We study black holes with a source that is almost point-like (blurred), rather than exactly point-like, which could be caused by the noncommutativity of 3-space. Depending on its mass, such object has either none, one or two event horizons. It possesses new properties, which become important on microscopic scale, in particular the temperature of its Hawking radiation does not increase infinitely as its mass goes to zero, but vanishes instead. Such frozen, extremely dense pieces of matter are good dark matter candidate. In addition, we introduce an object oscillating between frozen black hole and naked (softened) singularity, such objects can serve as constituents of dark matter too. We call it gravimond.

**Keywords:** Noncommutative quantum mechanics; microscopic black holes; dark matter.

**PACS numbers:**

1. **Introduction**

Quantum theory allowed us to merge three of the four (known) forces of nature within one unified theory. However, its relation with the last one - gravity is, to put it mildly, questionable. At least some of the problems with it are caused by infinitely large energies or equivalently, by zero distances. If the space we live in has some shortest possible distance, those problems would vanish.

Noncommutative (NC) theories are formulated in spaces whose coordinates do not commute with each other and therefore one cannot localize their points (this is similar to ordinary quantum mechanics where one cannot exactly know the phase space position of a particle). They could be viewed as effective theories to some higher theory which fuses quantum physics with gravity, yet they already possess a natural energy cut-off.

Black holes are important objects in both classical and quantum gravity which also posses a high-energy ill behavior. As discovered by Hawking, they radiate with a temperature inversely proportional to their mass, thus as they become infinitely small, they also turn infinitely hot.

*For example in [13] it has been shown that the spectrum of free Hamiltonian in a NC space has not only a lower boundary but also an upper one.*
When a black hole forms, its matter shrinks into a singular point. However, in NC theories there is nothing like a separate point, and hence the singularity cannot presumably arise in the course of the collapse. This restriction has only a negligible effect on huge black holes, however a question is whether it can modify the behavior of microscopic ones. The aim of this paper is to answer this question.

Instead of using a complete NC description of black holes we follow a method used in [16] - NC theory is used only to obtain the energy density of the black hole, rest of the study is done using the classical theory (this is dubbed as NC - inspired black holes). More details on NC inspired cosmology and gravity could be found in [3, 4, 15, 17, 18, 20, 24].

**Outline of the paper**

This paper is organized as follows. At first we briefly demonstrate construction of 3 dimensional NC space and derive a NC point-like ("blurred") density \( b \). Such matter density is completed into the stress-energy tensor \( T^\mu_\nu \), for which we write down and solve the Einstein field equations. Afterwards we analyze the solution, mostly focusing on the event horizons and temperature of Hawking radiation. Finally we point out some physical consequences of our theory for \( \lambda \sim l_{\text{Planck}} \) (and provide the scaling of results for different choices of \( \lambda \)).

2. NC inspired Black Holes

2.1. Noncommutative space, coherent states and almost point-like matter density

Ordinary quantum mechanics (QM) is defined by the famous Heisenberg uncertainty principle

\[
[\hat{x}, \hat{p}] = i\hbar,
\]

which states that one cannot exactly identify a phase space position of a particle. The idea of noncommutative (NC) theories is to have a space in which one cannot determine the exact position of a point - the smooth structure of space is abandoned. Therefore, NC theories are built upon a relation defining how the position operators do not commute

\[
[\hat{x}^i, \hat{x}^j] \neq 0.
\]

By choosing the RHS of this equation we define the properties of the corresponding NC space, including symmetries. A popular choice for the RHS is \( i\theta^{ij} \), where \( \theta^{ij} \)

\(^b\text{Something as close to point-like density as one can get to in NC space}\)
is some constant antisymmetric matrix. This option however lacks the rotational invariance of our space. A more appropriate option is

$$[\hat{x}^i, \hat{x}^j] = 2i\lambda x^k \varepsilon^{ijk},$$

(3)

where $\varepsilon^{ijk}$ is the Levi-Civita symbol and $\lambda$ is a constant with the dimension of length, defining the length scale on which NC effects become significant. $\lambda$ is not fixed within our model, but since it might be an artifact of quantum gravity, it is expected to be equal approximately the Planck’s length, $\lambda \sim l_{\text{Planck}} \sim 10^{-35} m$.

There are several ways how to satisfy (3) [5, 6, 7, 9, 14, 19], different approaches are equivalent and one is encouraged to switch between them whenever it is comfortable and makes calculations easier. We will employ the bosonic operator approach which was previously used in [8, 9, 10, 13].

Let us define two sets of bosonic creation and annihilation operators satisfying

$$[\hat{a}_\alpha, \hat{a}^\dagger_\beta] = \delta_{\alpha\beta}; \ \alpha, \beta = 1, 2,$$

(4)

and acting in an auxiliary Fock space $\mathcal{F}$ spanned on normalized states

$$|n_1, n_2> = (\hat{a}^+_1)^{n_1}(\hat{a}^+_2)^{n_2}|0, 0>,$$

(5)

where $|0, 0> = |0>$ is the vacuum state annihilated by both $\hat{a}_\alpha$.

NC coordinates defined with the help of Pauli matrices $\sigma^i$ as

$$\hat{x}^i = \lambda \sigma^i_{\alpha\beta} \hat{a}^+_\alpha \hat{a}_\beta,$$

(6)

satisfy (3) (their noncommutativity is inherited from the bosonic operators). The radial coordinate is defined as

$$\hat{r} = \lambda (\hat{a}^+_\alpha \hat{a}_\alpha + 1),$$

(7)

note that $\hat{r}^2 = \hat{x}^2 + \lambda^2$. Every $|n_1, n_2>$ is an eigenstate of $\hat{r}$ with an eigenvalue $\lambda(n_1 + n_2 + 1)$. The vacuum state $|0, 0> = |0>$ is the state with the minimal eigenvalue, so it should correspond to the origin of the coordinate system. This is as far as we need to go into the construction of NC space, for more details about constructing (NC) QM on it see the aforementioned references.

Coherent states play an important role in ordinary quantum mechanics and they have a crucial role in NC theories as well [12, 21, 22, 23, 25]. A coherent state is well localized wave packet which minimizes the uncertainty relation and is defined as annihilation operator eigenstate ($\hat{a}^+ |\alpha> = \alpha |\alpha>$). Such states can be generated as

$$|\alpha> = e^{-|\alpha|^2/2} e^{\alpha \hat{a}^+} |0>,$$

(8)
We can use them as a useful overcomplete sets of states in $F$, [1]. The overlap of two coherent states is

$$<\alpha|\beta> = e^{-|\alpha|^2 + |\beta|^2 + \bar{\alpha}\beta}.$$  

We are interested in the overlap of a general coherent state and the vacuum state (which corresponds to the origin of the coordinates),

$$\tilde{\rho}(\alpha) = <\alpha|0>|^2 = e^{-|\alpha|^2}.$$  

This represents a well localized state in the origin of coordinates, which however contains no information about the length scale $\lambda$. To overcome this we define new bosonic operators (no longer dimensionless) as

$$\hat{z}_\alpha = \sqrt{\lambda}\hat{a}_\alpha, \quad \hat{z}^+_\alpha = \sqrt{\lambda}\hat{a}^+_\alpha.$$  

With these operators, the entire construction [4] - [10] can repeated

$$[\hat{z}_\alpha, \hat{z}^+_\beta] = \lambda\delta_{\alpha\beta},$$

$$\hat{x}_i = \sigma^i_{\alpha\beta}\hat{z}^+_\alpha \hat{z}_\beta,$$

$$\hat{r} = \hat{z}^+_\alpha \hat{z}_\alpha + \lambda = \hat{x}^2 + \lambda.$$  

The overlap of coherents states, now defined as eigenstates of $z_\alpha$, with the state localized at the origin is

$$\tilde{\rho}(z) = |<z|0>|^2 = e^{-|z|^2/\lambda} = e^{-r/\lambda}.$$  

Let us pause for a moment to make a few remarks. First of all, we define $\lambda \to 0$ as the commutative limit (RHS of [3] vanishes, as in the ordinary QM). It is easy to see that in this limit the RHS of [13] vanishes everywhere but at the point $r = 0$, it becomes a point-like (particle matter) density. It is therefore natural to call $\tilde{\rho} \propto e^{-\frac{r}{\lambda}}$ an almost point-like density or a blurred point-like density.

Note that $\tilde{\rho}$ in [13] is dimensionless. The matter density with proper dimension will be denoted $\rho$ (without a tilde).

Since the rest of the calculations will be done using ordinary (not NC) calculus, we will normalize $\rho$ with respect to the ordinary integration instead of a trace norm. This yields an almost point-like mass density

$$\rho(r) = \frac{M}{8\pi\lambda^3}e^{-\frac{r}{\lambda}}.$$  

In the paper by P. Nicolini [16], which served as a main inspiration for ours, a similar line of reasoning was used. The starting point in [16] was a two dimensional NC space and the resulting density was generalized into three dimensional only afterwards, yielding $\rho \propto e^{-\frac{r^2}{\lambda^2}}$. As we have shown, a direct three dimensional derivation based on [3] leads to a different result.
2.2. Stress-energy tensor and energy conditions

The plan is to complete $\rho$ into a full stress-energy tensor, write down Einstein field equations, solve them and analyze their solution. Most of the work will be done analytically, yet some of the equations will be transcendent, so we will have to settle for less and find only a numerical approximation of the solutions.

We focus here only on uncharged nonrotating black holes, so we expect all of our results to recover the ordinary Schwarzschild black hole behavior in the $\lambda \to 0$ limit. This requirement also encourages us to use a "Schwarzschild-like" ansatz for the metric tensor $g_{00} = -g_{rr}^{-1}$, therefore our goal will be to find a single function $f(r)$ such that

$$g_{\mu\nu} = \begin{pmatrix} f(r) & 0 & 0 & 0 \\ 0 & -\frac{1}{f(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}.$$  \hspace{1cm} (15)

We will label the coordinates by $(0, r, \theta, \phi)$ and use the metric tensor signature $(-, +, +, +)$. We will also often omit writing (any of) arguments of functions.

We are expecting a diagonal $T_{\mu\nu}$, our starting point being the energy density component $T_{00} = -\rho(r)$ (we put $c = 1$ so that the mass and energy density coincide). Because of our ansatz $T_{r}^{r} = T_{0}^{0}$ is fixed as well (this can be seen from the Einstein field equations). The other two components follow from the conservation law $T_{\mu\nu}^{;\nu} = 0$. For $\mu = \theta$ we get $T_{\theta}^{\theta} = T_{\phi}^{\phi} =: p_{\perp}$, for $\mu = r$ we get

$$p_{\perp} = -\frac{r}{2}(\partial_{r}\rho + \frac{2}{r}\rho) = -\rho - \frac{r}{2}\partial_{r}\rho,$$

other equations are trivial. Our stress energy tensor therefore is

$$T_{\mu}^{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p_{r} & 0 & 0 \\ 0 & 0 & p_{\perp} & 0 \\ 0 & 0 & 0 & p_{\perp} \end{pmatrix}, \hspace{1cm} p_{r} = -\rho, \hspace{0.2cm} p_{\perp} = -\rho - \frac{r}{2}\partial_{r}\rho.$$  \hspace{1cm} (17)

Is such stress-energy tensor realistic or not? To decide on this we can use weak and strong energy conditions:

$$\text{weak} \quad T_{\mu\nu}X^{\mu}X^{\nu} \geq 0,$$

$$\text{strong} \quad \langle T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu} \rangle X^{\mu}X^{\nu} \geq 0,$$

where $X^{\mu}$ is a timelike vector. The weak condition can be interpreted as "energy is always positive" and the strong condition can be regarded as "matter gravitates towards matter" [2]. The weak energy condition reduces to the inequalities...
\[ \rho + p_r \geq 0, \]
\[ \rho + p_\perp \geq 0, \]
which are in our case always satisfied. However the strong energy condition, which takes the form
\[ \rho + p_r + 2p_\perp \geq 0. \]
is violated for \( r < 2\lambda \). This could be expected since the noncommutativity generates some sort of quantum repulsion which prevents the matter from collapsing into a singularity.

### 2.3. Einstein field equations and their solution

Now it is time to write down the Einstein field equations. In fact, because of the form of our ansatz (15) we only need one of them (and from now on, we set \( G = 1 \)). We can choose \( G^0_0 = 8\pi T^0_0 \) since our choice \( p_r = -\rho_r \) ensures that the equation \( G^r_r = 8\pi T^r_r \) is identical to it. The reads
\[ \frac{1 + f + rf'}{r^2} = \frac{M}{\lambda^3} e^{-\frac{r}{\lambda}}, \]
and has a solution
\[ f(r) = -1 - e^{-\frac{r}{\lambda}} \frac{M}{r} \left( \frac{r^2}{\lambda^2} + \frac{2r}{\lambda} + 2 \right) + \frac{C}{r}. \]

Recall that \( g_{00}(r) = f(r) \), therefore if we want the solution to approach Schwarzschild solution for \( r \gg \lambda \), we need to set \( C = 2M \). For the rest of this paper we will need only the time component of the metric tensor,
\[ g_{00}(r; \lambda, M) = -1 + \frac{2M}{r} - e^{-\frac{r}{\lambda}} \frac{M}{r} \left( \frac{r^2}{\lambda^2} + \frac{2r}{\lambda} + 2 \right). \]

### 2.4. Event horizon(s) and Hawking radiation

Event horizons are solutions of the equation
\[ g_{00}(r) = 0. \]

For an ordinary Schwarzschild black hole the solution is \( r = 2M \), however for our metric there are two, one or zero solutions, depending on the value of \( M \). This can be seen in Fig. 1 and one can easily prove it by doing a little mathematical analysis.
When the mass is large ($M \gg \lambda$), there are two horizons, one near the origin ($r_- \approx 0$) and the other near the classical horizon (at $r_+ \approx 2M$, see Fig. 2). As $M$ gets smaller, these two surfaces move towards each other and meet for some $M =: M_0$ at $r =: r_0$. We call a black hole with the mass $M_0$ and a single horizon at the radial coordinate $r_0$ extremal, since for any smaller $M$ there is no horizon at all, extremal black hole is the smallest possible black hole.

Obviously both $M_0, r_0$ depend on $\lambda$ and as can be seen from their physical dimensions the dependence is linear (without the absolute term, since they both vanish as $\lambda \to 0$). Eq. (24) is transcendental so we can obtain the linear coefficients only numerically,

![Figure 1. $g_{00}(r)$ for $\lambda = 1$ and different values of $M$.](image1)

![Figure 2. Radius of the outer horizon $r_+$ as a function of $M$, compared to the Schwarzschild value $2M$.](image2)
$M_0 = 2.57\lambda,$
\[ r_0 = 3.38\lambda. \]  

What happens to the Hawking radiation \[11\] as a black hole approaches the extremal mass $M_0$? The Hawking temperature is given as $T = \frac{\kappa}{2\pi}$, where $\kappa$ is the surface gravity at the horizon $r_+$ which is equal to $\kappa = -\frac{g_{00}(r_+)}{2}$. For an extremal black hole the function $g_{00}(r; M_0, \lambda)$ only touches the horizontal axis at $r = r_0$ (otherwise there would be two horizons), therefore $r = r_0$ is the point where it reaches its maximum and its first derivative vanishes there. Because of that there is no surface gravity at the horizon of an extremal black hole and the black hole has zero temperature - it becomes frozen and stops evaporating.

Note that infinite temperatures are avoided (Fig. 3). An interesting question is how does the maximal temperature depend on $\lambda$. From dimensional analysis we can see that $T_{\text{MAX}} \propto \lambda^{-1}$ and to get an (almost) exact relation let us first factorize out the mass from $g_{00}$,

$$g_{00}(r; \lambda, M) = -1 + M\tilde{g}(r; \lambda).$$

where $\tilde{g}(r)$ does not depend on $M$. At the (outer) horizon $g_{00}(r_+) = 0$, so that $\tilde{g}(r_+) = \frac{1}{2\kappa}$, and

$$g'_{00}(r_+) = M\tilde{g}'(r_+) = \frac{\tilde{g}'(r_+)}{\tilde{g}(r_+)}.$$  

Figure 3. The Hawking temperature as a function of black holes mass.
This is, up to a multiplicative constant, equal to the Hawking temperature. We may now ask for what size of the (outer) horizon $r_+$ does this achieve maximum. To answer this we need to solve one of the two following equations

$$\partial_r g'_{00}(r_+) = 0 \Leftrightarrow \tilde{g}(r_+) \tilde{g}''(r_+) = \tilde{g}'(r_+)^2.$$  \hspace{1cm} (28)

We choose $\lambda = 1$ and solve the numerical numerically to find that the extremal value is $g'_{00}(r_+ = 6.54) = -0.12$. The maximal temperature is (we recover all constants for a moment, $\tau = 0.18 \times 10^{-3} mK$)

$$T_{\text{max}} = \frac{\hbar c}{4\pi k_B} \frac{0.12}{\lambda}.$$  \hspace{1cm} (29)

As we have seen already, microscopical black holes (mBH) do not evaporate entirely, but stay frozen with the mass $M_0$ instead. When such extremal black hole consumes a particle with non-zero mass its own mass becomes larger then $M_0$ and the black hole is reignited (since for $M > M_0$ is the Hawking temperature nonzero). If we throw a particle with a small mass $\delta M \ll M_0$ into an extremal black hole, how much will its radius grow and at what temperature will it radiate?

To answer this question we use the decomposition (26). Let us denote the increment in radius $\delta r$. We can write down two conditions, one before and one after adding the mass $\delta M$, \hspace{1cm} (30)

$$-1 + M_0 \tilde{g}(r_0; \lambda) \frac{1}{M_0} = 0,$$

$$-1 + (M_0 + \delta M) \tilde{g}(r_0 + \delta r; \lambda) \frac{1}{M_0} = 0.$$  \hspace{1cm} (31)

Truncating the Taylor expansion of (31) we obtain

$$\tilde{g}(r_0 + \delta r; \lambda) = \tilde{g}(r_0; \lambda) + \delta r \partial_r \tilde{g}(r_0; \lambda) + \frac{1}{2} \delta r^2 \partial_r^2 \tilde{g}(r_0; \lambda),$$  \hspace{1cm} (32)

and inserting this back into (31) we get

$$\delta r = \pm \sqrt{\frac{-2\delta M}{M_0(M_0 + \delta M) \partial_r^2 \tilde{g}(r_0; \lambda)}} = \pm \sqrt{\frac{-2\delta M}{M_0^2 \partial_r^2 \tilde{g}(r_0; \lambda)}}.$$  \hspace{1cm} (33)

This expression might look a little hideous, but evaluating it for $M_0$ and $r_0$ as given in (25) we arrive at a simple equation

$$\delta r \simeq \pm 2.54 \sqrt{\lambda \delta M}.$$  \hspace{1cm} (34)

We have two symmetric solutions because we have truncated the Taylor expansion after the quadratic term. Now we can determine the temperature
\[
T(r_0 + \delta r) = T(r_0) + \partial_r T(r_0) \delta r \equiv \\
\frac{1}{4\pi} \frac{\delta M}{M_0} \frac{\sqrt{\delta M}}{2\pi 6.53 \lambda^{3/2}},
\]
where we have used (33) first, then (34). If we recover the constants again we get

\[
T(M_0 + \delta M) \equiv \frac{\sqrt{\delta M}}{41.01 \lambda^{3/2}} \frac{\hbar c}{k_B}.
\]

It is useful to express this with respect to \(T_{\text{max}}\)

\[
\frac{T(M_0 + \delta M)}{T_{\text{max}}} \equiv 2.55 \sqrt{\frac{\delta M}{\lambda}} \equiv 4.09 \sqrt{\frac{\delta M}{M_0}}.
\]

We can see that for \(\delta M \ll M_0\) the black hole does not reach its maximum temperature, only a small fraction of it. The last question of this section will be whether will the temperature reach the maximum value if we merge two extremal black holes together? As can be seen in Figure 3, if we have \(M = 2M_0\) we are in a region where we can safely take \(r_+ = 2M = 4M_0 = 10.28\lambda\). This is larger then the value \(r_+ = 6.54\lambda\) for which the temperature reaches maximum, therefore the maximum will be reached when the new black hole evaporates from the radius 10.28\(\lambda\) to 6.54\(\lambda\).

2.5. Physical implications

As we have seen, all of our results depend on \(\lambda\) - the scale of the space noncommutativity. The problem is that we do not know how large \(\lambda\) really is, we can only say it is beyond our experimental reach so far. However, since it should be an artifact of the spacetime structure, one expects that it could be approximated by the Planck’s length \(\lambda \sim l_{\text{Planck}} \equiv 1.62 \times 10^{-35} \text{m}\). In this section we provide some possible physical implications of the existence of “blurred” microscopic black holes (mBH).

The calculations are done assuming \(\lambda = l_{\text{Planck}}\) and accompanied with a scaling rule for different choices of \(\lambda\).

According to (25) an extremal mBH should have the radius \(r_0 = 5.48 \times 10^{-35} \text{m}\) (we can take the cross section to be \(\sigma = \pi r^2 = 9.43 \times 10^{-69} \text{m}^2\)) and the mass \(M_0 = 5.59 \times 10^{-8} \text{kg}\). Thus, such black holes are indeed minuscule, but still quite massive when compared to elementary particles (\(r_0, M_0\) scale as \(\lambda\)). Furthermore \(T_{\text{max}} = 1.33 \times 10^{30} \text{K}\) which is two orders below the Planck’s temperature (this scales as \(\lambda^{-1}\)).

"Blurred" referring to their nonsingular matter density.
Considering these numbers, mBH are perfect cold dark matter candidates. They are cold and absolutely dark (since their radiation froze out), extremely small and heavy enough so there does not need to be too many of them. To make up for the dark matter mass density \( \rho_{DM} \approx 2.38 \times 10^{-27} \text{kg m}^{-3} \) we would need mBH concentration \( n_{mBH} \approx 4.25 \times 10^{-20} \text{m}^{-3} \) in the universe (this scales as \( \lambda^{-1} \)), that means approximately one mBH in every cube with edges almost 3000 km long. Dark matter density is uniform only on cosmological scales, but there seems to be more of it in galaxies than in between them (by the factor \( 10^5 \cdots 10^6 \), see [26], possibly even more within the solar systems).

The cross section of extremal mBH is small enough for them not to interact with each other, however it is still possible for them to be hit by some other particles. Let us assume that a mBH gets hit by a proton and swallows it, what would happen? Since the mass of the proton is significantly smaller than \( M_0 \) we can use eq. (37), for this example \( \frac{\delta M}{M_0} \approx 2.98 \times 10^{-20} \). The resulting (non extremal) mBH will warm up just to \( 7.06 \times 10^{-10} \) of it the maximum temperature, that is \( 9.39 \times 10^{20} \text{K} \) or \( 8.09 \times 10^{16} \text{eV} \) (this scales as \( \lambda^{-2} \)), two orders below the energy of ultra-high-energy cosmic rays. Had the \( \lambda \) been shorter than the Planck’s length, a mBH radiation after consuming a proton could explain these rays. It should be noted here that it might be more correct to consider mBH-electron or mBH-quark collision instead\(^d\), since proton is significantly larger than mBH.

It is important to note that the energy of radiation exceeds the energy of the consumed particle. The possible scenario is that the energy will be radiated in one or t quanta and the mBH will end with \( M < M_0 \), it will have no horizons and stops being a black hole. Then it will be moving through space as an extremely dense chunk of matter and collect additional mass until it reaches the mass \( M_0 \) and becomes black hole again.

This object, let us name it a gravimond, lives in cycles: first it is an extremal black hole with mass \( M_0 \), and then, after it consumes a particle its radiation and is reignited as \( M > M_0 \), then it stops being a black hole since so much energy has been radiated that \( M < M_0 \), and it becomes an extremely dense object (almost a black hole) which needs to capture some mass to become (extremal) black hole again. The period of this cycles is unknown and probably largely depends on the location of such object (how often does it get to interact with other matter).

3. Conclusion

The paper is devoted to (microscopic) black holes with almost point-like (blurred) mass density, instead of singular one. Such mass density could be due to the noncommutativity of space on some small length scale \( \lambda \), however all calculations have been done using the ordinary calculus and general relativity\(^e\). Let us sum up the

\(^d\)Interesting questions about the confinement arise in that case

\(^e\)This is why the objects in question are sometimes referred to as NC-inspired black holes instead of just NC black holes
important results

- Depending on the mass $M$ there are none, one or two event horizons. The black hole with one event horizon (extremal black hole) has the mass $M_0$ and the radius of event horizon $r_0$ both equal to the NC constant $\lambda$, multiplied by dimensionless constants of order unity, see [25].
- The Hawking radiation of an extremal black hole has zero temperature so it does not evaporate anymore. Such frozen black holes are a good dark matter candidate. To make up for the observed mean dark matter energy density $\rho_{DM}$ there needs to be one such mBH in every volume of order $10^{19} m^3$.
- If an extremal black hole gains additional mass and therefore stops being extremal, for example by consuming a particle or colliding with another mBH, its radiation is reignited and it starts to emit extremely energetic quanta. The resulting radiation might be a possible candidate for ultra high cosmic rays, yet it seems to be 1-2 orders of magnitude too low. We also lack a better understanding of this mechanism.

During the radiation stage more energy is emitted than has been absorbed and the mBH ends up with $M < M_0$. Having no event horizons it is no longer a black hole, therefore we hypothesize that it turns into an object which we have called a *gravimond*) undergoing (theoretically infinite number of) the following life cycles:

- $M = M_0$, being a frozen extremal microscopic black hole,
- after consuming/collision with another particle the object acquires $M > M_0$ and a rapid radiation begins,
- emission of huge quanta of energy leads to $M < M_0$, the object has no event horizons anymore and stops being black hole. While moving through space, it gathers mass until it reaches $M = M_0$ again.

The time period of these cycles is unknown and depends on the local density of other particles.

Acknowledgment

I would like to thank Peter Prešnajder (my PhD supervisor) and Vladimír Balek for their valuable comments and corrections.

Preprint of an article submitted for consideration inInternational Journal of Modern Physics A] © 2015 [copyright World Scientific Publishing Company]

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