Analysis of Allowable Assembly Forces for Composite Laminates

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Abstract. Composite structures have been widely used in aerospace field due to their low density, high strength, and high stiffness-to-weight ratio. In the aerospace field, composite laminates are widely used in the skin of the airplane. Due to the complexity of the composite molding process, and the difficulty of machining composite laminates, the manufacturing precision of the skin is usually not very high. Hence the assembly force is applied to the skin to ensure the composite laminates fit the framework in the assembly process of an airplane. However, too large assembly force leads to the assembly out of tolerance owning to the over deformation and leads to the stress of the skin out of tolerance. Therefore, this paper proposes a method of analyzing the allowable assembly force. With the help of mechanics and physics, the mechanical relationship between the assembly forces and the deformation of a skin is modeled. On the basis of the model and the experimental data of a skin assembly process, the mathematical relationship between the assembly forces and the deformation of the skin is established. The allowable assembly forces are determined based on the mathematical relationship and the allowable deformation. This paper provides an optimization methodology of assembly force optimization.

1. Introduction
Composite structures have been widely used in practice due to their low density, high strength, and high stiffness-to-weight ratio [1]. In aerospace field, composite laminates are used in the skin of the airplane. Due to the complexity of the composite molding process, and the difficulty of machining composite laminates, the manufacturing precision of the skin is usually not very high. Hence the assembly force is applied to the skin to ensure the composite laminates fit the framework in the assembly process of an airplane. However, too large assembly force leads to the assembly out of tolerance owning to the over deformation and leads to the stress of the skin out of tolerance.

In recent years, research has been done in the area of assembly deviation modeling and analysis for the assembly of sheet metal compliant parts. Liu and Hu [2] proposed an offset element method to predict assembly deviation of sheet metal for a one dimensional model by combining engineering structural mechanics with statistical methods. Moreover, assembly tolerances in series and in parallel were evaluated by using linear mechanics [3], which showed that the assembly deviation can be less than the “stacked up” of component deviations in compliant part assembly due to the fact that assembly deviation was affected not only by the geometry of the components but also by the stiffness of the components in parallel assembly. Liu and Hu [4] expanded the model to compliant sheet metal parts with 2D or 3D free-form surfaces by using FEM to construct a sensitivity matrix relating the
incoming part deviation to the output assembly deviation. This method was called the method of influence coefficient (MIC). MIC was further extended to multi-station assembly processes with compliant parts by Camelio et al. [5] following the concepts of the stream-of-variation theory [6]. It is assumed that sources of variation are independent in the paper [5]. However, the independence assumption may not always be adequate [7]. Camelio et al. [7] presented the effect of geometric covariance of sources of variation on the calculation of assembly variation of compliant parts with consideration of the dependence of sources of variation. All the papers above are about the assembly variation.

The purpose of this paper is to develop a method of determining the allowable force. First of all, the mechanical relationship between the assembly forces and the deformation is modeled, on the basis of the mechanics and physics. And then, an experiment of a skin is done to obtain the assembly force and the deformation of the skin. In further, the mathematical relationship between the assembly force and the deformation of the skin is constructed. Finally, the allowable assembly force is determined based on the allowable deformation and the mathematical relationship.

2. Derivation of the Relationship between the Assembly Force and the Assembly Deformation

Please follow these instructions as carefully as possible so all articles within a conference have the same style to the title page. This paragraph follows a section title so it should not be indented.

A lot of papers study how to model the assembly deviation for the sheet metal assembly [2-7]. For an isotropic sheet metal assembly, the relationship between the assembly force in the z direction and the structure deformation in the z direction for isotropic sheet metal is represented in Eq. 1, which was developed in the MIC literature [5, 7].

\[ F = K \times V \]  

where matrix F is the force matrix, matrix V the deformation matrix, matrix K the stiffness matrix. Matrix K represents the force exerted to the sheet metal if the sheet metal produces unit deformation. It is noticed that the relationships between the assembly force and the structure deformation for isotropic sheet metal and anisotropic composite laminates are different. Because composite material is anisotropic material, coupling effects exist in non-symmetric composite laminates. For example, tension (or compression) deformation and shear deformation will be created when bending assembly load is exerted on a non-symmetric composite laminate [8].

To model dimensional deviation for the assembly process of composite laminates, a new finite element equation expressing the relationship between the external forces and the deformation, as shown in Eq.2, is derived on the basis of finite element theory. The derivation process is presented in [9].

\[
\begin{bmatrix}
K_{uu} & K_{uv} & K_{uu} \\
K_{vu} & K_{vv} & K_{vu} \\
K_{ww} & K_{ww} & K_{ww}
\end{bmatrix}
\begin{bmatrix}
u\\v\\\omega
\end{bmatrix}
= \begin{bmatrix}
u\\v\\\omega
\end{bmatrix}
\begin{bmatrix}
F^u \\
F^v \\
F^\omega
\end{bmatrix}
\]  

where each sub-matrix in Eq. 2 can be further represented as

\[
K_{rs} = \begin{bmatrix}
k_{11} & k_{12} & \Lambda & k_{1n} \\
k_{21} & k_{22} & \Lambda & k_{2n} \\
M & M & O & M \\
k_{n1} & k_{n2} & \Lambda & k_{nn}
\end{bmatrix}
\]  

\[
r = \begin{bmatrix}
r_1 \\
r_2 \\
r_n
\end{bmatrix}
\]  

\[
(r, s = u, v, \omega)
\]}
here the mechanics meaning of the term $K_{rs}^{rs}$ in the structure stiffness matrix is the force in the $r$ direction needed from the $i$th node to produce the unit displacement in the $s$ direction at the $j$th node, and $n$ is the total number of nodes.

In the assembly process, there are only forces in the $z$ direction in the assembly process. It indicates that

$$F^u = 0_{n 	imes 1} \text{ and } F^v = 0_{n 	imes 1}$$

Substitute Eq. 6 into Eq. 2, and with proper derivations, we get

$$K^{o\omega} \times \omega = F^{o\omega}$$

where $K^{o\omega}$ is called as the equivalent structure stiffness matrix to distinguish it from the structure stiffness matrix $K$. The expression of $K^{o\omega}$ can be presented as

$$K^{o\omega} = K^{o\omega} \times (K^{o\omega} \times (K^{o\omega})^{-1} \times K^{o\omega})^{-1} \times (K^{o\omega} \times (K^{o\omega})^{-1} \times K^{o\omega} - K^{o\omega}) \times (K^{o\omega} \times (K^{o\omega})^{-1} \times K^{o\omega})^{-1} \times (K^{o\omega} \times (K^{o\omega})^{-1} \times K^{o\omega} - K^{o\omega}) + K^{o\omega})^{-1} \times K^{o\omega}$$

Eq.7 indicates the mechanical relationship between the force in the $z$ direction and the deformation of the assembly in the $z$ direction.

### 3. Allowable Assembly Force of a Skin of an Airplane

The gap always exists between the framework of an airplane and the skin of an airplane due to the forming error of the skin and the machining error and the assembling error of the framework. To close the gap, the assembly force has to been applied to the skin. However, too large assembly forces can lead to the over deformation of a skin. Therefore, it is important to obtain the allowable assembly force. Since the allowable assembly force is the extreme value, it is harmful to a skin if the allowable assembly force is attained only by the experiment. Thereby, some experiments are done to monitor the values of the assembly forces and the assembly deformation which are much smaller than the extreme values. Substitute the values of the assembly forces and the assembly deformation to Eq. 7 to construct the mathematical relationship of the assembly force and the assembly deformation of a skin. On the basis of the mathematical relationship and the allowable deformation of a skin, the allowable assembly force can be obtained.

#### 3.1. Measuring of the Assembly Force and the Assembly Deformation of a Skin.

To measure the assembly force and assembly deformation of a skin, an measuring equipment is designed and manufactured. As shown in Figure 1, the measuring equipment consists of three parts: ① base, ② pressure mechanism, and ③ deformation measuring mechanism. The base is used to fix the skin of an airplane. The pressure mechanism can slide along the rail to measure the pressure applied to any region of the skin through the manometers, and the deformation measuring mechanism can also slide along the rail to measure the deformation of any region of the skin.
Based on Eq.10, the stiffness matrix $K^{eo}$ can be obtained as
3.3 Calculation of the Allowable Assembly Force

Since the allowable deformation of the skin is 5mm, the allowable assembly force is obtained as

\[
K^{\text{allow}} = \begin{bmatrix}
2.0033 & 1.4599 & 1.2247 & 1.2166 \\
1.1135 & 1.3042 & 1.1439 & 1.0182 \\
1.6369 & 1.4861 & 1.6225 & 1.5507 \\
0.9518 & 1.1915 & 1.3179 & 2.0634
\end{bmatrix}
\]

\[
\begin{bmatrix}
F_1^a \\
F_2^a \\
F_3^a \\
F_4^a
\end{bmatrix} = \begin{bmatrix}
k_{11}^{\text{allow}} & k_{12}^{\text{allow}} & k_{13}^{\text{allow}} & k_{14}^{\text{allow}} \\
k_{21}^{\text{allow}} & k_{22}^{\text{allow}} & k_{23}^{\text{allow}} & k_{24}^{\text{allow}} \\
k_{31}^{\text{allow}} & k_{32}^{\text{allow}} & k_{33}^{\text{allow}} & k_{34}^{\text{allow}} \\
k_{41}^{\text{allow}} & k_{42}^{\text{allow}} & k_{43}^{\text{allow}} & k_{44}^{\text{allow}}
\end{bmatrix} \times \begin{bmatrix}
5 \\
5 \\
5 \\
5
\end{bmatrix} \left(11\right)
\]

Therefore, the allowable assembly force of the skin is around 29N at point 1, around 23 N at point 2, around 31N at point 3, and around 28 N at point 4.

4. Conclusions

This paper developed a method of determining the allowable assembly forces and optimizing the assembly forces. In the method, an equivalent stiffness matrix is defined to construct the mechanical relationship between the assembly forces and structure deformation with the help of mechanics and physics. One case study of a skin assembly process is conducted. Depending on the experimental data of the assembly force and skin deformation and the mechanical relationship, the mathematical relationship between the assembly forces and the skin deformation is modeled. The allowable assembly forces are finally determined based on the allowable skin deformation.

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