Learning Particle Dynamics for Manipulating Rigid Bodies, Deformable Objects, and Fluids

Yunzhu Li, Jiajun Wu, Russ Tedrake, Joshua B. Tenenbaum, and Antonio Torralba
Computer Science and Artificial Intelligence Laboratory
Massachusetts Institute of Technology
{liyunzhu,jiajunwu,russt,jbt,torralba}@mit.edu

Abstract
Real-life control tasks involve matter of various substances—rigid or soft bodies, liquid, gas—each with distinct physical behaviors. This poses challenges to traditional rigid-body physics engines. Particle-based simulators have been developed to model the dynamics of these complex scenes; however, relying on approximation techniques, their simulation often deviates from real world physics, especially in the long term. In this paper, we propose to learn a particle-based simulator for complex control tasks. Combining learning with particle-based systems brings in two major benefits: first, the learned simulator, just like other particle-based systems, acts widely on objects of different materials; second, the particle-based representation poses strong inductive bias for learning: particles of the same type have the same dynamics within. This enables the model to quickly adapt to new environments of unknown dynamics within a few observations. Using the learned simulator, robots have achieved success in complex manipulation tasks, such as manipulating fluids and deformable foam. The effectiveness of our method has also been demonstrated in the real world. Our study helps lay the foundation for robot learning of dynamic scenes with particle-based representations.

1 Introduction
Objects have distinct dynamics. Under the same push, a rigid box will slide, modeling clay will deform, and a cup full of water will fall with water spilling out. The diverse behavior of different objects poses challenges to traditional rigid-body simulators used in robotics (Todorov et al., 2012). Particle-based simulators aim to model the dynamics of these complex scenes (Macklin et al., 2014); however, relying on approximation techniques for the sake of perceptual realism, their simulation often deviates from real world physics, especially in the long term. Developing generalizable and accurate forward dynamics models is of critical importance for robot manipulation of distinct real-life objects.

We propose to learn a differentiable, particle-based simulator for complex control tasks, drawing inspiration from recent development in differentiable physical engines (Battaglia et al., 2016; Chang et al., 2017). In robotics, the use of differentiable simulators, together with continuous and symbolic optimization algorithms, has enabled planning for increasingly complex whole body motions with multi-contact and multi-object interactions (Toussaint et al., 2018). Yet these approaches have only tackled local interactions of rigid bodies. We develop dynamic particle interaction networks (DPI-Nets) for learning particle dynamics, focusing on capturing the dynamic, hierarchical, and long-range interactions of particles (Figure 1a). DPI-Nets can then be combined with classic perception and gradient-based control algorithms for robot manipulation of deformable objects (Figure 1c).

Learning a particle-based simulator brings in two major benefits. First, the learned simulator, just like other particle-based systems, acts widely on objects of different materials. DPI-Nets have successfully captured the complex behaviors of deformable objects like fluids, rigid-bodies, and plasticine (Figure 1b). With learned DPI-Nets, our robots have achieved success in manipulation tasks that involve deformable objects of complex physical properties, such as molding plasticine to a target shape (Figure 1d).

Our project page: https://sites.google.com/view/learn-particle-dynamics
Figure 1: Learning Particle Dynamics for Control. (a) DPI-Nets learn particle interaction while dynamically building the interaction graph over time. (b) Simulation using learned DPI-Nets for rigid bodies, deformable objects, and fluids. (c) Perception and control with the learned model. Our system first reconstructs the particle-based shape from visual observation. It then uses gradient-based trajectory optimization to search for the actions that produce the most desired output. For clarity, we plot particles in 2D; in practice reconstruction and control are conducted on particles in 3D. (d) Control with learned DPI-Nets to manipulate a deformable foam (blue) into a target shape (red).

Second, the particle-based representation poses strong inductive bias for learning: particles of the same type have the same dynamics within. This enables the model to quickly adapt to new environments of unknown dynamics within a few observations. Experiments suggest that DPI-Nets quickly learns to adapt to characterize a novel object of unknown physical parameters by doing online system identification. The adapted model also helps the robot to successfully manipulate object in the real world.

We learn particle dynamics with a dynamic particle interaction network. A DPI-Net learns how particles interact, and uses the learned knowledge to predict how they move. It relies on an interaction graph, describing the pairs of particles that influence each other. As objects deform when robots interact with them, a fixed interaction graph over particles is insufficient for robot manipulating non-rigid objects. DPI-Nets introduce dynamic interaction graphs that are built on the fly throughout manipulation. The use of dynamic graphs allows neural models to focus on learning meaningful interactions among particles, increasing its simulation accuracy and manipulation success rate.

Experiments demonstrate that DPI-Nets work well, significantly outperforming interaction networks (Battaglia et al., 2016) and a few other baselines. When applied in downstream control tasks, our model helps to complete complex manipulation tasks for deformable objects and fluids. It also adapts to cases with unknown physical parameters that need to be identified online. We have also performed real-world experiments to demonstrate our model’s generalization ability.

2 RELATED WORK

Differentiable physics simulators. Researchers have developed many differentiable physics simulators (Ehrhardt et al., 2017; Degrave et al., 2016). In particular, Battaglia et al. (2016) and Chang et al. (2017) have explored approximating object interactions with neural networks. Li et al. (2018) proposed learning to propagate signals along the interaction graph. These methods mostly focus on modeling rigid body dynamics.

Differentiable simulators for deformable objects have been less studied. Recently, Schenck & Fox (2018) proposed SPNets for differentiable simulation of position-based fluids (Macklin & Müller, 2013). A concurrent work from Mrowca et al. (2018) explored learning to approximate particle dynamics of deformable shapes with a hierarchical representation. Compared with these papers, our model uses dynamic graphs to learn particle interaction of various materials (rigid bodies, deformable shapes, fluids) under complex behaviors. We also demonstrated how it can be used in control tasks in both simulation and real world.
Our approach is also complementary to some recent work on learning to discover the interaction graphs. Our model can also be naturally augmented with a perception module to handle raw visual input, as suggested by Watters et al. (2017); Wu et al. (2017); Fragkiadaki et al. (2016).

Model-predictive control with a differentiable simulator. Many recent papers have studied model-predictive control with deep networks (Lenz et al., 2015; Gu et al., 2016; Nagabandi et al., 2018; Farquhar et al., 2018; Srinivas et al., 2018). They often learn an abstract state transition function, instead of an explicit account of the environment (Silver et al., 2017; Oh et al., 2017), and then use the learned function to facilitate training of a policy network. A few recent papers have employed analytical, differentiable simulators (de Avila Belbute-Peres et al., 2018; Schenck & Fox, 2018) for control problems, such as tool manipulation and tool-use planning (Toussaint et al., 2018). Our model builds on and extends these approaches by learning a general physics simulator that takes raw object observations (e.g., positions, velocities) of each particle as input. We then integrate it into classic trajectory optimization algorithms for control. Compared with pure analytical simulators, our learned simulator can better generalize to novel testing scenarios where object and environment parameters are unknown.

A few papers have explored using interaction networks for planning and control. They often learn a policy based on interaction networks’ rollouts (Racanière et al., 2017; Hamrick et al., 2017; Pascanu et al., 2017). In contrast, our model learns a dynamics simulator and directly optimizes trajectories for continuous control. Recently, Sanchez-Gonzalez et al. (2018) have applied interaction networks for control, and Li et al. (2018) have further extended interaction nets to handle instance signal propagation for controlling multiple rigid bodies under partial observations. Compared with them, our dynamic particle interaction network simulate and control deformable, particle-based objects, using dynamic graphs to tackle scenes with complex object interactions.

3 Approach

3.1 Preliminaries

We first describe how interaction networks (Battaglia et al., 2016) represent the physical system; we then extend them for particle-based dynamics.

The interactions within a physical system are represented as a directed graph, \( G = (O, R) \), where vertices \( O = \{o_i\} \) represent objects and edges \( R = \{r_k\} \) represent relations. Specifically, \( o_i = (x_i, q_i^o) \), where \( x_i = (q_i, \dot{q}_i) \) is the state of object \( i \), containing its position \( q_i \) and velocity \( \dot{q}_i \). \( q_i^o \) denotes its attributes (e.g., mass, radius). For relation, we have \( r_k = (u_k, v_k, a^r_k) \), \( 1 \leq u_k, v_k \leq |O| \), where \( u_k \) is the receiver, \( v_k \) is the sender, and \( a^r_k \) is the type and attributes of relation \( k \) (e.g., collision, spring connection).

The goal is to build a learnable engine to capture the underlying physical interactions using function approximators \( \phi \). The learned model can then be used to infer the system dynamics and predict the future from the current interaction graph as \( G_{t+1} = \phi(G_t) \), where \( G_t \) denotes the scene state at time \( t \).

Interaction networks. Battaglia et al. (2016) proposed interaction networks (IN), a general-purpose, learnable physics engine that performs object- and relation-centric reasoning about physics. INs define an object function \( f_O \) and a relation function \( f_R \) to model objects and their relations in a compositional way. The future state at time \( t + 1 \) is predicted as \( e_{k,t+1} = f_R(o_{uk,t}, o_{vk,t}, a^r_k) \). The initial state \( G_t \) is encoded as \( \phi(G_t) \), where \( \phi(G_t) = (x_{i,t}, a^o_i) \) denotes object \( i \) at time \( t \), \( u_k \) and \( v_k \) are the receiver and sender of relation \( r_k \) respectively, and \( N_i \) denotes the relations where object \( i \) is the receiver.

Propagation networks. A limitation of INs is that at every time step \( t \), it only considers local information in the graph \( G \) and cannot handle instantaneous propagation of forces, which however is a common phenomenon in rigid-body dynamics. Li et al. (2018) proposed propagation networks to handle the instantaneous propagation of forces by doing multi-step message passing. Specifically, they first employed the ideas on fast training of RNNs (Lei & Zhang, 2017; Bradbury et al., 2017) to encode the shared information beforehand and reuse them along the propagation steps. The encoders for objects are denoted as \( f^\text{enc}_O \) and the encoder for relations as \( f^\text{enc}_R \), where we denote \( e^o_{i,t} = f^\text{enc}_O(o_{i,t}) \) and \( e^r_{k,t} = f^\text{enc}_R(o_{uk,t}, o_{vk,t}, a^r_k) \).
At time $t$, denote the propagating influence from relation $k$ at propagation step $l$ as $e_{k,t}^l$, and the propagating influence from object $i$ as $h_{i,t}^l$. For step $1 \leq l \leq L$, propagation can be described as

Step 0: $h_{i,0}^0 = 0, \quad i = 1 \ldots |O|$, (1)

Step $l = 1, \ldots, L$:

$$ e_{k,t}^l = f_R(e_{k,t}^{l-1}, h_{u_k,t}^{l-1}, h_{v_k,t}^{l-1}), k = 1 \ldots |R|, $$ (2)

$$ h_{i,t}^l = f_O(e_{i,t}^l, \sum_{k \in N_i} e_{k,t}^{l-1}, h_{i,t}^{l-1}), i = 1 \ldots |O|, $$ (3)

Output: $\hat{o}_{i,t+1} = f_O^{\text{output}}(h_{i,t}^L), \quad i = 1 \ldots |O|$, (4)

where $f_O$ denotes the object propagator at propagation step $l$ and $f_R$ denotes the relation propagator.

3.2 Dynamic Particle Interaction Networks

Particle-based system is widely used in physical simulation due to its flexibility in modeling various types of objects (Macklin et al., 2014). We extend existing systems that model object-level interactions to allow particle-level deformation. Consider object set $\{o_i\}$, where each object $o_i = \{o_i^k\}_{k=1,..,|o_i|}$ is represented as a set of particles. We now define the graph on the particles and the rules for influence propagation.

**Dynamic graph building.** The vertices of the graph are the union of particles for all objects $O = \{o_i\}_{i=1,..,|O|}$. The edges $R$ between these vertices are dynamically generated over time to ensure efficiency and effectiveness. The construction of the relations is specific to environment and task, which we’ll elaborate in Section 4. A common choice is to consider the neighbors within a predefined distance.

An alternative is to build a static, complete interaction graph, but it has two major drawbacks. First, it is not efficient. In many common physical systems, each particle is only interacting with a limited set of other particles (e.g., those within its neighborhood). Second, a static interaction graph implies a universal, continuous neural function approximator; however, many physical interactions involve discontinuous functions (e.g., contact). In contrast, using dynamic graphs empowers the model to tackle such discontinuity.

**Hierarchical modeling for long-range dependence.** Propagation networks (Li et al., 2018) require a large $L$ to handle long-range dependence, which is both inefficient and hard to train. Hence, we add one level of hierarchy to efficiently propagate the long-range influence among particles (Mrowca et al., 2018). For each object that requires modeling of the long-range dependence (e.g. rigid-body), we cluster the particles into several non-overlapping clusters. For each cluster, we add a new particle as the cluster’s root. Specifically, for each object $o_i$ that requires hierarchical modeling, the corresponding roots are denoted as $\delta_i = \{o_i^k\}_{k=1,..,|o_i|}$, and the particle set containing all the roots is denoted as $\tilde{O} = \{o_i^k\}_{i=1,..,|O|, k=1,..,|\delta_i|}$. We then construct an edge set $R_{\text{LeafToRoot}}$ that contains directed edges from each particle to its root, and an edge set $R_{\text{RootToLeaf}}$ containing directed edges from each root to its leaf particles. For each object that need hierarchical modeling, we add pairwise directed edges between all its roots, and denote this edge set as $R_{\text{RootToRoot}}$.

We employ a multi-stage propagation paradigm: first, propagation among leaf nodes, $\phi_{\text{LeafToLeaf}}(\langle O, R \rangle)$; second, propagation from leaf nodes to root nodes, $\phi_{\text{LeafToRoot}}(\langle O \cup \tilde{O}, R_{\text{LeafToRoot}} \rangle)$; third, propagation between roots, $\phi_{\text{RootToRoot}}(\langle \tilde{O}, R_{\text{RootToRoot}} \rangle)$; fourth, propagation from root to leaf, $\phi_{\text{RootToLeaf}}(\langle O \cup \tilde{O}, R_{\text{RootToLeaf}} \rangle)$. The signals on the leaves are used to do the final prediction.

**Applying to objects of various materials.** We define the interaction graph and the propagation rules on particles for different types of objects as follows:

- **Rigid bodies.** All the particles in a rigid body are globally coupled; hence for each rigid object, we define a hierarchical model to propagate the effects. After the multi-stage propagation, we average the signals on the particles to predict a rigid transformation (rotation and translation) for the object. The motion of each particle is calculated accordingly. For each particle, we also include its offset to the center-of-mass to help determine the torque.
- **Elastic/Plastic objects.** For elastically deforming particles, only using the current position and velocity as the state is not sufficient, as it is not clear where the particle will be
restored after the deformation. Hence, we include the particle state with the resting position to indicate the place where the particle should be restored. When coupled with plastic deformation, the resting position might change during an interaction. Thus, we also infer the motion of the resting position as a part of the state prediction. We use hierarchical modeling for this category but predict next state for each particles individually.

- **Fluids.** For fluid simulation, one has to enforce density and incompressibility, which can be effectively achieved by only considering a small neighborhood for each particle (Macklin & Müller, 2013). Therefore, we do not need hierarchical modeling for fluids. We build edges dynamically, connecting a fluid particle to its neighboring particles. The strong inductive bias leveraged in the fluid particles allows good performance even when tested on data outside training distributions.

For the interaction between different materials, two directed edges are generated for any pairs of particles that are closer than a certain distance.

### 3.3 Control on the Learned Dynamics

Model-based methods offer many advantages when comparing with their model-free counterparts, such as generalization and sample efficiency. However, for cases where an accurate model is hard to specify or computationally prohibitive, a data-driven approach that learns to approximate the underlying dynamics becomes useful.

Function approximators such as neural networks are naturally differentiable. We can rollout using the learned dynamics and optimize the control inputs by minimizing a loss between the simulated results and a target configuration. In cases where certain physical parameters are unknown, we can perform online system identification by minimizing the difference between the model’s prediction and the reality. An outline of our algorithm can be found in Section A.

**Model predictive control using shooting methods.** Let’s denote $G_g$ as the goal and $\hat{u}_{1:T}$ be the control inputs, where $T$ is the time horizon. The control inputs are part of the interaction graph, such as the velocities or the initial positions of a particular set of particles. We denote the resulting trajectory after applying $\hat{u}$ as $\hat{G} = \{G_i\}_{i=1:T}$. The task here is to determine the control inputs as to minimize the distance between the actual outcome and the specified goal $L_g(G, G_g)$.

Our dynamic particle interaction network does forward simulation by taking the dynamics graph at time $t$ as input, and produces the graph at next time step, $\hat{G}_{t+1} = \Phi(G_t)$, where $\Phi$ is implemented as DPI-Nets as described in the previous section. Let’s denote the the history until time $t$ as $\hat{G} = \{G_i\}_{i=1...t}$, and the forward simulation from time step $t$ as $\hat{G} = \{G_i\}_{i=t+1...T}$. The loss $L_g(\hat{G} \cup \hat{G}, G_g)$ can be used to update the control inputs by doing stochastic gradient descent (SGD). This is known as the shooting method in trajectory optimization (Tedrake, 2009).

The learned model might deviate from the reality due to accumulated prediction errors. We use Model-Predictive Control (MPC) (Camacho & Alba, 2013) to stabilize the trajectory by doing forward simulation and updating the control inputs at every time step to compensate the simulation error.

**Online adaptation.** In many real-world cases, without actually interacting with the environment, inherent attributes such as mass, stiffness, and viscosity are not directly observable. DPI-Nets can estimate these attributes on the fly with SGD updates by minimizing the distance between the predicted future states and the actual future states $L_s(\hat{G}_t, G_t)$.

### 4 Experiments

We evaluate our method on four different environments containing different types of objects and interactions. We will first describe the environments and show simulation results. We then present how the learned dynamics helps to complete control tasks in both simulation and the real world.

#### 4.1 Environments

**FluidFall** (Figure 2a). Two drops of fluids are falling down, colliding, and merging. We vary the initial position and viscosity for training and evaluation.

**BoxBath** (Figure 2b). A block of fluids are flushing a rigid cube. In this environment, we have to model two different materials and the interactions between them. We randomize the initial position of the fluids and the cube to test the model’s generalization ability.
Figure 2: **Qualitative results on simulation.** We show side-by-side comparisons between the ground truth (GT) and the rollout from our model (DPI-Net) in four difference environments. (a) FluidFall - Two drops of high-viscosity fluids are falling down and merging with each other. (b) BoxBath - A block of water is flushing a rigid cube. (c) FluidShake - Shaking a box of fluids. (d) RiceGrip - Grip an object that can deform both elastically and plastically (e.g., sticky rice).

| Methods                        | FluidFall | BoxBath | FluidShake | RiceGrip |
|--------------------------------|-----------|---------|------------|----------|
| IN (Battaglia et al., 2016)    | 2.74      | N/A     | N/A        | N/A      |
| DPI-Net w/o dynamic graphs     | 0.42      | 123.5   | 97.7       | 0.64     |
| DPI-Net w/o hierarchy          | 0.15      | 2.58    | 1.89       | 0.13     |
| DPI-Net                        | 0.15      | 2.58    | 1.89       | 0.13     |

(a) Rollout discrepancy

Figure 3: **Quantitative results on simulation.** The errors showed here are the MSE ($\times 10^{-2}$) between the ground truth and model rollout. (a) FluidFall and FluidShake involve no hierarchy, so DPI-Net performs the same as the variant without hierarchy. Fixing the graph (i.e., w/o dynamic graphs) severely damages the modeling of flowing fluids as indicated in BoxBath and FluidShake. (b) The gray bars denote the range of particle numbers that have been seen during training, which indicate interpolation performance. The blue bars indicate extrapolation performance that our model can generalize to cases containing two times more particles than cases it has been trained on.

**FluidShake** (Figure 2c). We have a box of fluids and the box is moving horizontally. The speed of the box is randomly selected at each time step. We vary the size of the box and volume of the fluids to test generalization.

**RiceGrip** (Figure 2d). We manipulate an object with both elastic and plastic deformation (e.g., sticky rice). We use two cuboids to mimic the fingers of a parallel gripper, where the gripper is initialized at a random position and orientation. During the simulation of one grip, the fingers will move closer to each other and then restore to its original positions. The model has to learn the interactions between the gripper and the “sticky rice”, as well as the interactions within the “rice” itself.

We use all four environments in evaluating our model’s performance in simulation. We use the rollout MSE as our metric. We further use the latter two for control, because they involve fully actuated external shapes that can be used for object manipulation. In FluidShake, the control task requires determining the speed of the box at each time step, in order to make the fluid match a target configuration within a time window; in RiceGrip, the control task corresponds to select a sequence of grip configurations (position, orientation, closing distance) to manipulate the deformable object as to match a target shape. The metric for performance in control is the Chamfer distance between the manipulation results and the target configuration.

4.2 **Physical Simulation**

We present implementation details for dynamics learning in the four environment and perform ablation studies to evaluate the effectiveness of the introduced techniques.
Implementation details. For FluidFall, we dynamically build the interaction graph by connecting each particle to its neighbors within a certain distance $d$. No hierarchical modeling is used.

For BoxBath, we model the rigid cube as in Section 3.2 using multi-stage hierarchical propagation. Two directed edges will be constructed between two fluid particles if the distance between them is smaller than $d$. Similarly, we also add two directed edge between rigid particles and fluid particles when their distance is smaller than $d$.

For FluidShake, we model fluid as in Section 3.2. We also add five external particles to represent the walls of the box. We add a directed edge from the wall particle to the fluid particle when they are closer than $d$. The model is a single propagation network, where the edges are dynamically constructed over time.

For RiceGrip, we build a hierarchical model for rice and use four propagation networks for multi-stage effect propagation (Section 3.2). The edges between the “rice” particles are dynamically generated if two particles are closer than $d$. Similar to FluidShake, we add two external particles to represent the two “fingers” and add an edge from the “finger” to the “rice” particle if they are closer than the distance $d$. As “rice” can deform both elastically and plastically, we maintain a resting position that helps the model restore a deformed particle. The output for each particle is a 6-dim vector for the velocity of the current observed position and the resting position. More training details for each environment can be found in Section C. Details for data generation are in Section B.

Results. Qualitative and quantitative results are in Figure 2 and Figure 3. We compare our method (DPI-Net) with three baselines, Interaction Networks (Battaglia et al., 2016), DPI-Net without hierarchy, and DPI-Net without dynamic graphs (whose interaction graph is fixed to that computed from the initial configuration).

Specifically, Interaction Networks (IN) consider a complete graph of the particle system. Thus, it can only operate on small environments such as FluidFall; it runs out of memory (12GB) for the other three environments. IN does not perform well, because its use of a complete graph makes training difficult and inefficient, and because it ignores influence propagation and long-range dependence.

Without a dynamic graph, modeling fluids becomes hard, because the neighbors of a fluid particle changes constantly. Figure 3 shows that for environments that involve fluids (BoxBath and FluidShake), the model without dynamic graphs does not perform well. Without hierarchy, it is hard to capture long-range dependence, leading to performance drop in environments that involve hierarchical object modeling (BoxBath and RiceGrip).

We also test our model’s ability to generalize outside the training distribution. Figure 3b shows the generalization performance in FluidShake, where the grey bars are for the number of particles have been used during training, and the blue bars for generalization. Our model handles extra particles effectively due to the inductive bias leveraged.

4.3 Control

We leverage dynamic particle interaction network for control tasks in both simulation and real world. Because trajectory optimization using shooting method can easily stuck to a local minimum, we first randomly sample $N_{\text{sample}}$ control sequences, and select the best performing one according to the rollouts of our learned model. We then optimize it via shooting method using our model’s gradients. We also use online system identification to further improve the model’s performance. Figure 4 and Figure 5 show qualitative and quantitative results, respectively. More details of the control algorithm can be found in Section D.

FluidShake. We aim to control the speed of the box to match the fluid particles to a target configuration. We compare our method (RS+TO) with random search over the learned model (without trajectory optimization - RS) and Model-free Deep Reinforcement Learning (Actor-Critic method optimized with PPO (Schulman et al., 2017) - RL). Figure 5 suggests that our model-based control algorithm outperforms both baselines with a large margin. Also RL is not sample-efficient, requiring more than 10 million time steps to converge while ours requires 600k time steps.

RiceGrip. We aim to select a sequence of gripping configurations (position, orientation, and closing distance) to mold the “sticky rice” to a target shape. We also consider cases where the stiffness of the rice is unknown and need to be identified. Figure 5 shows that our dynamic particle interaction network with system identification performs the best, and is much more efficient than RL (150K vs. 10M time steps).
Figure 4: **Qualitative results on control.** (a) FluidShake - Manipulating a box of fluids to match a target shape. The Result and Target indicate the fluid shape when viewed from the cutaway view. (b) RiceGrip - Gripping a deformable object and molding it to a target shape. The Result and Target indicate the shape when viewed from the top. (c) RiceGrip in Real World - Generalize the learned dynamics and the control algorithms to the real world by doing online adaptation.

Figure 5: **Quantitative results on control.** We show the control performance (as evaluated by the Chamfer distance ($\times 10^{-2}$) between the manipulated result and the target) for (a) FluidShake and (b) RiceGrip by comparing with baselines: RL - Model-free deep reinforcement learning optimized with PPO. RS - Random search the actions from the learned model and select the best one to execute. TO - Trajectory optimization augmented with model predictive control. ID - Online system identification by estimating uncertain physical parameters during run time.

**RiceGrip in the real world.** We generalize the learned model and control algorithm for RiceGrip to the real world. We first reconstruct object geometry using a depth camera mounted on our Kuka robot using TSDF volumetric fusion (Curless & Levoy 1996). We then randomly sampled $N_{fill}$ particles within the object mesh as the initial configuration for manipulation. Figure 4c and Figure 5b shows that, using DPI-Nets, the robot successfully adapts to the real world environment of unknown physical parameters and manipulates a deformable foam into various target shapes. The learned policy in RiceGrip does not generalize to the real world due to domain discrepancy, and outputs invalid gripping configurations.

5 CONCLUSION

We have demonstrated that a learned particle dynamics model can approximate the interaction of diverse objects, and can help to solve complex manipulation tasks of deformable objects. Our system requires standard open-source robotics and deep learning toolkits, and can be potentially deployed in household and manufacturing environment. Robot learning of dynamic scenes with particle-based representations shows profound potentials due to the generalizability and expressiveness of the representation. Our study helps lay the foundation for it.
REFERENCES

Peter W. Battaglia, Razvan Pascanu, Matthew Lai, Danilo Rezende, and Koray Kavukcuoglu. Interaction networks for learning about objects, relations and physics. In NIPS, 2016.

James Bradbury, Stephen Merity, Caiming Xiong, and Richard Socher. Quasi-recurrent neural networks. In ICLR, 2017.

Eduardo F Camacho and Carlos Bordons Alba. Model predictive control. Springer Science & Business Media, 2013.

Michael B Chang, Tomer Ullman, Antonio Torralba, and Joshua B Tenenbaum. A compositional object-based approach to learning physical dynamics. In ICLR, 2017.

Brian Curless and Marc Levoy. A volumetric method for building complex models from range images. In SIGGRAPH, 1996.

Filipe de Avila Belbute-Peres, Kevin A Smith, Kelsey Allen, Joshua B Tenenbaum, and J Zico Kolter. End-to-end differentiable physics for learning and control. In Neural Information Processing Systems, 2018.

Jonas Degrave, Michiel Hermans, and Joni Dambre. A differentiable physics engine for deep learning in robotics. In ICLR Workshop, 2016.

Sebastien Ehrhardt, Aron Monszpart, Niloy Mitra, and Andrea Vedaldi. Taking visual motion prediction to new heightfields. arXiv:1712.09448, 2017.

Gregory Farquhar, Tim Rocktäschel, Maximilian Igl, and Shimon Whiteson. Treeqn and atreec: Differentiable tree planning for deep reinforcement learning. In ICLR, 2018.

Katerina Fragkiadaki, Pulkit Agrawal, Sergey Levine, and Jitendra Malik. Learning visual predictive models of physics for playing billiards. In ICLR, 2016.

Shixiang Gu, Timothy Lillicrap, Ilya Sutskever, and Sergey Levine. Continuous deep q-learning with model-based acceleration. In ICML, 2016.

Jessica B Hamrick, Andrew J Ballard, Razvan Pascanu, Oriol Vinyals, Nicolas Heess, and Peter W Battaglia. Metacontrol for adaptive imagination-based optimization. In ICLR, 2017.

Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In ICLR, 2015.

Thomas N Kipf, Ethan Fetaya, Kuan-Chieh Wang, Max Welling, and Richard S Zemel. Neural relational inference for interacting systems. arXiv:1802.04687, 2018.

Tao Lei and Yu Zhang. Training rnns as fast as cnns. arXiv preprint arXiv:1709.02755, 2017.

Ian Lenz, Ross A Knepper, and Ashutosh Saxena. Deepmpc: Learning deep latent features for model predictive control. In RSS, 2015.

Yunzhu Li, Jiajun Wu, Jun-Yan Zhu, Joshua B Tenenbaum, Antonio Torralba, and Russ Tedrake. Propagation networks for model-based control under partial observation. arXiv preprint arXiv:1809.11169, 2018.

Miles Macklin and Matthias Müller. Position based fluids. ACM TOG, 32(4):104, 2013.

Miles Macklin, Matthias Müller, Nuttapong Chentanez, and Tae-Yong Kim. Unified particle physics for real-time applications. ACM TOG, 33(4):153, 2014.

Damian Mrowca, Chengxu Zhuang, Elias Wang, Nick Haber, Li Fei-Fei, Joshua B Tenenbaum, and Daniel LK Yamins. Flexible neural representation for physics prediction. arXiv preprint arXiv:1806.08047, 2018.

Anusha Nagabandi, Gregory Kahn, Ronald S Fearing, and Sergey Levine. Neural network dynamics for model-based deep reinforcement learning with model-free fine-tuning. In ICRA, 2018.

Junhyuk Oh, Satinder Singh, and Honglak Lee. Value prediction network. In NIPS, 2017.
Razvan Pascanu, Yujia Li, Oriol Vinyals, Nicolas Heess, Lars Buesing, Sebastien Racanière, David Reichert, Théophasne Weber, Daan Wierstra, and Peter Battaglia. Learning model-based planning from scratch. arXiv:1707.06170, 2017.

Sébastien Racanière, Théophasne Weber, David Reichert, Lars Buesing, Arthur Guez, Danilo Jimenez Rezende, Adrià Puigdomènec Badia, Oriol Vinyals, Nicolas Heess, Yujia Li, Razvan Pascanu, Peter Battaglia, David Silver, and Daan Wierstra. Imagination-augmented agents for deep reinforcement learning. In NIPS, 2017.

Alvaro Sanchez-Gonzalez, Nicolas Heess, Jost Tobias Springenberg, Josh Merel, Martin Riedmiller, Raia Hadsell, and Peter Battaglia. Graph networks as learnable physics engines for inference and control. In ICML, 2018.

Connor Schenck and Dieter Fox. Spnets: Differentiable fluid dynamics for deep neural networks. arXiv preprint arXiv:1806.06094, 2018.

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. arXiv:1707.06347, 2017.

David Silver, Hado van Hasselt, Matteo Hessel, Tom Schaul, Arthur Guez, Tim Harley, Gabriel Dulac-Arnold, David Reichert, Neil Rabinowitz, Andre Barreto, and Thomas Degris. The predictron: End-to-end learning and planning. In ICML, 2017.

Aravind Srinivas, Allan Jabri, Pieter Abbeel, Sergey Levine, and Chelsea Finn. Universal planning networks. In ICML, 2018.

Russ Tedrake. Underactuated robotics: Learning, planning, and control for efficient and agile machines course notes for mit 6.832, 2009.

Emanuel Todorov, Tom Erez, and Yuval Tassa. Mujoco: A physics engine for model-based control. In IROS, pp. 5026–5033. IEEE, 2012.

Marc Toussaint, K Allen, K Smith, and J Tenenbaum. Differentiable physics and stable modes for tool-use and manipulation planning. In RSS, 2018.

Sjoerd van Steenkiste, Michael Chang, Klaus Greff, and Jürgen Schmidhuber. Relational neural expectation maximization: Unsupervised discovery of objects and their interactions. In ICLR, 2018.

Nicholas Watters, Andrea Tacchetti, Theophane Weber, Razvan Pascanu, Peter Battaglia, and Daniel Zoran. Visual interaction networks. In NIPS, 2017.

Jiajun Wu, Erika Lu, Pushmeet Kohli, Bill Freeman, and Josh Tenenbaum. Learning to see physics via visual de-animation. In NIPS, 2017.
A CONTROL ALGORITHM

Algorithm 1 Control on Learned Dynamics at Time Step t

**Input:** Learned forward dynamics model $\Phi$
predicted dynamics graph $\hat{G}_t$
current dynamics graph $G_t$
goal $G_g$, current estimation of the attributes $A$
current control inputs $\hat{u}_{t:T}$
states history $\hat{G} = \{G_i\}_{i=1}^t$
forward simulation time $N$ and time horizon $T$

**Output:** Controls $\hat{u}_{t:T}$, predicted next time step $\hat{G}_{t+1}$

Update $A$ by descending with the gradients
$\nabla_A L_s(G_t, G_t)$
for $i = 1, \ldots, N$
do
Forward simulation using the current graph
$\hat{G}_{t+1} \leftarrow \Phi(G_t)$
Make a buffer for storing the simulation results
$G \leftarrow G \cup \hat{G}_{t+1}$
for $j = t + 1, \ldots, T - 1$
do
Forward simulation
$\hat{G}_{j+1} \leftarrow \Phi(G_j)$; $G \leftarrow G \cup \hat{G}_{j+1}$
end for
Update $\hat{u}_{t:T}$ by descending with the gradients
$\nabla_{\hat{u}_{t:T}} L_g(G, G_g)$
end for
Return $\hat{u}_{t:T}$ and $\hat{G}_{t+1} \leftarrow \Phi(G_t)$

B DATA GENERATION

The data is generated using NVIDIA FleX.

**FluidFall.** We generated 3,000 rollouts over 120 time steps. The two drops of fluids contain 64 and 125 particles individually, where the initial position of one of the drop in the 3 dimensional coordinates is uniformly sampled between $(0.15, 0.55, 0.05)$ and $(0.25, 0.7, 0.15)$, while the other drop is uniformly sampled between $(0.15, 0.1, 0.05)$ and $(0.25, 0.25, 0.15)$.

**BoxBath.** We generated 3,000 rollouts over 150 time steps. There are 960 fluid particles and the rigid cube is consist of 64 particles. The fluid particle block is initialized at $(0, 0, 0)$, and the initial position of the rigid cube is randomly initialized between $(0.45, -0.0155, 0.05)$ to $(1.0, -0.0155, 0.15)$.

**FluidShake.** We generated 2,000 rollouts over 300 time steps. The height of the box is 1.0 and the thickness of the wall is 0.025. For the initial fluid cuboid, the number of fluid particles is uniformly sampled between 10 and 12 in the $x$ direction, between 15 and 20 in the $y$ direction, 3 in the $z$ direction. The box is fixed in the $y$ and $z$ direction, and is moving freely in the $x$ direction. We randomly place the initial $x$ position between -0.2 to 0.2. The sampling of the speed is implemented as $v = v + \text{rand}(-0.15, 0.15) - 0.1x$, in order to encourage motion smoothness and moving back to origin, where speed $v$ is initialized as 0.

**RiceGrip.** We generated 5,000 rollouts over 30 time steps. We randomize the size of the initial “rice” cuboid, where the length of the three sides is uniformly sampled between 8.0 and 10.0. The material property parameters $\text{clusterStiffness}$ is uniformly sampled between 0.3 and 0.7, $\text{clusterPlasticThreshold}$ is uniformly sampled between 5e-6 and 1e-4, and $\text{clusterPlasticCreep}$ is uniformly sampled between 0.1 and 0.3. The position of the gripper is randomly sampled within a circle of radius 0.5. The orientation of the gripper is always perpendicular to the line connecting the origin to the center of the gripper and the close distance is uniformly sampled between 0.7 to 1.0.
Of all the generated data, 90% of the rollouts are used for training, and the rest 10% are used for validation.

C TRAINING DETAILS

The models are implemented in PyTorch, and are trained using Adam optimizer (Kingma & Ba (2015)) with a learning rate of 0.0001. The number of particles and relations might be different at each time step, hence we use a batch size of 1, and we update the weights of the networks once every 2 forward rounds.

For all propagation networks used below, the object encoder $f_{encO}$ is an MLP with two hidden layers of size 200, and outputs a feature map of size 200. The relation encoder $f_{encR}$ is an MLP with three hidden layers of size 300, and outputs a feature map of size 200. The propagator $f_{O}$ and $f_{R}$ are both MLP with one hidden layer of size 200, in which a residual connection is used to better propagate the effects, and outputs a feature map of size 200. The propagators are shared within each stage of propagation. The motion predictor $f_{output}$ is an MLP with two hidden layers of size 200, and output the state of required dimension. ReLU is used as the activation function.

FluidFall. For the propagation network $\phi$ used in this environment, the propagation step $L$ is chosen as 2. The distance for determining the neighborhood $d$ is 0.08, and the model is trained for 13 epoches. The output of the model is the 3 dimensional velocity, which is multiplied by $\Delta t$ and added to the current position to do rollouts.

BoxBath. In this environment, four propagation networks are used due to the hierarchical modeling and the number of roots for the rigid cube is set as 8. $\phi_{LeafToLeaf}$ uses $L = 2$, $\phi_{LeafToRoot}$ uses $L = 1$, $\phi_{RootToRoot}$ uses $L = 2$, and $\phi_{RootToLeaf}$ uses $L = 1$. We have two separate motion predictor for fluids and rigid body, where the fluid predictor output velocity for each fluid particle, while the rigid predictor takes the mean of the signals over all its rigid particles as input, and output a rigid transformation (rotation and translation). The distance for neighborhood search $d$ is 0.08, and the model is trained for 5 epoches.

FluidShake. Only one propagation network is used in this environment, the propagation step is $L = 2$. The distance for neighborhood search $d$ is 0.08, and the model is trained for 5 epoches.

RiceGrip. Four propagation networks are used due the hierarchical modeling, which have the same length of propagation steps as in BoxBath, and the number of roots for the “rice” is set as 30. The distance $d$ for determining the neighborhood is chosen as 0.12, and the model is trained for 20 epoches.

D CONTROL DETAILS

$N_{sample}$ is chosen as 20 for all three cases, where we sample 20 random control sequences, and choose the best performing one as evaluated using our learned model. The evaluation is based on the Chamfer distance between the controlling result and the target configuration.

FluidShake. In this environment, the control sequence is the speed of the box along the x axis. The method to sample the candidate control sequence is the same as when generating training data of this environment. After selected the best performing control sequence, we first use RMSprop optimizer to optimize the control inputs for 10 iterations using a learning rate of 0.003. We then use model-predictive control to apply the control sequence to the FleX physics engine using Algorithm1, where $N$ is selected as 3.

RiceGrip. In this environment, we need to come up with a sequence of grip configurations, where each grip contains positions, orientation, and closing distance. The method to sample the candidate control sequence is the same as when generating training data of this environment. After selected the best performing control sequence, we first use RMSprop optimizer to optimize the control inputs for 20 iterations using a learning rate of 0.003. We then use model-predictive control to apply the control sequence to the FleX physics engine using Algorithm1, where $N$ is selected as 5.
RiceGrip in Real World. In this environment, we need to come up with a sequence of grip configurations, where each grip contains positions, orientation, and closing distance. The method to sample the candidate control sequence is the same as when generating training data of RiceGrip, and $N_{\text{fill}}$ is chosen as 768. Different from the previous case, the physical parameters are always unknown and has to be estimated online. After selected the best performing control sequence, we first use RMSprop optimizer to optimize the control inputs for 20 iterations using a learning rate of 0.003. We then use model-predictive control to apply the control sequence to the real world using Algorithm 1, where $N$ is selected as 10.