Rotating black holes in the novel $4D$ Einstein-Gauss-Bonnet gravity

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Abstract

Recently, Glavan and Lin [Phys. Rev. Lett. 124, 081301 (2020)] formulated the $4D$ novel Einstein-Gauss-Bonnet gravity (EGB), by rescaling the Gauss-Bonnet (GB) coupling $\alpha/(D-4)$ and taking the limit $D \to 4$, which preserves the number of degrees of freedom thereby free from the Ostrogradsky instability. The theory admits spherically symmetric black holes generalizing the Schwarzschild black holes that are free form the singularity problems. We obtain the rotating counterparts of these spherically symmetric black holes and discuss their horizon properties and shadow cast. The effects of the GB coupling parameter on the shape and size of shadows are investigated in the context of recent M87* observations from the EHT. Interestingly, for a given parameter set, the apparent size of the shadow decreases and gets more distorted due to the GB coupling parameter. We find that within the finite parameter space, e.g. for $a = 0.1, \alpha \leq 0.00394$, and within the current observational uncertainties, the rotating black holes of the $4D$ novel EGB gravity are consistent with the inferred features of M87* black hole shadow.

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I. INTRODUCTION

The uniqueness of the Einstein’s field equations is based on the Lovelock’s theorem [1], which ensures that for a theory of gravity with i) four-dimensional spacetime, ii) metricity, iii) diffeomorphism invariance, and iv) second-order equations of motion, Einstein’s tensor along with a cosmological constant term is the only divergence-free symmetric rank-2 tensor constructed solely from the metric tensor $g_{\mu\nu}$ and its derivatives up to second differential order. However, in higher-dimensions (HD) spacetimes $D > 4$, the Einstein-Hilbert action is not unique, and one particularly interesting example in the $HD$ is the Einstein-Gauss-Bonnet (EGB) gravity, which is motivated by the heterotic string theory [2, 3]. Lanczos [2] and Lovelock [3], in their pioneering works, showed that the Einstein-Hilbert action of EGB gravity admits quadratic corrections constructed from the curvature tensors invariants with action given by

$$S_{\text{EGB}} = \int d^D x \sqrt{-g} (L_{\text{EH}} + \alpha L_{\text{GB}}),$$  

with

$$L_{\text{EH}} = R, \quad L_{\text{GB}} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2,$$

where $g$ is the determinant of $g_{\mu\nu}$, and $\alpha$ is the Gauss-Bonnet (GB) coupling constant regarded as the inverse string tension and positive-definite. The EGB gravity has been widely studied, because it can be obtained in the low energy limit of string theory [4] and also leads to the ghost-free nontrivial gravitational self-interactions [5]. In the EGB theories, one can explore several conceptual issues of gravity in a much broader setup than in general relativity and they are also shown to be free from instabilities when expanding about flat spacetime [6, 7]. The spherically symmetric static black hole solution for the EGB theory was first obtained by Boulware and Deser [7], later several interesting black hole solutions are obtained [8–10] for various sources including the regular ones [11].

For a $D$-dimensional spacetime with $D < 5$, the GB Lagrangian $L_{\text{GB}}$ is a total derivative and does not contribute to the gravitational dynamics. However, by re-scaling the coupling constant as $\alpha \to \alpha/(D - 4)$, the GB invariant makes a non-trivial contribution to the gravitational dynamics even in the $D = 4$, thus bypass the Lovelock’s theorem [12]. This evident by considering maximally symmetric spacetimes with curvature scale $\mathcal{K}$

$$\frac{g_{\mu\nu} \delta L_{\text{GB}}}{\sqrt{-g} \delta g_{\mu\nu}} = \frac{\alpha (D - 2)(D - 3)}{2(D - 1)} \mathcal{K}^2 \delta_{\mu}^\nu,$$
obviously the variation of the GB action does not vanish in $D = 4$ because of the re-scaled coupling constant \[12\].

The static spherically symmetric black hole solution of the novel EGB gravity in the $D \to 4$ limit reads as \[12\]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2),$$

(4)

with

$$f(r) = 1 + \frac{r^2}{32\pi \alpha G} \left[ 1 \pm \left(1 + \frac{128\pi \alpha MG^2}{r^3} \right)^{1/2} \right].$$

(5)

Here, $M$ is the black hole mass and $G$ is the Newton’s gravitational constant which is hereafter set to be unity. We consider $\alpha > 0$, because for the negative values of $\alpha$ Eq. (4) is not a valid solution at small distances \[12\]. At large distances Eq. (5) has two distinct branches

$$f(r)_- = 1 - \frac{2M}{r}, \quad f(r)_+ = 1 + \frac{2M}{r} + \frac{r^2}{16\pi \alpha},$$

(6)

however, we will be focusing only on the negative branch that asymptotically goes over to the Schwarzschild black hole with correct mass sign. Whereas at small distances, the metric function (5) for the negative branch reduces to

$$f(r) = 1 + \frac{r^2}{32\pi \alpha} - \sqrt{\frac{M}{8\pi \alpha}} r^{1/2},$$

(7)

which infers that the gravitational potential does not diverge at $r = 0$ \[12\]. These black holes in the novel 4D EGB gravity are free from the singularity problem as the gravitational force is repulsive at small distances thereby, unlike in general relativity, an infalling particle never reaches the singularity \[12\]. Interestingly, the black hole solution of the semi-classical Einstein’s equations with conformal anomaly \[13\], gravity theory with quantum corrections \[14\], and the third order regularized Lovelock gravity \[15\], have the same form as 4D EGB gravity black hole given in the metric (4) with (5).

Nevertheless, recently, interesting measures have been taken to get the solutions of this novel theory \[16–21\]. In particular, spherically symmetric black hole solution (4) was extended to include charge for a anti-de Sitter spacetime \[20\], and a Vaidya-like radiating black holes in Ref. \[22\]. Other probes in the theory includes study of the the innermost stable circular orbit (ISCO) \[17\], its stability and quasi-normal modes \[19\]. Interestingly, it turns out that the GB coupling parameter has been shown to strongly effect the stability of the black hole \[17, 19\].
It is the purpose of this paper to obtain Kerr-like rotating black holes in the 4D novel EGB theory of gravity, namely the rotating EGB black hole and discuss the horizons geometry and shadow cast by the rotating EGB black hole. Null geodesics equations of motions are obtained in the first-order differential form and the analytical expressions for the photon region are determined. Effects of the GB coupling parameter on the black hole shadow morphology is investigated, and it is found that rotating black holes in the EGB gravity cast smaller and much distorted shadows than those for the Kerr black holes. Shadow observables $A$ and $D$ are used to characterize the size and shape of the shadows, and in turn to uniquely determine the black hole parameters. The M87* black hole shadow results, inferred from the recent Event Horizon Telescope (EHT) collaborations observations, are further used to constrain the GB coupling parameter.

This paper is organized as follows. In the Sect. II, we derive the rotating EGB black holes and in turn discuss their horizon structure. Photons geodesics equations of motion and effects of the GB coupling parameter on the black hole shadow are subjects of Sect. III. Finally, we summarize our main findings in Sect. IV.

II. ROTATING BLACK HOLES

The rotating black hole, in four-dimension (4D) general relativity, is described by the Kerr solution [23], which is completely specified by the mass $M$ and angular momentum $a$. Beginning with a spherically symmetric metric (4) and applying the modified Newman-Janis algorithm [24], we constructed the stationary and axially symmetric (rotating) metric in the 4D novel EGB gravity, which in the Boyer-Lindquist coordinates reads

$$ds^2 = - \left( \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right) dt^2 + \frac{\Sigma}{\Delta} dr^2 - 2a \sin^2 \theta \left( 1 - \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right) dt d\phi$$

$$+ \Sigma d\theta^2 + \sin^2 \theta \left[ \frac{\Sigma + a^2 \sin^2 \theta \left( 2 - \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right)}{2} \right] d\phi^2,$$

(8)

with

$$\Delta = r^2 + a^2 + \frac{r^4}{32\pi\alpha} \left( 1 - \left( 1 + \frac{128\pi\alpha M}{r^3} \right)^{1/2} \right), \quad \Sigma = r^2 + a^2 \cos^2 \theta,$$

(9)

and $a$ is the spin parameter. The metric Eq. (8), in the limit $\alpha \to 0$ or large $r$, goes over to the Kerr black holes [23]. Whereas, the static spherically symmetric black hole solution (4)
can be retrieved in the limit $a = 0$. The rotating black hole (8), like the Kerr black hole, possesses two linearly independent Killing vectors $\eta_{(t)}^\mu$ and $\eta_{(\phi)}^\mu$, respectively, associated with the isometries along the time translation and rotational invariance.

Next, we investigate the nature of the metric (8) to show that its properties are similar to the ones of the Kerr black hole. In particular, we compute the horizon-like surfaces with the aim to discuss the effect of the GB coupling parameter $\alpha$ on the structure of such surfaces. The horizons of rotating EGB black hole can be identified as the zeros of

$$g^{\mu\nu} \partial_\mu r \partial_\nu r = g^{rr} = \Delta = 0,$$

which is also the coordinate singularity of the metric (8). Numerical analysis reveals that depending on the values of $M, a$ and $\alpha$, Eq. (10) can have two distinct real positive roots or degenerate roots, or no-real positive roots, which respectively, correspond to the nonextremal black holes, extremal black holes, and no-black holes configurations for metric (8). The existence condition of the horizons gives a bound on the black hole parameters $a$ and $\alpha$. In Fig. (1), the parameter space $(a, \alpha)$ is shown, for the parameters values within the gray region, metric (8) admits two distinct roots, whereas for those outside no horizons exist and metric (8) corresponds to the no-black hole spacetime. For the values of the parameters lying on the black solid line, black hole admits degenerate roots and called the extremal black hole. The two roots of Eq. (10) are identified as the inner Cauchy horizon $(r_-)$ and the outer event horizon $(r_+)$ radii, such that $r_- \leq r_+$ (cf. Fig. 2). For the non-rotating black hole $(a = 0)$, Eq. (10) admits solutions

$$r_\pm = M \pm \sqrt{M^2 - 16\pi\alpha}.$$  \hspace{1cm} (11)

The behavior of the horizon radii $r_\pm$ with varying spin parameter $a$ and GB coupling parameter $\alpha$ is shown in Fig. 2. For a fixed value of $a$, the event horizon radius $r_+$ decreases, while the Cauchy horizon radius $r_-$ increases with increasing $\alpha$. However, there exist an extremal value of $\alpha$, i.e., $\alpha = \alpha_E$, for which both horizons coincide $r_- = r_+$, such that for $\alpha < \alpha_E$, horizon radii are $r_- \neq r_+$ (cf. Fig. 2). Further, the presence of the GB coupling reduces the horizon size, as for the fixed values of $M$ and $a$, the event horizon radii for the rotating EGB black holes are smaller as compared to that for the Kerr black hole (cf. Fig. 2). The stationary observers, having zero angular momentum with respect to the distant observer at spatial infinity, outside the event horizon of the rotating black hole
FIG. 1: The parameter space \((a, \alpha)\) for the existence of the black hole horizons.

FIG. 2: The behavior of horizons with varying black hole parameters \(a\) and \(\alpha\). Black solid line corresponds to the extremal black hole with degenerate horizons.

Spacetime, can rotate along with the black hole rotation due to the frame-dragging effect [25]. The angular velocity \(\omega\) of the rotation reads

\[
\omega = \frac{d\phi}{dt} = - \frac{g_t\phi}{g_{\phi\phi}} = \frac{a(r^2 + a^2 - \Delta)}{[(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta]},
\]

(12)
FIG. 3: The behavior of SLS with varying parameters $a, \alpha$, and $\theta = \pi/4$. The Black solid curve in each plot corresponds to the degenerate SLS.

$\omega$ monotonically increase with decreasing $r$ and eventually takes the maximum value at $r = r_+$, and reads as

$$\Omega = \omega|_{r=r_+} = \frac{a}{(r_+^2 + a^2)} = \frac{32\pi \alpha a}{r_+^4 \left(-1 + \left(1 + \frac{128\pi \alpha M}{r_+^3}\right)^{1/2}\right)},$$

(13)

which can be identified as the black hole angular velocity $\Omega$. Though stationary observers can exist outside the event horizons, static observers, following the worldlines of timelike Killing vector $\eta_{(t)}$, can exist only outside the static limit surface (SLS) defined by $\eta_{(t)}^\mu \eta_{(t)}^{\mu} = g_{tt} = 0$ [25]. The radii of SLS are determined by the zeros of

$$r^2 + a^2 \cos^2 \theta + \frac{r^4}{32\pi \alpha} \left(1 - \left(1 + \frac{128\pi \alpha M}{r^3}\right)^{1/2}\right) = 0,$$

(14)

which apart from black hole parameters also depends on $\theta$ and coincides with the event horizon only at the poles. Equation (14) is solved numerically and the two real positive roots, corresponding to the two SLS, are shown with varying $a$ and $\alpha$ in Fig. 3. The radii of the outer SLS decreases with individually increasing $\alpha$ and $a$. For a fixed value of $a$, there
exist a particular value of $\alpha$ for which the two SLS get coincide. However, these extremal values of $\alpha$ for $\theta \neq \pi/2$ are different from those for the degenerate horizons. For fixed values of $M$ and $a$, the SLS radii for the rotating EGB black holes are smaller as compared to the Kerr black hole values. The region between the SLS and the event horizon is termed as the ergoregion. It has been shown that it is possible, at least theoretically via Penrose process [26], to extract energy from the ergosphere of the black hole as the region that lies outside of a black hole. In Fig. 4, we plotted the ergoregion of rotating EGB black holes, and it is evident that the size increase with $\alpha$. It will be interesting to see how the GB parameter $\alpha$ makes an impact on energy extraction. The surface gravity of the rotating EGB black hole takes the following form

$$\kappa = \frac{\Delta'(r)}{2(r^2 + a^2)} \bigg|_{r=r^+_+},$$

$$= \frac{1}{2(r^2 + a^2)} \left[ 2r^+_+ + r^2_{+}\frac{^3}{8\pi\alpha} - \frac{10M + r^2_{+}}{8\pi\alpha} \right] \left[ 1 + \frac{128M\pi\alpha}{r^2_{+}} \right]$$

which in the limit of $\alpha \to 0$, reduces to

$$\kappa = \frac{r^+ - M}{r^2_{+} + a^2},$$

and that corresponds to the Kerr black hole surface gravity [27].

III. BLACK HOLE SHADOW

The null geodesics describing the photon orbits around the black hole are especially interesting because of their observational importance in probing the gravitational impact of the black holes on the surrounding radiation. Photons originating from the light source behind the black hole arrive in the vicinity of the event horizon, and a part of it falls inside the horizon while another part scattered away to infinity. This along with the strong gravitational lensing results in the optical appearance of the black hole, namely the black hole shadow encircled by the bright photon ring [28–31]. The study of black hole shadow was led by the seminal work by Synge [28] and Luminet [29], who gave the formula to calculate the angular radius of the photon captured region around the Schwarzschild black hole by identifying the diverging light deflection angle. Later, Bardeen [30] in his pioneering
FIG. 4: The cross-section of event horizon (blue line), SLS (red line) and ergoregion (region between event horizon and SLS) for different values of parameters $a$ and $\alpha$.

work studied the shadow of the Kerr black hole in a luminous background and shown that the spin would cause the shape of shadows distorted. The photon ring, encompassing the black hole shadow, explicitly depends on the spacetime geometry and thus its shape and size is a potential tool to determine the black hole parameters and to reveal the valuable information regarding the near-horizon field features of gravity. Over the past decades,
a flurry of activities in the analytical investigations, observational studies and numerical simulation of shadows for large varieties of black holes have been reported [32, 33]. Black hole shadows have also been investigated in the context of black hole parameter estimations and in testing theories of gravity [34, 35].

We use the Hamilton Jacobi equation and Carter’s separable method [36] to determine the geodesics motion in the rotating black hole spacetime (8). The four integrals of motions, namely the particle rest mass \( m_0 \), energy \( E \), axial angular momentum \( L \) and the Carter constant \( K \) related to the latitudinal motion of the test particle, completely describe the geodesics equations of motion in the first-order differential form [25, 36]

\[
\sum \frac{dt}{d\tau} = \frac{r^2 + a^2}{\Delta} (E(r^2 + a^2) - aL) - a(aE \sin^2 \theta - L),
\]

\[
\sum \frac{dr}{d\tau} = \pm \sqrt{R(r)},
\]

\[
\sum \frac{d\theta}{d\tau} = \pm \sqrt{\Theta(\theta)},
\]

\[
\sum \frac{d\phi}{d\tau} = \frac{a}{\Delta} (E(r^2 + a^2) - aL) - \left(aE - \frac{L}{\sin^2 \theta}\right),
\]

where \( \tau \) is the affine parameter along the geodesics and

\[
R(r) = \left((r^2 + a^2)E - aL\right)^2 - \Delta((aE - L)^2 + K),
\]

\[
\Theta(\theta) = K - \left(\frac{L^2}{\sin^2 \theta} - a^2E^2\right) \cos^2 \theta
\]

The separable constant \( K \) is related to the Carter’s constant of motion \( Q = K + (aE - L)^2 \) [25, 36]. Let define the two dimensionless impact parameters

\[
\eta \equiv K/E^2, \quad \xi \equiv L/E,
\]

which parameterize the null geodesics. Unstable photon orbits, at constant Boyer-Lindquist coordinate \( r_p \), are determined by the vanishing radial potential and its radial derivative

\[
R\big|_{(r=r_p)} = \frac{\partial R}{\partial r}\big|_{(r=r_p)} = 0 \quad \text{and} \quad \frac{\partial^2 R}{\partial r^2}\big|_{(r=r_p)} > 0.
\]

Solving Eq. (24) yields the pair of critical values of impact parameters \( (\eta_c, \xi_c) \) for the unstable photon orbits

\[
\eta_c = \frac{r_p^2 \left(-16\Delta(r_p)^2 - r_p^2\Delta'(r_p)^2 + 8\Delta(r_p)\left(2a^2 + r_p\Delta'(r_p)\right)\right)}{a^2\Delta'(r_p)^2},
\]

\[
\xi_c = \frac{\left(r_p^2 + a^2\right)\Delta'(r_p) - 4r_p\Delta(r_p)}{a\Delta'(r_p)},
\]
which effectively separate the captured orbits from the scattered one. Planer circular orbits are possible only at the equatorial plane \((\theta = \pi/2)\) for \(\eta_c = 0\), whereas, generic spherical photon orbits exist for \(\eta_c > 0\). The radii \(r_p^\pm\) of co-rotating and counter-rotating circular orbits can be identified as the real positive zeros of \(\eta_c = 0\). These spherical photons orbits construct the photon regions, which are determined by Eqs. (22) and (25), and given by \((\Theta(\theta) \geq 0)\)

\[
(4r_p \Delta(r_p) - \Delta'(r_p) \Sigma)^2 \leq 16a^2 r_p^2 \Delta(r_p) \sin^2 \theta,
\]

the gravitationally lensed image of this photon region corresponds to the black hole shadow (cf. Fig. 5). Indeed rotating black holes have two distinct photon regions, one inside the Cauchy horizon and another outside the event horizon [37]. Black hole image observations morphology relies on photons that can reach the observer, therefore, we look only for the unstable photons orbits lying outside the event horizon, i.e., \(r_p > r_+\). We consider an observer residing far away from the black hole \((r_o, \theta_o)\) so that the observer’s neighborhood can be taken as asymptotically flat. The observer can pick a Cartesian coordinate system centered at the black hole, such that the projection of the spherical photon orbits on the celestial plane delineates a closed curve parameterized by the celestial coordinates \((X, Y)\).

Back tracing a photon trajectory from the observer’s position to the celestial plane marks a point on the image plane, described as follow [30, 31]

\[
X = -r_o \frac{p^{(\phi)}}{p^{(\ell)}} = \lim_{r_o \to \infty} \left( -r_o^2 \sin \theta_o \frac{d\phi}{dr} \right),
\]

\[
Y = r_o \frac{p^{(\theta)}}{p^{(\ell)}} = \lim_{r_o \to \infty} \left( r_o^2 \frac{d\theta}{dr} \right),
\]

where \(p^{(\mu)}\) are the tetrad components of the photon four-momentum with respect to a locally nonrotating reference frame. Equation (27) can be further simplified in terms of impact parameters as follow [30, 31]

\[
X = -\xi_c \csc \theta_o \frac{\theta_o=\pi/2}{\theta_o} - \xi_c,
\]

\[
Y = \pm \sqrt{\eta_c + a^2 \cos^2 \theta_o - \xi_c^2 \cot^2 \theta_o} \theta_o=\pi/2 \pm \sqrt{\eta_c}.
\]

The parametric curve \(Y vs X\) delineates the rotating EGB black hole shadow. For the non-rotating case, Eq. (28) yield

\[
X^2 + Y^2 = 2r_p \left( r_p + \frac{4\Delta(r_p)(2r_p - \Delta'(r_p))}{\Delta'(r_p)^2} \right) \bigg|_{a=0},
\]
FIG. 5: The photon region (shaded orange region) structure around the rotating EGB black hole (shaded black disk).

FIG. 6: Non-rotating EGB black hole shadows with varying coupling parameter $\alpha$. Black solid line corresponds to the Schwarzschild black hole shadow.

which shows that the static spherically symmetric EGB black hole cast a perfectly circular shadow (cf. Fig. 6). The size of the shadow decreases with increasing $\alpha$, such that the non-rotating EGB black holes shadows are smaller than the Schwarzschild black hole shadow as shown in Fig. 6. Rotating EGB black hole shadows with varying $a$ and $\alpha$ are shown in Fig. 7, which clearly infer that the presence of the GB coupling parameter has a profound
FIG. 7: Plot showing the rotating EGB black holes shadows with varying parameters $a$ and $\alpha$. Solid black curves in the upper panel are for the Kerr black holes.

Influence on the apparent shape and size of the shadow. For non-zero values of $a$, the rotating black holes shadows are not perfect circles. The size and the amount of deviation from the circularity are measurable quantities and therefore, we introduce the shadow observables, namely shadow area $A$ and oblateness parameter $D$, for the characterization of shadows [34, 38]

\[
A = 2 \int Y(r_p) dX(r_p) = 2 \int_{r_p}^{r_p'} \left( Y(r_p) \frac{dX(r_p)}{dr_p} \right) dr_p,
\]

\[
D = \frac{X_r - X_i}{Y_i - Y_b},
\]

\[
(30)
\]

\[
(31)
\]
FIG. 8: The shadow area $A$ and oblateness observables $D$ vs $a$ for the rotating EGB black holes, (solid Black curve) for the Kerr black hole $\alpha = 0.0$, (dashed Blue curve) for $\alpha = 0.005$, and (dotted Magenta curve) for $\alpha = 0.01$.

FIG. 9: The shadow area $A$ and oblateness observables $D$ vs $\alpha$ for the rotating EGB black holes, (dotted Magenta curve) for $a = 0.0$, (dotted Brown curve) for $a = 0.1$, (dashed Blue curve) for $a = 0.3$, (long-dashed Red curve) for $a = 0.6$, and (solid Black curve) for $a = 0.8$.

where $A$ and $D$, respectively, characterize the shadow size and shape; for a perfectly circular shadow $D = 1$. Figure 8 shows the behavior of the shadow observables $A$ and $D$ with varying $a$ for different values of $\alpha$. It is evident that the shadows of the rotating EGB black holes are smaller and more distorted as compared to the Kerr black hole shadow as depicted by the black solid curves in Fig. 8. The shadow size decrease and the oblateness increases with increasing $a$. In Fig. 9, the variation of the $A$ and $D$ with $\alpha$ is shown for various values of $a$. The shadow size monotonically decreases with $\alpha$. For the estimation of the rotating EGB black hole parameters, we plotted $A$ and $D$ as functions of $a$ and $\alpha$ in Fig. 10.
FIG. 10: Contour plots of the observables $A$ and $D$ in the plane $(a, \alpha)$ for the rotating EGB black hole. Each curve is labeled with the corresponding values of $A$ and $D$. Solid red curves correspond to the $A$, and dashed blue curves are for the oblateness parameter $D$. The curve corresponds to constant values of either $A$ or $D$. The point of intersection of the $A$ and $D$ curves gives the unique values of the black hole parameters $a$ and $\alpha$. In Table I, we summarize the extracted values of EGB black hole parameters from the known shadow observables. Hence, from Fig. 10 and Table I, it is clear that for a given set of 4D EGB black hole shadow observables, $A$ and $D$, we can determine information about black hole spin and GB coupling parameter.

We can parameterize the shadow boundary also by the radial and angular coordinates $(R(\varphi), \varphi)$ in a polar coordinate system with the origin at the shadow center $(X_C, Y_C)$. Figure (7) infers that the black hole shadow is symmetric under reflection around $Y = 0$. However, it is not symmetric under reflections around the $X$ axis and is shifted from $X = 0$. This ensure that the $X_C = (X_r - X_l)/2$, and $Y_C = 0$. A point on the shadow boundary has a radial distance $R(\varphi)$ from the shadow center and subtends an angle $\varphi$ on the $X$ axis at the geometric center $(X_C, 0)$, which reads as

$$R(\varphi) = \sqrt{(X - X_C)^2 + (Y - Y_C)^2}, \quad \varphi \equiv \tan^{-1} \left( \frac{Y}{X - X_C} \right).$$
TABLE I: Estimated values of rotating EGB black hole parameters $a$ and $\alpha$ from known shadow observables $A$ and $D$.

The shadow average radius $\bar{R}$ is defined as [39]

$$\bar{R} = \frac{1}{2\pi} \int_0^{2\pi} R(\varphi) d\varphi,$$

(32)

We describe the circularity deviation $\Delta C$ as a measures of the root-mean-square deviation of $R(\varphi)$ from the shadow average radius [39, 40]

$$\Delta C = 2 \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (R(\varphi) - \bar{R})^2 d\varphi},$$

(33)

$\Delta C$ quantifies the shadow deviation from a perfect circle, such that for a spherically symmetric black hole circular shadow, $\Delta C = 0$. We can use our numerical results to compare the latest observation of the black hole shadow of M87*. The EHT Collaboration [41] using the Very Large Baseline Interferometry technique has observed the shadow of M87* black hole residing at the galactic center of nearby galaxy M87 [42, 43]. Their measurement of the M87* black hole mass ($M = (6.5 \pm 0.7) \times 10^9 M_\odot$) is consistent with the prior mass measurement using stellar dynamics ($M = (6.2 \pm 0.8) \times 10^9 M_\odot$) but inconsistent with the gas dynamics measurement ($M = (3.2 \pm 0.6) \times 10^9 M_\odot$). Though the observed shadow is found to be consistent with the general relativistic magnetohydrodynamics simulated images of the Kerr black hole as predicted by the general relativity, various Kerr modified black
FIG. 11: Shadow asymmetry parameter $\Delta C$ (left panel) and the angular diameter $\theta_d$ (right panel) as a function of $(a, \alpha)$ for the rotating EGB black holes. Black solid lines correspond to the M87* black hole shadow bounds, namely $\Delta C = 0.10$ and $\theta_d = 39\mu\text{as}$ within the $1\sigma$ region.

Black hole models in general relativity as well in modified gravities could not be completely ruled out currently [44]. The measured circularity deviation, $\Delta C \leq 0.10$, for the M87* black hole shadow can be used to constrain the 4D EGB black hole parameters. $\Delta C$ is calculated for the rotating EGB black hole metric Eq. (8) and plotted as a function of $(a, \alpha)$ in Fig 11. The EHT bound for the M87* black hole shadow, shown as the black solid line, is used to constrain the $a$ and $\alpha$. It is clear that the $\Delta C$ merely puts a constrain on the EGB parameter $\alpha$.

Further, a far distant observer, at a distance $d$ from the black hole, measures the angular diameter $\theta_d$ for the black hole shadow

$$\theta_d = 2\frac{R_s}{d}, \quad R_s = \sqrt{A/\pi},$$  \hfill (34)

The inferred angular diameter for the M87* black hole shadow is $\theta_d = 42 \pm 3\mu\text{as}$ [42]. We calculated the angular diameter for the rotating EGB black hole shadow for $M = (6.5 \pm 0.7) \times 10^9 M_\odot$, and $d = 16.8 \text{Mpc}$, and plotted as a function of $a$ and $\alpha$, the region enclosed by the black solid line, $\theta_d = 39\mu\text{as}$, falls within the $1\sigma$ region of the M87* shadow angular diameter. For the given mass and distance, the Schwarzschild black hole cast the biggest shadow with angular diameter $\theta_d = 39.6192\mu\text{as}$. 
IV. CONCLUSION

The EGB theory has a number of additional nice properties than Einstein’s general relativity that are not enjoyed by other higher-curvature theories. The GB action is topological in $4D$ and hence does not contribute to gravitational dynamics in $4D$. However, if one re-scale the GB coupling to $\alpha/(D-4)$, the resulting theory leads non-trivial contribution for the $4D$ spacetimes in the limit $D \to 4$ while finding equations of motion [12]. The novel $4D$ EGB gravity extension to Einstein’s gravity bypass the Lovelock’s theorem and is free from the singularity problem. Further, it admits spherically symmetric black hole (4) in $4D$ and depending on critical mass it has two horizons [12]. We have constructed a rotating black hole in the $4D$ novel EGB gravity, which has an additional GB parameter $\alpha$ than the Kerr black hole, and it produces deviation from Kerr geometry but with a richer configuration of the event horizon and SLS. The rotating EGB black holes allow studying the effect of higher curvature on the Kerr black holes. It is found that the GB coupling parameter $\alpha$ makes a profound influence on the structure of the horizon by reducing its radius. For a fixed value of $a$, there always exists an extremal value of $\alpha = \alpha_E$, for which black hole has degenerate horizons i.e., $r_- = r_+ = r_E$, black hole with two distinct horizons for $\alpha < \alpha_E$, and no horizon for $\alpha > \alpha_E$. Similarly, for a given value of $\alpha$, one can obtain the extremal value of $a = a_E$ for which $r_- = r_+ = r_E$. The radius of horizons significantly decreases with increasing $\alpha$ and the ergosphere area is also affected, thereby can have interesting consequences on the astrophysical Penrose process.

This motivates us to reconsider the shadow cast by the rotating EGB black holes by discussing the photons geodesics equations of motion, which are analytically solved in the first-order differential form. Observables, namely area $A$ and oblateness $D$, are used to characterize the size and shape of the shadows. It is noticed that the rotating EGB black holes cast smaller and more distorted shadows than those for the Kerr black holes. The shadow size further decreases and the distortion increases with the increasing $\alpha$. It is shown that for a given set of shadow observables ($A, D$), we can explicitly determine the black hole parameters ($a, \alpha$). Shadow observational results for the M87$^*$ black hole are used to place constraints on the GB parameter in the supermassive black hole context. We modeled the M87$^*$ black hole as the rotating EGB black hole and used the inferred shadow angular diameter and the asymmetry parameter to determine the bound on $\alpha$. We have shown that,
within a finite parameter space, e.g. for $a = 0.1$, $\alpha \leq 0.00394$, the rotating EGB black hole is consistent with the M87* shadow results.

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**Note added in proof:** After this work was completed, we learned a similar work by Wei and Liu [18], which appeared in arXiv a couple of days before.

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