Matrix elements and Parton Shower in the event generator BABAYAGA

G. Balossini\textsuperscript{a}, C.M. Carloni Calame\textsuperscript{ba}, G. Montagna\textsuperscript{ab}, O. Nicrosini\textsuperscript{b}, F. Piccinini\textsuperscript{ba}

\textsuperscript{a}Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, via A. Bassi 6, Pavia (Italy)

\textsuperscript{b}Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, via A. Bassi 6, Pavia (Italy)

A new version of the event generator BABAYAGA is presented, which is based on an original matching of the Parton Shower approach with the complete exact $\mathcal{O}(\alpha)$ matrix element for the inclusion of the QED radiative corrections to the Bhabha process at flavour factories. The theoretical accuracy of the improved generator is conservatively estimated to be 0.2%, by comparison with independent calculations. The generator is a useful tool for precise luminosity determination at flavour factories, for center of mass energies below 10 GeV.

1. Introduction

The precise determination of the machine luminosity is an important ingredient for the successful achievement of the physics programme at the $e^+e^-$ colliders running with center of mass energy in the range of the low lying hadronic resonances.

One of the most important challenges is the precise measure of the $R$ ratio, by means of the energy scan or the radiative return method. The aim is to reduce the theoretical error on the hadronic contribution to the vacuum polarization, which reflects on the error of the anomalous magnetic moment of the muon $a_\mu$ and the QED coupling constant at the $Z$ peak $\alpha_{\text{QED}}(M_Z^2)$ \cite{1}. The precise measure of $R$ will give a stringent test of the Standard Model predictions.

In order to achieve a precise determination of the collider luminosity, high-precision calculations of the QED processes $e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \gamma\gamma$, and relative Monte Carlo generators, are required. The large-angle Bhabha process is, in particular, of major interest for its large cross section and its clean experimental signature.

The event generator BABAYAGA was developed \cite{3,4,5} to this end and it is widely used by experimental collaborations. The event generators MGPJ \cite{6}, BHAGENF \cite{7}, BHACO \cite{8} and BHWIDE \cite{9} are also employed by experimental collaborations.

The theoretical precision of the BABAYAGA event generator for the Bhabha process was estimated at the 0.5% level \cite{2}. The need of an improvement in the calculation of radiative corrections within BABAYAGA emerged in the last years, for a number of reasons. First, the total luminosity error quoted by KLOE is presently 0.6% \cite{7}, where the dominant source of uncertainty comes from the theory, i.e. from the 0.5% physical precision of the BABAYAGA generator. Secondly, the measurement of the hadronic cross section in the $\pi^+\pi^-$ channel at VEPP-2M has achieved a total systematic uncertainty of 0.6%, requiring an assessment of the collider luminosity at the level of 0.1%. Last but not least, precision measurements of $R$ through radiative return at KEK-B and PEP-II are already performed or foreseen in the near future.

Here, we describe the main features of a high-precision calculation of photonic radiative corrections to the Bhabha process, in order to improve the theoretical formulation of the original BABAYAGA generator up to $\mathcal{O}(0.1\%)$. The approach is based on the matching of exact next-to-leading ($\mathcal{O}(\alpha)$) order corrections with resummation through all orders of $\alpha$ of the leading contributions due to multiple radiation, taken into account according to a QED Parton Shower (PS).

2. Matching exact $\mathcal{O}(\alpha)$ matrix element and Parton Shower

The original version of BABAYAGA (3.5) is based on a PS in QED which allows to calculate the
photonic radiative corrections to the Bhabha process. The PS is a numerical algorithm which exactly solves the DGLAP equation for the electron Structure Function (SF) in QED, allowing to include essentially the leading logarithmic (LL) corrections to the cross section up to all orders of α. The main advantage of the PS approach is that, thanks to its Monte Carlo nature, the events can be generated exclusively, i.e. all the momenta of the final state particles (fermions and an indefinite number of photons) can be reconstructed. The PS can be improved to account also for interference effects between initial and final state radiation, allowing thus for a more accurate description of the radiative events.

The corrected differential cross section in the PS approach can be written (in a simplified form for the sake of clarity) as

$$\hat{d}\sigma = \Pi(Q^2,\varepsilon) F_{SV}\left\{d\sigma_0 + \sum_{n=1}^{\infty} \frac{d\hat{\sigma}}{n!} \times \prod_{i=1}^{n} \left[ \frac{\alpha}{2\pi} P(x_i) I(k_i) dx_i dc_i \theta(x_i - \varepsilon) F_{i,H}\right]\right\} \tag{1}$$

where $F_{SV}$ and $F_{i,H}$ are infrared safe quantities and they let the cross section $\sigma$ coincide with the exact $O(\alpha)$ expansion of $d\sigma$ in Eq. (1) coincide with the exact $O(\alpha)$ cross section. It is worth noticing that correcting each single photon emission with a factor $F_{i,H}$ is crucial to make the cross section independent from the $\varepsilon$ parameter and that the correction is larger in those phase space regions where the PS is more unreliable, typically where the photon is hard and/or not collinear to one of the charged particles. Notice that Eq. (1) is cast in a completely differential form, so that events can be generated exclusively as in the pure PS approach. A more detailed discussion of the matching procedure, as well as its implementation in BABAYAGA, is presented in Ref. [9].

3. Numerical results

In order to check the technical implementation and the physical accuracy of Eq. (1) in the new BABAYAGA [10], the generator has been compared with independent calculations, namely LABSPV [11], BHWIDE and the old 3.5 release, tuning the relevant input parameters. In LABSPV, which is not a true event generator, the corrected...
cross section is calculated as \( \sigma = (1 + C_{NL})\sigma_0^\infty \)SF, where \( C_{NL} = (\sigma_{\alpha,ex} - \sigma_{\alpha}^SF)/\sigma_0 \) and the Structure Functions in the strictly collinear limit are used; thus \( O(\alpha) \) corrections are included exactly and h.o. are included in the collinear limit with LL accuracy. BHWIDE is an event generator based on the YFS formalism to exponentiate exact \( O(\alpha) \) corrections. Here, \( O(\alpha) \) is included exactly and h.o. are included in the YFS approach.

The comparisons have been performed neglecting vacuum polarization effects, at 1.02 GeV of center of mass energy, requiring for the final state \( e^+ e^- \) \( E_{\pm} > 0.4 \times E_{CoM} \), the \( e^+ e^- \) acollinearity \( \xi \) lower than 10° and choosing two angular acceptances: a) \( 20^\circ < \vartheta_{\pm} < 160^\circ \) and b) \( 55^\circ < \vartheta_{\pm} < 125^\circ \).

The results of the comparisons on the integrated cross section are shown in Table 1, where also the Born and \( O(\alpha) \) predictions are added. In setup a) (b)), the \( O(\alpha) \) corrections reduce the Born cross section by 13% (17%) and the corrections beyond \( O(\alpha) \) raise the cross section by 0.4% (0.9%), showing that h.o. contributions are important at the level of the needed accuracy.

The agreement between BABAYAGA and BHWIDE is very good, being the discrepancies well below the 0.1%. The tiny differences with respect to LABSPV (0.08% in setup b)), where the impact of h.o. is larger) can be ascribed to the strictly collinear approximation of h.o. corrections in LABSPV. Finally, the differences between the new and old version of BABAYAGA are within the 0.5%, in agreement with the theoretical accuracy estimated for the old release of the event generator.

Besides the integrated cross section, also differential cross sections have been compared. In Fig. 1, the acollinearity distribution is plotted for setup b), as obtained with the old and the new BABAYAGA and at \( O(\alpha) \). The effect of h.o. corrections is clearly visible. The relative difference between the old and the new BABAYAGA is better pointed out in the smaller panel, being almost constant in the whole range at 0.5% level.

Figure 1. Acollinearity distribution, the new BABAYAGA compared with its old version.

In Fig. 2 the comparison of the acollinearity distribution obtained with BABAYAGA and BHWIDE is shown. The differences are very small and can be only appreciated in the panel, showing they reach the 0.4% level only in the distribution tail. This region is populated by events where hard photons are emitted and where the different treatment of the multiple-photon emission and h.o. corrections shows up, as can be expected.

4. Estimate of the theoretical accuracy

The results presented the previous section clearly demonstrate that the matching procedure of Eq. (1) has the desired behaviour of including the missing \( O(\alpha) \) contributions in the PS algorithm while preserving the resummation of all h.o. corrections. They also demonstrate that the not trivial Monte Carlo implementation of Eq. (1) is correctly realized and robust.
The theoretical error of BABAYAGA is now shifted at the two loop level, to terms of the order of $\frac{1}{\alpha^2}L (\simeq 3 \times 10^{-4}$ at 1 GeV), without any infrared enhancement.

In [9], a critical and systematic discussion of the remaining theoretical error will be presented. Once the vacuum polarization is (easily) included in the BABAYAGA formulation, the formulae can be expanded at order $\alpha^2$ and consistently compared with the recent calculation of Refs. [12], where relevant contributions to the complete two loop corrections to Bhabha scattering are computed. It is worth noticing that in Refs. [12] the real photon radiation is accounted only in the soft limit and that at two loop level also the emission of real $e^+e^-$ pairs has to be carefully considered. However, preliminary studies and tests show that the missing $O(\alpha^2)$ contributions have an impact not larger than the 0.1% on the integrated cross section if typical event selection criteria for luminometry at flavour factories are applied.

These considerations suggest that a conservative estimate of the theoretical error of the improved BABAYAGA can be fixed at 0.2%.

5. Conclusions

An improved version of the event generator BABAYAGA is now available [10], where an original matching of the QED PS and the exact $O(\alpha)$ matrix element is implemented in order to go beyond the LL approximation intrinsic to the PS approach. The BABAYAGA theoretical error for the Bhabha cross section calculation at flavour factories is reduced from the 0.5% down to 0.2% (conservatively estimated). A more detailed description of the implementation and a more robust estimate of the theoretical accuracy are discussed in Ref. [9].

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