NEUTRON STARS, SUPERNOVA EXPLOSIONS
AND THE TRANSITION TO QUARK MATTER

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Abstract. The transition to quark matter can take place in neutron stars. The structure of a hybrid star, containing a core made of quark matter is discussed. The maximum mass of the non-rotating hybrid star turns out to be $\sim 1.6 M_\odot$. Possible signatures of the quark phase as pulsar's timing, cooling rate etc. are briefly analyzed. The deconfinement transition can also take place during the pre-supernova collapse. This possibility is studied by introducing a finite temperature EOS. The dependence of the latter on the proton fraction is shown to be crucial. The softening of the EOS at densities just above nuclear matter saturation density for $Z/A \sim 0.3$ helps in obtaining an explosion. At the same time, at larger densities the EOS is stiff enough to support a neutron star compatible with observations.

1. Introduction

The major problem in the study of the transition to quark matter is the uncertainty in the choice of the relevant degrees of freedom at the densities and temperatures at which the transition is supposed to take place. In the present contribution I will restrict the discussion to the low temperatures regime, i.e. temperatures of the order of few tens MeV. In this regime, the transition to quark matter will be driven essentially by the density of the system, while the temperature will play a relatively minor role, shifting the critical densities to lower values.

The reasons why I will concentrate into the low temperature regime are the following: a) neutron stars (NS), except during the very first seconds of their life, are at temperatures of the order of few MeV or lower; b) during the pre-supernova collapse, temperatures not exceeding few tens MeV can be reached; finally c) quark models have been extensively studied at low temperatures, where the experimental value of several observables constrains the value of the models’ parameters. In this contribution I will try to show that, at least in the above described regime, a coherent scenario can be outlined.

2. Quark models

Most of the calculations of the transition to quark matter are based on the MIT bag model. The crucial parameter in this model is the so-called pressure of the vacuum $B$, namely the energy necessary to dig a hole in the non-perturbative QCD vacuum. Here are a few estimates of $B$:
• from the computation of the hadronic spectrum \[1\]
  \[B = 59 \text{ MeV/fm}^3 \text{ or } B^{1/4} = 145 \text{ MeV}\]
• from the computation of hadronic structure functions \[2\]
  \[B \sim 109 \text{ MeV/fm}^3 \text{ or } B^{1/4} \sim 170 \text{ MeV}\]
• from the comparison of MIT bag pure-gauge results with lattice QCD at finite temperature \[3\]; a critical temperature \(T_c = 240 \pm 20 \text{ MeV}\) for pure gauge SU(3) corresponds to
  \[B = 170 \pm 50 \text{ MeV/fm}^3 \text{ or } B^{1/4} = 190 \pm 15 \text{ MeV}\]

The conclusions one can obtain from the previous estimates are the following: a) there is a certain ambiguity among the various calculations of single hadron properties. The preferred value for \(B\) is in the range \(60 \text{ MeV/fm}^3 \leq B \leq 110 \text{ MeV/fm}^3\); b) the estimates based on calculations trying to reproduce lattice QCD results indicate a larger value for \(B\). It is anyway important to stress that at the moment there is no consensus about the relevant degrees of freedom at finite temperature. For instance, gluons can develop a thermal mass: if the latter is taken into account the lattice QCD data can be reproduced using a very small value for \(B\). It is also important to stress that for \(B \geq 65 \text{ MeV/fm}^3\) iron is the ground state of matter. Therefore one is not obliged to accept Witten’s strange matter hypothesis even using for \(B\) a value indicated by single hadron calculations.

Among the many studies of the structure of NS based on the MIT bag model, I would like to comment the most recent ones. Akmal et al. \[5\] have used, for the hadronic sector, a non-relativistic Equation Of State (EOS) based on the Argonne potential and incorporating three-body forces. In their analysis they consider two values for \(B\): 122 and 200 MeV/fm\(^3\). Using the smaller value the transition to quark matter starts at \(\rho_c \sim 3.5\rho_0\) and the bulk of a heavy NS is made of a mixed phase of hadronic and quark matter. Clearly, if a smaller value for \(B\) would have been used, the mixed phase would be present also in lighter NS.

A similar calculation has been performed by Iida and Sato \[6\]. In this case in the hadronic sector it has been used a relativistic EOS which takes into account also hyperonic degrees of freedom. The main result they obtain is that heavy NS contain quark matter essentially for all values of \(B\). Moreover, if \(B\) is smaller than \(\sim 90\) MeV/fm\(^3\) quark matter is present even in ‘standard’ NS having a mass \(M \sim 1.4 M_\odot\).

The calculations that will be presented in the remaining part of this contribution are not based on the MIT bag model, but on a non-topological soliton model called Color Dielectric Model (CDM) \[7, 8, 9\]. The main differences respect to the MIT bag model are the following:

- **MIT**
  - current masses for the quarks
  - large value of the pressure of the vacuum
  - rigid confinement in a sphere
  - ambiguous treatment of center of mass motion

- **CDM**
  - constituent masses for the quarks
  - small value of the pressure of the vacuum
  - soft confinement via interaction with a scalar field
  - center of mass motion removed in the non-relativistic limit
In the past it has been shown that in the CDM, using a fixed set of parameters’ values it is possible to study nucleon form factors, structure functions and to reproduce the main features of low energy spectroscopy. This same set of parameters’ values has been used to study the transition to quark matter.

3. Neutron stars

It has been investigated the structure of a hybrid star, which in the outer region is made of nucleonic matter and in the center contains quark matter [10].

3.1. Composition

In most of the calculations a naive Walecka-type relativistic model has been used to describe the hadronic phase. Considering a NS having a mass \( M = 1.4M_\odot \), half of the volume is occupied by pure quark matter. The mixed phase extends over 1.5 Km. The radius of the star is slightly larger than 10 Km. The maximum mass for a non-rotating star is \( M_{\text{max}} = 1.59M_\odot \). Looking to the mass-radius relation it appears that the star is more compact than a NS made only of nucleons. This can be relevant in the light of recent estimates of the radii of NS [11].

It is worth mentioning the recently discovered possibility of having, during the slow-down of millisecond pulsars, an effect similar to the back-bending in nuclear physics [12]. Since a millisecond pulsar is strongly deformed, its central density is reduced respect to a non-rotating star. It is therefore possible that quark matter is formed during the slow-down of the pulsar. Since quark matter is denser than normal matter, the moment of inertia of the star reduces and the pulsar re-accelerates to conserve the angular momentum. If detected, this anomalous behaviour of a millisecond pulsar would be the signal that a deep modification in the composition of the star is taking place.

3.2. Cooling

The problem of the cooling of a NS has become very unclear in the last years [13]. It has been pointed out recently [14] that the relation between the internal temperature of the NS, which depends on the composition of the interior of the star, and the external temperature, that can be measured using X-ray satellites, can be deeply modified by the presence of light nuclei on the surface of the star. The presence of light nuclei increases the heat transport and therefore the ratio between the internal and the external temperature could be smaller than the previously estimated factor one hundred.

It is anyway interesting to remark that, using the CDM, the direct URCA mechanism at the level of the quarks can take place, but is strongly suppressed due to the extreme smallness of the electron fraction in the interior of the NS. The computed cooling rate is only slightly faster than the modified URCA mechanism taking place in a traditional NS made of nucleons. Therefore, at variance with the MIT bag results, using the CDM is not necessary to invoke any re-heating mechanism to obtain a cooling rate of the right order of magnitude.
4. Supernova explosion

The problem of getting a successful supernova explosion is still open after many years of work. I will concentrate here on the so-called direct mechanism. It is characterized by the idea that the explosion is directly related to the shock wave generated by the bounce. During the last years this mechanism drop out of fashion, and most of the recent studies are related to the idea of a neutrino driven explosion, in which the initial shock stalls and the explosion is revitalized by the energy deposited by the neutrinos in the regions outside the forming proto-neutron star.

![Figure 1](image)

**Figure 1.** Boundaries separating hadronic matter from mixed phase (left lines) and the latter from quark matter (right lines). The labels indicate various values of $Y_{le}$, $s$ is for symmetric matter.

The possibility of a successful explosion via the direct mechanism is related to the softness of the EOS at densities just above nuclear matter saturation density $\rho_0$. This possibility has been ruled out for the only reason that seems to be inconsistent with the $1.44 M_\odot$ constraint coming from PSR 1913+16. On the other hand Cooperstein concluded that if a phase transition takes place during the collapse, ‘the presence of a mixed-phase region softens the EOS and leads to a direct explosion’.

The crucial problem to be analyzed is the dependence of the critical densities on the proton fraction $Z/A$. Clearly no quark matter has to be present in (nearly) symmetric nuclear matter at densities of the order of the saturation density $\rho_0$. On the other hand, it is exactly in this range of densities that the transition should take place, when $Z/A \sim 0.3$, to influence the collapse of the supernova. We have therefore explored the dependence of the critical densities on the proton fraction. It is also important to check the effect of the temperature on the transition.

In Fig. 1 are presented the boundaries separating hadronic matter from mixed phase and the latter from pure quark matter. The labels correspond to various values of the lepton fraction $Y_{le}$. Symmetric nuclear matter is also presented. The transition region depends on the lepton fraction $Y_{le}$. In symmetric matter at low temperatures the mixed phase forms at $\rho = 0.23\text{ fm}^{-3}$, therefore no quark matter is present in heavy
nuclei. Decreasing the value of \( Y_l \) the phase transition starts at lower densities. At any value of \( Y_l \) the mixed phase extends on a rather limited range of densities and even at zero temperature pure quark matter phase is reached at densities slightly larger than \( 2\rho_0 \). At higher temperatures the transition starts at lower densities.

![Figure 2](image-url)

**Figure 2.** Pressure (upper box) and adiabatic index (lower box) as function of the density in CDM (solid) and in BCK (dotted). The pressure in Walecka model is also shown (dashed).

To investigate our EOS in connection with the problem of supernova explosion, we compare with BCK EOS \[19\]. The latter is a totally phenomenological EOS which is soft enough to allow for supernova explosion, but gives a maximum mass smaller than the mass of PSR 1913+16.

In Fig. 2 are presented results for \( Y_l = 0.4 \) and entropy per baryon number \( S/R = 1 \). In the upper box we compare the pressure in the Walecka model, in our model and in BCK EOS. Due to the phase transition, our EOS is rather soft from \( \rho = 0.17 \text{ fm}^{-3} \) to \( \rho = 0.34 \text{ fm}^{-3} \). On the other hand, after \( \rho = 0.34 \text{ fm}^{-3} \) it is considerably stiffer than BCK, allowing higher masses for the proto-neutron star. These conclusions are strengthened by the computation of the adiabatic index, shown in the lower box of Fig. 2. Clearly in the mixed phase matter offers little resistance to collapse, but when pure quark matter phase is reached the collapse is halted. In the mixed phase region our adiabatic index is even smaller than in BCK. It is important to remark that similar results have been obtained using the Lattimer-Swesty EOS \[20\] to describe the hadronic phase.

5. Conclusions

The main results are the following. Calculations based on various quark models indicate the presence of quark matter at least in the center of heavy NS. Moreover, if the quark model’s parameters are fixed to reproduce basic hadronic properties then
a) the transition to quark matter takes place at low densities in \( \beta \)-stable matter and
b) neutron stars having a mass \( M \sim 1.4M_\odot \) contain quark matter. Hybrid stars are
characterized by a small $R/M$ ratio.

A signal of a transition to quark matter would be the spin-up of a millisecond pulsar, due to the modification of its moment of inertia.

Finally, the transition to quark matter could take place during heavy star collapse, helping supernova to explode.

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