Topography evolution in convergent flows of two liquid layers

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Abstract. We have recently developed a two–layer model that considers the convergent motion of two initially uniform liquid layer with different densities and viscosities and assumes that the flow is due to the basal traction that acts at the bottom of the lower layer. We have used this model to describe successfully the evolution of mountains belts (Perazzo & Gratton, Phys. Fluids \textbf{22}, 056603, 2010). In this work we discuss how to modify our model to also describe the formation of plateaus. To this end we assume that below of a given level the viscosity of the upper layer drops abruptly, and in consequence the flow of this layer becomes decoupled of the motion of the lower region of the system.

1. Introduction

The structure, properties and motions of the outer layers of the Earth determine the topography of its surface, that changes in processes that take place very slowly: the timescale $T_o$ of the evolution of mountain ranges is of the order of $10^6 − 10^7$ years. Rocks subject to stresses for such long times deform plastically, with a rate that increases rapidly with the temperature, that in turn increases with the depth. Let us now summarize some concepts relevant to our purpose in order to introduce the geophysical terminology.

The lithosphere (figure 1) is the outer solid layer of the Earth and comprises the crust and the lithospheric mantle, that are separated by the Mohorovičić discontinuity (also called Moho), where the composition and the density of the rocks changes abruptly. The lithosphere rests on the asthenosphere that is the remaining portion of the upper mantle and that in the timescale in which we are interested behaves as a liquid. The lithosphere–asthenosphere boundary (LAB in brief) corresponds to the 1500 °C isotherm, where the viscosity drops by 2–3 orders of magnitude. The upper crustal rocks are rigid and fracture if the stresses overcome their mechanical strength, but deform plastically at greater depths. The level of this transition depends on the strain rate and on the temperature and is closer to the surface for slow deformations.

The term isostasy denotes the state of gravitational equilibrium of the lithosphere in which the pressure at the LAB is uniform. It implies that underneath any visible relief there is a anti–relief of the Moho, called root (see figure 1 for the case of a continent). If $\rho_c$ ($\approx 2.7$ g/cm$^3$) is the density of the crust, $R$ the topographic elevation, $\rho_m = \rho_c + \rho'$ ($\approx 3.2$ g/cm$^3$) the density of the lithospheric mantle ($\rho'$ $\approx 0.5$ g/cm$^3$) and $R'$ the depth of the root, isostasy requires that

$$\rho' R' = \rho_c R,$$

(1)
that means that $R' \approx 5.4R$. Any departure from the condition (1) (as happens if the relief flattens due to erosion) produces an isostatic recovery that takes place on a timescale $T_i < T_o$, which means that mountain ranges are isostatically compensated. This compensation occurs on regional scale and not locally because the rigidity of the crust distributes stresses over a larger area.

The lithosphere is divided into a number of plates that are in relative motion. The formation of reliefs such as mountain ranges (the orogeny) is due to the shortening and thickening of the crust that occurs when two continental plates collide (like the India–Asia collision that produced the Himalaya and the Tibet) or when an oceanic plate is subducted beneath a continent (as happens in the case of the Andes).

Various physical models have been used to describe the build–up and the evolution of mountain belts. All are extensions and variations of the thin viscous sheet model of England and McKenzie [1; 2]. A discussion and a classification of these models has been made by Medvedev and Podladchikov [3; 4], to which the reader is referred for more details. Many numerical calculations based on these models have been performed to describe specific orogenies as realistically as possible, but little effort has been aimed to gain a deeper physical insight of the process.

Several years ago Gratton [5] derived scaling laws for the evolution of cordilleras, based on simple assumptions about the viscous flow due to the shortening of the crust. To achieve a better understanding about these kind of flows we recently investigated a simple model that consists of an initially uniform layer of a Newtonian fluid that rests over an horizontal substrate [6; 7]. This substrate is divided in two parts, that for $T > 0$ are pushed one against the other with constant velocity. This convergent motion drags the liquid and produces a ridge, and we showed that there are two self–similar regimes that occur in different space–time domains. These regimes and their corresponding scaling laws were obtained analytically. Next we extended these results for liquids with a power–law rheology [8] because it describes better the behavior of the lithosphere. However, these simple models do not take into account isostasy so that are nor adequate to describe the behavior of the lithosphere. For this reason, more recently we used a two–layer model to investigate the formation of mountain belts [9].

In this paper we discuss the hypotheses and approximations involved in the theoretical models that describe the hydrodynamics of the orogenic processes that take place in the lithosphere. In Section 2 we discuss the basic assumptions of the two–layer model and its implementation. In Section 3 we comment its application to the evolution of peaked ridges. In Section 4 we deal with the changes that must be introduced to describe flat–topped plateaus. Finally in Section 5 we summarize the main conclusions.
2. The lubrication approximation and the two–layer model

The simplest way to take into account isostasy and the convergent motion that leads to orogeny is to model the lithospheric plates as two–layer systems, to allow for the different properties of the crust and the lithospheric mantle (see figure 2). For simplicity we shall consider here that the problem depends on a single horizontal coordinate \( X \) (not all orogens are linear, but the generalization to two horizontal dimensions \( X,Y \) is straightforward). The crust has a viscosity \( \mu_c \) and a thickness \( H_c(X,T) \). The lithospheric mantle has a viscosity \( \mu_m \) and its thickness is \( H_m(X,T) \). In general \( \mu_c \neq \mu_m \) and for the moment we shall assume that both are uniform, but later on we shall discuss what happens if \( \mu_c \) depends on the depth. We shall also assume that initially both layers are uniform and there is no relief, so that \( H_c(X,0) = C \) and \( H_m(X,0) = M \), and that isostasy is maintained at any time, so that for \( 0 \leq Z \leq H_m \) the pressure does not depend on \( X \). The visible topography is then given by

\[
R = \frac{\rho'_c}{\rho_m} (H_c - C)
\]

and \( H_m \) can be expressed in terms of \( H_c \) as

\[
H_m = M + \frac{\rho_c}{\rho_m} (C - H_c).
\]

To model the basal traction that drives the convergent motion of the plates we assume that at \( T = 0 \) the bottom of the lithosphere \( (Z = 0) \) starts moving with a prescribed velocity \( U_b(X) = -U_0 \text{ sign}(X) \). The two–layer model conserves the mass of the crust, but not the mass of the lithospheric mantle. The flow \( Q \) (per unit transversal length) of lithospheric mantle material across the LAB is

\[
Q = 2U_0 M \left( 1 + \frac{\rho_c C}{\rho_m M} \right).
\]

Notice that \( Q \) is larger than the inflow of lithospheric mantle due to the convergence of the plates \( (2U_0 M) \).

The vertical, horizontal and temporal scales of the problem are, respectively:

\[
C, \quad X_0 = \rho \frac{\rho_c g M C^2}{\rho_m \mu_m U_0}, \quad T_0 = X_0 / U_0.
\]

A characteristic of all mountain ranges is that on the regional scale the slopes are gentle \( (X_0 \gg C) \) so that the vertical component of the velocity can be neglected and the pressure is hydrostatic. The theory (see [9]) is then based on the lubrication approximation [10–12], for details see [13]. Briefly, a single equation can be obtained for \( H_c \) in terms of the vertically averaged horizontal velocity in the crust:

\[
V_c = \frac{1}{H_c} \int_{H_m}^{H_m+C} UdZ.
\]

Introducing the dimensionless quantities

\[
h = H_c/C, \quad v = V_c/U_0, \quad x = X/X_0, \quad t = T/T_0,
\]

the equations of the model are

\[
v = \mp 1 - \left( 1 + \frac{\rho_c C}{\rho_m M} \right) h \frac{\partial h}{\partial x} - \frac{C}{M} \left( \frac{\mu_m - \rho_c}{3 \mu_c} \right) h^2 \frac{\partial h}{\partial x}
\]

\((-1 \text{ for } x > 0 \text{ and } +1 \text{ for } x < 0) \) and the continuity equation

\[
\frac{\partial h}{\partial t} + \frac{\partial (vh)}{\partial x} = 0.
\]
3. The two-layer model and the evolution of cordilleras

Omitting details (see [9]), it can be shown that there is a self-similar asymptotics of the phenomenon in the limit \( R \ll C \), that describes with good approximation the growth of ranges whose height does not exceed 5 km. According to this self-similar solution the height and the width of the range vary as \( t^{1/2} \), and the aspect ratio (width/height) is given by \( X_0/C \). This value is of the correct order of magnitude for actual mountain ranges, within the uncertainties of the parameters.

The scaling law \( t^{1/2} \) can be justified [14] by means of a dimensional argument based on isostasy, conservation of the crustal mass, and a balance between gravitational and viscous stresses, as was done by Gratton [5]. However, in that paper different scaling laws were obtained because the viscous stresses were not estimated correctly (it was assumed that the relevant velocity gradients occurred near the root since it was not realized that the whole lithospheric mantle participated in the dynamics).

In [9] we compared the results of the theory with the average profiles of 10 approximately straight segments of various mountain belts and we obtained a very good agreement.

We conclude that the two-layer model describes reasonably well the formation of peaked ranges, and that with good approximation their evolution is self-similar. Although the properties of the lithosphere involve many parameters, orogeny depends only on \( U_0 \), \( C \) and \( X_0 \).

These results have been obtained assuming a Newtonian rheology for both layers, but our previous results for a convergent flow of a non-Newtonian liquid on moving solid substrates [8] indicates that the \( t^{1/2} \) scaling does not depend on the rheology, although the profile of the ridge depends on it.

4. The two-layer model and the evolution of plateaus

The two-layer model with uniform viscosity in the crust used in [9] does not describe the formation of almost flat plateaus like the Tibetan and the Altiplano. It is generally believed (see for example the review of Hodges [15]) that a flat top appears when the upper crust becomes decoupled from the basal traction. The formation of plateaus has been investigated by several authors [3; 4; 16; 17] by means of the channel flow hypothesis. In these papers it is assumed that there is a low viscosity layer at the bottom of the crust, just above the Moho. It is also assumed that the basal traction acts on the Moho, so that the lithospheric mantle is taken into account only for isostasy, but does not participate in the dynamics of the process. For example, Royden [16] assumes that the crust consists of an upper part in which the viscosity is uniform, and a lower part where the viscosity falls exponentially with depth. Obviously, as long as the root is shallow the viscosity of the crust is uniform and convergence produces a peaked ridge. However, as the thickness of the crust increases, the root eventually reaches the depth where \( \mu_c \) becomes very small and then the basal traction decouples from the crust. In these places...
Figure 3. A plateau develops when the root arrives at the level $H_0$ where $\mu_c$ falls abruptly and the crust becomes decoupled.

Figure 4. Formation of a plateau: (a) in the sides $\mu_c$ is uniform and the profile can be calculated by means of the model of Ref. [9]; (b) when the root arrives at $H_0$ the crust decouples and the relief cannot grow. The figures show the dependence on $Z$ of $\mu_c$ and $\mu_m$ (thin, red) and of $U$ (thick, blue) in both cases.

the height of the relief ceases to grow and a flat plateau develops, whose width increases as the convergent motion accumulates cortical mass at its sides (figure 3).

According to the two–layer model of Ref. [9] a decoupling that might produce a plateau happens only if the root touches the LAB, but the resulting plateau would be absurdly high (about 20 km).

Clearly, to reproduce real plateaus it is necessary to modify the two–layer model and assume that $\mu_c$ depends on $Z$ and that falls abruptly when the root arrives at a given height $H_0$ above the LAB (figure 4). The height $R_0$ of the plateau depends on $H_0$:

$$R_0 = \frac{U_0}{\rho_c} (M - H_0).$$

5. Conclusiones
The two–layer model based on the lubrication approximation allows to describe the essential aspects of the hydrodynamics of orogeny, provided it is assumed that the viscosity of the crust may fall abruptly at a certain depth in order to produce the decoupling that is needed to generate a plateau. We are presently investigating these processes.

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