A Supersymmetric Composite Model
of Quarks and Leptons

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Abstract
We present a class of supersymmetric models with complete generations of composite quarks and leptons using recent non-perturbative results for the low energy dynamics of supersymmetric QCD. In these models, the quarks arise as composite “mesons” and the leptons emerge as composite “baryons.” The quark and lepton flavor symmetries are linked at the preon level. Baryon number violation is automatically suppressed by accidental symmetries. We give some speculations on how this model might be made realistic.

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1 Introduction

There has been a revival of interest in the old idea that quarks and leptons may be composite \[1, 2\] due to recent progress in the understanding of the non-perturbative low energy dynamics of $\mathcal{N} = 1$ supersymmetric gauge theories \[3, 4\]. In special cases such as $SU(N)$ gauge theory with $N + 1$ flavors, it is believed that the low-energy degrees of freedom are composite gauge-invariant chiral superfields interacting via a dynamical superpotential \[3\]. While this picture of the dynamics cannot be rigorously proven, it passes an impressive number of consistency checks, far beyond the 't Hooft anomaly matching conditions that are the main dynamical input in traditional composite models. For example, the models of Ref. \[3\] have moduli spaces of supersymmetric vacua, and the dynamics interpolates consistently between vacua in which the theory is strongly and weakly coupled. Also, by adding masses for some of the fields in the model, one can relate the dynamics in these models to models with fewer flavors, which are well understood \[4\]. This gives us confidence in the dynamical picture advocated in Ref. \[3\], and holds out the exciting possibility that one can construct composite models without ad hoc dynamical assumptions.

By gauging a subgroup of the global symmetry group of a strongly-coupled supersymmetric gauge theory with the standard model gauge group, one can find models with composite chiral superfields with the quantum numbers of the standard model fields. Such models generally do not break supersymmetry, and have unwanted particles and symmetries that must be removed by adding more fields and interactions to obtain a realistic model. Such a strategy has been successfully pursued in several recent papers by Nelson and Strassler \[6\]. In their models, the right-handed down quarks $\bar{D}$ and the lepton doublets $L$ are elementary whereas the remaining quark and lepton fields are composite.

In this note, we present a class of models where all standard model quarks and leptons emerge as bound states from the confining dynamics of an $SU(N)$ gauge theory. These models display an interesting unification between the quarks and leptons at the preon level, in the sense that quarks and leptons are composites of the same preons. In a multi-generation version of these models, the same preonic horizontal symmetries act on up-type (respectively down-type) quarks and charged leptons (respectively right-handed neutrinos). Because the quarks correspond to higher-dimension operators in the preonic theory, the lowest-dimension baryon number violating operators are dimension 7 in this model, and baryon number violation is highly suppressed.

The resulting models are far from realistic, and we only make some brief remarks on extending the models to make them more realistic. (For one application, see
Ref. [7].) We hope that some of the ingredients of these models will be relevant for the understanding of the flavor problem or other aspects of quark–lepton physics.

2 One Composite Generation

We first illustrate the basic ideas with a model for one generation of composite quarks and leptons. We describe the model by giving the representations of the fields under the group

$$SU(4)_H \times SU(3)_C \times SU(2)_W \times U(1)_Y \times [U(1)_B \times U(1)_L],$$

where $SU(4)_H$ is a strong “hypercolor” gauge group, $SU(3)_C \times SU(2)_W \times U(1)_Y$ is the usual standard model gauge group, and $U(1)_B \times U(1)_L$ are the global baryon and lepton number symmetries. We explicitly display the baryon and lepton number quantum numbers to show that baryon number and lepton number are anomaly-free conserved quantum numbers. The theory consists of the preon fields

$$P_Q \sim (\square, 1, 0) \times (\frac{1}{4}, \frac{1}{4}),$$
$$\bar{P}_U \sim (\square, 1, 1, -1) \times (-\frac{1}{4}, -\frac{1}{4}),$$
$$\bar{P}_D \sim (\square, 1, 1, 1) \times (-\frac{1}{4}, -\frac{1}{4}),$$
$$P_C \sim (\square, 1, \frac{1}{3}) \times (-\frac{1}{12}, \frac{1}{4}),$$
$$\bar{P}_C \sim (\square, 1, \frac{1}{3}) \times (\frac{1}{12}, -\frac{1}{4}),$$

where $\square$ denote the fundamental (antifundamental) representation of the relevant $SU(N)$ group. If we combine $\bar{P}_U, \bar{P}_D$ into an $SU(2)_R$ doublet, we see that the hypercharge assignments obey $Y = T_{3R} + (B - L)$, so this model can be embedded in a left-right symmetric model. If we ignore the weak standard model gauge couplings, this theory is $SU(4)$ supersymmetric QCD with 5 flavors. The analysis of Ref. [8] implies that the low energy degrees of freedom in this model have quantum numbers
follows. There are composite “meson” states

\[
Q \sim \bar{P}C_P \sim (1, \square, \frac{1}{3}) \times (\frac{1}{3}, 0), \\
\bar{U} \sim \bar{P}_U C_P \sim (1, \square, 1, -\frac{4}{3}) \times (-\frac{1}{3}, 0), \\
D \sim P_D C_P \sim (1, \square, 1, \frac{2}{3}) \times (-\frac{1}{3}, 0), \\
\Phi_U \sim \bar{P}_U C_P \sim (1, 1, \square, -1) \times (0, 0), \\
\Phi_D \sim \bar{P}_D C_P \sim (1, 1, \square, 1) \times (0, 0), \\
A \sim (\bar{P}_C P_C)_8 \sim (1, A_{d}, 1, 0) \times (0, 0), \\
T \sim (\bar{P}_C P_C)_1 \sim (1, 1, 1, 0) \times (0, 0),
\]

and composite “baryons”

\[
L \sim P_C^3 P_Q \sim (1, 1, \square, -1) \times (0, 1), \\
E \sim P_C^3 P_U \sim (1, 1, 1, 2) \times (0, -1), \\
\bar{N} \sim \bar{P}_C^3 \bar{P}_D \sim (1, 1, 1, 0) \times (0, -1), \\
X \sim P_C^2 P_Q^2 \sim (1, \square, 1, -\frac{2}{3}) \times (\frac{1}{3}, 1), \\
\bar{X} \sim \bar{P}_C^2 \bar{P}_U \bar{P}_D \sim (1, \square, 1, \frac{2}{3}) \times (-\frac{1}{3}, -1).
\]

We see that the composite fields include one generation of quarks and leptons (\(Q, \bar{U}, \bar{D}, L, E\), plus right-handed neutrinos \(\bar{N}\)) and several additional fields: \(\Phi_{U,D}\) are Higgs doublets, \(A\) is a color octet, \(T\) is a singlet, and \(X, \bar{X}\) are leptoquarks. The theory has a dynamical superpotential

\[
W_{\text{dyn}} = L\Phi_U \bar{E} + L\Phi_D \bar{N} + LQ\bar{X} + X\bar{U} \bar{E} + X\bar{D} \bar{N} + XA\bar{X} + XT\bar{X} \\
- \text{determinant},
\]

where “determinant” denotes terms proportional to 5 powers of composite meson fields.

This is a promising starting point for constructing a realistic model of composite quarks and leptons. We find it striking that the composite particles with standard model quantum numbers (including lepton and baryon numbers) emerge from such a simple model. In particular, the leptons are “unified” with the quarks (in the sense that they are composites of the same preons) without an \(SU(4)\) Pati–Salam symmetry.
3 Three Composite Generations

We can extend the model above to three generations in at least two ways. The simplest is to copy the above structure three times, resulting in a model with hypercolor group \( SU(4) \times SU(4) \times SU(4) \). A more interesting way is based on an \( SU(8) \) hypercolor group with 9 flavors, where part of the additional global symmetry is interpreted as a "horizontal" flavor symmetry.

We describe the second model by giving the representations of the fields under the group

\[
SU(8) \times SU(3)_C \times SU(2)_W \times U(1)_Y \\
\times [SU(3)_Q \times SU(3)_U \times SU(3)_D \times U(1)_B \times U(1)_L],
\]

where the groups in brackets are global symmetries, some of which will have to be broken in a realistic model. The preon fields are

\[
\begin{align*}
P_Q & \sim (\square, 1, \square, 0) \times (\square, 1, 1, \frac{1}{8}, \frac{1}{8}), \\
\bar{P}_U & \sim (\square, 1, 1, -1) \times (1, \square, 1, -\frac{1}{8}, -\frac{1}{8}), \\
\bar{P}_D & \sim (\square, 1, 1, 1) \times (1, 1, \square, -\frac{1}{8}, -\frac{1}{8}), \\
P_C & \sim (\square, \square, 1, -\frac{1}{3}) \times (1, 1, 1, -\frac{5}{24}, \frac{1}{8}), \\
\bar{P}_C & \sim (\square, \square, 1, \frac{1}{3}) \times (1, 1, 1, \frac{5}{24}, -\frac{1}{8}).
\end{align*}
\]

This theory has confining dynamics similar to the model in the previous section. The composite fields in this model consist of the "meson" fields

\[
\begin{align*}
Q & \sim \bar{P}_CP_Q \sim (1, \square, \square, \frac{1}{3}) \times (\square, 1, 1, \frac{1}{3}, 0), \\
\bar{U} & \sim \bar{P}_UP_C \sim (1, \square, 1, -\frac{4}{3}) \times (1, \square, 1, -\frac{4}{3}, 0), \\
\bar{D} & \sim \bar{P}_DP_C \sim (1, \square, 1, \frac{2}{3}) \times (1, 1, \square, -\frac{1}{3}, 0), \\
\bar{\Phi}_U & \sim \bar{P}_UP_Q \sim (1, 1, \square, -1) \times (\square, \square, 1, 0, 0), \\
\bar{\Phi}_D & \sim \bar{P}_DP_Q \sim (1, 1, \square, 1) \times (\square, 1, \square, 0, 0), \\
A & \sim (\bar{P}_CP_C)_8 \sim (1, \text{Ad}, 1, 0) \times (1, 1, 1, 0, 0), \\
T & \sim (\bar{P}_CP_C)_1 \sim (1, 1, 1, 0) \times (1, 1, 1, 0, 0),
\end{align*}
\]
and “baryon” fields

\[
\begin{align*}
L &\sim P_C^3 P_Q^5 \sim (1, 1, 1, 0, 1) \\
\bar{E} &\sim P_C^3 \bar{P}_U^2 P_D^3 \sim (1, 1, 1, 2) \times (1, 1, 0, 1), \\
\bar{N} &\sim P_C^3 \bar{P}_U^3 P_D^2 \sim (1, 1, 0, 1) \times (1, 1, 0, 1, 1), \\
X &\sim P_C^2 P_Q^6 \sim (1, 1, 1, 1, 1) \\
\bar{X} &\sim P_C^2 \bar{P}_U^3 P_D^3 \sim (1, 1, 1, 1, 1)
\end{align*}
\]

This model produces precisely three generations of quarks and leptons, plus right-handed neutrinos. The charged leptons transform under the $SU(3)_U$ horizontal symmetry, while the right-handed neutrinos transform under $SU(3)_D$. Note also that the Higgs multiplets $\bar{\Phi}_{U,D}$ transform under the horizontal symmetries, while the leptoquarks $X$ and $\bar{X}$ do not.

The dynamical superpotential in this model has the same form as the one-generation model:

\[
W_{\text{dyn}} = L\Phi_U E + L\Phi_D \bar{N} + LQX + \bar{E}U + \bar{N}D + X\bar{E} + X\bar{N} + XT \bar{X} - \text{determinant,}
\]

where “determinant” denotes terms proportional to 9 powers of composite meson fields.

## 4 Phenomenological Implications

We now discuss some possible phenomenological implications of the three-generation model. The model is far from realistic as it stands: supersymmetry is not broken, and there are exact horizontal symmetries that forbid quark and lepton masses.

From the dynamical superpotential in Eq. (3.5), we see that if $\langle \Phi_{U,D} \rangle \neq 0$, then the leptons would become massive. However, since $\langle \Phi_{U,D} \rangle$ is presumably of order the weak scale, this would give equal masses to all leptons of order the weak scale. If $\langle T \rangle \neq 0$, then the leptoquarks become massive, and lepton masses can induce quark masses through loop effects. However, this would result in up-type quarks that are proportional to charged lepton masses, with proportionality $\lesssim 1/(16\pi^2)$, which is clearly unrealistic.

To make the model more realistic, one must add additional fields and interactions to break supersymmetry, electroweak symmetry, and the horizontal symmetries. We will not attempt to construct a fully realistic model here, but we make some brief comments possible extensions.
4.1 Direct Bounds on Compositeness Scale

We now discuss the phenomenological bounds on the scale $\Lambda$ where the hypercolor interactions become strong. Bounds on flavor-conserving four-fermion interactions give a bound \[ \Lambda \gtrsim \text{few TeV}. \] The bounds on flavor-changing neutral currents can be more stringent, but their interpretation generally depends on the structure of the flavor symmetry breaking. In the present model, there are interesting flavor-changing neutral current constraints arising from the fact that the quarks and leptons transform under the same flavor groups. In the effective theory below the scale $\Lambda$, this allows interactions such as

\[ \delta \mathcal{L} = \frac{1}{\Lambda^2} \int d^2 \theta \, d^2 \theta^\dagger \left[ L_j^\dagger Q_j^t Q_k L_k + \bar{E}_j^\dagger \bar{U}_j^t \bar{U}_k \bar{E}_k + \cdots \right], \] \[ (4.2) \]

where we show flavor indices explicitly for clarity. This will give rise to flavor-changing neutral current processes conserving $B_j - L_j$ and $I_{3j}$, where the charges are nonzero only on generation $j$. The most stringent bound comes from the process $D^0 \to e^- \mu^+$, which gives a bound

\[ \Lambda \gtrsim 4 \text{ TeV} \left( \frac{f_D}{100 \text{ MeV}} \right)^{1/2} \left( \frac{m_c}{1 \text{ GeV}} \right)^{-1/2}. \] \[ (4.3) \]

There is also a bound from $\pi^\pm \to e^\pm \nu_e$ that comes from demanding consistency with $\pi^\pm \to \mu^\pm \nu_\mu$, which has no non-standard contribution in this model. This gives a bound $\Lambda \gtrsim 2$ TeV. Note that the highly sensitive process $K^0 \to e^\pm \mu^\mp$ is forbidden in this model.

4.2 Flavor Symmetries

Because this model naturally has horizontal symmetries, it is attractive to consider the possibility that these are spontaneously broken. If the horizontal symmetries are gauged, bounds on flavor-changing neutral currents imply that the scale of spontaneous flavor breaking is larger than $\sim 10^6$ GeV/g if the horizontal symmetries are gauged with gauge coupling $g$. If the horizontal symmetries are global symmetries, constraints from big-bang nucleosynthesis on horizontal Nambu–Goldstone bosons imply that the flavor-breaking scale is larger than $\sim 10^{10}$ GeV. Since we have been unable to find this bound in the literature, we explain it briefly. In order to sufficiently dilute the $3 \times 8 = 24$ Nambu–Goldstone bosons that occur when the flavor
symmetry is broken, they must decouple above the electroweak phase transition. (The dilution factor is \( \sim 1/50 \) for the minimal supersymmetric standard model.) If we assume that the observed pattern of fermion masses is due to a sequential breakdown of the flavor symmetries \( SU(3) \rightarrow SU(2) \rightarrow 1 \), then 10 of the Nambu–Goldstone bosons will be associated with the flavor breaking that gives rise to the order-1 top quark Yukawa coupling, and will have couplings \( \sim 1/f \), where \( f \) is the flavor breaking scale. Demanding that these decouple above \( T \sim 100 \text{ GeV} \) gives the quoted bound.

To generate lepton masses, we add to the preonic theory the interaction terms

\[
\delta W = \Phi_U P_Q \overline{P}_U + \Phi_U \Delta_U H_D + \Phi_D P_Q \overline{P}_D + \Phi_D \Delta_D H_U, \tag{4.4}
\]

where we have introduced new elementary fields

\[
\begin{align*}
\Phi_U &\sim (1, 1, \mathbf{1}, 1) \times (\mathbf{1}, 1, 0, 0), \\
\Phi_D &\sim (1, 1, \mathbf{1}, -1) \times (\mathbf{1}, 1, 0, 0), \\
\Delta_U &\sim (1, 1, 1, 0) \times (\mathbf{1}, 1, 0, 0), \\
\Delta_D &\sim (1, 1, 1, 0) \times (\mathbf{1}, 1, 0, 0), \\
H_U &\sim (1, 1, \mathbf{1}, 1) \times (1, 1, 0, 0). \\
H_D &\sim (1, 1, \mathbf{1}, -1) \times (1, 1, 0, 0).
\end{align*}
\tag{4.5}
\]

That is, \( \Phi_{U,D} \) are flavored Higgs doublets with quantum numbers conjugate to the composite \( \overline{\Phi}_{U,D} \) fields, \( H_{U,D} \) are elementary Higgs doublets, and \( \Delta_{U,D} \) are fields that transform only under the horizontal symmetries.

Below the compositeness scale, the first term in Eq. (4.4) gives rise to the effective superpotential term

\[
\delta W_{\text{eff}} = \Lambda \overline{\Phi}_U \Phi_U + \Lambda \overline{\Phi}_D \Phi_D, \tag{4.6}
\]

so we can integrate out the fields \( \Phi_{U,D} \) and \( \overline{\Phi}_{U,D} \). When we do this, the other terms in Eq. (4.4) give rise to

\[
\delta W_{\text{eff}} = \frac{1}{\Lambda} \Delta_U L H_D \overline{E} + \frac{1}{\Lambda} \Delta_D L H_U \overline{N}. \tag{4.7}
\]

Now we assume that there are additional interactions of \( \Delta_{U,D} \) that give \( \langle \Delta_{U,D} \rangle \neq 0 \), thereby breaking the horizontal symmetries. This will give masses to the charged leptons of order

\[
m_\ell \sim \frac{v f_\ell}{\Lambda}. \tag{4.8}
\]
where $f_\ell$ is the vacuum expectation value that breaks the flavor symmetry of the lepton of type $\ell$. Note that we have introduced no dimensionful couplings into the theory. Also, if $f_\ell \ll \Lambda$, this can explain why the charged leptons are lighter than the weak scale.

If we attempt to extend this mechanism to quarks, we immediately encounter a problem. The fact that the quarks and leptons transform under the same flavor symmetry apparently leads to the prediction that the charged lepton mass matrix is proportional to the up-type quark mass matrix, which is not even approximately correct. This conclusion may be avoided by noting that below the compositeness scale, the theory has a larger accidental global flavor symmetry under which the quarks and leptons transform separately. (In the effective theory below the scale $\Lambda$, this symmetry is broken by terms in the effective Kähler potential such as Eq. (4.2).) Therefore, flavor symmetry breaking for quarks and leptons can be independent.

For this to work, the compositeness scale must be larger than the scale of flavor symmetry breaking, so we have e.g. the bound $\Lambda \gtrsim 10^6$ GeV/g for gauged flavor symmetries. We believe that there are no insurmountable obstacles to constructing an explicit model along these lines, but we will not attempt this here.

4.3 Baryon Number Violation

One very attractive feature of this model is that baryon number violation is highly suppressed by accidental symmetries. Constraints on $B$ violation are very severe: even dimension-5 operators suppressed by $\sim 1/M_{\text{Planck}}$ give rise to unacceptably large $B$ violation. The lowest-dimension baryon number violating terms in the preonic theory are dimension-7 operators of the form

$$\delta W = \frac{1}{M^4}(P_UP_C)(P_DP_C)(P_DP_C).$$

(4.9)

Since lepton number is not violated, this does not lead to proton decay. The absence of observed neutron–antineutron oscillations gives a bound $M/\Lambda \gtrsim 50$.

We will not analyze the precise constraints on higher-dimension operators, since it is clear that the theory is safe from Planck-scale baryon number violation. This feature is not spoiled by additional interactions needed to make the model realistic as long as these do not involve fields that carry baryon or lepton number.

4.4 Supersymmetry and Electroweak Symmetry Breaking

This model does not break supersymmetry as it stands. If we take the compositeness scale to be large (as suggested by considerations of flavor symmetry breaking...
above), then the usual mechanisms of supersymmetry breaking can be used. That is, supersymmetry may be broken in a hidden sector and communicated to the observable sector through gauge, gravitational, or other interactions. (For an alternative approach, see Ref. [7].)

5 Conclusions

We have constructed a simple class of models that give rise to composite quarks, leptons, and Higgs bosons. The models are based on supersymmetric QCD, with a subgroup of the global symmetries gauged with the standard model gauge group. Composite fields with quantum numbers of quarks and leptons emerge in an interesting way in these models; for example, it is amusing that leptons are “baryons” from the point of view of the hypercolor dynamics. The models require additional interactions to break supersymmetry, electroweak symmetry, and generate fermion masses. We hope that this type of model will prove useful in constructing realistic composite models of quarks and leptons.

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