Confinement and anisotropy of ultrahigh-energy cosmic rays in isotropic plasma wave turbulence

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Abstract. The mean free path and anisotropy of galactic cosmic rays is calculated in weak plasma wave turbulence that is isotropically distributed with respect to the ordered uniform magnetic field. The modifications on the value of the Hillas energy, above which cosmic rays are not confined to the Galaxy, are calculated too. The original determination of the Hillas limit has been based on the case of slab turbulence where only parallel propagating plasma waves are allowed. We use quasilinear cosmic ray Fokker-Planck coefficients to calculate the mean free path and the anisotropy in isotropic plasma wave turbulence. In isotropic plasma wave turbulence the Hillas limit is enhanced by about four orders of magnitude to $E_c = 2.03 \cdot 10^5 A_n^{1/2} (L_{\text{max}}/10 \text{ pc})$ PeV resulting from the dominating influence of transit-time damping interactions of cosmic rays with obliquely propagating magnetosonic waves. Below the energy $E_c$ the cosmic ray mean free path and the anisotropy exhibit the well known $E^{1/3}$ energy dependence. At energies higher than $E_c$ both transport parameters steepen to a $E^3$-dependence. This implies that cosmic rays even with ultrahigh energies of several hundreds of EeV can be rapidly pitch-angle scattered by interstellar plasma turbulence, and are thus confined to the Galaxy.

1. Introduction

To unravel the nature of cosmic sources that accelerate cosmic rays to ultrahigh energies has been identified as one of the eleven fundamental science questions for the new century [1]. The most interesting are the cosmic rays with energies as large as $10^{20.5}$ eV. It has been argued that, due to the amplification of the magnetic field in the shock, the acceleration of cosmic rays in young supernova remnants is possible up to $\sim 10^{18}$ eV. This implies that such particles may have a Galactic origin. For ultrahigh-energy ($10^{18} - 10^{20.5}$ eV) cosmic rays an extragalactic origin is favored by many researchers. Extragalactic ultrahigh-energy cosmic rays (UHECRs) coming from cosmological distances $\geq 50$ Mpc should interact with the universal cosmic microwave background radiation (CMBR) and produce pions. For an extragalactic origin of UHECRs the detection or non-direction of the Greisen-Kuzmin-Zatsepin cutoff [2] resulting from the photopion attenuation in the CMBR will have far-reaching consequences not only for astrophysics but also for fundamental particle physics as e.g. the breakup of Lorentz symmetry [3] or the non-commutative quantum picture of spacetime [4].

For many years the theoretical development of the resonant wave-particle interactions has mainly concentrated on the special case that the plasma waves propagate only parallel or antiparallel to the ordered magnetic field – the so-called slab turbulence. In this case only cosmic ray particles with gyroradii $R_L$ smaller than the longest parallel wavelength $L_{\parallel,\text{max}}$ of
the plasma waves can resonantly interact. Obviously this condition is equivalent to a limit on the maximum particle rigidity $R$:

$$R = \frac{p}{Z} \leq eB_0L_{\parallel,max}.$$  

(1)

An alternative way to express the condition (1) is

$$E_{15}/Z \leq 40 \cdot \left( \frac{B_0}{4 \mu G} \right) \left( L_{\parallel,max}/10 \text{pc} \right),$$  

(2)

where $E_{15}$ denotes the cosmic ray particle energy in units of $10^{15}$ eV. The limit set by the right hand side of Eq. (2) is referred to as Hillas limit [5]. According to this limit, cosmic ray protons of energies larger than 40 PeV $= 4 \cdot 10^{16}$ eV cannot be confined or accelerated in the Milky Way, and an extragalactic origin for this cosmic ray component has to be invoked. Moreover, as the cosmic ray mean free path in case of spatial gradients is closely related to the cosmic ray anisotropy ([6], Eq. (94)), the Hillas limit (2) implies strong anisotropies at energies above 40 PeV which have not been observed by the KASKADE experiment [7].

It is the purpose of this work to investigate how the Hillas limit (2) is affected if we discard the assumption of purely slab plasma waves, i.e. if we allow for oblique propagation angles $\theta$ of the plasma waves with respect to the ordered magnetic field component. In order to estimate confinement and relevant transport parameters we have to investigate also influence of strong coupling of cosmic ray particles with damped plasma waves. When discussing the nature of interstellar turbulence, it is necessary to consider the fact that the interstellar medium contains a number of plasmas of very diverse characteristics, not only cold plasma. In this work, we include the temperature effects to the first order [8], in deriving the relevant Fokker-Planck coefficients.

2. Relevant magnetohydrodynamic plasma modes without damping

Most cosmic plasmas have a small value of the plasma beta $\beta_p = c_S^2/V_A^2$, which is defined by the ratio of the ion sound $c_S$ to Alfvén speed $V_A$, and thus indicates the ratio of thermal to magnetic pressure. For low-beta plasmas the two relevant magnetohydrodynamic wave modes are the

(1) incompressional shear Alfvén waves

(2) the fast magnetosonic waves

2.1. Resonant interactions of shear Alfvén waves

For shear Alfvén waves only interactions with $n \neq 0$ are possible. These are referred to as gyroresonances and corresponding the resonance parallel wavenumber is

$$k_{\parallel,A} = \frac{n\Omega_c}{\pm V_A - v_\mu},$$  

(3)

which apart from very small values of $|\mu| \leq V_A/v$ typically equals the inverse of the cosmic ray particle’s gyroradius, $k_{\parallel,A} \simeq n/R_L$ and higher harmonics.

2.2. Resonant interactions of fast magnetosonic waves

In contrast, for fast magnetosonic waves the $n = 0$ resonance is possible for oblique propagation due its compressive magnetic field component. The $n = 0$ interactions are referred to as transit-time damping, hereafter TTD

$$v_\mu = \pm V_A/\cos \theta$$  

(4)
as necessary condition which is independent from the wavenumber value \( k \). Apparently all super-Alfvenic \((v \geq V_A)\) cosmic ray particles are subject to TTD provided their parallel velocity \( v \mu \) equals at least the wave speeds \( \pm V_A \). Hence equation (4) is equivalent to the two conditions

\[
|\mu| \geq V_A/v, \quad v \geq V_A.
\]  

(5)

Additionally, fast mode waves also allow gyroresonances \((n \neq 0)\) at wavenumbers

\[
k_F = \frac{n\Omega_c}{\pm V_A - v \mu \cos \theta}.
\]  

(6)

2.3. Implications for cosmic ray transport

The simple considerations of the last two subsections allow us the following immediate conclusions:

(1) The momentum diffusion coefficient

\[
A_M = \frac{1}{2} \int_{-1}^{1} d\mu [D_{pp}(\mu) - \frac{D_{pp}^2(\mu)}{D_{\mu\mu}(\mu)}] = A_T + A_2
\]  

has contributions both from transit-time damping of fast mode waves,

\[
A_T \simeq \int_{V_A/v}^{1} d\mu D_{pp}^{TTD}(\mu),
\]  

(8)

and from second-order Fermi gyroresonant acceleration by shear Alfven waves \cite{6}

\[
A_2 = \frac{1}{2} \int_{-1}^{1} d\mu [D_{pp}^A(\mu) - \frac{[D_{pp}^A(\mu)]^2}{D_{\mu\mu}^A(\mu)}].
\]  

(9)

(2) On the other hand, the spatial diffusion coefficient

\[
\kappa = \frac{v^2}{8} \int_{-1}^{1} d\mu (1 - \mu^2)^2 D_{\mu\mu}^{-1}(\mu)
\]  

is given by the integral over the inverse of the Fokker–Planck coefficient \( D_{\mu\mu} \), so here the small values of \( D_{\mu\mu} \) due to gyroresonant interactions in the interval \(|\mu| < V_A/v\) determine the spatial diffusion coefficient and the corresponding parallel mean free path

\[
\kappa = v \lambda/3 \simeq \frac{v^2}{8} \int_{-V_A/v}^{V_A/v} \frac{d\mu}{D_{\mu\mu}^G(\mu)}.
\]  

(11)

The gyroresonances can be due to shear Alfven waves or fast magnetosonic waves. For relativistic cosmic rays the relevant range of pitch angle cosines \(|\mu| \leq v_A/v\) is very small allowing us the approximation \( D_{\mu\mu}^G(\mu) \simeq D_{\mu\mu}^G(0) \) so that

\[
\kappa = v \lambda/3 \simeq \frac{v^2}{4} \frac{\epsilon}{D_{\mu\mu}^G(0)} = \frac{v V_A}{4D_{\mu\mu}^G(0)}.
\]  

(12)

(3) According to Eq. (90) of Schlickeiser \cite{6} the streaming cosmic ray anisotropy due to spatial gradients in the cosmic ray density is given by

\[
\delta = \frac{F_{\text{max}} - F_{\text{min}}}{F_{\text{max}} + F_{\text{min}}} = \frac{1}{2F} \frac{v \partial F}{4 \varnothing} \int_{-1}^{1} d\mu (1 - \mu^2) D_{\mu\mu}^{-1}(\mu),
\]  

(13)
which also is determined by the smallest value of $D_{\mu \mu}$ around $\mu = 0$. Approximating again $D_{\mu \mu}(\mu) \simeq D_{\mu \mu}^G(0)$ for $|\mu| \leq \epsilon = V_A/v$ we derive the direct proportionality of the cosmic ray anisotropy with the parallel mean free path, i.e.

$$D \simeq \frac{v}{8} \frac{\partial F}{\partial \ln z} = \frac{V_A}{4} \frac{1}{D_{\mu \mu}^G(0)} \frac{\partial F}{\partial \ln z} = \frac{1}{3} \frac{\partial F}{\partial \ln z}. \quad (14)$$

Introducing the characteristic spatial gradient of the cosmic ray density $<z> \equiv \left( 1/F \right) |\partial F/\partial z|$, Eq. (14) reads

$$\delta \simeq \frac{\lambda}{3} < z >. \quad (15)$$

Cosmic ray gradients derived from diffuse galactic GeV gamma-ray emissivities suggest a value of $<z> \simeq 2 \text{ kpc}$.

2.4. Quasilinear cosmic ray mean free path and anisotropy isotropic plasma wave turbulence

Throughout this work we consider isotropic linearly polarized magnetohydrodynamic turbulence, and we adopt a Kolmogorov-like power law dependence (index $q > 1$) together with vanishing cross helicity of each plasma mode. According to [9] at particle pitch-angles outside the interval $|\mu| > \epsilon$ transit-time damping provides the dominant and overwhelming contribution to these Fokker-Planck coefficients. This justifies the approximations to derive Eqs. (12) and (14) for the cosmic ray mean free path and anisotropy, respectively. Both transport parameters are primarily fixed by the small but finite scattering due to gyroresonant interactions in the interval $|\mu| < \epsilon$. In the following, we consider both transport coefficients for positively charged cosmic ray particles with $\Omega > 0$ especially in the limit $k_{\min}R_L >> 1$.

2.4.1. Gyroresonant Fokker-Planck coefficients at $\mu = 0$

Comparing the Fokker-Planck coefficients from fast mode waves and Alfven waves we have found that the latter one is always smaller by the small ratio $\epsilon = V_A/v$ than the first one:

$$D_{\mu \mu}^A(\mu = 0) \simeq \epsilon D_{\mu \mu}^F(\mu = 0), \quad (16)$$

so that the gyroresonant contribution from Alfven waves can be neglected in comparison to the gyroresonant contribution from fast mode waves.

2.4.2. Cosmic ray mean free path

Neglecting $D_{\mu \mu}^A(\mu = 0)$ we obtain for the cosmic ray mean free path

$$\lambda(\gamma) \simeq \frac{3v\epsilon}{4D_{\mu \mu}^F(\mu = 0)} = \frac{1}{\pi(q-1)\left(\delta B\right)^2} \frac{B_0^2}{R_L(k_{\min}R_L\epsilon)^{1-q}} \sum_{n=1}^{\infty} n^{-(q+1)} H\left[n - \epsilon R_L k_{\min}\right], \quad (17)$$

which exhibits the familiar Lorentz factor dependence $\propto \beta \gamma^{2-q} \simeq \gamma^{2-q}$ at Lorentz factors $\gamma \leq \gamma_c$ below a critical Lorentz factor defined by

$$\gamma_c = k_c/k_{\min} \quad (18)$$

with $k_c = \Omega_{0,p}/V_A = \omega_{p,i}/c$ being the inverse ion skin length. The Lorentz factor dependence $\lambda \propto \gamma^{2-q}$ especially holds at rigidities $1 \leq k_{\min}R_L \leq \epsilon = c/V_A$, in a rigidity range where the slab turbulence model would predict an infinitely large mean free path.
Expressing $k_{\text{min}} = 2\pi/L_{\text{max}}$ in terms of the longest wavelength of isotropic fast mode waves $L_{\text{max}} = 10$ pc yields

$$\gamma_c = \frac{\omega_{p,i} L_{\text{max}}}{2\pi c} = 2.16 \times 10^{11} n_e^{1/2}(\frac{L_{\text{max}}}{10 \text{ pc}}).$$

(19)

The corresponding cosmic ray hadron energy is

$$E_c = A\gamma_c m_p c^2 = 2.03 \times 10^5 A n_e^{1/2}(\frac{L_{\text{max}}}{10 \text{ pc}}) \text{ PeV},$$

(20)

which is four orders of magnitude larger than the Hillas limit (2) for equal values of the maximum wavelength. This difference demonstrates the dramatic influence of the plasma turbulence geometry (slab versus isotropically distributed waves) on the confinement of cosmic rays in the Galaxy. With isotropically distributed fast mode waves, even ultrahigh energy cosmic rays obey the scaling $\lambda \gamma^{-2} = \text{const.}$

Only, at ultrahigh Lorentz factors $\gamma > \gamma_c$ or energies $E > E_c$ the mean free path approaches the much steeper dependence

$$\lambda(\gamma > \gamma_c) \simeq \frac{1}{\pi(q-1)} \frac{B_0^2}{(\delta B)^2} R_L(k_{\text{min}} R_L) \propto \beta \gamma^3 \simeq \gamma^3.$$  

(21)

independent from the turbulence spectral index $q$. Here the mean free path quickly attains very large values greater than the typical scales of the Galaxy.

2.4.3. Anisotropy  Because of the direct proportionality between mean free path and anisotropy, the cosmic ray anisotropy (15) shows the same behaviour as a function of energy.

3. Relevant magnetohydrodynamic plasma modes for damped waves

It has been pointed out by Schlickeiser and Miller [9] that this transit-time damping (TTD) contribution provides the overwhelming contribution to particle scattering because in this interaction the cosmic ray particle interacts with the whole wave spectrum, in contrast to gyroresonances that singles out individual resonant wave numbers. The inclusion of resonance broadening due to wave damping in the resonance function guarantees that this dominance also holds for cosmic ray particles at small pitch angle cosines $\mu \leq |V_a/v|$, unlike the case of negligible wave damping [10].

The damping of magnetosonic waves is caused both by collisionless Landau damping and collisional viscous damping, Joule damping and ion-neutral friction. The dominating contribution is provided by viscous damping with the rate calculated for plasma parameters of the diffuse intercloud medium at very small propagation angles

$$\gamma_F \simeq 2.9 \times 10^5 \beta V_A k^2 \sin^2 \theta.$$  

(22)

For the ratio of damping rate to real frequency we obtain

$$\frac{\gamma_F}{|\omega_R|} \simeq 2.9 \times 10^5 \beta V_A k \sin^2 \theta = 2.9 \times 10^5 \beta \Omega_{0,p} \sin^2 \theta.$$  

(23)

Because in the interstellar medium $\Omega_{0,p} = 3.6 \times 10^{-2} B(4\mu G)$ Hz, we see that in the far MHD-wave region $\omega_R \leq 10^{-4}\Omega_{0,p}/\beta$ the weak damping limit is fulfilled.
3.1. Fast magnetosonic plasma modes

In the MHD wave range that interests us here, the dispersion relation reads as

\[ \omega_R \simeq jkV_A \]  

(24)

describing forward \((j = 1)\) and backward \((j = -1)\) moving fast mode waves.

The mean free path:

\[ \lambda^0_F(T \gg 1) = \frac{15V_A\alpha}{8} \frac{q}{q - 1} \left( \frac{B_0}{\delta B} \right)^2 10^{14} T^3, \]

(25)

where \(T = k_{\text{min}}R_L = E\) and is normalized with respect to \(E_c\) (where \(E_c\) is defined as for undamped case (eq. 20)). At relativistic rigidities we find that \(\lambda^0 \sim T^3\).

\[ \lambda^F_0(T << 1) = 36V_A\alpha \left( \frac{B_0}{\delta B} \right)^2. \]

(26)

In this energy limit the mean free path is constant with respect to \(T\).

3.2. Slow magnetosonic plasma modes

Dispersion relation for slow magnetosonic waves in low-\(\beta\) plasma reads \[8\]

\[ \omega^2_R \simeq k^2 V_A^2 \left( \frac{\eta\beta}{1 + \beta} + \frac{\eta^2\beta^2}{(1 + \beta)^3} \right) \]

(27)

with \(\eta = \cos \theta\) and \(\beta\) is the ratio of thermal and magnetic pressure. In the last equation, in the first approximation, we neglect the second term in brackets since it is one order smaller than the first term.

The mean free path for slow mode waves:

\[ \lambda^0_S(T \gg 1) = \frac{3V_A\alpha}{8\sqrt{2}} \frac{q}{q - 1} \left( \frac{B_0}{\delta B} \right)^2 \pi \frac{1}{h} 10^{13} T^3, \]

(28)

where \(T = k_{\text{min}}R_L, h = 2\arctanh\sqrt{1 - \delta - 2\sqrt{1 - \delta}}\), with \(\delta << 1\).

At relativistic rigidities we find that \(\lambda^0 \sim T^3\).

\[ \lambda^S_0(T << 1) = 36V_A\alpha \left( \frac{B_0}{\delta B} \right)^2. \]

(29)

In this energy limit the mean free path is constant with respect to \(T\).

3.2.1. Anisotropy

According to eq. (20, 21, 22) of VS, we can calculate anisotropy for damped waves using eq. (26) and eq. (29) (fast and slow, respectively). Anisotropy dependence on energy is the same as for mean free path.

4. Summary

We have investigated the implications of isotropically distributed interstellar magnetohydrodynamic plasma waves on the scattering mean free path and the spatial anisotropy of high-energy cosmic rays. We demonstrate a drastic modification of the energy dependence of both cosmic ray transport parameters compared to previous calculations that have assumed that the plasma waves propagate only parallel or antiparallel to the ordered magnetic field (slab turbulence). In case of slab turbulence cosmic rays with Larmor radius \(R_L\) resonantly interact with plasma waves with wave vectors at \(k_{\text{res}} = R_L^{-1}\). If the slab wave turbulence power spectrum vanishes for wavenumbers less than \(k_{\text{min}}\), as a consequence then cosmic rays with Larmor radii larger than \(k_{\text{min}}^{-1}\) cannot be scattered in pitch-angle, causing the so-called Hillas limit for the maximum
energy $E_{15}^H = 4Z \cdot (B_0/4\mu G)(L_{\parallel,\text{max}}/\text{parsec})$ of cosmic rays being confined in the Galaxy. At about these energies this would imply a drastic increase in the spatial anisotropy of cosmic rays that has not been detected by KASKADE and other air shower experiments.

In case of isotropically distributed interstellar magnetohydrodynamic waves we demonstrated that the Hillas energy $E^H$ is modified to a limiting total energy that is about 4 orders of magnitude larger $E_c = 2.03 \cdot 10^4 A^{1/2} n^{1/2} (L_{\text{max}}/1 \text{ pc})$ PeV, where $A$ denotes the mass number and $L_{\text{max}}$ the maximum wavenumber of isotropic plasma waves. Below this energy the cosmic ray mean free path and the anisotropy exhibit the well known $E^{-q}$ energy dependence, where $q = 5/3$ denotes the spectral index of the Kolmogorov spectrum. At energies higher than $E_c$ both transport parameters steepen to a $E^3$-dependence. This implies that cosmic rays even with ultrahigh energies of several tens of EeV can be rapidly pitch-angle scattered by interstellar plasma turbulence, and are thus confined to the Galaxy.

Considering damped fast and slow mode waves caused by dominate viscous damping, we have determined energy dependence of the mean free path. We have found that for small energies it is approximately constant and for high energies is proportional to the third power of energy of the particle. The analysis for the influence of different types of damping, as well as the influence of different geometries of turbulence, will be the subject of further research.

5. References

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