Accelerated nonlinear discriminant analysis

Nikolaos Gkalelis and Vasileios Mezaris

1 Informatics and Telematics Institute, CERTH, 6th Km Charilaou-Thermi Road, P.O. BOX 60361, Thermi 57001, Greece, Tel. ++ 30 2311257774, Fax ++ 30 2310474128, email: {gkalelis, bmezaris}@iti.gr

TABLE I

| Nomenclature | Description |
|--------------|-------------|
| N−1; N+1 | non-zero natural numbers: \{1, \ldots, a\}, a ∈ N* |
| i, j, l, m | class counters; subclass counters; observation counters |
| x ∈ R^F | observation in F-dimensional real vector measurement space |
| X^i | n-th training observation of i-th class |
| X^i,j | n-th training observation of j-th subclass in class i |
| m, m_i, j_i | estimated total sample mean |
| N; N_i; N_i,j | total number of training observations; observations in i-th class; observations in j-th subclass of class i estimated prior of i-th class and (i,j) subclass |
| p_i, p_i,j | vector with all elements equal to L−1 |
| n_L ∈ R^L | vector with all elements equal to zero |
| L ∈ R^L | vector with all elements equal to one |
| J_{L×L} ∈ R^{L×L} | matrix with all elements equal to one |
| X_i ∈ R^{F×N_i} | matrix of observations in class i |
| X_i,j ∈ R^{F×N_i,j} | matrix of observations in subclass (i,j) |
| S_i ∈ R^{F×F} | block matrix of all training observations |
| S_{w} ∈ R^{F×F} | total scatter matrix |
| S_{b} ∈ R^{F×F} | within-class scatter matrix |
| S_{wb} ∈ R^{F×F} | between-class scatter matrix |
| B_i ∈ R^{N_i×N_i} | matrix satisfying S_i = XB_iX^T |
| B_w ∈ R^{N×N} | matrix satisfying S_w = XB_wX^T |
| B_b ∈ R^{N×N} | matrix satisfying S_b = XB_bX^T |
| B_{w, b} ∈ R^{N×N} | matrix satisfying S_{wb} = XB_{wb}X^T |
| [B]_{i, j, n, k, l, m} | element of matrix B corresponding to x^{n}_{i,j}, x^{m}_{k,l} |
| N(B) | null space of matrix B |

Abstract—In this paper, a novel nonlinear discriminant analysis is proposed. Experimental results show that the new method provides state of the art performance when combined with LSVM in terms of training time and accuracy.

Index Terms—Feature extraction, discriminant analysis, kernels, support vector machines, generalized eigenproblems.

I. INTRODUCTION

Nonlinear discriminant analysis (NDA) using kernel functions is a fundamental dimensionality reduction (DR) technique with applications in several domains such as machine learning, information retrieval, pattern detection, visualization and other. The NDA optimization problem is reformulated as generalized eigenproblem (GEP). The mathematical treatment of GEP is one of the most aesthetically pleasing problems in all numerical algebra. This fact contributed to the increasing interest of NDA-based techniques. The major advantage over linear DA (LDA) is that it captures data nonlinearities thus offering more informative feature representation in most real-world problems. Moreover, in comparison to other popular component analysis (CA) techniques (e.g., PCA, ICA, LLE) it offers a much more compact representation by projecting the data to a very low dimensional space while preserving the clustering structure of the specific task.

For instance, in classification tasks the final classifier in the projection subspace deals more effectively with the curse of dimensionality, thus increasing the generalization performance and speed up the classifier. The latter is very important in big data problems. In visualization tasks it offers more informative representations preserving valuable information by the extreme dimensionality reduction capabilities even in two or three dimensions.

However, the computational cost of computing the eigen-pairs of NDA has prohibited this technology from the widespread use in today’s big data problems. Recently, a number of methods have been proposed to speed up NDA techniques. Here we provide a general framework accelerating the generalized eigenvalue decomposition of NDA approaches. Moreover, we show that the combination of the proposed method provides improved detection rates and computational efficiency when combined with LSVM, over the use of LSVM and K SVM alone.

II. FUNDAMENTALS OF NONLINEAR DISCRiminANT ANALYSIS

A training set in the feature space may be described using a block data matrix

$$\Phi = [X_1, \ldots, X_C] \in R^{L \times N}$$

(1)

Let the block matrix $X = [X_1, \ldots, X_C] \in R^{L \times N}$ represents an annotated training set of $C$ classes and $N$ observations $x_n$, $i \in N_C^*$, $n \in N_N^*$, in the input space $R^L$, where the $i$th block $X_i$, contains the $N_i$ observations $x_n$, $n \in Y_i \subseteq N_N^*$ of class $i$, and $Y_i$ is the respective set of indices. In many real-world applications, class distributions are not linearly separable and the direct application of a linear classifier in the input space may provide a poor performance. Kernel-based techniques address this problem by utilizing a vector-valued function $\phi(\cdot)$ to map observations non-linearly from the input space into some feature space $F \subseteq R^F$ where the data are expected to be linearly separable

$$\phi(\cdot) : R^L \rightarrow F \subseteq R^F,$$

(2)

$$x \rightarrow \phi = \phi(x).$$

(3)

1The curse of dimensionality problem states that phenomena described in high dimensional spaces require much more parameters to capture their properties.
The problem is then reformulated in terms of dot products, which in turn are replaced with kernel function evaluations
\[ \phi_{k,n}(x) = K(x_n, x_k) = k_{n,s}, \]
where \( k(\cdot, \cdot) \) is a Mercer kernel \([2]\). That is, we additionally require that \( \phi(\cdot) \) satisfies \([3]\). This allows the use of traditional linear solvers in problems where classes are nonlinearly separable in the input space and additionally the large or even infinite. Also, exploiting the kernel trick the mapping is intrinsic and the problem can be solved in the feature space without even knowing the actual mapping.

Assuming that the block matrix representing the training set in the feature space is
\[ \Phi = \left[ \Phi_1, \ldots, \Phi_C \right] \in \mathbb{R}^{F \times N}, \quad \Phi_i \in \mathbb{R}^{F \times N}, \]
nonlinear DA methods seek the linear transformation \( \Psi \) that simultaneously optimize the following criteria in the feature space
\[ \argmin_{\Psi} \text{tr}(\Psi^T \hat{S} \Psi), \quad \argmax_{\Psi} \text{tr}(\Psi^T S_b \Psi), \]
where \( S_b \) is the between-class scatter matrix and \( \hat{S} \) is either the within-class scatter matrix \( S_w \) or the total scatter matrix \( S_t \).

For the reformulation of \([6]\) using dot products we need an appropriate factorization of \( \Psi, S_b \) and \( \hat{S} \) (i.e. \( S_w \) or \( S_t \)). Starting from \( \Psi \) its solution space in \( \mathbb{R}^F \) is restricted to \( \text{span}(\Phi) \) \([3],[4]\). This allows us to express each column of \( G \) as a linear combination of the mapped training data
\[ \Psi = \Phi W \]
where \( W \in \mathbb{R}^{N \times D} \) contains the expansion coefficients. Substituting \([7]\) in \([6]\) the optimization criterion becomes
\[ \argmin_W \text{tr}(W^T \hat{S} W), \quad \argmax_W \text{tr}(W^T S_b W), \]
where
\[ \hat{S}_b = \Phi^T S_b \Phi, \]
\[ \hat{S}_w = \Phi^T S_w \Phi, \]
\[ \hat{S}_t = \Phi^T S_t \Phi, \]
and \( \hat{S} \) is replaced by \( S_w \) or \( S_t \). The above matrices can be entirely expressed in terms of dot products. By replacing dot products with kernel evaluations, \( \hat{S}_b, \hat{S}_w \) and \( \hat{S}_t \) (called hereafter kernel scatter matrices) can be viewed as the scatter matrices of the Gram matrix
\[ K = \Phi^T \Phi, \]
where each column in \( K \) is considered as a data point in \( \mathbb{R}^N \) \([2],[3]\).

The matrix \( \hat{S} \) is positive semidefinite (PSD) \([3]\) (as shown also in Sections \( ?? \) and \( ?? \)), and, thus, the optimal solution of \([6]\) is commonly approximated using pseudoinverse criteria \([3]\). That is, we compute the transformation matrix \( \Psi \in \mathbb{R}^{F \times D} \) that maximizes \( \hat{S} \)
\[ \argmax_W \text{tr}\left((W^T \hat{S} W)^+ W^T \hat{S}_b W\right), \]
Considering that all kernel scatter matrices are symmetric PSD (SPSD), the above optimization problem is equivalent to solving the symmetric-semidefinite generalized eigenproblem
\[ \hat{S}_b W = \hat{S} W \Lambda. \]
where \( \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_D), W = [w_1, \ldots, w_D] \in \mathbb{R}^{D \times D} \) and \( \lambda_i, w_i \) are the \( D \) nonzero eigenpairs (EP) of the SPSD matrix pencil \( (\hat{S}_b, \hat{S}) \) with eigenvalue and eigenvector sets \( \lambda(S_b, S) = \{ \lambda_i \in \mathbb{R}^+_* \mid \text{det}(\hat{S}_b - \lambda_i \hat{S}) = 0; \lambda_i > \lambda_j \text{ if } i > j \} \)
\[ g(S_b, S) = \{ w_i \in \mathbb{R}^D | (S_b w_i) = \lambda_i (S w_i) \}. \]

Simultaneous diagonalization techniques are used to solve the problem in case of both SPD or SPSD matrix \( \hat{S} \) as explained in the following. Firstly, we note that it can be easily shown that \( \hat{S} = \text{SPSD} \) and \([6],[7]\)
\[ \hat{S}_i = \hat{S}_b + \hat{S}_w, \]
\[ \text{null}(\hat{S}_i) = \text{null}(\hat{S}_b) \cap \text{null}(\hat{S}_w). \]
Then according to Theorem 8.7.1 in \([8]\) there exists a matrix \( W \) that simultaneously diagonalizes the above scatter matrices. It has been shown that the matrix optimizing the above criterion is given by
\[ W = \Gamma \Upsilon \]
where \( \Upsilon \in \mathbb{R}^{D \times D} \) is any nonsingular matrix and \( \Gamma \in \mathbb{R}^{F \times D} \) provides the following diagonalization
\[ \Gamma^T S_b \Gamma = \text{diag}(I_{r \times r}, D_b) \]
\[ \Gamma^T S_w \Gamma = \text{diag}(0_{r \times r}, D_w) \]
\[ \Gamma^T S_t \Gamma = I_{D \times D} \]
\[ D_b = \text{diag}(b_{r+1}, \ldots, b_{r+s}) \]
\[ D_w = \text{diag}(w_{r+1}, \ldots, w_{r+s}) \]
and, \( 1 > b_{r+1} \geq \ldots \geq b_{r+s} > 0, 0 < w_{r+1} \leq \ldots \leq w_{r+s} < 1, b_1 + w_1 = 1, r = r_b - r_w, s = D + r_w - r_b, D_w = \text{rank}(S_w) \), \( r_b = \text{rank}(S_b) \). Thus, the optimal transformation is provided by
\[ \Psi = \Phi \Gamma \Upsilon \]
Different constraints on the transformation matrix characterize a set of different algorithms
\[ \Psi^T S_w \Psi = W^T S_w W = \text{diag}(0_{r \times r}, I_{s \times s}) \]
\[ \Psi^T S_b \Psi = W^T S_b W = I_{D \times D} \]
\[ \Psi^T \Psi = W^T K W = I_{D \times D} \]
where \([24]\) is the conventional KDA. \([25]\) is the uncorrelated kernel discriminant analysis (KUDA) and \([26]\) is the orthogonal kernel discriminant analysis (OKDA). To this end, many authors exploit the special structure of the kernel scatter matrices and retrieve suitable factorization of them.

We should note that all the above methods (ULDA, OLDA, NLDA) are equivalent when the following property holds
\[ \text{rank}(S_t) = \text{rank}(S_w) + \text{rank}(S_b), \]
which is usually the case for high-dimensional feature vectors. Indeed when the above is valid \( r = r_b, s = 0, \)
\[ \Sigma_b = \text{diag}(I_{r_b \times r_b}, 0_{r \times r_b}) \]
\[ \Sigma_w = \text{diag}(0_{r_w \times r_w}, I_{r_b \times r_b}) \].
This explains why the above methods provide equivalent performance in experimental evaluations with high dimensional data.

During classification a test sample

\[ \mathbf{S} \leftarrow \mathbf{S} + \epsilon \mathbf{I}_{N \times N} \] (30)

where \( \epsilon \in \mathbb{R}^+ \) is a regularization constant; or remove the null space of \( \mathbf{S}_w \). Then, one way to identify the optimal transformation is to use the inverse of \( \mathbf{S} \) and identify the nonzero EPs of the pencil \( (\mathbf{S}^{-1} \mathbf{S}_b, \mathbf{I}) \). However, exploiting the inverse breaks the symmetry and definiteness of the pencil and thus standard unsymmetric methods are necessary, which are slower and more susceptible to round off errors in comparison to symmetric one [8]. Moreover, when \( \mathbf{S} = \mathbf{S}_w \), the null space of \( \mathbf{S}_w \) may contain significant discriminant information.

When \( \mathbf{S} \) is SPD then a common way to solve this problem is by using the Cholesky factorization and symmetric QR factorization (Algorithm 8.7.2 in [8]). Specifically, perform the Cholesky factorization of \( \mathbf{S} = \mathbf{L} \mathbf{L}^T \), use the symmetric QR decomposition of \( \mathbf{C} = \mathbf{L}^{-1} \mathbf{S}_b \mathbf{L}^{-T} \) to compute the matrix \( \mathbf{G} \) and set \( \mathbf{W} = \mathbf{L}^{-T} \mathbf{G} \). It can be easily shown that \( \mathbf{W} \) provides the desired simultaneous diagonalization

\[ \mathbf{W}^T \mathbf{S}_b \mathbf{W} = \mathbf{\Lambda}, \] (31)

\[ \mathbf{W}^T \mathbf{S} \mathbf{W} = \mathbf{I}, \] (32)

and \( \mathbf{W}^T \mathbf{W} = \mathbf{I} \).

The above techniques may have large round-off error and may be time consuming.

B. Factorization HH\(^T\)

In [10, 9], the scatter matrices in the feature space are factorized as

\[ \mathbf{S}_b = \mathbf{H}_b \mathbf{H}_b^T, \] (33)

\[ \mathbf{S}_w = \mathbf{H}_w \mathbf{H}_w^T, \] (34)

\[ \mathbf{S}_t = \mathbf{H}_t \mathbf{H}_t^T, \] (35)

where, \( \mathbf{H}_b \in \mathbb{R}^{F \times C} \), \( \mathbf{H}_w, \mathbf{H}_t \in \mathbb{R}^{F \times N} \).

Then, the problem can be reformulated using the following factorization

\[ \mathbf{S}_b = \mathbf{\Phi}^T \mathbf{S}_b \mathbf{\Phi} = \mathbf{K}_b \mathbf{K}_b^T, \] (36)

\[ \mathbf{S}_w = \mathbf{\Phi}^T \mathbf{S}_w \mathbf{\Phi} = \mathbf{K}_w \mathbf{K}_w^T, \] (37)

\[ \mathbf{S}_t = \mathbf{\Phi}^T \mathbf{S}_t \mathbf{\Phi} = \mathbf{K}_t \mathbf{K}_t^T, \] (38)

where,

\[ \mathbf{K}_b = \mathbf{H}_b \mathbf{\Phi}, \] (39)

\[ \mathbf{K}_w = \mathbf{H}_w \mathbf{\Phi}, \] (40)

\[ \mathbf{K}_t = \mathbf{H}_t \mathbf{\Phi}. \] (41)

Then, the GEP problem is reformulated as

\[ \mathbf{\tilde{K}}_b \mathbf{K}_b^T \mathbf{W} = \mathbf{\tilde{K}} \mathbf{\tilde{K}}^T \mathbf{W} \mathbf{\Lambda} \] (42)

where \( \mathbf{K} \) equals to \( \mathbf{K}_w \) or \( \mathbf{K}_t \). The above problem is connected with the computation of the generalized singular value decomposition (GSVD) of the matrix pair \( (\mathbf{K}_b^T, \mathbf{K}_b^T) \). That is the right generalized eigenvectors and the square root of the generalized singular values of the matrix pair \( (\mathbf{K}, \mathbf{K}_b^T) \) are the generalized eigenvectors and generalized eigenvalues of the matrix pencil \( (\mathbf{K}_b^T \mathbf{K}_b, \mathbf{K}_b^T \mathbf{K}) = (\mathbf{\tilde{S}}_b, \mathbf{\tilde{S}}) \) (Theorem 8.7.4 in [8]). This are computed using the complete orthogonal decomposition of \( [\mathbf{K}_w \mathbf{K}_t]^T \) and SVD of the derived orthogonal matrix \( \mathbf{H} \). The solution of the GEP problem in (42) can be also computed by using the SVD of \( \mathbf{K}_t \) [9].

Differently from the above methods, in [10, 11] the cross-product algorithm is exploited to exploit the above factorization and provide efficient solutions. Specifically, the EVD of \( \mathbf{H}_b^T \mathbf{H}_b \) is first computed to derive the EPs \( \mathbf{V}_b, \mathbf{\Lambda}_b \). Then, the EVD of \( \mathbf{U}_b^T \mathbf{S}_w \mathbf{U}_b \) is computed to derive the eigenvector matrix \( \mathbf{P} \), where \( \mathbf{U}_b = \mathbf{V}_b \mathbf{\Lambda}_b^{-1/2} \). The projection matrix \( \mathbf{Q} = \mathbf{U}_b \mathbf{P} \) we can obtain \( \mathbf{Q}^T \mathbf{S}_w \mathbf{Q} = \mathbf{\Lambda}_w \). Then the final projection matrix is defined as \( \mathbf{\Gamma} = \mathbf{Q} \mathbf{\Lambda}_w^{-1/2} \). Note that \( \mathbf{\Gamma}^T \mathbf{S}_w \mathbf{\Gamma} = \mathbf{I} \) and \( \mathbf{\Gamma}^T \mathbf{S}_t \mathbf{\Gamma} = \mathbf{\Lambda}_w^{-1} \). The disadvantage of this method is that \( \mathbf{\Lambda}_w \) may contain near zero or zero values and it is difficult to select the correct eigenvectors. This effect may be alleviated with regularization on the within and total scatter matrices. This will add additional time to retrieve the correct parameter. However, the major disadvantage is that the the cross-product algorithm in the first step is very susceptible to round off errors.

C. Factorization KBK

Generalized discriminant analysis (GDA) is based on the assumption that the data are centered in the feature space. Under this setting the between- and total-scatter matrices are factorized as follows

\[ \mathbf{\tilde{S}}_b = \mathbf{\tilde{K}} \mathbf{\tilde{B}}_b \mathbf{\tilde{K}}, \] (43)

\[ \mathbf{\tilde{S}}_t = \mathbf{\tilde{K}} \mathbf{\tilde{K}} \] (44)

where \( \mathbf{\tilde{B}}_b \in \mathbb{R}^{N \times N} \) is a symmetric block diagonal matrix in the form

\[ \mathbf{\tilde{B}}_b = \text{diag}(\mathbf{B}_{b_1}, \ldots, \mathbf{B}_{b_C}) \] (45)

\[ \mathbf{\tilde{B}}_{b_i} = \frac{1}{N_i} \mathbf{B}_{N_i \times N_i} \] (46)

GDA computes the SVD of \( \mathbf{\tilde{K}} \) and solves an equivalent EP.

To speed up the above computation in SRKDA first the following EVD is computed identifying the orthonormal matrix \( \mathbf{V} \in \mathbb{R}^{F \times (C-1)} \)

\[ \mathbf{\tilde{B}}_b \mathbf{V} = \mathbf{V}_b \mathbf{\Lambda}_b, \] (47)

Next, the transformation matrix \( \mathbf{G} \in \mathbb{R}^{F \times D} \) is identified by solving the following linear system for \( \mathbf{G} \)

\[ \mathbf{K} \mathbf{G} = \mathbf{V}_b, \] (48)

It can be easily shown that the computed transformation matrix has the required properties

\[ \mathbf{G}^T \mathbf{\tilde{S}}_b \mathbf{G} = \mathbf{\tilde{A}}_b, \] (49)

\[ \mathbf{G}^T \mathbf{\tilde{S}}_t \mathbf{G} = \mathbf{I}_{D \times D}, \] (50)
The eigenvectors of $\bar{B}b$, can be efficiently obtained by the inspection of this matrix. Particularly, observing that $1_{N_j}$ is the eigenvector of $\bar{B}_b$, a set of $C$ orthogonal eigenvectors $\bar{B}b$ is obtained as $V = [v_{1,1}, \ldots, v_{1,D}]$ where

$$v_{i,j} = \begin{cases} 
0_{N_j} & \text{if } i \neq j, \\
1_{N_j} & \text{if } i = j.
\end{cases}$$

(51)

with repeated eigenvalue $1$.

However, note that the vector $1_N$ is in the null space of the training vector set $\Phi$ and thus in the null space of the Gram matrix $\Phi 1_N = \Phi 1_N = 0$. Therefore, the all ones vector is the trivial solution of the linear matrix system and thus useless. Since 1 is a repeated eigenvalue, we pick $1_N$ as our first eigenvector and use the as any other $C$ orthogonal vectors in the space spanned by $\{v_c\}$ and the Gram-Schmidt process is used to orthogonalize the set of eigenvectors $\{v_c\}_{c = 1, \ldots, C}$. The derived vectors are then denoted as

$$\{v_c, c = 1, \ldots, C-1\} | v_c^T 1_N = 0, v_i^T \bar{v}_i = 1, v_i^T \bar{v}_j = 0, i \neq j \}$$

(52)

Note that the Gram-Schmidt process is applied to the initial set of eigenvectors $\{v_c\}$ and not in any random vector in $\mathbb{R}^N$.

Concerning the computation of the solution of the linear matrix system problem, this can be efficiently solved, e.g., using Cholesky factorization. Thus, the computational cost of identifying $V$ is negligible, and is extremely fast in comparison to the application of other nonlinear DA algorithms.

III. CONCLUSIONS

In this paper we have proposed an efficient generalized eigendecomposition framework and showed how it can be applied to accelerate fundamental NDA techniques. Experimental results showed that the combination of LSVM with the accelerated methods achieve state of the art performance in terms of both accuracy and training time performance.

REFERENCES

[1] K. Muller, S. Mika, G. Ratsch, S. Tsuda, and B. Scholkopf, “An introduction to kernel-based learning algorithms,” IEEE Trans. Neural Netw., vol. 12, no. 2, pp. 181–202, Mar. 2001.
[2] J. Mercer, “Functions of positive and negative type, and their connection with the theory of integral equations,” Philos. Trans. Roy. Soc., London, vol. 209, pp. 415–446, 1909.
[3] S. Mika, G. Rätsch, J. Weston, B. Schölkopf, A. Smola, and K.-R. Muller, “Invariant feature extraction and classification in kernel spaces,” in NIPS 1999, Denver, CO, USA, Jun. 2000, pp. 526–532.
[4] C. H. Park and H. Park, “Nonlinear discriminant analysis using kernel functions and the generalized singular value decomposition,” SIAM J. Matrix Analysis Applications, vol. 27, no. 1, pp. 87–102, 2005.
[5] Z. Zhang, G. Dai, C. Xu, and M. I. Jordan, “Regularized discriminant analysis, ridge regression and beyond,” J. Mach. Learn. Res., vol. 11, pp. 2199–2228, Aug. 2010.
[6] R. Huang, Q. Liu, H. Lu, and S. Ma, “Solving the small sample size problem of lda,” in IEEE Int. Conf. on Pattern Recognition, vol. 3, Quebec, Canada, Aug. 2002, pp. 29–32.
[7] J. Ye and T. Xiong, “Computational and theoretical analysis of null space and orthogonal linear discriminant analysis,” J. Mach. Learn. Res., vol. 7, pp. 1183–1204, Dec. 2006.
[8] G. H. Golub and C. F. V. Loan, Matrix Computations, 4th ed. Baltimore, MD, USA: The Johns Hopkins University Press, 2013.
[9] T. Xiong, J. Ye, and V. Cherkassky, “Kernel uncorrelated and orthogonal discriminant analysis: A unified approach,” in CVPR 2006, vol. 1, New York, NY, USA, Jun. 2006, pp. 125–131.
[10] J. Lu, K. N. Plataniotis, and A. N. Venetsanopoulos, “Face recognition using kernel direct discriminant analysis algorithms,” IEEE Trans. Neural Netw., vol. 14, no. 1, pp. 117–126, Jan. 2003.
[11] J. Lu, K. Plataniotis, A. Venetsanopoulos, and J. Wang, “An efficient kernel discriminant analysis method,” Pattern Recognit., vol. 38, no. 10, pp. 1788–1790, Oct. 2005.