X-ray irradiation in low mass binary systems

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ABSTRACT
We calculate self-consistent models of X-ray irradiated accretion discs in close binary systems. We show that a point X-ray source powered by accretion and located in the disc plane cannot modify the disc structure, mainly because of the self-screening by the disc of its outer regions. Since observations show that the emission of the outer disc regions in low mass X-ray binaries is dominated by the reprocessed X-ray flux, accretion discs in these systems must be either warped or irradiated by a source above the disc plane, or both. We analyse the thermal-viscous stability of irradiated accretion discs and derive the stability criteria of such systems. We find that, contrary to the usual assumptions, the critical accretion rate below which a disc is unstable is rather uncertain since the correct formula describing irradiation is not well known.

Key words: accretion, accretion discs – instabilities – X-rays: general – binaries: close

1 INTRODUCTION

There is a convincing body of evidence that most of the optical light emitted by accretion discs in Low Mass X-ray Binaries (LMXBs) is due to reprocessed X-rays illuminating the disc surface (van Paradijs & McClintock, 1993). van Paradijs (1996) pointed out that this irradiation, observed both in steady LMXBs and during the outburst phases of LMXB transient sources (“Soft X-ray Transients”, SXTs) should influence the stability properties of accretion discs in such systems. X-ray irradiation increases the surface (and therefore the central) disc temperature and could, in principle, prevent the growth of the thermal instability that appears at a temperature for which hydrogen becomes partially ionized. This instability leads, under certain conditions, to a global thermal-viscous instability which is taken to be responsible for dwarf nova outbursts and SXT events (see e.g. Cannizzo 1993).

Models used to describe X-ray irradiated discs are often, as we show below, inconsistent and/or assume unverified hypotheses. Despite these drawbacks, results obtained in such studies seem to be in agreement with observations. In the present work, we construct self-consistent models of irradiated disc vertical structures and show what should be modified in the usual treatment to account for observations whilst being consistent.

This article is organized as follows: in Section 2 we re-discuss the basics of irradiated disc models. In Section 3 we present self-consistent models of irradiated disks and discuss their stability.

2 X-RAY IRRADIATION OF ACCRETION DISCS IN CLOSE BINARY SYSTEMS

2.1 Steady discs

In most studies of X-ray illuminated accretion discs in close binary systems it is assumed (explicitly or implicitly) that the disc is isothermal in regions in which reprocessed light is an important part of the disc emission (see e.g. Vrtilek et al., 1990). For example, de Jong, van Paradijs & Augusteijn (1996) use the Vrtilek et al. (1990) model to deduce various properties of X-ray irradiated discs in LMXBs, such as the albedo and the disc opening angle. It is sometimes said that the values obtained in this way are suggested by observations, and they are used even in the case where it is stated that the disc is not isothermal (King et al., 1997a).

To justify the assumption of an isothermal disc it is argued that if the irradiating flux is larger than the flux due to viscous dissipation in the disc, the viscous heating term can be neglected in the energy equation so that the central temperature is equal to the irradiation temperature. This reasoning is, in general, erroneous: irradiation dominates the internal disc structure only if the irradiation flux divided by the disc total optical depth is larger than the flux due to viscous dissipation (Lyutyi & Sunyaev, 1979, Tuchman, Mineshige & Wheeler, 1990, Hubeny, 1991, Hure et al., 1993). This is a simple consequence of energy conservation and of the radiative transfer equation and is independent of the details of disc models. Since there seems to be some confusion about
At the disc midplane, by symmetry, the flux must vanish: \( F(\tau_{\text{tot}}) = 0 \), whereas at the surface, \( (\tau = 0) \)
\[
F(0) \equiv \sigma T_{\text{eff}}^4 = F_{\text{vis}}
\]
Equation (6) states that the total flux at the surface is equal to the energy dissipated by viscosity (per unit time and unit surface). The solution of Eq. (6) is thus
\[
F(\tau) = F_{\text{vis}} \left(1 - \frac{\tau}{\tau_{\text{tot}}} \right)
\]
To obtain the temperature stratification one has to solve the transfer equation. Here we use the diffusion approximation
\[
F(\tau) = \frac{4}{3} \sigma dT^4 d\tau
\]
appropriate for the optically thick discs we are dealing with. The integration of Eq. (6) is straightforward and gives:
\[
T^4(\tau) - T^4(0) = \frac{3}{4} \tau \left(1 - \frac{\tau}{2\tau_{\text{tot}}} \right) T_{\text{eff}}^4
\]
The upper (surface) boundary condition is:
\[
T^4(0) = \frac{1}{2} T_{\text{eff}}^4 + T_{\text{irr}}^4
\]
where \( T_{\text{irr}}^4 \) is the irradiation temperature, which depends on \( R \), the albedo, the height at which the energy is deposited and on the shape of the disc (see Eq. (4)). In Eq. (11) \( T(0) \) corresponds to the emergent flux and, as mentioned above, \( T_{\text{eff}} \) corresponds to the total flux (see e.g. Hubeny 1991) which explains the factor 1/2 in Eq (11). The temperature stratification is thus:
\[
T^4(\tau) = \frac{3}{4} \frac{T_{\text{eff}}^4 \tau \left(1 - \frac{\tau}{2\tau_{\text{tot}}} \right) + \frac{2}{3} + T_{\text{irr}}^4}{\tau_{\text{tot}}}
\]
which is very similar to the formula obtained by Hubeny (1991) for a different viscosity prescription. For \( \tau_{\text{tot}} \gg 1 \) the first term on the rhs has the form familiar from the stellar atmosphere models in the Eddington approximation.

For \( \tau_{\text{tot}} \gg 1 \), the temperature at the disc midplane is
\[
T_{\text{c}}^4 \equiv T^4(\tau_{\text{tot}}) = \frac{3}{8} \tau_{\text{tot}} T_{\text{eff}}^4 + T_{\text{irr}}^4
\]
It is clear, therefore, that for the disc inner structure to be dominated by irradiation and the disc to be isothermal one must have
\[
\frac{F_{\text{irr}}}{\tau_{\text{tot}}} \equiv \frac{\sigma T_{\text{eff}}^4}{\tau_{\text{tot}}} \gg F_{\text{vis}}
\]
and not just \( F_{\text{irr}} \gg F_{\text{vis}} \) as is usually assumed. The difference between the two criteria is important in LMXBs since, for parameters of interest, \( \tau_{\text{tot}} \gtrsim 10^5 \) in the outer disc regions. In Section 3.1 we use self-consistent detailed vertical disc structure calculations in order to determine the
influence of irradiation on the inner disc structure. Note that we neglect here the possible presence of an X-ray irradiation generated corona and wind, described by Idan & Shaviv (1996).

2.2 Irradiation temperature

Condition (4) is not however a necessary condition for the disc emission properties to be dominated by irradiation. It is enough that $F_{\text{irr}} > F_{\text{vis}}$ for the disc observed luminosity to be dominated by reprocessed emission. Observations suggest (van Paradijs & McClintock, 1993) that the absolute visual magnitude $M_V$ of nova-like cataclysmic variables which are accreting at rates $\dot{M} \sim 2 - 5 \times 10^{-8}$ g s$^{-1}$ (Warner, 1995) are roughly two magnitudes fainter than the $M_V$'s of LMXBs of similar size (since one has to compare luminosities and not fluxes one should compare systems with the same emitting areas). This would imply $F_{\text{irr}} \approx 6F_{\text{vis}}$ so that, although the optical emission is dominated by reprocessed X-rays, the inner disc structure is not dominated by irradiation (Eq. 13). 

For a point source, the irradiation temperature can be written as (Shakura & Sunyaev 1973)

$$T_{\text{irr}} = \frac{\eta \dot{M} c^2 (1 - \varepsilon)}{4\sigma R^2 \varepsilon} \left( \frac{d \ln H_{\text{irr}}}{d \ln R} - 1 \right)$$

where $\eta$ is the efficiency of converting accretion power into X-rays, $\varepsilon$ is the X-ray albedo and $H_{\text{irr}}$ is the local height at which irradiation energy is deposited, or the height of the disc “as seen by X-rays”. We use here $H_{\text{irr}}$ and not $H$, the local pressure scale-height, as is usually written in the literature because, in general, $H_{\text{irr}} \neq H$.

In the usual approach (Vrtilek et al. 1990) one assumes that the disc is isothermal, which, for a Keplerian disc, implies that the pressure scale-height varies like $R^{0.7}$. de Jong et al. (1996) applied an isothermal model for X-ray reprocessing in neutron-star LMXBs and concluded that the disc opening angle is $\sim 12^\circ$, i.e. $H_{\text{irr}}(R_d)/R_d \sim 0.2$, where $H_{\text{irr}}(R_d)$ is measured at the outer disc radius $R_d$. It is easy to see that the value 0.2 cannot be the pressure scale-height to radius ratio since this would imply temperatures $> 10^7$ K at the outer disc radius. In addition, the value $H/R = 0.2$ was obtained by modeling two dipping sources in which the accreting object is a neutron star. It is not at all obvious that this value applies to all LMXBs. de Jong et al. (1996) also deduced a high value for the X-ray albedo $\varepsilon \gtrsim 0.9$, although they discussed possible lower values for this parameter.

Equation (4) has been extensively used in recent work on disc stability properties in LMXBs. van Paradijs (1996) suggested that X-ray illumination might modify the conditions under which accretion disks undergo dwarf novae type thermal-viscous instabilities. King and collaborators (King, Kolb & Burderi, 1996; King & Kolb, 1997; King et al., 1996; King et al., 1997a; King et al., 1997b) have considered different implications of this idea on the evolution of various classes of LMXBs. All these models used Eq. (4) with $\eta = 0.1$, $\varepsilon \approx 0.9$. Since the stability criterion applies to the outer disc radius the value 0.2 is assumed for $H_{\text{irr}}/R$ and $d \ln H_{\text{irr}}/d \ln R = 9/7$. In King et al. (1997a), the rhs of Eq. (4) is multiplied by $H/R$ for black hole LMXBs, supposedly to account for the non-point source character of the X-ray emitter in this case (Shakura & Sunyaev 1973).

The new stability criterion proposed by van Paradijs (1996) seems to correspond quite well to observed properties of LMXBs in the sense that most sources that should be stable according to this criterion are more or less steady and all transient sources are unstable. This new criterion is a sufficient condition for disc stability (or necessary for instability). The problem is that it is based on a model which cannot represent the discs to which it is applied. The isothermal disc model gives an incorrect description of X-ray irradiated discs in LMXBs. It is often argued that this is just a simple disc model. This is true. Unfortunately, like many other simple models, it does not apply to X-ray irradiated discs in LMXBs. It assumes, contrary to observed properties of accretion discs in LMXBs, that the irradiation temperature is equal to the midplane disc temperature. The midplane temperature at which a hot, steady, accretion disk becomes unstable is (this corresponds to the minimum surface density for hot stable discs, $\Sigma_{\text{min}}$)

$$T_{c, B} = 21700 \, K \alpha^{-0.21} \left( \frac{M_1}{M_{\odot}} \right)^{-0.02} \left( \frac{R}{10^{15} \, \text{cm}} \right)^{0.05}$$

which for standard parameters ($\alpha \sim 0.1$) is close to 30 000 K. Eq. (16) is based on numerical calculations by Hameury et al. (1998, hereafter H98); similar formulae were obtained by other authors (e.g., Cannizzo 1993b). Isothermal vertical structures are obtained only for irradiation temperatures corresponding to excessively high accretion rates and/or radii.

2.3 Stability of X-ray irradiated discs

However, X-ray irradiation does not have to dominate the vertical structure to play an important role in the disc properties. Also, as pointed out by King & Ritter (1995), X-ray irradiation may affect the temporal behaviour of transient LMXB outbursts.

It is instructive to see how irradiation modifies disc stability properties. The effective temperature at which a hot disc becomes thermally unstable is, according to H98, given by

$$T_{\text{eff}, B} = 7200 \, \alpha^{-0.002} \left( \frac{M_1}{M_{\odot}} \right)^{0.03} \left( \frac{R}{10^{15} \, \text{cm}} \right)^{-0.08} \, K$$

For parameters of interest, this leads to $T_{\text{eff}} \sim 10^3$ (see Eq. 16) and, as mentioned above, the vertical disc structure can then only be changed for very high irradiation fluxes. Much lower fluxes, however, are sufficient to modify its thermal stability properties, as will be shown in the next sections.

The disc is thermally stable if radiative cooling varies faster with temperature than viscous heating (see e.g., Frank, King & Raine 1992). In other words

$$\frac{d \ln \sigma T_{\text{eff}}^4}{d \ln T_c} > \frac{d \ln F_{\text{vis}}}{d \ln T_c}$$

Using Eq. (14) one obtains

$$\frac{d \ln T_{\text{eff}}^4}{d \ln T_c} = 4 \left( 1 - \left( \frac{T_{\text{irr}}}{T_c} \right)^4 \right)^{-1} - \frac{d \ln \kappa}{d \ln T_c} \tag{19}$$

In a gas pressure dominated disc $F_{\text{vis}} \sim T_c$ (see e.g., Frank et al. 1992). The thermal instability is due to a rapid change
of opacities with temperature when hydrogen begins to recombine. At high temperatures \( d\ln \kappa / d\ln T \approx -4 \). In the instability region, the temperature exponent becomes large and positive \( d\ln \kappa / d\ln T \approx 7 - 10 \), and in the end cooling is decreasing with temperature. As can be seen from Eq. (19), even a moderate \( T_{\text{irr}} / T_c \) ratio modifies the transition from stable to unstable configurations, thereby pushing it to lower temperatures. This effect is clearly seen in Figs. 1 and 2. One should also note that another effect of irradiation is to suppress or alter convection, even for moderate values of \( T_{\text{irr}} / T_c \) as can be seen in Figs. 1–3 (and discussed below). This effect is moderate, as the convective flux can be small compared to the radiative flux; it cannot be included in the simple analytic model developed here for illustrative purposes, but is taken into account in our numerical calculations.

3 SELF-CONSISTENT MODELS OF X-RAY IRRADIATED ACCRETION DISCS

Having demonstrated above that the simplest models taking into account X-ray irradiation of accretion discs in LMXBs give interesting results, we decided to undertake more rigorous and detailed calculations to investigate these effects further. In all cases, we rewrite Eq. (13) as:

\[ T_{\text{irr}}^4 = C \frac{M c^2}{4\pi \sigma R^2} \]

with

\[ C \equiv \eta (1 - \varepsilon) \frac{H_{\text{irr}}}{R} \left( \frac{d\ln H_{\text{irr}}}{d\ln R} - 1 \right) \]

and the results depend on the value of \( C \), which in turn is a function of \( \eta, \varepsilon \) and \( H_{\text{irr}}(R) \). In a self-consistent model \( H_{\text{irr}}(R) \) results from calculations so that \( \eta \) and \( \varepsilon \) are the only free parameters. There is not much freedom in \( \eta \) but the value of \( \varepsilon \) is rather uncertain, as discussed, for example by de Jong et al. (1996). We assumed that X-rays are deposited at the disc photosphere, \( H_{\text{irr}} = z_{\text{ph}}(R) \), whose height is in turn modified by irradiation. Our model is fully self-consistent, in the sense that this feedback mechanism is taken into account.

3.1 The vertical structure

We use the numerical code described in H98 to find the vertical structure of an X-ray irradiated disc.

The equations describing the vertical structure are the same as in H98, with the exception of the surface boundary condition which is modified to account for illumination. These equations are:

\[ \frac{dP}{dz} = -\rho g_z = -\rho \Omega_K^2 z, \]

\[ \frac{d\kappa}{dz} = 2\rho, \]

\[ \frac{d\ln T}{dz} = \frac{d\ln P}{d\nu}, \]

\[ \frac{dF_\nu}{dz} = 3 z \alpha_{\text{eff}} \Omega_K^2 \rho, \]

where \( g_z = \Omega_K^2 z \) is the vertical component of gravity, \( \alpha \) is the
surface column density between \( -z \) and \( +z \), \( \alpha_{\text{eff}} \) the effective viscosity coefficient (see H98) and \( \nabla \) the temperature gradient of the structure, which in the radiative case is

\[
\nabla_{\text{rad}} = \frac{\kappa P_z}{4P_{\text{rad}} c g_z},
\]

\( P_{\text{rad}} \) being the radiative pressure. In the convective case, we use a mixing length approximation (see H98 for more details).

Eq. (23) is the hydrostatic equilibrium equation, Eq. (24) represents mass conservation and Eqs. (25) and (26) are the energy transport and conservation equations.

The set of equations (22-24) is integrated between the disc midplane and the photosphere (\( \tau_0 = 2/3 \)) with the boundary conditions \( z = 0, F_z = 0, T = T_0, \zeta = 0 \) at the disc midplane. At the disc photosphere \( \zeta = \Sigma \) and

\[
T^4(\tau_0) = T_{\text{eff}}^4 + T_{\text{irr}}^4,
\]

where \( \sigma T_{\text{eff}}^4 = F_{\text{vis}} \).

Figures 4 and 5 are obtained using a fixed, constant irradiation temperature. In actual discs, the irradiation flux varies with the radius and inner mass accretion rate, so that irradiation modifies the “radial” disc structure. Here, we are interested in “self-irradiation”, i.e. the irradiation of the accretion disc by X-rays produced by the accretion flow itself.

El-Koury & Wickramasinghe (1998) have recently considered the effect of X-ray irradiation on accretion disc S-curves. Our calculations correspond to their \( \gamma = 1 \) case.

3.3 The radial structure

The irradiated radial disk structure is now computed using the time-dependent model of H98 to calculate stationary solutions in which we have zeroed the time derivatives. The disk evolution will be discussed in a future paper. Results are shown in Figures 6 and 7. We first used Eq. (23) for the irradiation temperature with values of albedo, efficiency and \( H_{\text{vis}}/R \) from de Jong et al. (1996). Results are shown in Figures 6 and 7.

We have checked that the results depend basically on the average value \( \zeta \) which is \( \approx 5 \times 10^{-4} \) in this case. In other words, the small radial dependence in the illumination formula used by de Jong et al. (1996) i.e. \( H_{\text{vis}}/R \propto R^{2/7} \) can be neglected. For this value for \( \zeta \), illumination keeps the disk on the hot branch at a significantly larger radius than in the unilluminated case, i.e. the stabilizing effect is strong. However, if one uses \( H_{\text{vis}} = z_0 \), or \( H_{\text{vis}} = H \) to calculate \( \zeta \) self-consistently, the resulting disc structure is not affected by irradiation, regardless of the values used for the albedo and efficiency: the outer disc regions that could be modified by irradiation are screened by the inner disc regions. The radial profiles of midplane temperature, surface density, etc. are, in Figs. 6 and 7, indistinguishable from the continuous line profiles of these quantities representing a non-irradiated disc.

Contrary to the often used assumption of a concave disc, a self-consistent disc model produces a convex disc (see Fig. 6) Tuchman et al. (1999). In the inner (radially), hot regions of a stationary disc, where the mean opacity is well described by the Kramers’ formula \( (\kappa \sim \rho T^{-3.5}) \) the disc height varies as \( R^{9/8} \), but, as temperature decreases with radius, the opacity gradually changes and this flattens the disc shape. Finally, at effective temperatures \( \lesssim 10^3 \) K, opacities decrease with temperature and \( d \ln z_0 / d \ln R \) becomes negative and the outer disc regions are screened from X-ray irradiation. Screening becomes effective when \( d \ln z_0 / d \ln R = 1 \), which occurs in regions which are still stable, as can be seen from Figs. 6 and 7.

Of course, the same opacity behavior leads to the disc thermal instability, the marginally stable and unstable outer disc regions are therefore screened from X-rays (see also Cannizzo 1998).

The abrupt change of slope of \( T_c, \Sigma \) and \( z_0/\rho \) profiles which is prominent in Figs. 6 and 7 results from the drastic change of opacities due to hydrogen recombination. In the non-irradiated case this happens at a midplane temperature \( T_c \lesssim 40000 \) K. Irradiation stabilizes the disc and the critical midplane temperature is \( \sim 20000 \) K. Note that at these temperatures the disc is still far from being isothermal.
Figure 4. $\Sigma - T_{\text{sur}}$, $\Sigma - T_c$, $\Sigma - T_{\text{eff}}$ and $\Sigma - \dot{M}$ curves for $r = 3 \cdot 10^{10}$ cm, $M = 1.4 M_\odot$, $\alpha = 0.1$, and $T_{\text{irr}} = [0, 3, 6, 9, 12] \times 10^3$ K.

Figure 5. Same as Fig. 4 but for $M = 10 M_\odot$.
Figure 6. Radial profiles of the midplane temperature, surface density, and photospheric height to radius ratio for an un-irradiated (continuous line) and irradiated disc (dashed line) around a 1.4 $M_\odot$ compact object. In the temperature diagram the continuous line represents the effective temperature. The accretion rate is $\dot{M} = 10^{18} \text{g s}^{-1}$, $\alpha = 0.1$. $T_{\text{irr}}$ is taken from Eq. (15), with $\eta = 0.1$, $\epsilon = 0.92$, $H_{\text{irr}}/R = 0.2(R/R_{\text{out}})^{2/7}$. Regions beyond the radius at which a break in the $T_c$ or $\Sigma$ curve is seen are unstable.

Figure 7. Same as in Fig. 6 but for $M = 10 M_\odot$. 
Figure 8. Stationary accretion disc surface density profiles for 4 values of accretion rate. From top to bottom: $\dot{M} = 10^{18}, 10^{17}, 10^{16}$ and $10^{15}$ g s$^{-1}$. $\dot{M} = 1.4 M_\odot$, $\alpha = 0.1$. The continuous line corresponds to the un-irradiated disc, the dotted lines to the irradiated configuration.

Figure 9. Same as Fig. 8 but for $\dot{M} = 10 M_\odot$

effect of irradiation is to extend the hot disc temperature profile ($\sim R^{-3/4}$) to larger radii. Note also the difference between the values of $z_0/R$ for 1.4 M$_\odot$ and 10 M$_\odot$ compact objects due to the stronger gravity (at a given radius) in the case of a larger mass central body.

Figs. 8 and 9 showed disc structures for a given accretion rate ($\dot{M} = 10^{18}$ g s$^{-1}$). In Figs. 8 and 9 we present surface density profiles for 4 values of the accretion rate, for a $\dot{M} = 1.4 M_\odot$ and $\dot{M} = 10 M_\odot$ compact object respectively. As expected, the change in the radial extension of the stable, hot branch is larger for higher mass transfer rates, i.e. stronger irradiation.

The conclusion that a planar (non-warped) stationary disc irradiated by a point X-ray source powered by accretion is unaffected by irradiation is unavoidable. Since observations suggest that accretion discs in LMXBs are affected by X-irradiation, either the assumption of a planar disc or that of a point source (or both) are wrong. Accretion discs in stationary LMXBs could be warped due to the Pringle irradiation instability (Pringle, 1991). In the framework of AGN accretion discs, a point source above the disc is often invoked to explain the data but its physical meaning is unclear.

In what follows we use Eq. (20) with $C$ constant, which should be a reasonable approximation of more accurate irradiation laws as discussed above and as argued in the next section.

### 3.4 Stability criteria

The hot stable solutions (slope $d \ln \Sigma/d \ln R \sim -1$) end at a surface density $\Sigma_{\text{min}}$ which in the unirradiated case is given by (H98):

$$\Sigma_{\text{min}} = 30.82 \left(\frac{R}{10^{10} \text{cm}}\right)^{1.11} \text{g cm}^{-2}$$

where $\alpha = 0.1$ and $M = 1.4 M_\odot$ was assumed. Irradiation extends the hot branch to lower critical surface densities at larger radii; the amplitude of this extension increases with accretion rate so that the slope of the $\Sigma_{\text{min}}(R)$ is flatter than in the unirradiated case. With $C$ constant in the irradiation law given by Eq. (20)

$$\Sigma_{\text{min}}^{\text{irr}} \approx 11.4 \left(\frac{C}{5 \times 10^{-4}}\right)^{-0.3} \left(\frac{R}{10^{10} \text{cm}}\right)^{0.8} \text{g cm}^{-2}$$

where $C$ is scaled by its “typical value” $5 \times 10^{-4}$.

In the irradiated case the critical accretion rate, below which no steady, stable solution can exist, is given by:

$$M_{\text{crit}}^{\text{irr}} \approx 1.5 \times 10^{15} \left(\frac{M_1}{M_\odot}\right)^{-0.4} \left(\frac{R}{10^{10} \text{cm}}\right)^{2.1} \times \left(\frac{C}{5 \times 10^{-4}}\right)^{-0.5} \text{g s}^{-1}$$

which can be compared to the unirradiated value of H98:

$$M_{\text{crit}} \approx 9.5 \times 10^{15} \left(\frac{M_1}{M_\odot}\right)^{0.89} \left(\frac{R}{10^{10} \text{cm}}\right)^{2.68} \text{g s}^{-1}$$

The curves corresponding to an irradiated disc also show a density maximum analogous to the $\Sigma_{\text{max}}$ appearing in the non-irradiated case. According to the thermal-viscous disc instability model, a quiescent disc in the low state must satisfy $\Sigma < \Sigma_{\text{max}}$ everywhere, and cannot be steady, for any reasonable value of the mass transfer rate (see e.g. Cannizzo 1993; H98). This condition is unchanged by irradiation because, in a quiescent disc $\Sigma(R) \sim R^{2.65}$ (H98), so that $F_{\text{vis}}/F_{\text{irr}} \sim R^{1.65}$ and self-irradiation is never important.

Eq. (21) provides the thermal-viscous stability criterion for X-ray irradiated discs. Assuming standard relations (King et al., 1997a) between the disc radius and orbital parameters of the binary one gets ($P_{\text{br}}$ is the binary orbital period in hours)

$$M_{\text{crit}}^{\text{irr}} \approx 2.0 \times 10^{15} \left(\frac{M_1}{M_\odot}\right)^{0.5} \left(\frac{M_2}{M_\odot}\right)^{-0.2} P_{\text{br}}^{1.4} \times \left(\frac{C}{5 \times 10^{-4}}\right)^{-0.5} \text{g s}^{-1}$$

whereas King et al. (1997) get

$$M_{\text{crit}}^{\text{irr}} \approx 3.6 \times 10^{14} \left(\frac{M_1}{M_\odot}\right)^{5/6} \left(\frac{M_2}{M_\odot}\right)^{-1/6} P_{\text{br}}^{1.33}$$
The values for the different systems have been taken from van Paradijs [1996]. Transient systems are represented by stars and persistent systems by triangles. The criteria are weakly dependent on the secondary mass and, following van Paradijs, we took $M_2 = 0.4 M_\odot$. The dotted line corresponds to the criterion for an un-irradiated disk ($M_1 = 1.4 M_\odot$), the upper and lower dashed line are the King et al. criterions respectively with and without the additional $H/R$ factor (Eq. 33). The solid line is the criterion deduced here (Eq. 32). The conventions are identical in the SXT diagram with $M_1 = 10 M_\odot$.

$$\times \left( \frac{C}{5 \times 10^{-4}} \right)^{-1} \text{g s}^{-1}. \quad (33)$$

King et al. assume that $C$ should be multiplied by an additional factor $H/R \sim 0.2$ to account for the non-source character of a black hole. Note that $M_1^{\text{KSS}}$ is inversely proportional to $C$ because the King et al. criterion is obtained simply by equating $T_{\text{irr}}$ with the temperature corresponding to hydrogen ionization (in this case $T_{\text{irr}} \sim 6500$ K).

Following van Paradijs [1996], we show in Fig. [4] the stability criterion in the different cases of neutron star LMXB and SXT (black hole LMXB). As already noticed by van Paradijs, the non-irradiated stability criterion cannot explain the important population of persistent neutron star LMXB. For similar values of $C$, our calculations result in less stable disks than what King et al. [1997a] found. Our result is very close to that of King et al. if their additional $H/R$ factor is taken into account. However, this amounts to comparing different values of $C$ and is probably coincidental.

The critical values do of course depend on $C$. We use $C \sim 5 \times 10^{-4}$ by comparison with a formula extensively used in the recent literature, but, as argued above, the numerical values of the various parameters (such as the albedo) entering $C$ could be different from those usually assumed. Furthermore, we want to stress that the assumptions (isothermal vertical structure) used in deriving this “standard” formula are not justified. The results obtained through this formula are reasonable probably only because it is equivalent to taking $C$ almost constant. It is clear that the geometry and physics of irradiation in disks are still poorly understood and that much more work is needed before a satisfying model can be proposed.

4 CONCLUSIONS

We calculated self-consistent models of X-ray irradiated accretion discs around stellar mass compact bodies. We showed that irradiation of a stationary, standard, planar (non-warped) accretion disc by a point-like source positioned near the equatorial plane cannot modify the disc structure. The outer disc regions, which one could expect to be modified by irradiation, cannot intercept the X-rays because of self-screening. Claims to the contrary must be based on the use of erroneous equations and/or inadequate assumptions about disc properties such as shape and vertical temperature structure.

We used the diffusion approximation to describe radiative transfer and a simple assumption about energy deposition by X-rays but our conclusions are general because they basically result from the law of energy conservation. The temporal evolution of unstable irradiated disks will be discussed in a forthcoming paper (Dubus et al., 1998).

Observations show, however, that outer regions of accretion discs in LMXBs are irradiated so they are intercepting X-rays. We conclude, therefore, that these discs could be non-planar (i.e. warped), and/or that the irradiating source could be not in the equatorial plane and/or could be not point-like (see e.g. Dumont & Collin-Souffrin [1999]). These types of irradiation can be summarized in a parameter for which we have few theoretical constraints; in view of the uncertainties in the models, observations will bring important constraints to this problem.

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Figure 10. Stability criterion applied to neutron star and black hole LMXB. The values for the different systems have been taken from van Paradijs [1996]. Transient systems are represented by stars and persistent systems by triangles. The criteria are weakly dependent on the secondary mass and, following van Paradijs, we took $M_2 = 0.4 M_\odot$. The dotted line corresponds to the criterion for an un-irradiated disk ($M_1 = 1.4 M_\odot$), the upper and lower dashed line are the King et al. criterions respectively with and without the additional $H/R$ factor (Eq. 33). The solid line is the criterion deduced here (Eq. 32). The conventions are identical in the SXT diagram with $M_1 = 10 M_\odot$. 

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