Multi-physics computation and deformation testing of a shell-type 1.5-GHz cavity

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Abstract. A copper prototype of a 1.5-GHz cavity was manufactured to simulate a superconducting radio-frequency cavity for technique development. Frequency tuning with longitudinal compression of this prototype and cryogenic cooling with liquid nitrogen were performed to examine the numerical results from finite-element models, mainly the corresponding shifts of the fundamental resonant frequency. An appropriate element option improved the accuracy of the resonant frequency and the distribution of the magnetic field. Effects of geometry distortion of an uneven length on the frequency shift of this shell-type cavity as loaded on longitudinal compression are also examined and discussed.

1. Introduction

Several superconducting radio-frequency (SRF) cavities have been developed to serve in particle accelerators and synchrotron light sources. Two 500-MHz SRF modules of KEK type are operated as the main accelerating cavities [1, 2] of the Taiwan Photon Source (TPS) in National Synchrotron Radiation Research Center (NSRRC). The installation of harmonic cavities in the near future is under consideration, mainly to increase the lifetime of the electron beam. Successful application of third-harmonic SRF cavity has been demonstrated at the Swiss Light Source (SLS) and Italian Synchrotron Light Laboratory (ELETTRA) [3].

Technical transfer from KEK [4] made available to NSRRC related engineering drawings of the KEK-type 500-MHz SRF cavity. This cavity is thus scaled down to approach its fundamental mode for target resonant frequency 1.499 GHz as the third-harmonic cavity of TPS [5]. Figure 1 shows the proposed geometry; the main dimensions are total length 320.3 mm, equatorial radius 87.6 mm, and...
iris radius 36.7 mm. The software ANSYS [6] was used in a previous work [5] to simulate not only the shift of its resonant frequency but also the required force with varied longitudinal compression. The entire main cavity surface was found to be subject to a large magnetic field, with small spikes appeared on the sloping sections and exhibited peak values that varied with meshes. This issue is first studied herein to show that this uncertainty can be solved with appropriate computational options and meshes.

Moreover, a copper prototype of this cavity was manufactured for testing and technique development. Longitudinally compression and cooling with liquid nitrogen were then applied to this prototype to investigate the frequency shift of its fundamental mode and to examine the accuracy of calculation of the numerical models.

2. FEM computation

2.1. Convergence test
To compute the EM field of the cavity with ANSYS, we select the 20-node hexahedral element HF120 because of the curved boundaries shown in figure 1. This element has an option for its degrees of freedom (DOF): 12 and 54 DOF for the 1\textsuperscript{st}-order and 2\textsuperscript{nd}-order element option, respectively. The 1st-order element option was adopted in previous work [5], in which the calculated magnetic field on the cavity surface had small spikes on the sloping sections near irises of the cavity. This inaccuracy might also affect the calculated resonant frequency. The 2\textsuperscript{nd}-order element option is thus here used to calculate the EM field of the same cavity as an examination. The first step is to test the convergence of its resonant frequency $f_0$ of the TM$_{010}$ mode.

Figure 2 shows that, with the 1st-order element the calculated resonant frequency $f_0$ varies from 1.4996 GHz to 1.4974 GHz as the total number N of elements increases from 45,111 to 318,045, whereas with the 2nd-order element it converges from 1.49682 GHz to 1.49665 GHz as N increases from 936 to 79,761. The resonant frequency $f_0$ calculated with the 2\textsuperscript{nd}-order element option is smaller and converges much more rapidly. The model with the 2\textsuperscript{nd}-order element option has the same resonant frequency of the TM$_{010}$ mode, $f_0=1.49665$ GHz, as the total element number N increases from 61,941 to 79,761; for further work this model with 61,941 elements thus seems satisfactory.

2.2. Distribution of electromagnetic field
The calculated electric field, $E_{Z}$, of the TM$_{010}$ mode along the central axis behaves as a region with large strength near the center of the cavity but vanishes rapidly at both beam pipe regions; only the field in the central region is thus presented in figure 3, while its maximum field strength at the cavity

![Figure 2. Calculated resonant frequency of the TM$_{010}$ mode converges much more rapidly for the model with the 2\textsuperscript{nd}-order element option.](image)
center, \( EF_{Zpk} \), serves to normalize the distribution. The computed strength of the electric field from the model of the 1st-order element at a fine mesh with \( N = 136,896 \) approaches the result from the model of the 2nd-order element with \( N = 61,941 \); this indicates that a model with the 1st-order element might yield an accurate electric field with a sufficiently fine mesh.

**Figure 3.** Distribution of normalized accelerating electric field \( EF_Z \) of the TM\(_{010}\) mode along the cavity axis.

The last item to examine is the magnetic field on the cavity surface, \( H_S \), of the TM\(_{010}\) mode. Figure 4 shows that \( H_S \) computed from the model of the 2nd-order element has a continuous and smooth distribution on the central part of the cavity, whereas the results of the 1st-order element show spikes at the side straight sections. The maximum strength of the magnetic field is used to normalize the corresponding distribution in figure 4. The 2nd-order element option evidently eliminates the computational inaccuracy of the magnetic field as reported in previous work [5, 7]. For comparison, a finer mesh with the 1st-order element option improves the accuracy of \( H_S \) only slightly. Obviously the 2nd-order element option should be adopted to compute the resonant frequency and magnetic field of an RF cavity.

**Figure 4.** Distributions of normalized magnetic field on the cavity surface \( H_S \) of the TM\(_{010}\) mode.

### 2.3. Multi-physics computation

For an SRF cavity, the tuning mechanism alters the length of the cavity to correct the resonance frequency. To predict this correlation in the design phase, a multi-physics computation is an essential numerical tool. The computation proposed in [5] is used here to calculate the shift of the resonant frequency of the cavity prototype under a longitudinal displacement, as well as for cooling from 295 K
to 77 K, for comparison with the measured results to verify the numerical accuracy. Briefly, the cavity wall is meshed with shell element SHELL93 and assigned to have appropriate mechanical properties, whereas with solid structural element SOLID95 the interior space of the cavity is assigned to have a small Young’s modulus, $1 \times 10^{-6}$ of the cavity material; all solid structural elements are consequently deformed as external loadings become applied to the shell elements so that the shape and node locations of the interior space thus become adjusted promptly [5]. The deformed solid structural elements are then transferred to compatible elements HF120 for the subsequent electromagnetic computations, with the 2nd-order element option. The cavity wall is simulated with shell elements to take into account the varying cavity thickness on simply altering the assigned thickness of the shell elements.

3. Experiment
A copper prototype of this cavity was manufactured for testing and technique development. Both the side cone sections linking the curved sections from the equator to the iris; because the diameter of this cone alters so much to become thinner during formation, they were then machined to 2 mm, as figure 1 shows, to attain a uniform thickness. Some strip specimens were cut from one original copper pipe for tensile tests, from which the average Young’s modulus 116.37 GPa and Poisson’s ratio 0.329 were obtained [8] and applied to the numerical models. Sections of this prototype were all welded manually by a skilled technician. Through the use of bimetallic rings, the stainless-steel flanges of 100 CF were directly welded on both ends of the SBP and LBP, but the thermal effect during the welding tilted the cavity; a length difference $\pm 1.5$ mm along the circumference of the cavity was observed. The effects of this tilt are discussed in this work.

Figure 5 illustrates the experiment on measuring the resonant frequency of the cavity prototype as a longitudinal displacement was applied with 12 long screws. A network analyzer was used to measure the resonant frequency through a RF-pickup with a vacuum barrier; a pumping system evacuated the cavity interior to eliminate the effect of the relative permittivity of the gas. A cylinder with 200 CF flanges, same as the big bottom flange was then used to surround the cavity to act as a container to fill with liquid nitrogen to cool the cavity to 77 K. The network analyzer was adjusted to have measurement bandwidth 3.2 MHz and 1600 measurement points; the resolution is thus 2 kHz, $1.33 \times 10^{-6}$ of the resonance frequency of the cavity.

![Figure 5. Experiment setup to measure the resonant frequency of a cavity prototype.](image-url)
4. Results and discussion

4.1. Longitudinal compression

The resonant frequency of the SRF cavity decreases on being axially compressed. Figure 6 shows the test and calculated results of the cavity prototype under various axial displacements. Frequency shift $\Delta f$ is here normalized with the initial resonance frequency $f_0$; a negative sign of longitudinal displacement $U_Z$ signifies a compression. The cavity prototype was previously evacuated, gradually compressed to displacement 1.25 mm as the loading path for test #1, and then gradually released for test #2. A residual deformation was observed, as shown in figure 6, indicating that the cavity structure was loaded to the elastoplastic range at an axial compression beyond 1 mm. Slopes of the fitted lines for tests #1 and #2 in a linear range are both $2.18 \times 10^{-3}$ mm$^{-1}$, but these two lines have an offset between them due to the plastic deformation. For a 1.5-GHz cavity, the slope indicates that the resonant frequency alters more than 300 kHz under longitudinal compression 1 mm. The tuner mechanism is thus expected to have a resolution better than 0.3 $\mu$m to achieve a frequency adjustment of 1 kHz.

![Figure 6. The resonant frequency of the SRF cavity decreases when the cavity is longitudinally compressed.](image)

As mentioned previously, this cavity prototype has a length difference $\pm 1.5$ mm. Elastic models with perfect geometry $\Delta L = 0$ and length difference $\Delta L = \pm 1.5$ mm were then both established to observe the effect. Figure 6 shows that the numerical results slightly differ from each other; the slopes of their fitted lines are $2.12 \times 10^{-3}$ and $2.15 \times 10^{-3}$ mm$^{-1}$, respectively, with a small discrepancy versus the test results. We thus conclude that the tilt of the cavity prototype has no perceptible effect on the shift of the resonant frequency when the cavity is uniformly compressed within the elastic range. Further investigation on the elastoplastic behaviour of this cavity is presented in [8].

4.2. Cooling to 77 K with liquid nitrogen

With the cavity interior evacuated, the cavity prototype has a dimensionless resonant frequency shift 0.31 on being cooled from 295 K to 77 K with liquid nitrogen. The corresponding mean coefficient of thermal expansion $\alpha_{avg}$ for this temperature range is $13.986 \times 10^{-6}$ K$^{-1}$ [9]. From 295 K to 77 K, Young’s modulus of copper would increase about 10% [9, 10], the value 128.51 GPa was thus assigned in numerical models. Under an assumption that the Poisson ratio also increases about 10% to 0.361, the calculated shift of the resonant frequency at varied $\alpha_{avg}$ is shown in figure 7, with a small difference from the test results. The calculation is thus proved promising to predict the frequency shift at cryogenic temperature.
5. Conclusion
The 2nd-order element option should be selected with use of the ANSYS HF120 element to compute the SRF cavity. An experimental setup on a prototype of the 1.5-GHz SRF cavity was established to examine the computation results. The multi-physics computation developed in previous work proved promising to predict the EM characteristics of the SRF cavity. The resonance frequency of this shell-type cavity decreases as being axially compressed. The tuner mechanism shall have a resolution better than 0.3 \( \mu \)m to achieve a frequency adjustment of 1 kHz.

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