Robust wideband adaptive beamforming based on covariance matrix reconstruction in the spatial-frequency domain

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Abstract
Without the response variation constraint, the main-lobe responses of wideband beamforming usually vary with the frequencies of interest, and the beam pointing may deviate from the incident angle of the desired signal. If the interference-plus-noise covariance matrix (INCM) is directly replaced by the sample covariance matrix, the desired signal in the sample data may result in self-null problem, which can lead to the degradation of the performance of wideband beamformers. Through frequency division, a novel wideband adaptive beamforming is proposed by reconstructing the INCM and solving the convex optimisation problem. Using fast Fourier transform (FFT), the time-domain samples are transformed into the frequency field, and the covariance matrix of each subband can be obtained from the frequency-domain data. In the spatial-frequency domain, the INCM and signal-plus-noise covariance matrix can be respectively reconstructed by integrating the stacked steering vector and the Capon spatial spectrum over the corresponding angular region. Then, we establish the optimisation criteria, that is, minimising the output interference-plus-noise power, minimising the main-lobe spatial response variation, and constraining the response deviation of the desired signal. A novel convex optimisation model can be established, and then the weight coefficients for time-domain wideband beamforming can be obtained by solving the optimisation problem. The simulation and experimental results demonstrate that the proposed beamformer can provide better performance compared with other tested beamformers and maintain good robustness in various applications.

1 | INTRODUCTION

With the widespread use of array antenna and the increasing bandwidth of the desired signal, wideband adaptive beamforming has found a range of applications, including radar, sonar, medical imaging, wireless communication and radio astronomy [1–5]. Some results have been reported for wideband beamforming, which can be divided into two major categories, that is, frequency-domain wideband beamforming and time-domain wideband beamforming. As shown in Figure 1, a classical wideband beamformer in the time domain employs a series of tapped delay lines (TDLs), which can form a frequency-dependent response for the received wideband signals to compensate for the phase differences for different frequency components [2,6]. Generally, this approach can be realised by designing a group of finite impulse response (FIR) filters to shape the temporal and spatial responses [7–9]. In Ref. [10], an equivalent multi-input and single-output system was treated as the wideband beamforming network, and the frequency-dependent optimal Riblet-Chebyshev weights can be obtained by system identification. By optimising the hardware components of array antenna, a complex domain radio frequency frontend was designed to accomplish adaptive beamforming [11]. Alternatively, an infinite impulse response (IIR) filter may be used, which can satisfy the design requirements with a lower filter order while having a smaller group delay [12–14]. In Ref. [15], similar to IIR filters, a new space-time beamforming technique based on Laguerre filters was proposed to achieve a low computational load with good performance and stability. For this method, the main-lobe
beam direction is decided by the linear constraint minimum variance (LCMV) criterion, which may lead to poor robustness against the signal direction error. In Ref. [16], a robust wideband beamforming was proposed, which invoked a response variation element to control the consistency of the beamformer’s response and utilised the eigenvector constraints to trade off the direction-of-arrival (DOA) mismatch. However, designing an accurate constraint set is difficult in practice. If the error constraints are too small, the algorithm may fail. On the contrary, the algorithmic performance may dramatically degrade.

In addition to the time-domain method, the frequency-domain approach generally decomposes the time-domain signals into several sets of narrowband data and then applies narrowband beamforming to each subband; the final wideband beamforming output can be obtained by synthesising the output data of all narrowband beamformers accordingly. Although general filter banks can be employed for subband decomposition of the wideband signals [17], Fourier transform implemented by fast Fourier transform (FFT) with higher computational efficiency is normally adopted [18,19]. In Ref. [20], a nonuniform decomposition method was proposed to design subband adaptive beamforming, which can dramatically improve the overall computational efficiency. However, without the frequency-invariant constraint on the main-lobe, the main-lobe width may vary with the frequency of signal of interest. Because the frequency-domain wideband beamforming is realised by a series of independent narrowband beamformers, a phase shift corresponds with the centre of each subband [21], and the phase errors in the radar system may have negative effects on subsequent signal processing, such as target recognition and coherent integration.

It is well known that adaptive beamformers are very sensitive to various model mismatches, such as array calibration errors, gain and phase perturbations, incoherent local scattering, wavefront distortion and DOA errors. For narrowband beamforming, many solutions have been proposed, such as the diagonal loading (DL) technique [22–24], the uncertainty set-based technique [25–27], and the eigenspace-based technique [28,29]. Recently, robust adaptive beamforming based on covariance matrix reconstruction has attracted substantial attention [15,30–32]. In Ref. [33], by transforming the response variation inequality constraint into a weighted part of the objective function, the FIR coefficient vector can be obtained by reconstructing the interference-plus-noise covariance matrix (INCM) and estimating the steering vector of the desired signal. However, multiple bandpass filters are used for subband segmentation of the samples, which results in the lack of flexibility. In addition, the DOAs of all interferences must be roughly estimated with a robust low-resolution method. In Ref. [34], a beamformer without TDLs was designed by reconstructing the INCM using a narrowband Capon spectral estimator. This method uses a pre-steering technique to compensate for the propagation delay of the desired signal, and its performance depends on the accuracy of the pre-steering delays. In Ref. [36], the worst-case performance optimisation criterion was used to design a robust wideband beamformer, which can achieve a better tradeoff between robustness and performance. However, this method must solve two optimisation problems: one is to find the optimal positive semi-definite Hermitian matrix, and the other is to find the final weight coefficients, which significantly increases its computational efficiency.

We propose a robust wideband adaptive beamformer by reconstructing the INCM and constraining the response variation of the desired signal. The main contributions of the work can be summarised as follows:

1. The sampling signals are decomposed into multiple subbands by FFT with high computational efficiency, and the subband covariance matrices can be constructed in frequency domain.
2. In the spatial-frequency domain, a novel reconstruction method of INCM and signal-plus-noise covariance matrix (SNCM) is proposed by utilising the frequency-domain covariance matrix and spatial integration.
3. Based on maximising the output signal-to-interference-plus-noise ratio (SINR), a new optimisation model is established by constraining the response deviation of the desired signal, minimising the output interference-plus-noise energy and main-lobe spatial response variation.

The remainder of this paper is organised as follows. In Section II, the signal model and problem background are briefly introduced. In Section III, the wideband adaptive beamformer based on covariance matrix reconstruction in the spatial-frequency domain and convex optimisation model is proposed. Simulation results are provided in Section IV, and conclusions are drawn in Section V.

2 | SIGNAL MODEL AND PROBLEM BACKGROUND

2.1 | Wideband signal decomposition

Consider that a uniform linear array (ULA) of N omnidirectional elements receives M far-field wideband signals
composed of one desired signal and M-1 interferers, which impinge from incident angles \( \theta = [\theta_1, \theta_2, \ldots, \theta_M]^T \). Assume that \( s_i(t) \) is the desired signal and \( s_m(t) (2 \leq m \leq M) \) are interferers. Hence, the received signal at the \( n \)-th sensor can be modelled as
\[
s_n(t) = \sum_{m=1}^{M} s_m(t - \tau(n, m)) + n_n(t),
\]
where \( \tau(n, m) \) denotes the time delay of the \( m \)-th impinging signal arriving at the \( n \)-th sensor, \( s_n(t) \) and \( n_n(t) \) denote the \( m \)-th far-field wideband signal and the system noise at the corresponding sensor, respectively. Assume that the desired signal, interferences and noise are statistically independent with each other. The received signal vector can be written as \( \mathbf{s}(t) = [s_1(t), s_1(t), \ldots, s_{N-1}(t)]^T \).

For the ULA with adjacent sensor spacing \( d \) of a half wavelength of the maximum frequency, the received signal \( s_n(t) \) on the \( n \)-th sensor is sampled with the frequency \( f_s \). Then, an \( F \)-point discrete Fourier transform (DFT) is applied to the sampled data \( s_n(q) \), and the array output data at the \( l \)-th frequency bin can be expressed as \([37]\).

\[
\hat{x}(l) = \mathbf{A}(l, \theta)\mathbf{x}(l) + \mathbf{n}(l),
\]
where \( \hat{x}(l) \) denotes the frequency-domain array signal vector at the \( l \)-th frequency bin, \( \mathbf{x}(l) = [x_1(l), x_2(l), \ldots, x_M(l)]^T \) denotes the signal vector consisting of \( M \) far-field signals at the \( l \)-th frequency bin, \( \mathbf{n}(l) \) denotes the noise vector at the \( l \)-th frequency bin, and \( \mathbf{A}(l, \theta) = [a[l, \theta_1], \ldots, a[l, \theta_M]] \) denotes the steering matrix at frequency \( f_l \) corresponding to the \( l \)-th frequency bin. At direction \( \theta_m \) and frequency \( f_a \), the steering vector \( a[l, \theta_m] \) can be written as
\[
a(f_l, \theta_m) = [e^{-j2\pi f_a(0,m)}, \ldots, e^{-j2\pi f_a(N-1,m)}]^T,
\]
where \( (\cdot)^T \) denotes the transpose operator, and \( \tau(n, m) = (n-1) \sin \theta_m / c \) with \( c \) represent the signal propagation velocity. In frequency domain, the frequency-domain data \( \hat{x}(l) \) \( (1 \leq l \leq F) \) are partitioned into \( K \) disjoint sub-blocks with the same size, and the \( k \)-th subband data can be expressed as
\[
\mathbf{X}_{k\hat{f}_k} = \begin{bmatrix} \mathbf{x}_{1f_k} \\ \mathbf{x}_{2f_k} \\ \vdots \\ \mathbf{x}_{N-1f_k} \\ \mathbf{x}_{nf_k} \end{bmatrix} \in \mathbb{C}^{N \times L},
\]
where \( 1 \leq k \leq K \), \( C \) denotes a complex number, \( \mathbf{x}_{nf_k} \) denotes the data vector of the \( n \)-th sensor at the \( k \)-th subband, \( L = [F/K] \) denotes the data length of each subband, with \( \lfloor \cdot \rfloor \) representing the round down operator. Therefore, each subband data can be considered as the frequency-domain form of a narrowband component. For the \( k \)-th subband, the centre frequency can be calculated by
\[
f_k = f_c + (k - 1) \frac{B}{K}, \quad k \leq \frac{K}{2}
\]
\[
f_k = f_c + (k - 1 - K) \frac{B}{K}, \quad k > \frac{K}{2}
\]
where \( f_c \) denotes the carrier frequency, \( B \) denotes the bandwidth of the signal of interest. Correspondingly, in frequency domain, the array output data at the \( k \)-th subband can be modelled as
\[
\hat{\mathbf{X}}_{k\hat{f}_k} = \mathbf{A}(f_k, \theta)\mathbf{x}_k + \mathbf{n}_k,
\]
where \( \mathbf{A}(f_k, \theta) = [a(f_k, \theta_1), \ldots, a(f_k, \theta_M)] \), \( \mathbf{x}_k = [x_1(f_k), \ldots, x_M(f_k)]^T \) denotes the time-domain signal vector of \( M \) incident signals at the \( k \)-th subband and \( \mathbf{n}_k \) denotes noise at the \( k \)-th subband. Assume that the noise at each subband follows the Gaussian distribution with zero mean and variance \( \delta_k^2 \). Hence, at the \( k \)-th subband, the covariance matrix \( \mathbf{R}_{f_k} \) can be expressed as
\[
\mathbf{R}_{f_k} = \frac{1}{L} \hat{\mathbf{X}}_{k\hat{f}_k} \mathbf{X}_{k\hat{f}_k}^H = \sum_{m=1}^{M} \sigma_{m,f_k}^2 \mathbf{a}(f_k, \theta_m) \mathbf{a}^H(f_k, \theta_m) + \delta_k^2 \mathbf{I}_N,
\]
where \( (\cdot)^H \) stands for Hermitian transpose, \( \sigma_{m,f_k}^2 = E(|x_{mf_k}(l)|^2) \) \( (1 \leq m \leq M) \) is the power of the \( m \)-th far-field signal at the \( k \)-th subband, \( \delta_k^2 \) is the power of noise at \( k \)-th subband, and \( \mathbf{I}_N \) is the \( N \times N \) identity matrix. Correspondingly, the Capon spatial spectrum estimator at the \( k \)-th subband can be written as \([32]\).
\[
\hat{P}_{f_k}(\theta) = \frac{1}{\mathbf{a}^H(f_k, \theta) \mathbf{R}_{f_k}^{-1} \mathbf{a}(f_k, \theta)}.
\]
With the Capon spatial spectrum, we can reconstruct the INCM in the spatial-frequency domain.

### 2.2 Time-domain wideband beamforming

Assume that in time domain, all interferers can be divided into \( K \) subband signals \( s_{m,k}(q), (1 \leq k \leq K) \) at different centre frequencies. At the \( k \)-th \( (k = 1, 2, \ldots, K) \) subband, the received data of interference-plus-noise in the discrete form at the \( q \)-th snapshot can be expressed as
\[
\hat{s}_{m,k}(q) = \sum_{m=2}^{M} a(f_k, \theta_m)s_{m,k}(q) + n_{nf_k}(q).
\]
Figure 1 is a time-domain wideband beamforming structure based on the FIR filter. In the structure of the time-domain wideband beamformer, the \( N \times J \) observation data matrix \( \mathbf{s}_{n,k}(q) \) at the \( q \)-th snapshot can be written as

\[
\mathbf{s}_{n,k}(q) = [\mathbf{s}_{n,k}(q), \mathbf{s}_{n,k}(q-1), \ldots, \mathbf{s}_{n,k}(q-J+1)],
\]

where \( \mathbf{s}_{n,k}(q) \) is the stacked observation vector at the time instant \( q \). Then we have the stacked vector

\[
\mathbf{s}_{n,k}(q) = \text{vec}(\mathbf{s}_{n,k}(q)) \in \mathbb{C}^{NJ \times 1},
\]

where \( \text{vec}\{\cdot\} \) denotes the vectorisation operator and \( \mathbb{C}^{NJ \times 1} \) denotes a \( NJ \)-dimensional complex vector. Accordingly, the \( NJ \times NJ \) INCM at the \( k \)-th subband can be written as

\[
\mathbf{R}_{i+n} = \frac{1}{Q} \mathbf{S}_{n,k}^{H} \mathbf{S}_{n,k} \in \mathbb{C}^{NJ \times NJ},
\]

where \( \mathbf{S}_{n,k} = [\mathbf{s}_{n,k}(1), \ldots, \mathbf{s}_{n,k}(Q)] \) is a \( NJ \times Q \) observation data matrix at the \( k \)-th subband, \( Q \) is the number of snapshots. Then, we have the INCM as follows

\[
\mathbf{R}_{i+n}^{H} = \int_{f_{i}}^{f_{i+}} \mathbf{R}_{i+n}^{H} df_{k},
\]

where \( f_{i} \) and \( f_{i+} \) are the lowest and highest frequencies of the desired signal, respectively. The output interference-plus-noise power of the time-domain wideband beamformer can be expressed as

\[
P_{i+n} = \mathbf{w}^{H} \mathbf{R}_{i+n} \mathbf{w},
\]

where \( \mathbf{w} = [\mathbf{w}_{0,0}, \ldots, \mathbf{w}_{N-1,0}, \ldots, \mathbf{w}_{0,f-1}, \ldots, \mathbf{w}_{N-1,f-1}]^{T} \) is the weight coefficients of the wideband beamformer. At time \( q \), the wideband beamforming output can be calculated by

\[
y_{k}(q) = \mathbf{w}^{H} \mathbf{s}(q),
\]

where \( \mathbf{s}(q) \) denotes the \( NJ \)-dimensional stacked observation vector at the time instant \( q \). Note that \( \mathbf{s}_{n,k}(q) \) in Equation (10) is just a component of \( \mathbf{s}(q) \) on the \( k \)-th subband, and in practice, all we can get is \( \mathbf{s}(q) \). The sample data vector \( \mathbf{s}(q) \) can be written as

\[
\mathbf{s}(q) = [\mathbf{s}_{0}(q), \mathbf{s}_{1}(q), \ldots, \mathbf{s}_{N-1}(q)]^{T},
\]

where \( \mathbf{s}_{n}(q) \) denote the sample data on the \( n \)-th sensor. The \( N \times J \) observation data matrix is expressed as

\[
\mathbf{s}(q) = [\mathbf{s}(q), \mathbf{s}(q-1), \ldots, \mathbf{s}(q-J+1)].
\]

By vectorising the matrix \( \mathbf{s}(q) \), we can obtain the stacked observation vector \( \mathbf{s}(q) \). The optimal weight vector \( \mathbf{w} \) can be obtained by maximising the output SINR,

\[
\text{SINR} = \frac{\mathbf{w}^{H} \mathbf{R}_{i+n} \mathbf{w}}{\mathbf{w}^{H} \mathbf{R}_{i+n} \mathbf{w}},
\]

where \( \mathbf{R}_{i+n} \) denotes the signal covariance matrix. The maximisation of Equation (18) is mathematically equivalent to the following optimisation problem

\[
\min_{\mathbf{w}} P_{i+n} = \mathbf{w}^{H} \mathbf{R}_{i+n} \mathbf{w},
\]

subject to \( \mathbf{w}^{H} \mathbf{u}(f_{k}, \theta_{i}) = 1, k = 1, \ldots, K, \)

where \( \mathbf{u}(f_{k}, \theta_{i}) \) denotes the \( NJ \)-dimensional stacked steering vector of the desired signal. According to the structure of the time-domain wideband beamformer, \( \mathbf{u}(f_{k}, \theta_{i}) \) can be written as

\[
\mathbf{u}(f_{k}, \theta_{i}) = \mathbf{a}(f_{k}, \theta_{i}) \otimes \mathbf{e}(f_{k}) = [e^{-2\pi f_{k}T_{e}}, \ldots, e^{-2\pi f_{k}(N-1)T_{e}}, \ldots],
\]

\[
\mathbf{e}(f_{k}) = [1, e^{-2\pi f_{k}T_{e}}, \ldots, e^{-2\pi f_{k}(J-1)T_{e}}]^{T} \in \mathbb{C}^{J \times 1},
\]

where \( T_{e} \) is the sampling interval. In practice, the interference-plus-noise data \( \mathbf{s}_{n,k}(q) \) are usually unavailable, and then INCM \( \mathbf{R}_{i+n} \) cannot be directly obtained. However, the presence of the desired signal in the sample data may cause self-null phenomena and result in dramatic performance deterioration, especially at high signal-to-noise ratios (SNRs) [32]. Through frequency segmentation, the covariance matrix at each subband can be reconstructed. Moreover, in the beampattern of wideband beamforming, the main-lobe width usually varies with the signal frequency. Therefore, the frequency-invariant constraints must be used to control the main-lobe beam response and keep the beamwidth constant over the frequency of interest.

3 | PROPOSED ALGORITHM

In this section, we will describe the proposed wideband adaptive beamforming in detail. The core idea of the proposed algorithm is to design a novel optimisation model by reconstructing the covariance matrix in the spatial-frequency domain and constraining the response variation of the desired signal. The coefficient vector of the FIR-filter-based wideband beamformer can be obtained by solving the optimisation problem.

3.1 | Minimising output interference-plus-noise energy

To design an efficient wideband beamformer, we actually constrain the response of the beamformer in the frequency range of the signal of interest and then design the wideband beamformer over the specified frequency range [38]. In this way, the operating band of the designed beamformer is limited to the bandwidth of the desired signal, which is beneficial for obtaining the optimal weight coefficients. With fewer constraints, the
process of solving the optimisation problem will be dramatically accelerated. Before being sent into the wideband beamformer, the sample data need to be preprocessed to be limited in the desired bandwidth. The data preprocessor consists of a low-noise amplifier, bandpass filter, down-conversion mixer and digital extraction. Through data preprocessing, we only need to design constraints within the frequencies of interest.

Since the desired signal is often contained in the sample data, the beamforming response may result in the self-null problem in the DOA of the desired signal and cause dramatic performance degradation. Correspondingly, to improve the robustness of the wideband beamforming, it is necessary to remove the desired signal from the sample covariance matrix, and the INCM reconstruction based on frequency segmentation is proposed. In practice, the covariance matrix $R_{f_k}$ at each subband cannot be directly obtained, but it can be reconstructed by the sample data. To obtain the covariance matrix at each subband, the filtered frequency-domain samples can be acquired by transforming the time-domain sample data into frequency-domain data through FFT. As shown in Figure 2, the sample data on each sensor are transformed into frequency domain by FFT, and then the frequency-domain data are divided into $K$ subbands, which are processed on each frequency block.

The specific process is as follows:

1. Perform FFT operation for transforming the preprocessed data into the frequency-domain form.
2. The frequency-domain data are divided into $K$ blocks, and the data length of each subband is $L$.
3. The frequency-domain matrix of each block can be expressed as given in Equation (4).
4. The central frequencies of the frequency blocks are $f_1, f_2, \ldots, f_K$ as given in Equation (5), respectively.
5. The covariance matrix $R_{f_k}$ at the $k$-th subband can be calculated by Equation (7).

Accordingly, the $NJ \times NJ$ INCM can be reconstructed by

$$\tilde{R}_{i+n} = \int_{f_1}^{f_N} \int_{\theta_\Theta} \tilde{P}_f(\theta_i)u(f, \theta_i)u^H(f, \theta_i)d\theta_i df + \tilde{\delta}_n^2I_{NJ}$$

$$\approx \sum_{k=1}^{K} \sum_{\eta=1}^{\Omega} \hat{u}(f_k, \theta_i)u^H(f_k, \theta_i) + \tilde{\delta}_n^2I_{NJ},$$

where $\Gamma$ is the total samples in the angular sector $\Theta$, $\tilde{\delta}_n^2$ is the estimation of noise power, $\Theta$ is the complement sector of $\Theta$, with $\Theta$ denoting an angular sector of the desired signal, $\theta_i \in \Theta$ is the possible incident angle of interference, $I_{NJ}$ is an $NJ \times NJ$ identity matrix. Note that $\Theta \cup \Theta^c$ covers the entire spatial domain, and the noise power $\tilde{\delta}_n^2$ can be estimated with the smallest eigenvalues of the time-domain covariance matrix $R = SS^H/Q$ with $S = [s(1), \ldots, s(Q)]$. By eigen-decomposition, the number of eigenvalues can be obtained as

$$\sum_{n=1}^{r} \sigma_n^2 \geq G_r,$$

where the eigenvalues $\sigma_1^2, \ldots, \sigma_r^2, \ldots, \sigma_N^2$ are arranged in descending order, and $G_r$ is a threshold slightly less than 1. Hence, the noise power can be estimated by

$$\tilde{\delta}_n^2 = \frac{1}{N - r} \sum_{n=r+1}^{N} \sigma_n^2.$$ (23)

Therefore, the output interference-plus-noise energy is expressed as

$$\tilde{P}_{i+n} = \tilde{w}_i^H \tilde{R}_{i+n} \tilde{w}. $$ (24)

By minimising the output interference-plus-noise power $\tilde{P}_{i+n}$, the proposed algorithm can achieve the best performance of interference suppression capability.

### 3.2 Minimising main-lobe spatial response variation

For the wideband beamformers, the main-lobe width is susceptible to the influence of signal frequency, and it is necessary to constrain the main-lobe spatial response variation. The frequency-invariant constraints are applied to the response variation in the main-lobe. Based on the frequency division, the covariance matrix of the main-lobe spatial response variation can be modelled as

$$V_f = \int_{f \in \rho(f, \theta)} \int_{\Omega} (u(f, \theta) - u(f_r, \theta))(u(f, \theta) - u(f_r, \theta))^H df d\theta$$

$$\approx \sum_{k=1}^{K} \sum_{\eta=1}^{\Omega} \tilde{u}(f_k, f_r, \theta_i)\tilde{u}^H(f_k, f_r, \theta_i),$$

with $\tilde{u}(f_k, f_r, \theta_i) = u(f_k, \theta_i) - u(f_r, \theta_i)$, where $f_r$ is the reference frequency, $\theta_i, 1 \leq i \leq \Omega$ is the spatial sampling in the angular sector $\Theta$. In general, the lowest frequency $f_\delta$ of the desired signal is used as the reference frequency $f_r$. By minimising
the main-lobe spatial response variation $V_f$, the frequency-invariant constraint in the main-lobe can be expressed as

$$C_v = \min \ w^H V_f w. \quad (26)$$

Based on the optimisation objective function given in Equation (26), the response of the proposed beamformer remains consistent in the frequency range of the signal of interest and the angular sector $\Theta$.

### 3.3 Response constraint of desired signal

In adaptive beamforming, the ultimate goal is to maximise the output SINR given in Equation (18), which is composed of two sub-objectives. The first goal is to minimise the output interference-plus-noise power, and the second one is to maximise the output energy of the desired signal. The output power of the signal of interest can be written as

$$P_s = w^H \tilde{R}_s w, \quad (27)$$

where $\tilde{R}_s$ denotes the covariance matrix of the desired signal. In the wideband beamforming, we can reconstruct $\tilde{R}_s$ by

$$\tilde{R}_s = \frac{1}{K} \int_{f_{\min}}^{f_{\max}} \int_{\Theta} \tilde{P}_f (\theta) u(f, \theta) u^H(f, \theta) d\theta df$$

$$\approx \sum_{k=1}^{K} \sum_{\eta=1}^{\Omega} u(f_k, \theta_\eta) u^H(f_k, \theta_\eta) R_{f_k, \theta_\eta}^{-1} a(f_k, \theta_\eta) \quad (28)$$

Because the covariance matrix $\tilde{R}_s$ is not positive definite, we can reconstruct the positive definite matrix SNCM as

$$\tilde{R}_{s+n} = \tilde{R}_s + \delta_n I_{NF}. \quad (29)$$

To design a convex optimisation model, maximising $w^H \tilde{R}_s w$ is approximately equivalent to minimising $w^H \tilde{R}_{s+n}^{-1} w$. The incident angle of the desired signal is assumed to be $\theta_1$, and the presumed steering vector can be written as $u(f, \theta_1)$. To maximise the output power of the desired signal, the following constraint must be met

$$w^H \tilde{R}_{s+n}^{-1} w \leq u^H(f, \theta_1) \tilde{R}_{s+n}^{-1} u(f, \theta_1). \quad (30)$$

### 3.4 Time-domain wideband beamformer

In this subsection, a novel convex optimisation model is designed for the time-domain wideband adaptive beamforming. From Equations (24), (26) and (30), the maximum of the output interference-plus-noise power determines the positions and depths of the nulls in the beampattern; main-lobe spatial response variation constraint makes the main-lobe responses consistent with frequencies, and maximising the output energy constraint aligns the beam in the DOA of the desired signal. By synthesising three optimisation objectives, a novel optimisation problem can be established.

Assume that $\bar{w}$ denotes the weight vector of the proposed wideband beamformer, and the mismatch vector $\bar{w} - u(f, \theta_1)$ can be decomposed into two components about $u(f, \theta_1)$, that is orthogonal component and parallel component, and the latter cannot affect the output signal energy [32]. Therefore, the wideband adaptive beamformer can be designed by constraining the desired signal response, minimising the output power of interference-plus-noise and main-lobe spatial response variation, as follows:

$$\min_{\bar{w}} \bar{w}^H (\tilde{R}_{s+n} + \alpha V_f) \bar{w}$$

subject to

$$\bar{w}^H \tilde{R}_{s+n}^{-1} \bar{w} \leq u^H(f, \theta_1) \tilde{R}_{s+n}^{-1} u(f, \theta_1),$$

$$(\bar{w} - u(f, \theta_1)) u^H(f, \theta_1) = 0, \quad (31)$$

where $\alpha$ denotes the regularisation parameter with a positive number to balance the weights of two optimisation objectives. The optimisation problem given in Equation (31) is a quadratically constrained quadratic programing problem. From Equations (7) and (21), the INCM $\tilde{R}_{s+n}$ is positive definite, and the covariance matrix $V_f$ in Equation (25) is positive semidefinite. From Equations (28) and (29), the SNCM $\tilde{R}_{s+n}$ is positive definite. Consequently, the optimisation problem shown in Equation (31) is convex, which can be solved by the interior point method or optimisation software, such as CVX [39]. Hence, the weight vector $\bar{w}$ of the proposed wideband beamformer can be directly obtained by solving the convex optimisation model.

Based on the above description, the proposed wideband adaptive beamforming is summarised as follows:

**Step 1:** Perform FFT processing on the sample data received by each array sensor;

**Step 2:** Reconstruct the INCM $\tilde{R}_{s+n}$ (21) with the covariance matrix $R_{f_k}$ (7) at each subband;

**Step 3:** Calculate the covariance matrix $V_f$ of the main-lobe spatial response variation using Equation (25);

**Step 4:** Reconstruct the SNCM $\tilde{R}_{s+n}$ via Equation (29);

**Step 5:** Construct the convex optimisation model in Equation (31);

**Step 6:** Solve the optimisation problem and obtain the estimation $\bar{w}$ of the optimal weight vector.

### 3.5 Computational complexity analysis

For the proposed beamformer, the main complexity consists of FFT, INCM reconstruction, and solving the
optimisation problem. The complexity of FFT is $O(Q \log Q)$. The complexity of the INCM reconstruction is $O(K_S N^2)$, where $S$ is the number of sampling points in the spatial domain. The convex optimisation problem can be efficiently solved using the interior point method with $O((NJ)^{3.5})$ computational complexity. In general, $N^{1.5} f^{1.5}$ is larger than $K_S$. Consequently, the algorithmic complexity of the proposed beamformer is $O((NJ)^{1.5})$. Although the wideband beamformers in Ref. [16,35,40,41] may cost less computational load, their performance will be compared with the proposed method in Section IV. In Ref. [33], the reconstruction-estimation-based beamformer with eigenspace technique (REWB-FI-1) is the latest wideband beamformer with good performance. The computational complexity of the REWB-FI-1 beamformer is $O(\max((NJ)^2 D L_s, (NJ)^2 D L_t))$, where $I$ is the number of interferences, $L_s$ is the number of angular grids in $\Theta$, $D_s$ is the total number of sampling angles, and $L_t$ is the number of angular grids in each interference section. The lower limit of the computational complexity of the REWB-FI-1 beamformers is equivalent to the upper limit of that of the proposed algorithm. In terms of computational complexity, the proposed wideband adaptive beamforming is superior to the REWB-FI-1 beamformer presented in Ref. [33]. In addition, the latter requires several bandpass filters for subband segmentation and the roughly estimated DOAs of interferences for the INCM reconstruction. Therefore, the proposed wideband beamformer costs relatively less computational complexity.

In the simulations, the proposed wideband beamformer is compared to several broadband beamformers, including the frequency-response-invariance-and-eigenvector-constraint-based beamformer (RAWB-FRI-EC) in Ref. [16], REWB-FI-1 beamformer in Ref. [33], fast reduced-rank Laguerre beamformer (FRR LB) in Ref. [35], space-time broadband beamformer (STBB-FC) in Ref. [40], and frequency-invariance-constraint-based beamformer (RWB-FC) in Ref. [41]. To demonstrate the differences in computational complexity of the tested algorithms, the run time in the simulations is tallied. The simulation computer with 8 GB memory is installed with 64-bit windows 7 operating system, and the central processing unit (CPU) is Intel Core(TM) i5-6500 with 3.2 GHz clock speed. The simulation programs of all tested algorithms are edited and run by MATLAB R2014a. The numbers of snapshots, elements and taps in the FIR filters are set to be 1600, 16 and 15, respectively. The run time of all tested methods is shown in Figure 3. Figure 3(a) depicts the run time of STBB-FC, RAWB-FRI-EC and RWB-FC, and Figure 3(b) displays the run time of FRR LB, REWB-FI-1 and the proposed algorithm. It can be seen from Figure 3 that due to solving the optimisation models, the last three algorithms need more run time than the previous three ones, and the proposed algorithm costs moderate computing resources in the last three methods. In the simulations, the average elapsed time of the proposed algorithm is 9.28 s. The simulation results are consistent with the previous computational complexity analysis.

Simulation results show that the INCM reconstruction of each subband costs about 0.2663 s, and the process of solving the optimisation problem requires about 4.662 s. In the above experiments, the efficiency of the proposed algorithm is insufficient. In the practical engineering, there are two main steps to improve the processing performance. The first step is to reduce the run time of the proposed algorithm. Multiple methods can be used to accelerate the solving process, such as optimal parameter settings, more efficient programming, better processors, parallel signal processing. With these methods, the run time of the proposed algorithm can be effectively reduced. If the processing speed still can not achieve the system requirements, the second step can be adopted. That is to design the parallel processing hardware, in which the update cycle of the weight vector can be speeded up by processing segmented data in parallel. The two steps can provide a feasible solution for the engineering implementation of the proposed algorithm.

![Figure 3](image-url)
4 | SIMULATION RESULTS

In this section, several simulation experiments are used to verify the performance of the proposed beamformer. Note that the desired signal and interferers are wideband signals with the centre frequency \( f_c \). In the simulations, a ULA consisting of \( N = 16 \) omnidirectional sensors is considered spaced a half wavelength of the highest frequency, and the number of taps in the FIR filters is \( J = 15 \). There are three wideband signals composed of the desired signal and two interferers impinging upon the ULA. The wideband signals are generated from bandpass-filtered white Gaussian noises, and they are statistically unrelated to each other. Before being sent into the tested beamformers, the sample data are processed by bandpass filtering to ensure that all signals are within the bandwidth of the beamformers, the sample data are processed by bandpass filtering to ensure that all signals are within the bandwidth of the beamformers. The angular sector of the desired signal is set to be \( \Theta = [\theta_1 - 6^\circ, \theta_1 + 6^\circ] \), and two interferers impinge on the array from both sides of the desired signal. It should be noted that the definition of a wideband signal and ultra wideband (UWB) signal is usually based on the fractional bandwidth \( B_f = \frac{f_{\text{sc}} - f_s}{0.5 f_s} \). If the fractional bandwidth meets the condition of \( 0.01 < B_f < 0.25 \), the desired signal is a wideband signal, which can be considered as a UWB signal while satisfying \( 0.25 < B_f < 1.50 \) [42]. When the value of \( B_f \) has been determined, the signal bandwidth and sampling frequency increase along with the increase of carrier frequency. To reduce the sampling frequency and storage space, the carrier frequency in the simulations is set to be \( f_s = 1.2 \) GHz, and the bandwidth of wideband signal and sampling frequency are set to \( B = 200 \) MHz and \( f_s = 800 \) MHz, respectively. With the statistical INCM and the stacked steering vectors of the desired signal and all interferers, the optimal wideband beamformer can be obtained according to the LCMV criterion, and the optimal weight vector \( \mathbf{w}_{\text{opt}} \) is calculated by

\[
\mathbf{w}_{\text{opt}} = \mathbf{R}_n^{-1} \mathbf{C}(\mathbf{C}^H \mathbf{R}_n^{-1} \mathbf{C})^{-1} \mathbf{C}_g,
\]

\[
\mathbf{C} = [\mathbf{C}_1, \mathbf{C}_2],
\]

\[
\mathbf{g} = [1_{1 \times K}, \theta_1, \theta_1] \mathbf{T},
\]

\[
\mathbf{C}_i = [\mathbf{u}(f_1, \theta_1), \ldots, \mathbf{u}(f_K, \theta_1)],
\]

\[
\mathbf{C}_i = [\mathbf{u}(f_1, \theta_2), \ldots, \mathbf{u}(f_K, \theta_2), \mathbf{u}(f_1, \theta_3), \ldots, \mathbf{u}(f_K, \theta_3)],
\]

where \( \mathbf{R}_n \) denotes the \( N \times N \) INCM of simulation data, \( \mathbf{g} \) denotes a \( 3K \times 1 \) vector composed of the constrained values, \( \mathbf{C} \) denotes the constraint matrix, with the signal matrix \( \mathbf{C}_s \) and the interference matrix \( \mathbf{C}_i \) composed of the stacked steering vectors at \( K \) subbands, \( 1_{1 \times K} \) denotes a row vector of all ones, \( \theta_1, \theta_2, \theta_3 \) are the DOAs with \( \theta_1 = 30^\circ \) and \( \theta_2 = -20^\circ \) and \( \theta_3 = 30^\circ \), respectively. The width of the main-lobe is consistent over all frequencies, and the nulls at different frequencies are always in the correct locations. Figure 4(b) indicates that the gains near two interferers is always lower than \(-62 \) dB, and the peak side lobe is about \(-32 \) dB. It has been demonstrated that the proposed algorithm can effectively control the beam direction and null positions, and maintain main-lobe responses with good consistency.

4.1 | Performance comparison of interference suppression

In the second subsection, we explore the performance of the tested beamformers in the case of suppressing wideband interferences. In the simulations, the DOA of the desired signal with the fixed SNR = 10 dB is set to \( \theta_1 = 10^\circ \), and the wideband interferences with INR = 30 dB impinge on the array from the directions of \( \theta_2 = -20^\circ \) and \( \theta_3 = 30^\circ \), respectively. Only the DOA of the desired signal is known in advance. When the number of snapshots is fixed to \( Q = 1200 \), the output SINR versus the input SNR is shown in Figure 5(a). The performance of the proposed beamformer is clearly superior to that of the other tested methods, and the output SINR is very close to the optimal value. Furthermore, the proposed algorithm maintains good robustness in the input SNR range. Figure 5(b) compares the output SINR with the number of snapshots for the fixed SNR = 10 dB. As the
number of snapshots increases, the output SINR of the proposed beamformer approaches the optimal value, and the proposed algorithm clearly outperforms the other tested methods.

### 4.2 Performance comparison in ultra wideband condition

In the third example, the influence of signal bandwidth on the performance of the tested beamformers is considered. The bandwidth of the desired signal is set to $B = 320$ MHz, which has met the requirements of UWB signal. The number of subbands is still set to $K = 12$. The input SNR and INR are set to 10 and 30 dB, respectively. The DOA of the desired signal is set as $0^\circ$, and the incident angles of two interferers are $-30^\circ$ and $30^\circ$, respectively. Figure 6 presents the beampattern when the number of snapshots is $Q = 2000$. As shown in Figure 6, the frequency range of the UWB signal is from 1040 to 1360 MHz with the frequency interval 80 MHz. Figure 6(a) demonstrates that the beam direction and null locations are accurate, and the main-lobe slightly widens with the decrease of signal frequency. Figure 6(b) demonstrates that the gain around the DOAs of two interferes is always below -60 dB, and the peak side lobe is lower than -33 dB. Under the UWB condition, the output SINR versus input SNR for the fixed number of snapshots $Q = 1200$ is shown in Figure 7(a). Compared to Figure 5(a), the performance of all tested beamformers degrades to a certain extent, and the proposed algorithm still outperforms other methods, whose performance is close to the optimal SINR. In Figure 7(b), the output SINR versus the number of snapshots is examined. The performance of the proposed beamformer is clearly better than that of other tested methods. Compared to Figure 5(b), the output SINR can approach the optimal value with more sampling snapshots. The proposed algorithm divides the wideband signals into $K$ subbands by FFT, which improves the INCM reconstruction accuracy and computational efficiency. Furthermore, the number of subbands can be changed flexibly according to the bandwidth of signal of interest. Therefore, the proposed beamformer outperforms other tested methods in the UWB condition.
4.3 | Signal look direction mismatch

In the fourth subsection, the impact of random signal look direction mismatch is examined. Assume that the random DOA errors of the desired signal and interferences are subject to a uniform distribution in $[-5^\circ, +5^\circ]$ and $[-10^\circ, +10^\circ]$, respectively. In the simulations, the actual DOA of the desired signal is uniformly distributed in $[5^\circ, +15^\circ]$, and two interferences are uniformly located in $[-30^\circ, -10^\circ]$ and $[20^\circ, 40^\circ]$, respectively. Note that the DOAs of the desired signal and interferences change with each simulation run while remaining constant from snapshot to snapshot. Figure 8(a) shows the output SINR of the tested beamformers versus the input SNR when the number of snapshots is $Q = 1200$. In the case of signal direction error, the proposed method can maintain good performance. In Figure 8(b), the output SINR versus the number of snapshots is shown with the fixed input SNR = 10 dB. The proposed beamformer clearly outperforms other tested methods, whose performance degrades to varying degrees. Compared to Figure 5(b), the output SINR of the proposed beamformer can converge to the optimal value with more snapshots. Due to the beam-pointing constraint in Equation (31), the signal direction mismatch has little effect on the performance of the proposed wideband beamformer.

4.4 | Incoherent local scattering

In the fifth subsection, the influence of incoherent local scattering is examined. Assume that there are four scattering signals, and the wideband signals on the $n$-th sensor can be modelled as

$$s_n(t) = s_1(t - \tau_0(\theta_1)) + \sum_{c=1}^{4} s'_c(t - \tau_c(\theta'_c)), \quad (33)$$

where $s'_c$ is the incoherent local scattering signal, $\theta'_c$, $1 \leq c \leq 4$ is the incident angle of the scattering signal, which follows the normal distribution in $N(\theta_1, 2^\circ)$. Note that the generating mode of incoherent local scattering signals $s'_1$, $s'_2$, $s'_3$ and $s'_4$ is the same as the previous desired signal and interferers with bandwidth $B = 200$ MHz and centre frequency $f_c = 1.2$ GHz. The desired signal impinges upon the array from $\theta_1 = 0^\circ$, and two wideband interferers with INR = 30 dB come from $\theta_2 = -30^\circ$ and $\theta_3 = +30^\circ$, respectively. With the number of snapshots fixed as $Q = 1200$, the output SINR versus the
input SNR is depicted in Figure 9 (a). As shown in Figure 9 (a), the performance of the proposed algorithm is close to that of the REWB-FI-1 beamformer, and outperforms the other tested beamformers. For the proposed algorithm, with the increase of input SNR, the beam-pointing accuracy is improved and the output SINR approaches to the optimal value. With the fixed input SNR = 10 dB, Figure 9 (b) shows the output SINR of the tested beamformers versus the number of snapshots. The performance of the proposed algorithm outperforms the remaining methods. Therefore, in the case of incoherent local scattering, the proposed wideband beamformer can keep better anti-interference capability.

5 | EXPERIMENTAL DATA PROCESSING

In this section, the effectiveness of the proposed wideband beamformer is verified by processing the experimental data in the anechoic chamber. The experimental data are come from the Automatic Target Recognition Laboratory (ATR Key Lab), National University of Defence Technology, China. Figure 10 (a)–10(c), respectively, demonstrate the array antenna, signal reflector, and multi-channel data acquisition module. The functional block diagram of the experimental system is shown in Figure 11. The signal processing flow includes the following steps: (1) Through frequency conversion and power amplification, the signal transmitter generates and transmits the wideband signals. (2) By the signal reflector, the feed spherical wave of the transmitted signal is reflected as a plane wave, and then impinges on the array antenna. (3) Through signal amplification and filtering, the received signals are sent into the multi-channel data acquisition module for digital sampling. (4) The field programmable gate array (FPGA) performs data preprocessing, including digital quadrature demodulation, array calibration, etc. (5) Through the optical fibre transmission system, the radar signal processor is used to store and process the experimental data. The ULA consisting of eight elements receives linear frequency modulation (LFM) signals from three different directions, and all of them have the same parameters including carrier frequency $f_c = 2.77$ GHz, pulse duration $T_p = 5$ μs, signal bandwidth $B = 500$ MHz, and modulated frequency rate $\kappa$. The incident angles of wideband LFM signals are, respectively, $-40.5$, $0.5$ and $29.5$. Other experimental
Parameters include the inter-element spacing \( d = 0.065 \, \text{m} \), sampling frequency \( f_s = 1.2 \, \text{GHz} \), sampling length \( Q = 8192 \) of each pulse. In the process of data acquisition, only one wideband LFM signal is sampled in each experiment, and the final sampling data are synthesised according to different SNRs. The second signal is considered as the signal of interest, and other two signals are assumed as the wideband interferers. Note that in the wideband digital receiver, digital down conversion is performed, and then zero intermediate frequency technique is applied to data acquisition. For the tested beamformers, the angular sector and the number of subbands are set to be \( \Theta = [-10^\circ, +10^\circ] \) and \( K = 16 \), respectively. In addition, other parameters remain unchanged.
By comparing the output SINR versus input SNR, the experimental results are shown in Figure 12. It can be indicated that the performance of the tested beamformers is similar to that of simulation results. When the input SNR is lower than 10 dB, STBB-FC and RWB-FIC outperform other tested beamformers, but their performance degrades at high SNRs due to the self-null problem. The proposed beamformer outperforms the other tested algorithms, and the performance of RAWB-FRI-EC is better than the simulation results. In general, the proposed wideband beamformer can achieve a near-optimal SINR performance when the input SNR varies from –30 to 50 dB.

6 | CONCLUSION

Based on covariance matrix reconstruction in the spatial-frequency domain and convex optimisation model with beam pointing constraint, we propose a novel wideband adaptive beamforming algorithm for the array antenna. After transforming the time-domain broadband signals into the frequency-domain form via FFT, both the INCM and SNCM can be reconstructed in the spatial-frequency domain, and then the optimisation objective is constructed according to minimising the output interference-plus-noise power and minimising the main-lobe spatial response variation. With the constraint of the output power of the desired signal, a novel optimisation model is established by synthesising two optimisation objectives. Therefore, the time-domain wideband adaptive beamformer can be obtained by solving the optimisation problem. Numerical simulations demonstrate that the proposed beamformer has better interference rejection performance, even to the UWB signals, and outperforms other tested beamformers in the case of signal look direction mismatch and incoherent local scattering. Due to the efficiency of FFT, the proposed algorithm can adapt flexibly to different signal bandwidths and require lower computational complexity while maintaining the performance advantages. Furthermore, the effectiveness of the proposed algorithm is verified by processing the experimental data in the anechoic chamber.

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CONFLICT OF INTEREST DISCLOSURE

The authors declare no conflict of interest.

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REFERENCES

1. Byrne, D., Craddock, I.J.: Time-domain wideband adaptive beamforming for radar breast imaging. IEEE Trans. Antenn. Propagat. 63(4), 1725–1735 (2015)
2. Wang, Y., et al.: An adaptive beamforming method for ultrasound imaging based on the mean-to-standard-deviation factor. Utrasound. 90, 32–41 (2018)
3. Al-Zuhairi, D.T., et al.: Phase-based window function and CD-DMAS beamforming for microwave breast cancer detection. IET Microw. Antennas Propagat. 14(7), 608–616 (2020)
4. Yang, X., Li, S., Sun, Y.: Robust wideband adaptive beamforming with null broadening and constant beamwidth. IEEE Trans. Antenn. Propagat. 67(8), 5380–5389 (2019)
5. Lee, C., Khattak, M.K., Kahng, S.: Wideband 5G beamforming printed array cluttered by LTE-A 4X4-multiple-input-multiple-output antennas with high isolation. IET Microw. Antenn. Propagat. 12(8), 1407–1413 (2018)
6. Liu, W.: Adaptive wideband beamforming with sensor delay-lines. Signal Process. 89(5), 876–882 (2009)
7. Yan, S.: Optimal design of FIR beamformer with frequency invariant patterns. Appl. Acoust. 67(6), 511–528 (2006)
8. Hossain, M.S., Godara, L.C., Islam, M.R.: Efficient robust broadband beamforming algorithms using variable loading. IEEE Latin America Trans. 10(3), 1697–1702 (2012)
9. Neihis, M., Solbach, K.: Finite impulse response-filter-based RF-beamforming network for wideband and ultra-wideband antenna arrays. IET Microw. Antenn. Propagat. 5(7), 844–851 (2011)
10. Wang, B.H., Hui, H.T., Leong, M.S.: Optimal wideband beamforming for uniform linear arrays based on frequency-domain MISO system identification. IEEE Trans. Antenn. Propagat. 58(8), 2580–2587 (2010)
11. Peng, Z., et al.: Radio frequency beamforming based on a complex domain frontend. IEEE Trans. Microw. Theory Tech. 64(1), 289–298 (2015)
12. Duan, H., et al.: Broadband beamforming using TDL-Form IIR filters. IEEE Trans Signal Process. 55(3), 990–1002 (2007)
13. Wijenayake, C., et al.: All-pass filter-based 2-D IIR filter-enhanced beamformers for AESA receivers. IEEE Trans. Circuits Syst. I: Regular Papers. 61(5), 1331–1342 (2014)
14. Nongpiur, R.C., Shpak, D.J., Antoniou, A.: Improved design method for nearly linear-phase IIR filters using constrained optimization. IEEE Trans. Signal Process. 60(4), 895–906 (2013)
15. Zheng, Z., et al.: Covariance matrix reconstruction with interference steering vector and power estimation for robust adaptive beamforming. IEEE Trans. Veh. Technol. 67(9), 8495–8503 (2018)
16. Yang, J., Zhang, Y., Chen, Z.: Robust adaptive wideband beamforming with combined frequency response invariance and eigenvector constraints. In: Progress in Electromagnetic Research Symposium (PIERS) (PIES), pp. 8–11.Shanghai (August 2016)
17. Liu, W., Langley, R.J.: An adaptive wideband beamforming structure with combined subband decomposition. IEEE Trans. Antennas Propagat. 57(7), 2204–2207 (2009)
18. Yang, K., et al.: Robust adaptive beamforming using an iterative FFT algorithm. Signal Process. 96, 253–260 (2014)
19. Elkamchouchi, H.M., Hassan, M.M.: A wideband direct data domain genetic algorithm beamforming approach to adaptive processing using fast Fourier transform. In: Proceedings of 2nd international Conference on Electronics and Communication Systems (ICECS), pp. 1640–1644. Cairo, Egypt (Sep. 2015)
20. Zhang, S., et al.: Low-complexity adaptive broadband beamforming based on the non-uniform decomposition method. Signal Process. 151, 66–75 (2018)
21. Kalia, S., et al.: Multi-Beam spatio-spectral beamforming receiver for wideband phased arrays. IEEE Trans. Circuits Syst. 6(8), 2018–2029 (2013)
22. Yang, J., et al.: Automatic generalized loading for robust adaptive beamforming. IEEE Signal Process. Lett. 16(3), 219–222 (2009)
23. Du, L., et al.: Fully automatic computation of diagonal loading levels for robust adaptive beamforming. IEEE Trans. Aerosp. Electron. Syst. 46(1), 449–458 (2010)

24. Hossain, M.S., et al.: Efficient robust broadband antenna array processor in the presence of look direction errors. IEEE Trans. Antenn. Propag. 61(2), 718–727 (2013)

25. Liao, B., et al.: Robust adaptive beamforming with random steering vector mismatch. Signal Process. 129, 190–194 (2016)

26. Liao, B., et al.: Robust adaptive beamforming with precise main beam control. IEEE Trans. Aerosp. Electron. Syst. 53(1), 345–356 (2017)

27. Yi, S., Wu, Y., Wang, Y.: Projection-based robust adaptive beamforming with quadratic constraint. Signal Process. 122, 65–74 (2016)

28. Huang, F., Sheng, W., Ma, X.: Modified projection approach for robust adaptive array beamforming. Signal Process. 92(7), 1758–1763 (2012)

29. Jia, W., et al.: Robust adaptive beamforming based on a new steering vector estimation algorithm. Signal Process. 93(9), 2539–2542 (2013)

30. Shen, Q., et al.: Underdetermined DOA estimation under the compressive sensing framework: a review. IEEE Access. 4, 8865–8878 (2016)

31. Zhao, Y., Liu, W., Langley, R.J.: Adaptive wideband beamforming with frequency invariance constraints. IEEE Trans. Antenn. Propag. 59(4), 1175–1184 (2011)

32. Grant, M., Boyd, S., Ye, Y.: CVX: Matlab software for disciplined convex programming. version 2.1’, Mar. 2017. [Online]. http://cvxr.com/cvx

33. Chen, P., et al.: Robust covariance matrix reconstruction algorithm for time-domain wideband adaptive beamforming. IEEE Trans. Veh. Technol. 68(2), 1405–1416 (2019)

34. Liu, Y., Zhao, Y.: Low-complexity robust adaptive wideband beamforming based on covariance matrix reconstruction. Electron. Lett. 54(16), 978–980 (2018)

35. Zhang, S., et al.: A low-complexity Laguerre wideband beamformer. Digit. Signal Process. 83, 35–44 (2018)

36. Bao, Y., Chen, H.: Design of robust broadband beamformers using worst-case performance optimization: a semidefinite programming approach. IEEE/ACM Trans. Audio Speech Lang. Process. 25(4), 895–907 (2017)

37. Shen, Q., et al.: Underdetermined DOA estimation under the compressive sensing framework: a review. IEEE Access. 4, 8865–8878 (2016)

38. Zhao, Y., Liu, W., Langley, R.J.: Adaptive wideband beamforming with frequency invariance constraints. IEEE Trans. Antenn. Propag. 59(4), 1175–1184 (2011)

39. Grant, M., Boyd, S., Ye, Y.: CVX: Matlab software for disciplined convex programming. version 2.1’, Mar. 2017. [Online]. http://cvxr.com/cvx

40. Ebrahimi, R., Seydnejad, S.R.: Elimination of pre-Steering delays in space-time broadband beamforming using frequency domain constraint. IEEE Commun. Lett. 17(4), 769–772 (2013)

41. Wang, W., et al.: A novel robust wideband beamformer with frequency invariance constraints. In: 2014 Proceedings of 4th International conference on Instrumentation and Measurement, Computer, Communication and Control (IMCCC), pp. 1–5 Harbin, China (2014)

42. Wicks, M.C., et al.: Principles of Waveform Diversity and Design. Sci-Tech Press, Raleigh, NC, USA (2010)

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