Ring-shear Test and Constitutive Model of Surfaces between Clay in Nanchang Area and Different Roughness Contact

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Abstract. The shear failure characteristics of soil-structure planes are the internal mechanism of slip deformation of foundation structures and strata, and it is an inevitable and important problem in engineering. In this paper, the shear tests of silty clay and structural planes with different roughness in Nanchang area are carried out by using the modified shear apparatus, and the statistical piecewise constitutive model of shear damage based on Weibull distribution is established. The results show that the shear stress-shear displacement curves of contact surfaces with different roughness exhibit strain hardening and strain softening phenomena; the peak shear strength and the final residual strength of the contact surface increase with the enhancement of the roughness of the contact surface and the normal stress applied on the contact surface; the statistical piecewise constitutive model of shear damage based on Weibull distribution achieves the elastic-plastic properties of the contact surface well. The strain softening phenomena at the deformation stage after the peak value is in good agreement with the experimental curves.

1. The introduction
The relationship between shear stress and shear displacement of contact surface determines the fundamental cause of deformation and failure of foundation structure and stratum. A large number of scholars have studied this problem. Clough and Duncan [1] found that the relationship between shear stress and shear displacement in direct shear test of contact surface with direct shear apparatus conforms to hyperbolic form, and established a nonlinear elastic constitutive model (hyperbolic model) suitable for describing the shear process of contact surface. Brandt [2] proposed the elastic-plastic constitutive model of contact surface based on the laboratory direct shear test and retaining wall observation data. Huizyuan Chen [3] considered that when the shear stress was less than the friction force, the relationship between shear stress and shear displacement of contact surface was the linear elastic relationship, otherwise it satisfied the plastic relationship. Zongze Yin [4] proposed a rigid plastic model of contact surface, and believed that the failure of contact surface was a process of gradual development from edge to interior. DESAI [5] first applied the concept of damage to the contact surface constitutive model to establish the damage constitutive model. Hu [6] established a contact surface constitutive model based on the basic principle of damage mechanics to describe the mechanical properties of the contact surface. Linde Yang [7] established the statistical damage constitutive model of the contact surface between soil and structure based on the randomness of the distribution of internal defects. Ga Zhang et al. [8-10] established the elastic-plastic damage model of the contact surface between coarse grained soil and structure. Wengui Cao [11] divided the stress and deformation curve of contact surface into four stages and established the damage constitutive model that could reflect residual strength characteristics by combining the statistical damage theory.
Based on the data obtained from the indoor ring shear test, the relation curve between shear stress and shear displacement of contact surface was fitted with nonlinear elastic model and hump softening model, the applicability and deficiency of them were analyzed. A statistical damage segmenting function based on Weibull distribution and strain softening curve is proposed.

2. ring shear test

The improved HJ-1 Ring Shear instrument was used in the test. Remove the permeable plate originally used to fix the soil sample in the lower box, load the steel ring with outer radius of 50mm, inner radius of 30mm and height of 20mm. Three kinds of sand grains with particle size ranges of 0.25mm-0.5mm, 0.5mm-1mm and 1mm-2mm were pasted by epoxy resin on the surface of steel ring respectively, and then a smooth ring without sticking sand grains was made to achieve four different structural surface roughness. The surface roughness of the four processed structures was measured and quantified by the surface roughness measuring instrument 0918. In the measurement process, 10 roughness data were measured from 10 points on each steel ring, and 5 largest data were selected to obtain the average, the average value was taken as the roughness value.

Fig 1. Steel rings with different roughness

![Steel rings with different roughness](image)

Table 1. Roughness measurement

| Number of measurement | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | Roughness |
|-----------------------|----|----|----|----|----|----|----|----|----|----|-----------|
| First                 | 1.839 | 1.893 | 2.384 | 1.624 | 2.076 | 2.264 | 1.629 | 1.732 | 1.855 | 2.345 | 2.1924 |
| Second                | 1.497 | 0.912 | 1.541 | 1.392 | 1.535 | 1.407 | 1.716 | 1.391 | 1.046 | 1.159 | 1.5392 |
| Third                 | 0.728 | 0.695 | 0.834 | 0.809 | 0.656 | 0.525 | 0.948 | 0.654 | 0.705 | 0.865 | 0.8368 |
| Fourth                | 0.071 | 0.008 | 0.067 | 0.048 | 0.045 | 0.038 | 0.013 | 0.083 | 0.018 | 0.049 | 0.0636 |

The soil material used in the test was red-brown clay in Nanchang area. The liquid limit was 38.38%, plastic limit was 19.23%, plastic index was 19.15, and the optimal moisture content was 19.87%. The soil sample used for the test was remolded soil which was prepared with the optimal moisture content of 19.87% after the silty clay was dried and maintained for one day to ensure even water content in the soil. By tamping, the density of the soil sample is 1.70g/cm³ cubed, and the annular soil sample with an outer radius of 50mm, an inner radius of 30mm and a height of 20mm is made.

In this test, the shear angular velocity V=3°/min was controlled, and the test stopped when the shear angle reached 30°. Ring shear tests were conducted on four roughness contact surfaces under six normal stresses of 30, 50, 100, 150, 200 and 300 kPa respectively, and the shear stress-shear displacement curves obtained are shown in figure 2.
The test results show the following conclusions:

1) The peak shear strength and the final residual strength of the contact surface increase with the enhancement of the roughness and the normal stress applied on the contact surface.

2) When the roughness is low, the shear stress-shear displacement curve shows obvious strain softening phenomenon after the peak value. Before the peak, the shear stress rises linearly with the shear displacement and then reaches the peak gradually. After the peak, under the action of normal stress of 100kPa, the shear stress of the contact surface with lower roughness will reduce in a shorter shear path and reach the residual stable shear stress. However, with the increase of roughness and normal stress, the shear stress decreases gradually and the residual stress keeps approaching the peak stress.

When researching the strength properties of the contact surface between soil and structure, most domestic scholars refer to the molar coulomb strength criterion and take the cohesion force and internal friction angle as the shear strength parameters of the contact surface between soil and structure, the construction of molar coulomb criterion is

\[ \tau_{\text{max}} = c + \sigma \tan \phi \]  

where, \( \tau_{\text{max}} \) is the peak shear stress when the contact surface fails, \( \sigma \) is the normal stress acting on the contact surface, \( \phi \) is the internal friction angle of the contact surface, \( c \) is the initial cohesion of the contact surface. The contact surface parameters and diagrams obtained by fitting the peak shear stress of the contact surface obtained from the experiment are shown in table 3 and fig 5.
Fig 3. Relation between peak shear strength and normal stress of contact surface

Fig 3 shows that there is a good linear relationship between the peak shear stress of the four-level roughness contact surface and the normal stress acting on it, and the correlation coefficients are all higher than 0.98, which proves that it conforms to the molar coulomb strength criterion.

The data in table 2 shows that the internal friction angles of the four roughness contact surfaces do not change too much, that is, the internal friction angles of the contact surface are less affected by the roughness changes, and the initial cohesion increases with the increase of the contact surface roughness, which indicates that the increase of the roughness can effectively improve the shear strength of the contact surface.

| Number of measurement | Initial cohesion $c$ /kPa | Internal friction angle $\phi$ (°) | The correlation coefficient |
|-----------------------|----------------------------|-----------------------------------|---------------------------|
| First                 | 51.85                      | 19.82                             | 0.98                      |
| Second                | 46.99                      | 20.05                             | 0.99                      |
| Third                 | 40.43                      | 20.10                             | 0.99                      |
| Fourth                | 27.16                      | 20.14                             | 0.99                      |

3. Constitutive model analysis of contact surface

In this paper, nonlinear elastic constitutive model, hump softening model and piecewise function combined with random distribution theory of damage are used to fit the test curve. Mainly from the line and change trend of the three fitting situation is discussed

3.1. Nonlinear elastic constitutive model (hyperbolic model)

Clough and Duncan studied the relationship between shear stress and shear displacement of the sand-smooth concrete contact surface and found that the average shear stress $\tau$ and average shear displacement $\omega$ of the contact surface presented a nonlinear hyperbolic model, that is:

$$\tau = \frac{\omega}{a + b\omega}$$

where $a$ and $b$ are parameters reflecting the properties of the contact surface.

In the test results, the shear stress-shear displacement curve of the second order roughness under 0.2MPa normal pressure which shows obvious hardening after the peak and the shear stress-shear displacement curve of the fourth order roughness under 0.1MPa normal stress which shows obvious softening after the peak were selected for analysis. The curves fitted according to the nonlinear elastic
The nonlinear elastic constitutive model are shown in Fig 4 and Fig 5. Figures show that the nonlinear elastic constitutive model is applied to the deformation hardening curve with a high degree of fitting, and the peak value and the displacement reaching the peak value can well correspond to the test value. However, when there is a large strain softening phenomenon after the peak value, it cannot restore the strain softening phenomenon after the peak value. When the fitting converges, there is a big error between the peak value of the fitting results and the test peak value.

3.2. Hump softening model fitting

It can be seen from the above analysis that the nonlinear elastic constitutive model cannot achieve the strain softening phenomenon after the peak value. In this experiment, all curves with obvious strain softening show a hump at the peak. This is consistent with the epsilon hump softening model [12] obtained in the soil triaxial test. The equation is:

\[
\sigma_1 - \sigma_3 = \frac{\varepsilon(a + c\varepsilon)}{(a + b\varepsilon)^2}
\]

where the parameters \(a\), \(b\) and \(c\) are related to the peak strength \(\sigma_f\), residual strength \(\sigma_r\) and axial deformation of soil when the peak strength is reached in the triaxial test \(\varepsilon_f\).

Compare hump softening model with \(\tau - \omega\) curve of contact surface, there is:

\[
\tau = \frac{\omega(a + c\omega)}{(a + b\omega)^2}
\]

where the parameters \(a\), \(b\) and \(c\) are related to the peak shear strength \(\tau_f\), residual strength \(\tau_r\) and shear displacement when the peak shear strength is reached \(\omega_f\). Similarly, the shear stress-shear displacement curve of the second order roughness under 0.2MPa normal stress and the shear stress-shear displacement curve of the fourth order roughness under 0.1MPa normal stress are selected for fitting analysis of construction (4). The results are shown in Fig 6 and Fig 7.
By comparing with the nonlinear elastic constitutive model, the hump cubic polynomial has higher fitting accuracy on both the typical hardening curve and the typical softening curve, and also solves the disadvantage that the former cannot achieve the strain softening phenomenon. However, although the strain softening phenomenon is retained in fitting the canonical strain softening curve, the strain softening law cannot be guaranteed. The fitting value of the hump cubic polynomial still does not reach the residual strength at the larger shear displacement, and the shear strength shows a decreasing trend, which is not consistent with the test value reaching the residual strength at the shorter shear displacement and tending to the stable state. Moreover, although the peak value of the fitting value and the peak value of the test value of the hump softening model are more accurate than the nonlinear elastic constitutive value, their errors cannot be ignored.

3.3. The piecewise function combined with the random distribution theory of damage
A large number of studies have proved that [7,13-16], the failure process of contact surface can be described by random damage theory, and its failure element body satisfies Hooke’s law. Based on this, the damage factor of contact surface is combined with the generalized Hooke’s law to get construction (5):

$$\tau = k_\omega (1-\varepsilon)$$  \hspace{1cm} (5)

where, $\tau$ is the shear stress, $\omega$ is the shear displacement, $k$ is shear modulus of contact surface, $\varepsilon$ is the damage factor.

In the study of damage statistics, it is necessary to use statistical methods to describe the damage infinitesimal. Weibull distribution function is a relatively reliable statistical function, and the probability density function of failure probability is

$$f(\omega) = \frac{m_\omega \eta^{m-1}}{\eta^m} \exp \left[-(\omega/\eta)^m\right]$$  \hspace{1cm} (6)

where, $m$ is the shape parameter of distribution, $\eta$ is the size parameter of distribution, $\omega$ is the shear displacement of contact surface element, $f(\omega)$ is the failure probability density of contact surface element when the shear displacement reaches $\omega$.

According to the definition of damage factor, the failure probability density function is integrated on the total deformation amount, and the relationship between $\varepsilon$ and the strength probability in the micro element body is

$$\varepsilon = \int_0^\omega f(\omega) d\omega = 1 - e^{-(\omega/\eta)^m}$$  \hspace{1cm} (7)

$$\tau = k_\omega \exp \left[-(\omega/\eta)^m\right]$$  \hspace{1cm} (8)

Substitute construction (6) into construction (7):
Construction (8) is the relationship between shear stress and shear displacement of contact surface damage. All parameters in the construction can be obtained by fitting test curves.

Before the contact surface reaches the peak shear stress, it is mainly filled into the gap of the lower structural plane by the continuous destruction of soil near the contact surface. When the gap of the lower structural plane is filled, the contact surface reaches the peak shear stress, this process is mainly characterized by elastic-plastic deformation. After the peak, residual shear stress \( \tau_f \) gradually tends to be stable. This is the result of the interaction of the upper soil, the lower structure plane and the soil filled in the structure plane. After the peak, the soil filled in the structural plane will reach a stable state in a very short time, which shows that the shear stress-shear displacement curve of the contact surface will show a hump-shaped peak area. Residual strength will be reached at a short shear displacement after the peak.

It can be seen from the above analysis that the shear deformation of the contact surface takes the peak value as the cut-off point. Before the peak value, the soil mass near the contact surface is destroyed and filled into the void of the structural plane. The variation rule of residual strength after peak value is influenced by the joint action of the upper soil, the lower structural plane and the soil in the structural plane. Therefore, this paper takes the peak point as the segmented point and adopts construction (8) to fit the test curve before reaching the peak stress. Aiming at the strain softening phenomenon after the peak stress, hyperbolic function was used to fit it according to its linear change. The whole construction is:

\[
\tau = \begin{cases} 
  k \omega \exp\left[-\left(\frac{\omega}{\eta}\right)^m\right] & \omega \leq \omega_f \\
  \frac{d}{\omega} + e & \omega > \omega_f 
\end{cases}
\]

(9)

While \( \omega \leq \omega_f \), \( k \) is the shear modulus of the contact surface, \( m \) and \( \eta \) are the Weibull distribution shape parameters related to the physical properties of the contact surface, which can be obtained by fitting with the curve of experimental data.

When \( \omega > \omega_f \), let \( \omega \) goes to infinity, there is

\[
\tau_{\omega \to \infty} = e + \tau_f
\]

(10)

Take \( \omega \) as the shear displacement \( \omega_f \) corresponding to the peak shear stress \( \tau_f \), there is

\[
\tau_f = \frac{d}{\omega_f} + \tau_f
\]

(11)

\[
d = (\tau_f - \tau_f)\omega_f
\]

(12)

Fig 8 is the fitting between construction (9) and the test value.
Fig 8. Piecewise function fitting with test value

Correlation fitting parameters are shown in table 3.

| Roughness series | normal stress /kPa | a     | b     | c     | d     | e     | $r^2$ (anterior segment, posterior segment) |
|------------------|-------------------|-------|-------|-------|-------|-------|------------------------------------------|
| first            | 30.00             | 75.28 | 2.03  | 0.69  | 40.66 | 51.94 | 0.99,0.96                                |
|                  | 50.00             | 439.45| 0.06  | 0.28  | 56.99 | 66.38 | 0.99,0.91                                |
|                  | 100.00            | 503.21| 0.06  | 0.27  | 45.06 | 83.37 | 0.99,0.91                                |
|                  | 150.00            | 155.13| 0.77  | 0.39  | 45.07 | 100.75| 0.99,0.91                                |
|                  | 200.00            | 108.65| 1.75  | 0.46  | 89.23 | 109.75| 0.99,0.94                                |
|                  | 300.00            | 52.89 | 7.98  | 0.75  | 277.21| 142.83| 0.99,0.91                                |
| second           | 30.00             | 20.75 | 5.79  | 2.84  | 41.56 | 48.87 | 0.99,0.98                                |
|                  | 50.00             | 55.26 | 2.85  | 0.63  | 41.21 | 56.35 | 0.98,0.90                                |
|                  | 100.00            | 79.11 | 2.65  | 0.69  | 55.12 | 73.28 | 0.99,0.99                                |
|                  | 150.00            | 48.40 | 5.26  | 0.67  | 175.09| 90.30 | 0.99,0.93                                |
|                  | 200.00            | 105.72| 1.89  | 0.48  | 96.07 | 18.74 | 0.99,0.99                                |
|                  | 300.00            | 38.64 | 11.22 | 1.02  | 234.46| 141.27| 0.99,0.94                                |
| third            | 30.00             | 36.70 | 3.56  | 0.93  | 35.73 | 39.69 | 0.99,0.95                                |
|                  | 50.00             | 68.96 | 2.29  | 0.77  | 52.04 | 49.70 | 0.99,0.93                                |
|                  | 100.00            | 27.53 | 6.69  | 2.35  | 67.75 | 58.48 | 0.96,0.97                                |
|                  | 150.00            | 24.41 | 7.30  | 4.26  | 116.92| 79.92 | 0.99,0.98                                |
|                  | 200.00            | 35.59 | 7.71  | 1.79  | 111.25| 97.26 | 0.96,0.91                                |
|                  | 300.00            | 42.83 | 9.38  | 1.15  | 115.71| 138.09| 0.99,0.90                                |
| fourth           | 30.00             | 112.99| 0.67  | 0.54  | 14.85 | 30.81 | 1.097                                    |
|                  | 50.00             | 24.95 | 4.40  | 2.21  | 32.63 | 39.13 | 0.98,0.97                                |
|                  | 100.00            | 24.28 | 4.48  | 5.99  | 84.20 | 44.86 | 0.99,0.98                                |
|                  | 150.00            | 31.92 | 6.03  | 1.93  | 117.29| 58.23 | 0.95,0.96                                |
|                  | 200.00            | 25.49 | 7.47  | 3.17  | 145.69| 72.25 | 0.95,0.97                                |
|                  | 300.00            | 31.00 | 8.64  | 3.49  | 83.63 | 127.38| 0.99,0.91                                |
Figure 7 and Table 2 demonstrate that Construction 9 can well restore the strain softening of the test curve, achieved the strain softening law that the contact surface reaches the residual strength under a short shear displacement and tends to be stable after the peak shear stress. Both the peak shear stress, the peak shear displacement corresponding to the peak shear stress and the residual shear stress of the fitted value are all have a high degree of agreement with the test values. In summary, the model is suitable for describing the shear characteristics of the contact surface.

4. Conclusion
Based on the ring shear test, the shear stress-shear displacement curves of four contact surfaces with different roughness under six normal stresses are obtained. The experimental curve was fitted with the nonlinear elastic constitutive model and the hump-type softening model, and the error in fitting accuracy was found considering the characteristics of curves. A new piecewise function is proposed based on the random damage statistical theory of contact surface and the linear characteristics of experimental curves. The main conclusions obtained are as follows.

(1) The shear stress-shear displacement curves of the contact surface mainly show two forms: strain hardening and strain softening.

(2) The peak shear strength of the contact surface and the final residual strength after the peak increase with the increase of the roughness of the contact surface and the normal stress applied on the contact surface;

(3) The piecewise constitutive model of shear damage based on Weibull distribution can well realize the elastic-plastic deformation stage of the contact surface and the rapid strain softening after the peak, which is in good agreement with the test value.

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