SEARCHING FOR COLOR COHERENT EFFECTS
AT INTERMEDIATE $Q^2$
VIA DOUBLE SCATTERING PROCESSES

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ABSTRACT

We propose that measuring the $Q^2$ dependence of the number of final-state interactions of the recoil protons in quasi-elastic electron scattering from light nuclei is a new method to investigate Color Coherent effects at intermediate values of $Q^2$ ($\sim$ few $(GeV/c)^2$). This is instead of measuring events without final-state interactions. Our calculations indicate that such measurements could reveal significant color transparency effects for the highest of the energies initially available at CEBAF. Measurements that detect more than one hadron in the final state, which require the use of large acceptance (4$\pi$) detectors, are required.
1 Introduction

How does quantum chromodynamics QCD work? Although the purely perturbative regime is well understood, much more needs to be done to answer the question. To be specific, consider the process of elastic electron-nucleon scattering. At high enough values of the momentum transfer $Q^2$, pQCD is valid and the dominant contributions to the form factor arise from components in which the quarks and gluons are closely separated [1]. Such components have been called point-like configurations PLC [2, 3]. However, it is not at all clear that this regime is relevant for the $Q^2$ for which experiments exist [4, 5].

Another idea [6] is that PLC may arise from non-perturbative effects. In this case the PLC can be thought of as precursors to the dominance of pQCD. Indeed, the work of Ref. [6] shows that realistic quark models of a nucleon which contain a Coulomb type interaction at small interquark distances and Skyrmion models both allow a PLC to form starting from a momentum transfer as small as $1 - 2 (GeV/c)^2$. The opposite behavior is expected in mean-field quark-models of a nucleon, and in Chiral Lagrangian models where a nucleon is considered as a structureless particle with a meson cloud.

Thus, the pressing problem now is to find experimental measurements that help to distinguish between the two classes of models. The ideas of pQCD lead to the suggestion [6, 8] that the $A$-dependence of quasi-exclusive processes

$$l(h) + A \rightarrow l(h) + p + (A - 1)$$

(1)

could be used to determine the configurations that dominate in hard two-body reactions. If PLC are produced, QCD color screening causes the ejectile-nucleon interaction to be small, and the ejected object would escape from the nucleus with no or reduced final-state interactions. Hence the cross section of reaction (1) would be proportional to $A$. This is the Color Transparency (CT) phenomenon. This prediction of a spectacular change of the $A$-dependence of the cross section of reaction (1) has led to a number of further theoretical analyses [9, 10, 11, 12, 13, 14] and to the first attempts to observe the phenomenon using the BNL proton beam [15] and ongoing experimental investigations.
at SLAC (NE18) \cite{16} and BNL \cite{17}.

But there is an impressive practical problem which emerges in looking for CT effects in actual experiments. The PLC expands rapidly to the size of a normal hadron while propagating through the nucleus, unless the Lorentz factor is very large \cite{9,10}. This does not occur for intermediate values of $Q^2$. Therefore, the observation of CT effects at intermediate values of $Q^2$ requires the suppression of wavepacket expansion effects. This can be achieved by using the lightest nuclear targets, where the propagation distances are small. In this case, the transparency is close to unity, so the effects of CT in $(e,e'p)$ reactions can not be large. However, if we select a process where the produced system interacts in the final state, a double scattering event, then the color coherent effects would manifest themselves as a decrease of the probability for final-state interactions with increasing $Q^2$. Then one would observe a $(e,e'pp)$ reaction with one proton having a momentum close to that of the virtual photon and a second proton other with a momentum $p_2$ of about 400 $MeV/c$, large enough so that its production is dominated by the effects of final-state interactions. The obvious advantage of looking for the processes with rescattering is that in this case it is possible to observe an effect decreasing from the value expected without CT (eikonal approximation) to zero. Thus, the measured cross section is to be compared with a vanishing quantity so that the relevant ratio of cross sections runs from 1 to infinity.

The value of $p_2 \approx 400$ $MeV/c$ is chosen as large enough so that its origin is the effects of nuclear correlations. This is also but small enough ($-t=0.15$ $GeV^2$) so that the forward peaked rescattering is allowed to occur. The effects of correlations are small, but may not be entirely negligible. Indeed, if it happens that the final state interactions are removed by the effects of color transparency, a non-zero cross section could be caused by the correlation effects. Moreover, the kinematics of scattering from a pair of nucleons causes the angular distributions for an correlation dominated $e,e'pp$ reaction to be substantially different from rescattering dominated ones. Thus one could eliminate this unwanted “background” using this signature and the $4\pi$ detector.
We consider two models to explore this new possibility to observe CT effects. The first calculation uses the quantum diffusion model of [3] and a Monte Carlo simulation. This is an intuitive calculation that allows us to consider a wide range of nuclei and to account also for the dependence of the rescattering amplitude slope on the size of a rescattered configuration. The second calculation employs the three-state model of Ref. [18]. This allows us to perform a quantum mechanical calculation for the $^3He$ target, though with an oversimplified wave function. Both models are schematic and do not allow us to reliably estimate the effects of the background of $(e,e'pp)$ events produced by nuclear correlations.

It is worth emphasizing that in both models the CT effect predicted for the $A(e,e'p)$ process is rather small and does not contradict preliminary data of the NE-18 experiment. We demonstrate that both models lead to quite substantial CT effects for the rescattering reaction of interest.

Our results emphasize the need for both experimental study of this reaction, feasible at CEBAF, and for more refined calculations including realistic wave functions of $^3,^4He$ and short range correlations for heavy nuclei.

2 Probabilistic estimation of rescattering in $(e,e'pp)$

In general, the calculation of $A(e,e'NN)$ processes is rather cumbersome. So we will use a simple probabilistic description to illustrate the importance of color screening (reduced interactions of small objects) and the space-time picture of a PLC evolving to a normal state. A quark-basis description is used to describe this evolution. This intuitive approach gives us the possibility of considering heavier targets than for the next section. Thus, in this approximation, the $(e,e'pp)$ cross section can be represented as:

$$
\frac{d^9\sigma}{dE_e d\Omega_e d^3p_f d^3p_r} = \int d\Omega_f \frac{d\sigma_{ge}^{PWIA}}{dE_e d\Omega_e d\Omega_f} \cdot \Phi_{A-1}^2(|\vec{p}_r|) \cdot \frac{2|\vec{p}_f|}{E_f} \cdot \langle \int \rho(l) \frac{d\sigma_{pp}(l, Q^2)}{dt} dl \exp \left(-\int_z^\infty \sigma_{tot}(l', Q^2)\rho(l')dl'\right) \rangle .
$$
\[
\delta(q_0 + M_A - E_{f'} - E_r - E_{A-2})
\]

where \((E_{f'}, \vec{p}_{f'})\) and \((E_r, \vec{p}_r)\) are the energy and momenta of the produced (by virtual photon with energy and momentum \((q_0, \vec{q})\)) and rescattered (by pp-elastic scattering) protons respectively, \(\vec{p}_{r0} = \vec{p}_r - (\vec{p}_f - \vec{p}_{f'})\), where \(\vec{p}_f\) is the momentum of produced protons before the rescattering. For the simple case of quasifree rescattering
\[
E_{A-2} \approx M_{A-2} + p_{A-2}^2 / 2M_{A-2},
\]
where \(\vec{p}_{A-2} = \vec{q} - \vec{p}_{f'} - \vec{p}_r\) (in calculations we specify the kinematics where \(\vec{p}_{A-2} \approx 0\)). The \(\frac{d\sigma_{PWIA}}{dE_e d\Omega_e d\Omega_f}\) represents the quasielastic \((e, e'p)\) cross section in the plane wave impulse approximation (PWIA), \(\Phi_A^2(|\vec{p}_{r0}|)\) is the momentum distribution of rescattered protons in \(A - 1\) residual nucleus. The term in the "\(\langle \rangle\)" gives the probability for the processes where final nucleon experience only one elastic collision. The brackets means that the integral over the transverse coordinates is taken. (Those coordinates are left implicit in Eq. (2) to simplify the notation.) The quantity \(\frac{d\sigma_{pp}(l,Q^2)}{dt}\) represents the elastic scattering between a proton and a PLC that has moved a distance \(l\) from its point of formation.

Eq. (2) accounts for the geometry of the process, but not the quantum mechanical averaging over the nuclear wavefunction. This may introduce uncontrollable errors in our calculations, especially in the case of heavy nuclei where interference effects between elastic rescattering and nucleon correlations could be large. However, the present calculation demonstrates the significant size of color transparency effects and the noticeable dependence on momentum transfer.

It is easy to check that after integrating the Eq. (2) by \(p_{f'}\) and \(p_r\) and summing over elastic rescattering states one recovers the conventional probabilistic description for the quasielastic \((e, e'p)\) cross section. Note that above equation is better suited for the case of quasifree \((e,e')\) scattering at \(x = \frac{Q^2}{2m_{q0}} \approx 1\), where Fermi motion effects are a correction.

To estimate the color coherent effects in Eq. (2) we introduce the differential cross section for the PLC elastic scattering with momentum transfer \(t\) and at the distance \(l\)
from the point where the photon has been absorbed:

\[
\frac{d\sigma_{pp}(l, Q^2)}{dt} = \frac{\sigma_{tot}^2(l, Q^2)}{16\pi} \cdot e^{bt} \cdot \frac{G_N^2(t \cdot \sigma_{tot}(l, Q^2)/\sigma_{tot})}{G_N^2(t)}
\]  

(3)

Here \(\sigma_{tot}\) is a proton-nucleon total cross section, \(b\) is the slope of elastic \(NN\) amplitude, \(G_N(t) \approx (1 - t/0.71)^2\) is the Sachs form factor. The last factor in Eq. (3) accounts for the difference between elastic scattering for point-like and average configurations, which is based on the observation that \(t\) dependence of \(d\sigma_{h+N \rightarrow h+N}/dt \sim G_h^2(t) \cdot G_N^2(t)\).

In Eq. (3) \(\sigma_{tot}(l, Q^2)\) is the effective total cross section of the interaction of the PLC at the distance \(l\) from the interaction point. The quantum diffusion model \([9]\) provides the estimate:

\[
\sigma_{tot}(l, Q^2) = \sigma_{tot}\left\{\left(\frac{l}{l_h} + \frac{\langle r_t(Q^2)^2 \rangle}{Q^2} \cdot (1 - l/l_h)\right) \cdot \Theta(l_h - l) + \Theta(l - l_h)\right\}
\]

(4)

where \(l_h = 2p_f/\Delta M^2\), with \(\Delta M^2 = 0.7 \text{ GeV}^2\). Here \(\langle r_t(Q^2)^2 \rangle\) the average transverse size squared of the configuration produced at the interaction point. In several realistic models considered in \([6]\) it can be approximated as \(\langle r_t(Q^2)^2 \rangle \sim 1 \text{ GeV}^2\) for \(Q^2 \geq 1.5 \text{ GeV}^2\). Note that the effects of expansion cause the results to rather insensitive to the value of this ratio whenever it is much less than unity.

It is tedious, but not difficult, to demonstrate that the semiclassical calculation which includes quantum diffusion gives the exact result in QCD for \(\sigma_{tot}(l, Q^2)\) at sufficiently large \(Q^2\) in the leading logarithmic approximation and beyond. At moderate \(Q^2\), quantum diffusion is a guess based on an analogy with pQCD and on the success of related approaches in the description of final states in \(e^+ e^- \rightarrow \text{hadrons}\). The success of dispersion sum rules in describing semi-hard processes indicates that matrix elements of correlators of currents can be effectively be calculated in either of the quark gluon or hadron bases.

Note that Eq. (2) has been obtained by using classical mechanics to calculate the rescattering processes. The difference between this and the semiclassical approximation (see e.g. \([3]\)) is that in the classical mechanical description one averages the absolute
square of the transition matrix element instead of averaging the amplitude and then squaring. The shortcoming of this approach to rescattering processes is that it gives no possibility to account properly for the correlations between nucleons. Thus, we use a simple one-body Wood-Saxon parametrization of the nuclear density function $\rho(r)$ appearing in Eq. (2). The authors hope to improve this in future publications.

We turn to the results. First, Fig. 1(a) and Fig. 2(a) show the predictions of probabilistic approach and independent particle model for the standard transparency $T$ defined as $T = \frac{\sigma(e,e'pp)^{exp}}{\sigma(e,e'p)^{exp}}$ (see e.g. Refs. [10, 19, 20]). We find that predictions of this simplified approach agree well with more conventional quantum mechanical optical model and CT calculations [11, 20] within a few percent. The $(e,e'pp)$ calculations presented in Figs. 1(b-d), 2(b-d) are performed for the kinematics where $\vec{p}_f \sim \vec{p}_f' \sim \vec{q}$ and $|\vec{q}| \gg |\vec{p}_r| \approx 0.4 \text{ GeV}/c$ to suppress Fermi motion and evaporation effects. For $pp$ rescattering this corresponds to the transferred momentum $t \approx -0.15 \text{ GeV}^2$. In Fig. 1(b) and Fig. 2(b) we present results of calculations of the transparencies defined as:

$$T_{el} = \frac{\frac{d\sigma^9}{dE_e'd\Omega_e'd^3p_{f'}d^3p_r}}{\frac{d\sigma^5}{dE_e'd\Omega_e'd\Omega_f}}^{PWIA}.$$

These are computed in Glauber approximation (solid curve) and including the effects of CT. The solid and dashed curves have a ratio of about 1.5 for $Q^2 \approx 7\text{GeV}^2/c^2$ so that the effects of CT are substantial.

Another useful quantity is the ratio of the cross sections of double scattering $(e,e'p_fp_r)$ and $(e,e'p_f)$ processes studied above. Note that the uncertainties in the estimate of PLC production cancel to large extent in this ratio. Another advantage is that there is more sensitivity to CT effects. Indeed using the $T$ and the $T_{el}$ defined above (see also [21]) one gets:

$$\frac{T_{el}}{T} = \frac{\sigma(e,e'pp)^{exp}}{\sigma(e,e'p)^{exp}} \approx \frac{\langle \int \rho(l) \frac{d\sigma^{pp}(l,Q^2)}{dt} dl \exp (-\int_{l}^{\infty} \sigma_{tot}(l',Q^2)\rho(l')dl') \rangle}{\langle \exp (-\int \sigma_{tot}(l,Q^2)\rho(l)dl) \rangle},$$

where the effects of CT cause the numerator to decrease and the denominator to increasing. As a result the difference between CT and Glauber calculations is more pronounced
for these quantities - compare the solid and dashed curves in Figs. 1(c), 2(c). Note that from the experimental point of view, Eq. (3) represents the ratios of quantities measured in the same run of experiment, that allow to avoid many problems (such as necessity to calculate a denominator in $T$, the radiative corrections, etc.).

This point is further demonstrated in the Fig. 1(d) and Fig. 2(d) where calculations of CT for $T, T_{el}, T_{el}/T$ are normalized by the corresponding quantities computed in the Glauber approximation. The effect of deviation from the Glauber calculation is the smallest for $T$ - solid curves, larger for $T_{el}$ - dashed curves, and the largest for the $T_{el}/T$ ratios - dotted curves. These figures demonstrate that color transparency is a significant effect for the case of double rescattering processes even at intermediate $Q^2$. Note that results of calculation in this section are consistent with the predictions of three state model considered in the next section.

The results displayed above are for double rescattering processes at fixed $t$ or at fixed relative transverse (to the direction of virtual photon) momenta of two final proton. These cross sections should tend to zero when the color transparency phenomena become important. Another observable is the $t$ ($p_t$) dependence of $(e,e'p)$ cross section at a fixed (large) value of $Q^2$. Here, one expects an enhancement of the $(e,e'pp)$ cross section starting at some transferred momenta $t$ for the CT cases compared to that of the Glauber approximation [2]. This is because at sufficiently large values of $t$ the elastic rescattering are dominated by the interaction of the nucleon PLC components. Here the underlying physics is that large angle scattering occurs anyway in a PLC, and if the initial state is in PLC one does not have to pay the price of finding the projectile in PLC.

The $t$-dependence of the cross section ratios is examined in Fig. 3. We use the parametrization of $pp$ cross sections in Eq. (3). The dashed curve shown in this figure indicates a significant enhancement of $\sigma^{CT}/\sigma^{Glauber} |t| \geq 1$ GeV$^2$ for sufficiently high $Q^2$ ($\geq 10$ GeV$^2$). The solid curve for $Q^2=4$ GeV$^2$ shows a slower $t$-dependence. Furthermore, there is not much of a change as $-t$ is changed from 0 to 0.16 GeV$^2$. This supports the approximations made in Sec. 3, where this dependence is neglected.
3 Calculation of $^3\text{He}(e,e'pp)$ in the three state model

The measurement of the $^3\text{He}(e,e'pp)$ reaction of our interest involves detecting a high momentum proton carrying almost all of the virtual photon’s three-momentum, and another of moderate momentum, about 400 $\text{MeV}/c$. The value 400 $\text{MeV}/c$ is chosen as small compared with virtual photon’s momentum and large compared with the momentum of a bound proton. If such a 400 $\text{MeV}/c$ proton is detected, we can be almost certain that it was produced as a result of a final state interaction. If CT occurs, there are no final state interactions and our $^3\text{He}(e,e'pp)$ reaction will not take place.

In order to make an illustrative calculation for such a process, we use the three-state model of Ref. [18]. In this model, the resonance and continuum regions excited in diffractive dissociation processes are each modelled as one state. Since the rescattering process involves only a small momentum transfer, and because the major contribution at moderate energies is given by the color fluctuations near the average size we shall ignore the variation of the $t$ dependence of the scattering amplitude with the distance from the point of hard the interaction. This approximation means that we neglect any experimentally observed difference between the form factors of different states. The possible effect of neglecting this $t$-dependence is examined in section 2, where the effects are found to be small.

The matrix element for our $(e,e',pp)$ process is given by

$$M_1(\vec p_1, \vec p_2, \vec p_3) = \langle N_1, \vec p_1; N_2, \vec p_2; N_3, \vec p_3 \mid \sum_{i\neq j} T_j G_0 T^{(i)}_H \mid ^3\text{He} \rangle,$$  \hspace{1cm} (7)

where the hard scattering operator $T^{(i)}_H$ acts on the $i$'th proton, which propagates a wavepacket, via the free Green’s function $G_0$, until a soft final-state interaction $T_j$ leads to the knockout of the $j$'th proton. Most of our interest here concerns the $(e,e'pp)$ reaction, but we also compute cross sections for the $(e,e'N^*p)$ and $(e,e'N^{**}p)$ processes. The expression for $N^*$ or $N^{**}$ production is similar to that of Eq. (7) except that the $N_1, \vec p_1$ is replaced by $m_1, \vec p_{1m}$ where $m$ is either of the two resonances. We shall evaluate Eq. (7) in some detail, but present the general form of the final result.
The above notation is general, our model is specified by defining the operators $T_H^{(i)}$ and $T_j$. We assert that the action of $T_H^{(i)}$ on a nucleon leads to the formation of a PLC which does not interact:

$$T_H |N\rangle = |\text{PLC}\rangle,$$

where only a single baryon is involved and the label $i$ is superfluous. The PLC is a coherent sum of three states so that

$$|\text{PLC}\rangle = \alpha |N\rangle + \beta |N^*\rangle + \gamma |N^{**}\rangle,$$

where $\alpha$, $\beta$ and $\gamma$ represent the elastic and inelastic transition form factors in this model. These coefficients depend on $Q^2$; here we assume each has the same functional form. Thus the $Q^2$ dependences of each of the electromagnetic form factors are the same and disappear when considering ratios of cross sections.

Next we discuss $T_j$. We treat this operator in lowest order so that it is the transition matrix $\hat{T}_S$ for the soft final state interaction between the ejected wavepacket $(G_0|\text{PLC}\rangle)$ and the second proton. This operator is defined by the condition that

$$\hat{T}_S |\text{PLC}\rangle = 0.$$

It is this condition that separates models with color transparency from other models which merely include use a matrix to include coupling to excited states. In particular, $\hat{T}_S$ is the most general $3 \times 3$ matrix that annihilates the PLC:

$$\hat{T}_S = \begin{pmatrix}
1 & -\frac{\alpha + \gamma*}{\beta} & -\epsilon \\
-\frac{\alpha^* + \gamma*\epsilon}{\beta^*} & \frac{\mu}{\beta^*\gamma} & \frac{|\alpha|^2 - \alpha^*\gamma\epsilon^* - \mu|\beta|^2}{\beta^*\gamma} \\
-\epsilon^* & \frac{|\alpha|^2 - \alpha^*\gamma\epsilon - \mu|\beta|^2}{\beta^*\gamma} & \frac{\mu|\beta|^2 - |\alpha|^2 + 2\text{Re}(\alpha^*\gamma^*)}{|\gamma|^2}
\end{pmatrix}.$$

The work of Ref. [18] showed how to use data for cross section fluctuations in proton-proton diffractive scattering to constrain the parameters of this matrix. We will use the parameter sets of that work.

It is useful to present the coordinate space representations for the operators $T_j$, $G$ and $T_H^{(1)}$. The final-state interaction is specified by

$$\langle N_1, \vec{R}_1; N_2, \vec{R}_2; N_3, \vec{R}_3 | T_1 | m_1, \vec{R}_1; m_2, \vec{R}_2; m_3, \vec{R}_3 \rangle =$$
\[
\left( \hat{T}_S \right)_{m_1,N_1} \delta_{m_2,N_2} \delta_{m_3,N_3} \delta^{(3)}(\vec{R}_1 - \vec{R}_2) \delta^{(3)}(\vec{R}_3 - \vec{R}_3)^{(3)} \delta\left(\vec{R}_1 - \vec{R}_2\right) \delta^{(3)}(\vec{R}_3 - \vec{R}_3); \tag{12}
\]

where the range of this two-body optical potential is assumed to be exactly zero, as specified by the above delta function.

The Green’s function operator, which is diagonal in the hadronic mass eigenstate basis, is given by:

\[
\langle m_1, \vec{R}_1; N_2, \vec{R}_2; N_3, \vec{R}_3 | G | m_1, \vec{R}_1'; N_2, \vec{R}_2'; N_3, \vec{R}_3' \rangle =
\]
\[-\frac{e^{ip_{1m}|\vec{R}_1 - \vec{R}_1'|}}{4\pi|\vec{R}_1 - \vec{R}_1'|} \delta^{(3)}(\vec{R}_2 - \vec{R}_2') \delta^{(3)}(\vec{R}_3 - \vec{R}_3'). \tag{13}\]

Here, the quantity \( p_{1m} \) is the momentum of the \( m'th \) component of the wavepacket. We take all of the final state nucleon wave functions as plane waves so that energy conservation yields

\[
|p_{1m}| = \sqrt{\left( \frac{Q^2}{2M_N} + 3M_N - \sqrt{p_2^2 + M_N^2} - \sqrt{p_3^2 + M_N^2} \right)^2 - M_N^2}. \tag{14}\]

In this sense, \( \vec{p}_1 \equiv \vec{p}_{1N} \). Lastly, the hard scattering operator is given by

\[
\langle m_1, \vec{R}_1; N_2, \vec{R}_2; N_3, \vec{R}_3 | T^{(1)}_H | \text{}^3He \rangle = F_{m,N}(Q^2)e^{i\vec{q} \cdot \vec{R}_1}\psi_{\text{}^3He}(\vec{R}_1, \vec{R}_2, \vec{R}_3). \tag{15}\]

Here, the quantities \( F_{m,N}(Q^2) \) are the elastic and inelastic transition form factors, \( \alpha, \beta \) and \( \gamma \). Since only ratios of these quantities ultimately appear, we neglect the possible dependence on \( Q^2 \). We have also introduced the position space \( \text{}^3He \) wavefunction, \( \psi_{\text{}^3He} \).

We put all of the pieces together, insert complete sets of states and arrive at the following expression for the scattering matrix element:

\[
\mathcal{M}_1(\vec{p}_1, \vec{p}_2, \vec{p}_3) = i |\vec{p}_1| \sum_m \left( \hat{T}_S \right)_{N,m} F_{m,N}(Q^2) \int (d^3R)_{cm} e^{-i\vec{p}_1 \cdot \vec{R}_1 - \vec{p}_2 \cdot \vec{R}_2 - \vec{p}_3 \cdot \vec{R}_3} e^{i\vec{q} \cdot \vec{R}_1}
\]
\[\times \frac{e^{ip_{1m}|\vec{R}_1 - \vec{R}_2|}}{4\pi|\vec{R}_1 - \vec{R}_2|} \psi_{\text{}^3He}(\vec{R}_1, \vec{R}_2, \vec{R}_3), \tag{16}\]

where \( d^3R = d^3R_1d^3R_2d^3R_3 \) and the subscript \( cm \) is to remind us that the center of mass of the \( \text{}^3He \) nucleus, which is irrelevant for these sorts of considerations, is factored out.
of the problem. To this end, we introduce the usual Jacobi coordinates,

\[
\vec{R}_1 = \vec{R}_{cm} - \sqrt{\frac{1}{2}} \vec{\rho} + \sqrt{\frac{1}{6}} \vec{\lambda},
\]

(17)

\[
\vec{R}_2 = \vec{R}_{cm} + \sqrt{\frac{1}{2}} \vec{\rho} + \sqrt{\frac{1}{6}} \vec{\lambda},
\]

(18)

\[
\vec{R}_3 = \vec{R}_{cm} - \sqrt{\frac{2}{3}} \vec{\lambda}.
\]

(19)

Now, the above matrix element can be expressed purely in terms of the relative coordinates, \(\vec{\rho}\) and \(\vec{\lambda}\). The last piece of information we need to specify before the calculation can proceed is the configuration space representation of the helium wavefunction we use. We take the simple parameterization

\[
\psi_{3He}(\rho, \lambda) = \frac{\alpha^3}{\pi^{3/2}} e^{-\alpha^2(\rho^2 + \lambda^2)/2}.
\]

(20)

Here, we choose the parameter \(\alpha = 1.5 \text{ fm}^{-1}\). Because of the delta functions which appear in Eq. (16), the 18 dimensional integral reduces to a simple integral over \(d^3 \rho d^3 \lambda\), a vast improvement. With the above choice for the helium wavefunction, the \(\lambda\) and \(\rho\) integrals separate. The resulting amplitude for producing a final state baryon \(m = N, N^*, N^{**}\) is given by

\[
\mathcal{M}_1(\vec{p}_1, \vec{p}_2, \vec{p}_3) = \sum_{m'} \left( \hat{T}_S \right)_{m,m'} F_{m',N}(Q^2) \frac{\alpha^3}{4\sqrt{2\pi^{5/2}}} I_\lambda(\vec{p}_1, \vec{p}_2, \vec{p}_3) I_\rho(\vec{p}_1, \vec{p}_2),
\]

(21)

where

\[
I_\lambda(\vec{p}_1, \vec{p}_2, \vec{p}_3) = \int d^3 \lambda e^{-\alpha^2 \lambda^2/2} e^{i\sqrt{1/6} \cdot (2\vec{\lambda} - \vec{p}_1 - \vec{p}_2 + \vec{q})},
\]

(22)

\[
I_\rho(\vec{p}_1, \vec{p}_2) = \int d^3 \rho \frac{1}{\rho} e^{-\alpha^2 \rho^2/2} e^{-i\sqrt{2\pi^2} \rho} e^{i\sqrt{1/2} \cdot (\vec{q} + \vec{p}_1 + \vec{p}_2)}.
\]

(23)

We can use these expressions to explain why the effects of the using a different \(t\)-dependence for the different amplitudes \(\alpha, \beta, \gamma\) are not very large. Including such these “finite range effects” would modify the integral \(I_\rho\). The factor \(1/\rho e^{-i\sqrt{2\pi^2} \rho}\) could be replaced by, for example, \(1/\rho e^{-i\sqrt{2\pi^{1/2}} M_{1/2} \rho} - 1/\rho e^{-i\Lambda_{1/2} \rho}\) where \(\Lambda_{1/2}\) represents the \(t\)-dependence of the relevant amplitudes. The finite size effects are determined by the differences
between the parameters $\gamma_m = \alpha \Lambda_m$. Each of the $\gamma_m$ is fairly small because the size of $^3\text{He}$ is larger than the range of the interactions in $\hat{T}_S$. The differences of small quantities are even smaller. Thus it seems safe to neglect such differences, at least if are considering small-$t$ rescattering with $-t \approx 0.15 \text{ GeV}^2$.

We return to the evaluation of $M_1$ by realizing both $I_\rho$ and $I_\lambda$ can be done analytically and the resulting matrix element can be put in closed form. The result for the $(e,e'pp)$ process is that

$$M_1(\vec{p}_1, \vec{p}_2, \vec{p}_3) = \frac{p_1\sqrt{\pi}}{2\alpha s} \sum_m \left(\hat{T}_S\right)_{N,m} F_{m,N}(Q^2) e^{-\frac{x^2}{2\alpha^2}}$$

$$\times \left[ e^{-x^2} \text{erfc}(-ix_+) - e^{-x^2} \text{erfc}(-ix_-) \right], \quad (24)$$

where $s = |\vec{s}|$, $v = |\vec{v}|$ and

$$\vec{v} = \frac{1}{\sqrt{6}} \left( 2\vec{p}_3 - \vec{p}_1 - \vec{p}_2 + \vec{q} \right), \quad (25)$$

$$\vec{s} = \frac{1}{2} \left( \vec{q} + \vec{p}_1 + \vec{p}_2 \right), \quad (26)$$

$$x_+ = \frac{p_{1m} + s}{\alpha}, \quad (27)$$

$$x_- = \frac{p_{1m} - s}{\alpha}, \quad (28)$$

$$i \text{erfc}(-ix) = i - \frac{2}{\sqrt{\pi}} \int_0^x dt \, e^{+t^2}. \quad (29)$$

The CT cross section for double scattering events is then calculated by taking the absolute square of this quantity, $\sigma^{(e,e'pp)} \sim |M_1|^2$. To see that this matrix element gives CT, we consider the very high energy limit. At very high energies, $x_- \to 0$ and $x_+$ gets big. The Gaussian factor of $e^{-x^2}$ kills the rapidly growing error function completely. Then, since erfc(0) = 1, the matrix element reduces to some numbers times the quantity

$$\sum_m \left(\hat{T}_S\right)_{N,m} F_{m,N} = 0$$

and CT is obtained.

This closed form is amusing, but does not provide much insight and is also difficult to evaluate. Therefore, we use Eq. (21) and evaluate $I_\lambda$ analytically and $I_\rho$ numerically. We need to specify the kinematics in order to proceed. We choose the case where the photon hits one proton which moves quickly through the remaining nuclear medium and
interacts with the other proton which leaves with momentum $|\vec{p}_2| = 400 \text{ MeV}/c$, while the neutron remains a spectator, $|\vec{p}_3| = 0$. With these kinematics, $\vec{s} = \vec{q}$ and $\vec{v} = 0$. As noted previously, the helium bound state parameter is taken to be $\alpha = 1.5 \text{ fm}^{-1}$.

Now, since we are calculating a cross section which should vanish in the limit of full transparency, we normalize our CT cross sections to the DWBA or Glauber cross sections. The Glauber process is defined by first using $\hat{T}_S$ for the amplitude to produce an excited state. Then one uses an optical potential (a diagonal operator) to describe the escape of the given excited state from the nucleus. Thus the Glauber amplitude $\mathcal{M}_1^G(\vec{p}_1m, \vec{p}_2, \vec{p}_3)$ is given by

$$\mathcal{M}_1^G(m, \vec{p}_1m, \vec{p}_2, \vec{p}_3) = \left(\hat{T}_S\right)_{m,m} F_{m,N}(Q^2) \frac{\alpha^3}{4\sqrt{2\pi}5/2} I_\lambda(\vec{p}_1, \vec{p}_2, \vec{p}_3) I_\rho(\vec{p}_1, \vec{p}_2). \quad (30)$$

This so-called distorted wave Born approximation to the amplitude leads to DWBA cross sections labelled as $\sigma_{\text{Glauber}}$. Thus, for the nucleon, the cross section ratio starting out at unity, where we know the DWBA works well, and decreasing to zero at high momentum transfers. The rate of decrease depends on the initial wavepacket or PLC.

The motivation for the calculation of the double scattering events is that the Glauber treatment of the single scattering events, $(e, e'p)$ events, is already close to the plane wave values. But we have argued that examining processes where the cross section vanishes in the CT limit and the $Q^2$ variation is rapid at moderate values of $Q^2$ increases the sensitivity to CT effects. Figures 1 and 2 (d) show that we can further increase the $Q^2$ variation by including these relatively slowly varying single scattering events by considering the ratio

$$\left(\frac{T_{el}}{T}\right)_{CT/G} = \frac{\sigma_{CT}^{e,e'pp}}{\sigma_{CT}^{e,e'p}} / \frac{\sigma_{Glauber}^{e,e'pp}}{\sigma_{Glauber}^{e,e'p}}. \quad (31)$$

This is the ratio of $T_{el}/T$ for CT to Glauber calculations shown in Figs. 1 and 2 (d). This expression is generalized for the case of resonance production by replacing the "$pp$" by $N^*p$ or $N^{**}p$.

In order to calculate the cross section for single scattering events, we must go back to the original amplitude and calculate the Born term. Thus, we write the full amplitude,
to first order in the interaction, as a sum of two terms

\[ \mathcal{M}(\vec{p}_1, \vec{p}_2, \vec{p}_3) = \mathcal{M}_0(\vec{p}_1, \vec{p}_2, \vec{p}_3) + \mathcal{M}_1(\vec{p}_1, \vec{p}_2, \vec{p}_3), \]  

(32)

where \( \mathcal{M}_1(\vec{p}_1, \vec{p}_2, \vec{p}_3) \) is given, generally, by Eq. (24). It is easy to calculate the Born term. The result is

\[ \mathcal{M}_0(\vec{p}_1, \vec{p}_2, \vec{p}_3) = \frac{8\pi^{3/2}}{\alpha^3} e^{-(w^2+v^2)/2\alpha^2} F_{NN}(Q^2), \]  

(33)

where \( v = |\vec{v}| \) in Eq. (25), \( w = |\vec{w}| \) and

\[ \vec{w} = \frac{1}{\sqrt{2}} (\vec{p}_1 - \vec{p}_2 - \vec{q}). \]  

(34)

Now, the \((e,e'p)\) cross section has different kinematics than the \((e,e'pp)\) one. In particular, for single nucleon knockout, we imagine that the detected proton is carrying all of the photon’s three-momentum, which leaves very little for the other two nucleons. Thus, we imagine that \( \vec{p}_1 = \vec{q} \) and \( \vec{p}_2 = \vec{p}_3 = 0 \). In these kinematics \( \vec{w} = \vec{v} = 0 \). The relevant ratio we want to consider then, is

\[ (T_{el}/T)_{CT/G} = \left| \frac{\mathcal{M}_1^{CT/G}(\vec{p}_1, |\vec{p}_2| = 400 \text{ MeV}, 0)}{\mathcal{M}_{CT/G}(\vec{p}_1, 0, 0)} \right|^2, \]  

(35)

where the above notation means

\[ \mathcal{M}_{CT/G}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \equiv \frac{\mathcal{M}_{CT}(\vec{p}_1, \vec{p}_2, \vec{p}_3)}{\mathcal{M}_{Glauber}(\vec{p}_1, \vec{p}_2, \vec{p}_3)}. \]  

(36)

We present results for the quasi-elastic production of nucleons, \( N^* \)’s and \( N^{**} \)’s for two sets of masses of the excited states, in the form of ratios \( \Sigma_{2/G;1/G} \). The generalization of Eq. (7) to the quasi-elastic production of nucleon isobars is straightforward. In Figure 4 we take \( M_{N^*} = 1.44 \text{ GeV} \) and \( M_{N^{**}} = 1.80 \text{ GeV} \) while in Figure 5 we take \( M_{N^*} = 1.80 \text{ GeV} \) and \( M_{N^{**}} = 3.0 \text{ GeV} \). The values of these masses control the rate of PLC expansion and therefore the energy dependence of the computed ratios of cross sections. The first set, with the lower values of masses, leads to a slow rate of expansion and larger CT effects, while the second set has a quicker rate of expansion. The parameters of the matrix \( \hat{T}_S \) are specified in the figures and in Ref. [18].
Fig. 4 shows that $\Sigma_{2/G;1/G}$ decreases by factors between 2 and 10, as $Q^2$ is varied from about 1.5 to 7 GeV$^2$/c$^2$. These factors are reduced somewhat by increasing the values of the excited state masses, an expected effect. The use of the parameter set shown on the lower right along with of higher values of the masses leads to a result with little variation with $Q^2$. This is the only such case. We also note that the $(e,e'p)$ Glauber cross section for the nucleon (which is essentially the same as the CT cross section at low energies), using the above equations, remains constant at approximately 0.68. This number is just controlled by the value of the $pp$ cross section $\sigma = 40 \text{mb}$.

The advantage of using double scattering events, as described here, to explore the effects of CT is clear upon looking at the figures. We see that there are significant effects, as large as an order of magnitude, for values of $Q^2$ as low as $Q^2 \approx 6$ and $8$ GeV$^2$/c$^2$.

4 Summary and Conclusions

We have shown that the effects of color transparency can be investigated at intermediate values of $Q^2 \geq 5 - 6$ GeV$^2$ by detecting a final quasielastic proton and another with momentum about 300-400 MeV/c. We have used oversimplified wave functions, which do not allow us to investigate the effects of the nuclear correlations. This could be corrected in future studies. Most important would be detailed experimental study of this reaction which is feasible at CEBAF.

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Figure Captions

Figure 1. The dependence of nuclear transparencies to $Q^2$ for $^4He$ target. (a) - transparencies for $(e,e'p)$ processes, solid – Glauber approximation, dashed – color transparency approximation. (b) – Transparency $T_{el}$ as defined in Eq. (5), for $(e,e'pp)$ reactions: notations same as in (a). (c) – ratio of transparencies ($\frac{T_{el}}{T}$) defined in Eq. (6): notations same as in (a). (d) – Ratios of $T$, $T_{el}$ and $\frac{T_{el}}{T}$ at the case of color transparencies to the corresponding quantities at Glauber approximations; Solid line - $\frac{T_{CT}}{T_{GA}}$, dashed line - $\frac{T_{CT}}{T_{el}}$ and dotted line - $\frac{\left(\frac{T_{el}}{T}\right)_{CT}}{\left(\frac{T_{el}}{T}\right)_{GA}}$.

Figure 2. Same as in Fig.1, for $^{12}C$.

Figure 3. The $t$ dependence of nuclear transparencies - $\frac{(\sigma(e,e'pp)_{CT})}{\sigma(e,e'pp)_{GA}}$ for $^4He$ at $(e,e'pp)$ reaction at $Q^2 = 4GeV^2$ (solid line) and $Q^2 = 10GeV^2$ (dashed line), provided that slope of $pp$ cross section is changed according to Eq. (3).

Figure 4. Cross sections ratios $\Sigma_{2/G:1/G}$ in three-state model for $^3He$. Solid: quasi-elastic proton production. Dashed: quasielastic $N^*$ (1.4 GeV) production. Dotted: quasielastic $N^{**}$ (1.8 GeV) production. The parameters $\alpha, \beta, \gamma$ and $\epsilon$ define the baryon-nucleon interaction, see Ref. [18].

Figure 5. The same as in Fig. 4 but the masses are now $(N^*,N^{**})=(1.8, 3.0)$ GeV.
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