Improved Estimators of Population Mean under Nonresponse in Successive Sampling

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It is well accepted that suitable additional information improves the efficiency of an estimator but at the same time, it faced the situation of nonresponse. The two-occasion successive sampling is useful to handle the absence of a full response from respondents. In this present study, we have developed exponential estimators for population mean using a subsampling nonrespondent procedure. To show the efficacy of the recommended estimators, several properties are derived, and respective optimum replacement strategies are inferred. To evaluate the performance of the recommended estimators empirically, a computational study is carried out as well. It was found that the recommended estimators outperform the existing ones under the nonresponse in successive sampling.

1. Introduction

Incomplete information is a well-known problem in sample surveys, especially in socio-economic surveys of households, in which individual data are collected. The reasons for missing information may be migration, refusal to respond, not being available at the time of surveys performed, etc.

Jessen [1] initially encountered the problem and suggested the method of estimation under the successive sampling with partial replacement of units utilizing the complete information at the previous occasion. Furthermore, Patterson [2]; Rao and Graham [3]; Feng and Zou [4] studied the properties of different estimators under successive sampling. Biradar and Singh [5] and Singh and Vishwakarma [6]; Singh and Pal [7]; Sanahulla et al. [8]; Javed et al. [9] and Pal et al. [10] used additional information for estimation under successive sampling.

The concept used in this paper is nonresponse. First time during the data collection, Hansen and Hurwitz [11] realize the problem of nonresponse at the estimation stage. He took this problem forward and realizes that while taking the subsamples from the nonrespondents group effect of nonresponse can be reduced. Furthermore, he developed the estimators utilizing the information from response and nonresponse groups together which was well accepted in the sampling theory.

Later on, several researchers including Chaudhary et al. [12]; Singh and Priyanka [13], and Pal and Singh [14] combined the concept of successive sampling and nonresponse and used it for estimation of population mean on current occasion on two-occasion successive sampling. Under different practical situations using the auxiliary variable, the aforesaid authors suggested some estimators when nonresponse was observed on the current occasion in two-occasion successive sampling.

The remainder of this paper is organized as follows: in section 2, terminology and notations are presented further the resent estimators and proposed estimators along with their properties are in sections 3 and 4, respectively. Section 5 provides the computational study, and concluding remarks are offered in section 6.
2. Terminology and Notations

Let $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_N)$ be a finite population of $N$ units, which has been sampled over two occasions. The character under study is denoted by $x(y)$ at the first (second) occasion. It is assumed that information on an ancillary variable $z$ (with unknown population mean), which is positively correlated with the study variable, is readily available and almost stable over both the occasions. A simple random sample (without replacement) $s_n$ of $n$ units is drawn on the first occasion. A random subsample $s_m$ of $m = m\lambda$ units is retained (matched) for its use on the second occasion. We assume that there is nonresponse at the current occasion, so that the population can be divided into two classes, those who will respond at the first attempt and those who will not respond. Let the sizes of these two classes be $N_1$ and $N_2$, respectively. At the current (second) occasion, a simple random sample (without replacement) $s_\nu$ of $\nu = (n - m) = m(1 - \lambda)$ units is drawn afresh from the entire population so that the sample size on the current (second) occasion is also $n$. $\mu$ and $\lambda$, $(\lambda + \mu = 1)$ are the fractions of matched and fresh samples, respectively, at the current (second) occasion. We assume that in the unmatched portion of the sample on the current (second) occasion, $\nu_1$ units respond and $\nu_2$ units do not respond. Let $s_{\nu_1} \cap s_{\nu_2}$ and $s_{\nu_1} \cap s_{\nu_2}$ be the sizes of the subsample $s_{\nu_2}$ (of $s_{\nu_2}$) drawn from the nonresponding units in the unmatched (fresh) portion $s_{\nu_2}$ of the sample ($s_{\nu_2}$) on the current (second) occasion (i.e., from the $\nu_2$ nonrespondents, on SRSWR of $s_{\nu_2}$ units is selected with the inverse sampling rates $f_2$, where $v_2 = (v_2/f_2)^2$, $f_2 > 1$.

The notations used in the research paper are shown in the Table 1.

3. Recent Developments of Estimators

For estimating the current population mean $\bar{Y}$, Singh et al. [15, 16] have given the estimators of set $s_\nu$ as

$$T_{1\nu} = \bar{Y}_\nu \exp \left[ \frac{(Z - \bar{z})}{(Z + \bar{z})} \right],$$

and

$$T_{2\nu} = \bar{Y}_\nu \exp \left[ \frac{(Z - \bar{z})}{(Z + \bar{z})} \right].$$

Singh et al. [15] also gave the estimators of set $s_m$ for estimating the current population mean $\bar{Y}$ as

$$T_m = \bar{Y}_m \left( \frac{Z}{Z + \bar{z}} \right) \exp \left[ \frac{(x_m - \bar{z})}{(x_m + \bar{z})} \right].$$

Singh et al. [15] suggested the following estimators of population mean $\bar{Y}$ at the current (second) occasion by combining the estimators of sets $s_\nu$ and $s_m$ as

$$T_i = \phi_i T_{1\nu} + (1 - \phi_i) T_{2\nu}; \quad (i = 1, 2),$$

where $\phi_i (0 \leq \phi_i \leq 1); \quad (i = 1, 2)$ are unknown constants (scalars) to be determined under certain criteria. For detailed properties of the estimators $T_i (i = 1, 2)$, readers are referred to Singh et al. [15].

4. New Development of Estimators

4.1. Estimators Based on the Unmatched Portion of the Ample $s_\nu$—Looking the formulation of the estimators $T_{1\nu}$ given by (1) due to Singh et al. [15], it is clear that the estimator $T_{1\nu}$ is defined in the situation, in which information on the auxiliary variable $z$ is obtained for all the sample units $\nu$ (drawn afresh from the entire population at the second occasion), and the population mean $\bar{z}$ of the auxiliary variable $z$ is known, but some sample units fail to supply information on the study variable $y$. We note that, when suggesting the estimators for the population mean $\bar{Y}$ on the current (second) occasion, Singh et al. [15] used only information on the sample mean $\bar{z}_\nu$ of the auxiliary variable $z$. However, one can also obtain the unbiased estimator $\bar{z}_\nu$ of the population mean $\bar{Z}$ (without any extra effort) while in the process of obtaining the unbiased estimator $\bar{Y}_\nu$ of the population mean $\bar{Y}$. Thus, in this situation, when information on the auxiliary variable $z$ is obtained for all the sample units $\nu$, we have two unbiased estimators $\bar{z}_\nu$ and $\bar{z}_\nu$, of the population mean $\bar{Z}$ of the auxiliary variable $z$ (see, Singh and Kumar [17]). With this background, we suggest the following estimators (using $\bar{z}_\nu$ and $\bar{z}_\nu$ together) for the population mean $\bar{Y}$ on the current (second) occasion based on unmatched portion of the sample $S_\nu$ of the size $\nu$.

$$P_{1\nu} = \bar{Y}_\nu \exp \left[ \frac{(Z - \bar{z})}{(Z + \bar{z})} \right] \exp \left[ \frac{(Z - \bar{z}_\nu)}{(Z + \bar{z}_\nu)} \right],$$

and

$$P_{2\nu} = \bar{Y}_\nu \exp \left[ \frac{(Z - \bar{z})}{(Z + \bar{z})} \right] \exp \left[ \frac{4(\bar{z} - \bar{z}_\nu)}{(Z + \bar{z}_\nu)(Z + \bar{z}_\nu)} \right].$$

where $(\bar{z}_\nu^*, s_\nu^*, R_\nu^*) = (s_\nu^* / s_\nu^*, R_\nu^* (\bar{z}_\nu^* / s_\nu^*))$ are, respectively, the estimates of $(\bar{z}_\nu^*, s_\nu^*, R_\nu^*)$.
For the simplicity in calculation, Reddy (1974, [18]) and Ruiz Espejo [19] initially assumed that the coefficient of variation of study and auxiliary variables are equal. It is further assumed that under incomplete information, coefficient of variation of class is equal to coefficient of variation of population. Following this, we state that

**Theorem 1.** The MSE of the estimator \( P_{1v} \) to the fda is given by

\[
\text{MSE}(P_{1v}) = S_y^2 \left[ 2 \lambda_v \left( 1 - \rho_{yz} \right) + \theta_v \left( \frac{5}{4} \right) \right].
\]  

(6)

**Theorem 2.** The MSE of the estimator \( P_{2v} \) to the fda is given by

\[
\text{MSE}(P_{2v}) = S_y^2 \left[ 2 \lambda_v \left( 1 - \rho_{yz} \right) + \theta_v \left( \frac{5}{4} \right) \right].
\]  

(7)

**Remark 1.** The MSE of the estimators \( P_{1v} \) and \( P_{2v} \) is given while considering the assumption made by Singh et al. [15, 16] that the population correlation coefficient is equal to the nonresponse class correlation coefficient \( (\rho_{yz(2)} = \rho_{yz}) \):

\[
\text{MSE}(P_{1v}) = S_y^2 \left[ 2 \lambda_v \left( 1 - \rho_{yz} \right) + \theta_v \left( \frac{5}{4} \right) \right],
\]  

(8)

\[
\text{MSE}(P_{2v}) = S_y^2 \left[ \lambda_v + \theta_v \right] \left( 1 - \rho_{yz}^2 \right).
\]

**4.2. Estimators Based on the Matched Portion of the Sample \( s_m \).** It is assumed that there is no nonresponse on the first occasion as well on the matched portion of the sample. Under the above assumption, we consider the following estimators based on the matched sample \( s_m \) of size \( m \):

\[ P_{1m} = \left[ \bar{y}_m + b_{yz(m)} (\bar{z} - \bar{z}_m) \right] \exp \left[ \frac{(\bar{x}_m - \bar{x}_m)}{\bar{x}_m + \bar{x}_m} \right], \]

(9)

where \( b_{yz(m)} \) are the estimates of the population regression coefficients \( \beta_{yz(2)} \), respectively, based on the sample of size \( m \).

To the \( fda \), the MSE of the proposed estimators \( P_{1m} \) is given by

\[
\text{MSE}(P_{1m}) = \tau \left[ \frac{1 - \lambda}{N} \right] \left[ \frac{1}{m} \right] \left( 1 - \rho_{yz}^2 \right) + \frac{1}{n} \left( 1 - \rho_{yz}^2 \right) \left( \frac{1}{m} \right),
\]

(10)

Under the assumption \( C_x = C_z = C_y \), the MSEs (10) reduce to

\[
\text{MSE}(P_{1m}) = S_y^2 \left[ \frac{1 - \lambda}{N} \right] \left( 1 - \rho_{yz}^2 \right) + \frac{1}{m} \left( 1 - \rho_{yz}^2 \right) \left( \frac{1}{m} \right).
\]  

(11)

**4.3. Covariance between \( s_u \) and \( s_m \).** The covariance between \( (P_{1u}, P_{1m}) \) is given by

\[
C_{11} = \text{Cov}(P_{1u}, P_{1m}) = \frac{S_y^2}{N} \left( 1 - \rho_{yz}^2 \right).
\]  

(12)

It is to be noted that the expression (11) has been derived under the assumptions \( C_x = C_z = C_y, \rho_{yz} = \rho_{yz(2)} \), and \( C_y = C_{y(2)} \).

\[
C_{21} = \text{Cov}(P_{2u}, P_{1m}) = \frac{S_y^2}{N} \left( 1 - \rho_{yz}^2 \right).
\]  

(13)

**4.4. Linear Combination of Estimators.** We have suggested the following estimators for estimating the population mean \( \bar{y} \) at the current occasion by combining the different estimators at the matched and unmatched portions of the samples, respectively:

\[
P_1 = \phi_1 P_{1u} + (1 - \phi_1) P_{1m},
\]

\[
P_2 = \phi_2 P_{2u} + (1 - \phi_2) P_{1m},
\]

(14)

where \( \phi_1 (1 \leq \phi_1 \leq 1) \) is suitably chosen constant to be determined under certain assumptions.

**4.5. MSEs of the Estimators \( P_1 \) and \( P_2 \).** The MSEs of the estimators are derived up to \( fda \), and under the assumption, \( C_x = C_z = C_y, \rho_{yz} = \rho_{yz(2)} \), and \( C_y = C_{y(2)} \).

**Theorem 3.** MSEs of \( P_i (i = 1, 2) \) to the \( fda \) are obtained as

\[
\text{MSE}(P_1) = \phi_1^2 \text{MSE}(P_{1u}) + (1 - \phi_1)^2 \text{MSE}(P_{1m}) + 2\phi_1 (1 - \phi_1) C_{11},
\]

(15)

\[
\text{MSE}(P_2) = \phi_2^2 \text{MSE}(P_{2u}) + (1 - \phi_2)^2 \text{MSE}(P_{1m}) + 2\phi_2 (1 - \phi_2) C_{21}.
\]

(16)

**4.6. MMSEs of the Estimators \( P_i (i = 1, 2) \).** Since the MSE of the estimators \( P_i (i = 1, 2) \) in (15) and (16) is functions of unknown constants \( \phi_1’s (i = 1, 2) \), therefore, the MSEs are minimized with respect to \( \phi_1 (i = 1, 2) \), respectively, for

\[
\phi_1(\text{opt}) = \frac{\text{MSE}(P_1) - C_{11}}{\text{MSE}(P_{1u}) + \text{MSE}(P_{1m}) - 2C_{11}},
\]

(17)

\[
= \frac{(1 - \mu_1) \left[ h_{0(1)} - \mu_1 f(k_5 - k_0) \right]}{h_{0(1)} + \mu_1 f_{2(1)}},
\]

\[
\phi_2(\text{opt}) = \frac{\text{MSE}(P_2) - C_{21}}{\text{MSE}(P_{2u}) + \text{MSE}(P_{1m}) - 2C_{21}},
\]

(18)

\[
= \frac{(1 - \mu_2) h_{0(2)}}{h_{0(2)} + \mu_2 f_{2(2)} + \mu_2^2 h_{2(2)}}.
\]
and thus, the resulting MMSEs of the estimators $P_i (i = 1, 2)$ are given as

$$
\text{MMSE}(P_i) = \frac{\text{MSE}(P_{i1})\text{MSE}(P_{i2}) - C_{ii}^2}{\text{MSE}(P_{i1}) + \text{MSE}(P_{i2}) - 2C_{ii}}
$$

where $C_{ii} = \Theta \times (f_2 - 1) \times ((5/4) - \rho_{yx})$, $\rho_{yx} = ((1/4) - \rho_{yx} + \rho_{yy}), \rho_{yx} = ((1/4) - \rho_{yx} - \rho_{yx})\rho_{yx}, \rho_{yx} = [1 + \Theta (f_2 - 1)], h_{01}(1) = (\rho_{yx} + \rho_{yy}), h_{11}(1) = [(1 - f)(\rho_{yx} - \rho_{yy}) - f], h_{21}(1) = [\rho_{yx} - \rho_{yy}], h_{02}(1) = [\rho_{yx} - \rho_{yy}], h_{12}(1) = [\rho_{yx} - \rho_{yy}], h_{22}(1) = [\rho_{yx} - \rho_{yy}], h_{03}(1) = [1 - f](\rho_{yx} + \rho_{yy}), h_{13}(1) = [(\rho_{yx} + \rho_{yy})], h_{23}(1) = [\rho_{yx} - \rho_{yy}], h_{04}(1) = [1 - f]\rho_{yx}, h_{14}(1) = [(\rho_{yx} + \rho_{yy})], h_{24}(1) = [\rho_{yx} - \rho_{yy}], h_{05}(1) = [\rho_{yx} - \rho_{yy}], h_{15}(1) = [(\rho_{yx} + \rho_{yy})], h_{25}(1) = [\rho_{yx} - \rho_{yy}], h_{06}(1) = [1 - f]\rho_{yx}, h_{16}(1) = [(\rho_{yx} + \rho_{yy})], h_{26}(1) = [\rho_{yx} - \rho_{yy}], h_{07}(1) = [1 - f]\rho_{yx}, h_{17}(1) = [(\rho_{yx} + \rho_{yy})], h_{27}(1) = [\rho_{yx} - \rho_{yy}], h_{08}(1) = [1 - f]\rho_{yx}, h_{18}(1) = [(\rho_{yx} + \rho_{yy})], h_{28}(1) = [\rho_{yx} - \rho_{yy}]

4.7. Optimum Replacement Policy. It is observed from (18) and (19) that the MMSEs of the estimators $P_i (i = 1, 2)$ are the functions of $\mu_i (i = 1, 2)$ (functions of sample to be drawn at the second occasion); therefore, the optimum values of $\mu_i$ are obtained to estimate the population mean $\bar{y}$ with minimum precision and lowest cost. To obtain the optimum values of $\mu_i (i = 1, 2)$, we minimize the MMSEs of the proposed estimators $P_i (i = 1, 2)$ given in (18) to (19), respectively, with respect to $\mu_i (i = 1, 2)$ which result in quadratic equations in $\mu_i (i = 1, 2)$ and the respective solutions of $\mu_i (i = 1, 2)$ say $\hat{\mu}_i (i = 1, 2)$ are given as follows:

$$
\psi_{12}(1)\hat{\mu}_i^2 + 2\psi_{02}(1)\hat{\mu}_i + \psi_{01}(1) = 0,
$$

$$
\hat{\mu}_i = \frac{-\psi_{02}(1) \pm \sqrt{\psi_{12}(1) - \psi_{01}(1)\psi_{12}(1)}}{D_{12}(1)},
$$

where

$$
\psi_{12}(1) = \frac{\psi_{12}^2}{\psi_{01}(1)}, \psi_{02}(1) = \frac{\psi_{02}^2}{\psi_{01}(1)}, \psi_{01}(1) = \frac{\psi_{01}^2}{\psi_{12}(1)}, \psi_{12}(1) = \frac{\psi_{12}^2}{\psi_{01}(1)}
$$

From (21) and (22), it is observed that real values of $\mu_i (i = 1, 2)$ exist, if the quantities under square roots are greater than or equal to zero, and for any combinations $\rho_{xy}, \rho_{yx}$, and $\rho_{xx}$, which satisfy the conditions of real situations, two real values of $\mu_i (i = 1, 2)$ are possible, and hence, while choosing the values of $\mu_i (i = 1, 2)$, it should be remembered that $0 \leq \mu_i \leq 1 (i = 1, 2)$. All other values of $\mu_i (i = 1, 2)$ are inadmissible. Substituting the admissible values of $\mu_i (i = 1, 2)$ from (21) and (22) in (17) and (18), respectively, we have the optimum values of mean squared errors of $P_i (i = 1, 2)$ which are shown as follows:

$$
\text{MMSE}(P_{i1})_{\text{opt}} = \frac{\text{MSE}(P_{i1})_{\text{opt}} - C_{ii}^2}{\text{MSE}(P_{i1})_{\text{opt}} + \text{MSE}(P_{i2})_{\text{opt}} - 2C_{ii}}
$$

$$
\text{MMSE}(P_{i2})_{\text{opt}} = \frac{\text{MSE}(P_{i2})_{\text{opt}} - C_{ii}^2}{\text{MSE}(P_{i1})_{\text{opt}} + \text{MSE}(P_{i2})_{\text{opt}} - 2C_{ii}}
$$

4.8. Efficiency Comparisons. The percent relative losses in efficiencies of the estimators $P_i (i = 1, 2)$ are obtained with respect to the similar estimator and natural successive sampling estimator when the nonresponse no observed on any occasion. The estimator $\Phi_i$ is for complete information and under the similar assumption as estimator $P_i (i = 1, 2)$. Whereas the estimator $\Phi_2$ is natural estimator under successive sampling, given by

$$
\psi_{12}(1)\mu_i^2 + 2\psi_{02}(1)\mu_i + \psi_{01}(1) = 0
$$

$$
\hat{\mu}_i = \frac{-\psi_{02}(1) \pm \sqrt{\psi_{12}(1) - \psi_{01}(1)\psi_{12}(1)}}{D_{12}(1)}
$$

where

$$
\psi_{12}(1) = \frac{\psi_{12}^2}{\psi_{01}(1)}, \psi_{02}(1) = \frac{\psi_{02}^2}{\psi_{01}(1)}, \psi_{01}(1) = \frac{\psi_{01}^2}{\psi_{12}(1)}, \psi_{12}(1) = \frac{\psi_{12}^2}{\psi_{01}(1)}
$$
### Table 2: Percent relative losses $L_{11}$ and $L_{12}$ with respect to $\Phi_1$ and $\Phi_2$ for $f = 0.1$.

| $\theta$ | $\rho_{yx}$ | $\rho_{yz}$ | $\mu^{(0)}_{11}$ | $L_{11}$ | $L_{12}$ | $\mu^{(0)}_{12}$ | $L_{11}$ | $L_{12}$ |
|----------|-------------|-------------|------------------|----------|----------|------------------|----------|----------|
| 0.50     | 0.80        | 0.78        | -32.63           | -119.12  | 0.90     | -26.42           | -108.85  |          |
|          | 0.85        | 0.63        | -49.64           | -176.93  | 0.72     | -42.89           | -164.44  |          |
|          | 0.90        | 0.53        | -79.78           | -277.20  | 0.60     | -70.70           | -258.16  |          |
|          | 0.95        | 0.43        | -152.55          | -510.50  | 0.49     | -134.20          | -466.16  |          |
|          | 0.80        | 0.78        | -28.02           | -110.43  | 0.90     | -22.03           | -100.58  |          |
|          | 0.85        | 0.63        | -44.13           | -165.96  | 0.72     | -37.62           | -153.96  |          |
|          | 0.90        | 0.53        | -72.77           | -262.25  | 0.60     | -64.05           | -243.96  |          |
| 0.60     | 0.95        | 0.43        | -142.10          | -486.31  | 0.49     | -124.52          | -443.73  |          |
|          | 0.80        | 0.78        | -23.16           | -99.14   | 0.90     | -17.39           | -89.81   |          |
|          | 0.85        | 0.63        | -38.33           | -151.69  | 0.72     | -32.09           | -140.33  |          |
|          | 0.90        | 0.53        | -65.44           | -242.81  | 0.60     | -57.09           | -225.51  |          |
| 0.10     | 0.95        | 0.43        | -131.22          | -454.85  | 0.49     | -114.42          | -414.55  |          |
|          | 0.80        | 0.78        | -18.01           | -84.13   | 0.90     | -12.48           | -75.51   |          |
|          | 0.85        | 0.63        | -32.26           | -132.71  | 0.72     | -26.30           | -122.22  |          |
|          | 0.90        | 0.53        | -57.83           | -216.97  | 0.60     | -49.86           | -200.97  |          |
|          | 0.95        | 0.43        | -119.99          | -413.02  | 0.49     | -104.01          | -375.76  |          |
| 0.50     | 0.80        | 0.84        | -29.55           | -114.03  | 1.02     | -20.03           | -98.30   |          |
|          | 0.85        | 0.67        | -46.26           | -170.69  | 0.80     | -36.15           | -151.98  |          |
|          | 0.90        | 0.56        | -75.19           | -267.58  | 0.66     | -62.02           | -239.96  |          |
|          | 0.95        | 0.46        | -143.08          | -487.61  | 0.55     | -118.03          | -427.07  |          |
|          | 0.80        | 0.84        | -25.05           | -105.55  | 1.02     | -15.87           | -90.45   |          |
|          | 0.85        | 0.67        | -40.88           | -159.96  | 0.80     | -31.14           | -142.00  |          |
|          | 0.90        | 0.56        | -68.36           | -253.01  | 0.66     | -55.71           | -226.48  |          |
| 0.60     | 0.95        | 0.46        | -133.02          | -464.33  | 0.55     | -109.01          | -406.18  |          |
|          | 0.80        | 0.84        | -20.30           | -94.52   | 1.02     | -11.46           | -80.23   |          |
|          | 0.85        | 0.67        | -35.22           | -146.01  | 0.80     | -25.87           | -129.01  |          |
|          | 0.90        | 0.56        | -61.22           | -234.07  | 0.66     | -49.10           | -208.96  |          |
|          | 0.95        | 0.46        | -122.55          | -434.04  | 0.55     | -99.62           | -379.02  |          |
|          | 0.80        | 0.84        | -15.27           | -79.86   | 1.02     | -6.80            | -66.64   |          |
| 0.70     | 0.85        | 0.67        | -29.28           | -127.47  | 0.80     | -20.35           | -111.75  |          |
|          | 0.90        | 0.56        | -53.80           | -208.89  | 0.66     | -42.24           | -185.67  |          |
|          | 0.95        | 0.46        | -117.74          | -393.79  | 0.55     | -89.92           | -342.91  |          |
| 0.80     | 0.80        | 0.90        | -26.42           | -108.85  | 1.14     | -13.56           | -87.61   |          |
|          | 0.85        | 0.72        | -42.89           | -164.44  | 0.89     | -29.51           | -139.68  |          |
|          | 0.90        | 0.60        | -70.70           | -258.16  | 0.73     | -53.76           | -222.62  |          |
|          | 0.95        | 0.49        | -134.20          | -466.16  | 0.60     | -103.66          | -392.32  |          |
| 0.50     | 0.85        | 0.90        | -22.03           | -100.58  | 1.14     | -9.62            | -80.18   |          |
|          | 0.90        | 0.60        | -64.05           | -243.96  | 0.73     | -47.77           | -209.84  |          |
|          | 0.95        | 0.49        | -124.52          | -443.73  | 0.60     | -95.23           | -372.81  |          |
| 0.60     | 0.85        | 0.90        | -17.39           | -89.81   | 1.14     | -5.46            | -70.51   |          |
|          | 0.90        | 0.60        | -57.09           | -225.51  | 0.73     | -41.50           | -193.21  |          |
|          | 0.95        | 0.49        | -114.42          | -414.55  | 0.60     | -86.46           | -347.44  |          |
| 0.20     | 0.85        | 0.90        | -12.48           | -75.51   | 1.14     | -1.04            | -57.66   |          |
|          | 0.90        | 0.60        | -26.30           | -122.22  | 0.89     | -14.47           | -101.41  |          |
|          | 0.95        | 0.49        | -104.01          | -375.76  | 0.60     | -77.40           | -313.71  |          |
| $\Theta$ | $\rho_{yx}$ | $\rho_{yz}$ | $\mu_1^{(0)}$ | $\mu_2^{(0)}$ | $f_1 = 1.5$ | $f_2 = 1.5$ | $f_3 = 2.0$ | $f_2 = 2.0$ |
|--------|------------|------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 0.50   | 0.80       | 0.58       | −41.77        | −134.22       | 0.69          | −37.11        | −126.52       |                |
|        | 0.85       | 0.52       | −57.61        | −191.68       | 0.59          | −52.94        | −183.04       |                |
|        | 0.90       | 0.46       | −88.47        | −295.43       | 0.50          | −83.28        | −284.56       |                |
|        | 0.95       | 0.38       | −169.46       | −551.39       | 0.40          | −162.49       | −534.54       |                |
|        | 0.80       | 0.58       | −36.85        | −124.94       | 0.69          | −32.35        | −117.55       |                |
|        | 0.85       | 0.52       | −51.80        | −180.12       | 0.59          | −47.30        | −171.82       |                |
|        | 0.90       | 0.46       | −81.12        | −279.76       | 0.50          | −76.14        | −269.32       |                |
| 0.60   | 0.80       | 0.58       | −31.65        | −112.87       | 0.69          | −27.33        | −105.87       |                |
|        | 0.85       | 0.52       | −45.70        | −165.09       | 0.59          | −41.38        | −157.24       |                |
|        | 0.90       | 0.46       | −73.43        | −259.38       | 0.50          | −68.66        | −249.50       |                |
|        | 0.95       | 0.38       | −146.70       | −492.00       | 0.40          | −140.32       | −476.69       |                |
| 0.10   | 0.80       | 0.58       | −39.31        | −145.11       | 0.59          | −35.18        | −137.84       |                |
|        | 0.85       | 0.52       | −65.46        | −232.29       | 0.50          | −60.91        | −223.15       |                |
|        | 0.90       | 0.46       | −134.72       | −447.37       | 0.40          | −128.65       | −433.22       |                |
| 0.70   | 0.80       | 0.64       | −39.48        | −130.44       | 0.80          | −32.17        | −118.36       |                |
|        | 0.85       | 0.55       | −55.28        | −187.38       | 0.65          | −48.20        | −174.27       |                |
|        | 0.90       | 0.48       | −85.86        | −289.97       | 0.54          | −78.20        | −273.88       |                |
|        | 0.95       | 0.39       | −165.93       | −542.87       | 0.42          | −155.82       | −518.42       |                |
|        | 0.80       | 0.64       | −34.64        | −121.31       | 0.80          | −27.58        | −109.71       |                |
|        | 0.85       | 0.55       | −49.56        | −175.99       | 0.65          | −42.74        | −163.40       |                |
|        | 0.90       | 0.48       | −78.62        | −274.52       | 0.54          | −71.25        | −259.07       |                |
| 0.15   | 0.80       | 0.64       | −29.53        | −109.43       | 0.80          | −22.74        | −98.46        |                |
|        | 0.85       | 0.55       | −43.55        | −161.18       | 0.65          | −37.00        | −149.27       |                |
|        | 0.90       | 0.48       | −71.04        | −254.42       | 0.54          | −63.98        | −239.80       |                |
|        | 0.95       | 0.39       | −143.47       | −484.26       | 0.42          | −134.22       | −462.05       |                |
| 0.50   | 0.80       | 0.64       | −24.11        | −93.64        | 0.80          | −17.60        | −83.50        |                |
|        | 0.85       | 0.55       | −37.25        | −141.49       | 0.65          | −30.99        | −130.48       |                |
|        | 0.90       | 0.48       | −63.17        | −227.70       | 0.54          | −56.44        | −214.18       |                |
|        | 0.95       | 0.39       | −131.65       | −440.22       | 0.42          | −122.84       | −419.68       |                |
| 0.60   | 0.80       | 0.69       | −37.11        | −126.52       | 0.90          | −27.04        | −109.89       |                |
|        | 0.85       | 0.59       | −52.94        | −183.04       | 0.71          | −43.44        | −165.46       |                |
|        | 0.90       | 0.50       | −83.28        | −284.56       | 0.58          | −73.22        | −263.44       |                |
|        | 0.95       | 0.40       | −162.49       | −534.54       | 0.44          | −149.45       | −503.01       |                |
| 0.20   | 0.85       | 0.59       | −47.30        | −171.82       | 0.71          | −38.15        | −154.93       |                |
|        | 0.90       | 0.50       | −76.14        | −269.32       | 0.58          | −66.47        | −249.04       |                |
|        | 0.95       | 0.40       | −151.63       | −509.39       | 0.44          | −139.13       | −479.11       |                |
|        | 0.80       | 0.69       | −27.33        | −105.87       | 0.90          | −17.98        | −90.75        |                |
|        | 0.85       | 0.59       | −41.38        | −157.24       | 0.71          | −32.60        | −141.25       |                |
|        | 0.90       | 0.50       | −68.66        | −249.50       | 0.58          | −59.40        | −230.31       |                |
|        | 0.95       | 0.40       | −140.32       | −476.69       | 0.44          | −128.38       | −448.03       |                |
|        | 0.80       | 0.69       | −22.00        | −90.35        | 0.90          | −13.04        | −76.37        |                |
|        | 0.85       | 0.59       | −35.18        | −137.84       | 0.71          | −26.78        | −123.07       |                |
|        | 0.90       | 0.50       | −60.91        | −223.15       | 0.58          | −52.07        | −205.41       |                |
|        | 0.95       | 0.40       | −128.65       | −433.22       | 0.44          | −117.29       | −406.72       |                |
where \( \Phi_{ij} = \psi_j \Phi_{jm} + (1 - \psi_j)T_{jm}; \quad (j = 1, 2), \)

\[
\Phi_{ij} = \psi_j \Phi_{jm} + (1 - \psi_j)T_{jm}; \quad (j = 1, 2),
\]

Proceeding in similar manner as discussed for the estimators \( P_i \) \((i = 1, 2)\), the optimum mean squared errors of the estimators \( \Phi_{ij} \) \((j = 1, 2)\) are derived as

\[
\text{MMSE}(\Phi_{ij}^{(0)})_{\text{opt}} = \frac{[A_1 + \mu_1(0)A_2 + \mu_1^{(0)2}A_1]}{[B_3 + \mu_1(0)B_2 + \mu_1^{(0)2}B_1]} \frac{S_n^2}{n},
\]

\[
\text{MMSE}(\Phi_{ij}^{(0)})_{\text{opt}} = \frac{1}{2}\left[1 + \sqrt{1 - 2\rho_{yz}}\right] - f \frac{S_n^2}{n},
\]

where \( \mu_1(0) = Q_2 + \sqrt{Q_2^2 - Q_1Q_3Q_4} \) \((\) fraction of the sample for the estimator \( \Phi_{ij} \)),

\[
Q_i = B_1A_2 - A_1B_2, \quad Q_2 = B_1A_3 - A_1B_3, \quad Q_3 = B_2A_3 - A_2B_3,
\]

\[
A_1 = \left[\frac{9}{16}f^2\kappa_0^2 - f^2\kappa_0\kappa_3 - f\kappa_2\kappa_3\right],
\]

\[
A_2 = f\kappa_0\kappa_3 + \kappa_2\kappa_3 - f\left(1 - f\right)\kappa_0\kappa_3 - \left(\frac{9}{16}f^2\kappa_0^2\right),
\]

\[
A_3 = (1 - f)\kappa_0\kappa_3,
\]

\[
B_1 = f\kappa_0 + \kappa_2 - \left(\frac{3}{2}\right)f\kappa_0 + f\kappa_3,
\]

\[
B_2 = (1 - f)\kappa_0 - (1 + f)\kappa_3 + \left(\frac{3}{2}\right)f\kappa_0,
\]

\[
B_3 = \kappa_3.
\]

Remark 2. For comparison of estimators \( P_i \) and \( \Phi_{ij} \), it was advisable by Cochran [3] & Feng and Zou [4] to assume the intraclass correlation coefficient equal, that is, \( \rho_{xz} = \rho_{yz} \).

5. Numerical Illustrations

For \( N = 5000, n = 500 \) and different choices of \( \rho_{yx} \) and \( \rho_{yz} \), Tables 2 & 3 give the optimum values of \( \mu_1^{(0)} \) and percent relative losses \( L_{ij} \) \((i = 1, 2; j = 1, 2)\) in the precision of estimators \( P_i \) \((i = 1, 2)\) with respect to \( \Phi_{ij} \) \((j = 1, 2)\), under their respective optimality conditions are given by

\[
L_{ij} = \frac{\text{MMSE}(\Phi_{ij}^{(0)})_{\text{opt}} - \text{MMSE}(\Phi_{ij}^{(0)})_{\text{opt}}}{\text{MMSE}(\Phi_{ij}^{(0)})_{\text{opt}}}, \quad (i = 1, 2; j = 1).
\]

The following inferences may draw from Tables 2 and 3. The above Table 2 depicts that

(i) Considering the constant value of \( \Theta, f_2, \) and \( \rho_{yz} \), we observe that values of \( \mu_1^{(0)}, L_{11}, \) and \( L_{12} \) decrease as \( \rho_{yx} \) increases. In other words, the intraclass correlation coefficient between \( y \) and \( z \) is inversely proportional to the new sample. That means the efficacy of the estimators under nonresponse depends positively on the auxiliary information. This attitude of the estimators is very important and extremely desirable.

(ii) Considering the constant value of \( \Theta, f_2, \) and \( \rho_{yz} \), we observe that values of \( \mu_1^{(0)} \) remain the same whereas \( L_{11} \) and \( L_{12} \) increase as \( \rho_{yx} \) increases.

(iii) Considering the constant value of \( \Theta, \rho_{yx}, \) and \( \rho_{yz} \), we observe that values of \( \mu_1^{(0)}, L_{11}, \) and \( L_{12} \) increase as \( f_2 \) increases.

(iv) Considering the constant value of \( f_2, \rho_{yx}, \) and \( \rho_{yz} \), we observe that values of \( \mu_1^{(0)}, L_{11}, \) and \( L_{12} \) increase as \( \Theta \) increases. Table 3 depicts that nonresponse rate and sample size at current occasion are directly proportional and therefore the cost of the survey increases as well.

(v) Considering the constant value of \( \Theta, f_2, \) and \( \rho_{yz} \), we observe that values of \( \mu_1^{(0)}, L_{21}, \) and \( L_{22} \) decrease as \( \rho_{yx} \) increases. In other words, the intraclass correlation coefficient between \( y \) and \( z \) is inversely proportional to the new sample. That means the efficacy of the estimators under nonresponse depends positively on the auxiliary information. This attitude of the estimators is very important and extremely desirable.

(vi) Considering the constant value of \( \Theta, f_2, \) and \( \rho_{yz} \), we observe that values of \( \mu_1^{(0)} \) remain the same whereas \( L_{21} \) and \( L_{22} \) increase as \( \rho_{yx} \) increases.

(vii) Considering the constant value of \( \Theta, \rho_{yx}, \) and \( \rho_{yz} \), we observe that values of \( \mu_1^{(0)}, L_{21}, \) and \( L_{22} \) increase as \( f_2 \) increases.

(viii) Considering the constant value of \( f_2, \rho_{yx}, \) and \( \rho_{yz} \), we observe that values of \( \mu_1^{(0)}, L_{21}, \) and \( L_{22} \) increase as \( \Theta \) increases. In other words, the nonresponse rate and sample size at the current occasion are directly proportional; consequently, the cost of the survey increases as well.

6. Conclusions

In this paper, we have proposed exponential type estimators for estimating the population mean under the unavailability of full response in two-occasion successive sampling using the additional information. An extensive theoretical and
computational study led to the conclusion that both the proposed estimators are better than the usual estimators available in the literature, and moreover, the estimator \( P_2 \) is better than the estimator \( P_1 \) between the self-comparison. Therefore, the authors recommend the use of estimator \( P_2 \) over the other estimators considered in this paper in practice. Furthermore, the researchers can use the composition of the proposed estimators for their use [20, 21].

Data Availability
No external data were used to support this study.

Conflicts of Interest
The authors declare that there are no conflicts of interest.

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