Infinity Cancellation, Type \( I' \) Compactification and String \( S \)-Matrix Functional

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Abstract

Nonvanishing tadpoles and possible infinities associated in the multiparticle amplitudes are discussed with regard to the disk and \( RP^2 \) diagrams of the Type \( I' \) compactification. We find that the infinity cancellation of \( SO(32) \) type \( I \) theory extends to this case as well despite the presence of tadpoles localized in the D-brane world-volume and the orientifold surfaces. Formalism of string \( S \)-matrix generating functional is presented to find a consistent string background as c-number source function: we find this only treats the cancellation of the tadpoles in the linearized approximation. Our formalism automatically provides representation of the string amplitudes on this background to all orders in \( \alpha' \).

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I. Introduction

There are a few important implications which the $D$-brane has brought to us \cite{1}. In this paper, we consider one of these which has been discussed relatively little so far. The $D$-brane can be introduced by taking a $T$-dual of type $I$ theory (or in bosonic string theory containing both closed and open string sectors) \cite{2}, which is sometimes referred to as type $I'$. The simplest nontrivial example which will be dealt with here is SO(32) Type $I'$ theory in flat ten dimensional space where $x^9$ is compactified on a circle with radius $R$. This model has 32 Dirichlet eight-branes at $x^9 = 0$ forming Dirichlet boundary condition of the disk geometry. In addition it has two orientifold surfaces at $x^9 = 0, \pi R$ where cross cap of the $RP^2$ geometry lies. The $n$-point amplitude of the disk diagram and that of the $RP^2$, which are next leading to the spherical geometry in string perturbation theory, depend on these locations respectively. These facts can be easily understood by the notion of boundary states in the closed string sector. (See Appendix A of this paper and \cite{3}.)

It has been observed in ref. \cite{4} that this model has nonvanishing tadpoles localized in spacetime. This, combined with dynamics captured partly by the low energy spacetime action \cite{4, 5}, exhibits breakdown of perturbation theory, avoiding an immediate conflict with the heterotic- type $I$ duality.

There are two issues on tadpoles which are related but which are found in this paper to be handled somewhat separately. Firstly, the conventional string lore tells us that nonvanishing tadpoles would in general introduce infinities in the multiparticle amplitudes. We must, therefore, examine how the cancellation of $SO(32)$ type $I$ theory stated in \cite{3} and established by explicit computation in \cite{7} may remedy the situation. In the next section, the $n$-point amplitude for the disk and the $RP^2$ diagrams is computed in the original flat background of the type $I'$ model defined above. We find that the potential infinities appear only in the momentum conserving part and the infinity cancellation of the Disk and the $RP^2$ diagrams in type $I SO(32)$ theory renders the type $I'$ case finite as well. This is the case despite the presence of the dilaton source localized at the position of the D-brane and that of the orientifold surfaces.

Secondly, it is well known that tadpoles are considered to be the source of vacuum instability and can be removed by shifting background geometry. This is, of course, the spirit of the mechanism \cite{8} illustrated by the bosonic string in \cite{8}. We find that a version of this mechanism can be implemented at the level of string amplitudes within the first quantized framework. In section 3, we present formalism of string models based on the generating func-

\footnote{2 The original mechanism has been discussed in the light-cone string field theory and the $\sigma$ model approach.}
tional $S$ for the $S$-matrix elements $[3]$. This is used to determine the background geometry as $c$-number source function by demanding the cancellation of tadpoles. We also check that this background does not introduce new short distance singularity of world sheet through self-contractions.

Expansion of $S$ around this background automatically produces the representation of the string amplitudes to all orders in $\alpha'$. This is, however, found to be the linearized approximation to the full nonlinear theory whose low energy behavior (leading in $\alpha'$) is captured by the spacetime action of $[3, 4]$. This and related aspects are discussed in section 4.

In Appendix $A$, we present the disk and the $RP^2$ geometries of type $I'$ theory as boundary states. In appendix $B$, we briefly discuss the bosonic example discussed in $[8]$ in our formalism.

It will be appropriate to introduce some of our notation here. We denote by \( \prod_{I=1}^{n} \zeta_{M_i N_i}^{M_i N_i} \) \( \tilde{A}_{M_1 N_1, \ldots, M_n N_n}^{(n)}(k_I^{M_i}) \) an $n$-particle amplitude for bosonic massless states of closed string with polarization tensor $\zeta_{M_i N_i}$ and momentum $k_I^{M_i}$ $I = 1 \sim n$. The contribution from the worldsheet geometry $M$ is indicated by the subscript $M$. The well-known formula reads

\[
\left( \prod_{I=1}^{n} \zeta_{M_i N_i}^{M_i N_i} \right) \tilde{A}_{M_1 N_1, \ldots, M_n N_n}^{(n)}(k_I^{M_i}) \\
\equiv \int_{(moduli)} d(WP)^M \frac{(det' P_1^i P_1^j)^{1/2}}{d(det' P_1^i P_1^j)^{1/2} vol(Ker P_1)} \left( \prod_{I=1}^{n} V(k_I^{M_i}, \zeta_{M_i N_i}^{M_i N_i}) \right)_{M} .
\]

(1.1)

Here the integral in front is the standard integrations over the moduli space of the Riemann surface $M$ and $V(k_I^{M_i}, \zeta_{M_i N_i}^{M_i N_i})$ denotes the vertex operator of the particle under question. We will not elaborate upon the notation on this formula here. (See $[10]$.) $< \ldots >_M$ means the functional averaging with respect to the action:

\[
S[X] = \frac{1}{2\pi \alpha'} \int_{M} d^2z d\theta d\bar{\theta} \frac{1}{2} \hat{E} \tilde{A}_X + S_{ct}
\]

\[
S_{ct} = \mu^2 \int_{M} d^2z \sqrt{\hat{g}} + \frac{\ln \kappa}{4\pi} \int_{M} d^2z \sqrt{\hat{g}} R^{(2)}
\]

(1.2)

The second term of $S_{ct}$ of course produces $(c\kappa)^{-\chi(M)}$. Here we take the loop counting factor $\kappa$ to be proportional to the string coupling $\kappa$.

In section 3, we find it more convenient to convert the expression for the amplitudes into position space $A_{M_1 N_1, \ldots, M_n N_n}^{(n)}(x_I)$ via Fourier transform albeit being formal:

\[
A_{M_1 N_1, \ldots, M_n N_n}^{(n)}(x_1, \ldots x_n) \equiv \int \ldots \int \prod_{I=1}^{n} \frac{d^Dk_I}{(2\pi)^D} e^{-ik_I \cdot x_I} \tilde{A}_{M_1 N_1, \ldots, M_n N_n}^{(n)}(k_I) .
\]

(1.3)

\[ \text{The proportionality constant $c$ will be set equal to 1 as this does not spoil the essence of our discussion.} \]
The cancellation of the tadpoles against the background is stated as
\[
\frac{\delta}{\delta j_{j}^{MN}(x)} \left( S[j_{j}^{MN}] \right)_{j_{j}^{MN} = \kappa_{j}} = 0 .
\] (1.4)

Here \( j_{j}^{MN}(x) \) is the source function we introduce and \( \kappa_{j} \) is the background geometry we find. The string amplitudes on this background is obtained by simply expanding \( S[j_{j}^{MN}] \) around this point.

We collect some of the earlier and the recent references on orientifolds and scattering off \( D \)-brane in \[11\] and in \[12, 13, 14\].

II. Type \( I' \) Disk and \( RP^2 \) Amplitudes in the Original Background and Infinity Cancellation

Recall that the graviton/dilaton vertex operator in Type \( I' \) and therefore Type \( I' \) theory is
\[
V(k^M, \zeta^{MN}) = \kappa \int_{M} d^{2}zd\theta d\bar{\theta} \zeta^{MN} \int d\bar{\eta}_{N} d\eta_{M} \exp \left( i(k - i\eta D - i\bar{\eta} \bar{D}) \cdot X + \frac{\alpha'}{4} \eta \cdot \bar{\eta} \theta R^{(2)} \right) .
\] (2.1)

The second term in the exponent is due to the anomalous contraction of the dilaton vertex and produces the correct coupling to the two-dimensional curvature \( R^{(2)}(z, \bar{z}) \)[15, 16].

We begin with representing \( \tilde{A}(n)_{\text{disk}/RP^2 M_i N_i \cdots N_n} (k_I^M) \) by the disk and \( RP^2 \) boundary states which we denote respectively by \( < B | \) and \( < C; R | \). The construction of these states is given in Appendix A. Write \( \tilde{A}(n)_{\text{disk}/RP^2 M_i N_i \cdots N_n} (k_I^M) \) as
\[
\prod_{J=1}^{n} \varepsilon_{J}^{M} \varepsilon_{J}^{N} \tilde{A}(n)_{\text{disk}/RP^2 M_i N_i \cdots N_n} (k_I^M) \equiv \frac{C'_{\text{disk}/RP^2}}{V_{SKV, \text{disk}/RP^2}} \langle B/C; R | \prod_{I=1}^{n} V(k^M_I, \zeta^{MN}) | 0 \rangle .
\] (2.2)

Let us first evaluate the zero mode part. In obvious notation, we obtain
\[
_{\text{zero}} \langle B | \prod_{I=1}^{n} e^{ik_I \cdot \hat{x}_{\text{zero}}} | 0 \rangle \equiv \prod_{\mu=0}^{8} \otimes_{\mu} \langle B; p^{\mu} = 0 | e^{\sum_{I=1}^{n} k^{\mu}_{I} x_{\mu}} | p^{\mu} = 0 \rangle \otimes_{\nu} \langle B; x_{\nu} = x_{\nu}^{9} = 0 | e^{\sum_{I=1}^{n} k^{\nu}_{I} x_{\nu}} | p^{\nu} = 0 \rangle = \left( \prod_{\mu=0}^{8} \delta \left( \sum_{I=1}^{n} k_{I}^{\mu} \right) \right) ,
\] (2.3)
where \( x_B^9 = 0 \) is the location of the D-brane world-volume. Similarly,

\[
\langle C; R | \prod_{I=1}^{n} e^{ik_I \cdot \tilde{x}_{zero}} | 0 \rangle 
= \prod_{\mu=0}^{8} \otimes \mu \langle C; p^\mu = 0 | e^{i \sum_{I=1}^{n} k_I^\mu \tilde{x}_9} | p^\mu = 0 \rangle 
\otimes \frac{1}{2} \left( g \langle C; x^9 = x_C^9 = 0 \rangle + g \langle C; x^9 = \tilde{x}_C^9 = \pi R | e^{i \sum_{I=1}^{n} k_I^9 \tilde{x}_9} \rangle \right) 
= \left( \prod_{\mu=0}^{8} \delta(\sum_{I=1}^{n} k_I^\mu) \right) \left( \frac{1}{2} \right) \left( 1 + e^{i \pi R \sum_{I=1}^{n} k_I^9} \right),
\]

where \( x_C^9 = 0 \) and \( \tilde{x}_C^9 = \pi R \) are the location of the orientifold surfaces. The boundary states \( \langle B | \langle C; R \rangle \) are eigenstates of the total momenta for \( M = \mu = 0 \sim 8 \) and those of the center of mass coordinate of string for \( M = 9 \). We find

\[
\tilde{A}^{(n)}_{disk/RP^2 M_1 N_1 \cdots M_n N_n} (k_1^M \cdots k_n^M) = \left( \prod_{\mu=0}^{8} \delta(\sum_{I=1}^{n} k_I^\mu) \right) 
\times \begin{cases} 
1 & \text{for } B \text{ or } \frac{1}{2} \left( 1 + e^{i \pi R \sum_{I=1}^{n} k_I^9} \right) & \text{for } C 
\end{cases}
\tilde{A}^{(n)}_{disk/RP^2 M_1 N_1 \cdots M_n N_n} (k_I^M)
\]

where

\[
\tilde{A}^{(n)}_{disk/RP^2 M_1 N_1 \cdots M_n N_n} (k_I^M) = \frac{C'_{disk/RP^2 \hat{r}^{n-1}}}{V_{SKV, disk/RP^2}} \prod_{I'=1}^{n} \int d^2 z_{I'} d\theta_{I'} d\bar{\theta}_{I'} \int d\tilde{\eta}_{I'} \tilde{\eta}_{I'} \tilde{\eta}_{M_{I'}}
\times \exp \left[ \pi \hat{\alpha}' \sum_{I,J}^{n} (k_I - i \eta_I D - i \bar{\eta}_I \bar{D})_M (k_J - i \eta_J D - i \bar{\eta}_J \bar{D})_N G_{disk/RP^2}^{MN}(I, J) 
- \frac{\hat{\alpha}'}{4} \sum_{I}^{n} \eta_I \bar{\eta}_I \eta_{I'} \bar{\eta}_{I'} \sqrt{\tilde{g} R^{(2)}} \right],
\]

where \( C' = C [\det' \Delta]^{-D/2} [\det' D]^{D/2} \) with \( C \equiv [\det' P_1^I P_1^I]^{1/2} [\det' P_1^{I/2} P_1^{I/2}]^{-1/2}/\tilde{d} \). \( \tilde{d} \) is the order of the group of diffeomorphism classes and \( \tilde{d}_{disk} = 2, \tilde{d}_{RP^2} = 1 \). The volume of the superconformal killing vector is denoted by \( V_{SKV, disk/RP^2} \). See \[4, 17\] for more detail.

It was noticed in ref. \[6\] that, in order to seek for a cancellation of infinities in the multiparticle amplitudes of \( SO(32) \) type \( I \) theory, we have to fix the odd elements of the
invariance group. This is needed as we would like to parametrize the integrand in terms of the superspace distance. In the operator language, this corresponds to $F_2$ picture. The leading divergence is then of the form $\int \frac{d\lambda}{\lambda} \cdots$ and we can address the question of finiteness via the principal value prescription. As in ref. [7], we fix the graded extension of $SU(1,1)/SU(2)$ symmetry of the integrand by setting $z_1$ and arg $z_2$ and $\theta_1$ zero. This offsets the volume of the superconformal killing vectors. The formula is then regarded as the supersymmetric extension of the Koba-Nielsen formula. In evaluating this expression, we allow on-shell condition as well as the transversality to put this in a preferable form.

The Green function $G_{M,N}(I,J) \equiv G_{M,N}(z_I, \bar{z}_I, \theta_I, \bar{\theta}_I; z_J, \bar{z}_J, \theta_J, \bar{\theta}_J)$ can be represented as

$$G_{M,N}^{\text{disk}/RP^2}(I,J) = G_{M,N}^{\text{sphere}}(I,J) + G_{im, \text{disk}/RP^2}^{MN}(I,J)$$

$$G_{M,N}^{\text{im, disk}}(I,J) = \frac{1}{2\pi} \ln |z_I - z_J + i\theta_I \theta_J|.$$

(2.7)

The Green function $G_{M,N}^{\text{disk}/RP^2}(I,J)$ can be represented as

$$G_{M,N}^{\text{im, disk}}(I,J) = \frac{1}{2\pi} \ln |z_I - z_J + i\theta_I \theta_J|.$$

(2.7)

The sign ambiguity denoted by $\pm$ disappears in the final expression. It is straightforward to evaluate the disk and $RP^2$ tadpoles from the formulas above. In position space, they read

$$A^{(n=1)\chi=1}_{MN}(x^M) = NA_{disk}^{(n=1)MN}(x^M) + A_{RP^2}^{(n=1)MN}(x^M), \quad N = 32,$$

$$A_{disk}^{(n=1)MN}(x^M) = C'_{disk} \frac{r_{disk}}{\pi} \frac{\alpha'}{2 (2\pi)^9} \delta(x^9 - x^9_B) (\eta_{\mu\nu} \pm -\eta_{99})$$

$$A_{RP^2}^{(n=1)MN}(x^M) = -C'_{RP^2} \frac{r_{RP^2}}{\pi} \frac{\alpha'}{2 (2\pi)^9} \frac{1}{2} \left\{ \delta(x^9 - x^9_B) + \delta(x^9 - x^9_C) \right\}$$

$$\left(\eta_{\mu\nu} \pm -\eta_{99}\right).$$

(2.8)

The constants $r_{disk}$ and $r_{RP^2}$ are defined in [1], [7] and $r_{disk}/r_{RP^2} = 2$. We also quote

$$C'_{disk}/C'_{RP^2} = 2^{-(D/2)-1} = 2^{-6}.$$

(2.9)

The qualitative feature of eq. (2.8) is as is given in ref. [1]. Eq. (2.8) may be associated with the process in which a dilaton/graviton located at $x^M$ gets absorbed into the vacuum. It is nonvanishing only when dilaton/graviton is located in the eight-brane world-volume or the orientifold surfaces.

Let us now turn to the issue of the infinity cancellation. First rescale the worldsheet Grassmann variables as $\theta_I = \lambda^{1/4} \tilde{\theta}_I$ in addition to $z_I = \sqrt{\lambda} w_I$, $I \geq 2$, $w_2 = 1$, so that

\footnote{Here we ignore a possible subtlety associated with the compactness of $V_{SKV,RP^2}$.}
\[
d\theta_I = \lambda^{-1/4} d\tilde{\theta}_I, \quad \frac{\partial}{\partial \theta_I} = \lambda^{-1/4} \frac{\partial}{\partial \tilde{\theta}_I}, \quad D_I = \lambda^{-1/4} \tilde{D}_I. \quad \text{Next, let } \eta_I = \lambda^{1/4} \tilde{\eta}_I \text{ so that } \eta_I D_I = \tilde{\eta}_I \tilde{D}_I \quad \text{and } \quad d\eta_I = \lambda^{-1/4} d\tilde{\eta}_I. \quad \text{After these rescalings, } \tilde{A}_n \text{ is expressible as}
\]

\[
\zeta^{(n)}_{\tilde{A}_{\text{disk}}/\text{RP}^2, M_1 N_1 \ldots M_n N_n}(k_1^M \ldots k_n^M) = \left( C'_{\text{disk}/\text{RP}^2} \right)^{n-1} r_{\text{disk}/\text{RP}^2} \int_0^1 d\lambda \left( \frac{\lambda^{-3/2} + \frac{\pi}{4} (\sum I=1^n k_I^2)^2}{\lambda} \right)
\]

\[
\times \int \prod_{I'=3} d^2 w_I' \int \prod_{J'=2} d\tilde{\theta}_{I'} d\theta_{J'} \int \prod_{K'=1} d\tilde{\eta}_{K'} N_{K'} d\tilde{\eta}_{K'} M_{K'}
\]

\[
\times \exp \left[ \pi \alpha' \sum_{I,J} (k_I - i\tilde{\eta}_I \tilde{D} - i\tilde{\eta}_J \tilde{D}) M_{k, j} - i\tilde{\eta}_J \tilde{D} - i\tilde{\eta}_J \tilde{D})_N G_{\text{sphere}}^M (I, J) \right]
\]

\[
- \frac{\pi}{4} \sum_{I} \tilde{\eta}_I \tilde{\eta}_I \theta_{i} \bar{\theta}_{i} \sqrt{g R^{(2)}}
\]

\[
\times \exp \left[ \pi \alpha' \sum_{I,J} (k_I - i\tilde{\eta}_I \tilde{D} - i\tilde{\eta}_J \tilde{D}) M_{k, j} - i\tilde{\eta}_J \tilde{D} - i\tilde{\eta}_J \tilde{D})_N G_{\text{im}, \text{disk}}^M (I, J; \lambda) \right] \bigg|_{w_1 = 0, w_2 = 1, \tilde{\theta}_1 = 0}
\]

\[
(2.10)
\]

The factor \( \lambda^{-1/4} \) comes from the part in \( G_{\text{sphere}}^M (I, J) \) which is rescaled to give \( \eta_{MN}^{\frac{1}{2} \ln \lambda} \). As there is no momentum conservation in the ninth direction, this produces \( \pi \alpha' \sum_{I,J} (k_I - i\tilde{\eta}_I \tilde{D} - i\tilde{\eta}_J \tilde{D}) M_{k, j} - i\tilde{\eta}_J \tilde{D} - i\tilde{\eta}_J \tilde{D})^M \frac{1}{4\pi} \ln \lambda = \frac{\alpha'}{4} (\sum I=1^n k_I^2)^2 \ln \lambda. \)

Let us discuss the Grassmann integrations over \( \tilde{\theta}, \tilde{\eta} \) (analytic variables) and \( \bar{\theta}, \bar{\eta} \) (antianalytic variables). The leading divergence comes from the case in which one picks up as many as possible terms from the first exponent containing \( G_{\text{sphere}}^M (I, J) \). After doing this, one is left with one analytic variable and one antianalytic variable, which need to be saturated by a term from the second exponent. This term contains \( \lambda^{1/2} \). We can therefore write eq. (2.10) as

\[
\zeta^{(n)}_{\tilde{A}_{\text{disk}} M_1 N_1 \ldots M_n N_n}(k_1^M \ldots k_n^M) = C'_{\text{disk}} K^{n-1} r_{\text{disk}} \left( \int_0^1 d\lambda \frac{1}{4\pi} \lambda^{-\frac{3}{2} + \frac{\pi}{4} (\sum I=1^n k_I^2)^2} \right)
\]

\[
F_{M_1 N_1 \ldots M_n N_n}^{(n)} (\lambda; k_1^M \ldots k_n^M) + \text{ higher order terms in } \lambda
\]
Here $F_{M_1 N_1 \cdots M_n N_n}^{(n)}$ is some function regular and nonvanishing at $\lambda = 0$. Similarly, we find for $RP^2$

$$\tilde{A}_{RP^2 M_1 N_1 \cdots M_n N_n}^{(n)}(k_1^M \cdots k_n^M) = C'_{RP^2} \kappa_{n}^{-1} r_{RP^2} \left( \int_0^1 d\lambda \lambda^{1 + \frac{2}{n}(\sum_{l=1}^{n} k_l^M)^2} F_{M_1 N_1 \cdots M_n N_n}^{(n)} (-\lambda; k_1^M \cdots k_n^M) + \text{higher order terms in } \lambda \right).$$

We find that the individual amplitude $\left( \prod_{I=1}^{n} \zeta_{M_I N_I}^{M_I} \right) \tilde{A}_{disk/\mathcal{N}}^{(n)}_{M_1 N_1 \cdots M_n N_n} (k_I^{M_I})$ is infinite only when the sum $\sum_{I=1}^{n} k_I^M$ vanishes. But in this region, the dependence on $x_C = \pi R$ coming from the zero mode integrations (see eq. (2.12)) becomes irrelevant. The original infinity cancellation of $SO(32)$ type $I$ theory persists under type $I'$ compactification. We can, therefore, write as

$$N \left( \prod_{I=1}^{n} \zeta_{M_I N_I}^{M_I} \right) \tilde{A}_{disk}^{(n)}_{M_1 N_1 \cdots M_n N_n} (k_I^{M_I}) + \left( \prod_{I=1}^{n} \zeta_{M_I N_I}^{M_I} \right) \tilde{A}_{RP^2 M_1 N_1 \cdots M_n N_n}^{(n)} (k_I^{M_I}) = C'_{disk} \kappa_{n}^{-1} r_{disk} \left[ N \int_0^1 \frac{d\lambda}{\lambda} F_n(\lambda) - 32 \int_0^1 \frac{d\lambda}{\lambda} F_n(-\lambda) + \text{finite terms} \right],$$

where $N = 32$, $F_n(\lambda)$ is some function nonvanishing and regular at $\lambda = 0$ and the factor 32 is accounted for by $(r_{disk}/r_{RP^2}) \cdot (C'_{disk}/C'_{RP^2}) = 2^{-5}$. Tadpoles are nonvanishing locally but, for infinity cancellation, it is sufficient for them to cancel in the whole space.

### III. String S-Matrix Functional and Consistent Background

We have seen that theory is finite despite the presence of the localized tadpole sources. The presence of nonvanishing tadpoles itself, however, implies instability of the vacuum we have chosen to work with, namely, the geometry of flat spacetime. This is supported by the spacetime action of [3, 4].

A nontrivial background must be found which is consistent with the propagation of strings and which offsets tadpoles in the original background. This shift of background can be discussed within the first quantized framework.
Let us first introduce the $S$ matrix generating functional:

$$S[j^{MN}] = \sum_{\text{topologies}} S_M[j^{MN}], \quad (3.1)$$

where

$$S_M[j^{MN}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \cdots \int d^Dx_1 \left( \prod_{i=1}^{n} j^{M_jN_j(x_J)} \right) A^{(n)}_{M_1N_1\cdots M_nN_n}(x_1 \cdots x_n). \quad (3.2)$$

The indices $(M, N, \cdots)$ run $0 \sim D-1$ with $D = 10$. Here $j^{MN}(x)$ is a source function which we introduce in place of a polarization tensor, which is an external wave function. Its Fourier transform is denoted by

$$\tilde{j}^{MN}(k) \equiv \int d^Dxe^{-ik \cdot x} j^{MN}(x). \quad (3.3)$$

The spherical topology has vanishing $n$-point amplitudes for $n \leq 2$. For our purpose, we find it necessary to add by hand to the original expression the two point amplitude

$$\tilde{A}_{\text{sphere} M_1N_1M_2N_2}^{(2)}(k_I) = \eta_{M_1N_1} \eta_{M_2N_2} k_I^2, \quad (3.4)$$

which is of course the inverse propagator and vanishes on-shell. This is, however, an important ingredient to our discussion.

From eqs. (3.2) and (2.1), we find

$$S_M[j^{MN}] = \int_{(\text{moduli})_M} d(WP)_M \frac{(\det' P_1^I P_1^I)^{1/2}}{d(\det' P_1^I P_1^I)^{1/2} vol(\ker P_1)} \times \left\{ \exp \left[ \kappa \int_{\mathcal{M}} d^2z \bar{d} \bar{\theta} \bar{\theta} \bar{E}_{j^{MN}}(X(z, \bar{z}, \theta, \bar{\theta})) \bar{D}X^N D\bar{X}^M \right. \right.$$  

$$\left. + \frac{\alpha' \kappa}{4} \int_{\mathcal{M}} d^2z \sqrt{g} j_{M}^{N}(X(z, \bar{z})) R^{(2)} \right\}_{\mathcal{M}}, \quad (3.5)$$

where

$$j^{MN}(X) \equiv \int \frac{d^Dk}{(2\pi)^D} \exp(ik \cdot X) \int d^Dxe^{-ik \cdot x} j^{MN}(x). \quad (3.6)$$

See [10] for the rest of the notation in eq. (3.5). The salient feature of eq. (3.5) is that the dynamical variable $X$ ends up with appearing in the argument of $j^{MN}$. That $X$ appears only through the exponential operator is in accordance with the paradigm of conformal field theory. The second term in the exponent originates from the curvature term in the dilaton vertex operator of eq. (2.1). This identifies $j_{M}^{N}(X(z, \bar{z}))$ with dilaton field. On-shellness can be implemented by inserting the on-shell delta function $\delta(k^2)$ in $j^{MN}(X)$:

$$j^{MN}_{(\text{mod})}(X) \equiv \int \frac{d^Dk}{(2\pi)^D} \delta(k^2) \exp(ik \cdot X) \int d^Dxe^{-ik \cdot x} j^{MN}(x). \quad (3.7)$$
Expansion around nonzero value of $j^{MN}$ is regarded as the one around the nontrivial string background: the expression inside $< \cdots >$ agrees with the standard $\sigma$ model expression.

The cancellation of the tadpole amplitude of the $SO(32)$ type I superstring up to $-\chi(M) \leq -1$ is stated as

$$\frac{\delta}{\delta j^{MN}(x)} \left( S_{\text{sphere}}[j^{MN}] + S_{\text{disk}}[j^{MN}] + S_{\text{RP}^2}[j^{MN}] \right) \bigg|_{j^{MN}=0} = 0 \ .$$

(3.8)

This implies that the flat ten-dimensional Minkowski space is perturbatively stable for $SO(32)$.

As stated in introduction, we are concerned with equation\footnote{We are assuming power counting by $\kappa$, so that the consideration here is genus by genus argument.}

$$\frac{\delta}{\delta j^{MN}(x)} \left( S_{\text{sphere}}[j^{MN}] + S_{\text{disk}}[j^{MN}] + S_{\text{RP}^2}[j^{MN}] \right) \bigg|_{j^{MN}(x)=\kappa j^{MN}(x)} = 0 \ .$$

(3.9)

In position space, eq. (3.9) is

$$\frac{\partial}{\partial x_L} \frac{\partial}{\partial x^M} j^{MN}(x^M) = A_{\text{disk}}^{MN}(x^M) + A_{\text{RP}^2}^{MN}(x^M) \ .$$

(3.10)

In momentum space, it reads as

$$k^2 \tilde{j}^{MN}(k) + \tilde{A}_{\text{disk}}^{MN}(k) + \tilde{A}_{\text{RP}^2}^{MN}(k) = 0 \ .$$

(3.11)

Note that we are extending, albeit minimally, the expression off-shell. In order to check the validity of this recipe, we consider in Appendix B the example \footnote{We are assuming power counting by $\kappa$, so that the consideration here is genus by genus argument.} of the bosonic string, where we reproduce the argument of \footnote{We are assuming power counting by $\kappa$, so that the consideration here is genus by genus argument.} for the cancellation of the torus tadpole by the background on spherical topology.

The solution to eq. (3.10) is

$$j^{MN}(x^9) = \alpha a_1 (\eta_{\mu\nu} \mp \eta_{99}) \{ x^9 \Theta(x^9) - (x^9 - \pi R) \Theta(x^9 - \pi R) \} \ ,$$

(3.12)

where $a_1 = C'_{\text{disk}} \frac{r_{\text{disk}}}{\pi} \frac{1}{(2\pi)^9}$ and $\Theta(x^9)$ is the step function. In momentum space,

$$\tilde{j}^{MN}(k^9) = -\alpha' a_1 (\eta_{\mu\nu} \mp \eta_{99}) \frac{1}{(k^9)^2} \left[ 1 - e^{\pm i\pi k^9 R} \right] \left( \prod_{\mu=0}^8 \delta(k^\mu) \right) \ .$$

(3.13)

Let us finally check that this background determined does not create any new short distance singularity off-shell which may arise from self-contractions of the operators.
is necessary to keep consistency with the conclusion of section 2. The \( n \)-point amplitude on this background is expressed as

\[
F.T. \frac{\delta^n}{\delta^2 j^{M_1 N_1}(x_1) \cdots \delta^2 j^{M_n N_n}(x_n)} \left( S[j^{MN}] \right)_{j^{MN} = \kappa j_{cl}},
\]

where \( F.T. \) denotes Fourier transforms over \( x_1, \ldots, x_n \). This contains the original sphere \( n \)-point amplitude of the flat spacetime at \( \kappa^{n-2} \). At \( \kappa^{n-1} \), it contains the disk and the \( RP^2 \) \( n \)-point of the flat spacetime and the sphere \( (n+1) \)-point where one of the spacetime arguments is integrated together with this background. We are concerned with this latter quantity which reads

\[
\begin{align*}
F.T. & \int d^{10} x_{n+1} A^{(n+1)}_{\text{sphere } M_1 N_1 \cdots M_{n+1} N_{n+1}}(x^M_1, \ldots, x^M_{n+1}) j^{M_{n+1} N_{n+1}}(x^M_{n+1}) \\
&= \int \left( d^9 k_{n+1} \sum_{k_{n+1}^9} \delta^{M_{n+1} N_{n+1}}(k_{n+1}) \left( \prod_{M=0}^9 \delta(\sum_{I=1}^{n+1} k_I^M) \right) \times \tilde{A}_{\text{sphere } M_1 N_1 \cdots M_{n+1} N_{n+1}}(k^M_1, \ldots, k^M_{n+1}) \right). \quad (3.15)
\end{align*}
\]

Carrying out the momentum integrations and the sum for \( k^M_{n+1} \), we find

\[
= -\alpha' a_1 (\eta^{\mu_{n+1} \nu_{n+1}} \oplus -\eta^{99}) \frac{1}{\left( \sum_{I=1}^{n} k_I^9 \right)^2} \left[ 1 - e^{-i\pi \left( \sum_{I=1}^{n} k_I^9 \right)} R \right] \times \left( \prod_{\mu=0}^{8} \delta(\sum_{I=1}^{n} k_I^\mu) \right) \tilde{A}_{\text{sphere } M_1 N_1 \cdots M_{n+1} N_{n+1}}(k^M_1, \ldots, k^M_{n+1}) = -\delta^{M,9} \sum_{I=1}^{n} k_I^9). \quad (3.16)
\]

There are now potential infinities coming from short distance singularity of the worldsheet. This is because the \( k^M_{n+1} \) is not put on shell. The divergent part of \( \tilde{A}_{\text{sphere } M_1 N_1 \cdots M_{n+1} N_{n+1}} \) multiplied by \( 1/(\sum_{I=1}^{n} k_I^9)^2 \) is proportional to

\[
\begin{align*}
\frac{1}{(\sum_{I=1}^{n} k_I^9)^2} & \exp \left( \pi \alpha' \sum_{I=1}^{n+1} k_I : k_I \frac{1}{2\pi} \ln \epsilon \right) \\
&= \frac{1}{(\sum_{I=1}^{n} k_I^9)^2} \epsilon^{\alpha'} \left( \sum_{I=1}^{n} k_I^9 \right)^2 = \alpha' \epsilon^{\alpha'} \frac{4}{\sum_{I=1}^{n} k_I^9} \int_0^\epsilon d\lambda \lambda^{-1 + \alpha' / 4} \left( \sum_{I=1}^{n} k_I^9 \right)^2. \quad (3.17)
\end{align*}
\]
Here $\epsilon$ is an invariant short distance cutoff. This expression is the same as the one seen in the last section and is singular as $\epsilon \to 0$ only when

$$\sum_{l=1}^{n} k_{l}^9 = 0 .$$

(3.18)

This regime, however, is killed by the prefactor $\left(1 - e^{-i\pi \left(\sum_{l=1}^{n} k_{l}^9 \right)}\right)$ seen in eq. (3.16) and this is what we wanted to show.

**IV. Discussion**

The cancellation of infinities and the removal of tadpoles are two basic constraints for consistent type I string vacua. In the case of type I', we find that these two are established separately without further condition. This tells us naturalness of introducing $D$-branes.

On the other hand, the nontrivial background we have just found in eq. (3.12) must be compared with the solution [4] from the low-energy spacetime action. This latter one reads

$$\frac{\partial}{\partial x^9} \frac{\partial}{\partial x^9} Z(x^9) = -24\sqrt{2} C \mu_8 \delta(x^9) ,$$

$$e^{\phi(x^9)} = Z(x^9)^{-5/6} , \quad \Omega(x^9) = CZ(x^9)^{-1/6} , \quad g_{MN} = \Omega \eta_{MN} ,$$

$$\mu_8 = (2\pi)^{-9/2}(\alpha')^{-5/2} . \quad (4.1)$$

We see that our solution from the string amplitudes is only a linearized approximation to eq. (4.1). To regain eq. (4.1), we will have to add infinite number of terms to $S$ which vanish on-shell $k^2 = 0$. This is of course consistent with the derivative self-interaction of the dilaton in the spacetime action [6].

The construction of string theory as is currently formulated provides a set of prescriptions to compute on-shell scattering amplitudes. The local Weyl invariance, which is a guiding principle of critical string theory, uses explicitly on-shellness. The point raised by eqs. (3.12) and (4.1), therefore, confronts ourselves to a limitation of our recipe to the string dynamics which we would like to formulate. The hope is that the issues discussed in this paper will, at the same time, navigate us somewhere, teaching what string theory ought to be.

\footnote{This is in contrast to the case of tachyon where the tree level string $S$ matrix functional in the zero-slope limit obeys the same non-linear equation as that of $\phi^3$ field theory. (See the third ref. of [3].)}
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Appendix A

The presence of D-brane as well as that of orientifold surfaces can be captured by the notion of boundary state. In type $I'$ theory with circle of radius $R$ in the ninth direction which is considered in the text, the disk and $RP^2$ diagrams are respectively associated with the boundary state $< B|$ and the boundary crosscap state $< C|$ in the closed string sector. We only need the bosonic sector here. They are defined by

$$< B| (X^9(z, \bar{z}) - 2\pi n R) | \tau = 0 = 0, \quad n \in \mathbb{Z}, \quad -\pi < \sigma < \pi,$$

$$< B| \frac{\partial}{\partial \tau} X^\mu(z, \bar{z}) | \tau = 0 = 0,$$  

(A.1)

$$< C| (X^9(z, \bar{z}) + X^9(-1/\bar{z}, -1/z) - 2\pi n R) | \tau = 0 = 0, \quad n \in \mathbb{Z}, \quad 0 < \sigma < \pi,$$

$$< C| (X^\mu(z, \bar{z}) - X^\mu(-1/\bar{z}, -1/z)) | \tau = 0 = 0.$$

(A.2)

where $z = e^{\tau + i\sigma}$. The mode expansion of closed string coordinate is as usual

$$X^M(z, \bar{z}) = X^M_R(z) + X^M_L(\bar{z})$$

(A.3)

where

$$X^M_R(z) = \frac{1}{2} \alpha^M_R - i \frac{\alpha'}{2} p^M_R \ln z + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \alpha^M_R \frac{z^{-n}}{n},$$

$$X^M_L(\bar{z}) = \frac{1}{2} \alpha^M_L - i \frac{\alpha'}{2} p^M_L \ln \bar{z} + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \tilde{\alpha}^M_L \frac{\bar{z}^{-n}}{n},$$

(A.4)

and $p^M = (p^M_R + p^M_L)/2$, $\ell^M = (p^M_R - p^M_L)/2$.

In terms of modes, eqs. (A.3) and (A.2) read

$$< B| \ell^9 = 0, < B|(x^9 - 2\pi n R) = 0, < B|(\alpha^9_n - \tilde{\alpha}^9_n) = 0,$$

$$< B|p^\mu = 0, < B|(\alpha^\mu_n + \tilde{\alpha}^\mu_n) = 0,$$

(A.5)

$$< C|\ell^9 = 0, < C|(x^9 - \pi n R) = 0, < C|(\alpha^9_n - (-1)^n \tilde{\alpha}^9_n) = 0,$$

$$< C|p^\mu = 0, < C|(\alpha^\mu_n + (-1)^n \tilde{\alpha}^\mu_n) = 0,$$

(A.6)

Note that, in contrast with type $I$ case, both $< B|$ and $< C|$ are eigenstates of the center of mass coordinate $x^9$ as opposed to the total momentum $p^9$. Their eigenvalues represent the position of D-brane and the orientifold surfaces respectively. Eqs.(A.3) and (A.6) are solved to give

$$\langle B \rangle = \text{zero} \langle B \rangle \otimes \text{oscil} \langle 0 \rangle \exp \left[ - \sum_{m > 0} \frac{1}{m} \alpha^\mu_m \tilde{\alpha}^\mu_m + \sum_{m > 0} \frac{1}{m} \alpha^9_m \tilde{\alpha}^9_m \right],$$
\[ \langle C \rangle = \text{zero} \langle C \rangle \otimes \text{osci} \langle 0 \rangle \exp \left[ - \sum_{m>0} \frac{(-1)^m}{m} \alpha_m \alpha_m^\dagger + \sum_{m>0} \frac{(-1)^m}{m} \alpha_9 \alpha_9^\dagger \right], \quad (A.7) \]

\[ \text{zero} \langle B/C \rangle \equiv \prod_{\mu=0}^{8} \otimes_{\mu} \langle B/C ; p^\mu = 0 \rangle \otimes 9 \langle B/C ; x^9 = x^9_{B/C} \rangle, \quad (A.8) \]

with \( x^9 = 0 \) and \( x_9 = 0, \pi R \).

The Green's functions used in the text are obtained by evaluating
\[ G_{\text{disk/RP}^2}^{MN}(z_I, \bar{z}_I, \theta_I, \bar{\theta}_I; z_J, \bar{z}_J, \theta_J, \bar{\theta}_J) = D_I D_I \langle \tilde{G}_{\text{disk/RP}^2}^{MN}(z_I, \bar{z}_I; z_J, \bar{z}_J) \rangle \]
where the bosonic part \( \tilde{G}_{\text{disk/RP}^2}^{MN} \) is given by
\[ (-2\pi \alpha') \tilde{G}_{\text{disk/RP}^2}^{MN} = \langle B/C | X^M(z_I, \bar{z}_I) X^N(z_J, \bar{z}_J) \rangle = \text{zero} \langle B/C | x^M x^N \rangle \]

**Appendix B**

The \( S \)-matrix functional of the closed bosonic string for the dilaton at tree and one-loop level reads as
\[ S_{\text{sphere}}[j] = -\frac{1}{2} \int d^D x j(x) \frac{\partial}{\partial x^M} \frac{\partial}{\partial x^M} j(x) \]
\[ + \sum_{n=0}^{\infty} \frac{1}{n!} \int \cdots \int d^D x_I \prod_{I=1}^{n} j(x_I) A^{(n)}_{\text{sphere}}(x_1 \cdots x_n), \quad (B.1) \]

\[ S_{\text{torus}}[j] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \cdots \int d^D x_I \prod_{I=1}^{n} j(x_I) A^{(n)}_{\text{torus}}(x_1 \cdots x_n), \quad (B.2) \]

Eq. (1.4) is
\[ \frac{\delta}{\delta j(x)} (S_{\text{sphere}}[j] + S_{\text{torus}}[j]) \bigg|_{j = \kappa_{\text{dil}} j_{cl}(x)} = 0, \quad (B.3) \]
which determines \( j_{cl}(x) \):
\[ -\kappa_{\text{dil}} \frac{\partial}{\partial x^M} \frac{\partial}{\partial x^M} j_{cl}(x) + A^{(1)}_{\text{torus}}(x) = 0 \]

The dilaton vertex operator is
\[ V(k; \hat{g}_{ab}) = \kappa_{\text{dil}} \int d^2 z \sqrt{\hat{g}} \{ g^{\hat{z} \hat{z}} \partial_{\hat{z}} X^M \partial_{\hat{z}} X^N \epsilon_{MN}^{\text{dil}} + \frac{\alpha'}{4} \epsilon_{M}^{\text{dil}} R^{(2)}(\hat{g}) \}. \quad (B.5) \]

The polarization tensor of the dilaton is \( \epsilon_{MN}^{\text{dil}} = (\eta_{MN} - k_M \bar{k}_N - k_N \bar{k}_M) / \sqrt{D - 2} \) where \( k \cdot k = \bar{k} \cdot \bar{k} = 0 \) and \( k \cdot \bar{k} = 1 \). The torus n-point amplitude is
\[ A_{\text{torus}}^{(n)}(k_1, \cdots, k_n) \]
\[\frac{1}{\text{Vol}(CKV)}(2\pi)^D \delta^{(D)} \left( \sum_{I=1}^{n} k_I \right) \int d^2 \tau \frac{1}{(\tau_2)^{14}} \prod_{m=1}^{n} \left| 1 - e^{2\pi i m \tau} \right|^{-48} e^{4\alpha' \tau_2} \]

\[
(k_{\text{dil}})^n \prod_{I=1}^{n} \int \sqrt{\tau_2^2} d^2 z_I \ e^{\text{dil} M_I N_I} \frac{\partial}{\partial \eta(I) N_I} \frac{\partial}{\partial \eta(I) M_I} \\
\exp \left[ \pi \alpha' \sum_{I,J}^{n} (k_I - \eta(I) \partial z_J - i \eta(I) \bar{\partial} z_J) \cdot (k_J - i \eta(J) \partial z_I - i \bar{\eta}(J) \bar{\partial} z_I) G_{\text{torus}}(z_I, z_J) \right] \] \quad \text{(B.6)}

where the Green's function on torus is given by

\[
G_{\text{torus}}(z_I, z_J) = \frac{1}{4\pi} \log \left| \frac{\theta(1 - z_I - z_J \tau)}{\theta(1 - \bar{z}_I - \bar{z}_J \tau)} \right|^2 + \frac{i}{2} \left\{ \frac{(z_I - z_J - \bar{z}_I + \bar{z}_J)^2}{\tau - \bar{\tau}} \right\} . \quad \text{(B.7)}
\]

From the above expression we find

\[
\tilde{A}_{\text{torus}}^{(1)}(k) = -(2\pi)^D \delta^{(D)}(k) \frac{\alpha'}{2\pi} \kappa_{\text{dil}} \epsilon \frac{M}{M} \int d^2 \tau \frac{1}{(\tau_2)^{14}} \prod_{m=1}^{n} \left| 1 - e^{2\pi i m \tau} \right|^{-48} e^{4\alpha' \tau_2} ,
\]

Fourier transform of which is constant:

\[
\tilde{A}_{\text{torus}}^{(1)}(x) = -\frac{\alpha'}{2\pi} \sqrt{D - 2} \kappa_{\text{dil}} \Lambda . \quad \text{(B.8)}
\]

Here

\[
\Lambda \equiv \int d^2 \tau \frac{1}{(\tau_2)^{14}} \prod_{m=1}^{n} \left| 1 - e^{2\pi i m \tau} \right|^{-48} e^{4\alpha' \tau_2} \quad \text{(B.9)}
\]

is the vacuum amplitude of torus [18]. The solution to eq. (B.4) is

\[
j_{\text{cl}}(x) = -\frac{\alpha'}{2\pi} (\sqrt{D - 2}) \Lambda \ x^M x_M \quad \text{(B.10)}
\]

to the order we are considering. The loop-corrected consistent background metric is

\[
G_{MN}(x^M) = \eta_{MN} \left( 1 - (\kappa_{\text{dil}})^2 \frac{\alpha'}{2\pi} (\sqrt{D - 2}) \Lambda \ x^M x_M \right) . \quad \text{(B.11)}
\]

This is in agreement with [18]. Although we will not discuss here, one can proceed to the cancellation of infinities in the \(n\)-point amplitude on this background which is given by

\[
F.T. \frac{\delta^n}{\delta j(x_1) \cdots \delta j(x_n)} (S_{\text{sphere}}[j] + S_{\text{torus}}[j]) \bigg|_{j = \kappa_{\text{dil}} j_{\text{cl}}(x)} . \quad \text{(B.12)}
\]
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