Evolution of black hole mass and spin in collapsars

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We investigate the collapsar scenario, in which, according to Woosley (1993) and Paczyński (1998), the long gamma ray bursts (LGRBs) can be explained. The total energetics of explosions observed in the $\gamma$-ray band are consistent here with the total binding energy of a progenitor star. The characteristic of events, e.g. duration times, lightcurve profiles, variability, and connection with supernovae of different spectral properties, are still subject of many studies. In our scenario, the evolved progenitor star is collapsing onto a compact core, which forms a spinning black hole. The process of accretion via rotationally supported torus powers the ejection of relativistic jets. Here, the rotational energy of the black hole is presumably transported to the remote jet, which is mediated by magnetic fields. The rotation of pre-supernova star is a key property of the model. In our study, we investigate different types of collapsing stars: magnetized and non-magnetized. We probe the distributions of angular momentum inside the collapsar and the process of spinning up the black hole. Both Kerr parameter, and increasing mass of the black hole, put a constraint for the existence of rotationally supported torus inside the collapsar’s envelope.

1 Tested pre-supernova models

We used pre-supernova models from Woosley et al. (2002), Heger et al. (2000), and Heger et al. (2005). For of all those stars $M_{\text{ZAMS}} \sim 25M_\odot$. First of them (NR) similarly to the one used in Janiuk & Proga (2008) and Janiuk et al. (2008) did not take into account rotation neither magnetic field during star evolution. Its initial metallicity was $10^{-4}$ solar. There is no mass-loss in this case. In the second star (R) rotation was introduced, hence this model is affected by the biggest mass-loss leading to pre-supernova mass of $5.62M_\odot$. In the third model (RMF) both rotation and magnetic field is incorporated in evolution. It leads to pre-supernova mass of $12.55M_\odot$. For the second and third model initial metallicity was equal to solar.

2 Models of accretion

Critical specific angular momentum $l_{\text{crit}}$ is given by the equation:

$$l_{\text{crit}} = \frac{2GM_c}{c}\sqrt{2-A+2\sqrt{1-A}},$$

where $M$ and $A$ denote mass and spin of black hole. Torus can be sustained when there is a matter with $l_{\text{spec}} > l_{\text{crit}}$. The original distribution of angular momentum in the stellar envelope was given by $l_{\text{spec}} = x l_{\text{crit}} f(r, \theta)$, where $f(r, \theta)$ depends on the model. We examined variety of $x$ values. Every step of black hole mass ($M$) and angular momentum ($J$) evolution in our model is given by following set of equations:

$$M^k = M^{k-1} + \Delta m \quad J^k = J^{k-1} + \Delta j \quad \Delta m^k = 2\pi \int_{r_k}^{r_k+\delta r} \int_0^\pi \rho r^2 \sin \theta d\theta dr \int_0^\pi \rho r^2 \sin \theta d\theta dr$$
Thereafter spin parameter $A$ is expressed by the formula: 

$$A = \frac{cJ}{GM^2}.$$ 

This evolution formula ensure fulfillment of the condition $A < 1$, but do not specify any mechanism of the angular momentum loss. We investigate two accretion scenarios (scenario 1 and 3 accordingly from Janiuk & Proga (2008)). Accretion proceeds through: both torus and the envelope in the first scenario, only torus in the second. Calculations were stopped when there was no matter with $l_{\text{spec}} > l_{\text{crit}}$. We present examination of four models, with combinations of accretion scenarios and $f(r, \theta)$ formula: $f(r, \theta) = 1 - |\cos\theta|$ (A models) and $f(r, \theta) = \sin^2\theta \sqrt{\frac{r}{r_{\text{in}}}}$ (D models).

### 2.1 Models with angular momentum distribution given by function A

In the left panel of Fig. 1 we present evolution of the spin parameter for the A1 model. The evolution depends heavily on the pre-supernova star type. Accretion in RMF and NR models ends at the same radii. For small $x$ their final $A$ is similar, but with increase of $x$ the difference between final spins grow, RMF leads to higher values. Pre-supernova model R allows the support of torus for higher $r$. Starting from $\log r \sim 10.5$ accretion almost doesn’t change $A$, due to the low density in outer most parts of the pre-supernova. For A3 scenario (i.e., only torus accretion), all examined $x$ values lead to rapid growth of $A$ to value $\sim 1$ and maintaining this value until the end of the accretion. In the right panel of Fig. 1 we present final mass of the black hole as a function of $x$. Higher $x$, which corresponds to higher specific angular momentum of the matter, leads to more massive black holes, because accretion in our model can proceed longer.

### 2.2 Models with angular momentum distribution given by function D

In case of model D1, we examined different range of $x$ values. Due to dependence of $l_{\text{spec}}$ on $\sqrt{r}$, the $x$ values from A1 model lead to very high initial $l_{\text{spec}}$. Therefore, now we examined values in the range $x = [0.05, 1.0]$. 

Fig. 1: Left panel: Evolution of the spin parameter for different pre-spernova models and values of $x$ parameter. Right panel: Final mass of the black hole as a function of $x$ parameter.

$$\Delta j^k = 2\pi \int_{r_k}^{r_k+\Delta r} \int_0^\pi \min[l_{\text{spec}}, l_{\text{crit}}] \rho r^2 \sin\theta d\theta dr.$$
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Fig. 2: Left panel: Evolution of the spin parameter for different pre-supernova models and values of x parameter. Right panel: Final mass of the black hole as a function of x parameter.

Dependence of the spin of BH on pre-supernova model is similar to the A1 model. For small $x$ values, the NR and RMF models are terminated after roughly the same part of the star’s envelope has accreted, and they give similar final $A$ values. Situation changes for faster rotation, with $x = 0.9$. For this $x$ value, both NR and RMF models sustain the torus until the outermost part of the star start accrete, and the difference in the radius of termination occurs due to different sizes of the stars. In case of pre-supernova model with rotation, the whole star accretes for $x$ larger than 0.5. Evolution of the Kerr parameter is presented in the left panel of Fig. 2. Final mass of the black hole as a function of $x$ is shown in the right panel of Fig. 2.

3 Summary

Final black hole masses are in the range of $\log\left(\frac{M_{BH}}{M_{\odot}}\right) = [0.6 - 1.05]$, depending on the accretion scenario and pre-supernova model. In case of spin evolution through consecutive accretion layers, models A1 and D1 give different shape of the $A$ evolution: in case of A1 scenario for every pre-supernova model we observe growth at first and then depending on exact model and $x$ value, the spin may drop or saturate, at the outer parts of the star. On the other hand, every D1 model gives drop of $A$ at first and then growth for high enough value of $x$.

References

Heger, A., Langer, N., Woosley, S. E., ApJ 528, 1, 368 (2000)
Heger, A., Woosley, S. E., Spruit, H. C., ApJ 626, 1, 350 (2005)
Janiuk, A., Moderski, R., Proga, D., ApJ 687, 1, 433 (2008)
Janiuk, A., Proga, D., ApJ 675, 1, 519 (2008)
Paczyński, B., in Gamma-Ray Bursts, 4th Hunstville Symposium, American Institute of Physics Conference Series, volume 428, 783–787 (1998)
Woosley, S. E., in American Astronomical Society Meeting Abstracts #182, American Astronomical Society Meeting Abstracts, volume 182, 55.05 (1993)
Woosley, S. E., Heger, A., Weaver, T. A., Reviews of Modern Physics 74, 4, 1015 (2002)