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ABSTRACT

By using the Hubbard operator Green’s function method, the spin-resolved transport properties of a quantum dot coupled to metallic leads and side-coupled to a topological superconductor wire hosting Majorana bound states (MBSs) are studied theoretically. Compared with the second quantization representation, the method can lead us to an analytical result for the retarded Green’s function with finite $U$. The spin-related current and conductance are discussed. In the case of zero Coulomb interaction and on-resonance, the MBS’s $1/2$ signature is recovered, and furthermore, there exists a $1/2$ negative differential spin conductance. In the case of infinite Coulomb interaction, the $1/2$ signature does not survive due to the Coulomb correlation reducing the current and conductance. Also due to this correlation, the MBS-induced symmetry of conductance peaks around zero energy is destroyed. In addition to this, we find that there are two MBS-induced negative differential spin conductance peaks. Theoretically, our work is supplementary and contrastive to the mainstream second quantization method, and these spin-resolved results may be observed in future experiments.

I. INTRODUCTION

Majorana fermions, with antiparticles being themselves, have been searched in high-energy physics for a long time. Recently, searches for Majorana fermions in condensed matter physics have attracted much attention, due to the fact that they have non-Abelian statistics and are promising in quantum computation. The predicted Majorana bound states (MBSs) or Majorana fermion zero modes correspond to the Majorana fermions in high-energy physics. They are coherent superpositions of hole and electron excitations of zero energy, which can be realized at the two ends of a topological superconductor wire.

With regard to MBSs’ application, the readout and manipulation of themselves in quantum dot-Majorana hybrid systems have been extensively studied and many interesting results are found. For example, the presence of MBSs leads to a quantized zero-bias conductance $G = 2e^2/h$, when the dot is on resonance and symmetrically coupled to the probing leads. This zero-bias conductance is usually robust. It appears not only in the topologically nontrivial regime but also in the topologically trivial regime where scattering sources and impurities are involved. For another example, a spatially separated pair of MBSs could induce nonlocal current cross correlation when they are coupled to mesoscopic circuits. Researchers are also interested in MBSs’ thermoelectric properties, and for instance, it is found that the sign of the thermopower can change due to the presence of Majorana zero-energy modes.

In this work, we extend those studies to the spin-resolved case. By using the Hubbard operator Green’s function method, the spin-resolved transport properties of a quantum dot coupled to metallic leads and side-coupled to a topological superconductor wire hosting Majorana zero-energy modes are studied theoretically. Compared with the second quantization representation, the method allows us to obtain an analytical result for the retarded Green’s function with finite $U$. When Coulomb interaction is considered, it usually has a significant influence on the transport properties of the quantum dot system. This motivates us to ask a question, namely, how robust are those results, for instance, the $1/2$ signature of the MBSs in the quantum dot-Majorana hybrid systems, in the presence of the Coulomb...
interaction? In order to study the MBSC's effect on the transport properties, we first analyze the spectral function and calculate the conductance spectrum of two spin components in the noninteracting case. Then, the Coulomb interaction effect on the conductance spectrum is investigated.

II. THE ANDERSON IMPURITY MODEL WITH A MAJORANA-FERMION ZERO MODE

For the description of a spinful quantum dot coupled to a Majorana-fermion zero mode, we use the Anderson impurity model. The Hamiltonian can be written as

\[ H = H_{\text{leads}} + H_F + \sum_{\sigma} t_{\sigma} d_{\sigma}^\dagger n_{\bar{\sigma}} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow} + i \frac{eM}{2} \gamma_{\uparrow} \gamma_{\downarrow} + \lambda (d_{\uparrow} - d_{\downarrow})^\dagger \gamma_1, \]

where \( H_{\text{leads}} = \sum_{k} \sum_{\alpha=L,R} \epsilon_{k\alpha} c_{k\alpha}^\dagger c_{k\alpha} \) describes the left and right metallic leads. \( \gamma_1 \) is the electronic creation (annihilation) operator with momentum \( k \), spin \( \sigma \), and energy \( \epsilon_{k\sigma} \) in the electrodes \( \alpha = L, R \). \( H_F = \sum_{i} \left( t \epsilon_{k\sigma} c_{i\sigma}^\dagger c_{i\bar{\sigma}} + H.c. \right) \) describes the coupling between the dot and the leads. \( t \) describes the tunneling amplitude between the quantum dot and the leads. The third and fourth terms in Eq. (1) describe the spinful quantum dot. The operators \( d_{\uparrow} \) \( d_{\downarrow} \) denote annihilation (creation) of a quantum dot electron with spin \( \sigma \) and gate tunable energy level \( \epsilon_0 \). \( U \) is Coulomb interaction that accounts for the electronic correlation. \( \gamma_{\uparrow} = f_{\uparrow} + f_{\downarrow}, \gamma_{\downarrow} = i(f_{\uparrow} - f_{\downarrow}) \). These Majorana zero modes can equivalently be described in terms of fermionic operators: \( \gamma_{\uparrow} = f_{\uparrow} + f_{\downarrow}, \gamma_{\downarrow} = i(f_{\uparrow} - f_{\downarrow}) \). These Majorana operators fulfill the algebraic relation \( \{\gamma_{\uparrow}, \gamma_{\downarrow}\} = 2\delta_{\theta} \).

The last two terms in Eq. (1) become

\[ H_{\text{MBS}} = \epsilon_{\text{MBS}} (f_{\uparrow} f_{\downarrow} - \frac{1}{2}) + \lambda d_{\uparrow}^\dagger (f_{\uparrow} + f_{\downarrow}) + \lambda (f_{\uparrow} + f_{\downarrow})^\dagger d_{\downarrow}. \]

The Hamiltonian of the dot coupled to topological superconductor (in the case of \( \Gamma = 0 \)) is easily diagonalized.\(^{21}\) In the basis states \( \{|00\}, \{|01\} = f_{\uparrow} f_{\downarrow}|00\rangle, \{|10\} = f_{\downarrow}^\dagger f_{\uparrow}|00\rangle, \{|11\} = d_{\uparrow}^\dagger d_{\downarrow}^\dagger|00\rangle, \{|20\} = d_{\uparrow}^\dagger|01\rangle, \{|21\} = d_{\uparrow}^\dagger f_{\downarrow}|00\rangle \), the Hamiltonian is not diagonal. One can find that these pairs of states ((|00\rangle, |11\rangle), (|01\rangle, |10\rangle), (|20\rangle, |1\rangle) and (|21\rangle, |\uparrow\rangle)) couple each other. The respective eigenenergies and unnormalized eigenstates are given in Table I.

| Eigenenergies | Eigenstates |
|--------------|-------------|
| \( \epsilon_1 = \frac{1}{2} (\epsilon_0 - \delta_\uparrow) \) | \( |\psi_1\rangle = (\epsilon_0 + \epsilon_{\text{MBS}} + \delta_\downarrow)|00\rangle + 2\lambda |1\rangle \) |
| \( \epsilon_2 = \frac{1}{2} (\epsilon_0 + \delta_\uparrow) \) | \( |\psi_2\rangle = (\epsilon_0 + \epsilon_{\text{MBS}} - \delta_\downarrow)|00\rangle + 2\lambda |1\rangle \) |
| \( \epsilon_3 = \frac{1}{2} (\epsilon_0 - \delta_\downarrow) \) | \( |\psi_3\rangle = (\epsilon_0 - \epsilon_{\text{MBS}} + \delta_\uparrow)|00\rangle + 2\lambda |1\rangle \) |
| \( \epsilon_4 = \frac{1}{2} (\epsilon_0 + \delta_\downarrow) \) | \( |\psi_4\rangle = (\epsilon_0 - \epsilon_{\text{MBS}} - \delta_\uparrow)|01\rangle + 2\lambda |1\rangle \) |
| \( \epsilon_5 = \frac{1}{2} (\epsilon_0 + \delta_\downarrow) \) | \( |\psi_5\rangle = (\epsilon_0 + \epsilon_{\text{MBS}} + \delta_\downarrow)|01\rangle + 2\lambda |1\rangle \) |
| \( \epsilon_6 = \frac{1}{2} (\epsilon_0 + \delta_\downarrow) \) | \( |\psi_6\rangle = (\epsilon_0 + \epsilon_{\text{MBS}} + \delta_\downarrow)|20\rangle + 2\lambda |1\rangle \) |
| \( \epsilon_7 = \frac{1}{2} (\epsilon_0 + \delta_\downarrow) \) | \( |\psi_7\rangle = (\epsilon_0 + \epsilon_{\text{MBS}} + \delta_\downarrow)|21\rangle + 2\lambda |1\rangle \) |
| \( \epsilon_8 = \frac{1}{2} (\epsilon_0 + \delta_\downarrow) \) | \( |\psi_8\rangle = (\epsilon_0 + \epsilon_{\text{MBS}} + \delta_\downarrow)|21\rangle + 2\lambda |1\rangle \) |

Correspondingly, the quantum dot’s on-site retarded Green’s function is expressed as \( G_0 (\omega) \equiv \langle \langle d_{\uparrow}^\dagger d_{\downarrow}^\dagger \rangle \rangle = \langle \langle \langle X^{0\uparrow} | d_{\uparrow}^\dagger \rangle \rangle + \langle \langle X^{0\downarrow} | d_{\downarrow}^\dagger \rangle \rangle \rangle \). The standard equation of motion\(^{15,35,36}\) of the Green’s function in energy space reads \( \omega (\langle A | B \rangle) = \langle \langle A, B | H | B \rangle \rangle \). We calculate the spin-up Green’s function first: \( \langle \langle d_{\uparrow}^\dagger | d_{\uparrow} \rangle \rangle = \langle \langle (X^{0\uparrow} | d_{\uparrow}^\dagger \rangle \rangle + \langle \langle (X^{1\uparrow} | d_{\uparrow}^\dagger \rangle \rangle \rangle \). The equation of motion of \( \langle \langle (X^{0\uparrow} | d_{\uparrow}^\dagger \rangle \rangle \) yields

\[ (\omega - \epsilon_0) \langle \langle (X^{0\uparrow} | d_{\uparrow}^\dagger \rangle \rangle = P_0 + P_1 - \lambda \langle \langle (X^{0\uparrow} + X^{1\uparrow}) f^\dagger d_{\uparrow}^\dagger \rangle \rangle + \sum_{k} t_{\uparrow} \langle \langle (X^{0\uparrow} + X^{1\uparrow}) c_{k\uparrow} \rangle \rangle + \sum_{k} t_{\downarrow} \langle \langle (X^{0\downarrow} + X^{1\downarrow}) c_{k\downarrow} \rangle \rangle, \]

where \( P_0 = \langle \langle (X^{0\uparrow} d_{\uparrow}^\dagger \rangle \rangle \) and \( P_1 = \langle \langle (X^{1\uparrow} d_{\uparrow}^\dagger \rangle \rangle \) are statistical averages of states \( |0\rangle \) and \( |1\rangle \), respectively. In order to close the infinite series of the equations of motion, we should use a proper approximation. The simplest approximation \( \sum_{k} t_{\uparrow} \langle \langle (A c_{k\uparrow} d_{\uparrow}^\dagger \rangle \rangle = \sum_{k} (A d_{\uparrow} c_{k\uparrow}^\dagger) \rangle \rangle \). The self-energy \( \Sigma = \sum_{k} (A c_{k\uparrow} d_{\uparrow}^\dagger) \rangle = -\pi \Omega \), where the symmetric coupling has been used \( \Omega = \Gamma = \Gamma \). This approximation takes into account sequential tunneling processes but neglects the spin-flip processes and Kondo resonance. In the above equation, \( \langle \langle (X^{0\uparrow} c_{k\uparrow}^\dagger \rangle \rangle \) is a spin-flip term and \( \langle \langle (X^{1\uparrow} c_{k\uparrow}^\dagger \rangle \rangle \) is a term involving the two-particle tunneling process; thus, they are neglected. Using this approximation, Eq. (4) is closed as
The simultaneous solution of Eqs. (9) and (10) gives us an analytical result for the spinless Green’s function,

\[ G_0'(\omega) = \frac{1}{\omega - \epsilon_0 - \Sigma_0 - 2\lambda^2 K(\omega)\left[1 + 2\lambda^2 K(\omega)\right]} \]  

To compare with the spinless case, in this section, we consider the case of uncorrelated quantum dot, i.e., \( U = 0 \) in Eq. (1). In this case, the spin-up and spin-down electrons do not affect each other. As in Ref. 11, one may also use the equation of motion method in the second quantization representation or equivalently start from Eqs. (11) and (12) with \( U = 0 \) to obtain an exact expression for the spin-up Green’s function,

\[ G_l'(\omega) = \frac{1}{\omega - \epsilon_0 - \Sigma_0 - 2\lambda^2 K(\omega)\left[1 + 2\lambda^2 K(\omega)\right]} \]  

Note that the pre-\( \lambda^2 \) factor "2" originates from the definition of Majorana operators. The spin-down Green’s function reads simply as follows:

\[ G_l'(\omega) = \frac{1}{\omega - \epsilon_0 - \Sigma_0 - 2\lambda^2 K(\omega)\left[1 + 2\lambda^2 K(\omega)\right]} \]  

We discuss the spectroscopic properties by analyzing the spectral function \( A(\omega) = -1\text{Im}G_0'(\omega) \). Excited states formally represent the poles of the Green’s function. To get some analytical results, we consider the case of infinitesimally weak coupling to the metallic electrodes, \( \Gamma \to 0^+ \). Under such a condition, the excited states become strictly resonant, i.e., they represent the quasiparticles of an infinite lifetime. For example, Eq. (16) yields the following spectral function:

\[ A_1(\omega) = u(\omega - \epsilon_0) \]  

which means it reaches unity at \( \epsilon_0 \) (i.e., the conductance reaches the unity of \( e^2/h \)). In Eq. (17), the unit function is defined as

\[ u(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \]  

After some algebra, Eq. (15) is factored as

\[ G_l'(\omega) = \frac{1}{\omega - \epsilon_0 - \Sigma_0 - 2\lambda^2 K(\omega)} \left[1 + 2\lambda^2 K(\omega)\right] \]  

with the following meaning of the symbols \( \tilde{\omega} \) and \( \tilde{\epsilon}_0 \):

\[ \tilde{\omega} = \omega - 2\lambda^2 K(\omega), \]  

\[ \tilde{\epsilon}_0 = \sqrt{\epsilon_0^2 + 4\lambda^2 K^2(\omega)}. \]  

Equation (19) has the same mathematical structure with Eq. (11) in Ref. 39. We argue that the pairing effect will generate this structure, either the Cooper pair or the Majorana-fermion pair. Equation (19) yields the following spectral function:

\[ A_1(\omega) = W_+ u(\tilde{\omega}) + W_- u(\tilde{\epsilon}_0), \]  

\[ (\omega - \epsilon_0 - \Sigma_0)((X^0|d_i^\dagger|) = P_0 + P_1 - \lambda((X^0 + X^\dagger)f|d_i^\dagger|) \]

\[ - \lambda((X^0 + X^\dagger)f|d_i^\dagger|). \]  

(5)

The last four terms are correlation functions involving three degrees of freedom; thus, they are neglected. Then, Eq. (6) is closed as

\[ (\omega - \epsilon_0 - \Sigma_0)\langle (X^0|d_i^\dagger|) = -\lambda((X^0|d_i^\dagger|) + \lambda((X^0|d_i^\dagger|) \]

\[ + \sum_{ka}(c_{ka}^\dagger X^0|d_i^\dagger|) \]

\[ - \sum_{ka}(X^0 c_{ka}|d_i^\dagger|) \]

\[ + \sum_{ka}(X^2 c_{ka}|d_i^\dagger|) \]

\[ - \sum_{ka}(c_{ka}^\dagger X^2|d_i^\dagger|). \]  

(6)

The simultaneous solution of Eqs. (9) and (10) gives us an analytical result for the spinless Green’s function,

\[ G_0(\omega) = \frac{1}{\omega - \epsilon_0 - \Sigma_0 - 2\lambda^2 K(\omega)\left[1 + 2\lambda^2 K(\omega)\right]} \]
with \( W_+ = (1 + \varepsilon_0/\tilde{E}_0)/2 \) and \( W_- = (1 - \varepsilon_0/\tilde{E}_0)/2 \) being their spectral weights. The excited energies (quasiparticle energies) represent the solutions of the following equation:

\[
\tilde{\omega} \pm \tilde{E}_0 = 0. \tag{23}
\]

After some algebra, we get the following solutions:

\[
\omega_{1,2,3,4} = \pm \frac{1}{2} (\delta_+ \pm \delta_-), \tag{24}
\]

where \( \delta_+ = \sqrt{(\varepsilon_0 \pm \varepsilon_M)^2 + 4\lambda^2} \). For different \( \varepsilon_0 \) and \( \varepsilon_M \), there are four cases: (a) \( \varepsilon_0 = 0 \) and \( \varepsilon_M = 0 \), (b) \( \varepsilon_0 = 0 \) and \( \varepsilon_M \neq 0 \), (c) \( \varepsilon_0 \neq 0 \) and \( \varepsilon_M = 0 \), and (d) \( \varepsilon_0 \neq 0 \) and \( \varepsilon_M \neq 0 \). For the first case, Eq. (24) reduces to \( \omega = 0, \pm 2\lambda \). The spectral weights are reduced to \( W_+ = W_- = 1/2 \), which means that in this special case, Majorana bound state's "1/2 signature" holds for \( \omega = 0, \pm 2\lambda \). For the second case, Eq. (24) reduces to \( \omega = 0, \pm \sqrt{\varepsilon_M^2 + 4\lambda^2} \). This three-peak structure of the spectral function corresponds to Fig. 2 in Ref. 11. It is tricky for the peak heights of the spectral function in this case. For \( \varepsilon_0 = 0 \), the spectral weights are also reduced to \( W_+ = W_- = 1/2 \). However, the solution of \( \omega = 0 \) is now populated in a two unit function, i.e., Eq. (15) is now factored as

\[
G_0(\omega) = \frac{1}{2} \left[ \frac{1}{\omega + \Omega} + \frac{1}{\omega - \Omega} \right], \tag{25}
\]

and the spectral function in Eq. (22) can be expressed as

\[
A_\sigma(\omega) = \frac{1}{2} [u(\omega - 0) + u(\omega + 0) + u(\omega - \delta) + u(\omega + \delta)], \tag{26}
\]

where \( \delta = \sqrt{\varepsilon_M^2 + 4\lambda^2} \). In general, the spectral function shows a four-peak structure, which is shown in Fig. 1.

Excited energies (poles of the Green’s function) usually relate with the difference values of eigenenergies. Thus, we compare Eq. (24) with Table I when \( U = 0 \). For \( U \neq 0 \), Table I is reduced to Table II. From Table II, one can see that the transport between states

| \( \varepsilon_1 \) | \( \varepsilon_2 \) | \( \varepsilon_3 \) | \( \varepsilon_4 \) | \( \varepsilon_5 \) | \( \varepsilon_6 \) | \( \varepsilon_7 \) | \( \varepsilon_8 \) |
|---|---|---|---|---|---|---|---|
| \( \frac{1}{2}(\varepsilon_0 - \delta_+) \) | \( \frac{1}{2}(\varepsilon_0 + \delta_+) \) | \( \frac{1}{2}(\varepsilon_0 + \delta_-) \) | \( \frac{1}{2}(\varepsilon_0 - \delta_-) \) | \( \frac{1}{2}(3\varepsilon_0 + \delta_+) \) | \( \frac{1}{2}(3\varepsilon_0 - \delta_-) \) | \( \frac{1}{2}(\varepsilon_0 + \delta_+) \) | \( \frac{1}{2}(\varepsilon_0 + \delta_-) \) |
| \( |\Psi_1\rangle = (\varepsilon_0 + \varepsilon_M + \delta_+)(00) + 2\lambda|\uparrow1\rangle \) | \( |\Psi_2\rangle = (\varepsilon_0 + \varepsilon_M - \delta_+)(00) + 2\lambda|\uparrow1\rangle \) | \( |\Psi_3\rangle = (\varepsilon_0 + \varepsilon_M - \delta_-)(01) + 2\lambda|\uparrow10\rangle \) | \( |\Psi_4\rangle = (\varepsilon_0 - \varepsilon_M - \delta_-)(01) + 2\lambda|\uparrow10\rangle \) | \( |\Psi_5\rangle = (\varepsilon_0 - \varepsilon_M - \delta_-)(20) + 2\lambda|\downarrow1\rangle \) | \( |\Psi_6\rangle = (\varepsilon_0 - \varepsilon_M + \delta_-)(20) + 2\lambda|\downarrow1\rangle \) | \( |\Psi_7\rangle = (\varepsilon_0 + \varepsilon_M + \delta_-)(21) + 2\lambda|\downarrow10\rangle \) | \( |\Psi_8\rangle = (\varepsilon_0 + \varepsilon_M + \delta_+)(21) + 2\lambda|\downarrow10\rangle \) |

Any experimental verification of the excited states is possible only indirectly by measuring the differential conductance of the tunneling current. In general, the spin-resolved current is expressed in the Landauer-type form

\[
I_\sigma = -\frac{e^2}{h} \int \left[ f_\sigma(\omega) - f_R(\omega) \right] \text{Im}G_\sigma(\omega) d\omega = \frac{e^2}{h} \int \left[ f_\sigma(\omega) - f_R(\omega) \right] A_\sigma(\omega) d\omega. \tag{27}
\]
In the above equation, \( f_s(\omega) = \{\exp[(\omega−\mu_0)/k_B T]+1\}^{-1} \) stands for the Fermi distribution function with \( \alpha = L, R \). Accordingly, the charge current is defined as \( I_c = I_1 + I_2 \), and the spin current is \( I_s = I_1 − I_2 \). Furthermore, the spin conductance is defined as \( G_s = G_1 − G_2 \). We set \( \mu_L = V, \mu_R = \text{constant} \), and then in the limit of weak coupling (\( T \to 0 \)) and low temperature (\( T \to 0 \)), the spin-resolved conductance \( G_s = e^2/h \partial V = (e^2/h)A_0(V) \). For instance, Eq. (16) yields simply \( G_1 = (e^2/h)u(V−\varepsilon_0) \), i.e., when the bias of the left lead crosses \( \varepsilon_0 \), the conductance reaches its maximum \( e^2/h \).

It is worth mentioning that in the case of on-resonance (\( \varepsilon_0 = 0 \) and \( \varepsilon_M = 0 \)), there is a 1/2 negative differential spin conductance: when \( V = 0, G_1 = e^2/2h, G_1 = e^2/h \) and hence \( G_s = −e^2/2h \). Thus, in Fig. 2, we plot the spin-related current and conductance. It is also worth pointing out that this correlation or competition between different spin components usually induce negative conductance, which can be a signature of MBS.

In addition to this, we have also plotted the probabilities of states (\( P_0, P_1, P_1, \) and \( P_2 \) ) and dot occupations (\( \mu_L = \mu_0 = 0 \) and \( \varepsilon_M = \text{constant} \)). These average values are related to the Green’s function as follows:\(^{34,35}\)

\[
\langle AB \rangle = -\frac{1}{\pi} \int d\omega \frac{f(\omega) + f_s(\omega)}{2} \text{Im}(A^0(B^0))' .
\]

For example, \( P_1 = (−1/\pi) \int d\omega f(\omega) \text{Im}(X^0(d_k^0)) \), where \( f(\omega) = [f(\omega) + f_{\text{th}}(\omega)])/2 \) is defined for abbreviation. In the case of \( U = 0 \), from Eqs. (11)–(13), we find the spin-dependent states

\[
C_1 P_0 + (C_1 − 1) P_1 = 0 , \quad (29)
\]

\[
C_1 P_0 + (C_1 − 1) P_1 = 0 , \quad (30)
\]

\[
C_1 P_1 + (C_1 − 1) P_2 = 0 , \quad (31)
\]

\[
P_0 + P_1 + P_1 + P_2 = 1 , \quad (32)
\]

where \( C_1 = (1/\pi) \int d\omega f(\omega) \text{Im}(\omega − \varepsilon_0 − \Sigma_0 − 2\lambda^2 I(1 + 2\lambda^2 K))^{-1} \) and \( C_1 = (1/\pi) \int d\omega f(\omega) \text{Im}(\omega − \varepsilon_0)^{-1} \). Figures 2(a) and 2(b) show the probabilities of states and occupations. Because we set \( \mu_0 = −2 \), when \( \mu_0 = V \) increases from \(-2 \) to \(-1 \), the first MBS-involved excited state (or transport channel) is reached and spin-down electrons begin to transport. In the region of \(-1 < V < 0 \), this situation holds. Thus, in this region, the occupations \( n_r = 0 \) and \( n_\nu = 0 \), and there is only spin-up current, as shown in Fig. 2(c). When \( V = 0 \), a spin-down channel enters into the bias window and hence spin-down electrons begin to transport. In the mean time, the second MBS-involved excited state also enters into the bias window [there are three of them, as shown in Fig. 1(a)]. In the region of \( 0 < V < 1 \), both the spin-up and spin-down electrons contribute to the current, but the probabilities of states and occupations are spin-resolved. In the region of \( V > 1 \), all three MBS-involved excited states enter into the bias window and the spin-up and spin-down currents become spin unresolved. In Fig. 2(e), the spin-up conductance \( G_1 \) is exactly the same as in Fig. 1(a), and in Fig. 2(f), one can see the 1/2 negative differential spin conductance, as mentioned above.

**FIG. 2.** The spin-resolved transport physics in the case of on-resonance for \( U = 0 \). (a) Probabilities of states, (b) dot occupations, (c) spin-resolved currents \( I_r, I_\nu \), (d) charge current \( I_c \) and spin current \( I_s \), (e) spin-resolved conductances \( G_1, G_\nu \), and (f) charge conductance \( G_c \) and spin conductance \( G_s \), as a function of bias voltage \( V \). The parameters used are \( \mu_0 \) = \( \alpha \mu_0 = −2, \varepsilon_0 = 0, \mu_u = 0, \lambda = 0.5, k_B T = 0.001 \), and \( \Gamma = 0.03 \). The parameters are measured in units of \( \text{meV} \).
When Coulomb interaction is considered, it induces correlation effects. In Secs. III B and III C, we will discuss two cases: single occupation (i.e., \( U \rightarrow \infty \)) and double occupation (finite \( U \)).

**B. \( U \rightarrow \infty \)**

The study of correlation effects can be built upon the above analysis. When \( U \rightarrow \infty \), the double occupation is forbidden and Eq. (15) is modified as

\[
G^r_\uparrow (\omega) = \frac{P_0 + P_1}{\omega - \varepsilon_0 - \Sigma_0 - 2\lambda^2 K(\omega)} \left[ 1 + 2\lambda^2 K(\omega) \right].
\]

(33)

Accordingly, the spin-up spectral function now reads

\[
A^r_\uparrow (\omega) = (P_0 + P_1) \left[ W_+ u(\bar{\omega} - \bar{\varepsilon}_0) + W_- u(\bar{\omega} + \bar{\varepsilon}_0) \right].
\]

(34)

In the case of on-resonance (\( \varepsilon_0 = 0 \) and \( \varepsilon_M = 0 \)), \( A^r_\uparrow (\omega) \) takes a simpler form

\[
A^r_\uparrow (\omega) = (P_0 + P_1) \frac{1}{2} \left[ u(\omega) + u(\omega - 2\lambda) + u(\omega + 2\lambda) \right].
\]

(35)

Thus, the correlation (i.e., the prefactor \( P_0 + P_1 \)) will decide if the MBS’s 1/2 signature survives. The results are plotted in Fig. 3. Figure 3(a) plots the probabilities of states. In the case of single occupation, \( P_0, P_1, \) and \( P_1 \) share the total probability and \( n_s = P_0 \). As shown in Fig. 3(a), the states \( P_1 \) and \( P_1 \) are still spin-resolved. In the region of \(-1 < V < 0\), there is a positive spin current, and in the region of \( 0 < V < 1 \), there is a negative spin current, as shown in Fig. 3(c). Compared with Fig. 2, the height of charge current is reduced because of Coulomb correlation. In addition, due to this correlation, in Fig. 3(d), the main result is that the MBS’s 1/2 signature at \( V = 0 \) does not survive. The three conductance peaks are orderly lowered from left to right, with the leftmost peak height equalling \( e^2/2h \).

**C. Finite \( U \)**

In the following, let us discuss the case of finite Coulomb interaction. Similarly, the analysis of the spectral function is given first. From Eqs. (11) and (12), the spin-up retarded Green’s function is factored as

\[
G^r_\uparrow (\omega) = \frac{P_0 + P_1}{\omega - \varepsilon_0} \left[ \frac{1 + \frac{\varepsilon_0}{U}}{\omega + i\Gamma - \bar{\varepsilon}_0} + \frac{1 - \frac{\varepsilon_0}{U}}{\omega + i\Gamma + \bar{\varepsilon}_0} \right] + \frac{P_1 + P_2}{\omega - \varepsilon_0} \left[ \frac{1 + \frac{\varepsilon_0 + U}{U}}{\omega + i\Gamma - \bar{\varepsilon}_0} + \frac{1 - \frac{\varepsilon_0 + U}{U}}{\omega + i\Gamma + \bar{\varepsilon}_0} \right],
\]

(36)

where \( \bar{\varepsilon}_0 = \sqrt{\varepsilon_0^2 + 4\lambda^4 K^2(\omega)} \). The above equation yields the spectral function

\[
A^r_\uparrow (\omega) = \frac{P_0 + P_1}{2} \left[ W_+ u(\bar{\omega} - \bar{\varepsilon}_0) + W_- u(\bar{\omega} + \bar{\varepsilon}_0) \right] + \frac{P_1 + P_2}{2} \left[ W^+ u(\bar{\omega} - \bar{\varepsilon}_0) + W^- u(\bar{\omega} + \bar{\varepsilon}_0) \right],
\]

(37)

where the spectral weights are \( W^\pm = |1 \pm (\varepsilon_0 + U)/\bar{\varepsilon}_0|/2 \). The above spectral function yields the solutions

\[
\omega = \frac{1}{2} (\delta^+ \pm \delta^-), \pm \frac{1}{2} (\delta^U \pm \delta^L),
\]

(38)

**FIG. 3.** The spin-resolved transport physics in the case of on-resonance for \( U \rightarrow \infty \). (a) Probabilities of states, (b) spin-resolved currents \( I^r_{\uparrow, \downarrow} \), (c) charge current \( I^\uparrow \), spin resolved conductances \( G^{\uparrow, \downarrow}_{\uparrow, \downarrow} \), and (e) charge conductance \( G^\uparrow \) and spin conductance \( G^\downarrow \) as a function of bias voltage \( V \). The parameters used are the same as in Fig. 2, which are measured in units of meV.
where one can refer to Table I for the definition of \( \delta_u \) and \( \delta_u^{\pm} \). Thus, in general, there are eight peaks in the spectral function, while in the case of on-resonance, there are five peaks. The corresponding transport physical quantities are plotted in Fig. 4. In Fig. 4, the probabilities of states are also given. When \( U \) is considered, Eq. (31) is modified as

\[
C_{\uparrow}U P_{\downarrow} + (C_{\uparrow}U - 1) P_{\downarrow} = 0,
\]

where \( C_{\uparrow}U = -(1/\pi) \int d\omega f(\omega) \text{Im}(\omega - \epsilon_0 - U - \Sigma_0 - 2\lambda^2 K(1 + 2\lambda^2 \bar{K}))^{-1} \). The other equations \{i.e., Eqs. (29), (30), and (32)\} stay the same. As discussed above, there are five peaks in the spectral function in the case of on-resonance. Their peaks are located at \( 0, \pm 2\lambda, \pm \delta_U \), where \( \delta_U = \sqrt{U^2 + 4\lambda^2} \). However, in conductance \( G_{\uparrow} \), as shown in Fig. 4(e), there are only four peaks. The peak at \( V = -\delta_U \) does not appear. This is also because of the effect of Coulomb correlation. Specifically, when \( V < -1, P_0 = 1, P_{\uparrow} = P_{\downarrow} = P_2 = 0 \), as shown in Fig. 4(a). Then, the numerator of Eq. (12) equals to zero. Although there is a pole in the denominator of Eq. (12), it will not appear in conductance. Here, it is worth mentioning that many correlation effects could induce the asymmetric conductance around the zero energy peak, such as Coulomb interaction or nonlinear band curvature for the hole and electron populations in the presence of exchange field shift in energies.\(^{16-20}\)

When \( U \) is finite, spin-up current shows four steps [see Fig. 4(c)], and there are two negative differential spin conductance peaks in Fig. 4(f).

**IV. CONCLUSION**

In summary, we have studied the spin-resolved transport properties of a quantum dot coupled to two metallic leads and side-coupled to a Majorana zero-energy mode. By using the Hubbard operator Green’s function method, an analytical result for the retarded Green’s function with finite \( U \) is obtained. Based on this result, the spectral function and the conductance spectrum of two spin components in the noninteracting case are analyzed first. Then, the Coulomb interaction effect on the conductance spectrum is discussed. In the case of zero Coulomb repulsion and on-resonance, the MBSs’ 1/2 signature is recovered. When Coulomb interaction is considered, the MBSs’ 1/2 signature does not survive and the MBS-induced symmetry of conductance peaks around zero energy is also destroyed. Furthermore, we found that there are MBS-induced negative differential spin conductance peaks.

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