Predictions for Proton Life-Time in Minimal Non-Supersymmetric SO(10) Models: An Update

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Abstract

We present our best estimates of the uncertainties due to heavy particle threshold corrections on the unification scale $M_U$, intermediate scale $M_I$, and coupling constant $\alpha_U$ in the minimal non-supersymmetric SO(10) models. Using recent CERN $e^+e^-$ collider LEP data on $\sin^2\theta_W$ and $\alpha_{\text{strong}}$ to obtain the two-loop-level predictions for $M_U$ and $\alpha_U$, we update the predictions for proton life-time in minimal non-supersymmetric SO(10) models.

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1 Introduction

The hypothesis of a single unified gauge symmetry describing all forces and matter at very short distances is a very attractive one from practical as well as aesthetic point of view. Right now there are several good reasons to think that this gauge symmetry may indeed be SO(10)\(^1\). The most compelling argument in favor of SO(10) comes from ways to explain\(^2\) the observed deficit\(^3\) of the solar neutrino flux compared to the predictions\(^4\) of the standard solar model in terms of a two flavor MSW\(^5\) neutrino oscillation. Consistent understanding of the data from all four experiments using the MSW oscillation hypothesis requires the neutrino masses and mixings to lie in a very narrow range of values. It was shown in a recent paper\(^6\), that the minimal SO(10) theory that implements the see-saw mechanism\(^7\) is a completely predictive theory in the neutrino sector and predicts masses and mixings between \(\nu_e\) and \(\nu_\mu\) that are in this range. In addition to this, there are many other highly desirable features of the SO(10) theory, such as fermion unification into a single \(\{16\}\)-representation, a simple picture of baryogenesis\(^8\), asymptotic parity conservation of all interactions, etc. In view of these, we have undertaken a detailed quantitative analysis of the symmetry breaking scales of this minimal model in order to pinpoint its predictions for proton life time, especially the uncertainties in it arising from unknown Higgs masses in the theory.

Since we are going to discuss the minimal SO(10) model, let us explain what we mean by the word ‘minimal’. It stands for the fact that a) the Higgs sector is chosen to consist of the smallest number of multiplets of SO(10) that is required for symmetry breaking and b) only those fine-tunings of the parameters needed to achieve the desired gauge hierarchy are imposed. The above fixes the Higgs mass spectrum of the model completely.

Before we proceed further, we wish to make the following important remark
about the minimal SO(10) model. For a long time, it was thought that this model
cannot be realistic since it predicts the relations among fermion masses such as
\( m_s = m_\mu \) and \( m_d = m_e \) at the GUT scale \( M_U \), which after extrapolation to the
weak scale, are in complete disagreement with experiment. However, it was shown
in Ref. [6] that in the minimal SO(10) models where the small neutrino masses
arise from the see-saw mechanism[7], there are additional contributions to charged
fermion masses, that solve this problem. They arise from the fact that the \((2,2,15)\)
submultiplet of the \{126\}-dim. Higgs multiplet used in implementing the see-saw
mechanism automatically acquires an induced VEV without additional fine-tuning.
These additional contributions correct the above mass relations in such a way as
to restore agreement with observations. The same theory, as mentioned above, also
predicts the interesting values for neutrino masses and mixings making the minimal
SO(10) models not only completely realistic but also testable by neutrino oscillation
experiments to be carried out soon.

Next, let us mention a word on our choice of non-supersymmetric version of the
model. While the question of gauge hierarchy certainly prefers a supersymmetric
(SUSY) SO(10) model, in the absence of any evidence of supersymmetry at low
energies as well as for the sake of simplicity alone, we believe that minimal non-
SUSY SO(10) should be thoroughly explored and confronted with experiments.

Another interesting point that needs to be emphasized is that for the minimal
set of Higgs multiplets, SO(10) automatically breaks to the standard model via only
one intermediate stage, that consists of the left-right symmetric gauge group with or
without the parity symmetry[10], depending on the Higgs multiplet chosen to break
SO(10). This leads to the following four possibilities for the intermediate gauge
symmetry:

A) \( G_{224D} \equiv SU(2)_L \times SU(2)_R \times SU(4)_c \times D \)

B) \( G_{224} \equiv SU(2)_L \times SU(2)_R \times SU(4)_C \)

3
Case A arises if the Higgs multiplet used to break is a single $\{54\}$-dimensional one\(^{11}\). Cases B and C arise if a single $\{210\}$-Higgs multiplet is used. Depending on the range of the parameters in the Higgs potential, either case B or case C arises as the intermediate symmetry\(^ {12}\). Case D arises when one uses a combination of $\{45\}$- and $\{54\}$-dimensional Higgs multiplets\(^{13}\). The rest of the symmetry breaking is implemented by a single $\{126\}$-dimensional representation to break $\text{SU}(2)_R \times \text{U}(1)_{B-L}$ as well as to understand neutrino masses and a single complex $\{10\}$ to break the electroweak $\text{SU}(2)_L \times \text{U}(1)_Y$ down to $\text{U}(1)_{em}$. These four cases therefore represent the four simplest and completely realistic minimal $\text{SO}(10)$ models. In the rest of the paper, we present calculations of the predictions for proton life-time ($\tau_p$) in these models as well as the uncertainties in these predictions due to unknown Higgs masses and the uncertainties in the low energy input parameters, in order to see if the next round of proton decay search at Super-Kamiokande (SKAM)\(^ {14}\) can test this model.

2 Computation of the Threshold Uncertainties in $M_U$ and $M_I$

The two main equations in our discussion are i) the two-loop renormalization group equation for the evolution of the gauge coupling, i.e.,

$$\frac{d\alpha_i}{dt} = \frac{a_i}{2\pi} \alpha_i^2 + \sum_j \frac{b_{ij}}{8\pi^2} \alpha_i^2 \alpha_j,$$

(1)

and ii) the matching formula at the mass scale where the low energy symmetry group enlarges\(^ {13}\).
\[
\frac{1}{\alpha_i(M_f)} = \frac{1}{\alpha_f(M_I)} - \frac{\lambda_i^f}{12\pi}. \tag{2}
\]

In Eqs. (1) and (2), \(\alpha_i\) is the “fine-structure” constant corresponding to the gauge group \(G_i\) and

\[
\lambda_i^f = \text{Tr} \theta_i^V^2 + \text{Tr} \theta_i^H^2 \ln \frac{M_H}{M_I}, \tag{3}
\]

where \(\theta_i^H\) is the representation of the gauge group \(G_i\) in the representation of the Higgs submultiplet \(H\). The expressions for \(a_i\) and \(b_{ij}\) for the four cases are given in Table I\[13, 17\]. In deriving the values of \(a_i\) and \(b_{ij}\) in various cases as well as to obtain the threshold corrections \(\lambda_i\), we need to know the order of magnitude of the mass of the various Higgs submultiplets in the models. We obtain these by invoking the survival hypothesis for the Higgs multiplets as dictated by the minimal fine tuning condition for gauge symmetry breaking\[18\]. Using this hypothesis, in Tables IIa-IId, we list the various Higgs multiplets whose masses are near the relevant symmetry scales along with their contributions to \(\lambda_i^f\).

We proceed as follows: first using the two-loop equation, we derive the mean values for the mass scales in various cases. These results already exist in the literature\[16, 17, 19, 20\] based on the earlier LEP results. In Table III, we have presented their values from Ref. [20], which uses the inputs \(\alpha_1(M_Z) = 0.16887 \pm 0.000040; \alpha_2(M_Z) = 0.03322 \pm 0.00025; \alpha_3(M_Z) = 0.120 \pm 0.007\), for further use in calculating \(\tau_p\). These values of \(M_U\) and \(M_I\) were obtained using analytic integration of Eq. (1) which has been done exactly for case A. For cases B, C, D, we have ignored terms whose effect in the final result of the renormalization group equation is smaller than the error coming from low energy LEP data by a factor of ten or more. We have also checked that inputting the most recent LEP [21] gives results for the mass scales which are within the level of accuracy of our calculations. For instance, for \(M_U\)
the changes are 10^{-08}, 10^{-01}, 10^{-06}, and 10^{-14} for cases A, B, C and D respectively. Then, we estimate the uncertainties in \( M_I \) and \( M_U \) due to both the experimental uncertainties in the low energy parameters \( \alpha_s, \sin^2 \theta_W \) and \( \alpha_{em} \) as well as the unknown Higgs boson masses. For the cases B and D, these were discussed in Ref. [22] although we refine these uncertainties somewhat, but the threshold uncertainties for cases A and C are new. These uncertainties are presented in Table IV. We have allowed the Higgs masses to be between \( 1/10 \) to \( 10 \) times the scale of the relevant symmetry breaking.

The formulas for the threshold effect on the mass scales \( M_I \) and \( M_U \), which were used to obtain Table IV, are given below for each symmetry breaking chain. We have defined \( \eta_i = \ln \left( \frac{M_{H_i}}{M} \right) \), where \( M \) is the relevant gauge symmetry breaking scale near \( M_{H_i} \).

Model A

\[
\begin{align*}
\Delta \ln \left( \frac{M_{C+}}{M_Z} \right) &= 0\eta_0 + 0\eta_{126} + 0\eta_{54} \\
&+ 0\eta_\phi + 0.431818\eta_R - 0.454545\eta_L \\
\Delta \ln \left( \frac{M_U}{M_Z} \right) &= -0.04\eta_0 - 0.08\eta_{126} - 0.04\eta_{54} \\
&+ 0.02\eta_\phi - 0.106818\eta_R + 0.194545\eta_L \\
&\tag{4A}
\end{align*}
\]

Model B

\[
\begin{align*}
\Delta \ln \left( \frac{M_C}{M_Z} \right) &= +0.0518135\eta_0 + 0.103627\eta_{126} + 0.0518135\eta_{210} \\
&- 0.0259067\eta_\phi + 1.12953\eta_R - 1.29534\eta_{\Delta L} \\
\Delta \ln \left( \frac{M_U}{M_Z} \right) &= -0.0621762\eta_0 - 0.124352\eta_{126} - 0.0621762\eta_{210} \\
&+ 0.0310881\eta_\phi - 0.40544\eta_R + 0.554404\eta_{\Delta L} \\
&\tag{4B}
\end{align*}
\]

Model C
\[ \Delta \ln \left( \frac{M_{R+}}{M_Z} \right) = +0.0952381 \eta_{10} - 0.0952381 \eta_{26} + 0 \eta_{210} 
- 0.047619 \eta_{\phi} + 0.190476 \eta_{R1} - 0.142857 \eta_{L1} \]

\[ \Delta \ln \left( \frac{M_U}{M_Z} \right) = -0.0544218 \eta_{10} - 0.159864 \eta_{126} - 0.0357143 \eta_{210} 
+ 0.0272109 \eta_{\phi} + 0.0340136 \eta_{R1} + 0.117347 \eta_{L1} \]  

(4C)

Model D

\[ \Delta \ln \left( \frac{M_R}{M_Z} \right) = +0.124138 \eta_{10} - 0.0827586 \eta_{26} + 0.00689655 \eta_{45} 
- 0.062069 \eta_{\phi} + 0.22069 \eta_{R1} - 0.193103 \eta_{H \Delta} \]

\[ \Delta \ln \left( \frac{M_U}{M_Z} \right) = -0.0781609 \eta_{10} - 0.170115 \eta_{126} - 0.0413793 \eta_{45} 
+ 0.0390805 \eta_{\phi} + 0.0091954 \eta_{R1} + 0.158621 \eta_{H \Delta} \]  

(4D)

In obtaining the above equations, we have assumed that the particles from a single SO(10) representation which have masses of the same order are degenerate. This is the same assumption as in Ref. [22]. Before proceeding to give our predictions for proton life-time, few comments are in order.

a) We want to clarify how we get the uncertainties presented in Table IV. First, as already mentioned, we chose \( M_H/M_I \) or \( M_H/M_U \) to vary between \( 10^{-1} \) and \( 10^{+1} \). The maximum values of the uncertainties are obtained by allowing the different \( \eta \)'s to vary independently to their extreme values that lead to the largest positive or negative uncertainty. The only exception to this is the two parameters \( \eta_{\Delta L} \) and \( \eta_{H \Delta} \), which are always kept negative. (See below.) Secondly, in chains A and C we do not assume that the left- and right-handed Higgs submultiplets to have same mass (that would lead to \( \eta_L = \eta_R \)). The reason is that since the masses are close to the intermediate scale, where left-right symmetry is broken, the multiplets need not necessarily be degenerate. If we assumed the degeneracy, there would be cancellation.
between $\eta_L$ and $\eta_R$ terms reducing the threshold uncertainties\cite{23}; the uncertainties we present in table IV are, therefore, most conservative.

b) In cases B and D, since D-parity is broken at the GUT scale, the masses of $\Delta_L$ in Table IIb-(1) and $H_\Delta$ in Table IIId-(1) are always above the scale $M_I$, but below $M_U$\cite{22}. Although \textit{a priori} $M_{\Delta_L}$ could be bigger than $M_U$, we have kept it smaller in presenting the uncertainty in $\tau_p$. Therefore, $\eta_{\Delta_L}$ and $\eta_{H_\Delta}$ in Eqs.(4B) and (4D) are always negative, since we use $M = M_U$ to define them. In any case, from an experimental point of view, the upper value of the uncertainty is not too relevant.

c) The first set of entries in Table IV is obtained by maximizing the uncertainty in $M_I$ whereas the second set is obtained by reversing this procedure.

d) Note that, in case A, the intermediate mass scale $M_I$ and the unification scale $M_U$ are so close that one might think of this as an almost single step breaking. This is similar to the D-parity broken scenario (case B) recently discussed in Ref. \cite{24}. For proton life-time estimate, this is inconsequential.

### 3 Predictions for Proton Life-Time

Now, we present our predictions for proton life-time in the four SO(10) models A - D. For this purpose, we need the values of $M_U$ and $\alpha_U$ and remember that in SO(10) there are extra gauge bosons contributing to proton decay compared to the SU(5) model. We use the following formula from the review by Langacker\cite{25}, where the original literature can be found. We write

$$\tau_p = \tau_p^{(0)} F_p,$$
where \( F_p \) denotes the uncertainty arising from threshold corrections as well as the experimental errors in \( \alpha_s, \alpha_{em}, \) and \( \sin^2\theta_W \). From Ref. [25], we get for \( \tau_{p}^{(0)} \)

\[
\tau_{p \rightarrow e^+\pi^0}^{(0)} = \frac{5}{8} \left( \frac{\alpha_{SU(5)}^U}{\alpha_{SO(10)}^U} \right)^2 \times 4.5 \times 10^{29 \pm 7} \left( \frac{M_U}{2.1 \times 10^{14} \text{GeV}} \right)^4 \text{yrs.} \quad (5)
\]

Including the \( F_p \)-factors, we present below the predictions for proton life-time in SO(10) (noting that \( \alpha_{SU(5)}^U \approx \alpha_{SO(10)}^U \)). The first uncertainty in the predictions below arises from the proton decay matrix element evaluation whereas the second and the third ones come from LEP data and threshold correction, respectively [26].

**Model A**

\[
\tau_{p \rightarrow e^+\pi^0} = 1.44 \times 10^{32.1 \pm 1.0 \pm 1.9} \text{yrs.}
\]

**Model B**

\[
\tau_{p \rightarrow e^+\pi^0} = 1.44 \times 10^{37.4 \pm 1.0 \pm 5.0} \text{yrs.}
\]

**Model C**

\[
\tau_{p \rightarrow e^+\pi^0} = 1.44 \times 10^{34.2 \pm 7 \pm 1.7} \text{yrs.}
\]

**Model D**

\[
\tau_{p \rightarrow e^+\pi^0} = 1.44 \times 10^{37.7 \pm 7 \pm 2.7} \text{yrs.}
\]

4 Conclusion

In conclusion, we have computed the threshold uncertainties in both the intermediate and the unification scales for all four possible minimal non-supersymmetric SO(10) models A - D. We then update the predictions for proton life-time in all these cases including the most conservative estimates for the threshold uncertainties.
in it. We see that for case A, \( \tau_p \) is very much within the range of Super-Kamiokande search even without threshold corrections. On the other hand, for cases B and C, the threshold uncertainties have the effect of bring it within the range of SKAM search.

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[26] The predictions for cases B and D are different from those given in Ref.[22], because of different input LEP data.
Table Caption

Table I: One- and two-loop $\beta$-function coefficients for models A - D.

Table IIa: The heavy Higgs content of the model A. The $G_{224}$ submultiplets in (1) acquire masses when SO(10) is broken, while the $G_{123}$ submultiplets in (2) become massive when $G_{224D}$ is broken. The multiplets $\phi$, $R_i$, and $L_i$ in (2) arise from $\phi(2,2,0)$ in $\{10\}$, from $\Delta_R(1,3,10)$, and $\Delta_L(3,1,\overline{10})$ in $\{126\}$, respectively. Also listed, in the extreme right column of the tables, are the threshold contributions $\lambda_i$ of the different multiplets.

Table IIb: The heavy Higgs particles in the model B whose intermediate symmetry is $G_{224}$. The particles whose masses are on the order of $M_U$ are listed in (1), and the particles on the order of $M_I$ are listed in (2). Also listed are their threshold contributions.

Table IIc: All the heavy Higgs particles in the model C whose intermediate symmetry is $G_{2213D}$. The submultiplets with masses of order $M_U$ are presented in (1), and the particles with masses of order $M_I$ are listed in (2). The entries in the extreme right column denote their threshold contributions.

Table IID: All the heavy Higgs particles in the model D whose intermediate symmetry is $G_{2213}$. The submultiplets with masses of order $M_U$ are presented in (1), and the particles with masses of order $M_I$ are listed in (2). The entries in the far right column give their $\lambda_i$ contributions.

Table III: The values of $M_U$, $M_I$ and $\alpha_U$ obtained by solving the two-loop renormalization group equation Eq. (1) for models A - D. The results were taken from Ref. [20].

Table IV: The threshold uncertainties due to the difference between symmetry break-
ing scale and masses of Higgs bosons on the order of that scale. For the cases where
threshold effects in $M_I$ and $M_U$ are maximized, corresponding threshold uncertain-
ties are given in the first two lines and the last two lines, respectively.
Table I.

| Model | $a_i$ | $b_{ij}$ |
|-------|-------|----------|
| A     | $\left\{\frac{11}{3}, \frac{11}{3}, -\frac{14}{3}\right\}$ | $\left\{\left\{\frac{584}{3}, 3, \frac{765}{2}\right\}, \left\{3, \frac{584}{3}, \frac{765}{2}\right\}, \left\{\frac{153}{2}, \frac{153}{2}, \frac{1739}{6}\right\}\right\}$ |
| B     | $\left\{-3, \frac{11}{3}, -\frac{23}{3}\right\}$ | $\left\{\left\{8, 3, \frac{45}{2}\right\}, \left\{3, \frac{584}{3}, \frac{765}{2}\right\}, \left\{\frac{9}{2}, \frac{153}{2}, \frac{643}{6}\right\}\right\}$ |
| C     | $\left\{-\frac{7}{3}, -\frac{7}{3}, 7, -7\right\}$ | $\left\{\left\{\frac{80}{3}, 3, \frac{27}{2}, 12\right\}, \left\{3, \frac{80}{3}, \frac{27}{2}, 12\right\}, \left\{\frac{81}{2}, \frac{81}{2}, \frac{115}{2}, 4\right\}, \left\{\frac{9}{2}, \frac{9}{2}, \frac{1}{2}, -26\right\}\right\}$ |
| D     | $\left\{-3, -\frac{7}{3}, \frac{11}{2}, -7\right\}$ | $\left\{\left\{8, 3, \frac{3}{2}, 12\right\}, \left\{3, \frac{80}{3}, \frac{27}{2}, 12\right\}, \left\{\frac{9}{2}, \frac{81}{2}, \frac{61}{2}, 4\right\}, \left\{\frac{9}{2}, \frac{9}{2}, \frac{1}{2}, -26\right\}\right\}$ |

Table IIa-(1).

| SO(10) representation | G_{224} submultiplet | $\left\{\lambda_{2L}^U, \lambda_{2R}^U, \lambda_{4C}^U\right\}$ |
|------------------------|----------------------|-----------------------------------------------|
| 10                     | H(1,1,6)             | $\{0, 0, 2\}$                               |
| 126                    | $\zeta_0(2,2,15)$    | $\{30, 30, 32\}$                            |
|                        | S(1,1,6)             | $\{0, 0, 2\}$                               |
| 54                     | S_{\Sigma}(3,3,1)    | $\{6, 6, 0\}$                               |
|                        | S_{\zeta}(1,1,20')   | $\{0, 0, 8\}$                               |
|                        | S_{+}(1,1,1)         | $\{0, 0, 0\}$                               |
Table IIa-(2).

| SO(10) representation | $G_{123}$ submultiplet | $\{\lambda_1^Y, \lambda_2^L, \lambda_3^C\}$ |
|------------------------|------------------------|----------------------------------|
| 10                     | $\phi(-\frac{1}{2}\sqrt{\frac{3}{5}}, 2, 1)$ | $\{\frac{3}{5}, 1, 0\}$ |
| 126                    | $R_1(-2\sqrt{\frac{3}{5}}, 1, 1)$ | $\{\frac{24}{5}, 0, 0\}$ |
|                        | $R_2(+\frac{1}{3}\sqrt{\frac{3}{5}}, 1, 3)$ | $\{\frac{2}{5}, 0, 1\}$ |
|                        | $R_3(-\frac{4}{3}\sqrt{\frac{3}{5}}, 1, 3)$ | $\{\frac{32}{5}, 0, 1\}$ |
|                        | $R_4(-\frac{1}{3}\sqrt{\frac{3}{5}}, 1, 6)$ | $\{\frac{4}{5}, 0, 5\}$ |
|                        | $R_5(+\frac{2}{3}\sqrt{\frac{3}{5}}, 1, 6)$ | $\{\frac{16}{5}, 0, 5\}$ |
|                        | $R_6(-\frac{4}{3}\sqrt{\frac{3}{5}}, 1, 6)$ | $\{\frac{64}{5}, 0, 5\}$ |
|                        | $L_1(+\sqrt{\frac{3}{5}}, 3, 1)$ | $\{\frac{18}{5}, 4, 0\}$ |
|                        | $L_2(+\frac{1}{3}\sqrt{\frac{3}{5}}, 3, 3)$ | $\{\frac{6}{5}, 12, 3\}$ |
|                        | $L_3(-\frac{1}{3}\sqrt{\frac{3}{5}}, 3, 6)$ | $\{\frac{12}{5}, 24, 15\}$ |

Table IIb-(1).

| SO(10) representation | $G_{224}$ submultiplet | $\{\lambda_2^U, \lambda_2^R, \lambda_4^C\}$ |
|------------------------|------------------------|----------------------------------|
| 10                     | $H(1,1,6)$             | $\{0, 0, 2\}$                    |
| 126                    | $\zeta_0(2,2,15)$      | $\{30, 30, 32\}$                |
|                        | $S(1,1,6)$             | $\{0, 0, 2\}$                    |
|                        | $\Delta_L(3,1,\overline{10})$ | $\{40, 0, 18\}$                |
| 210                    | $\Sigma_L(3,1,15)$    | $\{30, 0, 12\}$                  |
|                        | $\Sigma_R(1,3,15)$    | $\{0, 30, 12\}$                  |
|                        | $\zeta_1(2,2,10)$     | $\{10, 10, 12\}$                |
|                        | $\zeta_2(2,2,\overline{10})$ | $\{10, 10, 12\}$               |
|                        | $\zeta_3(1,1,15)$     | $\{0, 0, 4\}$                    |
|                        | $S'(1,1,1)$            | $\{0, 0, 0\}$                    |
Table IIb-(2).

| G_{123} submultiplet |
|-----------------------|
| \phi, R_1, R_2, R_3, R_4, R_5, R_6 in Table IIa-(2) |
| SO(10) representation | $G_{2213}$ submultiplet | $\{\lambda_{2L}^U, \lambda_{2R}^U, \lambda_{1X}^U, \lambda_{3C}^U\}$ |
|------------------------|-------------------------|---------------------------------|
| $10$                   | $T_1(1,1,+,\frac{1}{3}\sqrt{2},\frac{3}{2})$ | $\{0, 0, 1, 1\}$ |
|                        | $T_2(1,1,+,\frac{1}{3}\sqrt{2}, 3)$          | $\{0, 0, 1, 1\}$ |
| $126$                  | $H_{1R}(1,3,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 6)$ | $\{0, 24, 6, 15\}$ |
|                        | $H_{1L}(3,1,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 6)$ | $\{24, 0, 6, 15\}$ |
|                        | $H_{2R}(1,3,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{0, 4, 3, 3\}$ |
|                        | $H_{2L}(3,1,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{0, 3, 3\}$ |
|                        | $H_3(2,2,+,\frac{2}{3}\sqrt{2}, 3)$           | $\{6, 6, 16, 4\}$ |
|                        | $H_4(2,2,+,\frac{2}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{6, 6, 16, 4\}$ |
|                        | $H_5(2,2,0,8)$                                   | $\{16, 16, 0, 24\}$ |
|                        | $H_6(2,2,0,1)$                                   | $\{2, 2, 0, 0\}$ |
|                        | $H_7(1,1,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{0, 0, 1, 1\}$ |
|                        | $H_8(1,1,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{0, 0, 1, 1\}$ |
| $210$                  | $B_{L1}(3,1,0,8)$                               | $\{16, 0, 0, 9\}$ |
|                        | $B_{R1}(1,3,0,8)$                               | $\{0,16, 0, 9\}$ |
|                        | $B_{L2}(3,1,+,\frac{2}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{6, 0, 6, 6\}$ |
|                        | $B_{R2}(1,3,+,\frac{2}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{0, 6, 6, 6\}$ |
|                        | $B_{L3}(3,1,+,\frac{2}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{6, 0, 6, 6\}$ |
|                        | $B_{R3}(1,3,+,\frac{2}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{0, 6, 6, 6\}$ |
|                        | $B_{L4}(3,1,0,1)$                               | $\{2, 0, 0, 0\}$ |
|                        | $B_{R4}(1,3,0,1)$                               | $\{0, 2, 0, 0\}$ |
|                        | $B_5(2,2,+,\frac{1}{3}\sqrt{2}, 6)$            | $\{6, 6, 4, 10\}$ |
|                        | $B_6(2,2,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 6)$ | $\{6, 6, 4, 10\}$ |
|                        | $B_7(2,2,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{3, 3, 2, 2\}$ |
|                        | $B_8(2,2,+,\frac{1}{3}\sqrt{2}, \frac{3}{2}, 3)$ | $\{3, 3, 2, 2\}$ |
|                        | $B_9(2,2,+,\frac{1}{3}\sqrt{2}, 1)$             | $\{1, 1, 6, 0\}$ |
|                        | $B_{10}(2,2,+,\frac{1}{3}\sqrt{2}, 1)$         | $\{1, 1, 6, 0\}$ |
|                        | $B_{11}(1,1,0,8)$                               | $\{0, 0, 0, 3\}$ |
|                        | $B_-(1,1,0,1)$                                  | $\{0, 0, 0, 0\}$ |
|                        | $B_+(1,1,0,1)$                                  | $\{0, 0, 0, 0\}$ |
Table IIc-(2).

| G_{123} submultiplet | \phi, R_1, L_1 in Table IIa-(2) |

Table IIId-(1).

| SO(10) representation | G_{2213} submultiplet | \{\lambda^U_{2L}, \lambda^U_{2R}, \lambda^U_{1X}, \lambda^U_{3C}\} |
|-----------------------|------------------------|--------------------------------------------------|
| 10                    | T_1, T_2               |                                                  |
| 126                   | All H’s in Table IIc-1 |                                                  |
|                       | \text{H}_{\Delta}(3,1,\sqrt{\frac{7}{2}},1) | \{4, 0, 9, 0\}                                    |
| 45                    | S_1(1,1,0,8)            | \{0, 0, 0, 3\}                                   |
|                       | S_2(3,1,0,1)            | \{2, 0, 0, 0\}                                   |
|                       | S_3(1,3,0,1)            | \{0, 2, 0, 0\}                                   |
|                       | S_{-}(1,1,0,1)          | \{0, 0, 0, 0\}                                   |

Table IIId-(2).

| G_{123} submultiplet | \phi, R_1 in Table IIa-(2) |

Table III.

| Model | M_I (GeV) | M_U (GeV) | \alpha^{-1} |
|-------|-----------|-----------|-------------|
| A     | 10^{13.64} | 10^{15.02\pm.25} | 40.76 \pm.16 |
| B     | 10^{10.70} | 10^{16.26\pm.25} | 46.35 \pm.23 |
| C     | 10^{10.16} | 10^{15.55\pm.20} | 43.86 \pm.18 |
| D     | 10^{9.08}  | 10^{16.42\pm.23} | 46.12 \pm.15 |
| Threshold Uncertainty | Model A       | Model B        | Model C       | Model D       |
|-----------------------|---------------|----------------|---------------|---------------|
| \( M_I/M^0_I \)      | \( 10^{\pm 0.886} \) | \( 10^{\pm 2.658} \) | \( 10^{\pm 0.571} \) | \( 10^{\pm 0.690} \) |
| \( M_U/M^0_U \)      | \( 10^{\pm 0.481} \) | \( 10^{\pm 0.131} \) | \( 10^{\pm 0.031} \) | \( 10^{\pm 0.128} \) |
| \( M_I/M^0_I \)      | \( 10^{\pm 0.886} \) | \( 10^{\pm 2.658} \) | \( 10^{\pm 0.000} \) | \( 10^{\pm 0.303} \) |
| \( M_U/M^0_U \)      | \( 10^{\pm 0.481} \) | \( 10^{\pm 0.131} \) | \( 10^{\pm 0.429} \) | \( 10^{\pm 0.479} \) |