The past few years have seen many interesting theoretical developments in lattice QCD. This talk (which is intended for non-experts) focuses on the problem of non-perturbative renormalization and the question of how precisely the continuum limit is reached. Progress in these areas is crucial in order to be able to compute quantities of phenomenological interest, such as the hadron spectrum, the running quark masses and weak transition matrix elements, with controlled systematic errors.

1 Introduction

This year’s lattice conference attracted more than 300 physicists from all over the world. Lattice QCD remains to be the dominant subject at these conferences, but other topics are also being addressed, such as the electroweak phase transition, quantum gravity and supersymmetric Yang-Mills theories. Much of the work done in QCD is spent to improve the computations of hadron masses, decay constants and weak transition matrix elements. Calculations of moments of structure functions, the running coupling and quark masses and many other physical quantities have also been reported. A good place to look for specific results are the proceedings of the 1996 lattice conference and more recent contributions can be found in the hep-lat section of the Los Alamos preprint server.

1.1 Numerical simulations

Quantitative results in lattice QCD are almost exclusively obtained using numerical simulations. Such calculations proceed by choosing a finite lattice, with spacing \( a \) and linear extent \( L \), which is sufficiently small that the quark and gluon fields can be stored in the memory of a computer. Through a Monte Carlo algorithm one then generates a representative ensemble of fields for the Feynman path integral and extracts the physical quantities from ensemble averages. Apart from statistical errors this method yields exact results for the chosen lattice parameters and is hence suitable for non-perturbative studies of QCD.

Numerical simulations require powerful computers and a continuous effort for algorithm and software development. Technical expertise is also needed to be able to cope with the systematic errors incurred by the finiteness of the lattice and by the data analysis. In practice such calculations are being performed by collaborations of (say) 5–15 physicists. For these groups to remain competitive it is vital that they have access to dedicated computer
systems. An adequate amount of computing power would otherwise be difficult to obtain over a longer period of time.

1.2 Computers

Until recently leading edge numerical simulations of lattice QCD have been performed on computers with sustained computational speeds on the order of 10 Gflops. The community is now moving to the next generation computers which deliver several 100 Gflops for QCD programs.

One of these machines, the CP-PACS computer\textsuperscript{2} has been installed last year at the Center for Computational Physics in Tsukuba. With its 2048 processing nodes, a total memory of 128 GB and a theoretical peak speed of 614 Gflops, this computer is a unique research facility for lattice QCD.

Other machines that will be available for dedicated use by the lattice theorists include the QCDSP and the APEmille computers\textsuperscript{3, 4}. The first of these has been designed by a consortium of physicists in the US. Machines of various size are being assembled and will be installed in the course of this year at different places, totalling more than 1000 Gflops of peak computational power. The APEmille grew out of a long-term effort of INFN, now also supported by DESY-Zeuthen, to construct affordable computers optimised for lattice gauge theory applications. It has a scalable massively parallel architecture with the largest system delivering more than 1000 Gflops. A fully operational, medium-size machine is expected to be available next summer.

It should be said at this point that the theoretical peak speed of a computer is a relatively crude measure of its performance. The sustained computational speed that can be attained also depends on many other parameters such as the bandwidths for memory-to-processor and node-to-node communications, for example. All computers mentioned here are well balanced in this respect and achieve a high efficiency for QCD programs.

1.3 Lattice QCD at 100 Gflops

At present most calculations in lattice QCD neglect sea quark effects, because the known simulation algorithms slow down dramatically when they are included. If one is willing to make this approximation (which is called “quenched QCD”), the new computers are good for lattice sizes up to about $128 \times 64^3$. Such a lattice may be arranged to have a spacing $a = 0.05$ fm, for example, in which case its spatial extent will be 3.2 fm. This is a very comfortable situation for calculations of the light hadron masses and many other quantities of interest. In general the increased computer power allows one to explore a greater range of lattices with higher statistics and thus to achieve better control on the systematic errors.

When a doublet of sea quarks is included in the simulations, lattice sizes up to $64 \times 32^3$ are expected to be within reach. This will be quite exciting, because studies of full QCD on large lattices have been rare so far, leaving many basic questions unanswered. The physics programme is essentially the same as in quenched QCD, but since one cannot afford to perform simulations at very many different values of the parameters, and since the
generated ensembles of field configurations tend to be smaller, the results will be generally less precise.

1.4 Topics covered in this talk

While the progress in computer technology is impressive, one cannot ignore the fact that the accessible lattices are too small to accommodate very large scale differences. This talk addresses two important issues which arise from this limitation and which must be resolved if one is interested in results with reliable error bounds.

One of the problems is that the lattice spacing cannot be made arbitrarily small compared to the relevant physical scales (the confinement radius for example). Taking the continuum limit thus is a non-trivial task and a lot of work has recently been spent to answer the question of how precisely the limit is approached and whether the lattice effects are negligible at current values of the lattice spacing.

The other topic that will be discussed goes under the heading of non-perturbative renormalization. In physical terms the problem is to establish the relation between the low-energy properties of QCD and the perturbative regime. Hadronic matrix elements of operators, whose normalization is specified at high energies through the \( \overline{\text{MS}} \) scheme of dimensional regularization, are an obvious case where this is required. Again a large scale difference is involved which makes a direct approach difficult, but promising ways to solve the problem have now been found.

2 Lattice effects and the approach to the continuum limit

2.1 Perturbation theory

In lattice QCD one is primarily interested in the non-perturbative aspects of the theory. Perturbation theory can, however, give important structural insights and it has proved useful to study the nature of the continuum limit in this framework. A remarkable result in this connection is that the existence of the limit has been rigorously established to all orders of the expansion\(^5\).

The Feynman rules on the lattice are derived straightforwardly from the chosen lattice action. Compared to the usual rules, an important difference is that the propagators and vertices are relatively complicated functions of the momenta and of the lattice spacing \( a \). In particular, at tree-level all the lattice spacing dependence arises in this way. A simple example illustrating this is the quark-gluon vertex. Using the standard formulation of lattice QCD (which goes back to Wilson’s famous paper of 1974\(^6\)), one finds

\[
\begin{align*}
\begin{array}{c}
\text{\( g_0 \lambda^a \{ \gamma_\mu + \frac{i}{2} a(p + p')_\mu + O(a^2) \} \)}
\end{array}
\end{align*}
\]

where \( g_0 \) denotes the bare gauge coupling and \( \lambda^a \) a colour matrix. It is immediately clear from this expression that the leading lattice corrections to the continuum term can be quite large even if the quark momenta \( p \) and \( p' \) are well below the momentum cutoff \( \pi/a \).
Moreover the corrections violate chiral symmetry, a fact that has long been a source of concern since many properties of low-energy QCD depend on this symmetry.

Lattice Feynman diagrams with \( l \) loops and engineering dimension \( \omega \) can be expanded in an asymptotic series of the form

\[
a^{-\omega} \sum_{k=0}^{\infty} \sum_{j=0}^{l} c_{kj} a^k [\ln(a)]^j.
\]

(2)

After renormalization the negative powers in the lattice spacing and the logarithmically divergent terms cancel in the sum of all diagrams. The leading lattice corrections thus vanish proportionally to \( a \) (up to logarithms) at any order of perturbation theory.

### 2.2 Cutoff dependence of hadron masses

Sizeable lattice effects are also observed at the non-perturbative level when calculating hadron masses, for example. An impressive demonstration of this is obtained as follows. Let us consider QCD with a doublet of quarks of equal mass, adjusted so that the mass of the pseudo-scalar mesons coincides with the physical kaon mass. This sets the quark mass to about half the strange quark mass and one thus expects that the mass of the lightest vector meson is given by

\[
m_V \simeq m_{K^*} = 892 \text{ MeV}.
\]

(3)

Computations of the meson masses using numerical simulations however show that this is not the case at the accessible lattice spacings (see Figure 1). Instead one observes a strong dependence on the lattice spacing and it is only after extrapolating the data to \( a = 0 \) that one ends up with a value close to expectations.
2.3 Effective continuum theory

In phenomenology it is well known that the effects of as yet undetected substructures or heavy particles may be described by adding higher-dimensional interaction terms to the Standard Model lagrangian. From the point of view of an underlying more complete theory, the Standard Model together with the added terms then is a low-energy effective theory. A similar situation occurs in lattice QCD, where the momentum cutoff may be regarded (in a purely mathematical sense) as a scale of new physics. The associated low-energy effective theory is a continuum theory with action

\begin{equation}
S_{\text{eff}} = \int d^4x \left\{ L_0(x) + aL_1(x) + a^2L_2(x) + \ldots \right\},
\end{equation}

where \( L_0 \) denotes the continuum QCD lagrangian and the \( L_k \)'s, \( k \geq 1 \), are linear combinations of local operators of dimension \( 4+k \) with coefficients that are slowly varying functions of \( a \) (powers of logarithms in perturbation theory).

Through the effective continuum theory, the lattice spacing dependence is made explicit and a better understanding of the approach to the continuum limit is achieved. In particular, neglecting terms that do not contribute to on-shell quantities, or which amount to renormalizations of the coupling and the quark masses, the general expression for the leading lattice correction is

\begin{equation}
L_1 = c_1 \overline{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi,
\end{equation}

with \( F_{\mu\nu} \) being the gluon field strength and \( \psi \) the quark field. The lattice thus assigns an anomalous colour-magnetic moment of order \( a \) to the quarks. Very many more terms contribute to \( L_2 \) and a simple physical interpretation is not easily given. The pattern of the lattice effects of order \( a^2 \) should hence be expected to be rather complicated.

2.4 \( O(a) \) improvement

The effective action, Eq. (4), depends on the physics at the scale of the cutoff, i.e. on how precisely the lattice theory is set up. By choosing an improved lattice action one may hence be able to reduce the size of the correction terms and thus to accelerate the convergence to the continuum limit\[^{10}\]. Different ways to implement this idea are being explored and there is currently no single preferred way to proceed. At last year’s lattice conference the subject has been reviewed by Niedermayer\[^{11}\] and many interesting contributions have been made since then.

\( O(a) \) improvement is a relatively modest approach, where the leading correction \( L_1 \) is cancelled by replacing the Wilson action through

\begin{equation}
S_{\text{Wilson}} + a^5 \sum_x c_{sw} \overline{\psi}(x) \frac{i}{2} \sigma_{\mu\nu} F_{\mu\nu}(x) \psi(x)
\end{equation}

and tuning the coefficient \( c_{sw} \). At the time when Sheikholeslami and Wohlert published their paper\[^{12}\], the proposition did not receive too much attention, because systematic studies of lattice effects were not feasible with the available computers. The situation has now changed.
and there is general agreement that improvement is useful or even necessary, particularly in full QCD where simulations are exceedingly expensive in terms of computer time.

An obvious technical difficulty is that $c_{sw}$ (which is a function of the bare gauge coupling) needs to be determined accurately. The problem has only recently been solved by studying the axial current conservation on the lattice\cite{13}. Chiral symmetry is not preserved by the lattice regularization and the PCAC relation satisfied by the $\bar{u}d$ component of the axial current,

$$\partial_{\mu}(\bar{u}\gamma_{\mu}\gamma_{5}d) = (m_u + m_d)\bar{u}\gamma_{5}d + \epsilon(a),$$

thus includes a non-zero error term. In general $\epsilon(a)$ vanishes proportionally to $a^2$, but after improvement the error is reduced to order $a^2$ if $c_{sw}$ has the proper value. Conversely this may be taken as a condition fixing $c_{sw}$, i.e. the coefficient can be computed by minimizing the error term in various matrix elements of Eq. (7). Proceeding along these lines one has been able to calculate $c_{sw}$ non-peturbatively in quenched QCD\cite{13,14} and now also in full QCD with a doublet of massless sea quarks\cite{15}.

2.5 Impact of $O(a)$ improvement on physical quantities

Once the improvement programme has been properly implemented, the question arises whether the lattice effects on the quantities of physical interest are significantly reduced at the accessible lattice spacings. Several collaborations have set out to check this\cite{16,17,18}, but it is too early to draw definite conclusions. The status of these studies has recently been summarized by Wittig\cite{19}.

For illustration let us again consider the calculation of the vector meson mass $m_{V}$ discussed in Subsection 2.2. A preliminary analysis of simulation results from the UKQCD collaboration gives, for the $O(a)$ improved theory, $m_{V} = 924(17)$ MeV at $a = 0.098$ fm and $m_{V} = 932(26)$ MeV at $a = 0.072$ fm. These numbers do not show any significant dependence on the lattice spacing and they are also compatible with the value $m_{V} = 899(13)$ that one obtains through extrapolation to $a = 0$ of the results from the unimproved theory (left-most point in Figure 1).

Other quantities that are being studied include the pseudo-scalar and vector meson decay constants and the renormalized quark masses. The experience accumulated so far suggests that the residual lattice effects are indeed small if $a \leq 0.1$ fm. Most experts would however agree that further confirmation is still needed.

2.6 Synthesis

At sufficiently small lattice spacings the effective continuum theory provides an elegant description of the approach to the continuum limit. Whether the currently accessible lattice spacings are in the range where the effective theory applies is not immediately clear, but the observed pattern of the lattice spacing dependence in the unimproved theory and the fact that $O(a)$ improvement appears to work out strongly indicate this to be so. Very much smaller lattice spacings are then not required to reliably reach the continuum limit. It is evidently of great importance to put this conclusion on firmer grounds by continuing and extending the ongoing studies of $O(a)$ improvement and other forms of improvement.
3 Non-perturbative renormalization

3.1 Example

We now turn to the second subject covered in this talk and begin by describing one of the standard ways to compute the running quark masses in lattice QCD. The need for non-perturbative renormalization will then become clear. Any details not connected with this particular aspect of the calculation are omitted.

A possible starting point to obtain the sum $\overline{m}_u + \overline{m}_s$ of the up and the strange quark masses is the PCAC relation

$$m_K^2 f_K = (\overline{m}_u + \overline{m}_s) \langle 0 | \bar{u} \gamma_5 s | K^+ \rangle.$$  

(8)

Since the kaon mass $m_K$ and the decay constant $f_K$ are known from experiment, it suffices to evaluate the matrix element on the right-hand side of this equation. On the lattice one first computes the matrix element of the bare operator $(\bar{u} \gamma_5 s)_{\text{lat}}$ and then multiplies the result with the renormalization factor $Z_P$ relating $(\bar{u} \gamma_5 s)_{\text{lat}}$ to the renormalized density $\bar{u} \gamma_5 s$.

Some recent results for the strange quark mass obtained in this way or in similar ways are listed in Table 1 (further results can be found in the review of Bhattacharya and Gupta [24]). The sizeable differences among these numbers have many causes. An important uncertainty arises from the fact that, in one form or another, the one-loop formula

$$Z_P = 1 + \frac{g_0^2}{4\pi} \left\{ \left( \frac{2}{\pi} \right) \ln(a\mu) + C \right\} + O(g_0^4)$$  

(9)

has been used to compute the renormalization factor, where $g_0$ denotes the bare lattice coupling, $\mu$ the normalization mass in the $\overline{\text{MS}}$ scheme and $C$ a calculable constant that depends on the details of the lattice regularization. Bare perturbation theory has long been known to be unreliable and various recipes, based on mean-field theory or resummations of tadpole diagrams, have been given to deal with this problem [25,26]. Different prescriptions however give different results and it is in any case unclear how the error on the so calculated values of $Z_P$ can be reliably assessed.

3.2 Intermediate renormalization

An interesting method to compute renormalization factors that does not rely on bare perturbation theory has been proposed by Martinelli et al. [27]. The idea is to proceed in two steps,

| $\overline{m}_s$ [MeV] | reference |
|-----------------------|-----------|
| 122(20)               | Allton et al. (APE collab.) [28] |
| 112(5)                | G"ockeler et al. (QCDSF collab.) [29] |
| 111(4)                | Aoki et al. (CP-PACS collab.) [30] |
| 95(16)                | Gough et al. [31] |
| 88(10)                | Gupta & Bhattacharya [32] |

Table 1: Recent results for $\overline{m}_s$ (quenched QCD, $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV)
first matching the lattice with an intermediate momentum subtraction (MOM) scheme and then passing to the \( \overline{\text{MS}} \) scheme. The details of the intermediate MOM scheme do not influence the final results and are of only practical importance. One usually chooses the Landau gauge and imposes normalization conditions on the propagators and the vertex functions at some momentum \( p \). In the case of the pseudo-scalar density, for example, the renormalization constant \( Z_{\text{P}}^{\text{MOM}} \) is defined through

\[
Z_{\text{P}}^{\text{MOM}} = Z_{\text{MOM}}^{2}/Z_{\text{MOM}}^{P} \times (10)
\]

where \( Z_{\text{MOM}}^{2} \) denotes the quark wave function renormalization constant and the diagrams represent the full and the bare vertex function associated with this operator.

On a given lattice and for a range of momenta, the quark propagator and the full vertex function can be computed using numerical simulations \(^{27,28}\). \( Z_{\text{MOM}}^{2} \) and \( Z_{\text{MOM}}^{P} \) are thus obtained non-perturbatively. The total renormalization factor relating the lattice normalizations with the \( \overline{\text{MS}} \) scheme is then given by

\[
Z_{\text{P}}(g_0, a\mu) = Z_{\text{P}}^{\text{MOM}}(g_0, a\mu)X_{\text{P}}(\overline{g}_{\overline{\text{MS}}}, p/\mu),
\]

with \( X_{\text{P}} \) being the finite renormalization constant required to match the MOM with the \( \overline{\text{MS}} \) scheme. \( X_{\text{P}} \) is known to one-loop order of renormalized perturbation theory and could easily be worked out to two loops.

While this method avoids the use of bare perturbation theory, it has its own problems, the most important being that the momentum \( p \) should be significantly smaller than \( 1/a \) to suppress the lattice effects, but not too small as otherwise one may not be confident to apply renormalized perturbation theory to compute \( X_{\text{P}} \). On the current lattices the values of \( 1/a \) are between 2 and 4 GeV and it is hence not totally obvious that a range of momenta exists where both conditions are approximately satisfied\(^{29,30}\). A simple criterion which may be applied in this connection is that the calculated values of \( Z_{\text{P}} \) should be independent of \( p \).

3.3 Non-perturbative renormalization group

It should now be quite clear that further progress depends on whether one is able to make contact with the high-energy regime of the theory in a controlled manner. As has been noted some time ago, this can be achieved through a recursive procedure\(^{31}\). A general solution of the non-perturbative renormalization problem is then obtained\(^{32,33,34,35}\).

The basic idea of the method can be explained in a few lines. One begins by introducing a special intermediate renormalization scheme, where all normalization conditions are imposed at scale \( \mu = 1/L \) and zero quark masses, \( L \) being the spatial extent of the lattice. We could choose a MOM scheme, for example, and set \( p \) equal to \( 2\pi/L \), the smallest non-zero momentum available in finite volume. But this is not the only possibility and other schemes are in fact preferred for technical reasons.
\[
\Lambda_{\text{MS}} = k\Lambda, \ M \quad \leftarrow \quad \Lambda, \ M
\]
\[
\downarrow \quad \text{perturbative evolution} \quad \downarrow
\]
\[
\alpha_{\text{MS}}(\mu), \overline{m}_{\text{MS}}(\mu) \quad \uparrow \quad \text{perturbative evolution} \quad \uparrow
\]
\[
\overline{g}, \overline{m} \text{ at } \mu = 100 \text{ GeV}
\]
\[
\uparrow \quad \text{non-perturbative evolution} \quad \uparrow
\]
\[
f_\pi, m_\pi, m_K, \ldots \quad \rightarrow \quad \text{finite-volume scheme}
\]
\[
\overline{g}, \overline{m} \text{ at } \mu = 0.6 \text{ GeV}
\]

Figure 2: Strategy to compute the running coupling and quark masses, taking low-energy data as input and using the non-perturbative renormalization group to scale up to high energies.

In such a scheme the scale evolution of the renormalized parameters and operators can be studied simply by changing the lattice size \( L \) at fixed bare parameters. One usually simulates pairs of lattices with sizes \( L \) and \( 2L \). Up to lattice effects the running couplings on the two lattices are then related through

\[
\overline{g}^2(2L) = \sigma(\overline{g}^2(L)), \quad (12)
\]

where \( \sigma \) is an integrated form of the Callan-Symanzik \( \beta \)-function. Similar scaling functions are associated with the renormalized quark masses and the local operators. An important point to note is that these functions can be computed for a large range of \( \overline{g}^2 \) without running into uncontrolled lattice effects, because the lattice spacing is always much smaller than \( L \), on any reasonable lattice, no matter how small \( L \) is in physical units.

Once the scaling functions are known, one can move up and down the energy scale by factors of 2. With only a few steps a much larger range of scales can be covered in this way than would otherwise be possible.

3.4 Application

So far the recursive procedure described above has been used to compute the running coupling in quenched QCD\(^{32,33}\) and first results are now also being obtained for the running quark masses\(^{34,35}\). The calculation follows the arrows in the diagram shown in Figure 2, starting at the lower-left corner. In this plot the energy is increasing from the bottom to the top while the entries in the left and right columns refer to infinite and finite volume quantities respectively.
The computation begins by calculating the renormalized coupling $\bar{g}$ and quark masses $\bar{m}$ in the chosen finite-volume scheme at some low value of $\mu$, where contact with the hadronic scales can easily be made using numerical simulations. In the next step one takes these results as initial values for the non-perturbative renormalization group and scales the coupling and quark masses to high energies. At still higher energies the perturbative evolution equations apply and the $\Lambda$-parameter and the renormalization group invariant quark masses

$$M = \lim_{\mu \to \infty} \bar{m} \left(2b_0 \bar{g}^2\right)^{-d_0/2b_0}$$

may be extracted with negligible systematic error ($b_0$ and $d_0$ denote the one-loop coefficients of the $\beta$-function and the anomalous mass dimension). The renormalization group invariant quark masses $M$ are scheme-independent and thus do not change when we pass from the finite-volume to the $\overline{\text{MS}}$ scheme, while the matching of the $\Lambda$-parameters involves an exactly calculable proportionality constant $k$ (top line of Figure 2). The perturbative evolution in the $\overline{\text{MS}}$ scheme, which is now known through four loops, finally yields the running coupling and quark masses in this scheme.

Figure 3 shows the scale evolution of the running coupling in the SF scheme, which is the particular finite-volume scheme that has been employed. The data points are separated by scale factors of 2, i.e. the recursion has been applied 8 times. At the higher energies the scale dependence of the coupling is accurately reproduced by the perturbative evolution, which has recently been worked out to three loops in this scheme\cite{[36,37,38]}. The perturbative region has thus safely been reached and, using the 3-loop evolution in the range $\alpha \leq 0.08$, 

![Figure 3: Simulation results for the running coupling $\alpha = \bar{g}^2/4\pi$ in the SF scheme (full circles). The solid (dashed) lines are obtained by integrating the perturbative evolution equation, starting at the right-most data point and using the 3-loop (2-loop) expression for the $\beta$-function.](image)
one obtains

\[ \Lambda_{\text{MS}}^{(0)} = 251 \pm 21 \text{ MeV}. \]  (14)

The index \(^{(0)}\) reminds us that this number is for quenched QCD which formally corresponds to zero flavours of light sea quarks. Otherwise no uncontrolled approximations have been made and Eq. (14) thus is a solid result.

The scale evolution of the quark masses in the SF scheme is also accurately matched by perturbation theory and the renormalization group invariant masses are hence easily obtained [cf. Eq. (13)]. Some preliminary simulation results for the flavour-independent ratio \( \overline{m}/M \) are plotted in Figure 4. The left-most data point corresponds to a normalization mass \( \mu \) around 290 MeV and a ratio \( M/\overline{m} = 1.18(2) \). This factor provides the required link between the low-energy and the perturbative regime of the theory. To complete the computation of (say) the strange quark mass in the \( \overline{\text{MS}} \) scheme, one still needs to go through a few steps, but the renormalization problem has been solved at this point and what is left to do are some standard calculations of meson masses and of the vacuum-to-kaon matrix element of the unrenormalized pseudo-scalar density.

The fact that the curves in Figures 3 and 4 agree so well with the data down to very low energies should not be given too much significance. Rather than a general feature of the theory, the absence of large non-perturbative corrections to the scale evolution should be taken as a property of the chosen renormalization scheme. Other schemes behave differently in this respect and there is usually no way to tell in advance at which energy the perturbative scaling sets in.
4 Concluding remarks

The theoretical developments described in this talk lead to a better understanding of the continuum limit and of the parameter and operator renormalization in lattice QCD. In particular, using improved actions and the new techniques for non-perturbative renormalization, one will be able to obtain more reliable results and to approach difficult problems such as the calculation of moments of structure functions \(^{40}\) and \(K \rightarrow \pi\pi\) decay rates \(^{41,42}\) with greater confidence.

Most of the examples and results that have been mentioned here refer to quenched QCD, but the theoretical discussion also applies to the full theory with any number of sea quarks. At this point the bottleneck are the simulation algorithms, which remain to be rather inefficient when sea quark effects are included. Continuous progress is however being made \(^{43}\) and the new generation of dedicated computers will no doubt allow the lattice theorists to move a big step forward in this area too.

While there are many indications that \(O(a)\) improvement and other forms of improvement are successful, the conclusion that this will lead to dramatic savings in computer time, ultimately allowing the solution of lattice QCD on a PC \(^{44,45,46}\), is not justified. Fast computers are indispensible if one is interested in obtaining good control on the systematic errors. They are also needed to tackle the more complicated physics issues mentioned before and the inclusion of sea quark effects is clearly beyond the capabilities of present-day PC’s.

Acknowledgements

I would like to thank Guido Martinelli, Hubert Simma, Stefan Sint, Rainer Sommer, Hartmut Wittig and Tomoteru Yoshie for helpful correspondence and Peter Weisz for critical comments during the preparation of this talk.

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