Massless QCD$_2$ From Current Constituents

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Abstract

We discuss the spectra of multi-flavor massless QCD$_2$. An approximation in which the Hilbert space is truncated to two currents states is used. We write down a 't Hooft like equation for the wave function of the two currents states. We solve this equation for the lowest massive state and find an excellent agreement with the DLCQ results. In addition, the 't Hooft model and the large $N_f$ limit spectra are re-derived by using a description in terms of currents.
1 Introduction

Two dimensional quantum chromodynamics ($\text{QCD}_2$) is a useful toy model for the real world QCD. The large $N_c$ model was solved by ’t Hooft[1] and shows confinement of quarks with an approximately linear Regge trajectory of states. Other issues, such as the baryonic spectrum at strong coupling[2] and questions of screening versus confinement[3, 4] can also be addressed in this framework.

The two-dimensional model with fermions in the adjoint representation is also interesting and attracted a lot of attention in recent years[5, 6, 7, 8, 9, 10, 11, 12, 13]. In particular it was shown in [8] that the adjoint fermions model is equivalent to $\text{QCD}_2$ model with level $N_f = N_c$, for the massive part of the spectrum, in the case of massless fermions. Another attempt to address the adjoint fermions model, was by using the currents as building blocks of the spectrum [9].

The idea of the present work is to study the spectrum of $\text{QCD}_2$ at arbitrary level $N_f$ using states built from two currents, for the case of massless fermions.

Our goal in this work is two folded: (i) To derive a ’t Hooft like equation for the wave function of the “currentball” states for $\text{QCD}_2$ models at arbitrary level $N_f$. The equation should interpolate between the description of a single flavor model (’t Hooft model), the adjoint fermions model $N_f = N_c$ and the large $N_f$ model. (ii) To solve the equation for the lowest massive state.

Whereas the ’t Hooft model $N_f = 1$ is exactly solvable, the multi-flavor case $N_f > 1$ is not solvable model even in the Veneziano limit when both $N_c$ and $N_f$ are taken to infinity (with a fixed ratio), since pair creation and annihilation is not suppressed. In the present work, we use an approximation in which we restrict ourselves to two currents states. We cannot justify a-priori such an approximation for arbitrary level. However the numerical solutions for the lowest massive state admit a very close resemblance to the DLCQ results where such a truncation was not used [11, 12, 13]. A justification can be given for $\frac{N_f}{N_c}$ very small or very large.

The obtained equation suggests the following picture: the underlying degrees of freedom in the problem are interacting “gluons” with mass $\frac{\alpha^2 N_f}{\pi}$. Actually, these are really quanta of the the color currents. As it is well known, there are no independent gluon degrees of freedom in two dimensions.
The organization of the manuscript is as follows. In section 2 the bosonization of multi-flavor QCD is reviewed. In section 3 we state the problem of diagonalizing the mass operator in terms of currents and arrive at a 't Hooft like equation for the currents wavefunction. Section 4 is devoted to a solution of the differential equation and for a discussion in some specific cases, such as the 't Hooft limit, the $N_f \gg N_c$ case and the adjoint fermions ($N_f = N_c$) case. Section 4 is a summary and a discussion.

2 Massless QCD$_2$ and Bosonization

Massless multi-flavor QCD$_2$ with fermions in the fundamental representation of $SU(N_c)$ is described by the following action

$$S = \int d^2 x \, \text{tr} \left( -\frac{1}{2\epsilon^2} F_{\mu\nu}^2 + i\bar{\Psi} \not{D} \Psi \right)$$

where $\Psi = \Psi_a^i$, $i = 1 \ldots N_c$, $a = 1 \ldots N_f$.

It is natural to bosonize this system, since bosonization in the $SU(N_c) \times SU(N_f) \times U_B(1)$ scheme decouples color and flavor degrees of freedom (in the massless case). The bosonized form of the action of this theory is given by (14)

$$S_{\text{bosonized}} =$$

$$N_f S_{\text{WZW}}(h) + N_c S_{\text{WZW}}(g) + \int d^2 x \, \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \int d^2 x \, \frac{1}{2\epsilon^2} F_{\mu\nu} F^{\mu\nu}$$

$$- \frac{N_f}{2\pi} \int d^2 x \, \text{tr} \left( i h^\dagger \partial_+ h A_- + i h \partial_- h^\dagger A_+ + h A A^\dagger - A_+ A_- \right)$$

where $h \in SU(N_c)$, $g \in SU(N_f)$, $\phi$ is the baryon number and $S_{\text{WZW}}$ stands for the Wess-Zumino-Witten action, which for complex fermions reads

$$S_{\text{WZW}}(g) = \frac{1}{8\pi} \int_{\Sigma} d^2 x \, \text{tr} \left( \partial_\mu g \partial^\mu g^{-1} \right) +$$

$$\frac{1}{12\pi} \int_B d^3 y \epsilon^{ijk} \text{tr} \left( g^{-1} \partial_i g \right)\left( g^{-1} \partial_j g \right)\left( g^{-1} \partial_k g \right),$$

Since we are interested in the massive spectrum of the theory and the flavor degrees of freedom are entirely decoupled from the system and they are massless, we can put aside the $g$ and $\phi$ fields (There is a residual interaction
of the zero modes of the $g, h$ and $\phi$ fields, but it is not important to our discussion[8]).

Upon choosing the light cone gauge $A_-=0$ and integrating $A_+$ we arrive to the following action

$$ S = N_f S_{ZW}(h) - \frac{1}{2} e^2 \int d^2 x \; \text{tr} \left( \frac{1}{\partial_-} J^+ \right)^2, \quad (3) $$

where $J^+ = \frac{i N_f}{2\pi} h \partial_- h^\dagger$. In terms of $J = \sqrt{\pi} J^+$, the light-cone momentum operators $P^\mu$ take the following simple form

$$ P^+ = \frac{1}{N_c + N_f} \int dx^- : J^a(x^-, x^+ = 0) J^a(x^-, x^+ = 0) :; \quad (4) $$

namely, the Sugawara form, and

$$ P^- = -\frac{e^2}{2\pi} \int dx^- : J^a(x^-, x^+ = 0) \frac{1}{\partial_-} J^a(x^-, x^+ = 0) :. \quad (5) $$

Our task will be to solve the eigenvalue equation

$$ 2P^+ P^- |\psi\rangle = M^2 |\psi\rangle. \quad (6) $$

We write $P^+$ and $P^-$ in terms of the Fourier transform of $J(x^-)$ defined by $J(p^+) = \int \frac{dx^-}{\sqrt{2\pi}} e^{-ip^+ x^-} J(x^-, x^+ = 0)$. Normal ordering in the expression of $P^+$ and $P^-$ are naturally with respect to $p$, where $p < 0$ denotes a creation operator. To simplify the notation we write from now on $p$ instead of $p^+$. In terms of these variables the momenta generators are

$$ P^+ = \frac{1}{N_c + N_f} \int_0^\infty dp J^a(-p) J^a(p) \quad (7) $$

$$ P^- = \frac{e^2}{\pi} \int_0^\infty dp \frac{1}{p^2} J^a(-p) J^a(p) \quad (8) $$

Recall that the light-cone currents $J^a(p)$ obey a level $N_f$, $SU(N_c)$ affine Lie algebra

$$ [J^a(p), J^b(p')] = \frac{1}{2} N_f \, p \, \delta^{ab} \delta(p + p') + i f^{abc} J^c(p + p') \quad (9) $$

We can now construct the Hilbert space. The vacuum $|0, R\rangle$ is defined by the annihilation property:

$$ \forall p > 0, \; J(p) \; |0, R\rangle = 0 \quad (10) $$

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Where $R$ is an “allowed” representations depending on the level. Therefore, a physical state in Hilbert space is $|\psi\rangle = \text{tr} J(-p_1) \ldots J(-p_n) |0, R\rangle$. Note that this basis is not orthogonal.

3 't Hooft like equation for the two currents wave-function

Let us restrict ourselves to the 2-currents sector of the Hilbert space

$$|\Phi\rangle = \frac{1}{N_c N_f} \int_0^1 dk \; \Phi(k) J^a(-k) J^a(k-1) |0\rangle,$$

namely to states which are color singlets of two currents with total $P^+ = 1$ momentum and a distribution of $P^−$ momentum $\Phi(k)$. Note that $\Phi$ is a symmetric function

$$\Phi(k) = \Phi(1 - k).$$

Our task now is to find the eigenvalue (Schrödinger) equation for the wavefunction $\Phi(k)$. Let us act with the “Hamiltonian” $P^−$ on the state $|\Phi\rangle$.

The commutator of $P^−$ with a current $J^b(-k)$ yields the following result

$$\left[ \int_0^\infty \frac{dp}{p^2} J^a(-p) J^a(p), J^b(-k) \right] =$$

$$\left( \frac{1}{2} N_f - N_c \right) \frac{1}{k} J^b(-k) +$$

$$\int_0^\infty dp \left( \frac{1}{p^2} - \frac{1}{(p-k)^2} \right) i f^{abc} J^a(-p) J^c(p-k) +$$

$$\int_0^k \frac{dp}{p^2} i f^{abc} J^c(p-k) J^a(-p).$$

(13)

The above expression (13) contains 3 terms on the R.H.S. The first term contains a single creation operator. The second term contains an annihilation current and therefore should be again commuted with $J^b(k-1)$. The third term contains two creation currents and it would lead to a 3-currents state. Namely, the affine Lie algebra created a higher state. This is a manifestation of the fact that pair creation is, generically, not suppressed in multi-flavor QCD$_2$, as expected in general in QFT.
Note that while deriving eq. (13) we get an “infinite” contribution $N_c \frac{1}{2} J^b(-k)$. This contribution will be canceled by a counter contribution which comes from the regime $p \sim k$ in the first integral on the R.H.S. of (13), as below.

The commutator of the second term in the R.H.S of (13) with $J^b(k - 1)$ yields

$$\left[ \int_k^\infty dp \left( \frac{1}{p^2} - \frac{1}{(p-k)^2} \right) i f^{abc} J^a(-p) J^c(p-k), J^b(k-1) \right] =$$

$$N_c \int_k^\infty dp \left( \frac{1}{p^2} - \frac{1}{(p-k)^2} \right) (J^a(-p) J^a(p-1) - J^a(p-k) J^a(k-p-1)).$$

Our results can be summarized by the following set of equations

$$M^2 |\Phi\rangle =$$

$$\frac{1}{N_c N_f} \int_0^1 dk \, \tilde{\Phi}(k) J^a(-k) J^a(k-1) |0\rangle +$$

$$\frac{1}{(N_c N_f)^{\frac{3}{2}}} \int_0^1 dk \, dp \, dl \, \delta(k + p + l - 1) \Psi(k, p, l) i f^{abc} J^a(-k) J^b(-p) J^c(-l) |0\rangle$$

with

$$\Psi(k, p, l) = \frac{2e^2(N_c N_f)\frac{1}{2}}{\pi} \left( \frac{\Phi(l) - \Phi(k)}{p^2} \right)$$

and

$$\tilde{\Phi}(k) = \frac{e^2}{\pi} \left( (N_f - N_c) \left( \frac{1}{k} + \frac{1}{1-k} \right) \Phi(k) + \frac{2N_c}{\epsilon} \Phi(k) \right.$$  

$$- N_c \int_0^{k-\epsilon} dp \frac{\Phi(p)}{(p-k)^2} - N_c \int_{k+\epsilon}^1 dp \frac{\Phi(p)}{(p-k)^2} + N_c \left( \frac{1}{k^2} - \frac{1}{(1-k)^2} \right) \int_0^k dp \, \Phi(p) \right).$$

Ignoring the 3-currents term (see below), we get that $\Phi(k)$ obeys the following eigenvalue equation

$$\frac{M^2}{e^2/\pi} \Phi(k) = (N_f - N_c) \left( \frac{1}{k} + \frac{1}{1-k} \right) \Phi(k)$$

$$- N_c \mathcal{P} \int_0^1 dp \frac{\Phi(p)}{(p-k)^2} + N_c \left( \frac{1}{k^2} - \frac{1}{(1-k)^2} \right) \int_0^k dp \, \Phi(p).$$
For general $N_c$ and $N_f$ discarding the 3-currents term is unjustified. However, since the length of $\Psi$ is $|\Psi(k, p, l)| \sim e^2(N_c N_f)^{1/2}$, in the limit of large $N_c$ with fixed $e^2 N_c$ and fixed $N_f$, or large $N_f$ with fixed $e^2 N_f$ and fixed $N_c$, the 3-currents contribution is indeed negligible, as compared with the 2-currents term, the latter being of order 1. Note also that while deriving eq. (17) we assumed that $\int_0^1 dp \Phi(p) = 0$. We will justify this assumption in the following.

Another remark is that the first integral in (17) should be calculated as a principal value integral. The divergent part of this integral (arising from the regime $p \sim k$) cancels the previously mentioned infinity.

In order to make contact with the ordinary 't Hooft equation, it is useful to integrate equation (17) with respect to $k$ and rewrite the equation in terms of $\varphi(k) \equiv \int_0^1 dp \Phi(p)$.

$$\frac{M^2}{e^2 \pi} \varphi(k) = (N_f - N_c) \left( \frac{1}{k} + \frac{1}{1 - k} \right) \varphi(k) - N_c \mathcal{P} \int_0^1 dp \frac{\varphi(p)}{(p - k)^2} + N_f \int_0^k dp \frac{\varphi(p)}{p^2} + N_f \int_k^1 dp \frac{\varphi(p)}{(1 - p)^2}$$

(18)

The derivation goes as follows. First, integrating eq. (12) we get

$$\varphi(k) = -\varphi(1 - k) + \text{const.}$$

Taking $\varphi(1) = 0$ we get

$$\varphi(k) = -\varphi(1 - k).$$

(19)

Now $\varphi(1) = 0$ implies $\int_0^1 dk \Phi(k) = 0$, which was our assumption above. Then, differentiating (18) we do get (17), and by (19) we also get that there is no extra integration constant.

We would like to comment on the issue of the Hermiticity of the 'Hamiltonian' $M^2$. Naively, it seems that $M^2$ is not Hermitian with respect to the scalar product $<\psi|\varphi> = \int_0^1 dk \psi^*(k) \varphi(k)$, since the Hermitian conjugate of (18) is

$$\left( \frac{M^2}{e^2 \pi} \right)^{\dagger} \varphi(k) = (N_f - N_c) \left( \frac{1}{k} + \frac{1}{1 - k} \right) \varphi(k) - N_c \mathcal{P} \int_0^1 dp \frac{\varphi(p)}{(p - k)^2} - N_f \frac{1}{k^2} \int_0^k dp \varphi(p) - N_f \frac{1}{(1 - k)^2} \int_k^1 dp \varphi(p)$$

(20)

However, as we shall see in the next section, the numerical solution yields real eigenvalues and eigenfunctions. Therefore, at least on the subspace which
is spanned by the eigenfunctions, namely real functions which are zero at
\( k = 0,1 \) and anti-symmetric with respect to \( k = \frac{1}{2} \), the operator \( M^2 \) is
Hermitian. Note that (20) is “more regular” than (18), as in (18) it is \( \varphi(p)/p^2 \)
that appears in the integration from zero.

Equation (18) is similar to ’t Hooft equation for a massive single flavor
large \( N_c \) QCD, with \( m^2 = \frac{\pi^2 N_f}{3} \). It differs from ’t Hooft’s equation by having
two additional terms (two last terms in (18)). It suggests that the dynamics
which governs the lowest state of the multi-flavor model is given, approxi-
mately, by a model of massive “glueball” with an \( SU(N_c) \) gauge interaction
and additional terms which are proportional to \( N_f \).

Before we present our solution of (18) it is important to note that it is only
an approximated solution. We neglected the 3-currents state with, a-priori,
no justification. We shall see, however, that the restriction to the truncated
2-currents sector is an excellent approximation for the lowest massive meson.

4 The spectrum - numerical results

The most convenient way to solve (18) is to expand \( \varphi(k) \) in the following
basis (see, however [15], a different interesting choice of basis)

\[
\varphi(k) = \sum_{i=0}^{\infty} A_i (k - \frac{1}{2}) (k(1 - k))^{\beta+i}
\]  

(21)

The value of \( \beta \) chosen such that the Hamiltonian will not be singular near
\( k \to 0 \) (or \( k \to 1 \)) [1],[14]. This consideration leads to the following equation

\[
\left( \frac{N_f}{N_c} - 1 \right) - \frac{N_f}{\beta + 1} \frac{N_c}{N_f} + \beta \pi \cot \beta \pi = 0.
\]  

(22)

This comes from eq.(21). Had we started with (18), it would have been \(-\beta \)
replacing \( \beta \) in (22), and constrained to \( \beta \) larger than 1. Upon truncating
the infinite sum in (21) to a finite sum, the eigenvalue problem reduces to a
diagonalization of a matrix. So, the problem can be reformulated as follows

\[
\lambda N_{ij} A_j = H_{ij} A_j,
\]  

(23)

with

\[
N_{ij} = \int_0^1 dk (k - \frac{1}{2})^2 (k(1 - k))^{2\beta+i+j},
\]  

(24)
and

\[ H_{ij} = \left( \frac{N_f}{N_c} - 1 \right) \int_0^1 dk (k - \frac{1}{2})^2 (k(1 - k))^{\beta+i+j-1} \]

\[ - \frac{N_f}{N_c} \int_0^1 dk (k - \frac{1}{2}) (k(1 - k))^{\beta+i} \frac{1}{k^2} \int_0^k (p - \frac{1}{2}) (p(1 - p))^{\beta+j} \]

\[ - \frac{N_f}{N_c} \int_0^1 dk (k - \frac{1}{2}) (k(1 - k))^{\beta+i} \frac{1}{(1 - k)^2} \int_1^k (p - \frac{1}{2}) (p(1 - p))^{\beta+j} \]

\[ - \int_0^1 dkdp \frac{(k - \frac{1}{2}) (k(1 - k))^{\beta+i} (p - \frac{1}{2}) (p(1 - p))^{\beta+j}}{(k - p)^2} \]  

(25)

Hence

\[ N_{ij} = \frac{B(2\beta + i + j + 2, 2\beta + i + j + 2)}{2(2\beta + i + j + 1)}, \]  

(26)

and

\[ H_{ij} = \left( \frac{N_f}{N_c} - 1 \right) \frac{B(2\beta + i + j + 1, 2\beta + i + j + 1)}{2(2\beta + i + j)} \]

\[ - \frac{N_f B(2\beta + i + j + 1, 2\beta + i + j + 1)}{N_c 2(2\beta + i + j)(\beta + j + 1)} \]

\[ + \frac{(\beta + i)(\beta + j)B(\beta + i, \beta + i)B(\beta + j, \beta + j)}{8(2\beta + i + j)(2\beta + i + j + 1)} \]  

(27)

where \( B(x, y) \) is the Beta function

\[ B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x + y)}. \]  

(28)

In practice, the process converges rapidly and a 5 × 5 matrix yields the ‘continuum’ results.

The lowest eigenvalues of (18) as a function of the ratio \( \frac{N_f}{N_c} \) are listed in table 1 below (see also figure 1). Note that by \( \beta = 0, N_f/N_c = 0 \) we mean the limit \( \beta \to 0, N_f/N_c \to 0. \)
Table 1: The mass of the lowest massive meson, in units of $\frac{e^2 N_c}{\pi}$, as a function of $N_f/N_c$ and $\beta$.

These values are in excellent agreement with recent DLCQ calculations. For comparison see [11], [12] and especially [13] for a recent work. The typical error is less than 0.1%!

An interesting observation is that the eigenvalues depend linearly on $N_f$ (see figure 1). The dependence is

$$M^2 = \frac{e^2 N_c}{\pi}(5.88 + 5\frac{N_f}{N_c}).$$

We do not have a good understanding of this observation. It is not clear why the lowest eigenvalue sits on a straight line. It is not clear even why, as an eigenvalue equation (18) exhibits such a behavior.

In the following sections we will consider some special cases.

4.1 $N_f = 1$, The ’t Hooft Model

The limit $N_c \to \infty$ with $e^2 N_c$ fixed and $N_f \ll N_c$ corresponds to the well known ’t Hooft model. In this limit QCD$_2$ was solved exactly long time ago by ’t Hooft[1]. Let us see how our approach coincides with the fermionic basis in this case. In the limit $N_f \ll N_c$ we can neglect terms which are proportional to $N_f$. Equation (18) (or (20)) takes the following form

$$\frac{M^2}{e^2/\pi} \varphi(k) = -N_c \left(\frac{1}{k} + \frac{1}{1-k}\right) \varphi(k) - N_c \mathcal{P} \int_0^1 dp \frac{\varphi(p)}{(p-k)^2},$$

(30)
which is just 't Hooft equation for the massless case. Note that (30) is exact, since in the small $N_f$ limit the 3-currents state is suppressed by $N_c^{-\frac{1}{2}}$ with respect to the 2-currents state and therefore we can neglect it.

Since the wavefunction $\varphi(k)$ is anti-symmetric, we will recover only the odd states in the spectrum of QCD$_2$ (the even states can be recovered by considering other sectors of the Hilbert space which decouple from the 2-currents state).

Though equation (30) is formally the same as 't Hooft equation, the interpretation of $\varphi(k)$ should be different. $\varphi(k)$ is the integral of the function $\Phi(k)$ which corresponds to 2-currents state, namely to a mixture of 4-fermions and 2-fermions. What is the relation between the states that we find here and the mesons in 't Hooft’s model?

In order to answer this question let us expand the currents in terms of fermions. It is useful to denote the current in double index notation

$$J^a(k) \rightarrow J^i_j(k) = \int_{-\infty}^\infty dq \left( \bar{\Psi}^i(q)\Psi_j(k-q) - \frac{1}{N_c}\delta^i_j\bar{\Psi}^k(q)\Psi_k(k-q) \right)$$ (31)

We do not bother about normal ordering, as no problem for $k$ non zero, and we have to treat the $k = 0$ part in a limiting way. The state $|\Phi\rangle$ can be written as

$$|\Phi\rangle = \frac{1}{2N_c} \int_0^1 dk \Phi(k)J^i_j(-k)J^i_j(k-1) |0\rangle =$$

$$\frac{1}{2N_c} \int_0^1 dk \Phi(k) \int_{-\infty}^\infty dq \int_{-\infty}^\infty dp \left( \bar{\Psi}^i(-q)\Psi_j(-k+q) - \frac{1}{N_c}\delta^i_j\bar{\Psi}^k(-q)\Psi_k(-k+q) \right) \times$$

$$\left( \bar{\Psi}^j(-p)\Psi_i(k+p-1) - \frac{1}{N_c}\delta^j_i\bar{\Psi}^k(-p)\Psi_k(k+p-1) \right) |0\rangle.$$ (32)

Note that the above expression (32) contains annihilation and creation fermionic operators. Written in terms of creation operators only, (32) reads

$$|\Phi\rangle = \frac{1}{2N_c} \int_0^1 dk \int_0^k dq \int_0^{1-k} dp \Phi(k)\bar{\Psi}^i(-q)\Psi_j(-k+q)\bar{\Psi}^j(-p)\Psi_i(k+p-1) |0\rangle$$

$$- \frac{1}{2N^2_c} \int_0^1 dk \int_0^k dq \int_0^{1-k} dp \Phi(k)\bar{\Psi}^i(-q)\Psi_i(-k+q)\bar{\Psi}^j(-p)\Psi_j(k+p-1) |0\rangle$$

$$- (1 - \frac{1}{N_c}) \int_0^1 dk \int_0^k dq \Phi(k)\bar{\Psi}^i(-q)\Psi_i(q-1) |0\rangle.$$ (33)
The last term in (33) corresponds to a meson. It can be written also as

\[ \int_0^1 dq \int_q^1 dk \, \Phi(k) \overline{\Psi}^i(-q) \Psi_i(q-1) \left| 0 \right> = - \int_0^1 dq \, \varphi(q) \overline{\Psi}^i(-q) \Psi_i(q-1) \left| 0 \right> \]

which is exactly the 't Hooft meson. We conclude that the 2 currents state has an overlap with the 't Hooft meson and this is why (18) reproduces exactly the (odd part of the) spectrum of the 't Hooft model.

4.2 Large \( N_f \gg N_c \) limit

In the limit \( N_f \gg N_c \), with \( e^2N_f \) fixed, the truncation to 2-currents state should again predict exact results. The reason is that the 3-currents state is suppressed by \( N_f^{-\frac{3}{2}} \) with respect to the 2-currents state.

In this limit eq.(17) takes the form

\[ M^2 = \frac{e^2N_f}{\pi} \left( \frac{1}{k} + \frac{1}{1-k} \right) \]

It describes a continuum of states with masses above \( 2m \), where \( m^2 = \frac{e^2N_f}{\pi} \). The interpretation is clear: in this limit the spectrum of the theory reduces to a single non-interacting meson (or “currentball”) with mass \( m \). This phenomena was already observed in [9] and in [17] by using a different approach.

4.3 \( N_f = N_c \), The Adjoint Fermions Model

The case \( N_f = N_c \) is the most interesting one. It was shown that the massive spectrum of this model is equivalent to the massive spectrum of a model with a single adjoint fermion, due to 'universality' [8]. Since this model is not exactly solvable, it is interesting to see how our approach reproduces, almost accurately, previous numerical results.

The mass of the lowest massive meson, predicted by (18), is \( M^2 = 10.86 \times \frac{e^2N_c}{\pi} \). For comparison, the recent values reported in the literature are \( M^2 = 10.8 \) [11] and \( M^2 = 10.84 \) [13], in units of \( \frac{e^2N_c}{\pi} \).

This agreement is very surprising. In the regime \( N_f \sim N_c \), the 3-currents state is not suppressed by factors of color or flavor with respect to the 2-currents state. Why, thus, is our approach so successful? The reason seems
to be that as in the fermionic basis\cite{7}, the lowest massive state is an almost pure 2 currents state. However, the present approach is much more successful than the fermionic basis, where the prediction for the mass of the lowest massive boson of the adjoint model is twice as much as the lowest massive boson of ’t Hooft model. It seems that the ’correct’ underlying degrees of freedom are currents and not fermions, as predicted by the authors of \cite{8}.

5 Summary

In this work we used a description of massless QCD$_2$ in terms of currents. With this basis we wrote down a ’t Hooft like equation (17) for the wave function of the two currents states.

The equation interpolates smoothly between the description of a single flavor model with large $N_c$ (’t Hooft model), the adjoint fermions model $N_f = N_c$ and the large $N_f$ model. The equation is derived by using an a-priori unjustified suppression of the three currents coupling. Nevertheless, we observe an excellent agreement with the DLCQ results for the first excited state. For higher excited states the agreement deteriorates and it is of the order of 20%.

The accuracy of the results for the first excited state, which implies that for this state the truncation of the “pair creation terms” is harmless, deserves further investigation.

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Figure 1: The mass of the lowest massive meson, in units of $\frac{e^2 N_c}{\pi}$, as a function of $N_f/N_c$. 