Summary: The moduli space of planar polygons with generic side lengths is a closed, smooth manifold. Mapping a polygon to its reflected image across the $X$-axis defines a fixed-point-free involution on these moduli spaces, making them into free $\mathbb{Z}_2$-spaces. There are some important numerical parameters associated with free $\mathbb{Z}_2$-spaces, like index and coindex. In this paper, we compute these parameters for some moduli spaces of polygons. We also determine for which of these spaces a generalized version of the Borsuk-Ulam theorem holds. Moreover, we obtain a formula for the Stiefel-Whitney height in terms of the genetic code, a combinatorial data associated with side lengths.

MSC:

55M30 Lyusternik-Shnirel’man category of a space, topological complexity à la Farber, topological robotics (topological aspects)
55P15 Classification of homotopy type
57R42 Immersions in differential topology

Keywords:
free $\mathbb{Z}_2$-space; coindex; index; Stiefel-Whitney class; tidy spaces; planar polygon spaces

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