Establishment and analysis of coupled dynamic model for dual-mass silicon micro-gyroscope

Zhanghui Wang¹, Anping Qiu¹*, Qin Shi¹ and Taoyuan Zhang²

¹School of Mechanical Engineering, Nanjing University of Science and Technology, Nanjing, China
²Shanghai Aerospace Control Technology Institute, Shanghai, China

*E-mail:apqiu@mail.njust.edu.cn

Abstract. This paper presents a coupled dynamic model for a dual-mass silicon micro-gyroscope (DMSG). It can quantitatively analyze the influence of left-right stiffness difference on the natural frequencies, modal matrix and modal coupling coefficient of the DMSG. The analytic results are verified by using the finite element method (FEM) simulation. The model shows that with the left-right stiffness difference of 1%, the modal coupling coefficient is 12% in the driving direction and 31% in the sensing direction. It also shows that in order to achieve good separation, the stiffness of base beam should be small enough in both the driving and sensing direction.

1. Introduction

The Dual-mass Silicon Micro-gyroscopes (DSMG) are employing electrostatic actuation and capacitive detection. Limited by the current MEMS fabrication technology, the stiffness of the support beams for the left and right structures of DSMG will be different with the fabrication error [1]. The left-right stiffness difference (stiffness difference) will change the mechanical properties especially the bias and vibration sensitivity of the gyroscope. Therefore, it is important to analyze the impact of stiffness difference on bias and vibration sensitivity of the DMSG.

The fabrication error impacting on the driving direction of the DSMG had been analyzed in some literatures [2-4]. Take into considered of both the drive and sense mode, a non-ideal dynamic model was studied in the literatures [5-6]. However, the stiffness difference of the DSMG in the above mentioned papers was only considered in its driving direction and the modal coupling coefficient was not be proposed.

In this paper, firstly, the coupled dynamic model is proposed for a DMSG. Second, the natural frequencies and modal matrix are calculated. Then the modal coupling coefficient is deduced from the proposed modal matrix. Additionally, the coupled dynamic model can be solved by modal decoupling method. At last, the reliability of the theory is verified by finite element method (FEM) simulation.

2. Structure and operation principle

Fig.1 is the schematic of a dual-mass silicon micro-machined gyroscope. By applying the alternating voltage on the driving comb capacitor, the left and right mass will move in opposite direction along X-axis (driving direction). When the sensor rotates about Z-axis, the resulting Coriolis force causes the left and right mass to move in opposite direction along Y-axis (sensing direction). The relative motion
between the movable detection comb and the fixed detection comb forms the differential capacitance for detection. Ideally, the amount of differential detection capacitance is proportional to the input angular rate.

**Figure 1.** The schematic of a dual-mass silicon micro-machined gyroscope

### 3. Modeling

Fig. 2 shows the dynamic model in the drive direction and sensing direction. The motion equations are listed as below.

\[
\begin{align*}
    m_{dl}(x_{dl}(t) + b_{dl}(t)x_{dl} + k_{dl}(x_{dl} - x_{l})) &= F_{dl}(t) \\
    m_{s}(x_{s} + k_{s}(x_{s} - x_{l}) - k_{b}(x_{l} - x_{s}) &= 0 \\
    m_{s}(y_{s}(t) + b_{s}(y_{s}) + k_{s}(y_{s} - y_{l}) - \partial L_{\psi}) &= -2\Omega_{z}x_{l}(t) \\
    I_{b} \dddot{\theta} + k_{bd} L_{\psi} \dddot{\theta} + k_{bs}(y_{s} - \partial L_{\psi})L_{\psi} &= 0
\end{align*}
\]

Herein, \(m_{dl}\) and \(m_{sr}\) are the left and right proof masses, \(m_{dl}\) and \(m_{dr}\) are the left and right drive comb masses, \(m_{b}\) and \(I_{b}\) are the proof mass of the base beam, \(k_{dl} = k_{d} + \Delta k_{d}/2\) and \(k_{dr} = k_{d} - \Delta k_{d}/2\) are the bending stiffness of the left and right drive beams, \(k_{dl} = k_{d} + \Delta k_{d}/2\) and \(k_{sr} = k_{d} - \Delta k_{d}/2\) are the bending stiffness of the left and right sense beams. \(k_{bd}\) and \(k_{bs}\) are the bending stiffness of the base beam in its driving and sensing direction. The parameters of the DMSG designed are list in the Tab. 1.

**Figure 2.** The dynamic model in the drive direction and sensing direction

**Table.1** The parameters of the DMSG designed

| Parameter Name | \(k_{b}\) | \(k_{s}\) | \(\Delta k_{b}\) | \(\Delta k_{s}\) | \(k_{bd}\) | \(k_{bs}\) | \(m_{b}\) | \(m_{s}\) |
|---------------|-----------|-----------|----------------|----------------|-----------|-----------|--------|--------|
| Value         | 626       | 674       | 6.26           | 6.74           | 17400     | 54040     | 1.08   | 0.97   |
| Unit          | N/m       | N/m       | N/m            | N/m            | N/m       | N/m       | um     | um     |


The base beam mass is much less than the proof mass. Therefore, set \( m_b \) and \( I_b \) to zero for simplification. Take into account the mass matrix \( [m] \) and the stiffness matrix \( [k] \). Supposing \( z(\alpha_\theta) = [k-m\alpha_\theta^2] \), the equations can be written as below.

\[
[z(\alpha_\theta)]v = \{p\}.
\] (5)

Then, the natural frequencies can be obtained by setting \( z(\alpha_\theta) = 0 \). And the modal matrix \( [A] \) defined as the modal matrix is calculated from the eigenvectors of matrix \( [z(\alpha_\theta)] \).

\[
[A] = \begin{bmatrix} r_1 & r_2 \end{bmatrix}.
\] (6)

Here \( r_1 \) and \( r_2 \) are defined as the mode function. Now, the modal coupling coefficient which reflects the coupling between the in-phase and anti-phase motion can be express as

\[
\alpha_{1,2} = \frac{1}{\sqrt{|r_{1,2}|} + 1}.
\] (7)

Finally, the system can be decoupling by applying the modal matrix. As a result the motion equations change to

\[
[M_p]\ddot{\{q\}} + [K_\theta]\{q\} = \{p\},
\] (8)

where \( [M_p] = [A]^T[MA][A] \) and \( [K_\theta] = [A]^T[K][A] \) are the modal mass and modal stiffness. \( \{q\} = [A]^T\{u\} \) is the modal freedom. As a consequence, the motion equations can be solved independently.

3.1. Drive mode

The natural frequencies and mode functions in its driving direction are shown as:

\[
\omega_{nd}^2 \approx \left(2k_d k_{bd} + 2k_d^2 \pm \left[4k_d^4 + \Delta k_d^2 k_{bd} (2k_d + k_{bd})\right]^{1/2}\left(2(2k_d + k_{bd})M_d\right)^{-1}\right),
\] (9)

\[
r_{d(1,2)} = 2k_d \left(k_{bd}\Delta k_d \pm \left[4k_d^4 + (k_{bd}^2 - 2k_d^2)\Delta k_d^2\right]^{1/2}\right)^{-1}.
\] (10)

3.2. Sense mode

The natural frequencies and mode functions in its sense direction are shown as:

\[
\omega_{sn}^2 = \left(2k_s k_{bs} + 2k_s^2 \pm \left[4k_s^4 + \Delta k_s^2 k_{bs} (2k_s + k_{bs})\right]^{1/2}\left(2(2k_s + k_{bs})M_s\right)^{-1}\right),
\] (11)

\[
r_{s(1,2)} = -2k_s \left(k_{bs}\Delta k_s \pm \left[4k_s^4 + (k_{bs}^2 - 2k_s^2)\Delta k_s^2\right]^{1/2}\right)^{-1}.
\] (12)

According to (9)-(12), the natural frequencies and mode functions without the stiffness difference are show as below.

\[
\omega_{d,\text{in-phase}}^2 = k_d k_{bd} (2k_d + k_{bd})^{-1} M_d^{-1}, \omega_{d,\text{anti-phase}}^2 = k_d / M_d
\] (13)

\[
\omega_{s,\text{in-phase}}^2 = k_s / M_s, \omega_{d,\text{anti-phase}}^2 = k_s k_{bs} (2k_s + k_{bs})^{-1} M_s^{-1}
\] (14)

\[
|r_{1d}| = |r_{2d}| = 1, |r_{1s}| = |r_{2s}| = 1.
\] (15)

Obviously, the relation in (15) means that the in-phase motion and the anti-phase motion are complete independence. According to (9)-(12), \( \Delta k_d \) and \( \Delta k_s \) are determined by the processing precision. \( k_d \) and \( k_s \) determine the drive mode and sense mode. As a result, in order to achieve good separation, the stiffness of base beam should be small enough in both the driving and sensing direction. However, these low-rigidity beams present challenges and often introduce other problems.
4. Fem simulation
The Finite element software ANSYS was employed to verify the coupling dynamic model of the DSMG. The schematic of a dual mass silicon micro-machined gyroscope is shown in Fig. 3. First, the modal analysis of the DSMG structure with stiffness difference is done to verify the natural frequencies. The natural frequency variation due to the stiffness difference of 1% is shown in Tab. 2.

The data in Tab. 2 shows that the natural frequency variations due to the stiffness difference of 1% are 0.5Hz in its driving direction and 1.5Hz in its sensing direction. It also shows that the natural frequency variation can be accurately represented by the coupling dynamic model.

![Schematic of a dual mass silicon micro-machined gyroscope](image)

**Figure 3.** The schematic of a dual mass silicon micro-machined gyroscope.

**Table 2.** The natural frequency variation.

|                   | Theoretic frequency change (Hz) | Simulated frequency change (Hz) | Error (%) |
|-------------------|--------------------------------|--------------------------------|-----------|
| In-phase drive mode | 0.49                           | 0.43                           | 13.9      |
| Anti-phase drive mode | 0.55                          | 0.58                           | 5.1       |
| In-phase sense mode   | 1.54                          | 1.38                           | 11.5      |
| Anti-phase sense mode   | 1.52                          | 1.45                           | 4.8       |

Then the harmonic response simulation is done to verify the modal coupling coefficient. Fig. 4 shows the frequency response of the anti-phase drive mode. The frequency response of other modes has the similar results. Fig. 4 shows that the in-phase and anti-phase displacements are not independent from each other. Fig. 5 shows the modal coupling coefficient of theory and simulation in each mode. According to Fig. 5, with the stiffness difference of 1%, the modal coupling coefficient is 12% in the driving direction and 31% in the sensing direction. It also shows that the theoretical result and simulation result are in good agreement.
Figure 4. The frequency response of the anti-phase drive mode.

Figure 5. The mode coupling coefficient of theory and simulation in each mode (the theoretical and the simulation value are located in the left and right of each column).
5. Conclusion
Base on the structural characteristics, a coupling dynamic model is proposed and established for the dual mass silicon micro-gyroscope in this paper. The effect of the stiffness difference on the natural frequencies and mode functions are demonstrated. The modal coupling coefficient is calculated based on the mode functions. The natural frequency variations due to the stiffness difference of 1% are 0.5Hz in its driving direction and 1.5Hz in its sensing direction. The modal coupling coefficient is 12% in the drive direction and 31% in the sense direction due to the stiffness difference of 1%. These theoretical results are verified by the FEM simulation. The good agreement between the theory result and simulation result prove the accuracy of our model and its theory.

Acknowledgement
This work is supported by Department of Instrument Science and Engineering at Nanjing University of Science and Technology. The authors also thank the personnel of Department of Instrument Science and Engineering at Nanjing University of Science and Technology for designing, fabricating, and packing the device for this study.

References
[1] M S Weinberg and A Kourepenis Jun. 2006 “Error sources in in-plane silicon tuning-fork MEMS gyroscopes,” J. Microelectromech. Syst. vol 15 no. 3 pp 479–91
[2] S W Yoon, S W Lee, N C Perkins and K Najaﬁ Oct. 28–31, 2007 “Vibration sensitivity of MEMS tuning fork gyroscopes,” in Proc. IEEE Sensors, Atlanta GA, USA pp. 115-19
[3] L Huang, Y Ni and Y Yin Sept. 16-18, 2011 “Fabrication error analysis on dual-mass silicon micro-gyroscope,” International Conference on Electrical and Control Engineering pp 3931-34
[4] A R Schofield, A A Trusov and A M Shkel Oct. 28–31, 2007 “Multi-degree of freedom tuning fork gyroscope demonstrating shock rejection,” in Proc. IEEE Sensors Atlanta GA USA pp 120–23
[5] B Yang, X Wang, D Hu and L Wu 2017 “Research on the non-ideal dynamics of a dual-mass silicon micro-gyroscope,” Microsystem Technologies, vol 23 no 1 pp 151-62
[6] B Yang, D Hu, Y Deng and X Wang Feb. 2016 “An improved dual-mass decoupled micro-gyroscope for the non-ideal decoupled error suppression”. in Proc. IEEE Int. Symp. Inertial Sens. Syst pp 58-61