PROPERTIES OF THE $B_c$ SYSTEM

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ABSTRACT

I summarize a comprehensive account of the energies, splittings and electromagnetic decays of the low-lying states of the bottom charmed meson system. Richardson’s potential is used to include running coupling constant effects in the central potential, and the full radiative one-loop expressions derived by Panteleone, Tye and Ng are used for the spin-dependent potentials. Our predicted result for the ground state energy is $6286^{+15}_{-6}$ MeV. The ground state lifetime is found to be $\tau_{B_c} = 0.36 \pm 0.05$ ps. Both of these numbers are consistent with the measured values recently announced by the CDF collaboration at Fermilab.

1. Introduction

The CDF collaboration at the Fermilab Tevatron has recently reported the discovery of $B_c$ mesons. Their announced value for the mass of the ground state is $6400 \pm 390 \pm 130$ MeV, and their value for the ground state lifetime is $\tau_{B_c} = 0.46^{+0.18}_{-0.16} \pm 0.03$ ps. This state should be one of a number of relatively stable states below the threshold for the emission of B and D mesons, and thus I want to present some results from a comprehensive calculation of the energies, splittings and electromagnetic decay rates of the low-lying states of the $B_c$ system.

In 1991 Kwong and Rosner predicted the masses of the two lowest states of this system from an empirical mass formula and a logarithmic potential. They obtained a range for the ground state, $6194 \leq M_{B_c} \leq 6292$ MeV. Eichten and Quigg gave a more comprehensive account of the properties of the $B_c$ system. Using several potential models to obtain error estimates, they quoted $6258 \pm 20$ MeV for the ground state energy. In anticipation of the experimental interest in the $B_c$ system, there have been a number of other potential model calculations.

2. Richardson’s potential and the spin-dependent potentials

Richardson’s potential contains a confining potential and a short range piece,

$$ V(r) = Ar - \frac{8\pi f(Ar)}{(33 - 2n_f)r}, \quad f(t) = \frac{4}{\pi} \int_0^\infty \frac{\sin tx}{x} \left[ \frac{1}{\ln(1 + x^2)} - \frac{1}{x^2} \right] dx, \quad (1) $$
where the string constant will be treated as a free parameter instead of a function of \( \Lambda \) in order to achieve a separation of the long and short distance parameters.

To specify the spin-dependent potentials, we will use Pantaleone, Tye and Ng’s (PTN) generalization \(^\text{7}\) of the Eichten-Feinberg formalism. Thus

\[
V_{SD} = \frac{L \cdot S_1}{2m_1^2} \left[ \frac{1}{r} \frac{d}{dr} (E(r) - Ar + 2V_1) + 2V_5 \right] + \frac{L \cdot S_2}{2m_2^2} \left[ \frac{1}{r} \frac{d}{dr} (E(r) - Ar + 2V_1) - 2V_5 \right]
+ \frac{L \cdot (S_1 + S_2)}{m_1 m_2} \frac{1}{r} \frac{d}{dr} V_2 + \frac{L \cdot (S_1 - S_2)}{m_1 m_2} V_5 + \frac{(S_1 \cdot \hat{r} S_2 \cdot \hat{r} - \frac{1}{3} S_1 \cdot S_2)}{m_1 m_2} V_3 + \frac{S_1 \cdot S_2}{3m_1 m_2} V_4,
\]

where the component potentials \( E, V_1, V_2, V_3, V_4 \) and \( V_5 \), include both the tree-level and the full radiative one-loop contributions. These may be expressed in terms of the strong coupling constant \( \alpha_{\bar{S}} \), the renormalization scale \( \mu \) and the constituent masses.

As an example, we list the second spin-orbit potential,

\[
V_2(r) = -\frac{4\alpha_{\bar{S}}}{3r} \left[ 1 + \frac{\alpha_{\bar{S}}}{6\pi} \left( 33 - 2n_f \right) (\ln \mu r + \gamma_E) + \frac{39}{2} - \frac{5n_f}{3} - 9(\ln \sqrt{m_1 m_2 r} + \gamma_E) \right],
\]

where \( \gamma_E \) is Euler’s constant. All of the PTN component potentials are given in the modified minimal subtraction scheme. The central potential parameters are assumed to be flavor independent. These parameters and the constituent masses are determined from the low-lying states of charmonium and the upsilon system. Their values are

\[
A = 0.152 \text{ GeV}^2, \ \Lambda = 0.431 \text{ GeV}, \ m_b = 4.889 \text{ GeV}, \ m_c = 1.476 \text{ GeV},
\]

and \( n_f \) is taken to be 3. For the upsilon system the value of the coupling constant \( \alpha_{\bar{S}} = 0.30 \), and the value of the renormalization scale \( \mu = 1.95 \text{ GeV} \). For charmonium these values are \( \alpha_{\bar{S}} = 0.486 \) and \( \mu = 0.80 \text{ GeV} \). In both cases the universal QCD scale \( \Lambda_{QCD} = 0.190 \text{ GeV} \), a value consistent with other determinations. These parameters give an average deviation of 4.3 MeV for the upsilon states below the continuum threshold and an average deviation of 19.9 MeV for the corresponding charmonium states. The one-loop potentials give a substantial advantage in simultaneously accounting for the fine structure and the hyperfine structure of charmonium and the upsilon system. Thus, it is reasonable to expect they should produce better results for the spin-dependent effects in the \( B_c \) system.

3. Results and conclusions

Since the central potential parameters are assumed to be strictly flavor independent and the constituent masses are not allowed to run, the only decision that one has to make to treat the \( B_c \) system is to choose a value for \( \alpha_{\bar{S}} \). We choose \( \alpha_{\bar{S}} = 0.393 \), the average of the values for charmonium and the upsilon system. Then preserving the value of the universal QCD scale requires that \( \mu = 1.12 \text{ GeV} \). Our calculated results for the three lowest levels of the \( B_c \) system are shown in Figure 1, where they are compared with earlier potential model calculations \(^\text{4,5}\) and recent lattice calculations. The 190
MeV error associated with uncertainty of the overall energy scale of the lattice calculation is not shown. In Figure 1 FUI98 denotes the one-loop results and the FUI98 results are based on tree-level expressions for the spin-dependent potentials, and one can see substantial differences for both the S and the P states. The FUI98 results are very close to the results of Eichten and Quigg, as expected.

An important difference between the spectrum of the $B_c$ system and those of charmonium and the upsilon system arises because the total spin $S = S_1 + S_2$ no longer commutes with the Hamiltonian. Thus one must compute a mass-mixing matrix for P states, and it convenient to use the j-j coupling scheme for this purpose. After computing the matrix representatives of each of the spin-dependent operators in the spin-dependent potential, one finds that the $J = 2$ states and the $J = 0$ state are the same as in the L-S basis, but that the $J = 1$ states are mixtures. Our result for the lowest $J = 1$ state is

$$\psi_{1m}(1^+) = 0.118\psi_{1m}(3/2, 1/2) + 0.993\psi_{1m}(1/2, 1/2),$$

which is very near the j-j coupling limit, a result consistent with the lattice calculations. In this regard we differ from Eichten and Quigg whose tree-level results were much closer to the L-S limit. This difference is reflected in the character of the spectrum of photons following the dipole radiative decays of the 1P state. Assuming an equal distribution of initial populations, our results for the photon spectrum of this state contains 6 distinct lines, as shown in Figure 2, instead of the 4 one would expect in the L-S limit.
To estimate the uncertainty of our approach, we have allowed $\alpha_S$ to vary from its charmonium value (0.486) to its upsilon value (0.30). Holding $\Lambda_{QCD} = 0.190 \, GeV$ allows an error estimate of the ground state, that is, $M_{B_c} = 6286^{+15}_{-6} \, MeV$. In order to determine the lifetime of the ground state, we have considered $\bar{b}$-quark decay, where the charmed quark is a spectator, and c-quark decay, where the antibottom quark is a spectator, and the annihilation channels reached by the $B_c$ decay constant. Thus $\Gamma_{tot}(B_c) = 1.85 \pm 0.24 \times 10^{-3} \, eV$, where most of the error arises from the uncertainty in the Kobayashi-Maskawa matrix element $V_{bc}$.

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