A prediction model of wellbore temperature and pressure distribution in hydrocarbon gas injection well

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Abstract. For the sake of solving the problem that it is difficult to dynamically monitor the wellbore temperature and pressure of hydrocarbon gas injection well and accurately predicting the bottom hole injection pressure, a mathematical model to describe wellbore temperature and pressure distribution is presented according to the heat transfer principle, the energy conservation law and the momentum theorem. The Shiu-Beggs wellbore temperature calculation method is used in this model to avoid errors caused by inaccurate thermodynamic parameters, and the pressure gradient equation is modified by introducing correction coefficients to improve the calculation precision of the model in practice. The fourth-order Runge-Kutta method was used to solve the model, and the new model was analyzed and verified by actual field data. The results show that the wellbore temperature is nonlinearly distributed, while the wellbore pressure is linearly distributed, and the bottom hole pressure calculated by this model is more accurate than that calculated by other methods. This study can play a certain guiding role for the injection scheme design of hydrocarbon gas injection well.

1. Introduction

Theoretical research and field practice show that hydrocarbon gas flooding can greatly improve oil field recovery [1-2]. In the process of hydrocarbon gas injection, the change of wellbore temperature will lead to the change of hydrocarbon gas physical properties which will affect the wellbore pressure distribution, and the bottom hole pressure will affect the development effect of hydrocarbon gas flooding. Therefore, it is particularly important to dynamically monitor the wellbore temperature and pressure distribution of hydrocarbon gas injection well. Because of the high pressure of the hydrocarbon gas injection well, the test instrument is vulnerable to damage, which makes it difficult to measure the dynamic wellbore temperature and pressure. Hence, it is necessary to forecast the temperature and pressure distribution of the wellbore with corresponding theoretical methods.

In engineering, the average temperature and compressibility factor method is usually used to calculate the wellbore pressure distribution by the wellhead parameters [3]. This method takes the temperature and the hydrocarbon gas compressibility factor of the whole wellbore as constants for calculation, so the calculation result is not accurate enough. Cullender and Smith [4] used numerical integration to calculate the pressure distribution of gas-phase steady pipe flow in the wellbore. But this method ignores the pressure loss caused by the change of kinetic energy, as a result, the calculation accuracy of this method needs to be improved. Ramey [5] proposed the heat transfer model of injection well, which greatly promoted the study of temperature and pressure coupling model of gas injection well. Some scholars established wellbore temperature-pressure coupling models respectively combined with the wellbore heat transfer theory to describe the wellbore temperature and pressure...
distribution when non-hydrocarbon gas such as CO$_2$ or N$_2$ is injected [6-7]. Although these models have certain reference significance for the study of wellbore temperature and pressure distribution of hydrocarbon gas injection well, the solution of the temperature-pressure coupling model requires thermodynamic parameters of fluid, cement sheath and formation whose values are often difficult to obtain accurately in practice, so it is difficult to get accurate wellbore temperature and pressure distribution. In addition, in the process of solving these models, empirical formulas are usually used to calculate intermediate parameters such as friction coefficient and injected gas physical properties, but these calculation formulas of intermediate parameters need to be modified by the actual test data, so the calculation result is not accurate enough.

In this study, for the purpose of providing an accurate calculation method of wellbore temperature and pressure distribution in hydrocarbon gas injection well, a mathematical model is presented based on the principle of heat transfer, the energy conservation law and the momentum theorem. The effect of wellbore temperature and pressure on the hydrocarbon gas physical properties is considered, and the correction coefficients which can be obtained by fitting the actual test data are introduced in this model. Therefore, this model not only conforms to the actual situation, but also can obtain the calculation accuracy better than other methods.

2. Model establishment

2.1. Basic assumptions of the model
The wellbore of hydrocarbon gas injection well is mainly composed of tubing, casing and cement sheath, and its bottom is set with a packer. The bottom hole pressure prediction model of hydrocarbon gas injection well meets the following basic assumptions:

(1) The fluid flow in the wellbore is a one-dimensional steady flow, and the physical parameters of the fluid in the same section are the same.

(2) Only radial heat transfer occurs between the wellbore and the surrounding formation, while the heat transfer along the flow direction is ignored. One-dimensional steady heat transfer occurs from the tubing inner wall to the outer edge of the cement sheath, while one-dimensional transient heat transfer occurs from the outer edge of the cement sheath to the formation.

(3) The geothermal gradient is constant, and the formation temperature around the wellbore is symmetrically linear.

(4) The tubing and casing are concentric rings. The pipe string is in good seal condition and there is no leakage.

(5) Only the thermal resistance of annulus fluid and cement sheath is considered, while the thermal resistance of water film and metal on the inner wall of tubing is ignored.

2.2. Calculation model of wellbore temperature distribution
In a hydrocarbon gas injection well, radial heat transfer between hydrocarbon gas and surrounding formation is carried out through tubing, annulus, casing and cement sheath. In engineering, the temperature distribution of a hydrocarbon gas injection well is usually simplified as a linear distribution. Whereas in the process of gas injection, once the wellbore temperature changes, the physical parameters of the hydrocarbon gas will also change, and the physical parameters of hydrocarbon gas will affect the calculation of bottom hole pressure of injection wells. The simple treatment of wellbore temperature distribution as a linear distribution is bound to cause large errors in the calculation of bottom hole pressure of hydrocarbon gas injection well. At present, the temperature distribution of wellbore fluid is generally studied based on Ramey wellbore heat transfer theory [8-9].

For the purpose of solving the Ramey wellbore heat transfer model, the relaxation distance $A$ should be determined first. The calculation of relaxation distance $A$ requires a series of thermodynamic parameters of fluid, cement sheath and formation [5, 10], but these thermodynamic parameters are difficult to accurately obtain in practice [11], which leads to high uncertainty in the calculation results. Moreover, the calculation process of solving the Ramey wellbore heat transfer model is too
complicated to be popularized in engineering. To solve this problem, the Shiu-Beggs wellbore temperature calculation method is used to avoid the error caused by inaccurate thermodynamic parameters and meet the engineering calculation accuracy requirements.

Based on the Ramey wellbore heat transfer theory, Shiu and Beggs [11] took physical parameters such as relaxation distance, specific heat capacity at constant pressure and geothermal gradient as constants, and derived a formula for calculating the temperature at any depth in the wellbore. For gas injection wells, there is:

$$T_f(z) = T_{wh} + g_T z - g_T A + g_T A e^{-\frac{z}{A}}$$  \hspace{1cm} (1)

Where $T_f$ is the flow temperature of hydrocarbon gas, °C; $T_{wh}$ is the wellhead temperature, °C; $g_T$ is the geothermal gradient, °C·m$^{-1}$; $A$ is the relaxation distance, m; $z$ is the depth in the wellbore, m.

Shiu and Beggs considered the relaxation distance as a function of wellbore fluid mass flow rate, tubing diameter, fluid physical properties and wellhead pressure, and derived a formula for the relaxation distance by regression fitting. For gas injection wells, the relaxation distance is calculated as follow:

$$A = 52.562 W_g D^4 \gamma_g p_{wh}$$  \hspace{1cm} (2)

Where $W_g$ is the mass flow rate of injected gas, kg/s; $D$ is the tubing inner diameter, m; $\gamma_g$ is the relative density of hydrocarbon gas, dimensionless; $p_{wh}$ is the wellhead pressure, MPa; $c_1$, $c_2$, $c_3$ and $c_4$ are regression coefficients, dimensionless.

The regression coefficient values in equation (2) are shown in table 1:

|     | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|-----|-------|-------|-------|-------|
|     | 0.4882 | -0.3476 | 4.7240 | 0.2219 |

2.3. Calculation model of wellbore pressure distribution

Take a microelement $dz$ on the tubing and establish the coordinate system as shown in figure 1. Take the direction of the tubing axis as $Z$-axis, the direction of fluid flow as the positive direction of $Z$-axis, and the angle between $Z$-axis and the horizontal direction is $\theta$.

According to the momentum theorem and the gas equation of state, for the wellbore microelement $dz$, the pressure gradient equation is:

$$\frac{dp}{dz} = \rho g \sin \theta - \frac{f \rho v^2}{2D} + \frac{\rho v^2}{p} \frac{dp}{dz}$$  \hspace{1cm} (3)

Where $\rho$ is the hydrocarbon gas density, kg/m$^3$; $g$ is the acceleration of gravity, m/s$^2$; $f$ is the friction coefficient, dimensionless; $v$ is the velocity of hydrocarbon gas, m/s; $p$ is the pressure, Pa.
The three terms at the right end of equation (3) indicate that the total pressure gradient of the wellbore flow in the hydrocarbon gas injection well consists of gravity pressure gradient, friction pressure gradient and acceleration pressure gradient, and the gravity pressure gradient, friction pressure gradient and acceleration pressure gradient are related to intermediate parameters such as hydrocarbon gas density $\rho$ or friction coefficient $f$. The density of the injected hydrocarbon gas can be calculated by the following equation:

$$\rho = \frac{\rho_{sc} T_{sc}}{Z_g p_{sc}}$$

(4)

Where $\rho_{sc}$ is the density of hydrocarbon gas under standard condition, kg/m$^3$; $T_{sc}$ is the temperature of hydrocarbon gas under standard condition, °C; $Z_g$ is the compressibility factor of hydrocarbon gas, dimensionless; $p_{sc}$ is the pressure of hydrocarbon gas under standard conditions, Pa.

The compressibility factor $Z_g$ of injected hydrocarbon gas can be calculated by the empirical formula proposed by Hall and Yarborough [12]. The viscosity of injected hydrocarbon gas can be calculated according to the empirical formula proposed by Lee et al. [13], and then the Reynolds number $Re$ can be obtained.

In the process of gas injection, the gas injection rate is usually large, and the viscosity of injected gas is small. The Reynolds number $Re$ in the process of gas injection is far greater than 4000, the flow of gas in the wellbore is turbulent. Therefore, the friction coefficient $f$ can be solved by the empirical formula by proposed Jain [14]:

$$\frac{1}{\sqrt{f}} = 1.14 - 2 \log \left( \frac{\varepsilon}{D} + \frac{21.25}{Re^{0.8}} \right)$$

(5)

Where $\varepsilon/D$ is the relative roughness, dimensionless.

According to the solution method of the intermediate parameters in the above model, physical quantities such as hydrocarbon gas density $\rho$ or friction coefficient $f$ are calculated directly or indirectly by empirical formulas. These empirical formulas, which are based on the statistical regression of experimental data or field test data, have good calculation accuracy in the oilfield blocks with conditions similar to those of the experiments or field tests, while there are some errors in the calculation of other blocks. Therefore, the calculation results of gravity pressure gradient, friction pressure gradient and acceleration pressure gradient are not accurate enough, and these calculation formulas need to be modified by the measured data.

To improve the calculation result accuracy of the model and establish a calculation method of bottom hole pressure with strong applicability in practice, three fitting correction coefficients, $\alpha$, $\beta$ and $\gamma$, are introduced to correct the gravity pressure gradient, friction pressure gradient and acceleration pressure gradient in equation (3) respectively. The values of $\alpha$, $\beta$ and $\gamma$ can be obtained by fitting the field data with the least square method. The corrected wellbore pressure gradient equation is:

$$\frac{dp}{dz} = \frac{\alpha g \sin \theta - \beta f \rho v^2}{2D} \left(1 - \gamma \frac{v^2}{p}\right)$$

(6)

3. Model solution
The initial condition is that the pressure at the wellhead $z_0$ is $p_0$, and the right end of equation (6) is denoted as $F(z, p)$, so:

$$\left\{ \begin{array}{l} \frac{dp}{dz} = F(z, p) \\ p(z_0) = p_0 \end{array} \right.$$ 

(7)

For such initial value problems of ordinary differential equations, the fourth-order Runge-Kutta method can be used to solve iteratively. Take the step length $h$ for $z$, and calculate the following values from the known initial values $(z_0, p_0)$ and the function $F(z, p)$:
The pressure value at the node \( z_1 = z_0 + h \) is:

\[
k_1 = F(z_0, p_0)
\]

\[
k_2 = F\left(z_0 + \frac{h}{2}, p_0 + \frac{h}{2} k_1\right)
\]

\[
k_3 = F\left(z_0 + \frac{h}{2}, p_0 + \frac{h}{2} k_2\right)
\]

\[
k_4 = F\left(z_0 + h, p_0 + hk_3\right)
\]

The pressure value at the node \( z_1 = z_0 + h \) is:

\[
p_1 = p_0 + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4)
\]

If \( z_1 \) fails to reach the expected wellbore depth, then the calculated \((z_1, p_1)\) should be used as the initial value of the next calculation step to repeat the above process and continue to recurse until the expected wellbore depth.

### 4. Case study

Taking Well X, a hydrocarbon gas injection well in Reservoir A, as an example. The temperature and pressure distribution of wellbore and bottom hole pressure are predicted and analyzed. The parameters required in the calculation are shown in the table below:

| Parameter                        | Value       |
|----------------------------------|-------------|
| Formation temperature/°C         | 129.44      |
| Geothermal gradient/°C∙m\(^{-1}\) | 0.031       |
| Drilling depth/m                 | 2866.03     |
| Tubing diameter/mm               | 100.3       |
| Tubing absolute roughness/mm     | 0.01524     |
| Gas injection rate/10\(^4\)m\(^3\)∙d\(^{-1}\) | 16.54 |
| Hydrocarbon gas relative density | 0.8004453   |
| Measured wellhead pressure/MPa   | 29.63       |
| Measured bottom hole pressure/MPa| 38.38       |
| Measured wellhead temperature/°C | 40.56       |

According to the bottom hole pressure test data of Well X, \( \alpha = 1.1101, \beta = 1.1918, \gamma = 0.9719 \) by least-square fitting. The fitting result is shown in figure 2. \( R^2 \) is 0.99962, so the fitting effect is good.

![Figure 2. Fitting result.](image-url)
By solving the wellbore temperature equation and the revised wellbore pressure gradient equation, the wellbore temperature distribution curve and the pressure distribution curve are shown in figure 3 and figure 4 respectively, and then the bottom hole pressure value of Well X is obtained from figure 4.

As can be seen in figure 3, the wellbore temperature increases with the increase of the well depth, and the overall wellbore temperature distribution of Well X is nonlinear. The flow temperature of hydrocarbon gas near the wellhead is mainly controlled by the injection wellhead temperature. With the increase of the well depth, the influence of wellhead temperature on the flow temperature of hydrocarbon gas decreases, so the temperature distribution shows a nonlinear distribution feature near the wellhead. While the temperature distribution in the middle and lower part of the wellbore is mainly affected by the geothermal gradient, so it is basically linear. Furthermore, because the hydrocarbon gas has low thermal conductivity near the bottom hole and creates a thermal insulating effect, the calculated bottom hole temperature is 125.74°C, lower than the formation temperature.

As is shown in figure 4 that the wellbore pressure increases with the increase of the well depth, and the wellbore pressure distribution of Well X has linear characteristics. The total pressure gradient in the wellbore is composed of gravity pressure gradient, friction pressure gradient and acceleration pressure gradient. Since the density change rate under high wellbore pressure is much lower than that under low-pressure conditions, the gravity pressure gradient and the acceleration pressure gradient change little. Because of the small hydrocarbon gas viscosity, the friction pressure gradient is small, and its influence on the total pressure gradient is not obvious. In summary, the wellbore pressure distribution is basically linear.

The measured bottom hole pressure of Well X is 38.38MPa. Table 3 shows the calculation results and errors of the new bottom hole pressure prediction model proposed in this paper, average temperature and compressibility factor method and temperature-pressure coupling method. Compared with these three methods, the model proposed in this paper has higher accuracy and more practical value.

| Method                               | Value/MPa | Error/% |
|--------------------------------------|-----------|---------|
| Average temperature and compressibility factor method | 36.49 | 4.92 |
| Temperature-pressure coupling method  | 37.47 | 2.37 |
| The new model proposed in this paper  | 38.39 | 0.03 |

5. Conclusion
In this study, according to the momentum theorem, the energy conservation law and the heat transfer theory, considering the influence of temperature and pressure on the physical properties of hydrocarbon gas and the change of pressure caused by the change of kinetic energy, the new model is
established which can describe the wellbore temperature and pressure distribution of hydrocarbon gas injection well. The case study result shows that the wellbore temperature and pressure of the hydrocarbon gas injection well increase with the increase of the well depth, the wellbore temperature presents a nonlinear distribution, and the wellbore pressure presents a linear distribution. The bottom hole pressure calculated by this model is more accurate than the conventional method for gas injection well, which proves that the model has a certain application value.

This work, which can solve the problem of predicting the temperature and pressure in the wellbore of hydrocarbon gas injection well, is of great significance to the design of the hydrocarbon gas flooding scheme.

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