Fun with colours

The standard model with two-colour QCD has radically different long-distance physics

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Abstract: In our world, the standard model of particle physics contains within it the fairly intractable theory called QCD. A toy version with two colours is often studied as a model confining and chiral symmetry breaking field theory. Here, we investigate the cascade of changes at various distance scales if we make this change within the standard model. It is possible to limit the changes at the hadronic scale. However, the minor changes that occur actually cascade down to the far infrared, into nuclear and atomic physics, and chemistry. Through this, it also possibly affects the evolution of stars and galaxies. We remark on this unexpected sensitivity of the universe to physics at the scale of quarks.

Keywords: Two-colour QCD; Standard model; Effective field theory

1. Introduction

In our universe, the standard model (SM) of particle physics [1] contains strong interactions described by a gauge group SU($N_c$) with $N_c = 3$ and with six flavours of quarks in the fundamental representation of the gauge group. This is quantum chromodynamics, QCD. The action has a global chiral symmetry, and quarks are, additionally, in the fundamental representation of this symmetry. The Glashow Iliopoulos Maiani (GIM) mechanism organizes these quark flavours into three generations, each of two flavours. The currents built out of the quark doublets, and the corresponding lepton doublets are coupled to the SU(2) × U(1) gauge group of the electroweak interactions. A matching of the number of generations is required by the GIM anomaly cancellation mechanism. The Higgs mechanism does not depend on QCD1.

The electroweak part of the theory is reasonably well described in a weak coupling expansion, but the strong interactions are not, in general. In particular, the external states of the SM are not quarks and leptons but leptons and hadrons. This absence of quarks from the external states is said to be due to a property of QCD called confinement, whose understanding remains incomplete in spite of a million dollar prize for a proof. Since the strong interaction is resistant to a weak coupling expansion, methods such as lattice field theory, the large $N_c$ (number of colours) expansion, and effective field theories (EFTs) have been developed to deal with it.

Our current understanding is that confinement holds for all $N_c$ when the number of light quark flavours is small enough. As a result, one may study arbitrary $N_c$ for the insights it gives into the central problem of the confinement of quarks inside hadrons, and the lack of asymptotic quark states. We argue here that although these insights and arguments are quite valid, a world with different values of $N_c$ can behave quite differently.

We take the case of $N_c = 2$ as an example. This was once a popular toy model in lattice studies of the strong interactions [6–16]. Not only does the smaller number of components of the gauge and quark fields make it easier to work with, the fundamental representation of SU(2) is self-contragradient. As a result, the integration measure in the path integral at finite baryon density is real. This allows direct numerical simulation at finite chemical potential, something which is not possible for SU(3) colour. The realization that the phase diagram and physics of QCD$_2$ is

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1 This can change if the hierarchy of scales in the standard model is modified drastically [2–5], which we do not plan to do.
quite different from that of QCD has dampened some of the enthusiasm. However, these results have recently leaked out into the wider world of nuclear physics [17–19], and this is where some caution needs to be exercised.

Here we explore the physics of QCD\(_2\), the theory of strong interactions with \(N_c = 2\) for the gauge group, with \(N_f\) quark flavours in the fundamental of the colour gauge group, and an unchanged electroweak sector, over many different length scales. This note is meant to serve multiple purposes. First, one can have fun constructing the physics of a world with two colours, in a long tradition of speculative fiction [20–23]. Second, it sharpens the questions one can (and cannot, yet) answer about our world by examining this counterfactual universe. An example is the answer to the question: given our knowledge of weak interactions, what is the simplest low energy experiment that shows that our SM has \(N_c = 3\) and not \(2\)? One surprising answer is that \(N_c\) is an odd number since we know that Hydrogen atoms exist. Third, an exploration like this may reveal technical niceties about the real world which are worth exploring. Pushan Majumdar was involved with the authors in an unfinished computation which led on from such speculation, and we present this work as a discursive memorial to a friend. Finally, our exploration illustrates a larger technical issue for the effective field theory program, since the physics at the nuclear, atomic, molecular, and longer length scales seems to know about changes at the length scale of quarks. Through this exploration, we probe how to interpret decoupling theorems in quantum field theory.

2. Particle physics

We state the model before starting\(^3\). The lepton and electroweak sector of the SM remains unchanged. We will introduce quark flavours in pairs, so that the GIM mechanism continues to operate, and take \(u\) and \(d\) to be light. The SU(2) anomaly [24] forces us to have two or four generations of fermions [25]. Where the other flavours are concerned, one may take a slightly heavier \(s\), and significantly heavier other quarks. Apart from this rough hierarchy, we allow the masses to be generic. When we consider the chiral limit, we take two or three lightest flavours to vanish. The substantive change is that \(N_c = 2\), so the strong interactions are described by QCD\(_2\) and not QCD.

2.1. Quarks

We have made a minimal change to the SM. The charged weak current connecting quarks and leptons continues to allow decays such as \(s \to u\tau_l\) in QCD\(_2\), through the usual Cabibbo Kobayashi Maskawa mixing of quark flavours. This also requires the charges of all up-type and down-type quarks to be equal, and the two charges to differ by unity, i.e.

\[
Q_u - Q_d = 1
\]

The Adler Bell Jackiw (ABJ) anomaly cancellation [1] also gives the relation

\[
Q_u^2 - Q_d^2 = \frac{1}{N_c}.
\]

These two together give \(Q_u = (1 + N_c)/(2N_c)\) and \(Q_d = (1 - N_c)/(2N_c)\) [25–28]. The \(u\) is positively charged, and the \(d\) is negative for all \(N_c\). For \(N_c = 2\) we find that \(Q_u = 3/4\) and \(Q_d = -1/4\). It is interesting that in the large \(N_c\) limit \(Q_u = -Q_d = 1/2\).

We will assume that confinement holds in QCD\(_2\). Then all meson and baryon states are colour singlet. Mesons can be built from quark antiquark pairs, exactly as in QCD. At large \(N_c\), baryons are built using \(N_c\) quarks [29]. So the baryon number of each quark is taken to be \(1/N_c\). It is natural to extend this counting to \(N_c = 2\). Then a baryon in QCD\(_2\) is a diquark state. The fact that in QCD\(_2\) the baryon is a boson turns out to be consequential.

The first difference between our world and this alternative occurs at this point. All the baryons that one can build from the first generation of quarks, i.e. the QCD\(_2\) analogues of the nucleon, have fractional charge. Confinement does not rule them out. In fact, the observational absence of fractionally charged particles in our world is enough to rule out all even values for \(N_c\). The argument is the following. Take a baryon with \(n_u\) quarks and the remaining \(d\) quarks. Its charge is

\[
nQ_u + (N_c - n)Q_d = n - (N_c - 1)/2\]

This is an integer only for odd \(N_c\).

2.2. Hadrons

A basic organizational tool in hadronic physics is the quark model. The chiral symmetry of QCD, for \(N_f\) flavours of massless quarks, is described by a global symmetry group \(G(N_f)\). It is spontaneously broken to the flavour subgroup \(H(N_f)\) by a flavour-singlet quark condensate. As a result, the particle spectrum is classified by \(H(N_f)\). In particular, the Goldstone bosons all lie in the same representation of \(H(N_f)\). In QCD \(G(N_f)\) is \(SU_f(N_f) \times SU_R(N_f) \times U_V(1)\), where the two \(SU(N_f)\) factors act on the left- and right-handed components of the quarks, the vector flavour singlet \(U_V(1)\) corresponds to the baryon number symmetry, and the quantum theory has no axial \(U_A(1)\) symmetry. This is broken to \(H(N_f)\), which is \(U_V(N_f)\). The \(N_f^2 - 1\) Goldstone bosons are pseudoscalar mesons which all lie in the same

\(^3\) We follow the metric and other conventions of Weinberg [1].
representation of \( SU_V(N_f) \). \( N_f = 3 \) gives Gell-Mann’s eighfold way.

When the gauge group of the strong interactions is \( SU(2) \), it turns out that a Pauli–Gürsey mechanism enlarges the global symmetry group to \( SU(2N_f) \) (see [30] for details). This fact has been used for model building in the past [31]. The largest subgroup, \( H(N_f) \), which results from a spontaneous symmetry breaking through the formation of a flavour singlet condensate is the unitary group \( USp(N_f) = SU(2N_f) \cup Sp(2N_f) \). This contains the usual flavour group \( U_V(N_f) \) as a subgroup. The number of Goldstone bosons is \( (N_f - 1)(2N_f + 1) \), and they all lie in one irrep of \( USp(N_f) \). They are \( N_f^2 - 1 \) pseudoscalar mesons and \( N_f(N_f - 1) \) baryons. The fact that they would lie in different irreps of \( U_V(N_f) \) is consistent with the fact that \( USp(N_f) \) is larger and contains generators which connect them.

One can proceed to write down an effective field theory of the interacting Goldstone bosons. In such a theory, all operators which are allowed by symmetry should be included. This means that operators which change the baryon number could also appear. This is another difference between this world and ours.

Forbidding baryon number changing currents from the symmetry broken theory is equivalent to asking \( H(N_f) \) to be \( U_V(N_f) \). In this case, the number of Goldstone bosons is \( 3N_f^2 - 1 \). This requires a much larger irrep of \( SU(N_f) \) than the adjoint which contains the pseudoscalar mesons. One also sees that an irrep as large as this cannot be made either of \( p\bar q \) mesons or \( qq \) baryons, since both these constructions give irreps no larger than \( N_f^2 \). As a result, this kind of symmetry breaking would also yield a particle physics very different from that in the SM.

The origin of these oddities is the Pauli–Gürsey construction allowed in QCD$_2$ [30]. The fact that the fundamental representation of \( SU(2) \) is self-contragradient allows one to construct the following global transformation which commutes with the gauge symmetry

\[
\psi_f \rightarrow a\psi_f - b\psi_f^c, \quad \text{where} \quad |a|^2 + |b|^2 = 1, \tag{3}
\]

with the notation that \( \psi^c \) is the charge conjugate of the quark field \( \psi \), and \( f \) is a flavour index. The corresponding current allows us to violate baryon number conservation. If one wants a conserved baryon number, then one can legislate that such transformations are not allowed. The global symmetry group then remains what it was for \( N_c = 3 \), and this is broken to the usual flavour group, \( U_V(N_f) \) by the condensate. The Goldstone bosons are then exactly the same pseudoscalar mesons, which lie in an adjoint representation of \( SU(N_f) \). The baryon multiplets are distinct, and Weingarten’s theorem [32] tells us that all of them are massive.

The simplest way to impose baryon number conservation is to introduce it as a constraint into the path integral of the theory. As usual, this can be implemented in the form of a term in the Lagrangian that breaks the larger \( SU(2N_f) \) symmetry to the desired one explicitly. The extra term is

\[
L_B = B \sum_{f=1}^{N_f} \bar{\psi}_f \gamma_0 \psi_f \tag{4}
\]

whose coefficient, \( B \), is analogous to a magnetic field which breaks the \( SU(2N_f) \) symmetry. We will call this a baryomagnetic field. A side effect of this term is to lift the degeneracy between the baryon and the pion, and to make the former more massive [33]. The scalar baryon mass is proportional to \( B \) when this parameter is not too large.

For \( N_f = 2 \) the Goldstone bosons are an isotriplet of pions, with the usual charges. The scalar baryon is a isosinglet \( uu \) combination and has charge 1/2. There is an isosinglet of vector baryons, with charges 3/2, 1/2, and \(-1/2 \). The isotriplet of baryons is heavier than the isosinglet for generic values of \( B \).

For \( N_f = 3 \), the Goldstone bosons are the usual octet of pseudoscalar bosons, the \( \pi^0, \pi^\pm, K^0, K^- \), and \( \eta \). The scalar baryons are in a \( \mathbf{3} \) of \( \text{ud}, \text{us}, \text{sd} \) states such as glueballs could be as light as (or lighter than) the baryons. The answer is hard to give without a lattice computation. However, one could argue [29] that when one changes \( N_c \) and fixes the scale of QCD by an independent scale such as the glueball mass, then the baryon mass is proportional to \( N_c \). For sufficiently large \( N_c \), glueballs could become lighter than baryons. However, for \( N_c = 3 \) lattice computations indicate that the glueball mass is about 50% more than the baryon mass. As a result, baryon

\[
\text{Fig. 1} \quad \text{The lowest multiplets of baryons in QCD}_2, \text{ with 3 flavours of quarks, when the chiral symmetry group obeys baryon number conservation. The non-strange baryons are given names which are used in the discussion of the nuclear physics of this world}
\]


masses could be expected to be significantly smaller than the glueball mass for \( N_c = 2 \). This counting could be suspect at \( N_c = 2 \), but the argument through the breaking of Pauli–Gürsey symmetry also leads us to believe this.

We close this part of the discussion with a brief remark on another interesting issue. The chiral anomaly of QCD persists into QCD\(_2\). The amplitude of the decay \( \pi \rightarrow 2\gamma \) involves the ABJ anomaly term and contains the coefficient \( N_c (Q_u^2 - Q_d^2)/f_x \), which goes as \( 1/\sqrt{N_c} \), using Eq. (2) and taking \( f_x \propto \sqrt{N_c} \) [34]. The Wess–Zumino–Witten term gives the amplitude for the reaction \( K^+ K^- \rightarrow \pi^+ \pi^- \pi^0 \) to be order \( N_c \). Taking into account the appropriate power of \( f_x \), the amplitude for this reaction is proportional to \( 1/N_c^{3/2} \). This power counting assumes the dominance of planar graphs and could be inaccurate in scaling from \( N_c = 2 \) to \( 3 \).

2.3. Phase diagram

The discussion of the phase diagram of QCD\(_2\) in the literature is in the context of the breaking of SU(2\(N_f\)) symmetry. This has a conserved current which mixes quark fields with their charge conjugates. As a result, the diquark condensate and the chiral condensate become related. These studies are not relevant to the version of QCD\(_2\) we discuss.

Lattice studies of the phase diagram of the model we are interested in would have to include the term in \( B \) in the action. In the Euclidean action, this would appear as an imaginary part to the baryon chemical potential, \( \mu \). It would have to be tuned to fix the nucleon mass at a fixed lattice cutoff and then left unchanged for a scan of the real part of the chemical potential. In this set-up, the path integral of QCD\(_2\) at finite baryon chemical potential is no longer real. In spite of the sign problem, working with an SU(2) gauge group takes about a third of the CPU time that SU(3) takes. So we can write the NN potential is attractive in QCD\(_2\), as it is in QCD.

3. Low energy physics

3.1. Nuclear physics

Is there nuclear physics in this world? If the masses of the quarks are similar to those in our world, then the only beta-stable non-strange baryons are the isosinglet scalar \( ud \) (which we will call the nucleon, \( N \)), and the vector \( uu \) (which we will call \( \Delta^{++} \)). It is interesting to note that once baryon number conservation is imposed on the theory, and confinement holds, the lightest baryon must be stable. The vector \( ud \) (\( \Delta^{+} \)) can go to \( \Delta^{++} \) by a weak decay, and the \( dd \) (\( \Delta^{-} \)) can decay to \( N \) or \( \Delta^{+} \) by the weak interactions, and more rarely to \( \Delta^{++} \). Also possible are the strong decays, \( \Delta^{++} \rightarrow N \pi^{+} \), \( \Delta^{+} \rightarrow N \pi^{0} \), and \( \Delta^{-} \rightarrow N \pi^{-} \). As a result, we expect the vector baryons to play a minor role in the structure of nuclei, just as the spin \( 3/2 \) \( \Delta \)s do in our world [34]. The chart of nuclei will be simple; it will contain integer baryon number \( B \) and charge \( Z = B/2 \). Since the isosinglet \( N \) is the only component of nuclei, there can be no isotopes. Also, since \( N \) has half integer charge, it might be possible to have nuclei which do not form neutral atoms.

Let us examine the question of nuclear physics in slightly more detail. Our understanding of the long-range nuclear force in our world as due to pion-nucleon interactions is related to QCD through baryon chiral perturbation theory. The leading piece of this action has an Yukawa interaction between pions and nucleons which is non-vanishing even when the nucleon is at rest (see [36] for a review). Since baryons remain massive in QCD\(_2\), there will be non-vanishing interactions between the bosonic nucleon and pions. Their origin is interesting enough to merit some discussion.

When the baryomagnetic field \( B \) vanishes, the SU(2\(N_f\)) symmetry breaking gives us a set of Goldstone fields \( \phi \), which contain both nucleons and pions. As a result, the interactions between them vanish in the static limit. Interaction terms such as \( \phi^4 \) have coefficients which are linear in \( B \) and vanish in the chiral limit. So the dimension-4 terms which give rise to nucleon-nucleon forces are of the form

\[
L_{\text{NN}} = \lambda \sum_{\pi, a} N_u^\dagger N_d \pi_a^\dagger \pi_a
\]

where \( N \) is the scalar baryon field, \( \pi \) is the pseudoscalar meson field, and \( \pi \) and \( a \) are flavour indices taking values in the appropriate multiplet. For \( N_f = 2 \), \( N \) is isoscalar, so \( \pi \) is redundant, and \( a \) takes value between 1 and 3. \( M \), the mass of \( N \), is proportional to \( B \), as is \( \lambda \). So we can write \( \lambda \propto M \). This parameter controls the strength of the NN interaction. The range of the interaction is given by the
pion mass, \( m \). Taking \( m_q \) to be the light quark mass, one has, as usual, \( m \propto \sqrt{m_q} \). So in this version of the strong interactions, there are independent parameters to tune the range and strength of the NN forces. The sign of \( \lambda \) can be arranged to give attractive interactions at long distances.

At very short distances one should expect repulsion if stable tetraquark (the QCD\(_2\) analogue of dibaryon), triquark-antiquark (the analogue of pentaquark) or other exotic baryon states are absent in the light quark sector. Then nuclei can form, and nuclear matter away from the chiral limit is roughly incompressible, as in our world\(^3\). It is noteworthy that in this world the spectrum of the \( B = 2 \) nucleus can be tuned by changing \( M \) and \( m \). One can have a situation with a complex nuclear spectrum for \( B = 2 \), or a single shallow bound state, as in our world, or even a situation where \( B = 2 \) is unbound, but the \( B = 3 \) state is a Borromean nucleus [37]. More generally, studies of pairwise interacting bosons show [38, 39] that the \( B = 3 \) and \( B = 4 \) nuclei are likely to exist in QCD\(_2\).

Since \( N \) is spinless, one may first think that the shell structure of nuclei is irrelevant in QCD\(_2\), and all nuclei have spin zero. However, for large enough \( B \), the Coulomb repulsion between the \( N \)s would eventually make it favourable to populate a higher quantum state in the effective NN potential. Consider the nuclear Hamiltonian

\[
H = \sum_{i=1}^{B} \left[ \frac{p_i^2}{2M} + V(r_i) \right] + \frac{e^2}{4} \sum_{i>j} \frac{1}{|r_i - r_j|},
\]

where \( M \) is the mass of the \( N \), \( e \) is the charge of the electron, and \( V \) is a short range potential which arises in the usual way through the nuclear mean field. For simplicity, choose the simplest shell model potential, \( V(r) = kr^2/2 \).

The single particle level spacing is \( \Omega = \sqrt{k/M} \). Taking the Coulomb energy term as a perturbation, one sees that when \( B(B-1)/2 \) is large enough, the contribution of this term may exceed \( \Omega \). When this happens, it is energetically favourable to start filling the next shell. As a result, there is a rudimentary shell structure of the nucleus. Clearly, this argument is not specific to the choice of \( V(r) \).

There is an interesting piece of physics one can investigate here. The mean field potential \( V(r) \) arises from the NN potential. Since it is short ranged, the number of bound states is finite. As a result, when \( B \) is larger than some critical \( B_d \), the Coulomb repulsion will become so large that there will not be a bound state which accommodates all \( B \) nucleons. This is the QCD\(_2\) analogue of drip lines in our world. Since the NN potential is tunable by changing \( M \) and \( m \), one may be able to investigate the dependence of \( B_d \) on \( M \) and \( m \). It is particularly interesting to ask whether it is possible to have macroscopic charge neutral objects which are not bound by gravity, but by the strong interactions.

In our world, one can describe the binding energies of nuclei in a liquid drop model [40]. This depends mainly on the existence of a short range effective central potential, \( V \) in Eq. (6). So it should continue to work for nuclear physics built over QCD\(_2\). The volume term is due to the mean-field averaged nuclear potential, \( V(r) \) in eq. (6), the surface term is due to remnant two body nuclear forces which are not shown in eq. (6), and the origin of the Coulomb repulsive term is clear. The asymmetry and pairing terms, which account for isotopes, are absent. The binding energy per nucleon will therefore first increase slightly with \( B \) because of the remnant two body terms and then decrease due to the Coulomb term.

Is there radioactivity in this world? Nuclei will have excited states, which could decay radiatively. Weak decays are ruled out, since \( N \) is stable and isoscalar. Is there an analogue of \( \alpha \) decay? In our world, this is due to two factors. The first is that there is an unique light nucleus, \(^4\)He, which is extremely tightly bound. This is because two nuclear shells close with \(^4\)He, and the next stable nucleus has mass number 6. The second factor is that heavy nuclei become less tightly bound as the baryon number increases. In QCD\(_2\), the shell structure sets in at much higher values of \( B \). As a result, there is no tightly bound light nucleus which could be emitted to stabilize a heavy nucleus. Can a heavy nucleus emit a single \( N \) to stabilize itself? This is possible, since the first nucleus which has a nucleon in an upper shell may be able to lower its energy significantly by ejecting a nucleon. So a rudimentary version of radioactivity is not ruled out.

This radioactive decay mode also opens up the possibility of spontaneous, as well as induced, fission. Such a nucleus with a \( B \) just large enough to send a few nucleons into a new shell could tunnel into a lower energy configuration with two nuclei, both with all constituents in a lower shell. Chain reactions are also possible.

Fusion is possible in spite of the absence of an unusually tightly bound light nucleus like \(^4\)He. As long as the binding energy per nucleon increases with \( B \), it is possible to have a chain of nuclear reactions which produce heavier nuclei. This chain would generally stop when increasing \( B \) by unity no longer increases the binding energy per nucleon.

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\(^3\) Both the intermediate distance regime, where multi-meson exchanges and the tower of higher spin mesons have to be accounted for, and the chiral limit, where intermediate states with infinite number of pions are not suppressed by energy considerations, require a more detailed analysis. If nuclear physics and bulk nuclear matter in QCD\(_2\) were of greater interest, these effects would have to be computed.
3.2. Atomic physics and chemistry

Let us move on to even lower energy. Uncharged atoms have integer $Z$, increasing from unity, with baryon number, $B$, increasing in steps of 2. There are no isotopes, so one always has $B = 2Z$. There is no analogue of the hydrogen atom of our world, with $B = 1$ and $Z = 1$. However the atom with $Z = 1$ and $B = 2$, which we denote as $^2H$, has a spectrum which is like that in our world, with a value of the Rydberg constant which is the same as ours. In fact this sector of nuclei gives rise to atomic physics which closely parallels that in our world, except for a near absence of hyperfine structure, since nuclei have vanishing spin (except when higher shells are occupied). This sector of the periodic table is the same as in our world, and its chemistry is nearly identical. The main difference is that the change in atomic masses will change reaction rates somewhat.

On the other hand, there is also exotic atomic and molecular physics. There is, for example, a charged bound state of $N$ and $e$, which is an exotic version of hydrogen with half integer negative charge, $^1H^-$. It has a spectrum similar to that of $^2H$ with energy levels at half the value, so that the spectrum is shifted further to the infrared. Our world has a positively charged ion $H_1^+$, which is a Hydrogen molecule with one electron removed. In the QCD2 world, one has a molecule $\left( ^1H \right)_2^0$ which has one electron orbiting two separate nuclei N. Positive and negative ions of different elements with half integer charges are also possible. The change in the effective value of the Rydberg in this sector changes reaction rates drastically. Redox reactions with $^1H$ will proceed much faster than those with $^2H$, for example.

Matter in the bulk could consist of uncharged molecules, either of the kind which are familiar to us (like $\left( ^2H \right)_2$ or more complex molecules) or the exotic molecules which we discussed (such as $\left( ^1H \right)_2$ or more complex molecules) or molecules formed of familiar and exotic atoms. (Could $\left( ^1H \right)_2^2H$ exist in this world?) Electromagnetic plasmas are also possible as bulk matter, as long as they are overall electrically neutral. Either state of matter at the scale of $10^{23}$ molecules will behave in a manner more or less familiar to us, albeit with detailed properties which could be very different.

3.3. Cosmology and astrophysics

The physics of the inflationary and immediate post-inflationary epoch of the universe is determined by physics beyond the SM, and we have little understanding of it. Often this is dealt with by writing a low-energy limit of this extended theory as an effective field theory built out of only the SM fields. Much work has been done with this so-called SMEFT, and many results could be taken over to this case. Since it is still an open problem, we do not consider its effects on the imaginary universe we are creating.

The physics of somewhat shorter cosmological length scales is determined largely by the structure of the electroweak theory. One interesting question here is the baryon asymmetry of the universe. Could the QCD2 universe also have a baryon asymmetry? In the SM $B - L$ (where $L$ is the lepton number) is conserved due to the GIM mechanism and the consequent cancellation of the ABJ anomaly. This generalizes to SM with any $N_c$ in the QCD sector. Unless the SMEFT is radically different, one would expect that the mechanism which gives rise to baryon asymmetry of our universe continues to operate in this one.

The history of this universe could diverge from that in ours a little after the time of the QCD2 phase transition. Primordial nucleosynthesis (BBN) would certainly be different because of differences in the baryon multipoles. For $N_f = 3$, weak decays of the $su$ scalar baryon must first give a $D^+$, which then decays into the $N$ by emitting a pion. This weak decay rate depends on the relative masses of the scalar and vector baryons. So it may be possible to create hypernuclei at BBN. Even otherwise the difference in the process of nuclear fusion will change BBN.

The most interesting subsequent physics is the decoupling of photons and matter. In QCD2, this proceeds differently. The decoupling temperature is a balance between the photon density, matter density, and the Rydberg energy. When there are sufficient number of photons per atom in our universe to ionize a good fraction of hydrogen, one has a plasma, and photons undergo Thompson scattering with ease. When the temperature falls to a point that the probability of an atom being ionized by a photon is small enough, then matter becomes largely uncharged, and the sky becomes transparent. If the conditions at very early times are similar, then the only difference between QCD2 and our world is in the atomic physics, assuming that we have tuned the nuclear interactions so that the spectrum of $^2H$ in this world is similar to our Deuteron. Primordial nucleosynthesis would produce a small amount of $^3\bar{H}$, leaving most nucleons as they are. When the temperature falls to 3000 K, then $^2H$ atoms bind, but that still leaves a highly charged plasma. When the temperature falls further, the remaining electrons will bind into $^1H^-$, but there would

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4 We adapt the standard notation for the nucleus of element X as $^{Z}\_X$ and drop the index Z, since it is always $B/2$.

5 It is interesting that specifically for $N_c = 2$ it is easy to write terms in SMEFT which violate $B$ and $B - L$, through dimension 6 operators. This could allow a different path for baryon asymmetry to be generated.
still be a charged plasma. Since the Thompson scattering cross section, $\sigma \propto e^2/m$, where $e$ is the charge of the lightest charged species and $m$ its mass, the mean free path of photons could increase substantially at this point, but still remain much smaller than the Hubble radius. It is only when the temperature falls sufficiently that neutral $^1H_2$ molecules form would the universe become transparent. Relic radiation in this world would have a different temperature at a comparable age of the universe, and the fluctuations in it could carry signatures of a time between the formation of $^1H^-$ and decoupling.

The subsequent evolution of the universe involves gravity, and we know no reason that at long distances it knows about $N_c$. As a result, the overall shape of the universe would remain the same. However, gas clouds made of cold charged atoms will interact differently with radiation than in our universe. The chemistry of gas clouds would be different. Their collapse into galaxies and smaller objects is unlikely to change. Stars in this universe would still be in self-regulating hydrostatic equilibrium because there are analogues of nuclear fusion. However, the details of stellar evolution will be very different since differences in nuclear structure would change thermonuclear pathways radically. The flip side of hydrostatic equilibrium is core collapse. Collapsed objects like black holes, and analogues of white dwarfs and neutron stars are possible. So although the stars, galaxies and gas clouds behave quite differently, the overall appearance of the universe may not be very different from ours.

4. Conclusion

We examined the physics of a world where the standard model changes in a single respect: the value of $N_c$ goes from 3 to 2. We minimized the consequent changes in particle physics by insisting on retaining baryon number conservation. In spite of this, a seemingly minor change at the Fermi scale induces a whole cascade of changes in the physics at long distances. The changes in particle physics are expected. We argued that the physics of the phase diagram of QCD$_2$, with baryon number conservation, may differ from that in QCD only in minor (but interesting) ways. However, at longer scales, the EFT which describes NN interactions changes. At even longer length scales, nuclear and atomic physics, as well as chemistry and astronomy, are significantly different.

This change in the physics at very long distances due to a single change at short distances may at first be surprising to a quantum field theorist. On examining this in detail, one sees that it is due to two facts: first that baryons are bosons, and second that baryons are fractionally charged (in units of the electron charge). These are the open windows through which UV physics escapes into the IR world. This is therefore a cautionary tale about making too strong an assumption about UV/IR decoupling in quantum field theory. It is not only the global symmetries that are important, but also the nature of the fields which represent them.

From our discussions, it is clear that fractionally charged bosonic baryons are characteristic of all QCD-like theories with even $N_c$, coupled to an unchanged remainder of the standard model. The long-distance physics of all these model worlds will differ from ours in the ways that we outlined for $N_c = 2$. For odd $N_c$, baryons are fermions and carry integer charges. It is also possible to construct a nucleon isodoublet of spin 1/2 in all these models, and arrange them to have charges which are the same as in our world. These worlds will differ only subtly from ours. The existence of galaxies and stars, stable nuclei and complex chemistry means that the anthropic principle (in the sense used in [41]) cannot be invoked to rule out any value of $N_c$. a priori. In this context at least, if the anthropic principle is used as a tool to decide on the viability of theories, then it is a pretty blunt tool.

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