A Semiclassical ANEC Constraint on Classical Traversable Lorentzian Wormholes

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Abstract

The present article lies at the interface between gravity, a highly nonlinear phenomenon, and quantum field theory. The nonlinear field equations of Einstein permit the theoretical existence of classical wormholes. One of the fundamental questions relates to the practical viability of such wormholes. One way to answer this question is to assess if the total volume of exotic matter needed to maintain the wormhole is finite. Using this value as the lower bound, we propose a modified semiclassical volume Averaged Null Energy Condition (ANEC) constraint as a method of discarding many solutions as being possible self-consistent wormhole solutions of semiclassical gravity. The proposed constraint is consistent with known results. It turns out that a class of Morris-Thorne wormholes can be ruled out on the basis of this constraint.

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The static, spherically symmetric traversable Lorentzian wormholes are special classes of theoretical solutions of the highly nonlinear gravitational field equations of Einstein. The solutions can be interpreted as objects connecting ("handle") two distant regions of spacetime. These objects (i.e., wormholes) are threaded by what is called "exotic" matter. The notion of such has found a justification also in the wider context of cosmology where one deals with dark matter or classical phantom field. However, this article is concerned with a local problem described by static, spherically symmetric wormhole spacetimes in which it is known that several pointwise and average energy conditions are violated. We

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shall discuss here the violation of the weakest of all energy conditions, viz., the Averaged Null Energy Condition (ANEC), and refer to the ANEC violating matter as exotic. An immediate question to be asked is this: How to quantify the total amount of exotic matter present in a wormhole spacetime? The question has significant relevance to some global results such as the singularity (see, e.g. [1-3]) or positive mass theorems of general relativity [4]. Normally, the ANEC is stated as an integral of the stress tensor averaged along a complete null geodesic being non-negative. This is a line integral with dimensions (mass)/(area), and hence is not very useful in providing information about the total amount of exotic matter. Considering this, Visser, Kar and Dadhich [5] proposed a volume integral quantifier that has been properly modified recently [6] on physical grounds. We are going to use this in what follows and consider traversable wormholes joining two asymptotically flat regions.

The object of the present paper is to propose a constraint on the ANEC violating matter in the form of an inequality in which the classical ANEC volume integral appears as the lower bound to the generalized ANEC integral following from Quantum Field Theory. That is the reason why we termed our constraint as semiclassical and its importance lies in the fact that it could be used to test the physical viability of any given static spherically symmetric wormhole. (We call any wormhole physically viable if the classical ANEC violation is finite. This finiteness is the primary condition for Morris-Thorne type of wormholes to be threaded by any quantum field. For simplicity, we consider here only the quantum Klein-Gordon field.) We show that a well known class of Morris-Thorne wormholes is not quantum mechanically viable.

As a first step, we shall provide the classical volume ANEC integral. As the next step, we shall consider the generalized quantum ANEC suggested by Yurtseven [7]. He concluded that the Klein-Gordon stress energy of semiclassical gravity can support wormholes that are only roughly of the Planck size. His arguments are based on the convergence of line integrals but we will see that the volume ANEC integral does not always preserve this convergence. In the final step, we apply the constraint to Morris-Thorne wormholes.

Let us consider the wormhole solution in the general form

$$ds^2 = -\exp[2\Phi(l)]dt^2 + dl^2 + r^2(l) \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right).$$

The throat of the wormhole occurs at $l = 0$. The volume ANEC integral to be calculated in an orthonormal frame is [6] (We take $G = c = \hbar = 1$):

$$\Omega_{ANEC} = \int_{\mathcal{I}} \int \left[ T_{\mu\nu} k^\mu k^\nu \right] \sqrt{-g_4} d^5x$$

for null $k^\mu$ and $g_4 = \det \left| g_{\mu\nu} \right|$. One might notice that the volume measure is just the one appearing in the Tolman-Komar integral [8] with the difference that the radial integration is from the throat to $\infty$ for one side of the wormhole. The integral resembles the usual definition of quasi-local energy. The quantity $T_{\mu\nu} k^\mu k^\nu$ is a general covariant scalar but depends on the congruence $C$ of null geodesics filling the entire region of space [9]. For further details and physical justification of Eq.(2), see Ref.[6]. The energy and pressure densities in that region as measured in the local orthonormal Lorentz frames ($^\hat{\cdot}$) for the
metric (1) are:

\[ \rho = T_{\hat{t}\hat{t}} = -\frac{2r''}{r} + \frac{1 - r'^2}{r^2} \] (3)

\[ p_t = T_{\hat{t}\hat{t}} = \frac{2\Phi' r'}{r} - \frac{1 - r'^2}{r^2} \] (4)

\[ p_\theta = T_{\hat{\theta}\hat{\theta}} = p_\phi = T_{\hat{\varphi}\hat{\varphi}} = \frac{1}{2} \left[ \Phi'' + (\Phi')^2 + \frac{\Phi' r' + r''}{r} \right] \] (5)

where \( X' \equiv dX/dl \). It is argued in Ref.[5] that the transverse components are associated with “normal” matter and only the remaining components are to be considered for investigating the volume ANEC violation. Therefore, Eq.(2) translates into

\[ \Omega_{ANEC} = \frac{1}{2} \times \int_{-\infty}^{+\infty} (\rho + p_t) e^{\Phi r^2} dl. \] (6)

There is a factor of \( 4\pi \) multiplying the integral coming from the \( \theta, \varphi \) integration, but \( \rho \) and \( p_t \) each has \( (1/8\pi) \) as factors. Hence the resulting factor of \( (\frac{1}{2}) \) that actually cancels out when we compute \( \Omega_{ANEC} \) for two mouths of the symmetric wormhole. We say that ANEC is satisfied if \( \Omega_{ANEC} \) is non-negative, but for classical wormhole spacetime, ANEC is always violated, that is, \( \Omega_{ANEC} \) is negative. We advocate using the volume integral (6) (multiplied by 2) for assessing the total amount of ANEC violating matter in preference to the conventional line integral, viz.,

\[ V = \frac{1}{8\pi} \int_{\gamma} T_{\mu\nu} k^\mu k^\nu dv = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} e^{-\Phi} \left( \frac{r'}{r} \right)^2 dl \] (7)

where \( dv = e^{\Phi} dl \), \( v \) being the affine parameter along the null geodesic \( \gamma \).

The next step is to consider the quantum picture, that is, the quantum field theory in curved spacetime, or semiclassical relativity: \( G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle \). The situation here is far more complex than the classical picture: No complete characterization of the stress tensor in the semiclassical Einstein equations is available as yet. Early works [10-12] have shown that, under certain asymptotic regularity conditions, the ANEC is satisfied by minimally coupled scalar field in four dimensional Minkowski spacetime and conformally coupled field in the curved two-dimensional spacetime in all quantum states forming a subset of the standard Fock space. These results have been strengthened by an analysis based on an algebraic approach devised by Wald and Yurtsever [13]. If the null geodesic is achronal, then the ANEC is satisfied when the Casimir vacuum contribution is subtracted from the stress energy resulting into Ford-Roman difference inequalities [14]. Yurtsever has provided a proof of this inequality in globally hyperbolic two-dimensional spacetimes [15].

However, crucial for our analysis is the suggestion of a generalized ANEC by Yurtsever [7] which might hold, unlike the conventional quantum ANEC, in a four-dimensional curved spacetime given by

\[ \beta(k) = \inf_{\omega} \int_{\gamma} \langle \omega | T_{\mu\nu} | \omega \rangle k^\mu k^\nu dv. \] (8)
The quantum stress tensor $\langle T_{\mu\nu} \rangle$ satisfies the generalized ANEC along a null geodesic $\gamma$ if the one-form $\beta(k) > -\infty$. The infimum is taken over all Hadamard states $\omega$ of the quantum field and the tangent vector is defined by $k^\alpha = \frac{dx^\alpha}{ds}$. The integral on the left is further refined into $\beta_e(k)$ by introducing a weighting function $c(x)$ but, with Yurtsever [7], we assume that $\beta_e(k) = [c(0)]^2 \beta(k)$. The value of $\beta(k)$ can be obtained by using the scaling argument. Hereafter, we shall closely follow the arguments in Ref.[7]. Given any arbitrary four-dimensional spacetime $(\mathbf{M}, g)$ in which the massless Klein-Gordon field $\phi$ satisfies generalized ANEC along the null geodesic $\gamma$, the scaling argument requires us to consider a new spacetime $(\mathbf{M}, g_{\kappa^2}^2)$ where $\kappa > 0$. The renormalization procedure (See Refs.[7,16]) involving the two-point function contributes, apart from the simply scaled term $\kappa^2 \langle \omega | T_{\mu\nu} | \omega \rangle$, two additional terms to the value of $\langle \omega | T_{\mu\nu} | \omega \rangle$ that are of the form $[a^{(1)} H_{\mu\nu} + b^{(2)} H_{\mu\nu}] \kappa^{-2} \ln \kappa$ where $a$ and $b$ are dimensionless constants having values of the order of $10^{-4}$ in Planck units,

$$\begin{align*}
(1) H_{\mu\nu} & \equiv 2 R_{\mu\nu} + 2 R R_{\mu\nu} - g_{\mu\nu} (2 \Box R + \frac{1}{2} R^2) \\
(2) H_{\mu\nu} & \equiv R_{\mu\nu} - \Box R_{\mu\nu} + 2 R^\alpha R_{\alpha\mu\nu} - \frac{1}{2} g_{\mu\nu} (\Box R + R^{\phi\delta} R_{\phi\delta}).
\end{align*}$$

For a general spacetime, $\beta(k)$ scales as [7]

$$\overline{\beta(k)} = \frac{1}{\kappa^3} \beta_0(k) + \frac{\ln \kappa}{\kappa^3} \int_\gamma (a^{(1)} H_{\mu\nu} k^\mu k^\nu + b^{(2)} H_{\mu\nu} k^\mu k^\nu) dv. \tag{11}$$

We shall replace the line integral measure above by the volume integral measure as in (2) such that $\overline{\beta(k)}$ changes to

$$\overline{\beta_1(k)} = \frac{1}{\kappa} \beta_0(k) + \frac{\ln \kappa}{\kappa} \int_{-\infty}^{\infty} (a^{(1)} H_{\mu\nu} k^\mu k^\nu + b^{(2)} H_{\mu\nu} k^\mu k^\nu) e^{\Phi} k^2 dl. \tag{12}$$

for the spacetime (1) where $\beta_0(k)$ is the volumized version of Eq.(8). The expressions for $(1) H_{\mu\nu} k^\mu k^\nu$ and $(2) H_{\mu\nu} k^\mu k^\nu$ have been computed by Yurtsever [7]. Assuming that the same null congruences fill the scaled and unscaled spacetimes, we shall compare the value of $\overline{\beta_1(k)}$ with $\overline{\Omega_{ANEC}}$ which is just the scaled value of $\Omega_{ANEC}$. If the ANEC violating matter is to be supportable by the renormalized quantum stress tensor, then, we conjecture, on dimensional grounds, that the following inequality

$$\overline{\beta_1(k)} \leq \overline{\Omega_{ANEC}} \Rightarrow \overline{\beta_1(k)} \geq \overline{\Omega_{ANEC}} \tag{13}$$

be satisfied. That is, the classical quantity $\overline{\Omega_{ANEC}}$ plays the role of a finite lower bound to $\overline{\beta_1(k)}$. In this sense, the inequality (13) may be regarded as a modification to the generalized ANEC. In order that the conjectured inequality makes sense, it is necessary that $\overline{\Omega_{ANEC}} < \infty$ be negative but finite. Any classical asymptotically flat traversable wormhole satisfying this condition is defined in this paper as physically viable. The constraint (13) is not only nontrivial, as the latter arguments will show, but is also of sufficiently general nature in that we can apply it to any given spherically symmetric wormhole spacetime.
With Eqs. (6) and (12) at hand, let us consider the general form of a near-Schwarzschild traversable wormhole of mass $M$, asymptotically ($l \to \pm \infty$) described by the metric functions

$$r(l) \simeq |l| - M \ln(|l|/r_0), \Phi(l) \simeq -M/|l|$$

(14)

where $r_0 \sim 2M$ is the throat radius. To proceed with the integration (6) using Eqs. (3) and (4), we note that $r(l) \simeq |l|[1 - (M/|l|) \ln(|l|/r_0)] \simeq l$ without much error since the functions $1/|l|$ and $\ln(|l|/|l|)$ almost compensate each other in the intervals $|l| \geq 2M$.

Using this fact, and computing $r'$, $r''$ etc we can integrate Eq. (7) over $|l| \geq 2M$ to find that

$$V \simeq -\frac{0.17}{\pi M}$$

Under the same approximation, we find from Eq. (6) that

$$\Omega_{ANEC} \simeq -0.39M$$

(15)

and hence ANEC is violated. An examination of the integrand in Eq. (6) reveals that, under the scaling $\bar{g} = \kappa^2 g$ ($\kappa > 0$), we have $\bar{r}_0 = \kappa r_0$, and consistent with this, $\bar{\Omega}_{ANEC} = \kappa \Omega_{ANEC}$. This immediately gives $M = \kappa M$. Thus, $|\bar{\Omega}_{ANEC}| < \infty$ showing that the wormhole could be supported by a quantum scalar field. Using the expressions given in Ref. [7], we find that the extra renormalization contributions work out to finite negative values

$$\int_{-\infty}^{+\infty} (1) H_{\mu\nu} k^\mu k^\nu e^\Phi r^2 dl \simeq -\frac{23.29}{M}$$

(16)

$$\int_{-\infty}^{+\infty} (2) H_{\mu\nu} k^\mu k^\nu e^\Phi r^2 dl \simeq -\frac{9.04}{M}$$

(17)

Let $B(M)$ and $B(\bar{M})$ denote respectively the values of $\beta(k)$ and $\bar{\beta}(k)$ such that we can rewrite Eq. (12) as

$$B(\kappa M) = \frac{1}{\kappa^3} B(M) + \frac{\ln \kappa}{\kappa^3} (10^{-3} c/M)$$

(18)

where the numerical constant $|c| \sim 1$. Assuming that $|B(1)| \sim 1$ for a Planck mass $M \sim 1$, we have, $\bar{M} = \kappa$ and

$$B(\bar{M}) \simeq \left(\frac{1}{\bar{M}^3}\right) [c_1 + 10^{-3} c_2 \ln \bar{M}]$$

(19)

where, again, $|c_1| \sim |c_2| \sim 1$. For reasonable values of $\bar{M}$, the logarithmic term in the square bracket can be ignored compared to the first term and so, dropping bars, we are left with $|B(M)| \sim 1/M^3$. Using the condition (13), viz., $|B(M)| \geq |\Omega_{ANEC}|$, and putting in the corresponding values, we see that $M^3 \leq c_3$ where $|c_3| \sim 1$. That is, a wormhole to be supportable by a massless quantum Klein-Gordon field must only be of the order of a Planck mass. We see that the volume integral approach also supports the Planck size constraint in the case of near Schwarzschild wormholes imposed by the earlier consideration of line integrals in the form of the constraint $|\beta(k)| \geq |V|$. A similar constraint on size is provided also by the Ford-Roman Quantum Inequality (FRQI) stated in the form somewhat similar to the “energy-time” inequality [17]. The FRQI has been recently discussed in connection with several classical wormhole solutions from the minimally coupled theory [18].
The modification suggested in (13) is not trivial. The difference is that the volume integral (6) and the line integral (7) do not necessarily lead to similar results in the case of solutions deviating from the near-Schwarzschild metric (14). To illustrate our point, we consider first the case where the two integrals (line and volume ANEC) do lead to finite values. This is given, for instance, by the widely discussed class of “zero tidal force” traversable wormholes [19]. One typical member is given by
\[ \Phi(l) = 0, r(l) = \sqrt{l^2 + b^2}. \] (20)
The throat occurs at \( l = 0 \), or at \( r = b \) where the parameter \( b > 0 \). It immediately follows that the integrals (6) and (7) are finite. In particular, \( \Omega_{\text{ANEC}} = -\frac{\pi b^2}{2} \), and thus ANEC is violated. Also, the integrals (16) and (17) easily work out to finite values that are of the order of \( |c_4/b| \) where \( |c_4| \sim 1 \). The inequality (13) is satisfied only if \( b^2 \leq 1 \), that is, the radius of the wormhole is of the Planck size and conversely. It is clearly a physically viable wormhole. Similar considerations apply to many other kinds of wormholes [20] with different expressions for \( r(l) \) for which (6) and (12) converge. All these wormholes are physically realistic.

Consider next another class of Morris-Thorne solutions given by [17,21]
\[ \Phi(l) = 0, r(l) \simeq |l| - M \ln (|l|/r_0). \] (21)
For this, the line integral of Eq.(7) for \( V \) converges to a finite negative value. Assuming that the GANEC holds, the arguments surrounding Eqs.(18) and (19) (but without the consideration of the volume integral) would lead to the conclusion that these wormholes are also of Planck size and physically realistic. However, this is not necessarily the case. The \( \Omega_{\text{ANEC}} \) of Eq.(6) [and hence \( \Omega_{\text{ANEC}} \)] produces a logarithmic divergence in the asymptotic region since \( \rho + p_l \sim O(l^{-3}) \) indicating that any quantum field is unlikely to support this huge quantity of exotic matter. The divergence on the left hand side renders the inequality (13) physically meaningless. The example clearly illustrates that the conclusions based on the volume integrals could be radically different from those based on line integrals. Wormholes of the type Eq.(21) may thus be physically unrealistic.

Eq.(6) provides a new classical volume ANEC in the form \( \Omega_{\text{ANEC}} \geq 0 \) (or, which is the same, \( \Omega_{\text{ANEC}} \geq 0 \)) with similar volume measures adopted for \( \beta_1(k) \) in the scaled spacetime. The constraint imposed by the quantum field theory on \( \Omega_{\text{ANEC}} \) is that it must be negative and finite, but not too large (not more than \( \sim 10^4 M \)) knowing that \( \beta_1(k) \) is finite for asymptotically flat wormholes. The finiteness of \( \Omega_{\text{ANEC}} \) is guaranteed via the local classical conservation law \( T_{\mu\nu} = 0 \) that describes the exchange of energy between (exotic) matter and gravitation together with the fall-off \( \rho \sim O(l^{-4}) \). In the geodesic orthonormal coordinates, the law leads to a conserved quantity \( P_\mu = \int T_{\hat{\mu}\hat{\nu}} \sqrt{-g_{\hat{\mu}\hat{\nu}}} d^3x \), and we obtain finite values for total energy \( P_\mu \). (Note that \( P_\mu \) is similar to \( \Omega_{\text{ANEC}} \) except in the radial integration limits). Note further that stress tensors of well known classical fields (like in the minimally or conformally coupled theories) satisfy \( T_{\mu\nu} = 0 \) independently of the Bianchi identities \( G_{\mu\nu} \equiv 0 \) together with the desired fall-off for general spherically symmetric configurations. However, for arbitrary choice of metric functions, one can compute from the Einstein tensor some expressions for \( T_{\mu\nu} \), and that \( T_{\mu\nu} = 0 \) follows only as a result of the Bianchi identities \( G_{\mu\nu} \equiv 0 \). It is not guaranteed that these local
conservation laws, in turn, would provide the desired decay law $\rho + p_l \sim O(t^{-4})$. An illustration is provided by the example in (21), for which $\rho + p_l \sim O(t^{-3})$. Although there is asymptotic decay, it is quite unlikely that the wormhole is threaded by a finite quantity of ANEC violating matter.

Conversely, in the context of quantum field theory, an interesting example is this: The natural in-vacuum states of any scalar field in the Minkowski spacetime will have a Casimir energy density $\rho = \text{negative constant} = -a$ (say) over all space. This essentially represents the vacuum solution of semi-classical relativity, having no classical gravity counterpart. (Note parenthetically that the “Casimir vacuum” Morris-Thorne-Yurtsever quantum wormholes require a plate separation smaller than the electron Compton wavelength [22].) Nonetheless, a naive integration in (6) over the Minkowski space does give $\Omega_{\text{ANEC}} = -\infty$. Now, it is understood that there is no unique way of making the transition from classical to quantum regime. The quantum system may contain interesting aspects of the true situation which disappear in the correspondence-principle limit. It is not clear if there is any classical curved spacetime counterpart to the Minkowski spacetime quantum field theory. Clearly, the semiclassical ANEC constraint can not be meaningfully applied in this case. In a related context, it might be of interest to note that Popov [23] has obtained, under a “subtraction” scheme, an analytic approximation of the stress energy tensor of quantized massive scalar fields in static spherically symmetric spacetimes with topology $R^2 \times S^2$. The stress tensor supports a Morris-Thorne wormhole if the curvature coupling parameter $\xi \geq 0$. 2538.

Recall that the situation is different in the near-Schwarzschild wormhole case. The massless quantum Klein-Gordon field does have a classical analogue in the sense that the wormhole Eq. (14) is an approximation to a physically meaningful wormhole solution in the minimally coupled field theory with a negative kinetic term in the Lagrangian, viz., $L_{\text{matter}} = -(1/8\pi)\partial_\mu \phi \partial^\mu \phi$ [24]. (This result is important as the explicit existence of a classical scalar field is useful for the regularization program and the correspondence limit.) Interestingly, the example in Eq. (20) too describes an exact extremal zero total mass wormhole solution in that theory [20]. It has been shown that zero mass wormholes, including slightly massive ones, are stable [25]. Physically realistic stable wormholes following from Hilbert-Einstein action with a well defined scalar field matter Lagrangian such as above have $\Omega_{\text{ANEC}} < -\infty$. Microscopic quantum wormholes also require this classical condition to hold and stability of such wormholes lends support to the guess (not a proof) that the condition could also be a key ingredient in a general classical stability analysis. We do not argue here that macroscopic wormholes can not occur in nature just because of the stability criterion. In fact, if one includes classical matter field $T^C_{\mu\nu}$ in addition to quantum field $T^Q_{\mu\nu}$, one could have macroscopic wormholes supported by quantum field [26]. What we do argue is that the validity of the generalized ANEC in the entire spacetime giving a finite $\beta_1(k)$ requires that $\Omega_{\text{ANEC}} < -\infty$, and this condition is sufficient to rule out many classical macroscopic configurations. The requirement of a correspondence limit could be an additional condition on the quantum scenario, but we do not emphasize it.

To summarize, we saw, by employing the volume integrals, that the scaling argument holds and that classical ANEC violation can be supported by semiclassical gravity if the wormhole is microscopic. However, for non-Schwarzschild $\Phi = 0$ wormholes that abound
in the literature, the constraint $\Omega_{ANEC} < -\infty$ can rule out well known macroscopic configurations (such as the one in Eq.(21)) as being physically unrealistic.

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