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Investigation of critical material removal transitions in compliant machining of brittle ceramics

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HIGHLIGHTS
- The proposed model reveals the "three zones" in compliant processing mechanism of ceramics.
- The critical transition pressures at zone boundaries are derived in association with ceramics properties.
- Lower hardness and higher tensile strength of material contribute to enlarged plastic removal zone for high manufacturability.
- The model can well describe the material removal behaviors with predicted removal rates under different conditions.

ABSTRACT
Compliant machining processes, such as bonnet polishing, can be used on hard and brittle ceramic materials such as alumina and silicon carbide, to produce ultra-precise surfaces with sub-micron form accuracy and nanometric surface roughness. However, a comprehensive understanding of the removal mechanism in such process is lacking. In this paper, an analytical model is proposed that is based on the existence of "three zones" in compliant machining process, namely elastic recovery, plastic removal and brittle fracture. The inherent relationships of the three critical pressures with actual pressure, due to compression of the elastic bonnet tool and asperity effect, are established and analyzed in association with different material removal behaviors. Analysis indicates that pad asperity plays an important role in material removal and that lower material hardness combined with higher tensile strength contributes to enlarged plastic removal zone, and thus higher manufacturability. Removal footprints and polishing tests were then generated to verify accurate prediction of the material removal rate under different conditions and demonstrate effectiveness of the proposed model.

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1. Introduction
With ever more stringent requirements from science and technology, there is an increasing demand for high performance products with complex surfaces and super-fine roughness to be made from materials with difficult-to-machine properties such as high hardness and brittleness [1,2]. This challenges conventionally used machining technologies, such as single point diamond turning, ultra-precision raster milling, electrochemical polishing and chemical mechanical lapping which are generally for planar surface finishing [3–6].

During the mechanical polishing methods, spherical lapping (Fig. 1(a)) by using spherical polishing solid pad can achieve very smooth surface, it is however limited to the spherical surface due to
the full-aperture polishing principle. By comparison, bonnet polishing (BP), an emerging technology originally developed by Walker et al. [7], is a computer controlled polishing process (Fig. 1(b)) where the position and orientation (precess angle) of a spinning, inflated or solid elastic tool are actively controlled as it traverses the surface of a workpiece [8]. Benefiting from the compliant contact in sub-aperture region, the workpiece may have general freeform shape. Besides, a flexible contact spot can be achieved even without an attempt to actively control the Z position of the tool relative to the workpiece surface. With those advantages, fabrication of ultra-precision surfaces with complex freeform shapes becomes feasible for many promising applications with enhanced optical performances, e.g. hard X-ray molds [9], infrared mirror for optical telescope [10]. However, typical process development in bonnet polishing consists of trial-and-error when confronting any new material not machined previously. To achieve stable and deterministic polishing process control, a comprehensive understanding of the material removal mechanism is of fundamental importance.

In the past decades, much effort went to studying the material removal characteristics based on the famous Preston’s equation [11], namely \( \frac{dt}{d} = kPV \), where \( dt \) is the removal depth per unit time and \( k \) is the Preston coefficient. It considers that \( dt \) has a linear relationship with the pressure \( P \) and velocity \( V \), though discrepancies have been found between this assumption and experimental results. To address this problem, nonlinear pressure dependence of material removal rate was proposed, for example, in the form of \( \frac{dt}{d} = kP^{2/3} \) [12], or \( \frac{dt}{d} = k\sqrt{PV} \) [13]. Lately, the interfacial friction coefficient as a function of tool spinning speed was introduced in the Preston’s equation based on observation from experiments [14]. Although a better match can be achieved after such modification, a large number of experiments is essential and the agreement is usually limited to the narrow conditions of a few specific experiments. This is because, apart from the pressure and velocity, there are many other factors such as material properties of polishing pad, geometry of contact area, abrasive grit size, workpiece material properties, which have a large influence on the material removal process.

To address limitations of Preston’s law, some researchers have developed more detailed models to understand the fundamental mechanism for conventional chemical mechanical polishing (CMP). Based on the assumption of plastic contact over wafer-grit and pad–grit interfaces, Luo and Dornfeld established a comprehensive model for predicting the material removal rate (MRR) of planar polishing [15]. Fu et al. proposed a plasticity-based mechanistic model to explore the effect of various factors (e.g., grit shape, size and concentration, and pad stiffness) on MRR [16]. Rabbinowicz et al. [17] and Samuels et al. [18] found that the polishing force per particle is insufficient to generate elastic deformation, and the force through asperity contacts may approach the hardness of the work material, resulting in plastic deformation in CMP process. Based on a two-body model with elastic indentation fracture mechanics, Evans et al. investigated the effect of grit size and concentration in planar lapping of brittle materials [19]. A great amount of efforts has been devoted in the past decades to model the conventional CMP process [3], but the conceptual differences of bonnet polishing as a compliant adaptive machining process means that these models cannot be directly applied to deterministic prediction of material removal rate in the BP process.

Compared with the CMP process, much fewer theoretical contributions have been devoted to BP since its original development [8,20,21]. To mention a few, Li et al. built a macroscopic model based on finite element simulation of pressure distribution and calculation of velocity distribution in the contact region [10], and accurately predicted tool influence functions (removal footprints) for segment manufacture of an extremely large telescope. Likewise, Wang et al. simulated pressure distribution in the circular contact area and computed static tool influence functions for BP process.

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**Fig. 1.** (a) Full-aperture polishing on spherical workpiece, (b) sub-aperture polishing on freeform workpiece.
[22]. Nevertheless, these two models were based on macro-scale finite element simulation and did not consider pressure variations at the grit scale. Cheung et al. presented another model based on the assumption of Gaussian distribution across the contact pressure and demonstrated fabrication of structured surfaces [23], but this empirically based model is not able to provide insights into the actual removal mechanism in grit-workpiece interaction. Zeng et al. built an analytical model and have empirically established the link between material removal rate and process parameters based on the Preston equation and experimental results [24]. However, to obtain the coefficient $k$ for a given material, a large number of experiments is required. More recently, a multi-scale model was established by Cao et al. to investigate the material removal characteristics in the BP process, where the active number of grits is associated with pad asperities [25]. However, the inherent effect of asperities on the actual pressure applied to the grit was not considered. Besides, due to near Gaussian distribution of pressure, the different material removal behaviors and especially critical transitions in grit-workpiece interaction have not been revealed yet.

Targeted at providing comprehensive insights into the fundamental material removal mechanism of compliant polishing process, this paper presents a new analytical model and associated experimental investigation. Quite distinctly from other existing models, the basic concept in this new model is to divide the contact area into three concentric regions corresponding to three different material removal behaviors, namely elastic recovery, plastic removal and brittle fracture, respectively. The critical transitions between these three behaviors and associated threshold pressures in ceramics processing can be theoretically derived and validated. On the basis of this model, removal footprint predictions can be generated in multi-scale that match very well with experiments, demonstrating effectiveness of the model in processing of brittle ceramics (e.g. low thermal expansion ceramics, alumina, silicon carbide, etc.). Furthermore, analysis of the machinability by compliant polishing can be derived from the model and offer helpful guidance to optical designers in the selection of substrate materials with mechanical properties (e.g. hardness, tensile strength, etc.) that facilitate super-fine fabrication of the final product.

2. Theoretical modeling and analysis

2.1. Modeling of macroscopic tool-workpiece contact pressure

The compliant bonnet polishing tool is composed of elastic rubber and a thin pad on the top. As illustrated in Fig. 2(a), with a geometric offset of $Q_i$, the macroscopic contact force is mainly due to the elastic deformation of the rubber. The corresponding force component can be calculated according to the theory of Hertz elastic contact, namely:

$$ F_1 = \frac{4}{3} E_t R_1^{1/2} Q_i^{3/2} $$

(1)

where $E_t$ and $R_1$ are the Young’s modulus and radius of the rubber; $Q_i$ is the offset value of the tool. $E_t$ is the Young’s modulus of the rubber, which can be estimated from the international rubber hardness (I.R.H.) $H$ of the rubber using the formula by Refs. [26,27]:

$$ E_t = \frac{0.0984(56 + 7.62(H - 4))}{0.1374(254 - 2.54(H - 4))} $$

(2)

The corresponding pressure in the circular contact region can be calculated by:

$$ P = \frac{2E_t}{\pi} \sqrt{\frac{Q_i}{R_1 + t_h}} \sqrt{1 - \frac{r}{r_0}} $$

(3)

where $r_0 = \sqrt{Q_i(R_1 + t_h)}$ is the radius of the contact area, and $t_h$ is the pad thickness.

Due to the pad asperities, the actual contact area will be smaller than the nominal contact area. Asperities, as illustrated in Fig. 2(b), imply that the pad surface is uneven rather than ideally smooth, thus the actual asperity pressure $P_a(i,j)$ at the position of the $(i,j)$ asperity is higher than the pressure obtained from Eq. (4), which can be derived as [15,28]:

$$ P_a(i,j) = \frac{1}{b_1} E_t^3 P(i,j)^{1/3} $$

(4)

with

$$ b_1 = \pi \left( \frac{3R_0}{4} \right)^{2/3} D_{sum}^{1/3} $$

(5)

where $v_p$ and $v_w$ are the Poisson ratios of the pad and workpiece, respectively. $E_p$ and $E_w$ are the Young’s moduli of the pad and workpiece, respectively. $R_0$ is the typical radius of pad asperity. $D_{sum}$ is the number of asperities per unit of area, which is geometrically related to the spacing $d_\alpha$ of adjacent asperities.

2.2. Modeling of material removal mechanisms

Typically, the tool is tilted by the precess angle $\alpha_0$, to make the zero velocity point of the spindle axis outside of the contact area during polishing process. At any time instant, from Eq. (3), it can be understood that pressure is maximum at the center of the contact region, and decreases as a spherical power function of the spot radius according to Hertz contact theory. In this study, a “three zone” concept is proposed to understand the material removal behavior for brittle materials, based on which the contact area is divided into three concentric zones, corresponding to elastic recovery, plastic removal and brittle cracking, respectively, as shown in Fig. 3. If the critical transitions between different removal behaviors can be predicted at any position of the contact spot, the boundary of three continuous zones will be identified. Therefore, the following studies involve the elastic/plastic/cracking transitions as well as their associations with threshold pressures.

2.2.1. Elastic recovery and plastic removal

In this study and according to previous observations in the literature [15], the slurry particles are assumed to be spherical. The process can be described by the interaction of the spherical particles with the workpiece, which will generate an elastic stress field inside the workpiece. With a very low force applied on the particle, the material is elastically deformed even in the case of ceramics, resulting in a contact radius based on Hertz elastic contact mechanics at the microscopic scale:

$$ a = \left( \frac{3F_d d_g}{8E_w} \right)^{1/3} $$

(7)

where $F_d$ is the generated force on the particle due to the pressure. $d_g$ is the average diameter of grit. $E_w$ is the effective Young’s modulus of the grit and workpiece, given by:
\[
\frac{1}{E_w} = \frac{1 - v_w^2}{E_w} + \frac{1 - v_g^2}{E_g}
\]  

(8)

where \(v_g\) and \(E_g\) are the Poisson ratio and Young’s modulus of the grit, respectively.

Based on the stress field generated by the spherical indenter on the workpiece [29], the material starts to yield when the force is increased above:

\[
F_{\text{ductile}} = \left( \frac{1.1 \pi}{c} \right)^3 \left( \frac{3(1 - v_w)^2}{4} \right)^2 \frac{H_v^3}{E_w^2} \left( \frac{d_g}{2} \right)^2
\]

(9)

where \(H_v\) is the hardness of the material. \(c = \frac{H_v}{\sigma_y}\) is usually evaluated as 3, in accordance with well-known results from Tabor’s work on hardness of solid materials [30]. It is the ratio between the yield stress \(\sigma_y\) and indentation hardness.

For grit-workpiece interaction to occur above the yield point, based on Eq. (9), the pressure \(P_a\) should reach a critical value, corresponding to the elastic threshold pressure:

\[
P_{\text{elastic}} = \frac{F_{\text{ductile}}}{\pi \left( \frac{d_g}{2} \right)^2} = \left( \frac{1.1 \pi}{c} \right)^3 \left( \frac{3(1 - v_w)^2}{4} \right)^2 \frac{H_v^3}{E_w^2}
\]

(10)

When the asperity pressure \(P_a\) exceeds the elastic threshold pressure, material is removed in ductile mode within the plastic range of the ceramic. In this case, as shown in Fig. 4(a), force in the normal direction (z) on individual particle can be derived by the
integration of yield stress across the spherical grit:

$$F_z = k' \cdot \int_0^{\Phi_y} \frac{\sigma_y \cos \theta \cdot R^2 \cos \theta \cdot d\theta \cdot d\phi}{2} \left( \frac{H_0}{c} \right) d\theta h_1$$

(11)

where $h_1$ is the penetration depth in plastic mode shown in Fig. 4(a).

The correction coefficient $k'$ is introduced as the stress cannot be perfectly distributed on the grit-workpiece interface. The value $(0 < k' < 1)$ is derived by setting $F_z = F_{\text{ductile}}$, which also guarantees force continuity at the elastic/plastic transition boundary.

Similarly, the force in tangential direction ($x$) can be obtained:

$$F_x = \sqrt{2k' \sigma_y \frac{d_g}{2}} \left( h_1 - h_{\text{elastic}} \right)^{1/2}$$

(12)

By substituting Eq. (9) to Eq. (11), the elastic recovery corresponding to the elastic/plastic transition depth can be obtained in a simplified form:

$$h_{\text{elastic}} = \frac{11.98 \pi^2 (1 - \nu_w)^4 H_0^2 d_g}{32 \sigma^2 E_w}$$

(13)

Therefore, grits working in the elastic zone are not responsible for removal of material. When the actual asperity pressure $P_a$ reaches the elastic threshold pressure $P_{\text{elastic}}$, grits start removing the material plastically. The material removal rate of a grit can be calculated by multiplying the cross section area and grit velocity:

$$V_i = A_i V_r = \frac{4}{3} \sqrt{2} \left( \frac{d_g}{2} \right)^{1/2} \left( h_1 - h_{\text{elastic}} \right)^{1/2} V_r$$

(14)

where $A_i$ is the cross section area of the grit-workpiece interaction illustrated in Fig. 4. $V_r$ is the grit velocity. As shown in Fig. 5, assuming the coordinates of the grit are $(x_{ij}, y_{ij})$ in the O-xyz coordinate system of the contact region, the velocity can be expressed according to the kinematics of the process:

$$\begin{align*}
V_{rx} &= (R_1 + t_h - Q_h) \sin \alpha_q W_H - y \cdot \cos \alpha_q W_H \\
V_{ry} &= x \cdot \cos \alpha_q W_H \\
V_r &= \sqrt{V_{rx}^2 + V_{ry}^2}
\end{align*}$$

(15)

2.2.2. Brittle cracking

It is well known that when the penetration depth is larger than a critical value, the ductile to brittle transition occurs in ceramics processing [31]. To predict the threshold pressure value in the BP process, brittle fracture relates to the tensile component of the elastic stress field. The maximum tensile stress is given by Ref. [16]:

$$\sigma_m = \frac{(1 - 2\nu_w)F_g}{2\pi a^2}$$

(16)

The brittle fracture occurs when $\sigma_m > k_t \sigma_t$, where $\sigma_t$ is the tensile strength of the material and $k_t$ is a factor considering the size effect.

By combining Eqs. (7), (11) and (16), the critical force that will generate fracture can be derived:

$$F_{\text{brittle}} = \frac{2\pi k_t \sigma_t}{1 - 2\nu_w} \left( \frac{3(1 - \nu_w^2)}{8E_w} \right)^2$$

(17)

Since the force $F_g$ of the particle on the $(i, j)$ asperity equals to $0.25\pi r (r_{ij} \cdot d_g)^2$, it can be found that the plastic threshold pressure that generates fracture is independent on the particle size:

$$P_{\text{brittle}} = \frac{4}{\pi} \left( \frac{2\pi k_t \sigma_t}{1 - 2\nu_w} \right)^3 \left( \frac{3(1 - \nu_w^2)}{8E_w} \right)^2$$

(18)

The corresponding plastic/cracking transition depth will be:

$$h_{\text{plastic}} = \frac{9c (\pi k_t \sigma_t)^3 (1 - \nu_w^2)^2}{4\pi k^* (1 - 2\nu_w)^3 H_c (E_w)^2}$$

(19)

On condition that the force $F_g$ of the grit is beyond $F_{\text{brittle}}$, cracks will be formed, as depicted in Fig. 4(b). The depth of crack initiation $C_h$ is equal to the width $b$ of the plastic zone, given by Ref. [32]:

$$C_h = \left[ \frac{3 \cdot ss \cdot (1 - \nu_w^2)}{8E_w} \right]^{1/3} \left( \frac{E_w}{H_c} \right)^{m} (F_g)^{1/3}$$

(20)

where $m = 0.5$; $ss$ is chosen by adapting the strain generated by a spherical indenter [33], expressed as:

$$ss = 3 \tan \frac{\theta}{2} + \tan^2 \frac{\theta}{2}$$

(21)

where $\theta = \sin^{-1} \frac{d_g}{R}$. Given the fracture toughness $k_f$, the lateral crack $C_l$ satisfies [34]:

![Fig. 4. (a) Plastic removal, (b) brittle cracking.](image)
Therefore, the removal volume due to material fracture under the cracking will be:

\[ V_i = A_i V_r = 2 C_0 C_i V_r \]  

(23)

2.2.3. Trapping threshold pressure determination

Normally, the asperity pressure is directly applied on the grit as shown in Fig. 2(b). However, if the pad is sufficiently soft (low Young's modulus value), the grit will be probably trapped inside the pad [35], constituting a three-body (pad/ting/pad) contact, as illustrated in Fig. 2(c). For the pad/ting pair, it can be considered that a rigid spherical ert indents into the pad, because the pad is far softer than the polishing ert. The critical indentation depth in elastic mode can be determined according to the contact mechanics:

\[ h^*_g = \left( \frac{3 \pi k_p H_p}{4 E_p} \right)^{1/2} \frac{d_g}{2} \]  

(24)

where \( H_p \) is the hardness of the pad, \( k_p \) is the contact factor set to be 0.4, as it is believed that when the contact pressure is beyond 0.4\( H_p \), the initial yielding occurs [36]. Thus, the contact force between ert and pad depends on elastic or plastic contact:

\[ F_g = \begin{cases} 
\frac{4 \pi k_p H_p}{3 E_p} \left( \frac{d_g}{2} \right)^2 h_g^2, & h_g < h^*_g \\
H_p \cdot \pi \cdot d_g \cdot h_g, & h_g \geq h^*_g 
\end{cases} \]  

(25)

For the trapping condition, based on Eq. (11) and Fig. 2(d), the following equations in terms of force balance and displacement should be satisfied:

\[ \begin{align*}
F_g &= \frac{\pi k^*_g H_g}{C} d_g h_t \\
h_t + h_g &= d_g
\end{align*} \]  

(26)

Thus, for the elastic contact between grid and pad, by combining Eqs. (25-26), the penetration depth \( h_t \) can be obtained by solving the following equation:

\[ h_t^3 + \left( \frac{9 \pi^2}{32} \cdot \frac{H_g^2}{C^2 E_p^2} - 3 \right) \cdot d_g h_t^2 + 3 d_g^2 h_t - d_g^3 = 0 \]  

(27)

By substituting \( h_t \) into Eq. (11), the critical trapping pressure for elastic pad/ting contact can be obtained:

\[ p_t = \frac{2 k^*_g \cdot H_g \cdot h_t}{d_g \cdot c} \]  

(28)

For the plastic pad/ting contact, the critical trapping pressure can be derived by combining Eqs. (11), (25) and (26):

\[ p_t = \frac{4 k^*_g \cdot H_g \cdot H_v}{2 H_p \cdot c + k^*_g \cdot H_v} \]  

(29)

2.3. Material removal rate calculation

Assume \( G \) is the number of grids per unit of volume, it relates to the density of the slurry:

\[ w\% = \frac{1}{6} \pi \rho_g d_g^3 \cdot G \]  

(30)

where \( \rho_g \) is the density of the particle.

According to Ref. [15], the deformation of asperity makes the grids captured in the actual contact region. Thus, the number of active grids corresponding to the \((i, j)\) asperity can be estimated by the number of grids in the volume of asperity deformation, given by:

\[ N(i,j) = G \cdot b \cdot A_a \cdot l_a = G \cdot \frac{P_0(i,j)}{P(i,j)} \cdot \pi d_a^2 \cdot l_a \]  

(31)

where \( A_a \) is the planar area of the asperity, \( b \) is the actual contact area of the asperity over the asperity area \( \pi d_a^2 \) on the workpiece.

By substituting Eq. (14) or (23), the material removal rate on the \((i, j)\) asperity region can be calculated:

\[ MRRas(i,j) = N(i,j) \cdot A_i \cdot V_r \]  

(32)

Accordingly, the material removal rate in the contact region can be numerically integrated by:
where \( \delta_{xy} \) is the resolution of mesh grid, namely \( \Delta h \).

3. Simulation and validation

3.1. Validation by finite element analysis

To check the validity and accuracy of the proposed analytical model, finite element analysis using the COMSOL software was implemented. A low thermal expansion ceramic material (Type CD107, Krosaki Harima Corporation, Japan) was used in this study. The mechanical properties of adopted ceramics are listed in Table 1 from the manufacturer. The slurry is cerium oxide with Young's modulus of 180 GPa and Poisson ratio of 0.29. Equivalently for simulation, rigid grit was used and thus the effective Young's modulus of workpiece was set to be 86 GPa in the software. By means of increasing the penetration depth, the maximum stress at the contact interface was found approximately to be \( \sigma_y = \frac{K_{p}}{K_{t}} \). The corresponding penetration depth is regarded as the elastic/plastic transition depth from simulations. Three different spherical grit sizes were tested to evaluate the effectiveness of the model. Through simulations, on the basis of reaching the yield stress, the transition depths were identified to be 1.7 nm, 3.3 nm and 13 nm for grits size of 1.5 \( \mu \)m, 3 \( \mu \)m, and 9 \( \mu \)m, respectively. These critical values are in high consistence with that obtained from Eq. (13), which are compared and plotted in Fig. 6(b). The corresponding forces applied on the grits to balance the contact force were also derived, and compared in Fig. 6(c) showing the agreement in all three cases. The matched results confirm that the proposed plasticity-based model could be adopted to accurately predict the grit-workpiece interaction in grit scale.

3.2. Plastic/cracking transition of ceramics

Actual scratching tests with increasing depth of cut were conducted to identify the transition points, through experiments which are presented in this section. For the elastic/plastic transition point, since elastic recovery occurs [37], it is quite difficult to identify the start point experimentally from the observation of scratched surface. For the plastic/cracking transition identification, as an alternative method, the diamond grit with 9 \( \mu \)m in average size, which has been deposited on a substrate, was adopted to scratch on the ceramics. The scratched groove was measured and shown in Fig. 7, from which a clear transition from plastic scratching to brittle cracking can be observed. The maximum depth of plastic removal was experimentally determined to be approximately 105 \( \mu \)m, which is close to the predicted value of about 122 nm on the basis of tensile strength (\( \sigma_t \)) of 230 MPa from the adopted ceramics specification. According to studies on ceramics tensile property [38,39], the tensile strength is always 2–3 times higher than the specified value when the tested specimen is of small size, therefore in this study the factor (\( k_f \)) is chosen as 2.75. It indicates that brittle cracking occurs when the penetration depth is above 122 nm. The corresponding asperity pressure is 26 MPa according to Eq. (19). This implies that, when the abrasive grit interacts with the ceramics, high actual pressure beyond this value will lead to fracture failure and crack propagation (Fig. 5(b)) in the ceramic material. This can be avoided, since the proposed model predicts the threshold pressure. As the grits are loose abrasives, unlike the fixed grit in grinding, brittle cracking rarely occurs as asperity pressure using the common polyurethane pad is far below the threshold value.

3.3. Five critical pressure analysis

It is noticed that there are five critical pressures involved in the model: 1) rubber pressure \( P_r \), 2) asperity pressure \( P_a \), 3) trapping pressure \( P_t \), 4) elastic threshold pressure \( P_{th} \), and 5) plastic threshold pressure \( P_{brittle} \). The relationships are investigated analytically based on the proposed model to have a clear understanding of the contact situation from macro to micro/nano-scale perspective, and the material removal behavior of grits as well. Here the grit size was chosen to be 1.5 \( \mu \)m, and the polyurethane was employed as the pad material with Young's modulus of 33.3 MPa and hardness of 90 MPa [35]. The asperity parameters of the polyurethane pad are listed in Table 3, and are experimentally identified in section 4.2. By substituting pad material parameters into Eqs. (24-27), the elastic deformation of pad occurs when the grit is fully trapped inside the pad, and the trapping pressure is calculated to be 39.1 MPa. Based on the grit-workpiece interaction and Eqs. (10) and (18), the elastic and plastic threshold pressures are found to be 2.4 MPa and 26 MPa, respectively. That means cracking might occur before the full trapping of grit into the pad, otherwise cracking never happens. On the other hand, when the asperity pressure (dependent on the rubber pressure) is lower than the elastic threshold, there will be no material removal as all the grits are only rubbing on the workpiece with full elastic recovery. It can be seen from Fig. 8(a), using rubber hardness of 44° (I.R.H.), that the asperity pressure is slightly higher than the elastic threshold, leading to a low material removal rate (MRR). This is also the reason that lower hardness rubber is adopted, when the aim is to improve the surface roughness without removing much material in practice. With the increase in rubber hardness, it is reasonable to find that the rubber pressure could exceed the elastic threshold in the absence of asperities, due to the harder contact. However, the resultant high MRR will easily degrade the surface form, thus rubber with medium hardness of 64° is commonly used to reach a compromise between high MRR and polishing quality.

With a rubber hardness of 64° and same conditions as above, the critical pressures were also investigated in terms of pad Young's modulus. According to Eq. (4), the increasing pad Young's modulus results in a rise in trapping pressure. Especially for high pad value of 1 GPa, the trapping pressure is calculated to be up-to 304 MPa, whereby plastic contact between pad and grit occurs according to Eq. (29). It can be predicted that the significantly increased asperity pressure, as shown in Fig. 8(b) resulting from the high Young's modulus of pad material, will be beyond the plastic threshold and lead to cracking behavior in the polishing process. However, this phenomenon should be avoided in polishing, as it is usually the final processing stage in the manufacturing chain. Therefore, pad

### Table 1

Mechanical properties of adopted ceramics.

| Young's modulus (GPa) | Poisson ratio | Hardness (GPa) | Tensile strength (MPa) | Fracture toughness (MPa m\(^{1/2}\)) |
|-----------------------|--------------|----------------|-----------------------|-------------------------------------|
| 140                   | 0.32         | 8.0            | 230                   | 1.4                                 |

\[ \text{MRR} = \sum_{j=1}^{M} \sum_{i=1}^{M} \text{MRR}_{ij}(i,j) \]
material with relatively small Young’s modulus value is usually adopted, such as the polyurethane pad. Nevertheless, for much softer pad with Young’s modulus of 18 MPa and same asperity parameters, the asperity pressure dips below even the elastic

Table 2

| Tool conditions | Process parameters |
|-----------------|--------------------|
| Tool conditions |                  |
| Pad             | Slurry size       |
| Type            | 1.5 μm            |
| Spindle speed   | 800 rpm           |
| #1 64° Poromeric | CeO₂              |
| #2 64° Polyurethane | 40 g/L           |
| #3 94° Plastic | Precess angle 20° |
|                 | Offset 0.3 mm     |
|                 | Dwell time 5 s    |

Table 3

| Properties of adopted polishing pads. |
|---------------------------------------|
| Material | h₀ (μm) | Std. dev (μm) | δ₀ (μm) | Rₐ (μm) | Eₚ (MPa) |
| Polyurethane | 10.8 | 2.5 | 52.0 | 130.6 | 33.3 [12] |
| Poromeric | 6.8 | 1.1 | 30.1 | 69.6 | 29 [33] |

Fig. 6. (a) Finite element analysis for identifying elastic/plastic transition, (b) elastic/plastic transition depth, (c) corresponding force applied on the grit.

Fig. 7. Plastic/cracking transition identification.
threshold, indicating that no material removal occurs in this condition. In addition, considering the smooth pad without asperities, the asperity pressure in this case should be equal to the rubber pressure. Hence, with extreme hard rubber of 94°, it can realize plastic removal of material, according to Fig. 8(b). Consequently, the model could well explain why rubber hardness of 64° is much preferred and the importance of pad asperities in a quantitative manner based on the critical pressures.

4. Experimental demonstration

4.1. Experimental setup

The experiments were carried out on a 7-axis polishing machine (IRP50, Zeeko), as shown in Fig. 9. The spindle speed was 800 rpm with a precess angle of 20° and offset of 0.3 mm when generating removal footprints. The dwell time was set to 5 s, and the adopted slurry was cerium oxide with grit size 1.5 \( \mu \)m and density 40 g/L. The detailed information is listed in Table 2. The surface form after polishing was measured by Fizeau interferometer (Wyko NT4100).

4.2. The effect of polishing conditions on MRR

Experimental validation of the model and analysis were conducted from three aspects, namely three specially selected conditions: #1 poromeric pad (Uninap 13, Universal Photonics) with rubber hardness of 64° (I.R.H.), #2 polyurethane pad (LP66, Universal Photonics) with rubber hardness of 64°, #3 smooth plastic pad with higher rubber hardness of 94°. The polishing pads were observed by optical microscope and measured by stylus profilometer (Taylor Hobson) according to the method described in Ref. [40]. The 2D images of polyurethane and poromeric pad, and their average asperity height \( h_a \) are reasonably characterized by the waviness of the measured line profile, as shown in Fig. 10. With reference to Ref. [15], the heights of asperities are assumed to be close to each other so that all asperities contact the workpiece and deform under the pressure. This assumption is reasonable since the elastic modulus of pad is small so that all asperities can contact the workpiece. Based on the observed images and eight measured profiles for each pad, the asperity geometries including the asperity height \( h_a \) with standard deviation and the spacing \( \Delta d_a \) were evaluated. Also, the corresponding radii of asperities were calculated accordingly. The identified parameters for two adopted polishing pads are listed in Table 3.

Using the experimental conditions described in section 4.1, three removal footprints (RF) were generated. It can be observed from Fig. 11 that the experimental RFs match well with theoretical predictions produced in accordance with the method described in section 2.3, where the Young's modulus of poromeric was set to be 29 MPa (comparable to soft leather [41]) and that of polyurethane was 33.3 MPa as mentioned. By comparing conditions #1 and #2 for the same rubber hardness, the use of poromeric pad leads to the larger area in elastic rubbing zone and much lower removal depth, which can be observed from Fig. 11(a)&(b). Note that the simulated RF images were obtained for the average grit size to especially emphasize the investigation of the critical transitions and the effects on material removal behaviors. To consider the nature of grit size variation [42], one can randomize the grit size with a stochastic distribution initially in Eq. (7).

![Fig. 8. Critical pressures with respect to (a) rubber hardness, (b) pad Young's modulus.](image)

![Fig. 9. Experimental setup.](image)
For comparison, a smooth plastic thin film was attached on rubber hardness of 94° (the same as used in simulation of section 3.2), to produce a hard tool without asperities. According to the prediction of section 3.2 and Fig. 8(b), the asperity pressure in this case is below that for 64° rubber with asperities, but slightly higher than the elastic threshold. In other words, more grits are working in the rubbing stage even with a hard tool, as predicted in Fig. 11(c). It can be obviously seen that it has a large flat rubbing area in the outer region for both experimental and theoretical RFs. The sectional profiles through the RF center are extracted and basically match with the predicted ones shown in Fig. 11. Quantitative evaluation of the volumetric material removal rate will be presented in the next section. In addition, the theoretical analysis and experiments has some implications for the design of materials with an emphasis on manufacturability with high material removal rate. Specifically, aiming for a lower hardness within the acceptable product tolerances leads to lower elastic threshold pressure $P_{\text{elastic}}$ according to Eq. (10), which minimizes the elastic recovery zone and boosts material removal rate. Meanwhile, aiming for the largest possible tensile strength (Eq. (18)) increases the plastic threshold

Fig. 10. Optical observations and profiles over 1 mm length for (a) polyurethane, and (b) poromeric pad.

Fig. 11. Experimental and theoretical removal footprints and section profiles under conditions: (a) #1 poromeric pad with rubber hardness of 64° (IRH), (b) #2 polyurethane pad with rubber hardness of 64°, (c) #3 smooth plastic pad with rubber hardness of 94°.
pressure $P_{\text{brittle}}$, and reduces the incidence of material fracture.

4.3. Prediction of Preston coefficient

To quantitatively characterize the material removal behavior, the removal rates for three conditions were calculated theoretically based on Eq. (33), showing a good agreement with the experimental values, as shown in Fig. 12(a). Maximum MRR of about 0.015 mm$^3$/min in average was obtained for condition #2 (LP66 pad, rubber hardness of 64°). Lowest average MMR was obtained for condition #3 (Smooth pad), which indicates the important role of pad asperities in polishing. Then, the Preston coefficient based on Eq. (34) is calculated with respect to different conditions and compared with the experimentally obtained values based on the general Preston equation. The corresponding Preston coefficients under three conditions are plotted in Fig. 12(b), showing the high consistency between values obtained from the proposed model and those from the experiments. The maximum deviation with experimental results is about 14%. By setting the same parameters as condition #2 with different spindle speeds of 400 rpm, 800 rpm and 1600 rpm, the experimentally obtained material removal rates with respect to spindle speeds are drawn in Fig. 12(c). The corresponding Preston coefficient shows a slightly decreasing trend from $2.3 \times 10^{-14}$ to $1.55 \times 10^{-14}$/Pa, as shown in Fig. 12(d). However, the experimental values are quite close to the theoretical prediction of $1.65 \times 10^{-14}$/Pa in this study. The discrepancy is attributed to the increased slurry flow likely causing a hydrodynamic effect between the pad and workpiece, thus influencing the force equilibrium at different speeds [43].

4.4. Prediction and validation of polishing topography

To further validate prediction capability of the model for polishing conditions other than the above experiments, a polishing test on the same ceramic material with size $50 \times 50$ mm was carried out, as illustrated in Fig. 13(a). In this test, a raster path was planned on the ceramic workpiece. The feed direction was along x direction at feed speed of 300 mm/min, and the spindle speed was set to be 1000 rpm. The precess angle was varied progressively from 0 to 30° as the tool moved in x direction, while the offset was increased from 0.1 mm to 0.4 mm along y direction. The maps of process variables in $50 \times 50$ mm area are plotted in Fig. 13(b). After subtracting measurements of the ceramic surface form before and after polishing, the experimental material removal map in the polishing area is shown in Fig. 13(c). On the other hand, by setting same parameters as in the experiments, the surface form was predicted by convolution of dwell time (obtained from the feed speed) and the continuously predicted removal footprint based on the variable maps, as shown in Fig. 13(d). The predicted removal form agrees well with experimental results. The maximum MRR is achieved under the extreme condition of 0.4 mm offset and 30° precess angle, because large offset contributes to the increased asperity pressure while the high precess angle results in high removal speed per grit.

By matching with the prediction map, the root mean square (rms) deviation in the removal depth over the entire area is

![Fig. 12.](image-url)
evaluated to be 0.103 μm. Through integral evaluation, the total removal amount of ceramic material after this procedure is evaluated to be 0.4269 mm³ for the experiment, while the close value of 0.3977 mm³ with deviation of 6.8% is obtained by model prediction. Such agreement especially in material removal rate confirms that the model could not only provide a scientific insight into the compliant polishing process, but is also able to produce accurate predictions for the entire process. The surface quality in the polishing area was also evaluated by white light interferometer with 50x objective. The characterized surface roughness was about 3.3 nm in average.

5. Conclusions

In this paper, theoretical and experimental investigations were conducted to reveal the material removal mechanism and critical transitions in compliant machining of brittle ceramics such as alumina and silicon carbide. The basic idea is to divide the circular tool contact region into “three zones”, corresponding to elastic recovery, plastic removal and brittle cracking, respectively. The critical transition pressures at the zone boundaries were analytically derived and validated by finite element simulations at the microscopic scale. Experiments were conducted that validate effectiveness of the proposed model, by means of removal footprints and characterizing surface topography when polishing under different conditions. This work implies that in design of materials with an emphasis on manufacturability, the designer should aim for hardness with the lowest value within acceptable product tolerance to decrease the elastic threshold pressure, which helps enlarge the plastic recovery zone and promotes higher material removal rate. Meanwhile, the tensile strength should be designed with a large value to reach a high plastic threshold pressure, and thus avoid material fracture. In cases where product tolerances cannot easily be modified, the designer can at least use the model to provide guidance to the product manufacturer regarding the range of process parameters that should be used in ceramics polishing, such that highest possible productivity is achieved whilst avoiding damage to the substrate.

CRediT authorship contribution statement

Wu-Le Zhu: Writing - original draft. Beaucamp Anthony: Writing - review & editing.

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