Study of collective flows of protons and $\pi^-$-mesons in p+C, Ta and He+Li, C collisions at momenta of 4.2, 4.5 and 10 AGeV/c

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Received: 15 February 2016 / Revised: 23 October 2016
Published online: 30 November 2016 – © Società Italiana di Fisica / Springer-Verlag 2016
Communicated by P. Salabura

Abstract. Collective flows of protons and $\pi^-$-mesons are studied at the momenta of 4.2, 4.5 and 10 AGeV/c for p+C, Ta and He+Li, C interactions. The data were obtained from the streamer chamber (SKM-200-GIBS) and from the Propane Bubble Chamber (PBC-500) systems utilized at JINR. A method of Danielewicz and Odyniec has been employed in determining a directed transverse flow of particles. The values of the transverse flow parameter and the strength of the anisotropic emission were defined for each interacting nuclear pair. It is found that the directed flows of protons and pions decrease with increasing the energy and the mass numbers of colliding nucleus pairs. The $\pi^-$-meson and proton flows exhibit opposite directions in all studied interactions, and the flows of protons are directed in the reaction plane. The Ultra-relativistic Quantum Molecular Dynamical Model (UrQMD) coupled with the Statistical Multi-fragmentation Model (SMM), satisfactorily describes the obtained experimental results.

1 Introduction

One of the central goals of the high-energy heavy-ion collision research is a determination of the properties of nuclear matter at densities and temperatures higher than those in the ground-state nuclei. The asymmetry of the produced particle emission relative to the reaction plane observed in nucleus-nucleus interactions is explained by the collective flows of the particles. Theoretically, those flows can be linked to the fundamental properties of nuclear matter and, in particular, to the equation of state (EOS) [1]. Two types of asymmetries were identified. The former is a directed flow [2] in the reaction plane, associated with the matter “bouncing-off” within the hot participant region of overlap between colliding nuclei. The latter is a squeeze-out [3] of the hot matter directed perpendicular to the reaction plane within the participant region. When energy increases into ultra-relativistic values the squeeze-out turns into an in-plane elliptic flow.

Many different methods have been proposed for studying the flow effects in relativistic nuclear collisions, of which the most commonly employed ones are the directed transverse momentum analysis technique developed by Danielewicz and Odyniec [4] and the method of the Fourier expansion for azimuthal distributions proposed by Demoulins et al. and Voloshin and Zhang [5]. Quantitatively, the anisotropic flow is characterized by coefficients in the Fourier expansion of the azimuthal dependence of the invariant yield of particles relative to the reaction plane [6]. A first coefficient, $v_1$, is usually called a directed flow parameter, and a second coefficient, $v_2$, is called an elliptic flow parameter. To improve anisotropic flow measurements, advanced methods based on multi-particle correlations (cumulants) have been developed to suppress non-flow contributions, which are not related to the initial geometry [7]. Most of the data below 4 AGeV in the literature were, in fact, analyzed by the method of Danielewicz and Odyniec [4]. The advantage of this method is that it can be employed even at small statistics, which is typical for film detectors. That is why we have chosen the method of Danielewicz for the analysis of our data from film detectors in order to investigate the directed flow of protons and $\pi^-$-mesons in the colliding systems p+C, Ta and He+Li, C.

The collective flow of charged particles has been first observed at the Bevalac by the Plastic Ball and Streamer Chamber Collaborations. The flow continued to be explored at Berkeley and at GSI, and further at AGS and
at CERN/SPS accelerators. By now, the collective flow effects were investigated in nucleus-nucleus interactions over a wide range of energies, from hundreds of MeV up to 5.02 TeV [8–16]. Measurements of multi-particle azimuthal correlations (cumulants) for charged particles in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and in p+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV have been carried out at ALICE, ATLAS and CMS LHC experiments. They help address the question of whether there is an evidence for global, flow-like, azimuthal correlations in these systems.

In order to study the properties of nucleus-nucleus interactions, the collective flows of protons, pions and $Λ$ hyperons were previously investigated [17–21] by the authors of the present paper at beam energies of 3.4 and 3.7 AGeV. According to the study, the flow parameters were determined and provided for comparison at different energies.

In each rapidity bin the statistical errors were 3–4 times larger than the pointed systematic uncertainty. Thus, the protons with momentum larger than 150 MeV/c were not detected within the PBC-500 as far as their track lengths are less than 2 mm (p+C interactions), and protons with $p < 200$ MeV/c were absorbed in the Ta target plate (the detector biases). Thus, the protons with momentum larger than 150 MeV/c were registered in p+C interactions, and the protons with $p \gtrsim 250$ MeV/c in p+Ta collisions.

The following factors were considered in estimations of systematic errors for both the set-ups: the contamination of non-beam particles (contamination due to interactions of other projectile nuclei with a charge less than required, $Z < Z_{proj}$), the choice of the effective area for the registration of collisions, secondary interactions in the target, absorption of slow $π^-$-mesons in the target, contamination of $K^\pm$ and $Σ^\pm$ particles, visual identification of $π^+$-mesons, scanning losses (including tracks with large angles at the plane of photography). For the SKM-200-GIBS experiment the corrections due to the trigger, selecting inelastic and central collisions, were added, while for PBC-500 the corrections for the selection of interactions with carbon nuclei from the collisions with propane nuclei were taken into account. The most important corrections were connected with the scanning losses which mainly reduce the number of the observed secondary particles. All the factors led to a systematic uncertainty of the $π^-$-meson average multiplicity on the level 2–4.5%. For the analysis presented in our paper, we study kinematic properties of $π^-$-mesons and protons in narrow intervals of rapidity. In each rapidity bin the statistical errors were 3–4 times larger than the pointed systematic uncertainty. Thus, we will not consider the systematic errors further.

A study of the collective flow phenomenon needs an “event-by-event” analysis, which requires the exclusive analysis of each individual collision. In this connection, there has been a necessity to perform the identification of $π^+$-mesons in order to separate $π^+$-mesons from the positive charged particles. The procedure has been done as described below.

For what concerns He+Li, C collisions, identification of positive charged particles has never been done yet, and we have carried out our effective $π^+$-mesons identification for the first time. It was assumed that $π^+$ and $π^-$-mesons hit a given cell of the plane ($P_L, P_T$) with equal probability in collisions of isospin-symmetrical nuclei. Thus [13], it was assumed that a proton yield in a given cell ($ij$) of the $P_L$-$P_T$ plane is presented as

$$N_{ij}^{prot} = N_{ij}^{pos} - N_{ij}^{neg},$$

where $N_{ij}^{prot}$ is the number of protons in the cell ($ij$), and $N_{ij}^{pos}$ and $N_{ij}^{neg}$ are the number of positively and negatively charged particles in the cell ($ij$), respectively. The analysis produced 4020 events of He+Li and 2127 events of He+C collisions.

The 2-meter long Propane Bubble Chamber (PBC-500) was placed in the magnetic field of 1.5 T. The procedures for separating out the p+C collisions in propane ($C_3H_8$) and the data processing details, including particle identification and corrections, were described in [28–30]. The analysis produced 5882 (10775 events in $C_3H_8$) and 16509 (28703 events in $C_3H_8$) events of p+C interactions at the momenta of 4.2 and 10 GeV/c, respectively, and 2342 events of p+Ta (at 10 GeV/c) collisions. The protons with momentum $p < 150$ MeV/c were not detected within the PBC-500 as far as their track lengths are less than 2 mm (p+C interactions), and protons with $p < 200$ MeV/c were absorbed in the Ta target plate (the detector biases). Thus, the protons with momentum larger than 150 MeV/c were registered in p+C interactions, and the protons with $p \gtrsim 250$ MeV/c in p+Ta collisions.

The 2 Experimental data

The data were obtained from the SKM-200-GIBS streamer chamber.

The SKM-200-GIBS experiment is based on a 2 m streamer chamber placed in the magnetic field of 0.8 T and on a triggering system. An inelastic trigger was used to select the events. The streamer chamber [26, 27] was exposed by a beam of He nuclei accelerated up to 4.5 AGeV/c in the JINR synchrophasator. The thickness of Li and C solid targets (in the form of a disc) were 1.5 and 0.2 g/cm², respectively. The analysis produced 4020 events of He+Li and 2127 events of He+C collisions.
where \( N_{ij}^{\text{pos}} \) is the number of positively charged tracks in the cell, and \( N_{ij}^{\text{neg}} \) is the number of negative charged tracks in the cell. As known [13], the procedure works quite well for inclusive distributions. In our case, we randomly choose how to treat a given positive charged track. We treated the track as a proton one with a probability \( W_p = N_{ij}^{\text{prot}}/N_{ij}^{\text{pos}} \), and as a \( \pi^+ \)-meson one with the probability \( W_{\pi^+} = N_{ij}^{\text{neg}}/N_{ij}^{\text{pos}} \).

Since \( \pi^+ \)-mesons and protons in the Propane Bubble Chamber (p+C, Ta collisions) have been identified in the narrow interval of momenta (up to 0.7 GeV/c), the additional “identification” of particles with higher momenta was necessary. It was done as follows: The momentum distributions (spectra) of positive and negative particles were divided into 10 intervals for \( p > 0.750 \) GeV/c. The distributions were created for isospin-symmetrical collisions (\(^2\text{H}, ^4\text{He, C + C}\)). Using them, the probabilities \( W_p \) and \( W_{\pi^+} \) were determined. According to the probabilities, the positive charged tracks with \( p > 0.750 \) GeV/c were subdivided into “protons” and “\( \pi^+ \)-mesons”. The results of the “identification” for He+Li, C and p+Ta collisions are presented in figs. 1, 2 for inclusive distributions in a comparison with the UrQMD model predictions. One can see that the spectrum of \( \pi^+ \)-mesons well agrees with the experimental spectrum (distribution) of \( \pi^- \)-mesons and with the simulated spectrum of \( \pi^+ \)-mesons.

For the flow analysis, a minimum of four participant protons, \( N_{\text{part}} \geq 4 \), (the remaining protons after the “identification”) was required in an event. In the experiment, the positively charged projectile fragmentation products were identified as those characterized by the momentum \( p > 3.5 \) GeV/c (4.2 AGeV/c, 4.5 AGeV/c) or \( p > 7 \) GeV/c (10 AGeV/c) and emission angle \( \theta < 3.5^\circ \).
and the target fragmentation products by the momentum $p < 0.25 \text{GeV/c}$ in the target rest frame.

3 The directed flows of protons and $\pi^-$-mesons

It has been investigated that the directed flow of protons and $\pi^-$-mesons in the colliding systems $p+C$, Ta and He+Li, C employed a method of Danielewicz and Odyniec [4]. The method relies on summation over transverse momenta of selected particles in the events. In an experiment, no determination of the impact parameter $b$ is possible. Therefore, instead of $b$, a vector sum of transverse momenta of projectile and target nuclear fragments or participant protons is used. The fragmentation regions of projectile and target nuclei were outside the acceptance regions of experimental setups in some experiments and, therefore, the reaction plane was defined by the second approach using the participant protons. The second approach is preferable also for the light nuclear systems, because the multiplicity of the participant protons is larger than the number of detected nuclear fragments. Therefore, the participant protons were used in our study for the reaction plane determination.

The analysis was carried out in the laboratory system. To eliminate the correlation of a particle with itself (autocorrelations) in the study of proton flows, we estimated the reaction plane for each proton in the event with a contribution of that proton removed from the definition of the reaction plane. The reaction plane is spanned by the impact parameter vector $\mathbf{b}$ and the beam axis. Within the transverse momentum method, the direction of $\mathbf{b}$ is estimated event-by-event in terms of the vector constructed from the proton transverse momenta $\mathbf{p}_i^\perp$,

$$Q_j = \sum_{i=1,i\neq j}^{n} \omega_i \mathbf{p}_i^\perp,$$  \hspace{1cm} (1)

where $\omega_i$ is a weight factor of the $i$-th particle, $\omega_i = y_i - y_c$, $y_i$ is the rapidity of the $i$-th participating proton, and $y_c$ is the average rapidity of the participant protons in an event [31]. The values of $y_c$, averaged over all events are shown in figs. 3, 4 by arrows.

The projection of a transverse momentum of a particle onto the estimated reaction plane is

$$p_{xj}^\perp = \left\{ Q_j \cdot \mathbf{p}_j^\perp / |Q_j| \right\}.$$  \hspace{1cm} (2)

The dependence of the projection on the rapidity $y$ was constructed for each collision systems. For further analysis, the average transverse momentum in the reaction plane, $\langle p_{xj}^\perp(y) \rangle$, is obtained by averaging over all events in the corresponding intervals of rapidity. Due to the finite number of particles used in the construction of the vector $\mathbf{Q}$ in (1), the estimated reaction plane fluctuates in the azimuth around the direction of the true reaction plane. Because of those fluctuations, the component $p_x$ of a particle momentum in the true reaction plane is systematically larger than the component $p_x^\perp$ in the estimated plane,

$$\langle p_x \rangle = \langle p_{xj}^\perp \rangle / \langle \cos(\Phi) \rangle,$$  \hspace{1cm} (3)

where $\Phi$ is the angle between the true and estimated reaction planes. The overall correction factor $k = 1 / \langle \cos(\Phi) \rangle$ is a subject of a large uncertainty [4,19], especially for low-multiplicity events. In [4] the method for the definition
Fig. 4. The dependence of \( p_x(y) \) on the rapidity \( y \) in He+Li,C collisions at 4.5 AGeV/c and p+Ta interactions at 10 GeV/c for protons and \( \pi^- \)-mesons in experimental (\( \bullet, \triangle \)) and UrQMD+SMM generated (\( \circ, \triangle \)) data, respectively. Arrows indicate average \( y_c \) over the interactions.

Table 1. The numbers of experimental and UrQMD+SMM events before and after applying the cuts (see text) and the values of flow parameters for protons and \( \pi^- \)-mesons.

|            | \( A_P + A_T \) | p+C 4.2 GeV/c | p+C 10 GeV/c | He+Li, C | p+Ta  |
|------------|----------------|--------------|--------------|----------|-------|
| \( N_{\text{exp}}^{\text{before}} \) | 5882           | 16509        | 6147         | 2342     |
| \( N_{\text{exp}}^{\text{after cut}} \) | 891            | 2890         | 786          | 1141     |
| \( N_{\text{mod}}^{\text{before}} \) | 20000          | 60000        | 23000        | 7230     |
| \( N_{\text{mod}}^{\text{after cut}} \) | 1428           | 4858         | 10435        | 6333     |
| \( \langle \cos \Phi \rangle \) exp. | 0.633          | 0.638        | 0.662        | 0.642    |
| \( \langle \cos \Phi \rangle \) mod. | 0.657          | 0.658        | 0.670        | 0.702    |
| \( P_{x,\pi^-}^{\text{exp}} \) (MeV/c) | 125.2 ± 7.2    | 85.7 ± 5.8   | 80.9 ± 16.2  | 76.1 ± 5.3 |
| \( F_{x,\pi^-}^{\text{mod}} \) (MeV/c) | 116.2 ± 4.9    | 94.3 ± 3.7   | 86.2 ± 1.9   | 79.8 ± 1.9 |

of the correction factor has been proposed. Each event is randomly divided into two almost equal sub-events and the vectors \( Q_1 \) and \( Q_2 \) are constructed.

Then the distribution of the azimuthal angle \( \Phi \) between these two vectors was obtained, and the average \( \langle \cos \Phi \rangle \) was determined. (\( \langle \cos \Phi \rangle \) for all considered interactions are given in table 1. Figures 3, 4 show dependencies of the average in-plane transverse-momentum components on the rapidity for protons and \( \pi^- \)-mesons in p+C, Ta and He+Li, C collisions. The values of the flow parameter \( F \), which are the slopes of \( p_x(y) \) at its mid-rapidity crossover, are given in table 1. All presented error bars are only statistical ones.

The flow analysis was done also for the changed requirement for the determination of the reaction plane, namely for \( N_{\text{part}} \geq 3 \). In this case the absolute values of the directed flow parameter \( F \) for protons have been obtained, \( 82.7 ± 4.3 \text{MeV/c} \) (\( N_{\text{event}} = 1720 \) after cut) for He+Li, C and \( 76.8 ± 4.9 \text{MeV/c} \) (\( N_{\text{event}} = 1541 \) after cut) for p+Ta collisions. One can see from table 1 that the change of the selection criterion of the events for the flow analysis does not affect significantly the values of the flow parameters, while the errors of \( F \) increases for the criterion \( N_{\text{part}} \geq 4 \) due to the decrease of statistics: for He+Li, C collisions \( 80.9 ± 16.2 \text{MeV/c} \) (\( N_{\text{event}} = 786 \) after cut) and for p+Ta collisions \( 76.1 ± 5.3 \text{MeV/c} \) (\( N_{\text{event}} = 1141 \) after cut).

In view of the strong coupling between the nucleon and pion, it is interesting to know if pions also have a collective flow behaviour in the presented nuclear systems and how the pion flow is related to the nucleon one. An answer to this question is given in table 1. The pion flow has been observed with respect to the reaction plane determined by participant protons.

One can see from table 1 and figs. 3, 4 that the \( \pi^- \)-mesons flows are smaller than the proton ones. It is
apparent that the \( \pi^- \)-mesons and protons flow in the opposite in-plane directions, in the forward and backward hemispheres, in all nuclear systems. The flow parameter absolute values for \( \pi^- \)-mesons and protons decrease with increasing the projectile momentum and mass numbers \( A_T \) of target nuclei, which is the opposite of the results in nucleus-nucleus collisions (\(^2\text{H}+\text{C}, \text{He}+\text{C}, \text{C}+\text{C}, \text{He}+\text{Ta}, \text{He}+\text{Ta}, \text{and C}+\text{Ta} \) collisions) obtained at the same energy and on the same setup [19,21]. Interestingly, the proton and \( \pi^- \)-mesons collective flow parameters in \( \text{He}+\text{C} \) collisions of SKM-200-GIBS with the pure, solid thin targets are comparable with those from collisions with \( C_3\text{H}_8 \) (PBC-500, our earlier results) [21]. Both results are in agreement within errors.

The obtained experimental results from \( p+\text{C}, \text{Ta} \) and \( \text{He}+\text{Li}, \text{C} \) collisions are compared with predictions of the UrQMD model coupled with the SMM model. Detailed descriptions of the UrQMD and SMM models can be found in [23,24] and [25], respectively. The UrQMD model is a microscopic transport model based on the covariant propagation of all hadrons on classical trajectories in combination with stochastic binary scatterings, colour string formation and resonance decay. We added the SMM model to the UrQMD model to improve simulations of a baryon production in target and projectile fragmentation regions. There is not a strong subdivision between the participating protons and protons created at the de-excitation of residual nuclei. The SMM allows a simulation of the evaporated proton production, and UrQMD generates the participating protons. Thus, we have in the model events all processes presented in the experiment.

The UrQMD model is designed as a multi-purpose tool for studying a wide variety of heavy-ion-related effects ranging from multi-fragmentation and collective flow to particle productions and correlations in the energy range from SIS to RHIC. In the used version of the UrQMD model (1.3) with addition of the SMM, we consider potential interactions between nucleons and excitations of the residual nuclei. These allowed one to determine the reaction plane by the participant protons.

The incorporation of UrQMD and SMM was done as it was proposed in refs. [32,33]. The UrQMD calculation was carried out up to the transition time \( t_{tr} = 100 \text{fm}/c \). After that the minimum spanning tree method [34] was employed for the determination of prefragments. It was assumed that two nucleons form a cluster, if a distance between their centers is lower than \( R_{clus} = 3 \text{fm} \). The choice of the parameters \( t_{tr} \) and \( R_{clus} \) was considered in refs. [32,33]. The procedure allowed to determine mass numbers, charges, energies and momenta of pre-fragments. The energy and momentum of a prefragment were transformed to the prefragment rest frame using the Lorentz transformation. The excitation energy of the hot prefragment was calculated as a difference between the binding energy of the pre-fragment and the binding energies of this pre-fragment in its ground state. Skyrme, Yukawa and Coulomb potentials were used in the calculation of the binding energy. The Pauli potential was not accounted. A “hard” Skyrme-type EoS with the nuclear incompressibility \( K = 300 \text{MeV} \) was used. The decay of the prefragments was simulated by SMM.

We have checked out that the UrQMD model without the potential interactions (cascade mode of UrQMD) does not predict any anisotropic flows for proton interactions with light nuclei. An inclusion of the potential interactions leads to the flows, and allows one to estimate the excitation energy of the nuclear remnant. Evaporated protons, neutrons and nuclear fragments are produced at a de-excitation of the remnants. The average kinetic energy of the evaporated protons is below 30 MeV. An accounting of the protons does not have an essential influence on our results.

20000 (4.2 GeV/c) and 60000 (10 GeV/c) events have been generated for \( p+\text{C} \) interactions, 15000 (4.5 AGeV/c), 8000 (4.5 AGeV/c) and 7230 (10 GeV/c) events for \( \text{He}+\text{Li}, \text{C} \) and \( p+\text{Ta} \) collisions, respectively, by using the enlarged UrQMD model (UrQMD+SMM). The experimental conditions and selection criteria are applied to the generated events. For UrQMD+SMM events the projection of the transverse momentum onto the true reaction plane was determined. The values of the flow parameter \( F \) are extracted for protons and \( \pi^- \)-mesons from the dependencies of \( \langle p_x(y) \rangle \) on the rapidity \( y \) for each nuclear pair (table 1). As seen, there is a good agreement between the experimental and theoretical values (figs. 3, 4).

The reduced magnitude of the average pion momentum component in the reaction plane compared to the proton one was seen before at Bevalac, GSI-SIS, CERN-SPS and STAR [13,35,36]. Historically, the pattern of pion emission relative to the reaction plane has been first studied at Bevalac by the Streamer Chamber Group and later by the EOS Collaboration [37]. In the investigations at Bevalac and Saturne, the projection of the pion transverse momentum onto the reaction plane was examined. The direction of the pion flow opposite to the proton flow, the so-called anti-flow, was seen before in either asymmetric or symmetric systems [13,37,38]. However, we are unaware of observation of the pion anti-flow in a strongly asymmetric system such as our \( p+\text{Ta} \) one.

The anti-correlation of nucleons and pions was explained in [39] by the effect of multiple \( \pi \)N scattering. However, in [40] it is shown that the anti-correlation is a manifestation of the nuclear shadowing of the target and projectile spectators through both pion re-scattering and re-absorptions. Quantitatively, the shadowing can produce the in-plane transverse momentum components comparable to the momenta itself and, thus, much larger than the components due to the collective motion for pions [41]. In our opinion, our results indicate that the flow behaviour of \( \pi^- \)-mesons is a result of the nuclear shadowing effect.

4 Proton elliptic flow

We have studied a proton elliptic flow in \( p+\text{C} \) (4.2 and 10 GeV/c), \( \text{He}+\text{Li} \), C (4.5 AGeV/c) and \( p+\text{Ta} \) (10 GeV/c) collisions. The azimuthal \( \phi \) distributions of the protons were obtained and presented in figs. 5, 6 where \( \phi \) is the angle between the transverse momentum of each particle in
Fig. 5. The azimuthal distributions with respect to the reaction plane in p+C collisions at the momenta of 4.2 GeV/c and of 10 GeV/c for protons. The curves are the result of the approximation by $dN/d\phi = a_0 [1 + a_1 \cos(\phi) + a_2 \cos(2\phi)]$.

Fig. 6. The azimuthal distributions with respect to the reaction plane in He+Li, C collisions at 4.5 AGeV/c and p+Ta interactions at 10 GeV/c for protons in experimental (●) and UrQMD+SMM generated (○) data, respectively. The curves are the result of the approximation by $dN/d\phi = a_0 [1 + a_1 \cos(\phi) + a_2 \cos(2\phi)]$.

The azimuthal angular distributions show maxima at $\phi = 90^\circ$ and $270^\circ$ with respect to the event plane. The maxima are associated with a preferential particle emission perpendicular to the reaction plane (squeeze-out). To treat the data in a quantitative way, the azimuthal distributions were fitted with the Fourier cosine expansion (given the system invariance under reflections with respect to the reaction plane).

$$\frac{dN}{d\phi} = a_0 [1 + a_1 \cos(\phi) + a_2 \cos(2\phi)].$$

The squeeze-out signature is a negative value of the coefficient $a_2$, which is a measure of the strength of the anisotropic emission. The elliptic anisotropy, quantified in terms of the $a_2$ coefficient ($a_2 = 2v_2$), extracted from the azimuthal distributions of the protons with respect to the reaction plane at mid-rapidity is given in table 2.

The obtained experimental results have been compared with the calculations of the UrQMD+SMM model. The experimental selection criteria were applied to the generated events. The elliptic flow parameters with respect to the true reaction plane were calculated (table 2) for UrQMD+SMM events too. A good agreement between
Table 2. Characteristics of proton elliptic flow in the experimental and UrQMD+SMM collisions including event number ($$N_{\text{exp}}$$, $$N_{\text{UrQMD}}$$) before and after applying the cuts.

| $$A_T + A_T$$ | $$N_{\text{exp}}$$ before cut | $$N_{\text{mod}}$$ before cut | $$a_2$$ before cut | $$N_{\text{exp}}$$ after cut | $$N_{\text{mod}}$$ after cut | $$a_2$$ after cut |
|---------------|-------------------------------|------------------------------|---------------------|-----------------------------|-----------------------------|------------------|
| 4.2 GeV/$c$   | 5882                          | 20000                        | −0.053 ± 0.020      | 891                         | 1428                        | −0.052 ± 0.013   |
| p+C           | 16509                         | 60000                        | −0.071 ± 0.013      | 2890                        | 4858                        | −0.072 ± 0.008   |
| 10 GeV/$c$    | 6147                          | 23000                        | −0.051 ± 0.019      | 786                         | 10435                       | −0.052 ± 0.007   |
| He+Li, C      | 2342                          | 7230                         | −0.071 ± 0.016      | 1141                        | 6333                        | −0.072 ± 0.007   |
| p+Ta          | 2342                          | 7230                         | −0.071 ± 0.016      | 1141                        | 6333                        | −0.072 ± 0.007   |

The directed transverse collective flows of protons and $$\pi^-$$-mesons and elliptic flow of protons emitted from p+C (4.2 and 10 GeV/$c$), He+Li, C (4.5 AGeV/$c$) and p+Ta (10 GeV/$c$) collisions have been studied. In more detail:

1) The p+C system is the lightest studied one, and the p+Ta system is an extremely asymmetrical system in which collective flow effects (directed and elliptic) are detected for protons and pions. As shown, the $$\pi^-$$-mesons exhibit an opposite directed flow with respect to that for protons in all colliding systems. The absolute value of the directed flow parameter $$F$$ decreases with increasing of projectile momenta in p+C collisions for the protons from 125.2 ± 7.2 MeV/$c$ at 4.2 GeV/$c$ down to 85.7 ± 5.8 MeV/$c$ at 10 GeV/$c$. The values for $$\pi^-$$-mesons are −21.6 ± 11.1 MeV/$c$ at 4.2 GeV/$c$ and −16.1 ± 5.4 MeV/$c$ at 10 GeV/$c$. Also, $$F$$ decreases with increasing the mass numbers of the target $$A_T$$ nuclei, for protons from 85.7 ± 5.8 MeV/$c$ (p+C, 10 GeV/$c$) down to 76.1 ± 5.3 MeV/$c$ (p+Ta, 10 GeV/$c$), and almost does not change for $$\pi^-$$-mesons, −19.7 ± 19.4 MeV/$c$ (p+C, 10 GeV/$c$) and −18.8 ± 18.8 MeV/$c$ (p+Ta, 10 GeV/$c$). The results for nucleon-nucleus collisions are opposite to the results in nucleus-nucleus interactions obtained at the same energy and on the same experimental setup.

2) It should be mentioned that no change of the sign of the proton elliptic flow has been observed in p+C, Ta nucleon-nucleus collisions in the projectile momentum range of 4–10 GeV/$c$. The absolute value of the proton elliptic flow parameter $$a_2$$ in p+C collisions increases with projectile momentum from −0.053±0.020 at 4.2 GeV/$c$ up to −0.071 ± 0.013 at 10 GeV/$c$. Also, $$a_2$$ almost does not change with increasing mass numbers of the target $$A_T$$ nuclei at 10 GeV/$c$: −0.071±0.013 (p+C) and −0.071±0.016 (p+Ta).

An agreement between experimental and theoretical (UrQMD+SMM) collective flow distributions has been obtained for particles in the above-mentioned interactions.

5 Conclusion

The directed transverse collective flows of protons and $$\pi^-$$-mesons and elliptic flow of protons emitted from p+C (4.2 and 10 GeV/$c$), He+Li, C (4.5 AGeV/$c$) and p+Ta (10 GeV/$c$) collisions have been studied. In more detail:

1) The p+C system is the lightest studied one, and the p+Ta system is an extremely asymmetrical system in which collective flow effects (directed and elliptic) are detected for protons and pions. As shown, the $$\pi^-$$-mesons exhibit an opposite directed flow with respect to that for protons in all colliding systems. The absolute value of the directed flow parameter $$F$$ decreases with increasing of projectile momenta in p+C collisions for the protons from 125.2 ± 7.2 MeV/$c$ at 4.2 GeV/$c$ down to 85.7 ± 5.8 MeV/$c$ at 10 GeV/$c$. The values for $$\pi^-$$-mesons are −21.6 ± 11.1 MeV/$c$ at 4.2 GeV/$c$ and −16.1 ± 5.4 MeV/$c$ at 10 GeV/$c$. Also, $$F$$ decreases with increasing the mass numbers of the target $$A_T$$ nuclei, for protons from 85.7 ± 5.8 MeV/$c$ (p+C, 10 GeV/$c$) down to 76.1 ± 5.3 MeV/$c$ (p+Ta, 10 GeV/$c$), and almost does not change for $$\pi^-$$-mesons, −19.7 ± 19.4 MeV/$c$ (p+C, 10 GeV/$c$) and −18.8 ± 18.8 MeV/$c$ (p+Ta, 10 GeV/$c$). The results for nucleon-nucleus collisions are opposite to the results in nucleus-nucleus interactions obtained at the same energy and on the same experimental setup.

2) It should be mentioned that no change of the sign of the proton elliptic flow has been observed in p+C, Ta nucleon-nucleus collisions in the projectile momentum range of 4–10 GeV/$c$. The absolute value of the proton elliptic flow parameter $$a_2$$ in p+C collisions increases with projectile momentum from −0.053±0.020 at 4.2 GeV/$c$ up to −0.071 ± 0.013 at 10 GeV/$c$. Also, $$a_2$$ almost does not change with increasing mass numbers of the target $$A_T$$ nuclei at 10 GeV/$c$: −0.071±0.013 (p+C) and −0.071±0.016 (p+Ta).

An agreement between experimental and theoretical (UrQMD+SMM) collective flow distributions has been obtained for particles in the above-mentioned interactions.

One of us (LC) would like to thank the board of directors of the Laboratory of Information Technologies (LIT) of JINR for the warm hospitality. This work was partially supported by the Georgian Shota Rustaveli National Science Foundation under Grant DI/38/6-200/13. The authors are thankful to heterogeneous computing (HybriLIT) team of the Laboratory of Information Technologies (LIT) of JINR for support of our calculations.

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