CONTROVERSY OF THE GRO J1655-40 BLACK HOLE MASS AND SPIN ESTIMATES AND ITS POSSIBLE SOLUTIONS

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Received 2016 February 23; revised 2016 April 18; accepted 2016 April 18; published 2016 June 27

ABSTRACT

Estimates of the black hole mass $M$ and the dimensionless spin $a$ in the microquasar GRO J1655-40 implied by strong gravity effects related to the timing and spectral measurements are controversial, if the mass restriction determined by the dynamics related to independent optical measurements, $M_{opt} = (5.4 \pm 0.3) M_{\odot}$, is applied. The timing measurements of twin high-frequency (HF) quasiperiodic oscillations (QPOs) with the frequency ratio 3:2 and the simultaneously observed low-frequency (LF) QPO imply spin in the range $a \in (0.27-0.29)$ if models based on the frequencies of geodesic epicyclic motion are used to fit the timing measurements, and the correlated creation of the twin HF QPOs and the LF QPO at a common radius is assumed. On the other hand, the spectral continuum method implies $a \in (0.65-0.75)$, and the Fe-line-profile method implies $a \in (0.94-0.98)$. This controversy can be cured if we abandon the assumption of the occurrence of the twin HF QPOs and the simultaneously observed LF QPO at a common radius. We demonstrate that the epicyclic resonance model of the twin HF QPOs is able to predict the spin in agreement with the Fe-profile method, but no model based on the geodesic epicyclic frequencies can be in agreement with the spectral continuum method. We also show that the non-geodesic string loop oscillation model of twin HF QPOs predicts spin $a > 0.3$ under the optical measurement limit on the black hole mass, which is in agreement with both the spectral continuum and Fe-profile methods.

Key words: methods: analytical – stars: black holes – X-rays: individual (GRO J1655-40)

1. INTRODUCTION

Strong gravity affecting an accretion disk in the vicinity of a black hole horizon governs three observationally significant phenomena, enabling us to determine the black hole mass $M$ and the dimensionless spin $a$. The phenomena are related to the timing effects, namely to the frequencies of the high-frequency (HF) quasiperiodic oscillations (QPOs) and connected low-frequency (LF) QPOs, to the spectral continuum of the accretion disk, and the Fe spectral lines profiled by the influence of the black hole spacetime.

The most interesting (and precise) information is connected to the twin HF QPOs observed with the fixed frequency ratio 3:2 in the microquasars GRS 1915+105, XTE J1550-56, and GRO J1655-40. Such twin HF QPOs can be explained by the so-called geodesic oscillation models using frequencies of the geodesic epicyclic motion in the field of Kerr black holes, i.e., the orbital frequency, $\nu_o$, and the epicyclic radial, $\nu_r$, and latitudinal, $\nu_\phi$, frequencies. However, the twin HF QPOs in the three microquasars cannot be explained by a fixed oscillation model, if we assume a Kerr black hole (Török et al. 2011). A unique, epicyclic resonance (ER) model exists (Kotilová et al. 2014) if we assume central Kerr naked singularities that demonstrate special properties of the prograde circular motion (Stuchlík 1980; Stuchlík & Schée 2012).

In the case of the GRS 1915+105 microquasar, the 3:2 HF QPOs can be explained by the ER model (Török et al. 2005), while for the XTE J1550-56 and GRO J1655-40 microquasars, the twin HF QPOs and the related LF QPO can be explained by the relativistic precession (RP) model (Stella et al. 1999) that is by definition combined with the relativistic nodal model of the LF QPOs (Stella & Vietri 1998).

In the case of the microquasar GRS 1915+105, the limits on the black hole mass and spin implied by the models of the twin HF QPOs are in agreement with the limits implied by the spectral measurements (McClintock & Remillard 2006, pp. 157–213), and agreement on the limits implied by the models of the QPOs and the models of spectral measurements has been demonstrated also in the case of the XTE J1550-56 microquasar (Motta et al. 2014b). On the other hand, in the case of the microquasar GRO J1655-40, the spin limits of the spectral (continuum and Fe-line) measurements (Shafee et al. 2006; Miller et al. 2009) contradict each other, and, moreover, they both contradict the spin limits implied by the geodesic models of QPOs (Motta et al. 2014a; Stuchlík & Kolosi 2016), if the limits on the black hole mass determined by the dynamical studies based on optical measurements (Beer & Podsiałowski 2002) are considered.

In the present paper we consider the controversial restrictions on the GRO J1655-40 black hole mass and spin. We keep the relevance of the mass restrictions implied by the weak gravity dynamical models based on the optical measurements of the binary system that have high relevance and are quite independent of the timing and spectral measurements connected with the strong gravity near the black hole horizon. We first discuss the possibility of obtaining an agreement of the twin HF QPO geodesic models and any of the spectral methods, and then we test the possibility of the agreement of the string loop oscillation model of twin HF QPOs with the predictions of the spectral measurements.

2. THE CONTROVERSY OF THE MASS AND SPIN LIMITS ON THE GRO J1655-40 BLACK HOLE

The GRO J1655-40 low-mass-X-ray-binary (LMXB) source is one of the most extensively studied Galactic microquasars,
i.e., the sources where the accreting central object is assumed to be a black hole. However, controversial estimates of the GRO J1655-40 black hole gravitational mass $M$ and dimensionless spin $a$ have been reported recently (Motta et al. 2014a; Stuchlík & Kolos 2016). We first summarize the origin of this controversy related in the strong gravity regime to models of X-ray spectral measurements, implying limits on the black hole spin, and the so-called geodesic models of QPOs, implying in the strong gravity regime limits on the black hole mass and spin that are confronted with the limits on the black hole mass implied by dynamical studies in the weak gravity regime.

The mass of the GRO J1655-40 black hole is restricted by dynamical methods related to spectro-photometric optical measurements (Beer & Podsiadlowski 2002) that are not related to the timing studies of QPOs based on X-ray measurements, and have a high degree of credibility as they are related to weak field gravity techniques. The range of allowed values of the black hole mass implied by the optical measurements reads (Beer & Podsiadlowski 2002)

$$M_{\text{opt}} = (5.4 \pm 0.3) \, M_\odot.$$  

Note that there exists an earlier and larger estimate of the GRO J1655-40 black hole mass determined in Orosz & Bailyn (1997) that reads

$$M_{\text{bh}} = (7.02 \pm 0.22) \, M_\odot.$$  

However, the later estimate presented in Beer & Podsiadlowski (2002) is in recent papers considered to be the relevant one.

The X-ray spectroscopy techniques give controversial results in the case of the GRO J1655-40 microquasar. The spectral continuum measurements (Shafee et al. 2006) predict the black hole spin in the range

$$0.65 < a < 0.75.$$  

The measurements of the Fe spectral lines profiled by the strong gravity of the black hole predict (Miller et al. 2009)

$$0.94 < a < 0.98.$$  

Clearly, there is a strong discrepancy between the results of spectral measurements related to radiation processes in the strong gravity regime. We could expect that the timing effects related to QPOs occurring in the strong gravity regime could help to determine the correct spectral measurements due to the spin estimates.

The Rossi XTE observatory provides many timing measurements of the X-rays emitted by the GRO J1655-40 source, which are summarized in Motta et al. (2014b). Interesting are those related to the twin HF QPOs and the LF QPOs. The LF QPOs were observed between 0.1 and 30 Hz, usually independently of the HF QPOs. However, most important is the observation of twin HF QPOs with frequencies stabilized at the lower frequency $\nu_L \sim 300$ Hz and the upper frequency $\nu_U \sim 450$ Hz that were observed simultaneously; it is natural, then, to assume that the twin HF QPOs are related to a common radius where both observed oscillatory modes occur. The twin HF QPOs were reported for the first time in Strohmayer (2001) where, simultaneously with the twin HF QPOs, a LF QPO was reported at $\nu_{\text{LF}} \sim 17$ Hz. The magnitude of the observed frequencies and the character of the oscillations indicate strongly that the twin HF QPOs should occur in close vicinity to the black hole horizon, being related to the orbital motion.

The special simultaneous observation of the twin HF QPOs and the LF QPO enables us to obtain stringent restrictions on the mass and dimensionless spin of the central black hole, if we assume that all of these QPOs arise at a given radius of the accretion disk (Motta et al. 2014a). The Monte Carlo technique, applied to the observational data and the RP model, belongs to the geodesic models of twin HF QPOs, with the frequency identification $\nu_L = \nu_0 - \nu_t$ and $\nu_U = \nu_0$, along with the related relativistic nodal precession model of the LF QPO with the frequency identification $\nu_{\text{rad}} = \nu_0 - \nu_t$, implied for the black hole mass and spin the limits (Motta et al. 2014a)

$$M = (5.31 \pm 0.07) \, M_\odot, \quad a = 0.290 \pm 0.003.$$  

The mass limit is in very good agreement with the mass limit given by the optical measurements. However, there is a clear discrepancy with the spin limits given by both spectral measurements, as the RP model predicts the spin $a < 0.3$.

Recently, the standard nodal precession model of the LF QPOs and a variety of the geodesic models of twin HF QPOs, i.e., models using the frequencies of the oscillatory modes combined from the frequencies of the geodesic epicyclic motion, have been tested for matching to the observational data of the simultaneously observed twin HF QPOs and LF QPO in the microquasar GRO J1655-40; models originally proposed to explain only the twin HF QPOs were generalized to interpret also the LF QPO by the relativistic nodal precession (Stuchlík & Kolos 2016). Instead of the Monte Carlo technique, the method of frequency relations introduced in Stuchlík et al. (2013) has been used (Stuchlík & Kolos 2015a), and the fitting has been done to the data of the twin HF QPOs and simultaneous LF QPO presented as Sample B1 in Table 2 of Motta et al. (2014a). The peak frequencies with the measurement error of the peak frequencies (centroid frequencies dominated by the statistics of the frequencies) were used (Motta et al. 2014a):

$$\nu_L = 298 \pm 4 \, \text{Hz}, \quad \nu_{\text{low}} = 17.3 \pm 0.1 \, \text{Hz},$$
$$\nu_U = 441 \pm 2 \, \text{Hz}.$$  

For all the considered models of twin HF QPOs it was assumed that both the twin HF QPOs and the LF QPO arise at a common radius. Then the frequency relation technique enables us to obtain a mass–spin relation $M_{\text{HF}}(a, \rho)$ for the twin HF QPOs with the frequency ratio parameter

$$p = \left(\frac{\nu_L}{\nu_U}\right)^2,$$  

and due to the assumption of the common radius a mass–spin relation $M_{\text{HF}}(a, \rho)$ can be determined for the LF QPO. The two relations imply limits on the black hole mass and spin related to the QPO measurements (Stuchlík & Kolos 2016). It has been found in Stuchlík & Kolos (2016) that three geodesic models can predict the mass in agreement with the optical measurement limit. Along with the RP models when the frequency relation method implies the limits

$$M = (5.3 \pm 0.1) \, M_\odot, \quad a = 0.286 \pm 0.004,$$  

in agreement with the estimates given by the Monte Carlo technique results in Motta et al. (2014a), only two other twin HF QPOs models can fit the optical mass limit. The first is the so-called total precession model, with the frequency identification
given by \( \nu_U = \nu_0 \) and \( \nu_L = \nu_0 - \nu_r \), implying the limits

\[
M = (5.5 \pm 0.1) \, M_\odot, \quad a = 0.276 \pm 0.003. \tag{9}
\]

The second is the resonance epicyclic model, demonstrating a beat frequency, with the frequency identification \( \nu_U = \nu_0 \) and \( \nu_L = \nu_0 - \nu_r \), implying the limits

\[
M = (5.1 \pm 0.1) \, M_\odot, \quad a = 0.274 \pm 0.003. \tag{10}
\]

Clearly, all the models predict the black hole spin \( a < 0.3 \). We conclude that, unfortunately, none of the considered geodesic oscillatory models of twin HF QPOs combined with the nodal precession model of LF QPOs can predict values of the black hole spin that could be matched to the predictions of the X-ray precession model of LF QPOs. We conclude that, unfortunately, none of the considered geodesic models predict values of the black hole spin.

3. GEODESIC MODELS OF QPOS MATCHING THE SPIN LIMITS BY SPECTRAL MEASUREMENTS

In the Kerr spacetimes, circular geodesics exist only in the equatorial plane \((\text{Bardeen et al. 1972; Stuchlík 1980})\). The radial epicyclic frequency \( \nu_r \), and the vertical epicyclic frequency \( \nu_0 \) of the near-circular epicyclic motion are given by the relations \((\text{Kato et al. 1998; Stella & Vietri 1998; Stuchlík & Schee 2012})\)

\[
\nu_r^2 = \alpha_r \nu_0^2, \quad \nu_0^2 = \alpha_0 \nu_0^2, \tag{11}
\]

where the orbital (azimuthal) frequency \( \nu_0 \), sometimes called the Keplerian frequency, and the related dimensionless epicyclic frequencies are given by the formulae

\[
\nu_0 = \frac{1}{2\pi} \left( \frac{c^3}{GM} \right) \frac{1}{x^{3/2} + a},
\]

\[
\alpha_r = 1 - \frac{6}{x} + \frac{8a}{x^{3/2}} - \frac{3a^2}{x^2},
\]

\[
\alpha_0 = 1 - \frac{4a}{x^{3/2}} + \frac{2a^2}{x^2}. \tag{12}
\]

The dimensionless radius \( x = r/r_g \) is introduced, where the gravitational radius of the black hole \( r_g = GM/c^2 \). The radial profiles of the epicyclic frequencies are illustrated in Figure 1.

Since all the frequencies \( \nu_{0r}, \nu_{0v}, \nu_r \) have the same mass scaling, it is clear that in the geodesic models of the twin HF QPOs containing only linear combinations of these frequencies, the frequency ratio of the lower and upper frequencies will be independent of the mass parameter \( M \), and are dependent only on the spin parameter \( a \) \((\text{Stuchlík et al. 2013})\). This fact enables an effective application of the frequency relation method in the case of the geodesic models of the twin HF QPOs.

3.2. Models Based on the Epicyclic Geometric Motion

The geodesic models of twin HF QPOs can be separated into three classes: the hot spot models (the RP model and its variants \((\text{Stella & Vietri 1999; Stuchlík et al. 2013})\)), the tidal precession model \((\text{Kostíč et al. 2009})\), resonance models \((\text{Török et al. 2005; Stuchlík et al. 2011})\), and disk oscillation (diskoseismic) models \((\text{Rezzolla et al. 2003; Montero & Zanotti 2012})\). These models were applied to match the twin HF QPOs and the LF QPO in the microquasar GRO J1655-40 \((\text{Stuchlík & Kološ 2016})\). We have tested in the present study all the models considered in Stuchlík & Kološ \((2016)\). Here we briefly summarize properties of the resonance models that were shown to be the only successful models in matching at least one of the spin limits predicted by the spectral measurements, along with the mass limits implied by the dynamical restrictions due to optical measurements.

The LF QPO remains related to the relativistic nodal (Lense–Thirring) precession with frequency \( \nu_{\text{nod}} = \nu_0 - \nu_r \). The observations demonstrate that LF QPOs given by the Lense–Thirring effect can occur independently of the twin HF QPOs, and only in exceptional cases are their simultaneous occurrence observed \((\text{Motta et al. 2014a})\). Therefore, we are allowed to use the assumption of the twin HF QPOs and the LF QPO simultaneously observed, but physically uncorrelated.

The ER models \((\text{Abramowicz & Kluzniak 2001; Török et al. 2005})\) consider the resonance of the axisymmetric
oscillation modes of accretion disks. The accretion disks can be geometrically thin, having the geodetical (Keplerian) profile of angular velocity (Novikov & Thorne 1973, pp. 343–450; Page & Thorne 1974), or they can be toroidal, geometrically thick, with the angular velocity profile determined by pressure gradients (Abramowicz et al. 1978; Kozlowski et al. 1978; Stuchlík et al. 2009). The frequencies of the disk oscillations are related to the orbital and epicyclic frequencies of the circular geodesics for both Keplerian disks (Kato et al. 1998; Nowak & Lehr 1998, pp. 233-253; Kato 2004) and slender tori (Rezzolla et al. 2003; Montero & Zanotti 2012). The resonance can be of two kinds: the internal, parametric resonance of the radial and vertical epicyclic oscillatory modes, representing the basical resonance epicyclic model; or the parametric resonance governed by the Mathieu equation, which predicts the strongest resonant phenomena for the frequency ratio 3:2 (Landau & Lifshitz 1969; Nayfeh & Mook 1979; Stuchlík et al. 2013). The forced nonlinear resonance admits slight scatter of the resonant frequencies, i.e., this kind of resonance can occur while the oscillating modes in the resonance have a frequency ratio slightly different from the exact rational ratio; the width of the resonance scatter decreases with an increasing order of resonance (Landau & Lifshitz 1969). For forced resonances, the scatter of the frequency ratio from the rational ratio is governed by nonlinear effects (Nayfeh & Mook 1979). Therefore, we consider as relevant all the frequency ratios given by the measured HF QPO frequencies with their errors.

3.3. Matching the Observed QPO Frequencies to the Geodesic Oscillation Models

Considering a twin HF QPOs geodesic model we determine for a given frequency ratio parameter $\rho$ the frequency relations

$$a = \alpha_{v R} / \alpha_{v L} (x, \rho)$$

2 Variants of the resonance model with beat frequencies are presented in Stuchlík & Kološ (2016)—all the variants were considered in the present study.

**Figure 1.** Restrictions on the GRO J1655-40 black hole parameters $M$ and $a$ given by the ER and ER5 models due to the matching of the twin HF QPOs. In the upper row, the mass–spin relation due to the geodesic models demonstrates satisfactory matching to the optical mass limit (shaded area) and the spectral continuum dimensionless spin limits (dark shaded regions). The edges of the cross region are denoted as points A, B, and C for the ER model, and D for the ER5 model. In the lower row, the radial profiles of the upper and lower frequencies in the ER and ER5 models are confronted with the nodal precession frequency radial profile, and the radii where the twin HF QPOs and the LF QPO occur are given.
and (by a numerical procedure) the corresponding frequency relations governing the radius of occurrence
\[ x = \frac{\sqrt{x \nu_0 / (r, \theta)}}{\nu_0 (r, \theta)} (a, p); \] (14)

the twin oscillations with the upper (lower) frequency \( \nu_U (\phi, r, \theta) \) \( \nu_L (\phi, r, \theta) \) are determined by the concrete geodesic model. The radius of occurrence have to satisfy the condition \( x = \nu_0 (a, p) \), where radius of the marginally stable orbit \( x_{ms} (a) \) is implicitly given by (Bardeen et al. 1972; Stuchlík et al. 2013)
\[ a = a_{ms} = \sqrt{\frac{x}{3}} (4 - \sqrt{3x - 2}). \] (15)

Since the assumption of the coincidence of the radii where the twin HF QPOs and the simultaneously observed LF QPO were observed is abandoned now, we have to use the frequency relation technique developed in Stuchlík & Kološ (2016) in slightly modified form where we consider separately the fitting of the twin HF QPOs giving the dependence \( M_{HF} (a, p) \) and the dependence \( M_{LF} (a, x) \) for an arbitrarily fixed radius \( x \) where the LF QPO can occur.

We have tested all the geodesic models of the twin HF QPOs studied in Stuchlík & Kološ (2016), demonstrating that none of the models can be in accord with the limits implied by the spectral continuum measurements \((0.65 < a < 0.75)\). On the other hand, two geodesic models of the twin HF QPOs are in (partial) agreement with the Fe-line spectral measurements \((0.94 < a < 0.98)\). We focus attention on these two satisfactory models. We give detailed description of the method for one of the models, namely the ER model. The method of matching the observational data constitutes from the following successive steps.

3.3.1. Matching Procedure for the ER Model

First, we determine the frequency ratio interval of the lower and upper centred frequencies of the measured twin HF QPOs with the related errors. Assume that the interval reads \( p_1 < p < p_2 \). For each ratio from the interval, we have to find the \( M_{HF} (a, p) \) relation. To find the spin-radius and mass–spin relations with errors corresponding to the measurement errors of the twin HF QPOs frequencies, it is enough to make the calculations for the frequency ratio parameters \( p_1 \) and \( p_2 \)—these errors represent the maximal errors, as opposed to the statistical errors (Stuchlík & Kološ 2016).

Second, we use the frequency relation of the ER model
\[ a = a^{\theta r} (x, p) \]
\[ = \sqrt{\frac{x}{3}} (2p + 2) \]
\[ - \sqrt{1 - p} [3x (p + 1) - 2(2p + 1)] \] (16)

and determine by numerical procedure the inverse frequency relation for the radius where the twin HF QPOs have to occur \( x^{\theta r} (a; p) \); the spin parameter is assumed in the interval \( 0 \leq a \leq 1 \).

Third, using the relation
\[ \nu_U = \nu_0 = \frac{1}{2\pi GM} \left( \frac{c_i}{x^{3/2}} (1 - 4a x^{-3/2} + 3a^2 x^{-2})^{1/2}}{a^{3/2} + a} \right), \] (17)

where for a given spin \( a \) we apply the numerically determined relation \( x = x^{\theta r} (a; p) \), we obtain the mass–spin relation \( M_{HF}^\theta (a, p) \). We construct the HF mass–spin relation for the limiting values of \( p = p_1 \), \( p = p_2 \) reflecting the error in determining the mass–spin relation connected to the measurement errors of twin HF QPOs. The mass–spin relation \( M_{HF}^\theta (a, p) \) related to the ER model of twin HF QPOs is illustrated in Figure 1 (left column). We can see that the function \( M_{HF}^\theta (a, p) \) really satisfies simultaneously the optical mass limit and the spin limit of the Fe-line spectral measurements, but not for the whole intervals given by the optical measurements of mass and the Fe-line limits on spin. In fact the ER model introduces additional restrictions: \( M > 5.4 M_\odot \) and \( a < 0.96 \).

Fourth, the frequency of the relativistic nodal precession in the Kerr geometry reads (Stella & Vietri 1998)
\[ \nu_{nod} (x; M, a) = \frac{1}{2\pi GM} \left[ 1 - \left(1 - \frac{4a}{x^{3/2}} + \frac{3a^2}{x^2}\right)^{1/2} \right]. \] (18)

Under the assumption of the coincidence of the radii of occurrence of the twin HF QPOs and the LF QPO, we used the condition \( x = x_{HF} (a, p) \) (Stuchlík & Kološ 2016). Here we fix the radius \( x = \text{const.} \) and use the condition
\[ \nu_{LF} = \nu_{nod} (x, M, a) \] (19)

in order to determine the LF mass–spin relation \( M_{LF} (a, x) \) governed by the relativistic nodal precession. We numerically determine the radii \( x \) for which the mass–spin relations \( M_{LF} (a, x) \) cross the region of the mass–spin parameter space corresponding to the optical measurement limits on the mass and the spectral limits on the spin of the GRO J1655-40 black hole. This procedure is now independent of the geodesic models of the twin HF QPOs. The results given in units of gravitational mass \( M \), or the radius \( x_{ms} \) are represented in Figure 2. Of course, they can be expressed also in terms of the corresponding \( x_{HF} (a, p) \).

3.3.2. Matching Procedure for the ER5 Model with Beat Frequency

We have found one model with beat frequencies that meets the optical limits on the black hole mass and the Fe-line limits on the black hole spin. The identification of the ER5 model\(^5\) with beat frequencies reads
\[ \nu_U = \nu_0 - \nu_\tau, \quad \nu_U = \nu_\tau. \] (20)

The frequency relation of the ER5 model reads
\[ a = a^{\theta r} (x, p) = a^{\theta r} (x, p') \] (21)

where
\[ p' = \frac{1}{(1 + \sqrt{b})^2}. \] (22)

Following the first three steps of the procedure of determination of the mass–spin relation presented above, we arrive to \( M_{HF}^{\theta r} (a, p) \). The resulting curve is presented in Figure 1. We can see that the predictions of the ER5 model only touch the mass–spin region given by the optical

\(^5\) We keep the notation of the models as presented in Stuchlík & Kološ (2016).
measurement and the Fe-line fittings at the point $M = 5.1 \, M_\odot$ and $a = 0.98$.

The resulting regions of the mass–spin parameter space allowed by the ER and ER1 geodesic models are presented in Table 1, along with the radii $r_{\text{HF}}$ and $r_{\text{LF}}$. Note that none of the considered twin HF QPO models is in agreement with the mass estimate $M = (7.02 \pm 0.22) \, M_\odot$.

4. STRING LOOP OSCILLATION MODEL

The current-carrying string loops (Larsen 1993, 1994; Jacobson & Sotiriou 2009; Kološ & Stuchlík 2010) represent one of the models based on non-geodesic phenomena that could reflect plasma exhibiting a string-like behavior due to dynamics of the magnetic field lines (Christensson & Hindmarsh 1999; Semenov et al. 2004), or due to the thin flux tubes of magnetized plasma described as 1D strings (Semenov & Bernikov 1991; Cremaschini & Stuchlík 2013; Cremaschini et al. 2013; Kovář 2013; Kološ et al. 2015; Tursunov et al. 2016). The high-energy string loops can serve as a model of formation and collimation of ultra-relativistic jets in the field of black holes or naked singularities located in active galactic nuclei or Galactic microquasars (Stuchlík 1983; Stuchlík & Hledík 1999; Stuchlík & Kološ 2012a, 2012b; Kološ & Stuchlík 2013; Tursunov et al. 2014), while the low-energy string loops can serve as a model of twin HF QPOs occuring in accretion disks orbiting black holes or neutron stars (Stuchlík & Kološ 2012b, 2014, 2015a, 2015b).

4.1. Frequency of the String Loop Radial and Vertical Oscillatory Modes

Dynamics of the axisymmetric string loops in the axisymmetric Kerr geometry is governed by the parameters of the gravitational field, the energy parameter $E$ of the string loop, and the two parameters, $J$, $\omega$, governing the combined effects of the string tension and its angular momentum (Kološ & Stuchlík 2013; Stuchlík & Kološ 2014). Small harmonic or quasi-harmonic oscillations of the string loops can occur around stable equilibrium positions in the equatorial plane of the Kerr geometry. For the radial and latitudinal (vertical) harmonic oscillatory string loop motion in the Kerr spacetimes,
the dimensionless angular frequencies read

\[
\Omega_\nu^2(r) = \frac{J_{\text{Eex}}(r)}{2r(a^2 + r^3)^2(2a\omega(a^2 + r^3) + \sqrt{(\omega^2 + 1)(r^3 - a^3)}^2),
\]

\[
\Omega_\theta(r) = \frac{2a\omega(\sqrt{(\omega^2 + 1)(r^3 - a^3)}^2 + (\omega^2 + 1)(a^3r^3 - a^2r^3 + r^3 + 2r^2))}{r^2(a^2 + r^3)^2(2a\omega(a^2 + 3r^2)\Delta^{1/2} + (\omega^2 + 1)(r^3 - a^3))},
\]

where

\[
J_{\text{Eex}}(r) \equiv H(\omega^2 + 1)(r - 1)(6a^2r^3 - 3a^2r^2 - 6a^2 - 5r^4 + 12r^3) + (\omega^2 + 1)(2F(2a^2 + 3r^2)(1 - r) - FH) + 8a\omega(\sqrt{(\omega^2 + 1)(a^2 + 3r^2)}^2 + 4a\omega\Delta^{-1/2}H) \times [(a^2 + 3r^2)\Delta - (r - 1)^2 - 6r\Delta(r - 1)].
\]

4.2. Fitting the Twin HF QPO Frequencies

We assume that the resonance phenomena in the string loop oscillatory motion, governed by the Kolmogorov–Arnold–Moser theory (Arnold 1978), the 3:2 frequency ratio, is observed at the twin HF QPOs observed in the GRO J1655-40 microquasar. We directly identify the observed frequencies \(\nu_0\) and \(\nu_r\) with the \(\nu_0\) and \(\nu_r\) frequencies. In the case of the string loop oscillation model the additional nodal frequency model can be attributed to a physically independent relativistic nodal precession of a hot spot related to the LF QPO.\(^4\) The fitting of the string loop oscillation frequencies to the observed frequencies introduced in Stuchlík & Kološ (2014) will be used here. The string loop oscillation model implies a triangular limit on the spacetime parameters \(M, a\)—see Figure 3.\(^5\) The limiting values of the black hole mass are presented in Table 2.

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\(^4\) For the possibility of obtaining low-frequency string loop oscillations, see Stuchlík & Kološ (2015b).

\(^5\) For the string loop oscillation model, the mass limit \(M = (7.02 \pm 0.22) M_\odot\) introduces no restriction on the black hole spin, and only the stringy parameter is restricted in this case.
agreement with both restrictions implied by the spectral measurements. The matching of the twin HF QPOs by the string loop model puts restrictions on the string loop parameters, namely the parameter $\omega$ is restricted to negative values. The LF QPO can be matched by the relativistic nodal precession model for any black hole parameters limited by the string loop model.

We can conclude that in order to select between the successful models of the timing effect related to the QPOs observed in the GRO J1655-40 microquasar, giving coherent limits on the black hole mass and spin, additional data from measurements of the spectral continuum and pro"led spectral lines in the microquasar, and more precise data of the timing measurements of QPOs, are necessary. Such precise measurements could finally exclude the validity of the geodesic models for the microquasar GRO J1655-40.

We stress that the con"rmation or falsi"cation of the presented QPO models can be expected due to observations of the QPOs at the microquasar GRO J1655-40 by the planned space X-ray observatory LOFT, which promises detection of the timing effects related to the QPOs with precision by one order higher than those obtained recently by the ROSSI X-ray detector; even the temporal evolution of QPOs during the measurements is expected in Feroci et al. (2012a, 2012b). For example, in the case of the LF QPOs with frequencies around 10 Hz even lag effects could be estimated, while for the twin HF QPOs observed in hundreds of Hz the expected precision of the frequency measurements could enable us to determine the character and details of the assumed resonant phenomena.

Z.S. acknowledges the Albert Einstein Centre for Gravitation and Astrophysics supported by the Czech Science Foundation.
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