A Scalable Algorithm for Tracking an Unknown Number of Targets Using Multiple Sensors

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Abstract—We propose a method for tracking an unknown number of targets based on measurements provided by multiple sensors. Our method achieves low computational complexity and excellent scalability by running belief propagation on a suitably devised factor graph. A redundant formulation of data association uncertainty and the use of “augmented target states” including binary target indicators make it possible to exploit statistical independencies for a drastic reduction of complexity. An increase in the number of targets, sensors, or measurements leads to additional variable nodes in the factor graph but not to higher dimensions of the messages. As a consequence, the complexity of our method scales only quadratically in the number of targets, linearly in the number of sensors, and linearly in the number of measurements per sensor. The performance of the method compares well with that of previously proposed methods, including methods with a less favorable scaling behavior. In particular, our method can outperform multisensor versions of the probability hypothesis density (PHD) filter, the cardinalized PHD filter, and the multi-Bernoulli filter.

Index Terms—Multitarget tracking, data association, belief propagation, message passing, factor graph, sensor network.

I. INTRODUCTION

A. Multitarget Tracking Using Multiple Sensors

Multitarget tracking is important in many applications including surveillance, autonomous driving, biomedical analytics, robotics, and oceanography [1–4]. Multitarget tracking aims at estimating the states—i.e., positions and possibly further parameters—of moving objects (targets) over time, based on measurements provided by sensing devices such as radar, sonar, or cameras [3]. Often information from multiple sensors is required to obtain satisfactory reliability and accuracy. The number of targets is usually unknown [4] and there is a data association uncertainty, i.e., an unknown association between measurements and targets [3].

Traditional methods for multitarget tracking model the target states as a random vector, i.e., an ordered list of random variables, and estimate them jointly with the random association variables. Examples are the joint probabilistic data association (JPDA) filter [3] and the multiple hypothesis density tracker (MHT) [5] and their extensions to multiple sensors [6–8]. Most of these methods assume that the number of targets is fixed and known, which is typically not true in practice. Because of this assumption, most traditional methods do not solve the track management problem, i.e., they are unable to create or cancel a track when a target appears or disappears, respectively. Track management extensions of the single-sensor JPDA filter and single-sensor MHT include the joint integrated probabilistic data association (JIPDA) filter [9], the joint integrated track splitting (JITS) filter [10], and the search-initialize-track filter [11].

Even more recently, FISST-based multitarget tracking methods using labeled random finite sets have been proposed. These filters track an unknown number of targets that are identified by an (unobserved) label, and thus are able to estimate individual target tracks. In particular, the labeled multi-Bernoulli (LMB) filter [18] and the generalized LMB filter [19] achieve good estimation accuracy with a computational complexity that is similar to that of the CPHD filter. Alternatively, the track-oriented marginal Bernoulli/Poisson (TOMB/P) filter and the measurement-oriented marginal Bernoulli/Poisson (MOMB/P) filter proposed in [20] can estimate individual target tracks by integrating probabilistic data association into FISST-based sequential estimation; they are not based on labeled random finite sets.

In the case of low-observable targets, i.e., targets leading to measurements with a low signal-to-noise ratio, reliable detection and tracking using a single sensor may be impossible. Theoretical results [21] suggest that the probability of detection can be strongly improved by increasing the number of sensors. Unfortunately, the computational complexity of optimum multisensor-multitarget tracking scales exponentially in the number of sensors, number of targets, and number of measurements per sensor [22–25]. Computationally feasible
multisensor-multitarget tracking methods include the iterator-corrector (C)PHD or briefly IC-(C)PHD filter [26], the approximate product multisensor (C)PHD filter [27], and the partition-based multisensor (C)PHD (MS-(C)PHD) filter [23], [24]. These methods either use approximations of unknown fidelity and thus may not be able to fully realize the performance gains promised by multiple sensors, or they still scale poorly in relevant system parameters. A further disadvantage of the IC-(C)PHD filter is the strong dependence of its performance on the order in which the sensor measurements are processed [23], [24], [26], [27]. We note that the methods in [18]–[20] have been formulated only for a single sensor.

B. The Proposed Method and Other Message Passing Methods

Here, we propose a multisensor method for multitarget tracking with excellent scalability in the number of targets, number of sensors, and number of measurements per sensor. Our method allows for an unknown, time-varying number of targets (up to a specified maximally possible number of targets), i.e., it implicitly performs track management. These advantages are obtained by performing ordered estimation using belief propagation (BP) message passing, based on the sum-product algorithm [28]–[31]. Contrary to most FISST-based methods, which calculate an approximation of the joint posterior multijoint pdf, BP provides accurate approximations of the marginal posterior pdfs for the individual targets. These are then used to perform Bayesian detection and estimation of the target states.

The proposed BP method is derived by formulating a detection-estimation problem involving all the target states, existence variables, and association variables—which are modeled via random vectors—for all times, targets, and sensors. We use a redundant formulation of data association uncertainty in terms of both target-oriented and measurement-oriented association vectors [32], [33], and “augmented target states” that include binary target existence indicators. In contrast to FISST-based techniques, the joint augmented target state is ordered and has a fixed number of components.

By this new formulation of the multisensor-multitarget detection-estimation problem, the statistical structure of the problem can be described by a factor graph, and the problem can be solved using loopy BP message passing. The advantage of the BP approach is that it exploits conditional statistical independencies for a drastic reduction of complexity [28]–[31]. We use a “detailed” factor graph in which each target state and each association variable is modeled as an individual node. Because this factor graph involves only low-dimensional variables, the resulting BP algorithm does not perform high-dimensional operations. As a consequence, the complexity of our method scales only quadratically in the number of targets, linearly in the number of sensors, and linearly in the number of measurements per sensors (assuming a fixed number of message passing iterations). In addition, because our method uses particle-based calculations of all messages and beliefs, it is suited to general nonlinear, non-Gaussian measurement and state evolution models.

Simulation results in a challenging scenario with intersecting targets demonstrate that our method exhibits excellent scalability and, at the same time, its performance compares well with that of previously proposed methods. This includes methods with a less favorable scaling behavior, namely, cubic in the number of measurements and in the number of targets. In particular, our method can outperform the IC-PHD and IC-CPHD filters [26], the IC-MB filter [17], and the MS-PHD and MS-CPHD filters [23], [24]. Furthermore, its performance does not depend on an assumed order of processing the measurements of the different sensors.

To the best of our knowledge, previously proposed BP methods for multisensor-multitarget tracking are limited to the method presented in [34] and our previous method in [35]. In [34], all target states and association variables at one time step are modeled as a joint state. This results in a tree-structured factor graph for which BP is exact but also in an unappealing scalability in the number of targets. In contrast, our method is based on a detailed (but loopy) factor graph that gives rise to low-dimensional messages and, in turn, results in the attractive scaling properties described above. Furthermore, both methods [34], [35] assume that the number of targets is known, whereas our present method is suited to an unknown number of targets.

BP-based methods have also been proposed for the problems of data association alone or data association within a multitarget tracking scheme where the tracking itself is not done by BP. In particular, BP has been used in [32] and [33] to calculate approximate marginal association probabilities for a single sensor; in [32], [33] to calculate exact marginal association probabilities for a single sensor; in [37] to calculate approximate association probabilities for multiple sensors with overlapping regions of interest; and in [20] to calculate approximate association probabilities for a single sensor. In contrast to these methods, our method uses BP for the overall multisensor-multitarget tracking problem, of which data association is only a part.

C. Paper Organization

This paper is organized as follows. The system model and the multisensor-multitarget tracking problem are described in Section II and a statistical formulation of the problem is presented in Section III. In Section IV, we briefly review the framework of factor graphs and BP message passing. Section V develops the proposed multisensor-multitarget tracking method. A particle-based implementation is presented in Section VI. Section VII proposes a scheme for choosing the birth and survival parameters. In Section VIII, relations of the proposed method to existing methods are discussed. Finally, simulation results in a scenario with intersecting targets are reported in Section IX. We note that this paper advances over the preliminary account of our method provided in our conference publication [38] by adding a particle-based implementation, a detailed discussion of relations to existing methods, additional performance results, and an experimental verification of scaling properties.

II. SYSTEM MODEL AND PROBLEM STATEMENT

In this section, we describe our system model and formulate the multitarget detection-estimation problem to be solved.
A. Potential Targets and Sensor Measurements

We consider at most $K$ targets with time-varying states. We describe this situation by introducing potential targets (PTs) $k \in K \triangleq \{1, \ldots, K\}$. The existence of the PTs is modeled by binary variables $r_{n,k} \in \{0,1\}$, i.e., PT $k$ exists at time $n$ if and only if $r_{n,k} = 1$. We also define the vector $r_n \triangleq [r_{n,1}, \ldots, r_{n,K}]^T$. The state $x_{n,k}$ of PT $k$ at time $n$ consists of the PT’s position and possibly further parameters. It will be convenient to formally consider a PT state $x_{n,k}$ also if $r_{n,k} = 0$. We define the augmented state $y_{n,k} \triangleq [x_{n,k}^T, r_{n,k}]^T$ and the joint augmented state $y_n \triangleq [y_{n,1}^T, \ldots, y_{n,K}^T]^T$.

There are $S$ sensors $s \in S \triangleq \{1, \ldots, S\}$ that produce “thresholded” measurements resulting from a detection process (as performed, e.g., by a radar or sonar device). Let $z_{n,m}$, $m \in M_n \triangleq \{1, \ldots, M_n\}$ denote the measurements produced by sensor $s$ at time $n$. We also define the stacked measurement vectors $z_n \triangleq [z_n^{(1)} \cdots z_n^{(S)}]^T$ and $z_n \triangleq [z_n^{(1)} \cdots z_n^{(S)}]^T$. Because the measurements are thresholded, the multitarget tracking problem is complicated by a data association uncertainty: it is unknown which measurement $z_{n,m}$ originated from which PT $k$, and it is also possible that a measurement $z_{n,m}$ did not originate from any PT (false alarm, clutter) or that a PT did not generate any measurement of sensor $s$ (missed detection) [3], [4]. We make the usual assumption that at any time $n$, an existing target can generate at most one measurement at sensor $s$, and a measurement at sensor $s$ can be generated by at most one existing target [3], [4]. The PT-measurement associations at sensor $s$ and time $n$ can then be described by the target-oriented association variables

$$ a_{n,k}^{(s)} \triangleq \begin{cases} m \in M_n^{(s)}, & \text{at time } n, \text{ PT } k \text{ generates measurement } m \text{ at sensor } s \\ 0, & \text{at time } n, \text{ PT } k \text{ is not detected by sensor } s \end{cases} $$

We also define $a_n^{(s)} \triangleq [a_{n,1}^{(s)} \cdots a_{n,K}^{(s)}]^T$ and $a_n^{(s)} \triangleq [a_{n,1}^{(1)} \cdots a_{n,K}^{(S)}]^T$. Following [3] and [4], we also use an alternative description of the PT-measurement associations in terms of the measurement-oriented association variables

$$ b_{n,m}^{(s)} \triangleq \begin{cases} k \in K, & \text{at time } n, \text{ measurement } m \text{ at sensor } s \text{ is generated by PT } k \\ 0, & \text{at time } n, \text{ measurement } m \text{ at sensor } s \text{ is not generated by a PT} \end{cases} $$

We also define $b_n^{(s)} \triangleq [b_n^{(1)} \cdots b_n^{(S)}]^T$ and $b_n^{(s)} \triangleq [b_n^{(1)} \cdots b_n^{(S)}]^T$. The description in terms of $b_n^{(s)}$ is redundant in that $b_n^{(s)}$ cannot be derived from $a_n^{(s)}$ and vice versa.

B. Target Detection and State Estimation

The problem considered is detection of the PTs $k \in K$ (i.e., of the binary target existence variables $r_{n,k}$) and estimation of the target states $x_{n,k}$ from the past and present measurements of all the sensors $s \in S$, i.e., from the total measurement vector $z \triangleq [z_1^T \cdots z_S^T]^T$. In a Bayesian setting, this essentially amounts to calculating the marginal posterior existence probabilities $p(r_{n,k} = 1 | z)$ and the marginal posterior pdfs $f(x_{n,k}|r_{n,k} = 1, z)$. Target detection is performed by comparing $p(r_{n,k} = 1 | z)$ to a threshold $P_D$, i.e., PT $k$ is considered to exist if $p(r_{n,k} = 1 | z) > P_D$ [39, Ch. 2]. For the detected targets $k$, an estimate of $x_{n,k}$ is then produced by the minimum mean-square error (MMSE) estimator [39, Ch. 4]

$$ x_{n,k}^{\text{MMSE}} \triangleq \int x_{n,k} f(x_{n,k}|r_{n,k} = 1, z) dx_{n,k}. \quad (2) $$

This Bayesian two-stage detection-estimation procedure is also employed by the JITS method [10], the JIPDA filter [9], and certain FISST-based algorithms, e.g., [4], [20]. The main problem to be solved now is to find a computationally feasible recursive (sequential) calculation of $p(r_{n,k} = 1 | z)$ and $f(x_{n,k}|r_{n,k} = 1, z)$.

III. STATISTICAL FORMULATION

Next, we present a statistical formulation of the system model and the multitarget detection-estimation problem.

A. Target States

While in our model a PT state $x_{n,k}$ is formally defined also if $r_{n,k} = 0$, the states of nonexistent PTs are obviously irrelevant. Accordingly, all pdfs and BP messages defined for an augmented state, $\phi(y_{n,k}) = \phi(x_{n,k}, r_{n,k})$, have the property that for $r_{n,k} = 0$,

$$ \phi(x_{n,k}, 0) = \phi_{n,k}^D(x_{n,k}), \quad (3) $$

where $f_D(x_{n,k})$ is a “dummy pdf.” The form [3] must be consistent with a message multiplication operation (such as Equation (24) in Section IV), in the sense that the resulting message product can still be expressed as in [3]. This implies that the dummy pdf $f_D(x_{n,k})$ satisfies $f_D(x_{n,k}) = f_D(x_{n,k})$ for all values of $x_{n,k}$. Because $f_D(x_{n,k})$ must also integrate to one, it follows that $f_D(x_{n,k})$ is 1 on an arbitrary support of area/volume 1 and 0 outside that support. Let

$$ \phi(r_{n,k}) \triangleq \int \phi(x_{n,k}, r_{n,k}) dx_{n,k}. \quad (4) $$

We then have for $r_{n,k} = 0$

$$ \phi(0) = \int \phi(x_{n,k}, 0) dx_{n,k} = \phi_{n,k}^D dx_{n,k} = \phi_{n,k}. \quad (5) $$

Furthermore, using [4] and [5] yields

$$ \sum_{r_{n,k} \in \{0,1\}} \int \phi(x_{n,k}, r_{n,k}) dx_{n,k} = \int \phi(x_{n,k}, 0) dx_{n,k} + \int \phi(x_{n,k}, 1) dx_{n,k} = \phi_{n,k} + \phi(1). $$

Hence, if $\sum_{r_{n,k} \in \{0,1\}} \int \phi(x_{n,k}, r_{n,k}) dx_{n,k} = 1$, i.e., if $\phi(x_{n,k}, r_{n,k})$ is a true pdf in the sense of being normalized, then $\phi_{n,k} + \phi(1) = 1$. In that case, $\phi_{n,k} = \phi(0)$ can be
interpreted as a probability of nonexistence of PT $k$, i.e., of the event $r_{n,k} = 0$, and $\phi(1)$ can be interpreted as a probability of existence of PT $k$, i.e., of the event $r_{n,k} = 1$.

The augmented target states $y_{n,k} = [x_{n,k}^T, r_{n,k}]^T$ are assumed to evolve independently according to Markovian dynamic models $[3, 4]$, and at time $n = 0$, they are assumed statistically independent across $k$ with prior pdfs $f(0, k) = f(x_{0,k}, r_{0,k})$. Thus, the pdf of $y$ is $f(y) = \prod_{k=1}^K f(y_{0,k}) n \prod_{n'=1}^n f(y_{n,k}|y_{n'-1,k})$. (6)

Here, the single-target augmented state transition pdf $f(y_{n,k}|y_{n-1,k}) = f(x_{n,k}, r_{n,k}|x_{n-1,k}, r_{n-1,k})$ is given as follows. If PT $k$ did not exist at time $n - 1$, i.e., $r_{n-1,k} = 0$, then the probability that it exists at time $n$, i.e., $r_{n,k} = 1$, is given by the birth probability $p_{b}^{s,k}$, and if it does exist at time $n$, its state $x_{n,k}$ is distributed according to the birth pdf $f_s(x_{n,k})$. Thus, for $r_{n-1,k} = 0$, we have

$$f(x_{n,k}, r_{n,k}|x_{n-1,k}, r_{n-1,k}, 0) = \begin{cases} (1 - p_{b}^{s,k}) f_s(x_{n,k}), & r_{n,k} = 0 \\ p_{b}^{s,k} f_s(x_{n,k}), & r_{n,k} = 1. \end{cases}$$

(7)

If PT $k$ existed at time $n - 1$, 1, then the probability that it still exists at time $n$, i.e., $r_{n,k} = 1$, is given by the survival probability $p_{s}^{n,k}$, and if it still exists at time $n$, its state $x_{n,k}$ is distributed according to the state transition pdf $f(x_{n,k}|x_{n-1,k})$. Thus, for $r_{n-1,k} = 1$, we have

$$f(x_{n,k}, r_{n,k}|x_{n-1,k}, r_{n-1,k}, 1) = \begin{cases} (1 - p_{s}^{n,k}) f_s(x_{n,k}), & r_{n,k} = 0 \\ p_{s}^{n,k} f_s(x_{n,k}), & r_{n,k} = 1. \end{cases}$$

(8)

A possible strategy for choosing $p_{b}^{s,k}$, $p_{s}^{n,k}$, and $f_s(x_{n,k})$ is presented in Section [VII]. We note that our previous work in [5], which assumed that the number of targets is known, is a special case of this setup that uses survival probabilities $p_{s}^{n,k} = 1$ (existing targets always survive), birth probabilities $p_{b}^{n,k} = 0$ (no targets are born), and initial prior pdfs $f(x_{0,k}, r_{0,k})$ with $\int f(x_{0,k}, 1 | dx_{0,k} = 1$ (at time $n = 0$, all targets exist with probability 1).

B. Sensor Measurements

An existing target $k$ is detected by sensor $s$ (in the sense that the target generates a measurement $z_{n,m}^{s(k)}$ at sensor $s$) with probability $f_d^{s(k)}(x_{n,k})$, which may depend on the target state $x_{n,k}$. The number of false alarms at sensor $s$ is modeled by a Poisson probability mass function (pmf) with mean $\mu^{s}(s)$, and the distribution of each false alarm measured at sensor $s$ is described by the pdf $f_{FA}(z_{n,m}^{s(k)})$ $[3, 4]$.

The dependence of the measurement vector $z = [z_1^T \ldots z_n^T]^T$ on the numbers-of-measurements vector $m = [m_1^T \ldots m_n^T]^T$, the augmented state vector $y = [y_1^T \ldots y_n^T]^T$, and the association vector $a = [a_1^T \ldots a_n^T]^T$ is described by the global likelihood function $f(z|y, a, m)$. With the commonly used assumption $[3, 4]$ that given $y$, $a$, and $m$, the measurements $z_{n}^{s(k)}$ are conditionally independent across time $n$ and sensor index $s$, the global likelihood function factorizes as

$$f(z|y, a, m) = \prod_{n'=1}^n \prod_{s=1}^S f(z_{n,m}^{s(k)}|y_{n',a_{n'}^{s(k)}, m_{n'}^{s(k)}}).$$

(9)

Assuming in addition that the different measurements $z_{n,m}^{s(k)}$ at sensor $s$ are conditionally independent given $y_{n'}, a_{n'}^{s(k)}$, and $M_{n'}^{s(k)}$, we have the further factorization $[3, 4]$.

$$f(z_{n,m}^{s(k)}|y_{n'}, a_{n'}^{s(k)}, M_{n'}^{s(k)}) = \prod_{m=1}^{M_{n'}^{s(k)}} f_{FA}(z_{n,m}^{s(k)}) \times \prod_{k \in A_{n,m}^{s(k)}} \frac{f(z_{n,m}^{s(k)}|x_{n,k})}{f_{FA}(z_{n,m}^{s(k)})}. \quad (10)$$

Here, $D_{n,m}^{s(k)}$ denotes the set of existing targets detected at sensor $s$ and time $n$, i.e., $D_{n,m}^{s(k)} = \{ k \in K : r_{n,k} = 1, r_{n,k} \neq 0 \}$. If $z_{n,m}^{s(k)}$ is observed and thus fixed, $M_{n'}^{s(k)}$ is fixed as well and (10) can be written as

$$f(z_{n,m}^{s(k)}|y_{n'}, a_{n'}^{s(k)}, M_{n'}^{s(k)}) = C(z_{n,m}^{s(k)}) \prod_{k=1}^{K} g(x_{n,k}, r_{n,k}, a_{n,k}^{s(k)}, z_{n,k}^{s(k)}), \quad (11)$$

where $C(z_{n,m}^{s(k)})$ is a normalization factor that depends only on $z_{n,m}^{s(k)}$ and $g(x_{n,k}, r_{n,k}, a_{n,k}^{s(k)}, z_{n,k}^{s(k)})$ is defined as

$$g(x_{n,k}, r_{n,k}, a_{n,k}^{s(k)}, z_{n,k}^{s(k)}) = \begin{cases} \frac{f(z_{n,m}^{s(k)}|x_{n,k})}{f_{FA}(z_{n,m}^{s(k)})}, & a_{n,k}^{s(k)} = m \in M_{n}^{s(k)} \\ 1, & a_{n,k}^{s(k)} = 0 \end{cases}$$

(12)

Inserting (11) into (3) yields

$$f(z|y, a, m) = C(z) \prod_{n'=1}^n \prod_{s=1}^S K \prod_{k=1}^{K} g(x_{n,k}, r_{n,k}, a_{n,k}^{s(k)}, z_{n,k}^{s(k)}), \quad (13)$$

where $C(z)$ is a normalization factor that depends only on $z$.

C. Joint Prior Distribution of Association Variables and Numbers of Measurements

Under the assumption that given $y$, the $a_{n}^{s(k)}$ and the $M_{n}^{s(k)}$ are conditionally independent across $n$ and $s$ $[3, 4]$, the joint prior pmf of the association vector $a$ and the numbers-of-measurements vector $m$ given $y$ factorizes as

$$p(a, m|y) = \prod_{n'=1}^n \prod_{s=1}^S p(a_{n,s}^{s(k)}, M_{n,s}^{s(k)}|y_{n'}). \quad (14)$$

Assuming a random permutation of the measurements $z_{n,m}^{s(k)}$, $m \in M_{n}^{s(k)}$ at sensor $s$, with each permutation equally likely, it is shown in $[3, 11]$ that
where
\[
\psi\left(a_{n,k}^{(s)}\right) \triangleq \begin{cases} 0, & \exists k, k' \in \mathcal{K} \text{ such that } a_{n,k}^{(s)} = a_{n,k'}^{(s)} \neq 0, \\ 1, & \text{otherwise}, \end{cases}
\]
and \(1(a)\) denotes the indicator function of the event \(a = 0\) (i.e., \(1(a) = 1\) if \(a = 0\) and 0 otherwise). Note that the factor \(\psi\left(a_{n,k}^{(s)}\right)\) enforces the exclusion assumptions stated in Section II-A, i.e., that each existing target can generate at most one measurement at sensor \(s\) and each measurement at sensor \(s\) can be generated by at most one target. We can express (15) as
\[
p(a_{n}^{(s)}, M_{n}^{(s)} | y_{n}) = C(M_{n}^{(s)}) \psi\left(a_{n}^{(s)}\right) \prod_{k=1}^{K} h(x_{n,k}, r_{n,k}, a_{n,k}^{(s)}, M_{n}^{(s)}),
\]
where \(C(M_{n}^{(s)})\) is a normalization factor depending only on \(M_{n}^{(s)}\) and \(h(x_{n,k}, r_{n,k}, a_{n,k}^{(s)}, M_{n}^{(s)})\) is defined as
\[
h(x_{n,k}, 1, a_{n,k}^{(s)}; M_{n}^{(s)}) = \begin{cases} \frac{P_{d}^{(s)}(x_{n,k})}{\mu}\mu_n^{(s)}, & a_{n,k}^{(s)} \in M_{n}^{(s)} \\ 1 - P_{d}^{(s)}(x_{n,k}), & a_{n,k}^{(s)} = 0 \end{cases}
\]
\[
h(x_{n,k}, 0, a_{n,k}^{(s)}; M_{n}^{(s)}) = 1(a_{n,k}^{(s)}).
\]
(18)

Using the measurement-oriented association vectors \(b_{n}^{(s)}\) defined in (11) alongside with the target-oriented association vectors \(a_{n}^{(s)}\), the exclusion-enforcing function \(\psi\left(a_{n}^{(s)}\right)\) in (16) can be formally replaced by the function \(\Psi\) in (19)
\[
\psi\left(a_{n}^{(s)}, b_{n}^{(s)}\right) = \prod_{k=1}^{K} \prod_{m=1}^{M_{n}^{(s)}} \Psi(a_{n,k}^{(s)}, b_{n,m}^{(s)}),
\]
with
\[
\Psi(a_{n,k}^{(s)}, b_{n,m}^{(s)}) \triangleq \begin{cases} 0, & a_{n,k}^{(s)} = m, b_{n,m}^{(s)} \neq k \text{ or } b_{n,m}^{(s)} = k, a_{n,k}^{(s)} \neq m \\ 1, & \text{otherwise}. \end{cases}
\]
(19)

Using this redundant reformulation, and defining \(b \triangleq [b_{1}^{T} \cdots b_{K}^{T}]^{T}\), the prior pmf \(p(a, m | y)\) in (14) can be formally rewritten as
\[
p(a, b, m | y) = \prod_{n=1}^{N_S} \prod_{k=1}^{K} p(a_{n,k}^{(s)}, b_{n,m}^{(s)}, M_{n}^{(s)} | y_{n}) \prod_{n=1}^{N_S} p(a_{n,k}^{(s)}, b_{n,m}^{(s)}, M_{n}^{(s)} | y_{n}),
\]
(20)

with the single-sensor prior pmfs (cf. (17) and (19)).

Fig. 1. Factor graph representing the factorization \(f(x | z) \propto \psi_{1}(x_{1}, x_{2}) \psi_{2}(x_{2}) \psi_{3}(x_{2}, x_{3})\).

\[
p(a_{n}^{(s)}, b_{n}^{(s)}, M_{n}^{(s)} | y_{n}) = C(M_{n}^{(s)}) \prod_{k=1}^{K} h(x_{n,k}, r_{n,k}, a_{n,k}^{(s)}, M_{n}^{(s)}) \prod_{m=1}^{M_{n}^{(s)}} \psi(a_{n,k}^{(s)}, b_{n,m}^{(s)}).
\]
(21)

Thus, Equation (20) can be expressed as
\[
p(a, b, m | y) = C(m) \prod_{n'=1}^{N} \prod_{k=1}^{K} h(x_{n',k}, r_{n',k}, a_{n',k}^{(s)}, M_{n'}^{(s)}) \prod_{m=1}^{M_{n'}^{(s)}} \Psi(a_{n',k}^{(s)}, b_{n',m}^{(s)}),
\]
(21)

where \(C(m)\) is a normalization factor depending only on \(m\). The factorization in (21) constitutes an important basis for our development of the proposed BP method in Section IV.

IV. REVIEW OF BP MESSAGE PASSING

We briefly review factor graphs and the generic BP message passing scheme, which constitute the main methodological basis of the proposed multisensor-multitarget tracking method. Consider the problem of estimating parameter vectors \(x_{k}\), \(k \in \{1, \ldots, K\}\) from a measurement vector \(z\). Bayesian estimation of \(x_{k}\) relies on the posterior pdf \(f(x_{k} | z)\) [40]. This pdf is a marginal pdf of the joint posterior pdf \(f(x | z)\), where \(x = [x_{k}]_{K=1}^{K}\); however, direct marginalization of \(f(x | z)\) is usually infeasible. An efficient marginalization can be achieved if the posterior pdf \(f(x | z)\) factorizes, i.e.,
\[
f(x | z) \propto \prod_{q=1}^{Q} \psi_{q}(x^{(q)}).
\]
(22)

Here, each factor argument \(x^{(q)}\) comprises certain parameter vectors \(x_{k}\) (each \(x_{k}\) can appear in several \(x^{(q)}\)) and \(\propto\) indicates equality up to a normalization factor. Note that for compactness, our notation does not indicate the dependence of the factors \(\psi_{q}(x^{(q)})\) on \(z\).

The factorization structure (22) can be represented by a factor graph [41]. As an example, for \(x = [x_{1}^{T} x_{2}^{T} x_{3}^{T}]^{T}\), the factor graph representing the factorization \(f(x | z) \propto \psi_{1}(x_{1}, x_{2}) \psi_{2}(x_{2}) \psi_{3}(x_{2}, x_{3})\) is shown in Fig. 1. In a factor graph, each parameter variable \(x_{k}\) is represented by a variable node and each factor \(\psi_{q}(\cdot)\) by a factor node (depicted in Fig. 1 by a circle and a square, respectively). Variable node "\(x_{k}\)" and factor node "\(\psi_{q}\)" are adjacent, i.e., connected by an edge, if the variable \(x_{k}\) is an argument of the factor \(\psi_{q}(\cdot)\).

Belief propagation (BP), also known as the sum-product algorithm [28], is based on a factor graph and aims at computing the marginal posterior pdfs \(f(x_{k} | z)\) in an efficient
way. For each node, certain messages are calculated, each of which is passed to one of the adjacent nodes. For each variable node, the incoming and outgoing messages are functions of the corresponding variable. More specifically, consider a variable node \(x_k\) and an adjacent factor node \(\psi_q\), i.e., the variable \(x_k\) is part of the argument \(x^{(q)}\) of \(\psi_q\). Then, the message passed from factor node \(\psi_q\) to variable node \(x_k\) is given by

$$\zeta_{\psi_q \rightarrow x_k}(x_k) = \int \psi_q(x^{(q)}) \prod_{k \neq k'} \eta_{x_{k'} \rightarrow \psi_q}(x_{k'}) \, dx_{k'},$$

(23)

where \(\prod_{k \neq k'} \eta_{x_{k'} \rightarrow \psi_q}(x_{k'})\) denotes the product of the messages passed to factor node \(\psi_q\) from all adjacent variable nodes except \(x_k\), and \(\int \ldots \, dx_k\) denotes integration with respect to all constituent vectors of \(x^{(q)}\) except \(x_k\). For example, the message passed from factor node \(\psi_1\) to variable node \(x_2\) in Fig. 1 is \(\zeta_{\psi_1 \rightarrow x_2}(x_2) = \int \psi_1(x_1, x_2) \eta_{x_1 \rightarrow \psi_1}(x_1) \, dx_1\); note that \(x^{(1)} = [x_1^T \, x_2^T]^T\). The message \(\eta_{x_2 \rightarrow \psi_1}(x_2)\) passed from variable node \(x_2\) to factor node \(\psi_1\) is given by the product of the messages passed to variable node \(x_2\) from all adjacent factor nodes except \(\psi_1\). For example, in Fig. 1, the message passed from variable node \(x_2\) to factor node \(\psi_1\) is \(\eta_{x_2 \rightarrow \psi_1}(x_2) = \zeta_{\psi_2 \rightarrow x_2}(x_2) \zeta_{\psi_3 \rightarrow x_2}(x_2)\). Message passing is started at variable nodes with only one edge (which pass a constant message) and factor nodes with only one edge (which pass the corresponding factor). Note that BP can also be applied to factorizations involving discrete variables by replacing integration with summation in (23).

Finally, for each variable node \(x_k\), a belief \(\tilde{f}(x_k)\) is calculated as the product of all incoming messages (passed from all adjacent factor nodes) followed by a normalization such that \(\int f(x_k) \, dx_k = 1\). For example, in Fig. 1

$$\tilde{f}(x_2) \propto \zeta_{\psi_1 \rightarrow x_2}(x_2) \zeta_{\psi_2 \rightarrow x_2}(x_2) \zeta_{\psi_3 \rightarrow x_2}(x_2).$$

(24)

If the factor graph is a tree, i.e., without loops, then the belief \(\tilde{f}(x_k)\) is exactly equal to the marginal posterior pdf \(f(x_k|z)\). For factor graphs with loops, BP is applied in an iterative manner, and the beliefs \(\tilde{f}(x_k)\) are only approximations of the respective marginal posterior pdfs \(f(x_k|z)\); these approximations have been observed to be very accurate in many applications [28–30]. In this iterative “loopy BP” scheme, there is no canonical order in which the messages should be calculated, and different orders may lead to different beliefs. The choice of an appropriate order of message calculation will be an important aspect in our development of the proposed method.

V. THE PROPOSED BP-BASED MULTISENSOR-MULTITARGET TRACKING METHOD

The marginal posterior existence probability \(p(r_{n,k} = 1|z)\) underlying target detection as discussed in Section II.B can be obtained from the marginal posterior pdf of the augmented target state, \(f(y_{n,k}|z) = f(x_{n,k}, r_{n,k}|z)\), according to

$$p(r_{n,k} = 1|z) = \int f(x_{n,k}, r_{n,k} = 1|z) \, dx_{n,k},$$

(25)

and the marginal posterior pdf \(f(x_{n,k}, r_{n,k}|1, z)\) underlying MMSE state estimation (see (2)) can be obtained from \(f(x_{n,k}, r_{n,k}|z)\) according to

$$f(x_{n,k}|r_{n,k} = 1, z) = \frac{f(x_{n,k}, r_{n,k} = 1|z)}{p(r_{n,k} = 1|z)}. \quad (26)$$

An efficient approximate calculation of \(f(x_{n,k}, r_{n,k}|z)\) can be obtained by performing BP message passing on a factor graph that expresses the factorization of the joint posterior pdf involving all relevant parameters. This factor graph will be derived next.

A. Joint Posterior pdf and Factor Graph

The marginal posterior pdf \(f(y_{n,k}|z) = f(x_{n,k}, r_{n,k}|z)\) is a marginal density of the joint posterior pdf \(f(y, a, b|z)\), which involves all the augmented states, all the target-oriented and measurement-oriented association variables, and all the measurements of all sensors, at all times up to the current time \(n\). In the following derivation of \(f(y, a, b|z)\), the measurements \(z\) are observed and thus fixed, and consequently \(M_{1}^{(s)}\) and \(M_{2}^{(s)}\) are fixed as well. Then, using Bayes’ rule and the fact that a implies b, we obtain

$$f(y, a, b|z) = f(y, a, b, m|z) \propto f(z|y, a, b, m) f(y, a, b, m) = f(z|y, a, m) p(a, b, m|y) f(y).$$

Inserting (6) for \(f(y, a, b, m|z)\), and (21) for \(p(a, b, m|y)\) then yields the final factorization

$$f(y, a, b|z) \propto \prod_{k=1}^{K} f(y_{0,k}) \prod_{n'=1}^{n} f(y_{n',k}|y_{n'-1,k})$$

$$\times \prod_{s=1}^{S} v(y_{n',k}, a_{n',k}^{(s)}; z_{n'}^{(s)}) \prod_{m=1}^{M_{1}^{(s)}} \Psi (a_{n',k}^{(s)}; b_{n',m}^{(s)}),$$

(27)

with

$$v(y_{n',k}, a_{n',k}^{(s)}; z_{n'}^{(s)}) \triangleq g(x_{n',k}, r_{n',k}, a_{n'}^{(s)}; z_{n'}^{(s)}) \times h(x_{n',k}, r_{n',k}, a_{n'}^{(s)}; z_{n'}^{(s)}). \quad (28)$$

This factorization can be represented graphically by a factor graph as explained in Section IV This factor graph is depicted for one time step in Fig. 2; it provides the starting-point for our development of the proposed BP method.

B. BP Method

As discussed in Section IV approximations \(\tilde{f}(y_{n,k})\) of the marginal posterior pdfs \(f(y_{n,k}|z)\) can be obtained in an efficient way by running iterative BP message passing [28, 30, 31] on the factor graph in Fig. 2. Since this factor graph is loopy, we have to decide on a specific order of message computation. We choose this order according to the following rules: (i) Messages are not sent backward in time [41] (ii)

This is equivalent to the approximative assumption that the target states are conditionally independent given the past measurements, as is done in the derivation of the JPDA filter [41].
Iterative message passing is only performed at each time step and at each sensor separately—i.e., in particular, for the loops connecting different sensors we only perform a single message passing iteration—and only for data association. With these rules, the generic BP procedure for calculating messages and beliefs as summarized in Section IV yields the following BP message passing operations at time $n$.

First, a prediction step is performed for all PTs $k \in K$, i.e.,

$$
\alpha(x_{n,k}, r_{n,k}) = \sum_{r_{n-1,k} \in \{0,1\}} \int f(x_{n,k}, r_{n,k}|x_{n-1,k}, r_{n-1,k}) \times \tilde{f}(x_{n-1,k}, r_{n-1,k}) \, dx_{n-1,k}.
$$

Here, $\tilde{f}(x_{n-1,k}, r_{n-1,k})$ was calculated at the previous time $n-1$. Inserting (7) and (8) for $f(x_{n,k}, r_{n,k}|x_{n-1,k}, r_{n-1,k})$, we obtain the following expressions of $\alpha(x_{n,k}, r_{n,k})$: for $r_{n,k} = 1$,

$$
\alpha(x_{n,k}, 1) = p^b_{n,k} \tilde{f}(x_{n,k}) \tilde{f}(x_{n-1,k}, 1) \, dx_{n-1,k},
$$

and for $r_{n,k} = 0$,

$$
\alpha(x_{n,k}, 0) = (1 - p^b_{n,k}) \tilde{f}(x_{n-1,k}) \, dx_{n-1,k}.
$$

We note that $\tilde{f}(x_{n-1,k}) = \int \tilde{f}(x_{n-1,k}, 0) \, dx_{n-1,k}$ and

$$
\alpha(x_{n,k}, 0) = \int \alpha(x_{n,k}, 0) \, dx_{n,k} \quad \text{(cf. (5)).}
$$

Furthermore, since $\tilde{f}(x_{n,k}, r_{n,k})$ is normalized, so is $\alpha(x_{n,k}, r_{n,k})$, i.e.,

$$
\sum_{r_{n,k} \in \{0,1\}} \alpha(x_{n,k}, r_{n,k}) \, dx_{n,k} = 1.
$$

Thus, we have $\alpha(x_{n,k}) = 1 - \int \alpha(x_{n,k}) \, dx_{n,k}$.

After the prediction step, the following steps are performed for all PTs $k \in K$ and all sensors $s \in S$ in parallel:

1) **Measurement evaluation:**

$$
\beta(a^{(s)}_{n,k}) = \sum_{r_{n,k} \in \{0,1\}} \int v(x_{n,k}, r_{n,k}, a^{(s)}_{n,k} ; z^{(s)}_{n}) \times \alpha(x_{n,k}, r_{n,k}) \, dx_{n,k}
$$

In the last expression, we used $v(x_{n,k}, 0, a^{(s)}_{n,k} ; z^{(s)}_{n}) = 1(a^{(s)}_{n,k})$, which follows from (28) with (12) and (13).

2) **Iterative data association** (this part of the BP method closely follows [22], [33], [42]). In iteration $p \in \{1, \ldots, P\}$, the following calculations are performed for all measurements $m \in M^{(s)}_n$:

$$
\nu^{(p)}_{m \rightarrow k}(a^{(s)}_{n,k}) = \sum_{b^{(s)}_{n,m}} \Psi(a^{(s)}_{n,k}, b^{(s)}_{n,m}) \prod_{r \in K \setminus \{k\}} \zeta^{(p-1)}_{r \rightarrow m}(b^{(s)}_{n,m})
$$

and

$$
\zeta^{(p)}_{k \rightarrow m}(b^{(s)}_{n,m}) = \sum_{a^{(s)}_{n,k}} \beta(a^{(s)}_{n,k}) \Psi(a^{(s)}_{n,k}, b^{(s)}_{n,m})
$$

Here, $\sum_{a^{(s)}_{n,k}}$ is short for $\sum_{b^{(s)}_{n,m} \in \{0,\ldots,K\}}$ and $\sum_{a^{(s)}_{n,k}} \beta(a^{(s)}_{n,k})$ is short for $\sum_{a^{(s)}_{n,k} \in \{0,\ldots,M^{(s)}_n\}}$. The operations (32) and (33) constitute an iteration loop, which is initialized (for $p = 0$) by

$$
\zeta^{(0)}_{k \rightarrow m}(b^{(s)}_{n,m}) = \sum_{a^{(s)}_{n,k}} \beta(a^{(s)}_{n,k}) \Psi(a^{(s)}_{n,k}, b^{(s)}_{n,m}).
$$

An efficient implementation of (32) and (33) is described in [42] and [33]. After the last iteration $p = P$, the messages $\nu^{(P)}_{m \rightarrow k}(a^{(s)}_{n,k})$, $m \in M^{(s)}_n$ are multiplied, i.e.,

$$
\eta(a^{(s)}_{n,k}) = \prod_{m=1}^{M^{(s)}_n} \nu^{(P)}_{m \rightarrow k}(a^{(s)}_{n,k}).
$$

3) **Measurement update:**

$$
\gamma^{(s)}(x_{n,k}, 1) = \sum_{a^{(s)}_{n,k}} v(x_{n,k}, 1, a^{(s)}_{n,k} ; z^{(s)}_{n}) \eta(a^{(s)}_{n,k})
$$

Finally, beliefs $\tilde{f}(y_{n,k}) = \tilde{f}(x_{n,k}, r_{n,k})$ approximating the marginal posterior pdfs $f(y_{n,k}|z_{n}) = f(x_{n,k}, r_{n,k}|z_{n})$ are obtained as the products

$$
\tilde{f}(x_{n,k}, 1) = \frac{1}{C_{n,k}} \alpha(x_{n,k}, 1) \prod_{s=1}^{S} \gamma^{(s)}(x_{n,k}, 1)
$$

with the normalization constant

$$
C_{n,k} = \int \alpha(x_{n,k}, 1) \prod_{s=1}^{S} \gamma^{(s)}(x_{n,k}, 1) \, dx_{n,k} + \alpha_{n,k} \prod_{s=1}^{S} \gamma^{(s)}(x_{n,k}, 1).
$$

Note that $\tilde{f}^{\beta}(x_{n,k}) = \tilde{f}^{\beta}_{0}(x_{n,k})$ (cf. Section III-A) was used to obtain (38). Because the belief $\tilde{f}(x_{n,k}, 1)$ in (37) approximates
\[
\{ (x_{n-1,k}^{(j)}, w_{n-1,k}^{(j)}) \}^J_{j=1} \sim \tilde{f}(y_{n-1,k}), \ k \in K \quad \text{(from the previous time step } n-1) \\
\]

Prediction step maps \( \{ (r_{n-1,k}^{(j)}, w_{n-1,k}^{(j)}) \}^J_{j=1} \sim \tilde{f}(y_{n-1,k}) \) to \( \{ (x_{n,k}^{(j)}, w_{n,k}^{(j)}) \}^J_{j=1} \sim \alpha(y_{n,k}), \ k \in K \) (see Section VI-A).

For all sensors \( s \):

- Particle-based measurement evaluation provides \( \tilde{\beta}(a_{n,k}^{(s)}) \), \( k \in K \) (see (12)).
- Iterative data association calculates \( \tilde{\eta}(a_{n,k}^{(s)}) \) from \( \tilde{\beta}(a_{n,k}^{(s)}), \ k \in K \) (see (32)–(35)).

Importance sampling calculates \( \{ (x_{n,k}^{(j)}, w_{n,k}^{(j)}) \}^J_{j=1} \sim \tilde{f}(y_{n,k}), \ k \in K \) from \( \{ (x_{n,k}^{(j)}, w_{n,k}^{(j)}) \}^J_{j=1} \sim \alpha(y_{n,k}) \) and \( \tilde{\eta}(a_{n,k}^{(s)}), \ s \in S \) (see Section VI-C).

The quantities \( p_{n,k}^{(s)} \) and \( x_{n,k} \) are determined from \( \{ (x_{n,k}^{(j)}, w_{n,k}^{(j)}) \}^J_{j=1} \sim \tilde{f}(y_{n,k}), \ k \in K \) (see (43) and (45), respectively).

Resampling maps \( \{ (x_{n,k}^{(j)}, w_{n,k}^{(j)}) \}^J_{j=1} \sim \tilde{f}(y_{n,k}), \ k \in K \) (see Section VI-D) to \( \{ (x_{n,k}^{(j)}, w_{n,k}^{(j)}) \}^J_{j=1} \sim f(y_{n,k}), \ k \in K \) (for processing at the next time step \( n+1 \)).

Fig. 3. Flowchart of a particle-based implementation of the proposed BP method. The symbol \( \sim \) expresses the fact that a set of particles and weights represents a certain distribution.

\( f(x_{n,k}, r_{n,k} = 1|z) \), it can be substituted for \( f(x_{n,k}, r_{n,k} = 1|z) \) in (25) and (26), thus providing the basis for Bayesian target detection and state estimation as discussed in Section [I-B]. A particle-based implementation of the above BP method that avoids the explicit evaluation of integrals and message products will be presented in Section [VI].

The “data association” iteration loop (32)–(35) involves solely messages related to discrete random variables. Being based on loopy BP, it does not perform an exact marginalization [28], [30], [31]. However, its high accuracy has been demonstrated numerically [32], [33] (see also Section [IX-B]), and its convergence has been proven [33], [42].

C. Scalability

The main advantage of the BP message passing method described in Section [V-B] is its scalability. Assuming a fixed number \( P \) of message passing iterations, the computational complexity of calculating the (approximate) marginal posterior pdfs of all the target states is only linear in the number of sensors \( S \) (see (37), (38)). Moreover, the complexity of the operations (31)–(36) is performed for a given sensor \( s \in S \) scales as \( O(KM_s^{(s)}) \), where \( M_s^{(s)} \) increases linearly with the number of PTs \( K \) and with the number of false alarms. Thus, the overall complexity of our algorithm scales linearly in the number of sensors and quadratically in the number of PTs. Note that in practical implementations, measurement gating [3] can be used to further improve scalability.

Such favorable scaling is a consequence of the “detailed” factorization [27]. This factorization, in turn, is due to the redundant formulation of the joint state-association estimation task in terms of both target-related and measurement-related association variables as described in Sections [II-A] and [II-C].

Using this factorization, an increase in the number of PTs, the number of sensors, or the number of measurements leads to additional variable nodes in the factor graph (see Fig. 2) but not to higher dimensions of the messages passed between the nodes. The scalability of the proposed algorithm will be further analyzed in Section [IX-C].

VI. PARTICLE-BASED IMPLEMENTATION

For general nonlinear and non-Gaussian measurement and state evolution models, the integrals in (2), (29), (30), and (31) as well as the message products in (37) and (38) typically cannot be evaluated in closed form and are computationally infeasible. Therefore, we next present an approximate particle-based implementation of these operations. In this implementation, each belief \( \tilde{f}(x_{n,k}, r_{n,k}) \) is represented by a set of particles and corresponding weights \( \{ (x_{n,k}^{(j)}, w_{n,k}^{(j)}) \}^J_{j=1} \). More specifically, \( \tilde{f}(x_{n,k}, 1) \) is represented by \( \{ (x_{n,k}^{(j)}, w_{n,k}^{(j)}) \}^J_{j=1} \), and \( \tilde{f}(x_{n,k}, 0) \) is given implicitly by the normalization property of \( \tilde{f}(x_{n,k}, r_{n,k}) \), i.e., \( \int f(x_{n,k}, 0) = 1 - \int f(x_{n,k}, 1)dx_{n,k} \). Contrary to conventional particle filtering [43], [44], the particle weights \( w_{n,k}^{(j)}, j \in \{1, \ldots, J\} \) do not sum to one; instead,

\[
p_{n,k}^{(s)} \triangleq \sum_{j=1}^{J} w_{n,k}^{(j)} \approx \int \tilde{f}(x_{n,k}, 1)dx_{n,k} \\
\]

Note that since \( \int \tilde{f}(x_{n,k}, 1)dx_{n,k} \) approximates the posterior probability of target existence \( p(r_{n,k} = 1|z) \), it follows that the sum of weights \( p_{n,k}^{(s)} \) is approximately equal to \( p(r_{n,k} = 1|z) \).

The particle operations discussed in the remainder of this section are performed for all PTs \( k \in K \) in parallel. The resulting particle-based implementation of the overall BP method is summarized in Fig. 3.
A. Prediction

For $n \geq 1$ and for each PT $k \in \mathcal{K}$, $J$ particles and weights
\[ \{(x_{n-1,k}^{(j)}, \tilde{w}_{n-1,k}^{(j)} = p_{n-1,k}^{(j)} / Z_{j}) \}_{j=1}^{J} \]
representing the previous belief $\hat{f}(x_{n-1,k}, r_{n-1,k})$ were calculated at the previous
time $n - 1$ as described further below. Weighted particles
\[ \{(x_{n,k}^{(j)}, \tilde{w}_{n,k}^{(j)}) \}_{j=1}^{J} \]
describing the message $\alpha(x_{n,k}, 1)$ in (29) are now obtained as follows.\footnote{Note that $\alpha_{n,k}$ in (30) is again given implicitly by these weighted particles since $\alpha(x_{n,k}, r_{n,k})$ is normalized.}

First, for each particle $x_{n,k}^{(j)}$, $j \in \{1, ..., J\}$, one particle $x_{n,k}^{(j)}$ is drawn
from $\hat{f}(x_{n,k}, x_{n-1,k})$. Next, $I$ additional “birth particles”
\[ \{x_{n,k}^{(j)}\}_{j=J+1}^{J+I} \]
are drawn from the birth pdf $f_{b}(x_{n,k})$. Finally, weights $w_{n,k}^{(j)}$, $j \in \{1, ..., J + I\}$ are obtained as
\[ w_{n,k}^{(j)} = \begin{cases} p_{n,k}^{(j)} \tilde{w}_{n-1,k}^{(j)}, & j \in \{1, ..., J\} \\ p_{n,k}^{(j)}(1 - p_{n-1,k}^{(j)}) / I, & j \in \{J + 1, ..., J + I\}. \end{cases} \] 

(B. Measurement Evaluation)

For each sensor $s \in \mathcal{S}$, an arbitrary proposal distribution can be found in (16). A more general expression for particle-based prediction with
an arbitrary proposal distribution can be found in (16).

C. Data Association, Measurement Update, Belief Calculation

The approximate messages $\hat{\beta}(a_{n,k}^{(s)})$ obtained in (42) are used in the data association loop, i.e., they are substituted for the messages $\beta(a_{n,k}^{(s)})$ in (33) and (34). After convergence of the data association loop, approximate messages $\hat{\eta}(v_{n,k}^{(s)})$ (approximating the messages $\eta(v_{n,k}^{(s)})$ in (35)) are then available for all PTs $k$ and all sensors $s$.

Next, the measurement update step (36) and the belief calculation step (37), (38) are implemented by means of

D. Target Detection, State Estimation, Resampling

The weighted particles $\{(x_{n,k}^{(j)}, u_{n,k}^{(j)})\}_{j=1}^{J+I}$ can now be used for target detection and estimation. For each PT $k$, an approximation $p_{n,k}^{(s)}$ of the existence probability $p(r_{n,k} = 1 | x_{n,k})$ is calculated from the particle weights $\{w_{n,k}^{(j)}\}_{j=1}^{J+I}$ as in (45). PT $k$ is then detected (i.e., considered to exist) if $p_{n,k}^{(s)}$ is above a threshold $p_{th}$ (cf. Section 11.B). For the detected targets $k$, an approximation of the MMSE state estimate $x_{n,k}^{\text{MMSE}}$ in (2) is calculated according to

[Equation]

Finally, as a preparation for the next time step $n + 1$, a resampling step (33), (34) is performed to reduce the number of particles to $J$ and to avoid degeneracy effects. The resampling
results in equally weighted particles \( \{ \tilde{x}_{n,k}^{(j)} \}_{j=1}^{J} \); the corresponding weights are given by 
\[
\tilde{w}_{n,k}^{(j)} = \tilde{w}_{n,k} = \frac{1}{J} \sum_{j=1}^{J} w_{n,k}^{(j)}, 
\]
\( j \in \{ 1, \ldots, J \} \).

VII. CHOICE OF BIRTH AND SURVIVAL PARAMETERS

We next present a scheme for choosing the birth pdfs \( f_b(x_{n,k}) \), birth probabilities \( p_{n,k}^b \), and survival probabilities \( p_{n,k}^s \) (see Section III-A). This scheme is heuristic but results in scalability with respect to the number of sensors \( S \) and, as demonstrated in Section IX-B, in good detection and tracking performance. It is based on the standard assumption that the number of newly born targets obeys a Poisson distribution with mean \( \mu_b \), and existing targets survive with a fixed, specified probability \( p_s^* \).

We first distinguish between “reliable” and “unreliable” PTs at time \( n-1 \) by comparing the PT existence probabilities \( p_{n-1,k}^e \) with a reliability threshold \( R_b \). PT \( k \) is considered reliable at time \( n-1 \) if \( p_{n-1,k}^e > R_b \) and unreliable otherwise. Let \( K_{n-1}^r \) and \( K_{n-1}^u \) denote the sets of indices \( k \) of reliable and unreliable PTs at time \( n-1 \), respectively. For PTs \( k \in K_{n-1}^r \), we set the birth and survival probabilities at time \( n \) to \( p_{n,k}^b = 0 \) and \( p_{n,k}^s = p_s^* \), respectively. Since \( p_{n,k}^b = 0 \), no birth pdf is needed at time \( n \) (cf. (7)).

For PTs \( k \in K_{n-1}^u \), we set \( p_{n,k}^b = 0 \) and \( p_{n,k}^s = \mu_b / |K_{n-1}^u| \), and we construct the birth pdf \( f_b(x_{n,k}) \) as follows. Consider an arbitrary sensor \( s_0 \), and let 
\[
Z_{n-1}^{(s_0)} = \{ Z_{n-1,m}^{(s_0)} \}_{m \in M_{n-1}^{(s_0)}}
\]
denote the set of measurements of that sensor at time \( n-1 \). (Note that \( Z_{n-1}^{(s_0)} \) corresponds to the measurement vector \( z_{n-1} = [z_{n-1,m}]_{m \in M_{n-1}^{(s_0)}} \), with the difference that the elements \( z_{n-1,m}^{(s_0)} \) are unordered in \( Z_{n-1}^{(s_0)} \). We partition \( Z_{n-1}^{(s_0)} \) into disjoint subsets 
\[
Z_{n-1,k}^b, k \in K_{n-1}^u
\]
that satisfy the cardinalities of the \( Z_{n-1,k}^b \) differ at most by 1, i.e., 
\[
|Z_{n-1,k}^b| - |Z_{n-1,l}^b| \leq 1 \text{ for any } k, l \in K_{n-1}^u.
\]
Then, based on the \( k \)th measurement set \( Z_{n-1,k}^b \), we construct a corresponding “adaptive birth pdf” as 
\[
f_b(x_{n,k}) \triangleq \int f_b(x_{n-1,k} | Z_{n-1,k}^b) d(x_{n-1,k}),
\]
where the pdf \( f_b(x_{n-1,k} | Z_{n-1,k}^b) \) is constructed using \( Z_{n-1,k}^b \) and prior knowledge (e.g., about the target velocity) as discussed in [45].

Particles representing \( f_b(x_{n,k}) \) are obtained by drawing particles from \( f_b(x_{n-1,k} | Z_{n-1,k}^b) \) and performing particle-based prediction [44].

VIII. RELATION TO EXISTING METHODS

Several aspects of the proposed method are related to existing methods, as discussed next.

- The hybrid model for data association using both target-oriented and measurement-oriented association variables was previously proposed in [22] and [33]. In [22], BP is used to estimate optical flow parameters. In [33], BP is used for data association (without multitarget tracking).
- Our model for target existence was previously used by the search-initialize-track filter in [11], which, however, is not BP-based, considers only a single sensor, does not employ the hybrid model for data association, and uses a different track initialization scheme.
- In the case of a single target and a single sensor, the proposed method reduces to the particle-based implementation of the Bernoulli filter [16] (which is derived using the FISST framework).
- The TOMB/B filter [20], which is effectively a FISST-based variant of the JPDA filter that uses BP and the hybrid data association model, differs from the proposed method in the following respects: it is restricted to a single sensor and to linear-Gaussian state evolution and measurement models—see [46] for an extension to nonlinear, non-Gaussian models—and the number of PTs (tracks) varies over time.

- If the parameters of the proposed method are chosen such that all targets exist at all times (this special case was mentioned in Section III-A) and was considered in our previous work in [35], then the method becomes similar to the Monte Carlo JPDA filter [8] in that it uses a similar particle-based processing scheme. However, contrary to the Monte Carlo JPDA filter, the proposed method performs data association by means of BP, is based on the hybrid model for data association, and can also be used when the number of targets is unknown.

IX. SIMULATION RESULTS

Next, we report simulation results assessing the performance of our method and comparing it with that of five previously proposed methods for multisensor-multitarget tracking.

A. Simulation Setting

We simulated up to five actual targets whose states consist of two-dimensional (2D) position and velocity, i.e., \( x_{n,k} = [x_{1,n,k}, x_{2,n,k}, \dot{x}_{1,n,k}, \dot{x}_{2,n,k}]^T \). The targets move in a region of interest (ROI) given by \([-3000, 3000] \times [-3000, 3000] \) according to the constant-velocity motion model, i.e., \( x_{n,k} = x_{n-1,k} + W_{n,k} \) where \( A \in R^{4 \times 4} \) and \( W \in R^{4 \times 2} \) are chosen as in [47] and \( u_{n,k} \sim N(0, \sigma_u^2 I_2) \) with \( \sigma_u^2 = 0.025 \) is an independent and identically distributed (iid) sequence of 2D Gaussian random vectors. The birth distribution \( f_b(x_{n,k}) \), the birth probabilities \( p_{n,k}^b \), and the survival probabilities \( p_{n,k}^s \) were chosen as described in Section VII using the global birth probability \( p_b = 0.01 \) and the global survival probability \( p_s = 0.999 \). The number of PTs was set to \( K = 8 \).

We considered a challenging scenario where the five target trajectories intersect at the ROI center. The target trajectories were generated by first assuming that the five targets start from initial positions uniformly placed on a circle of radius 1000 and move with an initial speed of 10 toward the ROI center, and then letting the targets start to exist at times \( n = 5, 10, 15, 20, \) and 25.

The sensors are located uniformly on a circle of radius 3000 and perform range and bearing measurements within a measurement range of 6000. More specifically, within the measurement range, the target-generated measurements are given by 
\[
z_{n,m}^{(s)} = \left[ \| \tilde{x}_{n,k} - p^{(s)} \| \right. 
\]
\[
\left. \varphi(\tilde{x}_{n,k}, p^{(s)}) \right] + v_{n,m}^{(s)},
\]
where \( x_{n,k} \triangleq [x_{1,n,k}, x_{2,n,k}]^T \), \( p^{(s)} \) is the position of sensor \( s \), \( \varphi(x_{n,k}, p^{(s)}) \) is the angle (in degrees) of the vector \( x_{n,k} \) relative to the vector \( p^{(s)} \), and \( v_{n,m}^{(s)} \sim \mathcal{N}(0, C_v) \) with \( C_v = \text{diag}(10^2, 0.5^2) \) is an iid sequence of 2D Gaussian random vectors. The false alarm pdf \( f_{FA}\) is linearly increasing on \([0, 6000]\) and zero outside \([0, 6000]\) with respect to the range component, and uniform on \([0^\circ, 360^\circ]\) with respect to the angle component. In Cartesian coordinates, this corresponds to a uniform distribution on the sensor’s measurement area. The mean number of false alarm measurements is \( \mu_{FA} = 2 \) if not noted otherwise.

Our implementation of the proposed method used \( J = 3000 \) particles and \( I = 3000 \) birth particles for each PT. We performed \( P = 20 \) BP iterations for iterative data association. The threshold for target detection was \( P_{th} = 0.5 \), and the reliability threshold was \( R_{th} = 10^{-3} \). We simulated 150 time steps \( n \).

### B. Performance Comparison

We compare the proposed BP method with particle implementations of the IC-PHD filter \([4, 13, 26]\), the IC-CPHD filter \([4, 15, 26]\), the IC-MB filter \([4, 17]\), and the partition-based MS-PHD and MS-CPHD filters \([23, 24]\). The “IC-” filters are straightforward multisensor extensions performing a single-sensor update step sequentially for each sensor \([4, 6, 26]\). The partition-based MS-(C)PHD filters approximate the exact multisensor (C)PHD filters; they can outperform the “IC-” filters but have a higher computational complexity. Since the trellis algorithm used for partition extraction in the original formulation of the MS-(C)PHD filter \([23, 24]\) is only suitable for a Gaussian mixture implementation of the filter, it was adapted to a particle-based implementation. We note that the exact multisensor (C)PHD filters are not computationally feasible for the simulated scenario since their complexity scales exponentially in the number of sensors and in the number of measurements per sensor \([22, 25, 48]\). The performance of the various methods is measured by the Euclidean distance based OSPA metric with a cutoff parameter of 200 \([49]\).

The (C)PHD-type filters use 24000 particles to represent the PHD of the target states, and they perform kmeans++ clustering \([50]\) for state estimation. The IC-MB filter uses 3000 particles to represent each Bernoulli component. The maximum numbers of subsets and partitions used by the MS-(C)PHD filter are 120 and 720, respectively, similarly to \([23, 24]\). With the above-mentioned parameters, the runtime per time scan for a MATLAB implementation without gating on a single core of an Intel Xeon E5-2640 v3 CPU was measured as 0.07s for the proposed method, 0.11s for the IC-PHD filter, 0.14s for the IC-CPHD filter, 0.27s for the IC-MB filter, 13.21s for the MS-PHD filter, and 13.82s for MS-CPHD filter. The high runtimes of the MS-(C)PHD filters are due to the fact that in our particle-based implementation, the trellis algorithm used for partition extraction is computationally intensive. We note that the efficient extraction of high-quality partitions in a particle-based implementation of the MS-(C)PHD filter is still an open problem.

Fig. 4 shows the mean OSPA (OSPA) error—averaged over 400 simulation runs—of all methods versus time \( n \), assuming \( S = 3 \) sensors with a detection probability of \( P_d = 0.8 \). The error exhibits peaks at times \( n = 5, 10, 15, 20, \) and 25 because of target births. However, very soon after a target birth, the proposed method as well as the IC-CPHD, MS-PHD, and MS-CPHD filters are able to reliably estimate the number of targets. The proposed method is seen to outperform all the other methods. In particular, it outperforms the IC-CPHD, MS-PHD, and MS-CPHD filters mainly because particle implementations of (C)PHD filters involve a potentially unreliable clustering step. This clustering step is especially unreliable for targets that are close to each other. This fact explains the higher OSPA error of the IC-CPHD, MS-PHD, and MS-CPHD filters around \( n = 100 \), i.e., around the time when the target trajectories intersect in the ROI center. Finally, the OSPA error of the IC-PHD and IC-MB filters is seen to be significantly larger than that of the other methods; this is caused by the inability of these filters to reliably estimate the number of targets. We note that for sensors with different probabilities of detection, the performance loss of IC-(C)PHD filters relative to MS-(C)PHD filters tends to be higher than in Fig. 4 \([23, 24]\).

Fig. 5 shows the time-averaged OSPA error versus \( P_d \) for \( S = 3 \) sensors. For all methods, as expected, the OSPA error decreases with decreasing \( P_d \). Fig. 6 shows the time-averaged OSPA error versus the number of sensors...
S for $P_d = 0.6$. It can be seen that the MOSPA error of the MS-PHD and MS-CPHD filters increases for $S$ larger than 5; this is because the chosen maximum numbers of subsets (120) and partitions (720) are too small for that case. (We note that choosing larger maximum numbers of subsets and partitions leads to excessive simulation times.) Finally, Fig. 7 shows the time-averaged MOSPA error versus the mean number of false alarms $\mu^{(s)}$. As expected, the MOSPA error of all methods increases with growing $\mu^{(s)}$. In addition, Figs. 5-7 again show that the proposed method outperforms the other methods. We note that the poor performance of the IC-CPHD filter is due to the approximation used by that filter, which is accurate only for a high $P_d$ and a very low $\mu^{(s)}$ \cite{17}.

### C. Scalability

Finally, we investigate how the runtime of our method scales in the number of sensors $S$ and the number of actual targets, and how it depends on the mean number of false alarms $\mu^{(s)}$. Fig. 8 shows the average runtime per time step $n$ versus $S$, $\mu^{(s)}$, and the number of actual targets for a MATLAB implementation on a single core of an Intel Xeon E5-2640 v3 CPU. The runtime was averaged over 150 time steps and 400 simulation runs. The probability of detection was set to $P_d = 0.6$. These results confirm the linear scaling of the runtime in $S$ and in the mean number of measurements per sensor (which grows linearly with $\mu^{(s)}$). The scaling in the number of actual targets is seen to be roughly quadratic. Further investigation showed that the scaling in the number of actual targets is linear if the number of PTs is held fixed, and similarly, the scaling in the number of PTs is linear if the number of actual targets is held fixed. The low absolute complexity of the proposed method is evidenced by the fact that for 20 actual targets, $S = 3$ sensors, and $\mu^{(s)} = 2$, the computations per time step $n$ require less than 0.4s.

### X. Conclusion

We developed and demonstrated the application of the belief propagation (BP) scheme to the problem of tracking an unknown number of targets using multiple sensors. The proposed BP-based multitarget tracking method exhibits low complexity and excellent scaling properties with respect to all relevant systems parameters. This is achieved through the use of “augmented target states” including binary target indicators and the establishment of an appropriate statistical model involving a redundant formulation of data association uncertainty \cite{33}. The complexity of our method scales only quadratically in the number of targets, linearly in the number of sensors, and linearly in the number of measurements per sensor. Simulation results in a challenging scenario with intersecting targets showed that the proposed method can outperform previously proposed methods, including methods with a less favorable scaling behavior. In particular, we observed significant improvements in OSPA performance relative to various multisensor extensions of the PHD, CPHD, and multi-Bernoulli filters.

Promising directions for future research include extensions of the proposed BP method that adapt to time-varying en-
vironmental conditions, e.g., to a time-varying probability of detection \cite{51}, and distributed variants for use in decentralized wirel

cessors with communication constraints \cite{52}. A direction of theoretical interest would be a FISST-based derivation of multisen

sor-mutltarget tracking algorithms using BP for data association.

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