Dynamical Analysis for the INS Vibration Control System Used in UAV

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Abstract. A six-degree-of-freedom mechanical dynamic model and its equations of the damping system are established for the offset installation environment of the UAV. The influence parameters of the damping system on the vibration coupling state is studied based on the model. And the optimal parameters of the damping system are obtained by doing the mechanical dynamic calculation. And then the vibration isolator of the UAV strap-down inertial measurement unit is designed and manufactured. Finally, the damping system was tested and verified on the shaking table and in the flight environment of the UAV. It is shown that the damping system could implement the tri-axial equal rigid design when the shaft-to-radius stiffness of the isolator is about 0.8. And the experiments also prove this point. The damping system weakens the coupling problem of the strap-down inertial measurement unit effectively, and it reaches the quasi-decoupled state. The amplitude of the acceleration decreases by more than 50% after vibration isolation.

1. Introduction

The strap-down inertial navigation system (INS) is the key equipment in aeronautics and astronautics, that mainly used for positioning and attitude measurement of carriers [1]. Its performance and reliability could directly affect the flight safety and working state of the carriers. However, its measurement accuracy could be reduced by the severe vibration environments. The vibration decrease has been the key problems in the safety and stability of the INS.

Yu et al [2] developed a hybrid mode tri-axial magnetorheological elastomer isolators and the phase-based control to suppress the vibration of the IMU in three directions. Chen et al [3] proposed a vibration isolation system (VIS) structural and established the decoupled six-degree-of-freedom dynamic model to eliminate the motion coupling errors. Tuo et al [4] studied a method for designing a IMU damping system and made the modal analysis of dithered ring laser gyroscope IMU damping system using the finite element analysis. Tu et al [5] proposed a novel dynamic analysis and experimental verification method for the deviation angles in dual-axis rotational INS in order to accurately analyse the deviation angles caused by rubber dampers deformation. The proposed model and dynamic analysis method were validated by comparing it against the real deviation angles measured by rotation experiments for a turntable.

Based on the research of Doctor Tu et al [5], a six-degree-of-freedom mechanical dynamic model and its equations of the damping system would be established for the offset installation environment of the UAV. The influence parameters of the damping system on the vibration coupling state was studied
using the model. The optimal parameters of the damping system were obtained by doing the mechanical dynamic calculation.

2. Establishment of the damping system model

In order to simplify the model, the vibration isolator could be considered as a joint with a certain stiffness and damping. Then the damping system which is composed of the isolators can be regard as control by four matrixes: the translational stiffness matrix $\mathbf{K}$, the translational damping matrix $\mathbf{C}$, the rotational stiffness matrix $\mathbf{T}$ and the rotational stiffness matrix $\mathbf{Ξ}$. And each isolator could be simplified and decomposed to one stiffness damping system which is shown in figure 1 along $x$, $y$, and $z$ axes. It is assumed that the axial stiffness and damping of the isolator are $K_a$ and $C_a$, and the radial stiffness and damping are $K_r$ and $C_r$. Considering that the vibration isolator is a structure that is symmetrical along the axis rather than the radial direction, and the installation direction of the isolators in the damping system is parallel to the $x$ direction. Thus

$$
K_x = K_a, \quad K_y = K_z = K_r,
$$

$$
C_x = C_a, \quad C_y = C_z = C_r.
$$

(1)

![Figure 1. Model of isolator of the damping system.](image)

The dynamic state of the whole IMU damping system is shown in figure 1. The mass centre of the IMU is at the geometric centre of the damping system. If the damping system is completely symmetrical in three directions, the stiffness centre of the damping system can be coincided with the mass centre that we could decouple the whole dynamic system.

![Figure 2. Dynamical mechanical coupling state of the model.](image)
In practice, the heading direction \((y)\) and lateral direction \((z)\) are absolutely identical and symmetrical that the vibration state of the two directions is exactly the same. But vibration state of the vertical direction \((x)\) is different from the two other direction since the stiffness centre was not coincided with the mass centre along axial direction of the isolator \((x\), namely vertical direction\). We assume \(\Psi\) is the offset vector between the stiffness centre of the IMU damping system and mass centre. Analysis above shown that the components along \(y\) and \(z\) direction are both zero and we could take \(\Psi = [\psi, 0, 0]\).

The figure 2 shows that the components of \(\Psi\) along \(y\) and \(z\) direction are both zero so that the damping system of the IMU is decoupled along \(x\) direction when the system was vibrated along vertical direction (namely \(\Lambda_x\)). Nevertheless, the \(x\) components of \(\Psi\) is not zero so that the linear vibration along \(y\) direction was coupling with the angular vibration along \(z\) direction and the linear vibration along \(z\) direction was also coupling with the angular vibration along \(y\) direction when the system was vibrated alongside direction \((\Lambda_y\) or \(\Lambda_z\)).

We assume that the IMU is one isotropic cube with a side length of \(2l\), that is, \(L_y=L_z=l\). Moreover, it’s known that \(L_x=\psi\). Thus

\[
\begin{align*}
+m(\ddot{x} - g) + 4C_a \dot{x} + 4K_a x &= 0 \\
m\ddot{y} + 4C_y \dot{y} + 4K_y y + 4C_y \psi \dot{\psi} + 4K_y \psi \dot{\psi} &= 0 \\
m\ddot{z} + 4C_z \dot{z} + 4K_z z - 4C_z \psi \dot{\psi} - 4K_z \psi \dot{\psi} &= 0 \\
\frac{1}{6} m l^2 \dddot{\alpha} + 8C_l l^2 \dot{\alpha} + 8K_l l^2 \alpha &= 0
\end{align*}
\]

(2)

3. Decoupling the damping system model

3.1. Coupling analysis of \(x\) direction
Firstly, one linear sine excitation \(\Lambda_x\) was exerted along \(x\) direction. Thus

\[
m(\ddot{x} - g) + 4C_x (\dot{x} - \Lambda_x) + 4K_x (x - \Lambda_x) = 0. \tag{3}
\]

Then the linear and angular transfer function (namely \(H_L\)) of the damping system along \(x\) direction are

\[
H_L = (1 + (\frac{C_x}{K_x})^2)(1 - \frac{m\omega^2}{4K_x} + (\frac{C_x}{K_x})^2)^{-\frac{1}{2}}. \tag{4}
\]

3.2. Coupling analysis of \(y\) direction
Firstly, one linear sine excitation \(\Lambda_y\) was exerted along \(y\) direction. Thus

\[
\begin{align*}
m\ddot{y} + 4C_y (\dot{y} - \Lambda_y) + 4K_y (y - \Lambda_y) + 4C_y \psi \dot{\psi} + 4K_y \psi \dot{\psi} &= 0 \\
\frac{1}{6} m l^2 \dddot{\gamma} + 4(C_l l^2 + C_y \psi^2) \dot{\gamma} + 4(K_y l^2 + K_y \psi^2) \gamma + 4C_y \psi \dot{\gamma} + 4K_y \psi \gamma &= 0
\end{align*}
\]

(5)

Transforming them into the matrix forms, one obtains

\[
\begin{bmatrix}
mI \end{bmatrix} \begin{bmatrix}
\ddot{\gamma} \\
\dot{\gamma}
\end{bmatrix} + \begin{bmatrix} C_x \\
\end{bmatrix} \begin{bmatrix}
\dot{\gamma} \\
\dot{\gamma}
\end{bmatrix} + \begin{bmatrix} K_x \\
\end{bmatrix} \begin{bmatrix}
\gamma \\
\gamma
\end{bmatrix} = \begin{bmatrix} \Lambda_x \\
\Lambda_y
\end{bmatrix}. \tag{6}
\]
Where

\[
[M_L] = \begin{bmatrix} m & 0 \\ 0 & \frac{1}{6} ml^2 \end{bmatrix}
\]

\[
[C_L] = \begin{bmatrix} 4C_r & 4C_r \psi \\ 4C_r \psi & 4(C_r l^2 + C_r \psi^2) \end{bmatrix}
\]

\[
[K_L] = \begin{bmatrix} 4K_r & 4K_r \psi \\ 4K_r \psi & 4(K_r l^2 + K_r \psi^2) \end{bmatrix}
\]

\[
[K_{li}] = \begin{bmatrix} 4K_r & 4C_r \\ 4K_r \psi & 4C_r \psi \end{bmatrix}
\]

Thus the impedance \([Z_L]\) [7] is

\[
[Z_L] = [K_L] - \omega^2 [M_L] + i\omega [C_L].
\]

\[
[U_L] = [K_L] \begin{bmatrix} 1 \\ i\omega \end{bmatrix}
\]

Then the transfer function \([H_L]\) is

\[
[H_L] = \begin{bmatrix} H_{L11} \\ H_{L21} \end{bmatrix} = [Z_L]^{-1} [U_L] = \frac{\text{Adj}[Z_L][U_L]}{\text{det}[Z_L]}.
\]

4. Design of the damping system and Experiments

The 1st passage frequency of the UAV in this article is 12 Hz. In order to avoid the coupling resonance between the IMU and engine of the UAV, the resonance frequency of the IMU damping system should be placed in the middle of the first two passage frequencies. So the resonance frequency of the damping system is designed as 30 Hz. Four identical isolators were installed on the flange of the UAV. The structure of the system was shown in figure 1(c) and parameters of the IMU shown in table 1. The IMU had the offset vector \(\Psi\) since the installation environment restrictions.

| Table 1. Parameters of the IMU. |
|-----------------------------|
| Parameters                  | Value       |
| Mass of IMU\((m)\)          | 0.58 kg     |
| Installation distance of isolators\((l)\) | 120 mm |
| Offset vector\((\Psi)\)      | [50, 0, 0] |

Figure 3 shows the influence of axial damping \(C_a\) and radial damping \(C_r\) on coupling vibration of the damping system when the resonance frequency is about 30 Hz. Therefore, we could conclude that the main contribution of \(C_r\) is to reduce the amplification of the linear resonance frequency and \(C_a\) can effectively eliminate the influence of coupling vibration. When \(C_a = 7\) kg s\(^{-1}\), the influence on coupling vibration was eliminated basically. And \(C_r = 7\) kg s\(^{-1}\), the amplification of the system is 4.8 which could also meet the requirement. So that we are setting \(C_a\) and \(C_r\) both equal 7 kg s\(^{-1}\) in the preliminary calculation.
Substituting the values of \( C_a \) and \( C_r \) into equation (4), one obtains that the axial stiffness \( K_a \) equals 5320 N/m when the resonance frequency was 30 Hz. Then substituting the values of \( C_m \), \( C_r \), and \( K_a \) into equation (8), one obtains that the radial stiffness \( K_r \) equals 6650 N/m.

In a word, the ratio of axial stiffness to radial stiffness is

\[
K_a / K_r = \frac{5320 \text{ N/m}}{6650 \text{ N/m}} = 0.8.
\]

5. Results and discussions

Figure 4 shows the test results of the damping system on shaking table, that the solid line is the test result and the dashed line is calculation result. Figure 5 shows the test results of the damping system on the UAV, that the grey part is the vibration response on the flange of the UAV and the pure black part is the vibration response on the IMU.

One observes that the damping system was both decoupled along axial direction \((x)\) and side direction \((y)\). The 2\(^{nd}\) resonance frequency peak of the angular coupling vibration had been largely eliminated. And the axial and side resonance frequencies were basically the same, so the damping system could be considered to be in the state of tri-axial equal stiffness.

Furthermore, the acceleration amplitude on the IMU decreased significantly. The peak-to-peak value was reduced by more than 50 percent compared with that before the vibration isolation, and the rotor frequency amplitude reduced to one third of the original value.
6. Conclusion
We studied the dynamic coupling state under the offset installation of the damping system, and concluded that

- The offset installation of the damping system will cause the coupling between the angular vibration and the linear vibration, and could be designed to achieve the quasi-decoupling state. When the ratio of axial stiffness to radial stiffness is around 0.8 and damping ratio is above 0.125, the 2nd resonance frequency peak caused by angular vibration could be eliminated basically and the damping system would be in the state of tri-axial equal stiffness.
- According to the design and calculation of the damping system, the vibration problems had been solved effectively. And the working state of the system is consistent with the theoretical analysis. The peak-to-peak value decreased by more than 50 percent compared with before the vibration isolation after filtering the output data.

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Figure 5. Results of the vibration test.