A study of colour field distributions in the baryon

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The distributions of chromo-electric and chromo-magnetic field associated with flux tubes in the baryon are studied in SU(3) lattice QCD. Maximal Abelian projection is used to reduce the statistical fluctuations. For a fixed source geometry, many different string configurations are possible. We investigated whether the string configuration, that is the choice of operator, biases the observed flux distribution.

1. Introduction

Recently, the problem of how chromo-electric and chromo-magnetic fields are distributed in a baryon has been of considerable interest. It has been investigated in terms of the potential \cite{1,2}. But the flux distribution can be calculated directly using lattice QCD. There are many studies for a meson \cite{3,4,5}, but only a few for a baryon. The first attempt to do this was by Flower\cite{6} but it was very noisy and not conclusive. However, in the past year, Ichie et al.\cite{7} have presented very nice results using Abelian projected fields.

Generally speaking, all lattice simulations have some systematic errors and it is important to have some estimate of these errors in order to assess the results. What are the sources of systematic error in the map of flux distribution?

One is interested in the shape of the flux distribution. However, in order to do the calculation some choice for the three quark operator has to be made. This operator consists of three quarks connected by strings of gauge field links which assume some shape. The systematic effect which we study here is whether the shape of the operator influences the measured flux distribution.

Of course, the ground state property in a lattice simulation should be independent of the operator. But the conditions needed to obtain this ideal situation may be different for different quantities and may be difficult to achieve in practice. This can be seen by considering the time dependence of the two-point and three-point correlation functions.

For the two-point function

\[
\langle \Omega | O(T) O(0) | \Omega \rangle = \sum_n e^{-E_n T} \langle \Omega | O(0) | n \rangle \langle n | O(0) | \Omega \rangle \rightarrow e^{-E_0 T} \langle \Omega | O(0) | 0 \rangle \langle 0 | O(0) | \Omega \rangle. \tag{1}
\]

For the three-point function \((T > t)\)

\[
\langle \Omega | O(T) P(t) O(0) | \Omega \rangle = \sum_{n,n'} e^{-E_n (T-t)} e^{-E_{n'} t} \times \langle \Omega | O(0) | n \rangle \langle n | P(0) | n' \rangle \langle n' | O(0) | \Omega \rangle \rightarrow e^{-E_0 T} \langle \Omega | O(0) | 0 \rangle \langle 0 | P(0) | 0 \rangle \langle 0 | O(0) | \Omega \rangle. \tag{2}
\]

The generic form of the correlation function which gives the field distribution is

\[
\frac{\langle \Omega | O(T) P(t) O(0) | \Omega \rangle}{\langle \Omega | O(T) O(0) | \Omega \rangle} = \langle 0 | P(0) | 0 \rangle - \langle \Omega | P(0) | \Omega \rangle \rightarrow (0 | P(0) | 0) - \langle \Omega | P(0) | \Omega \rangle. \tag{3}
\]

In this case, the condition \(T >> t >> 0\) is required to isolate the ground state contribution. For the two point function, which yields the potential, one only needs \(T >> 0\). If a simulation...
Figure 1. The three quark Wilson loop (a) T-shape string configuration (b)L-shape string configuration.

shows a dependence on the choice of operator this indicates that there are non-ground state contributions.

2. Simulation

We studied the three quark potential and the field distributions. The three quark Wilson loop is defined as

\[ W_{3Q} = \frac{1}{3!} \epsilon_{abc} \epsilon_{a'b'c'} U_1^{ab} U_2^{b'c'} U_3^{c'} \]  

with the path-ordered link variables

\[ U_j \equiv P \exp \left\{ ig \int_{\Gamma_j} dx A^\mu(x) \right\} \quad (j = 1, 2, 3). \]  

The path is denoted by \( \Gamma_j \) in Fig. 1.

The correlation function can be described as

\[ P_{\mu\nu}(x, t) = \frac{\langle P_{\mu\nu}(x, t) W_{3Q}(T) \rangle}{\langle W_{3Q}(T) \rangle} - \langle P_{\mu\nu}(t) \rangle. \]  

Here, \( P_{\mu\nu} \) denotes an unsmeared plaquette, used as a probe of the chromo-electric or chromo-magnetic field. In our simulation the plaquette insertions were made at \( t = T/2 \).

The simulation is done in quenched QCD using the Wilson plaquette action with \( \beta = 6.2 \) on \( 24^4 \) lattices. In order to be able to study systematic effects it is necessary to control statistical errors. The multi-hit procedure was used for temporal links in order to reduce the statistical fluctuations in the potential calculation. In addition, smearing of spatial links is used to enhance the ground state. The flux distribution calculation is very noisy. Ichie et al. [7] have shown that maximally Abelian projection, which preserves the correct long distance behavior of the Wilson loop, can be used very effectively to investigate the flux tube in the baryon. We adopt this method here.

Fig. 1 shows the choice of the operator. Three quarks are located on the \( x - y \) plane with the distance \( k \) from the origin, \( (Q_1(x, y), Q_2, Q_3) = ((-k, 0), (0, k), (k, 0)) \) in (a), \( ((0, 0), (0, k), (k, 0)) \) in (b). The operator (b) has not been considered before.

3. Results and Discussion

Fig. 2 shows the three quark potential for different operators. \( L_{\text{min}} \) describes the total minimum length from the physical junction to each quark. The circles denote the case of three quarks located on the axes at the same distance \( k \) from
the origin, while the triangles and the squares show the quark geometries of Fig.1 (a) and (b) respectively. There is no evidence for any operator dependence.

Figure 3. The action density (a) T-shape string configuration (b) L-shape string configuration.

Fig. 3 shows the action density for operators with different string configurations. The three quark positions, ((-6,0),(0,6),(6,0)) in (a) and ((8,0),(0,0),(0,8)) in (b) are chosen since the inter-quark separations are similar. \( T = 8 \) is considered since it is large enough to extract the ground state for the potential. There a visible difference between the shapes of the distributions which are still biased by the form of the operator. This operator dependence suggests that excited states are still contributing.

Ideally, the physical ground state property in a lattice simulation should be independent of the choice of the operator. For the three quark potential, this seems to be readily achievable. However, since the flux distribution contains the three-point function, it requires large time separation not just between the quark source and sink but also between the source and sink and the flux probe. These conditions are not easy to meet and in our simulations, operator dependence in the flux distribution is visible. It is important for future calculations to check for this systematic effect, especially if it is claimed that the measured flux distribution has the same shape as the strings contained in the baryonic operator.

To reduce the systematic error discussed here, even more effective techniques to reduce statistical fluctuations would be helpful. This would allow calculations at larger time separations. Alternatively, operators without strings, as used, for example, by the MILC Collaboration\[8\] to calculate the quark-antiquark potential, might be useful.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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