Output from an atom laser: theory vs. experiment

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Atom lasers based on rf-outcoupling can be described by a set of coupled generalized Gross-Pitaevskii equations (GPE). We compare the theoretical predictions obtained by numerically integrating the time-dependent GPE of an effective one-dimensional model with recently measured experimental data for the $F=2$ and $F=1$ states of Rb-87. We conclude that the output of a rf-atom laser can be well described by this model.

The discovery of Bose-Einstein condensation (BEC) in trapped atomic gases [1,2] and the subsequent demonstration of macroscopic coherence of Bose condensed gases [3] have raised a lot of interest in creating coherent atomic beams using such condensates. Experimental setups based on Bose-Einstein condensates that produce coherent atomic beams are now called atom lasers. The first experimental realization [4] was based on a pulsed outcoupler: high Raman-frequency pulses were used to flip magnetically trapped atoms to untrapped states such that they could leak out of the trap. The method of rf-outcoupling has been extensively investigated theoretically [5,6], whereas the possibility of using Raman transitions [7] has not been treated in such detail. Most recently, the outcoupling process was treated taking also thermal excitations within the trap into account [8].

Experiments have also made some remarkable progress. In [9], a Bose condensate is released from a magnetic trap into an optical standing wave leading to mode-locked like pulses of atoms falling downwards. A method using Raman pulses is demonstrated in [7]. There, the outcoupled atoms are moving horizontally and are pulsed with a very high frequency resulting in a quasi-continuous beam. Finally, Bloch et al. [10] use a rf-outcoupler within a very stable magnetic setup, allowing to operate the atom laser in a regime close to the weak-coupling regime dealt with in [9]. As in all other cases there is no refill mechanism for the Bose condensate in the trap that balances the outcoupling loss. Nevertheless, the outcoupling rate is so low that the output is similar to a cw-laser with a slowly decreasing intensity.

In the following, we want to model the situation in [10]. The trap is elongated horizontally. As the atomic beam is directed downward, we approximate the trap with a 1D model in the vertical $z$-direction taking account of the other two dimensions in a way explained in the next section.

I. THEORY OF COUPLED GROSS-PITAEVSKII EQUATIONS

Condensates of weakly interacting Bose gases at zero temperature are well described by the Gross-Pitaevskii equation (see [9] and references therein). As it is a nonlinear Schrödinger equation for a macroscopic wavefunction it not only accounts for the density properties but also for the coherence inherent in Bose condensates. In [11] a generalization of the GPE was introduced to treat a system of trapped and untrapped states of a Bose gas coupled via an rf field with a frequency $\omega_{rf}$. Here, we want to model an atom laser for Rb-87 in its $F=2$ or $F=1$ hyperfine-manifold. The $2F+1$ Zeeman sublevels are represented by macroscopic wavefunctions $\psi_m (m \in \{-F, \ldots, F\})$ so that the system of generalized GPEs after applying the transformation $\psi_m(t) \rightarrow e^{-i m \omega_{rf} t} \psi_m(t)$ and the rotating wave approximation reads

$$i \hbar \frac{\partial}{\partial t} \psi_m(\vec{r},t) = \left( - \frac{\hbar^2 \nabla^2}{2M} + V_m(\vec{r}) + \hbar \omega_{rf} + U||\psi(\vec{r},t)||^2 \right) \psi_m(\vec{r},t)$$

$$+ \hbar \Omega \sum_{m'} \left( c_{m'} \delta_{m,m+1} + c_m \delta_{m,m-1} \right) \psi_{m'}(\vec{r},t). \quad (1.1)$$

Thereby we have assumed that all Zeemann levels interact with the same s-wave scattering length $a_0 = 110 a_{\text{Bohr}}$ so that $U = 4\pi \hbar^2 a_0^2 N/M$. The total density devided by the number $N$ of atoms in the system is denoted by $||\psi(\vec{r},t)||^2 = \sum_m ||\psi_m(\vec{r},t)||^2$ (we normalize to 1, not to $N$). The Rabi frequency is determined by the strength $B_{rf}$ of the rf-field and given by $\hbar \Omega = g_F \mu_{\text{Bohr}} B_{rf}/2$. It is modified by the matrix elements of the angular operators $F_{\pm}$ represented by $c_m = \sqrt{F(F+1)-m(m+1)}$. For $F=1$, this results in $c_m = \sqrt{2}$; for $F=2$, $c_m$ takes values 2 and $\sqrt{6}$ depending on $m$ so the Rabi frequencies differ for different $m \rightarrow m'$ transitions.

Ioffe-type traps often used in experiments are usually elongated in the horizontal plane being axial symetric in the remaining directions. As mentioned above, we want to model an atom laser based on such traps by 1D GPEs where the axis under consideration is the vertical axis of the trap. We choose coordinates where the $z$-axis points downwards, the long axis of the trap is denoted by $y$, the short horizontal one by $x$. Taking gravity into account,
the total effective potentials in $z$-direction then are given by
\begin{equation}
V_{m,\text{eff}}(z,t) = \text{sgn}(g_F) m M \omega_z^2 z^2/2 + m\hbar \Delta - M g_z + g_{1D}||\psi(z,t)||^2.
\end{equation}
(1.2)

$\Delta = \hbar \omega_{\text{rf}} - V_{\text{eff}}$ is simply the detuning of the transitions ($V_{\text{eff}} = -g_F \mu_{\text{Bohr}} B_{\text{eff}}, \omega_{\text{rf}} > 0$ for $F = 1$ and $\omega_{\text{rf}} < 0$ for $F = 2$).

The effective interaction strength $g_{1D}$ is determined in such a way that the chemical potential in Thomas-Fermi approximation in the 1D model equals that for the full 3D situation for $N$ particles in a trapped state $m_{\text{trap}}$. It turns out to be equal to
\begin{equation}
g_{1D} = \frac{2}{3} \left( \frac{\omega_y}{\omega_z} \right) \left( \frac{15NA_0}{a_z} \right) \frac{|m_{\text{trap}}|}{z_{\text{trap}}}.
\end{equation}
(1.3)

with $a_z = \sqrt{\hbar/(M \omega_z)}$. The 1D version of Eq. (1.1) now reads
\begin{equation}
\begin{aligned}
\hbar \frac{\partial}{\partial t} \psi_m(z,t) &= \left( -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial z^2} + V_{m,\text{eff}}(z,t) \right) \psi_m(z,t) \\
&+ \hbar \Omega \sum_{m'} \left( c_{m'} \delta_{m,m'+1} + c_m \delta_{m,m'-1} \right) \psi_{m'}(z,t).
\end{aligned}
\end{equation}
(1.4)

Gravitation leads to a sag of the trapped atomic cloud. The shift of the minimum of the potentials away from the trap center at $z = 0$ depends on $m$ and is given by $z_{m,\text{sag}} = g/|m|\omega_z^2$. The effective potentials always cross at $z_{\text{res}} = \pm a_z \sqrt{2\Delta/(\hbar \omega_z)}$. Usually, only the positive sign plays a role because the negative value is outside the condensate (cf. inset in Fig. (1)).

For weak coupling $\Omega < \omega_z$ the trapped atoms are only disturbed slightly by the outcoupling process \cite{10}, after a certain propagation time the wavefunctions are quasi-stationary. In Fig. (2) we plot the densities of the $m = -1,0$ states for $F = 1$ in such a situation. The outcoupling takes place primarily at the resonance point $z_{\text{res}}$. The untrapped state can be approximated by an Airy function, its strength being determined by the outcoupling rate which is known analytically \cite{10,20}.

Why is the location of maximal outcoupling identical to $z_{\text{res}}$? This is not so obvious; if one considers the same situation without the mean-field contribution in $V_{m,\text{eff}}$, the density maximum of the untrapped state is located at the classical turning point in its potential, which depends on the energy (analogous to photo-dissociation of molecules). If the mean-field contribution is present like in Eq. (1.4) the classical turning point is shifted to the resonance point $z_{\text{res}}$ (see inset in Fig. (1)), and hence the point of maximum density coincides with $z_{\text{res}}$.

II. NUMERICAL RESULTS

In this section we present some numerical results obtained by propagating the set of coupled GPEs Eq. (1.4) and compare them to experimental data for the same set of parameters. The time propagation was accomplished by a usual split-operator method using FFT on a 1D grid. The starting condition is always the condensate ground state obtained by imaginary time propagation.

The basic parameters in our calculations are taken from \cite{18}, the $|F = 2, m = 2\rangle$-state is trapped with frequencies $\omega_{\perp} = 2\pi \times 180 \text{ Hz}$ and $\omega_{\parallel} = 2\pi \times 19 \text{ Hz}$. Throughout the paper we use harmonic oscillator units with respect to the $|m\rangle = 1$ trapping frequency for the vertical motion $\omega_{\text{trap}} = 2\pi \times 127 \text{ Hz}$ which happens to be the same for both $F = 1$ and $F = 2$ ($g_{F=1} = -1/2$, $g_{F=2} = 1/2$). Thus, the length unit is $a_z = 0.95 \mu \text{m}$, time is measured in $1/\omega_{\text{trap}} = 1.3 \text{ ms}$.

The Rabi frequencies used in \cite{18} are all bigger than $\omega_{\text{trap}}$, so we are not in a weak coupling regime. During the outcoupling process, the condensate is depleted significantly. In Fig. (2), one can see how fast this takes place. The populations of the states show some small oscillations that are reminiscent of the Rabi oscillations occurring at very strong RF fields where there is hardly no outcoupling \cite{11,12}. Such oscillations can be seen in experiment as a sort of pulses \cite{21}.
FIG. 2. Time evolution of the populations for $F = 1$ atom laser. The condensate in the $m = -1$-state is depleted very fast ($\Omega/\omega_{\text{trap}} \approx 3.5$), the time axis extends over 12.5 ms. The period of the small oscillations corresponds to the Rabi frequency $\Omega$. 

In [18] Bloch et al. presented measurements of the number of particles remaining in the $|F = 2, m = 2\rangle$-state after a fixed time of outcoupling depending both on the strength and the frequency of the radio frequency. Figures (3) and (4) display our numerical data for their parameters and the experimental values. In the calculations we took only the $m = 2, 1, 0$-states into account. Tests with more states show that this is enough, as the missing $m = -1, -2$-states are populated only very weakly.

FIG. 3. Number of atoms remaining in $F = 2, m = 2$ after 20 ms of outcoupling. Initially there are $N = 7 \times 10^5$ atoms. The detuning $\Delta$ is tuned to maximal outcoupling, so $z_{\text{res}} \approx z_{\text{zag}}$. $B_{\text{rf}}$ is varied from 0.1 to 1.0 mG leading to Rabi frequencies from 0.44 to 4.4 kHz. Lines are drawn for guiding the eye only.

FIG. 4. Same as in Fig. (3), but now $\Omega = 2\pi \times 700$ Hz $= 5.5\omega_{\text{trap}}$ is kept fixed and the detuning $\Delta$ (resp. radio frequency) is varied. The experimental data is shifted in frequency to match the minimum of the theoretical curve; this is necessary because the offset field $B_{\text{off}}$ of the trap is not known with high enough precision. Again, lines are there for guiding the eye.

The theoretical values fit the experimental data at least qualitatively. The differences are most probably due to dimensional effects: in the 1D model there is only a resonance point at $z_{\text{res}}$, whereas in 3D the outcoupling takes place on a resonance surface defined by the distance $r_{\text{res}} = z_{\text{res}}$ to the trap center. Accordingly, the outcoupling rate in 3D depends on the density of the trapped state averaged over the resonance surface which is smaller than the density on the $z$-axis. Thus, the outcoupling yield in 3D is smaller than in 1D because there is only the “on-axis” density.

This effect is even stronger for the $F = 1$ results shown in Fig. (5); the experimental data are again from the Hänisch group [21]. In this case, all three Zeeman sublevels were propagated because they are all significantly populated. (see e.g. Fig. (2)). We have tried to quantify the aforementioned difference between the 1D- and 3D-situation including gravity at least in the weak coupling limit in the spirit of the perturbational rates from [10,20] but have not found a conclusive answer.
FIG. 5. Same as in Fig. (4) but for $F = 1$, $m = -1$, and $\Omega = 2\pi \times 312$ Hz = $2.4 \omega_{\text{trap}}$. The curve at weaker coupling ($\Omega = 1.1 \omega_{\text{trap}}$) clearly exhibits an asymmetry reflecting the density distribution of the trapped state (see text). Mind also the bump around $\Delta = 120 \omega_{\text{trap}}$ in the “theory” curve.

A closer look on the spectral data in Figs. (4) and (5) reveals two further details: from a weak coupling analysis one expects the curves in these figures to reflect directly the density distribution of the trapped states transferred to the frequency domain via $z = a_z \sqrt{2\Delta / (\hbar \omega_{\text{trap}})}$. This leads to an asymmetry around the minimum that can be seen in the curve at weaker coupling shown in Fig. (5).

The second detail concerns the small bump visible in the “theory”-curve in Fig. (5) around $\Delta = 120 \omega_{\text{trap}}$. It occurs only for intermediate couplings. We have no explanation for this feature.

III. CONCLUSIONS AND OUTLOOK

In this paper, we have described a numerical analysis for an RF-atom laser. The output yield of such a device can be determined in a satisfactory way though the restriction to one dimension does not allow for an exact quantitative comparison with the real world data form experiment.

What is lacking is a more analytical treatment of the outcoupling process for intermediate coupling strengths, which would probably also explain the bump in Fig. (5). Furthermore, it would be nice to have a at least 2D picture for the outcoupled wavefunction to discuss things like the beam divergence. A 2D treatment would also be able to account for the long axis of the trap where collective excitations of the condensate can occur much easier than in the other two directions (cf. [13]).

Another interesting subject is an atom laser driven by two radio frequencies. This may lead to a pulsed operation mode like in a mode-locked laser. We are investigating such setups, results will be published elsewhere.

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