On the Physical Mechanisms that Cause the Full Load Instability in Francis Turbines

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Abstract. Today, hydropower receives increased attention for the safe integration of volatile renewable energies as it is able to stabilize the electrical grid. For that reason, turbines are more often operated at off-design conditions with the drawback that undesirable flow phenomena like the full load instability can occur. To assess their hazard potential, it is of great importance to understand those phenomena. Even though the physical mechanisms behind the full load instability have been analyzed in the past, they are not yet fully understood. The goal of this study is to close this gap and to develop a complete understanding of the full load instability. Two-phase simulations of a Francis turbine at model scale are performed for a full load operating point that becomes unstable when the cavitation number falls below a certain threshold. The unsteady simulations are performed using the Zwart cavitation model and the SST turbulence model with applied curvature correction. For simulations at different cavitation numbers it is investigated how characteristic quantities like swirl number or cavitation volume differ between stable and unstable conditions. This allows getting a deeper insight into characteristic changes of the flow for the transition from stable to unstable conditions. It can be identified that the interaction between cavitation on the runner blades and the cavitating draft tube vortex plays a key role in the development process of the full load instability. All in all, a complete explanation for the development of the full load instability can be given. The new findings of this study allow proposing an adapted 1D model, which considers the occurrence of blade cavitation.

1. Introduction

The increasing share of renewable energies requires technologies like Francis turbines that are able to stabilize the electricity grid. However, this requires that the turbines are often operated in off-design conditions with the disadvantage that complex flow phenomena like the full load instability can occur. Even though there have been experimental (e.g. [1, 2]) and numerical (e.g. [3, 4]) research activities on the full load instability in the past, the phenomenon is not yet fully understood. For a safe turbine operation it is, however, crucial to understand this behavior. With this profound comprehension it is then possible to develop suitable models and have a more accurate prediction of this phenomenon. Based on this, measures can be derived that eventually can prevent the occurrence of the full load instability.

2. Theory on the Full Load Instability

At certain conditions, the full load instability can develop in hydraulic turbines, which shows characteristics of a self-oscillation. Müller linked the development of the instability to a reduced swirl at the runner outlet that results from the occurrence of cavitation on the runner blades...
but pointed out that this theory “should be verified or disproved” [1]. Research from Dörfler highlights that the full load instability can only develop due to a swirl transport delay that results from the fact that swirl is transported with the flow, while pressure propagates with speed of sound [5, 6]. This agrees well to theory on self-oscillations as it requires a time delay [7].

2.1. Standard 1D Model

Today, the range of stable turbine operation at full load is often investigated using 1D models. These models are typically based on the two key quantities mass flow gain factor $\chi$ and cavitation compliance $C_c$ that were first introduced by Brennen and Acosta [8]. According to Alligné et al. [9] the continuity equation for an oscillating cavitation volume $V_{c,DT}$ in a control volume in the draft tube cone can be described by the following equation:

$$Q_1 - Q_2 = -\frac{dV_{c,DT}}{dt} = \epsilon \frac{dU_1}{dt} + \chi \frac{dQ_1}{dt} + C_c \frac{dh}{dt} \quad (1)$$

There $Q_1$ and $Q_2$ denote the discharge at the inlet and outlet of the draft tube, $\epsilon$ the rotational speed gain factor, $U_1$ the runner outlet peripheral velocity and $h$ the piezometric head state variable. Typically, the first term on the right side is neglected as the rotational speed is constant in most cases.

2.2. Cavitation Noise

When vapor bubbles collapse massive noise production can be observed. It has been derived in literature (e.g. [10, 11]) that the radiated acoustic pressure $p_a$ is proportional to the second time derivative of the vapor volume:

$$p_a \sim \frac{d^2V_c}{dt^2} \quad (2)$$

This correlation has been experimentally confirmed on a cavitating propeller for surge instability [12]. Based on equation 2 it is found that due to the acceleration of a cavitation volume acoustic pressure is radiated. This change of pressure can have an impact on another cavitation region, which is some distance away from the cavitation volume that experiences the acceleration.

3. Setup

3.1. Geometry and Operating Point

For the present study, a Francis turbine with a specific speed $n_q \approx 43 \text{ min}^{-1}$ is investigated. It consists of 20 stay and guide vanes and 16 runner blades. For the investigated full load operating point the discharge factor is $Q_{ED} = 0.260$ and the speed factor is $n_{ED} = 0.288$. Compared to the best efficiency point the discharge is increased by 30 %.

For the selected operating point the full load instability develops when the cavitation number $\sigma$ falls below a certain threshold. Measurement results for the transition from stable to unstable conditions and back to stable show that for $\sigma < 0.155$ pressure fluctuations are significantly increased (see figure 1), which is characteristic for the full load instability.

In the scope of this study, simulation results are evaluated at different points and planes that are displayed in figure 2. The four planes in the draft tube cone are located around 0.05, 0.39, 0.73 and 1.08 times the external runner outlet diameter below the runner exit. For the investigation of the relative flow angle $\beta$ at the runner exit, the monitor point DTin08 is set downstream of the runner blades. This point is located at midspan. Furthermore, an averaging ribbon (see figure 2 right) is defined, which allows to assess the correlation between relative flow angle and the occurrence of blade cavitation. In this averaging ribbon an average cavitation volume is determined.
3.2. Numerical Setup

The simulation setup that is described here has been validated against experimental data in previous studies [13, 14]. The simulation domain ranges from the inlet of the spiral case to the draft tube outlet. All two-phase simulations are performed with the homogeneous multi-phase modeling approach and the Zwart model [15] with standard model constants. For turbulence modeling the SST model with curvature correction is applied. A high resolution scheme is used for spatial discretization and a second order backward Euler scheme for temporal discretization.

A constant mass flow is set at the inlet of the spiral case. This is contrary to the findings from a previous study [16] but has been extensively discussed in [14]. At the outlet the static pressure is prescribed so that the following values for the cavitation number are obtained: 0.15, 0.19, 0.21, 0.22, 0.23 and 0.25. The applied mesh consists of 14 million cells and has an average $y^+$-value of approximately 50.

All simulations are performed unsteady with a time step that corresponds to 0.96° of one runner revolution. For investigations of the full load instability an appropriate number of coefficient loops is crucial [14, 16]. In this study seven coefficient loops are set. Time-averaged quantities are calculated from 20 runner revolutions for stable conditions. For unstable conditions the averaging period consists of 12 instability cycles. This means that 22.2 runner revolutions for $\sigma = 0.15$, 21.1 runner revolutions for $\sigma = 0.19$ and 20.0 runner revolutions for $\sigma = 0.21$.

4. Results

The occurrence of the full load instability is accompanied by strong pressure fluctuations (see figure 1) and oscillations of the cavitation volume in the draft tube [2]. In figure 3 the time-averaged cavitation volume in the draft tube and the standard deviation are presented for the investigated cavitation numbers. It can be observed that the transition from stable to unstable conditions is in the range $0.21 < \sigma < 0.22$ as between these cavitation numbers a sudden increase of the standard deviation occurs. Furthermore, it is conspicuous that within stable conditions the averaged cavitation volume in the draft tube increases with decreasing $\sigma$, which is not the case for unstable conditions. In this study, the expected behavior that the cavitation volume increases with decreasing $\sigma$ is called effect 1. The fact that for unstable conditions the cavitation volume is not following effect 1 indicates that another mechanism is important for the development of the full load instability.

Based on the findings from Müller [1], the time-averaged cavitation volume in the runner and
the swirl number $S$ in the different planes in the draft tube cone are investigated (see figure 4). It can be observed that the cavitation volume in the runner is close to zero for $\sigma = 0.25$ and increases in approximately a parabolic shape with decreasing cavitation number. The highest swirl numbers occur for stable conditions. While for plane 1 and 2, the swirl number already decreases with decreasing $\sigma$ for stable conditions, in plane 3 and 4 the swirl number remains constant within the stable regime. For unstable conditions a clear correlation can be found: The swirl number decreases with increasing blade cavitation. By analyzing the standard deviation of the swirl number in plane 2, it can be observed that the swirl number shows significantly higher variations in the unstable regime. As displayed in figure 5, only a small cavitation volume can be observed on the runner blades for $\sigma = 0.22$. Contrary to that for $\sigma = 0.21$ at some time interval of one instability cycle a cavitation region is forming on the runner blade close to the trailing edge. This region ranges approximately from midspan to the shroud. Due to the fact that this connected region is not present for $\sigma = 0.22$ it can be expected that the occurrence of cavitation at this location is linked to the instability mechanisms.

![Figure 3. Time-averaged cavitation volume in the draft tube and standard deviation for different cavitation numbers.](image)

![Figure 4. Time-averaged swirl number in the evaluation planes and cavitation volume in the runner for different cavitation numbers.](image)

![Figure 5. Visualization of the cavitation volume in the runner and upper draft tube with two isosurfaces of the vapor volume fraction at different times of one instability cycle for $\sigma = 0.21$ and one snapshot for $\sigma = 0.22$.](image)

The connection between the swirl number and the occurrence of blade cavitation can be
visualized by velocity triangles at the runner exit for a full load operating point (see figure 6). Neglecting effects due to reduced deflection, the relative flow angle $\beta$ equals the blade angle at the runner outlet for non-cavitating conditions. Caused by the occurrence of blade cavitation, the flow at the runner exit is slightly deflected and consequently the relative flow angle is increased ($\beta_c > \beta$). Assuming that mass transfer is small compared to the discharge through the turbine, the meridional component of the absolute velocity remains constant ($c_{m,c} = c_m$). The consequence is that the circumferential component of the absolute velocity decreases ($c_{u,c} < c_u$). As a result, the swirl number reduces:

$$S = \frac{\int_0^R r^2 c_u c_m \, dr}{\int_0^R r c_m^2 \, dr} > \frac{\int_0^R r^2 c_{u,c} c_{m,c} \, dr}{\int_0^R r c_{m,c}^2 \, dr} = S_c \quad (3)$$

A reduced swirl results in a weaker vortex and as a consequence a smaller cavitation volume in the draft tube can be expected. This mechanism, which is called effect 2 in the following, acts opposite to effect 1 and explains why decreasing the cavitation number does not necessarily lead to a reduced cavitation volume in the draft tube (see figure 3).

![Velocity triangles for full load operating point. Left: Non-cavitating conditions. Right: Cavitating conditions. [14]](image)

Due to the oscillation of the cavitation volume on the runner blades, it can be expected that the relative flow angle varies over time. This can be confirmed by the simulation results that are displayed in figure 7. For better clarity variations of $\beta$ due to rotor stator interaction are cancelled out by using the moving average. It can be observed that for stable conditions ($\sigma = 0.22$) both, relative flow angle and cavitation volume in the corresponding averaging ribbon, show only minor fluctuations. Contrary to that, for unstable conditions ($\sigma = 0.21$) these quantities are facing significant oscillations. Furthermore, a clear correlation can be identified between the occurrence of cavitation in the averaging ribbon and relative flow angle.

Based on equation 3 it can be expected that the swirl number is facing oscillations when the blade cavitation volume varies over time. In figure 8, this is exemplarily displayed for $\sigma = 0.21$. The phase averaged results show that the minimum cavitation volume in the runner occurs at 0° and the maximum around 200°. Based on equation 3 it can be expected that for large cavitation volumes in the runner a small swirl number is present and vice versa. This correlation is met with a small phase shift of around 25° for plane 1. However, significantly higher phase shifts are present for the other planes, where the phase shift increases with increasing distance to the runner. This highlights that the swirl is transported with flow, which is in agreement with the findings from Dörfler et al. [5].

So far it has been identified that the reduction of cavitation number results in two effects that work contrarily. On the one hand, due to effect 1 pressure reduction should result in a larger cavitation volume in the draft tube. On the other hand, a lower cavitation number results in a
larger cavitation volume in the runner, which leads to a decreased swirl number. Consequently, a smaller cavitation volume in the draft tube can be expected due to effect 2.

The superposition of these two counteracting effects does not inevitably result in the development of an instability. The key for the development of the full load instability is that there is a time delay between effect 1 and 2. While pressure propagates with speed of sound, which is infinite for incompressible simulations, swirl variations need some time to travel with the flow into the draft tube. This results in the time delay between effect 1 and 2, which is necessary for a self-oscillation to develop [7].

Figure 9 displays the cavitation volume in the draft tube as a function of the swirl number for the different planes. For stable conditions (shades of blue), the swirl number shows only minor fluctuations within 20 runner revolutions and also the cavitation volume oscillation is significantly lower compared to unstable conditions (shades of red). The presented phase averaged results for unstable conditions show much higher variations of the swirl number and the cavitation volume. Furthermore, the results of twelve instability cycles are shown exemplarily in plane 1 for $\sigma = 0.15$ (grey line). It can be observed that the individual cycles differ from each other but in general all cycles show a similar behavior.
Figure 9. Cavitation volume in the draft tube as function of swirl number in the four evaluation planes for different cavitation numbers. The results for unstable conditions are presented phase averaged. Exemplarily, the full results of 12 instability cycles are displayed in plane 1 for $\sigma = 0.15$ (grey line). [14]

The above stated time delay between effect 1 and 2 can be identified by finding the representative plane in the draft tube where the cavitation volume of the vortex rope grows with increasing swirl number. For the three investigated cavitation numbers this location can be found to coincide with reasonable agreement with plane 3. A closer look at the results of this plane indicates some dependency on the cavitation number. While the results for $\sigma = 0.19$ and 0.21 can with reasonable agreement be approximated by a line, this is partly violated for $\sigma = 0.15$ in the range $0.27 < S < 0.305$. In that range, the cavitation volume remains constant even though the swirl number changes. This might be caused by the difference in the average cavitation volume of the vortex rope (see figure 3). For $\sigma = 0.15$, a smaller and shorter cavitation volume is present, which justifies a change in representative location.

Due to a phase shift, the correlation that the cavitation volume in the draft tube increases with increasing swirl number is not fulfilled for the other planes. For plane 1 the results even show that the cavitation volume increases with decreasing swirl number. The necessary phase shift that needs to be applied can be derived from figure 8, for example by determining the difference of the swirl number maximum between the different planes. As the representative location has the correct phase, the phase shift is $0^\circ$ for plane 3 and the other phase shifts have to be determined related to plane 3. In figure 10 the results are presented when the described procedure is applied. It can be observed that including the phase shift all planes show the expected correlation that the cavitation volume of the vortex rope increases with increasing swirl number.

All the presented findings still cannot completely describe the development of the full load instability. A small pressure reduction could still result in the following behavior: Caused by the pressure reduction, the cavitation volumes in runner and draft tube increase. After the swirl reduction that results from the increased cavitation volume in the runner arrives in the draft tube, the current explanations would allow the occurrence of a new equilibrium of cavitation volume in the draft tube if no significant pressure oscillations occur. To better understand the
As described in section 2.2, the change of cavitation volume causes the emission of radiated acoustic pressure. To be more precise: The radiated pressure is a result of the acceleration of cavitation volume $d^2 V_c / dt^2$ (see equation 2), which is called effect 3 in the following. For a pressure reduction this correlation can explain the feedback to the size of cavitation volume in the runner. When the decreased swirl has arrived in the draft tube cone, the cavitation volume tries to adapt to its new equilibrium from the superposition of effect 1 and 2. However, the accompanied acceleration of cavitation volume causes a pressure radiation, which results in reduced cavitation volume in the runner. Consequently, due to effect 2 swirl number changes, which again affects the cavity size of the vortex rope. Due to the time delay the self-oscillation develops and equilibrium can never be reached.

5. Discussion on the Cause of the Full Load Instability

The proposed mechanism for the development of the full load instability differs from some explanations in state of the art literature. There, it is argued that variations of the discharge at the draft tube inlet (mass flow gain factor) are the cause of the instability. Indeed, the swirl reduction due to cavitation that is visualized in figure 6 could also be caused by either a reduction of discharge or increased rotational speed. All explanations have in common that they result in a changed swirl that arrives with some time delay in the draft tube. However, only for the mechanism that is proposed in this study it is guaranteed that the interaction of cavitation in runner and draft tube results in the excitation of oscillations due to effect 3. For possible variations of discharge or rotational speed, these variations might have a phase that excites instability but it might also be possible that they have a phase that dampens oscillations.

Furthermore, for discharge variations that seem to be more likely than rotational speed variations the whole mechanism would be caused by a system instability. Consequently, the occurrence of the instability would highly depend on the upstream pipe system of a hydropower plant or test rig because the traveling time of the pressure wave depends on the pipe length. Contrary to this, explaining the cause of the full load instability with the occurrence of blade cavitation, there is always a direct feedback due to effect 3, which is expected to be almost independent from the system. For the investigated Francis turbine the full load instability can also occur on prototype [17]. For that reason a system dependency cannot be found between...
model and prototype. This supports the findings from this study. A further argument for the proposed mechanism is that for suppressed cavitation in the runner, by locally setting the model constant for evaporation in the cavitation model to zero, the full load instability does not develop [14]. This clearly indicates that the full load instability develops due to the interaction of the cavitation regions in runner and draft tube, which is in agreement with the findings from Müller [1].

6. Revised 1D Model

Based on the discussion in section 5, the present standard 1D model (see section 2.1) of the cavitation volume oscillation caused by the full load instability that is based on the parameters mass flow gain factor and cavitation compliance needs some adaption. To include all relevant effects of the full load instability the following equation is proposed:

$$Q_1 - Q_2 = -\frac{dV_{c,DT}}{dt} = \chi \frac{dQ_1}{dt} + \epsilon \frac{dU_1}{dt} + \psi \frac{dh}{dt} + C_c \frac{dh}{dt}$$  \hspace{1cm} (4)

The cavitation gain factor $\psi$ that considers the connection between cavitation volume in the draft tube and the changed flow angle due to blade cavitation is defined as follows:

$$\psi = -\frac{\partial V_{*,DT}^*}{\partial h}$$  \hspace{1cm} (5)

The special character of the cavitation gain factor can be found in the fact that even though the change of cavitation volume in the draft tube is caused by a change of pressure, it results from the changed swirl. It is important to notice that although cavitation compliance and cavitation gain factor have the same definition, they describe different phenomena and for that reason have different values (marked by the * in equation 5). Furthermore, it should be noted that in the 1D model a time delay has to be included for the terms of the different gain factors as proposed by Dörfler et al. [5].

Cavitation compliance and the different gain factors can be determined with 3D simulations. However, it is important that these parameters have to be identified separately. For the determination of mass flow gain factor and rotational speed gain factor a constant pressure level at the outlet can be set and the discharge or rotational speed, respectively, has to be varied. It is crucial to separate the effects of cavitation compliance and cavitation gain factor as both are resulting from pressure variations. This separation can be obtained by suppressing cavitation in the runner for the determination of cavitation compliance. Likewise, cavitation gain factor can be determined by suppressing cavitation in the draft tube. Then, a pseudo cavitation volume in the draft tube can be obtained by calculating the volume from the region where pressure falls below vapor pressure. The cavitation region can be suppressed locally by setting the model constant for evaporation in the cavitation model in this region to zero.

7. Conclusion and Outlook

A detailed description of the physical mechanisms that cause the full load instability in a Francis turbine is given by means of numerical simulations. Two-phase simulations were performed, due to the necessity to capture the interaction between cavitation in runner and draft tube. The assessment of the full load instability was made by comparing one operating point at six different cavitation numbers, three at stable conditions and three at unstable conditions. For stable conditions, the cavitation volume in the runner is of negligible size and fluctuations of pressure and cavitation volume in the draft tube are small. At instability onset, a small reduction of the cavitation number results in a significant increase of pressure fluctuations and oscillations of the cavitation volume in the draft tube. A cavitation region forms at the trailing edge of the runner blades that ranges approximately from midspan to the shroud.
All in all, three main mechanisms could be identified that result in the development of the full load instability. Effect 1 stands for the correlation that the cavitation volume increases when the pressure is reduced. This effect acts instantaneously for incompressible simulations, as pressure is traveling with speed of sound. The response of the cavitation volume in the draft tube due to changes of the cavitation volume in the runner is described by effect 2. For a pressure reduction, the increased cavitation volume on the trailing edge of the runner results in a changed velocity triangle that leads to a reduced swirl number. After the changed swirl has travelled into the draft tube with the flow, the cavitation volume in the draft tube reduces because of effect 2. Due to the different transport velocities of the phenomena, a time delay occurs between the counteracting effects 1 and 2. Finally, effect 3 represents the correlation that the second time derivative of the cavitation volume is proportional to the radiated acoustic pressure. This results in a feedback of cavitation volume oscillations in the draft tube on changes of the cavitation volume in the runner. The findings highlight that the full load instability is caused by the interaction of cavitation in the runner and the draft tube. With the new knowledge an adapted 1D model is developed that considers the effects resulting from blade cavitation. This new model needs some discussion in the community in terms of applicability.

For future work it is planned to implement a 1D-3D coupling, which allows investigating effects that result from discharge variations caused by the instability. Furthermore, it would be desirable to investigate whether the findings of this study are transferable to other Francis turbines.

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