QCD-inequality analyses on pion condensate at real and imaginary isospin chemical potentials under finite imaginary quark chemical potential

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By employing QCD inequalities, we discuss appearance of the pion condensate for both real and imaginary isospin chemical potentials. In our discussion, imaginary quark chemical potential is also taken into account. We show that the charged pion can condense for real isospin chemical potential, but not for imaginary one. Furthermore, we evaluate the expectation value of the neutral-pion field \( \langle \pi^3 \rangle \) for imaginary isospin chemical potential by using framework of the twisted mass. As a result, it is found that \( \langle \pi^3 \rangle \) becomes zero for the finite current-quark mass, whereas the expression of \( \langle \pi^3 \rangle \) gives the Banks-Casher relation in the massless limit.

I. INTRODUCTION

Mass spectrum of hadronic degree of freedoms is a key to understand properties of the QCD in the low-energy regime. QCD inequalities provide a powerful framework to deduce the relation between various hadron masses, directly from the QCD \([1-4]\). The inequalities are also useful to see which symmetries are spontaneously broken or not \([5]\), and which kind of meson condensates can occur under various external variables, such as isospin chemical potential \([6-8]\). As a review, for example, see Ref. \([9]\). Application of QCD inequalities is extended to analyses on free energy of the QCD \([10]\), the QCD in the large \( N_c \) limit \([11]\), and hadron interactions \([12]\).

In applying QCD inequalities, it is necessary that the measure in the QCD grand-canonical partition function has positivity \([9]\). This is closely related to whether the fermion determinant possesses positivity or not. It is well-known that the fermion determinant becomes complex for non-zero quark chemical potential \((\mu_q)\) \([13]\). Meanwhile, positivity is ensured for imaginary \(\mu_q\) \([14]\) and hence QCD inequalities are applicable there. The introduction of imaginary \(\mu_q\) also plays a critical role in lattice QCD (LQCD) simulations \([15-22]\), since usual Monte-Carlo technique can be applied.

At finite isospin chemical potential \((\mu_I)\) and imaginary \(\mu_q\), positivity is also realized whether \(\mu_I\) is real \([23-25]\) or imaginary \([26]\). LQCD simulations are thus feasible in both the cases. Indeed, various quantities were calculated by using LQCD simulations so far \([24,27,29]\). These results may give a hint to understand behavior of highly isospin asymmetric matter that exists in the interior of neutron stars \([30]\).

The studies on finite real \(\mu_I\) are also seen in Refs. \([6,8]\), based on QCD inequalities and the chiral perturbation theory in which \(\mu_q\) is set to zero. It was proved in Ref. \([8]\) that the charged-pion condensate occurs for real \(\mu_I\), which is starting at \(\mu_I = m_\pi/2\) with the pion mass \(m_\pi \sim 138\) MeV. Meanwhile, Sakai et al. studied the imaginary \(\mu_I\) region in Ref. \([26]\) with the chiral perturbation theory and the Polyakov-loop extended Nambu–Jona-Lasinio model \([31-34]\). They demonstrated that there is no pion condensate in the entire region of imaginary \(\mu_I\). It is interesting to clarify the reason why the pion condensate does not take place at imaginary \(\mu_I\) in the view point of QCD inequalities, considering the contributions of imaginary \(\mu_q\) simultaneously.

In this paper, we employ QCD inequalities to the real or imaginary \(\mu_I\) regions, taking also into account imaginary \(\mu_q\). We first investigate the \(\gamma_5\)-hermiticity of the fermion matrix. It is shown that positivity of the fermion determinant is guaranteed for both cases, but expression of the \(\gamma_5\)-hermiticity is different between them. Next, we derive QCD inequalities and prove that the charged-pion condensate can take place for real \(\mu_I\), whereas there is no charged-pion condensate for imaginary \(\mu_I\). This suggests that the result in Ref. \([8]\) holds even if imaginary \(\mu_q\) switches on.

As for the neutral pion \(\pi^3\), QCD inequalities are not available since the \(\pi^3\) channel has a disconnected piece in its correlator. Therefore, the expectation value \([\langle \pi^3 \rangle]\) of the neutral-pion field is evaluated directly. To do so, we use the twisted-mass technique \([35-38]\). From the analysis, we find that \([\langle \pi^3 \rangle]\) vanishes for the finite current-quark mass. Meanwhile, the Banks-Casher relation \([39]\) is deduced in the massless limit.

The rest of this paper is organized as follows. In Sec. II, we discuss the \(\gamma_5\)-hermiticity of the fermion matrix and positivity of the measure. In Sec. III, we formulate QCD inequalities for the pion channel and study the possibility of appearance of the pion condensate. In Sec. IV we evaluate the expectation value of the neutral-pion field. Section V is devoted to a summary.

II. FERMION DETERMINANT AND \(\gamma_5\)-HERMITICITY

Our starting point is the two-flavor QCD Lagrangian with finite \(\mu_q\) in Euclidean space-time:

\[ \mathcal{L}_{\text{QCD}} = \bar{q} \gamma_\mu D_\mu + \bar{q} - \mu_q \gamma_4 q + \frac{1}{4 g^2} F_{\mu \nu}^a F^a_{\mu \nu}, \]

where \( q = (u,d)^T \) is the quark field, \( D_\mu = \partial_\mu + i A_\mu \) is the covariant derivative, and \( F_{\mu \nu}^a \) is the field strength of the gluon field \( A_\mu \). The current quark-mass matrix \( \hat{m} \) is given by \( \hat{m} = \text{diag}(m_u, m_d) \) with current \( u \) - and \( d \)-quark masses. Here, the condition \( m_u \neq m_d \) is imposed unless otherwise
stated. In the following discussion, we do not consider the \( \theta \) term that breaks CP symmetry \([40, 52]\) since it causes the sign problem \([5, 43]\), even if the fermion determinant has positivity.

From Eq. (2.1), we can define the QCD action and the QCD grand-canonical partition function as

\[
S_{\text{QCD}} = \int_0^\beta d\tau d^3x L_{\text{QCD}},
\]

\[
Z_{\text{QCD}} = \int DA_\mu DqD\bar{q} \exp \{-S_{\text{QCD}}\},
\]

where \( \beta = 1/T \) is an inverse temperature (T). The gluon and quark fields satisfy the boundary conditions,

\[
A_\mu(\tau + \beta, x) = A_\mu(\tau, x), \quad q(\tau + \beta, x) = -q(\tau, x),
\]

for Euclidean-time (\( \tau \)) direction. In Eq. (2.3), the quark field appears only as a bilinear form and can be integrated out:

\[
Z_{\text{QCD}} = \int DA_\mu \text{Det}\mathcal{M}(\mu_q)e^{-S_G} \equiv \int DA(A),
\]

\[
\mathcal{D}_\mu(A) = DA_\mu \text{Det}\mathcal{M}(\mu_q)e^{-S_G},
\]

where \( S_G \) is the pure gauge action and \( \mathcal{M}(\mu_q) \) is the two-flavor fermion matrix defined by

\[
\mathcal{M}(\mu_q) = \gamma_\mu D_\mu + \tilde{m} - \mu_q \gamma_4.
\]

The symbol “Det” in Eq. (2.6) stands for the determinant for flavor, Dirac, and color indices.

For \( \mu_q = 0 \), the fermion determinant \( \text{Det}\mathcal{M}(\mu_q) \) and the measure \( (2.5) \) have positivity \([44]\) because the fermion matrix has the following \( \gamma_5 \)-hermiticity

\[
\gamma_5 \mathcal{M}(0) \gamma_5 = (\mathcal{M}(0))^\dagger.
\]

For finite real \( \mu_q \), however, the \( \gamma_5 \)-hermiticity is lost due to the relation \([13]\)

\[
\gamma_5 \mathcal{M}(-\mu_q) \gamma_5 = (\mathcal{M}(\mu_q))^\dagger,
\]

which induces the sign problem and positivity of the measure \( (2.5) \) is not assured any longer.

One of the solutions to recover positivity is an introduction of imaginary chemical potential \( \mu_q = i\theta_q T \) with dimensionless quark chemical potential \( \theta_q \). Indeed, the relation

\[
\gamma_5 \mathcal{M}(i\theta_q T) \gamma_5 = (\mathcal{M}(i\theta_q T))^\dagger
\]

(2.10)

guarantees positivity of the measure \([44]\).

Now, let us consider the case of finite isospin chemical potential, i.e. \( \mu_1 > 0 \) or \( \mu_1 < 0 \). In this case, \( \mu_q \) and \( \mu_I \) are given by

\[
\mu_q = \frac{\mu_u + \mu_d}{2}, \quad \mu_I = \frac{\mu_u - \mu_d}{2},
\]

where \( \mu_u \) and \( \mu_d \) are the \( u \)- and \( d \)-quark chemical potentials. Inversely, \( \mu_u \) and \( \mu_d \) are

\[
\mu_u = \mu_q + \mu_I, \quad \mu_d = \mu_q - \mu_I,
\]

respectively. For finite \( \mu_I \), the QCD Lagrangian is changed into

\[
\hat{\mathcal{L}}_{\text{QCD}} = \mathcal{L}_{\text{QCD}} - \mu_I \hat{q} \gamma_4 \tau^3 q
\]

(2.13)

and the isospin SU(2) symmetry is explicitly broken to U(1)_I, where \( I_3 = \tau^3/2 \) for the third component \( \tau^3 \) of the Pauli matrix. The fermion determinant is thus rewritten into

\[
\mathcal{M}(\mu_q, \mu_I) = \mu_I D_\mu + \tilde{m} - \mu_q \gamma_4 - \mu_I \gamma_4 \tau^3.
\]

(2.14)

We first consider the case that \( \mu_I \) is real. Under the setting of \( m_u = m_d = m_0 \), Eq. (2.14) satisfies the relation

\[
\tau^a \gamma_5 \mathcal{M}(i\theta_q T, \mu_I) \gamma_5 \tau^a = \mathcal{M}^I(i\theta_q T, \mu_I) \quad (a = 1, 2),
\]

(2.15)

where \( \tau^a \) means the first or the second component of the Pauli matrix. Here, the summation for \( a \) is not taken. From Eq. (2.15), it can be proved that the fermion determinant \( \text{Det}\mathcal{M}(i\theta_q T, \mu_I) \) possesses positivity \([45]\) because

\[
\{ \text{Det}\mathcal{M}(i\theta_q T, \mu_I) \}^* = \{ \text{Det}\mathcal{M}'(i\theta_q T + \mu_I) \}^* = \text{Det}\mathcal{M}'(i\theta_q T + \mu_I) \]

(2.16)

which is the one-flavor fermion matrix with \( i\theta_q T \pm \mu_I \) and the symbol “det” denotes the determinant only for Dirac and color indices. Note that

\[
\gamma_5 \mathcal{M}'(i\theta_q T \pm \mu_I) \gamma_5 = \mathcal{M}'(i\theta_q T \mp \mu_I)
\]

(2.17)

and hence

\[
\{ \text{Det}\mathcal{M}'(i\theta_q T \mp \mu_I) \}^* = \text{Det}\mathcal{M}'(i\theta_q T \mp \mu_I).
\]

(2.19)

The measure

\[
\mathcal{D}\tilde{\mu}(A) = DA_\mu \text{Det}\tilde{\mathcal{M}}(i\theta_q T, \mu_I)e^{-S_G}
\]

(2.20)

thus maintains positivity. Along this line, we also call Eq. (2.15) the \( \gamma_5 \)-hermiticity.

Now, we return to the condition \( m_u \neq m_d \) and show that the fermion determinant also keeps positivity for finite imaginary isospin chemical potential, i.e. \( \mu_1 = i\theta_1 T \) with dimensionless isospin chemical potential \( \theta_1 \). For \( \mu_1 = i\theta_1 T \), the fermion matrix does not satisfy Eq. (2.15), but rather fulfills

\[
\gamma_5 \tilde{\mathcal{M}}(i\theta_q T, i\theta_1 T) \gamma_5 = \tilde{\mathcal{M}}^I(i\theta_q T, i\theta_1 T).
\]

(2.21)

It should be noted that the Pauli matrix \( \tau^a \) and the condition \( m_u = m_d \) are not needed to prove Eq. (2.21).
TABLE I: In this table, we present whether positivity exists or not for each case. The word “not” means that positivity of the measure does not exist in the corresponding case.

|  | non-zero real \( \mu_q \) | imaginary \( \mu_q \) |
|---|---|---|
| real \( \mu_I \) | not | has positivity for \( m_u = m_d \) |
| imaginary \( \mu_I \) | not | has positivity for any \( m_u \) and \( m_d \) |

From this, its determinant

\[
\text{Det} \mathcal{M}(i \theta_q T, i \theta_f T) = \text{det} \mathcal{M}'(i \theta_q T)\text{det} \mathcal{M}'(i \theta_f T) \tag{2.22}
\]

have positivity, since the relation

\[
\gamma_5 \mathcal{M}'(i \theta_f T) \gamma_5 = (\mathcal{M}'(i \theta_f T))^\dagger \tag{2.23}
\]

is satisfied for \( f = u, d \) and this type of \( \gamma_5 \)-hermiticity guarantees positivity [44]. Here, we have used Eq. (2.17) and introduced \( \theta_u, \theta_d \) as

\[
\theta_u = \theta_q + \theta_1, \quad \theta_d = \theta_q - \theta_1. \tag{2.24}
\]

Positivity of the corresponding measure is thus ensured.

From the discussions mentioned above, we can apply QCD inequalities to the cases of imaginary \( \mu_q \) and real \( \mu_I \), or imaginary \( \mu_q \) and imaginary \( \mu_I \); see Table I. In the next section, we formulate QCD inequalities and discuss what is different for real or imaginary \( \mu_I \).

III. QCD INEQUALITY AND PION CONDENSATE

We derive QCD inequalities for the general meson correlator. Hereafter, we impose the condition \( m_u = m_d = m_0 \). The meson operator is defined by

\[
M(x) = \bar{q}(x) \Gamma q(x), \tag{3.1}
\]

where \( \Gamma \) is a product of the \( \gamma \)-matrix and the Pauli matrix. The meson correlator then can be written as

\[
\langle M(x)M^\dagger(0) \rangle_{q,A} = - \langle \text{Tr} [S(x, 0) \Gamma S(x, 0) \bar{F}] \rangle_A + \langle \text{Tr} [S(x, x) \Gamma] \rangle_A \langle \text{Tr} [S(0, 0) \bar{F}] \rangle_A \tag{3.2}
\]

with \( \bar{F} = \gamma_3 \Gamma \gamma_4 \) [8, 9]. Here, \( \langle \cdots \rangle_{q,A} \) and \( \langle \cdots \rangle_A \) mean the full average and the average over the gauge field, respectively. The propagator \( S(x, y) \) is defined by \( \langle x|M^{-1}|y \rangle \) from an inverse fermion matrix.

Now, we take \( \mathcal{M}(i \theta_q T, \mu_I) \) as a fermion matrix, i.e. the fermion matrix with imaginary \( \mu_q \) and real \( \mu_I \). This matrix satisfies Eq. (2.15) and hence Eq. (3.2) can be transformed into

\[
\langle M(x)M^\dagger(0) \rangle_{q,A} = \langle \text{Tr} [S(x, 0) \Gamma^{\dagger} \Gamma_5 S^\dagger(x, 0) \Gamma_5 \bar{F}] \rangle_A + \langle \text{Tr} [S(x, x) \Gamma] \rangle_A \langle \text{Tr} [S(0, 0) \bar{F}] \rangle_A \leq \langle \text{Tr} [S(x, 0) S^\dagger(x, 0)] \rangle_A \langle \text{Tr} [S(0, 0) \bar{F}] \rangle_A \tag{3.3}
\]

in the case of imaginary \( \mu_I \).

Let us take \( \Gamma = i \gamma_5 \tau^a \) \( (a = 1, 2) \) and consider the correlator of the pions \( \pi^a \). Note that the linear combination of \( \pi^1 \) and \( \pi^2 \) gives the charged-pion channel. In this case, the contribution of a disconnected piece vanishes for both real and imaginary \( \mu_I \), because

\[
\langle \text{Tr} [S(x, x) i \gamma_5 \tau^a] \rangle_A = \langle \text{Tr} [S(x, x) i \gamma_5 \tau^a (\tau_3)^2] \rangle_A = - \langle \text{Tr} [S(x, x) i \gamma_5 \tau^a] \rangle_A \tag{3.5}
\]

holds for \( \pi^a \). Here, we have used \( [S(x, x), \tau_3] = 0 \). Therefore, the inequality (3.3) is saturated for \( \pi^a \), while not for \( \pi^3 \). The charged-pion condensate can thus come out for real \( \mu_I \). On the contrary, the inequality (3.4) is not saturated for \( \pi^a \), and hence at least there is no charged-pion condensate for imaginary \( \mu_I \). These statements suggest that the results in Refs. [8, 26] still hold even when imaginary \( \mu_q \) is finite.

IV. CONDENSATE OF NEUTRAL PION

The discussion mentioned above is not applicable for neutral pion, \( \pi^3 = q_i \gamma_5 \tau^3 q_i \), since it is isoscalar meson and a disconnected piece does not vanish. The inequality (3.3) is saturated only for \( \pi^1 \) or \( \pi^2 \), and hence the \( \pi^3 \) condensate does not occur for real \( \mu_I \) [8]. To prove that \( \pi^3 \) does not condense also for imaginary \( \mu_I \), we should evaluate the expectation value \( \langle \pi^3 \rangle \) directly. In this section, we use the framework of twisted mass [35][53].

We first define the QCD Lagrangian with imaginary \( \mu_q \) and \( \mu_I \), together with the twisted mass:

\[
\mathcal{L}_{\text{twist}} = \bar{q}(\gamma_{\mu} D_{\mu} + m_0 - i \gamma_5 J_5)q - i \theta_4 T \bar{q} \gamma_4 q - i \theta_4 T \bar{q} \gamma_4 \tau^3 q + \frac{1}{4 g^2} F_{\mu \nu}^a F_{\mu \nu}^a, \tag{4.1}
\]

where \( J_5 \) is the real parameter determining a twist angle. Note that the term \( \bar{q} i \gamma_5 \tau^3 q J_5 q \) does not change the \( \gamma_5 \)-hermiticity (2.21). From a generating functional \( Z[J_5] \) defined by

\[
Z[J_5] = \int Dq D\bar{q} D\mu \exp \left[ - \int d^4 x \mathcal{L}_{\text{twist}} \right], \tag{4.2}
\]

the expectation value of \( \pi^3 \) is calculated by

\[
\langle \pi^3 \rangle = \lim_{J_5 \to 0} \lim_{V \to \infty} \frac{\delta}{\delta J_5} \log Z[J_5], \tag{4.3}
\]

Here, we have employed the Schwartz inequality to the right-hand side of the first equality.

For imaginary \( \mu_I \), the inequality differs from Eq. (3.3) since the fermion matrix \( \mathcal{M}(i \theta_q T, i \theta_f T) \) satisfies Eq. (2.21), rather than Eq. (2.15). Adopting the same procedure, we can obtain

\[
\langle M(x)M^\dagger(0) \rangle_{q,A} = \langle \text{Tr} [S(x, 0) \Gamma_5 S^\dagger(x, 0) \Gamma_5 \bar{F}] \rangle_A + \langle \text{Tr} [S(x, x) \Gamma] \rangle_A \langle \text{Tr} [S(0, 0) \bar{F}] \rangle_A \leq \langle \text{Tr} [S(x, 0) S^\dagger(x, 0)] \rangle_A \langle \text{Tr} [S(0, 0) \bar{F}] \rangle_A \tag{3.4}
\]

for imaginary \( \mu_I \).
where $V$ is a volume.

Now, we first take $m_0 \neq 0$ and rewrite the mass term in Eq. (4.1) as

$$m_0 - i\gamma_5 \tau^3 J_5 = M(J_5) e^{-i\gamma_5 \tau^3},$$

(4.4)

where $M(J_5) = \sqrt{m_0^2 + J_5^2}$ and $\alpha = \tan^{-1}(J_5/m_0)$ is the twist angle. Furthermore, we perform the axial $U(1)_A$ transformation to the quark field:

$$q \to e^{\phi \gamma_5 \tau^3} q.$$  

(4.5)

Here, $\phi$ is a rotational angle. Under this transformation, the twisted mass is changed into

$$M(J_5) e^{-i\gamma_5 \tau^3} \rightarrow M(J_5) e^{-i(\alpha - \phi) \tau_5 \tau^3},$$

(4.6)

while the other terms and the measure in $Z[J_5]$ keep the same form. If we choose $\phi = \alpha$, the twisted-mass term becomes $M(J_5)$ [37] [38], i.e. no phase factor, and can evaluate $\langle \pi^3 \rangle$ easily.

The fermion matrix we consider is given by

$$\hat{\mathcal{M}}(i\theta_q, i\theta_I; J_5) = \gamma_\mu D_{\mu} + M(J_5) - i\theta_q \gamma_4 - i\theta_I \gamma_4 \tau^3 = D + M(J_5),$$

(4.7)

where $D = \gamma_\mu D_{\mu} - i\theta_q \gamma_4 - i\theta_I \gamma_4 \tau^3$. The operator $D$ is an anti-hermitian and hence its eigenvalue is pure imaginary. Note that the eigenvalues of the corresponding operator for real $\mu_I$ are not purely imaginary in general, and thereby the discussions presented below cannot be applied.

Since the matrix [4.7] is diagonal in flavor space and the mass $M(J_5)$ is isospin symmetric, we can reach the expression

$$\langle \pi^3 \rangle = \lim_{\mu_I \to 0} \frac{J_5}{M(J_5)} \sum_{f=u,d} \int_0^\infty d\lambda_f \frac{\rho(\lambda_f, m_0)}{i\lambda_f + M(J_5)}.$$  

(4.8)

where $\rho(\lambda_f, m_0)$ is a spectral function and $\lambda_f$ are its eigenvalues for each flavor $f$. It is thus found for $m_0 \neq 0$ that the $\pi^3$ condensate does not take place since Eq. (4.8) vanishes. For $m_0 = 0$,

$$\langle \pi^3 \rangle = \lim_{J_5 \to 0} \sum_{f=u,d} \int_0^\infty d\lambda_f \frac{\rho(\lambda_f, 0)}{i\lambda_f + J_5}.$$  

(4.9)

is deduced, instead of Eq. (4.8). This relation is equivalent to the Banks-Casher relation [39] with flavor dependence and gives the chiral condensate. For $\mu_I = i\theta_I T = 0$ and $m_0 = 0$, Eq. (4.9) returns to isospin symmetric Banks-Casher relation [38].

V. SUMMARY

In this paper, we have investigated appearance of the pion condensate for real and imaginary $\mu_I$, introducing imaginary $\mu_q$. The fermion matrix with imaginary $\mu_q$ has positivity for both the cases of real and imaginary $\mu_I$, but the $\gamma_5$-hermiticity is different from each case.

QCD inequalities for the pion correlator were derived, and the equalities were found to be saturated for the charged-pion channels when we consider real $\mu_I$. However, for imaginary $\mu_I$, the inequalities are not saturated for the charged pion, and hence at least any charged-pion condensate does not occur for imaginary $\mu_I$. This indicates that the results in previous works [8, 26] are also true, even when imaginary $\mu_q$ is finite.

Finally, we have evaluated the expectation value $\langle \pi^3 \rangle$ of the neutral pion directly by using the framework of the twisted mass, because QCD inequalities are not applicable for this channel. The inapplicability comes from the fact that the $\pi^3$ channel has a disconnected piece. We have proved that $\langle \pi^3 \rangle = 0$ under the condition $m_u = m_d = m_0 > 0$. For $m_0 = 0$, the Banks-Casher relation was derived with flavor dependence.

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