A \(q\)-rung orthopair fuzzy multiple attribute group decision making method based on generalized Maclaurin symmetry mean and Dombi \(t\)-norm and \(t\)-conorm

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Abstract. Generalized Maclaurin symmetry mean (GMSM) operator is an effective tool in the process of multi-attribute group decision making (MAGDM) with the characteristic of capturing the interrelationships between multiple arguments. At the same time, \(q\)-rung orthopair fuzzy set (\(q\)-ROFS) is a good tool to describe uncertainty and fuzziness. To effectively aggregate \(q\)-rung orthopair fuzzy information based on the extension of Dombi operations, some novel operators and ideal properties of the proposed operators are put forward in this study. Further, this research present a novel method to MAGDM based on the proposed operators. Finally, the applicability of new method can be proved by a numerical experiment. A detailed parametric analysis and a comparative analysis are also discussed to highlight the flexibility and superiority of the method proposed in this paper.

1. Introduction

Multi-attribute group decision-making (MAGDM) plays an important role in modern decision-making theory. In MAGDM, it is necessary to express the attribute information expressed by decision makers reasonably to eliminate the fuzziness and uncertainty of information. In order to solve this problem, Zadeh [1] first proposed the fuzzy set (FS) and Atanassov [2] introduced the intuitionistic fuzzy set (IFS) with membership function and non-membership function. Owing to this characteristic, IFS is more effective to handle fuzziness [3-5]. However, the sum of membership and non-membership degrees cannot be greater than 1 in IFSs. Thus, Yager [6] generalized the IFSs and introduced the concept of the Pythagorean fuzzy set (PFS), whose constraint is that the square sum of the membership degree and the non-membership degree cannot be greater than 1. Then on the basis of the theory of PFSs, some scholars have carried out extensive research [7,8]. Recently, Yager [9] further extended PFSs to a new concept, the \(q\)-rung orthopair fuzzy set (\(q\)-ROFS), whose constraint is that the sum of \(q\)th power of the membership degree and the \(q\)th power of the degree of non-membership is bounded by 1. Obviously, the \(q\)-ROFS provides more freedom and convenience than IFS and PFS. Therefore, the \(q\)-ROFS has also become a hot spot in current MAGDM research.

How to aggregate the attribute information is another important problem when we select the best one from all the alternatives. Therefore, the study of aggregation operator count for a great deal and some aggregation operators have been used [10-11]. However, these operators cannot consider the interrelationships between arguments. That means these operators assume that criteria are independent, which is somewhat inconsistent with the reality. So some extended aggregation operators have been applied to MAGDM in various fuzzy environments. The Bonferroni mean (BM) operator proposed by...
Bonferroni [12] and the Heronian mean operator proposed by Sykorá [13] can capture the relationship between any two parameters. Xu and Yager [14] put forward some intuitionistic fuzzy Bonferroni mean operators. Liang et al. [15] extended them and proposed the weighted Pythagorean fuzzy geometric Bonferroni mean (WPFGBM) to solve the Pythagorean fuzzy MAGDM problem. Liu PD and Liu JL [16] utilized BM operators to fuse q-rung orthopair fuzzy information. Yu [17] developed an approach for MAGDM based on the intuitionistic fuzzy geometric weighed Heronian mean (IFGWHM) operator. Wei et al. [18] studied the generalized Heronian mean operator under the q-rung orthopair fuzzy sets. With continuous development of theory, some extended forms of BM and Heronian mean operators were also developed for information aggregation [19-20].

The Maclaurin symmetric mean (MSM) operator proposed by Maclaurin [21] and the Hamy mean (HM) operator proposed by Hara [22] have the capacity of taking the interrelationship among multiple arguments into consideration, that makes them more flexible than the BM and Heronian mean operators. Xing YP et al. [23] combined Hamy mean operators with interaction operational laws to solve MAGDM problems. Li N et al. [24] combined the Hamy Mean operator with Schweizer-Sklar t-conorm and t-norm. In addition, Qin [25] presented that the MSM is a special circumstance of HM based on a comparative analysis. The MSM operator has achieved abundant research on fuzzy information aggregation, such as, Wei and Lu [26] used the MSM operator in Pythagorean fuzzy environment; To combine the merits of the MSM operator and interaction operations of IFSs, Liu PD and Liu WQ [27] proposed some intuitionistic fuzzy interaction Maclaurin symmetric mean (IFIMSM) operators. Wang et al. [28] proposed some q-rung orthopair fuzzy Maclaurin symmetric mean (q-ROFMSM). Wang JQ et al. [29] first proposed the MSM operator to a generalized form (GMSM) and extended a series of MSM aggregation techniques. GMSM operator is a more general operator, which can adjust the parameter values to transform into many existing simpler operators. Of course, like MSM, it can also capture the relationship between multiple input parameters. Liu P and Gao H [30] developed the GSM operator for multi-hesitant fuzzy linguistic term sets (MHFLTSs). Liu P and Li Y [31] extended the GMSM operator to probabilistic linguistic information (PLI). However, there are only a few research results on aggregating indeterminate information based on the GMSM operator. Until now, the GMSM operator has not been applied in intuitionistic fuzzy, Pythagorean fuzzy or q-rung orthopair fuzzy environments. As we all know, the IFS and PFS are special cases included in q-ROFS. Thus, it is necessary to develop some q-rung orthopair fuzzy GMSM operators.

The Dombi t-norm and t-conorm [32] have flexible parameters. Recently, Liu et al. [33] proposed the extension of the Dombi operations for IFSs. Chen and Ye [34] generalized the Dombi operations to single-valued neutrosophic numbers. Further, Shi and Ye [35] studied the Dombi operational rules under neutrosophic cubic environments and developed some neutrosophic cubic Dombi weighted arithmetic average operators for MADM problems similarly. Yang and Pang [36] developed some q-ROFBM operators based on the BM operator, Dombi t-norm and t-conorm. This paper utilizes GMSM to aggregate q-rung orthopair fuzzy (q-ROF) information based on the Dombi operational rules. Additionally, the effectiveness and superiority of the new operator can be proved by a numerical example of MAGDM.

The contributions of this study are: (1) to propose the dual form of GMSM, (2) to extend the GMSM in q-ROFS and propose a family of new operators to solve MAGDM problems, (3) to propose a new method to solve MAGDM problem based on the proposed operators. Section 2 is about some basic concepts related to q-ROFS, Dombi t-norm and t-conorm, GMSM and Dombi operational rules of q-ROFSs, and in this part, the dual form of GMSM are proposed. Section 3 develops a series of q-rung orthopair fuzzy Dombi generalized Maclaurin symmetry mean operators. In Section 4, we conduct a new method to MAGDM problems on the basis of the proposed operators. Section 5 presents a numerical instance to demonstrate the validity of method proposed in this article. A detailed parametric analysis and a comparative analysis are also discussed to further prove the flexibility and superiority. Conclusions and future works are presented in Section 6.
2. Preliminaries

In the present section, we will briefly recall some fundamental notions and propose the dual form of GMSM. These concepts will be applied to the analysis and discussion in the following chapters.

2.1. q-rung orthopair fuzzy set

**Definition 1** [9]. Let $X$ be an universe of discourse, then a q-ROFS $A$ on $X$ is defined as follows:

$$A = \{ (x, h_q(x), g_q(x)) | x \in X \},$$

where $h_q(x)$ and $g_q(x)$ denote the membership and non-membership degrees respectively, satisfying $0 \leq h_q(x) \leq 1$, $0 \leq g_q(x) \leq 1$ and $0 \leq h_q(x)^q + g_q(x)^q \leq 1$. The degree of indeterminacy is $\pi_q(x) = [h_q(x)^q + g_q(x)^q]^{1/q}$. Liu and Wang [23] called $(h_q(x), g_q(x))$ a q-ROFN, which can be denoted as $d = (h, g)$.

**Definition 2** [11]. Let $d = (h, g, \lambda)$ be a q-ROFN, then the score of $d$ can get by the function $S(d) = h^\lambda - g^\lambda$, the accuracy of $d$ is defined as $H(d) = h^\lambda + g^\lambda$. For any two q-ROFNs, $d_i = (h_i, g_i, \lambda)$ and $d_j = (h_j, g_j, \lambda)$, then a comparison method for q-ROFNs is defined in the following.

1. If $S(d_i) > S(d_j)$, then $d_i > d_j$,
2. If $S(d_i) = S(d_j)$, then $H(d_i) = H(d_j)$, then $d_i = d_j$.

2.2. Dombi $t$-norm and $t$-conorm

**Definition 3** [37]. The Dombi $t$-norm and $t$-conorm between $x$ and $y$ are defined as follows, $x$ and $y$ are any two real numbers:

$$D(x, y) = \frac{1}{1 + \left( \frac{(1-x)^{1/\lambda} + (1-y)^{1/\lambda}}{x + y} \right)^{\lambda}}$$

$$D^*(x, y) = \frac{1}{1 + \left( \frac{x^{1/\lambda} + y^{1/\lambda}}{1-x + 1-y} \right)^{\lambda}}$$

where $\lambda > 0$, $(x, y) \in [0,1] \times [0,1]$.

**Definition 4** [38]. Let $d_i = (h_i, g_i)$ and $d_j = (h_j, g_j)$, be two q-ROFNs, $\lambda > 0$, then the Dombi operational rules of q-ROFNs can be obtained below:

1. $d_i \oplus d_j = \left[ 1 - \left( \frac{h_i^{\lambda} + h_j^{\lambda}}{1-h_i^{\lambda}} \right) \right]^{1/\lambda}, \left[ 1 + \left( \frac{g_i^{\lambda} + g_j^{\lambda}}{1-g_i^{\lambda}} \right) \right]^{1/\lambda}$ (4)
2. $d_i \odot d_j = \left[ 1 + \left( \frac{h_i^{\lambda} + h_j^{\lambda}}{1-h_i^{\lambda}} \right) \right]^{1/\lambda}, \left[ 1 - \left( \frac{g_i^{\lambda} + g_j^{\lambda}}{1-g_i^{\lambda}} \right) \right]^{1/\lambda}$ (5)
3. $d_i \odot d_j = \left[ 1 - \left( \frac{h_i^{\lambda} + h_j^{\lambda}}{1-h_i^{\lambda}} \right) \right]^{1/\lambda}, \left[ 1 + \left( \frac{g_i^{\lambda} + g_j^{\lambda}}{1-g_i^{\lambda}} \right) \right]^{1/\lambda}$, $n > 0$ (6)
4 \text{d}_1^* = \left[ \left( 1 + \left( n \left( \frac{1-k^j}{k^j} \right)^{\frac{1}{\gamma}} \right) \right)^{\gamma} - 1 \right]^{-1} \left( 1 + \left( n \left( \frac{g_j^i}{1-g_j^i} \right)^{\frac{1}{\gamma}} \right) \right)^{\gamma}, n > 0 \quad (7)

\textbf{Theorem 1.} Let \text{d}_1 \text{ and } \text{d}_2 \text{ be two q-ROFNs, then we can get the following operations easily.}

1. \quad n(\text{d}_1 \oplus \text{d}_2) = nd_1 \oplus nd_2 ;
2. \quad n\text{d}_1 \oplus n\text{d}_2 = (n_1 + n_2)\text{d}_1 ;
3. \quad \text{d}_1^n \oplus \text{d}_2^n = (\text{d}_1 \oplus \text{d}_2)^n ;
4. \quad d^n_1 \oplus d^n_2 = (d_1 \oplus d_2)^n .

\textbf{2.3 Generalized Maclaurin symmetric mean}

\textbf{Definition 5 [39].} Let \text{d}_j (j = 1, 2, ..., n) be a collection of crisp numbers, and \text{m} = 1, 2, ..., n . If

\[ GMSM^{(\text{m}, \text{n}, \ldots, \text{n})}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) = \left( \sum_{j=1}^{n} \prod_{i=1}^{j} d_i^p \right)^{1/n} \quad (8) \]

then \( GMSM^{(\text{m}, \text{n}, \ldots, \text{n})} \) is called the generalized Maclaurin symmetric mean (GMSM), where \( p_1, p_2, \ldots, p_n \geq 0 \), \( (i_1, i_2, \ldots, i_m) \) traversal all the combination of \( (1, 2, \ldots, n) \) , \( C_n^m \) is the binomial coefficient.

Based on the Equation (4), the following desirable properties can be easily proved.

1. Idempotency: \( GMSM^{(\text{m}, \text{n}, \ldots, \text{n})}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) = \text{d}_1 ;
2. Monotonicity: \( GMSM^{(\text{m}, \text{n}, \ldots, \text{n})}(a_1, a_2, \ldots, a_n) \leq GMSM^{(\text{m}, \text{n}, \ldots, \text{n})}(b_1, b_2, \ldots, b_n) \) If \( a_i \leq b_i \) for all \( i(i = 1, 2, \ldots, n) ;
3. Boundedness: \( \min \{\text{d}_i\} \leq GMSM^{(\text{m}, \text{n}, \ldots, \text{n})}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) \leq \max \{\text{d}_i\}, i = 1, 2, \ldots, n .

Especially, if \text{m} = 1, \text{m} = 2 and \text{m} = 3, then the \( GMSM^{(\text{m}, \text{n}, \ldots, \text{n})}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) \) separately reduces to the generalized WA operator, the BM operator and the GBM operator as follows :

\[ GMSM^{(3)}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) = \left( \sum_{j=1}^{n} d_j^p \right)^{1/n} \quad (9) \]

\[ GMSM^{(2)}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) = \left( \sum_{j=1}^{n} \frac{d_j^p d_j^{p_2}}{C_n^p} \right)^{1/n} = \left[ \frac{2 \sum_{j=1}^{n} d_j^p d_j^{p_2}}{n(n-1)} \right]^{1/n} = BM^{(\text{m}, \text{n})}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) \quad (10) \]

\[ GMSM^{(1)}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) = \left( \sum_{j=1}^{n} \prod_{i=1}^{j} d_i^p \right)^{1/n} = \left( \frac{\sum_{j=1}^{n} d_j^p d_j^{p_2} d_j^{p_n}}{n(n-1)(n-2)} \right)^{1/n} = GBM^{(\text{m}, \text{n})}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) \quad (11) \]

In the following, we propose a dual form of GMSM, i.e. generalized dual Maclaurin symmetric mean (GDMSM).

\textbf{Definition 6.} Let \text{d}_j (j = 1, 2, ..., n) be a collection of crisp numbers, and \text{m} = 1, 2, ..., n . Then the generalized dual Maclaurin symmetric mean is defined as

\[ GDMSM^{(\text{m}, \text{n}, \ldots, \text{n})}(\text{d}_1, \text{d}_2, \ldots, \text{d}_n) = \frac{1}{p_1 + p_2 + \ldots + p_n} \left( \prod_{i=1}^{n} \frac{\sum_{j=1}^{m} d_j^p}{p_j} \right)^{1/n} \quad (12) \]
where \(p_1, p_2, \ldots, p_n \geq 0\), \((i_1, i_2, \ldots, i_n)\) traversal all the combination of \((1, 2, \ldots, n)\), \(C_n^m\) is the binomial coefficient.

3. Some \(q\)-rung orthopair fuzzy generalized Maclaurin symmetry mean operators based on Dombi t-norm and t-conorm

In this section, we use GMSM and GDMSM to aggregate \(q\)-ROF information based on Dombi t-norm and t-conorm and introduce the \(q\)-ROFDGMSM operator, the \(q\)-ROFDWGMMSM operator, and the \(q\)-ROFDWGDMMSM operator. In addition, we discuss some properties and special cases of newly proposed operators.

3.1. The \(q\)-rung orthopair fuzzy Dombi generalized Maclaurin symmetric mean operator

**Definition 7.** Let \(\{d_j = (h_j, g_j)\} (j = 1, 2, \ldots, n)\) be a set of \(q\)-ROFNs, \((i_1, i_2, \ldots, i_n)\) be all the \(m\)-tuple combination of \((1, 2, \ldots, n)\), and \(m = 1, 2, \ldots, n\). If

\[
q - \text{ROFDGMSM}^{(m, i_1, i_2, \ldots, i_n)}(d_1, d_2, \ldots, d_n) = \left( \frac{\prod_{j=1}^{n} d_j^{i_j}}{C_n^m} \right)
\]

then \(q - \text{ROFDGMSM}^{(m, i_1, i_2, \ldots, i_n)}\) is called the \(q\)-rung orthopair fuzzy \((q\)-ROF\) Dombi generalized Maclaurin symmetric mean operator, where \(C_n^m\) is the binomial coefficient.

We can obtain the following aggregation result on the basis of Dombi operational rules.

**Theorem 2.** Let \(\{d_j = (h_j, g_j)\} (j = 1, 2, \ldots, n)\) be a set of \(q\)-ROFNs, \((i_1, i_2, \ldots, i_n)\) be all the \(m\)-tuple combination of \((1, 2, \ldots, n)\), and \(m = 1, 2, \ldots, n\). Then the aggregated value by the \(q\)-ROFDGMSM operator is still a \(q\)-ROFN and

\[
q - \text{ROFDGMSM}^{(m, i_1, i_2, \ldots, i_n)}(d_1, d_2, \ldots, d_n) = \left( 1 + \left( \sum_{p_j} \left( \frac{h_j^{i_j}}{g_j^{i_j}} \right)^{1/(1-q)} \right) \right)^{1/(1-q)}
\]

**Proof.** Since

\[
d_j^{i_j} = \left( 1 + \left( \sum_{p_j} \left( \frac{h_j^{i_j}}{g_j^{i_j}} \right)^{1/(1-q)} \right) \right)^{1/(1-q)}
\]

then, we have

\[
\prod_{j=1}^{n} d_j^{i_j} = \left( 1 + \left( \sum_{p_j} \left( \frac{h_j^{i_j}}{g_j^{i_j}} \right)^{1/(1-q)} \right) \right)^{1/(1-q)}
\]

Subsequently, we have

\[
\sum_{i_1, i_2, \ldots, i_n} \prod_{j=1}^{n} d_j^{i_j} = \left( 1 + \left( \sum_{p_j} \left( \frac{h_j^{i_j}}{g_j^{i_j}} \right)^{1/(1-q)} \right) \right)^{1/(1-q)}
\]
Theorem 3. (Idempotency) Let \( d_j = (h_j, g_j) (j = 1, 2, ..., n) \) be a set of \( q \)-ROFNs, suppose \( d_j = d \) for all \( j \), then

\[
q - \text{ROFDGMSM}^{(m, n, \ldots, m)}(d_1, d_2, \ldots, d_n) = d.
\]  

(20)

Theorem 4. (Monotonicity) Let \( a_j = (h_j, g_j) \) and \( b_j = (h_j, g_j) \) \((j = 1, 2, ..., n)\) be two sets of \( q \)-ROFNs. If \( h_j \leq h_j \) and \( g_j \geq g_j \) holds for all \( j \), then
Definition 9. Let $d_j = (h_j, g_j) (j=1,2,...,n)$ be a set of q-ROFNs, and $d^* = \left( \max(h_j), \min(g_j) \right)$, then
\[
q - \text{ROFDGMSM}^{(m,n;p_1,....,p_n)} (d_1, d_2, ..., d_n) \leq q - \text{ROFDGMSM}^{(m,n;p_1,....,p_n)} (h_1, h_2, ..., h_n). \quad (21)
\]

Theorem 5. (Boundedness) Let $d_j = (h_j, g_j) (j=1,2,...,n)$ be a set of q-ROFNs, and $d^* = \left( \max(h_j), \min(g_j) \right)$, then
\[
d^- \leq q - \text{ROFDGMSM}^{(m,n;p_1,....,p_n)} (d_1, d_2, ..., d_n) \leq d^-. \quad (22)
\]

3.2. The q-rung orthopair fuzzy Dombi weighted generalized Maclaurin symmetric mean operator

Definition 8. Let $d_j = (h_j, g_j) (j=1,2,...,n)$ be a set of q-ROFNs, $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of $d_j (j=1,2,...,n)$, satisfying $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, $(i_1, i_2, ..., i_m)$ be all the m-tuple combination of $(1,2,...,n)$, and $m=1,2,...,n$. If
\[
q - \text{ROFDWGM}^{(m,n;p_1,....,p_n)} (d_1, d_2, ..., d_n) = \frac{\sum_{i=1}^{m} \prod_{j=1}^{n} \left( \frac{1}{w_j^{p_i}} - 1 \right)^{\frac{-1}{p_i}}}{\sum_{i=1}^{m}} \quad (23)
\]

then $q - \text{ROFDWGM}^{(m,n;p_1,....,p_n)}$ is called the q-ROF Dombi generalized Maclaurin symmetric mean operator, where $\binom{n}{m}$ is the binomial coefficient.

The q-ROFDGMSM operator also has the monotonicity and the boundedness.

3.3. The q-rung orthopair fuzzy Dombi generalized dual Maclaurin symmetric mean operator

Definition 9. Let $d_j = (h_j, g_j) (j=1,2,...,n)$ be a set of q-ROFNs, $(i_1, i_2, ..., i_m)$ be all the m-tuple combination of $(1,2,...,n)$, and $m=1,2,...,n$. If
\[
q - \text{ROFDGDMSM}^{(m,n;p_1,....,p_n)} (d_1, d_2, ..., d_n) = \frac{1}{p_1 + p_2 + ... + p_n} \prod_{i=1}^{p_i} \left( \sum_{j=1}^{n} d_j^{\frac{-1}{p_i}} \right)^{\frac{-1}{p_i}} \quad (24)
\]

then $q - \text{ROFDGDMSM}^{(m,n;p_1,....,p_n)}$ is the q-rung orthopair fuzzy Dombi generalized dual Maclaurin symmetric mean operator, where $\binom{n}{m}$ is the binomial coefficient.

Based on the Dombi T-norm and T-conorm operational rules of q-ROFNs, we can obtain the following aggregation result.

Theorem 6. Let $d_j = (h_j, g_j) (j=1,2,...,n)$ be a set of q-ROFNs, $(i_1, i_2, ..., i_m)$ be all the m-tuple combination of $(1,2,...,n)$, and $m=1,2,...,n$. By proving similar to Theorem 2, we can get that the aggregate value of q-ROFDGDMSM operator is still q-ROFN and
\[
q - \text{ROFDGDMSM}^{(m,n;p_1,....,p_n)} (d_1, d_2, ..., d_n) = \frac{1}{\sum_{i=1}^{p_i} \prod_{j=1}^{n} \left( \sum_{j=1}^{n} d_j^{\frac{-1}{p_i}} \right)^{\frac{-1}{p_i}}} \quad (25)
\]

The q-ROFDGDMSM operator also has the idempotency, monotonicity and the boundedness.
3.4. The q-rung orthopair fuzzy Dombi weighted generalized dual Maclaurin symmetric mean operator

**Definition 10.** Let $d_j = (h_j, g_j)$ ($j = 1, 2, ..., n$) be a set of q-ROFNs, $w = (w_1, w_2, ..., w_n)^\top$ be the weight vector of $d_j$ ($j = 1, 2, ..., n$), satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, $(i_1, i_2, ..., i_m)$ be all the m-tuple combination of $(1, 2, ..., n)$, and $m = 1, 2, ..., n$. If

$$q - ROFDWGMSM^{(n,p_1,p_2,..,p_m)}(d_1, d_2, ..., d_n) = \frac{1}{p_1 + p_2 + \ldots + p_m} \left\{ \prod_{j=1}^{m} \left( \sum_{i=j}^{n} (w_i d_i)^{p_j} \right)^{1/p_j} \right\}^{1/p_m}$$

then $q - ROFDG DSM^{(n,p_1,p_2,..,p_m)}$ is the q-ROF Dombi weighted generalized dual Maclaurin symmetric mean operator, where $C^m_n$ is the binomial coefficient.

Similarly, we can obtain the following aggregation result.

**Theorem 7.** Let $d_j = (h_j, g_j)$ ($j = 1, 2, ..., n$) be a set of q-ROFNs with a weight vector $\omega_j = (\omega_1, \omega_2, ..., \omega_n)$ satisfying $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, $(i_1, i_2, ..., i_m)$ be all the m-tuple combination of $(1, 2, ..., n)$, and $m = 1, 2, ..., n$. By proving similar to Theorem 2, we can get that the aggregate value of $q$-ROFDWGMSM operator is still a q-ROFN and

$$q - ROFDWGMSM^{(n,p_1,p_2,..,p_m)}(d_1, d_2, ..., d_n) = \frac{1}{c_{p_1}^{m-1}} \left( \sum_{i=1}^{n} \left( (w_i d_i)^{p_1} \right)^{1/p_1} \right)^{1/p_1}$$

The q-ROFDWGMSM operator also has the monotonicity and the boundedness.

4. A novel method for MAGDM with q-rung orthopair fuzzy information

**Step 1:** Normalize the decision matrix in the following method to eliminate the influence of different attribute types:

$$d_{ij} = (h_{ij}, g_{ij}) := \begin{cases} (h_{ij}, g_{ij}) & G_j \in I_1, \\ (h_{ij}, g_{ij}) & G_j \in I_2, \\ \end{cases}$$

where $I_1$ and $I_2$ represent the benefit-type criteria and the cost-type criteria respectively.

**Step 2:** Utilize the q-ROFDWGMSM operator

$$d_y = q - ROFDWGMSM^{(n,p_1,p_2,..,p_m)}(d_1, d_2, ..., d_n) ,$$

or the q-ROFDWGMSM operator

$$d_y = q - ROFDWGMSM^{(n,n,p_1,p_2,..,p_m)}(d_1, d_2, ..., d_n) ,$$

to fuse all decision makers’ evaluation values $a_{ij}$ in regard to each criteria value for each alternative.

**Step 3:** Utilize the q-ROFDWGMSM operator

$$d_y = q - ROFDWGMSM^{(n,n,p_1,p_2,..,p_m)}(d_1, d_2, ..., d_n) ,$$

or the q-ROFDWGMSM operator

$$d_y = q - ROFDWGMSM^{(n,n,p_1,p_2,..,p_m)}(d_1, d_2, ..., d_n) ,$$

to determine the collective overall preference value $a_i$ ($i = 1, 2, ..., m$).
Step 4: Calculate the score values $S(d_i)$ of the overall preference value $d_i (i = 1, 2, ..., m)$ by definition 2.

Step 5: Order the alternatives $\{x_1, x_2, ..., x_m\}$ and select the best one.

5. Numerical example

Next, we cite the examples of Liu PD and Liu JL [16] to illustrate the effectiveness and benefit of our approach. When an investment company faces five investment options, there is an investment company that wants to make some investments. Now this company has five investment options $\{x_1, x_2, x_3, x_4, x_5\}$. In order to make the best choice, the investment company invites three experts $\{D_1, D_2, D_3\}$ to be the decision-making committee to evaluate the five companies from four criteria $\{G_1, G_2, G_3, G_4\}$. The weight vector of the criteria is $\omega^T = [0.2, 0.1, 0.3, 0.4]$. Decision makers whose weight vector is $\lambda^T = [0.35, 0.40, 0.25]$ are required to use the $q$-rung orthopair fuzzy\ MAGDM to assess the five alternatives from four aspects respectively. Therefore, the decision matrices $A^k = [d^k_{ij}]$ can be obtained, which are shown in Tables 1-3.

Table 1. Decision matrix $R^1$ by $D_1$.

|   | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $x_1$ | (0.5, 0.4) | (0.5, 0.3) | (0.2, 0.6) | (0.4, 0.4) |
| $x_2$ | (0.7, 0.3) | (0.7, 0.3) | (0.6, 0.2) | (0.6, 0.2) |
| $x_3$ | (0.5, 0.4) | (0.6, 0.4) | (0.6, 0.2) | (0.5, 0.3) |
| $x_4$ | (0.8, 0.2) | (0.7, 0.2) | (0.4, 0.2) | (0.5, 0.2) |
| $x_5$ | (0.4, 0.3) | (0.4, 0.2) | (0.4, 0.5) | (0.4, 0.6) |

Table 2. Decision matrix $R^2$ by $D_2$.

|   | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $x_1$ | (0.4, 0.5) | (0.6, 0.2) | (0.5, 0.4) | (0.5, 0.3) |
| $x_2$ | (0.5, 0.4) | (0.6, 0.2) | (0.6, 0.3) | (0.7, 0.3) |
| $x_3$ | (0.4, 0.5) | (0.3, 0.5) | (0.4, 0.4) | (0.2, 0.6) |
| $x_4$ | (0.5, 0.4) | (0.7, 0.2) | (0.4, 0.4) | (0.6, 0.2) |
| $x_5$ | (0.6, 0.3) | (0.7, 0.2) | (0.4, 0.2) | (0.7, 0.2) |

Table 3. Decision matrix $R^3$ by $D_3$.

|   | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $x_1$ | (0.4, 0.2) | (0.5, 0.2) | (0.5, 0.3) | (0.5, 0.2) |
| $x_2$ | (0.5, 0.3) | (0.5, 0.3) | (0.6, 0.2) | (0.7, 0.2) |
| $x_3$ | (0.4, 0.4) | (0.3, 0.4) | (0.4, 0.3) | (0.3, 0.3) |
| $x_4$ | (0.5, 0.3) | (0.5, 0.3) | (0.3, 0.5) | (0.5, 0.2) |
| $x_5$ | (0.6, 0.2) | (0.6, 0.4) | (0.4, 0.4) | (0.6, 0.3) |

5.1. The decision making process

In this part, we use the proposed method for $q$-rung orthopair fuzzy MAGDM to solve the above problem.

Step 1: Since all the criteria are benefit type, there is no need for standardization.
**Step 2:** For each alternative, use the $q$-ROFDWGMSM operator to process the criteria values provided by decision maker $D_k$. Here we utilize Eq. (29) to aggregate decision makers’ preference. We assume $q=3$, $\lambda=2$, $m=2$ and $p_1=p_2=1$. So, we can obtain
\[
d_1 = (0.7779,0.8560), \quad d_2 = (0.7787,0.9908), \quad d_3 = (0.5027,0.3667), \quad d_4 = (0.5415,0.2647),
\]
\[
d_5 = (0.7924,0.9791), \quad d_6 = (0.8084,0.9881), \quad d_7 = (0.7419,0.9901), \quad d_8 = (0.5711,0.9901),
\]
\[
d_9 = (0.7779,0.8304), \quad d_{10} = (0.7694,0.8304), \quad d_{11} = (0.7786,0.9797), \quad d_{12} = (0.7707,0.3446),
\]
\[
d_{13} = (0.7925,0.9797), \quad d_{14} = (0.8105,0.9885), \quad d_{15} = (0.7705,0.8702), \quad d_{16} = (0.7787,0.9912),
\]
\[
d_{17} = (0.7062,0.9897), \quad d_{18} = (0.6892,0.9736), \quad d_{19} = (0.7333,0.7496), \quad d_{20} = (0.6892,0.2645).
\]

**Step 3:** Utilize Eq. (31) to aggregate decision makers’ preference, we can get
\[
d_1 = (0.4446,0.0000), \quad d_2 = (0.7752,0.0000),
\]
\[
d_3 = (0.3652,0.0002), \quad d_4 = (0.3642,0.0000), \quad d_5 = (0.6801,0.0000).
\]

**Step 4:** Calculate the scores $S(d_i)$ of $a_i$ based on the Definition 2 and we can get
\[
S(d_1) = 0.0696, \quad S(d_2) = 0.4495, \quad S(d_3) = 0.0359, \quad S(d_4) = 0.0341, \quad S(d_5) = 0.3136.
\]

**Step 5:** Therefore, the ranking result is $x_5 \succ x_1 \succ x_2 \succ x_1 \succ x_i$. So $x_5$ is the best option.

In Step 2, if we utilize the $q$-ROFDWGDMMSM operator to aggregate the criteria values provided by decision maker $D_k$ (We assume $q=3$, $\lambda=2$, $m=2$ and $p_1=p_2=1$). Thereafter, we utilize Eq. (32) to aggregate decision makers’ preference for each alternative. Thus, we have
\[
d_1 = (0.0006,0.1857), \quad d_2 = (0.2710,0.2008),
\]
\[
d_3 = (0.0006,0.2031), \quad d_4 = (0.0002,0.2209), \quad d_5 = (0.0003,0.1685).
\]

Then calculate the scores of the overall preference, we can get
\[
S(d_1) = -0.0062, \quad S(d_2) = 0.0129, \quad S(d_3) = -0.0075, \quad S(d_4) = -0.0108, \quad S(d_5) = -0.0047.
\]

Therefore, the ranking result is $x_5 \succ x_1 \succ x_2 \succ x_1 \succ x_4$. Thus, the company should invest its money to $x_5$ .we can know that the scores obtained by these two calculation methods are different, but the best choice is consistent, which preliminarily verifies the correctness of the methods.

5.2. The influence of the parameters on the final results

One of the prominent features of the proposed operators is that they have parameters $m$, $p$, $q$, and $\lambda$, leading to flexible and feasible aggregation processes. In this section, we consider the influence of these parameters on the overall values and the final ranking results according to different variables.

Firstly, we assign different values to $m$ and $p$ in the $q$-ROFDWGMSM operators when $\lambda=2$, $q=3$ and the scores and ranking orders are presented in Tables 4-5.

**Table 4.** Ranking results when $m=2$ by $q$-ROFDWGMSM operator.

| $P_1$ | $P_2$ | score function $S_i = S(d_i), i=1,2,3,4,5$ | ranking results in descending order |
|-------|-------|--------------------------------|-----------------------------------|
| 1     | 0     | 0.1391 0.4216 0.0203 0.0305 0.0296 | $x_2 \succ x_4 \succ x_5 \succ x_3 \succ x_1$ |
| 0     | 1     | 0.0438 0.3214 0.0168 0.0583 0.1637 | $x_2 \succ x_4 \succ x_5 \succ x_3 \succ x_1$ |
| 1     | 2     | 0.1725 0.4606 0.0302 0.0340 0.1219 | $x_2 \succ x_5 \succ x_4 \succ x_3 \succ x_1$ |
| 3     | 1     | 0.0366 0.4129 0.0302 0.0340 0.1219 | $x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$ |
| 1     | 4     | 0.0309 0.3755 0.0291 0.0317 0.0957 | $x_2 \succ x_5 \succ x_4 \succ x_1 \succ x_3$ |
| 5     | 1     | 0.0274 0.3364 0.0283 0.0298 0.0795 | $x_2 \succ x_5 \succ x_4 \succ x_1 \succ x_3$ |
| 2     | 1     | 0.0721 0.3476 0.0394 0.0316 0.2133 | $x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$ |
| 3     | 1     | 0.0684 0.2710 0.0395 0.0309 0.1531 | $x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$ |
| 4     | 1     | 0.1150 0.2283 0.0386 0.0305 0.1150 | $x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$ |
| 5     | 1     | 0.0603 0.1888 0.0372 0.0300 0.0894 | $x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$ |
0.5 0.5 0.0867 0.3276 0.0359 0.0268 0.4399 $x_2 > x_4 > x_1 > x_3 > x_5$
1 1 0.0696 0.4495 0.0359 0.0341 0.3136 $x_2 > x_3 > x_1 > x_4$
2 2 0.0544 0.3557 0.0338 0.0333 0.1413 $x_2 > x_5 > x_4 > x_1 > x_3$
3 3 0.0468 0.2901 0.0316 0.0313 0.0918 $x_2 > x_5 > x_3 > x_1 > x_4$
4 4 0.0419 0.2257 0.0298 0.0296 0.0701 $x_2 > x_3 > x_1 > x_5 > x_4$
5 5 0.0383 0.1692 0.0283 0.0281 0.0581 $x_2 > x_5 > x_3 > x_1 > x_4$

Table 5. Ranking results when $m = 3$ by $q$-ROFDWGMSM operator.

| $P_1$ | $P_2$ | $P_3$ | score function $S_i = S(d_i), i = 1, 2, 3, 4, 5$ | ranking results |
|-------|-------|-------|-------------------------------------------------|-----------------|
| 1     | 1     | 1     | $0.4699$ $0.0000$ $0.5540$ $0.4251$ $0.0280$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 1     | 1     | 2     | $0.2072$ $0.0001$ $0.6055$ $0.4999$ $0.0380$ | $x_3 > x_4 > x_1 > x_5 > x_2$ |
| 1     | 1     | 3     | $0.5363$ $0.0002$ $0.6418$ $0.5363$ $0.0462$ | $x_3 > x_4 > x_1 > x_5 > x_2$ |
| 1     | 1     | 4     | $0.5601$ $0.0004$ $0.6697$ $0.5619$ $0.0535$ | $x_3 > x_4 > x_1 > x_5 > x_2$ |
| 1     | 1     | 5     | $0.5801$ $0.0007$ $0.6922$ $0.5821$ $0.0607$ | $x_3 > x_4 > x_1 > x_5 > x_2$ |
| 1     | 2     | 1     | $0.5053$ $0.0000$ $0.6104$ $0.4729$ $0.0395$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 1     | 3     | 1     | $0.5331$ $0.0000$ $0.6487$ $0.5043$ $0.0475$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 1     | 4     | 1     | $0.5558$ $0.0000$ $0.6776$ $0.5285$ $0.0538$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 1     | 5     | 1     | $0.5749$ $0.0000$ $0.7006$ $0.5485$ $0.0800$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 2     | 1     | 1     | $0.5042$ $0.0000$ $0.5836$ $0.4372$ $0.0375$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 3     | 1     | 1     | $0.5315$ $0.0000$ $0.6024$ $0.4442$ $0.0471$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 4     | 1     | 1     | $0.5539$ $0.0000$ $0.6154$ $0.4490$ $0.0537$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 5     | 1     | 1     | $0.5727$ $0.0000$ $0.6254$ $0.4529$ $0.1140$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 2     | 1     | 1     | $0.5316$ $0.0000$ $0.6370$ $0.4868$ $0.0469$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 3     | 1     | 1     | $0.5726$ $0.0000$ $0.6880$ $0.5276$ $0.0611$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 4     | 1     | 1     | $0.6030$ $0.0000$ $0.7233$ $0.5588$ $0.1481$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 5     | 1     | 1     | $0.6269$ $0.0000$ $0.7495$ $0.5842$ $0.2536$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 2     | 2     | 1     | $0.5551$ $0.0001$ $0.6708$ $0.5519$ $0.0542$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 3     | 3     | 1     | $0.6049$ $0.0002$ $0.7312$ $0.6101$ $0.1400$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 4     | 4     | 1     | $0.6398$ $0.0005$ $0.7691$ $0.6464$ $0.2879$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |
| 5     | 5     | 1     | $0.6667$ $0.0009$ $0.7954$ $0.6722$ $0.4046$ | $x_3 > x_1 > x_4 > x_5 > x_2$ |

It can be seen from table 4-5 that when $m$ or $p$ takes different values, the ranking results may be different. When $m$ which representing the relationship between attributes changes, the number of $p$ values will change at first, and then if some of $p$ values change, the final result will inevitably change. This is because the relationship between attributes represented by different $m$ is different and the position of the parameter $p$ is in the exponential part, which will also affect the operation results. Therefore, when the relationship between attributes changes or the $p$ value of the exponential part changes, the ranking will change and the optimal selection will also change. This shows the flexibility of our method. Furthermore, Table 4-5 shows the ranking results of a series of alternatives after we change the parameters $m$ and $p$. When $m = 2$, we can know from Tables 4, with the increase of $1$ $p$, $2$ $p$, and $3$ $p$, the scores of alternatives produced by $q$-ROFDWGMSM operator decreases. And in this case the $q$-ROFDWGMSM operator reduces to the $q$-ROFDWBM. we can also know from Table 4 that with the increase of sum, the scores of $q$-ROFDWGMSM operators also increase. When $m = 3$, we can also know from Table 5 that with the increase of $1$ $p$, $2$ $p$, and $3$ $p$, the scores produced by the $q$-ROFDWGMSM operator increase.
Whether \( m = 2 \) or \( m = 3 \), we can get from the results in the table that when one of the \( p \) values is much larger than the other \( p \) values, if the maximum \( p \) value continues to increase, it will no longer affect the sorting results. On the contrary, if the maximum \( p \) value becomes smaller and smaller, and becomes close to or even equal to other \( p \) values, the sorting results will have some changes. Therefore, \( p \) can be regarded as an expert's assessment of risk, and changing the value of \( p \) will affect the result, which is also the biggest characteristic of GMSM that different from the traditional MSM operator.

In addition, the score values of the overall assessments become greater as \( p \) increases in the group which has the same balance. Therefore, the parameter \( p \) can reflect decision makers’ attitude.

In the following, we assign different \( \lambda \) to solve the same MAGDM example to discuss that how the parameter \( \lambda \) affects the scores of the overall values and study the final sorting results in proposed operators. Here, we assume \( \lambda = 3 \), \( m = 2 \) and \( p_1 = p_2 = 1 \). Details can be found in Figures 1 made by MATLAB software.

**Figure 1.** Score values of the alternatives when \( \lambda \in (1,10) \) based on the \( q \)-ROFDWGMSM operator.

Figure 1 is a change trend diagram of alternative score values obtained by the \( q \)-ROFDWGMSM operator. In order to explain the problem more clearly, we analyze the combination of Figure 1 and Table 4 and we can get the scores from Fig.1, which is also shown in Table 4. For example, we can get \( S(a_i) = 0.0696, S(a_j) = 0.4495, S(a_k) = 0.0359, S(a_l) = 0.0341, S(a_m) = 0.3136 \) when \( q = 3, m = 2, \lambda = 2 \) and \( p_1 = p_2 = 1 \) in the \( q \)-ROFDWGMSM operator. From Table 4 and Fig. 1, we can see that the scores of alternatives are different when assigning various parameters \( \lambda \) to the \( q \)-ROFDWGMSM operator. However, the best option is always the yellow one, which represents \( x_2 \). In addition, when the parameter \( \lambda \) ranges from 1 to 3, the score values of alternatives change sharply. Furthermore, when the parameter \( \lambda \) is more than 3, their images fluctuate regularly and periodically. Then, as the value of \( \lambda \) becomes greater and greater, the score values get closer and closer to a fixed value.

### 5.3. Comparative analysis

In the previous chapter, we explained the effectiveness of the proposed method. In this section, we will prove the superiority of our proposed method by comparing other methods. We conduct comparative analysis of the proposed methods based on the \( q \)-ROFDWGMSM and \( q \)-ROFDWGMSM operators in the present paper, with the \( q \)-ROFWBM operator proposed by Liu PD and Liu JL [16], the \( q \)-ROFWHM operator and the \( q \)-ROFWGHM operator proposed by Wei et al. [27], the \( q \)-ROFIWHM operator by Xing et al. [23], the \( q \)-ROFWMSM operator proposed by Wang et al. [28] for coping the above example. Results can be found in Table 6, and comparative analysis is shown in the followings.

**Table 6.** Score values and ranking results by using different methods

| Methods | Ranking results |
|---------|-----------------|
| \( q \)-ROFWHM operator [18] \( (s = 1, t = 1, q = 3) \) | \( S_1 = 0.5064, S_2 = 0.7660, S_3 = 0.4545 \) \( x_2 > x_4 > x_5 > x_1 > x_3 \) |
First of all, it is noted that the best option is always $x_1$, no matter what above method is selected. It can prove our methods’ effectiveness well. Furthermore, the $q$-ROFWHM, $q$-ROFWGHM, $q$-ROFWBM, and $q$-ROFWMSM operators are based on algebraic operations, while $q$-ROFDWHSM operators and our methods are based on Dombi operations. As we know, the Dombi operation includes algebraic operation, which is a more general form of algebraic operation. That is to say, the proposed operators are more powerful and general than above methods.

Xing et al.’s [23] method are on the basis of Hamy mean operator, Wang et al.’s [28] method are based on MSM. Both of them can consider the interrelationship among more arguments, decision makers’ attitude also can be reflected by $k$ and $q$. However, these parameter cannot reflect the attitude of the whole. In our methods, decision makers’ attitude can be showed more specifically by using the parameter $p$. Thus, the proposed methods are more general, feasible and reasonable than other methods.

6. Conclusions

In this paper, we proposed several new Dombi operations for $q$-ROFSs according to the Dombi t-norm and t-conorm. Subsequently, we extended the traditional generalized Maclaurin symmetric mean operator to $q$-ROFSs based on the Dombi operational rules, and proposed the $q$-ROFDGMSM, $q$-ROFDGDSM, and $q$-ROFDGDMSM operators. Further, we applied the operator proposed in this study to a new method, which is illustrated by an investment example, in which the criteria values are characterized by $q$-ROFSs. To better demonstrate the advantages and effectiveness of proposed method, we conducted some parametric and comparative analysis. Analysis results indicate that the main highlights of the proposed operators are: (1) The scores of the overall values and ranking results are different by assigning different values of four parameters $m,q,\lambda, p$ based on decision makers’ attitude and actual needs, which make information fusion processes more flexible. (2) The interrelationship between criteria values can be taken into account, which better reflects the actual situation. (3) The proposed operators provide a new method to aggregate $q$-ROFNs based on the Dombi t-norm and t-conorm, which is more general and powerful.

In future works, we will apply the proposed method to solve more practical MAGDM problems. In addition, considering the effectiveness and merits of the proposed Dombi generalized Maclaurin symmetric mean operators, we will study them under more fuzzy environments, such as hesitant fuzzy environment, uncertain linguistic environment.
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