Synthesis of Optimal Heat Exchanger Networks with Quantified Uncertainties and Non-isothermal Mixing

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Abstract- The primary objective of this study is to develop a simultaneous approach for the synthesis of flexible heat exchanger networks (HENs) with non-isothermal mixing assumptions. The HENs synthesis procedure presented in this study took into consideration quantified uncertainties in inlet temperatures and flow rates with an unpredictable time of shift. The proposed multi-period MINLP model was used to generate a HEN with optimized heat exchanger areas and total annualized costs attributed to utility duties. A framework for generating the flexible HEN over a specified range of variations in flow rates and stream temperature was proposed in this study. The framework was based on a two-stage strategy; a HEN design stage was first performed before the testing stage where the energy-saving potential of the synthesized HEN was established. The effectiveness of the proposed approach was tested for energy minimization using a case study in literature with variation in inlet temperature and flow rate. It was observed that the inclusion of non-isothermal parameters in the non-linear model resulted in a HEN that optimally works under fluctuating conditions without losing stream temperature targets while maintaining economically-optimal energy integration.

Keywords: Energy minimization, Heat exchanger networks, MINLP models, Non-isothermal mixing

1. Introduction

Energy is a vital commodity with increasing demand in most process industries [1]. The cost of energy in a particular industrial process usually determines how cost-effective the process is. High energy requirement associated with most industrial processes (e.g. distillation, CO₂ separation, etc.) is raising serious global concerns today. There is a need to minimize energy consumption in these industrial processes to guarantee their economic benefit. Researchers have shown that the development of state-of-the-art strategies for energy conservation is fundamental to reducing energy consumption in selected energy-intensive processes like absorptive CO₂ capture [2]. Most studies in the past proposed the use of external utilities to maintain energy efficiency during industrial applications [3]. However, it has been observed that although the use of external utilities has recorded significant improvements in maintaining energy efficiency in the past [4], it is expensive thereby increasing the economic cost of the process. In response to this, this study seeks to explore the use of internal process heat as a substitute to external heating and cooling utilities as a cost-effective heat recovery process which could solve the problem of energy efficiency, and subsequently cut-down process cost.
Heat exchanger networks (HENs) synthesis have been adequately investigated for energy minimization in selected industrial applications in the past. HEN synthesis was first reported in the literature by Broeck [5] and subsequently, comprehensive reviews on heat exchanger network synthesis emphasizing its application in decreasing energy consumption in most industrial processes have been documented in open literature by Furman and Sahinidis [6] and Yoro et al. [7]. The importance of HENs can be ascribed to its role in controlling the energy cost of a process while the target of every HEN synthesis task is to develop a network design that optimally minimizes investment cost in units alongside the operating cost with respect to utility consumption.

Most HENs synthesis techniques reported in the past used sequential approaches [8],[9],[10] while a few considered simultaneous approaches [11],[12]. Sequential synthesis methodologies involve the decomposition of a task into smaller tasks through temperature interval partitioning. In a sequentially-based synthesis technique, targets are set based on existing thermodynamics rules such that it either involves minimization of utility cost, the total number of heat exchangers or the heat transfer area of the network. Pinch-based approaches are the most common sequential techniques for HENs synthesis. On the other hand, simultaneous methodologies for HENs synthesis apply mathematical programming concepts through unconventional mathematical solvers to optimize an objective function that explores diverse competing variables at the same time. The famous paper of Yee and Grossman [12] has been the basis for most simultaneous-based techniques for HENs synthesis. The authors presented a technique based on the application of superstructures with mixed-integer nonlinear programs to minimize an objective function that considered fixed, area, and utility costs. Till date, most reports on the synthesis of flexible HENs develop their designs from the model and superstructure of Yee and Grossman [12].

Conventionally, syntheses of HENs have been consistently reported with operating parameters assumed to be fixed at nominal conditions. However, in a plant environment, there are significant uncertainties attributed to fluctuations in the plant operating parameters as well as other perturbations which distraught the system. In view of this, it is expected that a good HEN design should have the capacity to uphold a reasonable steady-state operation under indeterminate circumstances. Escobar et al. [13] published a report on the synthesis of flexible HEN with operability considerations. The authors were able to successfully develop a framework that incorporates both flexibility and controllability aspects in HENs. Nonetheless, the main limitation of their study was that they applied only steady-state linear models to achieve controllability which resulted in substantial deviations from the non-linear model, and this subsequently limits its applicability.

Before now, researchers have published different methods to solve problems with multi-period operations and multiple utilities for HENs [14],[15]. However, most of the methodologies reported in the literature used the theory of optimizing a condensed framework from a set of feasible matches to advance the generation of results and decrease the intricacy of the resultant network [16]. The main shortcoming identified in previously reported methodologies was that the resultant HENs always assume that the split streams at each period mix isothermally thereby preventing the superstructure and splitting streams from exchanging heat energy in a chain formation. In addition, more time is also needed in creating simple superstructure resolution grids, from which the condensed superstructure is produced. Again; there were no clear
benchmarks for defining likely matches with an exception that such matches occur in the finest set of primary superstructure resolution network.

In this study, the inclusion of parametric flexibility in the synthesis of HENs is presented using a modified SYNHEAT model of Yee and Grossman [12]; but in this case, it excludes the isothermal mixing assumption. Although Bjork and Westerlund [17] were the first set of researchers to suggest the exclusion of the isothermal mixing assumption of the Yee and Grossman model, the authors did not consider uncertain disturbances with fluctuations in operating parameters in their methodology. The model of Yee and Grossman [12] was considered in this study because of its simplicity and concurrent inclusion of trade-offs amongst the cost of utility, the number of heat exchanger units and cost of heat transfer areas. Uncertain parameters were defined here by a set of distinct operational points considering unequal duration and unpredictable time of shift. The major contribution of this study is that the proposed model considers variable stream flow rates and temperatures with non-isothermal mixing. The HEN in this study was generated in a unique step, and all fluctuating parameters were concurrently optimized.

1.1 Simultaneous synthesis of multi-period HENS

Different simultaneous methods for HENs synthesis in multi-period scenarios have been presented in literature before now, but not without some limitations. For instance, Isafiade et al. [14] modified a multi-period SWS model to handle tasks with multiple utility options. The researchers suggested a method where the model was solved several times and the common matches of the multi-period HENs were used to initialize a reduced superstructure before it was solved as an MINLP model. The major weakness observed in this approach is that the result attained is constrained to a pre-determined fixed period. Ahmad et al. [18] introduced a stochastic-based optimization technique to synthesize of flexible and multi-period HENs. However, their methodology did not consider the effect of uncertain parameters and unpredictable time of shift. Jiang and Chang [19] used a timesharing method through a common area allocation algorithm to identify the descriptive period depending on the period with the longest time interval while Kang et al. [20] developed a symbolic-period technique with a stepwise generalization method to generate multi-period HENs that include features of sub-periods. In both reports, the authors identified the characteristic period using a method proposed by Jiang and Chang [19]. In addition, Escobar et al. [21] suggested a novel Lagrangian disintegration methodology while El-Temtany and Gabr [22] modified the MINLP models of Floudas and Grossman [23] by introducing a method where the models were solved iteratively in a random manner. However, in the aforementioned reports, the methodologies were based on an automated sequential approach for HENs where each step depended on solutions from previous steps which makes it non-simultaneous in nature. Furthermore, Li and Land-Silva [24] developed a method that was used to synthesize flexible HENs in which the level of network flexibility was determined for a non-convex heat exchanger task using a simulated annealing algorithm. To account for the degree of change of fluctuating parameters in the proposed methodology, the authors suggested a direction matrix approach which is different from the approach presented in this study. Although most of the aforementioned literature reports discussed in this paper involved multi-period scenarios, it is worthy to note that fluctuations in process parameters are unpredictable in most industrial operations, and these fluctuations take
place around a nominal point. Hence, the need to consider an unpredictable time of shift in this study.

The main attention in this study is dedicated to improving and testing a simultaneous procedure to generate flexible multi-period heat exchanger networks (HENs) that consider quantified uncertainties, non-isothermal mixing and unpredictable time of shift at the same time. The main advantage of the simultaneous technique proposed in this study is its effectiveness in handling the balance between capital and operating costs of the synthesized HENs.

1.2 Objective function

The objective function proposed in this study is analogous to that of Yee and Grossman [12], but with the introduction of a non-isothermal mixing assumption while relaxing the original isothermal mixing assumption. The MINLP optimization model discussed here includes the overheads of hot and cold utility (energy cost), fixed and area costs cost of heat exchangers and cost of some augmentation device. The aim of the objective function is simply to reduce the annual cost of the synthesized HENs. The objective function is presented in Equation (1).

\[
\min \sum_{i \in \text{HP}} \text{CCU}_{qi} + \sum_{j \in \text{CP}} \sum_{k \in \text{CP}} \sum_{j \in \text{CP}} \left( \frac{q_{ijk}}{U_{en,ijklMTD_{ijkl}}} + C_{e,ij} \right) + \sum_{i \in \text{HP}} \left( \frac{q_{ij}}{U_{en,ijklMTD_{ijkl}}} + C_{e,ij} \right)
\]

For Equation (1), we assume that:

\[
U_{e,ij} = \frac{h_{Hi,CI}}{h_{Hi,CI} + h_{Cj}}
\]

\[
U_{CU,ij} = \frac{h_{Hi,CI}}{h_{Hi,CI} + h_{CU}}
\]

\[
U_{HU,ij} = \frac{h_{Hi,CI}}{h_{CU} + h_{Cj}}
\]

\[
\text{LMTD}_{ijkl} = \left( \frac{dt_{ijk} + dt_{ijk+1}}{2} \right)^{1/3}
\]

\[
\text{LMTD}_{CU,ij} = \left( \frac{dty_{ij} + T_{OUT,Hi} - T_{IN,CU}}{2} \right)^{1/3}
\]

\[
\text{LMTD}_{HU,ij} = \left( \frac{dth_{ij} + T_{IN,Hi} - T_{OUT,Cj}}{2} \right)^{1/3}
\]
Estimation of log mean temperature difference in this work is fairly analogous to the one suggested by Chen [25]. This estimation of the log mean temperature difference was adopted in this study as seen in Equations (5),(6) and (7) to avoid singularities while calculating the log mean temperature difference using the method suggested by Liang et al. [26].

2. Materials and methods

2.1 Materials

Resources used in this study consist of a 64-bit computer operating system (window 7 professional) with the following processor details; Intel(R) Core (TM) i5-4590 CPU@3.30GHz. The software used to run mathematical programs is a fully licensed GAMS (General Algebraic Modelling System) software with the following details; GAMSIDE build44386 / 46072, GAMS Release24.2.3 r46072 WEX-WEI x86_64/MS Windows. The DICOPT solver was used throughout this study to crack down the HENs problem. The approach used in this paper is purely simultaneous in nature because it handles the given task in one piece and addresses it unswervingly without breaking it down into smaller problems.

2.2 Methods

The technique used in this study primarily embroils the development of MINLP models for the heat exchanger problem. The mathematical formulation involves the modification of a stage-wise superstructure that considers all possible matches between hot and cold streams divided into an already known number of stages (k=2) with the inclusion of a non-isothermal mixing assumption as shown in Figure 1.

![Figure 1: The stage-wise superstructure as used in this study (Modified from Yee and Grossman [12])]
optimization model, the choice of stream matches and number of heat exchanger units for the network were also enhanced. To account for the non-isothermal behaviour, non-linearity was introduced into the model expressing logarithmic mean temperature difference proposed by Chen [25] while binary variables were stated using suitable model indices that denote network configurations with respect to selected matches which ensures proper calculation of fixed charges of the HENs.

**Definition of terms:**
- \( B_{CU} \) = Area cost of heater exponent
- \( B_e \) = Exponent of HE area cost
- \( B_{HU} \) = Exponent of cooler’s area cost
- \( C_{CU} \) = Hot utility unit cost
- \( C_e \) = Heat exchanger area cost coefficient
- \( C_{CU} \) = Unit charge of cooler (fixed)
- \( C_{HU} \) = Heat exchanger unit’s fixed charge
- \( C_{HU} \) = Heater unit’s fixed charge
- \( C_{HU} \) = Area cost coefficient for the heater
- \( C_{HU} \) = Unit cost of hot utility
- \( T, I_{CU} \) = Temperature of cold utility (inlet)
- \( T, I_{H} \) = Initial inlet temperature of hot stream
- \( T, I_{HU} \) = Inlet temperature of the hot utility
- \( T, O_{TU} \) = Hot Stream final outlet temperature
- \( T, O_{HU} \) = Hot utility temperature (outlet)
- \( U_{CU} \) = Overall heat transfer coefficient for match of hot stream \( i \) and cold utility
- \( U_{HU} \) = Overall heat transfer coefficient for match of cold stream \( j \) and hot utility
- \( EMAT \) = Estimated minimum approach temperature
- \( i \) = Hot process stream index
- \( IN \) = Inlet
- \( j \) = Cold process stream index
- \( k \) = Stage index
- \( LMTD \) = Log mean temperature difference
- \( OUT \) = Outlet
- \( q \) = quantity of exchanged heat between hot stream \( I \) and cold stream \( j \) in stage \( k \)
- \( q_{cu} \) = Heat exchanged between hot stream \( I \) and cold utility
- \( U \) = coefficient of transferred heat for each heat exchanger (overall)
- \( z \) = Binary variable showing existence of matches for hot and cold streams \( i \) and \( j \) in stage \( k \)
- \( z_{cu} \) = Variable showing presence of matches for cold utility (Binary)
- \( z_{HU} \) = Variable showing presence of matches for hot utility (Binary).
Heat balance for the exchanger in respective stages was calculated from Equations (8) and (9):

\[(t_{\text{p}_H,i,j,k,\text{IN}} - t_{\text{p}_H,i,j,k,\text{OUT}})F_{p_{H,i,j,k}} = q_{ij,k} , \quad i \in \text{HP}, \quad j \in \text{CP}, \quad k \in \text{KN},\]

\[(t_{\text{p}_C,i,j,k,\text{OUT}} - t_{\text{p}_C,i,j,k,\text{IN}})F_{p_{C,i,j,k}} = q_{ij,k} , \quad i \in \text{HP}, \quad j \in \text{CP}, \quad k \in \text{KN}\]

\(t_{\text{p}_H}\) is the temperature of the splitting stream (hot), while \(t_{\text{p}_C}\) is the splitting stream temperature (cold), \(F_{p_H}\) is the splitting stream (hot) heat capacity flow rate and \(F_{p_C}\) is the cold splitting stream heat capacity flow rate in each stage.

Each splitter has a heat balance; this was calculated using Equations (10) and (11):

\[F_{H,i} \cdot t_{j,k} = \sum_{j=1}^{CN} F_{p_{H,i,j,k}} \cdot t_{p_{H,i,j,k,\text{OUT}}} , \quad j \in \text{CP}, \quad k \in \text{KN}\]

\[F_{C,j} \cdot t_{C,j,k} = \sum_{i=1}^{HN} F_{p_{C,i,j,k}} \cdot t_{p_{C,i,j,k,\text{OUT}}} , \quad i \in \text{HP}, \quad k \in \text{KN}\]

For each splitter in the network, the mass balance was calculated using Equations (12) and (13):

\[F_{H,i} = \sum_{j=1}^{CN} F_{p_{H,i,j,k}} \quad (i = 1, 2, \ldots \text{HN}; \quad k = 1, 2, \ldots \text{KN})\]

\[F_{C,j} = \sum_{i=1}^{HN} F_{p_{C,i,j,k}} \quad (j = 1, 2, \ldots \text{CN}; \quad k = 1, 2, \ldots \text{KN})\]

The overall heat balance for each stream was calculated using Equations (14) and (15):

\[(T, \text{IN}_{H,i} - T, \text{OUT}_{H,i})F_{H,i} = \sum_{k \in \text{KN}, j \in \text{CP}} q_{ij,k} + q_{cu_i} , \quad i \in \text{HP}\]

\[(T, \text{IN}_{C,j} - T, \text{OUT}_{C,j})F_{C,j} = \sum_{k \in \text{KN}, i \in \text{HP}} q_{ij,k} + q_{hu_j} , \quad j \in \text{CP}\]

\(T, \text{IN}_{H}\) is the inlet temperature of the hot stream, \(T, \text{IN}_{C}\) is the cold stream inlet temperature, HP is the set of hot process streams, CP represents sets of cold process streams and KN is the set of stages for the superstructure. Heat balance for each stream per stage of the superstructure was determined using Equations (16) and (17):

\[(t_{H,i,k} - t_{H,i,k+1})F_{H,i} = \sum_{j \in \text{CP}} q_{ij,k} , \quad i \in \text{HP}, \quad k \in \text{KN}\]

\[(t_{C,j,k} - t_{C,j,k+1})F_{C,j} = \sum_{i \in \text{HP}} q_{ij,k} , \quad j \in \text{CP}, \quad k \in \text{KN}\]

Hot and cold utility loads for the network were determined from Equations (18) and (19) respectively:

\[(t_{H,i,N,k+1} - T, \text{OUT}_{H,i})F_{H,i} = q_{cu_i} , \quad i \in \text{HP}\]

\[(T, \text{OUT}_{C,j} - t_{C,j+1})F_{C,j} = q_{hu_j} , \quad j \in \text{CP}\]
Minimum approach temperature was calculated using Equation (20);
\[ d_{t_{i,j,k}} \leq \epsilon, \quad i \in \text{HP}, \quad j \in \text{CP}, \quad k \in \text{KN} \quad (20) \]

Inlet temperature for the superstructure was assigned using Equations (21) and (22);
\[ t_{H,i,1} = T_{i,IN_{H,i}} \quad i \in \text{HP} \quad (21) \]
\[ t_{C,j,N,K+1} = T_{j,IN_{C,j}} \quad j \in \text{CP} \quad (22) \]

The isothermal mixing assumption of the Yee and Grossman model was replaced with a non-isothermal assumption in this study by introducing non-linear and non-convex constraints in Equations (8),(9),(10),(11),(12) and (13).

The structure of heat exchanger area network was determined by building a set of binary variables that represent the existence of each match of the heat exchanger while the binary variables were randomly generated to define likely matches between streams in the two stages. Stream splitting initialization was carried out by initializing all heat capacity flow rates according to the mass balance in Equations (23) and (24);
\[ F_{P_{H,i,j,k}} = F_{H,i} - \sum_{j=1}^{N_{C}} F_{P_{H,i,j,t,k}} (j=N_{C}) \quad (23) \]
\[ F_{P_{C,i,j,k}} = F_{C,i} - \sum_{j=1}^{N_{H}} F_{P_{C,i,t,j,k}} (i=N_{H}) \quad (24) \]

In the temperature initialization stage, the outlet temperature of the cold splitting stream was isolated as an independent variable according to Equation (25) to make the outlet temperature of the splitting streams unequal.
\[ t_{p_{C,i,j,k,OUT}} = T_{i,IN_{H,j}} + \text{rand} (t_{p_{H,i,j,k,IN}} - EMAT - T_{j,IN_{C,j}}) \quad (25) \]

Heat load values were randomly generated in relation to the heat capacity flow rates. Violation of the constraints for minimum approach temperature at both ends of the heat exchanger was avoided in this study by setting a maximum limit of the heat load for the heat exchanger using Equation (26);
\[ q_{max_{i,j,k}} = \min \left( F_{P_{H,i,j,k}} (t_{p_{H,i,j,k,IN}} - T_{OUT_{H,i}}), (F_{P_{C,i,j,k}} (t_{p_{C,i,j,k,OUT}} - T_{IN_{C,j}})) \quad (26) \]

The area cost of the heat exchangers in this study was calculated using Equation (27);
\[ AC (\$) = 10000 + 350A \quad (27) \]

3. Results and discussions
3.1 Results
The problem solved in this study is presented thus; “Given are the stream data, the specified range for the uncertainties (e.g. inlet temperatures and heat capacity flow rates) and a minimum approach temperature”. The task is to simultaneously synthesize a flexible non-isothermal heat exchanger network with minimum Total Annualized Cost (TAC) that is able to operate optimally under the specified range of disturbance (uncertainties) and unpredictable time of
shift. A case study has been adapted from literature and solved according to the proposed technique in this study.

Case study: This problem considers synthesizing a HEN network to minimize energy usage in the absorption of CO₂ from a steam of flue gas (source). The absorption process involves 2 stages with 2 hot and 2 cold streams. Steam and cold water are used in the process as hot and cold utilities respectively. The cost of hot utility (steam) is $60/kW.yr while cold utility (cold water) is pegged at $6/kW.yr. The process data for this example is presented in Table 1, while Table 2 shows the result as compared with literature and the generated network is shown in Figure 2.

Table 1: Process data for the case study

| Streams | T, IN (°C) | T, OUT (°C) | F(kW/°C) | h (kW/m²°C) |
|---------|------------|------------|----------|-------------|
| H1      | 180        | 90         | 100      | 0.60        |
| H2      | 160        | 100        | 140      | 0.50        |
| H3      | 160        | 50         | 80       | 0.20        |
| H4      | 140        | 40         | 400      | 0.30        |
| C1      | 100        | 160        | 100      | 0.40        |
| C2      | 70         | 140        | 60       | 0.70        |
| C3      | 80         | 120        | 350      | 0.60        |
| C4      | 90         | 130        | 200      | 0.20        |
| Steam   | 170        | 170        | -        | 0.60        |
| Cold water | 20        | 80         | -        | 0.60        |

Figure 2: Structure of the synthesized HEN network for the case study considered.

Table 2: Results obtained compared with literature

| H1 | H2 | H3 | H4 | C1 | C2 | C3 | C4 |
|----|----|----|----|----|----|----|----|
| 180| 160| 160| 140| 160| 140| 120| 130|
| 90 | 100| 160| 140| 100| 70 | 80 | 90 |

Area cost = $13,661, Number of units = 7, Total area = 10.46 m², Annual cost = $1,866,100, qhu = 22.77 MW, qcù = 30.55 MW, Life span of the CO₂ absorption plant = 6 years.
### 3.2 Discussion of results

The HENs network generated in this study is presented in Figure 2 while the results obtained are summarized in Table 2 and compared with different methodologies reported by other researchers. Optimal results were achieved in a reasonable computation time of 3026 seconds which is slightly faster than the computation time reported by Liang et al. [26]. This signifies that the proposed MINLP model in this study is reliable, and the inclusion of non-linearity due to the non-isothermal assumption introduced in this work has no significant effect on the behaviour of the mixed integer nonlinear program. When compared with previous studies, it was observed that quantity of internal hot and cold utilities used by the network synthesized in this study alongside other variables was slightly lesser than most of the values obtained from previous reports with exception to the ones reported by Khorasany and Fesanghary [29] (See Table 2).

The slight discrepancies observed in the results presented in Table 2 is attributed to the different simultaneous techniques used by the researchers. For example, Liang et al. [26] used a simultaneous stage-wise synthesis model using initialization techniques in combination with an innovative two-level algorithm while Lewin [27] used a more generalized method that considered stochastic optimization. Although the technique proposed in this study and the approach reported by Khorasany & Fesanghary [29] resulted in a network with the least number of heat exchangers (7 units), the technique presented by Khorasany & Fesanghary [29] yielded more superior results in terms of the network’s annual cost, areas, hot and cold utility demand. This is because the authors used a hybrid methodology that was solved using a two-level algorithmic approach. This implies that if the technique used in this study is solved using a hybrid dualistic-optimization algorithm approach, better results than the ones reported herein may be obtained.

The moderate potential match of hot/cold streams in Figure 2 suggests a lesser number of heat exchanger units in the network which will eventually yield a better solution than a network with many heat exchanger units. This explains why the technique employed in this study yielded better results than most techniques presented in Table 2. The HEN structure in Figure 2 has 2 stages and stream splitting occurs in both stages. The number of stages and branches of each of the split stream in the network is very important because it determines the optimality of the HENs network. The results also signify that area cost for the HEN is proportional to the total area. This means that the higher the area of the synthesized network, the higher its area cost. A smaller number of heat exchanger areas yield cheaper HENs.

| No. of Units | qcu (MW) | qhu (MW) | Annual cost ($x10^6) | Reference                      |
|--------------|----------|----------|-----------------------|--------------------------------|
| 11           | 32.30    | 24.58    | 2.926                 | Liang et al. [26]              |
| 12           | 32.81    | 25.09    | 2.936                 | Lewin [27]                     |
| 13           | 33.03    | 25.31    | 2.960                 | Linhoff and Ahmad [28]         |
| 7            | 28.10    | 18.10    | 1.190                 | Khorasany & Fesanghary [29]    |
| 7            | 30.55    | 22.77    | 1.866                 | This study                     |
4. Conclusions
A model (MINLP) for optimal synthesis of heat exchanger networks that involve shifting source-stream temperatures, fluctuating flow rates, and non-isothermal mixing with an unpredictable time of shift has been proposed, developed and tested in this paper. Although the non-isothermal mixing assumption included in this model introduced non-linear terms in the constraints, no important HEN configuration was lost. The HEN generated has a total area of 10.46 m² and it requires 7 units of heat exchangers with an annual cost of about $1.9 million. Results obtained established that the proposed non-isothermal mixing model in this study can produce a feasible network for indeterminate supply temperatures and flow rates in a comparatively more efficient way with improved results. In addition, the MINLP can be used for process optimization which could enhance effective energy saving. For future studies that may involve increased variables with more non-linear constraints, an initialization strategy with a dualistic (two-level) optimization algorithm should be used to find a solution to the simultaneous HENs problem to obtain more improved results.

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