Influence of sensor tilts on bio-inspired polarized skylight orientation determination

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1. INTRODUCTION

Traditional navigation systems such as Inertial navigation system, Global Position System (GPS) and Geomagnetic Navigation System (GNS) play a key role in navigation for aircraft, robots, missiles, vehicles and so on. Inertial navigation system has many advantages, whereas the gyroscopes and accelerometers are usually prone to drifts and noises, which may cause errors to accumulate over time [1]. In addition, although GPS is a real-time and cheap locating system, GPS signal can be easily jammed due to the presence of disturbances [2, 3]. Besides, GNS is sensitive to electromagnetic interference [4]. With the fast development of human society, there is an urgent need to design a highly precise, autonomous, reliable and robust navigation system.

Animals' navigation behavior provides us with a new idea about navigation. Desert ants rely on the predictable pattern of polarized light in the sky to find their way back home in hostile environments [5, 6]. Honey bees are able to detect the polarization of skylight to journey from hives [7, 8]. Birds may use skylight polarization patterns to calibrate magnetic compasses during longer-range migratory [9]. Dorsal rim area (DRA) of locusts’ compound eyes is sensitive to the polarized skylight, thus can estimate its orientation [10, 11].

The reason why these animals can detect polarized light as a compass is that there is a polarization pattern in the sky [12, 13]. Unpolarized sunlight through the Earth's atmosphere produces the skylight polarization pattern [14-16]. Sunlight remains unpolarized until interacting with the atmospheric constituents, scattering sunlight causes a partial linear pattern of polarization in the sky, which can be well described by Rayleigh sky model [17-19].

Inspired by animals’ polarization navigation behavior, several orientation determination methods have been proposed based on Rayleigh sky model. Polarization orientation determination methods mainly include the following four typical approaches: Zenith approach, SM-ASM (solar meridian and anti-solar meridian) approach, Symmetry approach and Least-square approach. The heading angle is determined by measuring the angle of polarization (AOP) at the sky zenith, which is named Zenith approach [3, 14, 20-22]. The polarization E-vector along SM-ASM is consistently perpendicular to SM-ASM, so the heading angle can be calculated by extracting SM-ASM, which is named SM-ASM approach [23-26]. Because of the symmetry of the skylight polarization pattern, symmetry detection can be used to determine orientation, which is named Symmetry approach [27-29]. Polarization E-vector of Rayleigh sky model is consistently perpendicular to the solar vector, so the orientation can be determined by total least square method, which is then named Least-square approach [30-33]. However, all most of
these heading determination approaches require the polarization sensor point toward the zenith of sky dome [34]: Zenith approach needs to directly capture the polarization information at the sky zenith; SM-ASM and Symmetry approaches require that reference direction of AOP is converted to the local meridian using the sky zenith as a reference point; Least-square approach requires a known sky zenith dependent coordinate system to accurately determine orientation.

In actual navigation, the carriers moving in three-dimensional space such as aerial vehicles, aircraft, and rockets will tilt. Even the carriers moving on the ground such as vehicles and multi-legged robots will tilt when the ground is uneven [35, 36]. So, the influence of polarization sensor tilts on bio-inspired polarized sunlight heading determination urgently needs to be investigated and discussed in detail [13].

To summarize, this paper aims to make a profound study on the influence of sensor tilts for polarization orientation determination. Firstly, a polarization compass simulation system is designed. Secondly, based on this simulation system, numerical simulation experiments are carried out to investigate the influence of sensor tilt on the above four classical heading determination approaches. Finally, the results of field experiments are compared with digital simulation to further verify our conclusions.

2. POLARIZATION COMPASS SIMULATION SYSTEM

To investigate the influence of sensor tilts on orientation determination, a polarization compass simulation system is designed, which is shown in Fig. 1.

Fig. 1. Polarization compass simulation system, where AOP represents the angle of polarization, DOP represents the degree of polarization, E-vector represents the polarization electric field vector. Green words indicate input, the orange words indicate input or process data, pink words indicate process data and red word indicates output.

In addition, Sun azimuth coordinate frame is constructed to better describe this system. As shown in Fig. 2, $\alpha_{x,y,z}$ is the East-North-Up (ENU) geography coordinate frame. The $x_a$ axis of Sun azimuth coordinate frame $\alpha_{x_a,y_a,z_a}$ is aligned with the solar azimuth, the $z_a$ axis points to the zenith, and the $y_a$ axis completes the right-handed coordinate frame. The Sun azimuth coordinate frame rotates around $z_a$ axis when the solar azimuth changes, and the direction of $y_a$ axis is always aligned with the direction of solar azimuth.

The rotation matrix from ENU coordinate to Sun azimuth coordinate is

$$
C_s = \begin{bmatrix}
\cos \varphi_{\delta S} & -\sin \varphi_{\delta S} & 0 \\
\sin \varphi_{\delta S} & \cos \varphi_{\delta S} & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

where $\varphi_{\delta S}$ is the solar azimuth angle in the ENU coordinate.

According to the relevant formula of astronomy, the position of the Sun can be solved through three angles, which are solar declination angle $\delta_s$, solar hour angle $T_s$, and latitude $L_{\delta_s}$ of observing site. By solving the spherical triangle $S- O-N P$ in Fig. 3, the solar altitude angle $h_s \in [0^\circ, 90^\circ]$ and solar azimuth angle $\varphi_{\delta S} \in [0^\circ, 360^\circ]$ in ENU coordinate frame are given by
\[
\begin{align*}
\sin \delta_s &= \sin \delta_2 \sin L_0 + \cos \delta_2 \cos L_0 \cos T_s \\
\cos \varphi_{\delta_s} &= \sin \delta_3 - \sin h_s \sin L_0 \cos T_s \\
\end{align*}
\]

where the formula of solar declination angle \( \delta_s \) for a particular year 1985 is given by

\[
\delta_s = 0.3723 + 23.2567 \sin \sigma_y + 0.1149 \sin 2\sigma_y - 0.1712 \sin 3\sigma_y - 0.758 \cos \sigma_y + 0.3666 \cos 2\sigma_y + 0.0201 \cos 3\sigma_y
\]

where day angle \( \sigma_y = 2\pi(D - D_0)/365.2422 \), \( D \) is day of year, and the spring equinox time \( D_0 \) expressed in days from the particular year 1985 is

\[
D_0 = 79.6764 + 0.2422 \times (Y - 1985) - \text{INT}[ (Y - 1985)/4 ]
\]

where \( Y \) is the year and INT represents rounding down.

Calculation progress of the solar hour angle \( T_s \) is shown below, where the local standard time \( S_d \) of observing site can be calculated by

\[
S_d = S_o + \left( F_o - \frac{4(120 - \text{Lon}_o)}{60} \right)
\]

In Eq (7), for observing site \( O \), \( S_o \) and \( F_o \) are the hour and minute of Beijing time, \( \text{Lon}_o \) is the longitude of observing site \( O \).

Then, time error \( E_t \) is given by

\[
E_t = 0.0028 - 1.9857 \sin \sigma_y + 9.9059 \sin 2\sigma_y - 7.9924 \cos \sigma_y - 0.6882 \cos 2\sigma_y
\]

After that, \( S_d \) is corrected by \( E_t \) to obtain solartime \( S_i \).

\[
S_i = S_d + E_t / 60
\]

Finally, the solar hour angle \( T_s \) is given by

\[
T_s = (S_i - 12) \times 15
\]

In short, through the above formulae, the solar azimuth angle and solar altitude angle can be finally calculated and obtained.

### 2.2. Rayleigh Sky Model

Rayleigh sky model predicts the sky polarization properties degree of polarization (DOP) [3] as

\[
DOP = DOP_{\text{max}} \frac{\sin^2 \gamma_p}{1 + \cos^2 \gamma_p}
\]

where \( \gamma_p \) is the angle between observation point \( P \) and Sun, which is named scattering angle. \( DOP_{\text{max}} \) is the maximum detected DOP in the sky and \( DOP_{\text{max}} = 1 \) for an ideal sky.

Rayleigh sky model predicts the sky polarization properties \( AOP \) [23] as

\[
AOP = \arctan \left( \frac{\sin h_s \cos h_p - \cos h_s \sin h_p \cos \left( \varphi_{\delta_s} - \varphi_{\delta_P} \right)}{\sin \left( \varphi_{\delta_s} - \varphi_{\delta_P} \right) \cos h_s} \right)
\]

Then, the \( \sin AOP \) and \( \cos AOP \) are given by

\[
\begin{align*}
\sin AOP &= \frac{\sin h_s \cos h_p - \cos h_s \sin h_p \cos \left( \varphi_{\delta_s} - \varphi_{\delta_P} \right)}{\cos \gamma_p} \\
\cos AOP &= \frac{\sin \left( \varphi_{\delta_s} - \varphi_{\delta_P} \right) \cos h_s}{\sin \gamma_p}
\end{align*}
\]

Then, the sky polarization E-vector in ENU coordinate predicted by Rayleigh sky model is given by

\[
\vec{E}_{\varphi} = \vec{V}_{\varphi} \cos AOP + \vec{H}_{\varphi} \sin AOP
\]

where \( \vec{V}_{\varphi} \) represents the tangent direction of local meridian, and \( \vec{H}_{\varphi} \) represents the vector, which is perpendicular to \( \vec{V}_{\varphi} \) and parallel to plane \( \alpha_x, \gamma_z \). \( \vec{V}_{\varphi} \cos AOP \) represents the projection of polarization E-vector on \( \vec{V}_{\varphi} \), and \( \vec{H}_{\varphi} \sin AOP \) represents the projection of polarization E-vector on \( \vec{H}_{\varphi} \).

\[
\begin{align*}
\vec{V}_{\varphi} &= \left( -\sin h_p \sin \varphi_{\delta_P}, -\sin h_p \cos \varphi_{\delta_P}, \cos h_p \right)^T \\
\vec{H}_{\varphi} &= \left( -\cos \varphi_{\delta_P}, \sin \varphi_{\delta_P}, 0 \right)^T
\end{align*}
\]

The superscript \( T \) represents matrix or vector transpose. Substituting (14) into (15), \( \vec{E}_{\varphi} \) in ENU coordinate is given by

\[
\begin{align*}
\vec{E}_{\varphi} &= \left( \cos \varphi_{\delta_P} \sin h_s \cos h_p - \cos \varphi_{\delta_P} \sin h_s \sin h_p \right) / \sin \gamma_p \\
&\quad + \sin \gamma_p \left( \sin \left( \varphi_{\delta_s} - \varphi_{\delta_P} \right) \cos h_s \right)
\end{align*}
\]

In short, the polarization E-vector \( \vec{E}_{\varphi} \) in Sun azimuth coordinate can be given by

\[
\vec{E}_{\varphi} = C_{\gamma_p} \vec{E}_{\varphi}
\]

### 2.3. Hypothetical Polarization Imager

In order to construct a comprehensive and perfect simulation system, not only skylight polarization model, but also polarization imaging sensor need to be constructed [34]. In this section, a hypothetical polarization imager is designed and described in detail.
To construct hypothetical polarization imager, camera coordinate and pixel coordinate frames are established, as shown in Fig. 4. The \( z_c \) axis of camera coordinate frame \( o_c x_c y_c z_c \) is aligned with the optical axis of imager; the \( x_c \) and \( y_c \) axes of camera coordinate frame \( o_c x_c y_c z_c \) are aligned with the column and row directions of image, respectively. The \( x_c \) and \( y_c \) axes of pixel coordinate frame \( o_c x_p y_p \) are aligned with the column and row directions of image respectively and the unit of this coordinate is pixel.

In the camera coordinate frame, the vector of pixel \( P(x_p, y_p) \) is

\[
\overrightarrow{V_{ep}} = \left( D_c(x_p - \eta_1 + \frac{1}{2}), D_c(y_p - \eta_1 + \frac{1}{2}), f \right)^T
\]

where \( D_c \) and \( D_c \) represent the column and row pixel size, respectively, \( \eta_1 \) and \( \eta_2 \) indicate the polarization image has \( \eta_1 \times \eta_2 \) pixels, and \( f \) is the focal length of pixel-based polarization camera that we used.

Suppose three Euler angles of polarization imager are given, then the rotation matrix from camera coordinate to Sun azimuth coordinate can be described as

\[
C_E = \begin{bmatrix}
\cos \beta \cos \psi + \sin \beta \sin \alpha \sin \psi & \cos \beta \sin \psi - \cos \alpha \sin \beta \sin \psi & \sin \beta \\
\cos \alpha \sin \psi & -\cos \psi & \sin \psi \\
-\sin \beta \sin \psi & \cos \psi & \cos \beta \sin \beta \sin \psi & \cos \beta \sin \alpha \sin \psi & \cos \beta \sin \alpha \sin \psi & \sin \beta \\
-\sin \beta \sin \psi & \cos \psi & \cos \beta \sin \beta \sin \psi & \cos \beta \sin \alpha \sin \psi & \sin \beta \\
\end{bmatrix}
\]

where \( \psi, \alpha \) and \( \beta \) represent yaw, pitch and roll angle, respectively. Then, the shooting direction of pixel \( P' \) in Sun azimuth coordinate is

\[
\overrightarrow{V_{ep'}} = C_E \overrightarrow{V_{ep}}
\]

Azimuth angle \( \varphi_{ep'} \) of the shooting direction of pixel \( P' \) in Sun azimuth coordinate is

\[
\varphi_{ep'} = \arctan \left( \frac{V_{ep'}(1,1)}{V_{ep'}(2,1)} \right)
\]

Altitude angle \( h_{ep'} \) of the shooting direction of pixel \( P' \) is

\[
h_{ep'} = \arcsin \left( \frac{V_{ep'}(3,1)}{V_{ep'}} \right)
\]

where \( V_{ep'}(1,1), V_{ep'}(2,1) \) and \( V_{ep'}(3,1) \) are the components of \( \overrightarrow{V_{ep'}} \), \( \overrightarrow{V_{ep'}} \) is the mode of \( \overrightarrow{V_{ep'}} \). Then, the scattering angle \( \gamma_{ep'} \) of pixel \( P' \) is given by

\[
\gamma_{ep'} = \arccos \left( \frac{\sin h_{ep'} + \cos h_{ep'} \cos \varphi_{ep'} \cos \psi}{\cos h_{ep'} \sin \psi + \cos h_{ep'} \cos \psi} \right)
\]

Substituting Eq. (25) into Eq. (11), the DOP of pixel \( P' \) can be obtained and Fig. 5(a) shows a hypothetical DOP image.

Fig. 5. Hypothetical polarization images: (a) DOP image; (b) AOP image; (c) AOPLM image.

The azimuth angle \( \varphi_{ep'} \) of the shooting direction of pixel \( P' \) in ENU coordinate is

\[
\varphi_{ep'} = \varphi_{ep} - \varphi_{sl}
\]

So, substituting Eq. (24), Eq. (26) and Eq. (18) into Eq. (19), the polarization E-vector \( \overrightarrow{E_{ep'}} \) of pixel \( P' \) in Sun azimuth coordinate frame can be obtained. Thus, the polarization E-vector \( \overrightarrow{E_{ep'}} \) of \( P' \) in camera coordinate frame can be given by

\[
\overrightarrow{E_{ep'}} = C_E' \overrightarrow{E_{ep'}}
\]

where \( C_E' \) is the transpose of \( C_E \), which represents the rotation matrix from Sun azimuth coordinate to camera coordinate. As the AOP reference direction is aligned with \( y_s \) axis and the shooting direction of the hypothetical polarization imager is aligned with \( z_c \) axis, then AOP can be given by

\[
AOP = \arctan \left( \frac{E_{ep'}(1,1)}{E_{ep'}(2,1)} \right)
\]

where \( E_{ep'}(1,1), E_{ep'}(2,1) \) are the components of \( \overrightarrow{E_{ep'}} \), and a hypothetical AOP image is shown in Fig. 5(b).

For the classical four typical orientation determination algorithms described in Section 1, Zenith approach and Least-square approach can directly use AOP for orientation determination. However, for SM-ASM approach and Symmetry approach, further transformation of AOP is required, the reference direction of AOP needs to be converted to the local meridian; the AOP whose reference direction is local meridian can be defined as AOPLM.

\[
AOPLM = AOP - \xi
\]
where $\xi$ is the angle between $y$ axis and local meridian. When polarization imager points to the sky zenith, we have

$$\xi = \arctan \left( \frac{\eta_y - \eta_f + 1}{\eta_x - \eta_f + 1/2} \right)$$  \hspace{1cm} (30)$$

And a hypothetical AOPLM image is obtained and shown in Fig. 5(c).

### 3. SIMULATION

In order to investigate the influence of sensor tilts on orientation determination, we have carried out a lot of simulation experiments for four classical polarization orientation determination algorithms: Zenith approach, SM-ASM approach, Symmetry approach and Least-square approach, as shown in Fig. 6. According to Rayleigh sky model, the polarization E-vector at the sky zenith is perpendicular to the solar azimuth, so, Zenith approach determines the heading angle by measuring the AOP at the sky zenith [3, 14, 20-22]. The polarization E-vector along SM-ASM is consistently perpendicular to SM-ASM, so SM-ASM approach calculates the heading angle by extracting SM-ASM [23-26]. According to the symmetry of the skylight polarization pattern, Symmetry approach determines orientation by symmetry detection [27-29]. Polarization E-vector of Rayleigh sky model is consistently perpendicular to the solar vector, so, Least-square approach determines orientation by total least square of Polarization E-vectors [30-32].

![Rayleigh sky model and four typical orientation determination approaches](image)

**Fig. 6.** Rayleigh sky model and four typical orientation determination approaches, where the red point represents the Sun, the pink point represents the sky zenith, the red line represents solar meridian and anti-solar meridian (SM-ASM), the blue lines represent the polarization electric field vectors (E-vector).

| Table 1 Simulation Parameters. |
|--------------------------------|
| Symbol | Value | Units | Description                        |
|--------|-------|-------|------------------------------------|
| $DOP_{\text{max}}$ | 1 | / | Maximum DOP in the sky |
| $D_x$ | 3.45 | $\mu m$ | Pixel size in column direction |
| $D_y$ | 3.45 | $\mu m$ | Pixel size in row direction |
| $\eta_x$ | 2048 | pixel | Number of pixels in column direction |
| $\eta_y$ | 2448 | pixel | Number of pixels in row direction |
| $f$ | 4 | mm | Focal length of polarization imager |

Considering that the polarization imager needs to capture the skylight polarization pattern, the imager field of view should always be above the horizon, and the interference of buildings and obstacles should be eliminated. In our simulation and experiment, the imager angle of view is 108°, therefore, we set pitch and roll angles to be $|\alpha| + |\beta| \leq 30^\circ$.

The tilt state of the sensor in practice can be divided into three situations:

1. Only pitch tilt condition
2. Only roll tilt condition
3. Pitch and roll tilts condition

And the parameters of simulation are shown in Table 1.

#### 3.1. Only Pitch Tilt

This part discusses the error characteristics when the tilt is only the pitch angle with roll angle set to zero. By using the polarization compass simulation system in Section 2, more than $1.4 \times 10^5$ sets of simulation experiments were carried out, and the orientation errors of four typical approaches were obtained in the range of solar altitude angle $h_s \in [0^\circ, 50^\circ]$ , yaw angle $\psi \in [-180^\circ, 180^\circ]$ , pitch angle $\alpha \in [-30^\circ, 30^\circ]$ and roll angle $\beta = 0^\circ$. And the results under only pitch tilt are shown in Fig. 7.

![Orientation errors of four typical polarization orientation determination approaches under only pitch tilt](image)

**Fig. 7.** Orientation errors of four typical polarization orientation determination approaches under only pitch tilt: (a) Orientation error of Zenith approach; (b) Orientation error of SM-ASM approach; (c) Orientation error of Symmetry approach; (d) Orientation error of Least-square approach. The unit of the three axes is degree.

It can be observed in Fig. 7, when only pitch tilt exists, the variation trend of orientation error of the four typical approaches is the same. And there are three following similarities:

( I ). When the pitch angle is 0°, errors of the four approaches are all close to zero. With the increase in pitch angle, the errors of the four approaches all have a trend to increase.

( II ). When the solar altitude angle is 0°, the errors of the four approaches are always close to 0°. When the pitch angle is not 0°, the errors of the four approaches tend to increase with the increase in the solar altitude angle.
The errors of the four approaches are all symmetric with respect to the plane $\psi = 0^\circ$ and $\psi = 180^\circ(-180^\circ)$. And when the yaw angle is $0^\circ$ or $180^\circ(-180^\circ)$, no matter what the pitch and solar altitude angles are, errors of the four typical approaches are always close to zero. In addition, there are the following trends: Errors are close to zero at $\psi = -180^\circ$, and with the yaw angle increasing gradually, errors increase gradually and reach a maximum; Then, errors decrease and are close to zero at $\psi = 0^\circ$; After that, with the yaw angle increasing gradually, orientation calculating errors increase gradually and reach a maximum; Finally, the orientation errors decrease and are close to zero at $\psi = 180^\circ$.

The following is a detailed analysis of the reasons for the above three similarities:

For (Ⅰ), when the pitch angle $\alpha = 0^\circ$, there is no influence of sensor tilt, which means, under ideal conditions, all these four typical approaches can effectively determine the orientation. When pitch tilt increases, the tilt interference increases, which leads to the increase in orientation errors.

For (Ⅱ), the four approaches essentially use the solar azimuth information to determine orientation. When the solar altitude angle increases, the component of the solar vector projected on the plane $\alpha_x y_z (\alpha_x, y_x)$ decreases, so the stability and reliability of solar azimuth are weakened, which leads to the increase in orientation errors.

For (Ⅲ), The errors of the four approaches are all symmetric with respect to the planes $\psi = 0^\circ$ and $\psi = 180^\circ(-180^\circ)$. This manifests the symmetry of skylight polarization pattern with respect to the SM-ASM. When the yaw angle is $0^\circ$ or $180^\circ(-180^\circ)$ and only pitch tilt, the direction of polarization imager’s optical axis always points to SM-ASM which is parallel to $y_z$ axis, thus results in that when the yaw angle is $0^\circ$ or $180^\circ(-180^\circ)$, no matter what the pitch and solar altitude angles are, the errors of the four approaches are always close to zero.

In addition, to further compare the four approaches, we have drawn groups of simulation results of the four approaches on a graph, as shown in Fig. 8 and 9. It can be found that the variation trend of the four error curves is exactly the same, and the four curves almost coincide. Furthermore, under the same condition, the error difference of these four approaches is always less than 0.66°. Therefore, when there is only pitch tilt interference, it can be concluded that the error characteristics of the four approaches are consistent and the orientation errors of the four approaches are almost the same. Moreover, similarity (Ⅰ) can be clearly seen from Fig. 8 and similarities (Ⅰ) and (Ⅲ) can be partially reflected in Fig. 9.

![Fig. 8. Simulation orientation error curves of four typical polarization orientation determination approaches under only pitch tilt situation.](image-url)
3.2. Only Roll Tilt

This part discusses the error characteristics when it has only the roll angle tilt situation, with pitch angle set to zero. The orientation errors of four typical approaches were obtained in the range of solar altitude angle $\psi \in [0°, 50°]$, yaw angle $\psi \in [-180°, 180°]$, roll angle $\beta \in [-30°, 30°]$ and pitch angle $\alpha = 0°$. The results under only roll tilt are shown in Fig. 10, where the yaw range is $[-180°, 180°]$. For ease of observation and comparison, the range of yaw is converted to $[-90°, 270°]$, as shown in Fig. 11. By comparing Fig. 11 and Fig. 7, it can be seen that the two sets of graphs have exactly the same shape, with the only difference being the range of yaw angle. Therefore, the error characteristics of only roll tilt are very similar to that of only pitch tilt. The first two error similarities are the same as described in Section 3.1, the only difference is the third one.

For only roll tilt, errors of the four approaches are all symmetric with respect to the plane $\psi = -90°(270°)$ and $\psi = 90°$. And when the yaw angle is $90°$ or $-90°(270°)$, no matter what the roll and solar altitude angles are, the errors of the four typical approaches are always close to zero. In addition, there are the following trends: The errors are close to zero at $\psi = -90°$, and with the yaw angle increasing gradually, the errors increase gradually and reach a maximum; Then, the errors decrease and are close to zero at $\psi = 90°$; After that, with the yaw angle increasing gradually, the errors increase gradually and reach a maximum; Finally, the errors decrease and are close to zero at $\psi = 270°$.

In short, compared with the only pitch tilt case, the result of the only roll tilt case has a $90°$ shift in the yaw direction. The reason for this phenomenon is explained in detail below.

As shown in Figure 5, the direction of polarization imager’s optical axis is $(0, 0, 1)^T$. According to Eq. (22), the optical axis direction in Sun azimuth coordinate is given by

$$V_{at} = C_c^{-1} \begin{bmatrix} 0 \\ 0 \cos \beta \sin \psi - \cos \beta \sin \alpha \sin \psi \\ \sin \alpha \sin \psi \end{bmatrix}$$

(31)

Assume two sets of attitude $(\psi_1, \alpha_1, \beta_1)$ and $(\psi_2, \alpha_2, \beta_2)$, which satisfy $\psi_2 = \psi_1 + 90°$, $\beta_2 = \alpha_1$, $\beta_1 = 0°$ and $\alpha_2 = 0°$.

$$V_{at1} = \begin{bmatrix} \sin \beta_1 \cos \psi_1 - \cos \beta_1 \sin \alpha_1 \sin \psi_1 \\ -\sin \beta_1 \sin \psi_1 - \cos \beta_1 \sin \alpha_1 \cos \psi_1 \\ \cos \beta_1 \cos \alpha_1 \end{bmatrix} \quad V_{at2} = \begin{bmatrix} -\sin \alpha_1 \sin \psi_1 \\ -\sin \alpha_1 \cos \psi_1 \\ \cos \alpha_1 \end{bmatrix}$$

(32)

where $V_{at1}$ and $V_{at2}$ are the optical axis directions at $(\psi_1, \alpha_1, \beta_1)$ and $(\psi_2, \alpha_2, \beta_2)$ in Sun azimuth coordinate system, respectively. $V_{at1} = V_{at2}$ shows that the image’s optical axis direction at $(\psi_1, \alpha_1, \beta_1)$ and $(\psi_2, \alpha_2, \beta_2)$ are exactly the same, so the polarization information collected by the polarization imager at $(\psi_1, \alpha_1, \beta_1)$ and $(\psi_2, \alpha_2, \beta_2)$ corresponds to almost the same area of the sky. This is the reason that the results of only roll tilt have $90°$ shift in the yaw direction compared with that of only pitch tilt.

To further compare these four approaches under only roll tilt, we have drawn groups of simulation results of the four approaches on a graph, as shown in Fig. 12 and 13. It can be found that the variation trend of the four error curves is exactly the same, and the four curves almost coincide. Furthermore, as shown in Fig. 13, the error curve of only pitch tilt is drawn to compare with that of only roll tilt. It can be clearly seen, the results of only roll tilt have $90°$ shift in the yaw direction compared...
with that of only pitch tilt. Therefore, the properties of only roll tilt can be referred from that of only pitch tilt, which will not be repeated here.

Fig. 10. Orientation errors of four typical polarization orientation determination approaches under only roll tilt, where $\psi \in [-180^\circ, 180^\circ]$: (a) Orientation error of Zenith approach; (b) Orientation error of SM-ASM approach; (c) Orientation error of Symmetry approach; (d) Orientation error of Least-square approach. The unit of the three axes is degree.

Fig. 11. Orientation errors of four typical polarization orientation determination approaches under only roll tilt, where $\psi \in [-90^\circ, 270^\circ]$: (a) Orientation error of Zenith approach; (b) Orientation error of SM-ASM approach; (c) Orientation error of Symmetry approach; (d) Orientation error of Least-square approach. The unit of the three axes is degree.

Fig. 12. Simulation orientation error curves of four typical polarization orientation determination approaches under only roll tilt situation.
3.3. Pitch and Roll Tilts

In addition, the error characteristics under pitch and roll tilts are discussed. The orientation errors of four typical approaches were obtained in the range of solar altitude angle \( h_s \in [0^\circ, 50^\circ] \), yaw angle \( \psi \in [-180^\circ, 180^\circ] \), pitch angle \( \alpha \in [-30^\circ, 30^\circ] \), roll angle \( \beta \in [-30^\circ, 30^\circ] \) and \(|\alpha| + |\beta| \leq 30^\circ\). The results of Zenith approach under pitch and roll tilts are shown in Fig. 14.

It can be observed in Fig. 14, under pitch and roll tilts, the error of Zenith approach has the following characteristics:

(I). With the increase in pitch and roll angles, the error of Zenith approach has a tendency to increase.

(II). When the solar altitude angle is 0\(^\circ\), orientation error of Zenith approach is close to 0\(^\circ\). When the pitch and roll angles are not 0\(^\circ\), the error of Zenith approach tends to increase with the increase in the solar altitude angle.

(III). When the other conditions are the same, yaw angle difference also affects orientation errors obviously.

The following will be a detailed analysis of the reasons for the above three characteristics. The reasons are identical to that mentioned in Section 3.1.

For (I), when the pitch and roll tilts increase, the tilt interference increases, which leads to the increase in orientation errors.

For (II), the Zenith approach essentially uses the solar azimuth information to determine orientation. When the solar altitude angle increases, the component of the solar vector projected on the plane \( \alpha_x y_s (\alpha_x, y_s) \) decreases, so the stability and reliability of solar azimuth are weakened, which leads to the increase in orientation error.

For (III), with different yaw angles, the relative position between the Sun and polarization sensor is different, resulting in different orientation errors when the sensor tilts.

Under pitch and roll tilts, the error difference between the four approaches is always less than 0.77\(^\circ\). So, the error characteristics of the other three approaches are consistent with that of Zenith approach.
Fig. 14. Simulation orientation error of Zenith approach under pitch and roll tilts.
4. FIELD EXPERIMENT

To further verify the simulation results, field experiments were carried out to investigate the influence of polarization sensor tilts on orientation determination. In addition, the results of field experiments are compared with that of simulation.

Fig. 15. Polarization orientation determination experiment platform.

Our experiment platform is shown in Fig. 15. Two tripods are equipped with a Sony IMX250MZM polarization imager and a GPS/IMU (Global Position System/Inertial Measurement Units) integrated navigation system. The parameters of the actual polarization imager are also consistent with that of hypothetical polarization imager, as shown in Table 1. The GPS/IMU integrated navigation system is used to determine pitch and roll angles of polarization imager. The true North is determined by a double antenna GPS device as a benchmark (orientation resolution is 0.1° with a 2 m baseline). Field experiments were performed in Nanjing, China, on the roof of our laboratory (32°01′36.4″ N, 118°51′11.9″ E), from 15 November to 19 November 2019, meteorological conditions were stable. Fig. 16 shows a set of actual polarization images.

Fig. 16. Actual polarization images: (a) DOP image; (b) AOP image; (c) AOPLM image. The four corners of these polarization images cannot be properly imaged due to the short focal lens adopted by the imager, so they are not used in polarized skylight navigation and are set to 0.

Aiming at the influence of sensor tilt, field experiments were carried out, which include only pitch tilt condition, only roll tilt condition, pitch and roll tilts condition. The experimental results are shown in Fig. 17, 18 and 19. Note that the green curves in Fig. 17, 18 and 19 are the simulation results. According to Section 3, the orientation errors of simulation results of the four typical approaches are almost exactly the same, so only one curve is drawn here to facilitate the observation and comparison of simulation and experimental results. In addition, the dithering of the experiment orientation error curves is due to cloud interference, which is not the focus of this paper and would not be discussed in detail here.

Fig. 17. Field experiment under only pitch tilt condition on 15 November 2019, where the pitch angle is -20.0°.

Fig. 18. Field experiment under only roll tilt condition on 16 November 2019, where the roll angle is 29.1°.

Fig. 19. Field experiment under pitch and roll tilts on 19 November 2019, where the pitch angle is -16.3° and the roll angle is -99°.
As illustrated in Fig. 17, 18 and 19, comparing the simulation results with the experiment results of the four typical approaches, it is clear that:

I. There are some differences of the simulation results and the field experimental results, which are always maintained within a range; (II) The experiment error curves and simulation error curves have the same variation trend, and the experiment errors of the four typical approaches have the same variation trend compared with each other.

For further analysis, in (I) of the above paragraph, Rayleigh sky model is an ideal model which only considers a single scattering event, and has some differences from the actual skylight polarization pattern [41]. So, the experiment error curves do not coincide with the simulation error curves.

For (II) of the above paragraph, no matter only pitch tilt condition, only roll tilt condition, pitch and roll tilt condition, the orientation error curves of field experiments and simulation have the same variation, and the experiment errors of the four typical approaches have the same variation trend compared with each other. All these further showed that the orientation error characteristics of the four typical approaches are consistent under tilt interference. In simulation, the simulation error curves of the four typical approaches almost coincide. However, field experiment error curves of the four typical approaches do not coincide with each other. This is due to the fact that field experiments have the influence of not only sensor tilts, but also some other disturbances, such as measurement noise and clouds.

5. CONCLUSION

In this study, the influence of sensor tilts on polarized skylight orientation determination is investigated in detail. Four typical polarization orientation determination approaches are described and compared with each other under only pitch tilt condition, only roll tilt condition, pitch and roll tilt condition. Simulation based on Rayleigh sky model shows that the error characteristics of the four approaches are completely consistent and the error curves almost coincide when only affected by sensor tilt. With the increase in tilt and solar altitude, the orientation errors of the four approaches all tend to increase, and the orientation errors are also affected by yaw angle. In addition, field experiments show that the errors of the four approaches have the same variation trend. All these provide an important reference for the practical application of polarization orientation determination, especially for the installation error of sensor, the tilt of application platform, the change of three-dimensional attitude of carrier and so on.

Moreover, the results of this paper can also mitigate such orientation determination errors. Given pitch angle and roll angle, the error caused by tilts can be obtained based on the results of this paper. After that, the orientation can be calibrated by subtracting this error.

Based on the simulations and experiments, the influence of sensor tilts is investigated in detail. However, polarization orientation determination can be influenced not only by the sensor tilt, but also the measurement noise and clouds, how to eliminate these impacts for orientation determination would be the focus of our future research.

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