Magnetic properties of a two-dimensional electron gas strongly coupled to light

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Considering the quantum dynamics of 2DEG exposed to both a stationary magnetic field and an intense high-frequency electromagnetic wave, we found that the wave decreases the scattering-induced broadening of Landau levels. Therefore, various magneto electronic properties of two-dimensional nanostructures (density of electronic states at Landau levels, magnetotransport, etc) are sensitive to the irradiation by light. Thus, the elaborated theory paves a way to optical controlling magnetic properties of 2DEG.

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I. INTRODUCTION

The study of a two-dimensional electron gas (2DEG) exposed to a high-frequency electromagnetic field is one of the most excited areas in the modern physics of nanostructures. The permanent interest to this topic originates from rich fundamental and applied capabilities of two-dimensional electron systems (see, e.g., Refs. [1–3]). Particularly, the magneto electronic properties of 2DEG subjected to a microwave irradiation are actively studied during last years$^{4–16}$. However, the most attention on the subject was paid before to the simplest case of weak electromagnetic field which does not change electron states. Namely, the only effect of the weak field is the field-induced electron transitions between the unperturbed states. On the contrary, a strong electromagnetic field can substantially mix electron states. As a result of this mixing, the composite electron-field object “electron dressed by field” (dressed electron) appears.$^{17,18}$ The light-induced renormalization of physical properties of dressed electrons has been studied in various atomic systems$^{17–19}$ and condensed-matter structures, including bulk semiconductors$^{20–22}$, quantum wells$^{23–26}$, quantum rings$^{27–31}$, graphene$^{32–40}$, etc. In the present research, we develop the theory describing the magnetic properties of dressed 2DEG and demonstrate that they can be substantially modified by the dressing field.

The paper is organized as follows. In the second section, we solve the Schrödinger equation for a 2DEG subjected to both a stationary magnetic field and a high-frequency dressing field. In the third section, the found solutions of the Schrödinger problem are used to analyze various magneto electronic characteristics of dressed 2DEG, including density of electron states and magnetotransport. The last sections contain conclusions and acknowledgments.

II. SCHRODINGER PROBLEM FOR LANDAU LEVELS IN DRESSED 2DEG

Let us consider a two-dimensional electron gas (2DEG) confined in the $(x, y)$ plane, which is subjected to both a stationary magnetic field, $B = (0, 0, B)$, directed along the $z$ axis and a linearly polarized electromagnetic wave (dressing field) propagating along the same axis $z$ (see Fig. 1). The Hamiltonian of 2DEG reads as

$$\hat{H}_e = \frac{1}{2m_e} \left[ \hat{p} - e (A_0 + A_t) \right]^2,$$

(1)

where $m_e$ is the effective electron mass, $e$ is the electron charge, $A_0 = (-By, 0, 0)$ is the stationary vector potential of the magnetic field, $A_t = (0, |E/\omega| \cos \omega t, 0)$ is the time-dependent vector potential of the electromagnetic wave, $E$ is the amplitude of electric field of the wave, $\omega$ is the wave frequency, and $\hat{p} = (\hat{p}_x, \hat{p}_y, 0)$ is the operator of two-dimensional electron momentum, $p_{x,y}$. Solutions of the nonstationary Schrödinger problem with the
Hamiltonian (1) should be sought in the form

$$\psi(r, t) = \frac{1}{\sqrt{L_x L_y}} \exp \left[ \frac{p_x x}{\hbar} + i \frac{e E(y - y_0)}{\hbar \omega} \cos \omega t \right] \times \phi(y - y_0, t),$$

(2)

where $L_{x,y}$ are dimensions of the 2DEG plane, $r = (x, y, 0)$ is the radius-vector of electron in the 2DEG plane, and $y_0 = -p_x/eB$ is the center of cyclotron orbit along the $y$ axis. Substituting the wave function (2) into the Schrödinger equation with the Hamiltonian (1), $i\hbar \partial \psi / \partial t = \hat{H}_e \psi$, we arrive at the equation for the driven quantum oscillator,

$$\left[ \frac{m_e \omega_0^2 y^2}{2} - e E y \sin \omega t - \frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial y^2} - i \hbar \frac{\partial}{\partial t} \right] \phi(y, t) = 0,$$

which has the well-known exact solution (see, e.g., Refs. 11-13),

$$\phi(y, t) = \chi_N(y - \zeta(t)) \exp \left[ -i \frac{\varepsilon_N t}{\hbar} + i \frac{m_e \zeta(t)}{\hbar} (y - \zeta(t)) \right] + i \frac{\hbar}{\hbar} \int_0^t d't L(t'),$$

(3)

where $\chi_N(y)$ is the eigenfunction of the quantum harmonic oscillator, $\varepsilon_N = \hbar \omega_0 (N + 1/2)$ is the energy spectrum of the oscillator, $N = 0, 1, 2, \ldots$ is the number of Landau level, $\omega_0 = |e| B / m_e$ is the cyclotron frequency,

$$\zeta(t) = \frac{e E \sin \omega t}{m_e (\omega_0^2 - \omega^2)}$$

is the trajectory of the driven classical oscillator, and

$$L(t) = \frac{m_e \zeta^2(t)}{2} - \frac{m_e \omega_0^2 \zeta^2(t)}{2} + e E \zeta(t) \sin \omega t$$

is the Lagrangian of the classical oscillator.

It should be noted that the field-induced terms in the wave functions (2–3) do not depend on the Landau number level, $N$. This means that the dressing field does not change the structure of Landau levels. However, the dressing field produces exponential phase shifts in the wave functions (2–3). In the absence of a magnetic field, similar phase shifts strongly effect on transport characteristics of dressed 2DEG via the renormalization of electron scattering$^{26, 27}$. Since the phase shifts in Eqs. (2–3) depend on both the dressing field and the magnetic field, one can expect that magnetotransport properties of 2DEG will be renormalized by the dressing field as well. In order to describe this renormalization accurately, we have to solve the scattering problem for the dressed electron states (2–3).

Let an electron interact with scatterers in the presence of the same fields, $A_0$ and $A_r$. Then the wave function of the electron, $\Psi(r, t)$, satisfies the Schrödinger equation

$$i\hbar \frac{\partial \Psi(r, t)}{\partial t} = [\hat{H}_e + U(r)] \Psi(r, t),$$

(4)

where $U(r)$ is the total scattering potential of 2DEG arisen from macroscopically large number of scatterers. Since the wave functions (2) at any time $t$ coincide with the eigenfunctions of quantum harmonic oscillator, they form the complete basis. Therefore, one can seek solutions of the Schrödinger equation (4) as an expansion

$$\Psi(r, t) = \sum_j a_j(t) \psi_j(r, t),$$

(5)

where the different indices $j$ correspond to the different sets of all quantum numbers ($p_x$ and $N$) describing electron states of the considered system. It should be stressed that Eqs. (2–3) describe exact wave functions of a dressed electron. Therefore, the using of the complete basis (4) in the expansion (5) takes into account the interaction between the electron and the dressing field in full, i.e. non-perturbatively. As to the electron transition from a state $j$ to a state $j'$ due to the potential $U(r)$, we will describe this scattering process within the conventional perturbation theory.

Let an electron be in the state $j$ at the time $t = 0$ and, correspondingly, $a_j(0) = \delta_{j,j'}$. Substituting the expansion (5) into the Schrödinger equation (4) and restricting the accuracy by the first order of the perturbation theory (the Born approximation), we can write the amplitude of scattering to the state $j'$ as

$$a_{j'}(t) = -i \int_0^t dt \int_S d^2r \psi^*_j(r, t) U(r) \psi_j(r, t),$$

(6)

where the integration should be performed over the 2DEG area, $S = L_x L_y$. Applying the Jacobi-Anger expansion,

$$e^{i z \cos \theta} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in \theta},$$

to transform the time-dependent exponential terms in the wave functions (2–3), we arrive from the scattering amplitude (6) to the scattering probability

$$|a_{j'}(t)|^2 = \frac{|U_{j'j}|^2}{\hbar^2} \sum_{n=-\infty}^{\infty} i^n J_n \left( \frac{e E |y_0 - y| \omega_0^2}{\hbar \omega_0^2 - \omega^2} \right) \times e^{i(\varepsilon_{j'} - \varepsilon_j + n \hbar \omega) t / 2 \hbar} \int_{-t/2}^{t/2} dt' e^{i(\varepsilon_{j'} - \varepsilon_j + n \hbar \omega) t' / \hbar} \left| U_{j'j} \right|^2,$$

(7)

where

$$U_{j'j} = \langle \varphi_{j'}(r) | U(r) | \varphi_j(r) \rangle$$

(8)

is the matrix element of the scattering between the “bare” electron eigenstates,

$$\varphi_j(r) = \frac{e^{i p_x x / \hbar}}{\sqrt{L_x}} \chi_N(y),$$

which satisfy the Schrödinger equation with the Hamiltonian (1) in the absence of the dressing field ($A_r = 0$).
Since the integral in Eq. (7) for long time $t \to \infty$ turns into the delta function, the scattering probability (7) can be rewritten as

$$|a_k(t)|^2 = 4\pi^2 |U_{j'j}|^2 \sum_{n=-\infty}^{\infty} J_n^2 \left( \frac{eE|y_n - y_0|\omega_0^2}{\hbar \omega_0^2 - \omega^2} \right) \times \delta^2(\varepsilon_{j'}, -\varepsilon_j + n\hbar \omega). \quad (9)$$

To transform square delta functions in Eq. (9), we can apply the conventional procedure,

$$\delta^2(\varepsilon) = \delta(\varepsilon)\delta(0) = \frac{\delta(\varepsilon)}{2\pi \hbar} \lim_{t\to\infty} \int_{-t/2}^{t/2} e^{i\omega t'/\hbar} dt' = \frac{\delta(\varepsilon)t}{2\pi \hbar}.$$ 

Then the probability of the electron scattering between the states $j$ and $j'$ per unit time is

$$w_{j'j} = \frac{d|a_j(t)|^2}{dt} = |U_{j'j}|^2 \sum_{n=-\infty}^{\infty} J_n^2 \left( \frac{eE|y_n - y_0|\omega_0^2}{\hbar \omega_0^2 - \omega^2} \right) \times \frac{2\pi}{\hbar} \delta(\varepsilon_{j'}, -\varepsilon_j + n\hbar \omega). \quad (10)$$

It should be noted that the derivation of Eqs. (9)–(10) is done within the conventional time-dependent perturbation theory which is extended to the case of the oscillating basis (2). Physically, this extension is similar to the scattering theory developed recently for dressed electron states in various conductors. 

To avoid the energy exchange between a high-frequency field and electrons, the field should be purely dressing (nonabsorbable). In the considered electron system, there are the two mechanism of absorption of the field by electrons: (i) the resonant absorption of the field, which corresponds to electron transitions between different Landau levels; (ii) the collisional absorption of the field, which corresponds to electron transitions between different states within the broadened Landau level. To exclude the first mechanism, the field frequency, $\omega$, should be far from the resonant frequencies, $n\omega_0$ ($n = 1, 2, 3, \ldots$), corresponding to interlevel electron transitions. To exclude the second mechanism, the photon energy, $\hbar \omega$, should be much more than the scattering-induced broadening of Landau levels, $\Gamma = \hbar/\tau$ (i.e., $\omega \tau \gg 1$). Physically, the terms with $n \neq 0$ in Eq. (10) describe the electron scattering accompanied by the absorption (emission) of $n$ photons. It follows from the aforesaid that these terms can be neglected if the dressing field is both off-resonant and high-frequency. Therefore, the only effect of the dressing field on 2DEG is the renormalization of the probability of elastic electron scattering within the same Landau level ($\varepsilon_{j'} = \varepsilon_j$) which is described by the term with $n = 0$ in Eq. (10):

$$w_{j'j} = J_0^2 \left( \frac{eE|y_0 - y_0|\omega_0^2}{\hbar \omega_0^2 - \omega^2} \right) w_{j'j}^{(0)}, \quad (11)$$

where

$$w_{j'j}^{(0)} = \frac{2\pi}{\hbar} |U_{j'j}|^2 \delta(\varepsilon_{j'}, -\varepsilon_j). \quad (12)$$

is the probability of scattering of “bare” electron. As expected, the probabilities (11) and (12) are identical in the absence of the dressing field ($E = 0$). The formal difference between the scattering probability of dressed electron (11) and the scattering probability of “bare” electron (12) consists in the Bessel-function factor depending on both the dressing field and the stationary magnetic field. Just this factor is responsible for all effects discussed below. Particularly, the lifetime of dressed electron at the Landau level, $\tau$, is renormalized by the Bessel function as

$$\frac{1}{\tau} = \sum_{j'} w_{j'j} = \sum_{j'} J_0^2 \left( \frac{eE|y_0 - y_0|\omega_0^2}{\hbar \omega_0^2 - \omega^2} \right) w_{j'j}^{(0)} \quad (13)$$

In order to calculate the lifetime (13), let us rewrite the delta function, $\delta(\varepsilon_{j'} - \varepsilon_j)$, with using the well-known representation

$$\delta(\varepsilon) = \lim_{\Gamma \to 0} \frac{\Gamma}{\pi} \frac{\Gamma}{\Gamma^2 + \varepsilon^2}. \quad (14)$$

In the context of the discussed problem, the parameter $\Gamma = \hbar/\tau$ has the physical meaning of scattering-induced broadening of Landau level. For the considered case of elastic scattering within the same Landau level, we can write the delta function (14) as $\delta(\varepsilon_{j'} - \varepsilon_j) \approx 1/(\pi \Gamma)$ and, therefore, Eq. (13) takes the form

$$\frac{1}{\tau} = \left[ \frac{2}{\hbar^2} \sum_{j'} J_0^2 \left( \frac{eE|y_0 - y_0|\omega_0^2}{\hbar \omega_0^2 - \omega^2} \right) |U_{j'j}|^2 \right]^{1/2}, \quad (15)$$

where the summation is performed over electron states $j'$ within the same Landau level. To calculate the lifetime (15), let us approximate the scattering potential using the model of delta-function scatterers,

$$U (r) = \sum_{i=1}^{N_s} U_0 \delta (r - r_i),$$

which is commonly used to describe electronic transport in various two-dimensional systems. Assuming that the scatterers to be distributed randomly and the total number of scatterers, $N_s$, to be macroscopically large, we can obtain from Eq. (15) the final expression for the electron lifetime at the N-th Landau level,

$$\frac{1}{\tau} = \sqrt{\frac{n_s U_0^2}{\pi \hbar^2 \hbar^2}} \times \left[ \int_{-\infty}^{\infty} \chi_N^2 (y') \chi_N^2 (y + y') J_0^2 \left( \frac{eE|y_0|\omega_0^2}{\hbar \omega_0^2 - \omega^2} \right) dy dy' \right]^{1/2}, \quad (16)$$

where $n_s = N_s/S$ is the density of scatterers per unit area of 2DEG, and $l_0 = \sqrt{\hbar/|e|B}$ is the magnetic length. The argument of the Bessel function in the integrand of
Eq. (16) is the dimensionless parameter which describes the ratio of the characteristic energy of the electron-field interaction and the photon energy. Physically, it describes the strength of electron-photon coupling in the considered electron-field system. Since the dressing field, $E_d$, leads to decreasing the Bessel function, the scattering time, $\tau$, increases due to the field. Magneto-electronic effects following from this increasing are discussed below.

III. MAGNETOELECTRONIC CHARACTERISTICS OF DRESSED 2DEG

Since the scattering time $\tau$ depends on the dressing field, the scattering-induced broadening of Landau levels, $\Gamma = \hbar/\tau$, is also affected by the field. In order to describe the broadening accurately, it is convenient to rewrite Eq. (16) in the dimensionless form,

$$
\frac{\Gamma(N)}{\Gamma_0} = \left[ \int_{-\infty}^{\infty} \frac{\chi^2_N(y')}{2} \right]^{1/2},
$$

(17)

where $\Gamma(N) = \hbar/\tau$ is the broadening for the Landau level with the number $N = 0, 1, 2, \ldots$, and $\Gamma_0$ is the broadening of Landau levels in the absence of the dressing field (natural broadening). It should be noted that Eq. (17) does not depend on the density of scatterers, $n_s$, and the strength of scatterers, $U_0$. Therefore, Eq. (17) describes the dependence of the broadening of Landau levels on the dressing field in the most general form, where the broadening of “bare” Landau levels, $\Gamma_0$, should be treated as a phenomenological parameter which can be found from experiments. In the absence of the dressing field ($E = 0$), the broadening (17) is the same for all Landau levels, $\Gamma = \Gamma_0 \propto \sqrt{B}$, in complete agreement with the conventional theory of magneto-electronic properties of 2DEC. On the contrary, the dressing field leads to the different broadening (17) for different Landau levels (see Fig. 2). As to the density of electron states, it is described by the expression

$$
D(\varepsilon) = D_0 \sum_N \frac{\Gamma_0}{\Gamma(N)} \left[ 1 - \left( \frac{\varepsilon - \varepsilon_N}{\Gamma(N)} \right)^2 \right]^{1/2},
$$

(18)

where $D_0 = 1/(\sqrt{2\hbar\varepsilon_F})$. Substituting the broadening (17) into Eq. (18), one can calculate the density of states in dressed 2DEG (see the insert in Fig. 2). Since the dressing field decreases the broadening of Landau levels (17), this results in increasing the density of states at Landau level energies, $\varepsilon = \varepsilon_N$. As a consequence, all phenomena sensitive to the density of electronic states (magnetotransport, magneto-optical effects, etc) are affected by the dressing field. Particularly, the longitudinal magnetoconductivity of 2DEG at the temperature $T = 0$ is described by the conventional expression

$$
\sigma_{xx} \approx \sigma_0 \left( N + \frac{1}{2} \right) \left[ 1 - \left( \frac{\varepsilon - \varepsilon_N}{\Gamma(N)} \right)^2 \right],
$$

(19)

where $\sigma_0 = e^2/(\pi^2\hbar)$ is the elementary conductivity, and $N$ is the number of Landau level at the Fermi energy. Substituting the broadening (17) into Eq. (19), one can calculate the dependence of the conductivity on the dressing field. Experimentally, one can change the Fermi energy of 2DEG, $\varepsilon_F$, with a gate voltage. Then we arrive from Eq. (19) at the oscillating behavior of the conductivity (the Shubnikov-de Haas oscillations) plotted in Fig. 3.

It should be stressed that there is the crucial difference between the considered high-frequency dressing field and the low-frequency case. Since 2DEG absorbs a low-frequency field, the multiphoton-assisted scattering of electrons can increase the longitudinal conductivity. Particularly, this effect was proposed to explain the phenomenon of “zero resistance states” in 2DEG subjected to both a magnetic field and a low-frequency (microwave) irradiation. On the contrary, the considered high-frequency field cannot be absorbed by 2DEG. The only effect of the field is the suppression of electron scattering, which results in decreasing both the broadening of Landau levels and the longitudinal conductivity (see Figs. 2 and 3). Thus, a high-frequency irradiation and a low-frequency one lead to different behavior of the magneto-electronic properties of 2DEG.

It should be noted also that the magneto-electronic effects induced by a dressing field strongly depends on the kind of electron dispersion. In Dirac materials with linear electron dispersion, a dressing field changes the energy
FIG. 3: (Color online) The dependence of the longitudinal conductivity, $\sigma_{xx}$, on the Fermi energy, $\varepsilon_F$, in a GaAs-based quantum well at the magnetic field $B = 1.2$ T, irradiation frequency $\omega = 2 \cdot 10^{12}$ rad/s, and the natural broadening $\Gamma_0 = 1$ meV. The solid line describes the conductivity of unirradiated 2DEG, whereas the dotted one corresponds to the conductivity at the irradiation intensity $I = 600$ W/cm$^2$. The insert shows the difference of these two conductivities, $\Delta \sigma_{xx}$.

distance between Landau levels and, therefore, modifies all phenomena depending on the cyclotron frequency. On the contrary, in the considered case of 2DEG with the parabolic electron dispersion, a dressing field does not change the cyclotron frequency but influences on the electron scattering within Landau levels.

As to experimental observability of the discussed phenomena, all dressing effects increase with increasing the intensity of the dressing field. Particularly, the strong dressing field can turn the Bessel function in Eq. (10) into zero, what corresponds physically to the field-induced suppression of electron scattering. However, an intense irradiation can fluidize a semiconductor quantum well. To avoid the fluidizing, it is reasonable to use narrow pulses of a strong dressing field. This well-known methodology has been elaborated long ago and commonly used to observe various dressing effects — particularly, modifications of energy spectrum of dressed electrons arisen from the optical Stark effect — in semiconductor structures (see, e.g., Refs. [48][50]). Within this approach, giant dressing fields (up to GW/cm$^2$) can be applied to semiconductor structures.

IV. CONCLUSIONS

Summarizing the aforesaid, we can conclude that a strong high-frequency electromagnetic field (dressing field) decreases the electron scattering between different cyclotron orbits within the same Landau level. As a consequence, the field decreases the scattering-induced broadening of Landau levels in 2DEG. This results in the field-induced modification of various magnetoelectronic properties depending on the density of electron states (particularly, magnetotransport characteristics of 2DEG). Therefore, a dressing field can be considered as a perspective tool to manipulate the magnetoelectronic properties of various two-dimensional nanostructures. Since such nanostructures serve as a basis for nanoelectronic devices, the developed theory opens a way for optical control of their magnetoelectronic characteristics.

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