Precisely position- and angular-controllable optical trapping and manipulation via a single vortex-pair beam

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Optical trapping and manipulation using laser beams play a key role in many areas including biology, atomic science, and nanofabrication. Here, we propose and experimentally demonstrate the first use of a vortex-pair beam in optical trapping and manipulation. We successfully trap two spherical microparticles simultaneously by a single vortex-pair beam. Precisely position-controllable manipulation of the trapped spherical microparticles is realized by adjusting the off-axis distance of the vortices on the initial phase plane of the vortex-pair beam. Based on the feature of the vortex-pair beam, as an optical wrench, the high-precision angular-controllable rotation of the cylindrical microrod is achieved by rotating the initial phase structure. Our result provides a rich control on the trapping of microparticles and has greatly important applications in biological area, and optically driven micromachines or motors.

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I. INTRODUCTION

Optical trapping and manipulation have been widely used in a variety of areas including atom cooling \[1, 2\], molecular biology \[3\], nanotechnology \[4, 5\], as well as other disciplines. From 1970, Ashkin pioneered the investigation on optical force of the radiation pressure produced by the laser beam and the interaction with microparticles \[6\]. Later in 1986, Ashkin et al. realized optical trapping of dielectric particles by the gradient force from a single beam which was also named as optical tweezers \[7\]. From then on, optical tweezers become a powerful and flexible tool for trapping and manipulating the micrometre-sized objects. The initial method of trapping microparticles is often made use of normal Gaussian beam via its gradient force from the light field. With the invention of new kind of both scalar and vectorial optical beams, more methods for trapping microparticles are developed by using structured beams like Laguerre-Gaussian beams \[8, 9\], Bessel beams \[10, 11\], partially coherent light beams \[12\]. Airy beams \[13\], helico-conical beams \[14\], radially polarized beams \[15\], cylindrical vector beams \[16\], and Poincaré beams \[17\]. Moreover, not only the structured beams but also the laser pulse can be used to trap linear microparticles \[18, 19\] and nonlinear microparticles \[20, 21\]. More interestingly, the optical tweezers were also generalized to three-dimensional trapping of microparticles \[22\], transportation of the micro-objects in the air \[23\], and surface plasmon-based nano-optical tweezers \[24\].

In many applications, it is required to rotate the trapped microparticles. Due to the orbital angular momentum carried by vortex beams, it always makes particle rotate, and this technology was known as an optical spanner \[27\]. However, on the contrary, it is hard to realize the stable trapping of microparticles based on vortex beams, and this rotation cannot be well suppressed and controlled. People have also developed some other methods to rotate microparticles. Paterson et al. \[28\] demonstrated the rotation of optically trapped microscopic particles by changing the optical path to rotate the spiral interference pattern generated by interfering Laguerre-Gaussian beam with a reference beam. However, an interferometer of high accuracy is necessary \[28\]. O’Neil et al. achieved rotational control within optical tweezers by rotating a rectangular aperture inserted in the beam axis which results in a focused spot that has rectangular symmetry \[29\]. Moreover, Zhang et al. theoretically proposed optical doughnuts for optical tweezers using a modified vortex phase and rotated the beams directly by revolving phase \[30\].

On the other hand, as the shape complexity of microparticles increases, the problem of trapping and rotating microparticles becomes more difficult. In practice, the trapped particle is not always spherical shape, for example, some crystals and bacteria or other single cells. A typical research object is the cylindrical microrod, which is one interesting class of non-spherical microparticles being used in a wide range of practical applications such as scanning optical probes in force microscope \[31\], and the fabrication of nano electronic devices \[32\]. Given above-mentioned applications, there has been also increasing interest in the trapping of non-spherical microparticles especially the cylindrical microrod. For instance, Gauthier et al. theoretically predicted trapping properties of cylindrically shaped micro-objects and observed that cylinders can be manipulated and rotated in the transverse plane about the optical axis, however, the rotation of the cylinders is not controllable \[33\]. In 2008, Kreysing et al. reported on cell rotator using a dual-beam fiber trap with one non-rotationally symmetric trapping beam which is owing to the excitation of higher-order modes in a dual-mode optical fiber \[34\]. Lee et al. have used a line optical
trap to trap and rotate the nanowires. Phillips et al. presented an approach based on optical force through a shaped particle capable of acting as passive clamp. Although these proposals have been raised to trap and manipulate non-spherical particles, they are complex and not easy to operate. More effective methods for trapping of the non-spherical particles are urgently needed in practical application.

The vortex-pair beam, a kind of structured light field, contains a pair of vortices at the initial phase plane which is different from the normal vortex beam (e.g., Laguerre-Gaussian beam). It was first studied in 1993 by Guy. The propagation of the vortex-pair beams in the free space has been analyzed analytically. Vortex-pair beam containing oppositely charged vortices is also known as the vortex-dipole beam. In 2008, Chen et al. discussed on how to force the vortex dipole to annihilate by a background phase function. The transformation of the dipole vortex beam by an astigmatic lens was investigated by Yan et al. in 2009 and further Reddy et al. developed a noninterferometric technique to determine the charge of vortex-pair beam through an astigmatic system. Furthermore, Chen et al. theoretically analyzed the properties of the vortex-pair beam focused by high-aperture. He et al. gave explicit expressions for the vortex-pair beam diffracted by a half-plane. By knife-edge test, Singh et al. revealed the rotation of the internal energy of the vortex dipole beam through striking distinct intensity intrusions in the geometrical shadow region. More recently, Zhao et al. investigated the transverse focal shift of vortex pair beam in a high numerical aperture system and found that there are some works related to evolution properties of the vortex-pair beam as introduced above, the application of the vortex-pair beam to optical trapping has not been realized yet.

In this paper, we have experimentally realized the stable tapping of the spherical microparticles and cylindrical minicrotrod based on a single vortex pair beam with two positive topological charges. It shows great advantages of our proposal that precisely control of the spherical microparticle with its position in one direction is realized. Moreover, we have demonstrated the controllable rotation of cylindrical minicrotrod based on the rotation of the vortex-pair beam. To best our knowledge, it is the first time to adopt a vortex-pair beam to realize the controllable trapping and manipulation of the micrometre-sized objects. Our method may provide a high-precision and reliable control on the trapping of microparticles and is potentially useful for many other areas like biology.

II. THEORACRICAL DESCRIPTION

First, let us introduce the description of such vortex-pair beam. Let a Gaussian beam be incident on a designed phase plate with a pair of vortices. The light field at the initial plane can be expressed as,

$$E_i(u, v, z = 0) = \exp\left(-\frac{u^2 + v^2}{w_0^2}\right)\phi(u, v),$$

where $w_0$ is the beam width of the incident Gaussian beam, and $\phi(u, v)$ is the phase function of the vortex pair phase given by

$$\phi(u, v) = \left[\frac{u - a + iv}{\sqrt{(u-a)^2 + v^2}}\right]^{m_1} \left[\frac{u + a + iv}{\sqrt{(u+a)^2 + v^2}}\right]^{m_2},$$

where $m_1$ and $m_2$ are the integer topological charges of each vortex contained in the vortex pair phase, and $a$ is the initial off-axis distance of each vortex. It should be noted that the vortex pair phase is not rotationally symmetric which is different from a single integer vortex beam. When the light propagates in a linear optical system, under the paraxial approximation, the output field can be calculated by the Collins formula expressed as

$$E_o(x, y, z) = \frac{\exp(ikL)}{i\lambda B} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_i(u, v, z = 0) \exp\left(\frac{ik}{2B}\right) \times \left[A(u^2 + v^2) + D(x^2 + y^2) - 2(ux + vy)\right] dudv,$$

where $A$, $B$, and $D$ are the elements of a $2 \times 2$ ray transfer matrix $(\frac{A}{B} \frac{D}{B})$ describing a linear optical system, $L$ is the eikonal along the propagation axis, and $k$ is the wave number of light. It is hard to obtain the analytical results, thus we would numerically solve the Eq. (3) in the following results.

III. RESULTS AND DISCUSSIONS

Use this kind of beam to trap and manipulate microparticles. The setup is shown in Fig. 1(a). The initial Gaussian beam is incident on a phase-only spatial light modulator (SLM, Holoeye Pluto-2-VIS-056) with high resolution (1920×1080 pixel) and 8 μm pixel pitch. The incident Gaussian beam is expended by the expander with the beam width $w_0 \approx 1.5$ mm. The polarized beam splitter is used to make the incident beam be horizontally polarized. Combining with the half-wave plate, the incident intensity of the laser beam is well controlled. The designed vortex pair phase is loaded on the SLM. Two examples of the vortex pair phases with topological charges $m_1=1, m_2=1$ and $m_1=1, m_2=-1$ are shown in Fig. 1(b). The first order of the generated beam after modulating by SLM is selected by an aperture. The vortex-pair beam
FIG. 1: (a) Schematic optical trapping system based on vortex-pair beam. Other notations are: HWP, half-wave plate; PBS, polarized beam splitter; SLM, spatial light modulator; AP, Aperture; BS, beam splitter; M, mirror; L, lens. Inset in (a) shows the vortex-pair beams propagate in a 2-f lens system with the focal length f = 50 cm. (The dash line represents the position for measuring the power of the incident beam.) (b) Two examples of vortex pair phase with different topological charges (b1) \( m_1=1, m_2=1 \); (b2) \( m_1=1, m_2=-1 \). (c) Numerical result of the evolution of vortex-pair beam under parameter \( m_1=6, m_2=6, a=0.4w_0 \) in a 2-f system as shown in the inset of (a). Here, \( w_0 = 1.5 \) mm, \( z = f + s \).

then goes through a 0.6 numerical aperture, \( \times 40 \) objective lens and is focused on the sample which is placed on a three-dimensional adjustable platform. The trapping system is configured with vortex-pair beam directed upwards which allows easier access to the adjustable platform. The sample is illuminated by a white light source and imaged onto a camera. The filter is used to block the green light reflected or scattered from the cover slip which has high intensity during the trapping process.

Figure 2 shows the trapping and manipulation of spherical microparticles using the vortex-pair beam. Here, polystyrene spheres (3 \( \mu \)m particle diameter) are used as probe samples in this experiment. It is observed from Fig. 2(a) that in the case of the vortex-pair beam with positive topological charges \( m_1 = 6, m_2 = 6 \), two spherical microparticles are trapped in the horizontal direction simultaneously. It be understood by Fig. 1(c) that when the vortex-pair beam is focused by a lens, there exists two light spots at the focal plane, which can directly trap two microparticles. When parameter \( a \) contained in the SLM phase structure is adjusted, the position of the trapped spherical microparticles is also changed. For instance, the separation distance \( d \) of two trapped spherical microparticles is 9.52 \( \mu \)m with \( a = 0.4w_0 \) as shown in Fig. 2(a1) while it decreases to 4.82 \( \mu \)m under \( a = 0.8w_0 \) displayed in Fig. 2(a3). It seems that the larger \( a \) is, the closer the trapped spherical microparticles are. Therefore, one can precisely control the position and distance of the interested microparticles by this method. However, in the case of \( m_1 = 6, m_2 = -6 \), it is seen in Fig. 2(b) that only single spherical microparticle is trapped as compared with the case of \( m_1 = 6, m_2 = 6 \). Furthermore, adjusting \( a \) makes no difference on the trapping results and a single

FIG. 2: Camera snapshots of spherical microparticle with diameter 3 \( \mu \)m inside the optical trap via vortex-pair beam by adjusting the off-axis parameter (a1, b1) \( a = 0.4w_0 \), (a2, b2) \( a = 0.6w_0 \), (a3, b3) \( a = 0.8w_0 \) of the initial phase of vortex-pair beam with different topological charges (a) \( m_1 = 6, m_2 = 6 \), (b) \( m_1 = 6, m_2 = -6 \). Here, \( w_0 = 1.5 \) mm.
spherical microparticle is trapped all the time.

Figure 3 further shows the dependence of separation distance between two trapped spherical microparticles on the off-axis distance $a$ of the initial vortex phase with two positive topological numbers. It is clearly seen that for example when $m_1 = 6, m_2 = 6$, the separation distance $d$ between two trapped spherical microparticles decreases linearly as $a$ increases. In other words, it shows the great advantage for the realization of accurate position-controllable trapping of spherical microparticles. Moreover, the separation distance $d$ between two trapped spherical microparticles decreases as two positive topological charges decrease under the same off-axis distance $a$. It tells us that it is better to choose a vortex-pair beam with two larger topological charge numbers for realization of a large position-controllable manipulation range between two trapped samples in practice.

It is observed from Fig. 1(b) that the initial phase structure of the vortex-pair beam is rotationally asymmetric. Therefore, naturally, this kind of beam becomes a perfect light source for controllable rotation of the trapped microparticle by rotating the beam once the microparticle is trapped. Experimentally, we can rotate the initial phase loaded on the SLM which is equivalent to rotate the vortex-pair beam. Figure 4 gives the experimental results of the controllable rotation of a cylindrical microrod realized by rotating the initial phase. The rotated initial phase is calculated by $\mathbf{u}' = u \cos \theta(t) + v \sin \theta(t)$ and $\mathbf{v}' = -u \sin \theta(t) + v \cos \theta(t)$, which is the rotation transformation of rectangular coordinates. The rotation angle can be arbitrarily designed ranged from $0^\circ$ to $360^\circ$. Here, $t$ represents the elapsed time of encoding initial vortex pair phase with different rotated angle on the SLM.

Thus $\bar{\omega} = \Delta \theta(t)/\Delta t$ can be the average angular speed of rotation of a vortex-pair beam, where $\Delta \theta$ and $\Delta t$ represents the step of rotated angle of encoding phase image and encoding time step, respectively (e. g. 100 images of phase are designed with rotated angle uniformly from $0^\circ$ to $360^\circ$, thus $\Delta \theta = 3.6^\circ$, and elapsed time for encoding 100 pieces of phase image is set to 10 s which means $\Delta t = 100$ ms). In this experiment, a special designed cylindrical microrod with length 19 $\mu$m and diameter 4.5 $\mu$m made of silica is used. It is observed from Fig. 4(a2) that when the vortex pair beam is rotated by $30^\circ$ by rotating the initial phase on SLM, the trapped cylindrical microrod is also rotated by near $30^\circ$ as compared with the non-rotation case as seen Fig. 4(a1). As expected, Fig. 4(a3) also shows that the rotation angle of the trapped cylindrical microrod is consistent well with the rotation angle of the initial vortex pair phase. It can be concluded that controllable rotation of trapping cylindrical microrod is realized and the rotation process is precisely controlled. Moreover, continuous rotation of the trapped cylindrical microrod is experimentally performed (see Visualization 1). In Fig. 4(b), a tweezed cylindrical microrod can be seen to rotate between the frames which is extracted from the video (see Visualization 1) at some typical angles. Here the average angular speed $\bar{\omega} = 30^\circ$/sec.

Figure 5 shows the propagation dynamics of the vortex-pair beam in a 2-$f$ focusing system to explain why such a vortex-pair beam can realize the trapping results in Figs. 2-4. Here, we want to emphasize that although there are some previous works related to the vortex-pair beam [37, 41, 42, 44], strictly speaking, those beams with pair of vortices are different from our design since we use a pure phase function given by Eq. 2 and there is no amplitude modulation. Moreover, little work has been done experimentally on investigating the focusing property of the vortex-pair beam. Therefore, it is necessary to systematically demonstrate the evolution of the vortex-pair beam propagating in a 2-$f$ lens system as shown in the inset of Fig. 4(a), which is equivalent to a objective lens system in the optical trapping system. The ray transfer matrix of a 2-$f$ linear optical system is given by $\begin{pmatrix} 0 & f \\ -1/f & 0 \end{pmatrix}$. Substitute it into Eq. 3, the output light field is obtained numerically. A lens with focal length $f = 50$ cm is applied to both experiments and numerical calculations. Fig. 5 shows the intensity dynamics of the vortex-pair beam with $a = 0.6w_0$ propagating at different propagation distance behind the lens. It is observed from experimental results in the top row of Fig. 5(a) that two ring-like intensity distributions resulted from the vortex pair phase interfere with each other. As the beam propagates to the focal plane, in the case of two positive charged numbers $m_1 = 6, m_2 = 6$, both of two ring-like intensity distributions rotate together by $\pi/2$ (anticlockwise) due to Gouy phase [37] and finally intensity distribution of the beam is localized at two areas on $x$-axis which is symmetric about the $x$-axis and $y$-axis, respectively, at focal plane. It explains why such a single
FIG. 4: Experimental realization of the angular-controllable rotation of the cylindrical microrod. (a) rotated initial phase (left) of the vortex pair beam loaded on SLM and rotated cylindrical microrod (right) trapped by the vortex-pair beam with specific angle (a1) $\theta = 0^\circ$, (a2) $\theta = 30^\circ$, (a3) $\theta = 120^\circ$; (b) camera snapshots of continuously rotating cylindrical microrod trapped by the vortex-pair beams with positive topological charge $m_1 = 6, m_2 = 6$, off axis distance $a = 0.6w_0$, $w_0 = 1.5$ mm.

FIG. 5: The experimental (top) and numerical (bottom) results of the propagation dynamics of vortex-pair beams with topological charge (a) $m_1 = 6, m_2 = 6$, (b) $m_1 = 6, m_2 = -6$ propagating in a $2\cdot f$ lens system under off-axis distance $a = 0.6w_0$ where $w_0 = 1.5$ mm. Here, $z = f + s$, $f = 50$ cm.

A vortex-pair beam with two positive charged numbers can trap two microparticles simultaneously as shown in Fig. 2. It is also understandable that when a single vortex-pair beam with two positive charged numbers is applied to trap the cylindrical microrod, both ends of the cylindrical microrod are firmly trapped by the beam like an optical wrench. Due to the non-rotational symmetry of the intensity distribution, when the beam is rotated externally by rotating the initial phase on SLM, torque is generated to exert on the cylindrical microrod for rotation. However, in the case of the vortex-pair beam with two opposite sign of topological charges as shown in Fig. 5(b), two ring-like intensity distribution rotate in the opposite direction. Thus, the intensity distribution at focal
plane is mainly located at the $y$-axis which is symmetric about the $y$-axis but not $x$-axis. Although it can also rotate the cylindrical microrod by rotating the beam, the rotation of the cylindrical microrod is not as smooth and stable as the case of the beam with two positive topological charges (see Visualization 2).

**IV. CONCLUSION**

In summary, we experimentally realized a new kind of optical tweezers based on a single vortex-pair beam with two positive charged numbers. It shows the great advantage that two spherical shaped microparticles are trapped simultaneously using such kind of beams. The separation distance between the two trapped spherical shaped microparticles can be tuned by the off-axis distance of the vortex pair within the initial phase. Moreover, it shows that the separation distance between the two trapped spherical shaped microparticles varied linearly with off-axis distance. Owing to anisotropic properties of the intensity distribution of the vortex-pair beam resulted from the anisotropy of the initial phase, the controllable rotation of the cylindrical microparticle is realized by rotating the initial phase structure. Finally, the propagation dynamics of the vortex-pair beam through a $2\cdot f$ lens system are investigated systematically to explain the reason why the vortex-pair beam can have those advantages for optical trapping and manipulation. Our results, to best our knowledge, is the first time to use a vortex-pair beam to trap and manipulate microparticles (both spherical and cylindrical) which may have great importance to the study of the optical tweezers. It may also have potential applications in many areas like biological area and nanofabrication.

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