EXTENDING THE ΛCDM MODEL THROUGH SHEAR-FREE ANISOTROPIES

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If the spacetime metric has anisotropic spatial curvature, one can afford to expand the universe isotropically, provided that the energy-momentum tensor satisfy a certain constraint. This leads to the so-called shear-free metrics, which have the interesting property of violating the cosmological principle while still preserving the isotropy of the cosmic microwave background (CMB) radiation. In this work we show that shear-free cosmologies correspond to an attractor solution in the space of models with anisotropic spatial curvature. Through a rigorous definition of linear perturbation theory in these spacetimes, we show that shear-free models represent a viable alternative to describe the large-scale evolution of the universe, leading, in particular, to a kinematically equivalent Sachs-Wolfe effect. Alternatively, we discuss some specific signatures that shear-free models would imprint on the temperature spectrum of CMB.

Keywords: ΛCDM model; spatial anisotropies; perturbation theory; shear-free anisotropies.

1. Introduction

The standard concordance model of cosmology – or ΛCDM model – is based on three main ingredients: i) the validity of general relativity at cosmological scales, ii) the validity of the standard model of particle physics at all cosmological epochs and distances and iii) the cosmological principle, according to which our universe is, on average, spatially homogeneous, isotropic and infinite. While the first two ingredients have been largely tested, modified and scrutinized as an attempt to explain diverse phenomena such as dark matter and dark energy, attempts to question the validity of the cosmological principle happens at a slower pace, mainly because of our inabilities to collect data from regions other than our insignificant corner in an otherwise indifferent and colossal universe.

Notwithstanding the fact that the cosmic microwave background (CMB) radiation is isotropic at 0.001% level,¹ ² and that the distribution of matter at scales
above $100h^{-1}\text{Mpc}$ are compatible with the cosmological principle,\textsuperscript{3–5} one cannot take for granted a principle of central importance to the whole scientific endeavor. On the other hand, attempts to extend the cosmological principle have to cope with these very data suggesting isotropy and homogeneity.

There are two ways of bridging the extensions of the cosmological principle with observational data. One is to admit that small deviations of isotropy and homogeneity are hidden under current cosmological data, either lurking in the precision of our current experiments or, to take the example of CMB, in the form of large-scale statistical anomalies.\textsuperscript{6} The second alternative is to formulate symmetry-violating models that respect the data we have at hand.

In this work we explore the second of these alternatives and investigate a class of spatially anisotropic models which preserve, at the background level, the observed isotropy of CMB. In particular, we start from previous works in which the anisotropy of the universe results from the curvature of the spatial sections, and not from the kinematics of expansion.\textsuperscript{7–10} Such feature is implemented by metrics admitting shear-free expansion,\textsuperscript{7} and in this work we focus on two particular cases: Bianchi type III and Kantowski-Sachs metrics.

This work is organized as follows: in section 2.1 we describe a simple class of anisotropic metrics admitting anisotropic spatial curvature, and show that, for a specific choice of the energy-matter content, these metrics lead to an attractor solution in which the universe expands isotropically. Assuming that shear-free models go through a period of inflationary expansion, we show in section 2.2 that the theory of linear perturbations in these models is feasible, and leads to very specific signatures. In section 2.3 we explore a few signatures that shear-free models would imprint on the temperature spectrum of CMB. We conclude in section 3 with some perspectives of extensions of this work.

Throughout this work use metric signature ($-;++$) and adopt units such that $c = 1 = 8\pi G$. Space and spacetime indices are represented by Latin and Greek letters, respectively. The lower case letters ($a, b, c$) represent coordinates on two-dimensional manifolds.

2. Shear-free anisotropy

Once we are willing to admit our ignorance about the global symmetries of the universe, we find that there is much more to anisotropy than just anisotropic expansion.\textsuperscript{11–13} In fact, there exists anisotropic solutions of the Einstein field equations in which not only the expansion of the universe is anisotropic, but so is the curvature of spatial sections.\textsuperscript{14,15} In the standard four-dimensional description of the universe, one way\textsuperscript{a} of constructing a manifold with anisotropic curvature is by multiplying the (flat) one-dimensional real line $\mathbb{R}$ with a curved two-dimensional space

\textsuperscript{a}Evidently, there exist more sophisticated three-dimensional geometries, such as the Nil, Sol and $SL(2,\mathbb{R})$ geometries, which we will not consider here. For a recent cosmological study involving a Bianchi type II solution (which corresponds to the Nil geometry) see Ref. 16.
If we restrict, for the sake of simplicity, to maximally symmetric two-dimensional spaces, then there are only two possibilities: either $M$ is a sphere ($S^2$) or a pseudo sphere ($H^2$). The first case represents the known Kantowski-Sachs (KS) anisotropic solution, while the second gives the Bianchi type III (BIII) metric. In comoving cylindrical coordinates, these two solutions can be parameterized as follows:

$$ds^2 = -dt^2 + e^{2\alpha} \left[ e^{2\sigma} \left( dp^2 + \frac{1}{|\kappa|} \sin^2(\sqrt{|\kappa|} \rho) d\phi^2 \right) + e^{-4\sigma} dz^2 \right],$$

where $\alpha$ is the average scale factor and $\sigma$ measures the spatial shear. The number $\kappa$ measures the curvature of the two-dimensional spaces, and can be either $-1$ (BIII), $+1$ (KS). Incidentally, we note that $\kappa = 0$ also corresponds to the locally-rotationally-symmetric Bianchi I solution.

Given that (1) is already anisotropic at the level of the spatial curvature, it is natural to ask whether these models can evolve with a single scale factor. Indeed, it has been shown that for some specific choices of the energy-momentum content, these models admit a shear-free (SF) expansion, that is, there exist anisotropic cosmological solutions with $\sigma = 0$ in (1). Thus, before developing the observational signatures of SF models, it is important to investigate whether these solutions are dynamically stable.

2.1. Background dynamics

Let us consider the scenario of a universe with metric (1) and composed of a perfect fluid with energy density $\rho_f$ and pressure $p_f$, plus some anisotropic source of energy and momentum. To be more specific, let us model the latter by a two-form field $B_{\mu\nu}$, for which we know that shear-free solutions exist. The total energy-momentum tensor of the system is thus:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} + \pi_{\mu\nu}$$

$$= [(\rho_f + p_f)u_\mu u_\nu + p_f g_{\mu\nu}] + \left[ -3\gamma J_{\mu\alpha\beta} J_{\alpha\beta}^\gamma + \frac{1}{2} \gamma J_{\alpha\beta\gamma} J_{\alpha\beta\lambda} g_{\mu\nu} \right],$$

where $\gamma$ is a constant and the field strength $J_{\mu\nu\lambda} = 3! \partial_{[\mu} B_{\nu\lambda]}$ is such that $\partial_{\mu}(\sqrt{-g} J^{\mu\nu\lambda}) = 0$. In four dimensions, $J_{\mu\nu\lambda}$ has only four components, which means that it is dual to a four vector $V^\rho$. Moreover, since the $z$-direction has a distinct character in the coordinates adopted in (1), we will define

$$J_{\mu\nu\lambda} \equiv \epsilon_{\mu\nu\lambda\rho} V^\rho, \quad V^\rho \equiv V^\delta^\rho_3$$

For a recent application of two-form fields in the context of anisotropic cosmologies, see Ref. 17

We are assuming that the two-form field does not couple to the perfect fluid.
where $V$ is a function of time. The Einstein field equations resulting from (1)-(3) are:

$$ H^2 - \dot{\sigma}^2 = \frac{1}{3} \rho - \frac{R^{(3)}}{6} $$

$$ \dot{H} + 3H^2 = \frac{1}{2}(\rho - p) - \frac{R^{(3)}}{3} $$

$$ \ddot{\sigma} + 3H \dot{\sigma} = \pi_\perp - \frac{R^{(3)}}{6} $$

$$ \dot{R}^{(3)} = -2(H + \dot{\sigma})R^{(3)} $$

where $\rho = \rho_f + \rho_B$ and $p = p_f + p_B$ are the total energy density and pressure, respectively, $R^{(3)} = 2\kappa e^{-2\alpha - 2\sigma}$ is the three-dimensional Ricci scalar, and

$$ \rho_B = -3\gamma e^{2\alpha - 4\sigma}V^2, \quad p_B = \frac{1}{3}\rho_B, \quad \pi_\perp = \pi_1^\perp = 2\gamma e^{2\alpha - 4\sigma}V^2. $$

The fluid variables are also constrained by the equations

$$ \dot{\rho}_f + 3H(\rho_f + p_f) = 0, $$

$$ \dot{\rho}_B + 3H(\rho_B + p_B) = -6\dot{\sigma}\pi_\perp, $$

as follows from the Bianchi identities. Note that, from the positiveness of $\rho_B$, one requires $\gamma < 0$.

In order to analyze the linear stability of the system it is convenient to work with the following dimensionless variables

$$ \Omega_f = \frac{\rho_f}{3H^2}, \quad \Omega_B = \frac{\rho_B}{3H^2}, \quad \Omega_\kappa = -\frac{R^{(3)}}{6H^2}, \quad \Sigma = \frac{\dot{\sigma}}{H}. $$

Note that they are not all independent, but must obey the constraint $\Omega_f + \Omega_B + \Omega_\kappa + \Sigma^2 = 1$. Eliminating $\Omega_f$ in terms of the other variables, the dynamical system becomes

$$ \frac{d\Omega_B}{d\alpha} = 2\Omega_B \left[ 3\Sigma^2 + 2\Sigma + \Omega_B + \Omega_\kappa - 1 + \frac{3}{2}(1 + \omega)(1 - \Sigma^2 - \Omega_B - \Omega_\kappa) \right], $$

$$ \frac{d\Omega_\kappa}{d\alpha} = 2\Omega_\kappa \left[ 3\Sigma^2 - \Sigma + \Omega_B + \Omega_\kappa - 1 + \frac{3}{2}(1 + \omega)(1 - \Sigma^2 - \Omega_B - \Omega_\kappa) \right], $$

$$ \frac{d\Sigma}{d\alpha} = -2\Omega_B + \Omega_\kappa + \Sigma \left[ 3(\Sigma^2 - 1) + \Omega_B + \Omega_\kappa + \frac{3}{2}(1 + \omega)(1 - \Sigma^2 - \Omega_B - \Omega_\kappa) \right], $$

where we have assumed that the perfect fluid has an equation of state $\omega = p_f/\rho_f$.

The above system is quite general and can be applied to different scenarios with different perfect fluids. We are particularly interested to see whether an inflationary (more precisely, de Sitter) phase would produce shear-free expansion, which would then determine the metric during the following radiation and matter dominated eras, possibly affecting the formation of CMB anisotropies. We thus consider the case with $\omega = -1$, for which the point

$$ (\Sigma, \Omega_B, \Omega_\kappa) = (0, 1/3, 2/3), $$

where
is a stable fixed point \(^\text{18} \) – see Fig. (1). It is worth mentioning that, since by definition \( \Omega_\kappa \propto -\kappa \) (see (11)), the above result imply that only the BIII geometry is dynamically stable. However, this does not exclude the possibility that the KS geometry leads to a stable fixed points when couplings between the fluids are allowed. For a more sophisticated dynamical analysis in KS spacetimes, see.\(^ \text{19} \)

Fig. 1. Attractor behavior of the shear-free solution on the planes \( \Omega_B \times \Sigma \) (left) and \( \Omega_\kappa \times \Sigma \) (right).

Thus, cosmologies with metric (1) possess an attractor solution in which the universe, although anisotropic, will expand isotropically. Since the metric has only one scale factor, it can be brought to a conformally static form

\[
ds^2 = a^2(\eta) \left[ -d\eta^2 + \gamma_{ab}(x^c)dx^a dx^b + dz^2 \right], \tag{13}
\]

which implies that the CMB will be perfectly isotropic.\(^ \text{20,21} \) Moreover, provided that the stress tensor “balances” the spatial curvature\(^ \text{d} \) – see Eq. (6) – the shear decays and the background equations in conformal time will be given by

\[
H^2 = \frac{1}{3} a^2 \rho - \frac{\kappa}{a^2}, \tag{14}

H' = -\frac{1}{6} a^2 (\rho + 3p). \tag{15}
\]

These are exactly the Friedmann-Robertson-Walker (FRW) equations of universes with spatial curvature. Evidently, the anisotropy in the spatial curvature will lead to new signatures at the perturbative level, which we now explore.

\(^ \text{d} \)Rigorously speaking, the stress tensor has to equal the electric part of the Weyl tensor in these spacetimes.\(^ \text{7} \)
2.2. Perturbation Theory

As far as the machinery of gauge-invariant and linear cosmological perturbations is concerned, perturbation theory in anisotropic spacetimes\textsuperscript{22–25} is essentially the same as its isotropic cousin.\textsuperscript{26} Nonetheless, when one departs from FRW universes, there are three main aspects which require attention. These are:

(1) the dynamics of the background spacetime;
(2) the geometry of constant-time hypersurfaces;
(3) the determination of spatial eigenfunctions.

Thus, for example, perturbation theory in Bianchi I spacetimes is directly affected by item (1) since, at the background level, the anisotropy of expansion couples perturbative modes through the background shear, even if they are decoupled at some initial time.\textsuperscript{22,23} Consequently, one cannot track the evolution of each perturbative mode independently, which considerably complicates the analysis. SF models are obviously exempt from this difficulty, and in this regard they are much simpler to perturb. On the other hand, SF are directly affected by items (2) and (3), from which most of their distinctive observational signatures follows.

SF space-times are orthogonal models, which means that they admit a timelike vector field everywhere orthogonal to the spatial hypersurfaces. Thus, say, metric perturbations can be naturally split into time-time (or scalars, e.g. $\delta g_{00}$), space-time (or 3-vectors, e.g., $\delta g_{0i}$), and space-space components (or 3-tensors, e.g. $\delta g_{ij}$). Next, 3-vectors and 3-tensors can be further decomposed into their irreducible pieces, according to the symmetries of the spatial hypersurfaces where they live. In the case of isotropic FRW spacetimes, the spatial sections are invariant under the SO(3) group, which leads to the standard Scalar-Vector-Tensor (SVT) decomposition of perturbations.\textsuperscript{26,27} As we have seen, the spatial hypersurfaces of SF universes is a product manifold, and we thus implement an irreducible decomposition according to the symmetries of each submanifold.

| Space-time | Spacetime splitting | Irreducible pieces |
|-----------|---------------------|--------------------|
| FRW       | 1+(3)               | Scalar + Vector + Tensor |
| BII/KS    | 1+(2+1)             | Scalar + Vector + Scalar |

Thus, in the real line $\mathbb{R}$ one can only have scalars, while in the two-dimensional submanifold of metric (13), vectors and tensors are decomposed as:

\begin{align}
V^a &= D^a V + \tilde{V}^a, \\
\tilde{h}^{ab} &= 2S_{\gamma}^{\gamma ab} + D^a D^b U + D^a \tilde{E}^b,
\end{align}

where $(V, S, U)$ are scalars and $(\tilde{V}^a, \tilde{E}^a)$ are transverse vectors: $D^a \tilde{V}_a = 0 = D^a \tilde{E}_a$. Note that, in two dimensions, transverse vectors are essentially scalars. Moreover,
there are no transverse and traceless tensors in two dimensions. Of course, this does not mean that there are no gravitational waves, but rather that each polarization of the wave come from a different perturbative sector. However, this does imply that each polarization will have its own dynamics. Incidentally, this is a general feature of anisotropic spacetimes which can be relevant to the recently founded era of gravitational wave astronomy.

The last step to the implementation of perturbation theory is the determination of a complete set of spatial eigenfunctions. This step cannot be overlooked since, without it, crucial cosmological observables, like the primordial power spectrum, cannot be computed. The eigenfunctions $\phi_q$ that we are looking for are the solutions of the eigenvalue problem

$$\frac{1}{\sqrt{h}} \partial_i \left( \sqrt{h} \partial^i \phi_q \right) = -q^2 \phi_q,$$

where $h_{ij}$ is the metric on spatial sections of (13). In the cylindrical coordinates of metric (1), these eigenfunctions can be found by means of a simple separation of variables. They are, up to a normalization factor, given by:

$$\phi_q(x) \propto \begin{cases} P_{m-1/2,i}^m(\cosh \rho) e^{im\varphi} e^{ikz}, & \text{(BIII)} \\ P_{\ell}^m(\cos \rho) e^{im\varphi} e^{ikz}, & \text{(KS)} \end{cases}$$

The eigenvalues $\ell$, $m$ and $k$ are related to the wave-vector $q$ through the following dispersion relations:

$$q^2 = \begin{cases} \ell^2 + k^2 + 1/4, & \text{(BIII)} \\ (\ell + 1/2)^2 + k^2 - 1/4. & \text{(KS)} \end{cases}$$

Some general remarks about these results are in order: first, it is straightforward to show that, in the limit of small distances and large $\ell$, both eigenfunctions become

$$\phi_q(x) \propto \ell^{1/2} J_m(\ell \rho) e^{im\varphi} e^{ikz}.$$

As expected, these are the spatial eigenfunctions of the Laplacian on a flat FRW universe. Second, note that the eigenvalues $m$ do not appear explicitly in the dispersion relations (20), which reflects the residual rotational symmetry of (13). Finally, we note that in both BIII and KS cases there is an intrinsic lower limit to the “Fourier” mode $q$. In fact, the largest wave in BIII has $\ell = 0 = k$, whereas in KS it has $\ell = k = 0$. In both cases, thus

$$q \geq \frac{1}{|\text{curvature scale}|}.$$  

In other words, there can’t be a wave larger than the curvature scale in such universes. This feature offers an interesting observational window through the Grishchuk-Zel’dovich effect.

Note that $(\rho, \ell) \in \mathbb{R}^+$ in $H^2$, whereas $\rho \in [0, \pi)$ and $\ell \in \mathbb{N}$ in $S^2$. The case $\ell = 0$ in KS corresponds to a monopole, and can thus be neglected.
From the above recipes, it is a straightforward but rather tedious task to parameterize metric and matter perturbations, construct gauge-invariant variables and linearize Einstein equations. The reader interested in the details can check Ref. 29.

We now comment on the observational signatures that SF models would imprint in the CMB.

2.3. Observational signatures

In order to discuss observational signatures of SF models, let us thus focus on scalar perturbations. Assuming an inflationary period, the perturbations to the metric are found to be:

\[ ds^2 = a^2(\eta)[-(1 + 2\Phi)d\eta^2 + (1 - 2\Phi)dx_idx_j], \]

where \( dx_idx_j = \gamma_{ab}dx_adx_b + dz^2 \) and where \( \Phi \) is the only gauge-invariant scalar metric perturbation. Interestingly, the above line element is identical to the one we find from scalar metric perturbations in an inflationary FRW universe. Thus, from the dynamical point of view, \( \Phi \) has the same time evolution as the Newtonian gravitational of standard perturbation theory. A corollary of the above result is that, since the Sachs-Wolfe (SW) effect is purely kinematic, it has the same shape in SF universes. That is

\[ \Delta T(\hat{n}) = \frac{1}{3}\Phi(x, \eta). \]

Of course, SW effect will still lead to different signatures, since in the above relation \( x \) are the coordinates of a point in a manifold with anisotropic spatial curvature. In order see how these differences comes about, we can compute the two-point correlation function (2pcf),

\[
C(\hat{n}, \hat{n}') = \langle \Delta T(\hat{n})\Delta T(\hat{n}') \rangle, \]

under the assumption that the gravitational potential that we measure today is one realization of a Gaussian random variable. In “Fourier” space, this is implemented by the relation

\[
\langle \Phi(q)\Phi^*(q') \rangle = P(q) \times \begin{cases} \frac{(\tanh \ell \pi)^{-1}}{2\pi} \delta_{mm'}\delta(\ell - \ell')\delta(k - k') & \text{(BIII)}, \\ \delta_{mm'}\delta(\ell\ell')\delta(k - k') & \text{(KS)}, \end{cases}
\]

where \( P(q) \) is the (anisotropic) primordial power spectrum. Expanding \( \Phi \) in the eigenfunctions (19) and using some identities of Bessel functions, one can easily show that, in the BIII case

\[
C(\hat{n}, \hat{n}') = \frac{1}{(6\pi)^2} \sum_{\ell=1}^{\infty} \int_{-\ell}^{\ell} d\ell \int_{-\infty}^{\infty} dk P(\ell, k)P_{1/2+1}(\cosh \Delta\rho)e^{ik\Delta z}, \quad \text{(BIII)}
\]

\[
\sum_{\ell=1}^{\infty} (\ell + \frac{1}{2}) \int_{-\infty}^{+\infty} dk P(\ell, k)P_{2}(\cosh \Delta\rho)e^{ik\Delta z}, \quad \text{(KS)}
\]

where \( \cos(h)\Delta\rho = \cos(h)\rho \cos(h)\rho' \pm \sin(h)\rho \sin(h)\rho' \cos(\varphi - \varphi') \), with the minus\plus sign corresponding to the \( \cosh \Delta\rho \text{ \& } \cos \Delta\rho \) cases, respectively. We remind the reader that, in deriving these expressions, we have used the fact the \( P(q) = P(\ell, k) \) cannot depend on the eigenvalue \( m \), since the latter is associated with an angular variable of a rotationally symmetric (sub) space. It is important to
compare the above two-point functions with the same quantity in FRW universes. In cylindrical coordinates, the latter is

\[ C(\hat{n}, \hat{n}') = \frac{1}{(6\pi)^2} \int_0^\infty \ell d\ell \int_{-\infty}^{+\infty} dk \mathcal{P}(q) J_0(\ell \Delta \rho) e^{ik\Delta z}, \]  

(26)

where \( \Delta \rho^2 = \rho^2 + \rho'^2 - 2\rho \rho' \cos(\varphi - \varphi') \). A further integration\(^8\) reveals that, in this case, the 2pcf is only a function of \( \vartheta = \arccos(\hat{n} \cdot \hat{n}') \), as one expects from isotropy. However, we prefer to keep Eq. (26) in its present form to compare with Eqs. (25). There are two main differences between them. The first is obviously in the anisotropy of the primordial power spectrum \( \mathcal{P} \), which in the SF cases is a function of the modes \( \ell \) and \( k \). Rigorously speaking, one should fix \( \mathcal{P}(\ell, k) \) by canonically quantizing the inflaton perturbations in SF universes. However, since we are only interested in the general signatures of SF models, we can take a simpler route by demanding that the anisotropic 2pcf recovers \( C(\vartheta) \) in the limit of coincident points:

\[ C(\hat{n}, \hat{n}') \big|_{\hat{n} = \hat{n}'} = C(\vartheta) \]  

(27)

which completely fix \( \mathcal{P}(\ell, k) \) in terms of the isotropic power spectrum \( \mathcal{P}(q) \). The second difference between (26) and (25) is that the function \( J_0 \) in the kernel of the integral gets replaced by \( \mathcal{P}_\mu^\nu \), the latter being a function of the distance between two points in a curved two-dimensional space. As we know, CMB data suggest that the observable universe is spatially flat.\(^1\) Thus, in the light of current observations, shear-free models should be considered in the limit of large spatial curvature, where \( \mathcal{P}_\mu^\nu \to J_0 \). Evidently, we are interested in the next-to-leading order corrections, since those will lead to statistical anisotropies in CMB maps. Schematically, thus, we can write the large-curvature limit of (25) as:\(^3\)

\[ C(\hat{n}, \hat{n}') = C(\vartheta) \pm \mathcal{F}(\hat{n}, \hat{n}') \]  

(28)

where the plus\minus signs correspond to BIII\KS cases, respectively, and the function \( \mathcal{F} \) is of second order in the quantity \( (\Delta \eta/L) \), that is, the horizon distance \( \Delta \eta \) in units of the curvature scale \( L \). Since the latter is not known, this ratio appears as a free parameter in the model. One interesting aspect of the function \( \mathcal{F} \) is that it only couples multipoles of equal parities, which is a direct consequence of parity invariance of metric (1). In order to have an idea of the shape of this function, we define its angular power spectrum in terms of off-diagonal terms of the CMB covariance matrix:

\[ \mathcal{F}_{\ell+\Delta \ell} \equiv \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |\langle a_{\ell m} a_{\ell+\Delta \ell, m} \rangle|, \]  

(29)

which can in turn be directly computed from (28). The strongest signal is expected to come from the closest neighbors in multipole space. Since multipoles with separation \( \Delta \ell = 1 \) give zero signal due to parity, the next effect results from neighbors

\(^8\)If we let \( \ell = q \sin \psi \) and \( k = q \cos \psi \), the integral in \( \psi \) can be evaluated analytically, leading to the famous expression for the 2pcf in real-space. See Ref. 35.
with $\Delta \ell = 2$. We show in Figure (2) a plot of this quantity for two arbitrary choices of the parameter $\Delta \eta/L$. It is interesting to note that this function grows smoothly with growing $\ell$ – a feature which, in observational terms, might alleviate the cosmic variance of very low multipoles.

3. Conclusions and Perspectives

Unprecedented progress in observational cosmology compel us to explore theoretical possibilities beyond the simple scenario of an isotropic and spatially flat universe. However, since the framework of a vanilla $\Lambda$CDM cosmology becomes stronger after each new observational mission, it is important to develop models with new degrees of freedom that do not spoil known observational results.

In this work we have explored cosmological models which, starting from a general anisotropic configuration, are rapidly attracted to an FRW-like model, while still being anisotropic at the level of its spatial curvature. Although we have focused on the choice of a specific anisotropic source of matter to obtain this feature, the simplicity of our model suggest that it might hold in more general scenarios. Since the background dynamics of these models is exactly the one of a curved FRW universe, it represents an interesting counter example to our (unjustified) intuition that the isotropy of CMB requires an equally isotropic universe.

By developing a proper decomposition of perturbative modes in spaces with anisotropic curvature, we have shown that linear perturbation theory in shear-free models is perfectly doable, and leads to interesting phenomenological consequences – one of which is the existence of a geometrical upper bound to the wavelength of cosmological perturbations. Assuming that the curvature of the universe lurks just
beyond the current horizon radius, we have computed off-diagonal signatures that an anisotropically curved geometry would imprint on the temperature spectrum of CMB. Such effects could be responsible to some of the known CMB statistical anomalies, although further investigation is required to clarify this issue.

Finally, we comment on an interesting possibility to extend the results of this work. A general prediction of anisotropic cosmological metrics, and of shear-free metrics in particular, is that each polarization of gravitational waves should have its own dynamics. This will be an interesting signature to look for in the forthcoming measurements of primordial gravitational waves\(^b\). Moreover, due to the specificities of the eigenfunctions and mode decomposition in spaces with anisotropic curvature, we also expect the tensor-to-scalar ratio to be very different in these models.\(^3^6\) Thus, future measurements of the $B$-modes in the CMB polarization maps will offer a new window to constrain both the isotropy and the curvature of the universe.

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\(^b\)http://www.core-mission.org/
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