A mean field theory for the cold quark gluon plasma

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Abstract. A hot and strongly interacting quark gluon plasma (QGP) has been observed in heavy ion collisions at RHIC. Quarks and gluons in a deconfined phase might also exist in the core of compact stars. We shall assume here that the cold QGP is also a strongly interacting system. The strong interactions may be partly due to non-perturbative effects, which survive after the deconfinement transition and which can be related with the non-vanishing gluon condensates in the QGP. In this work, starting from the QCD Lagrangian we perform a gluon field decomposition in low (“soft”) and high (“hard”) momentum components, we make a mean field approximation with spatial and time dependence for the hard gluons and take the matrix elements of the soft gluon fields in the plasma. The latter are related to the gluon condensates. We derive an analytical expression for the equation of state, which is compared with the MIT bag model one. The effect of the condensates is to soften the equation of state whereas the hard gluons significantly increase the energy density and the pressure.

1. Introduction

From the study of heavy ion collisions we have learned one important lesson about the hot quark gluon plasma (QGP): it is a strongly interacting system. It is therefore natural to assume that the cold QGP is also strongly interacting. Presumably, this kind of system exists in the core of dense stars. Because of the limitations of lattice calculations in this domain and also because of the lack of experimental information, the cold QGP is less known than the hot QGP.

In [1] we have studied the non-perturbative effects in the cold QGP generated by the residual dimension two and dimension four gluon condensates. In the present work we extend the results of [1] and consider the inhomogeneities in the gluon fields, including their spatial and time derivatives.

In the vacuum, non-perturbative effects have been successfully understood in terms of the QCD condensates, i.e., vacuum expectation values of quark and gluon “soft” (low momentum) fields. These condensates can, in principle, be computed in lattice QCD. In practice, since they are vacuum properties and therefore universal, they can be extracted from phenomenological analyses of hadron masses, as it is customary done in QCD sum rules [2]. The condensates are expected to vanish in the limit of very high temperature or chemical potential. However, it has been suggested that they may survive after the deconfinement transition both in the high temperature [3, 4] and in the high chemical potential cases [5]. For our purposes the relevant gluon condensates are those of dimension four, \( \langle 0 | \frac{4}{\pi^2} F^2 | 0 \rangle = \langle F^2 \rangle \), and of dimension two \[6, 7, 8, 9\], \( \langle 0 | g^2 A^2 | 0 \rangle = \langle g^2 A^2 \rangle \).

In what follows we shall repeat the procedure outlined in [1] and derive an equation of state for the QGP but, in contrast to our previous work, we will not neglect the field derivatives.
2. The equation of state
In this section we introduce a mean field approximation for QCD, extending previous works along the same line [10, 11]. The Lagrangian density of QCD is given by:

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^{a} F^{a\mu\nu} + \sum_{q=1}^{N_f} \bar{\psi}_j^q \left[ i \gamma^\mu (\delta_\mu \partial_\mu - ig T^a_\mu G^{\mu a}_\mu) - \delta_{ij} m_q \right] \psi^q_j \]  \tag{1}

where

\[ F^{a\mu\nu} = \partial^\mu G^{\nu a} - \partial^\nu G^{\mu a} + g f^{abc} G^{a\mu b}_\nu G^{c_\nu} \]  \tag{2}

The summation on \( q \) runs over all quark flavors, \( m_q \) is the mass of the quark of flavor \( q \), \( i \) and \( j \) are the color indices of the quarks, \( T^a_\mu \) are the SU(3) generators and \( f^{abc} \) are the SU(3) antisymmetric structure constants. For simplicity we will consider only light quarks with the same mass \( m \). Moreover, we will drop the summation and consider only one flavor. At the end of our calculation the number of flavors will be recovered. Following [10, 11], we shall start writing the gluon field as:

\[ G^{\mu a} = A^{\mu a} + \alpha^{\mu a} \]  \tag{3}

where \( A^{\mu a} \) and \( \alpha^{\mu a} \) are the low (“soft”) and high (“hard”) momentum components of the gluon field respectively. We will assume that \( A^{\mu a} \) represents the soft modes which populate the vacuum and \( \alpha^{\mu a} \) represents the modes for which the running coupling constant is small.

Inserting (3) into (2) we obtain:

\[ F^{a\mu\nu} = (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a} + g f^{abc} A^{b\mu}_\nu A^{c_\nu}) + (\partial^\mu \alpha^{\nu a} - \partial^\nu \alpha^{\mu a} + g f^{abc} \alpha^{b\mu}_\nu \alpha^{c_\nu}) \]
\[ + g f^{abc} A^{b\mu}_\nu \alpha^{c_\nu} + g f^{abc} \alpha^{b\mu}_\nu A^{c_\nu} \]  \tag{4}

2.1. The mean field approximation
In a cold quark gluon plasma the density is much larger than the ordinary nuclear matter density. These high densities imply a very large number of sources of the gluon field. Assuming that the coupling constant is not very small, the existence of intense sources implies that the bosonic fields tend to have large occupation numbers at all energy levels, and therefore they can be treated as classical fields. This is the famous approximation for bosonic fields used in relativistic mean field models of nuclear matter [12]. It has been applied to QCD in the past and amounts to assume that the “hard” gluon field, \( \alpha^{\mu a}_\mu \), is simply a function of the coordinates [12]:

\[ \alpha^{\mu a}_\mu (\vec{x}, t) = \delta_{\mu 0} \alpha^{a 0}_0 (\vec{x}, t) \]  \tag{5}

with \( \partial_\nu \alpha^{\mu a}_\mu \neq 0 \). This space and time dependence goes beyond the standard mean field approximation, where \( \alpha^{a 0}_\mu \) is constant in space and time [12] and consequently \( \partial_\nu \alpha^{a 0}_\mu = 0 \). We keep assuming, as in [1], that the soft gluon field \( A^{\mu a}_\mu \) is independent of position and time and thus:

\[ \partial^\nu A^{\mu a} = 0 \]  \tag{6}

Substituting (5) and (6) into (4) and (1) we have:

\[ \mathcal{L}'_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^{a} \Gamma^{a\mu\nu} - \frac{f^{abc}}{2} \Gamma^{a\mu\nu} (g A^{b\mu}_\nu \alpha^{c_\nu} + g \alpha^{b\mu}_\nu A^{c_\nu} + g A^{b\mu}_\nu A^{c_\nu}) \]
\[ - \frac{f^{abc} f^{ade}}{4} g^{2} A^{b\mu}_\mu A^{c_\nu}_\nu A^{d\mu}_\nu A^{e_\nu} \]
\[ + g^{2} A^{b\mu}_\mu A^{c_\nu}_\nu A^{d\mu}_\mu A^{e_\nu} + g^{2} A^{b\mu}_\mu A^{c_\nu}_\nu A^{d\mu}_\mu A^{e_\nu} + g^{2} A^{b\mu}_\mu A^{c_\nu}_\nu A^{d\mu}_\mu A^{e_\nu} \]
\[
+ g^2 A^b_{\mu} \alpha^c_\nu \alpha^{d\mu} A^{e\nu} + g^2 A^b_{\mu} \alpha^c_\nu A^{d\mu} \alpha^{e\nu} + g^2 \alpha^b_\mu A^c_\nu A^{d\mu} \alpha^{e\nu} + g^2 \alpha^b_\mu A^c_\nu A^{d\mu} \alpha^{e\nu} \\
+ \overline{\psi}_i \left\{ \gamma^\nu [\delta_{ij} \partial_\mu - i T^a_{ij} (g_\mu \alpha^a_\mu + g_\mu \alpha^a_\mu)] - \delta_{ij} m \right\} \psi_j
\]

(7)

and

\[
\Gamma^{a\mu\nu} = \partial^\mu \alpha^{a\mu} - \partial^\nu \alpha^{a\mu} + g_h f^{abc} \alpha^{b\mu} \alpha^{c\nu} \tag{8}
\]

In the above expressions the coupling is running. When we have only soft fields \( A^a_\mu \) the corresponding \( g \) is therefore large. Accordingly, we shall call it \( g = g_v \). The interactions of the hard fields among themselves and also the interactions between \( \alpha^0_\mu (\vec{x}, t) \) and the quarks are dominated by high momenta and the hard coupling is small. We shall call it \( g = g_h \). In the mixed terms containing soft and hard fields the couplings could be soft or hard.

We next replace in (7) all the terms that contain soft gluon fields by their corresponding expectation values in the cold QGP:

\[
\mathcal{L}'_{QCD} = -\frac{1}{4} \Gamma^{a\mu\nu} \Gamma^{a\mu\nu} - \frac{f^{abc}}{2} \Gamma^{a\mu\nu} \left( \langle g A^{b\mu} \rangle \alpha^{c\nu} + \alpha^{b\mu} \langle g A^{c\nu} \rangle + \langle g_s A^{b\mu} A^{c\nu} \rangle \right) \\
- \frac{f^{abc} f^{ade}}{4} \left( \langle g^2 A^b_{\mu} A^c_{\nu} A^{d\mu} A^{e\nu} \rangle \\
+ \langle g^2 A^b_{\mu} A^c_{\nu} A^{d\mu} \rangle \alpha^{c\nu} + \langle g^2 A^b_{\mu} A^c_{\nu} A^{d\mu} \rangle \alpha^{d\mu} + \alpha^c_\nu \langle g^2 A^b_{\mu} A^{d\mu} A^{e\nu} \rangle + \alpha^d_\mu \langle g^2 A^c_{\nu} A^{d\mu} A^{e\nu} \rangle \\
+ \alpha^c_\nu \langle g^2 A^b_{\mu} A^{d\mu} A^{e\nu} \rangle \alpha^{d\mu} + \alpha^d_\mu \langle g^2 A^c_{\nu} A^{d\mu} A^{e\nu} \rangle \alpha^{c\nu} + \alpha^b_\mu \langle g^2 A^c_{\nu} A^{d\mu} A^{e\nu} \rangle \alpha^{d\mu} \right) \\
+ \overline{\psi}_i \left\{ i \gamma^\mu [\delta_{ij} \partial_\mu - i T^a_{ij} (g_s A^a_\mu + g_h \alpha^a_\mu)] - \delta_{ij} m \right\} \psi_j
\]

(9)

These expectation values have the following properties:

\[
\langle A^{a\mu} \rangle = 0 \tag{10}
\]

\[
\langle A^{a\mu} A^{b\nu} A^{c\rho} \rangle = 0 \tag{11}
\]

\[
\langle g_s^2 A^{a\mu} A^b_\mu \rangle \equiv \langle A^2 \rangle \neq 0 \tag{12}
\]

\[
\frac{\alpha_s}{\pi} \frac{F^{a\mu\nu} F_{a\mu\nu}}{g_s^2 A^4} = \frac{g_s^2}{4 \pi^2} \frac{F^{a\mu\nu} F_{a\mu\nu}}{g_s^4 A^4} = \frac{g_s^4}{4 \pi^2} A^4 \equiv \langle F^2 \rangle \neq 0 \tag{13}
\]

As in [10, 11] we have:

\[
\langle g_s^2 A^{a\mu} A^{b\nu} \rangle = -\frac{\delta^{ab}}{8} \frac{g_{\mu\nu}}{4} \mu_0^2 = -\frac{\delta^{ab}}{32} \frac{g_{\mu\nu}}{\mu_0^2} \tag{14}
\]

As a consequence:

\[
\langle g_s^2 A^{a\mu} A^b_\mu \rangle = \langle g_s^2 A^{a\mu} A^b_\mu g^{\nu}_\mu \rangle = \langle g_s^2 A^{a\mu} A^b_\mu \rangle g^{\nu}_\mu = \\
-\frac{\delta^{ab}}{32} \frac{g^{\nu}_\mu}{\mu_0^2} \mu_0^2 = -\frac{\delta^{ab}}{32} \frac{g^{\nu}_\mu}{\mu_0^2} \mu_0^2 = -\frac{\delta^{ab}}{8} \frac{g^{\nu}_\mu}{\mu_0^2} \tag{15}
\]

and also [11]:

\[
\langle g_s^2 A^a_\mu A^b_\nu A^{c\rho} A^{d\eta} \rangle = \frac{\phi_0^4}{(32)(34)} \left[ g_{\mu\nu} g^{\rho\phi} \delta^{ab} \delta^{cd} + g_{\mu} g^{\rho} \delta^{ac} \delta^{bd} + g_{\mu} g^{\rho} \delta^{ab} \delta^{cd} \right] \tag{16}
\]

The \( \langle g_s^2 A^2 \rangle \) condensate is associated with a dynamical gluon mass [10, 11] which is defined as:

\[
m_G^2 = \frac{9}{32} \mu_0^2 \tag{17}
\]
In spite of the recent progress in the field, still very little is known about the parameter \( \langle A^2 \rangle \) at finite (and high) density. In our approach, as in [13], we have \( \langle A^2 \rangle < 0 \) in (14) so \( m_G^2 \) is positive.

Inserting (10) to (17) into (8) and (9), using the fact that \(- (g^2 f ^{abc} f ^{ade } b_0 a_0 d_0 \delta^a_0) / 4 = 0\), because of the color symmetry, we obtain the following effective Lagrangian:

\[
\mathcal{L}_0 = - \frac{1}{2} \alpha_0^a (\bar{\psi} \gamma_5 \psi_0^a) + \frac{m_G^2}{2} \alpha_0^a \alpha_0^a - b \phi_0^4 + \bar{\psi} i \gamma^\mu \partial_\mu \psi + g_h \gamma^0 T^a_\alpha \alpha_0^a - \delta_{ij} \mu \psi_j
\]  
(18)

where the constant \( b \) is defined as \( b \equiv 9/(4 \times 34) \) and

\[
- \frac{1}{4} F^{\mu
u} F_{\mu
u} = - \frac{\pi^2}{g^2} \alpha_s \pi F^{\mu
u} F_{\mu
u} = - b \phi_0^4
\]  
(19)

2.2. Energy density and pressure

The equations of motion [14] are given by:

\[
\frac{\partial \mathcal{L}}{\partial \eta_i} - \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \eta_i)} \right) + \partial_\nu \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\nu \eta_i)} \right) = 0
\]  
(20)

Inserting (18) into (20) with \( \eta_1 = \alpha_0^a \) and \( \eta_2 = \bar{\psi} \) we find:

\[
- \sqrt{2} \alpha_0^a + m_G^2 \alpha_0^a = - g_h \rho^a
\]  
(21)

\[
( i \gamma^\mu \partial_\mu + g_h \gamma^0 T^a_\alpha \alpha_0^a - m ) \psi = 0
\]  
(22)

where \( \rho^a \) is the temporal component of the color vector current given by \( j^{\mu \nu} = \bar{\psi} i \gamma^\mu T^a_\alpha \psi \). From [14] the energy-momentum tensor is given by:

\[
T^{\mu \nu} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \eta_i)} (\partial_\nu \eta_i) - g^{\mu \nu} \mathcal{L} + \left( \partial_\beta \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \partial_\beta \eta_i)} \right) (\partial_\nu \eta_i) + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \partial_\beta \eta_i)} (\partial_\nu \eta_i)
\]  
(23)

The energy density is:

\[
\varepsilon = \langle T_{00} \rangle
\]  
(24)

which, with the use of (22), becomes:

\[
\varepsilon = \frac{1}{2} \alpha_0^a (\bar{\psi} \gamma_5 \psi_0^a) - \frac{m_G^2}{2} \alpha_0 \alpha_0^a + b \phi_0^4 + i \bar{\psi} \gamma^0 (\partial_0 \psi)
\]  
(25)

Rewriting the last term of the above expression [1] we arrive at:

\[
\varepsilon = \frac{1}{2} \alpha_0^a (\bar{\psi} \gamma_5 \psi_0^a) - \frac{m_G^2}{2} \alpha_0 \alpha_0^a + b \phi_0^4 - g_h \rho^a \alpha_0 \alpha_0^a + 3 \frac{\gamma_Q}{2 \pi^2} \int_0^{k_F} dk k^2 \sqrt{k^2 + m^2}
\]  
(26)

where \( \gamma_Q \) is the quark degeneracy factor \( \gamma_Q = 2(\text{spin}) \times 3(\text{flavor}) \). The sum over all the color states was already performed and resulted in the pre-factor 3 in the expression above. \( k_F \) is the Fermi momentum defined by the quark number density \( \rho \):

\[
\rho = \langle N | \psi_i | \psi_i | N \rangle = \frac{3}{V} \sum_{k, \lambda} \langle N | N \rangle = 3 \frac{\gamma_Q}{(2 \pi)^3} \int d^3k = 3 \frac{\gamma_Q}{2 \pi^2} \int_0^{k_F} dk k^2 = \frac{\gamma_Q}{2 \pi^2} k_F^3
\]  
(27)

In the above expression \( | N \rangle \) denotes a state with \( N \) quarks.
In a first approximation the field $\alpha_0^a$ may be estimated from (21). Neglecting the terms containing the derivatives $\partial_0^2 \alpha_0^a$ and $\vec{\nabla}^2 \alpha_0^a$ of (21) we have: [14]:

$$\alpha_0^a \cong -\frac{g_h}{m_G^2} \rho^a$$  \hspace{1cm} (28)

Inserting (28) back on the left-hand-side of (21) and then solving it for $\alpha_0^a$ we find:

$$\alpha_0^a = -\frac{g_h}{m_G^2} \rho^a - \frac{g_h}{m_G^4} \vec{\nabla}^2 \rho^a$$ \hspace{1cm} (29)

We can write the color charge density $\rho^a$ in terms of the quark number density $\rho$ through:

$$\rho^a \rho^a = 3\rho^2$$ \hspace{1cm} (30)

Analogously we have

$$\rho^a \vec{\nabla}^2 \rho^a = 3\rho \vec{\nabla}^2 \rho, \quad \rho^a \vec{\nabla}^4 \rho^a = 3\rho \vec{\nabla}^4 \rho, \quad \vec{\nabla}^4 \rho^a \partial_0^2 \rho^a = 3\vec{\nabla}^4 \rho \frac{\partial^2 \rho}{\partial t^2}, \ldots$$ \hspace{1cm} (31)

Inserting (29), (30) and (31) into (26) and performing the momentum integral we arrive at the final expression for the energy density:

$$\varepsilon = \left( \frac{3g_h^2}{2m_G^2} \right) \rho^2 + \left( \frac{3g_h^2}{2m_G^4} \right) \rho \vec{\nabla}^2 \rho + \left( \frac{3g_h^2}{2m_G^6} \right) \rho \vec{\nabla}^4 \rho + \left( \frac{3g_h^2}{2m_G^8} \right) \vec{\nabla}^2 \rho \vec{\nabla}^4 \rho + b\phi_0^4$$

$$+ \frac{3\gamma Q}{2\pi^2} \left[ \frac{k_F^3 \sqrt{k_F^2 + m^2}}{4} \right] + \frac{3m^2}{8} \frac{\gamma Q}{2\pi^2} \left[ k_F \sqrt{k_F^2 + m^2} - m^2 \ln \left( \frac{k_F + \sqrt{k_F^2 + m^2}}{m} \right) \right]$$ \hspace{1cm} (32)

The pressure is given by

$$p = \frac{1}{3} < T_{ii} >$$ \hspace{1cm} (33)

Repeating the same steps mentioned before we arrive at:

$$p = \left( \frac{3g_h^2}{2m_G^2} \right) \rho^2 + \left( \frac{2g_h^2}{m_G^4} \right) \rho \vec{\nabla}^2 \rho - \left( \frac{g_h^2}{m_G^6} \right) \rho \vec{\nabla}^4 \rho - \left( \frac{g_h^2}{2m_G^4} \right) \vec{\nabla}^2 \rho \vec{\nabla}^4 \rho$$

$$+ \left( \frac{g_h^2}{2m_G^6} \right) \vec{\nabla}^2 \rho \vec{\nabla}^2 \rho - \left( \frac{g_h^2}{m_G^8} \right) \vec{\nabla}^2 \rho \vec{\nabla}^4 \rho - \left( \frac{g_h^2}{2m_G^6} \right) \vec{\nabla}^2 \rho \vec{\nabla}^2 \rho - \left( \frac{g_h^2}{m_G^8} \right) \vec{\nabla}^2 \rho \vec{\nabla}^4 \rho$$

$$+ \frac{\gamma Q}{2\pi^2} \left[ \frac{k_F^3 \sqrt{k_F^2 + m^2}}{4} \right]$$

$$- \frac{3m^2}{8} \frac{\gamma Q}{2\pi^2} \left[ k_F \sqrt{k_F^2 + m^2} - m^2 \ln \left( \frac{k_F + \sqrt{k_F^2 + m^2}}{m} \right) \right]$$ \hspace{1cm} (34)

where the speed of sound $c_s$ is given by:

$$c_s^2 = \frac{\partial p}{\partial \varepsilon}$$ \hspace{1cm} (35)
3. Numerical results and discussion

Including the inhomogeneities of the gluon field (and consequently of its derivatives) was an important step in the restoration of gauge invariance, which was broken in [1] by the assumption of constant fields. Moreover these derivatives play an important role in the hydrodynamics of the cold QGP. It was shown in [15] that these derivatives, coupled to derivatives already present in the equations of Euler and of the continuity lead to the Korteweg - de Vries (KdV) equation for perturbations in the baryon density. The existence of KdV solitons in the QGP is a remarkable phenomenon.

In spite of their conceptual relevance, the derivatives in (32) and (34) are not very important for the bulk properties of the system, since they are corrections to the results found in [1]. Therefore, in what follows we compare our results (32), (34) and (35) without the gradients, with the corresponding results obtained with the MIT bag model for a gas of quarks at zero temperature [12, 16]. We choose $B = 110 \text{ MeV fm}^{-3}$, which lies in the range $(50 < B < 200 \text{ MeV fm}^{-3})$ used in calculations of stellar structure [17]. For the comparison we must rewrite our results as functions of the baryon density, which is $\rho_B = \frac{1}{3} \rho$.

We consider the MIT bag model with finite $B$ and our model with massless quarks and soft gluons but no hard gluons. In this case we can identify our gluonic term with the gluonic component of the MIT bag model, represented by the bag constant. We then obtain an expression for the bag constant in terms of the gluon condensate:

$$B_{QCD} = b_0^4 = \frac{1}{4} \langle F^{a\mu\nu} F_{a\mu\nu} \rangle$$

Fixing $B$ and choosing a reasonable value of the coupling of the soft gluons, $g_s$, appearing in (19) we can infer the value of the dimension four condensate, $< F^2 >$, in the deconfined phase. For $B_{QCD} = B = 110 \text{ MeV fm}^{-3}$ and $g_s = 2.7$ (which would correspond to $\alpha_s(soft) = g_s^2/4\pi = 0.6$) we find:

$$< F^2 > = \frac{\alpha_s}{\pi} \langle F^{a\mu\nu} F_{a\mu\nu} \rangle = \frac{g_s^2}{\pi^2} B_{QCD} = 0.0006 \text{ GeV}^4$$

In the lack of knowledge of the in-medium dimension two condensate, we use the factorization hypothesis, which, in the notation of Refs. [10] and [11], implies the choice $\mu_0^2 = g_s \phi_0^2$. As a consequence, (14), (16), (17) and (19) are related and we obtain:

$$\langle g_s^2 A^2 \rangle = -\sqrt{\frac{4(34)\pi^2}{9}} < F^2 > = -0.3 \text{ GeV}^2$$

which corresponds to a dynamical mass of $m_G = 290 \text{ MeV}$. This number is consistent with the values quoted in recent works [18, 19], which lie in the range $200 < m_G < 600 \text{ MeV}$. Finally, the numerical evaluation of (32), (34) and (35) requires the choice of $g_h$, the coupling of the hard gluons, and of the quark mass, $m$. We choose them to be $g_h = 0.35$ (corresponding to $\alpha_s(\text{hard}) = g_h^2/4\pi = 0.01$) and $m = 0.02 \text{ GeV}$.

For this set of parameters our EOS is harder than the MIT one. This can be seen in the Fig. 1 where we show the pressure as a function of the energy density, and also in Fig. 2 where we show the ratio of the speed of sound $c_s^2/c_0^2$, where $c_s^2$ is given by (35) and $c_0^2 = 1/3$ for MIT. In the same range of baryon densities, we have more energy, much more pressure and consequently a larger speed of sound. This behavior can be attributed to the first term of the equations (32) and (34), which comes from the hard gluons. This term is exactly the same both in (32) and (34) and in the limit of high densities becomes dominant yielding $p \simeq \varepsilon$ and hence $c_s \rightarrow 1$. Physically, this term represents the perturbative corrections to the MIT approach. Since the quark density is extremely large, even in the weak coupling regime (typical of the hard gluons) the field $\alpha_0^b$ is intense.
We now show the EOS for different choices of the condensates, which are now treated as independent from each other. In Fig. 3 we fix $\langle F^2 \rangle$ and vary $\langle g_s^2 A^2 \rangle$, starting from the central value $-0.3 \text{ GeV}^2$ and increasing its magnitude up to the lattice result $-2.56 \text{ GeV}^2$ [20] (or $\langle g_s^2 A^2 \rangle = -(1.6 \text{ GeV})^2$ which corresponds to $m_G = 848 \text{ MeV}$). In Fig. 4 we perform the complementary study keeping $\langle g_s^2 A^2 \rangle$ and increasing the magnitude of $\langle F^2 \rangle$. As it can be seen, increasing the condensates reduces the pressure and, in the case of $\langle g_s^2 A^2 \rangle$, softens the equation of state. This behavior could be anticipated from Eqs. (32), (34) and from equation of motion (21) without derivatives of the density. Indeed, keeping fixed the coupling and the quark density, when we increase the gluon mass, the field becomes weaker.

To summarize, we have derived an equation of state for the cold QGP, which may be useful for calculations of stellar structure. The derivation is simple and based on three assumptions: i) decomposition of the gluon field into soft and hard components; ii) replacement of the soft gluon fields by their expectation values (“in-medium condensates”) and iii) replacement of the hard gluon fields by their mean-field (classical) values. The effect of the condensates is to soften the EOS whereas the hard gluons significantly increase the energy density and the pressure.

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