TOP QUARK THEORY

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I review recent theoretical work on top-quark physics within the standard model, beyond the standard model, and at the Planck scale.

In this talk I begin with a discussion of the top quark within the context of the standard model, move on to consider top-quark physics beyond the standard model, and end with some speculative comments about the relation of the top quark to Planck-scale physics.

1 Top within the Standard Model

There are only a few fundamental parameters associated with the top quark in the standard model: the top-quark mass and the three CKM elements involving top. Below I discuss \( m_t \) and \( V_{tb} \); I will also discuss the top-quark cross section.

1.1 \( m_t \)

The world-average value of the top-quark mass from Run I at the Fermilab Tevatron was reported at this conference to be

\[
m_t = 175 \pm 6 \text{ GeV}
\]  

based on roughly 100 \( pb^{-1} \) of data.\(^1\) It is appropriate to ask how precisely we desire to know \( m_t \), and how precisely we can measure it.

This discussion is often carried out in the context of precision electroweak measurements. The indirect determination of the top-quark mass from these measurements is\(^2\)

\[
m_t = 179 \pm 8^{+17}_{-20} \text{ GeV},
\]

where the second uncertainty is obtained by varying the Higgs mass from 60 GeV to 1 TeV. The direct measurement of the top-quark mass is much more accurate than the indirect determination, so the measured mass can now be included in the precision studies to learn something about the Higgs mass (or, more generally, about electroweak symmetry breaking). Rather than pursue this line of thought further, I will leave it to another speaker,\(^2\) and take a different tack.

Figure 1 shows the quark mass spectrum on a logarithmic scale. These are the running \( \overline{\text{MS}} \) masses, evaluated at the quark mass (for \( c, b, t \)) or at 1 GeV (for \( u, d, s \)).\(^3\) The top-quark \( \overline{\text{MS}} \) mass is

\[
\overline{m}_t(m_t) = 166 \pm 6 \text{ GeV}.
\]

Figure 1: The quark mass spectrum. The bands indicate the running \( \overline{\text{MS}} \) masses, evaluated at the quark mass (for \( c, b, t \)) or at 1 GeV (for \( u, d, s \)), and the associated uncertainty.

The percentage uncertainty in the quark mass is proportional to the width of the band in the figure. It is evident that the larger the quark mass, the better known it is. Since the top quark is the SU(2) partner of the bottom quark, one can imagine...\(^a\)
ine that we will someday have a theory relating their masses, so it is desirable to know the top mass at least as well as we know the bottom mass.

The most accurate determination of the bottom mass comes from a comparison of the Upsilon mass with a lattice calculation, yielding

$$m_b(m_b) = 4.0 \pm 0.1 \text{ GeV},$$

an uncertainty of 2.5%. This corresponds to a measurement of the top mass to 4 GeV, which should be attainable in Run II at the Tevatron (2 fb$^{-1}$), based on roughly $10^3$ fully-reconstructed top events. The uncertainty in the bottom mass is entirely theoretical, coming from lattice perturbation theory, and once this calculation is carried out to next order the uncertainty in the mass will be reduced to perhaps 1%. This corresponds to a measurement of the top mass to about 2 GeV, within reach of the Tevatron with 10 fb$^{-1}$ of data (which requires additional running beyond Run II). The CERN Large Hadron Collider (LHC) will be such a prolific source of top quarks, about a million fully-reconstructed events per year (at “low” luminosity, $10^{33}/\text{cm}^2/\text{s}$), that one can imagine a measurement of the top mass to 1 GeV or even less. The issue is entirely one of systematics, as the statistical uncertainty is negligible.

A more ambitious goal is to find a theory that relates one or more quark masses to the gauge couplings. I argue in the final section that the top quark is our best candidate for such a relation to exist. We know the electroweak gauge couplings with an accuracy of about 0.1%. This corresponds to an uncertainty in the top mass of 200 MeV, which may be attainable with $e^+e^-$ and $\mu^+\mu^-$ colliders operating at the $t\bar{t}$ threshold.

Before leaving the topic of the top mass, I would like to mention an issue which must be resolved before Run II at the Tevatron. It is important to understand hard gluon radiation in top events, and to correct for this effect when extracting the mass. At present this effect introduces an uncertainty of 2.2 GeV in the mass. The gluon radiation is simulated with HERWIG. However, it has been shown recently that HERWIG produces too much gluon radiation in top events, despite the fact that it simulates gluon radiation from light quarks accurately. This problem occurred in a region of phase space where perturbation theory is reliable, so it is not a consequence of multiple-gluon radiation.

### 1.2 $V_{tb}$

Of the three CKM matrix elements associated with the top quark, only $V_{tb}$ appears to have the potential to be measured directly. Indirect measurements of $V_{ts}$ and $V_{td}$ via $K$ and $B$ decays are discussed in another talk.

If there are only three generations, $V_{tb}$ is already well known; in fact, it is the best-known CKM matrix element, although it has not been measured directly. Three-generation unitarity, along with the known small values of $V_{cb}$ and $V_{ub}$, yields $V_{tb} = 0.9989 - 0.9993$. Thus a direct measurement of $V_{tb}$ is only of real interest if we entertain the possibility of more than three generations. In this case, the value of $V_{tb}$ is essentially unconstrained: $V_{tb} = 0 - 0.9993$.

\[\sigma = \sigma(t\bar{b}) BR(t \rightarrow bW)\]  \(5\)

and to extract $BR(t \rightarrow bW)$ from $t\bar{t}$ production, amount of gluon radiation in top events; that turned out to be a bug, which has been fixed.

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\(^6\)A previous version of HERWIG underestimated the

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**Figure 2:** Single-top-quark production in hadron collisions: 
(a) quark-antiquark annihilation, (b) W-gluon fusion.

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The best way to measure $V_{tb}$ at a hadron collider is via single-top-quark production, shown in Fig. 2. There are two separate processes: (a) quark-antiquark annihilation, which is similar to the Drell-Yan process, and (b) W-gluon fusion, which is similar to heavy-flavor production via charged-current deep-inelastic scattering. Both of these processes should be observed in Run II at the Tevatron. They both involve the top-quark charged current, so their cross sections are proportional to $|V_{tb}|^2$. The strategy is to measure the single-top cross section, which is given by

$\sigma = \sigma(t\bar{b}) BR(t \rightarrow bW)$  \(5\)
given by
\[ \sigma = \sigma(t\bar{t})|BR(t \to bW)|^2. \] (6)

One thus obtains \( \sigma(t\bar{t}) \), which is proportional to \( |V_{tb}|^2 \). This procedure requires a theoretical calculation of \( \sigma(t\bar{t}) \), which is the topic of the next section.

Let’s consider the strengths and weaknesses of the two separate single-top processes. The \( q\bar{q} \) annihilation process has been calculated to next-to-leading order in QCD, and much of the technology to extend this calculation to next-to-next-to-leading order exists. The \( W \)-gluon-fusion process has only been calculated to leading order, but the technology for the next-to-leading order calculation exists. The \( q\bar{q} \) annihilation process involves the quark distribution functions, which are better known than the gluon distribution function in the \( W \)-gluon-fusion process. Furthermore, the \( q\bar{q} \) annihilation process benefits from its similarity to the Drell-Yan process, which can be used as a normalization. The \( W \)-gluon-fusion process has the advantage that it will be observable at the LHC, while \( q\bar{q} \) annihilation will not, due to backgrounds. The large rate of the \( W \)-gluon-fusion process at the LHC implies that the measurement of \( V_{tb} \) will have negligible statistical uncertainty.

\( V_{tb} \) should be measured to about 10% in Run II at the Tevatron, and to about 5% with additional data (10 \( fb^{-1} \)), via the \( q\bar{q} \) annihilation process (assuming \( V_{tb} \) is near unity), where the uncertainty is statistical. The measurement of \( V_{tb} \) at the LHC via the \( W \)-gluon-fusion process will be limited mostly by the uncertainty in the gluon distribution function: \( \Delta V_{tb} \sim \Delta g(x)/2 \). An uncertainty of 5% requires knowledge of the gluon distribution function to 10%. \( V_{tb} \) can be extracted from the top width measured from \( e^+e^- \) and \( \mu^+\mu^- \) colliders operating at the \( t\bar{t} \) threshold: \( \Delta V_{tb} \sim \Delta \Gamma/2 \). An uncertainty in the width of less than 10% may be possible, yielding an uncertainty in \( V_{tb} \) of less than 5%.

The measurement of \( V_{tb} \) at a hadron collider requires input from a variety of sources: deep-inelastic scattering (for the parton distribution functions), theory (for precise QCD calculations), and of course the actual experiment. It’s a good example of the coordinated effort that is often required to measure a fundamental parameter of the standard model.

1.3 \( \sigma(t\bar{t}) \)

Top-quark production at the Tevatron is dominated by quark-antiquark annihilation at moderate values of the parton momentum fraction, \( x \sim 2m_t/\sqrt{s} \sim 0.2 \), and thus should be calculable with high precision. As we have just seen, an accurate calculation of the cross section for top-quark pair production is a necessary ingredient for the measurement of \( V_{tb} \). More importantly, this cross section is sensitive to new physics in top-quark production and/or decay. A new source of top quarks (such as gluino production, followed by the decay \( \tilde{g} \to t\bar{t} \)) would appear as an enhancement of the cross section, and a new decay mode (such as \( t \to t\chi^0 \)) would appear as a suppression. Resonances in \( t\bar{t} \) production would also increase the top-quark cross section.

The top-quark cross section was calculated at next-to-leading order in QCD many years ago, and was recently updated in Ref. There are three sources of uncertainty in the calculation. First, the value of \( \alpha_s(M_Z) \), when varied by 10%, results in a change in the cross section of 6%. Since the cross section is proportional to \( \alpha_s^2 \) (at tree level), it is perhaps surprising that the sensitivity of the cross section is not 20%. The explanation is that there now exist parton distribution functions with a varying \( \alpha_s(M_Z) \), and it turns out that these largely compensate for the variation of the partonic cross section with \( \alpha_s \). Second, varying the parton distribution functions themselves (for fixed \( \alpha_s(M_Z) \)) leads to an uncertainty of at least 3%. This is judged by comparing the results using the MRS and CTEQ parton distribution functions; however, these are best fits to some subset of the world’s data, and are not meant to represent the possible range of values for the parton distribution functions. A set of parton distribution functions with built-in uncertainties is badly needed. Finally, the uncertainty due to the uncalculated higher-order QCD correction is estimated by varying the factorization and renormalization

\footnote{One cannot extract \( V_{tb} \) from top decay alone. For example, if \( BR(t \to bW) \) is close to unity, one only learns that \( V_{tb} \gg V_{ts}, V_{td} \).}
scales, and is about 7%. Adding these three uncertainties in quadrature yields a total uncertainty of at least 10%.

A comparison of the theoretical cross section with the measured value, as a function of the top-quark mass, is given in Fig. 3. The agreement is satisfactory at the one-sigma level, so there is no hint of new physics in top-quark production at this time.

![Figure 3: Cross section for $t\bar{t}$ production at the Tevatron vs. the top-quark mass. Theory curve, with error bands, is from the NLO QCD calculation of Ref. 24. The cross indicates the world-average mass and cross section.](image)

Given that the uncertainties in $\alpha_s(M_Z)$ and the parton distribution functions will decrease with time (an uncertainty of 10% in $\alpha_s(M_Z)$ is already conservative), the uncertainty due to the uncalculated higher-order QCD corrections will soon be dominant. The next-to-next-to-leading-order QCD correction is needed to reduce the uncertainty further. This is a technically challenging task, but the tremendous progress in two-loop, and one-loop multi-leg, QCD calculations gives hope that this calculation will be possible.

There are attempts to sum, to all orders in $\alpha_s$, the contributions from soft initial-state gluon emission. This is relevant to all hadron-collider processes at large invariant mass, not just to top-quark production. Single-gluon emission yields a correction proportional to $\alpha_s \ln^2 E_g$, where $E_g$ is the gluon energy. This is a large correction for small $E_g$, but the integral over gluon energies is convergent, and yields a finite result. Each additional gluon emission yields another such factor. One finds that these terms approximately exponentiate, yielding a correction factor of

$$e^{\alpha_s \ln^2 E_g}.$$  (7)

The integral over gluon energies is now divergent for small $E_g$. This is not necessarily unphysical; it could be related to the fact that QCD is non-perturbative for soft gluons. Refs. 27, 28 take this point of view, and cut the integral off at the lower limit. The result is that soft-gluon resummation increases the top-quark cross section at the Tevatron by about 10% above the next-to-leading-order cross section. However, Ref. 24 argues that the divergence in the integral is an artifact of the approximations made in deriving the correction factor of Eq. 27. An alternative summation procedure is proposed that does not diverge for small $E_g$. This yields a correction due to soft-gluon emission of only about 1% above the next-to-leading-order cross section.

2 Top beyond the standard model

Because top is so much heavier than the other fermions, it may be special in some sense. There are many explicit models which implement this idea, too many to consider in this talk. One such proposal, Topcolor, is discussed in another talk. Here I confine my remarks to a model-independent observation which demonstrates the role that mass might play in distinguishing the top quark from the other fermions.

Consider the gauge Lagrangian of a single quark doublet ($qL = (uL, dL)$),

$$\mathcal{L} = i\bar{q}_L \gamma \! \! \! \! = \bar{q}_L + i\bar{u}_R \gamma \! \! \! \! = u_R + i\bar{d}_R \gamma \! \! \! \! = d_R \tag{8}$$

where the covariant derivatives contain the gauge fields. The gauge interactions possess an exact $U(1) \times U(1) \times U(1)$ chiral symmetry, which independently rotates the $qL$, $uR$, and $dR$ fields by a phase. Quark mass terms (generated by electroweak symmetry breaking) explicitly violate this

$^c$A separate, unrelated, divergence occurs for small $E_g$ because the (running) $\alpha_s$ in the exponent is evaluated at the scale $E_g$.29
symmetry, but because quark masses are small compared to the weak scale (except for the top quark) we may regard the symmetry as being approximate. If a quark has non-standard interactions with gauge bosons or with itself, these can be described by higher-dimension operators in the Lagrangian. Some of these operators also explicitly violate the chiral symmetry. If the chiral symmetry is exact in the limit of vanishing quark mass, we expect such operators to be proportional to the quark mass. For example, an anomalous magnetic-dipole interaction term has the form

$$\frac{i m}{A^2} \tilde{q} \sigma^{\mu\nu} q R F_{\mu\nu} + H. c. \quad (9)$$

where $A$ is the scale of the physics responsible for the non-standard interaction. Since the top-quark mass is so much larger than the other fermions, its anomalous magnetic-dipole interaction, if it exists, could be much larger than that of the other quarks.

Because the top quark is so much heavier than the other fermions, it would be a mistake to assume that its interactions are identical to those of the other fermions. Among other things, top-quark interactions could involve large CP violation and flavor-changing neutral currents. A large amount of phenomenological work has been done detailing the signatures of new physics in top-quark studies at present and future colliders, and I cannot hope to review it all. At best I can review the phenomenological work which was presented at this conference.

There were several contributions related to CP violation in top physics. Ref. argues that the top quark is a sensitive probe of sources of CP violation beyond the standard model. Ref. considers $e^+e^- \to t\bar{t}$, including CP violation in both production and decay. Ref. considers the same process with the radiation of an additional gluon, as a probe of non-standard $g\bar{t}t$ (as well as $(\gamma, Z)\bar{t}t$) interactions. Ref. considers the process $e^+e^- \to th$ to test the CP nature of the Higgs boson, $h$. There was also a contribution on exploring the flavor-changing neutral current $Z\bar{t}c$ via $e^+e^- \to \bar{t}c$. Finally, an extended gauge model, based on the group $SU(2)_1 \times SU(2)_2 \times U(1)_Y$ was proposed in Ref. The first $SU(2)$ couples to the first two generations, and the second $SU(2)$ to the third generation. The two $SU(2)$ groups are spontaneously broken to the ordinary $SU(2)$ of the standard model at a high scale. The phenomenology of the $W', Z'$ associated with the extra $SU(2)$ is considered.

To complement the phenomenology at future $e^+e^-$ colliders mentioned above, I would like to spend some time on top phenomenology at hadron colliders, a topic I feel has not received as much attention as it should. It is well known that top quarks can be polarized at an $e^+e^-$ collider by polarizing the electron beam, and that this is a useful tool to study the weak decay properties of the top quark, because the top quark decays before the strong interaction has time to depolarize it. There is an analogue of this tool at hadron colliders. Although the top quarks are produced unpolarized in (unpolarized) hadron collisions, the spins of the $t$ and $\bar{t}$ are correlated. The spins are also correlated in unpolarized $e^+e^-$ collisions. This spin correlation can be used to study the weak decay properties of the top quark by observing the angular correlations between the decay products of the $t$ and $\bar{t}$. The spin correlation should be observed in Run II at the Tevatron.

Let’s consider the origin of the spin correlation in the process $q\bar{q} \to t\bar{t}$, the dominant top-quark production process at the Tevatron. At energies large compared with the top mass, chirality conservation implies that the $t$ and $\bar{t}$ are produced with opposite helicities (“helicity basis”). At the other extreme, the $t$ and $\bar{t}$ are produced with zero orbital angular momentum at threshold, so spin is conserved. Since the colliding quark and antiquark have opposite spins (due to chirality conservation), the $t$ and $\bar{t}$ have opposite spins along the beam axis (“beamline basis”). Remarkably, there exists a basis which interpolates at all energies between these two extremes (“diagonal basis”), such that the $t$ and $\bar{t}$ spins are always opposite.

The single top-quark processes discussed above are sources of polarized top quarks at hadron colliders, since they involve the weak interaction. Given the large numbers of top-quark pairs and single top quarks that will be produced at the Tevatron and LHC, the spin correlation and the single-top polarization should be powerful tools to analyze the properties of the top quark.
3 Top and the Planck scale

There are many ideas relating the top quark to the Planck scale. One of the prettiest is that the top quark is responsible for driving one of Higgs mass-squared parameters negative in supersymmetric grand-unified models, thus triggering electroweak symmetry breaking. This is a feature of the minimal supersymmetric standard model, which is reviewed in another talk.

Here I would like to consider attempts to understand the value of the top-quark mass from first principles, something more ambitious than the minimal supersymmetric model, although compatible with it in some cases. I first discuss two contributions to this conference, then end with my own personal favorite.

3.1 Top and the Higgs potential

The tree-level Higgs potential is given by the familiar expression

\[ V(\phi) = -\mu^2 \phi^2 + \lambda \phi^4 \]  
(10)

where \( \phi \) is the Higgs field. The Higgs field acquires a vacuum-expectation value, \( v \approx 250 \text{ GeV} \), at the minimum of the potential. The coupling \( \lambda \) may be expressed (at tree level) in terms of the Higgs mass and the vacuum-expectation value by \( \lambda = m_H^2/2v^2 \). At one loop, the top quark renormalizes \( \lambda \), making a negative contribution for large Higgs-field values. As a result, the Higgs potential can develop a second minimum, typically at a value much greater than \( v \), which has lower energy than the first minimum. To avoid this, the tree-level coupling \( \lambda \) must be sufficiently large that it is not driven negative by the one-loop top-quark contribution. This leads to a lower bound on \( \lambda \), and hence on \( m_H \). This is the well-known vacuum-stability bound on the Higgs mass, which was recently updated in Ref. [4].

The lower bound on the Higgs mass depends on the energy up to which the standard Higgs model is valid, because if the second minimum occurs at an energy greater than this, it should not be taken seriously. If one assumes that the standard Higgs model is valid up to the Planck scale, one obtains \( m_H > 135 \text{ GeV} \). However, if one trusts the model only up to 1 TeV, one finds \( m_H > 70 \text{ GeV} \), similar to the current experimental lower bound.

Froggatt and Nielsen [5] have argued that this second minimum should in fact be degenerate with the first minimum, and that it should occur at the Planck scale. These two conditions allow one to predict the Higgs mass and the top-quark mass. They find \( m_t = 173 \pm 4 \text{ GeV} \), compatible with the experimental measurement, and \( m_H = 135 \pm 9 \text{ GeV} \). They give a statistical-mechanics argument for the degeneracy of the vacuua based on the coexistence of two phases at a first-order phase transition.

3.2 Reduction of couplings

The couplings of a theory are in general unrelated to each other. However, symmetries can relate couplings. A well-known example is (supersymmetric) grand unification, which relates the strong, weak, and hypercharge couplings by unifying them into a single coupling, \( g_U \), at the GUT scale. This unification is a consequence of the gauge symmetry of the grand-unified group.

The top-quark Yukawa coupling to the Higgs field is \( y = \sqrt{2} m_t/v \approx 1 \), a natural number, unlike the other fermions which have very small Yukawa couplings. It is tempting to try to relate the top Yukawa coupling to the grand-unified gauge coupling, \( g_U \approx 0.7 \), since it also has a natural value. However, there is no way to relate a gauge coupling and a Yukawa coupling via a symmetry in conventional field theory.

If one attempts to simply relate the gauge and Yukawa couplings by imposing \( y = \kappa g_U \), where \( \kappa \) is a real number, one finds that in general this relation is not preserved by renormalization; the (running) couplings depend on a scale, such that the relation can be true only at one particular scale. However, it sometimes occurs that for special values of \( \kappa \), the relation happens to be true at all scales, despite the fact that there is apparently no symmetry enforcing the relation. Such a situation occurs in a (finite) SU(5) grand-unified model.

If one regards this special value of \( \kappa \) as being favored by nature, one obtains a prediction for the top-quark mass of \( m_t = 183 \pm 5 \text{ GeV} \), an acceptable value.

\footnote{An exception is \( N \geq 2 \) supersymmetric theories.}
3.3 Top and string theory

I now come to my own personal favorite for Planck-scale physics, and the favorite of many others: string theory. String theory is the leading candidate for a quantum theory of gravity, and one can ask if it has anything to say about the top-quark mass. Since string theory is not a mature subject, one cannot extract a definite prediction for the top-quark mass at this time, but I will argue that such a prediction is foreseeable.

String theory has just one coupling, the so-called string coupling, and it is naturally of order unity. Thus the effective field theory which arises from string theory should have couplings which are all of order unity. This is encouraging because, as remarked in the previous section, the grand-unified coupling and the top-quark Yukawa coupling are both of order unity.

Furthermore, string theory holds the promise of explaining why only the top-quark Yukawa coupling is of order unity, while all the other Yukawa couplings are small. The effective field theory which arises from string theory typically has a variety of discrete symmetries, which are the remnants of spontaneously-broken gauge symmetries (“discrete gauge symmetries”). These discrete symmetries restrict the Yukawa couplings. In models with three generations of quarks and leptons, one typically finds that there is at most one nonvanishing Yukawa coupling, which we interpret as the top-quark Yukawa coupling. The other Yukawa couplings arise from higher-dimension operators in the effective field theory, and are suppressed by powers of the small ratio \( <\phi>/M_{\text{Planck}}\sim 10^{-1} \sim 10^{-2}\), where \( <\phi>\) is the vacuum-expectation value of some scalar field.

4 Conclusions

The future holds increasingly-precise measurements of the fundamental parameters \(m_t\) and \(V_{tb}\). However, I believe the study of top-quark physics promises much more. I have argued that future study of the top quark will yield either

- New physics in the next generation of colliders
- Information on Planck-scale physics

Either way, we learn something of great importance.

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Questions

**J. Richman, University of California at Santa Barbara:**
You described various models that give predictions for the top-quark mass. Do these models also provide information on the values of CKM matrix elements?

**S. Willenbrock:**
The specific models I mentioned do not address the CKM matrix.

**H. Nielsen, Niels Bohr Institute, Copenhagen:**
Let me answer the just put question for the case of our own work. There is a very weak and indirect connection between the top-mass prediction and a fit we made for the other quark (and lepton) masses and the CKM matrix: the requirement of the degenerate minima or degenerate vacua we also used to predict the three fine structure constants using also a special gauge group hypothesis. We used this gauge group to a fermion mass-spectrum fit.

**S. Willenbrock:**
Thank you for reminding me of this work.

**B. Ward, University of Tennessee:**
In your discussion of the resummation results of Laenen et al. versus Catani et al., did you mean to imply that the results of Catani et al. were the correct ones?

S. Willenbrock:

I certainly find the argument of Catani et al. compelling. However, I believe this topic deserves further theoretical scrutiny.