Associated Production of Gauginos and Gluinos at Hadron Colliders in Next-to-Leading Order SUSY-QCD

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Abstract

We report a next-to-leading order (NLO) calculation of the production of gaugino-like charginos ($\tilde{\chi}^\pm$) and neutralinos ($\tilde{\chi}^0$) in association with gluinos ($\tilde{g}$) at hadron colliders, including the strong corrections from colored particles and sparticles. We predict inclusive cross sections at the Fermilab Tevatron and CERN LHC. The NLO cross sections are more stable against variations in the hard-scattering scale parameter and are greater than the LO values.

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Supersymmetry predicts the existence of supersymmetric partners for each of the particles of the standard model. The search for these sparticles is a principal motivation of the forthcoming Run II of the Fermilab Tevatron collider and of the CERN Large Hadron Collider (LHC) program. A potentially important, but heretofore largely overlooked, discovery channel is the associated production of a spin-1/2 gaugino ($\tilde{\chi}$) with a spin-1/2 gluino ($\tilde{g}$) or with a spin-0 squark ($\tilde{q}$). Color-neutral gauginos couple with electroweak strength, whereas the colored squarks and gluinos couple strongly. Associated production is therefore a semi-weak process in that it involves one somewhat smaller coupling constant than the pair production of colored sparticles. However, in popular models of SUSY breaking [1,2], the mass spectrum favors much lighter masses for the low-lying neutralinos and charginos than for the squarks and gluinos. This mass hierarchy means that the phase space for production of neutralinos and charginos, the corresponding partonic luminosities, and the production cross sections will be greater than those for gluinos and squarks. These advantages are potentially decisive at a collider with limited energy, such as the Tevatron. Furthermore, associated production has a clean experimental signature. For example, the lowest lying neutralino is the (stable) lightest supersymmetric particle (LSP) in supergravity (SUGRA) models [1], manifest as missing energy in the events, and it is the second lightest in gauge-mediated models [2]. In models with a very light gluino [3], there could be large rates for $\tilde{g}\tilde{\chi}$ production, with simple signatures, whereas $\tilde{g}\tilde{g}$ production suffers from large hadronic jet backgrounds.

Experimental investigations are facilitated by firm theoretical understanding of the expected sizes of the cross sections for production of the superparticles. In the case of hadron-hadron colliders, the large strong coupling strength ($\alpha_S$) results in potentially large contributions to cross sections from terms beyond leading order (LO) in a perturbative quantum chromodynamics (QCD) evaluation of the cross section. For accurate theoretical estimates, it is necessary to extend the calculations to next-to-leading order (NLO) or beyond. NLO contributions generally reduce and stabilize dependence on undetermined parameters such as the renormalization and factorization scales. To date, associated production has been
calculated only in LO \[4\], but NLO results exist for hadroproduction of gluinos and squarks \[5\], top squarks \[6\], sleptons \[7,8\], and gauginos \[8\]. Studies have begun to incorporate these NLO results into Monte Carlo simulations \[9\].

In this Letter we present the first NLO (in SUSY-QCD) calculation of hadroproduction of a $\tilde{g}$ in association with a $\tilde{\chi}$, including contributions from virtual loops of colored sparticles and particles and three-particle final states involving the emission of light real particles. We extract the ultraviolet, infrared, and collinear divergences by use of dimensional regularization and employ standard MS renormalization and mass factorization procedures. In the course of computing the virtual contributions, we encountered new divergent four-point functions. The contributions from real emission of light particles are treated with a phase space slicing method. We provide predictions for inclusive cross sections at Tevatron and LHC energies. Our reason to focus on the $\tilde{g} + \tilde{\chi}$ final state, rather than on the associated production $\tilde{q} + \tilde{\chi}$, is that at the energy of the Tevatron the LO cross sections for $\tilde{g} + \tilde{\chi}$ are 3 to 6 times greater than those for $\tilde{q} + \tilde{\chi}$ ($\tilde{\chi} = \tilde{\chi}_1^0, \tilde{\chi}_1^\pm$) when $m_{\tilde{g}} = m_{\tilde{q}} = 300$ GeV, and 6 to 15 times greater when $m_{\tilde{g}} = m_{\tilde{q}} = 600$ GeV. In obtaining the $\tilde{q}$ cross sections, we sum over five flavors of squarks and antishuarks.

In LO of SUSY-QCD, the associated production of a gluino and a gaugino proceeds through the subprocess $q\bar{q} \rightarrow \tilde{g}\tilde{\chi}$ with a $t$-channel or a $u$-channel squark exchange. We assume that there is no mixing between squarks of different generations and that the squark mass eigenstates are aligned with the squark chirality states, equivalent to the assumption that the two squarks of a given flavor are degenerate in mass. We ignore the $n_f = 5$ light quark masses in all of the kinematics and couplings. Under these assumptions, the massless incoming quarks and antiquarks have a particular helicity, and thus the Feynman diagrams in which a right-handed squark is exchanged cannot interfere with those mediated by a left-handed squark. In evaluating the Feynman diagrams involving Majorana and explicitly charge-conjugated fermions, we follow the approach of Ref. \[10\]. In the case of charged gauginos, only the left-handed squarks participate, whereas neutral gauginos receive contributions from both left- and right-handed squarks.
The LO matrix element summed (averaged) over the colors and helicities of the outgoing (incoming) particles has the analytic form

\[
|\mathcal{M}|^2 = \frac{8\pi \hat{\alpha}_S}{9} \left[ \frac{\hat{X}_t t_{\tilde{g}} t_{\tilde{\chi}}}{(t - m_{\tilde{g}}^2)^2} - \frac{2\hat{X}_{tu} s m_{\tilde{g}} m_{\tilde{\chi}}}{(t - m_{\tilde{g}}^2)(u - m_{\tilde{g}}^2)} \right] + \frac{\hat{X}_u u_{\tilde{g}} u_{\tilde{\chi}}}{(u - m_{\tilde{g}}^2)^2}.
\]

(1)

Here, \( m_{\tilde{g}_{t,u}} \) is the mass of the squark exchanged in the \( t \)- and \( u \)-channel; \( \hat{\alpha}_S = \hat{g}_S^2/4\pi \) is the Yukawa coupling between quarks, squarks, and gluinos. At leading order it is equal to the gauge coupling constant \( \alpha_S \). In Eq. (1) \( \hat{X}_{t,tu,u} \) stands for the Yukawa couplings of quarks, squarks, and gauginos. Variables \( s, t, \) and \( u \) are the usual Mandelstam invariants at the partonic level; \( t_{\tilde{g},\tilde{\chi}} = t - m_{\tilde{g},\tilde{\chi}}^2 \) and \( u_{\tilde{g},\tilde{\chi}} = u - m_{\tilde{g},\tilde{\chi}}^2 \).

At NLO the cross section receives contributions from virtual loop diagrams and from real parton emission diagrams. The virtual contributions arise from the interference of the Born amplitudes with the related one-loop amplitudes containing self-energy corrections, vertex corrections, and box diagrams. We include the full supersymmetric spectrum of strongly interacting particles in the virtual loops, i.e. squarks and gluinos as well as quarks and gluons.

Since the virtual loop contributions are ultraviolet and infrared divergent, we regularize the cross section by computing the phase space and matrix elements in \( n = 4 - 2\epsilon \) dimensions. We calculate the traces of Dirac matrices using the “naive” \( \gamma_5 \) scheme in which \( \gamma_5 \) anticommutes with all other \( \gamma_\mu \) matrices. This choice is justified for anomaly-free one-loop amplitudes. The \( \gamma_5 \) matrix enters the calculation through both the quark-squark-gluino and quark-squark-gaugino Yukawa couplings. We simplify the integration over the internal loop momenta by reducing all tensorial integration kernels to expressions that are only scalar in the loop momentum \( \cot \). The resulting one-, two-, three-, and some of the four-point functions were computed in the context of other physical processes \( \cot \). However, we compute two previously unknown divergent four-point functions; these new functions arise because the final state gluino and gaugino have different masses in general. We evaluate the scalar four-point functions by calculating the absorptive parts with Cutkosky cutting rules and the
real parts with dispersion techniques.

The ultraviolet (UV) divergences are manifest in the one- and two-point functions as poles in $1/\epsilon$. We remove them by renormalizing the coupling constants in the $\overline{\text{MS}}$ scheme at the renormalization scale $Q$ and the masses of the heavy particles (squarks and gluinos) in the on-shell scheme. The self-energies for external particles are multiplied by a factor of $1/2$ for proper wave function renormalization. A difficulty arises from the fact that spin-1 gluons have $n - 2$ possible polarizations, whereas spin-1/2 gluinos have 2, leading to broken supersymmetry in the $\overline{\text{MS}}$ scheme. The simplest procedure to restore supersymmetry is with finite shifts in the quark-squark-gluino and quark-squark-gaugino Yukawa couplings [12, 5].

In addition to the ultraviolet singularities, the virtual corrections have collinear and infrared singularities that show up as $1/\epsilon$ or $1/\epsilon^2$ poles in the derivatives of the two-point function and in the three- and four-point functions. These infrared singularities appear as factors times parts of the Born matrix elements. They can be separated into $C_F$ and $N_C$ color classes, depending on the color flow and the abelian or non-abelian nature of the correction vertices. They are cancelled eventually by corresponding soft and collinear singularities from the real three parton final state corrections.

The real corrections to the production of gluinos and gauginos arise from three parton final-state subprocesses in which additional gluons and massless quarks and antiquarks are emitted: $q\bar{q} \to \tilde{g}\tilde{\chi}g$, $qg \to \tilde{g}\tilde{\chi}q$, and $\bar{q}g \to \tilde{g}\tilde{\chi}\bar{q}$.

The $n$-dimensional phase space for $2 \to 3$ scattering may be factored into the phase space for $2 \to 2$ scattering and the phase space for the subsequent decay of one of the two final state particles with squared invariant mass $s_4 = (p_1 + p_3)^2 - m_1^2$ into two particles with momenta $p_1$ and $p_3$, parametrized in the rest frame of particles 1 and 3 [13]. We follow the procedure in Ref. [13] to reduce all of the angular integrals. The angular integrations involving negative powers of $t' = (p_b - p_3)^2$ and $u' = (p_a - p_3)^2$, where $p_a$ and $p_b$ are the four-momenta of the incoming partons, produce poles in $1/\epsilon$ which correspond to the collinear singularities in which particle 3 is collinear with particle $a$ or $b$. Because these singularities have a universal structure, they may be removed from the cross section and absorbed into
the parton distribution functions according to the usual mass factorization procedure [14].

In addition to the collinear singularities described above, the corrections involving real gluon emission also have infrared (IR) singularities arising when the energy of the emitted gluon approaches zero. These singularities appear as poles in $s_4$ in the cross section and must also be extracted so that they can be combined with corresponding terms in the virtual corrections and shown to cancel. In order to make this cancellation conveniently, we slice the gluon emission phase space into hard and soft pieces,

$$\frac{d^2\hat{\sigma}^R_{ij}}{dt_\hat{\chi}_i du_\hat{\chi}_j} = \int_0^\Delta ds_4 \frac{d^3\sigma^S}{dt_\hat{\chi}_i du_\hat{\chi}_j ds_4} + \int_{s_4^{max}}^\Delta ds_4 \frac{d^3\sigma^H}{dt_\hat{\chi}_i du_\hat{\chi}_j ds_4},$$

(2)

where $\Delta$ is an arbitrary cut-off between soft and hard gluon radiation. When the cut-off is much smaller than the other invariants, the $s_4$ integration for the soft term becomes simple and can be evaluated analytically, leading to explicit logarithms $\log \Delta/m^2$, $\log^2 \Delta/m^2$; $m = (m_\tilde{g} + m_\tilde{\chi})/2$. The hard term is free from infrared and, after mass factorization, also collinear singularities and can be evaluated numerically in four dimensions. This procedure leads to an implicit logarithmic dependence of the hard term on the cut-off $\Delta$ which cancels the explicit logarithmic dependence in the soft term.

To obtain numerical results for the cross sections, we work within a particular SUGRA scheme, but the cross sections depend principally on the masses of the $\tilde{\chi}$ and $\tilde{g}$ and are otherwise fairly independent of the details of the SUSY breaking. The physical gluino and gaugino masses as well as the gaugino mixing matrices are calculated from a default minimal SUGRA scenario [1]. We choose the common scalar and fermion masses at the GUT scale to be $m_0 = 100$ GeV and $m_{1/2} = 150$ GeV. The trilinear coupling $A_0 = 300$ GeV, and the ratio of the Higgs vacuum expectation values $\tan \beta = 4$. The absolute value of the Higgs mass parameter $\mu$ is fixed by electroweak symmetry breaking, and we choose $\mu > 0$. For this scenario, we find the neutralino masses $m_{\tilde{\chi}^0_{1-4}}$ to be 55, 104, 283, and 309 GeV with $m_\tilde{\chi}^0_3 < 0$ inside a polarization sum. The chargino masses $m_{\tilde{\chi}^\pm_{1,2}}$ are 102 and 308 GeV and therefore almost degenerate with the masses of the $m_{\tilde{\chi}^0_2}$ and $m_{\tilde{\chi}^0_4}$, respectively.

The total hadronic cross section is obtained from the partonic cross section through the
\[ \sigma^{h_1h_2}(S, Q^2) = \sum_{i,j=q,q,\overline{q}} \int_{\tau}^{1} dx_1 \int_{\tau/x_1}^{1} dx_2 \]
\[ f_i^{h_1}(x_1, Q^2)f_j^{h_2}(x_2, Q^2)\hat{\sigma}_{ij}(x_1x_2S, Q^2), \]  
(3)

where \( \tau = \frac{4m^2}{S} \), and \( S \) is the square of the hadronic center-of-mass energy (\( \sqrt{S} = 2 \) TeV for Run II at the Fermilab \( p\bar{p} \) collider Tevatron and 14 TeV at the CERN LHC \( pp \) collider). For the NLO predictions, we employ the CTEQ4M parametrization [15] for the parton densities \( f(x, Q^2) \) in the proton or antiproton and a two-loop approximation for the strong coupling constant \( \alpha_S \) with \( \Lambda(5) = 202 \) MeV. In LO we use the LO parton densities CTEQ4L and the one-loop approximation for \( \alpha_S \) with \( \Lambda(5) = 181 \) MeV.

In Fig. 1 we present predictions for total hadronic cross sections at the Tevatron (top) and the LHC (bottom), as a function of the physical gluino mass. To obtain these results, we use the average produced mass as the hard scale \( Q \) in Eq. (3), \( Q = (m_{\tilde{g}} + m_{\tilde{\chi}})/2 \). We vary the SUGRA parameter \( m_{1/2} \) between 100 and 400 GeV and keep the other SUGRA parameters fixed to the values listed above. As gluino mass increases over the range shown in the figure, the corresponding gaugino mass ranges are 31 to 163 GeV for \( \tilde{\chi}^0_1 \), 62 to 317 GeV for \( \tilde{\chi}^0_2 \) and \( \tilde{\chi}^\pm_1 \), 211 to 666 GeV for \( \tilde{\chi}^0_3 \), and 240 to 679 GeV for \( \tilde{\chi}^0_4 \) and \( \tilde{\chi}^\pm_2 \). The chargino cross sections are summed over positive and negative charges.

We observe that the cross sections for \( \chi_2^0 \) and \( \tilde{\chi}_1^\pm \) and those for \( \chi_4^0 \) and \( \tilde{\chi}_2^\pm \) are very similar in magnitude at the Tevatron, as are their respective masses. One might expect the largest cross section for the lightest gaugino \( \tilde{\chi}_1^0 \). However, its coupling is dominantly photino-like and smaller than the zino-like coupling of \( \tilde{\chi}_2^0 \) which therefore has a larger cross section at small \( m_{\tilde{g}} \) despite its larger mass. The heavier gauginos \( \tilde{\chi}_{3,4}^0 \) and \( \tilde{\chi}_{2}^\pm \) are dominantly higgsino-like and their cross sections are suppressed by over an order of magnitude with respect to those of the lighter gauginos.

At the Tevatron, the NLO contributions increase the cross sections by 5 to 15% at the hard scattering scale \( Q = (m_{\tilde{g}} + m_{\tilde{\chi}})/2 \), depending on the channel considered and the values of the masses. At the LHC, the increases are in the range of 15 to 35%. The
purely NLO $qg$ incident channel contributes significantly at the LHC, in addition to the $q\bar{q}$ channel, particularly for the lighter gauginos, whereas the $qg$ channel plays little role at the Tevatron. In the event sparticles are not observed, the predicted increases translate into more restrictive experimental mass limits.

The enhancements of the cross sections are modest and, as such, underscore the validity of perturbative predictions for the processes considered. A further important benefit of the NLO computation is the considerable reduction in theoretical uncertainty associated with variation of the renormalization and factorization scale $Q$. For the processes studied here, this dependence is typically $\pm 10\%$ at the Tevatron when $Q$ is varied over the interval $Q/m$ from 0.5 to 2, compared to $\pm 25\%$ in leading order. At the LHC, the dependences are $\pm 9\%$ at NLO and $\pm 12\%$ at LO.

In this Letter we limit ourselves to total cross sections. Differential distributions in the transverse momentum $p_T$ and the rapidity $\eta$ of the produced sparticles will be published elsewhere [16], along with figures of scaling functions, renormalization/factorization scale dependence, K-factors, and several appendices containing a detailed exposition of the calculation.

To summarize, we provide NLO predictions of the cross sections for the associated production of gauginos and gluinos at hadron colliders. If supersymmetry exists at the electroweak scale, the cross section for this process is expected to be large and observable at the Fermilab Tevatron and/or the CERN LHC. It is enhanced by the large color charge of the gluino and the (in many SUSY models) small mass of the light gauginos. The cross section for $\tilde{\chi}_2^0$ production is the largest, because of its zino-like coupling. The cross section for $\tilde{\chi}_1^\pm$ is about equal to that of the $\tilde{\chi}_2^0$. The NLO predictions are modestly larger than the LO values but considerably more stable theoretically.

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FIG. 1. Total hadronic cross sections for the associated production of gluinos and gauginos at Run II of the Tevatron (top) and the LHC (bottom). NLO results are shown as solid curves, and LO results as dashed curves. We vary the SUGRA scenario as a function of $m_{1/2} \in [100; 400]$ GeV and provide the cross sections as a function of the physical gluino mass $m_{\tilde{g}}$. The chargino cross sections are summed over positive and negative chargino charges.