SU(2)$_N$ Model of Vector Dark Matter with a Leptonic Connection

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Abstract

Models of fermion and scalar dark matter abound. Here we consider instead vector dark matter, from an SU(2)$_N$ extension of the standard model. It has a number of interesting properties, including a natural implementation of the inverse seesaw mechanism for neutrino mass. The annihilation of dark matter for calculating its relic abundance in this model is not dominated by its cross section to standard-model particles, but rather to other new particles which are in thermal equilibrium with those of the standard model.
Whereas the existence of dark matter is universally accepted, its nature remains unknown. It is usually assumed to be a single particle, but it may also be more than one \[1\]. In specific models, it is often considered to be a fermion or scalar. However, vector dark matter is certainly also possible \[2, 3, 4, 5, 6, 7, 8, 9\]. In this paper we consider a variant of an SU(2)\(_N\) model proposed previously \[5, 6\]. The difference is that in our present study, all standard-model (SM) fermions are singlets under SU(2)\(_N\), whereas in the earlier work, that was not the case.

The new particles of our model are the three neutral gauge bosons \(X_{1,2,3}\) of SU(2)\(_N\), three copies of a neutral Dirac fermion SU(2)\(_N\) doublet \((n_1, n_2)_{L,R}\), a neutral scalar SU(2)\(_N\) doublet \((\chi_1, \chi_2)\), and a scalar bidoublet
\[
\zeta = \begin{pmatrix} \zeta_1^0 & \zeta_2^0 \\ \zeta_1^- & \zeta_2^- \end{pmatrix},
\]
which transform (vertically) under SU(2)\(_L\) \(\times\) U(1) and (horizontally) under SU(2)\(_N\). The allowed Yukawa and trilinear scalar couplings are
\[
(p_L \zeta_1^0 + \bar{e}_L \zeta_1^-) n_{1R} + (\bar{\nu}_L \zeta_2^0 + \bar{e}_L \zeta_2^-) n_{2R}, \quad (1)
\]
\[
(\phi^0 \zeta_1^0 - \phi^+ \zeta_1^-) \chi_1 + (\phi^0 \zeta_2^0 - \phi^+ \zeta_2^-) \chi_2. \quad (2)
\]

In analogy to the earlier work \[5, 6\], we impose a global U(1) symmetry \(S'\) on the new particles so that \(n\) and \(\chi\) have \(S' = 1/2\) and \(\zeta\) has \(S' = -1/2\). As \(\chi_2\) breaks SU(2)\(_N\) completely, \(T_{3N} + S' = S\) remains exact, under which \(n_1, \chi_1 \sim +1, n_2, \chi_2, \zeta_2 \sim 0\), and \(\zeta_1 \sim -1\). As for the vector gauge bosons, \(X(X') = (X_1 \mp iX_2)/\sqrt{2} \sim \pm 1\) and \(Z' = X_3 \sim 0\). Note that \(S'\) distinguishes the bidoublet \(\zeta\) from its dual \(\bar{\zeta} = \sigma_2 \zeta^* \sigma_2\), and forbids certain terms in the Lagrangian which would be otherwise allowed. In Refs. \[5, 6\], the residual U(1) symmetry \(S\) is broken explicitly to \((-1)^S\). Here it remains exact. Note that without \(\zeta\), the dark sector would only communicate with the SM through the Higgs portal. Here \(\zeta\) serves another purpose, i.e. neutrino mass.
Consider first the neutrino mass matrix spanning \((\bar{\nu}_L, n_{2R}, \bar{n}_{2L})\), i.e.

\[
M_{\nu n} = \begin{pmatrix}
0 & m_D & 0 \\
m_D & 0 & M \\
0 & M & 0
\end{pmatrix},
\]

where \(m_D\) comes from \(\langle \zeta_2^0 \rangle\) and \(M\) is an allowed invariant mass. It is clear that this results in one heavy Dirac fermion and one massless neutrino corresponding to \(\cos \theta \nu_L - \sin \theta n_{2L}\), where \(\tan \theta = m_D/M\). As such, lepton number \(L\) is conserved, with \(n_{1,2}\) also having \(L = 1\).

To obtain a nonzero neutrino mass, a scalar triplet \(\Delta\) under \(SU(2)_N\) is required. Let

\[
\Delta = \begin{pmatrix}
\Delta_2/\sqrt{2} \\
\Delta_1 \\
-\Delta_2/\sqrt{2}
\end{pmatrix},
\]

with \(S' = -1\), then the terms

\[
n_1n_1\Delta_1 + (n_1n_2 + n_2n_1)\Delta_2/\sqrt{2} - n_2n_2\Delta_3,
\]

where each \(nn\) denotes either \(n_Ln_L\) or \(n_Rn_R\), break \(L\) to \((-1)^L\) with a nonzero \(\langle \Delta_3 \rangle\), without breaking \(S\). Note that whereas \(\langle \Delta_1 \rangle\) as well as \(\langle \Delta_3 \rangle\) must be nonzero (and large) in the model of Ref. \[6\], thus breaking \(S\) to \((-1)^S\), only \(\langle \Delta_3 \rangle\) is nonzero here and it is very small for the implementation of the inverse seesaw \[10\] \[11\] \[12\]:

\[
M_{\nu n} = \begin{pmatrix}
0 & m_D & 0 \\
m_D & m'_2 & M \\
0 & M & m_2
\end{pmatrix},
\]

where \(m_2\) comes from \(\langle \Delta_3^* \rangle\) and \(m'_2\) from \(\langle \Delta_3 \rangle\). This means that an inverse seesaw neutrino mass is obtained:

\[
m_\nu \simeq \frac{m'^2_2m_2}{M^2}.
\]

Let \(\langle \phi^0 \rangle = v_1, \langle \zeta_2^0 \rangle = v_2, \langle \chi_2 \rangle = u_2, \langle \chi_3 \rangle = u_3\), with \(g_N\) the \(SU(2)_N\) gauge coupling, then the masses of the vector gauge bosons of this model are

\[
m^2_W = \frac{1}{2} g^2(v_1^2 + v_2^2), \quad m^2_X = \frac{1}{2} g^2_N(u_2^2 + v_2^2 + 2u_3^2),
\]

\[
m^2_{Z, Z'} = \frac{1}{2} \begin{pmatrix}
(g_1^2 + g_2^2)(v_1^2 + v_2^2) & -g_N\sqrt{(g_1^2 + g_2^2)v_2^2} \\
-g_N\sqrt{(g_1^2 + g_2^2)v_2^2} & g^2_N(u_2^2 + v_2^2 + 4u_3^2)
\end{pmatrix},
\]
where $X_3$ has been renamed $Z'$. In this model, all the SM fermions obtain masses from $v_1$, except for the neutrinos which require $v_2$, i.e. $m_D$ in Eqs. (6) and (7). We assume $v_2$ (which causes $Z−Z'$ mixing) and $u_3$ (which breaks $L$ to $(-1)^L$) to be small.

Another important difference is that whereas the $X_{1,2}$ relic abundance is determined by their annihilation cross section to standard-model (SM) particles in Ref. [6], it is determined by $X\bar{X} \rightarrow \zeta_2 \bar{\zeta}_2$ here. Thermalization with SM particles is maintained through $\zeta_2 \bar{\zeta}_2 \rightarrow l^- l^+$, etc.

We now supply the details of this model. The Higgs potential is given by

$$V = \mu_1^2 \text{Tr}(\zeta \bar{\zeta}) + \mu_2^2 \Phi \bar{\Phi} + \mu_3^2 \chi \bar{\chi} + \mu_4^2 \text{Tr}(\Delta \bar{\Delta}) + (\mu_1 \Phi \bar{\zeta} \chi + \mu_2 \bar{\chi} \Delta \chi + \text{H.c.})$$
$$+ \frac{1}{2} \lambda_1 (\text{Tr}(\zeta \bar{\zeta}))^2 + \frac{1}{2} \lambda_2 (\Phi \bar{\Phi})^2 + \frac{1}{2} \lambda_3 \text{Tr}(\zeta \bar{\zeta} \zeta \bar{\zeta}) + \frac{1}{2} \lambda_4 (\chi \bar{\chi})^2 + \frac{1}{2} \lambda_5 [\text{Tr}(\Delta \bar{\Delta})]^2$$
$$+ \frac{1}{4} \lambda_6 \text{Tr}(\Delta \bar{\Delta} - \Delta \bar{\Delta})^2 + f_1 \chi \bar{\chi} \bar{\zeta} \tilde{\zeta} + f_2 \chi \bar{\chi} \tilde{\zeta} \bar{\zeta} + f_3 \Phi \bar{\zeta} \bar{\zeta} + f_4 \Phi \bar{\zeta} \bar{\zeta}$$
$$+ f_5 (\Phi \Phi) (\chi \bar{\chi}) + f_6 (\chi \bar{\chi}) \text{Tr}(\Delta \bar{\Delta}) + f_7 \chi \bar{\chi} (\Delta \bar{\Delta} - \Delta \bar{\Delta}) \chi + f_8 (\Phi \Phi) \text{Tr}(\Delta \bar{\Delta})$$
$$+ f_9 \text{Tr}(\zeta \bar{\zeta}) \text{Tr}(\Delta \bar{\Delta}) + f_{10} \text{Tr}[\zeta (\Delta \bar{\Delta} - \Delta \bar{\Delta}) \zeta]$$,

(10)

where

$$\Phi^\dagger = (\phi^0, -\phi^+), \quad \bar{\chi}^\dagger = (\chi_2, -\chi_1), \quad \zeta = \begin{pmatrix} \zeta_2^+ & -\zeta_1^+ \\ -\zeta_2^- & \zeta_1^- \end{pmatrix}.$$ (11)

It is the same as that of Ref. [6] but with two fewer terms. The reason is that $S$ is conserved in our model, whereas $S$ breaks to $(-1)^S$ in Ref. [6].

The spontaneous symmetry breaking of $SU(2)_N$ is mainly through $\langle \chi_2 \rangle = u_2$, where

$$u_2^2 \approx -\frac{\mu_2^2}{\lambda_4}.$$ (12)

As previously mentioned, this breaks both $SU(2)_N$ and $S'$, but the combination $T_{3N} + S' = S$ remains exact. The further breaking of $SU(2)_N$ by $\langle \Delta_3 \rangle = u_3$ is assumed to be small for the implementation of the inverse seesaw mechanism, i.e.

$$u_3 \approx -\frac{\mu_2 u_2^2}{\mu_4^2 + (f_6 - f_7) u_2^2}.$$ (13)
This also does not break $S$, but it breaks $L$ to $(-1)^L$ because of Eq. (5). The spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$ is mainly through $\langle \phi^0 \rangle = v_1$, where

$$v_1^2 \simeq \frac{-\mu_\phi^2 - f_5 u_2^2}{\lambda_2}.$$  \hspace{1cm} (14)

The further breaking of $SU(2)_L \times U(1)_Y \times SU(2)_N$ through $\langle \zeta_2^0 \rangle = v_2$ is assumed small, i.e.

$$v_2 \simeq \frac{-\mu_1 v_1 u_2}{\mu_\zeta^2 + f_2 u_2^2}.$$  \hspace{1cm} (15)

This also does not break $S$.

The resulting physical scalar particles of this model have the following masses:

$$m^2(\sqrt{2} Re \chi_2) \simeq 2 \lambda_4 u_2^2, \quad m^2(\sqrt{2} Re \phi^0) \simeq 2 \lambda_2 v_1^2, \quad m^2(\zeta_2^0) \simeq \mu_\zeta^2 + f_2 u_2^2 + f_4 v_1^2,$$

$$m^2(\zeta_2^-) \simeq \mu_\zeta^2 + f_2 u_2^2 + f_3 v_1^2, \quad m^2(\zeta_2^0) \simeq \mu_\zeta^2 + f_1 u_2^2 + f_4 v_1^2, \quad m^2(\Delta_3) \simeq \mu_\Delta^2 + (f_6 - f_7) u_2^2 + f_8 v_1^2,$$

$$m^2(\Delta_2) \simeq \mu_\Delta^2 + f_6 u_2^2 + f_8 v_1^2, \quad m^2(\Delta_1) \simeq \mu_\Delta^2 + (f_6 + f_7) u_2^2 + f_8 v_1^2.$$  \hspace{1cm} (16)-(19)

Of all the particles having nonzero $S$, i.e. $\zeta_1^0, \zeta_1^-, \Delta_2, \Delta_1, n_1$ and $X$, we assume that $X$ is the lightest. Of all the new particles having zero $S$, i.e. $\zeta_2^0, \zeta_2^-, \Delta_3, \sqrt{2} Re \chi_2, n_2$ and $Z'$, we assume that $\zeta_2^0, \zeta_2^-$ are lighter than $X$ so that $X \bar{X} \to \zeta_2^0 \zeta_2^- + \zeta_2^- \zeta_2^+$ is kinematically allowed. Since $\Delta_1$ has $S = -2$, if $m(\Delta_3) < 2 m_X$, it is also stable and may become a significant second component $[\Pi]$ of dark matter. We will explore this very interesting possibility elsewhere.

The annihilation of $X \bar{X} \to \zeta_2 \bar{\zeta}_2^\dagger$ proceeds via the diagrams of Fig. 1. Assuming $X$ and $\bar{X}$ to be at rest, this amplitude is given by

$$A = \frac{g_N^2}{2} \left[ \bar{\epsilon}_1 \cdot \bar{\epsilon}_2 + \frac{4(\bar{\epsilon}_1 \cdot \vec{k})(\bar{\epsilon}_2 \cdot \vec{k})}{m_{\zeta_2^0}^2 + m_X^2 - m_{\zeta_2^0}^2} \right],$$  \hspace{1cm} (20)

where $\vec{k}$ is the three-momentum of $\zeta_2$, and $\epsilon_{1,2}$ are the polarizations of $X$ and $\bar{X}$. Summing over $\zeta_2^0$ and $\zeta_2^-$, and averaging over the spins of $X$ and $\bar{X}$, the corresponding cross section $\times
their relative velocity is given by
\[
\sigma \times v_{\text{rel}} = \frac{g_N^4}{576 \pi m_X^2} \sqrt{1 - \frac{m_{\zeta_2}^2}{m_X^2}} \left( 2 + \left[ 1 + \frac{4(m_X^2 - m_{\zeta_2}^2)}{m_{\zeta_1}^2 + m_X^2 - m_{\zeta_2}^2} \right]^2 \right). \tag{21}
\]

Since \( m_{\zeta_1} > m_X \) by assumption, the above expression is bounded from below by \( m_{\zeta_1} \to \infty \) and from above by \( m_{\zeta_1} = m_X \). We plot in Fig. 2 the allowed region for \( m_X/g_N^2 \) as a function of \( r = m_{\zeta_2}^2/m_X^2 \) for the optimum value \[13\] of \( \sigma \times v_{\text{rel}} = 2.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \) implied by the observed relic abundance of dark matter in the Universe.

To detect \( X \) in underground experiments using elastic scattering off nuclei, the only possible connection is through \( \phi^0 - \chi_2 \) mixing. The 125 GeV particle \( h \) discovered \[14, 15\] at the Large Hadron Collider (LHC) is a linear combination of \( \sqrt{2} Re\phi^0 \), \( \sqrt{2} Re\zeta_0 \), and \( \sqrt{2} Re\chi_2 \). The induced \( hX\bar{X} \) interaction is approximately given by \( (g_N^2 v_1/\sqrt{2})(f_5/\lambda_4) \). Since \( h \) interacts with quarks through \( \sqrt{2} Re\phi^0 \) according to \( (m_q/\sqrt{2}v_1)\bar{q}q \), there is a small cross section for \( X \) to be detected in such experiments. Following Ref. \[16\], we obtain
\[
\frac{f_p}{m_p} = -0.075 \left[ \frac{g_N^2(f_5/\lambda_4)}{4m_\phi^2} \right] - 0.925(3.51) \left[ \frac{g_N^2(f_5/\lambda_4)}{54m_\phi^2} \right], \tag{22}
\]
\[
\frac{f_n}{m_n} = -0.078 \left[ \frac{g_N^2(f_5/\lambda_4)}{4m_\phi^2} \right] - 0.922(3.51) \left[ \frac{g_N^2(f_5/\lambda_4)}{54m_\phi^2} \right], \tag{23}
\]
with the spin-independent elastic cross section for \( X \) scattering off a nucleus of \( Z \) protons.
Figure 2: Allowed values of $m_X/g_N^2$ plotted against $r = m_{\zeta_1}^2/m_X^2$ from relic abundance.

and $A-Z$ neutrons normalized to one nucleon given by

$$\sigma_0 = \frac{1}{\pi} \left( \frac{m_N}{m_X + Am_N} \right)^2 \left| Z f_p + (A-Z)f_n \right|^2. \quad (24)$$

Using the recent LUX data [17], we plot in Fig. 3 the maximum allowed value of $g_N^2(f_5/\lambda_4)$ as a function of $m_X$ using $m_\phi = 125$ GeV. We also plot the allowed regions of $g_N^2$ versus $m_X$ for $r = 0.2$ and $r = 0.8$ from Fig. 2. We see that for moderate values of $m_X$, future improvement in direct detection will probe the allowed region from relic abundance if $f_5/\lambda_4$ is not too small.

In conclusion, we have discussed in this paper a simple and realistic model of vector dark matter based on $SU(2)_N$. It has a leptonic connection which is a natural framework for the
inverse seesaw mechanism to generate a small Majorana neutrino mass. It also accommodates a standard-model Higgs boson which mixes only slightly with its $SU(2)_N$ counterpart, which does not couple directly to any standard-model particle. Thus the observed 125 GeV particle behaves as the one Higgs boson of the standard model for all practical purposes. The scalar particle content of this model may also allow a stable second component of dark matter, because of kinematics, and not because the dark symmetry is extended. This is the first explicit example of how such a general situation may occur, a topic we will explore in another paper. The $SU(2)_N$ gauge symmetry is also suitable to be embedded in an $SU(7)$ model unifying matter and dark matter, as outlined in Ref. [18].
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