Orbital orientation evolution of massive binary black holes at the centres of non-spherical galaxies

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10 October 2013

ABSTRACT

At the centre of a spherical and kinematically isotropic galaxy, the orientation of a massive binary black hole (BBH) orbit (i.e., the direction of the BBH orbital angular momentum) undergoes a random walk. If the stars in a spherical system have a non-zero total angular momentum, the BBH orbital orientation evolves towards aligning with the total stellar angular momentum direction. In this paper, we show that a triaxial galaxy has an alignment-erasing effect, that is, the alignment of the BBH orientations towards the galaxy rotation axis can be decreased significantly or erased. We also show that in a non-rotating axisymmetric galaxy, the BBH orbital orientation evolves towards the axisymmetric axis and precesses about it in a retrograde direction. Our results provide a step towards understanding the spin orientations of the final merged BH (and hence probable orientation of any jet produced) within its host galaxy, and may help to constrain the recoiling velocity of the merged BH arose from gravitational wave radiation as well.

Key words: black hole physics – galaxies: evolution – galaxies: interactions – galaxies: kinematics and dynamics – galaxies: nuclei

1 INTRODUCTION

In the modern paradigm of hierarchical galaxy formation and evolution, formation of massive binary black holes (BBHs) is a natural consequence of mergers of galaxies with central massive black holes (BHs) (e.g., Begelman et al. 1980; Yu 2002). Study of the BBH orbital evolution and possible observational signals of existing or existed BBHs is one of the important steps to understand the formation and evolution of massive BHs, to answer the possibility of detection of gravitational wave radiation due to the BBH merger (e.g., Hughes 2009; Centrella et al. 2010), and to probe the hierarchical structure formation model (e.g., Volonteri et al. 2003; Micic et al. 2007; Yu et al. 2011). Orbital evolution of BBHs in merged galaxy remnants has been investigated in various aspects (such as evolution timescales, evolution of semi-major axes, eccentricities, and orbital orientations; e.g., Quinlan 1996; Yu 2002; Sesana et al. 2006; Sesana 2010; Merritt 2002). In this paper, we study the orbital orientation evolution of BBHs in purely stellar systems and investigate how the evolution is related to the BBH host galaxy properties such as triaxiality and/or rotation. The orbital orientation is one of the most basic physical properties of a BBH. It can influence the spin magnitude and direction of the merged BH (e.g., Flanagan & Hughes 1998), and the spin direction is believed to determine the direction of a jet launched from the BH inner accretion disk (Bardeen & Petterson 1975; Rees 1978). The BBH orbital orientation can also influence the gravitational wave radiation from the BBH merger, the recoiling velocity of the merged BH arose from the asymmetry of the gravitational wave radiation, and possibly the strength of any electromagnetic signature of the merger (e.g., Hughes 2009; Centrella et al. 2010; Bode et al. 2010, 2012; Bosdanović et al. 2007).

After a galaxy merger, the orbit of a BBH decays in the merged galaxy remnant. In a gas-poor environment, the orbital evolution of a BBH can be divided into several stages, according to the mechanisms that act on the different separation scales to drain its orbital energy and angular momentum. Initially, each BH (with mass denoted by $m_1$ or $m_2$, $m_1 \geq m_2$) sink

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independently towards the centre of the common gravitational potential under the action of dynamical friction, at separation scales ranging from several ten kpc to \( \sim 10 \) or 1 pc. As they migrate inward and form a bound BBH with orbital semimajor axis

\[
a \lesssim r_{\text{inf}} \equiv \frac{GM_1}{\sigma_c^2} \approx 10 \text{ pc} \left( \frac{M_1}{10^8 M_\odot} \right) \left( \frac{\sigma_c}{200 \text{ km s}^{-1}} \right)^{-2},
\]

where \( M_1 = m_1 + m_2 \) and \( \sigma_c \) is the one-dimensional velocity dispersion of the merged galaxy core, it continues to lose energy and angular momentum through dynamical friction. However, the influence of dynamical friction on the BBH orbit becomes less efficient as its orbital period decreases and its orbital velocity increases. After the BBH becomes hard at (Quinlan 1996)

\[
a \sim a_h \equiv \frac{Gm_2}{4\sigma^2} \approx 2.8 \left( \frac{m_2}{10^8 M_\odot} \right) \left( \frac{\sigma_c}{200 \text{ km s}^{-1}} \right)^{-2} \text{ pc},
\]

it loses energy mainly through three-body interactions with low-angular momentum stars passing by its vicinity. Finally, after the BBH orbit decays to some point (\( a \lesssim 10^{-7} \) pc), gravitational radiation becomes the dominant dissipative force (i.e., gravitational radiation stage) to make the BBH lose energy. The BBH at the different stages has different evolution timescales. The slowest evolution stage normally starts at \( a \sim a_h \) and ends at the gravitational radiation stage, and the evolution timescale at the bottleneck depends on how many and how fast low-angular momentum stars are available to interact with the BBH. The non-spherical gravitational potential of galaxies (e.g., highly flattened or triaxial) has been shown to be effective in having stars precessing from high-angular momentum orbits onto low-angular momentum ones so that the bottleneck timescales can be lower than the Hubble time (Yu 2002; see also Khan et al. 2013; Preto et al. 2011; Khan et al. 2013; Berczik et al. 2006). A BBH is also more likely to have merged in low-velocity dispersion ‘power-law’ galaxies (Yu 2002; Zier 2007).

At the BBH evolution bottleneck, each interaction of the BBH with a star passing by its vicinity may cause an exchange of the energy and the angular momentum, and lead to a slight change of the BBH orbital orientation. The slight orientation change of each interaction, denoted by \( \delta \alpha \), may accumulate, as the number of the stars passing by (denoted by \( N \)) increases. In spherical and isotropic systems, the orbital orientation of a BBH evolves like a Brownian motion with the accumulated orientation angle change \( \Delta \alpha \propto \sqrt{N(\delta \alpha^2)} \) (Merritt 2002; Gualandris & Merritt 2007). If the stellar system is rotating and the galactic core has a non-zero total angular momentum, Gualandris et al. (2012) find that the orbital orientation of a BBH evolves toward the direction of the total angular momentum of the stars. Observations reveal that realistic galactic spheroids are likely to be triaxial (e.g., Ryden 1992; Bakh & Statler 2000; Kimm & Yu 2007; Fasano et al. 2010). Some recent numerical simulations (e.g., Gualandris & Merritt 2012; Khan et al. 2013; Preto et al. 2011) also show that the stellar remnant of galaxy mergers is triaxial and rotating. In this paper, we show that the non-spherical gravitational potential of a stellar system may affect the kinematic distribution of the stars passing by the vicinity of the BBH. We investigate the orientation evolution of hard BBHs in gas-poor non-spherical stellar systems including rotating ones, and show how their evolution is affected by the kinematic distributions of the stars passing by.

The paper is organized as follows. We describe our model and method in Section 2. We present the simulated orbital orientation evolution of hard BBHs in different stellar systems (spherical, axisymmetric, triaxial, rotating) in Section 3. As a check for the method, we apply it first to spherical systems in Section 3.1 and find that the results are consistent with the analytical result and previous work well. Then we apply the method to non-spherical galaxies. We present the kinematic distribution of the stars passing by the vicinity of the BBH in Section 3.2. By generating a sample of BBHs with various initial orbital orientations, we illustrate the tendency of the BBH orientation evolution and distributions after the BBHs interact with a large number of the stars in Sections 3.3 and 3.4. The results obtained for non-spherical galaxies are presented along with the comparison with those obtained for spherical cases. Discussion is given in Section 3.5 and a summary of the conclusion is in Section 4.

## 2 Model and Computational Method

In this section, we first present the basic physical equations on the dynamical evolution of the system to be studied, and then describe the method to solve the equations. To expedite the calculations, we divide the evolution of the system into two stages and present them in detail in Sections 2.1 and 2.2, respectively.

Consider that a massive hard BBH with masses \( m_1 \) and \( m_2 \) is located in the centre of a gas-poor galaxy. We denote the galaxy gravitational potential field contributed by stars at position \( r \) by \( \Phi_G(r) \). The BBH interacts with stars passing by its vicinity through three-body gravitational interactions, which can be described through the following equations of motion:

\[
\begin{align*}
\ddot{r}_1 &= \frac{Gm_2}{|r_2 - r_1|^3} (r_2 - r_1) + \frac{Gm_1}{|r_2 - r_1|^3} (r_2 - r_1), \\
\ddot{r}_2 &= \frac{Gm_1}{|r_2 - r_1|^3} (r_2 - r_1) + \frac{Gm_2}{|r_2 - r_1|^3} (r_2 - r_1), \\
\ddot{r}_* &= -\nabla \Phi(r_*),
\end{align*}
\]
where \( \mathbf{r}_1, \mathbf{r}_2, \) and \( \mathbf{r}_* \) are the position vectors of the two BHs and the star, respectively,
\[
\Phi(\mathbf{r}_*) = -\frac{GM_1}{|\mathbf{r}_* - \mathbf{r}_1|} - \frac{GM_2}{|\mathbf{r}_* - \mathbf{r}_2|} + \Phi_G(\mathbf{r}_*),
\]
and the Newtonian mechanics is used. Note that the galactic potential field \( \Phi_G(\mathbf{r}) \) is included in the motion of the star, as the stars interacting with the hard BBH may come from a large distance in the galaxy and the stellar motion is affected by the galactic potential along the long ways to/from the hard BBH. The centre of the galactic potential, i.e., the minimum point of the potential, is always put at rest at the origin of the coordinate system in the calculation. The centre of the mass of the BBH is initially put at the centre of the galactic potential; and the translational Brownian motion of the BBH is ignored.

After each interaction of the BBH with a star, the BBH generally receives an energy loss and angular momentum change, which can be obtained by following their motion and numerically solving the differential equations \(6\)-\(9\) together. The total change of the BBH is an accumulative effect of the interactions. However, the numerical calculation to follow the stellar motion may be time-consuming, as the moving time of a star in the galaxy may be much longer than the orbital period of the BBH and the rotational motion of the BBH described in Equations \(3\) and \(4\) limits the time steps used in the calculation. Noting that the galactic potential \( \Phi_G \) dominates the stellar motion when the stars are significantly far away from the BBH, the BBH can be simplified as one object with mass \( M_* = m_1 + m_2 \). Thus, the combined potential in Equation \(6\) can be simplified as
\[
\Phi(\mathbf{r}_*) = -\frac{GM_*}{r_*} + \Phi_G(\mathbf{r}_*), \quad r_* = |\mathbf{r}_*|,
\]
which is independent of \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \). To expedite the calculations, we divide the stellar motion and our calculations into the following two stages:

- stellar precessing stage: the stellar orbits precess in the combined gravitational potential field described by Equations \(5\) and \(\Phi\), and we numerically trace the stellar orbits obtain the kinematic distributions of the stars that can come to a distance \( \lesssim a \) to the galactic centre;
- three-body scattering stage: we calculate the three-body scattering processes of the BBH with the stars that can come to its vicinity by using Equations \(3\) - \(\Phi\). The kinematic distribution of the stars coming to the BBH vicinity obtained in the above stellar precessing stage is used as the initial condition of the three-body interactions.

The galactic potential is spherical if it can be described in the form of \( \Phi_G(\mathbf{r}) \) \((r = |\mathbf{r}|)\) and triaxial if it can be described in the form of \( \Phi_G(x^2 + y^2/\xi^2 + z^2/\zeta^2) \) \((\zeta < \xi < 1)\). In our model, we use the following logarithmic potential as an example for triaxial galaxies:
\[
\Phi_G = \sigma_t^2 \log(R_c^2 + x^2 + y^2/\xi^2 + z^2/\zeta^2),
\]
where \( R_c \) represents the core radius. The \( \sigma_t \) can be set through the tight empirical correlation between the BH mass and the galactic velocity dispersion (e.g., Tremaine et al. 2002; Ferrarese & Ford 2005; Gültekin et al. 2009; McConnell & Ma 2013), and in this paper we adopted the following relation:
\[
M_* = 1.66 \times 10^8 M_\odot \left(\frac{\sigma_t}{200 \text{ km s}^{-1}}\right)^{4.86}.
\]
Although a self-consistent model of a realistic stellar distribution around a massive BH is not used in this paper, the above logarithmic potential used should be sufficient to display the effects of the triaxiality qualitatively.

### 2.1 Stellar precessing stage

We describe the dynamics of a star in the phase space of its specific energy and specific angular momentum \((E, J)\). In spherical potentials, the angular momenta of stars are conserved, and the stars that can come to the vicinity of the BBH \((\sim a\) from the centre) have the orbital angular momenta \( J < J_{lc} \), where \( J_{lc} \approx \sqrt{2GM_*a} \) and the corresponding region in the phase space \(a \approx \)
is called the “loss cone”. In axisymmetric and triaxial galaxies, there exist centrophilic orbits such as box orbits, which pass arbitrarily close to the centre and have low angular momentum, as well as centrophobic orbits such as loop orbits, which avoid centre and have high angular momentum. Here, we introduce $J_c$ to mark the transition from centrophilic ($J \lesssim J_c$) to centrophobic ($J \gtrsim J_c$) orbits. Stars on centrophobic orbits with $J < J_c$ can process into the loss cone. In non-spherical galaxies, the number of stars available to interact with the BBH can be large, and it has been shown that the BBH bottleneck evolution timescale can be decreased significantly (Yu 2002; see also Pretto et al. 2011; Khan et al. 2011).

We use Monte-Carlo simulations to obtain the characteristic angular momentum $J_c$ and the kinematic distribution of the stars that can come to the vicinity of the BBH. In our simulations, the initial kinematic settings for the stars are set as follows.

- Given the specific energy $E$ of a star, its specific angular momentum $J$ is randomly generated so that $J^2$ is uniformly distributed within the range $[0, J_c^2]$, where $J_c$ is the specific angular momentum of a circular orbit at energy $E$ (cf., Eq. 4.288 in Binney & Tremaine 2008, where the difference in stellar orbital periods with different $J$ is ignored).
- The test particle is initially put at its apocentre with zero radial velocity. Given its $(E,J)$, its apocentre distance to the central BH $r$ and velocity $v$ can be determined by approximating the potential as spherical through setting $\xi = \zeta = 1$ in the triaxial potential. The relative error of the initial $r$ and $v$ caused by approximating the potential as spherical is negligible for the purpose of this paper, as the orbital energy difference for the same initial $r$ and $v$ between the spherical and the triaxial/axisymmetric systems is at the most the galactic potential difference between the positions $r_\ast = (0,0,r)$ and $(0,0,r/\zeta)$. The initial direction of the orbital angular momentum of the star is parameterized by $(\theta_\ast, \phi_\ast)$, where $\theta_\ast$ is the angle between the angular momentum and the $z$-axis and $\phi_\ast$ is the azimuthal angle of the angular momentum direction in the $x-y$ plane. The directions are generated isotropically and randomly. The argument of its apocentre $\psi_\ast$ is generated randomly within $[0, 2\pi]$. Thus, the position and velocity vectors of stars can be derived and serve as the initial conditions for Equation [1].

We use the above initial settings and Equations [3] and [4] to trace the motion of the stars. The calculation for each star is terminated once its $r \lesssim a$ (centrophilic orbits) or its traveling time is longer than $2 \times 10^5 G M_\ast/\sigma_\ast^2 \simeq 1.4 \times 10^9 \text{yr} (M_\ast/10^9 M_\odot)^{0.38}$ (where Eq. 3 is used). We select those stars for which the calculation is terminated due to $r \lesssim a$, and obtain their kinematic distributions when they pass through $r \simeq r_{\text{inf}}$ at its last orbit of the calculation. The kinematic distributions of these stars are used as the initial conditions in the calculation for the three-body scattering stage of the stars interacting with the BBH. The $J_\ast$ can be determined from the initial angular momentum distribution of the stars (see Fig. 2 later).

### 2.2 Three-body scattering stage

Given a BBH with total mass $M_\ast$, mass ratio $q \equiv m_2/m_1$, we set its semi-major axis $a = a_\ast$. The unit orbital angular momentum of the BBH is denoted by $l_\ast = (l_\ast, b_\ast, l_\ast)$, and the orientation direction can also be parameterized by angles $(\theta_\ast, \phi_\ast)$, similarly as the angles $(\theta_\ast, \phi_\ast)$ described above for the stellar angular momentum. We randomly select a star from the kinematic distribution obtained in the stellar precessing stage and use Equations [3]–[5] to trace its three-body scattering processes with the BBH. The calculation is terminated when the distance of the star has $r \gtrsim 2r_{\text{inf}} \simeq 8(1 + q^{-1})a_\ast$ or its traveling time in the three-body scattering stage is longer than $10 G M_\ast/\sigma_\ast^2$, and the BBH orbital orientation $(\theta_\ast, \phi_\ast)$ is recorded and used as the initial condition in the scattering process with next star. The calculation is reiterated by scattering with $N$ number of stars. The number of the scattered stars is related to the BBH orbital decay by the following equation (see Eq. 12 in Yu 2002):

$$N \simeq 0.32 \frac{M_\ast}{m_\ast} \ln \left( \frac{a_\ast}{a} \right).$$

For simplicity and saving the calculation time, the semi-major axis and the eccentricity of the BBH $(a,e)$ is fixed during the reiterations; and the translational Brownian motion of the BBH induced after the scattering process with each star is ignored, with the center of mass of the BBH being reset to the centre of the galactic potential at the beginning of the scattering with the next star. We also assume that the stars have an identical stellar mass $m_\ast(=10^{-4} M_\ast)$. All of the simplifications made above do not affect our main conclusions (see discussion in Section 3.5).

Regarding the conditions on the traveling time that are set to terminate the numerical calculations in the above two stages, we have tested that the results below are not affected much if the time lengths are increased by one order of magnitude. With these settings in natural units of the dynamical system, the results below do not depend on the detailed values of $M_\ast$.

In the calculations, the motion of the stars and the BBH is traced by integrating the differential equations of their motion with an explicit Runge-Kutta method of order 8(5,3) (Hairer et al. 1993; Dormand & Prince 1978). In the three-body scattering stage, the total energy of the BBH and the star is conserved; and to ensure the accuracy of numerical calculations, the change in their total energy due to numerical errors must be much smaller than the change of the BBH energy due to each interaction with a star passing by, so that the change of the BBH energy obtained from the calculations is not caused
by numerical errors. The absolute relative error of the total energy achieved in our calculations is lower than \( \sim 10^{-9} \) for each three-body encounter, which is accurate enough, as the relative change in the BBH energy due to each interaction with a passing-by star is roughly order of \( m_s/m_1 \). We have also checked that our results are not affected much even by decreasing the accuracy by one order of magnitude. As to be mentioned in Section 3, the validity of our calculation methods is further supported by Figure 4 below.

3 RESULTS

3.1 Simple model test: spherical galaxies

We apply the model and the numerical method described in Section 2 to spherical galaxies. The obtained BBH orientation evolution serves as our model test, and we find that they are consistent with theoretical expectation well.

In our calculations for the example case, the related parameters are set as follows: \( \xi = \zeta = 1 \), \( R_e = 4r_{\text{inf}} \), the BBH mass ratio \( q \equiv m_2/m_1 = 0.01, 0.1, 1 \), \( m_s = 10^{-4}M_\odot \). The semi-major axis of the BBH is chosen to be \( a_h \), and the eccentricity \( e \) is set to \( 0, 0.1, \ldots, 0.9 \). All the stars are set to have the same specific energy (e.g., \( E = \Phi(10r_{\text{inf}}) \) here), and the results are not sensitive to the detailed value of \( E \) if \( E \) is high enough. In one hardening time \( t_h \equiv |a/\langle da/dt\rangle| \), the number of the scattered stars is \( N \sim 0.32\langle M_\odot/m_s \rangle \sim 320 \) (see Eq. 10). We simulate the orientation changes of the BBH within one hardening time. For each set of the BBH mass ratio and eccentricity, we do the Monte-Carlo simulation for 100 times and show the root mean square (rms) of the orientation change in Figure 4. As seen from Figure 4, our calculated BBH orientation changes and their dependence on \( q \) and \( e \) (especially at low \( e \)) is generally well consistent with the following analytical expectation (Gualandris & Merritt 2007):

\[
\Delta \alpha \sim q^{-1/2} \left( \frac{m_s}{M_\odot} \right)^{1/2} (1 - e^2)^{-1/2},
\]

although they have a relatively large deviation at small \( q \) and high \( e \). The relatively large deviation are unlikely to be caused by numerical errors in the calculations, as they do not change much by changing the numerical accuracy by one order of magnitude; and they are more likely to be due to some approximation in deriving the analytical formula. We find that the values of \( \Delta \alpha \) of the 100 BBHs scatters largely at small \( q \) and high \( e \), and the medians of the \( \Delta \alpha \) distributions are consistent with Equation (11) better.

For convenience, the parameter sets used in Figure 4 and some other figures below are listed in Table 1.

3.2 Dynamical distribution of stars that can precess onto the loss cone in non-spherical galaxies

3.2.1 Triaxial and axisymmetric galaxies

We use the model and the numerical method described in Section 2 to obtain the kinematic distribution of the stars that can precess onto the loss cone in triaxial galaxies. We illustrate our calculation results in Figures 5 and 6 where \( q \), \( R_e = 4r_{\text{inf}} \), and \( (\xi, \zeta) = (0.9, 0.8) \). The values of \( (\xi, \zeta) \) fall well into the observational distribution of the axis ratios of elliptical galaxies and brightest cluster galaxies (e.g., Pasano et al. 2011; Kimm & Yi 2007; Ryden 1992), where we note that the axis ratios used here are for the isopotential shape, not for the intrinsic isophotal shape of a galaxy, and the flattening in the potential is roughly a third of that in the density distribution (see eq. 2.72b in Binney & Tremaine 2008).

Figure 5(a) shows the cumulative distribution of the initial specific angular momenta of the stars, where different curves represent different initial energy of the stars. In the panel, the drop-off at the high-angular momentum end characterizes the transition of centrophobic orbits to centrophilic orbits, as \( J_\phi \) mentioned in Section 1. In spherical systems, we have \( J_\phi \simeq J_\zeta \), which is about \( 0.2G M_\odot /\sigma_c^2 \) for the example shown in Figure 2. In triaxial systems, \( J_\phi \) can be much larger than \( J_\zeta \), especially for stars with high energy (or large apocentre distances) whose motion is affected more significantly by the triaxiality of the potential. Figure 5(b) shows the cumulative distribution of the traveling time taken for the stars to precess into the loss cone. The traveling time of a significant fraction of stars is less than several thousand times of \( \tau_{\text{H}} \) for the example case, the related parameters are set as follows: \( J_\phi \simeq J_\zeta \), which is about \( 0.2G M_\odot /\sigma_c^2 \) generally shorter than the Hubble time.

For the stars shown in Figure 6, we show the initial and the final distributions of their orbital orientations reached at the stellar precessing stage in Figure 6 (see Section 2). The stellar orbital orientations are expressed through the unit vector of their angular momenta \( (l_{x,y}, l_{x,z}, l_{y,z}) \), and the angle \( \phi_* \) is defined as the azimuthal angle of the vector projected onto the \( x-y \) plane. The distributions are shown for different specific energies of the stars. In spherical distributions, the final

2 Note that although a full mapping of the orbits in a logarithmic potential needs a large number of orbits, the several thousand orbits used here should be sufficient to map the orbits at the given energy that can pass by the vicinity of the BBH. Even if there exist some delicate orbits which are not contained in these several thousand orbits, the \( 10^4 \) orbits used in Figs. 5-6 or the \( 2 \times 10^4 \) orbits used in Figs. 9 and 10, they should not affect the main results significantly and statistically.
from the case of GM expectation from Equation (11). As seen from this panel, our simulation results are well consistent with Equation (11), although it shows a relatively large deviation at small \( q \) and high \( e \). Panel (b): an example for the absolute relative error of the total energy achieved for each three-body scattering in our calculation, which is lower than \( \sim 10^{-9} \). The horizontal axis represents the traveling time of each star at the three-body scattering stage, in unit of \( GM_*/\sigma_c^2 \). For simplicity, we only show the errors of 100 simulation results randomly chosen from the case of \( q = 0.1 \) and \( e = 0 \). This panel serves as a supplementary support for the high accuracy in our calculation.

Figure 2. (a) Example for the cumulative distribution of the initial angular momenta of the stars that can precess to the vicinity of a BBH in a triaxial galaxy, with parameters \( q = 0.1 \), \( R_c = 4r_{\text{inf}} \), and \((\xi, \zeta) = (0.9, 0.8)\). The angular momentum is in unit of \( GM_*/\sigma_c \). The lower boundary of the \( x \)-axis marks the loss cone of the BBH at its hardening radius in a spherical system, i.e., \( \log(J_{\text{lc}}/GM_*/\sigma_c) \approx \frac{\sqrt{\frac{2}{\pi^2}}}{q} \approx -0.7 \). (b) The cumulative distribution of the traveling time for the stars to precess to the vicinity of the BBH, in unit of \( GM_*/\sigma_c^2 \). Different curves represent the stars with different specific energy. The number of the simulated stars used for each curve is 10000. The Hubble timescale is located at \( 5.2 - 0.38 \log(M_*/10^3 M_\odot) \) in the \( x \)-axis (where Eq. (3) is used). The figure indicates that the number of the stars in a triaxial system that can move into the vicinity of a BBH within a Hubble time can be much more than those in spherical systems, and thus the triaxiality of the stellar system can play an important role in shrinking the BBH orbit. See details in Section 3.2.1.

distributions should be isotropic if the initial distributions are isotropic, due to the conservation of the angular momentum; and the distribution curves should be flat. However, in triaxial systems, as seen from the figure, the curves in many cases are not flat and the anisotropy of stellar distributions are displayed in the following two aspects. First, the probability for stars with different initial orbital orientations to precess into the loss cone is not identical (see dotted line). Second, the distribution of the orbital orientation of the stars when they move to the vicinity of the central BHs is anisotropic (see solid line). For the low energy case \( (E \simeq \Phi(2r_{\text{inf}})) \); top panel), although this anisotropy appears mild, it does exist. For relatively high energy (middle panel), the anisotropy appears more obvious, and a significant fraction of stellar orbits orient close to the short axis \((z\text{-axis})\). The final distributions deviate significantly from the initial ones, which manifests the effect of the torque induced from the triaxial potential. However, for even higher energies or larger apocentre distances \((E \gtrsim \Phi(10r_{\text{inf}}))\); bottom panel), the distribution curves are close to be flat and the anisotropy decreases significantly. Note that the centrophilic
orbits with relatively low energy are regular and those with relatively high energy are stochastic; and the critical energy for that shown in the first row of the corresponding figure.

simulations. In all the figures, we set \(m\) randomly generated given all the other parameters; and in Figs. 6–7, only seven cases are shown by being randomly selected from 100 the scattered stars are those that can pass by the vicinity of the BBH. The last column represents the number of the BBH orientations

| Figure | Shape       | Rotation | \(q\)     | \(e\)     | \(E\)        | \(N\) | BBH orient. |
|--------|-------------|----------|------------|------------|-------------|------|-------------|
| 1      | spherical   | N        | 1,0,1,0,01 | 0–0.9      | \(\Phi(10|\inf\))| 3200 | 100         |
| 2      | triaxial    | N        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(5|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 3(top) | triaxial    | N        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 3(middle) | triaxial  | N        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 3(bottom) | triaxial  | N        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 4(top) | axisymmetric | N        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 4(middle) | axisymmetric | N        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 4(bottom) | axisymmetric | N        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 5(top) | triaxial    | Y        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 5(middle) | triaxial  | Y        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 5(bottom) | triaxial  | Y        | 0.1        | N/A        | \(\Phi(2|\inf\))\(\Phi(3|\inf\))\(\Phi(10|\inf\))| 10000| N/A         |
| 6(a)   | spherical   | N        | 0.1        | 0          | \(\Phi(10|\inf\))| 20000| 7           |
| 6(b)   | spherical   | Y        | 0.1        | 0          | \(\Phi(10|\inf\))| 20000| 7           |
| 7(a)   | triaxial    | N        | 0.1        | 0          | \(\Phi(3|\inf\))| 20000| 7           |
| 7(b)   | triaxial    | Y        | 0.1        | 0          | \(\Phi(3|\inf\))| 20000| 7           |
| 7(c)   | triaxial    | Y        | 0.1        | 0          | \(\Phi(10|\inf\))| 20000| 7           |
| 7(d)   | axisymmetric | N        | 0.1        | 0          | \(\Phi(10|\inf\))| 20000| 7           |
| 8(a)   | spherical   | Y        | 0.1        | 0          | \(\Phi(10|\inf\))| 20000| 1500       |
| 8(b)   | triaxial    | N        | 0.1        | 0          | \(\Phi(10|\inf\))| 20000| 1500       |
| 8(c)   | axisymmetric | N        | 0.1        | 0          | \(\Phi(10|\inf\))| 20000| 1500       |
| 9      | spherical   | Y, N     | 0.1        | 0          | \(\Phi(10|\inf\))| 32000| 100        |
| 9      | axisymmetric | N        | 0.1        | 0          | \(\Phi(10|\inf\))| 32000| 100        |

Table 1. Parameter sets used in some figures. The first column gives the figure number and the corresponding panel (if any). The second column gives the shape of the galactic gravitational potential used: ‘spherical’ corresponds to \((\xi, \zeta) = (1, 1)\), ‘triaxial’ corresponds to \((\xi, \zeta) = (0, 0.8)\) in Equation 8, and ‘axisymmetric’ corresponds to \((\xi, \zeta) = (1, 0.8)\). The third column represents the rotational property set to the galaxy, where the labels ‘Y’ and ‘N’ mean a rotating galaxy \((P_\phi = 7/8)\) and a non-rotating one \((P_\phi = 1/2)\), respectively. The \(q\) and \(e\) are the BBH mass ratio and eccentricity. For Figs. 2–5, the BBH eccentricity is not needed to obtain the kinematic distributions of the stars that can precess into the loss cone during the stellar precession stage, so it is labeled by ‘N/A’ (similar for the last column), but the value of \(q\) is needed to define the size of the loss cone by giving the semimajor axis of a hard BBH. The \(E\) represents the specific energy of the stars used in the simulation. Given the shape and rotational property of a galaxy, the \(N\) represents the number of the scattered stars used given each parameter set of the stellar energy and the BBH configuration in the simulations of each figure, where the scattered stars are those that can pass by the vicinity of the BBH. The last column represents the number of the BBH orientations randomly generated given all the other parameters; and in Figs. 6–7 only seven cases are shown by being randomly selected from 100 simulations. In all the figures, we set \(m_\ast = 10^{-4}M_\odot\) and \(R_c = 4r_{\text{inf}}\). The blank in the table means that the parameter is the same as that shown in the first row of the corresponding figure.

orbits with relatively low energy are regular and those with relatively high energy are stochastic; and the critical energy for the transition of the orbits depends on the detailed shape of the gravitational potential (e.g., the parameters \((R_c, \xi, \zeta)\) for the model used here). The degree of the anisotropy of the stellar kinematic distribution is related with the relative fraction of regular centrophilic orbits and stochastic ones. The angular momenta of regular orbits have their own regular precession patterns and are more likely to indicate the anisotropy in the distribution, in contrast to stochastic orbits. The anisotropy disappears at the high energy because of the dominance of the stochastic orbits.

Figure 4 shows an example for an axisymmetric potential with \((\xi, \zeta) = (1, 0.8)\), where the anisotropy of the distributions does not decrease at high energy. As seen from the middle and the bottom panels, the dotted line has a peak around \(l_z = 0\), as only the stars initially located in the loss wedge \((|J_2| < J_{\text{th}})\) can precess into the loss cone [Magorrian & Tremaine 1999]; and the solid line has peaks around \(l_z = \pm 1\), indicating that most of the stars have final orbital orientations along the axisymmetric axis (i.e., \(\pm z\)-axis here).

3.2.2 Rotational galaxies

As mentioned above, the remnant of a galaxy merger is likely to be rotating (e.g., see also Milosavljević & Merritt 2001). We generate the rotational property of a galaxy in the following way. The initial distribution of \(l_{x,z}\) have different fractions
Figure 3. Histograms of the angular momentum orientation distribution of the stars that can precess into the loss cone during the stellar precession stage in triaxial galaxies. The vertical axis represent the number fraction of the stars normalized by the average fraction in each horizontal axis bin. The parameters $(q, R_c, \xi, \zeta)$ are the same as those in Figure 2. The number of the stars used for the statistics in each row is 10000. The dotted lines represent the initial distributions and the solid lines represent the final distributions. Different rows show the results for the stars with different specific energies. As seen from the figure, in triaxial systems the final kinematic distribution at the stellar precessing stage can be isotropic for high-energy stars (bottom panel) and anisotropic for low or intermediate-energy stars (top or middle panel).

for retrograde orbits and prograde orbits, as done in Gualandris et al. (2012). The probability that the initial $\theta_\ast$ is randomly generated in the range of $[0, \pi/2]$ is denoted by $P_\ast^+$. As seen from the figure, the initial distributions of $l_\ast, z$ (the dot short-dashed curves) displays a strong asymmetry of alignment, and the stars with $l_\ast, z > 0$ outnumber those with $l_\ast, z < 0$ as the initial settings. As in Figure 3, the final distributions also have a dependence on stellar energy. When the energy is low (top panel), the influence of the triaxial potential is mild, and the final distributions follow the initial ones roughly. When the energy is high (bottom panel), the alignment in the final distribution of $l_\ast, z$ decreases significantly; and the triaxial potential erases the alignment of the initial stellar angular momenta because of stochastic orbits of the stars, for which we call the alignment-erasing effect of the triaxiality here. Note that the degree of the galaxy net rotation adopted in this study is probably stronger than that of realistic galaxies (e.g., Gadotti 2011); and the alignment-erasing effect of the triaxiality, of course, should still hold true for galaxies with smaller rotation.

3.3 BBH orbital orientation evolution

We use the method described in Section 2 to simulate the evolution of the BBH orbital orientation in the three-body scattering stage. For simplicity, we set the BBH eccentricity $e = 0$ in the calculation.

We generate the initial orbital orientation of the BBH isotropically and trace their evolution by interacting with a number of stars. We show the BBH orientation evolution as a function of the total mass of the interacting stars (i.e., $N m_\ast$), which is related to the BBH orbital decay by Equation (10). We express the BBH orientation evolution through the evolution of its angular momentum unit vector $(l_{b,x}, l_{b,y}, l_{b,z})$, the angles $\theta_b \equiv \cos^{-1} l_{b,z}$ and $\phi_b$ is the azimuthal angle of the vector projected...
Figure 4. Histograms of the angular momentum orientation distribution of the stars that can precess into the loss cone during the stellar precession stage in axisymmetric galactic potential with \((\xi, \zeta) = (1, 0.8)\). The lines have the same meanings as those in Fig. 3. As seen from the figure, in axisymmetric systems most of the stars have final orbital orientations along the axisymmetric axis (i.e., \(\pm z\)-axis).

Figure 5. Histograms of the angular momentum orientation distribution of the stars that can precess into the loss cone during the stellar precession stage in triaxial galaxies with rotation \(P_\phi = 7/8\). The initial distributions (dotted lines) are higher at \(l_{s,z} > 0\) than that at \(l_{s,z} > 0\), which indicates that the system has a net rotation. As seen from the final distribution in the bottom panel (solid line), the net rotation of high-energy stars is erased in triaxial galaxies.
onto the $x$-$y$ plane, and $\Delta \alpha$ is the angle of the orientation deviated from its initial angular momentum direction. The results are shown in Figures 6 and 7. In each figure, different curves display the results for different initial BBH orientations.

### 3.3.1 Spherical galaxies

Figure 6(a) shows the BBH orbital orientation evolution in spherical galaxies without rotation, and Figure 6(b) shows the results in those galaxies with rotation $P_+ = 7/8$. As seen from Figure 6(b), $\theta_b$ evolves toward 0, i.e., all of the orbital planes are reoriented toward the direction of the total stellar angular momentum, which is consistent with the result obtained by Gualandris et al. (2012). The BBH orientation change $\Delta \alpha$ is larger in panel (b) than that in panel (a).

### 3.3.2 Non-spherical galaxies

In Section 3.2, we find that the kinematic distribution of the stars that can come to the BBH vicinity can be anisotropic at the end of the stellar precessing stage in non-spherical galaxies, and the anisotropy depends on the stellar energy.
In triaxial galaxies, the distributions can be significantly anisotropic at intermediate energies, but nearly isotropic at high energies. The value of the energy where the anisotropy is significant depends on the non-spherical properties of galaxies (e.g., triaxiality). In reality, the stars that can come to the BBH vicinity have a distribution in energy, and the value of the energy where the stars are the most numerously distributed depends mostly on the radial distribution of the mass density of the galaxies, which may not be the same as the energy where the anisotropy is significant. If the number of the stars is dominated at an energy significantly higher or lower than the energy where the anisotropy dominates, the BBH orientation evolution should undergo random walks as that shown in spherical systems (e.g., Fig. 3). In case that the number of the stars is dominated at an energy where the anisotropy is significant, we get a chance to see the effect of the anisotropy of the stellar kinematic distributions on the BBH orbital orientation evolution.

Figure 7(a)–(c) illustrates our results for triaxial galaxies; and panels (b) and (c) are for those with rotating properties, with different stellar energy, respectively. The initial stellar kinematic distributions used for the three-body scattering stage correspond to the final distributions shown in Figures 6 and 4.

As seen from Figure 7(a), the orientation change of the BBHs increases significantly, compared to the random walks in non-rotating spherical systems shown in Figure 6(a). The total angle change in Figure 7(a) can be up to 0.4 rad when a total 2$M_\star$ mass of stars are scattered; and after scattering the stars with such a total mass, the BBH semimajor axis decays by a factor of $\sim 500$ (see Eq. 10). The large angle change comes from the relatively large change of $\theta_0$ and $\phi_0$, due to the anisotropy of the initial kinematic distributions of the interacting stars (see the middle panel of Fig. 5). The effect is more clearly shown for the axisymmetric case in Figure 7(d) below.

The BBH orientation changes for rotating triaxial galaxies shown in Figures 7(b)-(c) are smaller than those for rotating spherical systems shown in Figure 6(b). The reduced change originates from the alignment-erasing effect of the triaxial potential on the kinematic distributions of the interacting stars, as mentioned in Section 3.2 (see Fig. 5). The effect is stronger for the case with high stellar energy shown in Figure 7(c), where the final stellar kinematic distribution obtained at the stellar precessing stage is close to isotropic and the magnitude of the BBH orientation evolution is close to the random walk results shown in Figure 7(a). Although the alignment-erasing effect shown in Figure 7 is done for the BBH mass ratio $q = 0.1$, it is plausible to expect that it also exist for some other $q$, which is supported by our numerical tests (e.g., for $q = 0.3$).

Figure 7(d) shows the BBH orientation evolution in axisymmetric galaxies, which is much higher than that for non-rotating spherical galaxies shown in Figure 6(a). As seen from Figure 7(d), the orientation change comes from the following parts: (1) the motion of the orientation towards the axisymmetric axis ($\pm z$-axis); that is, $\theta_0$ evolves towards 0 if initially $0 \lesssim \theta_0 \lesssim \pi/2$ or towards $\pi$ if initially $\pi/2 \lesssim \theta_0 \lesssim \pi$; and (2) the retrograde precession in $\phi_0$ around the axisymmetric axis; that is, $\phi_0$ statistically tends to increase for $\pi/2 \lesssim \theta_0 \lesssim \pi$ and decrease for $0 \lesssim \theta_0 \lesssim \pi/2$. As seen from Figure 6(a), the stars passing by the vicinity of the BBH in axisymmetric potential are inclined to distribute in the plane perpendicular to the axisymmetric axis. The motion of the BBH orientation towards the axisymmetric axis ($\pm z$-axis) suggests that the average BBH orientation alignment magnitude towards the orbital angular momentum of a star caused by interacting with the star is statistically larger if the star is on a prograde orbit (with relative inclination to the BBH orbit, denoted by $\beta$ here; $0 < \beta < \pi/2$) than the alignment magnitude if the star is on a retrograde orbit (with relative inclination $\pi - \beta$). The retrograde precession of $\phi_0$ can be explained through the gravitational potential of the planar mass distribution of the passing-by stars, with dynamics analogical to the precession of a spinning top under the torque induced by the earth’s gravity. If the stars in the axisymmetric galaxies have a net rotation with non-zero total angular momentum along the axisymmetric axis, the BBH orbital orientation is expected to evolve towards the direction of the total angular momentum as shown in Figure 6(b); and for simplicity, we do not show the calculation results for this case here.

### 3.4 BBH Orbital Orientation Distributions

To see the effects of different potentials on the BBH orbital orientation distributions, we generate a sample of 1500 BBHs and obtain their orientation distributions after they interact with a large number of stars with total mass $\sim 2$ times the BBH mass. The initial orientations of the BBHs are assumed to distribute isotropically, for a large range of initial conditions instead of limiting to some specific directions. Figure 8 shows our results of the BBH orbital orientation distributions in different systems. Panel (a) is for rotating spherical systems, where the distribution of $l_z$ peaks at $l_z \simeq 1$, i.e., the reorientation effect is strong and most of the BBHs orient toward the rotating axis. Panel (b) shows the results for rotating triaxial galaxies, where the BBH orientations are isotropically distributed due to the alignment-erasing effect. Panel (c) is for the axisymmetric galaxies, where the distribution peaks at $l_{b,z} = \pm 1$ and most of the BBH orientations are aligned along the axisymmetric axis ($\pm z$-axis).

In triaxial galaxies, for the general case in which the kinematic distribution of the stars that can pass by the vicinity of the BBH is isotropic at the end of the stellar precessing stage (e.g., bottom panel of Fig. 5), the magnitude of the BBH orientation evolution is small as shown in Figure 7(c). Thus, the isotropic distribution of the BBH orientations shown in Figure 8(b) is due to the initial isotropy of the BBH orientations set in the calculation. If the BBH initial orientation distribution is anisotropic, the anisotropy of the distribution maintains as the orientation evolution has a small magnitude.

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Figure 7. The BBH orientation evolution in non-spherical galaxies. The labels and the curves have the same meanings as those in Fig. 6. (a) Triaxial galaxies without rotation. The parameters are the same as those in the middle panel of Fig. 3. (b) Triaxial galaxies with rotation. The parameters are the same as those in the middle panel of Fig. 5. (c) Same as panel (b), but with different stellar energy. The BBH orientation evolution in non-spherical galaxies. The labels and the curves have the same meanings as those in Fig. 6. (d) Axisymmetric galaxies, with parameters same as those in the bottom panel of Fig. 4. As seen from figure, the alignment effect of the BBH orbital orientations towards the rotational axis can be erased in triaxial systems (panel c), and the BBH orientations in axisymmetric systems evolves towards alignment with the axisymmetric axis (panel d).

In axisymmetric galaxies, the BBH orientation distribution shown in Fig. 8(c) is not affected by the precession in $\phi_0$, although the total orientation angle change $\Delta \alpha$ may be different by different precession rates.

3.5 Discussion

Below we discuss the results and some assumptions made in our method.

In our calculations above, we set a fixed stellar mass with $m_* = 10^{-4} M_\odot$ for simplicity. If the BBH orientation evolution is like random walks (e.g., Figs. 6(a) and 6(c)), the magnitude of the orientation change is expected to be proportional to $\sqrt{N}$, as indicated in Equation (11), where $N \propto M_* / m_*$ and $\langle \Delta \alpha^2 \rangle^{1/2} \propto m_* / M_*$ are used. For the orientation evolution in rotating spherical systems (e.g., Fig. 6(b)), the magnitude of the orientation change is expected to be insensitive to the value of $m_*$, as the BBH orbital alignment towards the rotational axis is approximately proportional to $N$ instead of $\sqrt{N}$ (see also Gualandris et al. 2012). For other cases (e.g., Fig. 7(a), (b), and (d)), our numerical tests also support that the magnitude of the orientation change is insensitive to the value of $m_*$ for $m_* / M_*$ ranging from $10^{-6}$ to $10^{-4}$.

In the cases that the BBH orientation evolves like random walks (e.g., Figs. 6(a) and 6(c)), the dependence of the orientation change on the BBH mass ratio $q$ and eccentricity $e$ is indicated in Equation (11). For the other cases, the quantitative dependence on $q$ and $e$ is not obvious, but it can be achieved by performing numerical simulations and qualitative analysis. The alignment-erasing effect discussed for triaxial systems above should still exist even for different $q$ and $e$. We expect that for sufficiently small $q$ and high $e$, the random walks of the BBH orientation evolution may surpass its alignment effect along the rotational axis even in spherical or axisymmetric galaxies.

Note that the net rotation property of a stellar system discussed here is different from the global pattern rotation of a galaxy shape. In the case that the triaxial galactic potential (e.g., Eq. 8) has a global pattern rotation around one of its axis.
Orbital orientation evolution of massive binary black holes

Figure 8. The simulated BBH orbital orientation distributions. Different rows are for different systems: (a) rotating spherical systems; (b) rotating triaxial galaxies with \((\xi, \zeta) = (0.9, 0.8)\) and \(E = \Phi(10r_{\text{inf}})\); and (c) axisymmetric galaxies with \((\xi, \zeta) = (1, 0.8)\). The dotted lines represent the initial isotropic distributions of the BBH orientations, and the solid lines in panels (a)–(c) represent the distributions obtained at the end of simulation shown in Figs. 6(b), and 7(c)–(d), respectively. The number of the simulated BBHs for each panel is 1500. The distributions of the solid lines illustrate the alignment of the BBH orientations along with the rotation axis in rotating spherical systems (panel a), the alignment-erasing effect in triaxial systems (panel b), and the alignment along with the axisymmetric axis in axisymmetric systems (panel c).

[e.g., Deibel et al. 2011], we expect that the effects on the the stellar kinematic distributions discussed in this study would be qualitatively close to the results obtained by approximating the rotating triaxial potential as a potential axisymmetric around the pattern rotation axis if the speed is fast enough, while the effects are little if the pattern speed is low.

In our method, the semimajor axis and the eccentricity of the BBH is fixed during the three-body scattering stage. In reality, the BBH orbit shrinks due to interactions with stars. By using the similar method above, we calculate the BBH orientation evolution at different fixed BBH semi-major axes. We find that the orientation changes at different \(a\) do not differ significantly in spherical systems (see circles in Fig. 9), which is consistent with Equation (11) that is not sensitive to the parameter \(a\). Figure 9 shows that the orientation changes at different \(a\) are also mild in axisymmetric systems (see crosses in the figure); and the changes result mainly from different precession speeds in \(\phi_b\), which do not affect the BBH orientation distribution (e.g., as shown in 8c). Regarding the BBH orientation evolution, the cases illustrated in Figure 9 are the extreme ones among the cases shown in Figures 6, 8 which suggests that our main conclusions should not be affected by the fixing of \(a\). Regarding the BBH eccentricity evolution, it is plausible to ignore it in an isotropic stellar system if the initial BBH eccentricity is low, as Quinlan (1996) shows that the change of the BBH eccentricity in such a case is not significant (see also the simulation results of different initial settings in Amaro-Seoane et al. 2010, Hensendorf et al. 2002). In kinematically anisotropic spherical systems with stars mostly counter-rotating with the BBH orbit, some simulations show that the BBH eccentricity could increase significantly [Sesana et al. 2011, Amaro-Seoane et al. 2011], and we expect that the significant increase of the BBH eccentricity can be decreased or erased if the systems are triaxial, by applying the same reason for the alignment-erasing effect obtained in this paper.

As mentioned in Section 1 before a BBH becomes bound and hard, the two BHs sink into the galactic centre through dynamical friction, and their orbital orientation is likely to be changed in a non-spherical galaxy during the dynamical friction stage. We construct a simple model to simulate the inspiraling of a relatively small BH from a large scale (e.g., \(10^4 R_g\)) into the galactic centre during the dynamical friction stage, as modeled in Section 4 in Yu (2002). We assume that the distribution of the orbital orientations of the small BHs is isotropic initially at the large scale, and we find that in triaxial galaxies (e.g., with triaxiality parameters as those in Fig. 8(b)) the BBH orbital orientation distribution is still isotropic when the two BHs...
becomes bound, as assumed for the initial BBH orientation distributions in Figure 8. In significantly flattened axisymmetric galaxies (e.g., $\xi = 1$, $\zeta = 0.7$), we find that the distribution tends to cluster toward the axisymmetric axis; and the BBH orientation distribution obtained from Figure 8(c) are not affected qualitatively, unless the shape of the galaxy changes with time. As seen from this figure, the rms of the orientation change of hard BBHs is not affected much by different $a$. See more in Section 3.5.

Figure 9. The rms of the simulated orientation change of hard BBHs obtained in one hardening time. The points are our simulation results at the different fixed BBH semimajor axis. The open and the solid circles are for spherical systems ($\xi = 1$, $\zeta = 1$) with rotation ($P_+ = 7/8$) and without rotation ($P_+ = 1/2$), respectively; and the crosses are for axisymmetric systems without rotation. Each point is the rms of orientation change of 100 hard BBHs. In the spherical systems, the initial orientations of the BBHs are generated randomly; and in the axisymmetric systems, we fix their values of $\theta_0 (= \pi/4)$ to see the effects solely due to the change of the semimajor axes. The error-bar of each point has the same meaning as those in Fig. 2. As seen from this figure, the rms of the orientation change of hard BBHs is not affected much by different $a$. See more in Section 3.5.

becomes bound, as assumed for the initial BBH orientation distributions in Figure 8. In significantly flattened axisymmetric galaxies (e.g., $\xi = 1$, $\zeta = 0.7$), we find that the distribution tends to cluster toward the axisymmetric axis; and the BBH orientation distribution obtained from Figure 8(c) are not affected qualitatively, unless the shape of the galaxy changes with time. As seen from this figure, the rms of the orientation change of hard BBHs is not affected much by different $a$. See more in Section 3.5.

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As mentioned in Section 2, the Newtonian mechanics is used in our calculations of the motion of the BBH and the stars. To see how the general relativistic effect could affect the calculation results in the three-body scattering stage, we estimate the potential contribution from the post-Newtonian terms by orders of magnitude as follows. Consider a star that starts with a low velocity passes by the BBH at a distance $r \approx a$. The energy and the angular momentum change of the star comes mainly from the interaction with the smaller BH if $q \ll 1$. First, with only using the Newtonian mechanics, the specific interaction force on the star $F \sim Gm_2/a^2$ acts for a time $\delta t \sim (a^3/GM_\bullet)^{1/2}$ to produce a velocity change $\delta v \sim F \delta t$ and thus result in an orientation change of the BBH $\langle \delta \alpha \rangle^2 \sim (v \delta v \delta t)^{1/2}/J_b$, where $v \sim (2GM_\bullet/a)^{1/2}$ and $J_b$ is the orbital angular momentum of the BBH. Then, if considering the contribution from the post-Newtonian terms, the relative change in $\langle \delta \alpha \rangle^2$ is $\sim \frac{1}{2} \left[ \frac{\Delta \delta v}{\delta v} + \frac{\Delta \delta \alpha}{\delta \alpha} - 2 \frac{\Delta J_b}{J_b} \right]$, where $\Delta v$, $\Delta \delta v$, and $\Delta J_b$ are the changes in $v$, $\delta v$, and $J_b$ due to the post-Newtonian terms, respectively. To approximate the contribution from the post-Newtonian terms, we replace the gravitational potential $-\frac{Gm_2}{a}$ of a particle with mass $m$ by $-\frac{Gm_2}{r_g}$, where $r_g = 2GM/c^2$ and $c$ is the speed of light (Paczynski & Wiita 1980). Thus we have $\frac{\Delta v}{\delta v} \sim \frac{GM_\bullet}{ac^2}$, $\frac{\Delta \delta \alpha}{\delta \alpha} \sim -\frac{2GM_\bullet}{ac^2}$, and $\frac{\Delta J_b}{J_b} \sim \frac{2GM_\bullet}{ac^2}$; and the absolute value of the relative change in $\langle \delta \alpha \rangle^2$ is $\sim \frac{5}{2} \frac{GM_\bullet}{ac^2} \sim 4.4 \times 10^{-6} q^{-1} (\sigma_c/200 \text{km s}^{-1} (a/a_\text{in})^{-1}$, where the definition of $a_\text{in}$ in Equation 2 is used. So the effect of the post-Newtonian terms is generally small enough to be negligible for the study of this paper.

Gas-poor mergers can occupy a significant fraction of galaxy mergers (e.g., Barausse 2012, Lin et al. 2008). However, a significant fraction of galaxy mergers can also be gas-rich. In a merger with sufficient gas, it is expected that the orbital orientation of the BBH always evolve towards co-alignment with the angular momentum direction of its circumbinary disk. However, the role of the gas depends on the amount of gas available. On a timescale short compared with the mass growth timescale of the BBH, the BBH orientation may maintain the counter-alignment if the BBH is initially sufficiently retrograde with the disk (Nixon et al. 2011); and each BH may be surrounded by its own accretion disk (e.g., Yu & Ly 2001, Dotti et al. 2007, Havasak et al. 2008), which does not necessarily co-align with the circumbinary disk.

4 SUMMARY

In this paper, we study the orbital orientation evolution of BBH systems, and mainly focus on their evolution in triaxial (and axisymmetric) systems. For spherical stellar systems, we have reproduced the result that the orientation of a BBH undergoes random walks in kinematically isotropic systems. And in rotating spherical galaxies where the stars have a non-zero total angular momentum, the BBH orientation change is larger and its orbital direction reorients toward the direction of that total angular momentum. In triaxial systems, the initial angular momenta of the stars that can precess to the very vicinity of a
BBH can be far larger than those of the stars initially in the loss cone. The directions of the angular momenta of the stars when they precess to the vicinity of a BBH can be anisotropic due to the torque induced by the triaxial potential, and the degree of the anisotropy depends on the stellar energy as well as the triaxiality or the flattening of the galactic gravitational potential. In axisymmetric galaxies, the angular momenta of the stars are inclined to align to the axisymmetric axis of the system. If the anisotropy of the kinematic distributions of the interacting stars is significant, the orientation evolution of a BBH is different from random walks obtained in non-rotating spherical galaxies. In axisymmetric galaxies, the evolution of the BBH orientation includes the alignment towards the axisymmetric axis and the precession around the axisymmetric axis in a direction retrograde to the BBH orbit.

We find that the triaxial potential has an alignment-erasing effect, if most of the stars that can come to the vicinity of the BBH have significantly high energy. In this case, the kinematic distribution of the stars when they come to the BBH vicinity is close to isotropic even if they are initially anisotropic (e.g., in a rotating system). Due to the alignment-erasing effect, the orientation of the BBH orientations towards the rotation axis of a system can be decreased significantly in triaxial galaxies.

If orbital orientation distributions of BBHs are isotropic when they become hard, their distributions may maintain isotropic after interacting with sufficiently numerous stars in significantly triaxial systems; however, they can be anisotropic in axisymmetric systems, and the number of BBHs with angular momentum directions near the axisymmetric axis is enhanced. If a BBH with comparable mass components (e.g., formed in a major merger of two galaxies) merges, the direction of the BBH orbital angular momentum is likely to dominate the spin direction of the merged BH remnant, and also dominate the jet direction if some gas sink to the vicinity of the merged BH and a jet is produced. If the galaxy is significantly triaxial, the alignment-erasing effect obtained in this paper would imply that there is no preferred orientation of host galaxies and radio jets, but a preferential alignment in relatively axisymmetric galaxies if the axisymmetric axis of the system does not change significantly with galactic radii.

Regarding the tendency for the axis of the radio emission to align with an axis of the galaxy starlight, a dependence on the galaxy properties was possibly revealed in observations. Such a tendency was found in weak radio-loud AGNs, but not among the radio-louder objects possibly hosted in triaxial elliptical galaxies (e.g., Browne & Battye 2010; Saripalli & Subrahmanyan 2009).

This preliminary study mainly uses simple hybrid models for the galactic gravitational potential, stellar distribution, and rotation of the system to illustrate the related effects; and the simplification of the model should not be a concern for the purpose of this paper, although a more sophisticated self-consistent model of these properties needs to be used in realistic galaxies.

We thank Youjun Lu for helpful discussion and comments. This research was supported in part by the National Natural Science Foundation of China under nos. 10973001, 11273004.

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