Observational constraints on complex quintessence with attractive self-interaction

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In this paper we consider that dark energy could be described solely by a complex scalar field with a Bose-Einstein condensate-like potential, that is, with a self-interaction and a mass term. We analyse a particular solution which in a fast oscillation regime at late times behaves as a cosmological constant. First, we will show that this proposal adequately describes the standard homogeneous Fridman dynamics. However, we show that the precision cosmological test using current surveys show that the equation of state (EoS) \( w \) of such model, which depends on a single parameter, fails to adequately describe a dynamical dark energy since the analysis in fact constrains the scalar field parameters within values ruled out by the theoretical model.

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I. INTRODUCTION

The inclusion of the dark components of the Universe in Einstein equations gives a consistent description of the current observed dynamics at a large scale [1,3]. This components are known as dark energy and, at a galactic level [4-8], dark matter. The nature of such dark components remains unknown. Dark energy, although in first order is modelled as repulsive gravitational term, such as the Cosmological Constant \( \Lambda \), certain observations have shown some tensions in the Hubble flow in the standard \( \Lambda CDM \) model [9], so that it seems that is not sufficient to describe the dark energy with a constant term; it is thus proposed to be modelled by different types of matter such that the relation between the spatial components of the corresponding stress energy tensor, to the temporal one, be consistent with the observed dynamics; that is, using an analogy with fluid dynamics, an equation of state \( T^i_j = -w \rho c^2 T^0_0 \) (i. e. \( p = w \rho \rho \) for the pressure and density of a fluid like description). The behaviour of the function \( w \) can be related to the observations, as described bellow, and its value at the present is close to minus one.

Regarding the modelling of the dark matter, several models have proposed that it should be considered as a weakly interactive particle. However, not strong evidence of such a particle has been detected in the current projects that have been created ex professo to have a detection either directly [10,13] or indirectly [14,16]. It must be faced the possibility that dark matter had zero interaction with the baryonic matter.

Indeed, as mentioned above, the Theory of General Relativity allows to describe several kinds of matter/energy, in comparison to the Newtonian case. In this way, once there are models of one type of matter or another, consistent with the observations, the next step is the determination of characteristic features generated on the baryonic matter by each kind of matter which could, in principle, be detected. Even supposing that there is not interaction of the baryonic matter with the dark components, other than gravitational, it can still be seen that the density distribution of the different kinds of matter/energy has a very distinctive feature which affects the distribution of baryonic matter, and that could tell at least what kind of matter better describes the observed density distribution, see e.g [17] for a discussion on the subject. As an example of the latter, in [18] was studied how the density perturbations evolve inside a dark matter halo considering that the matter was a collection of non-interactive particles, whose dynamics are described by the Vlasov equation. The main result in that work was that the final state indeed has a very distinct distribution in the coordinate space, a double peaked Gaussian in the density and, in the phase space, a volcano-like form in the distribution function, features that could in principle affect the baryonic density distribution, which is an observable quantity.

There are several considerations that must be taking into account regarding the dark energy. It is a component associated mainly with the cosmic acceleration [19, 21] which, as mentioned above, can be modelled with the simple inclusion of a properly tuned \( \Lambda \) in the Einstein’s equations, although this constant rules out the usual Minkowski’s solution, and the asymptotic limits of all the well established solutions to the Einstein’s equations need to be modified. It is an exciting fact that there is a new constant of Nature, see [22] for an interesting discussion on the subject, but the implications in the equations themselves enhance the need to prove the veracity of such model, a fact which is done proposing more general models to describe the cosmic acceleration. In addition to its modeling with different kinds of matter, another way to proceed, is to propose alternative gravity models of matter that can describe the current observed dynamics [23, 25].

Within the models proposed to describe the dark energy other than a constant term, in Einstein gravity, those considering a scalar field can be the simplest, well
motivated choice from a particle physics point of view. Nevertheless, the great challenge is to determine the appropriate scalar potential $V(\Phi)$ that could explain current cosmological observations. The main motivation for considering quintessence models is to reduce the so-called fine-tuning problem, issue that has been explored by the tracker solutions. However, the predicted values on these models for the EoS at the present epoch is not in good agreement with supernovae results [26, 27]. Another example is the exponential potential that focus on models and parameters which lead to inflation, nevertheless nucleosynthesis constraints require that the energy density of the scalar field be $\Omega_\phi \leq 0.2$, i.e., it would never dominate the Universe [28]. Another dynamical potentials proposed in [29] and [30] avoid successfully the fine-tuning and cosmic coincidence problem, but the values of the potential parameters can not be unambiguously determined in order to match the observations constraints.

The above models are made up of real scalar fields however, complex scalar fields should be considered since such fields (unlike the real case) have been invoked in many different sectors of particle physics [31] (such as the Higgs mechanism) and interestingly in the scene of ultra cold gases; they can be used to construct static distributions as Boson stars, configurations surrounding a black hole, so called wigs [32, 33], and they can even define static configurations with an associated angular momentum number [34, 35]. Furthermore, a real quantized scalar field yields the same field equations as those obtained by using a classical complex scalar field [36]. These reasons motivated us to consider a dark energy model described by a massive quintessence–complex scalar field with attractive self interaction. Such field was formerly studied by SuÅąrez et. al. in [37] and, in the present work, we revisited the idea focusing in the so-called peculiar branch solution of the Einstein-Klein-Gordon equations in order to obtain parameter restrictions of the potential consistent with the current precision observations. Although we are aware of the latest results regarding the possible dynamical behavior of the EoS [9] and the impossibility for a single canonical field to evolve crossing over $w = -1$ because of the no-go theorem [38], we are interested in exploring in detail the properties of the previously mentioned branch and in computing best fit values of their parameters, in order to have a quantitative description of the model and a clearer picture of what the model needs to be consistent with such a dynamical behavior of the dark energy.

This paper is organised as follows: in Sec. II we briefly describe the Fridman (we use the direct transliteration from the Russian) background considering a complex scalar field instead of the standard cosmological constant. In Sec. II we derive the Einstein-Klein-Gordon equations to describe dark energy based in the fact that the scalar potential can be proposed as an effective fluid, with the caution of not solving the EoS, but solving Klein-Gordon first, and with the field, and its derivative, compute the density and the scalar pressure, and then, compute the corresponding $w$. We consider the fast oscillation regime, where the pulsation $\omega = d\Phi/dt$ of the scalar field is assumed to be faster than the Hubble expansion. In Sec. III a generic EoS with a complex scalar field mimicking the dark energy term is presented. This EoS is obtained for a peculiar branch in the fast oscillation regime. We denote the model such as the one presented in this manuscript as Complex Scalar Field Dark Energy (CSFDE). A description of the current late-time observations are given in Sec. IV. These samplers will be employed to constrain the only free cosmological parameter that goes into the expression for the EoS of our CSFDE model. Also, we will discuss the cosmological constraints obtained. Finally, our conclusions are given in Sec. V.

\section{II. Complex Scalar Field in an Homogeneous Background}

In this section we derive first the evolution equations for a homogeneous and flat universe filled with radiation, baryonic and dark matter components and an effective density described by a complex scalar field, which will mimic the dark energy component. In the second part, we introduce the complex scalar field to obtain the corresponding Klein-Gordon equation.

\subsection{A. Fridman equations}

First, let us consider an homogeneous isotropic Universe, described by the Fridman-Lemître metric

\begin{equation}
    ds^2 = -c^2dt^2 + a^2(t) \left[ \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right],
\end{equation}

where $a(t)$ is the scale factor and $K$ the curvature scalar. From this point forward we consider spatial flatness. As it is standard, we can derive the Fridman equation and the energy conservation equation by introducing the above metric in the Einstein’s equations. Before continue with this straightforward calculation, let us establish our pivot model: the paradigmatic cosmological model, $\Lambda$CDM, which considers a total density of the Universe $\rho_T = \rho_r + \rho_b + \rho_{\text{cdm}} + \rho_{\text{DE}}$, normalised by the critical density given by $\rho_{\text{crit}} = 3H_0^2/8\pi G$, where $H_0$ is the Hubble parameter at present time and $G$ is the gravitational constant. According to this, we can derive the constriction equation from the Friedman evolution as

\begin{equation}
    \left( \frac{H}{H_0} \right)^2 = \Omega_m + \Omega_\Lambda,
\end{equation}

with

\begin{equation}
    \Omega_m = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{b,0}}{a^3} + \frac{\Omega_{\text{cdm,0}}}{a^3},
\end{equation}

where $\Omega_i = \rho_i/\rho_{\text{crit}}$ ($i = \text{cdm}, b, r$), represents the density parameter and the symbols $\text{cdm}, b, r$ correspond to
cold dark matter (CDM), baryonic matter and radiation, respectively.

According to Planck 2018 [39], the statistical values for the densities described above are: \( \Omega_{\text{cdm}} h^2 = 0.120 \pm 0.001 \), \( \Omega_B h^2 = 0.0224 \pm 0.0001 \), \( \Omega_L = 0.674 \pm 0.013 \) and \( \Omega_m = 0.315 \pm 0.007 \). Currently, this model has proved to be consistent with several observations, however, it has problems in regards to the tension on the value of some parameters like those of \( \sigma_8 \) and \( H_0 \) [40].

As indicated in the introduction, it is interesting to explore dynamic EoS since they alleviate tensions between certain cosmological parameters. Classical scalar fields are simple models for introducing time-dependent equations of state. In particular, the case of a scalar field minimally coupled to gravity, with a positive canonical kinetic term, is called quintessence [26, 41]. Extensions to this model have been widely considered, for example some by including non-canonical scalar fields or negative signed kinetic terms. However in this work, we take an example from a simpler case, which considers a rapidly oscillating minimally coupled complex scalar field [31] [37] [32].

B. The Klein-Gordon equation

We use the evolution described as a starting point, and introduce a complex scalar field in order to model dark energy. Our proposal is based in the fact that the scalar potential \( V(|\Phi|^2) \), has a quartic-form with a negative scattering length as

\[
V(|\Phi|^2) = \frac{m^2 c^2}{2\hbar^2} |\Phi|^2 - \frac{2\pi A_s m}{\hbar^2} |\Phi|^4, \quad (4)
\]

where \( m \) is the complex scalar field mass, \( A_s \) the absolute value of the scattering length and \( \hbar \) the reduced Planck constant. This scalar potential describes, for instance, a relativistic Bose-Einstein condensate at zero temperature with attractive self-interaction [43] [44], and it is also similar to the Higgs potential of particle physics but with an overall opposite sign.

The evolution of this complex scalar field in the cosmological scenario described above is given by the Klein-Gordon equation

\[
\frac{1}{c^2} \frac{d^2 \Phi}{dt^2} + \frac{3H}{c^2} \frac{d\Phi}{dt} + 2\frac{dV}{d|\Phi|^2} \Phi = 0, \quad (5)
\]

from where we can express the complex scalar field as

\[
\Phi = |\Phi| e^{i\theta}. \quad (6)
\]

In our proposal, we are going to follow the procedure given in [37]. Using (6) in (5), the Klein-Gordon equation can be divided into a real and an imaginary part, from which the second leads to the equation:

\[
Q = -\frac{1}{\hbar c^2} a^3 |\Phi|^2 \frac{d\theta}{dt}, \quad (7)
\]

where \( Q \) is constant and \( a \) the scale factor.

From the real part we obtain

\[
\frac{1}{c^2} \left[ \frac{d^2 |\Phi|}{dt^2} - |\Phi| \left( \frac{d\theta}{dt} \right)^2 \right] + \frac{3H}{c^2} \frac{d|\Phi|}{dt} + 2\frac{dV}{d|\Phi|^2} |\Phi| = 0. \quad (8)
\]

To compute the energy density and pressure of the complex scalar field, we consider the following expressions:

\[
\epsilon = \frac{1}{2c^2} \left( \frac{d|\Phi|}{dt} \right)^2 + V(|\Phi|^2), \quad (9)
\]

\[
P = \frac{1}{2c^2} \left( \frac{d|\Phi|}{dt} \right)^2 - V(|\Phi|^2). \quad (10)
\]

Notice how we can connect these equations to the ones presented in Sec.IIA where the quantity \( \epsilon \) will replace the \( \Lambda \text{CDM} \) quantity \( \rho_{\text{crit}} \Lambda \) in the Friedman equation.

From the equations (5), (9) and (10) we can obtain a useful equation for the energy density that resembles the continuity equation for a perfect fluid

\[
\frac{d\epsilon}{da} + \frac{3}{a} (\epsilon + P) = 0. \quad (11)
\]

With these equations, now we are ready to study particular solutions of the Einstein-Klein-Gordon system evolving with a complex scalar field mimicking the dark energy component.

III. DARK ENERGY SOLUTION IN THE FAST OSCILLATION REGIME

In [37] was found that in the fast-oscillation regime, i.e., when the oscillation frequency of the scalar field is much larger than the value of the Hubble function, the solution of the Einstein-Klein-Gordon equations for the case of a complex scalar field with a potential of attractive self interaction (4) splits into two. One solution (called normal branch) resembles to a dark matter scalar field, while the other solution (called peculiar branch) corresponds to a quintessence model. This solution remains in the fast oscillation regime, in which the scalar field suddenly emerges and behaves as dark energy at late times.

Following the same logic, in this work we propose a deduction of an exact solution for the equation of state of the quintessence field. Once with this equation, we explore their possible constraints by using current observational data.

\[\text{footnote}{1} \text{After integration, the imaginary part of the Klein-Gordon equation leads to a conserved quantity, which correspond to the conserved charge of a complex scalar field, given by } Q = \frac{1}{\pi} \int dx^3 \sqrt{-g} \text{Im}(\Phi \Phi^*).\]
A. Peculiar branch solution in the fast oscillation approximation

To establish the fast oscillation regime mentioned above, we consider the following condition which needs to be satisfied during its evolution

$$\omega = \frac{d\theta}{dt} \gg H.$$ \hspace{1cm} (12)

In addition to the latter condition, we will impose that the magnitude of the scalar field change slowly on time with respect to the angular frequency of oscillation $\omega$ as:

$$\frac{1}{|\Phi|} \frac{d|\Phi|}{dt} \ll \omega.$$ \hspace{1cm} (13)

Conditions (12)-(13) set the so-called fast oscillation regime of the Klein-Gordon equation (5). Following this prescription, (8) can be reduce to

$$\omega^2 = 2c^2 \frac{dV}{d|\Phi|^2}.$$ \hspace{1cm} (14)

This allows us to write the fast oscillation condition in terms of the charge $Q$ defined in (7), which becomes

$$\frac{Q^2 \hbar^2 c^4}{a^3 |\Phi|^4} = 2c^2 \frac{dV}{d|\Phi|^2}.$$ \hspace{1cm} (15)

Using the expression for the scalar field potential $\Phi$, we can approximate (9) using the fast oscillation condition (14) as

$$\epsilon = \frac{1}{2c^2} \left[ \left( \frac{d|\Phi|}{dt} \right)^2 + \omega^2 |\Phi|^2 \right] + \frac{m^2 c^2}{2 \hbar^2} |\Phi|^2 - \frac{2\pi A_s m}{\hbar^2} |\Phi|^4 \approx \frac{m^2 c^2}{\hbar^2} |\Phi|^2 - 6\pi A_s m \frac{|\Phi|^4}{\hbar^2},$$ \hspace{1cm} (16)

By a similar approach, the scalar pressure from (11) can take the approximate form

$$P \approx -\frac{2\pi A_s m}{\hbar^2} |\Phi|^4.$$ \hspace{1cm} (17)

Solving (16) for $|\Phi|^2$, we obtain two possible branches that correspond to solutions of the Einstein-Klein-Gordon system in the fast oscillation approximation

$$|\Phi|^2 = \frac{c^2 m}{12\pi A_s} \left( 1 \pm \sqrt{1 - 24\pi A_s \hbar^2 / m^3 c^4 - \epsilon} \right).$$ \hspace{1cm} (18)

Notice that this is a different result in comparison to the repulsive self-interaction case (15), where there was only one branch of the solution. Furthermore, in (37), it was showed that, when we take the negative sign, the scalar field undergoes to a matter-like phase (and even to an inflation epoch). While for the positive branch, the solution behaves as dark energy. From this point forward, we will take the negative sign, to focus on that particular branch.

Therefore, by using (18) in (17) we obtain

$$P(\epsilon) = -\frac{m^3 c^4}{72\pi A_s \hbar^2} \left( 1 + \sqrt{1 - \frac{24\pi A_s \hbar^2}{m^3 c^4} - \epsilon} \right)^2.$$ \hspace{1cm} (19)

Physical solutions of this latter equation correspond to those values of $\epsilon$ smaller than a certain $\epsilon_i$:

$$\epsilon_i = \frac{m^3 c^4}{24\pi A_s \hbar^2}.$$ \hspace{1cm} (20)

From the two latter expressions, notice that $P(\epsilon_i) = -\frac{m^3 c^4}{72\pi A_s \hbar^2}$, implies that $w_i = P(\epsilon_i) = -1/3$.

The scale factor for which the energy density takes the value $\epsilon_i$ can be calculated by inserting the value of $|\Phi|^2$ evaluated in $\epsilon_i$, and taking the result on the fast oscillation condition (15):

$$a_i = \sqrt{\frac{12\sqrt{3}\pi A_s \hbar^2 |Q|}{m^2 c^2 \epsilon_i^3}}.$$ \hspace{1cm} (21)

For convenience, we re-define a dimensionless quantity in terms of the differential equation for the energy density as

$$\bar{\epsilon} = \frac{\epsilon}{\epsilon_i},$$ \hspace{1cm} (22)

does (11) can be written as

$$\frac{d\bar{\epsilon}}{da} = -\frac{3}{a} \left[ \bar{\epsilon} - \frac{1}{3} \left( 1 + \sqrt{1 - \bar{\epsilon}} \right)^2 \right].$$ \hspace{1cm} (23)

Evaluating in $\epsilon = \epsilon_i$, we can see that $d\bar{\epsilon}/da$, takes a negative value of $-\frac{2}{3a}$, therefore for $a < a_i$ the solution is not valid. Furthermore, at $a \rightarrow \infty$, $\epsilon$ approaches to a constant value.

Now, taking the fast oscillation equation (15) and inserting $|\Phi|^2$ from (18) we obtain

$$\left( \frac{a_i}{a} \right)^6 = 3 \left( 1 + \sqrt{1 - \bar{\epsilon}} \right)^2 - 2 \left( 1 + \sqrt{1 - \bar{\epsilon}} \right)^3.$$ \hspace{1cm} (24)

In order to find the asymptotic value of $\epsilon$, when $a \rightarrow \infty$, we should consider the fast oscillation equation (15), which for potential (4) takes the form

$$\frac{Q \hbar c^2}{a^3} = \sqrt{2\bar{\epsilon}} |\Phi|^2 \sqrt{\frac{m^2 c^2}{2 \hbar^2} - \frac{4\pi A_s m}{\hbar^2} |\Phi|^2}.$$ \hspace{1cm} (25)
Since \( \epsilon \) decreases with \( a \), then \( |\Phi|^2 \) increases as \( a \to \infty \) as we can notice from \([18]\), therefore the term inside the square root in \([20]\) should vanish as \( a \to \infty \), leading to an asymptotic value of

\[
|\Phi|^2 = \frac{mc^2}{8\pi A_s}. \tag{26}
\]

Using \([16]\) and \([19]\) we can obtain

\[
\epsilon_A = \frac{m^3c^4}{32\pi A_s h^2} = \frac{3}{4} \epsilon_i \tag{27}
\]

\[
P(\epsilon_A) = -\epsilon_A. \tag{28}
\]

Notice how in the limit \( a \to \infty \), the scalar field has an EoS that corresponds to \( w_A = -1 \).

**B. Exact solution for the dark energy term-like**

In order to obtain an expression for \( \epsilon \) in terms of the scale factor, we have to solve the equation \([24]\). This can be obtained making the change of variable

\[
\zeta = \sqrt{1 - \epsilon} + \frac{1}{2}. \tag{29}
\]

The latter leads to an expression in terms of a cubic equation

\[
\zeta^3 - \frac{3}{4} \zeta + \frac{1}{2} \left( a_i^6 \frac{1}{a^6} - \frac{1}{2} \right) = 0, \tag{30}
\]

which has three real solutions. However, it must satisfy the conditions \( \zeta(a_i) = \frac{1}{2} \) and \( \zeta(a \to \infty) = 1 \). The only solution that satisfy this condition is

\[
\zeta(a) = \cos \left[ \frac{1}{3} \arccos \left( 1 - 2 \frac{a_i^6 }{a^6} \right) \right]. \tag{31}
\]

in terms of this function \( \zeta(a) \), the energy density and the EoS parameter are given by the following expressions

\[
\epsilon(a) = \left[ 1 - \left( \zeta(a) - \frac{1}{2} \right)^2 \right] \epsilon_i, \tag{32}
\]

\[
w(a) = -\frac{\left( \zeta(a) + \frac{1}{2} \right)^2}{3 - 3 \left( \zeta(a) - \frac{1}{2} \right)^2}. \tag{33}
\]

These solutions must have been taken into account only in certain region \( a_i < a < a_e \) of the evolution of the Universe and, from now on, \( a \) will only be referred to this range. First, we must make sure that the solution at \( a_i \), satisfy the fast oscillation approximation described in the latter section.

Under these ideas, the fast oscillation condition \( \omega \gg H \) is given by

\[
\frac{Q^2h^2c^4}{a^6|\Phi|^4} \gg \frac{8\pi G}{3c^2} (\rho_m + \epsilon). \tag{34}
\]

By performing the substitution of \( |\Phi|^2 \), re-writing it in terms of \( \epsilon \) and, finally, taking \( \epsilon \gg \rho_m \), we can obtain

\[
\left( \frac{a_i}{a} \right)^2 \gg \frac{mG}{3c^2 A_s} \epsilon (1 + \sqrt{1 - \epsilon})^2. \tag{35}
\]

This condition will be satisfied initially if

\[
\frac{3c^2 A_s}{mG} > 1. \tag{36}
\]

In order to compute the value \( a_e > a_i \) in where the solution is no longer valid, we will consider the end of the fast oscillation regime when \( \omega = NH \) (with \( N = 200 \) analogous to \([15]\) ). If \( a_e \gg 1 \) and also \( a_e \gg a_i \), in order to be able to make the approximations \( \epsilon \gg \rho_m \) and \( a_i/a_e \ll 1 \) in \([55]\), then the end value of the scale factor will be

\[
a_e \approx \sqrt[3]{\frac{768\pi^2 A_s^4 h^4 Q^2}{G m^6 c^2}}. \tag{37}
\]

In Fig. 1 we show the evolution of the equation of state parameter \( w \) between the values for the scale factor \( a_i \) and \( a_e \), determined by certain values of \( m, A_s \) and \( Q \). In this example we take \( a_i \) to be the value \( a_{min} = 0.1 < 1/(1 + z_{max}) \) where \( z_{max} = 2.3 \) corresponds to the maximum redshift used in the multiple data sets within the analysis described in the next section. In this way, we ensure that the scalar field is present throughout the \( a \) range of the analysis. We have restricted this example to the case where \( a_e = 1 \), thus ensuring that the limit of rapid oscillations and therefore the cosmological constant type behaviour continues to be valid today.

To give intuition about what is happening in the complete cosmological model where dark energy is described by the previous scalar field solution, we present in Fig. 2 the energy density fractions of the quintessence model with the same values \( a_i \) and \( a_e \) as in Fig. 1 additionally, for our example we have chosen the initial scalar field energy density to be \( \epsilon_i = \frac{4}{3} \epsilon_A = \frac{3}{4} \rho_{crit} \Omega_A \). In other words,

\[2\] This particular election of \( a_i \) and \( a_e \) in our example reduces the dimension of the free parameter space from 3 to 2, thus we can put \( Q \) and \( A_s \) in terms of \( m \): \( Q = \frac{4a_{crit} mc^2}{27\sqrt{3} \pi N GH}, A_s = \frac{a^{16} G m^2 c^2}{\pi} \).
FIG. 1: Evolution of the $w_\text{Eq. (32)}$ as a function of the scale factor, $a$ in the quintessence model.

FIG. 2: Evolution of the $\Omega_i$ as a function of the scale factor, $a$ in the quintessence model.

we have chosen that the asymptotic value for the energy density of the scalar field coincides with the current energy density for $\Lambda$ in the pivot model. Interestingly, it turns out that this condition on the example, fixes the three free parameters of our model, leading to a mass $m \sim 10^{-22}$ eV$/c^2$, frequently used in the ultralight models of dark matter [46–48]. This figure is almost indistinguishable from the corresponding figure for the $\Lambda$CDM model, this is because the discontinuity for $\epsilon$ appears in an epoch where the contribution to the total energy density of the scalar field is relatively small and also because $\epsilon$ quickly tends to the $\epsilon_\Lambda$ value, as can be inferred from Fig. 1 and equation (32).

C. Parametric Equation of State in the late cosmic acceleration approximation

Let us write explicitly (33) as an effective dark energy EoS described by a complex scalar field with a Bose-

Einstein condensate-like potential. By using the standard definition $a = 1/(1 + z)$ and expand the function $\zeta$ in (31) with the assumption $a \gg a_i$ we obtain:

$$w(z) = w_0 + w_a (1 + z)^6,$$  (38)

where $w_0 = -1$ and $w_a = \frac{16}{27} a_i^6$. Notice that this generic expression for the EoS impose directly on $w_0$ the cosmological constant value, while on $w_a$ we must give values of $a_i$ consistent with the following redshift range:

$$z \in \left[0, \frac{16}{27} (1 + z_{\text{max}})^{-6}\right] \approx [0, 0.0004588].$$  (39)

We should remark that (38) is not obtained as in the traditional derivation of the solution of the conservation equation, where an effective dark energy fluid needs to be consider and certain values over $w$ denote the different matter in the universe. Also, the addition of a $\Lambda$ is avoided, since from this methodology we got $w = -1$.

IV. OBSERVATIONAL CONSTRAINTS

In this section, we analize to what extend our model (38) is consistent, to some degree, with late and early time observations if we relax the $w_0$ parameter. To perform the statistical analyses of (38) and to understand current constraints in our model, we need to choose specific late-time data sets as SNeIa (Pantheon), Observational Hubble data (OHD) and Baryon Acoustic Oscillations (BAO), together with Planck 2018 (PL18) likelihood.

Each observational data has the following features:

- Pantheon SNeIa compilation: composed by 1048 SNeIa in a range $z \in [0.01, 2.3]$ [49]. This kind of sampler characterised by Type Ia supernovae can give us determinations of the distance modulus $\mu$, whose theoretical prediction is related to the luminosity distance $d_L$ as

$$\mu(z) = 5 \ln \left[ \frac{d_L(z)}{1\text{ Mpc}} \right] + 25,$$  (40)

where the luminosity distance is given in Mpc units. In the standard statistical analyses, one adds to the distance modulus the nuisance parameter $M$, an unknown offset sum of the supernovae absolute magnitude and other possible systematics, which is degenerate with the value of $H_0$. As we are assuming spatial flatness, the luminosity distance can be given by the comoving distance $D$ as

$$d_L(z) = \frac{c}{H_0} (1 + z) D(z).$$  (41)
where $c$ is the speed of light. Also, for this sampler we are taking the nuisance parameter $M$ inside the sample, for this we choose the respective values of $M$ from a statistical analysis of the ΛCDM model with a fixing $H_0$ from the Late Universe measurements (SH0ES + H0LiCOW) as $H_0 = 73.8 \pm 1.1$ km/s/Mpc with $M = -32.79$.

- Planck Legacy 2018 (PL18): we adopt the low-$l$ and high-$l$ likelihoods from [39].
- BAO measurements: we consider the sampler of 15 transversal measurements obtained in a quasi model-independent approach. This can be done by compute the 2-point angular correlation function tracers via $D_A(z; r_{\text{drag}})$ [50]. The sampler is given in a redshift range $[0.11, 2.225]$. These kind of observations contribute important features by comparing the data of the sound horizon today to the sound horizon at the time of recombination (extracted from the CMB anisotropy data). The BAO distances are given by $d_z \equiv \frac{r_s(z_d)}{D_V(z)}$, with $r_s(z_d) = \frac{c}{H_0} \int_{z_d}^{\infty} \frac{E(z)}{E(z_d)} dz$ and $r_s(z_d)$ being the comoving sound horizon at the baryon dragging epoch, $c$ the light velocity, $z_d$ is the drag epoch redshift and $c_s^2 = c^2 / 3(1 + (3\Omega_0/4\Omega_{\gamma 0})(1 + z)^{-1})$ the sound speed with $\Omega_0$ and $\Omega_{\gamma 0}$ the present values of baryon and photon density parameters, respectively. The dilation scale is given by

$$D_V(z, \Omega_m; \Theta) = \left[ \frac{c z (1 + z)^2 D_A^2(z)}{H(z, \Omega_m; \Theta)} \right]^{1/3},$$

where $D_A$ is the angular diameter distance

$$D_A(z, \Omega_m; \Theta) = \frac{1}{1 + z} \int_0^z \frac{c dz}{H(z, \Omega_m; \Theta)},$$

where $\Theta = \{w_0, w_a\}$. Through the comoving sound horizon, the distance ratio $d_z$ is related to the expansion parameter $h$ (defined such that $H \equiv 100h$) and the physical densities $\Omega_m$ and $\Omega_\gamma$. Here we use BAO data to connect SNeIa (Pantheon) to CMB data (PL18). This is possible by calibrating the $D_A$ from BAO with the $d_L$ from supernovae in a cosmology-independent way.

- Observational Hubble data (OHD): we consider a sample of 51 measurements in the redshift range $0.07 < z < 2.0$ [51]. This sample gives a measurement of the expansion rate without relying on the nature of the metric between the chronometer and us as observers. The normalised parameter $h(z)$ can be compute by considering the values by Planck 2018 and SH0ES + H0LiCOW given above. In this sample are content 31 data points from passive galaxies and 20 data points are estimated from
Using the data samplers described above, we can compute the best fit values for each cosmological parameter using the standard $\chi^2$-method as: $\chi^2_{\text{Total}} = \chi^2_{\text{Planck}} + \chi^2_{\text{SN}} + \chi^2_{\text{BAO}} + \chi^2_{\text{OHD}}$. In Table I we report the mean and best fits for the cosmological parameters and the model parameters, $w_0$ and $w_a$, for the following joint samplers Planck 2018, CC+BAO+Planck 2018+Pantheon.

| Parameters | Mean with errors | Best fit |
|-----------|------------------|----------|
| $100\,\omega_b$ | $2.223^{+0.025}_{-0.026}$ | 2.225 |
| $\omega_{cdm}$ | $0.119^{+0.0023}_{-0.0024}$ | 0.1183 |
| $w_0$ | $-1.034 \pm 0.18$ | $-1.002$ |
| $w_a$ | $-0.173 \pm 0.022$ | $-0.5468$ |
| $\Omega_m$ | $0.288 \pm 0.03$ | 0.3093 |
| $\tau$ | $0.057^{+0.008}_{-0.009}$ | 0.055 |
| $n_s$ | $0.976^{+0.004}_{-0.004}$ | 0.977 |
| $\sigma_8$ | $0.810^{+0.008}_{-0.008}$ | 0.804 |

TABLE I: Background best fits values for Eq.(38). For OHD+BAO+Planck 2018+Pantheon

Using the data samplers described above, we can compute the best fit values for each cosmological parameter using the standard $\chi^2$-method as: $\chi^2_{\text{Total}} = \chi^2_{\text{Planck}} + \chi^2_{\text{SN}} + \chi^2_{\text{BAO}} + \chi^2_{\text{OHD}}$. In Table I we report the mean and best fits for the cosmological parameters and the model parameters, $w_0$ and $w_a$, for the following joint samplers Planck 2018, CC+BAO+Planck 2018+Pantheon.

V. CONCLUSIONS

Complex scalar field theory has been used from a Bose-Einstein condensate point of view. In this line of thought, it is possible to construct quintessence–complex scalar field scenarios, which can mimic dark energy effects. Therefore, in this work we proposed, inside this later scenario, a study of the peculiar branch solution of the Einstein-Klein-Gordon equations in the fast oscillation regime, where the scalar field is modelled as an effective dark fluid. As it is standard, from these field equations it is possible to derive an effective equation of state \(w\), a model we called CSFDE. In this panorama, the cosmological parameters related with the model can be constrained using current observational surveys in order to study epochs where the dark energy (at $z = 0$) and dark matter ($z \approx 9$) domination occurs.

We can conclude that this observational approach discard the theoretical model since the best fit value for $w_a$ is negative as we can see on Table I. According to the Eq. (21), this would imply that $a_3^2$ is negative, which is only possible for $|Q|$ or $A_\Lambda$ (both positive definite quantities) negative. Consistency between the analysis performed and the theoretical model in the late cosmic acceleration approximation is given by the fact that the value $w_0$ reported in Table I is close to $-1$.

Furthermore, we can see that the CSFDE model cannot reproduce an oscillating behaviour associated with a dynamical dark energy. This can be consider as a natural derivation from the field equations, and by consequence from the action with a scalar field. Our result point out the necessity of more than one canonical scalar field to reproduce the transition dynamics between viable cosmological epochs, i.e. a radiation/matter/etc. domination era. Further investigation could require combinations of scalar fields like quintom scenarios or changes in the kinetic term. Also exact solutions for other scalar potentials in the fast oscillation regime could lead to models favored by Bayesian analyzes.

Based in these affirmations, it is possible to consider our method as a classification mechanism to discard equations of state within the scalar field description.

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