Electrons, New Physics, and the Future of Parity-Violation

M.J. Ramsey-Musolf\textsuperscript{a,b}

\textsuperscript{a} Department of Physics, University of Connecticut, Storrs, CT 06269 USA
\textsuperscript{b} Theory Group, Thomas Jefferson National Laboratory, Newport News, VA 23606 USA

Abstract. The study of parity-violation in semi-leptonic processes has yielded important insights into the structure of the Standard Model and the substructure of the nucleon. I discuss the future of semi-leptonic parity-violation and the role it might play in uncovering physics beyond the Standard Model.

I INTRODUCTION

In addition to celebrating the silver anniversary this year of electron scattering at the MIT-Bates Laboratory, we may also mark the passing of 25 years for another sub-field of physics: parity-violation (PV) in semi-leptonic neutral current interactions. Since the MIT-Bates Laboratory has made important contributions to the field of neutral current PV, it seems highly appropriate to consider the future of the field at this Symposium. Paul Souder has discussed in detail the history of neutral current PV in electron scattering, and Besty Beise has summarized the present program of strange-quark searches here at MIT-Bates, the Jefferson Lab, and Mainz. Consequently, I will focus on the future: where the field might go once the current round of parity-violating electron scattering (PVES) experiments are completed. I will also broaden the topic to include PV in atoms. Historically, atomic parity-violation (APV) has been at the forefront of the field, and it will undoubtedly continue to hold such a position in the future. In discussing the future, I hope to convey the following three points: (a) the forefront of neutral current PV will consist of searches for physics beyond the Standard Model; (b) APV and PVES can play complementary roles in this search for “new physics”; and (c) parity-violating, low-energy semi-leptonic processes and high-energy collider searches can, in principle, provide complementary insights as to what may lie beyond the Standard Model (SM). My guess is that this situation will persist for the better part of the next decade, until the LHC begins to produce significant physics results.

Before considering the next decade, it is useful to look back briefly at the last quarter century. One may trace the birth of this field to the Bouchiats, who pro-
posed in 1974 that studying PV atomic processes might produce evidence for the weak neutral currents of the SM in the semi-leptonic sector \cite{1}. The Bouchiats suggested a clever technique for enhancing the signal for these tiny neutral currents so that they might be observed in tabletop experiments. This technique, called “Stark mixing”, relies on the interference of a Stark-induced mixing of opposite parity states in an atom and the mixing caused by weak neutral currents. In effect, the Stark-induced amplitude functions as a lever arm to magnify the importance of the neutral current amplitude. The importance of this idea cannot be overstated. Following the Bouchiats’ proposal, a number of groups endeavored to search for weak neutral currents in APV, using either the Stark-mixing idea or by studying the rotation of plane-polarized light as it passes through a gas of atoms (see, \textit{e.g.}, Ref. \cite{2} and references therein). In fact, the recent, very precise result for cesium APV reported by the Boulder group was obtained using a variation on the Bouchiats’ original Stark-mixing idea \cite{3}. The result of these APV experiments has been to confirm the SM prediction for the structure of the weak neutral current in the low-energy domain at the few percent level. Given the scope of effort involved in testing the SM in high-energy collider experiments, the results of the APV measurements represent a significant triumph for tabletop physics.

Among noteworthy collider experiments are those involving semi-leptonic PVES. Results from the SLAC deep-inelastic PVES experiment on deuterium were reported in the late 1970’s \cite{4}. These results also confirmed the structure of the semi-leptonic weak neutral currents of the SM and yielded a value for the weak mixing angle with nine uncertainty. About a decade later, the collaboration at Mainz reported results on a quasi-elastic PVES experiment involving a \textsuperscript{8}Be target \cite{5}. This experiment tested a different combination of the neutral current parameters than tested by the SLAC experiment. Shortly after the appearance of the Mainz result, the results of the elastic PVES experiment on \textsuperscript{12}C performed at Bates were reported \cite{6}. Again, the results of the carbon experiment complemented those from quasi-elastic and deep inelastic measurements and confirmed the predictions of the SM. As discussed in more detail by Paul Souder, an important benefit of these PVES experiments was the development of experimental expertise and technology that is crucial to the success of the present program and the future prospects of PVES.

Turning back to APV, the Boulder group’s result for cesium dominates the present landscape. The group reports an experimental error of less than 0.4 \%. As with the earlier APV and PVES experiments, the goal of the cesium measurement was to test the SM. The cesium results deviate from the SM prediction by about 1.5\%, representing a 2.5\(\sigma\) difference. The potentially serious consequences of this deviation call for a repeat measurement. To that end, the Bouchiat group in Paris is currently involved in another Stark-mixing experiment with cesium, although the experimental uncertainty is not projected to be as small as in the Boulder measurement.

Over the last decade, the emphasis of PVES has shifted away from SM tests to the study of hadron structure. As Paul Souder and Besty Beise discussed, a well-defined program of measurements to determine the nucleon’s strange quark
vector current form factors is underway [7]. Instead of studying the structure of the lepton-quark weak neutral current interaction, these experiments rely on the present knowledge of that interaction in order to learn something new about the sea-quark structure of the nucleon. Results from the MIT-Bates backward angle experiments on the proton and deuterium have been reported by the SAMPLE collaboration [8], and the results of a forward angle measurement have been published by the HAPPEX collaboration at the Jefferson Lab [9]. The list of approved strange-quark experiments also includes the G0 experiment at the Jefferson, a Hall-C experiment on $^4$He, and an experiment on the proton at the MAMI facility in Mainz [7]. The HAPPEX collaboration has also been approved to run another forward angle proton measurement at $Q^2$ similar to that of the SAMPLE experiment. In addition, the G0 detector will be used to measure the axial vector $N \rightarrow \Delta$ transition form factor.

For both PVES and APV, the next generation of experiments are on the horizon. The groups in Seattle and Berkeley have undertaken measurements of APV observables for several atoms along the chain of isotopes. As I discuss below, ratios of such observables are less sensitive to atomic theory uncertainties than is the APV observable for a single isotope. One hopes that such measurements may provide an even more precise tool for uncovering new physics than the Boulder cesium experiment. In order to realize this goal, however, one requires a new level of insight into nuclear structure than required for the interpretation of a single isotope APV measurement. In the case of PVES, the interest of future experiments seems to be returning to studying the weak neutral current interaction at the elementary fermion level. To that end, a purely leptonic experiment involving PV Möller scattering has been approved for SLAC [10]. Similarly, a letter of intent to perform a precise, forward angle PV $\vec{e}p$ experiment at the Jefferson Lab has appeared [11]. Finally, the Jefferson Lab PAC is considering a proposal to carry out elastic PVES with a $^{208}$Pb target [12]. This experiment would provide the most precise information we have to date on the distribution of neutrons in a nucleus, something of considerable interest to nuclear structure physicists. At the same time, the $^{208}$Pb experiment may provide enough nuclear structure information to help with the interpretation of the APV isotope ratio studies in terms of new electroweak physics. In this respect, the lead experiment would solidify a unique marriage of tabletop and collider efforts having important consequences for atomic, nuclear, and particle physics.

In the remainder of this discussion, I consider these future APV and PVES experiments in detail. First, I review the motivation for searching for new physics at low-energies. I subsequently review the basics of the relevant PV observables and show how precise measurements of these observables can provide a window on physics at the TeV scale. I give a few examples of new physics scenarios that can be tested by low-energy PV and consider a possible connection with nuclear $\beta$-decay. Finally, I discuss the relationship between the APV isotope ratio studies, the nuclear neutron distribution $\rho_n(r)$, and the PVES experiment on $^{208}$Pb. For an in-depth discussion of these issues, I refer the reader to Refs. [13,14]
II SEARCHING FOR NEW PHYSICS

Although there exist a plethora of data confirming the electroweak sector of the Standard Model at the few \( \times 0.1\% \) level, there also exist strong conceptual reasons to believe that the SM is only a piece of some larger framework. A nice perspective from which to view the reasons for this belief is the so-called high-energy desert. The high-energy desert is the region in mass scale ranging from the weak scale \( M_{\text{Weak}} \sim 250 \text{ GeV} \) up to the Planck scale \( M_p \sim 1/\sqrt{8\pi G_\text{Newton}} = 2.4 \times 10^{18} \text{ GeV} \). The conceptual shortcomings of the SM appear at both edges of this desert. First, at the high-energy end, the SM does not appear to produce unification of the electroweak and strong interactions at any scale. If one perturbatively runs the \( SU(3)_C, SU(2)_L, \) and \( U(1)_Y \) couplings up from the weak scale, they never meet at a common point. This lack of unification is undesirable, particularly if one believes a common framework ought to describe the electroweak, strong, and gravitational interactions.

At the low-energy \((\mu << M_{\text{Weak}})\) edge of the desert, the SM is similarly less than satisfying. The most obvious shortcoming is the presence of 19 independent parameters (in the limit of zero neutrino mass) which must be determined from experiment. In addition, the violation of discrete symmetries, such as parity and CP, is put in by hand. The SM does not explain why nature violates these symmetries; it simply incorporates them into a unified framework. Similarly, the quantization of electric charge must be put in by hand; it does not follow naturally (at tree-level in the theory) as does, say, the quantization of isospin charge \([15]\). A particularly serious challenge for the SM is to account for the wide range of mass scales in the SM spectrum. A related aspect of this “hierarchy problem” has to do with quadratic divergences appearing in the renormalization of the Higgs mass. The presence of these divergences lead one to wonder why the Higgs mass should turn out to be at or below the weak scale without the aid of some fine tuning of electroweak parameters. In short, the SM leaves open many questions regarding the various mass scales governing low-energy physics.

Despite the phenomenological successes of the SM, then, one has good reason to believe there must exist some larger framework which contains the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) theory and which, presumably, provides answers to the conceptual puzzles of the SM. Hence, there exists intense interest these days in the search for new physics. In considering what such new physics might be, one faces two broad questions: (a) Which new physics scenarios are most viable, both conceptually and phenomenologically? (b) What are the mass scales associated with a given scenario? In the remainder of this discussion, I will illustrate the insight parity-violating processes involving electrons might play.
III PV OBSERVABLES

The basic quantity of interest in considering neutral current PV is the so-called weak charge, $Q_w$. This quantity is the weak neutral current analog of the EM charge. It gives the strength of the vector current coupling of $Z^0$-boson to an elementary fermion or system of fermions. For our purposes, it is useful to write the weak charge as

$$Q_w = Q_w^{(SM)} + \Delta Q_w^{(NEW)} + \Delta Q_w^{(MB)},$$

where the first term, $Q_w^{(SM)}$ is the contribution to the weak charge from the SM. This contribution can be computed precisely and compared with an experimental value for $Q_w$. Any significant deviation would signal non-zero values for the remaining terms. Of these, $\Delta Q_w^{(NEW)}$ represents contributions from possible physics beyond the SM, while $\Delta Q_w^{(MB)}$ denotes contributions from conventional many-body effects, such as strong interactions among quarks which interfere with the $Z^0$-quark interaction. The extent to which we can reliably compute the latter determines the confidence with which we can learn about $\Delta Q_w^{(NEW)}$ from a given measurement.

Presently, the most precise determination of $Q_w$ has been obtained with APV in cesium. In an APV process, the weak neutral current interaction between the electron and nucleus generates a PV atomic Hamiltonian which mixes states of opposite parity in the atom:

$$H_{PV} = H_{PV}^{(NSID)} + H_{PV}^{(NSD)}.$$

Here, “NSID” and “NSD” denote, respectively, nuclear spin-independent and nuclear spin-dependent components of the interaction. The former arises from the product of axial vector electron and vector nuclear currents, whereas the latter arises from a $V(e) \times A(nucleus)$ structure. These terms can be separated by measuring PV transitions between different hyperfine levels. The NSID term contains $Q_w$. The physics of the NSD term, which includes the effects of the nuclear anapole moment, is also interesting, though I will not consider it further here (for a general discussion, see Ref. [16]).

As pointed out by the Bouchiats, the small parity-mixing effects caused by $H_{PV}^{(NSD)}$ can be enhanced by applying an electric field, which also causes states of opposite parity to mix. Reversing the direction of the applied field can isolate terms in the transition rate which depend on the interference of the Stark and weak interaction amplitudes. In the end, one extracts a ratio such as

$$|A_{PV}|/|A_{STARK}| = \xi Q_w,$$

where $\xi$ is an atomic structure-dependent constant that must be computed by atomic theorists. Thus, any errors associated with atomic theory will propagate into uncertainties in $Q_w$ (we might associate these conventional, atomic structure
uncertainties with \( \Delta Q_w (\text{MB}) \). In fact, the dominant uncertainty in the present value for \( Q_w \) of cesium is from atomic theory.

An alternate method for determining \( Q_w \) is with PVEs. In PVEs, one scatters longitudinally polarized electrons from a target and compares the cross sections when the electron helicity is flipped. Any non-zero difference results from an interference of the PV neutral current electron-nucleus scattering amplitude and the more familiar, parity-conserving electromagnetic amplitude. The observable of interest in this case is the “left-right” asymmetry

\[
A_{LR} = \frac{N_+ - N_-}{N_+ + N_-} = a_0 Q^2 \left \{ \frac{Q_w}{Q_{EM}} + F(q) \right \} .
\]

(4)

Here, \( N_\pm \) denote the number of electrons detected for a given helicity of the incident beam; \( a_0 \) is a constant whose scale is set by the Fermi and EM fine structure constants; \( Q^2 \) is the square of the momentum transfer; \( Q_{EM} \) is the electromagnetic charge of the target; and \( F(q) \) is a term which depends on hadronic or nuclear form factors. In principle, one can separate the effects of \( F(q) \) from those of \( Q_w \) by exploiting the kinematic dependence of the former. The goal of the present strange-quark program is to determined the contribution made by strange quarks to \( F(q) \).

It is interesting to compare present and prospective determinations of \( Q_w \) with those of other low-energy electroweak observables. In Table I I list several of interest.

| Observable | Quantity | Present Value | Source |
|------------|----------|---------------|--------|
| Weak Charge | \( (Q_{wX}^E - Q_{wM}^S)/Q_{wM}^M \) | \(-0.016 \pm 0.0038(E) \pm 0.005(T)\) | Cesium APV |
| | \( \frac{N_+ - N_-}{N_+ + N_-} \) | \( \pm 0.07(E) \pm 0.03(T) \) | PV \( \vec{e}e \) |
| | \( \pm 0.03(E) \pm 0.03(T) \) | PV \( \vec{e}p \) |
| Isotope Ratios | \( (R_{EX}^E - R_{SM}^S)/R_{SM}^M \) | \( ? \pm 0.001(E) \pm 0.004(T) \) | APV on Ba,Yb |
| CKM Matrix | \( |V_{ud}|^2_{EX} - |V_{ud}|^2_{SM} \) | \(-0.0028 \pm 0.0013 \) | 0\(^+\) \( \rightarrow \) 0\(^+\) \( \beta \)-decay |
| | \( ? \pm 250 \times 10^{-11} \) | BNL E821 |
| Muon M.M. | \( \kappa^E - \kappa^M \) | \( (750 \pm 733) \times 10^{-11} \) | Present |
| EDM | \( |d| \) | \( \leq 4 \times 10^{-28} \text{ e} - \text{ cm} \) | electron |
| | | \( \leq 0.97 \times 10^{-25} \text{ e} - \text{ cm} \) | neutron |
| | | \( \leq 9 \times 10^{-28} \text{ e} - \text{ cm} \) | \(^{199}\text{Hg}\) |

The top line in Table I gives the present limits on the agreement of the cesium weak charge with the SM prediction. The Boulder group finds 2.5\( \sigma \) deviation.
(about 1.5%) from the SM value. The following rows give the expected precision on the weak charge of the electron expected in the SLAC Möller experiment and the weak charge of the proton in a prospective Jefferson Lab experiment. It is worth noting that the electron and proton weak charges are suppressed at tree level by $(1 - 4\sin^2 \theta_w) \approx 0.1$; the electron weak charge is further suppressed by SM radiative corrections [17]. Crudely speaking, then, a 10% determination of the proton or electron weak charge is equivalent to a 1% determination of the weak charge of the cesium atom. The fourth line gives the expected precision for the isotope ratio measurements at Berkeley and Seattle. Note that the prospective experimental error is much smaller than the present theoretical uncertainty – a point I address at the end of this discussion.

The other entries in Table I include the anomalous magnetic moment of the muon, the permanent electric dipole moments (EDM’s) of the electron, neutron, and mercury atom; and the square of the $u - d$ matrix element of the CKM matrix. Thus far, one has no evidence of a non-vanishing permanent EDM or of a muon anomalous moment which differs from the SM prediction. In the case of $|V_{ud}|^2$, however, an average over the results of nine superallowed, Fermi nuclear $\beta$-decays yields a deviation from the requirements of CKM unitarity at the 2.2$\sigma$ level (about 0.3%) [18,19]. It is intriguing that both semi-leptonic observables – $Q_W$ from cesium APV and $|V_{ud}|^2$ from superallowed $\beta$-decay – have the same relative sign for the experimental deviation from the SM prediction. If this discrepancy is due to new physics, this common sign may point to a common new physics scenario, as I discuss below.

### IV PV AND NEW PHYSICS

Before considering specific scenarios for physics beyond the SM, it is useful to consider the generic sensitivity of PV observable to such scenarios. In doing so, I follow the discussion of Ref. [13] restrict my attention to those scenarios which generate new effective, four-fermion interactions. Specifically, I write the PV fermion-fermion interaction as

$$\mathcal{L} = \mathcal{L}_{S.M.}^{PV} + \mathcal{L}_{NEW}^{PV},$$

where

$$\mathcal{L}_{S.M.}^{PV} = \frac{G_F}{2\sqrt{2}} g_A e \gamma_\mu \gamma_5 e \sum_f g_V^f \bar{f} \gamma^\mu f$$

(6)

(5)

gives the SM contribution and

$$\mathcal{L}_{NEW}^{PV} = \frac{4\pi\kappa^2}{\Lambda^2} e \gamma_\mu \gamma_5 e \sum_f h_V^f \bar{f} \gamma^\mu f$$

(7)
is the contribution from some new physics. Here, $g_A^e$ axial vector electron-$Z^0$ coupling and $g_f^V$ is the vector current coupling of the $Z^0$ to fermion $f$. In Eq. (7), $\Lambda$ denotes the mass scale associated with the new physics and $\kappa^2$ parameterizes the overall strength of the interaction. The $h_f^V$ give the scenario-specific couplings of the electron axial vector current to the vector current of fermion $f$. If the SM interaction in Eq. (6) determines the SM value of $Q_w$, the fractional shift induced by the new interaction in Eq. (7) is

$$\frac{\Delta Q_w}{Q_w(\text{SM})} = \frac{8\sqrt{2\pi}}{\Lambda^2 G_F},$$

(8)

assuming $g_A^e, g_f^V$ and $h_f^V$ have commensurate magnitudes. If an experiment is sensitive to shifts on the order of $\Delta Q_w/Q_w(\text{SM}) \sim 0.01$, then Eq. (8) implies one is probing new physics at the $\Lambda \sim 20\kappa$ TeV scale. For new physics of a strong-interaction character, one expects $\kappa^2 \sim 1$, while for new gauge interactions one expects $\kappa^2 \sim \alpha$. In either case, high-precision PV measurements are incredibly powerful probes of physics at the TeV scale.

It is instructive to consider how these general features apply in the case of specific new physics scenarios. One of the most interesting such scenarios is that of extended gauge symmetry. The basic of extended gauge symmetry is that the SM group structure is embedded in some larger group $G$. The full symmetry of $G$ may break down spontaneously at one or more scales $M_X$ above the weak scale, leaving the SU(3)$_C \times$SU(2)$_L \times$U(1)$_Y$ symmetry of the SM intact at $M_{W,E,A,K}$. In principle, the gauge bosons associated with the additional symmetries of $G$ will acquire masses commensurate with the symmetry breaking scales $M_X$. If one of these scales is not too much larger than $M_{W,E,A,K}$, then the additional massive gauge bosons could generate small effects in low-energy processes.

In addition to its phenomenological implications, extended gauge symmetry can provide resolution to some of the rough edges of the SM. For example, if $G$ contains an SU(2)$_R$ subgroup, then one has a natural explanation for PV at low-energies. At some high scale, one has exact parity symmetry. However, if the scale of symmetry breaking associated with the right-handed sector is much larger than $M_{W,E,A,K}$, the right-handed gauge bosons will be too heavy to compete effectively with the SM gauge bosons, so that low-energy processes favor the left-handed sector. Similarly, the electromagnetic charge can appear as a generator of $G$, in which case its quantization is natural. Even the apparent lack of SM coupling unification can be resolved by extended gauge symmetry. The presence of additional symmetry breaking scales implies that the running of the couplings will change as one crosses each scale. Thus, there exists sufficient room within different extended gauge group scenarios to bring about coupling unification near the expected grand unified scale.

Here, I concentrate on the neutral current phenomenology of extended gauge symmetry. Specifically, I consider a scenario in which spontaneous symmetry breaking of $G$ yields a second neutral gauge boson $Z'$ with mass not too different from the weak scale. To make life simple, I also consider the case in which this $Z'$ does
not mix with the SM $Z^0$. If it did mix, its effects would show up strongly in the $Z$-pole observables. In fact, the latter severely constrain the mass of a $Z'$ that does mix with the $Z^0$ [20]. In the language of Eq. (7), we have for this scenario $\kappa^2 = \alpha'$, the fine-structure constant associated with the $Z'$ interaction; $\Lambda = M_{Z'}$; and the $h_V^I$ to be specified by a particular scenario.

Given the experimental precisions listed in Table I, how sensitive would the different measurements be to extended gauge symmetry-induced new interactions? A detailed summary is given in Ref. [13]. Here, I quote a few illustrative examples. Extended gauge symmetry scenarios which fit naturally into the framework of heterotic strings live in a group called $E_6$. The factors of $E_6$ include two $U(1)$ groups called $U(1)_\chi$ and $U(1)_\psi$. The neutral gauge boson associated with the $U(1)_\chi$ would show up particularly strongly in low-energy PV if it had a sufficiently low mass; the $Z_\psi$, on the other hand, does not contribute to PV amplitudes at tree-level. Let $G_\chi$ denotes the Fermi constant associated with the interactions of the $Z_\chi$. We may characterize the sensitivity of various PV observable in terms of the ratio $r_\chi = G_\chi/G_F$. The present cesium APV is able to discern effects of the scale $r_\chi \sim 0.003$ or larger. The sensitivities of the SLAC Möller experiment, the proposed Jefferson Lab PV $ep$ experiment, and the isotope ratio measurements are comparable. We can turn this statement about Fermi constants into mass limits by assuming the breakdown of $E_6$ to the SM $\times U(1)_\chi$ occurs in one step, so that the coupling associated with the new $U(1)$ group is maximal. In this case, the cesium APV, isotope ratio, and PVES measurements would probe $M_{Z_\chi}$ at about the one TeV level or better. In contrast, the sensitivity of the cesium measurement to a neutral right-handed gauge boson vastly exceeds the corresponding sensitivities of the isotope ratio and PVES measurements. Thus, the use of different low-energy PV measurements could prove useful in sorting out among competing scenarios.

It is also interesting to compare the sensitivities of low-energy PV and high-energy collider experiments. In terms of mass limits, the “reach” of the present and prospective PV experiments exceeds that of the Tevatron by almost a factor of two. Even an up-graded Tevatron (Tev33) would only achieve comparable sensitivities. One must wait until the LHC has taken sufficient data before the PV sensitivities will be surpassed. In fact, the information provided by colliders and the PV measurements is complementary. The colliders are primarily sensitive to the mass scale associated with the new gauge boson relatively insensitive to the coupling strength $g'$ or detailed structure of the fermion-$Z'$ coupling. The PV observables, in contrast, depend on $(g'/M_{Z'})^2 (\kappa/\Lambda)$ in the language of Eq. (7) and on the effective couplings fermion-$Z'$ couplings ($h_V^I$ in Eq. (7)).

To illustrate, I again consider $E_6$ theories [21]. The phenomenology of neutral $E_6$ gauge bosons is essentially governed by three parameters: $M_{Z'}$; a parameter $\lambda_g$ which governs the overall coupling strength $g'$ and whose value depends on the number of symmetry breaking steps leading to a massive $Z'$; and an “extended” weak mixing angle $\phi$ which describes the structure of the additional “low-energy” $U(1)$ group. Specifically, if $Z_\chi$ and $Z_\psi$ are the gauge bosons associated with the $U(1)_\chi$ and $U(1)_\psi$ groups, respectively, then a general neutral $E_6$ gauge boson can
be written as

\[ Z' = \cos \phi Z\psi + \sin \phi Z\chi \quad . \]  

(9)

The couplings \( h_{V}^{d} \) of this \( Z' \) to electrons and light quarks are given by

\[
\begin{align*}
    h_{V}^{u} & = 0 \\
    h_{V}^{d} & = -h_{V}^{e} = \left[ \sin^{2} \phi - \sqrt{15} \sin \phi \cos \phi / 3 \right] / 20 .
\end{align*}
\]

(10) (11)

Note that for \( \phi = 0 \) or \( \pi \), \( Z' = Z\psi \) and all of the PV couplings vanish. The d-quark and electron couplings also vanish for \( \phi = \phi_{c} = \tan^{-1}(\sqrt{5}/3) \) and have opposite signs for \( \phi \) on either side of \( \phi_{c} \). Thus, the net effect of the \( Z' \) on \( Q_{W} \) can be either positive or negative, depending on the value of \( \phi \). The present cesium APV results favor \( \phi > \phi_{c} \), if an \( E_{6} \) gauge boson is responsible for the observed deviation from the SM value for \( Q_{W} \). This kind of information about the structure of the extended gauge sector is difficult to obtain from high-energy collider limits.

It is also amusing to combine information obtained from colliders and low-energy experiments. To do so, let’s assume the \( E_{6} \) gauge boson is responsible for the deviation of the cesium \( Q_{W} \) from the SM value (about a two \( \sigma \) effect). Under this assumption, one has a relationship between \( M_{Z'} \), \( \lambda_{g} \), and \( \phi \). A second condition derives from the CDF lower bounds, which are roughly 600 GeV with little dependence on the value of \( \phi \). Combining the two pieces of information, one obtains

\[
600 \begin{array}{c} \text{GeV} \\ \sim \end{array} M_{Z'} \begin{array}{c} \sim \\ \lesssim \end{array} 1.15\lambda_{g} \begin{array}{c} \text{TeV} \end{array} ,
\]

(12)

where \( \lambda_{g} \leq 1 \). This range is already rather narrow. If a future up-graded Tevatron found no evidence for extra neutral gauge bosons with a mass less than about one TeV, then a low-mass \( Z' \) would be ruled out as the culprit behind the cesium APV result.

Another popular extension of the Standard Model is supersymmetry. The literature on SUSY extensions of the SM is legion, so I will not discuss SUSY models in detail. The appeal of SUSY includes its solution to the hierarchy problem associated with mass renormalization. In addition, the gauge couplings in the minimal supersymmetric standard model (MSSM) unify at the GUT scale when run perturbatively up from the weak scale. Whether this coupling unification is fortuitous or reflects deeper physics can be debated. It is, nevertheless, intriguing. One important characteristic of the MSSM as far as low-energy phenomenology is concerned involves a quantity called R-parity. The R-parity quantum number is defined as

\[ P_{R} = (-1)^{3(B-L)+2S} , \]

(13)

where \( B, L, \) and \( S \) denote the baryon number, lepton number, and spin, respectively, of a given particle. Every SM particle has \( P_{R} = 1 \) while each superpartner has \( P_{R} = -1 \). The MSSM conserves total \( P_{R} \), which implies that every interaction involves an even number of superpartners. As a result, superpartners cannot
appear in low-energy processes involving SM particles at tree-level. They only contribute through loops. Their effects are correspondingly suppressed by loop factors, making them hard to see at low-energies.

It is possible, however, to write down simple extensions of the MSSM in which $P_R$ is not conserved. In such $B$ and/or $L$-violating theories, superpartner effects can appear at tree-level. To illustrate, consider a purely leptonic R parity-violating SUSY model. The relevant Lagrangian is [22]

$$L_{RPV} = \lambda_{ijk} (\tilde{e}_R^k)^* (\bar{\nu}_L^j) c \ e_L^i + \text{h.c.} , \quad (14)$$

where $\tilde{e}_R^k$ denotes the bosonic superpartner of a right-handed charged lepton of generation $k$ (the other superscripts denote generation). Since the interaction contains three leptons, $L$ (and $P_R$) are not conserved. Tree-level exchange of the $\tilde{e}_R^k$ between lepton currents can generate new four-fermion effective interactions, such as the following interaction relevant to $\mu$-decay:

$$L_{eff} = -(\lambda_{12k}/\sqrt{2}M_{\phi_{kr}})^2 \bar{e}_L^i \gamma_\alpha \nu_L^j \bar{\nu}_L^\mu \gamma^\alpha \mu_L . \quad (15)$$

The interaction of Eq. (14) may provide a partial explanation for both the cesium APV result and the apparent CKM unitarity violation inferred from the superallowed $\beta$-decays. The reason has to do with the Fermi constant. Both the $\beta$-decay amplitude and the PV amplitude of Eq. (6) are written in terms of the Fermi constant. The reason is that these amplitudes depend on $g^2/M_W^2$, which can be related to the Fermi constant as measured in $\mu$-decay. At tree-level, this relationship is given by

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} . \quad (16)$$

Because of the precision with which $\mu$-decay is measured, Eq. (16) must be modified to account for electroweak radiative corrections:

$$\frac{g^2}{8M_W^2}(1 + \Delta r) = \frac{G\mu}{\sqrt{2}} , \quad (17)$$

where $\Delta r$ contains the radiative corrections. Suppose now some new physics, such as the interaction of Eq. (15), contributes to $\mu$-decay. Then one must further modify Eq. (17) as

$$\frac{g^2}{8M_W^2}(1 + \Delta r + \Delta^{NEW}_\mu) = \frac{G\mu}{\sqrt{2}} , \quad (18)$$

where $\Delta^{NEW}_\mu$ gives the corrections from the new interaction. When writing down the amplitude for $\beta$-decay or PV, one needs $g^2/M_W^2$ in terms of $G\mu$:

$$\frac{g^2}{8M_W^2} = \frac{G\mu}{\sqrt{2}}(1 - \Delta r - \Delta^{NEW}_\mu) \quad (19)$$
to first order in the small corrections.

To make contact with the semi-leptonic observables, it is useful to consider the effective Fermi constants $G_{F}^{\beta}$ and $G_{F}^{p\nu}$ which govern them. In terms of other quantities, these effective Fermi constants are

$$G_{F}^{\beta} = G_{\mu}(1 - \Delta r + \Delta r_{\beta} - \Delta_{\mu}^{\text{NEW}} + \Delta_{\beta}^{\text{NEW}})|V_{ud}|^{2}$$  \hspace{1cm} (20)

$$G_{F}^{p\nu} = G_{\mu}(1 - \Delta r + \Delta r_{p\nu} - \Delta_{\mu}^{\text{NEW}} + \Delta_{p\nu}^{\text{NEW}})Q_{W}$$ \hspace{1cm} (21)

where $\Delta r_{\beta}$ and $\Delta r_{p\nu}$ denote SM radiative corrections to the $\beta$-decay and PV amplitudes, respectively, and $\Delta_{\beta}^{\text{NEW}}$ and $\Delta_{p\nu}^{\text{NEW}}$ are the corresponding contributions from new interactions.

The results of from the superallowed decays and cesium APV imply

$$G_{F}^{\beta,EX}/G_{F}^{\beta,SM} < 1$$ \hspace{1cm} (22)

$$G_{F}^{p\nu,EX}/G_{F}^{p\nu,SM} < 1$$ \hspace{1cm} (23)

where the $EX$ and $SM$ superscripts denote the experimental and SM values, respectively. From Eq. (20), we see that if the new physics contributions vanish, one obtains the conventional interpretation of the experimental results:

$$|V_{ud}|^{2}_{EX}/|V_{ud}|^{2}_{SM} < 1$$ \hspace{1cm} (24)

$$Q_{W}^{EX}/Q_{W}^{SM} < 1$$ \hspace{1cm} (25)

However, an equally acceptable explanation is to assume $|V_{ud}|^{2}$ and $Q_{W}$ assume their SM values and that

$$\Delta_{\beta}^{\text{NEW}} - \Delta_{\mu}^{\text{NEW}} < 1$$ \hspace{1cm} (26)

$$\Delta_{p\nu}^{\text{NEW}} - \Delta_{\mu}^{\text{NEW}} < 1$$ \hspace{1cm} (27)

In particular, if both $\Delta_{\beta}^{\text{NEW}}$ and $\Delta_{p\nu}^{\text{NEW}}$ vanish and if $\Delta_{\mu}^{\text{NEW}} > 0$, the measured effective Fermi constants in $\beta$-decay and cesium APV would be smaller in magnitude than the SM predictions.

The R parity-violating interaction of Eq. (15) generates just such a positive value for $\Delta_{\mu}^{\text{NEW}}$:

$$\Delta_{\mu}^{\text{NEW}} = \frac{\lambda_{12k}^{2}}{4\sqrt{2}G_{\mu}M_{\phi_{kR}}^{2}}$$ \hspace{1cm} (28)

Using the present experimental results and Eq. (20) one obtains

$$\lambda_{12k} = (0.027 \pm 0.007)(M_{\phi_{k}}/100 \text{ GeV})$$ \hspace{1cm} (29)

from superallowed decays and

$$\lambda_{12k} = (0.13 \pm 0.05)(M_{\phi_{k}}/100 \text{ GeV})$$ \hspace{1cm} (30)
from cesium APV. Although these results differ by more than one \( \sigma \), one should keep in mind that the cesium result is the first PV result to differ from the SM, whereas the superallowed results depend on an average of \( ft \) values for nine different decays, several of which have been measured more than once. In short, the precise magnitude of the deviation leading to Eq. (30) may not be as robust as that observed in \( \beta \)-decay. The primary point here is that the magnitudes of the results in Eqs. (29-30) are not too distinct, and the signs of the observed deviations are both consistent with the R parity-violating effects in Eqs. (15) and (28). It will be interesting to see whether future electron PV experiments also produce deviations from the SM predictions consistent with this SUSY scenario\(^1\).

V INTERPRETATION ISSUES AND NEUTRON DISTRIBUTIONS

In general, the interpretation of precision, low-energy measurements raises thorny issues not relevant to high-energy measurements. The PV processes discussed here are no exception. To illustrate, I consider the interpretation of atomic PV. As noted above, the dominant error in the cesium weak charge comes from atomic theory. Although this theory error appears to have been reduced in light of new measurements of parity-conserving atomic transitions, it is questionable whether further reductions can be achieved. A clever strategy for evading this atomic structure uncertainty is to measure ratios of APV observables along an isotope chain. A representative ratio is

\[
R = \frac{A_{NSID}^{PV}(N') - A_{NSID}^{PV}(N)}{A_{NSID}^{PV}(N') + A_{NSID}^{PV}(N)},
\]

where \( A_{NSID}^{PV}(N) \) is an APV nuclear spin-independent observable for an atom with neutron number \( N \). Since the atomic electronic structure contributions \( A_{PV}^{NSID}(N) \) and \( A_{PV}^{NSID}(N') \) are relatively constant (for a given \( Z \)), the atomic structure-dependence drops out of the ratio \( R \) and one has

\[
R \approx \frac{Q_W(N') - Q_W(N)}{Q_W(N) + Q_W(N')} \equiv R_{SM}(1 + \delta_R),
\]

where \( R_{SM} \) is the value of the ratio in the SM.

The correction \( \delta_R \) contains contributions from possible new physics. As first pointed out by Fortson, Wilets, and Pang, however, there is also a second effect due to the variation of the neutron density \( \rho_n(r) \) along the isotope chain [24]. To get an idea of the relative importance of these two contributions, one can model

\(^1\) Another constraint on R parity-violating SUSY may be obtained from relations among electroweak parameters. The constraints imposed by these relations on some types of new physics have been analyzed in Ref. [23]. The corresponding SUSY constraints will be discussed in a forthcoming publication.
the nucleus as a sphere of constant neutron and proton density out to radii $R_N$ and $R_P$, respectively. In this case, one has

$$\delta_{R} \approx \left(\frac{2Z}{N + N'}\right) \Delta Q_{W}^P - \left(\frac{N'}{\Delta N}\right) (3/7) (Z\alpha)^2 \delta(\Delta X_N)$$

(33)

where $\Delta Q_{W}^P$ is the shift in the proton’s weak charge due to new physics,

$$\Delta X_N = \frac{R_{N'} - R_N}{R_P}$$

(34)

is the shift in the mean square neutron radius (relative to the proton radius) along the isotope chain, and $\delta(\Delta X_N)$ is the uncertainty in this shift.

Several features of Eq. (33) are worth noting. First, the shift in the ratio $\mathcal{R}$ due to new physics depends primarily on the shift in the weak charge of the proton. The shift in the weak charge of the neutron largely cancels out of the ratio, to first order in small shifts. Whereas the weak charge of a single isotope is slightly more sensitive $\Delta Q_{N}^W$ than to $Q_{W}^P$, the sensitivity of $\mathcal{R}$ to new physics is dominated by $\Delta Q_{W}^P$. Second, the dependence of $\mathcal{R}$ on variations in neutron radii along the isotope shift is enhanced by a factor of $N'/\Delta N$. For a heavy atom like cesium or barium, for example, this enhancement factor can be on the order of 5. Thus, if one is going to use APV isotope ratio measurements to learn about $\Delta Q_{W}^P$, one must have extremely precise knowledge of the shift in neutron radii.

At present, there exist no high-precision experimental determinations of the neutron radii of heavy nuclei. Consequently, nuclear theory must be used to determine the second term on the RHS of Eq. (33). To set the scale of the level of accuracy nuclear theory must achieve to make the isotope ratio measurements useful, supposed we require the uncertainty in the neutron radius term to be as small as the prospective experimental uncertainty in the value of $\mathcal{R}$, namely, 0.1%. Pollock [25] and Chen and Vogel [26,27] have analyzed the nuclear model spread in $\Delta X_N$; from their analyses, we learn that nuclear theory is at least a factor of two away from achieving the requisite precision (for a summary of the theoretical situation, see Ref. [13]). In principle, this presents a stumbling block for the isotope ratio program.

There exist two strategies for overcoming this difficulty. One is to perform a direct measurement of $\Delta Q_{W}^P$, using PVES from a proton target. From Eq. (4), we may write the proton asymmetry as

$$A_{LR}^{(1)} = a_0 Q^2 \left[ Q_{W}^P + F^p(q) \right] ,$$

(35)

where $Q_{W}^P$ is the proton weak charge. The form factor term $F^p(q)$ vanishes in the forward angle limit. Thus, by going to forward angle kinematics, the $Q_{W}^P$ can be separated from $F^p(q)$. The form factor term is presently under study in the strange quark experiments. Upon completion of the strange quark program, this term should be known with sufficient precision over a large enough kinematic range.
to afford a precise separation of $Q^P_w$ in a future, forward angle measurement. A letter of intent for such a measurement has recently been issued [11]. The proposed measurement would employ a re-configured G0 apparatus in order to reach sufficient forward angle kinematics. It is hoped that this measurement will yield a 3-5% determination of $Q^P_w$. This level of precision would be comparable to a 0.1-0.2% determination of $R$, if the interpretation of the latter were not clouded by $\rho_n(r)$ uncertainties.

A second for getting around the $\rho_n(r)$ problem in Eq. (33) involves measuring the neutron distribution of a heavy nucleus using PVES. It is possible that a sufficiently precise determination of $\rho_n(r)$ on a single isotope would sufficiently constrain nuclear theory that the nuclear model-dependence in the isotope shifts, $\delta(\Delta X_N)$ would be reduced to an acceptable level. The idea for using PVES to determine $\rho_n(r)$ was first suggested by Donnelly, Dubach, and Sick [28]. These authors noted that the $Z^0$ preferentially sees neutrons over protons, since at tree-level in the SM, $Q^P_w = 1 - 4\sin^2 \theta_W \sim 0.1$ whereas $Q^N_w = -1$. Thus, the PV asymmetry for scattering from a heavy nucleus should be quite sensitive to the neutron distribution. To illustrate this idea, consider PVES from a $(J^\pi, T) = (0^+, 0)$ nucleus. The asymmetry has the form [28,14]

$$-\frac{4\sqrt{2}\pi\alpha}{|G_F|^2} A_{LR} = Q^P_w + Q^N_w \frac{\int d^3x \ j_0(qx) \rho_n(x)}{\int d^3x \ j_0(qx) \rho_p(x)}.$$  \hspace{1cm} (36)

Since $\rho_p(x)$ is typically known with very high accuracy, the PV asymmetry essentially becomes a “meter” of $\rho_n(q)$. This idea is being exploited in a proposal before the Jefferson Lab PAC [12].

It goes without saying that a precise determination of $\rho_n(q)$ for any heavy nucleus is of fundamental interest for nuclear structure physics. From this standpoint alone, the investment of effort in making the measurement is well-justified. It remains to be seen, however, whether the information gleaned from a precise determination of $\rho_n(q)$ for $^{208}$Pb at one or two kinematic points will suffice to reduce the nuclear structure uncertainty in Eq. (33). For example, it is unlikely that lead atoms will be used in the APV isotope ratios. The isotopes of Ba and Yb are currently under study in Seattle and Berkeley. Moreover, the interpretation of $R$ requires knowledge of $\rho_n(r)$ in more detail than implied by the simplified expression in Eq. (33). Whether knowledge of the momentum-space distribution at a few points will supply the necessary details about $\rho_n(r)$ is an open question. Finally, the constraints which knowledge of $\rho_n(r)$ for a single isotope would place on calculations of isotope shifts has yet to be quantified. In short, there exist several challenges for nuclear theory in making a PVES determination of $\rho_n(q)$ useful for the APV isotope ratios (for a recent discussion of these issues, see Ref. [29]). From this standpoint, a measurement of the PV $\vec{e}p$ asymmetry provides a cleaner and more direct window on $\Delta Q^P_w$. 
VI CONCLUSIONS

The field of parity-violation with electrons has made tremendous strides in 25 years. I hope this discussion has convinced the reader that its future prospects are just as exciting as its history. For the next decade at least, it is likely that PV with electrons will provide one of the most powerful probes of new physics at the TeV scale, complementing information to be gained from high-energy collider experiments. At the same time, it will remain a focal point for interdisciplinary activity, bringing together insights from particle, nuclear, and atomic physics. One may only speculate as to the new insights PV with electrons will provide for each field by the time a Bates-35 celebration is planned.

ACKNOWLEDGEMENTS

It is a pleasure to thank W.J. Marciano, D. Budker, R. Carlini, J.M. Finn, E.N. Fortson, S.J. Pollock, and P. Souder for useful discussions and S.J. Puglia for assistance in preparing the manuscript. This work was supported in part under U.S. Department of Energy contract #DE-AC05-84ER40150 and a National Science Foundation Young Investigator Award.
REFERENCES

1. M.A. Bouchiat and C. Bouchiat, Phys. Lett. B48, 111 (1974); J. Phys. (Paris) 35, 899 (1974).
2. D. Budker, “Parity Nonconservation in Atoms”, to appear in proceedings of WEIN-98, C. Hoffman and D. Herzceg, Eds., World Scientific, Singapore, 1998.
3. C.S. Wood et al., Science 275 (1997) 1759; S.C. Bennett and C.E. Wieman, Phys. Rev. Lett. 82, 2484 (1999).
4. C.Y. Prescott et al., Phys. Lett. B77 (1978) 347; Phys. Lett. B84 (1979) 524.
5. W. Heil et al., Nucl. Phys. B327 (1989) 1.
6. P.A. Souder et al., Phys. Rev. Lett. 65 (1990) 694.
7. MIT-Bates experiment 89-06 (1989), R.D. McKeown and D.H. Beck spokespersons; MIT-Bates experiment 94-11 (1994), M. Pitt and E.J. Beise, spokespersons; Jefferson Lab experiment E-91-017 (1991), D.H. Beck, spokesperson; Jefferson Lab experiment E-91-004 (1991), E.J. Beise, spokesperson; Jefferson Lab experiment E-91-010 (1991), M. Finn and P.A. Souder, spokespersons; Mainz experiment A4/1-93 (1993), D. von Harrach, spokesperson.
8. B. Mueller et al., SAMPLE Collaboration, Phys. Rev. Lett. 78 (1997) 3824; D.T. Spayde et al., SAMPLE Collaboration, [nucl-ex/9909010].
9. K.A. Aniol et al., HAPPEX Collaboration, Phys. Rev. Lett. 82 (1999) 1096, [nucl-ex/9810012].
10. SLAC Proposal E158 (1997), K. Kumar, spokesperson.
11. R. Carlini, J.M. Finn, and M.J. Ramsey-Musolf, Letter of Intent to the Jefferson Laboratory PAC, unpublished (1999).
12. Proposal to the Jefferson Laboratory PAC, R. Michels and P.A. Sounder, spokespersons (1998).
13. M.J. Ramsey-Musolf, Phys. Rev. C60, 015501 (1999).
14. M.J. Musolf et al., Phys. Rep. 239 (1994) 1.
15. R.N. Mohapatra, Unification and Supersymmetry, Springer-Verlag, New York, 1992.
16. M.J. Musolf and B.R. Holstein, Phys. Rev. D 43, 2956 (1991).
17. A. Czarnecki and W.J. Marciano, Phys. Rev. D53 (1996) 1066.
18. I.S. Towner and J.C. Hardy, “Currents and their Couplings in the Weak Sector of the Standard Model”, in Symmetries and Fundamental Interactions in Nuclei, W.C. Haxton and E.M. Henley, Eds., World Scientific, Singapore, 1995, p. 183.
19. E. Hagberg et al., in Non-Nucleonic Degrees of Freedom Detected in the Nucleus, T. Minamisono et al., Eds., World Scientific, Singapore, 1996, Singapore.
20. P. Langacker, “Tests of the Standard Model and Searches for New Physics”, in Precision Tests of the Standard Electroweak Model, P. Langacker, Ed., World Scientific, Singapore, 1995, p.883.
21. D. London and J.L. Rosner, Phys. Rev. D34 (1986) 1530.
22. V. Barger, G.F. Guidice, T. Han, Phys. Rev. D40 (1989) 2987.
23. W.J. Marciano, Phys. Rev. D60, 093006 (1999).
24. E.N. Fortson, Y. Pang, L. Wilets, Phys. Rev. Lett. 65 (1990) 2857.
25. S.J. Pollock, E.N. Fortson, L. Wilets, Phys. Rev. C46 (1992) 2587.
26. B.Q. Chen and P. Vogel, Phys. Rev. C48 (1993) 1392.
27. P. Vogel, “Atomic Parity Non-conservation and Nuclear Structure” in *Nuclear Shapes and Nuclear Structure*, M. Vergnes, D. Goute, P.H. Heenen, and J. Sauvage, Eds., Edition Frontieres, 1994, Gif-sur-Yvette.

28. T.W. Donnelly, J. Duback, I. Sick, Nucl. Phys. A503 (1989) 589.

29. C.J. Horowitz, S.J. Pollock, P.A. Souder, and R. Michaels [nucl-th/9912038] (1999).