Research on fault identification of gearbox bearing based on quantum theory

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Abstract: In view of the problem that the fault signal of bearing in automobile gearbox is easy to be covered by background noise, a fault signal recognition method based on quantum theory is proposed. This method will quantize the individual sampling signal, and take the vibration signal as the state superposition of fault signal and background noise, deeply study the randomness and local characteristics of sampling signal, and analyze the core information of sampling point, so that the amplitude characteristics of sampling point signal can be converted into the probability distribution of fault signal and noise signal in quantum domain, and change integrated attenuation of noise reduction for the fault signal, and realize the fine quantization noise as well as the highlight of the fault information. Experimental results show that the algorithm is feasible and efficient.

1. Introduction

The bearing noise signal of automobile gearbox carries rich running state information, and its analysis becomes an important means to realize fault diagnosis, online detection and remote monitoring [1]. However, the complex factors of gearbox bearing noise make it difficult to detect and analyze, which limits the timely identification of fault signals and reduces the efficiency of bearings. Therefore, how to effectively identify fault signals has become the focus of research.

In the algorithm of noise reduction of mechanical fault signal, the idea of wavelet transform is to extract different high and low frequency information by shrinking the wavelet coefficients of different scales, so it is widely used in noise analysis and control [2-7]. The idea of noise reduction based on mathematical morphology is to construct filters with different functions with preset structural elements to correct and match the geometric characteristics of vibration signals, which is widely used in image processing [8-12]. There are also neural network algorithm, genetic algorithm and so on, most of which have achieved good results [13, 14]. In recent years, some traditional algorithms improved based on quantum theory have developed rapidly in fault diagnosis and recognition [15-20].

Comprehensive analysis shows that these methods can be further improved in two aspects: 1. Traditional algorithms process all signals as a whole in signal feature extraction, without analyzing the core characteristics of each sampling point information, which is extremely easy to cause the suppression of fault signals; 2. Traditional algorithms based on quantum theory improvement, which partially draw on the quantum theory features, but most of them are partially improved, there is no independent quantum noise reduction algorithm, which can not give full play to the advantages of quantum theory signal processing.

To solve these problems, this paper proposes an algorithm based on quantum theory. According to the correlation of the fault signal sampling field and the randomness of the background noise signal,
using the state superposition principle of quantum mechanics, the research method of the micro world is used for reference in the vibration signal processing, and the inner core of each sampling point signal is studied with more precise means, and each sampling point signal is quantized into the state superposition of the fault signal and the background noise signal. The amplitude characteristic of the sampling point signal is transformed into the probability distribution of the specific distribution characteristic, and then the probability amplitude of the sampling point is studied, so as to determine whether the sampling point signal is the fault model or the background noise signal, and finally realize the separation and noise reduction while highlighting the fault signal. The advantages and applications of quantum theory in vibration signal processing are fully extended. The experimental results show that the method can successfully take into account the noise suppression and fault signal enhancement, and is a new noise reduction method for non-linear and non-stationary signals.

2. Mathematical methods

2.1 Quantization of vibration signal

In order to map the vibration signal in time domain to quantum domain, according to the relationship between Hadamard transform (HT) and quantum bit (QB) in quantum mechanics, it is set that the quantum bit length of a quantum system is \( n \), then the total number of state vectors is unitary matrix \( H \) of \( N = 2^n \), then the expression of HT in two-dimensional space system expansion is [7]:

\[
H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{1}
\]

Quantum bits can be expressed as:

\[
|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \tag{2}
\]

Among them, \( \psi \) is the state function, \(|0\rangle \) and \(|1\rangle \) is the ground state of quantum information, which represents the ground state of fault signal and the ground state of background noise respectively; \( \alpha \) and \( \beta \) are the quantum probability amplitude of ground state \(|0\rangle \) and \(|1\rangle \) respectively, \( |\alpha|^2 \) and \( |\beta|^2 \) are the quantum probability of ground state \(|0\rangle \) and \(|1\rangle \) respectively. Therefore, the Hadamard transformation of qubit can be obtained as follows:

\[
H|\psi\rangle = H(\alpha |0\rangle + \beta |1\rangle) = \frac{\alpha + \beta}{\sqrt{2}} |0\rangle + \frac{\alpha - \beta}{\sqrt{2}} |1\rangle \tag{3}
\]

In quantum mechanics \( \alpha, \beta \in [0,1] \), but in physical essence, the vibration signal can't be a fault signal or a background noise signal, so in fact \( \alpha, \beta \in (0,1) \). The Hadamard quantum probabilities of fault state \( \alpha |0\rangle \) and background noise state \( \beta |1\rangle \) are as follows:

\[
\left(\frac{\alpha + \beta}{\sqrt{2}}\right)^2 = 0.5 + \alpha \times \beta > 0.5, \quad \left(\frac{\alpha - \beta}{\sqrt{2}}\right)^2 = 0.5 - \alpha \times \beta < 0.5 \tag{4}
\]

It can be seen from the above equation that the value of Hadamard quantum probability is determined by \( \alpha \times \beta \) according to reference [8], it can be seen that:

\[
0 < \alpha \leq \sqrt{0.5}, \quad \sqrt{0.5} \leq \beta < 1 \tag{5}
\]

The value range of \( \alpha \) and \( \beta \) is determined by the above equation. The signal normalization processing for sampling points is as follows:

\[
z(k) = \text{abs}\left(\frac{s(k)}{\text{max}(\text{abs}(s(k)))}\right) \in [0,1] \tag{6}
\]
Where: \( s(k) \) is the sampling signal point, and it is defined that the quantized probability amplitude is halved from \( z(k) \), that is:

\[
sn(k) = \frac{1}{2} z(k) \in [0, 0.5]
\]  

(7)

Since each sampling signal contains fault signal and background noise signal, \( a, B \) can be expressed as:

\[
\alpha = \begin{cases} 
\sqrt{\xi}, & sn(k) = 0 \\
\sqrt{sn(k)}, & 0 < sn(k) \leq 0.5 
\end{cases}
\]

(8)

\[
\beta = \begin{cases} 
\sqrt{1-\xi}, & sn(k) = 0 \\
\sqrt{1-sn(k)}, & 0 < sn(k) \leq 0.5 
\end{cases}
\]

(9)

In equations (8), (9), \( \lim_{z \to 0} \sqrt{\xi} = 0 \), \( \lim_{z \to 0} \sqrt{1-\xi} = 1 \), so the quantization processing of sampling point signal can be expressed as follows:

\[
\left| s(k) \right| = \begin{cases} 
\sqrt{\xi z(k)}, & sn(k) = 0 \\
\sqrt{sn(k)} |z(k)|, & 0 < sn(k) \leq 0.5 
\end{cases}
\]

(10)

The equation (10) after HT, it can be expressed as follows:

\[
\left| s(k) \right| = \sqrt{sn(k)} |0| + \sqrt{1-sn(k)} |1|
\]

(11)

Analysis shows that:

\[
\alpha^2 + \beta^2 = 0.5 + \alpha \times \beta \in [0.5, 1], \quad \alpha^2 - \beta^2 = 0.5 - \alpha \times \beta \in [0, 0.5]
\]

(12)

Combined with the above equation, it can be analyzed that the probability of fault state \( |0| \) is positively correlated with that of \( abs(s(k)) \), and the probability of background noise state \( |1| \) is negatively correlated with that of \( abs(s(k)) \).

2.2 Feasibility analysis

In the bearing vibration signal, there is a strong correlation between the adjacent sampling points of the fault signal, and the background noise signal will destroy this correlation characteristic. Therefore, the concept of correlation function (CF) is proposed to discrete the fault signal and background noise signal. Figure 1 shows the location relationship of 3 neighborhood sampling points.

\[
\cdots sn(k-1) \ast sn(k) \ast sn(k+1) \cdots
\]

Figure 1. Location relationship of three neighborhood sampling points.

It can be seen from the previous content that \( \alpha(k) \) is the probability amplitude of the quantized vibration signal, then the correlation function can be expressed as:

\[
CF(k) = \alpha(k-1) \times \alpha(k) \times \alpha(k+1)
\]

(13)

It can be expressed as follows:

\[
CF(k) = \sqrt{sn(k-1) \times sn(k) \times sn(k+1)}
\]

(14)

It can be seen from the analysis that the value of correlation function \( CF(k) \) depends on the same direction correlation of three sampling points, that is, only when the value of three sampling points is large at the same time, the value of correlation function \( CF(k) \) can be large, which only appears near the peak value of fault signal, so that the fault signal is obviously highlighted with feasibility. On this
basis, for the randomness of positive and negative pulses of vibration signal, median filter can smooth the signal. Because impulse noise causes signal mutation and fault noise also causes vibration signal mutation, they are similar, so median filter can be used to extract fault impulse signal. In this paper, a median filter (MF) is used to determine the threshold $T(n)$. The specific process can be expressed as follows:

$$T(k) = Med(CF(sn(k-3),...,CF(sn(k+3))))$$ (15)

Analysis of the above equation shows that MF filter can achieve the purpose of eliminating noise, ensuring $T(K) \leq CF_{max}$ and highlighting fault signal.

2.3 Noise reduction method

According to the randomness of fault signal, there may be positive fault pulse and negative fault pulse. For the two cases, separate treatment. If the positive signal $p_+$ is a fault near point $k$, the larger the calculated value of $\alpha \times \beta$ is, so that $|z_+|$ becomes larger and $|z_-|$ becomes smaller. According to the prominent demand of fault signal, when $CF(k) > T(k)$, the signal should be strengthened, when $CF(k) \leq T(k)$, the signal should be weakened. Therefore, in the case of $CF(k) > T(k)$, the probability that the sampling point is the fault signal is large, the quantum probability of $|0\rangle$ can be used to enhance the signal, and the degree of signal enhancement is positively correlated with $s(k)$. The specific expression is as follows:

$$s_+(k) = sn(k) + \left(\frac{\alpha(k) + \beta(k)}{\sqrt{2}}\right)^2$$

$$= 0.5 + sn(k) + \alpha(k) \times \beta(k)$$ (16)

According to the above equation, the range of $sn(k)$ enhancement is $[0, 1.0]$.

In case of $CF(k) \leq T(k)$, the probability that the sampling point is the background noise is large, so the quantum probability of $|1\rangle$ can be used for noise reduction, and the degree of noise reduction is negatively related to $s(k)$. The specific expression is as follows:

$$s_-(k) = sn(k) - \left(\frac{\alpha(k) - \beta(k)}{\sqrt{2}}\right)^2$$

$$= -0.5 + sn(k) - \alpha(k) \times \beta(k)$$ (17)

According to the above equation, the weakening range is $[0, 0.5]$.

If the negative pulse signal $p_-$ is a fault near point $k$ and the sampling point is a fault signal in case of $CF(k) > T(k)$, the quantum probability of $|0\rangle$ can be used to enhance the signal, and the degree of signal enhancement is positively related to the $s(k)$ mode value. The specific expression is as follows:

$$s_+(k) = -sn(k) + \left(\frac{\alpha(k) + \beta(k)}{\sqrt{2}}\right)^2$$

$$= -0.5 - sn(k) + \alpha(k) \times \beta(k)$$ (18)

According to the above equation, the range of $sn(k)$ reduction is $[0, 1.0]$.

In case of $CF(k) \leq T(k)$, the probability that the sampling point is the background noise is large, so the quantum probability of $|1\rangle$ can be used for noise reduction, and the degree of noise reduction is negatively related to the $s(k)$ mode value. The specific expression is as follows:

$$s_-(k) = -sn(k) - \left(\frac{\alpha(k) - \beta(k)}{\sqrt{2}}\right)^2$$

$$= 0.5 - sn(k) - \alpha(k) \times \beta(k)$$ (19)

According to the above equation, the range of $-sn(k)$ reduction is $[0, 0.5]$.  


2.4 Noise reduction steps

In conclusion, the algorithm of the output bearing signal in time domain is obtained by quantum theory HT. Firstly, the sampling signal is quantized, then the probability amplitude of ground state quantization is calculated, and then the correlation function of three adjacent sampling points is obtained. Then, the median filter is used to determine the filter threshold, and the fault signal after noise reduction can be obtained by combining the positive and negative pulse signals after highlighting and noise reduction. The specific algorithm flow is shown in the figure below.

![Flow chart of quantum noise reduction algorithm](image)

3. Experiment and results

In order to verify the feasibility and effectiveness of the algorithm designed in this paper, the actual measured fault signal is taken from the output bearing of the automobile transmission. Set fault 1.2mm×0.8mm×0.4mm scratch on the inner and outer rings of the bearing respectively, and measure the rotation speed 800/r/min and the corresponding frequency 13.33Hz at idle speed. The characteristic frequency of the bearing can be calculated by the following equation [21]:

Characteristic frequency of inner ring fault $f_{ic}$:

$$f_{ic} = \frac{1}{2} N \left(1 + \frac{d}{D \cos \beta}\right) f_r$$  \hspace{1cm} (20)$$

Characteristic frequency of outer ring fault $f_{oc}$:

$$f_{oc} = \frac{1}{2} N \left(1 - \frac{d}{D \cos \beta}\right) f_r$$  \hspace{1cm} (21)$$

In the equations (20) and (21): $N$ is the number of rollers, $d$ is the diameter of roller, $D$ is the diameter of bearing, $\beta$ is the contact angle, and $f_r$ is the frequency conversion. Relevant parameters and failure frequency of bearing are shown in the table below:
Table 1. Rolling bearing model, parameters and fault characteristic frequency

| Bearing model | Bearing name       | Operation mode | Outer diameter | Inner diameter | Number of rollers | Fault characteristic frequency of outer ring | Fault characteristic frequency of inner ring |
|---------------|-------------------|----------------|----------------|----------------|-------------------|---------------------------------------------|---------------------------------------------|
| 6204          | Deep groove ball bearing | Inner fixation | 47mm           | 20mm           | 12                | 64.55Hz                                     | 95.26Hz                                     |

In the experiment, the acceleration sensor is fixed on the bearing seat with magnetic material, and the VIBXpert signal acquisition instrument is used to realize the real-time acquisition of vibration signal, and the data analysis software omnitrend is used for further reasonable analysis. The vibration sampling frequency is 1000Hz and the sampling time is 2 seconds. In order to facilitate the comparison, the same signal is processed by wavelet filtering, mathematical morphological filtering and the quantum filtering proposed in this paper, and the detailed noise reduction effect map of 0-300Hz is intercepted.

![Time domain waveform and spectrum of outer ring](image1)

![Wavelet denoising effect of outer ring fault signal](image2)

![Morphological denoising effect of outer ring fault signal](image3)

![Effect diagram of quantum denoising of outer ring fault signal](image4)

Figure 3. Noise reduction of outer ring fault signal
a. Inner ring vibration signal and spectrum  
b. Wavelet denoising effect of inner ring fault signal  
c. Morphological denoising effect of inner ring fault signal  
d. Quantum denoising result of inner ring fault signal  

Figure 4. Noise reduction of inner ring signal

4. Discussion

By comparing Figure 3 and Figure 4, we can see that:

1. From the spectrum diagram of outer ring in Figure 3 (a), it can be seen that the resonance peak is 300-1300Hz, and the spectrum diagram of inner ring in Figure 4 (a) shows that the resonance peak is 400-1200Hz, and the energy is relatively concentrated in this range, indicating that the vibration frequency of mechanical equipment itself is also in this range, which is caused by resonance;

2. Observe the effect of wavelet denoising and morphological filtering in Figure 3 and Figure 4, and it can be seen that the traditional filtering can get better denoising effect, the fault characteristic frequency and double frequency signal are obvious, and the modulation signal of frequency conversion is also clear;

3. Comparing the effect of wavelet denoising, morphological filtering and quantum filtering in Figure 3 and Figure 4, it can be seen that the amplitude of characteristic frequency signal in quantum filtering is significantly higher than that of the other two kinds of filtering, and the frequency conversion modulation is suppressed, which shows that quantum filtering overcomes the defect of unified processing in traditional filtering.
5. Conclusion
Aiming at the problem of fault signal recognition of automobile gearbox bearing, a set of fault signal recognition method based on quantum theory is established by quantizing the sampling point signal and analyzing the core information of sampling point. The results are as follows:

(1) The mechanical vibration signal is quantized, the kernel information of vibration signal is studied deeply, and the noise reduction is realized according to the autocorrelation of sampling signal. The experimental results prove the feasibility of this method;

(2) The result of quantum noise reduction is better than that of traditional noise reduction. The fault characteristic frequency and double frequency are clear, and the frequency conversion modulation is suppressed. This method not only reduces noise, but also highlights fault signal. The randomness and uniqueness of signal are considered and applied;

(3) This method extends the application scope of quantum theory and realizes independent quantum noise reduction. It is a new noise reduction method.

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