On the residual finiteness of outer automorphisms of relatively hyperbolic groups

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Abstract

We show that every virtually torsion-free subgroup of the outer automorphism group of a conjugacy separable hyperbolic group is residually finite. As a result, we are able to prove that the group of outer automorphisms of every finitely generated Fuchsian group and of every free-by-finite group is residually finite. In an addendum, we also generalise the main result for relatively hyperbolic groups.

1 Introduction

The mapping class group of a compact connected, possibly with boundary, surface $S$, $\text{Mod}_S$, is defined as the group of isotopy classes of homeomorphisms $S \to S$. The algebraic investigation of that group goes as far back as Dehn and Nielsen, who first showed that the mapping class group of an orientable surface is embedded into the group of outer automorphisms of the fundamental group of $S$, $\text{Out}(\pi(S))$. This embedding was later extended to a wider class of 2-orbifolds. Macalchan and Harvey [16] proved that $\text{Mod}_S \to \text{Out}(\pi(S))$ for a surface $S$ obtained from a compact surface of genus $k$ by deleting $s$ points, $t$ discs and $r$ marked points. Recently, Fujikawa [10] extended the above embedding to the case of hyperbolic 2-orbifolds (orientable or non-orientable) with finite volume.

Grossman [12] was the first to show that if $S_k$ is a compact orientable surface of genus $k$, then $\text{Mod}_S$ is residually finite. Recently, Allenby, Kim and Tang [11] extended the result of Grossman for non-orientable closed surfaces. Ivanov [14] gave a geometric proof of the result of Grossman which seems to easily be extended to the non-orientable case. The proofs...
of both results in [12] and [11] are using combinatorial group theory arguments. One of the key ingredients of both results is the fact that the group of conjugating automorphisms of the fundamental group of the surface coincides with the group of its inner automorphisms. If $G$ is a group, then an automorphism $f$ of $G$ is called a conjugating automorphism if $f(g)$ is a conjugate of $g$ for every $g \in G$. The conjugating automorphisms of a group $G$ from a subgroup of the group of automorphisms of $G$, $\text{Aut}(G)$, which we denote by $\text{Conj}(G)$. Moreover, $\text{Conj}(G)$ is normal in $\text{Aut}(G)$ and contains the subgroup of inner automorphisms of $G$, $\text{Inn}(G)$.

In the present note we investigate the residual finiteness of $\text{Out}(G)$ for hyperbolic groups $G$. Our study is, as well, based on the investigation of the group of conjugating automorphisms of $G$. In fact, using the powerful geometric ideas developed by Paulin [20] and Bestvina [3] in the context of ultralimits of metric spaces, we show that in a hyperbolic group $G$, the quotient group $\text{Conj}(G)/\text{Inn}(G)$ is always finite. The main theorem shows that every virtually torsion-free subgroup of the outer automorphism group of a conjugacy separable, hyperbolic group is residually finite.

As a result, we obtain the residual finiteness of the outer automorphism group of finitely generated Fuchsian groups and of free-by-finite groups. Consequently the mapping class groups of hyperbolic 2-orbifolds with finite volume are residually finite. Hence, we retrieve the results of Rossmann and of Allenby, Kim and Tang as special cases of our corollaries.

In an addendum at the end of the paper we show how to generalize the main results for relatively hyperbolic groups.

2 Main Results

The next lemma is what really proved in [12] Theorem 1]. We reproduce here the proof for the reader’s convenience.

Lemma 2.1. Let $G$ be a finitely generated, conjugacy separable group. Then the quotient group $\text{Aut}(G)/\text{Conj}(G)$ is residually finite.

Proof. Since $G$ is finitely generated, we can find a family $(K_i)_{i \in I}$ of characteristic subgroups of finite index in $G$ such that for each normal subgroup $N$ of finite index in $G$ there is a subgroup $K_i$ in the above family, contained in $N$. Every automorphism $f$ of $G$ induces an automorphism $f_i$ of $G=K_i$ which acts as a permutation on the conjugacy classes of $G=K_i$. Therefore, for each $i \in I$ we have a homomorphism $\varphi_i : \text{Aut}(G)/S_i$, where $S_i$ denotes the permutation group on the set of conjugacy classes of the finite group $K_i$. The homomorphism $\varphi_i$ is onto since...
\(G = K_1\). Since each conjugating automorphism preserves conjugacy classes, the group \(\text{Conj}(G)\) is contained in the intersection \(\bigcap_i \text{Ker}(\iota_i)\) of the kernels of the \(\iota_i\)'s. Now let \(f \in \bigcap_i \text{Ker}(\iota_i)\). Then for every \(g \in G\), the elements \(f(g)\) and \(g\) are conjugate in each quotient \(G = K_i\). In particular, the elements \(f(g)\) and \(g\) are conjugate in each \(i\) and thus they are conjugate in \(G\), by the conjugacy separability of \(G\). This means that \(f\) is a conjugating automorphism of \(G\) and hence \(\text{Conj}(G) = \bigcap_i \text{Ker}(\iota_i)\). \(\square\)

The following material on ultralimits of metric spaces can be found in detail in the paper of Kapovich and Leeb [15].

A iter or set of natural numbers \(N\) is a non-empty set of subsets of \(N\) with the following properties:

1. The empty set is not contained in \(!\).
2. \(!\) is closed under finite intersections.
3. If \(A \subset \subset B\), then \(B \subset \subset \).

A iter \(!\) is called ultra iter if it is maximal. An ultra iter is called non-principal if it contains the complements of the finite subsets of \(N\). Let \(!\) be an ultra iter on \(N\) and \(a : N \to (0, 1)\) a sequence of non-negative real numbers. Then there is a unique point \(l\), which we denote by \(\lim \ a\), in the one-point compactification \((0, 1]\) of \([0, 1]\) such that for each neighborhood \(U\) of \(l\), the inverse image \(\{a \in U \mid a \text{ is contained in } !\}\).

Let \((X; d_i; x_i^{0})_{i \in N}\) be a sequence of based metric spaces and let \(!\) be an ultra iter on \(N\). On the subspace \(Y\) of \((\bigcap_i X_{i})_{i \in N}\) consisting of all sequences \((x_i)\) for which \(\lim \ d_i(x_i; x_i^{0}) < 1\), we define a pseudo-metric \(d_i\) by \(d_i(x; y) = \lim \ d_i(x; y_i)\). The based ultralimit \((X; d; x)\), where \(x = (x_i^{0})_{i \in N}\), is the associated metric space.

Let \(X\) be a geodesic metric space and \(a\) a non-negative real number. The space \(X\) is called \(\alpha\)-hyperbolic if for every triangle \(X\) with geodesic sides, each side is contained in the \(\alpha\)-neighbourhood of the union of the two other sides.

Let \(G\) be a finitely generated group and let \(X = X(G; S)\) be the Cayley graph of \(G\) with respect to a finite generating set \(S\) for \(G\), closed under inverses. The group \(G\) is called \(\alpha\)-hyperbolic if \(X\) is a \(\alpha\)-hyperbolic space with respect to the word metric.

In the next lemma we use various results concerning actions of groups on \(\mathbb{R}\)-trees. For details, we refer the reader to the paper of M organ and Shalen [18].
The key point of Lemma 2.2 is the following statement. Assume that an action of a hyperbolic group \( G \) on a real tree \( Y \) is obtained as a limit of a sequence of actions of \( G \) on its Cayley graph \( X \), where each action in the sequence is the natural action of \( G \) on \( X \) twisted by an automorphism of \( G \). If the conjugacy class of an element \( g \) in \( G \) is periodic under these automorphisms, then \( g \) is elliptic when acting on the limit tree \( Y \).

As it was pointed out by the referee, the above statement seems to be well known to the experts (see [4]). Nonetheless, the authors failed to track down a reference for its proof. So, a proof of it, is included in the lemma for the reader's convenience and completeness.

Lemma 2.2. Let \( G \) be a hyperbolic group. Then the group \( \text{Inn}(G) \) of inner automorphisms of \( G \) is of finite index in \( \text{Conj}(G) \).

Proof. Suppose to the contrary that \( \text{Inn}(G) \) is of infinite index in \( \text{Conj}(G) \). Fix an infinite sequence \( f_1; f_2; \ldots; f_n; \ldots \) of conjugating automorphisms of \( G \) representing pairwise distinct cosets of \( \text{Inn}(G) \) in \( \text{Conj}(G) \). We apply the method of Bestvina and Paulin, using ultra limits, to construct an \( R \)-tree on which \( G \) acts by isometries.

Let \( X = X(G; S) \) be the Cayley graph of \( G \) with respect to a finite generating set \( S \) closed under inverses. Then \( X \) with the associated word metric \( d \) is a \( \delta \)-hyperbolic metric space for some \( \delta > 0 \). Each \( f_i \) gives an action \( \cdot f_i \) by isometries of \( G \) on \( X \) by \( f_i(g) \cdot x = f_i(g)x \). The outer automorphism group of a virtually cyclic group is finite. Thus, we may assume that \( G \) is non-elementary. In that case the action of \( G \) on the boundary \( \partial X \) of \( X \) is non-trivial and Lemma 2.1 in [20] applies to any action \( \cdot f_i \), yielding a sequence \( x^0_i \) of elements of \( X \) such that

\[
\max_{g \in S} d(x^0_i; f_i(g)x^0_i) = \max_{g \in S} d(x; f_i(g)x)
\]

for all \( x \in X \). Let \( i = \max_{g \in S} d(x^0_i; f_i(g)x^0_i) \). Then

\[
d(x^0_i; f_i(g)x^0_i) \leq kgk
\]

for all \( g \in G \), where \( kgk \) denotes the word-length of \( g \) with respect to \( S \). If the sequence (1) contains a bounded subsequence, then the argument in [20, Case 1, p. 338] shows that there are indices \( i \) and \( j \) with \( i \neq j \) such that the automorphisms \( f_i \) and \( f_j \) differ by an inner automorphism of \( G \) and consequently they give rise to the same coset of \( \text{Inn}(G) \), contradicting the choice of the \( f_i \). It follows that \( \lim_{i \to 1} i = 1 \). We consider the sequence...
(X; d_i; x_i^0) of based metric spaces, where X_i = X and d_i = \frac{d}{i}. Note that the space (X; d_i; x_i^0) is \(-\) hyperbolic for all i. For any non-principal ultra limit \(X_i\), let \((X_i; d_i; x_i)\) be the corresponding based ultra limit. The fact that the distance \(d_i(x; y)\) of two points \(x = (x_i)\) and \(y = (y_i)\) of \(X_i\) is approximated by the distances \(d_i(x_i; y_i)\) for infinitely many \(i\), implies that \(X_i\) is a 0-hyperbolic space (i.e., an R-tree), since \(\lim_{i \to \infty} d_i = 0\). The action of \(G\) on \(X_1\) is given by \(g \cdot (x_i) = f_i(g)x_i\). Inequality (2) ensures that the action is well-defined. We will show that the action of \(G\) on \(X_1\) is trivial (i.e., there is a global fixed point).

Let \(g\) be an element of \(G\) which acts as a hyperbolic isometry on \(X_1\), and let \(\text{lip}(g)\) denote its translation length. Fix \(x = (x_i)\) in \(X_1\) such that \(x_i\) lies on the axis of \(g\). Then

\[
\text{lip}(g) = d_i(xgx; x) = \lim_{i \to \infty} d_i(f_i(g)x_i; x_i) = \frac{d_i(g^nx; x)}{n}
\]

for all positive integers \(n\). In particular, \(d_i(xgx; x) = \frac{d_i(g^nx; x)}{2}\) and thus

\[
\lim_{i \to \infty} 2d_i f_i(g)x_i; x_i \quad d_i f_i(g)x_i; x_i = 0; \quad (3)
\]

For each \(i\), we \(x\) an element \(y_i\) of \(X\) on which the displacement function of \(f_i(g)\) attains its minimum \((\text{lip}(g))\), i.e., \(\text{lip}(g) = d\ f_i(g)y_i; y_i = \inf d\ f_i(g)y; y \geq 2\ X\ g^1\). Since \(X\) is a \(-\) hyperbolic space, there is a non-negative constant \(K(\ )\) depending only on \(\ )\) such that

\[
d f_i(g)x_i; x_i \quad 2d(x_i; y_i) + (f_i(g)) \ K(\ ); \quad (4)
\]

Then,

\[
A = 2d f_i(g)x_i; x_i \quad d f_i(g)x_i; x_i \quad 4d(x_i; y_i) + 2 (f_i(g)) \ 2K(\ ) \ d f_i(g)x_i; x_i
\]

\[
4d(x_i; y_i) + 2 (f_i(g)) \ 2K(\ ) \ 2d(x_i; y_i) \ 2 (f_i(g))
\]

\[
= 2d(x_i; y_i) \ 2K(\ )
\]

where the second inequality follows from the triangle one. Working in a similar way, we see that

\[
2d(x_i; y_i) \ K(\ ) \ d f_i(g)x_i; x_i \quad (f_i(g)) \ 2d(x_i; y_i)
\]

\[1\]The reader should not confuse this with the algebraic translation length of the elements of a group \(G\).
and hence we can say that
\[ d(f_i(g)x_i; y_i) + K(\cdot) = 0. \] (7)

Consequently,
\[ d(f_i(g)x_i) + 2d(x_i; y_i) + K(\cdot) = 0, \]
\[ d(f_i(g)x_i) + 2d(x_i; y_i) + K(\cdot) = 0, \]
\[ d(f_i(g)x_i) + 2d(x_i; y_i) + K(\cdot) = 0, \]
where the second inequality follows from (7) and the third one from (5).

The \( \lim \) of each term in the right-hand side of the above inequality is 0 (for the second term see (6)). Therefore
\[ \lim_{i} d(f_i(g)x_i) = 0; \]

since \( d(f_i(g)) = (g) \) for all \( i \) (\( f_i \) being a conjugating automorphism). Hence, each element \( g \) of the virtually generated group \( G \) acts as an elliptic isometry on \( X_i \). This means that the action of \( G \) on \( X_i \) has a global fixed point, say \( z = (z_i) \). It follows that for every \( \alpha > 0 \) and every finite subset \( F \) of \( G \), there is an \( 2^{-\alpha} \) such that
\[ d(z_i; f_i(g)z_i) < \alpha; \]
for all \( i \) and \( g \in F \). This contradicts the minimality of \( \alpha \) (see (6)). \( \square \)

We should mention here that this is the best possible result in that generality. Indeed, as shown by Burns [6] and subsequently by several other authors (see also Sah [21]), there are finite groups that possess non-trivial outer conjugating automorphisms (known also as outer, class preserving automorphisms). Therefore one can easily construct free-by-finite groups with outer conjugating automorphisms by considering the direct product of the above mentioned finite groups by free groups.

We are now able to show our main theorem.

Theorem 2.3. Let \( G \) be a conjugacy separable, hyperbolic group. Then each virtually torsion-free subgroup of the outer automorphism group \( \text{Out}(G) \) of \( G \) is residually finite.
Proof. It succeeds to show that each torsion-free subgroup $H$ of $\text{Out}(G)$ is residually finite. We consider the following short exact sequence

$$1 \to \text{Conj}(G) \to \text{Inn}(G) \to \text{Aut}(G) \to \text{Conj}(G) \to 1.$$ 

By Lemma 2.2 the first term is finite. This implies that the restriction of $H$ is a monomorphism. It follows that $H$ is residually finite being isomorphic to a subgroup of $\text{Aut}(G) \text{Conj}(G)$, which is residually finite by Lemma 2.1.

In view of the above theorem, given a group $G$ it is natural to seek conditions under which the outer automorphism group $\text{Out}(G)$ of $G$ is virtually torsion-free. Recently, Guirardel and Levitt [13] have shown that the outer automorphism group of a hyperbolic group $G$ is virtually torsion-free, provided that $G$ is virtually torsion-free. Thus, by Theorem 2.3, the outer automorphism group of a conjugacy separable hyperbolic group is residually finite, since a residually finite hyperbolic group is virtually torsion-free.

The next lemma is more or less known (see [13,17]).

Lemma 2.4. Let $G$ be a finitely generated group containing a normal subgroup $N$ of finite index whose center is trivial. If $\text{Out}(N)$ is virtually torsion-free, then so is $\text{Out}(G)$.

Proof. Let $\text{Aut}_N(G)$ denote the subgroup of $\text{Aut}(G)$ consisting of those automorphisms of $G$ which fix $N$ and induce the identity on $G/N$. The restriction map $\text{Aut}_N(G) \to \text{Out}(N)$ is an injection. Indeed, let $g \in G$ and $f \in \text{Aut}_N(G)$. Then $f(g) = gh$ for some $h \in N$. Suppose now that $f$ is in the kernel of the restriction map. Then for each $h \in N$ we have $ghg^{-1} = f(ghg^{-1}) = ghg^{-1}hghg^{-1}$. This implies that $h$ is in the center of $N$, which is trivial, and therefore $f$ is the identity. From the injectivity of we get $\text{Inn}_N(G) = \text{Inn}(N)$, where $\text{Inn}_N(G)$ denotes the (normal) subgroup of $\text{Inn}(G)$ consisting of all inner automorphisms of $G$ induced by elements of $N$. We conclude that the quotient group $\text{Aut}_N(G) = \text{Inn}_N(G)$ embeds into $\text{Out}(N)$. In particular, $\text{Aut}_N(G) = \text{Inn}_N(G)$ is virtually torsion-free.

Now the restriction of the natural projection $\text{Aut}(G) \to \text{Aut}_N(G)$ induces a map $\text{Out}(G) \to \text{Out}(N)$. The kernel of this map is $\text{Inn}_N(G)$, which is finite, since $N$ is of finite index in $G$, while its image has finite index in $\text{Out}(G)$, since $\text{Aut}_N(G)$ is of finite index in $\text{Aut}(G)$, by [17, Lemma 1]. It follows that each finite-index, torsion-free subgroup of $\text{Aut}_N(G) = \text{Inn}_N(G)$ embeds as a subgroup of finite index in $\text{Out}(G)$, which proves the lemma.

□
Corollary 2.5. The outer automorphism group of a finitely generated, free-by-finite group is residually finite.

Proof. It is well-known that every finitely generated, free-by-finite group $G$ is hyperbolic. Furthermore, $G$ is conjugacy separable by [3]. Therefore Theorem 2.3 applies to $G$. On the other hand, it is also known that the outer automorphism group of a free group is virtually torsion-free. If $G$ is virtually in finite cyclic, then $Out(G)$ is finite. In the case where $G$ is not virtually cyclic, the center of a free subgroup of finite index in $G$ is trivial, and the result follows from Lemma 2.4. □

Recall that a Fuchsian group is a discrete subgroup of the group of isometries of the hyperbolic plane $H^2$.

Corollary 2.6. The outer automorphism group of a finitely generated, Fuchsian group is residually finite.

Proof. Every finitely generated Fuchsian group $G$ contains a normal torsion-free subgroup of finite index, say $N$, which is either a free group or the fundamental group of an orientable surface group of genus $g \geq 2$. So $N$ is hyperbolic since it is either free or quasi-isometric to $H^2$. Moreover $N$ is conjugacy separable [22, Theorem 3.3] and its outer automorphism group $Out(N)$ is virtually torsion-free and so $N$ satisfies the hypotheses of Theorem 2.6. Hence, $Out(N)$ is residually finite.

If $N$ is cyclic then $G$ is virtually cyclic and so $Out(G)$ is finite. In all other cases, $N$ is a normal subgroup of finite index in $G$ with trivial center (since it is a non-cyclic torsion-free hyperbolic group) and so from the proof of Lemma 2.4 we have that $Aut_H(G) = Inn_H(G)$ is a subgroup of $Out(N)$. Hence, every subgroup of $Aut_H(G) = Inn_H(G)$ is residually finite. But, again from the proof of Lemma 2.4, there is a torsion-free subgroup of $Aut_H(G) = Inn_H(G)$ that embeds as a finite index subgroup in $Out(G)$. Therefore, $Out(G)$ is residually finite. □

The corollary below generalizes the results of Grossman [12] and of Allenby, Kim and Tang [1]. Notice that for the exceptional cases of the Torus and the Klein bottle it is easily verified that the result still holds.

Corollary 2.7. The mapping class group $Mod_S$ of a hyperbolic 2-orbifold $S$ with finite volume is residually finite.

Proof. The fundamental group of a hyperbolic 2-orbifold with finite volume is a finitely generated Fuchsian group. Hence, the proof is an immediate consequence of the results of Fujiwara [10], the above corollaries and the fact that subgroups of residually finite groups are residually finite. □
Addendum. Added, October 8, 2005.

Only recently the authors found out that their main result can be generalized to relatively hyperbolic groups by using [2, Theorem 1.1].

Relatively hyperbolic groups were introduced by Gromov in [11], in order to generalize notions such as the fundamental group of a complete, non-compact, finite volume hyperbolic manifold and to give a hyperbolic version of Stallings cancellation theory over free groups by adopting the geometric language of manifolds with cusps.

This notion has been developed by several authors and, in particular, various characterizations of relatively hyperbolic groups have been given (see [3,19] and [7] and references therein).

We recall here one of Bowditch’s equivalent definitions. A finitely generated group \( G \) is hyperbolic relative to a family of finitely generated subgroups \( G \) if \( G \) admits a proper, discontinuous and isometric action on a proper, hyperbolic metric space \( X \) such that \( G \) acts on the ideal boundary of \( X \) as a geometrically finite convergence group and the elements of \( G \) are the maximal parabolic subgroups of \( G \).

We should mention here that Farb [9] introduced a weaker notion of relative hyperbolicity for groups using constructions on the Cayley graph of the groups.

Except of the fundamental groups of hyperbolic manifolds of finite volume, another interesting family of relatively hyperbolic groups are the fundamental groups of graphs of finitely generated groups with finite edge groups which are hyperbolic relative to the family of vertex groups (since their action on the Bass-Serre tree satisfies Definition 2 in [3]).

The key lemma of the paper, Lemma 2.2, generalizes to relatively hyperbolic groups.

Lemma 2.2. Let \( G \) be a relatively hyperbolic group. Then the group \( \text{Inn}(G) \) of inner automorphisms of \( G \) is of finite index in \( \text{Conj}(G) \).

Sketch of proof. In this case the Cayley graph of \( G \) is replaced by a hyperbolic metric space \( X \) on which \( G \) acts by isometries.

Let \( \inf_{x \in X} \max_{f_i(s)}(x; f_i(s)x) \) where \( S \) is a fixed finite generating set \( G \) and let \( x_i^0 \in X \) such that \( \max_{f_i(s)}(x_i^0; f_i(s)x_i^0) \to \frac{1}{i} \).

By the proof of Theorem 1.1 in [2], the sequence \( x_i \) converges to infinity. Hence for every non-primitive ultrafilter \( \mathcal{U} \) on \( N \) the based ultralimit \( (X; d; x_0) \) of the sequence of based metric spaces \( (X; d; x_i^0) \) is an R-tree. Moreover, there is an induced non-trivial isometric \( G \)-action on \( (X; d; x_0) \), given by \( g \cdot x = f_i(g)x_i \).
Following step-by-step the proof of Lemma 2.2 we arrive again at the contradiction that the action has a global fixed point.

The reader should be careful in the following.

1. The elements $y_i$ of $X$ are chosen such that

$$ (f_i(g)) \ d((f_i(g)y_i; y_i)) \ (f_i(g)) + \frac{1}{i}; $$

2. The inequalities (5), (6) and (7) become

$$ A \ 2d(x_i; y_i) \ 2K ( ) \ 2 \ \frac{2}{i} \ (5') $$

$$ 2d(x_i; y_i) \ K ( ) \ d((f_i(g)x_i; x_i)) \ (f_i(g)) \ 2d(x_i; y_i) + \frac{1}{i} \ (6') $$

and

$$ j(f_i(g)x_i; x_i) \ (f_i(g))j \ 2d(x_i; y_i) + K ( ) + \frac{1}{i}; \ (7') $$

respectively.

3. Finally, the last inequality becomes

$$ (g) \ \frac{(f_i(g))}{i} \ j((f_i(g)x_i; x_i))j+ A j \ 3 \ K ( ) + \frac{3}{i}; \ (8) $$

Consequently, Theorem 2.3 generalizes as follows.

Theorem 2.3. Let $G$ be a conjugacy separable, relatively hyperbolic group. Then each virtually torsion-free subgroup of the outer automorphism group $\text{Out}(G)$ of $G$ is residually finite.

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