigma_{a \rightarrow \beta}
\]  

(37)

To estimate the size of the total cross section, it is easier to consider first the case of electromagnetic scattering, such as the scattering of microwaves from a pair of large drops with radii \( R \) and a separation \( r \) on the order of a microwave wavelength (but with \( r > 2R \)). The diameter \( 2R \) of the drops can be made to be comparable to their separation \( r \simeq \lambda \) (e.g., with \( 2\pi R = \lambda \) for the first Mie resonance), provided that many electrons are added on their surfaces, so that the “criticality” charge-to-mass ratio is maintained (this requires the addition of 20,000 electrons for the first Mie resonance at \( \lambda = 2.5 \) cm, where \( R = 4 \) mm).

For an incident EM wave of a particular circular polarization, even just a single, delocalized electron in the presence of a strong magnetic field is enough to produce specular reflection of this wave (see Appendix B). Therefore, for circularly polarized light, the two drops behave like perfectly conducting, shiny, mirror-like spheres, which scatter light in a manner similar to that of perfectly elastic hard-sphere scattering in idealized billiards. The total cross section for the scattering of electromagnetic radiation from this pair of large drops is therefore given approximately by the geometric cross-sectional areas of two hard spheres

\[
\sigma_{a \rightarrow \text{all } \beta} = \int d\sigma_{a \rightarrow \beta} \simeq \text{order of } \pi R^2
\]  

(38)

where \( R \) is the hard-sphere radius of a drop. This hard-sphere cross section is much larger than the Thomson cross section for the classical, localized single free-electron scattering of electromagnetic radiation.

However, if, as one might expect on the basis of the prevailing (but possibly incorrect) opinion that all gravitational interactions with matter, including
the scattering of gravitational waves from all types of matter, are completely independent of whether this matter is classical or quantum mechanical in nature on any scale of size, and that therefore the scattering cross section for the drops would be extremely small as it is for the classical Weber bar, then by reciprocity the total cross section for the scattering of electromagnetic waves from the two-drop system must also be extremely small. In other words, if “Millikan oil drops” were to be essentially invisible to gravitational radiation as is commonly believed, then by reciprocity they must also be essentially invisible to electromagnetic radiation. To the contrary, if it should turn out that the quantum Hall fluid on the surface of these drops should make them behave like superconducting spheres, then the earlier discussion in connection with Equation (24) would imply that the total cross section of these drops will be like that of hard-sphere scattering, so that they certainly would not be invisible.

11 A proposed preliminary experiment

To check the above hard-sphere scattering cross section result, we propose first to perform in a preliminary experiment a measurement of the purely EM scattering cross section for quadrupolar microwave radiation off of a pair of large “Millikan oil drops” (see Fig. 11). An oscillator at 12 GHz emits microwaves that are prepared in a quadrupolar TEM$_{11}$ mode and directed in a beam toward these drops, which are placed in a large magnetic field and cooled to ultra low temperatures. The intensity of the scattered microwave beam generated by the pair of drops is then measured by means of a 12 GHz heterodyne receiver, which receives a quadrupolar TEM$_{11}$ mode. The purpose of this experiment is to check whether the scattering cross section is indeed as large as the geometric cross section predicted by Equations (24), (38), and (67). As one increases the temperature, one should observe the disappearance of this enhanced scattering cross section above the quantum Hall transition temperature or the superfluid lambda point, whichever comes first.

12 A common misconception corrected

In connection with the idea that an EM wave incident on a pair of drops could generate a GR wave, a common misconception arises that the drops are so heavy that their large inertia will prevent them from moving with any appreciable amplitude in response to the driving EM wave amplitude. How can they then possibly generate copious amounts of GR waves? This objection overlooks the major role played by the principle of equivalence in the motion of the drops, as will be explained below.

According to the equivalence principle, two tiny inertial observers, who are undergoing free fall (i.e., who are freely floating near their respective centers of the two “Millikan oil drops”) would see no acceleration at all of the nearby surrounding matter of their drop (nor would they feel any forces) as a result of
Figure 4: Schematic of apparatus (not to scale) to measure the scattering cross section of quadrupolar microwaves from a pair of “Millikan oil drops” in a strong magnetic field at low temperatures.
the gravitational fields arising from a gravitational wave passing over the two drops. However, when they measure the distance separating the two drops, by means of laser interferometry, for example, they would conclude that the other drop is undergoing acceleration relative to their drop, as a result of the fact that the space between the drops is being periodically stretched and squeezed by the incident gravitational wave. They would therefore further conclude that the charges attached to the surfaces of their locally freely falling drops would radiate electromagnetic radiation, in agreement with the observations of the observer at infinity, who sees two charges undergoing time-varying relative acceleration in response to the passage of the gravitational wave.

According to the reciprocity principle, this scattering process can be reversed in time. Under time reversal, the scattered electromagnetic wave now becomes a wave that is incident on the drops. Again, the two tiny inertial observers near the center of the drops would see no acceleration at all of the surrounding matter (nor would they feel any forces) because of the electric and magnetic fields of the incident electromagnetic wave. Rather, they would conclude from measurements of the distance separating the two drops that it is again the space between the drops that is being periodically squeezed and stretched by the incident electromagnetic wave. They would again further conclude that the masses associated with their locally freely falling drops would radiate gravitational radiation, in agreement with the observations of the observer at infinity, who sees two masses undergoing time-varying relative acceleration in response to the passage of the electromagnetic wave.

From this general relativistic viewpoint, which is based on the equivalence principle, the fact that the drops might possess very large inertias is irrelevant, since in fact the drops are not moving at all with respect to the local inertial observer located at the center of drop. Instead of causing motion of the drops through space, the gravitational fields of the incident gravitational wave are acting directly on space itself by periodically stretching and squeezing the space in between the drops. Likewise, in the reciprocal process the very large inertias of the drops are again irrelevant, since the electromagnetic wave is not producing any motion at all of these drops with respect to the same inertial observer (see Appendix D). Instead of causing motion of the drops through space, the electric and magnetic fields of the incident electromagnetic wave are again acting directly on space itself by periodically squeezing and stretching the space in between the drops. The time-varying, accelerated motion of the drops as seen by the distant observer that causes quadrupolar radiation to be emitted in both cases is due to the time-varying curvature of spacetime induced both by the incident gravitational wave and by the incident electromagnetic wave. It should be remembered that the space inside which the drops reside is therefore no longer flat, so that the Newtonian concept of a radiation-driven, local accelerated motion of a heavy drop with a large inertia through a fixed and flat Euclidean space is therefore no longer valid.
13 The strain of space produced by the drops for a milliwatt of GR wave power

Another common objection to these ideas is that the strain of space produced by a milliwatt of an electromagnetic wave is much too small to detect. However, in the Hertz-like experiment, one is not trying to detect directly the strain of space (as in LIGO), but rather the power that is being transferred by the gravitational radiation fields from the source to the receiver.

Let us put in some numbers. Suppose that one succeeded in completely converting a milliwatt of EM wave power into a milliwatt of GR wave power at the source. How big a strain amplitude of space would be produced by the resulting GR wave? The gravitational analog of the time-averaged Poynting vector is given by [27]

$$\langle S \rangle = \frac{\omega^2 \epsilon^3}{32\pi G} h_+^2$$

where $h_+$ is the dimensionless strain amplitude of space for one polarization of a monochromatic plane wave. For a milliwatt of power in such a plane wave at 30 GHz focused by means of a Newtonian telescope to a 1 cm² Gaussian beam waist, one obtains a dimensionless strain amplitude of

$$h_+ \simeq 0.8 \times 10^{-28}$$

This strain is indeed exceedingly difficult to detect directly. However, it is not necessary to directly measure the strain of space in order to detect gravitational radiation, just as it is not necessary to directly measure the electric field of a light wave, which may also be exceedingly small, in order to be able to detect this wave. Instead, one can directly measure the power conveyed by a beam of light by means of bolometry, for example. Likewise, if one were to succeed in completely back-converting this milliwatt of GR wave power with high efficiency back into a milliwatt of EM power at the receiver, this amount of power would be easily detectable by standard microwave techniques.

14 Signal-to-noise considerations

The signal-to-noise ratio expected for the Hertz-like experiment depends on the current status of microwave source and receiver technologies. Based on the experience gained from the experiment done on YBCO using existing off-the-shelf microwave components [11, 12, 13], we expect that we would need geometric-sized cross sections and a minimum conversion efficiency on the order of parts per million per transducer to detect a signal. The overall system’s signal-to-noise ratio depends on the initial microwave power, the scattering cross section, the conversion efficiency of the quantum transducers, and the noise temperature of the microwave receiver (i.e., its first-stage amplifier).

Microwave low-noise amplifiers can possess noise temperatures that are comparable to room temperature (or even better, such as in the case of liquid-helium-cooled paramps or masers used in radio astronomy). The minimum
power $P_{\text{min}}$ detectable in an integration time $\tau$ is given by

$$P_{\text{min}} = \frac{k_B T_{\text{noise}} \Delta \nu}{\sqrt{\tau \Delta \nu}}$$  \hspace{1cm} (41)

where $k_B$ is Boltzmann’s constant, $T_{\text{noise}}$ is the noise temperature of the first stage microwave amplifier, and $\Delta \nu$ is its bandwidth. Assuming an integration time of 1 sec, a bandwidth of 1 GHz, and a noise temperature of $T_{\text{noise}} = 300$ K, one gets $P_{\text{min}}(\tau = 1 \text{ sec}) = 1.3 \times 10^{-16}$ W, which is much less than the milliwatt power levels of typical microwave sources.

## 15 Possible applications

If we should be successful in the Hertz-like experiment, this could lead to important possible applications in science and engineering. In science, it would open up the possibility of gravitational wave astronomy at microwave frequencies. One important problem to explore would be observations of the analog of the cosmic microwave background (CMB) in gravitational radiation. Because the universe is much more transparent to gravitational waves than to electromagnetic waves, such observations would allow a much more penetrating look into the extremely early Big Bang toward the Planck scale of time than the currently well-studied CMB. Different cosmological models of the very early universe give widely differing predictions of the spectrum of this penetrating radiation, so that by measurements of the spectrum, one could tell which model, if any, is close to the truth [28]. The anisotropy in this radiation would also be very important to observe.

In engineering, it would open up the possibility of intercontinental communication by means of microwave-frequency gravitational waves directly through the interior of the Earth, which is transparent to such waves. This would eliminate the need of communications satellites and would allow communication with people deep underground or underwater in submarines in the oceans. Wireless power transmission by gravitational microwaves would also be a possibility. Such a new direction of gravitational wave engineering could aptly be called “gravity radio.”

## 16 Appendix A:

### Wald’s derivation of the Maxwell-like equations

The Maxwell-like equations (Eqs. (15)–(18)) are a consequence of the derivation by Wald (see Ref. [9]) in section 4.4, which starts from the assumption that for weak gravitational fields, the metric of spacetime can be approximated by (in the notation of Misner, Thorne, and Wheeler [1])

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$$  \hspace{1cm} (42)
where \( g_{\mu\nu} \) is the metric tensor, \( \eta_{\mu\nu} \) is the Minkowski metric tensor for a flat spacetime, and \( h_{\mu\nu} \) are small perturbations of the metric tensor, such as those arising from gravitational radiation.

When the lowest order effects of the motion of the source are taken into account, but neglecting stresses, the linearized Einstein field equations, when also linearized in the nonrelativistic velocity of the matter, become (in units where \( G = c = 1 \))

\[
\partial^\mu \partial_\mu \overline{h_{0\lambda}} = 16\pi J_\lambda
\]

where \( \overline{h_{\mu\nu}} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \) and where \( J_\lambda \) is the mass current density four-vector of the source. If, following Wald, one defines the “vector potential” as:

\[
A_\mu \equiv -\frac{1}{4} \overline{h_{\mu\nu}} t^\nu
\]

where \( t^\nu \) is the four-velocity of a test particle (which for a nonrelativistic particle is time-like), one obtains

\[
\partial^\mu \partial_\mu A_\lambda = -4\pi J_\lambda
\]

These equations are equivalent to the Maxwell-like equations and have the form of Maxwell’s equations in the Lorentz gauge, with the consequence that the perturbations \( \overline{h_{0\lambda}} \) propagate with precisely the speed of light \( c \), and not at the speed \( c/2 \).

In contrast to this, using the PPN formalism, Braginsky, Caves, and Thorne [29] derived a set of Maxwell-like equations that yielded a speed of \( c/2 \), and not the speed of light \( c \), for time-varying perturbations of the fields. This difference in speeds arises from the fact that the PPN formalism describes the near fields as seen by an observer close to the source, but Wald’s formalism describes the far fields as seen by an observer in an asymptotically flat spacetime far away from the source.

The standard transverse-traceless (TT) coordinate system can be transformed into Wald’s coordinate system by means of a local Galilean coordinate transformation [2]. In the TT gauge, one of the gauge conditions is

\[
h_{0\mu} = 0
\]

An incorrect conclusion drawn from this gauge condition is that only the gravitoelectric components given by the strains \( h_{ij} \) of a gravitational plane wave exist, and that no gravitomagnetic components of radiation fields in the far field of sources exist. In Relativity on Curved Manifolds (Chapter 9), Felice and Clarke point out that the Riemann curvature tensor for gravitational waves propagating in a flat background can be separated into “electric” and “magnetic” parts, and that these Riemann curvature tensor components satisfy tensor Maxwell-like equations [30]. The wave speed that follows from these equations is again precisely \( c \). This gauge-invariant way of characterizing gravitational radiation shows that the “electric” and “magnetic” components of the Riemann curvature tensor for a monochromatic gravitational plane wave propagating in the vacuum are equal in magnitude to each other in natural units.

The earliest mention of Maxwell-like equations for linearized general relativity was perhaps made by Forward in 1961 [31].
17 Appendix B: Specular reflection of a circularly polarized EM wave by a delocalized electron moving on a plane in the presence of a strong magnetic field

Here we address the question, what is the critical frequency for specular reflection of an EM plane wave normally incident on a plane, in which electrons are moving in the presence of a strong B field? The motivation for solving this problem is to answer also the following questions: How can just a single electron on the outside surface of a “Millikan oil drop” generate enough current in response to an incident EM wave, so as to produce a reradiated wave that totally cancels out the incident wave within the interior of the drop, with the result that none of the incident radiation can enter into the drop? Why does specular reflection occur from the surface of such a drop, and hence why does a hard-sphere EM cross section result for a pair of “Millikan oil drops”?

To simplify this problem to its bare essentials, let us examine first a simpler, planar problem consisting of a uniform electron gas moving classically on a frictionless, planar dielectric surface. I start from a three-dimensional point of view, but the Coulombic attraction of the electrons to their image charges inside the dielectric will confine them in the direction normal to the plane, so that the electrons are restricted to a two-dimensional motion—that is, to frictionless motion in the two transverse dimensions of the plane. The electrons are subjected to a strong DC magnetic field applied normally to this plane. What is the linear response of this electron gas to a weak, normally incident EM plane wave? Does a specular plasma-like reflection occur below a critical frequency, even when only a single, delocalized electron is present on the plane? Let us first solve this problem classically.

Let the plane in question be the \( z = 0 \) plane, and let a strong, applied DC \( \mathbf{B} \) field be directed along the positive \( z \)-axis. The Lorentz force on an electron is given by

\[
\mathbf{F} = e \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right)
\]

where \( \mathbf{E} \), the weak electric field of the normally incident plane wave, lies in the \((x, y)\) plane. (I use Gaussian units only here in this appendix.) The cross product \( \mathbf{v} \times \mathbf{B} \) is given by

\[
\mathbf{v} \times \mathbf{B} = \begin{vmatrix} i & j & k \\ v_x & v_y & 0 \\ 0 & 0 & B \end{vmatrix} = iv_y B - jv_x B
\]

Hence, Newton’s equations of motion reduce to \( x \) and \( y \) components only:

\[
F_x = m \ddot{x} = e\dot{E}_x + \frac{v_y}{c} eB = e\dot{E}_x + \frac{\dot{y}}{c} eB
\]
\[ F_y = m \ddot{y} = e E_y - \frac{v_x}{c} e B = e E_y - \frac{\dot{x}}{c} e B \quad (50) \]

Let us assume that the driving plane wave is a weak monochromatic wave with the exponential time dependence

\[ E = E_0 \exp(-i\omega t) \quad (51) \]

Then assuming a linear response of the system to the weak incident EM wave, the displacement, velocity, and acceleration of the electron all have the same exponential time dependence

\[ x = x_0 \exp(-i\omega t) \quad \text{and} \quad y = y_0 \exp(-i\omega t) \quad (52) \]

\[ \dot{x} = (-i\omega) x \quad \text{and} \quad \dot{y} = (-i\omega) y \quad (53) \]

\[ \ddot{x} = -\omega^2 x \quad \text{and} \quad \ddot{y} = -\omega^2 y \quad (54) \]

which converts the two ODEs, Equations (49) and (50), into the two algebraic equations for \( x \) and \( y \)

\[ -m\omega^2 x = e E_x - \frac{i\omega y}{c} e B \quad (55) \]

\[ -m\omega^2 y = e E_y + \frac{i\omega x}{c} e B \quad (56) \]

Let us now add \( \pm i \) times the second equation to the first equation. Solving for \( x \pm iy \), one gets

\[ x \pm iy = e \left( \frac{E_x \pm iE_y}{-m\omega^2 \pm \omega \omega_{\text{cycl}}} \right) \quad (57) \]

where the upper sign corresponds to an incident clockwise circularly polarized EM and the lower sign to an anticlockwise one. Let us define as a shorthand notation

\[ z_{\pm} \equiv x \pm iy \quad (58) \]

as the complex representation of the displacement of the electron. Solving for \( z_{\pm} \), one obtains

\[ z_{\pm} = \frac{e E_{\pm}}{-m (\omega^2 \mp \omega \omega_{\text{cycl}})} \quad (59) \]

where the cyclotron frequency \( \omega_{\text{cycl}} \) is defined as

\[ \omega_{\text{cycl}} \equiv \frac{e B}{mc} \quad (60) \]

and where

\[ E_{\pm} \equiv E_x \pm iE_y \quad (61) \]

For a gas of electrons with a uniform number density \( n_e \), the polarization of this medium induced by the weak incident EM wave is given by

\[ P_{\pm} = n_e e (x \pm iy) = n_e e z_{\pm} = \frac{n_e e^2 E_{\pm}}{-m (\omega^2 \mp \omega \omega_{\text{cycl}})} = \chi_e E_{\pm} \quad (62) \]
where the susceptibility of the electron gas is given by
\[
\chi_e = \frac{n_e e^2}{-m (\omega^2 + \omega \omega_{cycl})} = -\frac{\omega^2_{\text{plas}}/4\pi}{\omega^2 + \omega \omega_{cycl}}
\] (63)
where the plasma frequency \( \omega_{\text{plas}} \) is defined by
\[
\omega_{\text{plas}} \equiv \sqrt{\frac{4\pi n_e e^2}{m}}
\] (64)
The index of refraction of the gas \( n(\omega) \) is given by
\[
n(\omega) = \sqrt{1 + 4\pi \chi_e(\omega)} = \sqrt{1 - \frac{\omega^2_{\text{plas}}}{\omega^2 + \omega \omega_{cycl}}}
\] (65)
Specular reflection occurs when the index of refraction becomes a pure imaginary number. Let us define the critical frequency \( \omega_{\text{crit}} \) as the frequency at which the index vanishes, which occurs when
\[
\frac{\omega^2_{\text{plas}}}{\omega^2_{\text{crit}} \pm \omega \omega_{cycl}} = 1
\] (66)
Because the index vanishes at this critical frequency, the Fresnel reflection coefficient \( R(\omega) \) from the planar structure for normal incidence at the critical frequency is given by
\[
R(\omega) = \left| \frac{n(\omega) - 1}{n(\omega) + 1} \right|^2 \to 100\% \text{ when } \omega \to \omega_{\text{crit}}
\] (67)
which implies specular reflection of the incident plane EM wave from the electron gas. This yields a quadratic equation for \( \omega_{\text{crit}} \),
\[
\omega^2_{\text{crit}} \pm \omega_{\text{crit}} \omega_{\text{cycl}} - \omega^2_{\text{plas}} = 0
\] (68)
The solution for \( \omega_{\text{crit}} \) is
\[
\omega_{\text{crit}} = \pm \omega_{\text{cycl}} \pm \sqrt{\omega^2_{\text{cycl}} + 4\omega^2_{\text{plas}}} \frac{2}{2}
\] (69)
The first \( \pm \) sign is physical and is determined by the sense of circular polarization of the incident plane wave. The second \( \pm \) sign is mathematical and originates from the square root. One of the latter mathematical signs is unphysical. To determine which choice of the latter sign is physical and which is unphysical, let us first consider the limiting case when the inequality
\[
\omega_{\text{cycl}} \ll \omega_{\text{plas}}
\] (70)
holds. This inequality corresponds physically to the situation when the magnetic field is very weak but the electron density is very high, so that the phenomenon
of specular reflection of EM waves with frequencies below the plasma frequency $\omega_{\text{plas}}$ occurs. Let us therefore take the limit $\omega_{\text{cycl}} \to 0$ in the solution given by Equation (69). Negative frequencies are unphysical, so that we must choose the positive sign in front of the surd as the only possible physical solution. Thus, in general, it must be the case that the physical root of the quadratic is given by

$$\omega_{\text{crit}} = \pm \frac{\omega_{\text{cycl}} + \sqrt{\omega_{\text{cycl}}^2 + 4\omega_{\text{plas}}^2}}{2} \quad (71)$$

Let us now focus on the more interesting case in which the magnetic field is very strong but the number density of electrons is very small, so that the plasma frequency is very low, corresponding to the inequality

$$\omega_{\text{cycl}} \gg \omega_{\text{plas}} \quad (72)$$

There are then two possible solutions, corresponding to clockwise-polarized and anticlockwise-polarized EM waves, respectively, viz.,

$$\omega_{\text{crit},1} = \omega_{\text{cycl}} \quad \text{and} \quad \omega_{\text{crit},2} = 0 \quad (73)$$

Note the important fact that these solutions are independent of the number density (or plasma frequency) of the electron gas, which implies that even a very dilute electron gas system can give rise to specular reflection. The fact that these solutions are independent of the number density also implies that they would apply to the case of an inhomogeneous electron density, such as that arising for a single delocalized electron confined to the vicinity of the plane $z = 0$ by the Coulomb attraction to its image. Both solutions of the quadratic Equation (73) are now physical ones and imply that whether the sense of rotation of the EM polarization corotates or counterrotates with respect to the magnetic-field-induced precession of the guiding center motion of the electron around the magnetic field determines which sense of circular polarization is transmitted when $\omega > \omega_{\text{crit},2} = 0$, or which sense of circular polarization is totally reflected when $\omega < \omega_{\text{crit},1} = \omega_{\text{cycl}}$, provided that the frequency of the incident circularly polarized EM wave is less than the cyclotron frequency $\omega_{\text{cycl}}$. The interesting solution is the one with the nonvanishing critical frequency, because it implies that one solution always exists where there is specular reflection of the EM wave, even when the number density of electrons is extremely low (i.e., even when the plasma frequency $\omega_{\text{plas}}$ approaches zero), and even when this number density becomes very inhomogeneous as a function of $z$.

In the extreme case of a single electron completely delocalized on the outside surface of superfluid helium, one should solve the problem quantum mechanically, by going back to Landau’s solution of the motion of an electron in a uniform magnetic field and adding as a time-dependent perturbation the weak (classical) incident circularly polarized plane wave. However, the above classical solution should hold in the correspondence principle limit, where, for the single
delocalized electron, the effective number density of the above classical solution is determined by the absolute square of the electron wave function, viz.,

\[ n_e = |\psi_e|^2 \]  

and

\[ \int n_e dV = \int |\psi_e|^2 dV = 1 \]  

Here we must take into account the fact that there is a finite confinement distance \( d_e \approx 80 \, \text{Å} \) in the \( z \)-direction of the electron’s motion in the hydrogenic ground state caused by the Coulomb attraction of the electron to its image charge induced in the dielectric, but the electron is completely delocalized in the \( x \)- and \( y \)-directions on an arbitrarily large plane (and hence over the large spherical surface of a large drop). The effective plasma frequency of the single electron may be extremely small; nevertheless, total reflection by this single, delocalized electron still occurs, provided that the frequency of the incident circularly polarized EM wave is below the cyclotron frequency. The fundamental reason why even just a single delocalized electron in a strong magnetic field can give rise to specular reflection is that the \( \mathbf{v} \times \mathbf{B} \) Lorentz force leads to a longitudinal quantum Hall resistance that is strictly zero, which shorts out the incident circularly polarized EM wave. Thus, one concludes that the hard-wall boundary conditions used in the order-of-magnitude estimate given by Equation \( \text{(38)} \) of the scattering cross section of microwaves from the drops are reasonable ones. This conclusion will be tested experimentally (see Fig. \( \text{[11]} \)).

The \( \mathbf{v} \times \mathbf{B} \) Lorentz force leads to a “gravito-quantum Hall effect,” in which an electron, when subjected to a gravitational field \( \mathbf{g} \) in a quantum Hall sample, moves with a velocity that is perpendicular to both \( \mathbf{g} \) and \( \mathbf{B} \) fields. For example, an electron in a vertically oriented, planar quantum Hall sample subjected to the Earth’s gravity field will move with a velocity at right angles to both the Earth’s \( \mathbf{g} \) field and a horizontal DC \( \mathbf{B} \) field applied normally to the sample. This then induces a Hall current that is directly proportional to, and perpendicular to, the applied \( \mathbf{g} \) field. Local, time-varying gravitational fields \( \mathbf{g}(t) \) arising from a gravitational wave impinging on the sample will induce time-varying transverse electrical currents in the quantum Hall sample in the DC magnetic field. Since each electron carries mass as well as charge with it when it moves, this radiation will also induce transverse, time-varying mass currents in this sample. The above analysis can be generalized to gravitational waves, once the quadrupolar pattern of these waves is taken into account. For one sense of circular polarization, a 180° phase shift between the transmitted and incident radiation fields leads to the destructive interference of the transmitted and incident radiation fields, independent of whether these fields are EM or GR in nature. The destructive interference of the transmitted wave with the incident wave in the forward direction leads to reflection of the incident wave in the backward direction. The longitudinal quantum Hall resistance in both EM and GR sectors vanishes, so that circularly polarized EM and GR radiation fields of one sense are both “shorted out,” leading to the specular reflection for both kinds of waves.
18 Appendix C:
Tinkham’s analysis of reflection from thin superconducting films

It may be objected that Equations (24), (31), and (67) are believed to apply only when the thickness $d$ of a sample is large compared with the relevant penetration depth $\ell_P$, whereas the opposite limit (appropriate for a thin-film sample) is assumed here. (In the case of superconductors, the penetration depth $\ell_P$ is the London penetration depth $\lambda_L$.)

Contrary to this common belief, for the case in which the film is thin compared to the penetration depth but the penetration depth is much less than the radiation wavelength—that is, $d \ll \ell_P$, but $\ell_P \ll \lambda$, where $\lambda$ is the free space wavelength—the reflectivity is not of the order of $(d/\ell_P)^2$, as one might naively expect. Rather, it is much higher, and in fact approaches unity as $\lambda$ becomes infinite. See Equation (3.128) of Tinkham’s book [14] for the transmissivity $T$ of superconducting thin films, which reads as follows:

$$T = \left[ \left( 1 + \frac{\sigma_1 Z_0 d}{n + 1} \right)^2 + \left( \frac{\sigma_2 Z_0 d}{n + 1} \right)^2 \right]^{-1}$$

where $\sigma = \sigma_1 + i \sigma_2$ is the complex conductivity of the thin film, $d$ is its thickness, $n$ is the index of refraction of its substrate, and $Z_0 = \sqrt{\mu_0/\varepsilon_0} = \mu_0 c$ is the characteristic impedance of free space for EM waves. Although this equation was derived by Tinkham in the context of superconductivity, it applies to all thin films with a complex conductivity $\sigma = \sigma_1 + i \sigma_2$. (It can also be readily generalized to the case of a complex conductivity tensor, which is applicable to the quantum Hall fluid.)

From this equation, we see that the transmissivity can vanish in the low-frequency limit $\omega \to 0$, since for superconductors $\sigma_2 \to 1/\omega \to \infty$, leading to a substantial reflection of these waves when there is a negligible dissipation within the superconducting film. This result can be understood in terms of an inductance per square element of the thin film

$$L = \mu_0 \ell_{\text{gap}}$$

where $\ell_{\text{gap}}$ is a characteristic energy-gap length scale of the superconductor or of the quantum Hall fluid. This leads to a reactance per square element of the film of

$$X_L = \omega L = \frac{1}{\sigma_2 d}$$

whose low value is responsible for the high reflectivity for waves with frequencies well below the relevant gap frequency. For details, see Refs. [2, 3].

However, in the derivation of Equation (3.128) in Tinkham’s book, it was assumed that the thin conducting film sample was transversely infinite, so that it is not immediately obvious that it can be applied to the electrons on a spherical “Millikan oil drop,” nor is it clear that the concept of a “penetration depth”
applies to the quantum Hall fluid on the surface of superfluid helium. Nevertheless, the only relevant length scales for this fluid are the magnetic length scale (in SI units) $\ell_B = (\hbar/eB)^{1/2}$ for the quantum Hall effect and the confinement distance scale $d_e$ of electrons on the superfluid drop surface discussed in Appendix B, both of which are on the order of 10 nm [15, 22, 23] (also see the footnote on page 15), whereas the radius of a typical drop is around 4 mm, which is much larger than both of these microscopic length scales.

Because a small patch on the surface of a large spherical drop looks planar on these length scales, one can still apply locally to this small patch, in the limit of long wavelengths $\lambda$, the discontinuous-jump boundary conditions for the tangential magnetic field that follows from the Maxwell equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ and from its gravitational analog $\nabla \times \mathbf{B}_G = \mu_0 \mathbf{J}_G$. It is these discontinuous-jump boundary conditions for the tangential components of both $\mathbf{B}$ and $\mathbf{B}_G$ that lead to nonnegligible reflections of both EM and GR waves from the quantum Hall fluid on the surface of a drop. They are also the basis for Equation (3.128) in Tinkham’s book.

Therefore, the planar model used in the derivation of Equation (3.128) in Tinkham’s book should be valid for the reflectivity of the spherical “Millikan oil drops” being considered for the proposed experiment. See Appendix B for a discussion of the physical origin of the surface currents responsible for the reflection in the case of GR waves. In the case of EM waves, the transmissivity of EM waves at low frequencies is given by

$$T \approx 4 \left( \frac{\omega L}{Z_0} \right)^2 = 4 \left( \frac{\omega \mu_0 \ell_{\text{gap}}}{\mu_0 c} \right)^2 = 4 \left( \frac{2\pi \ell_{\text{gap}}}{\lambda} \right)^2$$

(79)

where the approximation has been made that $n \approx 1$. Thus, $T$ is on the order of $(\ell_{\text{gap}}/\lambda)^2 = (\omega/\omega_{\text{gap}})^2 \approx (\omega/\omega_{\text{cycl}})^2$, since $\omega_{\text{gap}} \approx \omega_{\text{cycl}}$ in the case of the quantum Hall fluid. (See Appendix B.) Thus, the transmission $T$ both of a superconducting thin film and of a quantum Hall fluid film remains small, and therefore the reflectivity $R = 1 - T$ of these films remains high for all frequencies $\omega$ of an incident wave that are well below the relevant gap frequency $\omega_{\text{gap}}$.

Note that the permeability of free space $\mu_0$ cancels out of Equation (79) and therefore that $\mu_0$ will also cancel out of the analogous expression for the case of GR waves. Therefore, since the quantum Hall fluid is strictly dissipationless, a nonnegligible reflectivity results for both EM and GR waves from the “Millikan oil drops” for waves with frequencies well below the relevant gap frequency— that is, the cyclotron frequency $\omega_{\text{cycl}}$.

I thank an anonymous referee for pointing out to me Equation (3.128) of Tinkham’s book [14].

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19 Appendix D:

How can a spin-1 photon be converted into a spin-2 graviton?

The question of how a graviton (spin-2) can be produced from a photon (spin-1) is important to consider. (I must thank Tom Kibble for raising this important question.)

The principle of equivalence should apply to all charges and fields in curved spacetime [5, 8]. However, Maxwell’s equations for standard electromagnetism are expressed in terms of fields on a flat spacetime. They must be generalized to fields on a curved spacetime when interactions with gravitational radiation are considered.

The back-action of EM waves propagating in a curved spacetime on GR waves can in principle arise from the contribution of the Maxwell stress-energy tensor, which is quadratic in the EM field strengths, as a source term on the right-hand side of Einstein’s field equations. In the absence of DC fields, such quadratic terms would give rise to second harmonic generation in the conversion of EM to GR waves, but not to first harmonic generation. However, there can in principle arise a linear coupling of EM to GR waves when a DC magnetic (or DC electric) field is present, and Einstein’s equations are linearized in the weak EM and GR wave amplitudes. This linear coupling can arise from a cross term, which consists of a product of the DC field strength and the EM wave amplitude in the quadratic Maxwell stress-energy tensor that leads to first harmonic generation of GR waves at the same frequency as that of the incident EM waves in a linear scattering process.

The role of the “Millikan oil drops” is that they can greatly enhance the coupling between EM and GR waves due to their hard-wall boundary conditions and mesoscopic gravitational masses. The electrons on their surfaces tightly tie the local B field lines to these drops, so that these lines are firmly anchored to the drops. At very low temperatures when the system remains adiabatically in the ground state, the B field lines and the drops co-move rigidly according to a distant observer when the system is disturbed by the passage of a GR or an EM wave. A given drop, however, remains at rest with respect to a local inertial observer at the center of the drop, and the local B field lines also do not appear to move with respect to this local inertial observer. By contrast, to the distant inertial observer in an asymptotically flat region of spacetime far away from the pair of drops, where radiation fields become asymptotically well defined, the two objects appear to be in relative motion, and the system emits power in both GR and EM radiations.

Thus, a graviton (spin-2) can in principle be produced from a photon (spin-1) in the presence of a DC magnetic or a DC electric field (spin-1), in a scattering process from the two objects. See Ref. [32] for a quantum field theoretic treatment of such scattering processes.
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