Bringing Real Analysis Subject Into Real Life: an Experimental Research for Prospective Teacher of Mathematics Study Program Using Realistic Mathematics Education

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Abstract. This research focus on describing the method used by the lecturer in understanding Prospective Teacher about the definition of $V_\varepsilon(a)$. This research conducted in Mathematics Education Study Program, Cokroaminoto Palopo University. The data collected by using interview, observation sheet, and questionnaire. Realistic Mathematics Education (RME) used by the lecturer to deliver the material of $V_\varepsilon(a)$. In this case the lecturer introduced three principles of RME such as guided reinvention, didactical phenomenology, and self-developed model to the prospective teacher. The result shows that prospective teachers are able to understand the definition of $V_\varepsilon(a)$. The prospective teacher can solve or proof any problem and theorem related to definition of $V_\varepsilon(a)$.

Keywords: Bringing real analysis, real life, experimental, realistic mathematics

1. Introduction
Education is mentioned in the Preamble of 1945 Constitution of the Republic of Indonesia make it plays an important role in Indonesia. The effect is Indonesia has implemented a "twelve-year compulsory education". That twelve-year consist of primary education, junior high school, and senior high school. Now days Indonesia has implemented the Kurikulum 2013 in some schools. In its implementation, the 2013 curriculum expects a different class atmosphere than before. The 2013 curriculum emphasizes the dominance of students in learning that we commonly call student centered learning.

Teaching in higher education is different from teaching in high school and junior high school. Especially for the teaching faculty, lecturers must have more skills in delivering lecture material.

However, there are only a few references that can be used by lecturers to teach in universities. Higher education teachers have little pedagogical information to support their practice [1]. The result is that lectures are the most used method in teaching. This method is teacher-centered learning so that the role of students is limited. Teaching using the lecture method is an option for most lecturers because it is easy to apply. However, this method will make students tend to be passive and become true listeners.

Real analysis is one of the courses taught at the university especially in mathematics education study program. This course discusses some basic concepts of the nature of the field and the order of real numbers, the completeness of real numbers, the Archimedean Properties, the set theory in real number, the real number sequence, the limit theorem, the Cauchy sequence, series convergence and special convergence test. According to that material, real analysis subject is intended so that students understand...
several structures in the analysis and can use it to solve simple problems, and be able to think logically and reason mathematically in solving problems [2].

Referring to the purpose of the course, using the lecture method in my opinion is inappropriate. Teaching real analysts must make students participate in solving problems. As I suggest in my previous research that the lecturer must find the suitable learning method to improve the prospective mathematics teacher in solving real analysis problem [3]. One example proposed by [4] if a student is able to demonstrate that method A gives similar results to the learning outcomes of students using method B, but method A requires less mental effort during learning, then it is more student friendly to adopt method A in class.

In this study I used the Realistic Mathematics Education to teach certain material in real analysis that is $\varepsilon - \text{neighborhood}$. This material needs precise language to discuss the notion of one real number being “close to” another. If $a$ is given real number, then saying that a real number $x$ is “close to” $a$ should mean that the distance $|x - a|$ between them is “small.” “small”. A context in which this idea can be discussed is provided by the terminology of neighborhoods [5].

2. Realistic mathematics education

Realistic Mathematics Education was developed in Netherland based on the idea of Freudenthal and his colleagues at the Freudenthal Institute. Indonesian Realistic Mathematics Education was aimed to make learning Mathematics more interesting and meaningful for students by introducing teaching this subject through contextual problems where the problems were in the students’ knowledge and experience [6].

The three key principles of realistic mathematics education could be described as follows:

(1) Guided reinvention: Students should be given an opportunity to experience a process similar to the process in which mathematics was invented. The history of mathematics could be used as a source of inspiration. During the learning process, students should have an opportunity to build their own mathematical knowledge. Students’ informal strategies could be interpreted as anticipated more formal procedures. Contextual problems allowing a wide variety of solution procedures should be selected and preferably solution procedures could reflect a possible learning route by itself (Gravemeijer, 1997, Gravemeijer & Terwel, 2000) [6].

(2) Didactical phenomenology: Situations where a given mathematical topic is applied required investigation to reveal the sort of applications that have to be anticipated for instruction, and to consider their suitability as points of impact for a process of progressive mathematization. The goals of phenomenological investigation are to find problem situations in which situation-specific approaches can be generalized, and to find situations that can evoke paradigmatic solution procedures which can be taken as the basis for vertical mathematization (Freudenthal, 1983 as cited in Gravemeijer, 1997) [6].

(3) Self developed model: A self-developed model plays a vital role in bridging the gap between informal knowledge and formal mathematics. Models were developed by the students themselves. At first, the model was any model of a situation familiar to the students. By generalizing and formalizing, the model then becomes an entity of its own. This made it possible to use this model as a model for mathematical reasoning (Gravemeijer, 1994 as cited in Gravemeijer, 1997) [6].

3. Research method

The research take place in Program Studi Pendidikan Matematika, Fakultas Keguruan dan ilmu Pendidikan Universitas Cokroaminoto Palopo. The research method is descriptive qualitative. This study describes the implementation of learning using Realistic Mathematics Education which includes the use of the term "neighbor" in a literal manner to make prospective teacher more understand about material.

The data collected by using observation sheet and prospective teacher worksheet and the data analysis using qualitative descriptive.
4. Finding and Discussion
The material taught by lecturers is the definition of \( \varepsilon - \text{neighborhood} \) that is

Let \( a \in \mathbb{R} \) and \( \varepsilon > 0 \). Then \( \varepsilon - \text{neighborhood} \) of \( a \) is the set \( V_\varepsilon(a) = \{ x \in \mathbb{R} : |x - a| < \varepsilon \} \)

To explain this material the lecturer uses the word "neighbour" and applies the concept of "distance" previously studied. The lecturer ways to make prospective teacher understand about the definition is as follows.

The lecturer demonstrates that a house is located on the side of the road and directs students to determine who belongs to their "neighbour". Some students stated that their neighbour were houses close to his home. This is the meaning of the word "close to or \( |x - a| \)" and the meaning of the symbol "epsilon" which is very small.

The lecturer reiterates that the meaning of "distance" mentioned by students is "\( |x - a| \)" where \( a \) is "a real number" and \( x \) is another real number that will be determined as "neighbor" if the distance is less than epsilon.

Furthermore, students are directed to determine the distance of a house as a "neighbour". This command directs students to understand the meaning of "epsilon" which is a "small" number determined by themselves.

Thus, the lecturer can explain scientifically that the \( \varepsilon - \text{neighborhood} \) of \( a \) is symbolized \( V_\varepsilon(a) \) is a set of \( \varepsilon \) real numbers which are very small distances from \( a \in \mathbb{R} \). a number that states a small distance is symbolized as \( \varepsilon \).

![Figure 1. Lecturer Illustration in Teaching Definition of \( V_\varepsilon(a) \)](image)

By using this method, the lecturer more easily conveys the definition of \( V_\varepsilon(a) \) to prospective teacher. The choice of the word "neighbor" that is often encountered by prospective teacher in everyday life makes them able to imagine a real word that is close to other real numbers even though the number is very small.

Realistic mathematics make the atmosphere of the class more effective and comfortability. Bring the prospective teacher to learn the abstract matter by imagine it in their daily life is the best way because they build their own knowledge so that students hard to forget. This learning also emphasizes intellectual and affective activities where students are involved in efforts to explore their experiences in order to achieve new understanding and appreciation in mathematics learning [7]. The goal is not only to help the students elaborate their informal understanding and informal solution strategies in to the formal mathematical insights and strategies. The aim is to maintain the relationship between mathematical concepts and what these concepts explain [8].

Prospective teacher investigation in solving problem of Applications of the Supremum Property by using the problem in Bartle. We know that taking suprema and infima of sets is compatible with the algebraic properties of \( \mathbb{R} \), as an example in addition.

Let \( S \) be a nonempty subset of \( \mathbb{R} \) that is bounded above, and let \( a \) be any number in \( \mathbb{R} \). Define the set \( a + S := \{ a + s : s \in S \} \) We will prove that

\[
\sup(a + S) = a + \sup S
\]
Prospective teacher solves this problem by using bringing the statement of “$a + S$” in term of neighbourhood. Prospective teacher illustrates that “$S$ is a group of houses in one the straight road” and “$a$ is other house that will join to the group of $S$”. In this case they define that “Supremum is the house that located in the boundary of $S$”. They illustrate the problem as below.

![Figure 2. Student preliminary Illustration in solving problem](image)

The following sentences are the prospective teacher problem solving step (s) compare with Bartle problem solving steps (b) and (l) is the scaffolding from the lecturer.

(s) : $u$ is the very close boundary from the right house means $u = \sup S$. Means $s \leq u$

Because $a$ will join to $S$ so the boundary $u$ and $s$ will added by $a$ we get $a + s \leq a + u$

It means that $a + u$ will be boundary of $S$ with $a$ says it $a + S$

(l) : can we say “$a + u$” is the supremum of $a + S$? You must try to use the definition of supremum to get the solution

(b) : If we let $u = \sup S$, then $s \leq u$ for all $s \in S$, so that $a + s \leq u + a$. Therefore, $u + a$ is an upper bound for the set $a + S$. Consequently, we have $\sup(a + S) \leq a + u$. To find the solution student know

(s) : $v$ is any upper bound of the set $a + S$, then $a + s \leq v$ for all $s \in S$. Consequently $s \leq v - a$

For all $s \in S$, so that $v - a$ is an upper bound of $S$. Therefore, $u = \sup S \leq v - a$. If and only if $u \leq v - a$, consequently $u + a \leq v$

Because $u + a$ is an upper bound of $a + S$ and $v$ is another upper bound such that $u + a \leq v$ then $u + a = \sup(a + S)$. We have $u = \sup S$, then we can conclude that $\sup S + a = \sup(a + S)$.

(b) : now if $v$ is any upper bound of the set $a + S$, then $a + s \leq v$ for all $s \in S$. Consequently $s \leq v - a$ for all $s \in S$, so that $v - a$ is an upper bound of $S$. Therefore, $u = \sup S \leq v - a$, which give us $a + u \leq v$. Since $v$ is any upper bound of $a + S$, we can replace $v$ by $\sup(a + S)$ to get $a + u \leq \sup(a + S)$. Combining these inequalities, we can conclude that $\sup(a + S) = a + u = a + \sup S$.
The problem solving performed by prospective teachers differ with the Bartle Book show. First of all, the prospective teachers try to understand the problem by using real-life illustration. This is a transition from a traditional, technically oriented approach to mathematics education based on problem solving is a complex innovation [9].

5. Conclusion
This research concludes that:
1. Prospective teachers are able to prove some problem related to definition of \( v_\varepsilon(a) \) neighbourhood.
2. The real-life illustration used by the prospective teachers to help them to understand and solving the problem.
3. Realistic Mathematics Education make student allows prospective teacher to use real-life context such as “neighbourhood”, “epsilon” and “supremum” to solve problem related to definition of \( v_\varepsilon(a) \) neighbourhood.

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