Differential Game Research Based on Industrial Poverty Alleviation in E-Commerce Environment

Lulu Sun*
School of Management, Shanghai University, Shanghai, China

*Corresponding author e-mail: 762040894@qq.com

Abstract. This paper studies the decision-making of poverty alleviation investment in the supply chain system composed of agricultural suppliers and e-commerce sales platforms. Using the method of differential games, the changes of the equilibrium results of supply chain members in two cases are investigated and compared. The research shows that under the Nash non-cooperative game situation, government subsidies can improve the degree of poverty alleviation efforts and the optimal benefits of supply chain members. The government subsidy coefficient $\tau$ only affects the poverty alleviation efforts and total revenue of agricultural suppliers and e-commerce sales platforms, and has no effect on the promotion efforts of e-commerce sales platforms.

1. Introduction
With the rapid development of Internet information, e-commerce poverty alleviation plays an important role in promoting poverty alleviation. In order to enable the poor population and poor areas to enter an all-round well-off society together with the whole country, the state has issued a number of supporting policies to promote the development of rural e-commerce. Various provinces and cities have also issued various relevant policies to subsidize rural e-commerce activities. E-commerce poverty alleviation pays more attention to the long-lasting effect of poverty alleviation work, and makes outstanding contributions to the development of poverty-stricken areas. Therefore, in the current "Internet +" era, exploring e-commerce poverty alleviation can provide a certain policy basis for China to promote poverty alleviation.

In e-commerce poverty alleviation research, Anita kelles-viitanen [1] points out that advanced information technology can help the economy of poor areas achieve leapfrog development. Ping [2] elaborated on the current situation of poverty alleviation in Inner Mongolia, analyzed its problems in promoting e-commerce poverty alleviation, and made recommendations for promoting poverty alleviation in Inner Mongolia. Wen. [3] Constructed an intelligent electronics sales system of agricultural products, and proposed a new e-commerce mode of developing agricultural products. Li et al. [4] adopted the social network analysis method to study the influencing factors of rural e-commerce poverty alleviation, providing theoretical reference for improving the efficiency of rural e-commerce poverty alleviation. Wang [5] pointed out that e-commerce poverty alleviation mode innovation is imperative, and proposed policy Suggestions for the development of e-commerce poverty alleviation in poverty-stricken areas. Chen et al. [6] used game theory to analyze the impact of private enterprises' poverty alleviation, e-commerce sales and government poverty alleviation on rural production enterprises' income and poverty alleviation.
To sum up, the existing researches on e-commerce poverty alleviation are mostly discussed from the macro level, such as the development mode, countermeasures and Suggestions of e-commerce poverty alleviation, and few quantitative analysis is carried out from the mathematical model. In addition, few existing studies have incorporated government subsidies into the analysis model. In view of this, this paper constructs the differential game model of agricultural supplies suppliers and e-commerce sales platform, and puts government subsidies into the analysis framework to analyze the impact of government subsidies on the poverty alleviation strategies and benefits of supply chain members in the e-commerce environment, so as to provide reference and help for farmers to increase income and get out of poverty.

2. Problem description and model hypothesis

2.1. Problem Description

This paper chooses a supply chain consisting of a single agricultural supplier and a single e-commerce sales platform as the research object, and studies the impact of government subsidies on the poverty alleviation strategies and benefits of supply chain members in the e-commerce environment. The agricultural resource suppliers provide farmers with planting services and other poverty alleviation measures, which are acquired after the farmers collect the fruits, and then processed and sold through the e-commerce platform; E-commerce sales platform provides poverty alleviation resources for agricultural suppliers; Meanwhile, e-commerce sales platforms make use of the advantages of the Internet to publicize and promote poverty alleviation products, so as to attract consumers to buy poverty alleviation products and help the poor increase their incomes.

2.2. Model hypothesis

1) This paper assumes that the cost of poverty alleviation efforts is a convex function of the degree of poverty alleviation efforts. The cost of poverty alleviation efforts for agricultural suppliers and e-commerce sales platforms is

\[ C_M(t) = \frac{1}{2} \mu_M E_M^2(t), \quad C_R(t) = \frac{1}{2} \mu_R E_R^2(t). \]

Among them, \( C_M(t) \) and \( C_R(t) \) respectively represent the poverty alleviation effort cost of agricultural suppliers and e-commerce sales platforms; \( \mu_M, \mu_R > 0 \) respectively represent the poverty alleviation cost coefficient of agricultural suppliers and e-commerce sales platforms; \( E_M(t) \) and \( E_R(t) \) respectively represent the poverty alleviation efforts of agricultural suppliers and e-commerce sales platforms.

Meanwhile, we assume that the promotion efforts of e-commerce sales platforms cost as follows

\[ C_P(t) = \frac{1}{2} \mu_P E_P^2(t). \]

\( C_P(t) \) Represents the promotion effort cost of e-commerce sales platform; \( \mu_P > 0 \) represents the promotion cost coefficient of e-commerce sales platform; \( E_P(t) \) represents the promotion efforts of e-commerce sales platform.

2) This paper assumes that the poverty alleviation performance level of enterprises is jointly affected by the poverty alleviation efforts of agricultural supplies suppliers and e-commerce sales platforms; Therefore, using the variant of the Nerlove-Arrow [7] goodwill model to represent the changes in the level of corporate poverty alleviation performance, namely

\[ \dot{G}(t) = \alpha E_M(t) + \beta E_R(t) - \delta G(t). \]
Among them, $G(t)$ represents the level of poverty alleviation performance of the enterprise, and the initial poverty alleviation performance level $G(0) = G_0 \geq 0$; $\alpha, \beta > 0$ respectively indicate the influence coefficient of poverty alleviation efforts of agricultural suppliers and e-commerce sales platforms on poverty alleviation performance levels; $\delta > 0$ is the attenuation rate of poverty alleviation performance.

(3) Since the poverty alleviation efforts of agricultural materials suppliers and e-commerce sales platforms have an indirect impact on revenue, while the promotion efforts of e-commerce sales platforms have a direct impact on them. Therefore, this paper assumes that the total return function is as follows

$$S(t) = \varphi + \eta E_p(t) + (\gamma + \tau)G(t).$$

Among them, $\varphi > 0$ is the potential revenue without poverty alleviation input; $\eta > 0$ means electricity sales platform of promotional effort degree effect on the profit function coefficient; $\gamma > 0$ is the effect coefficient of poverty alleviation performance level on income function; $\tau$ is the government subsidy factor for poverty alleviation input based on the level of poverty alleviation performance, among them, $\tau \in [0,1]$.

(4) Assume that in the infinite time range, both the agricultural suppliers and e-commerce sales platforms have the same positive discount rate $\rho$.

3. Model establishment and solution

Based on the description and hypothesis of the above problems, firstly, non-cooperative Nash game with anarchic subsidies is studied. Then, there's the study of non-cooperative Nash games with government subsidies. On this basis, the equilibrium results of the two parties are compared and analyzed in two cases, and a clear reference can be given to the agricultural supplier or e-commerce sales platform.

3.1. Nash non-cooperative game (no government subsidy)

$$\max_{E_M} \int_0^\infty e^{-\rho t} \left[ \omega (\varphi + \eta E_p + \gamma G) - \frac{1}{2} \mu_M E_M^2 \right] \, dt, $$

$$\max_{E_R} \int_0^\infty e^{-\rho t} \left[ (1 - \omega) (\varphi + \eta E_p + \gamma G) - \frac{1}{2} \mu_R E_R^2 - \frac{1}{2} \mu_p E_p^2 \right] \, dt.$$ 

Proposition 1 Under the Nash non-cooperative game with no government subsidies, the equilibrium strategies of agricultural suppliers and e-commerce sales platforms are

$$E_M^N = \frac{\alpha \omega \varphi}{\mu_M (\rho + \delta)}; \quad E_R^N = \frac{\beta (1 - \omega) \gamma}{\mu_R (\rho + \delta)}; \quad E_p^N = \frac{(1 - \omega) \eta}{\mu_p}.$$

Proof: According to the optimal control theory, for any $G \geq 0, V_M^N(G)$ and $V_R^N(G)$ satisfy the Hamilton-Jacobi-Bellman (HJB) equation, ie

$$\rho V_M^N(G) = \max_{E_M} \left[ \omega (\varphi + \eta E_p + \gamma G) - \frac{1}{2} \mu_M E_M^2 + V_M^N(\alpha E_M + \beta E_R - \delta G) \right], \quad (2)$$

$$\rho V_R^N(G) = \max_{E_R, E_P} \left[ (1 - \omega) (\varphi + \eta E_p + \gamma G) - \frac{1}{2} \mu_R E_R^2 - \frac{1}{2} \mu_p E_p^2 + V_R^N(\alpha E_M + \beta E_R - \delta G) \right]. \quad (3)$$

Take the first partial derivative with respect to $E_M$ on the right side of (2) and set it equal to 0, then

$$E_M = \frac{\alpha V_M^N}{\mu_M}. \quad (4)$$
Take the first partial derivative of (3) with respect to $E_R$ and $E_P$, and set them equal to 0, then

$$E_R = \frac{\mu_R N'}{\mu_R},$$

$$E_P^* = \frac{(1-\omega)\eta}{\mu_P}.\tag{6}$$

Substitute (4), (5) and (6) into (2),

$$\rho V_M^N(G) = (\omega \Psi - \delta V_M^N)G + \omega \varphi + \frac{\omega(1-\omega)\eta^2}{2\mu_P} + \frac{\alpha^2(V_M^N)^2}{\mu_M} + \beta^2 V_M^N V_R^N \frac{V_R^N}{\mu_R}.\tag{7}$$

Substitute (4), (5) and (6) into (3),

$$\rho V_R^N(G) = [(1-\omega)\Psi - \delta V_R^N]G + (1-\omega)\varphi + \frac{(1-\omega)^2\eta^2}{2\mu_P} + \frac{\alpha^2 V_M^N V_R^N}{\mu_M} + \beta^2 (V_R^N)^2 \frac{1}{2\mu_R}.\tag{8}$$

According to the characteristics of the differential equation (7) and (8), so the linear expression of the function $V_M^N(G), V_R^N(G)$ is set for

$$V_M^N(G) = m_1 G + m_2,\tag{9}$$

$$V_R^N(G) = r_1 G + r_2.\tag{10}$$

Among them, $m_1, m_2$ and $r_1, r_2$ are undetermined coefficients.

Take the derivative of (9) and (10) with respect to $G$, then

$$V_M'^N(G) = m_1,\tag{11}$$

$$V_R'^N(G) = r_1.\tag{12}$$

Substitute (9), (10), (11) and (12) into (7) and (8), Comparing the coefficients of the same term at both ends of the equation

$$m_1^* = \frac{\omega \varphi}{\rho + \delta},\tag{13}$$

$$m_2^* = \frac{\omega \varphi}{\rho} + \frac{\omega(1-\omega)\eta^2}{\rho \mu_P} + \frac{\alpha^2 \omega^2 \eta^2}{\rho \mu_M (\rho + \delta)^2} + \frac{\beta^2 \omega (1-\omega)\eta^2}{\rho \mu_P (\rho + \delta)^2},\tag{14}$$

$$r_1^* = \frac{(1-\omega)\varphi}{\rho + \delta},\tag{15}$$

$$r_2^* = \frac{(1-\omega)\varphi}{\rho} + \frac{(1-\omega)^2\eta^2}{2\rho \mu_P} + \frac{\alpha^2 \omega (1-\omega)\eta^2}{\rho \mu_M (\rho + \delta)^2} + \frac{\beta^2 (1-\omega)^2 \eta^2}{2 \rho \mu_P (\rho + \delta)^2}.\tag{16}$$

By substituting (11) and (13) into (4), the optimal poverty alleviation efforts of agricultural supplies suppliers can be obtained in this case, ie

$$E_M^N = \frac{\alpha \omega \Psi}{\mu_M (\rho + \delta)}.\tag{17}$$
By substituting (12) and (15) into (5), the optimal poverty alleviation efforts of e-commerce sales platform can be obtained in this case, ie

$$E^*_R = \frac{\beta(1-\omega)\gamma}{\mu_R(\rho+\delta)}.$$  \hspace{5cm} (18)

Proposition 1 is established according to (17) and (18).

**Proposition 2** Under the Nash non-cooperative game situation with no government subsidies, the long-term optimal value functions of agricultural suppliers and e-commerce sales platforms are

$$V^*_M(G) = \frac{\omega\gamma}{\rho+\delta}G + \frac{\omega(1-\omega)\eta^2}{\rho\mu_p} + \frac{\alpha^2\omega^2\gamma^2}{2\rho\mu_M(\rho+\delta)^2} + \frac{\beta^2\omega(1-\omega)\gamma^2}{\rho\mu_R(\rho+\delta)^2},$$

$$V^*_R(G) = \frac{(1-\omega)\gamma}{\rho+\delta}G + \frac{(1-\omega)\varphi}{\rho} + \frac{(1-\omega)^2\eta^2}{2\rho\mu_p} + \frac{\alpha^2\omega(1-\omega)\gamma^2}{\mu_M(\rho+\delta)^2} + \frac{\beta^2(1-\omega)^2\gamma^2}{2\rho\mu_R(\rho+\delta)^2}.$$  \hspace{5cm} (21)

Proof: Substituting (13) and (14) into (9), the long-term optimal value function of the agricultural suppliers in this case can be obtained, ie

$$V^*_M(G) = \frac{\omega\gamma}{\rho+\delta}G + \frac{\omega(1-\omega)\eta^2}{\rho\mu_p} + \frac{\alpha^2\omega^2\gamma^2}{2\rho\mu_M(\rho+\delta)^2} + \frac{\beta^2\omega(1-\omega)\gamma^2}{\rho\mu_R(\rho+\delta)^2}.$$  \hspace{5cm} (19)

Substituting (15) and (16) into (10), the long-term optimal value function of the e-commerce sales platforms in this case can be obtained, ie

$$V^*_R(G) = \frac{(1-\omega)\gamma}{\rho+\delta}G + \frac{(1-\omega)\varphi}{\rho} + \frac{(1-\omega)^2\eta^2}{2\rho\mu_p} + \frac{\alpha^2\omega(1-\omega)\gamma^2}{\mu_M(\rho+\delta)^2} + \frac{\beta^2(1-\omega)^2\gamma^2}{2\rho\mu_R(\rho+\delta)^2}.$$  \hspace{5cm} (20)

Proposition 2 is established according to (19) and (20).

**Proposition 3** Under the Nash non-cooperative game situation without government subsidies, the optimal trajectory of poverty alleviation performance level is

$$G^*(t) = (G_0 - G^N_\infty) e^{-\delta t} + G^N_\infty.$$  \hspace{5cm} (22)

Among them, $G^N_\infty = \frac{\alpha^2\omega\gamma}{\delta\mu_M(\rho+\delta)} + \frac{\beta^2(1-\omega)\gamma}{\delta\mu_R(\rho+\delta)}$.

Proof: Substitute equilibrium strategies (17) and (18) into equation of state (1), then

$$\dot{G}(t) = \frac{\alpha^2\omega\gamma}{\mu_M(\rho+\delta)} + \frac{\beta^2(1-\omega)\gamma}{\mu_R(\rho+\delta)} - \delta G(t).$$  \hspace{5cm} (21)

Solving the differential equation (21) can obtain the optimal trajectory of poverty alleviation performance level under the non-government subsidy Nash non-cooperative game situation, ie

$$G^*(t) = (G_0 - G^N_\infty) e^{-\delta t} + G^N_\infty.$$  \hspace{5cm} (22)

Among them, $G^N_\infty = \frac{\alpha^2\omega\gamma}{\delta\mu_M(\rho+\delta)} + \frac{\beta^2(1-\omega)\gamma}{\delta\mu_R(\rho+\delta)}$.

Proposition 3 is established according to (22).
3.2. Nash non-cooperative game (with government subsidies)
Under this circumstance, in order to encourage enterprises to participate in poverty alleviation work, local governments will subsidize enterprises that participate in poverty alleviation and development and make outstanding contributions through various forms, namely \(0 < \tau \leq 1\).

Using superscript \(G\) to represent Nash non-cooperative game with government subsidies, the objective functions of agricultural suppliers and e-commerce sale platforms are respectively

\[
\max_{E^G_M} \int_0^\infty e^{-\rho t} \left[ \omega (\varphi + \eta E_p + (\gamma + \tau) G) - \frac{1}{2} \mu_M E^2_M \right] dt,
\]

\[
\max_{E^G_R} \int_0^\infty e^{-\rho t} \left[ (1 - \omega) (\varphi + \eta E_p + (\gamma + \tau) G) - \frac{1}{2} \mu_R E^2_R - \frac{1}{2} \mu_p E^2_p \right] dt.
\]

**Proposition 4** Under the Nash non-cooperative game with government subsidies, the equilibrium strategies of agricultural suppliers and e-commerce sales platforms are

\[
E^*_M = \frac{\alpha \omega (\gamma + \tau)}{\mu_M (\rho + \delta)}, \quad E^*_R = \frac{\beta (1 - \omega) (\gamma + \tau)}{\mu_R (\rho + \delta)}, \quad E^*_P = \frac{(1 - \omega) \eta}{\mu_p}.
\]

**Proposition 5** Under the Nash non-cooperative game situation with no government subsidies, the long-term optimal value functions of agricultural suppliers and e-commerce sales platforms are

\[
V^*_M (G) = \frac{\omega (\gamma + \tau)}{\rho + \delta} G + \frac{\omega (1 - \omega) \eta^2}{\rho \mu_p} + \frac{\alpha^2 \omega^2 (\gamma + \tau)^2}{2 \rho \mu_M (\rho + \delta)^2} + \frac{\beta^2 \omega (1 - \omega) (\gamma + \tau)^2}{\rho \mu_p (\rho + \delta)^2},
\]

\[
V^*_R (G) = \frac{(1 - \omega) (\gamma + \tau)}{\rho + \delta} G + \frac{(1 - \omega) \varphi}{\rho} + \frac{(1 - \omega)^2 \eta^2}{2 \rho \mu_p} + \frac{\alpha^2 \omega (1 - \omega) (\gamma + \tau)^2}{\rho \mu_M (\rho + \delta)^2} + \frac{\beta^2 (1 - \omega) (\gamma + \tau)^2}{2 \rho \mu_p (\rho + \delta)^2}.
\]

**Proposition 6** Under the Nash non-cooperative game situation with government subsidies, the optimal trajectory of poverty alleviation performance level is

\[
G^* (t) = (G_0 - G^*_M) e^{-\delta t} + G^*_M.
\]

Among them, \(G^*_M = \frac{\alpha^2 \omega (\gamma + \tau)}{\delta \mu_M (\rho + \delta)} + \frac{\beta^2 (1 - \omega) (\gamma + \tau)}{\delta \mu_p (\rho + \delta)}.\)

The proof process is similar to the proof process of Propositions 1, 2, and 3, so it is omitted.

4. Comparative analysis
Inference 1 can be obtained from Proposition 1, Proposition 2, Proposition 4, and Proposition 5.

When \(0 < \omega < \frac{2}{3}\), \(E^*_M < E^*_M, V^*_M, V^*_R < V^*_R, E^*_P = E^*_P\):

When \(0 < \omega < \frac{2}{3}\), \(V^*_M, V^*_R < V^*_R, V^*_M < V^*_M + V^*_R + V^*_R, V^*_R < V^*_M + V^*_R + V^*_R\).

Under the Nash non-cooperative game, government subsidies can effectively improve the poverty alleviation efforts of supply chain members. This is because the subsidies provided by local governments to poverty alleviation enterprises are mainly based on the poverty alleviation efforts of enterprises, which can better encourage enterprises to increase investment in poverty alleviation.
Under the Nash non-cooperative game, government subsidies can effectively improve the optimal income of supply chain members. This is because the government subsidy increases the enthusiasm of enterprises to increase the investment in poverty alleviation, and urges enterprises to make greater efforts to invest in poverty alleviation to improve the performance level of poverty alleviation. Thus, the overall benefit of the system is increased, and Pareto improvement can be achieved for both sides.

The government subsidy coefficient $\tau$ only affects the poverty alleviation efforts and total revenue of agricultural suppliers and e-commerce sales platforms, and has no effect on the promotion efforts of e-commerce sales platforms.

5. Conclusion

In this paper, the supply chain poverty alleviation system composed of agricultural supplies suppliers and e-commerce sales platforms is taken as the research object. By applying the differential game theory, two differential game dynamic models are established to study the influence of government subsidies on poverty alleviation strategies and benefits of supply chain members in the e-commerce environment. By comparing the equilibrium results in different situations, the main conclusions are as follows:

Under the Nash non-cooperative game situation, government subsidies can improve the degree of poverty alleviation efforts and the optimal benefits of supply chain members. The government subsidy coefficient $\tau$ only affects the poverty alleviation efforts and total revenue of agricultural suppliers and e-commerce sales platforms, and has no effect on the promotion efforts of e-commerce sales platforms.

In order to facilitate the solution, this paper assumes that the effect of poverty alleviation investment of enterprises is timely, but the effect of poverty alleviation investment in practice often has a certain lag, so the problem of poverty alleviation strategy of supply chain members under the influence of lag effect can be considered.

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