A path to build basic Quantum Mechanics ideas in the context of light polarization and learning outcomes of secondary students

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Abstract. There is consensus on the goal to introduce Quantum Mechanics in secondary school curricula for its paradigmatic role in the modern physics. In literature a wide spectra of proposals were developed and tested. With the goal to approach the basic ideas of quantum theory by means of active learning, we design a research based educational path in the context of light polarization, offering an open software environment for ideal experiments. Several Intervention Modules with secondary school students were performed, monitoring the learning paths by means of tutorials and pre-post-test for total about 900 students. Here a homogenous sample of 126 students is considered. Qualitative/quantitative data analysis shows coherent line of reasoning developed by students along the quantum concepts, coherent in some case with a hidden variables approach.

1. Introduction

Quantum mechanics (QM) is the theoretical paradigm of the modern physics, having relevant role in many scientific and technical fields. There is a wide consensus to include it in the high school curricula [1-2]. Different educational approaches were proposed in literature [2-5]. These are focused on specific aspects and on how the proposals produce some peculiar ideas relevant in quantum mechanics theory, as dualism, uncertainty principle, probabilistic nature of quantum prediction [6-7]. There is the need to implement coherent paths and to individuate the students’ reasoning profile, the conceptual trajectories and the relative coherence in student learning for the whole specific proposed path. The main conceptual based approaches are the two state approaches, where the formal description of the quantum entities are possible in a two dimensional vector space, as for spin [8-9], polarization [7, 10-11] and other approaches [3-4, 12-14]. In the perspective of that conceptual approach to the quantum theory on the personal building of ideas, we designed and implemented an active learning educational paths for secondary school students in the context of light polarization [15-17]. The strategic choices were to point towards an introduction of theoretical way of thinking through the treatment of crucial aspects, cardinal concepts and elements peculiar to quantum mechanics in the context of photon polarization. The main characteristics of the proposal are the Inquiry Based Learning Approach by means of simple hands-on experiments, ideal experiments in an open computer environment to build an educational reconstruction of the fundamental ideas and the relative formalism in a coherent path by means of tutorials [18] to support reasoning and peer discussion in learning progress, as detailed described elsewhere [19-21]. Experimentation with students were performed with 15-20 students per group for about 900 Italian students 17-19 aged in different geographical sites, school and type of level. Here, the rationale of the path will be presented, documenting the evolution of the ideas of a sample of 126 students from a classical to a quantum framework, monitoring the process by means of tutorials and pre/post-test [18-21].
2. The educational path on the pivotal concepts of quantum mechanics

On the disciplinary level, we chose to begin with the concept of state and the superposition principle. On the didactic level, we chose linear polarization as a quantum dynamical property of photons and we suggest to explore it in lab by means of simple hands-on experiments and of quantitative measurement with on-line light intensity sensors. Students develop, test and compare interpretative ideas on quantum concepts, performing ideal experiments with photons in an open simulated lab [22-23]. We suggest the analysis of simple situations, that students are able to analyze thanks to the concepts gained in the previous steps of the educational path. Quantum concepts are constructed and discussed in qualitative ways before the introduction of the formalism and its conceptual role, that acts as a synthesis and conceptual organizer at the end of each conceptual step in an integrated form. We profit of the light linear polarization context, that requires only the use of two dimensional vector space.

Table 1 shows the rationale of the educational path. Polarization is explored as a property of light produced by and explored with Polaroids. Malus’s law studied in lab suggests how to prepare polarized light and to detect it (fig.1). The polarization as property of the single photon is suggested by the validity of Malus’s law at very low light intensity and admits to interpret it as probability for the photon transmission. The interactions of polarized photons with Polaroids, having different permitted directions, individuate mutual exclusive and incompatible properties.

![Exploring light intensity](image)

**Figure 1.** Exploring light intensity by means of two or more Polaroids and a simple light source it emerges that: (a) rotating the analyzer polaroid, the light intensity decreases from the maximum (α=0° - Polaroids with parallel permitted direction), to zero (α=90° - Polaroids with crossed permitted direction). Using the simple setting shown in (b) where a light intensity sensor measures transmitted light in relationship with the angle between the permitted direction of two Polaroids, the obtained relationship (c) is identify by Malus’s law (d).

The uncertainty principle derives from the identification of incompatible properties. That principle, the quantum indeterminism, the identity of quantum systems are basic concepts discussed, considering
photon beams and single photons in a virtual experimental environment (fig.2). An iconographic representation (fig. 3) helps students to distinguish property (eigenvalue) and state (eigenvector) bridging them toward the vector representation of the quantum state and the use of linear operators. The analysis of the interaction of photons with birefringent crystals allows to conclude the impossibility of attributing a trajectory to a single photon (to a quantum system). The problems of the quantum measure, of describing macro-objects and non-locality are treated in the same context, with examples in the phenomenology of particle diffraction and analogies in the macroscopic world.

**Table 1.** The rationale of the educational path.

- Phenomenology of light (linear) polarization with Polaroid filters to recognize polarization as property of light produced by a polarizer. Active role of polarizer.
- Measurement of light polarization and Malus’s law. Real Polaroid vs ideal Polaroid results.
- Malus’s law for the single photon: polarization as property of each photon (not collective property); probabilistic interpretation of Malus’s law.
- Photons prepared by a Polaroid are always transmitted by another Polaroid with parallel permitted direction, therefore photons in a well define state, with a well define property (represented with an icon. i.e. $\Delta$, $\ast$, $\hat{0}$ for vertical, horizontal, 45° polarization).
- Mutual exclusive properties: properties producing a certain result (probability = 0 or 1) interacting with Polaroid with permitted direction parallel or orthogonal
- Superposition state vs statistical mixture and incompatible properties.
- Uncertainty, non-epistemic indeterminism, identity of quantum particle prepared in the same state
- Not classical behavior of quantum systems assuming co-existent properties (hidden variable)
- Vector of state
- Rules to evaluate the probability of transition
- Impossibility to attribute a trajectory to a quantum system analyzing state and measured properties for photons interacting with one and with two birefringent crystals
- Linear operator and quantum observables – eigenvectors and eigenvalue
- The problems of macro-object and measurement - non locality, entanglement
- Interference, diffraction

3. **The steps of the path**

Polarization is operatively introduced (fig.1a), exploring with an analyzer Polaroid the prepared light by a first Polaroid. Rotating the analyzer and measuring light intensity, polarization is recognized as a property of light that can be detected by light intensity but it is a different property living in the plane perpendicular to the propagation direction, produced by a polarizer according with its permitted direction of polarization. Students explore how different common processes produce polarized light, i.e. reflection, refraction, diffusion…. The Malus’s law is obtained in lab using light sensors (fig.1b-d). Its meaning is explored by means of JQM system (Fig.2) in ideal experiments even in the case of a single photon and written as follow

$$N = N_o \cos^2 \theta$$  \hspace{1cm} (1)

where $N_o$ is the number of photons prepared by a first polarizer and incident on the analyzer, $N$ is the number of photons transmitted by the analyzer, $\theta$ is the angle between the permitted direction of the two Polaroids. The polarization is recognized as a property of the single photon and the result of a measurement process is identified to have a genuine stochastic nature (fig.2).
Figure 2. Examples of ideal experiments realized in the virtual environment JQM [22-23]: a) single photons and b) a beam of ten photons prepared in the state D (corresponding to 45° polarization), interacting with a Polaroid with vertical permitted direction; c) a beam of ten 45° polarized photons interacting with two reverse birefringent crystals are transmitted by the analyzer and d) detected by the detector.

Students are free to explore different cases in JQM simulation, but three cases are suggested: 1) Analyzer with permitted direction parallel and 2) orthogonal to those of the polarizer preparing the photons; 3) Analyzer with permitted direction at 45° with those of the polarizer preparing the photons. Representing the light polarization properties by means of icons ∆, *, ♦ respectively for vertical, horizontal, 45° polarization. It emerges that the cases of 1 and 2 represent situations with certainty results, corresponding to mutual exclusive properties as result of prepared photons in the bases (h, v) of the polarization states analyzed. It emerges as well that the polarization properties in the case 3 are incompatible properties and the result is uncertain and previsions can be done only in probabilistic terms (fig.3).

At the same time a new question emerges: can we consider a photon beam with the property ♦ equivalent to a statistical mixture obtained mixing half photons with property ∆ and half photons with property *? A test with a 45° permitted direction of the Polaroid gives a negative answer to this question: all photons with property ♦ will pass a second Polaroid with a 45° permitted direction; only half of the photons of the mixture will pass it. The two ensembles are statistically different. A photon with property ♦ cannot possess also the property ∆ or *, but the probability of transition from 45° state to the state V or H is not 0. Therefore the property ∆ and ♦, or * and ♦ are called incompatible. When the polarization along a direction is well defined, the polarization along another direction is not defined. The existence of incompatible properties is the core of the uncertainty principle. Moreover, the process of measurement is genuinely stochastic and it is impossible (also as ideal hypothesis) to predict which photons will be transmitted in a test of polarization: all the photons prepared with the property ♦ are identical and indistinguishable (it is a sort of democracy of photons) and behave in a stochastic unpredictable way when subjected a measurement process.
Figure 3. (left) All the photons filtered by a Polaroid will be transmitted by a second Polaroid with permitted direction parallel to the first one. The result is certain for each single photon ($P=1$). Each photon is in a well-defined state and possess a well-defined property. This property is the same for all the prepared photons and is related to the direction of the last Polaroid orientation. It can be represented by an icon: $\Delta$ for vertical polarization; * for horizontal polarization; $\diamond$ for 45° polarization. (right) All photons prepared with property $\Delta$ (or *) will be absorbed by a horizontal (vertical) Polaroid. For that, the properties $\Delta$ and * are called mutually exclusive.

Another consequence of the incompatibility is the impossibility to attribute a trajectory to a quantum system. This point can be addressed considering a beam of photon with property $\diamond$, incident on two reverse aligned birefringent crystals, where the ordinary beam is associated to the vertical polarization and the extraordinary beam the horizontal polarization (or vice versa). All the photons transmitted beyond the two crystals will pass a 45° polarization test. It is possible to establish the path followed by each photon? To know the path of the photons, a measurement must be done after the first crystal, producing the precipitation of the state in a specific polarization state, that is each photon follows one of the possible paths, therefore the number of transmitted photons by the second crystal will be reduced to $\frac{1}{4}$ of the incident photons, being $\frac{1}{2}$ in each of the two paths after the first crystal.

Figure 4. All photons prepared in a pure state (pol. 45°) with a well define property ($\diamond$), traveling along two reverse aligned birefringent crystals will be transmitted by a Polaroid parallel to the first (45°). Photons do not travel along the ordinary or the extraordinary path, they are not divided, and they do not travel along other path: it is impossible to attribute a trajectory to photons (to a quantum system). Photons in a mixture of state travel along well defined (knowable) paths, but only half pass the polarization test.
This means that the photons do not follow the path of the ordinary beam, do not follow the path of the extraordinary path, do not follow any other path. Due to the unitary nature of the photon. The conclusion is the impossibility to attribute a trajectory to a photon (to a quantum system). Concepts offer the opportunity to discuss the two slit interference experiment as well as the problematic description of macro-systems and the measurement problem. Interesting is in this phase the discussion of other interpretative frame, well known in literature, as for example that the photons possess simultaneously two properties: the property attributed by the Polaroid; the property (or the properties) account for the experimental results (contextually depending). This line of thinking, that reflect a local hidden variable approach, can be discussed with experiments that are similar to those already discussed, showing that the photons behave in a not classical way also in a not orthodox frame.

The approach to formalism is done considering the transition probability of the state of photons passing two Polaroids. Describing by vectors the polarization states of each photon in the framework of Malus’s law, the transmission probability of a linear polarized photon is than \( \cos^2 \theta \). If \( \mathbf{u} \) is the vector state of the prepared photon and \( \mathbf{w} \) those after the transmission, the probability of transition is given by the square of the scalar product of the two vectors: \( P_{\mathbf{u} \rightarrow \mathbf{w}} = (\mathbf{u} \cdot \mathbf{w})^2 = \cos^2 \theta \). The crucial point is that changing the orientation of the analyser Polaroid, for the same preparation state \( \mathbf{u} \), the probability is always the square of the scalar product between the states (initial and resulting) of the quantum object (photon). In particular, a photon prepared in the \( \nu \) state with the property \( \Delta \) (or in the state \( \mathbf{h} \) and property \( \ast \)) will be always absorbed when a measurement is made with a horizontal (vertical) permitted direction Polaroid. Mutually exclusive properties are associated to mutually orthogonal states.

Assumed \( \nu \) and \( \mathbf{h} \) the base state vectors, each state vector \( \mathbf{u} \) can be expressed as a linear combination of \( \nu \) and \( \mathbf{h} \) (as we usually do with vectors even in classical physics):

\[
\mathbf{u} = \psi_1 \nu + \psi_2 \mathbf{h}
\]  
(2)

where \( \psi_1 \) and \( \psi_2 \) (that we can limit ourselves to consider real numbers) called probability amplitudes, because \( \psi_1^2 \) and \( \psi_2^2 \) give the probability that a photon prepared in the state \( \mathbf{u} \) makes a transition into the state \( \nu \) or \( \mathbf{h} \), respectively, when is measured with a Polaroid with permitted direction V or H. The simple expression (2) is the fundamental quantum linear superposition principle, expressed in the case of polarization states.

Figure 5. From Malus’s law to the vector representation of photon state.

As discussed before, the property attributed to the system in the state \( \mathbf{u} \) is incompatible to the properties attributed to the system in the component states. In other words, the incompatibility is automatically framed in the linear structure of the superposition principle.

The last step of formalization regards the representation of physical observables with linear operators. The scalar product giving the probability of transition can be rearranged as a projector operating on the state vector of preparation. Considering the expectation value of an observable measured, the more
general form of an operator representing a physical observable is introduced, showing the roles of eigenvalues (the mutually exclusive possible values assumed by the observable) and eigenvectors (the possible state detected after the measurement).

4. Research context, research questions and method
The outlined educational path was tested and modified in many different research experimentations. Here we consider the results of research experimentations performed in 5 Italian schools involving 126 students aged 17-19. The majority of students were of middle-high level, according to the evaluation of their own school teachers. They faced the quantum knots of the educational path in eight hours of group activity, using the tutorial developed as supporting material for the educational proposal [18]. Data were collected by means of the tutorials and the pre/post test, combining multiple choices questions and open motivations. Questions posed focus on the main conceptual goals of the educational path and on how students pass from a classical vision of phenomena to the quantum way of thinking [19-21]. The main research questions considered here are:

RQ1. How sort of reasoning characterizes students’ learning trajectories to the quantum ideas?
RQ2. Which sort of models do they develop?
RQ3. Which are the more problematic concepts?

Here we answer these questions, considering qualitative analysis of tutorial and test. Categories of students’ answers were operatively defined, selecting examples of students’ sentences. In particular, according to a phenomenographic methodology [20-21], we individuate profiles distinguishing between the classical point of view, the quantum way of thinking and also individuating when students developed ides coherent with a hidden variable approach to the microscopic phenomena. These perspectives can be defined a priori as in the following:

Clas – Classic profile. Microscopic systems and classical macroscopic systems have an analogous nature. All their observables are always well defined and assume well defined values, independently of a previous measurement. In order to describe their evolution, the concept of trajectory can be used, even if it is necessary to use a statistical approach for lack of information about the initial state or on the dynamic of the system under observation.

Hid – Hidden variables profile (local). Microscopic systems preserve some properties of the classic macroscopic systems. It is possible to attribute to it a trajectory, even if it is not knowable/detectable. Their not-classical behavior is due to uncontrollable disturbances, characterizing their behavior both in the normal evolution and in the measurement processes.

Quant – Quantum profile. Classic and quantum systems have different nature. It is possible to associate dynamic properties to quantum systems only by means of measurements. These properties are in general incompatible with those that characterize the state before the measurement itself. The description through states substitutes the description through position in time (trajectory) of systems evolution. The process of measure is genuinely stochastic and can be described only as a transition between an initial and a final state.

To assign each student answer to one of the profiles, an analysis was carried out by keywords and by the meaning attributed to them in the context of each sentence, carrying out a cross-check between at least two different researchers. To give an immediate view on the students’ conceptions changes, from classical to quantum profiles, a CQ index was constructed, modifying the index introduced by Müller and Wiesner [7], as follow: each answer to the eight items of the pre/post-test (Q1-Q8) are ranked -2 in the case of Clas profile, -1 in the case of Hid profile, +2 in the case of Quant profile. Correlated answers to the pair items Q2-Q4, Q6-Q8 and Q3-Q5 was ranked -1 both Clas, 0 both Hid, +1 both Quant. Another point was attributed when all the three pair ranks are equal. The CQ index is the sum of all the points attributed to each contribution, obtaining: -20 ≤ CQ ≤ +20.
To characterize better the three perspectives of students reasoning a three-dimension index CHQ was introduced, as following: each answer to the items Q1-Q8 was classified in the classical, hidden or quantum profile. The CHQ index is obtained as:

$$CHQ = (N_{cl} \times w_{cl}; N_{hid} \times w_{hid}; N_{quant} \times w_{quant})$$

where $N_{cl}$, $N_{hid}$, $N_{quant}$ are respectively the numbers of answers assigned to the classical, hidden, quantum profiles ($0 \leq N \leq 8$), and $w_{cl}$, $w_{hid}$, $w_{quant}$ ($1 \leq w \leq 2$) are the weight obtained according the correlations of answers. Change in distribution from pre to post test was also analyzed.

5. Results

Fig. 6 represents the CQ index for each student of a sub-sample ($N' = 60$), where the shifts from a classical point of view to a quantum one are quite evident and frequent. At the same time, some cases (as for instance students numbered 17, 25, 54) are also evident some CQ values remaining around 0. The students with these scores usually are classified as confused. But as we discussed presenting the path, different logical lines can be coherently developed. In particular, the difficulty to abandon the determinism and the concept of trajectory, as we seen also historically, pushes to adopt a hidden variables perspective. To evaluate the possibility that students develop ideas coherent with a hidden variable perspective the three profiles presented in previous paragraph are formulated.

![Figure 6. CQ index in the pre/post-test (black/grey): on the top - for each students of a subsample ($N' = 60$); on the bottom - distribution of the CQ values for all the sample ($N = 126$) ($P < .001$).](image)

The students’ answers classified according to these profiles show that some students effectively developed these ideas, as presented in fig. 7, resuming the phenomenographic analysis performed. It is possible to follow the evolution of students profiles showing that hidden variables profiles constituted both a bridge toward orthodox quantum concepts as well an isle where some students reinforce their
perspective. To describe the process of evolution of students’ conception about quantum concepts, the three dimensional index CHQ is introduced as discussed in the method section.

**Figure 7.** Students’ profiles and evolution obtained from their answers to the pre-post test.

Fig 8. shows the three dimensional space of the CHQ index, evidencing the evolution of all students and in particular the centroid of the distributions, the coordinates of which are the mean values of the coordinates each students. The paths of the students are also quite simple to follow from the classical perspective to the two alternative dimension of the quantum and hidden variables concepts: for the majority of students the CHQ index have a 0 value for the classical coordinate; the CHQ of students with conflictual conceptions remains in the plane Hid-Clas, with low values of Quant coordinate. Some examples concerning trajectory of a quantum system can clarify the distinction between Quant and Hid profiles, referring to the bibliography for further details [20-21]. Quant sentences are the following “if I hypothesize a trajectory I do not obtain the experimental results”, “we do not have any possibility to detect the way in which a particle is moving”. In Hid sentences the trajectory exists but is not detectable: “We cannot attribute for certain to each photon the followed trajectory”; “We cannot establish a priori the trajectory followed, because the quantum uncertainty”.

**Figure 8.** Three dimensional representation of the CHQ index related to a subsample of 31 students; • Centroid of the distrubutions (the coordinates are the mean values of the coordiantes of each students of the sample): in the pre-test (3,3; 2,8; 2,0); in the post test (1,3; 2,3; 5,9).

Finally, we can consider how students attributed meaning to the formalism. In tab 2 are reported the students sentences on the superposition principle, answering the request to summarize the physics meaning of the superposition principle and its formal expression. The two main ways to recall the principle are that of reporting its mathematical formulation (cat A) or describing in words the meaning (cat. C). Students of cat C acquired the meaning of the principle, but do not master its formal representation.
Table 2. The superposition principle according to the students

| Cat | Description | % |
|-----|-------------|---|
| A   | $U = \psi_1 + \psi_2$ | 40 |
| B   | Other (incomplete/incorrect) formula | 13 |
| C   | «The vector $U_{45}$ must be considered as a state of superposition of states represented by mutually exclusive vectors ($u$ and $v$)» | 35 |
| D   | NA | 12 |

6. Conclusion

Research implementations of the path developed with 126 students and documented in previous papers [19-21] evidenced that the majority students profit of the iconographic representation to discuss in a proper way on mutual exclusive (80% of students) and incompatible properties (55%) (RQ3). The employ of the iconographic representation (85%) and formalism as conceptual synthesizer (60%) facilitates rigorous reasoning in a coherent QM frame (80%), helping students in formulating testable predictions on new contexts (50%) (RQ2-3), even if in different conceptual perspectives. In fact, as here presented, the students learning paths show evolution toward quantum concepts, where, often, some typical hidden variables assumptions bridge them from classical to quantum way of thinking [20-21] (RQ1-2). Conceptual nuclei about a not-orthodox interpretation of QM mixed with classical ones, constitute the initial ideas with which the majority of students starts facing basic QM concepts. In several cases these nuclei can become stronger and lead logically to develop conceptions coherent with hidden variables framework of QM as those of hidden variables or to play the role as a bridge toward a quantum orthodox vision. Tendency in the evolution of students’ ideas, from a completely classic conception of phenomena, to hidden variable ideas, to a quantum picture. This process is not linear and regards part of the students of the current study. Incompatibility and the impossibility to attribute a trajectory to a quantum systems are the more difficult learning problems for the large majority of students. Finally, 75% of students acquired the concept of superposition, but only about half of these (40%) master also its formal representation (RQ3).

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