Lepton Flavor Violation and the Origin of the Seesaw Mechanism

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Abstract

The right–handed neutrino mass matrix that is central to the understanding of small neutrino masses via the seesaw mechanism can arise either (i) from renormalizable operators or (ii) from nonrenormalizable or super-renormalizable operators, depending on the symmetries and the Higgs content of the theory beyond the Standard Model. In this paper, we study lepton flavor violating (LFV) effects in the first class of seesaw models wherein the \( \nu_R \) Majorana masses arise from renormalizable Yukawa couplings involving a \( B - L = 2 \) Higgs field. We present detailed predictions for \( \tau \to \mu + \gamma \) and \( \mu \to e + \gamma \) branching ratios in these models taking the current neutrino oscillation data into account. Focusing on minimal supergravity models, we find that for a large range of MSSM parameters suggested by the relic abundance of neutralino dark matter and that is consistent with Higgs boson mass and other constraints, these radiative decays are in the range accessible to planned experiments. We compare these predictions with lepton flavor violation in the second class of models arising entirely from the Dirac Yukawa couplings. We study the dependence of the ratio \( r \equiv B(\mu \to e + \gamma)/B(\tau \to \mu + \gamma) \) on the MSSM parameters and show that measurement of \( r \) can provide crucial insight into the origin of the seesaw mechanism.
I. INTRODUCTION

With the evidence for neutrino masses and mixings from solar and atmospheric neutrino data getting more and more firm, the nature of new physics that could explain the observations is under a great deal of scrutiny [1]. There are several issues that need to be understood, notably (a) the smallness of neutrino masses, and (b) the origin of the large atmospheric neutrino as well as the solar neutrino mixing angles, the latter being favored [2] by the combined solar neutrino results [3] including the recent SNO neutral current data [4].

Seesaw mechanism [5] provides one of the simplest ways to understand the small neutrino masses. It assumes the existence of a set of three right–handed neutrinos ($\nu_R$) that have masses at the scale $v_{B-L}$ corresponding to a new (local) $B-L$ symmetry of weak interactions. Atmospheric neutrino observations suggest that the scale $v_{B-L}$ is much lower than the Planck scale, leading to a new threshold inferred solely from experiments. At low energies, these heavy right–handed neutrinos induce operators of the form $Y_\nu^2 (LH_u)^2 / v_{B-L}$, where $Y_\nu$ is the Dirac Yukawa coupling matrix connecting the $\nu_R$ with the left-handed neutrinos ($\nu_L$) and the Standard Model (SM) Higgs doublet $H_u$. $L$ denotes the lepton doublet which contains $\nu_L$. After electroweak symmetry breaking, these operators induce masses for the light $\nu_L$ of order $Y_\nu^2 v_{B-L}^2 / v_{B-L}$. Since $v_{B-L}$ is much larger than the weak scale $v_{wk}$, the induced $\nu_L$ masses will be extremely small compared to all other masses of the SM fermions. Present neutrino oscillation data suggests a scale $v_{B-L} \sim 10^{12} - 10^{15}$ GeV.

The precise seesaw formula for the light neutrino mass matrix with three generations is given by

$$M_\nu = -M_D^T M_R^{-1} M_D,$$  \hspace{0.5cm} (1)

where $M_D$ is the Dirac neutrino mass matrix and $M_R$ is the $\nu_R$ Majorana matrix. The neutrino mixing angles in such schemes would arise as a joint effect from two sources: (i) mixings among the right–handed neutrinos present in $M_R$ and (ii) mixings among different generations present in the Dirac mass matrix $M_D$. The (physical) neutrino oscillation angles will also receive contributions from mixings among the charged leptons.

A. Neutrino mixings and lepton flavor violation

The neutrino flavor mixings induced by the seesaw mechanism (as needed by oscillation data) can lead to lepton flavor violating (LFV) effects. However, within the SM extended minimally to accommodate the seesaw mechanism, such effects are extremely small in any process other than neutrino oscillation itself, owing to a power suppression factor $(1/v_{B-L})^2$, as required by the decoupling theorem. This situation is drastically different if there is low energy supersymmetry. The power suppression then becomes $(1/M_{SUSY})^2$ which is very much weaker and can lead to observable LFV effects at low energies, as noted in a number of papers [6,7,8]. The main difference is that with low energy SUSY, lepton flavor violation can be induced in the slepton sector, which can then be transferred to the leptons suppressed only by a factor $(1/M_{SUSY})^2$. The way this comes about is as follows. Consider the minimal $N = 1$ supergravity (mSUGRA) models [9]. At the fundamental scale $M_{Pl}$ where SUSY
breaking is communicated to the SM sector, all the soft SUSY breaking scalar masses are taken to be universal, and the trilinear $A$-matrices are taken to be proportional to the respective Yukawa matrices. Thus, at the Planck scale, there is no flavor violation anywhere except in the Yukawa couplings and SUSY–preserving mass terms in the superpotential. Now, the right–handed neutrinos have masses of order $v_{B-L}$ which is much lower than $M_{Pl}$, as inferred from atmospheric neutrino data. In the momentum regime $v_{B-L} \leq \mu \leq M_{Pl}$ where the $\nu_R$ fields are active, the soft masses of the sleptons will feel the effects of LFV in the neutrino Yukawa sector through the renormalization group evolution. At the scale $v_{B-L}$, the slepton mass matrix is no longer universal, and this non-universality will remain down to the weak scale. This LFV in the slepton sector is subsequently transferred to the leptons through one–loop diagrams involving the exchange of gauginos.

It should be noted that in the absence of neutrino masses, there is only one leptonic Yukawa matrix, $Y_\ell$ for the charged leptons, which can be diagonalized at $M_{Pl}$. $Y_\ell$ will remain diagonal to the weak scale, and would not induce any flavor violation in the slepton sector in mSUGRA models. The experimental evidence for neutrino oscillation is thus a strong indicator that there might very well be lepton flavor violation, assuming the validity of low energy SUSY. Searches for LFV processes such as $\tau \rightarrow \mu + \gamma$ and/or $\mu \rightarrow e + \gamma$ can therefore be an important source of information on the $\nu_R$ mixings in $M_R$ and/or family mixings in $M_D$. (There are some exceptional circumstances, which we point out later in this section.)

**B. Majorana LFV versus Dirac LFV**

It is possible to classify the seesaw models into two classes depending on the way the $\nu_R$ gets its Majorana mass $M_R$. In one case, $M_R$ arises from renormalizable Yukawa couplings involving a $B - L = 2$ Higgs field $\Delta$ through the superpotential terms

$$W \supset f \nu^c \nu^c \Delta + Y_\nu \nu^c L H_u.$$

(2)

Here $f$ is the Majorana Yukawa coupling matrix, $Y_\nu$ is the Dirac Yukawa matrix and we use the standard notation appropriate for supersymmetry with $\nu^c$ being the conjugate of $\nu_R$. In the second class of models, the matrix $M_R$ is either put in “by hand” as a bare mass term in the Lagrangian (such terms are super-renormalizable), or arises from non-renormalizable operators involving a $B - L = 1$ Higgs boson $\chi^c$ via the couplings $(\nu^c \chi^c)^2 / M$. In models of the second class, the flavor violating slepton mixings can arise (assuming mSUGRA) only from the Dirac Yukawa couplings of the neutrinos, $Y_\nu$. On the other hand, in the first class, which is the main focus of this paper, LFV can arise from both the Dirac Yukawa coupling $Y_\nu$ as well as the Majorana Yukawa coupling $f$.

For the purpose of the present paper, we will call the two lepton flavor violation alternatives as LFV in the Majorana case and LFV in the Dirac case.

The simplest example of models with renormalizable Majorana Yukawa couplings is provided by the minimal (SUSY) extension of the SM with a gauged $B - L$ symmetry. Bare masses for the $\nu_R$ are then forbidden by $B - L$ gauge invariance. If the Higgs sector contains a $B - L = 2$ field $\Delta$, the Yukawa coupling $f \nu^c \nu^c \Delta$ as in Eq. (2) will be allowed which will induce $M_R$ of order $v_{B-L} \equiv \langle \Delta \rangle$. (If instead of $\Delta$, a $B - L = 1$ Higgs field $\chi^c$
is used, $M_R$ can arise from a non-renormalizable coupling $(\nu^c \chi c)^2/M$. This belongs to the second class.) There are interesting extensions of this class of models with renormalizable Majorana couplings, such as $\text{SO}(10)$ with a 126 Higgs field or left-right symmetric model with a triplet Higgs field. In these models the Majorana Yukawa couplings may play a dominant role in generating the required neutrino mixings. The role of the Dirac Yukawa coupling $Y_\nu$ for LFV may be subleading and may even be negligible. To be concrete, we shall assume in our analysis that the matrix $Y_\nu$ is diagonal in a basis where the charged lepton Yukawa matrix $Y_\ell$ is also diagonal. For numerical purposes we shall further assume that $Y_\nu$ and $Y_\ell$ are proportional, a situation realized naturally if the gauge symmetry is left-right symmetric. The neutrino mixings will all arise from the Majorana coupling matrix $f$, which will be determined (up to an overall scale factor) from neutrino oscillation data. The proportionality assumption $Y_\nu \propto Y_\ell$ is not crucial to our main conclusions on LFV effects related to $M_R$, but it greatly simplifies the presentation of our results. This assumption may be viewed as analogous to the assumption often made in the case of Dirac LFV models that the super-renormalizable Majorana mass matrix $M_R$ is proportional to an identity matrix.

C. Summary of results

In an earlier paper we have presented results of our preliminary investigations on LFV involving the Majorana Yukawa matrix $f$, assuming proportionality of $Y_\nu$ and $Y_\ell$. (We called this relation up–down unification.) There we studied a general class of SUSY breaking models and did not adhere to mSUGRA. The analysis of Ref. was carried out in the context of left–right symmetric gauge theories with an enhanced gauge structure. In contrast to Ref., in this paper we analyze LFV effects induced by the $f$ matrix within mSUGRA models. This means that the only source of lepton flavor violation is in the neutrino Yukawa sector. Significant LFV effects are found even in this minimal scenario. As explained in Sec. II.C, a novel way of fitting the low energy neutrino oscillation data has the effect of enhancing the LFV in this minimal scheme. We also use a simplified gauge structure involving only an extra $B - L$ symmetry. Even this symmetry is not essential, the crucial ingredient is the Yukawa coupling matrix $f$ of Eq. (2). Thus the models studied here are more minimal compared to the left-right models and are likely to be the low energy limits of a wider class of unified models.

Our main result is that LFV effects associated with the Majorana Yukawa coupling $f$ are in the range accessible to forthcoming experiments for a large range of MSSM parameters consistent with constraints from dark matter, the Higgs boson mass and direct experimental search for supersymmetry. We also point out that the detailed dependence of the branching ratios in processes such as $\tau \to \mu + \gamma$ and $\mu \to e + \gamma$ on the MSSM parameters is different in this class of models compared to the case of flavor violation associated with the Dirac Yukawa coupling $Y_\nu$. With some information of the MSSM parameters, measurements of these radiative decays will enable one to pin down the nature of interactions that give rise to the right-handed neutrino masses. Obviously, this will have a profound impact on our understanding of the neutrino sector and can shed light on the origin of the seesaw mechanism.

It may be worth pointing out that there are other experimental predictions that may help to distinguish between these two possibilities. For instance, two of us have recently pointed
out that the Dirac LFV models can lead to an observable signal in the baryon number non-conserving process neutron-anti-neutron oscillation \( \bar{N}N \) whereas in the Majorana LFV case, the \( N-\bar{N} \) oscillation is unobservable.

**D. One caveat**

There is one caveat in the foregoing discussions that should be mentioned. It applies equally well to the Dirac and Majorana LFV alternatives. In a special class of seesaw schemes it is possible that no lepton flavor violation shows up in the next round of experiments without contradicting the current neutrino oscillation data even in the presence of low energy SUSY. While we feel that such situations would be non-generic, it should be born in mind, nevertheless. In the Dirac LFV alternative it occurs as follows. It is conceivable that all the flavor mixings arise from the mass matrix \( M_R \) and that \( Y_\nu \) is diagonal. Flavor violation in the bare mass parameter (or non-renormalizable term) \( M_R \) is not transmitted to the slepton sector via the RGE, implying highly suppressed LFV effects. One can easily fit all the neutrino oscillation data via the mixings present in \( M_R \). It is also possible that the scale \( M_R \) is much below \( 10^{12} \text{ GeV} \), in which case the elements of \( Y_\nu \) will be much smaller than unity so as to fit the neutrino data, again suppressing LFV effects. Similarly, in the Majorana LFV alternative, it might so happen that the elements of \( f \) are very small, compensated by a higher value of \( v_{B-L} \) so that \( M_R \) is unchanged, or that the elements of \( Y_\nu \) are small. Such a scenario will suppress lepton flavor violation. A null result in the rare lepton decay searches would point to a situation in which either of these two situations could be operative and we will have no way to tell which operators are inducing the \( \nu_R \) masses. On the other hand, if there is a positive signal in the rare decays, our results will help in probing the origin of \( M_R \). It appears to us natural to have at least the third family Yukawa coupling in \( Y_\nu \) of order one (analogous to the top quark Yukawa coupling being of order one) and similarly at least one element of \( f \) \((f_{33})\) to be of order one. Observable LFV effects follow in this case, which is what we analyze.

**E. Outline of the paper**

We have organized this paper as follows. In Section II, we present the basic model and the relevant renormalization group equations (RGE) that will be used in getting the slepton mixings and thus the lepton flavor violating rates for the case of Majorana LFV. Here we also present analytic approximations to the LFV effects by integrating the RGE. These expressions, while only approximate, are useful in gauging the significance of LFV in the Majorana case as well as in comparing it with the case of Dirac LFV. In Section II.A we present the relevant expressions for obtaining the \( f \) matrix by inverting the seesaw formula. In II.B this information is used to compare analytically the predictions for \( \tau \rightarrow \mu + \gamma \) and \( \mu \rightarrow e + \gamma \) branching ratios in the two cases. In Section II.C we present a convenient way to parametrize the Majorana Yukawa matrix \( f \) that can be compared directly against low energy neutrino oscillation data on the one hand and LFV effects on the other. Section III is devoted to our exact numerical results for LFV effects in the case of Majorana as well as Dirac Yukawa couplings. In III.A we present our numerical fits to the neutrino parameters,
III.B has our results for the branching ratios $\tau \to \mu + \gamma$ and $\mu \to e + \gamma$ as well as the ratio of the two branching ratios. Section IV has our conclusions.

II. SLEPTON MIXING AND LEPTON FLAVOR VIOLATION FROM MAJORANA YUKAWA COUPLINGS

In order to illustrate in detail the source of LFV for the case of Majorana Yukawa couplings generating the $\nu_R$ mass matrix (the Majorana LFV alternative), we work with a simple extension of the minimal supersymmetric standard model that accommodates the seesaw mechanism. It is based on the weak gauge group $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ with fermions assigned as follows: $Q(2, 0, + \frac{2}{3})$; $L(2, 0, -1)$; $u^c(1, -\frac{1}{2}, -\frac{1}{3})$; $d^c(1, +\frac{1}{2}, -\frac{1}{3})$; $c^c(1, +\frac{1}{2}, +1)$; $\nu^c(1, -\frac{1}{2}, +1)$. This is the simplest extension of the weak interaction sector that predicts the existence of the $\nu_R$ from gauge anomaly cancellations conditions and thus to small neutrino masses via the seesaw mechanism. The Higgs superfields are $H_u(2, +\frac{1}{2}, 0)$; $H_d(2, -\frac{1}{2}, 0)$; $\Delta(1, +1, -2)$, $\Delta(1, -1, +2)$. The superpotential for this theory is:

$$W = Y_u u^c Q H_u + Y_d d^c Q H_d + Y_\ell \ell^c L H_d + Y_{\nu} \nu^c L H_u$$

$$+ f \nu^c \nu^c \Delta + \mu H_u H_d + S(\Delta \Delta - M^2).$$

Here all the Yukawa couplings are $3 \times 3$ matrices; $S$ is a singlet field introduced to help facilitate the breaking of $U(1)_{B-L} \times U(1)_{I_{3R}}$ down to $U(1)_Y$ in the SUSY limit. One identifies the SM hypercharge as $Y/2 = I_{3R} + (B - L)/2$. This superpotential gives equal vacuum expectation values (VEVs) to $\langle \Delta \rangle = \langle \Delta \rangle = v_{B-L}$. The Yukawa coupling matrix $f$ will induce large mass for the $\nu^c$ field, leading to the seesaw mechanism. This model should be contrasted with the commonly discussed case in the literature where the heavy $\nu_R$ masses are either put in by hand or arise from a non-renormalizable term in the superpotential.

In order to study flavor violation, we will make the simplifying assumption that both $Y_\nu$ and $Y_\ell$ are diagonal at the Planck scale; therefore all neutrino mixings arise from the general flavor structure of the Majorana Yukawa couplings $f_{ij}$. Then from the approximate bi-maximal pattern for neutrino mixings deduced from oscillation data, we can obtain $f_{ij}$ for different patterns of neutrino masses. Since the model is supersymmetric, the flavor violation present in $f_{ij}$ will induce processes such as $\tau \to \mu + \gamma$ and $\mu \to e + \gamma$ decays. We investigate these predictions using simple analytic approximations in this section.

We need to write down the RGE for the Yukawa couplings and slepton masses for the class of models under consideration. Denote the left–handed slepton mass squared matrix as $m_L^2$ and the trilinear $A$ terms in the slepton sector as $A_\ell$, $A_\nu$, $A_f$ (in an obvious notation). The relevant RGE for $Y_\nu$ and $m_L^2$ are:

$$\frac{dY_\nu}{dt} = \frac{Y_\nu}{16\pi^2}[Tr(3Y_u Y_\nu^\dagger + Y_\nu Y_\nu^\dagger) + 3Y^\dagger_\nu Y_\nu + Y_\nu^\dagger Y_\nu + 4f^\dagger f - 3g_2^2 - g_R^2 - \frac{3}{2}g_{B-L}^2]$$

$$\frac{dm_L^2}{dt} = \frac{1}{16\pi^2}[2m_L^2 + 2m_{H_u}^2]Y_\nu^\dagger Y_\nu + 2m_L^2 Y_\nu^\dagger Y_\nu + 2Y_{\nu}^\dagger m_L^2 Y_\nu + Y_{\nu}^\dagger Y_\nu m_L^2]$$

$$+ 2Y_{\nu}^\dagger Y_{\nu}^\dagger Y_{\nu} + Y_{\nu}^\dagger Y_{\nu} m_L^2 + 2A_{\ell}^\dagger A_{\ell} + 2A_\nu^\dagger A_\nu - 6g_2^2 M_2^2 - 3g_{B-L}^2 M_{B-L}^2.$$
The RGE for the relevant $A$ parameters are:

\[
\frac{dA_\ell}{dt} = \frac{1}{16\pi^2} [A_\ell [Tr(3Y_d^\dagger Y_d + Y_e^\dagger Y_e) + 5Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu,
\text{Eq. (10)}
\]

\[
- 3g_2^2 - g_R^2 - \frac{3}{2}g_{B-L}^2] + Y_\ell [Tr(6A_d Y_d^\dagger + 2A_e Y_e^\dagger)]
\]

\[
+ 4Y_\ell^\dagger A_\ell + 2Y_\nu^\dagger A_\nu + 6g_2^2 M_2 + 2g_R^2 M_R + 3g_{B-L}^2 M_{B-L}]
\]

\[
\frac{dA_\nu}{dt} = \frac{1}{16\pi^2} [A_\nu [Tr(3Y_u^\dagger Y_u + Y_\nu^\dagger Y_\nu) + 5Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu,
\text{Eq. (10)}
\]

\[
+ 4f^\dagger f - 3g_2^2 - g_R^2 - \frac{3}{2}g_{B-L}^2] + Y_\nu [Tr(6A_u Y_u^\dagger + 2A_\nu Y_\nu^\dagger)]
\]

\[
+ 4Y_\nu^\dagger A_\nu + 8f^\dagger A_f + 2Y_\ell^\dagger A_\ell + 6g_2^2 M_2 + 2g_R^2 M_R + 3g_{B-L}^2 M_{B-L}]
\]

We are interested in LFV arising primarily from the matrix $f$. Adopting mSUGRA, we have at the Planck scale, $A_\ell = A_0 Y_\ell$, $A_\nu = A_0 Y_\nu$, $A_f = A_0 f$. Furthermore, at $M_{Pl}$, we have universality of all scalar masses (denoted by $m_0$) and a common gaugino mass (denoted as $m_{1/2}$).

Under the set of assumptions specified above, it is clear from Eqs. (4)-(7) that the off–diagonal elements of $f$ will not induce slepton flavor violation to the lowest order in a perturbative expansion, since $f$ does not enter directly in the RGE for $m_2^\ell$ or $A_\ell$. However, the one–loop solution for $m_2^L$ involves $Y_\nu^\dagger Y_\nu$, which receives a contribution from $f^\dagger f$ (see Eq. (4)). Therefore in second order in the parameter $\frac{1}{16\pi^2} ln \left( \frac{M_{Pl}}{M_{B-L}} \right)$, we do have flavor violating effects in $m_2^L$ and $A_\ell$. It is important to note that this is not a two–loop RGE effect, rather that it is a one–loop RGE improved effect, as indicated by the presence of the $[ln \left( \frac{M_{Pl}}{M_{B-L}} \right)]^2$ term.

We can estimate the strength of the slepton flavor violation by integrating the RGE analytically. We find from Eq. (4)-(7) that

\[
\Delta m_{ij}^2 (i \neq j) \approx \frac{-3(m_0^2 + A_0^2)}{32\pi^4}[Y_\nu^\dagger Y_\nu f^\dagger f + f^\dagger f Y_\nu^\dagger Y_\nu]_{ij} \left( ln \frac{M_{Pl}}{M_{B-L}} \right)^2
\]

\[
A_{ij} (i \neq j) \approx \frac{-3}{64\pi^4}[A_\ell (Y_\nu^\dagger Y_\nu f^\dagger f + f^\dagger f Y_\nu^\dagger Y_\nu)]_{ij} \left( ln \frac{M_{Pl}}{M_{B-L}} \right)^2
\]

The branching ratio for $\ell_j \to \ell_i + \gamma$ can be written approximately as

\[
\frac{B(\ell_j \to \ell_i + \gamma)}{B(\ell_j \to \ell_i + \nu_j + \bar{\nu}_i)} \approx \frac{3\alpha_{em}(c_1 g_2^2 + g_2^2)^2}{32\pi m_{3/2}^4 G_F^2} \left( \frac{\Delta m_{ij}^2}{m_{sl}^2} \right) \tan^2 \beta
\]

Here $m_{sl}^2$ is the average slepton mass at the weak scale, which may be quite different from $m_0^2$. Eq. (10) is obtained as an approximation to the chargino and neutralino exchange diagram proportional to LFV in Eq. (8) with the enhancement factor of $\tan \beta$ as indicated. $c_1$ in Eq. (10) is an $O(1)$ coefficient. LFV arising from Eq. (9) can be comparable, but it will not change our approximate results by much. We have found that Eq. (10) is a
reasonable approximation to the exact numerical results to within a factor of few, provided
that $\tan \beta$ is not too large and that there is no large hierarchy between the SUSY breaking
mass parameters ($\mu$, $m_d$, $m_{1/2}$).

Within mSUGRA, $m_{1/2}$ must be larger than about 250 GeV, to be consistent with the
Higgs boson mass constraint (viz., $m_h \geq 114$ GeV) and the $b \to s\gamma$ constraint. Since we
also want to identify the lightest neutralino (LSP) as the dark matter in the universe, the
relic abundance puts another constraint that the second lightest SUSY particle (NLSP), the
right–handed stau in our case, should be nearly degenerate with the LSP to within about
20-30 GeV. This condition can be satisfied if $m_{1/2} \simeq 4.4m_0$ is obeyed, which we shall adopt.
Take for example a relatively light SUSY spectrum with $m_{1/2} = 250$ GeV and $\tan \beta = 10$.
The charged Wino mass is then approximately 200 GeV, and the Bino mass is about 100
GeV. Choosing $m_0 = 55$ GeV and $A_0 = 0$ leads to a mass of about 190 GeV for the
left–handed sleptons and a mass of about 130 GeV for the right–handed sleptons, with the
right–handed stau being somewhat lighter around 115 GeV. Co-annihilation of LSP will be
efficient with such a spectrum.

Next we need to choose the neutrino parameters. With $Y_\nu \propto Y_\ell$, a good fit to the current
neutrino oscillation data can be obtained with the choice $(Y_\nu)_{33} = 0.66$ and $(f^\dagger f)_{23} \simeq 0.28$
(see detailed numerical fits later). Furthermore, $\ell n(M_{PL}/M_{B-L}) \simeq 9$ is quite plausible. With
this choice, we estimate $B(\tau \to \mu + \gamma) \simeq 2 \times 10^{-9}$ for $\tan \beta = 10$ from Eq. (10). As $A_0$
is increased from 0 to 300 GeV, this branching ratio increases to about $2 \times 10^{-6}$. Thus
we see that for interesting ranges of parameters, the process $\tau \to \mu + \gamma$ can be within
experimental reach. More careful examination of the decay is thus warranted. Similarly,
the decay $\mu \to e + \gamma$ can have a branching ratio of order $10^{-14} - 10^{-9}$ for reasonable ranges
of model parameters. Some of this parameter space is already excluded by data, there is a
good chance of being able to measure this decay in the planned experiments.

It is instructive to compare the branching ratios obtained here in the case of Majorana
LFV to that expected in the case of Dirac LFV. In the case of Dirac LFV, assuming that
the $\nu_R$ Majorana mass matrix is proportional to the identity matrix, we can write down the
analog of Eq. (8) as

$$\Delta m^2_{ij}(i \neq j) \simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu^\ast Y_\nu)_{ij}\left(\ell n\frac{M_{PL}}{M_{B-L}}\right).$$

Comparing Eq. (11) with Eq. (8), we find that numerically the two effects can be compa-
rrable. Which effect is more dominant will depend on the overall strengths of the coupling
matrices in the two cases. This will be discussed further in Sec. III.

**A. Flavor structure of the Majorana-Yukawa couplings**

We can invert the seesaw formula (Eq. (1)) and obtain the matrix elements of the
Majorana Yukawa coupling matrix $f$:

$$f = \frac{1}{v_{B-L}}M_D M^{-1}_\nu M^T_D.$$  \hspace{1cm} (12)

We also have the relation for the inverse of the light neutrino mass matrix $M^{-1}_\nu$ in terms of
the light neutrino mass eigenvalues ($m_1$, $m_2$, $m_3$) and the leptonic mixing matrix $U$ as
\[ \mathcal{M}_\nu^{-1} = U \begin{pmatrix} m_1^{-1} & 0 & 0 \\ 0 & m_2^{-1} & 0 \\ 0 & 0 & m_3^{-1} \end{pmatrix} U^T. \] (13)

We ignore CP violation and use the near bi-maximal approximation for \( U \) given by

\[ U \simeq \begin{pmatrix} \cfrac{c}{\sqrt{2}} & \cfrac{s}{\sqrt{2}} & \cfrac{1}{\sqrt{2}} \\ \cfrac{-s}{\sqrt{2}} & \cfrac{c}{\sqrt{2}} & \cfrac{1}{\sqrt{2}} \\ \cfrac{1}{\sqrt{2}} & \cfrac{-c}{\sqrt{2}} & \cfrac{1}{\sqrt{2}} \end{pmatrix}. \] (14)

Here \( U_{e3} \simeq \epsilon \leq 0.16 \) from the CHOOZ and Palo-Verde reactor experiments [13]. \( s \) is the solar neutrino mixing angle for which the allowed range after the SNO data is [2] \( 0.71 \leq \sin^2 2\theta_\odot \leq 0.94 \) at three \( \sigma \) confidence level. Multiplying all these together, we have

\[ M_{R,ij} = m_{D,i}^{-1} \mu_{ij}^{-1} m_{D,j} \] (15)

where we have assumed \( M_D \) to be diagonal with its entries denoted by \( m_{D,i} \). The definition \( \mu_{ij}^{-1} \equiv (\mathcal{M}_\nu^{-1})_{ij} \) has been used. We have then

\[
\begin{align*}
\mu_{11}^{-1} & = \frac{c^2}{m_1} + \frac{s^2}{m_2} + \frac{\epsilon^2}{m_3} \\
\mu_{12}^{-1} & = -\frac{c(s + c\epsilon)}{\sqrt{2}m_1} + \frac{s(c - s\epsilon)}{\sqrt{2}m_2} + \frac{\epsilon}{\sqrt{2}m_3} \\
\mu_{13}^{-1} & = \frac{c(s - c\epsilon)}{\sqrt{2}m_1} - \frac{s(c + s\epsilon)}{\sqrt{2}m_2} + \frac{\epsilon}{\sqrt{2}m_3} \\
\mu_{22}^{-1} & = \frac{(s + c\epsilon)^2}{2m_1} + \frac{(c - s\epsilon)^2}{2m_2} + \frac{1}{2m_3} \\
\mu_{23}^{-1} & = -\frac{(s^2 - c^2\epsilon^2)}{2m_1} - \frac{(c^2 - s^2\epsilon^2)}{2m_2} + \frac{1}{2m_3} \\
\mu_{33}^{-1} & = \frac{(s - c\epsilon)^2}{2m_1} + \frac{(c + s\epsilon)^2}{2m_2} + \frac{1}{2m_3}.
\end{align*}
\]

Using these expressions, for different scenarios of neutrino masses we can determine the values of \( f_{ij} \) which enter into the flavor changing slepton masses. We will focus primarily on the hierarchical neutrino mass schemes where \( m_1 \ll m_2 \ll m_3 \).

**B. Comparison between Majorana LFV and Dirac LFV alternatives**

Let us first discuss the case where the right–handed neutrino masses arise from a renormalizable Majorana Yukawa coupling \( f \). As in the previous section, let us assume that the Dirac neutrino mass matrix \( M_D \) is diagonal in the same basis where \( M_\ell \) is diagonal. One interesting pattern is provided by the normal hierarchy \( m_1 \ll m_2 \ll m_3 \). In this case, it is plausible, but not mandatory, that in Eq. (16), the terms with \( m_1 \) in the denominator dominate \( \mu_{ij}^{-1} \) with all other terms being negligible. In this approximation, we obtain for the process \( \tau \rightarrow \mu + \gamma \) process an amplitude proportional to
Similarly, the $\mu \to e + \gamma$ process has an amplitude given by

$$(\Delta m^2)_{12} \propto \frac{m_D m_{D,2}^5 c^3}{2\sqrt{2} m_1^2 w_k v_{B-L}^2}.$$  

We have set $\epsilon = 0$ for simplicity here. From Eq. (17)-(18) it might appear that the branching ratios can increase without limit as $m_1$ is decreased. However, this is not true. The coupling $f_{33} \approx \frac{m_D^2 s^2}{2m_1 v_{B-L}}$ should remain perturbative ($f_{33} \leq 1$). This means that for a given $m_1$, there is a lower limit on $v_{B-L}$. For example, if $m_1 = 10^{-3}$ eV, and for $m_{D,3} = 115$ GeV, $s = 0.52$, it must be that $v_{B-L} \geq 1.8 \times 10^{15}$ GeV so that $f_{33} < 1$. It turns out that with this constraint, the branching ratios (BR) for the decays $\tau \to \mu + \gamma$ and $\mu \to e + \gamma$ are suppressed within mSUGRA, mainly because the logarithm upon which the BR has a fourth power dependence, is small. We conclude that in this scenario with $1/m_1$ dominance in Eq. (16), in the case of Majorana LFV there is no significant LFV in lepton sector within mSUGRA.

We have also examined LFV effects if the mSUGRA assumption is relaxed somewhat for the case of $1/m_1$ dominance. If for example, $A_f = A_0 f$, $A_\ell = A_0 Y_\ell$, $A_\nu = A_0 Y_\nu$ at $M_{Pl}$ with $A'_0$ different from $A_0$, we found that the BR for $\tau \to \mu + \gamma$ can be in the range $10^{-7} - 10^{-9}$ for relative large $A'_0 \sim$ TeV. The BR for $\mu \to e + \gamma$ is extremely small ($\sim 10^{-16}$) with this set of boundary conditions. We shall not present details of this fit, and will focus solely on the mSUGRA case.

For comparison, let us also consider the case of Dirac LFV. For $M_R \propto 1$ there is an unambiguous connection between the LFV effects and the neutrino mass matrix. To see this more explicitly, note that in this case, the seesaw formula reduces to

$$-M_{\nu} M_{\nu} = M_D^T M_D.$$  

Thus the product $M_D^T M_D$ can be completely expressed in terms of the neutrino masses and mixings. Since for the CP conserving case, the flavor violating slepton mixings effects always involve $Y^\tau_f Y_\nu$, the LFV amplitudes can be expressed directly in terms of the neutrino mass matrix. Assuming again that $m_1 \ll m_2 \ll m_3$, we can write down the amplitude for the decay $\tau \to \mu + \gamma$ to be proportional to $(\Delta m^2)_{23} \propto \frac{m_3}{2}$, while that for the decay $\mu \to e + \gamma$ is proportional to $(\Delta m^2)_{12} \propto c sm_2/\sqrt{2}$. These amplitudes increase with the light neutrino masses, unlike the case of Majorana LFV, which scale inversely with $m_1$. Both decays have branching ratios accessible in the planned experiments in the Dirac LFV alternative, as will be further explained in Sec. III.

### C. Enhanced LFV from Majorana Yukawa couplings

The dominance of $1/m_1$ in $\mu_{ij}^{-1}$ and therefore in $f_{ij}$ is not the only possibility. We have found a large range of parameters where the LFV effects are significant while deviating from the $1/m_1$ dominance. To see this explicitly it is more convenient to define the light neutrino mass matrix $M_\nu$ in the gauge basis as a perturbative expansion in a small parameter $\epsilon \sim 0.1$.
\[ M_\nu = m_0 \begin{pmatrix} e\epsilon^n & h\epsilon^m & d\epsilon \\ h\epsilon^m & 1 + a\epsilon & 1 \\ d\epsilon & 1 & 1 + b\epsilon \end{pmatrix} \]  \hspace{1cm} (20)

Here \((a, b, d, e, h)\) are order one coefficients, \(\Delta m^2_{atm} \simeq 4m^2_0\) and the exponents \((n, m)\) in the \((1,1)\) and \((1,2)\) entries have to be at least 1, but can be larger. This matrix provides a good fit to all neutrino data. If \(m = 1\), and \(n = 1, 2\) it will reproduce the results of the previous section with \(1/m_1\) dominance. But for other values of \((n, m)\), which are also consistent with all neutrino oscillation data, we have found much larger LFV effects. Consider the case \(n = 2, m = 4\) for example. In this case the \((1,1)\) and the \((1,2)\) entries of \(M_\nu\) are not completely specified by neutrino data alone. Inverting Eq. (20) we obtain for the matrix \(f\),

\[ f = \frac{m^2_{D,3}}{d^2 m_0 v_{B-L}} \begin{pmatrix} (a + b)\epsilon^2 \epsilon^5 & c\epsilon^3 & -c\epsilon^2 \\ c\epsilon^3 & -d\epsilon^2 & d\epsilon^2 \\ -c\epsilon^2 & d\epsilon^2 & (e - h^2)\epsilon^2 \end{pmatrix} \]  \hspace{1cm} (21)

Here we have used the parametrization \(M_D = \text{Diag}(ce^3, \epsilon, 1)m_{D,3}\). All parameters in \(f\) are fixed, except for \((e, h)\). If \((e, h) \sim O(1)\), we have a situation where the off–diagonal entries as well as the diagonal entries of \(f\) are of the same order leading to large LFV effects. Contrast this with the case where \(m = 1\) is adopted in Eq. (20), in which case we would have the \((3,3)\) entry of \(f\) to be order 1, the \((2,3)\) to be order \(\epsilon\), the \((1,3)\) of order \(\epsilon^2\), and the \((1,2)\) entry of order \(\epsilon^3\). The choice (which is equivalent to \(1/m_1\) dominance in \(f\)) will lead to a suppression of the \(\tau \rightarrow \mu + \gamma\) branching ratio by a factor \(\epsilon^2 \sim 10^{-2}\) which would put it in an unobservable range. The decay \(\mu \rightarrow e + \gamma\) will be suppressed even further, by a factor \(\epsilon^4 \sim 10^{-5}\), compared to the case of \(m = 2\).

It is clear then that the most interesting case for lepton flavor violation is when \(n = 4, m = 2\) in Eq. (20). This is the choice we shall make for our numerical fits in Sec. III.

**III. NUMERICAL RESULTS**

**A. Neutrino mass fit**

We start with a basis where the charged lepton mass matrix is diagonal. For the Dirac neutrino mass matrix we assume that

\[ M_D = \gamma \tan \beta M_\ell \]  \hspace{1cm} (22)

where \(M_\ell = \text{Diag}(m_e, m_\mu, m_\tau)\). This assumption is well justified if the model is embedded into a SUSY left–right framework as we have shown earlier papers [14]. It is also possible that this assumption holds approximately in a wider class of models where the off–diagonal entries in \(Y_\ell\) and \(Y_\nu\) are small, although not strictly proportional. The proportionality constant \(\gamma\) is a parameter of the model. If we assume that the \(\nu_\tau\) Dirac Yukawa coupling is the same as the top–quark Yukawa couplings, as happens in some versions of minimal \(SO(10)\) models, then \(\gamma = 1\) for large values of \(\tan \beta \approx 60\) while \(\gamma \simeq 5 - 6\) for \(\tan \beta \approx 10\).

The light neutrino Majorana mass matrix is obtained as
\[ \mathcal{M}_\nu = \frac{\gamma^2 \tan^2 \beta}{v_{B-L}} M_\ell f^{-1} M_\ell. \]  \hfill (23)

In our analysis, we choose values of \( \tan \beta \geq 10 \), since smaller \( \tan \beta \) values are less preferred by the recent Higgs mass bounds from LEP [13] and the muon \( g - 2 \) anomaly [16], if interpreted as a SUSY effect, would prefer a moderately large \( \tan \beta \).

Following the analytic expression given in Sec. II.C, we obtain a fit for the light neutrino spectrum as follows. We choose \( \tan \beta = 10 \) and find a good fit to the solar and atmospheric neutrino data with hierarchical neutrino masses if the matrix \( f \) defined at \( v_{B-L} = 2 \times 10^{12} \) GeV has entries given by

\[
\begin{pmatrix}
-1.1 \times 10^{-4} & -0.015 & 0.29 \\
-0.015 & 0.50 & -0.57 \\
0.29 & -0.57 & 0.104
\end{pmatrix}. \hfill (24)
\]

The resulting neutrino masses at the weak scale (using the RGE evolution [17]) are:

\[
(m_1, m_2, m_3) = (-2.7 \times 10^{-3}, 6.4 \times 10^{-3}, 8.6 \times 10^{-2}) \text{ eV}. \hfill (25)
\]

The leptonic mixing matrix is given by:

\[
U = \begin{pmatrix}
0.85 & -0.52 & -0.053 \\
0.33 & 0.62 & -0.72 \\
-0.40 & -0.59 & -0.70
\end{pmatrix}. \hfill (26)
\]

Note that the mixing angles \( |U_{e2}| = 0.52 \) and \( |U_{\mu 3}| = 0.72 \) are in very good agreement with the solar and atmospheric oscillation data. Note also the prediction for the parameter \( |U_{e3}| \approx 0.05 \), which is in the range that can be tested in planned long baseline experiments.

For comparison purposes we will also present LFV effects associated with Dirac Yukawa couplings, assuming no flavor violation in the Majorana sector. It will be desirable to have a neutrino mass fit for this case as well, where \( M_R \) is proportional to an identity matrix and all flavor mixings arise from \( Y_\nu \). We have found such fits for the same set of light neutrino spectrum as in Eq. (25) - (26). Actually, there are discrete possibilities for \( Y_\nu \) since the equations are not linear in \( Y_\nu \). We have found that the predictions for LFV decays are essentially unchanged within these discrete possibilities. So we focus on one of them. At \( v_{B-L} = 9 \times 10^{13} \) GeV, we find that \( Y_\nu \) is given by

\[
Y_\nu = \begin{pmatrix}
0.04 + 0.074i & -0.073 + 0.029i & 0.025 - 0.034i \\
-0.073 + 0.029i & -0.22 + 0.011i & -0.35 - 0.013i \\
0.025 - 0.034i & -0.35 - 0.013i & -0.24 + 0.016i
\end{pmatrix}. \hfill (27)
\]

Although \( Y_\nu \) in Eq. (27) have complex elements, \( Y_\nu^T Y_\nu \) is real, as can be easily verified. Thus the light neutrino mass matrix \( \mathcal{M}_\nu \) is real (see Eq. (19)), and there is no CP violation in neutrino oscillations in this limit, as in the case of Majorana LFV. We shall use this fit for the case of Dirac LFV case in our numerical analysis.
FIG. 1. $B(\mu \to e + \gamma)$ vs $m_{1/2}$ for three values of $A_0$ corresponding to the case of Majorana LFV. The Yukawa coupling matrix $f$ is given in Eq. (24). The horizontal solid line shows the current experimental limit.

B. Predictions for lepton flavor violation

In our calculation we make the following assumptions about the supersymmetry breaking sector, adopting the mSUGRA framework. We assume universal scalar masses $m_0$, a common gaugino mass $m_{1/2}$ and trilinear soft SUSY breaking $A$ terms that are proportional to the respective Yukawa coupling matrices, with a common proportionality constant $A_0$. We take the fundamental scale at which these relations hold to be the GUT scale $= 2.4 \times 10^{16}$ GeV. We impose various experimental constraints on the SUSY spectrum. In particular, the Higgs boson mass limit ($m_h \geq 114$ GeV) along with the $b \to s + \gamma$ constraint requires that $m_{1/2} \geq 250$ GeV. The direct limits on SUSY searches are also taken into account. Furthermore, radiative electroweak symmetry breaking is demanded. We choose the sign of $\mu$ to be positive, which is preferred by the $b \to s + \gamma$ constraint. We do not explicitly impose constraints from muon $g - 2$, but we note that for the range of parameters we have chosen (positive $\mu$ and moderate to large $\tan \beta$), this anomaly can be explained via SUSY exchange, provided that $m_{1/2}$ is limited to less than about 800 GeV ($\tan \beta \leq 40$).

Additionally, we require that the lightest neutralino be the lightest supersymmetric particle (LSP) in the model. Furthermore, this LSP should constitute the dark matter of the universe. However, for generic SUSY parameters the neutralino annihilation cross section is not strong enough to bring its relic abundance to the required level from dark matter constraints (density in dark matter should be $\sim 30\%$ of the critical density in order for the LSP to form a good dark matter candidate). This remark applies for the range of parameters that we are interested in, viz., $m_{1/2} \geq 300$ GeV and $\tan \beta \approx 10$. The dark matter
constraint can be satisfied by invoking the co-annihilation mechanism involving the LSP and the second lightest SUSY particle (NLSP), which is the right–handed stau in these models [19]. In order for this co-annihilation to be efficient, we need the lighter stau mass to be within 25-30 GeV of the LSP mass. We choose the universal scalar mass $m_0$ such that this constraint on the lighter stau mass is satisfied. We allow the range $0.07 < \Omega_{\tilde{\chi}_1} h^2 < 0.21$. For example, for the case of $A_0 = 0$, $m_{1/2} \approx 4.4 m_0$ will be required to satisfy this constraint.

We choose $A_\ell = Y_\ell A_0$, $A_f = Y_f A_0$, $A_\nu = Y_\nu A_0$ at the GUT scale. For the common $A_0$, we choose three different values, $A_0 = (0, 300, 800)$ GeV. We find that the branching ratio increases with $A_0$, as can be inferred from the analytic approximation, Eq. (8).

In Fig. 1 we plot the branching ratio for $\mu \rightarrow e + \gamma$ in the Majorana LFV case as a function of $m_{1/2}$, varying it from its lower limit of 250 GeV to 1 TeV. Here we chose $A_0 = 0$. We find that the BR varies between $10^{-14}$ and $10^{-9}$. Part of the parameter space is already ruled out by the non–observation of $\mu \rightarrow e + \gamma$. This experimental lower limit on the BR is indicated by a thick solid line in Fig. 1. We conclude that the entire parameter space can be explored with the planned round of $\mu \rightarrow e + \gamma$ experiment [20].

In Fig. 2, we show the $\tan \beta$ dependence of the $B(\mu \rightarrow e + \gamma)$ for the same set of input parameters as in Fig. 1. Here we present results for two values of $\tan \beta$, $\tan \beta = 10$ and 30. The branching ratio increases with $\tan \beta$, as anticipated from Eq. (10). As in Fig. 1, we demand the dark matter constraint be satisfied in Fig. 2 as well.

In Fig. 3 we plot the branching ratio for $\tau \rightarrow \mu + \gamma$ for the case of $A_0 = 0$. The solid line corresponds to the Majorana LFV case. For comparison, we also present in the same graph the $\tau \rightarrow \mu + \gamma$ BR in the case of the Dirac alternative (fit of Eq. (27)). In Fig. 4 we plot the same quantities for a different value of $A_0 = 800$ GeV and $\tan \beta = 10$. We find

FIG. 2. $B(\mu \rightarrow e + \gamma)$ vs $m_{1/2}$ for two different values of $\tan \beta$. All other parameters are same as in Fig. 1.
FIG. 3. $B(\tau \to \mu + \gamma)$ vs $m_{1/2}$ (solid line) for the same set of parameters as in Fig. 2 corresponding to the Majorana LFV case. The dashed line corresponds to the Dirac LFV alternative given in Eq. (27).

FIG. 4. Same as in Fig. 3, but with $A_0 = 800$ GeV.
from Figs. 3 and 4 that the $\tau \to \mu + \gamma$ is typically smaller in the Majorana case compared to the Dirac case, although they can be comparable for some values of $m_{1/2}$ where there is an accidental cancellation between the chargino and the neutralino diagrams in the case of Dirac LFV (and not for the Majorana LFV for this input choice). The BR for the case of Majorana LFV is $< 10^{-9}$ for $A_0 = 0$, but can be as large as $2 \times 10^{-7}$ for $A_0 = 800$ GeV.

In Fig. 5, we show the $B(\mu \to e + \gamma)$ as a function of $m_{1/2}$ for the Majorana alternative (solid lines) and compare it with the Dirac case (dashed line and dot–sashed line). We have chosen $A_0 = 0$ and $\tan \beta = 10$ for illustration in this case. The dashed line is drawn for the $Y_\nu$ shown in Eq. (27). We see that the branching ratio is large in the Dirac case. However, if we scale down the Dirac neutrino coupling by a factor of 2.5, which does not alter the light neutrino spectrum (since the overall scale factor is not determined from low energy data), we find that the branching ratio (denoted by the dot–dashed line) decreases. Thus, depending on the overall scale factor, $B(\mu \to e + \gamma)$ can be larger in the Majorana case compared to the Dirac case (or vice versa).

As Fig. 5 shows, there is some uncertainty in the BR arising from the overall scale factor. If we consider ratios of branching ratios, this uncertainty disappears. We define the ratio

$$r \equiv \frac{B(\mu \to e + \gamma)}{B(\tau \to \mu + \gamma)} \ (28)$$

and study its dependence on the SUSY spectrum as well as on the Dirac/Majorana cases. In Figures 6 and 7, we plot $r$ for the cases of Dirac and Majorana alternatives for $A_0 = 0$ (Fig. 6) and $A_0 = 800$ GeV (Fig. 7) for $\tan \beta = 10$. We see from these figures that the ratio depends on the supersymmetry breaking parameters $A_0, m_0, m_{1/2}$ etc, in a way that is different for these two cases. This difference shows up in almost the entire parameter space. We also notice that the ratio $r$ is large for the Majorana case for large $A_0$. We conclude that with some information on the SUSY breaking parameters, measurement of $r$ can provide crucial insight into the origin of the seesaw mechanism.

IV. CONCLUSIONS

In this paper we have studied the consequences of implementing the seesaw mechanism by renormalizable Yukawa couplings involving a $B - L = 2$ Higgs field on lepton flavor violating processes. The $\nu_R$ Majorana masses arise as renormalizable Yukawa couplings, rather than bare mass terms, and therefore influence the renormalization group evolution of the soft SUSY masses of sleptons. This slepton flavor violation is transmitted to the lepton sector and induces the rare leptonic decays $\mu \to e + \gamma$ and $\tau \to \mu + \gamma$. We have emphasized that this case is qualitatively as well as quantitatively different from the case where the $\nu_R$ Majorana masses are directly put in by hand or are induced by non-renormalizable terms. We have found interesting structure for the Majorana Yukawa coupling matrix $f$ which is consistent with low energy neutrino oscillation data and that leads to observable lepton flavor violations in radiative lepton decays. We have adopted a SUSY breaking scenario with minimal flavor violation, the popular mSUGRA scheme, and have found even within this scheme that the radiative lepton decays are within reach of planned experiments. The parameter space that we have explored is consistent with acceptable abundance of neutralino...
FIG. 5. $B(\mu \to e + \gamma)$ vs $m_{1/2}$. The solid lines correspond to the Majorana LFV case and the dashed line line to the Dirac alternative. The dot–dashed line is obtained for the Dirac LFV case, but with a rescaling of $Y_\nu$ of Eq. (27) by a factor of 2.5. The horizontal solid line shows the current experimental limit.

FIG. 6. The ratio $r = B(\mu \to e + \gamma)/B(\tau \to \mu + \gamma)$ versus $m_{1/2}$ for $A_0 = 0$. The solid line correspond to the Majorana LFV case and the dashed line corresponds to the Dirac LFV alternative.
dark matter, as well as Higgs boson mass limit, $b \to s + \gamma$ constraint, muon $g-2$ constraint as well as the direct experimental limits on SUSY particles.

We have compared the predictions from the Majorana Yukawa coupling related LFV with that related to the Dirac Yukawa coupling. The ratio $r = B(\mu \to e + \gamma)/B(\tau \to \mu + \gamma)$, being insensitive to the overall scale of $B - L$ symmetry breaking, along with some information on the SUSY spectrum, can provide deep insight into the origin of the seesaw mechanism.

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