I present two disparate examples of isospin violation in hadronic B-decays. In the first, the presence of $\rho^0$-$\omega$ mixing in the decay $B^+ \to \rho^+\rho^0(\omega) \to \rho^+\pi^+\pi^-$ permits the extraction of $\text{sgn}(\sin \alpha)$, where $\alpha$ is the usual angle of the unitarity triangle, with minimal hadronic uncertainty. In the second, the presence of $\pi^0$-$\eta, \eta'$ mixing can obscure the extraction of $\sin 2\alpha$ from an isospin analysis in $B \to \pi\pi$ decays.

1 Introduction

In the standard model, CP violation is characterized by a single phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, rendering its elements complex. The CKM matrix of the standard model is unitary, so that determining whether or not this is empirically so is a central test of the standard model’s veracity. Thus, determining whether the angles of the unitarity triangle, $\alpha$, $\beta$, and $\gamma$, empirically sum to $\pi$ and whether its angles are compatible with the measured lengths of its sides lie at the heart of these tests of the standard model.

CP-violating effects in hadronic B-decays will play a crucial role in the determination of $\alpha$, $\beta$, and $\gamma$, and many clever methods have been devised to evade the uncertainties the strong interaction would weigh on their extraction. Irrespective of the efficacy of these methods, discrete ambiguities in the angles remain, for experiments in the neutral B sector which would measure an unitarity triangle angle, $\phi$, determine $\sin 2\phi$ and not $\phi$ itself. Nevertheless, removing discrete ambiguities is important, for standard model unitarity requires merely $\alpha + \beta + \gamma = \pi$, mod $2\pi$. Determining the precise equality yields another standard model test, for consistency with the measured value of $\epsilon$ and the computed $B_K$ parameter suggest that it ought be $\pi$.

The isospin-violating effects to be discussed here impact the extraction of $\alpha$, where $\alpha \equiv \text{arg}[\frac{-V_{td}V_{tb}^*}{(V_{ud}V_{ub}^*)}]$ and $V_{ij}$ is an element of the CKM matrix. The first exploits isospin violation to extract $\text{sgn}(\sin \alpha)$ from the rate asymmetry in $B^+ \to \rho^+\rho^0(\omega) \to \rho^+\pi^+\pi^-$, where $\rho^0(\omega)$ denotes the $\rho^0$-$\omega$ interference region, with minimal hadronic uncertainty, removing the

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mod(π) ambiguity in α consequent to a sin2α measurement. An asymmetry emerges only if both a weak and strong phase difference exists between two interfering amplitudes. The strong phase difference is typically small and uncertain, yet in the decay \( B^+ \to \rho^+ \rho^0(\omega) \to \rho^+ \pi^+ \pi^- \), the presence of the \( \omega \) resonance not only enhances the asymmetry to some 20% but also permits the determination of the strong phase from \( e^+ e^- \to \pi^+ \pi^- \) data for \( \pi^+ \pi^- \) invariant masses in the vicinity of the \( \omega \) resonance.

The second topic concerns the impact of isospin violation on the extraction of sin2α from an isospin analysis in \( B \to \pi\pi \) decays. Isospin is broken not only by electroweak effects but also by the \( u \) and \( d \) quark mass difference. The latter drives \( \rho^0 - \eta, \eta' \) mixing, which, in turn, generates an amplitude in \( B \to \pi\pi \) not included in the isospin analysis. Thus, although the effect of electroweak penguins is estimated to be small, when all the effects of isospin violation are included, the error in the extracted value of sin2α can be significant.

2 \( \rho^0-\omega \) Mixing and CP Violation in \( B^+ \to \rho^+ \rho^0(\omega) \to \rho^+ \pi^+ \pi^- \)

Here we consider the extraction of sgn(sinα) from \( B^+ \to \rho^+ \rho^0(\omega) \to \rho^+ \pi^+ \pi^- \), as proposed by Enomoto and Tanabashi. In this channel, the rate asymmetry, which is CP-violating, is also isospin forbidden. If isospin were a perfect symmetry, then the Bose symmetry of the \( J = 0 \) \( \rho^+ \rho^0 \) final state would force it to have isospin \( I = 2 \). The strong penguin amplitude is also purely \( \Delta I = 1 / 2 \), so that no CP violation is possible in this limit. If isospin violating effects are included, however, two effects occur. The penguin operators then possess both \( \Delta I = 1 / 2 \) and \( \Delta I = 3 / 2 \) character; the latter are generated by electroweak penguin operators and by the isospin-violating effects which distinguish the \( \rho^\pm \) from the \( \rho^0 \). Yet isospin violation also generates \( \rho^0-\omega \) mixing, so that a \( I = 1 \) final state is also possible. In our detailed numerical estimates, we find that the strong phase in the \( \rho^0-\omega \) interference region is driven by \( \rho^0-\omega \) mixing. The sign and magnitude of \( \rho^0-\omega \) mixing is fixed by \( e^+ e^- \to \pi^+ \pi^- \) data, so that we are able to interpret direct CP violation in this channel to extract sgn(sinα).

Resonances can play a strategic role in direct CP violation, for their mass and width can be used to constrain the strong phase and their interference can significantly enhance the CP-violating asymmetry. To see how these effects are realized here, consider the amplitude \( A \) for \( B^- \to \rho^- \pi^+ \pi^- \) decay:

\[
A = \langle \pi^+ \pi^- \rho^- | H^T | B^- \rangle + \langle \pi^+ \pi^- \rho^- | H^P | B^- \rangle,
\]

where \( A \) is given by the sum of the amplitudes corresponding to the tree and penguin diagrams, respectively. Defining the strong phase \( \delta \), the weak phase \( \phi \), and the magnitude \( r \) via \( A = \)
\[ \langle \pi^+ \pi^- | H^T | B^- \rangle [1 + re^{i\delta} e^{i\phi}] \]

the CP-violating asymmetry \( A_{\text{CP}} \) is

\[
A_{\text{CP}} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r \sin \delta \sin \phi}{1 + 2r \cos \delta \cos \phi + r^2},
\]

(1)

where \( \phi \) is \(-\alpha\). To express \( \delta \) in terms of the resonance parameters, let \( t_V \) be the tree amplitude and \( p_V \) be the penguin amplitude to produce a vector meson \( V \). Thus, the tree and penguin amplitudes become

\[
\langle \pi^+ \pi^- \rho^- | H^T | B^- \rangle = g_{\rho} s_{\rho} \rho^* \tilde{\Pi}_{\rho \omega} + g_{\rho} s_{\rho} \rho^* \beta e^{i\delta},
\]

(2)

\[
\langle \pi^+ \pi^- \rho^- | H^P | B^- \rangle = g_{\rho} s_{\rho} \rho^* \tilde{\Pi}_{\rho \omega} + g_{\rho} s_{\rho} \rho^* \beta e^{i\delta},
\]

(3)

where we have introduced \( \tilde{\Pi}_{\rho \omega} \), the effective \( \rho^0 \)-\( \omega \) mixing matrix element; \( g_{\rho} \), the \( \rho^0 \rightarrow \pi^+ \pi^- \) coupling; \( s_V / s_V \), the vector meson propagator, with \( s_V = s - m_V^2 + i m_V \Gamma_V \); and \( s \), the square of the invariant mass of the \( \pi^+ \pi^- \) pair.

Using \( re^{i\delta} e^{i\phi} = \frac{\langle \pi^+ \pi^- \rho^- | H^P | B^- \rangle}{\langle \pi^+ \pi^- \rho^- | H^T | B^- \rangle} \)

and \( p_{\omega} / t_{\rho} \equiv r' e^{i(\delta_q + \phi)} \), \( t_{\omega} / t_{\rho} \equiv \bar{\alpha} e^{i\delta_q} \), \( p_{\omega} / p_{\rho} \equiv \beta e^{i\delta_q} \),

one finds, to leading order in isospin violation,

\[
re^{i\delta} = \frac{r' e^{i\delta_q}}{s_{\omega}} \left\{ \tilde{\Pi}_{\rho \omega} + \beta e^{i\delta_q} \left( s_{\omega} - \tilde{\Pi}_{\rho \omega} \bar{\alpha} e^{i\delta_q} \right) \right\}.
\]

(6)

A \( J = 0, I = 1 \) \( \rho^\pm \rho^0 \) final state is forbidden by Bose symmetry if isospin is perfect, so that \( \beta \) is non-zero only if electroweak penguin contributions and isospin violation in the \( \rho^\pm \) and \( \rho^0 \) hadronic form factors are included. Numerically, \( |\tilde{\Pi}_{\rho \omega}| / (m_\omega \Gamma_\omega) \gg \beta \). Thus, as \( s \rightarrow m_\omega^2 \),

\[
re^{i\delta} \rightarrow \frac{r' e^{i\delta_q} \tilde{\Pi}_{\rho \omega}}{m_\omega \Gamma_\omega},
\]

(7)

so that \( \delta \rightarrow \delta_q - \pi / 2 \) and \( r \rightarrow r' \tilde{\Pi}_{\rho \omega} / m_\omega \Gamma_\omega \) in this limit. The asymmetry is thus determined by the resonance parameters \( \tilde{\Pi}_{\rho \omega} \), \( m_\omega \), and \( \Gamma_\omega \); the weak phase \( \phi \); and the “short distance” parameters \( r' \) and \( \delta_q \). The latter are calculable within the context of the operator product expansion if the factorization approximation is applied, though a ratio of hadronic form factors enters as
The CP-violating asymmetry, Eq. [1], in percent, plotted versus the invariant mass $q$ of the $\pi^+\pi^-$ pair in MeV for $[N_c, k^2/m_c^2]$. The asymmetries with $\tilde{\Pi}_{\rho\omega} = -3500$ MeV$^2$ and $\tilde{\epsilon} = -0.005$ are shown for $[2, 0.5]$ (solid line), $[3, 0.5]$ (long-dashed), $[2, 0.3]$ (dashed), and $[3, 0.3]$ (dot-dashed). The asymmetry for various $N_c$ and $k^2$, where $k$ is the momentum of the virtual boson, as a function of $s$ is shown in Fig. 1, where we have used the effective Hamiltonian of Deshpande and He in QCD to next-to-leading logarithmic (NLL) order, in the factorization approximation.

To extract the sign of $\sin \delta$, note that for $s \approx m_\omega^2$, we have

$$r \sin \delta \approx \frac{\tilde{\Pi}_{\rho\omega} r'}{|s_\omega|^2} \left[ (s - m_\omega^2) \sin \delta - m_\omega \Gamma_\omega \cos \delta \right];$$

the sign of $\sin \delta$ at $s = m_\omega^2$ is that of $-\tilde{\Pi}_{\rho\omega} \cos \delta$. The sign and magnitude of $\tilde{\Pi}_{\rho\omega}$ is determined from a fit to the time-like pion form factor, $F_\pi(s)$, as measured in $e^+e^- \rightarrow \pi^+\pi^-$, where

$$F_\pi(s) = F_{\rho}(s) \left[ 1 + \frac{1}{3} \frac{\tilde{\Pi}_{\rho\omega}(s)}{s - m_\omega^2 + im_\omega \Gamma_\omega} \right].$$

Fitting $e^+e^- \rightarrow \pi^+\pi^-$ data we find $\tilde{\Pi}_{\rho\omega} = -3500 \pm 300$ MeV$^2$. The error is statistical only; the value of $\tilde{\Pi}_{\rho\omega}$ is insensitive to the theoretical ambiguities in

Figure 1: The CP-violating asymmetry, Eq. [1], in percent, plotted versus the invariant mass $q$ of the $\pi^+\pi^-$ pair in MeV for $[N_c, k^2/m_c^2]$. The asymmetries with $\tilde{\Pi}_{\rho\omega} = -3500$ MeV$^2$ and $\tilde{\epsilon} = -0.005$ are shown for $[2, 0.5]$ (solid line), $[3, 0.5]$ (long-dashed), $[2, 0.3]$ (dashed), and $[3, 0.3]$ (dot-dashed).
the $\rho$ parametrization, $F_{\rho}(s)$. Finally, then, the determination of $\text{sgn}(\sin \alpha)$ relies on that of $\text{sgn}(\cos \delta_q)$. The latter cannot be determined without additional theoretical input; however, it is worth noting that, in our computations in the factorization approximation, $\cos \delta_q < 0$ for all $N_c$ and $k^\pm$. The situation, finally, is not dissimilar from that of the determination of $\text{sgn}(\sin 2\alpha)$ in neutral B decays to CP eigenstates, for fixing the sign of $\sin 2\alpha$ requires that of the parameter $B_B$. Nevertheless, the computation of $\cos \delta_q$ can be tested through the measurement of the “skew” of the asymmetry in Eqs. (11) and (12) from a Breit-Wigner shape, noting Fig. 1. Indeed, the shape of the asymmetry yields $\tan \delta_q$ and thus offers a sensitive test of factorization. This is possible as other effects which would skew the asymmetry are smaller still. For example, we have assumed $\Pi_{\rho\omega}$ to be both real and $s$-independent, yet if we include these effects in our fits to $e^+e^- \rightarrow \pi^+\pi^-$ data, the phase and $s$-dependence of $\Pi_{\rho\omega}$ in the $\rho^0-\omega$ interference region are statistically consistent with zero and, moreover, do not mar our interpretation of the skew.

In summary, the rate asymmetry in $B^+ \rightarrow \rho^+ \rho^0(\omega) \rightarrow \rho^+\pi^+\pi^-$ is large and robust with respect to the known strong interaction uncertainties. The presence of isospin violation in this decay permits the determination of $\text{sgn}(\sin \alpha)$ once the sign of $\cos \delta_q$ is known, noting $|\cos \delta_q| \sim \mathcal{O}(1)$. The latter can be calculated, yet the shape of the asymmetry yields a direct test of the suitability of our estimate. $\tan \delta_q$ can also be extracted from a comparison with the decay $B^\pm \rightarrow \rho^\pm \omega \rightarrow \rho^\pm \pi^0 \pi^0$; here the asymmetry goes as $\sin \delta_q$. The sign of $\sin \alpha$ can be thus extracted with minimal hadronic uncertainty, removing the mod($\pi$) ambiguity in $\alpha$ consequent to a $\sin 2\alpha$ measurement.

3 $\pi^0-\eta, \eta'$ Mixing in $B \rightarrow \pi\pi$ Decays

To review the isospin analysis in $B \rightarrow \pi\pi$ decays, due to Gronau and London, let us consider the time-dependent asymmetry $A(t)$:

$$A(t) = \frac{1 - |r_{f_{\text{CP}}}|^2}{1 + |r_{f_{\text{CP}}}|^2} \cos(\Delta m t) - \frac{2(\text{Im} r_{f_{\text{CP}}})}{1 + |r_{f_{\text{CP}}}|^2} \sin(\Delta m t) ,$$

(10)

where $r_{f_{\text{CP}}} = (V_{tb}V_{td}^*/V_{ub}V_{ud}^*) (A_{f_{\text{CP}}}/A_{f_{\text{CP}}}) = e^{-2\phi_m} \frac{\sqrt{A_{f_{\text{CP}}}}}{A_{f_{\text{CP}}}}$, $A_{f_{\text{CP}}} \equiv A(B^0 \rightarrow f_{\text{CP}})$, and $\Delta m \equiv B_H - B_L$. Denoting the amplitudes $B^+ \rightarrow \pi^+\pi^0, B^0 \rightarrow \pi^0\pi^0$, and $B^0 \rightarrow \pi^+\pi^-$ by $A^{+0}, A^{00}$, and $A^{+-}$, respectively, and introducing $A_I$ to denote an amplitude of final-state isospin $I$, we have

$$\frac{1}{2} A^{+-} = A_2 - A_0 ; \quad A^{00} = 2A_2 + A_0 ; \quad \frac{1}{\sqrt{2}} A^{+0} = 3A_2 ,$$

(11)

where analogous relations exist for $A^{-0}, A^{00}$, and $A^{+-}$ in terms of $A_2$ and $A_0$. If isospin is perfect, the Bose symmetry of the $J = 0$ $\pi\pi$ state permits
amplitudes of $I = 0, 2$, so that the amplitude $B^\pm \to \pi^\pm \pi^0$ is purely $I = 2$. Moreover, the strong penguin contributions are of $\Delta I = 1/2$ character, so that they cannot contribute to the $I = 2$ amplitude and no CP violation is possible in the $\pi^+ \pi^0$ final states. This is identical to the situation in $B^\pm \to \rho^\pm \rho^0$. The penguin contribution in $B^0 \to \pi^+ \pi^-$, or in $B^\to \pi^+ \pi^-$, can then be isolated and removed by determining the relative magnitude and phase of the $I = 0$ to $I = 2$ amplitudes. We have

$$r_{\pi^+ \pi^=} = e^{-2i\phi_m} \frac{(A_2 - A_0)}{(A_2 - A_0)} = e^{2i\alpha} \frac{(1 - z)}{(1 - z)},$$

where $z(\tau) \equiv A_0/A_2(A_0/A_2)$ and $A_2/A_2 = \exp(-2i\phi_t)$ with $\phi_t \equiv \arg(V_{6 d}V_{6 b}^*)$ and $\phi_m + \phi_t = \beta + \gamma = \pi - \alpha$ in the standard model. Given $|A^+|$, $|A^0|$, $|A^+|$, and their charge conjugates, the measurement of $\text{Im} \, r_{\pi^+ \pi^=}$ determines $\sin 2\alpha$, modulo discrete ambiguities in $\arg((1 - z)/(1 - z))$. The latter can be removed via a measurement of $\text{Im} \, r_{\pi^0 \pi^0}$ as well.

We examine the manner in which isospin-violating effects impact this extraction of $\sin 2\alpha$, for isospin is broken not only by electroweak effects but also by the $u$ and $d$ quark mass difference. Both sources of isospin violation generate $\Delta I = 3/2$ penguin contributions, but the latter also generates $\pi^0 - \eta, \eta'$ mixing, admitting an $I = 1$ amplitude. Although electroweak penguins are estimated to be small, other isospin-violating effects, such as $\pi^0$-and $\eta'$-mixing, can also be important.

To include the effects of $\pi^0$-and $\eta'$-mixing, we write the pion mass eigenstate $|\pi^0\rangle$ in terms of the $SU(3)$ perfect states $|\phi_3\rangle = |u\pi - d\eta|/\sqrt{2}$, $|\phi_8\rangle = |u\pi + d\eta|/\sqrt{6}$, and $|\phi_0\rangle = |u\pi + d\eta + s\eta'|/\sqrt{3}$. To leading order in isospin violation

$$|\pi^0\rangle = |\phi_3\rangle + \varepsilon (\cos \theta |\phi_8\rangle - \sin \theta |\phi_0\rangle) + \varepsilon' (\sin \theta |\phi_0\rangle + \cos \theta |\phi_0\rangle),$$

where $|\eta\rangle = \cos \theta |\phi_8\rangle - \sin \theta |\phi_0\rangle + O(\varepsilon)$, and $|\eta'\rangle = \sin \theta |\phi_8\rangle + \cos \theta |\phi_0\rangle + O(\varepsilon')$. Expanding QCD to leading order in $1/\Lambda_c$, momenta, and quark masses and diagonalizing the quadratic terms in $\phi_3$, $\phi_8$, and $\phi_0$ of the resulting effective Lagrangian determines the mass eigenstates $\pi^0$, $\eta$, and $\eta'$ and yields $\varepsilon = \varepsilon_0 \chi \cos \theta$ and $\varepsilon' = -2\varepsilon_0 \chi \sin \theta$, where $\chi = 1 + (4m_K^2 - 3m_\eta^2 - m_\pi^2)/(m_\eta^2 - m_\pi^2) \approx 1.23$, $\chi = 1/\chi$, $\varepsilon_0 \equiv \sqrt{3}(m_u - m_d)/(4(m_s - \tilde{m}))$, and $\tilde{m} \equiv (m_u + m_d)/2$. Thus the magnitude of isospin breaking is controlled by the SU(3)-breaking parameter $m_s - \tilde{m}$. The $\eta$-$\eta'$ mixing angle $\theta$ is found to be $\sin 2\theta = -(4\sqrt{2}/3)(m_K^2 - m_\eta^2)/(m_\eta^2 - m_\pi^2) \approx -22$°. The resulting $\varepsilon = 1.14\varepsilon_0$ is comparable to the one-loop-order chiral perturbation theory result of $\varepsilon = 1.23\varepsilon_0$ in $\eta \to \pi^+ \pi^- \pi^0$. Using $m_q(\mu = 2.5 \text{ GeV})$ of Ali et al., we find $\varepsilon = 1.4 \times 10^{-2}$ and $\varepsilon' = 7.7 \times 10^{-3}$. 
In the presence of isospin-violating effects, the $B \to \pi \pi$ amplitudes become
\begin{align}
A^{-0} &= (\pi^- \phi_3 | \mathcal{H}^{\text{eff}} | B^-) + \varepsilon_8 (\pi^- \phi_8 | \mathcal{H}^{\text{eff}} | B^-) + \varepsilon_0 (\pi^- \phi_0 | \mathcal{H}^{\text{eff}} | B^-) \quad (14) \\
\overline{A}^{00} &= \langle \phi_3 \phi_3 | \mathcal{H}^{\text{eff}} | B^0 \rangle + 2\varepsilon_8 \langle \phi_3 \phi_8 | \mathcal{H}^{\text{eff}} | B^0 \rangle + 2\varepsilon_0 \langle \phi_3 \phi_0 | \mathcal{H}^{\text{eff}} | B^0 \rangle, \quad (15)
\end{align}
where $\varepsilon_8 \equiv \varepsilon \cos \theta + \varepsilon' \sin \theta$ and $\varepsilon_0 \equiv \varepsilon' \cos \theta - \varepsilon \sin \theta$. The $B \to \pi \pi$ amplitudes satisfy
\begin{align}
\overline{A}^{++} + 2\overline{A}^{00} - \sqrt{2} A^{-0} &= 4\varepsilon_8 \langle \phi_3 \phi_8 | \mathcal{H}^{\text{eff}} | B^0 \rangle + 4\varepsilon_0 \langle \phi_3 \phi_0 | \mathcal{H}^{\text{eff}} | B^0 \rangle \\
&- \sqrt{2}\varepsilon_8 \langle \pi^- \phi_8 | \mathcal{H}^{\text{eff}} | B^- \rangle - \sqrt{2}\varepsilon_0 \langle \pi^- \phi_0 | \mathcal{H}^{\text{eff}} | B^- \rangle, \quad (16)
\end{align}
and thus the triangle relation implied by Eq. 11 becomes a quadrilateral. We ignore the small mass differences $m_{\pi \pm} - m_{\pi^0}$ and $m_{B \pm} - m_{B^0}$.

We proceed by computing the individual amplitudes using the $\Delta B = 1$ effective Hamiltonian resulting from the operator product expansion in QCD in NLL order, using the factorization approximation for the hadronic matrix elements. In this context, we can then apply the isospin analysis delineated above to infer $\sin 2\alpha$ and thus estimate its theoretical systematic error, incurred through the neglect of isospin violating effects. Numerical results for
The discrete ambiguity in the strong phase is resolved wrongly.

The matching procedure fails to choose a sin 2\( \alpha \) which is as close to the input value as possible.

The reduced amplitudes \( A_R \) and \( \overline{A_R} \), where \( \overline{A_R} \equiv 2\theta_0^0/(G_F/\sqrt{2})iV_{ub}V_{ud}^* \), 
\( \overline{A_R}^0 \equiv \overline{A^0}^0/(G_F/\sqrt{2})iV_{ub}V_{ud}^* \), and \( A_{R\pi}^0 \equiv \sqrt{2}A^0 /((G_F/\sqrt{2})iV_{ub}V_{ud}^*) \),
with \( N_c = 2, 3, \infty \) and \( k^2/m_\pi^2 = 0.3, 0.5 \) are shown in Fig. 2. \( A_{R\pi}^0 \) and \( A_{R\pi} \) are broken into tree and penguin contributions, so that \( A_{R\pi}^0 = T_{\pi+\phi_0} + P_{\pi+\eta_0} \) and \( A_{R\pi} \equiv T_{\pi+\phi_0} + P_{\pi+\eta_0} \), where \( P_{\pi+\eta_0} \) is defined to include the isospin-violating tree contribution in \( A_{R\pi}^0 \) as well. The shortest side in each polygon is the vector defined by the RHS of Eq. 16. The values of sin 2\( \alpha \) extracted from the computed amplitudes with \( N_c \) and \( k^2/m_\pi^2 = 0.5 \) are shown in Table 1 — the results for \( k^2/m_\pi^2 = 0.3 \) are similar and have been omitted. In the presence of \( \pi^0-\eta, \eta' \) mixing, the \( \overline{A_R}^0, \overline{A_{R\pi}}^0 \), and \( \overline{A_R}^0 \) amplitudes obey a quadrilateral relation as per Eq. 16. Consequently, the values of sin 2\( \alpha \) extracted from Im\( r_{\pi+\pi^-} \) and Im\( r_{\eta+\eta^-} \) can not only differ markedly from the value of sin 2\( \alpha \) input but also need not match. The incurred error in sin 2\( \alpha \) increases as the value to be extracted decreases; the structure of Eq. 16 suggests this, for as sin 2\( \alpha \) decreases the quantity Im\( (1 - z)/(1 - z) \) becomes more important to determining the extracted value. It is useful to constrain the impact of the various isospin-violating effects. The presence of \( \Delta I = 3/2 \) penguin contributions, be they from \( m_u \neq m_d \) or electroweak effects, shift the extracted value of sin 2\( \alpha \) from its input value, yet the “matching” of the sin 2\( \alpha \) values in Im\( r_{\pi+\pi^-} \) and

Table 1: Strong phases and inferred values of sin 2\( \alpha \) from amplitudes in the factorization approximation with \( N_c \) and \( k^2/m_\pi^2 = 0.5 \). The strong phase 2\( \delta_{\text{true}} \) is the opening angle between the \( \overline{A_R}^0 \) and \( A_{R}^0 \) amplitudes in Fig. 1, whereas 2\( \delta_{\text{GL}} \) is the strong phase associated with the closest matching sin 2\( \alpha \) values, denoted (sin 2\( \alpha \))\( GL \), from Im\( r_{\pi+\pi^-} \)/(Im\( r_{\eta+\eta^-} \))\( 0 \), respectively. The bounds |2\( \delta_{\text{GQI}} \)| and |2\( \delta_{\text{GQII}} \)| on 2\( \delta_{\text{true}} \) from Eqs. 4.14 and 2.15 of Ref. [26] are also shown. All angles are in degrees. We input a) sin 2\( \alpha \) = 0.0432[2-2.2] \( \pi \) b) sin 2\( \alpha \) = -0.233 \( \pi \) c) sin 2\( \alpha \) = 0.959 \( \rho \) = -0.12.

| case | \( N_c \) | 2\( \delta_{\text{true}} \) | 2\( \delta_{\text{GQI}} \) | 2\( \delta_{\text{GQII}} \) | 2\( \delta_{\text{GL}} \) | (sin 2\( \alpha \))\( GL \) |
|------|---------|----------------|----------------|----------------|----------------|----------------|
| a    | 2       | 24.4           | 26.1           | 15.8           | 16.6           | -0.0900/-0.0221 |
| a    | 3       | 24.2           | 16.9           | 16.1           | 16.2           | -0.0926/0.107  |
| a    | \( \infty \) | 23.8           | 59.4           | 25.1           | 23.6           | 0.0451/0.394   |
| b    | 2       | 19.6           | 23.4           | 12.1           | 12.9           | -0.343/-0.251  |
| b    | 3       | 19.4           | 13.5           | 12.9           | 13.0           | -0.719/-0.855(*)|
| b    | \( \infty \) | 19.2           | 59.9           | 23.6           | 0.76           | -0.550/-0.814(*)|
| c    | 2       | 28.3           | 36.5           | 20.4           | 21.0           | 0.917/0.915    |
| c    | 3       | 28.0           | 24.0           | 19.1           | 19.0           | 0.905/0.952    |
| c    | \( \infty \) | 28.3           | 36.5           | 20.4           | 21.0           | 0.917/0.915    |

* The matching procedure fails to choose a sin 2\( \alpha \) which is as close to the input value as possible.

† The discrete ambiguity in the strong phase is resolved wrongly.
\[ \text{Im} r_{\pi^0,\pi^0} \text{ is unaffected. The mismatch troubles seen in Table 1 are driven by } \\
\pi^0-\eta, \eta' \text{ mixing, though the latter shifts the values of } \sin 2\alpha \text{ in } \text{Im} r_{\pi^+\pi^-} \text{ as well.} \\
Picking the closest matching values of } \sin 2\alpha \text{ in the two final states also picks } \\
\text{the solutions closest to the input value; the exceptions are noted in Table 1. If } \\
|A_{00}| \text{ and } |A_{00}^0| \text{ are small, the complete isospin analysis may not be possible, so } \\
\text{that we also examine the utility of the bounds recently proposed by Grossman and Quinn, } \\
25 \text{ on the strong phase } 2\delta_{\text{true}} = \arg((1 - z)/(1 - z)) \text{ of Eq. 12. The bounds } \\
2\delta_{\text{GQI}} \text{ and } 2\delta_{\text{GQII}} \text{ given by their Eqs. 2.12 and 2.15, respectively, follow from Eq. 11, and thus can be broken by isospin-violating effects. As } \\
\text{shown in Table 1, the bounds typically are broken, and their efficacy does not } \\
\text{improve as the value of } \sin 2\alpha \text{ to be reconstructed grows large.} \\
\]

To conclude, we have considered the role of isospin violation in } B \rightarrow \pi\pi \text{ decays and have found the effects to be significant. Most particularly, the utility of the isospin analysis in determining } \sin 2\alpha \text{ strongly depends on the value to be reconstructed. The error in } \sin 2\alpha \text{ from a } \text{Im} r_{\pi^+\pi^-} \text{ measurement can be } \\
\text{50% or more for the small values of } \sin 2\alpha \text{ currently favored by phenomenology} \text{; however, if } \sin 2\alpha \text{ were near unity, the error would decrease to less than } 10\%. \text{ The effects found arise in part because the penguin contribution in } B^0 \rightarrow \pi^+\pi^-, \text{ e.g., is itself small; we estimate } |P|/|T| < 9%|V_{tb}V_{td}^*|/|V_{ub}V_{ud}^*|. \\
\text{Relative to this scale, the impact of } \pi^0-\eta, \eta' \text{ mixing is significant. Yet, were } \\
\text{the penguin contributions in } B \rightarrow \pi\pi \text{ larger, the isospin-violating effects considered would still be germane, for not only would the } \Delta I = 3/2 \text{ penguin contributions likely be larger but the } B \rightarrow \pi\eta \text{ and } B \rightarrow \pi\eta' \text{ contributions could also be larger as well.} \text{ To conclude, we have shown that the presence of } \\
\pi^0-\eta, \eta' \text{ mixing breaks the triangle relationship, Eq. 11, usually assumed, and can mask the true value of } \sin 2\alpha. \text{ } \\
\]

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