**Reduced order model of draft tube flow**

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**Abstract.** Swirling flow with compact coherent structures is very good candidate for proper orthogonal decomposition (POD), i.e. for decomposition into eigenmodes, which are the cornerstones of the flow field. Present paper focuses on POD of steady flows, which correspond to different operating points of Francis turbine draft tube flow. Set of eigenmodes is built using a limited number of snapshots from computational simulations. Resulting reduced order model (ROM) describes whole operating range of the draft tube. ROM enables to interpolate in between the operating points exploiting the knowledge about significance of particular eigenmodes and thus reconstruct the velocity field in any operating point within the given range. Practical example, which employs axisymmetric simulations of the draft tube flow, illustrates accuracy of ROM in regions without vortex breakdown together with need for higher resolution of the snapshot database close to location of sudden flow changes (e.g. vortex breakdown). ROM based on POD interpolation is very suitable tool for insight into flow physics of the draft tube flows (especially energy transfers in between different operating points), for supply of data for subsequent stability analysis or as an initialization database for advanced flow simulations.

1. **Introduction**

Hydro power plants (HPPs), especially pump storage power plants (PSPPs), are used for balancing of load fluctuations in electrical grids. Their fast response, ability to store energy and independence on current weather conditions predestine them to be an efficient and reliable control element of the transmission system operators. Boom of renewable energy sources and growth of electricity market put emphasize on even higher flexibility of hydro power plants. HPPs are required to provide their services in broader operation ranges, including both very low and high part load regimes. Most PSPPs are equipped with Francis type turbines, which have only one control mechanism: guide vanes. Operation of Francis turbine in best efficiency point (BEP) and its vicinity is safe and without any undesirable effects. However instabilities appear when going away from BEP. The instabilities are manifested by pressure pulsations and noise. So called draft tube surge propagates into whole hydraulic system eventually leading to fatigue cracks of the blades or power swing phenomena of the electrical generator. All these effects are result of the vortex rope – concentrated vortical structure with rotational frequency of about 20-25% of the runner rotation (so called low frequency pulsations). Vortex rope arises due to the instability of highly swirling flow, which appears downstream of the turbine runner. Significant circumferential velocity component is caused by imbalance between the
moment of momentum exerted by guide vanes and moment of momentum extracted by the turbine runner. This situation is inevitable, when only one control mechanism is available (i.e. guide vanes) and runner has fixed blades.

Substantial research effort was focused on description of draft tube flow field over last two decades. Extensive experimental investigation of pressure pulsations induced by vortex rope and measurement of velocity fields downstream of the turbine runner were done within FLINDT project [1], [2], reconstruction of vortex rope structure was done by Iliiescu et al. [3]. Computational 3D simulations were carried out for example by Rudolf and Skotak [4], Ruprecht et al [5] and Scherer et al. [6]. Comparison of URANS simulations with detailed experimental data were published by Ciocan et al [7], while LES computations including cavitation modeling were presented by Jost and Lipej [8]. Although numerical simulations of draft tube flows become common nowadays with use of HPC facilities, they still present extensive computational effort and any reduction of costly computational time is welcome.

Present paper focuses on construction of reduced order model (ROM) of the draft tube flow field, which would cover whole turbine operating range. Reduction is based on proper orthogonal decomposition (POD) and exploits ability of POD to describe flow fields with strong coherent structures with just a few eigenmodes, while still retaining the correct physics of the flow. Input for ROM building are axisymmetric CFD simulations in a limited number of operating points. ROM then “interpolates” among the operating points and constitutes a relatively simple database, which can be used for consequent stability analysis or as an initialization data matrix for advanced high resolution CFD simulations (e.g. LES).

2. Draft tube flow instability

Vortex rope in draft tube of Francis turbine is a result of the vortex breakdown of highly swirling flow. Vortex breakdown is characterized by decrease of axial velocity along the axis, leading to internal stagnation point or even backflow. Axisymmetric bubble with recirculating flow is formed, which is rather unstable and shear layers on the bubble edge are susceptible to Kelvin-Helmholtz instability. Increasing cross-section of the draft tube further enhances the adverse pressure gradient and promotes the backflow towards the runner cone, see figure 1. Vortex sheet is rolling up in the shear layer and the spiral form of vortex breakdown is triggered. Helical structure of the vortex breakdown and transition from bubble form to spiral form of vortex breakdown were described by Sarpkaya [9] for a divergent pipe flow. Situation with spiral vortex rope rolled around stagnant region (or dead water region) was also experimentally identified for draft tube flow corresponding to Francis turbine part load operation by Nishi et al [10].

![Figure 1. Vortex rope rolled around the stagnant region](image1.png)

![Figure 2. Hill chart of the draft tube pressure recovery coefficient (adopted from [12])](image2.png)
Resiga et al [11] realized that stagnant region is the necessary condition for the inception of the helical vortex rope. They proved that the velocity field obtained from axisymmetric (i.e. 2D) simulations is very similar to averaged 3D computational results and that the data from the axisymmetric simulations can be used for further stability analysis. Shift from full 3D to axisymmetric 2D domain obviously means substantial decrease of computational time.

3. Model reduction of draft tube flow field using proper orthogonal decomposition

Turbine characteristic is a function of two parameters: unit discharge and unit speed or alternatively discharge coefficient and specific energy coefficient. Hill chart of efficiency plotted above a plane defined by those two parameters describes performance of the turbine. Similar characteristic can be constructed for pressure recovery coefficient of the draft tube. Each of the points in the plane is characterized by unique inlet boundary conditions and unique velocity field within the draft tube. This chart is usually divided into regions or bands where helical vortex rope appears and where is no occurrence of concentrated spiral vortical structures.

To assess the velocity field a special computational simulation has to be carried out for each of the points. Every numerical simulation based on finite volume formulation presents thousands of degrees of freedom. Therefore an approach, which would reduce the model complexity would simplify description of the draft tube flow field. There are various model reduction approaches e.g. response surface method or methods, which use projection of the solutions onto some basis. Present paper focuses on the latter method with construction of the basis using proper orthogonal decomposition.

Proper orthogonal decomposition (POD) was introduced to fluid mechanics by Lumley, cited in [13]. However it is also widely spread in image processing analysis or structural mechanics. Nature of POD is very similar to that of Fourier decomposition: scalar or vectorial function is projected onto a basis, thus providing a set of coefficients which represent the function. POD ensures that the basis is optimal in a least square sense and describes the function better than any other available linear basis. POD is usually applied to unsteady data acquired either from CFD simulations or PIV experiments. POD enables to transform the time domain data into set of orthogonal spatial eigenmodes and temporal eigenfunctions. Application of POD on unsteady datasets from CFD simulations of draft tube flows were presented by Rudolf and Jizdny [14] and Rudolf and Stefan [15], where complete spatio-temporal description of the coherent vortical structure, i.e. vortex rope, was obtained. POD enabled to assess the most significant modes of the flow and associated frequencies. This new approach also provided information about energy transfer in between the modes during variation of the operating point.

In following a general scalar function $u_l(x, t)$ is assumed, which is decomposed into set of time-dependent eigen functions (temporal POD modes) $a_k(t)$ and time-independent eigenmodes $\phi^k_l$ (spatial POD modes). Approximation of $u_l(x, t)$ is then sought in form:

$$u_l(x, t) = \sum_{k=1}^{M} a_k(t) \phi^k_l(x) \quad (1)$$

Finding function $u_l(x, t)$ according to equation (1) presents a minimization:

$$\min_{\Omega, T} \int_{\Omega} \int_{T} \left[ u_l(x, t) - \sum_{k=1}^{M} a_k(t) \phi^k_l(x) \right]^2 dt dx \quad (2)$$

which can be solved as the eigen value problem formulated by Fredholm integral equation:

$$\int_{\Omega} R(x, x') \phi_l(x') dx' = \lambda_l \phi_l(x) \quad (3)$$
where the kernel $R(x,x')$ is a two-point cross-correlation tensor having following form for a set of discrete data:

$$R(x,x') = \frac{1}{M} \sum_{i=0}^{M} u_i(x; t)u_i(x'; t)dt$$

(Equation 4)

Especially in case of computational simulations, number of mesh nodes $M$ significantly exceeds number of times steps $N$ used for POD. It was Sirovich [16], who suggested a more efficient way of finding the eigen functions, based on the fact that $u_i(x; t)$ and $\phi_i^k(x)$ span the same linear space. Since matrix $R(x,x')$ is composed of $M$ linearly independent data functions $u_i(x; t)$, the eigen functions $\phi_i^k(x)$ can be written as a linear combination of these data functions.

$$\phi_i^k = \sum_{i=0}^{N} A_{ik}^l u_i$$

(Equation 5)

where $A_{ik}^l$ is the $i$-th component of the $k$-th eigenvector. Substituting (5) to (4) and using (3) yields eigenvalue problem:

$$C_{jk}A_{ik}^l = \lambda_i A_{il}^j$$

(Equation 6)

Matrix $C$ is defined as:

$$C_{ij} = \frac{1}{\text{vol}\Omega} \int_{\Omega} u(x; t_1)u(x; t_2)dx$$

(Equation 7)

Whereas solution of the original eigenvalue problem (3) is based on a matrix $M \times M$, solution of the problem (6), usually referred to as method of snapshots, employs matrix $N \times N$. The temporal coefficients $a_i(t)$ are calculated by projecting the dataset $u_i(x; t)$ onto the eigenmodes $\phi_i^k(x)$:

$$a_i(t) = \sum_{i=1}^{N} u_i(x; t)\phi_i^k(x) \text{ for } i = 0, \ldots, N$$

(Equation 8)

Present paper focuses on application of POD not on datasets form unsteady problems, but on a series of data from steady simulations (one steady simulation $\equiv$ one snapshot). The idea is to shift from time domain to parameter space. The parameters are for example discharge coefficient ($\phi$) and specific energy coefficient ($\psi$) of the turbine or draft tube characteristic curve/hill chart. Eigen function $a_i$ is not, in this case, a temporal function as in the classical POD, but a parameter, which represents the operating point.

### 4. Interpolation of the draft tube flow using POD

Resiga et al [12] presented in their paper hill chart of pressure recovery of the FLINDT draft tube. Theoretical investigation in [12], which is focused on operation of the draft tube in 11 points, also provides analytical description of the corresponding inlet velocity profiles.

Based on this paper a series of steady RANS axisymmetric simulations was carried out using commercial CFD code with Reynolds stress model. As already noted, appearance of internal stagnant region in axisymmetric simulation can be correlated with inception of the helical vortex rope [12]. This statement is also supported by comparison of axisymmetric simulations with full 3D URANS simulations and instantaneous photos of the draft tube flow, where vortex rope is visualized by cavitation [17]. It should be noted that for example for operating points $\psi = 1.18$ no stagnant zone
appears until $\varphi = 0.36$ (rather narrow stripe of backflow). Really remarkable stagnant region develops for $\varphi = 0.34$.

It is assumed that having sufficiently rich underlying snapshot database should be sufficient to build a reduced order model of the draft tube flow field spanning its whole operating range. For simplicity we rely on steady flows, since even steady axisymmetric flow can be indicator of vortex rope presence by appearance of the stagnant region. Unfortunately paper [12] only presents 11 operating points in relatively close vicinity of the draft tube best efficiency point and there is no other source of reliable boundary conditions for the draft tube flows.

*Figure 3.* Experimental visualization of the vortex for $\varphi = 0.34, \psi = 1.18$ [17]

*Figure 4.* Experimental visualization of the vortex for $\varphi = 0.368, \psi = 1.18$ [17]

*Figure 5.* Vortex rope visualized by constant pressure contour for $\varphi = 0.34, \psi = 1.18$ (3D URANS simulation [18])

*Figure 6.* Vortex rope visualized by constant pressure contour for $\varphi = 0.368, \psi = 1.18$ (3D URANS simulation [18])

*Figure 7.* Axisymmetric steady RANS simulation for $\varphi = 0.34, \psi = 1.18$

*Figure 8.* Axisymmetric steady RANS simulation for $\varphi = 0.368, \psi = 1.18$
4.1. Reduced order model for operating points $\psi = 1.18$

Line of constant energy coefficient representing 6 operating points was selected for demonstrating of the reduced order model construction. POD applied on velocity field resulted in 6 eigenmodes and 6 eigenfunctions, which describe the particular operating points.

![Contribution of POD modes, axial velocity](image1)

**Figure 9.** Magnitude of the eigenmodes of axial velocity field ($\psi = 1.18$)

![Eigenfunctions $a_i$ of axial velocity field ($\psi = 1.18$)](image2)

**Figure 10.** Eigenfunctions $a_i$ of axial velocity field ($\psi = 1.18$)

Smooth, almost linear, decline of eigenfunction $a_i$ is observed until $\varphi = 0.36$. Sudden drop marks change of the flow field and inception of the vortex breakdown, which is in 2D axisymmetric case marked by substantial stagnant region.

Eigenmodes represent different flow patterns, see figures 11-16. Similar pictures could be also plotted for radial and circumferential velocity components.

![1st mode of axial velocity, axisymmetric swirl FLINDT, $\psi=1.18$](image3)

**Figure 11.** Eigenmode of axial velocity $n=1$ ($\psi = 1.18$)

![2nd mode of axial velocity, axisymmetric swirl FLINDT, $\psi=1.18$](image4)

**Figure 12.** Eigenmode of axial velocity $n=2$ ($\psi = 1.18$)

![3rd mode of axial velocity, axisymmetric swirl FLINDT, $\psi=1.18$](image5)

**Figure 13.** Eigenmode of axial velocity $n=3$ ($\psi = 1.18$)

![4th mode of axial velocity, axisymmetric swirl FLINDT, $\psi=1.18$](image6)

**Figure 14.** Eigenmode of axial velocity $n=4$ ($\psi = 1.18$)
The first eigenmode is merely average of all snapshots, whereas the signs of unstable behaviour are contained in higher modes.

Reliability of ROM was proved by backward reconstruction of all six velocity fields.

4.2. Interpolation in operating region with no vortex rope

Reduced order model can be used for interpolating of missing points in the hill chart. Let us exclude operating point $\psi = 1.18 \varphi = 0.368$ from reduced order model (ROM) construction. Then this point will be interpolated using ROM and compared with the data from simulation. Interpolation is done via interpolating the eigenfunctions $a_i$ using spline curve.

It is apparent that although ROM had no information about this particular operating point it was able to recover the original axial velocity field, including the spots of low axial velocity along the axis. Only 5 eigenmodes (i.e. 5 degrees of freedom) were sufficient for rather accurate description.
It is well known that very sensitive indicator of any instabilities is radial velocity field. Although a minor difference is present between figures 19 and 20, still very good agreement is obtained.

4.3. Interpolation in operating range close to vortex rope region

Operating point $\psi = 1.18 \varphi = 0.36$ is now excluded from ROM construction. POD interpolation seeks for the velocity field in this particular point using ROM built from the other operating points.

Apparent discrepancy is observed between figures 21 and 22. Interpolated flow field is under excessive influence of the operating point $\varphi = 0.34$. The reason is in the interpolation error of the eigenfunctions $a_i$. Figure 25 illustrates change of the slope of the first three $a_i$ eigenfunctions (ROM) between operating points $\varphi = 0.34$ and $\varphi = 0.368$. The real flow field is not changing smoothly, but rather a sudden change in the flow pattern occurs close to operating point $\varphi = 0.34$. This can be concluded by comparing figures 10 and 25. It is believed that including another point close to $\varphi = 0.34$ would improve the interpolation. Practically the same conclusion can be drawn about the radial velocity field, see figures 23 and 25.
Figure 25. Eigenfunctions of axial velocity $a_i$ ($\psi = 1.18$)

Figure 26. Eigenfunctions of radial velocity $a_i$ ($\psi = 1.18$)

5. Conclusion

Methodology for interpolation of velocity fields in whole range of draft tube operation was proposed. The main advantage is in drastic reduction of degrees of freedom. While current computational simulations have thousands of degrees of freedom, POD interpolation relies on tens of eigenmodes, which are able to cover most of the operating range. Although presented example only showed quite narrow operating region due to limited availability of the boundary conditions, it is believed that decomposition of the flow field into eigenmodes poses faithful yet computationally cheap description of the draft tube flow field for most of the operating range.

While many positives are connected with POD interpolation, some problems also have to be mentioned. First, POD will only recover those features, which are already contained within the underlying snapshots. Second, the linear POD basis struggles with strongly nonlinear phenomena of the vortex breakdown type. In other words, whenever a sudden change of the flow field occurs a rich database of snapshots in vicinity of this nonlinearity has to be supplied to construct accurate reduced order model.

POD interpolation represents a suitable and efficient tool to provide data for subsequent stability analysis or multicriteria optimization process. It can also serve as a database of initial data for high resolution computational simulations, where substantial speed up is expected.

Authors of the paper propose a following strategy in study of stability of the draft tube flows, where present paper forms the second point:
1. To perform very fast axisymmetric steady CFD simulations of several tens of operating points, which span whole hill chart.
2. Application of POD and construction of the reduced order model, which describes axisymmetric flow fields within draft tubes over whole operating range.
3. Application of tailored and very fast stability analysis tool, e.g. [19], using data from ROM and construction of stability map providing information about stability/instability regions and about frequency of potential unstable phenomena.
4. Detailed but time consuming 3D unsteady CFD simulation of selected unstable operating points to recover shape of the vortex rope and amplitude of pressure pulsations.
5. Possible application of POD on 3D unsteady data to construct reduced order models of selected unstable operating points to obtain their complete spatio-temporal characterization.
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