Efficient emulator for solving three-nucleon continuum Faddeev equations with chiral three-nucleon force comprising any number of contact terms

H. Witała\textsuperscript{a}, J. Golak, R. Skibiński
M. Smoluchowski Institute of Physics, Jagiellonian University, 30348 Kraków, Poland

Received: 6 May 2021 / Accepted: 8 July 2021 / Published online: 20 July 2021
© The Author(s) 2021
Communicated by Vittorio Somà

Abstract We demonstrate a computational scheme which drastically decreases the required time to get theoretical predictions based on chiral two- and three-nucleon forces for observables in three-nucleon continuum. For a three-nucleon force containing N short-range terms all workload is reduced to solving N+1 Faddeev-type integral equations. That done, computation of observables for any combination of strengths of the contact terms is done in a flash. We demonstrate on example of the elastic nucleon-deuteron scattering observables the high precision of the proposed emulator and its capability to reproduce exact results.

1 Introduction

Since the birth of nuclear physics the nuclear force problem has been at the centre of experimental and theoretical studies. Extensive efforts based on purely phenomenological approaches or incorporating the meson-exchange picture have led to numerous nucleon-nucleon (NN) potentials, able to describe a vast amount of available data \cite{1} with high precision. A major breakthrough occurred with the emergence of the effective field theory (EFT) concept \cite{2}, which paved the way for developing precise nuclear forces \cite{3–6}. The progress in constructing nuclear forces within the EFT approach is presently documented by the availability of numerous high precision NN potentials developed by the Bochum-Bonn \cite{7,8}, Idaho-Salamanca \cite{9}, and Bochum \cite{10} groups. These forces provide a very good description of the NN data set.

Applications of the EFT approach in the form of chiral perturbation theory (ChPT) have resulted not only in the theoretically well grounded NN potentials but also for the first time have given a possibility to apply in practical calculations NN forces augmented by consistent 3N interactions, derived within the same formalism. Understanding of nuclear spectra and reactions based on these consistent chiral two- and many-body forces has become a hot topic of present day few-body studies \cite{11}.

The first nonvanishing contributions to the 3N force (3NF) appear at next-to-next-to-leading order of chiral expansion (N\textsuperscript{2}LO) \cite{3,12} and comprise in addition to the 2π-exchange term two contact contributions with strength parameters \(c_D\) and \(c_E\) \cite{13}. The chiral 3NF at N\textsuperscript{3}LO was derived in \cite{14,15}. At that order three components of long-range character \cite{14} supplemented by the short-range terms \cite{15} contribute to 3NF. The 3NF at N\textsuperscript{3}LO order does not involve any new unknown low-energy constants and depends only on two parameters, \(c_D\) and \(c_E\), that parameterize the leading one-pion-contact term and the 3N contact term present already at N\textsuperscript{2}LO. The \(c_D\) and \(c_E\) values need to be then fixed at this order, as at N\textsuperscript{2}LO, from a fit to few-nucleon data. At the higher order, N\textsuperscript{4}LO, in addition to long- and intermediate-range interactions generated by pion-exchange diagrams \cite{16,17}, the chiral N\textsuperscript{4}LO 3NF involves thirteen purely short-range operators, which have been worked out in \cite{18,19}.

Since the advent of numerically exact three-nucleon continuum Faddeev calculations the elastic nucleon-deuteron (Nd) scattering and the deuteron breakup reaction have been a powerful tool to test modern models of the nuclear forces \cite{20–22} and the question about the importance of 3NF has developed into the main topic of 3N system studies. That issue has been given a new impetus by the ChPT-based achievements and the possibility to apply consistent two- and many-body nuclear forces, derived within this framework, in 3N continuum calculations.

\textsuperscript{a} e-mail: henryk.witala@uj.edu.pl (corresponding author)
Using chiral 3NF in 3N continuum requires numerous
time consuming computations with varying strengths of the
contact terms in order to establish their values. They can be
determined for example from the $^3$H binding energy and the
minimum of the elastic Nd scattering differential cross sec-
tion at the energy ($E_{lab} \approx 70$ MeV), where the effects of 3NF
start to emerge in elastic Nd scattering [23,24]. Specifically
at N$^2$LO, after establishing the so-called ($c_D$, $c_E$) correlation
line, which for a particular chiral NN potential combined with
a N$^2$LO 3NF gives pairs of ($c_D$, $c_E$) values reproducing the
$^3$H binding energy, a fit to experimental data for the elas-
tic Nd cross section is performed to determine the $c_D$ and
$c_E$ strengths. Fine-tuning of the 3N Hamiltonian param-
eters requires an extensive analysis of available 3N elastic Nd
scattering and breakup data. That ambitious goal calls for
a significant reduction of computer time necessary to solve
the 3N Faddeev equations and to calculate the observables.
Thus finding an efficient emulator for exact solutions of the
3N Faddeev equations seems to be essential and of high pri-

In Ref. [25] we proposed such an emulator which enables
us to reduce significantly the required time of calculations.
We tested its efficiency as well as ability to reproduce
exact solutions of 3N Faddeev equations. In the present
study we introduce a new computational scheme, based on
the perturbative approach of [25], which even if by far more
reduces the computer time required to obtain the observ-
ables in the elastic nucleon-deuteron scattering and deuteron
breakup reactions at any energy, and which is well-suited for
calculations with varying strengths of the contact terms in a
chiral 3NF.

2 The new emulator

Before presenting this new emulator, for the reader’s conve-
nience we shortly outline the main points of the 3N Faddeev
formalism and of the perturbative treatment of Ref. [25]. For
details of the formalism and numerical performance we refer to [20, 26–28].

Neutron-deuteron (nd) scattering with nucleons interact-
ing via NN interactions $v_{NN}$ and a 3NF $V_{123} = V^{(1)} + V^{(2)} +
V^{(3)}$, is described in terms of a breakup operator $T$ satisfying
the Faddeev-type integral equation [20, 26, 27]

$$
T|\phi\rangle = tP|\phi\rangle + (1 + tG_0) V^{(1)}(1 + P)|\phi\rangle + tP G_0 T|\phi\rangle + (1 + tG_0) V^{(1)}(1 + P)G_0 T|\phi\rangle.
$$

(1)

The 2N t-matrix $t$ is the solution of the Lippmann-Schwinger
equation with the interaction $v_{NN}$. $V^{(1)}$ is the part of a
3NF which is symmetric under the interchange of nucleons
2 and 3: $V_{123} = V^{(1)}(1 + P)$. The permutation operator
$P = P_{12}P_{23} + P_{13}P_{23}$ is given in terms of the transposi-
tion operators, $P_{ij}$, which interchange nucleons $i$ and $j$. The
initial state $|\phi\rangle = |q_0\rangle|\phi_d\rangle$ describes the free motion of
the neutron and the deuteron with the relative momentum $q_0$ and
contains the internal deuteron wave function $|\phi_d\rangle$. $G_0$ is the
free three-body resolvent. The amplitude for elastic scatter-
ing leading to the final nd state $|\phi\rangle$ is then given by [20, 27]

$$
\langle \phi'|U|\phi\rangle = \langle \phi'|P G_0^{-1}|\phi\rangle + \langle \phi'|V^{(1)}(1 + P)|\phi\rangle + \langle \phi'|V^{(1)}(1 + P)G_0 T|\phi\rangle + \langle \phi'|P T|\phi\rangle,
$$

(2)

while the amplitude for the breakup reaction reads

$$
\langle pq|U_0|\phi\rangle = \langle pq|(1 + P)T|\phi\rangle,
$$

(3)

where the free breakup channel state $|pq\rangle$ is defined in terms
of the Jacobi (relative) momenta $p$ and $q$.

We solve Eq. (1) in the momentum-space partial-wave decompo-
sition in momentum space at a given energy $E$ requires taking
all 3N partial wave states up to the 2N angular momentum
$j_{max} = 5$ and the 3N angular momentum $J_{max} = 25$, with
the 3N force acting up to the 3N total angular momentum
$J = 7/2$. The number of resulting partial waves (equal to the
number of coupled integral equations in two continuous vari-
ables $p$ and $q$) amounts to 142. The required computer time
to get one solution on a personal computer is about $\approx 2$ h.
In the case when such calculations have to be performed for
a big number of varying 3NF parameters, time restrictions
become prohibitive. Fortunately, the perturbative approach
of Ref. [25] leads to a significant reduction of the required
computational time.

Let us consider a chiral 3NF at a given order of chiral expan-
sion with variable strengths of its contact terms. The
3NF at N$^2$LO has one parameter-free term (2π-exchange
contribution) and two short-range terms with strength par-
eters $c_D$ and $c_E$. At N$^3$LO there are more contributing
parameter-free parts but again only two contact terms. At
N$^4$LO parameter-free contributions are supplemented by fift-
teen short-range terms with strengths: $c_D$, $c_E$, $c_{E1}$, ..., $c_{E15}$. All
these contact terms are restricted to small 3N total angu-
lar momenta and to only few partial wave states for a given
total 3N angular momentum $J$ and parity $\pi$. For example
for $J^P = 7/2^±$ all matrix elements $<pq\tilde{a}|V^{(1)}|p'q'\tilde{a}'>$ proportional to $e_{E_i}$ and $e_{E_f}$ vanish, while the $c_D$ and $c_E$ terms are nonzero only for a restricted number of $\tilde{a}$, $\tilde{a}'$ pairs (mostly containing $1S_0$ and $3S_1-3D_1$ quantum numbers) [12, 13]. Bearing that in mind and taking into account the fact that contact terms yield a small contribution to the 3N potential energy compared to the leading, parameter-free part, it is possible to apply a perturbative approach in order to include the contact terms.

We split the $V^{(1)}$ part of a 3NF into a parameter-free term $V(\theta_0)$ and a sum of $N$ contact terms $c_i \Delta V_i$ with strengths $c_i$:

$$
V^{(1)} = V(\theta_0) + \Delta V(\theta) = V(\theta_0) + \sum_{i=1}^{N} c_i \Delta V_i ,
$$

with $\theta_0 = (c_i = 0, i = 1, \ldots, N)$ and $\theta = (c_i, i = 1, \ldots, N)$ being the sets of contact terms strength values, for which we would like to find solution of Eq. (1).

We divide the 3N partial wave states $\tilde{a}$ into two sets: $\beta$ and the remaining one, $\alpha$. The $\beta$ set contains states with nonvanishing elements of $\Delta V(\theta)$ and states most strongly coupled to them through the permutation operator $P$. This set of states is sufficient for the convergence of $\Delta T(\theta)$ in the second equation (7), also in the intermediate states. Introducing $T(\theta_0)$ and $\Delta T(\theta)$ such that $T \equiv T(\theta) = T(\theta_0) + \Delta T(\theta)$, and using the fact, that $\Delta V(\theta)$ has nonvanishing elements only for channels $|\beta\rangle$, one gets from Eq. (1) (omitting the Jacobi momenta in notation of partial wave states) two separate equations for $\langle \alpha|T(\theta_0)\rangle\phi$ and $\langle \alpha|\Delta T(\theta)\rangle\phi$ [25]:

$$
\langle \alpha|T(\theta_0)\rangle\phi = \langle \alpha|tP\rangle\phi + \langle \alpha|(1 + tG_0)V(\theta_0)(1 + P)\rangle\phi + \langle \alpha|tPG_0T(\theta_0)\rangle\phi + \langle \alpha|(1 + tG_0)V(\theta_0)(1 + P)G_0T(\theta_0)\rangle\phi
$$

$$
\langle \alpha|\Delta T(\theta)\rangle\phi = \langle \alpha|tPG_0\Delta T(\theta)\rangle\phi + \langle \alpha|(1 + tG_0)V(\theta_0)(1 + P)G_0\Delta T(\theta)\rangle\phi ,
$$

as well as for $\langle \beta|T(\theta_0)\rangle\phi$ and $\langle \beta|\Delta T(\theta)\rangle\phi$:

$$
\langle \beta|T(\theta_0)\rangle\phi = \langle \beta|tP\rangle\phi + \langle \beta|(1 + tG_0)V(\theta_0)(1 + P)\rangle\phi + \langle \beta|tPG_0T(\theta_0)\rangle\phi + \langle \beta|(1 + tG_0)V(\theta_0)(1 + P)G_0T(\theta_0)\rangle\phi
$$

$$
\langle \beta|\Delta T(\theta)\rangle\phi = \langle \beta|(1 + tG_0)\Delta V(\theta)(1 + P)\rangle\phi + \langle \beta|(1 + tG_0)\Delta V(\theta)(1 + P)G_0T(\theta_0)\rangle\phi + \langle \beta|(1 + tG_0)V(\theta_0)(1 + P)G_0\Delta T(\theta)\rangle\phi + \langle \beta|tPG_0\Delta T(\theta)\rangle\phi .
$$

The first equations in (6) and (7) are the Faddeev equations (1) for $T(\theta_0)$. The second equation in the set (7) for $\langle \beta|\Delta T(\theta)\rangle\phi$ can be solved within the set of channels $|\beta\rangle$ only. Using this solution, $\langle \alpha|\Delta T(\theta)\rangle\phi$ is then computed by:

$$
\langle \alpha|\Delta T(\theta)\rangle\phi = \langle \alpha|tPG_0\sum_{\beta} \int |p'q'\rangle \langle p'q'| \langle p'q'|\beta\Delta T(\theta)\rangle\phi + \langle \alpha|(1 + tG_0)V(\theta_0)(1 + P)G_0T(\theta_0)\rangle\phi + \langle \alpha|(1 + tG_0)V(\theta_0)(1 + P)G_0\Delta T(\theta)\rangle\phi
$$

$$
\sum_{\beta} \int |p'q'\rangle \langle p'q'| \langle p'q'|\beta\Delta T(\theta)\rangle\phi ,
$$

and finally, $\langle \tilde{a}|T(\theta)\rangle\phi$ is calculated.

The outlined above procedure constitutes the perturbative approach of Ref. [25]. In short, one solves the 3N Faddeev equation (1) exactly with the NN potential combined with the 3NF restricted to the parameter free term (set $\theta_0 = (0, \ldots, 0)$). The solution with the $\theta_0$ set forms a starting point in the perturbative treatment of Eqs. (6–8) and has to be calculated only once, regardless of how many variations of strength parameters are required. In the next step, the proper perturbative treatment is performed, solving first the second equation in set (7). Having determined $\langle \alpha|\Delta T(\theta)\rangle\phi$ from Eq. (8) the emulator solution $\langle \tilde{a}|T(\theta)\rangle\phi$ is calculated (in the following this emulator will be denoted by $E\Delta T$). That allows one to reduce the required computation time and to reproduce surprisingly well the exact predictions for neutron-deuteron (nd) elastic scattering as well as for nd breakup observables [25]. To be specific, taking set $|\beta\rangle$ which includes all 2N states with the total 2N angular momenta $j \leq 2$, leads to a reduction of the computing time by a factor of approximately 4 in comparison to the exact calculations. Note that it takes approximately 30 minutes on a personal computer to solve Eq. (1), provided that the $V(\theta_0)(1 + P)$ and $V(\theta_0)(1 + P)$ kernels, acting in $(1 + tG_0)V(\theta)(1 + P)G_0T(\theta)\phi$ term of Eq. (1), are prepared in advance, with the strengths $\theta_i = (c_i = 1, c_k \neq i = 0)$.

In spite of such a large reduction, the computational time can be even further decreased and calculation of 3N continuum observables made in a flash. This notion is based on the observation that among three kernel-terms in the second equation of set (7), it is possible (because of the smallness of $\Delta V(\theta)$) to neglect the term $\langle \beta|(1 + tG_0)\Delta V(\theta)(1 + P)G_0\Delta T(\theta)\rangle\phi$. The resulting integral equation for $\langle \beta|\Delta T(\theta)\rangle\phi$:

$$
\langle \beta|\Delta T(\theta)\rangle\phi = \langle \beta|(1 + tG_0)\Delta V(\theta)(1 + P)\rangle\phi + \langle \beta|(1 + tG_0)\Delta V(\theta)(1 + P)G_0T(\theta_0)\rangle\phi + \langle \beta|(1 + tG_0)\Delta V(\theta)(1 + P)G_0\Delta T(\theta)\rangle\phi + \langle \beta|tPG_0\Delta T(\theta)\rangle\phi
$$

permits one to transfer the linear dependence on the strengths $c_i$ from the $\Delta V(\theta)$ on the $\Delta T(\theta)$. Namely, let $\langle \beta|\Delta T_i\rangle\phi$ be a solution of Eq. (9) for a set $\theta_i = (c_i = 1, c_k \neq i = 0)$:

$$
\langle \beta|\Delta T_i\rangle\phi \equiv \langle \beta|(1 + tG_0)\Delta V_i(1 + P)\rangle\phi
$$
Multiplying (10) by $c_i$ and summing over $i$ one gets:

$$\langle \beta | \sum_i c_i \Delta T_i | \phi \rangle = \langle \beta | \sum_i c_i \Delta V_i (1 + P) | \phi \rangle$$

and the solution of Eq. (9) is given by:

$$\langle \beta | \Delta T (\theta) | \phi \rangle = \sum_{i=1}^{N} c_i \langle \beta | \Delta T_i | \phi \rangle.$$ (12)

In this way at a given energy the computation of observables in the elastic Nd scattering and deuteron breakup reaction for any combination of strengths $c_i$ of contact terms is reduced to solving once $N + 1$ Faddeev equations: one equation for $T (\theta_0)$ and $N$ equations for $\Delta T_i$. In the first step, solution for $| \alpha(\beta) | T (\theta_0) \rangle$ is found. Then Eq. (10) is solved for $| \beta | \Delta T_i \rangle$, from which the $| \alpha | \Delta T_i \rangle$ is calculated using Eq. (8). The above computations need to be done only once and then for any combination of the strengths $c_i$ the amplitude $| \tilde{\alpha} | T (\theta = (c_i, i = 1, \ldots, N)) \rangle$ is obtained by trivial summation.

For a calculation of elastic scattering observables the required sum of the second and the third term in Eq. (2) is obtained by:

$$\langle \beta | V^{(\pm)} (\pm) (1 + P) | \phi \rangle + \langle \beta | V^{(\pm)} (\pm) (1 + P) G_0 T (\theta_0) \rangle$$

$$= \langle \beta | V (\theta_0) (1 + P) | \phi \rangle + \langle \beta | V (\theta_0) (1 + P) G_0 T (\theta_0) \rangle$$

$$+ \sum_i c_i (\langle \beta | \Delta V_i (1 + P) | \phi \rangle$$

$$+ \langle \beta | \Delta V_i (1 + P) G_0 T (\theta_0) \rangle$$

$$+ \langle \beta | V (\theta_0) (1 + P) G_0 \Delta T_i \rangle$$

$$+ \sum_{i,k} c_i c_k \langle \beta | \Delta V_i (1 + P) G_0 \Delta T_k \rangle.$$ (13)

It is interesting to note that while the breakup amplitude (3) is linear in 3N contact strengths, the elastic scattering amplitude contains, as a consequence of the third term present in Eq. (2), also terms quadratic in these strengths.

3 Results and discussion

The above computational scheme forms the new emulator, which will be denoted in the following by $E \Delta T$. To check its quality and efficiency as well as to compare it with $E \Delta T$ we have chosen, as in Ref. [25], the SMS $N^4LO^+$ chiral potential of the Bochum group [10], with the regularization cutoff $\Lambda = 450$ MeV, and combined it with the chiral $N^2LO$ 3NF. We solved the 3N Faddeev equation (1) exactly at two incoming neutron energies $E = 70$ and 190 MeV with that combination as well as with this NN potential supplemented with the parameter free $2\pi$-exchange term of the $N^2LO$ 3NF (set $\theta_0 = (c_D = 0, c_E = 0)$). The first energy was taken from a region where 3NF effects start to appear in 3N continuum observables and the second one from a range with well-developed 3NF effects [23,24]. The solution with the $\theta_0$ set together with solutions $| \beta | \Delta T_i \rangle$ of N integral equations (10) with the set of 3N channels $| \beta \rangle$, comprising all 2N states with the total angular momentum $j \leq 2$, form a starting point in the proposed computational scheme and have to be calculated only once. Also $| \alpha | \Delta T_i \rangle | \phi \rangle$ is computed only once according to Eq. (8), and, in parallel with those calculations, all terms in Eq. (13) independent from $c_i$. Based on these quantities we calculated the emulator solution $| \tilde{\alpha} | T (\theta) \rangle | \phi \rangle$. Our new scheme performed for $N = 2$ leads to reduction by a factor of $\approx 50$ of the computer time required by the perturbative approach of Ref. [25]. Computation of all elastic scattering observables or observables in one exclusive breakup geometry for a particular set of strengths $\theta = (c_i, i = 1, \ldots, N)$, in the perturbative approach requires $\approx 8$ minutes of a personal computer time, while the new scheme requires only $\approx 10$ seconds. Since the calculation of $| \tilde{\alpha} | T (\theta) \rangle | \phi \rangle$ from $| \tilde{\alpha} | T (\theta_0) \rangle$ and $| \tilde{\alpha} | \Delta T_i \rangle \phi$ takes practically no time, the drastic reduction of time in the new scheme is independent from the number of the contact terms $N$.

The emulators $E \Delta T_i$ and $E \Delta T$ are approximations of the exact results. To estimate the quality of these approximations we show in Fig. 1 at both energies the distributions of percentage deviations from exact results $O_{exact}$ predictions of both emulators $O_{appr} = (O_{appr} - O_{exact}) / O_{exact} \times 100.0$, for all nonvanishing nd elastic scattering observables $O$ (cross section, nucleon and deuteron analyzing powers as well as spin correlation and spin transfer coefficients between all participating particles, altogether 51 observables). They were calculated on a uniform grid of 73 c.m. angles $\theta_{c.m.} \in (0^\circ, 180^\circ)$. These distributions encompass six sets of strengths values $(c_D, c_E) = (1, 1), (2, 1), (4, 1)$, $(6, 1), (8, 1)$, and $(10, 1)$ (according to the notation of Refs. [12,13]). The precision of both emulators at both energies is similar and amounts to $\approx 1 - 2\%$, with $E \Delta T$ being slightly more precise than $E \Delta T_i$.

To demonstrate the power of the new emulator we show in Fig. 2 the color map of $\chi^2$ values in the plane of strengths...
(c_D, c_E) for elastic scattering cross section at E = 70 MeV. That map was obtained with about 5000 values of strength combinations, both varied in step of 0.1. The minimum of $\chi^2$ valley is indicated by black squares. It crosses with $\chi^2$ values for the $^3$H correlation line, shown by yellow dots, in their minimum. The resulting values of strengths and their errors are $c_D = 2.910 \pm 0.140$ and $c_E = 0.385 \pm 0.015$. Using these strengths we demonstrate in Fig. 3 importance of contributions to the cross section from individual terms of the N^2LO 3NF. We show percentage changes in the cross section values calculated solely with the NN potential by adding the parameter-free term itself, and by the combination of that term with the D- or E-terms. The $2\pi$-exchange contribution as well as the short-range D-term are important at both energies. The contribution of the E term is negligible at $E = 70$ MeV but clearly visible at 190 MeV.

4 Summary and conclusions

In summary, we presented a new powerful calculational scheme which enables us to take efficiently into account any number of contact terms of a chiral 3NF in the 3N continuum Faddeev calculations. That approach facilitates a reduction of the time required to compute observables for a given set of strengths to seconds and is thus especially suited to repeated calculations with varying strengths of contact terms. We demonstrated that the proposed emulator reproduces very well the exact predictions for 3N continuum observables. It is conceivable that with the help of the constructed emulator of the exact solutions of the 3N Faddeev equation fine tuning of a 3N Hamiltonian parameters based on available 3N scattering data is feasible. To that aim not only existing neutron-deuteron elastic scattering and breakup data but also more abundant and precise proton-deuteron data should be used. However, before using the proton-deuteron data they should be corrected for the effects of the proton-proton Coulomb force acting in the proton-deuteron system. Such procedure involving estimation of the Coulomb corrections through the difference of calculations with and without Coulomb force included [21,30–32] have been already applied for example in [12].
Acknowledgements This study has been performed within Low Energy Nuclear Physics International Collaboration (LENPIC) project and was supported by the Polish National Science Center under Grant No. 2016/22/M/ST2/00173. The numerical calculations were performed on the supercomputer cluster of the JSC, Jülich, Germany.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: All relevant data are already given in the figures.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

1. R. Machleidt, Adv. Nucl. Phys. 19, 189 (1989)
2. S. Weinberg, Nucl. Phys. B 363, 3 (1991)
3. U. van Kolck, Phys. Rev. C 49, 2932 (1994)
4. E. Epelbaum, W. Glöckle, U.-G. Meißen, Nucl. Phys. A 747, 362 (2005)
5. E. Epelbaum, Prog. Part. Nucl. Phys. 57, 654 (2006)
6. R. Machleidt, D.R. Entem, Phys. Rep. 503, 1 (2011)
7. E. Epelbaum, H. Krebs, U.-G. Meißen, Eur. Phys. J. A 51, 53 (2015)
8. E. Epelbaum, H. Krebs, U.-G. Meißen, Phys. Rev. Lett. 115, 122301 (2015)
9. D.R. Entem, R. Machleidt, Y. Nosyk, Phys. Rev. C 96, 024004 (2017)
10. P. Reiners, H. Krebs, E. Epelbaum, Eur. Phys. J. A 54, 86 (2018)
11. E. Epelbaum, LENPIC Collaboration et al., Phys. Rev. C 99, 024313 (2019)
12. E. Epelbaum et al., Phys. Rev. C 66, 064001 (2002)
13. E. Epelbaum et al., Eur. Phys. J. A 56, (2020)
14. V. Bernard, E. Epelbaum, H. Krebs, U.-G. Meißen, Phys. Rev. C 77, 064004 (2008)
15. V. Bernard, E. Epelbaum, H. Krebs, U.-G. Meißen, Phys. Rev. C 84, 054001 (2011)
16. H. Krebs, A. Gasparyan, E. Epelbaum, Phys. Rev. C 85, 054006 (2012)
17. H. Krebs, A. Gasparyan, E. Epelbaum, Phys. Rev. C 87, 054007 (2013)
18. L. Girlanda, A. Kievsky, M. Viviani, Phys. Rev. C 84(1–8), 014001 (2011)
19. L. Girlanda, A. Kievsky, M. Viviani, Phys. Rev. C 102(E), 019903 (2020). arXiv:1102.4799v3 [nucl-th]
20. W. Glöckle, H. Witała, D. Hübner, H. Kamada, J. Golak, Phys. Rep. 274, 107 (1996)
21. A. Kievsky, M. Viviani, S. Rosati, Phys. Rev. C 52, R15 (1995)
22. A. Deltuva, K. Chmiulewski, P.U. Sauer, Phys. Rev. C 67, 034001 (2003)
23. H. Witała et al., Phys. Rev. C 63, 024007 (2001)
24. H. Witała, W. Glöckle, D. Hübner, J. Golak, H. Kamada, Phys. Rev. Lett. 81, 1183 (1998)
25. H. Witała, J. Golak, R. Skibiński, K. Topolnicki, Few-Body Syst. 62, 23 (2021)
26. H. Witała, T. Cornelius, W. Glöckle, Few-Body Syst. 3, 123 (1988)
27. D. Hübner, H. Kamada, H. Witała, W. Glöckle, Acta Phys. Pol. B 28, 1677 (1997)
28. W. Glöckle, The Quantum Mechanical Few-Body Problem (Springer, Berlin, 1983)
29. K. Sekiguchi et al., Phys. Rev. C 65, 034003 (2002)
30. A. Kievsky, M. Viviani, S. Rosati, Phys. Rev. C 60, 034001 (1999)
31. A. Deltuva, A.C. Fonseca, P.U. Sauer, Phys. Rev. C 71, 054005 (2005)
32. A. Deltuva, A.C. Fonseca, P.U. Sauer, Phys. Rev. C 72, 054004 (2005)