Delineating effects of tensor force on the density dependence of nuclear symmetry energy

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Abstract. In this talk, we report results of our recent studies to delineate effects of the tensor force on the density dependence of nuclear symmetry energy within phenomenological models. The tensor force active in the isosinglet neutron-proton interaction channel leads to appreciable depletion/population of nucleons below/above the Fermi surface in the single-nucleon momentum distribution in cold symmetric nuclear matter (SNM). We found that as a consequence of the high momentum tail in SNM the kinetic part of the symmetry energy $E_{\text{sym}}^{\text{kin}}(\rho)$ is significantly below the well-known Fermi gas model prediction of approximately $12.5(\rho/\rho_0)^{2/3}$. With about 15% nucleons in the high momentum tail as indicated by recent experiments at J-Lab by the CLAS Collaboration, the $E_{\text{sym}}^{\text{kin}}(\rho)$ is negligibly small. It even becomes negative when more nucleons are in the high momentum tail in SNM. These features have recently been confirmed by three independent studies based on the state-of-the-art microscopic nuclear many-body theories. In addition, we also estimate the second-order tensor force contribution to the potential part of the symmetry energy. Implications of these findings in extracting information about nuclear symmetry energy from nuclear reactions are discussed briefly.

1. Introduction

Nuclear symmetry energy $E_{\text{sym}}(\rho)$, which encodes the energy related to neutron-proton asymmetry in the nuclear matter Equation of State (EOS), is a vital ingredient in the theoretical description of neutron stars and of the structure of neutron-rich nuclei and reactions involving them. Since the density-dependence of $E_{\text{sym}}(\rho)$ is still the most uncertain part of the EOS of neutron-rich nucleonic matter especially at supra-saturation densities, to better determine the $E_{\text{sym}}(\rho)$ has become a major goal of both nuclear physics and astrophysics [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. While significant progress has been made recently in narrowing down the symmetry energy near normal nuclear matter density $\rho_0$, see, e.g., [15, 16, 17, 18, 19, 20, 21, 22, 23, 24], much more efforts are needed to pin down the $E_{\text{sym}}(\rho)$ at both sub- and supra-saturation densities. Moreover, it is now broadly recognized that essentially all of the constraints extracted from experimental data are model dependent. Thus, to make
further progress in the field, it is imperative to identify clearly the key physics ingredients determining the density dependence of nuclear symmetry energy in each model [9, 25]. Besides different techniques used in various nuclear many-body theories, several ingredients, such as, the spin-isospin dependence of the three-body force, tensor force induced high momentum tail in the single-nucleon momentum distribution of symmetric nuclear matter (SNM) and the associated isospin-dependence of short-range two-nucleon correlations, the isospin-dependence of nucleon pairing and clustering at low densities, are particularly known to affect significantly the $E_{sym}(\rho)$. Of course, these ingredients may be approximately equally important and interfere strongly at some densities but individually dominate at other densities in models where they are all considered. In reality, however, they are rarely all taken into account simultaneously in a given model. Also, among these ingredients effects of the tensor force are least known so far. For instance, in most of the Relativistic Mean-Field (RMF) models, the $E_{sym}(\rho)$ are determined by the coupling schemes and properties of the $\rho$ and $\delta$ mesons. Generally, no tensor coupling is considered. In phenomenological models, such as the Skyrme and/or Gogny Hartree-Fock approaches, the spin-isospin dependence of the three-body force is the most uncertain term determining the density dependence of the $E_{sym}(\rho)$ while effects of the tensor forces are normally not considered either. On the other hand, most of the more microscopic many-body theories using modern nucleon-nucleon interactions have incorporated all major ingredients affecting the $E_{sym}(\rho)$ albeit at different levels. Because of the different many-body approaches and interactions used, although all these models are well established and transparent, it has been hard to identify the main causes for their different predictions for the $E_{sym}(\rho)$. To our best knowledge, currently there is no community consensus regarding the underlying physics responsible for the uncertain density dependence of nuclear symmetry energy especially at supra-saturation densities.

Why is the density dependence of nuclear symmetry energy so uncertain and what are the effects of the tensor force? To help answer these questions, using simple and phenomenological approaches [26, 27], we have recently investigated effects of the tensor force on the kinetic ($E_{sym}^{kin}(\rho)$) and potential ($E_{sym}^{pot}(\rho)$) parts of the symmetry energy, separately. In this talk, we report the most important findings of these studies. The most striking finding is that, unlike the free Fermi gas model prediction $E_{sym}^{kin}(FG)(\rho) \equiv (2^{5} - 1) \left( \frac{3}{5} \frac{\hbar^{2}k_{F}^{2}}{2m} \right) \approx 12.5\rho^{2/3}$ that has been widely used in both nuclear physics and astrophysics, the tensor force induced high momentum tail in the single-nucleon momentum distribution in SNM reduces significantly the $E_{sym}(\rho)$ to values much small than the $E_{sym}^{kin}(FG)(\rho)$. In fact, the $E_{sym}^{kin}(\rho)$ can become zero or even negative if more than about 15% nucleons populate the high-momentum tail above the Fermi surface as indicated by the recent experiments done at the Jefferson National Laboratory (J-Lab) by the CLAS Collaboration [28]. It is very encouraging to note that not only this finding was very recently confirmed qualitatively by three independent studies using the state-of-the-art microscopic many-body theories [29, 30, 31, 32], our calculation of the direct but second-order tensor contribution to the $E_{sym}^{pot}(\rho)$ was also qualitatively verified by a more accurate calculation [33].

2. Tensor force induced short-range nucleon-nucleon correlation and its effect on single-nucleon momentum distribution in symmetric nuclear matter

Our work was largely stimulated by the recent progress in studying the tensor force induced nucleon short-range correlation (SRC) and its consequence in single-nucleon momentum distribution in SNM. Microscopic many-body theories indicate that the tensor force affects both the single-nucleon momentum distribution $n(k)$ and the two nucleon momentum distribution (as a function of their total and relative momenta). Both have been extensively studied theoretically and experimentally. For reviews, see, e.g., refs. [34, 35, 36]. Firstly, it is well known that both the short-range repulsive core and the tensor force acting in the isosinglet neutron-proton interaction
channel lead to SRC. Consequently, some nucleons are expected to be expelled from below to
above the Fermi surface leading to a high momentum tail in the single-nucleon momentum
distribution. Moreover, since the hard core is approximately the same for all nucleon pairs while
the tensor force only acts between isosinglet neutron-proton pairs, the difference in single-nucleon
momentum distributions in SNM and pure neutron matter (PNM) is mainly due to the tensor
force induced SRC. Since the isospin-dependence of SRC caused by the tensor force will affect
differently the EOSs of PNM and SNM especially at high densities, the tensor force is expected
to play an important role in determining the $E_{\text{sym}}(\rho)$ which can be written as the difference
between the energy per nucleon in PNM and SNM, i.e., $E_{\text{sym}}(\rho) = E_{\text{PNM}}(\rho) - E_{\text{SNM}}(\rho)$, within
the parabolic approximation of the EOS of isospin-asymmetric nuclear matter. On the other
hand, effects of the short-range repulsive core which is essentially isospin-independent on the $E_{\text{sym}}(\rho)$ largely cancel out. We noticed that because the local density of SRC pairs in nuclei is
estimated to reach that expected in the core of neutron stars, it has been repeatedly speculated
that the observed isospin-dependence of the SRC may have significant effects on the EOS of
cold dense neutron-rich nucleonic matter and thus properties of neutron stars [37, 38].

How does the SRC affect the single-nucleon momentum distribution $n(k)$? This question
has been studied both experimentally and theoretically for a long time. Fortunately, many
interesting results have already been well established. For a comprehensive review, we refer the
reader to the book by Antonov et al. [39]. Probably the most striking theoretical prediction
that was later experimentally verified is the universal high momentum tail in $n(k)$, i.e., the,
high momentum part of $n(k)$ is independent of the mass number $A$ for finite nuclei and it is
almost the same as for infinitely large SNM [40, 41, 42]. This feature clearly indicates the
short-range origin of the high momentum tail. Moreover, microscopic many-body calculations
have shown that the underlying tensor force is responsible for the universal high momentum
tail between about 300-600 MeV/c [41]. At even higher momentum a weak three-body SRC
will show up. Successful efforts have been made to extract experimentally from the scaling
of inclusive electron scattering cross sections on different targets the absolute per-nucleon
probability for nucleons to be in the high momentum tail. In particular, an analysis of the
experiments done by the CLAS Collaboration [28] have shown quantitatively that the absolute
values of the per-nucleon probability of two-nucleon SRCs due to the tensor force is about
$15.4 \pm 3.3\%$, $19.3 \pm 4.1\%$, $22.7 \pm 4.7\%$ for $^4\text{He}$, $^{12}\text{C}$ and $^{56}\text{Fe}$, respectively. Theoretically, to
investigate how the strength of the tensor force affects the high-momentum tail of $n(k)$, some
phenomenological methods have been particularly useful [39]. For instance, Dellagiachoma et. al
have derived formulas for $n(k)$ explicitly including the tensor force induced SRC for finite nuclei
[43, 44]. It was shown that the probability of finding nucleons above the Fermi surface increases
with the strength of the tensor force (or equivalently the percentage of D-wave mixture) [43, 44].

Another kind of experiments especially useful for exploring the SRC is the exclusive
measurement of two-nucleon knockout reactions induced by a high energy proton or electron.
Recently, the isospin-dependence of the SRC has been studied extensively both experimentally
[37, 38, 45, 46] and theoretically [47, 48, 49, 50, 51, 52]. For reviews, see, e.g., Refs. [35, 36].
The experimental finding that the $np$ SRC dominates over the $nn$ ($pp$) one indicates clearly that
the tensor force instead of the repulsive core is mainly responsible for the high-momentum tail
of $n(k)$. It is particularly exciting to note that experiments at J-Lab have shown that about
20% of nucleons in $^{12}\text{C}$ are correlated. This is quantitatively consistent with the finding by the
CLAS Collaboration from inclusive electron scattering experiments. Moreover, the strength of
the $np$ SRC is found to be about 20 times that of the $pp$ ($nn$) SRC [38]. This has been shown
as a direct consequence of the tensor force acting in the deuteron-like $np$ state in the targets
[37, 47, 48, 49].

As mentioned above, one of our main goals is to understand at least qualitatively effects of
the tensor force induced SRC on the $E_{\text{sym}}(\rho)$. Within a phenomenological model we can take
into account the main features of the $n(k)$ predicted by the microscopic many-body theories and confirmed by the experiments. To explore effects of the tensor force, we can also easily vary the percentage of nucleons in the high momentum tail and examine its effects on the $E_{\text{sym}}(\rho)$. More specifically, we parameterize the $n(k)$ of nucleons in SNM as

$$n(k) = a \quad (k \leq k_F)$$

$$= e^{bk} \quad (k > k_F).$$

The $a$ and $b$ are parameters determined by the normalization condition

$$\frac{3}{k_F^2} \int_0^{k_F} n(k)k^2dk = 1$$

and the percentage $\theta_{k \leq k_F} \ (\theta_{k > k_F})$ of nucleons below (above) the Fermi surface, i.e.,

$$\frac{3}{k_F^2} \int_0^{k_F} n(k)k^2dk \times 100\% = \theta_{k \leq k_F}, \quad \frac{3}{k_F^2} \int_{k_F}^{\infty} n(k)k^2dk \times 100\% = \theta_{k > k_F}.$$  

For PNM, the SRC is induced only by the repulsive core. The relatively weak n-n SRC indicated by the J-Lab experiments justifies the use of an ideal gas approximation for the $n(k)$ in PNM. Shown in the left panel of Fig.1 is the $n(k)$ in SNM at the saturation density for the ideal Fermi gas and $\theta_{k > k_F} = 5\%, \ 10\%, \ 15\%, \ \text{and} \ 20\%$, respectively. For a comparison, the $n(k)$ for PNM is shown in the right panel. It should be noted that the $n(k)$ for SNM with $\theta_{k > k_F} = 20\%$ is very close to the microscopic single-nucleon momentum distribution given by Ciofi degli Atti et.al. [42] obtained by fitting results of the variational many-body calculations [40].

3. Tensor force effects on nuclear symmetry energy

It has long been known that the tensor force influences the high density behavior of $E_{\text{sym}}(\rho)$ [53, 54, 55, 56, 57, 58]. However, predictions from different models diverge widely partially because the strength of the in-medium tensor force and its effects on the SRC were not so clearly known. The newly available and more quantitative information on the tensor force induced SRC may allow us to better understand effects of the tensor force on the $E_{\text{sym}}(\rho)$ of dense neutron-rich nucleonic matter. In the following, we explore separately effects of the tensor force on the kinetic and potential parts of the symmetry energy.

Figure 1. Single-nucleon momentum distribution in symmetric nuclear matter and pure neutron matter at normal density.
3.1. Tensor force effects on the kinetic part of nuclear symmetry energy

While the average kinetic energy per nucleon in a Fermi gas of independent nucleons is simply

\[ E_{\text{kin}} = \frac{3}{5} \frac{\hbar^2 k_F^2}{2m}, \]

with correlated nucleons it is given by

\[ E_{\text{kin}} = \alpha \int_0^\infty \frac{\hbar^2 k^2}{2m} n(k) k^2 dk, \]

where \( \alpha = \frac{3}{k_F^3} \) and \( \frac{3}{2k_F^3} \) for SNM and PNM, respectively. We compare in left window of Fig.2 the average kinetic energy as a function of Fermi momentum for PNM and SNM with the percentage of high momentum nucleons to be \( \theta_{k>k_F} = 0\%, 5\%, 10\%, 15\%, \) and 20\%, respectively. As one expects, the SRC increases the \( E_{\text{kin}} \) significantly for SNM. More quantitatively, for SNM at the saturation density corresponding to \( K_F = 1.33 \text{ fm}^{-1} \), the \( E_{\text{kin}} \) with \( \theta_{k>k_F} = 20\% \) (\( E_{\text{kin}}(k_F) \approx 40 \text{ MeV} \)) is about twice of that (\( E_{\text{kin}}(k_F) \approx 22 \text{ MeV} \)) for the free Fermi gas. However, the \( E_{\text{kin}} \) for PNM is the same as for the free Fermi gas. Consequently, the tensor force induced high momentum tail in SNM will affect the kinetic part of the nuclear symmetry energy. In particular, if about 15% nucleons in SNM are in the high momentum tail, it is seen that the average kinetic energy is about the same in PNM and SNM. This will then lead to an approximately zero kinetic symmetry energy.

Shown in right window of Fig.2 is the kinetic part of the symmetry energy as a function of Fermi momentum with different \( \theta_{k>k_F} \). Indeed, it is interesting to see that the tensor force induced SRC has a significant impact on the kinetic part of the symmetry energy, especially at supra-saturation densities. For instance, the kinetic contribution to the symmetry energy \( (E_{\text{kin}}^\text{sym}(\rho)) \) is negligibly small when 15% nucleons are considered to be correlated \( (\theta_{k>k_F} = 15\%) \). With \( \theta_{k>k_F} = 20\% \) when the tensor force is even stronger, the \( E_{\text{kin}}^\text{sym}(\rho) \) becomes negative at supra-saturation densities. We are encouraged that this new features first observed in our preliminary study [26] is also seen in very recent studies based on the Brueckner-Hartree-Fock approach (BHF) [29], the Self-Consistent Green’s Functions approach (SCGF) [30], and the Fermi-Hypernetted-Chain calculations (FHNC) [31]. While all these models were long established, the kinetic and potential contributions to the symmetry energy were always combined together in previous studies. A careful examination of their respective contributions was found very informative [29, 30, 31]. Since the well-known and widely used Fermi gas...
model prediction for the kinetic contribution $E^\text{kin}_{\text{sym}}(\rho)$ is always positive and increases with increasing density, the dramatic tensor force effects demonstrated by both the phenomenological and microscopic models are conceptually important and practically useful.

At this point, some discussions regarding the implications of the above results are in order. In many studies in both nuclear physics and astrophysics, it is customary to write the total symmetry energy as $E_{\text{sym}}(\rho) = 12.5(\rho/\rho_0)^{2/3} + E_{\text{pot}}^{\text{sym}}(\rho)$ where the first term is the Fermi gas prediction for the $E^\text{kin}_{\text{sym}}(\rho)$ and the $E^\text{pot}_{\text{sym}}(\rho)$ is the potential contribution. In doing so, however, one neglects completely effects of the tensor force on the $E^\text{kin}_{\text{sym}}(\rho)$. As shown above, the latter is approximately zero at normal density if indeed about 15% nucleons in SNM are above the Fermi surface as indicated by the analysis of experiments at the J-Lab. Then, noticing that the total symmetry energy at normal density is known to be about 30 MeV from analyzing the atomic masses and many other experiments, if one believes in the $E^\text{pot}_{\text{sym}}(\rho)$ extracted from nuclear reactions and the almost zero $E^\text{kin}_{\text{sym}}(\rho)$ at normal density, an interesting question arises: where is the remaining symmetry energy? As we shall show next, it is in the tensor contribution to the potential part of the symmetry energy.

### 3.2. Second-order tensor force contribution to the potential part of the symmetry energy

It is well known that the first-order (at the mean-field level) tensor contribution to the EOS vanishes in spin-saturated systems. In the best-studied phenomenology of nuclear forces, i.e., the one-boson-exchange model, the tensor interaction results from the exchange of an isovector $\pi$ and/or $\rho$ meson. For instance, the tensor part of the one-pion exchange potential (OPEP) can be written in configuration space as [59]

$$V_{\text{t}} = -\frac{f^2_\pi}{4\pi} m_\pi (\tau_1 \cdot \tau_2) S_{12} \left[ \frac{1}{(m_\pi r)^3} + \frac{1}{(m_\pi r)^2} + \frac{1}{3m_\pi r} \right] \exp(-m_\pi r)$$  \hspace{1cm} (4)

where $r$ is the interparticle distance and $S_{12} = 3(\sigma_1 \cdot r)(\sigma_1 \cdot r) - (\sigma_2 \cdot \sigma_2)$ is the tensor operator. The $\rho$-exchange tensor interaction $V_{1\rho}$ has the same functional form as the OPEP, but with the $m_\pi$ replaced everywhere by $m_\rho$, and the $f^2_\pi$ by $-f^2_\rho$. The magnitudes of both the $\pi$ and $\rho$ contributions grow quickly with decreasing $r$. A proper cancellation of these opposite contributions leads to a realistic strength for the nuclear tensor force. However, since the tensor coupling is not well determined consistently from deuteron properties and/or nucleon-nucleon scattering data, the tensor interaction is by far the most uncertain part of the nucleon-nucleon interaction [60]. In addition, due to both the physical and mathematical differences in methods used during construction [60], various realistic nuclear potentials usually have widely different tensor components at short range ($r \leq 0.8$ fm). For example, in the Paris potential [61], it is just described simply by a constant soft core. The Argonne V18 (AV18) uses local functions of Woods-Saxon type [62], while Reid93 applies local Yukawas of multiples of the pion mass [63]. While it is promising that new experiments, such as, $(p,d)$ reactions induced by high energy electrons [38, 46], may allow us to better constrain the short-range tensor force in the near future, currently the short-range behavior of the tensor force is still very uncertain. Here we use several typical and widely used tensor forces that are the same at long-range as the one used in the AV18 but have characteristically different short-range behaviors. This will allow us to examine effects of the short-range tensor force on the density-dependence of nuclear symmetry energy.

It is easy to see from Eq. 4 that the expectation value of the tensor force $\langle V_{\text{t}} \rangle$ is zero in spin-saturated systems. Thus, the first-order tensor force does not contribute to the symmetry energy unless one assumes that all isosinglet neutron-proton pairs behave as bound deuterons [9]. Thus, it is the second-order tensor contribution that is important for the binding energy of nuclear matter [66, 65] and also for the symmetry energy [67]. Using a second-order effective tensor
interaction obtained first by Kuo and Brown [66], see. e.g., ref. [68] for a review, the tensor contribution to the symmetry energy is approximately [67]

\[ \langle V_{\text{sym}} \rangle = \frac{12}{e_{\text{eff}}} \langle V_1^2(r) \rangle \]

(5)

where the \( e_{\text{eff}} \approx 200 \text{ MeV} \) around normal density. While this approximate expression may lead to symmetry energies systematically different from predictions of advanced microscopic many-body models using various interactions especially at high densities, it is handy to evaluate effects of the different short-range tensor forces within the same analytical approach. To evaluate the expectation value of \( V_{\text{sym}} \), we use the free single-particle wave function \((V^{-1}c_{\mathbf{k}}^\dagger \mathbf{r})\eta_\lambda \zeta_\tau\), where \( \eta_\lambda = \gamma_\lambda / 4 \) and \( \zeta_\tau = p_\tau / n \) is the spin and isospin wave function, respectively. The direct and exchange matrices are, respectively,

\[ \langle \mathbf{k}_\lambda \tau \mathbf{k}'_\lambda' \tau'|V_{\text{sym}}|\mathbf{k}_\lambda \tau \mathbf{k}'_\lambda' \tau' \rangle = \frac{1}{V} \int V_{\text{sym}}(\mathbf{r}) d^3r \]

(6)

and

\[ \langle \mathbf{k}_\lambda \tau \mathbf{k}'_\lambda' \tau'|V_{\text{sym}}|\mathbf{k}'_\lambda' \tau' \mathbf{k}_\lambda \tau \rangle = \frac{1}{V} \delta_{\mathbf{k}\lambda} \delta_{\mathbf{k}'\tau'} \int \exp[-i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{r}] V_{\text{sym}}(\mathbf{r}) d^3r. \]

(7)

The expectation value of \( V_{\text{sym}} \) in the \( S = 1, T = 0 \) channel is thus

\[ \langle V_{\text{sym}} \rangle = \frac{1}{16} \sum_{\mathbf{k}_\lambda \tau} \sum_{\mathbf{k}'_\lambda' \tau'} \left[ \langle \mathbf{k}_\lambda \tau \mathbf{k}'_\lambda' \tau'|V_{\text{sym}}|\mathbf{k}_\lambda \tau \mathbf{k}'_\lambda' \tau' \rangle - \langle \mathbf{k}_\lambda \tau \mathbf{k}'_\lambda' \tau'|V_{\text{sym}}|\mathbf{k}'_\lambda' \tau' \mathbf{k}_\lambda \tau \rangle \right] \]

\[ = \frac{V}{2} \frac{1}{(2\pi)^6} \int_{\mathbf{k}_F}^k \frac{d^3k}{\mathbf{k}_F} \int_{\mathbf{k}_F}^{\mathbf{k}_F} \frac{d^3k'}{\mathbf{k}_F} \left\{ \int V_{\text{sym}}(\mathbf{r}) d^3r - \frac{1}{4} \int \exp[-i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{r}] V_{\text{sym}}(\mathbf{r}) d^3r \right\}. \]

(8)

Noticing that the momentum integral \( \int_{\mathbf{k}_F}^k d^3k e^{i \mathbf{k} \cdot \mathbf{r}} = 4\pi \int_{0}^{k_F} k^2 j_0(kr) dk = \frac{4\pi k_F^3}{3} j_1(k_F r) \) and the particle number density \( \frac{dP}{d^3r} = \frac{2}{3\pi^2} k_F^3 \), we can write the tensor contribution to the symmetry energy as

\[ \frac{\langle V_{\text{sym}} \rangle}{A} = \frac{12}{e_{\text{eff}}} \frac{k_F^3}{12\pi^2} \left\{ \frac{1}{4} \int V_1^2(r) d^3r - \frac{1}{16} \int \left( \frac{3j_1(k_F r)}{k_F r} \right)^2 V_1^2(r) d^3r \right\}. \]

(9)

For large \( k_F \), the second integral in the above equation approaches zero, the first term is thus expected to dominate at high densities, leading to an almost linear density dependence.

To access quantitatively effects of the short-range tensor force on the density dependence of nuclear symmetry energy, we adopt here several tensor forces used by Otsuka et al. in their recent studies of nuclear structures [69]. The considered tensor forces, including the standard \( \pi + \rho \) exchange (labelled as \( a \)), the G-Matrix (GM) [69] (labelled as \( b \)), M3Y [70] (labelled as \( c \)), and the AV18 [62] (labelled as AV18), as shown in the left panel of Fig. 3, behave rather differently at short distance, but merge to the same AV18 tensor force at longer range. In addition, we add a case (\( d \)) where the tensor force vanishes for \( r \leq 0.7 \text{ fm} \). The \( \pi + \rho \) exchange interaction is fixed by the standard meson-nucleon coupling constants with a strong \( \rho \) coupling [68], and we use a short-range cut-off at \( r = 0.4 \text{ fm} \), i.e., \( V(r < 0.4 \text{ fm}) = V(r = 0.4 \text{ fm}) \). The resulting tensor contribution to the nuclear symmetry energy is shown as a function of density in the right panel of Fig. 3. As expected, they tend to grow linearly with increasing density. Since it is the square of the tensor force that determines its contribution to the symmetry energy, tensor forces having larger magnitudes at short distance affect more significantly the symmetry energy. It is seen that the variation of the tensor force at short distance affects significantly the high-density behavior of nuclear symmetry energy.
Figure 3. Left panel: different short-range tensor interactions with the same AV18 long-range part for $r > 0.7\text{fm}$; Right panel: potential part of the symmetry energy with the different short-range tensor interactions. The (filled) open (circles) squares are the 2-particle-2-hole calculations from ref. [33].

Around normal density where predictions based on Eq. 5 are most reliable, the tensor force contribution to the symmetry energy is about 7 to 15 MeV depending the interaction used. Generally speaking, this can largely compensate the tensor force induced reduction in the kinetic part of the symmetry energy. While in some microscopic models, the tensor force induced effects may have been treated self-consistently in calculating both the kinetic and potential parts of the symmetry energy, in almost all phenomenological models they are not considered at all. However, they can all be adjusted to give the correct symmetry energy at least at normal density. Our analysis here indicates that this is not surprising largely because of the approximate cancelation between the tensor force induced reduction of the $E_{\text{sym}}(\rho)$ and its second-order contribution to the $E_{\text{sym}}(\rho)$. In a recent study by Wang et al. [33], the tensor force contribution to the potential part of the symmetry energy due to the one-pion exchange was evaluated accurately at the 2-particle-2-hole level. While their results depend on the momentum cut-off parameter $\Lambda$ in the form factor, they demonstrated clearly that the potential part of the symmetry energy has a large tensor contribution. For a comparison, their results with $\Lambda = 0.85 \text{ GeV}$ and $\Lambda = 0.65 \text{ GeV}$ which are within the range of theoretical expectations for this parameter [33, 71] are also shown in Fig. 3. Qualitatively, their results are consistent with our results based on Eq. 5 in the sense that the direct, second-order tensor contribution to the potential part of the $E_{\text{sym}}(\rho)$ is large. However, as most other calculations in the literature, effects of the tensor force on the kinetic part of the symmetry energy was not considered in their work either. With the strong tensor force they used, the kinetic part of the symmetry energy is likely to become zero or negative as we discussed in the previous subsection. To this end, it is also worth noticing that in essentially all mean-field based models without considering high-order tensor contributions, the interaction parameters are normally adjusted to give the $E_{\text{sym}}(\rho_0)$ at normal density a value of about 30 MeV. The discussions above indicate that these parameters need to be re-adjusted if the tensor force contributions in either or both kinetic and potential terms are also considered.
3.3. Tensor force effects on the central force contribution to the symmetry energy

Figure 4. The average potential energy per nucleon $E_{\text{pot}}$ for symmetric nuclear matter with different percentages of correlated nucleons ($\theta_{k>k_F}$) as a function of Fermi momentum.

Because of the spin-isospin dependence of the central force, it also contributes to the symmetry energy. Moreover, if finite-range interactions are considered, the corresponding single-particle potential is momentum dependent and the potential energy density involves a double integration over the momenta of two interacting nucleons. Thus, the tensor force induced high-momentum tail may also affect the central force contribution to the EOS of SNM. The central force contribution to the symmetry energy may then also be affected. To our best knowledge, this effect was previously ignored in all phenomenological models. To evaluate this effect, we use here the MDI (Momentum-Dependent Interaction) potential which has been used extensively in transport models simulations of heavy-ion reactions [73]. The MDI potential energy per nucleon can be written as (with the parameter $x = 0$)[72]

$$E_{\text{pot}} = \frac{A}{2} \frac{\rho}{\rho_0} + \frac{B}{\sigma + 1} \frac{\rho^\sigma}{\rho_0^\sigma} + \frac{C}{\rho \rho_0} \int_0^\infty \int_0^\infty \frac{n(k_1)n(k_2)}{1+(k_1-k_2)^2/\Lambda_k^2} dk_1 dk_2,$$

where $\sigma = 4/3$ and the parameter $\Lambda_k = 1.0 k_F^0$ [72]. The $n(k_1)$ and $n(k_2)$ are the one-body momentum distribution of nucleon-1 and nucleon-2, respectively. The parameters $A$, $B$ and $C$ were determined by fitting the empirical properties of SNM without considering effects of the tensor force induced SRC as the MDI potential was based on a mean-field model (i.e., the Hartree-Fock with a modified Gogny force). In principle, with each tensor force leading to a different high momentum tail in SNM, one has to readjust all associated parameters to reproduce the empirical properties of SNM. However, for the purpose of this study it is actually advantageous to keep using the same original parameters so that effects of the SRC can be clearly revealed. Thus, in the present calculations, we do not modify the original parameters of the MDI interaction density functional.

Shown in Fig.4 is a comparison of $E_{\text{pot}}$ with different percentages $\theta_{k>k_F}$ of correlated nucleons for SNM. Similar to the case of the average kinetic energy $E_{\text{kin}}$, the SRC increases the potential energy $E_{\text{pot}}$ for SNM as one expects. Since the tensor force has no effect on the EOS of PNM,
Figure 5. The kinetic, central force and tensor force contributions to the symmetry energy with 5% and 20% high-momentum nucleons as a function of Fermi momentum.

4. Summary
Using phenomenological models, we explored effects of the tensor force on the density dependence of nuclear symmetry energy. The high momentum tail in symmetric nuclear matter induced by the tensor force acting between protons and neutron makes the kinetic part of the symmetry energy $E_{\text{sym}}^{\text{kin}}(\rho)$ significantly smaller than the Fermi gas model prediction. With about 15% nucleons in the high momentum tail in SNM as indicated by the recent experiments at the J-Lab, the $E_{\text{sym}}^{\text{kin}}(\rho)$ is negligibly small. It even becomes negative when more nucleons are in the high momentum tail in SNM. While at the mean-field level the tensor force has no contribution to the EOS, its second-order contribution to the potential part of the symmetry energy is large. To completely take into account effects of the tensor force, it is necessary to include not only its second-order potential contribution and effects on the kinetic part but also its effects on the central force contribution to the symmetry energy. Implications of these finding on extracting experimental constraints on the density dependence of nuclear symmetry energy are also discussed briefly.

5. Acknowledgement
We would like to thank Lie-Wen Chen and William G. Newton for helpful discussions. This work is supported in part by the US National Science Foundation grant PHY-0757839 and PHY-1068022, the National Aeronautics and Space Administration under grant NNX11AC41G issued through the Science Mission Directorate, the National Natural Science Foundation of China (Grants 10805026, 10905048 and 11175085) and the National Basic Research Program of China under Grant 2009CB824800.
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11th International Conference on Nucleus-Nucleus Collisions (NN2012) IOP Publishing
Journal of Physics: Conference Series **420** (2013) 012090
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