Stabilization of Hyperbolic Brane-World Scenarios.

Pedro J. Silva

Physics Department,
Syracuse University,
Syracuse, New York 13244-1130.

Abstract

In this talk we consider the issue of stabilization of compact hyperbolic brane-world scenarios from the point of view of 4-dimensional effective theories. The idea is to clarify the status of stabilization for these models. Possible ways to overcome a no-go theorem that appeared in a recent paper are shown invoking the holographic framework and type IIA*/IIB* theories. A brief discussion on supersymmetry is also given.
I. **INTRODUCTION**

In the last couple of years brane-world scenarios have received much attention. These constructions are effective models, where our 4-dimensional world is realized as a brane embedded in a bigger universe, usually called the bulk space-time. Ideally this picture should result from a low energy limit of a more fundamental theory like string theory. In this short note we address the problem of stabilization of the extra-dimensions for the particular case of compact hyperbolic brane-world scenarios (CHBS). In CHBS the bulk space-time is of the form $M_4 \otimes H_d/\Gamma$, where $M_4$ is identified with our 4-dimensional world, $H_d$ is a $d$-dimensional hyperbolic manifold (a homogenous Euclidean space with constant negative curvature) and $\Gamma$ is a discrete subgroup of SO($d,1$) acting freely. These types of constructions enjoy many appealing characteristics that make the investigation of the stability issue worthwhile. For information on the phenomenological implications of these models we refer the original articles \[1, 2, 3\].

The main purpose of this talk is to present a detailed analysis of radion stabilization in CHBS (see the original paper \[4\]). It has recently been demonstrated \[5\] that, in the context of general relativity in $4 + d$ dimensions, stabilization of large hyperbolic extra dimensions, leaving Minkowski space on our brane, requires a violation of the null dominant energy condition. Here we extend this argument to the case in which our brane is allowed to exhibit standard FRW expansion and comment on the regime of validity of this result. We then turn to possible ways in which stabilization may work due to a breakdown of the assumptions in the previous argument by considering holography and type IIA*/IIB* supergravities.

In the second section, we show the flat brane case and recover the results of \[4\]. In the third section, we generalize to FRW branes and in the fourth section, the inclusion of extra compact directions like a $n$-sphere is considered. The fifth section contains some comments on possible ways to bypass the no-go-theorem and a small discussion on supersymmetry.
II. FLAT BRANE-WORLD AND D-DIMENSIONAL HYPERBOLIC MANIFOLD

Our starting point is for the Einstein gravity in 4+d-dimensional space-time, with bulk matter.

\[
S_{d+4} = \int dX^{4+d} \sqrt{-G} \left( M^{d+2} R(G) - L_{\text{bulk}} \right),
\]

where \( M \) is the 4 + d plank mass, \( L_{\text{bulk}} \) stands for the bulk matter field, the geometry is described by the metric \( ds^2 = G_{AB} dX^A dX^B = \bar{g}_{\mu\nu} dx^\mu dx^\nu + r^2 \gamma_{ij} dx^i dx^j \), where capital Latin letters runs over all of the space-time dimensions, Greeks letters over the 4-dimensional brane-world and lower case Latin over the hyperbolic manifold. \( \bar{g} \) is the brane metric, \( \gamma \) is the hyperbolic metric of radius \( d(d+1) \) and \( r \) is the radion, that we want to stabilized at the value \( R_h \). Using that we are in the case of compact hyperbolic manifold with volume \( \alpha \), we define the 4-dimensional planck mass \( M_4^2 = M^{2+d} R_h^d e^\alpha \) and the field \( \phi \) by the equation

\[
r = R_h e^{\sqrt{1/d(d+2)\phi/M_4}}.
\]

Also we need a conformal rescaling on the brane metric \( \bar{g}_{\mu\nu} = g_{\mu\nu} e^{-\sqrt{d(d+2)\phi/M_4}} \), to decouple the new field \( \phi \) and the reduced Einstein tensor. After some algebra we get the 4-dimensional effective action,

\[
S_{d+4} = \int dx^d \sqrt{g} S_{\text{eff}} = \int dx^4 \sqrt{-\hat{g}} \left[ M_4^2 R(g) - \frac{1}{2} (\nabla \phi)^2 - W(\phi, g) \right],
\]

where

\[
W(\phi, g) = \frac{M_4^2}{M^{d+2}} e^{-\sqrt{d(d+2)\phi/M_4}} L_{\text{bulk}} + \frac{d(d-1)M_4^2}{R_h^2} e^{-\sqrt{(d+2)\phi/M_4}}
\]

and we have used that \( R(\gamma) = -d(d-1) \). The stabilization of the radion translates into the following system of equations

\[
\partial_\phi W|_{\phi=0} = 0 , \quad \partial^2_\phi W|_{\phi=0} > 0 \quad \left( g_{\mu\nu} W - 2 \frac{\partial W}{\partial g_{\mu\nu}} \right)|_{\phi=0} = 0.
\]

To obtain information about the energy conditions that the bulk matter has to obey in order to satisfy this equations, we have to rewrite these effective equations in terms of the stress energy tensor of the bulk space-time. Here we use that the radion filed is really part of the metric, therefore its field equations involve some combinations of the stress energy tensor of the bulk matter fields, i.e.

\[
\frac{\partial}{\partial \phi} = \frac{\partial G^{\mu\nu}}{\partial \phi} \frac{\partial}{\partial \phi} + \frac{\partial G^{ij}}{\partial \phi} \frac{\partial}{\partial \phi},
\]
taking into account the conformal transformation on the brane metric, part of the set of equations translates into

\[
T_{\mu\nu} = \frac{d(d-1)M^{d+2}}{2R_h^2} G_{\mu\nu},
\]

\[
dG^{\mu\nu} T_{\mu\nu} - 2G^{ij} T_{ij} = \frac{d(d+2)(d-1)M^{2+d}}{R_h^2},
\]

(6)

where we have assume that the stabilization is archive by the matter field on the bulk. Next, using the following form for the stress energy tensor,

\[
T_{tt} = \rho,
\]

\[
T_{\alpha\beta} = p \Sigma_{\alpha\beta},
\]

\[
T_{ij} = qR_h^2 \gamma_{ij},
\]

(7)

where \(\mu\) has been divided into time and the three space-like directions \((t, \alpha)\). Then we get the constraints

\[
\rho = -\frac{d(d-1)M^{2+d}}{2R_h^2},
\]

\[
p = \frac{d(d-1)M^{2+d}}{2R_h^2},
\]

\[
q = \frac{(d-1)(d-2)M^{2+d}}{4R_h^2}.\]

(8)

This type of matter field does not obey the null energy condition as can be easily seen by considering a general null-vector along the hyperboloid, like \(l = \partial_t + e_i\), where \(e_i\) are orthogonal vector basis on the hyperboloid. Then, contracting \(l\) twice with the stress energy tensor \(T_{AB}\) gives \(-(d-1)M^{2+d}/R_h^2\), a negative number violating the above energy condition. Therefore we conclude this part of the discussion saying that:

*Although in principle equation can be satisfied, the matter field required will not satisfy the null energy condition.*

To clarify the above general statement, let us consider a simple example of stabilization due to matter field violating the null energy condition. Consider a flat brane-world and as bulk matter a cosmological constant \(\Lambda = M^{d+4}\), also include a d-form over the hyperbolic manifold \(F_{[d]}\). The bulk lagrangian is \(L_b = \Lambda + F^2/(2d!\), and the field equation for \(F_{[d]}\) can be solved by the ansatz \(F_{45...d+4} = B\) with \(B\) independent of the hyperbolic coordinates.
Using that there is a trapped magnetic flux on the compact hyperbolic space, the constant $B$ is related to the “radion” by the equation $B = bM^{(d+4)/2}(R_h/r)^d$. Therefore the on-shell form of the bulk lagrangian is $L_b = M^{d+4}\lambda + M^{d+4}b^2/2(R_h/r)^d$, and the equation reduces to

$$\lambda + \frac{b^2}{2} + d(d-1)\beta^2 = 0,$$

$$b^2d + d(d-1)(d+2)\beta^2 = 0,$$

where $\beta = R_h M$. The corresponding solution is $\lambda = -(d-1)(d-2)\beta^2/2$, and $b^2 = -(d-1)(d+2)\beta^2$, but the on-shell stress energy tensor of the d-form is $T_{AB} = \frac{(d-1)(d-2)\beta^2}{8}(g_{AB} - \delta_{AB}g_{ij})$, therefore when contracted twice with a null vector along one of the hyperbolic directions (i.e. $l = t^0e_0 + t^i e_i$), we get $T \cdot l \cdot l = -\frac{(d-1)(d-2)\beta^2}{8}g_{ii}l^i l^i < 0$, violating the null energy condition.

### III. FRW BRANE-WORLD AND D-DIMENSIONAL HYPERBOLIC MANIFOLD

Let us assume now that the metric of the four dimensional space-time is a of the form of a FRW metric,

$$ds^2 = -dt^2 + a(t)^2d\sigma^2,$$

where $d\sigma^2$ stands for the spatial part of the metric, with curvature $k = (+1, -1, 0)$. We can repeat the calculations of the previous section with this new metric obtaining the following set of equations:

$$T_{\mu\nu} - \frac{d(d-1)M^{d+2}}{2R_h^2}G_{\mu\nu} = (R_{\mu\nu} - \frac{1}{2}RG_{\mu\nu})M^{d+2},$$

$$dG^{\mu\nu}T_{\mu\nu} - 2G^{ij}T_{ij} = \frac{d(d+2)(d-1)M^{2+d}}{R_h^2},$$

where $R_{\mu\nu}, R$ are the Ricci tensor and the Ricci scalar of the four dimensional space-time and derivative respect to time are wrote as dots. Then, using the usual form for the stress energy tensor (see equation 7) we get,

$$\rho = \left[3 \left( \frac{k}{a^2} + \left( \frac{\dot{a}}{a} \right)^2 \right) - \frac{d(d-1)}{2R_h^2} \right] M^{2+d},$$

$$p = \left[- \left( \frac{k}{a^2} + \left( \frac{\dot{a}}{a} \right)^2 + 2\frac{\ddot{a}}{a} \right) + \frac{d(d-1)}{2R_h^2} \right] M^{2+d},$$

$$q = \left[-3 \left( \frac{k}{a^2} + \left( \frac{\dot{a}}{a} \right)^2 + \frac{\ddot{a}}{a} \right) + \frac{(d-1)(d-2)}{4R_h^2} \right] M^{2+d}.$$
Then, using general null vectors, we have to impose the following system of inequalities in order to satisfy the null energy condition:

\[
\begin{align*}
\frac{k}{a^2} + \left(\frac{\ddot{a}}{a}\right)^2 - \frac{\dot{a}^2}{a^2} &\geq 0, \\
\frac{\ddot{a}}{a} &\leq -\frac{(d-1)}{3R_h^2}, \\
2\frac{k}{a^2} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} &\geq \frac{d(d-1)}{R_h^2}, \\
2\frac{k}{a^2} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} &\geq \frac{(d-1)^2}{3R_h^2}.
\end{align*}
\]

(13)  
(14)  
(15)  
(16)

Note that if the first inequality is saturated then the third is irrelevant since the third inequality comes multiplied by the first inequality originally. The same is truth for second and fourth. After some analysis we get the following conclusions:

- If the first is saturated also the second has to be saturated to get a solution, then the space-time is of the form \(AdS_4 \otimes H_d\), and the curvature of \(AdS_4\) is same as \(H_d\).

- If the second is saturated but not the first, the resulting space time is not geodesically complete, as a parts of space has to be cut off.

- If third or fourth are saturated, there is no solution.

- If none of the inequalities are saturated then the second inequality rules out any physical solution as the acceleration of the brane-world radius \(a(t)\) measure in natural units is huge, producing incompatibilities with phenomenological data.

*Although in principle equation (12) can be satisfied, the matter field required will not satisfy the null energy condition or the solutions will not be relevant in this context.*

### IV. FRW Brane-World, D-Dimensional Hyperbolic Manifold and N-Spheres

Let us add to the previous space-time an n-dimensional sphere of radius \(r_s\) i.e. \(d(sphere)^2_n = r_s^2d\omega_{ab}dx^adx^b\) where \(d\omega_{ab}dx^adx^b\) corresponds to the metric of an n-sphere of unit radius with volume \(\Omega_n\). We have to modify our previous definitions by: the 4-dimensional plank mass is given by \(M_4^2 = M_4^{2+d+n}(R_h^d e^\alpha)(R_s^n \Omega_n)\), we define the new field \(\psi\) by the equation \(r_s = R_4 e^{1/(n(n+2))\psi/M_4}\). Also, we need a new conformal rescaling on the
brane metric $\bar{g}_{\mu\nu} = g_{\mu\nu} e^{[-\sqrt{d/(d+2)}\phi/M_4 - \sqrt{n/(n+2)}\psi/M_4]}$, to decouple the fields ($\phi, \psi$) and the reduced Einstein tensor. After some algebra we get,

$$S_{4+d+n} = \int dx^{d+n} \sqrt{\Omega} \frac{\sqrt{\omega}}{e^{\alpha}} S_{\text{eff}},$$

(17)

where

$$S_{\text{eff}} = \int dx^4 \sqrt{-g} \left[ M_4^2 R(g) - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} (\nabla \psi)^2 - 2 \sqrt{\frac{nd}{(d+2)(n+2)}} \nabla \phi \nabla \psi - W(\phi, \psi, g) \right],$$

(18)

and the potential is given by the expression

$$W(\phi, \psi, g) = \frac{M_4^2}{M_{4+d+2}} C_{\text{bulk}} e^{[-\sqrt{d/(d+2)}\phi/M_4 - \sqrt{n/(n+2)}\psi/M_4]} + \left( \frac{d(d-1)}{R_h^2} - \frac{n(n-1)}{R_s^2} \right) M_4^2 e^{[-\sqrt{(d+2)/d}\phi/M_4 - \sqrt{(n+2)/n}\psi/M_4]}.$$

(19)

where we used that $R(\omega) = n(n-1)$. The stabilization of the two radions translates into the following system of equations:

$$\partial_\phi W|_{(\phi, \psi)=0} = 0 \quad \partial_\phi^2 W|_{(\phi, \psi)=0} > 0$$

$$\partial_\psi W|_{(\phi, \psi)=0} = 0 \quad \partial_\psi^2 W|_{(\phi, \psi)=0} > 0$$

$$\left(g_{\mu\nu} W - 2 \frac{\partial W}{\partial g_{\mu\nu}}\right)|_{(\phi, \psi)=0} = 0.$$ 

(20)

In terms of the stress energy tensor we get

$$T_{\mu\nu} + \frac{1}{2} \left( \frac{n(n-1)}{R_h^2} - \frac{d(d-1)}{R_s^2} \right) M^{n+d+2} G_{\mu\nu} = (R_{\mu\nu} - \frac{1}{2} R G_{\mu\nu}) M^{n+d+2},$$

$$d G^{\mu\nu} T_{\mu\nu} - 2 G^{ij} T_{ij} = \frac{d(d+2)(d-1) M^2}{R_h^2},$$

$$d G^{\mu\nu} T_{\mu\nu} - 2 G^{ab} T_{ab} = -\frac{n(n+2)(d-1) M^2}{R_s^2}.$$ 

(21)

Assuming the that $T_{ab} = e R_s^2 \omega_{ab}$ and equation [7] we the following set of equations,

$$\rho = 3 \left( \frac{k}{a^2} + \frac{\dot{a}}{a^2} \right) + \frac{1}{2} \left( \frac{n(n-1)}{R_s^2} - \frac{d(d-1)}{R_h^2} \right) M^{2+d+n},$$

$$p = - \left( \frac{k}{a^2} + \frac{\dot{a}}{a^2} + \frac{\ddot{a}}{a} \right) - \frac{1}{2} \left( \frac{n(n-1)}{R_s^2} - \frac{d(d-1)}{R_h^2} \right) M^{2+d+n},$$

$$q = -3 \left( \frac{k}{a^2} + \frac{\dot{a}}{a^2} + \frac{\ddot{a}}{a} \right) - \frac{1}{2} \left( \frac{2n(n-1)}{R_s^2} - \frac{d(d-1)}{R_h^2} \right) M^{2+d+n},$$

$$e = -3 \left( \frac{k}{a^2} + \frac{\dot{a}}{a^2} + \frac{\ddot{a}}{a} \right) - \frac{1}{2} \left( \frac{(n-1)(n-2)}{R_s^2} - \frac{2d(d-1)}{R_h^2} \right) M^{2+d+n}.$$ 

(22)
Then, the null energy condition impose the following system of inequalities:

\[ \frac{k}{a^2} + \left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a} \geq 0, \quad (23) \]

\[ 2 \frac{k}{a^2} + 2 \left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} \geq \frac{1}{2} \left( - \frac{n(n-1)}{R_s^2} + \frac{d(d-1)}{R_h^2} \right), \quad (24) \]

\[ \frac{\ddot{a}}{a} \leq -\frac{1}{6} \left( \frac{n(n-1)}{R_s^2} + \frac{2(d-1)}{R_h^2} \right), \quad (25) \]

\[ 2 \frac{k}{a^2} + 2 \left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} \geq \frac{1}{6} \left( - \frac{3n(n-1)}{R_s^2} + \frac{2(d-1)^2}{R_h^2} \right), \quad (26) \]

\[ \frac{\ddot{a}}{a} \leq \frac{1}{6} \left( \frac{2(n-1)}{R_s^2} + \frac{d(d-1)}{R_h^2} \right), \quad (27) \]

\[ 2 \frac{k}{a^2} + 2 \left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} \geq \frac{1}{6} \left( - \frac{2(n-1)^2}{R_s^2} + \frac{3d(d-1)}{R_h^2} \right). \quad (28) \]

Note that if the first inequality is saturated then the second is irrelevant since the second inequality comes multiplied by the first inequality originally. The same is truth for the third and fourth, and fifth and sixth. After some analysis we find that:

- If inequality (23) is saturated, the solution is not useful as the Ricci scalar is of the same order as the Hyperbolic manifold.

- If inequality (24) is saturated the resulting space time is not geodesically complete, as a parts of space have to be cut off.

- If inequality (27) is saturated, the Ricci scalar of the brane-world is bigger than the biggest curvature scalar of any of the compact spaces.

- If any of the reaming inequalities are saturated there are no solutions to the system.

- If none of the inequalities are saturated, although in principle there could be solutions to the system, by means of inequality (23) we get back to situation in the previous section, where the acceleration of the brane-world radius \( a(t) \) measured in natural units is huge, producing incompatibilities with phenomenological data.

Therefore the conclusion of the previous analysis is:

*Although in principle equation (22) can be satisfied, the matter field required will not satisfy the null energy condition or the solutions will not be relevant in this context.*
V. COMMENTS ON CHBS AND STRING THEORY

If you want to insist on the possibility of hyperbolic brane-world scenarios, some of the initial assumptions have to be relaxed to bypass the no-go theorem. Because these scenarios are thought to be induced from string theory or M-theory, there are a couple of generalization that can be introduced within the framework of the different string dualities.

First let us consider the possibility of more general geometries than the one used. Basically we can look for ten or eleven dimensional supersymmetric solutions with hyperbolic spaces built in. In general lets start with the action

$$S_{\text{eff}} = \int dx^D \sqrt{-G} \left( R - \sum_k \frac{1}{2k!} F_{[k]}^2 \right),$$

(29)
in $D$-dimensions, with a few k-form field strengths $F_k$. For example, assuming we have three fields $F_d, F_p, F_q$, where each one is proportional to the corresponding volume-form on the different subspace where the three field strength lives, we get the following space-time types of solutions:

$$AdS_d \otimes H_p \otimes S^q,$$

$$AdS_d \otimes H_p \otimes T^q,$$

$$AdS_d \otimes H_p \otimes H^q.$$

(30)

Where the different solutions correspond to different ratios between the proportionality constants of the fields $F_k$. The $AdS$ part of the solution can not be understood as our brane-world, as its curvature is proportional to the hyperbolic manifold curvature, giving no useful phenomenological models. Nevertheless, what you can certainly do is to pick up a solution of the form $AdS_5 \otimes H_d \otimes \text{something}$ and instead of thinking of it as a solution of low energy critical string theory, use it as a solution of a non-critical string theory, where one of the $AdS$ directions is identify with the Liouville field. In this case taking the usual holographic coordinates in $AdS_5$ we get a flat 4-dimensional brane world, with a hyperbolic manifold and a holographic direction giving information about the different energy scales probed by the theory. This type of interpretation is along the same line of reasoning of the confining string of Polyakov [8], and also in the Verlinde et. al. constructions [9]. Of course in the ultraviolet limit there will be no gravitation on the brane (only a flat brane with no gravitons propagating on it), but at intermediate energies there will be gravity on
the brane, and generalizations to curved branes are possible. This picture will look like a
Randall-Sundrum curved brane-world.

Another arena where to look for CHBS is within the context of type IIA*/IIB* supergravity. These supergravity theories have been argue to result from t-dualizing along the
time-like direction. As a result of this duality the signature of the space-time metric can
change, obtaining for example two time-like directions (see \[8\]). To illustrate the mechanism
that allows the bypassing of the theorem, let us consider F-theory \[9\] (just as an example
of a framework with two time-like directions). The bulk manifold has twelve dimensions in-
cluding two time-like directions. We will be using the signature \((-+++-++++++++)\).
In this framework we are in the presence of a 4-form \(F_4\) \[10\], therefore using the ansatz

\[
F_{[4]} = f^a_{[4]} + f^b_{[4]},
\]

\[
f^a_{[4]}_{4...8} = a \epsilon_{4...8},
\]

\[
f^b_{[4]}_{9...11} = b \epsilon_{9...11},
\]

and decomposing the 4,5,6,7 directions into \(\tau, u, x, y\), we get the following space-time solu-
tion,

\[
M_{(0,1,2,3)} \otimes H_{(x,y)} \otimes S^4 \otimes AdS_{(\tau,u)} \quad (a = b),
\]

\[
AdS_4 \otimes H_{(x,y)} \otimes S^4 \otimes AdS_{(\tau,u)} \quad (a > b),
\]

\[
dS_4 \otimes H_{(x,y)} \otimes S^4 \otimes AdS_{(\tau,u)} \quad (a < b).
\]

The first case has exactly the structure we are looking for, and also note that we have the
correct \(SL(2, R)\) symmetry in the part of the metric that should account for the correspond-
ing type IIB symmetry between the dilaton and the axion. Thus due to the presence of an
extra time-like direction the no-go theorem is bypass.

Finally, assuming that the above frameworks bring no other problems or conflicts with
phenomenological observations, a short discussion on the supersymmetric properties of
CHBS seems necessary.

A question of great interest when suggesting any compactification scheme is that of
low-energy supersymmetry. For the case of interest in this paper, we may begin with an
explicit construction of the Killing spinors of maximally symmetric spaces with negative
cosmological constant \[11, 12\]. For the space \(H^d\) we choose coordinates in the horospherical
frame in which the metric takes the form,

\[ ds^2 = e^{2r}\delta_{\alpha\beta}dx^\alpha dx^\beta + dr^2. \]  

(33)

In this frame, the killing spinors are given by,

\[ \xi = e^{\frac{1}{2}r\Gamma_r} \left[ 1 + \frac{1}{2}x^\alpha \Gamma_\alpha (1 - \Gamma_r) \right] \epsilon, \]  

(34)

where \( \epsilon \) is an arbitrary constant spinor (the cases \( d=2,3 \) are special and expressions can be found in ref [13]).

We can see from this expression that the number of supersymmetries of \( H^d \) is equal to the number of independent spinor components. Now, recall that the isometry group is \( SO(1,d) \) and that compact hyperbolic manifolds are obtained by quotient of \( H^d \) by a discrete subgroup \( \Gamma \) of \( SO(1,d) \), with no fixed points. Whether or not any killing spinors survive this quotienting process depends on \( \Gamma \) [14].

1. If \( d \) is even then the spinors are in an \( SO(1,d-1) \) representation, and all supersymmetries are broken.

2. If \( d \) is odd, then the spinors are in an \( SO(1,d) \) representation. In this case there are several possibilities.

   (a) If \( \Gamma \) is a subgroup of \( SO(1,d-1) \) some killing spinors may survive, since we can decompose the original killing spinors into Weyl spinors on the representation of this group.

   (b) If \( \Gamma \) is a not subgroup of \( SO(1,d-1) \) then all supersymmetries are broken.

   (c) In the special case \( d = 3 \) there also remain no supersymmetries.

Acknowledgments

The author would like to thank the organizers of the conference *Theoretical High Energy Physics: SUNY Utica/Rome*, where this talk was presented. This work was supported
in part by NSF grant PHY-0098747 to Syracuse University and by funds from Syracuse University.

[1] N. Kaloper, J. March-Russell, G. D. Starkman, M. Trodden, Phys. Rev. Lett. 85 (2000) 928-931, hep-ph/0002001.
[2] G. D. Starkman, D. Stojkovic and M. Trodden, Phys. Rev. D 63, 103511 (2001), hep-th/0012226.
[3] G. D. Starkman, D. Stojkovic and M. Trodden, Phys. Rev. Lett. 87, 231303 (2001), hep-th/0106143.
[4] S. Nasri, P. J. Silva, G. D. Starkman and M. Trodden, hep-th/0201063.
[5] S. M. Carroll, J. Geddes, M. B. Hoffman and R. M. Wald, hep-th/0110149.
[6] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Rev. Lett. B428 (1998) 105-114, hep-th/9802109.
A. Polyakov, Int. J. Mod. Phys. A14 (1999) 645-658, hep-th/9809057.
[7] E. Verlinde and H. Verlinde, JHEP 9912 (1999) 022, hep-th/9912018.
[8] C. M. Hull, "Duality and Strings, Space and Time" hep-th/9911080.
[9] Cumrun Vafa, Nucl. Phys. B469 (1996) 403-418, hep-th/9602022.
[10] Supriya Kar, Nucl. Phys. B497 (1997) 110-126, hep-th/9701117.
[11] H.Lu, C.N. Pope and P.K. Townsend, Phys. Lett. B391 (1997) 39-46, hep-th/9607164.
[12] H. Lu, C.N. Pope and J. Rahmfeld, J. Math. Phys. 40 (1999) 4518-4526, hep-th/9805151.
[13] Y. Fujii and K. Yamagishi, J. Math. Phys. 27 (1986) 979.
[14] A. Kehagias and J.G. Russo, JHEP 0007 (2000) 027 hep-th/0003281.