Abstract

MODFLOW is one of the most popular groundwater simulation tools available; however, the development of lake modules that can be coupled with MODFLOW is lacking apart from the LAK3 package. This study proposes a new approach for simulating lake-groundwater interaction under steady-state flow, referred to as the sloping lakebed method (SLM). In this new approach, discretization of the lakebed in the vertical direction is independent of the spatial discretization of the aquifer system, which can potentially solve the problem that the lake and groundwater are usually simulated at different scales. The lakebed is generalized by a slant at the bottom of each lake grid cell, which can be classified as fully submerged, dry, and partly submerged. The SLM method accounts for all lake sources and sinks, establishing a governing equation that can be solved using Newton’s method. A benchmarking case study was conducted using a modified model setup in the LAK3 user manual. It was found that when there is a sufficient number of layers at the top of the groundwater model, SLM simulates an almost identical groundwater head as the LAK3-based model; when the number of layers decreases, SLM is unaffected while LAK3 may be at a risk of giving unrealistic results. Additionally, the SLM can reflect the relationship between the simulated lake surface area and lake water depth more accurately. Therefore, the SLM method is a promising alternative to the LAK3 package when simulating lake-groundwater interaction.

Introduction

Lakes are relatively closed natural or artificial depressions capable of holding water on the ground surface. Most lakes are located in places where the groundwater table is shallow, and therefore exhibit significant hydraulic connections with groundwater (Stets et al. 2010; Kidmose et al. 2013; Zhou et al. 2013; Rosenberry et al. 2015). Studies of lake-groundwater interactions can have important practical implications, such as ensuring that the lake is sufficiently fed by groundwater (Kishel and Gerla 2002), or managing nutrient load from groundwater (Nakayama and Watanabe 2008; Meinikmann et al. 2015). One of the most promising tools to study lake water balance is through numerical hydrological modeling. However, the numerical simulation of lake-groundwater interaction is challenging because lakes interact with groundwater in a complex fashion where many different flow regimes and seepage patterns exist (Bobba 2012).

Previously, lake-groundwater interaction modeling underwent several stages of development: Winter (1976)
provided a general framework for understanding groundwater interaction with lakes without defining specific groundwater flow regimes; Nield et al. (1994) analyzed two-dimensional groundwater flow in the vertical direction near lakes using multiple flow regimes; and Townley and Trefy (2000) extended the numerical and analytical models to three-dimensional space. These studies laid the foundation for several lake-groundwater model codes including the Lake and Reservoir package in MODFLOW (Fenske et al. 1996; Merritt and Konikow 2000), the “high-K” technique (Anderson 2005; Chui and Freyberg 2008; Yihdego and Becht 2013), and the analytical element technique used by GFLOW (Hunt et al. 2003; Yager and Neville 2010).

The most widely used model code among the aforementioned approaches has been the LAK package because it is available in the popular groundwater model, MODFLOW (Merritt and Konikow 2000; Hunt et al. 2010; Yihdego et al. 2017). The LAK package has also undergone three significant updates from LAK1 (Cheng and Anderson 1993), LAK2 (Council 1998), and eventually the present version, LAK3 (Merritt and Konikow 2000). However, even with the most updated LAK 3 package, several problems still exist: First, LAK3 discretizes bathimetry into rectangular grids and cannot work with deformed aquifer grids. Therefore, the lake water surface changes abruptly when the lake water level moves from one layer to another. Second, if a more detailed lakebed bathimetry is needed, the top of the aquifer must be divided into many thin layers, resulting in higher computational costs. Third, closer to the lake, the computational grid cells may switch between wet and dry conditions, which can provoke numerical instability and model convergence difficulties.

To overcome these issues, we developed a new sloping lakebed method (SLM) to simulate lake water balance coupled with MODFLOW; similar ideas were used by Niswonger et al. (2006) in the unsaturated-zone flow (UZF) module. The present study is also our first attempt to couple SLM with MODFLOW to simulate lake-groundwater interaction under steady-state flow conditions. This manuscript is organized as follows: first, the SLM code is introduced. Second, several test simulations are performed using a slightly modified model setup in the LAK3 user manual as a benchmark. Third, the model results are compared with those of the corresponding LAK3 code. Finally, the applicability of the SLM method is discussed.

Method

Spatial Discretization of a Lake

Conceptualized lake bathimetry in its natural form curves continuously, as shown in Figure 1A1. However, to perform numerical calculations, grids are assigned, and a cross-section can be discretized. The SLM method assumes that the lake bottom elevation within the lake calculation cells is sloped. This is achieved as follows: First, the lake bathimetry is overlaid onto the groundwater model grids, and the averaged lakebed elevations are calculated where the lake bottom intercepts the groundwater model (Figure 1A2). If the lake bathimetry data are given as point values, they are interpolated to the grid size and location, the same as the underlying groundwater model. The interpolated values were taken as the averaged lakebed elevation. If the lake bathimetry data are given as spatially distributed grid values, spatial arithmetic averaging is performed to determine the averaged lakebed elevation in each calculation grid. Second, averaged lakebed elevations are ranked from top down for all calculation cells, for example, L1 to L7 in Figure 1A2. Third, the higher and lower points in each calculation grid are determined, represented as black dots (point-a to -m) in Figure 1A3. The bottom point of the upper rank is at the same elevation as the top point of the adjacent lower rank, namely, the lowest point of L1, point-b, is at the same elevation as the highest point of L2, point-c. The highest point of the whole setup is determined by the highest rank plus half of the distance between the highest rank and the second highest rank, for example, point-a = L1 + (L1-L2)/2. The very bottom point is determined by the lowest level rank minus half of the distance between the lowest elevation and the second lowest elevation, for example, point-m = L7-(L6-L7)/2. For every point in between, it is determined by the arithmetic average of the rank above and below, for example, point-b = (L1+L2)/2.

Conditions of the Lake Bottom Cells

In the SLM method, the status of a lake bottom cell can be classified into three types: fully submerged, dry, and partly submerged, as shown in Figure 2A, B, and C respectively, depending on the position of the lake water level. Note that because the bottom of a lake cell is sloped, the partly submerged condition in Figure 2C appears when the lake water level is between the upper and lower bounds of a cell, which is in contrast to the binary wet and dry conditions if the lake bottom cells are flat, as in Figure 1B. The exchange of water between the lake and the groundwater aquifer is based on Darcy’s law:

\[ q(h) = -C_m \Delta h \text{ and } C_m = \frac{K \cdot A}{L} \]

(1)

where \( C_m \) denotes the hydraulic conductance of the lakebed-aquifer interface, \( \Delta h \) is the hydraulic head gradient between the lake and the aquifer, \( K \) is the vertical hydraulic conductivity of the lakebed, \( A \) is the equivalent cross-section area of the lakebed perpendicular to the direction of flow, and \( L \) is the thickness of the lakebed. The hydraulic head in the aquifer underneath the lake is determined by the groundwater flow module, namely MODFLOW.

Under fully submerged conditions, the flow direction is shown either upward or downward in Figure 2A1-A4 depending on the juxtaposition of the lake water and groundwater levels. Under dry conditions, it is noted that if the groundwater head is below the lower bound of a
cell (Figure 2B3), there is no water exchange between the lake and groundwater; if the groundwater head is between the upper and lower bound, as in Figure 2B2, only the wetted part of the cell has water exchange. When a lake calculation cell is dry and the groundwater head is above the lowest point of the sloped lake bottom (Figure 2B1 and B2), the average lake bed elevation is used as the head in the lake calculation cell. For the partly submerged condition, it is a combination of the submerged and dry situations.

Lake Water Balance

All inflow and outflow terms in a lake are shown in Figure 1A. The lake is recharged from five sources during a given period.

\[
\text{Flow In} = P + Q_{si} + Rnf + G_{p}^{in} + G_{N}^{in} \tag{2}
\]

where \(\text{Flow In}\) denotes the total recharge rate to a lake in \(L^3/T\); \(P\) is precipitation, \(Q_{si}\) is the rate of the inlet flow, \(Rnf\) denotes the surface runoff from the unsubmerged area, \(G_{p}^{in}\) represents the groundwater inflow from the aquifers into the submerged area, and \(G_{N}^{in}\) represents the rate of groundwater inflow into the unsubmerged area. The water leaving a lake occurs in the following processes:

\[
\text{Flow Out} = E + G_{p}^{out} + W + Q_{so} \tag{3}
\]
where \( \text{FlowOut} \) is the total outflow rate of a lake in L\(^3\)/T; \( E \) is evaporation, \( G^\text{out}_{p} \) is the outflow from the submerged area into the aquifer, \( W \) denotes water withdrawal from the lake, and \( Q_{\text{so}} \) denotes the outlet discharge of a lake.

All recharge and discharge terms for a given lake can be expressed as functions of the lake water level \( h_l \). The lake water balance equation under steady-state flow can be obtained using Equations 2 and 3, as follows:

\[
\Delta S(h_l) = P(h_l) + R_{\text{nf}}(h_l) + G^\text{in}_{p}(h_l) + G^\text{in}_{N}(h_l) - G^\text{out}_{p}(h_l) - Q_{\text{so}}(h_l) - W = 0
\]

(4)

where \( \Delta S(h_l) \) is the lake water storage change which also has the unit of L\(^3\)/T.

**Solving the Water Balance Equation**

To solve the water balance equation, Newton’s iterative method is used. Thus, the calculated lake water level in the \( n \)th iteration, \( h^n_l \), is found as

\[
h^n_l = h^{n-1}_l - \frac{\Delta S(h^{n-1}_l)}{\Delta \dot{S}(h^{n-1}_l)}
\]

(5)

where \( \Delta \dot{S}(h^{n-1}_l) \) is the derivative value of the storage change at \( h^{n-1}_l \). It should be noted that all terms in Equation 4 are in units of volume per time. Suppose \( A_T \) is the total lake area, \( A_P(h_l) \) is the submerged area, referred to as the water ponded area, \( A_N(h_l) \) is the dry lakebed area. Both \( A_P \) and \( A_N \) are functions of the lake water level \( h_l \).

\[
A_T = A_P(h_l) + A_N(h_l).
\]

(6)

The precipitation term \( P(h_l) \) is a function of precipitation intensity \( p \) [L/T] and the area where the rainfall falls is \( A_P \):

\[
P(h_l) = p \cdot A_P(h_l).
\]

(7)

Note that \( A_P \) is used instead of \( A_T \) because rainfall that falls on the dry lakebed is considered runoff (\( R_{nf} \)). The expression is as follows:

\[
R_{nf}(h_l) = p \cdot \gamma \cdot A_N(h_l)
\]

(8)

where \( \gamma \) is the runoff coefficient (unitless) and \( E \) is the evaporation as given in Equation 9.

\[
E(h_l) = e_0 \cdot A_P(h_l)
\]

(9)

where \( e_0 \) is the evaporation rate. Incorporating Equation 6 to 9 into Equation 4, and differentiating both sides of the equation yields:

\[
\Delta S'(h_l) = [(1 - \gamma)P - e_0] \cdot A_P'(h_l) + G^\text{in}_p'(h_l) + G^\text{in}_N'(h_l) - G^\text{out}_p'(h_l) - Q_{\text{so}}'(h_l).
\]

(10)

In our study, the lake water level and groundwater level were used jointly to determine the direction and quantity of water exchange. This means that, if the groundwater numerical simulation does not converge, it is not possible for the lake water level calculation to converge, and vice versa. Therefore, implicit solutions are used when solving both the groundwater and lake water balance equations to ensure convergence, and the error limit used for the lake module is set to be the same as the groundwater module.

**Test Simulations**

To verify the feasibility of the proposed SLM package, we compared the results from SLM and the LAK3 package, the latter being a well-established method.
developed by the USGS. The test case we select is a modified example from Merritt and Konikow (2000).

System Description

The basic hydrogeological system setup was the same for the SLM-based model and the LAK3 based model, except for the shape of the lake bottom. The model domain was 13,000 m × 13,000 m, which was divided into 10 cells with 1000 m dimensions, and another 10 cells with locally refined 250 m dimensions at the center of the model domain in each direction, as shown in Figure 3. Taking a cross-section in the middle of the model domain A-A', it was seen that the model is divided into six vertical layers. The bottom layer was 30 m thick, and the top five layers were 4 m thick as shown in Figure 4A1 and A2. The lake bottom elevation was set to 97 m.

The top aquifer layer was unconfined with a hydraulic conductivity of 30 m/day, and the layers below can switch between an unconfined and confined condition. The leakage coefficient of the lakebed-aquifer interface was 0.1 per day. The outside model domain rim, shown in Figure 3, was defined as the constant-head boundary. The initial head at the west border was fixed at 116 m and the east at 114 m with a linear transition from west to east in between the north and south boundaries. The initial groundwater head condition was 115 m, uniformly distributed over the model domain.

Evaporation occurred at the groundwater phreatic surface and lake water surface. The maximum phreatic evaporation depth was set to 15 m. The maximum phreatic evaporation rate was set to 0.025 m/day (note that such intensive evaporation is not likely to occur in reality, and the value is used here only to test the model). Evaporation from the lake surface was calculated automatically after the lake water depth was calculated.

A fully implicit procedure was employed in the simulation to solve the numerical equations. The control threshold for groundwater convergence was 0.0001 m.

Simulation Results

The aforementioned system setup was taken as the baseline model, and the results are shown in Figure 5A. It can be seen in Figure 5A that when the model upper 20 m was divided into five layers, the simulated groundwater heads were almost identical between the LAK and SLM methods, and the difference between the average groundwater head was only 0.1 m. The lake water depth for the LAK3 method was 8.0 m with a 2.3 km² lake surface area, and with the SLM method, the lake water depth was 7.2 m with a 1.4 km² lake surface area (lake water depth refers to depth in the middle of the lake). This corresponded to a 10 and 40% difference in simulated lake water depth and lake surface area, respectively. Therefore, the difference between the simulated lake water was much more significant than that of the simulated groundwater head.

To demonstrate the difference between the SLM and LAK3 methods in further detail, the top 20 m of the baseline system setup was then combined into two layers, and then one layer, as seen in Figure 4B and C, respectively. The results are shown in Figure 5B and C, respectively. The simulated groundwater head, lake water depth, lake surface area, and lake water volume for scenarios A, B, and C are summarized in Table 1.
Figure 4. Test case study, side view of the model with different layer delineations: (A1) and (A2), the top 20 m is divided into five layers; (B1) and (B2), the top 20 m is divided into two layers; (C1) and (C2), the top 20 m is one layer. The left column denotes the lake bottom condition in LAK3, whereas the right column denotes the lake bottom in SLM.

It can be seen in Figure 5 and Table 1 that the simulations based on the SLM method are independent of how the groundwater model grids were discretized. In contrast, simulations based on the LAK3 method depended heavily on the geometry of the upper part of the groundwater model. With fewer groundwater model layers, the difference between the simulated groundwater head of the two methods increased from 0.1 to 3.6 m. The simulated lake water conditions had even larger differences between the two methods when the top layers were combined into one or two layers; the model-simulated results based on the SLM method were unaffected. The LAK3-based model simulates a dried-up lake, which indicates that the model may be at risk of giving unrealistic results. This was primarily due to the larger evaporation in the LAK3-based models.

Water balance calculations for both the lake and groundwater components are shown in Table 2. It was shown that both the SLM method and LAK3 can close the water balance successfully with close to zero error for both the lake and groundwater components. However, the SLM-based model simulated identical in-and-out flow regardless of the number of layers, while the LAK3-based model depended heavily on how the groundwater model was discretized.

Simulated Lake Surface Area

We selected the parameter Evaporation Rate ($e_0$ in Equation 6) and then changed its value from 0 to 0.05 m/day in increments of 0.005 m/day. Scenario A was used for the top groundwater model layers, so that 20 m was equally divided into five layers, as seen in Figure 5A1 and A2. The results are shown in Figure 6.

Figure 6 shows that both the LAK3- and SLM-based models reacted strongly to the change in evaporation rate, in that both the lake water depth and lake surface area decreased with increasing $e_0$. The difference between the two modeling approaches is within the margin of error. However, it is observed that with changing water depth, the lake surface area changes continuously (black dots) for the SLM-based model and stepwise (gray shaded dots) for the LAK3-based model. For instance, when $e_0$ increases from 0.015 to 0.03 m/day, the lake water level simulated by the LAK3 method is at the second layer from the bottom, and therefore, the corresponding lake water area remains at 2.25 km², which is equivalent to 36
Figure 5. Contour of simulated groundwater head under three different scenarios: (A) top 20 m is divided into five layers, (B) the top 20 m is divided into two layers, (C) the top 20 m is one layer.

Table 1
Simulation of Average Groundwater Head, Lake Water Level, Lake Surface Area, and Lake Water Volume

| Scenario | Number of Layers | Average Groundwater Table (m) | Lake Water Depth (m) | Lake Water Surface Area (km²) | Lake Water Volume (mill. m³) |
|----------|-----------------|-------------------------------|----------------------|-------------------------------|-----------------------------|
|          |                 | LAK3  | SLM   | LAK3 | SLM | LAK3 | SLM | LAK3 | SLM | LAK3 | SLM |
| A        | 6               | 111.6 | 111.5 | 8.0  | 7.2 | 2.25 | 1.4 | 5.1  | 5.0 |
| B        | 3               | 108.5 | 111.5 | 0    | 7.2 | 0    | 1.4 | 0    | 5.0 |
| C        | 2               | 107.9 | 111.5 | 0    | 7.2 | 0    | 1.4 | 0    | 5.0 |
Table 2
Water Balance for Lake and Groundwater Components in the Test Example

| Source/Sink                             | SLM               | LAK3              |
|-----------------------------------------|-------------------|-------------------|
|                                         | Number of Layers  | Number of Layers  |
| Lake component                          | 2                 | 2                 |
| In (m³)                                 | 3                 | 3                 |
| Precipitation                           | 6                 | 6                 |
| Channel flow to lake                    | 0                 | 0                 |
| Overland flow to lake                   | 0                 | 0                 |
| GW discharge to lake                    | 0                 | 0                 |
| Total                                   | 34,682            | 34,682            |
| Evaporation                             | 34,683            | 34,683            |
| Lake leakage to groundwater             | 0                 | 0                 |
| Water withdrawal from lake              | 0                 | 0                 |
| Channel flow from lake                  | 0                 | 0                 |
| Total                                   | 34,683            | 34,683            |
| GW component                            | 40,601            | 40,601            |
| In (m³)                                 | 40,601            | 40,601            |
| Boundary inflow (constant head)         | 40,601            | 40,601            |
| Recharge from lake leakage              | 0                 | 0                 |
| Total                                   | 40,601            | 40,601            |
| GW discharge to lake                    | 34,682            | 34,682            |
| Total                                   | 40,601            | 40,601            |

Error

Figure 6. Simulated lake water depth and lake surface areas under different evaporation rates based on the LAK3 and SLM methods. Simulated lake water depth based on the LAK3 method is shown on top of the bar chart.

Discussion

In the hydrological modeling community, surface water and groundwater are now considered as interconnected components within the framework of terrestrial water circulation (Ibrakhimov et al. 2018; Sterte et al. 2018). Several modeling tools have developed to account for surface and groundwater simultaneously, for example, MIKE SHE (Abbott et al. 1986) and SWAT (Arnold and Fohrer 2005). If groundwater is the primary interest, such that rivers and lakes are fed by groundwater baseflow, the obvious choice of model code is MODFLOW and its accessories, because it is widely used and proven to be valid in many cases (Xi et al. 2010; Yao et al. 2015). In contrast to the large demand for integrating lake and groundwater simulations, the development of a software package that can materialize this functionality under the umbrella of MODFLOW has been rather lagging. The most advanced code on the market, LAK3, can provide acceptable functionality, but still has several shortcomings. As noted by its developers, LAK3 has certain limitations in that the lateral and vertical groundwater model grid dimensions must be carefully defined so that the spatial extent and bathymetry of lakes are defined with the necessary accuracy (Merritt and Konikow 2000).

Here, we developed a new code named SLM, which worked seamlessly with MODFLOW and therefore can be considered as an alternative to LAK3 for simulating lake-groundwater interactions. One of the most distinguishing features of SLM compared to LAK3 is that it simulates the status of a lake without having to consider the underlying groundwater model grid structure. This feature is preferred because the scale of interest is quite different between the lake water and groundwater model, especially from computational grids. Meanwhile, the SLM-based model simulates lake water surface area changing from 2.0 to 1.2 km².

Another notable finding was that the simulated lake water depth based on the LAK3 method had a tendency to vary around at the boundary between two vertical groundwater model layers. For instance, when $e_0$ increased from 0.015 to 0.03 m/day, the simulated lake water depth based on the LAK3 method was approximately 8 m, which was the boundary between the second and third layers from the bottom; and when $e_0$ was increased from 0.04 to 0.05 m/day, the lake water depth simulated by the LAK3 method was approximately 4 m, which was the boundary between the first and the second layer from the bottom. On the contrary, SLM simulated lake water depth that changed smoothly with the increase in the evaporation rate.
a practical point of view. For instance, an area change of 2 km² can be very substantial when practical lake water management is concerned. However, such size may not be critically important when looking from a regional groundwater resources perspective. Thus, the model grid size must come to a common ground in areas close to the lake to satisfy both needs when coupling the lake and groundwater models. The only solution to this problem in LAK3 would be to increase the number of layers in the upper groundwater model to increase lake water simulation accuracy, but there is a risk of having heavier computational burden or even nonconvergence. Regarding the model nonconvergence problem, the main cause is usually that simulated heads are near the boundary between two model layers. Therefore, a calculation cell switches between wet and dry conditions constantly during the iterative solution (El-Zehairy et al. 2018; Szczepinski 2019). This indicates that, in LAK3, the more layers used, the higher the chance of encountering nonconvergence issues.

The proposed SLM method, however, can avoid the aforementioned problems that frequently occur in LAK3. The coupled model is much more likely to converge because the calculation cells in the lake no longer switch between just wet and dry, as there is an intermediate condition, namely, the partly submerged condition. Therefore, if the MODFLOW portion can converge, the coupled model can most likely converge when SLM is added later. It is also quite convenient if an off-the-shelf groundwater model developed in MODFLOW needs a new update with a lake module integration, because it is no longer necessary to subdivide the upper groundwater layers. Meanwhile, SLM does not sacrifice groundwater flow simulation quality because it gives basically the same result in simulated groundwater head in the SLM-based model in comparison to the LAK3-based model. Lastly, the simulated lake water surface area changes smoothly with the change in lake water depth, as shown in our study.

Conclusion

In the present study, a lake module was developed that can be coupled with MODFLOW to simulate lake-groundwater interaction. The new lake module, named SLM, used a sloped bottom at lake calculation grids, so that each lake cell had three conditions: fully submerged, dry, and partly submerged, in contrast to a wet and dry binary system when the lake grid bottom is flat. To demonstrate the validity of the new SLM method, a modified model setup from the LAK3 manual was used so that the simulation results of the two methods could be compared side by side. It was found that when the top part of the groundwater model was divided into a sufficient number of layers, the SLM-simulated groundwater head was almost identical to the LAK3-based model; there was a difference in the simulated lake water depth and lake surface area of 10 and 40% between the two methods. Second, when the top part of the groundwater model was not divided into a sufficient number of layers (e.g., one or two layers), the SLM was unaffected, but LAK3 held the risk of giving unrealistic results. Third, when the lake water depth changes, the SLM-based model generated smooth changes in lake surface area, while the LAK3-based model simulated abrupt changes in lake surface area. Fourth, it was, in principle, less likely to run into nonconvergence problems using the SLM method. Therefore, SLM was an improvement to the LAK3 module for simulating lakes that are closely linked to the underlying groundwater.

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Authors’ Note

The author(s) does not have any conflicts of interest.

Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article. Supporting Information is generally not peer reviewed.

Figure S1 Three-dimensional view of lake calculation cells.

References

Abbott, M.B., J.C. Bathurst, J.A. Cunge, P.E. Oconnell, and J. Rasmussen. 1986. An introduction to the European hydrological system—Systeme Hydrologique European, SHE. 1. History and philosophy of a physically-based, distributed modeling system. Journal of Hydrology 87, no. 1–2: 45–59.

Anderson, E.I. 2005. Modeling groundwater-surface water interactions using the Dupuit approximation. Advances in Water Resources 28, no. 4: 315–327.

Arnold, J.G., and N. Fohrer. 2005. SWAT2000: Current capabilities and research opportunities in applied watershed modeling. Hydrological Processes 19, no. 3: 563–572.

Bobba, A.G. 2012. Ground water-surface water interface (GWSWI) modeling: Recent advances and future challenges. Water Resources Management 26, no. 14: 4105–4131.

Cheng, X.X., and M.P. Anderson. 1993. Numerical-simulation of groundwater interaction with lakes allowing for fluctuating lake levels. Groundwater 31, no. 6: 929–933.

Chui, T.F.M., and D.L. Freyberg. 2008. Simulating a lake as a high-conductivity variably saturated porous medium. Groundwater 46, no. 5: 688–694.
