Bounce in GR and higher-order derivative operators

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Abstract

Recent progress seems to suggest that one must modify General Relativity (GR) to stably violate the null energy condition and avoid the cosmological singularity. However, with the higher-order derivative operators of scalar field (a subclass of the degenerate higher-order scalar-tensor theory), we show that at energies well below the Planck scale, fully stable nonsingular cosmologies can actually be implemented within GR.

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I. INTRODUCTION

It is well-known that General Relativity (GR) suffers the singularity problem, which indicates that our understanding about the gravity and the origin of the universe is incomplete [1, 2]. It is still a difficult and intangible task to look for a ultraviolet (UV)-complete theory to describe what happens at the “singularity”. However, implementing a fully stable nonsingular universe with the effective field theory (EFT), which captures low energy behaviors of the complete theory, might be a significant step to this task.

In the spatially flat nonsingular cosmologies, the Null Energy Condition (NEC) must be violated for a period. However, it is often accompanied by the (ghost, gradient) instabilities [3, 4], or else the singularities in the equations of background or perturbations, see also Refs.[5–7]. Recently, it has been found that the solutions of fully stable nonsingular cosmologies do exist in the EFT beyond Horndeski [8–13]. Degenerate higher-order scalar-tensor (DHOST) theory [14] actually is a rich pool for such EFTs [15]. However, it is noteworthy that in the nonsingular models built, the gravity has been no longer GR-like.

Recently, the LIGO Scientific and Virgo Collaborations have detected the gravitational wave (GW) signals of binary black hole (BH) [16] and binary neutron star mergers [17], which opened a new window to probe the gravity physics. The results of all tests performed in Refs.[18, 19] showed perfect agreement with GR, particularly in the strong-field regime. Currently, GR is still a well-established effective theory in the low energy regime of the UV-complete theory, though it must break down around the Planck energy.

How to implement the nonsingular bounce with GR? It is well-known that the $P(\phi, X)$ theory can hardly bring a stable NEC violation. To stably regulate such a violation, one may include the higher-order derivative operators $(\Box\phi)^2$, $(\phi^{\mu\nu})^2$· · · in the $P(\phi, X)$ theory, and set the EFT as, e.g.[20],

$$L \sim \frac{M_P^2}{2} (4) R + P(\phi, X) + \mathcal{O} ((\Box\phi)^2, (\phi^{\mu\nu})^2 \cdots ),$$  \hspace{1cm} (1)

Generally, integrating out the massive particles beyond the cutoff scale, one will have the higher-order correction $\mathcal{O} ((\Box\phi)^2, (\phi^{\mu\nu})^2 \cdots )$ [21, 22]. One frequently-used possibility is $\mathcal{O} \sim (\Box\phi)^2$, see e.g.[23–25]. However, the corresponding EFT must beg unknown physics in the sufficiently far past, or else the higher-order derivative operator will show itself the Ostrogradski ghost. It is noticed that such higher-order derivative operators can be actually
allowed in the DHOST theory [26, 27], see also [28], as the specific conditions are fulfilled so that the Ostrogradski ghost is avoided [29, 30].

Nevertheless, which operator in $\mathcal{O}((\square \phi)^2, (\phi^\mu)^2 \cdots)$ is indispensable for achieving a pathology-free bounce in GR is still not clear so far. In this paper, we will propose a consistent (1)-like EFT for the spatially-flat fully stable nonsingular cosmologies. We, with it, will discuss how to evade the No-go Theorem [3, 4] plaguing the cosmologists, and show a concrete example for the cosmological bounce.

II. DHOST THEORY WITH $c_T = 1$

A. Reducing to GR

We begin with the DHOST theory with $c_T = 1$ ($c_T$ is the speed of GWs) [31]

$$
L_{c_T=1}^{DHOST} = P + Q \Box \phi + A^{(4)} R + \frac{1}{A} \left( 6A_X^2 - (A - X A_X) B - \frac{X^2 B^2}{8} \right) \phi^\mu \phi_{\mu\nu} \phi_{\lambda\nu} + B \phi^\mu \phi_{\mu\nu} \Box \phi + \frac{B}{A} \left( 2A_X + \frac{XB}{2} \right) (\phi_{\mu\nu\rho} \phi_{\rho})^2,
$$

(2)

where $\Box \phi \equiv \phi_{\mu\nu} \equiv \Box \nu \nabla_{\mu} \phi$ and $X \equiv \phi_{\mu} \phi^\mu$. The coefficients $A$, $Q$ and $B$ only depend on $\phi$ and $X$. Generally, $B$ is independent of $A$. However, if $B = -\frac{4}{X} A_X$, $L_{c_T=1}^{DHOST}$ will reduce to the $c_T = 1$ beyond Horndeski theory $L_{c_T=1}^{bH}$ [32].

It is significant to notice that if setting $A = \text{const.}$ and $Q = 0$, in $L_{c_T=1}^{bH}$, we will get $B = -\frac{4}{X} A_X = 0$, and so (2) will reduce to GR, while in $L_{c_T=1}^{DHOST}$, we will have GR with extra DHOST operators (higher-order derivative operators), which is not in the beyond Horndeski theory but belongs to a subclass of the $c_T = 1$ DHOST theory. Degenerate conditions required by the DHOST theory suggest that such a combination of higher-order derivative operators is free of the Ostrogradsky ghost. Thus a (1)-like EFT will be Ostrogradsky ghost-free, only if it is a subclass of the DHOST theory.

B. Perturbation in DHOST theories with $c_T = 1$

We adopt the ADM metric,

$$
d s^2 = -N^2 d t^2 + h_{ij} (d x^i + N^i d t)(d x^j + N^j d t),
$$

(3)
where $N$ is the lapse, $N_i$ is the shift and $h_{ij}$ is the spatial metric. We set $\eta = \phi$ as the time coordinate. Dynamics of $\phi$ is absorbed into $N(\eta)$, as $\dot{\phi} \equiv d\phi/d\eta = 1$.

Defining
\[ B = -\frac{4}{X} A_X + A\tilde{B}, \]
we have
\[ L_{\text{DHOST}}^{bH} = L_{\text{cT} = 1}^{bH} + \Delta L, \]
where
\[ \Delta L = A\tilde{B}\phi^\mu \phi^\nu \phi_{\mu\nu} \Box \phi + \left(-A\tilde{B} + 2X A_X \tilde{B} - \frac{X^2 A\tilde{B}^2}{8}\right) \phi^\mu \phi_{\mu\mu} \phi^\lambda \phi_{\lambda\lambda} \]
\[ + \frac{X \tilde{B}}{2} \left(-\frac{4}{X} A_X + A\tilde{B}\right) (\phi_{\mu} \phi^{\mu\mu} \phi_{\nu})^2. \]

In the unitary gauge, one could rewrite $L_{\text{cT} = 1}^{bH} = \tilde{P} + \tilde{Q} K + A(\mathcal{R} - \mathcal{K}_2)$ [28], where $\mathcal{R} \equiv h^{ij} \mathcal{R}_{ij}$ is the Ricci scalar on the spacelike hypersurface, $K \equiv h^{ij} K_{ij}$ is the extrinsic curvature and $\mathcal{K}_2 \equiv K^2 - K_{ij} K^{ij}$. The DHOST operators follow
\[ \phi^\mu \phi^\nu \phi_{\mu\nu} = \frac{-1}{N^5} \left(\ddot{N} - N_i \partial_i N\right) \equiv \frac{-1}{N^5} N', \]
\[ \Box \phi = \frac{1}{N^3} \left(\ddot{N} - N_i \partial_i N\right) - \frac{1}{N^3} K \equiv \frac{1}{N^3} N' - \frac{1}{N} K, \]
\[ \phi^\mu \phi_{\mu\mu} \phi^\lambda \phi_{\lambda\lambda} = \frac{-1}{N^8} \left(\ddot{N} - N_i \partial_i N\right)^2 + \frac{1}{N^6} (\partial N)^2 \equiv \frac{-1}{N^8} N'^2 + \frac{1}{N^6} (\partial N)^2. \]

Thus we have
\[ \Delta L = -\frac{3A\tilde{B}^2}{8N^{12}} N'^2 + \frac{A\tilde{B}}{N^6} N' K - \frac{\tilde{B}}{N^5} \left(\frac{A}{N} + NA_N + \frac{A\tilde{B}}{8N^5}\right) (\partial N)^2 \]
where the equality $X = -1/N^2$ is used. Replacing $-\tilde{B}/(2N^5)$ with $\tilde{B}$, we get the ADM form of $L_{\text{cT} = 1}^{\text{DHOST}}$ (2)
\[ L_{\text{cT} = 1}^{\text{DHOST}} = \tilde{P} + \tilde{Q} K + A(\mathcal{R} - \mathcal{K}_2) - \frac{3A\tilde{B}^2}{2N^2} N'^2 - \frac{2A\tilde{B}}{N} N' K \]
\[ + \tilde{B} \left(\frac{2A}{N} + 2A_N - \frac{A\tilde{B}}{2}\right) (\partial N)^2. \]

We will work with (6). To study the stability of perturbations, we will expand $L_{\text{cT} = 1}^{\text{DHOST}}$ in (6) to second order. Defining the metric perturbation
\[ N^i = \delta^i j \partial_j \psi, \quad h_{ij} = a^2(\eta) e^{2\kappa} \delta_{ij}, \]
we have $L^{(2)} = 3a^3N\zeta\delta L + a^3\delta N\delta L + a^3N\delta_2 L$ at quadratic order, where $\delta_2 L$ refers to the expansion of $L$ at second order. To proceed, we first expand $K_i^j$ and $R$,

$$K_i^j = \frac{1}{N} \left[ \left( \mathcal{H} + \dot{\zeta} - \mathcal{H} \frac{\delta N}{N} \right) \delta_i^j - \frac{\delta^j_k}{a^2} \partial_i \partial_k \psi \right] + \mathcal{O}(\delta N^2),$$  \tag{7}

$$R = -\frac{2}{a^2} \left[ 2\partial^2 \zeta + (\partial \zeta)^2 - 4\zeta \partial^2 \zeta \right] + \mathcal{O}(\zeta^3),$$  \tag{8}

where $\mathcal{H} \equiv \frac{d\alpha}{d\eta} = NH$, and $H$ is the Hubble parameter. The quadratic term with the time derivative in $L_{{\text{DHOST}}=1}^{(2)}$ (6) is contributed by $-AK_2 - \frac{3\tilde{A}B^2}{2N^2}N'^2 - \frac{2\tilde{A}B}{N}N'K$. Considering (7) and (8), one finds that

$$L^{(2)}_{\text{kinetic}} = a^3A_N \left[ U\dot{\tilde{\zeta}}^2 - V(\partial \tilde{\zeta})^2 \right] = -\frac{6a^3A_N}{N} \left( \dot{\tilde{\zeta}} + \frac{\tilde{B}}{2} \frac{\delta N}{2} \right)^2$$

is diagonal for $\tilde{\zeta} = \zeta + \tilde{B}\delta N/2$. One of the degenerate conditions ($\beta_2 = -6\beta_1$, see e.g. Ref. [27]) in the DHOST theory actually suggests that the coefficients of the DHOST operators $N'^2$ and $N'K$ should satisfy a relation. As a result, $L^{(2)}_{\text{kinetic}}$ is necessarily diagonal. Confronting $\tilde{\zeta}$ with the constraint $\delta L/\delta (\partial^2 \psi) = 0$, we get

$$L^{(2)} = a^3NA \left[ U\ddot{\zeta}^2 - V(\partial \zeta)^2 \right]$$

with

$$U = \frac{\Sigma}{\gamma^2} + \frac{6}{N^2},$$  \tag{10}

$$V = \frac{2}{aA} \frac{d}{d\eta} (a\mathcal{M}) - 2,$$  \tag{11}

where

$$\gamma \equiv \left( \frac{1}{N} + N\alpha_B \right) \mathcal{H} + \frac{\tilde{B}}{2},$$  \tag{12}

$$\Sigma \equiv \mathcal{H}^2 \left[ \alpha_K + 6 \left( \alpha_B^2 - \frac{\gamma^2}{\mathcal{H}^2N^2} \right) + \frac{9\alpha_B\tilde{B}}{N} + \frac{3d(\alpha_B \mathcal{H} \tilde{A})/d\eta}{\mathcal{H}^2NA} \right],$$  \tag{13}

$$\mathcal{M} \equiv \frac{1}{\gamma} \left[ \left( \frac{A}{N} + A_N \right) - \frac{\tilde{A}\tilde{B}}{2} \right].$$  \tag{14}

Usually, one sets $\alpha_B$ and $\alpha_K$ as the coefficients of the operators $\delta K \delta N$ and $\delta N^2$, respectively,

$$\alpha_B = \frac{1}{4NA\mathcal{H}} \left( NL_{NK} + 2\mathcal{H}L_{NS} \right), \quad \alpha_K = \frac{1}{NA\mathcal{H}^2} \left( L_N + \frac{N}{2}L_{NN} \right),$$  \tag{15}

where $S \equiv K_{ij}R^{ij}$. 

III. BOUNCE IN GR

A. Expelling No-go with higher-order derivative operators

In the Horndeski theory, the fully stable nonsingular cosmological model is prohibited, the so-called No-go Theorem [3, 4], see also [33–36] for the relevant studies. One way out is going beyond Horndeski, as pointed out in Refs.[8, 9]. In particular, in a beyond-Horndeski subclass of the DHOST theory, the solutions of fully stable nonsingular cosmologies have been found [11–13, 15].

Setting $A = M^2_P/2 = \text{const.}$ in (6), we have

$$L_{c_T=1, A=M^2_P/2}^\text{DHOST} = \tilde{P} + \tilde{Q}K + \frac{M^2_P}{2}(\mathcal{R} - \mathcal{K}_2) - \frac{3M^2_P\tilde{B}^2}{4N^2}N'^2 - \frac{M^2_P\tilde{B}}{N}N'K$$

$$+ \tilde{B}\left(\frac{M^2_P}{N} - \frac{M^2_P\tilde{B}}{4}\right)(\partial N)^2, \quad (16)$$

which also belongs to a subclass of the DHOST theory. Recall the redefinitions (4) and $-\tilde{B}/(2N^5) \rightarrow \tilde{B}$ in Sect.II, thus the theory (2) with $A = M^2_P/2 = \text{const.}$ may be related to (16) by $B = -M^2_PN^5\tilde{B}$. It is also noticed that if $A = \text{const.}$, setting $\tilde{Q} = 0$, we will have $Q = 0$ and $P = \tilde{P}$ for $L_{c_T=1}^\text{DHOST}$ in (2). Thus if $\tilde{Q} = 0$, (16) is actually a (1)-like EFT.

The essence of the No-go proof is rewriting $V > 0$ ($c_S^2 > 0$) in (11) as the integral inequality, see [37] for a review,

$$a\mathcal{M}|_f - a\mathcal{M}|_i > \int_i^f a\mathcal{A}d\eta. \quad (17)$$

In the nonsingular models, the integral $\int_i^f a\mathcal{A}d\eta$ will diverge, thus $\mathcal{M}$ must cross 0 at a certain time. According to (14), we have

$$\mathcal{M} = \frac{M^2_P}{2\gamma}\left(\frac{1}{N} - \tilde{B}/2\right) \quad (18)$$

for (16). Thus we might get $\mathcal{M} = 0$ by adjusting $\tilde{B}(N, \eta)$, or equivalently $B(X, \phi)$ for $L_{c_T=1}^\text{DHOST}$ in (2). This suggests that it is possible to achieve the fully stable nonsingular cosmological solutions with (16) (equivalently, (1)-like EFT).

B. An example

To show that the observation made in Sect.III A is correct, we will present a concrete model for the nonsingular bounce, which might have significant applications in early universe
scenarios, e.g.\cite{38-41}.

We adopt

$$H = H/N = \frac{\eta}{3(1 + \eta^2)},$$

(also $N(\eta) = 1$ (equivalently $\phi = \eta$) as the background solution. When $\eta < 0$, the universe contracted with $H \sim 1/\eta < 0$. Cosmological bounce happened at $\eta = 0$. We might set $\tilde{P}(N, \eta)$ and $\tilde{B}(N, \eta)$ in $L_{c_T=1, A=\text{const.}}^{DHOST}$ (16) as

$$\tilde{P}(N, \eta) = \frac{g_1(\eta)}{2N^2} + \frac{g_2(\eta)}{N^4} + g_3(\eta),$$

(20)

and $\tilde{B}(N, \eta) = g_4(\eta)$, respectively. Here, since $\tilde{Q} = 0$, $\tilde{P}(N, \eta)$ is actually equivalent to $P(X, \phi) = g_1(\phi)X/2 + g_2(\phi)X^2 + g_3(\phi)$ in $L_{c_T=1}^{DHOST}$ (2).

One simple possibility for (13) is, see also \cite{15},

$$\Sigma = c_1(\eta)\gamma^2.$$  

(21)

According to Eq.(10), we will have $U > 0$ for a suitable $c_1(\eta)$. Combining Eq.(21) with the background equations (A1) and (A2) in Appendix A, we get the algebraical solutions of $g_1(\eta)$, $g_2(\eta)$ and $g_3(\eta)$, see Appendix B.

Inserting $\tilde{B}(N, \eta) = g_4(\eta)$ into Eq.(12), we have $\gamma = H + \dot{g}_4/2$. Thus

$$\mathcal{M} = \frac{M_P^2(1 - g_4/2)}{\dot{g}_4 + 2H}.$$  

(22)

Requiring that around $\eta = 0$, $1 - g_4/2 = 0$ (so $\mathcal{M} = 0$) and $\dot{g}_4 \sim H$, we consider such a $g_4$,

$$g_4(\eta) = \int_\eta^{+\infty} 2\mu H(s)e^{-s^2/\lambda^2} ds,$$

with $\lambda$ set by $g_4(0) = \mu e^{1/\lambda^2} \Gamma(0, 1/\lambda^2)/3 = 2$. Fig.1 plots the evolutions of $\dot{g}_4$ for $\mu = 0.9$ and $H$. When $|\eta| \gg \lambda$, $g_4 = 0$, we will have a $P(X, \phi)$ EFT with GR. Inserting (22) into (11), we have $V(\eta = 0) = \frac{2(2\mu - 1)}{-\mu + 1}$, so $c_S^2(\eta = 0) = V/U > 0$ suggests $0.5 < \mu < 1$.

As a concrete example, we plot Figs.2 and 3 with $c_1(\eta) = 150e^{-\eta^2/500}$. We see that the model is fully stable.

IV. DISCUSSION

Currently, GR is the well-tested effective theory of gravity. Based on the higher-order derivative operators, which might capture the physics of a UV-complete theory, we propose
a consistent EFT

\[
L = \frac{M_p^2}{2} (4) R + P(\phi, X) - \left( B + \frac{X^2 B^2}{4M_p^2} \right) \phi^{\mu} \phi_{\mu\nu} \phi^{\lambda\nu} + B \phi^{\mu} \phi^{\nu} \phi_{\mu\nu} \Box \phi \\
+ \frac{X B^2}{M_p^2} (\phi_{\mu} \phi^{\mu\nu} \phi_{\nu})^2, \tag{24}
\]

for the spatially-flat fully stable nonsingular cosmologies. It corresponds to a subclass \((A = M_p^2/2, Q = 0)\) of the DHOST theory \((2)\). It has been speculated that the higher-order derivative operators \(O((\Box \phi)^2, (\phi^{\mu\nu})^2 \cdots)\) in the EFT \((1)\) might play crucial roles in nonsingular cosmologies. Here, we clearly showed what kind of \(O((\Box \phi)^2, (\phi^{\mu\nu})^2 \cdots)\) is required for the full stability of nonsingular cosmologies.

We discussed how to evade the No-go Theorem with the EFT \((24)\) (its ADM Langrangian \((16)\)). In Refs.\([8, 9, 11, 12]\), the operator \(\mathcal{R} \delta g^{00}\) is added to GR to expel the No-go. However, besides the higher-order derivative operators, the corresponding covariant EFT also includes the derivative coupling of \(\phi\) to the gravity \(\sim X^{(4)} R\). Here, we found that expelling the No-go can be implemented only by the higher-order derivative operators \(O((\Box \phi)^2, (\phi^{\mu\nu})^2 \cdots)\) (the DHOST operators) in \((24)\) without modifying GR. A concrete model of the cosmological bounce have been presented in Sect.III B. It might be also interesting to apply the EFT \((24)\) to regulate the singularity of BH, e.g.\([42–44]\).

Recently, the well-posedness issue on the initial value has been promoted in the non-perturbative cosmologies \([45]\). An issue worthy of exploring is whether the initial value problem for \((24)\) is well-posed.

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FIG. 1: $\dot{g}_4$ is given by (23). We require $\dot{g}_4 \sim H$ for simplicity. We have set $\mu = 0.9$ and $M_P = 10$ in the plot.

FIG. 2: Coefficients $g_1$, $g_2$ and $g_3$ in $\tilde{P}(N, \eta)$ (20).

Appendix A: The background equations

Varying (6) with respect to $N$ and $\mathcal{H}$, respectively, we get

$$3B \left( \frac{AB\dot{N}}{N} + \frac{2A\dot{\mathcal{H}}}{N} \right) = \dot{\mathcal{N}}^2 \left( \frac{3AB^2}{2N^2} - \frac{3ABB_N}{N} - \frac{3A_{NN}B^2}{2N} \right) + \mathcal{H}^2 \left( -\frac{18AB}{N} - \frac{6A}{N^2} + \frac{6A_B}{N} \right) + \dot{\mathcal{N}} \left( -\frac{9AB^2\mathcal{H}}{N} - \frac{6ABB_\eta}{N} - \frac{3A_\eta B^2}{N} \right) + \mathcal{H} \left( -\frac{6AB_\eta}{N} - \frac{6A_\eta B}{N} - 3Q_N \right) - P_N N - P,$$

(A1)
FIG. 3: Throughout the whole evolution, $c_S^2 > 0$, while $c_S^2 \to 1$ as $|\eta| \to \infty$.

$$6 \left( \frac{AB \dot{N}}{N} + \frac{2A \dot{H}}{N} \right) = \dot{N} \left( \frac{12AH}{N^2} - \frac{12ANH}{N} - \frac{6AB\eta}{N} - \frac{6A_BB}{N} + 3Q_N \right)$$

$$+ \dot{N}^2 \left( \frac{9AB^2}{2N^2} + \frac{6AB}{N^2} - \frac{6AB_N}{N} - \frac{6A_NB}{N} \right)$$

$$- \frac{18AH^2}{N} - \frac{12ANH}{N} - 3PN + 3Q_N.$$  \hspace{1cm} (A2)

**Appendix B: Solutions of $g_1$, $g_2$ and $g_3$**

$$g_1 = -\frac{1}{8N} \left( 4c_1HM_p^2N^4\dot{g}_4 + 2c_1M_p^2g_4N^4\dot{g}_4\dot{N} + c_1M_p^2N^5\dot{g}_3^2 + 4c_1HM_p^2g_4N^3\dot{N} + c_1M_p^2g_4^2N^3\ddot{N}^2 \right.$$  

$$+ 4c_1H^2M_p^2N^3 - 12HM_p^2N^2\dot{g}_4 - 36M_p^2g_4N^2\dot{g}_4\dot{N} + 12M_p^2N\dot{g}_4\dot{N} + 6M_p^2N^3\dot{g}_4^2$$

$$- 108H^2M_p^2g_4N^2 - 36M_p^2g_4H^2N^2 - 72HM_p^2g_4^2N^2\dot{N} + 24HM_p^2g_4N\dot{N}$$

$$- 24M_p^2g_4\dot{N}\ddot{N} + 12M_p^2\dot{g}_4N\ddot{N} + 18M_p^2g_4^2N\dot{N}^2 - 12M_p^2g_4\dot{N}^2 + 24H^2M_p^2N$$

$$+ 24M_p^2\dot{H}N - 24HM_p^2\dot{N} \right).$$  \hspace{1cm} (B1)
\[ g_2 = \frac{1}{32} \left( 4c_1 \mathcal{H} M_p^2 N^5 \dot{g}_4 + 2c_1 M_{p}^2 g_4 N^5 \dot{g}_4 \dot{N} + c_1 M_{p}^2 N^6 \dot{g}_4^2 + 4c_1 \mathcal{H} M_p^2 g_4 N^4 \dot{N} + c_1 M_{p}^2 g_4 N^4 \dot{N}^2 + 4c_1 \mathcal{H}^2 M_p^2 N^4 + 12 \mathcal{H} M_p^2 N^3 \dot{g}_4 - 12 \dot{M}_p^2 g_4 N^3 \dot{g}_4 \dot{N} + 4 \dot{M}_p^2 N^2 \dot{g}_4 \dot{N} + 6 \dot{M}_p^2 N^4 \dot{g}_4^2 - 36 \mathcal{H}^2 M_p^2 g_4 N^3 - 12 \mathcal{H} M_p^2 g_4 \dot{N}^3 - 36 \mathcal{H} M_p^2 g_4 N^3 \dot{N} + 24 \mathcal{H} M_p^2 g_4 N^2 \dot{N} - 12 \dot{M}_p^2 g_4 N^3 \dot{N} + 4 \dot{M}_p^2 g_4 N^2 \dot{N} + 18 \dot{M}_p^4 g_4 N^2 \dot{N}^2 - 4 \dot{M}_p^2 g_4 N \dot{N}^2 + 24 \mathcal{H}^2 M_p^2 N^2 + 8 \mathcal{H} M_p^2 N \dot{N} \right), \quad (B2) \]

\[ g_3 = -\frac{1}{32 N^3} \left( -4c_1 \mathcal{H} M_p^2 N^4 \dot{g}_4 - 2c_1 M_{p}^2 g_4 N^4 \dot{g}_4 \dot{N} - c_1 M_{p}^2 N^5 \dot{g}_4^2 - 4c_1 \mathcal{H} M_p^2 g_4 N^3 \dot{N} - c_1 M_{p}^2 g_4 N^3 \dot{N}^2 - 4c_1 \mathcal{H}^2 M_p^2 N^3 \dot{g}_4 + 60 \dot{M}_p^2 g_4 N^2 \dot{g}_4 \dot{N} + 12 \dot{M}_p^2 N^2 \dot{g}_4 \dot{N} - 6 \dot{M}_p^2 N^3 \dot{g}_4^2 + 180 \mathcal{H}^2 M_p^2 g_4 N^2 + 60 \dot{M}_p^2 g_4 \dot{N} \dot{N}^2 + 108 \mathcal{H} M_p^2 g_4 N^2 \dot{N} - 24 \dot{M}_p^2 g_4 N^2 \dot{N} + 36 \dot{M}_p^2 g_4 N^2 \dot{N} + 12 \dot{M}_p^2 g_4 N \dot{N} - 42 \dot{M}_p^2 g_4 N \dot{N}^2 - 12 \dot{M}_p^2 g_4 N^2 \dot{N}^2 + 72 \mathcal{H}^2 M_p^2 N + 24 \mathcal{H} M_p^2 N \dot{N} - 24 \mathcal{H} M_p^2 \dot{N} \right), \quad (B3) \]

In Sect.III B, since \( N = 1 \), (B1), (B2) and (B3) will be simplified.
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