A new exchange flow computational method of quasi-one-dimensional flow between matrix and fractures in Fractured Reservoirs

Huan Zheng, Anfeng Shi*, Xiaohong Wang
Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei, Anhui 230026, P. R. China

*Corresponding author e-mail: zhhhappy@mail.ustc.edu.cn

Abstract. Natural fractured reservoirs exist widely in the world. The heterogeneous porous medium was idealized to be a uniform system of the fracture and matrix for the convenience of numerical simulation. The traditional method to describe the flow exchange between the matrix and fracture is using a function related to shape factor and the pressure difference between the matrix and fracture. However, this classical model cannot be applied for the incompressible fluid. A new exchange flow computational method between matrix and fractures of quasi-one-dimensional flow in fractured reservoirs is proposed in this article. Numerical simulations are performed to demonstrate that the proposed method can provide more accurate results.

1. Introduction
In the naturally fractured reservoir which is strong inhomogeneous, fractures are interconnected and provide the main flow path, when the matrix blocks are not highly interconnected and provide most storage space for oil or gas. In order to quantify the flow behavior of such systems, Warren-Root (1963) proposed a Dual-Porosity Model which used the superposition of fractures and matrix blocks to describe naturally fractured reservoir. It is assumed that each space point has two kinds of pressure: the average pressure in fracture and in the matrix blocks in this model. In order to describe the exchange flow flux between the matrix and fractures, a transfer function which is related to pressure difference between matrix and fractures and a shape factor is constructed.

For oil or gas single-phase flow, the fluid compressibility and viscosity have a great influence on the fluid flow. Depends on the assumption of a small and constant compressibility of the fluid, Aziz (1995) derived the transfer shape factor for dual-porosity simulation by combining analytical solutions of pressure diffusion for various flow geometries. In his deduction, the pseudo steady state assumption was not adopted and the exchange flow flux of a tiny compressible single-phase fluid can be determined. The compressibility is important when he determine the mass flow from the matrix to the fracture, or vice versa.

Apparently, the pressure difference between matrix and fracture will be zero in incompressible fluid. The exchange flow will be nearly zero if we use series model of Warren and Root no matter what kind of shape factor is adopt. This result will create an inaccurate impression that the exchange between the matrix and the fracture is very slow. Then, if imbibition is not so influential, we think flooding process of incompressible oil-water two-phase flow through matrix fracture interface under full-field pressure
gradient is the main mechanism and get a new exchange flow flux computational method for incompressible of quasi-one-dimensional oil-water two-phase flow in fractured reservoir.

2. Algorithm description

2.1. Traditional computational method

In fractured reservoirs with two different medium, the matrix with low permeability is the major space of fluid, while fractures with high permeability are the major ways of flowing. The continuity equation of \( \alpha \) phase in dual-porosity models is:

\[
\phi \frac{\partial (\rho \phi S_t^\alpha)}{\partial t} + \nabla \cdot (\rho \phi \mathbf{V}_t^\alpha) = q_t^\alpha
\]

(1)

where \( \phi \), \( \rho \) and \( S \) are porosity, density and saturation; \( \mathbf{V} \) is Darcy velocity, and \( q \) is the transfer term between matrix and fractures. The superscript \( \beta = m, f \) represents the variable in the fracture and in the matrix respectively. Obviously, \( q_t^m = -q_t^f \). In this paper, we consider only two phase: oil and water. Kazemi. et al(1976) proposed a computational method of exchange flux rate in two-phase dual-porosity model under pseudo steady state assumptions:

\[
q_t^f = \sigma \rho_a^{(m)} K_a^{(m)} \lambda_a^{(m)} \left[ p_a^{(m)} - p_a^{(f)} \right]
\]

(2)

\( \sigma \) is the shape factor, which relates to the size of the matrix.

In the original exchange flux rate computational method in, the exchange speed between matrix and fractures depends on the pressure difference between this two different medium. The exchange volume is the volume of elastic release in weakly compressible matrix. However, under incompressible condition, the pressure in matrix and fractures is same. If we insisted on the original exchange flux rate computational method in, the exchange flow will be zero. Then, the fluid in matrix can only flow very slowly because of the low permeability and cannot inject into fractures under incompressible condition, the results are obviously inaccurate. Thus, a new exchange flow flux algorithm for incompressible oil-water two-phase flow in fractured reservoir is necessary.

2.2. New computational method

In the new exchange flow flux algorithm, neglecting the influence of capillary force, we think the water which is rich in fractures will be driven into matrix by the pressure gradient and then displace the oil in the matrix because the phase flow ability of water and oil is different. In this method, the main mechanism of transfer between matrix and fractures is displacement. The exchange volume is the volume difference of entry and outflow in matrix. In this condition, liquid flow fast in fractures and there is no direct flow exchange between matrix and matrix. Matrix only exchange with the fractures which surround it and the exchange flow is considered as a source term.

Same assumption as traditional dual-porosity model, neglecting the influence of capillary force, the volume conservation in the 1D system for incompressible oil-water flow is written as:

\[
\phi_f \frac{\partial S_a^{f,f}}{\partial t} + \nabla \cdot \mathbf{V}_a + q_c^f = 0, \quad \phi_a \frac{\partial S_a^{a,m}}{\partial t} - q_c^a = 0
\]

(3)

According to Darcy’s Law:

\[
\mathbf{V}_a = -\lambda_a K_a \frac{\partial p}{\partial x}
\]

(4)
Where: \( \lambda_f^m = \frac{kr_f}{\mu_f} \), \( \lambda_m^m = \frac{kr_m}{\mu_m} \);

When fluid flow from fractures to matrix:
\[
v^m = v_m^m + v_m^f, \quad \frac{v_m^m}{v_m^f} = \frac{\lambda_f^m}{\lambda_m^f}
\]

When fluid flow from matrix to fractures:
\[
v^f = v_f^m + v_f^f, \quad \frac{v_f^m}{v_f^f} = \frac{\lambda_f^m}{\lambda_m^f}
\]

For the case of incompressible flow:
\[
v^m = v^f
\]

we get the exchange flow speed from difference of entry and outflow flow in matrix:
\[
V_c = v_m^m - v_m^f
\]

Then, we can get the exchange flow speed:
\[
V_c = \left( \frac{\lambda_f^m}{\lambda_f^m + \lambda_m^m} - \frac{\lambda_f^m}{\lambda_f^m + \lambda_m^m} \right) v_m^m
\]

Here, we use the flow speed from matrix to fractures as the total flow speed across the matrix. Then, we can get the exchange flow speed for two phase flow:
\[
V_c = \left( \frac{\lambda_f^m}{\lambda_f^m + \lambda_m^m} - \frac{\lambda_f^m}{\lambda_f^m + \lambda_m^m} \right) \left( \frac{\lambda_f^m}{\lambda_f^m + \lambda_m^m} \cdot \lambda_f^m \right) \cdot K_m \left( \frac{\partial p}{\partial x} \right)
\]

Then, we get the exchange flow volume and transform the flow volume into an average source item:
\[
q_c = \frac{K_m}{L_m} \left( \frac{\lambda_f^m}{\lambda_f^m + \lambda_m^m} \cdot \lambda_f^m \cdot \lambda_f^m \right) \cdot K_m \left( \frac{\partial p}{\partial x} \right)
\]

Where \( L_m \) is the dimension factor, which relates to the size of the matrix.

3. Numerical example
In this example, there are 9\( \times \)9 square matrix blocks separated by fractures uniformly in the simulation area. The width of each fracture is 0.1m and the representative matrix block here is rectangular with its size being \( L_x \times L_n = 0.5m \times 0.9m \), as shown in Figure 1. The original porosities and permeability of matrix and fracture and their mapped values in the dual porosity model are shown in Table 1. The relative permeability in matrix is given as \( k_{rw} = S_w^2 \), \( k_{ro} = (1 - S_w)^2 \), and in fracture, \( k_{rf} = S_w \), \( k_{ro} = 1 - S_w \). The shape factor in the original dual-porosity model takes the value: \( a = \pi \left( \frac{1}{L_x} \right) \left( \frac{1}{L_n} \right) ^2 \).
Two cases are considered here. In order to ensure the connectivity of the fractures, the pressure boundary conditions for these two cases are given in Figure 1. The pressure distributes linearly along each boundary side. In the first case, the flow is mainly in x direction. In the second case, the flow is mainly in y direction.

**Table 1.** The original and the mapped values of porosities and permeabilities for example.

| Related parameters | The value in the reference | The mapped value in dual porosity model |
|--------------------|---------------------------|----------------------------------------|
| Fracture porosity  | 1.0                       | 0.25                                   |
| Matrix porosity    | 0.35                      | 0.2625                                 |
| Absolute permeability of fracture | 1.0 D            | \((K_x) = 0.1 \text{D}, \ (K_y) = 0.1667 \text{D}\) |
| Absolute permeability of matrix | 0.001 D          | \((K_x) = 0.0009 \text{D}, \ (K_y) = 0.00083 \text{D}\) |

The total oil production for the case 1 and the case 2 are shown respectively in Figure 2a and Figure 2b.

**Figure 2.** Comparison of the calculated oil production for the case 1 (quasi-\(x\) direction flow) and the case 2 (quasi-\(y\) direction flow).
It can be seen that the original dual porosity model underestimate the oil production, and the proposed one can improve the accuracy greatly.

4. Conclusion
In this paper, a new exchange flow flux algorithm for incompressible quasi-one-dimensional oil-water flow is proposed. We consider the main mechanism of transfer between matrix and fracture is displacement, not the elastic release because of compressibility. Then, we get the exchange flow depends on the pressure gradient instead of the pressure difference between matrix and fractures. Examples indicate that the proposed algorithm in this paper can simulate incompressible oil-water two-phase flow in fractured reservoirs efficiently and accurately.

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