A theoretical investigation on tetra-quark states in the context of composite fermion model of diquark

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Abstract.

The masses of the tetra-quark states (qqq̅q̅) have been investigated in the framework of diquark [qq] anti-diquark [q̅q̅] configuration where the diquarks (anti-diquarks) are described by the Composite Fermion Model. The tetra-quark masses of several particles such as f0(1370), K (1460), D (2632), Y (4143), Zc(4430) and Tcc(3797) have been investigated. The higher state masses of these have also been studied using Mass Loaded Flux Tube Model. Their Regge trajectories for different angular momenta have been drawn indicating their masses at L=0. These estimated masses for L=0 compare favorably with their experimental ground state masses. The corresponding string tensions are estimated. The slope of the Regge Trajectories (RT) are extracted and it is observed that RT slope shows a non-linear behavior.

1. Introduction

The concept of multi-quark state is as old as usual quark model of baryon and meson. Multiquark states are new states of matter. Multiquark states are bound states of five valence quarks like pentaquarks and four valence quarks which are tetraquarks. These states are color neutral with configuration (qqqqQ) and (qqQ̅q̅) respectively. Since the discovery of X (3872) by Belle group [1] a number of multiquark states are reported by different groups. Recently a number of new exotic states which are tetra-quark states have been identified by LHC. In the past decade many tetraquarks are reported like Y (3940) [2], Y (4140) [3] [4], Z⁰(4430) [5], and Z⁺(4200) [6] which are charmonium-like states and Zb⁰(10610) and Zb⁺(10650) are bottomonium-like states [7]. The open-heavy flavor meson X(5568) [8] has stimulated extensive discussions on their exotic assignments. Si-Qiang Luo et al [9] have studied systematically the mass splittings of the qqQ̅Q̅ (q = u, d, s and Q = c, b) tetraquark states with the color-magnetic interaction by considering color mixing effects and estimated roughly their masses. They also discussed the possible decay patterns of the tetraquarks. A search for the exotic meson X(5568) has been done by Aaltonen et al [10].

Multi quark states can not be described in the framework of conventional model. Tetraquark states have been studied by Wilcox et al [11] in the framework of the Thomas-Fermi quark model. Deng et al [12] have investigated Y(4626) state as P-wave tetraquark state [cs][̅cs] in the context of multiquark color flux tube model considering multibody confinement potential and
one gluon exchange interaction. Low lying S-wave higher states of doubly heavy tetraquarks have been studied by Yang et al [13] in the context of chiral quark model. An excellent review on tetraquark and pentaquark states has been given by Liu et al [14]. Diquark correlation is one of the most important candidate for describing the mutiquark states. In hadron physics, two quarks which are anti-symmetric in colors and spins are attracted to each other forming a low energy configuration described as diquark. Exact nature of the diquark is not known. A number of theoretical models for diquark have been suggested to study such states. A composite model for diquark has been suggested by Ghosh et al [15] which is extensively used to study the properties of penta-quark states and yields very good agreement with the experimental predictions. In the current work tetraquark states are studied in detail using the model.

Composite Fermion (CF) is the bound state of an electron in the strong magnetic field. The electron in two dimensions absorbs a substantial amount of magnetic field transforming it to a new particle called composite fermion (CF) [16]. Composite fermions are quasi particles which are particle-like entities arising in some system of interacting particles. Quasi particles are low-lying excited states representing the properties of the system simulating many body interaction. In analogy with the two dimensional electron gas in high magnetic field a CF model for the diquark is suggested where it is described as Composite Fermion in the presence of chromo-magnetic nature of the QCD vacuum and acts as an independent entity like quasi-particle which interacts weakly within the system.

2. Formulation
Starting from the Hamiltonian of a composite fermion with a momentum cut off Λ, the expression for the quasi particle mass in a gauge invariant system can be expressed as (with potential V=0) [17]:

\[
\frac{1}{m^*} = \frac{1}{m} \left[ 1 + \frac{\Lambda^4}{2p_F^4} \right]
\] (1)

where \(m^*\) is the effective mass of the CF, \(m\) is the constituent mass, \(p_F\) is the Fermi momentum of the CF and \(\Lambda\) is a cut off parameter. The CF picture has been applied in computing the diquark mass. The effective mass of the diquark now can be expressed as following (1):

\[
\frac{1}{m_D^*} = \frac{1}{m_{q_i} + m_{q_j}} \left[ 1 + \frac{\Lambda^4}{2p_F^4} \right]
\] (2)

where \(m_D^*\) is the effective mass of the diquark, \(m_{q_i}\) and \(m_{q_j}\) are the constituent masses of the corresponding quark flavors. We have computed the masses of the diquarks of different flavors using the above expression (2). It is well known that the collective excitation which occurs in complicated microscopic system is equally applicable to both electrons and holes. We argue that the effective masses of the anti-diquarks can be obtained by using the same approach as in diquark. So diquark and anti-diquark can be treated in equal footing. The diquark masses are computed in CF model.

The semi classical Mass Loaded Flux Tube Model has been suggested by Selem and Wilczek [18] where two masses are connected by a relativistic string of string tension \(\sigma\), rotating with angular momentum \(L\). Considering the heavy tetraquark in the diquark-antidiquark picture in the context of flux tube model, the system can be represented as the heavy diquark at rest and the other diquark connected to it by flux tube which is in constant rotation with the heavy diquark.

The mass formula for the tetraquark can be written as:
\[ E = M_D + \sqrt{\frac{\sigma L}{2}} + 2^{\frac{1}{2}} K L^{-1} \mu_D^2 + a L \cdot S \]  \hspace{1cm} (3)

where \( \mu_D \) is the mass of the diquark which is connected by the flux tube and \( K = \frac{2 \pi \frac{1}{2}}{2} \). \( L \) represents the orbital angular momentum of the diquark and \( S \) is the total spin. \( L \cdot S \) is spin-orbit interaction. The orbital excitation of the particle can be described by the Regge trajectories and has been expressed by:

\[ E^2 = \alpha J \]  \hspace{1cm} (4)

where \( E \) is the energy at the higher state for different values of \( L \) and \( J \) is the total angular momentum. Regge slope (\( \alpha \)) can be expressed as:

\[ \alpha = \frac{1}{2\pi\sigma} \]  \hspace{1cm} (5)

The energies of the heavy tetra-quark system have been estimated for different \( L \) using (3). The calculated diquark masses in CF model which have been displayed in Table 1, have been used for estimation of these higher energy states of several tetra-quarks. The input values are string tension \( \sigma = 0.194 \text{GeV}^2 \) [19], \( \Lambda = 0.573 \text{ GeV} \) [20] for light sector and \( \Lambda = 0.6533 \text{ GeV} \) [21] for heavy sector, \( m_u = m_d = 0.360 \text{ GeV} \), \( m_s = 0.54 \text{ GeV} \), \( m_c = 1.71 \text{ GeV} \), \( m_b = 5.05 \text{ GeV} \) [22], \( a = 0.04 \text{ GeV} \) [23]. The Fermi momenta for different particles have been estimated using the work of Bhattacharya et al [24]. The radii for diquarks are given from the existing literatures [25], [26], [27]. The results have been displayed in Table 2. The tetra-quark masses have been determined using the scalar diquark masses. Scalar diquark mass corresponds to spin 0. Hence all the variations of \( E^2 \) versus \( L \) have been plotted for different tetra-quarks (Figure 1- Figure 6) and the value of Regge slopes \( \alpha \) have been extracted from their graphs. These Regge slopes are displayed in Table 3. This is to be mentioned that Regge trajectories indicate the particle masses at \( L=0 \) which corroborate to the ground state masses of the tetra-quark systems. These extracted masses of the tetra-quarks compare favorably with their experimental ground state masses and have been displayed in Table 2.
3. Tables and table captions

Table 1: Diquark masses computed in CF model

| Diquark content $[qq]$ | Scalar Diquark mass in GeV | Vector Diquark mass in GeV |
|------------------------|----------------------------|---------------------------|
| $[ud]$                 | 0.4848                     | 0.5957                    |
| $[us]$                 | 0.4287                     | 0.5913                    |
| $[uc]$                 | 0.9158                     | 1.4052                    |
| $[sc]$                 | 1.0363                     | 1.6958                    |
| $[ub]$                 | 3.568                      | 4.0775                    |
| $[sb]$                 | 4.0244                     | 4.5258                    |
| $[cc]$                 | 3.124                      | 3.1531                    |
| $[bc]$                 | 6.4575                     | 6.4599                    |
| $[bb]$                 | 9.9712                     | 9.9811                    |
### Table 2: Diquark masses computed in CF model

| Particle name | Quark content | Values of Angular Momentum | Calculated Mass in GeV | Mass obtained from graph at L = 0 in GeV | Experimental Mass at L = 0 in GeV |
|---------------|---------------|----------------------------|------------------------|----------------------------------------|----------------------------------|
| $K$           | $[ud][\bar{u}s]$ | 1                          | 1.4652                 | 1.414                                  | 1.460                            |
|               |               | 2                          | 1.4816                 |                                        |                                  |
|               |               | 3                          | 1.5237                 |                                        |                                  |
|               |               | 4                          | 1.5705                 |                                        |                                  |
| $f_0$         | $[ud][\bar{u}d]$ | 1                          | 1.6722                 |                                        |                                  |
|               |               | 2                          | 1.7286                 | 1.470                                  | 1.370                            |
|               |               | 3                          | 1.8107                 |                                        |                                  |
|               |               | 4                          | 1.8975                 |                                        |                                  |
| $D$           | $[cd][\bar{d}s]$ | 1                          | 2.1328                 |                                        |                                  |
|               |               | 2                          | 2.1205                 | 2.598                                  | 2.6325                           |
|               |               | 3                          | 2.1479                 |                                        |                                  |
|               |               | 4                          | 2.1851                 |                                        |                                  |
| $Y$           | $[cs][\bar{c}s]$ | 1                          | 3.9257                 |                                        |                                  |
|               |               | 2                          | 4.0174                 | 3.816                                  | 4.143                            |
|               |               | 3                          | 4.1582                 |                                        |                                  |
|               |               | 4                          | 4.3066                 |                                        |                                  |
| $T_{cc}$      | $[ud][\bar{c}c]$ | 1                          | 4.1605                 |                                        |                                  |
|               |               | 2                          | 4.177                  | 4.117                                  | 3.797                            |
|               |               | 3                          | 4.2190                 |                                        |                                  |
|               |               | 4                          | 4.2658                 |                                        |                                  |
| $Z_c$         | $[cd][\bar{c}u]$ | 1                          | 4.5331                 |                                        |                                  |
|               |               | 2                          | 4.5334                 | 4.337                                  | 4.430                            |
|               |               | 3                          | 4.6482                 |                                        |                                  |
|               |               | 4                          | 4.7941                 |                                        |                                  |
Table 3: Regge slopes extracted from the graphs

| Particle name | Quark content | Values of Angular Momentum | Value of $\alpha$ from Regge trajectory in $GeV^{-2}$ |
|---------------|---------------|----------------------------|-----------------------------------------------|
| $K$           | $[ud][\pi\bar{\pi}]$ | 1                          | 2                                             |
|               |               | 3                          | 0.09                                          |
|               |               | 4                          |                                               |
| $f_0$         | $[ud][ud]$    | 1                          | 2                                             |
|               |               | 3                          | 0.14                                          |
|               |               | 4                          |                                               |
| $D$           | $[cd][ds]$    | 1                          | 2                                             |
|               |               | 3                          | 0.4                                           |
|               |               | 4                          |                                               |
| $Y$           | $[cs][\tau\bar{\tau}]$ | 1                       | 2                                             |
|               |               | 3                          | 0.528                                         |
|               |               | 4                          |                                               |
| $T_{cc}$      | $[ud][\tau\bar{\tau}]$ | 1                       | 2                                             |
|               |               | 3                          | 0.58                                          |
|               |               | 4                          |                                               |
| $Z_c$         | $[cd][\tau\bar{\tau}]$ | 1                       | 2                                             |
|               |               | 3                          | 1.35                                          |
|               |               | 4                          |                                               |
4. Figures and figure captions

Figure 1: Variation of $E^2$ vs. L for $K[ud][us]$.  

Figure 2: Variation of $E^2$ vs. L for $f_0[ud][ud]$.  

Figure 3: Variation of $E^2$ vs. L for $D[cd][us]$.  

Figure 4: Variation of $E^2$ vs. L for $Y[cs][cs]$.  

Discussions
In the present work the masses of the tetraquarks states are estimated using composite fermion model of diquark. The higher state masses are estimated in Mass loaded Flux tube model. The masses of the tetraquark states corresponding to $J = 0$ have been extracted from the graph and found to have close agreement with the experimental values of masses of the respective exotic mesons. The Regge slopes show non-linear behavior and found to be less than the universal accepted value $\sim 1 GeV^{-2}$. The non-linear behavior of Regge slopes of hadrons is one of the most interesting topic of research now a days [28]. It is suggested that most of the ordinary resonances can be fitted in the linear Regge trajectories with universal slope $1 GeV^{-2}$ at least in the small angular momentum limit but a number of scattering data and models suggest non-linearity of RT. Pelaez et al [29] have studied the Regge slopes of meson resonances. They have observed that the Regge trajectories of $K_0(899)$ and $f_0(500)$ mesons are non linear with much smaller slopes and show a non ordinary behavior. Tang et al [30] have constructed the Regge trajectories with experimental data and have suggested that RT is non linear and and sometimes intersecting. Badalian et al [31] have studied $q\bar{q}$ interaction in relativistic string Hamiltonian with flattening confining potential and have observed 30% decrease in the slope of RT. The current investigation with our model suggests a non linearity for RT for the exotic mesons tetra-quark systems. The slope of RT is found to be non linear even at the small values of angular momentum except $Z_c$ where it is found to be more than unity. It may be noted that slope of RT is related to the string tension or the energy density of the tubes connecting the various quarks. The present work suggests that exotic tetra-quark systems appear to be of non ordinary nature with the slope of the trajectories scaling with the masses of the exotic mesons. The higher state tetra-quark masses are predicted theoretically but the experimental results are yet to be found out. It may be asserted that the theoretical masses of tetra-quarks predicted in the current work could be a useful guide for the determination of experimental higher state masses in near future.

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