The influence of the angles of the asymmetrical gear rack on the geometrical and functional parameters for the pinion and gear with asymmetric teeth

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Abstract. The paper presents some research on the design of the asymmetric gears, special gears with involute asymmetrical teeth. The aim of these researches is to establish the influence of the angles of the asymmetrical gear rack upon some geometrical and functional parameters of the gears. We have used, as design method, the direct gear design, followed by the gear rack design, considering that asymmetric gears are not conventional gears, so the design method it is not the standard one. The results that are presented was obtained by using a computer modelling program, for asymmetric gears, in MatLab, and for one can see the asymmetric gears and the zone of the tooth action, a computer program in AutoLISP is developed. Based on the computer applications developed for determinate the geometrical parameters and for representing the teeth and gears models, the asymmetric teeth pinion and gear can be manufactured. Between the design parameters established at starting the described process there are the angles of the asymmetrical gear rack.

1. Introduction

Involute gearing, with involute curves as teeth profiles, is the most common because it has a lot of advantages as related to other types of gearing.

The opposite flanks of an involute tooth function in a different way, for the majority of gearings. The load on the active flank is significantly larger and is applied over longer periods of time than on the inactive flank. These aspects led to the idea of using gears called asymmetric gears [3], for which in order to build the tooth profile two involutes are drawn from two different base circles, involutes that are at a certain distance one from another, in order to create the tooth body, which this time is no longer symmetrical as related to the radius that crosses the intersection point of the two involutes, radius which for the common teething was the tooth symmetry axis. Thus, an asymmetrical involute flanks tooth results with supplementary advantages.

Designing gears with asymmetrical teeth, is aimed at improving the performance of active flanks, by reducing the performance of inactive flanks, taking into consideration the fact that these are not loaded or rarely loaded and usually for short periods of time. Improving the performances can mean an increase in the loading capacity and weight, noise and vibration reduction.
The degree of asymmetry, determined by the two different pressure angles \( \alpha_a, \alpha_i \), on the active and inactive profile:

\[
k = \frac{\cos \alpha_i}{\cos \alpha_a}
\]  

(1)

can be chosen in accordance with the requirements of the particular application for which the gearing is designed.

The conventional gear design is generally based on the standard tools, in these cases dimensions are determined with the parameters of standard gear rack. This makes gearing quite simple, economical, reducing the costs with tooling and design, but in the same time it is known that standard design can’t obtain the optimum performance.

For some particular application, in many industries, it is necessary to reduce the noise, the vibrations, the weight and dimensions, by developing design method for more compact and higher capacity gear pairs. The direct asymmetrical gear design, used in the computer program, a part of which results are presented in this paper, is one of these methods.

The computer program, in MatLab, developed for determining the geometrical and functional parameters of asymmetric gears, makes possible to study the performance of many gear pairs with different coefficient of asymmetry and to establish for the initial data, the number of teeth and centre distance, which is the best one.

The program can also be used for analysing the performance of asymmetrical gears in relation with the symmetrical gears, considering those as asymmetrical gears with the coefficient of asymmetry equal to the unit. Thus, one can see how the computer design makes production of nonstandard gears as easy as production of standard ones.

2. **Asymmetrical generating gear rack**

Considering, in figure 1, that one of the gears of an asymmetrical gear pair, for example gear 2 with center \( O_2 \), has the base circles infinitely large, its active and inactive profiles will degenerate into lines tangential to the active and inactive gear 1 profiles respectively, the gear 2 rolling circle will also have an infinite radius and becomes a line tangential to the gear 1 rolling circle. Gear 2 thus obtained is a gear rack with asymmetrical flanks.

While generating the symmetrical teeth the opposite profiles are in pairs symmetrical to a line perpendicular on the reference line.

While generating asymmetrical teeth, the opposite profiles generating the active and inactive flanks respectively, are no longer symmetrical, having different angles in relation to the gear rack’s reference line [5]. The pressure angles for the active and inactive flanks at generation of the gear with asymmetrical rack are the generating rack angles \( \alpha_{ac} \) and \( \alpha_{ic} \), the angles of the cutting edges in relation with the perpendicular to the reference line of the rack.

![Figure 1. Generating with the gear rack with asymmetrical flanks](image-url)
If the numbers of teeth, for the gear pair, and the center distance are initial data of the design program, one can determine the sum of the pinion rack shift and gear rack shift, by the generating rack angles with the following relation:

\[
(X_1 + X_2) = \frac{p_c(z_1 + z_2)}{2\pi \cdot (\tan \alpha_{ac} + \tan \alpha_{ac})} \cdot (\text{inv} \alpha_{ac} + \text{inv} \alpha_{ic} - \text{inv} \alpha_{ac} - \text{inv} \alpha_{ic}).
\]

(2)

where \(p_c\) is the generating rack pitch.

Thus, the pinion’s teeth’s thickness and the gear’s one, is determined by the generating rack angle \(\alpha_{ac}\), from which the other \(\alpha_{ic}\) results.

If the generating angle \(\alpha_{ac}\) is equal with the pressure angle \(\alpha_a\), the values of the pinion rack shift and gear rack shift are equal, but with different signs. If the generating rack angles are chosen different from the pressure angles of the gear pair, the rack shifts are different.

In the booth case the shifts are established in the computer program, finding the values corresponding to the condition to avoid the interference, to avoid the undercutting, to ensure the contact ratio greater than one.

For a given centre distance, number of teeth and pressure angles, the sum of the shift rack of the pinion and the shift rack of the gear is determined by the angle of the rack, in one of Matlab [3] developed application. It is necessary to study how this angle influences the geometrical and functional parameters of the gears.

The maximum value for the active profile angle of the rack is equal to the active pressure angle and the minimum value is:

\[
\alpha_{ac\_min} = \text{arccos}(1/k)
\]

(3)

corresponding to \(\cos \alpha_{ac} = 1; \quad \alpha_{ac} = 0\).

The \(\alpha_{ac}\) angle of the rack results from the coefficient of asymmetry:

\[
\alpha_{ic} = \text{arccos}(k \cdot \cos \alpha_{ac})
\]

(4)

The design and modelling program make possible also study the influence of generating the pinion and the gear with different racks, or with the same rack.

Thus, with the computer program, one can obtain from a great number of possibilities of design, for the given initial date, which is the optimum set of design parameters for a particular product [1], [2].

3. The variation of some geometrical and functional parameters of the gears in relation with the asymmetrical rack’s angles

Because the rack’s angle for the active profile is considered as an initial date that one can choose between minimum and maximum values, and the rack’s angle for the inactive profile results in relation with the coefficient of asymmetry, we have obtained the diagrams of variation of gears parameters depending on the gear rack angle \(\alpha_{ac}\), corresponding to the generating profile of the generated gear active profile.

The initial data of the analysed gears are chosen with the following values:

\[ z_1=16, z_2=16, z_2=57, a=120\, \text{mm}, \alpha_a=40^\circ, \alpha_i=20^\circ, P=18\, \text{kW}, n=1000 \, \text{rot/min} \]

The gears can be used with greater pressure angle on the active profile, in which case the coefficient of asymmetry is bigger than one, or with smaller pressure angle on the active profile, in which case the coefficient of asymmetry is smaller than one. We have analysed both of these cases and we have taken into consideration also the generation with one or two racks. The variations of studied parameters, for the coefficient of asymmetry \(k>1\), are represented in the following diagrams:
Figure 2. The variation of the fillet radius $R_1$, $R_2$

Figure 2 presents the variations of radius of the fillet of the racks in relation with $\alpha_{ac}$ angle. One can see that the increase of $\alpha_{ac}$ determines the decrease of both radius $R_1$, $R_2$ of the pinion’s and gear’s racks.

The 2D gears models were obtained with two computer programs in AutoLisp [6], developed using the parametrical equations of the asymmetrical involute profiles and of the two joint profiles at the tooth base, and also an algorithm for determine the dimensions of the gear racks.

The zone of the tooth action represented for different generating angles $\alpha_{ac}$ pairs of gears (figures 3, figure 4), show the variations of the fillet radius.

The zone of the tooth action for an example of designed and manufactured gears, based on the developed applications, is presented in figure 5.

The dimension of the maximum cross section, at the bottom of the gear tooth, increases by increasing the $\alpha_{ac}$ angle and the maximum cross section of the pinion tooth decreases by increasing the $\alpha_{ac}$ angle (figure 6).

Figure 3. The zone of the asymmetric tooth action $\alpha_{ac} = 0.6960$, $R_1 = 0.1138$, $R_2 = 0.6564$

Figure 4. The zone of the asymmetric tooth action $\alpha_{ac} = 0.6640$, $R_1 = 0.2803$, $R_2 = 1.0847$
Figure 5. The zone of asymmetric tooth action for the manufactured gears.

Figure 6. Variation of the dimension of cross section.

Figure 7. The variation of maximum tensile bending stress in relation with $\alpha_{dc}$.

Figure 8. The variation of the maximum bending tensile stress, $k>1$, using the same one rack.
Therefore, the cross-sectional area and the modulus of tooth section will be also significantly increased at the gear and decreased at the pinion. That has a direct influence on the bending stress of the tooth. The dimension of the maximum cross section was determined with a method established using the equation of the joint profiles of asymmetric tooth. The equations of the joint profiles have been obtained considering those profiles as curves generated by the fillet profile of the gear rack. Increasing the $\alpha_{ac}$ angle, the maximum tensile bending stress, at the bottom of the active profile, increases for the pinion and decreases for the gear, in such a manner that the two very different values of the maximum stress, for smaller values of the generating gear rack angle $\alpha_{ac}$, may become equal for greater values for $\alpha_{ac}$ (figure 7). In the case of generating the pinion and the gear with the same one rack, the difference between the maximum bending stress values at the pinion’s tooth and at the gear’s tooth are larger in comparison with the case of generating with two racks, because the dimensions of the rack are calculated on the base of corresponding gear’s parameters (figure 8).

In this paper have been presented diagrams of variations of geometrical and functional parameters for gears having the coefficient of asymmetry $k>1$, with the pressure angle on the active profile bigger than the pressure angle on the inactive profile, but it have been developed also computer applications for analysing the behaviour of the asymmetric gears with coefficient of asymmetry smaller that one.

4. Conclusions
A small generating rack angle $\alpha_{ac}$, chosen as the initial date in the design program for asymmetric gears, ensures greater fillet radius, decreases the contact stress, decreases the maximum tensile bending stress at pinion tooth but increases the maximum tensile stress at the gear’s tooth in comparison to the pinion’s one. If the aim of the design is to obtain equal values for the bending stress at pinion and gear one, there must be used the generating with two different racks and as an initial date, a large $\alpha_{ac}$ angle for the active profile of the rack.

References
[1] Banica M 2004 The Optimization of the Relief Parameters of the Involute Spur Gearing with the View of Decrease the Noise Emission Annals of the Oradea University Fascicle of Management and Technological Engineering Vol. III (XIII) ISSN 1583-0691 6p
[2] Banica M 2006 Optimizarea Dinamicit Angrenajelor (Optimization of Gear Dynamics) Publisher Risoprint Cluj-Napoca ISBN 978-973-751-385-4
[3] Ghinea M and Fireteanu V 1995 MATLAB Calcul numeric - Grafica - Aplicatii ISBN 973-601-275-1
[4] Kapelevich A L 2000 Geometry and design of involute spur gears with asymmetric teeth Mechanism and Machine Theory 35 (Retrived on 1.06.2015 from www.akgears.com)
[5] Kapelevich A L and Kleiss R E 2002 Direct Gear Design for Spur and Helical Gears Gear Technology, September/October 2002 pp.29-35 (Retrived on 1.06.2015 from www.akgears.com)
[6] Tisan V 1997 Graphical representation of gears using the AutoLisp language Proceedings of the MicroCAD 97 Miskole, 26.02.1997