One-shot quantum measurement using a hysteretic DC-SQUID

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We propose a single shot quantum measurement to determine the state of a Josephson charge quantum bit (qubit). The qubit is a Cooper pair box (CPB) and the measuring device is a two junction superconducting quantum interference device (dc-SQUID). This coupled system exhibits a close analogy with a Rydberg atom in a high Q cavity, except that in the present device we benefit from the additional feature of escape from the supercurrent state by macroscopic quantum tunneling, which provides the final read-out. We test the feasibility of our idea against realistic experimental circuit parameters and by analyzing the phase fluctuations of the qubit.

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A Cooper pair box is a controllable macroscopic two level system, which is considered as a potential qubit in the context of quantum computing. Coherent Rabi oscillations have been observed in a CPB, using ultrafast pulses. Recently, in a superconducting device named "quantonium", coherent oscillations were observed using microwave pulses. The observed oscillations lived for almost a microsecond, making superconducting circuits promising for realizing quantum gates.

Despite this progress, quantum measurement on a CPB still remains a challenge. The charge read-out circuits, like the one in [2] or those using single electron tunneling transistors (see, e.g., [3]) are far from ideal due to decoherence induced by the unavoidable background charge fluctuations and by backaction during the measurement, e.g., in form of shot noise. The former problem can be avoided to a large extent if the qubit is operated at the degeneracy point of the two level system: at this optimum point linear coupling to charge fluctuations is absent. In the quantonium experiment, both problems were eliminated successfully by measuring the two quantum states of a Cooper pair transistor (CPT) at the optimum point by a hysteretic Josephson junction. This junction, working in its classical regime, measured the persistent current of the two quantum states of the CPT. This method cannot, however, be used to measure the quantum state of a CPB. Moreover, the measurement did not resolve the quantum state in one shot.

In this letter we propose a new quantum measurement procedure based on the entanglement between two quantum systems: a CPB coupled to a superconducting resonator. The dynamics of this coupled system has been theoretically investigated recently. Here we show that at the degeneracy the two quantum states of the CPB can be resolved in one shot. The read-out device is a current-biased dc-SQUID (the resonator) and it is controlled by adiabatic pulses of flux. Very high sensitivity and fast measurement can be reached with this method. A dc-SQUID is preferred over a Josephson junction because manipulations using external flux do not suffer from time limitations. In a Josephson junction, nanosecond pulses of bias current cannot be applied because low pass filters are necessary to exclude high-frequency noise.

The circuit of a Cooper pair box coupled to a current-biased dc-SQUID was inspired by experiments on a Rydberg atom in a high Q cavity. For a bias current I well below the critical current Ic, the CPB and the SQUID play the roles of the Rydberg atom and the high Q cavity, respectively. For example, Rabi oscillations are predicted to occur as a result of spontaneous emission and re-absorption by the CPB of a single oscillation quantum in the SQUID. The device can be considered as a two level system coupled to a harmonic oscillator. However, for a bias current very close to, but below the critical current, macroscopic quantum tunneling (MQT) in the SQUID can occur. This quantum escape phenomenon has no equivalent in high Q cavity experiments, and it introduces a new element into the dynamics of the system. We will describe how we can benefit from MQT in order to do a very fast one-shot quantum measurement on the CPB. We will discuss the performance of this measurement and its back-action on the CPB.

The superconducting quantum circuit that we consider is shown in Fig. 1. It contains three parts which we describe in detail below: a Cooper pair box, a hysteretic dc-SQUID, and a coupling capacitance between the two. The circuit is connected to the external classical circuits by three different couplings: a resistance R in parallel to a DC current source and voltmeter, a mutual inductance to a source of flux modulating current pulses, and a gate capacitance to a dc and pulse voltage source.

The Cooper pair box consists of a small superconducting island, coupled to a gate-voltage VS by a gate capacitance CG. The island is furthermore connected capacitively to a superconducting electrode via a Josephson junction with a capacitance CJ and Josephson energy Ej. We are interested in the limit of small Joseph-
son energy, \( E_S \ll E_{C,1} \), where \( E_{C,1} = 2e^2/C_{1,\text{eff}} \) is the elementary charging energy of the box with \( C_{1,\text{eff}} = C_1 + [1/C_S + 1/(C_c + C_g)]^{-1} \); \( C_S \) is the SQUID capacitance and \( C_c \) the coupling capacitance. Let us introduce the basis of charge states \( |n(q)| \), where \( n(q) \) corresponds to the number of excess Cooper pairs on the island. If the dimensionless gate charge \( N_g = -C_gV_g/(2e) \) is close to \( 1/2 \), the charge states \(|0(q)\rangle \) and \(|1(q)\rangle \) are almost degenerate and the relevant eigenstates of the CPB are superpositions of these charge states. Specifically, the ground state and the first excited state are, respectively: \(|\beta\rangle = (|0(q)\rangle + |1(q)\rangle)/\sqrt{2} \) and \(|\alpha\rangle = (|0(q)\rangle - |1(q)\rangle)/\sqrt{2} \). The corresponding eigenenergies are \( E_{\beta} = E_{C,1} + \frac{1}{2}E_1 \). We see that the CPB effectively behaves as a quantum-mechanical two-level system.\(^3\)

The hysteretic dc-SQUID consists of a superconducting loop with two underdamped Josephson junctions which both have ideally the same critical current \( I_0 \) and capacitance \( C_S/2 \). We neglect the loop inductance and effects due to asymmetry in the SQUID. The damping of the two junctions is also neglected now, but it will be discussed at the end. With these approximations, the SQUID equation of motion is similar to that of a single Josephson junction, describing a particle of mass \( m = C_{S,\text{eff}}(\Phi_0/2\pi)^2 \) in a tilted washboard potential \( U(\varphi) = E_S[-\varphi I_c - \cos(\varphi)] \), where \( \varphi \) is the phase difference of the SQUID, \( I \) the bias current through the SQUID, \( \Phi_0 = h/2e \) the superconducting flux quantum, \( \Phi_{\text{DC}} \) the external flux, and \( I_c = 2[\cos(\pi\Phi_{\text{DC}}/\Phi_0)]I_0 \), \( E_S = I_c\Phi_0/(2\pi) \) and \( C_{S,\text{eff}} = C_S + [1/C_1 + 1/(C_c + C_g)]^{-1} \) the effective critical current, Josephson energy and capacitance of the SQUID, respectively.

For values of bias current not too far below the critical current, the potential \( U(\varphi) \) can be well approximated by a cubic potential. The quantum dynamics of the SQUID using the reduced momentum and position operators \( \hat{P} = 1/(\sqrt{m}\hbar)p \) and \( \hat{X} = \sqrt{m}\hbar X \), respectively, is then described by \( \hat{H}_0 = \frac{1}{2}\hbar\omega_p(\hat{P}^2 + \hat{X}^2) + \sigma(I)\hbar\omega_p\hat{X}^3, \) where \( \sigma(I) \) is the phase difference, \( \sigma(I) \) its conjugate operator and \( \omega_p = [2\pi I_c/(\Phi_0C_{S,\text{eff}})]^{1/2}2(1 - I/I_c)^{1/4} \) the effective plasma frequency of the SQUID. The parameter \( \sigma(I) = -1/[6a(2(1 - I/I_c))]^{-3/4} \), where \( a = \hbar^{-1/2}[\Phi_0/(2\pi)]^{3/4}(C_{S,\text{eff}}I_c)^{1/4} \) gives the relative magnitude of the cubic term as compared to that of the harmonic oscillator term.

For values of \( I \) below \( I_c \) with \( \sigma(I) \ll 1 \), the potential barrier is high compared to \( \hbar\omega_p \), and the cubic term in \( \hat{H}_0 \) can be neglected. Many low-lying states are found near the minimum of the quadratic potential. The broadening of these states due to tunneling can be ignored. Hence these states are well approximated by harmonic oscillator eigenstates, denoted by \( |0\rangle, |1\rangle, |2\rangle, \ldots \), corresponding to the presence of 0, 1, 2, ... oscillation quanta in the SQUID, respectively. Thus, at low enough bias current, the SQUID behaves as a superconducting quantum resonator. Since in this limit the phase is localized in a well defined minimum of the potential \( U \), the time-averaged voltage \( V_m \) across the SQUID remains zero.

If the bias current is increased such that \( I \lesssim I_c, \sigma(I) \) is no longer negligible and the cubic term affects the SQUID dynamics. The barrier height, given by \( \Delta U = 4\sqrt{2}/3E_S(1 - I/I_c)^{3/2} \), and \( \omega_p \) decrease and vanish at the critical current. The number of localized states in a given well decreases. Moreover, the remaining energy levels broaden due to quantum tunneling from the metastable wells of the potential \( U \). The broadening of the ground state energy for low damping is given by the tunneling rate \( \Gamma_0 = \omega_p\sqrt{6/\pi}\sqrt{\Delta U}/(\hbar\omega_p) \). Since the excited states \(|n\rangle \) are located closer to the top of the barrier, the tunneling rates and hence the broadening of these states increase with increasing energy of the level as \( \Gamma_n \sim (\exp(-36/5))^{n}\Gamma_0 \). Specifically, the tunneling rates from states \(|0\rangle \) and \(|1\rangle \) of the SQUID are different by approximately a factor thousand.\(^{12, 13}\) After a tunneling event, the phase of the SQUID is no longer localized and the time-averaged voltage \( V_m \) across the SQUID is finite.

The magnetic flux through the SQUID affects its critical current \( I_c \), hence \( \omega_p \), \( \sigma(I) \) and therefore \( \hat{H}_0 \) depend on the flux. For small, time-dependent variations of flux, \( \delta\Phi(t) \ll \Phi_0 \), and for SQUID parameters such that \( a \gg 1 \), the total time-dependent Hamiltonian is given by \( \hat{H}(t) = \hat{H}_0 - \lambda(I, \Phi_{\text{DC}})(\delta\Phi(t)/\Phi_0)\hbar\omega_p\hat{X} \), where \( \lambda(I, \Phi_{\text{DC}}) = a[2(1 - I/I_c)]^{-3/8}(1/\pi)\tan(\pi\Phi_{\text{DC}}/\Phi_0) \). Below we consider the effect of a time-dependent perturbation \( \delta\Phi(t) \) on the dynamics of the coupled system.

The coupling capacitance \( C_c \) plays a crucial role in the circuit of Fig. 1 since it couples the qubit and the SQUID to each other. Physically, \( C_c \) couples the charge \( n(q) - N_g \) on the CPB to the charge on the SQUID. The coupling Hamiltonian can be written as \( \hat{H}_c = -iE_{\text{coupl}}(n(q))\hat{P} \). Here we introduced the characteristic coupling energy \( E_{\text{coupl}} = \sqrt{\hbar\omega_p/E_{C,S}E_{C,\text{eff}}/4} \) where \( E_{C,c} = 2e^2/C_{c,\text{eff}} \) and \( E_{C,S} = 2e^2/C_{S,\text{eff}} \) with \( C_{c,\text{eff}} = C_c + C_g + (1/C_1 + 1/C_S)^{-1}[(C_g + C_S)/(C_c + C_g)]/2 \). This coupling energy leads to full entanglement between the states of the CPB and the SQUID at the resonance condition \( E_3 = \hbar\omega_p \).\(^4\)

Having detailed the superconducting quantum circuit, we will now describe the measurement procedure. Suppose that, at time \( t = 0 \), the CPB is in a coherent superposition \( \alpha|\beta\rangle + \beta|\alpha\rangle \), as a result of quantum oper-
FIG. 2: The quantum measurement procedure is illustrated for an initially state $|+\rangle$.

The barrier is therefore significantly decreased and the tunneling rates $\Gamma_0$ and $\Gamma_1$ are drastically increased. During the time $\Delta t$, the SQUID can escape from the well to the non-zero voltage state by tunneling. If $\Delta t$ satisfies $1/\Gamma_1 \ll \Delta t \ll 1/\Gamma_0$, and relaxation between the levels is neglected, the SQUID is in its finite-voltage state if and only if the SQUID was in the state $|1\rangle$ at time $T_0$. In other words, the proposed measurement determines the state of the SQUID in one shot. The escape probability corresponds to the $|\beta|^2$ amplitude of the initial superposition in the CPB. The lack of perfect contrast between the escape rates from the two states and relaxation processes introduce an intrinsic error in the proposed quantum measurement procedure; the influence of this will be estimated later.

At $t = T_0 + \Delta t$, the flux is switched back to its initial value in order to prevent further tunneling. Because the dc-SQUID is hysteretic, the zero-voltage and finite voltage states are stable for sufficiently long a time to perform the read-out. The first two steps of duration $T_0$ and $\Delta t$ perform the quantum measurement. The last step provides the classical read-out measurement.

To check the feasibility of our measurement procedure, we use some typical values for parameters of an aluminum superconducting circuit. For the CPB, we choose $E_3 = 26.2$ μeV, $C_g = 10$ aF and $C_3 = 0.63$ fF, and for the SQUID, $I_0 = 1$ μA, $I_1 = 1.1$ μA, $C_S = 1$ pF and $\Phi_{DC}/\Phi_0 = 0.277$. For $\Phi(t)/\Phi_0 = 0.013$ during the first step, the resonance condition is satisfied with 5 levels in the well; the escape time from the level $|1\rangle$ is much longer than 1 ns. An additional increase of flux $\delta \Phi(t)/\Phi_0$ by the same amount is enough to change $\hbar \omega_p$ by 4 μeV ($\gg E_{coup}$) and the system is driven out of resonance. The escape time of level $|1\rangle$ drops to about 1 ns. Finally we choose $C_c = 0.1$ fF yielding $E_{coup} = 0.21$ μeV. Using the measuring times $T_0 = 4.5$ ns and $\Delta t = 5$ ns, the one-shot quantum measurement can be performed.

We now turn to the influence of relaxation in the SQUID on the measurement procedure. At low temperature the rate $\Gamma_R$ of relaxation between adjacent levels is dominated by interaction with the environment. Assuming $\Gamma_1 >> \Gamma_0, \Gamma_R$, the escape probability at $t = T_0 + \Delta t$ is given in the lowest order by

$$P_{\alpha(0),+}^{\beta(1)} = |\beta|^2 + |\alpha|^2 \left[ 1 - e^{-\Gamma_0 \Delta t} \right]$$

Neglecting the influence of relaxation in Eq. 1, and assuming an infinite contrast between $\Gamma_0$ and $\Gamma_1$, the escape probability gives the $|\beta|^2$ amplitude of the initial superposition in the CPB. The finite contrast between the two states introduces an intrinsic error in the proposed quantum measurement procedure which is about 0.8%. Taking into account the relaxation processes, the escape probability versus the pulse duration $\Delta t$ is plotted in Fig. 3 using the experimental parameters listed above for three different initial states: the two states $|\pm\rangle$ and $|+\rangle$ and the coherent superposition $(|\pm\rangle + |+\rangle)/\sqrt{2}$. The
relaxation rate is defined as $\Gamma_R = \omega_P/Q$ where the Q-factor was chosen to be $Q = 500$. The measurement of the escape during $\Delta t = 5$ ns is a direct measurement of the $|\beta|^2$ amplitude. For $Q = 500$, the total error of the one-shot measurement is less than 4%. The presented values of 0.8% and 4% error are overestimates because the semiclassical approximation of the tunnelling rate slightly underestimates tunnelling from the excited state in particular.

It is important to maintain coherence of the qubit. Specifically, we will discuss the backaction, i.e., dephasing of the qubit, due to the measuring circuit formed by the SQUID and its electrical environment. To analyze this we need to study the amplitude of the fluctuations of the phase across the CPB. The measurement environment consists of a parallel LRC circuit, where $L_S = \sqrt{2} \frac{h}{e} (1 - I/I_c)^{-1/2}$ is the Josephson inductance of the SQUID and $R$ describes the dissipative part of the circuit. It is straightforward to obtain the phase fluctuations by

$$\langle (\Delta \phi)^2 \rangle \approx \frac{2\pi}{R_Q} \sqrt{\frac{L_S}{C_{S,\text{eff}}}} \left( \frac{C_c + C_g}{C_c + C_g + C_b} \right)^2. \quad (2)$$

Here $R_Q \equiv \hbar/4e^2$ is the resistance quantum, and we assumed low temperatures $k_BT \ll \hbar/(L_SC_{S,\text{eff}})$. According to Eq. (2) the inductance, $L_S$, provides a protection against dephasing: the dissipative part of the environment does not affect the value of $\langle (\Delta \phi)^2 \rangle$. Using the typical experimental values given above we obtain $\sqrt{\langle (\Delta \phi)^2 \rangle} \approx 0.02$, which is much smaller than $\pi$, thereby demonstrating the weakness of the residual phase fluctuations. The low temperature condition is verified if $T \ll 1$ K. As we have shown, the SQUID provides protection of the qubit from the decoherence induced by the environment. Finally, the proposed measurement can be realized at the optimum point, where the background charge induced decoherence is largely suppressed.

In summary, we have shown theoretically that a two junction SQUID can perform a single shot quantum measurement of a Josephson charge qubit. We discussed the limits of this detector posed by the finite contrast in measuring the quantum state of the SQUID, the finite quality factor of the SQUID, and coupling of the qubit to environment noise.

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