A Muon-Ion Collider at BNL: the future QCD frontier and path to a new energy frontier of $\mu^+\mu^-$ colliders

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We propose the development and construction of a novel muon-ion collider (MuIC) at Brookhaven National Laboratory (BNL) in the USA as an upgrade to succeed the electron-ion collider (EIC) that is scheduled to commence in the early 2030s, by a joint effort of the nuclear and particle physics communities. The BNL facility could accommodate a muon storage beam with an energy up to about 1 TeV with existing magnet technology. When collided with a 275 GeV hadron beam, the MuIC center-of-mass energy of about 1 TeV will extend the kinematic coverage of deep inelastic scattering at the EIC (with polarized beams) by more than an order of magnitude in $Q^2$ and $x$, opening a new QCD frontier to address many fundamental scientific questions in nuclear and particle physics. This coverage is comparable to that of the proposed Large Hadron-Electron Collider (LHeC) at CERN, but with complementary lepton and hadron kinematics, ion species and beam polarization. Additionally, the development of a MuIC at BNL will focus the worldwide R&D efforts on muon collider technology and serve as a demonstrator toward a future muon-antimuon collider at $O(10)$ TeV energies, which is an attractive option to reach the next high energy frontier in particle physics at an affordable cost and a smaller footprint than a future circular hadron collider. We discuss here the possible design parameters of the MuIC, kinematic coverage, science cases, and detector design considerations including estimates of resolutions on DIS kinematic variables. A possible road map toward the future MuIC and muon-antimuon colliders is also presented.

I. INTRODUCTION

Lepton-hadron (nucleus) deep inelastic scattering (DIS) has been a powerful tool to understand the fundamental structure of nucleons and nuclei. Decades of DIS experiments have revealed the point-like substructure of quarks and gluons inside the nucleon, and how they share the longitudinal momentum of a fast-moving nucleon. To develop a deeper understanding of the quark-gluon structure and dynamics (especially in three dimensions) of matter, governed by quantum chromodynamics (QCD), a high energy and luminosity polarized electron-ion collider (EIC) has recently been endorsed to be built at Brookhaven National Laboratory (BNL) by the late 2020s [1] as a high priority on the agenda of the US nuclear physics community. The EIC is capable of carrying out deep inelastic electron-proton and electron-nucleus collisions with polarized beams at a center-of-mass energy $(\sqrt{s})$ up to 140 GeV [1, 2]. It will establish a new QCD frontier to address key open questions such as the origin of nucleon spin, mass, and the emergence of QCD many-body phenomena at extreme parton densities. At CERN, the Large Hadron-electron Collider (LHeC) [3] has been proposed as a possible extension to the Large Hadron Collider (LHC) to explore even higher energy regimes of $\sqrt{s} \approx 1$ TeV and beyond. As a potential long-term step at CERN, the Future Circular Collider (FCC) proposed to be built in a new 100 km tunnel close to CERN also includes a mode of electron-hadron collisions (FCC-he) at $\sqrt{s} = 3.5$ TeV [4].

We propose an alternative approach to achieve the next-generation lepton-hadron (ion) collider at the 1-TeV scale using high-energy muon beams, a Muon-Ion Collider (MuIC), based on existing facilities at BNL after the mission of EIC is completed by late-2030s. Possibilities of muon-hadron colliders and their scientific potential have been discussed previously, for example in Refs. [5–12]. A MuIC at BNL would serve as a major step toward a future muon-antimuon ($\mu^+\mu^-$) collider, which has received revived attention in the particle physics community in recent years because of its potential of reaching $\sqrt{s} = O(10)$ TeV in a relatively compact tunnel (e.g., the size of the LHC). It has been argued that for the production cross section of hard processes or heavy unknown particles, the discovery potential of this particle at a 14 TeV $\mu^+\mu^-$ collider matches that of a 100 TeV proton-proton collider (e.g., FCC) [13], as all the available beam energy is carried by the interacting muons. While it is in principle feasible to build the next electron/proton colliders based on established technologies and advanced magnets with the construction of a new, longer $O(100)$ km tunnel, affordability is a concern in terms of both cost and time. For this reason, exploring new accelerator and collider approaches provides attractive alternatives. The MuIC and $\mu^+\mu^-$ collider also share synergies with the proposed “Neutrinos from STORed Muons” facility [14].

Development of muon collider technology is still at an early stage, with many challenges to overcome (e.g., see details in Ref. [13]). For example, because of the short lifetime of the muon, even accelerated to TeV energies, the beam will constantly decay, and thus will require rapid acceleration and collisions to maintain sufficiently high luminosity. To achieve a high luminosity, cooling of muon bunches to reduce its phase space be-
fore acceleration is crucial [15]. Beam backgrounds from muon decays also pose challenges to both the accelerator and the detectors. Radiation hazards from interacting neutrinos would need to be mitigated, especially at $O(10)$ TeV energies. There is a consensus in the muon collider community that realizing a smaller-scale muon collider would be a necessary intermediate step to serve as a demonstrator before pursuing the ultimate $O(10)$ TeV $\mu^+\mu^-$ collider. Such a demonstrator still requires tremendous R&D efforts and significant cost, so a compelling science program that is not accessible by other proposed facilities is needed. For example, the physics of a $\mu^+\mu^-$ collider with $\sqrt{s}$ of several hundred GeV to 1 TeV may not be competitive with an $e^+e^-$ collider proposed with more established technology, such as the International Linear Collider (ILC) [16], the Compact Linear Collider (CLIC) [17], the Circular Electron Positron Collider (CEPC) [18], and the FCC-ee [4].

In this paper, we argue that a Muon-Ion Collider (MuIC) at BNL will fulfill the objectives of both establishing a new QCD frontier to succeed the EIC, and serve as a demonstrator of muon collider technology toward an ultimate $O(10)$ TeV $\mu^+\mu^-$ collider. Such an intermediate machine will be of great interest to both nuclear and particle physics communities worldwide. Re-using the existing facility and infrastructure is a key to make the project much more affordable. In the following sections, we discuss potential key parameters and designs of the proposed MuIC, including rough estimates of achievable luminosity in Section II, its scientific potential in Section III, the final-state kinematics in Section IV, and detector design considerations in Section V. Finally, a possible road map toward realizing the MuIC with a joint effort of the nuclear and particle physics communities is discussed in Section VI.

II. A MUON-ION COLLIDER AT BNL

A key merit of our proposal is to re-use the existing facility and infrastructure to open new avenues and advance new technologies in both nuclear and particle physics at a reasonable cost. Some key parameters of the proposed muon-ion collider based at BNL are listed in Tables I and II, taking muon-proton and muon-Au collisions as examples. These parameters were chosen based on proposed $\mu^+\mu^-$ colliders [13] and the EIC being planned at BNL [19].

A. Design Concept

The EIC will be constructed based on the existing Relativistic Heavy Ion Collider (RHIC). The RHIC tunnel is built in a hexagon with rounded corners. A new linac following by the Rapid Cycling Synchrotron (RCS) will be added in the RHIC tunnel for electron injection and acceleration. A new electron storage ring will be added

| Parameter                  | 1 (aggressive) | 2 (realistic) | 3 (conservative) |
|----------------------------|----------------|---------------|------------------|
| Muon energy (TeV)          | 1.39           | 0.96          | 0.73             |
| Muon bending magnets (T)   | 16 (FCC)       | 11 (HL-LHC)   | 8.4 (LHC)        |
| Muon bending radius (m)    | 290            |               |                  |
| Proton (Au) energy (TeV)   | 0.275 (0.11/nucleon) |          |                  |
| CoM energy (TeV)           | 1.24 (0.78)    | 1.03 (0.65)   | 0.9 (0.57)       |

or be converted from one of the two existing hadron rings for collision. To evolve the EIC into a MuIC, linac+RCS is among possible proposed technologies for muon acceleration so could be reused. The electron storage ring would be converted into that for muon beams by replacing with corresponding magnets (see discussions below). It is anticipated that additional shieldings are needed to protect against backgrounds from muon decays. To generate high intensity muon sources, there are two proposed approaches for the muon collider technology: the proton driver scheme followed by the muon cooling and the positron driver scheme (no cooling is needed) [13]. The proton driver scheme is presumably more suitable for BNL, taking advantage of existing high intensity proton sources.

In Table I, we discuss possible energy scenarios of the MuIC at BNL. Assuming no upgrades to the hadron beam, the maximum proton beam energy at the EIC is 275 GeV. The achievable muon beam energy can be estimated by $p^\mu = 0.3 Br$, where $r$ is the bending radius and $B$ is the bending magnetic field of the storage ring. The bending radius is 290 m for the electron storage ring at the EIC. We consider three possible scenarios of the dipole bending magnets for the muons: 8.4 T currently used at the LHC, 11 T being developed for the HL-LHC [20], and 16 T aimed for the future FCC [21] requiring significant R&D effort. All three scenarios provide a muon-nucleon center-of-mass energy ($\sqrt{s_{\mu p}}$) of about 1 TeV (within ±12%), a 10-fold increase to the EIC energy. A conservative approach would be to use the 8.4 T LHC dipole magnets, which give a muon beam energy of 730 GeV. Even considering a more conservative scenario of 3.8 T magnets being used by the RHIC hadron ring would still lead to $\sqrt{s_{\mu p}}$ that is several times that of the EIC. In the discussion below, we will focus on a scenario of 11 T bending magnets planned for the HL-LHC, which is realistically achievable and can accommodate a muon beam energy of 960 GeV. In this scenario, the maximum achievable $\sqrt{s_{\mu p}}$ for muon-Au collisions is 650 GeV with 197Au ion beam energy of 110 GeV per nucleon.
TABLE II. Rough estimates of achievable luminosity for the proposed MuIC at BNL with the muon beam energy of 960 GeV.

| Parameter                        | Muon | Proton |
|----------------------------------|------|--------|
| Energy (TeV)                     | 0.96 | 0.275  |
| CoM energy (TeV)                 | 1.03 |        |
| Bunch intensity (10^{11})        | 20   | 3      |
| Norm. emittance, ε_{x,y} (μm)   | 25   | 0.2    |
| β_{x,y} @IP (cm)                 | 1    | 5      |
| Trans. RMS beam size, σ_{x,y} (μm) | 5.2  | 5.8    |
| Muon repetition rate, f_{rep} (Hz) | 15   |        |
| Cycles/Collisions per muon bunch, N_{c} | 3279 |        |
| L_{μp} (10^{33}cm^{-2}s^{-1})    | 7    |        |

B. Luminosity Estimates

The instantaneous luminosity of a muon-proton collider is discussed in Ref. [11] and can be generally expressed as follows:

\[ \mathcal{L}_{μp} = \frac{N^{μ}N^{p}}{4π} \max\left[\sigma_{x}^{μ}, \sigma_{y}^{μ} \right] \max\left[\sigma_{x}^{p}, \sigma_{y}^{p}\right] \min[f_{c}^{μ}, f_{c}^{p}]H_{hg}, \]  

Here, \( N^{μ} \) and \( N^{p} \) represent the number of particles per respective beam bunch. The transverse RMS beam size in \( x \) and \( y \) for the muon and proton beam is \( σ_{x,y}^{μ, p} \), and \( f_{c}^{μ, p} \) is the bunch frequency. Typically, the bunch frequency of proton beams is 2–3 orders of magnitude larger than that of muon beams, so \( L_{μp} \) is largely determined by \( f_{c}^{p} \), which is equal to the muon bunch repetition frequency \( (f_{rep}) \) multiplied by the number of cycles \( (N_{c}) \) muons can make in a circular storage ring before decaying away. For simplicity, the hour-glass factor, \( H_{hg} \), is assumed to be unity.

We use proposed parameters of the proton driver scheme for our estimate of MuIC luminosity below. As listed in Table II, the muon bunch repetition frequency is taken to be 15 Hz, and the transverse RMS beam size, \( σ_{x,y} = \sqrt{β_{x,y}γ / γ} \), is estimated to be about 5–6 μm for the muon beam based on proposed muon colliders [13, 22] and the proton beam achieved at RHIC [19]. Each muon bunch will survive an average of about 300B(Tesla) cycles in a ring. These parameters lead to a \( \mathcal{L}_{μp} \) of 5–10\times10^{33} \text{ cm}^{-2}\text{s}^{-1}, comparable to the LHeC design luminosity. For simplicity, the estimated MuIC luminosity in this paper assumes a round transverse beam profile for both the muon and proton beams. However, note that the EIC design at BNL adopts a flat transverse beam profile with the horizontal dimension stretched much larger than the vertical dimension. At the interaction point, the two beams would intersect at a finite crossing angle of 25 mrad in the horizontal plane. The purpose of such a design is to maximize the luminosity and, at the same time, fulfill other requirements such as the possibility of detecting scattered protons with a transverse momentum as low as 200 MeV by Roman Pots inside the beam pipe. Since there may still be large uncertainties in muon collider parameters, the values of luminosity presented should be taken as an order of magnitude estimate, likely optimistic. More sophisticated considerations would need to be taken into account for a practical MuIC design.

To put the MuIC luminosity into context, it is anticipated that the EIC will deliver an integrated luminosity up to about 1.5 fb\(^{-1}\)/month with \( \mathcal{L}_{μp} \approx 10^{33} \text{ cm}^{-2}\text{s}^{-1}, \) and most of science cases studied at the EIC require a total integrated luminosity of 10 fb\(^{-1}\) (= 30 weeks of operations). Therefore, even with much less stringent requirements on the muon beam, e.g., a peak \( \mathcal{L}_{μp} \approx 10^{32} \text{ cm}^{-2}\text{s}^{-1} \) operating for several years, the MuIC will still be a novel facility that breaks new ground in science and technology of high energy nuclear and particle physics. Generally speaking, higher luminosity will help access physics at the high \( Q^2 \) and electroweak regimes.

C. Beam polarization

Both the electron and hadron beams at the EIC are designed to be highly polarized (≈ 70%), a unique capability for studying the origin of the nucleon’s spin. Muons produced from pion decays are naturally polarized. The level of polarization in the lab frame depends on the initial pion energy and decay angle. The average natural polarization of captured muons is about 20% [23–25]. It is possible to achieve higher polarization by, e.g., selecting on the energy of forward-going muons at the cost of luminosity. Studies have suggested that about 50% polarization can be achieved at a reduced luminosity of about 25% of the maximum value [24, 25]. Therefore, the MuIC will retain the unique capability of beam polarization at BNL, which is particularly important for the study of nucleon spin. To what extent the muon polarization can be maintained through the cooling and acceleration stages is a subject that requires future studies. It is suggested in Ref. [26] that depolarization in the ionization cooling element should be negligible.

D. Synergy with the nuSTORM facility

The nuSTORM facility is proposed as a concept of neutrino factory using a racetrack-like muon storage ring to deliver high-intensity beams of ν\(_{e}\) and ν\(_{μ}\) from muon decays. It will enable searches for light sterile neutri-
The MuIC with stored muons of $\approx 1$ TeV will provide high flux, high energy neutrino beams to enable a compelling neutrino physics program at the same time.

## III. THE SCIENCE OF A MUON-ION COLLIDER

To put potential scientific impacts of the proposed MuIC in perspective, Fig. 1 compares the kinematic coverage of the MuIC in the map of momentum transfer, $Q^2$, and Bjorken scaling variable, $x$, with the EIC at BNL, HERA at DESY, and the proposed LHeC at CERN, for lepton-proton (top) and lepton-nucleus (bottom) collisions. The boundaries of the $Q^2$-$x$ coverage are determined by $Q^2 = s x y$, where $s$ is the squared center-of-mass energy and $y$ is the inelasticity. Here, we take an inelasticity range of $0.01 < y < 0.95$ that is commonly assumed for $e p$ collisions (although it generally depends on the detector technology and method of reconstructing the DIS kinematics). The dashed line at $Q^2 = 1$ GeV$^2$ can be considered as the transition from the non-perturbative to perturbative regime of QCD, which is usually a lower limit required in DIS studies.

The red dashed lines in Fig. 1 indicate the gluon saturation scales inferred from the GBW model based on fits to HERA data [27], below which nonlinear QCD evolution effects are expected to become significant and the growth of the gluon density toward small $x$ values will saturate [28]. Other models on estimating the saturation scale can be found in Refs. [29–31], including those that consider the impact parameter dependence. The saturation scale is predicted to be enhanced by a factor $A^{1/3}$ ($= 6$ for $^{197}$Au) in nuclei, where $A$ is the atomic number, because of overlapping nucleon gluon fields induced by the Lorentz contraction in the longitudinal direction [32, 33].

As shown by Fig. 1, both the LHeC and MuIC will significantly extend the kinematic coverage of the EIC to much larger $Q^2$ and smaller $x$ regimes, by an order of magnitude in each compared to the previous HERA $e p$ collider. In lepton-proton collisions (Fig. 1, top), the saturation regime is clearly out of the reach for the EIC, but it becomes within reach at very small $x$ values at the LHeC and MuIC. In lepton-nucleus collisions (Fig. 1, bottom), considering the $^{197}$Au nucleus as a representative example (with a factor of 6 enhancement in the saturation scale), the EIC starts approaching the saturation regime, while the LHeC and MuIC should be well in the domain to explore gluon saturation and nonlinear QCD phenomena. Because of large overlaps of the proposed MuIC and LHeC in terms of the $Q^2$-$x$ coverage (although the kinematics of the final-state particles are quite different in the lab frame, as will be discussed later), they share a lot in common for their physics potential. The science program of the LHeC has been well presented in Ref. [3]. In addition, the MuIC at BNL would offer the unique advantage of providing a polarized beam, which is important for nucleon spin physics, as documented in the EIC white paper [2]. We briefly discuss the science opportunities enabled by the MuIC at BNL in this section. Quantitative studies will be left for future work.
A. Parton Distribution Functions

Precision measurements of the structure functions at the MuIC in its new kinematic regime would enable a precise determination of the underlying parton distribution functions (PDFs) of the proton and other nuclei in a way complementary to hadron colliders, with a cleaner decoupling of the effects of QCD and quark flavor. This would enable more precise cross section calculations to compare with measurements made at present and future hadron colliders like the LHC and the proposed FCC-hh, particularly for Higgs boson production where the gluon-gluon fusion process dominates and the cross section uncertainty from the PDFs+αs is 3.2% at the LHC [34]. In particular, the MuIC, like the LHeC, would directly probe the PDFs at the scale and x-values for Higgs production at the hadron colliders. This would significantly reduce the cross section uncertainty obtained from global fits to previous DIS and pp collider data, to the level of 0.4% for the gluon-gluon fusion process and similarly for the tTH process [3]. The MuIC also would further disentangle the flavor structure of the PDFs.

The measurements of the PDFs at the MuIC also would resolve some open issues in the proton content, as summarized in Ref. [3], such as the ratio of u/d as x→1 and the universality of the light quark (and antiquark) sea. Additionally, fits to the structure function data allow for the precise measurement of αs and its running over a large Q2 range.

B. Nucleon Spin and 3-D Structure

The nucleon spin is one of its fundamental properties. It was found that quark polarization inside a nucleon only contributes to about 30% of the total spin. Therefore, the majority rest of the nucleon spin must be carried by the gluon polarization and orbital motion of quarks and gluons. To determine the contribution of gluon polarization, a measurement of the helicity-dependent gluon distribution function, ∆g(x), especially in the small x region, is crucial. Evidence for a finite gluon polarization in the proton for x > 0.05 has been found by polarized pp collisions at RHIC [35]. However, the uncertainty on the overall gluon polarization is still rather large mainly because of the limitation in accessing the small x region. For x < 0.01 ∆g(x) is largely unconstrained. The EIC is projected to significantly improve the precision of gluon polarization by accessing x values down to 0.001 at Q2 ≈ 10 GeV2 [36], with an integrated luminosity of 10 fb−1. Assuming the same integrated luminosity, the MuIC will extend the reach in x down to 10−5, potentially providing a definitive answer to the gluon spin contribution. Furthermore, precise measurements of three-dimensional (3D) parton distribution functions, generalized parton distributions (GPDs) and transverse-momentum-dependent (TMD) distributions, over a wide range of x values could provide a complete picture of orbital angular momentum of quarks and gluons inside the nucleon.

C. Gluon Saturation at Extreme Parton Densities in proton and nucleus

The gluon density inside the nucleon increases dramatically toward small x values. At extreme gluon densities, the nonlinear QCD process of gluon-gluon fusion will start playing a key role to limit the divergence of the gluon density. At a certain dynamic scale of momentum transfer, known as Qs, gluon splitting and fusion processes reach an equilibrium such that the gluon density is saturated, resulting in novel universal properties of hadronic matter. Examples of gluon saturation scales inferred from fits to HERA data (known as the GBW model) [27] are shown Fig. 1, as discussed earlier, which is expected to be enhanced in a nucleus by a factor of A1/3. The large Qs scale predicted at small x values, especially in large nuclei, enables perturbative QCD calculations of nucleon structure functions, as proposed in the color-glass condensate (CGC) effective field theory [32]. Predictions of signatures of gluon saturation in large nuclei at EIC can be found in Ref. [2]. As shown in the kinematic coverage of Fig. 1, the EIC starts entering the domain of gluon saturation in gold nuclei at x ≈ 10−3, while the MuIC (and also LHeC) will be well in the saturation regime for x ≈ 10−4−10−5 at Qs of a few GeV. The MuIC will also probe the saturation regime in the proton and other light nuclei for the first time, which is not accessible by the EIC.

D. Electroweak Physics

The Q2 reach of the MuIC, like the LHeC, would extend well above M为何, allowing studies of electroweak physics in the space-like regime. At such high Q2 the charged-current cross section is comparable to the neutral current one, leading to many charged current interactions at high Q2 and x. One thing to note at the MuIC is that the final state leptons will be produced very forward, as discussed in Section IV. However, the transverse momentum can still be sizable (O(100) GeV) at high Q2 > M为何, so even with final state neutrinos in charged-current interactions the kinematics can be measured from the final state hadrons. For smaller Q2 this will be challenging experimentally, which also limits the applicability of charged-current events to study the flavor structure of the PDFs for x ≲ 0.01.

The MuIC also can be a facility to measure Higgs boson couplings, provided that the integrated luminosity is large enough given that the inclusive cross section is of the order 100 fb [3]. In particular, as Ref. [3] suggests, a lepton-proton collider provides complementarity to the measurements made at pp colliders. For example,
the Higgs boson decay to charm quarks is accessible at a lepton-hadron collider.

E. Physics Beyond the Standard Model

The MuIC will provide complementary capabilities of searching for many physics phenomena beyond the standard model, by providing a cleaner environment than pp colliders. Particularly, it provides a unique opportunity to search for new couplings to the second-generation leptons. For example, searches for charged lepton flavor violation processes via $\mu \rightarrow \tau$ at high energies can be carried out at the MuIC, which complements the searches for $e \rightarrow \tau$ or $e \rightarrow \mu$ processes at HERA, EIC, and LHeC. Another example is the search for leptoquarks, where the MuIC could be uniquely sensitive to leptoquarks that couple to muons and quarks (of any generation, as the lepton and quark generations do not need to match [37]). In particular, anomalies observed in lepton universality in $B$ meson decays (e.g. [38]) and in the measurement of the anomalous magnetic moment of the muon [39] could be explained by leptoquarks [40].

Other search areas to which the MuIC could potentially improve sensitivity beyond the LHC are discussed in the LHeC report [3]. This includes compressed supersymmetry scenarios, heavy neutrinos, dark photons, axion-like particles, excited fermions (particularly excited muons and excited muon neutrinos), and lepton-quark contact interactions.

IV. FINAL-STATE KINEMATICS

While the $Q^2$ and $x$ coverage determines the region of physics potential for a DIS facility, the final-state kinematics of the scattered lepton and produced hadrons are also important considerations. For perspective, in electron-proton collisions at the EIC (LHeC), a relatively low energy 20 GeV (50 GeV) electron is scattered off a much higher energy 275 GeV (7 TeV) proton. In contrast, the MuIC would collide a nearly 1 TeV muon off of a 275 GeV proton, a more symmetric collision with much more momentum in the lepton direction. This leads to complementary final state kinematics (not to mention a different lepton species), which poses different detector design considerations. We focus here on a discussion of the kinematics and general requirements for a MuIC detector, but leave the details of a detector design and simulation to future work.

Figure 2 shows the kinematics of scattered leptons and partons (hadrons) in the $Q^2$ and $x$ map for the proposed MuIC (top) and the LHeC (bottom) at their highest energies for the inelasticity range $0.01 < y < 0.95$. We define the initial proton direction as the forward direction. The dot-dashed lines indicate constant energy contours of the scattered lepton (left) and partons (hadrons, right). Likewise, the short-dashed lines indicate constant pseudorapidity contours of the scattered lepton and partons (hadrons), as determined by [41]:

$$Q^2(x, \eta^l_h) = \frac{sx}{1 + \frac{E^l_h \exp(-2\eta^l)}{E^l_H}}$$  \hspace{1cm} (1)

$$Q^2(x, \eta^h_f) = \frac{sx}{1 + \frac{E^l_h \exp(2\eta^h)}{E^l_H}}$$  \hspace{1cm} (2)

$$Q^2(x, E^l_f) = \frac{1 - \frac{E^l_f}{E^l_H}}{\frac{1}{s^2} - \frac{1}{4(E^l_H)^2}}$$  \hspace{1cm} (3)

$$Q^2(x, E^h_f) = \frac{sx(1 - \frac{E^l_f}{E^l_H})}{1 - \frac{E^l_f}{E^l_H}}$$  \hspace{1cm} (4)

where $E^l_h$ and $E^l_f$ are the energies of the lepton or hadrons (parton) before and after the scattering, respectively. The pseudorapidity of the final-state scattered lepton or hadrons (parton) is denoted by $\eta^l_h$. Representative values of the scattered muon and hadron quantities for several benchmark points in $Q^2$ and $x$ are shown in Table III: one at very high $Q^2$ and $x$, another at a medium $Q^2$ scale representative of Higgs boson production at the LHC, and a low $Q^2$ one near the expected gluon saturation scale.

| $Q^2$ (GeV$^2$) $x$ | $E^l_f$ (GeV) | $\eta^l_h$ | $E^l_h$ (GeV) | $\eta^l_h$ |
|---------------------|--------------|------------|--------------|------------|
| High $Q^2$          | 500,000      | 0.5        | 180          | 0.5        | 920        | -2.4       |
| Med $Q^2$           | 4000         | 0.01       | 600          | -3.2       | 370        | -2.7       |
| Low $Q^2$           | 3            | $5 \times 10^{-4}$ | 906    | -7.0       | 54         | -4.2       |

As shown in Fig. 2 (top), the small-$x$ region at the MuIC corresponds to scattered muons in the very backward (lepton-going) direction. Reconstruction of the DIS event kinematics would require the detection of muons at very small scattering angles from the beamline (down to a few mrad) as well measurements of momenta up to 1 TeV, both of which can be very challenging but not dissimilar to detecting final state muons in vector boson processes at a $\mu^+\mu^-$ collider. However, in DIS the kinematics are over constrained at leading order when one considers also the kinematics of the final state hadrons. In particular, one can avoid energy and momentum measurements entirely using the so-called “double angle” method using only the angles of the scattered lepton and hadrons [42], or rely entirely on the kinematics of the hadrons only, known as the “Jacquet-Blondel” method (JB). Moreover, unlike electrons, high energy muons will easily make it out of the beam pipe and any surrounding shielding or magnets without much energy loss. Therefore, it is possible to set up muon detectors in the far backward direc-
tion outside the beam pipe. Here a beam crossing angle helps (e.g., 25 mrad as for the EIC) to bring the most forward scattered muons at low $Q^2$ outside of the beam pipe in a longitudinal distance of meters rather than tens of meters. (An overall slight tilt of the muon storage ring also may help control the location and direction of where neutrino induced radiation emanates.) In contrast, the high $Q^2$ region of the MuIC is more comfortably accessible near the central rapidity region for both scattered muons and final-state hadrons.

Reconstruction of the kinematic variables ($Q^2$, $x$ and $y$) with good resolution is crucial to all DIS experiments. As pointed out earlier, the DIS kinematics are over constrained and can be reconstructed from the scattered beam lepton, the hadronic recoil, or from a mixture of various species of particles at MuIC.

TABLE IV. List of assumed detector resolutions for measuring various species of particles at MuIC.

| Particle         | Detector          | Resolution |
|------------------|-------------------|------------|
| (Forward) Muons  | e.g., MPGD        | 0.01% $p \otimes 1\%$ | $0.2 \times 10^{-3}$ |
| Charged particles| Tracker + PID     | 0.1% $p \otimes 1\%$ | $(\frac{2}{p} \otimes 0.2) \times 10^{-3}$ |
| Photons          | EM Calorimeter    | 10% $\frac{1}{\sqrt{E}} \otimes 2\%$ | $0.087 \frac{1}{\sqrt{12}}$ |
| Neutral hadrons  | Hadronic Calorimeter | 50% $\frac{1}{\sqrt{E}} \otimes 10\%$ | $0.087 \frac{1}{\sqrt{12}}$ |

V. DETECTOR REQUIREMENTS AND DESIGN CONSIDERATIONS

Future extensive simulation studies of the detector will be required to fully explore the feasibility and challenges of physics at the MuIC. In this section, we discuss general detector requirements and design considerations based on the final-state kinematics presented in Section IV, and present an estimate of DIS kinematic variable resolutions based on feasible detector technologies.
both. Six methods of reconstructing $Q^2$, $x$ and $y$ are summarized in Ref. [42]. To get an estimate of realistic resolutions at MuIC, we study the following three methods in this paper:

- **Lepton-only method:**
  
  $Q^2(E^f_\mu, \theta) = 2E^i_\mu E^f_\mu (1 + \cos \theta), \quad (1)$
  
  $y(E^f_\mu, \theta) = 1 - \frac{E^f_\mu}{2E^i_\mu} (1 - \cos \theta), \quad (2)$

  where $E^i_\mu$ and $E^f_\mu$ are incoming and scattered muon energy, and $\theta$ is the polar angle of the scattered muon.

- **Jacquet-Blondel (JB) method** based on hadronic activities:
  
  $Q^2(P, \gamma) = \frac{P^2 \sin^2 \gamma}{1 - y(P, \gamma)}, \quad (3)$
  
  $y(P, \gamma) = \frac{F(1 - \cos \gamma)}{2E^i_\mu}, \quad (4)$

  where,

  $P^2 = (\Sigma_h P^x_h)^2 + (\Sigma_h P^y_h)^2 + (\Sigma_h P^z_h)^2, \quad (5)$

  $\cos \gamma = \frac{(\Sigma_h P^x_h)^2 + (\Sigma_h P^y_h)^2 - (\Sigma_h(E_h - P^z_h))^2}{(\Sigma_h P^x_h)^2 + (\Sigma_h P^y_h)^2 + (\Sigma_h(E_h - P^z_h))^2}, \quad (6)$

  and $(E_h, P^x_h, P^y_h, P^z_h)$ is a hadron four-momentum vector.

- **Double Angle (DA) method** based entirely on angles of scattered lepton ($\theta$) and recoil hadrons ($\gamma$),

  $Q^2(\theta, \gamma) = 4(E^i_\mu)^2 \frac{\sin \gamma (1 + \cos \theta)}{\sin \gamma + \sin \theta - \sin(\gamma + \theta)}, \quad (7)$

  $y(\theta, \gamma) = \frac{\sin \theta (1 - \cos \gamma)}{\sin \gamma + \sin \theta - \sin(\gamma + \theta)}, \quad (8)$

  We make a set of assumptions on the detector resolutions for measuring various species of particles at the MuIC, listed in Table IV, and smear the kinematic variables of particles generated by the PYTHIA 8 [43] Monte Carlo event generator:

  - **Scattered muons:** as mentioned earlier, coverage of muon detectors to $\eta = -7$ in the far-backward region with excellent momentum and angular resolution is necessary. We assume an angular resolution of 0.2 mrad, which should be within the reach with the Micro Pattern Gas Detectors (MPGDs) [44] placed 10-20 m away from the interaction point. The momentum resolution will depend on details of the tracking system and magnetic field, so we simply assume a resolution factor similar to the performance of CMS muon systems [45], which is up to about 10% at 1 TeV.

  - **Charged particles:** we assume that they will be measured by a high precision tracker (e.g., silicon sensors) with a resolution comparable to the CMS tracker [46]. In addition, particle identification can be achieved by a time-of-flight (TOF) or a ring-imaging Cherenkov detector (RICH).

  - **Photons** are detected by electromagnetic calorimeters (EMCal) with an energy resolution assumed to be similar to that required for the EIC detectors [47] and an angular resolution similar to that of the CMS EMCal [48].

  - **Neutral hadrons**, such as neutrons and $K^0_s$, are detected by hadronic calorimeters (HCal) with an energy resolution assumed to be similar to that required for the EIC detectors [47] and angular resolution similar to that of the CMS HCal [49].

  The resolution of the reconstructed DIS kinematic variables, $Q^2$, $x$, and $y$ at the MuIC were studied using the PYTHIA 8 generator by smearing the final state particles by the resolution parameters listed in Table IV. The resulting resolutions (RMS) are presented in Fig. 3 in two dimensions as functions of $Q^2$ and $x$ for the three reconstruction methods. Regions of different $y$ values are also indicated by lines for $y=0.95$, $y=0.01$, and $y=0.001$. A hadron acceptance of $|y_h| < 5$ is assumed.

  The lepton-only method gives good resolution of 10–20% in $Q^2$ over most of the kinematic region, but performs poorly in determining the $x$ and $y$ values, mainly because of the very high muon momentum. The DA method relying on exclusively the measurement of angles, instead, provides much better resolutions of $x$ and $y$, except for $Q^2 < 10$ (GeV)$^2$. The JB method has excellent $y$ resolution down to very low $Q^2$ of 1 (GeV)$^2$ but underperforms in $Q^2$ and $x$ resolutions than the DA method for $Q^2 < 100$ (GeV)$^2$. The main limitation of JB and DA methods in fact comes from the detector acceptance coverage, while the energy and angular resolutions are found to have negligible impact. At high $y$ regions (smaller $x$ and $Q^2$), the recoil hadrons start shifting toward very backward direction outside $|y_h| < 5$. Therefore, a wide coverage of detectors at MuIC, especially in the backward direction for hadrons, is essential. By optimizing the DIS kinematic reconstruction based on measurements of both the scattered muon and hadronic activities, it should be feasible to achieve good resolutions at MuIC with conceivable detector technologies.

VI. A POSSIBLE ROAD MAP TOWARD FUTURE MUON COLLIDERS

We present our view on a possible road map to a future combined facility for nuclear physics (NP) and high energy physics (HEP) based on muon collider technology in Fig. 4. We assume that the muon collider community will need about 15 years to fully establish the feasibility.
of a muon collider via intense R&D at a testing facility in order to be ready to construct a muon collider demonstrator. We propose that this effort be carried out jointly between the NP and HEP communities, as this will attract more interest, resources, and be beneficial to both science programs.

In the USA, the EIC has been identified as the next QCD frontier by the NP community in the 2015 US Nuclear Physics Long Range Plan [50]. It has recently been endorsed by the US National Academy of Science [1] and the Department of Energy to be built at BNL by around 2029. The EIC is a unique machine, capable of colliding polarized electrons (up to 18 GeV) with polarized protons (up to 275 GeV) and a variety of nuclei with unprecedented high luminosity for the first time. The success of the EIC is built upon its reuse of an existing RHIC that has been in operation since 2000, in order to take advantage of the existing tunnel, hadron beam, and infrastructure to minimize costs. Following this approach, constructing a MuIC after the EIC toward late 2040 by replacing the electron beam would open up new territory for QCD physics at a reasonable cost and share a strong synergy with the HEP community in advancing future collider technology. Along the way, a muSTORM-like facility may also be hosted at BNL with stored muon beams.

At the high energy physics frontier, the upgrade of the High-Luminosity LHC (HL-LHC) planned for 2027 aims to increase the delivered integrated luminosity by a factor of at least 10 beyond the LHC’s initial design, bringing the promise of new discoveries in the next 10–15 years. To plan for the next energy frontier era after the HL-LHC, there have been active discussions in the European Strategy Planning Group, in the U.S. “Snowmass” community planning exercise, and in Asia (Japan, China). There appears to be a consensus that a Higgs factory at an $e^+e^-$ collider (ILC, CLIC, CepC, FCC-ee) is a viable next step as a gateway to explore physics beyond the SM. After the $e^+e^-$ Higgs factory, muon collider technology would hopefully have matured with the MuIC as the cornerstone of a joint effort between the NP and HEP communities. Constructing a $\mu^+\mu^-$ collider at
A possible roadmap to future muon colliders in NP and HEP

O(10) TeV would then open the next energy frontier to directly probe new physics, possibly re-using the existing facility and infrastructure at CERN, which significantly reduces the civil engineering cost.

In summary, a bright future of high energy nuclear and particle physics requires more collaboration to develop new, innovative ideas and technology. We lay out a possible path toward a future high energy muon collider with a proposed muon-ion collider at BNL as an intermediate step. This approach has high scientific and technological merits, and synergies interests of nuclear physics and particle physics communities. We look forward to developing more concrete steps toward an exciting future program.

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