Macro-roughness height determination of teeth surfaces obtained by gear generating method

D T Babichev¹, D A Babichev² and S Yu Lebedev¹
¹Industrial University of Tyumen, Institute of Transport, Melnikayte Street 72, 625039 Tyumen, Russia
²Sibur Holding, ZapSibNeftekhim LLC, East Industrial District, Block 1, No. 6, Building 30, 626150 Tobolsk, Russia
E-mail: lebedevsergey1995@gmail.com

Abstract. The article describes three types of gear teeth surfaces: smooth (envelope), striped (with combs) and scaled (with pyramids and combs connecting the pyramids). The method for combs and pyramids height (above an envelope) determination based on “velocity and acceleration of penetration” conceptions is introduced. Acceleration of penetration allows determining an actual workpiece surface deviation from a theoretical envelope without generating surface knowing. The computer simulations have been carried out, and the correlations between combs and pyramids height and gear machining parameters are exposed.

1. Introduction
Teeth surfaces produced by gear generating method can be classified into three following categories: smooth, stripped and scaled surfaces.

Smooth surfaces have no machining marks – i.e., macro-roughness (initial roughness). The smooth surface is an envelope $\Sigma_2$ of a one-parametric family of generating surfaces $\Sigma_1$ if $\Sigma_1$ (i.e., a cutting tool initial surface (CTIS)) is an actual surface. Teeth surface obtained by gear knurling is an example of a smooth surface structure.

Striped surfaces have stripe-like (combs) machining marks, generally, of variable height and width (Fig.1). Stripped teeth surfaces can be obtained by generation through:

- a two-parametric family of generating surfaces when CTIS is an actual surface. The examples are: 1. Involute gears grinding with cylinder and straight wheels (stripes are along the tooth line). 2. Grinding with abrasive worm grinding wheel with a profile in a xial section close to a wheel profile in normal section (stripes are along the tooth profile – i.e., the lay pattern coincides with the generating direction).

- of a one-parametric family of generating surfaces when CTIS is a cutting edge mark. The example is cylindrical gears shaping (stripes are along the tooth line).

Fig. 1 demonstrates a helical involute gear tooth grinded with a straight wheel. Spur involute gear side surface is composed of multitude plane surfaces touching calculated involute surface along the entire plane surface length. In these conditions, adjacent surfaces intersect at lines, creating certain surface texture. For teeth number $z=18$ and the number of wheel motions for teeth space grinding $k_x=10$ the value of the surface deviation angle is equal to $\Delta a=2^\circ$.

Scaled surfaces are generally composed of parallelogram (upper image in Fig. 2) or hexagonal (lower image in Fig. 2) pits. Pit central points touch calculated tooth surface. Scaled surfaces were first
systematized by B.K. Shunaev [1] while investigating coarse-pitch cylindrical gears machining by diagonal involute gear hobbing cutters.

Smooth surfaces are called *envelope* whereas rough surfaces (stripped and scaled) were called by G.I. Shvelyova [2, p.144] *wraparound surfaces* (Fig. 3).

The stripped surface is divided into a set of stripe-like surfaces each of which touches envelope $\Sigma_2$ along a line. Compared to the stripped surface each scaled surface pit touches envelope at a point. The wraparound surface is always above the envelope surface. Determining the heights difference $- \Delta H$ in fig. 3 – for particular machining types is considered a quite challenging task.

Typically to complete the task the envelope and points on the intersection line of complex adjacent pits or stripes surfaces should be found. In the article another method for determining maximum macro-roughness value $\Delta H_{max}$ is proposed:

- $\Delta H_{max}$ is calculated simultaneously with the envelope determination by means of a computer program.
- Quite simple formulae based on stripes, pits and combs forms and dimensions research are used. The research and formulae are presented in the article.
- Kinematic method for determining surfaces and lines produced by generating process is used. The method is described in [3-5] and is different from all known in the classical theory of gearing methods.

The abovementioned method is based on «velocity of penetration» $V_N$ (proposed by V.A. Shyshkov [6]) and «acceleration of penetration» $a_N$ [3-4] conceptions. The following method features can be denoted: 1) for the conjugated surface $\Sigma_2$ points determination, simple formulae of kinematics of a point are used (solving of meshing equation is not required); 2) for the surface $\Sigma_2$ point coordinates determination it is necessary to know: $(x, y, z)$ coordinates of a point on a generating surface $\Sigma_1$; unit normal vector projections $n_x, n_y, n_z$ on $\Sigma_1$; penetration velocity $V_N$ and acceleration $a_N$ of the point on $\Sigma_1$. The above-described approximate kinematic method serves for determining envelope surface points if wraparound points are found. The application example of the method including calculations accuracy estimation is considered in [4]. In the work [5] advanced method is presented: another new geometric-kinematic surface movement index at a point of space (a derivative of penetration acceleration $a_N$ – i.e., the second derivative of penetration velocity $V_N$) was implemented for the first time. The new index allows increasing accuracy of the method by 2-3 orders of magnitude.

A brief account of «velocity and acceleration of penetration» terms that are fundamentals of the kinematic method described in [3] will be given further in the article.

2. Velocity of penetration

The velocity of penetration $V_N$ is a relative velocity vector projection $V_{12}$ on a normal $N$ to a cutting tool initial surface (CTIS). $V_N$ [6] is determined as a dot product of $V_{12}$ with a unit normal vector $n$ to CTIS. The unit normal vector is *directed away from CTIS*:

$$V_N = V_{12} \cdot n = V_{12x} \cdot n_x + V_{12y} \cdot n_y + V_{12z} \cdot n_z$$ (1)
if $V_n > 0$ – CTIS penetrates in workpiece; if $V_n < 0$ – CTIS moves off machining surface; if $V_n = 0$ – CTIS slides along machining surface. If $V_n = 0$, a cutting tool forms an envelope workpiece surface (at an envelope and a wraparound contact points).

### 3. Acceleration of penetration

The acceleration of penetration $a_N$ is an acceleration of CTIS penetration in the machining workpiece, i.e., $a_N$ – a time derivative of the penetration velocity $V_n$. Velocity and acceleration of penetration are scalar values: if $V_n > 0$ – CTIS comes closer to machining surface at a set point; if $V_n < 0$ – CTIS moves off machining surface; if $a_N > 0$ – penetration velocity increases. Differentiate (1) with respect to time $t$:

$$a_N = \frac{dV_n}{dt} = \frac{d}{dt}(V_{12} \cdot \mathbf{n}) = a_{12} \cdot \mathbf{n} + V_{12} \cdot \dot{\mathbf{n}}$$

where $a_{12}$ – relative acceleration of a point sliding along CTIS with “$-V_{12}$” velocity.

The process of $a_N$ design equations derivation from (2) is tedious. Therefore, final formulae for generalized meshing (fig. 4) are presented further in the article. In fig. 4 both meshing links are in helical motion around its axes while the axes are in helical motion around the interaxial perpendicular. The motion characteristics of the links are impact-free: motion parameters ($\omega_1, \omega_2, \omega_3, V_1, V_2, V_3$) can be variable or constant in time, can be positive, negative or equal to zero. All the parameters are considered a one or two independent motion parameters function ($t_1, t_2$) which stands for a one- or two-parametric motion (generation). CTIS is on the link 1, the machining surface is on the link 2; the fixed coordinate system is $X_hY_hZ_h$. Any manufacturing meshing can be inserted in the described coordinate system. Formulae for calculation of penetration acceleration $a_N$ in generalized meshing (Fig. 4):

The relative angular velocity vector $\mathbf{\omega}_{12}$ in the system of coordinates $X_hY_hZ_h$:

$$\mathbf{\omega}_{12}^{(6)} = \{\omega_3 \cdot \mathbf{i} - \omega_2 \cdot \sin \gamma \cdot \mathbf{j}, (\omega_1 - \omega_2 \cdot \cos \gamma) \cdot \mathbf{k}\}$$

- The relative linear velocity vector $V_{12}$ in the system of coordinates $X_hY_hZ_h$:
  $$V_{12x}^{(6)} = V_3 - \omega_1 \cdot y_h + \omega_2 \cdot y_p;$$
  $$V_{12y}^{(6)} = -V_2 \cdot \sin \gamma + \omega_1 \cdot x_h - \omega_2 \cdot x_p \cdot \cos \gamma - \omega_3 \cdot z_h;$$
  $$V_{12z}^{(6)} = V_1 - V_2 \cdot \cos \gamma + \omega_2 \cdot x_p \cdot \sin \gamma + \omega_3 \cdot y_h. $$

- The relative acceleration vector $a_{12}$:
  $$a_{12} = [(a_1 - a_2 - a_3) + (\epsilon_1 - \epsilon_2 - \epsilon_3) \times r_1 + \epsilon_2 \times S_3 - (\epsilon_2 + \epsilon_3) \times S_1] +$$
  $$+[(\mathbf{\omega}_1 \times (\mathbf{\omega}_2 \times r_1)) - \mathbf{\omega}_2 \times (\mathbf{\omega}_2 \times S_2) - (\mathbf{\omega}_3 + \mathbf{\omega}_4) \times ((\mathbf{\omega}_2 + \mathbf{\omega}_3) \times r_2)] -$$
  $$-2 \cdot (\mathbf{\omega}_3 \times V_2) - (\mathbf{\omega}_3 \times \mathbf{\omega}_2) \times r_1 - 2 \cdot (\mathbf{\omega}_2 + \mathbf{\omega}_3) \times V_{12} - \mathbf{\omega}_2 \times V_{12}.$$ (5a)

Excluding from (5a) zero summands for cylindrical, bevel and worm gears we get:

$$a_{12} = \mathbf{\omega}_1 \times (\mathbf{\omega}_1 \times r_1) - \mathbf{\omega}_2 \times (\mathbf{\omega}_2 \times r_2) + (\mathbf{\omega}_1 - \mathbf{\omega}_2) \times V_{12}$$

(5b)

- Acceleration of penetration $a_N$:
  $$a_N = a_{12} \cdot \mathbf{n} + V_{12} \cdot \dot{\mathbf{n}} = a_{12} \cdot \mathbf{n} + V_{12} \cdot \mathbf{c} \cdot \omega_K + V_{12} \cdot (\mathbf{c} \times \mathbf{n}) \cdot \omega_K.$$ (6)
where \( \omega_K \) – angular rolling velocity of generating surface moving over machining surface towards the vector \( \mathbf{c} \); \( \omega_{\perp} \) – angular rolling velocity of generating surface moving over machining surface in a plane perpendicular to the vector \( \mathbf{c} \). These parameters are calculated according to the following formulae:

\[
\omega_K = (\mathbf{n} \times \mathbf{c}) \cdot \omega_{12} + \frac{\mathbf{V}_{12} \cdot \mathbf{c}}{R^1_K} + \frac{\mathbf{V}_{12} \cdot (\mathbf{n} \times \mathbf{c})}{R^1_K} \tag{7a}
\]

\[
\omega_{\perp} = \mathbf{c} \cdot \omega_{12} + \frac{\mathbf{V}_{12} \cdot (\mathbf{n} \times \mathbf{c})}{R^2_K} - \frac{\mathbf{V}_{12} \cdot \mathbf{c}}{R^1_K} \tag{7b}
\]

where \( \mathbf{c} \) – unit tangent vector setting normal profile direction. \( R^1_K \) and \( R^2_K \) – CTIS normal profiles radii of curvature corresponding to the profile along the tangent line \( \mathbf{c} \) and the profile perpendicular to the unit normal vector \( \mathbf{c} \) (if \( R^1_K > 0 \) – body is convex). \( R_K \) – radius of geodesic torsion of a line on a surface along the tangent line \( \mathbf{c} \) (if \( R_K > 0 \) – twist is counterclockwise).

All the three formulae (7) components are rotating components in a normal plane section. The first component of (7a) is a relative angular velocity projection \( \omega_{12} = \omega_1 - \omega_2 \) on a normal to a cutting plane, i.e., rotation of the normal is caused by links relative motion. The second component of (7a) comprises rotation of the normal caused by bodies sliding in a normal plane section due to its surface curvature. The third component of (7a) comprises rotation of the normal caused by bodies sliding toward perpendicular to a normal plane section due to geodesic torsion of a line on a surface.

4. Macro-roughness height

“Combs” height \( \Delta H \) above an envelope surface (see fig. 2) determination formulae were obtained in [3]. In the article, the formulae are given without derivation and proofs.

4.1. Height \( \Delta H \) in case of one-parametric generation

The following two cases can be specified:

Case 1 – Generating by a cutting tool initial surface (CTIS) – an actual surface (gears grinding and honing) or a cutting edge mark (cylinder gear wheels cutting by shaping cutters, bevel wheels milling, etc.). Stripes and combs are directed along a tooth line.

Case 2 – Generating by a cutting tool initial surface (CTIS) – CTIS is a surface produced by a forming cutting edge motion in template-cutting method (gear wheels machining by involute side and end milling cutters and also toolheads). Stripes and combs are directed along a tooth profile.

In both cases “combs” height \( \Delta H \) above an envelope surface can be determined according to the formula:

\[
\Delta H = -0.125 \cdot a_N \cdot \Delta t^2 \tag{8}
\]

where \( a_N \) – generating tool penetration acceleration, calculated at a contact point of an instrument (its CTIS) with machining surface; \( \Delta t \) – period of time between two moments of adjacent stripes cutting determined according to one of the formulae:

\[
\Delta t = \frac{2 \cdot \pi}{\omega_b \cdot z_0 \cdot k_X} = \frac{2 \cdot \pi}{\omega_1 \cdot z_1 \cdot k_X} = \frac{2 \cdot \pi}{\omega_0 \cdot z_0 \cdot k_Z} \tag{9}
\]

where \( \omega_0 \), \( \omega_1 \) – angular tool and workpiece velocities respectively wherein acceleration of penetration \( \omega_N \) is calculated; \( z_0 \), \( z_1 \) – shaping cutter and workpiece teeth number respectively; \( k_X \) – number of shaping cutter or rack-type tool strokes per one teeth machining; \( k_Z \) – involute gear cutter chip grooves number or toolheads cutters number.

4.2. Height \( \Delta H \) in case of two-parametric generation

The example of two-parametric generation is cylindrical gears machining by involute gear hobbing cutters. In this case, pits can be parallelogram (or rectangular, which is a particular case) or hexagonal in shape. Parallelogram pit shape can be only obtained when pits form longitudinal and transverse...
rows directed precisely along generation and feed directions respectively – see fig. 5 and 6. Even if a single one of the conditions is not fulfilled, pits will be hexagonal in shape (fig. 6).

Shapes and heights of all the corners of parallelogram pits (as a projection on a tangent plane) are equal. The corners are tetrahedral pyramids (pyramidal asperities) in shape (fig. 5 and 6). In general case, pits and pyramids are “bevelled”, i.e., angle $\theta \neq 90^\circ$ (see fig. 5). Tetrahedral pyramids height above an envelope surface – maximum faceting height – is determined as a sum of two heights ($\Delta = \Delta_1 + \Delta_2$ in fig. 5) produced by generating and feeding respectively:

$$\Delta H_{\text{MAX}} = -0.125 \cdot (a_{NV} \cdot \Delta t_v^2 + a_{NS} \cdot \Delta t_s^2)$$  \hspace{1cm} (10)

where $a_{NV}, a_{NS} – generation$ and feed acceleration of penetration respectively; $\Delta t_v – time$ required for shaping an adjacent pit set along the generation direction; $\Delta t_s – time$ required for shaping an adjacent pit set along the feed direction.

Shapes of hexagonal pits (as a projection on a tangent plane) can vary significantly depending on the coefficient $k_L$ – pits displacement $\Delta L$ (along a row) in adjacent rows to distance between pits $L$ (in a row) ratio (shown on the right of fig. 6). In the case of hexagonal pits, pyramids are triangular. Figures 5 and 6 indicate that: three surfaces converge at all vertices of hexahedron angles, whereas at the vertices of parallelogram pits four surfaces converge.

For hexagonal pits obtained by a two-motions (generating and feeding) gear machining method it was analytically determined and computationally proven in [3] that: all the six vertices heights are equal and comprise 75-100% of $\Delta_{\text{MAX}}$ calculated according to the formula (10). It was also proven that macro-roughness height $\Delta \text{ depends only on the five following parameters: } \Delta t_v$ and $\Delta t_s – time$ periods required for shaping adjacent pits in a row and in two adjacent rows; $a_{NV}, a_{NS} – generation$ and feed acceleration of penetration; $k_L – coefficient$ of pits displacement in adjacent rows. Neither distances between adjacent pits ratio nor the angle $\theta$ between generating and feeding motion vectors nor cutting edge position on a generating surface have an impact on macro-roughness height.

For all pit types the following formula for pyramids height $\Delta H – i.e., maximum$ faceting height – determination was obtained:
\[ K_Z = 1 - \frac{k_{HL}}{1 + k_H} \cdot (2 - k_{HL}) \]  
\[ \Delta H = K_Z \cdot \Delta H_{MAX} \]  

where \( \Delta H_{MAX} \) is calculated according to the formula (10), and

\[ k_H = \frac{\Delta_1}{\Delta_2} = \frac{a_{SV} \cdot \Delta t_{L}^2}{a_{SS} \cdot \Delta t_{S}^2}, \quad k_{HL} = k_H \cdot k_L \cdot (1 - k_L) \]  

Formulas (12-13) can be applied if \( 0 \leq k_H \leq 4 \) and if \( 0 \leq k_L \leq 1 \).

\( \Delta H \) to \( \Delta H_{MAX} \) ratio denominated as \( K_Z \), we will call a macro-roughness height reduction coefficient. обозначенное через \( K_Z \), назовем коэффициентом уменьшения высоты макронеровностей.

While studying scaled surfaces forms (rectangular, parallelogram and hexagonal), all adjacent pits are considered symmetrical and equal though it is possible only if the value of acceleration \( a_N \) is equal at all the points of machining surface. Each pit “central” point \( K \) is situated on a surface, which is a generating surfaces family envelope.

### 5. Pits shape and pyramidal asperities height in case of two-parametric generation

Algorithms and computer programs based on formulae (1) – (13) and generalized gearing (shown in fig. 4) formulae of point coordinates and vector projections transformation were developed to investigate the influence of machining type parameters on macro-roughness types and sizes.

#### 5.1. Hexagonal pit shape

Fig. 7 and 8 illustrate how the pit transformation process is influenced by two key machining type parameters: the angle \( \theta \) between generation and feed direction vectors (fig. 5) and \( k_L = \Delta L/L \) coefficient which defines the value of pits displacement in adjacent rows along the generation direction (fig. 6). The result of computer simulations for pit dimensions ratio determination: \( k_S = L_1/L_2 = 2 \) (where \( L_1 \) – dimension along generation direction and \( L_2 \) – dimension along feed direction) is shown in fig. 7 and 8. In this case displacement coefficient \( k_L = \Delta L/L \) changes its value from 0 to 0.5 as when \( k_L \) is equal to 0.5-1.0, the symmetrical hexagonal pit transforms into parallelogram pit. Accepting any value of \( k_L \) exceeding 1.0 is meaningless as the shape of adjacent pits does not change. Moreover, a new pit near the considered one will be formed not by the current cutting edge of a cutter, but one of the following edges. Fig. 7 illustrates the results of the computer modelling in case the angle \( \theta = 90^0 \) (fig. 5), fig. 8 – in case the angle \( \theta = 70^0 \) (pits and pyramids are “beveled”).

**Figure 7.** Hexagonal pit transformation into parallelogram pit as \( k_l \) value changes: higher row is a projection on a tangent plane at the pit center; lower row is an axonometric projection.
5.2. Dependence of pyramid height on generation parameters

After calculating pits and pyramids sizes and shapes it was estimated how the macro-roughness height $\Delta$ can decrease depending on generation parameters. The ratio of macro-roughness height $\Delta$ to maximum macro-roughness height $\Delta_{MAX}$ cannot be less than 0.75 which is the significant result of the macro-roughness height analytical research. It also was discovered that $k_{\Delta} = \Delta/\Delta_{MAX}$ ratio is a function of the two parameters: $k_{\Delta} = f(k_L, k_H)$—(fig. 9).

It should be noted that:
1. the maximum value of macro-roughness height $\Delta$ is obtained if $k_L = 0.0$ ($k_H$ can take any value) which is the least preferable pit arrangement in adjacent rows according to surface roughness criterion;
2. the maximum value of macro-roughness height $\Delta$ can be obtained if $k_L = 0.5$ and $k_H = 2$ (the most preferable pit arrangement in adjacent rows);
3. macro-roughness height $\Delta$ decreases if $k_H$ increases from 0 to 2 while $k_L$ can take any value;
4. macro-roughness height $\Delta$ increases slightly if $k_H$ increases from 2 to 4 while $k_L$ can take any value;
5. $\Delta = \text{const} = 0.8 \Delta_{MAX}$ if $k_H = 4$ and $k_L > 0.25$.

Conclusion

The study of macro-roughness forms and sizes of surfaces machining by edge tools in case of two-parametric generation and by any tools in case of one-parametric generation with the use of “acceleration of penetration” term allows us to obtain:

- two general and simple formulae for determining maximum faceting height (“combs” height above an envelope surface) in case of one-parametric and two-parametric generation;
- the formula for calculating faceting height decrease coefficient $k$ (depending on coefficient of pits displacement in adjacent rows). The $k$ coefficient change its value from 1 to 0.75, i.e., generation parameters $k_L$ и $k_H$ have a slight impact on roughness.
The study also shows that:

- in case of two-motion gear machining method the shape of pits can be: parallelogram, rectangular or hexagonal. Moreover, heights of all 4 or 6 pyramids surrounding the pit are equal;
- the use of velocity and acceleration of penetration can simplify the process of resolving other gear generation analysis issues [3]: determining the curvature of surfaces formed by generating methods; estimating the cutting area and calculating the thickness of the cut layer; determining points coordinates on envelope surface through points coordinates on wraparound surface, etc. In other words, velocity and acceleration of penetration are the key parameters of generating process that should be calculated and used in computer programs along with $V_{12}$, $\omega_{12}$, $n$, $r$ vectors. It is a crucial part of the current research conclusion.

Acknowledgments

This work was supported by grant (project № 9.6355.2017/БЧ) of government order of Ministry of education and science of Russia Federation for the period 2017–2019 in Tyumen Industrial University.

References

[1] Shunayev B K 1965 Geometry of lateral surface with diagonal gear milling, Design and technology in heavy engineering: proceedings of UPI, Sverdlovsk, USSR, Sat 14
[2] Sheveleva G I 1999 Theory of formation and contact of moving bodies, Mosstankin, 494 p
[3] Babichev D T 2005 Development of theory of gearing and shaping of surfaces on basis of new geometric-kinematic representations, Tyumen, Doctoral Thesis
[4] Babichev D T, Babichev D A, Pankov D N, Panfilova E B 2016 Kinematic method of finding points on envelope, knowing points on enveloping, Bulletin of NTU "KhPI" 26 9-21
[5] Babichev D A, Babichev D T, Serebrennikov A A 2010 Using the derivative of acceleration of introduction when finding points on envelope, knowing points on enveloping, News of NTU "KPI" 27 20-25
[6] Shishkov V A 1951 Formation of surfaces by cutting by generating method, Mashgiz