Radion in Multibrane World

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Abstract

The radion dynamics related to the presence of moving branes with both positive or negative tensions is studied in the linearized approximation. The radion effective Lagrangian is computed for a compact system with three branes and in particular we examine the decompactification limit when one brane is sent to infinity. In the non-compact case we calculate the coupling of the gravitational modes (graviton, dilaton and radion) to matter on the branes. The character of gravity on the two branes for all possible combinations of brane tensions is also discussed. It turns out that one can have a normalizable dilaton mode even in the non-compact case. Finally, we speculate on the role of moving branes as a possible source of radion emission.

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1 Introduction

The brane universe scenario has been a quite old idea [1, 2, 3, 4] but recently it has been subject of renewed interest [5, 6, 7] with the realization that such objects are common in string theory. In particular, there has been a lot of activity on warped brane constructions in five spacetime dimensions, motivated by heterotic M-theory [8, 9, 10] and its five dimensional reduction [11, 12]. In the context of these constructions, one can localize gravity on the brane world having four dimensional gravity even with an extra dimension of infinite extent [13, 14] (RS2), or can generate an exponential mass hierarchy on a compact two brane model as it was done in the Randall - Sundrum (RS1) model [15], providing a novel geometrical resolution of the Planck hierarchy problem. For more details on warped models see an excellent review of [16]. It was soon realized that similar multibranes constructions could provide hints to address other long standing puzzles of particle physics as fermion mass hierarchies [17, 18, 19] or neutrino phenomenology [20, 21, 22, 23, 24, 25, 26] (both in warped and unwarped approaches) by considering bulk fields with non-trivial profiles in the extra dimension. Graviton loop effects on brane observables have been studied in [27]. Moreover, multibrane constructions can entirely change our usual conception about gravity by the possibility of having ultra-light massive gravitons contributing substantially to intermediate distance gravity, a scenario called multigravity [28, 29, 30, 31, 32]. An other interesting class of models with massive gravitons was studied in [33, 34]. Observation of modifications of gravity at ultra-large scale could be a striking signal of such a possibility [35, 36]. The multigravity possibility is intimately connected with the multilocalization of gravity in multibrane constructions, a property which can be (with appropriate mass terms) common in fields of all spins as we will review in a forthcoming publication [37].

In this paper we are interested in describing the dynamics of the scalar gravitational perturbations for a generic system with three flat 3-branes embedded in $AdS_5$. These excitations describe the effect of the fluctuation of the size of the extra dimension and/or of the relative positions of the branes. We will distinguish these two kinds of modes by calling the former dilaton [38] and the latter radions [38, 39]. An orbifold symmetry $Z_2$ acting on the extra dimensional coordinate as $y \rightarrow -y$ is also imposed. When the topology of the extra dimension is $S^1$, the compact case, the $Z_2$ action has two fixed points $y = 0, L$ and two of the three branes are sitting on fixed points. As a result of the $Z_2$ symmetry the branes in $y = 0, L$ are frozen and we are left with just one radion field corresponding to the fluctuation of the position of the third brane and the dilaton corresponding to the
fluctuation of the size of the orbifold.

Radion excitations play an important role in the context of multigravity models. A generic problematic feature of multigravity models with flat branes is that massive gravitons have extra polarization states which do not decouple in the massless limit [41], this is known as the van Dam - Veltman - Zakharov discontinuity [11, 12]. However, an equally generic characteristic of these models is that they contain moving branes of negative tension. In certain models the radion can help to recover 4D gravity on the brane at intermediate distances. Indeed, the role of the radion associated with the negative tension brane is precisely to cancel the unwanted massive graviton polarizations and recover the correct tensorial structure of the four dimensional graviton propagator [43, 39], something also seen from the bent brane calculations of [44, 45]. This happens because the radion in this case is a physical ghost because it has a wrong sign kinetic term. This fact of course makes the construction problematic because the system is probably quantum mechanically unstable. Classically, the origin of the problem is the fact that the weaker energy condition is violated in the presence of moving negative tension branes [46, 47].

A way out of this difficulty is to abandon the requirement of flatness of the branes and consider curved ones. A particular example was provided in the " + + " model of [48] where no negative tension brane was needed to get multigravity. The decompactification limit when one of the two branes is sent to infinity was discussed at the same time at [49] (see also [50, 51] for the KK spectrum analysis) and revealed that gravity was localized on the brane although a graviton zero mode was absent. Moreover, due to the fact that the branes where $\text{AdS}_4$ one could circumvent at least at tree level the van Dam - Veltman - Zakharov theorem [52, 53] and the extra polarizations of the massive gravitons where practically decoupled. Of course one can ask the question about the resurrection of these extra polarizations in quantum loops. One loop effects in the massive graviton propagator in $\text{AdS}_4$ were discussed in [54, 55]. Of course, purely four-dimensional theory with massive graviton is not well-defined and it is certainly true that if the mass term is added by hand in purely four-dimensional theory a lot of problems will emerge as it was shown in the classical paper of [56]. If however the underlying theory is a higher dimensional one, the graviton(s) mass terms appear dynamically and this is a different story. All quantum corrections must be calculated in a higher-dimensional theory, where a larger number of graviton degrees of freedom is present naturally (a massless five-dimensional graviton has the same number of degrees of freedom as a massive four-dimensional one).

Moreover, the smoothness of the limit $m \to 0$ is not only a property of the $\text{AdS}_4$
space but holds for any background where the characteristic curvature invariants are non-zero \[57, 58, 59\]. For physical processes taking place in some region a curved space with a characteristic average curvature, the effect of graviton mass is controlled by positive powers of the ratios \(m^2/R^2\) where \(R^2\) is a characteristic curvature invariant (made from Riemann and Ricci tensors or scalar curvature). A very interesting argument supporting the conjecture that there is a smooth limit for phenomenologically observable amplitudes in brane gravity with ultralight gravitons is based on a very interesting paper \[60\]. In that paper it was shown that there is a smooth limit for a metric around a spherically symmetric source with a mass \(M\) in a theory with massive graviton with mass \(m\) for small \(i.e.\) smaller than \(m^{-1}(mM/M_p^2)^{1/5}\) distances. The discontinuity reveals itself at large distances. The non-perturbative solution discussed in \[60\] was found in a limited range of distance from the center and it is still unclear if it can be smoothly continued to spatial infinity (this problem was stressed in \[56\]). Existence of this smooth continuation depends on the full nonlinear structure of the theory. If one adds a mass term by hand the smooth asymptotic at infinity may not exit. As far as we know this question is still open and the only other reference about which we are aware is \[62\]. However, it seems plausible that in all cases when modification of gravity at large distances comes from consistent higher-dimensional models, the global smooth solution can exist because in this case there is a unique non-linear structure related to the mass term which is dictated by the underlying higher-dimensional theory. In a forthcoming paper \[63\] an example of a 5d cosmological solution will be discussed which contains an explicit interpolation between perturbative and non-pertubative regimes: a direct analog of large and small distances in the Schwarschild case\[6\].

An interesting feature of the above "++" model, which only has a dilaton, is that the dilaton survives in the decompactification limit \[49, 50, 51\] when one of the two branes is sent to infinity (see \[34, 53\] for detailed analysis for the dilaton). Indeed, in \[49\] where this limit was firstly discussed it was found that there exists a massive scalar mode in the gravity perturbation spectrum. Although it seems strange to have a dilaton in an infinite extra dimensional model, it is clear that this mode is precisely the remnant of the decompactification process of the compact "++" model. This happens as we will show

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\[5\] Unfortunately this paper was unknown (at least to us) when the debate about possibility to have modification of gravity at large scales started more than a year ago and we became aware about it only recently from the preprint \[61\].

\[6\]IIK is grateful to A. Vainshtein for a very interesting discussion on this subject and for informing us about \[53\].
also to multibrane models with flat branes and is related to the fact that the radion has opposite localization properties compared to the ones of the graviton.

The above multibrane constructions in order to be physically acceptable should incorporate a mechanism which will stabilize the moduli (dilaton, radions) when they have positive kinetic energy and will give them some phenomenologically acceptable mass. This can be achieved by considering for example a bulk scalar field \([66, 67, 68, 69]\) with non trivial bulk potential (for the effect of the Casimir force between the branes see \([70]\)). When the radion has negative kinetic energy it is still not clear whether one can speak about stabilization of these systems. They are probably unstable at the quantum level and no one has attempted to estimate their life-time. A general condition that guarantees stabilization of the dilaton in the case of maximally symmetric branes was derived in \([64]\) and restricts the sum of the effective tensions of the branes and the leftover curvature of the brane. The moduli stabilization has greater importance in the context of brane cosmology where it was found that it played a crucial role in deriving normal cosmological evolution on the branes \([71, 72, 73]\). A non-perturbative analysis of the dilaton two brane models can be found in \([74]\).

In the present paper we will discuss about the dynamics of the dilaton and radions in flat brane models. Firstly, we will examine them in a three brane orbifold model which is a generalization of the "++" multigravity model \([28]\). The decompactification limit is reached moving one of the orbifold fixed points to infinity. For a certain range of tension of the moving brane, the dilaton decouples and we recover the result of \([39]\) for the radion field. However, there is a range of tensions of the moving brane where both the dilaton and the radion are dynamical. In the decompactification limit we study the coupling the dilaton and the radion to matter on the branes for all possible combinations of tensions. Finally, we will speculate about the effect of the motion of the brane and propose that is source of the radion field.

2 The general three three-Brane system

We will consider a three three-brane model on an \(S^1/Z_2\) orbifold. Two of the branes sit on the orbifold fixed points \(y = y_0 = 0, y = y_2 = L\) respectively. A third brane is sandwiched in between at position \(y = y_1 = r\) as in Fig.(1). In each region between the branes the space is \(AdS_5\) and in general the various \(AdS_5\) regions have different cosmological constants.
\( y = 0 \)

\( y = r \)

\( y = L \)

\[ \Lambda_1, \Lambda_2. \]

The action describing the above system is:

\[
S = \int d^4 x dy \sqrt{-G} \left[ 2M_5^3 R - \Lambda(y) - \sum_i V_i \delta(y - y_i) \frac{\sqrt{-g}}{\sqrt{-G}} \right].
\] (1)

where \( M_5 \) is the five dimensional Planck mass, \( V_i \) the tensions of the gravitating branes and \( \hat{g}_{\mu\nu} \) the induced metric on the branes. The orbifold symmetry \( y \to -y \) is imposed.

We seek a background static solution of Einstein equations for the following 4D Poincaré invariant metric ansatz:

\[
d s^2 = a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2.
\] (2)

The solution for the warp factor has the usual exponential form:

\[
a(y) = \begin{cases} 
e^{-k_1 y}, & 0 < y < r \\
e^{-k_2 y + r(k_2 - k_1)}, & r < y < L \end{cases}
\] (3)

where \( k_1 \) and \( k_2 \) are the curvatures of the bulk in the two regions and are related to the bulk cosmological constants as:

\[
k_1^2 = -\frac{\Lambda_1}{24M_5^3}, \quad k_2^2 = -\frac{\Lambda_2}{24M_5^3}.
\] (4)
Moreover, the Einstein equations impose the following fine tuning between the brane tensions and the bulk cosmological constants:

\[ V_0 = 24M_5^2 k_1, \quad V_1 = 24M_5^2 \frac{(k_2 - k_1)}{2}, \quad V_2 = -24M_5^2 k_2. \]  \(\text{(5)}\)

It is straightforward to recover some special models that have been considered in the literature. The RS1 model is obtained for \(k_1 = k_2\) where the intermediate brane is absent (zero tension). For \(k_1 > 0\) and \(k_2 = -k_1\) we get the "\(^{-}-\)" multigravity model \([28, 29]\). For \(k_1 > 0\) and \(k_2 > k_1\) we get the "\(^{-}+\)" brane model considered in \([32]\). In the decompactification limit where \(L \to \infty\) we get also two interesting models: For \(k_1 > 0\) and \(k_2 = 0\) we obtain the Gregory - Rubakov - Sybiriakov (GRS) model \([30]\) and for \(k_1 > 0\) and \(k_2 = 0 > k_1\) the non-zero tension version \([39]\) of the model considered in \([75]\).

3 Effective action

Our purpose is to study fluctuations of the background \((2)\). The first important observation is that there exists a generalization of Gaussian normal coordinates such that in the perturbed geometry the embedding of branes is still described by \(y = 0, y = r\) and \(y = L\), and the metric has the form (see for instance the appendix of \([39]\)):

\[ ds^2 = g_{\mu\nu}(x, y) \, dx^\mu dx^\nu + g_{yy}(x, y) \, dy^2. \]  \(\text{(6)}\)

When analyzed from a 4D point view, in each region, perturbations are of of three types.

- **Spin two:**
  
  Tensor-like perturbation \(h_{\mu\nu}(x, y)\) corresponding to massive (massless) 4D gravitons

\[ ds^2 = a^2(y) \left[ \eta_{\mu\nu} + h_{\mu\nu}(x, y) \right] \, dx^\mu dx^\nu + dy^2. \]  \(\text{(7)}\)

- **Spin zero: Dilaton**
  
  Scalar perturbation \(f_1(x)\) corresponding to an overall rescaling of distances \([38]\)

\[ ds^2 = a^2(y) \left[ 1 + Q(y) f_1(x) \right] \eta_{\mu\nu} \, dx^\mu dx^\nu + \left[ 1 + q(y) f_1(x) \right] \, dy^2. \]  \(\text{(8)}\)

- **Spin zero: Radion**
Scalar perturbation $f_2(x)$ corresponding to a fluctuating distance of the branes\cite{39}:

$$ds^2 = a^2(y) [(1 + B(y)f_2(x)) \eta_{\mu\nu} + 2\epsilon(y) \partial_\mu \partial_\nu f_2(x)] dx^\mu dx^\nu + [1 + 2A(y) f_2(x)] dy^2. \quad (9)$$

The function $\epsilon(y)$ is needed in order that the metric satisfies the Israel junction conditions in the presence of moving branes. As it will be shown later, the values of $\partial_y \epsilon$ at $y = 0$, $r$, $L$ are gauge invariant. When $k_1 \neq k_2$ a non-trivial $\epsilon$ is required.

The generic perturbation can be written as

$$ds^2 = a^2(y) \{[1 + \varphi_1(x, y)] \eta_{\mu\nu} + 2\epsilon(y) \partial_\mu \partial_\nu f_2(x) + h_{\mu\nu}(x, y)\} dx^\mu dx^\nu + [1 + \varphi_2(x, y)] dy^2; \quad (10)$$

where

$$\varphi_1(x, y) = Q(y) f_1(x) + B(y) f_2(x)$$

$$\varphi_2(x, y) = q(y) f_1(x) + 2A(y) f_2(x). \quad (11)$$

Given the expression (3) for $a$, Israel junctions conditions at $y = 0$, $L$, simply require that $A, B, \partial_y \epsilon, Q, q$ are continuous there\cite{39}. The 4D effective action $S_{eff}$ for the various modes is obtained inserting the ansatz (11) in the action (1) and integrating out $y$. So far the functions $A, B, \epsilon, Q, q$ haven’t been specified, however imposing that $S_{eff}$ contains no mixing terms among $h$ and $f_i$ one determines $Q, q$ and $A$ is expressed in terms of $B$ (see appendix) which satisfies

$$\frac{d}{dy} (Ba^2) + 2a^{-1} \frac{da}{dy} \frac{d}{dy} (a^4 \partial_y \epsilon) = 0; \quad (12)$$

$$\int_{-L}^{L} dy \ a \left(\frac{da}{dy}\right)^{-1} \frac{dB}{dy} = 0. \quad (13)$$

As a consequence of the no-mixing conditions the linearized Einstein equations for (11) will consist in a set independent equations for the graviton and the scalars.

The effective Lagrangian reads (see appendix)

$$S_{eff} = \int d^4x L_{eff} = \int d^4x (L_{Grav} + L_{Scal})$$

$$L_{eff} = 2M_5^3 \int_{-L}^{L} dy \left\{a^2 L_{PF}(h) + \frac{a^4}{4} [(\partial_y h)^2 - \partial_y h_{\mu\nu} \partial_y h^{\mu\nu}] + L_{Scal} \right\}; \quad (14)$$

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with
\[ \mathcal{L}_{\text{Scal}} = \mathcal{K}_1 f_1 \Box f_1 + \mathcal{K}_2 f_2 \Box f_2 \]
\[ \mathcal{K}_1 = 2M_5^3 \frac{3}{2} a^2 \int_{-L}^{L} a^{-2} dy, \quad \mathcal{K}_2 = -2M_5^3 \frac{3}{4} \int_{-L}^{L} a \left( \frac{da}{dy} \right)^{-1} \frac{d}{dy} (B^2 a^2) dy. \] (15)

In (14), the spin-2 part, as expected, contains the 4D Pauli-Fierz Lagrangian \( \mathcal{L}_{PF}(h) \) for the graviton (see Appendix) plus a mass term coming from the dimensional reduction. In the scalar part \( \mathcal{L}_{\text{Scal}} \) the mass terms are zero since \( f_i \) are moduli fields. Notice that after the no-mix conditions are enforced, the effective Lagrangian contains the undetermined function \( \epsilon \).

The metric ansatz \( G_{MN} \) in (11) is related to a special coordinate choice (generalized Gaussian normal), nevertheless a residual gauge (coordinate) invariance is still present. Consider the class of infinitesimal coordinate transformations \( X^M \to X'^M = X^M + \xi(X)^M \) such that the transformed metric \( G'_{MN} = G_{MN} + \delta G_{MN} \) retains the original form (10) up to a redefinition of the functions \( q, Q, A, B, \epsilon \) and the dilaton and the radion field. Consistency with the orbifold geometry and the requirement that the branes in \( y = 0, r \) and \( L \) are kept fixed by diffeomorphisms lead to \( \xi^5(x,0) = \xi^5(x,r) = \xi^5(x,L) = 0 \). From
\[ \delta G_{MN} = -\xi^A \partial_A G_{MN} - \partial_m \xi^A G_{AM} - \partial_N \xi^A G_{MA}, \] (16)

one can show that \( \xi^M \) has to be of the form
\[ \xi^\mu(x,y) = \hat{\xi}^\mu(x) - W(y) \eta^{\mu\nu} \partial_\nu f_2(x), \quad \xi^5(x,y) = a^2 W'(y) f_2(x) \]
\[ \text{with} \quad W'(0) = W'(r) = W'(L) = 0. \] (17)

The case \( W = 0 \) corresponds the familiar pure 4D diffeomorphisms, under which \( h_{\mu\nu} \) transforms as spin two field, \( f_i \) as scalars and \( q, Q, A, B, \epsilon \) are left unchanged. On the contrary the case \( \hat{\xi}^\mu = 0, W \neq 0 \) is relic of 5D diffeomorphisms and one can check that \( q, Q, A, B, \epsilon \) are not invariant and in particular \( \delta \epsilon = W \). As a result, the values of \( \partial_y \epsilon \) in 0, \( r \) and \( L \) are gauge independent and this renders \( \mathcal{L}_{\text{eff}} \) free from any gauge ambiguity.
4 Scalars Kinetic Energy

4.1 The compact case

In this section we will focus on the part of the effective Lagrangian involving the scalars and concentrate on the dilaton and radion kinetic term coefficients $K_1$ and $K_2$. In particular we are interested in the cases when the radion becomes a ghost field, i.e. $K_2 < 0$. Firstly, it is trivial to obtain the dilaton kinetic term $K_1$ by integrating (15):

$$K_1 = 3c^2 M_5^3 \left[ \frac{a^{-2}(r) - 1}{k_1} + \frac{a^{-2}(L) - a^{-2}(r)}{k_2} \right]$$  \hspace{1cm} (18)

It turns out that for any possible values of $k_1$, $k_2$ and $r$, $L$ the above quantity is positive definite. The radion kinetic term on the other hand is more involved. Integrating (12) we get the radion wavefunction for the regions ($y > 0$):

$$B(y) = \begin{cases} 
    c_1 a^{-2} + 2k_1 \partial_y \epsilon a^2, & 0 < y < r \\
    c_2 a^{-2} + 2k_2 \partial_y \epsilon a^2, & r < y < L
\end{cases} \hspace{1cm} (19)$$

where $c_1$ and $c_2$ are integration constants. The orbifold boundary conditions demand that $\partial_y \epsilon(0) = \partial_y \epsilon(L) = 0$ since $\epsilon$ is an even function of $y$. From the non-mixing conditions for radion and dilaton (13) and the continuity of $B$ we are able to determine $c_2$ and $\partial_y \epsilon(r)$ as the following:

$$c_2 = c_1 \frac{k_2}{k_1} \frac{a^2(r) - 1}{\left( \frac{a(r)}{a(L)} \right)^2 - 1}$$  \hspace{1cm} (20)

$$\epsilon'(r) = \frac{c_1 k_2}{2k_1(k_2 - k_1)a^4(r)} \left[ \frac{k_1}{k_2} - \frac{a^2(r) - 1}{\left( \frac{a(r)}{a(L)} \right)^2 - 1} \right].$$  \hspace{1cm} (21)

Therefore, the values of the radion wavefunction $B$ at the brane positions are given by the
following expressions:

\[
B(0) = c_1 ;
\]

\[
B(r) = \frac{c_1 k_2}{(k_2 - k_1)} \frac{1 - a^2(L)}{a^2(L) \left[ \left( \frac{a(r)}{a(L)} \right)^2 - 1 \right]} ;
\]

\[
B(L) = \frac{c_1 k_2}{k_1} \frac{a^2(r) - 1}{a^2(L) \left[ \left( \frac{a(r)}{a(L)} \right)^2 - 1 \right]} .
\]

Thus we can carry out the integral in (15) to find the radion kinetic term coefficient:

\[
\mathcal{K}_2 = 3M_5^3 \left[ \left( \frac{1}{k_1} - \frac{1}{k_2} \right) B^2(r) a^2(r) + \frac{1}{k_2} B^2(L) a^2(L) - \frac{1}{k_1} B^2(0) \right]
\]

\[
= \frac{3M_5^3 c_1^2}{k_1} \left\{ \frac{k_2}{(k_2 - k_1)} \frac{a^2(r)(a^2(L) - 1)^2}{a^4(L) \left[ \left( \frac{a(r)}{a(L)} \right)^2 - 1 \right]^2} + \frac{k_2}{k_1} \frac{(a^2(r) - 1)^2}{a^2(L) \left[ \left( \frac{a(r)}{a(L)} \right)^2 - 1 \right]^2} - 1 \right\}.
\]

The above quantity is not positive definite. In particular, it turns out that it is positive whenever the intermediate brane has positive tension and negative whenever the intermediate brane has negative tension. This result is graphically represented in Fig.(2) where the \((k_1, k_2)\) plane is divided in two regions.

### 4.2 The non-compact limit

It is instructive to discuss the decompactification limit in which the third brane is sent to infinity, \(L \to +\infty\). To examine this limit we distinguish two cases:

**The case \(k_2 > 0\)**

In this case we have \(a(\infty) \propto \lim_{L \to \infty} e^{-k_2L} = 0\). The dilaton kinetic term is trivial since from (18) we obtain \(\mathcal{K}_1 \to \infty\). In other words the dilaton decouples from the 4D effective theory and the condition of absence of kinetic mixing between the scalars (13) plays no role. The radion kinetic term coefficient can be read off from (25):

\[
\mathcal{K}_2 = 3M_5^3 c_1^2 \left[ e^{2k_1 r} \frac{k_2}{(k_2 - k_1)} - 1 \right].
\]
Figure 2: Sign of the radion kinetic term in the $(k_1, k_2)$ plane. In the regions $A, B', C'$ the moving brane has positive tension and the radion positive kinetic energy. In the regions $B, C, A'$ the moving brane has negative tension and the radion negative kinetic energy. We show the GRS line for the non-compact case. The dashed line corresponds to $k_1 = k_2$, i.e. a tensionless moving brane.

This result agrees with the computation of [39]. It is easy to see that the radion has positive kinetic energy when the moving brane has positive tension and negative kinetic energy when the tension is negative. Indeed, for $0 < k_1 < k_2$, or for $k_2 > 0$ and $k_1 < 0$ we have a positive brane and positive kinetic energy. On the other hand for $0 < k_2 < k_1$ we have a negative tension brane and negative kinetic energy. In the limit $k_2 \to 0$ we get the GRS model with negative kinetic energy as in [39].

The case $k_2 < 0$

In this case $a(\infty) \propto \lim_{L \to \infty} e^{-k_2 L} \to \infty$. This time the dilaton plays in the game since $\mathcal{K}_1$ is finite as seen from (18) and has the value:

$$\mathcal{K}_1 = \frac{3M_5^2 c^2}{2k_1|k_2|} \left[e^{2k_1 r}(|k_2| + k_1) - |k_2|\right]$$

which is manifestly positive definite. The presence of dilaton mode is a bit surprising since it describes the fluctuations of the overall size of the system which is infinite. Something similar happens in the model of [49] that has a dilaton mode although it is non-compact. The dilaton is a remnant of the decompactification process of the "++" model [48] and
enters in the game because the inverse of the warp factor is normalizable.

The radion kinetic term coefficient can be obtained from taking the limit \( L \to +\infty \) in \(^{28}\). We get

\[
K_2 = \frac{3M_5^3 c_1^2}{k_1} \left[ e^{-2k_1 r} \frac{k_2}{(k_2 - k_1)} - 1 \right]
\]

(28)

The same considerations for the compact case applies here. When \( k_1 < k_2 < 0 \) we have a positive tension brane and a positive kinetic energy. On the other hand when \( k_2 < k_1 < 0 \), or for \( k_2 < 0 \) and \( k_1 > 0 \) we have a negative tension brane and a negative kinetic energy. In the GRS limit \( k_2 \to 0 \), the radion has negative kinetic energy.

5 Gravity on the branes

In this section we will study how the moduli and graviton(s) couple to matter confined on the branes. For simplicity we will study the non-compact case. For this purpose, we consider a matter Lagrangian \( \mathcal{L}_m(\Phi_i, \hat{g}) \), where we denote generically with \( \Phi_i \) matter fields living on the branes; \( \hat{g} \) is the induced metric on the branes. Our starting point is the following action

\[
S_{5D} = \int d^4xdy \sqrt{G} \left[ 2M_5^3 R - \Lambda(y) \right] - \sum_i \int d^4x \sqrt{\hat{g}} V_i + \int d^4xdy \sqrt{\hat{g}} \mathcal{L}_m(\Phi_i, \hat{g})
\]

(29)

We have already calculated the effective action for the gravity sector for the perturbation \(^{10}\). It is useful to decompose the perturbation \( h(x, z) \) in terms of a complete set of eigenfunctions \( \Psi^{(n)}(z) \) of the graviton kinetic operator (a suitable 4D gauge fixing like de Donder is understood)

\[
h(x, z) = \sum_n \Psi^{(n)}(z) h^{(n)}_{\mu\nu}(x) + \int dm \Psi(y, m) h^{(m)}_{\mu\nu}(x)
\]

(30)

having taken into account both the discrete and the continuum part of the spectrum. The effective 4D Lagrangian has the following form

\[
\mathcal{L}_{4D} = \mathcal{L}_m(\Phi_i, \eta) + 2M_5^3 \sum_n \mathcal{L}_{PF}(h^{(n)}(x)) + K_1 f_1 \Box f_1 + K_2 f_2 \Box f_2
\]

\[
- \sum_n \frac{\Psi^{(n)}(y_{br})}{2} h^{(n)}_{\mu\nu}(x) T^{\mu\nu} - \frac{Q(y_{br})}{2} f_1 T - \frac{B(y_{br})}{2} f_2 T;
\]

(31)
The matter fields have been rescaled $\Phi_i \rightarrow \Phi_i^c$, to make them canonically normalized and the energy momentum tensor $T_{\mu\nu}$ is defined with respect to the rescaled fields $\Phi_i^c$. By construction the induced background metric on the branes is the flat 4D Minkowski metric. The asterix denotes that the sum has to be converted into an integral for the continuum part of the spectrum. Finally, defining the canonical normalized fields

$$\bar{h}^{(n)}_{\mu\nu}(x) = \sqrt{2M_5^2}h^{(n)}_{\mu\nu}(x), \quad \bar{f}_i = \sqrt{2|\mathcal{K}_i|}f_i,$$

the Lagrangian reads

$$\mathcal{L}_{4D} = \mathcal{L}_m(\Phi_i^c, \eta) + \sum_n^* \mathcal{L}_{PF}(\bar{h}^{(n)}(x)) + \frac{1}{2} \tilde{f}_1 \Box \tilde{f}_1 + \frac{1}{2} \text{sgn}(\mathcal{K}_2) \tilde{f}_2 \Box \tilde{f}_2$$

$$- \sum_n \frac{\Psi^{(n)}(y_{br})}{2\sqrt{2M_5^2}} \bar{h}^{(n)}_{\mu\nu}(x) T^{\mu\nu} - \frac{Q(y_{br})}{2\sqrt{2\mathcal{K}_1}} \tilde{f}_1 T - \frac{B(y_{br})}{2\sqrt{2|\mathcal{K}_2|}} \tilde{f}_2 T.$$  (33)

Thus the dilaton, the radion and the graviton (whenever it is normalizable) have the following couplings respectively:

$$C_D = \frac{Q(y_{br})}{2\sqrt{2\mathcal{K}_1}}, \quad C_R = \frac{B(y_{br})}{2\sqrt{2|\mathcal{K}_2|}}, \quad C_G = \frac{\Psi^{(0)}(y_{br})}{2\sqrt{2M_5^2}}.$$  (34)

For convenience we will define $C_N \equiv \sqrt{\frac{2 \mathcal{K}_1}{k_2}}$ and write the above quantities as:

$$C_D^{(i)} = C_N \frac{g_D^{(i)}}{\sqrt{3}}, \quad C_R^{(i)} = C_N \frac{g_R^{(i)}}{\sqrt{3}}, \quad C_G = C_N \frac{\Psi^{(0)}(y_{br})}{\sqrt{|k_1|}}.$$  (35)

where the dimensionless radion couplings are:

$$g_D^{(0)} = \frac{\theta(-k_2)}{\sqrt{e^{2k_1 r k_1 + |k_2|/|k_2|} - 1}}, \quad g_D^{(r)} = e^{2k_1 r} g_D^{(0)}$$

$$g_R^{(0)} = \frac{1}{\sqrt{e^{2k_1 r \text{sgn}(k_2) k_2/|k_2|} - 1}}, \quad g_R^{(r)} = e^{2k_1} \frac{k_2}{k_2 - k_1} g_R^{(0)}.$$  (36)

We should note at this point that there are certain cases as we will see in the following that multigravity is realized and four dimensional gravity in intermediate distances is due
Figure 3: The function $\sigma(y) = -\log[a(y)]$ for all possible combinations of $k_1, k_2$. The regions are named in accordance with the Fig.(2) phase diagram.

to more than one mode. In that case we will denote by $C_G$ the coupling of the “effective zero mode” even though there might not be a genuine zero mode at all.

We will now discuss how these couplings behave is the six distinct combinations of $k_1$ and $k_2$ shown in Fig.(3).

**Region $A$**

In this case, we have the non-compact system of two positive tension branes discussed in [75, 39, 32]. The volume of the extra dimension is finite and therefore we have a normalizable graviton zero mode. The KK tower will be continuum and its coupling to matter on either branes will be exponentially suppressed for reasonably large $r$ as can be easily seen by the analysis of [32]. The graviton wavefunction in the conformal gauge is constant, i.e. $\Psi^{(0)}(y) = A$. Thus the universal coupling of the zero mode will be $C_G = C_N$.

The dilaton in this region will simply be absent, or equivalently it will have zero coupling $C_D = 0$ to the branes because $K_1 \to \infty$. The radion in this region has positive kinetic term and from (37) we find that its coupling to matter on the $y = 0$ and $y = r$ branes is bounded as following (see Fig. 4):

$$0 \leq g^{(0)}_R \lesssim e^{-k_1 r}, \quad g^{(r)}_R \gtrsim e^{k_1 r} \quad (38)$$

The radion on the central brane is always weakly coupled and decouples in the tensionless moving brane limit. On the other hand the radion is always strongly coupled on the moving brane and diverges in the tensionless limit where $k_2 \to k_1$. 

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**Region B**

The volume of the extra dimension is still finite and therefore we have a normalizable graviton zero mode. When $k_1 = k_2$ the brane is tensionless and as soon as $k_2 < k_1$ the KK continuum starts to develop a resonance. This resonance is initially rather broad as $k_2 \to k_1$ and its width decreases until it coincides with the GRS width $\Gamma \sim k_1 \exp(-3k_1 r)$ as $k_2 \to 0$. In this region four dimensional gravity at intermediate distances is the net effect of the massless graviton and the lower part of the KK continuum that contributes more and more as $k_2 \to 0$.

The dilaton is again absent, $C_D = 0$, since still we have $\mathcal{K}_1 \to \infty$. The radion in this region is a ghost field, to compensate for the presence of the extra polarization states of the contributing massive gravitons, and its coupling to the branes is bounded as (see Fig. 4):

$$0 \leq g^{(0)}_R \leq 1, \quad g^{(r)}_R \leq 0 \quad (39)$$

The radion on the central brane interpolates between the decoupling limit of the tensionless moving brane to the GRS limit where it couples with strength equal to the one of the graviton. On the other hand the radion coupling on the moving brane interpolates between the infinitely strong limit of the tensionless (strictly speaking there is no brane in $y = r$ and we are left with RS2) brane to the GRS limit where it decouples.

**Region C**

In this region we have a infinite analogue of the $''+--''$ model with no normalizable zero mode present. The graviton spectrum for $k_2 \to 0$ is approximately equally spaced and there is a resonance which coincides with the GRS width and gets more and more broad as $k_2$ gets more negative. Soon enough a special light state is singled out as in the $''+--''$ model, a behaviour that persists for all values of $k_2$ in this region. From the above behaviour we deduce that four dimensional gravity at intermediate distances is generated by the lower part of the discrete spectrum as $k_2 \to 0$, whereas only by the special state for all other values of $k_2$.

The dilaton in this region is present and its coupling to the branes is bounded as following (see Fig. 4):

$$0 \leq g^{(0)}_D \lesssim e^{-k_1 r}, \quad 0 \leq g^{(r)}_D \lesssim e^{k_1 r} \quad (40)$$
It is always weakly coupled to matter on the central brane and becomes strongly coupled but saturated on the moving brane.

The radion in this region is again a ghost field to cancel the unwanted extra massive graviton polarization states and its coupling to the branes is bounded as (see Fig. [4]):

\[
g^{(0)}_R \approx 1, \quad 0 \leq g^{(r)}_R \lesssim e^{k_1 r} \tag{41}\]

On the central brane it couples always with strength equal to the one of the “effective graviton” and on the moving brane it interpolates between the decoupling limit of the GRS case to a strongly coupled region with saturated coupling as the tension of the second brane gets infinite.

Region A’

In this region we have no normalizable zero mode and the two negative tension brane system resembles an inverted version of the RS2 model. The KK spectrum is discrete and all excitations lie above the characteristic curvature scale \(k_1\). Thus the low energy effective theory does not have four dimensional gravity at all.

The dilaton in this system is present and the coupling to the branes is approximately constant (see Fig. [4]):

\[
g^{(0)}_D \approx 1, \quad g^{(r)}_D \approx 1 \tag{42}\]

The radion field is a ghost and its coupling to matter on the branes is bounded as (see Fig. [4]):

\[
g^{(0)}_R \gtrsim e^{-3|k_1| r}, \quad 0 \leq g^{(r)}_R \lesssim e^{-2|k_1| r} \tag{43}\]

The radion on the central negative tension brane is always weakly coupled and vanishes in the limit of \(k_2 \to k_1 = -|k_1|\) of the tensionless moving brane. On the other hand the coupling on the moving brane is bounded from below in the limit of infinite negative tension brane and diverges at the limit of the tensionless brane.

Region B’

In this region there is again no normalizable zero mode and the system of the negative and positive tension branes still resembles the inverted RS2 model. The KK spectrum is
Figure 4: The dimensionless couplings of the radion (upper) and the dilaton (lower) to matter on the $y = 0$ or the $y = r$ branes. The left diagrams correspond to $k_1 > 0$ while the right ones for $k_1 < 0$. The regions $A, B, \ldots$ are in accordance with the ones of the phase diagram in Fig.(2). The diagrams are not in scale.

almost identical with the one of the previous region except for the limit $k_2 \to 0$ when the spectrum drops below the curvature scale $k_1$ and a continuum develops.

The dilaton coupling to the branes decreases from the constant value of the previous region, to zero as $k_2 \to 0$ (see Fig. 4):

$$0 \leq g^{(0)}_D \lesssim 1, \quad 0 \leq g^{(r)}_D \lesssim e^{-2|k_1|r}$$  \hspace{1cm} (44)

The radion has positive kinetic energy and its coupling to the branes is bounded as following (see Fig. 4):

$$0 \leq g^{(0)}_R \leq 1, \quad g^{(r)}_R \leq 0$$  \hspace{1cm} (45)
It is weakly coupled in the central brane and the coupling interpolates between zero from the previous region to one in the inverted GRS case where \( k_2 \to 0 \). On the other hand it is divergent as \( k_2 \to k_1 = -|k_1| \) and vanishes as \( k_2 \to 0 \).

**Region C’**

In this region we again have a system of a negative and a positive tension brane, but gravity can be localized on the moving positive tension brane. There is a normalizable zero mode that mediates four dimensional gravity and a KK continuum with suppressed couplings on the branes.

The dilaton in this region will be absent, or equivalently it will have zero coupling \( C_D = 0 \) to the branes because \( K_1 \to \infty \).

The radion will have again positive kinetic term and its coupling will be bounded as following (see Fig. 4):

\[
0 \leq g_R^{(r)} \lesssim e^{-2|k_1|r}, \quad g_R^{(0)} \approx 1
\]  

(46)

It is approximately constant on the central brane with strength equal to the one of the graviton and is weakly coupled to the moving brane with coupling interpolating between zero in the inverted GRS case with \( k_2 \to 0 \) to a central value as the tension of the moving brane becomes infinitely large.

**6 Discussion and conclusions**

In the previous sections we have considered the infinitesimal branes motion consistent with our linearized treatment of Einstein equations. The ”small” motion takes place as a fluctuation around the unperturbed position \( y = r \). It is interesting to ask what would happen in the generic situation when the small perturbation condition is relaxed. In this case the radion excitations will be defined as the perturbation around the a priori determined time varying positions of the branes in the bulk.

The motion of the brane will be an additional source of energy and momentum in the bulk and will excite all gravity excitations between which the radion as well. It would be interesting to find a actual solution of a moving brane in the bulk and study the dynamics of the radion field. In [74] the analogous idea for the dilaton was studied. In the case of the radion there is not a simple solution preserving Poincaré or (A)dS invariance on the branes.
when they are moving. The dynamics of the radions in a system of the moving branes is more subtle than the ones of the dilaton and we hope to address this problem in the future.

In conclusion, in this paper we presented the dilaton and radion dynamics in a flat brane system in warped bulk. We showed how one could calculate the effective action for these modes for a general three brane compact model. In the following we examined the kinetic term coefficients for these modes and for the sake of simplicity we sent the third brane at infinite distance. The physics was in accordance with our intuition that a positive tension brane has positive kinetic term and a negative tension one gives rise to a ghost radion. We calculated the radion and studied it for all possible cases of brane tension combinations. Additionally, we presented how the tensor gravity excitations behave in the above regions. Finally, we speculated about the motion of the brane being a source for the radion field.

Acknowledgments: We would like to thank Thibault Damour, Panagiota Kanti, Graham G. Ross and Arkady Vainshtein for very stimulating discussions. S.M.’s work is supported by the Hellenic State Scholarship Foundation (IKY) No. 811781027. A.P.’s work is supported by the Hellenic State Scholarship Foundation (IKY) No. 8017711802. This work is supported in part by the PPARC rolling grant PPA/G/O/1998/00567, by the EC TMR grants HRRN-CT-2000-00148 and HPRN-CT-2000-00152.

Appendix

It is convenient to define a new variable \( z \) defined by

\[
\frac{1}{a(y)} = \frac{dz}{dy} \quad (A.1)
\]

In the coordinates \((x, z)\) the metric \((10)\) is conformal to a flat perturbed metric \( \bar{G}_{AB} = \eta_{AB} + H_{AB} \)

\[
ds^2 = \alpha^2 \left[ \bar{G}_{\mu\nu} dx^\mu dx^\nu + \bar{G}_{zz} dz^2 \right] = \alpha^2 \left[ \eta_{MN} + H_{MN} dx^M dx^N \right]
\]

\[
H_{\mu\nu} = \phi_1(x, z) \eta_{\mu\nu} + 2\epsilon(z) \partial_\mu \partial_\nu f_2(x) + h_{\mu\nu}(x, z) \quad (A.2)
\]

\[
H_{zz} = \phi_2(x, z)
\]

Inserting \((A.2)\) in \((10)\) and taking into account the equation of motion satisfied by \( a \) one
gets
\[ S_{\text{eff}} = \int d^4x \mathcal{L}_{\text{eff}} = \int d^4x 2M_0^3 \int dz \left[ a^3 \mathcal{L}_{\text{PF}}(h) + \frac{a^3}{4} [ (\partial z h)^2 - \partial_z h_{\mu\nu} \partial_z h^{\mu\nu} ] + \mathcal{L}_\phi + \mathcal{L}_{h\phi} \right] . \]
\[ \mathcal{L}_\phi = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_{12} \quad ; \]

(A.3)

Where
\[ \mathcal{L}_{h\phi} = \left[ a^3 \left( \varphi_1 + \frac{1}{2} \varphi_2 \right) + f_2 \frac{d}{dz} (\epsilon' a^3) \right] \partial_\mu \partial_\nu h^{\mu\nu} \]
\[ - \left[ a^3 \left( \varphi_1 + \frac{1}{2} \varphi_2 \right) + f_2 \frac{d}{dz} (\epsilon' a^3) \right] \Box h + \frac{3}{2} \frac{d}{dz} \left( a^2 a' \varphi_2 - a^3 \varphi_1' \right) h \quad ; \]

(A.4)

\[ \mathcal{L}_1 = \partial_\mu f_1 \partial^\mu f_1 \frac{3}{2} a^3 \left( Q^2 + Q q \right) + f_1^2 \left( 3a^3 Q'^2 + 3a^2 a q^2 - 6a^2 a' Q' q \right) ; \]

(A.5)

\[ \mathcal{L}_2 = \partial_\mu f_2 \partial^\mu f_2 \left( \frac{3}{2} a^3 B^2 + 3a^3 AB - 3a^3 B' \epsilon' + 6a^2 a' \epsilon' A \right) \]
\[ + f_2^2 \left( 3a^3 B^2 + 12a^2 a A^2 - 12a^2 a' AB' \right) ; \]

(A.6)

\[ \mathcal{L}_{12} = \partial_\mu f_1 \partial^\mu f_2 \left( \frac{3}{2} a^3 B q + 3a^3 A Q + 3a^3 B Q - 3a^3 \epsilon' Q' + 3a^2 a' \epsilon' q \right) \]
\[ + f_1 f_2 \left[ 6a^3 B' Q' + 12a^2 a A q - 6a^2 a' \left( 2A Q' + q B' \right) \right] ; \]

(A.7)

and \( \mathcal{L}_{\text{PF}}(h) \) is the 4D Pauli-Fierz Lagrangian for \( h \)
\[ \mathcal{L}_{\text{PF}}(h) = \frac{1}{2} \partial_\nu h_{\mu\alpha} \partial^\alpha h^{\mu\nu} - \frac{1}{4} \partial_\mu h_{\alpha\beta} \partial^\mu h^{\alpha\beta} - \frac{1}{2} \partial_\alpha h \partial_\beta h^{\alpha\beta} + \frac{1}{4} \partial_\alpha h \partial^\alpha h . \]

(A.8)

Differentiation with respect of \( z \) is denoted with a prime. The absence of mixing terms in \( \mathcal{L}_{\text{eff}} \) yields the following constraints
\[ A(z) = \frac{a B'}{2a'} , \quad \frac{d}{dz} \left( B a^2 \right) + \frac{2a'}{a^2} \frac{d}{dz} \left( a^3 \epsilon' \right) = 0 \]
\[ Q(z) = c a^{-2} , \quad q(z) = -2c a^{-2} , \quad c \text{ is a constant} ; \]
\[ \int dz a A(z) = 0 . \]

(A.9)

(A.10)

(A.11)
Eqns. (A.9)-(A.11) give

\[
L_{\text{eff}} = 2M_5^3 \int dz \left\{ a^3 L_{PF}(h) + \frac{a^3}{4} \left[ (\partial_z h)^2 - \partial_z h_{\mu\nu} \partial_z h^{\mu\nu} \right] - \frac{3}{2} c^2 a^{-1} \partial_z f_1 \partial^\mu f_1 + \frac{3}{4} a^2 \frac{d}{dz} \left( B^2 a^2 \right) \partial_{\mu} f_2 \partial^\mu f_2 \right\} .
\]

(A.12)

In particular the effective Lagrangian \( L_{\text{Scal}} \) for the dilaton \( f_1 \) and the radion \( f_2 \) is

\[
L_{\text{Scal}} = K_1 f_1 \Box f_1 + K_2 f_2 \Box f_2
\]

\[
K_1 = 2M_5^3 \frac{3}{2} c^2 \int_{-L}^{L} a^{-2} dy
\]

\[
K_2 = -2M_5^3 \frac{3}{4} \int_{-L}^{L} a \left( \frac{da}{dy} \right)^{-1} \frac{d}{dy} \left( B^2 a^2 \right) dy
\]

(A.13)

with

\[
\frac{d}{dy} \left( B a^2 \right) + 2a^{-1} \frac{da}{dy} \frac{d}{dy} \left( a^4 \partial_y \epsilon \right) = 0 ;
\]

\[
\int_{-L}^{L} dy a \left( \frac{da}{dy} \right)^{-1} \frac{dB}{dy} = 0 .
\]

(A.14)

As a result only \( \partial_y \epsilon(0) \), \( \partial_y \epsilon(r) \) and \( \partial_y \epsilon(L) \) enter the radion effective action.

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