Capacitated set-covering model considering the distance objective and dependency of alternative facilities

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Abstract. We propose a new model of facility location to solve a kind of problem that belong to a class of set-covering problem using an integer programming formulation. Our model contains a single objective function, but it represents two goals. The first is to minimize the number of facilities, and the other is to minimize the total distance of customers to facilities. The first goal is a mandatory goal, and the second is an improvement goal that is very useful when alternate optimum solutions for the first goal exist. We use a big number as a weight on the first goal to force the solution algorithm to give first priority to the first goal. Besides considering capacity constraints, our model accommodates a kind of either-or constraints representing facilities dependency. The either-or constraints will prevent the solution algorithm to select two or more facilities from the same set of facility with mutually exclusive properties.

A real location selection problem to locate a set of wastewater treatment facility (IPAL) in Surakarta city, Indonesia, will describe the implementation of our model. A numerical example is given using the data of that real problem.

Keywords: set-covering, integer programming, either-or constraint, wastewater treatment facility, distance goal

1. Introduction

The location model basically is a model of the relationship between the point of demand and the location point of the service facility. The decision variable in the location model is generally to determine where the optimal locations for service facilities are built. The assumptions and objective functions of the location model vary according to the characteristics of the related problem. Daskin [1] classified the location models based on mathematical modelling (gravity model) into 4 large groups, that is, analytic, continuous, network, and discrete models.

The network models and the discrete models, both assume that the facility locations and demand points are both discrete, that is, they are only present at certain points within the area. The network model assumes a network or path connecting the demand points with facility location points while the discrete model does not require such assumptions.

One kind of the most widely known discrete location models is the set-covering model. The decision variable for the set-covering model is where the optimal locations for service facilities are built so that the objective function is achieved. The objective function of the set covering model is to minimize the cost of the facility location such that a specified level of coverage is obtained. Popular special case of set-covering models is to minimize the number of service facility location points but all demand points are served. In other words, where to build the facilities such that all customers are
served with minimum cost is the common goal of the set covering model. Because of its discrete nature, most set-covering models are formulated using integer programming [2, 3].

The following is the integer linear programming model of a basic set-covering problem [4]:

Minimize \( \sum_{j} c_j X_j \)  
Subject to: \( \sum_{j \in N_i} X_j \geq 1 \) \( \forall i \),  
\( X_j \in \{0,1\} \) \( \forall j \).

Where,
- \( c_j \) = construction cost of facility at point \( j \)
- \( S \) = coverage radius of the facility
- \( N_i \) = a set of facility location \( j \) that can serve demand point \( i \),  
  \[ N_i = \{ j | d_{ij} \leq S \} \] \( \forall i \),  
- \( d_{ij} \) = the distance from demand point \( i \) to facility location \( j \)
- \( X \) = \[ \begin{cases} 1, \text{if point } j \text{ is selected as a facility location} \\ 0, \text{otherwise} \end{cases} \]

The above model, equation (1)-(4), explains that there is only one type of demand spread across multiple locations of point and also only one type of facility that can be placed at some points \( j \) to serve the demand. Equation (1) is the goal of the model, which is to minimize the facility location cost. Equation (2) assures that all demand points are served by at least one facility. Equation (3) is a logical binary constraint. Equation (4) explains the definition of coverage. However, the above standard model is not suitable to be applied in the case of determining the location of IPAL facilities. The model does not include the capacity and facilities dependency constraints that our model considers. In addition, this standard model does not consider the distance objective in its objective function.

A number of related works have been done to develop or modify the standard set-covering problem (SCP) model so that it's suitable for solving different cases. Rajagopalan et al. [5] developed an SCP model applied to emergency medical service (EMS) by considering demand fluctuations at certain time intervals. Suletra et al. [6] proposed a new constraint in the SCP model to represent the Indonesian government rule that each facility is used by at least two providers in the case of cellular telecommunication. The model is used to optimize the location of the joint base station. Karimi and Bashiri [7] presented a numerical example of a hub of airport location based on the hub covering location model developed in the study. Yaghini et al. [8] proposed a heuristic algorithm to solve a SCP model applied to train driver scheduling problems. Furthermore, Lutter et al. [9] and Zhang et al. [10] developed a new constraint in the SCP model to account an uncertain availability of facility applied to the case of emergency service facilities.

Although the research on SCP has been extensively discussed in the literature, there is no model suitable for solving the problem discussed in this study. The goal of the problem is to minimize the number of facilities and the total travel distance in the condition that the capacity of each facility is limited and there is the dependency property among facilities.

2. Proposed set covering model
The model proposed in this study is an integer programming model that refers to SCP. The basic model of SCP requires that each demand point is served by at least one facility (supply point). In this study, the requirement is rather different, each IKM (Industri Kecil dan Menengah) or SME’s (Small and Medium Enterprises) represented by a demand point must be served by only one IPAL facility. The goal is to minimize the number of IPAL facility that can serve all demand points. Besides
minimizing the number of IPAL facility, the objective function is formulated to minimize the total distance between IKM and IPAL facility. A big number is used as a weight on the number of facility minimization goal to force the solution algorithm to give first priority to this goal. Therefore, minimization of the total distance is a second priority goal that is very useful when alternate optimum solutions for the main goal exist.

\[
\text{Minimize} \quad \sum_i cX_i + \sum_i \sum_j d_{ij}Y_{ij} \quad (5)
\]

Subject to:

\[
\sum_j Y_{ij} = 1 \quad \forall j, \quad (6)
\]

\[
Y_{ij} - X_i \leq 0 \quad \forall i, j, \quad (7)
\]

\[
\sum_j Y_{ij} \leq p_i \quad \forall i, \quad (8)
\]

\[
\sum_i X_i = 1 \quad \forall i \in M, \quad (9)
\]

\[
X_i \in \{0,1\} \quad \forall i, \quad (10)
\]

\[
Y_{ij} \in \{0,1\} \quad \forall i, j. \quad (11)
\]

where,

- \(c\) = a big number to prioritize the first goal
- \(d_{ij}\) = the distance of IPAL \(i\) to IKM \(j\)
- \(l_j\) = the volume of wastewater produced by IKM \(j\)
- \(p_i\) = the capacity of IPAL \(i\)
- \(M\) = a set of IPAL with mutually exclusive selection (dependency property)
- \(X_i\) = biner number, 1 if alternative IPAL \(i\) is selected to build, and 0 is otherwise
- \(Y_{ij}\) = biner number, 1 if alternative IPAL \(i\) to serve IKM \(j\), and 0 is otherwise

The first term of the equation (5) is the first goal of the proposed model which minimizes the number of facilities, and the second term in the same equation is the second goal which minimize the total distance from IPAL \(i\) to IKM \(j\) in the condition that the first goal must be achieved first. A big number, \(c\), is used to force the model to assign the priority to the first goal. The second goal is an improvement goal. It will work when the alternate optimum solutions for the first goal exist. Equation (6) is used to model the requirement that each IKM must be served by only one IPAL facility. Equation (7) is a logical constraint to ensure that if the IPAL \(i\) is allocated to serve the IKM \(j\) then the IPAL \(i\) must be selected to build. Equation (8) explains that the capacity of each IPAL cannot be violated, i.e. the total waste distributed to the IPAL \(i\) from all IKM is less than the capacity of IPAL \(i\). Equation (9) describes the mutually exclusive selection representing the dependency property among IPAL. Equation (10) and (11) are logical binary constraints.

3. Numerical implementation

To describe the computation and the applicability of the proposed model, we use the data from a real problem, that is, location selection of IPAL facilities in Mojosongo, Surakarta, Indonesia (figure 1). The facilities serve the surrounding SME’s tahu tempe. An IKM is a demand point and the IPAL is the facility that serves the demand points. Having observed in site, there are 41 demand points \((j=1,2,\ldots,41)\) and 7 alternative IPAL locations \((i=1,2,\ldots,7)\). The map of IKM points and IPAL points are depicted in figure 1.
The first goal is to minimize the number of IPAL facilities and the second is to minimize the total distance of IKM to IPAL facilities. The constraints are capacity of IPAL, each IKM must be served only by one IPAL facility (and all IKM must be served), and facilities dependency constraints. Appendix A shows the distance measure from IKM to IPAL alternatives and appendix B shows the volume of wastewater produced by each IKM.

```
[1] model: 
[2] sets: 
[3]      ipal/S1..S7/:kapasitas,dipilihtidak; 
[4]      ikm/C1..C41/:volumelimbah; ... 
[21]tempe.xls','kapasitas','volumelimbah','jaraktempuh'); 
[22]enddata 
[23]end
```

Figure 1. Dispersion of 41 IKM locations and 7 IPAL alternatives.

Figure 2. Lingo code implemented in the problem
Branch-and-Bound algorithm is used to implement the proposed model to the real problem of IPAL at Mojosongo, Surakarta. Lingo software on a notebook i5 2.6GHz with 8Gb Ram and SSD 256Gb run the model in 5 second to get the global optimum solution. The code listed in figure 2 is the lingo code implemented to solve this problem.

The link between the proposed model and the lingo code can be explained as follows. \( Y_{ij} \) is represented by the code ‘alokasi’, \( X_i \) by the code ‘dipilihtidak’, \( c \) by the number 10000 (big number), \( d_{ij} \) by the code ‘jaraktempuh’, \( p_i \) by the code ‘kapasitas’, \( l_j \) by the code ‘volumelimbah’. The objective function of the proposed model, equation (5), is represented by the row [7] and [8] of the lingo code list. The first constraint of the proposed model, equation (6), is represented by the row [9] and [10], the second constraint or equation (7) by the row [13] and [14], the third constraint or equation (8) by the row [11] and [12], the fourth constraint or equation (9) by the row [15] and [16], the fifth constraint or equation (10) by the row [18], and the sixth constraint or equation (11) by the row [17]. The row [20] and [21] show that the data of \( p_i, l_j, \) and \( d_{ij} \), which are imported from MS Excel file using the Lingo function @OLE.

The optimum solution is depicted by figure 3 and the size of the problem is shown by the figure 3. Branch-and-Bound algorithm needs 169,273 iterations to get the global optimal solution in 5 seconds. Four IPAL facilities are selected to serve 41 IKM, i.e. IPAL 2, IPAL 3, IPAL 5, and IPAL 7. The optimum allocation is as follows. IPAL 2 serves 11 IKM, i.e. IKM 1, IKM 2, IKM 3, IKM 4, IKM 5, IKM 6, IKM 8, IKM 9, IKM 10, IKM 11, and IKM 12. IPAL 3 serves 11 IKM, i.e. IKM 7, IKM 14, IKM 17, IKM 20, IKM 22, IKM 23, IKM 24, IKM 25, IKM 28, IKM 34, and IKM 35. IPAL 5 serves 13 IKM, i.e. IKM 13, IKM 16, IKM 18, IKM 19, IKM 21, IKM 26, IKM 27, IKM 29, IKM 30, IKM 31, IKM 32, IKM 33, and IKM 36. IPAL 7 serves 6 IKM, i.e. IKM 15, IKM 37, IKM 38, IKM 39, IKM 40, and IKM 41.

![Figure 3](#)

**Figure 3.** Summary of problem and Lingo output.
Figure 4. Optimum allocation of 41 IKMs to 4 selected IPAL alternatives.

4. Concluding remark
We propose a kind of set-covering model to minimize two different goals using the single objective function. The problem is formulated as a single objective linear integer programming. The standard set covering model is a special case of our model by setting it to a single goal of minimizing the number of facilities and removing the capacity and facilities dependency constraints. We impose several assumptions about the alternative facilities and demand points. We use Branch-and-Bound Algorithms to solve the integer model and it needs just 5 second to solve the numerical example. The future research can look into accommodating a multiple objective approach to represent a more realistic problem, i.e. a problem considering objective and subjective criteria simultaneously. Further, the relaxation of assumptions about the facility and demand points may also be interesting to be studied.
### Appendix A. Distance between IKM and IPAL alternatives (in meters).

|       | IPAL1 | IPAL2 | IPAL3 | IPAL4 | IPAL5 | IPAL6 | IPAL7 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| IKM1  | 281   | 233   | 651   | 820   | 652   | 950   | 956   |
| IKM2  | 79    | 31    | 989   | 455   | 659   | 555   | 899   |
| IKM3  | 233   | 156   | 570   | 453   | 705   | 553   | 763   |
| IKM4  | 218   | 141   | 697   | 922   | 766   | 684   | 591   |
| IKM5  | 197   | 120   | 681   | 439   | 490   | 786   | 864   |
| IKM6  | 65    | 14    | 699   | 605   | 784   | 785   | 697   |
| IKM7  | 553   | 601   | 50    | 479   | 667   | 610   | 607   |
| IKM8  | 141   | 93    | 707   | 879   | 830   | 680   | 978   |
| IKM9  | 5     | 79    | 434   | 537   | 502   | 514   | 415   |
| IKM10 | 241   | 164   | 418   | 906   | 655   | 691   | 682   |
| IKM11 | 308   | 260   | 435   | 611   | 430   | 644   | 463   |
| IKM12 | 176   | 128   | 763   | 704   | 817   | 795   | 579   |
| IKM13 | 610   | 926   | 491   | 63    | 30    | 555   | 446   |
| IKM14 | 960   | 966   | 61    | 611   | 491   | 656   | 879   |
| IKM15 | 647   | 418   | 851   | 897   | 849   | 15    | 70    |
| IKM16 | 514   | 682   | 572   | 130   | 92    | 717   | 448   |
| IKM17 | 748   | 887   | 37    | 409   | 499   | 958   | 665   |
| IKM18 | 665   | 559   | 817   | 75    | 57    | 992   | 659   |
| IKM19 | 536   | 490   | 721   | 45    | 75    | 599   | 483   |
| IKM20 | 654   | 758   | 68    | 623   | 942   | 646   | 804   |
| IKM21 | 489   | 914   | 754   | 60    | 76    | 630   | 762   |
| IKM22 | 577   | 894   | 105   | 516   | 792   | 522   | 867   |
| IKM23 | 964   | 411   | 52    | 747   | 700   | 454   | 766   |
| IKM24 | 666   | 493   | 54    | 890   | 512   | 827   | 502   |
| IKM25 | 890   | 428   | 61    | 957   | 741   | 641   | 909   |
| IKM26 | 647   | 891   | 898   | 50    | 56    | 828   | 705   |
| IKM27 | 660   | 923   | 506   | 40    | 70    | 979   | 629   |
| IKM28 | 896   | 420   | 45    | 671   | 420   | 533   | 852   |
| IKM29 | 830   | 626   | 449   | 47    | 77    | 895   | 578   |
| IKM30 | 430   | 861   | 872   | 28    | 35    | 614   | 822   |
| IKM31 | 634   | 411   | 669   | 34    | 10    | 620   | 470   |
| IKM32 | 810   | 760   | 536   | 103   | 85    | 975   | 604   |
| IKM33 | 893   | 611   | 414   | 55    | 15    | 657   | 818   |
| IKM34 | 494   | 969   | 36    | 621   | 543   | 944   | 880   |
| IKM35 | 419   | 427   | 61    | 731   | 495   | 545   | 597   |
| IKM36 | 622   | 751   | 624   | 51    | 33    | 483   | 572   |
| IKM37 | 913   | 609   | 429   | 594   | 784   | 305   | 120   |
| IKM38 | 982   | 552   | 670   | 811   | 615   | 250   | 65    |
| IKM39 | 965   | 420   | 651   | 533   | 412   | 80    | 20    |
| IKM40 | 936   | 904   | 845   | 529   | 742   | 203   | 10    |
| IKM41 | 575   | 478   | 994   | 876   | 756   | 219   | 188   |
## Appendix B. Wastewater produced by IKM

| IKM  | Processed soy bean (Kg/day) | Daily Waste (litres) | Waste for 3 days dwell time (litres) |
|------|---------------------------|---------------------|-------------------------------------|
| 1    | 100                       | 950                 | 2,850                               |
| 2    | 200                       | 1,900               | 5,700                               |
| 3    | 250                       | 2,375               | 7,125                               |
| 4    | 40                        | 380                 | 1,140                               |
| 5    | 300                       | 2,850               | 8,550                               |
| 6    | 100                       | 950                 | 2,850                               |
| 7    | 200                       | 1,900               | 5,700                               |
| 8    | 25                        | 238                 | 713                                 |
| 9    | 100                       | 950                 | 2,850                               |
| 10   | 100                       | 950                 | 2,850                               |
| 11   | 50                        | 475                 | 1,425                               |
| 12   | 60                        | 570                 | 1,710                               |
| 13   | 150                       | 1,425               | 4,275                               |
| 14   | 150                       | 1,425               | 4,275                               |
| 15   | 90                        | 855                 | 2,565                               |
| 16   | 70                        | 665                 | 1,995                               |
| 17   | 50                        | 475                 | 1,425                               |
| 18   | 50                        | 475                 | 1,425                               |
| 19   | 150                       | 1,425               | 4,275                               |
| 20   | 50                        | 475                 | 1,425                               |
| 21   | 125                       | 1,188               | 3,563                               |
| 22   | 40                        | 380                 | 1,140                               |
| 23   | 125                       | 1,188               | 3,563                               |
| 24   | 50                        | 475                 | 1,425                               |
| 25   | 200                       | 1,900               | 5,700                               |
| 26   | 100                       | 950                 | 2,850                               |
| 27   | 400                       | 3,800               | 11,400                              |
| 28   | 100                       | 950                 | 2,850                               |
| 29   | 150                       | 1,425               | 4,275                               |
| 30   | 200                       | 1,900               | 5,700                               |
| 31   | 500                       | 4,750               | 14,250                              |
| 32   | 100                       | 950                 | 2,850                               |
| 33   | 50                        | 475                 | 1,425                               |
| 34   | 100                       | 950                 | 2,850                               |
| 35   | 500                       | 4,750               | 14,250                              |
| 36   | 100                       | 950                 | 2,850                               |
| 37   | 250                       | 2,375               | 7,125                               |
| 38   | 100                       | 950                 | 2,850                               |
| 39   | 80                        | 760                 | 2,280                               |
| 40   | 50                        | 475                 | 1,425                               |
| 41   | 50                        | 475                 | 1,425                               |
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