Multi-Objective Robust Optimization of Hybrid AC/DC Distribution Networks Considering Flexible Interconnection Devices

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ABSTRACT Hybrid AC/DC distribution networks with the high-efficiency consumption and high-proportion access of new energy have become a crucial trend for future modern distribution networks. High penetration of renewable energy brings various controllable resources along with uncertainties to hybrid AC/DC distribution networks, making conventional single objective optimization fail to meet optimal operation requirements of flexibility and reliability. In this paper, considering uncertainties of distribution generations and loads, a multi-objective robust optimization model based on various controllable devices is proposed. To increase resource utilization, the proposed method comprehensively and properly models the full variety of possible control means (i.e., flexible distribution switch, voltage source converter, energy storages, et al). Utilizing the abundant control means, a multi-objective optimization which minimizes network losses, voltage deviations and operation cost simultaneously is modeled. Then, based on the multi-objective model, a two-stage robust optimization based on second order cone method and column constraint generation algorithm is proposed to achieve the solution, which can deal with the fluctuation of loads and renewable energy quickly and effectively. Outdoing traditional single objective robust optimization, our multi-objective robust optimization utilizes various controllable resources, obviously improving the safety, flexibility and economy simultaneously. Finally, compared with existing robust optimization, the proposed method is tested in numerous case studies to verify its effectiveness and advantages in a modified IEEE 33-node system.

INDEX TERMS Hybrid AC/DC distribution networks, Multi-objective optimization, Two-stage robust optimization, Flexible interconnection devices, Column constraint generation algorithm.

I. INTRODUCTION 
With concerns on carbon emission and cost reduction of distributed generations (DG), the utilization rate of renewable energy is increasing [1], in which photovoltaic (PV) and wind turbines have been greatly utilized [2,3]. Moreover, energy storage systems (ESS), flexible electrical loads, and almost DC devices, are continuously connected to distribution networks. These changes make hybrid AC/DC distribution networks a crucial trend for future distribution networks. Modern electronic devices, such as DG, ESS and flexible interconnection devices, have the advantages in fast response, wide control range and low operation cost, which bring flexible controllable means to distribution system operation control systems, as well as high uncertainties.

Therefore, to increase energy efficiency and operation flexibility, it is necessary to study a novel operation method in hybrid AC/DC distribution systems considering multiple control means and uncertainties [4].

Due to uncertainties of solar energy, wind and consumer behavior, the predicted data cannot be accurate ideally in practice, especially in the future distribution networks with high proportion of DGs [5]. This phenomenon leads to the fact that the traditional deterministic optimization is no longer applicable [6]. To make the optimization more accurate, robust optimization has become an indispensable method in distribution network, microgrid and other fields, including voltage regulation [7], network reconfiguration [8], energy storage equipment optimization [9] and reducing...
operation cost [10]. In [11], a two-stage robust optimization model is proposed for microgrid system to ensure safe operation in the worst scenario by optimizing the active and reactive power of voltage source converter (VSC). In [12], “budget of uncertainty” is set to overcome the conservatism caused by only considering the worst scenarios in the system, and also improves the economy. However, most robust optimization only uses the single objective model, which cannot fulfill the requirements of flexibility, economy and safety in the AC/DC hybrid distribution network with multiple uncertainties [13]. In [14], a robust optimization model is established to co-optimize the slopes of active and reactive power droop control of VSCs with the aim to minimize the total network losses, whose objective function only includes distribution network losses. In [15], a robust optimization method is proposed to address the short-term variations of renewable energy generation and loads for the strategic investment problem. In the existing literature, a number of multi-objective optimization methods have emerged to deal with the safe, reliable and economic operation in distribution networks [16]. Therefore, to cope with the uncertainties from various renewable resources and controllable resources, the combination of multi-objective model and robust optimization can be considered as a promising methodology to solve the real operation optimization for future distribution networks under high uncertainties [17].

The continuous development of power electronic technology brings flexible controllable electronic devices into distribution networks, including flexible interconnected devices, DGs and reactive power compensation devices [18]. In hybrid AC/DC distribution networks, VSCs, as the interconnected devices between AC and DC distribution networks, can also regulate active/reactive power in AC networks and active power in DC networks. In [19], a coordinated real-time voltage regulation method is proposed to utilize converter-based controllable devices in hybrid AC/DC medium distribution networks, such as DGs, ESSs, and VSCs. Moreover, flexible distribution switch (FDS) which can regulate power between AC and AC distribution networks is a novel flexible switch with many advantages. Compared with traditional “hard” switch, FDS with fast response can accurately control the power flow of active power and reactive power between different sections in distribution networks, and can regulate power flow smoothly. FDS makes the distribution system more flexible and reliable [20]. At present, the model, control and optimization of FDS have been in-depth research in AC distribution networks [21]. In [22], the operation optimization based on FDS is modeled and analyzed, and the advantages of FDS in improving power quality of distribution networks and reducing network losses are verified. Therefore, as flexible interconnected devices, FDS and VSC can achieve flexible energy transmission between AC and DC networks, realizing real-time, fast, sensitive and smooth power control. Moreover, DGs and ESSs are also important control means in operation and control in distribution networks. However, the joint optimization of FDS, VSC and other control means is rarely considered in hybrid AC/DC distribution networks, which will certainly greatly improve the flexibility, economy and reliability of hybrid AC/DC distribution networks by the coordination of various control means.

In this paper, to target the above research gaps, a multi-objective robust optimization method in hybrid AC/DC flexible distribution networks is proposed. To resist the risk of fluctuation of renewable energy, the robust method optimizes operation cost, voltage deviation and network losses simultaneously by multiple control means. Its major contributions are as follows: (1) Various control means especially flexible interconnection devices are applied in our coordinated optimization to regulate power flow between different sections in hybrid AC/DC flexible distribution networks. (2) Considering uncertainties of DGs and loads, a multi-objective robust optimization method is proposed to operation cost, voltage deviation and network loss simultaneously. (3) A two-stage robust optimization method based on second order cone method and column constraint generation (C&CG) algorithm is proposed to deal with the fluctuation of load and renewable energy quickly and effectively.

The reminder of this paper is organized as follows: the multi-objective optimization model considering flexible interconnection devices in hybrid AC/DC distribution networks is proposed in Section 2; Section 3 presents a two-stage robust optimization model and solution algorithm. Case studies are presented in Section 4, while Section 5 concludes this paper.

II. MULTI-OBJECTIVE OPTIMIZATION MODEL IN HYBRID AC/DC DISTRIBUTION NETWORKS

To increase energy utilization, flexible interconnection devices are attracted significant attractions in the proposed multi-objective optimization model in this paper. Therefore, FDS, VSC and ESS are optimized to pursue the safety and economy of hybrid AC/DC distribution networks. This section aims to present the overall objective and constraints of the proposed multi-objective optimization model.

A. OBJECTIVE

In order to optimize network losses, voltage deviation and operation cost simultaneously, the objective function can be written as follows:

\[
f = \min(\lambda_1 P_{loss} + \lambda_2 \Delta V + \lambda_3 C_{op})
\]  

(1)

\[
P_{loss} = \sum_{i=1}^{24} price(t) \sum_{i=1}^{N_c} \sum_{j=1}^{N_r} \frac{1}{2} R_{ij} x I_i^2(t)
\]  

(2)

\[
\Delta V = \sum_{i=1}^{24} dev(V_i(t))
\]  

(3)
\[
\begin{align*}
\text{dev}(V_i(t)) &= \begin{cases} 
0 & \text{if } V_i(t) \in [V_{\text{min}}^{V}, V_{\text{max}}^{V}] \\
|V_i(t)| & \text{otherwise} 
\end{cases} \\
C_{op} &= \sum_{i=1}^{24} \text{Price}(t) \left[ \sum_{k=\text{K}}^{24} P_{\text{loss}, k}^{\text{VSC}}(t) + \sum_{n=\text{FDS}}^{\text{loss,n}}(t) - \sum_{j=\text{FDS}}^{\text{ESS}} P_{\text{ESS}}^{j}(t) + \sum_{j=\text{ESS}}^{\text{dis}} P_{\text{ESS}}^{j}(t) \right] \\
&+ K_{\text{ess}} \sum_{i=1}^{24} \sum_{j=\text{FDS}}^{\text{ESS}} (I_{\text{dis}, j}(t) + P_{\text{ESS}}^{j}(t)) 
\end{align*}
\]

where \( f \) is the objective function value; \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are the weight factors of network losses (\( P_{\text{loss}} \)), voltage deviation (\( \Delta V \)) and operation cost (\( C_{op} \)) in all day and \( \lambda_1 + \lambda_2 + \lambda_3 = 1 \). \( N_{\text{ac}} \) and \( N_{\text{dc}} \) are the number of nodes of AC and DC distribution systems respectively.

Power losses \( P_{\text{loss}} \): \( \Omega \) is the set of all nodes connected to node \( i \). \( R_q \) is the resistance of the branch between node \( i \) and node \( j \). \( I_q \) is the current from node \( i \) to node \( j \). \( \text{price}(t) \) is the electricity price at time \( t \).

Voltage deviation \( \Delta V \): \( V_i(t) \) is the nodal voltage of node \( i \) at time \( t \). \( V_{\text{min}}^{V} \) and \( V_{\text{max}}^{V} \) are the maximum and minimum voltage of voltage optimization range [23].

Operation cost \( C_{op} \): \( K, N \) and \( J \) are the number of VSC, FDS and ESS, respectively. \( P_{\text{VSC}}^{\text{loss,n}}(t) \) and \( P_{\text{FDS}}^{\text{loss,n}}(t) \) are the power losses of \( \text{th} \) VSC and \( \text{th} \) FDS at time \( t \) respectively. \( P_{\text{ESS}}^{j}(t) \) and \( P_{\text{ESS}}^{j}(t) \) is the charge/discharge power of the \( j \)th ESS. The active power value means that ESS absorbs power from the grid and vice versa. \( K_{\text{ess}} \) is the unit power cost of ESS which depends on investment cost and maintenance cost. In this paper, the electricity price and feed-in tariffs are the same at time \( t \).

\section*{B. CONSTRAINTS}

The multi-objective optimization model established in this paper needs to satisfy the constraints of power flow, VSC, FDS, ESS as well as constraints of secure operation. The constraints can be described as follows.

1) \textbf{POWER FLOW CONSTRAINTS}

The hybrid AC/DC distribution network adopts the DistFlow model. The power flow constraints are [24]:

\[
\begin{align*}
\sum_{\text{bus}} P_{\text{in}}(t) &= P_i(t) + \sum_{\text{branch}} (P_q(t) - R_q(I_q(t))^2) \\
\sum_{\text{bus}} Q_i(t) &= Q_i(t) + \sum_{\text{branch}} (Q_q(t) - X_q(I_q(t))^2) \\
I_q(t)^2 &= \frac{P_i(t)^2 + Q_i(t)^2}{V_i(t)^2} \\
V_j(t)^2 &= V_i(t)^2 - 2R_q P_q(t) + X_q Q_q(t) \\
&+ (R_q^2 + X_q^2) \times I_q(t)^2 
\end{align*}
\]

where, \( \phi_i \) is the branch set with node \( i \) as the head node. \( \phi_i \) is the branch set with node \( i \) as the end node. \( R_q \) and \( X_q \) are the resistance and reactance of branch \( ij \). \( P_i(t) \) and \( Q_i(t) \) are the injected active and reactive power of node \( i \) at time \( t \). \( P_q(t) \) and \( Q_q(t) \) are the active and reactive power flow from node \( i \) to node \( j \) at time \( t \). In the DC system, the reactive power and reactance value are equal to zero in (6)-(8).

2) \textbf{VSC CONSTRAINTS}

The equivalent model of VSC is shown in Figure 1, which is equivalent to the form of a converter and impedance in series.

\[
\begin{align*}
P_{\text{PSC}}^{\text{ac,k}}(t) + P_{\text{PSC}}^{\text{dc,k}}(t) - P_{\text{loss,k}}^{\text{VSC}}(t) &= 0 \\
-P_{\text{ac,k,\text{max}}}^{\text{PSC}} \leq P_{\text{ac,k}}^{\text{PSC}}(t) \leq P_{\text{ac,k,\text{max}}}^{\text{PSC}} \\
Q_{\text{ac,k}}^{\text{PSC}}(t) - Q_{\text{ac,k,\text{max}}}^{\text{PSC}} \leq Q_{\text{ac,k}}^{\text{PSC}}(t) \leq Q_{\text{ac,k,\text{max}}}^{\text{PSC}} \\
\sqrt{(P_{\text{PSC}}^{\text{ac,k}}(t))^2 + (Q_{\text{PSC}}^{\text{ac,k}}(t))^2} &\leq S_{\text{ac,k}}^{\text{PSC}} \\
P_{\text{PSC}}^{\text{ac,k}} + jQ_{\text{PSC}}^{\text{ac,k}} &= A_{\text{ac,k}}^{\text{PSC}} \times \sqrt{(P_{\text{PSC}}^{\text{ac,k}}(t))^2 + (Q_{\text{PSC}}^{\text{ac,k}}(t))^2}
\end{align*}
\]

where, \( P_{\text{PSC}}^{\text{ac,k}} \) and \( Q_{\text{PSC}}^{\text{ac,k}} \) are the injected active power and reactive power of the \( k \)th VSC’s AC side at time \( t \). \( P_{\text{PSC}}^{\text{dc,k}} \) is the injected active power of the \( k \)th VSC’s DC side at time \( t \). \( R_{\text{ac,k}}^{\text{PSC}} \) and \( X_{\text{ac,k}}^{\text{PSC}} \) are the equivalent resistance and reactance of the \( k \)th VSC. \( S_{\text{ac,k}}^{\text{PSC}} \) is the capacity limit of \( k \)th VSC. \( A_{\text{ac,k}}^{\text{PSC}} \) is the upper limit of active and reactive power of \( k \)th VSC.

\section*{3) \textbf{FDS CONSTRAINTS}}

FDS is a soft switching device to regulate energy transfer between AC and DC systems [25]. The equivalent model of the multi-terminal FDS is shown in Figure 2. The constraints of the \( n \)th multi-terminal FDS are:

\[
\begin{align*}
\sum_{m=1}^{M} P_{\text{FDS},m}^{\text{Tm}}(t) + \sum_{m=1}^{M} P_{\text{loss,m}}^{\text{Tm}}(t) &= 0 \\
\sqrt{(P_{\text{FDS},m}^{\text{Tm}}(t))^2 + (Q_{\text{FDS},m}^{\text{Tm}}(t))^2} &\leq S_{\text{m,\text{max}}}^{\text{Tm}}, \forall m \\
P_{\text{FDS}}^{\text{Tm}}(t) &= \sum_{m=1}^{M} A_{\text{m}}^{\text{Tm}} \sqrt{(P_{\text{FDS},m}^{\text{Tm}}(t))^2 + (Q_{\text{FDS},m}^{\text{Tm}}(t))^2}
\end{align*}
\]

where, \( M \) is the number of terminals of the FDS. \( Tm \) means the \( m \)th terminal of the FDS. \( P_{\text{FDS},m}^{\text{Tm}}(t) \), \( Q_{\text{FDS},m}^{\text{Tm}}(t) \) and \( P_{\text{loss,m}}^{\text{Tm}}(t) \) are the active power, reactive power and active power losses of \( n \)th FDS’s \( m \)th terminal at time \( t \). \( S_{\text{m,\text{max}}}^{\text{Tm}} \) is the capacity limit of \( m \)th terminal of \( n \)th FDS. \( A_{\text{m}}^{\text{Tm}} \) is the loss coefficient of \( n \)th FDS.
In this paper, the above model is transferred into second order cone model based on linearization and relaxation technique for rapid solution. Firstly, to realize the linearization, define

\[
\begin{align*}
V_{i,2}(t) &= V_i^2(t) \\
I_{y,2}(t) &= I_y^2(t)
\end{align*}
\]

(21)

(2), (4), (6-8) and (20) can be replaced by (21). To replace the absolute value item, an auxiliary variable \(\mu_i(t)=|V_{i,2}(t)-1|\) and some constraints are added:

\[
\begin{align*}
\mu_i(t) &
\geq 0 \\
\mu_i(t) &
\geq V_{i,2}(t)-(V_{op}^2)^2 \\
\mu_i(t) &
\geq -V_{i,2}(t)+(V_{op}^2)^2
\end{align*}
\]

The power flow constraint (7), VSC loss constraint (13) and FDS loss constraint (16) are nonlinear quadratic constraints, which can be further relaxed to the following second-order cone constraints:

\[
\begin{align*}
\begin{bmatrix}
P_{ac,k}^SC(t) \\
Q_{ac,k}^SC(t)
\end{bmatrix}
\leq
\begin{bmatrix}
P_{loss,k}^SC \\
Q_{loss,k}^SC
\end{bmatrix}
\end{align*}
\]

(23)

\[
\begin{align*}
\begin{bmatrix}
P_{FDS,Tm}^n(t) \\
Q_{FDS,Tm}^n(t)
\end{bmatrix}
\leq
\begin{bmatrix}
P_{FDS,Tm}^a_n(t) \\
Q_{FDS,Tm}^a_n(t)
\end{bmatrix}
\end{align*}
\]

(24)

\[
\begin{align*}
2P_{i,j}(t) \\
2Q_{i,j}(t)
\end{bmatrix}
\leq
I_{y,2}(t)+V_{i,2}(t)
\]

(25)

The circle constraints (12) and (15) can be transferred into (26) and (27) based on positive octagonal constraint.

\[
\begin{align*}
\begin{bmatrix}
P_{FDS,Tm}^n(t) \\
Q_{FDS,Tm}^n(t)
\end{bmatrix}
\leq
\begin{bmatrix}
P_{FDS,Tm}^a_n(t) \\
Q_{FDS,Tm}^a_n(t)
\end{bmatrix}
\end{align*}
\]

(26)

Now, using convex relaxation and linearization, the original multi-objective optimization model is reformulated as the following SOCP model, as shown in (28).

\[
\begin{align*}
\min f
\quad \text{s.t. (2), (3), (5), (6), (8), (11), (14), (17), (20), (22), (27)}
\end{align*}
\]

III. MODEL AND SOLUTION PROCEDURE OF MULTI-OBJECTIVE ROBUST OPTIMIZATION

Considering the significant spatial and temporal uncertainties of DG and load output, the model and algorithm should be improved to cope with uncertainties in the practical application. In this section, a multi-objective robust optimization model based on C&CG is developed to solve the robust optimization problem of hybrid AC/DC flexible distribution networks.

A. MODEL

The deterministic multi-objective optimization model proposed in Section 2 can be expressed in a compact form, shown as (29).

\[
\begin{align*}
\begin{bmatrix}
\min e^Ty
\end{bmatrix}
\quad \text{s.t. } \begin{bmatrix}
Ax+By=a \\
C+Dy \geq d
\end{bmatrix}
\end{align*}
\]

(29)

\[
\begin{bmatrix}
I_yu=0 \\
\|Fy\|_2 \leq f^Ty
\end{bmatrix}
\]

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where $x$ represents the binary variable vector, including ESS charge/discharge state. $y$ is the continuous variable vector, including the active and reactive power of FDS, ESS and VSC. In (29), $A$, $B$, $C$, $D$, $F$ and $I_x$ are coefficient matrices of variables corresponding to the constraints. $a$, $c$, $d$, $f$ and $u$ are constant column vectors. The 2nd line in (29) corresponds to (2), (3), (5), (6), (8)-(9) and (14), the 3rd line involves (9), (11), (17)-(20), (22) and (26)-(27), the 4th line indicates that the uncertainty is ignored in deterministic model, the output power of DGs and loads at each time step are equal to the predicted value, and the last line involves (23)-(25).

The deterministic optimization model (29) can be solved by second order cone optimization. However, the solution may be inappropriate due to the predicted errors. Therefore, in this paper, the uncertainties of DG and load are considered in (30):

$$
\begin{align*}
    u &= [u_{DG}(t), u_L(t)]^T \\
    u_{DG}(t) &\in [\hat{u}_{DG}(t) - \Delta u_{DG}^{\text{max}}(t), \hat{u}_{DG}(t) + \Delta u_{DG}^{\text{max}}(t)] \\
    u_L(t) &\in [\hat{u}_{L}(t) - \Delta u_{L}^{\text{max}}(t), \hat{u}_{L}(t) + \Delta u_{L}^{\text{max}}(t)]
\end{align*}
$$

where, $u_{DG}(t)$ and $u_L(t)$ are the actual output power of DG and load at time $t$. $\hat{u}_{DG}(t)$ and $\hat{u}_{L}(t)$ are the predicted output power of DG and load at time $t$. $\Delta u_{DG}^{\text{max}}(t)$ and $\Delta u_{L}^{\text{max}}(t)$ are the maximum forecast deviation of DG and load power at time $t$.

Then, the two-stage robust optimization model is formulated to handle the uncertainties in (31), to seek optimal scheme in the worst scenario. $u$ denotes the uncertainty variable of DG and load.

$$
\begin{align*}
    \min_{x} \quad & \max_{u \in \Lambda} \quad \min_{y \in \Omega(x,u)} \quad c^T y \\
\text{s.t.} & \quad A x + B y = a \\
& \quad C x + D y \geq d \\
& \quad I_x y = u' \\
& \quad ||F y'|| \leq f^T y'
\end{align*}
$$

where $x$ is the first-stage variable in the outer layer minimization problem, $y$ and $u$ are the second-stage variables in the inner layer maximization and minimization problem. The variable compositions of $x$ and $y$ are the same as (29).

**B. SOLUTION PROCEDURE**

C&CG is applied to solve the two-stage robust optimization model, denoted as equation (31). Similar to Benders-dual algorithm, the method decomposes the model into a master-problem and a sub-problem and then solves them iteratively. The formulation of the master-problem is expressed as:

$$
\begin{align*}
    \min_{x} \quad & \alpha \\
\text{s.t.} & \quad \alpha \geq c^T y' \\
& \quad A x + B y' = a \\
& \quad C x + D y' \geq d \\
& \quad I_x y' = u' \\
& \quad ||F y'|| \leq f^T y'
\end{align*}
$$

where, $\alpha$ represents the objective function value of the master-problem. $l$ is the iteration counter; $y'$ denotes the variables of sub-problem introduced to the master-problem in the $l$th iteration. $u'$ is the realization of $u$ in the worst scenario in the $l$th iteration.

The sub-problem is described as:

$$
\begin{align*}
    \max_{u} \quad & \min_{y} \quad c^T y \\
\text{s.t.} & \quad A x + By = a \quad \rightarrow \gamma \\
& \quad Cx + Dy \geq d \quad \rightarrow \lambda \\
& \quad I_x y = u \quad \rightarrow \nu \\
& \quad \|F y'\|_2 \leq f^T y' \quad \rightarrow \omega_l \varphi_l
\end{align*}
$$

where $\gamma$, $\lambda$, $\nu$, $\omega_l$ and $\varphi_l$ are the dual variables corresponding to the constraints. $x'$ denotes the variables of master-problem introduced to the sub-problem in the $l$th iteration.

For each set of given $u$, equation (33) can be simplified to deterministic optimization model. The above formulation has a max–min form, of which the inner minimization problem is linear for a fixed $x$ and $u$. Thus (33) can be recast as (34) based on strong duality theory:

$$
\begin{align*}
    \max_{u \in \mathcal{U}(\gamma, \lambda, \nu, \omega_l, \varphi_l)} & \quad (a - Ax')^T \gamma + (d - Cx')^T \lambda + u^T \nu \\
\text{s.t.} & \quad B \gamma + B' \lambda + I_x' \nu + \sum_{t} (F^T \omega_l + f \varphi_l) \leq c \\
& \quad \omega_l \leq \varphi_l \\
& \quad \lambda, \varphi_l \geq 0
\end{align*}
$$

According to [27], the optimal solution of (34) is obtained only when $u$ is an extreme point of the uncertainty set. In our paper, the worst scenario occurs in the upper bounds of loads and the lower bounds of DGs. Thus, the uncertainty set of DGs and loads can be rewritten as (35).

$$
\begin{align*}
    u_{DG}(t) &= \hat{u}_{DG}(t) - B_{DG}(t) \Delta u_{DG}^{\text{max}}(t) \\
    u_L(t) &= \hat{u}_L(t) + B_L(t) \Delta u_{L}^{\text{max}}(t) \\
    \sum_{t} B_{DG}(t) &\leq \Gamma^{DG} \\
    \sum_{t} B_L(t) &\leq \Gamma^L
\end{align*}
$$

where, $\Gamma^{DG}$ and $\Gamma^L$, named as the “budget of uncertainty”, are integers valued between 0 and $T$, which are used to adjust the conservatism degree of the optimal solutions. Usually, we make

$$
\begin{align*}
    \Delta u_{DG}^{\text{max}}(t) &= \xi_{DG} \times \hat{u}_{DG}(t) \\
    \Delta u_{L}^{\text{max}}(t) &= \xi_{L} \times \hat{u}_L(t)
\end{align*}
$$

where, $\xi_{DG}$ and $\xi_{L}$ are the fluctuation interval coefficient of DG and load.

After the above transformations, (34) becomes a form with binary variables and continuous variables, which can be converted by linearization techniques. Then, the max form of the sub-problem will be transferred into the following mixed-integer linear optimization model:
\[
\begin{align*}
\max_{\lambda, \gamma, v, \pi, \alpha} & \quad d^T\gamma + k^T\lambda + (h - Fx^c) ^T v + \bar{u}^T\pi + \Delta u^T B' \\
\text{s.t.} & \quad D^T\gamma + K^T\lambda + G^Tv + I^T\pi + \sum_i (F^T\omega_i + f\varphi_i) \leq c \\
& \quad \|\omega\|_1 \leq \varphi_i \\
& \quad 0 \leq B' \leq \bar{B}B \\
& \quad \pi - \bar{\pi}(1-B) \leq B' \leq \pi \\
& \quad \gamma, v, \varphi, \geq 0
\end{align*}
\]

where, \( \bar{\pi} \) is the upper bounds of dual variables \( \pi \), which may take the values large enough.

After the above process, the two-stage robust optimization model is decomposed into the master-problem (MP) and sub-problem (SP), and solved by C&CG algorithm. The flow chart is shown in Figure 3, where \( U_B \) and \( L_B \) are the upper and lower bounds of the loop, and (38) is as follows:

\[
\begin{align*}
\text{s.t.} & \quad \alpha \geq c^T y^{l+1} \\
& \quad Ax + By^{l+1} = a \\
& \quad Cx + Dy^{l+1} = d \\
& \quad I_y y^{l+1} = u^{l+1} \\
& \quad \|Fy^{l+1}\|_2 \leq f^T y^{l+1}
\end{align*}
\]

IV. CASE STUDIES

A. TEST SYSTEM PARAMETERS

The proposed method is verified on a modified IEEE33 node hybrid AC/DC distribution network, as shown in Figure 4. The network is divided into AC network and DC network by VSC. The reference voltage of both AC and DC parts is 12.66 kV. Node1 is the slack node, and the optimization interval of voltage per-unit value is [0.985,1.015]. In our paper, DGs are considered as PVs. There are four PVs in the system, which are connected to Node 11, 13, 20 and 31, respectively, three ESSs with the upper power limit of 1MW and the rated capacity of 5MW·h, which are connected to Node 15, 24 and 30, respectively. Moreover, a multi-terminal FDS is connected to Node 21, 25 and 33. Three 24-hour varying loads are connected to Node 18, 23 and 32. It is assumed that the power direction of the VSC is from AC part to DC part, which is positive. The sampling power of the system is 1 hour, and the system implements time-of-use electricity price. The electricity price in each period is shown in Figure 5. The power profiles of PVs and loads are shown in Figure 6. The parameters of VSC and FDS are shown in Table 1.

![Figure 3](image-url)  
**Figure 3.** Flow chart of the proposed robust optimization method.

![Figure 4](image-url)  
**Figure 4.** Structure of the modified IEEE 33-node hybrid AC/DC distribution network.

![Figure 5](image-url)  
**Figure 5.** Time-of-use electricity price in hybrid AC/DC distribution networks.
AC/DC distribution networks can increase economy and stability of the hybrid system. Adding FDS makes the power flow mode more flexible, and can better distribute the power between AC and DC networks. During this period, the improvement of voltage profiles is more obvious because FDS transmits power from terminal T1 and T2 to T3.

Compared with Case 1, voltage profiles in Case 2 get improved during 1-8h and 19-24h because FDS transmits power from terminal T1 and T2 to T3 during this period. The addition of FDS decreases the system operation cost, the regulation effect of ESSs on voltage deviation is obvious. This is because ESSs can absorb power when the power is relatively sufficient to make up for the serious voltage drop caused by power defect. It can be seen that VSCs, ESSs and FDS can greatly improve the system optimization effect efficiency.

In order to verify the effectiveness of the two-stage robust optimization method, the worst scenario of the above 5 cases is selected for verification. The minimum nodal voltage value in 5 cases is shown in Figure 8. It should be noted that Case 3 is a deterministic case which is the same as the situation that the predicted deviation is equal to 0 in Case 4. As shown in Figure 8 and Table II, comparison between Case 3, Case 4 and Case 5 shows that ignoring the predicted deviation, the optimization performance of Case 3 is better than that of Case 4 and Case 5. In other words, the strategies of robust optimization are more conservative than that of deterministic optimization. Comparison between Case 4 and Case 5 shows that the greater budget of uncertainty causes the more conservative optimization result and a certain increase in operation cost, the regulation effect of ESSs on voltage deviation is obvious. This is because ESSs can absorb power when the power is relatively sufficient to make up for the serious voltage drop caused by power defect. It can be seen that VSCs, ESSs and FDS can greatly improve the system optimization effect efficiency.

**B. SIMULATION ANALYSIS**

A 24h simulation has been carried out here. In order to verify the performance of the proposed two-stage robust optimization method in our paper, simulations are carried out in five different cases. The five cases are set as follows:

- **Case 1:** Only VSCs are optimized;
- **Case 2:** VSCs and FDS are optimized;
- **Case 3:** VSCs, FDS and ESSs are optimized but without considering the uncertainties of PVs and loads;
- **Case 4:** VSCs, FDS and ESSs are optimized, the uncertainties of PVs and loads are considered with the predicted fluctuation range $\xi_{PV} = \xi_{DG} = \xi_{L} = 15\%$, $P_{PV}^{f} = 6$ and $P_{L}^{f} = 6$;
- **Case 5:** VSCs, FDS and ESSs are optimized, the uncertainties of PVs and loads are considered with the predicted fluctuation range $\xi_{PV} = \xi_{DG} = \xi_{L} = 15\%$, $P_{PV}^{f} = 6$ and $P_{L}^{f} = 12$;

Daily operation optimization results of different cases are shown in Table II. Comparing the results of Case 1 and Case 2, it can be found that the addition of FDS decreases the network losses and improves voltage profiles significantly. Taking Node 33 as an example, the voltage profiles with and without FDS (Case 1 and Case 2) are shown in Figure 7. Compared with Case 1, voltage profiles in Case 2 get improved during 1-8h and 19-24h because FDS transmits power from terminal T1 and T2 to T3 during this period. The addition of FDS makes the system power flow mode more flexible, and can better distribute the power between AC and DC networks to increase economy and stability of the hybrid AC/DC distribution networks.

![Figure 6](image6.png)

**FIGURE 6.** Predicted profiles of PVs and loads.

**TABLE I**

| VSC/FDS | VSC1 | VSC2 | FDS |
|---------|------|------|-----|
| Working mode | $U_{C}Q$ | $PQ$ | - |
| Upper limit of $P$ (MW) | 1.5 | 1.5 | 1.0 |
| Lower limit of $P$ (MW) | -1.5 | -1.5 | -1.0 |
| Upper limit of $Q$ (MVAr) | 1.0 | 1.0 | - |
| Lower limit of $Q$ (MVAr) | -1.0 | -1.0 | - |
| $S$ (MVA) | 1.5 | 1.5 | 1.0 |
| Loss coefficient | 0.2 | 0.2 | 0.2 |

**TABLE II**

| Case  | 1     | 2     | 3     | 4     | 5     |
|-------|-------|-------|-------|-------|-------|
| Loss cost ($*10^{3}$ RMB) | 3.6886 | 2.8253 | 2.2619 | 2.1912 | 2.0979 |
| VSC FDS ESS | 0.9519 | 0.8051 | 0.7874 | 0.7863 | 0.7354 |
| ESS arbitrage ($*10^{3}$ RMB) | 0 | 0 | 8.0527 | 9.0463 | 9.4172 |
| Voltage deviation | 11.3388 | 2.1245 | 0.1009 | 0.0666 | 0.0656 |
| Objective | 9.9991 | 2.5058 | 1.1960 | 1.3509 | 1.3968 |

**FIGURE 7.** Voltage profiles of Node 33 and power flow of FDS. (a) Voltage profiles of Node 33 in Case 1 and Case 2; (b) Power flow of FDS.
The performance of the proposed robust optimization under different predicted deviations is shown in Figure 9. Comparing deterministic optimization and robust optimization, the larger the predicted deviation is, the more significant the effect of robust optimization is. Moreover, under the same predicted deviation, the case with higher uncertainty budget has higher objective value, meaning that its optimization strategies are more conservative. In practical applications, it can be considered to comprehensively select the budget of uncertainty according to the actual situation, while ensuring economy and safety.

In order to highlight the advantages of the proposed multi-objective robust optimization, different scenarios are compared between our proposed optimization and traditional single objective robust optimization [14, 15]. Firstly, whether voltage deviations are considered or not in the optimization objective influences voltage profiles obviously. As shown in Figure 11, taking Case 3 and Case 5 as examples, it can be seen that the minimum voltage is significantly lower at 1-6 and 23-24 hours when ignoring voltage deviations. Although sacrificing the network economy to some extent when considering voltage deviations, voltage profiles have been significantly improved. From the perspective of safety, especially in the hybrid AC/DC distribution networks with high penetration of renewable energy, strong volatility makes it necessary to keep voltage value in a safe and reasonable range with a certain margin.
Table III. In future distribution network electricity market, it is an important way for distributed resources, especially ESS, to get more profits by optimizing power outputs according to the electricity price.

| Objective Function | ESS Loss | ESS Cost | ESS Arbitrage | Total Cost of ESS |
|--------------------|----------|----------|---------------|------------------|
| Case 3 proposed [28] | 8.0527   | 6.9997   | 1.0530        |                  |
| Case 4 proposed [28] | 12.4188  | 4.5872   | 7.8316        |                  |
| Case 5 proposed [28] | 9.0463   | 6.1539   | 2.6924        |                  |
| Case 6 proposed [28] | 9.4172   | 6.2216   | 3.1956        |                  |
| Case 7 proposed [28] | 13.3725  | 4.1592   | 9.2133        |                  |

V. CONCLUSION

In this paper, a multi-objective optimization model of hybrid AC/DC distribution networks considering DGs and loads uncertainties is established based on a two-stage robust optimization algorithm. To optimize operation cost, voltage deviation and network losses simultaneously, a multi-objective coordinated optimization model based on various controllable means is proposed. The considered means in our paper are PVs, loads, FDS, ESS, and VSCs. Compared with the previous single objective optimization, the proposed multi-objective robust optimization takes into account both economy, flexibility and security of hybrid AC/DC distribution systems. FDS and VSCs are utilized to transfer energy between AC and DC distribution systems flexibly and to increase energy efficiency in hybrid AC/DC flexible distribution networks. Then, the proposed robust model considers the uncertainties of DGs and loads. Through solving the two-stage robust optimization model, the distribution networks are able to obtain an optimal solution under the "worst" scenario. Compared with the traditional deterministic method, the larger the actual deviations are, the more obvious the control effect of the proposed robust method is. Moreover, the proposed scheme has stronger robustness and can effectively resist the risk of fluctuation of renewable energy.

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