Extended non-chiral quark models confronting QCD

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Abstract

We discuss the low energy effective action of QCD in the quark sector. When it is built at the CSB (chiral symmetry breaking) scale by means of perturbation theory it has the structure of a generalized Nambu-Jona-Lasinio (NJL) model with CSB due to attractive forces in the scalar channel. We show that if the lowest scalar meson state is sufficiently lighter than the heavy pseudoscalar \( \pi' \) then QCD favors a low-energy effective theory in which higher dimensional operators (of the Nambu-Jona-Lasinio type) are dominated and relatively strong. A light scalar quarkonium (\( m_\sigma = 500 \div 600 \) MeV) would provide an evidence in favor to this NJL mechanism.

Thus the non-chiral Quasilocal Quark Models (QQM) in the dynamical symmetry-breaking regime are considered as approximants for low-energy action of QCD. In the mean-field (large-\( N_c \)) approach the equation on critical coupling surface is derived. The mass spectrum of scalar and pseudoscalar excited states is calculated in leading-log approach which is compatible with the truncation of the QCD effective action with few higher-dimensional operators. The matching to QCD based on the Chiral Symmetry Restoration sum rules is performed and it helps to select out the relevant pattern of CSB as well as to enhance considerably the predictability of this approach.
INTRODUCTION: DEFINITION OF QQM

The low energy effective action of QCD in the quark sector has a qualitatively different structure depending on whether it is built at the CSB scale by means of perturbation theory or below the CSB scale when the major chiral symmetry breaking effect - the formation of light pseudoscalar mesons - is implemented manifestly. In the first case the models [1] extend the Nambu-Jona-Lasinio one [2] (see the reviews [3, 4, 5, 6, 7, 8, 9, 10, 11]) with chiral symmetry broken due to strong attractive 4-fermion forces in the color-singlet scalar channel. In the second case the resulting model[12] is a generalization of the chiral quark model with a built-in constituent quark mass and the non-linear realization of chiral symmetry. This type of QCD effective action is not discussed in our talk although it may be more relevant [12, 13, 14] if the quarkonium scalar meson is sufficiently heavy [15, 16].

The quasilocal approach of [1] (see also [17, 18, 19]) represents a systematic extension of the NJL model towards the complete effective action of QCD where many-fermion vertices with derivatives are incorporated with the manifest chiral symmetry of interaction, motivated by the soft momentum expansion of the perturbative QCD effective action. For sufficiently strong couplings, the new operators promote the formation of additional scalar and pseudoscalar states. These models allow an extension of the linear $\sigma$ model provided by the NJL model, with the pion being a broken symmetry partner of the lightest scalar meson just as before, and with excited pions and scalar particles coming in pairs. In particular, when only scalar and pseudoscalar color-singlet channels are examined and dynamical quark masses are supposed to be sufficiently smaller than the CSB cutoff one may derive the minimal two-channel lagrangian of the QQM in the separable form [1, 19]:

$$L^{QQM} = \bar{q}i\gamma^\mu \partial_\mu q + L^I;$$

$$L^I = \frac{1}{4 N_f N_c \Lambda^2} \sum_{k,l=1}^{2} a_{kl} \left[ \bar{q}f_k(\hat{s})q \, \bar{q}f_l(\hat{s})q - \bar{q}f_k(\hat{s})q \, \tau^a \gamma_5 q \, \bar{q}f_l(\hat{s})q \, \tau^a \gamma_5 q \right], \quad (1)$$

where $\hat{q} \equiv \gamma^\mu \partial_\mu$, $a_{kl}$ represents a symmetric matrix of real coupling constants and polynomial formfactors are chosen as follows:

$$f_1(\hat{s}) = 2 - 3\hat{s}; \quad f_2(\hat{s}) = -\sqrt{3}\hat{s}; \quad \hat{s} \equiv -\frac{\partial^2}{\Lambda^2}. \quad (2)$$

As this model interpolates the low-energy QCD action it is supplied with the cutoff $\Lambda \sim 1$ GeV which bounds virtual quark momenta in quark loops. We restrict ourselves with consideration of two-flavor case, thus $\tau_a$ denote Pauli matrices.

A somewhat different, nonlocal approach to describe excited meson states was developed in [20, 21].

EFFECTIVE POTENTIAL AND MESON SPECTRUM

For strong four-fermion coupling constants $a_{kl} \sim 8\pi^2\delta_{kl}$ the lagrangian (1) reveals the phenomenon of dynamical chiral symmetry breaking. This phenomenon can be de-
scribed with the help of the effective potential for the attractive scalar channel where scalar mesons arise as composite states. Indeed its non-trivial minimum gives rise to a dynamical quark mass and the perturbative fluctuations around this minimum characterize the mass spectrum of meson states. To derive the required effective potential one should bosonize the quark action, i.e. incorporate auxiliary bosonic variables: \( \sigma_k \sim i\bar{q}_k(\hat{s})q, \quad \pi^a_k \sim \bar{q}_k(\hat{s})\pi^a\gamma_5 q \) and integrate out fermionic degrees of freedom. At the first step we introduce the bosonic variables in two channels:

\[
L_I = \sum_{k=1}^{2} i\bar{q}_k (\sigma_k + i\gamma_5 \pi^a_k \tau^a) f_k(\hat{s})q + N_f N_c \Lambda^2 \sum_{k,l=1}^{2} \left( \sigma_k a^{-1}_{kl} \sigma_l + \pi^a_k a^{-1}_{kl} \pi^a_l \right). \tag{3}
\]

Let us parametrize the matrix of coupling constants in a close vicinity of tricritical point:

\[
8\pi^2 a^{-1}_{kl} = \delta_{kl} - \frac{\Delta_{kl}}{\Lambda^2}, \quad |\Delta_{kl}| \ll \Lambda^2. \tag{4}
\]

The last inequality turns out to be equivalent to require the dynamical mass to be essentially less than the cutoff.

After integrating out the quark fields one comes to the bosonic effective action \( \mathcal{W}(\sigma_k, \pi^a_k) \) and therefrom, for constant meson variables, to the effective potential:

\[
V_{eff} = \frac{N_c N_f}{8\pi^2} \left( -\sum_{k,l=1}^{2} \sigma_k \sigma_l \Delta_{kl} - (\pi^a_2)^2 \Delta_{22} + 8(\sigma_1)^4 \left( \ln \frac{\Lambda^2}{4(\sigma_1)^2} + \frac{1}{2} \right) \right.
\]

\[
- \frac{159}{8} (\sigma_1)^4 - \frac{5\sqrt{3}}{2} \sigma_1^3 \sigma_2 + \frac{9}{4} \sigma_1^2 \sigma_2^2 + \frac{\sqrt{3}}{2} \sigma_1 \sigma_2^3 + \frac{9}{8} (\sigma_2)^4
\]

\[
+ \left( \frac{3}{4} \sigma_1^3 + \frac{\sqrt{3}}{2} \sigma_1 \sigma_2 + \frac{9}{4} \sigma_2^2 \right) (\pi^a_2)^2 + \frac{9}{8} (\pi^a_2)^4 \right) + O \left( \frac{\ln \Lambda}{\Lambda^2} \right), \tag{5}
\]

for the fixed direction of chiral symmetry breaking \( \pi^a_1 = 0 \).

The QCD inspired action should not, of course, induce the isospin symmetry breaking and therefore a non trivial solution for v.e.v is expected to be in the scalar channel, \( <\pi^a_2> = 0 \). It implies the following inequality to hold:

\[
\frac{3}{4} \sigma_1^3 + \frac{\sqrt{3}}{2} \sigma_1 \sigma_2 + \frac{9}{4} \sigma_2^2 > \Delta_{22}.
\]

The conditions on extremum of the effective potential (3), the mass-gap equations,

\[
\Delta_{11} \sigma_1 + \Delta_{12} \sigma_2 = 16 \sigma_1^2 \ln \frac{\Lambda^2}{4\sigma_1} - \frac{159}{4} \sigma_1^3 - \frac{15\sqrt{3}}{4} \sigma_1^2 \sigma_2 + \frac{9}{4} \sigma_1 \sigma_2^2 + \frac{\sqrt{3}}{4} \sigma_2^3
\]

\[
\Delta_{12} \sigma_1 + \Delta_{22} \sigma_2 = - \frac{5\sqrt{3}}{4} \sigma_1^3 + \frac{9}{4} \sigma_1^2 \sigma_2 + \frac{3\sqrt{3}}{2} \sigma_1 \sigma_2^2 + \frac{9}{4} (\sigma_2)^3, \tag{6}
\]

allow to find the relations between the components of dynamical mass function and (reduced) coupling constants \( \Delta_{kl} \). In practice, one uses the v.e.v.’s of scalar fields as input parameters, in particular, \( 2\sigma_1 = m_{dyn} = 200 \div 300 \text{ MeV} \), and determines the required \( \Delta_{kl} \).
The second variation of effective action in the vicinity of above v.e.v.,

\[
\begin{align*}
\frac{\delta^2 W}{\delta \sigma_k(p) \delta \sigma_l(p')} &= \frac{N_c N_f}{8\pi^2} (A_{kl}^\sigma p^2 + B_{kl}^\sigma) \delta^{(4)}(p + p'); \\
\frac{\delta^2 W}{\delta \pi_k(p) \delta \pi_l(p')} &= \frac{N_c N_f}{8\pi^2} (A_{kl}^\pi p^2 + B_{kl}^\pi) \delta^{(4)}(p + p'),
\end{align*}
\]

(7)

brings both the kinetic terms \(\sim A_{kl}^{\sigma,\pi}\) and the mass matrix \(B_{kl}^{\sigma,\pi}\) which represents the second derivative of the effective potential (5) (see their general structure in [22]).

The kinetic matrices \(A_{kl}^{\sigma,\pi}\) take the form:

\[
A_{kl}^\sigma \approx \begin{pmatrix} 4 \ln \frac{\Lambda^2}{4\sigma_1^2} - \frac{23}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{3}{2} \end{pmatrix},
\]

(8)

Let us now display the matrix of second variations \(B_{kl}^{\sigma,\pi}\):

\[
\begin{align*}
B_{11}^\sigma &= -2\Delta_{11} + 96\sigma_1^2 \ln \left( \frac{\Lambda^2}{4\sigma_1^2} \right) - \frac{605}{2} \sigma_1^2 - 15\sqrt{3}\sigma_1\sigma_2 + \frac{9}{2} \sigma_2^2, \\
B_{12}^\sigma &= -2\Delta_{12} - \frac{15\sqrt{3}}{2} \sigma_1^2 + 9\sigma_1\sigma_2 + \frac{3\sqrt{3}}{2} \sigma_2^2, \\
B_{22}^\sigma &= -2\Delta_{22} + \frac{9}{2} \sigma_1^2 + 3\sqrt{3}\sigma_1\sigma_2 + \frac{27}{2} \sigma_2^2, \\
B_{11}^\pi &= -2\Delta_{11} + 32\sigma_1^2 \ln \left( \frac{\Lambda^2}{4\sigma_1^2} \right) - \frac{159}{2} \sigma_1^2 - 5\sqrt{3}\sigma_1\sigma_2 + \frac{3}{2} \sigma_2^2, \\
B_{12}^\pi &= -2\Delta_{12} - \frac{5\sqrt{3}}{2} \sigma_1^2 + 3\sigma_1\sigma_2 + \frac{\sqrt{3}}{2} \sigma_2^2, \\
B_{22}^\pi &= -2\Delta_{22} + \frac{3}{2} \sigma_1^2 + \sqrt{3}\sigma_1\sigma_2 + \frac{9}{2} \sigma_2^2.
\end{align*}
\]

(9)

Their diagonalization allows to find the masses of meson states. One recovers two states in the scalar channel and two triplet states in the pseudoscalar one. The lightest multiplet consists of the massless pion, \(m_\pi = 0\) and the NJL scalar meson, \(m_\sigma = 4\sigma_1 = 2m_{\text{dyn}}\) in the leading-log approach. The masses of heavier mesons depend essentially on the pattern of CSB in the vicinity of tricritical point. But in the leading-log approach they are approximately equal, \(m_\sigma^2, \sim m_\pi^2, \sim -\frac{4}{3}\Delta_{22} = \mathcal{O}(\log \Lambda)\). The last estimation follows from the mass-gap eqs.(6). A more precise relation takes places:

\[
\begin{align*}
m_\sigma^2 &\approx -\frac{4}{3}\Delta_{22} + \sigma_1^2 + \frac{2\sqrt{3}}{3}\sigma_1\sigma_2 + 3\sigma_2^2; \\
m_\pi^2 &\approx -\frac{4}{3}\Delta_{22} + \sigma_1^2 + \frac{4\sqrt{3}}{3}\sigma_1\sigma_2 + 6\sigma_2^2 > 0.
\end{align*}
\]

(10)

CSR RULES

Let us employ the constraints based on Chiral Symmetry Restoration (CSR) in QCD at high energies. We consider two-point correlators of color-singlet quark currents in
Euclidean space-time,

$$\Pi_C(p^2) = \int d^4x \exp(ipx) \langle T(\bar{q}\Gamma q(x) \bar{q}\Gamma q(0)) \rangle,$$

restricting ourselves in this talk with

$$C \equiv S, P; \quad \Gamma = i, \gamma_5 \tau^a.$$ (11)

In the chiral limit the scalar correlator $\Pi_S$ and the pseudoscalar one $\Pi_P^{\alpha \alpha}$ approach to each other rapidly as their O.P.E.’s [23, 24] coincide at all orders in perturbation theory and, as well, in the non-perturbative, purely gluonic part [12, 25],

$$(\Pi_P^{\alpha \alpha}(p^2) - \Pi_S(p^2)) \sim \frac{\Delta_{SP}}{p^4} + \mathcal{O}\left(\frac{1}{p^6}\right), \quad \Delta_{SP} \simeq 24\pi \alpha_s < \bar{q}q >^2,$$ (13)

where $< \bar{q}q >$ stands for the quark condensate and the vacuum dominance hypothesis [23] is exploited for the estimation of four-quark condensates as we follow the large-$N_c$ limit. Meantime, in the latter limit the correlators are well saturated by narrow resonances,

$$\Pi_P^{\alpha \alpha}(p^2) - \Pi_S(p^2) = \sum_n \left[ \frac{Z^P_n}{p^2 + m^2_{P,n}} - \frac{Z^S_n}{p^2 + m^2_{S,n}} \right].$$ (14)

As the difference decreases rapidly, one can assume that the lower lying resonances will dominate in the above sum.

The outcoming CSR rules in the two-channel model (1) read:

$$Z_\sigma + Z_{\sigma'} = Z_\sigma + Z_{\sigma'}; \quad Z_\sigma m^2_\sigma + Z_{\sigma'} m^2_{\sigma'} = Z_{\sigma'} m^2_{\sigma'} + \Delta_{SP}.$$ (15)

The first relation can be fulfilled in the (one-channel) NJL model which corresponds to the one-resonance ansatz, $Z_{\sigma',\sigma'} = 0$, whereas the last one can be valid only in a two-resonance model, for the $\Delta_{SP}$ defined in (13) (see [13]).

**CONSTRAINTS ON MESON PARAMETERS**

The relevant correlators and the values of residues $Z_i$ can be found by variation of the external sources $S_k, P^a_k$ which couple to the scalar and pseudoscalar quark densities. The structure of the corresponding operators in the quark lagrangian is completely analogous to the Yukawa vertex in (3). Then the effect of external sources can be separated by shifting the scalar fields in the quark vertex of (3) and further on by integrating out the quark fields. As a result the effective action for generating of two-point correlators is parametrized in terms of the second variation matrix (7):

$$\mathcal{W}^{(2)} \simeq \frac{N_c N_f \Lambda^2}{8\pi^2} \sum_{k,l=1}^2 \left( S_k \Gamma^{(\sigma)}_{kl} S_l + P^a_k \Gamma^{(\pi)}_{kl} P^a_l \right),$$

$$\Gamma^{(\sigma)}_{kl} = \delta_{kl} - 2\Lambda^2 \left( A^\sigma p^2 + B^\sigma \right)_{kl}^{-1}; \quad \Gamma^{(\pi)}_{kl} = \delta_{kl} - 2\Lambda^2 \left( A^\pi p^2 + B^\pi \right)_{kl}^{-1}. $$ (16)
In particular, the strictly local quark densities can be presented as a superposition of two currents:
\[ \bar{q}\Gamma q = \frac{1}{2} \left( \bar{q}f_1 q - \sqrt{3} \bar{q}f_2 q \right); \quad \Gamma \equiv i\gamma_5 \tau^a. \] (17)

Respectively, their two-point correlator in the scalar channel, \( \Pi_s(p^2) \) reads,
\[ \Pi_s(p^2) = -\frac{N_c A^2}{2\pi^2} + \frac{Z_\sigma}{p^2 + m_\sigma^2} + \frac{Z_{\sigma'}}{p^2 + m_{\sigma'}^2}; \]
\[ Z_\sigma \simeq -\frac{N_c A^4}{12\pi^2 m_\sigma^2} \ln \frac{A^2}{4\sigma_1^2} \left[ -48\sigma_1^2 \ln \frac{A^2}{4\sigma_1^2} + 3\Delta_{11} + 2\sqrt{3}\Delta_{12} + \Delta_{22} \right]; \]
\[ \simeq -\frac{N_c A^4 \Delta_{22}(\sigma_1 - \sqrt{3}\sigma_2)^2}{12\pi^2 m_\sigma^2 \sigma_1^2 \ln \frac{A^2}{4\sigma_1^2}}; \]
\[ Z_{\sigma'} + Z_\sigma = \frac{N_c A^4}{2\pi^2} \equiv Z_0. \] (18)

These relations are derived with the help of mass-gap eqs.(6).

To the first order in the leading-log approach, the weak decay coupling constant for pion can be found from the gauged second variation:
\[ F_\pi^2 \simeq \frac{N_c \sigma_1^2}{\pi^2} \ln \frac{A^2}{4\sigma_1^2}, \] (19)
and it coincides with that one of the NJL model. Respectively the value of quark condensate can be expressed in terms of v.e.v. of \( \sigma_i \),
\[ <\bar{q}q> \simeq -\frac{N_c A^2}{8\pi^2} (\sigma_1 - \sqrt{3}\sigma_2). \] (20)

Thus taking these equations into account and remembering the leading order of the \( \pi' \)-meson mass \( (19) \) one arrives to the remarkable relation:
\[ Z_\sigma \simeq 4 \frac{<\bar{q}q>^2}{F_\pi^2}. \] (21)

Let us examine the two-point correlator of local densities \( (17) \) in the pseudoscalar channel:
\[ \Pi_\pi(p^2) = -\frac{N_c A^2}{2\pi^2} + \frac{Z_\pi}{p^2} + \frac{Z_{\pi'}}{p^2 + m_{\pi'}^2}; \]
\[ Z_\pi \simeq Z_\sigma \quad \text{for} \quad m_{\pi'}^2 \simeq m_{\sigma'}^2; \]
\[ Z_{\pi'} + Z_\pi = Z_0; \quad \text{for} \quad Z_{\pi'} \simeq Z_{\sigma'}. \] (22)

The equality of \( Z_\pi \) and \( Z_\sigma \) in eq.(21) realizes both the approximate restoration of chiral symmetry in each multiplet and the fulfillment of PCAC requirement \( (21) \) for the residue in the pion pole.

We stress that the residues in poles are of different order of magnitude:
\[ Z_\sigma \sim Z_\pi = \mathcal{O} \left( \frac{Z_0}{\ln \frac{A^2}{4\sigma_1^2}} \right) \ll Z_{\sigma'} \sim Z_{\pi'} = \mathcal{O} (Z_0). \] (23)
Now we are able to impose and check the CSR constraints (15). The leading asymptotics represents the generalized $\sigma$-model relation and is automatically fulfilled:

$$Z_\sigma + Z_{\sigma'} = Z_\pi + Z_{\pi'} = Z_0,$$

that in fact reflects the manifest chiral symmetry of the QQM lagrangian.

As to the second constraint the possibility to satisfy it depends on the value of the QCD coupling constants $\alpha_s$. Indeed, it can be written by means of (10) as follows:

$$m_{\sigma'}^2 - m_{\sigma''}^2 \simeq \frac{\Delta_{SP}^2}{Z_0};$$

$$\sigma_1^2 + \frac{2\sqrt{3}}{3} \sigma_1 \sigma_2 + 3 \sigma_2^2 \simeq \frac{3N_c \alpha_s}{8\pi} (\sigma_1 - \sqrt{3} \sigma_2)^2. \quad (25)$$

As the left part is always positive there exists a lower bound for $\alpha_s \geq \frac{8\pi}{9N_c}$ providing solutions of the constraint. The lowest value of $\alpha_s \simeq 0.9$ is given by $\sigma_1 = -\sqrt{3} \sigma_2$ and for these v.e.v.’s one obtains the following splitting between the $\sigma'$- and $\pi'$-meson masses:

$$m_{\sigma'}^2 - m_{\sigma''}^2 \simeq \frac{8}{3} \sigma_1^2 = \frac{1}{6} m_\sigma^2; \quad (26)$$

i.e. for $m_\sigma = 500 \div 600 MeV$ these masses practically coincide, $m_{\sigma'} \simeq m_{\pi'} = 1300 MeV$ and such a $\sigma'$-meson may be identified [26] with $f_0(1300)$. The above value of $\alpha_s$ lies in the region of rather strong coupling where next-to-leading corrections to the anomalous dimension of four-quark operator in (13) are not negligible, $\sim \frac{\alpha_s}{\pi} \sim 0.3$ and should be systematically taken into account to obtain a reasonable precision. However the very fact that one has to match to QCD asymptotics at a scale $\mu \sim 600 MeV \sim m_\sigma$ lower than the masses of heavy resonances is not troublesome as it relates just coefficients of $1/p^2$ expansion irrespectively of how high is the momentum. On the other hand the matching should be performed in the region where one can neglect even more heavier resonances, i.e. at a scale $\leq 1$ GeV.

**CONCLUSIONS**

1. We have shown that the quasilocal quark models truncating (perturbative) low-energy QCD effective action can serve to describe the physics of heavy meson resonances. The matching to nonperturbative QCD based on the chiral symmetry restoration at high energies improves the predictability of such models. QQM extend the NJL model and inevitably contain a rather light scalar meson which is however not excluded by the particle phenomenology [26].

2. The fast convergence in QQM of mass spectra and other characteristics of heavy parity doublers entail their decoupling from the low-energy pion physics. For instance, let us calculate the dim-4 chiral coupling constant [27, 28]:

$$L_8 = \frac{F_\pi^4}{64 < \bar{q}q >^2} \left( \frac{Z_\sigma}{m_\sigma^2} + \frac{Z_{\sigma'}}{m_{\sigma'}^2} - \frac{Z_{\pi'}}{m_{\pi'}^2} \right) \simeq \frac{F_\pi^2}{16 m_\sigma^2} \left( 1 - \frac{6 \alpha_s \pi F_\pi^2 m_\sigma^2}{m_{\pi'}^2} \right). \quad (27)$$
The second term represents the net effect of heavy resonances after the CSR constraints \([15]\) have been imposed. It is easy to find that its relative contribution is less than 2%. Therefore this constant is essentially determined in QQM by the lightest scalar meson. Its value, \(L_8 = (0.9 \pm 0.4) \times 10^{-3}\) from \([27, 29, 30]\) nearly accepts \(m_\sigma \simeq 600\text{MeV}\).

3. Some disadvantage of QQM as well as of the original NJL model is that they presuppose the large, critical values of four-quark coupling constants which is difficult to justify with perturbative calculations in QCD. As well the CSR matching has to be performed at a scale where the QCD coupling constant is rather large and the perturbation theory is unreliable. The Extended Chiral Quark Models \([12, 13]\) seem to be free of these shortcomings and are able to adjust the light scalar meson mass to be of order 1 GeV. However the final choice between them may be done by the fit of a larger variety of meson characteristics which is in progress.

4. Let us comment the approximations used to derive the meson characteristics: namely, the large \(N_c\) and leading-log approximations. The first one is equivalent \([31, 32]\) to the neglect of meson loops. The second one fits well the quarks confinement as quark-antiquark threshold contributions are suppressed in two-point functions in the leading log approximation. The accuracy of this approximation is controlled also by the magnitudes of higher dimensional operators neglected in QQM, \(i.e.\) by contributions of heavy mass resonances not included into QQM. All these approximations are mutually consistent. In particular, in the effective action without gluons the quark confinement should be realized with the help of an infinite number of quasilocal vertices with higher-order derivatives. Then the imaginary part of quark loops can be compensated and their momentum dependence can eventually reproduce the infinite sum of meson resonances in the large-\(N_c\) limit. If the effective action is truncated with a finite number of vertices and thereby deals with only a few resonances one has to retain only a finite number of terms in the low-momentum expansion of quark loops in the CSB phase, with a non-zero dynamical mass.

5. There are more possibilities to bind the phenomenological constants of QQM based on CSR constraints for three- and four-point correlators and also in the vector and axial-vector channels. Some of these CSR rules have been explored in \([12, 33, 34, 35, 36, 37]\).

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