Applications of Laplace Transform in science and technology

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Abstract: In the present paper, we discussed applications of Laplace Transform to solve the equations that occur in mathematical modeling of various engineering problems like simple electric circuit, in analysis and modeling of Mechanical system, in population growth, in conduction of heat equation (with example), in Bessel’s function and in economics. We interpreted a relation between beta and gamma function and applications to simultaneous differential equations with suitable example using Laplace Transform Method.
Keywords: Laplace Transform, Inverse Laplace Transform, Beta and Gamma Functions.

1. Introduction

The important field of Mathematical Analysis is Laplace transformation is referred as integral transforms with wide applications in various fields like engineering technology, basic sciences, and mathematics and in economics. It is also used to find the solution of differential equations at boundary value. Mathematical formulations of most of the engineering problems are in the form of differential equations.

In day to day life we commonly use mathematical models and its applications. Population growth model plays a very important role (see, [3]). Daci [4] used Laplace transform in Mathematical model on population projection in Albania and he experimentally verified how Laplace transform is used in population growth. The simple applications of Laplace transform in engineering fields related to transfer function of mechanical system, nuclear as physics are discussed by Sawant [9]. In [1], Patil explained how present discounted value in finance related to Laplace transforms and the application of time derivative property using Laplace transforms. The Laplace transform theory violates a very fundamental requirement of all engineering systems that is in control system (see. Das [2]).

Laplace transform is also used in the theory of Partial fraction. Thakur and et al [5] used Laplace transform method for fractional differential equation. Laplace transform theory is related to other transform (see Rani and Devi [6], Anumaka [7]) and Number theory (see. Aleksandar [13]). Many authors namely Ananda and Gangadharaiah[8], Novozhilovb [10], Duz [14], Sedletskii [11], Stankovi [12], Bhullar [15], subramanian [16], Wadkar and et al [17] done work on applications of Laplace transform in different fields. Now before going to discuss some applications of Laplace Transform we will see some preliminaries.
2. Preliminaries

**Definition 2.1:** The Laplace Transform (LT) of \( f(t), \ t > 0, \) is defined by
\[
L[f(t)] = \int_0^\infty e^{-st} f(t) dt = F(s),
\]
where \( s \) is real or complex parameter.

**Definition 2.2:** The inverse Laplace transform of \( F(s) \) is defined as
\[
L^{-1}[F(s)] = f(t).
\]

3. Main Results

3.1 Relation between Gamma and beta function

Since beta and gamma functions are definite integrals called Euler’s integral of first and second kind respectively. They are used to solve definite integrals which we cannot solve or reduce to standard form easily. Relation between beta and gamma function is also helps us to solve definite integrals. The relation between beta and gamma function is
\[
B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}
\]
is obtained with the help of Laplace transform.

By definition of beta function \( B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx \) \( m,n >0 \)

Let \( f(t) = t^{m-1} \) and \( g(t) = t^{n-1} \) then the convolution of \( f(t) \) and \( g(t) \) is given by
\[
f(t) \ast g(t) = \int_0^t (t-u)^{m-1}u^{n-1} du \]
(3.1.2)

Put \( tr = u, tdr = du \), we have
\[
f(t) \ast g(t) = \int_0^1 t^{m-1}r^{m-1}(1-r)^{n-1} dr = \int_0^1 t^{m+n-1} (1-r)^{n-1} tdr
\]
\[
f(t) \ast g(t) = t^{m+n-1}B(m,n)
\]

\[
L[t^{m+n-1}B(m,n)] = L[f(t) \ast g(t)] \Rightarrow L[t^{m+n-1}B(m,n)] = L[f(t)]L[g(t)] = \frac{\Gamma m \Gamma n}{sm \sn} \Rightarrow \frac{\Gamma m \Gamma n}{sm \sn} = \frac{\Gamma m \Gamma n}{s^{m+n}}
\]

Now taking inverse LT we can find
\[
t^{m+n-1}B(m,n) = \Gamma m \Gamma n \frac{t^{m+n-1}}{\Gamma(m+n)} \Rightarrow B(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}
\]

3.2 Application in solving simultaneous Differential equations

In engineering, the mathematical formulations of some of the models / problems are not only in terms of single differential equations but also in terms of simultaneous linear differential equations.

**Example 3.2.1:** suppose a particle is moving through the plane path. At any time \( t, \) a particle having coordinates \( (x, y) \) is moving along the path is expressed by \( \frac{dy}{dt} + 2x = \sin 2t \) and \( \frac{dx}{dt} - 2y = \cos 2t, \) \( (t > 0). \) If at \( t = 0, x = 1, \ y = 0, \) then by Laplace transform we prove the particle moves through the path \( 4x^2 + 4xy + 5y^2 = 4. \)

Taking Laplace transform of given simultaneous equations, we have
\[ sY(s) - y(0) + 2X(s) = \frac{2}{s^2 + 4} \quad \text{or} \quad 2X(s) + sY(s) = \frac{2}{s^2 + 4} \]  
(3.2.1)

and \[ sX(s) - x(0) - 2Y(s) = \frac{s}{s^2 + 4} \quad \text{or} \quad sX(s) - 2Y(s) = \frac{s}{s^2 + 4} + 1 \]
(3.2.2)

Solving for \( X(s) \) and \( Y(s) \), multiply result (3.2.1) by \( s \), result (3.2.2) by 2 and taking the difference, we get,

\[ 2Y(s) = \frac{-2}{s^2 + 4} \quad \text{and hence by inverse Laplace} \quad y(t) = -\sin 2t \]  
(3.2.3)

From (3.2.1) \( X(s) = \frac{1}{2} \left[ \frac{2}{s^2 + 4} + \frac{2s}{s^2 + 4} \right] \).

Taking its inverse Laplace transform, we obtain \( x(t) = \frac{1}{2} (\sin 2t + \cos 2t) \).

Hence \( 2x = \sin 2t + \cos 2t \quad \Rightarrow \quad 2x + y = 2\cos 2t \)
\( \Rightarrow (2x + y)^2 = 4(1 - \sin^2 2t) = 4 - 4y^2 \)
\( \Rightarrow 4x^2 + 4xy + 5y^2 = 4 \)

Hence the desired particle moves through the path \( 4x^2 + 4xy + 5y^2 = 4 \).

\section*{3.3 Application of Laplace transform in Heat conduction equation}

To determine the flow of heat in semi-infinite solid \( x > 0 \), when initially the solid is at zero temperature and at \( t = 0 \) the boundary \( x = 0 \) is raised to a temperature \( u_0 \) and maintained at \( u_0 \). Equation of conduction of heat in one dimension is,

\[ c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad x > 0, t > 0 \text{ with initial conditions } u = 0 \text{ when } t = 0 \quad (x \geq 0) \quad \text{and } u = u_0 \text{ when } x = 0 \quad (t > 0) \]

Now multiply given partial equation by \( e^{-st} \) and integrating with respect to \( t \) from 0 to \( \infty \) and using given initial conditions we get,

\[ c^2 \frac{\partial^2 u}{\partial x^2} = s\tilde{u}(x, s) - u(x, 0) = \tilde{s}(x, s) \].

Second condition gives \( \tilde{u} = \frac{u}{s} \) when \( x = 0 \). The general solution of \( c^2 \frac{\partial^2 u}{\partial x^2} = s\tilde{u}(x, s) \) is \( \tilde{u} = Ae^{x^2t/4c} + Be^{-x^2t/4c} \).

To find a solution which remains finite, as \( x \to \infty \), we must take \( A = 0 \). Hence \( \tilde{u} = Be^{-x^2t/4c} \). Now \( \tilde{u} = \frac{u}{s} \Rightarrow B = \frac{u_0}{s} \) therefore \( \tilde{u} = \frac{u_0}{s} e^{-x^2t/4c} \), taking inverse Laplace transform we get

\[ u = u_0 \left( 1 - \text{erf} \frac{x}{2\sqrt{ct}} \right) \].

In view \( L^{-1} \left[ \frac{e^{-c\sqrt{s}}}{s} \right] = 1 - \text{erf} \left( \frac{c}{2\sqrt{t}} \right) \) where \( \text{erf}(x) \) denotes the error function defined by \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du \)

\textbf{Example 3.3.1:} An infinitely long string whose one side at \( x = 0 \) is at rest. The end \( x = 0 \) is given a transverse displacement \( f(t) \), \( t > 0 \). Then to obtain the movement at particular point of the string at any time \( t \) if the
displacement \( y(x, t) \) is bounded, we have \( c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \), \( x > 0, t > 0 \) with conditions: \( y(x, 0) = 0, \frac{\partial}{\partial t} y(x, 0) = 0 \), \( y(x, t) \) is bounded and \( u(0, t) = f(t) \). Multiply given partial differential equation by \( e^{-st} \) and integrating with respect to \( t \) from \( 0 \) to \( \infty \), we get

\[
\left( \frac{\partial^2 \bar{y}}{\partial x^2} \right) - s \frac{\partial \bar{y}}{\partial x} = \frac{c^2}{\partial^2 \bar{y}} \Rightarrow \frac{c^2}{ \partial^2 \bar{y}} = c^2 \frac{\partial^2 \bar{y}}{\partial x^2}
\]

But \( u(0, t) = f(t) \Rightarrow \bar{y} = F(s) \) at \( x = 0 \), \( \bar{y}(x, s) \) is bounded

\[
c^2 \frac{\partial^2 \bar{y}}{\partial x^2} = \bar{y}(x, s) = Ae^{sx/c} + Be^{-sx/c}
\]

Since \( \bar{y}(x, s) \) is bounded, \( A \) must be zero and \( B = F(s) \) in view of \( \bar{y} = F(s) \) at \( x = 0 \).

Hence \( \bar{y}(x, s) = F(s)e^{-sx/c} \). Using the inversion formula we get

\[
y(t) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} F(s)e^{(s-x)c} ds = f\left(t - \frac{x}{c}\right).
\]

### 3.4 Laplace Transform in the analysis of Electric Circuits

Consider \( R, L \) and \( C \) are connected in series with electromotive power of voltage \( E \), where \( R \) is resistance, \( L \) is an inductance and \( C \) is capacitance. A switch is connected in the circuit. By Kirchhoff’s law, we get

\[
L \frac{di}{dt} + Ri + \frac{q}{c} = E
\]

**Example:** If \( R \) is 16 ohms, \( I \) is 3-henry, and \( C \) is 0.02 F are joined in series having an supply of 300v (See. figure1). At time \( t \) is equal to zero, the charge on the capacitor and current flowing in the closed circuit is zero. To obtain charge and current at any time \( t > 0 \), we have

\[
L \frac{di}{dt} + Ri + \frac{q}{c} = E \quad \text{i.e.} \quad \frac{d^2q}{dt^2} + \frac{8dq}{dt} + 25q = 150
\]

Initial conditions are: \( q = 0 \) and \( i = 0 \). at time \( t = 0 \)

Applying Laplace transform on both sides of above differential equations, we have

\[
L \left[ \frac{d^2q}{dt^2} \right] + 8L \left[ \frac{dq}{dt} \right] + 25L[q] = 150L[1]
\]

\[
s^2Q(s) - sq(0) - q'(0) + 8\left(sQ(s) - q(0)\right) + 25Q(s) = 150 \frac{1}{s}
\]

Applying initial conditions we get

\[
s^2Q(s) + 8\left(sQ(s)\right) + 25Q(s) = 150 \frac{1}{s}
\]
That is \( Q(s) = \frac{150}{s(s^2 + 8s + 25)} \). Taking inverse transform on both sides we get
\[
q = 6 - 6e^{-4t}\cos 3t - 8e^{-4t}\sin 3t \quad \text{and} \quad i = 50e^{-4t}\sin 3t
\]
These are required expressions for charge \( q \) and current \( i \) respectively at any time \( t > 0 \).

3.5 Laplace transform in the analysis and modeling of Mechanical system

Example 3.5.1: Laplace transform in Transfer function

For calculating the transfer function of that certain system. Let’s consider a big Pot as shown in figure 2. Initially the pot is empty at \( t = 0 \). Let \( F_i \) be the constant rate of flow added for \( t > 0 \) and \( F_0 = BH \) be the rate at which flow leaves the Pot. Let \( A \) be the cross sectional area of the Pot. Now we define the differential equation for head \( H \), find the time-constant and using Laplace transform we obtain the transfer function of system.

We know that \( F_0 = BH \) (3.5.1.1)

![Figure 2](image)

Let’s consider the fluid of mass \( m \) and fluid density \( \sigma \). Since Mass is product of velocity and density. Hence \( Mass = Velocity \times density \) i.e. \( M = V \times \sigma = HA \times \sigma \)

Now mass flow rate is given by \( \dot{M} = \frac{dM}{dt} = A\sigma \frac{dH}{dt} \) and
\[
\sigma A \frac{dH}{dt} = \sigma F_i - \sigma F_0, \quad \text{which gives,} \quad A \frac{dH}{dt} = F_i - F_0. \quad \text{Hence} \quad F_i = A \frac{dH}{dt} + BH
\]

This is differential equation for heat flow and its solution is easily obtained using Laplace Transform method. Taking LT we have
\[
L(F_i) = A_i L\left(\frac{dH}{dt}\right) + B_i L(H) \quad \text{Or} \quad F_i(s) = A_i \{H(s) - H(0)\} + B_i H(s)
\]

\( F_i(s) = A_i \{H(s)\} + B_i H(s) \), \( H(0) = 0 \) and can be written as
\[
\frac{H(s)}{F_i(s)} = \frac{1}{A_i + B_i}
\]

(3.5.1.2)

Now taking Laplace transform of equation (3.5.1) we get
\[
F_0(s) = B_i H(s)
\]

(3.5.1.3)

From (3.5.1.2) and (3.5.1.3) we get
\[
\frac{F_0(s)}{F_i(s)} = \frac{\frac{A_i}{B_i}}{1 + \frac{A_i}{B_i}}
\]
and this is equation of transfer function of the system, where

the time constant is given by \( \tau = \frac{A_i}{B_i} \)

Example 3.5.2: Laplace transform in vibrating mechanical system

Let’s consider the vehicle of mass \( m \). The key factors for investigating the suspension system of vehicle are mass of the vehicle, the damper and springs which are used to join vehicle body to links of suspension. Translational Mechanical structures involve three key elements: mass \( (M \text{ kg}) \), springs stiffness \( K \), in \text{Nm}–1 and dampers \( D \), in \text{Nsm}–1.

The related parameters are:
1) $y(t)$, the displacement which is measured in m and
2) $F(t)$, the force which is measured in N.

Figure 3

Figure 3 shows the Mass–spring–damper system. By Newton’s and Hooke’s law, we can develop mathematical model for Mass–spring–damper system. The represented system shown in figure 3 is governed by the differential equation

$$\frac{d^2 y}{dt^2} + D \frac{dy}{dt} + Ky = F(t) \quad \text{or} \quad \frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 5y = 2\sin\omega t$$

(3.5.2.1)

Taking Laplace transforms throughout in (3.5.2.1) gives Incorporating properties of Laplace transform, we get

$$L\left[\frac{d^2 y}{dt^2}\right] + 2L\left[\frac{dy}{dt}\right] + 5L(y) = 2L(\sin\omega t)$$

i.e. $(s^2 + 2s + 5)L\left[y(t)\right] = [sy(0) + y'(0) + 6y(0)] = \frac{2w}{s^2 + w^2}$

Taking initial conditions $y(0) = y'(0) = 0$ we can write $Y(s) = L\left[y(t)\right] = \frac{2w}{(s^2 + 2s + 5)(s^2 + w^2)}$

on resolving into partial fractions for different values of $w$ (say $w = 2$) we get

$$Y(s) = \frac{A}{s^2 + 2s + 5} + \frac{Bs + C}{s^2 + w^2}$$

On Taking inverse Laplace transforms

$$y = \frac{1}{4} \left[ -\frac{3}{2} \sin t - \cos t \right] + e^{-t} \left[ \frac{23}{8} \sin 2t - \frac{1}{4} \cos 2t \right]$$

Laplace transform is not limited to particular branch of Engineering it is used in civil Engineering and also in Economics problems.

3.6 Laplace Transform and Population Growth

We use Laplace transform for calculating growth of population. A group of the same species of crops, animals or other organisms which live together and reproduces is termed as Population. The logistic equation is in variable separable form, by applying transformation of Laplace and its properties to establish the exponential expression. In population growth rate of birth, death rate as well as rate of migration and immigration rate plays vital role. We know that as birth rate and death rate varies inversely i.e one increases other decreases. The above mentioned mechanism mathematically can be formulated as $\Delta P = B - D + I - M$, where $P, B, D, M$ and $I$ are rate of change population with respect to time $t$, rate of birth, death, migration and immigration respectively.

If we assume population is ‘closed’, that is migration and immigration are equal then

$$\Delta P = B - D$$

Considering the density-independent pollution the simple model is that pollution size combined with freedom in density does not affect birth and death rates. Therefore, number of people are proportional to birth and death.

$$P'(t) = rP(t)$$
The variation of change per capita \( r = \frac{p'(t)}{p(t)} \) for this model is continuous, positive when it increases and negative when \( P \) decreases. Rewriting above equation we have \( P'(t) = rP(t) \) and hence
\[
L[p'(t)] = L[rP(t)]
\]
Now by Laplace transform of derivative property we have \( sP(s) - P(0) = rP(s) \). Here the population at time \( t=0 \) is represented by \( P(0) \). Hence \( P(s) = \frac{P(0)}{s-r} \). Now taking inverse Laplace transform on both sides of above equation, we get
\[
L^{-1}[P(s)] = L^{-1}\left[\frac{P(0)}{s-r}\right] = P(0)e^{rt}
\]
Hence \( P(t) = P(0)e^{rt} \) \( (3.6.1) \)
Now using equation (3.6.1) we can determine \( r \) and the population can be evaluated for different years. We will apply the equation (3.6.1) for Population Albanian.

### Table 1

| Year  | Actual    | Predicted |
|-------|-----------|-----------|
| 2010  | 2,918,674 | 2918674   |
| 2011  | 2,918,674 | 2908622   |
| 2012  | 2,907,368 | 2898604   |
| 2013  | 2,903,008 | 2888622   |
| 2014  | 2,897,770 | 2878673   |
| 2015  | 2,892,394 | 2868759   |
| 2016  | 2,885,796 | 2858878   |
| 2017  | 2,875,592 | 2849032   |
| 2018  | 2,876,591 | 2839220   |
| 2019  | 2,870,324 | 2829442   |

Applying the data for the year 2010 with \( t = 0 \), we have \( P(0) = 2918674 \), we can evaluate \( r \) using the fact that \( P = 2819697 \) which is the year 2000. By using (3.6.1) \( r = -0.345 \times 10^{-2} \). The general solution is
\[
P(r) = 2916764e^{-0.00345}
\]
Now we calculate the population in later years and compare it with actual data. In between 2010 and 2019, we have a good agreement in between predicted and actual data. To predict our future it also helps us. Therefore we can apply this model to estimate the millions of population that year.
\[
P(2030) = 2918674 \times e^{-0.00345 \times 20} = 2714694
\]

3.7 Relation between Laplace transforms and Bessel’s functions

The differential equation \( x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0 \) is known as Bessel’s differential equation or Bessel’s equation of order \( n \). When \( n \) is not zero or an integer, the complete solution is given by
\[
y = A J_n(x) + B J_{-n}(x) \quad and \quad J_n(x) \quad and \quad J_{-n}(x)
\]
are independent and \( J_n(x) = \sum_{r=0}^{\infty} (-1)^r \frac{x^{n+2r}}{r!(n+r+1)} \)
i.e. \( J_n(x) = \frac{x^r}{2^{n+1}(n+1)!} \left[ 1 - \frac{x^2}{2(2n+2)} + \frac{2A(2n)(2n+2)}{4(2n+2)(2n+4)} - \cdots \right] \) is called first kind of Bessel’s function whose order is \( n \). When \( n \) is not integer then \( J_n(x) \) and \( J_{-n}(x) \) are independent \( J_{-n}(x) = (-1)^n J_n(x) \) and complete solution is \( y = A J_n(x) + B J_{-n}(x) \) where \( J_n(x) \) is Bessel’s function of order \( n \) or Neumann function.
Solution of Bessel’s equation for n=0: The equation \( \frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + y = 0 \) is known as Bessel’s equation for n=0 and solution is \( J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots \), \( J_0(x) \) is known as zero order Bessel’s function.

Now we find out Laplace transform of \( J_0(t) \).

\[
L[J_0(t)] = \frac{1}{\sqrt{s^2 + a^2}}
\]

Similarly, by using the theorems and properties of Laplace transform we can prove that:

i) \( L[J_0(at)] = \frac{1}{\sqrt{s^2 + a^2}} \)

ii) \( L[J_0(at)] = \frac{s}{(s^2 + a^2)^{3/2}} \)

iii) \( \int_0^\infty J_0(at) dt = 1 \)

iv) \( L[e^{-at} J_0(at)] = \frac{1}{(s^2 + 2as + a^2)^{1/2}} \)

### 3.8 Laplace Transform in Nuclear Physics

Consider the example centered on ideas from nuclear physics. Let linear differential equation of first order be \( \frac{dN}{dt} = -\lambda N \) which shows basic relationship that describes radioactive decay, where \( N = N(t) \) gives number of undecayed atoms remaining in a sample of a radioactive isotope at any time \( t \) and \( \lambda \) the decay constant.

Rewriting the above equation and by applying Laplace Transform, we get \( L\left\{ \frac{dN}{dt} \right\} + \lambda L(N) = 0 \) i.e. \( sL(N) - N(0) - \lambda L(N) = 0 \)

Hence \( L(N) = \frac{N(0)}{s + \lambda} \). Now, applying Inverse Laplace Transform, we get \( N(t) = N(0)e^{-\lambda t} \). which is really useful for radioactive decay.

### 3.9 Application in economic problems

#### Representation of present value in Laplace transform

In the investment project, for various alternatives one wishes to determine the present value of series of cash receipts and transactions. The Present value of a sequence of payments given by,

\[
[PV] = \sum_{t=1}^{T} \frac{C(t)}{(1+s)^t}
\]  

(3.9.1)

Where at time \( t \), the present discounted value is \( [PV] \), the flow of cash is \( C(t) \), the rate of discount is \( s \) and \( t \) be the time period. Let us assume the current value with constant compounding which is the current value of cash flow. In other words, this is the amount which we need to pay today in order to get a cash flow or a series of them in the future.

Now with the help of exponential series we derive equation (3.9.1) can be written as

\[
[PV] = \sum_{t=1}^{T} C(t)e^{-st}
\]

In the continuous time case replacing summation to an integral, equation (3.9.1) can be written as

\[
[PV] = \int_{t=1}^{T} C(t)e^{-st}dt
\]

Again here \( T \) is finite quantity. So let \( T \to \infty \), in above equation we have

\[
[PV] = \int_{t=1}^{\infty} C(t)e^{-st}dt
\]
By Laplace transform definition, above equation can be expressed as \([\mathcal{P}V]\) = \(L(C(0))\)

**Laplace transforms and presents value for cash flows**

Consider an example of constant cash payment \(K\) paid at the rate of interest \(r\) at the end of each year represented in time line given below.

Here the cash flows continuously forever. Therefore the present value can be expressed by an infinite geometric series:

\[
[\mathcal{P}V] = \frac{K}{1+s} + \frac{K}{(1+s)^2} + \frac{K}{(1+s)^3} + \cdots \quad (3.9.2)
\]

Dividing both side by \((1+s)\)

\[
[\mathcal{P}V] \frac{1+s}{1+s} = \frac{K}{(1+s)^2} + \frac{K}{(1+s)^3} + \frac{K}{(1+s)^4} + \cdots \quad (3.9.3)
\]

Subtracting equation (3.9.3) from (3.9.2), \([\mathcal{P}V] \frac{s}{1+s} = \frac{K}{1+s}\)

On evaluation, we obtain \([\mathcal{P}V] = \frac{K}{s} = L(k)\)

By Laplace transform equation if we assume constant cash flow say \(K\), the Present discounted value at interest rate \(r\) of the stream is given by

\[
L(k) = k \int_{t=0}^{\infty} e^{-st} dt = \frac{k}{s}
\]

Which is same result as above.

**Example 3.9.1:**

The insurance company has launched a protection that will pay Rs.1000 indefinitely, starting its first payment from next year. If the right product is 20% how much this security worth today?

Using time line we get,

\[
\mathcal{P}V = \frac{k}{s} = \frac{1000}{0.2} = 5000
\]

**4. Conclusion**

Throughout the paper, we have discussed some applications of Laplace Transform in various fields of Engineering.

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