Neutrino Mass Difference Induced Oscillations in Observed Muon Decays

Y.N. Srivastava \textsuperscript{a}, S. Palit and A. Widom \textsuperscript{b}, E. Sassaroli \textsuperscript{c}

\textsuperscript{a}Physics Department & INFN, University of Perugia, Perugia, Italy
\textsuperscript{b}Physics Department, Northeastern University, Boston MA 02115 U.S.A.
\textsuperscript{c}Laboratory for Nuclear Science, MIT, Cambridge MA U.S.A.

The recent real time KARMEN anomaly for the electron neutrino counting rates (obtained from stopped muon decays) is analyzed by employing a neutrino flavor rotation model in which the muon neutrino is in a superposition of only two mass eigenstates. On the basis of experimental electron neutrino counting oscillations, we find a neutrino mass splitting $|m_2^2 - m_1^2| = (0.22 \pm 0.02)(eV/c^2)^2$ and a flavor rotation angle $\phi = 0.34 \pm 0.10$, both within a 95\% confidence interval.

PACS numbers: 13.35.+s, 12.15.Ff, 14.60.Ef, 14.60.Gh

Recent real time observations of the electron neutrino $\nu_e$, produced from a $\mu^+$ decay

$$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e,$$  \hspace{1cm} (1)

indicate an oscillatory counting rate\cite{1}. These data provide experimental evidence for a neutrino mass matrix with flavor rotations. The oscillations of interest are made manifest in the experimental KARMEN anomaly\cite{1}. Our purpose is to discuss this fact, employing a model in which the muon neutrino is in a superposition of two mass eigenstates

$$< \nu_\mu | = \cos \phi < \nu_1 | + \sin \phi < \nu_2 |,$$  \hspace{1cm} (2)

with mass eigenvalues of $m_1$ and $m_2$, respectively.

One may view Eq.(1) as occurring in two stages: (i)

$$\mu^+ \to W_{\text{eff}}^{+} + \bar{\nu}_\mu,$$  \hspace{1cm} (3)

and (ii)

$$W_{\text{eff}}^{+} \to e^+ + \nu_e.$$  \hspace{1cm} (4)

The virtual “effective” $W_{\text{eff}}^{+}$ standard electroweak charged Boson is far off the $M_W$ mass shell, and appears in the lowest order tree diagram for Eq.(1) via the charged Boson propagator $D_{\alpha\beta}$. The effective wave function is given by

$$W_{\text{eff},\alpha}^{+}(x) = \int D_{\alpha\beta}(x-y)J^{+\beta}(y)d^4 y,$$  \hspace{1cm} (5)

where the charged current $J^{+} = gW\left(\frac{\hbar}{M_W c}\right)^2 \bar{\Psi}\gamma^\alpha(1 - \gamma_5)N_\mu$, \hspace{1cm} (6)

Eq.(6) employs units for which the $W$ fine structure constant $\alpha_W = (g_W^2/hc)$ is related to the Fermi coupling fine structure constant by $(\sqrt{2})\alpha_W = (G_F M_W^2/hc)$.

In Eq.(6), the muon wave function $\Psi$ is the one particle to vacuum matrix element of the muon field operator $\bar{\Psi}$. The muon wave function is

$$\Psi(x) = <0|\bar{\Psi}(x)|\mu^+ >.$$  \hspace{1cm} (7)

The muon neutrino wave function $N_\mu$ is the vacuum to one particle matrix element of the muon neutrino field operator,

$$\nu_\mu(x) = \cos \phi \nu_1(x) + \sin \phi \nu_2(x).$$  \hspace{1cm} (8)

The neutrino wave function is

$$N_\mu(x) = <\bar{\nu}_\mu|\nu_\mu(x)|0 >.$$  \hspace{1cm} (9)

From Eqs.(2), (8) and (9), one finds that the muon neutrino wave function has the oscillation form

$$\nu_\mu(x) = \cos^2 \phi N_1(x) + \sin^2 \phi N_2(x).$$  \hspace{1cm} (10)
Perhaps more surprising, but unambiguously true from Eqs. (6) and (10), is the notion that the decay Eq. (3) of a muon with fixed four momentum \( P_\mu \) throws the other particle (here the \( W_{eff}^+ \)) into an oscillation superposition of amplitudes \( \Psi \).

\[
W_{eff}^+(x) = \cos^2 \phi W_1^+(x) + \sin^2 \phi W_2^+(x). \quad (11)
\]

In Eq. (11), \( W_{eff}^+ \propto (\bar{\Psi}_\alpha (1 - \gamma_5) N_j) \) for \( j = 1, 2 \).

The four momentum kinematics may now be written as follows: Let \( P_0 \) be the initial four momentum of the muon, and \( p_j \) for \( j = 1, 2 \) the four momentum of the possible neutrino mass eigenstates. Then

\[
P_\mu^2 = -(M_\mu c)^2, \quad p_j^2 = -(m_j c)^2, \quad (j = 1, 2). \quad (12)
\]

Since total four momentum is conserved at each vertex of a Feynman diagram, even for internal virtual particles, one then associates two possible four momenta for the \( W_{eff}^+ \),

\[
P_{effW,j} = (P_\mu - p_j), \quad (j = 1, 2) \quad (13)
\]

whose effective (virtual off shell) mass \( M_{effW} \),

\[
(M_{effW})^2 = -P_{effW,1}^2 = -P_{effW,2}^2, \quad (14)
\]

is much smaller than the on shell mass of the \( W^+ \); i.e. \( M_{effW} \ll M_W \). From Eqs. (12), (13) and (14) it follows that

\[
2(p_2 - p_1) \cdot P_\mu = (m_2^2 - m_1^2) c^2. \quad (15)
\]

In the rest frame of the muon, the two possible neutrino energies, \( \epsilon_1 \) and \( \epsilon_2 \) have a Bohr transition frequency \( \omega \) determined by Eq. (15); It is

\[
h \omega = |\epsilon_2 - \epsilon_1| = \left( \frac{m_2^2 - m_1^2}{2M_\mu} \right) c^2. \quad (16)
\]

The central result of this work now follows from Eq. (6) for the virtual \( W_{eff}^+ \) wave function. This wave function contains a product of a neutrino wave function \( N_\mu(x) \) and a muon wave function \( \bar{\Psi}(x) \). After absolute value squaring the \( W_{eff}^+ \) decay amplitude, one finds (from the neutrino wave function) the usual neutrino oscillation probability

\[
| \cos^2 \phi e^{-ix_1 t / h} + \sin^2 \phi e^{-ix_2 t / h} |^2 = (1 - \sin^2(2\phi)\sin^2(\omega t / 2)), \quad (17)
\]

and (from the muon wave function) the usual muon decay probability \( \exp(-\Gamma t) \). Thus, for the probability \( P(t) \) of the \( W_{eff}^+ \) decay survival, we have the differential equation

\[
- \frac{dP(t)}{dt} = Z \Gamma e^{-\Gamma t} (1 - \sin^2(2\phi)\sin^2(\omega t / 2)). \quad (18)
\]

In Eq. (18), the constant \( Z \) is found by normalizing the total \( W_{eff}^+ \) survival probability; i.e.

\[
\int_0^\infty \frac{dP}{dt} dt = P(0) - P(\infty) = P(0) = 1. \quad (19)
\]

To test the above notion of observing oscillations in the muon neutrino \( \bar{\nu}_\mu \), we analyzed the KARMEN anomaly data using our central Eq. (18). In the KARMEN experiment \( \bar{\nu}_e \), \( \bar{\nu}_\mu \), \( \bar{\nu}_\tau \) the electron neutrinos from \( \mu^+ \to e^+ + \bar{\nu}_e + \nu_e \) were detected at times \( \{ t_i \} \) (after the muons were stopped) within time bins of width \( \Delta t_i = 0.5\mu s \).

One expects (using our theory) a mean number of detected electron neutrinos (from the muon decays) in each bin to be given by

\[
\mu_i(\omega, \phi) = -N \left( \frac{dP(t_i; \omega, \phi)}{dt_i} \right) \Delta t_i, \quad (20)
\]

where \( N \) is chosen so that \( \sum_i \mu_i \) is the (total) number of observed \( \nu_e \) events. If the events in each time bin obey Poisson statistics, then the probability distribution for a counting sequence of \( n_i \) events in the \( i^{th} \) bin is given by

\[
P[n; \omega, \phi] = \prod_i \left( \frac{e^{-\mu_i(\omega, \phi)} \mu_i(\omega, \phi)^{n_i}}{n_i!} \right). \quad (21)
\]

The likelihood function (for the theoretical parameters \( \omega \) and \( \phi \)) is then determined by experimental KARMEN data \( n_i, data \) via

\[
\Lambda(\omega, \phi) = K P[n_{data}; \omega, \phi], \quad (22)
\]

where \( K \) is an arbitrary constant. Under the best circumstances, the likelihood function exhibits a single maximum in the theoretical parameter space \( (\omega, \phi) \).

We employed this method of determining the theoretical parameters \( \omega \) and \( \phi \), thus analyzing the KARMEN data from the viewpoint of the \( \bar{\nu}_\mu \).
Figure 1. The likelihood as a function of the parameters $\omega$ and $\phi$.

induced oscillating $W_{\mu f}^+ \rightarrow f$ decays. The likelihood function is exhibited in Fig.1. A single maximum likelihood peak was found. The derived values of $\omega$ and $\phi$ were determined to be

$$\omega = (1.60 \pm 0.16)/\mu s, \quad \text{(23)}$$

and

$$\phi = 0.34 \pm 0.10, \quad \text{(24)}$$

within a 95% confidence interval obtained from the likelihood function. In terms of the neutrino mass squared differences,

$$|m_2^2 - m_1^2| = (0.22 \pm 0.02)(eV/\mu s)^2, \quad \text{(25)}$$

also within a 95% confidence interval.

In Fig.2, we compare the theoretical real time oscillation in the $\nu_e$ counting rate (from the $\mu^+$ decays) with the *experimental* oscillation for the $\nu_e$ counting rate in each time bin. The theoretical parameters ($\omega$, $\phi$) were chosen from the maximum of the likelihood function, The over all fit may be described by the statistical ($\chi^2/dof$) $\approx 1.4$.

In summary, the real time KARMEN anomaly for the electron neutrino counting rates (obtained from muon decay) has been analyzed. The data provides evidence in favor of a neutrino flavor rotation model in which the muon neutrino is in a superposition of two mass eigenstates. The neutrino mass splitting fits the data employing Eq.(25), and the flavor rotation angle fits the data employing Eq.(24). The theoretical fit to the experimental KARMEN anomaly appears quite satisfactory.

REFERENCES

1. B. Ambruster, et. al, *Phys. Lett.* B348, 19 (1995).
2. Y. N. Srivastava, A. Widom and E. Sassaroli. *Phys. Lett.* B344, 436 (1995).
3. Y. N. Srivastava, A. Widom and E. Sassaroli. *Z. Phys.* C66, 601 (1995).
4. Y. N. Srivastava, A. Widom and E. Sassaroli. *Proc. Conf. “Results and Perspectives in Particle Physics”*, Editor M. Greco, *Les Ren-*
countres de Physique de la Valee d’Aoste, La Tuile (1996).
5. Y. N. Srivastava, A. Widom and E. Sassaroli. *Euro. Phys. J.* C2, 769 (1998).
6. G. Drexlin, et. al. (KARMEN Collaboration), *Nuc. Inst. and Meth. in Phys.* A289, 490 (1990).
7. B. Ambruster, et. al, *Phys. Rev.* C57, 3414 (1998).
8. B. Ambruster, et. al. [hep-ex/9801007](http://arxiv.org/abs/hep-ex/9801007)