Research Article

Vibration Characteristics and Reduction Analysis of Lifting Pipe under Ocean Current

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1.Introduction

The development of renewable energy and the application of high and new technology need the support of new infrastructure that will consume a different mix of minerals from current applications, including not only "critical" metals such as rare earths, but also vast amounts of common commodities such as copper, steel, and cement [1]. The huge potential of marine mineral resources has been confirmed by many researchers, and deep ocean mining is being pushed to the next frontier of resource development [2]. Rich marine mineral resources will become irreplaceable resources for human development and utilization. Therefore, the development of marine mineral resources has become the strategic goal of all countries in the world. Companies in China, the UK, Belgium, Germany, France, and Japan have won contracts for exploration of deep-sea resources for three different mineral resources: seafloor massive sulphides, ferromanganese crusts, and poly metallic nodules [3]. In 2019, commercial deep-sea mining has begun off the coast of Papua New Guinea [4].

The solid mineral resources from the deep-sea bottom, such as ferromanganese nodules and cobalt-manganese crusts, most common ferromanganese nodules, are found typically at water depths of 4000–7000 m, and in the coastal areas of the shelf [5]. The deep-sea mining system consists of an ore collection system, a mother station, and a transportation system between the miner and mother station [6]. The transportation system is an important part of deep-sea mining system, which transports the collected manganese nodules from the miners to the mother station.

The research on the dynamic performance of the transportation system is of great significance to the design, performance evaluation, and control of the deep-sea mining system [7]. Under the influence of wind and waves in the complex marine environment, the mining vessel will produce surge, sway, heave, pitch, roll, and yaw motion. Obviously, the movement of the mining vessel will produce a motion excitation to the lifting pipe, which affects the reliability of the transportation system and the sea miner; especially the heave direction motion caused by ocean wave...
motion has a significant impact on the safety and stability of deep-sea mining system operation.

The longitudinal vibration of lifting pipe has been continuously researched and improved since deep-sea mining began to be studied. In order to reduce the longitudinal vibration of the lifting pipe, Aso et al. [8, 9] proposed a vibration absorber composed of a mass, springs, and dampers to the pump module as well as the buffer, and gave an active control system to control the longitudinal vibration of the pipe string [10]; the resonance amplitude at the top of the lifting pipe is greatly reduced. Kobayashi and Aso [11] installed a linear and a non-linear spring supports on the upper end of the lifting pipe, respectively; they found that linear spring supports often increase the resonance frequency of the string and the resonance amplitude of the buffer, and reduce the maximum axial load of the string. Cui and Aso [12] systematically studied the longitudinal vibration of the lifting pipe by Galerkin method, and proposed a method to reduce the amplitude and axial load of the lifting pipe by using additional vibration damper and sectional lifting. Using the method of separating variables, Erol [13] studied the overdamping and underdamping modes of the hoisting system, respectively, and obtained the exact analytical solutions of free response and forced response of the system damping under the heave motion of the mining vessel. Yu and Liu established a comprehensive dynamic model of oribit terrain interaction to adapt to China’s deep-sea mining system, and the simulation analysis of the vertical reciprocating motion process was carried out [14]. Oh et al. [15] designed a coupling device with rotational freedom for deep-sea mining flexible pipe and analyzed the motion characteristics of the coupling device with multi-body dynamic simulation method and finite element method. Kim et al. [16] used the subsystem synthesis method to establish the motion equations of the vessel lifting pipe system and the flexible mining robot subsystem, respectively, which improves the efficiency of the solution. Lee et al. [17] compared the finite difference method with lumped mass and the substructure method from the field of flexible multi-body dynamics in the deep seabed mining system, and it was reported that the finite difference method with lumped mass is more efficient than the substructure method if the same number of elements was used in the model. Ma et al. [18] compared the continuous line bucket lifting system and the pipe lifting system in terms of the lifting efficiency, the energy consumption, and the profitability and pointed out that the mining depth and the solid mineral production rate of the pipe lifting system have a wider application range compared with the continuous line bucket system. Yang and Liu [19] proposed a measuring method for solid-liquid two-phase flow in slurry pipeline for deep-sea mining based on differential capacitively coupled contactless conductivity detection. Yang and Liu [20] put forward a heave compensation system combined with a vibration absorber and accumulator, and the frequency domain characteristics and compensation rate of the system were analyzed and compared. Li et al. [21] used the discrete element method, computational fluid dynamics method, to study the solid-liquid two-phase flow in the deep-sea elevator pump and obtained the pressure distribution on the suction surface and pressure surface and the distribution of particles in the pump under different rotating speeds. Liu et al. [22] used Euler Lagrange two-phase flow model to simulate the turbulent flow of solid-liquid two-phase in the pump and used Finnie wear model to predict the wear rate of hydraulic components of deep-sea mining pump under different flow rate, rotation speed, and particle concentration. Cho et al. [6] studied the performance of the deep-sea manganese nodule experimental miner, designed the ore guiding system composed of chassis structure, buoyancy, crushing system, electric control system, track, and sea miner, and analyzed the coupling relationship of the ore guiding system by using the axiomatic design method. Liu et al. [23, 24] analyzed the longitudinal vibration characteristics of lifting pipe under different moving based on the finite element method. Song et al. [25] firstly used the Lagrangian method to establish the dynamic model of the rigid space stepped-pipe strings system and obtained the analytical solution by modal analysis method.

It can be seen from the above references that the current research on deep-sea mining system focuses on three aspects: mining sea miner [6, 16], pump set [21, 22], and lifting pipe [8–14, 18, 23–25]. The research on lifting pipe mainly focuses on two directions: reducing the longitudinal vibration of the lifting pipe with additional vibration damper [8–10], computer simulation with dynamic simulation method, and finite element analysis [14, 19].

At present, these researches are focused on the lifting pipe within one kilometer and did not consider that the lifting pipe will rotate at a certain angle when it is affected by the complex environmental load. However, most of the solid mineral resources in the deep sea occur in the depth of 4000–7000 meters [5]. Therefore, it is necessary to study the performance of the lifting system close to the actual sea depth. The goal of this paper is to offer a quantitative assessment of the influence of ocean current on the vibration characteristics of lifting pipe during deep-sea mining operation. Thus, according to the mechanism of deep-sea mining, physical model and analytical model of lifting system are designed. Second, the current velocity is divided into four cases by means of linear least-square approximate subsection method to solve the geometric nonlinear migration problem of ocean current, and the linear motion balance equation is deduced. Then, considering the effect of ocean current on the lifting subsystem, the dynamic models of the lifting subsystem with or without dynamic vibration absorber are established. Meanwhile, a finite element model for deep-sea mining with the OrcaFlex software is developed to simulate the dynamic responses of lifting pipe under the action of ocean current. Last, the dynamic model of lifting subsystem is verified by comparing the experimental curve and the simulation curve.

2. Model Setup and Linear Approximate Calculation under Current Force

2.1. Model Setup under Current Force. The physical model of the lifting system is a 1:80-scale prototype tested in National Ocean Technology Center of China, as shown in Figure 1. The mining vessel and lifting pipe in the model are
approximate to the objective entity. The wave load of the deep-sea mining system is provided by the wave making system and the wave height sensor. The motion state of the mining vessel and the axial force of the lifting pipe are obtained by using pull pressure sensor, attitude sensor, signal processing module, and signal acquisition software. In this study, the influence of ocean current on lifting pipe and the vibration reduction analysis of dynamic vibration absorber are mainly considered.

Figure 2 shows the general model of the lifting pipe subsystem in deep-sea mining under the action of ocean current, which is mainly composed of lifting pipe (four parts), lifting pumps (concentrated mass: \(M_1\)), and buffer (concentrated mass: \(M_2\)). The upper end of the lifting pipe is hinged with the mining vessel, the lower end of the lifting pipe is connected with the buffer, and the lifting pump set is installed at \(L_2\) from the top of the lifting pipe. \(L_{S_i}\) \((i = 1, 2, 3, 4)\) is the segment length of the pipe sting in still water, and \(L_{S1} = 500\) m, \(L_{S2} = 1000\) m, \(L_{S3} = 2000\) m, \(L_{S4} = 3500\) m, \(L_{S5} = 5000\) m. The current forces brought to bear on each segment are \(q_{S1}, q_{S2}, q_{S3}, q_{S4}\), and \(q_{S5}\). In addition, \(g\) is the acceleration of gravity and \(\gamma\) is the linear density of the lifting pipe.

As can be seen from Figure 2, the lifting pipe has four parts, the length of each part is \(L_s\), and the basic parameters of lifting pipe are shown in Table 1.

2.2. Linear Approximate Calculation under Ocean Current. In order to solve the geometric nonlinear problem of current velocity, a linear approximate solution of current velocity in deep-sea mining is proposed in this paper. In this linear approximation model, the lifting pipe string is regarded as a rigid body. When the string is placed in still water, the two-dimensional coordinate system \(W-O-V\) is established, and the \(O-W\) axis is vertical downward, and the positive direction of \(O-V\) axis is consistent with the wave propagation direction. When it is placed in ocean current, the string will shift angle \(\theta\), and its coordinate system is \(X-O-Y\).

The stability analysis of deep-sea mining lifting pipe involves ocean current, resistance coefficient, Reynolds number (Re), and other factors [26]. In particular, the velocity and direction of the ocean current vary significantly with depth and thus affect the dynamic characteristics of the lifting pipe. Using computer fitting methods, the current velocity is calculated depending on its depth below ocean surface [26]; one could obtain

\[
V_c = b + k \times \left(\frac{5000 - x}{5000}\right)^{12},
\]

where \(x\) is the depth below ocean surface, and the value is zero on the seawater surface. \(b\) and \(k\) are constants and coefficients related to current resistance and here we take 0.1 and 1.6, respectively.

The fitting current velocity is shown as A-curve in Figure 3. Obviously, the current velocity becomes smaller and smaller with the deepening of the seawater depth and it no longer increases with the deepening of the seawater depth, but presents a steady flow when the seawater depth exceeds 2000 meters. Since the nonlinear complex models are generally difficult to solve, the current velocity \(V_C\) can be linearized into four cases by means of linear least-square approximate method, and they can be expressed as

\[
\begin{aligned}
V_{C1} &= \left(-\frac{1.15}{500}\right) x + 1.7, \quad 0 \leq x \leq L_{S1}, \\
V_{C2} &= \left(-\frac{0.34}{500}\right) x + 0.89, \quad L_{S1} \leq x \leq L_{S2}, \\
V_{C3} &= \left(-\frac{0.055}{500}\right) x + 0.32, \quad L_{S2} \leq x \leq L_{S3}, \\
V_{C4} &= 0.1, \quad L_{S3} \leq x \leq L_{S5}.
\end{aligned}
\]

Figure 3 shows a comparison of the fitting and linearized current velocity with ocean depth. The red-A line is the fitting current velocity curve, and the blue-B line is the current velocity after linearization. It can be clearly seen that the linearized curve is very close to the actual curve.

The current force per unit length acting on the lifting pipe can be expressed by Morison equation as

\[
f_N(x) = \frac{1}{2} C_D \rho_w D |V_N| |V_N|, \tag{3}
\]

where \(C_D\) is the drag coefficient, \(C_D = 1.2, \rho_w\) is the seawater density, \(D_i\) \((i = 1, 2, 3, 4)\) is the external diameter of the lifting pipe, and \(V_N\) is normal current velocity of \(V_C\).

According to the linearization of the current velocity in equation (2), the linearization of the normal current force on the lifting pipe can be expressed as follows:

\[
q_{N_i}(x) = a_i x + b_i, i = 1, 2, 3, 4, \tag{4}
\]

where \(a_i\) and \(b_i\) are related to current velocity and current force per unit length of lifting pipe.

For \(a_i\)

\[
\begin{align*}
a_1 &= \frac{f_N(L_{S1}) - f_N(0)}{1000}, \\
a_2 &= \frac{f_N(L_{S2}) - f_N(L_{S1})}{1000}, \\
a_3 &= \frac{f_N(L_{S3}) - f_N(L_{S2})}{1000}, \\
a_4 &= a_5 = 0.
\end{align*}
\]
Table 1: Basic parameters of the lifting pipe subsystem.

| Pipe section | Length (m) | Internal diameter, d (mm) | External diameter, D (mm) | Wall thickness, h (mm) | Density, ρ (kg/m³) |
|--------------|------------|---------------------------|--------------------------|-----------------------|-------------------|
| $L_1$        | 1000       | 206                       | 254                      | 24                    |                   |
| $L_2$        | 1000       | 206                       | 240                      | 17                    |                   |
| $L_3$        | 1500       | 206                       | 232                      | 13                    |                   |
| $L_4$        | 1500       | 206                       | 226                      | 10                    | 7850              |

Figure 1: Physical model of the lifting subsystem.

Figure 2: The mechanical model of the lifting pipe subsystem under the action of ocean current. (a) Mechanical model. (b) Force microelement. (c) DVA.

Figure 3: The current velocity curves below ocean surface.
where \( q_{Ni}^w \) is the unit load of \( q_{Ni}(x) \) in \( L_{si} \), that is, \( q_{Ni}^w = q_{Ni}(L_{si}) \). Here, the \( R_{M1} \) and \( R_{M2} \) are constants related to the quality, and \( R_{M1} = M_1, R_{M2} = M_2 \) when the pipe is not equipped with a vibration absorber.

### 3. Vibration Analysis of the Lifting Subsystem without DVA

#### 3.1. Mathematical Model of Longitudinal Vibration

When a wave propagates forward, it moves sinusoidally and its shape remains unchanged \[27\], so the mining vessel is subjected to a harmonic motion with heave amplitude \( \eta_0 \) at frequency \( \Omega \), represented as \( \eta(t) = \eta_0 e^{i\Omega t} \). In the \( W-O-V \) coordinate system, the excitation motions of the \( O-V \) axis and the \( O-W \) axis can be represented by \( \eta_x(t) = \eta_0 e^{i\Omega t} \sin \theta \) and \( \eta_y(t) = \eta_0 e^{i\Omega t} \cos \theta \). When the lifting pipe is working, the mass per unit length of lifting pipe \( y_i \) is calculated by the following formula:

\[
y_i = \frac{\pi D_i^2 - d_i^2}{4} \rho - \frac{\pi D_i^2 \rho_w}{4}, \quad (i = 1, 2, 3, 4), \tag{7}
\]

where \( D_i \) and \( d_i \) are the external and internal diameters of each lifting pipe \( L_i \), respectively. \( \rho \) is the density of lifting pipe material, and \( \rho_w \) is seawater density.

The mechanical model of the lifting subsystem is shown in Figure 2. The lifting pipe has four parts, the length of each part is \( L_i \), the elastic modulus is \( E_i \), the cross-sectional area is \( S_i \), and the mass per unit length is \( y_i \). The viscous damping coefficient of the surrounding seawater is \( c_i \), and the relevant physical parameters are given in Table 2.

Under the action of ocean current, the lifting pipe will rigidly offset a certain angle \( \theta \) around the origin \( O \) when the system is balanced. According to the moment balance condition at the origin, the equation of moment balance can be deduced as

\[
\begin{align*}
\frac{L_{s1}}{3} & \left( \int_{t_0}^t q_{Ni} dx - q_{Ni}^w L_{s1} \right) + \frac{q_{Ni}^w L_{s1}^2}{2} + \sum_{n=1}^{3} \frac{2L_{s(n-1)} + L_{sn}}{3} \left( \int_{t_{n-1}}^{t_n} q_{Ni} dx - q_{Ni}^w L_{sn} + q_{Ni}^w L_{s(n-1)} \right) + \left( \frac{L_{s1}^2 - L_{s1(n-1)}^2}{2} \right) q_{Ni}^w \\
& - \left( \int_{t_0}^t \gamma_1 g \sin \theta dx + \sum_{n=4}^{5} \frac{L_{sn} + L_{s(n-1)}}{2} \int_{t_{n-1}}^{t_n} q_{Ni} dx - \sum_{n=2}^{4} \frac{L_{sn} + L_{s(n-1)}}{2} \int_{t_{n-1}}^{t_n} q_{Ni} dx \right) \gamma_1 g \sin \theta dx - [R_{M1} L_{s2} + R_{M2} L_{s5}] g \sin \theta = 0,
\end{align*}
\]

(6)

The axial displacement of each section is usually different because the lifting pipe is composed of infinite particles. Therefore, the displacement of any section \( x \) of the \( i \)-th pipe is a function of position and time, denoted as \( u_i(x,t) \). The equations of motion of the lifting pipe can be established as

\[
E_i S_i \frac{\partial^2 u_i}{\partial x^2} - \gamma_1 \frac{\partial^2 u_i}{\partial t^2} - c_i \frac{\partial u_i}{\partial t} = 0, \quad i = 1, 2, 3, 4. \tag{8}
\]

Set \( b_i = \sum_{j=1}^{i} L_j \) (\( i = 1, 2, 3, 4 \); then the boundary and the continuity conditions of lifting subsystem in the \( X-O-Y \) coordinate system can be formulated as

\[
\begin{align*}
& \quad u_1(0,t) = 0, \\
& u_2(b_i, t) = u_{i+1}(b_i, t),
\end{align*}
\]

(9)

The following equations are introduced to represent the force continuity condition:

\[
\begin{align*}
E_i S_i \frac{\partial^2 u_i}{\partial x^2} \bigg|_{x=b_i} + M_i \frac{\partial^2 u_i}{\partial t^2} \bigg|_{x=b_i} = E_2 S_2 \frac{\partial u_2}{\partial x} \bigg|_{x=b_i}, \\
E_i S_i \frac{\partial u_i}{\partial x} \bigg|_{x=b_i} = E_{i+1} S_{i+1} \frac{\partial u_{i+1}}{\partial x} \bigg|_{x=b_i}, \quad i = 2, 3, \\
E_4 S_4 \frac{\partial^2 u_4}{\partial x^2} \bigg|_{x=b_i} + M_4 \frac{\partial^2 u_4}{\partial t^2} \bigg|_{x=b_i} = 0.
\end{align*}
\]

(10)
The method of separating variables $u_i(x,t) = \phi_i(x) \times q(t)$ is introduced, and equation (8) is transformed into the following form:

$$\frac{E_iS_i}{y_i}\frac{d^2\phi_i(x)}{dx^2} = \left(1 - \frac{c_i}{y_i}\frac{dq(t)}{dt}\right)\frac{dq(t)}{dt} = a_i,$$

where $a_i$ are complex constants to be determined.

Letting $q(t)$ in equation (11) be represented by $e^{j\Omega t}$, here, $\lambda$ is an eigenvalue of vibration differential equation; then equation (11) takes the form

$$\lambda^2 + \frac{c_i}{y_i}\lambda = a_i, \quad i = 1, 2, 3, 4. \tag{12}$$

Setting $v_i^2 = (y_i/E_iS_i)a_i$, the first equation in the above equation (12) is expressed as

$$\frac{d^2\phi_i(x)}{dx^2} - v_i^2\phi_i(x) = 0, \quad i = 1, 2, 3, 4. \tag{13}$$

Suppose that the general solution of equation (13) is expressed as

$$\phi_i(x) = (A_ie^{v_i x} + B_ie^{-v_i x}), \quad i = 1, 2, 3, 4, \tag{14}$$

where $A_i$ and $B_i$ are complex constants to be determined; the equation is equivalently expressed by the following equation:

$$(A_1 + B_1)e^{j\Omega t} = \eta_0 e^{j\Omega t} \cos \theta. \tag{15}$$

According to the theory of solution of differential equation and [13], $\lambda$ must be equal to $j\Omega$ ($\lambda = j\Omega$). The boundary and the force continuity conditions given in equations (9)–(10) are transformed into the following forms as

$$\begin{align*}
\phi_1(b_i, t) &= \phi_{i+1}(b_i, t), \\
E_iS_1 \frac{d\phi_1(b_i)}{dx} - M_i\phi_1(b_i)\lambda^2 &= E_2S_2 \frac{d\phi_2(b_i)}{dx}, \\
E_iS_i \frac{d\phi_i(b_i)}{dx} - M_i\phi_{i+1}(b_i)\lambda^2 &= 0.
\end{align*} \tag{16}$$

Substituting equation (14) and the substitution of the expression $\lambda = j\Omega$ into equations (16) and (15) lead to

$$G \times [A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4]^T = [\eta_0 \cos \theta, 0, 0, 0, 0, 0, 0, 0, 0]^T, \tag{17}$$

where

The table below shows the relevant physical parameters of the lifting pipe subsystem.

| $i$ | $L_i$ (m) | $y_i$ (kg/m) | $S_i$ (m$^2$) | $M_i$ (kg) | $c_i$ (N·s/m) | $E_i$ (MPa) |
|-----|------------|--------------|--------------|------------|--------------|-------------|
| 1   | 1000       | 175.13       | 0.0173       | 8000       | 400          | 20600       |
| 2   | 1000       | 120.27       | 0.0119       | 30000      | 400          | 20600       |
| 3   | 1500       | 90.32        | 0.0089       | —          | 400          | 20600       |
| 4   | 1500       | 68.53        | 0.0068       | —          | 400          | 20600       |

\[ \text{Table 2: Relevant physical parameters of the lifting pipe subsystem.} \]
Note that the coefficients $\alpha_i$, $\beta_i$, and $\kappa_i$ are the expressions of $M_i$, $E_i$, $S_i$, $v_i$, and $\lambda_i$, as follows:

$$
\begin{align*}
\alpha_1 &= M_1 \lambda^2 + E_1 S_1 v_1, \\
\alpha_4 &= M_4 \lambda^2 + E_4 S_4 v_4, \\
\beta_1 &= M_1 \lambda^2 - E_1 S_1 v_1, \\
\beta_4 &= M_4 \lambda^2 - E_4 S_4 v_4, \\
\kappa_i &= E_i S_i v_i, \quad i = 2, 3, 4.
\end{align*}
$$

Program (code) was written in MATLAB R2013a environment to model the longitudinal vibration of lifting pipe in the deep-sea mining system using equations (8) to (17). The physical parameters required in the model are listed in Table 2, and the calculation results in equation (17) are shown in Table 3, where $j$ is the imaginary unit. Therefore, the general solution of equation (13) $\phi_i(x)$ and the forced longitudinal displacements of lifting pipe $u_i(x, t)$ are determined.

### 3.2. Influence of Ocean Current on the Displacement of the Lifting Pipe

This paper mainly studies the influence of the ocean current on the displacement of the lifting subsystem. The ocean current has little influence on the lifting pipe, but it has great influence on its lateral displacement, which is enough to affect the normal operation of the sea miner.

The Orcaflex software is used to calculate the longitudinal vibration displacement of lifting pipe at 1000 m, 2000 m, 3500 m, and 5000 m deep sea, as shown in Figure 6(a), and the corresponding longitudinal vibration curve is shown in Figure 6(b). At the same time, MATLAB is used to solve the differential equation of the lifting subsystem, and its longitudinal vibration displacement is shown in Figure 7. The comparison of longitudinal vibration simulation results is shown in Figure 8.

By carefully comparing Figures 6–8, it is not difficult to find that the numerical simulation results in MATLAB and Orcaflex show good agreement, and the longer the lifting pipe, the greater the longitudinal displacement. In addition, Table 4 lists the longitudinal vibration displacement errors at different times and locations. It was obvious that the calculating results obtained by MATLAB are in accordance with the simulation results obtained by Orcaflex finite element model, and the longitudinal amplitude error at the end of the lifting pipe is the largest, only 5.07%.

### 3.3. Axial Stress of the Lifting Pipe under Current Force

The total axial stress of the lifting pipe is composed of the gravity stress caused by its own weight and the dynamic stress caused by the vibration of the lifting pipe.

The total axial stress of the lifting pipe is determined by the structure, elastic modulus, cross-sectional area, linear weight, linear damping coefficient, and other factors. Using mechanical knowledge, the gravity stress of each stepped lifting pipe can be calculated as
Table 3: Calculation values of $\overline{A}_i$ and $\overline{B}_i$ in equation (17) under ocean current.

| $i$ | $\overline{A}_i$ | $\overline{B}_i$ |
|-----|------------------|------------------|
| 1   | $-0.3230 + 0.0648j$ | $1.8180 - 0.0648j$ |
| 2   | $-0.0103 + 0.0030j$ | $1.7687 - 0.8299j$ |
| 3   | $0.0001 - 0.0001j$ | $2.6000 - 0.5900j$ |
| 4   | $-0.0005 + 0.0002j$ | $4.3810 + 0.7707j$ |

Figure 4: Simulation model based on Orcaflex: (a) solid edge model, (b) ocean current parameter setting, and (c) heave motion RAO curve.
Figure 5: Longitudinal and lateral displacement of lifting pipe. (a) Longitudinal displacement. (b) Lateral displacement.

Figure 6: Simulation results of longitudinal vibration displacement based on Orcaflex: (a) calculation results and (b) longitudinal vibration curve.
Figure 7: Longitudinal vibration displacement based on MATLAB.

Figure 8: Comparison of longitudinal vibration simulation results.
Under the action of oceanic wave $\eta(w,t)$, the lifting pipe and the mining vessel heave at the same frequency, so the dynamic stress on the lifting pipe is only related to the longitudinal displacement $u_i(x,t)$, and then the axial dynamic stress produced in the pipe can be obtained by the following equation:

$$
\sigma_{gl}(x,t) = \frac{M_1 + M_2 + y_1(L_1 - x) + (y_2L_2 + y_3L_3 + y_4L_4)}{S_1}g\cos\theta, \quad 0 \leq x \leq L,
$$

$$
\sigma_{g2}(x,t) = \frac{M_2 + y_2(L_1 + L_2 - x) + (y_3L_3 + y_4L_4)}{S_2}g\cos\theta, \quad L_1 \leq x \leq L_1 + L_2,
$$

$$
\sigma_{g3}(x,t) = \frac{M_2 + y_3(L_1 + L_2 + L_3 - x) + y_4L_4}{S_3}g\cos\theta, \quad L_1 + L_2 \leq x \leq L_1 + L_2 + L_3,
$$

$$
\sigma_{g4}(x,t) = \frac{M_2 + y_4(L_1 + L_2 + L_3 + L_4 - x)}{S_4}g\cos\theta, \quad L_1 + L_2 + L_3 \leq x \leq L_1 + L_2 + L_3 + L_4.
$$

Under the action of oceanic wave $\eta(w,t)$, the lifting pipe and the mining vessel heave at the same frequency, so the dynamic stress on the lifting pipe is only related to the longitudinal displacement $u_i(x,t)$, and then the axial dynamic stress produced in the pipe can be obtained by the following equation:

$$
\sigma_{gl}(x,t) = E \frac{\partial u_i(x,t)}{\partial x},
$$

4. Vibration Analysis of the Lifting Subsystem with DVA

In the complex marine environment, the vibration and axial stress of the lifting subsystem caused by the motion of the mining vessel are easy to cause the fatigue failure and failure of the subsystem. Under the action of ocean current, with the increase of the length of the lifting pipe, the lateral displacement at different positions of the pipe increases significantly, which seriously affects the safety performance of the lifting pipe. By adding vibration absorber, the vibration amplitude can be reduced by changing the natural frequency of the lifting subsystem, so as to reduce the dynamic maximum axial stress and the lateral displacement of the pipeline. This dynamic vibration absorber model consists of a spring element $k_s$, a damping element $d_i$, and absorber concentrated mass $m_i$, as shown in Figure 2(c). In this model, the pump and the buffer of the lifting pipe are equipped with a dynamic vibration absorber, respectively.

In this section, the calculation will be demonstrated by using the DVA physical parameter given in Table 5 and the previous parameters given in Tables 1 and 2.

Applying the variable-separating method, let $z_i(x,t) = Z_i(x) \times q_i(t), \quad i = 1, 2$, be the displacement function of DVA; the motion differential equation of the first DVA can be calculated as

$$
m_1 \frac{\partial^2 z_1(x,t)}{\partial t^2} + k_1 \left[ z_1(x,t) - u_1(b_1,t) \right] + d_1 \left[ \frac{\partial z_1(x,t)}{\partial t} - \frac{\partial u_i(x,t)}{\partial t} \right]_{x=b_1} = 0.
$$

The second DVA’s motion differential equation is as follows:

$$
m_2 \frac{\partial^2 z_2(x,t)}{\partial t^2} + k_2 \left[ z_2(x,t) - u_4(b_4,t) \right] + d_2 \left[ \frac{\partial z_2(x,t)}{\partial t} - \frac{\partial u_4(x,t)}{\partial t} \right]_{x=b_4} = 0.
$$

When the two DVAs are equipped in pump and buffer respectively, the mathematical model of lifting subsystem can be established as.

| Time (s) | At 1000 m (m) | At 2000 m (m) | At 3500 m (m) | At 5000 m (m) |
|----------|---------------|---------------|---------------|---------------|
|          | OrcaFlex     | MATLAB        | OrcaFlex      | MATLAB        | OrcaFlex     | MATLAB        |
| 0        | 3.8999       | 4.0727        | 7.5398        | 7.8964        | 12.9997      | 13.4222       | 16.1196       | 16.9369       |
| 2        | 0.9537       | 0.996         | 1.8438        | 1.931         | 3.179        | 3.2823        | 3.942         | 4.1418        |
| 4        | -3.4231      | -3.5748       | -6.618        | -6.931        | -11.4104     | -11.7812      | -14.1488      | -14.8662      |
| 6        | -3.4673      | -3.6209       | -6.7035       | -7.0206       | -11.5578     | -11.9334      | -14.3317      | -15.0583      |
| 8        | 0.8926       | 0.9321        | 1.7257        | 1.8073        | 2.9753       | 3.072         | 3.6893        | 3.8764        |
| Error    | 4.43%        | 4.73%         | 3.25          | 5.07          |             |              |              |              |

Table 4: Longitudinal vibration displacements of lifting pipe at different positions.
respectively, \( \theta_2 \), and \( \theta_5 \), combined with deflection angle and equations (8)–(17), the coefficients obtained as equation (17). In the case of dynamic vibration absorbers, the coefficients are recombined as follows:

\[
E_i S_i \frac{d^2 \phi_i(x)}{dx^2} - M_i \phi_i(x) + M_i Z_i(x) \lambda^2 = E_2 S_2 \frac{d^2 \phi_i(x)}{dx^2} - M_2 \phi_i(x) + M_2 Z_2(x) \lambda^2 = 0,
\]

\[
m_i Z_i(x) \lambda^2 + d_i \lambda [Z_i(x) - \phi_i(b_i)] + k_i [Z_i(x) - \phi_i(b_i)] = 0,
\]

\[
m_2 Z_2(x) \lambda^2 + d_2 \lambda [Z_2(x) - \phi_2(b_2)] + k_2 [Z_2(x) - \phi_2(b_2)] = 0.
\]

(24)

Substituting the form of complex solution of differential equation into the second equation in equation (24), the displacement expression of DVA can be obtained as follows:

\[
Z_1(x) = \alpha_1 \frac{d_1 \lambda + k_1}{m_1 \lambda^2 + d_1 \lambda + k_1} e^{\nu b_1} + \beta_1 \frac{d_2 \lambda + k_2}{m_2 \lambda^2 + d_2 \lambda + k_2} e^{-\nu b_1},
\]

\[
Z_2(x) = \alpha_2 \frac{d_2 \lambda + k_2}{m_2 \lambda^2 + d_2 \lambda + k_2} e^{\nu b_1} + \beta_2 \frac{d_1 \lambda + k_1}{m_1 \lambda^2 + d_1 \lambda + k_1} e^{-\nu b_1}.
\]

(25)

When the buffer and pump are equipped with DVA, respectively, \( R_{M1} = M_1 + m_1 \), \( R_{M2} = M_2 + m_2 \) in equation (6). Numerical simulation has been performed with MATLAB to determine that the deflection angle \( \theta \) of the lifting pipe is 0.4307°. Combining equations (14) and (24), the equation for determining the unknown quantity is obtained as equation (17). In the case of dynamic vibration absorbers, the coefficients \( \alpha_i, \beta_i, \beta_3 \), and \( \beta_4 \) in equation (18) are recombined as follows:

\[
\alpha_1 = \left( M_1 + \frac{m_1 (d_1 \lambda + k_1)}{m_1 \lambda^2 + d_1 \lambda + k_1} \right) \lambda^2 + E_1 S_1 v_1,
\]

\[
\alpha_2 = \left( M_2 + \frac{m_2 (d_2 \lambda + k_2)}{m_2 \lambda^2 + d_2 \lambda + k_2} \right) \lambda^2 + E_2 S_2 v_2,
\]

\[
\beta_1 = \left( M_1 + \frac{m_1 (d_1 \lambda + k_1)}{m_1 \lambda^2 + d_1 \lambda + k_1} \right) \lambda^2 - E_1 S_1 v_1,
\]

\[
\beta_2 = \left( M_2 + \frac{m_2 (d_2 \lambda + k_2)}{m_2 \lambda^2 + d_2 \lambda + k_2} \right) \lambda^2 - E_2 S_2 v_2.
\]

(26)

Using physical model parameters given in Tables 1, 2, and 5, combined with deflection angle \( \theta = 0.4307° \) and equations (8)–(17), the coefficients \( \alpha_1, \beta_1 \), and \( \beta_2 \) in equation (17) are obtained by MATLAB, as shown in Table 6.

### 5. Results and Discussion

#### 5.1. Influence of Ocean Current

In the process of deep-sea mining operation, the lifting pipe will be offset due to the action of ocean current load. When the parameters of the lifting pipe have been determined, the mass of the buffer \( (M_2) \) and the force of the ocean current \( (q_{oc}) \) are the important factors affecting the deflection angle \( \theta \). According to Morison equation, the current force is a function about the relative velocity of vessel and ocean current. In order to illustrate the effects of ocean current on the lifting pipe dynamic characteristics during deep-sea mining operation, seven buffer masses are selected with different sailing velocities \( (V_0) \) of the mining vessel.

It is assumed that the mining vessel is considered to be dynamically stable and sailing against the current during towing. The comparison of the calculated results obtained from these states is conducted on aspects of deflection angle. As shown in Figure 9, under the condition of the same sailing velocity of the mining vessel, the deviation angle of the lifting pipe becomes smaller with the increase of the mass of buffer. Meanwhile, under the condition of the same buffer mass, the maximum deviation angle takes place at the maximum sailing velocity of the mining vessel; obviously the minimum deviation angle occurs when the sailing velocity is minimum. Moreover, the buffer mass has little effect on the deflection angle when the sailing velocity is smaller.

In order to intuitively understand the influence on the dynamic characteristics of the lifting pipe caused by the change of buffer mass and sailing velocity, the axial load and stress of the lifting pipe under aforementioned buffer mass and sailing velocity are simulated using the software OrcaFlex, as shown in Figure 10.

It can be seen from Figure 10 that the axial load decreases gradually with the increase of water depth. The axial load has a small abrupt change at 1000m due to the influence of the mass of the pump unit. For the whole lifting pipe, the axial load is continuous and is equal at the joint of any two-stage pipe. As shown in equation (19), the axial stress equation is a piecewise function related to the cross-sectional area, which is discontinuous due to the stepped structure of the pipe, so the axial stress is discontinuous. The dynamic load of lifting pipe is higher than its static load under the same sea condition. The maximum axial load takes place at the top of the pipe, the maximum static stress takes place at the top or 1000m of the lifting pipe, and the maximum axial dynamic stress takes place at 2000m of the pipe.

As a whole, the axial static load significantly increases with an increase in buffer mass, and the maximum axial static load at the top of it attains 4.73 × 10^6 N at 60T and increases 14.25% compared to the calculated value 4.14 × 10^6 N at 0T. The axial dynamic load of the lifting pipe

### Table 5: Physical parameters of DVA.

| \( i \) | \( m_i \) (kg) | \( k_i \) (N/m) | \( d_i \) (N·s/m) |
|---|---|---|---|
| 1 | 1660 | 119000 | 200 |
| 2 | 2600 | 119000 | 200 |

Shock and Vibration
Table 6: Values of $\bar{A}_i$ and $\bar{B}_i$ in equation (17) with DVA.

| $i$ | $\bar{A}_i$                  | $\bar{B}_i$                   |
|-----|------------------------------|------------------------------|
| 1   | $1.3633 - 0.3881j$           | $0.1317 + 0.3881j$           |
| 2   | $-0.0053 - 0.0269j$          | $1.7061 + 4.7149j$           |
| 3   | $0.0002 + 0.0002j$           | $0.8981 + 6.7834j$           |
| 4   | $-0.0003 - 0.0013j$          | $-2.9977 + 11.0151j$         |

Figure 9: Effect of buffer mass on deflection angle under different sailing velocities.

Figure 10: Continued.
decreases gradually with the increase of the buffer mass, and the maximum dynamic load at the top of it attains $15.6 \times 10^6$ N at 0T and increases 89.09% compared to the calculated value $8.25 \times 10^6$ N at 60T. Similarly, the axial stress changes in the same way as the axial load; when the mass of buffer is 0T and 60T, the maximum axial static stress is $2.39 \times 10^8$ Pa and $2.79 \times 10^8$ Pa, respectively, with a difference of 16.74%, and the maximum axial dynamic stress is $14.82 \times 10^8$ Pa and $7.65 \times 10^8$ Pa, with a difference of 93.73%. In addition, assuming that the mass of the buffer is a fixed value, the axial static load, axial dynamic load, axial static stress, and axial dynamic stress of the lifting pipe gradually decrease with an increase in the sailing velocity of the mining vessel.

Undoubtedly, the aforementioned results adequately demonstrate that obvious buffer mass and sailing velocity strongly influence not only lateral displacement of the lifting pipe, but also axial load and axial stress; among them, the influence of buffer mass on dynamic load and dynamic stress is more significant. Therefore, a reasonable selection of the buffer mass and the sailing velocity of the mining vessel in the practical engineering can effectively decrease the total axial load of the lifting pipe.

5.2. Comparison of the Lifting Pipe with and without DVA. In order to further explain the vibration characteristics of the lifting pipe after adding DVA, in this section, we compare the longitudinal vibration characteristics of the lifting pipe with and without DVA. Figure 11 is the comparison of the lifting pipe’s longitudinal vibration with and without DVA on time = 0.

It can be seen from Figure 11 that when the DVAs are attached at this subsystem, the longitudinal vibration amplitude is greatly reduced with the increase of the lifting pipe’s length. At 5000 m, the longitudinal vibration amplitude of the subsystem with DVAs is 13.24 m, while that of the subsystem without DVA is 16.75 m. After attaching DVAs, the longitudinal amplitude is reduced by 20.9%.

Figure 12 is the longitudinal displacements of the lifting pipe with DVAs from 0 to 20 seconds. From it, we can see that after attaching DVAs the vibration frequency of the lifting pipe is not changed, it still vibrates with the motion period of sea wave, and the vibration displacement amplitude is reduced when DVAs are installed.

When the DVAs is equipped, the deflection angle of the lifting pipe subsystem decreases from 0.5709° to 0.4307° under the action of sea waves, and the lateral displacement of the lifting pipe is reduced from 49.8196 to 37.5853. So, after equipping DVAs, the lateral displacement at the end of the pipe is reduced by 24.56%.

5.3. Comparison of Axial Force and Stress. In this section, we compare the axial force and stress characteristics of the lifting pipe with and without DVA. The MATLAB software was used to calculate equations (19)–(20), and the simulation results about gravity stress, dynamic stress, and total axial stress of the pipe with and without DVA are shown in Figures 13–16.

It can be seen from Figure 13 that the dynamic load is larger than the gravity load, and the trend is consistent with the total load. Figures 14–16 show a comparison between with and without dynamic vibration absorber. For gravity
load and gravity stress given in Figure 14, no matter with or without DVA, the downward trend of gravity load or gravity stress does not change but their values increase. At the top of the lifting pipe, the gravity stress increases from 0.261 GPa to 0.279 GPa. For dynamic load and dynamic stress, attaching DVAs can reduce the dynamic load and dynamic stress, as shown in Figure 15. And it is obvious that, at the top of the pipe, the dynamic stress is 0.624 GPa without DVA, and it decreases to 0.501 GPa when attaching DVAs. For the total axial load and axial stress, the same effect can also be observed from Figure 16 where the value of total axial stress of the lifting pipe with DVAs is 0.786 GPa at the top, and the value reaches 0.887 GPa without DVA. Therefore, attaching DVAs can effectively reduce the axial stress of the lifting pipe.

Using the experimental platform built in Figure 1, the experiment axial force of the connection point between the vessel and the lifting pipe is obtained. The simulation axial

Figure 11: Comparison of the lifting pipe’s longitudinal vibration with and without DVA. (a) $x: 0 \rightarrow 1000$ m. (b) $x: 1000 \rightarrow 2000$ m. (c) $x: 2000 \rightarrow 3500$ m. (d) $x: 3500 \rightarrow 5000$ m.
force can be calculated by MATLAB, and the experimental value and simulation value are compared to verify the usefulness and reliability of the mathematical model proposed. The data comparison is shown in Figure 17.

It can be seen from Figure 17 that the experimental curve is very consistent with the simulation curve. It can be claimed that this difference may be due to the axial force collection error and the difference
Figure 13: Axial load and stress with DVA. (a) Axial load. (b) Axial stress.

Figure 14: Gravity load and stress with/without DVA. (a) Axial load. (b) Axial stress.
**Figure 15:** Dynamic load and stress with/without DVA. (a) Axial load. (b) Axial stress.

**Figure 16:** Total axial load and stress with/without DVA. (a) Axial load. (b) Axial stress.
between the physical model and the prototype of the lifting system.

6. Conclusion

The main goal of the proposed model is to allow for more accurate dynamic model considering the influence of ocean current with DVAs in typical lifting pipe of deep-sea mining. Considering the geometrical features of the prototype, the physical model of lifting system is built at the National Ocean Technology Center of China, which simulates different marine environmental loads. Combined with the linearization method and current velocity, the equation of moment balance is derived, and then the deflection angle of lifting pipe is calculated by MATLAB. At the same time, a comprehensive deep-sea mining model is established by using Orcaflex software, and the influence of the buffer mass and sailing velocity on the deflection angle and the axial load and stress of the lifting pipe is studied. Applying the method of separation of variables, exact analytical solutions of the vibration and axial stress excited by the heave motion of mining vessel are determined with and without DVAs. The vibration process is modeled in MATLAB and Orcaflex to assess its feasibility, and the comparative analysis has shown that the proposed mathematical model of lifting pipe can be useful and reliable.

Since the solution of vibration and axial stress of the system equipped with DVAs is obtained according only to the experimental model in this work, the parameters combination of dynamic vibration absorbers cannot be directly used in production due to the different working environment of deep-sea mining. But this method has important guiding significance for the vibration and axial stress analysis of lifting pipe of deep-sea mining.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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