Study of Center of Mass Energy by Particles Collision in Some Black Holes

M. Sharif * and Nida Haider †
Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore-54590, Pakistan.

Abstract

This paper is devoted to the study of particles collision for two well-known black holes. We consider particles moving in equatorial plane and calculate their center of mass energy. Firstly, we explore center of mass energy of a regular black hole. In this case, acceleration and collision of particles lead to high center of mass energy which is independent of event horizon and naked singularity. Secondly, we investigate the center of mass energy of Plebanski and Demianski black hole (non-extremal) with zero NUT parameter. Here the center of mass energy depends upon the rotation parameter. We conclude that the center of mass energy becomes infinitely large for both black holes.

Keywords: Black hole; Particle collision; Center of mass energy.
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1 Introduction

Particle accelerators are devices that propel and accelerate charged particles (like protons) to high speed. Physicists use them to study the nature of
matter and energy. Tevatron and Large Hadron Collider are the particle accelerators which propel and produce collision between particles at the center of mass energy (CME) upto 10TeV. In particles collision, the CME is the energy required for the creation of new particles. The fascinating possibility to study this energy is to make use of naturally occurring processes in the vicinity of astrophysical objects. Black hole (BH) can behave as a particle accelerator and can accelerate the colliding particles to an unlimited CME. The CME of colliding particles can grow limitlessly in different situations, e.g., nature of colliding particles, BH/naked singularity, modified gravity theories etc. When two particles collide near horizon, the energy grows infinitely in the CM frame because particles are infinitely blueshifted near the horizon.

Banados, Silk and West (2009) discussed (BSW effect) the effect of infinite growth of energy in the CM frame due to collision of particles near the horizon. It was first studied in the locality of the Kerr (extremal) BH and was shown that two colliding particles propelling in equatorial plane can have arbitrarily large CME near the maximal BH spin. Lake (2010) explored the effect of particles collision near the horizon of the Kerr (non-extremal) BH and obtained finite CME. Grib and Pavlov (2011) suggested that CME can become infinite in the non-extremal case if two scattering particles with equal masses collide close to the horizon. The BSW effect has also been studied for Sen BH (Wei et al. 2010), Kerr-Taub-NUT BH (Liu et al. 2011) and Kaluza-Klein BH. Mao et al. (2011) found infinitely large CME for particles colliding at the horizon of charged non-rotating and extremal rotating Kaluza-Klein BH.

Piran et al. (1975, 1977) investigated collisions with infinite CME for two particles colliding in energy extraction process - known as collisional Penrose. Bejger et al. (2012) studied this process near the horizon of (extremal) Kerr BH. Wei et al. (2010) discussed the effect of charge on the CME for stringy and Kerr-NewmanBHs and obtained arbitrarily large CME. Harada et al. (2012) found this energy for colliding particles near the horizon of a BH with maximal rotation and concluded that it is arbitrarily large for the critical particles (with fine tuned angular momentum). Joshi and Patil (2011a, 2011b, 2012a) explored Reissner-Nordstrom (RN) and Kerr BHs and found high CME in the naked singularity. They concluded that the high energy collision can also be seen in the BH having naked singularity with no event horizon. The same authors (2012b) described that the high energy collision can take place near the naked singularity of Janis Newmann Winicor metric (found by adding a massless scalar field in the Schwarzschild BH),
while these collisions are not present in the Schwarzschild BH. They also proved that BH having no event horizon or singularity can also have high CME for particular values of parameters $m$ and $q$ (2012c). Hussain (2012) found that the CME of colliding particles in Plebanski and Demianski BH (extremal) with zero NUT parameter is unlimited at the acceleration and event horizons.

Zaslavskii (2010a, 2010b, 2011) studied the universal property for particle acceleration and generalized the BSW effect for dirty BHs with non-equatorial motion of colliding particles. Harada and Kimura (2011) studied the collision in non-equatorial plane with an unboundedly high CME for (extremal) Kerr BH. Yao et al. (2012) explored the CME for the collision of particles with non-equatorial motion in Kerr-Newman BH and discussed the effect of acceleration. Liu et al. (2011) obtained arbitrarily high CME for colliding particles with non-equatorial motion near horizon of (extremal) Kerr-Newman BH. Jacobson and Sotirious (2010) showed that it takes an infinite time to attain infinite CME.

In this paper, we study CME by particles collision in equatorial plane for two well-known BHs. The paper is organized as follows. In sections 2 and 3, we explore CME for regular BH and for charged accelerating and rotating BH near the horizon. The last section summarizes the results.

2 Particle Acceleration in Regular Black Hole

This section is devoted to study the particle acceleration in a static spherically symmetric BH coupled to a non-linear electrodynamics. The general form of a regular BH is

$$ds^2 = -Y(r)dt^2 + \frac{1}{Y(r)}dr^2 + r^2 d\Omega_2^2,$$

(1)

where $d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2$ is a 2-dimensional sphere and

$$Y(r) = 1 - \frac{2M(r)}{r}.$$

(2)
Ayon and Garcia (2000) proposed a non-singular BH solution coupled with nonlinear electrodynamics whose line element is

\[
 ds^2 = -(1 - \frac{2mr^2}{(r^2 + q^2)^{\frac{3}{2}}}) dt^2 + (1 - \frac{2mr^2}{(r^2 + q^2)^{\frac{3}{2}}}) \frac{q^2 r^2}{(r^2 + q^2)^2} \frac{1}{(r^2 + q^2)^{1/2}} dr^2 + r^2 d\Omega^2_2, 
\]

for which the electric field is

\[
 E(r) = qr^4 \left( \frac{r^2 - 5q^2}{(r^2 + q^2)^4} + \frac{15m}{2(r^2 + q^2)^{\frac{7}{2}}} \right). 
\]

Here \( m \) and \( q \) are the mass and magnetic charge parameter of BH, respectively. Comparing Eqs. (1) and (3), we have

\[
 Y(r) = 1 - \frac{2mr^2}{(r^2 + q^2)^{\frac{3}{2}}} + \frac{q^2 r^2}{(r^2 + q^2)^{\frac{1}{2}}},
\]

leading to

\[
 M(r) = \frac{mr^3}{(r^2 + q^2)^{\frac{3}{2}}} + \frac{q^2 r^3}{2(r^2 + q^2)^{\frac{7}{2}}},
\]

Expanding by Taylor’s expansion at \( r = 0 \), i.e., near the center, it follows that

\[
 M(r) = \left( \frac{m}{q^3} - \frac{1}{2q^2} \right) r^3 - \left( \frac{3m}{2q^3} - \frac{1}{q^4} \right) r^5 + O(r^7).
\]

The general expression is

\[
 M(r) = M_0 + M_1 r + M_2 r^2 + M_3 r^3 + \cdots.
\]

Comparing both these equations, we obtain \( M_0 = M_1 = M_2 = 0 \), while \( M_3 \neq 0 \). Using the condition (Joshi and Patil 2012c), it follows that the center is non-singular. Moreover, this solution asymptotically behaves as RN solution, i.e.,

\[
 g_{00} = 1 + \frac{2m}{r} + \frac{q^2}{r^2} + O(\frac{1}{r^3}).
\]

Thus we can write the metric near the center as

\[
 ds^2 = -(1 - 2M_3 r^2) dt^2 + (1 - 2M_3 r^2)^{-1} dr^2 + r^2 d\Omega^2_2.
\]
Consider a particle exhibiting geodesic motion in a static spherically symmetric spacetime. Let \( U^a = (U^t, U^r, U^\theta, U^\phi) \) be the four velocity of the particle, which is restricted to equatorial motion \((\theta = \frac{\pi}{2})\), hence \( U^\theta = 0 \). Thus the metric admits only two Killing vectors \( \partial_\phi \) and \( \partial_t \). Using these Killing vectors, we can define energy and angular momentum by

\[
E = -g_{\alpha\beta}(\partial_t)^a U^b = -g_{tt}U^t - g_{t\phi}U^\phi,
\]

\[
L = g_{\alpha\beta}(\partial_\phi)^a U^b = g_{t\phi}U^t + g_{\phi\phi}U^\phi,
\]

which are interpreted as constants of motion, i.e., energy and angular momentum are conserved throughout the motion. Using Eq. (1), these quantities turn out to be

\[
E = Y(r)U^t, \quad L = r^2U^\phi,
\]

leading to

\[
U^t = \frac{E}{Y(r)}, \quad U^\phi = \frac{L}{r^2}.
\] (6)

Using normalization condition, \( g_{\alpha\beta}U^a U^b = -1 \), the radial component becomes

\[
U^r = \pm \left[ E^2 - Y(r)(1 + \frac{L^2}{r^2}) \right]^{\frac{1}{2}}.
\] (7)

Here positive and negative signs correspond to ingoing and outgoing particles along the radial direction. We define the effective potential along radial direction as

\[
V_{\text{eff}}(r) = Y(r)(1 + \frac{L^2}{r^2}).
\] (8)

Thus Eq. (7) leads to

\[
(U^r)^2 + V_{\text{eff}}(r) = E^2.
\] (9)

The quantity \( U^r \geq 0 \) provides the necessary condition for a particle to reach the center. When velocity is positive, particle reaches the center but it turns back for zero velocity.

Now we evaluate the minimum of \( Y(r) \) graphically as shown in Figure 1. It is observed that the curve admits a minimum at \( x = 0.333 \), which gives \( r_{\text{min}} = 0.6q \) and the corresponding value of \( Y(r) \) is

\[
Y(r_{\text{min}}) = 1.19 - 0.453\frac{m}{q}.
\] (10)

When \( m > 2.6q \), we have \( Y(r_{\text{min}}) < 0 \). Moreover, we see that \( Y(r = 0) = 1 \) and \( Y(r) \to 1 \) as \( r \to \infty \). This implies that the function admits a zero for
two values of $r$, i.e., $Y(r) = 0$ for $r = r_1$ (say) and $r = r_2$ (say), where $0 < r_1 < r_{\text{min}}$ and $r_{\text{min}} < r_2 < \infty$. In this case, the metric has its inner and outer horizons at $r = r_1$ and $r = r_2$, respectively. For these values of mass and charge, the inner and outer horizons are defined by $-kk\mu = Y(r) = 0$. In this case metric corresponds to BH.

The curvature invariants $R_1 = R_{ab}g^{ab}$, $R_2 = R_{ab}R^{ab}$ and $K = R_{abcd}R^{abcd}$, yield

$$R_1 = \frac{1}{(q^2 + r^2)^2} \left[ 6q^2m(4q^4 + 3r^2q^2 - r^4) + 12q^4(q^2 - r^2)(q^2 + r^2) \right],$$

$$R_2 = \frac{1}{(q^2 + r^2)^4} \left[ 2q^4\{ 18q^8 + q^2r^4(81m^4 - 20r^2 - 168m(q^2 + r^2)^{\frac{1}{2}}) - 36q^6 \times (-2m^2 + r^2 + 2m\sqrt{q^2 + r^2}) + r^6(117m^2 + 2r^2 + 30m\sqrt{q^2 + r^2}) \} \right],$$

$$K = \frac{4}{(q^2 + r^2)^8} \left[ 6q^{12} + 12m^2r^{10} - 24q^2r^8(m + \sqrt{q^2 + r^2}) + q^6r^4(129m^2 \right.$$  

$$\left. - 44r^2 - 192m\sqrt{q^2 + r^2} - 12q^{10}(-2m^2 + r^2 + 2m\sqrt{q^2 + r^2}) + 2q^8r^2 \right. \times (6m^2 + 34r^2 + 15m\sqrt{q^2 + r^2}) + q^4r^6(105m^2 + 14r^2 + 90m\sqrt{q^2 + r^2}) \right].$$

Here $R_1$, $R_2$ and $K$ are finite for finite values of $r$ which show that all of them are bounded everywhere. Thus for $m > 2.6q$, the singularities appearing in the metric are only coordinate singularities describing the existence of horizon. As all the curvature invariants are finite everywhere and as $r \to \infty$, $2M(r) \to 2m$, which gives $2M(r) < r$, thus for $Y(r_{\text{min}}) > 0$, we neither have horizon nor singularity. In this case, the conditions $Y(0) = 1$ and
\( Y'(0) = 0 \) are also satisfied.

When \( q = 0.4m \), we have \( Y(r_{\text{min}}) = 0 \) and hence both inner and outer horizons coincide. In other words, these shrink into a single horizon, i.e., \( r_{\text{min}} = r_1 = r_2 \), which can be related to extremal BH as in the RN solution. When charge and mass satisfy the constraint \( q > 0.4m \), we obtain \( Y(r_{\text{min}}) > 0 \), which is the condition for the avoidance of horizon (Joshi and Patil 2012c). Thus we can avoid the singularities and event horizons.

Now we investigate CME of two particles having masses \( m_1 \) and \( m_2 \). In terms of four-momentum \( p_i^a, (i = 1, 2, a = t, r, \theta, \phi) \), the CME of two particles is (Liu et al. 2011)

\[
E_{\text{cm}}^2 = - p_i^a p_i^a,
\]

which yields

\[
E_{\text{cm}}^2 = 2m_1m_2 \left[ (m_1 - m_2)^2 + (1 - g_{ab} U_1^a U_2^b) \right].
\] (11)

When \( m_1 = m_2 = m \), using Eq.(8), it follows that

\[
V_{\text{eff}}(r_{\text{min}}) = Y(r_{\text{min}}), \text{ which leads to } V_{\text{eff}}(r_{\text{min}}) = 1.19 - 0.453 \frac{m}{q} = Y(r_{\text{min}}).
\] (13)

We consider the collision of particles whose motion is along radial geodesics with conserved energy \( E_1 = E_2 = E = 1 \) (implies that the ingoing particles asymptotically approach to the center) and angular momentum is taken to be \( L_1 = L_2 = L = 0 \) (particles reach the center). Consequently, Eq.(12) yields

\[
E_{\text{cm}}^2 = \frac{4m^2}{(1 - \frac{2mr^2}{(r^2 + q^2)^2} + \frac{q^2 r^2}{(r^2 + q^2)^2})}. \] (14)
The CME depends upon the chosen location of the colliding particles. This will be maximum when the effective potential is minimum. Inserting the value of $r_{\text{min}}$ in Eq. (14), it follows that

$$E_{\text{cm},\text{max}}^2 = \frac{4m^2}{1.19 - 0.453\frac{m}{q}}. \quad (15)$$

This shows that the maximum CME depends on the ratio of mass and charge of the regular BH, which is large for $q = 0.4m$. When charge is greater than mass by infinitesimally small amount, we introduce a new parameter $\epsilon$ as

$$\epsilon = 1.19 - 0.45\frac{m}{q}. \quad (16)$$

For $\epsilon \to 0$, the CME becomes infinite, i.e.,

$$\lim_{\epsilon \to 0} E_{\text{cm},\text{max}}^2 = \frac{4m^2}{\epsilon} \to \infty. \quad (17)$$

When we take a collision between particles with $E_1 = E_2 = E$ and angular momentum $L_1 = L_2 = 0$ moving along the same path, then

$$\frac{E_{\text{cm}}^2}{2m^2} = \frac{2E^2}{(1 - \frac{2mr^2}{(r^2+q^2)^2} + \frac{q^2r^2}{(r^2+q^2)^2})}. \quad (18)$$

$$U^r = \pm \sqrt{E^2 - (1 - \frac{2mr^2}{(r^2+q^2)^2} + \frac{q^2r^2}{(r^2+q^2)^2})}. \quad (19)$$

Since $Y(r_{\text{min}}) = \epsilon = 1.19 - 0.45\frac{m}{q}$, therefore, the condition for $U^r$ to be real leads to

$$E^2 - Y(r_{\text{min}}) \geq 0 \quad \Rightarrow \quad E \geq \sqrt{\epsilon}. \quad (19)$$

If $E = \sqrt{\epsilon}$, then $U^r = 0$, i.e., the particle remains at rest for $r = r_{\text{min}} = 0.6q$. If $E > \sqrt{\epsilon}$, then $U^r$ is real, i.e., an ingoing particle will come out as an outgoing particle either after getting bounced back or after crossing over the center. Thus the CME near the extremal limits is given by

$$\lim_{\epsilon \to 0} \frac{E_{\text{cm}}^2}{2m^2} = \frac{2E^2}{\epsilon} \to \infty. \quad (20)$$

Finally, we consider the collision between an ingoing particle with finite radial velocity as well as energy say $E_1$ and particle at rest, where $r = r_{\text{min}}$
(having energy $E_2 = \sqrt{\epsilon}$ and zero angular momentum). The CME of these colliding particles is

$$\frac{E_{\text{min}}^2}{2m^2} = 1 + \frac{E_1}{\sqrt{\epsilon}}$$

(21)

For extremal limits, this leads to

$$\lim_{\epsilon \to 0} \frac{E_{\text{cm}}^2}{2m^2} = 1 + \frac{E_1}{\sqrt{\epsilon}} \approx \frac{E_1}{\sqrt{\epsilon}} \to \infty,$$

(22)

which is divergent. Hence the high energy collisions are independent of horizon and singularities and can occur in the regular BH.

### 3 Particle Acceleration in Charged Accelerating and Rotating Black Holes

Here we investigate acceleration and particle collision in charged accelerating and rotating BHs (non-extremal). Plebanski and Demianski (PD) presented a class of type D BHs known as the family of PD BHs (Plebanski and Demianski 1976). The general form of the metric is

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 - 2H(r)dtd\phi + \Sigma(r)d\theta^2 + K(r)d\phi^2.$$ 

(23)

We consider a PD BH (Sharif and Javed 2012) with zero NUT parameter so that the metric is

$$ds^2 = -\frac{Q}{\rho^2}(dt - a \sin^2 \theta d\phi)^2 - \frac{\rho^2}{Q}dr^2 - \frac{P}{\rho^2}(adt - (r^2 + a^2)d\phi)^2 - \frac{\rho^2}{P} \sin^2 \theta d\phi^2).$$

(24)

with

$$\Omega = 1 - \frac{\alpha}{\omega} a \cos \theta, \quad \rho^2 = r^2 + a^2 \cos^2 \theta, \quad \tilde{P} = P \sin^2 \theta$$

$$Q = (\omega^2 k + e^2 + g^2 - 2Mr + \frac{\omega^2 k}{a^2}r^2)(1 - \frac{\alpha a}{\omega}r)(1 + \frac{\alpha a}{\omega}r).$$

Here $M$ and $a$ represent the mass and rotation of BH respectively, while the parameters $e$ and $g$ are the electric and magnetic charges, respectively, $\alpha$ represents acceleration of the BH. The rotation parameter $\omega$ in terms of $a$
and $k$ is given by $\frac{\omega^2}{\alpha} k = 1$. It is interesting to mention here that all the parameters $\alpha$, $M$, $e$, $g$ and $a$ vary independently but $\omega$ depends on the value of $a$. For $\alpha = 0$, the metric reduces to the Kerr-Newman BH, while $a = 0$ leads to C-metric. In the limit $a = 0 = \alpha$, this yields RN BH, while for $e = 0 = g$, the Schwarzschild BH can be obtained.

The horizons are found for $g(r) = 0$, leading to

$$\omega^2 k + e^2 + g^2 - 2Mr + \frac{\omega^2 k}{a^2} r^2 = 0,$$

which is quadratic in $r$ with roots

$$r_{\pm} = \frac{a^2}{\omega^2 k} \left( M \pm \sqrt{M^2 - \frac{\omega^2 k}{\alpha^2} (\omega^2 k + e^2 + g^2)} \right).$$

For the existence of horizon

$$M^2 \geq \frac{\omega^2 k}{\alpha^2} (\omega^2 k + e^2 + g^2), \quad (25)$$

where $r_{\pm}$ represent the inner and outer horizons. Also, $r_{\alpha} = -\frac{\omega}{a^2}$, $r_{a} = -\frac{\alpha}{a^2}$ are acceleration horizons. The angular velocity at outer horizon is $\Omega_H = \frac{\omega}{g_{\phi\phi}}$, which provides

$$\Omega_H = \frac{a}{r_{a}^2 + a^2} = \frac{a}{\frac{a^2}{\omega^2 k} \left( M + \sqrt{M^2 - \frac{\omega^2 k}{\alpha^2} (\omega^2 k + e^2 + g^2)} \right)^2 + a^2}. \quad (26)$$

The conserved energy and angular momentum along the geodesics are

$$E = \left( \frac{Q}{r^2} - \frac{Pa^2}{r^2} \right) U^t + \left( \frac{Pa(r^2 + a^2)}{r^2} - \frac{Qa}{r^2} \right) U^\phi, \quad (27)$$

$$L = -\left( \frac{Pa(r^2 + a^2)}{r^2} - \frac{Qa}{r^2} \right) U^t + \left( \frac{P(r^2 + a^2)^2}{r^2} - \frac{Qa^2}{r^2} \right) U^\phi. \quad (28)$$

These lead to the components of four velocity as follows

$$U^t = \frac{1}{PQ} \left\{ E \left( \frac{P(r^2 + a^2)}{r^2} - \frac{Qa^2}{r^2} \right) - L \left( \frac{Pa(r^2 + a^2)}{r^2} - \frac{Qa}{r^2} \right) \right\}, \quad (29)$$

$$U^\phi = \frac{1}{PQ} \left\{ E \left( \frac{Pa(r^2 + a^2)}{r^2} - \frac{Qa}{r^2} \right) + L \left( \frac{Q}{r^2} - \frac{Pa^2}{r^2} \right) \right\}. \quad (30)$$
The radial velocity component can be found using normalization condition

\[ U^r = \pm \left[ \frac{1}{P r^4} \{ -PQ + E^2(P(r^2 + a^2)^2 - Qa^2) - L^2(Q - Pa^2) \\ - 2EL(Pa(r^2 + a^2) - Qa) \} \right]^{\frac{1}{2}}. \]  

(31)

Here ± correspond to radially ingoing and outgoing particles, respectively.

We introduce the effective potential as

\[ U^r^2 + V_{\text{eff}}(r) = 0, \]  

(32)

where

\[ V_{\text{eff}}(r) = \frac{1}{Pr^4} \{ PQ - E^2(P(r^2 + a^2)^2 - Qa^2) + L^2(Q - Pa^2) \\ + 2EL(Pa(r^2 + a^2) - Qa) \}. \]  

(33)

The conditions for circular orbit are

\[ V_{\text{eff}}(r) = 0, \quad \frac{dV_{\text{eff}}(r)}{dr} = 0. \]

We know that the timelike component of four velocity is greater than zero \((U^t \geq 0)\), yielding (using Eq.(29))

\[ E \left( \frac{P(r^2 + a^2)^2}{r^2} - \frac{Qa^2}{r^2} \right) \geq L \left( \frac{Pa(r^2 + a^2)}{r^2} - \frac{Qa}{r^2} \right). \]  

(34)

which reduces (at horizon) to \( E \geq \frac{aL}{r_+^2 + a^2} \). The angular velocity of the BH (at \( r = r_+ \)) is \( \Omega_H = \frac{a}{r_+^2 + a^2} \). These lead to

\[ E \geq \Omega_H L. \]  

(35)

Now we explore the CME for two particles colliding with rest masses \( m_1 \) and \( m_2 \) moving in equatorial motion. The CME of these particles in the charged accelerating and rotating BH is given by

\[ \frac{E_{\text{cm}}}{\sqrt{2m_1 m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{2m_1 m_2} + \frac{m(r) - N(r)}{T(r)}}, \]  

(36)
Here \( E_i \) and \( L_i \) are the conserved energy and angular momentum for the \( i \)th particle. The above equations indicate that CME depends on rotation.

For the CME near the horizon, i.e., for \( r \to r_+ \), the term on the right side of Eq. (36) is undetermined. Using l'Hospital rule, it yields

\[
\frac{E_{\text{cm}}}{\sqrt{2m_1 m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{2m_1 m_2}} + \frac{M'(r) - N'(r)}{T'(r)}, \tag{37}
\]

with

\[
M'(r) |_{r=r_+} = -[PQ' - \frac{8Pa(r_+^2 + a^2)}{r_+^4} + \frac{4Pa(r_+^2 + a^2)(2Pa r_+ - Q')}{r_+^4}] \\
\times \left[ \frac{E_1E_2P^2(r_+^2 + a^2)^2}{r_+^4} + \frac{2P^2a^2(r_+^2 + a^2)^2}{r_+^4} \frac{E_1E_2}{r_+^3} (2P(r_+^2 + a^2)^2} \right. \\
- \frac{4Pa r_+^2 (r_+^2 + a^2) + Q' a^2}{r_+^4} + \frac{L_1L_2}{r_+^3} (2Pa^2 + r_+ Q')] - [E_1L_2] \\
- E_2L_2] \left. [\frac{6}{r_+^3} (2P^3 a^3 (r_+^2 + a^2)^3) - (2a^2 P^2 (r_+^2 + a^2)^2 (2Pa) + Q' a) + (Pa (r_+^2 + a^2))(PQ' (r_+^2 + a^2)^2 - 8a^2 P^2 r_+ (r_+^2 + a^2))] \right),
\]

\[
N'(r) |_{r=r_+} = \frac{1}{2 \sqrt{n_1(r_+)n_2(r_+)} \{n'_1(r_+)n_2(r_+ + n'_2(r_+)n_1(r_+)) \}}. \]
\[ n'(r) \big|_{r=r_+} = E_i^2 \left\{ \frac{2a^2 P^2 (r_+^2 + a^2)^2}{r_+^4} \right\} \frac{4P (r_+^2 + a^2)}{r_+^3} - \frac{2P (r_+^2 + a^2)}{r_+^3} - \frac{Q'a^2}{r_+^3} \]

\[ + \left[ \frac{P (r_+^2 + a^2)}{r_+^4} \right] \frac{4a^2 P (r_+^2 + a^2) (2P r_+ + Q')}{r_+^4} \]

\[ + \frac{PQ'}{r_+^4} \right\} - \left[ \frac{2a^2 P^2 (r_+^2 + a^2)^2}{r_+^4} \right] \frac{L_1^2 \left( \frac{Q'}{r_+^4} + \frac{2Pa^2}{r_+^3} \right) + 2E_i L_i \left( \frac{2Pr_a}{r_+^4} \right)}{r_+^4} \]

\[ - \left[ \frac{Q'a}{r_+^4} - \frac{2Pa (r_+^2 + a^2)}{r_+^4} \right] + \left[ \frac{2E_i L_i Pa (r_+^2 + a^2)}{r_+^4} - \frac{L_1^2 Pa^2}{r_+^4} \right] \]

\[ \times \left[ \frac{8a^2 P (r_+^2 + a^2)^2}{r_+^4} + PQ' - \frac{4a^2 P (r_+^2 + a^2)}{r_+^4} (2Pr_+ + Q') \right], \]

\[ T'(r) \big|_{r=r_+} = 2P^2 QQ' \big|_{r=r_+} = 0. \]

For \( m_1 = m_2 = m \), the value of \( E_{cm}^2 \) at \( r_+ \) turns out to be

\[ \lim_{r \to r_+} \frac{E_{cm}^2}{2m^2} = \infty. \] (38)

Thus the CME of colliding particles for charged accelerating and rotating BH moving in equatorial plane is infinite for the limiting case.

For \( a = 0 \), we have

\[ E_{cm} = \sqrt{2m \left\{ 1 + \frac{E_1 E_2 r^2}{Q} - \frac{L_1 L_2}{Pr^2} - \frac{1}{Q} \left( -Q + \frac{E_1^2 r^2}{P r^2} - \frac{L_1^2 Q}{Pr^2} \right)^{1/2} \left( -Q \right) \right\} + \frac{E_2^2 r^2}{P r^2} - \frac{L_2^2 Q}{Pr^2} \}}. \] (39)

Expanding this at \( Q = 0 \), it follows that

\[ E_{cm} = 2m \sqrt{1 + \frac{L_1 - L_2}{4Pr^2}} \]

which is the \( E_{cm} \) of the extremal Kerr-Newman BH with \( a = 0 \).

4 Conclusion

In order to discuss the nature of matter and energy in particles collision, the study of particle accelerators is of great significance. It is believed that BH can behave as a particle accelerator. In this paper, we have calculated the
CME by using equation of motion of particles moving in equatorial plane. We have discussed particles colliding near the horizon of Ayon and Garcia BH (regular BH) and Plebanski and Demianski BH with zero NUT (charged accelerating and rotating BH). The CME of the two colliding particles can be arbitrarily high, i.e., the collision can produce energetic particles. For regular black hole, we have taken different values of energy and angular momentum (conserved) and found that unlike the common belief, high energy collision between the particles can take place in a perfectly regular BH. It is found that CME depends on the mass to charge ratio and can become unlimited for some appropriate values of the parameters \((m, q)\). For the charged rotating and accelerating BH, the CME turns out to be arbitrarily high, which depends on rotation parameter only. We conclude that center of mass energy turns out to be infinitely large for both BHs. For \(\alpha = 0\), we have CME for the Kerr-Newmann BH and for \(a = 0 = \alpha\), results reduce to CME for the RN BH.

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