Abstract. The Sznajd model is an Ising spin model representing a simple mechanism of making up decisions in a closed community. In the model each member of the community can take two attitudes A or B represented by a spin up or spin down state respectively. It has been shown that, in one-dimension starting from a totally random initial state, three final fixed points can be obtained; all spins up, all spins down or an antiferromagnetic state in which each site take a state which is opposite from its two nearest neighbors. Here, a modification of the updating rule of the Sznajd model is proposed in order to avoid such antiferromagnetic state since it is considered to be an unrealistic state in a real community.

The Sznajd model is a successful Ising spin model describing a simple mechanism of making up decisions in a closed community. The model allows each member of the community to have two attitudes, to vote for option A or to vote for option B. These two attitudes are identified with the state of spins variables up or down respectively. A dynamic is established in the model in which a selected pair of adjacent spins influence their nearest neighbors through certain rules, applied in a random sequential manner. In several votes (units of evolution time) some difference $m$ of voters for A and against is expected. The dynamic rules of the Sznajd model are

\begin{align}
&\text{if } S_i S_{i+1} = 1 \text{ then } S_{i-1} \text{ and } S_{i+2} \text{ take the direction of the selected pair } [i,i+1], \\
&\text{if } S_i S_{i+1} = -1 \text{ then } S_{i-1} \text{ takes the direction of } S_{i+1} \text{ and } S_{i+2} \text{ the direction of } S_i,
\end{align}

being $S_i$ the state of the spin variable at site $i$. These rules describe the influence of a given pair of members of the community on the decision of its nearest neighbours.
In one dimension, the original rules give rise to three limiting cases in the evolution of the system:

(i) all members of the community vote for A (all spins up),
(ii) all members of the community vote for B (all spins down),
(iii) 50% vote for A and 50% vote for B (alternating state).

Here, attention is paid to the last limiting antiferromagnetic case (iii). This antiferromagnetic case, although possible in other spins systems, can be considered to be quite unrealistic in a model trying to represent the behavior of a community. To achieve exactly a 50-50 final state in a community is almost impossible, specially if it is composed by more than a few dozens of members. On the other hand such antiferromagnetic state implies that each member of the community is surrounded by a neighbor which has an opposite opinion. A quite “uncomfortable” situation, certainly.

From a simulational point of view, if the evolution of a one-dimensional Sznajd model is started from an antiferromagnetic state, i.e., a chain of neighbors with opposite opinions, the original dynamic rules does not give rise to any evolution at all.

In order to avoid the unrealistic 50-50 alternating final state, new dynamic rules are proposed:

- if \( S_i S_{i+1} = 1 \) then \( S_{i-1} \) and \( S_{i+2} \) take the same direction of the pair \([i, i+1]\), \( (r1) \)
- if \( S_i S_{i+1} = -1 \) then \( S_i \) take the direction of \( S_{i-1} \) and \( S_{i+1} \) take the direction of \( S_{i+2} \). \( (r2) \)

Using the new rules, in case of disagreement of the pair \( S_i - S_{i+1} \), rule \( r2 \) make the spin \( i \) to “feel more confortable” since it ends up with at least one neighbor having its own opinion.

Two samples of evolution of a system following the new rules and starting from an antiferromagnetic state are shown in Fig. 1, for a \( N = 100 \) lattice size. It can be seen that the 50-50 final state in completely avoided and that the other two types of total agreement (ferromagnetic) final states can be achieved, with equal probability, starting the systems from the same initial condition. Time \( t \) is advanced by one when each spin of the lattice has had one (probabilistic) opportunity to be updated.

Finally the scaling properties of the new model are tested by calculating the scaling exponent of the number of spins that does change their state with time. The value of this exponent has been shown to be \( 3/8 \) for the original Sznajd model. In Fig. 2 a log-log plot of the evolution of the number of spins in remaining the same state at time \( t \) is shown for the original Sznajd model and for the new model proposed here. Plots of Fig.
2 were obtained from simultaneous simulations of both models, using the same random numbers for update each lattice starting from the same initial condition. See figure caption for the parameters used in simulations. It can be seen that the model proposed here share the same type of scaling features as the original Sznajd model, but the value for the scaling exponent seems to be different. Although, more detailed simulations would be needed in order to verify exactly this last statement.

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Figure 1: Two sample of time evolution of the modified model using an alternating state as initial condition.
Figure 2: The number of sites with unchanged state follows a power law in both models. Lattice size $N=1000$ and total simulation time $T=2000$. 

Sznajd Original: Slope $= -0.36$

Modified Model: Slope $= -0.25$