Researching and solving a bicriteria supply management problem with the given volumes of batches

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Abstract.
We consider the supply management problem in which a product is supplied by batches. For all providers, the volumes of these batches are known. The total preference of the supply assignments is maximized and the number of providers for the consumer with a maximum volume of demand is minimized. For other consumers, the upper bounds on a number of providers depend on their demand. The NP-hardness of finding a feasible solution to this problem is shown. A bicriteria model of integer linear programming (ILP) is constructed for the problem under consideration. We showed that the cardinality of a complete set of alternatives (CSA) is polynomial. To search for solutions of CSA, we construct and investigate experimentally the single-criterion model of ILP. A heuristic algorithm for finding solutions close to the Pareto optimal is proposed. It is based on fixing values of a part of integer variables and solving the smaller-dimension ILP problem. This algorithm is implemented using the CPLEX solver. The results of the computational experiment for the heuristic algorithm on random instances are presented.

1. Introduction
The supply management problems are quite diverse in formulations and are of great practical importance. They arise in the areas of production planning, of shipment flows management, of scheduling the products processing, of the area of education, etc. In many cases, mathematical models of such problems are formulated as discrete programming problems. The problem of this class was considered in [5]. It differed from the well-known transport problem with fixed surcharges by the presence of lower and upper bounds on the quantity of each supply. In [5] for problems with one and several manufacturers, the properties of optimal solutions are established. On the basis of these properties, heuristic algorithms are proposed.

In [4] NP-hardness of finding a feasible solution was shown even in the case of one manufacturer. A pseudo-polynomial algorithm based on the dynamic programming technique was proposed. In [1] the problem of optimization of product delivery with linear delivery costs was considered. A fully polynomial time approximation scheme was suggested for this problem with single manufacturing unit, and its hardness was shown for greater numbers of manufacturing units. In [9] the same supply management problem with a set of manufacturing units was studied using the regular partitions method [8]. A parametric family of problems with exponential
$L_k$-coverings is constructed. Problems of this family are difficult for solving by many methods of integer linear programming (ILP). NP-hard expansion of the family of problems with the same property is formulated. In [7] was considered the supply management problem with a single consumer: to find a minimum cost delivery plan, given a set of admissible intervals for the shipment sizes of each supplier, lower-bounded demand, linear variable costs of delivery and fixed costs for opening each shipment. A fully polynomial time approximation scheme for this problem is proposed.

In this article, we consider a bicriteria supply management problem in which the one type product is shipped from a set of providers to several consumers. The product can be supplied only by batches, the sizes of which are given. Batches may differ as product quality and as size. For each consumer, the preference for each batch is known.

The total volume of delivered product to each consumer can exceed his given demand. In this case, the surplus product can not exceed the size of the consumer warehouse.

To reduce transportation and/or organizational costs, it is necessary to reduce the number of suppliers for each consumer. In our problem formulation, this number depends on the size of the demand, i.e., it is undesirable that a consumer with a small volume of demand has a large number of providers. It is necessary to find a product supply plan that maximizes the total preference and minimizes the number of suppliers for consumers, taking into account their demand. Note that considered problem is a special case of the academic load distribution problem [10].

For the considered problem, we construct a bicriteria model of ILP. One of the approaches to solving multicriteria problems is to search for all or part of the Pareto-optimal solutions (POS). We showed that a complete set of alternatives is polynomial. We construct and investigate experimentally the single-criterion ILP model for the search for such solutions. For solving this problem, a heuristic algorithm is proposed. The results of the computational experiment for this algorithm and CPLEX solver are presented.

2. Problem Formulation

Let $m$ be the number of providers of the one type product and let $n$ be the number of consumers. Denote the number of batches of $i$-th provider by $t_i$ and the volumes of these batches by $b_{ki}^i$, $k = 1, ..., t_i$. Values $c_j$ and $a_j$ define the minimum and maximum demand (or lower and upper bounds of demand) for $j$-th consumer, $j = 1, ..., n$. Here $a_j - c_j$ is the size of the warehouse of the $j$-th consumer, $j = 1, ..., n$. Let $l_{kj}^i$ denotes the coefficient of the preference by the $j$-th consumer of the $k$-th batche of $i$-th provider, $j = 1, ..., n$, $k = 1, ..., t_i$, $i = 1, ..., m$.

It is required to find a plan of supply, in which the quantity of product sent to each consumer satisfies his demand, and the surplus of the supplied product does not exceed the size of his warehouse. It is necessary to maximize the total consumer’s preference and to minimize the number of providers for the consumer with the greatest demand. For other consumers, the number of providers should not be more than the value proportional to the volume of its maximum demand. The last condition we will be called the uniformity of the number of providers.

We introduce Boolean variables $z_{kj}^i$ and $x_{ij}$, where $i = 1, ..., m$, $j = 1, ..., n$, $k = 1, ..., t_i$. Here $z_{kj}^i = 1$ if the $i$-th provider supplies the $k$-th batch to the $j$-th consumer and $z_{kj}^i = 0$, otherwise; $x_{ij} = 1$ if the $i$-th provider supplies the product to the $j$-th consumer and $x_{ij} = 0$, otherwise. Let $a_{\text{max}} = \max_{j=1, ..., n} a_j$. In this case, we suggest modeling the condition of “uniformity” in the number of providers, as follows. The number of providers assigned to the consumer should not exceed the value of proportional to the number of providers assigned to the consumer with the maximum demand with a coefficient of proportionality $p_j = a_j / a_{\text{max}}$.

Let $y$ be a non-negative integer variable. Denote the vector of variables $z_{kj}^i$ by $\vec{z}$. Then the supply management problem with given volumes of product batches can be formulated as a bicriteria ILP problem in this way:
minimize $y$  

$$
\text{maximize } L(\bar{z}) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{t_i} l_{ij} k_{ij} \quad (2)
$$

subject to

$$
c_j \leq \sum_{i=1}^{m} \sum_{k=1}^{t_i} b_{i}^{k} z_{ij}^k \leq a_j, \quad j = 1, \ldots, n, \quad (3)
$$

$$
\sum_{j=1}^{n} z_{ij}^k = 1, \quad i = 1, \ldots, m, \quad k = 1, \ldots, t_i, \quad (4)
$$

$$
x_{ij} \leq \sum_{k=1}^{t_i} \sum_{j=1}^{n} z_{ij}^k \leq t_i x_{ij}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n, \quad (5)
$$

$$
\sum_{i=1}^{m} x_{ij} \leq p_j y + q, \quad j = 1, \ldots, n, \quad (6)
$$

$$
y \geq 0, \quad y \in \mathbb{Z}, \quad x_{ij}, z_{ij}^k \in \{0, 1\}, \quad k = 1, \ldots, t_i, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n. \quad (7)
$$

Here, the optimization criterion (1) means minimizing the number of providers that are assigned to the consumer with the maximal upper bound of demand and the criterion (2) maximizes the total preference by consumers of batches $L(\bar{z})$. Restrictions (3) ensure the assignment to $j$-th consumer of batches with total volume which satisfies the minimal and maximal demands. Equations (4) show that each batch of any provider must be assigned to only one customer. The inequalities (5) describe the relationship of the variables $x_{ij}$ and $z_{ij}^k$. The variable $x_{ij} = 1$ if and only if there exists the index $k$ such that $z_{ij}^k = 1$, i.e., the provider is assigned to the consumer if and only if at least one his batch is assigned to this consumer. Restrictions (6) guarantee that the number of providers assigned to the consumer will not exceed the amount proportional to the number of providers assigned to the consumer with the maximal upper bound of demand with a coefficient of proportionality $p_j$. Since the variables $x_{ij}$ are Boolean then we introduce the constant $q \in [0, 1]$ which controls the rounding of the quantities on the right-hand side, for example, $q = 0.5$.

It should be noted that this problem has another actual interpretation. It can be considered as the academic load distribution problem of teachers without restrictions on the number of discipline’s blocks assigned to each teacher [10].

It is obvious that the following condition is a necessary condition of the solvability of problem (1)–(7):

$$
\sum_{j=1}^{n} c_j \leq \sum_{i=1}^{m} \sum_{k=1}^{t_i} b_{i}^{k} z_{ij}^k \leq \sum_{j=1}^{n} a_j.
$$

In the investigated problem, the given volumes of batches can be considered as a special case of conditions of interval supplies in problem from [7]. In our case, all intervals have lengths equal to zero. On the other hand, we consider a more general case where there is not one but many consumers. Besides, the demand of each consumer has not only the lower but also the upper bound. Besides, the functions of supply preferences (i.e. costs) do not depend on their volume.

The considered problem is NP-hard since the search for its feasible solution is reduced to the bin packing problem which is NP-hard.
One of the approaches to solving of bicriteria problem is finding all Pareto-optimal solutions or a part of this solutions. We recall the definition of a complete set of alternatives (CSA)(see, for example, [6]). We consider the bicriteria discrete optimization problem

\begin{equation*}
\begin{aligned}
\text{minimize} & \quad F(x) = (f_1(x), f_2(x)) \\
\text{subject to} & \quad x \in X,
\end{aligned}
\end{equation*}

where $X$ is some finite set of feasible solution, $f_1(x)$, $f_2(x)$ are functions from $X$ to $\mathbb{R}$.

We denote the set of Pareto-optimal solutions by $\tilde{X}$, and a complete set of alternatives by $X_0$. For $X' \subseteq X$, we denote $F(X') = \{F(x) | x \in X'\}$. CSA is any set $X_0 \subseteq \tilde{X}$ which has the minimal cardinality and $F(X_0) = F(\tilde{X})$. It is obvious that $X_0 \subseteq \tilde{X} \subseteq X$.

Note that the cardinality of a complete set of alternatives for the constructed problem (1)–(7) is polynomial because the value of the second criterion does not exceed $m$.

To search for the part of CSA, we construct a single-criterion problem:

\begin{equation*}
\begin{aligned}
\text{maximize} & \quad L(\bar{z}) \\
\text{subject to} & \quad (3)–(7) \text{ and } y \leq y_{\text{max}}.
\end{aligned}
\end{equation*}

To obtain CSA, it is necessary to solve a series of problems (2)–(8) for $y_{\text{max}} = 1, ..., m$. Solutions corresponding to small values $y_{\text{max}}$ are the most interesting. Note that the optimal solution of problems (2)–(8) can satisfy conditions (6) for $y < y_{\text{max}}$. The smallest of these values is denoted by $y_{\text{real}}$. It is clear that the optimal solutions of problem (2)–(8) for $y_{\text{max}}$, $y_{\text{max}} - 1$, ..., $y_{\text{real}} - 1$ are not Pareto-optimal.

3. Computational experiment

3.1. The results of applying the bicriteria model

Series of tasks with random instance were generated. To search for CSA, we used CPLEX solver for ILP model (2)–(8). The calculations were performed on a PC with an Intel Core i3 processor (3.30 GHz).

Series D consisted of 10 tasks of medium dimension with the following parameters: $m = 16$, $n = 8$, $t_j \leq 4$. The number of batches in tasks was on average 53. The corresponding ILP tasks had on average 550 Boolean variables and 340 constraints. Values of another parameters were generated uniformly from the intervals: $a_j \in [400, 800]$, $b_{ik} \in [40, 120]$, $l_{ij}^k \in [1, 10]$.

Series S included 10 tasks of greater dimension: $m = 45$, $n = 22$, $t_j \leq 9$, $a_j \in [300, 1400]$, $b_{ik} \in [40, 120]$, $l_{ij}^k \in [1, 10]$. The number of batches was on average 225. In both series for the consumer $j$, the volume of the warehouse was 5% of $a_j$. The dimension of the corresponding ILP tasks was on average 5900 × 2300.

Let $h_i = \lfloor p_i y_{\text{max}} + r \rfloor$, where $\lfloor \alpha \rfloor$ is the lower integer part of number $\alpha$. As the results for the series D showed, with decreasing the value $y_{\text{max}}$, the number of providers for each consumer in the optimal solution tends to $h_i$, i.e. for small values $y_{\text{max}}$, the number of providers satisfies the "uniformity" condition. For example, Table 1 shows the results for task D8. The number of providers which coincides with the corresponding $h_i$, $i = 1, 2, ..., 8$, is indicated in bold type.

For $y_{\text{max}} \leq 8$ the obtained solutions are Pareto-optimal. Really, for $y_{\text{max}} = 30$ a solution with $y_{\text{real}} = 8$ was obtained. Starting with $y_{\text{max}} = 8$, the value $L^*$ decreases at decreasing $y_{\text{max}}$. In Table 1, $\tilde{L} - L^*$ is the duality gap.
Table 1. The results of solving task D8 for various $y_{max}$

| $y_{max}$ | POS | $L^*$ | $\bar{L} - L^*$ | Time (sec) | Amount of providers for consumers: |
|-----------|-----|-------|-----------------|------------|-----------------------------------|
| 30        | -   | 467   | 2,23            | 0,2        | 6 4 4 5 4 5 8 5 8               |
| 8         | +   | 467   | 2,23            | 0,5        | 6 4 4 5 4 5 8 5 8               |
| 7         | +   | 464   | 5,53            | 0,9        | 6 4 4 5 5 5 7 4 7               |
| 6         | +   | 458   | 11,53           | 4,7        | 5 3 4 6 4 4 6 4 6 4 6           |
| 5         | +   | 446   | 23,53           | 4,8        | 4 3 4 5 3 4 5 3 5               |
| 4         | +   | 426   | 43,53           | 67         | 4 2 3 4 3 3 4 3 4               |
| 3         | +   | 388   | 81,53           | 139        | 3 2 2 3 2 2 3 2 2               |
| 2         | 0,1 | no solution |

In series D for 8 tasks, the optimal solutions were obtained by the CPLEX solver (see Table 2). The tasks D1 and D5 were difficult to solving. For them, the column $L^*$ contains the largest value of the objective function for the obtained solutions and its upper bound. In these tasks, the sum of batches sizes is close to the sum $a_i$.

In the series S for $y_{max} = 9$, the optimal solution was obtained by the CPLEX solver only for one task. For other tasks, optimal solutions failed to obtain for an acceptable time. Feasible solutions are obtained in 6 tasks for $y_{max} = 9$. The time of obtaining the first feasible solution varies from 34 to 18500 seconds.

In this connection, there is interest to construction of heuristic algorithms for finding feasible solutions close to Pareto-optimal solutions.

3.2. Searching of approximate Pareto-optimal solutions

We propose a heuristic algorithm $A$ for finding an approximate solution of problem (2)–(8). It is based on the use of CPLEX solver.

Initially the relaxation problem of mixed-integer programming is solved. It is obtained from the problem (2)–(8) by replacing the Boolean condition for the variables $z_{i,j}^k$ by constraints $0 \leq z_{i,j}^k \leq 1$ for all $i,j,k$. Let $\bar{w} = (\bar{x}, \bar{y}, \bar{z})$ be the optimal or feasible solution of relaxation problem.

Then the following actions are performed $K_{iter}$ times. If $\bar{z}_{i,j}^k = 1$ then in problem (2)–(8) $z_{i,j}^k = 1$ with probability $p_1$. Similarly, if $0 < \bar{z}_{i,j}^k < 1$ then $z_{i,j}^k = 1$ with probability $p_2$ if it does not violate condition (3). Further the smaller-dimension ILP problem (2)–(8) is solved using CPLEX solver for given time $T$. Depending on the obtained result, the parameters $p_1$, $p_2$ and $T$ change.

The results of algorithm $A$ for series D are presented in Table 2. They show that the feasible solutions for all tasks was finded in a time not more than 500 seconds. An average deviation of the objective function from the known optimal solutions is 2.04%. Feasible solutions for tasks D1 and D5 was obtained with a deviations 3.58 % and 2.71 % from the upper bound of objective function.

In series S by algorithm $A$, the feasible solutions were obtained for 7 tasks for $y_{max} = 9$. For example, for task S8, algorithm $A$ found an approximate solution in 936 seconds. The CPLEX solver did not find a feasible solution for this problem in 6000 seconds. Average deviation of the objective function of best found solutions from the upper bounds obtained by CPLEX solver did not exceed 1,2 percent.

Therefore, it becomes necessary to construct algorithms for finding an approximate solution
of the the problem of supply management problem.

### Table 2. Results for series D at $y_{max} = 6$

| Task | CPLEX solver | Algorithm A |
|------|--------------|-------------|
|      | $L^*$ | Time (sec) | $L$ | Time (sec) | Deviation (%) |
| D1   | 524/530,23 | 25000 | 511 | 367 | 3,58 |
| D2   | 450 | 1322 | 439 | 276 | 2,44 |
| D3   | 467 | 42,5 | 456 | 18 | 2,36 |
| D4   | 429 | 48 | 422 | 17 | 1,63 |
| D5   | 462/477,04 | 25000 | 466 | 500 | 2,3 |
| D6   | 458 | 47 | 453 | 32 | 1,09 |
| D7   | 5469 | 278 | 456 | 48 | 2,77 |
| D8   | 458 | 4,7 | 451 | 13 | 1,53 |
| D9   | 460 | 1 | 455 | 13 | 1,09 |
| D10  | 480 | 1518 | 472 | 78 | 1,67 |

### 4. Conclusion

A new variant of the supply management problem was considered. In this problem, a product is supplied by batches and for all providers, the batches volumes are given. The total preference of the supply assignments is maximized and the number of providers for the consumer with a maximum volume of demand is minimized. The NP-hardness of finding a feasible solution for this problem was shown.

A bicriteria ILP model of the problem was proposed. It is established that the cardinality of the complete set of alternatives is polynomial. To search for Pareto-optimal solutions, a single-criterion ILP problem was constructed. The results of computational experiment for this model showed, that for small values of $y_{max}$, the number of suppliers satisfies the condition of ”uniformity”. For CPLEX solver, the time of getting the optimal solution increases substantially at decreasing $y_{max}$. It should be noted that the generated tasks have a small duality gap, but many of them are difficult for CPLEX solver.

The heuristic algorithm for finding the approximate Pareto-optimal solutions was proposed. The results of the computational experiment on random instances showed that algorithm A makes it possible to find good solutions for problem with medium dimension.

In the future, the application of known heuristic approaches to solving the problem under investigation seems to be a promising direction.

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### References

[1] Chauhan S S Eremeev A V, Romanova A A and Servakh V V 2004 Approximation of linear cost supply management problem with lower-bounded demands Proc. of Discrete Optimization Methods in Production and Logistics (DOM-2004) (Omsk: Nasledie Dialog-Sibir Pbs) pp 16–21

[2] Chauhan S S Eremeev A V, Romanova A A and Servakh V V 2007 Approximate Solution of the Supply Management Problem J. of Appl. and Industrial Math. 1 4 pp 433-441
[3] Chauhan S S, Eremeev A V, Romanova A A, Servakh V V and Woeginger G J 2005 Approximation of the
supply scheduling problem Oper. Res. Lett. 33 pp 249-254
[4] Chauhan S S, Eremeev A V, Kolokolov A A and Servakh V V 2005 Concave cost supply management
problem for single manufacturing unit vol 94, ed A Dolgui, J Soldek and et al Supply Chain Optimisation
(Product/Process Design, Facility Location and Flow Control Series: Applied Optimization: Berlin
Heidelberg New York: Springer) pp 167–174
[5] Chauhan S S, Proth J M 2003 The concave cost supply problem European J. of Oper. Res. 148 2 pp 374–383
[6] Emelechev V A, Perepelitsa V A 1994 The complexity of discrete multicriteria problems Discrete Math. Appl.
4 2 pp 89–117
[7] Eremeev A V and Kuznetsov P M 2006 Approximate solution of the problem of supply management with
many intervals (in Russian) Herald of Omsk University 3 pp 26–28
[8] Kolokolov A A (1996) Regular partitions and cuts in integer programming 1996 Discrete analisis and operations
research. Mathematics and its application vol 355, ed A D Korshunov (Dordrecht: Springer)pp 59–79
[9] Zaozerskaya L A 2006 Analysis of Fractional Covering of Some Supply Management Problems J Math Model
Algor 5 Issue 2 pp 201–213
[10] Zaozerskaya L A, Plankova V A, Devyaterikova M V Modeling and Solving Academic Load Distribution
Problem 2018 Proc. Int. Conf. OPTA-SCL (Omsk) Vol 2098 (CEUR-WS) pp 438-445