Low-Energy Supersymmetry Breaking and Fermion Mass Hierarchies

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Abstract

In models with low-energy supersymmetry breaking, an anomalous Abelian horizontal gauge symmetry can simultaneously explain the fermion mass hierarchy and the values of the $\mu$ and $B$ terms. We construct an explicit model where the anomaly is cancelled by the Green-Schwarz mechanism at the string scale. We show that with our charge assignments, the breaking of the horizontal symmetry generates the correct order of magnitude and correct hierarchy for all Yukawa couplings.

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I. INTRODUCTION

Models of dynamical low-energy supersymmetry (SUSY) breaking [1–3] can provide a predictive and testable framework for weak-scale supersymmetry. In these models all mass scales arise via dimensional transmutation; the soft SUSY-breaking parameters are calculable in terms of only a few parameters. These low-energy SUSY-breaking models solve in a natural way the flavour problem of hidden sector supergravity theories—the soft masses of the quarks and leptons are proportional to their gauge quantum numbers and are therefore flavour neutral. At the weak scale these models resemble the minimal supersymmetric standard model, with a constrained parameter space.

A class of such models has been considered recently [1]. One difficulty with these models is the generation of appropriate \( \mu \) and \( B \) terms at low energies, without significant fine-tuning. In Ref. [1] this problem is circumvented by the introduction of an extra singlet field with higher order non-renormalizable couplings to the SUSY-breaking sector, which can provide a mechanism for generating a \( \mu \) term of appropriate magnitude.

In this paper we will illustrate another method for obtaining effective couplings of the right magnitude to generate acceptable \( \mu \) and \( B \) terms, without explicit fine-tuning. As an added bonus, the same mechanism can generate the fermion mass hierarchy, as has been recently explored elsewhere [4–9]. We propose to introduce an anomalous, generation dependent, \( U(1)_X \) gauge symmetry, under which the quark and lepton superfields of the supersymmetric standard model transform nontrivially. In addition the “messenger” singlet field responsible for communicating the SUSY breaking is charged under the \( U(1)_X \) symmetry. The anomaly is cancelled by the Green-Schwarz mechanism. When the \( U(1)_X \) symmetry is broken, the necessary Yukawa couplings are induced. The charge assignments of the various fields control the induced coupling hierarchy in a manner similar to that of the Froggatt-Nielsen approach [10].

The outline of this paper is as follows. We begin in Sec. 2 with a brief review of the sector responsible for communicating SUSY breaking to the SUSY standard model. We explain the difficulty in obtaining \( B \) and \( \mu \) terms of order the electroweak scale without recourse to significant fine tuning or to the introduction of additional structure in the Higgs sector. In Sec. 3 we outline the proposed solution of the fine-tuning problem via an anomalous horizontal symmetry and state the conditions necessary for the Green-Schwarz cancellation of the anomaly. In Sec. 4 we give an existence proof for models of this type, by presenting explicit solutions of the Green-Schwarz anomaly cancellation conditions. The given charge assignments explain the fermion mass hierarchy and the generation of acceptable \( \mu \) and \( B \) terms. Sec. 5 contains a summary of our results.

II. LOW-ENERGY SUSY-BREAKING MODEL

We will consider the model of low-energy SUSY breaking introduced in Ref. [1]. For our purposes the full structure of the SUSY-breaking model will be inessential. We will only consider the sector responsible for communicating SUSY breaking to the SUSY standard model (“messenger sector”). This sector consists of a pair of charged (under “messenger
hypercharge”) superfields $\phi_{\pm}$, a gauge singlet superfield $X$, and a pair of additional vectorlike quark and lepton superfields, the “messenger” quarks $q, \bar{q}$, and leptons $l, \bar{l}$. The superpotential of the messenger sector is given by

$$W_M = k_1 X \phi_+ \phi_- + k_2 X \bar{q} \bar{l} + k_3 X \bar{q} q + \frac{1}{3} \lambda X^3.$$  \hspace{1cm} (1)

Due to their messenger hypercharge interaction with the “supercolor” sector (the sector that dynamically breaks SUSY), the $\phi_{\pm}$ fields acquire a negative mass squared, $(-m^2_\phi)$, related to the scale of dynamical SUSY breaking [3]. The scalar potential of the messenger sector is

$$V_M = -m^2_\phi (|\phi_+|^2 + |\phi_-|^2) + \sum_\varphi \left| \frac{\partial W}{\partial \varphi} \right|^2,$$  \hspace{1cm} (2)

where $\varphi$ denotes all the fields in the messenger sector. A minimum of Eq. (2) with nonvanishing vacuum expectation values (vevs) for $\phi_{\pm}, X$, and the $F$-component $F_X$ of the messenger singlet $X$ occurs when $k_1 < \lambda$ [1] at the values,

$$|X|^2 = \frac{1}{k^2_1} \frac{\lambda - k_1}{2\lambda - k_1} m^2_\phi,$$  \hspace{1cm} (3)

$$|\phi_+|^2 = |\phi_-|^2 = \frac{1}{k^2_1} \frac{\lambda^2}{2\lambda - k_1} m^2_\phi,$$  \hspace{1cm} (4)

where the $\phi_{\pm}$ fields are constrained to be equal by the messenger hypercharge D-term. The messenger quark and lepton superfields do not obtain vevs, provided $\lambda \ll k_{2,3}$ [1]. The $F_X$ expectation value is of order

$$F_X \simeq \lambda \langle X \rangle^2.$$  \hspace{1cm} (5)

This SUSY-breaking vacuum expectation value of the gauge singlet superfield $X$ splits the masses of the superpartners in the messenger quark and lepton supermultiplets. This splitting is further transmitted to the standard model sector by loops involving the messenger quarks and leptons. In this class of models, gaugino masses appear at one loop order, while squark and slepton masses appear at two-loop order. These soft SUSY-breaking parameters are approximately

$$\tilde{m} \simeq \frac{g^2}{16\pi^2} \lambda \langle X \rangle \equiv \frac{g^2}{16\pi^2} \Lambda,$$  \hspace{1cm} (6)

where $g$ is a standard model gauge coupling, and we have introduced the dimensionful parameter $\Lambda$ as in Ref. [2]. The scale of all soft supersymmetry breaking parameters in the low-energy theory is determined by $\Lambda$. To obtain soft masses of order the electroweak scale, $\tilde{m} \simeq 10^2 \text{ GeV}$, we require $\Lambda \simeq 10^4 \text{ GeV}$.

Finally, there is a coupling between the gauge singlet messenger field $X$ and the Higgs doublets

$$W_\mu = \lambda' X H_U H_D,$$  \hspace{1cm} (7)
which generates a $B$ term in the scalar potential, $B = m_{12}^2 H_U H_D$, and a $\mu$ term in the superpotential, with parameters

$$m_{12} = \sqrt{\lambda' \lambda} \langle X \rangle = \sqrt{\frac{\lambda'}{\lambda}} \Lambda$$

$$\mu = \lambda' \langle X \rangle = \frac{\lambda'}{\lambda} \Lambda.$$

A $\mu$ term of order 100 GeV requires $\lambda'/\lambda \approx 10^{-2}$, which then determines the $B$ term to be of order 1 TeV. Notice that if one requires a 100 GeV $B$ term, then the $\mu$ term becomes unacceptably small for the simple prescription of Eq. (7). This problem can be overcome, as was done in Ref. [1], by extending the Higgs sector with the addition of extra gauge singlet fields which also have nonrenormalizable interactions with the supercolor sector. Since a $B$ term parameter $m_{12}$ of order 1 TeV is not ruled out experimentally we will consider the simpler scenario of Eq. (7), which yields the relations of Eq. (8).

As we saw above, in order to avoid colour and electroweak-breaking vevs of the messenger quarks and leptons, we require $\lambda \ll k_{2,3} \approx 1$. On the other hand, obtaining soft mass parameters $\tilde{m}$, $\mu$, and $B$ of electroweak order of magnitude requires fine tuning of the ratio $\lambda'/\lambda$. In the following sections, we propose an explanation of this hierarchy of couplings in the superpotentials, Eqs. (1) and (7), by assuming that they arise from higher dimension terms, constrained by a horizontal anomalous $U(1)$ symmetry. This symmetry, broken at some high energy scale (e.g. the string scale, $M_S$), may also be used to explain the fermion mass hierarchy. Thus the hierarchies of the fermion masses, as well as the correct order of magnitude of the soft supersymmetry breaking terms are explained by the same mechanism.

### III. STRING-INSPIRED HIERARCHY

In this section we will introduce the anomalous horizontal $U(1)_X$ symmetry, review its application for generating the fermion mass hierarchy, and extend it to explain the order of magnitude of the $\mu$ and $B$ terms in the low-energy supersymmetry breaking scenario. We assume the existence of a standard model gauge singlet field, $\theta$, which is charged under the $U(1)_X$ symmetry. This field couples to the standard model and messenger sector fields only through nonrenormalizable interactions, suppressed by the string scale $M_S$.

The total superpotential in which we will be interested is the sum of the messenger sector, Eq. (1), and the SUSY standard model superpotentials

$$W = W_M + W_{SM},$$

where

1. Recently, anomalous horizontal symmetries have been invoked to explain the value of the $\mu$ term in hidden sector supergravity models [7][8].

2. In the next section we will also consider a model with two standard model gauge singlet fields.
\[ W_M = \xi_\phi \left( \frac{\theta}{M_S} \right)^{q_X+q_{\phi_+}+q_{\phi_-}} X \phi_+ \phi_- + \left( \frac{\theta}{M_S} \right)^{q_X} \left[ \xi_i X \bar{l}l + \xi_q X \bar{q}q \right] \]
\[ + \frac{\xi_X}{3} \left( \frac{\theta}{M_S} \right)^{3q_X} X^3 + \xi_H \left( \frac{\theta}{M_S} \right)^{q_H+q_X} X H_1 H_D. \]

The various powers of \( \theta \) are determined by the \( U(1)_X \) charge assignments, where the \( \theta \) field has charge \(-1\). The coefficients \( \xi_\alpha \) are assumed to be numbers of order one and \( q_H = q_{H_U} + q_{H_D} \). The charge of \( X \) is \( q_X = 1 \) and \( \phi_\pm \) have charges \( q_{\phi_+} = q_{\phi_-} = 1 \). With these charge assignments we need \( \xi_\phi < \xi_X \) in order to obtain the required minimum, Eq. (2). We assume that the messenger quarks and leptons are in vectorlike representations with respect to the \( U(1)_X \) symmetry as well, and therefore they do not contribute to the various mixed anomalies considered below. The standard model Yukawa couplings are generated by

\[ W_{SM} = \xi^{U}_{ij} \left( \frac{\theta}{M_S} \right)^{p^{U}_{ij}} Q_i H_U u_j + \xi^{D}_{ij} \left( \frac{\theta}{M_S} \right)^{p^{D}_{ij}} Q_i H_D d_j + \xi^{L}_{ij} \left( \frac{\theta}{M_S} \right)^{p^{L}_{ij}} L_i H_D e_j, \]

where \( i, j = 1, 2, 3 \) are generation indices.

As shown in Ref. [1], in order to explain the fermion mass hierarchy, the \( U(1)_X \) symmetry must be anomalous. The anomaly is assumed to be cancelled by the Green-Schwarz mechanism. The mixed anomaly coefficients for \( U(1)_X \) and the standard model gauge groups are given by

\[ C_1 = \frac{1}{6} \left( 3 [q_{H_U} + q_{H_D}] + 3 \sum_{i=1}^{3} [q_{Q_i} + 8q_{u_i} + 2q_{d_i} + 3q_{L_i} + 6q_{e_i}] \right), \]
\[ C_2 = \frac{1}{2} \left( q_{H_U} + q_{H_D} + \sum_{i=1}^{3} [3q_{Q_i} + q_{L_i}] \right), \]
\[ C_3 = \frac{1}{2} \sum_{i=1}^{3} (2q_{Q_i} + q_{u_i} + q_{d_i}), \]
\[ C_{XXY} = q_{H_U}^2 - q_{H_D}^2 + \sum_{i=1}^{3} \left[ q_{Q_i}^2 - 2q_{u_i}^2 + q_{d_i}^2 - q_{L_i}^2 + q_{e_i}^2 \right]. \]

Here \( C_1, C_2, C_3 \) are the coefficients of the mixed \( U(1)_X U(1)_Y^2, U(1)_X SU(2)^2, U(1)_X SU(3)^2 \) anomalies, respectively, and \( C_{XXY} \) is the \( U(1)_X^2 U(1)_Y \) anomaly. The Green-Schwarz mechanism [11] for anomaly cancellation requires that

\[ \frac{C_1}{k_1} = \frac{C_2}{k_2} = \frac{C_3}{k_3} = \frac{C_X}{k_X} = \frac{C_{grav}}{k_{grav}}, \]

where \( k_i \) denotes the Kac-Moody levels of the corresponding gauge algebras, \( C_{grav} = (1/24) \text{Tr} Q \) (the sum of all the \( U(1)_X \) charges in the model), and \( C_X \) is the coefficient of the \( U(1)_X^3 \) anomaly. The \( U(1)_X^2 U(1)_Y \) anomaly cannot be cancelled by the Green-Schwarz mechanism. We will therefore only consider charge assignments for which

\[ C_{XXY} = 0. \]
We will consider Kac-Moody levels with the values \( k_1 = \frac{5}{3}, k_2 = k_3 = k_{\text{grav}} = 1 \) and \( C_X \) will not be used because there may be other \( U(1)_X \)-charged fields in the theory which are singlets under the standard model gauge groups. Similarly we do not consider the mixed anomalies with the supercolor sector gauge group(s), but simply assume that these can be made consistent with the Green-Schwarz mechanism by means of appropriate charge assignments. The gravitational anomaly \( C_{\text{grav}} \) also depends upon the \( U(1)_X \) charges of any additional fields, however by the Green-Schwarz mechanism it is proportional to \( C_3 \). This will be important below for determining the vev of \( \theta \).

String loop effects induce a Fayet-Iliopoulos term for the anomalous \( U(1)_X \) symmetry \cite{12}. The \( U(1)_X \) D-term is then given by

\[
D = \frac{g_S^2 M_S^2}{192\pi^2} \text{Tr}Q + \sum_i q_i |\phi_i|^2,
\]

where \( g_S \) and \( M_S \) are the string coupling and string scale, respectively, and the sum is over all fields carrying \( U(1)_X \) charge. Requiring unbroken supersymmetry at the string scale, with our choice of charge for \( \theta \), we find that the vev of \( \theta \) is then given by

\[
\langle \theta \rangle \frac{M_S}{g_S^2} = \sqrt{\frac{g_S^2}{192\pi^2} \text{Tr}Q} = \sqrt{\frac{g_S^2}{8\pi^2} C_3},
\]

where the last equality holds for our choice of Kac-Moody levels. Using the tree-level relation \( 1/g_S^2 = k_{\text{GUT}}/g_{\text{GUT}}^2 \) at the unification scale\(^3\) and taking \( k_{\text{GUT}} = 1 \) and the typical value \( g_{\text{GUT}}^2/4\pi \approx 1/25 \), we obtain

\[
\langle \theta \rangle \approx 0.08 \sqrt{C_3}.
\]

In the next section we will study two specific examples in which the hierarchies are parameterised by various powers of this ratio, with the powers determined from \( U(1)_X \) charge assignments of the various fields.

IV. EXAMPLES OF \( U(1)_X \) MODELS

A. One \( \theta \)-field case

Consider the case of one \( \theta \) field which couples to all terms in the superpotential. In order to determine the allowed couplings one can perform a computer search, along the lines of Ref. \cite{9}, for charges that obey the conditions of Eqs. (16) and (17) and that allow for \( q_H > 0 \), which leads to an acceptable fermion mass hierarchy. However, in this paper we will restrict ourselves to giving an existence proof for simple analytic solutions. For left-right symmetric

\(^3\)We assume string-type unification. For a discussion of the phenomenology of this model in an \( SU(5) \)-unified framework, see Ref. \cite{13}.
$U(1)_X$ charge assignments, one can analytically solve the $U(1)_X^2 U(1)_Y$-anomaly condition \[7\] \([\overline{7}]\). However, in the one $\theta$ field case, it was shown in Ref. \([\overline{7}]\) that the fermion mass hierarchy cannot be explained by assuming left-right symmetric $U(1)_X$ charge assignments. A solution which exhibits the required asymmetry is given by the following quark and lepton $U(1)_X$ charge assignments

$$
\begin{array}{cccccc}
1 & qQ_i & q_u & q_d & qL_i & q_e \\
3 & \frac{33}{40} & \frac{403}{240} & \frac{401}{240} & \frac{143}{120} & \frac{73}{240} \\
2 & \frac{13}{40} & \frac{43}{240} & \frac{161}{240} & \frac{83}{120} & -\frac{47}{240} \\
3 & -\frac{27}{40} & -\frac{197}{240} & \frac{401}{240} & \frac{83}{120} & -\frac{47}{240} \\
\end{array}
$$

where the two Higgs superfields carry charges $q_{H_u} = \frac{359}{240}$ and $q_{H_d} = \frac{121}{240}$, while $X$ and $\theta$ have charges $q_X = \frac{1}{2}$ and $q_\theta = -\frac{1}{2}$. In addition one needs to charge the $\phi_{\pm}$ fields, $q_{\phi_{\pm}} = \frac{1}{2}$ so that the correct SUSY-breaking minimum is achieved \([3]\). The charge assignments \[(21)\] lead to $C_3 = 3$ and from \[(20)\] we obtain $\epsilon \equiv \langle \theta \rangle / M_S \simeq 0.1$.

As noted earlier, the relation \[(20)\] depends on the choice of Kac-Moody levels, which we have fixed. In addition, there are couplings in the superpotential, $\xi_{ij}$, of order one. Here we assume that the values of these couplings (and $k_{\text{grav}}$) are such that $\epsilon \simeq 0.2$ in order to agree with previous parameterizations of the fermion mass hierarchies \([3, 7]\). The important point to note is that the correct order of magnitude for $\epsilon$ is obtained.

Thus we obtain $\lambda \simeq \epsilon^3, \lambda' \simeq \epsilon^5$, and a ratio $\lambda'/\lambda \simeq \epsilon^2 \simeq 10^{-2}$, as required earlier in Sec. 2. The Yukawa coupling matrices for the quarks are determined to be

$$
Y^u \simeq \begin{pmatrix}
\epsilon^8 & \epsilon^5 & \epsilon^3 \\
\epsilon^7 & \epsilon^4 & \epsilon^2 \\
\epsilon^5 & \epsilon^2 & 1
\end{pmatrix}, \quad Y^d \simeq \begin{pmatrix}
\epsilon^6 & \epsilon^4 & \epsilon^6 \\
\epsilon^5 & \epsilon^3 & \epsilon^5 \\
\epsilon^3 & \epsilon^1 & \epsilon^3
\end{pmatrix},
$$

while the lepton Yukawa coupling matrix is given by

$$
Y^l \simeq \begin{pmatrix}
\epsilon^4 & \epsilon^3 & \epsilon^3 \\
\epsilon^3 & \epsilon^2 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & \epsilon^2
\end{pmatrix}.
$$

These Yukawa coupling matrices give the correct phenomenological mass ratios \([14]\) for the quarks

$$
m_u/m_c \simeq \epsilon^4, \quad m_c/m_t \simeq \epsilon^4, \quad m_d/m_s \simeq \epsilon^2, \quad m_s/m_b \simeq \epsilon^2
$$

and the leptons

$$
m_e/m_\mu \simeq \epsilon^2, \quad m_\mu/m_\tau \simeq \epsilon^2.
$$

It should be pointed out that a moderate amount of tuning ($O(\epsilon)$) of the $\xi_{ij}$ coefficients is implicit in order to obtain these mass ratios, since otherwise the rank of these matrices would be generically nonmaximal. This is typically the case for these types of models.
Note that any phase of the $\theta$ field can be rotated into the $\Theta$-angles of the corresponding gauge groups by field redefinitions of all $U(1)_X$-charged fields (including those in the supercolor sector). Thus in order to incorporate CP-violation, it is necessary to assume that the coefficients $\xi_{ij}$ in Eq. (11) are complex. In the next example we will consider a solution with two $\theta$-fields which can have symmetric charge assignments and nontrivial phases in the Yukawa couplings.

**B. Two $\theta$-field case**

Consider a solution with two $\theta$-fields, $\theta$ and $\theta'$, with $U(1)_X$ charges $q_\theta = -1/2$ and $q_{\theta'} = -3/4$. We will use the analytic parametrization presented in Ref. [7] for the solutions of the Green-Schwarz anomaly cancellation conditions (16), (17) with left-right symmetric charge assignments. The most general superpotential allowed by the symmetries is now given by (11) and (11), where either $\theta$ or $\theta'$ (or a product of both) can appear in place of the single field $\theta$, in a way consistent with the $U(1)_X$ charge assignments. We will impose a $Z_2 \times Z_2$ discrete symmetry permitting only an even number of $\theta$ and $\theta'$ fields.

The solution which we describe below has a coefficient of the mixed $U(1)_X SU(3)^2$ anomaly $C_3 = 4$. Generalizing Eq. (20) to the case of two fields, we find that the D-term vanishes for

$$\frac{1}{2} \langle \theta \rangle^2 + \frac{3}{4} \langle \theta' \rangle^2 \simeq (0.2)^2. \quad (26)$$

We will assume that the vevs of the two fields are of the same order of magnitude, $\epsilon = \langle \theta \rangle / M_S \simeq \langle \theta' \rangle / M_S \simeq 0.2$.

The charges of the quark and lepton superfields under the $U(1)_X$ symmetry are as follows

$$\begin{align*}
    i & & q_{Q_i} & & q_{u_i} & & q_{d_i} & & q_{L_i} & & q_{e_i} \\
    1 & & \frac{29}{15} & & \frac{29}{15} & & \frac{11}{5} & & \frac{9}{20} & & \frac{71}{60} \\
    2 & & \frac{13}{30} & & \frac{13}{30} & & \frac{7}{10} & & -\frac{3}{10} & & \frac{13}{60} \\
    3 & & -\frac{17}{30} & & -\frac{17}{30} & & -\frac{3}{10} & & -\frac{11}{20} & & \frac{11}{60} \end{align*} \quad (27)$$

The two Higgs fields have charges $q_{H_U} = 17/15$ and $q_{H_D} = 28/15$. The sum of their charges is therefore $q_H = 3$ and allows for the generation of a $\mu$ term from the superpotential. The messenger field $X$ has charge $q_X = 1$. With these charge assignments, we obtain $\lambda \simeq \epsilon^4, \lambda' \simeq \epsilon^6$, and a ratio $\lambda'/\lambda \simeq \epsilon^2 \simeq 10^{-2}$, as discussed in Sec. 2.

The charge assignments (27) lead to the following quark Yukawa matrices,

\footnote{For supercolor sectors like the ones considered in Ref. [4] (with a single nonperturbative term in the superpotential) this phase will dynamically relax to zero.}
\[ Y^u \simeq \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \quad Y^d \simeq \epsilon^2 \begin{pmatrix} \epsilon^8 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}. \] (28)

Similarly the lepton Yukawa matrix is given by
\[ Y^l \simeq \begin{pmatrix} \epsilon^8 & 0 & \epsilon^4 \\ 0 & \epsilon^4 & 0 \\ \epsilon^4 & 0 & \epsilon^2 \end{pmatrix}. \] (29)

The above Yukawa matrices give the correct mass ratios for the quarks (24) and leptons (25). Note that, in order to reproduce the correct mass ratios for the down type quarks, \( Y^d_{22}, Y^d_{23} \) and \( Y^d_{32} \) must be tuned slightly. The required tuning of the \( \xi \)'s is at most of order \( \epsilon \), so this is acceptable. Also, we presume there is some freedom in the adjustment of \( \langle \theta \rangle / \langle \theta' \rangle \), which affects the \( Y^d_{ij} \) as well as order one.

Finally we would like to note the possibility of spontaneous CP violation in models with anomalous horizontal symmetries with more than one \( \theta \) field. In the example above with two fields, \( \theta \) and \( \theta' \), one linear combination of the two phases\(^5\) can be rotated away into the \( \Theta \) angles of the corresponding gauge groups, and this combination is expected to relax to zero due to nonperturbative effects in the supercolor sector. The other linear combination of the two phases can be rotated away from the SUSY-breaking parameters in the standard model (by field redefinitions of the messenger quarks and leptons and the two Higgs doublets) and appears in the Yukawa couplings and the QCD \( \Theta \) angle. However, this phase is also expected to appear in the supercolor-sector interactions; in order to dynamically determine its vev, one needs to specify in more detail the supersymmetry breaking dynamics. This possible source of CP violation requires further investigation.

\section*{V. CONCLUSION}

Models of low-energy supersymmetry breaking offer the hope for new physics at intermediate energy scales. It is therefore worthwhile to understand what is required in order to make these models realistic in detail. We have indicated here one option for building realistic models of this type; other options have also been studied in the literature \cite{3,1}. We have shown that the fermion mass hierarchy and the values of the \( \mu \) and \( B \) terms in supersymmetric extensions of the standard model with low-energy supersymmetry breaking can have a common origin. The values of all Yukawa couplings are determined by a spontaneously broken, anomalous \( U(1) \) horizontal symmetry, the anomaly being cancelled by the

\(^5\)To be precise, we should note that one linear combination of the string axion and the phases of \( \theta \) and \( \theta' \) becomes the longitudinal component of the \( U(1)_X \) gauge field, which obtains a mass of order the string scale \([12]\). Since possible kinetic-term mixing is not important for our considerations, we can parameterize the two remaining linear combinations of phases in the low energy theory as the phases of \( \theta \) and \( \theta' \).
Green-Schwarz mechanism. As an existence proof we have constructed explicit models of this type with both one and two $\theta$-fields.

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