Inside an Enhançon: Monopoles and Dual Yang-Mills Theory

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Abstract
In this paper we study wrapped brane configurations that give rise to three dimensional pure Yang-Mills theory with eight supercharges. The corresponding supergravity solution is singular and it was conjectured that the singularity is removed by an enhançon mechanism. Instead, we incorporate non-perturbative gauge fields into supergravity and find a smooth solution for these configurations. Along the way we derive a non-abelian supergravity Lagrangian for type IIA on K3 and explicit formulae for the toroidal reduction of the heterotic string with non-abelian gauge fields. We proceed to analyse the duality with Yang-Mills theory and find that the dual background is a monopole configuration in little string theory.
1. Introduction

1.1. Overview

The basic ingredient of gauge/gravity dualities is the existence of two pictures representing the same physical system, a supergravity picture and a D-brane picture. After the original dualities found by Maldacena and others \([1]\), recently a lot of attention has turned to extending the list of examples to settings with less than maximal supersymmetry. In one such attempt, Johnson, Peet and Polchinski \([2]\) considered a brane setup that could potentially lead to a useful gravity dual for pure \(N = 4\) Yang-Mills theory in three dimensions and related cases with eight supercharges. Curiously the candidate gravity solution representing these branes had a naked singularity. This problem was addressed in \([2]\) by showing that an unexpected phenomenon had not been taken into account: at a special radius, called the enhançon radius, there are extra massless fields that enhance the abelian gauge symmetry of the naive solution to \(SU(2)\). It was then proposed that the brane constituents could not be moved below the enhançon radius, but instead should be spread on a spherical shell at this radius. Inside the shell, the geometry with the singularity is excised and replaced by flat space.

In this article we directly incorporate the effect of gauge symmetry enhancement. We introduce light non-abelian fields in the supergravity Lagrangian, which we derive in appendix B, and ask for a BPS solution to the equations of motion of this Lagrangian. Such a solution exists and has the correct asymptotic behaviour to represent the system of wrapped D-branes. This new gravity solution is automatically non-singular, and differs from the shell of branes that was argued to be the correct supergravity solution in \([2]\).

We explain in section two how one may find this new solution. It turns out there is a different solution, the Harvey-Liu monopole \([3,4]\), that can be mapped to the desired one via non-abelian toroidal reduction and S-duality. In order to perform the reduction, we have worked out explicit formulae in appendix A which generalise those of Maharana and Schwarz \([5]\). There is a surprise along the way, since the Harvey-Liu monopole has an excited \(H\)-field, yet the solution we seek is not an \(H\)-monopole.

In section three, we analyse the limits that are involved in decoupling the Yang-Mills theory living on the D-branes. We then perform the corresponding limits on the supergravity solution, and state the duality we arrive at. The gravitational modes turn out to be a red herring; they can be decoupled without affecting the Yang-Mills modes. Finally in section four we summarise and add some more relevant remarks.
For the decoupling limit we consider a configuration of wrapped branes that is different from \[2\]. Although they are related by T-duality, there appears to be a discrepancy between the values of the three dimensional Yang-Mills coupling in the two cases. However, they can be reconciled by adding a correction to the D-brane action. We discuss this correction in section four also.

1.2. Review

One particular setup considered in \[2\] is that of \(N\) D6 branes in type IIA string theory. To reduce the number of supercharges from sixteen to eight, these D6 branes are wrapped on a K3 surface. In an appropriate low energy limit one can decouple the low energy Yang-Mills theory living on the D6 branes. If one also decouples the Kaluza-Klein modes on the K3 one is left over with a three dimensional \(\mathcal{N} = 4\) gauge theory along the non-compact directions of the branes. An \(\mathcal{N} = 4\) vector multiplet has three real scalars, which are accounted for by reduction of the three scalars in the seven dimensional Yang-Mills multiplet living on a D6-brane. In the zero instanton sector the gauge field has no zero modes on the K3 and therefore there are no additional scalars, hence no hypermultiplets. So the theory of interest is three dimensional \(\mathcal{N} = 4\) pure Yang-Mills theory. One can get additional matter multiplets by considering non-zero instanton number in the D6-brane gauge theory, which is the same as adding extra D2 branes, or by adding D4 branes wrapped on a curve inside the K3 with genus greater than one. As a further generalisation, other pure gauge theories with eight supercharges, such as \(\mathcal{N} = 2\) Yang-Mills theory in 3+1 dimensions, may be obtained by wrapping branes of different dimensionalities on the K3 surface. However we restrict to the case of three dimensional pure Yang-Mills theory in this paper.

The compactification on the K3 surface yields an interesting effect \[3\]: because of a tr\((R \wedge R)\) term in the Chern-Simons action of the D-branes, which is non-zero when the K3 is part of the worldvolume, the D6 brane acts as a source for the three form Ramond field. Based on the relation with the heterotic string on \(T^4\) it was conjectured that we should associate an anti D2 brane charge to it, rather than a D2 brane charge. We will refer to the wrapped D6 as D6\(-\overline{D2}\). Similarly a D4 brane will also carry an induced charge when wrapped on the K3 surface, and we write it as D4\(-\overline{D0}\).

There is another known \(R^2\) term in the D-brane action, written down explicitly in \[7\]. Its physical effect is a correction to the tension of any brane which is wrapped on a K3. In fact for, say, the D6 brane considered above, this correction reduces its tension exactly by
the tension of a D2 brane, as one would have expected for a BPS object with the charges of D6-D2.

A supergravity solution for D6-D2 was written down in [2] using the harmonic function rule. The solution exhibits an attractor flow as a function of the radius: as one comes in from infinity, the volume of the cycle on which the branes are wrapped, in this case the K3 itself, shrinks. The volume first reaches the “self-dual” value $(2\pi\sqrt{\alpha'})^4$, and after that the metric coefficient $g_{tt}$ blows up at finite radius as the volume shrinks to zero size. This singularity is referred to as the repulson, for the repulsive behaviour it exhibits at small radii [8]. It defies an explanation in terms of D-brane physics, for instance it does not satisfy the criterion of [9] for it to be interpreted in terms of a dual field theory, and the force between a massive uncharged particle and a D6 brane ought to be attractive. The presence of this singularity therefore casts doubt on whether the repulson geometry is the correct supergravity representation of a collection of wrapped D6 branes.

An important observation in [2] was that the D4 brane becomes massless when wrapped on a K3 surface of self-dual volume due to the cancelling tensions of the D4 brane and the anti D0 brane. In fact the wrapped D4 brane is a W-boson and, as can be seen by duality with the heterotic string on $T^4$, its masslessness signals that the abelian gauge symmetry under which the D6 is charged is enhanced to SU(2). The value of the radius where the K3 is at self-dual volume is then fittingly called the enhançon radius.

The appearance of new massless fields violates the basic premise that the supergravity Lagrangian incorporates all the light fields. We therefore need a more general Lagrangian, for which the equations of motion are more likely to have a finite energy solution without a singularity, or at least with a reasonable singularity. It is not known how to write such a Lagrangian for type IIA supergravity by compactifying on a K3 surface. We in fact work it out by S-duality with the heterotic string, where the compactification is on a $T^4$ and the non-abelian fields are perturbative.

To find the correct supergravity solution, we use the observation in [2] that a wrapped D6 brane must be represented by a monopole, since it is the electric-magnetic dual of a wrapped D4 brane, which is a W-boson. Again it would be easier to appeal to S-duality and find a supergravity monopole in heterotic string theory. In fact, a promising candidate was written down by Harvey and Liu [3] based on the gauge fivebrane Ansatz. We will write this solution in terms of the toroidally reduced six dimensional fields and then S-dualise it. The resulting supergravity solution is a smooth monopole and represents the wrapped D6 branes.
The authors of [2] suggested a different modification of the geometry. They noticed that a probe brane in the repulsion background acquires a potential that prevents it from moving inside the enhançon radius. Therefore they proposed a supergravity solution in which the D6 branes are still present, except that they are no longer placed at the origin but on a spherical shell at the enhançon radius. While in our geometry the energy density is not localised at the origin, we do not see it localised on such a spherical shell either.

The relation between the enhançon and BPS monopoles has also been explored in [10], although from a different point of view.

2. A supergravity monopole

2.1. The Harvey-Liu monopole

As explained in section one, we would like to start with a monopole configuration in heterotic string theory. We will use normalisations and conventions of [11]. The ten dimensional string frame action is in this case

$$S = \frac{2\pi}{(2\pi \sqrt{\alpha'})^8} \int d^{10}x \sqrt{-g} e^{-2\Phi} \left\{ R + 4(\partial\Phi)^2 - \frac{1}{12} H^2 - \frac{\alpha'}{8} \text{tr}(F^2) + \ldots \right\}. \quad (2.1)$$

The ellipses indicate that we have dropped fermionic and higher order terms. The definition of the three-form $H$ is such that it includes the Chern-Simons term:

$$dH = -\frac{\alpha'}{4} \text{tr}(F \wedge F). \quad (2.2)$$

The gauge group in the heterotic theory is well known to be restricted by the anomaly to be either $SO(32)$ or $E_8 \times E_8$. The traces above should be taken in the vector representation of those groups. In what follows only an $SU(2)$ subgroup will play a role and therefore we will focus on it as our gauge group and forget the rest of the generators of $SO(32)$ or $E_8 \times E_8$.

Following earlier work on the gauge fivebrane, Harvey and Liu [3] (see also [4]) found a supergravity monopole by starting with a flat space BPS monopole and dressing it up with a non-trivial metric, dilaton and $H$-field. The similarity with the heterotic gauge fivebrane becomes clear if one adopts the convention of writing the Bogomol'nyi equations for BPS monopoles as Yang-Mills self-duality equations on $R^3 \times S^1$, so that $A_6$ plays the
rôle of the Higgs field. It was found in [12] that the heterotic BPS equations are solved by
the following Ansatz, written in terms of the ten dimensional fields:
\[ F_{pq} = \pm \frac{1}{2} \epsilon_{pqrs} F_{rs} \]
\[ H_{pqr} = \mp 2 \epsilon_{pqr}^s \partial_s \Phi \]
\[ g_{\mu\nu} = \text{diag} (-1, e^{2\Phi}, e^{2\Phi}, e^{2\Phi}, 1, 1, 1, e^{2\Phi}, 1, 1, 1) \]
\[ \nabla_i \nabla^i \Phi = \mp \frac{\alpha'}{8} \epsilon_{pqrs} \text{tr} F_{pq} F_{rs}. \]  
(2.3)

In these equations indices are raised with the metric \( g_{\mu\nu} \). We have introduced a number
of conventions that we will use through the rest of the paper. We summarise them below:

- \( \mu, \nu, \lambda \) are space-time indices in ten and, later, six dimensions;
- \( \alpha, \beta, \gamma \in \{ 6, 7, 8, 9 \} \) are the internal indices for the four-torus;
- \( a, b, c \) or \( \parallel \) will be used for 0, 4 and 5, the directions along the soliton worldvolume;
- \( i, j, k \) or \( \perp \) are for the transverse directions 1, 2 and 3;
- \( p, q, r, s \) stand for 1, 2, 3 and 6, in order to write the self-duality equation;
- \( I, J, K \) are indices for the adjoint representation of the gauge group SU(2).

Apart from \( A_1, A_2, A_3 \) and \( A_6 \) all components of the SU(2) gauge field are set to zero. This
solution preserves half of the sixteen supercharges. It is not an exact solution of string
theory; the metric has to be adjusted order by order in \( \alpha' \). To understand what charges
the soliton carries we have to write down the solution in terms of the six dimensional fields,
which is done in the next subsection.

For later use let us write down the expressions for a single monopole from Harvey and
Liu [9]. The vector \( \vec{r} \) will denote \((x_1, x_2, x_3)\) and \( r \) is its length. The gauge fields are then
just those of a flat space monopole
\[ A_I^I(\vec{r}) = \epsilon_{ijk} x^k \frac{K}{r^2} (K(Cr) - 1), \quad A_6^I(\vec{r}) = \frac{x^I}{r^2} H(Cr) \]  
(2.4)

The functions \( K(x) \) and \( H(x) \) are given by
\[ H(x) \equiv x \coth(x) - 1, \quad K(x) \equiv \frac{x}{\sinh(x)}. \]  
(2.5)

Here \( C = \sqrt{\langle A_6^I A_6^I \rangle} \) is the expectation value of the Higgs field at infinity. The dilaton can
be found by solving a Laplace equation with a flat space Laplacian
\[ \partial_i \partial_i e^{2\Phi} = -\frac{\alpha'}{8} F_{pq} F_{pq} \]
\[ = -\frac{\alpha'}{2r^4} \left( 2H^2 K^2 + (K^2 - 1)^2 \right), \]  
(2.6)
which results in
\[
e^{2\Phi} = e^{2\Phi_0} + \frac{\alpha'}{4r^2}(1 - K(Cr)^2 + 2H(Cr))
\]
\[= e^{2\Phi_0} + \frac{\alpha'}{4}(C^2 - \frac{H(Cr)^2}{r^2}).\]  (2.7)

For higher charge monopoles explicit expressions are hard to come by. But one can do the next best thing by assuming a solution to the flat space Bogomol’nyi equations for monopoles. For this purpose, let us introduce a fiducial flat space BPS monopole solution of charge \(N\) labelled by \(A(\vec{r})\) and \(h(\vec{r})\), such that \(h^2(\infty) = 1\), and try to express all the ten dimensional fields in terms of these. Notice that \(A\) and \(h\) depend on \(N\) together with the choice of a point on the monopole moduli space. But in order to simplify notation we do not show this dependence explicitly. The ten dimensional gauge and Higgs fields for the Harvey-Liu monopole can be written as
\[
A_i(\vec{r}) = C A_i(C\vec{r}), \quad A_6(\vec{r}) = C h(C\vec{r}).\]  (2.8)

Next, we can get a useful expression for the dilaton by using the flat space BPS equations \(D_i A_6' = B_i = -(1/2)\epsilon_{ijk} F_{jk}^I\), the Bianchi identity \(D_i B_i = 0\) and the Ansatz (2.3):
\[
\partial_i \partial_i e^{2\Phi} = -\frac{\alpha'}{8} \text{tr}(F_{pq} F_{pq})
\]
\[= -\frac{\alpha'}{2} \text{tr}(D_i A_6 D_i A_6) = -\frac{\alpha'}{2} \partial_i \text{tr}(A_6 D_i A_6)
\]
\[= -\frac{\alpha'}{4} \partial_i \partial_i \text{tr}(A_6 A_6).\]  (2.9)

It follows that \(e^{2\Phi}\) and \(-\alpha' A_6^2/4\) can only differ by a constant. Since \(A_6^2 \to C^2\) as \(r\) goes to infinity, the relation between the dilaton and the Higgs field in general is
\[
e^{2\Phi} = e^{2\Phi_0} + \frac{\alpha' C^2}{4} \left(1 - \frac{A_6^2(\vec{r})}{C^2}\right)
\]
\[= e^{2\Phi_0} + \frac{\alpha' C^2}{4} \left(1 - h^2(C\vec{r})\right).\]  (2.10)

This indeed reproduces the result for a single monopole listed in (2.7).

Knowing the dilaton \(\Phi\), from (2.3) we immediately know the metric and the \(H\) field. However, for the later use we also need the \(B\) field. First, from (2.3) we compute the only non-zero component of \(H\) :
\[
H_{ij6} = -\epsilon_{ijk} \partial_k e^{2\Phi} = \frac{\alpha'}{4} \epsilon_{ijk} \partial_k \text{tr}(A_6 A_6) = -\frac{\alpha'}{2} \text{tr}(A_6 F_{ij})
\]
\[= -\frac{\alpha'}{4} \omega_{ij6} - \frac{\alpha'}{4} \partial_i \text{tr}(A_j A_6) + \frac{\alpha'}{4} \partial_j \text{tr}(A_i A_6).\]  (2.11)
Here $\omega^{cs}_{ij6}$ is the non-abelian Chern-Simons form built from the gauge and Higgs fields entering the definition of $H$, see (A.3). Then we may take

$$B_{i6} = -\frac{\alpha'}{4} \text{tr}(A_iA_6).$$

(2.12)

Furthermore, from (2.3) we find that $H_{ijk} = 0$, hence

$$dB_{ijk} = \frac{\alpha'}{4} \omega^{cs}_{ijk}.$$  

(2.13)

From this one may write an integral expression for $B_{ij}$ in terms of the gauge fields, but we will have no need for it here.

The Higgs expectation value breaks the $SU(2)$ gauge symmetry, and so the mass of the W-boson in six dimensions depends on the choice of $C$. The W-boson is a BPS state of the heterotic string and its mass is therefore determined by the rightmoving momentum of the string. The latter in turn can be expressed in terms of the background fields on the torus and the charges it carries under the remaining $U(1)$’s. The W’s have charge $\pm 1$ under the $U(1)$ that remains after $SU(2)$ is broken, and they get a mass due to non-zero Wilson lines, so we obtain ([11], Vol. II, Pg. 77)

$$M^2_W = k^2_R = (\pm 1)^2 G^{\alpha\beta} A_\alpha^I A_\beta^I.$$  

(2.14)

Since only $A_6$ is nonzero and $A_6^2 \to C^2$ as $r$ goes to infinity, we get

$$M_W = e^{-\Phi_0} C,$$  

(2.15)

a relation that will be useful later on.

2.2. Non-abelian toroidal reduction

Our next step is to compactify the ten-dimensional heterotic theory on a four-torus and rewrite the monopole solutions of the previous subsection in terms of the six-dimensional fields. Because of the nature of the monopole, this involves a toroidal reduction in the presence of non-abelian gauge fields. Following Maharana and Schwarz [3] we perform such a reduction in appendix A.

There we introduce the following notation for the fields of the resulting six-dimensional theory. From the ten-dimensional metric we obtain the six dimensional metric $g_{\mu\nu}$, four

1 $B$ is not gauge independent and this equation amounts to choosing a specific gauge.
$U(1)$ gauge fields $A^{(1)}_{\mu}^{\alpha}$ and a $4 \times 4$ symmetric matrix of scalars $G_{\alpha\beta}$. From the dilaton we obtain its six-dimensional cousin, $\phi$. The ten-dimensional two-form field gives rise to a two-form $B_{\mu\nu}$ in six dimensions, four more $U(1)$ gauge fields $A^{(2)}_{\mu\alpha}$ and a $4 \times 4$ antisymmetric matrix of scalars $B_{\alpha\beta}$. Finally, the ten-dimensional gauge fields produce the non-abelian gauge fields in six dimensions $A^{(3)}_{\mu I}$ and four adjoint scalars $a^I_\alpha$.

We choose the torus directions to run from 6 to 9. Now we use formulas from the appendix and write down the six-dimensional monopole solution.

The metric in ten dimensions is diagonal (2.3), which makes it easy to choose a vielbein. Then from (A.4) and (A.5) we have in six dimensions

\[
g_{\mu\nu} = \text{diag}(-1, e^{2\Phi}, e^{2\Phi}, e^{2\Phi}, 1, 1), \quad G_{\alpha\beta} = \text{diag}(e^{2\Phi}, 1, 1, 1), \quad A^{(1)}_{\mu\alpha} = 0.
\]

(2.16)

For the six-dimensional dilaton we have from (A.6)

\[
e^{-2\phi} = \frac{V_{T^4}}{(2\pi\sqrt{\alpha'})^4} e^{-2\Phi} = \frac{(\int_{T^4} dy) e^\Phi}{(2\pi\sqrt{\alpha'})^4} e^{-2\Phi} = \lambda e^{-\Phi},
\]

(2.17)

where we have denoted $\lambda = \int_{T^4} dy/(2\pi\sqrt{\alpha'})^4$ the “coordinate” volume of the internal torus in $\alpha'$ units. This specific dilaton normalization will turn out very convenient later, when we will apply S-duality.

The ten-dimensional gauge fields reduce to the expected monopole configuration for the $A^{(3)}_{\mu I}$ while the role of the Higgs field is played by $a^I_6$, see (A.7)

\[
A^{(3)}_i(C\vec{r}) = C \mathcal{A}_i(C\vec{r}), \quad a^6_6(C\vec{r}) = C h(C\vec{r}).
\]

(2.18)

The reduction of the $B$ field is the most interesting. In ten dimensions the Harvey-Liu monopole has non-vanishing $H$ field (2.3) and carries an $H$-monopole charge. Since $H_{ij6}$ is non-zero one might have expected the six-dimensional solution to carry an $H$-monopole charge also. However we are looking for a pure non-abelian monopole; we do not want the soliton to carry charge under a second gauge field.

In order to see if there is a six dimensional $H$-charge or not we need to examine the corresponding six dimensional gauge fields, $A^{(2)}_{\mu\alpha}$. It was already found by Sen [13] for the Harvey-Liu monopole compactified to four dimensions that there is no charge under $A^{(2)}$ gauge fields. In our case we can also see explicitly from (A.8) in appendix A and (2.12) that all contributions to $A^{(2)}_{\mu\alpha}$ cancel:

\[
A^{(2)} = 0, \quad F^{(2)} \equiv 0.
\]

(2.19)
So we conclude that the soliton in fact has no six dimensional $H$-monopole charge.

Finally, from (A.8) it is also clear that the six-dimensional scalars $B_{\alpha\beta}$ vanish, while the two-form $B_{\mu\nu}$ is still determined by (2.13). As a result, the contributions due to $dB$ and the Chern-Simons term to $H_{\mu\nu\lambda}$ also cancel after compactification, and we have $H_{\mu\nu\lambda} = 0$ in six dimensions.

2.3. S-duality between type IIA on K3 and heterotic on $T^4$

In this subsection we will use S-duality between heterotic string theory compactified on $T^4$ and type IIA string theory on K3. First we will obtain the Lagrangian of the latter theory near the point of enhanced gauge symmetry and then we will rewrite the monopole solutions of the previous subsection in the type IIA language. Let us begin by explaining how we are going to apply the duality.

S-duality is believed to be an exact equivalence of the two theories. Its simplest manifestation arises at the level of the low-energy effective actions. Usually these actions are written at the generic points on the moduli space, far away from the loci where enhanced gauge symmetry occurs. In particular it means that on the heterotic side the gauge group is maximally broken by Wilson lines along the internal directions of the torus and in addition none of the torus radii are close to the self dual value. On the type IIA side it means that the volumes of all the homology cycles should be large in $a'$ units. At such points in the moduli space the low energy effective action involves only the massless abelian fields. It can be expanded in series in the number of derivatives and also in the powers of the coupling constant - the dilaton.

At the two-derivative level the actions can be completely determined either by a direct compactification from ten dimensions or from supersymmetry alone. Then a simple change of variables brings the two-derivative effective actions into one another [14,15]. Under this change of variables the dilaton changes sign and the coupling constant is inverted. The string metric is rescaled by the power of the dilaton and because of that the masses measured by “natural” observers in the two theories are also rescaled. The six-dimensional two-form field $B_{\mu\nu}$ goes into its Hodge dual up to a power of the dilaton. Finally, all the scalar and gauge fields are the same in both theories.

Beyond the two-derivative level, the identification of the effective actions is much more problematic. Since the coupling constants are inverted under S-duality, the corresponding expansions run around different points. Therefore to match a given term on one side one
may potentially need to know all the terms on the other. Nevertheless, certain very special
terms have been computed and perfectly matched on both sides [16].

Apart from the effective actions, the duality can be further probed by comparing
charges and masses of the BPS states in the two theories. The virtue of the BPS states is
that they exist at all values of the coupling and we often know their exact masses. Therefore
a BPS state found at weak coupling in one theory may be continued into strong coupling
and then compared with the BPS states in the other theory, again at weak coupling. Many
states have been matched in this way between the type IIA on $K3$ and heterotic on $T^4$
[17,6,18].

The masses of the BPS states vary with the moduli of the theory. In particular, at
certain points in the moduli space they may become massless. For example, it is well
known that certain perturbative heterotic string states become massless at the points
of the enhanced gauge symmetry in the moduli space. Away from these points their
masses are interpreted simply as arising from the ordinary Higgs mechanism. On the
type IIA side exactly the same points correspond to singular K3 manifolds [15], where the
singularity arises from some holomorphic curves inside K3 shrinking to zero size. However,
the particles that become massless are completely non-perturbative now: they are $D$-branes
wrapped around the collapsing cycles [19].

From the perspective of the low-energy effective action, the points of enhanced gauge
symmetry are not regular points in the moduli space. In order to describe the theory in
their vicinity we have to add to the action new fields that are responsible for the appearance
of the light particles. On the heterotic string side this is not difficult since the new fields
are present in the string perturbative expansion. But on the type IIA side we cannot do
the same because the light particles are not perturbative string states. Instead, we will
use the duality to deduce the type IIA action from the heterotic side.

The extra fields that we have to add to the action are the non-abelian gauge bosons
and scalars. In appendix A we have obtained the two-derivative part of the effective action
for the heterotic string on $T^4$ that includes these non-abelian fields. In order to apply
the duality transformations we need to know their action on the extra fields. But, as we
mentioned, in the abelian case the gauge fields and scalars do not transform under duality;
we therefore assume the same to be true in the non-abelian case as well. In appendix B
we perform the duality transformations and obtain the two-derivative part of the effective
action for the type IIA string theory compactified on K3 near the point of enhanced gauge
symmetry.
Let us comment on the result, given in (B.9). First, we already know that it is fixed by supersymmetry and the field content of the theory. But by employing S-duality we have written the effective action in the variables that are simple to interpret for the type IIA compactification. It also makes it easy for us to transform known heterotic monopole solutions to the type IIA side. In addition, we were able to consider only the bosonic sector and avoid the complications of checking supersymmetry variations.

Second, having found the two-derivative part of the effective action, we need to find the domain of its validity, and for that we need to understand the higher-order corrections to the action. From dimensional analysis the derivative expansion is suppressed by powers of $\alpha'$, as always. However, what is playing the role of the coupling constant now? Ordinarily, in the low-energy effective action obtained from string theory all interaction vertices contain a power of the string coupling constant. But it can be easily seen from (B.9) that extra vertices coming from the non-abelian terms do not contain the dilaton. This is in agreement with the observation in [19] that quantum loops of non-perturbative states should not be suppressed by the string coupling. Hence the non-abelian part of our action does not have an expansion in the dimensionless coupling constant and would remain interacting even if the dilaton is taken to zero.

Using the rules from appendix B, we are now ready to write down a supergravity monopole which satisfies the equations of motion of the six dimensional type IIA action at the two derivative level. Type IIA fields will be primed in the remainder.

Applying S-duality to the fields in the string frame, we get the following six dimensional type IIA solution:

$$
\begin{align*}
\lambda e^{-\Phi} dx^2_+ + \lambda e^{\Phi} dx^2_- \\
e^\phi' = e^{-\phi} = \sqrt{\lambda e^{-\Phi/2}}
\end{align*}
$$

(2.20)

The only non-zero gauge field is $A^{(3)}$, which is the same as on the heterotic side

$$
A_i^{(J)}(\vec{r}) = A_i^{(J)}(C\vec{r}) = C a^{(J)}_i(C\vec{r}).
$$

(2.21)

For the anti-symmetric tensor field we simply find $H'_{ijk} = 0$. Of the various moduli, $B'_{\alpha\beta}, a_7', a_8'$ and $a_9'$ are all constants, whereas

$$
a_6' = a_6 = C h(C\vec{r}), \quad G_{\alpha\beta}' = G_{\alpha\beta} = \text{diag}(e^{2\Phi}, 1, 1, 1).
$$

(2.22)

Again $a_6'$ should be interpreted as the Higgs field. Furthermore, an expression for $e^\Phi$ in terms of the Higgs field was given in (2.10).
In order to put the solution in a more recognizable form let us redefine coordinates as
\[
x_{\parallel} \rightarrow \frac{1}{\sqrt{\lambda}} e^{\Phi_0/2} x_{\parallel}, \quad x_{\perp} \rightarrow \frac{1}{\sqrt{\lambda}} e^{-\Phi_0/2} x_{\perp}.
\] (2.23)

The metric and dilaton are then brought to
\[
ds^2 = Z^{-1/2} dx_{\parallel}^2 + Z^{1/2} dx_{\perp}^2 \\
e^{2\phi'} = e^{2\phi_0} Z^{-1/2}
\] (2.24)

with the warp factor
\[
Z = 1 + \frac{\alpha' C^2 e^{-2\Phi_0}}{4} (1 - h^2 (\frac{1}{\sqrt{\lambda}} e^{-\Phi_0/2} C \vec{r}))
\] (2.25)

which goes to one as \( r \) goes to infinity. We would like to express the solution entirely in terms of type II parameters. The most convenient way of getting an equation for \( C \) is to notice that it is related to the mass of the \( W \)-boson on the heterotic side by \( M_W = e^{-\Phi_0} C \), as in (2.15). Now apply S-duality to get the type II mass:
\[
M'_W = e^{-\phi'_0} M_W = e^{-\phi'_0 - \Phi_0} C = \frac{1}{\sqrt{\lambda}} e^{-\Phi_0/2} C.
\] (2.26)

We will use this equation to eliminate \( C \). Substituting for \( C \) and \( \Phi_0 \) yields the final form of the warp factor:
\[
Z = 1 + \frac{\alpha' M'^2_W e^{2\phi'_0}}{4} (1 - h^2 (M'_W \vec{r})).
\] (2.27)

The gauge and Higgs fields can also be obtained from the fiducial solution by:
\[
A'^I_i (\vec{r}) = M'_W \mathcal{A}_i^I (M'_W \vec{r}), \quad a'_6 (\vec{r}) = M'_W h^I (M'_W \vec{r})
\] (2.28)

where \( a'_6(\infty) = M^2_W \). Notice that we have rescaled \( a_6 \) by a factor of \( e^{-\Phi_0/2}/\sqrt{\lambda} \) here, so in order to use these formulae with the Lagrangian in appendix B one should also rescale any field with \( \alpha \) or \( \beta = 6 \) correspondingly. In particular, \( G'_{\alpha \beta} \) becomes
\[
G'_{\alpha \beta} = \text{diag}(e^{-2\phi'_0} Z, 1, 1, 1, 1).
\] (2.29)

To summarise, in analogy with the Harvey-Liu monopole, we can start with a flat space BPS monopole \( \mathcal{A}_i^I (\vec{r}), h^I (\vec{r}) \) and use equations (2.24) and (2.27) to dress it up with a dilaton and metric and get a supergravity monopole. The soliton we have written down lives in low energy supergravity obtained from compactifying type II string theory on a K3 surface, preserves half of the sixteen supercharges and makes essential use of the non-abelian gauge fields which in this case arise from massless D-branes at a special corner of the moduli space of the compactification.
2.4. Comparison with the enhançon

As discussed in the introduction, Johnson, Peet and Polchinski [2] wrote down a supergravity solution for the D6-D2 system using the harmonic function rule. When reduced to six dimensions, their solution reads

\[
ds_6^2 = Z_2^{-1/2}Z_6^{-1/2}dx_\parallel^2 + Z_2^{1/2}Z_6^{1/2}dx_\perp^2
\]

\[e^{2\phi'} = e^{2\phi_0'}Z_2^{-1/2}Z_6^{-1/2} \tag{2.30}\]

with the following relations:

\[Z_2 = 1 - \frac{(2\pi)^4gN\alpha'^5/2}{2rV}\]

\[Z_6 = 1 + \frac{gN\alpha'^{1/2}}{2r}\]

\[e^{2\phi_0'} = \frac{(2\pi\sqrt{\alpha'})^4g^2}{V}. \tag{2.31}\]

Here \(V\) is the asymptotic volume of the K3, \(g\) is the ten dimensional string coupling, and \(N\) is the number of D6-D2’s.

What is the relation to the metric we have found above? Let us first consider the case of a charge one monopole. Then from (2.7) we get the warp factor

\[Z = 1 + \frac{\alpha'e^{2\phi_0'}}{4r^2}(1 - K^2 + 2H)(M'_W r). \tag{2.32}\]

An expression for the six dimensional dilaton in terms of ten dimensional quantities was given in (2.30). The \(W\)-boson in this case can be identified with a D4-brane wrapped on the K3 surface. As explained in the introduction, it carries an induced unit of anti-D0-brane charge and its mass is reduced by the mass of the D0 brane. So we can find the mass immediately:

\[M'_W = \frac{2\pi}{2\pi\sqrt{\alpha'}g} \left( \frac{V}{(2\pi\sqrt{\alpha'})^4} - 1 \right). \tag{2.33}\]

Finally, we would like to drop the exponential terms in \((1 - K^2 + 2H)(M'_W r)\), which then becomes \(2M'_W r - 1\). Substituting these in (2.32) we find

\[Z = 1 + \frac{\alpha'e^{2\phi_0'}}{4r^2}(2M'_W r - 1)\]

\[= 1 - \frac{\alpha'(2\pi\sqrt{\alpha'})^4g^2}{4Vr^2} + \frac{g\sqrt{\alpha'}}{2r} \left( 1 - \frac{(2\pi\sqrt{\alpha'})^4}{V} \right) \tag{2.34}\]

\[= \left( 1 + \frac{g\sqrt{\alpha'}}{2r} \right) \left( 1 - \frac{g\sqrt{\alpha'}}{2r} \frac{(2\pi\sqrt{\alpha'})^4}{V} \right)\]

\[= Z_2 Z_6 |_{N=1}.\]
So we conclude that our solution is identical to the one found in [2] up to exponential corrections of the form $e^{-M' \nu r}$, which smooth out the singularity in the core.

To see what happens for the general solution with arbitrary charge let us expand $g_{tt} = Z^{-1/2}$ near infinity in powers of $1/r$ where $r$ is the distance from the centre of mass of the monopoles. Then the $1/r$ and $1/r^2$ terms in the expansion of $g_{tt}$ are proportional to the tension of the soliton and the square of its charge respectively. This implies that in the case of $N$ solitons the leading terms in $Z$ and $Z_2 Z_6$ must agree if they agree for $N = 1$.

Next we would like to briefly discuss the shape of large $N$ monopole solutions. It is impossible to construct a spherically symmetric, finite energy Ansatz unless $N = 1$ [21], but an algorithm for the construction of axially symmetric monopoles has been proposed [21]. Let us call the axis of symmetry the $x_3$-axis. The locations of zeros of the Higgs field are usually thought of as the positions of the monopoles. The solutions of [21] have a Higgs field that vanishes only at the origin, so the monopoles are coincident. Actually, the Higgs field has a zero of order one along the $x_3$-axis and a zero of order $N$ along the $x_3 = 0$ plane. As a result one expects the flat region in the interior of the enhançon for $N > 1$ to be of a “pancake” type of shape, rather than a sphere. We have computed $h^2$ along the $x_3$-axis and on the $x_3 = 0$ plane for these axially symmetric monopoles using expressions from [22] for several $N$, and the results appear to confirm the above picture. Since other monopole solutions are lacking, we do not know how this picture is affected if some of the remaining moduli are used to deform away from axial symmetry and whether there are solutions with energy approximately localised on a thin spherical shell.

3. Gauge theory

3.1. D-brane picture

Instead of the D6-D2 system studied in [2], we will consider a T-dual situation by wrapping $N$ D4-branes on a two-sphere inside the K3. On a flat D4-brane the low-energy theory is a pure SYM theory with 16 supersymmetries. When the brane is wrapped on a sphere inside the K3, the theory becomes twisted [3] thereby allowing preservation of 8 supersymmetries. The gauge fields on the sphere and the complex scalar describing the normal direction to it inside the K3 have no zero modes, so the massless fields are just a 2+1 dimensional vector multiplet with no hypermultiplets. For $N$ branes we obtain vector multiplets forming the adjoint of $U(N)$. 


As usual, we would like to take a limit in the parameter space such that only the 2+1 dimensional SYM survives on the brane. In addition we would like to decouple it from the theory in the bulk. For this we first have to take $\alpha'$ to zero to get rid of the massive open string states. Then in order to avoid Kaluza-Klein modes of the massless fields on the sphere we have to take its area, $A$, to zero. For the gauge coupling of the resulting 2+1 dimensional theory we have

$$\frac{1}{g_{YM,3}^2} = \frac{A}{(2\pi)^2 g \sqrt{\alpha'}} , \quad \text{(3.1)}$$

which we want to keep constant. Then in order to prevent the string coupling from blowing up we need to take $A$ to zero as $\sqrt{\alpha'}$ or faster, but not faster than $\alpha'$, or higher-derivative corrections in the background fields along the sphere may spoil the gauge theory. We also want to avoid higher-derivative corrections in the directions transverse to the sphere. It means that in these directions the K3 should be flat on a scale of order $\sqrt{\alpha'}$. Therefore its volume should be bounded below as

$$V \geq \alpha' A . \quad \text{(3.2)}$$

Now let us discuss decoupling of the brane and the bulk theories. It implies that there can be no energy transfer between the two. In fact it is enough to require that the excitations on the brane cannot “leak” out into the bulk. Then by CPT invariance the bulk excitations cannot influence the brane either. Now, in the limit $\alpha' \rightarrow 0$ the mass of most of the states in the bulk theory goes to infinity. But the finite energy brane excitations that we are interested in cannot leak into infinitely massive states. Such bulk states are therefore always decoupled from the brane. These include massive closed string states as well as non-perturbative D-brane states, though we have to be careful with the latter. While their tension does go to infinity, some of the states obtained by wrapping D-branes on very small cycles might remain light. An obvious suspect is the very same cycle we are putting the D4-brane on. By wrapping a D2-brane on it we obtain a W-boson. Fortunately, its mass is given by

$$M'_W = \frac{2\pi A}{g(2\pi \sqrt{\alpha'})^2} = \frac{1}{\alpha' g_{YM,3}^2} , \quad \text{(3.3)}$$

which is still infinite in the limit. Volumes of the other cycles in K3 are determined by the other moduli of the theory. They do not enter in the field theory on the D4-brane.
or the monopole solution in the bulk. We therefore can make them arbitrary and the corresponding D-brane states decouple from the brane. What we still need is the decoupling of the massless states as well as the states with masses remaining finite in the limit $\alpha' \to 0$. The former are described by the supergravity Lagrangian in six dimensions and the latter can only be the higher Kaluza-Klein modes of the ten-dimensional supergravity on K3.

Let us first analyze the interactions of the massless six-dimensional supergravity fields with the brane. For this we look at the string scattering amplitudes. In our limit the open string diagrams with no closed string (bulk) vertex operator insertions will be more and more dominated by the diagrams coming from the 2+1 dimensional gauge theory, albeit regulated by stringy effects. This is just a statement that the open string theory on the brane reduces to the gauge theory. In particular, since $g_{\text{YM,3}}$ is kept fixed in the limit, we expect the scattering amplitudes to be finite. The interaction with the bulk is described by inserting bulk vertex operators into the open string diagrams. However, every such an insertion brings with it the corresponding coupling constant. This coupling constant comes from the six dimensional supergravity that describes the massless bulk states. For the gravity multiplet it is the six-dimensional Newton’s constant:

$$G_N \sim \frac{g^2 \alpha'^4}{V},$$

where $V$ is the volume of K3. Clearly, it vanishes in our limit. From (B.9) the coupling constant for the gauge fields is just $(2\pi)^3 \alpha'$ and also goes to zero. Now, if we take an open string diagram that is finite and add a number of closed string vertices with the coupling constants tending to zero we would find that the scattering tends to zero, too.

For the higher Kaluza-Klein modes of the ten-dimensional supergravity on K3 we cannot make the same argument since we do not really know their couplings. However, when the volume of K3 is small these modes are heavy and therefore automatically decoupled. When the volume of the K3 is large we can look at the problem from ten dimensions. The ten-dimensional Newton’s constant and Ramond-Ramond coupling go to zero in our limit. We will assume that this is enough to decouple the Kaluza-Klein modes from the D4-brane at arbitrary K3 volume.
3.2. Soliton picture

We now want to discuss applying the limits considered in the previous subsection to the supergravity solution from section two. What we expect to find are again two decoupled systems: the free supergravity in the bulk and the theory describing excitations around the core of the soliton. In the spirit of the AdS/CFT correspondence [1] we hope to identify the latter with the gauge theory.

To begin we rewrite the solution in terms of the gauge coupling and other simple physical parameters. For the gravity multiplet fields we obtain:

\[ ds^2 = Z^{-1/2} dx^2_{\|} + Z^{1/2} dx^2_{\perp} \]
\[ e^{2\phi'} = \alpha' g_{\text{YM},3}^4 \frac{A^2}{V} Z^{-1/2}, \quad H = 0 \]
\[ Z = 1 + \frac{A^2}{4V} \left( 1 - h^2 \left( \frac{\vec{r}}{\alpha' g_{\text{YM},3}} \right) \right). \]  

(3.4)

Among the scalars and U(1) gauge fields in our solution that are not charged with respect to the SU(2) gauge group the only non-trivial field is \( G'_{66} = e^{-2\phi'} Z \). Finally, for the SU(2) gauge multiplet fields we have:

\[ A^I_i(\vec{r}) = \frac{1}{\alpha' g_{\text{YM},3}^2} \phi^I_i \left( \frac{\vec{r}}{\alpha' g_{\text{YM},3}} \right) \]
\[ a^I_6(\vec{r}) = \frac{1}{\alpha' g_{\text{YM},3}^2} h^I \left( \frac{\vec{r}}{\alpha' g_{\text{YM},3}} \right) \]
\[ a^I_7 = a^I_8 = a^I_9 = 0. \]  

(3.5)

The first question we should ask ourselves is whether we can trust the above solution in the limit \( \alpha' \to 0 \). Unless \( A^2/V \) goes to infinity as \( \alpha'^{-2} \) or faster, this is clearly not the case. The reason is that the higher-derivative corrections to the supergravity Lagrangian spoil the solution. Near the core of the soliton each transverse derivative of the fields above brings with it a factor of \( (\alpha' g_{\text{YM},3}^2)^{-1} \) while it is suppressed by \( \sqrt{\alpha'} \) from the derivative expansion and by a factor from the metric. However, this factor will not help unless \( A^2/V \geq \alpha'^{-2} \). Therefore the higher the number of derivatives a term in the supergravity Lagrangian has the bigger its potential contribution to the action becomes as we take \( \alpha' \to 0 \).

Taking \( A^2/V \) to infinity as \( \alpha'^{-2} \) or faster corresponds either to making the volume of the K3 too small, in violation of (3.2), or to taking \( A \) to infinity. In both cases we do not have a clear description of the theory on the D-brane side since the brane is either
embedded in a space highly curved on the scale of $\sqrt{\alpha'}$ or the Kaluza-Klein modes on the sphere do not decouple.

The conclusion is that despite having improved on the solution of [2] we have not been able to find a weakly-coupled dual to the three-dimensional $\mathcal{N} = 4$ pure gauge theory. This is independent of whether we take the number of branes, or colors in the gauge theory, to be large, as the derivative expansion diverges anyway. However, we can still try to elaborate on what is it we find dual to the gauge theory.

From the above solution we see that the behavior of the fields in the $SU(2)$ gauge multiplet depends only on $g_{YM,3}^2$ and $\alpha'$, which is consistent with the gauge theory duality. However, the behavior of all the fields neutral under $SU(2)$ depends on the ratio $A^2/V$ that can be adjusted, subject to (3.2), independently of the gauge theory coupling, which suggests that neutral fields have little to do with the gauge theory.

To investigate, let us consider a limit where we first take the volume $V$ of the K3 to infinity while keeping the area $A$ of the sphere constant and only then take $\alpha'$ to zero. From the D-brane side this is a good limit. The difference is that the bulk theory is now 10-dimensional type IIA string theory on an infinite-volume K3 manifold. Clearly, only the piece of the K3 containing the sphere on which the D-brane is wrapped is of interest to us. We can therefore think of the resolved $A_1$ ALE space instead of the K3. Since the 10-dimensional couplings vanish as $\alpha' \to 0$, we expect the bulk theory to decouple from the brane as before.

Taking the volume to infinity in our 6-dimensional supergravity solution at first does not seem to make much sense. To describe the 10-dimensional geometry via the 6-dimensional Lagrangian we have to include in it an infinite number of harmonics in the expansion of the 10-dimensional fields over the transverse space. However, the Lagrangian that we have only contains the lowest harmonics. Its only benefit is that it includes inherently 6-dimensional fields which represent non-perturbative D-brane states responsible for the gauge symmetry enhancement to $SU(2)$.

The harmonics on the transverse space can be heuristically divided into two classes: those that have support in the vicinity of the sphere inside the resolved ALE and those that spread over the entire space. For instance, the perturbative $U(1)$ part of the $SU(2)$ gauge multiplet in our Lagrangian contains scalars describing the metric on the sphere inside the ALE and a gauge field arising from the 3-form in 10 dimensions reduced using the 2-form dual to the sphere. These harmonics naturally have support in the vicinity of the sphere. The non-perturbative part of the $SU(2)$ gauge multiplet can also be thought of
as having support near the sphere. On the other hand, the 6-dimensional gravity multiplet represents 10-dimensional fields spread over the entire ALE.

Later we intend to take $\alpha' \to 0$. According to our limit, we will simultaneously take the string coupling $g$ and the area of the sphere $A$ to zero as well. In a flat 10-dimensional space taking $\alpha'$ and $g$ to zero would cause the string theory to become free. In our case the space far away from the sphere is flat and the harmonics supported over the entire ALE are just plane waves there. Although we do not know their behavior near the sphere, its vanishing area suggests that the “overlap” between them and the harmonics supported in the vicinity of the sphere vanishes. We then expect that the theory in the bulk of the ALE becomes free and decouples from the 6-dimensional theory supported in the vicinity of the sphere. The scale below which the bulk theory is free and decoupled is set by the 10-dimensional Newton’s constant.

Now, the advantage of taking the volume to infinity is that $Z$ goes to 1 and all the fields in our solution that are uncharged under the $SU(2)$ gauge group become constant. All the excited fields are in the $SU(2)$ gauge multiplet. The soliton therefore belongs entirely in the theory supported in the vicinity of the sphere. The fact that the metric is not excited means that the soliton is below the decoupling scale. Indeed, the 10-dimensional Newton’s constant is $\sim g^2 \alpha'^{7/4}$ while the tension of the soliton is $\sim (\alpha' g_{YM,3})^{-2}$. Hence we find that the theory in the bulk of the ALE is not relevant to the description of the dual gauge theory.

Altogether, the three-dimensional $\mathcal{N} = 4$ $SU(N)$ pure gauge theory is dual to an $N$-monopole configuration in the six-dimensional theory living in the vicinity of the sphere in the resolved $A_1$ ALE space. We know that the low-energy limit of the latter theory is six dimensional $\mathcal{N} = 2$ $SU(2)$ gauge theory. The parameters of the theories are related as follows. The six-dimensional gauge coupling is just $(2\pi)^3 \alpha'$, from (B.9), and the three-dimensional gauge coupling is related to the W-boson mass in the six-dimensional vacuum as in (3.3). The moduli space of the three-dimensional theory is the moduli space of $N$ monopoles in six dimensions, with four centre of mass degrees of freedom removed.

Let us emphasize that we do not know the true monopole configuration. The solution in (3.5) suffers from $\alpha'$ corrections whenever $\sqrt{\alpha' g_{YM,3}^2}$ is small. However, we know that the monopole configuration must exist because it carries a topological charge. Also note that (3.5) is a good solution for large $\sqrt{\alpha' g_{YM,3}^2}$ and being BPS it must survive taking $\alpha'$ to zero.
Now, it is well known \[23\] that the theory at the singularity of the \(A_{n-1}\) ALE space in type IIA string theory is T-dual to the little string theory \[24\] arising on the worldvolume of \(n\) NS5-branes in type IIB string theory. Note that at low energies this little string theory is also described by the \(\mathcal{N} = 2\) \(SU(n)\) gauge theory. Its gauge coupling, by duality with the D5-brane, is again \((2\pi)^3 \alpha'\).

The picture advocated above is then supported by duality with the construction of the three-dimensional \(\mathcal{N} = 4\) pure gauge theory due to Hanany and Witten \[25\]. In that construction the \(SU(N)\) gauge theory is realized in type IIB string theory on \(N\) D3-branes stretched between two NS5-branes. If the distance between the fivebranes is \(l\), the gauge coupling in three dimensions will be \(g_{YM,3}^2 = l/(2\pi)^2 g\), while the mass of the W-boson in six dimensions, which is just a D1-brane stretched between two NS5-branes, is \(M_W = l/(2\pi \sqrt{\alpha'}) g\), giving the relation between the two precisely as in (3.3).

In the “supergravity” picture each D3-brane stretched between the NS5-branes looks like a magnetic monopole. Taking the string coupling to zero while keeping \(g_{YM,3}\) constant we achieve decoupling of the little string theory on the NS5-branes from the type IIB string theory in the bulk \[24\]. This yields exactly the same description of the three-dimensional gauge theory as we have obtained.

4. Discussion

In this paper we have argued that one can find the correct non-singular supergravity description of the enhançon by solving supergravity equations with extra light fields added. The resulting solution is a smooth supergravity monopole. We have also commented on the fact that known solutions do not correspond to a spherical shell, even for large \(N\).

Subsequently we took a decoupling limit in order to isolate the Yang-Mills theory living on the D-brane. Unfortunately we have found that this limit is out of reach for supergravity. Specifically we found that the derivative expansion diverges. This situation appears to be generic; in attempts to describe pure Yang-Mills theories with four supercharges in 3+1 dimensions by means of supergravity it was also found that \(\alpha'\) corrections were large in the decoupling limit \[26,27\]. For instance in \[27\] it was found that in order to decouple the QCD scale from the scale of unwanted Kaluza-Klein modes, one needed to go beyond the supergravity approximation.

Despite these difficulties, we have been able to distill the duality in question. We found that the gauge theory is dual to a monopole configuration in \((1,1)\) little string theory, which
is corroborated by the Hanany-Witten construction. But it is notable that, unlike previous
cases considered in the literature, gravity turned out to be completely irrelevant to this
duality. The only relevant modes are the non-gravitational modes that live close to the
two sphere inside the ALE and make up little string theory with a defect.

It is tempting to formulate the result as open/closed duality for little strings. In the
limit $A \to 0$, $g \to 0$ with $A/g$ fixed and $\alpha'$ fixed, one reduces to (1,1) little string theory
with an object of finite tension. Then there would be two pictures of this object, inherited
from the full string theory: in the “open little string” picture it is a “D2” brane while in
the “closed little string” picture it can be described as a Yang-Mills monopole. In the low
energy limit $\alpha' \to 0$, the theory living on this “D2” brane is three dimensional Yang-Mills
theory with eight supercharges. Unfortunately, keeping the three dimensional Yang-Mills
coupling fixed in this limit causes the divergence of $\alpha'$ corrections in the closed string
picture.

It would be interesting to find out whether the holographic description of little string
theories sheds more light on this situation. Since far away from the monopoles the little
string theory is Higgsed, the double scaling limit of [28] may be applicable. There the
coupling constant for the dual string description is given by

$$\frac{1}{x} \sim \frac{1}{M'_W \sqrt{\alpha'}} = \sqrt{\alpha' g^2_{YM,3}}.$$ 

It becomes small in the gauge theory limit we are interested in.

In discussing the gauge theory we have not used quite the same system as in [3]. Instead of the D6-$\overline{D2}$ system of [2] we have decided to study the D4 brane wrapped on the
sphere inside K3. The two configurations are related by the $SO(20, 4; \mathbb{Z})$ T-duality group.
And while T-duality is supposed to be an exact equivalence of string theories, naively we
do not obtain the same descriptions of the two cases. Namely, if we look at the relation
between the mass of the W-boson and the 3-dimensional gauge coupling for the D6-$\overline{D2}$
system we do not obtain the same result (3.3) as for the wrapped D4 brane. The reason
is that in the former case the mass of the W-boson (2.33), which is a D4 brane wrapped
on the entire K3, receives a correction due to the known $R^2$ term in the D-brane action
[7]. However, the 3-dimensional gauge coupling for the D6 brane wrapped on K3 is naively
just

$$\frac{1}{g^2_{YM,3}} = \frac{V}{2\pi g (2\pi \sqrt{\alpha'})^3} = \alpha' M'_W \left(1 - \frac{(2\pi \sqrt{\alpha'})^4}{V}\right)^{-1}.$$ 

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The discrepancy may be interpreted as the T-duality prediction for an additional $R^2 F^2$ term in the D-brane action. When integrated over K3 such a term would produce a shift in the 3-dimensional Yang-Mills coupling that would restore consistency with T-duality. One can in fact explicitly show the presence of this term and other similar ones [29].

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Appendix A. Non-abelian toroidal reduction

In this appendix we present formulae for the dimensional reduction of the theory with the Lagrangian of the form (2.1) with non-abelian gauge fields. We will closely follow the work of Maharana and Schwarz [3].

We consider a theory in $D + d$ dimensions compactified on a $d$-dimensional torus. All fields will be independent of the coordinates $y^\alpha$ along the torus. To avoid confusing notation in this appendix only we will add a hat to all $(D + d)$-dimensional quantities while their cousins in $d$ dimensions remain intact. Then the $(D + d)$-dimensional index $\hat{\mu}$ splits into a $D$-dimensional $\mu$ and an internal $d$-dimensional $\alpha$.

For the purposes of dimensional reduction let us explicitly rewrite (2.1) in the new notation

$$S = \frac{2\pi}{(2\pi \sqrt{\alpha'})^8} \int dx \int_{T^d} dy \sqrt{-\hat{g}} \ e^{-2\hat{\Phi}} \left\{ \hat{R}(\hat{g}) + 4\hat{g}^{\hat{\mu} \hat{\nu}} \hat{\partial}_\hat{\mu} \hat{\Phi} \hat{\partial}_\hat{\nu} \hat{\Phi} - \frac{1}{12} \hat{g}^{\hat{\mu} \hat{\nu} \hat{\rho}} \hat{g}^{\hat{\alpha} \hat{\beta} \hat{\gamma}} \hat{H}_{\hat{\mu} \hat{\nu} \hat{\rho}} \hat{H}_{\hat{\alpha} \hat{\beta} \hat{\gamma}} - \frac{\alpha'}{8} \hat{g}^{\hat{\mu} \hat{\nu}} \hat{g}^{\hat{\alpha} \hat{\beta}} \text{tr} \left( \hat{F}_{\hat{\mu} \hat{\nu}} \hat{F}_{\hat{\alpha} \hat{\beta}} \right) \right\}. \quad (A.1)$$

For completeness we also write down our conventions. The non-abelian gauge field strength is

$$\hat{F}_{\hat{\mu} \hat{\nu}} = \hat{\partial}_{\hat{\mu}} \hat{A}_{\hat{\nu}} - \hat{\partial}_{\hat{\nu}} \hat{A}_{\hat{\mu}} + [\hat{A}_{\hat{\mu}}, \hat{A}_{\hat{\nu}}], \quad (A.2)$$

and $\hat{H}_{\hat{\mu} \hat{\nu} \hat{\rho}}$ includes Chern-Simons form as follows:

$$\hat{H}_{\hat{\mu} \hat{\nu} \hat{\rho}} = \hat{\partial}_{\hat{\mu}} \hat{B}_{\hat{\nu} \hat{\rho}} - \frac{\alpha'}{4} \text{tr} \left( \hat{A}_{\hat{\mu}} \hat{F}_{\hat{\nu} \hat{\rho}} - \frac{1}{3} \hat{A}_{\hat{\mu}} [\hat{A}_{\hat{\nu}}, \hat{A}_{\hat{\rho}}] \right) + (\text{cyc. perms.}) \quad (A.3)$$
Now we give definitions of the $d$-dimensional fields. The $(D + d)$-dimensional metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ gives rise to the metric $g_{\mu\nu}$ in $D$ dimensions, $d$ of $U(1)$ gauge fields $A_{\mu}^{(1)\alpha}$ and a $d \times d$ symmetric matrix of scalars $G_{\alpha\beta}$. We define these fields as in [5]; first we use the local Lorentz invariance to bring the $(D + d)$-dimensional vielbein into a triangular form:

$$\hat{e}_{\hat{r} \mu} = \begin{pmatrix} e_{\mu}^r & A_{(1)\beta}^{(1)\alpha} E_{\alpha}^a \\ 0 & E_{\alpha}^a \end{pmatrix},$$  \hspace{1cm} (A.4)

where $\hat{r} = \{r, a\}$ are vielbein indices. In the above relation we have already defined the gauge fields. The internal metric of the torus, $G_{\alpha\beta} = E_{\alpha}^a \delta_{ab} E_{\beta}^b$, becomes the symmetric matrix of scalars in $D$ dimensions while the “spacetime” metric is $g_{\mu\nu} = e_{\mu r} e_{\nu r} e^s$. Then the complete $(D + d)$-dimensional metric can be written as

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu} + A_{(1)\gamma}^{(1)\alpha} A_{\nu(1)\delta} & A_{(1)\gamma}^{(1)\alpha} G_{\gamma\beta} \\ A_{(1)\gamma}^{(1)\alpha} G_{\gamma\alpha} & G_{\alpha\beta} \end{pmatrix}.$$  \hspace{1cm} (A.5)

Next we define the $D$-dimensional dilaton field $\phi$

$$e^{-2\phi} = \frac{V}{(2\pi \sqrt{\alpha'})^4} e^{-2\hat{\phi}} = \frac{\int_{T^d} dy \sqrt{G}}{(2\pi \sqrt{\alpha'})^4} e^{-2\hat{\phi}},$$  \hspace{1cm} (A.6)

where $V$ is the volume of the internal torus. We have absorbed the fourth power of $(2\pi \sqrt{\alpha'})$ in the above for our later convenience in the main text.

The $(D + d)$-dimensional gauge fields $\hat{A}_{\hat{\mu}}^{I}$ upon compactification give rise to the non-abelian gauge fields $A_{\mu}^{(3)I}$ together with $d$ adjoint scalars $a_{\alpha}^{I}$. Their definitions are exactly the same as in the abelian case [5]

$$a_{\alpha}^{I} = \hat{A}_{\alpha}^{I},$$

$$A_{\mu}^{(3)I} = \hat{A}_{\mu}^{I} - a_{\alpha}^{I} A_{\mu}^{(1)\alpha}.$$  \hspace{1cm} (A.7)

Finally, from the $\hat{B}_{\hat{\mu}\hat{\nu}}$ field we obtain in $D$ dimensions the $d \times d$ antisymmetric matrix of scalars $B_{\alpha\beta}$, $d$ of $U(1)$ gauge fields $A_{\mu\alpha}^{(2)}$ and the $D$-dimensional two-form field $B_{\mu\nu}$. Their definitions differ only slightly from those in [5]

$$B_{\alpha\beta} = \hat{B}_{\alpha\beta},$$

$$A_{\mu\alpha}^{(2)} = \hat{B}_{\mu\alpha} - B_{\beta\alpha} A_{\mu(1)\beta} + \frac{\alpha'}{4} \text{tr}(A_{\mu(3)\alpha} a_{\alpha}),$$

$$B_{\mu\nu} = \hat{B}_{\mu\nu} + \frac{1}{2} (A_{(1)\alpha}^{(1)\alpha} A_{\nu\alpha}^{(2)} - A_{(1)\beta}^{(1)\alpha} A_{\mu\alpha}^{(2)}) + \frac{\alpha'}{4} (\text{tr}(A_{\mu(3)\alpha} a_{\alpha}) A_{\nu(1)\gamma} - \text{tr}(A_{\nu(3)\alpha} a_{\alpha}) A_{\mu(1)\gamma}).$$  \hspace{1cm} (A.8)
As for the $D$-dimensional field strength $H$, it now includes Chern-Simons terms for all the gauge fields

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{\alpha'}{4} \text{tr}(A^{(3)}_\mu F^{(3)}_{\nu\rho}) - \frac{1}{3} A^{(3)}_\mu [A^{(3)}_\nu, A^{(3)}_\rho]$$

$$- \frac{1}{2} (A^{(1)}_\mu F^{(2)}_{\nu\rho} + A^{(2)}_{\mu\alpha} F^{(1)}_{\nu\rho}^\alpha) + (\text{cyc. perms.}) .$$

With the above definitions after a somewhat tedious calculation we find the following action for the dimensionally reduced theory

$$S_D = \frac{2\pi}{(2\pi\sqrt{\alpha'})^4} \int dx \sqrt{-g} e^{-2\phi} \left\{ R(g) + 4g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{4} g^{\mu\nu} \partial_\mu G_{\alpha\beta} \partial_\nu G^{\alpha\beta} - \frac{1}{4} g^{\mu\rho} g^{\nu\lambda} G_{\alpha\beta} F_{\mu\nu}^{(1)\alpha} F_{\rho\lambda}^{(1)\beta} - \frac{1}{12} (H_{\mu\nu\rho})^2 - \frac{1}{4} (H_{\mu\nu\alpha})^2 - \frac{1}{4} (H_{\mu\alpha\beta})^2 - \frac{1}{12} (H_{\alpha\beta\gamma})^2 - \frac{\alpha'}{8} \text{tr}(F_{\mu\nu}^{(3)}) + a_{\alpha} F_{\mu\nu}^{(1)\alpha})^2 - \frac{\alpha'}{4} \text{tr}(D_{\mu} a_{\alpha})^2 - \frac{\alpha'}{8} \text{tr}([a_{\alpha}, a_{\beta}])^2 \right\} ,$$

Appendix B. S-duality transformations

In this appendix we will explicitly apply S-duality [14,15] to the action (A.10) specialized for the heterotic string on a four-torus. In our conventions the duality transformations are particularly simple [11]. First, the string tension is the same for the two theories, which can be checked in the end by transferring to the heterotic side the tension of the string obtained by wrapping the NS5-brane on K3. Next, the dilaton and the string metric change as

$$\phi' = -\phi, \quad g' = e^{-2\phi} g ,$$

$$D_{\mu} = \partial_\mu + [A^{(3)}_\mu, \cdot] \quad \text{denotes the gauge covariant derivative and the implied raising and lowering of the indices is done with } (g^{-1})^{\mu\nu} \text{ and } (G^{-1})^{\alpha\beta} .$$

We have also defined the following quantities

$$H_{\mu\nu\alpha} = F_{\mu\nu}^{(2)} - \frac{\alpha'}{2} \text{tr}(a_{\alpha} F_{\mu\nu}^{(3)}) - C_{\alpha\beta} F_{\mu\nu}^{(1)\beta}$$

$$H_{\mu\alpha\beta} = \partial_\mu B_{\alpha\beta} + \frac{\alpha'}{4} \{ \text{tr}(a_{\alpha} D_{\mu} a_{\beta}) - \text{tr}(a_{\beta} D_{\mu} a_{\alpha}) \}$$

$$H_{\alpha\beta\gamma} = -\frac{\alpha'}{2} \text{tr}(a_{\alpha} [a_{\beta}, a_{\gamma}]) ,$$

$$C_{\alpha\beta} = B_{\alpha\beta} + \frac{\alpha'}{4} \text{tr}(a_{\alpha} a_{\beta}) .$$

where

$$A^{(3)}_\mu = B_{\alpha\beta} + \frac{\alpha'}{4} \text{tr}(a_{\alpha} a_{\beta}) .$$
where we have denoted the type IIA six-dimensional fields with a prime.

Making this change of variables in the heterotic action we obtain

\[ S_{\text{IIA}} = \frac{2\pi}{(2\pi \sqrt{\alpha'})^4} \int d^6x \sqrt{-g} \left\{ e^{-2\phi'} \left( R(g') + 4 \partial_\mu \phi' \partial^\mu \phi' \right) + \frac{1}{4} e^{-2\phi'} \left( \partial_\mu G_{\alpha\beta} \right)^2 
\right. 
\left. - \frac{1}{4} \left( F^{(1)\alpha}_{\mu\nu} \right)^2 - \frac{1}{12} e^{2\phi'} \left( H_{\mu\nu}\rho \right)^2 - \frac{1}{4} \left( H_{\mu\nu\alpha} \right)^2 - \frac{1}{12} e^{-2\phi'} \left( H_{\alpha\beta}\gamma \right)^2 
\right. 
\left. - \frac{\alpha'}{8} \text{tr} \left( F^{(3)}_{\mu\nu} + a_\alpha F^{(1)\alpha}_{\mu\nu} \right)^2 - \frac{\alpha'}{4} e^{-2\phi'} \text{tr} \left( D_\mu a_{\alpha I} \right)^2 - \frac{\alpha'}{8} e^{-4\phi'} \text{tr} \left( [a_\alpha, a_\beta] \right)^2 \right\} . \]

(B.2)

All the quantities here are defined by the same expressions (A.9), (A.11) and (A.12) as in the Heterotic compactification. That is so because none of these definitions depend on the metric.

Now we need to dualize the six-dimensional three-form field strength \( H \). Recall its definition from (A.9)

\[ H = dB - \omega^{cs} , \]

where \( \omega^{cs} \) denotes a full Chern-Simons form from (A.9). Then we have

\[ dH = -d\omega^{cs} = -\frac{\alpha'}{4} \text{tr} \left( F^{(3)} \land F^{(3)} \right) - F^{(1)\alpha} \land F^{(2)}_\alpha . \]

(B.4)

Note that unlike \( \omega^{cs} \) itself, its exterior derivative is globally well-defined and gauge-invariant. We can then treat \( H \) as an independent field if we add to the action a Lagrange multiplier that will enforce (B.4) as a constraint, namely

\[ \delta S_{\text{IIA}} = \frac{2\pi}{(2\pi \sqrt{\alpha'})^4} \int B' \land (dH + d\omega^{cs}) , \]

(B.5)

where \( B' \) is another 2-form field.

Now we will focus only on those terms in the action that contain \( H \):

\[ S_{\text{IIA}|H} = -\frac{2\pi}{(2\pi \sqrt{\alpha'})^4} \int \left( \frac{1}{2} e^{2\phi'} H \land \star' H - B' \land (dH + d\omega^{cs}) \right) . \]

(B.6)

Integrating out \( B' \) just gives back the original action (B.2). Instead, integrating out \( H \), we need to set it to the solution of the equations of motion

\[ e^{2\phi'} \star' H - dB' = 0 . \]

(B.7)

Substituting this back in (B.6) we obtain the action in terms of the dual field \( B' \)

\[ S_{\text{IIA}|H} = -\frac{2\pi}{(2\pi \sqrt{\alpha'})^4} \int \left( \frac{1}{2} e^{-2\phi'} dB' \land \star' dB' - B' \land d\omega^{cs} \right) . \]

(B.8)
The above is the correct action for the $B$ field of the type IIA compactified on K3, including the normalization of the kinetic term and the periodicity of the $B$ field \[30\].

Altogether, dropping the primes on the six dimensional type IIA fields, we have the following action

$$S_{\text{IIA}} = \frac{2\pi}{(2\pi \sqrt{\alpha'})^4} \int d^6 x \sqrt{-g} \left\{ e^{-\phi} \left( R(g) + 4 \partial_{\mu} \phi \partial^{\mu} \phi + \frac{1}{4} (\partial_{\mu} G_{\alpha \beta})^2 - \frac{1}{12} (H_{\mu \nu \rho})^2 ight) - \frac{1}{4} (H_{\mu \alpha \beta})^2 - \frac{\alpha'}{4} \text{tr}(D_{\mu} a^I_{\alpha})^2 - \frac{1}{4} (F^{(1)}_{\mu \nu})^2 - \frac{1}{4} (H_{\mu \nu \alpha})^2 ight.$$\[B.9\]

$$- \frac{\alpha'}{8} \text{tr}(F^{(3)}_{\mu \nu} + a_{\alpha} F^{(1)}_{\mu \nu} a^I_{\alpha})^2 - \frac{1}{12} e^{-4\phi} (H_{\alpha \beta \gamma})^2 - \frac{\alpha'}{8} e^{-4\phi} \text{tr}([a_{\alpha}, a_{\beta}])^2 \right\}$$

$$+ \frac{2\pi}{(2\pi \sqrt{\alpha'})^4} \int B \wedge \left( \frac{\alpha'}{4} \text{tr}(F^{(3)} \wedge F^{(3)}) + F^{(1)} \wedge F^{(2)} \right).$$

On this side the definition of $H$ does not include the Chern-Simons term anymore

$$H_{\mu \nu \rho} = \partial_{\mu} B_{\nu \rho} + (\text{cyc. perms.}) \quad \text{or} \quad H = dB ,$$

while the definitions of $H_{\mu \nu \alpha}$, $H_{\mu \alpha \beta}$ and $H_{\alpha \beta \gamma}$ are given by the same expressions as before, \[A.11\] and \[A.12\].
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