ON THE ORIGIN OF PLANETARY NEBULA K648 IN GLOBULAR CLUSTER M15

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Received 1996 September 25; accepted 1997 February 6

ABSTRACT

We examine two scenarios for formation of the planetary nebula K648: a prompt scenario in which the planetary nebula is ejected and formed immediately after a helium shell flash and a delayed scenario in which a third dredge-up occurs and the envelope is ejected during the following interpulse phase. We present models of both scenarios and find that each can produce K648-like systems. We suggest that the prompt scenario is more favorable but cannot rule out the delayed scenario.

Subject headings: globular clusters: individual (M15) — planetary nebulae: individual (K648) — stars: AGB and post-AGB — stars: evolution — stars: interiors

1. INTRODUCTION

The globular cluster M15 contains the well-studied planetary nebula (PN) K648. This is one of the few Galactic PNs with a reasonably well-determined distance. Therefore, fundamental properties such as the stellar luminosity can be determined with some confidence. Because of its globular cluster membership, many of the progenitor properties, such as the zero-age main-sequence (ZAMS) mass, can be inferred reliably.

Because of the importance of K648 as a halo PN, it has been the focus of several abundance studies, and all of these show the abundances of most metals to be depleted relative to the Sun, consistent with a progenitor of low metallicity. Carbon is an exception; studies that determine the ratio (by number) of C/O in K648 infer values that range 4–11 (Adams et al. 1984; Henry, Kwitter, & Howard 1996; Howard, Henry, & McCartney 1997), which is far above a C/O of 0.43 in the Sun (Anders & Grevesse 1989, hereafter AG89). This is in fact much higher than the average C/O ratio of ≈0.8 for solar neighborhood PNs (Rola & Stasińska 1994).

Low- and intermediate-mass stars that have left the main sequence, ascended the giant branch, and passed through the horizontal branch then enter a thermally unstable phase in which energy is generated by shell He- and H-burning. This phase is called the thermally pulsing asymptotic giant branch (TP-AGB) stage, and it is a very important, yet not well-understood, phase (detailed reviews of this stage can be found in Iben 1995; Lattanzio 1993; and Iben & Renzini 1983). During the TP-AGB stage, the star alternates between a long stage in which the luminosity is generated mostly by quiescent hydrogen shell burning, with a helium-burning layer producing a minority of the energy, and a thermal runaway stage in the unstable helium-burning layer (Schwarzschild & Härm 1965, 1967; Weigert 1966). The second stage results in an expansion of the outer layers and an extinguishing of the H-burning shell. This short stage, characterized by rapid changes, with helium burning dominating the energy generation, is known as a thermal pulse or an He shell flash.

TP-AGB stars exhibit large mass-loss rates ranging from $10^{-7}$ to $10^{-4} \, M_\odot \, yr^{-1}$. Indeed, such high mass-loss rates are predicted to result in the ejection of the envelope, at which point the star leaves the AGB and becomes a planetary nebula central star (CSPN). The first models of CSPN tracks were made by Paczynski (1971), who showed that the CSPNs evolve horizontally on the H-R diagram when nuclear burning is still taking place, and then as they cool, the luminosity and temperature decrease. Härm & Schwarzschild (1975) showed that a CSPN could leave the AGB as either a helium-burning or a hydrogen-burning star. The observational consequences of hydrogen and helium burning studied in the more refined models including mass loss showed that the subsequent evolution of the central star depends on whether the star leaves the AGB as a helium or a hydrogen burner (Schönberner 1981, 1983; Iben 1984).

Low-mass stars ($M \lesssim 3 \, M_\odot$) can experience two mixing episodes, or dredge-ups. During dredge-up, material that has been processed by nuclear burning is mixed into the surface layers. At the entrance to the giant branch, the convective region can extend into the core, leading to mixing of CNO products into the outer layers. Similarly, as shown by Iben (1975), after a thermal pulse on the AGB, the convective region can extend into the core, mixing He-burning products into the outer layers. These two mixing events are known as the first and third dredge-ups, respectively (the second dredge-up will not concern us here). Therefore, a third dredge-up is a natural explanation of the high carbon abundance found in K648. On the other hand, no carbon stars have been observed in M15 or in any other globular cluster, although such stars should be the immediate progenitors of objects such as K648 if a third dredge-up occurs. Thus, the lack of carbon stars in M15 weakens the argument for a third dredge-up event.

One possible explanation for the absence of carbon stars is a delayed scenario in which the third dredge-up of carbon-rich material changes the structure of the envelope during the following interpulse phase, ultimately increasing the mass-loss rate significantly and driving off the stellar envelope (Iben 1995). Thus, envelope ejection is delayed until the interpulse phase following this dredge-up of carbon-rich material.

Another explanation supposes that the envelope is removed during the quiescent He-burning stage that follows a thermal pulse (Renzini 1989; Renzini & Fusi Pecci 1988).
The carbon then originates in a fast wind from the central star (CSPN). In addition, the wind produces shock heating in the nebula, which, if not properly accounted for during an abundance analysis, may lead to the inference of a spuriously high C/O ratio. In this case, the envelope would be ejected immediately after a thermal pulse, while helium shell burning still dominates the luminosity. We refer to this mechanism as the prompt scenario.

In this paper we calculate detailed envelope models of thermally pulsing asymptotic giant branch star envelopes to test the predictions of the delayed mechanism, perform other calculations relevant to the prompt mechanism, and compare the output of each with observations of K648. Section 2 describes the envelope code, § 3 presents the observational data and the results for the delayed and prompt models, and a brief discussion of our findings is given in § 4.

2. MODELS

The computer code used to calculate the delayed models is a significantly updated and modified version of a program kindly provided to us by A. Renzini for modeling the envelope of TP-AGB stars during the interpulse phase. Many of the basic details of the method are enumerated in Iben & Truran (1978) and Renzini & Voli (1981) and references therein; in this section we concentrate on those features that are different. In a future paper (Buell, Henry, & Baron 1997) we will provide a more detailed description of the code.

The mass of the hydrogen-exhausted core \( M_H \) at the first thermal pulse is given by the expression found in Lattanzio (1986). During each interpulse phase, the code follows the mass of the hydrogen-exhausted core and envelope and the evolution of envelope abundances of \(^{4}\text{He}\), \(^{12}\text{C}\), \(^{13}\text{C}\), \(^{14}\text{N}\), and \(^{16}\text{O}\) and determines \( T_{\text{eff}} \) by integrating the equations of stellar structure from the surface to the core.

Envelope abundances at the first pulse are determined by combining published main-sequence values with changes due to the first dredge-up. The former are established by scaling the AG89 solar abundances of all metals except the alpha elements, i.e., oxygen, neon, and magnesium, to the appropriate metallicity and then setting \( [N/\text{Fe}] = 0.4 \), where \( N \) is the number abundance of O, Ne, and Mg. This last value is chosen from an examination of the trends in the data of Edvardsson et al. (1993) for \([\text{Fe/H}] < -1.0\) and with the assumption that neon and oxygen vary in lockstep in PNs as shown by Henry (1989). The abundance changes due to the first dredge-up are calculated from the formulae of Groenewegen & de Jong (1993).

The mass loss both before and during the TP-AGB phase is very important, although the parameters are poorly understood. The pre-TP-AGB mass loss is a free parameter, while during the TP-AGB phase, mass loss is determined with the expression of Vassiliadis & Wood (1993, hereafter VW93), which can be written as

\[
\log \dot{M} = -11.43 + 1.0467 \times 10^{-4} \left( \frac{R}{R_\odot} \right)^{1.94} \left( \frac{M}{M_\odot} \right)^{-0.9} M_\odot \text{ yr}^{-1}.
\]

The above rate is used until \( \log \dot{M} = -4.5 \), and then it is held fixed. Equation (1) is a \( \dot{M} \)-period relation based on mass loss from Population I stars. However, recent calculations by Wilson, Bowen, & Struck (1995) suggest that the mass-loss rates of low-metallicity AGB stars are also strongly dependent on radius. There is considerable uncertainty in this equation. For example, predicted mass-loss rates from other equations with a similar form (see, e.g., Bazan 1991) differ from predictions of equation (1) by up to a factor of 5.

The luminosity of TP-AGB stars after the first few pulses can be described by a linear relation between core mass and luminosity as first discovered by Paczynski (1970). Models of TP-AGB stars have shown that for \( M \leq 3.0 M_\odot \), this relation depends on metallicity (Lattanzio 1986; Hollowell & Iben 1988; Boothroyd & Sackmann 1988b). At the first pulse, the luminosity of TP-AGB stars is less than the asymptotic core mass–luminosity relation (CML). The luminosity at the first pulse in our models is found by linearly extrapolating in metallicity from the expressions found in Boothroyd & Sackmann (1988b). After the first pulse, the luminosity of the AGB star rises steeply until it reaches a value predicted by the CML of Boothroyd & Sackmann (1988b). This relation predicts luminosity primarily as a function of core mass, although it has a weak dependence on helium and metal mass fractions.

Carbon-rich material can be dredged from the core into the envelope following a thermal pulse. We assume that when the mass of the hydrogen-exhausted core exceeds a minimum mass \( M_{\text{min}} \), a dredge-up occurs. The amount of material dredged up, \( \Delta M_{\text{dredge}} \), is determined by the free parameter \( \lambda \), where

\[
\lambda = \frac{\Delta M_{\text{dredge}}}{\Delta M_c}.
\]

In equation (2), \( \Delta M_c \) is the amount of core advance during the preceding interpulse phase. We determine the composition of the dredged-up material from the formulae in Renzini & Voli (1981), with \(^{4}\text{He} \approx 0.75\), \(^{12}\text{C} \approx 0.23\), and \(^{16}\text{O} \approx 0.01\) as the approximate mass fractions.

Finally, the code uses the opacities of Rogers & Iglesias (1992) supplemented by the low-temperature opacities of Alexander & Ferguson (1994).

3. RESULTS AND DISCUSSION

3.1. Observational Parameters

Numerous observed and inferred parameters for K648 are listed in Tables 1 and 2, where the symbols in the first column are explained in the table notes. We comment here on the method of determination for several of them.

Radio images of K648 have been made by Gathier, Pottasch, & Goss (1983), and optical images were made by Adams et al. (1984) and recently by Bianchi et al. (1995) using the HST. The HST data called into question the small size for the nebula inferred in the radio studies of Gathier et al. (1983) and the optical studies of Adams et al. (1984), since HST was able to resolve the structure of the nebula. This leads to, e.g., a larger planetary mass and smaller electron density. In Tables 1 and 2, we quote all results.

\( M_{\text{PN}} \) was computed with equation V-7 in Pottasch (1984), while the dynamical age was estimated by dividing the nebular radius by the expansion velocity \( v_{\text{exp}} \). Since no \( v_{\text{exp}} \) is available for K648, we use a range that represents typical values for PNs. The central star mass for K648 was estimated by linearly interpolating/extrapolating with both hydrogen-burning and helium-burning post-AGB tracks of Vassiliadis & Wood (1994) in the log \( L \)-log \( T \) plane. The
H-burning tracks and 0.56 \( \dot{M} \) tracks. We adopt a final core mass of 0.58 \( M_\odot \) mass of K648, e.g., the hydrogen-burning interpolating in log metallicity considered by & Wood Thus, Vassiliadis (1994).

Table 1

| Parameter          | Value         | Reference |
|--------------------|---------------|-----------|
| \( T_{\text{eff}} \) (K) | 36000 ± 4000  | 1, 2      |
| \( d \) (kpc)      | 10.0 ± 0.8    | 3         |
| \( v_{\text{exp}} \) (km s\(^{-1}\)) | 1.0–2.5  | 1, 2, 5   |
| \( n_e \) (cm\(^{-3}\)) | 15–25        |           |
| \( T_e \) (K)      | 1700–8000     | 1, 2      |
| \( \log F_{\text{H}\beta} \) | −12.10 ± 0.03 | 4         |

Note.—This table is a summary of the observed and inferred parameters for PN K648. The effective temperature, \( T_{\text{eff}} \), refers to the central star, while the distance, \( d \), is the adopted distance to K648. The following nebular parameters are also listed: the angular size of the nebula, \( \theta \); the expansion velocity, \( v_{\text{exp}} \); the electron density, \( n_e \); the ionized gas temperature, \( T_e \); and the log of the measured H\( \beta \) flux in ergs cm\(^{-2}\) s\(^{-1}\). The large range in \( \theta \) and \( n_e \) arises from differences between newer HST data and ground-based data. The HST data give higher values of \( \theta \) and a lower value for \( n_e \).

Table 2

| Parameter          | Observed Value | Reference |
|--------------------|----------------|-----------|
| \( L \) | 3200–4700      | 1, 2      |
| \( M_{\text{en}} \) (\( M_\odot \)) | 0.015–0.090    | 1, 2      |
| \( M \) (\( M_\odot \)) | 0.58 ± 0.03    |           |
| \( \tau_{\text{dyn}} \) (yr) | 2000–8000   | 1200      |
| \( \text{He/H} \) | 0.083–0.10    | 1, 3, 4   |
| \( \text{C/O} \) | 4–11           | 1, 3, 4   |
| \( \text{N/O} \) | 0.05–0.20      | 1, 3, 4   |

Note.—This table compares the observed and predicted parameters for PN K648. The observed luminosity, \( L \), refers to the central star, while the predicted luminosity is the luminosity on the AGB, but since the tracks are nearly horizontal they should be comparable. The following nebular parameters are also listed: the mass of ionized gas in the nebula, \( M_{\text{en}} \); the mass of the central star, \( M \); the dynamical timescale, \( \tau_{\text{dyn}} \); and the abundance ratios He/H, C/O, and N/O by number. The abundances for the prompt scenario are calculated with the assumption that 0.00014 \( M_\odot \) of helium- and carbon-rich material is removed by mass loss from the CSPN. The observed value for the dynamical timescale, \( \tau_{\text{dyn}} \), corresponds to an upper limit for the age of the nebula. The theoretical values correspond to evolutionary timescales required to reach a given central star temperature. The large range in \( L \) and \( M_{\text{en}} \) arise from differences between HST data and ground-based radio and optical data. The HST data give higher values for \( L \) and \( M_{\text{en}} \).

References.—(1) Adams et al. 1984; (2) Bianchi et al. 1995; (3) Henry et al. 1996; (4) Howard et al. 1997.

The adopted abundances of K648 for He/H, C/O, and N/O ratios represent a range of values in recent literature. Howard et al. (1997) find that in six of the nine halo PNs they studied, the C/O ratio exceeds the solar value. Many of these nebulae have stellar temperatures much higher than that of K648, which implies that they are older and more evolved. That the high C/O ratios persist into the later stages of PN evolution suggests that the inferred C/O is not influenced by the presence of shock heating in the nebula.

The mass-loss rate at the tip of the AGB was determined by dividing the nebular mass by the dynamical age. This is in reality a lower limit, since it assumes that the nebula has a filling factor of 1, which is unrealistic. By this procedure we calculate that the lower limit to the mass loss is \( 9 \times 10^{-6} M_\odot \) yr\(^{-1}\). The upper limit is assumed to be \( 10^{-4} M_\odot \) yr\(^{-1}\).

The composition of the central star is uncertain; two recent papers do not agree. McCarthy et al. (1996) find that the central star has a normal helium abundance, whereas Heber, Dreizler, & Werner (1993) find that the central star is helium and carbon rich.

3.2. Delayed Scenario

We have calculated several low-mass, low-metallicity models, but here we focus on the two models listed in Table 3, where we present the model input parameters: the ZAMS mass (\( M \)), the core mass at PN ejection (\( M_{\text{en}} \)), the mass of the PN (\( M_{\text{PN}} \)), the ZAMS [Fe/H] ratio, the adopted ratio of the interpulse luminosity, the stellar radius, the mass-loss rate, and the core mass as a function of total mass. All quantities are expressed in solar units. Figure 2 shows the evolution of the interpulse luminosity, the stellar radius, the mass-loss rate, and the core mass as a function of total mass. Table 2 compares the observed quantities with our predicted ones.

We note in Figure 1 that the interpulse radius of each model star increases dramatically after the final pulse, compared with the preceding interpulse phase. The increase in radius leads to a large increase in the mass-loss rate in each
model during the final interpulse phase; the mass-loss rate increases by almost a factor of 100 in model 2 and by a factor of 5 in model 1. This is a consequence of the steep dependence of our mass-loss law on the stellar radius. The significant increase in the mass-loss rate causes the star to lose its envelope in a few thousand years. The mass-loss rate for model 1 is clearly too small relative to the observationally derived value. However, we have found that by reducing the mixing length ($\alpha$) by a factor of 2, as we have done in model 2, we can make a model that essentially reproduces the observed AGB tip mass-loss rate.

The significant event that occurs during the final pulse is a dredge-up of helium- and carbon-rich material. The mass of material dredged up is a few times $10^{-5} \, M_\odot$. However, given the mass of the envelope and the low initial abundances, the amount of carbon dredged into the envelope is significant enough to increase the carbon mass fraction by a large factor in each case. Consequently, the envelope opacity rises, causing a dramatic increase in the stellar radius.

The envelope of each model at the last thermal pulse is only a few times $10^{-2} \, M_\odot$ and, after the final carbon-dredging pulse, is ejected on a timescale of a few hundred years. Each model star is a carbon star for only a few hundred years, because of the rapid mass loss after a dredge-up of carbon.

This short lifetime, coupled with the relatively low incidence of PNs in globular clusters (two confirmed and three possible candidates; Jacoby et al. 1995), perhaps explains why carbon stars have not been observed in globular clusters.

An important check on our models is to compare the predicted AGB tip luminosity with its observed value. The predicted luminosity of our models at the top of the AGB agrees fairly well with the tip of M15's red giant branch (Adams et al. 1984). Our models suggest that the observed AGB tip will actually correspond to the second-to-last pulse, since after the dredge-up event the star is predicted to remain an AGB star for only $\sim 1000$ yr. The luminosity of K648 in Adams et al. (1984) appears to be 0.1 dex higher than the tip of the giant branch; this may be a result of the metallicity enhancement due to the dredge-up. As noted earlier, the core mass–luminosity relationship depends on metallicity, with the luminosity at a set core mass increasing with increasing metallicity. If we lower the luminosity still further to $\sim 2000 \, L_\odot$ to match the tip of the giant branch, we believe that the addition of carbon to the envelope will still cause envelope ejection.

There is some question about whether dredge-up can occur at the low values of $M_{c,\min}^{\text{DUP}}$ indicated by our models (see Table 3). While Lattanzio (1989) found that dredge-up can occur at a core mass above 0.605 $M_\odot$, the same study also found a dependence of the minimum dredge-up mass on metallicity, with lower metallicities giving lower mass dredge-ups. Boothroyd & Sackmann (1988c) found that if they increased the mixing-length parameter $\alpha$ from 1 to 3, they were able to cause a dredge-up in a model with $Z = 0.001$, $M_c = 0.566 \, M_\odot$, and $M = 0.81 \, M_\odot$, although it is unclear if a mixing length this large is justified. In addi-

\begin{table}
\centering
\caption{Input parameters and results for delayed models}
\begin{tabular}{cccccccc}
Number & $M$ & $M_c$ & $M_{\text{PN}}$ & $[\text{Fe/H}]$ & $\alpha$ & $M_{\text{FTP}}$ & $\lambda$ & $M_{\text{OUT}}^{\text{DUP}}$\\
\hline
1 & 0.88 & 0.56 & 0.037 & -2.1 & 1.6 & 0.62 & 0.10 & 0.55 \\
2 & 0.85 & 0.58 & 0.060 & -2.1 & 0.8 & 0.72 & 0.02 & 0.56 \\
\end{tabular}
\begin{flushright}
\textbf{Note.}—Masses are in $M_\odot$.
\end{flushright}
\end{table}
tion, to match the low-luminosity end of the carbon star luminosity function of the LMC, Groenewegen & de Jong (1993) had to set \( M^{DU}_{\text{min}} = 0.58 \, M_{\odot} \). From these studies, it appears that our values for \( M^{DU}_{\text{c,min}} \) are not unreasonable.

Finally, we point out that each of the delayed models gives a very natural explanation of the high carbon abundance of K648 and the lack of carbon stars. Each model also predicts the observed mass of the ionized gas to be a few times \( 10^{-2} \, M_{\odot} \). The C/O ratios of each model range from 4 to 25, with model 2 giving the best fit, which agrees reasonably well with the observed values of 4–11. The He/H ratios of the model stars also agree with the observed value of 0.09. The high N/O ratio inferred in the models may be an artifact of our choice of initial O abundance and, hence, could be reduced with a higher O abundance, which would also slightly reduce the C/O ratio. Thus, our delayed models are consistent with several important observed properties of the K648 system.

3.3. Prompt Scenario

An alternative scenario results if we apply our mass-loss formulation to the secondary luminosity peak (SLP) that follows the helium shell flash of the 1 \( M_{\odot} \), \( Z = 0.001 \) model of Boothroyd & Sackmann (1988a, hereafter BS88a). The metallicity of the BS88a model is a factor of \( \sim 5 \) higher than M15; however, no models of the appropriate metallicity exist, and we attempted to use the closest one in terms of \( Z \), \( M_{\star} \), and \( M \). The SLP corresponds to the region between point C and the vertical dashed line in Figure 2 of Boothroyd & Sackmann (1988a), i.e., the same place at which Renzini (1989) and Renzini & Fusi Pecci (1988) predict this event to occur when the star expands. The SLP occurs when the excess luminosity produced in the helium shell flash reaches the surface. This peak can be seen in most models of low-mass AGB stars (Iben 1982; BS88a; VW93).

It should be noted that this is the point at which dredge-up can occur, although it does not necessarily do so. This scenario does not require a dredge-up of carbon-rich material for envelope ejection. We define the prompt scenario as ejection at the SLP without the dredge-up of carbon-rich material.

The adopted parameters of this model are shown in Table 4. The luminosity and radius are the eyeballed lower limits from the SLP of BS88a, while the mass and core mass are parameters stated in their text. The mass-loss rate calculated from our prescription (i.e., eq. [1]) is \( \sim 10^{-5} \, M_{\odot} \, \text{yr}^{-1} \) (essentially the Eddington limit), which will remove the 0.03 \( M_{\odot} \) envelope in a few thousand years. This model is similar to K648 in terms of core mass and envelope mass. Values for luminosity, radius, mass-loss rate, and core mass for the prompt scenario are indicated with filled diamonds in Figure 1, and observed quantities are also compared with those predicted for this scenario in Table 2.

A carbon-rich nebula could be formed by the prompt mechanism if a sufficient amount of helium- and carbon-rich material is ejected during the post-AGB phase and mixed with the ejected hydrogen-rich envelope. The fast wind overtaking the slower wind will produce a shock that would likely be Rayleigh-Taylor unstable, causing the nebula to mix. Only \( 5 \times 10^{-5} \, M_{\odot} \) of material with mass fractions of \(^{12}\text{C} = 0.75 \) and \(^{13}\text{C} = 0.23 \) need to be mixed into the envelope to match the C/O ratio of K648. Carbon-rich material can be ejected into the nebula during the post-AGB phase. As a star moves horizontally across the H-R diagram from the AGB stage to CSPN position, the mass-loss rate will decrease as the wind speed increases, so the material ejected during this transition can be mixed with the slower hydrogen-rich envelope. And since the currently observed mass-loss rate of the K648 central star is \( 10^{-9} \) to \( 10^{-10} \, M_{\odot} \, \text{yr}^{-1} \) (Adams et al. 1984; Bianchi et al. 1995), the nebula is no longer being polluted. Examination of the models of Vassiliadis & Wood (1994) suggests that as the star moves from the AGB phase to the CSPN phase, the mass-loss rate drops from \( 10^{-5} \, M_{\odot} \, \text{yr}^{-1} \) to \( 10^{-10} \, M_{\odot} \, \text{yr}^{-1} \), which indicates that during this transition the mass loss rate was higher in the past, and possibly high enough to account for the carbon enrichments in K648.

In the prompt scenario, the envelope is ejected when the star is burning helium, and as a result, the resulting CSPN will follow a helium-burning track (Schönberner 1981, 1983; Iben 1984).

Thus, in the prompt scenario, the evolved star ejects sufficient carbon into a slower moving hydrogen-rich shell to produce the PN we observe today. Mixing is assumed to occur because of shock-induced instabilities. Since the prompt scenario postulates the removal of the entire H-rich envelope during the He-burning stage, we expect K648 to follow an He-burning track because the H-rich shell has been extinguished during the thermal pulse. Ultimately, a white dwarf of type DB will be produced.

4. DISCUSSION

One additional scenario is also a delayed one, but one in which the CSPN is a helium burner. We have not as yet performed calculations relevant to it. In this case, if a dredge-up occurs, it does so at the SLP. The stellar envelope will be enriched in carbon, and the added opacity may allow an even greater expansion during the SLP, making it more likely that the envelope will be ejected during this phase. The resulting PN would be carbon rich and have a helium-burning CSPN. We feel that this is also a promising model, although proper calculations of this scenario need to be done.

Both the prompt and delayed scenarios can be made to match many of the observed features of K648. With each mechanism, the radius increases dramatically, in the prompt scenario because of the increase in luminosity of the star after a thermal pulse and in the delayed scenario because of an increase in the opacity due to an infusion of

| Parameter | Value |
|-----------|-------|
| Luminosity (\( L_{\odot} \)) | 4000 |
| Radius (\( R_{\odot} \)) | 400 |
| Mass (\( M_{\odot} \)) | 0.58 |
| Core mass (\( M_{\odot} \)) | 0.54 |
| Mass-loss rate (\( M_{\odot} \, \text{yr}^{-1} \)) | \( 3.2 \times 10^{-5} \) |
| Time in stage (yr) | 2000 |

NOTE.—The values in this table are estimated from Fig. 2 of BS88a for a 1.0 \( M_{\odot} \), \( Z = 0.001 \) model. All values are appropriate between point C and the vertical dashed line on that figure. The radius and luminosity are the estimated lower limits. The mass and core mass are taken from their listed values. The mass-loss rate is calculated from our mass-loss prescription. The time in this stage is estimated from the BS88a graph.
carbon-rich material. In addition, both mechanisms produce $^{12}\text{C}$ in sufficient amounts to explain the observed C/O ratio.

The most serious difficulty with the prompt scenario is that it can explain the enhancement of the carbon and helium abundances only by essentially ad hoc means, in this case by assuming that the central star wind pollutes the rest of the nebula or by shocks and carbon-rich pockets due to this wind. This may not be an unreasonable assumption, since the mass of K648 is low compared with that of a "typical" PN ($\sim 0.1 \, M_{\odot}$). Testing the prompt scenario would require a detailed model following the star from the horizontal branch to the central star phase with attention to the details of mass loss to see if the central star wind can truly enhance the carbon and helium abundances of the PN and multidimensional hydrodynamics to test the mixing hypothesis.

The difficulty with the delayed scenario is that it predicts that the CSPN should be an H burner. The dynamical age of K648 favors an He-burning CSPN, which is more likely to occur in the prompt scenario as the envelope is ejected during a phase in which helium burning is dominant. Since we assume that for a given metallicity only one of these scenarios will be operative, a strong observational test to determine the correct scenario would be to search for white dwarfs in M15. If they are found to be type DB, this would favor the prompt scenario, and if they are type DA, the delayed scenario is more likely.

A point favoring the prompt scenario is that it naturally accounts for the dynamical age. On the other hand, this scenario requires the assumption of efficient mixing, and there is some evidence (see § 3.1) that signatures of the requisite shocks are not actually observed. However, until detailed models are produced, both remain viable evolutionary scenarios for K648 and similar systems.

We thank the anonymous referee for pointing out some missing references in the original manuscript. This work was supported in part by NASA grant NAG 5-2389 and by NSF grant AST-9417242.

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