The hyperfine structure of highly charged $^{238}_{92}$U ions with rotationally excited nuclei

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Abstract

The hyperfine structure (hfs) of electron levels of $^{238}_{92}$U ions with the nucleus excited in the low-lying rotational $2^+$ state with an energy $E_{2^+} = 44.91$ keV is investigated. In hydrogenlike uranium, the hfs splitting for the $1s_{1/2}$-ground state of the electron constitutes 1.8 eV. The hyperfine-quenched (hfq) lifetime of the $1s2p\ ^3P_0$ state has been calculated for heliumlike $^{238}_{92}$U and was found to be two orders of magnitude smaller than for the ion with the nucleus in the ground state. The possibility of a precise determination of the nuclear $g_r$ factor for the rotational $2^+$ state by measurements of the hfq lifetime is discussed.

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Atomic hfs experiments and pure nuclear measurements (pionic scattering etc.) are two supplementary ways for obtaining informations on nuclear moments. However, up to now the atomic hfs experiments have been performed exclusively for atoms or ions with nuclei in the ground state. In this paper we point out that experiments are also feasible for highly charged ions with rotationally excited even-$A$ nuclei.

The low-lying excited rotational state of the $^{238}_{92}$U nucleus with excitation energy $E_{2^+} = 44.91$ keV plays an important role in recent accurate calculations of the Lamb shift for highly charged uranium ions. In particular, for these ions the contribution of the $2^+$ state dominates in calculations of nuclear polarization shifts [1–4]. However, in such calculations this state enters as a virtual nuclear excitation. In the present work we consider the situation of a real excitation of the $2^+$ state due to the interaction of the incoming atomic uranium beam with a target in beam-foil experiments.

The empirical energy spectrum of a rotationally excited nucleus in the ground-state band fits well to the formula $(\hbar = c = 1)$ [5,6]

$$E_I = \mathcal{A}I(I + 1). \quad (1)$$

In Eq. (1), $I$ denotes the total angular momentum of the rotating nucleus and $\mathcal{A}$ is the rotational constant. The latter can be related to the moment of inertia $\mathcal{I}$ of the nucleus according to $\mathcal{A} = 1/2\mathcal{I}$. The lowest rotational excitation in $^{238}_{92}$U is the electric quadrupole transition $0^+ \rightarrow 2^+$ within the ground-state band with an excitation energy $E_{2^+} = 44.91$ keV. Fitting this energy to Eq. (1) yields the rotational constant $\mathcal{A} \simeq 7.5$ keV [7].

A rotationally excited nucleus should have a magnetic moment $\mu = \mu'^n + \mu_Ng_r(I - \Omega n)$ associated with its total angular momentum $I$ [5,6]. Here $\mu'^n$ denotes the magnetic moment of the nonrotating nucleus, $n$ represents the unit vector directed along the nuclear axis, and $\mu_N$ is the nuclear magneton. The ratio $g_r = \mathcal{I}_p/\mathcal{I}$ defines the gyromagnetic factor for the rotation of the nucleus with $\mathcal{I}_p$ being the protonic part of the total moment of inertia $\mathcal{I}$. The projection $\Omega = (I n)$ characterizes the various rotational bands. After averaging over rotations the magnetic moment is directed along the conserving vector $\hat{I}$:

$$\hat{\mu} = \frac{\mu'}{I} \hat{I} = \mu'^n + \mu_Ng_r(\hat{I} - \Omega n). \quad (2)$$

Multiplying Eq. (2) by $\hat{I}$ and passing over to eigenvalues, we obtain

$$\mu = \mu' \frac{\Omega}{I + 1} + \mu_Ng_r \left( I - \frac{\Omega^2}{I + 1} \right). \quad (3)$$

Thus in highly charged ions with $\mu' = 0$ but rotationally excited nuclei a hyperfine structure splitting of levels should arise.
In the point-nucleus approximation the hfs magnetic-dipole interaction operator \( \hat{H}_{\text{hfs}} \) can be written in the form:

\[
\hat{H}_{\text{hfs}}(r) = e \hat{\mu} \frac{[\alpha \times r]}{r^3},
\]

(4)

where the nuclear magnetic moment is defined by Eq. (3), \( e \) is the electron charge (\( e < 0 \)), and \( \alpha \) and \( r \) are the Dirac matrices and the spatial coordinate for the atomic electron, respectively. For a spinless nucleus in the ground-state band (\( \Omega = 0 \)) the expression (3) yields \( \mu = \mu_N g_r I \).

The hfs correction to the energy levels of hydrogen-like \(^{238}_{92}\text{U}\) ion is defined by the standard Landé expression \( \Delta E_{\text{hfs}}(F) = Ca/2 \), where the cosine factor is \( C = F(F + 1) - I(I + 1) - j(j + 1) \), \( j \) is the total electron angular momentum, \( F \) is the total angular momentum of an ion, and the hfs constant \( a \) is determined by

\[
a = \frac{\alpha}{m_p j(j + 1)} \int_0^\infty \frac{dr}{r^2} P_{nlj}(r)Q_{nlj}(r).
\]

(5)

Here \( \alpha = e^2 \) is the fine-structure constant, \( m_p \) is the proton mass, and \( P_{nlj}(r) \) and \( Q_{nlj}(r) \) are the upper and lower radial components of the electron wave function characterized by the principal quantum number \( n \) and the relativistic quantum number \( \kappa = (l - j)(2j + 1) \). Employing analytical results from Refs. [8,9], one finds

\[
a = \alpha(Z/A)^3 \frac{m_e^2}{m_p} \frac{\kappa[2\kappa(\gamma + n_r) - N]}{j(j + 1)N^4\gamma(4\gamma^2 - 1)(1 - \delta_{nlj})},
\]

(6)

where \( \gamma = \sqrt{\kappa^2 - (\alpha Z)^2} \), \( N = \sqrt{n_r(2\gamma + n_r) + \kappa^2} \), \( m_e \) is the electron mass, \( Z \) is the number of protons, and \( n_r \) is the radial quantum number (\( n = n_r + |\kappa| \)). The correction \( \delta_{nlj} \) accounts for the finite nuclear charge distribution. The nuclear magnetization distribution correction as well as QED corrections are rather small and therefore negligible. For electron states with \( j = 1/2 \), the hfs splitting \( \Delta E_\mu \) between the states with \( F = I + 1/2 \) and \( F = I - 1/2 \) is just \( \Delta E_\mu = (I + 1/2)a \). Assuming homogeneous nuclear charge and mass distributions one obtains \( g_r = Z/A \) for a nucleus with mass number \( A \). Thus from atomic hfs experiments with rotationally excited nuclei one can deduce directly the deviation of the empirical \( g_r \) factor from the \( Z/A \) approximation. To our knowledge, \( g_r \) factors for \(^{238}_{92}\text{U}\) have been determined only for rotational states with spin \( I = 6 \) and higher by means of measurements of the precession angles in transient magnetic fields by \( \gamma \)-ray - particle coincidences [10]. These measurements cannot be performed in the case of the highly converted, low-lying \( 2^+ \) and \( 4^+ \) nuclear states.

For the \( 1s_{1/2} \)-ground state of hydrogenlike \(^{238}_{92}\text{U}\) the splitting is indicated in Fig. [11]. The value of \( E_{1s_{1/2}}^{(0)} \) corresponds to the ground-state energy level of the uranium ion with the
unexcited nucleus. The uncertainty of the value $E^{(0)}_{1s_{1/2}}$ is determined by the Lamb shift calculations. The present theoretical value for the Lamb shift is $\Delta E^{(th)}_{1s_{1/2}} = 464.7 \pm 1.0$ eV [11], while the experimental value is $\Delta E^{(exp)}_{1s_{1/2}} = 470 \pm 16$ eV [12]. The evaluation of the hfs constant (6) for the ground state of $^{238}_{92}$U$^{91+}$ with $g_r = Z/A$ yields $a = 0.89$ eV for a point nucleus and $a = 0.72$ eV for an extended one. In the latter case, the finite-size correction $\delta_{1s} = 0.19$ has been approximated according to results obtained in Ref. [9]. Then one finds a hyperfine splitting $\Delta E_\mu = 1.8$ eV ($\Delta \lambda = 0.69$ $\mu$m), which is well resolvable within the accuracy of 1 eV envisaged for the near-future level shift measurements in hydrogenlike uranium [12].

The lifetime $\tau_{2^+}$ of the excited rotational state $2^+$ of the $^{238}_{92}$U nucleus can be obtained from the known empirical value for the reduced transition probability $B(E2; 0^+ \rightarrow 2^+) = 12.3 \ e^2b^2$ [9], where $b$ denotes barn. This leads to $\tau_{2^+} \simeq 10^2$ ns. One should point out that the much smaller value $\tau_{2^+} \simeq 10^2$ ps given in the literature [7] corresponds to neutral uranium atoms and is due to the internal conversion process. The latter decay channel is absent in H- and He-like $^{238}_{92}$U ions (see also [13,14]). The time $\tau_{2^+}$ is large enough to consider the magnetic interaction between the electron in hydrogenlike uranium and the rotating nucleus as a stationary problem. The time of revolution $\tau_{\text{rot}}$ associated with this nuclear excitation can be deduced from

$$E_{2^+} = \frac{1}{2} I \omega_{\text{rot}}^2 = \frac{1}{4A} \left( \frac{2\pi}{\tau_{\text{rot}}} \right)^2$$

yielding $\tau_{\text{rot}} \simeq 10^{-4}$ fs, which is negligibly small compared to $\tau_{2^+}$.

Let us now discuss the possibility of an experimental verification of this effect using the SIS/ESR facility at GSI in Darmstadt. We consider a beam of bare uranium ions with typical kinetic energy $E_{\text{kin}} \simeq 320$ MeV/u which corresponds to a velocity $v \simeq 0.67c$. From the lifetime $\tau_{2^+} \simeq 10^2$ ns it follows that the decay length of the $2^+$ state is larger than 25 m behind the foil. The number of ions $n_i$ with rotationally excited nuclei that can be prepared per second is given by $n_i = J\sigma \mathcal{N}$, where $J$ denotes the intensity of the ion beam, $\sigma$ is the cross section for exciting the nucleus inside the foil, and $\mathcal{N}$ is the number of foil atoms per unit area.

The Coulomb excitation cross section $\sigma$ for uranium nuclei in collisions with nuclei of the carbon foil can be estimated within the framework of the equivalent photon method as described in Ref. [13]. Assuming that only the rotational $2^+$ state of $^{238}$U with the energy $E_{2^+} = 44.91$ keV is excited, the photonuclear absorption cross section may be approximated by $\sigma_\gamma^{E2}(\varepsilon) \simeq \frac{4\varepsilon^3}{\hbar^2} \left( \frac{\varepsilon}{\hbar c} \right)^3 B(E2) \delta(\varepsilon - E_{2^+})$. This approximation is legitimized by the huge value of the reduced transition strength $B(E2)$ of the rotational $2^+$ state as the most dominant
collective nuclear excitation of the $^{238}$U isotope. Then the total cross section $\sigma$ results as

$$\sigma \sim \int \frac{d\varepsilon}{\varepsilon} n^{E2}(\varepsilon) \sigma_{E2}(\varepsilon),$$

where $n^{E2}(\varepsilon)$ denotes the number of equivalent photons. The adiabaticity parameter involved in the problem is equal to $\xi = \frac{b_{\min} E_{2+}}{\hbar c \beta \gamma}$, where $\beta = v/c \simeq 0.67$, $\gamma = (1 - \beta^2)^{-1/2}$, and $b_{\min}$ is the minimum impact parameter taken as the sum of the two nuclear radii. Since $\xi \simeq 2.6 \times 10^{-3}$ is quite small, the number of equivalent photons can be approximated by

$$n^{E2}(E_{2+}) \simeq \frac{4\alpha Z_f^2}{\pi \gamma^2 \beta \xi}$$. 

Finally, we obtain $\sigma \simeq 10.7$ fm$^2$. Taking the value $N \simeq 0.5 \times 10^{20}$ ions/cm$^2$ for a typical carbon foil density $\rho = 1$ mg/cm$^2$ together with a characteristic intensity $J \simeq 10^{10}$ ions/s, one finds $n_i \simeq 0.5 \times 10^5$ ions/s. Only a fraction of about $0.5 \times 10^{-5}$ of the primary ions are ions with excited nuclei in the rotational $2^+$ state. However, even if all electrons captured in the foil are supposed to decay to the $1s_{1/2}$-ground state by the emission of Lyman radiation, a direct measurement of the hfs splitting is at present prohibited by the low efficiency ($\sim 10^{-8}$) of the required high-resolution spectrometers.

Still there exists another possibility for the observation of the effect. It is based on the measurement of the hfq lifetime of the metastable $2^3P_0$ state in $^{238}U^{90+}$ ions. This effect was observed in Refs. [16–21] for the isoelectronic sequence of heliumlike ions with non-zero nuclear spin. The level scheme of the first excited states in the $^{238}U^{90+}$ ion without taking into account the hfq decay channels is depicted in Fig. 2. The energy values and partial transition probabilities were calculated within the framework of the multiconfigurational Dirac-Fock method (MCDF) [22]. The energies of the electron levels include the radiative [23,24] (electron self-energy and vacuum polarization) and the exact one-photon exchange corrections. The E1M1 two-photon transition rate has been calculated by Drake [25] and the 2E1 decay rate is taken from Ref. [26].

Hyperfine quenching of the metastable $2^3P_0$ state results from a mixing with the short-lived $2^3P_1$ state by the hyperfine interaction (4). The partial widths $\Gamma_J$ ($J = 0, 1$) for $2^3P_J$ levels due to the radiative E1 transitions to the ground $1^1S_0$ state are related by

$$\Gamma_0 = \eta^2 \Gamma_1,$$

where the mixing coefficient $\eta$ is defined by

$$\eta = \sum_{i=1}^{2} \frac{\langle 2^3P_0 | \hat{H}_{\text{hfs}}(r_i) | 2^3P_1 \rangle}{E_{2^3P_0} - E_{2^3P_1}},$$

and the rotationally-induced hyperfine interaction operators $\hat{H}_{\text{hfs}}(r_i)$ are given by Eq. (4). The coefficient $\eta$ can be expressed directly through the $g_r$ factor. Performing the integrations over the angles, the matrix element in expression (8) reads
\[ \langle 2^3 P_0 \mid \sum_{i=1}^{2} \hat{H}_{\text{hfs}}(r_i) \mid 2^3 P_1 \rangle = g_r \frac{2\alpha}{3m_p} \sqrt{I(I+1)} \int_0^\infty \frac{dr}{r^2} \left[ P_{1s}(r)Q_{1s}(r) + P_{2p_{1/2}}(r)Q_{2p_{1/2}}(r) \right] \]

\[ = g_r \alpha (\alpha Z)^3 \sqrt{I(I+1)} \frac{m_e^2}{m_p} \left[ \frac{(2\gamma + 2 - \sqrt{2\gamma + 2})}{(2\gamma + 2)^2 \gamma (4\gamma^2 - 1)} (1 - \delta_{2p}) - \frac{(1 - \delta_{1s})}{\gamma (2\gamma - 1)} \right], \]

where \( \gamma = \sqrt{1 - (\alpha Z)^2} \).

For the \( ^{238}_{92}\text{U}^{90+} \) ion, the mixing coefficient has been calculated for \( g_r = Z/A \) in the framework of the MCDF approach to yield

\[ \eta = -\frac{0.764 \text{ eV}}{E_{2^3 P_0} - E_{2^3 P_1}} = 0.696 \times 10^{-2}. \]

It leads to the appearance of an additional contribution to the radiative width of the \( 2^3 P_0 \) level, that turns out to be \( 0.147 \times 10^{13} \text{ s}^{-1} \). As a result, the lifetime of the \( 2^3 P_0 \) level is diminished from 56 ps to 0.67 ps, which corresponds to a decay length of about 0.18 mm in the laboratory. We should emphasize that there will be no background from the ions with unexcited nuclei, since in those ions the one-photon transition \( 2^3 P_0 \rightarrow 1^1 S_0 \) with the energy \( \omega_0 \simeq 96.271 \text{ keV} \) is absolutely forbidden. The lifetime of 56 ps for ions in the \( 2^3 P_0 \) state arises from the transition \( 2^3 P_0 \rightarrow 2^3 S_1 \) with the energy \( \omega_1 \simeq 256.21 \text{ eV} \), which is far away from \( \omega_0 \) (70% of the total width) as well as from the two-photon transition \( 2^3 P_0 \rightarrow 1^1 S_0 \) (30% of the total width). Both of these transitions cannot give any background contribution in the proposed experiment. The transition \( 2^3 P_1 \rightarrow 1^1 S_0 \) with the energy of about 96.162 keV does also not contribute to the background, since the level \( 2^3 P_1 \) has a lifetime of 0.033 fs and hence decays already inside of the foil. In order to obtain clean spectra without loss of efficiency, a measurement of coincidences between heliumlike ions and photons should be performed. Thus, the observation of the predicted effect becomes feasible utilizing the beam-foil time-of-flight technique. In view of Eq. (7), the accuracy of the determination of the \( g_r \) factor is even two times better than the accuracy of the measured hfq lifetime, which in this region can be expected to be at the level of about 1%.

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FIG. 1. The hyperfine splitting $\Delta E_\mu$ of the ground state of hydrogenlike $^{238}$U with the nucleus in the rotationally excited $2^+$ state.
FIG. 2. Energy level scheme of the first excited states of heliumlike $^{238}\text{U}$ with the nucleus in the ground state. The partial probabilities of radiative transitions are given in s$^{-1}$. Numbers in brackets indicate powers of 10. The large radiative width for the $1s2p\,^3P_1$ state is indicated as a bold line. The double lines denote two-photon transitions. Numbers on the right-hand side indicate the binding energies in eV.
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