Toponium Tests Of Top-Quark Higgs Bags

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Abstract

Recently it has been suggested that top quarks, or very massive fourth generation quarks, might surround themselves with a Higgs “bag” of deformation of the Higgs expectation value from its vacuum magnitude. In this paper we address the question of whether such nonlinear Higgs-top interaction effects are subject to experimental test. We first note that if top quarks were necessarily accompanied by Higgs “bags”, then top quark weak decay would involve the sudden disruption of the Higgs “bag”, with copious production of physical Higgs particles accompanying the decay. We then examine the effects that such Higgs “bags” would produce on the spectrum of toponium, where the two bound top (anti)quarks, and their “bags”, overlap. We numerically evaluate the effects that the nonlinear feedback in the Higgs-toponium system would have on the energy level splittings of the toponium bound states, and find that for allowed values of the top and Higgs mass the effect is negligible, thus indicating that even in this favourable circumstance Higgs “bag” formation around top quarks does not observably occur. Finally, we consider the case of a second Higgs doublet, allowing the possibility of enhanced couplings for one of the physical Higgs to top. Even in this nonstandard scenario the effects are minimal, and we infer the general absence of observable effects at any level that might suggest the utility of considering top quarks to be accompanied by Higgs “bags”.
In the standard model the Higgs field acts, through its vev, as the source of mass for all particles, with the mass obtained depending on the strength of the particle’s coupling to the Higgs. Of the particles in the standard model, the only one with potentially very large mass, and hence large coupling to the Higgs, is the top quark. This opens the possibility that there are nonperturbative, strong-coupling effects, with Higgs particles, that will occur uniquely in interaction with the top quark. The idea that a fermion which is strongly coupled to an order parameter may locally deform that order parameter, and surround itself with a “bag” of field deformation, dates at least as far back as Feynman’s treatment of polarons \[1\], and more recently has been generally explored in relativistic field theories of scalars and spinors \[2, 3, 4, 5, 6, 7, 8\]. Recently it has been suggested \[9\] that for large values of the top quark mass just such nonlinear effects occur, with the top quark digging a hole in the Higgs vev, and surrounding itself with a “bag” or “dimple” of deformation of the Higgs field (a posteriori such a possibility would also appear for very massive quarks, or leptons, of a hypothetical fourth generation). More detailed quantitative examinations of this proposal have come to the conclusions that: semiclassical “bag” formation implies couplings sufficiently strong to jeopardize vacuum stability, or imply a breakdown of perturbation theory at energies not too far above the top quark mass range \[10\]; perturbative couplings result in “dimples” that as quantum superpositions involve on average a fraction of a quantum \[11\]; strong non-perturbative couplings result in quantum fluctuations that tend, at least in a large N expansion, to “deflate” the “bag” \[12\]. In this paper we adopt a slightly different approach to the problem; we ask what would be the observable signatures of formation of Higgs “bags”, both for individual top quarks, and also for toponium bound states. We then evaluate the magnitude of these effects for top quarks of moderate mass, where we may treat the Higgs-top coupling in perturbation theory, and examine where the nonlinear higher-order effects should begin to dominate, giving observable signatures of “bag” formation. In agreement with the previous analyses \[10, 11, 12\] we find for standard model Higgs masses in the range allowed by vacuum stability, and perturbative non-triviality, that the effects of Higgs “bag” formation are not strong enough to be significant. We then extend our analysis to the case of two Higgs doublets, where one of the Higgs may have enhanced coupling to the top, to examine whether in this case observable effects of Higgs “bag” formation may occur.

The possibility of the formation of Higgs “bags” around heavy quarks is suggested by simple energetic considerations. A heavy quark obtains its large mass by virtue of a large Yukawa coupling
to the Higgs field vev. If the value of that vev could be locally diminished in the vicinity of the top quark, then the mass of the top quark could be lowered. Provided that the gain in energy from decreasing the mass of the top quark can more than compensate for the kinetic and potential energy invested in deforming the Higgs field around the top quark, and the kinetic energy localizing the top quark, then the top quark will dig a hole for itself in the Higgs vev, and inhabit the region of diminished vev. For this to be energetically favourable, we need the energy saved from lowering the quark mass to dominate, which means the possibility depends on a large Yukawa coupling, and so it may occur only for (very) heavy quarks. If this scenario is correct, then a heavy quark such as the top should be thought of not as an isolated fermion, but rather as a structured object consisting of a fermion surrounded by a coherent superposition of Higgs bosons representing the deformation of the Higgs vev.

Since this coherent superposition of Higgs quanta is supported by the energy saved in reducing the mass of the heavy quark source, the disappearance of that quark would of necessity result in the dispersal of the Higgs quanta. In the case of top quarks, this means that their normal charged current weak decay, via $t \to bW^+$ would remove the source of the Higgs ‘‘bag’’ (the $b$ being too weakly coupled to the Higgs), and hence lead to the sudden disruption of the ‘‘bag’’. This would in turn mean that the dominant decay modes of such a top quark (with ‘‘bag’’) would involve a copious shower of Higgs bosons from the disruption of the ‘‘bag’’, as well as the $b$ and $W$. Decay to the $bW$ mode (without Higgs) would be strongly suppressed by the small wave function overlap of the ‘‘bag’’ state with the final state absence of Higgs. This means that the observation of the standard decay mode of the top would provide prima facie evidence against the formation of Higgs ‘‘bags’’. Conversely, a fermion strongly enough coupled to the Higgs field to engender ‘‘bag’’ formation, may be expected to have complex decay modes, that display the complexity of the coherent Higgs superposition in which it reposes.

A second way that one might imagine obtaining experimental evidence concerning the possibility of Higgs ‘‘bag’’ formation, is by examining toponium bound states. A priori, these seem like ideal systems to probe the possibility of ‘‘bags’’: first they represent already localized top quark sources for the Higgs; second the bound state spectrum provides a sensitive test of the structure of the potential well in which the $\bar{t}t$ find themselves, and should surely be sensitive to as qualitatively distinct a feature as Higgs ‘‘bag’’ formation. To reduce the problem to its essential form, let us
consider a $\bar{tt}$ bound state, held together by the QCD potential, which for the heavy toponium we may consider to be approximately Coulombic, and which interacts with the Higgs field via a Lagrangian of the form: (we ignore everything else in the standard model, as we expect it to be quantitatively insignificant in our considerations)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m_H^2}{2} \phi^2 + \bar{\Psi} (i \not{\partial} - g \phi) \Psi - m_t \bar{\Psi} \Psi - \frac{\lambda}{4!} \phi^4$$  \hspace{1cm} (1)

The interaction term $\mathcal{L} = -g \bar{\Psi} \phi \Psi$ will cause a minor deformation of the Higgs field in the presence of the top quarks. Moreover, due to the assumed heaviness of top, one can apply non-relativistic quasi-classical methods to bound state systems (toponium) composed of (anti)top quarks and a Higgs field. For quasi-static (anti)top quarks in a toponium bound state, which act as a source of deformation of the Higgs field from its vev, we have classically for the Higgs deformation:

$$(\nabla^2 - m_H^2) \phi = g \bar{\psi} \psi$$  \hspace{1cm} (2)

where in the preceding equation (and hereafter) the $\psi$ represents the “large” components of the non-relativistic spinor $\Psi$. Here the time dependence has been disregarded as the lowest energy state of the toponium is stationary. Further, the scalar coupling of the Higgs to the top quark, and the non-relativistic treatment of the toponium, implies that the spin degrees of freedom of the top can be neglected, and the $\bar{\psi} \psi$ can be treated as a scalar source for the Higgs vev deformation.

We consider the S-wave fermion wave functions of our toponium bound states as Higgs sources. In view of the spherical symmetry of the S-wave states, the source term composed of the $\bar{tt}$ can be written in terms of the top wave function, expressed in polar coordinates, centred on the toponium. Assuming that the QCD binding potential is approximately Coulombic, then over the distance scale probed by the toponium wave function, these wave functions are exponential in nature; they act as an exponentially falling (radially) Higgs source term; and the 1S and the 2S wave functions represent strong, localized Higgs sources. For Coulombic toponium, the Higgs field source terms are:

$$1S : \quad g \bar{\psi} \psi = \frac{g}{\pi a_0} e^{-\frac{2r}{a_0}}$$
$$2S : \quad g \bar{\psi} \psi = \frac{g}{32 \pi a_0^3} (2 - \frac{r}{a_0}) e^{-\frac{r}{a_0}}$$  \hspace{1cm} (3)

with $a_0 = \frac{1}{2 \alpha_s \bar{m}_t}$ as the Bohr radius of the unperturbed toponium bound state. $\alpha_s$ is the effective strong coupling constant on scales corresponding to the size of the toponium bound state, which
we take to have a value of \( \alpha_s \simeq 0.32 \), in approximate agreement with the values used by Athanasiu et al. [13] in their study of the \( \bar{t}t \) system. The deformation of the Higgs field in the neighborhood of the toponium source causes a decrease in the observed top (and hence toponium) mass.

To solve for the deformation of the Higgs vev, one uses the three dimensional Green’s function associated with the equation of motion of the Higgs field.

\[
G(\tilde{r}_1, \tilde{r}_2) = \frac{-1}{4\pi |\tilde{r}_1 - \tilde{r}_2|} e^{m_H|\tilde{r}_1 - \tilde{r}_2|}
\]  

(4)

Utilising this, one then can then analytically obtain the first order position dependent deviation of the Higgs vev from its asymptotic value of \( v = 246 \text{ GeV} \). For the 1S and 2S toponium wave function sources the form of the Higgs vev deviation is

\[
\phi_{1S}^{1st}(r) = \frac{\alpha_s^2 m_t^3 (2 \alpha_s e^{m_H r} m_t - 2 \alpha_s e^{\alpha_s m_H r} m_t - e^{m_H r} m_t^2 r + \alpha_s^2 e^{m_H r} m_t^2 r)}{8 e^{(m_H + \alpha_s m_H) r} (m_H - \alpha_s m_t)^2 (m_H + \alpha_s m_t)^2 \pi r}
\]  

(5)

\[
\phi_{2S}^{1st}(r) = \alpha_s e^{-(m_H r)} \frac{-\alpha_s m_H r}{512 (-2 m_H + \alpha_s m_t)^4 \pi r} \left( -128 e^{m_H r} m_H^3 + 128 e^{\frac{\alpha_s m_H r}{2}} m_H^3 m_t + 256 \alpha_s e^{m_H m_t} m_t^2 - 256 \alpha_s e^{\alpha_s m_H r} m_t^2 - 128 \alpha_s^2 e^{m_H r} m_H m_t^2 \\
+128 \alpha_s^2 e^{\alpha_s m_H r} m_H m_t^2 + 64 \alpha_s^3 e^{m_H r} m_t^3 - 64 \alpha_s^3 e^{\alpha_s m_H r} m_t^3 - 64 \alpha_s e^{m_H r} m_H m_t^3 r \right) \\
+64 \alpha_s^2 e^{m_H r} m_H m_t^3 r - 64 \alpha_s^3 e^{m_H r} m_H m_t^3 r + 24 \alpha_s^4 e^{m_H r} m_t^4 r \\
+16 \alpha_s^2 e^{m_H r} m_H m_t^4 r^2 - 12 \alpha_s^3 e^{m_H r} m_H m_t^4 r^2 + 4 \alpha_s^5 e^{m_H r} m_t^5 r^2 - 8 \alpha_s^3 e^{m_H r} m_H m_t^3 r^3 \\
+12 \alpha_s^4 e^{m_H r} m_H m_t^4 r^3 - 6 \alpha_s^5 e^{m_H r} m_H m_t^5 r^3 + \alpha_s^6 e^{m_H r} m_t^6 r^3 \right)
\]  

(6)

To see the effect of the coupling on the mass of the toponium bound state, we examine the change in the splitting between the 2S and 1S energy levels; we focus on the energy level splitting as it is a physical observable, and may reasonably be expected to be sensitive to Higgs “bag” formation, in as much as the 1S and 2S states represent Higgs sources with a different degree of localization, so they should be deformed differently by the formation of a Higgs “bag”. Our strategy is to determine the ratio of the leading perturbative correction to the 2S-1S splitting, to corrections that appear at second order, after the feedback of the Higgs field on the toponium source wave function has recorrected the energies of the toponium states. We would interpret second order corrections to

4
the splitting that were a significant fraction of the first order correction, as evidence of a nonlinear feedback in the Higgs-toponium system, representing the onset of “bag” formation.

To examine the effect of the interaction term, first consider as the zeroth order approximation, a QCD toponium bound state. The energy level for the \( nS \) state of such a system is given approximately by the Coulombic QCD binding potential for heavy quarkonium

\[
E_{nS}^0 \simeq -\frac{4}{3} \frac{(4\pi\alpha_s)^2 m_t}{4\pi^2} n^2 
\]

Here \( \frac{4}{3} \) is the colour factor. For the 2S-1S splitting, \( \Delta E^0 \) this gives \( \Delta E^0 \simeq -1.7 \text{ GeV} \) with our assumed value of the effective QCD coupling. The modification of the splitting due to the presence of the Higgs-top interaction is given by the change in the energy level splitting, \( \Delta E \), for which time independent non-degenerate perturbation theory is used. The perturbing Hamiltonian is given by

\[
\mathcal{H}_1 = -g\phi
\]

The first order correction to the energy levels due to the presence of the condensate \( \phi \) is then:

\[
E_{nS}^{1st} = \langle \psi_0^{nS} | -g\phi | \psi_0^{nS} \rangle
\]

Applying equation 4 one can obtain numerical values for the first order correction to the 2S-1S splitting for various values of \( m_H \) and \( m_t \). Figure 1(a) shows the ratio of the first order correction to the 2S-1S splitting to the zeroth order splitting, as a function of the top quark mass and the mass of the Higgs. In Figure 1(b) contour lines are shown corresponding to first order fractional shifts in the splitting of 5% and of 1%; also shown on the figure is the top and Higgs mass parameter range allowed in the standard model by the constraints of vacuum stability, and perturbative non-triviality up to the Planck scale \[14\]. Clearly, a measurable shift in the splitting from first order corrections is restricted to a small region of the allowed \( m_H \) and \( m_t \) parameter space. To test for evidence of “bag” formation, one has to consider the higher order corrections to the energy perturbation. In particular, Higgs “bag” effects would be observable if the non-linear feedback in the Higgs-toponium system, represented by the second order correction, was large in comparison with the first order correction (say of the same order or more). A large second order correction implies that the fermion wave function is pulled in tighter, giving stronger binding to the toponium, and thereby indicating
strong binding in a Higgs “bag” potential well. This in turn would increase the influence of the
source term in equation 2, and so result in a significant increase in the deviation of the Higgs field
around the toponium which would then cause a further correction to the splitting. This nonlinear
feedback would proceed to dig a hole in the Higgs field, and produce observable Higgs “bag” effects.

Using the first order perturbations, and maintaining the top normalisation, one has

\[
\begin{align*}
\psi_{1S} &= \psi_{1S}^0 + \psi_{1S}^1 \\
\psi_{2S} &= \psi_{2S}^0 + \psi_{2S}^1
\end{align*}
\]

for the first order corrected top wave functions. It should be noted that for \( m_t \) in the range 0 to
250 GeV the adjustment is slight. Given these corrected wave functions, the correction to the Higgs
field can be computed. The correction to \( \phi_0 \) is given by

\[
(\nabla^2 - m_H^2)\phi_1 = g\psi_{nS}^0\psi_{nS}^1
\]

As this equation only differs from equation 2 in the inhomogenous term, the Green’s function is
unaltered, and the \( \phi_1 \) can be found. This then allows one to evaluate the second order correction
to the toponium energy levels. For the nS top wave functions, the second order energy correction is

\[
E_{2nd}^{nS} = \langle \psi_{nS}^0 | - g\phi_0 | \sum_m b_m \psi_{mS}^0 \rangle + \langle \psi_{nS}^0 | - g\phi_1 | \psi_{nS}^0 \rangle
\]

where the \( b_m \) are the coefficients of the first order correction to the wave function. The ratio of
concern is

\[
R = \frac{\Delta E_{2nd}^{2nd}}{\Delta E_{1st}^{2nd}} = \frac{E_{2S}^{2nd} - E_{1S}^{2nd}}{E_{2S}^{1st} - E_{1S}^{1st}}
\]

If R (plotted in Figure 2(a)) is large then the feedback will have a significant effect on the toponium
bound state. On the other hand if R is negligible, then then feedback is insignificant. Figure 2(b)
displays the values of the mass parameters required to give a 0.1% and 1% value for R. Clearly,
the \( m_H \) and \( m_t \) for even such slight feedback are not physically acceptable as they lie outside the
range of the allowed mass parameters. Also, for any larger value of R the predicted values of \( m_H \)
and \( m_t \) fall further away from the acceptable region. The smallness of R in the mass parameter
range allowed by the standard model tells one that the feedback corrections to the energy splitting
are negligible, and thus Higgs “bag” are experimentally unobservable. The only circumstance with
marginally significant feedback is the case where the top mass is large \( m_t \approx 150 \text{ GeV} \) and the Higgs mass is of the order of a few GeV: this situation is already ruled out by LEP limits on the Higgs mass \([15]\).

If the Higgs sector is extended to a non-minimal content consisting of two Higgs doublets, then there will be extra physical scalar Higgs fields, each of which must be considered. Consider the possibility that one or more of the physical scalars in this non-minimal scenario has enhanced coupling to the top quark. Such an enhancement will in general result in an increase in the corrections to the energy level splittings, thus reopening the possibility for detectable Higgs “bag” effects. In Figure 3(a) the feedback ratio has been plotted for a coupling that has been enhanced by a factor of 5 over the standard model Higgs coupling, while Figure 3(b) indicates the mass parameters required for \( R \) to reach the 1% level. Clearly, while enhancement of the coupling increases the second order correction, even a factor of five increase in the coupling has not resulted in significant nonlinear feedback. As such, we do not find evidence for Higgs “bag” formation around toponium, even with substantially enhanced couplings that could appear in models with non-minimal Higgs content.

In conclusion, we have considered the possible observable effects of formation of a Higgs “bag” around toponium, as has been recently suggested. For values of the Higgs and top mass expected in the standard model, the potentially observable effects that could occur in toponium bound states are sufficiently small, that no indication of non-linear feedback characteristic of “bag” formation has appeared. This conclusion remains essentially unaltered, even with the ad hoc enhancement of the top-Higgs coupling by a factor of five, as might occur in a model with a non-minimal Higgs sector.

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Figure Captions

Figure 1: (a) The ratio of the first order correction to the 2S-1S splitting to the zeroth order splitting, as a function of the top quark mass and the mass of the Higgs; (b) Contour lines corresponding to first order fractional shifts in the splitting of 5% (dashed line) and of 1% (dot dashed line).

Figure 2: (a) The ratio $R$ of the second order correction to the first order correction for the 2S-1S energy splitting as a function of the top quark mass and the mass of the Higgs; (b) Contour lines corresponding to .1% (dot dashed line) and 1% (dashed line) in $R$.

Figure 3: (a) The $R$ ratio for normal Higgs coupling (upper surface), and for a coupling enhanced by a factor of 5 (lower surface) as a function of the top quark mass and the mass of the Higgs; (b) Contour line corresponding to 1% (dashed line) in $R$, with the enhanced coupling.