Combined Electroweak and QCD Fit to HERA Data

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Abstract

A simultaneous Electroweak and QCD fit of electroweak parameters and parton distribution functions to HERA data on deep inelastic scattering is presented. The input data are neutral current and charged current inclusive cross sections measured by the H1 and ZEUS collaborations at the $ep$ collider HERA. The polarisation of the electron beam was taken into account for the ZEUS and H1 data recorded between 2004 and 2007. Results are presented on the vector and axial-vector couplings of the $Z$ boson to $u$- and $d$-type quarks. The values are in agreement with Standard Model predictions. The results on $a_u$ and $v_u$ represent the most precise measurements from a single process.
1 Introduction

Data on deep inelastic electron–proton, ep, scattering (DIS) have been used in analyses within the framework of Quantum Chromo Dynamics (QCD) for many years [1] and have formed the basis of investigations of the structure of the proton. The data from the ep collider HERA extended the reach in the four-momentum-transfer squared, $Q^2$, and in Bjorken $x$, by several orders of magnitude with respect to previous fixed-target experiments [2]. At HERA, values of $Q^2$ of up to 50 000 GeV$^2$ were reached, a regime where the contribution of $Z$ exchange becomes comparable to the contribution from photon exchange.

During the HERA II running period, the HERA collider provided a significant amount of data with beams longitudinally polarised to an average level between 25% and 35%. This facilitates detailed studies of electroweak (EW) effects. Recently, the ZEUS collaboration published a combined QCD and electroweak analysis [3] exploiting the ZEUS neutral current (NC) and charged current (CC) $e^+p$ and $e^-p$ inclusive cross sections for polarised beams [4–7]. For the analysis presented here, cross sections published by the H1 collaboration [8] for polarised beams were also considered. These data sets, together with data sets for unpolarised beams originally published by H1 [9–15] and ZEUS [16–23], were used as input to a combined QCD and EW fit, HH-EW-Z. This fit was used to determine the couplings of the $Z$ boson to $u$- and $d$-type quarks.

2 QCD and EW Combined Analysis

The analysis presented here follows closely the method described in detail in the ZEUS publication [3]. It uses the next-to-leading-order (NLO) DGLAP [25–29] formalism to describe the evolution of the parton distribution functions (PDFs) with $Q^2$ and the on-shell definition of $\sin^2 \theta_W = 1 - M_W^2 / M_Z^2$, where $\sin^2 \theta_W$ is the electroweak mixing angle, and $M_W$ and $M_Z$ are the mass of the $W$ and $Z$ boson, respectively. The EW part of the analysis was performed at leading order with partial higher-order corrections in the on-shell scheme. The RT variable-number heavy-flavour scheme [30–32] was employed and the values of PDG14 [33] were used for all masses and couplings throughout the analysis, unless they were free parameters in a fit.

The PDFs of the proton were parameterised with 13 free parameters as

$$xg(x) = A_g x^B \left( 1 - x \right)^{C_g} - A'_g x^B \left( 1 - x \right)^{C'_g},$$

$$xu_e(x) = A_{u_e} x^B \left( 1 - x \right)^{C_{u_e}} \left( 1 + E_{u_e} x^2 \right),$$

$$xd_e(x) = A_{d_e} x^B \left( 1 - x \right)^{C_{d_e}},$$

$$x\bar{U}(x) = A_{\bar{U}} x^B \left( 1 - x \right)^{C_{\bar{U}}},$$

$$x\bar{D}(x) = A_{\bar{D}} x^B \left( 1 - x \right)^{C_{\bar{D}}},$$

where $x$ is the fraction of the proton momentum carried by the quark. The normalisation parameters, $A_{u_e}, A_{d_e}, A_g$, are constrained by the quark-number sum rules and the momentum sum

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1 In this paper, the word “electron” refers to both electrons and positrons, unless otherwise stated.

2 As used as input to the data combination presented by the H1 and ZEUS collaborations [24].
The strange-quark distribution is expressed as an $x$-independent fraction, $f_s$, of the $d$-type sea, $x\bar{s} = 0.4 \times \bar{D}$ at the starting scale $\mu_0^2 = 1.9 \text{ GeV}^2$. The parameter $C'_{g}$ is fixed to $C'_{g} = 25$ [34].

The PDF parameters were fitted to the HERA inclusive cross sections together with the axial-vector and vector couplings of the $Z$ boson to the $u$- and $d$-type quarks, $a_u$, $a_d$, $v_u$ and $v_d$, respectively. For this fit, called HH-EW-Z, the ZEUSfitter package was used. The results were cross-checked with the HERAFitter package. All cross sections for unpolarised beams were used as originally published by H1 [9–15] and ZEUS [16–23]. The H1 cross sections for polarised beams were also used as published by H1 [8]. The ZEUS cross sections for polarised beams [4–7] were used as originally published, but with updated values of the polarisation as published in the ZEUS EW analysis [3]. In addition, for the present analysis, extra uncertainties were added to the uncorrelated systematic uncertainties on these ZEUS data. In their original publications the ZEUS collaboration did not consider systematic uncertainties on EW corrections, whereas the H1 collaboration included such uncertainties [8]. The uncertainties now added to the ZEUS data are equivalent to the uncertainties on the EW corrections included by H1.

## 3 The HH-EW-Z Fit and the Z Couplings

The PDFs of the HH-EW-Z fit are shown in Fig. 1 with experimental/fit, model and parameterisation uncertainties, determined according to the prescriptions of the HERAPDF2.0 analysis [24]. Also shown are the central values for the PDFs of HERAPDF2.0 NLO. The PDFs are very similar. The PDF parameters of HH-EW-Z are only weakly correlated to the $Z$ couplings. The full correlation matrix for the 13 PDF parameters and the four $Z$ couplings is given in Table 1.

The $\chi^2$ per degree of freedom for HH-EW-Z is 3556/3231=1.10. This can be compared to 1.12 for ZEUS-EW-Z [3] and 1.20 for HERAPDF2.0 NLO [24]. The description of the data is very good. The predictions of HH-EW-Z are compared to the high-precision $e^+p$ NC data from H1 [8] and ZEUS [5] in Figs. 2 and 3, respectively.

The result of HH-EW-Z for the couplings of the $Z$ boson to $u$- and $d$-type quarks are

\[
\begin{align*}
  a_u & = +0.532^{+0.081}_{-0.058} \text{ (experimental/fit)} +0.036^{+0.022}_{-0.014} \text{ (model)} +0.060^{+0.006}_{-0.008} \text{ (parameterisation)} , \\
  a_d & = -0.409^{+0.327}_{-0.199} \text{ (model)} +0.112^{+0.140}_{-0.071} \text{ (parameterisation)} , \\
  v_u & = +0.144^{+0.065}_{-0.050} \text{ (model)} +0.013^{+0.002}_{-0.014} \text{ (parameterisation)} , \\
  v_d & = -0.503^{+0.168}_{-0.090} \text{ (model)} +0.031^{+0.006}_{-0.028} \text{ (parameterisation)} .
\end{align*}
\]

These values are compared to the results from ZEUS-EW-Z [3] in Table 2. They agree within uncertainties. Also listed are SM predictions and values obtained from fits which were performed as cross-checks:

- a fit with the PDFs fixed to those of a 13-parameter QCD-only fit, HH-13p;
- a fit with the PDFs fixed to those of HERAPDF2.0.

\[\text{3 The package was also used in the combined ZEUS electroweak and QCD analysis [3].}\]
Only experimental/fit uncertainties were considered for these cross checks. The values agree within the experimental uncertainties with the result from HH-EW-Z.

Profile likelihood contours at 68 % C.L. for the couplings were obtained as described in the ZEUS publication [3]. They are shown for \( a_u, v_u \) and \( a_d, v_d \) in Fig. 4 and for \( a_u, a_d \) and \( v_u, v_d \) in Fig. 5. These figures demonstrate very clearly that the HERA data constrain the couplings of the Z boson to the \( u \) quark significantly better than the couplings to the \( d \) quark. This is due to the larger \( u \) valence content of the proton and the larger charge of the \( u \) quark. The couplings as determined by HH-EW-Z are compatible with the SM. Figure 6 shows the 68 % C.L. contours from HH-EW-Z, together with the contours from ZEUS-EW-Z [3] and the measurements from LEP+SLC [36], the Tevatron [37,38] and HERA I (H1) [39]. The fits HH-EW-Z and ZEUS-EW-Z are based both on HERA I and HERA II data and were not included in the combinations for PDG14 [33]. The PDG values and all measurements are compared in Fig. 7. The HH-EW-Z results on the axial-vector and vector couplings to \( u \)-type quarks are the most precise results published from a single process. The vector couplings from HH-EW-Z are significantly more accurate than from ZEUS-EW-Z. This reflects the importance of the information on the polarisation of the beams for the vector couplings [3]. Thus, the inclusion of the H1 data for polarised beams is the reason for the improvement in these couplings.

The ZEUS collaboration also presented [3] measurements of the electroweak mixing angle and \( M_W \). These results do not depend strongly on the beam polarisation. Two fits were performed as cross-checks with the 13 PDF parameters fixed and either \( \sin^2 \theta_W \) or \( M_W \) as free parameters. The results are compatible with those of the ZEUS EW fits within experimental/fit uncertainties:

\[
\begin{align*}
\sin^2 \theta_W &= 0.2255 \pm 0.0011 \text{ (experimental/fit)} \quad \text{HH EW}, \\
\sin^2 \theta_W &= 0.2252 \pm 0.0011 \text{ (experimental/fit)} \quad \text{ZEUS EW},
\end{align*}
\]

\[
\begin{align*}
M_W &= (80.74 \pm 0.28 \text{ (experimental/fit)}) \text{ GeV} \quad \text{HH EW}, \\
M_W &= (80.68 \pm 0.28 \text{ (experimental/fit)}) \text{ GeV} \quad \text{ZEUS EW}.
\end{align*}
\]

A simultaneous fit to the 13 PDF parameters and both \( \sin^2 \theta_W \) and \( M_W \) also yielded results compatible with the results presented by ZEUS [3]. Since the sensitivity with respect to the ZEUS EW fits was not significantly increased, the detailed studies on \( \sin^2 \theta_W \) and \( M_W \) presented in the ZEUS paper were not repeated.

## 4 Summary and Conclusions

The results of a combined electroweak and QCD fit to all available HERA inclusive DIS cross sections, taking into account beam polarisation for both the H1 and ZEUS data, have been presented. The results on the couplings of the Z boson to \( u \)- and \( d \)-type quarks are:

\[
\begin{align*}
a_u &= +0.532 \pm 0.081 \text{ (experimental/fit)} \pm 0.036 \text{ (model)} \pm 0.060 \text{ (parameterisation)}, \\
a_d &= -0.409 \pm 0.327 \pm 0.071 \text{ (model)} \pm 0.112 \pm 0.025 \text{ (parameterisation)}, \\
v_u &= +0.144 \pm 0.060 \pm 0.013 \text{ (model)} \pm 0.025 \text{ (parameterisation)}, \\
v_d &= -0.503 \pm 0.168 \pm 0.093 \text{ (model)} \pm 0.031 \pm 0.028 \text{ (parameterisation)}.
\end{align*}
\]

\(^4\)Numerical information is available as additional material for this publication.
These results are compatible with the Standard Model. The exploitation of all available data for polarised beams provides very accurate determinations of the $Z$-boson couplings. The couplings to the $u$-type quarks are the most precise values published for a single process.

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Table 1: The correlation matrix of all parameters of the HH-EW-Z fit.
HH-EW-Z  
\[ +0.532 \pm 0.017 \pm 0.076 \]  
\[ -0.506 \pm 0.063 \]  
\[ -0.509 \pm 0.059 \]  
\[ +0.144 \pm 0.065 \pm 0.066 \]  
\[ -0.503 \pm 0.168 \pm 0.171 \]  
\[ -0.41 \pm 0.24 \pm 0.25 \]  
\[ -0.41 \pm 0.16 \pm 0.20 \]  

ZEUS-EW-Z  
\[ +0.50 \pm 0.09 \pm 0.12 \]  
\[ -0.56 \pm 0.14 \pm 0.15 \]  
\[ +0.14 \pm 0.08 \pm 0.09 \]  
\[ -0.41 \pm 0.24 \pm 0.25 \]  

PDF parameters fixed to  
HH-13p  
\[ +0.530 \pm 0.076 \pm 0.052 \]  
\[ -0.407 \pm 0.193 \pm 0.164 \]  
\[ +0.145 \pm 0.063 \pm 0.050 \]  
\[ -0.500 \pm 0.166 \pm 0.190 \]  

HERAPDF2.0  
\[ +0.507 \pm 0.073 \pm 0.047 \]  
\[ -0.473 \pm 0.284 \pm 0.253 \]  
\[ +0.155 \pm 0.062 \pm 0.053 \]  
\[ -0.479 \pm 0.173 \pm 0.190 \]  

SM  
\[ +0.500 \pm 0.500 \pm 0.202 \]  
\[ +0.202 \pm 0.351 \]  

Table 2: The results from HH-EW-Z on the axial-vector and vector couplings of the Z boson to \( u \)- and \( d \)-type quarks. Given are the experimental/fit (exp) and total (tot) uncertainties. For comparison, the results of ZEUS-EW-Z are also listed. In addition, results of fits with the PDFs fixed to HH-13p and HERAPDF2.0, for which only the couplings of the Z were free parameters, are given. Also listed are the SM predictions.
Figure 1: The NLO PDF set HH-EW-Z with cumulative experimental/fit, model and parameterisation uncertainties at the factorisation scale $\mu_r^2 = 10 \text{ GeV}^2$. All positive and negative uncertainties in the model were added separately in quadrature. The parameterisation uncertainty represents an envelope of all individual parameterisation uncertainties. Also shown are the central values of HERAPDF2.0 NLO.
Figure 2: The NLO predictions of HH-EW-Z compared to the H1 $e^+ p$ NC DIS reduced cross-sections $\sigma^+_{r,NC}$ for positively and negatively polarised beams plotted as a function of $x_{Bj}$ at fixed values of $Q^2$. The closed (open) circles represent the H1 data for positive (negative) polarisation. The bands indicate the full uncertainties on the predictions of HH-EW-Z.
Figure 3: The NLO predictions of HH-EW-Z compared to the ZEUS $e^+p$ NC DIS reduced cross-sections $\sigma_{e^+NC}$ for positively and negatively polarised beams plotted as a function of $x_{Bj}$ at fixed values of $Q^2$. The closed (open) circles represent the ZEUS data for positive (negative) polarisation. The bands indicate the full uncertainties on the predictions of HH-EW-Z.
Figure 4: The 68 % C.L. contours for $a_d, v_d$ and $a_u, v_u$ obtained for the HH-EW-Z fit. Also shown are the 68 % C.L. contours for the ZEUS-EW-Z fit with total uncertainties.
Figure 5: The 68 % C.L. contours for $a_u$, $a_d$ and $v_u$, $v_d$ obtained for the HH-EW-Z fit. Also shown are the 68 % C.L. contours for the ZEUS-EW-Z fit with total uncertainties.
Figure 6: The 68% C.L. contours for $a_d, v_d$ and $a_u, v_u$ obtained for the HH-EW-Z fit. Also shown are results from ZEUS-EW-Z, HERA I (H1), LEP (ALEPH, OPAL, L3 and DELPHI) plus SLC (SLD) combined, and the Tevatron (CDF and D0).
Figure 7: The values from the HH-EW-Z fit for $a_d, a_u, v_d$ and $v_u$ compared to the values from ZEUS-EW-Z and the results from LEP (ALEPH, OPAL, L3 and DELPHI) plus SLC (SLD) combined, the Tevatron (CDF and D0), HERA I (H1). The PDG14 world average is also shown; this does not contain the measurements from the HH-EW-Z and ZEUS-EW-Z fits based on all HERA data. All results are given with total uncertainties. Vertical black lines in each box indicate central values, the long gray vertical lines indicate the SM predictions.