Numerical study on the attenuation of high-amplitude stress wave in rock bar along with rock damage evolution

Tianqi Zhai¹ and Jianbo Zhu¹,²,*

¹ State Key Laboratory of Hydraulic Engineering Simulation and Safety, School of Civil Engineering, Tianjin University, Tianjin, China
² Guangdong Provincial Key Laboratory of Deep Earth Sciences and Geothermal Energy Exploitation and Utilization, Institute of Deep Earth Sciences and Green Energy, College of Civil and Transportation Engineering, Shenzhen University, Shenzhen, China

* Corresponding author’s e-mail: jianbo.zhu@szu.edu.cn

Abstract: The attenuation of high-amplitude stress waves in the rock and the rock failure caused by the stress waves have significance for vibration source analysis and rock engineering support design, where the interactions between the two deserve further study. In this paper, a FEM based numerical investigation on high-amplitude stress wave attenuation and rock damage evolution in one-dimensional rock bar is performed, aiming to understand the interaction between the variation of rock damage and wave attenuation. The numerical model is calibrated by experimental results. The spatial distribution of damage in rock bar is quantified by statistical windows. The effects of wave amplitude and heterogeneity index of rock on wave attenuation as well as the distribution of damage are analysed in details. The results indicate that the high-amplitude stress wave decays exponentially in the rock bar, and the attenuation of wave becomes significant as the incident wave amplitude increases. The incident wave amplitude corresponding to the lower limit of the amplitude effect is approximately 30% of the static compressive strength of rock, while that corresponding to the upper limit is the dynamic compressive strength of rock. Under the same incident wave amplitude, the attenuation of stress wave is more obvious in the rock with higher heterogeneity. The rock damage is quantified in a specific window, and its spatial distribution is described along with the movement of the window. The rock damage is concentrated in a region near the incident end. The damage development in space is consistent with that of high-amplitude stress waves and is more obvious in the models with higher heterogeneity. Findings in this work is helpful for the understanding of interaction between high-amplitude stress wave attenuation and rock damage and facilitate design and protection of rock engineering.

1. Introduction

Researches on wave attenuation in rocks and the damage evolution of rocks under dynamic loadings are of significant scientific interest and engineering applications. On the one hand, the mechanical properties of rocks under dynamic loadings are the basic parameters for studying the propagation and attenuation of waves in rock masses. On the other hand, when studying the dynamic response of rocks under shock loads, the analysis of stress wave propagation plays a key role [1].

[1]
The mechanical behaviour of rocks under dynamic loading is strain rate-dependent. Mukherje et al. [2] established a damage-plasticity model to capture the rate-dependent behaviour of rocks. Li and Shi [3] proposed a new dynamic damage model to further consider the combined effect of strain rate and confining pressure. Based on crack slip and crack growth, Ashby and Sammis [4] established a damage model to describe the mechanical behaviour of brittle rocks. Bhat et al. [5, 6] extended it further to allow for a more generalized stress state with high loading rates. The quasi-brittle model developed by Zeng et al. considered the plastic deformation of rocks under dynamic loading [7]. These damage constitutive models considering the strain rate effect contribute to determining the physical and mechanical parameters of the rock under dynamic loadings.

Wave attenuation in damaged rock is also an interesting topic and has received considerable attentions. The influence of the size or arrangement of cracks in damaged rock on the wave propagation characteristics has been investigated by theoretical or experimental methods [8, 9]. The effect of discontinuities, such as joints, on wave propagation was studied by using the improved Split Hopkinson Pressure Bars [10, 11]. With advances in computer technology, numerical methods have been developed to study the effects of different scale defects on wave attenuation [12, 13]. Large-scale underground explosion test could be performed to analyse stress wave propagation with the help of numerical methods [14]. In these studies, variations of wave-related parameters, such as wave velocity and amplitude, are generally used to assess the potential damage of rocks [15].

Although many efforts have been devoted to studying the damage of rocks under dynamic loading and attenuation of stress wave in rock masses, existing researches often only focus on one of the two aspects. The study of dynamic mechanical characteristics under high strain rate ignores the inertial effect of materials, while the investigation of stress wave propagation in damaged rock is based on the hypothesis of low-amplitude wave [16]. In addition, the heterogeneity of rock has not been seriously considered in above researches, which may cause the results of studies to differ from engineering practice.

In this paper, the FEM-based numerical tests were performed to investigate the attenuation of stress wave in a one-dimensional rock bar and the development of damage induced by the high-amplitude wave. Through comparison with analytical solutions and experimental data, the accuracy of numerical model was verified. Subsequently, the effects of stress wave amplitude and heterogeneity on wave attenuation and damage distribution in the model were studied. The findings in this paper could contribute to better understanding of the relationship between damage evolution and wave attenuation in rocks, and is significant for rock engineering protection.

2. Numerical methods and Verification

The FEM-based numerical code RFPA3D [17] is employed in this study to investigate the wave attenuation and damage evolution in rock bars. The heterogeneous characteristics of rocks is taken into consideration in this code. The properties of elements such as elastic modulus, strength and density are assumed to follow the Weibull distribution [18], which is expressed as:

\[ \phi(x) = \frac{m}{x_0} \left( \frac{x}{x_0} \right)^{m-1} \exp \left( -\left( \frac{x}{x_0} \right)^m \right) \]

where \( x \) is the mechanical parameter of one element, \( x_0 \) is the mean value of the whole elements corresponding to the parameter, and \( m \) defines the shape of the distribution function representing the degree of material heterogeneity, and is called the heterogeneity index. The material is more heterogeneous when the value of \( m \) is smaller.

The schematic diagram of numerical model is shown in Figure 1. The length and diameter of the cylindrical rock bar is 1000 and 50 mm, respectively. A triangular stress wave with a duration of 60 \( \mu \)s is applied normal to the left boundary of the rock bar. The right end surface is set as non-reflecting boundaries to eliminate the reflected waves. 10 measuring points are equally spaced on the central axis of the cylindrical bar. The distance between the first measuring point and the incident end is 25 mm. Previous findings [15] have shown that the ratio of the mesh size to the wavelength should be less than
0.02 to guarantee the accuracy of the simulation results. This rule serves as the basis for mesh generation in this paper.

Figure 1. Schematic diagram of the numerical model. Measuring points are equally spaced on the central axis; the duration of incident wave is 60 μs and the peak stress of the incident wave, A, is variable.

The heterogeneity index, $m$, is related to the mechanical parameters in the laboratory. Therefore, the value of $m$ and the other input parameters of model need to be determined by inverse analysis based on experimental measurements. In this paper, the study of Zhao [19, 20] on granite was selected to calibrate the model. The mechanical parameters of granite were listed in Table 1. After a lot of trial-and-error simulations, the mean values of the mechanical parameters of the model were determined and summarized in Table 1. Figure 2 shows the propagation characteristics of stress wave in the model. It could be found that when the amplitude of the incident stress wave is much lower than the compressive strength of material, the attenuation of the stress wave in the model is negligible, which is in line with the one-dimensional longitudinal wave propagation theory [1] and some experimental observations [21]. The wave velocity in the model (ratio of distance between the first and the last measuring points to the travel time of wave through the corresponding points) is 5882 m/s, which is consistent with the experimental measurement.

Table 1. Mechanical parameters of experimental measurement and the corresponding mean value of heterogeneous model.

| Mechanical parameters          | Experimental measurements [20] | Mean value of heterogeneous model |
|--------------------------------|--------------------------------|-----------------------------------|
| Elastic modulus (GPa)          | 84                             | 114.6                             |
| Compressive strength (MPa)     | 186                            | 580                               |
| Tensile strength (MPa)         | 11.4                           | 58                                |
| Poisson’s ratio                | 0.25                           | 0.25                              |
| Density (kg/m$^3$)             | 2610                           | 2610                              |
| Wave velocity (m/s)            | 5790                           | -                                 |
| Heterogeneity index            | -                              | 4                                 |
Figure 2. Stress waveforms at different measuring points.

Figure 3 presents the stress history of the first measuring point under the loading conditions of high-amplitude stress wave. It could be found that when the peak stress of incident wave is less than 250 MPa, the peak stress of the first measuring points is always slightly lower than that of incident wave, and the stress history at measuring point could match the incident wave signal. When the peak stress of incident wave further increases, the stress drop appears in the stress history curves. Figure 4 illustrates the distribution of damaged elements in the model when the peak stress of incident wave is 300 MPa. Near the incident end, almost all elements are damaged. It could be understood as that when the dynamic loading reaches the dynamic strength of the material, a large number of elastic elements are transformed into damage element and thus the stress drops sharply. Therefore, when the peak stresses of the incident wave are 300 and 350 MPa, the corresponding dynamic strengths of the model are 283.79 and 284.27 MPa, respectively. The simulated dynamic strengths are similar to the extrapolated results (the corresponding dynamic strengths are 284.78 and 285.58 MPa, respectively) based on the experimental measurements [20]. The results of Figure 2 to Figure 4 show that the model established in this paper combined with the input parameters determined can accurately describe the propagation characteristics of one-dimensional stress waves in the rock bar and capture the damage evolution of the rock.

Figure 3. Stress history of the first measuring point under different incident amplitude.

Figure 4. Distribution of damaged elements in the model when the incident amplitude equals to 300 MPa.
3. Results

3.1. Effects of incident amplitude

The propagation of stress waves with different amplitude in rock bar is studied in this section. The input parameters in Table 1 are employed, and the duration of the incident wave is always 60 µs. The amplitude of incident wave varies from 0.1 UCS to 1.9 UCS (the UCS is the uniaxial compressive strength and equals to 186 MPa). Figure 5 shows the attenuation of incident waves with different amplitudes in rock bar. When the amplitude of incident wave is less than 0.5 UCS, the peak stresses remain unchanged with the increase of the propagation distance. As the amplitude of incident wave increases, the attenuation of peak stress becomes significant. When the amplitude of incident wave is higher than 1.5 UCS, the dynamic strength of the rock (around 283 MPa) is reached. Therefore, the peak stress at the first measuring point no longer increases with the increase of the amplitude of incident wave, and the attenuation of the peak stress in the bar is independent of the amplitude of incident wave. In summary, the incident wave amplitude corresponding to the lower limit of the amplitude effect is approximately 30% of the static strength of rock, while that corresponding to the upper limit is the dynamic strength of rock.

It can be seen from Figure 4 that when the end of rock bar is failure, a large number of damaged elements develop in a narrow area. Therefore, along the wave propagation direction, a cylinder with a length of 20 mm is taken as a statistical window. The damage is quantified by the ratio of the number of damaged elements to the whole elements in one window, so that the spatial distribution of the damage in the bar can be obtained through the movement of the window. Figure 6 shows the damage caused by incident stress waves of different amplitudes in the rock bar and its variation with the propagation distance. When the amplitude of incident wave exceeds the dynamic strength, the damage near the incident end is 1. The damage decreases rapidly between the incident end and around 100 mm, and then decreases slowly as the propagation distance increases. Note that when the amplitude of incident wave is 1.5 UCS, the damage near the incident end is less than 1, yet at the distance of 60–300 mm from the incident end, the damage is slightly larger than that under the amplitude of 1.7 and 1.9 UCS (as shown in the partial enlarged view of Figure 6). In addition, there is no damage detected in the rock bar, when the amplitude of incident wave is less than 0.5 UCS.

![Figure 5](image5.png)  
**Figure 5.** Variation of peak stress with propagation distance under different incident amplitude.

![Figure 6](image6.png)  
**Figure 6.** Evolution of damage under different incident amplitude.

3.2. Effects of heterogeneity
The effects of heterogeneity are studied in this section while the amplitude and duration of the incident wave are always 1.3 UCS and 60 μs. The heterogeneity index, $m$, is set to 1.5, 2, 3, 4 and 5. It should be noted that when $m$ changes, the corresponding input parameters need to be redetermined through trial-and-error simulation. The criteria for the correctness of the input parameters are described in Section 2. Figure 7 shows the attenuation of stress wave in the heterogeneous models. With the decrease of the heterogeneity index, the peak stress of the stress wave obtained at the same measuring points decreases. In other words, the attenuation of stress wave is more significant in the model with a lower heterogeneity index. Figure 8 shows the distribution of damage in different heterogeneous models. The general variation of damage with propagation distance is similar to that in Figure 6. The partial enlarged view of Figure 8 shows the damage when the propagation distance exceeds 400 mm. It is clear that the damage is greater in the model with lower heterogeneity index. The reason is that in the model with lower heterogeneity index, the number of elements with strength or elastic modulus lower than the mean value is more. Therefore, more elastics elements would transform into damage elements when the stress wave passes, even though the stress has decreased with the increasing propagation distance.

**Figure 7.** Variation of peak stress with propagation distance in the model of different heterogeneity index.

**Figure 8.** Evolution of damage in the model of different heterogeneity index.

4. Discussion

The attenuation of stress wave in rock can be divided into geometric attenuation and physical attenuation [22]. The former is due to the geometric expansion of the wavefront, and the latter is due to the transformation of mechanical energy to thermal energy or surface energy caused by lattice dislocation or micro-crack propagation. In this paper, the geometric attenuation is ignored by carefully designing the ratio of mesh size to the wavelength and the diameter to the wavelength (as evidenced by the results in Figure 2). Therefore, the attenuation of stress wave is dominated by physical attenuation, which is quantified as damage. The findings in this paper just reflect the close connection between damage and wave attenuation. With the increase of the propagation distance, the variations of damage and wave amplitude are coordinated, and can be divided into two phases, i.e., rapid decrease and slow decrease. The length of the first phase is limited to 0–100 mm. The transition from the first to the second phase of damage precedes that of wave attenuation.

The increase of the incident amplitude leads to the increase of the damage. However, the opposite results appear when the incident amplitude is higher than the dynamic loading strength. Note that the wave attenuation in the cases of incident amplitude of 1.7 and 1.9 UCS is more obvious than that in the case of incident amplitude of 1.5 UCS. This phenomenon is not consistent with the relationship between damage and wave attenuation summarized from the cases of other incident amplitude in Figure 5 and Figure 6. It could be explained by the decrease in wavelength caused by stress drop (as...
shown in Figure 2). A smaller wavelength means a higher frequency. The high-frequency filtering properties of the rock will intensify the wave attenuation [15]. However, due to the shortening of the wavelength, the mechanical energy carried by the stress wave is reduced, and the damage caused by the stress wave consequently decreases. This result implies that the wavelength could affect the wave attenuation and the evolution of damage. The effect of wavelength would be studied in the future.

5. Conclusion
In this paper, the relationship between wave attenuation and damage evolution are undertaken using RFPA3D software. Numerical models were established and verified based on published experimental results. The effects of incident amplitude and heterogeneity of rock on wave attenuation and damage evolution were considered. The main conclusions are listed as follows:

1. There is a close relationship between stress wave attenuation and the evolution of damage in rock. The high-amplitude stress waves would first experience a rapid decay in the rock bar, followed by a relatively slow decrease. The variation of damage caused by stress wave in the one-dimensional bar has the same trend as the attenuation of wave.

2. The increase in the amplitude of the incident wave leads to the increase of damage, which in turn makes the attenuation of wave more obvious. The incident wave amplitude corresponding to the lower limit of the amplitude effect is approximately 30% of the static strength of rock, while that corresponding to the upper limit is the dynamic strength of rock.

3. The increase of rock heterogeneity could result in the increase of damage, but this effect mainly acts on the region far away from the incident end.

6. References
[1] Wang L L 2007 Foundations of Stress Waves (Oxford, UK: Elsevier)
[2] Mukherjee M, Nguyen G D, Mir A, Bui H H, and Shen L 2017 Int. J. Impact Eng. 110 208-18
[3] Li H Y and Shi G Y 2016 Int. J. Impact Eng. 89 38-48
[4] Ashby M F and 1990 Pure and Applied Geophysics. 133 489-521
[5] Bhat H S, Rosakis A J and Sammis C G 2012 J. Appl. Mech-T. Asme. 79(3) 031016
[6] Bhat H S, Sammis C G and Rosakis A J 2011 Pure. Appl. Geophys. 168(12) 2181-98
[7] Zeng Q L, Tonge A L and Ramesh K T 2019 J Mech Phys Solids. 130 370-92
[8] Aggelis D G, Polyzos D and Philippidi T P 2005 J. Mech. Phys. Solids. 168 (4) 857-83
[9] Huirong A A and Ahrens T J 2007 J. Geophys. Res-Sol. Ea. 112(B1)
[10] Li J C and Ma G W 2009 Int. J. Rock. Mech. Min. 46(3) 471-8
[11] Li J C, Rong L F, Li H B and Hong S N. 2019 Rock. Mech. Rock. Eng. 52(2) 403-20
[12] Liu K W, Hao H and Li X B 2017 Int. J. Rock. Mech. Min. 92 30-9.
[13] Zhu J B, Li Y S, Peng Q, Deng X F, Gao M Z and Zhang J G 2021 Tunn. Undergr. Sp. Tech. 108. 103648
[14] Deng X F, Chen S G, Zhu J B, Zhou Y X, Zhao Z Y and Zhao J 2014 Rock. Mech. Rock. Eng. 48(2) 737-47
[15] Zhu J B, Zhai T Q, Liao Z Y, Yang S Q, Liu X L and Zhou T 2020 Rock. Mech. Rock. Eng. 53(9) 3983-4000
[16] Jin J F, Yuan W, Wu Y and Guo Z Q 2020 J. Cent. South. Univ. 27(2) 592-607
[17] Liang Z Z, Tang C A, Li H X, Xu T, Zhang Y B 2004 Int. J. Rock. Mech. Min. 41 323-8
[18] Weibull W 1951 J. Appl. Mech. 13
[19] Zhao J 1996 Tunn. Undergr. Sp. Tech. 11(1) 65-72
[20] Zhao J 2000 Int. J. Rock. Mech. Min. 37(7) 1115-21
[21] Zhang J Y, Ren H Q, Han F, Sun G J, Wang X, Zhao Q and Zhang L 2020 Mech. Mater. 141 103273
[22] Jaeger J C, Cook N, and Zimmerman R W 2007 Fundamentals of rock mechanics (Malden: Blackwell Publ)
Acknowledgments

This research is financially supported by the National Key R&D Program of China (No. 2018YFC0407002), the Program for Guangdong Introducing Innovative and Entrepreneurial Teams (No. 2019ZT08G315) and the National Natural Science Foundation of China (No. 51974197).