Macroscopic properties of neutron stars including deformation

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Abstract. We used the equations of state of dense nuclear matter to construct the macroscopic properties of neutron stars and test them using available observational results. The Dirac-Brueckner-Hartree-Fock mean field calculations approximated by their parameterized form are the basis of our calculations. We calculated the central pressure and density and correspondingly the possible radius and mass both with and without allowance for hyperons first, and compared these results with recent astronomical observations, and, finally, we included effect of deformation and rotation.

1. Introduction
Neutron stars play important role in different areas of modern physics. We devote our attention on the possibility that phenomena related to neutron stars can be used as tests of equation of state (EoS) of asymmetric nuclear matter with allowance for hyperons.

A wide spectrum of different equations of state of nuclear matter and their applications to astrophysical problems has been reported in the literature (see, e.g., [1–7]). All these EoS yield (nearly) the same properties close to the standard nuclear density \(\rho_N \approx 0.16 \text{ nucleon/fm}^3 \approx 2.7 \times 10^{14} \text{ g/cm}^3\), but when one is far off this value, s/he has to rely more on underlying principles than on possible experimental verification of predicted physical observables.

Our calculations use the Dirac-Brueckner-Hartree-Fock (DBHF) mean field approach [8, 9] in its parameterized form [10, 11] which reproduces the full nuclear matter calculations[12–14].

Frequently reported and/or calculated quantities of neutron stars are their mass \(M\) and radius \(R\), but sometimes also quantities like the radiation radius \(R_\gamma\), ratio of baryon to gravitational mass and others [1, 4, 7, 15–28].

2. Equations of state of neutron star matter
We follow the DBHF mean field approach (see e.g. [8, 9, 29]), which allows to consider various compositions of the neutron star matter, and also inclusion of non-nucleonic degrees of freedom. The full mean-field DBHF calculations of nuclear matter [12–14] have been parameterized [10]. We choose three parameterizations, which were shown to yield the best fits to the well-known properties of nuclear matter, namely: HA corresponding to the best RMF fit to results of Huber \textit{et al} [13]; LA to those of Lee \textit{et al} [14]; MA to the results of [12], respectively. For details as well as for the values of coupling constants for the individual sets of parameterization, we refer
Figure 1. Inertia moment (left) and the ratio of the equatorial to the polar radii (right) vs. compactness for three parameterizations of the equation of state, both with and without hyperons.

to the original paper [10]. With the inclusion of hyperons, the full Lagrangian density becomes extended [11].

3. Other ingredients of the model

The matter in neutron stars is in β-equilibrium, which determines the particle fractions at each density.

Whereas the nuclear EoS are the dominant input for the calculations in the high-density region, namely \( \rho \geq 10^{14} \text{ g/cm}^3 \). For lower densities, the EoS used are: Feynman-Metropolis-Teller EoS [30] for \( 7.9 \text{ g/cm}^3 \leq \rho \leq 10^4 \text{ g/cm}^3 \) where matter consists of \( \text{e}^− \) and \( \frac{56}{26}\text{Fe} \); Baym-Pethick-Sutherland EoS [31] for \( 10^4 \text{ g/cm}^3 \leq \rho \leq 4.3 \times 10^{11} \text{ g/cm}^3 \) with Coulomb lattice energy corrections; and Baym-Bethe-Pethick one [32] for \( 4.3 \text{ g/cm}^3 \times 10^{11} \leq \rho \leq 10^{14} \text{ g/cm}^3 \).

The hydrostatic equilibrium is given by the Tolman-Oppenheimer-Volkoff equation relating the pressure \( P(r) \) and the energy density \( \rho(r) \) [33, 34],

\[
\frac{dP}{dr} = -(\rho + P) \frac{m(r) + 4\pi r^3 P}{r(r - 2m(r))},
\]

(1)

We refer here to our previous papers on the properties of neutron stars calculated in this way [25, 37], and concentrate ourselves on the neutron star deformation and rotation.

The centrifugal force in a star depends on the local spin frequency \( \bar{\omega} \) measured in a local inertial reference frame. To the lowest order, \( \bar{\omega} \) satisfies differential equation (see, e.g., [35])

\[
\frac{1}{r^4} \frac{d}{dr} \left( r^4 e^{-\Phi}\frac{d\bar{\omega}}{dr} \right) + \frac{4}{r} \left( \frac{d}{dr} e^{-\Phi} \bar{\omega} \right) = 0,
\]

(2)

where \( \Phi \) and \( \lambda \) are metric functions of a non-rotating star.

The total moment of inertia \( I = J/\Omega \), where \( J \) is the total stellar angular momentum, is given by

\[ I = \frac{8}{3} \pi \int_0^R r^4 (\rho + P/c^2) \frac{d\bar{\omega}}{dr} \exp(-\Phi - \lambda) dr \] [35]. In the Newtonian limit, \( \bar{\omega} = \Omega, \lambda = \Phi = 0 \), \( P \ll \rho c^2 \), what gives \( I_{\text{Newton}} = \frac{8}{3} \pi \int_0^R r^4 \rho dr \).

The resulting inertia moment vs. compactness for six parameterizations of the equation of state (see [25, 37]) and the ratio of the equatorial to the polar radii vs. compactness, as calculated for the rotation frequency 716 Hz (the most rapidly rotating star) are presented in Fig. 1. We see that all calculated curves lie very close one to each other (especially in the case of the inertia moment), what suggests that these quantities are generally very weakly dependent on the exact form of the equation of state, even if hyperons are included.
4. Conclusions
We calculated the properties of neutron stars stemming from the parametrized DBHF approach. In addition to usual quantities, like mass, radius, etc., we calculated also the neutron star deformation and moment of inertia vs. compactness. The nonzero deformation, which was obtained also for small compactness, suggests that the deformation is a common feature for these stellar objects and that it will be possible to study creation of gravitational waves.

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