On some hybrid-types of $Q$ balls in the gauge-mediated supersymmetry breaking

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We revisit the new-type of the $Q$ ball (the gravity-mediation type of the $Q$ ball in the gauge-mediation), in order to clarify its properties and correct some misunderstandings found in the recent literature. In addition, we investigate the feature of the other kind of the hybrid-type of the $Q$ ball, which was considered in the context of the $Q$-ball capture by the neutron star.

I. INTRODUCTION

A $Q$ ball is a nontopological soliton, the minimum energy configuration of the (complex) scalar field, whose existence is guaranteed by non-zero charge $Q$\cite{1}. It appears naturally in supersymmetric theories\cite{2,3}. In particular in the gauge-mediated supersymmetry (SUSY) breaking, the $Q$ ball is stable against the decay into fermions and other scalars for large enough charge $Q$, and can be the dark matter of the universe\cite{2}. Such large $Q$ balls are naturally produced in the early universe as byproducts of the Affleck-Dine mechanism for baryogenesis\cite{2,4,5,6}. They could be detectable and/or constrained by various experiments\cite{6,7,8} and by considering astrophysically such as the capture by neutron stars\cite{9,10}.

The properties of the $Q$ ball is determined by the shape of the scalar potential. In the gauge-mediated SUSY breaking, the potential is flat beyond the messenger scale, and the mass grows as $M \propto Q^{3/4}$\cite{11}. As the field amplitude becomes large, the potential will be dominated by the effect of gravity-mediation, and the features of the $Q$ ball change such as $M \propto Q$, for example. We called this kind of the $Q$ ball the new-type $Q$ ball\cite{12}. As we mentioned in\cite{12}, the ‘metamorphosis’ of the $Q$ ball should take place smoothly.

In this article, we revisit the properties of the new-type $Q$ balls, with special attention to clarify the transition region between the gauge and gravity mediation. This is partly because we must correct some misunderstandings found in\cite{13}, where they claim the new-type $Q$ ball disintegrates into the gauge-type ones.

In addition, we also investigate the features of the other kind of hybrid type of the $Q$ ball considered in\cite{10}, where the gauge-type $Q$ ball changes to the thin-wall-type $Q$ ball\footnote{In Ref.\cite{10}, it is called the ‘curved direction’ $Q$ ball. In this article, we call it the ‘thin-wall-type’ $Q$ ball because of its profile as shown in Fig.\ref{fig2}.} as it fattens in the interior of the neutron star. This happens since the field value inside the $Q$ ball cannot grow further when the potential is lifted by nonrenormalizable operators at large field values\footnote{This thin-wall-type $Q$ ball is not created through the Affleck-Dine mechanism, since the scalar field does not feel spatial instabilities while it stays on the nonrenormalizable potential.}.

II. $Q$ BALL SOLUTION

Let us first review the general properties of the $Q$ ball in the scalar theory with a global $U(1)$ symmetry. In the context of SUSY $Q$ balls, the charge is usually the baryon and/or lepton numbers. The energy and charge are given respectively by

$$E = \int d^3x \left[ \partial_\mu \phi \partial^\mu \phi^* + V(\phi) \right], \quad (1)$$

$$Q = \frac{1}{i} \int d^3x (\dot{\phi} \phi^* - \phi \dot{\phi}^*), \quad (2)$$

where $V(\phi)$ is the potential. Since the $Q$ ball is the energy minimum configuration of the scalar field with finite charge $Q$, using a lagrange multiplier $\omega$, we can write the energy as\cite{14}

$$\mathcal{E}_\omega = E + \omega \left[ Q - \frac{1}{i} \int d^3x (\dot{\phi} \phi^* - \phi \dot{\phi}^*) \right]. \quad (3)$$
Energy minimum configuration is obtained when the solution is spherical symmetric and rotating: \( \phi(x) = \varphi(r)e^{i\omega t}/\sqrt{2} \). Then, the energy is rewritten as

\[
E_\omega = 4\pi \int_0^\infty dr r^2 \left[ \frac{1}{2} \left( \frac{d\varphi}{dr} \right)^2 + V(\varphi) - \frac{1}{2} \omega^2 \varphi^2 \right] + \omega Q. \tag{4}
\]

In order to obtain the energy minimum solution, we just have to solve the equation

\[
\frac{d^2\varphi}{dr^2} + \frac{2}{r} \frac{d\varphi}{dr} + \left[ \omega^2 \varphi - \frac{dV}{d\varphi} \right] = 0, \tag{5}
\]

with boundary conditions \( \varphi(\infty) = 0 \) and \( \varphi'(0) = 0 \).

In the next sections, we apply the above argument and solve the equation (5) numerically for two kinds of hybrid-type \( Q \) balls: One is the gauge-type and new-type \( Q \) balls, and the other is the gauge-type and thin-wall-type \( Q \) balls.

### III. NEW-TYPE \( Q \) BALLS

The potential is written as \[12\]

\[
V(\phi) = m_\phi^4 \log \left( 1 + \frac{\phi^2}{m_\phi^2} \right) + m_{3/2}^2 \phi^2 \left( 1 + K \log \frac{\phi^2}{M_*^2} \right), \tag{6}
\]

where the first (second) term comes from the gauge- (gravity-)mediation effect. Here \( m_\phi \) is the scalar mass in the vacuum, \( m_{3/2} \) the gravitino mass, \( K < 0 \) the coefficient of the one-loop effect, and \( M_* \) a renormalization scale. When the each term of the potential dominates, the properties of each type of the \( Q \) ball are well known. For the gauge-type \( Q \) ball, the energy \( E \), the size \( R \), the rotation speed \( \omega \), and the field value at the center \( \varphi_c \) are given by \[2, 6, 11\]

\[
E \sim m_\phi Q^{3/4}, \\
R \sim \omega^{-1} \sim m_\phi^{-1} Q^{1/4}, \\
\varphi_c \sim m_\phi Q^{1/4}, \tag{7}
\]

while for the new-type \( Q \) ball \[3, 12\],

\[
E \sim m_{3/2}^2 Q, \\
R \sim |K|^{-1/2} m_{3/2}^{-1}, \\
\omega \sim m_{3/2}, \\
\varphi_c \sim m_{3/2} Q^{1/2}. \tag{8}
\]

To look also for the transition region, we solve numerically Eq.(5) for the potential (6). In Fig. 1 the energy, size, the field value, and energy per charge as a function of the charge are shown. In these figures, we set \( m_{3/2}/m_\phi = 10^{-5}, K = -0.01, M_\phi/m_\phi = 10^{10} \). One can see the features of both gauge-type and new-type \( Q \) balls, i.e., Eqs.(7) and (8), are reproduced in Fig. 1. Moreover, those parameters are smoothly connected in the transition region, as it should be. In particular, the energy per charge \( E/Q \) always decreases as the charge \( Q \) increases, which implies that the larger \( Q \) ball is energetically favored. This shows that the new-type \( Q \) ball is stable against disintegration into smaller gauge-type \( Q \) balls, so that it can be the dark matter of the universe, contrary to the claim in Ref. \[13\].

### IV. THIN-WALL-TYPE \( Q \) BALLS

Now let us consider the following potential,

\[
V = m_\phi^4 \log \left( 1 + \frac{\phi^2}{m_\phi^2} \right) + \frac{\lambda^2 \phi^{2(n-1)}}{M_*^{(n-3)}}. \tag{9}
\]
Although the thin-wall-type $Q$ ball is not created in the early universe, it could be formed through charge accumulation from the gauge-type $Q$ ball when the latter is swallowed by the neutron star \cite{10}. The properties of the thin-wall-type $Q$ ball are also well known as \cite{1, 10}

$$E \sim \mu Q,$$
$$R \sim \left(\frac{\mu Q}{m_\phi}\right)^{1/3},$$
$$\omega \sim \mu \approx \frac{m_\phi^2}{\varphi_c},$$
$$\varphi_c \approx \left(\frac{m_\phi^2 M_{P}^{n-3}}{\lambda}\right)^{1/(n-1)}.$$ 

To see the transition region as well, we solve numerically Eq. (5) for the potential (9). In Fig. 2, we show the profile of the $Q$ balls, where one can see the growth of the thin-wall-type $Q$ ball as well as the deformation from the gauge-type $Q$ balls as the charge is accumulating. $Q$-ball properties are shown in Fig. 3. They coincide to analytical estimates \cite{7} and \cite{11} for the gauge-type and thin-wall-type $Q$ balls, respectively, and they are smoothly connected in between.

V. CONCLUSIONS

We have revisited the new-type of the $Q$ ball, and clarified its properties. In particular, we have focused on the energy per charge $E/Q$ as a function of the charge $Q$, and reconfirmed the stability of the new-type $Q$ ball to be the
dark matter of the universe. This corrects some misunderstandings in Ref. [13]. In addition, we have investigated the feature of the $Q$ ball, transiting from the gauge-type to thin-wall-type $Q$ balls, which was considered in the context of the $Q$-ball capture by the neutron star in Ref. [10]. Since the energy per charge $E/Q$ decreases as the charge $Q$ increases through the transition region, we confirm the ‘metamorphosis’ of the gauge-type to thin-wall-type $Q$ balls.

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FIG. 3: Energy, size, field value at the center, and energy per charge of the $Q$ balls. Green and blue lines show the $Q$-dependence estimated analytically for the gauge-type and thin-wall-type $Q$ balls, respectively.