Optimal Observing Strategies for Velocity-Suppressed Dark Matter Annihilation

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Road Map: The whole talk in 20 seconds

Indirect Detection is a powerful probe of DM

Velocity-dependent DM annihilation requires different theoretical tools

Outcome: Optimizing the signal-to-noise ratio implies non-trivial observation strategies
Dark Matter Exists!
Indirect Detection

Looking for Standard Model particles produced from the annihilation or decay of dark matter.
Where should we point our telescopes?

Common sense: Look at where the DM density is highest!

M31, M87, dSphs

Galactic Center
Annihilation Flux – Quantifying the Signal

Typically, the photon flux for DM annihilation is decomposed as

$$\Phi = \frac{<\sigma v> dN_\gamma}{8\pi m_\chi^2 \frac{dE}{dl}} \int dl \, \rho[r(l, b)]^2$$

Particle Physics

J-Factor
Annihilation Flux – Quantifying the Signal

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$$\Phi = \frac{\langle \sigma v \rangle dN_{\gamma}}{8\pi m_{\chi}^2} \int dl \frac{dE}{dE} \rho [r(l, b)]^2$$

Particle Physics \hspace{2cm} J-Factor

Relies on the assumption that the cross section is velocity-independent!
Velocity-Dependent J-factor

\[ \langle \sigma v \rangle = \langle \sigma v \rangle_0 S(v/c) \]

\[ S(v/c) \overset{\text{def}}{=} (v/c)^n \]

\[ J(b) = \int dl \int d\nu^3 f(\nu) (v/c)^n \rho[r(l, b)]^2 \]

e.g. Board et al. 2101.06284
Velocity-Dependent J-factor

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\[ S(v/c) \overset{\text{def}}{=} (v/c)^n \]

\[ J(b) = \int dl \int d\mathbf{v}^3 f(v) (v/c)^n \rho[r(l, b)]^2 \]

- n = 0: s-wave
- n = 2: p-wave
- n = 4: d-wave

e.g. Board et al. 2101.06284
Velocity-Dependent J-factor

Think simple! Assume Maxwell-Boltzmann distribution

\[ f(v) \propto (\sigma_v^2)^{-3/2} e^{-v^2/\sigma_v^2} \]

From equipartition theorem:

\[ \sigma_v^2 = \frac{\langle v^2 \rangle}{3} \]

Where (virialized)

\[ <v^2>^{1/2} \propto v_c(r) = \sqrt{\frac{2GM(<r)}{r}} \]
Velocity-Dependent J-factor

Average relative velocity

\[
\langle v_{\text{rel}}^2 \rangle = \langle (v - v')^2 \rangle = \int d^3v \int d^3v' (v - v')^2 f(v) f(v') = 2\langle v^2 \rangle = 2v_c^2
\]

So in this case,

\[
J_n(b) \propto \int dl \ (v_c/c)^n \rho \left[ r(l, b) \right]^2
\]
Velocity-Dependent J-factor

\[ J_n(b) \propto \int dl \left( \frac{v_c}{c} \right)^n \rho[r(l,b)]^2 \]
Velocity-Dependent J-factor - Calculation

\[ r = \sqrt{r_d^2 - 2lr_d \cos(b) + l^2} \]

\[ \rho(r) = \frac{\rho_0}{\left( \frac{r}{r_s} \right)^2 (1 + \frac{r}{r_s})^2} \]

\[ M(< r) = 4\pi \rho_0 r_s^3 \left( \frac{r_s}{r_s + r} - 1 + \log \left( 1 + \frac{r}{r_s} \right) \right) \]

\[ v_c(r) = \sqrt{\frac{2GM(< r)}{r}}. \]

\[ J_n(b) \propto \int dl (v_c/c)^n \rho [r(l, b)]^2 \]

Everything is known once density profile is defined!
Velocity-Dependent J-factor

Velocity-dependent channels are less sharply peaked at the center!
Signal to noise ratio

Photon counts: independent, random events at a constant rate. Well described by a Poisson distribution!

\[ \sigma = N^{1/2} \]

Define optimal field of view as the one that maximizes the quantity

\[ \frac{\mathcal{J}}{N^{1/2}} \]
Gamma Ray Background – Diffuse, Isotropic

Bremsstrahlung

Inverse Compton

$N_{Itot} \propto \text{Field of View}$
Gamma Ray Background – Point Sources

\[ N_G(b) \propto \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{b^2}{\sigma^2} \right)} \]

\( \sigma \): Width due to instrument-dependent point spread function
Field of View Geometry

$\theta_1 = 0$: Disk

$\theta_1 \neq 0$: Annulus

$\theta_2 - \theta_1$: Thickness
Extragalactic Results

\[ \frac{J}{N^{1/2}} \]

EG s-wave

\[ \theta_2 - \theta_1 \] [deg]

\[ \theta_1 \] [deg]

[Color bar with values 0.1 to 0.9]

\[ \theta_1 \]

\[ \theta_2 \]
Extragalactic Results

\[
\frac{J}{N^{1/2}}
\]

EG s–wave

EG p–wave

EG d–wave

Dots: Fermi-LAT Surveys (Mauro et al.; Feng et al.; Abddo et al.)
Caveats and Future Work

No two sources are alike (e.g. dSphs). Results are dependent on background, $\rho$, and $v$.

Oman et al. MNRAS 452, 3650-3665
Caveats and Future Work

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Need to account for DM Substructure and baryons

Andrey Kravtsov, arXiv: 0906:3295

Oman et al. MNRAS 452, 3650-3665
Thank you!

Wonderful Collaborators

Gabby Huckabee

Stefano Profumo

Our recent paper on this subject:

arXiv: 2105.03438
nwsmyth@ucsc.edu
Bonus Slides
Velocity-Suppressed Cross Sections

Example: Majorana DM annihilating to fermion/antifermion pairs

\[ \mathcal{M} \propto \frac{m_f^2}{m_\chi^2} \]

Outgoing fermions must have same helicity (opposite chirality). Coupling must vanish in chiral limit. To get the correct final state spin, the s-wave cross section is chirality suppressed by

\[ L, S, J = 0 \]

Kumar and Light 1612.00773
Thermal Average + Velocity Expansion

\[
\langle \sigma v \rangle = \frac{\int \sigma v \, d\bar{n}_1^{eq} \, d\bar{n}_2^{eq}}{\int d\bar{n}_1^{eq} \, d\bar{n}_2^{eq}} = \frac{\int \sigma v \, e^{-E_1/T} \, e^{-E_2/T} \, d^3p_1 \, d^3p_2}{\int e^{-E_1/T} \, e^{-E_2/T} \, d^3p_1 \, d^3p_2}.
\]

\[
\langle \sigma v \rangle = \frac{1}{8m^4TK_2^2(m/T)} \int_{4m^2}^{\infty} \sigma(\bar{s} - 4m^2) \sqrt{\bar{s}} K_1(\sqrt{\bar{s}}/T) \, ds
\]

Expand \( \langle \sigma v \rangle \) in powers of \( v \):

\[
\langle \sigma v \rangle = \langle \sigma v \rangle_0 S(v/c)
\]
Instrumental Angular Resolution

FERMI-LAT:
~0.15 degrees for >10 GeV
~1 degree for ~1 GeV

AdEPT:
<0.1 degrees for >1 GeV
Gamma Ray Background – Bulge

Hooper, Goodenough. arXiv:1010.2752
Gamma Ray Background – 2 cases

Extra-galactic
Isotropic + Point Source

Galactic Center
Bulge + Point Source
Galactic Center – Weak Point Source

\[ \frac{J}{N^{1/2}} \]

**GC s-wave**

**GC p-wave**

- **Green:** Johnson et al. 1904.06261;  
- **Red:** Leane and Slatyer. 1904.08430
Galactic Center – Strong Point Source

\[ \frac{J}{N^{1/2}} \]

**GC s-wave**

**GC p-wave**

Green: Johnson et al. 1904.06261; Red: Leane and Slatyer. 1904.08430
Signal to Noise Ratio – Extra-galactic, Core

\[ \frac{J}{N^{1/2}} \]

\[ \theta_1, \theta_2 \]

EG Core s-wave

EG Core p-wave

\[ \theta_1 \text{[deg]} \]

\[ \theta_2 - \theta_1 \text{[deg]} \]

\[ \theta_1 \text{[deg]} \]
Properties of Dark Matter

Interacts through gravity

Invisible

Stable on long timescales