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--- Does an ideal memristor truly exist?

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ABSTRACT After a decade of research, we developed a prototype device and experimentally demonstrated that the direct $q$-$\phi$ interaction could be memristive, as predicted by Chua in 1971. With a constant input current to avoid any parasitic “inductor” effect, our device meets three criteria for an ideal memristor: a single-valued, nonlinear, continuously differentiable, and strictly monotonically increasing constitutive $q$-$q$ curve, a pinched $v$-$i$ hysteresis loop, and a charge-only-dependent resistance. Our work represents a step forward in terms of experimentally verifying the memristive flux-charge interaction but we have not reached the final because this prototype still suffers from two serious limitations: 1. a superficial but dominant “inductor” effect (behind which the above memristive fingerprints hide) due to its inductor-like core structure, and 2. bistability and dynamic sweep of a continuous resistance range. In this article, we also discuss how to make a fully-functioning ideal memristor with multiple or an infinite number of stable states and no parasitic inductance, and give a number of suggestions, such as “open” structure, nanoscale size, magnetic materials with cubic anisotropy (or even isotropy), and sequential switching of the magnetic domains. Additionally, we respond to a recent challenge from arXiv.org that claims that our device “is simply an inductor with memory” since our device did not pass their designed capacitor-memristor circuit test. Contrary to their conjecture that “an ideal memristor may not exist or may be a purely mathematical concept”, we remain optimistic that researchers will discover an ideal memristor in nature or make one in the laboratory based on our current work.

Keywords memristor; circuit element with memory; neuromorphic computing; brain-inspired computer; artificial intelligence.

I. Introduction

Chua defined the ideal memristor in 1971, which directly interacts physical flux $\phi$ and physical charge $q$ [1], analogous to the resistor, which directly interacts physical voltage and physical current; the capacitor, which directly interacts physical voltage and physical charge; and the inductor, which directly interacts physical current and physical flux.

From a rigorous physics-theoretical perspective, the HP memristor [2] is too incomplete (no magnetic flux), too complex (a “sandwich” structure), and too special (even a chemical reaction in the memory-holding oxygen vacancies) to be fundamental.

The HP memristor lacks a magnetic flux term in the original memristor definition. Williams from HP thought the actual definition of memristance would be more general. They argued, “Linking electric charge and magnetic flux is one way to satisfy the definition, but it’s not the only one. In fact, it turns out you can bypass magnetic interaction altogether.” [3].

Can we truly bypass magnetic interaction to define a memristor?

Unfortunately, thus far, it was somewhat popular to define “$\phi$” and “$q$” in a purely mathematical way without giving “$\phi$” and “$q$” any physical interpretations: “$\phi$” was defined as the time integral of voltage “$v$” and “$q$” was defined as the time integral of current “$i$”. Most people simply “abused” Chua’s suggested fingerprint [4] “If it (the $v$-$i$ hysteresis loop) is pinched, it is a memristor” by ignoring its prefixing (from an experimental perspective) and its context (“This definition greatly broadens the scope of memristive devices ... into three classes: Ideal Memristors, Generic Memristors, and Extended Memristors.”) [4].

At least three examples can be given to demonstrate that it is highly risky to define a memristor in the $v$-$i$ plane.

![Fig.1 The conformal differential transformation is similar to the 3D object projection in terms of projecting something from a “high-dimensionality” space into a “low-dimensionality” space.](image-url)
The first example is the indefinite integration itself. Starting from \(v\), we have \(\int v(t)dt = \varphi(t) + C\), where \(C\) is an arbitrary constant [meaning that any value for \(C\) makes \(\varphi(t) + C\) a valid antiderivative of \(v(t)\)]. This constant expresses an ambiguity inherent in the construction of integration; there is an indefinite number of antiderivatives of \(v(t)\), and therefore, a given \(v-i\) curve (as postulated by Chua [1], it’s a pinched hysteresis loop (looking like a diagonal “∞”)) cannot uniquely determine the \(ϕ-q\) curve, as shown in the inset of Fig.1.

In other words, the \(ϕ-q\) plane is a space with “high-dimensionality” (if the flux-charge relationship of a memristor is a high-order polynomial as imagined by Chua [5]), whereas the \(v-i\) plane is its “dimensionality-reduced” space. The conformal differential transformation [4][5] projecting the \(ϕ-q\) curve into the \(v-i\) plane just plays the role of dimensionality reduction (e.g. \(\frac{dx^n}{dx} = nx^{n-1}\)). Imagine that we are shining a light from above a 3D object and looking at the shadow it casts on a 2D screen. The transformation from 3D to 2D is unique, but the opposite is not; we cannot work out whether the original 3D object is a ball, cylinder or cone when our viewed 2D projection is a circle. A schematic (Fig.1) best illustrates this. We cannot simply restore the lost information in the projection in a single “anti-projection”.

The second example is that, as shown in Fig.2, no hysteresis can be seen in the \(v-i\) curve if the \(ϕ-q\) curve is odd-symmetric, although it complies with all the three criteria [1] for the ideal memristor’s \(ϕ-q\) curve: 1. nonlinear, 2. continuously differentiable, and 3. monotonically increasing. The \(ϕ-q\) curve projects to the corresponding \(v-i\) curve via the so-called conformal differential transformation (for a periodic input current): obtain the angle of incline \(α\) of the tangent line at an operating point in the \(ϕ = \hat{ϕ}(q)\) curve and draw a straight line through the origin in the \(v-i\) plane whose angle of incline is also \(α\).

The third example is that, as shown in Fig.3, a pinched \(v-i\) curve that is asymmetric even results from a double-valued \(ϕ-q\) curve, which indicates that the corresponding memristor is not ideal at all. It is concluded that an ideal memristor that is originally defined in the constitutive \(ϕ-q\) plane should not be characterized in the \(v-i\) plane.

What we have observed from the above three examples is that “even if it’s pinched it may not be an ideal memristor”. In our opinion, it is bad to define the ideal memristor in the \(v-i\) plane, it is worse to have no physical magnetic flux, and it is even worse to pretend to have a magnetic flux that was virtual and calculated from other physical attributes. Strictly speaking, it is incorrect to bypass or replace the direct, physical flux-charge interaction with a “virtual” interaction between the charge (that is normally physical, e.g., oxygen vacancies as ionic current in the HP memristor [2]) and the integral over the voltage (no choice due to the lack of magnetic flux).

Other skeptics also expressed a similar concern regarding the lack of a flux-charge interaction in the HP memristor. As mentioned in the same issue of IEEE Spectrum, as early as in 2009, skeptics argued that the HP memristor is not a fourth fundamental circuit element but an example of bad science [7]. In 2015, Vongehr even declared, “The Missing Memristor Has Not Been Found” (the title of their Nature Scientific Reports paper) in the sense that an ideal memristor device should be grounded in fundamental symmetries of basic physics, here electromagnetism and that the “ideal/real/perfect/… memristor” needs magnetism [8]. It is worth mentioning that the Vongehr work [8] is only one facet of the opposition to the ideality of a new passive fundamental electrical component. There are other reputed and more recent studies [9][10][11].
II. Designs
In order to design an artificial device with a direct interaction between physical magnetic flux and electric charge and then prove that the interaction is memristive, first principles originated by Aristotelians more than 2300 years ago [12] may help. That is to say, we should start directly at the level of the established memristor concept based on magnetic flux and electric charge without making any assumption such as an empirical model (e.g., a pinched hysteresis voltage-current loop) and parameter fitting (e.g., voltage as a derivative of flux, current as a derivative of the charge).

As our first attempt of putting together magnetic flux and electric charge, the so-called “Corbino disc” [13] was investigated around 2009, which allows the observation of Hall-effect-based magnetoresistance without the associated Hall voltage. It is a two-terminal device with the direct flux-charge interaction, as shown in Fig.4. Nevertheless, the absence of the free transverse boundaries and thereby the charge accumulation in this structure renders the memory-holding mechanism (essential for a memristor) unclear. Another practical problem with this device is that the magnitude of the magnetic field cannot be adjusted to investigate the flux-charge relationship. After a few years of intermittent research, no progress was made with this structure.

![Fig.4 The Corbino disc [13] as a candidate for the ideal memristor based on the direct flux-charge interaction. Dashed curves represent logarithmic spiral paths of deflected electrons subjected to a magnetic field perpendicular to the plane of the disc.](image)

In 2012, cathode-ray tubes [14] were tested, in which an external magnetic field is used to generate the Lorentz force and electric charges of the opposite sign appear on the opposing lateral surfaces in an electric-current carrying sample. A mathematical model describes the cycloid trajectory of the charge carriers and proves that such a spin/charge accumulation process in the Hall devices is memristive [15]. An experimental setup (Fig.5) was built, which is similar to Thompson’s cathode ray tube (he used it to discover the electron in 1897 [16]). However, no anticipated memristive effect was observed. Some theories demonstrate that such a spin/charge accumulation would be finished instantaneously \((10^{-12} \sim 10^{-14} \text{ s})\). Such a time scale is too tiny to be observed. This structure was abandoned at the end of 2016.

![Fig.5 The Hall memristor schematic and experimental setup [12]. The accumulated charge \(Q(t)\) at the top/bottom sides generates an electrostatic field \(E(t)\), which forces the electrons to follow a cycloid curve rather than a circular curve, defining a varying volume of the charge contributing to the current and resulting in a memristance dependent on the history of the current.](image)

After numerous failures, the magnetic core memory [17] was eventually chosen as another candidate to introduce the direct flux-charge interaction. Through the wire that carries a current and the magnetic core that hosts a magnetization rotation. The flux can be adjusted by the magnetization rotation. However, although the flux-charge interaction was present, one serious problem needed to be solved: a standard magnetic core memory cell has several wires (typically two X/Y lines for coincident-currents plus one sense/inhibit line), but an ideal memristor only needs two-terminals, i.e., one port. In order to pick up a signal from the flux-charge coupling with only one port, a magnetic core with one wire going through it was designed [Fig.6(a)]. This wire can simultaneously play two roles; on the one hand, it carries a current to switch the magnetization, on the other hand, it can also sense the possibly induced voltage by the switched flux.

III. Model
The resulting equivalent resistance, the ratio between the sensed voltage and the driving current, of the above structure is possibly memristive. It would be highly unlikely to have a linear flux-charge interaction due to the existence of magnetic material with rich hysteresis. That is, it is the magnetic core material that may provide a source of nonlinearity necessary for such an ideal memristor.

In order to describe the flux-charge interaction in a
mathematically and physically rigorous way, the Landau–Lifshitz–Gilbert equation [18][19] was used. Assuming $m(t) = \frac{M_z(t)}{M_s}$, in which $M_z$ is the component of the saturation magnetization $M_s$ in the $Z$ axis, and a magnetic field $H$ is applied along $Z$. The model is expressed as below:

$$m(t) = \tanh \left[ \frac{q(t)}{S_W} + C \right].$$  (1)

in which $S_W$ is a switching coefficient and $C$ is a constant of integration such that $C = \tanh^{-1}m_0$ ($m_0$ is the initial value of $m$) if $q(t=0)=0$ (no accumulation of charge at any point). See Appendix I for details.

By Faraday's law, the induced voltage $v(t)$ is:

$$\mu_0 S \frac{dM_z}{dt} = S \frac{dB_z}{dt} = \frac{d\phi_z}{dt} = -v(t),$$  (2)

where $\mu_0$ is the permeability of free space and $S$ is the cross-sectional area.

From Eq.2, we obtain

$$\dot{\varphi} = \mu_0 S M + C' = \mu_0 S M_5 m + C',$$  (3)

where $C'$ is another constant of integration.

Assuming $\varphi(t=0) = 0$, we have $C' = -\mu_0 S M_5 m_0$, so

$$\varphi = \mu_0 S M_5 \left[ \tanh \left( \frac{q}{S_W} + \tanh^{-1}m_0 \right) - m_0 \right] \pm \dot{\varphi}(q).$$  (4)

Eq.4 complies with the new three criteria [6] for the ideal memristor: 1. Nonlinear; 2. Continuously differentiable; 3. Strictly monotonically increasing. Fig.7 shows a typical $\varphi$-$q$ curve with $m_0=-0.964$ (such a value reflects the intrinsic fluctuation otherwise $M$ will stick to the stable equilibriums $m_0 = \pm 1$).

Fig.7 agrees with those experimentally observed $\varphi$-$q$ curves in the magnetic cores [21][22][23]. Therefore Eq.4 is used as a constitutive curve in this case. As shown in Fig.7, it is dramatically different from that of Chua’s “fictitious” memristor with a flux-charge relationship $\varphi = \frac{1}{3}q^3 + q$ in his tutorial [5]. $\lim_{q \to \pm \infty} \tanh(q) = \pm 1$ for a hyperbolic function whose output range is normalized from -1 to 1 (no matter how big the input is) whereas $\lim_{q \to \pm \infty} \left( \frac{1}{3}q^3 + q \right) = \pm \infty$ for a polynomial function (that diverges to infinity). In other
words, such a device’s operation range is finite (where \( M(q) = \frac{dp}{dq} \neq 0 \)) whereas the “fictitious” memristor’s range is infinite. The hyperbolic tangent function is more natural than other functions. In artificial neural networks, the \( tanh \) activation function is biologically reflected in the neuron in the sense that it stays at zero until input current is received, increases the firing frequency quickly at first, but gradually approaches an asymptote at 100% firing rate [24]. There is a clear physical explanation for the saturation of this device: it is because the magnetization vector is as aligned as the magnetic field allows it to be. The change in the magnetization alignment is negligible on increasing the field above this.

![Fig.7](image)

**Fig.7 Intuitively, the S-shaped \( \phi-q \) curve (sigmoid curve) of this device vividly depicts the self-limiting electricity-magnetism interaction in a circuit element.** It complies with the following four criteria for ideality: a. single-valued; b. nonlinear [6]; c. continuously differentiable [6]; d. strictly monotonically increasing [6].

An ideal memristor should be characterized by a time-invariant \( \phi-q \) curve complying with the following four criteria:

1. Single-valued;
2. Nonlinear;
3. Continuously differentiable;
4. Strictly monotonically increasing.

Criteria 2-4 for ideality were extracted from references [1][6]. The only exception is that we added the “single-valued” criterion to avoid the nonideal case as shown in Fig.3.

If a step-function excitation current (with a rise time short enough in approaching constant \( I \)) is applied, the induced voltage is:

\[
v(t) = \frac{dp}{dt} = \frac{\mu_0SM_T}{s_W} sech^2\left(\frac{q}{s_W} + tanh^{-1}m_0 \right).
\]  

Eq.5 is depicted in Fig.6 (b).

From Eq.4, the memristance \( M(q) \) of this device is:

\[
M(q) = \frac{dp}{dq} = \frac{\mu_0SM_T}{s_W} sech^2\left(\frac{q}{s_W} + tanh^{-1}m_0 \right) \geq 0. \tag{6}
\]

So far a state equation is established for this (charge-controlled) device: Eq.6 is viewed as the state-dependent resistance of the associated Ohm’s Law and \( dq/dt=I \) is its state equation.

The authors of Ref. [25] wrote that pinched hysteresis loops alone (“If it’s pinched, it’s a memristor” [4]) cannot serve as a good indicator of memristors since this feature is shared by many memristive devices, and there are systems with memory whose resistance depends on some internal degrees of freedom, other than charge [25]. They suggested that the only criterion for an ideal memristor is that its resistance depends only on the charge \( q \) that has flowed through the device from an initial moment of time [25]. As shown in Eq.6, our device meets this definition. Therefore, the flux-charge interaction in our prototype is indeed intrinsically memristive according to Eq.6.

The memristance \( M(q) \) is continuous as shown in Eq.6 and, when \( \frac{q}{s_W} + tanh^{-1}m_0 = \frac{1}{s_W}t + tanh^{-1}m_0 = 0 \), reaches its peak value:

\[
M_{max} = \frac{\mu_0SM_T}{s_W}. \tag{7}
\]

This maximum point occurs at the switching time \( t_s \), which is the correct time point to measure the (maximum) memristance (for potential memory/switch applications):

\[
t_s = -\frac{s_W}{I} tanh^{-1}m_0. \tag{8}
\]

As shown in Fig.6(b), the switching time \( t_s \) in Eq.8 is inversely proportional to the amplitude of the input current \( I \).

Eqs. 4-8 illustrate vividly the direct flux-charge interaction in Fig.6: a flux in the lump needs to be switched by a minimum amount of the charge. It is the charge \( q \) that determines the switching. A sole current or magnetic field (no matter how large it is) cannot switch the lump without a minimum time interval. Beyond the switching time \( t_s \), the magnetization precesses with decreasing amplitude until the magnetization has reversed.

The above formulas are directly derived from the LLG equation [18][19] (see Appendix I for details). We did not use the so-called rotational model [21][22][23] implemented in Ref. [26]. That publication was unfortunately retracted in 2021 because, according to the JAP Editors, “This model determines only the equilibrium states of \( m \) and not the time evolution of the switching at the critical fields. The switching process can be indeed described by the LLG equation.” [27] Notably, the JAP Editors did not challenge our experiments, which showed that our device exhibited a number of fingerprints for a memristor.

Putting aside the above argument, we use Eqs.4 to reproduce the \( m-H \) loops for a sine-wave input current and a step-function input current in Fig.8.
Fig. 8 The $m$-$H$ hysteresis loops (from a to c) simulated by Eq. 4 derived from the LLG equation [18][19] with both a sine-wave input current and a step-function input current. As a comparison, the bottom one (d) is a typical $m$-$H$ loop of real-world magnetic materials [28].

The above simulations reproduced those typical $m$-$H$ loops of real-world magnetic materials for sine-wave and step-function inputs with reasonable accuracy [28][29][30].

IV. Experiments
In principle, the magnetic lump in the above designed device [Fig. 6(a)] can take any shape, e.g., a ball, a slab, a ring or a barrel. Nevertheless, the shape matters in terms of the efficiency of the coupling between magnetism and electricity. As can be imagined, any “closed” shape (e.g., a ring) encircling the conductor can interact magnetism and electricity more efficiently than those “open” shapes (e.g., a ball). As its first embodiment, a “ring” memristor was made by extracting core components (Mn-Cu ferrite) from a very rare 64x64 former USSR military magnetic memory built in the 1960s. The wire going through the cores is an enamelled copper wire. Intuitively, this device is a reduced magnetic memory core with only one wire.

Fig. 6(a) shows the experimental apparatus to verify the ideal memristor concept. A current source (Tektronix MHS-5200P High Precision Digital Dual-channel Signal Generator with attached power amplifiers) delivers $i(t)$ which is independent of the voltage across this device. The two channels are connected in series to double the output voltage of a single channel. The current source is in series with a cement resistor of 47Ω 30W and the equivalent resistance $v/i$ of this device is less than 1Ω. In order to measure the possibly induced voltage $v(t)$ between the two terminals, the two fixed probes are used.

This device is working. When applying a current through a bare wire without any magnetic core, no voltage is observed. As shown in Fig. 6(c), a voltage is observed that was induced through the reversal of the magnetization in the core and the flux-charge interaction is intrinsically memristive as our theory anticipates. It is the magnetization in the core that induces a voltage through its reversal. Fig. 6(c) shows the induced voltage waveform measured from the same wire carrying an input current. The voltage response exhibits a steep but continuous “state-dependent” resistance, just as predicted in Eq. 6 (the power of sech). A reasonable agreement between the model [Fig. 6(b)] and the measured waveform [Fig. 6(c)] can be seen here.

The device studied in this work is judged to exhibit a direct electricity-magnetism interaction, in the sense that it is a device in which the change of resistance is dependent only on the charge, i.e., the integral of the current. This is directly evidenced by Fig. 6(c). If the resistance would depend only on the integral of the current, doubling the (effective) pulse width $t_p$ should have the same effect as doubling the current strength $i$. This is reasonably satisfied. In Fig. 6(c), it is seen that increasing current pulse width has the same effect on the
switching than increasing the current strength.

As mentioned in the Model section, the authors of Ref. [20] suggested that the only criterion for an ideal memristor is that the resistance depends only on the charge \( q \) that has flowed through the device [20]. As shown in the experiments in Fig. 6(c), our device meets this definition.

The authors of Ref. [20] also introduce a test to check experimentally if a resistor with memory is indeed an ideal memristor [20]. Their test is based on a capacitor-memristor circuit as a charge-tracking device [20][25]. Not surprisingly, our device cannot pass such a test, in which a sawtooth, a sinusoidal or another “nonflat” periodic input current needs to flow through a tested candidate device [31]. In our device with an inductor-like core structure, a parasitic “inductor” effect exists inevitably, which was observed as sharp transient spikes caused by the sudden change of the input current \( V_L(t) = L \frac{d{i(t)}}{dt} \neq 0 \). Fortunately, this effect is narrowly constrained at the rise/fall edges of the step function. In our own test, this high-frequency “noise” is removed by simply using the “compensation adjustment” function (a low-pass filter, LPF) of an oscilloscope (GW Instek GDS-1072B). In the most time of the cycle (especially in the memristive region of our research interest), no “inductor” exists as \( V_L(t) = L \frac{d{i(t)}}{dt} = 0 \).

We stress that this prototype device is neither “an inductor with memory” as claimed in Ref. [24][26] nor a mem-inductor [29][31] although there exists a parasitic “inductor” effect even if the parasitic effect becomes dominant at the macroscopic scale. We researchers are expected to reveal or catch those deeply hidden effects or phenomena behind the superficial ones. This is analogous to the fact that a real-world resistor is still thought to be a resistor in spite of the existence of the (inevitable) parasitic inductance and/or capacitance. Most importantly, our demonstration shows that the electricity-magnetism interaction could be memristive, which addresses a missing gap since Chua defined memristor 50 years ago [7][8].

In our previous adaptive neuromorphic architecture (ANA), a mem-inductor (whose inductance is a function of the charge) is used to adapt the time constant \( \sqrt{L(q(t))C} \) of the circuit to the changing environment [32]. This device differs from the mentioned mem-inductor and other circuit elements with memory [33] since its resistance is simply a function of the charge (Eq.6). after taking the following two measures, no capacitive or inductive effects was observed otherwise there should be a phase shift between the current and voltage: 1. Using a constant input current (such as a step-function or a sequence of square wave pulses); 2. Using a low-pass filter that passes signals with a frequency lower than a certain cut-off frequency and attenuates noises with frequencies higher than the cut-off frequency.

Fig. 9 shows an experimental electricity-magnetism interaction and its corresponding conformal differential transformation. Two memristor fingerprints can be seen: 1. Lissajous figure; 2. Zero-crossing. Note that a step-function excitement current is used here since a (continuous) sinusoidal current cannot be used due to the parasitic “inductor” effect in the magnetic core. A careful examination of the photos reveals that path (①-②) is not ideally flat because the rise of the current is not ideally sharp and path (①-②) is not ideally vertical to the i(t) axis because the current is not ideally flat. Scales in photos: horizontal: 50 mA/div; vertical: 30 mV/div.

As the key mechanism of this device, the memristive electricity-magnetism interaction is experimentally verified based on the observed Lissajous figure, which reflects a nonlinear relation between the memristor’s charge and flux, as we anticipated in the Model section. Moreover, both the current \( i(t) \) and the voltage \( v(t) \) are pinched at the origin as expected, which is another fingerprint of a memristor [1][4]. As a result, the memristive effect is experimentally observable and even a simple combination of a magnetic lump and a conductor placed in a close proximity to each other (Fig.6) could be memristive. This new device is memristive in the sense that it involves only the flux-charges interaction in a classic way (no quantum effect, no spin effect), and is therefore not constrained by its physical size.
Fig. 10 Continuous operation of the device. The measured voltage response (in blue) against a sequence of square-wave input current pulses (in yellow) in the up/up/down/down pattern. The purpose of using such a constant signal is to avoid any parasitic “inductor” effect. This voltage response exhibits a switching/no-switching/no-switching/switching pattern as anticipated. Horizontal scale is 1 μs/div; Vertical scales are 50 mA/div (yellow) and 50 mV/div (blue), respectively.

Fig. 11 Geometry of a magnetic core for the inductance calculation.

If a fixed aspect ratio \( D = 2d = 2h \) is taken, Eq. 9 can be simplified to:

\[
L = 0.4\pi\mu N^2 \frac{h}{3} \times 10^{-5} \text{ (mH)},
\]

which implies that the inductance of a magnetic core scales with its physical size. This is truly encouraging in the sense that, in principle, the ideal memristor on the nanoscale is expected to have a negligible parasitic inductance.

Notably, as shown in Eq. 10, the parasitic inductance in this device is not a function of the charge (otherwise it will become a mem-inductor [33]), whereas its resistance (memristance) is a function of the charge as described in Eq. 6.

An antiferromagnet (AFM)/ferromagnet (FM) heterostructure [35] was found to exhibit a stepwise memristance (due to the sequential switching of the domains in the ferromagnetic layer), in which one can freeze the resistance statically at any intermediate time point. It exhibits a continuous “state-dependent Ohm’s law” and provides a solution of Chua’s Enigma: all non-volatile memristors have continuum memories [36].

Such a structure (with carefully chosen parameters) facilitates the direct electricity-magnetism interaction. On the one hand, the anti-parallel neighboring spins of the AFM layer results in a zero total magnetization [35]. On the other hand, the AFM layer is nearly 10 times thicker than the FM layer in this structure [35] (the resistance of the bilayers is assumed to be the summation of the parallel resistance of each layer and determined by the smaller one). For the above two reasons, the AFM layer can be approximately viewed as a carrier of the charge in contrast to the FM layer (with a net magnetic moment due to spontaneous magnetization) as a carrier of the flux. There is a similarity between this bilayer (AFM/FM) structure on the nanoscale and the two-component (wire/core) structure of the device on the macroscale (Fig. 6). However, this bilayer itself is nonideal since its resistance is a function of some internal state variables including the temperature [35], other than a function of the charge only [20].

As illustrated in Fig. 12, both structures (two-components and bilayers) can be simplified as two electrons: one electron representing a charge carrier and the other representing a
magnetization carrier. The fundamental scientific question of this work is whether the interaction between (the charge of) the 1st electron and (the flux of) the 2nd one is memristive.

Fig.12 Both structures (two-components in Fig.6 and bilayers [29]) can be simplified as two electrons: one electron representing a charge carrier and the other representing a magnetization carrier. The fundamental scientific question of this work is whether the interaction between (the charge of) the 1st electron and (the flux of) the 2nd one is memristive. Magnetic moment (flux) and electric charge of an electron must be seen as two inseparable, intrinsically basic physical properties. Our work proves that \( \frac{d\phi}{dq} = \frac{d\phi}{dt} = \frac{v}{i} (\Omega) \), where \( \phi \) denotes flux, \( v \) voltage and \( i \) current.

The device on the macroscale is a binary memristor with only two stable states since the magnetic lump is magnetically uniaxial anisotropic [36], whereas the bilayer structure has two terminals (as an ideal memristor must have) and multistable states due to the sequential switching of domains on the nanoscale. The physical size of a memristor matters in the sense that the nanoscale is essential for IC chips, whereas the macroscale is essential for off-the-shelf circuit applications.

V. Conclusions & Discussion
To date, there are three ways to define an ideal memristor. Let us check the satisfaction of our device one by one as follows:

1. Its \( \phi-q \) curve is single-valued, nonlinear [6], continuously differentiable [6], strictly monotonically increasing [6]. As shown in Fig.7, our device meets this definition.
2. If it’s pinched, it’s a memristor [4]. As shown in Fig.9, our device meets this definition as well. Nevertheless, the authors of Ref. [20] claimed that a pinched I-V hysteresis loop does not definitively determine whether a resistor with memory is ideal because other resistors also exhibit this behaviour although their memory depends on additional internal state variables other than charge [20].
3. Its resistance depends on only the charge \( q \) that has flowed through the device from an initial moment of time [20]. As shown in Fig.6(c), our device meets this definition as well.

Our work represents a step forward in terms of experimentally verifying the memristive flux-charge interaction but we have not reached the final. Our prototype device has two serious limitations:

1. The aforementioned three fingerprints hide behind a superficial but dominant inductor effect due to its inductor-like core structure. It was necessary to apply a constant input current (such as a step-function or a sequence of square-wave pulses) to depress the inductor effect. For this reason, our device failed to pass the capacitor-memristor circuit test (as a charge-tracking device), in which a sawtooth, a sinusoidal or another “nonflat” periodic input current is applied [31]. Nevertheless, it is unfair to conclude that our device is “simply an inductor with memory” [31]. As illustrated above, when we applied a constant input current, our device exhibited all the fingerprints of an ideal memristor. Despite the existence of a parasitic inductance, our device displayed memristivity; similarly, a real-world resistor is still thought to be a resistor despite the existence of an (inevitable) parasitic inductance and/or capacitance. Most importantly, our device demonstrates that its electricity-magnetism interaction could be memristive.
2. Our device is bistable and dynamically sweeps a continuous range of resistances. This “dynamical continuity” results from the uniaxial magnetic anisotropy of the prototype, which contains magnetic core material with only one easy axis. A fully-functioning ideal memristor should have multiple or an infinite number of stable states so its static memristance can be “frozen” at any intermediate point in time.

Limitation 1 (the dominant inductor effect) may be overcome by introducing other “open” structures, rather than the “closed” or inductor-like structure we used in our current device. In our original JAP paper [26], we said “a current-carrying wire strung through a magnetic core is memristive with a parasitic inductor effect” whereas the authors of the Comment [29] said “it is simply an inductor with memory”. Both us and the Comment authors recognized the coexistence of the memristivity (or memory) and the inductivity in the core structure. Therefore, the ultimate argument is: which effect is dominant, memristivity or inductivity? Our calculation shows that this parasitic inductor effect decreases with decreasing size, which implies that such an unwanted effect may not be a major concern in a nanoscale device [30].

Limitation 2 (“dynamical continuity”) may be overcome by using magnetic materials with cubic anisotropy (three or four easy axes) or even a magnetically isotropic material (no preferential direction for its magnetisation) [37]. In addition, an antiferromagnet/ferromagnet heterostructure exhibited stepwise memristance due to sequential switching of the domains in its ferromagnetic layer [35]. However, this bilayer is nonideal since its resistance is a function of several internal state variables, including temperature, and it is not a function of only charge [35].

A fully-functioning flux-charge-interaction-based ideal memristor with multiple or an infinite number of stable states and no parasitic inductance is still highly in demand in terms of filling the gap of 50 years [7][8]. The existence of such a fundamental circuit element may appeal to many researchers.
in the memristor field within the context of the theoretical circuit innovations that depend on charge-flux linkage [38][39][40][41]. We are still optimistic that researchers will discover an ideal memristor in nature or make one in the laboratory, although some researchers felt that an ideal memristor may not exist or may be a purely mathematical concept [25].

It is worth mentioning that Chua still identified those resistance-switching memories as memristors [5], e.g., the spintronic memristors [42] exhibiting bi-stable states and a dynamically continuous range of resistances. For memory/logic applications that require only two states, the memristor needs to exhibit only two equilibrium states while switching swiftly between these two states [5]. However, an experimental proof was supplied to refute the claim that all resistance-switching memories are memristors [20].

Basically, our device is defined via Oersted’s law [43] discovered by Danish physicist Hans Christian Oersted in 1820. It states that an electric current creates a magnetic field that deflects the needle of a compass next to the wire from the magnetic north, which was the first connection found between electricity and magnetism. From a historical perspective, Oersted’s compass/wire apparatus could be viewed as a memristor (if he had measured the voltage induced by the deflection of the needle simultaneously when he was flowing the current, as shown in Fig.13), which predates the resistor built by Georg Ohm in 1827 [44] and the inductor by Michael Faraday in 1831 [45] but postdates the capacitor by Ewald Georg von Kleist in 1745 [45].

Fig.13 Oersted’s compass/wire apparatus would have been the 1st memristor in the world if he had measured the voltage induced by the deflection of the compass needle simultaneously when he was flowing the current.

Our memristor-based neuromorphic computing paradigm is moving towards the brain direction, with a view that brains are made of memristors [46][47]. According to recent studies, there are magnetic particles or magnetite scattered around the human brain, which enables humans to sense the Earth's magnetic field [48]. Another recent research indicates that applying a magnetic field will cause current to flow through the neurons and this can alter their activity because an action potential as an electrical impulse traveling through a neuron means neurons are electrically charged and can conduct electricity [49]. The operation of this device involving magnetic flux is therefore scientifically natural and sensible in the brain research.

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Appendix I The LLG model of flux reversal for a magnetic lump/conductor combination

In this appendix, we will develop a theory to describe the physical flux-charge interaction as shown in Fig. 1(a).

**Fig. 1** (a) The interaction between a magnetic lump and a current-carrying conductor in close proximity to each other. The Oersted field generated by the current $i$ rotates or switches the magnetization $M$ inside the lump and consequently the switched flux $\varphi$ induces a voltage $v$ across the conductor, resulting in a changed (equivalent) memristance. (b) The LLG model of flux reversal for the above magnetic lump/conductor combination. If the magnetic field $H$ is applied in the $Z$ direction, the saturation magnetization vector $M_S(t)$ follows a precession trajectory (blue) from its initial position ($\Theta_0=\pi$, $m_0=1$) and the angle $\Theta$ decreases with time continuously until ($\Theta=0$, $m=1$), i.e., the magnetization $M_S(t)$ reverses itself and is eventually aligned with the magnetic field $H$.

Suppose that the lump is a single-domain cylinder with uniaxial anisotropy in the approximate sense: the magnetization is uniform and rotates in unison [1]. In an ideal case, it is assumed that there is a negligible amount of eddy current effect.

The corresponding LLG model of flux reversal for the above magnetic lump/conductor combination is shown in Fig. 1(b), where the easy axis is along the $Z$ axis, $M_Z$ is the component of the saturation magnetization $M_S$ in the $Z$ axis, and the magnetic field $H$ is applied in the $Z$ direction.

Let’s write the Landau–Lifshitz–Gilbert equation [2][3] as follows:

$$(1 + g^2) \frac{dM_S(t)}{dt} = -\gamma |M_S(t) \times H| \frac{M_S(t)}{M_S} - \gamma |M_S(t) \times (M_S(t) \times H)|,$$

where $g$ is the Gilbert damping parameter and $\gamma$ is the gyromagnetic ratio.

The 1st term of the right-hand side can be re-written as:

$$-\gamma |M_S(t) \times H| = -\gamma |(M_S sin \theta \sin \psi H \tau - M_S \sin \theta \cos \psi H \tau)|$$

Note this term does not have any $\kappa$ component (along the Z axis) and does not contribute to $M_Z$.

The 2nd term of the right-hand side can be re-written as:

$$-\frac{g \gamma |M_S(t) \times (M_S(t) \times H)|}{M_S}$$
\[ \begin{align*}
&= \frac{g|\gamma|}{M_S} \left( M_S\sin\theta\cos\phi + M_S\sin\theta\sin\phi + M_S\cos\theta \right) \times [M_S\sin\theta\sin\phi H\tilde{k} - M_S\sin\theta\cos\phi H]\tilde{k} \\
&= -\frac{g|\gamma|}{M_S} \left[ -M_S\sin\theta\cos\phi M_S\sin\theta\cos\phi H - M_S\sin\theta\sin\phi M_S\sin\theta\sin\phi H \right] \tilde{k} \\
&= g|\gamma|M_S\tilde{H}\left[ \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi \right] \tilde{k} = g|\gamma|M_S\tilde{H}\sin^2\theta \tilde{k} \\
&= g|\gamma|M_S\tilde{H}(1 - \cos^2\theta) \tilde{k} = g|\gamma|M_S\tilde{H}[1 - \left( \frac{M_S}{M_S} \right)^2] \tilde{k}.
\end{align*} \] (3)

From the above, we should obtain the following equation:

\[ (1 + g^2) \frac{dM_S(t)}{dt} = g|\gamma|M_S\tilde{H}[1 - \left( \frac{M_S}{M_S} \right)^2]. \] (4)

Assuming \( m(t) = \frac{M_S(t)}{M_S} \), we obtain

\[ \frac{dm(t)}{dt} = \frac{g|\gamma|H}{(1+g^2)} [1 - m^2(t)] = \frac{1}{S_W} i(t)[1 - m^2(t)]. \] (5)

where \( S_W \) is the switching coefficient \([1]\). The threshold for the magnetization switching is automatically taken into account because the switching coefficient is defined based on the threshold field \( H_0 \), which is of one to two times the coercive force \( H_C \)[1].

The hyperbolic function \( \tanh \) has \( \frac{d}{dx} \tanh x = 1 - \tanh^2 x \) and the derivative of a function of function has \( \frac{du}{dx} = \frac{du}{dy} \frac{dy}{dx} \); therefore it is reasonable to assume:

\[ m(t) = \tanh \left[ \frac{q(t)}{S_W} + C \right], \] (6)

where \( \frac{d}{dt} q(t) = i(t) \) and \( C \) is a constant of integration such that \( C = \tanh^{-1} m_0 \) if \( q(t=0) = 0 \) (assuming the charge does not accumulate at any point) and \( m_0 \) is the initial value of \( m \).

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