Supplementary Materials for

**In situ stiffness manipulation using elegant curved origami**

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The PDF file includes:

- Sections S1 to S5
- Figs. S1 to S12
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Other Supplementary Material for this manuscript includes the following:

(available at advances.sciencemag.org/cgi/content/full/6/47/eabe2000/DC1)

- Movies S1 to S8
Section S1. Modelling of Reaction Force of a Quadrilateral Rigid Origami Cell upon Compression

This model is based on the theory of rigid origami, where the strain energy is only stored in linear elastic creases. Fig. S1A shows the folding pattern of a quadrilateral rigid origami unit cell. The angle between crease 1 and crease 2 is $\beta$. The summed length of the creases 2 and 4 is $2c$. The length of crease 1 (or crease 3) is $b$ and can be expresses by $\beta$ and $c$ as

$$b = \frac{c}{\sin(\beta)}$$

By compressing the origami through the top and bottom vertices, the origami will deform into a configuration as shown in Fig. S1B. The spatial geometry of the origami is very similar to the Miura pattern, as shown in Fig. S1C, and the same angle $\beta$ is used. The geometry of Miura pattern during folding has been studied somewhere else$^1$, which will be used to study the relationship between force and displacement. The same notations$^1$ are adopted here in Fig. S1C. The projection angle between two ridges is $\phi$. The dihedral angles $\alpha_1$ at crease 4 and $\alpha_2$ at crease 3 can be expressed as functions of $\phi$ as (30):

$$\alpha_1(\phi) = \cos^{-1}\left[1 - 2\frac{\sin^2\left(\frac{\phi}{2}\right)}{\sin^2(\beta)}\right]$$

$$\alpha_2(\phi) = \cos^{-1}\left[1 - 2\cot^2(\beta) \cdot \tan^2\left(\frac{\phi}{2}\right)\right]$$

$l$ is the distance between the top and bottom vertices and can characterize the deformation with $l = 2c$ for undeformed state. Angle $\phi$ can be expressed as a function of $l$,

$$\phi(l) = 2\sin^{-1}\left(\frac{l}{2b}\right)$$
Given the crease modulus $H$ (unit: N), the strain energy of the unit cell $U$ can be expressed as

$$U = \frac{1}{2} \left[ 2cH \left( \pi - \alpha_1 \right)^2 + \frac{1}{2} 2bH \left( \pi - \alpha_2 \right)^2 \right] = H \left[ c \left( \pi - \alpha_1 \right)^2 + b \left( \pi - \alpha_2 \right)^2 \right]$$  \hspace{1cm} (5)

The reaction force $F$ can be calculated as a derivative of $U$ with respect to $l$,

$$F = \frac{\partial U}{\partial l} = 2H \left[ \left( \alpha_1 - \pi \right) \frac{\partial \alpha_1}{\partial \phi} \sin \beta + \left( \alpha_2 - \pi \right) \frac{\partial \alpha_2}{\partial \phi} \right] \frac{1}{\sqrt{1 - \left( \frac{l}{2b} \right)^2}},$$  \hspace{1cm} (6)

The force $F$ can be normalized as

$$\overline{F}(l) = \frac{F}{H} = 2 \left[ \left( \alpha_1 - \pi \right) \frac{\partial \alpha_1}{\partial \phi} \sin \beta + \left( \alpha_2 - \pi \right) \frac{\partial \alpha_2}{\partial \phi} \right] \frac{1}{\sqrt{1 - \left( \frac{l}{2b} \right)^2}},$$  \hspace{1cm} (7)

The normalized reaction force due to creases 1 and 3 $\overline{F}_{1,3}$ is expressed as the second part of Eq. (7),

$$\overline{F}_{1,3}(l) = \frac{2 \left( \alpha_2 - \pi \right) \frac{\partial \alpha_2}{\partial \phi}}{\sqrt{1 - \left( \frac{l}{2b} \right)^2}}$$  \hspace{1cm} (8)

The normalized reaction force due to creases 2 and 4 $\overline{F}_{2,4}$ is expressed as the first part of Eq. (7),

$$\overline{F}_{2,4}(l) = \frac{2 \left( \alpha_1 - \pi \right) \frac{\partial \alpha_1}{\partial \phi} \sin \beta}{\sqrt{1 - \left( \frac{l}{2b} \right)^2}}$$  \hspace{1cm} (9)

Define the collapsing ratio as $1 - \frac{l}{2c}$. Figures S1D, E, and F show the normalized forces versus the collapsing ratio.
Section S2. Finite Element Simulations

We model the creases as linear elastic perfectly plastic materials in finite element simulations using ABAQUS. Using this model, the reaction moment per unit crease length as a function of rotation angle is shown in Fig. S2A. The slope of the curve in the elastic range (unit: $N \cdot m / m = N$) is the crease modulus $H$. The yield rotation angle is set as 1 rad, which is verified by the following experiments as shown in Fig. S2B, in which a square-shaped origami with a straight crease in the middle is used, with the right part of the origami is constrained and the left part is free to rotate about the straight crease. A tweezer is used to rotate the left part of origami to a specific angle (i.e., 130° in Fig. S2C) and then released (i.e., 80° in Fig. S2D). We then used the linear elastic and perfect plastic model to theoretically predict the relaxed angle. For example, as shown in Fig. S2E, for a given rotation angle 130°, the predicted relaxed angle is 73°, which agrees reasonably well with the experiments (Fig. S2D). The comparison between the experiment results and the predictions for different angles is shown in Fig. S2F, where the experimental and the predicted results show a same trend. Therefore, a linear elastic perfect plastic model with yield angle of 1 rad is verified to be reasonable for the simulation.

As shown in Fig. S3, the curved origami is undeformed and has zero energy at the initial state (step 0). The curved origami is bent for 250° in step 1, and released in step 2. In step 3, the curved origami is compressed under a displacement load $u = 0.2$. In step 4, the displacement load is gradually released to zero. In the following steps, the displacement load is applied and removed periodically as steps 3 and 4. The plastic dissipation energy $U_p$ increased during the 1st and 3rd step, and keeps constant after the step 4, which means no dissipation exists during the repeated loading and unloading steps of curved origami afterwards.
Section S3. Switching the Activation of Creases of Curved Origami

We used finite element simulation to show that the crease switching of curved origami is a multi-stable process, and the stable states exist when only one crease is activated. In the initial state, the curved origami model in Fig. 1D is constrained at $\alpha = 30^\circ$ ($\alpha$ is defined in Fig. 1E), i.e., only crease $\textcircled{1}$ activated. Then a rotational load is applied to activate crease $\textcircled{2}$ and deactivate crease $\textcircled{1}$ by decreasing the rotation angle of surface A with respect to x-axis. Contour plots of the rotation angle along x-axis is used to identify the state of curved origami. As shown in Fig. S6A, when the rotation angle of surface A reaches $0^\circ$, the curved origami is switched to creased $\textcircled{2}$. The strain energy of the curved origami increases and then decreases during this process, indicating it is a bi-stable process and the stable states are that either crease $\textcircled{1}$ or $\textcircled{2}$ is activated. In Fig. S6B, similar simulation is conducted for switching the curved origami from crease $\textcircled{2}$ to $\textcircled{3}$, and a similar bi-stable behavior is observed. Thus, the activation of a single curved crease is ensured.
Section S4. Normalization of the Parameters

Three basic parameters, i.e., unit cell length $a$, thickness $t$, and panel modulus $E$, are used for normalization. The displacement is normalized as $\frac{u}{a}$. Curvature is normalized as $k = \kappa a$. The energy, force, and stiffness are normalized by a bending deformation of panel. The bending energy of the panel for normalization $U_{\text{norm}}$ has the unit of $U_{\text{norm}} \sim EI_x \kappa_{\text{panel}}^2 a \sim Et^3 a \left(\frac{1}{a}\right)^2 a = Et^3$, where the second moment of cross-sectional area about $x$ axis $I_x \sim t^3 a$, and curvature of panel $\kappa_{\text{panel}} \sim \frac{1}{a}$. Therefore, the energy of the curved origami unit cell can be normalized as $U = \frac{U}{Et^3}$. The reaction force due to bending has the unit of $F_{\text{norm}} \sim \frac{U_{\text{norm}}}{a} \sim \frac{Et^3}{a}$. Thus, the reaction force of the curved origami unit cell $F$ is normalized as $\frac{F}{Et^3}$. Since crease modulus has the same unit as force, it is also normalized as $H = \frac{Fa}{Et^3}$. Stress has the unit of $\sigma_{\text{norm}} \sim \frac{F_{\text{norm}}}{at} = \frac{Et^2}{a^2}$, so normalized stress $\bar{\sigma}$ is expressed as $\bar{\sigma} = \frac{\sigma a^2}{Et^3}$. 
Section S5. Mechanical Characterizations

Mechanical Characterization of Plastic Film

The width of the tested plastic film (Fig. S8A) is \( w = 10 \text{ mm} \), and the thickness \( t \) is 0.125 mm. Therefore, the Young’s modulus of the plastic film can be estimated as

\[
E = \frac{\Delta F}{\Delta \varepsilon \cdot t \cdot w} = 3.54 \text{GPa}
\]  

(10)

Estimation of the Crease Modulus

We performed compression test for a simple straight crease origami made of the plastic in Fig. S8B. The depth of the crease is 100 \( \mu \text{m} \). The experimental result is shown in Fig. S8B. The same relationship between normalized reaction force \( \bar{F} = \frac{F}{H} \) and compressive strain was also theoretically studied and shown in Fig. S1F. By fitting the experimental result with theoretical result (Fig. S8C), we have \( H = 0.01667 \text{ N} \). For the plastic film used in this study, the film Young’s modulus \( E = 3.54 \text{ GPa} \) (Fig. S8A). For a crease of 100 \( \mu \text{m} \) cut depth, film thickness \( t = 125 \mu \text{m} \) and unit cell length \( a \), the normalized crease stiffness can be expressed as

\[
\bar{H} = \frac{H a}{E t^3} = \frac{0.01667 \text{N} \times a}{3.54 \times 10^9 \text{N} \cdot \text{m}^2 \times (1.25 \times 10^{-4} \text{m})^3} = (2.41 \text{m}^{-1}) \cdot a.
\]

For example, if \( a = 30 \text{ mm} \) (Fig. 3A), normalized crease stiffness is \( \bar{H} = (2.41 \text{m}^{-1}) \cdot (0.03 \text{m}) = 0.072 \).

Mechanical Characterization of Soft Tofu

We conducted a compression test to measure the mechanical properties of soft tofu. A piece of soft tofu in cylindrical shape with height \( h = 22 \text{ mm} \) and diameter \( d = 43 \text{ mm} \) was used. The photographs of the tofu before and after compression are shown in Fig. S10A and B, respectively. The force-displacement relationship is shown in Fig. S10C. The true strain is calculated as
\[ \varepsilon = \ln \left( \frac{h-u}{h} \right), \]  
\[ (11) \]

where \( u \) is the vertical displacement. The true stress is calculated as

\[ \sigma = \frac{4(h-u)F}{h \pi d^2} \]  
\[ (12) \]

assuming the tofu is uncompressible. The true stress-true strain relationship is shown in Fig. S10D. The Young’s modulus of soft tofu is calculated as

\[ E_{\text{tofu}} = \frac{\Delta \sigma}{\Delta \varepsilon} = 8.005 \text{kPa} \]  
\[ (13) \]

The strength of the soft tofu is 3.298 kPa. The toughness of soft tofu is calculated as

\[ T_{\text{tofu}} = \int_{\sigma}^{\varepsilon_f} \sigma d\varepsilon = 875 \text{J/m}^2 \]  
\[ (14) \]

where \( \varepsilon_f = 0.4 \) is the fracture strain.
Fig. S1. Theoretical and experimental results of quadrilateral rigid origami cell upon compression.

(A) Pattern of a quadrilateral rigid origami cell with creases 1, 2, 3, and 4, and an angle $\beta$ between creases 1 and 2. (B) Configuration of the unit cell upon a compressive load on the top and bottom vertices. (C) The
similarity between the quadrilateral rigid origami cell and a Miura pattern unit cell. (D) The relationship between normalized reaction force $\vec{F}$ and collapsing ratio $1 - \frac{l}{2c}$ for different $\beta$. (E) The relationship between normalized force $\vec{F}_{1,3}$ (i.e., reaction force due to creases 1 and 3) and collapsing ratio $1 - \frac{l}{2c}$ for different $\beta$. (F) The relationship between normalized force $\vec{F}_{2,4}$ (i.e., reaction force due to creases 2 and 4) and collapsing ratio $1 - \frac{l}{2c}$, which is independent with $\beta$. (G) Force-displacement relationship of quadrilateral rigid origami cells upon compression with different $\beta = 70^\circ$, $60^\circ$, and $50^\circ$. The results of $\beta = 70^\circ$ and $60^\circ$ show negative stiffness and the result of $\beta = 50^\circ$ shows positive stiffness. The inconsistent between experimental and theoretical results of $\beta = 50^\circ$ can be explained by the deformation of panels in origami. All the results show significant hysteresis.
Fig. S2. Rationale of the linear elastic perfectly plastic model of creases used in simulation. (A) A linear elastic and perfectly plastic model to describe the reaction moment per unit crease length as a function of rotation angle of crease. The yield angle of crease is set as 1 rad for all simulations. (B) Experiment setup to measure the plastic behavior of a crease with cutting depth 100 µm. The right side of the origami is constrained to a plane, and the left side is free to rotate. (C) Bending the left side of the origami to 130° with a tweezer. (D) The origami at relaxed state of 80° when the tweezer is removed. (E) Prediction of the test in (C) using the linear elastic and perfectly plastic model, which gives the result of 73° relaxed angle. (F) Comparison between the experiments results and the prediction with rotation angle of 60°, 90°, 130°, and 180°. Photo credit: Zirui Zhai, Arizona State University.
Fig. S3. Energy distribution of curved origami unit cell during loading steps. Loading steps in the finite element simulations and the external work $W$, elastic strain energy $U_E$, and plastic dissipation $U_P$ as functions of step number.
Fig. S4. Curvature contour of curved origami during compression. Curved origami with crease $\bigcirc_1$, $\bigcirc_2$, and $\bigcirc_3$ activated, respectively, under compression strain $u = 0.05$, 0.1, and 0.15, are plotted normalized curvature contour. The curvature distribution is more inhomogeneous with larger crease curvature $\kappa$ and under larger compression strain.
Fig. S5. Simulation results for an activated crease $\odot$ or $\heartsuit$ upon compression. (A) Normalized total energy $\bar{U}_{\text{tot}}$, bending energy $\bar{U}_b$, and folding energy $\bar{U}_f$ of the curved origami with crease $\odot$ activated as a function of normalized displacement $\bar{u}$. (B) Normalized total force $\bar{F}_{\text{tot}}$, force due to bending $\bar{F}_b$, and force due to folding $\bar{F}_f$ of the curved origami with crease $\odot$ activated as a function of normalized displacement $\bar{u}$. (C) Normalized total energy $\bar{U}_{\text{tot}}$, bending energy $\bar{U}_b$, and folding energy $\bar{U}_f$ of the curved origami with crease $\heartsuit$ activated as a function of normalized displacement $\bar{u}$. (D) Normalized total force $\bar{F}_{\text{tot}}$, force due to bending $\bar{F}_b$, and force due to folding $\bar{F}_f$ of the curved origami with crease $\heartsuit$ activated as a function of normalized displacement $\bar{u}$. 
**Fig. S6.** Switching the activated creases of curved origami. (A) Contour plot of rotation angle along $x$-axis of the curved origami and the normalized energy during activation switching from crease $\circled{1}$ to $\circled{2}$. (B) Contour plot of rotation angle along $x$-axis of the curved origami and the normalized energy during activation switching from crease $\circled{2}$ to $\circled{3}$. 
Fig. S7. Force-displacement relationship of single-crease curved origami for varied crease moduli.

Force-displacement relationship of single crease curved origami for a given normalized curvature $\kappa = 0.6$ and varied crease moduli $\bar{H} = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06,$ and $0.07$. 
Fig. S8. Mechanical characterizations of plastic film and curved crease. (A) Force-strain relationship of a stripe of plastic film. (B) Compression force of a single straight crease origami as a function of compressive strain. (C) Comparison between theoretical and experimental normalized forces as functions of compressive strain. Photo credit: Zirui Zhai, Arizona State University.
Fig. S9. Mechanical characterization of the gripper. (A) Photographs of the handler in ON mode being tested. (B) Photographs of the handler in OFF mode being tested. (C) Photographs of the clipper in being tested. A brick of Lego is gripped during the test. (D) Force-displacement relationships for the handler in ON mode, handler in OFF mode, and the clipper. Photo credit: Zirui Zhai, Arizona State University.
Fig. S10. Mechanical characterization of soft tofu. (A) Soft tofu before compression test. (B) Soft tofu after test. (C) True stress-true strain relationship of the soft tofu. Photo credit: Zirui Zhai, Arizona State University.
**Fig. S11. Compression test of a homogeneous curved Miura patterns.** (A) Photograph of a curved Miura pattern with crease [1] under compression load when the concave side is constrained by a tweezer. (B) Photograph of a curved Miura pattern with crease [1] under compression when the constrain is released. (C) Force-displacement relationships of the curved Miura of pattern [1] with constrain applied on concave side at beginning and then removed, compare with the result without the constrain. The constrain on concave side results a much higher critical load of the curved Miura, and the load drops suddenly when the constrain is removed. (D) Photographs of curved origami patterns with creases [1], [2], and [3] (see Fig. 5 in the main text) before and during compressive deformation. (E) Force-displacement relationship of curved origami patterns [1], [2], and [3] upon compressive load on A-A direction. The loading setup is shown in Fig. 5B. Photo credit: Zirui Zhai, Arizona State University.
Fig. S12. Design of the swimming robot. (A) 3D design of the assembly of the swimming robot, including a frame, two paddles, and a sliding trench. (B) Design of the frame, with an air inlet for balloon, and two sliding holes for pairing with the paddles. (C) Design of the paddle, with a sliding bar for pairing with the holes in frame, and a sliding track for pairing with the sliding bars of trench. (D) Design of the sliding trench, with two sliding bars to pair with the track of paddle, and bars for fixing the curved Miura pattern.
Supporting Movies:

Movie S1. Switching the gripper between ON and OFF modes.

Movie S2. Gripping different objects using the curved origami gripper.

Movie S3. Switching the curved origami isolator between modes A and B.

Movie S4. Vibration tests of the curved origami isolator.

Movie S5. Compression of the homogeneous curved Miura.

Movie S6. Switching the inhomogeneous curved Miura into different modes.

Movie S7. The swimming robots at different modes.

Movie S8. The swimming robots at the other three modes.