Masses, Beaming and Eddington Ratios in Ultraluminous X–ray Sources

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ABSTRACT

I suggest that the beaming factor in bright ULXs varies as $b \propto \dot{m}^{-2}$, where $\dot{m}$ is the Eddington ratio for accretion. This is required by the observed universal $L_{\text{soft}} \propto T^{-4}$ relation between soft–excess luminosity and temperature, and is reasonable on general physical grounds. The beam scaling means that all observable properties of bright ULXs depend essentially only on the Eddington ratio $\dot{m}$, and that these systems vary mainly because the beaming is sensitive to the Eddington ratio. This suggests that bright ULXs are stellar–mass systems accreting at Eddington ratios of order 10 – 30, with beaming factors $b \gtrsim 0.1$. Lower–luminosity ULXs follow bolometric (not soft–excess) $L \propto T^4$ correlations and probably represent sub–Eddington accretion on to black holes with masses $\sim 10 M_\odot$. High–mass X–ray binaries containing black holes or neutron stars and undergoing rapid thermal– or nuclear–timescale mass transfer are excellent candidates for explaining both types. If the $b \propto \dot{m}^{-2}$ scaling for bright ULXs can be extrapolated to the Eddington ratios found in SS433, some objects currently identified as AGN at modest redshifts might actually be ULXs (‘pseudoblazars’). This may explain cases where the active source does not coincide with the centre of the host galaxy.

Key words: accretion, accretion discs – binaries: close – X-rays: binaries – black hole physics – galaxies: active – BL Lac objects

1 INTRODUCTION

There are currently two models proposed for ultraluminous X–ray sources (ULXs). In one they are identified as intermediate–mass black holes (IMBH) accreting at rates below their Eddington limits. In the alternative model, ULXs represent a very bright and unusual phase of X–ray binary evolution, in which the compact object is fed mass at a rate $\dot{M}$ well above the usual Eddington value $\dot{M}_E$. In the picture proposed by Shakura & Sunyaev (1973) radiation pressure becomes important at the spherization radius $R_{\text{sph}} \simeq 27(\dot{M}/\dot{M}_E)R_s/4$, where $R_s = 2GM_1/c^2$ is the Schwarzschild radius of the accreting black hole of mass $M_1$ (Shakura & Sunyaev, 1973; see also Begelman et al., 2006; Poutanen et al., 2007). Within this radius the disc remains close to the local radiation pressure limit. Matter is therefore blown away so that the accretion rate decreases with disc radius as $\dot{M}(R) \simeq \dot{M}(R/R_{\text{sph}}) \simeq \dot{M}_E(R/R_s)$. As the disc wind has the local escape velocity at each radius, we see from mass conservation that the wind is dense near $R_{\text{sph}}$ and tenuous near the inner disc edge, and there is a vacuum funnel along the central disc axis through which radiation escapes.

In this model the large apparent X–ray luminosity $L_X = 10^{46} L_{46}$ erg s$^{-1}$ results from two effects of super–Eddington accretion (Begelman et al., 2006; Poutanen et al., 2000). First, the bolometric luminosity is larger than the usual Eddington limit by a factor $\sim 1 + \ln(\dot{M}/\dot{M}_E)$, which can be of order 5 – 10 for the high mass transfer rates encountered at various stages of the evolution of a compact stellar–mass binary. Second, the luminosity of a ULX is collimated by a beaming factor $b$ via scattering off the walls of the central funnel. (Note that here and throughout this paper, ‘beaming’ simply means geometrical collimation, and not relativistic beaming.) These conditions could occur in a state of high mass transfer (cf King, 2001, Rappaport et al., 2005) or conceivably a transient outburst (King, 2002). In this picture one would expect on physical grounds that the Eddington ratio $\dot{m} = \dot{M}/\dot{M}_E$ should determine the beaming factor $b$. However current modelling has not yet derived this connection, allowing a spurious extra degree of freedom in comparing this picture with observations.

A clue here comes from the fact that bright ULXs have spectra consisting of a power law plus a soft ($kT \sim 0.1 – 0.3$ keV) excess which can be modelled as a blackbody. This is usually taken as a multicolour disc with the
maximum disc temperature as the reference value, but the fitted temperature is not very different if the blackbody is assumed uniform. Feng & Kaaret (2007) show that the luminosity $L_{\text{soft}}$ and temperature $T$ of the blackbody component vary as $L_{\text{soft}} \propto T^n$ with $n = -3.1 \pm 0.5$ in the ultraluminous source NGC 1313 X-2. Kajava & Poutanen (2008, hereafter KP) have recently extended this result to a sample of nine ULXs (including NGC 1313 X-2) which have a power law continuum with a soft excess. These are essentially all the sources with inferred luminosities permanently above $\sim 3 \times 10^{39}$ erg s$^{-1}$. Strikingly, KP find that all of these soft–excess objects cluster around the line

$$L_{\text{soft}} = 7 \times 10^{40} T_{0.1\text{keV}}^{-4} \text{ erg s}^{-1}$$

(1)
at all epochs (see the right–hand panel of their Figure 3). Here $T_{0.1\text{keV}}$ is the temperature in units of 0.1 keV. KP caution that the agreement for the coolest and brightest may be affected by an incorrect subtraction of the hard emission component, but the overall trend (1) is clear.

KP also identify a distinct class of ‘non–power–law’ (or thermal) type ULXs whose medium energy spectra are fitted by harder multicolour disc blackbodies with reference temperatures $kT_{\text{medium}} \sim 0.5 – 2$ keV rather than power laws plus a soft excess. These systems all have inferred luminosities permanently below $\sim 10^{39}$ erg s$^{-1}$. They do not obey (1), but instead follow individual luminosity – temperature correlations $L_{\text{medium}} \propto T_{\text{medium}}^n$, just like standard (non–ULX) black hole binaries (Gierliński & Done 2004).

At first sight, as KP remark, the correlation (1) for the soft excesses of bright, power–law ULXs seems counterintuitive, as one might expect a blackbody to vary as $L_{\text{soft}} \propto T^4$. However this assumes that the characteristic radius $R$ of the blackbody remains fixed as other parameters vary, and indeed that the inferred $L_{\text{soft}}$ is not affected by beaming, which could itself also vary systematically.

I shall show here that in the picture of ULXs as super–Eddington accretors suggested by Begelman et al (2006) and Poutanen et al., (2007), the correlation $L_{\text{soft}} \propto T^{-4}$ is actually expected, and results from a tight relation between the beaming factor $b$ and the Eddington ratio $\dot{m}$ of the form $b \sim \dot{m}^{-2}$. Using the observed relation (1) we find $b$ and $m_1 = M_1 / M_\odot$ as functions of the Eddington ratio $\dot{m}$ for a given inferred disc luminosity. With $\dot{m}$ taking values giving only modest beaming factors $b \gtrsim 0.1$ we find that the accretors in ULXs with soft components all have stellar masses $m_1 \lesssim 25$. I suggest also that the non–power–law ULXs obeying $L_{\text{soft}} \propto T^2$ actually have black hole masses sufficiently high ($\gtrsim 10M_\odot$) that they are sub–Eddington.

2 THE $L \propto T^{-4}$ CORRELATION FOR BRIGHT ULXS

King & Puchnarewicz (2002) developed a general formalism for treating blackbody emission from the vicinity of a black hole. They allowed for geometrical collimation of this emission, but assumed that this did not change the blackbody luminosity $L$ or temperature $T$. This is true for example of radiation subject to scattering by nonrelativistic electrons. King & Puchnarewicz’ main result (their eqn (5)) is

$$L_{\text{ph}} = 2.3 \times 10^{44} T_{0.1\text{keV}}^{-4} \frac{I^2}{pbr^2} \text{ erg s}^{-1}.$$  

(2)

Here $L_{\text{ph}}$ is the blackbody luminosity an observer would infer from the observed flux by assuming that it is isotropic, even though in reality it is collimated by a factor $b$. The quantity $l$ is the ratio of the intrinsic luminosity to the Eddington limit $L_E$ (which can exceed unity by the factor $(1 + \ln \dot{m})$ mentioned above), $p$ is a factor allowing for deviations from spherical symmetry in the source (e.g. that it is actually plane and inclined to the line of sight) and $r = R/R_\bullet$ is the blackbody radius in units of the Schwarzschild radius.

The derivation of the relation (2) is simple. We express the intrinsic (pre–collimated) blackbody luminosity as

$$L \propto R^2 T^4 p \propto M_r^2 T^4 r_p^2 \propto L^2 T^4 \frac{r_p^2}{E},$$  

(3)

where one writes the radius as $R = rR_\bullet \propto rM$ at the first step, and the mass $M$ as $M \propto L_E \propto L t^{-1}$ at the second. Solving this equation for $L$ we find $L \propto T^{-4}$. An observer assuming that the flux is isotropic with the observed value, rather than collimated, now infers a total blackbody luminosity $L_{\text{ph}} = b^{-1} L$, i.e.

$$L_{\text{ph}} \propto T^{-4} \frac{l^2}{pbr^2},$$  

(4)

which gives (2) when the proportionality constants are included.

King & Puchnarewicz (2002) used (2) to argue that any source exceeding the normalization on the rhs must either be super–Eddington for its mass ($l > 1$), or emit from a region much smaller than the Schwarzschild radius ($r < 1$), or emit anisotropically ($pb < 1$). Ultrasoft quasars and some ULXs lie close to this regime on the $L - T$ plane. Here, setting $L_{\text{ph}} = L_{\text{soft}}$, we see that the $L_{\text{soft}} \propto T^{-4}$ correlation (1) for ULX soft excesses is reproduced provided that

$$\frac{l^2}{pbr^2} = 3 \times 10^{-4}.$$  

(5)

Observation thus strongly suggests that $b \propto r^{-2}$. If the power of $T$ in (1) were not precisely 4, e.g. the value $n = -3.1 \pm 0.5$ found by Feng & Kaaret (2007), this would introduce a $T$–dependence into the relation between $b$ and $r$, i.e. $b \propto T^{4-n} r^{-2} \sim T^{0.9} r^{-2}$. Since the fitted value of $T$ varies by a factor $\lesssim 3$, while (as we shall see) the inferred beaming factor $b$ varies much more, we would make only a small error in adopting the approximate dependence $b \propto r^{-2}$ here too.

This scaling of $b$ thus seems to be required by observation. Theoretically, a simple argument suggests that a $b \propto r^{-2}$ dependence follows from the picture of ULXs proposed by Begelman et al (2006) and Poutanen et al., (2007), in which a wind from the accretion disc surface keeps the local accretion rate at the radiation pressure limit at each disc radius, as originally suggested by Shakura & Sunyaev (1973). The outflowing wind is densest near $R_{\text{ph}}$, and has large optical depth both outwards along the disc plane, and in the vertical direction. Thus most of the disc radiation emitted within $R_{\text{ph}}$ must diffuse inwards by scattering, until it escapes through the central funnels parallel to the disc axis. The collimation results from the fact that the funnel is tall and thin, and has scattering walls.

To apply the formalism leading to (2) we identify the blackbody radius $R$ as $R \sim R_{\text{ph}} = rR_\bullet$, with $r = 27\dot{m}T/4$,
where \( \tilde{r} \sim 1 \). The blackbody luminosity emitted by the disc within \( R_{\text{sph}} \) is the intrinsic luminosity \( L \). This diffuses inwards and is collimated by the funnels.

For the beaming factor \( b \) we consider a simple cylindrical funnel around the central disc axis. If the typical cylindrical radius of the funnel is \( R_0 \), and its height is \( H_0 \gg R_0 \), the half-angle over which radiation escapes is \( \theta \sim \sin^{-1} R_0/H_0 \approx R_0/H_0 \). Then the beaming fraction \( b \) is simply the total fractional area of the two funnels, i.e. \( b \sim (1 - \cos \theta) \sim R_0^2/2H_0^2 \). Close to the disc plane, the structure of the central region of the disc wind (and thus the funnel radius \( R_0 \)) is independent of the value of Eddington ratio \( \dot{m} \gg 1 \) at large \( b \), so that the funnel height is only a few percent of the characteristic size of the central region of the disc wind (and thus the funnel height \( H_0 \)) is sensitive to \( \dot{m} \), or equivalently \( R_{\text{sph}} = \mu R_0 \), as at points far from the disc plane the wind flow pattern is set by \( \dot{m} \), which is equivalent to saying that the large-scale flow pattern is self-similar and scaled by \( r \). In particular this requires \( H_0 \sim \mu R_{\text{sph}} \propto \mu r \), where \( \mu < 1 \), so that finally

\[
b \simeq \frac{\chi^2}{2\tilde{r}^2} \simeq \frac{\lambda^2}{4\mu^2 \dot{m}^2 \mu R_0} \tag{6}
\]

where \( x \) stands for the dimensionless combination

\[
x = \frac{\mu}{\dot{m} R_0} \tag{7}
\]

We see the observational requirement \( \lambda/\mu \sim 58 \) implies \( \dot{m} \ll 1 \), so that the funnel height is only a few percent of \( R_{\text{sph}} \), i.e. \( \mu \ll 1 \). We get finally

\[
b \sim \frac{73}{\mu R_0} \tag{8}
\]

The reasoning of this paragraph assumes that \( \dot{m} \) is large enough that the two scales \( R_0 \) and \( R_{\text{sph}} \) are very different. The scaling of \( b \) with \( \dot{m} \) is clearly more complex for smaller \( \dot{m} \). In particular, unless \( H_0 \gg R_0 \), which requires \( \dot{m} \gg 1 \), one would formally get \( b \gg 1 \).

### 3 THE \( L \sim T^4 \) CORRELATIONS FOR NON–POWER–LAW ULXS

The last Section dealt with those ULXs (the majority) for which soft components are seen, and obey the \( L_{\text{sph}} \propto L_{\text{sph}} \) relation. We noted above that KP show that the remaining (non-power–law) ULXs follow opposed correlations \( L \sim T^4 \) for the medium–energy X–rays. Here the normalization differs for each individual system. The luminosities of the two groups differ sharply: the power–law–soft–excess systems have inferred luminosities permanently above \( 3 \times 10^{39} \text{erg s}^{-1} \) while the non–power–law systems are permanently below \( 10^{39} \text{erg s}^{-1} \). It seems likely that these fainter systems correspond to sub–Eddington accretion on to black holes with masses \( \geq 10M_\odot \). It is clear that such systems must exist, and that there is no reason to expect the collimation leading to the opposite \( L_{\text{sph}} \propto T^{-4} \) behaviour of the bright ULXs. The normalizations of the \( L \sim T^4 \) correlations are then fixed by the system inclinations and the inner disc radii. The latter are indeed of order a few Schwarzschild radii for black holes of \( \sim 10M_\odot \).

![Figure 1. Beaming factor \( b \) (dashed curve) and accretor mass \( M_1 \) (solid curve, in \( M_\odot \)) as functions of the Eddington ratio \( \dot{m} \) for ULXs. Here \( L_{\text{sph}} \) is the inferred isotropic bolometric luminosity in units of \( 10^{40} \text{erg s}^{-1} \) and \( x \sim 1 \) a dimensionless quantity given by equation (7)](image-url)

### 4 MASSES, BEAMING AND EDDINGTON RATIOS

We can now check whether the inferred behaviour of the beaming factor leads to sensible parameters for observed bright ULXs. Although the relation (8) for \( b \) was derived using the inferred blackbody disc emission, its geometrical nature and the fact that electron scattering is independent of photon energy makes it probable that it holds for all forms of ULX luminosity, and indeed even in ULXs where no blackbody disc component has been identified, provided only that these correspond to super–Eddington accretion. In particular we can use the \( b \propto \dot{m}^{-2} \) scaling in considering medium–energy X-rays, which are generally assumed to carry most of the bolometric luminosity of a ULX.

The effect of beaming is to cause an observer to infer a spherical luminosity

\[
L_{\text{sph}} \approx \frac{L_{\text{E}}}{b}(1 + \ln \dot{m}) \tag{9}
\]

(cf Shakura & Sunyaev, 1973; Begelman et al., 2006). Eliminating \( b \) using (5) (or (5) gives

\[
L_{\text{sph}} = 2.2 \times 10^{36} \dot{m}_1 \dot{m}^2 (1 + \ln \dot{m}) \text{erg s}^{-1}. \tag{10}
\]

We can re-express this as

\[
\frac{m_4}{L_{40}} = \frac{4500}{\dot{m}^2 (1 + \ln \dot{m})} \tag{11}
\]

where \( L_{40} = L_{\text{sph}}/10^{40} \text{erg s}^{-1} \).

Figure 1 shows \( M_1/L_{40} \) and \( b \) as functions of \( \dot{m} \) (using equations (11) and (5)). We see that Eddington ratios in the range \( 8.5 < \dot{m} < 20 \) imply stellar masses \( 1M_\odot \lesssim m_1 \lesssim 20 \) for the accretors if the disc luminosity is \( \lesssim 10^{41} \text{erg s}^{-1} \), and
beaming factors in the range $1 > b > 0.2$. Hence stellar–mass binaries with moderate Eddington ratios and consequently modest beaming provide very good candidates for explaining ULXs.

We note from (10) that the inferred luminosity $L_{sph}$ varies essentially only because of the sensitivity of the beaming factor $b$ to $\dot{m}$. Thus the bolometric luminosity varies only logarithmically above $L_E$ (assuming that the Eddington ratio always exceeds unity), but is spread over a smaller or greater solid angle as $\dot{m}$ increases or decreases, significantly altering the inferred luminosity.

Since $\dot{M} = \dot{m} M_E \propto (M_1/\eta) \dot{m}$, where $\eta$, the radiative efficiency, is similar $(\sim 0.1)$ for black holes and neutron stars, I note that that some ULXs could contain neutron stars, and could even have lower absolute accretion rates for the same inferred luminosity. From Fig. 1 we see that a 10$M_\odot$ black hole with $\dot{m} = 15$ and a neutron star with mass $\lesssim 2M_\odot$ and $\dot{m} = 30$ produce similar inferred luminosities, with the neutron–star system having an absolute accretion rate $\dot{M}$ lower by a factor $\sim 2.5$ than the black hole. The origin of this apparently paradox is that the latter system has a smaller beaming factor. (Put another way, on Fig. 1 the curves of constant $\dot{M}$ are hyperbolic which cross the hyperbola describing $b$.) For ultrasoft ULXs with no detectable medium–energy X–ray component, even white dwarf accretors are possible, particularly since for them $\eta$ can be enhanced over the pure accretion yield by nuclear burning of the accreting matter (cf Fabbiano et al., 2003).

5 ULX POPULATIONS

Population studies of ULXs have until now faced the difficulty that the beaming factor $b$ was not determined, introducing a spurious degree of freedom. Given the connection (3), we can now remove this. We consider a population of ULXs with host galaxy space density $n_g$ Mpc$^{-3}$ and assume that each host contains $N$ ULXs, with radiation beams oriented randomly. To be in the beam of one such object one has to search through $\sim 1/Nb$ galaxies, i.e. a space volume $\sim 1/n_g Nb\sph$ The nearest observed ULX is thus at a distance

$$D_{\text{min}} \sim \left( \frac{3}{4\pi n_g Nb} \right)^{1/3} \sim 0.7(n_gN)^{-1/3} \dot{m}_1^{2/3} \text{ Mpc}, \quad (12)$$

where $\dot{m}_1 = \dot{m}/10$. The apparent luminosity of the ULX is

$$L_{sph} = 2.2 \times 10^{39} m_4 \dot{m}_1^2 \text{ erg s}^{-1} \quad (13)$$

where $m_4 = M_1/10M_\odot$, giving a maximum apparent bolometric flux

$$F_{\text{max}} = \frac{L_{sph}}{4\pi D^2} = 4.0 \times 10^{-11} m_4 \dot{m}_1^{2/3}(n_gN)^{2/3} \text{ erg s}^{-1} \text{ cm}^{-2}. \quad (14)$$

These relations, together with the results of the previous Section, agree with the fact that ULXs of apparent luminosity few $\times 10^{39} - 10^{41} \text{ erg s}^{-1}$ are observed in the Local Group, and suggest that the typical intrinsic number $N$ per host galaxy is at most a few. This is in line with estimates of the numbers of high–mass X-ray binaries in phases of rapid mass transfer on thermal or nuclear timescales (King et al., 2001; Rappaport et al., 2005), suggesting that these systems offer good candidates for explaining most if not all ULXs. Ultimately one needs a population synthesis calculation to verify that this picture produces the right numbers of systems with the required moderate Eddington ratios to produce the nearby ULXs.

6 PSEUDOBLAZARS?

It is unclear to what value of $\dot{m}$ one may safely extrapolate the $b \propto \dot{m}^{-2}$ dependence inferred here. This is an interesting question, as we know (cf Begelman et al, 2006; King & Begelman, 1999) that the well–studied object SS433 has $\dot{m} \sim 3000 - 10^4$. Such values are typical for both thermal–timescale and nuclear–timescale mass transfer from massive donor stars (Rappaport et al., 2005).

From the work of the previous Section, now scaling $\dot{m}$ as $\dot{m} = 10^4 \dot{m}_4$, the nearest such object would be at a distance

$$D_{\text{min}} \sim \left( \frac{3}{4\pi n_g Nb} \right)^{1/3} \sim 660N^{-1/3} \dot{m}_4^{2/3} \text{ Mpc}, \quad (15)$$

where I have taken $n_g \sim 0.02 \text{ Mpc}^{-3}$ as appropriate for $L^*$ galaxies. The apparent isotropic luminosity of such an object would be

$$L_{sph} = 2.2 \times 10^{45} m_4 \dot{m}_4^2 \text{ erg s}^{-1}. \quad (16)$$

Hence in distance and apparent luminosity the object would appear as an AGN. However, unlike a genuine AGN, there is no requirement that it should lie precisely in the nucleus of the host galaxy.

A possible candidate for such an object is the BL Lac system PKS 1413+135 (Perlman et al., 2002). With redshift $z = 0.24671$ it has distance $D \sim 1000 \text{ Mpc}$ and isotropic luminosity $\sim 10^{44} \text{ erg s}^{-1}$, but lies at $13 \pm 4 \text{ mas}$ from the centre of the host galaxy.

7 DISCUSSION

The work of this paper suggests that the beaming factor in super–Eddington accretion varies as $b \propto \dot{m}^{-2}$. This seems to be required by observations of the $L_{\text{soft}} - T$ correlation, and is reasonable on general geometrical grounds. The existence of this scaling means that observable properties of ULXs depend essentially only on the Eddington ratio $\dot{m}$. If this conclusion is valid, this removes the spurious degree of freedom allowing one to choose $b$ independently of $\dot{m}$ which has made systematic parameter estimates difficult in the past (e.g. King, 2008, where these two quantities are not connected).

It appears that most ULXs correspond to stellar mass systems accreting at Eddington ratios of order $10 - 30$, with corresponding beaming factors $b \gtrsim 1$. High–mass X–ray binaries containing black holes or neutron stars are excellent candidates, although population synthesis studies need to check this. The scaling inferred here suggests that ULXs vary mainly because the beaming factor is sensitive to the Eddington ratio. If the scaling can be extrapolated to the Eddington ratios found in SS433, some objects currently identified as AGN at modest redshifts might actually be ULXs. This may explain cases where the AGN does not coincide with the centre of the host galaxy.
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