Simulations of Q-Ball Formation

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Abstract

The fragmentation of the Affleck-Dine condensate is studied by utilizing 3+1 dimensional numerical simulations. The 3+1 dimensional simulations confirm that the fragmentation process is very similar to the results obtained by 2+1 dimensional simulations. We find, however, that the average size of Q-balls in 3+1 dimensions is somewhat larger that in 2+1 dimensions. A filament type structure in the charge density is observed during the fragmentation process. The resulting final Q-ball distribution is strongly dependent on the initial conditions of the condensate and approaches a thermal one as the energy-charge ratio of the Affleck-Dine condensate increases.

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1 Introduction

The Affleck-Dine condensate [1] offers an interesting possibility for baryogenesis at a high energy scale. The Minimal Supersymmetric Standard Model (MSSM) has several flat directions in its scalar potential along which squark, slepton and higgs fields fluctuate during inflation forming AD-condensates [2]. The formation of an Affleck-Dine condensate is then natural in theories with supersymmetry.

The evolution of the AD-condensate is not as simple as one might envision at first, but includes highly nonlinear dynamics. In [3, 4] it was realized that the condensate will fragment into Q-balls [5] which represent the true ground state instead of the condensate. This fragmentation process has then been studied in a number of works [6]-[8], both in the case of gravity and gauge mediated supersymmetry breaking. Three dimensional simulations have been performed in both cases [6, 7], but only for a very limited range of the energy-charge ratio of the condensate. In two dimensions, Q-ball formation has been studied in detail in [8], where it was found that at high initial value of the energy-charge ratio of the condensate, the resulting Q-ball charge distribution is well represented by a thermal distribution. On the basis of the analytical arguments presented in [8], it is expected that the three dimensional results should be well represented by the two dimensional simulations. In order to verify this assumption and to study the fragmentation process in more detail, we have studied the fragmentation process by utilizing full three dimensional simulations.

In this letter we present results from full 3+1 dimensional simulations of the fragmentation of the Affleck-Dine condensate in the MSSM with gravity mediated supersymmetry breaking. The numerical results are presented in Section 2 along with illustrations of the fragmentation process. In Section 3 we study the Q-ball distribution statistically. The letter is concluded in Section 4.

2 Numerical results

2.1 Preliminaries

The numerical work is performed by utilizing the parallel computing resources available at the Finnish center for high-performance computing (CSC). The simulations are run on a Cray T3E parallel machine.

In the simulations, the lattice is divided into cubes of $40^3$ and data transfer between the cubes is achieved by using the Message Passing Interface (MPI)-library. The lattice size used was typically $120^3$ but also larger lattice sizes up to $200^3$ were studied. In all simulations, periodic boundary conditions were used.

As an initial condition we add uniform noise to the amplitude and the phase of the uniformly rotating condensate field,

$$\Phi = \Phi_0 e^{i\omega t} + \delta\Phi.$$  \hspace{1cm} (1)

The amplitude of the condensate is chosen to have the same value as in the 2D-case, $\Phi_0 = 10^9$ GeV. The type of noise added does not affect the results [9]. The amplitude of the noise was typically $|\delta\Phi|/\Phi_0 \sim 10^{-10}$, but it was also varied in order to study its effects on the results. Decreasing the amplitude of the noise simply delays the beginning of the fragmentation process but the essential features of the dynamics are unchanged.
The simulations are done with varying values of $\omega$ to study the effect of varying the initial energy-charge ratio, $x \equiv \rho/(mq)$, of the condensate. As $\omega$ is decreased the condensate rotates more slowly and $x$ increases. Assuming that $V(\Phi_0) \ll \omega^2|\Phi_0|^2$, $x$ and $\omega$ are related simply by $x \approx m\omega^{-1}$.

The equation of motion of the AD-condensate is

$$\ddot{\Phi} + 3H\dot{\Phi} - \frac{1}{a^2} \nabla^2 \Phi + m^2 \Phi [1 + K \log(\frac{|\Phi|^2}{M^2})] - gH^2 \Phi + \frac{3\lambda^2}{M_{Pl}^2} |\Phi|^4 \Phi = 0,$$

(2)

where $M = 10^{14}$ GeV, $a$ is the scale factor of the universe and $H$ is the Hubble parameter, $H = \dot{a}/a$. The field was decomposed into real and imaginary parts, $\Phi = \phi_1 + i\phi_2$. Field and space-time were also rescaled according to

$$\phi = \frac{\phi}{m}, \quad h = \frac{H}{m}, \quad \tau = mt, \quad \xi = mx.$$  

The parameter values chosen for the simulations were $m = 10^2$ GeV, $K = -0.1$ and $\lambda = \frac{1}{2}$ (the $gH^2$-term can be omitted since $gH^2 \ll m^2$). The universe is assumed to be matter dominated so that $a = a_0 t^{\frac{1}{2}}$ and $H = \frac{2}{3t}$. The spatial and temporal lattice units were varied in order to verify that their exact values do not affect the results. The calculations were done in a comoving volume so that the physical size of the lattice increases with time. The initial time is $\tau_0 = 100$ and $a = 0.1 \times \tau^\frac{2}{3}$. The simulations were run up to $\sim 10^6$ time steps.

### 2.2 Results

We have simulated the evolution of the AD-condensate for different values of $x$ (or $\omega$) to check for similar and differing features in two and three dimensions. The increase of phase space and somewhat larger dissipation in three dimensions are expected to modify some features of final Q-ball systems.

We have presented the essential features of the simulations in Figs. 3-8. Figs. 3-5 illustrate the evolution condensate with $x \approx 1$, while Figs. 6-8 show the $x \approx 10^5$-case\footnote{Color versions of the figures are available at www.utu.fi/~tuomul/qballs/}. Here we have plotted just one $40^3$-box from the whole lattice. Note that we plot comoving volume, i.e. the physical size of the box grows with time. In the Figs. 3-8, $\rho_c$ indicates the absolute value of the equal charge density surface drawn to each figure.

In both cases, the initial part of the condensate evolution follows along similar lines: The fastest growing mode starts to dominate the initially stochastic perturbation spectrum, Fig. 3. Linear growth continues until non-linear effects begin to dominate and perturbations begin rapid growth. The regions of largest charge density form filament type structures which then fragment into lumps of charge, Fig. 4.

If $x \approx 1$, no further qualitative development is visible after the filaments fragment. The energetic lumps are excited Q-balls which relax as the universe (box) expands, leaving a distribution consisting of Q-balls.

If $x \gg 1$, a further development, however, takes place, as is shown in Figs. 6-8. After the filaments have fragmented into energetic charge lumps, a rapid growth of high density regions with negative charge takes place (negative charge is plotted as light gray in the figures), Figs. 6-7. After this stage a large number
of charged lumps which carry positive and negative charge are present in the box. The lumps relax with time and the spatial distribution freezes as the box grows.

Note that in the depicted $x \approx 1$ case the amplitude of the initial perturbation is smaller than in the $x \approx 10^5$ case, which explains why the initial charge lumps appear more quickly in the large $x$ case. With equal initial perturbations, the fragmentation process takes place at the same time in both cases.

3 Discussion

The qualitative features in the three dimensional case are very similar to the previously studied two dimensional case, which is exactly what we expect on the basis of analytical arguments [8]. To verify that here the Q-ball anti-Q-ball distribution in the large $x$ case is thermal, we utilize statistical means.

The one-particle partition function for a thermal distribution is given by

$$Z_1 = \int_{V_D} \frac{d^D x}{(2\pi)^D} \int_{-\infty}^\infty dQ \, e^{-\beta E + \mu Q}$$

where $\mu$ is the chemical potential related to the charge of Q-balls. The cumulative distribution function $F(Q, \mu, \beta)$ in terms of the probability distribution function $g(Q, \mu, \beta)$ is then

$$F(Q, \mu, \beta) = \int_{-\infty}^Q dQ' g(Q') = \int_{-\infty}^Q dQ' e^{\mu Q' - \beta |Q'|} (\beta |Q'| + 1),$$

where we have omitted the fugacity of the distribution.

Before we can fit the cumulative distribution function to the Q-ball distribution seen in the simulations, we need to count and classify the lumps of charge. Our approach is two-fold: first we search for local maxima in the charge (anti-charge) density. Then we classify the local maxima, $\phi_{max}$, by using the criteria

$$\phi_{max} > M \exp\left(\frac{1 - (\omega/m)^2}{2|K|}\right),$$

which follows from the fact that inside a Q-ball, $V(\phi) - \omega^2 \phi^2 < 0$. In other words we require that the complex phase of a charge maximum must rotate quickly enough for it to be classified as a Q-ball.

The number of local maxima that pass the Q-ball criterion is only a fraction of the total number, typically the number of points is reduced by 80–90%. This is illustrated in Fig. 1, where a plot of the local maxima is presented before (dotted line) and after (solid line) the Q-ball criterion is applied. Clearly, there are both small and large charge lumps do not satisfy the criterion (6). The existence of these lumps that are not Q-balls is expected since the remains of the perturbed initial condensate are still present in the system. Also some of the lumps may be excited Q-balls which do not yet satisfy the condition (6).

After selecting which charge lumps to consider, we then fit the cumulative distribution function to the Q-ball charge distribution. The values of $m\beta$ at different $\tau$ are shown in Fig. 2 (corrected for the expansion of the box). Just like in the two dimensional case, we see that the Q-ball anti-Q-ball -system reaches a
thermal equilibrium soon after negative charge enters the system. The chemical potential, \( \mu \), is also again much smaller than \( m\beta \). Due to geometrical considerations, the number density of Q-balls is slightly larger in three dimensions. Because the radii of Q-balls are essentially equal in both dimensions [10], the higher the dimension is, the more Q-balls the system can support.

If the criterion (3) is not applied and (3) is fitted to the whole distribution, we find that the value of \( m\beta \) is practically unchanged. On this basis, we expect that the value of \( m\beta \) obtained from the whole distribution in two dimensions [8], is a good approximation to the distribution with the Q-ball criterion applied.

The value of \( m\beta \) in the three dimensional simulations performed here is smaller than in the two dimensional case i.e. the average energy of a Q-ball is larger. One can understand this by considering the total energies in the system in the two cases (omitting \( \mu \)) [8], \( E_D = (D+1)N_D/\beta_D \), where \( D \) is the dimension and \( N \) is the number of Q-balls. From this we get \( \beta_3 = (4E_2N_3)/(3E_2N_2)\beta_2 \). The energies in the two cases are related by \( (E_2/E_3) = (m/\text{GeV})^{-1}V_2/V_3 \), where \( V_D \) is the volume in \( D \) dimensions in lattice units. The number densities are found to be \( N_3/V_3 \approx 2N_2/V_2 \), hence \( \beta_3 \approx 8/3(\text{GeV}/m)\beta_2 \), which is in good agreement with the numerical results.
4 Conclusions

We note that the results fully support the expectations that the two dimensional simulations capture the essential features of the fragmentation process, both at qualitative and quantitative level. The final Q-ball distribution is strongly dependent on the initial charge-energy ratio $x$: with $x \approx 1$, only positive charge appears due to the fact that Q-balls in this scenario have a linear energy dependence on charge, $E \approx mQ$ [1], so that all energy and charge can be accounted for by creating Q-balls. As $x$ grows, the excess energy left after storing the charge in Q-balls grows and, as it must be accounted for, leads to the production of negative charge. In the $x \approx 10^5$ case, we have again seen that $|Q_+| + |Q_-| \gg Q_+ + Q_-$ i.e. that the absolute amount of charge created is much larger than the net charge in the box. This fact also presents itself in the form of a small chemical potential $\mu$.

The charge distribution in the large $x$ case is, like in two dimensions, well represented by a thermal distribution. After the initial formation process, the distribution thermalizes slightly more slowly than in two dimensions, which is natural due to the increased phase space. The value of $\beta$ is also compatible with the analytical and numerical expectations.

An interesting feature seen in the simulations is the formation of a filament network in early stages of the condensate evolution. This was also seen in two dimensions but the observation of this effect here confirms that this is not an artifact of two dimensions. Numerical analysis in two dimensions also showed that the final distribution of large Q-balls was not spatially random. Small Q-balls have a large kinetic energy component, which effectively will randomize the distribution, but the spatial distribution of large Q-balls may well have some structure also in three dimensions. If Q-balls survive thermal erosion [1, 2] and decay into baryons after the electroweak phase transition, some remains of this structure may still be present during nucleosynthesis. Note, however, that any remaining spatial correlation is very small: assuming a high reheat temperature, the universe can expand between reheating and nucleosynthesis by a factor of $\sim 10^8$. Further assuming a large correlation length of $\sim 10$ GeV$^{-1}$ (the size of the box when filaments appear), we see that during nucleosynthesis, spatial correlations are very small unless the universe expands significantly between fragmentation and reheating.

To conclude, we have verified that the two dimensional simulations capture all of the essential features of the dynamics of the Affleck-Dine condensate. Depending on the initial conditions, the AD-condensate fragments into a large number of Q- or anti-Q-balls. A period during which the charge density forms filament like structures is observed, which is likely to leave a spatial correlation between large Q-balls.

Acknowledgments

We thank S. Kasuya for discussions. This work has been partly supported by the Magnus Ehrnrooth Foundation. The Finnish center for high-performance computing and networking (CSC) is gratefully acknowledged for providing the computational resources.
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Figure 3: $\tau = 875 (\rho_\phi^c = 10^{12}), 1000 (\rho_\phi^c = 8 \times 10^{11})$, for $x \approx 1$.

Figure 4: $\tau = 1875, 2000 (\rho_\phi^c = 10^{11})$, for $x \approx 1$.

Figure 5: $\tau = 2250, 3000 (\rho_\phi^c = 10^{11})$, for $x \approx 1$. 
Figure 6: $\tau = 1500 (\rho^c_\phi = 2 \times 10^6), \ 3000 (\rho^c_\phi = 3 \times 10^7)$, for $x \approx 10^5$.

Figure 7: $\tau = 4500, \ 5250 (\rho^c_\phi = 8 \times 10^8)$, for $x \approx 10^5$.

Figure 8: $\tau = 7500, \ 1.5 \times 10^4 (\rho^c_\phi = 8 \times 10^8)$, for $x \approx 10^5$.