Strong Decays of Baryons

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Abstract

A Poincaré-invariant description of mesonic baryon resonance decays is presented following the point form of relativistic quantum mechanics. In this contribution we focus on pionic decay modes. It is found that the theoretical results in general underestimate the experimental ones considerably. Furthermore, the problem of a nontrivial normalization factor appearing in the definition of the decay operator is investigated. The present results for decay widths suggest a normalization factor that is consistent with the choice adopted for the current operator in the studies of electroweak nucleon form factors.

Introduction

Constituent quark models (CQMs) provide an effective tool to describe the essential hadronic properties of low-energy quantum chromodynamics. Recently, in addition to the traditional CQM, whose hyperfine interaction derives from one-gluon exchange (OGE) \cite{1}, alternative types of CQMs have been suggested such as the ones based on instanton-induced (II) forces \cite{2,3} or Goldstone-boson-exchange (GBE) dynamics \cite{4}. In particular, the GBE CQM aims to include the effective degrees of freedom of low-energy QCD, as they are suggested by the spontaneous breaking of chiral symmetry (SB\chi_S).

Over the years, a number of valuable insights in strong decays of baryon resonances have been gained by various groups, e.g., in refs. \cite{5,6,7,8,9}. Nonetheless, one has still not yet arrived at a satisfactory explanation especially of the \(N\) and \(\Delta\) resonance decays. This situation is rather disappointing from the theoretical side, especially in view of the large amount of experimental data accumulated over the past years \cite{10}.

Here, we study the mesonic decays of baryon resonances along relativistic, i.e. Poincaré-invariant, quantum mechanics \cite{11}. This approach is a-priori distinct from a field-theoretic treatment. It assumes a finite number of degrees of freedom (particles) and relies on a relativistically invariant mass operator with the interactions included according to the Bakamjian-Thomas construction \cite{12} thereby fulfilling all the required symmetries of special relativity. We assume a decay operator in the point-form spectator approximation (PFSA) with a pseudovector coupling. The PFSA has already been applied to the calculation of electromagnetic and axial form factors of the nucleons \cite{13,14,15} and electric radii as well as magnetic moments of all octet and decuplet baryon ground states \cite{16}. In all cases the experimental data are described suprisingly well within this approach.

Covariant results for the strong decays of \(N\) and \(\Delta\) resonances have already been presented in ref. \cite{17} for the relativistic GBE and OGE CQMs. They show a dramatically
different behaviour as compared to previous non-relativistic calculations \[18, 19\]. Specifically, it turns out that the theoretical results, in general, underestimate the experimental ones considerably. This behaviour has also been observed in the relativistic calculation based on the Bethe-Salpeter equation using instanton-induced dynamics \[20\]. Up till now all relativistic approaches face the problem of defining appropriate decay operators. Usually one has resorted to simplified versions such as the spectator model.

**Theory**

Generally, the decay width \( \Gamma \) of a resonance is defined by the expression

\[
\Gamma = 2\pi\rho_f |F(i \to f)|^2 ,
\]

where \( F(i \to f) \) is the transition amplitude and \( \rho_f \) is the phase-space factor. In eq. (1) one has to average over the initial and to sum over the final spin-isospin projections. Previous calculations, based on nonrelativistic approximations of the transition amplitude encountered an ambiguity in the proper definition of the phase-space factor \([7, 21, 22]\). Here, we present a Poincaré-invariant definition of the transition amplitude, thereby resolving this ambiguity. In particular, we adhere to the point-form of relativistic quantum mechanics \([11]\), because in this case the generators of the Lorentz transformations remain purely kinematic and the theory is manifestly covariant \([23]\). The interactions are introduced into the (invariant) mass operator following the Bakamjian-Thomas construction \([12]\). The transition amplitude for the decays is defined in a covariant manner, under overall momentum conservation \((P'_\mu - P_\mu = Q_{\pi,\mu})\), by

\[
F(i \to f) = \langle P, J, \Sigma | \hat{D}_\alpha | P', J', \Sigma' \rangle .
\]

(2)

Here \( \langle P, J, \Sigma \rangle \) and \( \langle P', J', \Sigma' \rangle \) are the eigenstates of the decaying resonance and the nucleon ground state, respectively. Inserting the appropriate identities leads to the reduced matrix element

\[
F(i \to f) \sim \sum_{\sigma_i, \sigma'_i, \mu_i, \mu'_i} \int d^3k_2 d^3k_3 d^3k'_2 d^3k'_3 \Psi^*_M J \Sigma \left( \vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3 \right) \Psi_M^{J' \Sigma'} \left( \vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3 \right) \prod_{\sigma_i} D^{\pm}_{\sigma_i \mu_i} \left[ R_W (k_i; B (v_{in})) \right] \langle p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 | \hat{D}_\alpha | p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 \rangle \prod_{\sigma'_i} D^{\pm}_{\sigma'_i \mu'_i} \left[ R_W (k'_i; B (v_{f})) \right] ,
\]

(3)

where the rest-frame baryon wave functions \( \Psi^*_M J \Sigma \) and \( \Psi_M^{J' \Sigma'} \) stem from the velocity-state representations of the baryon states \( \langle P, J, \Sigma \rangle \) and \( \langle P', J', \Sigma' \rangle \), respectively. These wave functions depend on the quark momenta \( \vec{k}_i \) for which \( \sum_i \vec{k}_i = \vec{0} \). They are related to the individual quark momenta by the Lorentz boost relations \( p_i = B(v) k_i \). The main challenge lies in the definition of a consistent and reasonable momentum-space
representation of the decay operator \( \hat{D}_\alpha \). Here, we adopt the PFSA and proceed in analogy to previous studies of the electroweak nucleon structure \([13, 14, 15]\) but use a pseudovector coupling at the quark-pion vertex:

\[
\langle p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 | \hat{D}_\alpha | p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 \rangle
= \sqrt{\frac{M^3 M'_{\sigma}^3}{(\sum \omega_i)^3 (\sum \omega'_i)^3}} 3 i g_{q\pi} \bar{u} (p_1, \sigma_1) \gamma^5 \gamma^\mu \vec{F} u (p'_1, \sigma'_1) 2 p'_2 \delta (\vec{p}_2 - \vec{p}'_2) 2 p'_3 \delta (\vec{p}_3 - \vec{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3} Q_{\pi, \mu}. \tag{4}
\]

The overall momentum conservation, \( P'_\mu - P_\mu = Q_{\pi, \mu} \), together with the two spectator conditions define the relation between all incoming and outgoing quark momenta. In particular, the momenta of the active quark are related by \( \vec{p}_1 - \vec{p}'_1 = \vec{Q} \), where \( \vec{Q} \) is completely determined. Thus the momentum transferred to the active quark is different from the momentum transfer to the baryon as a whole. This is a consequence of translational invariance which thereby introduces effective many-body contributions into the above definition of the spectator-model decay operator. Furthermore, in eq. (4) there appears an overall normalization factor

\[
\mathcal{N} = \sqrt{\frac{M^3 M'_{\sigma}^3}{(\sum \omega_i)^3 (\sum \omega'_i)^3}}. \tag{5}
\]

Through the \( \omega_i \) and the on-mass-shell condition of the quarks it depends on the individual quark momenta. This choice of \( \mathcal{N} \) is consistent with the one used in the definition of the electromagnetic and axial currents in the PFSA calculations of the nucleon electroweak form factors by the Graz-Pavia collaboration \([13, 14, 15]\). It guarantees for the correct proton charge. However, it is not a unique choice. Any other normalization factor of the asymmetric form

\[
\mathcal{N} (y) = \left( \frac{M^3}{(\sum \omega_i)^3} \right)^y \left( \frac{M'_{\sigma}^3}{(\sum \omega'_i)^3} \right)^{1-y} \tag{6}
\]

would do the same. In order to study the effects of these further choices we investigate the dependence of the decay widths on the parameter range \( 0 \leq y \leq 1 \).

**Results**

The decay widths of the PFSA calculation with the decay operator given in eq. (4), with the symmetric normalization factor, are shown in table II for the GBE and OGE CQMs. It is immediately seen that only the \( N_{1535}^* \) and \( N_{1710}^* \) predictions are within the experimental range. All other decay widths are underestimated, some of them considerably. In this regard, it is noteworthy that in the case of the \( N_{1535}^* \) the \( \Delta \pi \) branching ratio is exceptionally small (\( < 1\% \)). Therefore we found it interesting to look at the results with a view to the measured \( \Delta \pi \) branching ratios. In fact, one can observe a striking relation between these branching ratios and the sizes of the theoretical decay widths, expressed as percentage fractions of the experimental values in the last three columns of table II. The larger the \( \Delta \pi \) branching ratio of a resonance, the bigger the underestimation of the
Table 1: PFSA predictions for $\pi$ decay widths of the relativistic GBE [4] and OGE [9] CQMs in comparison to the Bethe-Salpeter results of the II CQM [20] and experimental data [24]. In the last three columns the theoretical results are expressed as percentage fractions of the (best-estimate) experimental values in order to be compared to the measured $\Delta\pi$ branching ratios.

| Decays $\rightarrow N\pi$ | Experiment [MeV] | Rel. CQM | $\Delta\pi$ | % of Exp. Width |
|---------------------------|------------------|----------|-------------|-----------------|
| $N_{1440}^*$              | $(227 \pm 18)_{-59}^{+70}$ | 33        | 53          | 38              | 20 – 30%        | 14  | 24  | 17  |
| $N_{1520}^*$              | $(66 \pm 6)_{-9}^{+5}$     | 17        | 16          | 38              | 15 – 25%        | 26  | 24  | 58  |
| $N_{1535}^*$              | $(67 \pm 15)_{-17}^{+28}$  | 90        | 119         | 33              | $< 1\%$         | 134 | 178 | 49  |
| $N_{1650}^*$              | $(109 \pm 26)_{-3}^{+36}$  | 29        | 41          | 3               | 1 – 7%          | 27  | 38  | 3   |
| $N_{1675}^*$              | $(68 \pm 8)_{-4}^{+14}$    | 5.4       | 6.6         | 4               | 50 – 60%        | 8   | 10  | 6   |
| $N_{1700}^*$              | $(10 \pm 5)_{-3}^{+3}$     | 0.8       | 1.2         | 0.1             | $> 50\%$        | 8   | 12  | 1   |
| $N_{1710}^*$              | $(15 \pm 5)_{-5}^{+30}$    | 5.5       | 7.7         | $n/a$           | 15 – 40%        | 37  | 51  | $n/a$|
| $\Delta_{1232}$          | $(119 \pm 1)_{-5}^{+5}$    | 37        | 32          | 62              | $-$             | 31  | 27  | 52  |
| $\Delta_{1600}$          | $(61 \pm 26)_{-10}^{+26}$  | 0.07      | 1.8         | $n/a$           | 40 – 70%        | $\approx 0$ | 3   | $n/a$|
| $\Delta_{1620}$          | $(38 \pm 8)_{-6}^{+8}$     | 11        | 15          | 4               | 30 – 60%        | 29  | 39  | 11  |
| $\Delta_{1700}$          | $(45 \pm 15)_{-10}^{+20}$  | 2.3       | 2.3         | 2               | 30 – 60%        | 5   | 5   | 4   |

This observation hints to a possible systematic problem in the description of mesonic decay widths within (relativistic) CQMs. It calls for a more complete treatment of baryon resonances with a more realistic coupling to decay channels. In fig. 1 we demonstrate the dependence of the PFSA predictions (for the case of the GBE CQM) on the possible asymmetric choice of the normalization factor $N$ (see eq. (6)). In the range $0 \leq y \leq 1$ all decay widths grow rapidly with increasing $y$. In this way one could enhance the theoretical predictions considerably. However, if one wants neither one of the decay widths to exceed its experimental range, one is limited to a value of $y \leq 0.5$. Any $y$ lower than 0.5 would lead to decay widths much too small in most cases. Consequently, a symmetric normalization factor as in eq. (4) seems to be the preferred and most reasonable choice also in the context of hadronic decay widths.

### Summary

We have presented a Poincaré-invariant description of strong baryon resonance decays in point form within relativistic CQMs. Covariant predictions have been given for $\pi$ decay widths. They are considerably different from previous nonrelativistic results or results with relativistic corrections included. The covariant results calculated with a spectator-model decay operator show a uniform trend. In almost all cases the corresponding theoretical predictions underestimate the experimental data considerably. This is true
in the framework of Poincaré-invariant quantum mechanics (here in point form) as well as in the Bethe-Salpeter approach \cite{20}. Indications have been given that for a particular resonance the size of the underestimation is related to the magnitude of the $\Delta \pi$ branching ratio. This hints to a systematic defect in the description of the decay widths.

The investigation of different possible choices for a normalization factor in the spectator-model decay operator has led to the suggestion that the symmetric choice is the most natural one. It is also consistent with the same (symmetric) choice that had been adopted before for the spectator-model current in the study of the electroweak nucleon form factors.

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References

[1] S. Capstick and N. Isgur, Phys. Rev. D 34, 2809 (1986).
[2] U. Loering, K. Kretzschmar, B. C. Metsch, and H. R. Petry, Eur. Phys. J. A10, 309 (2001).
[3] U. Loering, B. C. Metsch, and H. R. Petry, Eur. Phys. J. A10, 395 (2001).
[4] L. Y. Glozman, W. Plessas, K. Varga, and R. F. Wagenbrunn, Phys. Rev. D 58, 094030 (1998).
[5] F. Stancu and P. Stassart, Phys. Rev. D 39, 343 (1989).
[6] S. Capstick and W. Roberts, Phys. Rev. D 47, 1994 (1993).
[7] P. Geiger and E. S. Swanson, Phys. Rev. D 50, 6855 (1994).
[8] E. S. Ackleh, T. Barnes, and E. S. Swanson, Phys. Rev. D 54, 6811 (1996).
[9] L. Theussl, R. F. Wagenbrunn, B. Desplanques, and W. Plessas, Eur. Phys. J. A12, 91 (2001).

[10] S. A. Dytman, and E. S. Swanson (Eds.): NSTAR 2002 (Proc. of the Workshop on the Physics of Excited Nucleons, Pittsburgh, Pennsylvania, 2002). Singapore: World Scientific 2003.

[11] B. D. Keister and W. N. Polyzou, Adv. Nucl. Phys. 20, 225 (1991).

[12] B. Bakamjian and L. Thomas, Phys. Rev. 92, 1300 (1953).

[13] R. F. Wagenbrunn et al., Phys. Lett. B511, 33 (2001).

[14] L. Y. Glozman et al., Phys. Lett. B516, 183 (2001).

[15] S. Boffi et al., Eur. Phys. J. A14, 17 (2002).

[16] K. Berger, R. F. Wagenbrunn, and W. Plessas, nucl-th/0407009 (2004).

[17] T. Melde, W. Plessas, and R. F. Wagenbrunn, Few-Body Syst. Suppl. 14, 37 (2003).

[18] A. Krassnigg et al., Few Body Syst. Suppl. 10, 391 (1999).

[19] W. Plessas et al., Few Body Syst. Suppl. 11, 29 (1999).

[20] B. Metsch, hep-ph/0403118 (2004).

[21] S. Kumano and V. R. Pandharipande, Phys. Rev. D 38, 146 (1988).

[22] R. Kokoski and N. Isgur, Phys. Rev. D 35, 907 (1987).

[23] W. H. Klink, Phys. Rev. C 58, 3587 (1998).

[24] S. Eidelman et al., Phys. Lett. B592, 1 (2004).