Reliability analysis of box-shaped steel column subjected to blast loading

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Abstract—Structural collapse caused by explosion and terrorist attacks often occurs. Steel columns, as the main load-bearing members of steel structures, often cause continuous structural collapse due to their damage. In order to study the reliability of steel column subjected to blast loading, box-shaped steel columns are taken as the research object, with considering the uncertainties associated with blast loading and material properties. Damage is determined according to damage index based on the ratio of the overall displacement and the local displacement to the maximum displacement. In this paper, P-I curves evaluation method and Monte-Carlo simulation method are utilized to obtain the failure probability and cumulative distribution functions of the damage index. The results show the importance of considering the uncertainties in assessing the damage index of the columns against explosion. Also, sensitivity analyses show that damage index has the highest sensitivity to peak reflected pressure.

1. INTRODUCTION

Nassr et al. [1] experimentally studied the dynamic response of wide flange steel beams subjected to blast loading. However these methods fail to consider the influence of uncertainties of the problem parameters including blast load and material properties. Therefore, reliability analyzing methods are better to consider uncertainty parameters and to research behavior.

Various studies have been conducted to determine the blast loading uncertainty for structural reliability analysis. Low and Hao [2] conducted a reliability analysis of reinforced concrete simply-supported beams under arbitrary explosive loads, and suggested the function relationship between the bending failure probability and the positive overpressure of explosive loads. Shi and Stewart [3] adopted Monte-Carlo simulation method and finite element analysis method to analyze the reliability of reinforced concrete columns and floors. However, there are relatively few studies on steel columns under blast loads using the reliability theory. In addition, the related research methods either require massive numerical simulations, leading to lower calculation efficiency, or use a single-degree-of-freedom system, which fails to consider local deformation.

Based on the Monte-Carlo method and the P-I damage curve evaluation method, with the randomness of explosion load and material properties taken into consideration, this paper studies the reliability of box-shaped columns under blast loads, calculates the cumulative distribution functions of the damage index of steel column, and proposes the calculation method for the failure probability. To
determine the amount of damage, the criterion of damage index is used based on the ratio of the overall displacement and the local displacement to the maximum displacement. The effect of probability of failure is investigated and impact of random variable volatility is performed to evaluate the reaction of the model.

2. NUMERICAL SIMULATION OF EXPLOSION AND P-I DAMAGE CURVE

2.1. Finite element model
This paper uses ABAQUS to build a finite element model of box-shaped steel columns. As shown in Figure 1, B refers to the cross-section width of a square steel column, t is the cross-section thickness, and H is the column height. The steel column is consolidated at the bottom, and its top constrains rotation and horizontal displacement. With the effect of initial bending taken into consideration, the model introduced an initial defect of 0.1% column height. In order to avoid unrealistic stress concentration at the end of the column, rigid blocks are set up at both ends of the column. The model uses shell elements and takes the grid size as 25mm. A vertical load of 20% Fp (the ultimate bearing capacity of the steel column) is applied to the top of the column to represent the vertical force transmitted by the superstructure to the column. At the same time, the blast load is assumed to be a triangular load that uniformly affects one side of the box-shaped column.

![Figure 1 Finite element model of steel column](image)

Jama et al. [4] carried out a series of field tests to study the deformation of 53 columns under different working conditions. In this section, the author takes the sample No. 14 in the test as the research object, and adopts the above finite element model to compare and analyze the results. The test delivers the overall and the local deformation of 0.012m and 0.036m, while the deformations calculated using numerical simulation are 0.014m and 0.040m, with the error of 16.7% and 11.1%, respectively. The results are in good agreement, indicating the effectiveness of adopting numerical analysis method in structural anti-blast analysis.

2.2. P-I curve evaluation method for steel column
The deformation at the height of 1/2 of the steel column is taken as the basis for determining the overall deformation damage of the box-shaped steel column [5]. The maximum deformation at 1/2 height of steel columns of different sizes can be calculated using the following formula:

\[
\Delta_{\text{max}} = \delta \cdot \frac{H}{2}
\]  

(1)

In the formula, H is the height of the column, and \(\delta\) is the rotation angle of the column end.

Due to the local deformation inward bending of load flange of the box-shaped steel column under the blast load, this paper takes \(u\), the vertical deformation at the highest point of the cross section, as the criterion for determining local deformation damage[6]. Its maximum deformation is:

\[
\Delta_{\text{max}} = u_{\text{max}} = \frac{a - 1}{1 - \cos a} \cdot \frac{B}{2}
\]

(2)
In the formula, B is the cross-sectional width of the box-shaped steel column, and α is the radian of the arc of bent side wall.

Based on the damage determination index, the global deformation and local deformation damage are expressed as:

$$D = \frac{\Delta}{\Delta_{\text{max}}}$$  \hspace{1cm} (3)

In the overall deformation damage, Δ is the deformation at a height of 1/2 of the steel column; in the local deformation damage, Δ is the vertical deformation at the highest point of the cross section of the column.

The pressure-impulse (P-I) curve used to evaluate the damage degree of the target under impact load. It has been widely used in the safety assessment of building structures and components under explosive loads. Shi [7] proposed the unified formula for P-I damage curves of component columns with different damage indices through massive simulation trials:

$$P - P_0 + I - I_0 = A \left( \frac{P_0 + I_0}{2} \right)^{\beta}$$  \hspace{1cm} (4)

In the formula, P0 and I0 are the overpressure asymptote and impulse asymptote of the P-I curve; A and β are parameters that control the curve shape, and are usually constant values.

After many numerical simulations, the author obtained the prediction formulas for the overpressure asymptotic line (P0) and the impulse asymptotic line (I0) of the global and local deformations with different damage indices of the P-I curve based on formula (4):

$$I_{g0}(D) = n_{x1} \cdot n_{z1}, \quad P_{g0}(D) = n_{x2} \cdot n_{z2}$$  \hspace{1cm} (5)

$$I_{l0}(D) = n_{x3} \cdot n_{z3}, \quad P_{l0}(D) = n_{x4} \cdot n_{z4}$$  \hspace{1cm} (6)

In the formula, Ig0 and Pg0 are the global deformation impulse and the overpressure asymptote, respectively; Il0 and Pl0 are the local deformation impulse and the overpressure asymptote. nξ and nλ are the influencing factors of the dimensionless size of the box-shaped column, and the fitting results are all quadratic polynomials. ξ and λ are the width-thickness ratio and slenderness ratio of column.

In order to verify the validity and accuracy of the prediction formula of the P-I damage curve, the paper lists the range of damage index and numerical simulations of the steel columns determined by the prediction formula under three groups of explosive loads, as shown in Table 1, so as to obtain the corresponding damage index of the steel column. As can be seen from the table, the assessment results obtained using the P-I curve is in good agreement with that using numerical simulation.

| Group | Explosive load | Range of damage index obtained using P-I damage curve | Numerically obtained damage index |
|-------|----------------|------------------------------------------------------|----------------------------------|
|       | Overpressure/MPa | Impulse/MPa*ms | Overall | Local | Overall | Local |
| 1     | 100             | 10          | 0.2~0.3 | 0.4~0.5 | 0.256   | 0.455 |
| 2     | 20              | 15          | 0.3~0.4 | 0.8~0.9 | 0.329   | 0.881 |
| 3     | 4               | 100         | 0.5~0.6 | 0.5~0.6 | 0.623   | 0.583 |
3. RANDOM VARIABLES

3.1. Statistical variables of blast load
Low [8] suggested that the explosion load parameters, such as \( P_r \) and \( t_d \), statistically obey the normal distribution based on the research on the uncertainty of the explosion load variables. Figure 2 and Figure 3 shows the best fitted curve of the average \( \mu \), the standard deviation \( \sigma \), and coefficient of variable for peak reflection overpressure \( P_r \) and positive overpressure duration \( t_d/W^{1/3} \) at different scaled distances \( Z \).

![Figure 2](image1)

**Figure 2** Average, standard deviation and coefficient of variation for peak reflection overpressure

![Figure 3](image2)

**Figure 3** Average, standard deviation and coefficient of variation for normalized positive overpressure duration

3.2. Random variables of box-shaped steel column
In reality, due to the differences in construction quality control and environmental conditions, parameters such as the size and material strength of box-shaped steel columns will inevitably show fluctuations. Table 2 lists the statistical characteristics of the random variables of the size of the steel column (B, H, t).

| Parameters of steel column | Average value | Coefficient of variation | Probability density          |
|----------------------------|---------------|--------------------------|------------------------------|
| B  | 350mm         | 0.03                    | Normal distribution          |
| H  | 3600mm        | 0.03                    | Normal distribution          |
| t  | 16mm          | 0.03                    | Normal distribution          |
4. RELIABILITY ANALYSIS

4.1. Reliability analysis method

This paper uses Monte-Carlo simulation and P-I damage curve method to conduct reliability analysis. The steps are as follows:

1) Based on the specific actual distance $R$ and explosive mass $W$, this paper uses the explosion load statistical model to fit the curve (Figure 2 and Figure 3) so as to obtain the average and standard deviation of $P_r$ and $I_d$.

2) Generate a random set of dimensional parameters for the box-shaped steel column, including section width ($B$), column height ($H$), and section thickness ($t$). The statistical characteristics of the box-shaped steel column are shown in Table II. Generate a random group of explosive loads based on the mean value and standard deviation obtained in 1).

3) Substitute the section size of the random steel column into the fitting formulas (5) and formulas (6) to obtain the overpressure asymptotic line ($P_0$) and the impulse asymptotic line ($I_0$) under different damage indices, and then substitute $P_0$ and $I_0$ into the P-I curve equation (4) to get the critical P-I damage curve, and then obtain the P-I damage curve graph;

4) Mark the random explosion load parameters $P_r$ and $I_d$ in the P-I damage curve graph, and then obtain the corresponding damage index $D$;

5) Repeat 2) ~ 4) for $n$ times, and then count the number of different damage indexes

6) Calculate the cumulative distribution functions and failure probability of different damage indexes (the judging criteria of failure is $D > 0.8$).

Broding et al. [9] performed 300 and 1000 times of iteration analyses to ensure the accuracy of Monte-Carlo numerical calculation of failure probability. They pointed out the similarity between the results of the two iterations, indicating that 300 times of iteration can ensure result accuracy. Therefore, the random sample number ($n$) can be taken as 300.

Figures 4 to 6 show the cumulative distribution functions curves of the steel column damage index under different explosion conditions. The slope of each curve segment represents the proportion of range of the damage index. When the explosive mass ($W$) remains the same and the damage index is within the range of 0-0.2, as the actual distance ($R$) increases, the slope of cumulative distribution functions curve also gradually increases; when the damage index falls between 0.8 and 1, the slope of the cumulative distribution functions curve gradually decreases. This shows that when the explosive mass remains the same, as the actual distance decreases, the probability of a box-shaped steel column will decrease when the damage index is lower, while that will increase when the damage index is higher, and the structural components tend to fail.

![Figure 4. W = 20kg cumulative distribution functions of damage index](image-url)
In order to investigate the impact of the scaled distance and explosive quality on the failure probability, this paper calculates the failure probability of the box-shaped steel column at different proportional distances and with different quality of explosives, as shown in Figure 7. When the explosive mass remains the same, as the proportional distance increases, the failure probability of the steel column
tends to decrease. When the proportional distance is less than 0.3m/kg$^{1/3}$, the failure probability of the steel column under any explosive mass conditions reaches 100%, which indicates that even a small amount of explosive in the case of a small proportional distance can lead to the column failure. On the other hand, with the increase of explosive mass, the failure probability of the column at the same proportional distance shows an increasing trend. This shows that the quality of explosives also has an important effect on the failure probability of the steel column.

4.3. Impact of random variable volatility

In order to evaluate the changing scale of the model response under the fluctuations of random variables, this paper compares the situations where Pr fluctuation is not considered, td fluctuation is not considered, section size fluctuation is not considered, and where variable fluctuation is considered, as shown in Figure 8.

It can be seen that size variable fluctuation has almost no effect on the cumulative damage probability. Peak overpressure (Pr) variable fluctuation has a great influence on the cumulative damage probability, and it can even change the shape of the curve. The cumulative distribution probability is concentrated in a certain damage area. In addition, td fluctuation leads to a slight decline of cumulative damage distribution probability, indicating that when the td fluctuation is taken into consideration, the probability at a lower damage index will decrease, while that at a higher damage index will increase, and the box-shaped steel column will be more easily damaged. Based on the above analysis, the reliability study on the box-shaped steel column under blast loads must consider the uncertainty of blast load so as to obtain the ideal calculation results.

5. CONCLUSION

This paper proposes a calculation method for evaluating the reliability of the box-shaped steel column under blast loads with the fluctuation of explosion load and structural size taken into consideration by adopting the P-I damage curve evaluation method and Monte Carlo simulation method. The conclusion is listed as follows:

1) Explosive mass and proportional distance are two important factors affecting the failure probability of structural components under explosive loads. When the explosive mass remains constant, the failure probability of the box-shaped steel column decreases as the proportional distance increases.

2) The impact of the blast load fluctuations, especially the peak reflection overpressure Pr, on the cumulative damage probability of components is far greater than that of component size. Therefore, the structural reliability analysis under blast load must take into consideration the fluctuation of blast load.

Figure 8 Cumulative distribution function with random variables taken or not taken into consideration
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