GENERATION OF DIFFERENTIAL GPS CORRECTIONS ALONG WITH PERFORMANCE OF QUALITY CONTROL

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KEY WORDS differential GPS corrections; quality control; Kalman filter

ABSTRACT The DGPS technique can provide considerably better relative positioning accuracy than the stand-alone GPS positioning, but the improvement depends on the distance between the user and the reference station (spatial correlation), the latency of differential corrections (temporal correlation), and the quality of differential corrections. Therefore, how to correctly generate differential corrections as well as their precision is very important to the DGPS positioning technique. This paper presents a new algorithm for generating differential GPS corrections. This algorithm directly uses code and carrier observations in the measurement model of a Kalman filter, so that it is possible to use a simple stochastic model and to use the standard algorithm of the Kalman filter. The algorithm accounts for biases like multipath errors and instrumental delays in code observations and it shows how differential corrections are differently affected by code biases when dual or single frequency data is used. In addition, the algorithm can be integrated with a real time quality control procedure. As a result, the quality of differential corrections can be guaranteed with a certain probability.

1 Introduction

Currently, there already exist several algorithms for the generation of differential corrections, for instance, the algorithm based on carrier filtered code observations (van Dierendonck, 1993; Landau, 1993) and the algorithm based on code observations and sequential differences of carrier observations (Loomis, 1986; Loomis et al., 1989). But all these algorithms at least have difficulties to correctly specify stochastic models, since their input measurements are either correlated in time or correlated with dynamic noises of states (Jin, 1995a).

We will first derive a new algorithm for generating differential corrections and next show how to integrate it with the DIA quality control procedure, as detailed in (Teunissen, 1990a, 1990b). Finally we will discuss some properties of the algorithm.

2 Measurement model and dynamic model

At time $t_k$, a general form of GPS code and carrier observation equations are used as follows

$$P_k = \rho_k + c(dT_k - dt_k) + E_k + I_k + T_k + \eta_k$$

$$\epsilon_k = \rho_k + c(dT_k - dt_k) + E_k - I_k + T_k - \lambda N + \epsilon_k$$

where $P_k$ is code observation (m); $\epsilon_k$ is carrier observation (cycles); $\rho_k$ is satellite-receiver range computed from ephemeris data and station coordinates (m); $c$ is speed of light (m/s); $dT_k$ is receiver clock bias (s); $dt_k$ is satellite clock bias (including SA clock error) (s); $E_k$ is effect of ephemeris error (including SA orbit error) plus a priori coordinate error of station (m); $I_k$ is ionospheric delay (m); $T_k$ is tropospheric delay (m); $b_k$ is code observation bias (m); $\epsilon_k$ is code observation noise (m); $\lambda$ is wave length corresponding to one of the GPS carri-
ers (m); \(N\) is carrier ambiguity (cycles) which is a real value and \(\eta_k\) is carrier observation noise (m).

Note that the parameter \(b_k\) in the code observation equation is new. It was shown in (Jin, 1995a) that without introducing this parameter, code observation noises were biased and their time series appeared to have linear and/or periodic behaviour. Whereas after introducing the bias parameter \(b_k\) in the code observation equation, code observation noises became unbiased random noises of zero mean. The code bias \(b_k\) is a combination of all systematic errors in the code observation, which could have resulted from, for example, multipath and instrumental delay. For the same satellite, the sizes and behaviours of code biases on different frequencies may not be the same. For different channels or different observation environments, the bias may also behave differently. It was found that in quite good observation environments the variation of the bias could be about 0.5 metre or more for a period of one hour and it appeared more clearly in Trimble 4000 SSE receivers than in TurboRogue SNR-8000 receivers. With the improvement of GPS receiver and antenna designs, this bias may become less and less significant in the future. For more details on the behaviour of the code bias, please refer to Refs. [4] and [2].

By using the broadcast navigation data, the approximate value of \(dT_k\) for \(\delta T_k\) can be computed. Denote the correction to \(dT_k\) by \(\delta T_k\) and represent the combination of receiver-clock bias, correction to the approximate value of satellite-clock bias, tropospheric delay and effect of ephemeris error by \(S_k\), i.e.

\[
S_k = c(dT_k - \delta T_k) + T_k + E_k
\]

Consider the case that four types of observables, \(L_1\) and \(L_2\) code and carrier, are available. In the following, we use \(P_k, b_k, e_k, \lambda_1, \delta k\) and \(N\) to represent the quantities related to the \(L_1\) frequency and \(P_k, \delta k, e_k, \lambda_2, \delta k\) and \(\tilde{N}\) for those related to the \(L_2\) frequency. In GPS observation equations, the ionospheric delay depends on the frequency of the transmitted signal. Generally, when dual frequency data is processed, only the first-order ionospheric delay is accounted for. The first-order \(L_2\) ionospheric delay is equal to the first-order \(L_1\) delay multiplied by the squared ratio of the \(L_1\) and \(L_2\) frequencies. Therefore, if \(I_k\) denotes the first order ionospheric delay for the \(L_1\) frequency and \(r\) the squared ratio of the \(L_1\) and \(L_2\) frequencies \((r \approx 1.647)\), we arrive at the following system of code and carrier observation equations:

\[
\begin{bmatrix}
P_k - \rho_k + c \cdot dT_k^t \\
\lambda_1 \delta k - \rho_k + c \cdot dT_k^t \\
\lambda_2 \delta k - \rho_k + c \cdot dT_k^t \\
\bar{P}_k - \rho_k + c \cdot dT_k^t
\end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -r & -1 \\ 1 & r & 1 \end{bmatrix} \begin{bmatrix} S_k \\ I_k \\ \lambda_1 N \\ \lambda_2 \tilde{N} \\ b_k \\ \tilde{b}_k \\ e_k \\ \tilde{e}_k \end{bmatrix}
\]

The ambiguities \(N\) and \(\tilde{N}\) are constants in time, provided that no cycle slips occur in the \(L_1\) and \(L_2\) observations respectively. All other parameters are generally changing in time and have to be solved for every epoch. Eq. (4) cannot be used directly because the number of unknowns is larger than the number of observations, regardless of the number of epochs for which data are available. Therefore, we estimate the variations of \(b_k\) and \(\tilde{b}_k\) with respect to \(b_0\) and \(\tilde{b}_0\), respectively.

In order to improve the precision of the estimates of the parameters and to estimate the rate of change of the differential corrections as well as the corrections themselves and to provide redundancy for quality control, we introduce dynamic models for \(S_k, I_k, b_k\) and \(\tilde{b}_k\). It has been shown in (Ref. [4]) that the third order time derivative of \(S_k\) and the second order time derivative of \(I_k\), denoted by \(\dot{S}_k\) and \(\ddot{I}_k\), respectively, can be modelled as zero-mean white noise processes with constant spectral densities \(q_{\dot{S}} (m^2/s^3)\) and \(q_{\ddot{I}} (m^2/s^3)\). The second order time derivatives of \(b_k\) and \(\tilde{b}_k\), denoted by \(\dot{b}_k\) and \(\ddot{b}_k\), can be modelled as zero-mean white noise processes with the same constant spectral density \(q_e (m^2/s^3)\). Therefore, the state vector related to e-
poch \( k (k \geq 0) \) reads
\[
\begin{bmatrix}
\triangledown_k + b_0 \\
\triangledown_k \\
S_k \\
I_k + \frac{\lambda_1 N + b_0}{2} \\
I_k + \frac{\lambda_2 \tilde{N} + b_0}{r + 1} \\
b_k - b_0 \\
\tilde{b}_k - \tilde{b}_0 \\
\hat{b}_k
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
r + 1 \\
r - 1 \\
r - 1 \\
r - 1 \\
r - 1 \\
r - 1
\end{bmatrix}
\begin{bmatrix}
b_k \\
\tilde{b}_k \\
\hat{b}_k
\end{bmatrix}
\begin{bmatrix}
\n_k \\
\n_k \\
\lambda_1 N \\
\lambda_2 \tilde{N} \\
b_k \\
\tilde{b}_k \\
\hat{b}_k
\end{bmatrix}
\begin{bmatrix}
S_k \\
S_k \\
I_k \\
I_k \\
\lambda_1 N \\
\lambda_2 \tilde{N} \\
b_k \\
\tilde{b}_k
\end{bmatrix}
\end{equation}

results in the measurement model
\[
\begin{bmatrix}
\n_k \\
\n_k \\
\lambda_1 n_k \\
\lambda_2 \bar{n}_k \\
\bar{\n}_k \\
\n_k + c \cdot d\theta_k \\
\n_k + c \cdot d\theta_k \\
\n_k + c \cdot d\theta_k
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
1 \\
1 \\
1 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
r + 1 \\
r - 1 \\
r - 1 \\
r - 1 \\
r - 1 \\
r - 1
\end{bmatrix}
\begin{bmatrix}
x_k \\
\eta_k \\
\tilde{\eta}_k \\
\tilde{\eta}_k
\end{bmatrix}
\begin{bmatrix}
\varepsilon_k \\
\varepsilon_k
\end{bmatrix}
\end{equation}

with
\[
E \| e_k \|^2 = 0
\]
\[
\begin{bmatrix}
1 & \frac{1}{2} \Delta t_k^5 \\
1 & \Delta t_k^4 \\
1 & \Delta t_k^3 \\
1 & \Delta t_k^2 \\
1 & \Delta t_k \\
1 & 1
\end{bmatrix} \Phi_{k,k-1} = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\vdots & \vdots \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
x_k = x_{k-1} + T \int_{t_{k-1}}^{t_k} \left\{ \begin{array}{c}
\frac{1}{2} (t_k - t)^2 S(t) \\
(t_k - t) S(t) \\
S(t) \\
(t_k - t) I(t) \\
0 \\
0 \\
(\dot{t}_k - t) \ddot{b}_k \\
\ddot{b}_k \\
0 \\
0
\end{array} \right\} dt
\]

with

\[
E \left[ d_k \right] = 0 \quad (11)
\]

\[
E \left[ d_k^T d_k \right] = \begin{bmatrix}
\delta t_k^T \\
\delta b_k^T
\end{bmatrix} \begin{bmatrix}
\delta Q_{11} & 0 \\
0 & \delta Q_{22}
\end{bmatrix} \begin{bmatrix}
\delta t_k \\
\delta b_k
\end{bmatrix}
\]

where

\[
\delta Q_{11} = \begin{bmatrix}
\frac{\Delta t_k^5}{20} & \text{SYM.} \\
\frac{\Delta t_k^4}{8} & \frac{\Delta t_k^3}{3} \\
\frac{\Delta t_k^3}{6} & \frac{\Delta t_k^2}{2} \\
\Delta t_k & 1
\end{bmatrix}
\]

\[
\delta Q_{22} = \begin{bmatrix}
\frac{\Delta t_k^5}{3} & \text{SYM.} \\
\frac{\Delta t_k^4}{3} & \frac{\Delta t_k^3}{2} \\
\Delta t_k & 1
\end{bmatrix}
\]

Incidentally, an alternative derivation of the above dynamic model was given (Jin, 1995a).

On the basis of the above dynamic and measurement models, the recursive prediction and estimation equations for the states read

\[
\dot{x}_{k|k-1} = \Phi_{k,k-1} \dot{x}_{k-1|k-1} + Q_{d_k} \quad (16)
\]

corresponding covariance matrices for \( \dot{x}_{k|k-1} \) and \( \dot{x}_{k|k} \) respectively

\[
P_{k|k-1} = \Phi_{k,k-1} P_{k-1|k-1} \Phi_{k,k-1}^T + Q_{d_k}
\]

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\]

and

\[
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\]

and

\[
P_{k|k-1} = \Phi_{k,k-1} P_{k-1|k-1} \Phi_{k,k-1}^T + Q_{d_k}
\]

where \( K_k = P_{k|k-1} A^T (Q_y + AP_{k|k-1} A^T)^{-1} \) (18) is the so-called Kalman gain matrix. It will be shown later that the predicted residual vector which is defined as

\[
v_k = y_k - A \dot{x}_{k|k-1} \quad (19)
\]

can play an important role in the process of model testing. Its covariance matrix reads

\[
E \left[ v_k v_k^T \right] = \delta Q_y Q_{v_k} = Q_y + AP_{k|k-1} A^T
\]

(20) (Kailath, 1968), (Teunissen and Salzmann, 1989).

The initial values of the filter state and their covariance matrix can be determined by solving the first three epochs simultaneously by least squares.

As can be seen from the dynamic and measurement models, when \( L_2 \) code observations are not available, this algorithm can still be applied by simply removing the columns and rows related to \( P_k \), \( \ddot{b}_k, \ddot{b}_0 \), or \( \ddot{b}_k \) from the models. Most reference stations are probably equipped with dual frequency receivers, as differential corrections need to be generated, but there may be cases when differential corrections have to be generated by using only single frequency data. Since in the case of only \( L_1 \) code and carrier observables being available it is no longer possible to individually estimate the rates \( \nabla_k, I_k \) and \( b_k \), the above dynamic and measurement models should be adapted by combining the bias \( b_k \) with the differential correction \( \nabla_k \). More
specifically, if the types of observables available are $L_1$ code and carrier, Eqs. (6), (7), (9), (10) and (12) should be replaced by the following ones respectively.

$$
\begin{bmatrix}
\nabla_k + b_k \\
\nabla_k + b_k \\
\mathcal{S}_k \\
I_k + \frac{b_k + \lambda_1 N}{2} \\
I_k + \frac{1}{2} b_k
\end{bmatrix}^T
= 
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
S_k \\
S_k \\
I_k \\
I_k \\
b_k
\end{bmatrix}
$$

$$
\begin{bmatrix}
P_k - \rho_k + c \cdot dt_k^2 \\
\lambda_1 \rho_k - \rho_k + c \cdot dt_k^2
\end{bmatrix} = 
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -2 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\nabla_k + b_k \\
\nabla_k + b_k \\
\mathcal{S}_k \\
I_k + \frac{b_k + \lambda_1 N}{2} \\
I_k + \frac{1}{2} b_k
\end{bmatrix} + 
\begin{bmatrix}
e_k \\
e_k
\end{bmatrix}
$$

$$
E\{e_d e_d^T\} = \delta_k Q_d, Q_d = [\sigma_e^2 \sigma_d^2]
$$

$$
x_k = 
\begin{bmatrix}
1 & \Delta t_k & \frac{1}{2} \Delta t_k^2 \\
1 & \Delta t_k \\
1 & \Delta t_k
\end{bmatrix}
x_{k-1} + T_k^{t_h} \begin{bmatrix}
1 & \frac{1}{2} (t_k - t)^2 S(t) \\
(t_k - t) S(t) \\
\cdots \\
S(t) \\
(t_k - t) I(t) \\
I(t) \\
0 \\
(t_k - t) \tilde{b}_k \end{bmatrix} dt
$$

As is easy to see, this algorithm is also applicable, when only $L_1$ code is available. Therefore, the type of observable used in the algorithm can be $L_1$ code alone or along with other code and carrier observables.

### 3 Performance of quality control

The Kalman filter produces recursively optimal estimators of the state vector with well defined statistical properties. When the input data of the filter are normally distributed, the state estimators are unbiased and Gaussian distributed, and have minimum variance within the class of linear unbiased estimators. It is important to realize, however, that optimality is only guaranteed as long as the assumptions underlying the mathematical model hold. Misspecifications in the model will invalidate the results of the estimation and thus also any conclusions based on them. It is therefore of importance to have ways to verify the validity of the working hypothesis, denoted by $H_0$, made for the dynamic model and measurement model. The DIA testing procedure (Teunissen, 1990a) consists of
three steps:

i) Detection: An overall model test is carried out to detect if an unspecified model error has occurred.

ii) Identification: In case an unspecified model error is detected, various alternative hypotheses are evaluated to identify the most likely error source.

iii) Adaptation: After identification of the most likely error source, the filter is adapted.

3.1 Detection of model errors

Assume that the observation noises are normally distributed and the validation of the null hypothesis \( H_0 \) has been verified up to time \( k - 1 \). What we need to detect at time \( k \) is whether an unspecified model error has occurred at the present time. Therefore, the null and alternative hypotheses are:

\[
\begin{align*}
\mathcal{H}_0^k: E \mid y_k & = A_{k} x_k \\
\mathcal{H}_a^k: E \mid d_k & = x_k - \Phi_{k-1 \mid k-1} x_{k-1} \\
\mathcal{H}_a^k: E \mid y_k & = A_{k} x_k + \nabla y_k \\
\mathcal{H}_a^k: E \mid d_k & = x_k - \Phi_{k-1 \mid k-1} x_{k-1} + \nabla d_k
\end{align*}
\]

or, when expressed in terms of the predicted residuals

\[
\begin{align*}
\mathcal{H}_0^k: v_k & \sim N(0, Q_v) \\
\mathcal{H}_a^k: v_k & \sim N(\nabla y_k, Q_v)
\end{align*}
\]

with \( \nabla v_k = \nabla y_k \) in the case of model errors in the measurement model or \( \nabla v_k = A_k \nabla d_k \) in the case of model errors in the dynamic model.

Assume that the \( m_k \)-dimensional vector \( \nabla v_k \) can be written as

\[
\nabla v_k = C_v \nabla
\]

where \( C_v \) is a known \( m_k \times l \) matrix of full rank \( l \) and \( \nabla \) is an unknown model error vector of dimension \( l \). The appropriate test statistic for testing \( H_0 \) against \( H_a \) reads then (Teunissen and Salzmann, 1989)

\[
T^k = v^T Q_v^{-1} C_v \left[ C_v^T Q_v^{-1} C_v \right]^{-1} C_v^T Q_v^{-1} v_k
\]

which is distributed under \( H_0 \) and \( H_a \) as

\[
H_0: T^k \sim \chi^2(l, 0) \quad \text{and} \quad H_a: T^k \sim \chi^2(l, \lambda)
\]

with noncentrality parameter

\[
\lambda = \nabla^T C_v^T Q_v^{-1} C_v \nabla
\]

In most cases, it is impossible to affirm if the class of the alternative hypotheses specified by \( C_v \) indeed contains the true hypothesis. In order to test the overall validity of the local hypothesis \( H_0^k \), the mean \( \nabla v_k = C_v \nabla \) of \( v_k \) under \( H_0^k \) should remain completely unspecified. This implies mathematically that the matrix \( C_v \) should be chosen to be a square and regular matrix. Thus \( C_v \) can be eliminated from Eq. (24), which results in the local overall model (LOM) test statistic

\[
T^k = v^T Q_v^{-1} v_k
\]

In case

\[
T^k \geq \chi^2(m_k, 0)
\]

we may reject the null hypothesis \( H_0 \) of Eq. (21) at the confidence level of \( 1 - \alpha \) and consider that an unspecified local model error is present at time \( k \).

3.2 Identification of model errors

The next step after detection is the identification of the most likely alternative hypothesis. As with detection, identification is based on the test statistic Eq. (24). For identification, however, candidate alternative hypotheses need to be specified explicitly. In the following, the discussion is restricted to model errors in the measurement model. The theory is, however, applicable for the case of model errors in the dynamic model as well. For the case that the local alternative hypothesis \( H_a^k \) of Eq. (21) is restricted to the measurement model, we denote \( C_v \) by \( C_k \). It follows from Eq. (21) that the local alternative hypothesis reads

\[
H_a^k: E \mid y_k = A_{k} x_k + C_k \nabla
\]

This class of alternative hypotheses can be considered to model a slip in the mean of the vectors of observables at time \( k \). The dimension of the \( l \)-dimensional vector \( \nabla \) in Eq. (29) depends on the alternative hypotheses considered and can range from 1 to \( m_k \) for identification purposes. Here we consider the case \( l = 1 \), that is, the case of a single model error. But the theory is also applicable to the case of multiple model errors (Teunissen, 1990b).

With \( l = 1 \), the model error vector \( \nabla \) reduces to a scalar and the matrix \( C_k \) reduces to an \( m_k \)-dimensional vector. Choose

\[
C_i = (0 \cdots 0 1 0 \cdots 0)^T
\]

element \( i \)

In follows from taking the square-root of Eq. (24) that
Generation of Differential GPS Corrections along with Performance of Quality Control

This is the local slippage test statistic for the identification of a single local model error. The identification step is based on the so-called conventional alternative hypothesis or data snooping (Baarda, 1968) in combination with the local slippage test statistic. The model error which is tested is

$$\nabla_i = c_i\nabla$$

(32)

For $m_k$ predicted residuals, we have $m_k$ local slippage test statistics Eq. (31). The alternative hypothesis for which $|t^i|$ is at a maximum is then considered as the one that contains the most likely model error. The $j$ is determined by

$$j = \{i \mid |t^i| = \max \{ |t^1|, \ldots, |t^m_k| \} \}$$

(33)

and note that $t^i$ has the following distribution

$$t^i \sim \begin{cases} N(0,1) & \text{under } H_0 \\ N(\nabla \cdot \sqrt{c_i^T \mathbf{Q}_{i-k}^{-1}} , 1) & \text{under } H_0 \end{cases}$$

(34)

In case

$$|t^j| \geq N_{2_2} (0,1)$$

(35)

where $N_{2_2} (0,1)$ is the upper $\frac{\alpha}{2}$ -percentage point of the standard normal distribution, one may accept the alternative hypothesis $H_e$ of Eq. (29) at confidence level of $1 - \alpha$ and consider predicted residual $j$ in $v_k$ to be the most likely predicted residual containing the detected model error. Since other residuals may also contain unspecified model errors, remove predicted residual $j$ and corresponding covariances from $v_k$ and $\mathbf{Q}_{v_k}$, respectively, and repeat the same procedure as above until Eq. (35) is not fulfilled for any of the remaining predicted residuals any more.

### 3.3 Adaptation

After identification of the most likely alternative hypotheses, the recursive filter needs to be adapted. Consider the general case that $L_1$ and $L_2$ code and carrier observables are available, i.e. $m_k = 4$. It will be shown later that the adaptation procedure can be applied to any case, as long as $L_1$ code observable is available alone or along with other observables.

Depending on the accepted alternative hypotheses, see Eq. (34), the adaptation can be carried out as follows.

- $j = 1$ or $j = 4$ This is most likely corresponding to the situation that the $L_1$ and/or $L_2$ code measurement contains an outlier. In this case one can simply eliminate predicted residual $j$ from the residual vector $v_k$ and then continue the measurement update by using the remaining residuals.

- $j = 2$ or $j = 3$ This happens most likely due to the occurrence of a cycle slip or an outlier in the $L_1$ and/or $L_2$ carrier measurement. When a cycle slip occurs in a carrier measurement, a new ambiguity is introduced. As can be seen from the measurement model, the $L_1$-carrier and $L_2$-carrier related states are

$$I_k + \frac{\lambda_1 N + b_0}{2}$$

and

$$I_k + \frac{\lambda_2 N + b_0}{r - 1}$$

which contain their own ambiguities respectively. In this case the $L_1$-carrier or $L_2$-carrier related states should be reinitialized and the filter can continue the measurement update by using the other unbiased predicted residuals. It can be expected that if the unspecified model error is an outlier instead of a cycle slip, this adaptation will be repeated in the following epoch.

- $j = 1, j = 2, j = 3$ and $j = 4$ Since all predicted residuals are considered to have an unspecified model error, it is probably caused by a slip in the state of the differential correction $\nabla_k$. In this case, the states $\nabla_k + b_0, I_k + \frac{\lambda_1 N + b_0}{2}$, and $I_k + \frac{\lambda_2 N + b_0}{r - 1}$, $b_k - b_0$ and $\tilde{b}_k - b_0$ should be reinitialized.

We have discussed how to adapt the recursive filter in such three cases that an unspecified model error occurs in the code measurement, in the carrier measurement, and in all measurements. Apparently if unspecified model errors occur in some of the code and carrier measurements, the adaptation of the recursive filter can still be done in the same way. In short, if an unspecified model error is identified in a code predicted residual, then exclude the residual in the measurement update; if an unspecified model error is identified in a carrier predicted residual, then initialize its carrier ambiguity related state; but if an unspecified model error is identified in all measurements, then reinitialize all states related to the measurements.

### 3.4 Numerical results

Cycle slips are probably the most likely model er-
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To appear in carrier observations. In order to see if such errors can be indeed successfully detected and identified in the model testing procedure in practice, we conducted five testing experiments with different size and combinations of simulated errors in $L_1$ and $L_2$ carrier observations. The data used in the experiments was collected by a TurboRogue SNR-8000 receiver on 30 June 1993 at a known station in Kootwijk, The Netherlands. It consists of four types of observations: P code and carrier on $L_1$ and $L_2$. The sampling interval of the data was one second and an external rubidium clock with stability of $10^{-11}$ was used during the data collection. Based on the processing results of the data, it was concluded that SA was present in most Block II satellites during data collection; Anti-Spoofing (AS) was not active. In the processing of the data, the priori standard deviations of code and carrier observations were chosen to be 0.3 and 0.003 m, and the spectral densities of the dynamic noises of ionosphere-free differential corrections, ionospheric delays and code biases were chosen to be $10^{-5} \text{m}^2/\text{s}^5$, $10^{-8} \text{m}^2/\text{s}^3$ and $10^{-3} \text{m}^2/\text{s}^3$, respectively.

Table 1 shows the information on the simulated error(s) and the LOM testing result of each experiment as well as a LOM testing result without any simulated errors. Experiment 1 is concerned with a single model error with one cycle slip in the $L_1$ carrier observation, while experiments 2 and 3 are related to two model errors of the same or opposite signs with one in the $L_1$ carrier and the other in the $L_2$ carrier. Since the wave lengths of $L_1$ and $L_2$ carrier are not the same due to the ratio of $L_1$ and $L_2$ frequencies being $77/60$, experiments 4 and 5 were designed for testing a special combination of cycle slips in the $L_1$ and $L_2$ so that they could result in model errors with the same size and the same or opposite signs. The last experiment was a test without any simulated errors. As can be seen from Table 1, all the simulated cycle-slip model errors were indeed correctly detected, no matter they occurred as single model errors, multi model errors, or some special combinations. Actually, they were also successfully identified. Therefore, the real time model testing approach can indeed work well with cycle slip model errors.

| experiment number | simulated errors (cycles) L_1 | L_2 | $T^*_\text{LOM}$ |
|------------------|-------------------------------|-----|-----------------|
| 1                | 1                             | 0   | 477.57          |
| 2                | 1                             | 1   | 298.53          |
| 3                | 1                             | -1  | 2211.69         |
| 4                | -77                           | 60  | 1 214 956.50    |
| 5                | 77                            | 60  | 10 016 474.62   |
| 6                | 0                             | 0   | 0.02            |

Note: with the choices of $\alpha=0.001$ and $\gamma=0.8$, the critical value $F_{0.001}(4, \infty, 0) = 3. 38$

4 Discussion

Compared to other published algorithms for generating differential corrections, this new algorithm has some distinct features. First of all, this algorithm directly uses code and carrier observations in the measurement model of a Kalman filter, so that the measurements do not become correlated in time, as is the case when differences of observations at subsequent epochs are formed or when carrier filtered code observations are used. This makes it possible to use a simple stochastic model and to use the standard algorithm of the Kalman filter. Secondly, the algorithm accounts for biases like multipath errors and instrumental delays in code observations. It can be applied in the case that code biases are significantly present or absent. In the former case, the effect of a time-independent code bias at the initial epoch on the estimates of differential corrections can clearly be shown, whereas in the latter case, the algorithm can generate unbiased estimates of differential corrections. Thirdly, the algorithm can be integrated with a quality control procedure. Therefore, the quality of the estimated states including differential corrections can be guaranteed with a certain probability. Finally, all of its state estimates are not affected by the opposite influence of the ionosphere on code and carrier observations.

The three ionospheric-delay related states $I_k = I_1 + \frac{\lambda_1 N + b_0}{2}, I_k + \frac{\lambda_2 N + b_0}{r + 1},$ and $I_k - \frac{b_0 - b_0}{r - 1}$ in the state vector $x_k$ can be used to analyze the variation of ionospheric delays. Often dual-frequency GPS code observations are used to estimate the first or-
nder ionospheric delay. Strictly speaking, it should only be done in the case that \( L_1 \) and \( L_2 \) code biases are absent or the same or negligible. In general, by using dual frequency GPS code observations, one can only estimate the ionospheric delay contaminated by code biases \( b_k \) and \( \bar{b}_k \) (or \( b_0 \) and \( \bar{b}_0 \)), like the state \( I_k = \frac{b_0 - \bar{b}_0}{r - 1} \).

The time-varying states \( S_k, I_k, b_k \) and \( \bar{b}_k \) are affected by different factors. The ionosphere-free differential correction \( S_k \) is related with receiver and satellite clock biases, ephemeris error and tropospheric condition. The ionospheric delay \( I_k \) is related with the condition of the ionosphere. The code biases \( b_k \) and \( \bar{b}_k \) are related with receiver, antenna and the observation environment.

The greater the number of observables available, the better the other information like the variations of ionospheric delays and code biases can be obtained along with differential corrections. There are some essential differences in the estimation of differential corrections and their rates of changes when using dual frequency data and using single frequency data. In the former case, the estimates of differential corrections at all epochs are biased by a constant which is the initial value of \( L_1 \) code bias (see Eq. (7)); those of the rates of change of differential corrections are not biased. In the latter case, the estimates of differential corrections and their rates of changes are biased by \( L_1 \) code biases and their rates of changes at the same epochs (see Eq. (7')), respectively. Thus, the quality of the estimates of rates of change of differential corrections based on dual frequency data is much better than that based on single frequency data. It should be noted that when code biases become negligible, this algorithm is still usable after simply removing the columns and rows related to code biases and their rates of changes. The estimates of both differential corrections and their rates of change are then no longer biased.

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(Continued on page 78)