Multiple Electromagnetic Excitation in Fast Peripheral Heavy Ions Collisions

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Abstract

We study the corrections of first order electromagnetic excitation due to higher order electromagnetic interactions. An effective operator is introduced which takes these effects into account in the sudden approximation. Evaluating the matrix-elements of this operator between the relevant states corrections to the first order result are obtained in a simple way. As an example we discuss the excitation of the first excited state in $^{11}$Be. It tends to improve the agreement between experiment and theory.
Electromagnetic Excitation in the energy domain of several tens of MeV/u up to relativistic energies is a growing field of study. The cross-section can become large and irreducible nuclear effects can be kept under control. With increasing beam energy the equivalent photon spectrum becomes harder, and also particle-unstable states can be reached. The Coulomb dissociation $^{11}\text{Li} \rightarrow ^{9}\text{Li} + 2\text{n}$ and $^{14}\text{O} \rightarrow ^{13}\text{N} + \text{p}$, which is also astrophysically relevant, are examples [1]. Recently, bound states were also excited and their (Doppler shifted) de-excitation $\gamma$-rays were measured. A large deformation of the neutron-rich nucleus $^{32}\text{Mg}$ was recently deduced from a measurement of the $2^+ \rightarrow 0^+$ 885 keV transition to the ground state after medium energy electromagnetic excitation [2]. The 320 keV $^{1}\frac{1}{2}^- \rightarrow ^{1}\frac{1}{2}^+$ $\gamma$-transition in $^{11}\text{Be}$ was recently observed. The measured cross-section for the $^{11}\text{Be} \left(^{1}\frac{1}{2}^+ \rightarrow ^{1}\frac{1}{2}^- \right)$ Coulomb excitation was found to be noticeably less than expected from the known lifetime and 1st order pure Coulomb excitation [3].

Apart from possible nuclear and Coulomb-nuclear interference effects, a possible reason for this discrepancy is the influence of higher order electromagnetic interaction. It is the purpose of this letter to describe a framework suitable for fast projectiles. E.g. the rather loosely bound $^{11}\text{Be}$ in its 1st excited $\frac{1}{2}^-$ state could easily be excited electromagnetically into the continuum in a second step [4].

Electromagnetic excitation is mainly characterized by two parameters, the adiabaticity parameter

$$\xi = \frac{\omega b}{\gamma v}$$

(1)
and the strength parameter

\[ \chi_{fi}^{(E\lambda)} = \frac{Ze \langle f | M(E\lambda\mu) | i \rangle}{\hbar v b^\lambda}. \]  

(2)

The excitation energy is given by \( \hbar \omega \), the impact parameter in a straight-line approximation is denoted by \( b \), and \( \gamma = \sqrt{1 - \left(\frac{v}{c}\right)^2} \), where \( v \) is the projectile velocity. The target charge number is denoted by \( Z \), and \( M(E\lambda\mu) \) the electric multipole operator.

In Coulomb excitation below the barrier, multiple electromagnetic excitation is usually treated in a coupled channels approach using the relevant states from appropriate nuclear models, like the harmonic vibrator or rigid rotor. For a review see Ref. [5]. The situation for Coulomb excitation above the barrier becomes simpler, because the excitations tend to be sudden. While \( \hbar \omega \) is restricted to a few MeV, the adiabaticity parameter \( \xi \) is typically less than 1, for the important range of impact parameters \( b > R_1 + R_2 \), where \( R_1 \) and \( R_2 \) are the nuclear radii of projectile and target. Thus fast collisions become the domain of the sudden approximation [5], or of a recently developed low-\( \xi \) approximation [6, 7]. In this case it can be advantageous to construct operators which take into account the influence of intermediate states.

For simplicity, let us use the straight-line approximation and the dipole approximation. The first order excitation amplitude is given by [5]

\[ a_{fi}^{(1)}(E1, \xi) = -i \vec{q}(\xi) \cdot \langle f | \vec{r} | i \rangle \]  

(3)
with

\[ \vec{q}(\xi) = \frac{2ZZ_{eff}^{(1)}e^2}{\hbar vb} \begin{pmatrix} \xi K_1(\xi) \\ 0 \\ i\gamma \xi K_0(\xi) \end{pmatrix} \]  

(4)

where the projectile moves in the z-direction and the impact parameter points to the x-direction. The dipole effective charge is given by

\[ Z_{eff}^{(1)} = \frac{Z_b m_c - Z_c m_b}{m_b + m_c} \]  

(5)

using a model of two pointlike inert clusters \( b \) and \( c \) with charge numbers \( Z_b, Z_c \) and masses \( m_b \) and \( m_c \). \( K_0 \) and \( K_1 \) are the modified Bessel functions. For \( \xi \to 0 \) we get the classical Coulomb push

\[ \vec{q}(0) = \frac{2ZZ_{eff}^{(1)}e^2}{\hbar vb} e_x. \]  

(6)

In the limit \( \xi \ll 1 \), the excitation amplitude can be evaluated easily to all orders in the sudden approximation. It is given by

\[ a_{fi}^{sudden}(E1) = \langle f|e^{-i\vec{q}(0)\cdot \vec{r}}|i \rangle. \]  

(7)

This could be generalized to higher multipoarities and trajectories corrected for Coulomb deflection. By comparing \( a_{fi}^{(1)}(E1, \xi = 0) \) and \( a_{fi}^{sudden}(E1) \) the influence of multiple electromagnetic excitation can be assessed. This is a remarkably simple procedure, all intermediate states are included. The reduction of the excitation probability due to higher order effects is given by

\[ r = \left| \frac{a_{fi}^{sudden}(E1)}{a_{fi}^{(1)}(E1, \xi = 0)} \right|^2. \]  

(8)
For loosely bound states, e.g. in $^{11}\text{Be} = ^{10}\text{Be} + n$, we use simple model wave functions to reveal the characteristic parameters. We choose two models, with the correct asymptotic behaviour. Their differences give a feeling about the model dependence. For the wave functions we make the ansatz for the radial part in the initial state (with orbital angular momentum $l_i = 0$)

$$u_i(r) = \sqrt{2\alpha_i} \frac{\exp(-\alpha_i r)}{r}$$

which corresponds to the solution of the Schrödinger equation with a $\delta$-like potential. For the final state ($l_f = 1$) we choose

$$u_f^I(r) = \sqrt{2\alpha_f} \frac{\exp(-\alpha_f r)}{r}$$

and the more extended wave function

$$u_f^{II}(r) = 2\sqrt{\alpha_f^3} \exp(-\alpha_f r),$$

respectively. The constants $\alpha_z$ ($z = i, f$) are calculated from the binding energies $E_z = \frac{\hbar^2\alpha_z^2}{2\mu}$. With these wave functions the calculated mean lifetimes of the $\frac{1}{2}^-$ state are 110 fs and 97 fs, respectively. They are smaller than the experimental value of $(166 \pm 15)$ fs \[^8\]. Despite this difference, the ratio of higher order to first order effects can be given with some confidence, where, e.g., common spectroscopic factors cancel out. The absolute value depends on more sophisticated details of the nuclear model (see, e.g., Ref. \[^8\]). The reduction factor $r$ can be calculated analytically. We get

$$r' = \left( \frac{3}{z^2} \left( 1 - \frac{1}{z} \arctan z \right) \right)^2$$

(12)
and

$$r^{II} = \left( \frac{3}{2z^3} \left( \arctan z - \frac{z}{1+z^2} \right) \right)^2,$$

resp., where we have introduced the parameter

$$z = \frac{q(0)}{\alpha_i + \alpha_f}.$$  \hspace{1cm} (14)

The parameter $z$ is directly related to $\chi_{fi}^{(E1)}$ (Eq. 2). It describes the ratio of the strength of the Coulomb push $q$ and the “looseness” of the system, and is a measure of the importance of higher order effects. The quantity $r$ is plotted in Fig. 1 for the two model wave-functions described above.

For large $b$ the sudden approximation fails ($\xi \to \infty$), on the other hand, higher order effects diminish due to the decrease of the strength parameter $\chi_{fi}^{(E1)}$. The product

$$\chi_{fi}^{(E1)} \cdot \xi = \frac{Z e \langle f | M(E1 \mu) | i \rangle \omega}{\hbar \gamma v^2}$$  \hspace{1cm} (15)

is a very small number for low excitation energies $\hbar \omega$ and high projectile velocities $v$. The ranges of validity for the first order calculation ($\chi$ small, $\xi$ arbitrary) and the sudden approximation ($\xi$ small, $\chi$ arbitrary) overlap. In a convenient and accurate interpolation procedure we calculate the total cross section in the following way

$$\sigma^{(\infty)} = 2\pi \int_{b_{\text{min}}}^{\infty} r(b) |a_{fi}^{(1)}(E1, \xi(b))|^2 b \, db$$

which should be an accurate expression for all values of $b$ in the integrand. For small impact parameters we have $\xi \approx 0$ and the first order approximation cancels
out in the calculation of $\sigma^{(\infty)}$. We compare it with the total cross section in the first order calculation

$$
\sigma^{(1)} = 2\pi \int_{b_{\text{min}}}^{\infty} |a_{f_i}^{(1)}(E1, \xi(b))|^2 b \, db.
$$

(17)

Both cross sections depend on the value of the minimum impact parameter $b_{\text{min}}$. The change in the two cross sections will be similar so that their ratio is less affected by a change in $b_{\text{min}}$. If the sudden approximation is not well enough fulfilled, one could use the low-$\xi$ approximation [6, 7] which takes second order electromagnetic effects into account.

For the $^{11}\text{Be}$ Coulomb excitation a reduction of the cross section from $(490 \pm 50)$ mb (expected from the first order calculation) to the measured $(191 \pm 26)$ mb was recently found in an experiment at GANIL [3]. Indeed, the parameter $z$ can become substantial in this case, and higher order effects are not negligible. For a collision with $b = 15$ fm and an energy of $45 \cdot A$ MeV ($v/c \approx 0.32$) we have $z \approx 0.38$ and from Fig. 1 we see a substantial reduction of the excitation probability. In a recent experiment at RIKEN [9], which is currently evaluated, an energy of about $65 \cdot A$ MeV ($v/c \approx 0.38$) for the $^{11}\text{Be}$ was used. This leads to a value of $z \approx 0.32$ for the same impact parameter corresponding to a smaller reduction.

The product $\chi_{f_i}^{(E1)} \cdot \xi$ takes on the small values $0.00375(J_iM_i1\mu|J_fM_f)$ and $0.00399(J_iM_i1\mu|J_fM_f)$, respectively, for the two models and the GANIL conditions. At $b = 10$ fm we get an excitation probability of 3.4% and 3.9% in the
first order calculation, decreasing with $b^{-2}$. For impact parameters larger than $\frac{2\nu}{\omega} \approx 200$ fm the excitation becomes adiabatic and the excitation probability drops off exponentially. The excitation probability is small compared to 1 and the non-conservation of unitarity in the first order approximation will not affect the calculation of the cross section.

The apparent reduction of the B(E1)-value (due to higher order effects) is given by

$$R = \frac{\sigma^{(\infty)}}{\sigma^{(1)}} \quad (18)$$

which is plotted in Fig. 2 as a function of the projectile velocity $v$ for the two model wave functions described above. We assume a minimum impact parameter $b_{\text{min}} = 10$ fm corresponding to a grazing collision of the projectile and target. This will give an estimate of the largest possible effect to be expected from the higher order contributions. We obtain a reduction, depending on the particular model chosen, of 5.5% or 10.1% for the GANIL energy and 3.7% or 6.9% for the RIKEN energy. The range of the reduction R in the two models gives a feeling of the reliability of the results. The use of more realistic wavefunctions is outside the scope of the present work. The reduction of the excitation probability for the $\frac{1}{2}^-$ state by higher order effects will be accompanied by an increase of the cross section in the breakup channel.

In the dipole approximation, the lowest order correction was of 3$^{rd}$ order. E1-E2 excitation contributes already in second order. Nevertheless, it is smaller
than the third order E1-E1-E1 correction. For low $\xi$ we can estimate the ratio of the two amplitudes as

\[
\frac{\text{E1-E2 amplitude}}{\text{E1-E1-E1 amplitude}} \approx \frac{1}{Z_{1}^{2}} \frac{v}{Z_{2}^{2}} \frac{Z_{eff}^{(2)}}{Z_{eff}^{(1)^{2}}} \approx 0.13 \tag{19}
\]

for $Z = 82$, $Z_{eff}^{(2)} = 4/121$, and $Z_{eff}^{(1)} = 4/11$ at the GANIL energy. Thus the dipole approximation is reasonable, at least for a first exploration.

Possible higher order effects in the $^{32}\text{Mg}$ intermediate energy Coulomb Excitation \cite{4} can also be estimated. Assuming a rigid rotor model, high energy Coulomb excitation was calculated in the sudden approximation in Ref. \cite{10}. The characteristic strength parameter is

\[
C = \frac{Ze^{2}Q_{0}}{\hbar v 2b^{2}}. \tag{20}
\]

Using the $B(E2)$-value found in Ref. \cite{2} we have (see eq. 6 of Ref. \cite{10}, where a factor $\pi$ is missing on the rhs.)

\[
Q_{0}^{2} = \frac{16\pi}{5e^{2}} B(E2, 0^{+} \rightarrow 2^{+}) \tag{21}
\]

and

\[
C \approx 0.37 \tag{22}
\]

with $b = 15$ fm for an energy of $49.2 \cdot A$ MeV. From Fig. 1a of Ref. \cite{10} it can be seen that higher order effects are negligible for the value of the strength parameter $C$. This is in agreement with the result found in the coupled channel calculation of Ref. \cite{2}.
In conclusion, we provide a framework to apply corrections to 1st order electromagnetic excitation. It is appropriate for fast collisions. We constructed operators which take the influence of intermediate states into account. This can lead to a great simplification as compared, e.g., to the coupled channels approach. In this approach, a set of states, considered to be relevant, has to be chosen with known electromagnetic matrix-elements. In the present approach, of course, the model dependence cannot be altogether avoided; it enters when the corresponding matrix-elements of the operator has to be calculated, or at least estimated.

As an example we studied the excitation of the $\frac{1}{2}^-$ state in $^{11}$Be. The importance of higher order effects for this case of an extremely loosely bound nucleus was established. The estimate for the reduction of the cross section can only partly explain the observed reduction of the B(E1)-value in the GANIL experiment. However, for a final analysis, more accurate calculations with improved wave functions, including E2 and nuclear effects in the excitation, should be performed.

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Note added in revision: In the meantime we got to know about a coupled channel study of the $^{11}$Be Coulomb excitation by C. A. Bertulani, L. F. Canto, and M. S. Hussein. They get very similar conclusions as compared to our findings.
References

[1] see e.g. G. Baur and S. Typel, in: Proceedings of the 6th international conference on clusters in nuclear structure and dynamics, 6 – 9 Sept. 1994, ed. by F. Haas, (Strasbourg, 1995) and further references given there.

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Figure Captions

**Figure 1:** Reduction factor $r$ for the excitation probability as a function of the characteristic parameter $z$ defined in Eq. [14] for the two model wave functions (I: $-\cdash-\cdash$, II: $-\cdash-\cdash$).

**Figure 2:** Reduction $R$ of the total cross section for the excitation of the $\frac{1}{2}^{-}$ first excited state from the $\frac{1}{2}^{+}$ ground state of $^{11}$Be as a function of the projectile velocity $v$ for the two model wave functions (I: $-\cdash-\cdash$, II: $-\cdash-\cdash$).
Figure 2

The diagram shows the relationship between $R$ and $v/c$. The graph depicts three curves, each representing a different scenario or parameter. The x-axis represents $v/c$, ranging from 0.1 to 1, while the y-axis represents $R$, ranging from 0.4 to 1. The curves illustrates how $R$ increases as $v/c$ increases, approaching an asymptote near $R = 1$.

[Graphical representation of Figure 2 with axes labeled as $R$ for y-axis and $v/c$ for x-axis.]

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**Note:** The specific equations or data points used to generate this graph are not mentioned in the image, and the interpretation is based solely on the visual representation provided.