Effect of the buoyancy force on natural convection in a cubical cavity with a heat source of triangular cross-section

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Abstract. Numerical analysis of laminar natural convection inside a cubical cavity with a local heat source of triangular cross-section has been conducted. The mathematical model formulated in dimensionless variables such as “vector potential functions – vorticity vector” has been solved by the finite difference method of the second order accuracy. The three-dimensional temperature fields, 2D streamlines and isotherms in a wide range of the Rayleigh number from $10^4$ to $10^6$ have been presented illustrating variations of the fluid flow and heat transfer.

1. Introduction
Natural convection in three-dimensional cavities has received considerable attention in recent years. It is well known that this heat transfer mechanism has many applications in thermal engineering involving cooling of electronic devices, heat transfer in solar collectors and energy storage systems [1]. For example, Martyushev and Sheremet [2] numerically analyzed natural convection combined with thermal surface radiation in a cubical cavity bounded by solid walls of finite thickness and conductivity in the presence of a heat source of constant temperature. They developed an in-house computational code using vector potential functions and a vorticity vector for numerical simulation of natural convective heat transfer. It was found that less intensive cooling of the domain of interest takes place in case of the three-dimensional model in comparison with the 2D-model. Fontana et al. [3] studied numerically mixed and natural convection in a partially open cubical cavity with an internal heat source. They found that the buoyancy forces are dominant at low Reynolds numbers and high Rayleigh numbers. For higher Reynolds numbers recirculation zones in the upper part of the cavity can also appear due to the influence of the horizontal flow. For intermediate values of these dimensionless parameters, an interaction between the buoyancy and inertial forces generates a complex flow and temperature structures with periodic and quasiperiodic regimes. Oosthuizen and Paul [4] have analyzed an unsteady free convective flow in a 3D rectangular enclosure with two heat sources located at the bottom wall. They revealed that the flow, which is steady at low Rayleigh numbers, becomes unsteady at intermediate Rayleigh numbers and then again becomes steady at higher Rayleigh numbers. The conditions for unsteady flow were studied.

The purpose of the present study is to numerically investigate the effect of the Rayleigh number on the convective heat transfer inside a cubical cavity with a heat source of the triangular cross-section.

2. Physical and mathematical models
The computational domain is shown in the Figure 1. Two opposite vertical walls ($x = 0$ and $x = L$) are kept at constant low temperature $T_c$, while the heat source located at the bottom wall is kept at constant
high temperature $T_h$. The rest walls are adiabatic. The process of convective heat and mass transfer is described by the system of unsteady three-dimensional Boussinesq equations using the dimensionless variables of ‘vector potential functions – vorticity vector – temperature’ type [2, 4].

The mathematical model has been formulated in following form:

\[
\frac{\partial \omega_x}{\partial \tau} + u \frac{\partial \omega_x}{\partial x} + v \frac{\partial \omega_x}{\partial y} + w \frac{\partial \omega_x}{\partial z} - \omega_y \frac{\partial u}{\partial x} - \omega_z \frac{\partial u}{\partial z} = \frac{\Pr}{\text{Ra}} \left( \frac{\partial^2 \omega_x}{\partial x^2} + \frac{\partial^2 \omega_x}{\partial y^2} + \frac{\partial^2 \omega_x}{\partial z^2} \right) + \frac{\partial \theta}{\partial y}, \tag{1}
\]

\[
\frac{\partial \omega_y}{\partial \tau} + u \frac{\partial \omega_y}{\partial x} + v \frac{\partial \omega_y}{\partial y} + w \frac{\partial \omega_y}{\partial z} - \omega_z \frac{\partial v}{\partial x} - \omega_x \frac{\partial v}{\partial x} = \frac{\Pr}{\text{Ra}} \left( \frac{\partial^2 \omega_y}{\partial x^2} + \frac{\partial^2 \omega_y}{\partial y^2} + \frac{\partial^2 \omega_y}{\partial z^2} \right) - \frac{\partial \theta}{\partial x}, \tag{2}
\]

\[
\frac{\partial \omega_z}{\partial \tau} + u \frac{\partial \omega_z}{\partial x} + v \frac{\partial \omega_z}{\partial y} + w \frac{\partial \omega_z}{\partial z} - \omega_x \frac{\partial w}{\partial x} - \omega_y \frac{\partial w}{\partial y} = \frac{\Pr}{\text{Ra}} \left( \frac{\partial^2 \omega_z}{\partial x^2} + \frac{\partial^2 \omega_z}{\partial y^2} + \frac{\partial^2 \omega_z}{\partial z^2} \right). \tag{3}
\]

\[
\frac{\partial^2 \psi_x}{\partial x^2} + \frac{\partial^2 \psi_y}{\partial y^2} + \frac{\partial^2 \psi_z}{\partial z^2} = -\omega_x, \quad \frac{\partial^2 \psi_x}{\partial x^2} + \frac{\partial^2 \psi_y}{\partial y^2} + \frac{\partial^2 \psi_z}{\partial z^2} = -\omega_y, \quad \frac{\partial^2 \psi_x}{\partial x^2} + \frac{\partial^2 \psi_y}{\partial y^2} + \frac{\partial^2 \psi_z}{\partial z^2} = -\omega_z, \tag{4}
\]

\[
\frac{\partial \theta}{\partial \tau} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{1}{\text{Ra} \cdot \text{Pr}} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right). \tag{5}
\]

Here $x$, $y$, $z$ are the dimensionless Cartesian coordinates; $\tau$ is the dimensionless time; $u$, $v$, $w$ are the dimensionless velocity components; $\omega_x$, $\omega_y$, $\omega_z$ are the dimensionless vorticity components; $\psi_x$, $\psi_y$, $\psi_z$ are the vector potential functions; $\theta$ is the dimensionless temperature; Ra is the Rayleigh number; Pr is the Prandtl number.

The initial conditions for formulated governing equations (1) – (5) can be written as follows:

$\psi_x = \psi_y = \psi_z = \omega_x = \omega_y = \omega_z = 0, \quad \theta = 0.5$

The utilized boundary conditions are:

$\psi_x = \psi_y = \psi_z = 0, \quad \omega_x = 0, \quad \omega_y = -\frac{\partial^2 \psi_x}{\partial x^2}, \quad \omega_z = -\frac{\partial^2 \psi_z}{\partial z^2}, \quad \theta = 0$ on $x = 0$ and $x = 1$;

$\psi_x = \psi_y = \psi_z = 0, \quad \omega_x = -\frac{\partial^2 \psi_x}{\partial y^2}, \quad \omega_y = 0, \quad \omega_z = -\frac{\partial^2 \psi_z}{\partial z^2}, \quad \frac{\partial \theta}{\partial y} = 0$ on $y = 0$ and $y = 1$;

$\psi_x = \psi_y = \psi_z = 0, \quad \omega_x = -\frac{\partial^2 \psi_x}{\partial z^2}, \quad \omega_y = -\frac{\partial^2 \psi_y}{\partial x^2}, \quad \omega_z = 0, \quad \frac{\partial \theta}{\partial z} = 0$ on $z = 0$ and $z = 1$;

$\theta = 1$ on heat source surface.
Governing equations (1) – (5) with corresponding initial and boundary conditions have been solved using the finite difference method. A detailed description of the developed numerical method has been done earlier in [2, 5]. The developed computational code has been verified for several benchmark problems [2, 5].

3. Results and discussion

The numerical analysis has been conducted in a wide range of the Rayleigh number: $Ra = 10^4$–$10^6$, $Pr = 0.7$. Figure 2 shows a three-dimensional velocity and temperature fields for different values of the Rayleigh number. For a small value of the dimensionless buoyancy force ($Ra = 10^4$) one can find two main convective rolls with clockwise and counterclockwise rotation reflecting the formation of the ascending flow over the heat source and descending flows near the isothermal vertical surfaces. It should be noted that intensive motion can be observed in the central part of the cavity where the effect of the surrounding adiabatic walls ($y = 0$ and $y = 1$) is not essential. Also the temperature field for $Ra = 10^4$ illustrates development of the heat conduction. An increase in the Rayleigh number leads to an intensification of convective flow and formation of a stable thermal plume.

Figure 2. Three-dimensional velocity and temperature fields
for different values of the Rayleigh number

Figure 3 demonstrates streamlines (the top row) and isotherms (the bottom row) for 3D (solid lines) and 2D (dashed lines) models for different values of the Rayleigh number. An increase in Ra leads to a displacement of the convective cores to the upper part of the cavity. The thickness of the thermal boundary layers is reduced with Ra. There is an intensification of the convective flow and heat transfer inside the cavity with the dimensionless buoyancy force. Taking into account the results for 2D model it is possible to find some weak differences between streamlines and isotherms for the middle cross-section of the cubical cavity. The main reason for the observed differences is the presence of the third coordinate in 3D case.
4. Conclusion
A numerical simulation of the natural convection in a cubical cavity with a local heat source of triangular cross-section has been carried out. The effect of the Rayleigh number on three-dimensional velocity and temperature fields and on the streamlines and isotherms in the middle cross-section has been studied for Pr = 0.7. It has been revealed that an increase in Ra leads to an intensification of convective flow and heat transfer. A comparison of streamlines and isotherms with 2D data shows weak differences that can be explained by the presence of the third coordinate in 3D case.

Acknowledgments
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