Computational model of a micropolar lubricant with a non-standard support profile and a metal coating at incomplete filling of the working gap

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Abstract. The authors propose an asymptotic and exact self-similar solution for zero (without considering the melt) and the first (considering the melt) approximation of a wedge-shaped sliding support with a profile adapted to friction and a fusible metal coating of the guide surface. The solution is based on the equation of a micropolar liquid flow for a "thin layer", the continuity equation, as well as the equation describing the profile of the molten contour of a guide coated with a fusible metal alloy. The authors have taken into account the formula of the rate of mechanical energy dissipation as well as rheological properties of the lubricant and the melt, which have micropolar properties in the laminar flow mode at incomplete filling of the working gap. Analytical dependences have been obtained for the profile of the molten surface of the guide coated with a low-melting metal alloy, as well as for the velocity and pressure fields at zero and first approximation. In addition, the main operating characteristics of the friction pair under consideration have been determined: the bearing capacity and the friction force. The article contains estimation of the influence of the parameters conditioned by coating melt and adapted to the friction conditions of the support profile, and the parameter characterizing the rheological properties of the lubricant, as well as the length of the loaded area in terms of bearing capacity and friction force.

1. Introduction
A sufficient number of works have been devoted to development of a computational model of thrust plain bearings with a fusible coating of movable and fixed contact surfaces [1-13]. However, the process of coating lubrication on melts is not a self-sustaining process [14-19]. To ensure a self-sustaining lubrication process of sliding bearings, it becomes necessary not only to have fusible coatings on one of the working contact surfaces, but also to have a constant supply of lubricant, or an adapted profile of the support surface of the slider.

The proposed paper presents a mathematical computational model of the hydrodynamic flow regime of a lubricant and a coating melt with micropolar rheological properties in the working gap of a wedge-shaped sliding support with an adapted profile of the support surface of the slider.

2. Materials and methods
The article deals with the steady-state flow of an incompressible liquid and a coating melt in a working gap with micropolar properties. The wedge-shaped sliding support with a non-standard
support surface profile is stationary, and the support ring with a fusible coating moves at a speed of $u^*$ (Fig. 1).

The contours shown in the calculation scheme in the coordinate system $x' o' y'$ are: $C_0$ – contours of an inclined slider; $C_1$ – contours of an inclined slider with an adapted profile; $C_2$ – contours of the molten coating of the support ring surface, which are designated as follows:

$$C_0: y' = h_0 + x' \tan \alpha, \quad C_1: y' = h_0 + x' \tan \alpha - a' \sin \omega x', \quad C_2: y' = -\lambda f'(x'),$$

where $h_0$ – thickness of the lubricant; $h_0^*$ – thickness of the melt; $\alpha$ – angle of the inclined slider with the axis $Ox$; $a'$ – amplitude of the disturbance; $\omega'$ – parameter of the adapted slider profile; $\lambda f'(x')$ – function that determines the profile of the molten coating contour.

To solve this problem, we have used the well-known equations for the "thin layer" of micropolar lubricant movement, the continuity equation, as well as the equation that determines the profile of the molten coating contour taking into account the rate of mechanical energy dissipation. A number of additional factors has also been taken into account: wedge shape of a sliding support, the slider support profile adapted to friction conditions and the fusible metal coating of the surface of the support ring at incomplete filling of the working gap with lubricant:

$$\int_{C_0}^{C_1} \left( \frac{\partial^2 u'}{\partial y^2} + \frac{\kappa}{\partial y} \frac{\partial v'}{\partial y} - 2 \kappa v' \right) dy' = 0, \quad \frac{\partial v'}{\partial y} + \frac{\partial u'}{\partial x} = 0,$$

$$\frac{d\lambda f'(x')}{dx} \cdot u'^* L' = -2\mu \int_{-\lambda f'(x')}^{\lambda f'(x')} \left( \frac{\partial u'}{\partial y} \right)^2 dy'.$$

By generally accepted simplifications, the boundary conditions for this problem will be written as follows:

$$u' = -u'^*, v' = 0, \quad v' = 0 \text{ at } y = -\eta f'(x');$$

$$u' = 0, v' = 0, \quad v' = 0 \text{ at } y = h_0 + x' \tan \alpha - a' \sin \omega x'; \quad p(0) = p'(L) = p_0.$$

To move to dimensionless variables, we apply the standard method:
Taking into account (4) in the system of equations (2) and the boundary conditions (3), we obtain a system of equations and boundary conditions for it:

\[ u' = u^* u; \quad v' = u^* v; \quad p' = p^* p; \quad y' = h_0 y; \]

\[ N^2 = \frac{\kappa}{2\mu + \kappa}; \quad N_1 = \frac{2\mu l^2}{\kappa h_0^2}; \quad l^2 = \frac{y}{4\mu}; \]

\[ \varepsilon = \frac{h_0}{l}; \quad u^* = \frac{u'}{2h_0}; \quad p^* = \frac{(2\mu + \kappa)\mu v}{2h_0^2}; \quad x' = Lx. \] (4)

Taking into account the small size of the gap, as well as the equality \( v = 0 \) on moving and stationary surfaces, we average the second equation of the system (5) over the thickness of the lubricant layer. As a result, we obtain a system of equations:

\[ \frac{\partial^2 u}{\partial y^2} + N^2 \frac{\partial v}{\partial y} = \frac{\partial p}{\partial x}; \quad \frac{\partial^2 v}{\partial y^2} = \frac{\nu}{N_1} + \frac{1}{N_1} \frac{\partial u}{\partial y}, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \] (5)

\[ \frac{d\Phi(x)}{dx} = -K \int_{-\Phi(x)}^{\Phi(x)} \left( \frac{\partial u}{\partial y} \right)^2 dy, \]

where \( K = \frac{2\mu u^*}{h_0 l}; \quad \eta = \frac{\mu \eta}{h_0}; \quad \eta_1 = \frac{a}{h_0} \).

\[ v = 0, \quad v = 0, u = 0 \text{ if } y = 1 + \eta x - \eta_1 \sin \omega x = h(x); \]

\[ v = 0, \quad v = 0, u = -1 \text{ at } y = -\Phi(x); \]

\[ p(x_1) = p(x_2) = 0. \] (6)

Taking into account the small size of the gap, as well as the equality \( v = 0 \) on moving and stationary surfaces, we average the second equation of the system (5) over the thickness of the lubricant layer. As a result, we obtain a system of equations:

\[ \frac{\partial^2 u}{\partial y^2} + N^2 \frac{\partial v}{\partial y} = \frac{\partial p}{\partial x}; \quad \frac{\partial^2 v}{\partial y^2} = \frac{\nu}{2N_1 h} (2y - h), \quad v = \frac{1}{2N_1 h} (y^2 - hy), \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{d\Phi(x)}{dx} = -K \int_{-\Phi(x)}^{\Phi(x)} \left( \frac{\partial u}{\partial y} \right)^2 dy. \] (7)

The direct solution is found in the form of an asymptotic solution with respect to the parameter due to the melt and an exact self-similar solution. As a small parameter, taking \( K \) due to the melt and the rate of energy dissipation, we search for the function \( F(x) \) in the form:

\[ \Phi(x) = -K\Phi_1(x) - K^2\Phi_2(x) - K^3\Phi_3(x) - \ldots = H, \] (8)

where \( H = -K\Phi_1 - K^2\Phi_2 - K^3\Phi_3 - \ldots \).

The boundary conditions for the velocity components \( u \) and \( v \) on the contour \( y = 0 - \Phi(x) \) are given in the form

\[ v(0 - H(x)) = v(0) - \left( \frac{\partial v}{\partial y} \right)_{y=0} \cdot H(x) - \left( \frac{\partial^2 v}{\partial y^2} \right)_{y=0} \cdot H^2(x) + \ldots = 0; \]

\[ u(0 - H(x)) = u(0) - \left( \frac{\partial u}{\partial y} \right)_{y=0} \cdot H(x) - \left( \frac{\partial^2 u}{\partial y^2} \right)_{y=0} \cdot H^2(x) + \ldots = -1. \] (9)

The asymptotic solution of system (5), taking into account (6) and (9), can be found in the form

\[ v = \nu_0(x, y) + Ku_1(x, y) + K^2\nu_2(x, y) + \ldots ; \]

\[ u = u_0(x, y) + Ku_1(x, y) + K^2u_2(x, y) + \ldots ; \]

\[ \Phi(x) = -K\Phi_1(x) - K^2\Phi_2(x) - K^3\Phi_3(x) - \ldots ; \]

\[ p = p_0 + Kp_1(x) + K^2p_2(x) + K^3p_3(x) \ldots \] (10)

Deriving (10) from (7), we obtain a system of equations and boundary conditions for it:
For the zeroth-order approximation:
\[
\frac{\partial^2 u_0}{\partial y^2} + \frac{N^2}{2N_1h} (2y - h) = \frac{dp_0}{ax}, \quad \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} = 0,
\]
(11)
\[u_0 = 0, \quad v_0 = 0, u_0 = 0 \text{ at } y = 1 + \eta x - \eta_1 \sin \omega x;\]
(12)
\[v_0 = 0, \quad u_0 = -1, v_0 = 0 \text{ at } y = 0; \quad p_0(x_1) = p_0(x_2) = 0;\]
– for the first approximation:
\[
\frac{\partial^2 u_1}{\partial y^2} = \frac{dp_1}{dx}, \quad \frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} = 0, \quad -\frac{\partial \Phi_1(x)}{ax} = \frac{K}{\int_{-\Phi_1(\phi)}^{\Phi_1(\phi)} \left( \frac{\partial u_1}{\partial y} \right)^2 dy;\]
(13)
\[v_1 = \left. \frac{\partial u_0}{\partial y} \right|_{y=0} \cdot \Phi_1(x); u_1 = \left. \frac{\partial u_0}{\partial y} \right|_{y=0} \cdot \Phi_1(x);\]
(14)
\[v_1 = 0, \quad v_1 = 0, u_1 = 0 \text{ at } y = h(x) + \Phi; \quad p_1(x_1) = p_1(x_2) = 0.\]

We search for a self-similar solution of the system (11) and (12) in the form
\[
\psi_0(x, y) = \Phi_0(\xi); \quad \xi = \frac{y}{h(x)};
\]
(15)
\[V_0(x, y) = -\partial(\Phi) \cdot h'(x); \quad U_0(x, y) = \tilde{u}_0(\xi).\]

Given (15), we obtain a system of equations and boundary conditions for (11) and (12):
\[
\tilde{\psi}_0'' = \tilde{C}_2, \tilde{u}_0'' = \tilde{C}_1 - \frac{N^2}{2N_1h} (2\xi - h) \quad \tilde{u}_0^0
\]
(16)
\[
\frac{dp_0}{dx} = \frac{\tilde{C}_1}{h^3(x)} + \frac{\tilde{C}_2}{h^3(x)};
\]
\[\tilde{v}_0''(0) = 0, \quad \tilde{v}_0'(1) = 0, \quad \tilde{u}_0(1) = 1, \quad \tilde{u}_0(0) = 0, \quad \int_{0}^{1} \tilde{u}_0(\xi) d\xi = 0; \quad p(0) = p(1) = \frac{p_0}{p}.\]
(17)

Performing the solution (16) and (17), we obtain the calculation formulas
\[
\tilde{\psi}_0(\xi) = \frac{\xi^2}{2} (\xi^2 - \xi), \quad \tilde{C}_1 = 6.
\]
\[\tilde{u}_0(\xi) = \tilde{C}_1 \frac{\xi^2}{2} - \frac{N^2}{2N_1h} \left( \frac{\xi^2}{3} - \frac{\xi^2}{2} \right) - \left( \frac{N^2}{12N_1h} + \frac{\xi}{2} \right) + 1 \xi + 1.
\]
Taking into account \(p_0(x_1) = p_0(x_2) = 0\) to an approximation of \(O(\eta^2)\) for \(\tilde{C}_2\), we obtain the formula:
\[
\tilde{C}_2 = -6 \left( 1 + \frac{\eta}{2} (x_2 + x_1) - \frac{\eta_1}{\omega(x_2-x_1) \cos \omega x_2 - \cos \omega x_1} \right).
\]
(19)
Taking into account (19) for \(p_0\), we obtain:
\[
p_0 = 6 \left( \frac{\eta}{2} (x - x_1)(x - x_2) + \frac{\eta_1}{\omega} (\cos \omega x - \cos \omega x_1) + \frac{\eta_1 x - x_1}{\omega(x_2-x_1)} (\cos \omega x_2 - \cos \omega x_1) \right).
\]
(20)
For the function determining the molten contour of the reference ring, taking into account (18), we obtain:

$$\frac{d\Phi(x)}{dx} = h(x) \int_0^1 \left( \frac{\psi(x)}{h'(x)} + \frac{\bar{u}_0(x)}{h(x)} \right)^2 d\xi. \quad (21)$$

Performing the solution (21), we obtain the calculated formula

$$\Phi_1(x) = x - \frac{1}{2} \eta x^2 - \frac{\eta_1}{\omega} \cos \omega x + \frac{N^2}{720 N_1^2} \left( x - \frac{\eta_1}{2} x^2 - \frac{\eta_1}{\omega} \cos \omega x \right) + h_0. \quad (22)$$

We will search for the self-similar solution of (13) and (14) in the same way as for (11) and (12). As a result, we obtain the calculated formulas for the velocity and pressure fields:

$$\bar{u}_1(\xi) = \bar{C}_1 \xi^2 - \left( \frac{\bar{C}_1}{2} + M \right) \xi + \bar{C}_1; \quad \bar{p}_1 = \frac{6 M}{(1 + \Phi)^2} \left( \frac{\bar{\eta}}{2} (x - \bar{x}) (x_2 - \bar{x}) + \frac{\bar{\eta}_1}{\omega} (\cos \omega x_2 - \cos \omega x) + \frac{\bar{\eta}_1 (x_2 - \bar{x})}{\omega (x_2 - x_1)} (\cos \omega x_2 - \cos \omega x_1) \right), \quad (23)$$

where $\bar{\eta} = \frac{\eta}{1 + \Phi}; \quad \bar{\eta}_1 = \frac{\eta_1}{1 + \Phi}; \quad \overline{\Phi} = \sup_{x \in [x_1; x_2]} \Phi_1(x); \quad M = \sup_{x \in [x_1; x_2]} \frac{\partial u_0}{\partial y} \bigg|_{y=0} \cdot \Phi_1(x) = \sup_{x \in [x_1; x_2]} \left( 1 - 4 \eta x + \frac{3}{2} \eta (x_2 + x_1) + \frac{\eta_1}{\omega (x_2 - x_1)} (\cos \omega x_2 - \cos \omega x_1) + \frac{N^2}{4 N_1^2} (1 + \eta x - \eta_1 \sin \omega x) \right) \cdot \overline{\Phi}.$

3. The results

As a result of the study, we have obtained equations for the velocity and pressure fields in the lubricant layer for the zero and first approximation. We have also determined the function that characterizes the profile of the molten contour of the support ring surface, taking into account the adapted slider profile and the rheological properties of the lubricant used and the coating melt having micropolar properties. New multiparametric formulas have been developed for the load capacity and the friction force at incomplete filling of the working gap.

Taking into account (11), (13), (20) and (23) for the bearing capacity and the friction force, we obtain:

$$W = p^* L \int_{x_1}^{x_2} (p_0 + K p_1) dx = \frac{12 (\mu + \kappa) L^2 u}{h_0^2} \left[ \frac{1}{12} (x_1 - x_2) (x_1 + x_2)^2 \left( \eta (1 + \Phi)^2 + KM \bar{\eta} \right) \right] +$$

$$+ \left( \eta_1 (1 + \Phi)^2 + KM \bar{\eta}_1 \right) \left[ \frac{\sin \omega x_2 - \sin \omega x_1}{\omega^2} + \frac{2 x_1 \cos \omega x_1}{\omega} \right] +$$

$$+ \frac{\eta_1}{\omega} (\cos \omega x_2 - \cos \omega x_1) - \frac{7 \eta_1}{\omega} (\cos \omega x_2 - \cos \omega x_1) + K \left[ \frac{6 M \bar{\eta}_1}{\omega (1 + \Phi)^2} (\cos \omega x_2 - \cos \omega x_1) -$$
\[ -\frac{1}{2} \left( x_2 - x_1 + \frac{6N_2}{\omega} \cos{\omega x_2} - \cos{\omega x_1} \right) - \frac{N^2}{4N_2} (x_2 - x_1) \cdot \vec{F} \] \quad \text{(24)}

4. Numerical analysis
As a result of the numerical analysis, the dependencies presented in Fig. 2 have been constructed, which allows us to draw the following conclusions.

\begin{align*}
\textit{a)} & \quad \text{Fig. 2. Effect of the parameter } \omega \text{ characterizing the adapted profile,} \\
\text{and the thermal parameter } K, \text{ which characterizes the melt, on the values:} \\
\textit{a)} & \quad \text{load-bearing capacity and } \textit{b)} \quad \text{friction force}
\end{align*}

Theoretical studies have shown that the bearing capacity of the guide surface rail with fusible metallic coating and the adapted profile of the slide increase by \(~11-15\%\) with the increase of the parameter \(\omega\) characterizing the adapted profile and length \((x_2 - x_1)\) of the loaded area. At the same time, the coefficient of friction decreases by \(~14-16\%\). These processes happen due to the overall rheological properties of lubricant and melt fusible metal coatings with the viscoelastic rheological properties.

5. Experimental studies
In an experimental study, we consider wedge-shaped sliding support with a fusible metal coating made of Wood alloy (see Table). According to the results of the experiments, value of the friction coefficient has been determined. This allows us to reveal the presence of a hydrodynamic friction mode during operation of the bearing both with a lubricant having micropolar properties and with a melt of a fusible surface coating. The temperature regime and the transition of the hydrodynamic friction regime to boundary friction have also been determined. The analysis of experimental studies shows that the melt of a fusible coating affects the coefficient of friction more intensely than the rheological properties of the liquid lubricants used.

Theoretical studies have shown that the bearing capacity of the guide surface rail with fusible metallic coating and the adapted profile of the slide increase by \(~11-15\%\) with the increase of the parameter \(\omega\) characterizing the adapted profile and length \((x_1 - x_2)\) of the loaded area. At the same time, and coefficient of friction decreases by \(~14-16\%\). These processes happen in the account based on the overall rheological properties of lubricant and melt fusible metal coatings with the viscoelastic rheological properties.
Table 1. Comparison of the results of theoretical and experimental studies

| No. | Theoretical studies | Experimental studies |  
|-----|---------------------|----------------------|  
|     | without coating     | with coating          | slides with a fusible metal coating made of Wood alloy |
|-----|---------------------|----------------------|-----------------------------------------------------|
| 1   | 0.0047              | 0.0028               | 0.0032                                              |
| 2   | 0.0048              | 0.0027               | 0.0033                                              |
| 3   | 0.0049              | 0.0024               | 0.0029                                              |
| 4   | 0.0051              | 0.0026               | 0.0030                                              |
| 5   | 0.0053              | 0.0028               | 0.0031                                              |

The experimental study has been carried out on the basis of the computational models obtained in the theoretical part. As a result, the area of prospective operation of the developed tribosystem has been determined.

The determined tribotechnical characteristics allow us to determine duration of the hydrodynamic friction regime and the reliability of the developed theoretical computational models and numerical analysis data.

6. Conclusion

New multiparametric formulas have been developed for the main performance characteristics (bearing capacity and friction force) of a wedge-shaped sliding support, taking into account the rheological properties of a micropolar lubricant at the incomplete filling of the working gap, as well as taking into account the melt of the guide surface coated with a fusible metal alloy.

The influence of the parameters of variable factors caused by the guide surface melting coated with a fusible metal alloy with incomplete filling of the working gap has been estimated.

The obtained refined computational models of wedge-shaped sliding supports allow for adjustment of the ratio of bearing capacity of the fusible metal coating and the coefficient of friction through varying its coating.

A satisfactory convergence of theoretical and experimental studies has been established in support of the theoretical conclusions made.

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