Rapid cooling of magnetized neutron stars

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Abstract

The neutrino emissivities resulting from direct URCA processes in neutron stars are calculated in a relativistic Dirac-Hartree approach in presence of a magnetic field. In a quark or a hyperon matter environment, the emissivity due to nucleon direct URCA processes is suppressed relative to that from pure nuclear matter. In all the cases studied, the magnetic field enhances emissivity compared to the field-free cases.

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Neutron stars are born in the aftermath of supernova explosions with interior temperatures $T \gtrsim 10^{11}$ K, but cool rapidly in a few seconds by predominant neutrino emission to $T < 10^{10}$ K. Neutrino cooling then dominates and lasts for $t \sim 10^5 - 10^6$ yr and subsequently photon emission takes over when $T \gtrsim 10^8$ K. Since the long term cooling of the young neutron stars ($T \sim 10^8 - 10^{10}$ K) proceeds via emission of neutrinos primarily from matter at supranuclear densities within the core, the study of the cooling of neutron stars by examination of neutrino emissivities may provide considerable insight into their interior structure and composition.

For a long time the dominant neutrino cooling mechanism has been the so-called standard model based on the modified URCA processes
\begin{align}
(n, p) + n \rightarrow (n, p) + p + e^- + \bar{\nu}_e,
(n, p) + p + e^- \rightarrow (n, p) + n + \nu_e.
\end{align}

The ROSAT detection of thermal emission from neutron stars indicates the necessity of faster cooling mechanism in some young neutron stars, in particular the Vela pulsar. Faster neutrino emission than the standard model was proposed by invoking pion or kaon condensates which have neutrino emissivities comparable to that from the $\beta$-decay of quarks in quark matter (consisting of $u$, $d$ and $s$ quarks)
\begin{align}
d \rightarrow u + e^- + \bar{\nu}_e, \quad u + e^- \rightarrow d + \nu_e.
\end{align}

A similar relation for $s$-quark $\beta$-decay may occur and is obtained by replacing $d$ by $s$ quark in Eq. (2). The most powerful energy losses, expected to date, are produced by the so-called direct URCA mechanism involving nucleons
\begin{align}
n \rightarrow p + e^- + \bar{\nu}_e, \quad p + e^- \rightarrow n + \nu_e.
\end{align}

The threshold density of this process is however considerably larger than that of the modified URCA process.

Observations of pulsars predict large surface magnetic field of $B_m \sim 10^{14}$ G. In the core the field may be considerably amplified due to flux conservation from the original weak field of the progenitor during its core collapse. In fact, the scalar virial theorem predicts large interior field $B_m \sim 10^{18}$ G or more, and these fields are frozen in the highly conducting core. It has been demonstrated that when the field $B_m$ is comparable to or above a critical field $B^{(c)}_m$, the energy of a charged particle changes significantly in the quantum limit; the quantum effects are most pronounced when the particle moves in the lowest Landau level. The phase space modifications stemming from the strong magnetic field in the core are expected to influence the neutrino emission rate from young neutron stars.

In this communication we evaluate the neutrino emissivity for the nucleon direct URCA process of Eq. (3) in presence of a magnetic field $B_m$, and demonstrate that it would lead to more rapid cooling in the core. (A straightforward extension of the emissivity for nucleons into the quark sector may also be obtained in a magnetic field.) For this purpose, we consider a $npe$ matter in $\beta$-equilibrium within a relativistic Dirac-Hartree approach in the linear $\sigma$-$\omega$-$\rho$ model.
At the neutron star core at temperatures well below the typical Fermi temperature of \( T_F \sim 10^{12} \) K, the nucleons and electrons participating in neutrino producing processes are all degenerate (the \( \nu_e \) and \( \nu_e \) are free) and have their momenta close to the Fermi momenta \( p_{F_i} \), where \( i = n, p, e \). Since neutrino and antineutrino momenta are \( \sim kT/c \ll p_{F_i} \), the nucleon direct URCA process is allowed by the momentum conservation when \( p_{F_p} + p_{F_e} \geq p_{F_n} \). Since matter is very close to \( \beta \)-equilibrium, the chemical potentials of the constituents satisfy the condition \( \mu_n = \mu_p + \mu_e \). (Henceforth we set \( \hbar = c = k = 1 \).

Employing the Weinberg-Salam theory for weak interactions, the interaction Lagrangian density for the charged current reaction (3) may be expressed as \( \mathcal{L}_{\text{int}} = (G_F/\sqrt{2}) \cos \theta_c \bar{\nu} \gamma_\mu \nu \), where \( G_F \simeq 1.435 \times 10^{-49} \) erg cm\(^{-3}\) is the Fermi weak coupling constant and \( \theta_c \) the Cabibbo angle. The lepton and nucleon charged weak currents are respectively, \( l_\mu = \bar{\psi}_3 \gamma_\mu (1 - \gamma_5) \psi_2 \) and \( j_W^\mu = \bar{\psi}_3 \gamma_\mu (g_\nu - g_A \gamma_5) \psi_1 \). Here, and in other formulae to follow, the indices \( i = 1 - 4 \) refer to the \( n, \nu_e, p \) and \( e \), respectively. The vector and axial-vector coupling constants are \( g_\nu = 1 \) and \( g_A = 1.226 \).

The emissivity due to the antineutrino emission process in presence of a uniform magnetic field \( B_m \) along z-axis when both the electrons and protons are Landau quantized is given by

\[
\varepsilon_{\bar{\nu}}(B_m) = 2 \int \frac{V d^3 p_1}{(2\pi)^3} \int \frac{V d^3 p_2}{(2\pi)^3} \int^{q_{B_m} L_z/2}_{-q_{B_m} L_z/2} \frac{L_y dp_{3y}}{2\pi} \int^{p_{F_p}}_{-p_{F_p}} \frac{L_z dp_{3z}}{2\pi} \int^{q_{B_m} L_z/2}_{-q_{B_m} L_z/2} \frac{L_y dp_{4y}}{2\pi} \int^{p_{F_e}}_{-p_{F_e}} \frac{L_z dp_{4z}}{2\pi} \times \sum_{\eta = 0}^{\eta_{\text{max}}} \sum_{\eta' = 0}^{\eta'_{\text{max}}} E_2 W_{fi} f(p_1) [1 - f(p_3)] [1 - f(p_4)],
\]

where \( \eta_{\text{max}} \) and \( \eta'_{\text{max}} \) are respectively the maximum number of Landau levels populated for protons and electrons. The prefactor 2 takes into account the neutron spin degeneracy. The \( p_i \equiv (E_i, p_i) \) are the 4-momenta and \( E_2 \) the antineutrino energy. The functions \( f(E_i) \) denote the Fermi-Dirac functions for the \( i \)th particle. The transition rate per unit volume due to the antineutrino emission process may be derived from Fermi’s golden rule and is given by \( W_{fi} = \langle \mathcal{M}_{fi} \rangle^2 / (4V) \). Here \( t \) represents time and \( V = L_x L_y L_z \) the normalization volume. \( |\mathcal{M}_{fi}|^2 \) is the squared matrix element and the symbol \( \langle \cdot \rangle \) denotes an averaging over initial spins and a sum over final spins. The matrix element for the \( V - A \) interaction is given by

\[
\mathcal{M}_{fi} = \frac{G_F}{\sqrt{2}} \int d^4X \bar{\psi}_3(X) \gamma_\mu (g_\nu - g_A \gamma_5) \psi_3(X) \bar{\psi}_2(X) \gamma_\mu (1 - \gamma_5) \psi_4(X).
\]

In presence of a uniform magnetic field \( B_m \), the normalized proton wave function is \( \psi_3(X) = \left( 1/\sqrt{L_y L_z} \right) \exp \left( -i E_3 t + i p_{3y} y + i p_{3z} z \right) f_{p_{3y} p_{3z}}(x) \), where \( f_{p_{3y} p_{3z}}(x) \) is the 4-component spinor solution [11]. The form of the spinor in a magnetic field (see Ref. [12]) restricts the analytical evaluation of the neutrino emissivity to fields strong enough so as to populate only the ground state for electrons and protons, i.e. \( \eta = \eta' = 0 \). The only positive energy spinor for protons in the chiral representation is then [11,13]

\[
f_{p_{3y} p_{3z}}^{\eta = 0}(x) = N_{\eta = 0} \begin{pmatrix} E_3 + p_{3z} \\ 0 \\ -m^* \\ 0 \end{pmatrix} I_{\eta = 0, p_{3y}}(x),
\]
where \( N_{\eta=0} = 1/\sqrt{2E_3^2(E_3^2 + p_{3z})} \), and \( E_3^* = E_3 - U_{0,p}^H = (p_{3z}^2 + m^*{}^2)^{1/2} \) is the effective relativistic Hartree energy. The function \( I_{\eta=0,p_{3z}}(x) \) is similar in form as in Ref. [11]. The nucleon effective and rest masses are respectively, \( m^* \) and \( m = m_n = m_p = 939 \text{ MeV} \). In presence of the magnetic field, the wave functions for free electrons \( \psi_i(X) \) have the same form as those for protons, but with \( m^* \) and \( E_3 \) for protons replaced by the bare mass \( m_e \) and kinetic energy for electrons, respectively. The neutrons and neutrinos/antineutrinos being unaffected by \( B_m \), have plane wave functions.

Using these wave functions, it is straightforward to calculate the transition rate per unit volume and is given by

\[
W_{fi} = \frac{G_F^2}{E_1^2 E_2^3 E_4^3} \frac{1}{V^3 L_y L_z} \exp \left[ -\frac{(p_{1x} - p_{2x})^2 + (p_{3y} + p_{4y})^2}{2qB_m} \right] \\
\times \left[ (g_V + g_A^2)(p_{1} \cdot p_{2})(p_{3} \cdot p_{4}) + (g_V - g_A^2)(p_{1} \cdot p_{4})(p_{3} \cdot p_{2}) - (g_V^2 - g_A^2)m^*{}^2(p_{4} \cdot p_{2}) \right] \\
\times \delta(E_1 - E_2 - E_3 - E_4) \delta(p_{1y} - p_{2y} - p_{3y} - p_{4y}) \delta(p_{1z} - p_{2z} - p_{3z} - p_{4z}). \tag{7}
\]

Substituting Eq. (7) in the expression (4) for emissivity, and by the change of variable \( (p_{3y} + p_{4y}) \to p_{3y} \), the integration over \( dp_{3y} \) can be performed to yield a factor \( qB_m L_x \). The rest of the integrals of Eq. (4) can then be performed in the standard manner [4]. Electron capture gives the same emissivity as neutron decay, although in neutrinos, and thus the total emissivity (relativistically) for the direct URCA process in nuclear matter (NM) in a magnetic field \( B_m \) is \( \varepsilon_{\text{URCA}}^{\text{NM}}(B_m) = 2\varepsilon_\nu(B_m) \) i.e.

\[
\varepsilon_{\text{URCA}}^{\text{NM}}(B_m) = \frac{457\pi}{5040} G_F^2 \cos^2 \theta_c (qB_m) \left[ (g_V + g_A^2) \left( 1 - \frac{p_{F_F}}{\mu_n^*} \right) + (g_V - g_A^2) \left( 1 - \frac{p_{F_n}}{\mu_n^*} \cos \theta_{14} \right) \right] \\
- (g_V^2 - g_A^2 \frac{m^*{}^2}{\mu_n^* \mu_p^*}) \exp \left[ \frac{(p_{F_F} + p_{F_n})^2 - p_{F_F}^2}{2qB_m} \right] \mu_n^* \mu_p^* \mu_e T^6 \Theta, \tag{8}
\]

where \( \mu_n^* = (p_i^2 + m^*{}^2)^{1/2} \) and \( \cos \theta_{14} = (p_{F_n}^2 + p_{F_e}^2 - p_{F_F}^2)/2p_{F_n} p_{F_e} \). The threshold factor is \( \Theta = \theta(p_{F_F} + p_{F_e} - p_{F_n}) \), where \( \theta(x) = 1 \) for \( x > 0 \) and zero otherwise. For \( B_m = 0 \), the relativistic expression for the neutrino emissivity from the nucleon direct URCA process is

\[
\varepsilon_{\text{URCA}}^{\text{NM}}(B_m = 0) = \frac{457\pi}{10080} G_F^2 \cos^2 \theta_c \left[ (g_V + g_A^2) \left( 1 - \frac{p_{F_n}}{\mu_n^*} \cos \theta_{34} \right) \right] \\
+ (g_V - g_A^2) \left( 1 - \frac{p_{F_n}}{\mu_n^*} \cos \theta_{14} \right) - (g_V^2 - g_A^2 \frac{m^*{}^2}{\mu_n^* \mu_p^*}) \mu_n^* \mu_p^* \mu_e T^6 \Theta. \tag{9}
\]

It was shown [4] that quark matter (QM), if present, the \( \beta \)-decay (i.e. direct URCA process) of \( d \) quarks is kinematically allowed through reaction (2) if finite mass (and/or quark-quark interaction) is incorporated. The relativistic expression of the neutrino emissivity for the direct URCA process involving \( u \) and \( d \) quarks for \( B_m = 0 \) and without quark-quark interaction is given by [4]

\[
\varepsilon_{\text{URCA}}^{\text{QM}}(B_m = 0) = \frac{457\pi}{840} G_F^2 \cos^2 \theta_c (1 - \cos \theta_{34}) \mu_d \mu_u \mu_e T^6, \tag{10}
\]

in the usual notation [4]. The emissivity for the \( \beta \) decay of \( s \) quark for \( B_m = 0 \) is similar to Eq. (10) with \( \cos \theta_c \) replaced by \( \sin \theta_c \). The emissivity for the \( \beta \) decay of free \( d \) quark
in a magnetic field $B_m$ may be obtained from Eq. (8) by substituting $g_V = g_A = 1$ with $\mu^e_n \rightarrow \mu_d$, $\mu^e_p \rightarrow \mu_u$, and multiplying a color factor 3 for $d$ quark:

$$\varepsilon_{\text{URCA}}^\text{QM}(B_m) = \frac{457\pi}{420} G_F^2 \cos^2 \theta_c \left(qB_m\right) \left(1 - \frac{p_{F_u}}{\mu_u}\right) \exp\left[\frac{(p_{F_u} + p_{F_c})^2 - p_{F_d}^2}{2qB_m}\right] \frac{\mu_d\mu_u\mu_e}{p_{F_u}p_{F_c}} T^6. \quad (11)$$

Similar expression is obtained for $s$ quark, but is Cabibbo suppressed. The decay of $d$ and $s$ quarks is feasible if they satisfy the respective inequality conditions $p_{F_u} - p_{F_c} \leq p_{F_d} \leq p_{F_u} + p_{F_c}$ and $p_{F_u} - p_{F_c} \leq p_{F_u} \leq p_{F_u} + p_{F_c}$.

To estimate numerically the various neutrino emissivities for the direct URCA processes with and without magnetic field in a neutron star, we describe the nuclear matter and electrons within the relativistic Hartree approach in the linear $\sigma$-$\omega$-$\rho$ model [1,13]. The values for the dimensionless coupling constants for the $\sigma$, $\omega$ and $\rho$ mesons are adopted from Ref. [14] which are determined by reproducing the nuclear matter properties at a saturation density of $n_0 = 0.16 \text{ fm}^{-3}$. The variation of magnetic field with density $n_b$ from center of the star is parametrized by the form [13]

$$B_m(n_b/n_0) = B^\text{surf}_m + B_0 \left[1 - \exp\left\{-\beta(n_b/n_0)^\gamma\right\}\right], \quad (12)$$

where the parameters are chosen to be $\beta = 10^{-4}$ and $\gamma = 6$. The maximum field prevailing at the center is taken as $B_0 = 5 \times 10^{18} \text{ G}$ and the surface field is $B^\text{surf}_m \simeq 10^8 \text{ G}$. The number of Landau levels populated for a given species is determined by the $B_m$ and $n_b$ [1].

In Fig. 1, we show the neutrino emissivity as a function of baryon density at $B_m = 0$ (see Eq. (9)) for the direct URCA process in nuclear matter (denoted by NM) at an interior temperature $T = 10^9 \text{ K}$. Due to momentum conservation, the threshold density at which this process occurs, is at $n_t = 0.346 \text{ fm}^{-3}$. The variation of emissivity with $n_b$ in presence of magnetic field $B_m$ (see Eq. (8)) as seen in the figure may be explained as follows: At very low densities $n_b \sim 0.35 - 0.73$ the field $B_m$ (as given in Eq. (12)) is rather small $\lesssim 10^{18}$ G, and consequently a large number of Landau levels are populated. This gives essentially field-free results. At densities $n_b \geq 0.75 \text{ fm}^{-3}$, the field is strong enough to populate only the ground levels of both electrons and protons [11], and would have pronounced quantization effects; the critical field for electron is $B^\text{c}(e) = 4.144 \times 10^{13} \text{ G}$. The emissivity then rapidly increases with density and could have values as high as $\sim 2$ orders of magnitude larger than $B_m = 0$ case at $n_b \approx 1.2 \text{ fm}^{-3}$. Hereafter, $B_m$ saturates to a maximum of $5 \times 10^{18} \text{ G}$ so that for $n_b > 1.2 \text{ fm}^{-3}$, higher level states start to populate, and, as in the low density situation, results in field-free emissivity values. The central densities $n_c$ of neutron stars with maximum masses are also shown in Fig. 1 with (open circles) and without (solid circles) the magnetic field. For $B_m \neq 0$ star, $n_c = 1.448 \text{ fm}^{-3}$ and thus falls above the kernel of enhanced emissivity leading to faster cooling compared to the field-free case.

The neutrino energy losses from direct URCA processes of quark matter composed of free $u$, $d$ and $s$ quarks and $e$ are estimated in the bag model. The current masses of the quarks are taken as $m_u = 5 \text{ MeV}$, $m_d = 10 \text{ MeV}$ and $m_s = 150 \text{ MeV}$, and the bag constant as $B = 250 \text{ MeV fm}^{-3}$. In Fig. 1, we display the neutrino emissivity from the $\beta$-decay of $d$ and $s$ quarks at $B_m = 0$ (see Eq. (10)) in quark matter (denoted by QM). The $d$ quark $\beta$-decay reactions are kinematically allowed if $n_b \gtrsim n_0$. At densities $n_b \approx 0.85 \text{ fm}^{-3}$ and above when $s$ quark decay is allowed, the emissivity is increased to about an order of
magnitude. This is caused by the large $s$ quark mass which allows the momenta of the free particles to deviate appreciably from collinearity which tends to increase the matrix element. It was, however, shown \cite{6} that by the inclusion of quark-quark interaction, the neutrino emissivities from $d$ and $s$ quark $\beta$-decay are comparable in magnitude. The emissivities for the quark direct URCA processes in presence of the magnetic field (see Eq. (11)), remain virtually unaltered from the field-free case due to the population of a large number of levels in all the quark species. In either case, it is found that $\varepsilon_{\text{QM}}/\varepsilon_{\text{URCA}}^{\text{NM}} \approx 10^{-3}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The neutrino emissivities as a function of baryon density from the direct URCA process for a magnetic field $B_m = 0$ (solid line) and for $B_m = 10^8 - 5 \times 10^{18}$ G (dashed line) for: nucleons in nuclear matter (NM); quarks in quark matter (QM); a nucleon-quark phase transition (NQP); nucleons in nuclear matter with hyperons (NM(HY)); a nuclear matter with hyperons to quark phase transition (NQP(HY)). The maximum masses of the stars $M_{\text{max}}$ with these various compositions are given for $B_m = 0$ and those in the parentheses are for $B_m = 10^8 - 5 \times 10^{18}$ G. The corresponding central densities are indicated by solid and open circles, respectively.}
\end{figure}

In a realistic situation, if quarks at all exist, a star with increasing density from the surface to the center would have a pure nucleon phase at the inner crust and core with a possible pure quark phase at the center and a mixed nucleon-quark phase (NQP) in between. The mixed phase of nucleons and quarks is described following Glendenning \cite{14}. The conditions of global charge neutrality and baryon number conservation are imposed through the relations $\chi Q^p + (1 - \chi) Q^n = 0$ and $n_b = \chi n_b^p + (1 - \chi) n_b^q$, where $\chi$ represents the fractional volume occupied by the hadron phase. Furthermore, the mixed phase satisfies the Gibbs’ phase rules: $\mu_p = 2\mu_u + \mu_d$ and $P^m = P^q$. The neutrino energy loss rate in this phase is given
by $\varepsilon_{\text{URCA}}^{\text{NQP}} = \chi\varepsilon_{\text{URCA}}^{\text{NM}} + (1 - \chi)\varepsilon_{\text{URCA}}^{\text{QM}}$. The neutrino emissivities for the nucleon-quark phase transition are shown in Fig. 1 (denoted by NQP). For $B_m = 0$ case, with the appearance of the quarks at $n_b = 0.533$ fm$^{-3}$, the emissivity decreases from the corresponding NM case. Apart from the reduced emissivity of the quark phase (which being, however small at large $\chi$), the reduction in the chemical potentials of the nucleons and electrons resulting from the requirement of the global charge neutrality and baryon number conservation conditions in the mixed phase, primarily causes the decrease in emissivity. For stars with $B_m \neq 0$, the emissivities in the mixed phase are enhanced, particularly in the regime dominated by nucleons (i.e. for $\chi > 0.5$). The central densities of maximum mass stars fall within the mixed phase, and consequently such stars would have faster cooling than pure quark stars. The maximum mass NQP stars with and without magnetic field, however, have much smaller emissivities than that of the corresponding NM stars, while NQP stars with $B_m$ have nearly identical cooling as that of field-free NM stars even though their maximum masses are very distinct.

Since quark matter furnishes both baryon number and negative charge, intuitively, the trends exhibited by the emissivities for NQP stars may be anticipated by invoking strange baryons, namely hyperons ($\Lambda$'s, $\Sigma$'s and $\Xi$'s). The $\beta$-equilibrium conditions then generalize to $\mu_i = b_i\mu_n - q_i\mu_e$, where $b_i$ and $q_i$ are the baryon number and charge for the $i$th particle. Since the hyperons are more massive than the protons, the effect of the magnetic field on their direct URCA processes is negligible. Because of the large uncertainties in the hyperon-nucleon interactions even at nuclear density, for a conservative estimate of the emissivities we set the nucleon-meson and hyperon-meson coupling constants equal. Furthermore, the critical density for nucleon direct URCA process is nearly identical to the hyperon threshold density in the relativistic mean field model, and the emissivities from the hyperon direct URCA processes are about 5-100 times less than that from the nucleons [15]. Therefore, we shall present emissivity vs $n_b$ results only for the nucleon direct URCA process in presence of hyperons. This is shown in Fig. 1 and denoted by NM(HY). With the appearance of hyperons, the reduction in the chemical potentials of the nucleons and electrons required by the baryon number conservation and charge neutrality condition causes a substantial reduction of the emissivity compared to that from NM. In fact, with increasing density when hyperon abundances grow rapidly, the emissivities gradually decrease. For $B_m \neq 0$, only the ground Landau levels for $e$ and $p$ are populated over a considerable density range in this matter. Consequently, the emissivities with $B_m$ in NM(HY) stars are significantly larger than that for the corresponding field-free stars.

Allowing now baryon to quark phase transition, the emissivity displayed in Fig. 1 (denoted by NQP(HY)) for $B_m = 0$ is larger than that from the NQP matter. This is caused by the delayed appearance of quarks in hyperon rich matter, so that the total emissivity is primarily dominated by the nucleons. In presence of the field, the total emissivity of NQP(HY) matter is about an order of magnitude larger for the maximum mass star and therefore leads to faster cooling compared to the corresponding field-free star.

Within the non-relativistic (but interacting) approximation for the specific heat $c_v$ and neutrino emissivity, the time for the center of a NM star to cool by the direct URCA process to a temperature $T_9$ at $B_m = 0$ may be estimated to be $\Delta t = -\int(c_v/\varepsilon_{\text{URCA}}^{\text{NM}})dT \sim 10T_9^{-4}$ s. In contrast, for $B_m \neq 0$, the NM star’s center cools faster with $\Delta t \sim 0.5T_9^{-4}$ s. By invoking quarks and/or hyperons, the decrease in emissivity is much more compared to that of the
specific heat resulting in slow cooling of the stars center. The typical time scale associated with the propagation of thermal signals through the outer core and the crust to the surface before the sudden temperature drop is quite high $\sim 1$ to $100$ yr, depending on the crustal composition and relative sizes of the crust and the core and thus upon the equation of state. Therefore, it seems to be quite difficult to distinguish observationally from the effects of direct URCA process, the interior constitution of a star.

Throughout our discussion we have assumed that the electron is the only lepton. If the triangle inequality $p_{F_p} + p_{F_\mu} \geq p_{F_n}$ is satisfied then nucleon direct URCA process with muons will occur; the threshold density for this process is higher than electrons since $m_\mu > m_e$. The $\beta$-equilibrium condition $\mu_e = \mu_\mu$ moreover implies that the emissivity for URCA process with muons is same as that for the corresponding process with electrons. In the present model the nucleon direct URCA processes are not permitted at densities $n_b < 0.34$ fm$^{-3}$. In this outer core region, the dominant neutrino emission process are then the modified URCA processes of reaction (1) for which the emissivity is smaller by a factor $\sim (T/T_F)^2$ than the direct URCA processes.

At certain densities and temperature $T < T_c \approx 10^8 - 10^{10}$ K, the nucleon superfluidity may set in. The specific heat and direct URCA rate are then reduced by a factor $\sim \exp(-\Delta/T)$, where $\Delta$ is the larger of the neutron and proton gaps. The modified URCA rates are, however, reduced by a factor $\sim \exp(-2\Delta/T)$. In presence of a magnetic field, the superfluid protons are believed to form a type II superconductor in the outer core within the density range $0.7n_0 < n_b < 2n_0$, and the estimated lower and upper critical magnetic fields are respectively $H_{c1} \sim 10^{15}$ G and $H_{c2} \sim 3 \times 10^{16}$ G [16]. With the choice of variation of $B_m$ with $n_b$ (see Eq. (12)), the field is $10^{14} - 10^{16}$ G at the bulk of the outer core and could form a superconducting region at $T < T_c$, while the inner core and center with $B_m \sim 10^{18}$ G is in the normal state without superconductivity.

In conclusion, the neutrino emissivities for all the cases studied here are found to be dramatically enhanced in a magnetic field compared to that from the non-magnetized stars. However, for certain stars unambiguous determination of the interior constituents may be difficult. There can be stars with the same composition, as for example the NM stars in a magnetic field but with slightly different masses of $1.55M_\odot$ and $1.60M_\odot$ having their central densities at $0.72$ fm$^{-3}$ and $0.88$ fm$^{-3}$ residing below and within the kernel of fast cooling respectively, and thus have completely different emissivities. On the other hand, a NM star with $B_m = 0$ and a NQP star in a magnetic field, though possessing different interior compositions, have nearly identical emissivities. It is also found that when pure nuclear matter is injected with nonleptonic negative charges, namely hyperons and quarks, the emissivities turn out to be smaller than that from the nuclear matter. It has been already demonstrated [17] that nonleptonic negative charges cause a softening of the equation of state. We thus arrive at a general result that when matter contains nonleptonic negative charges, the maximum masses of the stars are smaller with a suppression of the neutrino emissivity than that of the pure nuclear matter with and without a magnetic field.
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