An Adaptive Predictive Control Method Based on State-space Model

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Abstract. An adaptive state-space model predictive control strategy is proposed for complex industrial processes with nonlinear, time-varying and constrained characteristics. The state-space model obtained by on-line identification algorithm is used as the system model, and the indirect form is used to design the adaptive predictive controller. The controller includes quadratic programming solution to the constraint problem. The effectiveness of the proposed control strategy is verified by the simulation experiment of 2-CSTR process control.

1. Introduction
In recent decades, Model Predictive Control (MPC) has attracted much attention in the field of control theory. It has become a major advanced control method and has been widely used in industrial processes. Traditional industrial predictive control adopts input-output model, but in order to further improve the control performance, academia and industry generally believe that state-space model should be adopted. In the early stage, due to the limitation of identification algorithm, it is difficult to obtain an accurate state-space model for complex industrial processes. However, with the further study of identification algorithms and the deepening understanding of multivariable control systems, MPC technology based on state-space model has gradually emerged in engineering applications and greatly surpasses the traditional MPC technology based on input-output model in performance[1].

The general predictive control method uses a fixed linear model to design the controller, but in the actual industrial process, there are strong nonlinearity and time-varying characteristics, which makes the control effect of this method not ideal. The existence of time-varying characteristics in the process is an important reason for online updating of the system model.

The ability to handle physical constraints is an important feature of MPC. In the production process of petroleum and chemical industry, in order to meet the requirements of product purity and operation feasibility, there are generally cases with constraints.

Therefore, this paper proposes an adaptive state-space model predictive control method based on online identification algorithm. The state-space model of the system is obtained by an on-line identification algorithm, and then an indirect adaptive predictive controller is designed. The controller includes the solution of physical constraint problems.

2. On-line Identification Algorithm
The following linear time-invariant systems are considered:

\[
\begin{align*}
x(k+1) &= Ax(k) + Bu(k) + Ke(k) \\
y(k) &= Cx(k) + Du(k) + e(k)
\end{align*}
\]

(1)

(2)
where \( u(k) \in \mathbb{R}^m \) is the input measurement value of the system, \( y(k) \in \mathbb{R}^r \) is the output measurement value of the system and \( x(k) \in \mathbb{R}^s \) is the process state of the system. \( K \) is a stable state Kalman gain and \( e(k) \) is zero mean white noise. \((A,B,C,D)\) are the system matrices of the corresponding dimension.

To obtain \((A,B,C,D)\), firstly, an initial matrix is constructed:
\[
U_0 = I_{(2mi + 2i + 2il + 2j) \times 2l} \quad \text{and} \quad S_0 = 0_{(2mi + 2i + 2il + 2j) \times 2l}
\]
where \( 2i \) is the row block size of the Hankel matrix \( H \) of the input and output data. \( H \) is defined as
\[
H = \begin{bmatrix}
    u(k) & u(k+1) & \cdots & u(k+j-1) \\
    y(k) & y(k+1) & \cdots & y(k+j-1) \\
    u(k+1) & u(k+2) & \cdots & u(k+j) \\
    y(k+1) & u(k+2) & \cdots & y(k+j) \\
    \vdots & \vdots & \ddots & \vdots \\
    u(k+2i-1) & u(k+2i) & \cdots & u(k+2i+j-2) \\
    y(k+2i-1) & u(k+2i) & \cdots & y(k+2i+j-2)
\end{bmatrix}
\]

SVD decomposition can be carried out on \( H \):
\[
H = USV^T = \begin{bmatrix} U_{i1} & U_{i2} \end{bmatrix} \begin{bmatrix} S_{ii} & 0 \\ 0 & 0 \end{bmatrix} V^T \quad (3)
\]

At each sampling time, new input and output vector \( H^+ \) is added to the \( H \), and forgetting factor \( \lambda \) is introduced to reduce the influence of past data on the identification results without deleting the old data. Calculate SVD decomposition of \( H \) on time \( k \):
\[
H_k = U_k S_k V_k^T = \lambda U_{k-1} S_{k-1} H_k^+ \quad (4)
\]

Decomposition equation (4)
\[
U_k S_k = \begin{bmatrix} U_{k11} & U_{k12} \\ U_{k21} & U_{k22} \end{bmatrix} \begin{bmatrix} S_{k11} & 0 \\ 0 & 0 \end{bmatrix} \quad (5)
\]

Calculate SVD decomposition of \( U_{k12} U_{k11} S_{k11} \)
\[
U_{k12} U_{k11} S_{k11} = \begin{bmatrix} U_q^T U_q^+ \\ U_q \end{bmatrix} \begin{bmatrix} S_q & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_q^T \\ V_q \end{bmatrix} \quad (6)
\]

where \( U_q^+ \) is the orthogonal complement space of \( U_q \) row space, i.e. \( \left(U_q^+\right)^T U_q = 0 \). Similarly, there is a similar definition of \( V_q^+ \). The following linear inequalities are used to solve the system state-space matrix \((A,B,C,D)\) at time \( k \). The specific derivation process can be seen in [2].
\[
\begin{bmatrix} U_q^T U_{k12} U_k(m + l + 1: (i + 1)(m + l),:) S_k \\ U_k(m + l + m + 1: (m + l)(l + 1),:) S_k \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_q^T U_{k12} U_k(1: (m + l),:) S_k \\ U_k(m + l + l + m + 1: (m + l)(l + 1),:) S_k \end{bmatrix} \quad (7)
\]

Therefore
\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} U_q^T U_{k12} U_k(m + l + 1: (i + 1)(m + l),:) S_k \\ U_k(m + l + m + 1: (m + l)(l + 1),:) S_k \end{bmatrix}^{-1} \begin{bmatrix} U_q^T U_{k12} U_k(1: (m + l),:) S_k \\ U_k(m + l + l + m + 1: (m + l)(l + m),:) S_k \end{bmatrix} \quad (8)
\]

At the next time \( k + 1 \), new data vectors are added and updated \( H \), thus continuing to solve the state-space model of the system at time \( k + 1 \) and realizing the online update of the system model.
3. Indirect Adaptive Predictive Control

Adaptive predictive control has been successfully applied in industrial processes. Adaptive predictive control can be divided into direct adaptive predictive control and indirect adaptive predictive control. Indirect adaptive predictive control needs to obtain the system model first, and then design the predictive controller.

Typical process constraints are as follows:

\[
\begin{align*}
    u_{\text{min}} & \leq u & \leq u_{\text{max}} \\
    \Delta u_{\text{min}} & \leq \Delta u & \leq \Delta u_{\text{max}} \\
    y_{\text{min}} & \leq y & \leq y_{\text{max}} \\
    \Delta y_{\text{min}} & \leq \Delta y & \leq \Delta y_{\text{max}}
\end{align*}
\]  

(9)

Set \( F_m = [I_n \ I_u \ I_u \ I_u]^T \), \( F_i = [I_i \ I_i \ I_i]^T \), Equation (9) can be changed into the form of a predictive controller:

\[
\begin{align*}
    F_m u_{\text{min}} & \leq U_f & \leq F_m u_{\text{max}} \\
    F_m \Delta u_{\text{min}} & \leq \Delta U_f & \leq F_m \Delta u_{\text{max}} \\
    F_i y_{\text{min}} & \leq Y_f & \leq F_i y_{\text{max}} \\
    F_i \Delta y_{\text{min}} & \leq \Delta Y_f & \leq F_i \Delta y_{\text{max}}
\end{align*}
\]  

(10)

where \( f \) is the future action moment. The control objective is to optimize the objective function through future control variable \( \Delta U_f \). First of all, the constraints of \( \Delta U_f \) are determined. These constraints can be combined into the form of a matrix inequality:

\[
\begin{bmatrix}
    T_m \\
    -T_m \\
    I_m \\
    -I_m
\end{bmatrix}
\begin{bmatrix}
    \Delta U_f
\end{bmatrix}
\leq
\begin{bmatrix}
    F_m u_{\text{max}} - F_m u_k \\
    F_m u_k - F_m u_{\text{min}} \\
    F_m \Delta u_{\text{max}} \\
    -F_m \Delta u_{\text{min}}
\end{bmatrix}
\]  

(11)

where \( T_m \) and \( T_i \) are subspace predictor matrix, which can be obtained by solving the least squares problem:

\[
\begin{bmatrix}
    I_n \ 0 \ \cdots \ 0 \\
    I_n \ I_n \ \cdots \ 0 \\
    \vdots \ \vdots \ \cdots \ \vdots \\
    I_n \ I_n \ I_n \ I_n
\end{bmatrix}
\begin{bmatrix}
    I_f \ 0 \ \cdots \ 0 \\
    I_f \ I_f \ \cdots \ 0 \\
    \vdots \ \vdots \ \cdots \ \vdots \\
    I_f \ I_f \ I_f \ I_f
\end{bmatrix}
\begin{bmatrix}
    \Delta y_{k-1} \\
    \Delta y_k \\
    \Delta y_{k-1} \\
    \Delta y_{k+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \Delta y_{k-1} \Delta y_{k} \\
    \Delta y_{k} \Delta y_{k+1}
\end{bmatrix}
\]

(12)

where \( \Delta Y_f \) is the future output variable. The following cost function is used:

\[
J = \frac{1}{2} \Delta U_f^T h \Delta U_f + \Delta U_f^T f
\]

(13)
where \( h = (T^TL)Q(T^TL) + R \), \( f = (T^TL)Q(T^L \Delta W + F, y(k) - Y_d) \), \( Q \) and \( R \) are weighted matrices and \( Y_d \) is reference outputs. Equation (11) may be expressed as

\[
A\Delta U_f \leq b
\]  

Inequality (14) can be solved by quadratic programming (QP) algorithm to get \( \Delta U_f \), and the first row \( \Delta u(k) \) of \( \Delta U_f \) is added to the current input \( u(k) \) to obtain a new control input \( u(k+1) \). Measure the new model output \( y(k+1) \), update \( \Delta W_f \), and then re-solve the control input.

4. Simulation and Result Analysis
The 2-CSTR system is a complex process object with nonlinear, time-varying and constrained characteristics. The detailed nonlinear differential equation of the system can be seen in [3-4]. The control objective is to keep the temperature in the two reactors at the set value. Input \( u = [Q_{cw1}, Q_{cw2}]^T \), which represents the two inflow coolant flows and output \( y = [T_{1t}, T_{2t}]^T \) represents the temperatures inside the two reactors. The system input meets the following physical constraints:

\[
0.1 \leq Q_{cw1} \leq 0.8 \text{[m}^3/\text{s]} \\
0.1 \leq Q_{cw2} \leq 0.8 \text{[m}^3/\text{s]} 
\]  

The parameters of adaptive state-space model predictive controller (ASMPC) are as follows: predictive time domain \( P = 10 \), control time domain \( M = 5 \), weighting matrix \( Q = I_{20} \), and \( R = I_{10} \). The sampling length is set to 2000 and the sampling time is 0.1s. For comparison, the PID controller is introduced to control the system. In the comparison of output \( T_{ct} \) tracking effect, it can be seen from Fig. 1 that ASMPC shows good control characteristics and tracking effect is obviously better than PID.

![Fig.1 The comparison of output tracking performance](image)

5. Conclusion
In this paper, an indirect adaptive state-space model predictive controller is designed. The real-time state-space model of the system is obtained through on-line identification algorithm to achieve the purpose of self-adaptation. Then the predictive control method under constraints is used to control the system. The controller is successfully applied to the process simulation test of the 2-CSTR system, and the simulation results verify the effectiveness of the controller.
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