Neutrino Production via $e^-e^+$ Collision at $Z$-boson Peak

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The production of the three normal neutrinos via $e^-e^+$ collision at $Z$-boson peak (neutrino production in a $Z$-factory) is investigated thoroughly. The differences of $\nu_e$-pair production from $\nu_\mu$-pair and $\nu_\tau$-pair production are presented in various aspects. Namely the total cross sections, relevant differential cross sections and the forward-backward asymmetry etc for these neutrinos are presented in terms of figures as well as numerical tables. The restriction on the room for the mixing of the three species of light neutrinos with possible externals (heavy neutral leptons and/or sterile) from refined measurements of the invisible width of $Z$-boson is discussed.

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I. INTRODUCTION

An electron-positron collider, running at the peak of $Z$-boson i.e. at the center mass energies around $Z$-boson mass, is called as a $Z$-factory, so neutrino production in such a $Z$-factory is just the production via $e^-e^+$ collision at $Z$-boson peak. In fact, since at a $Z$-factory the production of all kinds of Standard Model (SM) fermion-antifermion pair (except $t$-quark pair) is mainly via $Z$ boson s-channel annihilation, so it is greatly enhanced by the resonance effects of the $Z$-boson, i.e. the cross-sections of the production appear a peak at collision CM energy around $Z$-boson mass, moreover, the production at leading order is of two body in final state, so the produced fermion and anti-fermion have definite momentum and energy. Considering the characteristics, the production of the three kinds of normal light neutrinos is expected to be interesting, e.g. the restriction on the room for the mixing of the light neutrinos with externals, and to answer whether or with what a luminosity of the $Z$-factory the produced neutrinos may be detected with an accessible detector, even used as a mono-energy neutrino source etc.

Of the production of the three kinds of normal light neutrinos, the $\nu_e\bar{\nu}_e$ pair production is special: it is produced via the interference of the $Z$ boson s-channel annihilation and $W$ boson $t$-channel exchange, although of the rest two species, the production of $\nu_\mu\bar{\nu}_\mu$ and $\nu_\tau\bar{\nu}_\tau$, is via the $Z$ boson s-channel annihilation as the other fermions (quarks, muon and $\tau$-lepton). Moreover, if the invisible width of $Z$-boson measured at $Z$-factory indeed is due to the contributions from neutrino production and it may be measured accurately enough, then not only the number of the light neutrino species may be realized, but also one may see how big a ‘room’ still left in the invisible width for the mixing of the normal light neutrinos with possible external one(s), whereas the possible mixing is an absorbing topic on neutrino physics.$^{[1–4]}$

To determine the number of the light neutrino species and to examine the room left for mixing with externals both is to compare the invisible width of $Z$-boson measured at $Z$-factory experimentally with the contributions from production of each specie of the neutrinos, which are estimated theoretically. Thus we are now interested in studying the production of the three species of the neutrinos in a $Z$-factory theoretically with care.

The earlier $Z$-factories, such as LEP-I and SLC, run just under a luminosity of $10^{31}$ cm$^{-2}$s$^{-1}$ and via measuring the ‘invisible width’ of the $Z$ boson, the valuable conclusion that there are three species of light neutrinos is obtained. Namely the effective number of light neutrino species: $N_{e_{I}} = P_{inv}^{0}(\Gamma_{inv}^{SM}/\Gamma_{inv}^{0}) = 2.9840 \pm 0.0082$ is obtained via the experimental measurements of invisible width ratio of $Z$-boson: $P_{inv}^{0}$ = $\frac{\Gamma_{inv}^{0}}{\Gamma_{inv}^{SM}}$ = 5.943 ± 0.016 at the earlier $Z$-factories.$^{[2–4]}$ Here if ignoring the errors, here the effective number of light neutrino species being not an integer and smaller than three means the light neutrino species can be three but there is some room left for the mixture of the light neutrinos with external(s). If a new $Z$-factory, called as a super one, with a luminosity around $10^{35} \sim 10^{36}$ cm$^{-2}$s$^{-1}$ and proper improved detectors, that is accessible now under the present technology, is expected to be built, the invisible width of $Z$-boson will be measured more accurately (the errors are expected to be suppressed greatly), so the conclusion on the room left for light neutrinos mixing with externals will be improved. Bearing the possible progress in collider and detectors in mind, in this paper we will, based on SM, precisely calculate the neutrino production, and with the precise and fresh results we will discuss (explore) the meaning of refined measurements of the neutrino production in a super $Z$-factory with proper detectors in

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future.

The paper is organized as follows: after INTRODUCTION in section II, we present formulas of the neutrino production at a Z-factory; in section III we present the numerical results for the neutrino production; the section IV is to contribute to understanding the results and discussions.

II. NEUTRINO PAIR PRODUCTION AT Z-FACTORY

The neutrino production at a Z-factory:\footnote{Fig.1}:

\[ e^- e^+ \to \nu_i \bar{\nu}_i \quad (i = 1, 2, 3) \]

where

\[ \nu_i = \sum_{j=e,\mu,\tau} V_{ij} \nu_j , \]

where \( V_{ij} \) is the elements of neutrino mixing matrix. The Standard Model (SM) gives rise to the relevant interaction between Z-boson and leptons:

\[ \mathcal{L}_{\text{int}} = g Z_{\mu} J^\mu_Z, \tag{1} \]

where \( J^\mu_Z \), the so-called ‘weak neutral current’, is

\[ J^\mu_Z = \frac{1}{2 \cos \theta_w} \sum_{i=e,\mu,\tau} \left[ \bar{\nu}_i \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) \nu_i \right. \]
\[ \left. + \bar{\nu}_i \gamma^\mu \left( \frac{1 - 2 \sin^2 \theta_w + \gamma^5}{2} \right) \psi_i \right] . \]

Here another relevant interaction is that between \( W^\pm \) and leptons:

\[ \mathcal{L}_{\text{int}} = g (W^+_\mu J^\mu_W + W^-_\mu J^\mu_W) \tag{2} \]

where \( J^\mu_W \), the charged weak current, is

\[ J^\mu_W = \frac{1}{\sqrt{2}} \sum_{i=e,\mu,\tau} \bar{\psi}_i \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) \psi_i . \]

At the tree level, there are two relevant Feynman diagrams for the process \( e^- e^+ \to \nu_e \bar{\nu}_e \), which are shown in Fig.1. For the processes, \( e^- e^+ \to \nu_l \bar{\nu}_l \) \( (l = \mu, \tau) \), there is one Feynman diagram only which is shown in Fig.2. It is straightforward to write down the amplitudes according to the diagram(s). We have

\[ i\mathcal{M} = \frac{i^2 g^2}{2!} (I + \Pi) \]

where

\[ I = - \frac{D^{(Z)}_{\mu\nu}(q)}{2 \cos^2 \theta_w} \bar{u}(k) \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) v(k') \]
\[ \cdot \bar{v}(p') \gamma^\nu \left( \frac{1}{2} + 2 \sin^2 \theta_w + \frac{\gamma^5}{2} \right) u(p) ; \]

\[ \Pi = \frac{g^2}{2} \frac{D_{\mu\nu}^{(Z)}(q)}{2 \cos^2 \theta_w} \bar{u}(k) \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) v(k') \]
\[ \cdot \bar{v}(p') \gamma^\nu \left( \frac{1}{2} + 2 \sin^2 \theta_w + \frac{\gamma^5}{2} \right) u(p) . \]

Here to do the calculation, we take the unitary gauge for weak bosons, so the propagator for Z-boson:

\[ D^{(Z)}_{\mu\nu}(q) = \frac{i(-\eta_{\mu\nu} + q_\mu q_\nu/m_Z^2)}{q^2 - m_Z^2 + i\Gamma_Z m_Z} , \]

and that for W-boson:

\[ D^{(W)}_{\mu\nu}(q) = \frac{i(-\eta_{\mu\nu} + q_\mu q_\nu/m_W^2)}{q^2 - m_W^2 + i\Gamma_W m_W} . \]

Here \( \Gamma_Z, m_Z \) and \( \Gamma_W, m_W \) appearing in the denominators are the total widths, masses of Z and W boson respectively. Whereas the amplitude for the process \( e^- e^+ \to \nu_l \bar{\nu}_l \) \( (l = \mu, \tau) \) is shown

\[ i\mathcal{M} = \frac{g^2}{2} \frac{D_{\mu\nu}^{(Z)}(q)}{2 \cos^2 \theta_w} \bar{u}(k) \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) v(k') \]
\[ \cdot \bar{v}(p') \gamma^\nu \left( \frac{1}{2} + 2 \sin^2 \theta_w + \frac{\gamma^5}{2} \right) u(p) . \]

Hence the differential cross-sections for unpolarized incoming beams are related to the amplitudes via

\[ d\sigma = \frac{1}{2E_1 2E_2 |v_1 - v_2| \sum_{\text{spins}} |\mathcal{M}|^2} d\Pi ; \]
i.e. in C.M. system for the process $e^-e^+ \rightarrow \nu_{\ell}\bar{\nu}_{\ell}$ it is

$$
\frac{d\sigma}{d\Omega} = \frac{\alpha^2s}{16\sin^4\theta_w} \left[ \frac{\cos^4\frac{\theta}{2}}{(s\sin^2\frac{\theta}{2} + m_w^2)^2 + \Gamma_W^2m_W^2} - (2 - \frac{1}{\cos^2\theta_w}) \right] 
\frac{[\Gamma_Zm_Zm_W - (s - m_Z^2)(s\sin^2\frac{\theta}{2} + m_Z^2)]\cos^4\frac{\theta}{2}}{[(s - m_Z^2)^2 + \Gamma_Z^2m_Z^2]^2} \cdot \frac{[(s\sin^2\frac{\theta}{2} + m_W^2)^2 + \Gamma_W^2m_W^2]}{[(s - m_Z^2)^2 + \Gamma_Z^2m_Z^2]^2} 
\frac{1}{\cos^4\theta_w} \left[ \sin^4\theta_w\sin^2\frac{\theta}{2} + (\frac{3}{2} - \sin^2\theta_w)\sin^4\frac{\theta}{2} \right] \right] ; \quad (3)
$$

for the process $e^-e^+ \rightarrow \nu_{\ell}\bar{\nu}_{\ell}$ ($\ell = \mu, \tau$) it is

$$
\frac{d\sigma}{d\Omega} = \frac{\alpha^2s}{\sin^42\theta_w} \left[ \frac{\sin^4\theta_w\sin^2\frac{\theta}{2} + (\frac{1}{2} - \sin^2\theta_w)\sin^4\frac{\theta}{2}\cos^4\frac{\theta}{2}}{(s - m_{\ell}^2)^2 + \Gamma_{\ell}^2m_{\ell}^2} \right] , \quad (4)
$$

where

$$
g = \frac{e}{\sin\theta_w} \quad \text{and} \quad \alpha = \frac{e^2}{4\pi}
$$

are adopted. Moreover, from Eq. (3) one may see that the contributions to the cross-section from $t$-channel of $W$-scattering and $s$-channel of $Z$-annihilation are destructive.

The total cross-sections can be calculated by integrating the relevant differential ones respectively.

To calculate out the cross-sections numerically, we take the parameters, which appear in the formula, from PDG [2]:

$$
m_Z = 91.1876 \text{GeV} , \quad \Gamma_Z = 2.4952 \text{GeV} ,
m_W = 80.385 \text{GeV} , \quad \Gamma_W = 2.085 \text{GeV} ,
$$

$$
\sin^2\theta_W = 0.23116 \quad (\text{or} \quad 0.2231) , \quad \alpha = 1/127.944 ,
m_{\nu_{\ell}} \simeq m_{\nu_{\ell}} \simeq m_{\nu_e} \simeq 0.0 \text{eV} , \quad (5)
$$

i.e. the values of the parameters are taken to be renormalization at $Z$ pole.

Precisely the values of the total unpolarized cross-sections vs C.M.S. energies $\sqrt{s}$ are collected in TABLE I, and the curves of the cross-sections are plotted in FIG.3. In order to see the the difference between the production of $e^-e^+ \rightarrow \nu_{\ell}\bar{\nu}_{\ell}$ and that of $e^-e^+ \rightarrow \nu_{\ell}\bar{\nu}_{\ell}$ ($\ell = \mu, \tau$), we also plot $\Delta\sigma$, the difference in total cross-sections, vs $\sqrt{s}$ in FIG.4. From the table TABLE I and the figures FIG.3, FIG. 4 we may see that i). The difference caused by a different value of $\sin^2\theta_W$ is remarkable; ii). The differences between the total cross-sections for process $e^-e^+ \rightarrow \nu_{\ell}\bar{\nu}_{\ell}$ and for processes $e^-e^+ \rightarrow \nu_{\ell}\bar{\nu}_{\ell}$, $\ell = \mu, \tau$ are tiny at the $Z$-pole, but become sizable when C.M. energy $\sqrt{s}$ is away from the $Z$-pole. The differences are the consequences

![FIG. 3: The dependence of the total unpolarized cross-sections on the C.M.S. energies: the dished curve is those of the processes $e^-e^+ \rightarrow \nu_{\ell}\bar{\nu}_{\ell}$ ($\ell = \mu, \tau$) and the solid curve is that of the processes $e^-e^+ \rightarrow \nu_{\mu}\bar{\nu}_{\mu}$.

FIG. 4: The difference $\Delta\sigma$ in the total cross sections between those of $e^-e^+ \rightarrow \nu_{\ell}\bar{\nu}_{\ell}$ and $e^-e^+ \rightarrow \nu_{\mu}\bar{\nu}_{\mu}$ ($\ell = \mu, \tau$). Here $\Delta\sigma = 0.0$ is located at $\sqrt{s} \simeq 91.163 \text{GeV}$ ($m_Z$).](image)

1 Here we do the calculation only at tree level, thus to have comparatively better results we take the values renormalized at $Z$-pole for the parameters, but to see uncertainties and for comparison, when doing the calculation of the total cross-sections we take a different value of $\sin^2\theta_W$, i.e. $\sin^2\theta_W = 0.23116$ and $\sin^2\theta_W = 0.2231$ in different renormalization schemes in cases. Whereas in the rest calculations of the paper without declaration, we will take $\sin^2\theta_W = 0.23116$ only.
of the interference of $Z$-annihilation and $W$-exchange for $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$ (see FIG.1) and there is $Z$-annihilation only for $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$, $\ell = e, \mu, \tau$ (see FIG.2); iii). The interference of the $Z$-annihilation and the $W$-exchange in the process $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$ is ‘constructive’ when $\sqrt{s} \geq m_{Z}$, but it is ‘destructive’ when $\sqrt{s} \leq m_{Z}$ (FIG.4). To see

the characters of the production, we have also calculated the differential cross-sections at three C.M. energies for the processes $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$, $\ell = e, \mu, \tau$ numerically in C.M.S., and plot them vs the angle $\theta$ between the directions of the electron beam and the produced neutrinos in FIG.5. From the figure one may see that the differential cross-sections are clearly asymmetric in forward and backward. They are favoured in forward direction and have a minimum at $\theta \approx \pi/2$. To highlight the asymmetry, we plot $A_{FB} = \frac{\int_{1>\cos\theta>0} - \int_{1>\cos\theta>-1}}{\int_{1>\cos\theta>-1}} \frac{d\sigma}{d\sigma}$ vs the C.M. energy $\sqrt{s} = E_{cm}$ around $Z$-boson mass $m_{Z}$ in FIG.6. It is interesting that owing to the interference of $Z$-annihilation and $W$-exchange the forward-backward asymmetry $A_{FB}$ of the production $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$ changes sign from minus to plus as the collision energy increasing, but that of the production $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$ keeps constant (see FIG.6).

### III. CONCLUSIONS AND DISCUSSIONS

The differential cross-sections for the neutrino pair production: $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$, $e^{-}e^{+}$ → $\nu_{\mu}\bar{\nu}_{\mu}$ and $e^{-}e^{+}$ → $\nu_{\tau}\bar{\nu}_{\tau}$ around the CM energy region of $Z$-boson mass (at a $Z$-factory) are studied thoroughly. Since there are two Feynman diagrams: $t$-channel exchange and $s$-channel annihilation contributing to the first process (FIG.1), but there is only one Feynman diagram, $s$-channel annihilation contributing to the second and third ones (FIG.2), thus we have paid more attention on the differences among the processes. Since the processes being concerned around $\sqrt{S} \simeq m_{Z}$, the $t$-channel exchange diagram’s contribution is much smaller than that from the $s$-channel annihilation. The differences between the production $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$ and $e^{-}e^{+}$ → $\nu_{\mu}\bar{\nu}_{\mu}$ or $e^{-}e^{+}$ → $\nu_{\tau}\bar{\nu}_{\tau}$ are tiny at $Z$-boson peak, but become visible when off the peak (FIGs.3-5 and TABLE.I). Namely the shape of the $Z$-boson resonance for the production process $e^{-}e^{+}$ → $\nu_{\ell}\bar{\nu}_{\ell}$ around the $Z$-boson peak is distorted by the two diagram interference in certain degree. Therefore we think that the facts described here should be treated very carefully, especially when someone considers the room left for the mixing of the three light neutrinos with heavy neutral leptons and/or stereo via taking the invisible width of $Z$-boson into account. Precisely we mean that if one would like to obtain the effective number $N_{e eff}$ of the light neutrino species and to suppress the relevant errors, the way to obtain it via the data of the earlier $Z$-factories below:

\[N_{e eff} = R_{inv}^{0}(\frac{\Gamma_{Z}}{\Gamma_{\nu_{\ell}\bar{\nu}_{\ell}}})_{SM} = 2.9840 \pm 0.0082 ,\]

\[R_{inv}^{0} = \frac{\Gamma_{inv}}{\Gamma_{l}} = 5.943 \pm 0.016\]

should be added the distortion effects being affected carefully.

Since the neutrino-antineutrino pair production by electron-positron annihilation at $Z$-boson resonance is of a two-to-two body process and with resonance enhancement, so the produced neutrino and antineutrino at a $Z$-factory are productive and of mono-energy. Moreover $Z$-boson mass is quite heavy $m_{Z} \simeq 91.2$ GeV, so roughly the energy of the produced neutrino $E_{\nu} = \frac{m_{Z}}{2E_{\nu}} \simeq 45.6$ GeV (tens GeV order). Thus the produced neutrinos with the character: a quite high energy and mono-energy, seems may find some special usages in principle.
Based on the estimates of the production cross-sections in this paper (TABLE.I) and the differential cross-sections (FIG.5) quantitatively, one may realize that indeed the cross-section is greater in the forward direction than those in the other directions, thus if the produced neutrinos are considered as beams in directions then the beam intensity in forward direction is biggest. Namely, considering the cross-sections of neutrino with matters are very tiny, if one would like to detect the produced neutrinos directly and successfully, then the most hopeful way is to put the detector at the forward direction. Moreover, the production cross-sections at a $Z$-factory are in the order of a few $\text{nb}$ as shown in TABLE.I and FIG.5, therefore only when the luminosity of the $Z$-factory is higher than $10^{36}\text{ cm}^{-2}\text{s}^{-1}$, so that the flux of the produced neutrinos may be great enough for an accessible detector. The cross-sections of the energetic neutrinos (several tens GeV in energy) colliding with common matter are so small in the magnitude order of $10^{-36} \sim 10^{-37}\text{ cm}^{2}$, so even with a huge detector, e.g. a detector with 10km long (thickness), the neutrinos produced at a $Z$-factory can be detected only when the luminosity of the $Z$-factory is so high as pointed here, i.e., the intensity of the produced neutrinos reaches to the ability of the accessible detector to detect them.

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