Stability evaluation of the rod in triangular array predicted by CFD

S Upnere\textsuperscript{1,2}, N Jekabsons\textsuperscript{2,3}

\textsuperscript{1}Riga Technical University, Institute of Mechanics, Viskalu Street 36A, Riga, LV-1006, Latvia
\textsuperscript{2}Ventspils University College, Inzenieru Street 101, Ventspils, LV-3600, Latvia
\textsuperscript{3}Jekabsons Engineering Systems, Ozolciema Street, Riga, LV-1058, Latvia

E-mail: upnere@gmail.com

Abstract. Stability boundaries of structures consisting of circular cylinder arrays in the cross-flow are different from case to case depending on geometrical and mechanical variations of the rod bundle layout as well as depending on nature of the flow field, requiring an individual set of experiments for each characteristic case. In this study, close-packed staggered rod bundle with a pitch-to-diameter ratio of 1.1 is analysed. Numerical modelling has been done to check the stability threshold (critical velocity) of a flexibly mounted rod in an otherwise fixed rods array. The computational domain consists of the rod array with 6 rows of five cylinders. The unsteady flow through triangular cylinder array has been simulated using two-dimensional URANS computations with an open source Computational Fluid Dynamics (CFD) code. The CFD calculations are coupled with the six degree-of-freedom rigid body motion solver with reduced degree-of-freedom. Results were compared with the analytically determined threshold values.

1. Introduction

In heat exchangers, boilers or nuclear plants flow-induced vibrations (FIV) have been considered as one of the most destructive phenomena which can lead to structure integrity problems. Mechanical failures as a result of tube vibration can occur from fatigue, collision damages, baffle damage, or tube joint failures [1].

Three main FIV mechanisms in cross-flow in the rod bundle in a liquid medium are vortex shedding, turbulent buffeting and fluidelastic instability (FEI) [2]. Vortex shedding and turbulent buffeting are due to resonance phenomena when the excitation frequency is close to the oscillating rod natural frequency. They have a potential for long-term damages. Instability attains when the energy input to the rod mass-damping system exceeds the energy dissipated by the system. FEI generally occurs very abruptly and can cause tube failure in short time [3], therefore FEI is of high interest for investigation. Chen [4] was the first who formulates two basic dynamic instability mechanisms: instability controlled by fluid damping (velocity dependent, similar to classical galloping) and instability controlled by stiffness (position dependent, similar to aeroelastic flutter). Damping-controlled mechanisms require only one degree-of-freedom (DoF). An important term is the time lag between the motion-dependent fluid force acting on the tube and the resulting tube displacement. Fluid stiffness instability requires at least 2 DoF. Usually it is required at least two flexible tubes coupled through the fluid. However [5] notes that the stiffness-controlled instability can occur for a single flexible tube in...
an array by static divergence. In close-packed arrays, excitation mechanisms may be completely merged. A better understanding of vibration excitation mechanisms in the rod bundles allows improving the integrity of the complex structures and avoid costly damages or shutdowns of the plant.

Characteristic behaviours of the rod array are dependent on the bundle mechanical, geometrical (e.g. rod supports and configuration, the pitch-to-diameter ratio (\(P/d\)) and flow field (such as Reynolds number, turbulence intensity) variations. The stability boundaries strongly depend on the array geometry [6]. FIV in the rod bundle is a complex phenomenon with a number of unknown parameters. It leads to needing for a set of experimental data for each unique case to find requisite system features. The number of previews experiments are described in the literature, for example, Zukauskas et al. [7] identified above-mentioned mechanisms of instability excitation. Andjelić et al. [6] report on experiments using a normal triangular cylinder array with \(P/d = 1.25\). They found the existence of multiple stability boundaries for a group of flexibly mounted cylinders. The effects of upstream turbulence on the stability behaviour of a parallel triangular tube bundle with \(P/d = 1.375\) in the wind tunnel are analysed by Rottmann and Popp [8]. Bundles with various flexible tubes were investigated and stabilising effect of upstream turbulence has been found. Experimental investigation of FEI subjected by air cross-flow in array configurations of typical heat exchangers is given by Austermann and Popp [9].

The convenient approach is to combine experimental and mathematical methods using experimental data in order to create and validate a basic numerical and/or analytical models. After that, wide investigation of the system behaviour in different operational scenarios could be done with these models. Hassan et al. [10] used Computational Fluid Dynamics (CFD) combined with the unsteady flow model to predict fluidelastic forces in the tube bundle. They concluded that used approach provides an efficient means for rapidly estimating array stability maps. Simulation of the unsteady flow through normal triangular cylinder array with one flexible cylinder in the otherwise rigid array was done using optimised CFD methodology by Pedro et al. [11]. A finite volume solver based on Cartesian-staggered grid implemented for numerical prediction of FEI in normal triangular tube bundles with multiple flexible circular cylinders is described in [12].

In this paper, a closed-packed (\(P/d = 1.1\)) triangular cylinder array with one rigid, but flexibly-mounted cylinder, in a fixed cylinder bank has been analysed. The numerical calculations based on 2D unsteady flow computations were done with open source CFD toolkit OpenFOAM (OF). The 2D representation has been chosen due to lower computational costs, although it is known that turbulent flow has three-dimensional structure. However 2D computations are suitable for qualitative estimation. Connors’ equation variations using two sets of constants were applied as an analytical prediction of the instability threshold.

2. Methodology

The threshold of the fluidelastic instability has been predicted by numerical simulations of incompressible, unsteady, turbulent water cross-flow through a cylinder bundle in the triangular arrangement and compared with a simplified mathematical model.

2.1. Governing equations

To calculate two-dimensional incompressible water flow field in the flow domain, \(\mathcal{F}(t)\) Reynolds-Averaged Navier-Stokes (RANS) for steady-state cases and Unsteady RANS equations and for time depended cases have been used. The continuity and momentum equations are following:

\[
\frac{\partial \bar{U}_j}{\partial x_j} = 0 \text{ in } \mathcal{F}(t) \text{ for } t > 0, \quad (1)
\]
\[
\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_j \frac{\partial \bar{U}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 \bar{U}_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \{u_i \bar{u}_j\} \text{ in } \mathcal{F}(t) \text{ for } t > 0,
\]

where bar denotes averaged values. The Dirichlet (3) and Neumann (4) boundary conditions were applied for velocity and normal stresses, respectively:

\[
u = g_f \text{ in } \Gamma_{D,f} \text{ for } t > 0,
\]

\[
\sigma \cdot n_f = d_f \text{ in } \Gamma_{N,f} \text{ for } t > 0.
\]

The Reynolds number is \(5.6 \times 10^3\), therefore turbulence should be taken into account. Three two-equation turbulence models were selected for checking: standard k-epsilon, realizable k-epsilon and RNG k-epsilon [13].

It is assumed that there is no mechanical coupling between the x- and y-directions, thereby motion of the cylinder in the transverse direction can be written based on Newton’s second law as follows:

\[
m \ddot{y} + c \dot{y} + k y = F_y = F_L \cos \alpha + F_D \sin \alpha,
\]

where \(m\) is a mass of the oscillator including hydrodynamic added mass, \(c\) is damping coefficient, \(k\) is stiffness coefficient, \(\dot{y}\) is a derivation of \(y\) with respect to time, \(F_L\) and \(F_D\) are hydrodynamical forces in the lift and drag direction, respectively, and \(\alpha\) is an angle of the flow attack, see Fig. 1.

**Figure 1.** Movement diagram of oscillating cylinder.

### 2.2. Description of the analysed cylinder array and boundary conditions

In the numerical model, rods are described as 2D cylinders. From the point of view of studying FIV in the tube bundle, it is recommended that six tube rows be used [14]. In the particular case triangular array consists of 6 rows of five whole cylinders or four whole and two half-cylinders in each one (see Fig. 2). The pitch-to-diameter ratio of the array is 1.1. Experimental results of Weaver and El-Kashlan [14] show that the critical tubes for fluidelastic instability in the triangular array are in the third and fourth rows. Therefore spring-mounted cylinder \(c_0\) with diameter, \(d = 8\) mm is located in the middle of the fourth row. The inlet and outlet are positioned at a distance equivalent to 11 and 41 cylinder diameters upstream and downstream of the first and last row, respectively.

The computational domain is discretized by \(2 \times 10^6\) quadrilateral cells. Maximum non-dimensional distance \(y^+\) from the wall to the first mesh node is from 23 till 38.

At the inlet is defined constant uniform velocity, \(U_\infty\), which corresponds to the flow rate 1 \(\text{l/s}\). Turbulence intensity level of 5 % is assumed. At the outlet constant pressure, \(p_0\) is applied. On the cylinder walls and channel top and bottom walls are used no-slip conditions for velocity and standard OF wall functions for the turbulent kinetic energy, \(k\), the turbulent dissipation, \(\varepsilon\), and the turbulent viscosity, \(\nu_t\).
3. Results and discussion

3.1. Static cases

The quasi-steady-state modelling with all the cylinders being static are done to select the main calculation parameters such as turbulence model, cell size and boundary conditions. Based on the comparison of the numerically predicted and experimentally measured pressure drop in the channel with rigid rod bundle, standard $k-\varepsilon$ turbulence model was used in order to attain turbulence closure.

The SIMPLE algorithm is used to realise the pressure-velocity coupling. It is assumed that steady-state case convergence is achieved when residuals are less than $10^{-8}$. Converged quasi-static flow fields are used as initial conditions for time-dependent calculations. An example of the distribution of the time-averaged velocity field is shown in Fig. 3.

In Fig. 3 is seen that after the rod bundle, the flow is biased to the bottom wall although the system is symmetric and the solution is steady. The system of secondary vortexes takes a place on the top outlet part, while the main jet is located at the bottom. It can be speculated that the Coanda effect may arise in these steady-state simulations. However, as observed in some previews numerical simulations, this wake jet can be located in the middle of the channel as well. Also, it can be biased to the top wall - this seems to be a stochastic choice.

3.2. URANS simulations

To solve cylinder motion built-in OF 6-DoF rigid body motion solver was used. For simplicity, only y-direction movement of a single spring-mounted cylinder in an otherwise rigid cylinder array has been investigated. Rotation of the cylinder is not allowed.

Laplace’s equation for the motion displacement (6) has been solved to calculate the updated position of points:

$$\nabla \cdot (\gamma \nabla d_m) = 0,$$  \hspace{1cm} (6)

where $\gamma$ is the diffusion coefficient. Distance-based diffusivity model has been applied for a determination of points movement. Mesh modifications are performed after each time step.

It was defined a ring-shaped area around oscillating cylinder where mesh cells cannot be shrunk or expanded due to cylinder movement. Therefore boundary layer cells are without
undergoing deformations and a $y^+$ are not dependent on an instantaneous position of the vibrating cylinder.

For dynamic calculations, maximum Courant number is 0.1. The time step was chosen to correspond at least 270 steps per oscillation period. Suggested time step resolution is from $35 - 100$ steps per cycle [10], [15], thereby time step should be small enough to not affect the results.

It were numerically modelled 17 cases. Different combinations of masses and stiffness coefficients are studied to evaluate stability boundary of the given configuration for three mass-damping parameters: 2, 3.88 and 6.89. The typical graph of the mass center displacement in unstable case is shown in Fig. 4. Oscillation amplitude increases until collision of neighbour cylinders.

![Figure 3. Time averaged magnitude of velocity (m/s).](image)

![Figure 4. Displacement of the cylinder (in mm), unstable case.](image)
In the stable case, it is observed small (less than $2e - 4$ mm) cylinder oscillations around the equilibrium position, see Fig. 5.

\[ \frac{U_p}{f_n d} = K \left[ \frac{m^* \delta}{\rho d^2} \right]^\beta, \]

where the left side is reduced critical velocity parameter and the right hand side is mass-damping parameter multiplied by a constant $K$ and to the power of $\beta$. $U_p$ is the gap velocity, $m^*$ is the mass per unit length, and $\delta$ is the logarithmic decrement of damping. Constants with values 3.3 and 0.5 or 3.04 and 0.46 recommended by Pettigrew et al. [2] or Andjelić et al. [6], respectively, were used.

The stability diagram with analytical thresholds as lines are shown in Fig. 6. Locations of OF simulated cases are marked in the diagram by yellow squares and red circles. The logarithmic decrement of damping, $\delta$ is assumed to be 0.03.

From Fig. 6 follows that for a given array configuration when the mass-damping parameter is 2, OF calculations instability foresee earlier if compare with Pettigrew et al. and Andjelić et al. estimations. Also in other two cases when the mass-damping parameter is 3.88 and 6.89, OF results are conservative comparing with Pettigrew et al. prediction. Additional calculations are needed to more precisely find the stability boundary of analysed rod bundle configuration.

4. Conclusions
Numerical modelling in the six rows triangular cylinder array was done. As reference points for numerical model validation has been used experimental values and simplified analytical estimations for quasi-steady and dynamic cases, respectively.

Dynamic calculations with a single spring-mounted cylinder in a rigid cylinder bank were done using unsteady RANS equations combined with standard k-epsilon turbulence model for the system closure.

Numerically calculated cases better conform to analytical prediction of Andjelić et al than Pettigrew et al. For future research, cases located closer to analytical stability boundary should be modelled.
**Figure 6.** Stability diagram.

**References**

[1] Thulukkanam K 2013 *Heat Exchanger Design Handbook* (Boca Raton, London, New York: CRC Press).

[2] Pettigrew M J, Carlucci L N, Taylor C E, and Fisher N J, 1991 *Nuclear Eng. and Design* **131** 81 – 100.

[3] Hassan M, Weaver D S 2016 *ASME J. Pressure Vessel Tech.* **138**

[4] Chen S S 1983 *ASME J. Vibration, Acoustics, Stress, and Reliability in Design.* **105** 51 – 58

[5] Charreton C, Béguin C, Yu K R, Étienne S 2015 *J. Fluids and Struct.* **56** 107 – 123

[6] Andjelić M, Austermann R, Popp K 1992 *ASME J. Pressure Vessel Tech.* **114** 336 – 343

[7] Zukauskas A, Ulinskas R, Katinas V 1984 *Gidrodinamika i vibracii obtekaemyh puchkov trub* (Vilnus: Mokslas)

[8] Rottmann M, Popp K 2003 *J. of Fluids and Struct.* **18** 595 – 612

[9] Austermann R, Popp K 1995 *J. of Fluids and Struct.* **9** 303 – 322

[10] Hassan M, Gerber A, Omar H 2010 *ASME J. Pressure Vessel Tech.* **132**

[11] de Pedro B, Parrondo J, Meskell C, Oro J F 2016 *J. of Fluids and Struct.* **64** 67 – 86

[12] Jafari H H, Dehkordi B G 2013 *J. of Fluids Eng.* **135**

[13] Srinivasa Rao P 2010 Modeling of turbulent flows and boundary layer *Computational Fluid Dynamics (Electronic Material)* ed Hyoung Woo Oh (Vukovar: InTech) chapter 13 pp. 285-306

[14] Weaver D S, El-Kashlan M 1981 *J. Sound and Vibration.* **75** 265 – 273

[15] Lam K, Jiang G D, Liu Y, So R M C 2006 *J. Fluids Struct.* **22** 1113 – 1131