Quantal Definition of the Weak Equivalence Principle

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The present work analyzes the meaning of the Weak Equivalence Principle in the context of quantum mechanics. A quantal definition for this principle is introduced. This definition does not require the concept of trajectory and relies upon the phase shift induced by a gravitational field in the context of a quantum interference experiment of two coherent beams of particles. In other words, it resorts to wave properties of the system and not to classical concepts as the idea of trajectory.

I. INTRODUCTION

The modern idea of gravitation, at least in its classical description, is based upon the postulates of metric theories \[1\], which embody the so–called Weak Equivalence Principle (WEP). This principle postulates: If an uncharged test body is placed at an initial event of space–time and given an initial velocity, then its subsequent trajectory will be independent of its internal structure and composition \[1\].

This principle has its experimental foundation in the universality of free fall, an experiment first performed by Galileo \[2\], and it entails a relation between two different concepts of mass, i.e., inertial mass \((m_i)\) and passive gravitational mass \((m_p)\). Indeed, the classical equation of motion for a point–like particle immersed in a homogeneous gravitational field (along the \(z–\)axis) reads

\[
m_i \frac{d^2 \vec{r}}{dt^2} = -m_p g \hat{z}. \tag{1}
\]

WEP states that the solution to this equation depends upon initial conditions, but not on the internal structure of the considered particle. In other words, it is based upon the fact that

\[
\frac{m_i}{m_p} = \alpha, \tag{2}
\]

is a universal constant, i.e., the same for all particles.

At this point two remarks have to be done. Firstly, WEP requires as a necessary condition a particle–independent value of \(\alpha\). In other words, the particular value of \(\alpha\) is completely irrelevant. This may be understood if we introduce \[2\] into the motion equation

\[
\frac{d^2 \vec{r}}{dt^2} = \hat{g} \hat{z}. \tag{3}
\]

Here \(\hat{g} = g/\alpha\). A value of \(\alpha \neq 1\) would entail a redefinition of the acceleration of gravity. In an experiment we cannot detect \(g\) or \(\alpha\), only the effective acceleration of gravity, i.e., \(\hat{g} = g/\alpha\). This proves that the value of \(\alpha\) (assuming the validity of WEP) is irrelevant, it cannot be measured. Secondly, the geometrization of gravity requires WEP. In other words, if \(\alpha\) is particle–dependent, then WEP breaks down and therefore the geometrization of gravity would be a dubious issue, at least with a four–dimensional manifold. Finally, let us add that any experimental test of the...
validity, or invalidity, of WEP requires the use of more than one particle. Indeed, WEP is based upon the universal character of $\alpha$ and one particle cannot provide us this kind of information.

The mathematical version of WEP is done stating: The motion of an uncharged particle is provided by the geodesics of the corresponding manifold $[1]$. Notice that behind this phrase lies the concept of trajectory. The experimental verification of WEP in the classical level has been done in different approaches and devices $[2]$ but the quantum–mechanical version demands a careful analysis. This last remark acquires relevance since in quantum theory the concept of trajectory is not a necessary one, i.e., the theory can be formulated without it $[4]$. We may cast this last idea in the following manner: Is it conceptually correct to formulate WEP in the validity region of quantum theory in terms of a concept which happens to be not a fundamental one in quantum theory? This last comment leads us to the main question in the present work. Indeed, since WEP is formulated in terms of trajectories and quantum mechanics does not contain this notion as a central one, how could we formulate WEP in terms of ideas fundamental in quantum theory? The present work puts forward a definition in this sense. This will be done resorting to the idea of interference pattern, which is a wave–like property.

II. GRAVITY–INDUCED PHASE SHIFTS AND WEP

![Experimental device.](image)

A. Phase–Shifts and Gravity

The first experiments (usually denoted by COW) showing how gravity appears in the realm of quantum theory can be considered to begin in 1974 $[5, 6]$. The phenomenon dealt with in this series of experiments can be denoted a gravity–induced quantum interference. They embody an interference pattern of thermal neutrons induced by gravity. A brief description of this experiment is the following one $[5, 6]$. The idea is to use an almost monoenergetic beam of thermal neutrons. This last statement means a kinetic energy of about 20 MeV, which is tantamount to a speed of $2000 \text{ms}^{-1}$. The primary beam is split into two parts, such that each one of the new beams travels along different paths. These paths define a parallelogram of sides $l_1$ and $l_2$ (see Fig. 1). If these two paths lie in a horizontal plane, then, there is vanishing relative phase shift between the two beams. If we now rotate the plane, say an angle $\theta$ around any of its arms, then a non–vanishing relative phase shift appears, due to the fact that the paths, these secondary beams follow, are located at heights associated with different gravitational potential. It can be shown that the phase difference between the two secondary beams, $\Delta \phi$, has the following form $[5, 6, 7, 8, 9]$

$$\Delta \phi = \frac{m^2 g l_1 l_2 \lambda \sin(\theta)}{\hbar^2}. \hspace{1cm} (4)$$

In this last expression $m$ and $\lambda$ denote the mass of the neutrons and the de Broglie wavelength of the neutrons, respectively. Notice that in all these experiments no distinction is introduced between inertial and passive gravitational
mass \( m_i \). The dependence of (4) on the mass of the involved particles has originated a hot debate about the possible presence of a non–geometric element of gravity at the quantum realm, and in consequence the claim of a violation of WEP has been put forward \[10, 11\]. This ambiguity of COW in relation with WEP appears even in some textbooks on quantum mechanics, see, for instance, the last paragraph and footnote on page 129 in [3].

Of course, we may find in the literature the opposite position. For instance, already in the abstract of [12] we may find the following phrase: *We show that the COW experiment respects the equivalence principle.* This work [12] is an interesting one and sheds light upon some similarities between the classical and quantal versions of the experiment.

Other results within the context of neutron interferometry address different issues related to this experiment. For instance, the influence of spacetime torsion upon these experiments has already been analyzed [13], as well as the general relativistic description of them [14]. Nevertheless, these works do not tackle the relation between the mass dependency of the phase shift and WEP. As a matter of fact, a fleeting glance at these works allows us to confirm that none of them establishes a distinction between inertial and passive gravitational mass. In other words, they take by granted WEP.

Concerning this debate around the validity of WEP in the context of quantum theory we will take no sides. The appearance of mass in expression (4) seems to the authors neither enough to state that WEP is violate, nor fulfilled. The situation calls for some new solution that could lead to the disentanglement of the problem.

Our contribution lies along this direction. One possibility has already been considered, at least theoretically, i.e., free fall of quantum particles, with and without classical analogue [15]. Nevertheless, as pointed out the use of this idea seems to imply a redefinition of WEP for quantum theory since no quantum object can reproduce the classical concept of deterministic trajectory and the macroscopic version of WEP is formulated in terms of trajectories [15].

The present proposal is a slight generalization of COW, in order to consider the possibility of testing directly a quantal version for WEP. This definition does not require the concept of trajectory and relies upon the phase shift induced by a gravitational field in the context of a quantum interference experiment of two coherent beams of particles, i.e., it resorts to wavelike properties. Additionally, an experimental proposal designed to test it is also here put forward.

**B. Quantum WEP**

We will address the issue from its most fundamental aspect. Firstly, as mentioned in the introduction WEP is based upon the fact that \( m_i/m(p) \) is particle independent. This condition implies the universality of free fall. Secondly, quantum mechanics does not require the concept of trajectories. In other words, a quantal version of WEP based upon the idea of trajectory seems conceptually a mistake. Hence, the problem we face is to define in a consistent way the meaning of WEP.

A possibility is to define WEP operationally as follows. Let us re-write the phase shift introducing from square one the distinction between inertial and passive gravitational masses. Then we obtain

\[
\Delta \phi = \frac{m_i m(p) g l_1 l_2 \lambda \sin(\theta)}{\hbar^2} \quad \text{(5)}
\]

Introducing \( \frac{m_i}{m(p)} = \alpha \) we have

\[
\alpha = \frac{m_i^2 g l_1 l_2 \lambda \sin(\theta)}{\hbar^2 \Delta \phi} \quad \text{(6)}
\]

This expression allows us to put forward a quantal WEP definition. Indeed, a fleeting glance at (6) shows us that now we may distinguish between different particles, in the sense that \( \alpha \) could be particle–dependent. The usual expression, see [14], assumes that \( \alpha = 1 \) for all particles, and in consequence assumes from the very beginning the validity of WEP.

**Definition:** Let us consider a COW device, i.e., the geometry of it is already given. This means \( l_1, l_2 \) the lengths of the arms, and the tilt angle \( \theta \) are fixed parameters, and also \( g \). WEP in the quantum realm means that
\[ \alpha = \frac{m^{(1)}_{(i)} g l_1 l_2 \lambda \sin(\theta)}{\hbar^2 \Delta \phi}, \]  

(7)

is a universal parameter, i.e., independent of the mass of the employed particles. Before proceeding let us mention that this definition is motivated by two factors. Firstly, as mentioned before the core behind the classical WEP is encoded in a universal value for \( \alpha \), and in this sense our definition embodies the most substantial factor involved in the macroscopic version of this principle. In other words, our definition encompasses the main ingredient behind the macroscopic version of WEP. Secondly, the idea is to have a feasible experimental verification of it. Since our principle resorts to COW we need a slight generalization of it to have a possibility to falsify our definition.

C. Experimental Proposals

At our disposal, as experimental parameters, we have the arms length of the interferometer, the wavelength of the particle, and the tilt angle. This remark entails us to introduce three different variations of the same kind of experiment.

1. First Possibility: Wavelength

The first series of experiments modifies only the wavelength of the particle. With our arm lengths \( l_1 \) and \( l_2 \) and a certain non–vanishing tilt angle \( \theta \) we tune the velocity of our first type of particles (whose inertial mass will be denoted by \( m^{(1)}_{(i)} \)). Hence, since momentum and wavelength satisfy de Broglie’s relation, i.e., \( \lambda = h/p \), (6) becomes

\[ \alpha^{(1)} = \frac{m^{(1)}_{(i)} g l_1 l_2 \sin(\theta)}{h v \Delta \phi}. \]  

(8)

Here \( \alpha^{(1)} \) takes into account the possible particle–dependence of this parameter. Suppose now that we perform our experiment in such a way that our first case for the velocity, \( v^{(1)} \), provides a constructive interference pattern, hence \( \Delta \phi = 2\pi n \), where \( n \) is an integer. Our second experiment employs a velocity, \( v^{(2)} \), such that it provides the nearest destructive interference pattern to the first experiment (there are two velocities, one above \( v^{(1)} \) and one below it fulfilling this condition). Then we have the following two expressions

\[ 2\alpha^{(1)} \pi n = \frac{m^{(1)}_{(i)} g l_1 l_2 \sin(\theta)}{h v^{(1)}}, \]  

(9)

\[ \alpha^{(1)} \left[ 2\pi n \pm \pi \right] = \frac{m^{(1)}_{(i)} g l_1 l_2 \sin(\theta)}{h v^{(2)}}. \]  

(10)

With them we obtain

\[ \alpha^{(1)} \pi = \frac{m^{(1)}_{(i)} g l_1 l_2 \sin(\theta)}{h} \sqrt{\left[ \frac{1}{v^{(2)}} - \frac{1}{v^{(1)}} \right]^2}. \]  

(11)

We now carry out the same experiment with a different kind of particles (\( m^{(2)}_{(i)} \)) and obtain

\[ \alpha^{(2)} \pi = \frac{m^{(2)}_{(i)} g l_1 l_2 \sin(\theta)}{h} \sqrt{\left[ \frac{1}{w^{(2)}} - \frac{1}{w^{(1)}} \right]^2}. \]  

(12)
Here \( w(2) \) and \( w(1) \) are the corresponding velocities, which do not need to be equal to \( v(2) \) and \( v(1) \). Notice that in this way we may deduce the value of \( \alpha(n) \), \( n = 1, 2 \). We may now put forward an experimental proposal for testing the just introduced quantal WEP definition. Let us assume that we perform several times, resorting to different kind of particles, this experiment. According to our definition \( \alpha \) has to be particle-independent, hence, if WEP is to be fulfilled, then for different particles the same value for \( \alpha \) shall emerge. More than one value would imply the breakdown of WEP.

2. Second Possibility: Arm Length

If the tuning up of the velocity becomes a nuisance, then the possibility of having several samples of the silicon crystal interferometer could be exploited, and in this way we obtain an expression tantamount to (11). One of the arms, say \( l_1 \) takes to different values. From now on we write for these two values \( l_1(i) \), where, \( i = 1, 2 \). This means that in this situation we have (here the velocity \( v \) is the same for both experiments)

\[
\alpha^{(1)} = \frac{m_1 v g l_2 \sin(\theta)}{\hbar} |l_1(1) - l_1(2)|.
\]  

(13)

Clearly, in this manner we may compare the value of \( \alpha \) for different type of particles.

3. Third Possibility: Acceleration of Gravity

The third possibility could be the easiest to handle. It encompasses different values for the acceleration of gravity. This does not mean, necessarily, the use of satellites, though this is also an additional possibility. A fleeting glance to \( \text{[14]} \) shows us that in COW we have an effective acceleration of gravity given by

\[
\tilde{g} = g \sin(\theta).
\]

(14)

Then we may change the effective acceleration of gravity with \( \theta \). This means that now we have two different values for our effective acceleration \( \tilde{g}^{(i)} \), \( i = 1, 2 \) such that

\[
\alpha^{(1)} = \frac{m_1 v l_2}{\hbar} |\tilde{g}^{(1)} - \tilde{g}^{(2)}|.
\]

(15)

Once again we may use our derived expression with different kind of particles and test our quantal WEP definition.

We may wonder what information could be elicited from \( \text{[6]} \) in connection with the present model. Figure 2 on page 1473 allows us to estimate (using expression (2) on 1472) in a rough way the value of \( \alpha \) for neutrons. Indeed, consider the first maximum and minimum to the right of \( \phi = 0 \). The angle, in degrees, of the crest is, approximately, 2, while the valley is around 5 degrees. The experimental parameters are all given except the value of the acceleration of gravity, i.e., \( g \). Taking a value of \( g = 9.83 m/s^2 \) we obtain \( \alpha = 0.99951 \) for the case depicted in \( \text{[6]} \). Comparing against the result obtained in \( \text{[16]} \) (\( \alpha \in [0.99991, 1.00041] \)) we conclude that our rough procedure provides a reasonable value for \( \alpha \).

III. CONCLUSIONS

A simple definition for a quantal WEP version has been put forward, the one does not require the concept of trajectory and that also includes for macroscopic bodies the main element behind this principle. Our proposal is
connected with the phase shift appearing in COW and measures a possible particle dependency in the relation inertial mass–passive gravitational mass. Three different experiments have been also mentioned. Finally, let us address an additional issue which has to be contemplated in the context of the feasibility of the present proposal. As underlined before, different kinds of particles are required in order to test WEP, i.e., neutron interferometry does not suffice. Therefore, we must mention some other systems that could be employed in an interference experiment of this sort. The first atomic interferometry experiment is already an old experiment [17], though in this case it is a Young type–like device, i.e., it measures space coherence, and not time coherence as in a COW experiment, which can be catalogued as a Mach–Zehnder situation [18]. Additional results may already be found in the extant literature [19, 20, 21, 22]. These last works prove that atomic and even molecular interferometry are already a real possibility. These devices introduce different sort of particles, a factor which is the key element in our work. In other words, we may contemplate the possibility of resorting to atomic interferometry in the detection of gravity–induced phase shifts. Of course, this requires additional work, and at this point it defines an experimental problem.

Finally, modern elementary particle theories regard, for instance, the neutron as a system embodying more than one elementary particle called quarks [23]. In our experiment we do not take into account this structure. The reason for it is quite simple, namely, the quark confinement hypothesis, i.e., free quarks are never seen in isolation. Indeed, the quark confinement hypothesis, which can be understood as an implication of the asymptotic freedom, tells us that the force between two quarks increases at lower energies as they are separated more and more from one another. In other words, it seems that a possible detection of the quarks, defining a neutron, by COW could imply a violation of the quark confinement hypothesis.

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