Numerical study on the symmetric and asymmetric orientation of the crack branching in 2D plate

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Abstract. The phenomenon of crack branching is one of the typical fracture behaviours. The effect of crack branching orientation is investigated in this paper. By considering a static branched crack in a 2D plate under uniaxial traction, the numerical study is carried out for two study cases. The first study case is the symmetric crack branching in which the various crack branching length and orientation have the same value between both crack branching. The second case is the asymmetric case crack branching. In this case, both crack branching length has a particular constant value, Moreover, the orientation of first crack branching is constant and then the second one has various values. The stress intensity factors of the crack tips are calculated for both study cases. It is revealed for the symmetric case; the increasing of the crack branching length will increase the value of stress intensity factors $K_I$ for various orientations of crack branching. In contrast, the stress intensity factors $K_I$ will tend to decrease along with the increasing of the crack branching orientation. Moreover, the stress intensity factors $K_I$ of first crack branching will increase, but the stress intensity factors $K_I$ of second crack branching will decrease along with increasing of the second orientation crack branching for the asymmetric case. Furthermore, the direction the stress intensity factors $K_{II}$ will prone to change along with the increasing of the crack branching orientation. The stress intensity factor $K_{II}$ tends to increase along with the increasing of the crack branching orientation as well as the increasing of the crack branching length for the symmetric case. And then the increase of constant angle of first crack branching will increase significantly the stress intensity factors $K_{II}$ of the first crack tip along with the increasing of the second crack branching angle for the asymmetric case.

1. Introduction
The crack tends to branch out if cracks travel faster, especially in brittle solids such as glass, rocks, and rock-like materials. The mechanism of crack branching is a complicated process and has been usually treated dynamically. Many researchers have been considered crack branching under dynamic loading [1-4]. There are some methodologies used to predict the crack branching behaviour such as XFEM [5], Bond-particle model [2], Peridynamic modelling [4], multidimensional space method [6] and a modified displacement discontinuity method [7], the crack-tip displacement discontinuity element [8] and Pseudo-spring smoothed particle hydrodynamics [3]. Most of them, the stress intensity factors are considered as the crack growth parameter. However, Cheng [6] proposed the criterion of crack branching based on the energy release rate and the strain energy density by Theocaris and Andrianopoulos [9].
In this present study, the Finite Element Method (FEM) is proposed to study the effect of the Symmetric and Asymmetric of crack branching on the stress intensity factors. The Asymmetric case has been studied by Yan [8] using the crack branching model developed by Theocaris and Andrianopoulos [9]. They are introduced the 2D plate model having a center crack on the plate with crack branching. Therefore, three crack tips in the crack branching model are considered.

In this present analysis, the crack branching model proposed is the single crack edge with crack branching. Two crack tips are considered in this model. However, for the case of the Symmetric, only one crack tip is studied because in the symmetric case, it is assumed that two crack tips have similar behaviour. The stress intensity factor mode-I ($K_I$) and mode-II ($K_{II}$) is considered this study because in the crack branching the mode of fracture is not only mode-I and mode-II but also the mixed mode I/II at the crack tips [10].

2. Numerical modelling of crack branching
Consider the 2D plate with a 25 mm of main crack length ($a$) and the crack branching length ($L_1$ and $L_2$) having the angle of crack branching ($\theta_1$ and $\theta_2$). The plate has a 100 mm of width ($W$) and 100 mm of height ($h$) under the uniform tensile stress, $\sigma_0 = 10$ MPa, as shown in figure 1.

In this present analysis, there are two cases considered, namely the symmetric case and the asymmetric case. In the symmetric case, the stress intensity factors of the crack tips are studied for the variation of the crack branching length namely $L_1 = L_2 = 5$ mm, 10 mm, 15 mm and 20 mm along with the variation angle of crack branching, namely $\theta_1 = \theta_2 = 20^0, 25^0, 30^0, 35^0, 40^0, 45^0$ and $60^0$. Furthermore, in the case of asymmetric, the stress intensity factors for both of the crack tips (CT-1 and CT-2) are investigated with both of the given crack branching lengths ($L_1 = L_2 = 15$ mm). The variation angle of first crack branching ($\theta_1$) are $30^0$ and $45^0$ along with the variation angle of second crack branching ($\theta_2$) namely $20^0, 25^0, 30^0, 35^0, 40^0, 45^0$ and $60^0$. The crack branching process has been numerically modelled based on the LEFM principles by using the Finite Element Method (FEM). The meshing model used in FEM is shown in figure 2.
3. The results and discussion

The effect of the orientation of the branch crack length on the stress intensity factors $K_I$ for the Symmetric case is shown in figure 3. The figure shows that the lowest stress intensity factors $K_I$ occur at the $60^\circ$ of crack branching angle for all variation of the crack branching length. It shows that increasing of crack branching orientation will decrease the stress intensity factors $K_I$. The value of stress intensity factor depends on the direction of crack mode, increase the orientation will change the direction of crack mode. Therefore, the first mode direction will decrease along with increasing the angle of crack branching length. Moreover, the highest stress intensity factors $K_I$ occur in the 20 mm of the crack branching length. The increasing of the crack branching length will increase the stress intensity factors $K_I$. It means that stress intensity factors $K_I$ have a relationship that is directly proportional to the length of the branch crack.

The effect of the orientation of the crack branching length on the stress intensity factors $K_{II}$ for the Symmetric case is shown in figure 4. The figure shows that the direction of stress intensity factor changes along with the increasing of the crack branching orientation for both crack tips (CT). It shows that the direction of the stress intensity factors $K_{II}$ relates to the direction of the loading. Furthermore, the stress intensity factor tends to increase along with the increasing of the crack branching orientation as well as the increasing of the crack branching length. It reveals that stress intensity factors $K_{II}$ have a relationship that is directly proportional to the length of the branch crack.

Figure 2. Meshing model in FEM.
Figure 3. The effect of various crack branching orientation on the stress intensity factors $K_I$ for various crack branching length for the Symmetric case.

Figure 4. The effect of various crack branching orientation on the stress intensity factors $K_{II}$ for various crack branching length for the Symmetric case.
Figure 5. The effect of various crack branching orientation on the first mode of the stress intensity factor $K_I$ for various crack branching length for the Asymmetric case.

The effect of the angle of the branch crack length on the stress intensity factors $K_I$ for the Asymmetric case is shown in figure. 5. The figure shows that the stress intensity factors $K_I$ of the crack tip 1 (CT-1) having both the $30^\circ$ and $45^\circ$ of the constant angle ($\theta_1$) will prone to increase along with the increasing of the second crack branching angle ($\theta_2$). In contrast, the stress intensity factors $K_I$ of the crack tip 2 (CT-2) will tend to decrease along with the increasing of the second crack branching angle ($\theta_2$). Furthermore, the stress intensity factors $K_I$ of both of the crack tips (CT-1 and CT-2) will increasing along with the increasing of the constant angle of the first crack branching ($\theta_1$).

Figure 6. The effect of various crack branching orientation on the second mode of the stress intensity factors $K_{II}$ for various crack branching length for the Asymmetric case.

The effect of the orientation of the crack branching length on the stress intensity factors $K_{II}$ for the Asymmetric case is shown in figure. 6. The figure shows that in general, the direction of stress intensity factor changes along with the increasing of the crack branching orientation for both crack tips (CT)
except for CT-1 with 45° of the angle of crack branching. It is shown that increasing of the constant angle of first crack branching ($\theta_1$) will increase significantly the stress intensity factors $K_{II}$ of CT-1 along with the increasing of the second crack branching angle ($\theta_2$).

4. Conclusion
In this paper, the variation angle of crack branching on stress intensity factors $K_I$ and $K_{II}$ are investigated for both symmetric and asymmetric cases. It is revealed that for the symmetric cases, the increasing of the crack branching length will increase the value of stress intensity factors $K_I$ for various orientations of crack branching. In contrast, the stress intensity factors $K_I$ will tend to decrease along with the increasing of the crack branching orientation. Moreover, the stress intensity factors $K_I$ of first crack branching (CT-1) will increase, but the stress intensity factors $K_I$ of second crack branching (CT-2) will decrease along with increasing of the second orientation crack branching for the asymmetric case. Furthermore, the direction the stress intensity factors $K_{II}$ will prone to change along with the increasing of the crack branching orientation. The stress intensity factor $K_{II}$ tends to increase along with the increasing of the crack branching orientation as well as the increasing of the crack branching length for the symmetric case. And then the increase of constant angle of first crack branching ($\theta_1$) will increase significantly the stress intensity factors $K_{II}$ of CT-1 along with the increasing of the second crack branching angle ($\theta_2$) for the asymmetric case.

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