Heavy quarkonium in a holographic QCD model

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Abstract

Encouraged by recent developments in AdS/QCD models for light quark system, we study heavy quarkonium in the framework of the AdS/QCD models. We calculate the masses of $c\bar{c}$ vector meson states using the AdS/QCD models at zero and at finite temperature. Among the models adopted in this work, we find that the soft wall model describes the low-lying heavy quark meson states at zero temperature relatively well. At finite temperature, we observe that once the bound state is above $T_c$, its mass will increase with temperature until it dissociates at a temperature of around 494 MeV. It is shown that the dissociation temperature is fixed by the infrared cutoff of the models. The present model serves as a unified non-perturbative model to investigate the properties of bound quarkonium states above $T_c$. 

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1 Introduction

Recent developments in AdS/CFT \cite{1} find many interesting possibilities to study strongly interacting systems such as QCD. Confinement is assured with an IR cutoff in the AdS space \cite{2}, and flavors are introduced by adding extra probe branes \cite{3}. Phenomenological models were also suggested to construct a holographic model dual to QCD \cite{4, 5, 6}. However, the meson spectrum of the models does not follow the well-known Regge trajectories. In Ref. \cite{7}, a phenomenological dilaton background was introduced to improve the meson spectrum. Remarkably such a dilaton-induced potential gives exactly the linear trajectory of the meson spectrum: \( m_{n}^{2} \sim n \).

The model with a phenomenological dilaton is further improved by a finite UV cutoff \cite{8}.

In this work, we delve into the spectrum of heavy quarkonium states based on the AdS/QCD models \cite{4, 5}. The properties of heavy quark system both at zero and at finite temperature have been the subject of intense investigation for many years (for a review see, for example, \cite{9}). This is so because, at zero temperature, the charmonium spectrum reflects detailed information about confinement and interquark potentials in QCD \cite{10}, and at finite temperature, its change and dissociation will convey signals of the QCD deconfining phase transition from a relativistic heavy ion collisions (RHIC) \cite{11}. In addition, recent lattice calculations suggest that the charmonium states will remain bound at finite temperature to about 1.6 to 2 times the critical temperature \( T_{c} \) \cite{12, 13}. This suggests that analyzing the charmonium data from heavy ion collision inevitably requires more detailed information about the properties of charmonium states in QGP, such as the effective dissociation cross section and its dependence on the charmonium velocity \cite{14, 15}. Therefore, it is very important and a theoretical challenge to develop a consistent non-perturbative QCD picture for the heavy quark system both below and above the phase transition temperature. As a first step towards accomplishing these goals, we focus on the spectrum of heavy vector mesons at zero and at finite temperature. This is partly because, as it is well-known in the AdS/QCD models for light quarks \cite{4, 5}, the bulk vector field does not couple to the bulk scalar field, and so the vector sector of the models is relatively simple to analyze. We calculate the mass spectrum of the vector meson in three different models at zero temperature, and find that among the models considered, the soft wall model best fits the experimental value. We then introduce the black hole background on the AdS space and obtain the temperature-dependent mass spectrum.

It should be stressed that we are primarily interested in the spectrum and properties of the heavy quark system in the quark-gluon plasma (QGP), *i.e.*, above \( T_{c} \). Therefore, while a recent analysis \cite{16} of a Hawking-Page type transition \cite{17} in the AdS/QCD models claimed that the AdS black hole is unstable at low temperature, roughly \( T \leq 200\text{MeV} \), the use of the AdS black hole background at higher temperature, such as in the present work, is well justified. In general, using the AdS black hole background will not give any temperature dependence at low temperature, since the thermal AdS metric is just the AdS background with compactified Euclidean time. Moreover, according to the Hawking-Page transition, the AdS black hole is unstable in the confined phase below \( T_{c} \), so that we may not be able to use the black hole background to study the temperature dependence of light quark systems in AdS/QCD models. This means, that in the light of the Hawking-Page transition, the AdS/QCD models for light quarks may not be of much use, because those light states are expected to dissolve above the phase transition, and the only interesting temperature dependence is expected to occur below the phase transition temperature. As it is, for the light quark system, the AdS/QCD models are about the mesons and the baryons at zero temperature. However, given the present interest
in the properties of the heavy quark system and the validity of AdS black hole approach at high temperature, it is interesting to study the temperature dependence of some physical quantities in AdS/QCD models. In this respect heavy quark system at finite temperature, especially above $T_c$, may be a good theme for the AdS/QCD models.

2 Mass spectrum in AdS/QCD models

In the AdS/QCD models [4, 5], mesons made of light quarks, such as the pions and $\rho$-mesons, are described by gauged $\text{SU}(N_f)_L \times \text{SU}(N_f)_R$ chiral symmetry in AdS$_5$, where the metric is given by

$$ds_5^2 = \frac{1}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right). \tag{1}$$

In Ref. [4], gauged $\text{SU}(2)_L \times \text{SU}(2)_R$ chiral symmetry is adopted to study the mesons such as pions and rho-mesons, while with gauged $\text{SU}(3)_L \times \text{SU}(3)_R$ chiral symmetry [5] one could investigate the properties of mesons including strangeness. In the present study, we will attempt to generalize the approach to the charm sector and focus on the charmonium mass spectrum. Since the charm quark is much heavier than those in the light sectors and the symmetry is mainly broken by the explicit charm quark mass, we will adopt a separate $\text{U}(1)_L \times \text{U}(1)_R$ symmetry in AdS$_5$. In AdS$_5$, the chiral symmetry breaking, both explicit and spontaneous, is realized through a bulk scalar field profile near the boundary $z \to 0$, according to an AdS/CFT dictionary [18]. For instance, in [4, 5] the bulk scalar field, $\phi(x, z)$, which couples to the quark bilinear operator of $\bar{q}q$, takes the following form near the boundary:

$$\phi \sim c_1 z + c_2 z^3, \tag{2}$$

where $c_1 = m_q$ and $c_2 \sim \langle \bar{q}q \rangle$. In the light quark system, we could take the chiral-limit, $m_q = 0$. In the heavy quark system, we expect to have

$$\Phi \sim a_1 z + a_2 z^3, \tag{3}$$

where $a_1 = m_Q$ and $a_2 \sim \langle \bar{Q}Q \rangle$. Here $Q$ is for heavy quarks. In the present work we consider charm quark only and study the spectrum of $c\bar{c}$ at zero and at finite temperature. We describe the heavy quarkonium in AdS$_5$ with gauged $\text{U}(1)_L \times \text{U}(1)_R$ symmetry, which is explicitly broken by the $m_Q$-term in the bulk field profile near the boundary. Note that heavy quark and light quark systems have different IR cutoffs, as will be discussed below. In the potential model, the $1^- J/\psi(3097)$ and $0^- \eta_c(2980)$ can be described as s-wave charm quarks in the spin triplet and singlet states respectively, while the $1^+ \chi_{c1}(3510)$ and $0^+ \chi_{c0}(3415)$ with charm quarks in the p-wave. However, the present picture of treating the spectrum of heavy quarkonium in terms of explicit symmetry breaking is phenomenologically also acceptable, because the lowest parity partners have non degenerate mass, i.e. in the spin 1 channel $m(\chi_{c1}) - m(J/\psi) = 413$ MeV, and in the spin 0 $m(\chi_{c0}) - m(\eta_c) = 435$ MeV.

2.1 Hard-wall model

First we consider the hard-wall model, whose action is given by

$$S_{\text{HW}} = \int d^4x dz \sqrt{-g} L_5,$$
\[ \mathcal{L}_5 = -\frac{1}{4g_5^2}(L_{MN}L^{MN} + R_{MN}R^{MN}) + |D_M\Phi|^2 + 3|\Phi|^2, \]  

(4)

where \( D_M\Phi = \partial_M\Phi - iL_M\Phi + i\Phi R_M \) and \( g_5 \) is a 5D gauge coupling constant, \( L_{MN} = \partial_M L_N - \partial_N L_M - i[L_M, L_N] \) and \( L_M = L_MT_0 \) with \( T_0 \) being a \( U(1) \) generator. In the present work, we follow the convention in [4]. The scalar field is defined by \( \Phi = Se^{i\pi_0 T_0} \) and \( \langle S \rangle \equiv \frac{1}{2}v(z) \), where \( S \) is a real scalar and \( \pi_0 \) is a pseudoscalar. In this model, the 5D-AdS space is compactified such that \( z_0 < z < z_H^m \), where \( z_0 \to 0 \) and \( z_H^m \) is an infrared (IR) cutoff. In the light quark system \([4, 5]\), the value of \( z_L^m \) is fixed by the rho-meson mass \( (m_\rho) \) at zero temperature: \( m_\rho(\simeq 770 \text{ MeV}) \simeq 3\pi/(4z_L^m) \to 1/z_L^m \simeq 320 \text{ MeV} \). Note that the heavy quarkonium is blind to \( z_L^m \), since for heavy quarkonium \( z_L^m < z < z_H^m \) and \( z_H^m < z_L^m \). The vector meson mass spectrum from (4) is given by \([5]\]

\[ m_n \simeq (n - \frac{1}{4})\frac{\pi}{z_H^m}. \]  

(5)

Now we use this formula to calculate vector \( c\bar{c} \) mass spectrum. We use the lowest \( J/\psi \) mass of 3.096 GeV, as an input to fix \( z_H^m \). Then we obtain \( 1/z_H^m \simeq 1.315 \text{GeV} \), which is close to the value of \( c \)-quark mass. With this, the mass of the second resonance is predict to be \( \sim 7.224 \text{ GeV} \), which is quite different from the known mass of \( \psi' \) of 3.686 GeV.

2.2 Soft-wall model

In the soft wall model \([7]\), dilaton background was introduced for the Regge behavior of the spectrum, and we work mostly in this framework.

\[ S_{SW} = \int d^4xdz e^{-\Phi} \mathcal{L}_5, \]  

(6)

where \( \Phi = cz^2 \). Here the role of the hard-wall IR cutoff \( z_H^m \) is replaced by dilaton-induced potential, and the potential is given by \( V(z) \simeq cz^2 \) at IR. The equation of motion for the vector is \([7]\)

\[ \partial_z \left(e^{-B}\partial_z v_n\right) + m_n^2 e^{-B}v_n = 0, \]  

(7)

where \( B = cz^2 + \log z \) and \( m_n^2 \) is the 4D mass. After the following change, \( v_n = e^{B/2}V_n \), we obtain an exactly solvable Schrödinger type equation

\[ -\partial_z^2 V_n + V(z)V_n = m_n^2 V_n, \]  

(8)

where \( V(z) = cz^2 + 3/(4z^2) \). Then the mass spectrum of vector field in this model is given by \([7]\]

\[ m_n^2 = 4(n + 1)c, \]  

(9)

where \( \sqrt{c} \sim 1/z_H^m \). Again the lowest mode \( (n = 0) \) is used to fix \( c, \sqrt{c} \simeq 1.55 \text{ GeV} \). Then the mass of the second resonance is \( m_1 \simeq 4.38 \text{ GeV} \), which is 20% away from the experimental value of 3.686 GeV, and the third one \( m_3 \sim 5.36 \text{ GeV} \).
2.3 A braneless set-up

In [19], the IR cutoff is naturally replaced by the gluon condensate, in contrast to the hard-wall model, where the IR cutoff is introduced by hand. The background found in Ref. [19] is

\[ ds^2 = \left( \frac{R}{z} \right)^2 \left( \sqrt{1 - \left( \frac{z}{z_c} \right)^8} \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right). \]

(10)

The equation of motion reads

\[ \left( \partial_z^2 - \frac{1}{z} \left( \frac{4}{f_c} - 3 \right) \partial_z + \frac{m^2}{\sqrt{f_c}} \right) V_\mu = 0, \]

(11)

where \( f_c = 1 - (z/z_c)^8 \) and \( z_c^{-1} \sim m_c \). The value of \( z_c \) is fixed, again, by the lowest \( c\bar{c} \) mass, and is given by \( z_c^{-1} = 1.29 \text{ GeV} \). The mass of the second resonance is about 7GeV.

Judging from the observed vector \( c\bar{c} \) meson spectrum, we conclude that the soft wall model fits the low-lying charmonium masses better than the other models discussed in the present paper.

3 Finite temperature

To obtain the temperature dependence of the mass spectrum, we work on 5D AdS-Schwarzchild background, which describes the physics of the finite temperature in dual 4D field theory,

\[ ds^2 = \frac{1}{z^2} \left( f(z) dt^2 - (dx^i)^2 - \frac{1}{f(z)} dz^2 \right), \quad f(z) = 1 - \left( \frac{z}{z_h} \right)^4, \]

\[ i = 1, 2, 3. \]

(12)

where \( f_c = 1 - (z/z_c)^8 \) and \( z_c^{-1} \sim m_c \). The Hawking temperature is given by \( T = 1/\pi z_h \). A study [16] of a Hawking-Page type transition in the AdS/QCD models claimed that the AdS black hole is unstable at low temperature, roughly below \( T_c \sim 200 \text{ MeV} \). In the analysis, the contribution from mesons are not considered, since they are suppressed by \( 1/N_c \) compared to the gravitational part, and consequently, the estimated critical temperature could change slightly. Nevertheless, if we respect the observation made in [16], the background in Eq. (12) may not be relevant for describing the low-temperature regime, below \( T_c \). In the present work, however, we are mainly interested in the heavy quarkonium spectrum in the QGP, and so we will evaluate the temperature dependence of \( c\bar{c} \) states above the critical temperature of the Hawking-Page type transition calculated in [16]. In this respect, the AdS/QCD models, originally developed for light mesons like pions, may find their relevance in describing heavy quark system at finite temperature.

The equation of motion for the vector field at finite temperature in the soft-wall model is given by

\[ [\partial_z^2 - (2cz + \frac{4 - 3f}{zf}) \partial_z + \frac{m^2}{f^2}] V_i = 0. \]

(13)

If we take \( c = 0 \) and introduce the IR-cutoff \( z_m^H \), Eq. (13) is reduced to the equation of motion in the hard wall model. Here we set \( \partial_z V_i = 0 \), which corresponds to taking \( \vec{q} = 0 \), spatial
momentum to be zero, to define the mass at finite temperature in field theory. We first calculate the temperature dependent mass in the hard wall model, though it is not very successful in heavy quark system at zero temperature. We find that the heavy quarkonium masses decrease with temperature, similar to the behavior observed in the light meson system [20]. An interesting feature of the hard wall model at finite temperature for the heavy quark system is that we can roughly estimate the dissociation temperature in a simple and clear way. As it is nicely described in [20], when \( z_h < z_m^c \), the cutoff \( z_m^c \) is no longer an IR cutoff of the system, since \( 0 < z < z_h \), and the system is in a deconfined phase. In the light quark system, the critical temperature of the deconfinement is given by \( T_c = 1/(\pi z_m^c) \sim 102 \text{ MeV} \). On the analogy of the light meson system, we claim that a critical temperature defined by \( T_d^H = 1/(\pi z_m^H) (\sim 418 \text{ MeV}) \) can be identified as the dissociation temperature for the heavy quark system. The dissociation temperature of the heavy quark system is not the critical temperature of phase transition, but the point above which the bound heavy quark system in the quark gluon plasma dissociates into open heavy quarks. From lattice calculation of pure gauge theory, this dissociation is found to be around \( 1.6 T_c \) [12, 13], where for pure gauge theory, \( T_c \simeq 260 \text{ MeV} \) such that \( 1.6 T_c = 416 \text{ MeV} \). Therefore the predicted dissociation temperature from the hard wall model \( T_d^H \sim 418 \text{ MeV} \) is close to the lattice result.

Now we solve Eq. (13) in the soft wall model. To this end, we have to supply two boundary conditions. At UV, \( z = 0 \), we impose \( V_i(z = \epsilon) = 0 \) with \( \epsilon \to 0 \). The boundary condition at IR calls for more caution. At zero temperature, as in [20], the IR boundary condition (at \( z = \infty \)) is given by the requirement that the action should be finite. Now we require, at finite temperature, again that the action should be finite. Note that at finite temperature \( z_h \) plays a role of the IR cutoff. If the integrand of the action behaves, however, as \( \sim 1/(z - z_h)^n \), \( n \geq 1 \) near \( z_h \), then the action will not be finite. The relevant part of the action near \( z_h \) takes the following form:

\[
S_{SW} \sim \int_{\epsilon}^{z_h} (z_h - z)(\partial_z V_i)^2 dz.
\]  

(14)

Near \( z_h \), the vector field \( V_i \) will have the following profile: \( V_i(z) \sim (z_h - z)^n \). To make the action finite near \( z_h \), we require that \( a > 1/2 \). And so we impose the IR boundary at \( z_h \) as \( V_i(z = z_h) = 0 \). The results of the numerical solutions are shown in Fig. 1 where we used the thermal AdS metric below \( T_c \) and the AdS black hole above \( T_c \). Here we set \( T_c \simeq 250 \text{ MeV} \). As it is well-known, we have no temperature dependence with the thermal AdS background, and so the mass is temperature independent below \( T_c \). As in Fig. 1 we find that the mass will increase with temperature above \( T_c \). The sudden change of the mass spectrum near \( T_c \) is due to the use of different background below and above \( T_c \), namely, the thermal AdS and AdS black hole respectively. Finally we estimate the dissociation temperature of the heavy quarkonium system in the soft wall model. Since \( \sqrt{c} \) corresponds to \( 1/z_m^c \) in the hard wall model, we may define \( T_d^H = \sqrt{c}/\pi \), and so the predicted dissociation temperature in the soft wall model is \( \sim 494 \text{ MeV} \).

All the features appearing in Fig 1 namely the sudden decrease at \( T_c \) and its subsequent increase in the charmonium mass, can be qualitatively understood using lattice results on the color singlet part of the potential between a heavy quark and an anti-quark [22, 23].

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1 We note here that an analysis based on a Hawking-Page type transition [16] in the hard wall model predicts \( T_c = 2^{1/4}/(\pi z_m^c) \approx 1.189/(\pi z_m^c) \).

2 We refer to [21] for some discussion on the IR boundary condition in the soft wall model at finite temperature. It is shown that in some case, zero frequency and zero momentum limit, the infalling boundary condition can be effectively described by a Dirichlet boundary condition.
calculations of the heavy quark potential show a sudden flattening of the asymptotic value of the potential at $T_c$. This could be interpreted as the disappearance of the confining part of the potential, namely the string tension at the phase transition, which is first order for QCD with no dynamical quarks. Since the $J/\psi$ has a non trivial part of the wave function resting on the confining part, once this part vanishes, the mass is expected to decrease suddenly, as long as it is still bound. Such a sudden decrease of the $J/\psi$ mass just across the phase transition boundary is also found in a recent QCD sum rule analysis with condensate inputs from lattice gauge theory[24]. Furthermore, as the temperature increases further above $T_c$, the potential becomes shallow and the binding energy is expected to decrease[25]. Hence the mass of the bound state will increase until it dissociates into open charms with bare and thermal masses.

4 Summary

We have applied three AdS/QCD models to investigate properties of heavy quark system at zero and at finite temperature above $T_c$. We find that the soft wall model approaches the low-lying heavy quarkonium mass states at zero temperature better than the other two models considered in this work. At finite temperature, we observe that above $T_c$ the masses of quarkonium states increase with temperature, and the dissociation temperature, which is determined by the IR cutoff of a model, is around 494 MeV in the soft wall model. While we have limited the study to the calculation of mass spectrum so far, the results seem encouraging and consistent with expectations from lattice gauge theory calculation.

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