Production & Decay of Quarkonium

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Abstract. In this talk I review NRQCD predictions for the production of charmonium at the Tevatron. After a quick presentation of the NRQCD factorization formalism for production and decay I review some old results and discuss how they compare to recent data. Following this I discuss some recent work done with Adam Leibovich and Ira Rothstein.

INTRODUCTION

Heavy quarkonia have proven fruitful in helping us gain a better understanding of QCD. Early theoretical analyses of quarkonia decay were based on the color-singlet model (CSM) [1]. The underlying assumption of this model is that the heavy-quark–antiquark pair has the same quantum numbers as the quarkonium meson. (For example the $b\bar{b}$ that forms an $\Upsilon$ must be in a color-singlet $3S_1$ configuration.) One consequence of such a restrictive assumption is that theoretical predictions based on the CSM are simple, depending on only one nonperturbative parameter. However, the CSM does not provide a systematic approach to studying quarkonium. This is clear in $P$-wave decays where infrared divergences signal the breakdown of the CSM [2].

In order to systematically study nonrelativistic systems long distance physics needs to be separated from short distance physics. This can be accomplished with a proper effective field theory which provides a power counting that determines relevant operators. In most effective theories the power counting is based upon dimensional analysis, however, for non-relativistic QCD (NRQCD) [3, 4] this is not the case. Instead, it is an expansion in the parameter $\nu$, the relative velocity of the heavy quarks. This leads to the result that operators of the same dimension may be of different orders in the power counting. Inclusive decay rates and production cross sections are now understood within the framework of NRQCD factorization [3], where decay rates and production cross sections are predicted in a systematic double expansion in $\alpha_s$ and $\nu$. These predictions have met with varying degrees of success.

In the first half of my talk I give a quick review of NRQCD, and the NRQCD factorization formalism for production and decay. This gives the background needed to understand theoretical predictions for production and decay of $J/\psi$ and $\psi'$. I do not attempt to review all of the predictions, instead I focus on the transverse momentum distribution of $J/\psi$ and $\psi'$ at the Tevatron. For the reason that here lies the earliest success and the greatest challenge to NRQCD factorization. In Ref. [5] the $\psi'$ ‘anomaly’ (a factor of 30 discrepancy between the CSM prediction and date) was resolved using NRQCD. However, the initial data on the polarization of these states at large transverse momentum [6] seems to be at odds with the NRQCD prediction.
In the second half of the talk I will discuss a possible resolution to the charmonium polarization puzzle at the Tevatron. This is based on recent work [7] with Ira Rothstein and Adam Leibovich wherein we propose an alternative power counting for charmonium. I do not present all the pieces of evidence which seems to tell us that the effective field theory which best describes the $J/\psi$ system may not be the same theory which best describes the $\Upsilon$. For that the reader is directed to the literature. I merely give a quick review of the new power counting, and then proceed to discuss how this changes the predictions for $J/\psi$ and $\psi'$ polarization at the Tevatron.

**NRQCD**

The power counting depends upon the relative size of the four scales $(m, mv, mv^2, \Lambda_{\text{QCD}})$. If we take $m > mv > mv^2 \simeq \Lambda_{\text{QCD}}$ the bound state dynamics will be dominated by exchange of Coulombic gluons with $(E \simeq mv^2, \vec{p} = mv)$. This hierarchy has been assumed in the NRQCD calculation of production and decay rates and is probably a reasonable choice for the $\Upsilon$ system, where $mv \sim 1.5 \text{ GeV}$. However, whether or not it is correct for the $J/\psi$, where $mv \sim 700 \text{ MeV}$ remains to be seen.

The power counting can be established in a myriad of different ways. Here I will follow the construction of [4], which I now briefly review. There are three relevant gluonic modes [8]: the Coulombic $(mv^2, mv)$, soft $(mv, mv)$ and ultrasoft $(mv^2, mv^2)$. The soft and Coulombic modes can be integrated out leaving only ultrasoft propagating gluons. In the process of integrating out these modes large momenta must be removed from the quark field. This is accomplished by rescaling the heavy quark fields by a factor of $\exp(i \mathbf{p} \cdot \mathbf{x})$ and labeling them by their three momentum $\mathbf{p}$. The ultrasoft gluon can only change residual momenta and not labels on fields. This is analogous to HQET, where the four-velocity labels the fields and the nonperturbative gluons only change the residual momenta [9]. This rescaling must also be done for soft gluon fields [10] which, while they cannot show up in external states, do show up in the Lagrangian. After this rescaling a matching calculation leads to the following tree level Lagrangian [4]

$$
\mathcal{L} = \sum_{p} \bar{\psi}_p \left( iD^0 - \frac{p^2}{2m} \right) \psi_p - 4\pi \alpha_s \sum_{q, q', p, p'} \left\{ \frac{1}{q^0} \psi_{p'}^\dagger \left[ A_{q'}^{0}, A_{q}^{0} \right] \psi_p \\
+ g^{00}(q' - p + p')^\mu - g^{00}(q - p + p')^\nu - g^{00}(q - q')^0 \psi_{p'}^\dagger \left[ A_{q'}^{0}, A_{q}^{0} \right] \psi_p \right\} \\
+ \psi \leftrightarrow \chi, \ T \leftrightarrow \bar{T} + \sum_{p, q} \frac{4\pi \alpha_s}{(p - q)^2} \psi_q^\dagger T^A \psi_p \chi_{-q}^\dagger \bar{T}^A \chi_{-p} + \ldots
$$

(1)

where we have retained the lowest order terms in each sector of the theory. The matrices $T^A$ and $\bar{T}^A$ are the color matrices for the $3$ and $\bar{3}$ representations, respectively. Note the last term is the Coulomb potential, which is leading order and must be resummed in the four-quark sector, while the other non-local interactions arise from soft gluon scattering.

All the operators in the Lagrangian have a definite scaling in $v$, and the spin symmetry, which will play such a crucial role in the polarization predictions, is manifest. The two
subleading interactions which will dominate my discussion are the electric dipole \((E1)\) and the magnetic dipole \((M1)\)

\[ \mathcal{L}_{E1} = \frac{p \cdot A}{2m} \psi \psi^\dagger \]

\[ \mathcal{L}_{M1} = c_F g \frac{\sigma \cdot B}{2m} \psi \psi^\dagger \]

The \(E1\) interaction is down by a factor of \(v\) while the \(M1\) is down by a factor of \(v^2\).

**NRQCD FACTORIZATION FORMALISM**

In the NRQCD factorization formalism developed in Ref. [3] decay rates and production cross sections are written as a sum of products of Wilson coefficients encoding short distance physics and NRQCD matrix elements describing long distance physics. In this formalism a general decay process is written as

\[ \Gamma \psi \rightarrow J/\psi = \sum C_{2S+1L_J}(m, \alpha_s) \langle \psi \mid O^{(1,8)}(2S+1L_J) \mid \psi \rangle. \]

The matrix element represents the long distance part of the rate and may be thought of as the probability of finding the heavy quarks in the relative state \(n\), while the coefficient \(C_{2S+1L_J}(m, \alpha_s)\) is a short distance quantity calculable in perturbation theory. The sum over operators may be truncated as an expansion in the relative velocity \(v\). Similarly, production cross sections may be written as

\[ d\sigma = \sum_n d\sigma_{i+j \rightarrow Q\bar{Q}[n]+X} \langle 0 \mid O_n^H \mid 0 \rangle. \]

Here \(d\sigma_{i+j \rightarrow Q\bar{Q}[n]+X}\) is the short distance cross section for a reaction involving two partons, \(i\) and \(j\), in the initial state, and two heavy quarks in a final state, labeled by \(n\), plus \(X\). This part of the process is calculable in perturbation theory, up to possible structure functions in the initial state. The production matrix elements, which differ from those used in the decay processes, describe the probability of the short distance pair in the state \(n\) to hadronize, inclusively, into the state of interest. The relative size of the matrix elements in the sum are again fixed by the power counting which we will discuss in more detail below.

The formalism for decays is on the same footing as the operator product expansion (OPE) for non-leptonic decays of heavy quarks, while the production formalism assumes factorization, which is only proven, and in some applications of production this is not even the case, in perturbation theory \(^1\) [11]. The trustworthiness of factorization depends upon the particular application. I have reviewed these results here to emphasize the point that when the theory is tested one is really testing both the factorization hypothesis as well the validity of the effective theory as applied to the \(J/\psi\) system. Thus, care must be taken in assigning blame when theoretical predictions do not agree with data.

\(^1\) For a discussion of factorization in NRQCD see Refs. [3, 12].
### TABLE 1. Scaling of matrix elements relevant for $\psi$ production in NRQCD$_b$ and NRQCD$_c$.

|                  | $\langle O^W_{3S_1}\rangle$ | $\langle O^W_{S_0}\rangle$ | $\langle O^W_{3P_0}\rangle$ |
|------------------|-----------------------------|-----------------------------|-----------------------------|
| NRQCD$_b$        | $v^0$                       | $v^4$                       | $v^4$                       |
| NRQCD$_c$        | $(\Lambda_{QCD}/m_c)^0$    | $(\Lambda_{QCD}/m_c)^4$    | $(\Lambda_{QCD}/m_c)^2$    |

### CHARMONIUM PRODUCTION AT THE TEVATRON

Having introduced NRQCD and the factorization formalism I will now turn to theoretical predictions of $J/\psi$ and $\psi'$ production at the Tevatron. The leading order in $v$ contribution to $J/\psi$ production is through the color-singlet matrix element $\langle O^W_{3S_1}\rangle$, since the quantum numbers of the short distance quark pair matches those of the final state. All other matrix elements need insertions of operators into time ordered products to give a non-zero result, and are therefore suppressed compared to the color-singlet matrix element above. For instance, the matrix element $\langle O^W_{S_0}\rangle$ vanishes at leading order. The first non-vanishing contribution comes from the insertion of two $M_1$ operators into time ordered products, thus giving a $v^4$ suppression. The scaling of the relevant matrix elements for $\psi$ production are shown in Table 1 under NRQCD$_b$ (for reasons which I will explain later). It appears from just the $v$ counting that only the color-singlet contribution is important. However, other contributions can be enhanced by kinematic factors. At large transverse momentum, fragmentation type production dominates [13], and only the $\langle O^W_{3S_1}\rangle$ contribution is important. Without the color-octet contributions (i.e., the Color-Singlet Model), the theory is below experiment by about a factor of 30. By adding the color-octet contribution the fit to the data is very good [5].

Once the color-octet matrix elements are fit to the unpolarized date it is possible to make a parameter free prediction for the polarization of $J/\psi$ and $\psi'$ at the Tevatron: they are predicted to be transversely polarized at large $p_T$. This is because at large transverse momentum, the dominant production mechanism is through fragmentation from a nearly on shell gluon to the octet $3S_1$ state. The quark pair inherits the polarization of the fragmenting gluon, and is thus transversely polarized [14]. The leading order transition to the final state goes via two $E1$, spin preserving, gluon emissions. Higher order perturbative fragmentation contributions [15], fusion diagrams [16, 17], and feed-down for the $J/\psi$ [18] dilute the polarization some, but the prediction still holds that as $p_T$ increases so should the transverse polarization. Indeed, for the $\psi'$, at large $p_T \gg m_c$, we expect nearly pure transverse polarization. This prediction seems to be at odds with the initial data which seems to suggest that the $J/\psi$ and the $\psi'$ are unpolarized or slightly
longitudinally polarized as $p_T$ increases [6]. If, after the statistics improve, this trend continues one is left with two obvious possibilities: 1) The power counting of NRQCD does not apply to the $J/\psi$ system. 2) Factorization is violated “badly”, meaning that there are large power corrections.

NRQCD$_C$

In this section I discuss the work done with Adam Leibovich and Ira Rothstein in Ref.[7]. In that work we marshaled evidence that the NRQCD power counting might not apply to the $J/\psi$ system. We did not consider the second possibility mentioned at the end of the previous section: that factorization is violated. I will not be considering this possibility either.

The standard NRQCD methodology, which is based upon the hierarchy $m > mv > mv^2 \simeq \Lambda_{QCD}$, has been applied to the $J/\psi$ as well as the $\Upsilon$ systems. While it seems quite reasonable to apply this power counting to the $\Upsilon$ system, it is not clear that it should apply to the $J/\psi$ system. Indeed, I believe that the data is hinting toward the possibility that a new power counting is called for in the charmed system.

If NRQCD does not apply to the $J/\psi$ system, then one must ask: is there another effective theory which does correctly describe the $J/\psi$? One good reason to believe that such a theory does exist is that NRQCD, as formulated, does correctly predict the ratios of decay amplitudes for exclusive radiative decays. Using spin symmetry the authors of [14] made the following predictions:

$$\Gamma(\chi_{c0} \rightarrow J/\psi + \gamma) : \Gamma(\chi_{c1} \rightarrow J/\psi + \gamma) : \Gamma(\chi_{c2} \rightarrow J/\psi + \gamma) : \Gamma(h_c \rightarrow \eta_c + \gamma) = 0.095 : 0.20 : 0.27 : 0.44 \quad \text{(theory)}$$

$$= 0.092 \pm 0.041 : 0.24 \pm 0.04 : 0.27 \pm 0.03 : \text{unmeasured} \quad \text{(experiment)}. \quad (6)$$

Thus, an alternative formulation of NRQCD must preserve these predictions yet yield different predictions in other relevant processes.

Let me now consider the alternate hierarchy $m > mv \sim \Lambda_{QCD}$. One might be tempted to believe that in this case the power counting should be along the lines of HQET, where the typical energy and momentum exchanged between the heavy quarks is of order $\Lambda_{QCD}$. However, this leads to an effective theory which does not correctly reproduce the infra-red physics. With this power counting, the leading order Lagrangian would simply be

$$L_{HQET} = \psi_i^\dagger D_0 \psi_r,$$

where the fields are now labeled by their four velocity. This is a just a theory of time-like Wilson lines (static quarks) which does not produce any bound state dynamics. Thus I am forced to conclude that the typical momentum is of order $\Lambda_{QCD}$, whereas the typical energy is $\Lambda_{QCD}^2/m$, so that $D^2/(2m)$ is still relevant. I will call this theory NRQCD$_c$, and will refer to the traditional power counting as NRQCD$_b$, as I assume that it describes the bottom system.

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2 The data still has rather large error bars, so we should withhold judgment until the statistics improves.
The power counting of this theory is now along the lines of HQET where the expansion parameter is $\Lambda_{\text{QCD}}/m_Q$. However the residual energy of the quarks is order $\Lambda_{\text{QCD}}^2/m_Q$, while the residual three momentum is $\Lambda_{\text{QCD}}$. Thus one must be careful in the power counting to differentiate between time and spatial derivatives acting on the quark fields. As far as the phenomenology is concerned, perhaps the most important distinction between the power counting in NRQCD$_c$ and NRQCD$_b$ is that the magnetic and electric gluon transitions are now of the same order in NRQCD$_c$. This difference in scaling does not disturb the successes of the standard NRQCD$_b$ formulation but does seem help in some of its shortcomings.

**NRQCD$_c$ Predictions**

The relative size of the different matrix elements change in NRQCD$_c$. In particular, the $M1$ transition is now the same order as the $E1$ transition. The new scaling is shown in Table 1. Due to the dominance of fragmentation at large transverse momentum, we need to include effects up to order $(\Lambda_{\text{QCD}}/m_c)^4$, since the $\langle O_8^W(3S_1) \rangle$ matrix element will still dominate at large $p_T$.

Is this consistent? The size of the matrix elements is a clue. Extraction of the matrix elements uses power counting to limit the number of channels to include in the fits. Calculating $J/\psi$ and $\psi'$ production up to order $(\Lambda_{\text{QCD}}/m_c)^4$ in NRQCD$_c$ requires keeping the same matrix elements as in NRQCD$_b$. Previous extractions of the matrix elements only involve the linear combination

$$M_W^c = \langle O_8^W(1S_0) \rangle + \frac{r}{m_c} \langle O_8^W(3P_1) \rangle,$$

with $r \approx 3 - 3.5$, since the short-distance rates have similar size and shape. In the new power-counting, I can just drop the contribution from $\langle O_8^W(3P_1) \rangle$, since it is down by $(\Lambda_{\text{QCD}}/m_c)^2 \sim 1/10$ compared to $\langle O_8^W(1S_0) \rangle$. It is the same order as $\langle O_8^W(3S_1) \rangle$, but is not kinematically enhanced by fragmentation effects. The extraction from [18] would then give for the $J/\psi$ and $\psi'$ matrix elements

$$\langle O_8^W(1S_0) \rangle : \langle O_8^W(3S_1) \rangle = (6.6 \pm 0.7) \times 10^{-2} : (3.9 \pm 0.7) \times 10^{-3} \approx 17 : 1,$n$$

$$\langle O_8^W(1S_0) \rangle : \langle O_8^W(3S_1) \rangle = (7.8 \pm 3.6) \times 10^{-3} : (3.7 \pm 0.9) \times 10^{-3} \approx 2 : 1.$$ (9)

Other extractions have various values of the hierarchy, ranging from 3 : 1 to 20 : 1 [21]. While the relation of the color-octet matrix elements in the $J/\psi$ system is indeed in agreement with the NRQCD$_c$ power counting, the $\psi'$ does not look to be hierarchical. However, it should be noted that the statistical errors in the $\psi'$ extraction, quoted above, are quite large. Furthermore, there are also large uncertainties introduced in the parton distribution function. The above ratios used the CTEQ5L parton distribution functions. If we take the central values from [18] for the MRST98LO distribution functions, we

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3 These results reproduce those given in [20] when $\lambda$ is taken to be 1 in this reference.
find the ratio 3 : 1. On the other hand, the \( J/\psi \) extraction is much less sensitive to the choice of distribution function. Given the statistical and theoretical errors, it clear that the \( \psi' \) ratio is not terribly illuminating.

Let me now consider the extraction of these color-octet matrix elements in the \( \Upsilon \) sector [22], where according to NRQCD the choice of distribution function. Given the statistical and theoretical errors, it clear that the ratio for the \( \Upsilon(1S) \) color-octet matrix elements is 1 : 1 within the one sigma errors. Furthermore, these matrix elements are those extracted subtracting out the feed down from the higher states. While phenomenologically it is perfectly reasonable to define the subtracted matrix elements, I believe that, since the matrix elements are inclusive, one should not subtract out the feed down from hadronic decays when checking the power counting. In principle this subtraction should not change things by orders of magnitude, but nonetheless it can have a significant effect. Indeed, if one compares the ratios for inclusive matrix elements, which do not have the accumulated error, then the ratios come out to be 1 : 1, even for the \( \Upsilon(1S) \) [22].

With NRQCD c, the intermediate color-octet \( ^3S_1 \) states hadronize through the emission of either two \( E1 \) or \( M1 \) dipole gluons, at the same order in \( 1/m_c \). Since the magnetic gluons do not preserve spin, the polarization of \( \psi \) produced through the \( \langle O^\psi_Y(3S_1) \rangle \) can be greatly diluted. The net polarization will depend on the ratio of matrix elements

\[
R_{M/E} = \frac{\int \prod_x d^4x_T \langle 0 \left| \sum_{i=1}^3 T(M_1(x_1)M_1(x_2)\psi^+ T^a \sigma_i \chi) a_H^i a_H T(M_1(x_3)M_1(x_4)\chi^+ T^a \sigma_i \psi) \right| 0 \rangle}{\int \prod_x d^4x_T \langle 0 \left| \sum_{i=1}^3 T(E_1(x_1)E_1(x_2)\psi^+ T^a \sigma_i \chi) a_H^i a_H T(E_1(x_3)E_1(x_4)\chi^+ T^a \sigma_i \psi) \right| 0 \rangle}
\]

where

\[
a_H^i a_H = \sum_X |H + X \rangle \langle H + X |
\]

The leads to the polarization leveling off at large \( p_T \) at some value which is fixed by \( R_{M/E} \). In Fig. 1, we show the prediction for \( J/\psi \) and \( \psi' \) polarization at the Tevatron. The data is from [6]. The three lines correspond to different values for \( R_{M/E} = 0 \) (dashed), 1 (dotted), \( \infty \) (solid)). The dashed line is also the prediction for NRQCD b. The residual transverse polarization for \( J/\psi \) at asymptotically large \( p_T \) is due to feed down from \( \chi \) states. The non-perturbative corrections to our predictions are suppressed by \( \Lambda_{QCD}^4/m^4 \).
CONCLUSION

In this talk I have reviewed NRQCD, and the NRQCD factorization formalism which is used to make predictions for the production and decay of charmonium and bottomonium. I did not discuss all of these predictions. Instead I focused on what I believe to be the most important prediction of NRQCD factorization: the transverse momentum distribution of unpolarized and polarized $J/\psi$ and $\psi'$ produced at the Tevatron. Because the unpolarized data can be used to determine the unknown color-octet matrix elements, it is possible to make a parameter free prediction for polarized production. This provides a clean test of the NRQCD factorization formalism. Moreover the quality of the data for unpolarized production is good, and while the data for polarized production has large error bars it is expected to get better.

The NRQCD factorization formalism predicts the $J/\psi$ and $\psi'$ to be transversely polarized at large $p_T$. This is because at large transverse momentum, the dominant production mechanism is through fragmentation from a nearly on shell gluon to the octet $^3S_1$ state. The quark pair inherits the polarization of the fragmenting gluon, and is thus transversely polarized [14]. The leading order transition to the final state goes via two $E1$, spin preserving, gluon emissions. Various corrections dilute the polarization some, but the prediction still holds that as $p_T$ increases so should the transverse polarization. Indeed, for the $\psi'$, at large $p_T \gg m_c$, we expect nearly pure transverse polarization. The current experimental results [6] show no or a slight longitudinal polarization, as $p_T$ increases. If, after the statistics improve, this trend continues, then it will be the smoking gun that leads us to conclude that either NRQCD is not the correct effective field theory for charmonia, or that factorization fails in these processes.

The possibility that NRQCD is not the correct effective theory for charmonium leads me to ask: is there any reason to believe that there is any effective theory to correctly
describe the $J/\psi$? I believe that the spin symmetry predictions for the ratio of $\chi$ decays clearly answers this question in the affirmative. Assuming that such an effective theory exists, then is it NRQCD$_c$ or NRQCD$_b$? As I have shown the two theories do indeed make quite disparate predictions, which in principle should be easy to test.

However, these tests can be clouded by the issues of factorization and the convergence of the perturbative expansion. One would be justified to worry about the breakdown of factorization in hadro-production at small transverse momentum. However, for large transverse momentum one would expect factorization to hold, with non-factorizable corrections suppressed by powers of $m_c/p_T$. As far as the perturbative expansion is concerned, it seems that for most calculations the next-to-leading order results are indeed smaller than the leading order result [23, 24, 25], though, the NNLO calculation performed, in the leptonic decay width [26], is not well behaved at this order.

In the end I believe the data will be the final arbiter. The polarization measurement may fall in line with the NRQCD$_b$ prediction. Or the data may result in longitudinal polarization for $J/\psi$ and $\psi'$, in which case it may be that neither NRQCD$_c$ nor NRQCD$_b$ are the correct theory.

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