Optical MIMO Communication Using Holographic Spectral Multiplexing of Pulsed Ultrashort Laser

Alireza Khodaei
Department of Computer Science and Engineering
University of Lincoln-Nebraska
Lincoln, USA
khodaei@huskers.unl.edu

Jitender Deogun
Department of Computer Science and Engineering
University of Lincoln-Nebraska
Lincoln, USA
deogun@cse.unl.edu

Abstract—In this paper, we introduce Holographic Spectral Multiplexing (HSM) as a novel technique to enable multiple-input multiple-output (MIMO) communication in optical networks. HSM uses the spectral space of ultrashort laser pulses to create line codes in the form of 2D holograms. The pulse processing is performed in the temporal Fourier domain by spatially dispersing the pulse frequency components in a spectral processing device (SPD). The 2D holograms are composed of the patterns of intensity disparities that a SLM inscribes on the spectrally-decomposed Fourier plane of the pulse. The holographic line codes defined in this way transform the ultrashort laser pulses into high-entropy data symbols, hence, enhance the communication’s spectral efficiency. Unlike conventional optical multiplexing schemes (e.g., TDM, WDM, or SDM), HSM does not physically or abstractly separate the communication propagation space into subchannels. Rather, HSM realizes a MIMO communication paradigm by allowing the photonic waves under the pulse envelope to propagate in the same space so they scatter and interfere by chromatic dispersion. This allows HSM to form beams between the pixels of SLM at the sender and receiver sides and optimize the beam to adapt to channel scattering situation. In this way, HSM delivers a rate gain that in best case exponentially increases the information rate of communication.

Index Terms—Optical Communication Systems, MIMO Communication, Spectral Efficiency, Spatial Multiplexing, Ultrashort Laser Pulses

I. INTRODUCTION

The growth in demand for both quality and quantity of end-user Internet usage entail the need for an unprecedented capacity of optical links. In particular, development of next-generation mobile networks (5G) coupled with the explosive growth in the variety and fidelity of modern multimedia services such as video-on-demand, peer-to-peer communications, interactive games, and virtual reality require the backbone communication systems to deliver greater information rates. This need becomes more salient as the increasing use of advanced technologies like machine to machine communication (M2M), cloud computing, and Internet of Things (IoT) drives the demand for even bigger data rates. The rising number of user mobile devices also adds a new dimension to this great demand. For example, according to a prediction in [1] the global number of smartphone users and M2M communication devices are expected to grow from 4.3 billion to 6.7 billion in 2017 and 2022 respectively. Therefore, it is not surprising that we have reached 100 000 Pbit of monthly Internet usage in the US in 2020 from an amount of 10 Pbit in 20 years ago [1].

Optical communication systems (OCS) are the preferred choice for addressing the needs of futuristics applications due to their advantages over radio frequency (RF) counterparts in terms of reliability, white noise tolerance, lower power consumption and the potential for much higher channel capacity. However, current OCS have shown to be inadequate, especially in addressing the dramatic growth in the need for higher information rate, spectral efficiency, and intrinsic security mechanisms [2]. Accordingly, a modern solution that is capable of sustainably delivering rates beyond Tbit/s that features economic advantages in terms of using the existing infrastructure is highly needed and demanded.

A. The Problem

The conventional OCS have appeared as inadequate in addressing the increasing demands and future expectations of modern Internet users [2]. In particular, since the implementation of WDM in the early 90’s, the available wavelength spectrum has been rapidly populated by more channels until the early 2000s when OCS resort to use Time Division Multiplexing (TDM) with WDM subchannels. Thanks to using the WDM-TDM compound technique, OCS achieved information rates at Tbit/s scale though sill suffer from a low single-digit spectral efficiency ((bit/s)/Hz for individual channels [2]. Subsequently, OCS appealed to using multiple optical links (i.e., Spatial Division Multiplexing (SDM)) for their further growth to Pbit/s scale.

Nevertheless, any sustainable solution for increasing information rate without making changes in the specification or configuration of the current OCS infrastructure must involve enhancing the communication spectral efficiency. More specifically, the decomposition of information rate in the form of $B(\text{Hz}) \times SE((\text{bit/s})/\text{Hz})$ implies that any solution for enhancing information rate must involve either higher channel bandwidth $B$ or a higher spectral efficiency $SE$. Obviously, increasing the channel bandwidth is not a sustainable option due to the availability limitations and frequency band licensing issues.

On the other hand, increasing the communication spectral efficiency is a solution that promises an unbounded increase in...
information rate within the same bandwidth quote. However, spectral efficiency can not be made arbitrary large within the contemporary OCS design framework as it aims at aggregating information rate across multiple subchannels rather than increasing the efficiency of individual channels. Therefore, OCS state quo revolves around the idea of dividing the main channel into as many subchannels as possible. This idea is often executed by propagating continuous plane wave lights like CW\(^1\) laser (or even incoherent VL\(^2\) beams) in uncorrelated states namely contiguous time slots, diversified wavelengths, separated physical mediums, orthogonal waves, and constellation phases in TDM, WDM, SDM, OFDM, and QAM respectively.

In the meantime, modern communication system counterparts in RF domain like 5G realized multiple-input multiple output (MIMO) paradigm \(^3\) via spatial multiplexing to boost the communication spectral efficiency. Unlike SDM, the spatial multiplexing scheme in MIMO does not “divide” the waves propagation space into abstractly or physically separated subchannels. Rather, MIMO waves propagate through a shared physical medium. The scattering and interference between the waves create virtual channels with correlated signals that can be later decorrelated from one another through beam forming technique. The beam forming mechanism enables MIMO communication parties to optimize signal beams to conform to the instantaneous channel scattering properties (e.g., fading, shadowing, etc.), hence achieving the best spectral efficiency possible at the moment. In this way, MIMO achieves higher information rates than their single-input single output (SISO) counterparts given a fixed level of total transmission power and channel bandwidth.

As we will mention in Section I-C, few attempts have been made to port MIMO concept into OCS. However, they missed the key point of MIMO beam forming by running on lights that either make so much scattering (VL beam) or so little scattering (CW laser). The next subsection elaborates on our contribution in introducing an optical MIMO paradigm based on employing pulsed ultrashort laser beams and a providing novel technique for spatial multiplexing in the spectral domain.

**B. Our Contribution**

In this paper, we propose an innovative paradigm for optical MIMO communication based on using the spectral space of ultrashort laser pulses. The emergence of compact mode-locked ultrashort laser generators has sparked many interests in integrating them into the modern optical communication systems. These laser generators emit femtosecond laser pulses with Gaussian envelope at typical telecommunication wavelengths\(^1\)\(^5\). For example, the laser generator in \(^4\) emits 14 fs ultrashort pulses that each carries a huge wavelength spectrum of 500 nm around a central wavelength of 1250 nm. Due to their ultrashort temporal longitude, these pulses bear an ultrawide spectral bandwidth, hence, satisfy the time-bandwidth condition (\(\Delta t \Delta \nu \geq 0.5\) for a Gaussian envelop) for having high-resolution Fourier spectral decomposition. Furthermore, ultrashort laser pulses exhibit broadband coherence, that is, a specific and well defined frequency-dependent phase relation across the pulse spectrum. Other broadband sources such as light emitting diodes (LEDs) can emit broad spectra but cannot support such broadband coherence. The said broadband coherence together with the ultrawide bandwidth and the high-quality spectral decomposition potential of the pulses can be used to encode various types of optical data, as we did.

The most significant consequence of the explained broadband coherence is that ultrashort laser pulses represent the only possibility of realizing coherent spectral resolution of time-varying signals in optics. As is well known, in the actual (physical) Fourier transform analysis of a temporal signal, the frequency resolution \(\Delta \nu\) of the instrument by which the spectrum is formed must satisfy the condition \(\Delta \nu \ll 1/\tau\) where \(\tau\) is the duration of a signal. In optics, spectral decomposition at high resolution is usually based on the multibeam interference concept. In this case, the value of \(\Delta \nu\) is determined by the maximum path difference \(L\) of the interfering beams, i.e., \(\Delta \nu = 1/L\). Hence, the condition \(\Delta \nu \ll 1/\tau\) means that the longitudinal size of an optical wave packet has to be substantially less than the path difference \(L\). The largest path differences can be achieved using the Fabry-Perot interferometer (a.k.a etalon) which provides a frequency resolution about \(10^9\) GHz. Therefore coherent spectral decomposition of optical temporal signals is practicable for pulses whose length is much shorter than \(10^{-8}\) seconds.

Accordingly, we grasp the opportunity that contemporary laser generators like \(^4\) provide femtosecond (\(10^{-15}\) seconds) pulses to introduce Holographic Spectral Multiplexing (HSM). In HSM we exploit the high-resolution spectral space of the ultrashort pulses to create line codes in the form of 2D holograms. The processing for hologram generation is performed in the temporal Fourier domain by spatially dispersing the pulses’ frequency components in a spectral processing device (SPD) and masking 2D patterns of intensity disparities onto the spectrally decomposed waves of the pulse. The holographic approach taken in HSM is in contrast to the conventional pulse-code modulation (PCM) line coding scheme used in conventional optical multiplexing techniques (e.g., WDM, TDM, etc.) in that PCM suffers from very low spectral efficiency. This is because PCM needs a sequence of continuous line codes for transmitting a symbol while holographic line codes in HSM transfer a block of bit by a single symbol.

Furthermore, the spectral holographic line codes defined in HSM transform the ultrashort laser pulses into high-entropy symbols, hence, enhance the communication’s spectral efficiency. This intuitively makes sense as the message entropy is the maximum amount of information bits that can be transferred per unit time per unit channel bandwidth. We will formulate the relation between the two quantities in Section III. Also as we show in Section IV containing multiple bits in a single data symbol exponentially increases the spectral

---

\(^1\)CW: Continuous Wave  
\(^2\)VL: Visual Light
efficiency of the communication. Using holograms is a smart way of increasing the information rate without requiring extra bandwidth. In fact, at the time of finishing this paper, we just learned a parallel research have proposed a similar holographic approach to MIMO in RF domain very recently [6]. Moreover in HSM, the pixels of holograms at the sender and receiver sides act as individual light source points and sensor array elements respectively. As a result, HSM resembles a MIMO communication paradigm with multiple sender and receiver antennas that allows deriving an optimum rate gain by forming beams between the communication parties in a way that the beams adapt to channel scattering situation. For this purpose, HSM inspects the effects of a physical phenomenon called chromatic dispersion on the structure of the ultrashort pulses to learn the channel status information (CSI) at the receiver. As the effects of dispersion on individual spectral components of the pulse are different, dispersion creates spatial diversity in the pulse spectrum space. HSM models the channel spatial diversity by building a MIMO channel matrix with elements that reflect the degree of dispersion for the different cells in the spectrum space of the ultrashort pulse. The best rate gain is achieved when the the elements in HSM channel matrix are uncorrelated, which translates to high number of parallel transmissions, and so, high levels of spatial efficiency. In this paper, we investigate this case to demonstrate the best of what HSM is capable of achieving in terms of rate gain. We envision future works to obtain average and worst cases of rate gain improvement using HSM. Table I summarizes the important aspects of our proposed Holographic Spectral Multiplexing (HSM) and compares them with popular multiplexing techniques.

| Properties                              | HSM                | WDM                | 5G                  |
|-----------------------------------------|--------------------|--------------------|---------------------|
| Line Code Realization                   | Holographic Patterns | Optical Signals (PCM) | Electromagnetic Signals (PCM) |
| Symbol Formation                        | Single Line Code   | Sequential Line Code | Sequential Line Code |
| Carrier Signal Type                     | Discrete           | Continuous         | Continuous          |
| Diversity Gaining Approach              | Space              | Wavelength         | Space               |
| Rate Gaining Approach                   | Spectral Efficiency| Aggregation / Higher Symbol Rates | Spectral Efficiency |
| Parallel Transmission Approach          | Spatial Light Modulator | Wavelength Array | Antenna Array      |

TABLE I: A comparison of common multiplexing techniques with our proposed Holographic Spectral Multiplexing (HSM) technique.

C. Related Works

Pulsed ultrashort lasers have been considered to be a promising tool for increasing transmission rate in communications systems [7]. Several schemes have been suggested realize the benefits though they failed to demonstrate compatibility with existing telecom infrastructures. For example, the solution proposed in [8] is based on using two separate laser beams between the communication parties, and hence, it does not completely benefit fiber optical infrastructure. The literature also suggests several ways to use opportunities that the ultrashort pulses’ native properties bring to conventional optical communication systems. For instance, [9] suggests the long interval between pulses, ultrawide wavelength spectrum, and broadband coherence to be utilized in TDM, WDM, and code-division multiple access (CDMA) techniques respectively.

Modern RF communication systems like 5G widely use MIMO to increase the spectral efficiency of their limited available bandwidth [3]. In contrast to RF communication domain, a few research have been performed on optical MIMO communication. Nonetheless, the related literature can be divided into imaging and non-imaging approaches. The former approach is based on using video cameras to detect visible light carrier signals while the latter relies on the customary CW laser and sensor arrays. Notably both approaches use incoherent light beams which is in contrast to our approach based on coherent beams.

The imaging optical MIMO systems use pixelated imaging receivers [10] instead of typical sensor detectors. In this scheme VL beams from multiple senders collectively make an image that strikes anywhere on the receiver, so an alignment is not needed. However, due to scattering in propagation path, the constituent beams of the image become correlated, hence, the number of independent parallel transmissions is limited. It is known that non-imaging MIMO systems provide little diversity gain due to their ill-conditioned channel matrix that makes beamforming for MIMO communication almost impractical [11]. A reason for this is that parallel optical interconnections require a very precise alignment to line up pairs source and detectors that is impractical to achieve without using electronic signal processing methods [12].

D. Paper Organization

The rest of this paper is organized as the follows. In Section II we introduce HSM as our novel paradigm for realizing optical MIMO communication and explain how it delivers spectral efficiency and information rate gain. In the same section, we formulate the maximum rate gain obtained by HSM in noisy and noiseless channels. Section III formulates analytical solutions for HSM channel fading and capacity. Moreover in this section, we explain how HSM exploits chromatic dispersion as a means to determine the elements of the MIMO channel matrix to learn the channel status information (CSI) with regard to scattering properties. We provide quantitative results for the best case of rate gains achieved by HSM in Section IV and will conclude this paper.
with a short discussion on the novelty of our work and future endeavors in Section V.

II. Optical MIMO Communication

As we discussed earlier in Section I, the conventional OCS address the need for delivering higher information rates by implementing multiplexing schemes that physically or abstractly divide the propagation signal space into subchannels. In this section, we explain the working principals of our novel MIMO paradigm for optical communication and demonstrate how it exploits the peculiar physical characteristics of ultrashort laser pulses to deliver a fundamentally high information rates communication through sustaining very high spectral efficiencies and establishing spatial diversity without dividing the signals propagation space into separate subchannels. In particular, we demonstrate how our innovative spatial multiplexing scheme introduces space diversity to optical domain and realizes an optical MIMO communication paradigm with ultrahigh spectral efficiency.

A. Holographic Spatial Multiplexing (HSM)

The crux of HSM is in exponential augmentation of spectral efficiency through inaugurating the novel concept of holographic line codes made by forging patterns of intensity disparity on the large spectral space of ultrashort laser pulses. Figure 1 illustrates the general scheme of a spectral processing device (SPD) that we use to stretch the ultrashort laser pulses on the Fourier plane and process 2D holograms onto their spectra space. The scheme shown in Fig. 1 illustrates a uniform setup for Tx/Rx unit. In this scheme, the generator emits ultrashort laser pulses and propagates them to a diffractive grating element in order to get their wavelength content expanded in spatial dimensions. To prevent further spreading graded-index (GRIN) lens are used to collimate the beam into the spatial light modulator (SLM).

SLM is an electronically-controlled optical device that can be programmed to control the spectral bandwidth, intensity, phase and polarization of the light pulse. The most common example of a spatial light modulator is a transmission liquid crystal display (LCD) array. Individual pixels in the SLM can be set to on (for maximum transmission) or off (for minimum transmission) or a number of steps in between for intensity control. Furthermore, a properly selected group of pixels can form a slit for windowing a desired spectral wavelength range. In this manner, the SLM can be used to transmit certain wavelength subspectra with intensities at different levels. A second lens and a grating element undo the spectral spread to superpose them back into a Gaussian-enveloped pulse. Then the pulse gets focused into a single mode fiber. The receiver uses the same optical setup but with elements in reverse order.

The HSM hologram line code formation occurs on the pulses’ spectral space when it passes through the SLM. More specifically, as Fig. 2 illustrates, each hologram comprises a pattern of intensity disparities (shown by shady cells in the figure) across and along the wavelength subbands of the pulse spectrum space. A hologram in HSM acts as a 2D line code that accommodates a number of traditional pulse-code modulation (PCM) line codes correspond to several input channels. Using PCM line codes as channel input ensures HSM compatibility with conventional communication devices in the existing infrastructure. Figure 2 also demonstrates that the structure of an HSM line code hologram comprises a 2D grid of cells laid next to each other and separated by a narrow gap. The gaps in line code hologram transform to spectral band guard after the hologram got impressed on the pulse.

Each holographic cell displays the optical transduction of the instantaneous PCM line code of a certain input channel at anytime. The different levels of amplitude in PCM line codes transduce to the same number of the cell opacity degrees. For instance, HSM transforms the Polar Quaternary NRZ line codes of input channels shown in in Fig. 2 to holographic cells with 4 levels of optical opacity. We will explain later in this subsection how a SLM realizes such transduction. For a holographic pattern grid with $n$ rows and $m$ columns of cells with $l$ number of opacity levels, the size of patterns set $P$ (i.e., number of different holographic patterns can be made) can be obtained by:

$$|P| = l^{N_t},$$

(1)

where $N_t$ is the transmitter resolution (i.e., the number of cells inscribed on the line code hologram), so:

$$N_t = n \times m,$$

(2)

The exponential equation nature of Eq. (1) allows a large number of holograms to be made to create surplus against the number of needed holographic line codes. The extra line code holograms can be used for the purposes of information hiding and communication fault tolerance, though the details fall beyond the the scope of this paper.

Figure 2 illustrates how HSM uses SLM to inscribe line code holograms on the spectral space of the stretched ultrashort pulse. According to this figure, the process of impressing the line code holograms on the spectral space of the pulse involves displaying related holographic cells on the SLM, and then, getting the original pulse to pass through the SLM. The holograms formed in this way comprise of pixels grouped based on intensity disparities across and along the wavelength subbands. Because in HSM the processed pulse embeds a whole line code, the transmitted pulse realizes a data symbol. We will explain in Section II-B that a data symbol formed in this way needs a large number of bits to be specified (i.e., a large bit/symbol rate), and so, it greatly enhances spectral efficiency of communication.

In designing the pixel groups of hologram patterns, we need to make sure that every cell on SLM allows the same amount of light power to pass through. This uniform power scheme ensures a uniform fading is achieved for all MIMO channels. Using PCM line codes as channel input ensures HSM compatibility with conventional communication devices in the existing infrastructure. Figure 2 also demonstrates that the structure of an HSM line code hologram comprises a 2D grid of cells laid next to each other and separated by a narrow gap. The gaps in line code hologram transform to spectral band guard after the hologram got impressed on the pulse.
500 nm around the central wavelength of 1250 nm is divided into three columns of equi-power cells.

The calculation for dividing the spectral space of a pulse to equi-power bands can be done through a variety of optimization methods like linear programming though it is outside the scope of this paper. Note how the width of the columns reduces as they get closer to the central wavelength to take into account the increase in relational power $P/P_{\text{max}}$ on the way to the pulse peak. Also, a positive intensity offset is applied to upper cells in the same column to compensate for the lessening overlap of the pulse with the cells. The next subsection elaborates on the maximum gain that an HSM-based MIMO communication system provides over the conventional single-input single-output (SISO) communication.

**B. HSM Maximum Rate Gain**

In this subsection we formulate HSM rate gain that is the gain by which an optical MIMO system based on HSM enhances the data rate compared to a communication system with an underlying single-input single-output (SISO) channel. As mentioned in the previous subsection, a discrete pulse in HSM carriers a line code that entails a whole data symbol. The amount of information contained in such a symbol $S$ can be determined by calculating the amount of the entropy $H(S)$ contained in the outcome of selecting one symbol from a symbol set of size $|P|$ (see Fig. 2b), that is:

$$H(S) = \log_2|P|(\text{bit/symbol}),$$  

(3)

Therefore, HSM transmits $H(S)$ bits of information by a single use of the channel—through sending a single pulse. Therefore according to the Nyquist theorem, the data rate $r$ for a noiseless HSM channel can be formulated as:

$$r = f_{\text{rep}} H(S)$$

$$= 2B H(S), (\text{bit/s})$$

(4)

where $f_{\text{rep}}$(symbol/s) and $B$(Hz) are the pulse repetition rate and the channel bandwidth respectively. The spectral efficiency ($\eta$) normalizes the data rate $r$ with respect to the channel bandwidth $B$, therefore:

$$\eta = r/B, (\text{bit/s/Hz})$$

(5)

According to Eq. (5), given the bandwidth is fixed, data rate and spectral efficiency can be thought of interchangeably. Consequently, a gain in the spectral efficiency translates to the same amount of gain in the data rate. Therefore, assuming the channel is noiseless, the maximum rate gain $G_{\text{max}}$ that HSM delivers can be formulated as:

$$G_{\text{max}} = H(S) \log_2 2^{n'},$$

(6)

where $n'$ is the number of bits that a symbol carries and $2^n'$ is the signal level(s) used in the traditional PCM signaling for SISO channels (e.g., 2 and 4 respectively for the line codes illustrated in Fig. 2a). However in practice, communication channels are noisy. Accordingly, the effective rate gain $G_{\text{eff}}$ of HSM can be obtained by taking into account the signal to noise ratio $\rho$ as the follows:

$$G_{\text{eff}} = \lim_{\rho \to \infty} \frac{\eta(\rho)}{\log_2 \rho},$$

(7)

where $\eta(\rho)$ is the spectral efficiency as a function of signal to noise ratio. We will formulate $\eta(\rho)$ in the next section.

**III. CHANNEL ANALYSIS**

In this section, we analyze the characteristics of our HSM communication paradigm in term of channel fading and channel capacity. In general, an HSM channel can be modeled as the follow:

$$y = Hx + n,$$

(8)

where $y \in \mathbb{C}^{N_r \times 1}$, $H \in \mathbb{C}^{N_r \times N_t}$, and $x \in \mathbb{C}^{N_t \times 1}$ correspond to the received signal vector, the channel impulse response
matrix, and the transmitted signal vector respectively. Furthermore in in Eq. (6), \( n \sim N(0, \sigma^2) \) is the noise Gaussian vector. In this paper we assume that the total transmission power is constrained and is equally divided between MIMO subchannels through the scheme explained in II-A (also review Fig. 2c). In this subsection, we explain how the channel matrix \( H \) is formed in HSM.

A. Channel Fading

In this subsection we formulate the HSM channel fading properties through forming the channel impulse response matrix \( H \) (or the channel matrix in short). As we mentioned in Section I-B, our aim in this paper is to demonstrate the maximum rate gain that HSM can achieve. For this purpose, we need to assume channel matrix has diagonal and deterministic members. Accordingly, we consider the HSM channel matrix \( H \) with a diagonal structure which indicates the best behavior of the MIMO subchannels in maintaining zero cross-correlation. With the said assumption, the channel response matrix has the following form:

\[
H = \begin{bmatrix}
  h_{1,1} & 0 & 0 & \cdots & 0 \\
  0 & h_{2,2} & 0 & \cdots & 0 \\
  0 & 0 & h_{3,j} & \cdots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & 0 & \cdots & h_{N_r,N_t}
\end{bmatrix}_{N_r \times N_t}
\]  

(9)

The elements \( h_{i,j} \) of the channel matrix \( H \) represent the impulse response of the MIMO subchannel between the sender pixel \( j \) and the receiver pixel \( i \). Again, according to our best case assumption, we consider these elements to be deterministic.

As explained earlier, the impulse response of MIMO virtual channels appear as the channel matrix \( H \) elements. In HSM the virtual channels establish between the pixels of SLM of the sender and the receiver, the matrix has the same number of rows and columns accordingly. In general, HSM can use any amount of resolutions for the transmitter \( N_t \) and the receiver \( N_r \). However in this paper, we elaborate on a practical case in which \( N_t = N_r \), i.e., the case of the same resolution for sender’s SLM and the receiver’s sensor array. Accordingly, we simplify the index notation of \( h_{i,j} \) to \( h_i \) for the brevity of equations. In this case, an element \( h_i \) is known as channel gain and represents the impulse response of the channel medium to the spectral component regulated by the line code cell corresponding to the \( i \)th MIMO subchannel and can be formulated as the follow:

\[
h_i = R(\omega_i) \exp \left( -j\phi(\omega_i, z) \right),
\]

(10)

where \( \omega_i \) is the angular frequency of the central wavelength of the line code cell for \( i \)th MIMO subchannel, \( R(\omega_i) \) is the amplitude scaling factor which for a linear optical transparent medium can be approximated by \( R(\omega_i) \approx 1 \) [14]. Furthermore in Eq. (10), \( \phi(\omega_i, z) \) is the phase offset created due to chromatic dispersion effect on the wave component \( \omega_i \) of the ultrashort pulse during its propagation to the distance \( z \) from the transmitter. Chromatic dispersion is a property of coherent ultrashort laser pulses in which spectral component of the pulse propagate with different group velocities depending to their wavelength. Note that \( \phi(\omega_i, z) \) demonstrates how chromatic dispersion introduces spatial diversity to the spectral space of the pulse through differentiating phase response to the pulse spectral components with different wavelengths. The dispersion phase offset \( \phi(\omega_i, z) \) can be obtained as suggested in [15]:

\[
\phi(\omega_i, z) = \phi(\omega_i, z) - GD.\omega_i, \text{ (rad)}
\]

(11)

where \( \phi(\omega_i, z) \) is the function of phase evolution for the wave \( \omega_i \) in the communication channel and can be expressed relatively simply as the follow:

\[
\phi(\omega_i, z) = \omega_i n(\omega_i) z/C, \text{ (rad)}
\]

(12)
where \( C \approx 299,792,458 \text{ m/s} \) is the speed of light in vacuum and \( n(\omega_i) \) represents the optical conductance property of the channel medium for the wave component \( \omega_i \) of the pulse that is known as the refractive index function. Furthermore in Eq. (11), \( GD \) is the group delay of the pulse wave components and can be obtained as follows:

\[
GD = \varphi^{(1)} = \frac{\partial \varphi(\omega, z)}{\partial \omega} \bigg|_{\omega=\omega_i}, (8)
\]

As Eq. (10) demonstrates, the elements of channel matrix \( H \) for HSM do not include stochastic components that usually are used to model data carrier light scattering, which is in contrast to the models for typical optical communication paradigms based on continuous beams (e.g., CW or VL). The reason lies in the fact that in HSM scheme, a receiver can detect and discard the pulses that scatter and reflect back acquire different structures due to traveling longer distances \( z \). In the next subsection, we will formulate the channel capacity for HSM in general, and in particular for the situation of \( N_t = N_r \) that results in the maximum capacity.

**B. Channel Capacity**

In order to obtain the channel capacity of our MIMO, we need to decompose the channel matrix \( H \) into parallel subchannels in which each subchannel has a linearly independent gain. For this purpose, we need transform the channel matrix \( H \) into its single value decomposition (SVD) in the follow form:

\[
H = U \Sigma V^H, \quad (14)
\]

where \( U_{N_r \times N_s} \) and \( V_{N_r \times N_s} \) are the columns of \( U \) and \( V \) for the eigenvalues of \( HH^H \) and \( H^HH \) respectively. Also, \( (.)^H \) denotes the Hermitian matrix operator. Furthermore, \( \Sigma_{N_r \times N_s} = \text{diag}(\sigma) \forall i = 1, \ldots, R_H \) denotes the diagonal matrix with elements of \( \sigma_i \) being singular values of \( HH^H \). The rank of the channel matrix \( R_H \) represent the number of MIMO subchannels with linearly independent gains.

Using the Shannon capacity formula [16] and assuming that a uniform power distribution over all MIMO subchannels is in place as we discussed in Section II-A, the channel capacity for a number of \( R_H \) MIMO subchannels can be expressed as the follow:

\[
C = B \sum_{i=1}^{R_H} \log_2 \left( 1 + \frac{\delta_i P}{N_t\sigma_i^2} \right) = B \log_2 \Pi_{i=1}^{R_H} \left( 1 + \frac{\delta_i P}{N_t\sigma_i^2} \right) = B \log_2 \left( \text{det} \left( I_{R_H} + \rho \frac{HH^H}{N_t} \right) \right), \quad \text{bit/s} \quad (15)
\]

where \( B \) is the bandwidth of the channel, \( \delta_i = \sigma_i^2 \) are eigenvalues of \( HH^H \), \( P \) is the total transmission power, \( \sigma_i^2 \) is variance of the Gaussian noise, \( I_{R_H} \) is the \( R_H \times R_H \) identity matrix, \( \rho = P/\sigma_0^2 \) is the average signal to noise ration, and \( \text{det}(.) \) is the determinant matrix operation.

In general, \( R_H = \min(N_t, N_r) \), but in accordance to the assumption of \( N_t = N_r \) we made in Section III-A we consider \( R_H = N_t \). Furthermore, the channel capacity formulated Eq. (15) is maximized if \( H \) is an orthogonal matrix, i.e., when the following condition holds:

\[
H^HH = N_t I_{N_t}, \quad (16)
\]

The channel matrix \( H \) in HSM paradigm always satisfies the condition of Eq. (16) as it is always diagonal (See Section III-A) and every diagonal matrix is orthogonal. Having satisfied the condition in Eq. (16), an HSM approach to MIMO communication reduces Eq. (15) to following:

\[
C_{\text{max}} = B N_r \log_2 (1 + \rho), \quad \text{bit/s} \quad (17)
\]

In physical terms, Eq. (17) defines a MIMO communication system with orthogonal subchannels and a capacity to that of \( N_r \) independent single-input single-output (SISO) channels. Finally, by comparing Eq. (15) and Eq. (17) we can deliver the spectral efficiency as a function of the signal to noise ration as the follow:

\[
\eta(\rho) = N_t \log_2 (1 + \rho), \quad \text{bit/s/Hz} \quad (18)
\]

In the next section, we use the \( \eta(\rho) \) to perform a quantitative analysis on the maximum gain by which HSM enhances the data rate.

**IV. Numerical Results**

This section provides a quantitative analysis for the maximum rate gain \( G_{\text{max}} \) delivered by HSM. For the sake of brevity, we use the term rate gain to refer to the quantity maximum HSM rate gain \( G_{\text{max}} \). Figure 5 illustrates the rate gain for a noisy channel as a function of the transmitter resolution \( N_t \) and the channel signal to noise ratio \( \rho \). This function can be obtained by substituting Eq. (18) in Eq. (7). As Fig. 5 illustrates, at higher levels of signal to noise ratio...
per unit $N_t$ is exponential at the lower levels of the signal to noise ratio—e.g., $1 < \rho < 2$. The slower rate gain growth at the higher signal to noise ratios can be attributed to the higher transmission powers that cause cross-channel fading. This makes sense especially with regard to the cell-based holographic structure of HSM line codes in which a high intensity light cell can contaminate other adjacent cells. Figure 4 illustrates the rate gain for a noiseless channel as a function of the hologram column resolution $m$ and the number of intensity levels $l$ as listed in Eq. 6 with $n' = n = 2$. As the figure demonstrates the rate gain increases exponentially with diversifying intensity levels though the rate of increase is linear with higher levels of $m$.

Therefore, HSM rate gain increases linearly with enhancing transmitter resolution regardless of the channel noise. Also scaling up the number of intensity levels creates the opportunity for achieving higher rate gains. Note the high scale of rate gain in HSM even with very moderate values for transmitter resolution and intensity levels. In practice a transmitter resolution can be in the scale of millions of pixels using a typical off-the-shelf SLM. Also, the same SLM can regulate an arbitrary number of intensity levels. Therefore, HSM can achieve a higher amount of rate gain in reality.

V. CONCLUSION

In this paper, we introduced Holographic Spectral Multiplexing (HSM) as a novel paradigm for realizing MIMO communication in optical domain to address the growing needs of modern application to grow optical communication systems (OCS) information to Pbit/s scale. HSM enables optical MIMO communication with high spectral efficiency; similar to modern RF communication system counterparts. The novelty of HSM is in exploiting the massive spectral bandwidth of the ultrashort laser pulses through realizing spectral multiplexing in optical domain. For this purpose, we proposed a spectral processing device (SPD) that embeds line codes in the form of 2D holograms into the spectral structure of the ultrashort pulses. Our approach makes distinction from the conventional optical multiplexing techniques such as TDM and WDM in improving the spectral efficiency of multiplexed subchannels rather than aggregating information rate aggregated across multiple subchannels, as mentioned in Section I-A. We analytically formulated the maximum channel capacity delivered by HSM, and consequently, the maximum rate gain by which a MIMO communication system based on HSM enhance the channel capacity compared to a conventional SISO communication system. The quantitative analysis we performed showed that, fora lower and higher ratios of signal to noise, increasing the transmitter resolution will result in exponential and linear rate gains, respectively. Also, increasing the number of intensity levels creates the opportunity for HSM to achieve even higher data rates. We suggest future works to include analyzing the average and worst cases of HSM rate gain, the possibility of delivering a diversity gain for fault tolerance, and the opportunities that HSM brings forward for securing the optical physical layer without cryptography.

REFERENCES

[1] R.-J. Essiambre, R. W. Tkach, Capacity Trends and Limits of Optical Communication Networks, Proceedings of the IEEE 100 (5) (2012) 1035–1055. doi:10.1109/JPROC.2012.2182970
[2] P. J. Winzer, D. T. Neilson, A. R. Chraplyvy, Fiber-optic transmission and networking: The previous 20 and the next 20 years [Invited], Optics Express 26 (18) (2018) 24190–24239. doi:10.1364/OE.26.024190
[3] D. Gesbert, M. Shafi, Da-shan Shiu, P. J. Smith, A. Naguib, From theory to practice: An overview of MIMO space-time coded wireless systems, IEEE Journal on Selected Areas in Communications 21 (3) (2003) 281–302. doi:10.1109/JSAC.2003.809458
[4] K. Kieu, R. J. Jones, N. Peyghambarian, Generation of Few-Cycle Pulses From an Amplified Carbon Nanotube Mode-Locked Fiber Laser System, IEEE Photonics Technology Letters 22 (20) (2010) 1521–1523. doi:10.1109/LPT.2010.2065423
[5] D. J. Jones, S. A. Diddams, J. K. Ranka, A. Stentz, R. S. Windeler, J. L. Hall, S. T. Cundiff, Carrier-Envelope Phase Control of Femtosecond Mode-Locked Lasers and Direct Optical Frequency Synthesis, Science 288 (5466) (2000) 635–639. doi:10.1126/science.288.5466.635
[6] A. Pizzo, L. Sanguinetti, T. L. Marzetta, Holographic MIMO Communications, arXiv:2105.01535 [cs, eess, math] (May 2021). arXiv:2105.01535
[7] M. E. Ferrmann, A. Galvanauskas, G. Sucha (Eds.), Ultrafast Lasers: Technology and Applications, 1st Edition, Academic Press, Boston, 2002.
[8] G. S. Rogozhnikov, V. V. Romanov, N. N. Rukavishnikov, V. Y. Molchanov, K. B. Yushkov, Interference of phase-shifted chirped laser pulses for secure free-space optical communications, Applied Optics 57 (10) (2018) C98–C102. doi:10.1364/AO.57.000C98
[9] W. Knox, Ultrafast technology in telecommunications, IEEE Journal of Selected Topics in Quantum Electronics 6 (6) (2000) 1273–1278. doi:10.1109/97.894.902178
[10] J. M. Kuhn, R. You, P. Djangani, A. G. Weisbin, Beh Kian Teik, A. Tang, Imaging diversity receivers for high-speed infrared wireless communication, IEEE Communications Magazine 36 (12) (1998) 88–94. doi:10.1109/35.735884
[11] Y. Tang, S. Tao, W. Li, Z. Zha, Z. Shi, Indoor Visible Light Communication Networks for Camera-Based Mobile Sensing, in: X. S. Shen, X. Lin, K. Zhang (Eds.), Encyclopedia of Wireless Networks, Springer International Publishing, Cham, 2019, pp. 1–7. doi:10.1007/978-3-319-32903-1_270-1
[12] A. G. Kirk, Free-Space Optical Interconnects, in: L. Pavesi, G. Giuillot (Eds.), Optical Interconnects: The Silicon Approach, Springer Series in Optical Sciences, Springer, Berlin, Heidelberg, 2006, pp. 343–377. doi:10.1007/978-3-540-28912-8_13
[13] A. M. Weiner, Femtosecond pulse shaping using spatial light modulators, Review of Scientific Instruments 71 (5) (2000) 1929–1960. doi:10.1063/1.1150614

Fig. 4: The rate gain function for a noisy channel.
[14] J.-C. Diels, W. Rudolph, Chapter 2 - Femtosecond Optics, in: J.-C. Diels, W. Rudolph (Eds.), Ultrashort Laser Pulse Phenomena (Second Edition), Academic Press, Burlington, 2006, pp. 61–142. doi:10.1016/B978-012215493-5/50003-3

[15] A. Borzsonyi, A. P. Kovacs, K. Osvay, What We Can Learn about Ultrashort Pulses by Linear Optical Methods, 2013.

[16] C. E. Shannon, A Mathematical Theory of Communication, Bell System Technical Journal 27 (3) (1948) 379–423. doi:10.1002/j.1538-7305.1948.tb01338.x