Spin alignment of vector mesons in heavy ion and proton–proton collisions

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The spin alignment matrix element \( \rho_{00} \) for the vector mesons \( K^0 \) and \( \phi(1020) \) has been measured in RHIC at central rapidities. These measurements are consistent with the absence of polarization with respect to the reaction plane in mid-central \( \text{Au} + \text{Au} \) collisions whereas, when measured with respect to the production plane in the same reactions and in \( p + p \) collisions, a non-vanishing and \( p_{\perp} \)-dependent \( \rho_{00} \) is found. We show that this behavior can be understood in a simple model of vector meson production where the spin of their constituent quarks is oriented during hadronization as the result of Thomas precession.

The study of spin polarization of produced hadrons in reactions at high energies has opened a window to the understanding of the underlying dynamics of quark recombination. In the context of heavy-ion collisions, polarization studies can also help to understand the evolution of the system from its early stages [1–3].

Polarization analyses require to determine a given direction that serves as the spin quantization axis. From the experimental point of view, it is possible to determine two directions: the normal to the reaction and the normal to the production planes. The first plane is defined as the one containing the impact parameter and the beam direction vectors whereas the second one is defined as containing the hadron’s final momentum and the beam direction vectors.

Polarization studies have long been carried out for hyperons at lower energies and for smaller systems [5] and report mainly only average values for the polarization but not its \( p_{\perp} \)-dependence [6,7]. In summary, the existing data on the hyperon \( p_{\perp} \)-dependent polarization in \( A + A \) reactions is scarce.

On the other hand, the STAR collaboration has recently reported measurements of the \( p_{\perp} \)-dependence of the polarization of the vector mesons \( \phi \) and \( K^* \) [8]. These measurements refer to the \( 00 \) component of the so-called spin alignment density matrix \( \rho \), which is the density matrix for a two-spin one-half system in a triplet state, expressed in terms of the coupled basis [9]. Recall that a value \( \rho_{00} = 1/3 \), means that the spin of the vector meson is not aligned with respect to the chosen quantization axis. Deviations from this value indicate a degree of polarization of the vector spin which ultimately might reflect a polarization of the constituent quarks.

The experimental findings reported can be summarized as follows: When the spin alignment is referred to the reaction plane in \( \text{Au} + \text{Au} \) collisions at \( \sqrt{s_{NN}} = 200 \) GeV and measured at mid-rapidity, \( \rho_{00} \approx 1/3 \), and it remains constant both as a function of \( p_{\perp} \) in the range \( 0 < p_{\perp} < 5 \) GeV, for mid-central collisions, and as a function of the average number of participants in the same \( p_{\perp} \) range, for both \( \phi \) and \( K^* \). When the spin alignment is referred to the production plane and measured at mid-rapidity, both for \( p + p \) and mid-central \( \text{Au} + \text{Au} \) collisions at \( \sqrt{s_{NN}} = 200 \) GeV, \( \rho_{00} > 1/3 \) and it has a concave shape as a function of \( p_{\perp} \) in the range \( 0 < p_{\perp} < 5 \) GeV with minima at slightly different intermediate values of \( p_{\perp} \) for \( \phi \) and \( K^* \).

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An interesting observation that can be inferred from the above listed results is that the dynamics of hadron formation seems to play a role to orient the spin of quarks that form meson.

A possible origin of a quark polarization driving the polarization of vector mesons is discussed in Ref. [1] which study the transfer of local relative angular momentum in peripheral nuclear collisions to the quark spin polarization by means of rescattering during the reaction. However, this mechanism also predicts a small global polarization growing almost linearly with impact parameter which, according to the aforementioned results is not observed in data.

Another interesting possibility that has only been studied in the context of hyperon polarization [10,11] is the scenario where the spin of a quark is oriented during the recombination process. The semiclasical picture accounting for the polarization is the Thomas precession produced by the accelerating force that pulls a slow moving quark \(q_f\) to form a fast moving hadron [10]. This mechanism also predicts that if the quark is fast \(q^f\) and is decelerated to form the hadron, its polarization will be of opposite sign compared to the case when it is accelerated.

Recall that recombination is a main channel for hadron production spanning the intermediate \(p_{\perp}\) region \(2 \leq p_{\perp} \leq 5\). In this range, one can assume that the formed hadron is made up from the recombination of a slow and a fast quark. The polarization of the resulting hadron can be thus used as a testing ground of such scenario. Other scenario where fragmentation of a fast quark, instead of recombination is proposed as the main mechanism to produce the vector mesons is discussed Ref. [12]. For this model to work, the authors assume, based on the \(e^+e^-\) annihilation data, that the polarization of the anti-quark in the meson is proportional to that of the fragmenting one but opposite in sign. In contrast, as stated above, this is a natural consequence of Thomas precession in the recombination of slow and fast quarks.

For the Thomas precession to work, the pulling force is required to not be parallel to the original quark velocity since the Thomas precession frequency for a quark is oriented during the recombination process.

The pulling force is equal to the change in momentum \(\Delta p\) of the given quark, in the interval of time \(\Delta t\) for the recombination process to happen, that is

\[
F = \frac{\Delta p}{\Delta t}.
\]

Thus \(\omega_T\) for the given quark can be computed as the average over this time interval [10], namely

\[
\omega_T^{s,f} = \frac{\Delta \omega^{s,f}}{\Delta t}.
\]

where \(- (+)\) sign refers to the \(q^s (q^f)\). \(\omega_T^{s,f}\) is the magnitude of the Thomas precession frequency for \(q^s\) and \(q^f\), respectively and \(\Delta E\) is the change of energy in the process of hadron formation.

In this work we use the Thomas spin precession mechanism to describe the spin alignment of vector mesons produced at central rapidity in Au + Au and p + p collisions at \(\sqrt{s_{NN}} = 200\). We show that under very simple assumptions, data for \(\rho_{00}\) are well reproduced within this approach.

The physical picture we use is that of a fast quark that decelerates and a slow one that accelerates to form a fast moving hadron. In the process, Thomas precession makes the spin of the former to acquire a positive polarization whereas the latter acquires a negative one. For central rapidities, the large momentum component of the hadron will thus be its transverse momentum \(p_{\perp}^H\) whereas the small component will be its longitudinal one, \(p_{\parallel}^H\). We will assume that in the beam collision, a hard interaction produces a fast quark moving with a large transverse momentum \(p_{\perp}^f\) and, to simplify matters, a vanishing longitudinal momentum. This fast quark combines with the slow one, that we assume moves originally mainly in the longitudinal direction with momentum \(p_{\parallel}^s\) and, also for simplicity, take it with vanishing transverse momentum. This sharp difference in the original direction of motion of the recombining quarks is at the core of the produced polarization since, as we proceed to show, it gives rise to a distinct \(p_{\perp}\) dependence of \(\rho_{00}\) which seems to be also observed in data.

In order to form the hadron, which should move with an intermediate value of momentum, between that of the \(q^f\) and of the \(q^s\), the fast quark should slow down whereas the slow quark should speed up. Notice that for this mechanism to work, there is no need to assume that the process happens only in either a proton–proton or a nucleus–nucleus collision. Notice also that the momentum of the formed hadron provides a fixed direction to define that \(q^f (q^s)\) decelerates (accelerates), whereas, when referred to the reaction plane, no such fixed direction exists, since the direction of the impact parameter vector changes from one reaction to another and in such situation either quark can accelerate or decelerate.

The pulling force is equal to the change in momentum \(\Delta p\) of the given quark, in the interval of time \(\Delta t\) for the recombination process to happen, that is
\( x_\parallel = p_{\parallel}^{\perp/H} / p_{\parallel}^H \),
\( x_\perp = p_{\perp}^{\perp/H} / p_{\perp}^H \),
and have neglected the longitudinal hadron’s momentum with respect to its transverse one.

Similarly, for the \( q^f \) change of momentum when decelerating, notice that we have
\[
\Delta p^f = p_{\parallel}^{\perp/H} - p_{\perp}^{\perp/H} - p_{\perp}^f.
\]
(9)
Since \( \beta^f \) initially points along the perpendicular direction and, although this velocity vector changes so that the final hadron’s momentum eventually picks up a longitudinal component, the dominant component of the \( q^f \) in the hadron is the transverse one. Therefore, for the cross product of \( \Delta p^f \) with \( \beta^f \) we get
\[
\Delta p^f \times \beta^f = (1 - x_\parallel) p_{\parallel}^H (\sin \theta)^f
\]
where we have enforced momentum conservation \( x_\perp + x_\parallel = 1 \) and have approximated \( \beta^f \approx 1 \).

Now, we proceed to compute the change in energy. To compute \( (\sin \theta)^f \), notice that since \( \beta^f \) points along the parallel direction, its change is directly along the perpendicular one, the initial angle between these vectors is \( \pi /2 \) and the average one should be close to \( \pi /4 \). Thus \( (\sin \theta)^f \approx 1 / \sqrt{2} \). Rather than approximating \( \beta^f \) (and therefore \( y^f \)), we introduce a factor \( a \) for this polarization and let this vary such that \( 0 < a < 1 \).

Thus we write
\[
\omega^f = a \frac{\Delta p^f}{\Delta t},
\]
\[
a = \left( \frac{\gamma^s}{1 + \gamma^s} \right)^2 \beta^f (\sin \theta)^f.
\]
(11)
To compute \( (\sin \theta)^f \), notice that since \( \Delta p^f \) and \( \beta^f \) are almost perpendicular, \( (\sin \theta)^f \approx 1 \).

The change in energy is common to both the accelerating \( q^f \) and the decelerating \( q^f \)
\[
\Delta E = \left\{ \left[ (p_{\perp}^f)^2 + (p_{\perp}^f)^2 + (m^f)^2 \right]^{1/2}
+ \left[ (p_{\parallel}^f)^2 + (p_{\parallel}^f)^2 + (m^f)^2 \right]^{1/2}
- \left[ (p_{\parallel}^H)^2 + (p_{\perp}^H)^2 + (m^H)^2 \right]^{1/2} \right\}
\]
\[
\approx \left\{ \left[ (p_{\perp}^f)^2 + (m^f)^2 \right]^{1/2} + \left[ (p_{\parallel}^f)^2 + (m^f)^2 \right]^{1/2}
- \left[ (p_{\parallel}^H)^2 + (m^H)^2 \right]^{1/2} \right\}
\]
\[
= p_{\parallel}^f \left[ 1 + \frac{(m^f)^2}{2(p_{\perp}^f)^2} \right] + \left[ (p_{\parallel}^f)^2 + (m^f)^2 \right]^{1/2}
- p_{\parallel}^H \left[ 1 + \frac{(m^H)^2}{2(p_{\perp}^f)^2} \right],
\]
(12)
where we have set \( p_{\perp}^f = p_{\parallel}^f = 0 \), neglected \( (p_{\perp}^f)^2 \) compared with \( (p_{\parallel}^f)^2 \) and expanded the square roots assuming that the transverse momentum components are large. Introducing the relation between the hadron and \( q^f \) transverse momenta \( p_{\perp}^f = p_{\parallel}^f / z \), with \( 0 < z < 1 \), we get
\[
\Delta E = \left\{ \frac{p_{\parallel}^H}{z} \left[ 1 + \frac{z^2(m^f)^2}{2(p_{\perp}^f)^2} \right] + \left[ \frac{(m^f)^2}{2(m_{\perp}^f)^2} \right]^{1/2}
- p_{\parallel}^H \left[ 1 + \frac{(m^H)^2}{2(p_{\perp}^f)^2} \right] \right\}^{1/2}
\]
where we have made use of the assumption that the rapidity of the \( q^f \) coincides with that of the hadron. Notice that the above expression can be simplified. In particular, since \( p_{\parallel}^f = m_{\perp}^f \sin y^H \), we can write
\[
\Delta E = \left\{ \frac{(m^f)^2}{2(m_{\perp}^f)^2} \right\}^{1/2} = m^f \cosh y^H.
\]
(14)

Therefore, \( \Delta E \) can be expressed as
\[
\Delta E = \left\{ \left( 1 - z \right) p_{\parallel}^H + \left[ z(m_{\perp}^f)^2 -(m^H)^2 \right] \right\}^{1/2}
+ \frac{m^f \cosh y^H}{z}.
\]
(15)
We emphasize that the approximation to compute \( \Delta E \) is such that Eq. (15) is valid for \( p_{\perp} \gg m^H \) and that for \( z \lesssim 1 \) the validity of the approximation can be extended to lower values of \( p_{\perp} \).

It is now easy to compute the polarization for the slow and fast quarks by means of Eq. (2) and from them, the \( \rho_{00} \) density matrix element given by
\[
\rho_{00} = \frac{1 - P^f}{3 + P^f/3}.
\]
(16)

Let us first proceed to apply the model to compute \( \rho_{00} \) for \( \phi \). This case is the simplest one to treat within our approach since the quark content of this particle is \( s \) and thus either one of these quarks can be thought of as being the fast (\( q^f \)) or the slow (\( q^s \)) one. Rather than making an exhaustive search in the parameter space, we choose reasonable values for them. We first fix the \( \phi \) and strange quark masses to be \( m^f = 1.02 \) GeV, \( m^s = 0.5 \) GeV. The rapidity value we use is \( y^H = 1 \) and the formation time \( \Delta t = 1 \) fm.

For the fractions of longitudinal and transverse momenta that the slow quark has inside the \( \phi \) we take \( x_{\perp} = x_{\parallel} = 0.5 \). The fraction of the transverse momentum carried by the \( \phi \) from the fast quark is taken as \( z = 0.9 \).

Fig. 1 shows \( \rho_{00}^\phi \) as a function of \( p_{\perp}^\phi \) compared to data from STAR [8] for \( p + p \) and \( Au + Au \) collisions at centrality 20–60%. A good description is obtained for \( a = 0.25 \).

We now proceed to apply this model to the case of \( K^+ \), whose quark content is \( d \bar{s} \). However, in this case one needs to be careful since the symmetry between the masses, present in the description of \( \phi \), is absent. Consequently the spin alignment has to be treated in average. To this end, let us first take a simple scenario and consider the arithmetic average in the following manner
\[
\rho_{00}^{K^+} = \frac{1}{2} \left( \rho_{00}^{f,s,d = d} + \rho_{00}^{f = d,s = s} \right).
\]
(17)

Fig. 2 shows \( \rho_{00}^{K^+} \) as a function of \( p_{\perp}^{K^+} \) compared to data from STAR [8] for \( p + p \) and \( Au + Au \) collisions at centrality 20–60%, measured in the production plane. The curves are computed such that we employ the same set of parameters as in the computation of \( \rho_{00}^\phi \) (with \( m^d = 0.3 \) GeV) except for the value of \( z \). The reason is that, whereas in the case that \( q^f = s \) one can think that in order for this fast quark to pick up a \( d \), the momenta of \( s \) and \( K^+ \) are similar, when \( q^f = d \), its momentum must be larger, given the mass difference between \( K^+ \) and \( d \). Thus if \( q^f = s \) we choose \( z = 0.9 \) whereas when \( q^f = d \) we use \( z = 0.3 \). The upper dashed curve represents the case for \( \rho_{00}^{f,s,d = d} \), whereas the lower dashed curve is for \( \rho_{00}^{f = d,s = s} \). The intermediate solid curve represents \( \rho_{00}^{K^+} \) as the algebraic average of the above, as in Eq. (17).

An alternative approach is to consider that the \( \rho_{00}^{K^+} \) can be computed by the substitution
Fig. 1. (Color online.) $\rho_{00}$ as a function of $p_\perp$ for $\phi(1020)$ using the model parameters described in the text, compared to data from STAR [8] for $p + p$ and Au + Au collisions at centrality 20–60%, measured with respect to the production plane. For clarity, the $p_\perp$ for $p + p$ data has been displaced by 0.09 GeV with respect to the reported central value. The statistical and systematic errors have been added in quadrature. For comparison, we also draw the constant value 1/3 that represents the absence of polarization.

Fig. 2. (Color online.) $\rho_{00}$ as a function of $p_\perp$ for $K^*$ using the model parameters described in the text, compared to data from STAR [8] for $p + p$ and Au + Au collisions at centrality 20–60%, measured with respect to the production plane. For clarity, the $p_\perp$ for $p + p$ data has been displaced by 0.09 GeV with respect to the reported central value. The statistical and systematic errors have been added in quadrature. The upper dashed curve represents the case where $\rho_{00}$ is computed using the average product of polarizations. For comparison, we also draw the constant value 1/3 that represents the absence of polarization.

$$p^s p^f \rightarrow \frac{p^s=p f=d + p^s=d p^f=s}{2},$$  \hfill (18)

that is, by the average product of polarizations, in Eq. (16). Fig. 2 shows also this possibility represented by the intermediate dotted curve, using the same set of parameters for the cases were $q^f = s$, $q^s = d$ and $q^f = d$, $q^s = s$, as discussed above. As can be seen from the figure, no significant difference is found with the case where the average is taken with the functions $\rho_{00}$ and both approaches give a good description of data.

In conclusion, we have shown that data on the vector mesons $\phi$ and $K^*$ spin alignment with respect to the production plane in Au + Au and $p + p$ collisions are well described by assuming that these hadrons are produced by the recombination of a slow and a fast quark that in the process become polarized in opposite directions due to Thomas precession. In this plane, the momentum of the hadron provides a fixed direction to define whether a valence quark accelerates or decelerates. The mechanisms also sheds light on the fact that when no such fixed direction exists, the polarization vanishes. This could be the case when referring the spin alignment to the reaction plane, provided that no initial correlation between the impact parameter vector and the quark spins is present and the Thomas precession mechanism is indeed an important component of the polarization in the given kinematical regime. This is so because in this case, the impact parameter vector changes from one reaction to another and therefore so does the direction of the normal to the reaction plane. Thus the average polarization, which in the Thomas precession scenario involves the direction of the formed hadron, vanishes, since, when projecting this onto the normal to the reaction plane, it is proportional to the average of the cosine of the angle between these vectors, whose average vanishes. On the other hand, the polarization mechanism advocated in Ref. [1] relies on the existence of such an initial correlation arising from the presence of an orbital angular momentum in peripheral collisions, which is then transferred to the quark spin through scattering. Since this angular momentum is related to the direction of the impact parameter vector, such relation provides the correlation to possibly make the above average to not vanish.

Although data seem to be consistent with the absence of polarization in peripheral collisions with respect to the reaction plane [8], our work does not discard the scenario proposed in Ref. [1] but it merely points out that assuming the absence of an initial correlation between the impact parameter and quark spin vectors one can explain both the absence of polarization when measured with respect to the reaction plane and its presence when measured with respect to the production plane.

It is also possible that both scenarios may coexist, and the relative contributions of each deserve a close quantitative evaluation. This is work for the future to be reported elsewhere.

In the near future ALICE at the LHC will have the capability to measure and reconstruct $\phi$ and $K^*$ mesons with larger statistics [13]. In addition, its particle identification will allow these meson’s $p_\perp$ to be measured beyond 5 GeV, well into the region where energy losses become important and also where fragmentation (as opposed to the recombination picture we are using here), becomes the dominant particle production mechanism. It will thus be interesting to study how the polarization of vector mesons changes when including these effects. This is work for the future.

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