A Monte Carlo Implementation of Galactic Free–Free Emission for the EoR Foreground Models

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Abstract

The overwhelming foreground causes severe contamination on the detection of 21 cm signal during the Epoch of Reionization (EoR). Among various foreground components, the Galactic free–free emission is less studied, so that its impact on the EoR observation remains unclear. To better constrain this emission, we perform Monte Carlo simulation of Hα emission, which comprises direct and scattered Hα radiation from H II regions and warm ionized medium (WIM). The positions and radii of H II regions are quoted from the Wide-Field Infrared Survey Explorer H II catalog, and the WIM is described by an axisymmetric model. The scattering is off dust and free electrons that are realized by applying an exponential fitting to the HI4PI H I map and an exponential disk model, respectively. The simulated Hα intensity, the Simfast21 software, and the latest SKA1-Low layout configuration are employed to simulate the SKA “observed” images of Galactic free–free emission and the EoR signal. By analyzing the one-dimensional power spectra, we find that the Galactic free–free emission can be about $10^{5.3} - 10^{5.7}$, $10^{4.0} - 10^{4.7}$, and $10^{4.3} - 10^{4.1}$ times more luminous than the EoR signal on scales of $0.1 \text{ Mpc}^{-1} < k < 2 \text{ Mpc}^{-1}$ in the 116–124, 146–154, and 186–194 MHz frequency bands, respectively. We further calculate the two-dimensional power spectra inside the EoR window and show that the power leaked by Galactic free–free emission can still be significant, as the power ratios can reach about 110%–8000%, 30%–2400%, and 10%–250% on scales of 0.5 Mpc$^{-1} \lesssim k \lesssim 1$ Mpc$^{-1}$ in three frequency bands. Therefore, we indicate that the Galactic free–free emission should be carefully treated in future EoR detections.

Unified Astronomy Thesaurus concepts: Radiative transfer (1335); Intergalactic medium (813); H II regions (694); Reionization (1383); Population III stars (1285); Early universe (435); Astronomy data analysis (1858); Interferometry (808)

1. Introduction

The Epoch of Reionization (EoR) is a period after the Dark Ages ($z \sim 30–200$) and Cosmic Dawn ($z \sim 15–30$) that lasts from about 300 million to 1 billion years ($z \sim 5–15$; see Koopmans et al. 2015 and references therein), during which the baryonic matter was ionized by the ultraviolet and soft X-ray photons emitted from the first-generation celestial objects (e.g., first stars, and quasars), forming ionized bubbles that gradually grew larger and finally merged. Although the 21 cm emission line of neutral hydrogen (H I) is regarded as a decisive probe to directly explore the EoR (Fan et al. 2006; Furlanetto et al. 2006; Zaroubi 2013; Furlanetto 2016), its detection is currently precluded by the overwhelming foreground contamination. Among various foreground components, the impact of Galactic free–free emission is still poorly understood, and thus it is necessary to create as accurate an all-sky Galactic free–free emission map as possible in the low-frequency (50–200 MHz) radio band to guide the development of foreground removal techniques.

The Galactic free–free emission cannot be observed directly, because the Galactic synchrotron component dominates the emission at frequencies lower than 10 GHz, while the dust thermal emission becomes overwhelming at frequencies higher than 10 GHz. However, the Hα emission line (the 3–2 transition of the hydrogen atom at $\lambda = 656.28$ nm) provides a way to trace the Galactic free–free emission, since they share the same emission measure $EM = \int n_e^2 dl$ ($n_e$ is the electron density; e.g., McCullough 1997; Marcellin et al. 1998; Dickinson et al. 2003; Sims et al. 2016). For example, the brightness temperature of the Galactic free–free emission has been related to the Hα intensity by Valls-Gabaud (1998) and Reynolds & Haffner (2000). Dickinson et al. (2003) derived a 95% sky coverage (except the area $|b| < 5^\circ$, $l = 160^\circ–0^\circ–260^\circ$) of the Galactic free–free emission map at 30 GHz from the absorption-corrected Hα intensity map based on the Southern H-Alpha Sky Survey Atlas (SHASSA; Gaustad et al. 2001) data and Wisconsin H-Alpha Mapper (WHAM; Haffner et al. 2003) data. Note that these results may have been biased since the observed Hα intensities used in these works are often misunderstood, especially near or at the Galactic plane (e.g., Dennison et al. 1998; Dickinson et al. 2003), due to the absorption and scattering. On the other hand, the Galactic free–free emission can also be deduced from the radio recombination lines (RRLs; Alves et al. 2010, 2012). For example, a partial-sky ($|b| < 5^\circ$, $l = 52^\circ–0^\circ–192^\circ$) Galactic free–free emission map at 1.4 GHz was proposed by Alves et al. (2015) based on the observed RRLs map that was obtained via the H I observations of H I Parkes All-Sky Survey (Staveley-Smith et al. 1996) and Zone of Avoidance Survey (Staveley-Smith et al. 1998).

A large part (~50%–70%) of the Galactic Hα emission is contributed by the recombination process in the H II regions.

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Ionized by massive O and B stars, the H\textsc{ii} regions are mostly concentrated on the Galactic plane and become the brightest infrared and radio objects in the spiral arms (Shaver et al. 1983; Paladini et al. 2004; Balser et al. 2011; Anderson et al. 2014, 2015). Using the Wide-Field Infrared Survey Explorer (WISE) data, Anderson et al. (2014) compiled a most complete H\textsc{ii} region catalog that contains 8400 H\textsc{ii} regions (candidates) located at lower latitudes ($|b| < 8^\circ$), as well as five well-known H\textsc{ii} regions at middle latitudes. The rest ($\sim 30\%$–$50\%$) of the Galactic H\textsc{o} emission is attributed to the recombination process in the warm ionized medium (WIM), which is often known as diffuse ionized gas in extra-galaxies (e.g., Jura 1979; Miller & Cox 1993; Dove & Shull 1994; Haffner et al. 2009).

As one important phase of the diffuse interstellar medium (ISM) in our Galaxy, the WIM exhibits a scale height of $\sim 900$ pc, a characteristic temperature of $\sim 10,000$ K, and a specified volume-filling factor of $\sim 0.1$–$0.4$ (Wood & Reynolds 1999, hereafter Wood99). How the WIM is ionized and heated, as well as its relationship with other ISM phases, is still unclear (Miller & Cox 1993; Haffner et al. 1999, 2003; Reynolds et al. 1999; Dove et al. 2000). According to the axisymmetric ISM model proposed by Ferrière (1998), which provides the averaged hydrogen number densities for different ISM phases, as the Galactic height ($z$) measured from the Galactic plane increases from 0 to 5 kpc, the hydrogen nucleus number density of the WIM decreases from 1 to $10^{-4}$ cm$^{-3}$.

It is important to correct the absorption and scattering in the measurement of H\textsc{o} intensity in order to accurately obtain the Galactic free–free emission. Although many methods have been proposed to solve this problem (e.g., Reynolds 1990; Dickinson et al. 2003; Dong & Draine 2011), there still exists no standard solution owing to the limited dust data (Lehtinen et al. 2010; Witt et al. 2010; Brandt & Draine 2012). On the aspect of the theoretical modeling, Monte Carlo radiative transfer (MCRT) simulation has been proposed to be a useful tool to predict the scattered H\textsc{o} intensity and calculate the intrinsic (i.e., without absorption and scattering) H\textsc{o} emission in the Milky Way (e.g., Wood99) and extra-galaxies (e.g., Schiminovich et al. 2001; Lee et al. 2008; Seon 2009, 2015; Jo et al. 2012; Seon & Witt 2012, 2013; Seon et al. 2014). The MCRT algorithm regards the radiation field as a photon flow, in which photons move in the dusty medium (Steinacker et al. 2013). For each photon, its starting point, its initial direction of motion, and the place where it interacts with a dust grain are determined in a probabilistic way. Finally, the statistical analysis of the photons can be used to recover the radiation field. The MCRT method offers a variety of obvious advantages, i.e., take scattering into account properly, comparably simple computer programs (just a random number generator together with some basic loops), and easily parallel operation (Noebauer & Sim 2019). The stochastic fluctuation is unavoidable for the MCRT method, which can be reduced by increasing the number of test particles (e.g., Seon 2015; Murthy 2016). Since 1999, Wood and his collaborators have carried out a series of MCRT simulation works to predict the scattering property and the polarization of the H\textsc{o} emission in our Galaxy (e.g., Wood99, Wood et al. 2004, 2005, 2010; Barnes et al. 2015); these simulations and other observations (e.g., Reynolds 1988; Gordon et al. 2001; Dong & Draine 2011) show that at higher latitudes the scattered H\textsc{o} intensity may contribute up to $\sim 20\%$ of the total observed H\textsc{o} intensity, and its polarization is less than 1%.

In this work, we will focus on the MCRT implementation of Galactic free–free emission and estimate its impacts on the EoR detection by employing the latest configuration of SKA1-Low to incorporate the instrumental effects. The Galactic free–free emission is obtained by performing a three-dimensional (3D) MCRT simulation of Galactic H\textsc{o} emission, which comprises the direct and scattered H\textsc{o} radiation from H\textsc{ii} regions and the WIM. In the previous MCRT simulations (e.g., Wood99), the H\textsc{ii} regions are treated as simple “point sources,” and the dust is assumed to possess a smooth axisymmetric distribution along the Galactocentric distance. As an improvement we will adopt more realistic models, which are constrained by the multiband observations, to describe the 3D distributions of H\textsc{ii} regions and the dust. To be specific, each H\textsc{ii} region is modeled as a sphere with a radius of $R_e$ and is inserted into our simulation cube according to its Galactic coordinate and the distance to the Sun, which are provided in the WISE H\textsc{ii} catalog. To obtain the dust distribution, we employ the best exponential fitting to the newest observed HI4PI H\textsc{i} column density map (HI4PI Collaboration et al. 2016). The Thomson scattering of free electrons (see Section 2.3) is also taken into account by applying a plane-parallel exponential model to describe the distribution of the free electrons. Finally, by analyzing the one-dimensional (1D) and two-dimensional (2D) power spectra and EoR window, we quantitatively evaluate the contamination caused by Galactic free–free emission on the EoR detections.

This paper is organized as follows: In Section 2, we construct the physical components in the simulation box. In Section 3, we use the Simfast21 code to simulate the EoR signal and employ the latest SKA1-Low layout configuration to simulate the SKA “observed” images. In Section 4, we present the results of the simulated H\textsc{o} intensity and the corresponding Galactic free–free emission and also evaluate the contamination imposed by Galactic free–free emission on the EoR detection. We discuss the major uncertainties in our simulation and compare our results with the previous works of Wood99 and Finkbeiner (2003, hereafter F03) in Section 5. Finally, we summarize our work in Section 6.

2. Models

We calculate the full-sky Galactic H\textsc{o} intensity by carrying out a 3D MCRT simulation. To determine the scattered H\textsc{o} intensities from H\textsc{ii} regions and the WIM, we take the effects of absorption and scattering into account by filling the simulation box with dust and free electrons. The ingredients of the simulation box, H\textsc{o} emissivity, clumpy dust, scattering parameters, and radiation transfer algorithm are presented in Sections 2.1–2.5, respectively. We describe how to relate the Galactic free–free emission with the simulated H\textsc{o} intensities in Section 2.6.

2.1. Simulation Box

We carry out the MCRT simulation inside a 3D Cartesian box with $1000 \times 1000 \times 333$ cells, which covers a physical size of $30 \times 30 \times 10$ kpc$^3$ (i.e., the size of each cell is about $30 \times 30 \times 30$ pc$^3$), considering that the radial size of Galactic plane and Galactic height $z$ are $\pm 15$ kpc and $\pm 5$ kpc, respectively. The detector is assumed to be located at $(6.5, 15, 5)$ kpc (i.e., the position of Earth) inside the simulation box and can observe the entire sky. By following the work of Wood99, we insert an evacuated region with a radius of $30$ kpc–$10$ kpc to incorporate the instrumental effects. The Galactic free–free emission is obtained by performing a three-dimensional (3D) MCRT simulation of Galactic H\textsc{o} emission, which comprises the direct and scattered H\textsc{o} radiation from H\textsc{ii} regions and the WIM. In the previous MCRT simulations (e.g., Wood99), the H\textsc{ii} regions are treated as simple “point sources,” and the dust is assumed to possess a smooth axisymmetric distribution along the Galactocentric distance. As an improvement we will adopt more realistic models, which are constrained by the multiband observations, to describe the 3D distributions of H\textsc{ii} regions and the dust. To be specific, each H\textsc{ii} region is modeled as a sphere with a radius of $R_e$ and is inserted into our simulation cube according to its Galactic coordinate and the distance to the Sun, which are provided in the WISE H\textsc{ii} catalog. To obtain the dust distribution, we employ the best exponential fitting to the newest observed HI4PI H\textsc{i} column density map (HI4PI Collaboration et al. 2016). The Thomson scattering of free electrons (see Section 2.3) is also taken into account by applying a plane-parallel exponential model to describe the distribution of the free electrons. Finally, by analyzing the one-dimensional (1D) and two-dimensional (2D) power spectra and EoR window, we quantitatively evaluate the contamination caused by Galactic free–free emission on the EoR detections.

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200 pc centered on the Sun to represent the low-density “Local Bubble” (Cox & Reynolds 1987).

2.2. Hα Emissivity

The Hα emissivity $\frac{\varepsilon_{\text{HJ}}}{n_{\text{H}}}^{\text{WIM}}$ is contributed by the emissions from H II regions and the WIM. It is found that the Hα emissivity of WIM $\frac{\varepsilon_{\text{HJ}}}{n_{\text{H}}}^{\text{WIM}}$ is proportional to the square of its hydrogen density (see Equation 7 of Ferrière 1998). By following Wood99’s work, we adopt a total WIM emissivity of $10^{32} \text{HJ} \text{ photons s}^{-1}$. It is expected that the Hα emissivity near the Galactic plane is dominated by the bright H II regions. We adopt a total H II emissivity of $10^{32} \text{HJ} \text{ photons s}^{-1}$, same as that of the WIM, by following the work of Wood99, since observations show that the volume-averaged ratio of the total Hα emissivity of H II regions to that of the WIM is in the range of 0.47–0.70 (e.g., Veilleux et al. 1995; Ferguson et al. 1996). The H II regions are placed inside the simulation cube according to their coordinates and the distances to the Sun provided by the WISE H II catalog9 (except for some sources that lack accurate distance data; for more details, see Anderson et al. 2014). As illustrated in Figure 1, the 1546 H II regions possessing known radii and distances to the Sun are shown with black circles, and the other 6859 sources that lack the distances information are shown with blue circles, for which, by following Wood99’s work, we assign random positions by placing sources randomly in the molecular ring (Ferrière 1998) and the spiral arms (Nakanishi & Sofue 2016). We also display the spiral arms of our Galaxy given by Nakanishi & Sofue (2016) in Figure 1 for comparison.

To quantify the spatial distribution of Hα photons in each H II region, we adopt a $\beta$ profile $n_{\text{HJ}}^R = n_{\text{HJ}}^0 \left(1 + \frac{R}{R_s} \right)^{-3/2}$ ($\beta$ H II region model hereafter; Cavaliere & Fusco-Femiano 1976), where $R$ is the distance measured from the H II region center, $\beta = 0.7$ is the slope parameter, $R_s$ is the scale radius ($R_s/R_s = 0.1, 0.5$, and $1.0$ have been tested, where $R_s$ is the H II region’s radius provided by the WISE H II catalog), and $n_{\text{HJ}}^0$ is the density of Hα photons at the center of the H II region, whose value is about several times $10^{49} \text{HJ} \text{ photons s}^{-1}$ (the typical value of Orion Nebula; Wood99), which can be determined from the total Hα photons in each H II region. For comparison, we also test a uniform model of H II regions, i.e., the distribution of Hα photons in each H II region is uniform. The four types of H II region model, i.e., $\beta$ cases with $R_s = 0.1R_s$, $0.5R_s$, $1.0R_s$, and “Uniform” case, are shown in Figure 2.

2.3. Distributions of Dust and Free Electrons

We insert the dust and free electrons into our simulation cube to calculate the scattered Hα emission. Given that the dust optical depth at the Hα wavelength can be calculated via the H I column density (Bohlin et al. 1978), we employ an exponential fitting $N_{\text{HJ}} = N_{\text{HI}} e^{-D/125pc}$ to derive the 3D distribution of HI column density, where 125 pc is the scale length (Marshall et al. 2006). $D$ is the distance to the Sun, and $N_{\text{HI}}$ is the H I column density at the Galactic plane that can be derived from the 2D H IHI column density map. The H IHI is an all-sky 2D HI column density map, which is obtained from the observed data of the Effelsberg–Bonn H I Survey and Galactic All-Sky Survey (HI4PI Collaboration et al. 2016). A plane-parallel exponential model of free electrons $n_e(z) = n_{e,0} e^{-|z|/h}$ is applied to obtain the 3D distribution of free electrons, where $n_{e,0}$ is the free electron density at the Galactic plane and $h$ is the scale height of free electron density (Schnitzeler 2012). We adopt $n_{e,0} = 0.0165 \text{ cm}^{-3}$ and $h = 1.45 \text{ kpc}$, which are calculated based on Table 3 of Schnitzeler (2012).

2.4. Scattering Parameters

The scattered Hα emission is simulated by labeling each Hα photon and tracing its traveling routes in our simulation. To calculate the scattered Hα intensity, we employ the Henyey–Greenstein (HG) phase function (Henyey & Greenstein 1941)

$$\text{HG} (\theta) = \frac{1}{4 \pi} \frac{1 - g^2}{\left[1 + g^2 - 2g \cos(\theta)\right]^{3/2}},$$

where $\theta$ (in the range of $[0, \pi]$) is the scattering angle, so that $\theta = 0$ corresponds to forward scattering, $\theta = \pi$ means back scattering, $g (g \equiv \langle \cos(\theta) \rangle)$ is the phase function asymmetry factor, and $g > 0$ indicates forward scattering predominance. Three typical sets of scattering parameters, i.e., $g = 0.44, 0.50$, and 0.55, have been tested in our simulation (see also Table 1; Mathis et al. 1977; Weingartner & Draine 2001).

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9 http://astro.phys.wvu.edu/wise (version 1.5).
2.5. Monte Carlo Radiative Transfer

The design of our simulation code is similar to other MCRT programs in predicting the direct and scattered Hα intensities (e.g., Wood 1999; Gordon et al. 2001; Barnes et al. 2015). We will present the code flow in our simulation by tracing the motion of a single Hα photon.

(i) An Hα photon is emitted from either H II regions or the WIM according to their weighted distributions. Each photon begins with initial effective unit weight and is forced to send a fraction of weight $W_{\text{direct}}$ to the detector

$$W_{\text{direct}} = e^{-\tau}/4\pi d^2,$$

where $d$ is the distance from the point of emitter to the detector; $\tau$ is the optical depth of Hα emission along the distance of $d$, i.e., $\tau = \int_0^d (N_{\text{H I}} \sigma_{\text{Hα}} + N_e \sigma_e) dl$; $N_{\text{H I}}$ and $N_e$ are the H I column density and free electron column density, respectively; $\sigma_{\text{Hα}} = 3.801 \times 10^{-22} \text{ cm}^2$ is the scattering cross section at Hα wavelength (Draine 2003); and $\sigma_e = 6.652 \times 10^{-25} \text{ cm}^2$ is the Thomson cross section.

(ii) Next, two random numbers are generated to determine the direction of Hα photon motion, one for theta (in the range of $[0, \pi]$), measured from the $z$-axis of our simulation cube and the other for phi (in the range of $[0, 2\pi]$). To calculate the scattered Hα intensity, by following Murthy (2016), a third random number $\xi$ is generated from a uniform distribution $[0, 1]$ to determine a predetermined optical depth $\tau_{\text{pre}}$, which is sampled from $-\log(\xi)$. Then, the scattering location is determined by following the Hα photon’s motion until the cumulative optical depth $\tau_{\text{cum}}$ along the path equals $\tau_{\text{pre}}$. If this location is inside the simulation box, we apply the “peel-off” strategy to calculate the scattered weight received by the detector (e.g., Wood 1999; Yusef-Zadeh et al. 1984)

$$W_{\text{scatter}}^N = a \ W_{\text{rest}}^{N-1} \ (1 - e^{-\tau_{\text{pre}}}) \ e^{-\tau_{\text{cum}}},$$

Table 1

| Label | $a$   | $g$  | H II Model |
|-------|-------|------|------------|
| (a)   | 0.50  | 0.44 | $R_e = 0.1R_e$ |
| (b)   | 0.50  | 0.44 | $R_e = 0.5R_e$ |
| (c)   | 0.50  | 0.44 | $R_e = 1.0R_e$ |
| (d)   | 0.50  | 0.44 | Uniform     |
| (e)   | 0.67  | 0.50 | $R_e = 0.1R_e$ |
| (f)   | 0.67  | 0.50 | $R_e = 0.5R_e$ |
| (g)   | 0.67  | 0.50 | $R_e = 1.0R_e$ |
| (h)   | 0.67  | 0.50 | Uniform     |
| (i)   | 0.77  | 0.55 | $R_e = 0.1R_e$ |
| (j)   | 0.77  | 0.55 | $R_e = 0.5R_e$ |
| (k)   | 0.77  | 0.55 | $R_e = 1.0R_e$ |
| (l)   | 0.77  | 0.55 | Uniform     |

(2) $R_e = 10^{-20}$, Steinacker et al. 2013 for the effective weight of Hα photons or the maximum scattering number (e.g., 1000; Murthy 2016) to terminate the scattering process.

(iii) The total Hα intensity is the sum of the weights of the direct (Equation (2)) and multiple scattered (Equation (3)) photons. The received Hα photons are then used to construct the Hα intensity map using the Hierarchical Equal Area isoLatitude Pixelization (HEALPix)10 tessellation scheme with $N_{\text{side}} = 1024$ (pixel size $\approx 3.44'$; Góski et al. 2005).

2.6. Derivation of Free–Free Emission

The received Hα intensity depends on whether the emitting medium is optically thin (case A) or optically thick (case B), and it is found that case B is satisfied in the study of Galactic Hα emission (Osterbrock 1989; Dickinson et al. 2003). For case B, Valls-Gabaud (1998) proposed an analytical expression to describe the relation between the observed Hα intensity $I_{\text{Hα}}(r)$ and the emission measure $EM(r)$

$$EM(r) = 2.561 \ T_{\text{eff}}^{1.017}(r) 10^{0.029/T_{\text{eff}}} \ I_{\text{Hα}}(r),$$

where $T_{\text{eff}}(r) = T_e(r)/10^4$ ($r$ is the 2D position) is the electron temperature in units of $10^4$ K, $I_{\text{Hα}}(r)$ is in units of Rayleigh (R), and $EM(r)$ is in units of cm$^{-6}$ pc. Using the emission measure derived in Equation (4), we can calculate the optical depth of Galactic free–free emission $\tau_{\text{f}}(r)$ as

$$\tau_{\text{f}}(r) = 0.05468 \ g(r) \ T_e(r)^{-3/2} \ \nu_0^{-2} \ EM(r),$$

where $\nu_0 = \nu/10^9$ Hz is the frequency in units of GHz and $g(r)$ is the gaunt factor given by

$$g(r) = \log\{\exp[5.960 - \sqrt{3}/\pi \ \log(\nu_0 \ T_e(r)^{-3/2})] + e\},$$

where $e \approx 2.71828...$ is the natural constant (Draine 2011). The above three equations are valid in the 100 MHz–100 GHz frequency bands (Dickinson et al. 2003), which are often employed to deduce the brightness temperature of Galactic free–free emission $T_{\text{f}}^{\text{GR}}(r)$

$$T_{\text{f}}^{\text{GR}}(r) = T_e(r) \ [1 - e^{-\tau_{\text{f}}(r)}].$$

3. SKA Observation and EoR Signal

In order to incorporate the instrumental effects of radio interferometers, we have employed the latest SKA1-Low

10 http://healpix.sourceforge.net/

11 1 Rayleigh (R) $\equiv 10^9/4\pi$ photons s$^{-1}$ cm$^{-2}$ sr$^{-1}$ $\equiv 2.41 \times 10^{-7}$ erg cm$^{-2}$ s$^{-1}$ sr$^{-1}$. 
layout configuration\textsuperscript{12} to simulate the SKA “observed” images. The SKA1-Low interferometer layout includes 512 stations, with 224 stations randomly distributing within the “core” region (1000 m in diameter), and others scattering in “cluster” regions, which form three spiral arms up to a radius of \( \approx 35 \) km. Each station includes 256 antennas, which are randomly distributed in a circular region of 35 m in diameter with a minimum separation of \( d_{\text{min}} = 1.5 \) m (e.g., Mort et al. 2017).

We choose the sky maps centered at (R.A., decl.) = (0\(^\circ\), −30\(^\circ\)) with a sky coverage of 10\(^6\) \( \times \) 10\(^6\), which is located at a high galactic latitude (\( b = -78^\circ \)) and is expected to be an appropriate choice for this study. Moreover, this region passes through the zenith of the SKA1-Low telescope and is an ideal choice to simulate the SKA observation. We use the OSKAR\textsuperscript{13} (Mort et al. 2010) simulator to perform SKA observations for 6 hr to obtain the visibility data. The WSClean imager (Offringa et al. 2014) is employed to image the simulated visibility data using Briggs weighting with a zero robustness (Briggs 1995; Li et al. 2019). To avoid the problem of insufficient CLEAN in the marginal regions, we crop the created images and choose to keep their central regions of 6\(^\circ\) \( \times \) 6\(^\circ\), 5\(^\circ\) \( \times \) 5\(^\circ\), and 4\(^\circ\) \( \times \) 4\(^\circ\) in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands, respectively, given that the telescope’s field of view (FOV) is inversely proportional to the observing frequency (see the example maps in Figure 3). For each frequency band, the CLEAN algorithm with joined-channel deconvolution is adopted to create the foreground cube in order to ensure the spectral smoothness (Offringa & Smirnov 2017). We directly use the dirty image for the EoR signal because the CLEAN algorithm does not work well for such faint diffuse emission.

The seminumerical code Simfast21\textsuperscript{14} (Santos et al. 2010; Hassan et al. 2016) is employed to simulate the brightness temperatures of the 21 cm signal during the EoR by following our previous work (for more details about the Simfast21 simulation, see Lian et al. 2020). To construct the EoR signal cube, we assume a \( \Lambda \)CDM cosmology with parameters of \( \Omega_m = \Omega_{dm} + \Omega_b = 0.3089, \Omega_b = 0.0486, \Omega_{\Lambda} = 0.6911 \), Hubble constant \( H_0 = 67.74 \) km s\(^{-1}\) Mpc\(^{-1}\), power spectrum index \( n_s = 0.9667 \), and normalization \( \sigma_8 = 0.8159 \) (Planck Collaboration et al. 2016b). We initialize the Simfast21 at \( z_i = 100 \) on a 1024\(^3\) box with physical dimensions of 1.6\(^3\) comoving Gpc\(^3\), which corresponds to a field of \( \theta_i = \theta_h \approx 9.988 \), a pixel resolution of \( \Delta \theta = \Delta \theta_h \approx 0.588 \), and a frequency depth of \( \Delta \nu \approx 92.95 \) MHz. We then utilize the method of Mellema et al. (2006) to create the observable “light-cone” object using the outputs (the so-called “coeval cubes”) of Simfast21. From the derived “light-cone” object, we extract three subsets with a channel width of 160 kHz and construct our final tiled data cube with dimensions of \( (\theta, \theta, \Delta \nu) = (10^\circ, 10^\circ, 8 \) MHz in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands, among which each image is performed for the SKA “observed” simulation. We present the example SKA “observed” EoR images at 120 (\( z = 10.84 \)), 150 (\( z = 8.47 \)), and 190 MHz (\( z = 6.48 \)) in Figure 3. The rms brightness temperatures of \( \delta T_S^{21} \) are 21.03, 13.12, and 4.43 mK inside the central regions of 6\(^\circ\) \( \times \) 6\(^\circ\), 5\(^\circ\) \( \times \) 5\(^\circ\), and 4\(^\circ\) \( \times \) 4\(^\circ\) at 120, 150, and 190 MHz, respectively.

4. Results

We present the simulated H\(_{\alpha}\) intensity maps, including 12 cases of direct and scattered H\(_{\alpha}\) intensities and 4 cases of intrinsic H\(_{\alpha}\) intensities, in Section 4.1. Meanwhile, we derive the corresponding Galactic free–free emission maps from the simulated H\(_{\alpha}\) intensity maps in Section 4.2. Furthermore, by analyzing the 1D and 2D power spectra, we have quantitatively evaluated the contamination imposed by the Galactic free–free emission on the EoR detection in Section 4.3.

4.1. H\(_{\alpha}\) Intensity Maps

We perform 12 cases of simulations with diverse model parameters that are listed in Table 1. Each case is repeated 50 times to estimate the mean and the standard deviation (1\(\sigma\)) of the simulated total H\(_{\alpha}\) intensity (\( I_{\text{tot}}^{\alpha} \)) maps. The final mean \( I_{\text{tot}}^{\alpha} \) maps are Gaussian filtered and smoothed to 1\(^\circ\) to reduce the Poisson noise, as shown in Figure 4. We compare the results obtained with \( \beta \) H\(_{\text{II}}\) region models with the uniform H\(_{\text{II}}\) region model and find that when the uniform H\(_{\text{II}}\) region model is applied the highest H\(_{\alpha}\) intensity is obtained because fewer H\(_{\alpha}\) photons are concentrated on the Galactic plane, where the absorption is severest. We further compare the \( I_{\text{tot}}^{\alpha} \) maps with different scattering parameters and confirm that increasing \( a \) and \( g \) will increase the H\(_{\alpha}\) intensity. The

\textsuperscript{12} SKA1-Low Configuration Coordinates: [https://astronomers.skatelescope.org/wp-content/uploads/2016/09/SKA-TEL-SKO-0000422_02_SKA1_Low_ConfigurationCoordinates-1.pdf](https://astronomers.skatelescope.org/wp-content/uploads/2016/09/SKA-TEL-SKO-0000422_02_SKA1_Low_ConfigurationCoordinates-1.pdf) (released on 2016 May 31).

\textsuperscript{13} OSKAR: [https://github.com/OxfordSKA/OSKAR](https://github.com/OxfordSKA/OSKAR) (version 2.7.0).

\textsuperscript{14} [https://github.com/mariogrs/Simfast21](https://github.com/mariogrs/Simfast21)
averaged $I_{H\alpha}^{\text{tot}}$ of the whole sky for 12 cases and their corresponding standard deviations are listed in Table 2, with the values of $4.43 (\pm 0.30) - 11.24 (\pm 0.74) R$.

We then present the latitudinal distributions of the mean and 1σ uncertainty of the simulated $H\alpha$ intensities in Figure 5. We find that at lower latitudes ($|b| \lesssim 8^\circ$) the $I_{H\alpha}^{\text{tot}}$ obtained with $R_s = 0.5R_\odot$, $R_s = 1.0R_\odot$, and a uniform $H\text{II}$ region model show lower values than those obtained with $R_s = 0.1R_\odot$ by about 15%, 25%, and 40%, respectively. However, at middle and higher latitudes ($|b| \gtrsim 8^\circ$) a contrary tendency is found, as the corresponding simulated $H\alpha$ intensities become about 2, 3, and 5 times higher than those obtained with $R_s = 0.1R_\odot$. We also confirm that increasing $a$ and $g$ will enhance the averaged $H\alpha$ intensity at all latitudes. Compared with the $\beta H\text{II}$ region model, the $I_{H\alpha}^{\text{tot}}$ obtained with the uniform $H\text{II}$ region model shows about 15% lower $H\alpha$ intensities at lower latitudes ($|b| \lesssim 20^\circ$) but shows about 15% higher $H\alpha$ intensities at middle and higher latitudes ($|b| \gtrsim 20^\circ$). For each case, the 1σ uncertainty in our simulation is primarily caused by the method of setting random distances for $H\text{II}$ regions (see Section 2.2) and the process of random scattering, which is typically less than 10% (see Table 2).

We further present the scattered $H\alpha$ intensity ($I_{H\alpha}^{\text{sca}}$), including the scattered emission from the $H\text{II}$ regions ($I_{H\alpha}^{\text{sca-HII}}$) and that from the WIM ($I_{H\alpha}^{\text{sca-WIM}}$), which is realized by labeling the $H\alpha$ photon according to its behavior (i.e., scattered route) in the simulation. As presented in Figure 5, at middle and higher latitudes ($|b| \gtrsim 15^\circ$), $I_{H\alpha}^{\text{sca-HII}}$ increases with the scale radius $R_s$, which receives the highest value when the uniform $H\text{II}$ region model is applied. It is found that the scattering percentage is in the range of 15%–50%, which is very consistent with the previous observation results, depending on the $H\text{II}$ region model and the scattering parameters of $a$ and $g$. Note that the electron-scattered emission attributes less than 3% of the total scattered $H\alpha$ intensity since the cross section of free electrons is three orders of magnitude smaller than that of dust. Therefore, the contribution of scattering caused by the free electrons will no longer be discussed separately.

In addition, we have attempted to employ the cosecant law $I_{H\alpha}^{\text{tot}} = A_0 + A_1 / \sin(|b|)$ to fit the latitudinal distribution of the
Figure 5. Latitudinal distributions of the simulated total \textsc{H}_\alpha intensities, the scattered \textsc{H}_\alpha intensities from \textsc{H} \textsc{II} regions, and the scattered \textsc{H}_\alpha intensities from WIM. The solid lines and shaded regions show the mean values and the corresponding 1σ uncertainties estimated from 50 simulation runs, respectively.

Table 2

Averaged \textsc{H}_\alpha Intensities with the Cosecant Fitting Parameters of Offset (A_0) and Amplitude (A_1) and the Corresponding Averaged Galactic Free–Free Brightness Temperatures T_\textsc{a} at 120, 150, and 190 MHz

| Label | I_{\textsc{h}_\alpha} (R) | A_0 | A_1 | T_\textsc{a}(120 MHz) (K) | T_\textsc{a}(150 MHz) (K) | T_\textsc{a}(190 MHz) (K) |
|-------|-----------------|-----|-----|-----------------|-----------------|-----------------|
| (a)   | 4.30 ± 0.30     | -0.70| 0.80| 2.67 ± 0.18     | 1.67 ± 0.11     | 1.03 ± 0.07     |
| (b)   | 4.96 ± 0.35     | -0.60| 0.75| 3.00 ± 0.21     | 1.88 ± 0.13     | 1.15 ± 0.08     |
| (c)   | 5.44 ± 0.39     | -0.35| 0.70| 3.28 ± 0.24     | 2.06 ± 0.15     | 1.26 ± 0.09     |
| (d)   | 5.78 ± 0.40     | 0.15 | 0.55| 3.50 ± 0.24     | 2.20 ± 0.15     | 1.34 ± 0.09     |
| (e)   | 6.48 ± 0.46     | -0.90| 1.05| 3.91 ± 0.28     | 2.46 ± 0.17     | 1.50 ± 0.11     |
| (f)   | 7.06 ± 0.51     | -0.65| 0.95| 4.26 ± 0.31     | 2.68 ± 0.19     | 1.64 ± 0.12     |
| (g)   | 7.65 ± 0.55     | -0.25| 0.80| 4.63 ± 0.33     | 2.91 ± 0.21     | 1.78 ± 0.13     |
| (h)   | 8.13 ± 0.58     | 0.75 | 0.70| 4.92 ± 0.35     | 3.09 ± 0.22     | 1.89 ± 0.13     |
| (i)   | 9.62 ± 0.72     | -0.75| 1.15| 5.80 ± 0.43     | 3.65 ± 0.27     | 2.23 ± 0.17     |
| (j)   | 10.13 ± 0.74    | -0.65| 1.05| 6.13 ± 0.45     | 3.86 ± 0.28     | 2.36 ± 0.17     |
| (k)   | 10.81 ± 0.82    | 0.15 | 0.95| 6.54 ± 0.50     | 4.11 ± 0.31     | 2.52 ± 0.19     |
| (l)   | 11.24 ± 0.74    | 1.05 | 0.85| 6.80 ± 0.45     | 4.28 ± 0.28     | 2.62 ± 0.17     |
| (A)   | 17.00 ± 0.61    | -0.85| 2.85| 10.07 ± 0.36    | 6.37 ± 0.23     | 3.91 ± 0.14     |
| (B)   | 17.67 ± 0.80    | -0.15| 2.45| 10.58 ± 0.48    | 6.67 ± 0.30     | 4.08 ± 0.18     |
| (C)   | 17.76 ± 0.75    | 1.05 | 2.25| 10.66 ± 0.45    | 6.72 ± 0.28     | 4.11 ± 0.17     |
| (D)   | 18.04 ± 0.91    | 1.75 | 2.10| 10.84 ± 0.55    | 6.83 ± 0.34     | 4.18 ± 0.21     |

Simulated \textsc{H}_\alpha intensity, where A_0 is the offset, A_1 is the amplitude, and b is the Galactic latitude (Dickinson et al. 2003). In Figure 6, we present the best-fitting cosecant profiles of the latitudinal cuts of simulated \textsc{H}_\alpha intensities (averaged over \(-15^\circ \leq l \leq 15^\circ\)) for 12 cases. We find that the cosecant profiles agree well with the simulated total \textsc{H}_\alpha intensities at lower and middle latitudes (|b| \leq 70^\circ), but they are higher than the \textsc{H}_\alpha intensities at higher latitudes (|b| \geq 70^\circ), which are consistent with the simulation result of Wood99. We present the best-fitting cosecant (A_0, A_1) parameters for 12 cases in Table 2.
4.1.2. Intrinsic H α Intensities

Meanwhile, the intrinsic H α intensities are realized by removing the dust and free electrons from the simulation cube. The four cases of intrinsic H α intensities ($I_{\text{H}\alpha}^{\text{int}}$) labeled as (A), (B), (C), and (D) are simulated, which are only relevant to the H II region models (corresponding to $R_s = 0.1R_s$, 0.5$R_s$, 1.0$R_s$, and uniform H II region models, respectively.). We present each $I_{\text{H}\alpha}^{\text{int}}$ map in Figure 7, which is the mean value of 50 simulation runs. For four intrinsic cases, the 1σ uncertainties are mainly dominated by setting random distances for H II regions (see Section 2.2), with a typical value of ~5% (see the uncertainties listed in Table 2).

The latitudinal distributions of the means (red solid lines) and 1σ uncertainties (red shaded regions) of the intrinsic H α intensities are shown in Figure 8 (top panels), along with the best-fitting cosecant profiles (green solid lines). We find that the averaged intrinsic H α intensities of cases (B), (C), and (D) are about 1.5, 1.7, and 2.0 times more luminous than the $I_{\text{H}\alpha}^{\text{int}}$ of case (A). It is also found that the $I_{\text{H}\alpha}^{\text{int}}$ of cases (B), (C), and (D) are consistent with the cosecant law, but the $I_{\text{H}\alpha}^{\text{int}}$ of case (A) is lower than the cosecant model at middle latitudes ($10^\circ \lesssim |b| \lesssim 50^\circ$), since more H α photons are concentrated on the Galactic plane in case (A). The best-fitting cosecant parameters and the averaged intrinsic H α intensities are also given in Table 2. We further compare the $I_{\text{H}\alpha}^{\text{int}}$ with the $I_{\text{H}\alpha}^{\text{tot}}$ and present the results in the bottom panels of Figure 8. It is found that $I_{\text{H}\alpha}^{\text{int}}$ is more luminous than $I_{\text{H}\alpha}^{\text{tot}}$ by about 6.3, 3.2, 2.0, and 1.6 times when three β H II region models with $R_s = 0.1R_s$, 0.5$R_s$, 1.0$R_s$, and a uniform model of H II regions are adopted, respectively.

4.2. Galactic Free–Free Emission

We derive the Galactic free–free emission from the above simulated $I_{\text{H}\alpha}^{\text{int}}$ and $I_{\text{H}\alpha}^{\text{tot}}$ according to the equations given in Section 2.6. To obtain the Galactic free–free emission map, we employ an all-sky electron temperature map proposed by Planck Collaboration et al. (2016a), which is presented in the left panel of Figure 9 (reproduced with permission © ESO), and then we can derive the Galactic free–free brightness temperature map at any frequency (100 MHz–100 GHz). Meanwhile, an example optical depth map at 120 MHz is shown in the right panel of Figure 9. For each case, we present the averaged brightness temperatures of Galactic free–free emissions at 120, 150, and 190 MHz in Table 2.

By comparing the 12 cases of $I_{\text{H}\alpha}^{\text{int}}$ simulated in clumpy dust with the observed H α intensity of F03 ($I_{\text{H}\alpha}^{\text{F03}}$), we recommend the model parameters of case (f), i.e., $a = 0.67$, $g = 0.50$, and $R_s = 0.5R_s$ (for more detailed comparisons, see Section 5). Therefore, we derive the Galactic free–free emission from the intrinsic H α emission of case (B) to carry out our subsequent calculations. We present the example Galactic free–free emission maps at 120, 150, and 190 MHz in Figure 10. The latitudinal distributions of the mean and the corresponding 1σ uncertainties of the Galactic free–free emissions at 120, 150, and 190 MHz are presented in Figure 11 (top panel). The conversions between the Galactic free–free emissions and the
corresponding Hα intensities are illustrated in Figure 11 (bottom panel), which are 0.61, 0.38, and 0.23 [K/R] at 120, 150, and 190 MHz, respectively.

4.3. Contamination of Galactic Free–Free Emission

The 1D and 2D power spectra are calculated to estimate the contamination of Galactic free–free emission on the EoR signal. The EoR signals observed at different frequencies are expected to be a 3D image cube, where the two angular dimensions describe the transverse distances across the sky and the one frequency dimension depicts the line-of-sight distance. For each foreground component cube, its two angular dimensions describe the same sky coverage as the EoR signal, but its one frequency dimension depicts the emission distribution in the frequency space (i.e., spectrum), which is different from the EoR signal. The 3D power spectrum \( P(k_x, k_y, k_z) \) of the EoR signal should be spherical symmetry within a limited redshift range (e.g., \( \Delta z \sim 0.5 \)), corresponding to a frequency bandwidth of \( \sim 8 \text{ MHz} \) at 150 MHz, during which the evolution of the universe can be ignored and the H1 can be regarded as isotropic. The spherically averaged 1D k-space power spectrum \( P(k) \) can be calculated by averaging the \( P(k_x, k_y, k_z) \) to achieve a relatively higher signal-to-noise ratio. As adopted in both the theoretical studies (e.g., Morales & Hewitt 2004; Datta et al. 2010) and the low-frequency experiments (e.g., Li et al. 2019), the dimensionless variant of the 1D power spectrum \( \Delta^2(k) = P(k)k^3/(2\pi^2) \) is more commonly employed. The Blackman–Nuttall window function is applied to the frequency dimension before calculating the 3D power spectra to suppress the significant sidelobes in the Fourier transform (Trott & Tingay 2015; Chapman et al. 2016; Li et al. 2019).

We calculate the 1D power spectra \( \Delta^2(k) \) from the SKA "observed" image cubes of the Galactic free–free emission and the EoR signal. The comparisons of the power spectra \( \Delta^2(k) \) between the Galactic free–free emission and the EoR signal are presented in Figure 12. It is obvious that the contamination caused by Galactic free–free emission on the EoR signal is a function of position in the k-space. On large scales (\( k \lesssim 0.5 \text{ Mpc}^{-1} \)) the Galactic free–free emission has a greater impact on the EoR signal, while on small scales (\( k \gtrsim 0.5 \text{ Mpc}^{-1} \)) it causes relatively less contamination on the EoR signal. We find that, given the 1σ uncertainties, the Galactic free–free emissions are more luminous than the EoR signals by about \( 10^{5.4} \), \( 10^{5.0} \), and \( 10^{4.7} \) on scales of \( 0.1 \text{ Mpc}^{-1} < k < 2 \text{ Mpc}^{-1} \) in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands, respectively.

The 2D power spectrum \( P(k_{\parallel}, k_{\perp}) \) can be obtained by averaging the 3D power spectrum \( P(k_x, k_y, k_z) \) over the corresponding angular annuli, the radius of which is \( k_{\parallel} \equiv \sqrt{k_x^2 + k_y^2} \), for each line-of-sight plane \( k_{\parallel} \equiv k_z \). It is found that in the \((k_{\parallel}, k_{\perp})\) plane the spectral-smooth Galactic free–free emission dominates the low-\(k_{\parallel}\) region, but some purely angular \((k_{\perp})\) modes of the foreground signal can be thrown into the line-of-sight \((k_{\parallel})\) dimension (called mode mixing), due to the complicated instrumental and observational effects (e.g., chromatic primary beams, calibration errors). Consequently, an expanded wedge-like contamination region appears at the bottom right in the \((k_{\parallel}, k_{\perp})\) plane, which is known as the foreground wedge (Datta et al. 2010; Morales et al. 2012; Liu et al. 2014). The top left corner in the \((k_{\parallel}, k_{\perp})\) plane, on the other hand, is almost free from the

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**Figure 7.** All-sky Mollweide projections (\( N_{\text{side}} = 1024 \)) of the intrinsic Hα intensity maps simulated with \( R_e = 0.1R_e, 0.5R_e, 1.0R_e \), and uniform H II models, respectively, in Galactic coordinates with the same figure configuration as Figure 4. All panels show the mean values of 50 simulation runs and share the same logarithmic scale in units of R.
foreground contamination, namely, the EoR window, whose description is proposed by Thyagarajan et al. (2013)

\[ k_\parallel \geq \frac{H(z)D_M(z)}{(1 + z)c^2} \left[ k_\perp \sin \Theta + \frac{2\pi w f_{21}}{(1 + z)D_M(z)} \right] \]  

(8)

where \( H(z) \) is the Hubble parameter at redshift \( z \), \( D_M(z) \) is the transverse comoving distance, \( B = 8 \) MHz is the frequency bandwidth of the image cube, \( w (\propto B) \) is the number of characteristic convolution widths for the spillover region caused by the variations in instrumental frequency response, \( \Theta \) is the angular distance of the foreground sources from the field center, and \( f_{21} = 1420.4 \) MHz is the rest frequency of the 21 cm emission line.

We calculate the 2D power spectra \( P(k_\parallel, k_\perp) \) of the Galactic free–free emission and the EoR signal in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands and present

\[ \frac{R_s = 0.1R_s}{R_s = 0.5R_s}{R_s = 1.0R_s} \]

\[ \frac{10^4}{10^3}{10^2} \]

Figure 8. Top panels: latitudinal distributions of the mean (red solid lines) and the 1σ uncertainties (red shaded regions) of intrinsic Hα intensities estimated from the 50 simulation runs, as well as their corresponding cosecant fitting profiles (green solid lines). Bottom panels: comparisons between the simulated total Hα intensities (magenta, green, and blue solid lines) and the intrinsic Hα intensities (red solid lines).

Figure 9. All-sky Mollweide projections \( (N_{\text{side}} = 1024) \) of the electron temperature map (left panel; Planck Collaboration et al. 2016a, reproduced with permission © ESO) and an example optical depth map at 120 MHz (right panel) in Galactic coordinates with the same figure configuration as Figure 4. The color bars are in linear scales.
the results in Figure 13. We find that the spectral-smooth Galactic free–free emission dominates the low-$k_{\parallel}$ ($k_{\parallel} \lesssim 0.2$ Mpc$^{-1}$) regions, while the EoR signal distributes its power across all $k_{\parallel}$ modes, illustrating its rapid fluctuations along the line-of-sight dimension. Concerning the angular dimension, the powers of Galactic free–free emission and the EoR signal dominate on scales of $k_{\perp} \lesssim 0.2$ Mpc$^{-1}$.

To better constrain the contamination caused by Galactic free–free emission, we then calculate the 2D power spectrum ratio $R(k_{\perp}, k_{\parallel})$ defined as $R(k_{\perp}, k_{\parallel}) = P_{\text{Gff}}(k_{\perp}, k_{\parallel})/P_{\text{21cm}}(k_{\perp}, k_{\parallel})$, where $P_{\text{Gff}}(k_{\perp}, k_{\parallel})$ and $P_{\text{21cm}}(k_{\perp}, k_{\parallel})$ are the 2D power spectra of the Galactic free–free emission and the EoR signal, respectively. As presented in Figure 14, the EoR signal is almost free from the contamination of Galactic free–free emission on scales of $k_{\parallel} \gtrsim 0.17$ Mpc$^{-1}$ and $k_{\perp} \lesssim 0.5$ Mpc$^{-1}$, $k_{\parallel} \gtrsim 0.19$ Mpc$^{-1}$ and $k_{\perp} \lesssim 0.7$ Mpc$^{-1}$, and $k_{\parallel} \gtrsim 0.2$ Mpc$^{-1}$ and $k_{\perp} \lesssim 0.9$ Mpc$^{-1}$, in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands, respectively, while outside these regions, the Galactic free–free emission causes significant contamination, because the 2D power spectrum ratio is obviously greater than unity in these frequency bands.

To further quantify the contamination imposed by Galactic free–free emission, we define an EoR window (marked by white dashed lines in Figures 13 and 14) in the $(k_{\perp}, k_{\parallel})$ plane according to Equation (8) with a configuration of $w = 3$ and the SKA1-Low’s FOV (i.e., $\Theta = 6^\circ$, $5^\circ$, and $4^\circ$ in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands, respectively). We then calculate the 1D power spectrum ratio $R_{\text{EoR}}(k)$ of Galactic free–free emission to the EoR signal by averaging the modes within the EoR window. As shown in Figure 15, inside the EoR window, the impact induced by the leaked Galactic free–free emission on the EoR signal can be ignored on large scales ($k \lesssim 0.5$ Mpc$^{-1}$), while the leaked Galactic free–free emission causes severe contamination on the EoR detection on small scales ($k \gtrsim 0.5$ Mpc$^{-1}$). These results are consistent with the analysis of 2D power spectrum ratios (see Figure 14). We find that compared to Figure 12, the 1D power ratios inside the EoR window $R_{\text{EoR}}(k)$ are suppressed by about 3 orders of magnitude, which illustrates that the EoR window is a powerful tool in detecting the EoR signal. For example, on scales of $k \sim 0.5$ Mpc$^{-1}$, the $R_{\text{EoR}}(k)$ are generally about 12%, 5%, and 2% in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands, respectively. However, even inside the EoR window, the power leaked by Galactic free–free emission can still be significant, as the $R_{\text{EoR}}(k)$ can be up to about 110%–8000%, 30%–2400%, and 10%–250% when considering the 1σ uncertainties (shaded regions) on scales of 0.5 Mpc$^{-1}$ to 1 Mpc$^{-1}$ in the three frequency bands, respectively. These analyses further support that the Galactic free–free emission should be carefully removed in the EoR detections, especially toward the lower frequencies ($\sim$116 MHz).

5. Comparison and Discussion

To quantitatively verify our simulation, we compare the simulated $T_{\text{EoR}}^\text{obs}$ of cases (a), (b), (c), and (d) with the result of Wood99, given that they share the same scattering
parameters \((a = 0.50, g = 0.44)\). The black asterisks in Figure 16 mark the simulated Wood99 \(\text{H} \alpha\) intensity, which is about 25%, 35%, and 45% lower than \(I_{\text{H} \alpha}\) of cases (b), (c), and (d), respectively. The \(\text{H} \alpha\) intensity of case (a) is about 25% higher than that of Wood99 at lower latitudes (\(|b| \lesssim 10^\circ\)), but it is about 35% lower at middle and higher latitudes (\(|b| \gtrsim 10^\circ\)). The departures between them are due to the different \(\text{H} \ II\) region models and different dust models, as in our simulation the \(\text{H} \ II\) regions are modeled with detailed \(\beta\) or uniform structures (see Section 2.2) other than just simply “point sources.” Moreover, the clumpy dust is derived from the observed HI data rather than a simple axisymmetric model.

F03 derived an all-sky observed \(\text{H} \alpha\) intensity map\(^{15}\) by jointly studying three \(\text{H} \alpha\) surveys, i.e., Virginia Tech Spectral line Survey\(^{16}\) (Dennison et al. 1998), SHASSA\(^{17}\) (Gaustad et al. 2001), and WHAM\(^{18}\) (Haffner et al. 2003). For each case, we further compare the simulated total \(\text{H} \alpha\) intensity map with the observed F03 \(\text{H} \alpha\) intensity map and present the mean and \(1 \sigma\) uncertainty of \(I_{\text{H} \alpha}\), as well as the value and corresponding

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\(^{15}\) https://faun.rc.fas.harvard.edu/dfinkle/skymaps/

\(^{16}\) http://www.phys.vt.edu/~halpha

\(^{17}\) http://amundsen.swarthmore.edu/SHASSA

\(^{18}\) http://www.astro.wisc.edu/wham
The 2D power spectra ratios $R(k_\perp, k_\parallel)$ of Galactic free–free emission to the EoR signal in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands. The mean 2D power spectrum of 50 simulation runs for Galactic free–free emission is used. The white dashed lines mark the EoR window boundaries.

We also compare simulations with three different $\beta = 0.4, 0.7$ (case (f); see Figure 4), and 1.0 (take $R_e = 0.5R_c, a = 0.67$, and $g = 0.50$), for instance) to test the effect of $\beta$ on the simulated total H\textalpha intensity, and we present the results in Figure 17. The averaged $I_{\text{H\alpha}}^{\text{tot}}$ for simulations with $\beta = 0.4, 0.7,$ and 1.0 are $7.47 \pm 0.54, 7.06 \pm 0.51,$ and $6.79 \pm 0.48$, respectively. The comparisons of latitudinal distributions of the $I_{\text{H\alpha}}^{\text{tot}}$ simulated with $\beta = 0.4, 0.7,$ and 1.0 are shown in the right panel of Figure 17. Compared with case (f) ($\beta = 0.7$), we find that the $I_{\text{H\alpha}}^{\text{tot}}$ at lower latitudes ($|b| \lesssim 10^\circ$) increases slightly with the increase of $\beta$, while it shows a contrary tendency at middle and higher latitudes ($|b| \gtrsim 10^\circ$). In conclusion, we argue that the uncertainty of $I_{\text{H\alpha}}^{\text{tot}}$ caused by $\beta$ (when the $\beta$ changes from 0.7 to 0.4 or from 0.7 to 1.0) is less than 5%.

6. Summary

We have implemented an all-sky Galactic free–free emission map based on the Monte Carlo simulation of the H\textalpha intensity incorporating the direct and scattered emissions from H II regions and the WIM. Our simulation recovers the main structures of the Milky Way and reproduces the major characteristics of the observed H\textalpha intensity—the cosecant profile. We finally recommend a set of model parameters of $\beta = 0.7, R_e = 0.5R_c, a = 0.67,$ and $g = 0.50$ to match the current observation data. Based on the intrinsic H\textalpha intensity, we derive the Galactic free–free emission and evaluate its contamination on the EoR detection, for which we have incorporated the instrumental effects by utilizing the latest SKA1-Low layout configuration. By carrying out detailed comparisons of the power spectra between Galactic free–free emission and the EoR signal in the 116–124 MHz, 146–154 MHz, and 186–194 MHz frequency bands, we have shown that the contamination of Galactic free–free emission on the EoR signal is a function of position in the $k$-space, i.e., on large scales ($k \lesssim 0.5$ Mpc$^{-1}$) the Galactic free–free emission causes severe contamination, especially toward lower frequencies ($\sim 116$ MHz), while on small scales ($k \gtrsim 0.5$ Mpc$^{-1}$) it causes relatively less contamination on the EoR detection. Even inside the properly defined EoR window, the power leaked by Galactic free–free emission can still cause nonnegligible contamination on the EoR signal. Overall, we recommend that the Galactic free–free emission, as a severe contaminating source, needs serious treatment in the forthcoming deep EoR experiments.
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