Transferring Inductive Biases through Knowledge Distillation

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Abstract

Having the right inductive biases can be crucial in many tasks or scenarios where data or computing resources are a limiting factor, or where training data is not perfectly representative of the conditions at test time. However, defining, designing and efficiently adapting inductive biases is not necessarily straightforward. In this paper, we explore the power of knowledge distillation for transferring the effect of inductive biases from one model to another. We consider families of models with different inductive biases, LSTMs vs. Transformers and CNNs vs. MLPs, in the context of tasks and scenarios where having the right inductive biases is critical. We study how the effect of inductive biases is transferred through knowledge distillation, in terms of not only performance but also different aspects of converged solutions.

1 Introduction

Inductive biases are the characteristics of learning algorithms that influence their generalization behaviour, independent of data. They are one of the main driving forces to push learning algorithms toward particular solutions [26]. Having the right inductive biases is especially important for obtaining high performance when data or computing resources are a limiting factor, or when training data is not perfectly representative of the conditions at test time. Moreover, in the absence of strong inductive biases, a model can be equally attracted to several local minima on the loss surface; and the converged solution can be arbitrarily affected by random variations, for instance, the initial state or the order of training examples [35, 25, 8].

There are different ways to inject inductive biases into learning algorithms, for instance, through architectural choices, the objective function, curriculum strategy, or the optimization regime. In this paper, we exploit the power of Knowledge Distillation (KD) to transfer the effect of inductive biases between neural networks. KD refers to the process of transferring knowledge from a teacher model to a student model, where the logits from the teacher are used to train the student. KD is best known as an effective method for model compression [3, 16, 32] which allows taking advantage of the huge number of parameters during training, without losing the efficiency of a smaller model during inference. The question that we ask in this paper is: “In knowledge distillation, are the preferences of the teacher that are rooted in its inductive biases, also reflected in the dark knowledge [16] and can thus be transferred to the student?” We are interested in cases where the student model is efficient with respect to the teacher model, i.e., the student model can realize functions that are realizable by the teacher [4], while the teacher has a preference inductive bias so that the desired solutions are easily learnable for the teacher [33].

We consider two scenarios where the teacher and the student are neural networks with heterogeneous architectures, hence, with different inductive biases. We train the models, both independently and using KD, on tasks for which having the right inductive biases is crucial. In the first test case, we study RNNs vs. Transformers, on the subject-verb agreement prediction task [23]. In this task, we
use LSTMs as the most widely used RNN variant. LSTMs are shown to perform better than vanilla Transformers in this task and their superior performance is attributed to their recurrent inductive bias. The recurrent inductive bias of LSTMs enables them to model the hierarchical structure of the inputs, which is of crucial importance in the subject-verb agreement task [37]. In the second test case, we study CNNs vs. MLPs, in the context of MNIST-C (Corrupted MNIST) benchmark [28], which is designed to measure out-of-distribution robustness of models. We train our models on MNIST and evaluate it on the Translated/Scaled MNIST. The particular form of parameter sharing in CNNs combined with pooling mechanism brings them equivariance to these kinds of transformations [11], which leads to better generalization in these scenarios compared to MLPs.

We demonstrate that, when distilling the knowledge from a model with stronger inductive bias (that suits the task at hand) into a model with weaker inductive bias, the student model converges to a solution similar to its teacher’s, not only in terms of final performances, but also confidence calibration, the similarity of activations of the penultimate layer, and per sample behaviour. For example, in our second test case, i.e., MNIST-C dataset, when training an MLP model with KD using a CNN teacher, the student model explores the solution space more similar to its teacher. Figure 1 visualizes and compares the path that an MLP takes during training (Figure 1a), compared to a CNN (Figure 1b). The CNN model explores the surface completely different than the MLP, while the path of a student MLP distilled from the CNN model as the teacher (Figure 1c) is more similar to the CNN.

There is no “one size fits all” learning algorithm. Different algorithms vary in terms of the speed at training/inference or the ability to learn particular patterns. This makes them better at solving certain problems and worse for others. For instance, we might have an algorithm that has the right inductive bias for solving a task at hand but is inherently slow, or have an algorithm that is fast but lacks the right inductive bias. Hence, it is important to explore techniques that enable us to find better trade-offs. Our findings in this paper indicate the potential of knowledge distillation to transfer the effect of inductive biases and eventually benefit from the strengths of different learning algorithms. The codes for the input pipelines, models, analysis, and the details of the hyper-parameters used in our experiments is available at https://github.com/samiraabnar/Reflect.

2 Distilling LSTMs into Transformers

LSTMs [17] and Transformers [39] are the basic building blocks of many state-of-the-art models for sequence modeling and natural language processing. Transformers are an expressive class of models that do extremely well on many tasks where the training data is adequate in quantity [7,13,31]. Several studies, however, have shown that LSTMs can perform better than Transformers on tasks requiring sensitivity to (linguistic) structure, especially when the data is limited [37,6]. This is mainly due to the recurrent inductive biases of LSTMs that helps them better model the hierarchical structure of the inputs. We chose the subject-verb agreement prediction task introduced by Linzen et al. [23] as the test case since there is a meaningful difference between LSTMs and Transformers in this task [37]. We compare these two families of models and conduct experiments to emphasize the differences between them when trained independently and through knowledge distillation.

Recurrent Inductive Bias. Among sequence modeling architectures, models with recursion are in particular powerful for natural language processing due to their adequacy for modeling hierarchical
While theoretically, both recurrent neural networks (RNNs) and Transformers can deal with finite sequences, which the size of the training set is $\sim k$ examples and the size of the test set is $\sim m$. Succeeding at this task is a strong indicator that a model can learn syntactic structures and is therefore proposed as a proxy for assessing the ability of models to capture hierarchical structure in natural language. Previous studies have shown that RNNs have better inductive biases to learn this structure [23]. The recursion in a model can be implemented in different ways, like in Recurrent Neural Networks [9], Recursive Neural Networks [34, 22] and Universal Transformers [6, 15]. While theoretically, both recurrent neural networks (RNNs) and Transformers can deal with finite hierarchical structures, empirical results indicate the superiority of RNNs over Transformers [37, 6] [4].

In the literature [35, 6], the inductive bias of RNNs is referred to as the recurrent inductive bias. Here, we distinguish between three main sources of this bias: (1) The sequential processing of the input: There is an inherent notion of order in the architecture that forces the model to access next tokens in the input one by one; (2) No direct access to the past tokens: The model has to compress all the information from past tokens in a hidden state, which is accessible when processing a new token; (3) Recursion: The model recursively applies the same function on the varying input at every time step.

Transformers [39], in contrast, process the input in parallel. Although a weak notion of order is encoded by positional embeddings, no explicit assumption is made in the connectivity structure of the architecture. Moreover, they have a global receptive field and can access all tokens through self-attention. Finally, standard Transformers are not recursive. However, the standard Transformer can be modified to have an architecture with specifications that are similar to RNNs. We provide empirical results to demonstrate the benefits of these different sources of inductive biases of RNNs. For this purpose, we do not only study standard Transformers and LSTMs, but also design experiments with variants of Transformers in which we attempt to approximate some of the RNNs’ assumptions.

### Task and Models.

We study the performance of LSTMs and variants of Transformers on the task of predicting number-agreement between subjects and verbs in English sentences. We investigate the quality of the solutions they converge to when they are trained independently and when they are trained through distillation. We use the subject-verb agreement dataset of Linzen et al. [23], for which the size of the training set is $\sim 121k$ examples and the size of the test set is $\sim 1m$. Succeeding at this task is a strong indicator that a model can learn syntactic structures and is therefore proposed by Linzen et al. [23] as a proxy for assessing the ability of models to capture hierarchical structure in natural language. Previous studies have shown that RNNs have better inductive biases to learn this compared to standard Transformers [37, 6]. In this task, examples are grouped into different levels of difficulty based on the number of “agreement attractors” and distance between the verb and its subject. Hence, we report both micro accuracy ($\mu$-Accuracy) and macro accuracy over different groups in terms of distance (D-Accuracy) and numbers of attractors (A-Accuracy).

Similar to Tran et al. [37], we follow two setups: 1) when the learning objective is next word prediction, i.e., language modelling (LM) setup; 2) when we directly optimize for predicting the verb number, singular or plural, i.e., classification setup. In the LM setup, we look at the probabilities predicted when the target of the prediction is the verb of interest, and see whether the probability of the correct form of the verb is higher than the other form (singular vs plural). In the classification setup, the input to the model is a sentence up to the position of the verb of interest and the model predicts whether the verb at that position is singular or plural.

Table 1: Performance (mean±std over 4 trials) of different LSTM and Transformer models trained independently with the LM objective.

| Model          | Perplexity $\downarrow$ | D-Accuracy $\uparrow$ | A-Accuracy $\uparrow$ |
|----------------|-------------------------|------------------------|------------------------|
| Transformer    | 57.50 ± 0.1199         | 0.9417 ± 0.0017        | 0.9191 ± 0.0018        |
| Small Transformer | 55.31 ± 0.0847       | 0.9467 ± 0.0012        | 0.9261 ± 0.0020        |
| LSTM           | 56.68 ± 0.0906         | 0.9510 ± 0.0012        | 0.9400 ± 0.0024        |
| Small LSTM     | 58.05 ± 0.1141         | 0.9491 ± 0.0006        | 0.9366 ± 0.0015        |

Table 2: Performance (mean±std over 4 trials) of different LSTM and Transformer models trained independently with the classification objective.

| Model                | $\mu$-Accuracy $\uparrow$ | D-Accuracy $\uparrow$ | A-Accuracy $\uparrow$ |
|----------------------|--------------------------|------------------------|------------------------|
| Transformer-seq      | 0.954 ± 0.0016           | 0.901 ± 0.0037         | 0.717 ± 0.0244         |
| Transformer          | 0.964 ± 0.0010           | 0.909 ± 0.0037         | 0.742 ± 0.0121         |
| Universal Transformer-seq | 0.969 ± 0.0004   | 0.932 ± 0.0055         | 0.806 ± 0.0153         |
| LSTM                 | 0.977 ± 0.0001           | 0.970 ± 0.0003         | 0.928 ± 0.0007         |

1 Agreement attractors are intervening nouns with a different number than the number of the subject. E.g., given the input “The keys to the cabinet (is?/are?)”, the word “cabinet” is an agreement attractor.
In the LM setup, we employ two two-layer unidirectional LSTMs with different sizes, \textit{LSTM} and \textit{Small LSTM}, and two six-layer Transformers, \textit{Transformer} and \textit{Small Transformer}. In this setup, corresponding LSTMs and Transformers have roughly the same number of parameters.

In the classification setup we employ a standard two-layer unidirectional LSTM and three different variants of Transformers: (1) Transformer: a standard six-layer Transformer encoder with a class token (CLS) for classification (BERT \[7\] style), (2) Transformer-seq: a standard six-layer Transformer encoder with future masking where the classification is done using the representation of the last token, (3) UniversalTransformer-seq: a six-layer Universal Transformer \[6\] encoder, in which the parameters are shared in depth, with future masking. Among these variants of Transformer, Transformer-seq implements sequential access to tokens, and UniversalTransformer-seq has both sequential access to tokens and a form of recursion. Appendix \[D\] provides more details on the architectures.

### 2.1 The Importance of Recurrent Inductive Bias

In this section, we report results without distillation that illustrate the merits of the recurrent inductive bias. Table \[1\] shows the performance of the models when trained with the LM objective. A first important observation, in line with the results of Tran et al. \[37\], is that LSTMs achieve better accuracy on the subject-verb agreement task compared to Transformers. Even for instances of Transformer language models that achieve better (lower) perplexity, the accuracy on this task is worse compared to LSTM instances. Since both models achieve good scores on the training set (Appendix \[B\]), this suggests that LSTMs better capture relevant patterns, such as the hierarchical structure of the input, which leads to better generalization.

Figure 2 illustrates the accuracy versus perplexity of several instances of Transformers and LSTMs, in the LM setup. Note that although perplexity is an indicator of how well the model is optimized given the objective function, the accuracy is the metric that matters and shows models’ generalization in subject-verb agreement task. The plot in Figure 2 also illustrates different bias-variance trade-offs of Transformers and LSTMs, each with a large and a small variant (as measured by the number of trainable parameters). The richer hypothesis space of the Transformers, in fact, hurts their generalization, as the variance increases and becomes a source of error. In contrast, adding more capacity leads to slightly better accuracy in LSTMs as their stronger inductive biases control the generalization error.

In Table 2 we show the results of models trained on the classification objective. We compare LSTM with variants of Transformers with different inductive biases. The table shows that similar to the LM results, LSTM achieves the best performance. Interestingly, comparing all four models, we find that the performance steadily increases as more aspects of the recurrent inductive bias are included. This is illustrated in Figure 3a, with the filled circles on the black, dashed line (no distillation).

As another indicator of the quality of the solutions that different models converged to in the classification setup, we look into their confidence calibration. Confidence calibration captures how well likelihood (confidence) of the prediction of the model predicts its accuracy \[13\]. For a well-calibrated model, if we bin the confidence scores and compute the accuracy for each bin, the accuracies are perfectly correlated with the confidence values. The Expected Calibration Error (ECE) is computed as the distance between the calibration curve of the model and the perfect calibration curve \[5\]. In Figure 3b, we plot the ECE \[13\] of the models in the classification setup, with the filled circles on the black dashed line (no distillation). In line with the trends in the performances of these models, the expected calibration error decreases as we move from Transformer toward LSTM.

In summary, the results from this section support the importance of recurrence for solving this task \[37, 6\]. Additionally, as shown in Figures 3a and 3b we find a decreasing trend in the variance

\footnote{Note that future tokens are masked out by default when using a transformer in the decoder mode, e.g., in a language modeling setup.}
of the models, i.e., adding more inductive biases to the models decreases their variance. This is empirical evidence that supports the relation between variance of the solutions a model converges to and its inductive biases.

2.2 Transferring the Effect of Recurrent Inductive Bias

In this section, we show distilling knowledge from LSTM to Transformer can close the gap between their performances by pushing the Transformer to converge to solutions more similar to LSTM’s, by exposing it to the dark knowledge of an LSTM.

Table 3 and Table 4 summarize the distillation results, when the training objective is language modeling and classification respectively. A first general observation is that, for these tasks and setups, distilling a model into an identical model could result in a decrease in the performance. Note that whether self-distillation results in improved performance could potentially depend on many different factors such as the architecture of the model, optimization algorithm and details of the distillation process [10]. Despite no significant changes in the performance with self-distillation, we can improve the performance of the Transformers through distillation from LSTM teachers.

To check whether this improvement is due to the transfer of the effect of inductive biases through distillation and whether distillation helps students to converge to solutions similar to their teachers, we run a series of analyses. In Figure 4, we see how teacher LSTMs pull student Transformers toward solutions with higher accuracy on the subject-verb agreement task in the LM setup. This happens even when the perplexity of the student Transformer is higher (worse) than the independent Transformer.
Table 4: \( \mu \)−Accuracy ↑ (mean±std over 4 trials) of different LSTM and Transformer models with classification objective when we apply pure distillation with \( \tau = 1 \).

| Student Model      | Teacher Model       |
|--------------------|---------------------|
| Transformer        | Transformer         |
| 0.9555 ± 0.0013    | 0.9556 ± 0.0006     |
| Transformer-seq    | Transformer-seq     |
| 0.9599 ± 0.0006    | 0.9629 ± 0.0008     |
| UTransformer-seq   | UTransformer-seq    |
| 0.9611 ± 0.0006    | 0.9635 ± 0.0004     |
| LSTM               | LSTM                |
| 0.9682 ± 0.0002    | 0.9690 ± 0.0004     |
| 0.9679 ± 0.0005    | 0.9688 ± 0.0008     |
| 0.9741 ± 0.0004    | 0.9748 ± 0.0003     |

Figure 5: Calibration plots for independent and distilled Transformer for the classification setup. Note that since the task is binary classification, accuracy for confidences lower than 0.5 is not defined.

Figure 3 also shows the effects of distillation on each of the four models we study in the classification setup. In Transformer-based models, we get the most significant improvement both in accuracy and ECE when the teacher is an LSTM. As the recurrent inductive biases of the teacher get weaker, the amount of improvement in the performance of student models decreases. Figure 5 shows the effect of KD on the calibration, given a student Transformer and an LSTM teacher.

**Is the improvement in calibration merely the product of using soft targets?** Mueller et al. [29] shows training neural networks with soft targets (e.g. through label smoothing) results in models that are better calibrated. On the other hand, KD has a regularization effect similar to label smoothing [41, 36]. Given the lack of significant improvement in ECE in the self-distillation experiments (Figure 3b), it is more likely that the cause of the improvement in ECE when distilling LSTMs into Transformers is beyond the label smoothing effect of KD.

To further explore and better understand the effects of KD, we compare the internal representations of these models besides their final output. Figure 6 shows the 2D projection of the representational similarity [20] between the activations in the penultimate layer of the models (Check Appendix A for more details on the visualization). We see that, in the LM setup, the student Transformers that are distilled from LSTMs have different internal representations compared to independent Transformers and are closer to the LSTM models. For the classification objective, we also see that the distilled models are further away from their independent versions. This supports the idea that the effect of distillation goes beyond the output of the models and their final performances.

Figure 6: 2D projection of representational similarity of the activations from the penultimate layers for 1000 examples from the validation set (check Appendix A for more details). We use the notation of \( a \to b \) to refer to the student model \( b \) distilled from teacher model \( a \).
3 Distilling CNNs into MLPs

We study convolutional neural networks (CNN) vs. multilayer perceptrons (MLP) as two families of models with different inductive biases. CNNs are the de facto choice for processing data with grid-like topology. Sparse connectivity and parameter sharing in CNNs make them an effective and statistically efficient architecture. The particular form of parameter sharing in the convolution operation makes CNNs equivariant to translation [11]. Note that, we can view CNNs as MLPs with an infinitely strong prior over their weights, which says that first of all the weights for each hidden unit are identical to the weights of its neighbor with a shift in space, second, the weights out of the spatially continues receptive field assigned to each hidden unit are zero.

### Task and Models

We study these two models in the context of the Corrupted-MNIST dataset (MNIST-C) [28], which aims at benchmarking out-of-distribution robustness. We train the models on the original MNIST training set and evaluate them on the Translated and Scaled MNIST test sets from MNIST-C. In this scenario, the inductive biases of CNNs help them generalize better than MLPs.

Our CNN architecture is a stack of convolutions and pooling layers. Combining convolution and pooling over spatial regions results in invariance to translation. In order to have CNNs that can learn to be invariant to other transformations like changes in the scale, we can use cross-channel pooling [12], where we pool over separately parametrized convolutions that have learned to detect different transformed versions of the same underlying features. Our MLP is a stack of fully-connected layers on top of the flattened input. More details on the architectures are in Appendix D.

#### 3.1 The Importance of Translation Equivariance

Table 5 presents the accuracy and ECE of CNNs and MLPs when trained independently. All models are trained on the original MNIST training set and tested on the Scaled and Translated sets from MNIST-C. Even though CNNs’ accuracy and ECE on the original MNIST test set are only slightly better than MLPs, there is a rather large gap between their performances on the Translated and Scaled test sets. Moreover, the variance of the results from the CNNs is much less compared to MLPs. Both these observations are consistent with our expectations since CNNs have stronger and more suitable inductive biases for this scenario.

#### 3.2 Better Out of Distribution Generalization with KD

We also study the performance of these models when they are trained through pure distillation. We see in Table 6 that distilling from a CNN into an MLP improves its performance both in terms of
accuracy and ECE for all the three test sets. We also see a lower variance in the performance of MLP models that are trained through KD with CNN teachers. We compare the results of all possible pairs of models as teachers and students, to take into account different effects of KD that can potentially lead to a better performance in the student model. We see that self-distillation results in a slightly better performance in MLPs. This could be due to the regularization effect of distillation [27, 36]. However, the improvement in the performance of MLPs with an MLP teacher is much less compared to when the teacher is a CNN. Regardless of the teacher (MLP or CNN), KD results in slightly lower performances in student CNNs compared to CNNs trained independently (similar to results of an LSTM student in test case 1).

Furthermore, in Figure 7, we compare the representational similarity of the penultimate layers of independently trained CNNs and MLPs as well as their distilled versions. First of all, as expected based on our assumptions about the inductive biases of these models, MLPs have more variance than CNNs. Second, distilling from a CNN to an MLP results in representations that are more similar to the representations learned by CNNs, while this is not the case with the MLP student and the CNN teacher. Moreover, for both CNNs and MLPs, self-distillation does not significantly change the representations they learn.

Finally, we compare the paths the models follow during training until they converge to a solution. To plot the training path of a model, we compute the pairwise representational similarity between different stages of training of the model. Figure 1 illustrates the training path for an independent MLP, an independent CNN, and an MLP that is distilled from a CNN. While MLP and CNN seem to have very different behaviour during training, the student MLP with a CNN as its teacher behaves differently than an independent MLP and more similar to its teacher CNN. This is interesting, in particular, since the student model is only exposed to the final solution the teacher has converged to and no information about the intermediate stages of training is provided in the offline KD.

4 Conclusions

The no free lunch theorem states: for any learning algorithm, any improvement on performance over one class of problems is balanced out by a decrease in the performance over another class [40]. Neural networks with different architectures have different inductive biases and this is reflected in their performance across different tasks. In this paper, we investigate the power of knowledge distillation to enable benefiting from the advantages of different models at the same time. We first demonstrate having the right inductive bias can be crucial in some tasks and scenarios. We further show that when a model has the right inductive bias, we can transfer its knowledge to a model that lacks the needed inductive bias and indicate that solutions that the student model learns are not only quantitatively but also qualitatively reflecting the inductive biases of the teacher model.

We focus on offline distillation using the commonly used cross-entropy loss. But we recognize that the details of the distillation processes itself might have a major impact on its effects. Hence, a next step is to look into different distillation strategies, such as online distillation [2], relational KD [30], or similarity preserving KD [38], to better understand their effectiveness for transferring the effect of inductive biases. Another interesting direction to explore is to have multiple teachers, each with different inductive biases that are useful for different tasks and see if the student learns a more generalized solution than the teachers and can learn to combine all their benefits in one model.
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A Visualisation of representational similarity of the activations from the penultimate layer

In order to compare and visualize the state of \( m \) different models with respect to each other (at convergence or any stage of training), we propose using representational similarity\(^{[21,1]}\) of the activations from their penultimate layer. This is particularly useful when these models do not have the same architecture and their parameter space is not directly comparable.

To do so, given a sample set of size \( n \) from the validation/test set (e.g. 1000 examples), we feed them to the forward pass of each model to obtain the representation from the penultimate layer of the models. Then, for each model, we calculate the similarity of the representations of all pairs from the sample set using dot product which leads to a matrix of size \( n \times n \). We use the samples similarity matrix associated with each model to compute the similarity between all pairs of models. To do this, we compute the dot product of the corresponding rows of these two matrices after normalization and average all the similarity of all rows, which leads to a single scalar. Given all possible pairs of models, we then have a model similarity matrix of size \( m \times m \). We then apply a multidimensional scaling algorithm\(^{[3]}\) to embed all the models in a 2D space based on their similarities.

The code for projecting the representational similarity of the activations from the penultimate layer to a 2D space can be found in \( \text{https://github.com/samiraabnar/Reflect/tree/master/notebooks/viz} \).

B Performance Scores on the Training data

In the paper, for our first test case, we report the performance of LSTM and different Transformer models on the test set, when trained independently and with knowledge distillation. We observe that LSTMs achieve better accuracy on test set compared to Transformers due to their inductive biases. Here, we also report the performance of all the models, for both classification and LM setup, on the training set, which confirms that Transformer models have enough capacity to achieve good scores on the training data.

This solidifies the narrative that the inductive bias of LSTMs is helping with generalization and rules out, for example, the possibility that LSTMs have a higher capacity or are trained better.

| Model             | Perplexity ↓ | \( D^-\) Accuracy ↑ | \( A^-\) Accuracy ↑ |
|-------------------|-------------|----------------------|----------------------|
| Transformer       | 29.62 ± 0.10| 0.956 ± 0.001        | 0.936 ± 0.004        |
| Small Transformer | 33.02 ± 0.05| 0.959 ± 0.001        | 0.948 ± 0.005        |
| LSTM              | 28.92 ± 0.08| 0.964 ± 0.003        | 0.955 ± 0.003        |
| Small LSTM        | 31.03 ± 0.11| 0.964 ± 0.001        | 0.952 ± 0.006        |

Table 7: Performance (mean±std over 4 trials) of different LSTM and Transformer models trained independently with the LM objective on the training set.

| Model              | Train \( \mu^-\) Accuracy ↑ |
|--------------------|----------------------------|
| Transformer        | 99.57                      |
| Transformer-seq    | 99.57                      |
| UniversalTransformer-seq | 99.66                  |
| LSTM               | 98.62                      |

Table 8: Performance (mean±std over 4 trials) of different LSTM and Transformer models trained independently with the classification objective on the training set.
C  Per-sample Behaviour

In order to compare the models with each other and better understand how distillation affects the student models, we take a closer look at their per sample behaviour and investigate if the errors a student model makes are more similar to its teacher’s errors. Here, we look into the error overlap of the students and teachers, which reflects their similarity in terms of their behaviour per data example. This similarity can be another proxy to measure the similarity of the solutions learned by the models, with and without distillation. Figures 8, 9, and 10 illustrate the error overlap between different models as Venn diagrams when they are trained independently and when we use distillation.

In Figure 8, we observe that when the Transformer and LSTM models are trained independently, two independent LSTMs behave more similarly compared to two Transformers (Figures 8b and 8a). Given a similar number of trainable parameters, i.e., similar capacity for LSTMs and Transformers, this again supports the claim that models with stronger inductive biases converge to more similar solutions (Also shown in Figure 3a).

When we apply KD in a cross-architecture setting, with an LSTM teacher and a student Transformer, Figures 8c and Figure 8d, the student Transformer behaves more similarly to the LSTM teacher and an independent LSTM, compared to the independent version of itself. This confirms that through distillation the way the student model solves the task becomes more similar to the way the teacher model solves the task.

We have similar observations in Figures 9, and 10; where errors of a student MLP are less and more similar to the errors the teacher CNN compared to an independently trained MLP.

Figure 8: Error overlap for LSTM and Transformer models trained with the classification objective on SVA task. These Venn diagrams show the intersections of the sets of examples miss-classified by the models. In (a) we compare two independent LSTMs (LSTM#1 and LSTM#2) and an independent Transformer; in (b) we compare two independent Transformers (Transformer#1 and Transformer#2) and an independent LSTM; in (c) we compare a student Transformer and a teacher LSTM with an independent Transformer; and in (d) we compare a student Transformer and a teacher LSTM with an independent LSTM.

Figure 9: Error overlap for CNN and MLP models trained on MNIST and tested on Scaled-MNIST set from MNIST-C dataset. These Venn diagrams show the intersections of the sets of examples miss-classified by the models. In (a) we compare two independent CNN (CNN#1 and CNN#2) and an independent MLP; in (b) we compare two independent MLP (MLP#1 and MLP#2) and an independent CNN; in (c) we compare a student MLP and a teacher CNN with an independent MLP; and in (d) we compare a student MLP and a teacher CNN with an independent CNN.
Figure 10: Error overlap for CNN and MLP models trained on MNIST and tested on Translated-MNIST set from MNIST-C dataset. These Venn diagrams show the intersections of the sets of examples miss-classified by the models. In (a) we compare two independent CNN (CNN#1 and CNN#2) and an independent MLP; in (b) we compare two independent MLP (MLP#1 and MLP#2) and an independent CNN; in (c) we compare a student MLP and a teacher CNN with an independent MLP; and in (d) we compare a student MLP and a teacher CNN with an independent CNN.

D Detailed Models Architectures and Training setup

For the subject-verb agreement task, we study Transformers and LSTMs. In the LM setup, we use two sizes for each architecture: LSTM: two-layer uni-direction LSTM, with a hidden size of 1024. Small LSTM: two-layer uni-direction LSTM, with a hidden size of 512. Transformer: six-layer Transformer decoder with a hidden size of 512 and 8 heads. Small Transformer: Transformer: six-layer Transformer decoder with a hidden size of 256 and 8 heads.

In the classification setup, we employ an LSTM and three variants of Transformer, where the LSTM has a two-layer with a hidden size of 256, and the Transformers have 6 layers, 8 heads and a hidden size of 128. We use a hidden size of 256 for the UniversalTransformer-seq since its parameters are shared in depth and with the same hidden size as other Transformers, it will have fewer parameters.

On the MNIST-C dataset, we study CNNs and MLPs. Our CNN has two $3 \times 3$ convolutions, followed by a max-pooling layer over spatial dimensions, followed by another $3 \times 3$ convolution and a maxout (max-pooling over channel dimension) layer 12. Finally a global averaging is done over spatial dimensions, before the projection layer. The MLP model simply has three fully connected layers.

For training the independent models we use the Adam optimizer 19 with exponential decay learning rate scheduler and for the student models in the distillation process, we use Adam optimizer with cosine decay restart 24 learning rate scheduler. The hyperparameters related to the regularization and learning rate schedulers are tuned separately for each model/experiment. For each model, we report the set of hyper-parameters that gives the best average performance across multiple trials with different random seeds for initialization.

The code and the details of the hyper-parameter sets used in our experiments are available at https://github.com/samiraabnar/Reflect, to facilitate the replication of all the experiments.