d-Wave Pairing in an Ensemble of Spin Polaron Quasiparticles in the Spin-Fermion Model of the Electronic Structure of the CuO₂ Plane

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It is demonstrated for the first time that the strong coupling between spin moments of copper ions and oxygen holes, which arises upon hybridization mixing of two hole subsystems in the Emery model, not only affects the formation of spin polaron quasiparticles but also ensures effective attraction between them via the exchange interaction. This results in the Cooper instability with d-wave pairing in a 2D ensemble of spin polaron quasiparticles. The $T-x$-phase diagram obtained using this approach agrees well with the available experimental data.

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1. Introduction

Study of high-temperature superconductors (HTS) revealed the importance of the interactions between charge and spin degrees of freedom \cite{1}. These interactions manifest themselves in the properties of both the normal HTS phase, leading, e. g., to the pseudogap behavior \cite{2}, and the superconducting state, ensuring, in particular, d-type order-parameter symmetry \cite{3}.

Many theoretical works on the spin-fermion interactions were based on the Hubbard model \cite{4, 6} and $t - J$- and $t - J^*$- models \cite{7, 10} in which both the charge and spin subsystems were formed by the same electrons.

Meanwhile, in the real structure of the HTS’s CuO\textsubscript{2}-plane, the spin moments of copper ions and holes that move over oxygen ions are spatially separated. In addition, the presence of two oxygen ions in the unit cell causes a number of features.

As is known, the multi-band character of the Emery model \cite{11, 12} makes the theoretical consideration of the Copper instability extremely intricate. Therefore, the conditions for implementation of the superconducting d-phase with regard to the above-mentioned features of the CuO\textsubscript{2}-plane structure require further investigations.

The aim of this study was to develop the theory of the 2D superconducting phase with d-type order-parameter symmetry that would be free of the above-said limitations. It is important that the conditions for implementation of the Copper instability will be determined for an ensemble of the spin polaron quasiparticles formed by the strong coupling between the spin moments of copper ions and oxygen holes.

2. Hamiltonian of the spin-fermion model of the CuO\textsubscript{2}-plane

As is known, smallness of parameter $t_{pd}$ of mixing between the p-states of oxygen ions and d-states of copper ions as compared with the difference $\Delta_{pd} = \varepsilon_p - \varepsilon_d$ between the energies of these states in the strong correlation regime allows to obtain the SU(2)-invariant spin-fermion model \cite{14, 15}:

\begin{equation}
\hat{H} = \varepsilon_p \sum_{l, \rho} c_{l \rho}^+ c_{l \rho} - t \sum_{l, \rho, \rho'} c_{l \rho}^+ c_{l+\rho \rho'} + \\
+ \sum_{f, \delta, \delta'} c_{f \delta}^+ \left[ \frac{\tau}{2} + \tilde{S}_f \tau_+ \right] c_{f+\delta'} + \hat{H}_{exch},
\end{equation}

where

$\tau_{\pm} = \tau(1 \pm \eta), \quad \tau = \left( \frac{t_{pd}}{\Delta_{pd}} \right)^2, \quad \eta = \frac{\Delta_{pd}}{U_d - \Delta_{pd}}, \quad \tilde{S}_f = \tilde{S}_f \tilde{\sigma}.$

The first term in Eq. (1) describes the energy of coupling between a doped hole and an oxygen ion. Hereinafter, energy $\varepsilon_p$ is assumed to be counted from chemical potential $\mu$. The operator $c_{l \rho}^+$ is the spinor representation describes creation of a hole on an oxygen ion with site number $l$.

The second term in $\hat{H}$ corresponds to direct hoppings of holes between the nearest oxygen ions coupled by vectors $\rho$. The intensity of these hoppings is determined by the tunneling integral $t > 0$. Hereinafter, hybridization parameter $t_{pd}$ is assumed to exceed the tunneling integral, $t_{pd} > t$.

The third term in (1) is caused by the account for the second-order processes by hybridization parameter $t_{pd}$. The arising operator describes hoppings of a hole between oxygen ions adjacent to the copper ion. Operator $\tilde{S}_f$ is determined as a scalar product of vector operator $\tilde{S}_f$ of the spin moment on the copper ion in the site with index $f$ and the vector $\tilde{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$ that consists of the Pauli matrices. The most important feature of this operator is that it contains contributions of hoppings of holes accompanied by the spin-flip processes. At such hoppings, spin projections of both the hole and the copper ion change. The account for these contributions strongly affects the structure of the spin polaron elementary excitation spectrum. Vectors $\delta$ and $\delta'$ independently acquire the four values $\{ \pm a_x, \pm a_y \} = \{ \pm g_x, \pm g_y \},$ where
\( \{ \pm g_x, \pm g_y \} \) are the vectors of the nearest neighbors of the copper lattice.

The last term in (11) describes the exchange interaction between spins of copper ions. Hereinafter, we limit the consideration to the interactions of the spins located within two coordination spheres:

\[
\hat{H}_{\text{exch}} = \frac{I_1}{2} \sum_{fg} \vec{s}_f \cdot \vec{s}_{f+g} + \frac{I_2}{2} \sum_{fd} \vec{s}_f \cdot \vec{s}_{f+d}.
\] (2)

Here, \( I_1 \) and \( I_2 \) are the exchange integrals for the nearest and next-nearest \((d = \pm g_x, \pm g_y)\) spins, respectively. The exchange constants are convenient to express via frustration parameter \( p \) and effective exchange integral \( I \):

\[
I_1 = (1 - p)I, \quad I_2 = pI, \quad 0 \leq p \leq 1, \quad I > 0.
\] (3)

Quantity \( p \) can be related to concentration of holes \( x \) per copper atom [16].

Note that in reality the hopping integrals in the first and second terms of Hamiltonian (11) can have different signs for different hopping directions. It is easy to demonstrate that these signs can be taken into account by introducing the factors \( \exp\{iQ(l - l')\} \), where \( Q = (\pi, \pi) \). After the unitary transformation \( e^{iQl}c_l \rightarrow c_l \), these factors vanish and the spectrum can be reconstructed by the shift \( k \rightarrow k + Q \) in the \( k \)-space.

Below we use the following commonly accepted parameter values: \( \varepsilon_{pd} = 1.3 \) eV, \( \Delta_{pd} = 3.6 \) eV, \( U_d = 10.5 \) eV, and \( t = 0.1 \) eV [17]. For these values, we have \( \tau = 0.47 \) eV and \( \eta = 0.52 \).

3. Fermi quasiparticles in the strong coupling regime.

To clarify the nature of the Fermi quasiparticles that arise in the CuO_2-plane upon light doping, we consider the solution of the Schrödinger equation for one hole by the variational technique. We will take into account that, in accordance with the Mermin-Wagner theorem [18], without doping at an arbitrarily low temperature the 2D subsystem of the localized spin moments is in the state \( |G\rangle \) without the long-range magnetic order. At the antiferromagnetic exchange interaction, this state is characterized by the properties [19]:

\[
\bar{\mathcal{S}}^2_{\text{tot}}(G) = 0|G\rangle, \quad \langle G|\bar{\mathcal{S}}^2_{j,y,z}|G\rangle = 0, \quad \bar{\mathcal{S}}_{\text{tot}} = \sum_j \bar{\mathcal{S}}_j. \] (4)

The assumption about the singlet character of the state of the considered 2D system at a finite temperature is based on the results reported in [20], where it was mathematically strictly demonstrated that the ground state of a system of an arbitrarily large yet finite number of localized spins in the sites of the square lattice that antiferromagnetically interact with one another is singlet (the Marshall theorem).

Taking into account the symmetrical properties of the Hamiltonian, we obtain that for each irreducible representation \( k \) of the translations group, the state with one hole \( |\psi_{k\sigma}\rangle \) with spin moment projection \( \sigma \) can be written as:

\[
|\psi_{k\sigma}\rangle = \sum_j \alpha_{jk} A^+_{jk\sigma}|G\rangle,
\] (5)

where \( A^+_{jk\sigma} \) are, defined below, basis operators.

According to the stationarity condition of the energy functional under the additional condition \( \langle \psi_{k\sigma}|\psi_{k\sigma}\rangle = 1 \), with the use of the Lagrange technique, we obtain that the excitation energies \( \varepsilon_k = E_k - EG \), where \( E_k \) and \( E_G \) are the energies of the \( |\psi_{k\sigma}\rangle \) and \( |G\rangle \) states, respectively, and coefficients \( \alpha_{jk} \) are determined by the system of the linear homogeneous equations:

\[
\sum_j \left[ D_{ij}(k) - \varepsilon_k K_{ij}(k) \right] \alpha_{jk} = 0,
\] (6)

where

\[
D_{ij}(k) = \langle G|\left[ A_{ik\sigma}, \hat{H} \right]|G\rangle, \quad K_{ij}(k) = \langle G|\left[ A_{ik\sigma}, A^+_{jk\sigma} \right]|G\rangle.
\] (7)

The numerical calculations showed that the states of the one-hole sector can be described optimally, in terms of reaching the lowest energy at the minimum set of basis operators, when we limit the consideration to the three families of operators:

\[
A_{1(2)f\sigma} = c_{f+a_{x(y),\sigma}}, \quad A_{3f\sigma} = \frac{1}{2} \sum_{\delta} \left( \bar{\mathcal{S}}_f c_{f+\delta} \right)_{\sigma},
\] (9)

used in building the operators in the quasimomentum representation

\[
A_{jk\sigma} = N^{-1/2} \sum_f e^{-ikf} A_{jf\sigma}, \quad (j = 1, 2, 3).
\]

Note that the further increase in the number of basis operators almost does not affect the spin polaron branch of the spectrum. The calculations yield (\( K_{ij} = \delta_{ij} K_{ii} \)):

\[
K_{11}(k) = K_{22}(k) = 1, \quad K_{33}(k) = \frac{3}{4} + C_1 \gamma_1(k),
\] (10)

\[
D_{11(22)} = \varepsilon_\sigma + \tau_+ \left( 1 + \cos k_{x(y)} \right),
\]

\[
D_{12} = D_{21} = \left( \frac{T_\tau - t}{2} - \varepsilon_\sigma \right) \left( 1 + \cos k_{x(y)} \right),
\]

\[
D_{13} = D_{31} = 2\tau_+ K_{33} \left( 1 + e^{ik_{x(y)}} \right),
\]

\[
D_{33} = \left( \varepsilon_\sigma - 2t + \frac{5}{2} \tau_+ - 4\tau_- \right) K_{33} + \left( \tau_+ - 2t \right) \left( C_1 \gamma_{1k} + C_2 \gamma_{2k} \right) + \left( \tau_+ - 2t \right) \left( C_1 \gamma_{1k} + C_3 \gamma_{3k} \right) + \tau_+ C_1 (1 - 4\gamma_{1k}) + \left( I_1 C_1 (1 + 4\gamma_{1k}) - 4I_2 C_2. \right)
\] (11)

Here, \( \gamma_{jk} \) \((j = 1, 2, 3)\) are the square lattice invariants: \( \gamma_{1k} = (\cos k_x + \cos k_y)/2, \gamma_{2k} = \cos k_x \cos k_y, \gamma_{3k} = (\cos 2k_x + \cos 2k_y)/2. \)
Spin correlators $C_1$, $C_2$ and $C_3$ are averaged by the $|G\rangle$ state of the SU(2)-invariant products of spin operators corresponding to different cells: $C_j = \langle G|S_j^x S_{j+r}^x|G\rangle$, where $r_j$ is the radius of the $j$-th coordination sphere. The SU(2) invariance of the $|G\rangle$ state leads to the equality:

$$C_j = 3\langle S_j^x S_{j+r}^x \rangle = 3\langle S_j^y S_{j+r}^y \rangle = 3\langle S_j^z S_{j+r}^z \rangle. \quad (12)$$

As can be seen from the results of the numerical calculation (Figs. 1 and 2), it is important to take into account the interaction between the spin and charge degrees of freedom, as well as the spin polaron character of the lower branch of the one-hole state spectrum. The left part in Fig. 1 shows the one-hole state energy spectrum obtained with the use of the only two operators, $A_{1k\sigma}$ and $A_{2k\sigma}$, for the quasimomenta located on the main diagonal of the Brillouin zone. These branches describe, in fact, the spectrum of holes that do not interact with the subsystem of the spin moments of copper ions.

Addition of the third operator, $A_{3k\sigma}$, to the variational procedure leads to important qualitative transformations, which is demonstrated by the one-hole state spectrum obtained in the basis of three operators (Fig. 1 on the right). The main difference is the arising split branch with the minimum near $(\pi/2, \pi/2)$. The energy of such one-hole states lowers due to the Hamiltonian term $\sim \tau_\perp$, which describes both the exchange interaction between the hole and the nearest copper ions and the spin-correlated hoppings. Inclusion in the operator basis of the operators that explicitly take into account this strong spin-fermion correlation yields a significant energy gain. In addition, this is accompanied by renormalization of the two bare branches of the spectrum.

![FIG. 1: One-hole state energies $\varepsilon_{jk}$ vs quasimomentum along the main diagonal of the Brillouin zone for the parameters $\tau = 0.47$ eV, $\eta = 0.52$, $t = 0.1$ eV, and $I = 0.2\tau$. (a) Energy spectrum obtained using two operators $A_{1k\sigma}$ and $A_{2k\sigma}$, and (b) one-hole state energies calculated in the basis of three operators $A_{1k\sigma}$, $A_{2k\sigma}$ and $A_{3k\sigma}$. The lower branch of the spectrum corresponds to the spin polaron states.](image)

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It should be noted that, physically, the occurrence of the spin polaron states is analogous to their occurrence in the exactly solved problem on one electron with the spin flip in a ferromagnetic matrix at the antiferromagnetic s-d exchange coupling between the electron spin and the localized spin moment $|21\rangle$.

The above-mentioned qualitative modifications in the energy spectrum are kept at the quasimomentum variation in other directions of the Brillouin zone.

![FIG. 2: Partial contributions of the basis states to the one-hole state corresponding to the lower branch of the spectrum in Fig. 1. The model parameters are as in Fig. 1. $\Gamma = (0, 0)$, $M = (\pi, \pi)$, $X = (0, \pi)$, and $X' = (\pi, 0)$.](image)

FIG. 2: Partial contributions of the basis states to the one-hole state corresponding to the lower branch of the spectrum in Fig. 1. The model parameters are as in Fig. 1. $\Gamma = (0, 0)$, $M = (\pi, \pi)$, $X = (0, \pi)$, and $X' = (\pi, 0)$.

Let us consider the structure of the one-hole state illustrated by the lower branch of the spectrum in Fig. 1b. Weight contributions $P_{1k}$ and $P_{2k}$ of bare hole states $A^+_{1k\sigma}(G)$ and $A^+_{2k\sigma}(G)$ are determined as $P_{1k} = |\alpha_{1k}|^2$ and $P_{2k} = |\alpha_{2k}|^2$. The weight contribution of the spin polaron basis state is $P_{3k} = K_{33} |\alpha_{3k}|^2$.

Figure 2 shows the partial contributions for the quasimomenta lying on the four directions of the Brillouin zone. It can be seen that $P_{3k}$ (upper curves) multiply exceeds $P_{1k}$ and $P_{2k}$, which proves the spin polaron nature of the one-hole state corresponding to the lower split branch of the spectrum.

The dependencies presented in Fig. 1 were calculated with the use of the dispersion equation

$$\det_k(\omega) = |D(k) - \omega K(k)| = 0, \quad (13)$$

obtained from the condition of nontriviality of the solutions of system (14). Developing the determinant, we arrive at:

$$\det_k(\omega) = (\omega - \varepsilon_p)^3 - Q_k (\omega - \varepsilon_p)^2 + B_k (\omega - \varepsilon_p) + R_k,$$

where we made the following designations:

$$Q_k = 2\tau(1 + \gamma_1 + \Lambda_k),$$
$$B_k = (2\tau - \Lambda - 16\tau_+ K_{33}^2) (1 + \gamma_1 + 4t (\tau_+ - t) \chi_k),$$
$$R_k = 4t\chi_k [8\tau_+^2 K_{33} - \Lambda_k (\tau_+ - t)] ,$$
$$\Lambda_k = \frac{P_{33}}{K_{33}} - \varepsilon_p, \quad \chi_k = 1 + 2\gamma_1 + \gamma_2 k. \quad (14)$$
For the set of parameters used, it is easy to obtain the approximate solutions of the dispersion equation that describe the one-hole state spectrum with high accuracy. In this case, the spin polaron spectrum is determined as:

$$
\epsilon_{sp}(k) = \epsilon_p + x_k, \quad x_k = \frac{Q_k}{2} - \sqrt{\frac{Q_k^2}{4} - B_k - \frac{R_k}{x_{k0}}} \tag{15}
$$

where \(x_{k0} = Q_k/2 - \sqrt{Q_k^2/4 - B_k}\). For the two upper branches, obtain:

$$
\epsilon_{1k} = \epsilon_p + \frac{Q_k - x_k}{2} - \nu_k, \quad \epsilon_{2k} = \epsilon_p + \frac{Q_k - x_k}{2} + \nu_k,
\nu_k = \sqrt{(Q_k - x_k)^2/4 + R_k/x_k}. \tag{16}
$$

4. Equations of motion for the spin polaron Green’s functions in the superconducting phase

Developing of the theory of the superconducting state in a system of oxygen holes strongly coupled with the subsystem of the localized spin moments of copper ions suggests expanding of the set of basis operators that would allow introducing anomalous averages. Taking into account the results reported in Section 3, we can easily see that the system problem can be solved by adding operators \(\hat{G}_{1-k,\sigma}, \hat{G}_{2-k,\bar{\sigma}}\) and \(\hat{A}_{3-k,\bar{\sigma}}\) to operators \(\hat{A}_{1k\sigma}, \hat{A}_{2k\sigma}, \hat{A}_{3k\sigma}\) used previously.

To obtain self-consistency equations in the superconducting phase, we use the introduced basis of six operators and apply the Zwanzig–Mori technique [1, 2, 22, 23].

Let us introduce the retarded two-time temperature Green’s functions (GPs) \((i, j = 1, \ldots, 6)\):

$$
G_{ij}(k, t) = \langle \langle A_{ik\sigma}(t) | A_{jk\sigma}^+(0) \rangle \rangle = -i\theta(t) \langle \langle A_{ik\sigma}(t) | A_{jk\sigma}^+(0) \rangle \rangle. \tag{17}
$$

The system of 6 × 6 equations of motion for these GPs is:

$$
\omega \langle \langle A_{ik\sigma} | A_{jk\sigma}^+ \rangle \rangle = K_{ij}(k) + \langle \langle A_{ik\sigma} , \hat{G} \rangle | A_{jk\sigma}^+ \rangle \rangle, \quad i, j = 1, \ldots, 3, \tag{18}
$$

where the free term \(K_{ij}(k) = \langle \langle A_{ik\sigma} , A_{jk\sigma}^+ \rangle \rangle\) is determined analogously to expression (3), but with averaging over the ensemble of spin polaron quasiparticles. The aforesaid is valid also to the matrix elements \(D_{ij}(k)\) of expressions (11) and (11).

In the projection technique, the Green’s functions obtained by the commutation \([A_{ik\sigma}, \hat{G}]\) are written as the linear superposition of basis GPs [17], i.e.:

$$
\langle \langle A_{ik\sigma}, \hat{G} | A_{jk\sigma}^+ \rangle \rangle = \sum_l L_{il}(k) \langle \langle A_{ik\sigma} | A_{jk\sigma}^+ \rangle \rangle, \tag{19}
$$

where \(L(k) = \Gamma(k)K^{-1}(k)\). The obtained system of equations for the GPs \(\langle \langle A_{ik\sigma} | A_{jk\sigma}^+ \rangle \rangle\) is closed and can be presented, for the sake of brevity, in the matrix form:

$$
\left(\omega \cdot \hat{I} - D(k)K^{-1}(k)\right) G(k, \omega) = K(k), \tag{20}
$$

where \(\hat{I}\) is the unit matrix. Then, the energy spectrum of quasiparticles \(E_{jk}\) in the superconducting phase is determined by poles of GP \(G\) and can be obtained from the six-order dispersion equation:

$$
det \left| \omega \cdot \hat{I} - D(k)K^{-1}(k) \right| = 0. \tag{21}
$$

Matrix elements \(K_{ij}(k)\) and \(D_{ij}(k)\) with the indices \(i, j = 1, \ldots, 3\) are known (see (10) and (11)). Matrix \(K\) is still diagonal, and besides \(K_{j+3,j+3}(k) = K_{jk}(k)\) for all \(i, j = 1, 2, 3\). Matrix \(D(k)\) is convenient to be written in the block representation. The left upper block \(3 \times 3\) in size is composed only from the normal averages \(D_{ij}(k)\) for all \(i, j = 1, 2, 3\).

Furthermore \(D_{i+3,j+3}(k) = -D_{ij}(k)\), and Eq. (22) reduces to Eq. (21).

5. Self-consistency equation for the superconducting order parameter

Anomalous average \(D_{36}(k)\) is expressed via a sum of a large number of terms that can yield the solutions of the integral self-consistency equation with different types of symmetry of the superconducting order parameter (SOP). In particular, the terms proportional to parameter \(\tau_s\) yield s-wave pairing. In view of the experimental data, we limit the consideration to the d-type SOP. In this case, we may use the truncated expression:

$$
D_{36}(k) = I_1 \sum_\delta e^{ik_2\delta} \left[ -\langle \langle A_{6f,\sigma}^+ A_{3f,2\delta,\sigma} \rangle \rangle + \frac{C_k}{4} \sum_{\delta'_{\delta_1}} \langle \langle C_{f+2\delta+\delta_1, \sigma} \rangle \rangle \langle \langle C_{f+\delta', \sigma} \rangle \rangle \right]. \tag{24}
$$
We obtained this expression by applying the decoupling procedure to the averages with the products of operators that cannot be reduced to the basis operators. This causes the occurrence of magnetic correlator $C_1$ in front of the second term expressed via a sum of the averages of the introduced basis operators.

From expression (24), obtain

$$D_{36}(k) = \Delta_0 (\cos k_x - \cos k_y). \quad (25)$$

The amplitude of SOP $\Delta_0$ is determined from the equation:

$$1 = \frac{I}{N} \sum_k \frac{(\cos k_x - \cos k_y)^2}{2E_k (E_k^2 - \epsilon_{1k}^2) (E_k^2 - \epsilon_{2k}^2)} \tanh \left( \frac{E_k}{2T} \right) \times [\varphi_k(E_k) \varphi_k(-E_k) - 16C_1 r_s^2 \Psi_k(E_k) \Psi_k(-E_k)], \quad (26)$$

where

$$\Psi_k(\omega) = (\omega - \varepsilon_p) (1 + \gamma_1(k)) - 2t\chi(k), \quad (27)$$

and the Fermi excitation spectrum $E_k$ in the superconducting phase is:

$$E_k = \sqrt{\epsilon_{sp}^2(k) + D_{36}^2(k)}. \quad (28)$$

This formula demonstrates that in the superconducting phase the Fermi excitation spectrum is based on the spectrum of the spin polaron states. Therefore, we may state that the investigated Cooper instability describes the instability of the spin polaron ensemble relative to the Cooper pairing.

6. Effect of doping on the Cooper instability of the spin polaron ensemble

The concentration dependence of the critical temperature of the transition to the superconducting phase with the d-type order-parameter symmetry was calculated using expression (26) with regard to the equation for the chemical potential

$$\frac{x}{4} = \frac{1}{N} \sum_k J_k(E_k)f(E_k/T) - J_k(-E_k)f(-E_k/T) \times 2E_k \left( E_k^2 - \epsilon_{1k}^2 \right) \left( E_k^2 - \epsilon_{2k}^2 \right), \quad (29)$$

where the function $J_k(\omega)$ is expressed as

$$J_k(\omega) = \left( -\omega + \varepsilon_p + \tau_-(1 + \gamma_1k) \right) \varphi_k(\omega) \frac{\left| D_{36}(k) \right|^2}{K_{33}(k)} - \left[ (\omega - \varepsilon_p - \tau_-(1 + \gamma_1k)) (\omega - D_{33}(k)/K_{33}(k)) - 8\tau_2^2 K_{33}(k)(1 + \gamma_1k) \left| \text{det}_k(-\omega) \right| \right] f(z) = \frac{1}{\exp(z) + 1} \text{is the Fermi–Dirac function.} \quad (30)$$

In the numerical calculations, it was taken into account that $K_{ij}$ and $D_{ij}$ depend on spin correlation functions $C_j$ for the first three coordination spheres $j = 1, 2, 3$.

These correlation functions, as well as the gap in the magnetic excitation spectrum in the vicinity of point $Q$ of the Brillouin zone were jointly determined within the spherical symmetric self-consistent approach for a frustrated ferromagnet [19]. Determination of the spin correlation functions with the use of the model considered here was described in detail in [23].

The change in hole concentration $x$ was taken into account via both modification of spin correlators $C_j$ and shifting of chemical potential $\mu$.

The calculated data are presented in Fig. 3. It can be seen that the region of implementation of the superconducting phase with the d-type order-parameter symmetry depends on the exchange interaction value. This results from the fact that the Cooper instability manifests itself in the ensemble of quasiparticles that are spin polarons. Thus, the exchange interaction between spins of copper ions induces attraction between these complex Fermi quasiparticles. Note that for the characteristic values of the exchange integral, the obtained phase diagram is in good agreement with the available experimental data on both hole concentration $x$ and critical temperature $T_c$.

Figure 4 shows the gap variation in the spectrum of elementary excitations of spin polaron quasiparticles on the Fermi contour in the superconducting phase. It can be seen that the dependence of the gap on the quasimomentum in the first Brillouin zone is characterized by the d-type symmetry.

Figure 4 presents concentration dependences of order parameter $|D_{36}(k)|$ and critical temperature $T_c$. As follows from this figure, the amplitude of the order parameter turns to zero at approaching $T_c$ by the second-order phase transition.

8. Conclusions

In this study, we obtained the following results.

(i) Using the spin-fermion model that takes into account the strong correlation between charge and spin degrees of freedom and the real structure of the CuO$_2$-plane with two oxygen ions per unit cell, it was demonstrated...
for the first time that with a decrease in temperature the ensemble of spin polaron quasiparticles passes to the superconducting state with the d-type symmetry of the order parameter.

(ii) The mechanism ensuring Cooper pairing of spin polarons is the exchange interaction, which transforms to effective attraction between spin polarons as a result of the strong spin-charge coupling. In this regard it should be emphasized the important role of the spin-flip processes in the formation of the Cooper instability.

(iii) Despite the complexity of the initial self-consistency equations, simple analytical expressions were obtained for both the spin polaron spectrum in the normal phase and the Fermi excitation spectrum in the superconducting phase.

(iv) The $T-x$-diagram obtained using the spin polaron concept is in good agreement with the experimental data for copper oxides.

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