Band Gap Characteristics of Piezoelectric/Piezomagnetic Layered Periodic Structures with Magnetolectric Interlayer

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Abstract. Coupled propagation of elastic and electromagnetic waves in a piezoelectric/piezomagnetic layered periodic structure with magneto-electro-elastic interlayer is studied using the transfer matrix method. We find that magneto-electro-elastic interlayer can change the elastic wave band gap, electromagnetic wave phase velocity and phonon-polariton band gap position in piezoelectric / piezomagnetic composite structure.

1. Introduction
Piezoelectric/piezomagnetic layered composite structure has the ability of force-electricity-magnetism transformation, which shows a broad application prospect in practical applications. Generally, the parameters of piezoelectric materials and piezomagnetic materials differ greatly, which makes the bond of piezoelectric/piezomagnetic composite devices prone to fatigue damage during use. A magneto-electro-elastic interlayer is inserted at the junction of the piezomagnetic layer to avoid interface damage [1].

Coupled propagation of elastic and electromagnetic waves in one-dimensional piezoelectric/piezomagnetic layered periodic structures with magneto-electro-elastic interlayer is studied under the condition of dynamic Maxwell equations. Based on Bloch theorem, the dispersion relation between the coupled elastic wave and the electromagnetic wave can be given, and the effects of the thickness and the volume fraction are analyzed and discussed.

2. Problem description
As shown in Fig. 1, the thickness of the unit cell is 1mm, and the composite structure is polarized along the Oz direction. Coupling elastic wave and electromagnetic wave propagating along the z axis in the y-z symmetric plane.
Figure 1. Piezoelectric/piezomagnetic composite structure with magneto-electro-elastic interlayer

The constitutive equation of magnetoelectric elastic material can be expressed as

$$\sigma = c \frac{\partial u}{\partial z} - eE - hH, D = e \frac{\partial u}{\partial z} + eE + aH, B = h \frac{\partial u}{\partial z} + aE + \mu H$$

(1)

Where \(c, e, \) and \(\mu\) are elastic constant tensor, dielectric constant vector and permeability vector, respectively. \(e, h\) and \(a\) are piezoelectric coefficient, piezomagnetic coefficient and magnetoelectric coefficient tensor, respectively. \(u, \sigma, E, H, D\) and \(B\) are elastic displacement vector, stress tensor, electric field vector, magnetic field vector, electric displacement vector and magnetic induction vector, respectively. The constitutive equations of piezoelectric and piezomagnetic materials can be expressed by substituting \(h=0=a\) or \(e=0=a\) in equation (1) respectively.

The elastic wave and electromagnetic wave couple with Maxwell wave equation of electrodynamics and Newton equilibrium equation

$$\nabla \cdot \sigma = \rho \frac{\partial^2 u}{\partial t^2}, \nabla \times H = \frac{\partial D}{\partial t}, \nabla \times E = -\frac{\partial B}{\partial t}, \nabla \cdot B = 0, \nabla \cdot D = 0$$

(2)

Where \(\rho\) is the material density, and \(t\) express the time, combine Eq. (1) and (2).

$$u(z) = m_{11}a(z) + m_{13}a_2(z), \sigma(z) = m_{22}b_1(z) + m_{24}b_2(z)$$

$$H(z) = m_{31}a_1(z) + m_{32}b_1(z) = m_{33}a_2(z) + m_{34}b_2(z)$$

$$E(z) = m_{41}a_1(z) + m_{43}a_2(z) + m_{44}b_2(z)$$

(3)

The state vector of a single cell is a matrix.

$$V = MA$$

(4)

Where the elements of matrix \(M\) and \(A\) as follows

$$m_{11} = p^2 - \frac{\omega^2}{c^2}, m_{13} = q^2 - \frac{\omega^2}{c^2}, m_{22} = c_{44}p^3 - \frac{p\omega^2}{c^2}, m_{24} = c_{44}q^3 - q\frac{\omega^2}{c^2},$$

$$m_{31} = p^2e\omega, m_{32} = p(d\epsilon - e\alpha)\omega^2, m_{33} = q^2e\omega, m_{34} = q(d\epsilon - e\alpha)\omega^2,$$

$$m_{41} = p^2d\omega, m_{42} = p(e\mu - d\alpha)\omega^2, m_{43} = q^2d\omega, m_{44} = q(e\mu - d\alpha)\omega^2,$$
\[ m_{12} = m_{14} = m_{21} = m_{23} = 0 \quad \rho = o_0 \sqrt{\frac{\varepsilon_2 - \varepsilon_2^2 - 4}{2c_0c_1}}, \quad q = o_0 \sqrt{\frac{\varepsilon_2 + \varepsilon_2^2 - 4}{2c_0c_1}}, \quad \varepsilon = \frac{c_1}{c_0}, \quad \varepsilon_0 = \sqrt{\frac{c_{44}}{\rho}}, \]

\[ c_0 = \sqrt{\frac{c_{44}}{\rho}}, \quad c_i = \frac{1}{\sqrt{\varepsilon\mu - \alpha^2}}, \quad \varepsilon_{44} = c_{44}(1 + r), \quad r = \frac{h^2 + c^3 + \mu - 2h\varepsilon}{c_{44}(\varepsilon\mu - \alpha^2)}. \]

\[ a_i(z) = C_i \exp(ipz) + C_2 \exp(-ipz), \quad b_1(z) = i[C_i \exp(ipz) - C_2 \exp(-ipz)] \]

\[ a_s(z) = C_3 \exp(ipz) + C_4 \exp(-ipz), \quad b_2(z) = i[C_3 \exp(ipz) - C_4 \exp(-ipz)]. \]

C1, C2, C3 and C4 are constants.

The left and right state vectors of material layer satisfy the following relations

\[ V_n^+ = T_n V_n^- \]  

(5)

Where "-" and "+" represent the left and right sides of the sublayer, \( T_n \) is transfer matrix

\[ T_n = MH(M)^{-1} \]  

(6)

The total transfer matrix of single cell can be expressed as

\[ T_{cell} = T_{n+1} T_n^{mee} T_n^{mee} T_n^e \]  

(7)

In the transfer matrix, the superscript "e", "m" and "mee" denote piezoelectric, piezomagnetic and magnetoelectric shells respectively. According to the Bloch theorem

\[ |T_{cell} - \lambda I| = 0 \]  

(8)

Where \( \lambda = \exp(ikl) \), obtained according to formula (8) and (9).

\[ F(\lambda, \omega) = \lambda^4 + f(\omega)\lambda^3 + g(\omega)\lambda^2 + f(\omega)\lambda + 1 = 0 \]  

(9)

The dispersion relation between SH elastic wave and electromagnetic wave can be obtained by substituting \( \lambda + 1/\lambda = e^{ikl} + e^{-ikl} = 2\cos(kl) \) to equation (9).

\[ \cos(kl) = \frac{1}{4}(-f + \sqrt{f^2 - 4g + 8}), \quad \cos(kl) = \frac{1}{4}(-f - \sqrt{f^2 - 4g + 8}) \]  

(10)

### 3. Numerical results

#### Table 1. Material coefficients [2].

| Material     | \( c_{11} \) | \( c_{01} \) | \( \mu_{11} \) | \( e_{15} \) | \( h_{15} \) | \( \rho \) |
|--------------|--------------|--------------|----------------|-------------|-------------|-----------|
| PZT-7A       | 2.51         | 40.7         | 5              | 9.2         | 0           | 7.6       |
| PZT-5A       | 2.11         | 81.1         | 5              | 12.3        | 0           | 7.75      |
| PZT-6B       | 3.55         | 36           | 5              | 4.6         | 0           | 7.55      |
| CoFe_{2}O_{4} | 4.53         | 0.8          | 157            | 0           | 550         | 5.3       |
\[ P_{\text{mee}} = P_e V_e + P_m V_m \]

\( P_e \) is the magnetoelectric material parameter, \( P_m \) is piezoelectric material parameter, \( P_m \) is piezomagnetic material parameter, \( V_e \) and \( V_m \) are the volume fraction of piezoelectric material and piezomagnetic material in magnetoelectric material, \( \alpha_1 = 4 \alpha_{\text{max}} V_e V_m \), \( \alpha_{\text{max}} = 5 \times 10^{-12} \) Ns/VC.

From Fig. 2a, we can know that when the thickness of magnetoelectro-elastic interlayer increases, the change of elastic wave band gap is small at low frequency, and the position of elastic wave band gap move to high frequency, obviously. The width of some band gaps is affected, and the fourth, sixth and eighth band is obviously changed. Fig. 2b show the dispersion curves of SH elastic wave and electromagnetic wave, which confirms the coupling between elastic and electromagnetic wave. When the thickness of magnetoelectroelastic interlayer increases, the band gap and location of phonon-polariton change. The magnetoelectroelastic layer changes the physical parameters of the composite structure. Affects the position of elastic wave band gap and phonon-polariton position.

![Figure 2a. Influence of magnetoelectric interlayer thickness on band structure](image1)

![Figure 2b. The zoomed profile show near the center of the Brillouin zone](image2)

![Figure 3a. Influence of volume fraction of piezoelectric material on band structure](image3)

![Figure 3b. The zoomed profile show near the center of the Brillouin zone](image4)
It can be seen from Fig.3a that the elastic band gap of SH wave is obviously affected by the volume fraction of piezoelectric material. When the volume fraction of piezoelectric material increases from 0 to 1, the position of elastic band gap move to low frequency, and part of the band gap width is affected, but the variation of each band gap width is not the same. As we can see Fig.3b, the coupling between SH elastic wave and electromagnetic wave (phonon-polariton) changes, and the band gap position of phonon-polariton moves horizontally. It is found that when the piezoelectric volume fraction is 0 or 1, the phase velocity of electromagnetic wave is smaller. When the piezoelectric volume fraction is 0.5, the phase velocity of electromagnetic wave is obviously higher than that of the former two cases. It is precisely because of the change of phase velocity of electromagnetic wave that the position of polariton changes horizontally when the elastic wave band gap changes little. The results show that when the piezoelectric volume fraction increases, the piezoelectric parameter increases, and the piezomagnetic parameter decreases, the material parameter difference of each component changes, and the periodic structure changes, so that the structure dispersion relation changes.

4. Conclusion
The effects of magneto-electro-elastic interlayer on coupled elastic and electromagnetic waves in piezoelectric/piezomagnetic layered periodic structures are discussed. The results show that the elastic wave band gap and phonon-polariton band gap can be changed by the magneto-electro-elastic interlayer. When the thickness of the magneto-electro-elastic interlayer increases, the center frequency of the elastic wave band gap increases, the electromagnetic wave phase velocity increases, and the phonon-polariton position changes; when the volume fraction of the piezoelectric material in the magneto-electro-elastic interlayer increases, the position of the elastic wave band gap move to the low frequency direction, the width of part of the band gap changes, and the phonon-polariton band gap and position changes.

Acknowledgments
This work was financially supported by the Army Armored Forces Academy Innovation Fund (Project No. 2016CJ07).

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