Design steps for bulk micro machined single axis silicon capacitive accelerometer with optimised device dimensions

Vivek Agarwal, Tarun K Bhattacharayya and Subhadeep Banik

Department of Electronics & EC Engineering, Indian Institute of Technology, Kharagpur, India

E-mail: viveksrijan@yahoo.co.in and tkb@ece.iitkgp.ernet.in

Abstract. Design methodology of MEMS device for reducing the product development time and cost have been proposed in this paper. Design process starts with development of mathematical models and validation of the models by the finite element analysis using the CoventorWare software. Device dimension optimisation was done using the evolutionary optimisation techniques.

1. Introduction
Depending on the application and specification, the sensor structure and the pick off technique are decided. MEMCAD tools facilitate the design of sensor based on finite element analysis (FEA) [1]. To achieve the targeted specifications a number of computational intensive simulations are done. Each simulation requires building a three-dimensional model and meshing of the structure. The present design approach for MEMS devices is time consuming with high computational cost. Use of simplified mathematical models and the evolutionary optimization techniques, reduce the development time and costly fabrication reruns.

Proposed design approach is presented for bulk micro machined single axis silicon capacitive accelerometer and is shown in the form of flow chart attached as Appendix ‘A’.

2. Capacitive accelerometer
Device is fabricated using three wafers (glass, silicon, glass) bonded together one on top of the other. The top and bottom wafer form the fixed electrode and the middle wafer constitutes the movable electrode by means of a proof mass attached to the frame by beams. Acceleration acting on the device will result in an inertial force, causing the proof mass to move up or down depending on the direction of the applied acceleration. Restoring force by the beams decide the deflection of the proof mass. For low cross-axis sensitivity, structures selected for analyses are symmetrical along all the three axes (x, y and z). Normally the proof mass thickness is of an order higher than the thickness of the beams therefore in order to achieve structural symmetry in the z-axis the beams are fabricated in the central plane of the proof mass [2]. Two commonly used configurations, the cantilever-supported mass and the bridge supported mass are shown in figure 1. The targeted specifications are given in Table 1.
Table 1. Targetted specification for the design of bulk micro machined capacitive accelerometer.

| Parameter          | Value                                      |
|--------------------|--------------------------------------------|
| Range              | Input Range ±10g                           |
| Non linearity      | 0 to FS ±1% of FS                          |
| Natural Frequency  | 100Hz (min)                                |
| Damping Ratio      | 0.7 to 1.2                                 |
| Operating Temperature Range | -40ºC to +85ºC                            |

3. Mathematical modelling

Differential change of capacitance due to deflection of proof mass can be used to estimate the sensitivity, resolution and non-linearity of the sensor. Deflection of beams and the proof mass can be estimated by double integration or successive integration method [3]. The dynamic behavior is analyzed by determining the natural frequency and the damping force [4].

The proof mass shape is hexagonal due to anisotropic etching from both sides of the wafer. Its mass can be obtained by integration resulting in

\[ m = \iiint_{V} \rho \, dxdydz = \frac{\rho \, h_2 \, (l_2^3 - l_1^3)}{6(l_2^3 - l_1^3)} \]

where, \( \rho \) is the density of the silicon (2.33x10^3 kg m^-3), \( h_2 \) is the thickness of the silicon wafer, \( l_1 \) is the side length of the proof mass at the truncated top and \( l_2 \) is the side length of the proof mass at the center where the beams are attached to the proof mass.

3.1. Cantilever Structure

When the accelerometer is subjected to acceleration in the sensitive direction movement of the proof mass is not parallel to the fixed electrodes because of tilting. Assuming that the mass of the beam is much negligible and that the proof mass is rigid i.e. no bending takes place. For small deflections the deflection of the proof mass end in a single beam structure can be expressed as

![Figure 1. Different structures analysed for design of bulk micro machined capacitive accelerometer](image)

(a) Single beam cantilever structure  (b) Two beam cantilever structure
(c) Two beam bridge structure  (d) Four beam bridge structure
(e) Eight beam cross bridge structure
where, $E$ is the modulus of elasticity for silicon $(1.7 \times 10^{11}$ Pa), $a$ is the normal acceleration, $h_1$ is the thickness of beam, $b_1$ is the width of the beam, $a_1$ is the length of beam, $a_2$ is the distance of the proof mass end from the frame and $L$ is the distance of the proof mass center from the support. As can be seen, the deflection of proof mass end is linearly related to the force applied to the mass.

For a two-beam symmetrical cantilever structure the proof mass structure can be visualized as suspended by two springs in parallel. Hence, the deflection will be half of that of a single beam cantilever structure for a given inertial force and can be estimated from the above expression.

In case of a single beam cantilever structure lateral acceleration in the direction perpendicular to the beam direction will cause the proof mass to rotate thereby twisting the beam suspension. For low cross axis sensitivity two-beam structure is recommended. The beams should be located close to the corners of the proof mass. Separation of the beams is limited by the corner compensation techniques used in fabrication.

### 3.2. Bridge structure

When the device is subjected to acceleration the movement of the proof mass in a bridge structure is parallel to the fixed electrode. As a result the sensitivity of the bridge structures is expected to be higher than the cantilever structures. Assuming that the mass of the beam is much negligible and that the proof mass is rigid i.e. no bending takes place. For small deflections the deflection of the proof mass in a two-beam structure can be expressed as

$$W_2(a_2) = \frac{2ma}{E b_1 h_1} \left[ (5L a_1) - \left( \frac{5}{2} a_1^2 \right) - \left( \frac{12}{5} L^2 \right) a_1 \right]$$

From the above expression it can be seen that the deflection of proof mass is linearly related to the applied acceleration.

To avoid rotation of the proof mass due to lateral acceleration four beam or eight-beam structure is recommended. Four-beam structure can be as shown in figure 1 or in a crossbeam form.

### 3.3. Sensitivity and nonlinearity

In a cantilever structure when the device is subjected to acceleration in the sensitive direction the movement of the proof mass is not parallel to the fixed electrodes because of tilting. The new value of capacitance between the movable electrode and the top ($C_1$), bottom ($C_2$) fixed electrodes can be found out by integration along the length of the proof mass and expressed as

$$C_1 = \int_{a_1}^{a_2} \frac{e E_0 b_2}{d_0 - W_2(x)} dx$$

and

$$C_2 = \int_{a_1}^{a_2} \frac{e E_0 b_2}{d_0 + W_2(x)} dx$$

where, $d_0$ is the distance of the movable electrode from the fixed electrodes at rest and $b_2$ is the width of the proof mass. However, in a bridge structure the proof mass movement is parallel to the fixed electrode and the differential change of capacitance can easily be obtained from the deflection of the proof mass. Taking the linear part of the relationship between the differential change in capacitance ($\Delta C$) with acceleration, mathematical expression for sensitivity in terms of device dimensions can be found.

For the present work nonlinearity is defined as deviation of the straight line connecting the two end points over the operation range from the input output calibration curve. Mathematically nonlinearity at a specific acceleration ($a_i$) for the maximum input acceleration ($a_m$) can be expressed as

$$NL_i = \left( \frac{\Delta C(a_i) - \Delta C(a_m)}{\Delta C(a_m)} \right) \times 100\%$$
From the above expression value of applied acceleration for which the maximum value of nonlinearity occurs is found. The expression for this value of \(a_i\) is used for optimization runs.

3.4. Natural frequency and damping force

Bridge structure can be approximated as a single degree of freedom system. Assuming the deflection of the proof mass is small, so that the spring force is in the linear region of operation. From the expression of deflection of proof mass the value of spring constant (k) can be found. From the value of k and m natural frequency of vibration (f) can be expressed as

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{2Eb_lh_1^3}{m\bar{a}_l}}
\]

Simple relations cannot be used for finding the vibration frequency of a cantilever structure. For a multi degree of freedom system the procedure known as Rayleigh method was used to find the natural frequency. Usually, the frequencies of higher harmonics are much higher than those of the fundamental vibration and their effects can be neglected.

For the structure of micro-transducers, the ratio of surface area to volume is large so that air damping often plays an important role in determining the damping ratio of the system. The basic mechanism for air damping for bulk micro machined accelerometers is squeeze film air damping. The damping force developed is against the motion of the proof mass. The mathematical model for damping ratio has been reported earlier [5]. The measures used to reduce squeeze-film damping for MEMS device includes:

- Increasing the gap between the electrodes.
- Packaging the structure in rarefied air / medium.
- Perforating the proof mass.

4. Finite element analysis

For FEA silicon properties were taken as an-isotropic (table 2) whereas for the hand calculations silicon is approximated as an isotropic material with a value of elastic constant (E) as 1.7E+11 Pa [6]. The value of ‘E’ taken for hand calculations is quite accurate as the beam direction is expected to be (110) after fabrication. Density of Silicon has been taken as 2330 Kg/m³ both for simulation and hand calculations. One of the results of verification on a test sample using FEA is given in table 3 and 4. Since the results matched quite closely the building of number of three-dimensional models, meshing and simulation was greatly reduced. The next step was optimization of the different structure dimensions using the mathematical models discussed earlier.

| Orientation | Young’s Modulus (E) (GPa) |
|-------------|---------------------------|
| [100]       | 129.5                     |
| [110]       | 168.0                     |
| [111]       | 186.5                     |

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Table 3. Verification of mathematical models for single beam cantilever structure.

| Acceleration (m/s²) | Deflection of proof mass end using FEA (μm) | Deflection of proof mass end using models (μm) |
|---------------------|---------------------------------------------|---------------------------------------------|
| 10                  | 1.96x10⁰                                   | 1.99x10⁰                                   |
| 20                  | 3.92x10⁰                                   | 3.98x10⁰                                   |
| 30                  | 5.87x10⁰                                   | 5.97x10⁰                                   |

Table 4. Verification of mathematical models for dynamic behavior of single beam cantilever structure

| Property                      | Using FEA  | Using mathematical model |
|-------------------------------|------------|--------------------------|
| Natural frequency of vibration| 842 Hz     | 854 Hz                   |
| Damping coefficient           | 3.83       | 3.87                     |
5. Optimisation

The challenge to minimize the die area on the wafer without sacrificing the performance and reduce design cycle time necessitated the use of evolutionary optimization technique. Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) are used as design tools and problem solvers. For the present work particle swarming optimization (PSO) technique was used [7]. PSO is based upon social swarm behavior. With the mathematical models as input the cantilever and bridge structures were optimized to achieve the targeted specifications. The solution space was defined as per the fabrication limits and also to keep the die size within reasonable limits. For example the minimum beam thickness was kept as 40 µm for ease of wafer handling during fabrication. Based on the wafer thickness of the proof mass thickness was fixed as 575 µm. During the many optimization runs it was observed that the square proof mass was an optimal solution. The optimized dimensions of the different structures are given in table 5 and the expected device performance is given in table 6.

| Table 5. Optimized device dimensions for cantilever and bridge structure |
|---------------------------------------------------------------|
| Device Dimensions | Two Beam Cantilever | Eight Beam Bridge |
| Length of the beam | 700 µm | 1400 µm |
| Width of the beam | 250 µm | 250 µm |
| Thickness of the beam | 45 µm | 40 µm |
| Length of the mass | 2500 µm | 5300 µm |
| Width of the mass | 2500 µm | 5300 µm |
| Thickness of the mass | 575 µm | 575 µm |
| Distance between electrodes | 19 µm | 3 µm |

| Table 6. Expected device performance |
|---------------------------------------|
| Parameters | Two Beam Cantilever | Eight Beam Cross Bridge |
| Sensitivity | 0.00122 | 0.0155 |
| Non-Linearity | 0.927% | 0.934% |
| Natural Frequency | 934 Hz | 2.31 kHz |
| Damping Ratio | 0.876 | 404 |
| Deflection at 10g | 4.1 µm | 0.466 µm |

6. Result

Comparing the expected performance of two structures in table 6 it can be deduced that,

- The damping ratio is excessively high in a bridge structure. To reduce the damping ratio following are the recommended design steps:
  (a) The proof mass needs to be perforated.
  (b) The device needs to be hermetically sealed to control the pressure inside.
  (c) The interface circuitry needs to be forced feedback type.
- The die size in the bridge structure is quite large.
- Distance between the electrodes in a bridge structure is 3 µm, fabrication process for which could be quite challenging especially during bonding of the three wafers.
- The sensitivity of the bridge structure is high compared to the cantilever structure. This was expected as seen while developing the mathematical models.

Cantilever structure is expected to meet the targeted specifications with considerable ease of fabrication steps. Further, the associated interface circuitry design is relaxed which can be of open loop. The structure finalized was two-beam cantilever with no perforations in the proof mass.

7. Conclusion

The proposed approach helped in identifying the structure meeting the targeted specifications with minimum simulation effort. Relationship between device dimension and functional specifications was
visible with mathematical models. Device dimensions were optimized for the given foundry constraints. Hermetical sealing of device could be avoided thereby relaxing the fabrication process.

Appendix A. Accelerometer design flow chart

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