**Abstract.** Maintaining the position that the wave function $\psi$ provides a complete description of state, the traditional formalism of quantum mechanics is augmented by introducing continuous trajectories for particles which are sample paths of a stochastic process determined (including the underlying probability space) by $\psi$. In the resulting formalism, problems relating to measurements and objective reality are solved as in Bohmian mechanics (without sharing its weak points). The pitfalls of Nelson’s stochastic mechanics are also avoided.
We have yet to understand
How rich
And deep
The quantity $\psi$ is.

1 Introduction

This work concerns the following three deficiencies of the traditional formalism of quantum mechanics (QM): failure in describing individual systems (it deals with ensembles), in giving a concrete description of systems as objectively real entities and in giving a satisfactory account of measurements free of the problem of macroscopic superpositions.

As a remedy, a (minimal) completion of the formalism is proposed which is based on the following simple idea: The wave function $\psi(r,t)$ (of, say, a nonrelativistic spinless particle) defines not only a (time-dependent) probability measure on the configuration space $\mathbb{R}^3$ [thus giving a parametrised family of probability spaces $\mathcal{M}_t = (\mathbb{R}^3, \mathcal{B}(\mathbb{R}^3), |\psi(r,t)|^2d^3x)$ where $\mathcal{B}(\mathbb{R}^3)$ is the Borel $\sigma$-algebra of $\mathbb{R}^3$] but also a stochastic process $X(t)$ [see Eq.(18) below] (say, for $t \geq 0$) for which the underlying probability space is $\mathcal{M}_0$. The traditional formalism of QM is proposed to be supplemented with the following additional postulate:

Postulate $\mathcal{P}$. The quantum mechanical system whose dynamics is represented by the Schrödinger equation for the wave function $\psi$ [Eq.(9) below] is a concrete physical entity—a particle—having well defined position at every instant of time; the allowed configuration space trajectories for it are the (continuous) sample paths of the stochastic process $X(t)$ [labelled by the initial positions of the particle (points of $\mathcal{M}_0$)].

The theory permits the experimental realization of these paths arbitrarily closely but not exactly (the situation in this respect is similar to that for the absolute zero of temperature). The formalism evolved will have the good points of the de Broglie-Bohm theory or Bohmian mechanics (BM) (de Broglie 1926, 1927; Bohm 1952; Bohm and Vigier 1954; Bohm and Hiley 1993; Bell 1987; Holland 1993; Goldstein 1998) without sharing its weak points (and retaining all the good features of the traditional formalism) and also avoids the pitfalls of Nelson’s stochastic mechanics (Nelson 1966,67; Guerra 1981; Nelson 1985; Blanchard et al 1987). (Both of these theories
propose to do QM with particles moving along physically well defined trajectories.)

In section 2, we cover some background relating to the Hamilton-Jacobi theory and its probabilistic extension, de Broglie-Bohm theory and Nelson’s stochastic mechanics. The main work mentioned above is presented in section 3. The last section contains some additional remarks.

2 Background

2.1 Hamilton-Jacobi theory and its probabilistic extension

We shall recall a few essential points relating to this theory (Guerra 1981; Blanchard et al 1987; Holland 1993; Dass 2002). Given a classical system $S$ with a Lagrangian $L(q, \dot{q}, t)$ having configuration $q_0$ at the initial time $t_0$, the Hamilton-Jacobi function

$$S(q, t) = \int_{t_0, q_0}^{t, q} dt' L(q(t'), \dot{q}(t'), t')$$

(1)

where the integration is along the physical trajectory between $q(t_0) = q_0$ and $q(t) = q$ (assumed, for simplicity, unique) satisfies the Hamilton-Jacobi equation

$$\frac{\partial S(q, t)}{\partial t} + H(q, \frac{\partial S(q, t)}{\partial q}, t) = 0$$

(2)

where $H(q, p, t)$ is the Hamiltonian. A solution $S(q, t)$ of Eq.(2) with the initial condition $S(q, t_0) = S_0(q)$, when supplemented with the initial condition

$$q(t_0) = q_0$$

(3)

can be used to obtain the (unique) dynamical trajectory $(q(t), p(t))$ in phase space given by the initial condition

$$(q(t_0), p(t_0)) = (q_0, p_0)$$

where

$$p_{0\alpha} = \left( \frac{\partial S_0(q)}{\partial q^\alpha} \right)_{q=q_0}$$
as follows: Define momentum and velocity fields $p(q,t)$ and $v(q,t)$ by

$$p_\alpha(q,t) = \frac{\partial S(q,t)}{\partial q^\alpha}; \quad v_\alpha(q,t) = \left(\frac{\partial H(q,p,t)}{\partial p_\alpha}\right)_{p=p(q,t)}. \quad (4)$$

The differential equation

$$\dot{q}_\alpha(t) = v_\alpha(q(t),t) \quad (5)$$

with the initial condition (3) gives a unique solution $q(t)$. Finally $p_\alpha(t) = p_\alpha(q(t),t)$.

In this description of classical dynamics, the state at time $t$ is given by $q^\alpha(t)$ and the field $S(q,t)$; their time evolution is governed by the equations (5) and (2). For a particle in potential $V(x)$, we have $H = \frac{1}{2m}p^2 + V(x)$ and

$$v^j(x,t) = \frac{1}{m} \frac{\partial S(x,t)}{\partial x^j}; \quad j = 1, 2, 3. \quad (6)$$

If, instead of the condition (3), we are initially given a probability distribution $\rho(q,t_0) = \rho_0(q)$, we shall obtain, instead of the configuration space trajectories $q^\alpha(t)$, the probability density function $\rho(q,t)$ satisfying the continuity equation

$$\frac{\partial \rho(q,t)}{\partial t} + \frac{\partial}{\partial q^\alpha}[v^\alpha(q,t)\rho(q,t)] = 0. \quad (7)$$

The state of the system at time $t$ is now given by the pair of fields $\rho(q,t)$ and $S(q,t)$ whose evolution is governed by the equations (7) and (2).

A point worth emphasising is that, even in this probabilistic version of Hamilton-Jacobi theory, the concept of configuration space trajectories (which are particle trajectories if $S$ is a system of particles) governed by Eq.(5) remains meaningful; we only do not have complete information about them. Probabilistic questions relating to (families of) such trajectories can be answered in the formalism. Taking the system $S$, for convenience, to be a nonrelativistic particle, the trajectories are now the sample paths of the stochastic process $x(t)$ governed by the stochastic differential equation (SDE)

$$dx^j(t) = \frac{1}{m} \left(\frac{\partial S(x,t)}{\partial x^j}\right)_{x=x(t)} dt. \quad (8)$$
The underlying probability space for this process can be taken as $\mathcal{M}_0^d = (\mathbb{R}^3, \mathcal{B}(\mathbb{R}^3), \rho_0(r) d^3x)$ (points of the sample space $\mathbb{R}^3$ being possible initial positions of the particle).

Note that, to have the probabilistic treatment of particle trajectories along the lines mentioned above, we do not need to compromise with the fact that the fields $\rho(r,t)$ and $S(r,t)$ constitute a complete description of state at time $t$. These functions, determined as solutions of Eqs.(7) and (2) with initial conditions $\rho(r,t_0) = \rho_0(r)$ and $S(q,t_0) = S_0(q)$ determine the probability space $\mathcal{M}_0^d$ as well as the SDE (8) (and, therefore, the family of sample paths of this SDE which are particle trajectories). In particular, it is not true that, to have Eq.(8) in the formalism, the state at time $t$ must be taken to be given by $\rho(r,t), S(r,t)$ and the trajectory function $r(t)$.

In the limiting case $\rho_0(r) = \delta(r - r_0)$, we recover the deterministic trajectories. If we insist on $\rho$ being a well behaved function, such trajectories can be realised arbitrarily closely but not exactly.

### 2.2 Bohmian mechanics

Bohmian mechanics (BM) treats particles as concrete physical entities moving in a highly non-Newtonian manner along well defined trajectories. Each particle has associated with it a complex physical field $\psi(r,t)$. The complete description of state of the system (particle plus its associated field) at time $t$ is given by the field $\psi(r,t)$ and the trajectory function $r(t)$. Temporal evolution of state is assumed to be given by the Schrödinger equation

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \psi(r,t)$$

(9)

and an equation identifying the velocity of the particle with the velocity of the Schrödinger field $\psi(r,t)$ at its location:

$$\frac{dr(t)}{dt} = v^{(\psi)}(r(t),t)$$

(10)

where

$$v^{(\psi)}(r,t) = \left( \frac{J^{(\psi)}}{\psi^* \psi} \right)(r,t) \equiv \frac{\hbar}{m} \left[ \frac{Im(\psi^* \nabla \psi)}{\psi \psi^*} \right](r,t).$$

(11)
Eq. (9) implies the well-known continuity equation

\[ \frac{\partial |\psi(r,t)|^2}{\partial t} + \nabla J^{(\psi)}(r, t) = 0. \]  

(12)

Note that Eqs. (9) and (10) constitute a deterministic system admitting, with initial conditions \( r(0) = r_0, \psi(r, 0) = \psi_0(r) \), a unique solution (valid in an appropriate domain).

In a statistical ensemble of particles having a common associated field \( \psi \) (but possibly different initial positions), the probability density of position is assumed to be given by

\[ \rho(r, t) = |\psi(r, t)|^2. \]  

(13)

In fact, it is adequate to assume Eq. (13) at a single instant, say, \( t = 0 \); Eq.(12) then guarantees its validity for all \( t \).

The last assumption [Eq. (13)] was criticised by Pauli and others on the ground that such a hypothesis is not appropriate for a theory aimed at giving a causal explanation of QM. To counter this criticism, Bohm and Vigier (1954) proposed a hydrodynamic model taking the wave function to represent a conserved fluid with density \( |\psi|^2 \) and local stream velocity \( v^{(\psi)} \) [which, on writing \( \psi = \tilde{R} \exp[i\tilde{S}/\hbar] \), equals \( (\nabla \tilde{S})/m \)]. The particle is treated as an inhomogeneity which moves with the local stream velocity. By introducing a hypothesis of a very irregular and effectively random fluctuation in the motion of the fluid, they were able to prove that an arbitrary probability density ultimately decays into \( |\psi|^2 \). (For a relatively recent treatment of this ‘quantum equilibrium’, see Dürr et al 1992.) Eq. (10) is now replaced by

\[ \frac{dr(t)}{dt} = v^{(\psi)}(r(t), t) + \xi(t) \]  

(14)

where \( \xi \) represents the chaotic contribution to the particle velocity. Apart from some qualitative statements about it, no detailed treatment of this quantity was given.

The theory does not have the traditional quantum mechanical observables in its initial formulation; they can, however, be introduced through analysis of experiments (Dürr et al 2003). For observers not having knowledge of precise initial positions of particles, BM can be shown to make the same predictions as the standard quantum theory. (For such observers, BM is
effectively a hidden variable theory, the position variables of particles serving as the hidden variables.)

Major plus points of BM are:
(i) It gives the simplest and most appealing account of the wave function reduction. In the macroscopic superposition of possible outcomes resulting from von Neumann’s treatment of measurement (von Neumann 1955, Dass 2005), the one term representing the ‘channel’ in which the system trajectory happens to be is naturally picked up.
(ii) It permits additional insights into various quantum phenomena like barrier penetration, interference etc in terms of particle trajectories.
(iii) It makes quantum nonlocality manifest (by exhibiting the ‘quantum potential’ term in the treatment of particle motion).
(iv) The theory describes quantum systems as concrete physical systems having objective reality.

The somewhat overenthusiastic support of Bell notwithstanding, the theory is not without some weak points; the major ones are:
(i) Serious problems in the relativistic domain and quantum field theory;
(ii) Rather ‘painful’ treatment of spin in the nonrelativistic domain;
(iii) Some unappealing/unconvincing aspects of the physics of the $\psi$-field. (For example, it influences particle motion but is not influenced by it; there is no discussion of the energy, momentum etc of this field, etc.)

The troubles of BM basically arise from the fact that it tries to overdo a couple of (related) things:
(i) It assigns physical status to not only the trajectory functions $r(t)$ but also to $\psi$ and has to struggle with problems resulting from it.
(ii) The complete specification of state at time $t$ includes not only the wave function $\psi(r,t)$, but also the trajectory function $r(t)$. This is analogous to including, in the probabilistic version of Hamilton-acobi theory, $r(t)$ besides $\rho(r,t)$ and $S(r,t)$ in the definition of state at time $t$ which, as we have seen above, is not necessary.

As a result, there is an internal tension and awkwardness/uncleanliness in the formalism. This, in the author’s opinion, is at least partly responsible for the theory’s problems in the relativistic domain. Only a theory having a reasonably clean formalism at a certain level can be expected to have a straightforward generalization to a higher level.
2.3 Nelson’s stochastic mechanics

According to stochastic mechanics (SM), quantum effects in the dynamics of microscopic systems arise due to the influence of a background field. It is assumed that this field causes the particles to undergo stochastic motion (described in terms of a stochastic process $X_t$) which is (i) Markovian, (ii) diffusion, which is (iii) conservative (i.e., on the average, there is no energy transfer between the system and the background field). In this kinematic framework, the dynamics is given (for, say, a nonrelativistic particle) in terms of a stochastic version of Newton’s second law which is either assumed or derived from a stochastic version of Hamilton’s principle. From the equation for this law and the continuity equation for the probability density $\tilde{\rho}$ for the stochastic process $X_t$, it follows that the quantity

$$\psi = \sqrt{\tilde{\rho}} \exp\left[i\tilde{S}/\hbar\right]$$

(15)

[where $\tilde{S}$ is a quantity whose gradient (plus a known function of $\tilde{\rho}$) defines the drift term in the SDE for the stochastic process $X_t$] satisfies the traditional Schrödinger equation.

Observations on the system are analysed in terms of quantities related to the diffusion process $X_t$; typically, these are random variables of the type $f(X_{t_1},...,X_{t_n})$. For position observables, the predictions of SM and the traditional QM are identical. Taking the view that, in principle, all measurements can be analysed in terms of position measurements, there is a broad agreement between the predictions of the two theories.

Detailed investigations, however, show that SM predicts undesirable correlations between distant systems which made Nelson himself conclude (Nelson 1984) that ‘Markovian stochastic mechanics is untenable as a realistic physical theory’.

Here is a simple argument why Markov processes cannot provide the appropriate framework for the description of dynamics of objects whose wave functions satisfy Schrödinger equation. Such a formalism is ‘overconstrained’ in the sense that it has not only the Schrödinger evolution semigroup of quantum dynamics, but also the evolution semigroup of the Markov process $X_t$ (related to the Chapman-Kolmogorov equation). Normally the former does not imply the latter. [One simple way to see this is to note that Schrödinger evolution in general mixes the diagonal and off-diagonal terms in the density matrix $w(x,y) = \psi(x)\psi^*(y)$; the Markovian semigroup mixes only the...
diagonal terms."

The idea of starting with a background field hypothesis and producing a quantity \( \psi \) satisfying Schrödinger equation is very appealing; however, to do justice to this theme and produce a viable formalism, one needs to operate in a framework broader than the Markovian one.

3 Particle trajectories in quantum mechanics

We shall present the proposed completion of the traditional formalism of QM in the context of a nonrelativistic spinless particle whose wave function satisfies the Schrödinger equation (9). Some remarks relating to the generalization to other systems will be made at an appropriate stage.

The last part of section 2.1 makes it almost inevitable to invoke the postulate \( \mathcal{P} \) which we presently do and proceed to explore the consequences. Given a solution \( \psi(r,t) \) of Eq.(9) [supplemented with the initial condition \( \psi(r,0) = \psi_0(r) \)], we introduce, in the probability space \( \mathcal{M}_0 \), a stochastic process \( X(t; [\psi_0]) \) (the square bracket in \( [\psi_0] \) reflects the fact that the objects \( X(\cdot, \cdot) \) are functionals of \( \psi_0 \)) whose sample paths \( X(t; [\psi_0], r_0) \) (where \( r_0 \) labels points of the configuration space \( \mathbb{R}^3 \) considered at time \( t = 0 \)) are supposed to represent particle trajectories. Two obvious conditions to impose on these functions are:

\[
X(0; [\psi_0], r_0) = r_0; \quad (16)
\]

and \( X(t; [\psi_0], r_0) = 0 \) for those values of \( r_0 \) for which \( \psi_0(r_0) = 0 \).

We shall often suppress the arguments \( [\psi_0] \) and \( r_0 \) and restore them whenever clarity demands it.

To ensure consistency with the traditional operator formalism, we demand that the expectation value of \( X(t) \) (in the context of the probability space \( \mathcal{M}_0 \)) must be equal to the expectation value of the position operator \( \hat{X}_H(t) \) [notation: the subscripts H and S refer to Heisenberg and Schrödinger pictures (identified at time \( t = 0 \)); \( < r | \Psi_S(t) > = \psi_S(r,t) \); we shall often suppress the subscript S in \( \psi_S(r,t) \)] :

\[
E[X^j(t)] = \int_{\mathbb{R}^3} X^j(t; [\psi_0], r_0)|\psi_0(r_0)|^2 dx_0
\]

\[
= < \Psi_H | \hat{X}^j_H(t) | \Psi_H >
\]
\[
\langle \Psi_S(t) | \hat{X}^j_S | \Psi_S(t) \rangle \\
= \int_{\mathbb{R}^3} x^j |\psi_S(x, t)|^2 dx \quad j = 1, 2, 3.
\] (17)

We next try to set up, for the process \( \mathbf{X}(t) \), a stochastic differential equation (SDE) of the form

\[
d\mathbf{X}^j(t) = V^j(\mathbf{X}(t), t) dt + d\zeta^j(t) \quad (18)
\]

where the first term on the right (the traditional drift term) represents the mean motion during the time interval \((t, t+dt)\) and the last term represents fluctuations. We impose the reasonable demand that the fluctuations average out to zero:

\[
E(d\zeta^j(t)) = 0. \quad (19)
\]

This gives [recalling, for the intermediate steps, Eqs.(10) and (12)]

\[
E(V^j(\mathbf{X}(t), t)) dt = dt \int_{\mathbb{R}^3} V^j(x, t)|\psi(x, t)|^2 dx \\
= E(d\mathbf{X}^j(t)) \\
= \int x^j [|\psi(x, t+dt)|^2 - |\psi(x, t)|^2] dx \\
= dt \int x^j [-\nabla \cdot J^{(\psi)}(x, t) dx \\
= dt \int J^j(x, t) dx
\]

which in turn gives (upto an additive term of zero mean)

\[
V^j(\mathbf{r}, t) = J^j(\mathbf{r}, t)|\psi(\mathbf{r}, t)|^2 = v^{(\psi)j}(\mathbf{r}, t). \quad (20)
\]

We absorb the possible additive term with zero mean in the last term in Eq.(18). To do it transparently, we define \( \zeta^j \) as

\[
\zeta^j(t; [\psi_0], \mathbf{r}_0) \equiv X^j(t; [\psi_0], \mathbf{r}_0) - x^j_0 - \int_0^t dt' v^{(\psi)j}(\mathbf{X}(t'; [\psi_0], \mathbf{r}_0), t'). \quad (21)
\]

With \( V^j \) given by Eq.(20), Eq.(18) is now equivalent to Eq.(14) [making allowance for the fact that \( \xi(t) \) may be a generalized stochastic process (white noise, for example)]. We have, therefore, got the main desirable equation of BM essentially for free!
In fact, we are in an even better position. To see this, it is useful to recall the velocity path integral for the transition amplitude (Kleinet, 2004, p 172):

\[
\langle x_b, t_b | x_a, t_a \rangle = \int D^3v \delta(x_b - x_a - \int_{t_a}^{t_b} dt v(t)).
\]

\[
\exp \left\{ \frac{i}{\hbar} \int_{t_a}^{t_b} dt \left[ \frac{m}{2} |v|^2 - V \left( x_b - \int_{t'}^{t_b} dt' v(t') \right) \right] \right\} \tag{22}
\]

where the path integrator \( D^3v \) satisfies the normalization condition

\[
\int D^3v \exp \left[ \frac{i}{\hbar} \int_{t_a}^{t_b} dt \left( m \frac{1}{2} |v|^2 \right) \right] = 1. \tag{23}
\]

If, in Eq.(23), we put \((m\hbar^{-1})^{1/2}v = \tilde{v}\), and define \( D^3\tilde{v} = D^3v \), the resulting equation will have no dimensional parameters. Accordingly, writing \( \xi = (\hbar m^{-1})^{1/2} \tilde{\xi} \) and noting that the velocity path integral in Eq.(21) is an integral over velocity fluctuations of the particle, one expects \( \tilde{\xi} \) to be a quantity with standard normalization. The resulting equation is important enough to deserve a separate display:

\[
\frac{dX(t)}{dt} = v^{(\psi)}(X(t), t) + \sqrt{\frac{\hbar}{m}} \frac{\tilde{\xi}}{\tilde{\xi}}. \tag{24}
\]

With \( \hbar \neq 0 \) (as is the case with the real world), the last term in Eq.(24) is insignificant for particles of large mass. This is quite consistent with observations.

Recalling Eq.(15), if the limits

\[
\rho(r, t) = \lim_{\hbar \to 0} \tilde{\rho}(r, t); \quad S(r, t) = \lim_{\hbar \to 0} \tilde{S}(r, t) \tag{25}
\]

exist (and are smooth functions), the real and imaginary parts of Eq.(9) go over, in the limit \( \hbar \to 0 \), to Eqs. (7) and (2). In this limit, Eq.(24) goes over to the special case of Eq.(5) corresponding to the velocity field of Eq.(6) [see Eq.(8)]. The limiting trajectories are smooth.

The augmented formalism [to be referred to as CQM (Completed QM)] now has the plus points of BM given above. In particular, the explanation of wave function collapse can be given as, for example, in Ch.(6) of Bohm and Hiley [described briefly in the plus point (i) in section 2.1].

To clear some lingering doubts as to why, in a formalism admitting trajectories (in configuration space), the traditional quantum mechanical observables (self adjoint operators) – and not what Bell calls be-ables – should be
the proper choice for mathematical objects representing measurable quantities, note the following points:

(i) The trajectories in CQM are continuous but not smooth. There is, therefore, no concept of velocity (as a well defined function of time) at a general point of a trajectory. It follows that be-ables are not a natural choice of observables in this theory.

(ii) In CQM, trajectories are derived objects; the central objects are the wave functions which give a complete description of state. A straightforward logical analysis of measurements (Araki 1999, Dass 2002) on systems on which repeatable experiments can be performed (to facilitate frequency interpretation of probability), leads to the dual relationship between states and observables. When the description of state is given in terms of vectors in a Hilbert space (or, more generally, in terms of density operators), the proper choice of observables, according to this analysis, is an appropriate family of self-adjoint operators. This is by no means inconsistent with the picture of particles (or more general systems) moving along continuous trajectories in configuration space.

4 Additional remarks

(i) Two points of contrast between the present work and Nelson’s stochastic mechanics are:

(a) Whereas, in SM, the stochastic process $X_t$ is defined on an abstract (or background) probability space, we employ only the dynamical probability spaces $\mathcal{M}_t$ provided by the wave function. (Hence the term ‘dynamical probability’ in the title of the paper.)

(b) We do not make the Markovian assumption about the stochastic process $X(t)$.

(ii) In SM, one has to distinguish between the forward differential $d^{(+)}X(t) = X(t + dt) - X(t)$ and the backward differential $d^{(-)}X(t) = X(t) - X(t - dt)$ and correspondingly, in the drift term in the relevant SDEs, forward and backward velocities $v^{(\pm)}(x,t)$; the velocity $v^{(\psi)}$ appearing in Eq.(23) is the mean of the two. Formally, therefore, our differential $dX^j(t)$ must be taken to correspond to the symmetric differential

$$\circ X^j(t) = \frac{1}{2}[X^j(t + dt) - X^j(t - dt)].$$

(26)
In fact, trajectories in CQM are somewhat smoother objects than those in SM. For example, the calculational steps before Eq. (20) give the same result whether we use the forward, backward or symmetric differential.

(iii) An equation analogous to (23) appears in Roncadelli [1993; Eq.(21)]. Instead of $v^{(\psi)}$, however, Roncadelli has the classical velocity term (6) (where $S$, in his work, is a general solution of the Hamilton-Jacobi equation).

(iv) The functions $\zeta^j(t)$ (or, equivalently, $X^j(t)$) need to be determined completely (if necessary, by strengthening the postulate $P$). The author hopes to come back to this point in a future publication. Meanwhile, we give here a relation worth putting on record.

From Eq.(17), one can write [for those $r$ for which $\psi_0(r) \neq 0$]

$$X^j(t; [\psi_0], r) = \frac{x^j|\psi(r, t)|^2}{|\psi_0(r)|^2} + G^j(t; [\psi_0], r)$$  \hspace{1cm} (27)

where [remembering Eq.(16)]

$$G^j(0; \ldots) = 0$$  \hspace{1cm} (28)

$$\int_{R^3} G^j(t; [\psi_0], x)|\psi_0(x)|^2 dx = 0.$$  \hspace{1cm} (29)

Comparing Eqs.(27) and (21), we see that the functions $G^j$ and $\zeta^j$ are related. It is, therefore, adequate to determine one of them.

(v) The formalism of CQM, presented in section 3 in the context of a spinless nonrelativistic particle, admits straightforward generalization (with due alteration of details) to (at least) all systems with finite dimensional configuration spaces. In the statement of the postulate $P$, the term ‘particle’ is to be replaced by the relevant system and, in Eq.(24), the velocity $v^{(\psi)}$ is to be replaced by the velocity of the $\psi$-field appearing in the relevant continuity equation replacing Eq.(12). [See Eq.(7) for the form of the general continuity equation.] Whereas the wave function $\psi$ describes an ensemble, the trajectory functions [the quantities $X^j(t; [\psi_0], r_0)$ and their analogues in the more general case] describe, for a given $\psi$, the configuration space trajectories of individual systems in the ensemble.

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