On the limiting energy of the collision of elementary particles close to horizon of the rotating black hole

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Abstract. It is shown that the energy of collision of two ultrarelativistic elementary particles due to gravitational radiation cannot exceed the Planck value. Comparison of the gravitational and electromagnetic radiation for charged particles close to the horizon of Kerr black hole is made.

1. Introduction

Particle collision in the vicinity of the horizon of the Kerr black hole was considered in \cite{1,2}. In \cite{2} a new resonance in two particle collision for the case of extremal Kerr black hole was found. The authors claimed that the energy in the centre mass reference frame can be “arbitrarily large” growing up to infinity at the horizon \cite{2}. In our papers \cite{3,4} the same effect was found for nonextremal Kerr black holes in case of multiple scattering. In \cite{3,4,5} it was shown that not only the coordinate time but the proper time also needed to get the infinite energy must be infinity. Later \cite{6,7} it was shown that such resonance occurs for more general case of the charged and dirty rotating black holes.

In \cite{8} another mechanism of getting the resonance due to negative but large absolute value of the orbital momentum was proposed for any point of the ergosphere. In \cite{9} the resonance was obtained due to collision close to the event horizon of the particle falling inside the black hole with the particle moving from the black hole on the white hole geodesic. The need to account for gravitational radiation of a particle moving around a rotating black hole in the calculation of collision energy was indicated in the work \cite{5}. In \cite{10} gravitational radiation of the graviatom was considered.

In this paper we give the answer to the question: “Can the resonance energy be made arbitrary high?” The answer occurs to be negative if one takes into account the role of the gravitational radiation of the colliding particles. This radiation being negligible at low energy has the same order as the resonance energy at the Planck energy and plays the role of the “bremsstrahlung” radiation in electromagnetic case. Electromagnetic radiation of colliding charged particles is shown to be much smaller than the gravitational radiation at high energies.

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2. The energy of collision and the relative velocity.

Let us find the energy $E_{\text{c.m.}}$ in the centre of mass system of two colliding particles with rest masses $m_1$ and $m_2$ in arbitrary gravitational field. It can be obtained from

$$(E_{\text{c.m.}}, 0, 0, 0) = m_1c^2u_{(1)}^i + m_2c^2u_{(2)}^i,$$  \hspace{1cm} (1)

where $c$ — light velocity, $u^i = dx^i/ds$ — 4-velocity. Taking the squared (1) and due to $u^iu_i = 1$ one obtains

$$E_{\text{c.m.}}^2 = m_1^2c^4 + m_2^2c^4 + 2m_1m_2c^4u_{(1)}^i u_{(2)i}.$$  \hspace{1cm} (2)

Let us find the expression of the energy in the centre of mass frame through the relative velocity $v_{\text{rel}}$ of particles at the moment of collision [11]. In the inertial reference frame with the first particle at rest at the moment of collision the four-velocity components are

$$u_{(1)}^i = (1, 0, 0, 0), \quad u_{(2)}^i = \left(\frac{c}{\sqrt{c^2 - v_{\text{rel}}^2}}, \frac{v_{\text{rel}}}{\sqrt{c^2 - v_{\text{rel}}^2}}\right).$$  \hspace{1cm} (3)

So

$$u_{(1)}^iu_{(2)i} = \frac{1}{\sqrt{1 - v_{\text{rel}}^2/c^2}}, \quad \frac{v_{\text{rel}}}{c} = \sqrt{1 - \frac{1}{(u_{(1)}^iu_{(2)i})^2}}.$$  \hspace{1cm} (4)

These expressions evidently don’t depend on the coordinate system.

From (2) and (4) one obtains

$$E_{\text{c.m.}}^2 = m_1^2c^4 + m_2^2c^4 + \frac{2m_1m_2c^4}{\sqrt{1 - v_{\text{rel}}^2/c^2}},$$  \hspace{1cm} (5)

and the nonlimited growth of the collision energy in the centre of mass frame occurs due to growth of the relative velocity to the velocity of light.

3. Gravitational radiation in particle scattering

At the moment of scattering of ultrarelativistic particles they move with acceleration in variable gravitational field of each other. At this moment the gravitational radiation appears. In case of scattering of pointless classical massive particles S.Weinberg formula (10.4.23) from [12] gives the following expression for the total energy per unit frequency interval emitted in gravitational radiation in collision

$$\frac{dE}{d\omega} = \frac{G}{2\pi c} \sum_{N,M} \eta_N \eta_M m_N m_M \frac{1 + \beta_{NM}^2}{\beta_{NM}(1 - \beta_{NM}^2)^{1/2}} \ln \left(\frac{1 + \beta_{NM}}{1 - \beta_{NM}}\right),$$  \hspace{1cm} (6)

where $G$ is the gravitational constant, $N, M$ correspond to numeration of particles in initial and final states and the factors $\eta_N$ are defined as

$$\eta_N = \begin{cases} +1, & N \text{ in final state}, \\ -1, & N \text{ in initial state}. \end{cases}$$
$\beta_{NM} = |\mathbf{v}_{NM}|/c$, $\mathbf{v}_{NM}$ is the relative velocity of particles with indexes $N$ and $M$.

Evaluate the full energy in collision of two particles with ultrarelativistic energies

$$E_{\text{c.m.}} \gg m_1 c^2, m_2 c^2.$$ 

Then $\beta_{12} \sim 1$ and from (5) one obtains

$$\sqrt{1 - \beta_{12}^2} \approx \frac{2m_1m_2c^4}{E_{\text{c.m.}}^2},$$

$$1 - \beta_{12} \approx \frac{2 \left( \frac{m_1m_2c^4}{E_{\text{c.m.}}^2} \right)^2}{1}.$$ (8)

Consider particles to be classical pointlike particles. In the sum (6) take $N, M = 1, 2$ and $\beta_{12}, \beta_{21}$. For the energy of gravitational radiation in the unit frequency interval in such collision one obtains

$$\frac{dE}{d\omega} = \frac{2G}{\pi c^5} E_{\text{c.m.}}^2 \ln \left( \frac{E_{\text{c.m.}}^2}{m_1m_2c^4} \right).$$ (9)

If one calculates the full radiated energy taking the integral of (9) one obtains as it is mentioned in [12] the result divergent as $\int_0^\omega d\omega$. This occurs due to the fact that formula (6) is obtained (see [12]) in approximation of immediate collision. In reality scaterrings occur at some finite time interval $\Delta t$ and so the integral in the frequency is cut at some value of $\omega$ of the order of $1/\Delta t$. If the collision occurs at the energy much less than the Planck energy $E_{\text{Pl}} = \sqrt{\hbar c^5/G} = 1.22 \cdot 10^{19}$ GeV, one can take as such frequency

$$\omega \approx \frac{E_{\text{c.m.}}}{\hbar}.$$ (10)

To get the full radiation energy multiply (9) on the frequency (10) and obtain

$$E = \frac{2G}{\pi c^5 \hbar} E_{\text{c.m.}}^3 \ln \left( \frac{E_{\text{c.m.}}^2}{m_1m_2c^4} \right) = \frac{4}{\pi} \frac{E_{\text{c.m.}}^3}{E_{\text{Pl}}^2} \ln \left( \frac{E_{\text{c.m.}}}{M_{\text{Pl}}} \frac{M_{\text{Pl}}}{\sqrt{m_1m_2}} \right),$$ (11)

where $M_{\text{Pl}} = \sqrt{\hbar c^5/G} = 2.18 \cdot 10^{-8}$ kg is Planck mass.

If the value of $E_{\text{c.m.}}$ is given the value of the logarithm in the right hand side of (11) is not large even for such light particles as electrons $\ln(M_{\text{Pl}}/m) < 52$. So the role of the gravitational radiation in collisions of elementary particles with the energy in the center of mass frame less than Planck energy is negligible

$$E_{\text{c.m.}} \ll E_{\text{Pl}} \Rightarrow \frac{E}{E_{\text{c.m.}}} \ll 1.$$ (12)

If the energy in the center of mass frame is large then due to formula (11) the loss of energy thanks to gravitational radiation grows as the cube of the energy and becomes comparable to the energy of particles for $E_{\text{c.m.}} \sim E_{\text{Pl}}$.

Note that the value of the gravitational radiation equal in order with (11) can be obtained also from the well known formula for the quadrupole radiation of gravitational waves (see (110.16) in [13])

$$-\frac{dE}{dt} = \frac{G}{45c^5} \bar{D}_{\alpha\beta}^2.$$ (13)
Take as the effective time $\Delta t = \hbar/E_{c.m.}$ and the effective distance $\Delta r = \hbar c/E_{c.m.}$.

Such choice corresponds to the collision of particles taking into account the De Broglie wave length for such energies. Putting in formula (13) $m \rightarrow E_{c.m.}/c^2$, $D_{\alpha\beta} \rightarrow (E_{c.m.}/c^2)(\Delta r)^2/(\Delta t)^3 = E^2_{c.m.}/\hbar$, one obtains for the energy of radiation of gravitational waves

$$E = \Delta t \left| \frac{dE}{dt} \right| = \frac{1}{45} \frac{E^3_{c.m.}}{E_{Pl}^2}. \quad (14)$$

So the conclusion about the growth proportional to $E^3_{c.m.}$ of the radiation energy if value $E_{c.m.}$ close to Planck energy is valid in this approach also.

Note that the situation for the electromagnetic radiation is different. To evaluate the effect use the formula for the intensity of the electromagnetic radiation with terms of the second order in order to take into account not only the dipole but also the quadrupole terms to compare it with gravitational radiation (13). Using formula (71.5) from [13] one obtains

$$-\frac{dE_{em}}{dt} = \frac{2}{3c^2} \mathbf{d}^2 + \frac{1}{180c^3} D_{\alpha\beta}^2 + \frac{2}{3c^3} \mathcal{M}^2, \quad (15)$$

where $\mathbf{d}$ is electric dipole moment, $D_{\alpha\beta}$ is electric quadrupole moment, $\mathcal{M} = \sum q[\mathbf{r}, \mathbf{v}]/(2c)$ is the magnetic moment of the radiating system, $q$ is the particle charge. Calculating the electromagnetic radiation of the system of two colliding ultrarelativistic particles with the energy $E_{c.m.}$ again take time and distance $\Delta t = h/E_{c.m.}$, $\Delta r = h c/E_{c.m.}$. Take the electric dipole moment as $\mathbf{d} \sim e \Delta r$, the electric quadrupole moment as $D_{\alpha\beta} \sim e (\Delta r)^2$, the magnetic moment as $\mathcal{M} \sim e \Delta r/2$, where $e$ is the elementary charge. Then from (15) one obtains

$$-\frac{dE_{em}}{dt} = \frac{2}{3c^2} \left( \frac{e}{\Delta t} \right)^2 + \frac{1}{180c^3} \left( \frac{e}{\Delta t} \right)^2 + \frac{1}{6c} \left( \frac{e}{\Delta t} \right)^2 = \frac{151}{180c} \left( \frac{eE_{c.m.}}{\hbar} \right)^2. \quad (16)$$

Multiplying both sides on the effective time one obtains full energy of the electromagnetic radiation in ultrarelativistic collision of particles with size of the De Broglie wave length at such energies

$$E_{em} = \Delta t \left| \frac{dE_{em}}{dt} \right| = \frac{151}{180} \frac{e^2}{\hbar c} E_{c.m.} \approx \frac{151}{137} \frac{151}{180} E_{c.m.}. \quad (17)$$

So the relation of the energy of electromagnetic radiation to the energy of collision in the center of mass frame has the approximate value equal to the value of the fine structure constant $e^2/(\hbar c) \approx 1/137$.

For small energies the electromagnetic radiation is larger than the gravitational in many times. However due to the growth of the gravitational interaction in relativistic region proportional to the energies of colliding particles the gravitational radiation grows in energy greater than the electromagnetic one and as it is seen from our estimates becomes dominant close to the Planck energy. Comparing formulas (11) and (17) one obtains that the dominance of the gravitational radiation over the electromagnetic one begins from values $E_{c.m.} \approx \sqrt{e^2/(\hbar c)} E_{Pl}$.

Evaluate the power of radiation of gravitational waves taking the time $\Delta t = h/E_{c.m.}$ in (11). Then one obtains

$$\frac{\Delta E}{\Delta t} = \left( \frac{E_{c.m.}}{E_{Pl}} \right)^4 \frac{4}{\pi} \ln \left( \frac{E_{c.m.}}{E_{Pl}} \frac{M_{Pl}}{\sqrt{m_1 m_2}} \right), \quad (18)$$
where $L_{Pl} = c^5/G \approx 3.6 \cdot 10^{52}$ W is the Planck power. So for $E_{c.m.} \ll E_{Pl}$, the radiating power is much smaller than the Planck one but when the energy of collision grows close to the Planck value the power of radiation also grows to the Planck power $L_{Pl}$.

In case when the energy of elementary particles in the center of mass frame is larger than the Planck value our estimates must be changed because it is impossible to take time intervals smaller that the Planck time $t_{Pl} = \sqrt{\hbar G/c^5} = 5.39 \cdot 10^{-44}$ s, the frequencies larger than $\omega_{Pl} = E_{Pl}/\hbar = \sqrt{c^5/\hbar G} = 1.85 \cdot 10^{43}$ s$^{-1}$ and the power larger than the Planck one. So the extrapolation of formulas (11) and (18) on the region $E_{c.m.} \geq E_{Pl}$ shows that for such energies of collisions intensive gravitational radiation (creation of gravitons) takes place with the Planck power $L_{Pl}$. For the time of the order of $E_{c.m.}/L_{Pl}$ the energy of colliding particles in the center of mass frame falls to values smaller than $E_{Pl}$. So for all energies of particles in the center of mass frame the energy which can be used on nongravitational particle interaction occurs to be smaller than the Planck value.

From our formula (5) one can see that if $E_{c.m.}$ becomes smaller due to gravitational radiation then the velocity $v_{rel}$ also becomes smaller. This means that radiation due to gravitational wave plays the role of some “bremsstrahlung” as it occurs in electromagnetic case.

Surely one can reasonably note that consideration of Planck time and Plank power lead to quantization of gravity and then there is no sense in such notions as the critical trajectory etc., leading to “superPlanck high energy resonance”. This is again the argument for existence of the limit of the energy of the resonance discussed in this paper.

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