Variable Chaplygin gas: constraints from supernovae, GRB and gravitational wave merger events

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Abstract
We investigate the cosmological constraints on the Variable Chaplygin gas model from the latest observational data: SCP Union 2.1 compilation dataset of Type Ia supernovae (SNe Ia), Pantheon sample of SNe Ia, Platinum Sample of Gamma Ray Bursts (GRB), and GWTC-3 of gravitational wave merger events. Variable Chaplygin gas is a model of interacting dark matter and dark energy, which interpolates from a dust-dominated era to a quintessence-dominated era. The Variable Chaplygin gas model is shown to be compatible with Type Ia Supernovae and gravitational merger data. We have obtained tighter constraints on cosmological parameters $B_s$ and $n$ using the Pantheon sample. Using the Markov chain Monte Carlo (MCMC) method on the Pantheon sample, we obtain $B_s = 0.108 \pm 0.034$, $n = 1.157 \pm 0.513$ and $H_0 = 70.020 \pm 0.407$. For GRBs, we get $B_s = 0.20 \pm 0.11$, $n = 1.45 \pm 1.40$ and $H_0 = 70.41 \pm 0.67$, and on GWTC-3, we obtain $B_s = 0.130 \pm 0.076$, $n = 0.897 \pm 1.182$ and $H_0 = 69.838 \pm 3.007$. The combined constraints from the above data sets are $B_s = 0.11 \pm 0.03$, $n = 1.14 \pm 0.36$ and $H_0 = 70.34 \pm 0.61$.

Keywords Gravitational wave merger events · Dark energy · Maximum likelihood estimation

1 Introduction

Most of the time, the observations have the power to push the well-established field to its most stringent tests. This is especially true in modern cosmology, and one such observation was of the supernovae type-Ia (SNe Ia) by two independent groups in 1998 (Perlmutter et al. 1999; Riess et al. 1998), which established that the expansion of the universe is accelerating, and two-thirds of total energy density is of dark energy component with negative pressure. This was followed by massive experimental efforts to find other independent observational means to verify the expansion of the universe theory. Through a series of independent findings such as the Baryon Acoustic Oscillations (BAO), Cosmic Microwave Background Radiation (CMBR) (Spergel et al. 2003; Miller et al. 1999), large-scale structure (LSS) (Bahcall et al. 1999), and Gamma-ray Bursts (GRBs), the accelerated expansion of the universe and the existence of dark energy gained traction in cosmology.

Regardless of the limitations of cosmological models based on the cosmological principle, numerous cosmological models have been extremely successful in explaining the observed universe. The simplest of such models, the Friedmann–Lemaître–Robertson–Walker (FLRW) model, assumes cosmological principle, Einstein’s general relativity, and the Universe is composed of Baryonic matter and radiation. Assuming the universe is homogeneous and isotropic, the FLRW metric is,

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

where $k$= curvature of the space that can take values $k = 0, \pm 1$, and $a(t)$ is the scale factor. Even though this model was extremely successful, there are many aspects of our observed universe that cannot be captured by such a sim-
ple model. For instance, it is not possible to explain the accelerated expansion if assumptions of the FLRW model, i.e., cosmological principle, Einstein’s general relativity, and only Baryonic matter and radiation are taken into consideration. Subsequently, other models were explored, which can be broadly classified into two classes. One class of models involves reforming the geometry part of the Einstein equations. This includes generalization of the gravitational action such as f(R)-gravity and higher dimension spacetime. The other class of models alters the matter component of the Universe in the Einstein equations. If it is assumed that the universe is composed only of Baryonic matter and radiation as it was assumed in the FLRW model, due to the gravitational pull of these components expansion rate of the universe would have slowed down; therefore, the need for an additional entity to explain the accelerated expansion of the Universe was realized. In this approach, exotic matter with equation of state such that they do not follow strong energy conditions is added to the mass distribution of the Universe. This conclusion is easily followed by the second Friedmann equation. One possible approach to constructing a viable model for dark energy is to associate it with a slowly evolving and spatially homogenous scalar field called “quintessence” (Ratra and Peebles 1988; Zlatev et al. 2002). Over a large class of potentials, the Quintessence model gives the energy density convergent to its present value for a wide range of initial conditions in the past and exhibits tracker behavior (Sahni and Starobinsky 2000; Padmanabhan 2003). Some of the models based on adding exotic matter are quintessence, k-essence, tachyons, barotropic fluid, etc. Quintessence, k-essence are scalar field models, whereas Dark Energy models include barotropic fluids whose pressure is a function of energy density, $P = f(\rho)$. The relationship between pressure and energy density determines the dynamics of the fluid.

Observational results of SN Ia, as well as the anisotropy of the CMBR power spectrum and clustering estimates, show that our universe is also composed of dark matter, in addition to dark energy, Baryonic matter, and radiation. Among the various models, the one that is most favorable is the standard model of cosmology or Λ-Cold Dark Matter (ΛCDM) model, where Λ represents the cosmological constant accounting for the vacuum energy or the energy density of space. However, this model faces some serious problematic issues such as the fine tuning (unconventional small value) (Peebles and Ratra 2003; Weinberg 1989) and cosmic coincidence problems (Why the dark matter and the dark energy are of the same order today although the universe is in a speedy expansion phase?). Similarly, the “Quintessence” models also suffer from fine-tuning problem.

The cosmological constant model has been transformed into a dynamical form in several ways to remove these problematic issues. We can describe an interaction between dark matter and dark energy to execute the dynamical form of the cosmological constant dark energy (Barrow and Clifton 2006; Gonzalez et al. 2006; Boehmer et al. 2008; Jamil and Rashid 2009). Hence, as an alternative to both the cosmological constant and quintessence, the accelerating expansion of the universe may be explained by introducing a cosmic fluid component with an exotic equation of state known as a Chaplygin gas (CG) (Kamenshchik et al. 2001), which is an example of barotropic fluid. Moreover, dark matter and dark energy have no direct laboratory evidence for their existence, forcing us to find a model in which these two dark components are different manifestations of a single cosmic fluid. The CG model unifies the CDM and the Λ models’ features into a single component with an exotic equation of state. This versatility of the models built on the Chaplygin gas becomes an attractive feature as these models can explain both dark energy and dark matter with a single component, which makes it unified dark matter/energy (UDME) model. The CG model has been further extended also to the generalized (Bilic et al. 2002; Saadat and Pourhassan 2014), modified (Debnath et al. 2004), Variable (Chimento and Jakubi 1996), and the extended (Kahya et al. 2015) forms. The CG model arises from the string Nambu–Goto action in the light-cone coordinate (Jackiw 2000; Pedram and Jalalzadeh 2008). For the generalized Chaplygin gas (GCG) model, the action can be written as a generalized Born–Infeld form (Bento et al. 2002). These models have been found to be consistent with the SNe Ia data (Gong and Duan 2004), CMB peak locations (Carturan and Finelli 2003), and other observational tests like gravitational lensing, cosmic age of old high redshift objects, etc. (Dev et al. 2004), as also with some combination of some of them (Bean and Doré 2004). This model has been shown to fit within the standard structure formation scenarios (Bilic et al. 2008; Bento et al. 2002; Fabris et al. 2002). The generalized Chaplygin gas model is shown to be consistent with the constraints provided by the 21-cm absorption line from EDGES detection along with CMB probe (Yang et al. 2019).

Therefore, the generalized Chaplygin gas model seems to be a good alternative to explain the accelerated expansion of the universe. As the original Chaplygin gas model produced oscillations or exponential blowup of matter power spectrum that are inconsistent with observations, various modified models have been considered. In this paper, we consider the variable Chaplygin gas model that was proposed (Guo and Zhang 2007) and constrained using SNeIa “gold” data (Sethi et al. 2006). There are also hybrid models, like viscous generalized Chaplygin gas (VGCG), that were proposed to tackle the late accelerated expansion of the Universe. This kind of hybrid models was originally proposed by Zhai et al. (2006) and is able to avoid causality problems that arise when only dissipative fluids are considered.

The parameters of the variable Chaplygin gas (VCG) model have been constrained using statistical analysis of

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SNeIa (Supernova Type Ia) Union 2.1 dataset from Supernova Cosmology Project, Pantheon+ Dataset from SH0ES collaboration and with the 03 run of GWTC. Formal introduction to the VCG model is provided briefly in Sect. 2. The description of the Pantheon and GWTC-3 datasets used for constraining the parameters are in Sect. 3. The statistical formalism to constrain parameters is given in Sect. 4. The description of GRBs os in Sect. 5. The analysis of the performance of the VCG model with the dataset and the constraints obtained on the parameters of the model is discussed in Sect. 6, followed by conclusions in Sect. 7.

2 Variable Chaplygin gas model

The equation of state for the Chaplygin gas is \( P = -\frac{A}{\rho} \), where \( A \) is a positive constant. A generalized Chaplygin gas model is characterised by an equation of state

\[ P_{ch} = -\left( \frac{A}{\rho_{ch}} \right)^{\alpha} \]  

(2)

where \( \alpha \) is a constant such that \( 0 < \alpha \leq 1 \). The Chaplygin gas model corresponds to \( \alpha = 1 \). Taking time component of the energy-momentum conservation equation \( T_{\mu}^{\alpha} = 0 \), we obtain the continuity equation,

\[ \frac{\partial \rho}{\partial t} + 3H(\rho + \rho) = 0 \]  

(3)

where \( H \) is the Hubble parameter, \( H = \frac{\dot{a}}{a} \), \( \rho = \text{total density of the Universe. Using equation (3), the energy density of the Chaplygin gas evolves as (Bento et al. 2002)} \n
\[ \rho_{ch} = \left( A + \frac{B}{a^{3(1+\alpha)}} \right)^{\frac{1}{1-\alpha}} \]  

(4)

where \( a \) is the scale factor in the current epoch of the universe, and \( B \) is the constant of integration. In the earlier epochs, i.e., \( a \ll 1 \), equation (4) translates to \( \rho \propto a^{-3} \); therefore, the Chaplygin gas behaves like Cold Dark Matter (CDM), and for late epochs, where the scale factor of the Universe is \( a \gg 1 \), equation (4) translates to \( \rho = -\rho = \text{constant} \); therefore, the behavior of the Chaplygin gas is like the cosmological constant at alter times thus leading to an accelerated expansion of the Universe. Thus, the Chaplygin gas equation of state leads to a component that behaves as non-relativistic matter at early times and as a cosmological constant equal to \( 8\pi GA^{1/(1+\alpha)} \) at a later stage.

The models based on the Chaplygin gas have not faltered the interest it gained among researchers as they have shown promise in the past. However, the Chaplygin gas model produces oscillations or an exponential blowup of matter power spectra inconsistent with observations (Sandvik et al. 2004). This instability may be avoided by considering the combined effect of shear and rotation, which slows down the collapse with respect to the simple spherical collapse model (Del Popolo et al. 2013). Subsequently, a modification of the Chaplygin gas as proposed is the VCG (Guo and Zhang 2007). The VCG is a modification of the Chaplygin gas model where the equation of the state of the Chaplygin gas is free-flowing across the epochs.

The VCG emerges from the dynamics of a generalized d-brane in a \( (d + 1, 1) \) space-time and can be described by a complex scalar field \( \phi \) whose action can be written as a generalized Born–Infeld action (Bento et al. 2002). Considering a Born–Infeld Lagrangian (Sen 2002)

\[ L_{BI} = V(\phi)\sqrt{1 + g_{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi} \]  

(5)

where \( V(\phi) \) is the scalar potential. In a spatially flat FLRW universe, the energy density and pressure are given by

\[ \rho = V(\phi) \left( 1 - \frac{\phi^2}{2} \right)^{-1/2} \]  

and \( P = -V(\phi) \left( 1 - \frac{\phi^2}{2} \right)^{1/2} \), respectively. Thus, the corresponding equation of state of the CG is given by

\[ P = -\frac{V^2(\phi)}{\rho} \]  

(6)

One can rewrite the self-interaction potential as a function of the cosmic scale factor: \( V^2(\phi) = A(a) \). Thus, the VCG is characterised by the equation of state:

\[ P_{ch} = -\frac{A(a)}{\rho_{ch}} \]  

(7)

where \( A(a) = A_0 a^{-n} \) is a positive function of the cosmological scale factor \( a \), \( A_0 \), and \( n \) are constants. Using the energy conservation equation, equation (3) in a flat Friedmann–Robertson–Walker universe and equation (7), the VCG density evolves as,

\[ \rho_{ch} = \sqrt{\frac{6}{6-n} \frac{A_0}{a^n} + \frac{B}{a^n}} \]  

(8)

where \( B \) is a constant of integration. Using Einstein field equations, \( G_{\alpha\beta} = 8\pi GT_{\alpha\beta} \), and FLRW metric, we obtain Friedmann equations

\[ H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} \]  

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \]  

(9)

where \( \rho \) is the baryonic matter density. If the dark energy component is taken into consideration, the first Friedmann equation gives the expansion rate of the Universe in terms of matter and radiation density, \( \rho \), curvature, \( k \), and the cosmological constant, \( \Lambda \), as

\[ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3} \]  

(10)
Assuming critical density \( \rho_c = \frac{3H_0^2}{8\pi G} \) and density parameter \( \Omega = \frac{\rho}{\rho_c} \), the first Friedmann equation reads as,

\[
\Omega_b(a) + \Omega_k(a) = 1
\]  
(11)

This relation extends directly to other models with several components. Here, taking baryonic matter and radiation into account, the Friedmann equation reads as,

\[
H^2 = H_0^2(\Omega_{m,0}a^{-3} + \Omega_{r,0}a^{-4} + \Omega_{k,0}a^{-2})
\]  
(12)

where \( \Omega_{m,0} + \Omega_{r,0} + \Omega_{k,0} = 1 \). Every other component can be added when its behavior with scale factor is known.

After reducing Eq. (10) to the case of the spatially flat Universe,

\[
H^2 = \frac{8\pi G}{3} \rho
\]  
(13)

where \( H \equiv \dot{a}/a \) is the Hubble parameter. Therefore, the acceleration condition \( \ddot{a} > 0 \) is equivalent to

\[
\left( \frac{12 - 3n}{6 - n} \right) a^{6-n} > \frac{B}{A_0}
\]  
(14)

To incorporate accelerated expansion of the Universe, the necessary condition is \( n < 4 \).

For \( n = 0 \), the original Chaplygin gas behavior is restored. The gas initially behaves as dust-like matter \((\rho_{ch} \propto a^{-3})\) and later as a cosmological constant \((\rho = -\rho = constant)\). However, in the present case of VCG, the universe tends to be a quintessence-dominated \((n > 0)\) (Hannestad and Mörtsell 2002; Guo et al. 2005) or phantom-dominated one \((n < 0)\) (Caldwell et al. 2003) with the constant equation of state parameter \( w = -1 + n/6 \). The first term on the right-hand side of Eq. (8) is initially negligible so that Eq. (8) can approximately be written as \( \rho \sim a^{-3} \), which corresponds to a universe dominated by dust-like matter.

The present value of energy density of the VCG

\[
\rho_{ch0} = \sqrt{\frac{6}{6-n}} A_0 + B
\]  
(15)

where \( a_0 = 1 \). Defining a parameter, \( B_s \),

\[
B_s = \frac{B}{6A_0/(6-n) + B}
\]  
(16)

the energy density becomes

\[
\rho_{ch}(a) = \rho_{ch0} \left[ \frac{B_s}{a^6} + \frac{1-B_s}{a^n} \right]^{1/2}
\]  
(17)

### 2.1 Model

The Friedmann equation, using equation (17) and \( a = \frac{1}{1+z} \) for the VCG, becomes

\[
H^2 = \frac{8\pi G}{3} \left\{ \rho_r(1+z)^4 + \rho_b(1+z)^3 + \rho_{ch0} \left[ B_s(1+z)^6 + (1-B_s)(1+z)^n \right]^{1/2} \right\}
\]  
(18)

where \( \rho_r \) and \( \rho_b \) are the present values of energy densities of radiation and baryons, respectively. Using\(^1\)

\[
\frac{\rho_r}{\rho_{ch0}} = \frac{\rho_r}{\rho_{ch0}} = \frac{\rho_{r0}}{1 - \Omega_{r0} - \Omega_{b0}}
\]  
(19)

and

\[
\frac{\rho_b}{\rho_{ch0}} = \frac{\rho_{b0}}{1 - \Omega_{r0} - \Omega_{b0}}
\]  
(20)

Equation (18) becomes,

\[
H^2 = \Omega_{b0} H_0^2 a^{-4} X^2(a),
\]  
(21)

where

\[
X^2(a) = \frac{\Omega_{r0}}{1 - \Omega_{r0} - \Omega_{b0}} + \frac{\Omega_{b0} a}{1 - \Omega_{r0} - \Omega_{b0}} + a^4 \left( \frac{B_s}{a^6} + 1 - B_s \right)^{1/2}
\]  
(22)

### 3 Datasets

SNe Ia are crucial to understanding the expansion of the universe. We examine the parameters of the Variable Chaplygin gas model using a statistical analysis of the most recent SNe Ia data from the Pantheon Sample (Scolnic et al. 2018), the Supernova Cosmology Project (SCP) Union 2.1 compilation (Suzuki et al. 2012) and GWTC-3 gravitational waves dataset. Pantheon sample is composed of 1048 SNe Ia data whose redshift span from \( z = 0.01 \) up to \( z = 2.26 \). SCP Union 2.1 compilation is composed of 580 SNe Ia data whose redshift span from \( z = 0.623 \) up to \( z = 1.415 \) providing redshift, distance moduli and associated errors in distance moduli. The gravitational merger events are obtained from the GWOSC (Gravitational Wave Open Science Center), which has the events obtained from detectors at LIGO Hanford, LIGO Livingston and LIGO Virgo. The events are collected across the three runs: O1 (from 12 September 2015 to 19 January 2016), O2 (from 30 November 2016 to 25 August 2017), and the O3 runs, O3a (from 1 April 2019 to

\(^1\)We have used the fact that for a flat Universe, \( \Omega_{b0} + \Omega_{r0} + \Omega_{ch0} = 1 \), i.e., the total matter density sums up to unity.
30 September 2019) and O3b (from 1 November 2019 to March 2020). The data set consists of 90 confirmed events in the GWTC-3 (The LIGO Scientific Collaboration, the Virgo Collaboration, the KAGRA Collaboration 2021a,b). The catalog contains events whose sources are black hole binary mergers up to a redshift of 0.90. The merger events from the GWOSC dataset (from O1 run to the recent O3 run) are used to constrain the parameters of the Variable Chaplygin gas model. The redshift data from these events were taken to predict luminosity distance and distance modulus using the VCG model and compared with the luminosity distance obtained from the merger events using Bayesian inference.

4 Statistical methodologies

In this work, we analyze luminosity distance cosmological observable. Luminosity distance shows an explicit dependence on the cosmological model under consideration. Hence with this observable, we can compare the model with the experimentally observed value of such an observable. We have used the SNe-Ia and gravitational waves to constrain the parameters of the VCG model. Using the Friedmann equation, in a flat Universe, the luminosity distance is expressed as

\[ d_L(z, p) = c(1+z) \int_0^z \frac{dz'}{H(z', p)} \]  

where, \( \{p\} \) denotes the set of all parameters describing the cosmological model, and \( H(z, p) \) is the Hubble parameter as defined in the model chosen. In our case, we have considered the contributions from radiation and baryons in addition to the Chaplygin gas. Luminosity distance can also be expressed using (21) and \( a = \frac{1}{1+z} \) as

\[ d_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{\Omega_{\text{ch}0}^{1/2} X(z)} \]  

The theoretical predicted value of distance modulus is obtained by

\[ \mu_p(i) = 5 \log \left[ \frac{d_L(z_l)}{\text{Mpc}} \right] + 25 \]  

We can determine a best fit to the set of parameters using a \( \chi^2 \) goodness fit test

\[ \chi^2(p) = \sum_i \left[ \frac{\mu_p(i) (z_l) | p) - \mu_{0}(i) (z_l) }{\sigma_i^2} \right]^2 \]  

where \( \mu_p(i) (z_l) | p) \) is the theoretically predicted value of distance modulus at redshift \( z_l \), given the set of parameters \( p \), and the sum is over all the observed data. Here \( \sigma_i \) is the dispersion of the distance modulus due to intrinsic and observational uncertainties. In the case of SNe Ia, the errors \( \sigma_i \) are Gaussian and hence probability density function for the parameters \( (B_s, n) \) is

\[ p(s) \propto \exp \left( -\frac{\chi^2}{2} \right) \]  

with the parameter limits \( 0 \leq B_s \leq 1 \) and \( n \leq 4 \). The probability density function of a parameter \( p_i \) is obtained by integrating over all possible values of other parameters. To reduce the computation time, we can analytically integrate over the Hubble constant, and hence with modified \( \chi^2 \) statistics the fit is obtained by minimizing

\[ \chi^2 = \sum_i \left[ \frac{\mu_{0}(i) (z_l) - \mu_{\text{th}}(i) (z_l) }{\sigma_i} \right]^2 \]  

\[ C_1 \equiv \sum_i \frac{\mu_{0}(i) (z_l) - \mu_{\text{th}}(i) (z_l) }{\sigma_i^2} \]  

where \( \sigma_i \) is the error associated with observed distance modulus \( \mu_{\text{obs}} \). The modified distance modulus in the above relation is

\[ \mu_{\text{th}} = 5 \log \left( \frac{H_0 d_L}{ch} \right) + 42.38 \]  

\( H_0 \) is the value of the Hubble parameter at present, \( c \) is the speed of light. The probability distribution function of the estimated parameters (excluding \( h \)) is now obtained using the modified \( \chi^2 \). It is straightforward to check that the derivative of equation (28) with respect to \( h \) is zero; hence our results are independent of the choice of \( h \). We take \( h = 0.70 \). By minimizing the modified \( \chi^2 \) statistics, we have found the best fit for \( B_s \) and \( n \). We have obtained \( \chi^2 \) contour plot with confidence intervals for the Pantheon sample, see Fig. 1. As we are fitting for two parameters, we use \( \chi^2_{0.68} = \chi^2_{\text{min}} + 2.30, \chi^2_{0.95} = \chi^2_{\text{min}} + 4.61, \chi^2_{0.99} = \chi^2_{\text{min}} + 9.21 \) for the value corresponding to 1 \( \sigma \), 2 \( \sigma \), 3 \( \sigma \), respectively. We have also obtained \( \chi^2 \) contour plots with confidence intervals for the Pantheon sample, SCP Union 2.1 compilation and GWTC-3 together, see Fig. 2. We are able to constrain the parameters of the model more tightly with the Pantheon sample than SCP Union 2.1 compilation. Therefore, for further analysis in this paper, we will use the Pantheon sample
only. For the gravitational wave dataset, the errors in the observables are non-Gaussian, but we applied the above analysis for the gravitational wave dataset taking the average of the upper and lower error bars. As the errors on the gravitational waves observables are too large, we are not able to constrain the parameters of the model with it. Though, we are able to show that parameter values using $\chi^2$ by the Pantheon sample and GWTC-3 are consistent.

The theoretical luminosity distance and distance modulus from the equation (24) and equation (31) are compared to the observed values of distance luminosity and distance modulus from the Pantheon, SCP Union 2.1 and GWTC-3 dataset, respectively, see Fig. 3. We have shown that the model fits well and the viability of the model with all three samples. This shows Variable Chaplygin gas model (interacting dark energy and dark matter model) can explain the observed expansion of the universe.

Equation (24), can also be written as

$$d_L = \frac{X_1}{H_0}$$  \hspace{1cm} (32)

where

$$X_1 = c(1 + z) \int_0^z \frac{dz}{\Omega_\chi^{1/2}} X(z)$$ \hspace{1cm} (33)

From the above equation, we inferred the value of the Hubble constant for all three observational samples, see Fig. 4. For the case of the GWTC-3 dataset, errorbars are associated with both the variables, redshift and luminosity distance; hence we use the orthogonal distance regression method for linear regression by taking the average of the upper and lower error bar. The best fit is obtained between the observed and the theoretical value by minimizing

$$\sum_{i=1}^{n} \left[ \frac{(y_i - \alpha - \beta X_i)}{\eta} + (x_i - X_i) \right]$$ \hspace{1cm} (34)

where $y_i$ is the observed value of distance modulus or the luminosity distance. $\alpha - \beta X_i$ is the value of distance modulus or luminosity distance obtained by the VCG model with applied constraints. $X_i$ is the absolute value of the other variable in the study, the redshift associated with the merger event and $x_i$ is the redshift value, including the error range. $\eta$ is the ratio between the variance of the errorbars associated with the variables under consideration. By accounting for the variance in the errorbars associated with the response variable, luminosity distance or the distance modulus of the event and the predictor variable, the redshift of the merger event, the regression method is more sensitive towards the precision of both the predictor and the variables and minimizes the discrepancies brought in by the outliers in the dataset.

However, the Orthogonal Regression can only be used when the errors are normally distributed. In the case of the GWOSC dataset, the observed luminosity distance and, consequently, the distance modulus of the gravitational merger event is accompanied by non-Gaussian errors; therefore, the orthogonal regression can not provide accurate constraints on parameter $B_s$ and $n$. The non-Gaussian nature of the errorbars associated with the merger event makes Maximum Likelihood Estimation an ideal method to constrain the parameters under study.
Fig. 3 Viability of the variable Chaplygin gas model with Pantheon, SCP Union 2.1, and GWTC-3 datasets. Variation of theoretical prediction of distance modulus (DM) (left) and luminosity distance (DL or LD) (right) from variable Chaplygin gas model with redshift where the parameters \((B_s, n)\) are determined by the \(\chi^2\) goodness test. This shows that model fits well with the Pantheon sample and GWTC-3 dataset.

4.1 Maximum likelihood estimation (MLE)

The equation (28) used for determining the \(\chi^2\) goodness fit value depends on the \(H_0\), which is fixed. This results in substituting the Hubble parameter value from previous results into the equation. Therefore, \(\chi^2\) goodness value will be affected by any potential errors associated with the Hubble
parameter value from the previous results. This can not be avoided in the modified \( \chi^2 \) statistics analysis as explained in the previous section.

Maximum Likelihood Estimation (MLE) method can be utilized to infer the parameters \( B_s, n \) and \( H_0 \) together, unlike modified \( \chi^2 \) statistics equation (28), where we obtained parameters \( B_s \) and \( n \), and was independent of \( H_0 \). This is extremely useful as even the \( h_0 \) parameter can also be determined using the MLE along with \( B_s \) and \( n \), and therefore the result becomes self-contained. The assumption that has to be taken is that the dataset are obtained from the same distribution family of the given parameters of the dataset, and each individual data point on the dataset is independent of other data points to hold its current value. This is a classic i.i.d. (Independent and Identically Distribution) assumption. The MLE determines the combination of parameters by maximizing:

\[
\left( B_s, n, h_0 \right)_{\text{MLE}} = \arg \max_{B_s, n, h_0} \prod f(a_i \mid (B_s, n, h_0))
\]

where \( \prod f(a_i \mid (B_s, n, h_0))_{\text{MLE}} \) is a summation of probability function of every individual data point in the observed dataset. In general, the probability function for the total dataset under a given combination of parameters, \( f(x_1, x_2, \ldots, x_n \mid (B_s, n, h_0)) \) is required in equation (35). Since, the dataset follows i.i.d. distribution, the probability of combined data is equivalent to the summation of probability of each data point in the given dataset.

4.2 Maximum likelihood estimation using cobaya

Cobaya (COde for BAYesian Analysis) (Torrado and Lewis 2021, 2019) is a coding framework built for statistical modeling to find arbitrary posteriors from the given sets of parameters, especially for Cosmology. Cobaya determines Maximum Likelihood Estimation for the dataset from the given parameters using high advanced Monte Carlo samplers like MCMC from CosmoMC, nested samplers from PolyChord or a wide range of samplers incorporated in cobaya. Cobaya removes the dependency on the Hubble parameter value obtained from previous results and also significantly reduces the computing time to find the combination of the parameters \( B_s, n \) and \( H_0 \) for which the probability for the given data distribution under the MLE is maximum. Contour plot of the two-dimensional posterior distribution of cosmological parameters obtained using the Pantheon sample is shown in Fig. 5. The \( \chi^2 \) goodness fit algorithm takes a much longer duration to find the combination of the parameters for which the \( \chi^2 \) goodness fit value is minimum although the parameters that are kept arbitrary are \( B_s \) and \( n \).

From Figs. 6 and 7, and Table 1, we can conclude that cosmological parameters values from Gaussian and non-Gaussian likelihoods fall within 1 sigma; hence, we would be using Gaussian likelihoods in further analysis for GWTC-3 dataset. We used Gaussian priors to calculate cosmological results for the Pantheon sample and GWTC-3 dataset as shown in Figs. 6 and 8. For Gaussian priors, we considered the mean value as the expectation value from the \( \chi^2 \) contour plots, see Fig. 1 and 2, and we doubled the standard deviation value \( \sigma \) value which is then considered the new
standard deviation for the MCMC. To find these best-fitting parameters, we use the D’Agostini (2005) Bayesian method, which takes into account the error bars on all the axes. We are not considering SCP Union 2.1 compilation here as the

Pantheon sample has tighter constraints, as discussed before using \( \chi^2 \) statistics. We have used a modified likelihood function for the case of asymmetric errors in the GWTC-3 dataset. We can again show that the cosmological parameter values using the Pantheon sample and GWTC-3 dataset are consistent, and we are able to constrain the parameters using both datasets independently.

5 Gamma-ray bursts 3D fundamental plane correlation as a cosmological tool

Gamma-ray bursts (GRBs) are incredibly powerful phenomena. They are the brightest objects after the Big Bang and some of the farthest astrophysical objects ever detected (Paczyński 1986; Planck Collaboration 2018; Kumar and Zhang 2015). These features make GRBs promising cosmological tools, similar to supernovae type Ia (SNe Ia). GRBs are extremely luminous, which allows them to be observed at very large distances, corresponding to high redshifts. In-
deed, GRBs have been observed up to redshifts of 8.2 and 9.4 (Tanvir et al. 2009; Cucchiara et al. 2011), while SNe Ia have only been observed up to redshift 2.26 (Rodney et al. 2015). However, using GRBs as cosmological tools requires a full understanding of their physical mechanisms. Both their energy emission mechanisms and progenitors are still being studied by the scientific community.

There are two main scenarios for the birth of GRBs. The first scenario is the explosion of a very massive star at the end of its lifetime (Narayan et al. 1992; Woosley et al. 1993; MacFadyen and Woosley 1999; Nagataki et al. 2007; Nagataki 2009). This is followed by a core-collapse SNe (Stanek et al. 2003; MacFadyen et al. 2001). The second scenario is the coalescence of two compact objects, like black holes (BHs) or neutron stars (NSs) (Lattimer and Schramm 1976; Eichler et al. 1989; Li and Paczyński 1998; Rowlinson et al. 2014; Rea et al. 2015). The most probable frameworks for the central engine that powers the GRB consider the following astrophysical objects: BHs, NSs, or fast spinning newly born highly magnetized NSs magnetars (Usov 1992; Liang 2018; Ai 2018). GRBs can be divided into two main categories: short and long. Short GRBs are associated with the merging of two compact objects, while long GRBs are associated with the core collapse of a very massive star. GRB light curves (LCs) can be divided into two main phases: prompt emission and afterglow. The prompt emission is a rapid burst of gamma rays, while the afterglow is a longer-lasting emission that can be detected in X-ray, optical, and radio wavelengths. The plateau phase of GRBs is a flat part of the LC that follows the prompt emission. It was discovered by the Neil Gehrels Swift Observatory (Swift) and typically ranged from $10^2$ to $10^5$ seconds in duration. The plateau phase is thought to be caused by the spin-down of a newly born magnetar or the external shock model.

In the past decades, many efforts have been made to find possible correlations between the physical features of GRBs. One of these correlations is the so-called Dainotti relation, which links the time at the end of the plateau emission measured in the rest frame, $T_{X}^*$, with the corresponding X-ray luminosity of the LC, $L_X$ (Dainotti 2008). This correlation is theoretically supported by the magnetar model. Its extension in three dimensions has been discovered by adding the prompt peak luminosity, $L_{peak}$, and is known as the fundamental plane correlation or the 3D Dainotti relation (Dainotti 2016, 2017). It has also shown that GRBs can be used as cosmological probes. The fundamental plane relation has the following form:

$$\log L_X = c + a \cdot \log T_{X}^* + b \cdot (\log L_{peak}),$$

where $a$ and $b$ are the best-fit parameters given by the D’Agostini procedure (D’Agostini 2005) linked to $T_{X}^*$ and $L_{peak}$, respectively, while $c$ is the normalization. In this study, we follow the same approach as in (Dainotti et al. 2023) and use the Platinum Sample of GRBs as a cosmological probe. The Platinum Sample of GRBs is a compilation of the highest-quality GRBs with well-measured redshifts. The GRBs in this sample are well-suited for cosmological studies because they are bright and can be seen to high redshifts. Fig. 9 shows a corner plot of cosmological parameters obtained from the GRB Platinum sample. We further combine the GRB Platinum sample with the Supernovae Ia Pantheon sample and GWTC-3 Catalog using a Bayesian framework and compute the cosmological parameters. This facilitates the examination of whether the inclusion of GRBs in the analysis would corroborate the findings observed when studying individual samples independently, as well as the extent to which it could augment the accuracy of the cosmological parameters. Fig. 10 shows a corner plot of cosmological parameters obtained from the combined analysis of the GRB Platinum sample, Pantheon Sample and GWTC-3 Catalog.

### 6 Analysis and result

We used $\chi^2$ minimization to obtain the parameters $B_s$ and $n$ of the VCG model using the Pantheon sample, SCP Union 2.1 compilation and through MLE for GWTC-3 dataset. Subsequently, we used the obtained parameters to infer the Hubble constant. Values of these parameters can be found in Table 2. For comparison and completeness, we also tested our $\chi^2$ program with the Pantheon sample and obtained for
the Standard Model of Cosmology, $\Omega_m = 0.298 \pm 0.0087$ and $H_0 = 71.985 \pm 0.663$.

From Fig. 3, we can conclude that the VCG model is a viable model capable of explaining the observed expansion of the universe using the Pantheon sample, SCP Union 2.1 compilation, and GWTC-3 dataset. As the GWTC-3 dataset has asymmetrical errorbars, we cannot use the $\chi^2$ minimization method to obtain the VCG model parameters for GWTC-3 dataset. Hence, we used cobaya for MCMC computation using the modified likelihood function and the Gaussian priors. We used results obtained from $\chi^2$ in Table 2 using the Pantheon sample as the priors, and we let all the parameters vary simultaneously. We performed the analysis using SNe Pantheon sample, GWTC-3 catalog, GRBs Platinum sample, and a combination of all three. We find the best-fit results of the constants in the fundamental plane relation are: $a = -0.88 \pm 0.06$, $b = 0.49 \pm 0.01$, and $\sigma_{int} = 1.06 \pm 0.06$, where $\sigma_{int}$ is the intrinsic scatter of the correlation. The results are shown in Table 4. To complete our analysis in Table 3, we also present a comparison of our results with the VGCG and the GCG models.

7 Conclusions

The variable Chaplygin gas model is indeed able to explain the evolution of the universe in account for the gravita-
Fig. 10 Cosmological parameters contours for the combined samples (SNIe Pantheon+ GRBs Platinum + GWTC-3) using the equation of distance modulus and Gaussian priors. Results are shown in Table 4.

Table 3 Results of constraining the cosmological parameters with Pantheon dataset in other dissipative fluids models of cosmology like the VGCG and the GCG (Hernández-Almada et al. 2021)

| Dataset | $\chi^2_{min}/dof$ | $h$ | $\Omega_0 h^2$ | $B_s$ | $\alpha$ | $\xi_0$ |
|---------|-------------------|-----|----------------|-------|----------|--------|
| VGCG    |                   |     |                |       |          |        |
| SnIa    | 1036.4/1043       | 0.70$^{+0.18}_{-0.20}$ | 0.02242$^{+0.00014}_{-0.00014}$ | 0.50$^{+0.18}_{-0.21}$ | 0.93$^{+0.69}_{-0.61}$ | 0.15$^{+0.00}_{-0.00}$ |
| GCG     |                   |     |                |       |          |        |
| SnIa    | 1036.3/1044       | 0.71$^{+0.20}_{-0.18}$ | 0.02242$^{+0.00014}_{-0.00014}$ | 0.96$^{+0.05}_{-0.04}$ | 1.13$^{+0.44}_{-0.28}$ | –      |

Table 4 Constraints imposed on the parameters ($B_s$, $n$ & $H_0$) of the VCG model from Supernovae events, GRB events, GW merger events, and combination of all three datasets using the distance modulus equation, Gaussian priors, and the Gaussian likelihood

| Dataset   | $B_s$       | $n$         | $H_0$      |
|-----------|-------------|-------------|------------|
| Pantheon  | 0.11 ± 0.03 | 1.16 ± 0.51 | 70.02 ± 0.41 |
| GWTC-3    | 0.13 ± 0.08 | 0.90 ± 1.18 | 69.84 ± 3.01 |
| GRBs      | 0.20 ± 0.11 | 1.45 ± 1.40 | 70.41 ± 0.67 |
| Combined  | 0.11 ± 0.03 | 1.14 ± 0.36 | 70.34 ± 0.61 |

As the VCG model speculates, the Chaplygin gas is expected to behave like a non-relativistic entity and later evolve to account for the accelerated expansion observed in the current epoch of the Universe. The best fit obtained from the VCG model with the GWTC-3 Dataset lies within confidence levels obtained in analysis conducted by Sethi et al. (2006) and Guo and Zhang (2007) and also is compatible with the constraints obtained from GRBs. The GWTC-3 dataset has also brought in further constraints on the parameters $B_s$ and $n$. The constraints obtained using GWTC-3 are expected to improve as the dataset becomes bigger with more detections. These results are indeed limited to the fact that locating sources of Grav-
itional waves is still subject to improvement in the detection techniques.

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Declarations

Competing interests The authors declare no competing interests.

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