Energy-Aware Relay Selection and Power Allocation for Multiple-User Cooperative Networks

Sabyasachi Gupta*, Ranjan Bose†

* Bharti School of Telecommunications, Indian Institute of Technology Delhi, New Delhi, India
† Department of Electrical Engineering, Indian Institute of Technology Delhi, New Delhi, India

Email: sabyasachi.gupta@dbst.iitd.ac.in, rbose@ee.iitd.ac.in

Abstract—This paper investigates the relay assignment and power allocation problem for two different network power management policies: group lifetime maximization (GLM) and minimum weighted total power (MWTP), with the aim of lifetime maximization in symbol error rate (SER) constrained multiple-user cooperative network. With optimal power allocation solution obtained for each policy, we show that the optimal relay assignment can be obtained using bottleneck matching (BM) algorithm and minimum weighted matching (MWM) algorithm for GLM and MWTP policies, respectively. Since relay assignment with BM algorithm is not power efficient, we propose a novel minimum bottleneck matching (MBM) algorithm to solve the relay assignment problem optimally for GLM policy. To further reduce the complexity of the relay assignment, we propose suboptimal relay selection (SRS) algorithm which has linear complexity in number of source and relay nodes. Simulation results demonstrate that the proposed relay selection and power allocation strategies based on GLM policy have better network lifetime performance over the strategies based on MWTP policy. Compared to the MBM and SRS algorithms, relay assignment based on BM algorithm for GLM policy has inferior network lifetime performance at low update interval. We show that relay assignment based on MBM algorithm achieves maximum network lifetime performance and relay assignment with SRS algorithm has performance very close to it.

I. INTRODUCTION

User cooperation is a promising approach to achieve spatial diversity in wireless networks, where multiple antennas at nodes of the network are not available [1]. With nodes helping each other to transmit message through multiple independent fading paths, user cooperation reduces the probability of erroneous message reception significantly, thus reducing the transmit power consumption in energy limited wireless network. To further achieve power efficiency in cooperative networks, several sum power minimization based power allocation strategies for a quality of service (QoS) constraint have been investigated [2], [3]. In many wireless networks maximizing network lifetime is the main design objective. Minimizing overall power in the network does not necessarily maximize network lifetime, since lifetime depends upon power minimization as well as energy balancing. Studies on lifetime maximization with relay selection and power allocation in single user cooperative network is available in [3]-[7]. Recently lifetime maximization in multiple-user network is studied in [8], [9]. The study in [8] consider relay power allocation problem for lifetime maximization in rate constrained network. Relay assignment problem in multiple-user network without power allocation optimization is considered in [9].

In this paper we study the relay assignment and power allocation optimization problem for two network power management policies: group lifetime maximization (GLM) and minimum weighted total power (MWTP) to maximize lifetime of symbol error rate (SER) constrained multiple-user cooperative network. The relay selection and power allocation according to these policies are solved in two steps. At first, according to a policy the optimal power allocation solution is derived and a weight is assigned for each source relay pair. Then using this weight, relay assignment problem is solved optimally and suboptimally. The main contribution of this paper are threefold. Firstly, we consider relay assignment along with source relay power allocation for lifetime maximization in multiple-user network which has not been studied before to the best of our knowledge. Secondly, we solve relay assignment problem in multiple-user network optimally and suboptimally with lower complexity compared to the proposed method in [9]. Thirdly we show that compared to the MWTP policy proposed in [4], [8] for relay selection and power allocation for lifetime maximization, GLM policy based strategies achieve higher network lifetime.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

Consider a wireless network consisting of set of M source nodes $S = \{s_1, \ldots, s_M\}$ and N relay nodes $R = \{r_1, \ldots, r_N\}$ randomly distributed in an area. Each node is equipped with a single omnidirectional antenna and a battery energy supply. Each source node sends data to the base station (BS), denoted by $d$, with the help of a relay node selected for the source node. Each relay can at most be selected by only one source node. Therefore, we need $N \geq M$. To avoid interference, each source data transmission is assigned orthogonal channels using frequency-division multiple access. The source-BS, source-relay and relay-BS channels undergo independent quasi-static Rayleigh fading and path loss. The channel variances of source node $s_i \in S$ to BS, source node $s_i$ to relay node $r_j \in R$ and relay node $r_j$ to BS are modelled as $\sigma^2_{s_i,d} = \eta D^{-\alpha}_{s_i,d}$, $\sigma^2_{s_i,r_j} = \eta D^{-\alpha}_{s_i,r_j}$ and $\sigma^2_{r_j,d} = \eta D^{-\alpha}_{r_j,d}$ respectively where $D_{s_i,d}$, $D_{s_i,r_j}$ and $D_{r_j,d}$ are the respective link distances, $\alpha$ is path loss exponent, $\eta$ is propagation environment dependent constant. The additive white Gaussian noise (AWGN) power...
at the relay nodes and BS are $N_0$. In cooperative mode, the source node $s_i$ broadcasts messages with transmit power $P_{s_i}$ in the first phase of cooperation. During the second phase, a selected relay node $r_j$, transmits messages with transmit power $P_{r_j}$ in AF mode \[1\]. Upon receiving two copies of the message, the BS uses maximal ratio combining (MRC) to detect the transmitted symbols. Thus SER at BS at moderate power source relay pair of the source set $\Pi$ and disjoint source relay pair. For example, with $\Pi = \{(s_1, r_1), (s_2, r_2)\}$, the source node $s_i$-to-BS, source node $s_i$-to-relay node $r_j$ and relay node $r_j$-to-BS links, respectively. The average SNR terms can be expressed as \[\gamma_{s_i,d} = G_0 P_{s_i} \sigma^2_{s_i,d} / N_0\] and $\bar{\gamma}_{r_j,d} = G_0 P_{r_j} \sigma^2_{r_j,d} / N_0$ where $G_0$ as the power gain factor at reference distance 1 m. Substituting the values of average SNR terms in \[1\], end-to-end SER can be expressed as

$$p_e = \frac{A_{s_i,r_j}}{P_{s_i}} + \frac{B_{s_i,r_j}}{P_{s_i} P_{r_j}}$$

where $A_{s_i,r_j} = 3N_0^2 / 4K^2G_0^2\sigma^2_{s_i,r_j}$ and $B_{s_i,r_j} = 3N_0^2 / 4K^2G_0^2\sigma^2_{s_i,r_j} d^2$

**B. Problem Formulation**

For cooperative network, the lifetime definition considered in literature are: end-to-end QoS lifetime \[5\], \[6\], \[8\], \[9\], first node death lifetime \[3\], \[4\]. The end-to-end QoS lifetime for multiple-user network is defined as the time interval within which end-to-end QoS of all the source messages is maintained through their respective relay nodes \[9\]. Since we consider both source and relay nodes as energy limited, according to this definition the network expires if any of the source or relay nodes fails to transmit data with available residual energy. Therefore when all nodes are energy limited, end-to-end QoS lifetime is same as first node death lifetime which is defined as the time interval until first node in the network is depleted of energy \[3\], \[4\]. Throughout the paper we use first node death lifetime as network lifetime definition. This definition is useful in a network where all the nodes are equally important, e.g., surveillance applications. The GLM policy aims to maximize time interval of first node death occur in the network according to the current network state, i.e., residual energy level, location of the nodes. Let $T_{s_i} = E_{s_i} / P_{s_i}$, $T_{r_j} = E_{r_j} / P_{r_j}$ are the lifetime of the source node $s_i$ and relay node $r_j$ with residual energies $E_{s_i}$ and $E_{r_j}$ respectively. Therefore lifetime of the source relay pair $(s_i, r_j)$ can be expressed as

$$T_{g(s_i,r_j)} = \min\{T_{s_i}, T_{r_j}\}$$

Let $\Pi$ denotes the set of all possible source relay assignments of the source set $S$ and relay set $R$ such that every set $\pi \in \Pi$ is the source relay assignments that contains $M$ disjoint source relay pair. For example, with $S = \{s_1, s_2\}$ and $R = \{r_1, r_2\}$ we have two different source relay assignments partition $\{{(s_1, r_1), (s_2, r_2)}, \{(s_1, r_2), (s_2, r_1)\}$$ and $\Pi = \{(s_1, r_1), (s_2, r_2)\}$, \{(s_1, r_2), (s_2, r_1)\}$. Then the relay assignment and power allocation problem for GLM strategy can be formulated as

$$\max_{\pi \in \Pi, (s_i, r_j) \in \pi} \min_{T_{g(s_i,r_j)}}$$

$$s.t. \frac{A_{s_i,r_j}}{P_{s_i}} + \frac{B_{s_i,r_j}}{P_{s_i} P_{r_j}} \leq p^s_{th}, \forall (s_i, r_j) \in \pi$$

$$P_{s_i} > 0, \forall (s_i, r_j) \in \pi$$

where $P_{s_i} = \{P_{s_i} \mid P_{s_i} \in \mathbb{R}_+\}$ is the transmit power vector of source relay pair $(s_i, r_j)$, $p^s_{th}$ is the end-to-end SER constraint for source node $s_i$. Note that relay assignment and power allocation according to GLM policy is optimal for network lifetime maximization if the allocated transmit power and relay selection remains fixed and not updated till the network expires. To compare with network lifetime performance of GLM policy, we propose another residual energy aware relay selection and power allocation policy referred as MWTP policy. In MWTP policy, we aim to minimize residual energy weighted total power, i.e.,

$$\min_{\pi \in \Pi, (s_i, r_j) \in \pi} \frac{\sum_{(s_i, r_j) \in \pi} \frac{P_{s_i}}{E_{s_i}} + \frac{P_{r_j}}{E_{r_j}}}{\pi}$$

$$s.t. \frac{A_{s_i,r_j}}{P_{s_i}} + \frac{B_{s_i,r_j}}{P_{s_i} P_{r_j}} \leq p^s_{th}, \forall (s_i, r_j) \in \pi$$

$$P_{s_i} > 0, \forall (s_i, r_j) \in \pi$$

**III. POWER ALLOCATION FOR A FIXED PAIR**

**A. GLM policy**

The power allocation problem for GLM policy is given by

$$\max_{T_{g(s_i,r_j)}}$$

$$s.t. \frac{A_{s_i,r_j}}{P_{s_i}} + \frac{B_{s_i,r_j}}{P_{s_i} P_{r_j}} \leq p^s_{th}, P_{s_i} > 0, P_{r_j} > 0$$

By replacing $T_{g(s_i,r_j)} = 1/V$ and using (3), the power allocation problem becomes

$$\min_{V}$$

$$s.t. P_{s_i} \leq E_{s_i} V, P_{r_j} \leq E_{r_j} V,$$

$$\frac{A_{s_i,r_j}}{P_{s_i}} + \frac{B_{s_i,r_j}}{P_{s_i} P_{r_j}} \leq p^s_{th}, P_{s_i} > 0, P_{r_j} > 0$$

The optimization problem is convex and has a unique optimal solution \[11\]. From the Karush-Kuhn-Tucker (KKT) conditions we have

$$P_{s_i} = E_{s_i} V, P_{r_j} = E_{r_j} V,$$

$$\frac{A_{s_i,r_j}}{P_{s_i}} + \frac{B_{s_i,r_j}}{P_{s_i} P_{r_j}} = p^s_{th}.$$  

Hence, using (8), (9), the optimal transmit power of $s_i$ and $r_j$ becomes

$$P_{s_i}^{\text{glm}} = \sqrt{\frac{A_{s_i,r_j} E_{r_j} + B_{s_i,r_j} E_{s_i}}{E_{r_j} P_{th}}}, P_{r_j}^{\text{glm}} = \frac{P_{s_i}^{\text{glm}} E_{r_j}}{E_{s_i}}$$

(10)
and optimal lifetime of the source relay pair \((s_i, r_j)\) is
\[
\tau_{\text{glm}}^{\text{glm}} = \frac{E_{s_i}}{P_{s_i}^{\text{glm}}} = \frac{E_{r_j}}{P_{r_j}^{\text{glm}}}
\]
Hence, lifetime of the pair is maximized when both source and relay nodes die at same time. Let weight of source relay pair \((s_i, r_j)\) for GLM policy is
\[
\omega(s_i, r_j) = \frac{1}{\tau_{\text{glm}}^{\text{glm}}} = \sqrt{\frac{A_{s_i, r_j} E_{r_j} + B_{s_i, r_j} E_{s_i}}{E_{s_i} E_{r_j} p_{th}^s}}.
\]

**B. MWTP policy**

The power allocation problem for MWTP policy is
\[
\begin{align*}
\min_{P_{s_i}, P_{r_j}} & \quad P_{s_i} + P_{r_j} \\
\text{s.t.} & \quad \frac{A_{s_i, r_j}}{P_{s_i}^2} + \frac{B_{s_i, r_j}}{P_{s_i} P_{r_j}} \leq p_{th}^s, \quad P_{s_i} > 0, P_{r_j} > 0. \tag{13}
\end{align*}
\]
Since the objective function is linear and the constraint is convex function, the optimization problem is convex problem and has a unique optimal solution \([11]\). Since end-to-end SER decreases with increase of \(P_{s_i}, P_{r_j}\), to minimize the objective function, \(P_{s_i}, P_{r_j}\) must satisfy
\[
\frac{A_{s_i, r_j}}{P_{s_i}^2} + \frac{B_{s_i, r_j}}{P_{s_i} P_{r_j}} = p_{th}^s. \tag{14}
\]
Therefore \(P_{r_j}\) can be expressed in terms of \(P_{s_i}\) as
\[
P_{r_j} = \frac{B_{s_i, r_j} P_{s_i}}{p_{th}^s P_{s_i}^2 - A_{s_i, r_j}} \triangleq f(P_{s_i}) \tag{15}
\]
The optimal transmit power can be obtained by deriving the derivative on \(P_{s_i}/E_{s_i} + f(P_{s_i})/E_{r_j}\) to be zero, as
\[
P_{\text{mwtp}}^{\text{smwtp}} = \sqrt{\frac{A_{s_i, r_j} (2 + C_{s_i, r_j}) + \sqrt{C_{s_i, r_j}^2 + 8 C_{s_i, r_j}}}{2 p_{th}^s}},
\]
\[
P_{\text{mwtp}}^{\text{smwtp}} = f(P_{\text{mwtp}}^{\text{smwtp}}), \tag{16}
\]
where \(C_{s_i, r_j} = B_{s_i, r_j} E_{r_j}/A_{s_i, r_j} E_{s_i}\). Then weight of source relay pair \((s_i, r_j)\) for MWTP policy can be expressed as
\[
\omega(s_i, r_j) = \frac{P_{\text{mwtp}}^{\text{smwtp}}}{E_{s_i}} + \frac{P_{\text{mwtp}}^{\text{smwtp}}}{E_{r_j}}. \tag{17}
\]

**IV. RELAY SELECTION SCHEMES**

Consider first the relay assignment and power allocation problem for MWTP policy as given in \([3]\). With use of weight \(\omega(s_i, r_j)\) assigned to each source relay pair \((s_i, r_j)\) according to the optimal power allocation solution, the optimization problem of MWTP policy as given in \([3]\) reduces to
\[
\min_{\pi \in \Pi} \sum_{(s_i, r_j) \in \pi} \omega(s_i, r_j) \tag{18}
\]
where \(\omega(s_i, r_j)\) is defined in \([17]\) for MWTP policy.

With weight assigned to each source relay pair \((s_i, r_j)\) in \([12]\) according to the optimal power allocation solution for GLM policy, the optimization problem for GLM policy given in \([4]\), becomes
\[
\min_{\pi \in \Pi} \max_{(s_i, r_j) \in \pi} \omega(s_i, r_j) \tag{19}
\]
Both the relay assignment problems given in \([18]\) and \([19]\), can be solved with the exhaustive search over all possible relay assignments with complexity \(O(MN^M) \sim O(M^2N^M)\) or exhaustive search over all possible source priority vector with complexity of \(O(NM^{M+1})\) as proposed in \([9]\). In this section we propose graph theoretic approach to solve optimal relay assignment problem with lower complexity compared to these strategies. We further propose suboptimal relay selection method with lower complexity.

**A. Optimal Relay Selection with Bipartite Matching Approach**

Before we proceed, we review some preliminary concepts of bipartite graph theory matching \([12]–[14]\). A graph \(G\) comprising of vertex set \(V_G\) and edge set \(E_G\) is bipartite if \(V_G\) can be partitioned into two sets \(V_G^1\) and \(V_G^2\) (the bipartition) such that every edge in \(E_G\) connects a vertex in \(V_G^1\) to one in \(V_G^2\). A complete bipartite graph is a bipartite graph where every vertex of \(V_G^1\) is connected by an edge to every vertex of \(V_G^2\), i.e., \(|E_G| = |V_G^1||V_G^2|\) where \(|E_G|\) is the number of edges and \(|V_G^1|, |V_G^2|\) are the number of vertices in \(V_G^1, V_G^2\) of the graph, respectively. If the two sets of vertices have the same cardinality, i.e., \(|V_G^1| = |V_G^2| = |V_G|/2\), then the bipartite graph is a balanced bipartite graph. A matching in a graph \(G\) is a subset of \(E_G\) such that every vertex \(v \in V_G\) is incident to at most one edge of the matching. Maximum matching \(M^*\) in \(G\) is a matching that contains the largest possible number of edges. The maximum matching \(M^*\) is perfect matching if each vertex \(v \in V_G\) belongs to an edge in \(M^*\). Clearly, for a balanced bipartite graph, a maximum matching \(M^*\) is perfect matching if \(|M^*| = |V_G|/2\). Also note that, a balanced, complete bipartite graph always has perfect matching.

Now, if the network is represented as a complete bipartite graph \(G\) such that \(V_G^1 = \{s_1, \ldots, s_M\}\) and \(V_G^2 = \{r_1, \ldots, r_N\}\) are the vertex sets of \(G\) and weight of each edge \((s_i, r_j)\) between vertices \(s_i\) and \(r_j\), \(i = 1, \ldots, M, j = 1, \ldots, N\) is \(\omega(s_i, r_j)\) as given in \([17]\), the relay assignment problem in \([13]\) can be described as finding maximum matching in the graph \(G\) such that sum of edge weights in the matching has minimum value. This problem is known as minimum weighted matching (MWM) problem in a bipartite graph. The MWM algorithm in \([13]\) (also known as Hungarian algorithm) can solve the MWM problem optimally for a balanced bipartite graph. Therefore if \(M < N, N - M\) dummy vertices that have zero edge weights with each vertex in \(V_G^2\), require to be added in \(V_G^1\) of the graph \(G\) \([12]\). Then, relay assignment problem in \([13]\) can be solved in time \(O(N^3)\) using MWM algorithm on the graph \([13]\).

The bottleneck matching (BM) problem in a bipartite graph is defined as finding maximum matching in the graph such that the largest edge weight of the matching is as small as possible. If the network is represented as a complete bipartite graph \(G\) such that \(V_G^1 = \{s_1, \ldots, s_M\}\) and \(V_G^2 = \{r_1, \ldots, r_N\}\) are the vertex sets of \(G\) and weight of each edge \((s_i, r_j)\) is \(\omega(s_i, r_j)\)
as given in (12), the relay assignment problem in (19) can be described as BM problem of the graph G. The BM problem for balanced bipartite graph has been optimally solved in [14]. Therefore if \( M < N \), \( N - M \) dummy vertices that have zero edge weights with each vertex in \( V^D_i \) are added in \( V^S_i \) of G (12). Then, the relay assignment problem in (19) can be solved in time \( O(N^{2.5}) \) by applying BM algorithm on the graph [14].

The bottleneck matching in a bipartite graph is not necessarily unique. For example, consider the bipartite graph with edge weights as shown in the Fig 1. The graph has two different possible bottleneck matching \( B_1 = \{(s_1,r_1),(s_2,r_2),(s_3,r_3)\} \) and \( B_2 = \{(s_1,r_1),(s_2,r_3),(s_3,r_2)\} \). Note that, both the bottleneck matching \( B_1 \) and \( B_2 \), has same bottleneck edge \((s_1,r_1)\), with weight \( \omega(s_1,r_1) = 0.5 \).

Definition 1. An edge \( e \) of a bipartite graph G is unique bottleneck edge if the edge \( e \) corresponds to the maximum weight in every bottleneck matching of the graph G.

Therefore \((s_1,r_1)\) is the unique bottleneck edge of the graph. Now the question is, if relay selection according to one bottleneck matching will have same network lifetime performance always compared to relay selection according to another bottleneck matching in the network. To explain this, consider the example of the network corresponding to the graph shown in the Fig 1. The network has three source nodes and three relay nodes. The weight \( \omega(s_i,r_j) \) of each edge \((s_i,r_j)\) where \( i,j \in \{1,2,3\} \) shown in Fig 1 is actually weight of source relay pair \((s_i,r_j)\), calculated using (12) at time \( t = 0 \). Let the initial energies of each source node and relay node is \( E_{s_i} = E_{r_j} = 1J \). Therefore, average transmit power of nodes \( s_i, r_j \) is \( \omega(s_i,r_j) \) and lifetime of nodes \( s_i, r_j \) is \( 1/\omega(s_i,r_j) \) when \( s_i \) is paired with \( r_j \). Therefore lifetime of the network is same as lifetime of source relay pair \((s_1,r_1)\) which is \( 1/\omega(s_1,r_1) = 2 \) seconds when relay selection is according to the bottleneck matching \( B_1 \) or \( B_2 \). However the power consumption of source nodes \( s_2, s_3 \) relay nodes \( r_2, r_3 \) is lower if the relay selection at \( t = 0 \) is according to \( B_1 \) rather than \( B_2 \). Therefore at \( t = t' \) where \( 0 < t' < 2 \), the residual energy of source nodes \( s_2, s_3 \) relay nodes \( r_2, r_3 \) is higher if the initial relay selection at \( t = 0 \) was according to \( B_1 \) instead of \( B_2 \). Now if the edge weights are updated again at time \( t = t' \), to update the relay selection according to the residual energy state in the network at \( t = t' \), the weight of all edges except \((s_1,r_1)\) is higher if the relay selection at \( t = 0 \), was according to \( B_1 \) rather than \( B_2 \). Thus, if the initial relay selection at \( t = 0 \) was according to \( B_1 \) instead of \( B_2 \), there is a possibility of obtaining a bottleneck matching with lower bottleneck weight (i.e., higher network lifetime) when relay selection is updated at time \( t = t' \).

Definition 2. The minimum bottleneck matching of a bipartite graph G is defined as maximum matching in G such that largest edge weight, second largest edge weight, ..., \( M \) th largest edge weight of the matching is as small as possible where \( M \) is number of edges in the maximum matching.

Note that choice of the edge corresponding to \( i \)th largest edge weight of the matching with minimum possible value where \( i \in 2, ..., M \), depends upon previously selected edges corresponding to the minimum possible largest edge weight, second largest edge weight, ..., \((i-1)\)th largest edge weight of the matching. For example, in Fig 1, among all possible maximum matching in the graph, \((s_1,r_1)\) is the edge corresponding to minimum possible largest edge weight, i.e., the bottleneck edge. Then, among the maximum matching in which \((s_1,r_1)\) is the edge with largest edge weight (i.e., \( B_1 \) and \( B_2 \), second largest edge weight in \( B_1 \) is smaller than of \( B_2 \). Therefore, \( B_1 \) is the minimum bottleneck matching of the graph. As discussed above, the relay selection with minimum bottleneck matching may have better network lifetime performance compared to other bottleneck matching when the relay selection and power allocation are updated periodically. However, the traditional bottleneck matching algorithm [14] do not ensure to find minimum bottleneck matching but finds any of the bottleneck matching for the graph. Inspired by all the above observations, we propose a novel algorithm for relay selection as shown in Algorithm 1 which is referred to as the minimum bottleneck matching (MBM) algorithm. The algorithm runs for maximum \( M \) iterations for the graph G. At each iteration, a bottleneck matching with corresponding bottleneck edge is obtained. Then, if the bottleneck edge is unique bottleneck edge, the bottleneck edge is stored and the graph is updated by removing vertices corresponding to the bottleneck edge. If the graph do not have unique bottleneck edge during any of the iterations, the algorithm terminates with output bottleneck matching \( B_o \) as bottleneck matching obtained in the first iteration and acknowledgement metric \( I = 0 \), which states that the output bottleneck matching \( B_o \) is not necessarily a minimum bottleneck matching. This is because, if the obtained bottleneck edge at any of the iteration is not unique bottleneck edge, it can not be decided which of the bottleneck edges is an edge of minimum bottleneck matching and finding minimum bottleneck matching is difficult in such case. If the graph has unique bottleneck edge at each iteration, after \( M \) successful iterations, the algorithm returns the minimum bottleneck matching which is a set of all unique bottleneck edges obtained in each iterations, and the acknowledgement metric \( I = 1 \) stating that the output bottleneck \( B_o \) matching is the minimum bottleneck matching. To test if a bottleneck edge corresponding to a bottleneck matching is unique bottleneck edge in the graph, the following steps are used in the algorithm. Firstly, remove the bottleneck edge and the edges with weight greater than bottleneck weight from the graph and check if the graph has a perfect matching. If the perfect matching exists, then the bottleneck edge is not an unique bottleneck edge, otherwise it is unique bottleneck.
edge. This is because, the bipartite graph has perfect matching since it is balanced, complete and the maximum weight in every perfect matching of the graph is greater than or equal to the bottleneck weight. Now since the bottleneck edge and the edges with weight greater than bottleneck weight are removed, the perfect matching exist only if maximum weight of the perfect matching is equal to bottleneck weight. Therefore the perfect matching is actually a bottleneck matching of the original graph with different bottleneck edge and the removed bottleneck edge is not unique bottleneck edge. Note that, in a graph corresponding to a random network, weight of two or more different edges to be exactly same is rare as we have tested in the simulations. In case of a graph with non-identical weight edges, minimum bottleneck matching is always available and MBM algorithm can successfully find it since the bottleneck edge obtained for the graph at each iteration is unique bottleneck edge. The MBM algorithm at each iteration run the bottleneck matching algorithm in time $O(N^{w.5})$ [14] and maximum matching algorithm in time $O(N^w)$ where $w < 2.38$ [15]. Therefore, time complexity of the MBM algorithm is $O(MN^{2.5})$.

**Algorithm 1 Minimum bottleneck matching algorithm**

1: Construct graph $G$ with vertex sets $V_G^1 = \{s_1, ..., s_N\}$, $V_G^2 = \{r_1, ..., r_N\}$. Weight of each edge between vertices $s_i, r_j$, $i = 1, ..., M$, $j = 1, ..., N$, $\omega_{(s_i, r_j)}$ is according to [12] and weight of each edge between vertices $s_i, l \in M + 1, ..., N$ (i.e., dummy vertex) and $r_j, \omega_{(s_i, r_j)} = 0$.
2: Initialize acknowledgement metric $I = 1$, $H = G$, a bottleneck matching of $G$ is $B_G$ using [14] and $B_o = \emptyset$.
3: for $i = 1 : M$ do
4: Find bottleneck matching $B_H$ of the graph $H$ and corresponding bottleneck edge $b_H$ using [14].
5: $H' = H - \{b_H, E'\}$ where $E' = \{e \in E_H | \omega_e > \omega_{b_H}\}$
6: Find maximum matching $M^*$ of the graph $H'$. 
7: if $|M^*| = |V_H|/2$ then
8: $B_o = B_G$
9: $I = 0$
10: Break
11: end if
12: $B_o = B_o \cup b_H$
13: $H = H - v_{b_H}$ where $v_{b_H}$ is vertex set of the edge $b_H$.
14: end for

The algorithm can be further explained with the example of how it works for the graph given in Fig[1]. The graph at each iteration of the algorithm has been shown in Fig[2] At the first iteration, the bottleneck matching is either $B_1$ or $B_2$. Therefore in either case bottleneck edge is $(s_1, r_1)$ is to be tested if it is unique bottleneck edge. For this purpose, the edges $(s_1, r_1)$, $(s_1, r_2)$ and $(s_2, r_1)$ are removed from the graph and maximum matching is obtained on the resultant graph. Now, the vertices $s_1, r_1$ of the resultant graph are not connected to any edge. Therefore, the maximum matching on the resultant graph is not a perfect matching and edge $(s_1, r_1)$ is unique bottleneck edge. The edge $(s_1, r_1)$ is stored and the graph is updated by removing the vertices $s_1, r_1$ (and corresponding edges). Using similar steps at second and third iteration, the unique bottleneck edges for the graph for the corresponding iteration are obtained as $(s_2, r_2)$ and $(s_3, r_3)$, respectively. Therefore the minimum bottleneck matching $B_1$ is obtained finally after three successful iterations.

**B. Suboptimal Relay Selection**

Now we propose suboptimal relay selection (SRS) algorithm for the multiple-user network. The low complexity relay selection algorithm in multiple-user network can be solved in two steps: firstly to find a low complexity suboptimal source priority metric, then selecting best relay nodes sequentially according to the order of source priority metric [9]. [16]. Source priority can be found based on weights of source relay pairs. For example, higher priority is given to the source node that has larger weight with its best relay [16] or worst relay [9]. However in these cases, complexity of relay selection are still high. In the proposed SRS algorithm, source priority is decided based on source node average channel condition (distance from the BS) and its residual energy level. To decide the source priority, we use a priority metric $\varphi_{s_i}$. The source node $s_i$ with higher value of $\varphi_{s_i}$ has higher source priority. For GLM and MWTP policies we propose the source priority metric as

$$\varphi_{s_i} = \frac{1}{E_{s_i}} \sigma^2_{s_i,d}, \ s_i \in S \quad (20)$$

i.e., a source node with lower residual energy or located far from BS (i.e., higher power consumption) is given higher priority to select its best relay since the node is more likely to die out early.

**Algorithm 2 Suboptimal relay selection (SRS) algorithm**

1: Let $S = \{s_1, s_2, ..., s_M\}$ and $R = \{r_1, r_2, ..., r_N\}$.
2: Compute the priority metric $\varphi_{s_i}$ for source node $s_i \in S$.
3: Initialize $m_{s_i} = -1$, for $s_i \in S$.
4: while $|S| > 0$ do
5: $s^*_i = \arg \min_{s_i \in S} \varphi_{s_i}$
6: $S = S \setminus s^*_i$
7: for all $r_j \in R$ do
8: Calculate $\omega_{s^*_i, r_j}$ according to the policy.
9: end for
10: $r^*_j = \arg \min_{r_j \in R} \omega_{s^*_i, r_j}$
11: $m_{s^*_i} = r^*_j$
12: $R = R \setminus r^*_j$
13: end while
The proposed SRS algorithm is described in Algorithm 2. According to this algorithm, a source node is chosen from the unpaired source set according to source priority metric and then the relay with minimum source relay weight from the unpaired relay set is found and paired with the source node. This process continues repeatedly until each source node is allocated a relay node. For each source node $s_i$, $m_{s_i}$ denotes its assigned relay. Compared to the suboptimal algorithm proposed in [9] with complexity $O(NM^2)$ which is quadratic in number of source node, our proposed algorithm complexity behaves as $O(MN)$, linear in the number of source node and relay nodes. This is because of the fact that in SRS algorithm, source priority is found with lower complexity.

Therefore, for GLM policy, we have three relay selection and power allocation strategies: GLM BM, GLM MBM and GLM SRS, with time complexity $O(N^{2.5})$, $O(MN^{2.5})$ and $O(MN)$, respectively. The GLM BM, GLM MBM and GLM SRS strategies use bottleneck matching algorithm [14]. Algorithm 1 and Algorithm 2 respectively with weight of each source relay pair according to (12) to obtain the relay assignment in the network and assign the transmit power to each selected source relay pair according to the power allocation solution in (10). Similarly we have MWTP MWM and MWTP SRS strategies with time complexity $O(N^3)$ and $O(MN)$, respectively, for relay selection and power allocation for MWTP policy. Note that, GLM BM and GLM MBM strategies optimally solve (4). Hence these strategies are optimal for network lifetime maximization if relay selection and power allocation is not updated before the network expires.

V. PERFORMANCE EVALUATION

In this section, we present the simulation results to evaluate the network lifetime performance of the proposed strategies. Consider a network with six source nodes. Let the source nodes and relay nodes are distributed randomly in a square area of size $100 \, \text{m} \times 100 \, \text{m}$ and BS is located at $(50, 50)$. Each source node and relay node has initial energy 10 J. The system parameters for all simulations are data rate 10 Kbps, $G_0=70$ dB, $N_0=134$ dBm, $\alpha=3.5$, $\eta = 1$, $K=2$ i.e., BPSK modulation. SER constraint for each source node is $10^{-4}$. The network lifetime is measured in terms of total number of data packet received at the BS since this is equivalent to the time the network is operating. The results are averaged over 300 randomly generated network topologies.

Fig 3 depicts the lifetime performance of the proposed strategies when number of relay nodes in the network varies from 10 to 30 and SER constraint is $10^{-4}$. The relay selection and power allocation for each strategy is periodically updated after all six source nodes transmit 10 packets. Therefore the update interval is $T_u = 60$ packets. As relay number increases, the network lifetime increases for all the strategies. This is because adding more relay nodes in the network increases total energy of the network along with the increase of availability of more efficient relay nodes. The GLM policy based strategies perform better than MWTP policy based strategies and GLM MBM achieves maximum network lifetime. The GLM SRS and MWTP SRS perform close to GLM MBM and MWTP MWM strategies, respectively. The network lifetime of GLM BM is lower compared to the GLM MBM and GLM SRS since GLM BM is not power efficient when $T_u$ is low. With 20 relay nodes, GLM MBM has 1.03 and 1.05 times more network lifetime than GLM SRS and GLM BM, respectively while GLM SRS achieves 1.11, 1.12 times more network lifetime than MWTP MWM, MWTP SRS, respectively.

In Fig 4 we compare the lifetime performance of these strategies for different update interval $T_u$ with six source nodes and twenty relay nodes. As $T_u$ increases, the network lifetime decreases for all strategies. The reason is that with the increase of $T_u$, update of the relay nodes become more infrequent and few relay nodes carry heavier traffic compared to other which results in early die out of these relay nodes. It can be observed that for low $T_u$, the GLM BM strategy has lower network lifetime performance compared to GLM MBM as well as GLM SRS. A cross over occurs at $T_u = 1.7 \times 10^5$ packets, beyond which the GLM BM strategy has better network lifetime performance compared to the GLM SRS strategy. As
discussed in Section IV if the network expires before an update of relay selection and power allocation, GLM BM and GLM MBM are optimal for lifetime maximization and these strategies have same network lifetime performance. This situation is shown in Fig. 4 for \( T_u = 3 \times 10^4 \) packets which is higher than network lifetime for all the strategies.

To further understand the network lifetime performance of these strategies with \( T_u \), comparison of power efficiency and energy balancing in the network with update interval as 60 and \( 3 \times 10^3 \) packets are presented in Fig. 5 and Fig. 6, respectively. Fig. 5 shows the comparison of energy consumption per packet for different strategies with \( T_u \). In Fig. 6 network wasted energy, which is the total remaining energy in the network after network expires, has been compared for different strategies with \( T_u \). While strategies based on GLM policy require higher transmit power compared to MWTP MWM, MWTP SRS, network wasted energy for GLM policy based strategies is significantly low and therefore these strategies achieve higher network lifetime performance than MWTP MWM, MWTP SRS. The transmit energy consumption of GLM BM is high at low \( T_u \) which accounts for its lower lifetime performance compared to GLM MBM, GLM SRS at low \( T_u \).

VI. CONCLUSION

In this paper, we have investigated the relay assignment and power allocation problem for GLM and MWTP policies to maximize lifetime of the SER constrained multiple-user cooperative networks. Optimal power allocation for source relay pair for each policy has been derived. The optimal relay assignment for MWTP and GLM policies have been solved using MWM algorithm with time complexity \( O(N^3) \) and BM algorithm with time complexity \( O(N^{2.5}) \), respectively. Since relay assignment with BM algorithm is not power efficient, we propose MBM algorithm which solves relay assignment optimally in time \( O(MN^2) \) for GLM policy. To reduce the complexity of relay assignment further, a suboptimal relay selection algorithm has been proposed with time complexity \( O(MN) \). Simulation results demonstrate that the strategies based on GLM policy, achieve better network lifetime compared to the strategies based on MWTP policy. The network lifetime has been shown to improve if the relay selection and power allocation of the proposed strategies are updated frequently. The performance of GLM BM strategy is worse as compared to GLM SRS, GLM MBM strategies at low update interval. GLM MBM strategy has the best network lifetime performance and GLM SRS strategy performs close to it.

REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” IEEE Trans. Inform. Theory, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
[2] Y. R. Tsai and L. C. Lin, “Optimal power allocation for decode-and-forward cooperative diversity under an outage performance constraint,” IEEE Commun. Lett., vol. 14, no. 10, pp. 945–947, Oct. 2010.
[3] Z. Zhou, S. Zhou, J.-H. Cui, and S. Cui, “Energy-efficient cooperative communication based on power control and selective single-relay in wireless sensor networks,” IEEE Trans. Wireless Commun., vol. 7, no. 8, pp. 3066–3078, Aug. 2008.
[4] F. Ke, S. Feng, and H. Zhuang, “Relay selection and power allocation for cooperative network based on energy pricing,” IEEE Commun. Lett., vol. 14, no. 5, pp. 396–398, May 2010.
[5] H. Hui, S. Zhu, and G. Lv, “Relay selection for lifetime extension in amplify-and-forward cooperative networks,” in Proc. IEEE Int. Conf. Commun., May 2010, pp. 1–5.
[6] F. Zuo and M. Dong, “Prediction-based energy-aware relay cooperation for lifetime maximization,” IEEE Wireless Commun. Lett., vol. 1, no. 4, pp. 352–355, Aug. 2012.
[7] B. Maham, A. Hjørungnes, and M. Debbah, “Power allocations in minimum-energy SER constrained cooperative networks,” Annals of Telecommun., vol. 64, no. 7, pp. 545–555, Aug. 2009.
[8] M. Hajiaghayi, M. Dong, and B. Liang, “Maximizing lifetime in relay cooperation through energy-aware power allocation,” IEEE Trans. Signal Process., vol. 58, no. 8, pp. 4354–4366, Aug. 2010.
[9] F. Zuo and M. Dong, “Energy-aware relay selection for multiuser relay networks,” in Proc. IEEE Int. Conf. Commun., June 2012, pp. 4621–4625.
[10] A. Ribeiro, X. Cai, and G. B. Giannakis, “Symbol error probabilities for general cooperative links,” IEEE Trans. Wireless Commun., vol. 4, no. 3, pp. 1264–1273, May 2005.
[11] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge Univ. Press, 2004.
[12] C. Papadimitriou and K. Steiglitz, Combinatorial Optimization: Algorithms and Complexity. New York: Dover, 1998.
[13] H. W. Kuhn, “The Hungarian method for the assignment problem,” Naval Research Logistics Quarterly, vol. 2, pp. 83–97, 1955.
[14] A. P. Punnen and K. Nair, “Improved complexity bound for the maximum cardinality bottleneck bipartite matching problem,” Discrete Applied Mathematics, vol. 55, no. 1, pp. 91–93, 1994.
[15] M. Mucha and P. Sankowski, “Maximum matchings via Gaussian elimination,” in Proc. FOCS, Oct. 2004, pp. 248–255.
[16] S. Atapattu, Y. Jing, H. Jiang, and C. Tellambura, “Relay selection and performance analysis in multiple-user networks,” IEEE J. Select. Areas Commun., vol. 31, no. 8, pp. 1517–1529, Aug. 2013.