Correspondence between Electro-Magnetic Field and other Dark Energies in Non-linear Electrodynamics

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In this work, we have considered the flat FRW model of the universe filled with electro-magnetic field. First, the Maxwell’s electro-magnetic field in linear form has been discussed and after that the modified Lagrangian in non-linear form for accelerated universe has been considered. The corresponding energy density and pressure for non-linear electro-magnetic field have been calculated. We have found the condition such that the electro-magnetic field generates dark energy. The correspondence between the electro-magnetic field and the other dark energy candidates namely tachyonic field, DBI-essence, Chaplygin gas, hessence dark energy, k-essence and dilaton dark energy have been investigated. We have also reconstructed the potential functions and the scalar fields in this scenario.

I. INTRODUCTION

Recent observations of type Ia supernovae (SNIa) indicate that our universe is now undergoing an accelerating expansion [1]. The main responsible candidate for the cosmic acceleration is generally dubbed as ‘dark energy’, a mysterious exotic energy with negative pressure. Present cosmological observational data suggest that universe is dominated by this dark energy with 70 percent of the total. So the feature of the universe naturally depends on the nature of the dark energy. the simplest candidate of dark energy is Cosmological Constant with fixed equation of state (EOS) \(w = -1\). If it is quintessence then \(-1 < w < -1/3\) and if it is phantom then \(w < -1\). The constant EOS \(w = -1\) is called phantom divide. There are some dark energies which can cross the phantom divide from both sides.

In present years the standard cosmological model based on Friedmann-Robertson-Walker (FRW) with Maxwell electrodynamics has got much attention and many interesting results are obtained. This leads to a cosmological singularity at a finite time in the past and result the energy density and curvature arbitrary large in the very early epoch [2]. This singularity breaks the laws of physics with mathematical inconsistency and physical incompleteness of any cosmological model. There are some proposals to handle this primordial singularity such as cosmological constant [3], non minimal couplings [4], modifications of geometric structure of space-time [5], non-equilibrium thermodynamics [6], Born-Infeld type nonlinear electromagnetic field [7] and so on. Recently a new approach has been taken to avoid the cosmic singularity through a nonlinear extension of the Maxwell electromagnetic theory. Another interesting feature can be viewed that an exact regular black hole solution has been recently obtained proposing Einstein-dual nonlinear electrodynamics [8, 9]. Exact solutions of the Einstein’s field equations coupled with nonlinear electrodynamics (NLED) reveal an acceptable nonlinear effect in strong gravitational and magnetic fields. Also the General Relativity (GR) coupled with NLED effects can explain the primordial inflation.

In this work, we have briefly discussed the Maxwell’s electrodynamics in linear and non-linear forms and then the modified Lagrangian in accelerated universe has been considered. The energy density and pressure for non-linear electro-magnetic theory have been calculated. The purpose of the present work is to investigate the correspondence between electromagnetic field and other dark energy candidates namely tachyonic field [10], DBI-essence [11], Chaplygin gas [12], hessence dark energy [13], k-essence [14] and dilaton dark energy [15]. The potentials have been reconstructed in terms of electric field and magnetic field in above types of dark energy models.

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II. ELECTRO-MAGNETIC THEORY IN NON-LINEAR ELECTRODYNAMICS

Lagrangian density in Maxwell’s electrodynamics can be written as [16]

\[ \mathcal{L} = -\frac{1}{4\mu_0} F^\mu\nu F_{\mu\nu} = -\frac{1}{4\mu_0} F^2 \]  

where \( F^\mu\nu \) is the electromagnetic field strength tensor and \( \mu_0 \) is the magnetic permeability. The canonical energy-momentum tensor is then given by

\[ T_{\mu\nu} = \frac{1}{\mu_0} \left( F_{\mu\alpha} F^\alpha_{\nu} + \frac{1}{4} F g_{\mu\nu} \right) \]

Since the spatial section of FRW geometry are isotropic, electromagnetic fields can generate such a universe only if an averaging procedure is performed [17]. Applying standard spatial averaging process for electric field \( E_i \) and magnetic field \( B_i \), set

\[ < E_i > = 0, \quad < B_i > = 0, \quad < E_i E_j > = -\frac{1}{3} E^2 g_{ij}, \quad < B_i B_j > = -\frac{1}{3} B^2 g_{ij}, \quad < E_i B_j > = 0. \]

So one gets,

\[ < F_{\mu\alpha} F^\alpha_{\nu} > = \frac{2}{3} \left( \epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) u_\mu u_\nu + \frac{1}{3} \left( \epsilon_0 E^2 - \frac{B^2}{\mu_0} \right) g_{\mu\nu} \]

where \( u_\mu \) is the fluid 4-velocity. Now comparing with the average value of energy momentum tensor

\[ < T_{\mu\nu} > = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}, \]

the energy density and pressure have the forms

\[ \rho = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{B^2}{\mu_0} \right), \quad p = \frac{1}{3} \rho \]

This implies the Maxwell’s electro-magnetic fields generate effectively the radiation fluid.

The modified Lagrangian in non-linear electrodynamics for accelerated universe is considered as [18]

\[ \mathcal{L} = -\frac{1}{4} F + \alpha F^2 + \beta F^{-1} \]

where \( \alpha \) and \( \beta \) are arbitrary (constant) parameters. As seen this Lagrangian contains both positive and negative powers of \( F \). The second (quadratic) term dominates during very early epochs of the cosmic dynamics, while the Maxwell term (first term above) dominates in the radiation era. The last term is responsible for the accelerated phase of the cosmic evolution [19]. The above Lagrangian density yields a unified scenario to describe both the acceleration of the universe (for weak fields) and the avoidance of the initial singularity, as a consequence of its properties in the strong-field regime.

The energy density and pressure for electro-magnetic (EM) field are given by

\[ \rho_B = -\mathcal{L} - 4E^2 \mathcal{L}_F \]

and

\[ p_B = \mathcal{L} - \frac{4}{3} (2B^2 - E^2) \mathcal{L}_F \]
Now, the electro-magnetic field has the expression $F = 2(B^2 - E^2)$, so using (7) - (9) we get

$$\rho_B = \frac{1}{2}(B^2 + E^2) - 4\alpha(B^2 - E^2)(B^2 + 3E^2) - \frac{\beta}{2(B^2 - E^2)}$$  \hspace{1cm} (10)

and

$$p_B = \frac{1}{6}(B^2 + E^2) - \frac{4\alpha}{3}(B^2 - E^2)(5B^2 - E^2) + \frac{\beta(7B^2 - 5E^2)}{6(B^2 - E^2)^2}$$  \hspace{1cm} (11)

The electro-magnetic field generates dark energy if the strong energy condition is violated i.e., $\rho_B + p_B < 0$. So from (10) and (11), we get

$$(B^2 + E^2) - 8\alpha(B^2 - E^2)(3B^2 + E^2) + \frac{\beta(3B^2 - 2E^2)}{(B^2 - E^2)^2} < 0$$  \hspace{1cm} (12)

In particular, if $\alpha = 0$ and $\beta = 0$, then the expression in the l.h.s of (12) is $(B^2 + E^2)$ which cannot be negative. So only Maxwell’s electro-magnetic field (linear) cannot generate the dark energy. So for getting dark energy, we require non-linear electro-magnetic field.

The metric of a homogeneous and isotropic flat universe in the FRW model is

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right]$$  \hspace{1cm} (13)

where $a(t)$ is the scale factor and $k (= 0, +1, -1)$ is the curvature scalar. The Einstein field equations for electro-magnetic Universe are

$$3\frac{\dot{a}^2}{a^2} = \rho_B$$  \hspace{1cm} (14)

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_B + p_B)$$  \hspace{1cm} (15)

where $\rho_B$ and $p_B$ are energy density and pressure corresponding to electromagnetic field given by the equations (10) and (11) (choosing $8\pi G = c = 1$). Now the energy-conservation equation for electro-magnetic field is given by

$$\dot{\rho}_B + 3\frac{\dot{a}}{a} (\rho_B + p_B) = 0$$  \hspace{1cm} (16)

Since non-linear electro-magnetic field generates dark energy, so in the following sections, we shall discuss the correspondence between the electro-magnetic (EM) field and the other types of dark energies like tachyonic field, DBI-essence, Chaplygin gas, hessence, k-essence and dilaton dark energies and hence find the nature of the potentials and scalar fields.

### III. CORRESPONDENCE BETWEEN EM FIELD AND TACHYONIC FIELD

The present section aims to investigate the conditions under which there is a correspondence between EM field and the tachyonic field, in the flat FRW Universe. That is, to determine an appropriate potential for tachyonic field which makes the two dark energies to coincide with each other. Let us first consider the energy density $\rho_T$ and the pressure $p_T$ for the tachyonic field as [10]
Fig. 1 shows the variation of $V$ against $B$ and $E$ for different values of $\alpha = 1$, $\beta = 2$ when a correspondence between EM field and tachyonic field is considered.

\[ \rho_T = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \quad (17) \]

and

\[ p_T = -V(\phi)\sqrt{1 - \dot{\phi}^2} \quad (18) \]

where $\phi$ is the tachyonic field and $V(\phi)$ is the corresponding potential. Comparing the energy density and pressure corresponding to EM field and tachyonic field we have the expressions for the tachyonic field and the potential as

\[ \phi = \int 2 \sqrt{\frac{(B^2 - E^2)^2(B^2 + E^2 + 8(B^4 - 6B^2E^2 + 5E^4)\alpha - 5B^2\beta + 4E^2\beta)}{\alpha(-B^4 + E^4 + 8(B^2 - E^2)^2(B^2 + 3E^2)\alpha + \beta)^2}} da \quad (19) \]

and

\[ V = \frac{1}{2\sqrt{3}} \left[ \left( B^2 + E^2 - 40B^4\alpha + 48B^2E^2\alpha - 8E^4\alpha + \frac{(7B^2 - 5E^2)\beta}{(B^2 - E^2)^2} \right) \times \right. \\
\left. \left( -B^2 - E^2 + 8(B^4 + 2B^2E^2 - 3E^4)\alpha + \frac{\beta}{B^2 - E^2} \right) \right]^{\frac{1}{2}} \quad (20) \]

In figure 1, we have drawn the potential function $V$ against magnetic field $B$ and electric field $E$. From 3 dimensional figure, we have seen that $V$ always increases with $E$ increases but slightly increases with increase in $B$.

IV. CORRESPONDENCE BETWEEN EM FIELD AND DBI-ESSENCE

There have been many works aimed at connecting the string theory with inflation. While doing so, various ideas in string theory based on the concept of branes have proved themselves fruitful. One area which has been
well explored in recent years, is inflation driven by the open string sector through dynamical Dp-branes. This is the so-called DBI (Dirac-Born-Infield) inflation, which lies in a special class of K-inflation models. The energy density and pressure of the DBI-essence scalar field are respectively given by \[11\]

\[ \rho_D = (\gamma - 1)T(\phi) + V(\phi) \]  
and

\[ p_D = \frac{\gamma - 1}{\gamma}T(\phi) - V(\phi) \]

where \(\gamma\) is given by

\[ \gamma = \frac{1}{\sqrt{1 - \frac{T}{V}^2(\phi)}} \]  

(23)

where \(V(\phi)\) is the self interacting potential and \(T(\phi)\) is the warped brane tension.

Comparing the energy density and pressure corresponding to EM field and DBI-essence scalar field we have the expressions for potential, wrapped brane tension and \(\gamma\) for DBI-essence as

\[ V(\phi) = \frac{1}{\gamma + 1}(\rho_D - \gamma p_D) \]  
and

\[ T(\phi) = \frac{\rho_D + p_D}{2\gamma - 1} \]  

where

\[ \gamma = \frac{2B^6 - 2B^4E^2 - 2B^2E^4 + 2E^6 - 32B^8\alpha + 64B^6E^2\alpha - 64B^2E^6\alpha + 32E^8\alpha + 2B^2\beta - E^2\beta}{3(B^2 - E^2)^2\phi^2} \]  

(26)

Here we see that \(V(\phi), T(\phi)\) and \(\gamma\) are functions of \(B^2, E^2\) and \(\phi^2\). To overcome the complexity we consider here two cases \(\gamma = \text{constant}\) and \(\gamma \neq \text{constant}\) [20].

**Case I:** \(\gamma = \text{constant}\).

In this case, for simplicity, we assume \(T(\phi) = n\dot{\phi}^2\ (n > 1)\) and \(V(\phi) = m\ddot{\phi}^2\ (m > 0)\). So we have \(\gamma = \sqrt{n - 1}\).

In this case the expression for \(V(\phi)\) and \(T(\phi)\) are given by

\[ V = m \frac{2B^6 - 2B^4E^2 - 2B^2E^4 + 2E^6 - 32B^8\alpha + 64B^6E^2\alpha - 64B^2E^6\alpha + 32E^8\alpha + 2B^2\beta - E^2\beta}{3(B^2 - E^2)^2\sqrt{n - 1}} \]  

(27)

\[ T = m \frac{2B^6 - 2B^4E^2 - 2B^2E^4 + 2E^6 - 32B^8\alpha + 64B^6E^2\alpha - 64B^2E^6\alpha + 32E^8\alpha + 2B^2\beta - E^2\beta}{3(B^2 - E^2)^2\sqrt{n - 1}} \]  

(28)

**Case II:** \(\gamma \neq \text{constant}\).

In this case let us assume \(\gamma = \dot{\phi}^2\). So from equation (23) we have \(T(\phi) = \frac{\ddot{\phi}^2 + \dot{\phi}^2}{\dot{\phi}^2 - 1} > \dot{\phi}^2\). Since \(\gamma > 1\) so \(\dot{\phi}^4 > 1\).

Let us also assume \(V(\phi) = T(\phi)\). In this case we have the expression for \(\phi\) and \(V(\phi)(= T(\phi))\) for a particular case when \(s = -4\) as :

\[ \phi = \int \frac{3\sqrt{2(B^2 - E^2)}}{\alpha \sqrt{-B^4 + E^4 + 8(B^2 - E^2)^2[B^2 + 3E^2\alpha + \beta]/(2(B^2 - E^2)^2[B^2 + E^2](-1 + 16[B^2 - E^2]\alpha) + (-2B^2 + E^2)])} \, da \]  

(29)
Figs. 2 and 3 show the variation of $V$ and $T$ respectively against $B$ and $E$ for different values of $\alpha = 1$, $\beta = 2$, $m = 6$, $n = 5$ when a correspondence between EM field and DBI-essence scalar field is considered in Case I.

Fig. 4 shows the variation of $V$ or $T$ against $B$ and $E$ for different values of $\alpha = 2$, $\beta = 3$ when a correspondence between EM field and DBI-essence scalar field is considered in Case II.

$$V = T = \frac{(2(B^2 - E^2)^2(B^2 + E^2)(-1 + 16(B^2 - E^2)\alpha)(-2B^2 + E^2)\beta)^3}{27(B^2 - E^2)^6(1 - \frac{(21(B^2 - E^2)^2(B^2 + E^2)(-1 + 16(B^2 - E^2)\alpha)(-2B^2 + E^2)\beta)^3}{81(B^2 - E^2)^6})^5}$$

Figures 2 and 3 show the variation of $V$ and $T$ respectively against $B$ and $E$ for different values of parameters in the solution of case I. From 3 dimensional figures, we have seen that $V$ and $T$ always increase with $E$ increases but slightly increases with increase in $B$. Similarly, figure 4 shows the variation of $V$ or $T$ against $B$ and $E$ in the solution of case II. In this case we see that $V$ or $T$ always decreases with $E$ increases but slightly increases with increase in $B$. 
V. CORRESPONDENCE BETWEEN EM FIELD AND CHAPLYGIN GAS

A candidate for Q-matter is exotic type of fluid - the so called Chaplygin gas which obeys the EOS [12] \( p_{ch} = -\frac{B_1}{\rho_{ch}}(B_1 > 0) \), where \( p_{ch} \) and \( \rho_{ch} \) are respectively the pressure and energy density. Later on the above equation was generalized to the form \( p_{ch} = -\frac{B_1}{\rho_{ch}}(0 \leq \alpha_1 \leq 1) \) and recently it was modified to the form \( p_{ch} = A\rho_{ch} - \frac{B_1}{\rho_{ch}}(A > 0) \), which is known as Modified Chaplygin Gas [21].

Now comparing the energy density and pressure with electromagnetic field we have a relation between \( E \) and \( B \) in implicit form as

\[
\frac{1}{6} \left[ 8(-5+3A)B^4\alpha - 8(1+9A)E^4\alpha + E^2 \left( 1 - 3A - \frac{(5+3A)\beta}{(B^2 - E^2)^2} \right) + B^2 \left( 1 - 3A + 48E^2\alpha + 48AE^2\alpha + \frac{(7+3A)\beta}{(B^2 - E^2)^2} \right) \right. \\
\left. + 3 \times 2^{1+\alpha_1} B_1 \left( B^2 + E^2 - 8(B^4 + 2B^2E^2 - 3E^4)\alpha + \frac{\beta}{B^2 + E^2} \right)^{-\alpha_1} \right] = 0 \tag{31}
\]

The above expression is very complicated forms in \( B \) and \( E \). We can not write \( B \) in terms of \( E \) or \( E \) in terms of \( B \) explicitly. So for some particular values of the parameters, we have plotted the magnetic field \( B \) against electric field \( E \) in figure 5. From the figure, we see that \( B \) increases as \( E \) increases. So in this scenarios, electro-magnetic field behaves like modified Chaplygin gas model.

VI. CORRESPONDENCE BETWEEN EM FIELD AND HESSENCE DARK ENERGY

Wei et al [13] proposed a novel non-canonical complex scalar field named “hessence” which play the role of quintom. In the hessence model the so-called internal motion \( \dot{\theta} \) where \( \theta \) is the internal degree of freedom of hessence plays a phantom like role and the phantom divide transitions is also possible. The pressure and energy density for the hessence model are given by

\[
p_h = \frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) - V(\phi) \tag{32}
\]
Fig. 6 shows the variation of $V$ against $E$ and $B$ for different values of $\alpha = 2$, $\beta = 3$ when a correspondence between EM field and hessence scalar field is considered.

and

$$\rho_h = \frac{1}{2}(\dot{\phi}^2 - \dot{\theta}^2) + V(\phi)$$  \hspace{1cm} (33)$$

with

$$Q = a^3 \dot{\phi}^2 \theta = \text{constant},$$  \hspace{1cm} (34)

where $Q$ is the total conserved charge, $\phi$ is the hessence scalar field and $V$ is the corresponding potential.

Comparing the energy density and pressure corresponding to EM field and hessence field we have the expressions for scalar field for the hessence and potential as

$$\phi = \int \frac{\sqrt{3}}{a} \sqrt{1 + \frac{B^2 + E^2 + 3(7B^2 - 5E^2)}{(B^2 - E^2)} - 16(5B^4 - 6B^2E^2 + E^4) + \frac{60^2}{(B^2 - E^2)^2} da}$$  \hspace{1cm} (35)$$

and

$$V = \frac{1}{6} \left( B^2 + E^2 + 8B^4\alpha - 48B^2E^2\alpha + 40E^4\alpha + \frac{-5B^2\beta + 4E^2\beta}{(b^2 - E^2)^2} \right)$$  \hspace{1cm} (36)$$

In figure 6, we have drawn the hessence potential function $V$ against magnetic field $B$ and electric field $E$. From 3 dimensional figure, we have seen that $V$ always increases with $E$ increases but slightly decreases with increase in $B$.

**VII. CORRESPONDENCE BETWEEN EM FIELD AND K-ESSENCE**

Another type of model responsible for late time acceleration having non canonical kinetic term in Lagrangian called k-essence which is originating form Born-Infeld string theory as a possible model of inflation called k-inflation. The energy density and pressure of k-essence scalar field $\phi$ are given by [14]

$$\rho_k = V(\phi)(\chi - 3\chi^2)$$  \hspace{1cm} (37)$$
Fig. 7 shows the variation of $V$ against $E$ and $B$ for different values of $\alpha = 2$, $\beta = 3$ when a correspondence between EM field and k-essence scalar field is considered.

and

$$p_k = V(\phi)(\chi - \chi^2)$$  \hfill (38)

where $\phi$ is the scalar field having kinetic energy $\chi = \frac{1}{2}\dot{\phi}^2$ and $V(\phi)$ is the k-essence potential.

Comparing the energy density and pressure corresponding to EM field and k-essence field we have the expressions for k-essence field and the potential as

$$\phi = \int \frac{2}{a} \sqrt{\frac{- (B^2 - E^2)^3(B^2 + E^2 + 8(B4 - 6B^2E^2 + 5E^4)\alpha + (5B^4 - 9B^2E^2 + 4E^4)\beta)}{(-B^4 + E^4 + 8(B^2 - E^2)^2(B^2 + 3E^2)\alpha + \beta)(16(B^2 - E^2)^4\alpha - 4B^2\beta + 3E^2\beta)^2}} da \hfill (39)$$

$$V = \frac{3(16(B^2 - E^2)^4\alpha - 4B^2\beta + 3E^2\beta)^2}{2(B^2 - E^2)^4(B^2 + E^2 + 8(B4 - 6B^2E^2 + 5E^4)\alpha - 2(5B^2 - 4E^2)(B^2 - E^2)^2\beta} \hfill (40)$$

In figure 7, we have drawn the k-essence potential function $V$ against magnetic field $B$ and electric field $E$. From 3 dimensional figure, we have seen that $V$ always increases with $E$ increases but slightly increases with increase in $B$.

VIII. CORRESPONDENCE BETWEEN EM FIELD AND DILATON DARK ENERGY

String theory widely explain the present acceleration of the universe and provides candidate for cold dark matter. We consider here a dark energy model named dilaton dark energy which is basically motivated from low energy limit of string theory. Dilaton leads an initial inflationary phase followed by a kinetic energy dominated phase. In the Einstein frame, the coefficient of the kinematic term of the dilaton can be negative to behave as a phantom-type scalar field.

The energy density and pressure of the dilaton dark energy model are given by [15]

$$\rho_d = -X + Ce^{\lambda X}X^2$$  \hfill (41)
and

\[ p_d = -X + 3Ce^{\lambda \phi}X^2 \]  \hspace{1cm} (42)

where \( \phi \) is the dilaton scalar field having kinetic energy \( X = \frac{1}{2} \dot{\phi}^2 \), \( \lambda \) is the characteristic length which governs all non-gravitational interactions of the dilaton and \( C \) is a positive constant. Comparing the energy density and pressure corresponding to EM field and dilaton dark energy we have the expressions for scalar field as

\[ \phi = \int \frac{1}{a} \sqrt{\frac{2(B^2 - E^2)^2(B^2 + E^2 + 8(B^4 - 6B^2E^2 + 5E^4)\alpha - 10B^2\beta + 8E^2\beta)}{(B^2 - E^2)(-B^4 + E^4 + 8(B^2 - E^2)^2(B^2 + 3E^2)\alpha + \beta)}} da \]  \hspace{1cm} (43)

**IX. DISCUSSIONS**

In this work, we have considered the flat FRW universe filled with electro-magnetic field. First, the Maxwell’s electro-magnetic field in linear form has been discussed and after that the modified Lagrangian in non-linear form for accelerated universe has been considered. The corresponding energy density and pressure for non-linear electro-magnetic field have been calculated. We have found the condition such that the electro-magnetic field generates dark energy. Other types of dark energy candidates namely tachyonic field, DBI-essence, Chaplygin gas, hessence dark energy, k-essence and dilaton dark energy have been discussed shortly. The correspondence between the electro-magnetic field and the above types of dark energies have been investigated. We have reconstructed the potential functions and the scalar fields in this scenario.

In the tachyonic field model, the potential function \( V \) against magnetic field \( B \) and electric field \( E \) have been drawn in figure 1. From 3 dimensional figure, we have seen that \( V \) always increases with \( E \) increases but slightly increases with the increase of \( B \). In DBI-essence model, figures 2 and 3 show the variation of \( V \) and \( T \) respectively against \( B \) and \( E \) for different values of parameters in the solution of case I. From the figures, we have seen that \( V \) and \( T \) always increase with \( E \) increases but slightly increases with increase in \( B \). Similarly, figure 4 shows the variation of \( V \) or \( T \) against \( B \) and \( E \) in the solution of case II. In this case we see that \( V \) or \( T \) always decreases with \( E \) increases but slightly increases with increase in \( B \).

In modified Chaplygin gas model, we have found the relation between electric field and magnetic field if the electro-magnetic field behaves like modified Chaplygin gas equation of state. We have found an implicit relation between \( E \) and \( B \). We have also plotted the magnetic field \( B \) against electric field \( E \) in figure 5. From the figure, we see that \( B \) increases as \( E \) increases. In hessence dark energy model, we have drawn the hessence potential function \( V \) against magnetic field \( B \) and electric field \( E \) in figure 6 and have seen that \( V \) always increases with \( E \) increases but slightly decreases with increase in \( B \). For k-essence dark energy model, the k-essence potential function \( V \) against magnetic field \( B \) and electric field \( E \) have been drawn in figure 7 and have seen that \( V \) always increases with \( E \) increases but slightly increases with increase in \( B \). In every dark energy models, the expressions of scalar fields have been calculated in terms of electric field and magnetic field.

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