Aberration by gravitational lenses in motion

Simonetta Frittelli*
Department of Physics, Duquesne University, Pittsburgh, PA 15282, USA

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ABSTRACT
It is known that a fully relativistic integration of the null geodesics of a weak perturbation of flat space–time leads to a correction of order \( v/c \) to the bending angle and time delay due to a gravitational lens in slow motion with small acceleration. The existence of the \( v/c \) correction was verified by the very long baseline interferometry experiment of the bending of light by Jupiter on 2002 September 8. Here the \( v/c \) correction is interpreted by means of standard aberration of light in an optically active medium with an effective index of refraction induced by the gravitational field of a lens in motion.

Key words: gravitation – gravitational lensing – relativity.

1 INTRODUCTION
On 2002 September 8, the close alignment of Jupiter with the quasar J0842 + 1835 provided the opportunity for an extraordinary measurement that consisted of the delay between the observations of the quasar as recorded by two radio antennas. The observation was conducted by the National Radio Astronomical Observatory (USA) and the Max Planck Institute for Radio Astronomy. The time delay refers to the difference in arrival time of two light-rays leaving the quasar simultaneously at a single emission event. The observation (Fomalont & Kopeikin 2003) verified the existence of a correction of order \( v/c \) to the time delay of a gravitational lens in motion: the fact that the time delay of light signals by a point lens in motion carries a factor of \((1 + v/c)\) with respect to the Shapiro time delay – which, in turn, represents the additional time imposed on a light signal due to the slowdown by the gravitational field of a point mass at rest. Here \( v \) is the component of the velocity of the deflector along the line of sight.

The \( v/c \) correction of first order to the bending angle was predicted (to the best of my knowledge) by Pyne & Birkinshaw (1993), and subsequently confirmed by Kopeikin & Schäfer (1999), Frittelli, Kling & Newman (2002) and Frittelli (2003). The recent observational verification strongly suggests that such a correction needs to be incorporated into the fitting of gravitational lensing events. The size of the correction is extremely small and may be negligible at cosmological scales. Nevertheless, the effect may not only be important but may even be controllable in microlensing events such as nearby stars with high proper motions acting as lenses for distant stars in the Milky Way or in the Magellanic Clouds (Paczyński 1996; Frittelli 2003).

Here it is shown that the \( v/c \) effect admits a natural interpretation in the standard framework of gravitational lensing. In this framework, the bending of light is interpreted in terms of light propagating within a medium with an effective index of refraction, rather than in terms of a metric field (Petters, Levine & Wambsganss 2001; Schneider, Ehlers & Falco 1992). Formally, the bending is analogous to that induced by a piece of glass with a non-uniform index of refraction. The value of this interpretation resides in its potential to integrate the \( v/c \) effect with other standard practices in gravitational lensing.

2 ABERRATION BY GRAVITATIONAL LENSES
It is well known (Bergmann 1942) that the apparent angular position of distant stars wobbles as the Earth turns in its orbit around the Sun. The wobble is the aberration caused by the observation of a light-ray in two different frames: the Sun’s rest frame, and the inertial frame instantaneously attached to the Earth at the moment of observation.

In the Sun’s rest frame at the observation event, the direction of a light-ray and the velocity of Earth define a plane where coordinates \((x', y', z', ct')\), with

\[
\begin{align*}
  r' &= r - vt + v(y-1)\left(\frac{v \cdot r}{v^2} - t\right), \\
  t' &= \gamma \left( t - \frac{v \cdot r}{c^2} \right),
\end{align*}
\]

where \( \gamma = [1 - (v/c)^2]^{-1/2} \). With the convention that \( dx'^a = \eta_{ab} + dx^a \cdot dx^b = -d(ct)^2 + dx^2 + dy^2 + dz^2 \), the Lorentz transformation applied to the null vector \( k^a = (k^x, k^y, 0, 1) \) in the Sun’s rest frame, where \((k^x)^2 + (k^y)^2 = 1 \) yields the null vector \( k'^a = k^a \eta_{ab} x'^b \) as seen in the Earth’s frame of reference, the spatial part of which is

\[
  \mathbf{k}' = \mathbf{k} - \frac{\mathbf{v} \cdot \mathbf{r}}{c} + (\gamma - 1) \frac{\mathbf{v}}{c} \left( \frac{\mathbf{c} \cdot \mathbf{v}}{v^2} - 1 \right). 
\]
The observed angle with the negative y-axis on Earth is \( \tan \theta' = -k^y/k^x \). We are only interested in the leading correction of order \( v/c \), so we may neglect the term proportional to \( (v/c)^2 \) to get

\[
\frac{k^y}{k^x} = \frac{k^x - v^x/c}{k^x - v^x/c} + O((v/c)^2)
\]

\[
\left(1 + \frac{v^y}{ck^y}\right) \frac{k^x}{k^y} = \frac{v^x}{ck^x} + O((v/c)^2).
\]

By definition we have \( k^y = \sin \theta \) and \( k^x = -\cos \theta \), so this relationship reads

\[
\tan \theta' = \left(1 - \frac{v^y}{c} \cos \theta \right) \tan \theta - \frac{v^x}{c} \cos \theta,
\]

i.e. the standard aberration formula for angles \( \theta \) of any size between 0 and \( \pi/2 \), correct up to second order in \( v/c \) (Bergmann 1942). For small angles \( \theta \) this simplifies to

\[
\theta' = \left(1 - \frac{v^y}{c} \right) \theta - \frac{v^x}{c},
\]

which shows that the two components of the Earth’s velocity have completely different aberration effects. The ‘longitudinal’ component, \( v^y \), which at this level of accuracy is indistinguishable from the component of the velocity along the line of sight, affects the apparent angle as a pre-factor, whereas the ‘transverse’ component \( v^x \) simply adds a fixed shift to all small angles.

Next let us consider gravitational lensing in its simplest form, that is as the propagation of a light-ray within an optically active medium. A point mass at rest effectively slows down a light-ray, inducing an apparent angle as a pre-factor, whereas the ‘transverse’ component \( v^x \) in its simplest form, that is as the propagation of a light-ray within an optically active medium. A point mass at rest effectively slows down a light-ray, inducing an apparent angle as a pre-factor, whereas the ‘transverse’ component \( v^y \) simply adds a fixed shift to all small angles.

The observed time delay \( \Delta t \) of a light-ray following the direction of the unit vector \( \hat{k} \) that minimizes the travel time (see, for instance, Rossi 1965):

\[
\frac{dk}{dt} = \hat{k} \times \left(\nabla \frac{n}{n} \times \hat{k}\right).
\]

where \( \frac{dk}{dt} \) is the Euclidean element of length along the path. Since \( \hat{k} \times (\nabla \frac{n}{n} \times \hat{k}) = \nabla n \cdot (\hat{k} \cdot \nabla n)\hat{k} \), we have, equivalently,

\[
\frac{dk}{dt} = \nabla \frac{n}{n} \frac{n}{n},
\]

where the symbol \( \perp \) indicates the local plane perpendicular to \( \hat{k} \). From emission to reception, the light-ray changes direction by

\[
\alpha = \theta_{\text{in}} - \theta_{\text{out}},
\]

where the subscripts indicate the directions in which the light-ray effectively enters and leaves the gravitational field of the point mass. Since the path of the light-ray stays on a plane, we can restrict attention to the magnitude of \( \alpha \) which thus represents the change between the angles that the incoming and exiting light-rays make with the negative y-axis:

\[
\alpha = \theta_{\text{in}} - \theta_{\text{out}}.
\]

In the case of a point mass at rest, the bending angle is

\[
\alpha = \frac{4GM}{c^3 \xi},
\]

where \( \xi \) is the distance of closest approach of the light-ray to the point mass. This would be the bending of light that a deflector would induce on a light-ray, in the rest frame of the deflector, since, during the fast transit of a light-ray, the deflector is well approximated as an inertial frame. However, since the deflector is in motion with velocity \( v_2 \) relative to the observer, which is approximately constant for the duration of the transit of the light-ray in its vicinity, then from the point of view of the observer both angles \( \theta_{\text{in}} \) and \( \theta_{\text{out}} \) as well as the entire path during the transit are affected by aberration according to equation (5) where \( v = -v_2 \). In the rest frame of the observer we have

\[
\alpha' = \theta'_{\text{in}} - \theta'_{\text{out}}.
\]

\[
\left(1 + \frac{v^y}{c}\right) (\theta_{\text{in}} - \theta_{\text{out}}) = \left(1 + \frac{v^y}{c}\right) \alpha.
\]

Therefore the bending angle by a deflector moving with speed \( v \) along the line of sight carries a pre-factor of \( (1 + v/c) \) with respect to the bending angle by the same deflector at rest. The component of the velocity of the deflector transverse to the line of sight does not affect the bending angle (it affects both the incoming and exiting directions by the same shift, hence not their difference).

Now let us see what this implies for the time delay by a deflector in motion with respect to the same deflector at rest. By definition we have

\[
\alpha = -\int \frac{dk}{dt} dt.
\]

Since \( n \) differs from 1 only in terms that are of the order of the Newtonian potential (the mass), then

\[
\int \frac{dk}{dt} dt = \nabla \left(n - 1\right) + O(U^2)
\]

and

\[
\int \frac{dk}{dt} dt = \nabla \int \left(n - 1\right) dt = \nabla \xi (\xi \Delta t),
\]

where \( \Delta t \) is the Shapiro time delay by the lens at rest:

\[
\Delta t = -\frac{4GM}{c^3} \ln \xi.
\]

So the bending angle and the gravitational time delay are related by a gradient – that is, the time delay acts as a potential for the deflection:

\[
\alpha = -\nabla \xi (\xi \Delta t).
\]

The same is true if the lens is moving so far as the motion of the lens is encoded in the corresponding index of refraction \( n' \) resulting in a corresponding gravitational time delay \( \Delta t' \): that is

\[
\alpha' = -\nabla \xi (\xi \Delta t').
\]

It follows that

\[
\Delta t' = \left(1 + \frac{v^y}{c}\right) \Delta t,
\]

up to an additive constant. In this picture of gravitational lensing, the time delay by a deflector in motion with speed \( v \) along the line of sight carries a pre-factor of \( (1 + v/c) \) with respect to the standard Shapiro time delay, essentially due to the constancy of the speed of light.

One may question whether general relativity was used at all in this simplified scheme. The answer is yes, in two ways. First, the factor of 2 in the effective index of refraction \( n = 1 - 2U/c^2 \) is a direct effect of the notion that light travels along null geodesics of the metric

\[
dx^2 = -(1 + 2U/c^2) dx^2 + (1 - 2U/c^2) dx \cdot dx.
\]

In Newtonian fashion, in contrast, conservation of energy (per unit mass) for an
orbit with instantaneous speed $u$ that tends asymptotically to speed $c$ leads to $c^2/2 = u^2/2 + U$, which yields an effective index of refraction $n = R_{\text{Newton}} = 1 - U/c^2 + O(U^2)$. This difference in index of refraction leads to a difference of a factor of 2 in the bending angle, and, as is well known, constitutes the main and only difference between Newtonian and relativistic bending of light in the weak-field regime of a point mass at rest, being the object of the classical test of 1919. This is discussed, for instance, in section 1.1.1 of Schneider et al. (1992).

The second way in which general relativity is used in the present derivation of the $v/c$ effect, however, is in the fact that the gravitational physics involved is Lorentz-invariant. The physics resides in the index of refraction $n$. It is known that, under a Lorentz transformation, the index of refraction of an optical medium transforms as $n^{-1} = n_s^{-1} - (v/c)(1 - n_s^{-2})$, where $n_s$ is the index of refraction of the same medium at rest, as demonstrated by Fizeau in the classic experiment of light traversing a moving fluid in a pipe (Bergmann 1942). In gravitational terms, where $n_s = 1 - 2U/c^2$, this translates directly to $n = 1 - 2U/c^2 - 4(v/c)U/c^2$. This index of refraction is entirely consistent with the metric $ds^2 = -(1 + 2U/c^2)c^2 dt^2 - 8V/c^2 \cdot dx \, dx + (1 - 2U/c^2) dx \cdot dx$ with $V \equiv U/v/c$ (Schneider et al. 1992), which is related by a Lorentz transformation to the metric $ds^2 = -(1 + 2U/c^2)c^2 dt^2 + (1 - 2U/c^2) dx \cdot dx$ of a body at rest. Because the linearized Einstein equations are wave equations, which are Lorentz-invariant, both metrics are solutions (for exactly the same reason that the electric field of a uniformly moving charge coincides with the Lorentz-transformed field of a charge at rest). The verification of the $v/c$ effect is thus a direct check of the Lorentz invariance of the linearized Einstein equations, in a way that the Eddington solar eclipse expedition was not. The Jupiter experiment is, thus, nothing short of remarkable.

The significance of the Jupiter experiment has been overshadowed by a controversy as to whether or not the experiment directly measured the 'speed of gravity' (hence beating the gravitational wave detectors in the race for the first direct detection of gravitational waves). Because the experiment actually found the field equations of general relativity to be Lorentz-invariant in the weak limit, there is hardly any question that the experiment actually measured the speed of the characteristics of the field equations and found it consistent with the prediction: the speed of light. What this means is that, since the characteristics are light-like, when gravitational waves occur, they must travel at the speed of light. This does provide a foundation to interpret $c$ in the pre-factor of $(1 + v/c)$ in the time delay as the 'speed of gravity'. This does not mean, however, that gravitational waves emitted by Jupiter affected the measurement. The effect of gravitational waves in the experiment is practically negligible, as thoroughly demonstrated by Will (2003). The experiment did not directly measure the gravitational waves emitted by Jupiter. This is the foundation of Will's technical claim that $c$ in the pre-factor of $(1 + v/c)$ is not the 'speed of gravity' (Will 2003). It would seem as if semantics had been playing a part in fuelling the controversy. It is to be hoped that the present interpretation of the effect by simple aberration will help to settle the significance of the Jupiter experiment and provide a strong case for corrections to microlensing.

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REFERENCES

Bergmann P. G., 1942, Introduction to the Theory of Relativity. Prentice-Hall, New York, pp. 21, 37
Fomalont E. B., Kopeikin S., 2003, astro-ph/0302294
Frittelli S., 2003, MNRAS, 340, 457
Frittelli S., Kling T. P., Newman E. T., 2002, Phys. Rev. D, 65, 123007
Kopeikin S. M., Schäfer G., 1999, Phys. Rev. D, 60, 124002
Paczyński B., 1998, ARA&A, 36, 419
Petters A. O., Levine H., Wambsganss J., 2001, Singularity theory and gravitational lensing. Birkhäuser, Boston
Pyne T., Birkinshaw M., 1993, ApJ, 415, 459
Rossi B., 1965, Optics. Addison-Wesley, Reading
Schneider P., Ehlers J., Falco E. E., 1992, Gravitational Lenses. Springer-Verlag, New York
Will C. F., 2003, ApJ, 590, 683

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