Electrical implementation of a complete synchronization dynamic system

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Abstract. This work presents an electrical implementation of complete synchronization systems, proposing a master/slave synchronization of two identical particle-in-a-box electronic circuits, exhibiting a rich chaotic behaviour. This behaviour was measured, and also emulated, and the results were compared. Just a few works in literature describe experimental measurements of chaotic systems. The master/slave electronic circuits employed have a very simple electronic implementation and results show a complete synchronization of the system.

1. Introduction

Synchronization of chaotic systems is a phenomenon that occurs when chaotic oscillators are coupled, or when a chaotic oscillator drives another one. In several situations, these systems can exhibit exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions. However, synchronization of coupled or driven chaotic oscillators is a phenomenon well established experimentally, and well understood theoretically. It has been found that chaos synchronization is a rich phenomenon wich presents a variety of forms.

Synchronization between dynamical systems is being extensively explored in electronic circuits [1, 2]. There are many types of circuits with potential applications to generate chaotic signals, like Chua’s circuit [3] and Lorenz-based chaotic circuit [4]. Pecora and Carroll proposed the drive-response scheme and determined conditions of synchronization by means of conditional Lyapunov exponents[5].

Dynamical systems can exhibit several types of synchronization, among them: complete synchronization; generalized synchronization, phase synchronization and lag synchronization.

Complete synchronization occurs when there is coincidence of states of two interacting subsystems, as described by equation (1):

\[ x_c(t) = x_r(t) \]  \hspace{1cm} (1)

This regime appears only if the both interacting systems are identical [6].

Generalized synchronization has also been described in the literature [7], and introduced for drive-response systems, as the presence of a relation between the states of response and drive, thus:

\[ x_r(t) = \mathbb{S}[x_c(t)] \]  \hspace{1cm} (2)
Considering the behavior of two unidirectionally coupled chaotic systems:

\[ \dot{x}_d = H(x_d, g_d) \]  
\[ \dot{x}_r = G(x_r, g_r) + \varepsilon P(x_d, x_r) \]

Where \(x_d, x_r\) are the state vectors of the drive and response systems, respectively, and \(H, G\) define the vector fields of these systems, \(g_d\) and \(g_r\) are the controlling parameters, \(P\) represents the coupling term, and \(\varepsilon\) is the scalar coupling parameter.

The generalized synchronization (GS) regime can be observed for two identical systems, with equal or mismatched parameters, and unidirectional coupling. In this particular case, the systems are identical, so the dimensions of drive and response oscillators are equal. The system is described as follows:

\[ \dot{x}_d = H(x_d, g_d) \]  
\[ \dot{x}_r = G(x_r, g_r) + \varepsilon A(x_d - x_r) \]

Where \(A = \{ \delta_{ii} \} \) is the coupling matrix, and \(\delta_{ii} \neq 0\) or 1, and \(\delta_{ij} = 0\), if \(i \neq j\). The dynamics of the response system may be considered as the no autonomous dynamics of a modified system [7]:

\[ \dot{x}_m = H(x_m, g, \varepsilon) \]

Under the external force \(\varepsilon A x_d\):

\[ \dot{x}_m = H(x_m, g, \varepsilon) + \varepsilon A x_d \]

Where \(H'(x, g) = H(x, g) - \varepsilon A x_d\). The term \(-\varepsilon A x_d\) brings the dissipation into the modified system.

The regime of complete synchronization is a particular case of generalized synchronization [7].

In this paper, it was investigated synchronization between two identical particle-in-a-box circuits, exhibiting a rich chaotic behavior. Although the great interest in chaotic systems, just a few works in the literature show also experimental realization of these systems [1]. The great advantage of this work is to obtain synchronism with robust coupling by using simple electronic circuits [8, 9], in a closed loop scheme. In Section 2 we present the purpose of this work. In Section 3, the particle-in-a-box mechanical system is described, as well as the master-slave configuration proposed by using two particle-in-a-box identical circuits, and experimental methods employed. In Section 4, and experimental results are shown. Section 5 discusses results, and conclusions are given on Section 6.

2. Methods

The electronic circuit proposed emulates a particle-in-a-box mechanical system, described as follows, on figure 1.

Figure 1-(a) Particle inside an oscilate box. (b)Force applied by the springs. In this system, a particle with mass \(m\) can collide with the box walls in \(x\)-direction. The walls are supported by linear springs with coefficient \(k\).

In this system, a particle with mass \(m\) can collide the box walls in the \(x\)-direction. The term \(x_i\) is the distance between the walls. The walls are supported by linear springs with coefficient \(k\), and the collisions between the particle and the walls are totally inelastic. The walls and springs have no mass and dimension. The contribution of the forces acting in the particle of Fig. 1(a) results in the equation:
\[ m \frac{d^2x}{dt^2} + \mu \frac{dx}{dt} + f(x) = mg \sin \alpha \quad (9) \]

Where \( \mu \) represents the viscous coefficient and \( f(x) \) is the force applied by the springs. The angle \( \theta \) represents the term \( mg \sin \alpha \) and the term \( f(x) \) describes the external force acting on the particle due to the gravitational acceleration \( g \).

The particle-in-a-box circuit is described on figure 2. This circuit simulates the mechanical behavior of the system of the figure 1(a).

![Particle-in-a-box circuit](image)

Figure 2 – Particle-in-a-box circuit. The component values are:
- Resistors \( (51 \Omega; 10 \Omega; 47 \Omega; 21 \Omega) \),
- Capacitors \( (12 \text{nF}, 2 \text{nF}) \),
- Diodes \( D_1 = D_2 = 2N4148 \),
- Operational amplifiers 741 or equivalents.

The particle-in-a-box electronic circuit has a sinusoidal input \( V_e = V_{\text{max}} \sin(\omega t) \), and the contribution of the currents in the point S, shown in figure 2, results in the differential equation:

\[ R_C C_{\text{i}} \frac{d^2V_2}{dt^2} + \frac{R_C C_{\text{i}}}{C_{\text{i}}} \frac{dV_2}{dt} + I_e(V_2) = -\frac{V_e}{R_e} \quad (10) \]

In equation (10), the sign \( V_2 \) is the position \( x \), and the signal is the velocity \( \frac{dx}{dt} \).

The key devices of this circuit are the anti-parallel diodes \( D_1 \) and \( D_2 \) in association with the resistor \( R_L \). They emulate the nonlinearity of this circuit, as shown on figure 3.

![Current versus voltage for anti-parallel diodes](image)

Figure 3- Current versus voltage for anti-parallel diodes \( D_1 \) and \( D_2 \). They emulate the collisions of the particle with the walls.

A master-slave configuration was proposed by using two identical systems as shown on figure 4, in a closed-loop scheme, by using \( R_{RE} = 470 \Omega \).
The circuits were applied to closed-loop scheme. The chaotic output signals $V_i$ obtained from the drive and response systems were taken from a digital oscilloscope and then analyzed.

A master-slave configuration was implemented by using two identical systems as shown on figure 5, in which a closed-loop scheme was applied, as described on figure 4. Measurements were taken by using a digital oscilloscope Minipa MO 2061, two function generators Minipa MGF 4201A 2MHz, a digital electronic font Minipa MPL 3303, and data were transferred to a personal computer, as shown on figure 6.

Figure 5 – Experimental realization of the proposed dynamical system. Master (left) and slave (right) particle-in-a-box electronic circuits. Detail shows the closed-loop scheme, $R_{EE} = 470\text{\,\Omega}$. 

Figure 4-The circuits were applied to closed-loop scheme. The chaotic output signals $V_i$ obtained from the drive and response systems were taken from a digital oscilloscope and then analyzed.
3. Results

3.1. Numerical simulations results

Simulated results of synchronization between chaotic drive and response subsystems using the same parameters of experimental conditions are shown in this section.

Figure 7 exhibits the phase diagram of the particle-in-a-box circuit ($V_{1d}$ and $V_{2d}$), in a frequency of $f=1096\text{Hz}$.

Figure 7 – Numerical simulation of phase diagram of the particle-in-a-box circuit ($V_{1d}$ and $V_{2d}$), by using a frequency $f=1096\text{Hz}$. As shown, chaotic behaviour is exhibited.

Figure 8 exhibits the results obtained from the numerical simulation of the complete synchronization regime between output signals of drive and response subsystems.
3.2. Experimental results

Experimental results of measurements taken from a particle-in-a-box circuit are shown on this section. The periodic chaotic behavior can be obtained by the variation of the parameter of bifurcation of the circuit shown on figure 2, which controls the circuit. The variation of the parameter frequency of the input sinusoidal signal $v_x = V_{\text{max}} \sin(\alpha)$ can be used as the bifurcation parameter. Phase diagrams are shown on figure 9.

At first, the master-slave configuration of the proposed system was obtained by using two identical particle-in-a-box circuits. The circuits were applied to closed-loop scheme, by using $R_{RE}=470\Omega$. The output signals $V_z$ obtained from master and slave circuits were taken from a digital oscilloscope and then analyzed. It was observed complete synchronism between drive and response signals, as shown on figure 10.
Figure 9– Phase diagrams of the particle-in-a-box circuit for the fixed amplitude 2V, and variable frequency as bifurcation parameter: (a) Period 1; \( f = 958 \) Hz; (b) Period 2; \( f = 963 \) Hz; (c) \( f = 965 \) Hz; (d) Chaos; \( f = 1096 \) Hz.

Figure 10– (a) Output signals of drive and response subsystems. (b) Complete synchronism exhibited between output signals of drive and response subsystems.
Complete synchronism can be also observed analyzing the phase diagram obtained from output signals of drive and response subsystems in a closed-loop scheme, by using $R_{RE} = 470 \, \Omega$. Results are shown on figure 11. Phase diagram exhibits the relation $x_d = x_r$, which means that complete synchronism was achieved.

![Phase diagram synchronism obtained from output signals of drive and response subsystems, $R_{RE}=470\Omega$. Complete synchronism was observed, according to relation $x_d = x_r$.](image)

Finally, a configuration with a master and two slave particle-in-a-box circuits was proposed to verify robustness of the coupling. The circuits were applied to closed-loop scheme, by using $R_{RE}=470\Omega$. The output signals $v_r$ obtained from master and slaves circuits were taken from a digital oscilloscope and then analyzed. It was observed complete synchronism between drive and response signals, as shown on figure 12.

![Output signals of slaves subsystems](image)

![Phase diagram synchronism obtained from output signals of slaves subsystems. As shown, complete synchronism between slaves subsystems was achieved. Phase diagram shows mathematical relation $x_d = x_r$.](image)
4. Discussion

Figure 7 shows the phase diagram for an only particle-in-a-box circuit emulated according to equation (10), for the fixed amplitude 2V, and frequency \( f = 1096 \text{Hz} \). Figure 8 exhibits the results obtained from the numerical simulation of the complete synchronization regime between output signals of drive and response subsystems.

Phase diagrams of the particle-in-a-box circuit for the fixed amplitude 2V, by using variable frequency as bifurcation parameter, are shown on figure 9: (a) Period 1- \( f = 958 \text{Hz} \); (b) Period 2- \( f = 963 \text{Hz} \); (c) Period 4- \( f = 965 \text{Hz} \); and (d) Chaos- \( f = 1096 \text{Hz} \).

Two identical particle-in-a-box circuits were connected by using a closed-loop scheme, where \( R_{RE} = 470 \Omega \). They exhibit a complete synchronization regime. Figure 10 shows output signals of drive and response subsystems. Phase diagram obtained from output signals of drive and response subsystems is shown on figure 11. Both results are in agreement to mathematical relation \( x_d = x_r \).

At last, a configuration with a master and two slave particle-in-a-box circuits, to verify robustness of the coupling. The circuits were applied to closed-loop scheme, by using \( R_{RE} = 470 \Omega \). The output signals \( y \),obtained from master and slaves circuits were taken from a digital oscilloscope and then analyzed. As shown on figure 12, complete synchronism between both slaves subsystems was achieved. Phase diagram shows mathematical relation \( x_d = x_r \). Experimental results showed good agreement with the simulation results.

5. Conclusion

Just a few works in literature describe experimental measurements of chaotic systems, and most employ complex electronic circuits. The master/slave electronic circuits employed in this work have a very simple electronic implementation.

Numerical results show agreement with experimental results, and show the robustness of the coupling in this system. Analyzing preliminary results, it may be concluded that the proposed system emulates the chaotic behavior of a particle-in-a-box circuit, and results show complete synchronization of the system. Due to robustness of the coupling between these systems, we intend to apply this system in chaotic secure communication. Results suggest the system could be employed in chaotic mask successfully.

6. References

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