CARBON-POOR STELLAR CORES AS SUPERNOVA PROGENITORS

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ABSTRACT

Exploring stellar models which ignite carbon off-center (in the mass range of about 1.05–1.25 $M_{\odot}$, depending on the carbon mass fraction), we find that they may present an interesting Type 1 supernova (SN I) progenitor scenario, since whereas in the standard scenario runaway always takes place at the same density of about $2 \times 10^9$ g cm$^{-3}$, in our case, due to the small amount of carbon ignited, we get a whole range of densities from $1 \times 10^9$ up to $6 \times 10^9$ g cm$^{-3}$. These results could contribute to resolving the emerging recognition that at least some diversity among SNe I exists, since runaway at various central densities is expected to yield various outcomes in terms of the velocities and composition of the ejecta, which should be modeled and compared to observations.

Subject headings: stars: evolution — supernovae: general — white dwarfs

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1. INTRODUCTION

As soon as they were identified as a distinct type of supernovae, Type Ia supernovae (SNe Ia) proved to have a relatively small dispersion in their luminosity, and hence they have been used as distance indicators (“standard candles”), with particular significance for the effort to determine the cosmological parameters of our universe.

After some years, as the observational database of supernovae increased and became more detailed and accurate, it was recognized that the “standard candle” picture is inaccurate, since a certain scatter in SN Ia luminosities exists, and several empirical correlations where found, which connect the maximum luminosity with light curve shape, color evolution, spectral appearance, and host galaxy morphology. However, a physical understanding of the origin of this luminosity variation is still lacking. Moreover, some supernovae remain super- or subluminous even after these corrections are applied.

On the theoretical side, the almost unanimously accepted explanation of the phenomenon is the explosive burning of degenerate carbon in the core of a carbon-oxygen white dwarf (WD), which becomes unstable as it grows to the Chandrasekhar mass by accretion from a binary companion overflowing its Roche lobe (single degenerate scenario), or a merger of two WDs, following the angular momentum loss from the system by gravitational radiation (double degenerate scenario). However, theoretical models are still far from accurately reproducing crucial features of the observational data, such as the composition of the ejected matter. For a detailed review of the above see, e.g., Leibundgut (2000) and Hillebrandt & Niemeyer (2000).

As for the diversity in the luminosity, several explanations have been suggested, such as variations in the metallicity of the progenitor, in the carbon to oxygen ratio at its center, or in the central density at the time of ignition (Timmes et al. 2003; Röpke et al. 2005, 2006; Lesaffre et al. 2006). The variation of latter two parameters is expected to be an outcome of the variation in the initial WD mass and in the accretion history.

In this work we suggest an evolutionary scenario which could lead to a rather unusual, albeit possibly rare, type of progenitor, and thus add somewhat to the diversity. We shall look at WDs in the mass range where they undergo an off-center nondegenerate carbon-burning episode before the onset of accretion, leaving behind a star mostly, but not completely, depleted of carbon, having about 2% carbon in the central region of about 0.5 $M_{\odot}$. At reaching the Chandrasekhar mass, the central density of these progenitors could vary over a comparatively broad range, depending on several factors, as will be discussed later.

Let us first review the standard single degenerate scenario. In this scenario we have a carbon-oxygen white dwarf below the Chandrasekhar mass ($M_{\odot}$), which is a remnant of an intermediate-mass AGB star that has lost its hydrogen-rich envelope through binary evolution. At a certain stage the white dwarf begins accreting mass from its secondary companion at an appropriate rate ($M$). The increase in mass raises the density and temperature at the center, so that at a certain stage their values cross the carbon ignition line (hereafter IG), and carbon is ignited. The immediate outcome is an increase of the entropy at the center, leading to the growth of a convective region. If the size of the convective region reaches a certain value, further increase in entropy leads to expansion of the center (a decrease of the density at the center, hereafter DEC), in opposition to the accretion, tending to lead to contraction. Thus, between IG and DEC the density at the center increases, and after DEC it decreases. The temperature nevertheless continues to increase due to nuclear reactions, and with it the nuclear reaction rate and the convective flux are also increasing. When the reaction rate reaches a point where the convection can no longer compete with the entropy production rate, nuclear burning continues at almost constant density, and could reach a “dynamic” regime, where the nuclear time (defined as the time needed to exhaust all the fuel, including oxygen, at constant density) is shorter than the dynamic time (which can be defined for example as a pressure scale height divided by the speed of sound). We will refer to this situation as a “runaway” (RA), and it is clear that under these circumstances hydrostatic equilibrium can no longer be assumed. It is worth noting that the runaway does not necessarily lead to explosion, since electron capture behind the explosion front could produce a rarefaction wave which might convert the explosion into a collapse. Figure 1 shows the evolution of the density and temperature at the center

\footnote{The ignition line is defined as the locus in the $\rho_c$, $T_c$ plane, where the energy production rate from nuclear reactions equals the neutrino losses. We will refer to this point as IG.}
of a carbon-oxygen star model of mass $M = 1.18 \, M_\odot$ and carbon mass fraction $X_C = 0.05$, displaying also the relevant carbon ignition line and the points IG, DEC, and RA.

Quantitatively speaking, it is clear that the IG point is dependent on the carbon mass fraction ($X_C$), and will be located at a higher density for a lower carbon mass fraction. It also depends on the reaction rate, including the screening factor, and on the neutrino loss rate, but the sensitivity to these parameters is weaker. The DEC point is also dependent on $X_C$. For a low enough $X_C$ the amount of carbon is insufficient to raise the temperature to the RA regime, and even not to the DEC point.

If we take into account the $Q$-value of carbon burning ($\approx 4 \times 10^{17}$ erg g$^{-1}$) and the specific heat in the region under discussion $[(\partial T/\partial e)_C \approx 10^{-7}$ K erg$^{-1}$], we can estimate that the temperature will rise by about $4 \times 10^8$ K for a mass fraction of 1% carbon ($X_C = 0.01$). Since for RA the threshold temperature of oxygen ignition should be reached, i.e., about $1.4 \times 10^8$ K is needed, it is clear that if IG is reached at $T_C \approx 4 \times 10^8$ K, we need a carbon mass fraction of about 0.025 to reach RA. Clearly this is a rough estimate, since part of the energy produced by the burning of carbon is transported away, while on the other hand convection supplies fresh fuel. As we shall see below, this estimate turns out to be pretty good.

To the above, we should add the role which electron capture processes might play during evolution. A detailed discussion is given in § 2.3.3; here we just mention that the relevant processes are:

1. Electron capture on Mg$^{24}$, which is a product of carbon burning and is quite abundant, and Na$^{24}$, which cause a decrease of $Z/A$, and thus a decrease of the effective Chandrasekhar mass, leading to an accelerated compression and local heating.

2. Urea processes of two kinds, thermal and convective.

In this way we also check the influence of these processes.

Regarding the astrophysical scenario, it is usual to deal with white dwarfs that are the remnants of planetary nebula (PN) formation. In this scenario, the growth of the carbon-oxygen core of the star causes a luminosity increase, leading to envelope instability and ultimately to its ejection.

The typical mass of such white dwarfs is about $0.6 \, M_\odot$, although more massive ones do exist. According to Liebert et al. (2005), some 6% of the white dwarfs have masses above $1 \, M_\odot$.

Regarding their composition, the carbon mass fraction generally lies within the range $0.25 \leq X_C \leq 0.55$, due to some uncertainties.

As mentioned before, the key point of the standard scenario is mass accretion from a binary companion. Clearly, the accretion rate depends on the structure and evolutionary history of the binary couple. Theoretical surveys have been done in the literature, where the accretion rate, the initial mass of the accreting white dwarf, and the composition of the accreted matter served as free parameters (e.g., Nomoto & Sugimoto 1977).

In this work, we refer to the WD mass range where the original carbon core is large enough to ignite carbon before it grows toward $M_{\text{CH}}$, but small enough that ignition takes place off-center. Alas, investigating the required mass range is especially difficult, due to the need for extremely fine zoning in order to follow the burning shell and the convective region created above it. In § 3.2 we discuss this subject in detail.

In the following, we begin by first describing the computational methodology of our work, including the numerical algorithms and input physics in § 2. Our results are given in § 3. Finally, in § 4 we will discuss our results and present our conclusions, including observational predictions and suggestions for further research.

2. METHOD

2.1. Initial Models

For the sake of simplicity, rather than following the complicated evolution of the primary star in order to get the remaining carbon-oxygen white dwarf, we began our calculation with a carbon-oxygen star of appropriate mass and composition, with low enough central density at hydrostatic equilibrium, letting it contract along the well-known knee-shaped path on the central density versus central temperature diagram. This approach, which has been used by many authors since, was justified by Barkat (1971), stating that at each point along the evolutionary track of a carbon-oxygen core, growing as a result of a burning shell, the central conditions (density and temperature) are very close to the evolutionary track of a carbon-oxygen star of corresponding mass.

2.2. Stellar Evolution Code

The evolution was followed using the quasi-static Lagrangian one-dimensional evolution code ASTRA, which is an extensively improved version of the ASTRA evolution code first described by Rakavy et al. (1966), and used with some modifications many times. The quasi-static assumption, meaning that the star is dynamically stable, and the timescales for the three main processes that govern the evolution of the star, namely, the hydrodynamic motion, the convective mixing, and the exchange of energy due to thermonuclear reactions and radiative transport obeying the relation $\Delta V \ll \Delta V_{\text{conv}} \ll \Delta V_{\text{nuc}}$, is valid for most stages of stellar evolution, with the possible exception of very violent nuclear burning phases, where special care has to be taken. Since these stages are indeed in the scope of our interest, a special treatment of these stages was devised, and will be described in § 2.4.

Convective regions are treated as isentropic and fully mixed. This assumption greatly simplifies the program, as it eliminates the need to calculate the convective energy flux, and is valid (i.e., in good agreement with the mixing-length model of convection) wherever the pressure scale height is large compared to the size of the convective region of interest. In the stellar models of interest to us, this kind of condition occurs throughout the stellar core.
2.3. Input Physics

2.3.1. Nuclear Reaction Rates

Nuclear reactions were treated via two different sets of reaction rates, which were then compared to each other: (1) using an \( \alpha \) network of 13 elements from He\(^4\) up to Ni\(^{26}\), with reaction rates based on the “NON-SMOKER” tables by Rauscher & Thielemann (2001); and (2) using the reaction rates according to Caughlan & Fowler (1988) for multiple \( \alpha \) nuclei. The screening factor was treated according to Itoh et al. (1979, 1980), but a comparison was made with those of Dewitt et al. (1973).

2.3.2. Neutrino Losses

Neutrino losses were taken according to Itoh & Kohyama (1983).

2.3.3. Electron Capture

Three different processes of electron capture are considered: thermal Urca (TU), convective Urca (CU), and electron capture (EC) on carbon-burning products.

2.3.3.1. Thermal Urca (TU)

Thermal Urca is a situation in which at some Lagrangian mass point in the star (\( m_{\text{arc}} \)) the Fermi energy fulfills \( E_\text{F} = E_\text{th} \), so that throughout a mass shell (hereafter referred to as the Urca shell, or US), lying in the range of \( E_\text{F} = E_\text{th} \pm 1 \text{ keV} \), processes of emission and capture of electrons by some trace nuclei take place. Both processes are accompanied by production of neutrinos, which leave the star and create an effective local heat sink. Since the star continues to shrink and \( E_\text{F} \) increases, it is clear that \( m_{\text{arc}} \) increases and the US moves outward. The magnitude of the process depends on the mass fraction of suitable nuclei (Ergma & Paczynski 1974). In our context, the dominant nuclei are Na\(^{23}\), with a threshold of \( E_\text{th} = 4.4 \text{ MeV} \) and consequently \( \rho_\text{th} \approx 1.7 \times 10^6 \text{ g cm}^{-3} \), and afterward Ne\(^{21}\), with a threshold of \( E_\text{th} = 5.7 \text{ MeV} \), and thus \( \rho_\text{th} \approx 3.5 \times 10^6 \text{ g cm}^{-3} \).

We modeled the effect of the TU by means of the prescription by Tsuruta & Cameron (1970), while checking the sensitivity of the result to the reaction rate by varying the mass fraction of the relevant nuclei.

2.3.3.2. Convective Urca (CU)

In the convective Urca case, convection transfers nuclei through the US. These nuclei pass through a region where the difference \( E_\text{F} - E_\text{th} \) is an order of magnitude below 1 MeV, and certainly \( \Delta E \gg kT \). Since in this case the average electron capture occurs below the Fermi level, a hole is created, which is filled by an electron from above the Fermi level, and the energy surplus is emitted as a \( \gamma \) photon, causing local heating.

The effect of CU on the relevant evolution has been discussed for a long time in the literature, with contradictory conclusions (e.g., Paczynski 1972; Bruenn 1973; Couch & Arnett 1975; Lazareff 1975; Regev & Shaviv 1975; Barkat & Wheeler 1990; Mochkovitch 1996; Lesaffre et al. 2006). Lately it has been suggested that the main effect of CU is to hinder the extension of the convective region above \( m_{\text{arc}} \) (Stein et al. 1999; Bisnovatyi-Kogan 2001). Hence, in this work we try to examine the practical significance of CU, using a simplistic approach, by artificially forbidding convection above \( m_{\text{arc}} \).

2.3.3.3. Electron Capture (EC) on Carbon-burning Products

As Miyaji et al. (1980) have already shown, processes of electron capture by certain burning product nuclei, when the Fermi level \( (E_\text{F}) \) exceeds a relevant threshold \( (E_\text{th}) \), can have a major importance during the stage preceding RA, when the density, and consequently the Fermi energy, are rising. In our context, the dominant nuclei from among the carbon burning products are \(^3\)Mg\(^{24}\), with a threshold \( E_\text{th} = 5.52 \text{ MeV} \) and consequently \( \rho_\text{th} \approx 3.5 \times 10^6 \text{ g cm}^{-3} \), and afterward Na\(^{23}\), which is the electron-capture product of Mg\(^{24}\), with a threshold of \( E_\text{th} = 6.59 \text{ MeV} \), and thus \( \rho_\text{th} \approx 5.25 \times 10^6 \text{ g cm}^{-3} \).

Note that lately (Gutierrez et al. 2005) there have been claims that Na\(^{23}\) is much more abundant than previously thought, and is more abundant than even Mg\(^{24}\). Since the threshold for EC on Na\(^{23}\) is lower than on Mg\(^{24}\), it should be considered; however, Gutierrez et al. (2005) have shown that the effect on the evolution is small.

In order to calculate the effect of EC, it is necessary to know the mass fraction of the relevant nuclei, which is a result of carbon burning and the capture rate, as well as the neutrino loss rate and the resulting heating rate.

As will be evident from our results, this process has an important effect in our cases, so it was included as a standard in all our models, except when explicitly mentioned otherwise. Due to the existing uncertainties (cf. Gutierrez et al. 2005), in line with our basic approach, we checked the sensitivity of the results to both the mass fraction and the capture rate by introducing fudge factors.

We took the capture rates from Miyaji et al. (1980), but the range of variations we checked also covers the differences versus the rates given by Oda et al. (1994), which differ from the former by as much as an order of magnitude.

Mochkovitch (1984) pointed out that in the presence of EC, due to its effect on the gradient of the electron mole number \( Y_e \), the Ledoux criterion for convection has to be used, and hence the extension of the convective region is slower. In our case, it turned out that the effect of using the Ledoux criterion is quite small.

Another important caveat (J. Stein 2005, private communication) is that since it is clear that EC must start as TU (which is locally cooling), and can become exothermic only when occurring below the threshold by more than \( kT \), the question arises as to whether a convective zone broad enough to ensure heating can be formed. In our case, where EC starts when significant carbon burning is already present, this problem may not be severe. However, more careful analysis is needed to find out whether the local cooling might force the base of convection to move outward.

2.3.4. Opacities

Radiative opacities were calculated according to the OPAL opacity tables (see Iglésias & Rogers 1996), using the tables and interpolation subroutine provided at the OPAL Web site.\(^4\) The tables were extended to lower temperatures according to Alexander & Ferguson (1994). Electron conduction was calculated according to Iben (1975).

2.3.5. Equation of State

The equation of state takes into account ionizations to all available levels of the different atomic species in the composition. The distribution function of the various ionization levels is computed using a method similar to the one described in Kovetz & Shaviv (1994). The resulting electron density is then used together with the temperature in order to extract the pressure, energy, chemical potential, and their derivatives respective to the electron density and temperature from a table computed in advance by solving the

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\(^3\) Although the mass fraction of Ne\(^{20}\) in the burning products of carbon is high, we did not need to include EC on it, since the threshold in its case is high, and corresponds to \( \rho > 6 \times 10^6 \text{ g cm}^{-3} \), which is above our limit of interest.

\(^4\) Available at http://www-phys.llnl.gov/Research/OPAL.
2.4. Treatment of Deviations from the Quasi-Static Assumption

As mentioned above, our evolution code assumes that the timescales for the three main processes that govern the evolution of the star, namely, the hydrodynamic motion, the convective mixing, and the exchange of energy due to thermonuclear reactions and radiative transport, obey the relation: $t_{\text{rad}} \ll t_{\text{conv}} \ll t_{\text{nuc}}$.

However, as we see below, as the star approaches explosion, this relation between the timescales is no longer valid. As the reaction timescale $t_{\text{nuc}}$ becomes shorter, it first becomes comparable to the convective timescale $t_{\text{conv}}$, so in order to achieve a reasonable modeling, we have to appropriately restrict convection when this occurs. Hence, it is important to estimate these two timescales, and give an adequate treatment when the quasi-static assumption fails. In practice, as a very crude model, we restrict the outer boundary of the convective zone (the inner boundary in our case) to not be above the innermost Lagrangian zone, which fulfills $L_{\text{conv}}(r) > \alpha t_{\text{nuc}}$, where $r$ is the radius of the zone, and $\alpha$ is a fudge factor we use to check the sensitivity of the results. Note that $1/\alpha$ is in fact the number of convective turnover times during an interval of one thermonuclear timescale. The method for calculating the timescales $t_{\text{nuc}}$ and $t_{\text{conv}}$ is given below.

2.4.1. The Convective Timescale

The widely used mixing length theory gives an explicit relation between the convective luminosity and the convective velocity,

$$v_{\text{conv}} = \left( \frac{\lambda P (\partial \ln \rho / \partial \ln T)_{p} L_{\text{conv}}}{c_{p} T_{\rho}^{2}} \right)^{1/3},$$

where $\lambda$ is the mixing length, $c_{p}$ is the specific heat at constant pressure, and $L_{\text{conv}}$ is the convective flux through a unit area.

We do not use the mixing length theory, but rather assume an isentropic convective region. Nevertheless, we can estimate the convective luminosity as follows. Let $L_{\text{conv}}$ be the convective luminosity and $L_{\text{rad}}$ the radiative luminosity. At any point we have

$$\frac{\partial S}{\partial t} = q - \frac{\partial L_{\text{rad}}}{\partial m} - \frac{\partial L_{\text{conv}}}{\partial m}.$$  

(2)

However, we assume that $\partial S/\partial t$ is uniform throughout the convective region, thus

$$\int T \frac{\partial S}{\partial t} \, dm = \int T \frac{\partial S}{\partial t} \, dm = \int q \, dm - \Delta L_{\text{rad}} - \Delta L_{\text{conv}}.$$  

(3)

But $\Delta L_{\text{conv}} = 0$, since the convective flux vanishes at the boundaries of the convective region, so we have

$$\frac{\partial S}{\partial t} = \frac{\int q \, dm - \Delta L_{\text{rad}}}{\int T \, dm}.$$  

(4)

Together with equation (2), this gives

$$\frac{\partial L_{\text{conv}}}{\partial m} = q - \frac{\partial L_{\text{rad}}}{\partial m} - T \frac{\int q \, dm - \Delta L_{\text{rad}}}{\int T \, dm}.$$  

(5)

From the above equation we can derive the convective luminosity $L_{\text{conv}}(m)$ for each point in the convective region, and then use equation (1) to get the convective velocity $v_{\text{conv}}(m)$ at each point in the region.

The time needed for the convective flux to cover a distance $\Delta r$ in the vicinity of a Lagrangian point $m$ is of course

$$\Delta t = \frac{\Delta r(m)}{v_{\text{conv}}(m)}.$$  

(6)

Consequently, the time to reach a distance $r$ from the inner boundary of the convective region will be

$$\tau(r) = \int dt = \int \frac{dr}{v_{\text{conv}}}.$$  

(7)

The “convective timescale” will be defined as the time $\tau(r)$ to the outer boundary of the convective region, i.e., the time needed to cross the entire length of the region.

It is clear that this is only a rough estimate; nevertheless a comparison with models of convective envelopes calculated for the same conditions with the mixing length theory using the method described by Tuchman et al. (1978) give an agreement better than a factor of 3.

2.4.2. The Nuclear Timescale

As the nuclear timescale, we define the time needed to burn all the carbon and afterward the oxygen at a given point (with initial temperature $T$, density $\rho$, and carbon mass fraction $X_C$), under the assumption of constant density. Clearly this is a lower limit to the time, since in reality there are also energy losses.

To get a quantitative estimate, we used the analytical approximation of Woosley & Weaver (1986) for the reaction rates of carbon and oxygen burning in the relevant range (i.e., $2 < T_{9} < 6$, $\rho_{9} < 4$)

$$\text{carbon}: \quad q_{C}(\rho, T) = 8.25 \times 10^{15} X_{C}^{2} \rho_{9}^{0.79} T_{9}^{22} \text{ erg s}^{-1},$$

$$\text{oxygen}: \quad q_{O}(\rho, T) = 8.25 \times 10^{15} X_{O}^{2} \rho_{9}^{0.79} T_{9}^{36} \text{ erg s}^{-1}.$$  

(8)

They also approximated the heat capacity using

$$\frac{\partial T}{\partial e} = 1.57 \times 10^{16} \rho_{9}^{-0.26} T_{9}^{0.76}.$$  

(9)

Combining the last two equations, we get (for carbon)

$$\frac{\partial T}{\partial e} = q \frac{\partial T}{\partial e} = 1.3 \times 10^{32} X_{C}^{2} \rho_{9}^{2.53} T_{9}^{22.76}.$$  

(10)

A similar equation can be obtained for the oxygen. Accordingly, the time for raising the temperature from $T_{1}$ to $T_{2}$ is given by

$$t_{1 \rightarrow 2} = \frac{1}{1.3 \times 10^{32} X_{C}^{2} \rho_{9}^{2.53} T_{9}^{22.76}} \int_{T_{1}}^{T_{2}} \frac{dT}{T_{9}}.$$  

(11)

Given an initial temperature $T_{1}$, we can estimate the final temperature $T_{2}$ of carbon burnout, since

$$\int_{T_{1}}^{T_{2}} \left( \frac{\partial e}{\partial T} \right)_{\rho} dT = X_{C} Q_{e},$$  

(12)

where $Q_{e}$ is the $q$-value. After we get the time $t_{1 \rightarrow 2}$ for carbon burnout and the final temperature $T_{2}$, we use $T_{2}$ as the initial temperature of oxygen burning, and through the same method we can get the time $t_{2 \rightarrow 3}$ of oxygen burnout.

Note that we do not take into account the fact that $X_{C}$ is also a function of time, but this turns out to have a small effect. In fact,
in our calculations we checked the validity of these estimates by artificially preventing the density from changing, and actually measuring these times. We found excellent agreement.

Estimating the timescale of oxygen burning is important, since if the carbon mass fraction is very low, it might burn out without igniting the oxygen.

3. RESULTS

3.1. Overview

The evolution of carbon-oxygen stars is determined by their mass \( M \) and composition. We assume that the composition of the star is homogeneous, as a result of the helium burning being convective. Therefore, it is sufficient to specify the mass fractions of the relevant elements, \( C^{12} \) and \( O^{16} \), since the mass fractions of Ne and Mg are negligible. These mass fractions are determined at the end of helium burning, for which there is a well-known uncertainty, due to uncertainties in the cross-section for the reaction \( C(\alpha, \gamma)O \), which takes place during helium burning. It is common to assume the mass fraction of the carbon ranges between \( 0.25 < X_C < 0.55 \), and the mass fraction of the oxygen is its complement to unity (Umeda et al. 1998).

We can identify four fundamentally different ranges. Above the Chandrasekhar mass \( M_{Ch} \approx 1.4 \, M_{\odot} \) carbon is ignited at the center, and the stellar center evolves toward increasing temperatures and densities, igniting heavier fuels.

Below \( M_{Ch} \), but above a certain limit \( M_2 \), carbon is ignited at the center, but no heavier fuels are ignited, and finally evolution proceeds toward a white dwarf. The value of \( M_2 \) depends on the carbon mass fraction; for our example of \( X_C = 0.54 \) it lies around \( M_2 \approx 1.17 \, M_{\odot} \).

Below \( M_2 \), but above a lower limit \( M_1 \), carbon is ignited off-center, at a point depending on both the mass of the star and the carbon mass fraction (see Figs. 2 and 3). Subsequently, carbon burning propagates both inward toward the center and outward, and after almost all the carbon in the core is exhausted, evolution returns to its original path toward a white dwarf.

As we already suggested in §1, and will show in detail later, these stars are the focus of our interest, since the off-center burning might leave behind enough carbon, which will survive the subsequent evolutionary phases and finally ignite explosively after the mass reaches \( M_{Ch} \) through accretion. Figure 4 shows a typical example of an off-center carbon-igniting star with mass of \( M = 1.17 \, M_{\odot} \).

We can see that at the point of ignition the density and temperature at the center decrease, as a result of the expansion induced by the carbon-burning shell above. As we explain in detail in §3.2.3, this shell is extinguished, subsequently causing the center to contract and heat again, but soon another off-center burning shell is ignited, causing the center to expand and cool a second time. Finally the burning reaches the center, causing it to rise to carbon-burning temperature with almost no change in density. After the burning ceases, the center returns to its original path, as if no carbon burning had taken place.

Below the limit \( M_1 \) the star evolves toward a white dwarf without the carbon being ignited. The value of \( M_1 \) depends of course on the carbon mass fraction. For a carbon mass fraction of \( X_C = 0.54 \), we have \( 0.95 \, M_{\odot} < M_1 < 1.05 \, M_{\odot} \), while for a lower carbon mass fraction of \( X_C = 0.25 \), the limit is slightly higher, standing at \( 1.05 \, M_{\odot} < M_1 < 1.08 \, M_{\odot} \).

3.2. Off-Center Carbon Burning

We limit our interest to the range \( M_1 < M_C < M_2 \). In order to point out the major points of interest in this range, we first describe as a specific example a model with mass \( M = 1.17 \, M_{\odot} \) and a homogeneous mass fraction profile, i.e., \( X_C(m) = X_C(0) = 0.25 \).

We see that at the point of ignition the density and temperature at the center decrease, as a result of the expansion induced by the carbon-burning shell above. As we explain in detail in §3.2.3, this shell is extinguished, subsequently causing the center to contract and heat again, but soon another off-center burning shell is ignited, causing the center to expand and cool a second time. Finally the burning reaches the center, causing it to rise to carbon-burning temperature with almost no change in density. After the burning ceases, the center returns to its original path, as if no carbon burning had taken place.

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We investigate the evolution by looking at various physical quantities. Figure 5 presents a Kippenhahn diagram, which shows the history of convective regions in the star. As can be seen, the star goes through various evolutionary stages, each one presenting a burning shell topped by a convective region.

We will describe the three burning stages shown above by following the evolution of the carbon mass fraction profile.

3.2.1. Stage I

The evolution of the carbon mass fraction is shown in Figure 6, and we can see that in this case carbon ignites off-center, at \( m \approx 0.145 \, M_\odot \). The nuclear reaction rate \( q_n \) grows rapidly, and when it exceeds the energy-loss rate by neutrino emission \( q_\nu \), a gradually growing convective burning zone develops, while the carbon mass fraction \( X_C \) gradually decreases. At a certain stage (line 4 in Fig. 6) the convective region begins to shrink, retreating both its inner and outer boundaries. This evidently leaves behind a gradient in the carbon mass fraction \( X_C \). It is notable that immediately below the inner boundary of the convective region lies a narrow radiative burning shell, which locally exhausts the carbon at a relatively higher rate. This shell slightly penetrates inward due to conduction. The decline of \( X_C \), together with the expansion caused by the rise in entropy of the convective region, finally extinguishes \( q_n \). It is important to realize that due to \( q_\nu \), the nuclear burning \( q_n \) is extinguished, even though \( X_C \) has not completely vanished. This is a major point that will have important repercussions in what follows.

3.2.2. Stage II

After the nuclear burning is extinguished, the star continues to contract, leading momentarily to ignition of carbon in the region of the \( X_C \) gradient remaining at the base of the former convective burning zone. Again, \( q_n \) rapidly rises above \( q_\nu \), and a burning zone penetrates inward, forming a convective region above it. The evolution of the carbon mass fraction during this stage is shown in Figure 7.

The behavior of \( X_C \) in the convective region is complex. Although nuclear burning obviously decreases \( X_C \), following the growth of the convective region, variations of \( X_C \) due to incorporation of zones richer or poorer in carbon have to be taken into account. And indeed, at certain stages \( X_C \) in the convective region increases, while at other stages it decreases. This behavior varies from star to star, since it is dependent on the details of the preceding evolutionary stage. It repeats itself several times during subsequent evolutionary stages, and cannot be described in general terms, or as a phenomenon monotonically dependent on the stellar mass or the initial carbon mass fraction \( X_C(t = 0) \).

Several authors have described the case of an inward-advancing carbon-burning shell topped by a convective region. Kawai et al. (1987) followed a carbon burning shell that has been ignited at \( m = 1.07 \, M_\odot \), while mass was accreted on the white dwarf at a rate of \( 2.7 \times 10^{-6} \, M_\odot \, yr^{-1} \), while Saio & Nomoto (1998) found a similar ignition at \( m = 1.04 \, M_\odot \) for an accretion rate of \( 1 \times 10^{-5} \, M_\odot \, yr^{-1} \). As already mentioned, the conclusion of these two papers is that the shell penetrates inward only up to a certain point, where it diminishes as a result of the expansion induced by the rise of entropy in the convective region above the shell. After extinction of the shell, the star contracts, and a new shell is ignited at the point where the former one stopped, this possibly reoccurring several times. We did not explicitly interest ourselves in shells...
ignited far from the center, since in our case the waning of the shell which ignites far from the center is a result of the lack of fuel depleted by previous burning episodes. However, in the case described in §3.2.1 as "stage I," the expansion also contributes to extinguishing the shell and in particular its radiative leading part. We mention that in stage II the expansion is minimal, since the extent of the convective region is small.

Timmes et al. (1994) estimated, based on reasonable assumptions, the velocity of the shell as a function of the density, temperature, and carbon mass fraction, giving a tabulation of their results. A comparison with our results reveals a good agreement. For a case where \( \rho = 6.4 \times 10^5 \text{ g cm}^{-3}, T = 7.3 \times 10^8 \text{ K}, X_C = 0.32, \) the velocity of the shell is about \( 1.3 \times 10^{-3} \text{ cm s}^{-1}. \) A look at the relevant table in Timmes et al. (1994) shows a reasonable agreement, since for \( X_C = 0.30 \) they have a velocity between \( 8.47 \times 10^{-4} \text{ cm s}^{-1} \) (for \( T = 7 \times 10^8 \text{ K} \)) and \( 6.10 \times 10^{-3} \text{ cm s}^{-1} \) (for \( T = 8 \times 10^8 \text{ K} \)). In the work of Gil-Pons & Garcia-Berro (2001), dealing with the formation of oxygen-neon dwarfs during mass exchange in binaries, as well as in the series of papers by Garcia-Berro & Iben (1994), Ritossa et al. (1996), Garcia-Berro et al. (1997), Iben et al. (1997), and Ritossa et al. (1999), exploring the evolution of intermediate mass stars between 9 and 11 \( M_\odot, \) at a certain stage carbon is ignited off-center in a manner very similar to that expected in our case, and even the resulting carbon profile is very much like ours (see, e.g., Fig. 12 in Gil-Pons & Garcia-Berro 2001).

After the nuclear burning flame reaches the center of the star (line 15 in Fig. 7), the reaction rate starts to weaken with the decrease of \( X_C \) and the reduction of the convective region.

Figure 8 displays the advance of the flame inward, and it can be seen that it behaves like a regular almost self-similar front. Note, however, that close to the center the flame width narrows, and thus special care must be taken, or else the flame might incorrectly die out, leaving an inner zone of unburned carbon.

### 3.2.3. Stage III

In the next stage, as a result of the contraction of the star, the carbon reignites, usually in a relatively carbon-rich zone, above the extent of the convective regions of the previous stages.

Also in this stage a convective region develops, which grows up to a certain extent, and then the reactions are extinguished again. At the same time, a radiative burning front develops, which advances inward to the carbon-poor region along the composition gradient, which has remained there due to the previous burning. We find (see Fig. 9) that this flame gradually decays, and is only able to penetrate slightly. The evolution of the carbon mass fraction during this stage is shown in Figure 10.

In some cases, there are more stages where carbon is ignited in carbon-rich outer areas (Fig. 5). Eventually, further contraction does not result in carbon ignition, and the star evolves toward a white dwarf, with a "frozen" carbon profile. Figure 11 displays this final profile for the main elements present in the star: carbon, oxygen, neon, and magnesium.

Figure 12 displays the carbon mass fraction profile at the end of carbon burning for various masses and initial carbon mass fraction of \( X_C(t = 0) = 0.54. \) Figure 13 displays the carbon mass fraction profile at the end of carbon burning for various initial carbon mass fractions and total mass of \( M = 1.22 M_\odot. \) We can see that the typical profile has a "bump" around the center, above it a region almost devoid of carbon, and in most cases a small carbon-rich zone near the outer boundary.

**Fig. 8.** Inward-advancing carbon burning front during stage II in a 1.10 \( M_\odot \) carbon-oxygen model. The nuclear reaction rate (solid line) and neutrino loss rate (dashed line) are shown at various times. [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 9.** Decay of the inward-advancing carbon-burning front during stage III in a 1.17 \( M_\odot \) carbon-oxygen model. The nuclear reaction rate (solid line) and neutrino loss rate (dashed line) are shown at various times. [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 10.** Evolution of the carbon mass fraction profile in a 1.17 \( M_\odot \) mass carbon star model during stage III of off-center carbon burning. Each line represents the mass fraction profile at a different point in time, in the order of numbering in the legend, which corresponds to the numbering in Fig. 5. [See the electronic edition of the Journal for a color version of this figure.]
The widely accepted scenario for the onset of a SN I explosion is accretion of matter by a white dwarf; therefore, in the following sections we discuss the various possibilities of accretion onto the white dwarfs resulting from the evolutionary scenarios described above.

3.3. Accretion

For the purpose of understanding the various possibilities, we use the standard technique to investigate the outcome of accretion as a function of the relevant parameters. In a realistic case, the accreted matter could be hydrogen, helium, carbon, or a mixture of these. In the case of hydrogen or helium accretion, these also ignite and form a burning shell advancing outward, leaving behind a newly made layer of carbon-oxygen. Thus, the composition of the accreted matter influences the effective rate of carbon-oxygen accretion. Variations in the entropy of the accreted matter can also be represented by dictating a suitable effective accretion rate. As we will show, the carbon mass fraction in the accreted matter can be of importance, but even then the main parameter is $\dot{M}$.

It is known (Nomoto & Sugimoto 1977; Nomoto et al. 1984) that the relevant $\dot{M}$ for a supernova has to be

$$\dot{M} \geq 7.5 \times 10^{-7} \ M_\odot \ yr^{-1} \quad (13)$$

This rate results from the mass-luminosity relation given originally by Paczynski (1970) for a double shell of hydrogen and helium burning.

$$\frac{L}{L_\odot} = 59250 \left( \frac{\dot{M}}{M_\odot} - 0.52 \right).$$

Translating this relation to $\dot{M}$ gives

$$L = q = \dot{M}XQ,$$  \quad (15)

where $X$ is the hydrogen mass fraction and $Q$ is the $Q$-value of hydrogen burning.

It is worth noting that in the case of helium burning, $\dot{M}$ is expected to be, and indeed is, higher by at least a factor of about 10, since $Q$ is lower.

We decided to map the results for a wide range of $\dot{M}$:

$$7.5 \times 10^{-7} \ M_\odot \ yr^{-1} \leq \dot{M} \leq 3 \times 10^{-4} \ M_\odot \ yr^{-1}. \quad (16)$$

It is important to note that for rates in this range, the initial mass of the white dwarf is insignificant, due to the “convergence feature” of the evolutionary paths. This argument is not to be confused with the fact that when dealing with much smaller accretion rates, the initial mass is of much more importance.

In order to compare our results to the literature, we chose from among the many publications on this subject the extensive survey by Nomoto & Sugimoto (1977), followed by a study of the dependence on the accretion rate by Sugimoto & Nomoto (1980). It was important for us to verify that the evolutionary paths we get are similar to the literature, and thanks to the uniqueness of this path (for given accretion and neutrino loss rates) as mentioned above, this is indeed the case. It is obvious that the onset of carbon burning depends also on the mass fraction, and we are especially interested in the cases where this mass fraction is particularly low.

Figure 14 shows the results for a model with mass of $1.18 \ M_\odot$, for various accretion rates in the range explained above.

As mentioned, all the above results are for accretion of matter devoid of carbon. This is because, as previously mentioned, in the case of carbon accretion at high $\dot{M}$ above a certain threshold,
which depends on the mass fraction of carbon in the accreted matter, carbon ignition occurs in the accreted layer, and a burning front develops, advances inward, and as mentioned in \(3.2.2\), might reach the center before explosion occurs. We did not follow the front in this case, but we mapped the combinations of accretion rates and carbon mass fractions which lead to such a case. Figure 15 demonstrates this for a carbon-oxygen model of mass \(1.12 \, M_{\odot}\). We can see that for an accretion rate of \(7 \times 10^{-7} \, M_{\odot} \, yr^{-1}\), carbon does not ignite in the accreted layer, even if the carbon mass fraction in the accreted matter is as high as 0.5. For a higher accretion rate of \(7 \times 10^{-6} \, M_{\odot} \, yr^{-1}\), such ignition does not occur for a carbon mass fraction of 0.01, but does occur for 0.05 and above.

### 3.4. The Influence of Various Parameters beyond Ignition

Beyond ignition a convective region is formed, which goes on growing while supplying fuel to the nuclear flame, but at the same time also inducing expansion. Clearly, convection will continue as long as the convective turnover timescale, \(t_{\text{conv}}\), is shorter than the fuel exhaustion timescale, \(t_{\text{max}}\). Similarly, it is clear that the transition to a dynamic regime, i.e., to a situation where assuming hydrostatic equilibrium is no longer valid, will occur when the dynamic timescale \(t_{\text{dy}}\) is such that \(t_{\text{dy}} > t_{\text{conv}}\) and \(t_{\text{dy}} > \alpha t_{\text{max}}\), where \(\alpha\) is a fudge factor. Figure 16 shows the sensitivity to this factor \(\alpha\), and we can see that the smaller it is (i.e., convection is turned off earlier), the earlier runaway occurs, i.e., at higher mass and lower \(T_0\), since at this stage convection causes expansion and thus hinders runaway.

### 3.5. The Effect of Electron-Capture Processes

#### 3.5.1. Electron Capture on Carbon-burning Products

Figure 17 compares the evolution with and without taking into account the effect of electron capture onto Mg\(^{24}\). A considerable effect can be seen.

Keeping in mind the existing uncertainties for this process, we checked the sensitivity of the results by modifying the mass fraction of Mg\(^{24}\), which in our standard models came out to be about 0.19. Figure 18 shows the results, and we can see that lowering length divided by the speed of sound) is no longer short compared to \(t_{\text{max}}\).

In \(2.4\) we discuss the subject of timescales in detail, including the definition of these quantities and treatment of the relevant scenarios. As we already mentioned there, our treatment implies comparing \(t_{\text{conv}}\) and \(t_{\text{max}}\), and turning convection off where \(t_{\text{conv}} > \alpha t_{\text{max}}\), where \(\alpha\) is a fudge factor. Figure 16 shows the sensitivity to this factor \(\alpha\), and we can see that the smaller it is (i.e., convection is turned off earlier), the earlier runaway occurs, i.e., at higher \(\rho_0\), and lower \(T_0\), since at this stage convection causes expansion and thus hinders runaway.

### Fig. 17.—Effect of electron capture onto Mg\(^{24}\) on the evolution of accreting carbon star models toward explosive ignition of carbon. Initial mass is 1.18 \(M_{\odot}\), the accretion rate is \(5 \times 10^{-7} \, M_{\odot} \, yr^{-1}\). [See the electronic edition of the Journal for a color version of this figure.]
the mass fraction of Mg$^{24}$ has a small effect, causing earlier ignition, and thus runaway at lower $\rho_c$.

3.5.2. Thermal Urca (TU)

Figure 19 shows the influence of the TU process for our model of 1.18 $M_\odot$, for various mass fraction of the Urca nucleus Na$^{23}$. We get a minor perturbation only, which is manifested as a deviation in the $\rho_c$, $T_c$ path to lower temperatures due to the local cooling. As convection continues, the US moves outward, together with the local heat sink. When the US is far enough from the center, the density and temperature at the center start rising again, and shortly thereafter the path coincides with that of the TU-less case. Comparison with results from the literature (Gutierrez et al. 2005) shows a satisfying agreement.

3.5.3. Convective Urca (CU)

As discussed above, we modeled CU by artificially forbidding convection above the Urca shell, and examined its effect by varying the Fermi energy threshold at which the Urca shell is located. This works in two opposite directions. On the one hand, the entropy and with it the temperature increase faster (which promotes approaching RA), but on the other hand there is a smaller supply of fresh fuel, which can suppress burning, and might prevent the RA. Clearly, the lack of a fresh fuel supply might have cardinal importance in a case where the mass fraction of fuel is low to begin with.

We find that a feedback mechanism exists. When the star contracts, the US has to move farther away from the center (due to increase in $E_F$), and thus the outer boundary of the convection moves outward as well, so that the effectiveness of limiting the extent of the convective region on diminishing the fuel supply is small in all the cases we checked. It is interesting to note that when the star goes through an expansion phase, the situation is the opposite; the US approaches the center, and may limit the convective region. If this limiting is significant enough while the temperature is already high, it might induce an earlier runaway, which will thus occur at a higher density.

Figure 20 presents a comparison between the undisturbed case, where $E_F$ peaks at about 6.26 MeV$^8$, and cases where convection is limited at $E_F = 4.4$ MeV (the threshold of Na$^{23}$), $E_F = 5.7$ MeV (the threshold of Ne$^{21}$), and $E_F = 5.2$ MeV (an intermediate value).

We can see that the higher the threshold, the earlier RA occurs, i.e., at a higher density and lower temperature. It is clear that this tendency is limited, since should we raise the threshold to the vicinity of 6.26 MeV, the Fermi energy throughout our model would be below the threshold, and we would effectively be back to the case with no CU at all.

Obviously the above conclusions are only a rough estimate, and should be in fact examined by means of multidimensional simulations, which are still difficult to undertake.

4. DISCUSSION AND CONCLUSIONS

Exploring stellar models which ignite carbon off-center (in the mass range of about 1.05–1.25 $M_\odot$, depending on the carbon mass fraction), we find that they may present an interesting SN I progenitor scenario, since while in the standard scenario runaway always takes place at the same density of about $2 \times 10^9$ g cm$^{-3}$, in our case, due to the small amount of carbon ignited, we get a whole range of densities from $1 \times 10^9$ up to $6 \times 10^9$ g cm$^{-3}$.

$^8$ At the same time the US of Ne$^{21}$ was located at 0.09 $M_\odot$, and that of Na$^{23}$ at 0.45 $M_\odot$. 

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**Fig. 18.** Influence of the mass fraction of Mg$^{24}$ on the ignition and runaway of accreting carbon star models. All models have an initial mass 1.18 $M_\odot$ and accrete matter at a rate of $4 \times 10^{-6} M_\odot$ yr$^{-1}$. [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 19.** Influence of the mass fraction of Na$^{23}$ for the thermal Urca process on the ignition and runaway of accreting carbon star models. All models have an initial mass 1.18 $M_\odot$ and accrete matter at a rate of $4 \times 10^{-6} M_\odot$ yr$^{-1}$. [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 20.** Influence of the Fermi energy of the convective Urca shell on the ignition and runaway of accreting carbon star models. All models have an initial mass 1.18 $M_\odot$ and accrete matter at a rate of $4 \times 10^{-6} M_\odot$ yr$^{-1}$. [See the electronic edition of the Journal for a color version of this figure.]
These results could contribute to resolving the emerging recognition that at least some diversity among SNe I exists, since runaway at various central densities is expected to yield various outcomes in terms of the velocities and composition of the ejecta, which should be modeled and compared to observations.

Several issues which were beyond the scope of this work call for further investigation:

1. A deeper treatment of the question whether thermal Urca can hinder the formation of a convective zone when electron capture on Mg$^{24}$ sets in is needed.
2. Our work provides initial models that can apparently reach explosive runaway. However, our treatment of the onset of explosion, involving a very crude treatment of convection and of the convective Urca process, can only be regarded as a preliminary guideline, setting the stage for a much more profound study. The real value of our results could be judged by fitting the results of the dynamical simulations to the observational data.
3. According to Liebert et al. (2005), some 6% of the white dwarfs have masses above 1 $M_{\odot}$ (and below the Chandrasekhar mass of 1.4 $M_{\odot}$). Since the carbon-oxygen stars igniting carbon off-center lie between about 1.05 and 1.18 $M_{\odot}$, i.e., span about one-quarter of the above range, we can roughly estimate that their incidence among the white dwarf population should be on the order of 1%. Of course, a sounder estimate of the occurrence of this type of SNe needs a more thorough investigation, including modeling of binary evolution.

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