A SCALAR-TENSOR COSMOLOGICAL MODEL WITH DYNAMICAL LIGHT VELOCITY

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Abstract. The dynamical consequences of a bimetric scalar-tensor theory of gravity with a dynamical light speed are investigated in a cosmological setting. The model consists of a minimally-coupled self-gravitating scalar field coupled to ordinary matter fields in the standard way through the metric: \( g_{\mu\nu} + B \partial_\mu \phi \partial_\nu \phi \). We show that in a universe with matter that has a radiation-dominated equation of state, the model allows solutions with a de Sitter phase that provides sufficient inflation to solve the horizon and flatness problems. This behaviour is achieved without the addition of a potential for the scalar field, and is shown to be largely independent of its introduction. We therefore have a model that is fundamentally different than the potential-dominated, slowly-rolling scalar field of the standard models inflationary cosmology. The speed of gravitational wave propagation is predicted to be significantly different from the speed of matter waves and photon propagation in the early universe.

1. Introduction

Recently we have introduced a class of models that could loosely be described as a minimal introduction of dynamically evolving propagation speeds for different fields. These models \[1, 2, 3\] use a prior-geometric combination of fields to form a metric that is universally coupled to matter fields, thereby not introducing violations of the weak and Einstein equivalence principles. They are essentially a dynamical realization of the earlier work \[4, 5, 6\] by one of the present authors on the possibility of a phase transition in the speed of light in the very early universe solving the horizon and flatness problems of cosmology.

The model considered herein is that introduced in \[2\], and uses a scalar field \( \phi \) that is minimally coupled to a gravitational field described by the metric \( g_{\mu\nu} \), to construct the matter metric:

\[
\hat{g}_{\mu\nu} = g_{\mu\nu} + B \partial_\mu \phi \partial_\nu \phi.
\]

It is this metric that is used to construct the matter action and can be said to be the geometry on which matter fields propagate. Throughout we will refer to \( g_{\mu\nu} \) as the gravitational metric, \( \hat{g}_{\mu\nu} \) as the matter metric, and since \( \phi \) is being used to introduce a bimetric structure through \( \hat{g} \), we shall refer to \( \phi \) as the biscalar field in order to distinguish it from other scalar fields that may appear in the matter model. It is the combination of the gravitational metric and the biscalar field that we consider as being the gravitational fields of our theory.

This type of model has attracted some interest in the field \[3, 7, 8\], and similar models that introduce the prior-geometric structure in a different way have also appeared \[11, 12\]. Since a varying light velocity (in 3+1 spacetime) is a fairly generic feature of higher-dimensional models \[12, 13, 14\], they are also of interest as effective or toy models that may result from such a theory. There is also the more phenomenologically-based ‘varying constants’ theories \[15, 16, 17, 18\], and investigation of the quantum implications of such models in the very early universe \[19, 20, 21\]. Lest one think that these models are purely of theoretical interest, an analysis of quasar spectra \[22, 23\] and of the cosmic microwave background \[24\] have both indicated the possibility of time-variation.

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of the fine structure constant, and placed constraints on the variation of the speed of light since last scattering [25, 26].

In this work we will explore the possibility that our model contains cosmological solutions with sufficient inflation to solve the horizon and flatness problems. We will begin in Section 2 by reviewing the model introduced in [2] and its reduction to a homogeneous and isotropic spacetime. As we shall see, the resulting field equations are quite different than those of standard Einstein gravity plus matter or scalar-tensor models, and in Section 3 we present an (implicitly defined) solution for vanishing matter. We find that prior to a time \( t_{pt} \), (massless) matter fields will continue to propagate with a velocity \( c \) (we will work in a comoving frame of the matter metric \( \hat{g}_{\mu\nu} \)), whereas the speed at which gravitational and biscalalar field disturbances propagate, which is determined by \( g_{\mu\nu} \), will become vanishingly small. This ‘splitting’ of the two light cones causes the universe to experience exponential inflation, without requiring the existence of a self-interaction potential for the biscalalar field. At \( t_{pt} \) the restoration of Lorentz symmetry occurs, and the light cones of all fields coincide. Assuming that \( t_{reh} \geq t_{pt} \) (the former is the reheating time), we show that we can have sufficient inflation to solve the horizon and flatness problems.

In Section 4 we re-introduce matter with a radiation equation of state, and find an approximate solution that, in addition to the symmetry restoration time \( t_{pt} \), has an earlier period \( t < t_{pt} \) during which the light cones of the gravitational and biscalalar field are also ‘split’.

One fundamental issue that we will encounter is how to define “the Planck scale”. Combinations of quantities such as, for example, \( \ell_P = \sqrt{G\hbar/c^3} \), depend on the speed of light \( c \), gravitational coupling \( G \) and Planck’s constant \( \hbar \). Since in our model we have different propagation speeds for different fields (which are also dynamically varying) and we can expect that the effective gravitational coupling to matter also changes with time, we should expect that the scale at which quantum effects become important will be different for different fields. It is also normally assumed that \( h \) is a fixed constant, but clearly if we are allowing fundamental constants to vary with time, then it is not unreasonable to consider it to be represented by a field: \( h(t, \vec{x}) \). Although we will not completely resolve these issues, we will motivate the choice of the parameter \( B \) appearing in (1) as

\[
B \approx \frac{1}{32\pi} \ell_P^2.
\]

In Section 3 we show that including matter with a radiation equation of state will not alter the de Sitter behaviour significantly, but it will introduce another time scale \( t_i \) at which the light cone of the biscalalar field deviates from that of the gravitational field \( g_{\mu\nu} \). We therefore see that in general we will have two time scales in the model, each associated with the bifurcation of light cones associated with different fields. In Section 3, we end with concluding remarks.

### 2. The Model

The model that we introduced in [2] consisted in a self-gravitating scalar field coupled to matter through the matter metric \( (\hat{g}) \), with action

\[
S = S_{grav} + S_\phi + \hat{S}_M,
\]

where

\[
S_{grav} = -\frac{1}{\kappa} \int d\mu (R[\hat{g}] + 2\Lambda),
\]

\[
\kappa = 16\pi G/c^4, \quad \Lambda \text{ is the cosmological constant, and we employ a metric with signature } (+, -, -, -).
\]

We will write, for example, \( d\mu = \sqrt{-\hat{g}} \, d^4x \) and \( \mu = \sqrt{-g} \) for the metric density related to the gravitational metric \( g_{\mu\nu} \), and similar definitions of \( d\hat{\mu} \) and \( \hat{\mu} \) in terms of the matter metric \( \hat{g}_{\mu\nu} \). The
minimally-coupled scalar field action is:

\[ S_\phi = \frac{1}{\kappa} \int d\mu \left[ \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right], \tag{5} \]

where the scalar field \( \phi \) has been chosen to be dimensionless. The energy-momentum tensor for the scalar field that we will use is given by

\[ T^{\mu\nu}_\phi = \frac{1}{\kappa} \left[ g^{\mu\alpha} g^{\nu\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + g^{\mu\nu} V(\phi) \right], \tag{6} \]

and is the variation of the scalar field action with respect to the gravitational metric: \( \delta S_\phi / \delta g_{\mu\nu} = -\frac{1}{2} \mu T^{\mu\nu}_\phi \).

Instead of constructing the matter action \( \hat{S}_M \) using the metric \( g_{\mu\nu} \), we use the combination \( \hat{g}_{\mu\nu} \) resulting in the identification of \( \hat{g}_{\mu\nu} \) as the physical metric that provides the arena on which matter fields interact. That is, the matter action \( \hat{S}_M[\psi^I] = \hat{S}_M[\hat{g}, \psi^I] \), where \( \psi^I \) represents all the matter fields in spacetime, is one of the standard forms, and therefore the energy-momentum tensor derived from it by

\[ \frac{\delta S_M}{\delta \hat{g}_{\mu\nu}} = -\frac{1}{2} \mu \hat{T}^{\mu\nu}, \tag{7} \]

satisfies the conservation laws

\[ \hat{\nabla}_\mu \left[ \hat{\mu} \hat{T}^{\mu\nu} \right] = 0, \tag{8} \]

as a consequence of the matter field equations only \( \hat{g}_{\mu\nu} \). It is the matter covariant derivative \( \hat{\nabla}_\mu \) that appears here, which is the metric compatible covariant derivative determined by the matter metric: \( \hat{\nabla}_\alpha \hat{g}_{\mu\nu} = 0 \).

As described in \( \text{[2]} \), the gravitational field equations for this model can be written as

\[ G^{\mu\nu} = \Lambda g^{\mu\nu} + \frac{\kappa}{2} T^{\mu\nu}_\phi + \frac{\kappa}{2} \hat{\mu} \hat{T}^{\mu\nu}, \tag{9} \]

and that for the scalar field (written here in terms of matter covariant derivatives) as:

\[ \hat{g}^{\mu\nu} \hat{\nabla}_\mu \hat{\nabla}_\nu \phi + KV'[\phi] = 0. \tag{10} \]

In the latter, we have defined the biscalar field metric

\[ \hat{g}^{\mu\nu} = \hat{g}^{\mu\nu} + \frac{B}{K} \hat{\nabla}_\mu \phi \hat{\nabla}_\nu \phi - \kappa \frac{\hat{\mu}}{\mu} B K \hat{T}^{\mu\nu}, \tag{11} \]

and

\[ K = 1 - B \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \tag{12} \]

In this work we will assume a perfect fluid form for the matter fields:

\[ \hat{T}^{\alpha\beta} = \left( \rho + \frac{p}{c^2} \right) \hat{u}^\alpha \hat{u}^\beta - p \hat{g}^{\alpha\beta}, \tag{13} \]

with \( \hat{g}_{\mu\nu} \hat{u}^\mu \hat{u}^\nu = c^2 \).

We will now specialize to a cosmological setting, imposing homogeneity and isotropy on spacetime and writing the matter metric in comoving form as:

\[ \hat{g}_{\mu\nu} = \text{diag}(c^2, -R^2(t) \gamma_{ij}), \tag{14} \]

\footnote{Note that our earlier publication \( \text{[2]} \) contained a typo in equation (5); the rest of the letter is correct as published.}
with coordinates \((t, x^i)\) and 3-metric \(\gamma_{ij}\) on the spatial slices of constant time. The matter stress-energy tensor (using \(u^0 = c\)):

\[
\dot{T}^{00} = \rho, \quad \dot{T}^{ij} = \frac{p}{R^2} \gamma^{ij},
\]

then leads to the conservation laws (an overdot indicates a derivative with respect to the time variable \(t\), and \(H = \dot{R}/R\) is the Hubble function)

\[
\dot{\rho} + 3H(\rho + \frac{p}{c^2}) = 0.
\]

Since we are interested in the very early universe, we will assume a radiation equation of state:

\[
p = \frac{1}{3}c^2 \rho,
\]

which leads to \(\rho \propto 1/R^4\).

It is useful at this point to introduce the following quantities derived from the constant \(B\) which will appear throughout this work:

\[
H_B^2 = \frac{c^2}{12B}, \quad \rho_B = \frac{1}{2\kappa c^2 B},
\]

the latter comes from \(H_B^2 = \frac{1}{6\kappa c^4} \rho_B\). From the definition (12) we have

\[
K = 1 - \frac{\dot{\phi}^2}{12H_B^2},
\]

and using the relation (1) the gravitational metric is found to be

\[
g_{\mu\nu} = \text{diag}(Kc^2, -R^2 \gamma_{ij}),
\]

and so \(\mu = \sqrt{K} \mu\). From the definition (11) we find:

\[
\bar{g}^{\mu\nu} = \text{diag}\left(\frac{1}{Kc^2} I_\rho, -\frac{1}{R^2} I_p \gamma^{ij} I_p\right),
\]

where we have defined

\[
I_\rho = 1 - K^{3/2} \frac{\rho}{2\rho_B}, \quad I_p = 1 + \sqrt{K} \frac{\rho}{6\rho_B}.
\]

In this work, we will find that neither \(I_\rho\) nor \(I_p\) vanish in the solutions of interest, so the metric (11) is degenerate only when \(R = 0\) or \(K = 0\).

The Friedmann equation \((R_{00} + Kc^2 \gamma^{ij} R_{ij})\) is given by

\[
H^2 + \frac{k c^2 K}{R^2} = \frac{1}{3} Kc^2 \Lambda + H_B^2(1 - K) + 2H_B^2 K V_B + \frac{1}{6} \kappa c^4 K^{3/2} \rho.
\]

The remaining equation from (1) is

\[
2\dot{H} + 3H^2 - H \frac{\dot{K}}{K} + \frac{k c^2 K}{R^2} = c^2 \Lambda K - 3H_B^2(1 - K) + 6H_B^2 K V_B - \frac{1}{6} \kappa c^4 \sqrt{K} \rho,
\]

which can be shown to be redundant given the scalar field equation (below), the matter conservation laws (18) and the Friedmann equation. The scalar field equation (10) in this gauge is

\[
I_\rho \ddot{\phi} + 3KHI_\rho \dot{\phi} + c^2 K^2 V' = 0,
\]

which, using (19) can also be written as

\[
I_\rho \dot{K} - 6K(1 - K)HI_\rho - 2K^2 V_B = 0.
\]

We have introduced the dimensionless potential: \(V_B = BV\) and \(\dot{V}_B = V_B' \dot{\phi}\).
These combined equations suggest the following definitions motivated by the varying constants theory \[15, 16\]:

\[
c_{vc}(t) = \sqrt{Kc}, \quad G_{vc}(t) = K^{3/2}G,
\]

in which case the Friedmann equation becomes

\[
H^2 + \frac{k_{vc}^2}{R^2} = \frac{1}{3}c_{vc}^2\Lambda + \frac{1}{12}\dot{\phi}^2 + \frac{1}{6}c_{vc}^2KV + \frac{8\pi}{3}G_{vc}\rho.
\]

Aside from the biscalar potential term (which for simple polynomial potentials can be re-written as a potential with time varying mass and coupling-constants) we have a form that is identical to the Friedmann equation for matter and a scalar field minimally coupled to Einstein gravity. This suggests the definition of a Planck scale that is no longer a constant quantity, but varies as, for example:

\[
\rho_{P,vc}(t) = \frac{c_{vc}^5(t)}{hG_{vc}^2(t)} = \frac{1}{\sqrt{K}}\rho_P,
\]

where \(\rho_P = c^5/(hG^2)\).

We should note that in \[26\] it is stated that varying constants theories require a violation of the strong energy condition in order to solve the horizon and flatness problems. Since we will be setting the biscalar field potential to zero, one might ask how it is possible to claim to solve these problems with matter that satisfies a radiation equation of state? The answer to this is that the Friedmann equation \(23a\) does in fact display coupling to sources that appear to violate the strong energy condition. Note though that to a material observer, matter fields will obey the strong energy condition but couple to the gravitational field in a way that does not. Thus it is the reaction of the gravitational field that sees such violations, which is precisely what is necessary for inflation.

3. EMPTY SPACE, BISCALAR FIELD MODEL

The simplest possible scenario is one in which there is no matter present, and the biscalar field is solely responsible for the dynamics of the very early universe. As we will see in the following section, the presence of matter will have a nontrivial effect at earliest times, but not in a way that will affect the scenario significantly.

From \(24\) we see that both \(K = 0\) and \(K = 1\) are solutions, the first corresponding to an exact de Sitter solution and the latter to a model in which the biscalar field plays no active role. It is not hard to show that the \(K = 0\) solutions are unstable, with deviations \(K \propto R^6\), and so we end up with a strong relationship between how small \(K\) is at the beginning of the universe and the amount of inflation achieved. This fixed point corresponds to degenerate \(g_{\mu\nu}\) and \(\bar{g}_{\mu\nu}\), and so physical solutions will have \(K > 0\) except possibly at a point, which could be considered a physical singularity. What we are essentially going to find here are solutions that interpolate between these two limits, beginning near \(K = 0\) and making a transition (at roughly \(t_{pt}\)) to a minimally-coupled scalar field model.

Setting \(\rho = 0 = p\) and \(V_B = 0\) in \(24\), we find the solution

\[
K = \left(1 + 2\frac{R_{pt}^6}{R^6}\right)^{-1},
\]

where we have defined a scale \(R_{pt}\) to correspond to the end of the inflationary phase, at which point we will have \(K_{pt} = 1/3\). This is justified by using \(23\) and \(24\) to derive

\[
\ddot{R} = RH^2(1-K)(1-3K),
\]

which demonstrates that we have inflation (\(\ddot{R} > 0\)) when \(K < 1/3\). The subscript “pt” indicates that this is the scale at which the phase transition between an earlier ‘broken phase’ where the
propagation speeds for matter and gravitation are very different, and a later ‘restored phase’ where these speeds become approximately equal.

Using (28) in the Friedmann equation (23a), we find the implicit solution

$$\sqrt{2(2 + y)} - \ln \left(\frac{\sqrt{2(2 + y)} + 2}{\sqrt{2(2 + y)} - 2}\right) - y_c = 6H_B(t - t_{pt}),$$

(30)

where

$$y = \left(\frac{R}{R_{pt}}\right)^6, \quad y_c = \sqrt{6} - \ln \left(\frac{\sqrt{6} + 2}{\sqrt{6} - 2}\right) \approx 1.57,$$

(31)

with $y_c$ resulting from fixing the constant of integration so that $R = R_{pt}$ at $t = t_{pt}$.

For $R \ll R_{pt}$ we can expand (30) in $y$ to find

$$R \approx R_{pt} \exp \left[\frac{1}{6}(y_c + \ln 8 - 2)\right] e^{H_B(t - t_{pt})} \approx R_{pt} e^{H_B(t - t_{pt})},$$

(32)

and since $\exp \left[\frac{1}{6}(y_c + \ln 8 - 2)\right] \approx 1.27$ we use this approximation right up to $t_{pt}$. In the opposite limit ($R \gg R_{pt}$), we find that

$$R \approx R_{pt} \left[\frac{z}{\sqrt{2}} + 3\sqrt{2}H_B(t - t_{pt})\right]^{1/3} \approx R_{pt} \left[1 + 3\sqrt{2}H_B(t - t_{pt})\right]^{1/3},$$

(33)

where in this case we are making only a slighter larger error since $y_c/\sqrt{2} \approx 0.111$.

In the very early universe ($t \ll t_{pt}$) the Friedmann equation has a source that is effectively a constant energy density $\rho_B$, so it is reasonable to assume that $\rho_B \approx \rho_P = c^5/(\hbar G^2)$. At $t_{pt}$ the effective energy density has fallen to $2/3$ of this value (since $K_{pt} = 1/3$) and at some scale following inflation we expect reheating to take place. It is likely that in order to have reheating we will need to consider a non-vanishing biscalon scalar field potential, which for concreteness we will assume is a simple quadratic form $V(\phi) \approx 16\pi l^{-2}\phi^2$ where $l$ is a length scale characterizing the mass of the biscalon field. The dimensionless potential is then $V_B = 16\pi Bl^{-2}\phi^2$ with $32\pi Bl^{-2} \approx (t_{pt}/l)^2$ a dimensionless parameter.

Reheating will begin when the biscalon field begins to oscillate about $\phi = 0$, which is characterized by the potential dominating the Hubble damping. It is possible that this could happen prior to $t_{pt}$, in which case the analysis becomes quite complicated, since one would have to consider nonlinear contributions to (24). If it happens at or after $t_{pt}$, then we can approximate the scalar field equation by

$$\ddot{\phi} + 3H\dot{\phi} + 32\pi(c/l)^2\phi \approx 0,$$

(34)

and find that reheating begins at a scale $R_{reh}$ where $H_{reh}^2 \approx 32\pi \times 4c^2/(9l^2)$. From this we see that reheating begins at or after $t_{pt}$ if $l^2 \geq 8(32\pi B) \approx 8t_{pt}^2$. We will leave the details of reheating to future work, and assume for simplicity that reheating takes place at $t_{reh} = t_{pt}$ and is instantaneous. This means that $\rho_{reh}$, the radiation energy density produced at the end of reheating, should satisfy $H_{reh}^2(1 - K_{pt}) = \frac{1}{6}KcA\rho_{reh}$, which gives $\rho_{reh} = \frac{5}{2}\rho_B$. With this assumption, we would have the reheating temperature not too different from the Planck temperature: $T_{pt} \approx T_{reh} \approx T_P$.

The horizon problem is solved, if the size of the cosmological horizon

$$r_c = \frac{cR_0}{RH},$$

(35)

at some time in the history of the universe is much greater than its value at present [27], that is: $\max_t r_c(t) \gg r_c(t_0)$. During inflation ($t < t_{pt}$) the cosmological horizon is decreasing, and it will have a maximum value at earlier times. As it stands our purely classical model can be traced back to $t = -\infty$ and in that limit $r_c \to \infty$. More realistically, the model becomes a good
approximation to a more fundamental quantum model after a time we will call \( t_{qg} \). We then must have \( r_c(t_{qg}) \gg r_c(t_0) \), which yields the condition \( R_0H_0 \gg R_{qg}H_{qg} \), or

\[
\frac{R_0H_0}{R_{pt}H_{pt}} \gg \frac{R_{qg}H_{qg}}{R_{pt}H_{pt}}.
\]

Assuming that the universe undergoes \( \mathcal{N} \) e-folds in this time, we have \( R_{pt}/R_{qg} = \exp(\mathcal{N}) \), \( K_{qg} \approx \frac{1}{2}e^{-6\mathcal{N}} \) and \( H_{pt}/H_{qg} \approx \sqrt{2/3} \). Using standard physics following \( t_{pt} \approx t_{reh} \), this constraint then becomes

\[
\mathcal{N} \gg \ln\left(\frac{T_{pt}}{T_P} \sqrt{10^{60}z_{eq}}\right) \in (32, 60).
\]

The redshift at matter-radiation equality is \( z_{eq} \approx 4.3 \times 10^4 \Omega_{h}^2 \approx 4 \times 10^4 \), and \( 1 < T_{mt}/T_P < 10^{-5} \). Thus we have the usual statement that \( \mathcal{N} \approx 60 \) will solve the horizon problem.

The demonstration that this also solves the flatness problem proceeds in a manner similar to that described in [3]. Identifying the physically meaningful measure of the flatness of a spacelike hypersurface as the 3-curvature measured by a material observer, we have (see also [28])

\[
|\Omega - 1| = \frac{kc^2}{R^2H^2}.
\]

We see that this increases without bound as \( R \to 0 \), but relating the flatness at \( t_{qg} \) and \( t_{pt} \) we find

\[
|\Omega - 1|_{qg} = \frac{R_{pt}H_{pt}^2}{R_{qg}H_{qg}^2} \approx \frac{2}{3}e^{2\mathcal{N}},
\]

and we see that solving the horizon problem implies that we have also solved the flatness problem, as expected.

Using the definition [19] and the solution [28] we can determine how ‘far’ the scalar field travels between two scales \( R_i \) and \( R_f \):

\[
|\Delta \phi| = 2\sqrt{3}\ln(R_f/R_i),
\]

and so between \( t_{qg} \) and \( t_{pt} \) we have \( |\Delta \phi| = 2\sqrt{3}\mathcal{N} \approx 200 \). Note though, that even if the scalar field has a relatively large value at \( t_{qg} \), the effect of the potential is essentially removed by the factor involving \( K \) that multiplies it. Thus the effect of the potential is delayed until after \( t_{pt} \), and the magnitude of the scalar field will have an effect on reheating rather than the dynamics of inflation. We therefore see that in this model the biscal scalar can in fact ‘roll’ quite far, but since the potential (and therefore the value of \( \phi \)) has essentially no effect on the de Sitter behaviour, we do not need an extraordinarily flat potential to achieve sufficient inflation. (Because it is essentially the nonstandard kinetic terms for the biscal field that are driving the de Sitter phase, our model shares some similarities with pre-big bang string cosmology [29].) This makes us believe that this model may have something rather significant to say about the cosmological constant problem, since vacuum energy in the very early universe should be irrelevant to inflation in this model.

On the other hand, we do require that at \( t_{qg} \) the value of \( K \) is extremely small: \( K_{qg} \approx \exp(-6\mathcal{N}) \), which could be viewed as a form of fine-tuning. There are a couple of ways of looking at this problem. As an initial conditions problem, we really only need to argue that in some more fundamental theory we expect that the universe should enter this classical regime close to a vacuum where the light cones are quite different. This is probably not unreasonable, for we have seen in the literature dimensionally-reduced models with light cone variability similar to that of our model. Tracing our model back to this other vacuum we are forced towards \( K = 0 \), and so the issue becomes one of how this more fundamental theory reduces to a model similar to ours in the very early universe. Alternatively, we can transform the model back to a frame (with time coordinate \( T \)) in which the gravitational metric \( g_{\mu\nu} \) is in comoving form, using \( 3H_BT \approx \exp(3H_BT) \). From this we find that
\( \partial_t \phi \approx T \partial_T \phi \), and we see that the apparent fine-tuning \( \partial_t \phi \) is at least partially alleviated by the fact that \( T \to 0 \) in the very early universe.

Although we have tentatively identified \( \rho_B \approx \rho_P \), it is not at all clear that this is appropriate. The reason is that since we have fields that have a varying propagation speed, it is perhaps more reasonable to define the Planck scale as (assuming that \( h \) remains constant), for example:

\[
\rho_P = \frac{\bar{c}^5(t)}{h G^2(t)},
\]

where \( \bar{c}(t) \) and \( \bar{G}(t) \) are the effective speed of propagation and gravitational coupling of the field in question. We would then use this (or similar definitions for other Planck scale quantities) to argue that quantum corrections become important at this scale.

We know from the field equation (10) that the metric \( \bar{g}_{\mu \nu} \) determines the speed of propagation of the biscal field, which reduces to

\[
\bar{c} = \frac{I_p}{I_p} \sqrt{Kc} \approx \sqrt{Kc}.
\]

Since the Friedmann equation looks identical to that of a minimally-coupled scalar field, we assume that \( G = G \), and from (11) we would therefore have \( \rho_P = K^{5/2} \rho_P \). Since this vanishes as \( R \to 0 \) we conclude that we must have a scale above which quantum gravity dominates. Assuming that at \( t_{qs} \) we have \( \frac{1}{6} kc^5 \rho_P(t_{qs}) = H_B^2 (1 - K_{qs}) \), we find that

\[
\rho_B = \frac{K_{qs}^{5/2}}{1 - K_{qs}} \rho_P \approx K_{qs}^{5/2} \rho_P \approx 2^{-5/2} e^{-15N} \rho_P,
\]

which would require \( \rho_B \) to be incredibly small, if the horizon problem is to be satisfied. The varying constants definition (27) yields a result that goes in the opposite direction:

\[
\rho_B = \frac{1}{\sqrt{K_{qs}(1 - K_{qs})}} \rho_P \approx \frac{1}{\sqrt{K_{qs}}} \rho_P \approx \sqrt{2} e^{3N} \rho_P,
\]

which would indicate that \( \rho_B \) is greatly in excess of the usual Planck scale. Note that this identification would also result in quantum corrections becoming important after \( t_{qs} \), which is not reasonable.

On the other hand, because in the gravitational frame the system is an ordinary minimally-coupled scalar field, we would perhaps expect that the standard definition of the Planck scale would be appropriate—as we have assumed here. That is not to say that this issue is closed. Once we include matter into the system, we could legitimately claim that the relevant Planck scale is different for matter fields. We expect that a perturbation calculation will not only provide the seeds for structure formation, it will also illuminate when quantum corrections will dominate the very early universe (and therefore pin down \( t_{qs} \) more completely).

4. The Model with Non-Vanishing Radiation

In the last section we described a solution for which the matter contributions were set to zero. In such a limit we could have reverted back to the field equations in the “gravitational frame”, and chosen coordinates such that \( g_{\mu \nu} \) is of comoving form. We would then find a simple scalar field equation: \( \partial^2_T \phi + 3H \partial_T \phi = 0 \), with solution \( \partial_T \phi \propto 1/R^3 \), using which, the Friedmann equation would yield \( R \propto T^{1/3} \). To relate this solution to that which we found in the previous section requires making the coordinate transformation: \( dt = \sqrt{1 + 1/(3H_B T^2)} dT \) which puts \( \bar{g}_{\mu \nu} \) into comoving form. This coordinate transformation leaves the time coordinate essentially untouched for \( T > 1/(3H_B) \), and for \( T \ll 1/(3H_B) \) we see that \( t \propto \ln T \), so that whereas \( T \in (0, \infty) \) we have \( t \in (-\infty, \infty) \). If we want to retain primordial matter, this simple solution is no longer available to us since the matter stress-energy tensor appears in the scalar field equation (10). We have chosen to
work directly in a comoving matter frame since the interpretation of the resulting metric is clearer, and because the field equations with matter are slightly more manageable.

We now wish to retain the coupling to matter, choosing a radiation equation of state appropriate to the very early universe. From (23a) we see that if \( K \approx 0 \) and \( K^{3/2} \rho \approx 0 \) as \( R \to 0 \), then the Friedmann equation is still dominated by the biscalalar field kinetic terms, and we have the same de Sitter behaviour for the scale factor. In fact what we will find is that, at least when the scalar field potential may be neglected in the very early universe, the \( K = 0 \) fixed point is approached as \( R \to 0 \) in such a way that \( \sqrt{K} \rho \to \text{constant} \).

Let us begin by parameterizing the radiation energy density \( \rho \) as

\[
\rho = \rho_{sr} \left( \frac{R_{sr}}{R} \right)^4,
\]

where subscript “sr” indicates that the quantity is evaluated at the scale \( R_{sr} \) where the biscalalar and radiation contributions to the Friedmann equation would be equal. Defining the ratio

\[
\sigma = \frac{\rho_{sr}}{\rho_B},
\]

this implies that we have

\[
(1 - K_{sr}) \approx \sigma K_{sr}^{3/2},
\]

which has the small-\( \sigma \) solution:

\[
K_{sr} \approx 1 - 2\sigma + O(\sigma^2).
\]

Note that the constant \( \sigma \) defined in (46), although a very convenient parameter to use in making approximations at later times, cannot in any way be considered as a fundamental parameter of the model. Since we are interested in retaining the de Sitter phase, it is reasonable to assume that \( t_{sr} > t_{eq} \), and since we expect that \( \rho \) could possibly be as large as \( \rho_B \) at \( t_{eq} \) but no larger, then it will be redshifted significantly by inflation in order to make \( \sigma \) a very small parameter. In turn (48) tells us that \( t_{sr} > t_{pt} \), so that reheating happens well before biscalalar-radiation equality.

It is convenient to define the variable

\[
Z = \frac{1}{2} \sqrt{K} \left( \frac{R_{sr}}{R} \right)^4,
\]

so that we have

\[
I_p = 1 - \sigma KZ, \quad I_p = 1 + \frac{1}{3} \sigma Z.
\]

Setting the biscalalar field potential to zero, we have the following equations for \( K \) and \( Z \):

\[
I_p \dot{K} = 6K(1 - K)H(1 + \frac{1}{3} \sigma Z),
\]

\[
I_p \dot{Z} = -H(1 + 3K)Z(1 - \sigma Z).
\]

The first of these equations tells us that \( K \) is a nondecreasing function of time provided that \( H \geq 0 \), which is certainly the case at present and from the form of the Friedmann equation (23a) must also be true everywhere in the past (recall that \( K \in (0, 1) \)). We therefore see that \( K \) should begin near \( K \approx 0 \) and increase towards \( K = 1 \) as the universe expands. Since we expect that \( R \gg R_{sr} \) and \( K \approx 1 \) at present, \( Z \) should be small and decreasing towards zero at present. We also see from (48) that \( Z \) must be decreasing everywhere in the past, and as \( R \to 0 \) we must have \( Z \to 1/\sigma \). This is important, since it shows that near \( R = 0 \) we must have \( K \propto R^8 \) rather than the \( K \propto R^6 \) found in Section 3. This guarantees that the constant \( H_B^2 \) is still the only important term in the Friedmann equation as \( R \to 0 \).
Since $Z$ is an increasing variable, we can write $\partial_t = \dot{Z}\partial_Z$, and using (54) we find
\[
\frac{\partial K}{\partial Z} = -\frac{6K(1-K)(1+\frac{1}{3}\sigma Z)}{Z(1+3K)(1-\sigma Z)},
\] (52)
which can be integrated to find ($A_\sigma$ is a constant of integration):
\[
\frac{(1-K)^4(1-\sigma Z)^8}{K} Z^6 = 2^6 A_\sigma^4 > 0.
\] (53)
Using the definition (49) this becomes
\[
(1-K)\left[\left(\frac{R}{R_{sr}}\right)^4 - \frac{1}{2}\sigma\sqrt{K}\right]^2 = A_\sigma K\left(\frac{R}{R_{sr}}\right)^2,
\] (54)
and since at $R = R_{sr}$ we have $y = 1$, we must have
\[
(1-K_{sr})(1-\frac{1}{2}\sigma\sqrt{K_{sr}})^2 = A_\sigma K_{sr},
\] (55)
and combining this with (48) we obtain
\[
A_\sigma = 2\sigma\sqrt{K_{sr}}(1-\frac{1}{2}\sigma\sqrt{K_{sr}})^2 \approx 2\sigma + O(\sigma^2).
\] (56)
We shall now seek an asymptotic (in $\sigma$) solution to (54), and, re-summing the first term, we find
\[
K \approx \left[1 + 2\sigma\left(\frac{R_{sr}}{R}\right)^6\right]^{-1},
\] (57a)
which is identical to (28) up to rescaling. Taking $K_{pt} = 1/3$ as before, this tells us that $R_{sr}/R_{pt} = \sigma^{1/6}$. Note that this is not a uniform expansion (the higher-order corrections diverge as $R \to 0$), and we perform a series expansion in $R/R_{pt}$ to find (once again re-summing the leading contribution):
\[
K \approx \left(\frac{R}{R_{sr}}\right)^8\left[\frac{1}{2}\sigma - \sqrt{2\sigma}\left(\frac{R}{R_{sr}}\right)^2\right]^2.
\] (57b)
These two approximations match at the scale $R_\rho \approx \frac{1}{7}\sqrt{2} R_{sr}$ at a value $K_\rho \approx 4^{-8}\sigma^2$, and we therefore use the form (57a) for $t < t_\rho$ and (57b) for $t_\rho < t < t_{sr}$.

Since we expect that $\sigma$ is a small parameter, we can now determine the evolution of the comoving scale factor $R$ throughout the early universe. In fact, the only new feature is that for $t < t_\rho$ we have $K \propto R^8$, which does not alter the de Sitter behaviour of the scale factor since we still have $K \approx 1$.

It is also useful to show that for small $\sigma$ we have $I_\rho \approx 1$ throughout the history of the universe. To begin with, we take a time derivative of $I_\rho$ to find (retaining the scalar field potential for the sake of completeness):
\[
I_\rho \dot{I_\rho} = -\sigma K\dot{Z}H[5 - 9K + (3 - K)\sigma Z] - 3\sigma ZK^2\dot{V}_B.
\] (58)
Near $R = 0$ we have $Z \approx 1/\sigma$ and we easily see that $I_\rho$ begins near $I_\rho \approx 1$ and decreases until a time $t_m$ where the right-hand side vanishes (ignoring the potential), at which point we find
\[
\sigma Z_m = \frac{9K_m - 5}{3 - K_m}.
\] (59)
After this $I_\rho$ will increase again and end up at $I_\rho \approx 1$ as $Z \to 0$. Therefore (53) corresponds to the minimum value that $I_\rho$ will attain. By combining (29) with (54) it is not difficult to see that $K_m \approx \frac{5}{3} + K_0\sigma^{-1/3}$ where $K_0 \approx 0.989$. This gives $\sigma Z_m \approx 3.64\sigma^{1/3}$ and finally $I_\rho(t_m) \approx 1 - 2.02\sigma^{1/3}$. Clearly the correction can be neglected for small enough $\sigma$.

The scenario that emerges is the following. Since we have chosen to work in comoving coordinates of the matter metric $\bar{g}_{\mu\nu}$, (massless) matter fields propagate with a velocity $c$ throughout the history of the universe. For times $t < t_\rho$ we have $I_\rho \approx 4/3$ and so from (42) we have the speed of biscalar wave propagation is $(4/3)\sqrt{Kc}$, whereas that of gravitational modes contained in $g_{\mu\nu}$ is $\sqrt{K}c$. 

Following this time there is a period \((t_1 < t < t_{pt})\) in which the biscalar and gravitational speeds coincide at \(\sqrt{K}c\), and later \((t_{pt} < t)\) all three speeds coincide and Lorentz symmetry is completely restored.

It is interesting that it is the radiation pressure (rather than the energy density) that is ultimately determining the transition at \(t_1\). That is, it is \(\kappa B\sqrt{K}p \approx 1/3\) that characterizes the very earliest phase of the universe, and, while it is responsible for the second splitting of the light cones, it has very little effect on the de Sitter behaviour of the scale factor. Consequently, although the matter energy density \((15)\) evaluated at \(t_1\) is \(\rho_1 \approx 4^5\rho_B/\sigma\) and is therefore larger than the Planck density, the effective gravitational coupling to matter (from \((23a)\)) is \(K^{3/2}/2G\). Using \((41)\) we have the effective Planck density for matter is \(\rho_P/K^3\), and evaluated at \(t_1\) this becomes \(4^{24}\rho_P/\sigma^6\), and we see that the matter energy density is much less than this.

5. Conclusions

We have introduced a bimetric structure in a scalar-tensor gravitational theory coupled to matter, resulting in different propagation speeds for gravitational waves and matter waves. This can in principle be detected in gravitational wave experiments, such as LIGO, VIRGO and LISA [30, 31].

Within this model we have developed an alternative scenario to the standard inflationary models of cosmology, demonstrating the existence of a solution to the horizon and flatness problems without resorting to generic slow-roll scalar field potentials. In fact, we have shown that the de Sitter phase in the very early universe is largely independent of the introduction of a scalar field potential. Given ongoing concern about the cosmological constant problem in cosmology (see, for example [32]), we feel that this is a very significant feature of our model.

The model is characterized by a time \(t_{pt}\) before which the light cone of fields in the gravitational sector is contracted with respect to matter. (In the presence of matter fields, there will be an earlier time \(t_t\) before which the light cone of the biscalar field bifurcates from that of the rest of the gravitational sector.) Prior to \(t_{pt}\) the universe expands exponentially, driven by the kinetic terms of the scalar field, and is an effect that is attributable entirely to the differing light cones of matter and gravity. As time passes through \(t_{pt}\) a phase transition occurs, the light cones collapse back onto themselves, and the universe is effectively described by a minimally-coupled scalar field.

Matter is coupled to this extended scalar-tensor gravitational sector in a manner which preserves the Einstein equivalence principle. Therefore the matter model lives on a geometry described by a metric \(\hat{g}_{\mu\nu}\), and we do not expect that our model will be in conflict with any direct equivalence principle tests, nor will it violate charge conservation in the manner described in [33]. The strong equivalence principle is violated, and it is the gravitational reaction to the presence of matter that is altered. This is precisely what provides the apparent equation of state that violates the strong energy condition, despite the fact that the matter and scalar field actions are of the standard forms.

Due to the stretching of the perturbative scalar field modes in the early universe in the inflationary period, we expect that a scale invariant microwave background spectrum will be predicted, once the initial conditions for the scalar field and the metric are adopted. This issue will be explored in a later publication.

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