Discrete Black-Hole Radiation and the Information Loss Paradox

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Abstract

Hawking’s black hole information puzzle highlights the incompatibility between our present understanding of gravity and quantum physics. However, during the last three decades evidence has been mounting that, in a quantum theory of gravity black holes may have a discrete line emission. A direct consequence of this intriguing prediction is that, black-hole radiance may carry a significant amount of information. Using standard ideas from quantum information theory, we calculate the rate at which information can be recovered from the black-hole spectral lines. We conclude that the information that was suspected to be lost may gradually leak back, encoded into the black-hole spectral lines.

Black-hole radiation, first predicted by Hawking [1], imposes a great challenge to our understanding of the interface between quantum theory and gravity. As pointed out by Hawking, when a pure initial quantum state has collapsed to form a black hole, it will later evolve into a high entropy mixed state of radiation. This contradicts one of the basic principles of quantum mechanics – unitary evolution, according to which a pure state should always remain pure. This incompatibility is often discussed in terms of information theory [2]: Since fully thermal radiation cannot convey detailed information about its source, the
information hidden in the black hole about the state of the collapsed matter remains se-
questered as the black hole radiates, and it finally lost forever with the complete evaporation
of the black hole. This is the so-called black hole information (loss) paradox.

Several different reactions to the black-hole information puzzle have been suggested, for
reviews see e.g., [3–6]:

- **Information loss.** This point of view implies that the quantum theory of gravity in-
evitably violates the basic quantum mechanical principle of unitary evolution. Hawk-
ing [7] himself suggested a generalization of quantum mechanics which allows for an
information loss, namely a one which permits the evolution of a pure state into a
mixed state. However, specific schemes for such a generalization have been found to
be incompatible with either locality of energy-momentum conservation [8,9].

- **Remnants.** This point of view suggests that the black-hole evaporation stops as the
black hole approaches the Planck scale (where Hawking’s semi-classical analysis is
expected to break down), and the remnant holds the information in question. The
solution implies that there should be an infinite number of remnants species so that
they can encompass the information associated with an arbitrarily large black hole [6].
But an infinite variety of remnant may imply an infinite production rates for remnants
in processes like Hawking radiation, or Schwinger pair production (if the remnants are
charged) [6]. Moreover, a Planck mass remnant can hold only a few bits of information
[10,11], much less than the entropy of a large evaporating black hole.

- **Information leak.** Another alternative is that the information which is supposed to
be lost is encoded into the black-hole radiation and manages to leak back over the
full duration of the evaporation process. In particular, Bekenstein [3] stressed the
fact that Hawking’s radiation departs from a blackbody radiation due to the mode
dependence of the barrier penetration factor. This implies that Hawking’s radiation
is less entropic as compared with blackbody radiation (with the same power), and
may therefore carry information with it. Bekenstein [2] has estimated that (at least) 1% – 5% of the sequestered information could come out in principle.

For information leak to be a physically reasonable resolution of the puzzle one must show that information of the required magnitude can be encoded into the black-hole radiation. In the present paper we put forward the idea that, when quantum properties of the black hole itself are properly taken into account, the information outflow rate may indeed be large enough to allow a resolution of the paradox.

According to Hawking’s result, the black hole emits quanta of all frequencies, distributed according to the usual black-body spectrum (with a gray-body factor which represents the imprint of passage through the curvature potential surrounding the black hole).

However, Hawking’s prediction of black-hole evaporation is at a semiclassical level in the sense that the matter fields are treated quantum mechanically, but the spacetime (and the black hole itself) are treated classically. One therefore suspects some modifications of the character of the radiation when quantum properties of the black hole itself are properly taken into account.

The quantization of black holes was proposed long ago by Bekenstein [12]. Based on the remarkable observation that the horizon area of a nonextremal black hole behaves as a classical adiabatic invariant, and in the spirit of Ehrenfest principle [13], any classical adiabatic invariant corresponds to a quantum entity with discrete spectrum, Bekenstein [12] conjectured that the horizon area of a quantum black hole should have a discrete eigenvalue spectrum of the form

\[ A_n = \gamma \ell_P^2 \cdot n \quad ; \quad n = 1, 2, \ldots, \]

where \( \gamma \) is a dimensionless constant, and \( \ell_P = \left( \frac{G}{c^2} \right)^{1/2} \hbar^{1/2} \) is the Planck length (we use gravitational units in which \( G = c = 1 \)). This type of quantization-law has since been revived on various grounds [14–35] (most of these derivations have been made in the last few years). In particular, Mukhanov and Bekenstein [14–16] used a combination of thermodynamic (the
area-entropy relation $S_{BH} = A/4\hbar$ for black holes) and statistical physics (the Boltzmann-Einstein formula) arguments, to find that the dimensionless constant $\gamma$ in Eq. (1) should be of the form $\gamma = 4\ln \beta$, with $\beta = 2, 3, \ldots$ (this corresponds to a degeneracy factor of $\beta^n$ for the $n$th area level). Using Bohr’s correspondence principle Hod [27] has recently given evidence in favor of the value $\beta = 3$.

The discrete mass (area) spectrum implies a discrete line emission from a quantum black hole; the radiation emitted by the black hole will be at integer multiples of the fundamental frequency $\omega_0 = \ln \beta/8\pi M$ [13]. The broadness of the lines will be discussed at the end of the paper.

We note that a direct consequence of the discrete spectrum is that, compared with blackbody radiation (or even with Hawking’s semi-classical continues radiation), black-hole radiance is less entropic. The entropic deficiency may permit an information outflow of the required magnitude (In fact, we shall show below that the rate of information outflow increases as the spacing between the discrete energy levels increases.) Thus, information about the quantum state of the collapsed matter may in principle be encoded into the discrete black-hole spectral lines. From the point of view of quantum communication theory (for reviews see [36,37]), the maximum rate at which information can be recovered from the radiation is $\dot{I}_{max} \equiv \dot{S}' - \dot{S}$, where $\dot{S}$ is the actual entropy outflow rate and $\dot{S}'$ is the maximum rate for entropy outflow corresponding to the actual power under the boundary conditions of the physical system [2].

The probability for a black hole to emit a specific quantum should be proportional to the degeneracy of the final black-hole quantum state (and thus should be proportional to $\beta^{-k}$, $k$ being the level spacing between the initial and final quantum states), to the gray-body factor $\Gamma$ (representing a scattering of the quantum off the spacetime curvature surrounding the black hole), and to the square of the matrix element. In the spirit of the original treatment of Bekenstein and Mukhanov [15] we assume that the matrix element does not vary much as one goes from a nearest neighbor transition to one between somewhat farther neighbors. Thus, aside from an overall normalization factor, the matrix element does not
enter into our simple estimate. This assumption is further supported by a recent analysis of Massar and Parentani [38]. Thus, the probability \( p_k \) to jump \( k \) steps in the mass (area) ladder is proportional to \( \Gamma(k)\beta^{-k} \).

We consider the emission of a canonical set of three species of neutrinos, photon, and a graviton. This is the set of particles considered in former analyses of black-hole evaporation [39,2,40]. (The physical implications of a massive neutrino field were discussed in [41].) For the power emitted by the quantized black hole we write \( \dot{E} = \epsilon \hbar \omega_0 / \tau \), where \( \epsilon \hbar \omega_0 \) is the mean energy carried away by an emitted quanta, and \( \tau \) is the mean time between quantum leaps. Later on we shall estimate a lower bound on \( \tau \). The coefficient \( \epsilon \) depends on the gray-body factors and should therefore be evaluated numerically. One finds \( \epsilon \omega_0 = 0.185M^{-1} \) for \( \beta = 2 \), with a deviation of less than 1\% for \( \beta = 3 \). This last result also implies that the mean decrease in black-hole entropy with each quanta emitted, which is given by \( \epsilon \ln \beta \), is 4.65 for \( \beta = 2 \) (again, with a deviation of less than 1\% for \( \beta = 3 \)). [Each quanta emitted from the quantum black hole decreases its mass by \( \Delta M = -\epsilon \hbar \ln \beta / 8\pi M \) on the average. Using the relation \( S_{BH} = 4\pi M^2 / \hbar \), one finds that the mean decrease in black-hole entropy with each quanta emitted is \( \epsilon \ln \beta \), or \( \epsilon \omega_0 8\pi M = 4.65 \).

The entropy of a system measures one’s lack of information about its actual internal configuration [42–44]. Suppose that all that is known about the system’s internal configuration is that it may be found in any of a number of states, with probability \( p_n \) for the \( n \)th state. Then the entropy associated with the system is given by Shannon’s well-known relation \( S = -\sum p_n \ln p_n \). The ratio \( R = |\dot{S}_{rad} / \dot{S}_{BH}| \) of entropy emission rate from the quantum black hole, to the rate of black-hole entropy decrease is therefore given by

\[
R = -\frac{\sum_{i=1}^{N_s} \sum_{k=1}^{\infty} C\Gamma(k)\beta^{-k} \ln[C\Gamma(k)\beta^{-k}]}{\sum_{i=1}^{N_s} \sum_{k=1}^{\infty} C\Gamma(k)\beta^{-k} k \ln \beta},
\]

where \( N_s \) is the effective number of (massless) particle species emitted (\( N_s \) takes into account the various modes emitted), and \( C \) is a normalization factor [Note that the denominator in Eq. (2) is simply \( \epsilon \ln \beta \).] The ratio \( R \) was calculated in [41]: \( R = 1.119 \) for \( \beta = 2 \) and \( R = 1.016 \) for \( \beta = 3 \). Thus, the mean entropy carried away by an emitted quanta is 4.65R,
and the corresponding rate of entropy emission from the quantum black hole is given by

\[ \dot{S}_{\text{rad}} = 4.65R/\tau. \tag{3} \]

One now has to compare \( \dot{S}_{\text{rad}} \) with the maximally entropic (blackbody) distribution \( \dot{S}' \) whose power \( \dot{E}' \) equals the actual power \( \dot{E} \) of the black-hole radiation. According to the Boltzmann formula, a black body at temperature \( T \) and of area \( A \) emits power \( \dot{E}' = N\pi^2AT^4/60\hbar^3 \), where \( N \) is the effective number of particle species emitted. Photons and gravitons contribute 1 each to \( N \), while a neutrino contributes \( 7/16 \). Thus, we should take \( N = 37/8 \) for the canonical set. Now, for blackbody radiation flowing in three space dimensions, \( \dot{S}' = \frac{4}{3}\dot{E}'/T \). Thus, one has the relation

\[ \dot{S}' = \frac{4N^{1/4}A^{1/2}\dot{E}'^{3/4}}{4860^{1/4}\hbar^{3/4}}. \tag{4} \]

The effective radiating area of a black hole (the “photosphere”) was estimated by Bekenstein [2] as \( A_{\text{phot}} = 108\pi M^2\xi \), with \( \xi > 1 \). This is a reasonable estimation because in the high frequency regime, all quanta hitting within a cross section \( 27\pi M^2 \) will be captured [45,46] (this suggests a photospheric area four times larger). The factor \( \xi \) takes into account the fact that the gray-body factors \( \Gamma(\omega) \) vanish only as a power-law of \( \omega \) in the \( \omega \to 0 \) limit, and thus quanta of a fairly large impact parameter are sometimes absorbed by the black hole, and therefore may be emitted sometimes [2]. Substitution of the numerical factors in Eq. (4) yields

\[ \dot{S}' = 1.507\xi^{1/4}M^{-1/4}\tau^{-3/4}. \tag{5} \]

Taking cognizance of Eqs. (3) and (5) one finally finds that the maximum rate at which information can leak out from the quantum black hole (carried away by the black-hole spectral lines) is

\[ \dot{I}_{\text{max}} \equiv \dot{S}' - \dot{S}_{\text{rad}} = (1.507\xi^{1/4}\alpha^{1/4} - 4.65R)/\tau, \tag{6} \]

where \( \alpha \equiv \tau/M \).
If the rate at which information leaks out from the black hole amounts to $4.65 R/\tau$ (which is the actual entropy emission rate), than given an appropriate quantum mechanism this information may reduce one’s uncertainty (lack of information) about the actual internal configuration of the system, in accord with the general formula $\Delta I = -\Delta S$ relating information and entropy \cite{12, 14}. Thus, the radiation can end up in a pure state. We should therefore evaluate the ratio $T \equiv \dot{I}/\dot{S}_{\text{rad}}$, which is given by

$$T \equiv \frac{\dot{I}_{\text{max}}}{\dot{S}_{\text{rad}}} = \begin{cases} 
0.289\xi^{1/4}\alpha^{1/4} - 1, & \beta = 2 \\
0.319\xi^{1/4}\alpha^{1/4} - 1, & \beta = 3
\end{cases} \quad (7)$$

(Here we used the values of $R$ calculated in \cite{11}.)

We shall now estimate a lower bound on $\alpha$ (which would yield a lower bound on the physically interesting quantity $T$). To that end we use Page’s \cite{39} semi-classical result for the emitted power and write $\dot{E} = 2.829 \cdot 10^{-4}\eta\hbar/M^2$. The factor $\eta$ takes into account the fact that the emission is in discrete lines, and thus part of the frequency spectrum is blocked. This should suppress the power emitted from the quantized black hole as compared with the semi-classical continuous spectrum. One therefore expects the coefficient $\eta$ to be smaller than unity. On the other hand, we have already found that the mean energy carried by a quanta emitted from the quantum black hole is $0.185\hbar M^{-1}$, and the emitted power therefore equals $\dot{E} = 0.185\hbar M^{-1}/\tau$. Comparing the two expressions one finds $\tau \simeq 654\eta^{-1}M$, which implies $\alpha \simeq 654\eta^{-1}$. The corresponding values of $T$ are given in Table \ref{table}, from which we learn that the information outflow rate may be of just the required magnitude ($T \geq 1$) if $\sim 50\% - 70\%$ of the power (or more) is blocked \cite{17}.

Note that the value of $T$ derived in this paper (for a quantum black hole) is higher than the corresponding one given in \cite{2} (which was derived for a classical black hole). In other words, the radiation emitted from a quantum black hole carries with it more information than the one coming out of a classical black hole. This result is a direct consequence of the fact that the spectral emission from a quantum black hole is in discrete lines, whereas a classical black hole emits radiation in the form of a smooth continuum. This implies a
larger deviation from a perfect (maximally entropic) blackbody radiation in the case of a quantum black hole, as compared with a classical one.

For our analysis to be self-consistent, the natural broadening of the spectral lines should not smear the spectrum into a smooth continuum. The question of natural broadening of the black-hole (discrete) spectral lines has been discussed previously by Bekenstein and Mukhanov [15] and by Mäkelä [48]. These analyses are, however, semi-qualitative and in particular do not take fully into account the central role of the gray-body factors. Here we take these into account. We recall that, according to Heisenberg’s quantum uncertainty principle the natural broadening $\delta \omega$ is of the order of $\tau^{-1}$, which yields the ratio

$$\frac{\delta \omega}{\omega_0} \sim 0.035\eta,$$

for $\beta = 3$, and $\delta \omega/\omega_0 \sim 0.055\eta$ for $\beta = 2$. This implies that the emission lines are un-blended, in agreement with the semi-qualitative estimation of Bekenstein and Mukhanov [15] (However, the numerical ratio $\delta \omega/\omega_0$ is different because in [15] only one emitted specie was considered, and the gray-body factors were not taken into account.)

In summary, we have shown that if quantum properties of the black hole itself are properly taken into account, the information which was suspected to be lost may in fact be encoded into the black-hole spectral lines. Thus, with the help of an appropriate quantum mechanism to encode the information, the reconstruction of a pure radiation state seems physically reasonable. For the advocates of the “information leak” resolution the task remains to identify the appropriate quantum mechanism.

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[47] Note that, as previously conjectured, the rate of information outflow increases as the spacing between the (discrete) energy levels increases (see Table I). This is caused by the fact that the entropy of the radiation should be maximal when the various transitions have equal probabilities, but the fundamental transition \( n \rightarrow n - 1 \) becomes more and more dominant as the value of \( \beta \) increases. Thus, \( \dot{S} \) is a decreasing function of \( \beta \), which implies that \( \dot{I}_{\text{max}} \) increases with \( \beta \).

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TABLE I. The ratio $T \equiv \dot{I}/\dot{S}$ of the maximal *information* outflow rate to the actual *entropy* emission rate, and the natural broading $\delta \omega/\omega_0$ of the spectral lines.

| $\beta$ | $T$ | $\delta \omega/\omega_0$ |
|---------|-----|-------------------------|
| 2       | $1.461 \xi^{1/4} \eta^{-1/4} - 1$ | 0.055$\eta$ |
| 3       | $1.613 \xi^{1/4} \eta^{-1/4} - 1$ | 0.035$\eta$ |