Neutron stars and strange stars in the chiral SU(3) quark mean field model

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We investigate the equations of state for pure neutron matter and strange hadronic matter in $\beta$-equilibrium, including $\Lambda$, $\Sigma$ and $\Xi$ hyperons. The masses and radii of pure neutron stars and strange hadronic stars are obtained. For a pure neutron star, the maximum mass is about 1.8$M_{\text{sun}}$, while for a strange hadronic star, the maximum mass is around 1.45$M_{\text{sun}}$. The typical radii of pure neutron stars and strange hadronic stars are about 11.0-12.3 km and 10.7-11.7 km, respectively.

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I. INTRODUCTION

Hadronic matter under extreme conditions has attracted a lot of interest in recent years. On the one hand, many theoretical and experimental efforts have been devoted to the
discussion of heavy ion collisions where the temperature is high. On the other hand, the physics of neutron stars has become a hot topic which connects astrophysics with high density nuclear physics. In 1934, Baade and Zwicky [1] suggested that neutron stars could be formed in supernovae. The first theoretical calculation of a neutron star was performed by Oppenheimer and Volkoff [2], and independently by Tolman [3]. Observing a range of masses and radii of neutron stars will reveal the equations of state (EOS) of dense hadronic matter. Determination of the EOS of neutron stars has been an important goal for more than two decades. Six double neutron-star binaries are known so far, and all of them have masses in the surprisingly narrow range $1.36 \pm 0.08M_{\text{sun}}$ [4,5]. A number of early theoretical investigations on neutron stars were based on the non-relativistic Skyrme framework [6]. Since the Walecka model [7] was proposed and applied to study the properties of nuclear matter, the relativistic mean field approach has been widely used in the determination of the masses and radii of neutron stars. These models lead to different predictions for neutron star masses and radii [8,9]. For a recent review see Ref. [10]. Though models with maximum neutron star masses considerably smaller than $1.4M_{\text{sun}}$ are simply ruled out, the constraint on EOS of nuclear matter (for example, the density dependence of pressure of hadronic system) has certainly not been established from the existing observations.

In the process of neutron star formation, $\beta$-equilibrium can be achieved. As a consequence, hyperons will exist in neutron stars, especially in stars with high baryon density. These hyperons will affect the EOS of hadronic matter. As a result, the mass-radius relationship of strange hadronic stars will be quite different from that of pure neutron stars. The simplest way to discuss the effects of hyperons is to study strange hadronic stars including only $\Lambda$ hyperons. This is due to the fact that $\Lambda$ is the lightest hyperon and the $\Lambda$-N interaction is known better than other hyperon-nucleon interactions. However, one must also consider hyperons with negative charge in neutron stars because the negatively charged hyperons can substitute for electrons. There have been many discussions of strange hadronic stars including $\Lambda$ hyperons, $\Lambda$ and $\Sigma^-$ or even the whole baryon octet [11] - [20].

At high baryon density, the overlap effects of baryons are very important and the quark
degrees of freedom within baryons should be considered. There are some phenomenological models based on the quark degrees of freedom, such as the quark meson coupling model [21], the cloudy bag model [22], the quark mean field model [23] and the NJL model [24]. Several years ago, a chiral $SU(3)$ quark mean field model was proposed [25,26]. In this model, quarks are confined within baryons by an effective potential. The quark-meson interaction and meson self-interaction are based on $SU(3)$ chiral symmetry. Through the mechanism of spontaneous symmetry breaking the resulting constituent quarks and mesons (except for the pseudoscalars) obtain masses. The introduction of an explicit symmetry breaking term in the meson self-interaction generates the masses of the pseudoscalar mesons which satisfy the partially conserved axial-vector current (PCAC) relations. The explicit symmetry breaking term in the quark-meson interaction gives reasonable hyperon potentials in hadronic matter. This chiral $SU(3)$ quark mean field model has been applied to investigate nuclear matter [27], strange hadronic matter [25], finite nuclei, hypernuclei [26], and quark matter [28]. Recently, we improved the chiral $SU(3)$ quark mean field model by using the linear definition of effective baryon mass [29]. This new treatment is applied to study the liquid-gas phase transition of asymmetric nuclear system and strange hadronic matter [30,31]. By and large the results are in reasonable agreement with existing experimental data.

In this paper, we will study the neutron star and strange star in the chiral $SU(3)$ quark mean field model. The paper is organized in the following way. In section II, we briefly introduce the model. In section III, we apply this model to investigate the neutron star and strange hadronic star. The numerical results are discussed in section IV. We summarize the main results in section V.

**II. THE MODEL**

Our considerations are based on the chiral $SU(3)$ quark mean field model (for details see Refs. [25,26]), which contains quarks and mesons as the basic degrees of freedom. In the chiral limit, the quark field $\Psi$ can be split into left and right-handed parts $\Psi_L$ and $\Psi_R$: 
\[ \Psi = \Psi_L + \Psi_R. \] Under \( SU(3)_L \times SU(3)_R \) they transform as
\[ \Psi_L \rightarrow \Psi'_L = L \Psi_L, \quad \Psi_R \rightarrow \Psi'_R = R \Psi_R. \] (1)
The spin-0 mesons are written in the compact form
\[ \frac{M}{M^\dagger} = \Sigma \pm i \Pi = \frac{1}{\sqrt{2}} \sum_{a=0}^{8} (s^a \pm i p^a) \lambda^a, \] (2)
where \( s^a \) and \( p^a \) are the nonets of scalar and pseudoscalar mesons, respectively, \( \lambda^a (a = 1, \ldots, 8) \) are the Gell-Mann matrices, and \( \lambda^0 = \sqrt{\frac{2}{3}} I \). The alternatives indicated by the plus and minus signs correspond to \( M \) and \( M^\dagger \), respectively. Under chiral \( SU(3) \) transformations, \( M \) and \( M^\dagger \) transform as \( M \rightarrow M' = LMR^\dagger \) and \( M^\dagger \rightarrow M'^\dagger = R \Psi L^\dagger \). The spin-1 mesons are arranged in a similar way as
\[ l^\mu _\mu \rightarrow l'^\mu _\mu = L l^\mu _\mu L^\dagger, \quad r^\mu _\mu \rightarrow r'^\mu _\mu = R r^\mu _\mu R^\dagger. \] (3)
with the transformation properties: \( l^\mu _\mu \rightarrow l'^\mu _\mu = L l^\mu _\mu L^\dagger, \quad r^\mu _\mu \rightarrow r'^\mu _\mu = R r^\mu _\mu R^\dagger \). The matrices \( \Sigma, \Pi, V^\mu _\mu \) and \( A^\mu _\mu \) can be written in a form where the physical states are explicit. For the scalar and vector nonets, we have the expressions
\[ \Sigma = \frac{1}{\sqrt{2}} \sum_{a=0}^{8} s^a \lambda^a = \begin{pmatrix} \frac{1}{\sqrt{2}} (\sigma + a^0_0) & a^+_0 & K^{*+} \\ a^-_0 & \frac{1}{\sqrt{2}} (\sigma - a^0_0) & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \zeta \end{pmatrix}, \] (4)
\[ V^\mu _\mu = \frac{1}{\sqrt{2}} \sum_{a=0}^{8} v^a_\mu \lambda^a = \begin{pmatrix} \frac{1}{\sqrt{2}} (\omega^0_\mu + \rho^0_\mu) & \rho^+_\mu & K^{*+}_\mu \\ \rho^-_\mu & \frac{1}{\sqrt{2}} (\omega^0_\mu - \rho^0_\mu) & K^{*0}_\mu \\ K^{*-}_\mu & \bar{K}^{*0}_\mu & \phi^-_\mu \end{pmatrix}. \] (5)
Pseudoscalar and pseudovector nonet mesons can be written in a similar fashion.

The total effective Lagrangian is written:
\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_{qM} + \mathcal{L}_{\Sigma \Sigma} + \mathcal{L}_{VV} + \mathcal{L}_{\chi \Sigma} + \mathcal{L}_{\Delta m_s} + \mathcal{L}_{h_s} + \mathcal{L}_c, \] (6)
where \( \mathcal{L}_0 = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi \) is the free part for massless quarks. The quark-meson interaction \( \mathcal{L}_{qM} \) can be written in a chiral \( SU(3) \) invariant way as
\[ \mathcal{L}_{qM} = g_s \left( \bar{\Psi}_L M \Psi_R + \bar{\Psi}_R M^\dagger \Psi_L \right) - g_v \left( \bar{\Psi}_L \gamma^\mu t_\mu \Psi_L + \bar{\Psi}_R \gamma^\mu t_\mu \Psi_R \right) \]
\[ = \frac{g_s}{\sqrt{2}} \bar{\Psi} \left( \sum_{a=0}^8 s_a \lambda_a + i \gamma^5 \sum_{a=0}^8 p_a \lambda_a \right) \Psi - \frac{g_v}{2\sqrt{2}} \bar{\Psi} \left( \gamma^\mu \sum_{a=0}^8 U_\mu^a \lambda_a - \gamma^\mu \gamma^5 \sum_{a=0}^8 a_\mu^a \lambda_a \right) \Psi. \]  

(7)

From the quark-meson interaction, the coupling constants between scalar mesons, vector mesons and quarks have the following relations:

\[ \frac{g_s}{\sqrt{2}} = g_{a_o}^u = -g_{a_o}^d = g_{a}^u = g_{a}^d = \ldots = \frac{1}{\sqrt{2}} g_{s}^x, \quad g_{a_o}^s = g_{a}^s = g_{a}^u = g_{a}^d = 0, \]  

(8)

\[ \frac{g_v}{2\sqrt{2}} = g_{a_o}^u = -g_{a_o}^d = g_{a}^u = g_{a}^d = \ldots = \frac{1}{\sqrt{2}} g_{s}^x, \quad g_{a_o}^s = g_{a}^s = g_{a}^u = g_{a}^d = 0. \]  

(9)

In the mean field approximation, the chiral-invariant scalar meson \( \mathcal{L}_{\Sigma \Sigma} \) and vector meson \( \mathcal{L}_{VV} \) self-interaction terms are written as [25,26]

\[ \mathcal{L}_{\Sigma \Sigma} = -\frac{1}{2} k_0 \chi^2 \left( \sigma^2 + \zeta^2 \right) + k_1 \left( \sigma^2 + \zeta^2 \right)^2 + k_2 \left( \frac{\sigma^4}{2} + \zeta^4 \right) + k_3 \chi \sigma^2 \zeta \]
\[ - k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0^4} + \frac{\delta}{3} \chi^4 \ln \frac{\sigma^2 \zeta}{\sigma_0^2 \zeta_0}, \]  

(10)

\[ \mathcal{L}_{VV} = \frac{1}{2} \chi_0^2 \left( m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_\phi^2 \phi^2 \right) + g_4 \left( \omega^4 + 6 \omega^2 \rho^2 + \rho^4 + 2 \phi^4 \right), \]  

(11)

where \( \delta = 6/33; \sigma_0 \) and \( \zeta_0 \) are the vacuum expectation values of the corresponding mean fields \( \sigma, \zeta \) which are expressed as

\[ \sigma_0 = -F_\pi, \quad \zeta_0 = \frac{1}{\sqrt{2}} (F_\pi - 2F_K). \]  

(12)

The vacuum value \( \chi_0 \) is about 280 MeV in our numerical calculation. The Lagrangian \( \mathcal{L}_{\chi SB} \) generates the nonvanishing masses of pseudoscalar mesons

\[ \mathcal{L}_{\chi SB} = \frac{\chi_0^2}{\chi_0^2} \left[ m_\pi^2 F_\pi \sigma + \left( \sqrt{2} m_K^2 F_K - \frac{m_\pi^2}{\sqrt{2}} F_\pi \right) \zeta \right], \]  

leading to a nonvanishing divergence of the axial currents which in turn satisfy the partial conserved axial-vector current (PCAC) relations for \( \pi \) and \( K \) mesons. Pseudoscalar, scalar mesons and also the dilaton field \( \chi \) obtain mass terms by spontaneous breaking of chiral symmetry in the Lagrangian of Eq. (10). The masses of \( u, d \) and \( s \) quarks are generated by the vacuum expectation values of the two scalar mesons \( \sigma \) and \( \zeta \). To obtain the correct constituent mass of the strange quark, an additional mass term has to be added:

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\[ \mathcal{L}_{\Delta m_s} = -\Delta m_s \bar{q} Sq \]  

where \( S = \frac{1}{3} \left( I - \lambda_8 \sqrt{3} \right) = \text{diag}(0, 0, 1) \) is the strangeness quark matrix. Based on these mechanisms, the quark constituent masses are finally given by

\[ m_u = m_d = -\frac{g_s}{\sqrt{2}} \sigma_0 \quad \text{and} \quad m_s = -g_s \zeta_0 + \Delta m_s. \]  

(15)

The parameters \( g_s = 4.76 \) and \( \Delta m_s = 29 \text{ MeV} \) are chosen to yield the constituent quark masses \( m_q = 313 \text{ MeV} \) and \( m_s = 490 \text{ MeV} \). In order to obtain reasonable hyperon potentials in hadronic matter, we include an additional coupling between strange quarks and the scalar mesons \( \sigma \) and \( \zeta \) [25]. This term is expressed as

\[ \mathcal{L}_h = [h_1(\sigma - \sigma_0) + h_2(\zeta - \zeta_0)] \bar{s}s. \]  

(16)

Therefore, the strange quark scalar-coupling constants are modified and do not exactly satisfy Eq. (8). The hyperon potentials were listed in our previous paper [29]. In the quark mean field model, quarks are confined in baryons by the Lagrangian \( \mathcal{L}_c = -\bar{\Psi} \chi_c \Psi \) (with \( \chi_c \) given in Eq. (17), below). We note that this confining term is not chiral invariant. Possible extensions of the model which would restore chiral symmetry in this term have been discussed in Ref. [32].

The Dirac equation for a quark field \( \Psi_{ij} \) under the additional influence of the meson mean fields is given by

\[ \left[ -i\vec{\alpha} \cdot \vec{\nabla} + \beta \chi_c(r) + \beta m_i^* \right] \Psi_{ij} = e_i^* \Psi_{ij}, \]

(17)

where \( \vec{\alpha} = \gamma^0 \vec{\gamma} \), \( \beta = \gamma^0 \), the subscripts \( i \) and \( j \) denote the quark \( i \) (\( i = u, d, s \)) in a baryon of type \( j \) (\( j = N, \Lambda, \Sigma, \Xi \)); \( \chi_c(r) \) is a confinement potential, i.e. a static potential providing the confinement of quarks by meson mean-field configurations. In the numerical calculations, we choose \( \chi_c(r) = \frac{1}{2} k_c r^2 \), where \( k_c = 1 \text{ (GeV fm}^{-2} \text{)} \), which yields baryon radii (in the absence of the pion cloud [33]) around 0.6 fm. The quark mass \( m_i^* \) and energy \( e_i^* \) are defined as

\[ m_i^* = -g^i_\sigma \sigma - g^i_\zeta \zeta + m_{i0} \]  

(18)
and

\[ e_i^* = e_i - g^i_\omega \omega - g^i_\rho \rho - g^i_\phi \phi, \]  

(19)

where \( e_i \) is the energy of the quark under the influence of the meson mean fields. Here \( m_{i0} = 0 \) for \( i = u, d \) (nonstrange quark) and \( m_{i0} = \Delta m_s \) for \( i = s \) (strange quark). The effective baryon mass can be written as

\[ M_j^* = \sum_i n_{ij} e_i^* - E_j^0, \]  

(20)

where \( n_{ij} \) is the number of quarks with flavor “\( i \)” in a baryon with flavor \( j \), with \( j = N \{ p, n \}, \Sigma \{ \Sigma^\pm, \Sigma^0 \}, \Xi \{ \Xi^0, \Xi^- \}, \Lambda \) and \( E_j^0 \) was found to be only very weakly dependent on the external field strength. We therefore use Eq. (20), with \( E_j^0 \) a constant, independent of the density, which is adjusted to give a best fit to the free baryon masses. Compared with the earlier square root ansatz, here we use the linear definition of effective baryon mass. As we have explained in Ref. [29] the linear definition of effective baryon mass has been derived using a symmetric relativistic approach [34], while to the best of our knowledge, no equivalent derivation exists for the square root case.

III. HADRONIC SYSTEM

Based on the previously defined quark mean field model the thermodynamical potential for the study of hadronic systems is written as

\[ \Omega = \sum_{j=N,\Lambda,\Sigma,\Xi} \frac{-2k_B T}{(2\pi)^3} \int_0^\infty d^3k \left\{ \ln \left( 1 + e^{(-E_j^*(k) - \nu_j)/k_B T} \right) + \ln \left( 1 + e^{-(E_j^*(k) + \nu_j)/k_B T} \right) \right\} - \mathcal{L}_M, \]

(21)

where \( E_j^*(k) = \sqrt{M_j^{*2} + k^2} \) and \( M_j^* \) is the effective baryon mass. The quantity \( \nu_j \) is related to the usual chemical potential \( \mu_j \) by \( \nu_j = \mu_j - g^j_\omega \omega - g^j_\rho \rho - g^j_\phi \phi \). The mesonic Lagrangian

\[ \mathcal{L}_M = \mathcal{L}_{\Sigma\Sigma} + \mathcal{L}_{VV} + \mathcal{L}_{\chi SB} \]

(22)
describes the interaction between mesons which includes the scalar meson self-interaction $\mathcal{L}_{\Sigma\Sigma}$, the vector meson self-interaction $\mathcal{L}_{VV}$ and the explicit chiral symmetry breaking term $\mathcal{L}_{\chi SB}$ defined previously in Eqs. (10), (11) and (13). The Lagrangian $\mathcal{L}_M$ involves scalar ($\sigma$, $\zeta$ and $\chi$) and vector ($\omega$, $\rho$ and $\phi$) mesons. The interactions between quarks and scalar mesons result in the effective baryon masses $M_j^*$. The interactions between quarks and vector mesons generate the baryon-vector meson interaction terms. The energy per volume and the pressure of the system can be derived as $\varepsilon = \Omega - \frac{1}{T} \frac{\partial \Omega}{\partial T} + \nu_j \rho_j$ and $p = -\Omega$, where $\rho_j$ is the density of baryon $j$. At zero temperature, $\Omega$ can be expressed as

$$\Omega = - \sum_{j=N,\Lambda,\Sigma,\Xi} \frac{1}{24\pi^2} \left\{ \nu_j \left[ \nu_j^2 - M_j^{*2} \right]^{1/2} \left[ 2\nu_j^2 - 5M_j^{*2} \right] + 3M_j^{*4} \ln \left[ \frac{\nu_j + \left( \nu_j^2 - M_j^{*2} \right)^{1/2}}{M_j^*} \right] \right\} - \mathcal{L}_M, \quad (23)$$

The equations for mesons $\phi_i$ can be obtained by the formula $\frac{\partial \Omega}{\partial \rho_i} = 0$. Therefore, the equations for $\sigma$, $\zeta$ and $\chi$ are

$$k_0 \chi^2 \sigma - 4k_1 \left( \sigma^2 + \zeta^2 \right) \sigma - 2k_2 \sigma^3 - 2k_3 \chi \sigma \zeta - \frac{2\delta}{3\sigma} \chi^4 + \frac{\chi^2}{\chi_0^2} m_\pi^2 F_\pi$$

$$- \left( \frac{\chi}{\chi_0} \right)^2 m_\omega \omega^2 \frac{\partial m_\omega}{\partial \sigma} + \sum_{j=N,\Lambda,\Sigma,\Xi} \frac{\partial M_j^*}{\partial \sigma} < \bar{\psi}_j \psi_j > = 0, \quad (24)$$

$$k_0 \chi^2 \zeta - 4k_1 \left( \sigma^2 + \zeta^2 \right) \zeta - 4k_2 \zeta^3 - 3k_3 \chi \sigma^2 - \frac{\delta}{3\zeta} \chi^4 + \frac{\chi^2}{\chi_0^2} \left( \sqrt{2}m_k^2 F_k - \frac{1}{\sqrt{2}}m_\pi^2 F_\pi \right)$$

$$- \left( \frac{\chi}{\chi_0} \right)^2 m_\rho \rho^2 \frac{\partial m_\rho}{\partial \zeta} + \sum_{j=N,\Lambda,\Sigma,\Xi} \frac{\partial M_j^*}{\partial \zeta} < \bar{\psi}_j \psi_j > = 0, \quad (25)$$

$$k_0 \chi \left( \sigma^2 + \zeta^2 \right) - k_2 \sigma^2 \zeta + \left[ 4k_4 + 1 + 4\ln \frac{\chi}{\chi_0} - \frac{4\delta}{3\ln \sigma_0^2 \zeta_0} \chi^3 \right]$$

$$+ \frac{2\chi}{\chi_0} \left[ m_\pi^2 F_\pi \sigma + \left( \sqrt{2}m_k^2 F_k - \frac{1}{\sqrt{2}}m_\pi^2 F_\pi \right) \zeta - \frac{\chi}{\chi_0} \left( m_\omega \omega^2 + m_\rho \rho^2 + m_\phi \phi^2 \right) \right] = 0, \quad (26)$$

where $< \bar{\psi}_j \psi_j >$ is expressed as

$$< \bar{\psi}_j \psi_j > = \frac{M_j^*}{\pi^2} \int_0^{k_{F_j}} dk \frac{k^2}{\sqrt{M_j^{*2} + k^2}} \quad (27)$$

$$= \frac{M_j^*}{2\pi^2} \left[ k_{F_j} \sqrt{M_j^{*2} + k_{F_j}^2} - \ln \left( \frac{k_{F_j}}{M_j^*} + \sqrt{1 + \frac{k_{F_j}^2}{M_j^{*2}}} \right) \right].$$
with $k_{F_j} = \sqrt{\nu_j^2 - M_j^2}$.

For the $\beta$-equilibrium, the chemical potentials for the baryons satisfy the following equations:

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n = \mu_p + \mu_e = \mu_p + \mu_\mu,$$

(28)

$$\mu_{\Sigma^+} = \mu_p,$$

(29)

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e.$$  

(30)

There are only two independent chemical potentials which are determined by the total baryon density and neutral charge:

$$\rho_B = \rho_p + \rho_n + \rho_\Lambda + \rho_{\Sigma^+} + \rho_{\Sigma^0} + \rho_{\Sigma^-} + \rho_{\Xi^0} + \rho_{\Xi^-},$$

(31)

$$\rho_p + \rho_{\Sigma^+} - \rho_{\Sigma^-} - \rho_{\Xi^-} - \rho_e - \rho_\mu = 0.$$  

(32)

In order to get the mass-radius relation, one has to resolve the Tolman-Oppenheimer-Volkoff (TOV) equation:

$$\frac{dp}{dr} = -\frac{[p(r) + \varepsilon(r)] [M(r) + 4\pi r^3 p(r)]}{r(r - 2M(r))},$$

(33)

where

$$M(r) = 4\pi \int_0^r \varepsilon(r)r^2dr.$$  

(34)

With the equations of state, the functions, such as $M(r)$, $\rho(r)$ and $p(r)$, etc. can be obtained.

**IV. NUMERICAL RESULTS**

The parameters of this model were determined by the meson masses in vacuum and the saturation properties of nuclear matter which were listed in the table 1 of Ref. [29]. The improved linear definition of effective baryon mass is chosen in our numerical calculations.
We first discuss the equations of state of neutron matter and strange hadronic matter which are needed for the calculation of neutron stars. For pure neutron stars, there are only neutrons present. For strange hadronic stars, with increasing baryon density, other kinds of baryons will appear. In Fig. 1 we show the fractions of octet baryons versus density with $\beta$-equilibrium. With the increasing of baryon density, the neutron fraction decreases slowly from 1. If the density is lower than about 0.19 fm$^{-3}$, the fraction of electrons is the same as that of protons which makes the system charge neutral. The muon appears when the density is in the range 0.19 - 0.98 fm$^{-3}$. The maximum fractions of muons and electrons appear at $\rho_B \simeq 0.4$ fm$^{-3}$. Their fractions decrease with the increasing fractions of hyperons. When the density is larger than about 0.4 fm$^{-3}$, the $\Sigma^-$ hyperons appear and the fraction of neutrons decreases faster. After the density is larger than about 0.57 fm$^{-3}$, $\Lambda$ hyperons start to appear. The fraction of $\Sigma^-$ hyperons decreases with the increasing density after $\Xi^-$ hyperons appear where the density is about 0.84 fm$^{-3}$.

![FIG. 1. The fractions of proton, neutron, $\Lambda$, $\Sigma$ and $\Xi$ of strange hadronic stars versus baryon density with $\beta$-equilibrium.](image)
FIG. 2. The effective baryon masses and meson mean fields versus baryon density with $\beta$-equilibrium.

The density dependence of the effective baryon masses and scalar mean fields are shown in Fig. 2. The $\sigma$ field decreases quickly with the increasing baryon density when the density is small, $\rho_B < 0.4 \text{ fm}^{-3}$. This is because at small baryon density, the nucleon is dominant and there are no hyperons. With the increasing of density, more and more hyperons appear. As a result, the $\zeta$ field decreases quickly. At a broad range of densities, the value of $\chi$ changes little.

In Fig. 3, the pressure versus baryon density is shown. The dashed and solid lines are for the pure neutron star and the strange hadronic star with $\beta$-equilibrium, respectively. When the density is low, the two curves are close to each other. With the increasing of baryon density, the contributions of protons and hyperons are not negligible. The inclusion of hyperons will soften the equation of state of hadronic matter. As a result, at a given baryon density the pressure of strange hadronic matter is smaller than the corresponding pressure of pure neutron matter. The pressure $p$ versus energy density $\varepsilon$ is shown in Fig. 4. Again, one can see that the equation of state of strange hadronic matter is softer than that of pure neutron matter.
FIG. 3. The pressure of hadronic matter $p$ versus baryon density $\rho_B$. The dashed and solid curves are for pure neutron stars and strange hadronic stars with $\beta$-equilibrium, respectively.

FIG. 4. The pressure of hadronic matter $p$ versus energy density $\varepsilon$. The dashed and solid curves are for pure neutron stars and strange hadronic stars with $\beta$-equilibrium, respectively.

We now study neutron stars with the obtained EOS. By solving the TOV equation, the baryon density versus radius can be obtained which is shown in Fig. 5. The central densities $\rho_c$ are chosen to be $3\rho_0$ and $5\rho_0$ where $\rho_0$ ($0.16$ fm$^{-3}$) is the saturation density of symmetric nuclear matter. The dashed and solid lines are for pure neutron stars and strange hadronic stars with $\beta$-equilibrium, respectively. With the increasing radius, the density of strange hadronic stars decreases a little faster than that of pure neutron stars which results in a
smaller radius. The radii of stars are not sensitive to their the central density. For example, for $\rho_c$ of $3\rho_0$ and $5\rho_0$, the radii are both around 11-12 km.

![Graph showing baryon density versus radius](image)

FIG. 5. The baryon density of a hadronic star versus radius. The dashed and solid curves are for pure neutron stars and strange hadronic stars with $\beta$-equilibrium, respectively.

We plot the star mass ratio $M/M_{\text{sun}}$ versus central baryon density in Fig. 6. The maximum mass of pure neutron stars is about $1.8M_{\text{sun}}$ with a central density $1.05 \text{ fm}^{-3}$. After the central density is larger than $1.05 \text{ fm}^{-3}$, the star will become unstable. The maximum mass changes to $1.45M_{\text{sun}}$ when hyperons are included. In the range $3\rho_0 < \rho_c < 6\rho_0$, the masses of pure neutron stars and strange hadronic stars are $1.48M_{\text{sun}} < M < 1.8M_{\text{sun}}$ and $1.23M_{\text{sun}} < M < 1.45M_{\text{sun}}$, respectively. Our results are reasonable compared with the observation of the six known stars with masses in the range $1.36 \pm 0.08M_{\text{sun}}$, since the “neutron star” is in fact a strange hadronic star with $\beta$-equilibrium. We should also keep in mind that there are some heavy stars reported in recent years. For PSR J0437-4715, the mass is found to be $1.58 \pm 0.18M_{\text{sun}}$ [35]. For Vale X-1, Cygnus X-2 and 4U 1820-30, their masses are determined to be $1.87^{+0.23}_{-0.17}M_{\text{sun}}$ [36], $1.8 \pm 0.4M_{\text{sun}}$ [37] and $\simeq 2.3M_{\text{sun}}$ [38,39]. The rotation of a star can increase its mass by $\sim 10\%$ [40]. Therefore, the calculated maximum mass of strange hadronic stars can be as large as $1.6M_{\text{sun}}$. If the heavy stars such as 4U 1820-30 are confirmed, the strange hadronic star would be ruled out if this model is a good description of Nature. It is possible to increase the maximum star mass by making the EOS stiffer at
higher densities. Whether the inclusion of a quark core in the strange star will result in a large-maximum mass is an interesting topic.

![Graph showing masses of hadronic stars versus central baryon densities. The dashed and solid curves are for pure neutron stars and strange hadronic stars with $\beta$-equilibrium, respectively.](image)

**FIG. 6.** The masses of hadronic stars versus their central baryon densities. The dashed and solid curves are for pure neutron stars and strange hadronic stars with $\beta$-equilibrium, respectively.

In Fig. 7, the masses of stars versus their radii are shown. For pure neutron stars, when their masses are in the range $0.5M_{\text{sun}} < M < 1.8M_{\text{sun}}$, their radii are about 11.0-12.3 km. For the strange hadronic stars, when the masses are in the range $0.5M_{\text{sun}} < M < 1.45M_{\text{sun}}$, the radii are about 10.7-11.7 km. With the same mass ($M > 0.2M_{\text{sun}}$), strange hadronic stars have smaller radii compared with pure neutron stars. Because the size of neutron stars is small, it is very difficult to observe and measure their radii directly. Different indirect methods lead to different values of radii with large errors. For example, for the RX J1856-3754, the radius varies from 5 km to 15 km with a mass of $1.4M_{\text{sun}}$ [5]. More accurate values are needed to obtain a more strict constraint on the EOS of hadronic matter.
V. SUMMARY

We have investigated pure neutron stars and strange hadronic stars in the chiral $SU(3)$ quark mean field model. The $\Lambda$, $\Sigma$ and $\Xi$ hyperons are included in the model. The proton and hyperon contributions to the system are important at high baryon density when $\beta$-equilibrium is achieved, and soften the EOS of hadronic matter. The maximum pure neutron star mass is about $M = 1.8M_{\text{sun}}$ with a corresponding radius $R = 11.0$ km and central density $\rho_c = 1.05$ fm$^{-3}$. For the strange hadronic stars, the maximum masses are about $1.45M_{\text{sun}}$ and the corresponding radii and central density are $R = 10.9$ km and $\rho_c = 1.0$ fm$^{-3}$. When the central densities are between $3\rho_0$ and $6\rho_0$, the masses of stars are in the range $1.23M_{\text{sun}} < M < 1.45M_{\text{sun}}$ (strange hadronic stars) and $1.48M_{\text{sun}} < M < 1.8M_{\text{sun}}$ (pure neutron stars). If the masses of stars are larger than $0.5M_{\text{sun}}$, the typical values of radii are 10.7-11.7 km (strange hadronic stars) km and 11.0-12.3 km (pure neutron stars).

Our results are reasonable compared with astrophysical observations where the six known neutron stars have masses in the narrow range $1.36 \pm 0.08M_{\text{sun}}$. Accurate values of radii for neutron stars are needed to get a more strict constraint on the EOS of hadronic matter. As for the heavy stars, for example 4U 1820-30, if its mass $M \simeq 2.3M_{\text{sun}}$ is confirmed,
then strange hadronic stars are obviously ruled out if the model explored herein is a good
description of Nature. It is therefore of interest to see whether including quark degrees of
freedom can lead to this large mass.

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