Coulomb dissociation, a tool for nuclear astrophysics

G Baur
Forschungszentrum Jülich, Institut für Kernphysik, D-52425 Jülich, Germany
E-mail: g.baur@fz-juelich.de

S Typel
GANIL, Bd. Henri Becquerel, BP 55027, F-14076 Caen Cedex 05, France
E-mail: typel@ganil.fr

Abstract. A short status report on Coulomb dissociation, an indirect method for nuclear astrophysics is given. An analytically solvable approach to study electromagnetic excitation in $^{11}$Be, the archetype of a halo nucleus, is proposed.

PACS numbers: 21.10.-k, 25.60.-t, 26.50.tx

1. Introduction

Cross sections for nuclear reactions of astrophysical interest can be very small, too small to be directly accessible experimentally. Sometimes indirect methods can be useful to determine astrophysical S-factors. The ANC (Asymptotic Normalization Constant) method [1], the Trojan horse method [2, 3, 4, 5] and the Coulomb dissociation method [6] were discussed at the present conference. A more recent minireview is given in [7]. In all these methods, some theoretical interpretation of the experimental results is necessary in order to arrive at an astrophysical S-factor.

In the Coulomb dissociation method a fast projectile passes through the Coulomb field of a heavy nucleus. The time-varying electromagnetic field experienced by the projectile acts like a spectrum of equivalent photons which can dissociate the projectile. The equivalent photon spectrum can be computed from the kinematics of the process. In this way one can determine photodissociation cross-sections, essentially based on quantum electrodynamics. Radiative capture cross sections leading to the ground state of the final nucleus (the projectile) can be deduced by detailed balance. The projectile can be an exotic nucleus. Thus one has the unique possibility to study the interaction of unstable nuclei with photons [8, 9]. In particle physics the Primakoff effect [10] has been used for some time to study the interaction of photons, pions, Λ's, etc. with photons. Another method to study the interaction of photons with exotic nuclei are e-A colliders [11]. Coulomb dissociation is the only method for many years to come.
The theoretical description of intermediate energy Coulomb excitation and dissociation has been developed over the past decades, it is reviewed in \[12\]. The main characteristics are given in Sect. 2. We propose as a ‘homework problem’ a quasirealistic simple model, which can be solved analytically. It could also serve as a benchmark for tests of more sophisticated or involved models like the Continuum Discretized Coupled Channels model (CDCC). A major result of radioactive beam physics is the discovery of low-lying electric dipole strength in neutron-rich nuclei, see, e.g.,\[13\]. This discovery was made possible by the ‘working horse’ Coulomb dissociation. This is discussed in Sect. 3. Low lying dipole strength is directly related to the halo structure of these nuclei. The effective-range approach to low lying E1 strength and simple scaling laws are discussed. In some astrophysical scenarios for the r-process it is vital to know the low lying electric dipole strength. With the future radioactive beam facilities it will be possible to access these questions.

2. Theory of intermediate energy Coulomb excitation and dissociation

One of the basic parameters in Coulomb excitation is the ratio of the collision time to the nuclear excitation time, the so-called adiabaticity parameter

$$\xi = \frac{\omega b}{\gamma v}$$

where $\omega$ is the excitation energy, $b$ is the impact parameter and $v$ the beam velocity. The corresponding Lorentz factor $\gamma$ is typically not much larger than one. For $\xi \ll 1$ the process is sudden, and excitation is possible; for $\xi \gg 1$ the system follows adiabatically the time varying field and the excitation probability tends to zero. The strength of the excitation is measured by the strength parameter

$$\chi(\lambda) = \frac{Z_T e^2 \langle f|M(E\lambda)|i \rangle}{\hbar v b^\lambda}.$$  

The projectile, characterized by the electromagnetic matrixelements $\langle f|M(E\lambda)|i \rangle$ is excited on its passage through the Coulomb field of the (heavy) target nucleus with charge $Z_T$. The parameter $\chi(\lambda)$ can be regarded as the number of exchanged photons. In electromagnetic excitation it is a good approximation that the nuclei do not penetrate each other. In this case, the interaction is encoded in the electromagnetic matrixelement $\langle f|M(E\lambda)|i \rangle$ between the relevant states. The Coulomb parameter $\eta$ is the multipole strength parameter, i.e. $\lambda = 0$ in (2), and the multipole matrixelement is replaced by the charge $Ze$ of the nucleus. For higher beam velocities higher order effects tend to be small. For not too light nuclei $\eta$ is still $\gg 1$, and the semiclassical description is appropriate.

2.1. A quasirealistic and analytically solvable model of Coulomb excitation of neutron halo nuclei

An archetype of a halo nucleus is $^{11}\text{Be}$ with a $^{10}\text{Be}$ core and a single halo neutron in the $2s_{1/2}$-state. There is a strong E1 transition to the $1/2^-$ bound state, the only
bound excited state of the system. This dipole transition was studied by Coulomb excitation at GANIL, RIKEN and MSU \[14, 15, 16, 17\]. In Ref. \[17\] higher order effects are studied in the 'XCDCC' approach. We take this as an opportunity to revisit the theoretical study of higher order effects in intermediate energy Coulomb excitation. In \[18\] electromagnetic excitation of $^{11}\text{Be}$ is studied in the sudden limit of the semiclassical method. Higher order effects are treated to all orders. In \[19\] an analytically solvable model for higher order electromagnetic excitation effects of neutron halo nuclei was presented. In that work, there was only the s-wave bound state. Now we consider the case where there is, in addition, a p-wave bound state, as it is the case in $^{11}\text{Be}$. In the sudden approximation the dipole excitation amplitude is given by

$$a_{\text{sudden}} = \langle f | \exp (-i \vec{q}_{\text{Coul}} \cdot \vec{r}) | i \rangle$$  \hspace{1cm} (3)

where $\vec{q}_{\text{Coul}} = \frac{2Ze^2}{\hbar b} \vec{e}_x$. The impact parameter $b$ is chosen to be in the x-direction. (For the actual calculation in polar coordinates it is convenient to change this to the z-direction.) The dipole approximation is quite well fulfilled, the dipole effective charge $Z_{\text{eff}}^{(1)} = \frac{Z_n m_n}{m_n + m_e}$ is much larger than the corresponding quadrupole charge. The neutron and core mass are denoted by $m_n$ and $m_c$ respectively, the charge of the core is given by $Z_c$. The sudden approximation is applicable for $\xi \ll 1$. Even for the comparatively low GANIL energies of about 40 MeV/nucleon this is reasonably well fulfilled. The most important intermediate states are expected to be in the low energy continuum, where the dipole strength has a peak, at around 1 MeV excitation energy. The sudden approximation has the advantage that intermediate states are treated by closure, thus one only needs a model for the initial and final states, and not for all the intermediate states. In lowest order in $q_{\text{Coul}}$ the first order dipole approximation is obtained. It is shown in \[18\] that third order E1 excitation is more important than second order E1-E2 excitation. The matrix elements are dominated by the exterior contributions. In a pure single particle model the radial wave functions of the $2s_{1/2}$ and $1p_{1/2}$ states are given by

$$f_0(r) = C_0 q_0 r h_0(i q_0 r)$$  \hspace{1cm} (4)

and

$$f_1(r) = C_1 q_1 r h_1(i q_1 r)$$  \hspace{1cm} (5)

Both states are halo states and the normalization constants are given by $C_0 = \sqrt{2q_0}$ and $C_1 = \sqrt{2q_1^3 R/3}$ in the halo limit respectively \[20\]. The bound state parameters $q_i$ ($i = 0, 1$) are related to the binding energies by $E_i = \frac{\hbar^2 q_i^2}{2\mu}$ where $\mu$ is the reduced mass of the core-neutron system. We have $E_0 = 504$ keV and $E_1 = 184$ keV. The radius of the core is denoted by $R$. With these model assumptions we can calculate the $B(E1)$ value for the $1/2^+ \rightarrow 1/2^-$-transition as well as the higher order effects in electromagnetic excitation. Whereas in \[17\] quite sophisticated models are used, our approach is simple, transparent and at the same time close to reality. We propose this to be a model study and leave the spectroscopic factors equal to one. (They could be adjusted, which would result in a quasi-realistic description of the $^{11}\text{Be}$ system for our purpose.) The XCDCC calculations are quite involved, with many parameters. It would be very useful to check
the method by comparing to a simple case, such as this one, where analytical results are possible. In order to avoid nuclear effects a sharp cutoff at a minimum impact parameter $b_{\text{min}}$ can be introduced.

The $B(E1)$ value is given by $B(E1) = (Z_{\text{eff}}^{(1)})^2 |R_{01}|^2/(4\pi)$, where $R_{01}$ is the radial dipole integral. The integrals are elementary. Using $\int_0^\infty r^n e^{-ar}dr = n!/a^{n+1}$ we find

$$R_{01} = \frac{2\gamma_0^{1/2}(\gamma_0 + 2\gamma_1)}{3^{1/2}(\gamma_0 + \gamma_1)^2} R,$$

where $\gamma_0 = q_0 R$ and $\gamma_1 = q_1 R$. We note that we extended the radial integral over the exterior wave function from $R$ to zero. For $R \to 0$ the radial dipole integral goes to zero because the normalization of the p-wave function tends to zero in this limit. Thus $R$ must be kept finite, say $R = 2.78 \text{ fm}$ [21], this value determines the asymptotic normalization of the p-wave bound state. We find $B(E1) = 0.193 \text{ e}^2\text{fm}^2$, to be compared to the value of $B(E1) = 0.105(12) \text{ e}^2\text{fm}^2$ obtained from an analysis of the GANIL data, see [17], consistent with other Coulomb dissociation experiments at RIKEN and MSU and the value obtained by the Doppler shift attenuation method [22].

We expand the excitation amplitude \(\mathcal{B} \) in terms of the dimensionless strength parameter $y = q_{\text{Coul}}/(q_0 + q_1)$. (Cf. (2), we take $e/(q_0 + q_1)$ as a convenient measure for the order of magnitude of the dipole matrix-element.) The excitation probability is given by $P(b) = |a_{\text{sudden}}|^2$. The lowest order term is proportional to $y^2$. The most important higher order contribution comes from the third order in $q_{\text{Coul}}$. It can be calculated analytically. Its interference with the lowest order term leads to the next term in the expansion in $y$, of the order of $y^4$. We have $P(b) = P_{LO} + P_{NLO} + \ldots$. The lowest order term is given by

$$P_{LO} = y^2 \frac{4\gamma_0(\gamma_0 + 2\gamma_1)^2}{27(\gamma_0 + \gamma_1)^2} \equiv C_2/b^2 \quad (7)$$

The next term is found to be

$$P_{NLO} = -y^4 \frac{8\gamma_0^3(\gamma_0 + 2\gamma_1)(\gamma_0 + 4\gamma_1)}{45(\gamma_0 + \gamma_1)^2} \equiv -C_4/b^4 \quad (8)$$

Total cross sections are obtained by integration over the impact parameter, starting from a minimum impact parameter $b_{\text{min}}$. The sudden approximation fails for large impact parameters, and an adiabatic cut-off $b_{\text{max}} = \gamma v/\omega$ has to be introduced for the lowest order result. We put $\omega = 320 \text{ keV}$, the energy of the 1/2$^-$ state in $^{11}$Be. (For the higher order terms this is not necessary, the convergence in $b$ is fast enough.) We get

$$\sigma_{LO} = 2\pi C_2 \ln \frac{b_{\text{max}}}{b_{\text{min}}} \quad (9)$$

and

$$\sigma_{NLO} = -\pi C_4 \frac{b_{\text{max}}}{b_{\text{min}}}^2. \quad (10)$$

We note that the strength parameter $y$ is proportional to $1/v$, i.e. the leading order term decreases like $1/E$, the next-to-leading order term like $1/E^2$, where $E$ is the beam energy. We think that this analytical model could serve as a benchmark for tests of more
Indirect methods

involved reaction models. We hope to publish a more detailed account of the present approach in the future.

3. Low-lying electric dipole strength in neutron rich nuclei

An effective-range approach to low lying E1-strength for one-neutron halo nuclei was developed in [20, 21]. There is a small parameter

\[ \gamma \equiv qR = \frac{R_{\text{halo}}}{R}. \tag{11} \]

In lowest order, the dipole strength is independent of \( \gamma \). The \( B(E1) \)-strength function is proportional to the shape function \( S_{l_f}^{l_i} \) and scales with the parameter \( x^2 \equiv E/E_{\text{bind}} = q^2/q_0^2 \), where \( E \) is the c.m. energy in the continuum. For s-p transitions it is given by

\[ S_0^l = \frac{x^3}{(1 + x^2)^2} \left[ 1 - a_1 q^3 (1 + 3x^2) \gamma^3 + \ldots \right]. \tag{12} \]

This remarkably simple result can be applied to deuteron photodisintegration, and \( s_{1/2} \)-neutron halo nuclei like \(^{11}\text{Be}, \, ^{15}\text{C}, \, ^{19}\text{C}, \ldots \) The interaction of the final state p-wave neutron with the core can usually be neglected. Thus low lying strength due to transition to a structureless continuum is found. It may look like a resonance, but it has nothing to do with a resonance. This was recognized long time ago [23, 24]. We quote from a recent review of low lying dipole strength [26]: ’...the onset of dipole strength in the low-energy region is caused by nonresonant independent single-particle excitations of the last bound neutrons’. In general there are characteristic effects of the core-neutron interaction in the continuum state, usually more pronounced for states with \( l_f = l_i - 1 \). For the s-p transitions this term is proportional to \( \gamma^3 \), which is quite small for a halo nucleus. For the low energies relevant here this interaction can be parametrized in terms of the scattering length. An interesting effect of this type was found by analysing the high precision data of \(^{11}\text{Be} \) Coulomb dissociation [27]. A large scattering length \( a_{l=1/2}^{1/2} = 456 \text{ fm}^3 \) was found [21]. It is due to the \( p_{1/2} \)-subthreshold state. A treatment of two-neutron halo nuclei in the effective range method for low lying strength of halo nuclei is given in [25].

4. Conclusion and Outlook

Electromagnetic excitation is a powerful tool to investigate the interaction of (quasireal) photons with unstable nuclei. It will continue to play a prominent role at the future radioactive beam facilities. A good theoretical understanding of the process and its interplay with nuclear excitation is mandatory, see, e.g., [6]. In the future rp-process nuclei will come into focus. The possibility of 2p-capture is also discussed. It will never be possible to study this process in the laboratory. However, the time-reversed process of Coulomb dissociation with two protons in the final state is well within reach. An
example is Coulomb dissociation of $^{17}\text{Ne}$ [28], where the soft dipole mode in this proton-rich nucleus is discussed. By a suitable Coulomb dissociation experiment valuable information on the 2p-capture cross section on $^{15}\text{O}$ at astrophysical conditions appropriate for explosive burning in novae and X-ray bursts may be obtained.

At future radioactive beam facilities r-process nuclei will become available. In certain scenarios (see, e.g., [26]) it will be important to know the low lying E1 strength, which will decisively influence the r-process abundances.

References

[1] Mukhamedzhanov A 2007 these proceedings
[2] Romano S 2007 these proceedings
[3] LaCognata M 2007 these proceedings
[4] Typel S and Baur G 2003 *Ann. Phys., NY* **305** 228
[5] Typel S 2005 *Eur. Phys. J. A* **25** 665
[6] Typel S 2007 these proceedings
[7] Baur G and Typel S 2006 *Proceedings of the DAE-BRNS 50th Symposium on Nuclear Physics (Bhabha Atomic Research Centre, Mumbai, India, December 12-16, 2005)* (Preprint nucl-th/0601004)
[8] Sonnabend K 2007 these proceedings
[9] Boretzki K 2007 these proceedings
[10] Primakoff H 1951 *Phys. Rev.* **31** 899
[11] Bertulani C A 2007 *Phys. Rev. C* **75** 024606
[12] Baur G, Hencken K and Trautmann D 2003 *Prog. Part. Nucl. Phys.* **51** 487
[13] Aumann T 2005 *Eur. Phys. J. A* **26** 441
[14] Anne R et al. 1995 *Z. Phys. A* **352** 397
[15] Nakamura T et al. 1997 *Phys. Lett. B* **394** 11
[16] Fauerbach M et al. 1997 *Phys. Rev. C* **56** R1
[17] Summers N C 2007 *Phys. Lett. B* **650** 124
[18] Typel S and Baur G 1995 *Phys. Lett. B* **356** 186
[19] Typel S and Baur G 2001 *Phys. Rev. C* **64** 0246101
[20] Typel S and Baur G 2005 *Nucl. Phys. A* **759** 247
[21] Typel S and Baur G 2004 *Phys. Rev. Lett.* **93** 142502
[22] Millener D J et al. 1983 *Phys. Rev. C* **28** 497
[23] Hansen P G and Jonson B 1987 *Europhys. Lett.* **4** 409
[24] Bertulani C A and Baur G 1988 *Nucl. Phys. A* **480** 615
[25] Baur G and Typel S 2006 *Proceedings of the Erice School on Radioactive Beams, Nuclear Dynamics and Astrophysics (Erice, 16-24 September, 2006)* to be published in *Prog. Part. Nucl. Phys.* (Preprint nucl-th/0610082)
[26] Paar N, Vretenar D, Khan E and Colo G 2007 *Preprint* nucl-th/0701081
[27] Palit R et al. 2003 *Phys. Rev. C* **68** 034318
[28] Grigorenko L V et al. 2006 *Phys. Lett. B* **641** 254