Electromagnetic radiation from superconducting string cusps

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Abstract

Cusps in superconducting cosmic strings produce strongly beamed electromagnetic radiation. To calculate the energy emitted requires taking into account the effect of the charge carriers on the string motion, which has previously been done only heuristically. Here, we use the known exact solution to the equations of motion for the case where the current is chiral to update previous calculations for the total energy, spectrum and angular distribution in that case. We analyze the dependence of the radiated energy on the cusp parameters, and discuss which types of cusp dominate the total radiation emitted from an ensemble.

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I. INTRODUCTION

Cosmic strings are one dimensional topological defects which may have been left behind by phase transitions in the early universe \[1,2\]. In the zero width approximation we can describe the dynamics of the strings by the Nambu-Goto (NG) action \[3,4\], namely

\[
S = -\mu \int d^2\zeta \sqrt{-\gamma}
\]

where \(\mu\) is the energy per unit length of the string and \(\gamma\) the determinant of the worldsheet metric parameterized by the coordinates \(\zeta^0\) and \(\zeta^1\). Due to the reparameterization invariance of the Nambu-Goto action, we have the freedom to impose several conditions on the functions that denote the string position, in other words to fix the gauge. The most convenient gauge for ordinary strings in flat spacetime is the conformal gauge, in which we impose

\[
|x'(\sigma, \tau)|^2 + |\dot{x}(\sigma, \tau)|^2 = 1 \quad \text{(2a)}
\]

\[
x'(\sigma, \tau) \cdot \dot{x}(\sigma, \tau) = 0 \quad \text{(2b)}
\]

where we have fixed \(\zeta^0 = \tau = x^0\) and \(\zeta^1 = \sigma\), and primes and dots denote differentiation with respect to \(\sigma\) and \(\tau\) respectively. With these conditions the equations of motion become

\[
x''(\sigma, \tau) = \ddot{x}(\sigma, \tau) \quad \text{(3)}
\]

The general solution is

\[
x(\sigma, \tau) = \frac{1}{2}(a(\sigma - \tau) + b(\sigma + \tau)) \quad \text{(4)}
\]

where \(a\) and \(b\) are arbitrary functions constrained by the gauge conditions to satisfy \(|a'| = |b'| = 1\). From Eq. (4) we see that

\[
x' = \frac{1}{2}[a'(\sigma - \tau) + b'(\sigma + \tau)] \quad \text{(5)}
\]

and

\[
\dot{x} = \frac{1}{2}[-a'(\sigma - \tau) + b'(\sigma + \tau)] \quad \text{(6)}
\]

If there is a point along the string at which \(a' = -b'\), then \(x' = 0\) and even more importantly \(|\dot{x}| = 1\). This means that there are points on the string that move at the speed of light and where the string doubles back on itself. (See Fig. 1.) These events are called cusps \[5\].

It is clear that this kind of dynamics falls outside of the domain of validity of the assumptions involved to obtain the Nambu-Goto action. Nevertheless it has been shown by field theory simulations that the motion of the string is quite accurately described by the NG solution up to the point of interaction between the two branches of the string \[6\].

The violent motion of the string at the cusps suggests the possibility of a range of effects which could have prospects of detection \[5,13\].

In 1985 Witten \[16\] showed that certain particle physics models could lead to the formation of strings with superconducting behavior. It was soon realized that these models could
in principle have a much richer phenomenology due to the inertia of the charge carriers and their coupling to the long-range electromagnetic field.

In particular we can think that if the current on the string is not very large, then the string motion would not be affected, and calculate the electromagnetic radiation from a Nambu-Goto prescribed motion for the string. However, the power emitted by radiation near the cusp in this approximation is divergent due to the infinite Lorentz factor of the NG cusp \[8,17\]. This infinite result suggests that electromagnetic back-reaction is crucial in order to understand the superconducting string cusp.

The short-range effects of the electromagnetic field can be taken into account by renormalization of the charge carrier inertia and charge \[16,18,2\]. The charge carrier inertia term prevents the appearance of an infinite Lorentz factor, and thus the calculation yields a finite result. In principle, this result could be further modified by back-reaction due to long-range fields (i.e. radiation), but for generic values of the parameters we expect this correction to be small \[19\]. Further approximating that the current is chiral, at least in the vicinity of the cusp, we are able to do a proper calculation of the total power radiated.

II. SUPERCONDUCTING STRINGS

The equations of motion for a superconducting string can be written \[16,8\]

\[\partial_a \left( \sqrt{-\gamma} \left( \mu \gamma^{ab} + \theta^{ab} \right) x^\nu_b \right) = - \sqrt{-\gamma} F^\nu_{\sigma} x^\sigma_a J^a \]  

\[\partial_a \left( \sqrt{-\gamma} \gamma^{ab} \phi_{b} \right) = - \frac{1}{2} q \epsilon^{ab} F_{\mu\nu} x^\mu_a x^\nu_b \]

\[\partial_{\nu} F^{\mu\nu} = - 4 \pi j^\mu \]

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where \( \mu \) is the energy per unit length of the string, \( q \) the electromagnetic coupling of the fields on the string, \( \theta^{ab} \) the energy-momentum tensor of the charge carriers,

\[
\theta^{ab} = \gamma^{ac} \gamma^{bd} \phi_c \phi_d - \frac{1}{2} \gamma^{ab} \gamma^{cd} \phi_c \phi_d ,
\]

\( \phi \) the auxiliary scalar field in terms of which we can express the conserved worldsheet current

\[
J^a = q \frac{1}{\sqrt{-\gamma}} \epsilon^{ab} \phi_b ,
\]

and \( j^\mu \) the 4-dimensional current,

\[
j^\mu(x) = \int d^2 \zeta \sqrt{-\gamma} J^a x^\mu_{\alpha} \delta^{(4)}(x - x(\zeta)).
\]

These equations of motion are obtained from a two-dimensional effective action for the superconducting string and they are valid as long the current, measured in an invariant way, is not too large,

\[
J_a J^a \ll |J_{\text{crit}}|^2,
\]

where \( J_{\text{crit}} \) is the limiting current at which charge carriers can be ejected.

The leading effect of electromagnetic self-interaction of the current-carrying string can be taken into account by a redefinition of the auxiliary field \( \phi \) and its charge [16,18,2]. This enables us to include the near-field effect on the left-hand side of the equations of motion. If we take this as our only back-reaction effect, then we can rewrite the equations above in a much simpler way by taking \( q \) and \( \phi \) to be the renormalized quantities, to get

\[
\begin{align}
\partial_a \left( \sqrt{-\gamma} \left( \mu \gamma^{ab} + \theta^{ab} \right) x^\nu_{b} \right) &= 0 \quad (12a) \\
\partial_a \left( \sqrt{-\gamma} \gamma^{ab} \phi_b \right) &= 0 \quad (12b) \\
\partial_\nu F^{\mu\nu} &= -4\pi j^\mu \quad (12c)
\end{align}
\]

This procedure decouples the first two equations from the third one, and they now give the motion of a ‘neutral’ superconducting string. In the last equation, \( F^{\mu\nu} \) is the external field emitted by the current moving according to the first two equations.

Numerical simulations [20,21] have suggested that as a superconducting cosmic string loop contracts under its own tension, it would approach the ‘chiral’ limit, in which the current on the string is null, in other words, \( J_a J^a = 0 \). This, in turn, means that the auxiliary scalar field fulfills the condition,

\[
\gamma^{ab} \phi_{b} \phi_{a} = 0 .
\]

Furthermore, it can be shown [18] that if the current is not chiral it would become supercritical around the cusp and charge carriers would be ejected. We therefore expect that this process would drive this region of string around the cusp to the chiral limit. These arguments suggest that most of the electromagnetic radiation would come from chiral current carrying string cusps.
If the current is chiral, the set of equations describing the string and the ‘neutral’ current can be solved completely [22,19,23]. In this case, the general solution for the scalar field is

$$\phi(\sigma, \tau) = F(\sigma + \tau),$$

(14)

where $F$ is an arbitrary function, and the string position is given by

$$x^0 = \tau$$

(15a)

$$x = \frac{1}{2}[a(\sigma - \tau) + b(\sigma + \tau)],$$

(15b)

with constraints for the otherwise arbitrary functions $a’$ and $b’$,

$$|a’|^2 = 1$$

(16a)

$$|b’|^2 = 1 - \frac{4F'^2}{\mu}.$$  

(16b)

This description of the chiral strings makes evident that they behave very similarly to the Nambu-Goto case. As before, the functions $a’$ and $b’$ live on the surface of spheres (if the current is constant), but now their radii are different for the two cases. This change, however, brings important consequences for the string motion. As the magnitude of the current increases, the average velocity of the loop decreases, arriving at the limiting stationary case when $4F'^2/\mu = 1$, the chiral vorton [20].

This also means that the string can only reach a finite Lorentz factor at the cusp-like regions, in contrast with the Nambu-Goto case. This effect can be easily understood in terms of the resistance of the charge carriers to being accelerated by the string tension. Equations (10) also imply that $|x’| > 0$ at the cusp so the actual shape of the string around the cusp is rounded off by the effect of the charge carriers, as shown in Fig. 1. (Another way for the string to be rounded off, producing a self intersection, is discussed in [24].)

## III. ELECTROMAGNETIC RADIATION FROM THE CUSP

Using the solution for the string motion from the previous section, which as discussed is free from divergences in its Lorentz factor, we can now compute, using Eq. (12c), the electromagnetic radiation from the cusp-like region. Throughout this section we follow closely the calculation of the radiation by Spergel et al. [17], and introduce the necessary modifications for the chiral string motion.

Since the radiation is mainly produced by the region with a large Lorentz factor, we can estimate the energy output from the position of the string expanded in a Taylor series around the cusp, which we take as $\sigma = 0$ and $\tau = 0$. We also assume throughout the calculation that $2|F’|/\sqrt{\mu} \ll 1$, since otherwise the string would not reach high Lorentz factors at all.

Using the light-like variables $\sigma_+ = \sigma + \tau$ and $\sigma_- = \sigma - \tau$, we can expand the functions $a$ and $b$ near $\sigma_\pm = 0$ as

$$a(\sigma_-) = a’_0\sigma_- + \frac{1}{2}a''_0\sigma_-^2 + \frac{1}{6}a'''_0\sigma_-^3 + \cdots$$

(17a)

$$b(\sigma_+) = b’_0\sigma_+ + \frac{1}{2}b''_0\sigma_+^2 + \frac{1}{6}b'''_0\sigma_+^3 + \cdots,$$

(17b)
where we center our coordinate system at the position of the cusp, so that \( x_0 = 0 \). In analogy with the Nambu-Goto case, we consider the cusp to be the place at which \( a' \) and \( b' \) are anti-parallel,

\[
\frac{a'_0}{|a'_0|} = -\frac{b'_0}{|b'_0|}
\]  

which corresponds to the point of maximum concentration of energy and the minimum value of \( |x'| \). Assuming that the current remains constant over the region of interest, we can see that the constraint equations impose a constant magnitude for the vectors \( a' \) and \( b' \). We can choose for simplicity

\[
a'_0 = -\hat{x}
\]

\[
b'_0 = \Delta \hat{x}
\]

where \( \Delta = 1 - \epsilon \), with \( \epsilon \ll 1 \) and constant. Due to the fact that \( |a'| \) and \( |b'| \) are constants we have the following relations,

\[
a'_0 \cdot a''_0 = b'_0 \cdot b''_0 = 0
\]  

\[
a'_0 \cdot a''_0 = -|a'_0|^2
\]  

\[
b'_0 \cdot b''_0 = -|b'_0|^2
\]

Using Eqs. (19, 20) we can express the most general expansion of the string position near the cusp up to third order in \( \sigma \) in the form

\[
a(\sigma) = \left( \frac{1}{6} \alpha^2 \sigma^2 - \sigma \right) \hat{x} + \left( \frac{1}{2} \alpha \sigma^2 + \frac{1}{6} \varrho \sigma^3 \right) \hat{y} + \left( \frac{1}{6} \delta \sigma^3 \right) \hat{z}
\]

\[
b(\sigma) = \left( \Delta \sigma + \frac{6 \Delta}{\beta^2 + \gamma^2} \sigma^3 \right) \hat{x} + \left( \frac{1}{2} \beta \sigma^2 + \frac{1}{6} \varpi \sigma^3 \right) \hat{y} + \left( \frac{1}{6} \gamma \sigma^3 \right) \hat{z}.
\]

The parameters \( \alpha, \beta, \gamma \) are of order \( L^{-1} \), with \( L \) denoting the typical length scale of the loop, and \( \delta, \varpi, \varrho, \eta \) are of order \( L^{-2} \).

Using standard results in electromagnetic theory, we can calculate the energy radiated per unit frequency and per solid angle for a source given by Eq. (10) using the expression

\[
d^2 E_{rad} = \frac{\omega^2}{4\pi^2} \left| \int d\sigma \int d\tau (n \times c) \exp[-i\omega(n^\mu x_\mu)] \right|^2
\]

where \( n^\mu \) is the unit null vector from the source point to the observation point,

\[
n^\mu = \left( 1, \cos \theta \cos \varphi, \sin \theta \cos \varphi, \sin \theta \sin \varphi \right) \simeq \left( 1, \frac{\theta^2}{2}, \theta \cos \varphi, \theta \sin \varphi \right).
\]  

This last approximation is justified since the radiation is highly beamed in the direction of movement of the cusp, in this case, \( \hat{x} \). The four-vector \( e^\mu \) is the source current, which in our case takes the form
\[ c^\mu = q e^{ab} \partial_\alpha \phi \partial_b x^\mu \]  

(24)

where \( a, b = 1, 2 \) stand for \( \sigma \) and \( \tau \). In the chiral case, the four-vector current becomes

\[ c^\mu = q \frac{d\phi}{d\sigma} (\frac{\partial x^\mu}{\partial \sigma} - \frac{\partial x^\mu}{\partial \tau}) = 2qF' \frac{\partial x^\mu}{\partial \sigma_-} \]  

(25)

Keeping only the lowest terms in the angle \( \theta \), we get

\[
\frac{d^2 E_{\text{rad}}}{d\omega d\Omega} = \frac{\omega^2}{16\pi^2} \left| \int d\sigma_- C(\sigma_-, \theta, \varphi, F') \exp[i \omega (J\sigma_- + K\sigma_-^2 + L\sigma_-^3)] \right|^2 
\times \left| \int d\sigma_+ \exp[-i \omega (S\sigma_+ + T\sigma_+^2 + U\sigma_+^3)] \right|^2
\]  

(26)

with

\[ C(\sigma_-, \theta, \varphi, F') = qF' \left( -\alpha \theta \sin \varphi \sigma_-, -\theta \sin \varphi, \alpha \sigma_- + \theta \cos \varphi \right) \]  

(27)

and

\[ J = \frac{\theta^2}{4} \]  

(28a)

\[ K = \frac{\theta}{4} \alpha \cos \varphi \]  

(28b)

\[ L = \frac{\alpha^2}{12} \]  

(28c)

and

\[ S = \frac{1}{2} \left( 1 - \Delta (1 - \frac{\theta^2}{2}) \right) \]  

(29a)

\[ T = -\frac{\theta}{4} (\beta \cos \varphi + \gamma \sin \varphi) \]  

(29b)

\[ U = \frac{1}{12\Delta} \left( \beta^2 + \gamma^2 \right) \]  

(29c)

The two integrals in Eq. (26) can be evaluated, first changing variables to get rid of the term quadratic in \( \sigma_\pm \) in the exponent, and then using the following forms of the Airy integrals:

\[ \int_0^\infty dx \cos \left[ \frac{3}{2} \xi \left( x + \frac{1}{3} x^3 \right) \right] = \frac{1}{\sqrt{3}} K_{\frac{1}{2}}(\xi) \]  

(30)

and

\[ \int_0^\infty dx \sin \left[ \frac{3}{2} \xi \left( x + \frac{1}{3} x^3 \right) \right] = \frac{1}{\sqrt{3}} K_{\frac{1}{2}}(\xi) \]  

(31)

where \( K_\nu \) are the modified Bessel functions of order \( \nu \). With a little bit of algebra we get to the result,
\[
\frac{d^2 E_{\text{rad}}}{d\omega d\Omega} = \frac{\omega^2}{16\pi^2} \left( qF'(\sin^2 \varphi) \left( \frac{2}{3} K_{\frac{1}{3}}(\xi_+ \chi_+) \right)^2 \right)
\times \left[ \theta^2 \sin^2 \varphi \left( \frac{2}{3} K_{\frac{1}{3}}(\xi_+ \chi_+) \right)^2 + \alpha^2 \left( \frac{2}{3\sqrt{3}} K_{\frac{1}{3}}(\xi_+ \chi_-) \right)^2 \right] 
\]

(32)

where

\[
\chi_+(\epsilon) = \sqrt{\frac{3SU - T^2}{3U^2}} = \left( \frac{3(2\epsilon + \theta^2 Y)}{\beta^2 + \gamma^2} \right)^{1/2} 
\]

(33a)

\[
\xi_+(\epsilon) = \frac{2}{3\sqrt{3}} \omega \left( S - \frac{T^2}{3U} \right) \chi_+ = \frac{1}{6} \omega \left( \frac{2\epsilon + \theta^2 Y}{\beta^2 + \gamma^2} \right)^{1/2} 
\]

(33b)

and where we have defined the dimensionless quantity,

\[
Y = \frac{(\beta \sin \varphi - \gamma \cos \varphi)^2}{\beta^2 + \gamma^2} = \sin^2(\varphi - \varphi_0), 
\]

(34)

where \(\varphi_0\) is the angle between \(a_0''\) and \(b_0''\), so that \(\tan \varphi_0 = \gamma/\beta\). On the other hand the expression for the same type of parameters for the integral in \(\sigma_-\) are much simpler,

\[
\chi_- = \sqrt{\frac{3JL - K^2}{3L^2}} = \sqrt{3} \theta \left| \frac{\sin \varphi}{\alpha} \right| 
\]

(35a)

\[
\xi_- = \frac{2}{3\sqrt{3}} \omega \left( J - \frac{K^2}{3L} \right) \chi_- = \frac{1}{6} \omega \theta^3 \left| \frac{\sin^3 \varphi}{\alpha} \right| . 
\]

(35b)

It is important to note that the different behavior of the superconducting chiral string cusp is encoded in the presence of the parameter \(\epsilon\). If we take the limit \(\epsilon = 0\), we would recover the pure Nambu-Goto cusp.

Using the above definitions, we can simplify Eq. (32) to

\[
\frac{d^2 E_{\text{rad}}}{d\omega d\Omega} = \left( qF' \sin^2 \varphi \right)^2 \left( \omega \theta^2 \right)^2 \left( 2\epsilon + \theta^2 Y \right) \left[ K_{\frac{1}{3}}(\xi_+ \chi_+) \right]^2 \left[ \left( K_{\frac{1}{3}}(\xi_- \chi_-) \right)^2 \right] 
\]

(36)

We can now integrate over frequencies with the change of variables \(z = \xi_-\) to get,

\[
\frac{dE_{\text{rad}}}{d\Omega} = 24 \left( qF' \pi \right)^2 \theta^{-3} \left| \sin^{-3} \varphi \right| \frac{|\alpha|^{1/3}}{(\beta^2 + \gamma^2)^{2/3}} \mathcal{F}(\xi_+/\xi_-) 
\]

(37)

where we have defined the function \(\mathcal{F}\) as

\[
\mathcal{F}(a) = \int_0^\infty dz a^{2/3}z^2 \left[ \left( K_{\frac{1}{3}}(az) \frac{1}{2} \right)^2 + \left( K_{\frac{1}{3}}(az) \frac{1}{2} \right)^2 \right] . 
\]

(38)

The integral can be done exactly in hypergeometric functions. The function \(\mathcal{F}\) has asymptotic behavior

\[
\mathcal{F}(a) \rightarrow \begin{cases} \text{const} & a \to 0 \\ \text{const}/a & a \to \infty \end{cases} 
\]

(39)

(with different constants).
In order to integrate Eq. (37) with respect to the angle $\theta$, we first note that we can extend the range of integration to infinity since only the region around the direction of the cusp contributes. In order to extract the dependence on the parameter $\epsilon$, we perform another change of variables to $\theta' = \theta / \sqrt{\epsilon}$, so the final expression for the energy emitted at the cusp becomes

$$E_{\text{rad}} = A(\alpha, \beta, \gamma) (qF')^2 \epsilon^{-1/2}, \quad (40)$$

where $A(\alpha, \beta, \gamma)$ is a constant which can be written in terms of the magnitudes of $a''_0$ and $b''_0$ and the angle between them, $\phi_0$, as

$$A = \frac{24}{\pi^2} \frac{|a''_0|^{1/3}}{|b''_0|^{4/3}} \int_0^{2\pi} d\varphi \int_0^\infty d\theta' \theta'^{-2} |\sin^{-3} \varphi| F \left[ \frac{|a''_0|}{|b''_0|} \left( 2 + \theta'^2 \sin^2(\varphi - \phi_0) \right) \right]^{3/2} \sim O(L). \quad (41)$$

Figure 2 plots the value of $A/L$ versus $\varphi_0$ for an $m = 1, n = 2$ Burden loop, which has $|a''_0| = 2\pi/L$ and $|b''_0| = 4\pi/L$ at the cusp.

We can understand the form of $A$ as follows. When $\theta' \ll 1$ (i.e., $\theta \ll \sqrt{\epsilon}$), the argument to $F$ goes as $\theta'^{-3} \sin^{-3} \varphi \gg 1$, and the integrand becomes proportional to $\theta'$, which is just the geometrical factor from the integration. Thus the radiated power per solid angle is uniform for small angles. Now we consider the case with $\theta'$ large. When $\sin(\varphi - \phi_0)$ is far from zero, then $\theta' \gg 1$ makes the argument of $F$ constant. In such directions, the radiated power is suppressed by $\theta'^{-2}$, so their contribution is small. If $\sin(\varphi - \phi_0)$ is small, then the argument of $F$ goes to zero, and thus the value to a constant, unless $\sin \varphi$ is also small. Thus, as long as $\phi_0$ is far from 0 and $\pi$, the radiation is confined to a beam with radius $\theta \sim \sqrt{\epsilon}$.

However, if $\phi_0$ is near 0 (or, equivalently, near $\pi$), then it is possible to have $\sin(\varphi - \phi_0)$ and $\sin \varphi$ small simultaneously. In that case, there is a range of $\theta'$ up to about $\phi_0^{-1}$, in which
large contributions to the integral are still possible. This is the cause of the divergence seen in Fig. 2. $A$ is proportional to $\varphi_0^{-1}$ for $\varphi_0 \ll 1$. In this case, the beam can be greatly elongated in the plane of the cusp. In the limiting case, $\phi = 0$ or $\phi = \pi$, the Burden loop has a cusp at all times, which rotates around the string. In cases close to this, we see that the cusp is longer lived than usual and thus has the opportunity to beam radiation through a wider angle.

In this regime, Eq. (40) is not correct, because we cannot extend the $\theta$ integration to infinity. Instead, we should have a cutoff at $\theta' \sim \epsilon^{-1/2}$. If we include this effect, then the growth of $A$ with $\varphi_0^{-1}$ is cut off at $\varphi_0 \sim \sqrt{\epsilon}$.

Now suppose that we have a set of loops with $\varphi_0$ distributed evenly between 0 and $\pi$. To find the contribution due to the loops with $\varphi_0$ near 0, we can integrate

$$\int_0 d\varphi_0 E(\varphi_0) \sim \int_{-1/2} \phi_0^{-1} \sim -\ln \epsilon .$$

(42)

Thus if $\epsilon$ is small, loops with $a_0'$ and $b_0'$ nearly parallel or antiparallel dominate the total electromagnetic radiation, but the effect is only logarithmic.

Returning to the generic case, let’s now remember that we have defined $\epsilon$ to be the deviation of the magnitude $b'$ from unity,

$$|b'| = \Delta = 1 - \epsilon$$

(43)

so in the chiral string model, for a current much smaller than the energy per unit length of the string,

$$|b'| = \sqrt{1 - \frac{4|F'|^2}{\mu}} \approx 1 - \frac{4|F'|^2}{2\mu}$$

(44)

so

$$\epsilon \approx \frac{4|F'|^2}{2\mu}$$

(45)

It can be shown [19] that the parameter $\epsilon$ determines the maximum Lorentz factor reached near the cusp, namely,

$$\Gamma_{\text{max}} = (2\epsilon)^{-1/2}$$

(46)

so we can rewrite Eq (40) as

$$E_{\text{rad}} \sim q^2 \sqrt{\mu} |F'| L \sim L (qF')^2 \Gamma_{\text{max}}$$

(47)

in agreement with earlier estimates [8,17]. We can also write the total energy output in terms of the physical current as

$$E_{\text{rad}} \sim q j \sqrt{\mu} L .$$

(48)

Equations (47,48) are valid when $\phi_0$ is far from 0 and $\pi$. 
IV. DISCUSSION

Electromagnetic radiation from superconducting cosmic strings cusps has traditionally been considered a distinctive signature of superconducting string models. Recently this interest has been revived by as possible connection with gamma ray bursts [14,15] and ultra-high energy cosmic rays. The total energy output from a pure Nambu-Goto string cusp (i.e. no back-reaction included) yields an infinite result due to the infinite Lorentz factor of the tip of the string, where the charge carriers get concentrated. It is therefore necessary to impose some sort of cutoff for this process. Early calculations [8,17] assumed the cutoff to be where the energy in the charge carriers was comparable with the energy of the string. This procedure gives a linear dependence of the total energy with the maximum Lorentz factor. We redo this calculation in the context of a chiral current on the string. This is the most interesting case since loops are driven to chirality by charge carrier ejection [20,21], especially near the cusps [19].

The full system of equations for a superconducting string is very complicated. Nevertheless in the case of a chiral current they are much simpler and can be solved exactly [22,19,23]. This allows us to compute the electromagnetic radiation from a chiral-current string trajectory taking into account in an exact way the back-reaction of the charge carriers on the string motion. In order to do this we closely follow the procedure described in Spergel et al. [17] and we extract the dependence of the total energy on the maximum Lorentz factor that the string has in the pseudo-cusp region. We found the same linear dependence in the maximum Lorentz factor as earlier estimates. This suggests that the intuitive picture of thinking about the back-reaction of the charge carriers as a process that only becomes important when their energy density is comparable to the energy of the string is correct, making it possible to extend this result to all types of currents.

In the case of cusps where the parameters $a_0''$ and $b_0''$ are nearly parallel or antiparallel, the radiation can be much larger than in the generic case. For small currents, the radiation from such cusps dominates in the total flux, but only by a logarithmic factor.

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