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Haar wavelet collocation approach for the solution of fractional order COVID-19 model using Caputo derivative

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Abstract This article is devoted to study a compartmental mathematical model for the transmission dynamics of the novel Coronavirus-19 under Caputo fractional order derivative. By using fixed point theory of Schauder’s and Banach we establish some necessary conditions for existence of at least one solution to model under investigation and its uniqueness. After the existence a general numerical algorithm based on Haar collocation method is established to compute the approximate solution of the model. Using some real data we simulate the results for various fractional order using Matlab to reveal the transmission dynamics of the current disease due to Coronavirus-19 through graphs.

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1. Introduction

In last few months a threatening outbreak called novel Coronavirus-2019 disease (nCOVID-19) has attacked the whole globe. The researchers claim that the aforementioned virus was initiated in Wuhan city of China, (see[1]) at the end of 2019. The researchers believe that the concerned virus was transmitted from animal to human, because majority of infected cases occurred had been to local wild and fish market of Wuhan city in November 2019 according to a source (see[2]). Immediately, after that it was confirmed by researchers that the concerned viruses also transmitted from human to human too, ([3]). According to the latest report of WHO, nearly 6 million confirmed cases was registered in laboratories all over the world including 3.5 million deaths up to mid of May, 2020. In some countries like USA, Spain, Italy, the ratio of death rate is...
higher than 0.086. From aforementioned data, we can say that nCOVID-19 has highly infective and dangerous disease. The common symptoms of nCOVID-19 are cough, sick, high fever and breathing difficulty. The main transmission sources of the virus are droplets from nose releasing during sneezes and coughing of an infected person. A person will be exposed in the chances of getting infection, if he breathes the droplets from infected person.

In this regard, the best tools to prevent the spreading of disease in community is to avoid social distances among the humans. For this purpose, government of all of the affected countries and territories taken some precautionary measures and implemented lockdown the effected region of their respective areas/cities. They closed the markets, educational institutions, cut off the transportation systems, such as public and private transport, airplane and trains. Although, these precautions are taken for the benefits of mankind, but on the other side these precautions badly affected the communities around the whole world. Its produces a great deals of disruption and uncertainty. Also the same economical systems of many countries were nearly to collapsed. The spreading of disease is a biggest challenge for human society and economy. The proper understanding of dynamical behaviors could play a key role to prevent the spreading and elimination of disease from community. In this connection, the researchers paid a keen interest to control the spreading of diseases further and implement their own measure of precautions, (see [4–6]).

A powerful tools that described the real world situation in mathematical concept and terminology is known as mathematical modeling. The different aspects for the majority of biological and general dynamics are well described via aforementioned techniques of mathematics. In this regards, the researchers use the tools of mathematical modeling to study the transmission and make further plan to prevent the mankind form the effects of mentioned infectious disease. In this regards, many researchers developed different mathematical models for the current nCOVID-19, for detail see ([6–9]). From recent research work, it was observed that proper understanding and implementation for the control strategies against the transmission of spreading disease in the community is the biggest challenge for mankind. However, to some extend the aforementioned techniques play a key role to plan, prevent and eliminate the viral diseases form the community. It is worth-mentioning that mostly biological and as well as general dynamic are investigated by the researchers via mathematical modeling. The aforesaid techniques enable them to control and handle the spread of diseases in society, (see [10–12]).

The nCOVID-19 and many other epidemic model in existence literature are actually model via conventional derivative. Which is local in nature and cannot describe the dynamics comprehensively. Therefore to investigate a detail dynamics fractional calculus play a key roles in this regards. Since there have been various definitions introduced by researchers including Caputo, Riemann–Liouville, Caputo-Fabrizo, etc. In last few years for mathematical modeling Caputo derivative has been frequently used because it can be deal easily like conventional derivative. Infact fractional derivatives have the ability to explain heredity property and memory effects with more detail. The aforementioned operators, gives the freedom of choose to the researchers to select the best one among them, that elaborate the current situation properly. The remarkable results were obtained by researchers to utilize these operators.

Due to great degree of freedom, reliability and accuracy the demand of use these operators rapidly increases recently [13,25]. Further calculus of arbitrary order has verities of applications in various fields of engineering and social science, such as dynamical systems, fluid flow, traffic flow, pattern origination, single and image processing, etc, (see [17–20]). In present decade, the researchers paid considerable attention to use the mentioned operators in modeling of biological and epidemic models, (see [4]). Due the verities of aforementioned advantages of the fractional differential operators, their use rapidly increase for modeling of complex real world situations as compared to the natural order derivatives, we refers [14–16].

Recently, the concerned area of research got much attention of researchers and they frequently use the fractional order integrals and derivatives. Researchers investigate different aspects of differential equations of fractional order (FODEs), such as qualitative analysis, optimization and numerical approximation, for detail study we refers ([21–24]). Majority of the problems involving nonlinearity are some time too complicated to solve analytically, therefore the researchers use various numerical techniques to established the solution of such problems, we refer [26–28]. Recently, the aforementioned numerical techniques were extended to investigate FODEs under the Caputo derivative, see [29–32,36–40].

According to current status of the outbreak of nCOVID-19, the researchers have presented different mathematical models, see [41,46–48]. Currently, the area devoted to the investigation of mathematical modeling of nCOVID-19 attended the attention of researchers. Therefore, the authors investigate the following model with three compartment including susceptible population $H(t)$, the infected population $I(t)$ and the recovered class $R(t)$ (including death due to corona or natural) at time $t$ as

$$
\frac{dH}{dt} = -aH(t) - CI(t)H(t)(1 + \gamma I(t)) - dH(t) + sR(t),
$$

$$
\frac{dI}{dt} = CI(t)H(t)(1 + \gamma I(t)) - (\mu + d + D - B)I(t),
$$

$$
\frac{dR}{dt} = \mu I(t) - (x + \alpha)R(t),
$$

where $\alpha, x$ denote the immigration rate of infected individuals and healthy individuals, respectively.

subject to initial conditions

$$
H(0) = H_0, \quad I(0) = I_0, \quad R(0) = R_0.
$$

The parameters involve in the model (1) are described as in Table 1 as

Some necessary assumption, we impose on the model are that: all the involve parameters in the model (1) are nonnegative. The above model is investigated from three different

| Table 1 Description of the parameters used in model (1). |
|---------------------------------------------------------|
| Parameters          | The physical interpretation               |
|---------------------|-------------------------------------------|
| $H(t)$              | Susceptible compartment in millions       |
| $I(t)$              | Infected compartment millions             |
| $R(t)$              | Recovered compartment in millions         |
| $a$                 | The immigration rate of healthy individuals |
| $d$                 | Represent natural death                    |
| $D$                 | Represent death due to corona             |
| $B$                 | The immigration rate of infected individuals |
| $\mu$               | Represent infected population goes to recovered |
| $C$                 | Proportionality constant                  |
| $\gamma$            | Rate at which an individuals lose immunity |
| $x$                 | Recovered rate                            |
Further the involve state functions of the model satisfy
the results. Finally the results are displayed against real data
rarely used in the case of Coronavirus-19 mathematical mod-

3. Qualitative analysis of the considered model

It is natural phenomena that before the biological analysis of
any model it needs to be investigated that whether the model
is well posed or not. In this contact, we are implemented the
tools of fixed point theory to investigate the solution of our
proposed problem. Further, we are expressed the right hand
side of the proposed problem (1) as
\[ F_1(t, H, I, R) = aH(t) - CI(t)H(t)\left(1 + \gamma I(t)\right) - \delta H(t) + zR(t), \]
\[ F_2(t, H, I, R) = CI(t)H(t)\left(1 + \gamma I(t)\right) - \left(\mu + d + D + b\right)I(t), \]
\[ F_3(t, H, I, R) = \mu I(t) - (\mu + d + D + b)R(t). \]

Assume that \( X = C\left([0, T] \times \mathbb{R}^3, \mathbb{R}\right), \) with \( 0 \leq t \leq T < \infty \) be the Banach spaces, such that
\[ \|\varphi\|_X = \sup_{t \in [0, T]}\|H(t)\| + \|I(t)\| + \|R(t)\|, \]
where
\[ \varphi(t) = \begin{bmatrix} H \\ I \\ R \end{bmatrix}(t), \quad \varphi_0 = \begin{bmatrix} H_0 \\ I_0 \\ R_0 \end{bmatrix}, \quad Y(t, \varphi(t)) = \begin{bmatrix} F_1(t, H, I, R) \\ F_2(t, H, I, R) \\ F_3(t, H, I, R) \end{bmatrix}(t). \]

(3)

With the help of (2), the proposed system (1) can be expressed in the form of
\[ ^{\gamma}D_t^\alpha \varphi(t) = Y(t, \varphi(t)), \quad t \in [0, T], \]
\[ \varphi(0) = \varphi_0. \]

In view of Lemma 2.3, Eq. (4) yields
\[ \varphi(t) = \varphi_0 + \int_{0}^{t} \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \dot{Y}(s, \varphi(s)) ds, \quad t \in [0, T] \]

(5)

We assumed that the following assumptions holds for the exis-
tence of proposed problem.

(H1) There exist constants \( g_Y \) and \( M_Y \), such that
\[ |Y(t, \varphi(t))| \leq g_Y|\varphi(t)| + M_Y, \quad \forall \varphi \in X. \]

(H2) There exist a constant \( h > 0 \), for every \( \varphi, \varphi^* \in X \) such that
\[ |Y(t, \varphi) - Y(t, \varphi^*)| \leq h|\varphi - \varphi^*|. \]

Further, we use Schauder fixed point theorem to check the
existence of the solution of the proposed system.

Theorem 3.1. Under the assumption \( \{H_1\} \) and continuity of
\( Y : [0, T] \times \mathbb{R}^3 \to \mathbb{R} \), the integral Eq. (5) has at least one
solution. Consequently the proposed model (1) has at least one
solution with \( \forall g_Y < 1, \) where \( \gamma = \frac{\alpha}{\Gamma(\alpha+1)}. \)
Proof. Assume, that \((H_1)\) holds and define
\[
\Lambda = \{\varphi(t) \subseteq X : ||\varphi||_X \leq \zeta, \ t \in [0, T]\},
\]
a closed convex subset of \(X\) and \(\zeta \geq \frac{3}{2} \frac{V}{1+2B}\).

Also defined an operator \(\mathcal{F} : \Lambda \rightarrow \Lambda, \ \forall \varphi \in \Lambda\) and \(||\varphi_0|| = v_0\).

Let
\[
|\mathcal{F}\varphi(t)| = |\varphi_0 + \frac{1}{\Gamma(r)} \int_0^t (t-s)^{r-1} Y(s, \varphi(s))ds|
\]
\[
\leq |\varphi_0| + \frac{1}{\Gamma(r)} \int_0^t (t-s)^{r-1} ||Y(s, \varphi(s))||ds, \ |\mathcal{F}\varphi(t)|
\]
\[
\leq v_0 + \frac{1}{\Gamma(r)} \int_0^t (t-s)^{r-1} ||Y(s, \varphi(s))||ds,
\]
\[
= v_0 + 6\eta_1 \zeta + 92 R_1,
\]
\[
\leq \zeta, \ \text{which implies that} \ |\mathcal{F}\varphi||_X \leq \zeta.
\]

Hence \(\mathcal{F}(\Lambda) \subseteq \Lambda\) and \(\mathcal{F}\) is continuous.

Now consider that \(t_1 < t_2 \in [0, T]\), we are going to show that \(\mathcal{F}\) is completely continuous operator. For this let
\[
|\mathcal{F}\varphi(t_2) - \mathcal{F}\varphi(t_1)| = \left| \left( \varphi_0 + \int_0^{t_2} \frac{(t-s)^{r-1}}{\Gamma(r)} Y(s, \varphi(s))ds \right) - \left( \varphi_0 + \int_0^{t_1} \frac{(t-s)^{r-1}}{\Gamma(r)} Y(s, \varphi(s))ds \right) \right|
\]
\[
= \left| \int_0^{t_2} \frac{(t-s)^{r-1}}{\Gamma(r)} Y(s, \varphi(s))ds - \int_0^{t_1} \frac{(t-s)^{r-1}}{\Gamma(r)} Y(s, \varphi(s))ds \right|
\]
\[
= \left| \int_0^{t_2} \frac{(t-s)^{r-1}}{\Gamma(r)} (\varphi_0 + 6\eta_1 \zeta + 92 R_1)(t_2 - s)^{r-1} \right|
\]
\[
\leq \frac{6\eta_1 \zeta + 92 R_1}{\Gamma(r+1)} (t_2 - t_1)^{r-1}.
\]

If \(t_2 \rightarrow t_1\), then right side of (6) tends to Zero, so one has
\[
||\mathcal{F}\varphi(t_2) - \mathcal{F}\varphi(t_1)||_X \rightarrow 0, \ \text{as} \ t_2 \rightarrow t_1.
\]

Thus \(\mathcal{F}\) is bounded and equi-continuous. Hence by Arzilá-Ascoli theorem \(\mathcal{F}\) is relatively complete. So \(\mathcal{F}\) is completely continuous. Thanks to Schauder fixed point theorem the concerned system (1) has at least one solution.

Theorem 3.2. If assumption \((H_2)\) holds and \(T^r U_Y < \Gamma(r+1)\), then the consider system has unique solution.

Proof. If \(\varphi, \varphi^* \in X\), where \(\mathcal{F} : X \rightarrow X\) be the operator previously defined. Consider that
\[
||\mathcal{F}\varphi - \mathcal{F}\varphi^*||_X = \max_{t \in [0, T]} \left| \int_0^t \frac{(t-s)^{r-1}}{\Gamma(r)} Y(s, \varphi(s))ds - \int_0^t \frac{(t-s)^{r-1}}{\Gamma(r)} Y(s, \varphi^*(s))ds \right|
\]
\[
\leq \max_{t \in [0, T]} \int_0^t \frac{(t-s)^{r-1}}{\Gamma(r)} ||Y(s, \varphi(s)) - Y(s, \varphi^*(s))||ds,
\]
\[
\leq \frac{T^r}{\Gamma(r+1)} U_Y ||\varphi - \varphi^*||_X.
\]

Thus the operator \(\mathcal{F}\) is continuous. Hence the system (1) has unique solution. □

4. Stability result for Model (1)

To obtain stability results for the proposed model, we recall some definitions from [51,52]. Let \(\mathcal{F} : X \rightarrow X\) be an operator satisfying
\[
\mathcal{F}\varphi = \varphi, \ \text{for} \ \varphi \in X.
\]

Definition 4.1. The Eq. (7) is Ulam-Hyers stable, if there exist \(\epsilon > 0\) and \(\varphi \in X\) be any solution, such that the following inequality holds:
\[
||\varphi - \mathcal{F}\varphi||_X \leq \epsilon, \ \text{for} \ t \in [0, T].
\]
\[
\exists \text{ at most one solution} \ \bar{\varphi} \text{ of (7) with} \ \epsilon > 0 \text{ satisfying}
\]
\[
||\varphi - \bar{\varphi}||_X \leq \epsilon, \ \text{for} \ t \in [0, T].
\]

Definition 4.2. Further, if \(\exists \varphi \in C([\alpha, \beta], \mathcal{Y})\ \text{with} \ \varphi(0) = 0, \ \text{for any solution} \ \varphi \text{ of (8) and} \ \bar{\varphi} \ \text{be at most one solution of (7) with}
\]
\[
||\varphi - \bar{\varphi}||_X \leq \epsilon, \ \text{for} \ t \in [0, T].
\]

then (7) is generalized Ulam-Hyers stable.

Remark 4.3. If \(\exists \xi(t) \in C([0, T], \mathcal{Y})\), then \(\varphi \in X\) satisfies (8) if
\(\text{ (i)} \ \xi(t) \leq \epsilon, \ \forall \ t \in [0, T].\)
\(\text{ (ii)} \ \mathcal{F}\varphi(t) = \bar{\varphi} + \xi(t), \ \forall \ t \in [0, T].\)

For further analysis, we need the following relation. Consider the corresponding perturb equation of problem (4) as
\[
\left\{ \begin{array}{l}
\int_0^t \frac{\varphi(s)}{\Gamma(r)}(-L_\varphi(s) + \zeta(t))
\end{array} \right.
\]
\[
\varphi(0) = \varphi_0.
\]

Lemma 4.4. The following result hold for (11).
\[
||\varphi - \mathcal{F}\varphi||_X \leq \zeta \epsilon, \ \text{where} \ \zeta = \frac{T^r}{\Gamma(r+1)}.
\]

Proof. This is the simple consequences of Lemma 2.3 and Remark 4.3. □

Theorem 4.5. Under the Lemma 4.4, the solution of the considered problem (4) is Ulam-Hyers stable and also generalized-Ulam-Hyers stable if \(\frac{T^r}{\Gamma(r+1)} < 1\).

Proof. Let \(\varphi \in X\) be any solution and \(\bar{\varphi} \in X\) be at most one solution of (4), then
\[
||\varphi(t) - \bar{\varphi}(t)|| \leq ||\varphi(t) - \mathcal{F}\bar{\varphi}(t)||
\]
\[
\leq ||\varphi(t) - \mathcal{F}\varphi(t)|| + ||\mathcal{F}\varphi(t) - \mathcal{F}\bar{\varphi}(t)||
\]
\[
\leq \zeta \epsilon + \frac{T^r}{\Gamma(r+1)} ||\varphi(t) - \bar{\varphi}(t)|)
\]
\[
\leq \frac{T^r}{1 - \frac{T^r}{\Gamma(r+1)}} \epsilon.
\]
Hence, problem (4) is Ulam-Hyers stable. Consequently it is generalized Ulam-Hyers stable. □

Definition 4.6. Eq. (7) is Ulam-Hyers-Rassias stable for $\phi \in C([0, T], \mathbb{H})$ if for $\epsilon > 0$ and let $\varphi \in X$ be any solution of inequality

$$\|\varphi - \varphi(t)\| \leq \varphi(t)\epsilon, \; \text{for} \; t \in [0, T].$$

(14)

Lemma 4.9. The following result hold for (11).

$$|\varphi(t) - \mathcal{F}(\varphi(t))| \leq \varphi(t)\epsilon, \; \text{where} = \frac{T^r}{\Gamma(r + 1)}.$$

(17)

Proof. One can easily get the required relation by using Lemma 2.3 and Remark 4.8.

Theorem 4.10. Under the Lemma 4.9, the solution of the considered problem (4) is Ulam-Hyers-Rassias stable and also generalized-Ulam-Hyers-Rassias stable if $\frac{T^r}{\Gamma(r + 1)} < 1$.

Proof. Let $\varphi \in X$ be any solution and $\tilde{\varphi} \in X$ be at most one solution of (4), then

$$|\varphi(t) - \tilde{\varphi}(t)| = |\varphi(t) - \mathcal{F}(\tilde{\varphi}(t))| \leq |\varphi(t) - \mathcal{F}(\varphi(t))| + |\mathcal{F}(\varphi(t)) - \mathcal{F}(\tilde{\varphi}(t))| \leq \varphi(t)\epsilon + \frac{T^r}{\Gamma(r + 1)}|\varphi(t) - \tilde{\varphi}(t)|$$

which gives $\|\varphi - \tilde{\varphi}\| \leq \frac{\varphi(t)\epsilon}{1 - \frac{T^r}{\Gamma(r + 1)}}$.

(18)

Hence, problem (4) is Ulam-Hyers-Rassias stable. Consequently it is generalized Ulam-Hyers-Rassias stable. □

5. Numerical analysis of the considered model

In this section, implementation of the Haar technique is discussed to find solution of the proposed model (1). The derivative of the unknown function in the nonlinear system is approximated using Haar functions and the expression for the unknown function is obtained by integration. By applying the collocation technique to these equations, we get a system of algebraic equations by putting the nodal points. These nonlinear equations is solved by using Broyden’s technique to find the unknown coefficients. At last, the approximate solution at nodal points is obtained using these unknown coefficients.

5.1. Numerical scheme

Consider $H(t), I(t)$ and $R(t)$ are in $L^2[0,1]$, space of square integrable functions and hence can be written as a Haar series as:

$$H(t) = \sum_{i=1}^{N} a_i h_i(t), \; I(t) = \sum_{i=1}^{N} b_i h_i(t) \; \text{and} \; R(t) = \sum_{i=1}^{N} c_i h_i(t),$$

(19)

where $H(0) = H_0$ initial susceptible compartment, $I(0) = I_0$ initial infected compartment and $R(0) = R_0$ initial recovered compartment. Integration yields the following relation.

$$H(t) = H_0 + \sum_{i=1}^{N} a_i p_{i1}(t),$$

$$I(t) = I_0 + \sum_{i=1}^{N} b_i p_{i1}(t), \; \text{and}$$

$$R(t) = R_0 + \sum_{i=1}^{N} c_i p_{i1}(t).$$

(20)

By using Caputo derivative, we have

$$\int_{(0-\epsilon)}^{1} \frac{1}{(\tau-\epsilon)^r} \int_{(0-\epsilon)}^{1} R^{(r)}(t) (t-\tau)^{r-1} dt = \mu I(t) - (\alpha + \delta) R(t).$$

As we have assumed that $0 < r < 1$, therefore $n = 1$, and we have

$$\int_{(0-\epsilon)}^{1} \frac{1}{(\tau-\epsilon)^r} \int_{(0-\epsilon)}^{1} R^{(r)}(t) (t-\tau)^{r-1} dt = \mu I(t) - (\alpha + \delta) R(t).$$

Applying Haar approximations, we have

$$\int_{(0-\epsilon)}^{1} \frac{1}{(\tau-\epsilon)^r} \int_{(0-\epsilon)}^{1} R^{(r)}(t) (t-\tau)^{r-1} dt =$$

$$a \left( H_0 + \sum_{i=1}^{N} a_i h_i(t) \right) - C \left( I_0 + \sum_{i=1}^{N} b_i p_{i1}(t) \right) \left( H_0 + \sum_{i=1}^{N} a_i p_{i1}(t) \right)\left( 1 + \frac{1}{\gamma} \left( I_0 + \sum_{i=1}^{N} b_i p_{i1}(t) \right) \right) - d \left( H_0 + \sum_{i=1}^{N} a_i p_{i1}(t) \right)$$

$$+ \alpha \left( R_0 + \sum_{i=1}^{N} c_i p_{i1}(t) \right)$$

$$= C \left( I_0 + \sum_{i=1}^{N} b_i p_{i1}(t) \right) \left( H_0 + \sum_{i=1}^{N} a_i p_{i1}(t) \right)$$
The integral in the above system are approximated using the Haar integration formula [43] as

\[
\int_{a}^{b} f(t) dt \approx \frac{b - a}{N} \sum_{k=1}^{N} f(t_k) = \sum_{k=1}^{N} \left( a + \frac{(b - a)(k - 0.5)}{N} \right).
\]

Upon simplification, we have

\[
\frac{1}{1 + \gamma a} \sum_{i=1}^{N} b_i(t_i)(t - \tau_i)^{-d} dt - C \left( I_0H_0 + I_0 \sum_{i=1}^{N} a_{p1}(t_i) + H_0 \sum_{i=1}^{N} b_{p1}(t_i) \right) + \gamma \left( I_0H_0 + I_0 \sum_{i=1}^{N} a_{p1}(t_i) + H_0 \sum_{i=1}^{N} b_{p1}(t_i) \right) + \gamma \left( I_0H_0 + I_0 \sum_{i=1}^{N} a_{p1}(t_i) + H_0 \sum_{i=1}^{N} b_{p1}(t_i) \right) + \gamma \left( I_0H_0 + I_0 \sum_{i=1}^{N} a_{p1}(t_i) + H_0 \sum_{i=1}^{N} b_{p1}(t_i) \right) = 0.
\]

1 \Gamma(1 - r) \int_{0}^{t} \sum_{i=1}^{N} c_i h_i(t_i)(t - \tau_i)^{-d} dt - \mu \left( I_0 + \sum_{i=1}^{N} b_{p1}(t_i) \right) + \gamma \left( I_0H_0 + I_0 \sum_{i=1}^{N} a_{p1}(t_i) + H_0 \sum_{i=1}^{N} b_{p1}(t_i) \right) = 0.

The integral in the above system are approximated using the Haar integration formula [43] as

\[
\int_{a}^{b} f(t) dt \approx \frac{b - a}{N} \sum_{k=1}^{N} f(t_k) = \sum_{k=1}^{N} \left( a + \frac{(b - a)(k - 0.5)}{N} \right).
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\]
Haar wavelet collocation approach

\[ F_{1j} = \frac{t_j}{\lambda(r+1)} \sum_{m=1}^{N} \sum_{i=1}^{N} a_{ji}(t_m)(t_j - t_m)^r - aH_0 - a \sum_{i=1}^{N} a_{ji}(t_j) \]
\[ + C(1 + \gamma I_0) \left( I_0H_0 + \sum_{i=1}^{N} a_{ji}(t_j) + H_0 \sum_{i=1}^{N} b_{ji}(t_j) \right) \]
\[ + \sum_{i=1}^{N} a_{ji}(t_j) \sum_{j=1}^{N} b_{ji}(t_j) + \gamma \sum_{i=1}^{N} a_{ji}(t_j) \left( \sum_{j=1}^{N} b_{ji}(t_j)^2 \right) + dH_0 \]
\[ - \sum_{i=1}^{N} b_{ji}(t_j) - \alpha R_0 - 2 \sum_{i=1}^{N} c_{ji}(t_j), F_{2j} \]
\[ - \frac{t_j}{\lambda(r+1)} \sum_{m=1}^{N} \sum_{i=1}^{N} c_{ji}(t_m)(t_j - t_m)^r - \mu \left( I_0 + \sum_{i=1}^{N} b_{ji}(t_j) \right) + (\alpha + d) \left( R_0 + \sum_{i=1}^{N} c_{ji}(t_j) \right) \].

The solution of this system gives the values of unknown coefficients \(a_i\)'s, \(b_i\)'s and \(c_i\)'s. The required solution \(H(t), I(t)\) and \(R(t)\) at nodal points is calculated by putting \(a_i\)'s, \(b_i\)'s and \(c_i\)'s in Eq. (20). Here we provide a formula for computation of the experimental rate of convergence \(r_c(N)\) which is describe as from [55]

\[ r_c(N) = \frac{1}{\log 2} \log \left[ \frac{\text{Maximum absolute error at } N}{\text{Maximum absolute error at } N/2} \right]. \]

5.2. Numerical results and simulation

Now we simulate the results corresponding to the given values of parameters as.

Case I: We simulate the results first for \(a = 0.00073, B = 0.078601\) that let immigration is allowed. First of all the results of existence, uniqueness given as Theorem 3.1, 3.2 clearly holds. Further also the stability results

\[ r_c(N) = \frac{1}{\log 2} \log \left[ \frac{\text{Maximum absolute error at } N}{\text{Maximum absolute error at } N/2} \right]. \]

Fig. 1 The plot shows the dynamics of susceptible class in model (1) at various values of fractional order \(r\) at \(N = 40\).
given in Theorem 4.5 and 4.10 are obvious from graphs corresponding to the given data of Table 2.

From Figs. 1–3, we see that as immigration’s of both infected and susceptible people is allowed, the population of susceptible people will decrease and consequently the infection will increase up. Also the death cases will increase and hence the recovered population will suffered. This fall and up is different at different fractional order. At lower order the decay is faster and vice versa. Similar at smaller order stability occurs afore than larger order.

**Case II:** Here in the absence of immigration that is $a = 0, B = 0$, we simulate the results. We see from Figs. 4–6 in the absence of immigration, the infection is going on decreasing as susceptibility is also decreasing. As results the recovery is going on increasing. The dynamical behavior of different compartment is same as discussed earlier.

6. Conclusion

In this article, we have studied a novel Coronavirus-19 disease model for qualitative as well as numerical analysis. By using Schauder’s and Banach fixed point theorems, we have proved the existence of the considered model under Caputo fractional derivative. Further for numerical simulations, we have used Haar collocation method together with Broyden’s technique. The concerned numerical results have been presented by plots corresponding to different fractional order under some given numerical values. We see that Haar wavelet method can also be used as a powerful tools to handle mathematical models of infectious disease.
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