Comparison of all-side risk and down-side risk constrained probabilistic dispatch models considering wind power

T. Y. Ji¹, D. Y. Hong¹, M. S. Li¹ and Q. H. Wu¹

¹ School of Electric Power Engineering, South China University of Technology, Guangzhou, 510641, China.

Corresponding author: Dr. M. S. Li, E-mail: mengshili@scut.edu.cn.

Abstract. This paper, for the first time, compares the probabilistic optimal power flow (POPF) models targeting different objectives -- minimizing the expected generation cost (EGC) and all-side risk (AR) or down side risk (DR) simultaneously. In the POPF models, wind power generation is considered in dispatching, the uncertainty of which can be explained by building a model to describe the stochastic characteristic of the forecast error. The distribution of the forecast error can be obtained by sampling from the historical error time series. The POPFs are solved using group search optimizer with multiple producers (GSOMP), and a decision-making method TOPSIS is used to select the most suitable solution. Simulation studies are conducted on a modified IEEE 118-bus power system with wind farms integrated, and the results show that the EGC-DR model finds better solution compared to the EGC-AR model.

1. Introduction

With the increasing amount of wind power generation used in power systems, the uncertainty of the system dispatch ability is increasing [1]. To forecast wind power accurately is therefore extremely important because it enables better unit commitment and dispatch [2]. To consider the stochastic characteristic of wind power, probabilistic optimal power flow (POPF) is calculated to determine the dispatch strategy.

The wind power forecast models can be classified into deterministic and probabilistic forecast. Due to the deterministic prediction result is a point value, it is called point forecast as well. Efforts have been made to improve the accuracy in point forecast [3, 4]. Reference [3] proposes a mean trend detector to decompose the wind power data and a local predictor based on mathematical morphology to predict the fluctuation component accurately. Reference [4] predicts the wind speed first and then gains the wind power forecast result through the fitted speed-power curve. The result of probabilistic prediction turns out to be a series of intervals with their probabilities [5], or the possible values corresponding to their probability distributions [6]. In reference [5], the conditional copula function is suggested to describe the relationship among the historical wind power internals. In [6], the error series is forecasted by support vector machine (SVM) to correct the original SVM forecast value.

In order to improve the accuracy of the model, the probability distribution of wind power forecast error is obtained from the kernel density estimation of the time series of historical error, instead of assuming its probabilistic distribution function.

As wind power is widely and increasingly used in power systems, system operators pay more attention to the influence of wind power penetration to the safety and economy of power system operation. There are a number of studies focusing on weakening the negative impact of wind...
power forecast error on power system operation [7, 8].

Considering the stochastic characteristic of wind power, POPF is proposed and found to be an effective tool in power system operation [9]. The probabilistic optimal dispatch model usually has multi-objectives of minimizing the expected generation cost (EGC) [10] and risk simultaneously, where the risk is usually manifested by the standard deviation of generation cost in an uncertain environment [11, 12]. The risk index, standard deviation, was proposed by Markowitz to deal with the portfolio optimization problem [11]. However, in this case, the variance, which is the so-called all-side risk (AR) in this paper, treats the gain (the up-side risk) and the potential loss (the down-side risk) as equal, i.e., it weighs the gains that investors like the same to the potential losses that investors dislike [11]. As a matter of fact, investors concern more about potential loss. In this case, it is no longer appropriate to use the variance as the measure. Therefore, the down-side risk (DR), which is usually measured by the lower semi-variance, was proposed by Roy to emphasize the potential loss [13], which has now become the industry standard for risk management and is regarded as a better measure of investment risk, compared with the variance [13].

To solve the multi-objective optimization function, group search optimizer with multiple producers (GSOMP) is employed. A decision-making method, the technique for order preference similar to an ideal solution (TOPSIS) [14], is used to select the optimal solution between the EGC and risk. Moreover, the differences between the dispatch strategies aiming at EGC-AR minimization and EGC-DR minimization are analyzed.

2. The modified IEEE 118-bus test system

Fig. 1 shows a modified IEEE 118-bus test system, where a wind power farm is added to bus 7. The wind power data is obtained from the Elia database [15], which is the total wind power of Belgium collected once per hour. Other parameters remain the same as in the traditional IEEE 118-bus test system.

![Figure 1](image-url)

**Figure 1.** The modified IEEE 118-bus test system.

3. Probabilistic forecast model

The probabilistic forecast result is the summary of the point forecast result and the error sample of the point forecast error.

Firstly, the point result of the $N^{th}$ wind power is predicted using the Persistence model
\[ S_t(N) = S(N-1). \]  

Secondly, predict the wind power time series \( S(t) \), \( t=1,2,...,N-1 \) by the Persistence model, and obtain the error time series \( e(t) \): 
\[ \hat{S}_t = S(t-1) \text{ and } e(t) = S(t) - \hat{S}_t. \]  

According to probability statistics, an estimation curve of the kernel density \( o(t) \) can be obtained, from which the error samples \( E(N, i), i=1, 2, \ldots, M \), can be obtained using linear sampling, and their probability distribution follows the distribution of \( e(t) \). Finally, the samples of wind power, i.e., the possible values of wind power, can be obtained by adding the samples of errors to the point forecast value, which is expressed to be 
\[ \hat{S}(N, i) = \hat{S}_t(N) + E(N, i) = S(N-1) + E(N, i). \]  
The probability distribution of \( S(N) \) obeys the distribution of \( e(t) \).

4. Dispatch model
In the conventional OPF problem, one set of objective value is obtained on the base of the determined control values. As for the POPF, there are \( M \) sets of objective values, where \( M \) is the sampling size. Thus, statistical properties of the \( M \) sets of objective values should be considered as new objectives, such as the mean and variance, which refer to the expectation and risk of the fuel cost, respectively. However, the risk includes the all-side risk (the whole curve in Fig. 3) and the down-side risk (only the down side in Fig. 3). With different objectives, EGCA-AR and EGC-DR may have different dispatch solutions, which will be studied in the next.

4.1 Expected generation cost all-siderisk (EGC-AR) model
We aim to minimize the expected generation cost and all-siderisk simultaneously by adjusting the control variables while satisfying a set of operational constraints, which can be formulated as:
\[ \min\{E[F(x, u, P_w)], V[F(x, u, P_w)]\} \]
\[ \text{s.t.} \]
\[ h(x, u) < 0 \]  
where \( P_w \) represents the active power outputs by wind farms, \( P_w=S(N) \); \( x \) denotes the state variables including the slack bus active power \( P_{G_1} \), the load bus voltages \( V_{L} \), and generator reactive power \( Q_{G} \), which can be formulated by (5); \( u \) denotes the control variables including the active power of generators \( P_{G_i} \) (except \( P_{G_1} \) ), generator voltages \( V_{G} \), and reactive power outputs of VAR sources \( Q_{C} \), which can be formulated by (6); \( E[F(x, u, P_w)] \) and \( V(x, u, P_w) \) which can be expressed by (7) and (8), denote the expectation and all-side risk of the fuel cost, respectively.
\[ x = [P_{G_1}, V_{L}, \ldots, V_{L_{N}}, Q_{G_1}, \ldots, Q_{G_{N}}]^{T} \]  
\[ u = [P_{G_1}, \ldots, P_{G_{N}}, V_{G_1}, \ldots, V_{G_{N}}, Q_{C_1}, \ldots, Q_{C_{N}}]^{T} \]  
\[ E[F(x, u, P_w)] = \frac{1}{M} \sum_{i=1}^{M} F(x_i, u_i, P_w) \]  
\[ V[F(x, u, P_w)] = \left\{ \frac{1}{M} \sum_{i=1}^{M} [F(x_i, u_i, P_w) - E[F(x, u, P_w)]]^2 \right\}^{1/2} \]  
where \( F(x, u, P_w) \) is the total fuel cost in the system which changes with the wind power sample, and it is modeled by the following quadratic function that takes the valve point effects into consideration:
\[ F = \sum_{i=1}^{N_G} (a_i P_{G_i}^2 + b_i P_{G_i} + c_i) \]
where \( N_G \) is the number of the conventional generators in the oral IEE 118-bus power system, \( P_{G_i} \) is the active power output of generator \( i \), and \( a_i, b_i, c_i \) are fuel cost coefficients of the \( i^{th} \) generator, and \( N_D \) is the number of loads.

4.2 Expected generation cost down-side risk (EGC-DR) model

Many studies agree that all-side risk (AR) makes no distinction between gains and potential losses of investments, i.e., it regards gains that investors like as equally undesirable as potential losses that investors dislike. Indeed, investors concern more about the potential losses. Therefore, the down-side risk (DR) is put forward to replace the all-side risk (AR), and the objective functions in (4) are changed to

\[
\begin{align*}
\text{min}_x \mathbb{E}[F(x,u,P_w)] & , \mathbb{V}[F(x,u,P_w)] \\
\text{s.t.} & g(x,u,P_w) = 0 \\
& h(x,u) < 0
\end{align*}
\]

(10)

where \( \mathbb{V} \) means that only the fuel cost which is lower than the average generation cost will be taken into account, i.e., (8) is changed to

\[
\mathbb{V}[F(x,u,P_w)] = \left\{ \frac{1}{M} \sum_i \left[ F(x_i,u_i,P_w) - \mathbb{E}[F(x,u,P_w)] \right] \right\}^{\frac{1}{2}}
\]

(11)

for all \( F(x_i,u_i,P_w) < \mathbb{E}[F(x,u,P_w)] \)

where \( M \) is the total number of the fuel costs which are lower than the average generation cost.

The flowchart of Fig. 2 shows the computing procedure of the sampling and optimization.

5. Simulation results and discussion
The probability distribution of wind power is obtained according to the kernel density estimation described in Section 3 and an example is shown in Fig. 3. In this paper, the length of historical wind power data is N=1000, and the sampling number is M=50.

The EGC-AR model concerns all the possible wind power output which occurs in the whole curve from (116, 0) to (133, 0), while the EGC-DR model pays attention to the down side of the curve from the peak to (133, 0).

As stated in Section 4, the objectives that we aim to minimize are the expectation $\mathbb{E}[F(x,u,P_w)]$ and the all-side risk $V[F(x,u,P_w)]$ or down-side risk $V_-(F(x,u,P_w))$ of the fuel cost, respectively.

As for the optimization algorithm of GSOMP, the population size is set to be 100. The Pareto fronts obtained by GSOMP of the EGC-AR and EGC-DR models are shown in Fig. 4, among which 5 typical solutions whose EGC values spread from $6.55 \times 10^4$ $\$/h to $6.75 \times 10^4$ $\$/h with an interval of $0.05 \times 10^4$ $\$/h are listed in Table 1.

![Figure 3. Probability distribution of wind power.](image)

![Figure 4. Pareto solutions obtained by GSOMP.](image)

| Pareto solutions | $\text{PS}_1$ | $\text{PS}_2$ | $\text{PS}_3$ | $\text{PS}_4$ | $\text{PS}_5$ |
|------------------|--------------|--------------|--------------|--------------|--------------|
| EGC ($\times 10^4$/h) | 6.55 | 6.6 | 6.65 | 6.7 | 6.75 |
| AR | 3881 | 3704 | 3672 | 3640 | 3599 |
| DR | 2298 | 2217 | 2203 | 2178 | 2163 |

It can be seen from Fig. 4 and Table 1 that with the EGC grows, the AR and DR are declining. The solutions to the two Pareto fronts which have the minimum EGCs and highest risks demonstrate that the dispatch solution cannot adapt to the uncertain wind power environment well enough. On the other hand, the solutions with the minimum risks indicate that the corresponding dispatch solution is adaptable, even robust, to the uncertain environment. However, their values of EGC are high. Therefore, to determine the most suitable solution, the fuzzy TOPSIS is engaged. For the EGC-AR model, the best solution is $(6.6404 \times 10^4$/h, 3672), and the best solution for the EGC-DR model is $(6.5879 \times 10^4$/h, 2216). Obviously, the final expected generation cost of the EGC-DR model is lower, which demonstrates the effectiveness of considering the down-side risk only.

6. Conclusion

This paper pays attention to the POPF problem in a power system with wind power integrated. The uncertainty of wind power is explained through describing the probabilistic distribution function of forecast error with historical data. Besides, two POPF models with different objectives -- minimizing the expected generation cost (EGC) with the all-side risk (AR) or with the down-side risk (DR), are
simulated and compared. The POPFs on a modified IEEE 118-bus power system with wind farm integrated are solved using the multi-objective optimization algorithm of GSOMP. The results have shown that the EGC-DR model obtains a lower expected generation cost and lower risk than the EGC-AR model.

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