THE STRUCTURE OF DARK MATTER HALOS.
OBSESSION VERSUS THEORY

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The rotation curves of dark matter dominated dwarf galaxies are analysed. The observations show that dark matter halos represent a one-parameter family with self similar density profiles. The global halo parameters, like total mass and scale length are coupled by simple scaling relations. The inner halo regions resemble non-singular, isothermal spheres with constant density cores. The observations are compared with dark matter halos, resulting from cosmological cold dark matter simulations. The theoretical models predict that dark matter halos represent a one-parameter family in agreement with the observations. However, in contradiction to the observations, the calculations lead to dark matter halos with $r^{-1}$ density cusps in the center and non-isothermal velocity dispersion profiles. Processes which might affect the inner halo structure, resulting in isothermal, constant density cores are discussed.

1 Introduction

Structure formation in the universe is strongly coupled with dark matter (DM). Current cosmological models assume that there exists a non-baryonic cold dark matter (CDM) component which consist of non-relativistic particles that interact with the baryonic component only through gravity. Given a primordial density fluctuation spectrum, cosmological models investigate the formation of dark matter structures and compare the results with observations of the distribution of galaxies into clusters and superclusters. This comparison provides important information on cosmological parameters as well as on the initial dark matter fluctuation spectrum and by this on the origin and nature of dark matter.

Dark matter can also be studied on galactic scales. On these scales which are of order a few kpc, dark matter structures are in general much older than their internal dynamical timescales. They therefore have reached a dynamical equilibrium state, a virialized dark matter halo. These halos could however still retain valuable information about the initial conditions from which they formed, if the assumption is valid, that dark matter consists of collisionless and dissipationless particles. Dark matter halos often host galaxies in their inner regions. Studying the dynamical properties of these galaxies we gain insight into the inner density structure of dark matter halos and by this into the origin and nature of dark matter.
2 The structure of simulated dark matter halos

Assuming spherical symmetry, the radial DM mass distribution \( M(r) \) can be described by the DM rotation curve, that is the circular velocity profile \( V_c(r) = (GM(r)/r)^{1/2} \). The observations of constant circular velocities in the outer regions of many spiral galaxies\(^1\) have lead to the conclusion that DM halos are virialized isothermal spheres with an \( r^{-2} \) density profile in the observable radius range. Cosmological models indeed lead to dark matter halos which produce constant outer rotation curves, in general agreement with these observations. Early cosmological calculations did not have enough resolution in order to resolve the density structure of DM halos in detail. Recent high-resolution simulations\(^{11,12,14}\) however have shown that in the inner and outer regions dark matter halos depart significantly from an \( r^{-2} \) power-law distribution. All halo density profiles can be fit accurately by the simple formula,

\[
\rho(r) = \frac{\bar{\rho}}{(r/r_s)(1 + r/r_s)^2}
\]

where \( \bar{\rho} \) and \( r_s \) are two free parameters. It is very interesting that Navarro et al\(^{12,14}\) find a strong correlation between \( \bar{\rho} \) and \( r_s \). Dark matter halos seem to represent a one-parameter family, characterized completely by their virial mass \( M_{200} \) which is the total mass inside the virial radius \( r_{200} \). \( r_{200} \) is the characteristic radius inside which the mean DM density is \( 200 \times \rho_{\text{crit}} \), where \( \rho_{\text{crit}} = 3H^2/8\pi G \) is the critical density.

3 The structure of observed dark matter halos

Unfortunately it is difficult to observationally verify these numerical results as galaxies are in general gravitationally dominated by their visible baryonic components in the inner regions, while in the outer regions there is not enough visible material in order to measure accurately a rotation curve. In the inner region the inferred DM profiles will depend strongly on how much baryonic mass is subtracted, which in turn depends on the assumed baryonic mass-to-light ratio. The situation becomes even more complicated by the fact that a dominating baryonic component will gravitationally affect and change the cold dark matter density profile\(^3\).

3.1 Observed dark matter mass profiles

This situation has changed with the discovery of a new class of low surface brightness dwarf spirals and irregulars, which are strongly dominated by dark matter, even in their innermost regions. High-quality rotation curves have
Figure 1: Dark matter mass profiles are shown for the following dwarf spiral galaxies: DDO154 (open triangle), DDO105 (open square), NGC3109 (open circle) and DDO170 (starred). The errorbar at the innermost triangle represents the observational uncertainty in the inner region. The isothermal fit with core radius \( r_0 \) is shown as dashed curve, the solid line shows the revised profile, given in the text. The dotted and dot-dashed curve show the mass profiles as predicted from CDM calculations with formation redshifts of \( z = 0.6 \) and \( z = 1.5 \), respectively.
become available in the past few years, which provide insight into the detailed structure of dark matter halos.

Figure 1 shows the dark matter mass profiles of four dwarf galaxies with high signal-to-noise ratio HI rotation curves. All four profiles indeed follow the same universal mass relation. The dot-dashed and dotted curves show DM profiles as predicted from cosmological simulations (equation 1). The dot-dashed curve corresponds to a dark matter halo with virial radius \( r_{200} = 5 \times r_s \). The best fit through the data, using equation 1, is achieved with the dotted line which assumes \( r_{200} = 17.5 \times r_s \). Clearly, the halo profiles resulting from numerical simulations, are too massive at small radii when compared with the observations. This is a result of the central \( r^{-1} \) density cusp. The apparent contradiction between observation and theory has been discussed in detail by Flores and Primack, Moore, and Burkert.

A nice fit through the observed profiles over the whole observable radius range is achieved with the simple density distribution

\[
\rho_{\text{DM}}(r) = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)}
\]

where \( \rho_0 \) and \( r_0 \) are free parameters which represent the central DM density and a scale radius, respectively. Equation 2 resembles an isothermal profile with a constant-density core at small radii \((r < r_0)\). At large radii the density decreases faster than expected for an isothermal distribution, in agreement with the predictions from CDM calculations.

### 3.2 Dark matter scaling relations

Navarro et al. have shown that the two scale parameters of equation 1 are strongly correlated. Small halos are significantly denser than large halos as a result of the fact that small, low-mass halos formed at higher collapse redshifts when the density of the universe was higher.

Whether DM halos indeed represent a one-parameter family, being described completely by their total mass, can be investigated by looking for a correlation between the free parameters \( \rho_0 \) and \( r_0 \) in the observational fit formula (equation 2). Instead of using \( \rho_0 \), which cannot be observed directly, Fig. 2 shows the rotational velocity \( v_0 \) of observed DM rotation curves at \( r_0 \) as a function of \( r_0 \). \( r_0 \) is determined by fitting the data with a velocity curve as predicted by equation 2. We find indeed a very strong correlation between \( r_0 \) and \( v_0 \). The slope agrees well with the predictions from cosmological models. Using equation 2 and assuming spherical symmetry one can derive the following scaling relations for the observed DM halos:
Figure 2: The scaling relation between the rotational velocity $v_0$ measured at $r_0$ is shown for the DM halos, investigated by Burkert\textsuperscript{2}. Open circles represent the four DDO galaxies which have been used also in Fig. 1. The filled circles show three additional galaxies: NGC55, NGC300 and NGC1560. The dashed line is a fit through the data points.
\[ v_0 = 17.7 \left( \frac{r_0}{\text{kpc}} \right)^{2/3} \frac{\text{km}}{\text{s}} \]
\[ M_0 = 7.2 \times 10^7 \left( \frac{r_0}{\text{kpc}} \right)^{7/3} M_\odot \]  
\[ \rho_0 = 2.7 \times 10^{-2} \left( \frac{r_0}{\text{kpc}} \right)^{-2/3} \frac{M_\odot}{\text{pc}^3} \]

where \( M_0 \) is the total dark matter mass inside \( r_0 \). These relations indicate that dark matter halos indeed represent a one-parameter family, in agreement with cosmological models.

4 On the origin of isothermal dark matter cores

As shown in the last section, the shape of the cosmologically predicted universal dark matter density profiles disagrees with the observations. Whereas the observations indicate isothermal dark matter cores with constant density \( \rho_0 \) and constant velocity dispersion \( \sigma_0 \), the cosmological models lead to cuspy cores with density profiles \( \rho \sim r^{-1} \) and velocity dispersion profiles \( \sigma(r) \sim r \). The simulated dark matter cores are dynamically cold and dense. The observed dark matter cores are hotter and less dense. In order to explain this difference a mechanism has to be found which heats dark matter cores, increasing their velocity dispersion and by this decreasing the central dark matter density.

4.1 Cosmological initial conditions

Navarro et al.\(^1\) have investigated in detail how the structure of DM halos depends on the adopted cosmological model. They find that the profiles are always well fitted by equation 1, independent of halo mass, of the adopted initial density fluctuation spectrum, and of the values of the cosmological parameters. Thus the problem cannot be solved by selecting a certain cosmological model.

This result is not surprising. It is well known that the violent gravitational relaxation of collisionless particle systems leads to universal equilibrium profiles, independent of the initial conditions.\(^7\) The final profiles of such systems can be well described by a Hernquist profile,\(^9\)

\[ \rho_h(r) = \frac{M}{2\pi r(r + a)^3} \]
where $M$ is the total mass of the system and $a$ is its scale length. The Hernquist profile, for example, gives a good description of the surface brightness profiles of elliptical galaxies, collisionless stellar systems which have gone through a stage of violent relaxation. Note, that the simulated halo profiles (equation 1) are very similar to the profiles described by equation (3), with the main difference being a less steeply decreasing density distribution in the outermost regions. This results from the fact that the Hernquist profile describes systems with finite mass, whereas in cosmological models with $\Omega = 1$ halos will always accrete dark matter, leading to a mass profile that should diverge logarithmically for large radii.

4.2 Warm dark matter

Dark matter is assumed to consist of collisionless particles, which interact only by gravity. In this case, the 6-dimensional, microscopic phase space distribution function (DF) $f(\vec{x}, \vec{v})$ is a conserved quantity. The cuspy dark matter cores with $\rho \sim r^{-1}$ and $\sigma \sim r$ are characterized by a DF which diverges as $f \sim \rho/\sigma^3 \sim r^{-4}$. Given a critical phase space density $f_{\text{crit}}$, there always exists a finite radius $r_{\text{crit}}$, inside which $f > f_{\text{crit}}$. If the maximum phase space density of dark matter would be finite ($f < f_{\text{crit}}$), the dark matter density profile should flatten inside $r_{\text{crit}}$.

CDM particles formed with negligible initial velocity dispersion and therefore with an infinitely large $f_{\text{crit}}$. No phase space limitations are imposed on CDM cores. The situation is different in the case of warm dark matter, which starts with a finite initial velocity dispersion and therefore with a finite $f_{\text{crit}}$. In this case, DM cores might become isothermal inside $r_{\text{crit}}$, where $f$ approaches a constant and universal value $f_{\text{crit}}$, that is determined by the initial dark matter temperature. This idea can be tested. According to the equations 3, the central phase space density of observed dark matter cores scales as $f_0 \sim \rho_0/v_0^3 \sim r_0^{-8/3}$. It decreases steeply with increasing core radius. The centers of DM halos are not limited by a universal and finite maximum phase space density which rules out warm dark matter as origin for isothermal dark matter cores.

4.3 Secular dynamical processes

As the problem cannot be solved by varying the initial conditions or the nature of dark matter we have to focus on secular dynamical processes that might affect the central parts of the dark matter halos, after the halo formation phase.
Navarro et al. have proposed a scenario, where a gaseous disk forms in the centers of dark matter halos. The disk potential dominates the central gravitational potential. The authors assume that after a vigorous episode of star formation a large fraction of the total baryonic component is expelled from the galaxy through supernova-driven winds. They show that a sudden loss of a large fraction of the total gravitational mass from the inner region would result in an expansion of the dark matter core, decreasing the central DM density. This scenario seems at first very attractive. The observed scaling relations for dark matter cores and the fact that dark matter halo profiles are universal would however require significant fine tuning between the early cosmological collapse phase and the secular energetic processes. It is unlikely that DM halos would have self-similar density profiles if their inner structure is subsequently changed by dynamical processes which are not related to the collisionless relaxation process which determined the outer DM profiles. In this case, we would expect that dark matter halos are described by two independent parameters. The first parameter determines their inner structure. It will depend on the violence of the secular processes. The second parameter determines the outer DM structure and depends on the cosmological merging history of the halo.

Substantial mass loss also seems unlikely in the case of the DDO154, a dwarf spiral galaxy with a dark matter halo of total mass $M_{DM} \approx 3 \times 10^9 M_\odot$, containing an extended HI disk with total mass $2.5 \times 10^8 M_\odot$. Navarro et al. estimate that, prior to the mass loss epoch, the mass of the gaseous disk should be of order 6 per cent of the total dark matter mass. This ratio is in agreement with the disk-to-halo mass ratio in DDO154, demonstrating that no substantial mass loss has yet occurred in this system. On the other hand, the rotation curve of DDO154 clearly shows that its dark matter halo has an isothermal DM core, which must have formed by a different mechanism.

5 Summary

The observations indicate that dark matter halos are self-similar, being described completely by one free parameter. This surprising universality makes it unlikely that the isothermal cores of observed DM halos result from secular processes. The observed shallow central density profiles probably formed as a direct result of the same processes, which lead to the dark matter halos in the first place. As discussed above, DM calculations do not produce isothermal halo cores. We therefore have to conclude that some important, yet unknown physical features, related to the nature and origin of dark matter, are still missing in cosmological models.
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