Accuracy improvement for GNSS -based compass

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Abstract. To calculate relative position of two navigation antennas with a relatively short baseline phase measurements are commonly used. This approach is associated with technical difficulties related to the synchronization of receivers and resolution of phase ambiguities. Double phase differences of eliminate these errors and therefore allow to calculate the azimuth with better accuracy. Proposed algorithm for phase ambiguities resolution consists of three steps: applying double phase difference model insensitive to full phase due to the usage of cosine function, secondly using differential evolution method to fit model to observation by varying orientation angles and finally computing phase ambiguities from roughly estimated angles. To reduce the phase centre shift influence on azimuth computation error, the correction function depending on navigation signal arrival angles was introduced. To find its numerical values, the experiment with rigidly fixed antennas was carried out. Its idea was to take into account the relative movement of navigation satellites in fixed antenna system. Development of a multi-antenna navigation receiver with an additional channel for calculating phase ambiguities is described.

1. Introduction
Currently, accurate azimuth determination is an important task, necessary to calculate orientation of a vessel and build an optimal trajectory, providing more safety and less fuel consumption and travel time. In addition it allows to correct the readings of sensors such as depth meters and radars.

Traditionally, gyro compasses and magnetic compasses are used for this type of navigation. The use of magnetic compasses is difficult in the areas with geomagnetic anomalies, like polar regions, due to the necessity of additional corrections. The main disadvantage of gyro compasses is the accumulation of errors during operation, requiring, uninterrupted power supply. The most promising are the compasses able to determine azimuth (and maybe pitch and roll) angles using differential schemes. Such a compass, unlike a traditional gyro compass, does not accumulate errors and provides an indication of the geographical North Pole.

Navigational errors in GNSS systems can be classified by position of its source [1]:

- satellite errors (clock error \(dt^k\));
- errors added by a navigation signal propagation path (ionospheric \(T_i^k\) and tropospheric \(T_i^k\) delays);
- receiver errors (clock error \(dt_i\)).

Errors of a propagation path and satellite related errors can be compensated by calculating single difference between two receivers located on a short base line. To compensate receiver related errors the double difference (difference between measurements for reference satellite and
a certain one) can be calculated. The satellite with the highest elevation angle is commonly used as the reference one.

\[
\Phi_{k} = \rho_{k} + cdt_{k} - cdt_{k} - T_{k} + T_{k} + \lambda N_{k} - \epsilon_{k}
\]  

\[
\Phi_{ij} = (\Phi_{i} - \Phi_{j}) - (\Phi_{i} - \Phi_{j})
\]

**Figure 1.** Test setup.

For recording test data, the test setup shown in figure 1 was build. Two Ublox M8T navigation receivers were connected to antennas placed on the roof (figure 2) on a 0.5 m base line. The navigation receivers used provide two types of raw measurements: pseudo-range and phases. Their double differences are shown in figures 3 and 4. A pseudo-range double difference do not have ambiguities like phase measurements however, higher noise level does not allow using it for base line smaller than few meters [1].

**Figure 2.** Antennas of test setup.

**Figure 3.** Double pseudo-range difference.

**Figure 4.** Double phase difference.


2. Synchronization error

Double difference phase observations for two receivers: \( i \) and \( j \) for two satellites: \( l \) and reference \( k \) can be calculated in a following way:

\[
\phi_{ij}^{kl}(t) = f_{RF}^k \left( (\tau_i^k - \tau_j^k) - (\tau_i^l - \tau_j^l) \right) - (f_d^l - f_d^k + \Delta f^{lk})(\tau_i^l - \tau_j^l) + f_{d}^k (\tau_i^k - \tau_j^k) - f_{d}^l (\tau_i^l - \tau_j^l) + \Delta f^{lk}(\tau_i^l - \tau_j^l)
\]

where \( \tau \) - time delay between receiver and satellite, \( f_{RF} \) - carrier frequency, \( f_d \) - Doppler frequency shift, \( \Delta f^{lk} \) - clock offset between satellites, \( \delta t \) - receiver clock offset.

For 0.5 m base line the maximum time delay \((\tau_i^l - \tau_j^l)\) is 1.7 ns. The maximum Doppler shift for a static receiver on the Earth surface is 5 kHz [2]. The instability of GPS satellite oscillators does not exceed \(2 \times 10^{-11} \) s of Allan deviation, which corresponds to the instability of L1 carrier frequency in range of 33.1 mHz [3].

Therefore, the last three terms of the equation (3) are negligible (4) and can be omitted.

\[
f_{d}^l (\tau_i^l - \tau_j^l) - f_{d}^k (\tau_i^l - \tau_j^l) + \Delta f^{lk}(\tau_i^l - \tau_j^l) \approx \pm 0.17 \times 10^{-6}
\]

\[
\phi_{ij}^{kl}(t) = f_{RF}^k \left( (\tau_i^k - \tau_j^k) - (\tau_i^l - \tau_j^l) \right) - (f_d^l - f_d^k + \Delta f^{lk})(\tau_i^l - \tau_j^l) + \Delta f^{kl}_{ij}
\]

where \(|e^{kl}_{ij}| < 1.2 \times 10^{-6} \) for time shift 0.1 s between receivers and \(|e^{kl}_{ij}| < 6.1 \times 10^{-6} \) for synchronous measurements.

Thus, using double difference approach with synchronized receivers drastically improves accuracy.

3. Orientation angles and ambiguities calculation

The task of orientation determination can be reduced to finding angles \((\alpha, \beta)\) at which the difference between measured double phase difference values and the model values is minimal for all satellites:

\[
\sqrt{\frac{1}{n-1} \sum_{i=1}^{n-1} E_i^2} \rightarrow \min_{\alpha, \beta}
\]

\[
E_i = (\Phi_{ij}^{kl} + N_{ij}^{kl}) - \frac{1}{X} \left( \|r^k - r_i\| - \|r^k - r_i - b \begin{bmatrix} \cos \beta \cos \alpha \\ \cos \beta \sin \alpha \\ \sin \beta \end{bmatrix} \| \right) - \frac{1}{X} \left( \|r^l - r_i\| - \|r^l - r_i - b \begin{bmatrix} \cos \beta \cos \alpha \\ \cos \beta \sin \alpha \\ \sin \beta \end{bmatrix} \| \right)
\]

where \(r\) - satellite and receiver coordinates, \(b\) - base line length, \(\Phi_{ij}^{kl}\) - double phase difference, \(N_{ij}^{kl}\) - integer ambiguity.

There are several methods for finding ambiguities:

- least-squares method with further rounding to integer numbers;
- integer least-squares method;
- LAMBDA method - implying decorrelation of the double difference ambiguities and a least-squares based search [4].

The task of adding information about orientation from other sources or previous iteration is rather complicated. Evaluating ambiguities with subsequent computation of orientation angles and restoring ambiguities is much easier way.
The ambiguity can be evaluated by applying cosine function. In this case angle searching task is transformed into finding maximum value of the function (8).

\[
E_{\cos}^i = \Phi_{kl}^{ij} - \left\{ \frac{1}{\lambda} \left( \left\| r^k - r_i \right\| - \left\| r^k - r_i - b \left[ \begin{array}{c} \cos \beta \cos \alpha \\ \cos \beta \sin \alpha \\ \sin \beta \end{array} \right] \right\| \right) - \\
- \frac{1}{\lambda} \left( \left\| r^l - r_i \right\| - \left\| r^l - r_i - b \left[ \begin{array}{c} \cos \beta \cos \alpha \\ \cos \beta \sin \alpha \\ \sin \beta \end{array} \right] \right\| \right) \right\}
\]  

(9)

Figure 5. Surface of the error function \(E^i\).  
Figure 6. Surface of the likelihood function \(E_{\cos}^i\).

The surface of the function (8) has many maxima as shown in figure 6. To find a global one the differential evolution optimization method can be used [5].

After rough estimations of the angles, it is possible to compute phase ambiguity \(N\) as an integer number minimizing error function (7) for each satellite. The last step is the computation of accurate orientation angles using known ambiguity. The surface of the function (6) is shown in figure 5. The minimum value can be found with gradient descent method [6].

4. Antenna phase center offset

After processing the experimental record, the repetition interval of 12 hours was found (figure 7). Since the repetition interval is very close to the constellation repetition time, the error likely depends on satellite positions. One of the possible sources is displacement of antenna phase centres depending on incoming directions of navigation signals [7].

To correct phase centre offset it is possible to use function \(P(\alpha, \beta)\) (10) depending on angles \(\alpha\) and \(\beta\).

\[
\Phi_{ij}^{kl} = \left( \Phi_{ij}^k - P(\alpha^k, \beta^k) \right) - \left( \Phi_{ij}^l - P(\alpha^l, \beta^l) \right)
\]

(10)

To estimate \(P(\alpha, \beta)\) all incoming directions should be connectid to mesh nodes. The nodes should be evenly distributed over unit sphere surface to minimize number of points for slow
changing phase centre offset [8]. The equilateral triangle approach [9] was used as shown in figure 8. The mesh with 2562 elements with 4.73° distance was used (table 1).

Table 1. Distance between nodes.

| Nodes number | Distance between nodes |
|--------------|------------------------|
| 162          | 18.70°                 |
| 642          | 9.44°                  |
| 2562         | 4.73°                  |
| 10242        | 2.37°                  |

To find exact correction values the 48 hour-long record was made for static setup. Based on fixed azimuth value over all time of the experiment, the computed mean value can be used as true one. Based on the satellite positions it is possible to calculate discrepancies (error) between observations and calculate values for all the satellites at any moment. Due to the satellite movement over time (a trajectory is shown in figure 9) the signal incoming direction is also changing. Depending on incoming direction all errors can be grouped to mesh nodes. If there are several errors in a node, mean value used for aggregation. Dependence on these errors can be connected to the grid nodes and computed mean values. For each node where at least one satellite passes grid cell, correction function $P$ can be calculated. A sky plot of $P$ is shown in figure 10.

Orientation angles’ standard deviation with and without phase centre correction for the same record is shown in table 2.

Table 2. Standard deviations of orientation angles.

| RMS       | without correction | with correction |
|-----------|---------------------|-----------------|
| Azimuth   | 0.51°               | 0.39°           |
| Pitch     | 1.11°               | 0.78°           |
5. Position and time errors
According to equation (7), angle errors can be caused by position error of the base receiver as it was directly used in calculation along with the time error. Time is not included directly in equations, but it was used to compute satellite positions from ephemerides.

To simulate position error, the offset with random direction on East-North plane was added to calculate base receiver coordinates. The simulation results are shown in figure 11. Values without offsets were used as true values.

Figure 11. Sensitivity to position error.

Figure 12. Sensitivity to time error.

Figure 12 displays similar results for time error. Additional time offset was added to computed value before satellite position computation.

If the receiver position offset error is less than 500 m, the error does not significantly affect the determination of heading angles.

Time accuracy do not play significant role either, unlike synchronization error, discussed above.

6. Triple antenna receiver
When the length of the base line between the antennas decreases, the error in determining the course angles increases (figure 14), but when the base line is less than the navigation signal wavelength, the phase ambiguity is removed.

Using a three-antenna system can simplify the task of removing phase ambiguities without accuracy degradation. If a third (additional) antenna is placed between two antennas within the base line of 0.5 m, so that the distance to the leading antenna is 0.19 m, as shown in figure 13,
then the procedure of resolving phase ambiguities between main antennas can be replaced by calculating them using the following equation:

$$\Phi_{12}^{kl} = \phi_{12}^{kl} + \phi_{13}^{kl} \frac{b_{12}}{b_{13}}$$  \hspace{1cm} (11)$$

where $b_{12}$ - long base line length; $b_{13}$ - short base line length.

To implement a multi-channel receiver able to process signals from several antennas, a synchronous front-end such as NT1066 or SDRNAV can be used [10, 11].

It is possible to use one channel as the master (for the correction of the carrier and code generators) and use local generator signals from main channel in sub channels as shown in figure 16. Then using discriminators of additional channels, it is possible to obtain phase differences between main channel and sub channels.

This approach automatically solves the problem of synchronization between channels.

7. Conclusion

After processing of 24 hour data record with applying time correction based on solving navigation task for each receiver and correcting phase centre displacement, the RMS of azimuth angle on is 0.34° and 0.78° for pitch angle. The part of azimuth computation results is shown in figure 17.

Based on experimental data, to obtain pitch error less than 0.1° pitch error, position should not exceed 500 m. For 0.1° azimuth error these limits are 500 m.

Due to substantial influence of desynchronization on double phase difference, time offset between receivers should be thoroughly estimated and used for correction.

Using a multichannel front-end can solve synchronization problem. Proposed tracking module computing phase difference between multiple channels can be applied in configuration with three antennas, allowing to resolve phase ambiguity.
Figure 16. Tracking module block diagram of triple antenna receiver.

Figure 17. Calculated azimuth plot.

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