Developing the teaching materials of algebra matrix based on M-APOS model to develop students’ autonomous learning on math

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Abstract. This study aims to develop Matrix Algebra course teaching materials based on the M-APOS learning model to improve students' mathematical self-regulated learning with valid, practical, and effective criteria. This is a research and development (research and development) using the ADDIE method (Analysis, Design, Development, Implementation, and Evaluation). Validity test of teaching material was by expert judgment. The practicality test of teaching material was conducted by evaluating 5 small-scale test subjects and 24 large-scale trials. The effectiveness test of teaching materials is carried out by implementing the Matrix Algebra course by analyzing self regulated learning. The stages in this study include the Analysis, Design, Development and Evaluation stages. 1) The analysis phase includes analyzing the needs, characteristics of students, and curriculum. 2) The design phase includes activities to determine the product design (teaching material) to be developed, an outline of the presentation of the material, collecting references and compiling assessment instruments. 3) Development phase, namely the activity of making and testing teaching materials. 4) The evaluation phase shows that the development of teaching materials in the form of Matrix Algebra modules based on the M-APOS learning model can improve students' mathematical self regulated learning in a valid, practical, and effective criteria.

1. Introduction
Matrix Algebra is a basic course which focuses the study on computational skills, although is some parts, it requires students to use pure deductive logical thinking. In some sub-courses of it, such as the concepts of determinant and matrix equivalence, the natural deductive would be stressed such as the proofment of theorem and others. This is the reason why the concepts of Matrix Algebra should be mastered well to make the students’ understanding on Match related to this course better.

Students are required to design paper in group based on the subject matters that have been carefully considered, they would present the paper in front of the class and others would give comment. The routine skills in teaching describes that a lecturer can apply a set of knowledge and skill that have been well defined to solve problems. Skills on routine teaching would be cultivated to help students of teacher training department as a way to master some knowledge and skills on teaching and apply them to solve problems in general classrooms. Generally, knowledge and skill are identified through research or teaching model [1]. However, in some learning processes, a lecturer may take control the class due to the inability of the students to actively participate in the classroom discussion, as most of
the students have less eagerness to communicate ideas or to raise questions to the teachers of the presenters in front of the class. Only some of the students who have the better abilities and eagerness are willing to communicate their ideas and raise questions.

Autonomous learning is an important aspect for the students. [2] stated that autonomous is an effort to deepen and manipulate the associative network in a particular subject, observe and develop one’s process to master particular thing. More on it, Hargis (2000) supported fact that one who has high autonomous learning tends to study and learn better, able to observe, conduct evaluation and design the rhythm of learning effectively to gain the better achievement. The characteristics of autonomous learning [3] are as follows: (1) one designs his own learning based on the need and purposes that have been carefully considered; (2) he will employ particular learning strategies and conduct it appropriately; and (3) he would observe the learning development, evaluate the outcomes and compare it by using particular standards.[4] stated that there are 3 aspects related to this kind of autonomy, those are independent, autonomy and self-reliance.[5] stated that one can be assumed independent whenever he works and thinks physically independent, utter expression and idea which is understandable and other activities that are conducted enthusiastically. For those purposes, the researchers attempt to develop teaching materials based on M-APOS learning model.

APOS theory is the adaptation from the ideas of Piaget to learn Mathematical knowledge development on human through steps of Act, Process, Object and Scheme. sHowever, this gives perspectives that are possibly to be analyzed and explained [8] Using APOS as the theoretical perspective,[9] provides examples on how act and gradient can play a role on students’ understanding about function and covariative logical thinking. [10] employed the theory of APOS. He stated his perspective that one’s knowledge and understanding is his tendency to respond towards mathematical situation and reflect it to the social contexts. Next, he would construct and reconstruct mathematic ideas through acts, processes and mathematical objects, in which later, they would be organized into particular scheme as a way to solve problems. The problems would be solved by using both high and low order of thinking [11][12] The theory of APOS in the learning process would be conducted by the using ACE (activities, class discussion and exercises) [13]. In every step of activity, students do the exercises by using computer. Next, in the step of class discussion, students discuss the results of the exercises. Finally, the step of do the exercises, students are given the problems of exercises to do either at home or in the classroom. There are a lot of obstacles when the implementation of APOS using computer was implemented, so that the modification is required to do by giving students the exercise sheet. The modification of giving assignment in the frame of learning based on the theory of APOS is called M-APOS learning model [14].

The role of giving assignments to students based on M-APOS model is to guide students to study the material, solve the math problems and other activities before teachers give the course. Besides, the purpose of giving assignments is to develop the quality of students’ learning processes to make them becomes more engaged. By giving this assignment, it is expected that students would find and get any needed information to get students to gain knowledge and information by themselves, not only from the lecturers. As the result, students by themselves would find information and knowledge that should be studied. In simple words, it is expected that the students’ autonomy would develop. This would be beneficial whenever the act of giving assignments is aimed at developing students’ math concepts and learning autonomous in the Matrix Algebra Course. According to those descriptions, the researchers would develop learning materials based on M-APOS learning model. It is highly expected that the learning materials would develop autonomous learning effectively.

2. Methods

2.1. Research Types and Procedures
This is a research and development. In the process of developing the learning material, this R and D employs the ADDIE (Analysis, Design, Development, Implementation and Evaluation).
2.2. Research Instrument and Data Analysis Technique
The instruments that were employed in this research are observation sheet, students’ respond questionnaire and students’ questionnaire on learning autonomous related to the problems on Matrix Algebra. The data analysis in this research include 1) instrument testing analysis, 2) research data analysis. Instrument testing results on data analysis are used to reveal the content validity and instrument reliability. The instrument content validity is based on expert judgement of two lecturers and moment product, besides, the reliability is by the formula of Alpha Cronbach. Research data analysis were through three steps such as validity analysis, practicality analysis and effectivity analysis. 1) Validity analysis is used for the data which were collected from two validators that have filled 2 sheet questionnaire, next, the data analysis would be conducted. The learning material or the modul that has been developed would be considered as valid when the total mean score from a few experts judged as ‘fair’ criteria. 2) the practical analysis modul that was developed was gained from students’ questionnaire on responds and the observation sheet of the learning process. The modul would be considered as practical if the fair criterion has been achieved from the students’ questionnaire respond and the learning process questionnaire sheet. 3) The effectiveness analysis of modul development based on M-APOS learning model can be observed from the questionaire of students’ Mathematical learning autonomy. The modul development based on M-APOS learning modul in this research is considered effective to increase the students’ mathematical learning autonomous if not less than 70% students gain ‘high scores’ on autonomous learning and on the understanding of the Mathematical concepts.

3. Results and Discussion
This research generates the product of module of Matrix Algebra based on M-APOS learning model to develop students’ autonomous learning on math. The results would be explained below.

3.1. Analysis
The results of analysis step of this research is as follows: a) Need Analysis, Based on the research on matrix Algebra learning that has been conducted in the Departement of Mathematical of Universitas PGRI Yogyakarta; b) Curriculum analysis, On of the learning goals of the Matrix Algebra is the student will have the ability to understanding related to the matrix Algebra. The learning process, that has been conducted, is not totally the learning goals oriented; c) Students’ Characteristic Analysis. The analysis reveals us that student actually have the ability to be given the better understanding related to the matrix algebra.

3.2. Design
In the step of Designing, the researchers designed the product of the module on Matrix Algebra based on the M-APOS learning model and the developed assessment instruments. The arrangement of the module, including the making of cover, title, formula of the learning goals, title of the learning materials, learning model, assessment tools, the outline of the material presentation, collection of the references and the instrument of module assessment. The instruments of module assessment that would be developed are the assessment sheet of module validation, students’ questionnaire on respond, the observation sheet of the learning processes, questionnaire on learning autonomy and test of the students level of understanding on math concepts.

3.3. Development
The steps of development are the instrument development, module development, validation and module revision. The description of those are as follows.

3.3.1. The Instrument Development. The instruments of assessment that have been designed in the previous step would be validated by two validators. It aims at revealing the appropriateness of the instrument before it is used as the tool of assessment.

3.3.2. The Module Development. The Matrix Algebra module developed in this study follows a design that has been created that includes cover, identity, introduction, content table, material title,
learning objectives, discussion materials, discussion sheets, conclusions, exercises, assessments and references.

3.3.3. **The Module Validity.** Mathematical module as the development product in this research would be assessed in terms of the quality based on the expert judgement. The assessment was conducted by two validators. When they state that it is valid, then the module can be used for small and large-scale testing. The suggestion and critics from the validators can be used as the guidance for the sake of improvement. The result of data analysis related to the Matrix Algebra based on M-APOS learning model that have been developed by the validators are as follows.

| Table 1. The Module Assessment Analysis Result based on Expert Judgement |
|--------------------------------|----------------|--------------|---------------|----------------|
| Aspects that are assessed    | Assessment    | Mean (%)     | Percentage (%)| Criteria       |
|-------------------------------|---------------|--------------|---------------|----------------|
|                               | Validator 1   | 43           | 43            | 95.56 Excellent|
|                               | Validator 2   | 43           | 43            | 95.56 Excellent|
| Content Appropriateness      | 43            | 43           | 95.56         | Excellent      |
| Way of Presenting            | 35            | 27           | 88.57         | Excellent      |
| Language                     | 15            | 14           | 96.67         | Excellent      |
| Illustration                 | 9             | 8            | 85.00         | Excellent      |
| Completeness                 | 10            | 9            | 95.00         | Excellent      |
| The Module layout            | 9             | 9            | 90.00         | Excellent      |
| Feasibility                  | 15            | 15           | 100.00        | Excellent      |
| Mean Score                   | 92.97         | 92.97        | 92.97         | Excellent      |

The module assessment by the validators reveals us that the total mean is 92.97% with the criteria of excellent. Based on that assessment, the researchers conclude that the developed module of Matrix Algebra is valid.

3.3.4. **The Module Revision.** The result of module revision on mathematics which is based on validators’ assessment is related to the miss-writing of alphabeticals and numbering symbols. All of these mistakes are directly corrected by the two validators.

3.4. **Implementation**

There are some activities in the step of implementation, those are the testing module that is being developed, the spreading of questionnaire on students’ respond, filling the observation sheet on the learning processes, and giving the questionnaire on math learning autonomy after using module of Matrix Algebra that has been developed. The explanation of the implementation step is as follows:

3.4.1. **The Product Testing.** The first testing on product was conducted in small scale. It was conducted to the 5 students who joined Matrix Algebra to know the students’ assessment on content apropriateness, presentation, language, illustration, completeness, Physical appearance, modul feasibility that is developed by giving questionnaire to students. The results are as follows.

| Table 2. Students’ Respond Result in the Small Scale Testing |
|-------------------------------------------------------------|
| The assessed Aspects                                       | Mean (%) | Percentage (%) | Criteria |
|-------------------------------------------------------------|----------|----------------|----------|
| Content Appropriateness                                    | 41       | 91.11          | Excellent|
| Presentation                                               | 31.4     | 89.71          | Excellent|
| Language                                                   | 13.6     | 90.67          | Excellent|
| Illustration                                               | 9.2      | 92.00          | Excellent|
| Completeness                                               | 8.4      | 84.00          | Excellent|
| Physical Appearance                                        | 9.4      | 94.00          | Excellent|
| Feasibility                                                | 13.2     | 88.00          | Excellent|
| Mean score                                                 | 89.93    | 89.93          | Excellent|

Ideal Score = 100

After this small-scale product testing, the large scale one would be conducted. It would be done to the whole 20 students in the class of Matrix Algebra.
Table 3. Students’ Respond Result in the Large Scale Testing

| Assessed Aspects       | Mean (%) | Percentage (%) | Criteria       |
|------------------------|----------|----------------|----------------|
| Content Appropriateness| 86,78    | 86,78          | Excellent      |
| Presentation           | 85,71    | 85,71          | Excellent      |
| Language               | 89       | 89             | Excellent      |
| Illustration           | 85       | 85             | Excellent      |
| Completeness           | 85       | 85             | Excellent      |
| Physical appearance    | 86       | 86             | Excellent      |
| Feasibility            | 89,67    | 89,67          | Excellent      |
| Mean score             |          | 86,75          | Excellent      |

Ideal Score = 100

It can be seen from the questionnaire of students’ respond in the small-scale testing, it reaches the mean score of 90.14 from the ideal score 100 with the criteria of ‘excellent’. From the large-scale testing, it reaches the mean score of 86.75 from the ideal score 100 with the criteria of excellent. Based on the data analysis on students’ questionnaire on respond, it can be concluded that the module of Matrix Algebra based on M-APOS learning model fulfills the criteria of practical.

3.4.2. The Fulfilment of Observation Sheet on the Learning Feasibility. The observation sheet on learning feasibility by using the module of Matrix Algebra based on M-APOS learning model was fulfilled by the observers based on the result of observation results during the learning process. The observers in this research are the researchers’ colleagues. The results are as follows:

Table 4. The Result of Analysis on Observation Sheet on the Learning Feasibility

| Aspect            | Percentage of the meeting number: |
|-------------------|-----------------------------------|
|                   | 1       | 2       |
| Introduction      | 100%    | 100%    |
| Content/Core      | 100%    | 100%    |
| Closing           | 86.67%  | 100%    |
| Mean              | 95.56%  | 100%    |
| Total Mean Score  | 97.78%  |         |
| Criteria          |         | Excellent |

Ideal Score = 100%

Based on the analysis on observation sheet towards the learning feasibility above, it generates result that the module of Matrix Algebra based on M-APOS fulfills the practicality criterion due to the 97.78% percentage of total learning model from the ideal score of 100% with the criteria of excellent.

3.5. Evaluation
In this part, the evaluation was conducted towards the students’ learning autonomous after using the module which is based on M-APOS learning model. It aims at revealing the module effectiveness based on M-APOS learning on the determinant material which has been developed. The analysis result of the questionnaire on students’ learning autonomy are as follows:

Table 5. The Recapitulation of Analysis on Students Learning Autonomy

| Mean Score of Students Learning autonomy | Criteria | Deviation Standard |
|----------------------------------------|----------|--------------------|
| 80,31                                  | High     | 4,98               |

Moreover, the recapitulations of amount of students, which are based on the criteria of learning autonomy, are as follows:

Table 6. Recapitulations of amount of students for each

| Criteria | Amount of Students | Percentage (%) |
|----------|--------------------|----------------|
| High     | 20                 | 83.33          |
| Fair     | 4                  | 6.67           |
| Less     | 0                  | 0              |
| Low      | 0                  | 0              |
Based on Table 5 and 6, it can be concluded that the mean score of students’ learning autonomy is 80.31 from the ideal score 100 with the criteria of ‘excellent’ and the standard of deviation of 4.98. Meanwhile, there are 83.33% of excellent score and 6.67% of fair criteria. Furthermore, the excellent-score-students gain 70% successfulness indicator. It means, the conclusion is that the Matrix Algebra based on M-APOS learning model on the determinant material can increase students’ learning autonomy effectively. As the consequence, the researchers concluded that Matrix Algebra based on M-APOS learning model to develop ability to understand the Mathematical concepts and students learning autonomy on determinant concept fulfill the criteria of validity, practicality and effectiveness.

In this research, the development of Matrix Algebra based on the learning model of M-APOS is aimed at developing students’ learning autonomy on math. The learning autonomy can develop after learning process on Matrix Algebra which is based on M-APOS learning model which is considered from students’ Mathematical learning autonomy questionaire result. The successfulness of M-APOS learning model as the alternative learning model enables it to be the learning which is student-centered. [14] who stated that the steps of M-APOS learning model are ACE (activity, classroom discussion, and exercise) gives chances to students to be more active to gain information which is needed to do the exercise independently and or in group from various learning sources. In accordance with the statement [13], he stated that the implementation of M-APOS learning model through the steps of ACE such as assignment and exercise given to students are able to develop learning autonomy compared to the traditional methods.[15] in their research, stated that students who studied in group based on steps of M-APOS learning model enables the material to be internalized in students’ memory. Besides, the steps in M-APOS help students to prepare learning material in the next meeting.

Based on the experts’ statement, basic theories from some other relevant researchers with this research, and the result of this research that has been described, it can be concluded that this development research generates module on Mathematics Algebra based on M-APOS learning model on the material of determinant. It is valid, practical and effective which enables students to develop autonomy on Mathematical learning process.

4. Conclusion
The conclusion of this research are: 1) The development of teaching material in form of module on Matrix Algebra based on M_APOS learning model can develop students’ learning autonomy to understand mathematical concepts which is valid, practical and effective. 2) the mean score of ability to understand the mathematical concepts reaches 91.04 from the ideal score of 100 with the category of excellent and 9.44 of deviation standard. Meanwhile, there are a lot of students gained 91.67% of excellent score and students who have 8.33% of fair level. Furthermore, the mean score on students’ autonomy learning is 80.31 from the ideal score of 100 with the category of excellent with 4.98 of standard deviation. Meanwhile, there are a lot of students gain the excellent level of 83.33% and 6.67% of fair level.

According to that conclusion, it implies that the development of teaching material in Matrix Algebra based on M-APOS is able to increase students’ level of mathematical understanding together with the level of autonomous learning. For that, the students and lecturers can use the modul as tool to develop and increase the learning autonomy and level of understanding on Math concepts. The suggestions from this research are 1) The module on Matrix Algebra based on M-APOS learning model can be used by the lecturers as one of the teaching materials in that course since it develops students’ level of understanding and the learning autonomy, 2) It is needed to develop module on Matrix Algebra based on M-APOS learning model for other Math concepts, not just the determinant.

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