A state of matter with a multi-component order parameter can give rise to a vestigial order\textsuperscript{1,2}. In the vestigial phase, the primary order is only partially melted, leaving a remaining symmetry breaking behind, an effect driven by strong classical or quantum fluctuations. Vestigial states due to primary spin and charge-density-wave order have been discussed in the context of iron-based and cuprate materials. Here we present the observation of a partially melted superconductor in which pairing fluctuations condense at a separate phase transition and form a nematic state with broken Z\textsubscript{3}, i.e. three-state Potts-model symmetry. High-resolution thermal expansion, specific heat and magnetization measurements of the doped topological insulator Nb\textsubscript{x}Bi\textsubscript{2}Se\textsubscript{3} reveal that this symmetry breaking occurs at $T_{nem} \approx 3.8\,\text{K}$ above $T_c \approx 3.25\,\text{K}$, along with an onset of superconducting fluctuations. Thus, before Cooper pairs establish long-range coherence at $T_c$, they fluctuate in a way that breaks the rotational invariance at $T_{nem}$ and induces a distortion of the crystalline lattice.

Nematic electronic phases are known from iron-based superconductors and cuprates, where it has been suggested that the nematic phase and the nearby spin- and charge-density wave states are not independent competing but intertwined electronic phases\textsuperscript{1-10}. The density-wave states are the primary electronic phases and characterized by a multi-component order parameter. The nematic phase is a fluctuation-driven phase and characterized by a composite order parameter. Then the spin- or charge density-wave order melts partially, but leaves an Ising, i.e. Z\textsubscript{2}-nematic state as a vestige. Vestigial order, where the primary order is superconductivity, has not been observed. Such partially molten superconductivity requires a material with unconventional, multi-component order parameter and strong pairing fluctuations.

When the topological insulator Bi\textsubscript{2}Se\textsubscript{3} is doped with electrons, e.g. by intercalation of Cu, Sr, Nb or other metal ions in its layered structure, a superconducting state is formed\textsuperscript{11,12}. The presence of a strong spin-orbit coupling, which also manifests itself in a topological surface state of the parent insulator\textsuperscript{13}, led to the proposal of an unconventional pairing with an odd-parity symmetry
and topological superconductivity\textsuperscript{14}. The low carrier concentration, the layered structure and the low ratio $\varepsilon F/\lambda_F$ of the superconducting coherence length and the Fermi wavelength\textsuperscript{11,12}, strongly enhance fluctuation effects. In addition, numerous experiments have shown that the superconducting state is accompanied by a spontaneous breaking of rotational symmetry with a pronounced two-fold anisotropy within the Bi$_2$Se$_3$ basal plane\textsuperscript{15-22}; see Ref. 23 for a recent review. The two-fold symmetry can be observed in field-angle resolved experiments where a magnetic field is rotated in the plane with respect to the crystalline axes and the corresponding physical quantity (e.g. spin susceptibility, specific heat, magneto-resistance, upper critical field, magnetization, magnetic torque) is represented as a function of angle\textsuperscript{15-23}. This behavior directly reflects the anisotropy of the superconducting state. Thus, doped Bi$_2$Se$_3$ is an unconventional nematic superconductor with a pairing wave function in either the two-component $E_u$ or $E_g$ point group representation, the only pairing states that spontaneously break the trifold crystal symmetry within the basal plane. The temperature dependence of the penetration depth of Ref. 24 supports point nodes, consistent with $E_u$ odd-parity pairing.

In this letter we report on high-resolution thermal expansion experiments on a superconducting mono-crystalline Nb-doped Bi$_2$Se$_3$ sample in combination with electrical transport, DC magnetization, and specific heat data demonstrating a $Z_3$-vestigial nematic phase with enhanced superconducting fluctuations. We have measured the linear thermal expansion $\alpha_\mu = \partial \varepsilon_\mu / \partial T$ in three different directions in the Bi$_2$Se$_3$ basal plane and observed a strong anisotropic expansion occurring at a temperature of ~0.5 K above the superconducting transition. While the transition detected via thermal expansion is not visible at first sight in the other measurements, our high-resolution magnetization and specific heat data - after zooming near $T_c$ - show that an anomaly with increasing superconducting fluctuations occurs at the nematic transition. As we will explain below, these observations are perfectly consistent with a vestigial nematic phase of symmetry-breaking pairing fluctuations, recently predicted in Ref. 25. This observation of a genuine symmetry breaking of pairing fluctuations above $T_c$ is qualitatively distinct from the gradual onset of order-parameter fluctuations in the disordered phase\textsuperscript{26} or the crossover to Bose-Einstein condensation of pairs\textsuperscript{27}. It corresponds to a sharply defined state of matter that might, e.g. undergo a separate quantum phase transition when a magnetic field is applied in the plane at low temperatures.

The advantage of the Nb-doped Bi$_2$Se$_3$ system is that single crystals with a high superconducting volume fraction and a complete zero resistance can be found, as the results presented here show. All our bulk thermodynamic data (thermal expansion, magnetization and specific heat) show relatively large anomalies at the superconducting transition.

Fig. 1a shows magnetoresistance data recorded at 0.35 K with the magnetic field applied strictly parallel to the Bi$_2$Se$_3$ basal plane for different directions in the plane with respect to the trigonal crystalline axes. We determine the approximate $H_{c2}$ values from the fields in which 80% of the normal state resistance is reached and plot these values in Fig. 2b in a polar diagram as a function of angle $\phi$. A significant angular variation of $H_{c2}$ can be observed, ranging from ~0.6 T at 90° where $H_{c2}$ is minimal to 0.95 T at 180° with a clear maximum of $H_{c2}$. The data show a pronounced two-fold symmetry, at odds with the trifold crystalline symmetry. This is the characteristic property of nematic superconductivity in Nb$_x$Bi$_2$Se$_3$,\textsuperscript{19,20} also known from Cu$_x$Bi$_2$Se$_3$\textsuperscript{15,16,22} and Sr$_x$Bi$_2$Se$_3$\textsuperscript{17,18,21}. A fit with a theoretical model for nematic SC of the form\textsuperscript{20}
\[ H_{c2}(\phi) = \frac{H_{c2}(0)}{\sqrt{\cos^2 \phi + \Gamma^2 \sin^2 \phi}} \]  

yields \( \Gamma \approx 1.59 \) as a measure of the anisotropy in the basal plane. It should be noted that the normal state resistance well above the upper critical field has no variation for the different orientations of the magnetic field in the plane, indicating an isotropic normal state within the trigonal basal plane.

**Figure 1** | Dependence of the upper critical field \( H_{c2} \) at 350 mK of Nb_{0.25}Bi_{2}Se_{3} on the angle \( \phi \) in the plane determined from field-angle-resolved magnetoresistance measurements. a, Magnetoresistance data for various alignments of the magnetic field in the Bi_{2}Se_{3} basal plane. The additional symbols mark the characteristic fields in which the magnetoresistance reaches 60, 70, 80, 90 and 99% of the normal state value. The data were shifted vertically by 0.02 m\( \Omega \) cm for better clarity, except for the \( \phi = 90^\circ \) data. b, Polar plot of characteristic fields where the magnetoresistance reaches 80% of the normal state value, depending on the field direction in the Bi_{2}Se_{3} basal plane. Only the full squares are real data, while the open squares are the same data shifted by 180° to better illustrate the full angular dependence. In the center, the corresponding crystal structure is added. The lines are theoretical expectations of a superconductor with trigonal symmetry without (green) and with (red) vestigial nematic order.

Thermal expansion experiments provide us with a highly sensitive bulk thermodynamic proof of nematic and superconducting order. Fig. 2a shows the linear thermal expansion \( \Delta L/\mu /L_0 = \int_{T_0}^{T} \alpha_\mu(T')dT' \) measured along the three directions within the Bi_{2}Se_{3} basal plane of 90°, 155° and 215°. All data fall perfectly on each other in the normal state, but begin to deviate gradually from each other below 3.8 K. In the following we will refer to this characteristic temperature as \( T_{nem} \), because here the onset of a two-fold crystalline distortion and thus nematicity occurs. The two-fold crystalline distortion with a relative length change \( \Delta L/L_0 = 0.2 \times 10^{-5} \) amounts to a distortion of less than a femtometer within the unit cell. Still, these minute changes comparable
to the size of the proton, are clearly resolvable in our measurements. The distortion is correlated with the upper critical field, with a small negative length change along 90° where the $H_{c2}$ minimum occurs (Fig 1b), and large positive anomalies at 155° and 215°, both near the mean $H_{c2}$ value. At 3.25 K, much smaller anomalies are visible that can be identified as the superconducting transition, as the comparison with the specific heat (shown in the same graph) reveals. In the specific heat, no anomaly is visible at $T_{\text{nem}}$, while the temperature derivative of $C/T$ (Fig. 2b) hints at a small anomaly at $T_{\text{nem}} = 3.8$ K. The anomalies in $\Delta L/L_0$ at $T_{\text{nem}}$ show up as a somewhat broadened step. As a first-order derivative of free energy, a step-like transition in $\Delta L/L_0$ is the characteristic signature of a first-order transition, while a second-order transition would appear as a kink. In distinction, the superconducting transition at $T_c$ remains the standard second-order transition, as evidenced by the jump in the specific heat. Fig. 2c shows the Meissner signal in the zero-field cooled (ZFC) and field-cooled (FC) DC magnetization, which agrees with $T_c \approx 3.25$ K obtained from the specific heat. We also show the same data, but with a magnification of $10^5$, to illustrate that an enhanced diamagnetic response, signaling superconducting fluctuations, already sets in at $T_{\text{nem}}$.

![Figure 2](image)

**Figure 2 | Thermal expansion, specific heat and DC magnetization of of Nb$_{0.25}$Bi$_2$Se$_3$.** a, Linear thermal expansion $\Delta L(T)/L_0$ (units on the left axis) measured in three directions in the Bi$_2$Se$_3$ basal plane corresponding to 90°, 155° and 215°, together with the specific heat $C/T$ for comparison (units on the right axis). $C/T$ shows that the superconducting transition is at $\sim T_c = 3.25$ K, with small kink-like anomalies occurring in the thermal expansion. A pronounced anisotropy appears in $\Delta L(T)/L_0$ at a higher temperature below $T_{\text{nem}} = 3.8$ K. b, First order derivative $d(C/T)/dT$ of specific heat. A linear background fitted in the range above 4 K was subtracted for reasons of clarity. The large dip centered at $\sim 3.25$ K marks $T_c$, while at $T_{\text{nem}}$ a tiny step-shaped anomaly is visible. c, DC magnetization showing the total Meissner signal and a magnification of $10^5$ to demonstrate that the onset of superconducting fluctuations is at $\sim 3.8$ K.
Our data thus show that nematic superconductivity occurs in the form of a two-stage transition, see also Fig.3 for an illustration. The distortion forms near the higher onset temperature $T_{\text{nem}}$, where superconducting fluctuations in the magnetization and the first-order derivative of the specific heat are visible. The superconducting transition occurs at a lower temperature $T_c$, which corresponds to the formation of a global phase-coherent superconducting state. The signs and magnitudes of the length changes for the three measuring directions are consistent with the indicated distortions, see methods section.

![Figure 3](image)

**Figure 3** | Lattice distortion (schematic and strongly exaggerated) and real space image of the superconducting order parameter for Nb-doped Bi$_2$Se$_3$ with an intermediate nematic state. Purple dashed lines below $T_{\text{nem}}$ indicate the high-temperature atomic positions. Arrows in the nematic phase indicate the directions of the thermal expansion measurements. In the latter, strong superconducting fluctuations break the discrete lattice symmetry without broken U(1) symmetry and superconducting coherence. A globally coherent superconducting state only sets in at the superconducting transition temperature $T_c$.

It could be argued that thermal expansion reveals a separate structural transition at $T_{\text{nem}}$ that has nothing to do with superconductivity. Such a sequence of independent or competing transitions would be allowed within the Landau theory of phase transitions. Obviously, the anomaly in the diamagnetic response at $T_{\text{nem}}$ is strong evidence that this is not the case. Furthermore, in Fig. 4 we show magnetostriction data for the $155^\circ$ direction, along which we observed the greatest change in length in Fig. 2a. Here, $\Delta L/L_0$ was measured at a fixed temperature of 350 mK as a function of the magnetic field. A broad step-like transition occurs with a total length change of $\Delta L/L_0 \approx 0.22 \times 10^{-5}$ with onset at ~0.9 T, which is close to the resistively determined $H_{\text{c2}}$ for this direction. The field-induced length change at low temperature shown in Fig. 4 corresponds to the temperature-induced length change at zero field shown in Fig. 2a. This observation provides further evidence that the nematic distortion is closely linked to the superconducting state.
Figure 4 | Magnetostriction $\Delta L(T)/L_0$ measured along the $155^\circ$ direction at a fixed temperature of 350 mK as a function of the magnetic field. The field was applied along the direction of the measurement. The crystalline distortion is removed at $H_{c2}=0.9$ T. A weak linear normal state background was subtracted for clarity.

Our findings can be explained in terms vestigial order due to superconducting fluctuations. In fact, recently it has been suggested that such vestigial order should emerge from the superconducting phase in doped Bi$_2$Se$_3$. On the one hand, the superconducting order parameter of either the $E_g$ or the $E_u$ representation has two components $^{14,29-31}$

$$ (\Delta_x, \Delta_y) = \Delta_0 e^{i\varphi}(\cos \theta, \sin \theta), $$

that are characterized by the overall amplitude $\Delta_0$, the global U(1) phase $\varphi$ and three distinct values of the angle $\theta = \{\pi, \pi \frac{5\pi}{6}, \frac{5\pi}{6}\}$ that select a specific crystalline axis. Superconducting fluctuations will then induce a phase transition to a vestigial nematic state at a temperature $T_{nem}$ above $T_c$. While superconductivity is signaled by a finite expectation value of $\Delta_x$ or $\Delta_y$, the nematic phase is characterized by a finite expectation value of the composite order parameter

$$ Q = \begin{pmatrix} |\Delta_x|^2 - |\Delta_y|^2 & \Delta_x^*\Delta_y + \Delta_y^*\Delta_x \\ \Delta_x^*\Delta_y + \Delta_y^*\Delta_x & |\Delta_y|^2 - |\Delta_x|^2 \end{pmatrix}. $$

Upon increasing the temperature, superconducting fluctuations continue to break the rotational symmetry, even after restoration of global U(1) symmetry at $T_c$. The composite order parameter $Q_{\mu\nu}$ is made up of combinations of the superconducting order parameter, similar to charge-4e superconductivity proposed within the context of pair-density wave order in cuprate superconductors $^{32,33}$. As a traceless second-rank tensor, $\langle Q_{\mu\nu} \rangle = Q_0(n_\mu n_\nu - \frac{1}{2}\delta_{\mu\nu})$ behaves, however, like a nematic order parameter with director $\mathbf{n} = (\cos \theta, \sin \theta)$ $^{34}$ and strongly couples to the strain tensor $\varepsilon_{\mu\nu}$ via $\kappa tr(Q\varepsilon)$ with nemato-elastic coupling constant $\kappa$. A nonzero $Q_0$ then induces a lattice distortion $\varepsilon_{\mu\nu} \propto \kappa Q_{\mu\nu}$, see Fig.3. Thus, the lattice can be utilized to detect this
unconventional electronic order. The point group analysis further yields a first-order transition at \( T_{nem} \) into a state with \( Q_0 \neq 0 \), since it is in the three-state Potts model, i.e. the \( Z_3 \) universality class. The superconducting transition continues to be of second order, all in agreement with our experimental findings. In Fig. 5a we show the nematic and superconducting order parameters and in Fig. 5b the diamagnetic susceptibility obtained within the theory of Ref. 25. The susceptibility is compared with the data of Fig. 2c, where the logarithmic axis is used to illustrate the rapid growth of diamagnetic fluctuations below \( T_{nem} \). The anisotropy of \( H_{c2}(\phi) \) shown in Fig.1b (red line) was also obtained within the same theory and is compared with the behavior without nematic order (\( Q_0 = 0 \)) where \( H_{c2} \) should have six-fold symmetry\(^{24,25} \). Without nematic phase above \( T_c \), the superconducting order parameter directly at \( H_{c2}(\phi) \) is infinitesimal and no two-fold rotational symmetry breaking should be visible, in clear contrast to experimental observations. In Ref. 30, the two-fold symmetric behavior of \( H_{c2}(\phi) \) only occurred after an additional symmetry-breaking strain was added. Vestigial nematic order offers a natural explanation for this strain field.

![Diagram](image)

**Figure 4** Superconducting and nematic order parameters (a) and diamagnetic susceptibility (b) obtained within the theory of Ref. 25. We also show the experimental magnetization data (c) on a log-linear scale to compare with the theory.

To summarize, our data demonstrate that a separate nematic transition occurs in the doped topological insulator \( \text{Nb}_{0.25}\text{Bi}_2\text{Se}_3 \) at \( T_{nem} = 3.8 \text{ K} \), i.e. about 0.5 K above \( T_c \), with a distinct crystalline distortion occurring in the \( \text{Bi}_2\text{Se}_3 \) basal plane. \( T_{nem} \) coincides with the onset temperature of superconducting fluctuations. The direction of the crystalline distortion is correlated with the direction of the two-fold symmetry of the superconducting order parameter.
and is removed together with the superconductivity at or near the upper critical field $H_{c2}$. The two transitions are thus interconnected. Our observations are perfectly consistent with vestigial nematic order and a sequential restoration of U(1) and rotational symmetry. The new nematic phase is a state of matter in which Cooper pairs have lost their off-diagonal long-range order, yet fluctuate in a way that breaks the rotational symmetry of the crystalline lattice.

After this work was completed, we learned about Ref. 35, where a two-fold symmetry breaking above $T_c$ is reported. Our results agree with those of Ref. 35 and make evident that the high temperature phase is separated by an actual first order transition where superconducting fluctuations are enhanced.

## Methods

Methods, including statements on data availability, are available in the online version of this paper.

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Author contributions

This work was initiated by R.L., J.Y.S. carried out the magneto-transport measurements, C.w.C. carried out the thermal expansion measurements, C.w.C. and R.L. carried out the specific heat and DC magnetization measurements with help of J.L.; the single crystal sample was provided by S.H.L. and Y.S.H., M.H. and J.S. provided the theoretical simulations and further theoretical support. The manuscript was prepared by R.L. and J.S. with help of C.w.C. and M.H. All authors were involved in discussions and contributed to the manuscript.

Competing financial interests

The authors declare no competing financial interests.

Methods
The detailed growth method and characterization of $\text{Nb}_{0.25}\text{Bi}_2\text{Se}_3$ in the mono-crystalline form can be found in Ref. 19. The high-resolution linear thermal expansion was measured with a capacitive technique using a dilatometer, in which the sample is pressed by a fine screw mechanism against a cantilever forming one of the two plates of a capacitor. A change in the sample length leads to a change in the separation of the capacitor plates, which can be determined with a General Radio 1615A capacitance bridge in combination with a Stanford Research SR830 digital lock-in amplifier. We have measured the thermal expansion as a function of temperature in three different directions within the basal $\text{Bi}_2\text{Se}_3$ plane ($90^\circ$, $155^\circ$ and $215^\circ$). $0^\circ$ is the direction normal to the mirror plane and corresponds to the magnetic-field direction in the plane providing the maximum upper critical field. The other directions were chosen as representative of other characteristic directions in the plane, but largely dictated by the crystal shape, which allowed a stable mounting of the sample only for certain directions. Fig. 6 shows a photograph of the mounted crystal for the three different orientations in the dilatometer.

![Figure 6](https://via.placeholder.com/150)

**Figure 6** | Photographs of the $\text{Nb}_{0.25}\text{Bi}_2\text{Se}_3$ single crystal mounted in the capacitive dilatometer along the three measured directions within the $\text{Bi}_2\text{Se}_3$ basal plane. The inset provides a schematic sketch of the crystal shape and dimensions, illustrating the measured directions in the plane.

The specific heat was measured with a home-made calorimeter, which can be used either in AC modulated temperature mode or in long relaxation mode. The long relaxation mode provides high accuracy in the absolute value of 1% precision, while the AC mode provides high relative resolution with a high density of data points of 1000 points per K. The data presented in this letter has been acquired using the AC technique, but the absolute value has been calibrated using the relaxation technique. DC magnetization was measured using a commercial Quantum Design Vibrating Sample SQUID magnetometer and the electrical resistance was measured using a standard 4-probe technique with a Keithley 6221 AC current source combined with a SR830 digital lock-in amplifier. For the latter, a low temperature piezo rotator was used to precisely align the magnetic field along the different crystalline directions and to study the $H_{c2}$ anisotropy that reflects the nematic superconductivity.

Note that the magnetic field induced resistive transition at $H_{c2}$ is slightly less sharp than in the sample in our previous work, where we showed similar data for another $\text{Nb}_x\text{Bi}_2\text{Se}_3$ single crystal\textsuperscript{19}. This is due to small crystallites of different orientation and anisotropy, found and removed in subsequent measurements. The apparent two-fold anisotropy is extracted from the region of steepest slope, which corresponds to the $H_{c2}$ transition of the majority phase. We
selected this sample because of its particularly high superconducting volume fraction and the large $T_c$ anomalies in bulk thermodynamic quantities.

The sketch of the distorted unit cell in Fig. 3 is deduced from the linear coupling term $\kappa \text{tr}(Q\varepsilon)$ between the strain tensor and the composite order parameter. Thus the nematic order parameter acts in the same way as an applied external stress field in the $E_g$ symmetry channel and hence distorts the unit cell. Moreover, the linear coupling term does not entail a change of the unit cell volume, which is assumed to be unaltered in the following. Figure 7 shows the relative length changes in the three directions 90°, 155°, and 215° caused by an $E_g$ unit cell deformation as a function of the lattice parameter $a/a_0$. For the strongly exaggerated value $a/a_0 = 1.1$ we also show the corresponding distorted hexagon in the inset, which is quantitatively similar to the distorted unit cell in Fig. 3. While the computed relative length changes qualitatively capture the measured thermal expansion behavior, the calculated magnitude of the 90° direction is larger than that observed in experiment when compared to the other two directions. This could be due to the higher-order coupling that gives rise to a change of the unit cell volume at $T_{\text{nem}}$.

Figure 7 | Relative length changes in the three directions 90°, 155°, and 215° caused by an $E_g$ unit cell deformation as a function of the lattice parameter $a/a_0$. The inset shows a distorted hexagon for an exaggerated value of $a/a_0 = 1.1$. 