NATURALNESS AND ELECTRO-WEAK SYMMETRY BREAKING

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Abstract

The Principle of Naturalness of small parameters of a theory is reviewed. While quantum field theories constructed from gauge fields and fermions only are natural, those containing elementary scalar fields are not. In particular the Higgs boson mass in the Standard Model of electro-weak forces is not stable against radiative corrections. Two old canonical solutions of this problem are: (i) where the Higgs boson is a fermion-antifermion composite (technicolour solution) or otherwise (ii) we need supersymmetry to protect the mass of elementary Higgs boson from possible large radiative corrections. In recent years some other mechanisms for electroweak symmetry breaking have been under intense investigation. These include the little Higgs models and the gauge-Higgs unification models where the Higgs boson is the zero mode of the extra-dimensional component of a higher dimensional gauge field. Naturalness issues of such models are also briefly reviewed.

1 Naturalness dogma

When confronted with experiments, the Standard Model (SM) of fundamental forces has proven to be very robust both in its general structure as well as in every detail tested so far. However, a satisfactory understanding of the origin of electro-weak symmetry breaking mechanism has been an ever elusive problem. In the SM electro-weak symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken to $U(1)_{EM}$ through a non-zero vacuum expectation value of an $SU(2)$ doublet of elementary scalar fields. There are also other possibilities of achieving this goal. Besides the obvious requirement that the weak gauge bosons $W^\pm_\mu$ and $Z^0_\mu$ acquire the requisite masses, there is

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one guiding principle, known as the *Naturalness Principle*, which is largely adopted in working out any new mechanism for the symmetry breaking.

The dogma or principle of naturalness expresses the belief that a small parameter in Nature can not be an accident. It must be associated with a symmetry. This is in contrast to an anthropic principle.

The naturalness principle is best formulated through what can be called ’t Hooft’s doctrine of naturalness [1]:

At any energy scale $\mu$, a set of parameters, $\alpha_i(\mu)$ describing a system can be small, if and only if, in the limit $\alpha_i(\mu) \to 0$ for each of these parameters, the system exhibits an enhanced symmetry.

Weakly broken symmetry ensures that the smallness of a parameter is preserved against possible perturbative disturbances.

Let us analyse naturalness of various parameters in some of the quantum field theories that we come across in particle physics:

(1) **Quantum electrodynamics is a perfectly natural theory.** This theory describes electromagnetic interaction of charged fermions, electron $\lambda_e$, muon $\lambda_\mu$, etc:

$$
\mathcal{L}_{\text{QED}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{f=e,\mu,...} \bar{\lambda}_f \left[ i\gamma^\mu \left( \partial_\mu - ieq_f A_\mu \right) - m_f \right] \lambda_f
$$

Here the electromagnetic coupling $e$, the electron mass $m_e$, the muon mass $m_\mu$, etc. can all be independently small. The smallness of $m_e$ (or $m_\mu$) is protected by the fact that, in the limit $m_e \to 0$ (or $m_\mu \to 0$), we have an additional symmetry, the chiral symmetry which corresponds to separate conservation of the left- and right-handed electron-like leptons. No surprise that all the perturbative corrections to electron mass due to quantum fluctuations are small. These are in fact just proportional to $m_e$ itself; the self-energy diagrams have only logarithmic divergence. Also, $e \to 0$ enhances the symmetry; it implies no interaction; hence the particle number of each type is conserved in this limit. One-loop quantum correction to the electromagnetic coupling $e$ is indeed logarithmically divergent and is proportional to $e^2$.

Same discussion is valid for electromagnetic interaction of all other charged fermions and their masses, in particular for quarks.

(2) **Quantum Chromodynamics (QCD) is also a perfectly natural theory.** It describes the colour dynamics of $SU(3)$ triplet quarks $\lambda^i$ ($i = 1, 2, 3$) and an octet of gluons $A^a_\mu$ ($a = 1, 2, 3, ...8$):

$$
\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu} + \bar{\lambda}^i \left[ i\gamma^\mu \left( \delta^a_{ij} \partial_\mu - \frac{ig}{2} (T^a)_{ij} A^a_\mu \right) - m_\delta_{ij} \right] \lambda^j
$$

where $T^a$ are the eight generators of $SU(3)$ algebra in triplet representation. Here again the colour coupling constant $g$ is natural, because in the limit
$g \to 0$, there is enhanced symmetry. It reflects no interaction, hence particle number of each type in conserved in this limit. One-loop perturbative quantum corrections to the coupling constant $g$ have only logarithmic divergence and are proportional to $g^2$. Also mass parameter $m$ is natural because in the limit $m \to 0$ we have the chiral symmetry which is preserved by perturbative quantum corrections; such corrections to $m$ are logarithmically divergent and are proportional to $m$ itself.

(3) Quantum theories involving interacting elementary scalar fields are not natural. This has to do with the fact that the mass of an elementary scalar field is not associated with any approximate symmetry. Consider a self-interacting theory of a real scalar field:

$$L_{\text{scalar}} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4$$

At the classical level, the limit mass $m \to 0$ does lead to scale invariance; but at quantum level scale symmetry is broken. Thus smallness of the scalar mass can not be protected against perturbative quantum corrections. In fact such corrections appear with quadratic divergences. For example, at one loop level, such a correction comes from the diagram in Fig 1:

$$\delta m^2 \sim \lambda \int \frac{dk^2}{k^2 - m^2} \sim \lambda \Lambda^2$$

On the other hand, the other parameter of this theory, namely the $\phi^4$ coupling $\lambda$ is natural. This is so because, in the limit $\lambda \to 0$, we have a free scalar theory, which indeed has higher symmetry.

2 Naturalness of electro-weak model

Next let us consider a more general theory containing an elementary charged scalar field where local $U(1)$ gauge invariance is spontaneously broken through a nonzero vacuum expectation value of the scalar field. The theory describes a complex scalar field $\phi$ and a left-handed fermion $\psi_L$ interacting with a $U(1)$ gauge field $A_\mu$ and a right-handed fermion $\psi_R$ which is neutral with respect to this gauge symmetry and also Yukawa coupling of the fermions with the
scalar field:
\[ \mathcal{L} = -\frac{1}{4} F^\mu\nu F_{\mu\nu} + i\bar{\psi}_L \gamma^\mu (\partial_\mu - ieA_\mu) \psi_L + i\bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + (\partial^\mu + ieA^\mu) \phi^* (\partial_\mu - ieA_\mu) \phi \\
+ \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - Y [\phi \bar{\psi}_L \psi_R + \phi^* \bar{\psi}_R \psi_L] \]

This theory is invariant under the following gauge transformations:
\[ A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \theta, \quad \phi' = e^{i\theta} \phi, \quad \psi'_L = e^{i\theta} \psi_L, \quad \psi'_R = \psi_R. \]

The field theory described by the Lagrangian density (1) has many of the features of the Standard Model of electro-weak interactions. Besides a spontaneous breaking of symmetry and a Higgs mechanism it also has a Yukawa coupling for fermion and scalar fields which leads to the mass for fermion like in the SM. Unlike the SM this theory has anomalies in the \( U(1) \) gauge current. This can be cured by adding another left-handed fermion with opposite \( U(1) \) charge to the already included fermion. However the discussion of naturalness issues below does not depend on this.

Of the four parameters, the dimensionless couplings \( e \) and \( Y \) are independently natural as in the limit \( e \to 0, Y \to 0 \) we have no gauge interaction and no Yukawa interaction respectively, and hence enhanced symmetries. Indeed perturbative quantum corrections to these parameters are proportional to themselves. But in the case of dimensionless parameter \( \lambda \) situation is different. Even in the limit \( \lambda \to 0 \) at tree level, presence of gauge and Yukawa interactions induce quantum corrections, say at one loop level, to generate a non-zero \( (\phi^* \phi)^2 \) interaction in the effective potential with a coefficient \( Me^4 + NY^4 \) (\( M \) and \( N \) are some constants) due to gauge and fermion fields in the loops. This puts restrictions on how small the effective coupling \( \lambda \) can be. It can not be very much smaller than the gauge and Yukawa couplings.

The dimensionful parameter \( m \) deserves special attention. Notice due to the wrong sign of \( \phi^* \phi \) term \( (\mu^2 > 0) \), the \( U(1) \) symmetry is broken. The potential \( V(\phi, \phi^*) = -\mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \) has a ring of minima given by \( \phi^* \phi = \frac{v^2}{2} \equiv \frac{\mu^2}{(2\lambda)} \). Thus expanding the scalar field about its minimum value as \( \phi = (v + H)e^{i\sigma}/\sqrt{2} \) and rotating the phase away by absorbing the field \( \sigma \) (would be Nambu-Goldstone mode) into the massive vector field as its longitudinal component through a change of variables: \( A_\mu \to A_\mu - \partial_\mu \sigma/e \) and also \( \psi_L \to e^{-i\sigma} \psi_L \), we obtain a theory of a massive vector field \( A_\mu \), a massive scalar field \( H \) (so called Higgs field) and a massive Dirac fermion \( \psi \) with their masses given by:

\[ m_A = ev, \quad m_H = \sqrt{2\lambda} v, \quad m_\psi = \frac{1}{\sqrt{2}} Yv. \]

Now in the limit \( v \to 0 \) (or equivalently \( \mu \to 0 \)), we do have enhancement of symmetry classically; in this limit we have (i) scale invariance,
(ii) restored $U(1)$ gauge symmetry and also, (iii) since Dirac fermion becomes massless, chiral symmetry corresponding to the separate conservation of left- and right-handed fermions. Yet this does not make the vacuum expectation value $v$ or the mass parameter $\mu$ of the original Lagrangian (I) natural. None of these classical symmetries can protect the vector field mass $m_A$, Higgs field mass $m_H$ or fermion mass $m_\psi$. This is so because quantum fluctuations, through the well known Coleman-Weinberg mechanism, break all these symmetries by inducing a non-zero vacuum expectation value of the scalar field even if it were zero to start with in this limit of the classical theory. Thus at quantum level, there is no enhancement of symmetry in the limit where classical vacuum expectation value $v$ tends to zero. It is important to contrast this situation with the case of QED discussed above where electron mass $m_e$ is protected because chiral symmetry in the limit $m_e \to 0$ is an exact quantum symmetry.

Indeed in the present context, perturbative quantum corrections to each of the masses $m_A$, $m_H$ and $m_\psi$ have quadratic divergences. That is, corrections to the $(\text{mass})^2$ of these fields are proportional to the square of cut-off $\Lambda$. The vacuum expectation value $v$ of the scalar field also receives radiative corrections which are quadratically divergent. These come from radiative diagrams of the type in Fig 2.1.

![Fig 2.1. One-loop radiative corrections to the vacuum expectation value](image)

This one-loop correction is given by: $v^2_{\text{1-loop}} = v^2 + \frac{1}{16\pi^2}(P\lambda + Qe^2 - R\gamma^2)\Lambda^2$ where the three terms come from three diagrams of Fig 2.1. Notice in the limit $v \to 0$, one-loop vacuum expectation value is not zero. This is Coleman-Weinberg mechanism of radiative symmetry breaking.

Next for the Higgs field $m^2_H$ receives a correction at one-loop level as: $\delta m^2_H \sim \alpha \Lambda^2$ with $\alpha = \frac{1}{16\pi^2}(A\lambda + Be^2 - CY^2)$ where $A$, $B$, and $C$ are numerical constants and respectively correspond to the Higgs field, gauge field and fermion field going through the loop as in Fig. 2.2.

![Fig 2.2. One-loop radiative corrections to the Higgs mass](image)

Clearly this discussion holds for the Standard Model of electro-weak forces also. Here $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken to the electromagnetic symmetry $U(1)_{EM}$ due to the vacuum expectation value of a doublet scalar field resulting in a massive physical scalar field, the Higgs
field H. In the process the three weak gauge bosons $W^\pm$ and $Z^0$ also become massive and quarks and leptons acquire their masses through Yukawa couplings. Thus in this model also, radiative corrections to Higgs mass $m_H$ diverge quadratically as the internal momentum in the loop becomes large. Then our computations break down for loop momenta $p^2 \sim \Lambda^2$ where the cut-off $\Lambda$ is the energy scale up to which the SM is an adequate description of Nature. We may write, say the one-loop correction to Higgs boson mass due to quantum fluctuations of a size characterised by the scale $\Lambda$ as:

$$\delta m^2_H \sim \alpha \Lambda^2$$

(2)

Similar corrections obtain for the vacuum expectation value of the scalar field as well as for the masses of vector bosons $W^\pm$ and $Z^0$ and also fermions. Thus, for $m_H \sim 100$ GeV, and coupling $\alpha \sim (100)^{-1}$, the requirement that this mass does not receive large radiative corrections, $\delta m^2_H \sim m^2_H$, we have:

$$\Lambda^2 \sim \frac{\delta m^2_H}{\alpha} = \frac{(100 \ GeV)^2}{(100)^{-1}} = (1000 \ GeV)^2$$

That is, the cut-off $\Lambda \sim 1 \ TeV$. Naturalness of electro-weak theory breaks down at this scale. This is not any problem if there is no physics beyond 1 TeV; that is, if there are no elementary particles heavier than this scale, or there is no physical characteristic mass scale beyond this value. But in general, there is no reason to expect that this is so. For example, if the idea of grand unification for electro-weak and strong forces is valid, there is a physical scale, the grand-unification scale which is much larger than 1 TeV. In particular, in the $SU(5)$ Grand Unified Theory (GUT) we have two widely separated scales: the GUT scale $M_{GUT} \sim 10^{16}$ GeV where the GUT gauge group $SU(5)$ spontaneously breaks down to the SM group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the scale of electro-weak physics $M_{EW} \sim 100$ GeV where the electro-weak gauge group $SU(2)_L \times U(1)_Y$ breaks down to $U(1)_{EM}$. These two levels of symmetry breaking are achieved through vacuum expectation values of two sets of elementary scalar fields. It was in this context first that it was realized that the radiative corrections do not allow the two scales to be maintained at such widely separated values [2].

In fact, not only quadratically divergent radiative diagrams, but also some times certain kind of logarithmically divergent diagrams contribute large corrections to small masses. Such diagrams come with large coefficients proportional to the larger mass scale of the theory. These kind of large logarithmic divergent diagrams are typically present in GUTs. To understand the origin of such radiative corrections, consider a theory of two interacting scalar fields, $\Phi$ and $\phi$, which have two vastly separated vacuum expectation values generated by some suitable potentials: $<\Phi> = F$ and $<\phi> = f$ with $F >> f$. Expanding about their expectation values $\Phi = F + H$ and $\phi = f + h$, we have two massive scalars fields with their
masses $m_H \sim F$ and $m_h \sim f$. A possible interaction of the type $a \Phi^2 \phi^2$ in the original potential would lead, after shifting the fields by their vacuum expectation values, to effective three-point vertices of the form $a FHHh$ with a large dimensionful coupling $a F$. Such vertices will in turn lead to a large logarithmically divergent radiative correction to the mass of the light scalar field $h$ as

$$\delta m_h^2 \sim a^2 F^2 \ln \Lambda^2$$

from a radiative diagram shown in Fig.2.3:

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H

F

h

F

h
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Fig.2.3. Large logarithmically divergent radiative correction

Such corrections along with those large corrections from quadratically divergent diagrams would destabilise the mass of lighter scalar field. This is a generic feature of all such theories of scalar fields with two widely separated scales. In particular, it is true of GUTs: perturbative quantum corrections tend to draw the smaller electro-weak scale $M_W \sim 10^2 \text{GeV}$ towards the GUT scale $M_{GUT} \sim 10^{16} \text{GeV}$. This problem known as the gauge hierarchy problem of grand unified theories is due to the fact that field theories containing elementary scalars are not natural.

Not only this, there is yet another much larger and important physical mass scale $M_{\text{Planck}} = 10^{19} \text{GeV}$ in Nature which is associated with quantum gravity. This would imply that the radiative corrections would draw the masses of electro-weak theory to this high scale and hence their natural values would be $\sim 10^{19} \text{GeV}$ and not the physical values characterised by the SM scale of 100 GeV! What this quantum instability of the Higgs potential strongly suggests that there has to be some new physics beyond 1 TeV such that the SM with its characteristic scale of 100 GeV stays natural beyond this scale. Or otherwise there can not be any fundamental scale in Nature beyond 1 TeV; in particular the scale of four dimensional gravity, $M_{\text{Planck}}$ can not be a fundamental scale; but it should be derived from some new physics characterised by a fundamental scale of 1 TeV only.$^1$

$^1$Some of the higher dimensional theories exhibit such a feature. It may also be possible to achieve this, if starting from a classically scale invariant theory of all the interactions including gravity, both the weak scale and the Planck scale are generated by the quantum effects which break this invariance weakly, ensuring that any scalar fields which can acquire large vacuum expectation values are only weakly coupled to the SM sector.$^2$
3 Composite Higgs boson as a solution of naturalness problem

Earliest discovered solution to the naturalness problem of electro-weak theory is one where the Higgs particle is not an elementary particle but a spin-less composite of a fermion and anti-fermion. This is the Technicolour solution \[1,5\]. The constituents of the Higgs boson are to be held together by a new strong force. Known interactions like colour $SU(3)_C$ are not sufficiently strong at the electro-weak energy scale, a new much stronger interaction is needed to obtain the required bound state\[2\]. This interaction should exhibit confinement. For no better reason than the fact QCD is a confining gauge theory, the new interaction is postulated to be a QCD type theory, though, operative at a higher scale of about 1 TeV, where its running coupling constant becomes of order unity. The Higgs particle would then be built in the same fashion as pions are in QCD. If we were to probe the Higgs particle with energies greater than 1 TeV, we would see it not as an elementary scalar particle, but instead as a fermion and an anti-fermion. Except for a few, most of the physical states of this new theory would have high enough masses characterised by the high energy scale of this theory.

Historically a variety of names were proposed for these new fermions such as metafermions ('t Hooft), heavy fermions (Weinberg), hyperfermions (Eichten and Lane), technifermions (Susskind) and the new interaction experienced by them was called metacolour, heavy colour, hypercolour and technicolour respectively. The last nomenclature has survived over years.

Since theories with fermions and gauge fields like QED and QCD as indicated earlier, are natural, Technicolour theories do not suffer from Naturalness problem.

Besides technicolour forces, the techniquarks are also supposed to experience electro-weak interactions. Simplest technicolour model can be build in terms of an $SU(2)_L$ doublet $(U,D)_L$ of techniquarks and two right-handed singlets $U_R$ and $D_R$. Such theory exhibits an additional global flavour symmetry $SU(2)_L \times SU(2)_R \times U(1)_V$ which is broken to $SU(2)_{L+R} \times U(1)_V$ through the condensation of the techniquarks leading to three Nambu - Goldstone bosons (technipions) $\pi_{TC}^a$. These then in turn induce required masses for the weak interaction bosons $W^\pm$ and $Z^0$. These masses fix the analog of the pion decay constant $F_{TC\pi}$ of the new interaction, defined in terms of matrix elements of the associated spontaneously broken axial cur-

\footnote{While composites made up of the light quarks such as $u, d$ and $s$ are not adequate, much heavier top quark with its strong coupling to the electro-weak sector, may serve the purpose. It is worthwhile to ponder over this possibility of symmetry breaking where the Higgs boson would be a $t \bar{t}$ bound state. But unfortunately, this minimalist idea of a composite Higgs boson does not work because it requires a top quark of mass much heavier that the experimentally observed value $[6]$.}
rent $J^{\mu}_{5a}$ by:

$$< 0 | J^{\mu}_{5a} | \pi_{TCb}(q) > = i q^\mu \ F_{TC\pi} \ \delta_{ab}$$

The $W$ boson mass is related in a model independent way to the technipion decay constant as:

$$M_W = \frac{1}{2} \ g_2 \ F_{TC\pi}$$

where $g_2$ is the $SU(2)_L$ coupling constant. Because of the isospin symmetry of the strong interaction, the relation $M_W = M_Z \cos \theta_W$ holds as long as electric charge is conserved.

Now the technipion decay constant can be fixed by relating it to the Fermi constant $G_F$ of weak interaction as follows:

$$F_{TC\pi} = \frac{2}{g_2} \ M_W = \left( \sqrt{2} G_F \right)^{-\frac{1}{2}} = 250 \ GeV.$$

Since techni-QCD is only a scaled up QCD, the technipion decay constant $F_{TC\pi}$ and ordinary QCD pion decay constant $f_\pi$ are related as $F_{TC\pi}/f_\pi \sim \Lambda_{TC}/\Lambda_C$ where $\Lambda_C$ and $\Lambda_{TC}$ are the scales at which the QCD and techni-QCD coupling constants respectively become strong. Taking $\Lambda_C/f_\pi \sim 2$, we have the techni-QCD scale $\Lambda_{TC} \sim 0.5 \ TeV$.

This does yield a possible dynamical explanation of the origin of masses of electro-weak bosons $W^\pm$ and $Z^0$, but does not provide for the masses of ordinary quarks and leptons which in the Standard Model are generated through Yukawa couplings of these fermions with the elementary scalar field. In the technicolour framework, to generate these masses a new gauge interaction, the Extended Technicolour (ETC) is introduced. The gauge bosons of this new interaction, with their masses in the range 10 - 100 TeV, connect the ordinary quarks and leptons to techniquarks; thus providing a mechanism for masses for quarks and leptons.

There are some serious phenomenological difficulties with such scenarios. These include not enough suppression of flavour changing neutral current effects, heavier Higgs particle, presence of large anomalous contributions to the $Z b \bar{b}$ vertex and large contributions to the Peskin-Takeuchi $S$, $T$ and $U$ parameters. But these difficulties may be the problem of specific model for the dynamics of force responsible for holding the constituent fermions together in the composite Higgs boson. It may be that a QCD-type model for this force is not adequate. In particular, the relation $F_{TC\pi}/f_\pi \sim \Lambda_{TC}/\Lambda_C$ implied by the scaled up QCD-type model for the new interactions is too restrictive.

4 Supersymmetric solution

That supersymmetry also provides a solution to the naturalness problem of electro-weak theory was realized about 27 years ago [7, 8]. This option re-
tains the elementarity of the scalar field. While composite Higgs boson is a non-perturbative solution to the problem, supersymmetry provides a perturbative solution. An elementary property of quantum field theory which gives an extra minus sign for the radiative diagram with a fermionic as against a boson field going around in a loop allows for the possibility that naturalness violating effects due to bosonic and fermionic quantum fluctuations can be arranged to cancel against each other. For this to happen the various couplings of bosons and fermions have to be related to each other in a highly restrictive manner. Further, for such a cancellation to hold at every order of perturbation theory, a symmetry between bosons and fermions would be imperative. This is what supersymmetry indeed does provide.

In most of the supersymmetric field theories, troublesome quadratic divergences and also the large logarithmic divergences are independently absent [7]. This happens due to exact cancellation of such divergences between graphs with bosonic and fermion fields going around in the loops. While supersymmetric theories with a non-Abelian symmetry are always free of such divergences, those with $U(1)$ symmetry do have radiative corrections with quadratic divergence which are proportional to the sum of $U(1)$ charges of all the fields. So theories which have $U(1)$ charges adding up to zero do not have quadratic divergences. Supersymmetrized Standard Model is one such theory. This would, in particular, be also the case for any theory with an $U(1)$ symmetry that can be embedded in a non-Abelian group.

Also in supersymmetric field theories where gauge symmetry is broken spontaneously through non-zero vacuum expectation value of a scalar field, like in the SM, the limit when this vacuum expectation value goes to zero does indeed lead to an enhancement of symmetry even at the quantum level, provided, if there is a $U(1)$ symmetry present in the theory, $U(1)$ charges add up to zero. Quantum corrections do not spoil this symmetry; unlike the non-supersymmetric case, Coleman-Weinberg mechanism does not produce radiative violation of the symmetry. The Higgs boson mass is natural here and also so are the masses generated for gauge bosons through Higgs mechanism and the fermion masses generated through Yukawa couplings to scalar field with non-zero vacuum expectation value.

Supersymmetry requires that bosons and fermions come in families [9]: photon has a fermionic partner, the photino; electron’s bosonic partner is selectron; quarks have scalar partners squarks, etc. Similarly, if we are interested in gravity, spin 2 graviton has a fermionic super-partner spin $3/2$ gravitino.

Exact supersymmetry would imply that all properties except the spin of particles in a supermultiplet are the same. Thus, the masses and couplings of super partners would exactly be same. This, however, is not seen to be the case in Nature, otherwise we would have, for example, already observed the super partner of electron, selectron with same charge and mass as the electron. Supersymmetry has to be broken in Nature. But this breaking
should be such that the basic reason of naturalness does not get out of hand again. While particle and sparticle masses, $M_{\text{part}}$ and $M_{\text{spart}}$, have to be different, with $M_{\text{spart}}$ sufficiently high to have escaped detection till now, the cancellation of bosonic and fermionic radiative corrections need only be up to the naturalness breakdown scale of the SM:

$$\left| \int \frac{\Lambda^2}{k^2} \frac{dk^2}{k^2 - M_{\text{spart}}^2} - \int \frac{\Lambda^2}{k^2} \frac{dk^2}{k^2 - M_{\text{part}}^2} \right| \sim |M_{\text{spart}}^2 - M_{\text{part}}^2| \ln \Lambda^2 \leq (1 \text{ TeV})^2.$$ 

So the supersymmetry breaking has to be such that quadratically divergent parts of the radiative corrections cancel, but logarithmically divergent contributions need not cancel exactly. Such situations are obtained if supersymmetry is broken spontaneously or by what are called soft-terms in the action.

5 Naturalness of little Higgs models

In recent times, an alternative symmetry breaking mechanism for electroweak theory has been proposed where the Higgs boson, though an elementary scalar particle, is a pseudo-Nambu-Goldstone boson [10]. Such a Higgs boson is massless at the tree level. The symmetry is explicitly broken by weakly coupled operators in the theory so that the Higgs boson acquires mass without generating any quadratically divergent contributions at one-loop level; it has only a logarithmically divergent correction at this level. It is hoped that this mass is protected by the global symmetry with which the Higgs boson is associated as a Nambu-Goldstone boson when this symmetry is spontaneously broken. The electro-weak $SU(2)_L \times U(1)_Y$ is embedded in a larger gauge group, simplest being $SU(3) \times U(1)$. Models based on such an idea are called Little Higgs Models [3].

To understand the underlying structure let us consider an $SU(3)$ gauge theory with two scalar fields $\Phi_1$ and $\Phi_2$, each transforming as a complex triplet of the gauge group described by the Lagrangian density:

$$\mathcal{L} = |(\partial_{\mu} + igA_{\mu})\Phi_1|^2 + |(\partial_{\mu} + igA_{\mu})\Phi_2|^2 - \frac{1}{2} tr F_{\mu\nu} F^{\mu\nu} - V(\Phi_1, \Phi_2) \quad (3)$$

where the potential is:

$$V(\Phi_1, \Phi_2) = \frac{\lambda^2}{2} \left( \Phi_1^\dagger \Phi_1 - f^2 \right)^2 + \frac{\lambda^2}{2} \left( \Phi_2^\dagger \Phi_2 - f^2 \right)^2 \quad (4)$$

This potential has two global $SU(3)$ symmetries acting on the two triplets which are broken to two $SU(2)$ symmetries through the tree level vacuum

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3The contents of this Section have been developed with Gautam Bhattacharyya
expectation values $< \Phi_1^\dagger \Phi_1 > = < \Phi_2^\dagger \Phi_2 > = f^2$. This produces ten Nambu-Goldstone bosons, five each from the two spontaneous breakings $SU(3) \to SU(2)$. Further, the gauge couplings in the Lagrangian density above represent a weakly gauged vector $SU(3)$ such that the global $[SU(3)]^2$ is explicitly broken to a diagonal $SU(3)$. Five of the Nambu-Goldstone bosons become the longitudinal components of the five gauge fields through the Higgs mechanism, leaving behind other five Nambu-Goldstone bosons which are massless at the tree level.

Now let us understand the naturalness properties of this model: is the Higgs particle mass so generated natural? In the original Lagrangian density we have one dimensionful parameter, namely $f$ which is the tree level expectation value of the scalar fields $\Phi_1$ and $\Phi_2$. In the limit $f \to 0$, classically we do have enhanced symmetry. But at quantum level, like any theory of elementary scalar fields, this is not so due to the Coleman-Weinberg mechanism. Hence the vacuum expectation values of the scalar fields, $< \Phi_1 >$ and $< \Phi_2 >$, are not protected. These do receive large, quadratically divergent, corrections due to quantum fluctuations at one loop itself.

For definiteness, we write the one-loop correction to the effective potential as:

$$\Delta L = \lambda^2 \Lambda^2 \alpha(g^2, \lambda^2) \left( \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \right) + \beta(g^2) |\Phi_1^\dagger \Phi_2| \ln(\Lambda^2/F^2)$$

where $\alpha(g^2, \lambda^2) = \frac{1}{16\pi^2} (A g^2 + B \lambda^2)$ and $\beta(g^2) = \frac{1}{16\pi^2} C g^4$ with $A$, $B$ and $C$ as numerical factors. The first term with quadratic divergence comes from diagrams (a), (b), (c) and (d) and the last logarithmically divergent term comes from the diagram (e) of Fig.3 below:

![Fig.3. One-loop radiative corrections](image)

Adding this to the tree level potential, we have one-loop effective potential as:

$$V_{1\text{loop}} = \frac{\lambda^2}{2} \left( \Phi_1^\dagger \Phi_1 - F^2 \right)^2 + \frac{\lambda^2}{2} \left( \Phi_2^\dagger \Phi_2 - F^2 \right)^2 - \beta(g^2) |\Phi_1^\dagger \Phi_2| \ln(\Lambda^2/F^2)$$

where $F$ is the one-loop corrected vacuum expectation value of the scalar fields:

$$< \Phi_1^\dagger \Phi_1 >_{1\text{loop}} = < \Phi_2^\dagger \Phi_2 >_{1\text{loop}} = F^2 \equiv f^2 + \alpha(g^2, \lambda^2) \Lambda^2$$

Clearly the vacuum expectation values of the scalar fields are not protected; as expected their tree-level value $f$ does receive quadratically divergent corrections.
Now the triplet scalar fields can be expanded about their vacuum expectation value as:

$$\Phi_1 = e^{i\Theta^a T^a/F} \left( \begin{array}{c} 0 \\ 0 \\ F + \eta_1 \end{array} \right), \quad \Phi_2 = e^{-i\Theta^a T^a/F} \left( \begin{array}{c} 0 \\ 0 \\ F + \eta_2 \end{array} \right)$$

where $T^a$ are the $SU(3)$ generators and displaying only the left over five Nambu-Goldstone bosons in the exponent we write:

$$\Theta^a T^a = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc} 0 & 0 & h_1 \\ 0 & 0 & h_2 \\ h_1^* & h_2^* & 0 \end{array} \right) + \frac{\eta}{4} \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{array} \right)$$

The complex fields $h = (h_1, h_2)^T$ are to be identified with the Standard Model doublet of scalar fields and $\eta$ is a singlet neutral real scalar field. At tree level the vacuum expectation value of the scalar fields $\Phi_1$ and $\Phi_2$ leaves an $SU(2)$ gauge symmetry unbroken; fields $h_1$ and $h_2$ and also $\eta$ have zero vacuum expectation values. All the five Nambu-Goldstone bosons $h_1$, $h_2$ and $\eta$ are massless at this level. But quantum corrections involving the gauge interactions of (3) represented by the last logarithmically divergent term in the one-loop effective potential (5) above and also other such contributions from possible interactions with fermions of the theory which we have not displayed here, allows the doublet $h$ to pick up a non-zero vacuum expectation value and hence break the $SU(2)$ gauge symmetry in the standard way leading to three real fields becoming the longitudinal components of the $SU(2)$ gauge bosons leaving one massive real Higgs particle $H$. This follows immediately if we use the parametrisation (7) to write the expansion:

$$|\Phi_1^\dagger \Phi_2|^2 = F^2 \left[ -ah^\dagger h + \frac{b}{F^2}(h^\dagger h)^2 + ... \right]$$

where $a$ and $b$ are positive numerical coefficients. The first two terms of effective potential (5) do not contribute to the potential for $h$. Only contribution to this effective potential comes from the last term with logarithmic divergence so that:

$$V_{\text{1-loop}}(h^\dagger h) \sim -\beta F^2 \left[ -ah^\dagger h + \frac{b}{F^2}(h^\dagger h)^2 + ... \right] \ln \frac{\Lambda^2}{F^2}$$

In order to have the SM symmetry breaking $\beta$ has to be negative. This can be ensured by including fermions (for example a top quark) with large Yukawa couplings $Y$ to scalars $\Phi_1$ and $\Phi_2$ which will also then, through graphs with fermion loops, contribute to both the quadratically and logarithmically divergent terms in (5) and (6) above so that $\alpha = \frac{1}{16\pi^2}(Ag^2 + B\lambda^2 - EY^2)$ and $\beta = \frac{1}{16\pi^2}(Cy^4 - DY^4)$ in such a way that $\beta < 0$. Though
we have not displayed them explicitly here, these fermions are essential to trigger the electro-weak symmetry breaking and hence have to be included in a complete description of the theory. Negative $\beta$ allows the effective potential $V_{\text{loop}}(h^\dagger h)$ to have a minimum at a non-zero vacuum expectation value $< h^\dagger h > = [a/(2b)]F^2 \equiv v_H^2/2$. Writing $h^\dagger h \sim \frac{1}{2}(v_H + H)^2 + \ldots$, we find that SM symmetry breaking takes place leading to a mass for the standard Higgs particle $H$ as:

$$M_H^2 \sim 2(-a\beta) F^2 \ln \frac{\Lambda^2}{F^2} \quad (8)$$

where $\beta = \frac{1}{16\pi^2}(Cg^4 - DY^4) < 0$.

Though $(\text{mass})^2$ of Higgs boson has only a logarithmic divergence at one loop level, it is also proportional to $F^2$ which does contain a quadratic divergence, $F^2 = f^2 + \alpha \Lambda^2$. This brings in the quadratic divergences into the Higgs mass. Notice that this quadratic divergence comes with a factor of the coupling constant $\alpha$. This is yet another example of a large logarithmically divergent radiative correction in addition to the one that was mentioned earlier in the context of the GUTs with widely separated scales in Section 2.

Now in order the mass of Higgs particle be around 100 GeV, and for $(-a\beta) \sim (100)^{-1}$ and $\alpha \sim (100)^{-1}$, we have $f^2 \sim F^2 \sim (1 \text{ TeV})^2$ and the cut-off $\Lambda = 10 \text{ TeV}$.

There is a phenomenological difficulty with regard to the weak gauge boson mass in the little Higgs model we have outlined above. The quartic coupling of the Higgs field $(h^\dagger h)^2$ is too small and hence the weak gauge boson mass turns out to be too large, order $g_{EW}F$. However, there are other little Higgs models, like the ‘Littlest Higgs model’ [10] based on an $SU(5)/SO(5)$ sigma model, which do not suffer from such a limitation.

Like the SM, little Higgs models also suffer from naturalness problem. However, the scale of naturalness breakdown is about 10 $TeV$, an order of magnitude higher than that for the SM where, as discussed earlier in Section 2, it is only 1 $TeV$. This would mean that any new physics mass scale beyond 10 $TeV$, say that associated with the grand unification of strong and electro-weak forces, would destabilise this 10 $TeV$ scale thus giving the electro-weak Higgs boson a huge radiative correction characterised by the GUT scale. There is nothing to protect the mass of this Higgs boson from receiving such a large radiative correction. This is to be contrasted with the composite Higgs boson and supersymmetric cases discussed in Sections 3 and 4. For example, despite the large value of the GUT scale, in a supersymmetric framework, it does not destabilise the supersymmetry breaking scale at 1 $TeV$; the electro-weak Higgs boson does not receive huge radiative corrections from the GUT scale.

Finally, in the little Higgs models also, to have naturalness beyond this 10 $TeV$ scale, some other mechanism needs to be invoked at this scale. This
could be yet another little Higgs mechanism operative at about 10 \( TeV \)
to push the naturalness scale up by an order of magnitude to 100 \( TeV \).
There would have to be a ladder of such successive mechanisms. Otherwise,
supersymmetry or fermion-antifermion compositness of the Higgs particle
may appear at this scale. In particular, instead of 1 \( TeV \) supersymmetry for
the SM, Nature need have supersymmetry only at a higher scale of 100 \( TeV \).
Alternatively, even for Higgs boson mass around 100 \( GeV \), the compositness
scale for the forces binding new fermions into an effective Higgs boson can
be at a higher scale of about 10 \( TeV \) in contrast to the 1 \( TeV \) scale of
conventional technicolour models discussed in Section 3. This allows for the
possibilities of resolving the phenomenological difficulties faced by the old
technicolour composite Higgs models.

6 Higher dimensional theories

There is yet another framework that addresses the electro-weak symmetry
breaking with elementary scalar fields. In this framework, the four dimen-
sional scalar fields are identified with the zero modes of extra-dimensional
components of a higher dimensional gauge potential \cite{11, 12, 13}. When the
extra dimensional space is not simply connected (for example if it is \( S^1 \)),
there are Wilson line phases \( \theta_H \) associated with the extra dimensional com-
ponents of the gauge field, analog of Aharonov-Bohm phase in quantum
mechanics. The four dimensional fluctuations of these phases are identified
with Higgs scalar fields. These are massless at tree level and acquire masses
through radiative corrections as a finite-volume effect. When we probe the
Higgs boson with energies of the order of 1 \( TeV \), the extra dimensions open
up and we start seeing it as an extra-dimensional component of a higher di-
mensional gauge field. This framework is known as gauge-Higgs unification.

We could start with a five dimensional gauge theory containing fermions
on a manifold \( M^4 \times S^1 \) with size of the compactified fifth dimension as
2\( \pi R \). Since the extra-dimensional component \( A_5^a \) of the gauge fields are
in the adjoint representation of the gauge group, the starting gauge group
in the higher dimensional theory has to be large enough, say an \( SU(3) \)
or \( SO(5) \) or \( G_2 \), to accommodate the four dimensional Higgs field of the
\( SU(2)_L \times U(1) \) SM model. The fifth components of momentum \( p^5 \) of both the
gauge fields and fermions are discrete. These fields are expanded in terms
of their Kaluza-Klein (KK) modes. At low energies we have an effective
four dimensional theory obtained by integrating the action over the fifth
dimension. This is a theory of massless zero modes of the fields and a tower
of massive KK excitations for each field with their masses characterised by
the KK mass scale \( m_{KK} \sim 1/R \) and discrete values of the fifth component
of momentum.

Quantum effects induce an effective four dimensional potential for the ad-
joint representation extra-dimensional components of the gauge field thereby breaking the gauge symmetry. These calculations have to include the radiative effects from all the various KK excitations. The answer would have divergences, naively even quadratic divergences. But these require special care with regularization. We have to adopt a regularization which allows for the fact that the extra-dimensional components are the fifth components of five dimensional gauge fields and in the limit \( R \to \infty \), where we have five dimensional gauge symmetry, \( A_5^a \) should be massless. This is not any different from the fact the photon self energy graphs in four dimensional electrodynamics have to be regularised in a gauge invariant manner so that the photon does not acquire a quadratically divergent mass and thereby break the gauge symmetry; only a wave function renormalization is allowed.

Now in the present context, if we adopt a reasonable regularization, the effective potential for the extra-dimensional component of the gauge field turn out to be such that the bosonic (gauge) field loops tend to lead to a minimum of this effective potential at \( < A_5^a > = 0 \) whereas the fermionic loops tend to draw this vacuum value away from zero. So if we arrange sufficiently many fermions in the theory we can have a radiatively generated symmetry breaking potential with its minimum at a non-zero value of order \( 1/R \). This breaks the gauge symmetry. This way of breaking symmetry due to radiative corrections in a compactified higher dimensional gauge theory is known as Hosotani mechanism. The mass of Higgs boson so generated by the radiative corrections is of order \( 1/R \). Further it appears that the Higgs boson mass may be finite to all orders in five dimensions suggesting that it is independent of the physics of the cut-off scale.

It is important to note that in this gauge-Higgs unification framework, in the limit \( R \to \infty \), the Higgs boson mass goes to zero and we have the five dimensional gauge symmetry. It is this five dimensional gauge symmetry which protects the smallness of Higgs boson mass.

However there are two difficulties with this framework: (i) The first problem faced is that the fermions in higher dimensional theory lead to vector theories in the reduced effective four dimensional theory at low energies instead of a theory with chiral fermions. This can be cured making the extra dimension have a non-trivial topology or allow for non-vanishing flux in the extra dimensions. We do have chiral fermions if the extra dimensional space is an orbifold, say \( S^1/Z_2 \), instead of a circle \( S^1 \). The left-right asymmetry is achieved by appropriate orbifold boundary conditions so that the matter content of the SM is obtained in the effective low energy theory. (ii) Second problem is faced when contact is made with the required masses of the electro-weak theory. The mass scales of the theory, namely gauge boson masses \( m_W \) and fermion masses as well as KK modes mass scale \( m_{kk} \) are all related and are order \( 1/R \). The Higgs boson mass is down by a factor of the weak coupling constant, \( m_H \sim g_4^2/(4\pi) m_W \) where \( g_4 \) is the effective four dimensional gauge coupling which is related to the five dimensional gauge
coupling $g_5$ as $g_4 = g_5/\sqrt{2\pi R}$. This makes the KK energy scale rather low so that the masses of the low lying KK modes are same as that of weak gauge boson. In addition the mass of Higgs boson is too low. This phenomenological difficulty arises mainly because the framework has been set up in flat five dimensional spacetime and could be circumvented if the theory was instead set in curved spacetime.

Both these problems get resolved by setting up the theory in the Randall-Sundrum (RS) warped five dimensional spacetime \cite{14}. In this space time extra dimensional space has the topology of orbifold $S^1/Z_2$ of radius $R$. We have here a five dimensional anti-de Sitter space where the fifth dimension is an interval $|y| \leq \pi R$ and with $k^{-1}$ as the AdS curvature. The boundaries of the interval are at the fixed points $y = 0$ and $y = \pi R$ where two three-branes, the so called Planck or UV brane and TeV or IR brane respectively, are located. Induced metric on these boundaries differ by the exponential warp factor $e^{\pi k R}$ generating widely separated effective scales. At the Planck brane $y = 0$, the effective four dimensional mass scale is of order the Planck scale $k \sim M_{Pl}$. On the other hand, the effective mass scale at the other brane at $y = \pi R$ is $M_{Pl} e^{-\pi k R} \sim 1$ TeV. The low energy four dimensional effective action for the zero modes of the extra-dimensional gauge fields is such that these zero-modes are localised near the TeV-brane. The large warp factor $e^{\pi k R}$ relates the Planck scale $M_{Pl}$ to electro-weak scale $m_W$ through $M_{Pl}/m_W \sim e^{\pi k R} \sim 10^{17}$ for $kR \sim 12$ where $k \sim M_{Pl}$ and $-k^2$ is the cosmological constant in the bulk five dimensional space time. Thus RS spacetime provides a natural bridge between the Planck scale and weak scale through the warp factor. The radius of compactification $R$ here, unlike the case of flat spacetime where it is of order $(1$ TeV$)^{-1}$, is not large; it is instead as small as $\sim M_{Pl}^{-1}$.

This model has been studied for various five dimensional gauge groups like $SU(3)$ and $SO(5) \times U(1)_{B-L}$ \cite{15} with the latter having more satisfactory phenomenological properties, particularly those related to the physics of neutral currents of the electro-weak theory. The KK mass scale is given by $m_{KK} = \pi k e^{-\pi k R}$ which, for $kR \sim 12$ and $k \sim M_{Pl}$, is of order 1.5 TeV. The KK spectrum is not equally spaced unlike in the flat spacetime case. The gauge boson and Higgs boson masses are predicted to be \cite{15}: $m_W \sim 100$ GeV and $m_H \sim 120 - 290$ GeV. This makes the KK excitations to be sufficiently heavy and at the same time the gauge and Higgs boson masses of acceptable values.

In the RS framework there is an alternative proposal where Higgs boson is not the zero mode of a higher dimensional component of a gauge field but instead is an elementary scalar field. There are two versions of such theories: (i) the SM fields including the Higgs field live on the TeV brane, only gravity propagates in the five dimensional bulk \cite{14}; and (ii) all the SM fields live in the bulk \cite{16}.

In the former case, since effective momentum cut off near this brane is
warped down from the five dimensional cut off $\Lambda_5 \sim M_{Pl} \sim 10^{19} \text{ GeV}$ to $\Lambda_{eff} \sim \Lambda_5 e^{-\pi k R} \sim 1 \text{ TeV}$, radiative corrections to the Higgs boson mass are cut off by this value. However care needs to be taken when all the contributions of KK modes of the five dimensional graviton are added. The zero mode of the graviton is the standard four dimensional graviton which has the standard gravitational coupling to the matter fields on the TeV brane and this is small as $1/M_{Pl} \sim (10^{19} \text{ GeV})^{-1}$. However the couplings of the KK gravitons with their masses given by the characteristic KK scale $M_{KK} \sim k e^{-\pi k R}$, are enhanced by the wrap factor to $1/M_{Pl} e^{\pi k R}$ which is only $\sim (1 \text{ TeV})^{-1}$. It is this large coupling that allows for the possibility that such KK gravitons of 1 TeV mass may be observed at a TeV collider, LHC.

But these large couplings also create a possible problem for the radiative corrections to the Higgs boson mass from the tower of KK gravitons. Such corrections to the Higgs boson mass at the infrared brane are

$$\delta m_{H}^{2} = \sum_{KK} \left( \frac{e^{\pi k R}}{M_{Pl}} \right)^{2} \left( M_{Pl} e^{-\pi k R} \right)^{2} = \sum_{KK} (\text{TeV})^{-2} (\text{TeV})^{2} \sim \sum_{KK} 1$$

Indeed the effective four dimensional momentum cut off is $\Lambda_{eff}^{2} \sim (1 \text{ TeV})^{2}$. Each term in this correction is small, but there is a sum over all the KK modes. This sum has to be done in a meaningful way so that result does not become large again.

In the second case, where the SM fields including the Higgs field live in the bulk, the tree-level bulk mass of the scalar field has to be small so that gauge bosons get reasonable masses of order 100 GeV. But radiatively its natural value is $\sim k$. This brings in the gauge hierarchy problem back in to the game [16]. However introduction of supersymmetry again would protect mass of the Higgs mode far from the TeV-brane [17]. Such a model at low energy is more like the Minimal Supersymmetric Standard Model (MSSM).

This is indeed a very bold proposal which needs some special care. One problem that requires to be addressed is the mechanism for fixing the size of the extra dimensions. Particularly, this has to be done taking in to account the gravitational sector by including the five dimensional curved spacetime metric. The zero modes of this metric give four dimensional metric $g_{\mu \nu}(x)$ and the radion field $R(x)$ whose vacuum value is the size $R$ of extra dimension. A possible proposal for the stabilization of the size of extra dimension developed by Goldberger and Wise [18] involves a massive five dimensional scalar field. Equation of motion of this field is solved with appropriate boundary condition in the RS background. The solution in then put back into the action and extra dimension integrated out to get an effective potential for the modulus $R$. The minimum of the potential fixes the stable size of the modulus to a value $k < R > \sim 12$. However the back reaction of the scalar field need not be small and can spoil this
stabilization. Also scalar fields mass being prone to problems from large quantum corrections, this needs some extra care.

An additional problem is that the cosmological constant continues to be not a natural parameter and has to be fine tuned to its small value.

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