Constraint conditional finite element method for off-axial interfacial sliding of fiber reinforced composite

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Abstract. The main toughening mechanism of Ceramics Matrix Composite (CMC) is the fiber/matrix interfacial debonding and sliding. To simulate of the contact problem (such as interfacial sliding), penalty method etc have been used in general finite element method. But, these method need the repeated calculations, thus the CPU time is long and the accuracy depends on the number of repetitions. On the other hand, in actual CMCs, the fibers are not necessarily oriented along the loading direction, and the fiber diameter also fluctuates along the axis. Therefore, in this study, Constraint Conditional Finite Element Method (CC-FEM) was formulated to analyze the “off-axis interfacial sliding” with high precision and in a short time without repeated calculations. In CC-FEM, the equality of nodal displacements at the interface and the equilibrium of contact forces are assumed as constraint conditions, in which Coulomb's friction law and equivalent friction coefficient were taken into account. In addition, the validity was verified especially for “off-axis interfacial sliding” by comparing with general-purpose finite element software ANSYS. In the both cases of on- and off-axial interfacial sliding, the resultant stress distributions of the fiber and matrix agreed well with those of ANSYS. As compared to the case of on-axial interfacial sliding, the matrix stress recovered more steeply because of the higher equivalent frictional coefficient.

1. Introduction

Ceramics are excellent for their heat, abrasion and corrosion resistances, but their toughness and strength reliability are insufficient because of their brittle nature. On the other hand, fiber-reinforced ceramic matrix composites (CMCs) such as SiC fiber-reinforced SiC are damage-tolerant materials for which high toughness is achieved through damage mechanisms such as fiber bridging, fiber breakage, pullout, interfacial debonding followed by sliding, and crack deflection deriving from reinforcing materials such as ceramic fibers \cite{1}. From such excellent properties, CMC is anticipated for use as a high-temperature structural material applied for aerospace for which conventional metal are difficult to apply. Accordingly, the formulation for solving mechanical problems in this material is linked to deepened engineering significance.

In general, contact problems are solved through computational algorithms such as penalty method and enlarged Lagrange method, which are included in general finite element method (FEM) software. In these algorithms, stiffness between contact surfaces is expressed by contact spring. Computations are conducted repeatedly until the amounts of sliding and/or overlapping between nodal-points reach to a given allowable value. Therefore, the computation time and precision depend on the allowable value \cite{2}.
On the other hand, the constrained conditional finite element method (hereinafter, denoted as CC-FEM) is a mechanical modeling, in which the equivalence of nodal-point displacements and the equilibrium of contact forces are first given as contact conditions on the contact faces. Next, the global stiffness matrix is changed based on the given conditions, and solved numerically. According to this model, the exact numerical solution is obtainable by a single computation with no repetition. If this model is applied for the damage problems in CMCs, then even a complicated damage state including plural matrix cracks and fiber breaks could be solved by only once without divergence. In the previous papers, the formulation of CC-FEM was conducted in the case of on-axial interfacial debonding and sliding problem [1,3-5]. In actual CMCs, however, the fibers are often placed with an inclination along the loading direction; the fiber diameter varies along the fiber axis. Under such circumstances, the applicability of the CC-FEM is not adequate.

The purpose of this study is thus to formulate CC-FEM in the case of an “off-axial interfacial debonding and sliding” problem. The obtained numerical results were compared with the results obtained from ANSYS, a popular general-purpose FEM software. In addition, its validity was discussed.

2. Constrained conditional finite element method (CC-FEM)

2.1. Stiffness equation including contact forces

The present method deals with contact forces occurring at the interface, based on contact analysis. In general, the principle of virtual work including contact forces is given as follows [6,7].

\[
\iiint_V \sigma_{ij} \delta \varepsilon_{ij} \, dV - \left( \iint_V p_i \delta u_i \, dV + \iint_{S_c} T_i \delta u_i \, dS_c \right) - \iint_{S_c} R_i \delta u_i \, dS_c = 0
\]  

Where, \( \sigma_{ij}, \delta \varepsilon_{ij}, \delta u_{ij} \) are stress, virtual strain and virtual displacement. As shown in figure 1, \( p_i, T_i, R_i \) are body, surface and contact forces, \( S_c \) and \( S_c^\overline{C} \) are surface areas of contact surface \( C \) (such as fiber/matrix surface) and mechanical boundary except \( C \), while \( V \) is volume. Subscripts \( i \) and \( j \) are the \( r \) and \( z \)-directions. In equation (1), 1st-3rd terms show internal work, external work (sum of works by external force and body force), and work by contact force, respectively. By discretizing equation (1) and by defining the equivalent nodal force on nodes of each elements, the stiffness equation including contact forces can be derived. The incremental form is given as:

\[
[K][\Delta u] = \{\Delta f\} + \{\Delta R\}
\]  

Where, \([K]\) is the global stiffness matrix, derived from the first term of equation (1). \{\Delta u\} is nodal displacement increments, and \{\Delta f\} is load increments, derived from the 2nd term. \{\Delta R\} is contact force increments, derived from the 3rd term, which are treated as unknown variables.
2.2. Definition of interfacial contact states
As described in the Introduction, reinforcing fibers used for CMCs often have local variation in diameter, resulting in off-axial interfacial debonding and sliding. Therefore, the following interfacial damage process is presumed in the same way as the on-axial case: First, fiber breakage or matrix cracking occurs with subsequent interfacial debonding. Next, off-axial interfacial sliding continues until the fiber-matrix relation reaches the mechanical equilibrium state through interfacial friction. Under such circumstances, the interfacial contact states are defined as shown in figure 2. Every state comprises four nodal points, of which 1 and 3 are placed on the fiber element, and 2 and 4 are on the matrix one. As shown in figure 2(a), bonding state represents a non-damage state. On the other hand, figures 2(b)−2(d) show damage states after relative sliding at the fiber-matrix interface, which occurs at the angle $\alpha$ from the loading axis, where angle $\alpha$ is defined such that counterclockwise direction is positive.

![Figure 2](image)

Figure 2. Interfacial contact states for off-axial interface: (a) Bonding, (b) Steady state after interfacial debonding and sliding, (c) with matrix crack, and (d) with fiber breakage.

2.3. Interfacial contact conditions
Interfacial contact conditions in the present finite element model, i.e. constrained conditions, are based on the equivalence of displacement increments and the equilibrium of contact forces, as follows.

2.3.1. Bonding. All nodal points are bonded at the same position as shown in figure 2(a). Therefore, all the displacement increments must be identical. Furthermore, the sum of the contact force increments is zero at the stationary state. The conditions are therefore given as below.

\[
\begin{align*}
\Delta u_1 &= \Delta u_2 = \Delta u_3 = \Delta u_4 \\
\Delta R_1 + \Delta R_2 + \Delta R_3 + \Delta R_4 &= 0 \\
\Delta w_1 &= \Delta w_2 = \Delta w_3 = \Delta w_4 \\
\Delta R'_1 + \Delta R'_2 + \Delta R'_3 + \Delta R'_4 &= 0
\end{align*}
\] (3)

Where, $\Delta u$ and $\Delta w$ are displacement increments of nodes 1 to 4 along the r- and z-directions. $\Delta R$ and $\Delta R'$ are the contact force increments.

2.3.2. Steady states after interfacial debonding and sliding As shown in figures 2(b)−2(d), in the off-axis case, the direction of frictional forces differs from the direction of contact forces. In figure 2(b), contact forces of r- and z-directions are presented as dissolved to the sliding direction and the normal direction. Under such a situation, Coulomb’s law of friction is applicable as presented below.

\[
(\Delta R'_1 + \Delta R'_3)\cos \alpha - (\Delta R_1 + \Delta R_3)\sin \alpha = \mu((\Delta R'_1 + \Delta R'_3)\cos \alpha + (\Delta R'_1 + \Delta R'_3)\sin \alpha)
\] (4)

\[
\therefore (\Delta R'_1 + \Delta R'_3) = C(\Delta R_1 + \Delta R_3)
\] (5)

Therein, $\mu$ is the static friction coefficient, and $C$ is the equivalent friction coefficient:

\[
C = C_{\text{max}} = \frac{\mu + \tan \alpha}{1 - \mu \tan \alpha}
\] (6)
Equation (6), which means that the friction coefficient apparently varies as a function of $\alpha$, is established for the case in which the fiber and matrix elements are mutually pushed at the vertical direction of the interface. This equation engenders the relation: $C = C_{\text{max}}$. The frictional force increases concomitantly with increasing $\alpha$. In contrast, when the forces work such that the fiber and matrix elements are mutually separated, the equivalent frictional force is given as:

$$C = C_{\text{min}} = (\mu - \tan \alpha)/(1 + \mu \tan \alpha)$$

Equation (7) is valid when a radial compression works in advance by residual stresses. In equation (7), the frictional force is shown to decrease with increasing $\alpha$. From the above mention, the interfacial contact conditions of steady states after interfacial debonding and sliding (figure 2(b)) are given as:

$$\Delta u_1 = \Delta u_3 \quad \Delta u_2 = \Delta u_4 \quad \Delta w_1 = \Delta w_3 \quad \Delta w_2 = \Delta w_4$$

$$\tan \alpha = (\Delta u_2 - \Delta u_4)/(\Delta w_1 - \Delta w_2)$$

Equation (8) is valid when a radial compression works in advance by residual stresses. In equation (7), the frictional force is shown to decrease with increasing $\alpha$. From the above mention, the interfacial contact conditions of steady states after interfacial debonding and sliding (figure 2(b)) are given as:

$$\Delta u_1 = \Delta u_3 \quad \Delta w_1 = \Delta w_3$$

$$\tan \alpha = (\Delta u_2 - \Delta u_4)/(\Delta w_1 - \Delta w_2) = (\Delta u_3 - \Delta u_1)/(\Delta w_1 - \Delta w_3)$$

$$\Delta R'_2 = C_{\text{max}} \Delta R_2 \quad R'_4 = -C_{\text{min}} \Delta R_4 \quad \Delta R'_1 + \Delta R'_3 = 0$$

$$\Delta R_1 + \Delta R_3 + \Delta R_2 + \Delta R_4 = 0 \quad \Delta R'_1 + \Delta R'_3 + \Delta R'_2 + \Delta R'_4 = 0$$

2.3.3. Interfacial debonding and sliding with fiber breakage.

$$\Delta u_1 = \Delta u_3 \quad \Delta w_1 = \Delta w_3$$

$$\tan \alpha = (\Delta u_2 - \Delta u_4)/(\Delta w_1 - \Delta w_2) = (\Delta u_3 - \Delta u_1)/(\Delta w_1 - \Delta w_3)$$

$$\Delta R'_2 = C_{\text{min}} \Delta R_2 \quad R'_4 = C_{\text{max}} \Delta R_4 \quad \Delta R'_1 + \Delta R'_3 = 0$$

$$\Delta R_1 + \Delta R_3 + \Delta R_2 + \Delta R_4 = 0 \quad \Delta R'_1 + \Delta R'_3 + \Delta R'_2 + \Delta R'_4 = 0$$

2.3.4. Interfacial debonding and sliding with matrix cracking.

$$\Delta u_2 = \Delta u_4 \quad \Delta w_2 = \Delta w_4$$

$$\tan \alpha = (\Delta u_2 - \Delta u_4)/(\Delta w_1 - \Delta w_2) = (\Delta u_4 - \Delta u_2)/(\Delta w_1 - \Delta w_4)$$

$$\Delta R'_1 = -C_{\text{min}} \Delta R_1 \quad R'_3 = C_{\text{max}} \Delta R_3 \quad \Delta R'_2 + \Delta R'_4 = 0$$

$$\Delta R_1 + \Delta R_3 + \Delta R_2 + \Delta R_4 = 0 \quad \Delta R'_1 + \Delta R'_3 + \Delta R'_2 + \Delta R'_4 = 0$$

Equations (3) and (8)–(10) agree exactly with the on-axial interfacial contact conditions, when $\alpha = 0^\circ$ [1, 3-5]. In other words, the off-axial conditions include on-axial conditions.

2.4. Insertion of interfacial contact conditions into the global stiffness matrix

From contact conditions (such as equations (3), (8), (9), (10)), several unknown variables (displacements $\Delta u$, $\Delta w$, and contact forces $\Delta R$, $\Delta R'$) can be reduced. For example, in (b) Steady states after interfacial debonding and sliding (figure 1(b)), from constraint conditions (equation (8)), all variables can be represented by 3 displacements ($\Delta u_1, \Delta w_1, \Delta u_2$) and 5 contact forces ($\Delta R_2, \Delta R_3, \Delta R'_3, \Delta R_4, \Delta R'_4$), and the number of variables to be calculated can be reduced from 16 to 8.

Then, the 1st line of equation (1) is written out as:

$$K_{11} \Delta u_1 + K_{12} \Delta w_1 + K_{13} \Delta u_2 + K_{14} \Delta w_2 + K_{15} \Delta u_3 + K_{16} \Delta w_3 + K_{17} \Delta u_4 + K_{18} \Delta w_4 = \Delta f_{r1} + \Delta R_1$$

(11)
Using equation (8), equation (11) can be rewritten as:

\[
\begin{align*}
K_1 &\Delta u_1 + K_{12} \Delta w_1 + K_{13} \{(\Delta w_1 - \Delta w_2) \tan \alpha + \Delta u_2\} + K_{14} \Delta w_2 + K_{15}\Delta u_1 + K_{16} \Delta w_1 \\
+ K_{17} &\{(\Delta w_1 - \Delta w_2) \tan \alpha + \Delta u_1\} + K_{18} \Delta w_4 = \Delta f_{r1} - (\Delta R_2 + \Delta R_3 + \Delta R_4) \\
\end{align*}
\]

(12)

\[
\begin{align*}
\therefore (K_{11} + K_{13} + K_{15} + K_{17}) &\Delta u_1 + \{(K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha\} \Delta w_1 \\
+ (K_{14} + K_{18}) - (K_{13} + K_{17}) \tan \alpha &\Delta w_2 - \Delta R_2 - \Delta R_3 - \Delta R_4 = \Delta f_{r1}
\end{align*}
\]

(13)

Equation (13) contains only 8 variables \((\Delta u_1, \Delta w_1, \Delta u_2, \Delta R_2, \Delta R_3, \Delta R_3', \Delta R_4, \Delta R_4')\), and another lines can also be similarly represented by only these 8 variables. Consequently, the following global stiffness equation is ultimately given:

\[
[K_c] \{\Delta u_c\} = \{\Delta f\}
\]

(14)

Where, \([K_c]\) is the global stiffness matrix, which is changed by the interfacial contact conditions. Also, \(\{\Delta u_c\}\) consists of nodal displacement increments including contact force increments. For example, in (b) Steady state after interfacial debonding and sliding, the stiffness equations are given as:

\[
\begin{bmatrix}
K_{11} + K_{13} + K_{17} & (K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha & 1 & 0 & 0 & 0 & \Delta u_1 \\
K_{12} + K_{13} + K_{15} & (K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha & -1 & 0 & 0 & 0 & \Delta w_1 \\
K_{13} + K_{15} + K_{17} & (K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha & 0 & 1 & 0 & 0 & \Delta u_2 \\
K_{14} + K_{16} + K_{17} & (K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha & 0 & 0 & 1 & 0 & \Delta w_2 \\
K_{15} + K_{13} + K_{17} & (K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha & 0 & 0 & 0 & 1 & \Delta u_3 \\
K_{16} + K_{13} + K_{17} & (K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha & 0 & 0 & 0 & 0 & \Delta w_3 \\
K_{17} + K_{13} + K_{17} & (K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha & 0 & 0 & 0 & 0 & \Delta u_4 \\
K_{18} + K_{13} + K_{17} & (K_{12} + K_{16}) + (K_{13} + K_{17}) \tan \alpha & 0 & 0 & 0 & 0 & \Delta w_4
\end{bmatrix}
\begin{bmatrix}
\Delta f_{r1} \\
\Delta f_{r2} \\
\Delta f_{r3} \\
\Delta f_{r3}' \\
\Delta f_{r4} \\
\Delta f_{r4}' \\
\Delta f_{r5} \\
\Delta f_{r6}'
\end{bmatrix}
\]

(15)

Equation (15) is solvable numerically, and \(\{\Delta u_c\} = \{\Delta u_1, \Delta w_1, \Delta R_2, \Delta w_2, \Delta R_3, \Delta R_3', \Delta R_4, \Delta R_4'\}^T\) can be gotten directly. Subsequently, another unknown displacements \((\Delta u_2, \Delta u_3, \Delta w_3, \Delta w_4)\) and unknown contact forces \((\Delta R_1, \Delta R_2, \Delta R_2')\) can be also calculated using equation (8).

2.5. Finite element mesh and boundary conditions

As reported in the literature [8-12], the interfacial debonding size of CMCs is large-scale, which engenders an extremely small stress concentration around fiber break points or near the matrix crack plane, and which lowers the inter-fiber interaction. Therefore, the present study employed only the analysis of a single fiber composite in which the fiber was embedded in a cylindrical matrix. The finite element models are shown in figure 3. The fiber radius \(r_f\) and matrix radius \(r_m\) were 0.0055 and 0.011 mm, respectively, and the lengths were also 2.5 mm. The finite element model was composed of axisymmetric linear triangle elements. The respective numbers of nodal points and elements were 1,250 and 1,000. In ANSYS used for comparison, the model was composed of axisymmetric quadratic triangle elements. The respective numbers of nodal points and elements were 752 and 1,000. In these models, the constrained conditional state after interfacial debonding and sliding (figure 2(b)), was applied to area of 1.0 mm long from the matrix crack surface. The remaining area (1.5 mm long) was treated as the bonding state (figure 2(a)). The displacement boundary condition \(U\) was given only at the fiber end of compression. To simulate the off-axis interfacial debonding and sliding, a change in fiber diameter \(\Delta r\) and taper length \(\Delta z\) were assumed as shown in figure 3. For example, in \(\alpha = 0.113^\circ\) model, \(\Delta z = 0.77 mm, \Delta r = r_f/4 \ mm, \alpha = \tan^{-1}(\Delta r/\Delta z) = 0.113^\circ\), and equivalent frictional coefficient \(C\) is increased to \(C = C_{\text{max}} = (\mu + \tan \alpha)/(1 - \mu \tan \alpha) = 0.0520^\circ\), when \(\mu = 0.05\).
Results and discussion

3.1. Mechanical behavior caused by on-axial interfacial debonding and sliding

Figure 4 presents the axial stress distributions caused by on-axial interfacial debonding and sliding, calculated using CC-FEM and ANSYS. All stresses are normalized such that the axial fiber stress \( \sigma_z \) at the matrix crack surface is applied as \(-1\). The material and physical constants were given for simplicity as follows. Young’s modulus: \( E_f = E_m = 200 \, \text{GPa} \), Poisson’s ratio: \( \nu_f = \nu_m = 0.2 \), static frictional coefficient: \( \mu = 0.05 \). No residual stresses were included in the initial state. Results shown in figure 4(a) demonstrate that the stress distributions \( \sigma_z \) and \( \sigma_m \) mutually agree at the bonding area, i.e. the area more than 1.0 mm in the distance from matrix crack surface, and show a constant. On the other hand, \( \sigma_m \) recovers in compression nonlinearly in area less than 1.0 mm from the matrix crack surface, whereas \( \sigma_z \) increases greatly from the stress level of \(-1\). Such behavior means that, although no load is applied to the matrix, the matrix stress occurs by a frictional effect even at the debonding area. At the debonding end, \( \sigma_z \) and \( \sigma_m \) change intensely because the interfacial contact state is changed here from figure 2(b) to 2(a).

In figure 4(b), the normalized stress distributions along the radial and circumferential directions, \( \sigma_r \) and \( \sigma_\theta \) are shown. Both stresses show zero at the bonding area, because the same Poisson’s ratios are given to the fiber and matrix. At the sliding area, however, these distributions behave nonlinearly with compression along the \( r \)-direction in the fiber and matrix, because radial stresses occur by Poisson’s effect as a result of the difference between axial strains in the fiber and matrix. It is also noteworthy that \( \sigma_m \) and \( \sigma_\theta \) decrease in absolute value because distance \( r \) is separate from the interface. According to ANSYS the results (broken lines in figure 4), all stress distributions agree well with the CC-FEM results (solid lines). In other words, the CC-FEM validity is demonstrated in the on-axial interfacial state. In addition, the CPU times of ANSYS are about 4~5 sec, while CC-FEM one is about 0.75 sec and analysis can be carried out about 5.3~6.6 times faster.
Mechanical behavior caused by off-axial interfacial debonding and sliding

Figures 5(a) and 5(b) show axial stress distributions caused by off-axial interfacial debonding and sliding, calculated using CC-FEM and ANSYS. Results show that both distributions mutually agree well. Such agreement was also confirmed for \( \sigma_r \) and \( \sigma_\theta \) distributions. It is concluded that the present CC-FEM formulation is also valid in the case of off-axial interfacial debonding and sliding.

According to the distributions in figure 5, the changes of the fiber and matrix stresses become steeper with increasing off-axial angle \( \alpha \), because the equivalent friction coefficient \( C \) is larger, as described earlier, producing more efficient stress recovery in compression. Similar steeper changes in stress were confirmed for \( \sigma_r \) and \( \sigma_\theta \) distributions. Such stress recovery means a property improvement caused by a mechanical bond, the so-called ‘anchor effect’.

Figure 4. Comparison between stresses by CC-FEM and ANSYS (on-axis sliding). (a) \( \sigma_z \) and (b) \( \sigma_r \) and \( \sigma_\theta \).

Figure 5. Comparison between axial stresses by CC-FEM and ANSYS (off-axis sliding). (a) CC-FEM (\( \mu = 0.05 \)) and (b) ANSYS (\( \mu = 0.05 \)).

Figure 6. Effect of coefficients of static friction \( \mu \) on axial stress by CC-FEM. (a) CC-FEM (\( \mu = 0.03 \)) and (b) CC-FEM (\( \mu = 0.1 \)).
Figure 6 shows the effects of friction coefficient on the longitudinal stress distributions by CC-FEM. In the calculation, the same off-axial angles were used. Comparison between figures 5(a), 6(a) and 6(b) demonstrates that steeper changes of the fiber and matrix stresses occur with increasing \( \alpha \). As presented in figure 6(a), on the other hand, the smaller friction coefficient gives a larger difference in stress between \( \alpha=0.0788 \) and \( 0.158^\circ \). Because a smaller \( \alpha \) yields a larger ratio, i.e. \( C/\mu \), this difference appears more clearly. In other words, for smaller \( \mu \), the anchor effect appears more strongly.

4. Conclusions
Formulation of constrained conditional finite element method (CC-FEM) for the case of off-axial interfacial debonding and sliding occurring in ceramic matrix composites was proposed. The calculated stress distributions in longitudinal, radial, and circumferential directions were compared with those of general-purpose software ANSYS. Results showed that these stress distributions agreed well with those of ANSYS at compressive loading to a single fiber composite. Such agreement with ANSYS was also confirmed for the case of on-axial interfacial debonding and sliding.

Calculation results obtained using CC-FEM also demonstrated that the stress recoveries in the fiber and matrix were more efficient with increasing off-axial angle. Such a tendency appeared more strongly, even at the same off-axial angle, when a larger friction coefficient was given.

The proposed CC-FEM can resolve problems of on-axial and off-axial interfacial debonding and sliding by setting the off-axis angle and equivalent frictional coefficient and off-axis angle, appropriately, and establishing equations of displacement equivalence and contact force balance. Results suggest that CC-FEM can be used widely as an effective tool to clarify various physical problems related to any fibrous composite materials.

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