A New Model for the Spiral Structure of the Galaxy. Superposition of 2+4-armed patterns

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ABSTRACT

We investigate the possibility of describing the spiral pattern of the Milky Way in terms of a model of superposition 2- and 4-armed wave harmonics (the simplest description, besides pure modes). Two complementary methods are used: a study of stellar kinematics, and direct tracing of positions of spiral arms. In the first method, the parameters of the galactic rotation curve and the free parameters of the spiral density waves were obtained from Cepheid kinematics, under different assumptions. To turn visible the structure corresponding to these models, we computed the evolution of an ensemble of $N$-particles, simulating the ISM clouds, in the perturbed galactic gravitational field. In the second method, we present a new analysis of the longitude-velocity ($l - v$) diagram of the sample of galactic HII regions, converting positions of spiral arms in the galactic plane into locii of these arms in the $l - v$ diagram. Both methods indicate that the “self-sustained” model, in which the 2-armed and 4-armed mode have different pitch angles ($6^\circ$ and $12^\circ$, respectively) is a good description of the disk structure. An important conclusion is that the Sun happens to be practically at the corotation
circle. As an additional result of our study, we propose an independent test for localization of the corotation circle in a spiral galaxy: a gap in the radial distribution of interstellar gas has to be observed in the corotation region.

*Subject headings:* Galaxy: kinematics and dynamics – Galaxy: structure
1. Introduction

A good understanding of the large-scale spiral structure of the Galaxy has not been reached up to the present. Georgelin & Georgelin (1976), hereafter GG, derived a 4-armed pattern, based on an analysis of the distribution of giant H II regions. According to Vallée (1995) most researches support the 4-armed pattern, although there are discordant opinions; for instance Bash (1981) finds the pattern to be 2-armed, similar to the first spiral structure model proposed by Lin and Shu (1964). But even if we accept that there are strong observational evidences in favor of a 4-arms structure, theoretical and observational difficulties remain, and the observation of external galaxies strongly recommends that we look for more complex solutions, which should include arms with different pitch angles.

It is now accepted that the distance to the center is about 7.5 kpc (e.g. Olling & Merrifield, 1998), which implies a smaller rotation velocity of the Local Standard of Rest than previously estimated, and as a consequence, the rotation curve decreases beyond the solar radius (Amaral et al., 1996, Honma & Kan-Ya, 1998). From the rotation curve, the epicyclic frequency and the radius of the inner and outer Lindblad resonances can be derived. Amaral and Lépine (1997, hereafter AL) concluded that the 2-armed pattern could exist between 2.5 and 12 kpc, which is about the range of the observed spiral pattern, but a 4-armed pattern could only exist between about 6 to 11 kpc. AL proposed a representation of the galactic spiral structure as a superposition of 2- and 4-armed wave patterns, to account for the number of arms and for the existence of arms over a wider range of radius. In AL’s model, two arms of the 4-armed component are coincident with the 2-armed component, so that the Galaxy looks 4-armed. However, there are theoretical arguments discussed in the present paper showing that the pitch angle for the 4-armed pattern should be different from that of the 2-armed component. A Fourier analysis of external galaxies in terms of spiral modes, performed by Puerari & Dottori (1992) indicates as well that in the cases in which the 2-armed and 4-armed patterns are prominent, they indeed have different pitch angles. These arguments suggest that at least in a range of radius, the Galaxy might present a structure like 2-arms plus 4-arms with different pitch angles, which could look like 6 arms. An attractive aspect of such a model with arms of different pitch angles is that it naturally accounts for bifurcations of arms. In external galaxies branching of arms is a widespread phenomenon, e.g. the galaxy M 101, where there are both branching arms and bridges between them. The structure of the Milky Way is perhaps similarly complicated. Another fact that stimulates us to look for more refined solutions to the spiral structure is that there are observed arms in the Galaxy that do not fit in a simple 4-arms structure. For instance, outside the solar circle, around longitudes 210° - 260°, three arms are clearly visible, in HI and in IRAS sources (Kerr, 1969, Wouterloot et al., 1990), but they are not expected from a 4-arms model that fits the tangential directions of the inner parts of the Galaxy (e.g. Ortiz & Lépine, 1993).

The main goal of this paper is to investigate if a superposition of 2+4-armed wave harmonics is a good representation of the spiral structure of the Galaxy. Two different approaches are used. In one approach, we analyse the Cepheid kinematics, using a model that takes into consideration the perturba-
2. Method of estimation of structural parameters

The derivation of the spiral wave parameters is based on the statistical analysis of stellar motion in the Galaxy (see Crézé & Mennessier, 1973, Mishurov & Zenina, 1999, hereafter MZ, Mishurov et al., 1997, hereafter MZDMR and papers cited therein). Note that we look for the parameters of the structure of the galactic gravitational field, which is not directly visible. However, the gravitational field determines the stellar motion, and in particular, the spiral perturbations of the field deviate the stellar motion from rotation symmetry. Hence, analysing the stellar velocity field in the framework of a perturbation model, we can derive both the parameters of the density waves and those of the galactic rotation.

Unlike the above cited papers, let us represent the gravitational potential $\varphi_G$ of the Galaxy as the sum:

$$\varphi_G = \varphi_0 + \varphi_{s2} + \varphi_{s4}$$

(1)

where $\varphi_0$ is, as usual, the unperturbed axisymmetric part of the potential that determines the Galaxy equilibrium as a whole ($d\varphi_0/dR = \Omega^2 R$, $\Omega(R)$ is the angular rotation velocity of galactic disk, $R$ is the galactocentric distance) and $\varphi_{sm}$ ($m = 2$ or $m = 4$) are the $m-th$ harmonics of the perturbations due to spiral density waves. According to Lin et al (1969):

$$\varphi_{sm} = A_m \cos(\chi_m),$$

(2)

where $A_m$ is the amplitude of the $m-th$ harmonic, $\chi_m$ is the wave phase:

$$\chi_m = m \left[ \cot(i_m) \ln(R/R_\odot) - \vartheta \right] + \chi_\odot,$$

(3)

$m$ is the azimuthal wave number, i.e. the number of arms for a given harmonic, $i_m$ is
the corresponding pitch angle of the arms, $R, \vartheta$ are the polar coordinates with the origin at the Galactic center, $R_{\odot}$ is the solar galactocentric distance, $\chi_{\odot m}$ is the initial phase or the wave phase at the Sun position. From Eq.(3) it is seen that this last value fixes the $m-th$ harmonic position relative to the Sun. 

In accordance with Eq (1) we can write the perturbed stellar velocity for the radial (directed along galactocentric radius) $\tilde{v}_R$ and the azimuthal $\tilde{v}_\vartheta$ components as follows:

\begin{align}
\tilde{v}_R &= f_{R2} \cos(\chi_2) + f_{R4} \cos(\chi_4) , \\
\tilde{v}_\vartheta &= f_{\vartheta2} \sin(\chi_2) + f_{\vartheta4} \sin(\chi_4) ,
\end{align}

where $f_{Rm}$ and $f_{\vartheta m}$ are the amplitudes of $m-th$ harmonic. These quantities are related to the parameters of the density waves by some formulas (see Lin et al, 1969). 

Substituting Eqs (4-5) into Eqs.(1,2) of MZ and using the statistical method described by MZ and MZDMR, we derive the parameters of the rotation curve ($\Omega_{\odot}, A, R_{\odot} \Omega''_{\odot}$) and the parameters of the spiral waves ($f_{Rm}, f_{\vartheta m}, i_m, \chi_{\odot m}$) over the observed stellar velocity field. Then by means of the density wave theory (Lin et al, 1969) we compute the difference $\Delta \Omega_m$ between the angular rotation velocity of $m-th$ pattern $\Omega_{pm}$ and the rotation velocity of the Galaxy at the Sun position ($\Delta \Omega_m = \Omega_{pm} - \Omega_{\odot}$). By equating $\Omega(R_{cm}) = \Omega_{pm}$ we find the displacement $\Delta R_m$ of the Sun relative to the corotation radius $R_{cm}$ ($\Delta R_m = R_{cm} - R_{\odot}$, for computational details see MZ and MZDMR).

Notice here some features of our task. It is widely believed that the spiral structure of our Galaxy is generated by a bar in the galactic center (e.g., Marochnik & Suchkov, 1984). So, the angular rotation velocity of the pattern is determined by the bar rotation. Hence, the value $\Omega_{pm}$ does not depend on $m$, and the corotation radius does not depend on $m$ either.

Further, the nature of the perturbed gravitational field could be two-fold. First, in Lin et al.(1969) theory, the spiral waves are self-sustained and the disturbances of the gravitational field are mainly due to the density waves which propagate in the galactic disk. The corresponding dispersion relation imposes a connection between $i_2$ and $i_4$. Indeed, the solution of the dispersion relation for spiral waves relative to the radial wave number $k_m$ in the vicinity of corotation does not depend on $m$ (Shu, 1970, Mark, 1976). Since $\cot(i_m) = k_m R/m$, we have

\begin{equation}
\cot(i_2) = 2 \cot(i_4) .
\end{equation}

Therefore, the two-armed pattern is tighter wound (about twice for small pitch angles) than the four-armed one.

The above argument is strictly held in the vicinity of the corotation circle (according to Créze & Mennessier, 1973; MZ and MZDMR; AL, etc. the Sun is situated just near the corotation). However, since the radial wave number and consequently the pitch angle are slowly varying functions of $R$ (Lin et al, 1969) the relation (6) can be used for a sufficiently wide region around the corotation radius.

In the second approach the perturbations of the gravitational potential are mainly due to a bar in the galactic center. According to the models in which 2 of the 4-arms components coincide with the 2-arms components (eg. Englmaier & Gerhard 1999, Al):

\begin{equation}
\chi_4 = 2\chi_2; \quad i_4 = i_2 .
\end{equation}

In what follows we shall analyze both these approaches. We call them the self-sustained
model (approach 1) and the bar-dominated model (approach 2).

Another important peculiarity of our task is that some of the wave parameters: $f_R^2, f_\vartheta^2, f_R^4, f_\vartheta^4, i_2$ ($i_4$ is fixed automatically in the both models), $\chi_{\odot 2}$ and $\chi_{\odot 4}$, obey nonlinear statistics. In the self-sustained model $\chi_{\odot 2}$ and $\chi_{\odot 4}$ are considered to be independent quantities; in the bar-dominated approach, these values are connected by Eq (7). But if we fix the pitch angles and the initial wave phases, the task becomes a linear one over other the parameters. So, the strategy to localize the global minimum for the residual ($\delta^2$) in the second approach is just the same presented by MZ and MZDMR. However, in the self-sustained model it slightly changes. In this case we have to fix 3 independent quantities: $i_2, \chi_{\odot 2}$ and $\chi_{\odot 4}$, and look for the minimum of residual over other the parameters by means of the least square method (we denote this minimum by $\Delta = \min_{i_2, \chi_{\odot 2}, \chi_{\odot 4}}(\delta^2)$). Then we change values of $i_2, \chi_{\odot 2}$ and $\chi_{\odot 4}$ and again derive $\Delta$, and so on. After that, we construct the net $\Delta$ as a function of $i_2, \chi_{\odot 2}$ and $\chi_{\odot 4}$. Of course, we cannot imagine this function visually, but we can construct the surface, say, $\Delta(\chi_{\odot 2}, \chi_{\odot 4})$ for a set of values $i_2$ and check by eye localization of the global minimum of the residual. After the minimum is localized, by means of a linearization procedure (Draper & Smith, 1981) we define more exactly the searched parameters and the covariation matrix of errors (see details in MZ).

3. Results of Cepheid kinematics analysis

The above described procedure was applied to a sample of Cepheids which represent the best observational material for solving the above formulated problem. For the Cepheids both the line-of-sight velocities (Pont et al, 1994, Gorynya et al, 1996, Caldwell & Coulson, 1987) and proper motions (HIPPARCOS, ESA, 1997) are available. Let us discuss the results derived for the above approaches separately.

Self-sustained model. For illustration in Fig.1 the surfaces $\Delta(\chi_{\odot 2}, \chi_{\odot 4})$ are given for 3 values of $i_2$. One can easily see the global minimum in the vicinity of $i_2 \approx -6^\circ$. The final values of the parameters with their errors are given in Table 1.

First of all, we notice the significant decrease of the residual $\min(\delta^2)$ in this case in comparison with the one for the single harmonic $m = 2$ or $m = 4$ (see Table 1 of MZ, runs no 3 and 8). As it was shown in the cited paper, the inclusion of the spiral perturbation in stellar motion happens to be significant. However the authors could not make a choice between the two alternatives: pure 2- or 4-armed patterns.

Now by means of F-test we can show that the previous hypotheses that the pattern can be represented by only one harmonic ($m = 2$ or $m = 4$), should be rejected in favor of the alternative hypothesis that the pattern is well represented by superposition of 2+4-armed pattern. In other words, the representation of the galactic structure by a superposition of 2+4-harmonics is clearly preferable to the one that uses a single harmonic.

The quantities $\Delta\Omega_m$ and $\Delta R_m$ can both be calculated from the parameters obtained for $m = 2$ and for $m = 4$. It is not hitherto obvious that for different $m$ the quantities occur to be the same as we have supposed above. But our calculations lead to very close values for the corresponding quantities: $\Delta\Omega_2 = 0.15 \ km \ s^{-1} kpc^{-1}$ and $\Delta\Omega_4 = 0.18 km \ s^{-1} kpc^{-1}$; accordingly $\Delta R_2 = -0.03 kpc$
and $\Delta R_4=-0.04$ kpc, the standard (i.e. 68\%) confidence intervals being for $\Delta\Omega_2 : -0.61$ to $1.02 \text{ km s}^{-1}\text{kpc}^{-1}$; for $\Delta\Omega_4 : 0.13$ to $0.24 \text{ km s}^{-1}\text{kpc}^{-1}$; for $\Delta R_2 : -0.21$ to $0.13$ kpc; for $\Delta R_4 : -0.05$ to $0.03$ kpc (the confidence intervals were estimated by means of numerical experiments described by MZ). Hence, in the model under consideration the Sun is practically situated at the corotation circle, slightly beyond it.

We cannot estimate with any reasonable accuracy the values of the amplitudes of the spiral gravitational field $A_2$ and $A_4$ (this is expected from linear perturbation theory). However, their ratio is derived very precisely: $A_2/A_4 = 0.79$, the standard confidence interval being 0.77 to 0.80.

The locus of minima for $\varphi_{sm}$ are shown in Fig. 2; they are the lines of constant phase $\chi_m$ on the galactic plane corresponding to $\min \varphi_{sm}$. From this figure the pattern may be thought to be 6-armed one. However, this is not the case in our model, in which the potential perturbation is represented by a cosine function. Indeed, simple computation shows that for the above derived parameters the sum $\varphi_{s2} + \varphi_{s4}$ has at most 6 minima over $\vartheta$ for a fixed $R$. It would only be possible to obtain 6 minima (in the frame of a 2+4 armed model) if the potential were represented by some function presenting sharp minima, contrary to the cosine function. The visible structure derived by means of particle-cloud simulations and given in Sec. 4 supports this point of view. Of course, the actual structure of the Galaxy may occur to be more complicated e.g., due to higher wave harmonics or to special effects at the corotation (Mark, 1976). However, these possibilities are beyond the present investigation.

**Bar-dominated model.** In this case the results are quite different. The pattern rotation velocities are: for $m = 2$ $\Omega_{p2} = 35.0 \text{ km s}^{-1}\text{kpc}^{-1}$ and for $m = 4$ $\Omega_{p4} = 29.2 \text{ km s}^{-1}\text{kpc}^{-1}$. So that the main requirement of the model (independence of pattern rotation velocity and the corotation radius from $m$) is not held. Indeed, for parameters of rotation curve of Tables 1 and the above value for $\Omega_{p2}$ the corotation radius $R_{c2}$ does not exist as a real number at all (this is mainly because the second derivative $R^{''}\Omega$ is negative). On the other hand for $m = 4$ $R_{c4} = 5.4$ kpc. Further, in this case the ratio $A_2/A_4 \approx 8.21$ is very large. So, the visible pattern happens to be 2-armed!

At last, the residual $\delta^2 \approx 220$ is significantly greater than in the previous approach and occurs to be very close to the values for pure $m = 2$ and pure $m = 4$ solutions (see Table 1 of MZ 1, runs no 3 and 8). Hence, it is impossible to make a choice between pure $m = 2$, pure $m = 4$ or superposition of 2+4-modes in the bar-dominated approach.

The above statistical analysis of the large-scale stellar kinematics of Cepheid stars, leads us to the conclusion that the preferable solution for the spiral structure of the Galaxy is a superposition of self-sustained 2+4-harmonics of density waves, and that the Sun is situated very close to the corotation circle.

4. **Visible large-scale structure of the Galaxy**

In the previous Section we derived the structural parameters for the gravitational field of the Galaxy. However, as it was mentioned above, it is not directly seen. To make visible the above derived structure, the evolution of a gas cloud ensemble in the galactic gravitational field perturbed by spiral arms will be next considered. Following Roberts &
Hausman (1984), we simulate the interstellar clouds by ballistic particles moving in a given gravitational field with a potential $\varphi_G$ defined by Eqs (1-3). Let us briefly describe the formulation of the task.

At the initial moment of time ($t = 0$) the spiral perturbations are assumed to be absent ($\varphi_{sm} = 0$). $N$ particles ($N = 2 \cdot 10^4$) are uniformly distributed over a disk within $R < 13$ kpc. Each particle is given the local rotation velocity, disturbed by a chaotic velocity with one-dimensional dispersion $8 \text{ km s}^{-1}$. For $t > 0$ the spiral perturbation is ”switched on”. Our task is to compute the reaction of the system on this perturbation (the $N$-body problem for particles moving in an external field).

The parameters both for unperturbed and for perturbed potential were taken from Table 1. Since the derived rotation curve is valid in a restricted region, for $R > 9.4$ kpc we continue it by a flat part.

The self-sustained galactic waves are well known to exist between the inner and outer Lindblad resonances. Since we do not take into consideration a bar in the galactic center, the spiral perturbation was cut off for $R < 2$ kpc. For the spiral gravitational amplitude $A_2$ we assumed the ”standard” value: $2A_2 \cot(i_2)/\Omega_\odot^2 R_\odot^2 = 0.05$ (Lin et al, 1969). In the simulation, collisions between the interstellar clouds occur, the collisions being energy-dissipating. For the computation of this process, we used the method described in detail by Roberts & Hausman (1984). For the cloud cross-section we adopted a typical value for the H I clouds. Mutual clouds gravitation, or other effects like interaction of the clouds with expanding envelopes of supernovae, etc., were not taken into account. We also restrict ourselves to consideration of two-dimensional particle motion in the galactic plane. All computations are performed in a frame of reference corotating with spiral arms.

The result of our simulation of particle-cloud dynamics in the spiral gravitational field for the superposion of 2+4 self-sustained density wave harmonics is shown in Fig. 3. There is good agreement in major features with Fig. 3 of Efremov (1998). In a significant part of galactocentric distances the pattern looks like a 4-armed one. But we do not face with the problem of too short arms, as it would be in the case of pure $m = 4$ harmonic (see AL). Our pattern reflects the complicated picture often observed in external galaxies, e.g. arm bifurcation or their overlapping.

5. Gap in the galactic gaseous disk as an indicator for the corotation circle

One of the most important conclusion of Sec. 3 is that the Sun lies very close to the corotation radius. In this Section we present a new test which turns possible to localize directly the position of the corotation circle in a spiral galaxy.

Many years ago Kerr (1969) payed attention to a ring-like region which is markedly deficient in neutral hydrogen, with radius slightly greater than the solar distance from the galactic center (see also Simonson, 1970). This result was later supported in more detail by Burton (1976). He showed that there is a very clear gap in radial distribution of atomic hydrogen in our Galaxy at $R \approx 11$ kpc, whereas in the old scale, used in that paper, $R_\odot = 10$ kpc (see Fig. 6 of Burton, 1976). In general, the gap reminds the Cassini gap in Saturnian ring.

It is natural to connect this gap in the ra-
dial ISM distribution with the process occurring at corotation: the gas is pumped out from the corotation under the influence of the gravitational field of spiral arms (see also Suchkov, 1978). The qualitative explanations is as follows. It is well known, that when the gas flows through the galactic density waves, a shock arises in the medium (Roberts, 1969; Roberts & Hausman, 1984). Since the galactic disk rotates differentially and for $R < R_c$ $\Omega > \Omega_p$, the gas in this region overtakes the spiral wave, entering it from the inner side. In the shock the clouds are decelerated, and fall towards the galactic center. For $R > R_c$ $\Omega < \Omega_p$, and the process is inverse. Here the wave overtakes the gas and pushes it. So, the clouds pass to an orbit more remote from the galactic center (see also Goldreich & Tremaine, 1978 and Gor'kavyi & Fridman, 1994).

Our simulation of gas cloud dynamics in spiral gravitational field directly demonstrates this phenomenon. We show in Fig. 4 the radial gas distribution ($<n>$ is the particle concentration averaged over a circle) for $t = 0$ and $t = 3.0$ (the time is given in rotation period at the solar distance) for the best parameters of Sec 3. The gap in the ISM distribution at the corotation radius is well seen. Comparison of Fig.4 with Fig. 6 of Burton (1976) shows a close similarity between them.

This result enables us to explain another problem as well. It is well known that the rotation velocity of the disk presents a sharp minimum near the solar galactocentric distance; the minimum appears independently of the tracer being gaseous (eg. Honma & Kan-ya, 1998) or stellar (Amaral et al, 1996). This phenomenon, could be thought to be understood in terms of the velocity perturbation from the galactic density waves, or from the rising effect of a dark matter component at distances larger than the solar radius. Amaral et al. exclude these hypotheses, in their discussion of the nature of this minimum. Now, if there is a ring-like region devoid of gas, in principle the rotation velocity could not be measured in that region, using a gas tracer. Similarly, if one selects short-lived stars as tracers, these stars are not expected to form and to exist inside that region, because of the gas deficiency and of the very low velocity of the gas with respect to the spiral pattern, which turns the gas compression (the star-formation process) inefficient. It is therefore probable that the gas clouds and the stars that we observe inside the gap are objects with non-circular orbits that invade the gap; they are observed close to their maximum elongation and therefore, present smaller velocity than the circular one, in the direction of rotation.

So, a ring-like gap in the galactic gaseous disk, and possibly also a sharp minimum in the rotation curve, may serve as an independent indicator for localization of the corotation circle in a spiral galaxy.

6. New spiral structure of the Galaxy derived from H II data

The H II regions are the best tracers of the large-scale spiral structure, since they can be observed at large distances, and unlike H I, they are sharply concentrated in the arms (e.g. GG). We performed a new analysis of the sample of H II regions of the Galaxy, without making use of the results of the theoretical models discussed in this paper, except for one hypothesis, namely, that the structure can be represented by a superposition of 2+4-armed patterns. Therefore, the results of this section constitute an independent test of the previous
ones.

The procedure adopted is to trace spiral arms in the galactic plane, and to transform $X - Y$ positions along the arms into locus of the arms in the $l - v$ diagram, by means of the rotation curve. By varying the parameters of the arms, we looked for the best fit to the $l - v$ diagram. It is well known that the arms that are situated inside the solar circle, transform into narrow loops in the $l - v$ diagram; the extremity of a loop corresponding to a tangential direction in the galactic plane. The observed pattern is represented by the sum of the $m = 2$ system (two identical long arms with phase difference of $180^\circ$) and of the $m = 4$ system (four identical short arms each separated by $90^\circ$ in phase) shifted by some phase angle from the first system. The adjusted parameters are the pitch angles, the angle for the phase shift, the inner and the outer radii of each of the two systems.

We next discuss some of the features of the proposed structure. In the region $l = 340^\circ$ to $270^\circ$, which is the only region where clear arms were observed by GG, the structure resulting from our study closely resembles that of these authors, presenting about the same tangential directions. Remark that the longitudes of tangential directions are not affected by a change of distance scale (GG used $R_0 = 10$ kpc). Therefore, in this range of longitude, it is almost impossible to distinguish between the 2+ 4 arms model that results from our study, and the empirical model of GG. This is specially true if we adopt a smooth function to represent the potential, like we did in previous sections, since close potential minima tend to merge. A difference between GG and our work is that GG did not indicate the existence of a tangential direction at about $l=338^\circ$(our inner loop $e$), but obviously there are observed HII regions in that direction, at large negative velocities, that justify our model. In some other directions, the observations favor our model as well. Remark for instance that there are concentrations of HII regions near labels $a$ and $f$ in Fig. 5. These are well explained by a spiral arm that pass very close to the Sun, seen almost at $l \approx +90^\circ$ and then at $l \approx -90^\circ$, since the velocities are almost zero in these directions, for distances that are not too large, according to the well known expression $v \propto \sin(2l)$. The longitudes of the tangential directions indicate that it is an arm with small pitch angle (about 6°). On the contrary, the wide loop label $b$, can only be well reproduced with a larger pitch angle (12°). This emphasizes the need for arms with different pitch angles. The best fit of the $l - v$ diagram of the observed HII regions that we obtained with a simple 2 + 4 armed model, although not perfect, reproduces the main features of the diagram,
and in particular, the main tangential directions. This was obtained with pitch angles 6.6° for the 2-armed component and 12° for the 4-armed component.

Let us now discuss the theoretical $l - v$ diagrams, that were computed by means of our particle simulations in Sec. 4 and are shown in Figs. 7 to 9. Figs.7 and 8 are very similar to the observed diagram for H II regions, in many aspects. In particular, the observed loops like $b$, $c$, $d$, $e$ which are interpreted in Fig. 5 as empirical loci of spiral arms, with corresponding tangential directions to the arms, can be clearly seen in Fig. 7 and 8 as well. A difference that appears between the theoretical $l - v$ diagrams and the observed ones for H II regions is a strong concentration of points along the line from $l \approx -60°, v \approx +100 \text{ km s}^{-1}$ to $l \approx +60°, v \approx -100 \text{ km s}^{-1}$. This is an expected result since the points corresponding to H I (recall that the particles simulate the H I clouds, see Sec 4) are situated at large galactic radii, where H I is known to exist, whereas there is lack of H II regions at large radii.

It is not surprising that the $l - v$ diagrams in Figs. 7 and 8 are so similar, since both models contain similar 2-arms component, and this component is prominent over a wider galactocentric region than the 4-arms component of the 2+4 arms model. However, we can point out a number of differences. For instance, the observed loop $b$, which is due to a 4-arms component, is correctly reproduced by the particles of the self-sustained model (Fig. 7) and does not appear in Fig.8. On the other hand, the arm that passes very close to the Sun, as discussed above (loops $a$ and $f$) appear more clearly in the 2-arms model (Fig.8). We emphazise that the loops presented in Figs. 7-9 are eye-guides for comparison of theoretical diagrams with observed HII regions, but they are not perfect fits to the HII regions. In the HII regions diagram (Fig. 5) we can see many objects between loops $c$ and $d$, that are not well fitted by the loops. We can see many objects as well in this region in Figure 8, the theoretical self-sustained model. In other words, the theoretical model is closer to reality than the “empirical” fit represented by the lines. Around longitude 240° (or -120°), the theoretical $l - v$ diagram of the self-sustained shows many objects with velocities of the order of 30 km s$^{-1}$. These objects seem to delineate a spiral arm that is not seen in the HII regions diagram. However, a arm indeed exists at this position, as can be seen in the longitude-velocity diagram of IRAS sources given by Wouterlout et al. (1990). This arm is deficient in HII regions, probably due to the proximity of corotation.

On the contrary, the 4 arms model (Fig. 9) show stronger differences with observations. The particles do not show loops $a$ and $b$, but show a clear loop between loops $c$ and $d$, that is not present in the HII regions diagram.

In summary, the pure 2-arms model and the self-sustained 2+4 arms model produce theoretical $l - v$ diagrams that are similar between them and similar to that of observed HII regions. The only striking difference between the theoretical and observed diagrams is an expected one, due to the fact that we are comparing objects that behave differently (HI gas and HII regions). If we look into the details, the 2+4 arms self-sustained model is favored. The choice in favor of the 2+4 arms model, compared to the pure 2-arms, is more strongly dictated by the study of Cepheid kinematics.
7. Conclusion

In the present research a new approach to the problem of the galactic spiral structure was proposed in order to construct a more realistic picture like often seen in external galaxies: co-existing of different spiral systems in a galaxy, arm bifurcation and their overlapping. Our theoretical considerations shows that superposition of self-sustained spiral wave harmonics could explain some of the above features, since different azimuthal wave harmonics have different pitch angles.

In the framework of the simplest model of superposition of 2+4-armed spiral wave harmonics, we analysed the best up-to-date data on stellar kinematics, which is the sample of Cepheid stars, with proper motions and parallaxes determined by HIPPARCOS. We examined two models, the self-sustained and the bar-dominated waves. This study complements the previous studies of pure 2-arms and pure 4-arms presented by MZ ad MZDMR. Of the four models, clearly the one that gives the best fit to the Cepheid kinematics is the self-sustained model, which is a superposition two arms with pitch angle about 6° and four arms with pitch angle about 12°. We performed N-particle simulations to make visible the structure of the potential derived from the Cepheid kinetics for the four models, and to construct the corresponding \( l - v \) diagrams.

As an independent test of a 2+4 arms model, we performed a new analysis of the \( l - v \) diagram of the galactic HII regions, which are the best tracers of the large-scale spiral structure. We fitted the observed \( l - v \) diagram empirically with the locii of spiral arms, using a 2+4 arms model. Coincidentally, the best empirical fit was again found using pitch angles about 6° and 12°. We also compared the theoretical \( l - v \) diagrams from the particle simulations with the loci of arms derived from HII regions. Although the differences between \( l - v \) diagrams of the theoretical models (pure 2-arms, pure 4-arms, and 2 different models of 2+4 arms) are not striking, the 2-arms model and the self-sustained 2+4 arms models are the ones that produce theoretical \( l - v \) diagrams most similar to that of the HII regions.

Of all the arguments that we examined, the significantly better fit of the kinematics (the \( \delta^2 \) analysis) of Cepheids with the 2+4 armed, self-sustained model, is the most convincing one; but clearly the analysis of the HII regions sample gives support to our interpretation. Although the Galaxy probably shows some deviations from any simplified model, the 2+4 armed model with different pitch angles is the one that constitutes the best approach, being consistent with observations and with spiral waves theory. This is the simplest model that can be proposed, apart from pure harmonic modes. Our model does not exclude the existence of higher modes, that are allowed to exist near corotation, but these modes are probably less significant than the first harmonics. It is interesting to remark that our model is able to reconcile the first model of Lin & Shu (1964), which has 2 arms with pitch angle 6° similar to our 2-arms harmonic, with the need to satisfy Kennicut’s (1982) correlation between pitch angle and maximum rotation velocity, which predicts a pitch angle of about 14° for our Galaxy, not very different from that of our 4-arms harmonic.

As a by-product of the study, our particle simulation shows that a deficiency of interstellar gas must occur near corotation. This explains the gap in the HI distribution observed by Kerr(1969) and by Burton (1976). This ef-
fect is also possibly related to the sharp minimum in the rotation curve near the solar radius discussed by Amaral et al. (1996), also seen in the curve by Honma & Kan-ya (1998). In external galaxies the ring-like gap in H I distribution may directly show the localization of the corotation circle.

Our investigation of Cepheid kinematics points out that the Sun lies very close to the corotation circle. This conclusion is supported by the solar closeness to the ring-like region of H I deficiency in the Galaxy derived by Kerr (1969) and by Burton (1976), as well as by the position of Lindblad resonances (as discussed in Section 1) and by direct measurement of the pattern speed using open clusters (AL).

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Figure captions

Fig.1.- Surfaces of the $\Delta$ as a function of $\chi_{\odot 2}$ and $\chi_{\odot 4}$ for 3 values of pitch angle $i_{\odot}$.
Fig.2.- The locus of $\min \varphi_{S2}$ and $\min \varphi_{S4}$ on the galactic plane. The scale is indicated in kpc. As discussed in the text, this does not correspond necessarily to the visible structure.
Fig.3.- The visible structure of the Galaxy derived for the best model (superposition of 2+4 self-sustained wave harmonics) by means of particle-cloud simulation.
Fig.4.- The radial distribution of cloud concentration $< n >$ (averaged over a circle) for $t = 0$ (dashed line) and $t = 3.0$ (solid line). The gap in the gas distribution is clearly seen near the corotation radius $R_c$. The Sun is situated at $R_{\odot} = 7.5$ kpc.
Fig.5.- The observed $l - v$ diagram for H II regions (from Kuchar & Clark, 1997) and the loops that we fitted empirically with a 2+4 arms structure.
Fig.6.- The spiral structure of the Galaxy derived from H II data.
Fig.7.- The theoretical $l - v$ diagram computed by means of particle-cloud simulation for the best model: superposition of 2+4 self-sustained wave harmonics..The lines represent the fit to observed HII regions, from Fig. 5, for comparison.
Fig.8.- Same as in Fig. 7, but for the model of pure $m = 2$ wave harmonic (the parameters were taken from MZDMR).The lines represent the fit to observed HII regions, from Fig. 5, for comparison, and not a fit to the $m = 2$ model.
Fig.9.- Same as in Fig. 8, but for the model of pure $m = 4$ wave harmonic (the parameters were taken from MZ). The lines represent the fit to observed HII regions, from Fig. 5, for...
comparison, and not a fit to the $m = 4$ model.
Table 1: Model parameters and their errors derived by means of statistical analysis.

| approach | $\Omega$ \((km/s/kpc)\) | $A$ \((km/s/kpc)\) | $R_0\Omega'$ \((km/s)\) | $u_0$ \((km/s)\) | $v_0$ \((km/s)\) | $|i_2|$ \((^\circ)\) | $\chi_{12}$ \((^\circ)\) | $f_{R2}$ \((km)\) | $f_{v2}$ \((km)\) | $|i_4|$ \((^\circ)\) | $\chi_{14}$ \((^\circ)\) | $f_{R4}$ \((km)\) | $f_{v4}$ \((km)\) | min $\delta^2$ |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| self−    | 26.3            | 17.5            | 9.9             | −8.8            | 11.9            | 6.1             | 311.            | 0.4             | −14.0          | 12.0           | 122.            | 0.8             | −10.9          | 187.            |
| sust.    | ±1.3            | ±0.8            | ±1.9            | ±1.0            | ±1.1            | ±0.4            | ±11.            | ±3.0            | ±0.8           | ±15.           | ±3.3            | ±2.9            |
| bar−     | 25.7            | 9.8             | −6.8            | −13.6           | 9.3             | 12.6            | 184.            | 12.5            | −19.4          | 12.6           | 8.              | 6.6             | −10.1          | 220.            |
| domin.   | ±1.2            | ±2.3            | ±5.0            | ±2.4            | ±1.2            | ±0.5            | ±4.             | ±3.8            | ±4.4           | ±1.0           | ±7.             | ±2.4            | ±2.1            |
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