Finite time decoherence could be suppressed efficiently in photonic crystal

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(Dated: May 19, 2008)

The decoherence of two initially entangled qubits in anisotropic band gap photonic crystal has been studied analytically without Born or Markovian approximation. It is shown that the decoherence dynamics of two qubits in photonic crystal is greatly different from that of two qubits in vacuum or subjected to usual non-Markovian reservoir. The results also show that the finite time decoherence invoked by spontaneous emission could be suppressed efficiently and the entanglement of the Bell state possesses odd parity is more easily preserved in photonic crystal than that of the Bell state possesses even parity under the same condition. A store scheme for entangled particle pair is proposed.

PACS numbers: 03.67.mn,03.65.Yz,03.65.Ud,42.70.Qs

Entanglement is recognized as a global quantum-mechanical effect and has played a key role in quantum information[1,2], quantum computation[3,4], quantum cryptography[5], and so on. However, the phenomenon, termed as “entanglement sudden death” (ESD), has been found theoretically[6,7] and shown experimentally[8,9]. It is shown that spontaneous disentanglement may take only a finite-time to be completed, while local decoherence (the normal single-atom transverse and longitudinal decay) takes an infinite time[7]. And some issues have been devoted to extending the results in Markovian final decay) takes an infinite time[7]. And some issues have been devoted to extending the results in Markovian non-Markovian case[10,11].

On the other hand, photonic crystals form a new class of dielectric materials, in which the electromagnetic interaction is controllably altered over certain frequency[12]. The periodic dielectric structures leads to the formation of a photonic band gap (PBG), a range of frequencies for which no propagating electromagnetic modes are allowed[12]. The presence of the photonic band gap in the dispersion relation of the electromagnetic field results in a series of new phenomena, including the inhibition of the spontaneous emission[13], strong localization of light[14], formation of atom-photon bound states[15]. In this Letter, we will investigate how the entanglement of two qubits evolve in photonic crystal.

Now we restrict our attention to two noninteracting two-level atoms A and B coupled individually to two photonic crystal environment reservoirs which are initially in vacuum states. To this aim, we first consider the Hamiltonian of the subsystem of single qubit coupled to its reservoir as

$$
H = \omega_0|e\rangle\langle e| + \sum_k \omega_k a_k^\dagger a_k + \sum_k g_k \left(a_k^\dagger |g\rangle\langle e| + a_k|e\rangle\langle g|\right)
$$

where $\omega_0$ is the atomic transition frequency between the ground state $|g\rangle$ and excited state $|e\rangle$. The index $k$ labels the field modes of the reservoir with frequency $\omega_k$, $a_k^\dagger$ and $a_k$ are the modes’ creation and annihilation operators, and $g_k$ is the frequency-dependent coupling constant between the transition $e - g$ and the field mode $k$. For a single excitation of the subsystem, these states are

$$
\psi_1 = |e\rangle \bigotimes \prod_k |0_k\rangle
$$

$$
\psi_k = |g\rangle \bigotimes |1_k\rangle \prod_{k' \neq k} |0_{k'}\rangle
$$

(2)

where the ket $|0_k\rangle$ indicates the field mode $k$ is in vacuum state, $|1_k\rangle$ indicates the field mode $k$ is in the first excited state. The unexcited state

$$
\psi_0 = |g\rangle \bigotimes \prod_k |0_k\rangle
$$

(3)

is not coupled to any other state.

Now we could expand a general state vector of the subsystem as[10]

$$
\psi(t) = c_0\psi_0 + c_1 e^{-i\omega_0 t}\psi_1 + \sum_k c_k e^{-i\omega_k t}\psi_k
$$

(4)

in terms of the states (2) and (3) and insert this into the schrödinger equation $i(d/dt)\psi = H\psi$ to obtain the following set of coupled equations:

$$
\dot{c}_1 = -i \sum_k g_k e^{-i(\omega_k - \omega_0)t} c_k
$$

(5)

$$
\dot{c}_k = -ig_k e^{i(\omega_k - \omega_0)t} c_1
$$

(6)

The coefficient $c_0$ is constant in time. Now we can eliminate $c_k$ by integrating Eq.(5) with initial condition $c_k(0) = 0$ and substituting the result into Eq.(5), then we could obtain

$$
\dot{c}_1 = -\int_0^t d\tau G(t - \tau)c_1(\tau)
$$

(7)

where

$$
G(t - \tau) = \sum_k g_k^2 e^{-i(\omega_k - \omega_0)(t-\tau)}
$$

(8)

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is the delay Green’s function of the problem \cite{17, 18}.

In the effective mass approximation and long-time limit, the Green’s function \cite{3} takes the form \cite{17, 18}

\[ G(t - \tau) = -\alpha e^{i(\delta(t-\tau) + \pi/4)} \sqrt{(t-\tau)^2}, \quad \omega_c(t - \tau) \gg 1, \quad (9) \]

under the anisotropic photon-dispersion relation \cite{17, 18}

\[ \omega_k \approx \omega_c + A(k - k_0)^2 \quad (10) \]

where \( \omega_c \) is the upper band-edge frequency and \( k_0 \) is a constant characteristic of the dielectric material. \( \delta = \omega_\theta - \omega_c \) is the detuning of the atomic frequency with respect to the band-edge frequency and \( \alpha \approx \omega_0^2 d^2 / (8 \omega_c \epsilon_0 \pi A)^{3/2} \) is a constant that depends on the nature of the band-edge singularity, here \( d \) is the atomic dipole moment, \( \epsilon_0 \) is the dielectric constant in vacuum. By Laplace transform \cite{17}, we could finally obtain

\[ c_1(t) = c_1(0) e(t) \]

\[ c(t) = \epsilon [\lambda_+ e^{i\lambda_+^2 t} (1 + \Phi(\lambda_+ e^{i\pi/4})] - \lambda_- e^{i\lambda_-^2 t} (1 + \Phi(\lambda_- e^{i\pi/4})] \quad (13) \]

where

\[ \epsilon = \frac{e^{i\delta t}}{\sqrt{\alpha^2 - 4\delta}}, \]

\[ \lambda_+ = \left( -\alpha + \sqrt{\alpha^2 - 4\delta} \right) / 2, \]

\[ \lambda_- = \left( -\alpha - \sqrt{\alpha^2 - 4\delta} \right) / 2, \]

and \( \Phi(x) \) is the error function \cite{19}. So, we could obtain the density matrix of the subsystem as \cite{20}

\[ \rho(t) = \left( \begin{array}{cc} \rho_{ee}(0) |c(t)|^2 & \rho_{eg}(0) e(t) \\ \rho_{ge}(0) e^*(t) & \rho_{gg}(0) + \rho_{ee}(0)(1 - |c(t)|^2) \end{array} \right) \quad (14) \]

In order to investigate the entanglement dynamics of the bipartite system, we use Wooters concurrence \cite{21}. For simplicity, we set the two subsystems have the same parameters. Using the method in Ref. \cite{11}, we could obtain the concurrence of the whole system as

\[ C_\Phi = \max \{ 0, 2 \sqrt{1 - (\beta^2 |c(t)|^2) / 2} \} \quad (15) \]

\[ C_\Psi = \max \{ 0, 2 \sqrt{1 - (\beta^2 |c(t)|^2) / 2} \} \]

when the initial states are

\[ |\Phi\rangle = \beta |ge\rangle + \gamma |eg\rangle, \quad (17) \]

\[ |\Psi\rangle = \beta |gg\rangle + \gamma |\epsilon\epsilon\rangle, \quad (18) \]

respectively. Where \( \beta \) is real, \( \gamma = |\epsilon\epsilon| \) and \( \beta^2 + |\gamma|^2 = 1 \). Next, we will focus on the time behavior of the concurrence \( C_\Phi \) and \( C_\Psi \) as a function of \( \beta^2 \) and the dimensionless quantity \( \alpha^2 t \) \cite{17, 18}.

First, we investigate the decoherence of two qubits initially entangled in state \( |\Phi\rangle \). Fig. 1 shows that, with relatively large detuning of atom frequency outside the band gap with respect from the photonic band edge \( \delta = \alpha^2 \), the concurrence \( C_\Phi \) decreases exponentially to zero in time scale for all the value of \( \beta^2 \) except for \( \beta = 0 \) and \( \beta = 1 \), which corresponds to product states. The result is similar to that case in vacuum.

From Fig. 2 we can find that the concurrence \( C_\Phi \), with relatively large detuning of atom frequency inside the gap with respect from the photonic band edge \( \delta = -\alpha^2 \), decreases exponentially to a steady value bigger than zero for all the value of \( \beta^2 \) except for \( \beta = 0 \) and \( \beta = 1 \), as time increasing and then keep the value all the time.

Fig. 3 exhibits that, as the detuning of atom frequency inside the gap with respect from the photonic band edge increases to \( \delta = -4\alpha^2 \), the concurrence \( C_\Phi \) will decreases to a minimum value, and then return to a steady value. The bigger the detuing of atom frequency inside the gap with respect from the photonic band edge is, the bigger the steady value is.

Then, we investigate the decoherence of two qubits ini-
of the system will disappear at finite time as $0 < \beta^2 < 0.5$ because of the dominant influence of double excitation component $|ee\rangle$ in state $|\Psi\rangle$, while the disentanglement occurs asymptotically in time for the other value of $\beta^2$, which is similar to the results of that case in Markovian or usual non-Markovian regime [6, 7, 9, 10, 11].

Fig. 5 reveals that, for $\delta = 0$, the entanglement for the case of initial state $|\Psi\rangle$ is bigger than that for the case in Fig. 5 because of the existence of photonic band gap.

Last, we will compare the time behavior of the concurrence of maximum entanglement state $(|ge\rangle + |eg\rangle)/\sqrt{2}$ with that of $(|gg\rangle + |ee\rangle)/\sqrt{2}$. From Fig. 6, it is obviously that decoherence behavior for the case of initial state $(|ge\rangle + |eg\rangle)/\sqrt{2}$ is similar to that of $(|gg\rangle + |ee\rangle)/\sqrt{2}$ under the same parameter. As the atomic frequency is detuned inside the gap $\delta < 0$, the concurrence for (a) and (b) in Fig. 6 will decrease to steady value respectively, besides the steady value for the case of initial state $(|ge\rangle + |eg\rangle)/\sqrt{2}$ is bigger than that of $(|gg\rangle + |ee\rangle)/\sqrt{2}$ under the same parameter. The decay rate of concurrence for the case of initial state $(|gg\rangle + |ee\rangle)/\sqrt{2}$, as $\delta \geq 0$, is bigger than that for the case of initial state $(|ge\rangle + |eg\rangle)/\sqrt{2}$. So we could conclude that the state $(|ge\rangle + |eg\rangle)/\sqrt{2}$ is more robust than the state $(|gg\rangle + |ee\rangle)/\sqrt{2}$ against quantum noise in vacuum.

In summary, we have analytically derived the concur-
Concurrence of two initially entangled atoms coupled individually to its own anisotropic band gap photonic crystal environment without Born or Markovian approximation. The results show that the finite time decoherence invoked by spontaneous emission could be suppressed efficiently for the relatively large detuning of atom frequency inside the gap with respect from the photonic band edge, and the entanglement of Bell state $\beta|ge\rangle + \gamma|eg\rangle$ is more easily preserved in photonic crystal than that of $\beta|gg\rangle + \gamma|ee\rangle$ under the same condition.

In practice, we could store many particle pairs of maximum Bell entanglement state as $(|ge\rangle + |eg\rangle)/\sqrt{2}$ in photonic crystal individually. After an interval of time, all the particle pairs become partial entanglement particle pairs. However, we could recover the maximum entanglement by concentrating the partial entanglement with local operation [22]. Then, we could get entangled particle pair in any of the four maximum Bell entanglement states by local operation on particle pair in state of $(|ge\rangle + |eg\rangle)/\sqrt{2}$ [23].

Acknowledgments

This work was supported by the National Natural Science Foundation of China Grants No.60578055, the State Key Program for Basic Research of China under Grant No. 2007CB925204 and No.2007CB307001.

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