Formation and Evolution of Cosmic D-strings

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Abstract

We study the formation of \textit{D} and \textit{F}-cosmic strings in \textit{D}-brane annihilation after brane inflation. We show that \textit{D}-string formation by quantum de Sitter fluctuations is severely suppressed, due to suppression of RR field fluctuations in compact dimensions. We discuss the resonant mechanism of production of \textit{D} and \textit{F}-strings, which are formed as magnetic and electric flux tubes of the two orthogonal gauge fields living on the world-volume of the unstable brane. We outline the subsequent cosmological evolution of the \textit{D}−\textit{F} string network. We also compare the nature of these strings with the ordinary cosmic strings and point out some differences and similarities.

I. INTRODUCTION

In the brane inflation scenario \cite{1,2}, the inflationary expansion is driven by the attractive interaction potential between a stack of parallel \textit{D}-branes and a stack of anti-\textit{D}-branes evolving in a higher-dimensional bulk space. The two stacks are slowly pulled towards one another, until they collide and partially annihilate. Brane collision marks the end of inflation. We live on one of the surviving branes. Annihilation proceeds via tachyon condensation \cite{3}. During inflation, the tachyon is trapped in the false vacuum, and gets destabilized only after the branes approach one another at the string scale distance. So, in this respect brane-anti-brane inflation naturally realizes the idea of hybrid inflation \cite{4}, in which the role of the second field is played by the tachyon.

Brane annihilation can result in the formation of lower-dimensional \textit{D}-branes, which are seen as domain walls, strings, or monopoles by brane observers. This was discussed by Sarangi and Tye \cite{5} (ST) (see also \cite{6}), who concluded that cosmic strings are copiously produced in this scenario, while the formation of domain walls and monopoles is strongly suppressed. On the other hand, Majumdar and Davis \cite{7} (MD) have argued that all kinds of branes will be produced in large numbers, so the usual problems associated with cosmic overproduction of domain walls and monopoles may arise.

In the present paper we reexamine the problem of topological defect (such as cosmic string) production in brane annihilation, with conclusions somewhat different from those in ST and MD. The role of the defects can be played by lower-dimensional \textit{D}-branes, as...
well as by the fundamental ($F$) strings, and we shall consider both cases. The possibility of cosmic $F$-strings was suggested by Witten [8]. We discuss the production mechanisms and the cosmological evolution of $D - F$-string networks in the context of $D$-brane driven inflation and brane annihilation.

We begin in the next Section with a brief review of brane inflation. Different mechanisms of defect formation at the end of brane inflation are discussed in Sections III-VI. In Section VII we discuss some unusual features of $D$-strings and show that they share some properties of both global and gauge strings. The evolution of interacting $F$- and $D$-string networks is discussed in Section VIII. Our conclusions are summarized and discussed in Section IX.

During the completion of this work, we became aware of an independent work by Copeland, Myers and Polchinski [9], which also deals with cosmological implications of $D$ and $F$-strings.

II. BRANE INFLATION

We shall assume that all extra spatial dimensions are compactified. The parent branes fill the three large dimensions and may wrap around some of the compact dimensions. The two stacks of branes are separated in the remaining $d_{\perp}$ dimensions, which are orthogonal to the branes. The role of the inflaton in this scenario is played by the inter-brane separation. In generic models, the size of transverse extra dimensions $l_{\perp}$, the string scale $M_s$, and the Hubble expansion rate $H$ satisfy

$$H \ll l_{\perp}^{-1} \ll M_s.$$  \hfill (1)

The key idea of brane inflation is that the flatness of the inflaton potential may be protected by the locality in the extra space. That is, as seen from high-dimensions, the inflaton potential is simply an inter-brane potential. The latter is generated from the bulk (closed string) exchanges, which die-off as an inverse power of the distance $r$, in accordance with the high-dimensional Gauss law. Note that for a large separation $r \gg 1/M_s$, only massless modes (graviton, dilaton, and Ramond-Ramond fields) contribute. Hence, generically the inter-brane potential is

$$V(r) = M_s^4 [A - B(M_s r)^{-\alpha_{\perp} - 2}],$$  \hfill (2)

where the coefficients $A, B$ can be expressed in terms of the sizes of the longitudinal and transverse compact dimensions in units of $M_s^{-1}$. Their precise values depend on how much the system departs from the PBS limit. For instance, with only parallel $D$-branes and no anti-branes, the potential vanishes, since graviton-dilaton attraction is exactly compensated by RR-repulsion. This will change if one departs from the supersymmetry limit, which can be achieved, e.g., by rotating branes at angles, or by some other SUSY-breaking dynamics. In the present work, we shall only be interested in the case of maximal departure from the BPS limit, due to the presence of parallel brane-anti-brane pairs (their numbers can be different though).

A possible string theory implementation of inflation driven by a system of branes and anti-branes was studied in the interesting recent work [10,11]. Especially nontrivial is the issue of consistent stabilization of moduli during inflation. In the present work, we will
only be interested in the final stage of brane collision, and will simply assume that it was preceded by a stage of sufficiently long inflation, about which we shall only assume some very general constraints, such as, e.g., Eq. (1).

III. D-DEFECT FORMATION BY QUANTUM FLUCTUATIONS

Consider a stack of branes and a stack of anti-branes separated by a distance \( r \) and moving towards one another. When \( r \) becomes comparable to the string scale, \( r \sim M_s^{-1} \), a tachyonic instability develops. Tachyon is the state of an open string that connects brane and anti-brane. It has been conjectured that the brane annihilation can be described in terms of condensation of this tachyon [3]. The tachyon develops a vacuum expectation value \( \chi \), which takes values in the vacuum manifold \( \mathcal{M} \). The homotopy of the vacuum manifold can be conveniently described by \( K \)-theory [13]. For example, for \( N \) branes and \( N \) anti-branes, \( \mathcal{M} = U(N) \). The corresponding homotopy groups are

\[
\pi_{2k-1}(U(N)) = Z
\]

for \( N \geq k \), indicating that \( \mathcal{M} \) is topologically nontrivial. Thus, there must exist topological knots. These knots are the lower-dimension \( D \)-branes. From the point of view of the low energy four-dimensional theory, they can look as topological defects of different co-dimension, such as walls, strings, or pointlike charges. (It was suggested recently [14] that from the point of view of the low-energy supergravity theory, \( D \)-strings may be understood as BPS \( D \)-term strings, also indicating that the energy of the parent brane-anti-brane system is of the \( D \)-term type.)

Thus, in other words, daughter branes are formed as topological defects as a result of symmetry breaking in brane annihilation. The tachyon transforms as a bi-fundamental representation \( (N, \bar{N}) \) under the original symmetry group \( U(N) \times U(N) \). This group is broken to a diagonal subgroup \( U(N) \) by the tachyon condensation. Topologically non-trivial knots are the vortices formed by the tachyon. Take, for instance, a single pair of parent brane-anti-brane. The symmetry group is \( U(1)_1 \times U(1)_2 \) and the tachyon is charged under the linear combination \( A = A^{(1)} - A^{(2)} \), which gets Higgsed by the condensate. The elementary vortices are topologically stable configurations around which the phase of the tachyon VEV changes by \( 2\pi \). They carry one unit of \( A \)-magnetic flux, and are therefore charged under RR fields. Their charge can be inferred from the WZ couplings between close and open string modes on the world-volume of the parent branes,

\[
\int_{p+1} F \wedge C_{p-1},
\]

where \( F \) is the gauge field strength. This coupling accounts for the correct RR charge of the vortex under the \( C_{p-1} \)-form, equal to the RR charge of the \( D_{p-2} \)-brane.

Eq.(4) carries important model-independent information about the nature of daughter defects. It is immediately clear that daughter branes cannot be pointlike as viewed from the

\[1\text{All other homotopy groups are trivial.}\]
three non-compact dimensions. To see this, suppose the initial parent branes fill \( p = 3 + n \) space dimensions, \( n \) of which obviously have to be compact. The daughter branes are then charged under the \( n + 2 \) RR form. From the perspective of the 3 non-compact dimensions, the daughter branes will look as stable point-like defects (\( D_0 \)-branes) only if \( C_{n+2} \)-form has a single index in our \( 3 + 1 \)-dimensions. But this is impossible, since the number of the remaining \( n + 1 \) indices exceeds the number of the compact world-volume dimensions of the parent branes. Hence, irrespective of the cosmological context, 0-branes are never formed in D-brane annihilation after inflation.

Interestingly, formation of objects looking like strings in \( 3 + 1 \)-dimensions (effective \( D_1 \)-branes) is not forbidden. Indeed, strings are charged under the two-form fields, meaning that \( C_{n+2} \) form should have two indices in our dimensions. The remaining \( n \) indices are just enough to fill the compact world-volume dimensions of the parent branes.

Unfortunately, the same reasoning also permits the formation of domain wall type defects (i.e., \( D_2 \)-branes). Indeed, 2-branes will be stable if they couple to a 3-form RR field. Hence, \( C_{n+2} \) should have 3 indices in our dimensions and the rest in the compact ones. This is certainly possible, and implies that the gauge field strength \( F \) should have one index in the extra space and one in ours.

Hence, the formation of walls versus strings becomes a dynamical question, which we would like to address in the cosmological context. To make the problem more transparent, we shall first reduce it to bare essentials. For definiteness, let us do this explicitly for the \( n = 1 \) case, as the generalization to higher \( n \) is trivial. Let \( A \) be a world-volume gauge field Higgsed by the tachyon VEV. And let \( \theta \) be the phase of the tachyon \( \chi \). We shall expand \( A, \theta \) and the \( C_5 \) form in the Kaluza-Klein (KK) states with respect to the compact world volume coordinate, which we shall call \( x_5 \), and write down the effective \( 3 + 1 \)-dimensional couplings. The terms of our interest are

\[
m^2 (A_5^{(m)} - \theta^{(m)})^2 |\chi|^2 + (A_\mu^{(m)} - \partial_\mu \theta^{(m)})^2 |\chi|^2 + m^2 (A_\mu^{(m)} - \partial_\mu A_5^{(m)})^2 + m (A_\mu^{(m)} - \partial_\mu A_5^{(m)}) C_{\nu\alpha\beta}^{(m)} \epsilon^{\mu\nu\alpha\beta} + (\partial_\mu A_\mu^{(m)} - \partial_\nu A_\nu^{(m)}) C_{5\alpha\beta}^{(m)} \epsilon^{\mu\nu\alpha\beta}.
\]

(5)

Here, \( m \) is quantized in units of \( 1/l_5 \), where \( l_5 \) is the size of the compact \( x_5 \)-dimension. The usual kinetic terms of \( A_\mu \) and \( C \) are not displayed, and we have suppressed the relative powers of the string scale infront of the last two terms.

Notice that in the above notations \( A_5^{(m)} \) and \( \theta^{(m)} \) are dimensionless, so that the canonically normalized fields are \( m A_5^{(m)} \), and \( \theta^{(m)} |\chi| \) respectively. The meaning of each of the terms in (5) is clear. What we are dealing with is the usual Higgs effect. At each mass level \( A^{(m)}_\mu \) eats up a combination of \( A_5^{(m)} \) and the tachyon phase \( \theta^{(m)} \), and becomes a massive spin-1 field of mass \( m^2 + |\chi|^2 \). Another combination of \( A_5^{(m)} \) and \( \theta^{(m)} \) becomes a massive scalar of mass \( |\chi|^2 \). The important couplings are the last two terms in (5), which tell us how the objects charged under \( C \) can be created.

Let us first consider the first of these terms, which controls the formation of 2-branes. It is clear that in order to create a source for the \( C_{\nu\alpha\beta} \) 3-form, we either have to excite \( A^{(m)}_\mu \) (that is, create gradients of the gauge field in extra dimensions), or to produce gradients of \( A_5^{(m)} \) in our non-compact dimensions. For \( m \neq 0 \) such possibilities are costly in energy and cannot be achieved by de Sitter fluctuations, since \( H \ll 1/l_5 \). So the only option would be to create gradients in the zero mode \( A_5^{(0)} \). At the tree level this is a massless field, and such
gradients are possible, but they cannot lead to stable defects, as long as $A_5^{(0)}$ stays massless. For 2-branes to become stable, $A_5^{(0)}$ should get a potential with discretely-degenerate minima. For instance, a periodic potential of the form $\cos(A_5^{(0)})$ would lead to the formation of stable 2-branes similar to axionic-type domain walls. $A_5^{(0)}$ will change by $2\pi$ through such a wall, and so will the field strength of $C$, by the amount proportional to the change in $A_5^{(0)}$ [15]. Such potentials may indeed be generated from some non-perturbative dynamics, but this is highly model-dependent.

Let us now turn to the last term in Eq.(5) which is responsible for the existence of $D$-strings. We see that strings can be formed without exciting any of the KK states of either the gauge field or the tachyon phase. We could simply direct the magnetic flux of the zero mode gauge field $F_{\mu\nu}$ into the non-compact directions. As it is clear from (5), this flux would carry an effective two-form $C_{5\alpha\beta}$ RR charge, and be stable. Hence, we might conclude that formation of cosmic strings is not costly in energy and can be afforded by quantum fluctuations during brane inflation. This would be a false impression, however, since we would be forgetting the bulk nature of the RR fields. There are additional $6-n$ dimensions orthogonal to the brane, in which the $C_{n+2}$ RR field propagates. Directing the $F_{\mu\nu}$ flux in three non-compact dimensions necessarily implies the existence of RR gradients in transverse dimensions. In other words, there is a "zero sum" game: we can either avoid gradients of the world-volume fields (such as $A$ and $\theta$) in $n$ world-volume directions tangent to the parent branes, or the gradients of the RR fields in $6-n$ transverse dimensions, but we cannot avoid both! As a result, the formation of cosmic $D$-strings by de Sitter fluctuations is strongly suppressed.

**IV. THE ROLE OF THE RR FIELDS**

We wish to study the formation and the physical nature of $D$ and $F$ strings in the cosmological context. If the values of $\chi$ are uncorrelated beyond some correlation length $\xi$, daughter branes can be formed as topological defects by the usual Kibble mechanism [16]. The spatial variation of the tachyon may either be due to quantum de Sitter fluctuations during inflation, or to thermal or resonant excitation during brane annihilation. We shall first consider the possibility that quantum fluctuations are the dominant effect, as it was assumed in ST and MD. During the course of inflation, scalar fields of mass $m \ll H$ fluctuate by $\delta \phi \sim H/2\pi$ on the horizon scale $H^{-1}$ per Hubble time $H^{-1}$. The characteristic wavelength of the fluctuations is thus $\sim H^{-1}$.

The initial setup considered by MD [7] is that of space-filling $D9$ and anti-$D9$-branes, and the effects of compactification were ignored. By the standard Kibble argument it was concluded that all defects that can be formed will be formed as a result of parent brane disintegration, with a typical defect separation $\sim H^{-1}$. Note that it follows from Eq.(3) that the codimension of daughter branes with respect to the parent branes must be an even integer. MD comment that similar conclusions apply to the case of annihilating branes in the bran inflation scenario, but this case is not discussed in detail.

It was, however, noted by ST [5] that the effect of compactification on brane annihilation can be very significant. According to Eq.(1), the compactification radius is small compared to the horizon, $l_\perp \ll H^{-1}$, suggesting that the variation of $\chi$ in the compact dimensions
should be suppressed. If $\chi$ varies only along the three non-compact dimensions of the parent branes and not along their compact dimensions, then the resulting daughter branes will wrap around the same compact dimensions as the parent branes. The codimensions of the daughter branes should then lie within the large dimensions, and since the codimension should be even, it follows that the defects should be of codimension 2, that is, cosmic strings [5].

We believe, however, that this analysis does not fully account for the effects of compactification, particularly in the directions transverse to the parent branes. $D$-branes are bulk animals, and to analyze their formation it is not enough to consider only the dimensions within the parent branes. As discussed above, the crucial point is that the stable daughter $D$-branes carry conserved charges under the RR fields. These come from the closed string sector and their flux necessarily extends to all compactified directions, irrespective of the plane of the daughter brane localization. If the dominant source of fluctuations are de Sitter fluctuations, then daughter branes cannot be formed, unless these fluctuations prepare suitable gradients of the RR fields. As we shall see, for the values of the Hubble parameter compatible with $D$-brane-inflation, creation of such gradients is costly in energy, so that daughter $D$-strings formation is severely suppressed.

To be maximally general, let us consider the formation of $D_p$-branes, in annihilation of a $D_{p+n} - \bar{D}_{p+n}$ pair. $D_p$ branes are charged under the $p + 1$ form RR field $C_{p+1}$. Since $D$-brane inflation happens due to relative motion of branes, the initial branes cannot wrap all the compactified dimensions. The same is true about the daughter branes which are forced to lie entirely within the parent brane world-volume. Thus, in order for the daughter $D_p$-branes to form by quantum fluctuations, their $C_{p+1}$-flux has to vary by high-dimensional Gauss law at least within some of the compactified dimensions orthogonal to the parent branes. For instance, the variation needed to produce closely separated daughter branes, say by a distance $M_s^{-1} \ll L \ll l_{\perp}$, is unlikely to be achieved by de Sitter quantum fluctuations during brane inflation, since the characteristic fluctuation wavelength is greater than all the compactified dimensions, $l_{\perp} \ll H^{-1}$.

The same result can be easily understood in the language of the four-dimensional Kaluza-Klein (KK) expansion. In this language, the daughter branes act as sources for an infinite tower of massive KK excitations of the $C_{p+1}$ form field. $D_p$-branes cannot be formed by quantum fluctuations, unless these fluctuations excite massive KK states. In order for the RR fluctuations to be sufficient for the formation of daughter branes with separation $L$, the gradients of $C_{p+1}$ must be of the order of $1/L$. Translated into the KK language, the excited KK must fluctuate at least by $\sim 1/L$. However, the mass of the lightest KK state $m_{KK} \sim 1/l_{\perp}$ is much bigger that the value of the Hubble parameter; hence, their fluctuations are exponentially suppressed, at least as

$$\sim H e^{-(1/Hl_{\perp})}.$$  \hspace{1cm} (6)

From Eq. (1), $Hl_{\perp} \ll 1$, indicating that there will be essentially no daughter brane formation.

The only defects which are not covered by this argument are the branes that wrap around all compact dimensions, including the ones transverse to the parent branes. However, the formation of such branes is very costly in energy, since it requires extending the defect cores into the bulk. Such a process may occur as a rare quantum or thermal fluctuation, but it is
very strongly suppressed.\(^2\)

Tachyon variation and RR fields in the bulk can be avoided if the daughter branes form in close brane-anti-brane pairs, or in the form of closed loops or surfaces of size comparable to the brane thickness (which is set by the string scale \(M_s\)). However, this would require variation of \(\chi\) on a distance scale \(\sim M_s^{-1}\) along the large dimensions in the parent branes, which would in turn require expansion rates \(H \gtrsim M_s\). So high an expansion rate is impossible during the brane inflation [see Eq. (1)].

V. SOLITONIC ANALOGIES

Solitonic analogies are sometimes useful for understanding the complicated \(D\)-brane dynamics. Such analogies have been discussed in detail in [18]. Here we shall illustrate the points we made in the preceding Section with an example in which the role of the parent \(D\)-branes is played by a domain wall-anti-wall system, the role of the daughter defects is played by global cosmic strings, and the role of the bulk RR fields is played by the Goldstone phase.

Formation of low-dimensional daughter defects during the annihilation of higher dimensional ”parents” can occur in systems in which the order parameter (the Higgs field), responsible for the daughter brane stability, vanishes in the cores of the parent branes [19]. Just as for \(D\)-branes, the daughter defects are then the knots of the tachyon that is condensing during the parent brane annihilation.

In our example, the Higgs field \(\Phi\) responsible for the strings transforms under a global \(U(1)\) symmetry, \(\Phi \to e^{i\alpha}\Phi\). Outside the domain walls, the Higgs field is in the vacuum state, \(\Phi = \eta e^{i\theta}\), with \(\theta = \text{const}\), while in the wall cores \(\Phi = 0\). A cosmic string carries the topological charge

\[
n = \frac{1}{2\pi} \oint d\theta,
\]

where the integral is taken around a closed loop encircling the string. Thus, for the formation of a string it is necessary that the phase of the Higgs field VEV winds non-trivially along the loop. Formation of such configurations requires that there be a mechanism which supports zeros in the Higgs VEV at least in some regions of space, and at the same time allows the phase of the VEV to change randomly around these zeros. One example of such a mechanism is provided by a thermal phase transition with a spontaneous breaking of a \(U(1)\) symmetry (for a review see [20]). In this situation, the symmetry is restored at high temperatures and the Higgs VEV vanishes. This happens because the state with a zero Higgs VEV corresponds to a minimum of the free energy at sufficiently high temperatures. Hence, zeros in the Higgs VEV are supported by temperature effects. When the Universe cools below certain critical point, the minimum turns into a local maximum, the Higgs field

\(^2\)Spontaneous nucleation of defects by quantum tunneling in de Sitter space has been discussed in [17].
becomes tachyonic and condenses into the true vacuum with a broken symmetry. During the tachyon condensation, thermal effects force the phase of the Higgs VEV to randomly fluctuate over distances larger than the correlation length, and these fluctuations produce topological knots, the global strings. The typical distance between the strings is set by the correlation length of the phase fluctuations.

Let us now turn to the formation of strings in our example of wall-anti-wall annihilation. We assume the temperature is \( T = 0 \), so there are no thermal effects, and the Higgs VEV is in the vacuum everywhere outside the walls. That is, outside the walls the tachyon is already in the vacuum state. However, the Higgs field is forced to zero inside the domain wall cores. When wall and anti-wall are brought on top of each other, the tachyon starts condensing. However, for the tachyon to form topological knots, it is essential that the phase of the Higgs winds outside the walls. Thus, if we start with a configuration of a uniform phase everywhere outside the walls, we can only expect (at best) to form tiny string loops of size comparable to the wall width.

To make the analogy with the \( D \)-brane picture more transparent, let us rewrite the topological charge of global cosmic strings as the "electric" charge under a certain two-form field \( B_{\mu\nu} \), which plays the role analogous to that of the RR two-form for \( D \)-branes. The two-form field \( B_{\mu\nu} \) can be introduced by the following well-known identification:

\[
\epsilon_{\mu\nu\alpha\beta} \partial^\nu B^{\alpha\beta} = \partial_\mu \theta. \tag{8}
\]

The topological charge (7) under \( \theta \) now becomes an electric charge under \( B_{\mu\nu} \). As indicated above, in order for strings to form, it is not enough that the Higgs VEVs on the wall are uncorrelated, but it is essential that the phase also winds outside the wall. In the RR language, this means that \( B_{\mu\nu} \) must assume corresponding "electric field" configuration in the bulk.

Essentially the same remarks apply to daughter brane formation. We can think of the bulk space outside the parent branes as having the tachyon \( \chi \) already in its vacuum manifold \( \mathcal{M} \). Indeed, regions where \( \chi \) develops a VEV in \( \mathcal{M} \) become part of the bulk. Since the dimensions transverse to the parent branes are compact, the variation of \( \chi \) in those dimensions is strongly suppressed during the brane inflation.

VI. OTHER DEFECT FORMATION MECHANISMS

A. Bulk (p)reheating

We now consider some other defect formation mechanisms in brane inflation. The details of brane annihilation process are not well understood. The colliding brane and antibrane are expected to pass through one another and oscillate several times before finally annihilating. The whole process is likely to take time \( \tau \gg M_\star^{-1} \), and we may well have \( \tau \gtrsim l_\perp \). A substantial part of the energy released in this process will be carried away in the form of closed string states, that is, gravitino, RR, and dilaton waves. The resulting RR fluctuations in the bulk may induce the formation of daughter branes. The cores of these daughter branes will come from the core region of the annihilating parent branes.

An alternative version of this scenario is a multi-step brane annihilation. The colliding stacks of branes and anti-branes do not have to annihilate all at once. The energy released
in the first annihilation can generate RR fluctuations in the bulk, so that hedgehog field
configurations will appear with their centers lying within the cores of still existing parent
branes. The daughter branes can then be produced in subsequent annihilations.

To create an infinite network of cosmic strings with a characteristic length scale 

\[ L \lesssim l_\perp, \]

we need RR fluctuations of wavelength \( \sim L \) and magnitude \( dC \sim 1/L \). The corresponding
energy density (in 10 dimensions) is

\[ \rho_{RR}^{(L)} \sim M_s^{2+d_\parallel+d_\perp} L^{-2}. \tag{9} \]

On the other hand, the energy density obtained if the total energy of the annihilating branes
is uniformly spread over the transverse dimensions is

\[ \rho_{RR}^{(tot)} \sim M_s^{4+d_\parallel} l_\perp^{-d_\perp}, \tag{10} \]

Since only a fraction of this energy can go into modes of wavelength \( L \), we should have
\( \rho_{RR}^{(L)} \lesssim \rho_{RR}^{(tot)} \), and thus

\[ L \gtrsim l_\perp (M_s l_\perp)^{d_\perp-2}. \tag{11} \]

For daughter brane formation, we need \( L \lesssim l_\perp \), and it follows from (11) that this is possible
only for \( d_\perp \leq 2 \). This simple analysis suggests that, for \( d_\perp = 1 \) or 2, daughter \( D \)-brane
formation is energetically possible, but whether or not it actually happens depends on the
annihilation timescale \( \tau \) and on the power and spectrum of the emitted RR waves.

The situation may be different in models where the extra dimensions are strongly warped.
For example, in the KKLMMT model of Kachru et. al. [11] the brane annihilation occurs
in a deep gravitational potential well. The energy released in the annihilation may then be
localized in a region \( l_\perp^{(eff)} \ll l_\perp \). In the limit when \( l_\perp^{(eff)} \sim M_s^{-1} \), this region may heat up to
a temperature \( \sim M_s \) and defect formation may be efficient.

**B. Resonant formation of \( F \)-strings**

A completely different mechanism of defect formation in brane annihilation has been
discussed in [18]. There, it has been shown that \( U(1) \)-gauge fields which are massless on
the parent branes get resonantly excited in the process of brane annihilation. This mechanism
works as follows. When tachyon condenses, it higgses one combination of the original \( U(1) \times U(1) \) group, which we called \( A \). Magnetic flux tubes of this gauge field carry RR charge and
no NS-NS charge and are D-strings. It was argued in [18], that the tachyon condensation
must at the same time render the orthogonal gauge field \( \tilde{A} = A^{(1)} + A^{(2)} \) non-dynamical.
This is also clear from the fact that, first, this field is not Higgsed, and second, there must be
no open string excitations about the tachyonic vacuum. So the wave-function of \( \tilde{A} \) vanishes
as a result of tachyon condensation. In a sense, \( \tilde{A} \) becomes infinitely strongly coupled and
disappears from the spectrum. We have shown that during this process there is an instability
towards generation of an \( \tilde{A} \)-electric field. After the branes dissolve, this gauge field cannot
exist in the bulk, so it must be squeezed into electric (and possibly magnetic) flux tubes.
Electric flux tubes carry NS-NS 2-form \( B_{\mu\nu} \)-charge and are equivalent to the fundamental
strings (\( F \)-strings),
The gauge field amplification has been studied in Ref. [18] in a simple model of an exponential brane-anti-brane decay with a time constant $\tau$. It has been shown there that the energy density of the field (in the parent brane world volume) grows as

$$\rho(t) \sim (\tau t)^{-2-(d_\parallel/2)} e^{t/\tau},$$

(12)

and the main contribution to the energy is given by the modes of wavelength

$$l(t) \sim (\tau t)^{1/2}.$$  

(13)

The growth is cut off when $\rho(t) \sim M_s^{4+d_\parallel}$, and the back-reaction becomes important, that is, at $t \sim (4+d_\parallel)\tau \ln(M_s\tau)$. The characteristic length scale of field variation at that time is

$$l_* \sim \tau[(4+d_\parallel)\ln(M_s\tau)]^{1/2}.$$  

(14)

After brane-anti-brane annihilation, we expect a network of $F$ strings to be formed. These strings can of course be infinite or very long in the large dimensions. The initial scale $L$ of the networks will be set by the characteristic wavelength of the stochastic electric field, $l_*$. We expect the decay time $\tau$ to be at least a few times greater than $M_s^{-1}$; then it follows from Eq. (14) that $l_*$ is greater than $M_s^{-1}$ at least by an order of magnitude.

The character of the $F$-string network produced in this way may be different from the Brownian networks resulting from the Kibble mechanism [21]. If the typical electric flux through an area $\sim l_*^2$ is substantially greater than the unit flux trapped in a string, then the strings will be formed in bundles containing many strings each. This phenomenon has been observed in simulations of gauge string formation in the Higgs model where the dominant mechanism was due to fluctuations of the magnetic field [22,23].

We note that electric fields pointing into the $d_\parallel$ compactified directions of the parent branes get amplified as well. This leads to the formation of winding $F$-strings, wrapped around the compact dimensions. These winding modes correspond to particles of mass $m \sim M_s^2 l_\parallel$ and are dangerous if long-lived. They can be avoided if the parent branes are 3-branes with $d_\parallel = 0$, or if the compactified space of the branes has a trivial first homotopy group, $\pi_1(M_\parallel) = I$.

The gauge field $A$ orthogonal to $\tilde{A}$ can also be resonantly excited. The magnetic field of $A$ can then be squeezed into magnetic flux tubes. These are the $D$-strings. The growth of $A$ at the onset of brane annihilation is described by the same Eqs. (12),(13). However, as the tachyon expectation value grows, so does the mass of $A$. As a result, the amplification of $A$ is cut off at $t \sim \tau$. The corresponding energy density, $\rho \sim \tau^{-4-d_\parallel}$, may be insufficient for the $D$-string cores, but more energy may be pumped up if the parent branes oscillate a number of times prior to annihilation, or if the annihilation is very fast, $\tau \sim M_s^{-1}$. Formation of an infinite $D$-string network by this mechanism is likely to be model-dependent.

To summarize, there are two gauge fields, $A$ and $\tilde{A}$, living on the world volume of the unstable $D$-brane. One of them ($A$) is Higgsed by the tachyon condensate, and the corresponding magnetic flux tubes are $D$-strings. The other field $\tilde{A}$ is not Higgsed, but instead its norm gets diminished by the tachyon. This process creates electric flux tubes in $\tilde{A}$ which are the fundamental $F$-strings. Both $D$ and $F$-string networks can be formed as a result of brane annihilation. The evolution of these networks will be discussed in Section VII.
VII. THE NATURE OF D-STRINGS

In this Section we shall point out some peculiar properties of $D$-strings, and show that from the point of view of four-dimensional observers they share properties of both local and global strings. The role of a cosmic $D$-string in four dimensions can be played by any $D_{1+n}$-brane that is wrapped on $n$-dimensional compact manifold and has a one-dimensional projection on the three non-compact dimensions. For instance, it can be simply an un-wrapped $D_1$ brane; a $D_3$ brane wrapped on two compactified dimensions, and so on. The four-dimensional stability of such cosmic $D$-strings is due to the four-dimensional two-form projection $C_{\mu\nu}$ of the original $C_{2+n}$ form, where the remaining $n$ indices take values in the $n$-dimensional compact space on which the brane wraps. Since, according to (8), $C_{\mu\nu}$ in four dimensions is equivalent to a Goldstone field, cosmic $D$-strings can be thought of as global cosmic strings, of the axionic type, with some crucial differences.\(^3\)

To make this connection more explicit, consider first the field strength and the brane coupling of the RR form in ten dimensions (we set the string coupling equal to one),

$$M_s^8 \int_{10} (dC_{2+n})^2 + M_s^{2+n} \int_{2+n} C_{n+2}. \tag{15}$$

Reducing this to four dimensions, and keeping only the zero mode $C$, we get

$$M_p^2 \int_4 (dC)^2 + M_s^{2+n} V_\parallel \int_2 C, \tag{16}$$

where we have used the fact that the four-dimensional Planck mass is given by $M_p^2 = M_s^8 V_\parallel V_\perp$. Hence the effective charge of the cosmic string with respect to the canonically normalized $C$ field is related to the string tension,

$$T_s^{(D)} \sim M_s^{2+n} V_\parallel \tag{17}$$

This is in contrast with the ordinary global strings that are formed by spontaneous breaking of a global $U(1)$-symmetry at some scale $\eta$. Let $\phi = \eta e^{i\theta}$ be the scalar that forms such a string. Rewriting the Goldstone phase in terms of a dual two-form field $C_{\mu\nu}$ (we reserve the traditional notation $B_{\mu\nu}$ for the NS-NS 2-form field), the analog of the action (16) can be written as

$$\eta^2 \int_4 (dC)^2 + \eta^2 \int_2 C. \tag{18}$$

From here it follows that the $B$-charge of the global cosmic string is simply the square root of the string tension, $\sqrt{T_s} \sim \eta$, without additional $M_p$ suppression.

Comparison of the two actions, (16) and (18), reveals both the similarities and the crucial differences between the $D$ and ordinary global strings. Just like for ordinary global strings, the energy of the three-form field strength in transverse directions diverges logarithmically.

\(^3\)Another connection with axionic type string was made in [14], were it was conjectured that, from the point of view of 4D supergravity theory, $D$-strings are the $D$-term strings. In this language, the tachyon is the Higgs that compensates the non-zero $D$-term potential. A detailed analysis of this conjecture is given in [14].
\[ \int dC^2 \propto \ln R. \]  
(19)

For \( M_s \ll M_P \), however, this divergence is a negligible correction to the string mass on the present Hubble horizon, \( R \sim t_0 \), because the \( C_{\mu\nu} \) charge is suppressed by the four-dimensional Planck mass relative to the string tension,

\[ \frac{\Delta T_s^{(D)}}{T_s^{(D)}} \sim \frac{T_s^{(D)}}{M_p^2} \ln(M_s t_0) \ll 1. \]  
(20)

The above is the contribution of the zero mode RR axion to the effective four-dimensional string tension. The total contribution from the higher KK modes of the RR fields is greater, but only zero mode corresponds to a four-dimensional long range field.

The coupling of the string to the \( C_{\mu\nu} \) field is of the same strength as the one to the four-dimensional zero-mode graviton (and dilaton). The relevant ratio that controls the strength of the interaction is \( T_s^{(D)}/M_p^2 \), which is small for \( T_s^{(D)} \ll M_p^2 \). Because of this very small charge, there are other crucial differences from the ordinary global strings. The dominant energy loss mechanism for the conventional global cosmic strings is due to the Goldstone boson radiation [24,25]. According to (18), the Goldstone coupling to such strings is set by the symmetry breaking scale (i.e., the tension scale) \( \eta \), and for light strings is very much enhanced relative to the gravitational coupling. This is not the case for \( D \)-strings, for which the \( C_{\mu\nu} \) field couples with the gravitational strength. This is easy to understand if we recall that both the graviton and the RR fields (together with the dilaton) come as zero modes of closed strings, and in the BPS limit their exchanges exactly cancel each other. The rate of energy loss by oscillating string loops due to RR radiation is therefore comparable to that due to gravitational radiation. Hence, \( D \)-strings are sort of hybrid objects. On the one hand, like global strings (and unlike more conventional gauge strings), they exert long range RR fields, but on the other hand, the RR charge is gravitational and does not provide a dominant energy loss mechanism.

It is well known that in the presence of anomaly ordinary global cosmic strings become boundaries of domain walls. For cosmic \( F \)-strings in heterotic theory this is usually the case [8]. Such an effect as a possible source for \( D \)-string instability was pointed out in [9], and was analyzed in [14] from the 4D supergravity point of view. The bottom-line of the latter analysis is the following. As seen from the four-dimensional \( N = 1 \) supergravity theory, the \( U(1) \)-symmetry in question is non-linearly realized on two fields. One is the phase of the tachyon \( \theta \), and the other is the axion \( a \), which is dual to the (effective) RR two-form field \( C \). Under \( U(1) \), the axion shifts in the following way:

\[ a \to a + g Q_a \alpha, \]  
(21)

where \( \alpha \) is the gauge transformation parameter, \( g \) is the gauge coupling, and \( Q_a \) is the axionic charge, given by

\[ Q_a = \frac{\xi}{M_p^2}. \]  
(22)

Here, \( \xi \) is of order the \( D \)-string tension \( T \).\(^4\) Since axion is decoupled in the \( M_P \to \infty \) limit

\(^4\)More precisely, if the \( D \)-string is a \( D_1 \)-brane, then \( \xi = T/2\pi \).
(an infinite compactification volume), in this limit the $U(1)$-anomaly must cancel among the massless fermions. For finite $M_p$, however, the story is different, since the fermionic $U(1)$-charges may acquire a shift of order $\xi/M_p^2$, and may contribute to the chiral anomaly. In such a case, the consistency of the theory requires that the anomaly must be canceled by the axionic shift (via Green-Schwarz type mechanism), which implies that there is a coupling

$$a F \wedge F.$$  \hfill (23)

Hence, either the fermionic chiral anomaly vanishes on its own, in which case the coupling (23) is absent, or there is a fermionic chiral anomaly which is canceled by the axion through the coupling (23). In the former case strings have no axionic domain walls attached. In the latter case, the outcome depends on the presence of mixed chiral fermionic anomalies with other gauge groups. If such anomalies are present, then the anomaly cancellation will require couplings of the form (23) with all the gauge groups in question. The axion can then get a mass from instantons, and as a result the $D$-strings will become boundaries of domain walls.

Finally, let us point out that $D$-strings can be superconducting, and thus carry the observable properties of the ordinary superconducting cosmic strings [26]. This is because they may get connected to a surviving p-brane with open strings whose end points are charged under the gauge group living on the brane. Massless excitations of these open strings can carry a current along the $D$-strings. For charge carriers to be massless, the $D$-string must be on top of the surviving brane. This may (usually does) create a potential instability of breakage of the $D$-string on the surviving brane. If $D$-strings are stabilized away from the surviving brane, the charge carriers will be heavy, but they can still be excited by the cosmic magnetic field. In such a case the signatures are very different from the superconducting strings with massless charge carriers [46]. In this case the strings can still break by tunneling into monopole-anti-monopole pairs [9]. The charge carrier masses in this case will probably be too large for any appreciable currents to develop. The properties of strings in this case are similar to those of necklaces (see section VIII.B).

The tension of the fundamental ($F$) strings is $T^{(F)}_s \sim M_s^2$, and can be much smaller than that of $D$-strings if $l_{\parallel} \gg M_s^{-1}$. The $F$-strings are charged under the NS-NS two-form $B_{\mu\nu}$, and in this respect are similar to $D$-strings. An important property of $F$-strings is that they can end on the surviving $D$-branes. For instance, an $F$-string intersecting a $D$-string can break into two, with the ends of the two resulting strings attached to the $D$-string [27]. In the exact supersymmetric limit, for parallel $D$-brane and $F$-string, this is a dynamically preferred process: the energy of the system is substantially reduced when the $F$-string is swallowed by the brane. In the absence of supersymmetry, the situation may change, because the $D$-string tension gets renormalized, and the dilaton and RR fields may acquire masses, resulting in a non-trivial interaction potential between $F$ and $D$-strings. It is conceivable that as a result the $F$-string breaking process will be suppressed, so that intersecting $F$ and $D$-strings will simply pass through one another. (We note, however, that this would require a supersymmetry breaking scale comparable to the string scale.) Below, we shall consider both possibilities.
VIII. STRING EVOLUTION

In this Section, we assume that stochastic networks of $D$ and/or $F$-strings are formed in brane annihilation and study the subsequent evolution of these networks.

A. Independent networks

We shall first consider the evolution of an independent string network, disregarding the interaction of $D$ and $F$ networks with one another and with the surviving branes. This may be applicable, for example, if $F$-string formation is somehow suppressed and $D$-strings cannot break on the surviving branes. Another possibility is that $D$-string formation is suppressed and the $F$-network is localized by bulk gravitational forces in a region with no surviving branes.

The strings initially form a stochastic network localized in the plane of the parent branes. Later they may depart from that plane, under the action of bulk fields. Since the extra dimensions are compact, the gravitational effects of the strings will be the same as those of ordinary cosmic strings. However, the string evolution can be significantly modified.

A very important role in string evolution is played by reconnections of intersecting strings. These reconnections are responsible for closed loop formation and for the straightening of long strings on the horizon scale. Two string segments moving towards one another almost inevitably intersect in 3D. However, it is much easier to avoid intersection in higher dimensions. As an illustration, consider two pointlike particles moving towards one another in a 1D universe. If the universe is truly 1D, then the particles inevitably collide. Now suppose the universe is $(d-1)$-dimensional, with $d$ compact dimensions of size $l$. Then, if the effective size of the particle is $\delta \ll l$, the collision probability is

$$p \sim (\delta/l)^d \ll 1.$$  

Similarly, we expect the $D$-string reconnection probability to be given by (24), with $d$ being the number of dimensions transverse to the string world-volume and $\delta \sim M^{-1}$. 3-dimensional observers will not be aware that the strings often miss one another in extra dimensions. From their point of view, the strings will intersect, but fail to reconnect. The effect of extra dimensions on string evolution can therefore be simulated by simply introducing a reconnection probability $p \ll 1$.

For $F$-strings, Eq. (24) is replaced by

$$p \sim (M_{s}l)^{-d} (M_{s}l_{\parallel})^{-d}.$$  

Hence, the reconnection probabilities for $F$ and $D$-strings are generally different. Together with the different tensions, this may result in substantial differences in evolution.

Note that in models with strongly warped bulk geometries, like the KKLMMT model [11], strings may be confined to the bottom of the potential well, so the effective $l$ may be much smaller than the actual size of extra dimensions.

So far, numerical simulations of string evolution in an expanding universe have been performed assuming $p = 1$. They showed scale-invariant evolution, with the characteristic curvature radius of long strings and the typical inter-string separation both comparable to
the horizon, $L \sim t$, and a large number of small closed loops of sizes $\ll t$. We expect that a small reconnection probability will slow down the string evolution, so that the scale of the network $L$ will be smaller than $t$. If $v$ is the characteristic long string velocity, then the number of encounters of any given long string with other long strings per Hubble time is $N \sim vt/L$. In order for the strings to be able to keep up with the Hubble expansion and adjust their shape to the increasing size of the horizon, these encounters have to result in one or few reconnections. This means that $Np \sim 1$, and thus [28]

$$L \sim pt. \quad (26)$$

If the motion of strings is relativistic, as in the case of “usual” strings in $(3+1)$ dimensions, then $v \sim 1$ and $L \sim pt$. However, a small reconnection probability may result in accumulation of small-scale wiggles. This would increase the effective mass per unit length of strings and reduce their effective tension. The string velocity $v$ will then also be reduced.

The effect of a small reconnection probability on string evolution has been studied in flat-space simulations [29]. Surprisingly, the $p$-dependence obtained from the simulations is different from (26). A decrease of $p$ from 1 to about $1/3$ has little effect on the string scale $L$. As $p$ is decreased further, $L$ does go down, but not as fast as Eq. (26) suggests. A numerical fit to the data gives

$$L \propto \sqrt{p} \quad (27)$$

in the range $0.05 < p < 0.3$. A possible explanation suggested in [29] is that due to the presence of small-scale wiggles, long strings may have many opportunities to reconnect in each encounter. We expect though that the scaling (26) should set in when $p$ gets sufficiently small. It would be interesting to perform numerical simulations of string networks with $p \ll 1$ in an expanding universe and to analyze the impact of a small $p$ on the observational effects of strings.

Oscillating loops of string will lose their energy by gravitational, RR, and dilaton radiation. The energy loss rates in all three channels are comparable to one another, as long as the dilaton and the RR fields can be regarded as massless. However, the dilaton must have a nonzero mass $m_d$, and the dilaton radiation is suppressed after the typical loop size gets larger than $m_d^{-1}$. 5

We note finally that, since the evolving string network is no longer confined to the initial plane of the branes, string intersections will occasionally result in formation of small closed loops, winding modes, wrapped around the compact dimensions. 3D observers will perceive such loops as point particles of mass $m \sim T_s l_\perp$. It is important to estimate the rate of their production, since they can potentially overclose the universe. Winding modes do not exist if the extra dimensional manifold has trivial $\pi_1$.

5Constraints on the dilaton mass and string tension resulting from the observational bounds on dilaton decays have been discussed in [30]. These bounds may need a revision here, because of the modified string evolution.
B. Interacting networks

Suppose now that both $D$ and $F$-string networks are formed. $F$-strings will then break as they intersect with $D$-branes with the break points carrying gauge charges attached to the branes. Multiple intersections between Brownian $D$ and $F$-strings will result in chopping up of all $F$-strings into segments with their ends attached to the $D$-network. If this were the end of the story, the resulting network would resemble a $Z_3$ string network, with three strings joined at each vertex [31]. However, there is one further crucial point to consider.

In realistic brane inflation scenarios, some of the original 3-branes survive annihilation. In particular, there should be at least one such brane which we now inhabit. A string segment passing through such a brane will generally break into two segments, with their ends attached to the brane. The resulting $F$-segments will connect $D$-strings to the branes and will carry 3D-world gauge charges at the ends connected to the 3-branes. The charges will be pulled back and forth by the string tension, and the energy of the $F$-segments will be rapidly dissipated by radiation of gauge quanta. As a result, $D$-strings will be connected to the 3-branes by short $F$-segments of length $\sim l_\perp$. The tension in these segments will cause local $D$-string vibrations, and the energy will further be dissipated into gravitational, RR, and dilaton waves, until the $D$-strings become coincident with the 3-branes. In this limit, the $F$-segments become massless charged particles living on the strings, and $D$-strings become very similar to ordinary superconducting strings. Since $D$-strings are now held on top of the 3-brane, their reconnection probability is $p \sim 1$, as for ordinary strings in 3$D$.

In the above discussion we disregarded the force of interaction between $D$-strings and 3-branes. This interaction indeed vanishes in the BPS limit, but supersymmetry breaking should give rise to a non-trivial potential. It is possible, in particular, that the interaction force is attractive at large and repulsive at small distances, with an equilibrium position at some finite distance from the brane, $M_s^{-1} \lesssim r \lesssim l_\perp$. The connector $F$-strings will pull $D$-strings closer to the branes near the points of connection. In this scenario, the connectors act as massive point particles, with $m \gtrsim M_s$, from the 3$D$ point of view. The properties of $D$-strings are then similar to those of “necklaces”, where massive monopoles and antimonopoles play the role of beads on the strings [32,33].

IX. CONCLUSIONS AND DISCUSSION

We have examined possible mechanisms of defect formation at the end of brane inflation. As brane-anti-brane pairs collide and annihilate, lower-dimensional $D$-branes can be formed in their place. We argued that the crucial requirement for this to happen is that fluctuations of the tachyon expectation value and of the RR fields should be excited in the bulk on a length scale smaller than the size of the transverse dimensions, $l_\perp$. Quantum de Sitter fluctuations during inflation have characteristic wavelength $H^{-1} \gg l_\perp$ and fail to satisfy this requirement.

The required RR fluctuations can be produced by the RR waves excited during the brane annihilation process. Assuming that $l_\perp \gg M_s^{-1}$, we have shown that this is energetically possible only if the number of transverse dimensions is $d_\perp \leq 2$. The dynamics of this process and whether or not it actually yields an infinite string network remain to be investigated. In models with a strongly warped bulk, like the model of Kachru et. al. [11], the effective
value of \( l_\perp \) may be much smaller than the actual size of the extra dimensions and can be as small as \( l_\perp^{(\text{eff})} \sim M_s^{-1} \). Then the above bound on \( d_\perp \) does not apply and \( D \)-string formation can be very efficient.

We have pointed out that cosmologically interesting defect networks can be produced by the mechanism suggested in Ref. [18]. The crucial point is the following. Out of the two massless gauge fields \((A, \tilde{A})\) living in the world volume theory of an unstable brane-anti-brane pair, one \((A)\) is Higgsed by the tachyon condensate. The corresponding magnetic flux tubes are \( D \)-strings. The other field \((\tilde{A})\) is rendered non-dynamical by the tachyon VEV. During this process, \( \tilde{A} \)-electric field gets resonantly excited. This field is then squeezed into electric flux tubes, which are the \( F \)-strings. The resonant excitation of the magnetic component of the field \( A \) is less efficient, but can be enhanced if brane annihilation is preceded by the collision and multiple oscillations of the parent branes. Quantitative conditions for the formation of a \( D \)-string network require further study.

We have discussed the physical properties of \( D \)-strings and pointed out some important differences from the ordinary cosmic strings. The tension of \( D \)-strings, \( T_s^{(D)} \sim M_s^2 l_\parallel d_\parallel \) can be much greater than that of \( F \)-strings, \( T_s^{(F)} \sim M_s^2 \), if the compactified dimensions of the parent branes are large compared to the string scale, \( l_\parallel \gg M_s^{-1} \). Like ordinary (gauge) cosmic strings, \( D \)-strings are vortices of the tachyon carrying a unit magnetic flux of \( A \). At the same time, they carry a long-range RR field \( C \) and are in this respect similar to global strings. An important difference is that the coupling to \( C \) is suppressed by the ratio \( T_s^{(D)}/M_p^2 \) compared to the ordinary global strings. As a result, RR radiation power from an oscillating \( D \)-string loop is comparable to that of gravitational radiation. This is in contrast to global strings, for which the Goldstone boson radiation is the dominant energy loss mechanism.

Once the string networks are formed, their subsequent evolution crucially depends on the nature of interaction of \( D \) and \( F \) strings with one another and with the surviving 3-branes. If both types of networks are formed, then an \( F \)-string intersecting a \( D \)-string can break into two, with the ends of the two resulting strings attached to the \( D \)-string. The same kind of process occurs when \( F \)-strings collide with one of the surviving branes. As a result of these interactions, the \( F \)-network will be chopped into small segments connecting \( D \)-strings to the 3-branes. The energy of the segments will be rapidly dissipated, so the \( D \)-strings will become coincident with the 3-branes, and the segments themselves will turn into massless charged particles living on the strings. \( D \)-strings in this scenario will be very similar to ordinary superconducting strings. If there is an interaction potential that holds \( D \)-strings at a finite distance from 3-branes, the \( F \)-segments will appear as massive point particles of mass \( \gtrsim M_s \) attached to the strings, as in “necklaces”, where massive monopoles play the role of beads on strings.

We have also considered the evolution of a single string network, assuming that its interaction with the surviving branes is negligible. The strings then propagate in the higher-dimensional bulk and can avoid intersections much more easily than strings in 3 dimensions.\(^6\)

For macroscopic observers, string intersections will appear as frequent as usual, but on most

\(^6\)We should stress, however, that in a concrete model, whatever dynamics will prevent strings from breaking on the surviving brane, probably will also restrict their motion in full high-dimensional space. So we should be careful with applying the above argument.
occasions the intersecting strings will fail to reconnect. Thus, the effect of extra dimensions can be accounted for by introducing a small reconnection probability, $p \ll 1$. This is likely to result in the increased wiggliness of strings and in a smaller inter-string separation.

The string networks produced in brane annihilation are potentially observable through gravitational lensing [34], linear discontinuities on the microwave sky [35], gravitational wave bursts [36], or a stochastic gravitational wave background [37]. If $D$-strings are superconducting, the gravitational wave bursts they produce may be accompanied by gamma-ray bursts [38]. On the other hand, if $D$-strings behave as necklaces, the annihilations of heavy “beads” on the strings may produce ultrahigh-energy cosmic rays [39].

We finally note an intriguing recent observation of two nearly identical galaxies at redshift $z = 0.46$ with an angular separation of 1.9 arc seconds [40]. The most plausible interpretation appears to be lensing by a cosmic string with $T_s/M_p^2 \sim 3.7 \times 10^{-7}$ [40]. This estimate assumes a slowly moving string orthogonal to the line of sight at a relatively low redshift ($z \lesssim 0.1$). Increasing the string redshift or changing its orientation leads to a higher estimate for $\mu$, which may be in conflict with the microwave observation data. (The most recent constraint from string simulations is [41] $T_s/M_p^2 \lesssim 7 \times 10^{-7}$.) On the other hand, the estimate for $T_s$ can be decreased due to (relativistic) motion [42] or wiggliness [43] of the string.

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While this work was in progress, we learned that N. Jones, H. Stoica and S.-H. Tye have independently reached the conclusion that string reconnections may be suppressed in braneworld cosmology, resulting in a denser string network [44]. The string evolution with a low reconnections probability was then discussed in [45].

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