Using the Global Arrays Toolkit to Reimplement NumPy for Distributed Computation

Jeff Daily‡*, Robert R. Lewis§

Abstract—Global Arrays (GA) is a software system from Pacific Northwest National Laboratory that enables an efficient, portable, and parallel shared-memory programming interface to manipulate distributed dense arrays. Using a combination of GA and NumPy, we have reimplemented NumPy as a distributed drop-in replacement called Global Arrays in NumPy (GAiN). Scalability studies will be presented showing the utility of developing serial NumPy codes which can later run on more capable clusters or supercomputers.

Index Terms—Global Arrays, Python, NumPy, MPI

Introduction

Scientific computing with Python typically involves using the NumPy package. NumPy provides an efficient multi-dimensional array and array processing routines. Unfortunately, like many Python programs, NumPy is serial in nature. This limits both the size of the arrays as well as the speed with which the arrays can be processed to the available resources on a single compute node.

For the most part, NumPy programs are written, debugged, and run in singly-threaded environments. This may be sufficient for certain problem domains. However, NumPy may also be used to develop prototype software. Such software is usually ported to a different, compiled language and/or explicitly parallelized to take advantage of additional hardware.

Global Arrays in NumPy (GAiN) is an extension to Python that provides parallel, distributed processing of arrays. It implements a subset of the NumPy API so that for some programs, by simply importing GAiN in place of NumPy they may be able to take advantage of parallel processing transparently. Other programs may require slight modification. This allows those programs to take advantage of the additional cores available on single compute nodes and to increase problem sizes by distributing across clustered environments.

Background

Like any complex piece of software, GAiN builds on many other foundational ideas and implementations. This background is not intended to be a complete reference, rather only what is necessary to understand the design and implementation of GAiN. Further details may be found by examining the references or as otherwise noted.

NumPy

NumPy [Oli06] is a Python extension module which adds a powerful multidimensional array class ndarray to the Python language. NumPy also provides scientific computing capabilities such as basic linear algebra and Fourier transform support. NumPy is the de facto standard for scientific computing in Python and the successor of the other numerical Python packages Numarray [Dub96] and numeric [Asc99].

The primary class defined by NumPy is ndarray. The ndarray is implemented as a contiguous memory segment. Internally, all ndarray instances have a pointer to the location of the first element as well as the attributes shape, ndim, and strides. ndim describes the number of dimensions in the array, shape describes the number of elements in each dimension, and strides describes the number of bytes between consecutive elements per dimension. The ndarray can be either FORTRAN- or C-ordered. Recall that in FORTRAN, the first dimension has a stride of one while it is the opposite (last) dimension in C. shape can be modified while ndim and strides are read-only and used internally, although their exposure to the programmer may help in developing certain algorithms.

The creation of ndarray instances is complicated by the various ways in which it can be done such as explicit constructor calls, view casting, or creating new instances from template instances (e.g. slicing). To this end, the ndarray does not implement Python’s __init__() object constructor. Instead, ndarrays use the __new__() class method. Recall that __new__() is Python’s hook for subclassing its built-in objects. If __new__() returns an instance of the class on which it is defined, then the class’s __init__() method is also called. Otherwise, the __init__() method is not called. Given the various ways that ndarray instances can be created, the __new__() class method might not always get called to properly initialize the instance. __array_finalize__() is called instead of __init__() for ndarray subclasses to avoid this limitation.

The element-wise operators in NumPy are known as Universal Functions, or ufuncs. Many of the methods of ndarray simply invoke the corresponding ufunc. For example, the operator + calls ndarray.__add__() which invokes the ufunc add. Ufuncs are either unary or binary, taking either one or two arrays as input, respectively. Ufuncs always return the result of the operation as an ndarray or ndarry subclass. Optionally, an additional output parameter may be specified to receive the results of the operation. Specifying this output parameter to the ufunc avoids the sometimes unnecessary creation of a new ndarray.
Ufuncs can operate on ndarray subclasses or array-like objects. In order for subclasses of the ndarray or array-like objects to utilize the ufuncs, they may define three methods or one attribute which are __array_prepare__(), __array_wrap__() , __array__(), and __array_priority__, respectively. The __array_prepare__() and __array_wrap__() methods will be called on either the output, if specified, or the input with the highest __array_priority__. __array_prepare__() is called on the way into the ufunc after the output array is created but before any computation has been performed and __array_wrap__() is called on the way out of the ufunc. Those two functions exist so that ndarray subclasses can properly modify any attributes or properties specific to their subclass. Lastly, if an output is specified which defines an __array__() method, results will be written to the object returned by calling __array__().

**Single Program, Multiple Data**

Parallel applications can be classified into a few well defined programming paradigms. Each paradigm is a class of algorithms that have the same control structure. The literature differs in how these paradigms are classified and the boundaries between paradigms can sometimes be fuzzy or intentionally blended into hybrid models [Buy99]. The Single Program Multiple Data (SPMD) paradigm is one example. With SPMD, each process executes essentially the same code but on a different part of the data. The communication pattern is highly structured and predictable. Occasionally, a global synchronization may be needed. The efficiency of these types of programs depends on the decomposition of the data and the degree to which the data is independent of its neighbors. These programs are also highly susceptible to process failure. If any single process fails, generally it causes deadlock since global synchronizations thereafter would fail.

**Message Passing Interface (MPI)**

Message passing libraries allow efficient parallel programs to be written for distributed memory systems. MPI [Gro99a], also known as MPI-1, is a library specification for message-passing that was standardized in May 1994 by the MPI Forum. It is designed for high performance on both massively parallel machines and on workstation clusters. An optimized MPI implementation exists on nearly all modern parallel systems and there are a number of freely available, portable implementations for all other systems [Buy99]. As such, MPI is the de facto standard for writing massively parallel application codes in either FORTRAN, C, or C++.

The MPI-2 standard [Gro99b] was first completed in 1997 and added a number of important additions to MPI including, but not limited to, one-sided communication and the C++ language binding. Before MPI-2, all communication required explicit handshaking between the sender and receiver via MPI_Send() and MPI_Recv() in addition to non-blocking variants. MPI-2’s one-sided communication model allows reads, writes, and accumulates of remote memory without the explicit cooperation of the process owning the memory. If synchronization is required at a later time, it can be requested via MPI_Barrier(). Otherwise, there is no strict guarantee that a one-sided operation will complete before the data segment it accessed is used by another process. mpi4py

mpi4py is a Python wrapper around MPI. It is written to mimic the C++ language bindings. It supports point-to-point communication, one-sided communication, as well as the collective communication models. Typical communication of arbitrary objects in the FORTRAN or C bindings of MPI require the programmer to define new MPI datatypes. These datatypes describe the number and order of the bytes to be communicated. On the other hand, strings could be sent without defining a new datatype so long as the length of the string was understood by the recipient. mpi4py is able to communicate any serializable Python object since serialized objects are just byte streams. mpi4py also has special enhancements to efficiently communicate any object implementing Python’s buffer protocol, such as NumPy arrays. It also supports dynamic process management and parallel I/O [Dal05], [Dal08].

**Global Arrays and Aggregate Remote Memory Copy Interface**

The GA toolkit [Nie06], [Nie10], [Pnl11] is a software system from Pacific Northwest National Laboratory that enables an efficient, portable, and parallel shared-memory programming interface to manipulate physically distributed dense multidimensional arrays, without the need for explicit cooperation by other processes. GA compliments the message-passing programming model and is compatible with MPI so that the programmer can use both in the same program. GA has supported Python bindings since version 5.0. Arrays are created by calling one of the creation routines such as ga.create(), returning an integer handle which is passed to subsequent operations. The GA library handles the distribution of arrays across processes and recognizes that accessing local memory is faster than accessing remote memory. However, the library allows access mechanisms for any part of the entire distributed array regardless of where its data is located. Local memory is acquired via ga.access() returning a pointer to the data on the local process, while remote memory is retrieved via ga.get() filling an already allocated array buffer. Individual contiguous sets of array elements can be updated or retrieved using ga.scatter() or ga.gather(), respectively. GA has been leveraged in several large computational chemistry codes and has been shown to scale well [Apr09].

The Aggregate Remote Memory Copy Interface (ARMCII) provides general-purpose, efficient, and widely portable remote memory access (RMA) operations (one-sided communication). ARMCII operations are optimized for contiguous and non-contiguous (strided, scatter/gather, I/O vector) data transfers. It also exploits native network communication interfaces and system resources such as shared memory [Nie06]. ARMCII provides simpler progress rules and a less synchronous model of RMA than MPI-2. ARMCII has been used to implement the Global Arrays library, GPSHMEM - a portable version of Cray SHMEM library, and the portable Co-Array FORTRAN compiler from Rice University [Dot04].

**Cython**

Cython [Beh11] is both a language which closely resembles Python as well as a compiler which generates C code based on Python’s C API. The Cython language additionally supports calling C functions as well as static typing. This makes writing C extensions or wrapping external C libraries for the Python language as easy as Python itself.

**Previous Work**

GAiN is similar in many ways to other parallel computation software packages. It attempts to leverage the best ideas for
for a parallel environment can be a daunting task. One area of
parallel architectures is obvious, however, efficiently programming
The need for parallel programming and running these programs on
Design
Numeric Python [Asc99].

in a SPMD fashion. The CoArray was designed as an extension to
provided a local data structure existing on all processors executing
transparent, parallel processing found in current systems. The
following packages provided insight into how GAiN was to be
developed.

MITMatlab [Hus98], which was later rebranded as Star-P [Ede07],
provides a MATLAB binding to the GA toolkit, thus allowing for larger
problem sizes and parallel computation. GAMMA can be viewed as
as a GA implementation of MITMatlab and was shown to scale
well even within an interpreted environment like MATLAB.

IPython [Per07] provides an enhanced interactive Python shell
as well as an architecture for interactive parallel computing. IPython
supports practically all models of parallelism but, more
importantly, in an interactive way. For instance, a single interactive
Python shell could be controlling a parallel program running on a
supercomputer. This is done by having a Python engine running
on a remote machine which is able to receive Python commands.

distarray [Gra09] is an experimental package for the IPython
project. distarray uses IPython’s architecture as well as MPI exten-
sively in order to look and feel like NumPy ndarray instances.
Only the SPMD model of parallel computation is supported, unlike
other parallel models supported directly by IPython. Further, the
status of distarray is that of a proof of concept and not production
ready.

A Graphics Processing Unit (GPU) is a powerful parallel
processor that is capable of more floating point calculations per
second than a traditional CPU. However, GPUs are more difficult
to program and require other special considerations such as copying

research is to automatically parallelize otherwise serial programs
and to do so with the least amount of user intervention [Buy99].
GAiN attempts to do this for certain Python programs utilizing the
NumPy module. It will be shown that some NumPy programs can
be parallelized in a nearly transparent way with GAiN.

There are a few assumptions which govern the design of
GAiN. First, all documented GAiN functions are collective. Since
Python and NumPy were designed to run serially on workstations,
it naturally follows that GAiN, running in an SPMD fashion,
will execute every documented function collectively. Second, only
certain arrays should be distributed. In general, it is inefficient to
distribute arrays which are relatively small and/or easy to compute.
It follows, then, that GAiN operations should allow mixed inputs
of both distributed and local array-like objects. Further, NumPy
represents an extensive, useful, and hardened API. Every effort to
reuse NumPy should be made. Lastly, GA has its own strengths to
offer such as processor groups and custom data distributions. In
order to maximize scalability of this implementation, we should
enable the use of processor groups [Nie05].

A distributed array representation must acknowledge the du-
ality of a global array and the physically distributed memory of
the array. Array attributes such as shape should return the global,
coalesced representation of the array which hides the fact the array
is distributed. But when operations such as add() are requested,
the corresponding pieces of the input arrays must be operated
over. Figure 1 will help illustrate. Each local piece of the array
has its own shape (in parenthesis) and knows its portion of the
distribution (in square brackets). Each local piece also knows the
global shape.

Fig. 1: Each local piece of the gain.ndarray has its own shape
(in parenthesis) and knows its portion of the distribution (in square
brackets). Each local piece also knows the global shape.

A fundamental design decision was whether to subclass
ndarray or to provide a work-alike replacement for the entire
numpy module. The NumPy documentation states that ndarray
implements __new__() in order to control array creation via
constructor calls, view casting, and slicing. Subclasses implement
__new__() for when the constructor is called directly, and
__array_finalize__() in order to set additional attributes

Design
The need for parallel programming and running these programs on
parallel architectures is obvious, however, efficiently programming
for a parallel environment can be a daunting task. One area of

[Image 350x527 to 526x748]

\[
\begin{array}{cccc}
\text{[0:3,0:3]} & \text{[0:3,3,0:3]} & \text{[0:3,3,3,0:3]} & \text{[0:3,9,0:3]} \\
\text{(3,3)} & \text{(3,3)} & \text{(3,3)} & \text{(3,3)} \\
\text{[3,6,0:3]} & \text{[3,6,3,0:3]} & \text{[3,6,3,3,0:3]} & \text{[3,6,9,0:3]} \\
\text{(3,3)} & \text{(3,3)} & \text{(3,3)} & \text{(3,3)} \\
\text{[3,6,9,12,0:3]} & \text{[3,6,9,12,3,0:3]} & \text{[3,6,9,12,3,3,0:3]} & \text{[3,6,9,12,9,0:3]} \\
\text{(3,3)} & \text{(3,3)} & \text{(3,3)} & \text{(3,3)} \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{[0:6,0:12]} & \text{[0:6,6,0:12]} & \text{[0:6,9,0:12]} & \text{[0:6,9,12,0:12]} \\
\text{(3,3)} & \text{(3,3)} & \text{(3,3)} & \text{(3,3)} \\
\end{array}
\]

...
or further modify the object from which a view has been taken. One can imagine an ndarray subclass called gainarray circumventing the usual ndarray base class memory allocation and instead allocating a smaller ndarray per process while retaining the global shape. One problem occurs with view casting -- with this approach the other ndarray subclasses know nothing of the distributed nature of the memory within the gainarray. NumPy itself is not designed to handle distributed arrays. By design, ufuncs create an output array when one is not specified. The first hook which NumPy provides is __array_prepare__() which is called after the output array has been created. This means any ufunc operation on one or more gainarray instances without a specified output would automatically allocate the entire output on each process. For this reason alone, we opted to reimplement the entire numpy module, controlling all aspects of array creation and manipulation to take into account distributed arrays.

We present a new Python module, gain, developed as part of the main Global Arrays software distribution. The release of GA v5.0 contained Python bindings based on the complete GA C API, available in the extension module ga. The GA bindings as well as the gain module were developed using Cython. With the upcoming release of GA v5.1, the module ga.gain is available as a drop-in replacement for NumPy. The goal of the implementation is to allow users to write

```
import ga.gain as numpy
```

and then execute their code using MPI e.g.

```
mpiexec -np 4 python script.py
```

In order to succeed as a drop-in replacement, all attributes, functions, modules, and classes which exist in numpy must also exist within gain. Efforts were made to reuse as much of numpy as possible, such as its type system. As of GA v5.1, arrays of arbitrary fixed-size element types and sizes can be created and individual fields of C struct data types accessed directly. GAiN is able to use the numpy types when creating the GA instances which back the gain.ndarray instances.

GAiN follows the owner-computes rule [Zim88]. The rule assigns each computation to the processor that owns the data being computed. Figures 2 and 3 illustrate the concept. For any array computation, GAiN bases the computation on the output array. The processes owning portions of the output array will acquire the corresponding pieces of the input array(s) and then perform the computation locally, calling the original NumPy routine on the corresponding array portions. In some cases, for example if the output array is a view created by a slicing operation, certain processors will have no computation to perform.

The GAiN implementation of the ndarray implements a few important concepts including the dual nature of a global array and its individual distributed pieces, slice arithmetic, and separating collective operations from one-sided operations. When a gain.ndarray is created, it creates a Global Array of the same shape and type and stores the GA integer handle. The distribution on a given process can be queried using ga.distribution(). The other important attribute of the gain.ndarray is the global_slice. The global_slice begins as a list of slice objects based on the original shape of the array.

```
self.global_slice = [slice(0,x,1) for x in shape]
```

Slicing a gain.ndarray must return a view just like slicing a numpy.ndarray returns a view. The approach taken is to apply the key of the __getitem__(key) request to the global_slice and store the new global_slice on the newly created view. We call this type of operation slice arithmetic. First, the key is canonicalized meaning Ellipsis is replaced with slice(0, dim_max, 1) for each dimension represented by the Ellipsis, all slice instances are replaced with the results of calling slice.indices(), and all negative index values are replaced with their positive equivalents. This step ensures that the length of the key is compatible with and based on the current shape of the array. This enables consistent slice arithmetic on the canonicalized keys. Sliced arithmetic effectively produces a new key which, when applied to the same original array, produces the same results had the same sequence of keys been applied in order. Figures 4 and 5 illustrate this concept.

When performing calculations on a gain.ndarray, the current global_slice is queried when accessing the local data or fetching remote data such that an appropriate ndarray data block is returned. Accessing local data and fetching remote data is performed by the gain.ndarray.access() and gain.ndarray.get() methods, respectively. Figure 6 illustrates how access() and get() are used. The ga.access() function on which gain.ndarray.access() is based will always return the entire block owned by the calling process. The returned piece must be further sliced to appropriately match the
cur

current global_slice. The ga.strided_get() function on which gain.ndarray.get() method is based will fetch data from other processes without the remote processes’ cooperation i.e. using one-sided communication. The calling process specifies the region to fetch based on the current view’s shape of the array. The global_slice is adjusted to match the requested region using slice arithmetic and then transformed into a ga.strided_get() request based on the global, original shape of the array.

Recall that GA allows the contiguous, process-local data to be accessed using ga.access() which returns a C-contiguous ndarray. However, if the gain.ndarray is a view created by a slice, the data which is accessed will be contiguous while the view is not. Based on the distribution of the process-local data, a new slice object is created from the global_slice and applied to the accessed ndarray, effectively having applied first the global_slice on the global representation of the distributed array followed by a slice representing the process-local portion.

After process-local data has been accessed and sliced as needed, it must then fetch the remote data. This is again done using ga.get() or ga.strided_get() as above. Recall that one-sided communication, as opposed to two-sided communication, does not require the cooperation of the remote process(es). The local process simply fetches the corresponding array section by performing a similar transformation to the target array’s global_slice as was done to access the local data, and then translates the modified global_slice into the proper arguments for ga.get() if the global_slice does not contain any step values greater than one, or ga.strided_get() if the global_slice contained step values greater than one.

One limitation of using GA is that GA does not allow negative stride values corresponding to the negative step values allowed for Python sequences and NumPy arrays. Supporting negative step values for Gain required special care -- when a negative step is encountered during a slice operation, the slice is applied as usual. However, prior to accessing or fetching data, the slice is inverted from a negative step to a positive step and the start and stop values are updated appropriately. The ndarray which results from accessing or fetching based on the inverted slice is then re-inverted, creating the correct view of the new data.

Another limitation of using GA is that the data distribution cannot be changed once an array is created. This complicates such useful functionality as numpy.reshape(). Currently, Gain must make a copy of the array instead of a view when altering the shape of an array.

Translating the numpy.flatiter class, which assumes a single address space while translating an N-dimensional array into a 1D array, into a distributed form was made simpler by the use of ga.gather() and ga.scatter(). These two routines allow individual data elements within a GA to be fetched or updated. Flattening a distributed N-dimensional array which had been distributed in blocked fashion will cause the blocks to become discontiguous. Figure 7 shows how a 6 x 6 array might be distributed and flattened. The ga.get() operation assumes the requested patch has the same number of dimensions as the array from which the patch is requested. Reshaping, in general, is made difficult by GA and its lack of a redistribute capability. However, in this case, we can use ga.gather() and ga.scatter() to fetch and update, respectively, any array elements in any order. ga.gather() takes a 1D array-like of indices to fetch and returns a 1D ndarray of values. Similarly, ga.scatter() takes a 1D array-like of indices to update and a 1D array-like buffer containing the values to use for the update. If a gain.flatiter is used as the output of an operation, following the owner-computes rule is difficult. Instead, pseudo-owners are assigned to contiguous slices of the of 1D view. These pseudo-owners gather their own elements as well as the corresponding elements of the other inputs, compute the result, and scatter the result back to their own elements. This results in additional communication which is otherwise avoided by true adherence to the owner-computes rule. To avoid this inefficiency, there are some cases where operating over gain.flatiter instances can be optimized, for example with gain.dot() if the same flatiter is passed as both inputs, the base of the flatiter is instead multiplied together element-wise and then the gain.sum() is taken of the resulting array.
The success of GAiN hinges on its ability to enable distributed array processing in NumPy, to transparently enable this processing, and most importantly to efficiently accomplish those goals. Performance Python [Ram08] “perfpy” was conceived to demonstrate the ways Python can be used for high performance computing. It evaluates NumPy and the relative performance of various Python extensions to NumPy. It represents an important benchmark by which any additional high performance numerical Python module should be measured. The original program laplace.py was modified by importing ga.gain in place of numpy and then stripping the additional test codes so that only the gain(numpy) test remained. The latter modification makes no impact on the timing results since all tests are run independently but was necessary because gain is run on multiple processes while the original test suite is serial. The program was run on the chinook supercomputer at the Environmental Molecular Sciences Laboratory, part of Pacific Northwest National Laboratory. Chinook consists of 2310 HP DL185 nodes with dual socket, 64-bit, Quad-core AMD 2.2 GHz Opteron processors. Each node has 32 Gbytes of memory for 4 Gbytes per core. Fast communication between the nodes is obtained using a single rail Infiniband interconnect from Voltaire (switches) and Melanox (NICs). The system runs a version of Linux based on Red Hat Linux Advanced Server. GAiN utilized up to 512 nodes of the cluster, using 4 cores per node.

In Figure 8, GAiN is shown to scale up to 2K cores on a modest problem size. GAiN is also able to run on problems which are not feasible on workstations. For example, to store one 100,000 x 100,000 matrix of double-precision numbers requires approximately 75GB. During the evaluation, it was noted that a lot of time was spent within global synchronization calls e.g. ga.sync(). The source of the calls was traced to, among other places, the vast number of temporary arrays getting created. Using GA statistics reporting, the original laplace.py code created 912 arrays and destroyed 910. Given this staggering figure, an array cache was created. The cache is based on a Python dict using the shape and type of the arrays as the keys and stores discarded GA instances represented by the GA integer handle. The number of GA handles stored per shape and type is referred to as the cache depth. The gain.ndarray instances are discarded as usual. Utilizing the cache keeps the GA memory from many allocations and deallocations but primarily avoids many synchronization calls. Three cache depths were tested, as shown in Table 1. The trade-off of using this cache is that if the arrays used by an application vary wildly in size or type, this cache will consume too much memory. Other heuristics could be developed to keep the cache from using too much memory e.g. a maximum size of the cache, remove the least used arrays, remove the least recently used. Based on the success of the GA cache, it is currently used by GAiN.

Conclusion

GAiN succeeds in its ability to grow problem sizes beyond a single compute node. The performance of the perfpy code and the ability to drop-in GAiN without modification of the core implementation demonstrates its utility. As described previously, GAiN allows certain classes of existing NumPy programs to run using GAiN with sometimes as little effort as changing the import statement, immediately taking advantage of the ability to run in a cluster environment. Once a smaller-sized program has been developed and tested on a desktop computer, it can then be run on a cluster with very little effort. GAiN provides the groundwork for large distributed multidimensional arrays within NumPy.

Future Work

GAiN is not a complete implementation of the NumPy API nor does it represent the only way in which distributed arrays can be achieved for NumPy. Alternative parallelization strategies besides the owner-computes rule should be explored. GA allows for the get-compute-put model of computation where ownership of data is largely ignored, but data movement costs are increased. Task parallelism could also be explored if load balancing becomes an issue. The GA cache should be exposed as a tunable parameter. Alternative temporary array creation strategies could be developed such as lazy evaluation.

Acknowledgment

A portion of the research was performed using the Molecular Science Computing (MSC) capability at EMSL, a national scientific user facility sponsored by the Department of Energy’s Office
of Biological and Environmental Research and located at Pacific Northwest National Laboratory (PNNL). PNNL is operated by Battelle for the U.S. Department of Energy under contract DE-AC05-76RL01830.

REFERENCES

[Apr09] E. Apra, A. P. Rendell, R. J. Harrison, V. Tipparaju, W. A. deJong, and S. S. Xantheas. Liquid water: obtaining the right answer for the right reasons, Proceedings of the Conference on High Performance Computing Networking, Storage, and Analysis, 66:1-7, 2009.

[Asc99] D. Ascher, P. F. Dubois, K. Hinsen, J. Hugunin, and T. Oliphant. Numerical Python, UCRL-MA-128569, 1999.

[Beh11] S. Behnel, R. Bradshaw, C. Citro, L. Dalcin, D. S. Seljebotn, and K. Smith. Cython: The Best of Both Worlds, Computing in Science Engineering, 13(2):31-39, March/April 2011.

[Buy99] R. Buyya. High Performance Cluster Computing: Architectures and Systems, Vol. 1, Prentice Hall PTR, 1 edition, May 1999.

[Dai09] J. Daily. GAiN: Distributed Array Computation with Python, Master’s thesis, Washington State University, Richland, WA, August 2009.

[Dal05] L. Dalcin, R. Paz, and M. Storti. MPI for python, Journal of Parallel and Distributed Computing, 65(9):1108-1115, September 2005.

[Dal08] L. Dalcin, R. Paz, M. Storti, and J. D’Elia. MPI for python: Performance improvements and MPI-2 extensions, Journal of Parallel and Distributed Computing, 68(5):655-662, September 2005.

[Dot04] Y. Dotsenko, C. Coarfa, and J. Mellor-Crummey. A Multi-Platform Co-Array Fortran Compiler, Proceedings of the 13th International Conference on Parallel Architectures and Compilation Techniques, 29-40, 2004.

[Dub96] P. F. Dubois, K. Hinsen, and J. Hugunin. Numerical Python, Computers in Physics, 10(3), May/June 1996.

[Ede07] A. Edelman. The Star-P High Performance Computing Platform, IEEE International Conference on Acoustics, Speech, and Signal Processing, April 2007.

[Eit07] B. Eitzen. Gpupy: Efficiently using a gpu with python, Master’s thesis, Washington State University, Richland, WA, August 2007.

[Gra09] B. Granger and F. Perez. Distributed Data Structures, Parallel Computing and IPython, SIAM CSE 2009.

[Gro99a] J. Nieplocha, M. Krishnan, B. Palmer, V. Tipparaju, and Y. Zhang. Exploiting processor groups to extend scalability of the GA shared memory programming model, Proceedings of the 2nd conference on Computing Frontiers, 262-272, 2005.

[Gro99b] J. Nieplocha, M. Krishnan, B. Palmer, V. Tipparaju, and Y. Zhang. Exploiting processor groups to extend scalability of the GA shared memory programming model, Proceedings of the 2nd conference on Computing Frontiers, 262-272, 2005.

[Nie00] J. Nieplocha, J. Ju, and T. P. Straatsma. A multiprotocol communication support for the global address space programming model on the IBM SP, Proceedings of EuroPar, 2000.

[Nie05] J. Nieplocha, M. Krishnan, B. Palmer, V. Tipparaju, and Y. Zhang. Exploiting processor groups to extend scalability of the GA shared memory programming model, Proceedings of the 2nd conference on Computing Frontiers, 262-272, 2005.

[Nie06] J. Nieplocha, B. Palmer, V. Tipparaju, M. Krishnan, H. Trease, and E. Apra. Advances, Applications and Performance of the Global Arrays Shared Memory Programming Toolkit, International Journal of High Performance Computing Applications, 20(2):203-231, 2006.

[Nie10] J. Nieplocha, M. Krishnan, B. Palmer, V. Tipparaju, and J. Ju. The Global Arrays User’s Manual.

[Oli06] T. E. Oliphant. Guide to NumPy, http://www.tramy.us/, March 2006.

[Pal07] W. Palm III. A Concise Introduction to Matlab, McGraw-Hill, 1st edition, October 2007.

[Pan06] R. Panuganti, M. M. Baskaran, D. E. Hudak, A. Krishnamurthy, J. Nieplocha, A. Rountev, and P. Sadayappan. GAMMA: Global Arrays Meets Matlab, Technical Report. ftp://ftp.cse.ohio-state.edu/pub/tech-report/2006/TR15.pdf

[Per07] F. Perez and B. E. Granger. IPython: a System for Interactive Scientific Computing, Computing in Science Engineering, 9(3):21-29, May 2007.

[Ras04] C. E. Rasmussen, M. J. Sottile, J. Nieplocha, R. W. Numrich, and E. Jones. Co-array Python: A Parallel Extension to the Python Language, Euro-Par, 632-637, 2004.

[Zim88] H. P. Zima, H. Bast, and M. Gerndt. SUPERB: A tool for semi-automatic SIMD/MIMD Parallelization, Parallel Computing, 6:1-18, 1988.