Research Article

Robot Selection Using An Integrated MAGDM Model Based on ELECTRE Method and Linguistic q-Rung Orthopair Fuzzy Information

Hengjia Bao and Xuecheng Shi

School of Mathematical Sciences, Anhui University, Hefei 230601, China

Correspondence should be addressed to Hengjia Bao; a01914176@stu.ahu.edu.cn

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With the development of engineering sciences, many companies are using robots in different industries to improve the efficiency and accuracy of their work. However, the need for robots under different requirements is different, which makes the selection of robots complex. In this paper, a linguistic q-rung orthogonal fuzzy multiple attribute group decision-making (MAGDM) method based on ELECTRE is proposed for robot selection. First, the basic Hausdorff distance is extended to the linguistic q-rung orthopair fuzzy environment to measure the deviation between two linguistic q-rung orthopair fuzzy numbers and two linguistic q-rung orthopair fuzzy sets. Then, the properties of the linguistic q-rung orthopair fuzzy distance measure based on Hausdorff distance are investigated. In addition, two maximum deviation models for deriving the weights of decision-makers and attributes are proposed. Moreover, a new MAGDM method is proposed by extending the ELECTRE method to the linguistic q-rung orthopair fuzzy environment. Finally, the practicality as well as the effectiveness of the method is demonstrated through a case study of the robot selection problem. The linguistic q-rung distance measure is used to construct two maximum deviation models to objectively derive the weights of attributes and decision-makers, and the linguistic q-rung orthopair fuzzy ELECTRE method is used to complete the selection of robots for a clean energy company. Furthermore, the sensitivity analysis of the parameter in the proposed method is provided, and the superiority of the new method is illustrated by the comparison with existing MAGDM methods.

1. Introduction

Robots are extremely powerful components of industry these days, and the recent advancements in information technology and engineering sciences have been a major factor in the expanding use of robots in many modern manufacturing systems. Robots are popular because they can perform some tough and dangerous activities that humans cannot. The selection of a robot for a certain application and production environment, on the other hand, is a separate undertaking that involves balancing several qualitative and quantitative criteria. Furthermore, the differences in cognition of decision-makers from various specialty domains may raise the difficulty of robot selection in multi-attribute group decision-making (MAGDM) problems. Therefore, a suitable MAGDM method can improve the accuracy of robot selection and greatly reduce the time and economic cost.

Multiple attribute group decision-making (MAGDM) is a complex decision-making science that involves how to get the optimal choice by opinion expression, information synthesis, and analytical ranking of fuzzy information. Since human beings are always uncertain when facing complex issues, it is difficult for decision-makers (DMs) to provide precise evaluation results. To overcome this challenge, Zadeh [1] first proposed the concept of fuzzy sets (FSs) in 1965. However, the classical FSs can only be used to solve problems in which the membership function is limited to the interval [0,1]. Moreover, these methods consider only the degree of support, without considering the degree of
opposition. Therefore, Atanassov [2] extended FS by proposing intuitionistic fuzzy sets (IFSs). An IFS includes both a membership degree (MD) and a nonmembership degree (NMD), making it possible to describe fuzzy information more intuitively and accurately than a FS. Since then, IFSs have been widely used in project management [3], healthcare management [4], and multiple attributes decision-making (MADM) [5, 6]. Later, Das et al. [7] proposed group decision-making problem with fuzzy parameterized intuitionistic multi-fuzzy N-soft set and compared the ranking performance of the proposed method with existing methods, which greatly advanced the field of fuzzy decision-making.

However, the IFS can only handle uncertain information quantitatively. In real life, people often tend to use words to describe their views on uncertain problems. For example, when evaluating the importance of an item, one can describe it using a linguistic term set (LTS): $S = \{\text{Very bad, bad, medium, good, very good, extremely good}\}$. Zadeh [8–10] innovatively proposed the concept of linguistic variables (LVs) to solve the problem of imperfect expression of fuzzy information, and the related research has achieved fruitful results. For instance, based on intuitionistic fuzzy numbers (IFNs) and LVs, Chen et al. [11] proposed the concept of linguistic IFNs (LIFNs), using LVs to represent the MD and NMD of IFNs. LIFNs combine the advantages of both IFNs and LTSs, and have considerable achievements in basic theoretical research [12, 13], LIFN aggregation operators [14, 15], and MAGDM methods based on the extension of LIFNs [16, 17].

Although IFSs are widely used, their scope in expressing uncertain information is still limited, and the following conditions must be met: $0 \leq u + v \leq 1$ and $u, v \in [0,1]$, where $u$ represents the MD and $v$ represents the NMD. These constraints restrict the application of IFNs. When DMs provide assessment information beyond the scope of IFNs, that is, $u + v > 1$, we cannot use IFNs to express the evaluation information. To solve this problem, Yager [18] proposed the Pythagorean FSs (PFSs), in which the MD and NMD satisfy conditions $0 \leq u^2 + v^2 \leq 1$ and $u, v \in [0,1]$. For instance, if a DM gives the MD and NMD as 0.6 and 0.7, respectively, we cannot use PFSs because $0.6^2 + 0.7^2 > 1$; however, we can use PFNs as $0.6^2 + 0.7^2 < 1$. Since the introduction of PFS, many scholars have focused their research on MAGDM based on PFSs. Yager and Abbasov [19] characterized the relationship between Pythagorean fuzzy numbers (PFNs) and complex numbers. In [20, 21], Pythagorean fuzzy power weighted averaging operators were proposed, and an algorithm for solving MADM problems was proposed based on these operators. In [22, 23], scholars considered the relationship between the MD and NMD of a PFS and proposed several classes of Pythagorean fuzzy cross-integration operators. Garg [24] extended the PFS to interval-valued PFSs (IVPFSs) and proposed a series of algorithms based on IVPFSs. Ali and Granados [25] proposed CIVPFEOWG and CIV-PFEOWG operator based on the principle of a complex interval-valued Pythagorean fuzzy set, determined its consistency and reliability, and finally discussed the comparative analysis and graphical representation of the development principle. To capture the correlations between PFNs, Rani et al. [26] proposed the Pythagorean fuzzy weighted discrimination-based approximation (WDBA). Inspired by hesitating FSs, Garg [27] proposed hesitating PFSs and applied them to MAGDM. Tang and Wei [28] studied Pythagorean binary language sets and the corresponding aggregation operator and proposed a new MAGDM method.

However, the use of PFNs still has the limitation that it cannot describe more complex fuzzy information. For instance, if a DM gives the MD and NMD as 0.9 and 0.8, we cannot choose IFSs or PFSs to express this evaluation information. Therefore, to further expand the scope of expression of fuzzy information, Yager [29, 30] extended the PFS by proposing the q-rung orthopair FSs (q-ROFSs), which satisfy $u^q + v^q \leq 1 \ (q \geq 1)$ and $u, v \in [0,1]$, where $u$ represents the MD and $v$ represents the NMD. Conceptually, q-ROFSs are more general than IFSs or PFSs; by adjusting the value of parameter $q$, setting $q = 1$ or 2, q-ROFSs can be converted into IFs or PFSs, respectively. Thus, for the above instance, if we set $q = 6$, we can use q-ROFSs to express this assessment information (see Figure 1). Based on q-ROFS theory, Liu and Wang [31] proposed several aggregation operators and algorithms for q-rung orthopair fuzzy numbers (q-ROFNs) to solve MADM problems. Wei et al. [32] proposed a series of Heronian mean operators based on q-ROFSs. Although q-ROFS theory has been extensively studied and applied in daily life, it is often necessary to replace complex quantitative research with rapid qualitative assessments based on LVs for many emerging problems. Therefore, based on the LIFNs and linguistic PFNs, Liu and Liu [33] extended q-ROFNs to linguistic q-ROFNs (Lq-ROFNs).

However, for the MAGDM problem, it requires not only the expression of evaluation values but also the aggregation of evaluation information. To date, a number of methods have been used to solve the decision-making problem. In this paper, Table 1 is given to illustrate the MAGDM methods, weight solving methods, distance formulas involved, and applications so far in the Lq-ROFS setting. Among them, Liu [34] proposed Bonferroni mean operators and their application to solve the MADM problem. In MCDM, Mukhamedyanov [35] performed a comparative analysis of objective methods for determining the criteria weights and considering the specificity of the decision problem and proposed the EWM-Corr method for reallocating the weights among the relevant criteria. Although many researchers have used different algorithms to generate the weights between attributes, few have solved for the weights of DMs. Therefore, this paper introduces the maximum deviation model [36] to solve the weights of attributes and the weights of DMs to make the decision results more objective. In the process of weight solving, we believe that Hausdorff distance has the advantage of capturing the distance between each point in the set and the other set, and can solve the problem that the shortest distance contains too little information. Thus, this paper extends the Hausdorff distance to Lq-ROFNs based on Yang and Ding’s application [37] of the Hausdorff distance to q-rung orthopair fuzzy.

The benefits and weaknesses of existing methods are listed in Table 1.
As can be seen from the above table, no one has used the method of elimination and choice translating reality (ELECTRE) for decision-making in the Lq-ROFS environment. Therefore, in this paper, we innovatively propose ELECTRE method to solve the robot selection problem based on Lq-ROFN. Since the ELECTRE method introduction in 1966 [44], ELECTRE and its derivatives have played a prominent role in solving multiple-criteria decision-making (MCDM) problems. The main idea of this model, which is based on transcendental relations and the concepts of consistency and inconsistency [45], is to analyze ranking relations by using a consistency index and an inconsistency index [46]. Shortly after the original version, ELECTRE I was proposed, and the ELECTRE method evolved into various improved methods known as ELECTRE II, III, IV, IS, and ELECTRE TRI [44–50].

Furthermore, in the context of the rapid development of ELECTRE methods, ELECTRE has also been used to solve intuitionistic fuzzy problems. Wu [51] proposed an ELECTRE multi-criteria analysis problem based on the IFSs. Hashemi [52] improved the ELECTRE method and proposed the ELECTRE III method based on interval-valued IFSs to solve the MAGDM problem. Rashid [53] proposed an ELECTRE method based on hesitant intuitionistic LTSs. In recent years, based on the MAGDM and multiple-criteria group decision-making (MCGDM) methodologies, ELECTRE [54, 55] has also been compared with ViseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR) in an intuitionistic fuzzy environment. Although the VIKOR method [54, 55] has a decision mechanism coefficient, the decision-making result is both subjective and objective. The MABAC (multi-attributive
border approximation area comparison) method [56] is simple to calculate, and the results are stable. The MARCOS (measurement of alternatives and ranking according to compromise solution) method [4] can more precisely determine the degree of utility of the two sets of solutions, but the ELECTRE method has the advantage of solving the problem of attribute heterogeneity and unconditional compensation between attributes. Therefore, the ELECTRE method is used in this paper to solve the MAGDM problem for Lq-ROFNs and verifies its effectiveness as well as practicality by combining it with the robot selection problem.

The rest of this article is structured as follows. In the second section, some concepts and basic theories related to q-ROFNs, LTSs, Lq-ROFNs, and LSFs are introduced. In Section 3, the Hausdorff distance and maximum deviation model based on Lq-ROFNs are presented. In the fourth section, we introduce the steps of the ELECTRE method based on Lq-ROFNs to solve the MAGDM problem. In Section 5, an example is given to prove the effectiveness and superiority of the proposed method in solving MAGDM problems, and a sensitivity analysis is carried out. Section 6 is the conclusion of this paper.

2. Preliminaries

In this section, we briefly review some basic concepts related to q-ROFNs, LTSs, Lq-ROFNs, and LSFs.

2.1. q-Rung Orthopair Fuzzy Sets (q-ROFNs)

Definition 1 (see [31]). Let $X$ be a nonempty finite set. A q-ROFS can be defined as:

$$Q = \{x, u_Q(x), v_Q(x)\},$$

where the functions $u_Q: X \rightarrow [0,1]$ and $v_Q: X \rightarrow [0,1]$ denote the degrees of membership and nonmembership, respectively, of element $x$ with respect to $X$, satisfying $0 \leq u_Q(x)^q + v_Q(x)^q \leq 1$ ($q \geq 1$).

Accordingly, the indeterminacy of q-ROFSs is expressed as follows:

$$\pi_Q(x) = \left(1 - u_Q(x)^q - v_Q(x)^q\right)^{1/q},$$

where $\pi_Q(x) \in [0,1]$ expresses the hesitancy or uncertainty that $X$ of $x$ belongs to $Q$. For convenience, the simplified form $(u_Q,v_Q)$ can be used to represent an element of a q-ROFS, which is called a q-ROFN and satisfies $u_Q,v_Q \in [0,1]$ and $u_Q^q + v_Q^q \leq 1$.

2.2. Linguistic Term Sets (LTSs)

Definition 2 (see [57]). Let $\varphi = \{s_0, s_1, \ldots, s_k\}$ be a finitely ordered discrete LTS whose cardinality is odd and suppose that the semantics of $s_0,s_1,\ldots,s_k$ can correspond to different values in different cases. A LTS must satisfy the following conditions:

1. The set is monotonic: if $i > j$, then $s_i > s_j$, that is, $s_i$ is superior to $s_j$.
2. A negative operator: $\text{neg}(s_i) = s_j$ such that $j = k - i$.
3. A maximum operator: if $i \geq j$, then $s_i \geq s_j$, that is, $\max(s_i,s_j) = s_i$.
4. A minimum operator: if $i \leq j$, then $s_i \leq s_j$, that is, $\min(s_i,s_j) = s_i$.

Xu [58] proposed the concept of a continuous LTS $\varphi = \{s_i\}_{0 \leq i \leq k}$, $i \in [0,1]$ on the basis of the discrete LTS concept in order to better preserve the given information.

2.3. Linguistic q-Rung Orthopair Fuzzy Sets (Lq-ROFSs)

Definition 3 (see [33]). Let $X = \{x_1,x_2,\ldots,x_n\}$ be a universe of discourse. A linguistic q-ROFS (Lq-ROFS) defined in $X$ is expressed as

$$\tilde{\varphi} = \left\{\left(x, s_\alpha(x), s_\beta(x)\right) | x \in X\right\},$$

where $s_\alpha(x), s_\beta(x) \in S_{[0,1]}$ and $s_\alpha(x), s_\beta(x)$ represent the linguistic MD and linguistic NMD, respectively. For any $x \in X$, condition $0 \leq a^q + b^q \leq k^q$ ($q \geq 1$) is always true; then, $\tilde{\varphi} = (\alpha_\varphi, \beta_\varphi)$ is called a Lq-ROFN, and we can take $\varphi_\varphi$, which is expressed on the basis of $S_{[0,1]}$ as the set of all Lq-ROFNs.

$$\pi(x) = s(k^q - a^q - b^q)^{1/q}$$

called the degree of linguistic indeterminacy from $x$ to $\tilde{\varphi}$.

Let $\tilde{\varphi}_1 = (s_{\alpha_1}, s_{\beta_1}), \tilde{\varphi}_2 = (s_{\alpha_2}, s_{\beta_2})$ be the two Lq-ROFNs and $\lambda > 0$ be a real number, then the operational laws of Lq-ROFNs [33] are as follows:

1. The addition operation: $\tilde{\varphi}_1 + \tilde{\varphi}_2 = (s_{\alpha_1 + \alpha_2}, s_{\beta_1 + \beta_2})$
2. The multiplication operation: $\lambda \tilde{\varphi}_1 = (s_{\lambda \alpha_1}, s_{\beta_1})$
3. The scalar multiplication: $\lambda \tilde{\varphi}_1 = (s_{\lambda \alpha_1}, s_{\lambda \beta_1})$
4. The power operation: $\lambda^{1/q} = (s_{\lambda^{1/q} \alpha_1}, s_{\lambda^{1/q} \beta_1})$.

For the comparison of any two Lq-ROFNs, an intuitive method for size determination can be chosen. Let $\tilde{\varphi}_1 = (s_{\alpha_1}, s_{\beta_1}), \tilde{\varphi}_2 = (s_{\alpha_2}, s_{\beta_2})$ be two Lq-ROFNs, then

1. If $s_{\alpha_1} = s_{\alpha_2}$ and $s_{\beta_1} = s_{\beta_2}$, that is, $\tilde{\varphi}_1 = \tilde{\varphi}_2$
2. If $s_{\alpha_1} > s_{\alpha_2}$ and $s_{\beta_1} < s_{\beta_2}$, that is, $\tilde{\varphi}_1 > \tilde{\varphi}_2$

However, many Lq-ROFNs could not be compared in size intuitively, so Liu and Liu introduced a more precise comparison method to perform size comparison of Lq-ROFNs.

Definition 4 (see [33]). Let $\tilde{\varphi} = (s_\alpha, s_\beta) \in \Gamma_{[0,1]}$ be a Lq-ROFN. Then,

$$Ls(\tilde{\varphi}) = \left(\frac{k^q + a^q - b^q}{2}\right)^{1/q},$$

is called the score function $Ls$ for the Lq-ROFN $\tilde{\varphi}$, and

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…
is called the one-way linguistic q-rung orthopair fuzzy Hausdorff distance from point set \( \tilde{y}_1 \) to \( \tilde{y}_2 \) and

\[
 h(\tilde{y}_2, \tilde{y}_1) = \max_{s \in L_2, s \in L_1} |\tau(s_n) - \tau(s_m)|^q, \tag{10}
\]

is called the one-way linguistic q-rung orthopair fuzzy Hausdorff distance from point set \( \tilde{y}_2 \) to \( \tilde{y}_1 \).

**Theorem 1.** Let \( \tilde{y}_1 = (s_{\alpha_1}, s_{\beta_1}), \tilde{y}_2 = (s_{\alpha_2}, s_{\beta_2}), \) and \( \tilde{y}_3 = (s_{\alpha_3}, s_{\beta_3}) \) \((q \geq 1)\) be any three Lq-ROFNs. Then, the Hausdorff distances between \( \tilde{y}_1, \tilde{y}_2, \) and \( \tilde{y}_3 \) satisfy the following properties:

1. \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) \geq 0, H_{\text{had}}(\tilde{y}_1, \tilde{y}_1) = 0 \)
2. \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) = H_{\text{had}}(\tilde{y}_2, \tilde{y}_1) \)
3. \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) + H_{\text{had}}(\tilde{y}_2, \tilde{y}_3) \geq H_{\text{had}}(\tilde{y}_1, \tilde{y}_3) \).

**Proof.** (1) According to Definition 6, \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) \) satisfies \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) \geq 0 \). If \( \tilde{y}_1 = \tilde{y}_2 \), then when \( s_{\alpha_1} = s_{\alpha_2} \) and \( s_{\beta_1} = s_{\beta_2} \), \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) = \max(h(\tilde{y}_1, \tilde{y}_2), h(\tilde{y}_2, \tilde{y}_1)) = 0 \). Thus, condition (1) is proven. (2) For any two Lq-ROFNs \( \tilde{y}_1, \tilde{y}_2 \in [0, \infty] \) \((q \geq 1)\), \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) \) is monotonically nondecreasing with respect to \( \tilde{y}_1 \). Thus, condition (2) is proven. (3) \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) + H_{\text{had}}(\tilde{y}_2, \tilde{y}_3) = \max(h(\tilde{y}_1, \tilde{y}_2), h(\tilde{y}_2, \tilde{y}_1)) = \max(h(\tilde{y}_2, \tilde{y}_3), h(\tilde{y}_3, \tilde{y}_2)) = \max(h(\tilde{y}_1, \tilde{y}_2), h(\tilde{y}_2, \tilde{y}_3), h(\tilde{y}_3, \tilde{y}_2) = \max(h(\tilde{y}_1, \tilde{y}_2), h(\tilde{y}_2, \tilde{y}_3), h(\tilde{y}_3, \tilde{y}_2)). \) Hence, condition (3) is proven. 

Therefore, the theorem is proven. \( \square \)

**Example 1.** Let \( \tilde{y}_1 = (s_{\alpha_1}, s_{\beta_1}), \tilde{y}_2 = (s_{\alpha_2}, s_{\beta_2}), \) and \( \tilde{y}_3 = (s_{\alpha_3}, s_{\beta_3}) \) \((q \geq 1)\) be any three Lq-ROFNs. According to equation (8), we can calculate \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_2) \) \(= \max(h(\tilde{y}_1, \tilde{y}_2), h(\tilde{y}_2, \tilde{y}_1)) = 0.0527 \). Similarly, we can obtain \( H_{\text{had}}(\tilde{y}_1, \tilde{y}_3) = 0.2441 \) and \( H_{\text{had}}(\tilde{y}_2, \tilde{y}_3) = 0.4219 \).

**Definition 8.** Let \( \tilde{y}_1 = ((x_{s_{\alpha_1}}, s_{\beta_1})|x \in X) \) and \( \tilde{y}_2 = ((x_{s_{\alpha_2}}, s_{\beta_2})|x \in X) \) \((q \geq 1)\) be any two Lq-ROFSs; then,

\[
 d_{\text{ghd}}(\tilde{y}_1, \tilde{y}_2) = \left( \sum_{i=1}^{N} H_{\text{had}}^q(\tilde{y}_1^{(i)}, \tilde{y}_2^{(i)}) \right)^{1/q}, \tag{11}
\]

called the generalized Hausdorff distance \( d_{\text{ghd}}(\tilde{y}_1, \tilde{y}_2) \) between Lq-ROFSSs \( \tilde{y}_1 \) and \( \tilde{y}_2 \).

In particular, when \( \lambda = 1 \) and \( \lambda = 2 \), we can obtain the Hausdorff distance \( d_{\text{had}} \) and the Euclidean–Hausdorff distance \( d_{\text{had}} \). In this paper, only the case of \( \lambda = 1 \) is studied.

**Theorem 2.** Let \( \tilde{y}_1 = ((x_{s_{\alpha_1}}, s_{\beta_1})|x \in X) \) and \( \tilde{y}_2 = ((x_{s_{\alpha_2}}, s_{\beta_2})|x \in X) \) \((q \geq 1)\) be any three Lq-ROFSs. The generalized Hausdorff distances between \( \tilde{y}_1, \tilde{y}_2, \) and \( \tilde{y}_3 \) satisfy the following properties:

1. \( d_{\text{ghd}}(\tilde{y}_1, \tilde{y}_2) = 0, d_{\text{ghd}}(\tilde{y}_1, \tilde{y}_1) = 0 \)
2. \( d_{\text{ghd}}(\tilde{y}_1, \tilde{y}_2) = d_{\text{ghd}}(\tilde{y}_2, \tilde{y}_1) \)
(3) $d_{ghd}(\tilde{y}_1, \tilde{y}_2) + d_{ghd}(\tilde{y}_2, \tilde{y}_3) \geq d_{ghd}(\tilde{y}_1, \tilde{y}_3)$

3.2. Maximum Deviation Model to Derive the DM Weights and Attribute Weights. In a MAGDM problem, there are $m$ alternatives $A = \{a_1, a_2, \ldots, a_m\}$, $n$ attributes $C = \{c_1, c_2, \ldots, c_n\}$, and $t$ DMs $D = \{d_1, d_2, \ldots, d_t\}$. Let $\tilde{y}^d = (\tilde{y}_{ij})_{m \times n}$ be the decision matrix given by DM $D_d(x = 1, 2, \ldots, n)$, where $\tilde{y}_{ij} = (s_{ij}^x, s_{ij}^y)$ $(x = 1, 2, \ldots, t)$ is the Lq-ROFN of $D_j(1, 2, \ldots, n)$ with respect to alternative $A_i(1, 2, \ldots, m)$, consistent with the given LTS $S$. Let the DM’s weighting vector be $W = (w_1, w_2, \ldots, w_t)^T$, where $w_i \in [0, 1], \sum_{i=1}^n w_i = 1$, $(x = 1, 2, \ldots, t)$. And let $\Omega = (\omega_1, \omega_2, \ldots, \omega_t)^T$ be the weighting vector of attributes, where $\omega_i \in [0, 1], \sum_{j=1}^m \omega_j = 1, (j = 1, 2, \ldots, n)$. Suppose that $\tilde{y} = (\tilde{y}_{ij})_{m \times n}$ is a collective matrix of all DMs by using q-ROFLWA operator [31], where

$$\tilde{y}_{ij} = q - ROFLWA(\tilde{y}_{ij}^1, \tilde{y}_{ij}^2, \ldots, \tilde{y}_{ij}^n) = \phi_{x=1}^n w_x \tilde{y}_{ij}^x.$$  

In most cases, DM weights are unknown, so it is necessary to introduce a method to determine the DM weights to ensure the objectivity of the decision results. Stimulated by [36], we develop a maximum deviation model to determine the DM weights $W_x$ based on the distance measure for Lq-ROFNs. If there is little difference between the evaluations of DMS, the discrimination effect between the DMS is not significant, so it should be arranged a smaller weight. Otherwise, a larger weight will be given. Then, the following maximum deviation model is obtained:

$$\Psi = \max_{x=1}^n \max_{i=1}^m \max_{j=1}^n w_x H_{hd}(\tilde{y}_{ij}^x, \tilde{y}_{ij}),$$  

subject to $w_x \geq 0, \sum_{i=1}^m w_x^i = 1,$  

where $w_x \geq 0$ is the non-negative constraint of equation (13) and $\sum_{i=1}^m w_x^i = 1$ is the unitization constraint of equation (13) [36].

Similarly, it is also necessary to use the maximum deviation model to determine the attribute weights, and we consider the following maximum deviation model:

$$\Psi = \max_{x=1}^n \max_{j=1}^m \max_{i=1}^n \omega_j H_{d}(\tilde{y}_{ij}, \tilde{y}_{ij}),$$  

subject to $\omega_j \geq 0, \sum_{j=1}^n \omega_j^2 = 1 \forall j \in J.$

The linguistic function is constructed and solved to obtain

$$\omega_j^* = \frac{\sum_{i=1}^m \sum_{j=1}^n H_{d}(\tilde{y}_{ij}^x, \tilde{y}_{ij})}{\sqrt{\sum_{i=1}^m \left(\sum_{j=1}^n H_{d}(\tilde{y}_{ij}^x, \tilde{y}_{ij})\right)^2}} \forall j \in J.$$

The following is the normalization process for $\omega_1, \omega_2, \ldots, \omega_n$; thus, attribute weights $\omega_i$ that satisfy the condition $\sum_{j=1}^n \omega_j = 1$ are obtained:

$$\omega_j = \frac{\omega_j^*}{\sum_{j=1}^n \omega_j^*} = \frac{\sum_{i=1}^m \sum_{j=1}^n H_{d}(\tilde{y}_{ij}^x, \tilde{y}_{ij})}{\sum_{i=1}^m \left(\sum_{j=1}^n H_{d}(\tilde{y}_{ij}^x, \tilde{y}_{ij})\right)^2} \forall j \in J.$$  

4. A Linguistic q-Rung Orthopair Fuzzy MAGDM Method Based on ELECTRE

4.1. Linguistic q-Rung Orthopair ELECTRE Method. To better use the ELECTRE method to solve the MAGDM problem, we introduce concordance and discordance indices as follows.

Definition 9. Let $\bar{J} = \{ij | j = 1, 2, \ldots, n\}$ be the set of subscripts for all attributes. Then, the following sets are defined by the relationship between the alternatives $\tilde{y}_i$ and $\tilde{y}_j$.

1. $\bar{C}(\tilde{y}_i, \tilde{y}_j) = \{ij | 1 \leq j \leq n, LS(\tilde{y}_{ij}) > LS(\tilde{y}_{ji})\}$ is the set of subscripts for all attributes that satisfy the constraint $\tilde{y}_{ij} > \tilde{y}_{ji}$.
2. $\bar{D}(\tilde{y}_i, \tilde{y}_j) = \{ij | 1 \leq j \leq n, LS(\tilde{y}_{ij}) < LS(\tilde{y}_{ji})\}$ is the set of subscripts for all attributes that satisfy the constraint $\tilde{y}_{ij} < \tilde{y}_{ji}$.
3. $\bar{I}(\tilde{y}_i, \tilde{y}_j) = \{ij | 1 \leq j \leq n, LS(\tilde{y}_{ij}) = LS(\tilde{y}_{ji})\}$ is the set of subscripts for all attributes that satisfy the constraint $\tilde{y}_{ij} = \tilde{y}_{ji}$.

Definition 10. Consider the vector of weights $\omega_j$ related to the attributes; then,

$$c_d = \sum_{j \in \bar{C}(\tilde{y}_i, \tilde{y}_j)} \omega_j + \sum_{j \in \bar{D}(\tilde{y}_i, \tilde{y}_j)} \omega_j,$$

is called the comprehensive concordance index $c_d$ between $\tilde{y}_i$ and $\tilde{y}_j$, where $\bar{R}(\tilde{y}_i, \tilde{y}_j) = \bar{C}(\tilde{y}_i, \tilde{y}_j) \cup \bar{I}(\tilde{y}_i, \tilde{y}_j)$ and the concordance matrix is

$$\bar{C} = \begin{bmatrix} - & c_{12} & c_{13} & \cdots & c_{1(n-1)} & c_{1n} \\ c_{21} & - & c_{23} & \cdots & c_{2(n-1)} & c_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{(n-1)1} & c_{(n-1)2} & \cdots & \cdots & - & c_{(n-1)n} \\ c_{n1} & c_{n2} & c_{n3} & \cdots & \cdots & - \end{bmatrix}. $$

Definition 11. Consider the vector of weights $\omega_j$ related to the attributes; then,

$$d_d = \max_{j \in \bar{D}(\tilde{y}_i, \tilde{y}_j)} \left\{ \omega_j H_{d}(\tilde{y}_{ij}, \tilde{y}_{ij}) \right\}.$$
To enable the ranking of all alternatives, this paper gives the following definitions:

**Definition 12.** To enable the ranking of all alternatives, this paper gives the following definitions:

\[
\begin{align*}
    c_l &= \sum_{s=1,s \neq l}^{m} c_{ls} - \sum_{s=1,s \neq l}^{m} c_{dl}, \\
    d_l &= \sum_{s=1,s \neq l}^{m} d_{ls} - \sum_{s=1,s \neq l}^{m} d_{dl},
\end{align*}
\]

(22) is the formula for calculating the net advantage index \(c_l\), and (23) is the formula for calculating the net disadvantage index \(d_l\), where \(\Phi_l = c_l - d_l, \quad l = 1, 2, \ldots, m,\) (24) is the formula for calculating the net advantage \(\Phi_l\) of each alternative.

It can be seen from the above formula that the dominance index \(c_l\) reflects the degree of dominance of \(\tilde{y}_l\) among all alternatives. Similarly, the disadvantage index \(d_l\) reflects the degree of disadvantage of \(\tilde{y}_l\) among all alternatives. Therefore, for \(\tilde{y}_l\) to have a higher advantage among all alternatives, the following conditions need to be met: \(c_l\) must be larger, and \(d_l\) must be smaller.

**4.2. Procedure of the MAGDM Solution Based on the Linguistic q-Rung Orthopair Fuzzy ELECTRE Method.** The linguistic q-rung orthopair fuzzy ELECTRE method for MAGDM problem based on the Hausdorff distance involves the following steps (Figure 2):

**Step 1.** Obtain the decision matrices \(\tilde{y}^e\) of DMs.

**Step 2.** Use the maximum deviation models to calculate the DM weights and attribute weights.

**Step 3.** Transform the decision matrices into \(\tilde{y}\) in accordance with the DM weights using the linguistic q-rung orthopair fuzzy linguistic weighted average (Lq-ROFLWA) operator.

**Step 4.** Calculate the score function \(LS(\tilde{y}_{ij})\) for each element of \(\tilde{y}_{ij}\), and then obtain the set of attribute labels.

**Step 5.** Calculate the concordance matrix \(C\) and the discordance matrix \(D\) based on equations (19) and (20).

**Step 6.** Obtain the net dominance and disadvantage indices by using equations (21) and (22).

**Step 7.** To calculate the net advantage \(\Phi_l\) of each alternative based on (24).

**Step 8.** Rank the alternatives by \(\Phi_l\).

**5. A Practical Example**

To illustrate the use of the newly proposed ELECTRE method based on Lq-ROFNs, a practical example is given. In this section, the effectiveness and superiority of the new method are proven by calculating the results of the new method for comparison with other existing methods.

**5.1. Problem Description**

**Example 2.** (see [39]) To apply the new method to a practical example, we consider an example relevant to the robot selection MAGDM problem. Earlier, the analytic hierarchy process was used to evaluate waste treatment methods [60], and with the development of the automation field, energy companies are beginning to use robots for waste disposal. A clean energy company needs robots for waste sorting and disposal, so the energy company set up a team to select four different robots, which would be called \(A_1, A_2, A_3, \) and \(A_4\). To select the most suitable robot, the clean energy company invites three DMs \(D_1(x = 1,2,3)\) to perform a comprehensive assessment of the cost-effectiveness of the four robots. Considering the cost-effectiveness of each robot, the evaluation DMs discuss the following attribute to evaluate the four robots: robot performance \((C_1)\), robot cost \((C_2)\), programming flexibility \((C_3)\), and delivery time \((C_4)\). Three DMs \(D_1(x = 1,2,3)\) provide evaluation information \(y^e_{ij}\) for the alternatives \(A_i(i = 1,2,3,4)\) in terms of the attributes \(C_j(j = 1,2,3,4)\) using Lq-ROFNs \((s_{q,0}, s_{q,0})\) considering the following LTS: \(\varphi = \{s_0 = \text{extremely low}, s_1 = \text{very low}, s_2 = \text{slightly low}, s_3 = \text{general}, s_4 = \text{slightly high}, s_5 = \text{high}, s_6 = \text{very high}, s_7 = \text{extremely high}\}\). Tables 2–4 illustrate the decision matrices \(\tilde{y}^e = (\tilde{y}^e_{ij})_{4 \times 4}\) given by DMs \(D_1, D_2, \) and \(D_3\). The objective of this MAGDM problem is to select the most cost-effective robot among four alternatives with four attributes.

**5.2. Method Verification.** The steps of the ELECTRE method to solve the MAGDM problem are described below.

**Step 1.** Use the maximum deviation model to calculate the weights of the DMs and the weights of the attributes as shown in Tables 5 and 6, where \(q = 3\).

**Step 2.** Transform the decision matrices into \(\tilde{y}\) in accordance with the DM weights by means of the Lq-ROFLWA operator as shown in Table 7.

**Step 3.** Calculate the score function \(LS(\tilde{y}_{ij})\), and the results are shown in Table 8.

Accordingly, the sets of attribute labels are as follows:
The alternative set $A = \{a_1, a_2, \ldots, a_m\}$

The attribute set $C = \{c_1, c_2, \ldots, c_n\}$

The DMs set $D = \{d_1, d_2, \ldots, d_t\}$

Build the decision matrix $\tilde{y}^x$

Merge the decision matrix into $\tilde{y}$ through DMs' weight $\omega_x$ by the Lq-ROFLWA operator

Use Hausdorff distance and maximum deviation method to calculate the weight $\omega_x$ of DMs and the weight $\omega_j$ of attributes

Calculate the score function $LS_{\hat{y}}$ for each $\hat{y}_v$ and get the sets of criteria labels

Calculate the dominance index $c_i$ and disadvantage index $d_i$

Estimate the concordance matrix $\tilde{C}$ and discordance matrix $\tilde{D}$ by attribute weight

Calculate the net dominance $\Phi_i$

Rank the alternatives by ELECTRE method

Use ELECTRE method for decision making

End

**Figure 2:** Flowchart of the MAGDM solution method based on the ELECTRE method.

---

**Table 2:** Decision matrix $\tilde{y}^1$ given by $D_1$.

|     | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|-----|-------|-------|-------|-------|
| $A_1$ | $(s_{x_0}, s_1)$ | $(s_{x_5}, s_1)$ | $(s_{x_5}, s_2)$ | $(s_{x_3}, s_4)$ |
| $A_2$ | $(s_{x_5}, s_2)$ | $(s_{x_5}, s_3)$ | $(s_{x_4}, s_2)$ | $(s_{x_4}, s_3)$ |
| $A_3$ | $(s_{x_0}, s_2)$ | $(s_{x_6}, s_2)$ | $(s_{x_6}, s_1)$ | $(s_{x_2}, s_3)$ |
| $A_4$ | $(s_{x_4}, s_2)$ | $(s_{x_5}, s_3)$ | $(s_{x_4}, s_1)$ | $(s_{x_6}, s_2)$ |

**Table 3:** Decision matrix $\tilde{y}^2$ given by $D_2$.

|     | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|-----|-------|-------|-------|-------|
| $A_1$ | $(s_{x_5}, s_1)$ | $(s_{x_6}, s_2)$ | $(s_{x_4}, s_2)$ | $(s_{x_4}, s_3)$ |
| $A_2$ | $(s_{x_4}, s_2)$ | $(s_{x_4}, s_1)$ | $(s_{x_5}, s_3)$ | $(s_{x_4}, s_2)$ |
| $A_3$ | $(s_{x_7}, s_2)$ | $(s_{x_4}, s_2)$ | $(s_{x_4}, s_1)$ | $(s_{x_3}, s_3)$ |
| $A_4$ | $(s_{x_4}, s_3)$ | $(s_{x_5}, s_1)$ | $(s_{x_6}, s_2)$ | $(s_{x_4}, s_3)$ |

$\tilde{C}(\tilde{y}_1, \tilde{y}_2) = \{1, 3\}, D(\tilde{y}_1, \tilde{y}_2) = \{2, 4\}, ID(\tilde{y}_1, \tilde{y}_2) = \phi$

$C(\tilde{y}_1, \tilde{y}_3) = \{2, 4\}, D(\tilde{y}_1, \tilde{y}_3) = \{1, 3\}, ID(\tilde{y}_1, \tilde{y}_3) = \phi$

$C(\tilde{y}_1, \tilde{y}_4) = \{2\}, D(\tilde{y}_1, \tilde{y}_4) = \{1, 3, 4\}, ID(\tilde{y}_1, \tilde{y}_4) = \phi$

$C(\tilde{y}_2, \tilde{y}_3) = \{2, 4\}, D(\tilde{y}_2, \tilde{y}_3) = \{1, 3\}, ID(\tilde{y}_2, \tilde{y}_3) = \phi$

$C(\tilde{y}_2, \tilde{y}_4) = \{2\}, D(\tilde{y}_2, \tilde{y}_4) = \{1, 3, 4\}, ID(\tilde{y}_2, \tilde{y}_4) = \phi$

$C(\tilde{y}_3, \tilde{y}_4) = \{1, 3\}, D(\tilde{y}_3, \tilde{y}_4) = \{2, 4\}, ID(\tilde{y}_3, \tilde{y}_4) = \phi$.

(25)

Step 4. Calculate the concordance matrix $\tilde{C}$ and the discordance matrix $\tilde{D}$ based on the attribute weights.

For all DMs, the concordance matrix is
Table 4: Decision matrix $\bar{y}$ given by $D_3$.

|    | $C_1$  | $C_2$  | $C_3$  | $C_4$  |
|----|--------|--------|--------|--------|
| $A_1$ | $(z_5, z_3)$ | $(z_4, z_4)$ | $(z_3, z_5)$ | $(z_4, z_5)$ |
| $A_2$ | $(z_3, z_3)$ | $(z_6, z_4)$ | $(z_4, z_4)$ | $(z_3, z_5)$ |
| $A_3$ | $(z_6, z_2)$ | $(z_4, z_2)$ | $(z_5, z_1)$ | $(z_3, z_4)$ |
| $A_4$ | $(z_7, z_1)$ | $(z_5, z_2)$ | $(z_5, z_1)$ | $(z_3, z_5)$ |

$\mathbf{C} = \begin{pmatrix}
0 & 0.7358 & 0.2642 & 0.2286 \\
0.2642 & 0 & 0.2642 & 0.2286 \\
0.7358 & 0.7358 & 0 & 0.7358 \\
0.7714 & 0.7714 & 0.2642 & 0
\end{pmatrix}$

For all DMs, the discordance matrix is

$\mathbf{D} = \begin{pmatrix}
0 & 1 & 1.0000 & 1.0000 \\
0.8968 & 0 & 1.0000 & 1.0000 \\
0.0881 & 0.5364 & 0 & 0.1655 \\
0.2420 & 0.9602 & 1.0000 & 0
\end{pmatrix}$

Step 5. Obtain the net dominance and disadvantage indices.

For all DMs, the net dominance and disadvantage indices are as follows:

$c_1 = -0.5427, c_2 = -1.4860, c_3 = 1.4149, c_4 = 0.6138$

d_1 = 1.7731, d_2 = 0.4002, d_3 = -2.2101, d_4 = 0.0368.

(28)

Step 6. Calculate the net advantage $\Phi_i$ of each alternative.

Thus, the net advantages $\Phi_i$ are easily obtained as follows:

$\Phi_1 = -2.3158, \Phi_2 = -1.8862, \Phi_3 = 3.6250, \Phi_4 = 0.5770.$

(29)

Step 7. Rank the alternatives by $\Phi_i$:

$A_3 \succ A_4 \succ A_2 \succ A_1.$

(30)

Results indicate that the clean energy company should choose robot $A_3$ as a purchase target.

5.3. Sensitivity Analysis. Figure 3 shows that for different $q$ values, the rankings of the alternatives are not the same, but the difference is not large. When $q \geq 5$, $A_2$ is considered worse than $A_4$, and the ranking is $A_3 \succ A_4 \succ A_1 \succ A_2$. In addition, it can be seen from Figure 3 that as $q$ increases, the net advantages of alternative $A_3$ and $A_4$ both increase, but the net advantage of alternative $A_3$ increases significantly, while that of alternative $A_4$ increases only slightly. Similarly, as $q$ increases, the net advantages of alternatives $A_1$ and $A_2$ both decrease, but the net advantage of $A_2$ decreases significantly, while that of alternative $A_1$ decreases only slightly.

5.4. Comparative Analysis. The operators in [33, 39, 61] were applied to the case, and the comprehensive evaluation values of each alternative are calculated. The results are shown in Table 9.

Among the ranking results in Table 9, we can see that the ranking results $A_3 \succ A_4 \succ A_1$ of the ELECTRE method are effective and practical. By comparing the results of the different algorithms, we can conclude that the innovative points and advantages of the linguistic q-rung orthopair fuzzy ELECTRE method are as follows:

1. By introducing the Hausdorff distance into the linguistic q-rung environment, the DM weights and attribute weights between LVs are calculated by using the maximum deviation models.

2. In the decision-making process, it is generally assumed that a common scale can be established between attributes, as in the linguistic q-rung
orthopair fuzzy weighted power Muirhead mean (Lq-ROFWPMM) operator used in [39] and the linguistic intuitionistic fuzzy weighted power averaging (LIFWPA) operator used in [61]. However, in many practical problems, each attribute may be greatly heterogeneous in property, and even after data standardization, some attribute information in terms of time and distance may be lost. The ELECTRE method proposed in this paper can effectively solve this problem without causing information loss.

Concerning the linguistic q-rung orthopair fuzzy weighted power Bonferroni mean (Lq-ROFWPBM) operator used in [33], the influence on decision-making of the degree of correlation between attributes is considered, which complicates the decision-making process. However, the ELECTRE method does not accept unconditional compensation between attributes; that is, no additivity exists in the utility between attributes, which makes the decision-making process simple and clear.
(4) For general problems, slight differences in attribute estimation for at least one attribute will not significantly change the preference of DMs for an alternative; however, the accumulation of such slight differences in multiple attributes may significantly change the preference of DMs for a certain alternative. The maximum deviation models and the ELECTRE method proposed in this paper can effectively solve this problem.

6. Conclusions
In this paper, the traditional decision-making method ELECTRE is extended to the linguistic evaluation environment. At the same time, based on LIFNs and LPFNs, we have extended the Hausdorff distance and the maximum deviation models to the case of Lq-ROFNs. On this basis, we have combined the traditional ELECTRE method with Lq-ROFNs and accordingly proposed an ELECTRE method based on Lq-ROFNs. For MAGDM problem, the ELECTRE method can solve the problems of heterogeneous in property of the attributes themselves and the interaction and unconditional compensation among attributes well. Finally, this paper has presented the steps to solve the MAGDM problem based on the ELECTRE method, validated the proposed method with a robot-selected example, and compared it with existing methods. From the results, it can be seen that the proposed method is feasible and practical to solve the decision problem with complex fuzzy information.

In future research, the decision-making methodology presented in this paper can be more widely applied to practical MAGDM problems. In the case of unknown weights of DMs, the problem of more ambiguous decision information is solved by using the decision matrices as well as the maximum deviation model. However, the ELECTRE method is not resistant to the rank reversal problem, because it does not require prior ranking before selecting an alternative. Moreover, the ELECTRE method is insensitive to weak changes in attribute values. The maximum deviation model used in this paper is an objective method for solving weights, and there are many other subjective methods for solving weights, such as BWM, AHP, LBWA, and FUCOM, which can be extended to Lq-ROFNs in the future. Although the focus of this paper is on the discussion of Lq-ROFNs, the approach of this paper can also be extended to rough sets, neutrosophic, gray, etc. In addition, the robot usage decision for clean energy company in this paper can be extended to other fields, such as stock portfolio selection [62–64], railroad transportation business efficiency evaluation [65], and green supply chain management [25, 66].

Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest to report regarding the present study.

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