Comparison of some behaviors of planetary mechanism at internal and external gearing of satellite

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Abstract. The potential energy of a planetary type mechanism located in a vertical plane is analyzed for two different configurations with similar parameter values: with internal and external gearing of the satellite. The question of bifurcation of its oscillations is considered.

1. Introduction
Planetary mechanisms can be part of planetary gearboxes, on-board planetary gears of heavy vehicles [1]. In this paper, we study the change in the potential energy [2] of such mechanisms, and hence their movements, when the values of certain characteristic parameters vary.

2. Formulation of the Problem
The considered planetary-type mechanism is located in the vertical plane [3] (Figure 1). Satellite 1 of radius \( r \) and mass \( M_1 \) can roll without slipping on a fixed cylindrical surface of radius \( R \). Its axis \( A \) is attached to the rod 2, whose mass is \( M_2 \) and length is \( L \). The rod 2 can rotate around a fixed horizontal axis \( Oz \). In figure 1, the satellite has an internal attachment with a cylindrical surface, and in figure 2-an external engagement. A weightless rod 5 with a load 3 weighing \( M_3 \) is rigidly attached to the satellite. The load is fixed on the rod at a certain distance \( l_3 \) from the satellite axis. On the carrier 2 at a distance \( OD = l_4 \) there is a counterweight 4 with a mass of \( M_4 \). In the position of the mechanism when \( \varphi = 0 \) the axis \( A \) of the satellite lies on the same vertical and below the axis \( O \) of the carrier 2, the longitudinal axis of the rod 5 is located vertically and the load 3 (point \( B^* \)) is below the axis of the satellite \( A^* \) (Figure 2).

A mechanism has one degree of freedom. As the generalized coordinate was selected the angle \( \varphi \) (radian) of deflection of the rod 2 from the vertical.

The active forces acting on the mechanism are only the forces of gravity. Let's write out the potential energy of the system:

\[
\Pi(\varphi) = \frac{1}{2} \left( M_1 + M_2 + M_3 \right) \left( R \mp r - M_3 L / 2 - M_3 l_3 \right) \left( 1 - \cos \varphi \right) + gM_3 l_3 \left( 1 - \cos \left( \frac{R \mp r}{R} \varphi \right) \right) + \left(1\right)
\]

The upper "–" sign corresponds to the internal engagement of the satellite (Figure 1), and the lower "+" sign corresponds to the external attachment (Figure 2).
3. Investigation of the behavior of the potential energy of the system.

Consider the dependence of the potential energy of the mechanism on the radius of the satellite $r$ for its internal and external engagement. Select the following parameter values:

$M_1 = 0.2$, $M_2 = 0.5$, $M_3 = M_4 = 0.4 \ (kg)$, $R = 0.65$, $l_3 = 0.2$, $l_4 = 0.15$, $L = 1.8 \ (m)$.

Figure 3.a shows graphs of the potential energy at the internal attachment of the satellite, figure 3.b – at the external engagement of the satellite on the segment $\phi \in [0; \pi]$ (rad) of the angle of rotation of the rod 2 for some values of the radius $r$. With both engagements, when the point load 3 is located outside the satellite $(r << l_3)$, $\Pi(\phi)$ has several extremes on the interval $\phi \in [0; \pi]$.

As $r$ increases, the number of extremes decreases. And for the external attachment, their number is greater than for the internal one. This can be explained by the fact that with external engagement, the
trajectory of the center of mass $C$ of the satellite is longer. If $r = l_3$ (load 3 on the rim), the minimum of the potential energy $\Pi(\varphi)$ is only for $\varphi = 0$ and $\varphi \approx 2\pi$.

If $r \approx 0.29$ m for internal engagement the point $\varphi = 0$ changes from the minimum point to the maximum point of the $\Pi(\varphi)$, that is, it becomes an unstable equilibrium position. Near the point $\varphi = \pi$, there is either a maximum or a minimum, which depends on the ratio of radii $R$ and $r$. Moreover, for internal attachment, in contrast to external attachment, $r$ cannot exceed $R$.

Figure 4 shows graphs of the potential energy of the mechanism when the satellite is engagement internally. In the first three graphs, the angle of rotation of the rod 2 is taken as follows $\varphi \in [0;128]$ (rad). On the other charts $\varphi \in [0;384]$. In figure 5 – external attachment. On the first three graphs, the rotation angle $\varphi \in [0;32]$, on the rest – the angle $\varphi \in [0;128]$ (rad).

In both cases of satellite attachment, there is a periodicity in the behavior of potential energy. For internal engagement (figure 4), the period increases significantly with increasing $r$. So for $r=0.1$ m the period $T \approx 12.5$ rad, for $r=0.32$ m the period is $T \approx 204$ rad. Special situation for $r=R/2=0.325$ m, when $T=\pi$. In this case, the potential energy function (1) takes a simple form:
\Pi(\varphi) \approx 1.18 \cos \varphi - 0.711.

With external attachment (figure 5), the period with increasing \( r \) initially increases, but not so significantly: so for \( r=0.1 \) m the period \( T \approx 12.5 \) rad, for \( r=0.3 \) the period \( T \approx 37.5 \). And after the period decreases: for \( r=0.43 \) \( T \approx 12.5 \), for \( r=0.65 \) \( T \approx 6.12 \), but for \( r=0.1 \) \( T \approx 18.7 \).

For internal engagement (figure 4), for some values of the radius, for example, at \( r=0.32 \), there is a "beating" in the behavior of the function \( \Pi(\varphi) \) (1), which is explained by summation cosines with close values of arguments:

\[ \Pi(\varphi) \approx 1.44 \cos \varphi + 0.78 \cos(1,031 \varphi) - 0.66. \]

In formula (1), for \( r \approx \frac{R}{2} \), we have \( \frac{R-r}{r} \approx 1 \).

With external engagement (figure 5), we do not observe such behavior of the function \( \Pi(\varphi) \).

In both cases, a "snake" is observed in the graph of \( \Pi(\varphi) \) for certain ratios of radii. This happens when the function reduces to the multiplication of cosines (or cosine and sine) with comparable values of the arguments, but not equal argument values. For external: for \( r=0.1, r=0.3, r=1 \). For internal engagement, this is well observed at \( r=0.1 \) m and \( r=0.3; r=1.0 \).

Next, we consider the behavior of \( \Pi(\varphi) \) with the internal engagement of the satellite with minor changes in its radius \( r \) in the vicinity of the angle \( \varphi = 0 \). Figure 6 shows graphs of the potential energy in the interval \([-1, 1]\) of the angle \( \varphi \) (rad).

Point \( \varphi = 0 \) is a special point. At \( r \approx 0.2933 \), this is the point of stable equilibrium of the mechanism (\( \Pi(\varphi) \) has a local minimum). And for \( r > 0.2933 \), this is the point of an unstable equilibrium position (\( \Pi(\varphi) \) has a local maximum). That is, there is a bifurcation – a situation, when, passing through the parameter \( r^* = 0.2933 \) (m) (load 3 inside the satellite), the properties of potential energy change qualitatively. Periodic movements of the system in the vicinity of this point as a result of even minor changes in the size of the satellite become impossible and the system may then behave unpredictably.

Note that the position of stable equilibrium remains for the external engagement at the point \( \varphi = 0 \) for the considered radius changes (figure 5). A change in the behavior of potential energy \( \Pi(\varphi) \) during external engagement can be observed, for example, in the vicinity of \( \varphi = 0.445 \). For \( r=0.05 \) m, \( \Pi(\varphi) \) has a minimum here (figure 7). After a slight increase in the radius, the second derivative \( \Pi''(\varphi) \) becomes negative. So to the right is the position of stable equilibrium of the system. At \( r \approx 0.098 \) m, \( \Pi(\varphi) \) has a maximum at this point. At \( r \approx 0.098 \) m, \( \Pi(\varphi) \) has a maximum at this point.
When \( r > 0.1 \) \( m \) \( II'(\varphi) > 0 \). This means that to the right of this point is now the position of the unstable equilibrium of the system.

4. Conclusion
Numerical study of the potential energy of the mechanism has shown that even a slight change in the values of its characteristic parameters can significantly change its movement.

References
[1] Kraynev A 1987 Slovar'-spravochnik po mekhanizmam [Dictionary - a directory of mechanisms] (Moscow: Mechanical engineering)
[2] Babakov I 2004 Teoriya kolebaniy [Theory of oscillations] (Moscow: Drofa)
[3] Obnosov K, Bondarenko N and Panshina A 2015 Yestestvennyye i tekhnicheskiye nauki [Natural and technical Sciences] № 10(88) 18-22