Configuration Design and Motion Performance Analysis of a New Wheeled Rolling Robot

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Title page

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Abstract: A new wheeled rolling robot is designed based on planar 3-RRR parallel mechanism and spoke wheel with variable diameter, by adjusting the 3-DOF outputs of the planar 3-RRR parallel mechanism, the deformation and rolling motion of the rolling robot are realized, the rotation output of the parallel mechanism realizes the differential change of the diameter of the two supporting wheels of the rolling robot, and the moving output of the parallel mechanism changes the mass distribution of the system, so that the rolling robot can complete the forward, backward, turning and other motions. Based on the introduction of the performance parameters, driving system, variable diameter wheel configuration and motion mechanism of the rolling robot, the eccentric driving torque is analysis and the existing space of the eccentric torque is given, so as to further complete the selection of the driving motor of the robot. This article analysis three typical motions of robots and its performance parameters, such as straight movement, turn movement, climbing exercise, and builds a simple prototype under laboratory conditions to verify the feasibility of the three movements.

Keywords: Rolling robot; wheeled robot; cardiological adjustment device, mechanism; variable diameter; eccentric torque drive

1 Introduction

With the rise of the robot industry, various robot structures with different functions are emerging [1]. Rolling robot exists as a special mobile robot [2-5], through the continuous adjustment of the internal center of mass of the structure, the moment to the support point is changed, so as to realize various movements [6-8]. Because of its high flexibility, strong stability, and low energy consumption [9,10], it has developed rapidly in the field of investigation and exploration with rough roads and rugged environment, and has attracted the attention of major universities, scientific research institutions and high-tech enterprises [11-13].

Rolling robots are mostly spherical structures, HALME et al [14] developed Rollo for driving by using the single wheeled trolley with wall climbing motion in the spherical shell. Due to the unstable driving structure of the single wheeled trolley, the motion position accuracy of the robot is difficult to control [15]. Similarly, BERNSTEIN et al [16] also designed SPKR+ using trolley drive, in order to improve the stability, the robot uses a two-wheel trolley to drive, but the structural design complexity and control difficulty are increased. KARAVAEV et al [17] used Mecanum wheel as the support of the car inside the spherical shell and established a kinematic model to control the robot movement, but the offset will increase with the increase of speed, which greatly limits the application. CHEN et al [18] drive the roller in the spherical shell to rotate through two orthogonal arranged motors, so as to drive the outer spherical shell to roll. At the same time, they also confirmed the omni-directional motion performance of the robot. However, the above rolling robots suffer from low stability, unable to avoid obstacles independently, sideslip prevention, poor grip performance and so on.

In order to improve the stability of rolling robots, a varies of innovative rolling robots have been developed. Beijing Jiaotong University designed a multi-mode two wheeled mobile robot with a planar 6R single loop closed-chain linkage as the body [19]. By controlling the body deformation, the robot can carry out a variety of deformation according to its own environment and different set tasks, and switch to the corresponding walking system [20]. Harbin University of technology uses the continuous electrowetting effect [21] as the micro driving mechanism to drive the robot movement, effectively reducing its volume and...
expanding its application range. Megascout robot [22] designed by KRATOCHVI and his team can sense the difference of real-time road conditions and switch different action wheels for the adaption of road environment. Spherobot [23] developed by Ranjan Mukherhee's team adjusts the position of the center of mass through the movement of the counterweight along the metal spokes to make the robot move under the driving of heavy torque. In order to further improve the flexibility of heavy torque driving, Vrunda Joshi [24] and other scholars improved the spoke counterweight driving structure, but due to the weight limitation of counterweight, the speed of robot moving in this driving mode is relatively slow.

Considering the configuration and driving mode of various robots, the stability advantages of parallel mechanism [25] and the speed advantages of wheeled walking, a new type of wheeled rolling robot based on parallel mechanism is proposed, which has the advantages of small structure, light weight and flexible and rapid movement. The robot can realize accurate switching of wheel diameter, rapid adjustment of running path, high mobility and traffickability. Its research and development will provide new technologies and new choices for the innovative design of new robots in various fields such as investigation and exploration, military demining, cargo transportation, and mass entertainment.

2 Robot configuration design

2.1 conceptual design

The original intention of the scalable wheel-diameter rolling robot is to complete the reconnaissance, search and rescue tasks in a variety of terrains and road conditions. The conceptual model of the robot design is shown in Figure 1. It can be broadly divided into three parts: the frame part, the drive unit of the center of gravity adjustment device based on the parallel mechanism, and the spoke-type variable-diameter wheel.

The robot moves under the torsional torque generated by the center of gravity adjustment device carried by the 3-RRR biplane parallel mechanism [26], the power take-off shafts, telescopic universal joint couplings and intermediate rotating shafts at both ends are output to the coupling driving wheel, which causes the change of spoke angle, and then affects the concentric scaling of the outer ring circular arc, so as to change the wheel diameter of the robot, and realize two kinds of movements: straight walking on the same wheel and turning with different wheel diameter.

\[ \text{Figure 1} \quad \text{Robot conceptual model} \]

The working road conditions encountered by the robot in the application process are different. Therefore, the complexity of its own structure, the adaptability to the road surface, the flexibility of movement and the accuracy of positioning are the four important indexes to test whether it is competent for the reconnaissance work of ground mobile robot. Through a number of physical experiments on the robot platform, comprehensively test the mobile performance, mobility performance, climbing performance and balance performance of the robot, to evaluate the working environment in which the robot can walk smoothly, and analysis the effectiveness of the dynamic model and control strategy, so as to continue to improve the design of important parts of the robot and improve the performance of the robot.

2.2 Overall index parameter design

Combined with the overall design principles and functional requirements of wheeled rolling robot, referring to the structural parameters and design task requirements of various existing rolling robots, the index parameters of the robot prototype are preliminarily drawn up, as shown in Table 1 below.

\[ \text{Tab. 1} \quad \text{preliminary parameters of robot} \]

| Name                  | Symbol | Proposed parameters |
|-----------------------|--------|---------------------|
| Total weight of       | \( M_t \) | \( \leq 4Kg \)     |
2.3 Design of spoke variable diameter outer wheel

The design configuration of spoke scalable variable diameter outer wheel is shown in Figure 2, the round arc pieces of the same specification are staggered and stacked in front and back. One end of the collet is connected through the chute opened at both ends of the circular arc piece, and the other end is connected with spokes. The bottom end of the spokes is fixedly connected with the ball joint bearings evenly distributed along the coupling driving wheel, so as to realize the connection between the scalable changeable diameter outer wheel and the coupling driving wheel, which is the key to the realization of variable diameter.

The spokes pass through the perforated T-shaped column to connect the variable diameter outer wheel with the frame wheel. The coupling wheel rotates to drive the change of spoke angle. Under the constraint of T-shaped column, the spoke expands and contracts in the channel direction, and affects the external ring circular arc to make scaling movement, to enlarge and reduce the diameter of the contact wheel. As a component in direct contact with the ground, the circular arc pieces are always compactly arranged during the change of the diameter of the contact wheel, so that the robot can maintain a stable state during the movement, and the body will not tremble or lose stability due to the cooperation of the gear train components. Three sets of support springs are arranged between the linkage wheel and the fixed wheel in the frame to solve the balance problem of the robot’s zoomable outer wheel in the non-rolling state, and to prevent the spokes from shrinking under the action of gravity.

3 Construction of robot driving system

3.1 Design of workspace driven by eccentric torque

The driving principle of the cardiological adjustment device can be briefly summarized as destroying the internal balance state of the robot system through the cardiac of gravity offsets, so that the system moves to restore or find a new balance state. The structural schematic diagram of the cardiological adjustment device is shown in Figure 3, the workspace of the 3-RRR biplane parallel mechanism contains the movable range of the cardiac of gravity offset of the robot, and the analysis of the reachable motion space of the parallel mechanism is the basis of studying the existence space of eccentric torque of the robot.
When the diameters of the left and right wheels are consistent, the robot moves in a straight line driven by the center of gravity adjustment device. By adjusting the offset position of the center of gravity of the system, the robot can roll forward or backward. The workspace of 3-RRR parallel mechanism is analyzed and combined with its force, the effective workspace of the center of gravity adjustment device of rolling robot is obtained, which is defined as eccentric torque workspace.

In order to find out the optimal $L_1: L_2$ rod length ratio parameters and obtain the optimum eccentric torque driving space. According to the simulation operation test of the mechanism, take four groups of data with $L_1$ length of 50mm, 60mm, 70mm and 80mm respectively, the corresponding dimensions of $L_2$ are 110mm, 120mm, 90mm and 80mm, four groups of rod length ratio parameters are calculated (5:11), (3:5), (7:9), (1:1). By comparing the degree of spatial defects, the variation law of spatial defects of the driving force with the ratio coefficient of rod length is explored. By observing the movement space point set of the center of gravity adjustment device, it is found that within $\varphi \in [-30^\circ, 30^\circ]$, there are some irregular incomplete positions in the workspace, intercept the workspace plan of the working platform rotation angle $\varphi$ under the five positions of $\varphi=30^\circ$, $15^\circ$, $0^\circ$, $-15^\circ$, $-30^\circ$, as shown in Figure 4 below.

Based on the above analysis, the rod group coefficient size of center of gravity adjustment device is determined: $L_1=80\text{mm}$, $L_2=80\text{mm}$. Through the omni-directional space three-dimensional search method, in MATLAB, it is obtained that the eccentric torque exists in the workspace, as shown in Figure 5 below.

![Fig. 3 Structure schematic diagram of cardiological adjustment device](image)
According to the results, the eccentric moment workspace obtained by the optimization calculation of the rod length parameters is regular and continuous. This shows that cardiological adjustment device of the rolling robot has decent motion fluency and good eccentric torque transmission performance under this set of rod length design parameters, which shows that the robot can obtain uninterrupted driving torque in the rolling process.

3.2 Analysis of gravity center adjustment device platform

The working platform of the center of gravity adjustment device is used to carry equipment components such as battery pack, controller, driving circuit and series of sensors. The mass and volume of the battery pack are the main considerations in the design of the robot working platform. It can be seen from the eccentric torque formula generated by the cardiological adjustment device when the robot walks in a straight line that the structural size parameters of the robot are certain, the eccentric torque is positively correlated with the sinusoidal function of the swing angle of the parallel mechanism rod group. So, there is:

$$\beta \geq \arcsin \frac{\mu M_t}{m_t (R_L^0 - L_d)}$$

Let the weight ratio $K_m = \frac{m_s}{m_t}$ and bring in $(R_L^0 - L_d) = 90$mm. then the above formula can be expressed as:

$$\beta \geq \frac{180}{\pi} \arcsin \frac{\mu}{90K_m}$$

Under different rolling friction coefficients, the relationship between mass ratio and swing angle is shown in Figure 6 below.

It can be seen that the larger $K_m$, the smaller the swing angle $\beta$ of the rod group required for the movement of the wheeled rolling robot, and the better the maneuverability of the robot. It can be seen from the figure that improving the equivalent mass of the center of gravity adjustment device can make the system drive the robot when the swing angle is small.

3.3 Analysis of robot driving torque

The rolling of the robot is generated by the eccentric moment generated by the movement of the center of gravity adjustment device, which changes the position of the overall center of gravity of the system and overcomes the rolling friction couple moment \([28]\) between the variable diameter contact outer wheel and the ground. The power of the variable diameter wheeled robot is supplied by three double servo actuators evenly distributed in the middle of the frame, $T^*$ represents the driving torque.
the small driving force $F^*$ of the robot causes the robot to roll, the action wheel of the robot contacts and squeezes the ground to produce contact deformation, so that the constraint reaction force on the contact wheel is unevenly distributed.

As the driving force $F^*$ increases, the rolling friction couple moment $\mu_1F_N$ balances the rolling couple generated by the main driving force $F^*$ and the ground distributed force system $F$. The maximum rolling friction couple moment is expressed in $M_{max}^f$. When $\mu_1$ reaches the maximum value $\mu$, the system rolling friction couple moment reaches the maximum value, which increases the driving force of the system, and the robot will roll, and have:

$$M_{max}^f = \mu F_N$$

Take $\mu = 0.2$, the plane motion mechanical model of wheeled rolling robot is shown in Figure 8 below.

![Fig. 8](image_url)

**Fig. 8** motion force diagram of robot

According to the motion force analysis and the motor parameters of the driving system, the maximum rolling friction couple moment of the robot:

$$M_{max}^f = \mu F_N = \mu M_{t}g = 0.069N \cdot m$$

In addition, when the center of gravity adjustment device is located at the lowest position, the distance from the center to the ground is $L_d$, and there is the driving torque to drive the robot:

$$T^* \geq m_1g(R_1^2 - L_d)\sin\beta = M_{max}^f$$

The total driving torque of three annular uniformly distributed servo actuators is:

$$T \geq 0.023N \cdot m$$

The output power of driving steering gear is:

$$P = \frac{T_n}{9549} \approx 3.2w$$

In view of the power loss of the actuator in the actual situation of the robot, the DC servo actuator with an output power of 10W is selected.

### 4 Analysis on scaling relationship of robot wheel diameter

According to the connection principle of variable diameter gear train components, that is, the variable diameter gear train is connected with the gravity center adjustment driving unit based on the parallel mechanism through the symmetrical retractable universal joints arranged at both ends, and the coupling driving wheel and the frame wheel are connected through the spoke group to analyze the wheel diameter switching of the robot.[29,30]

Taking the left turning movement as an example, the diagram of wheel diameter changing process is shown in Figure 9 below. The middle yellow block represents the moving platform of the center of gravity adjustment device, which is connected with the coupling driving wheel in the variable diameter gear train through the middle blue rotating shaft, and the black vertical line represents the outer scaling wheel. When the robot is stationary with $R_1$ as the radius of the rolling wheel, the center of gravity adjustment device rotates an angle $\theta$, which is simultaneously transmitted to the coupling driving wheels on both sides through the universal shaft to drive the spokes to change. Due to the reverse arrangement of the spoke group of the variable diameter gear train on both sides, the spokes on one side are contracted and the outer wheel diameter is reduced to $R_2$ driven by the coupling driving wheel; The spokes on the other side open and the outer wheel diameter expands to $R_3$.

![Fig. 9](image_url)

**Fig. 9** schematic diagram of robot variable wheel diameter

The wheel diameter adjustment process of rolling robot is shown in Fig 10 and Fig 11. The Yellow wheel disc represents the moving wheel disc, and the primary color wheel disc is the fixed wheel (frame wheel). It is defined...
that the center of the moving wheel is \( O_d \), the fixed node between the bottom end of the spoke and the moving wheel is \( T_d \), the implicated point between the middle part and the \( T \)-pillar of the frame wheel is \( T_s \), and the connecting point between the top end and the circular arc piece of the moving wheel is \( T_p \). At the same time, the radius of coupling driving wheel is defined as \( R_d \), the radius of frame wheel is defined as \( R_s \), and the radius of variable wheel diameter outer wheel is defined as \( R_L \), \( R_R \). \( |T_dT_s| = m_1 \), total length of spokes: \( |T_dT_p| = m \).

**Fig 10** left variable wheel diameters pokes shrink state

When the robot rolls in a straight line \( \angle T_dO_dT_s = \alpha \), then in the triangle \( \Delta T_dO_dT_s \) it is obtained from the cosine theorem:

\[
m_1^2 = R_d^2 + R_s^2 - 2R_dR_s\cos \alpha
\]

Let \( \angle O_dT_dT_s = \beta \), then:

\[
\cos \beta = \frac{R_d - R_s\cos \alpha}{\sqrt{R_d^2 + R_s^2 - 2R_dR_s\cos \alpha}}
\]

In \( \Delta T_dO_dT_p \):

\[
\cos \beta = \frac{R_d^2 + m^2 - R_L^2}{2mR_d}
\]

Gain:

\[
\frac{R_d^2 + m^2 - R_L^2}{2mR_d} = \frac{R_d - R_s\cos \alpha}{\sqrt{R_d^2 + R_s^2}}
\]

Sorted out:

\[
R_L^2 = R_d^2 + m^2 - \frac{2mR_d^2 - 2mR_dR_s\cos \alpha}{\sqrt{R_d^2 + R_s^2 - 2R_dR_s\cos \alpha}}
\]

It can be seen that the outer wheel radius \( R_L \) of is only related to the input angle \( \alpha \) of the moving wheel. Because the parameters of gear trains on both sides are the same when the robot rolls in a straight line, in the right wheel variable diameter gear train, we have:

\[
R_L^2 = R_d^2 + m^2 - \frac{2mR_d^2 - 2mR_dR_s\cos \alpha}{\sqrt{R_d^2 + R_s^2 - 2R_dR_s\cos \alpha}}
\]

After the rotation of the robot center of gravity adjustment device is completed, the right figure in Figure 9 shows the state of the system angle of the robot left reducer gear train under the action of the rotation angle \( \theta \) of the moving wheel. The spoke rod group shrinks counterclockwise and is arranged compactly. Similarly:

\[
R_L^2 = R_d^2 + m^2 - \frac{2mR_d^2 - 2mR_dR_s\cos (\alpha + \theta)}{\sqrt{R_d^2 + R_s^2 - 2R_dR_s\cos (\alpha + \theta)}}
\]

In the formula, all are fixed values except angle.

Let \( R_L^2 + m^2 = A \), \( 2mR_d^2 = B \), \( 2mR_dR_s = C \), \( R_s^2 + R_d^2 = D \), \( 2R_dR_s = E \).

Among them, \( A, B, C, D, E \) are all positive, the above formula can be simplified as:

\[
R_L^2 = A - \frac{B - C\cos (\alpha + \theta)}{\sqrt{D - E\cos (\alpha + \theta)}}
\]

As shown in Figure 10, When the right variable diameter gear train of the rolling robot rotates along the angle \( \theta \), the spoke group swings reversely to make it radial, and \( \angle O_dT_dT_s \) increases. Thus:

\[
R_L^2 = R_d^2 + m^2 - \frac{2mR_d^2 - 2mR_dR_s\cos (\alpha - \theta)}{\sqrt{R_d^2 + R_s^2 - 2R_dR_s\cos (\alpha - \theta)}}
\]

Bring \( A, B, C, D, E \) in:

\[
R_L^2 = A - \frac{B - C\cos (\alpha - \theta)}{\sqrt{D - E\cos (\alpha - \theta)}}
\]
5 Three basic motions of robot

5.1 Linear motion analysis

The linear walking ability of wheeled rolling robot is determined by the eccentric torque provided by its center of gravity adjustment device and the scaling size of variable diameter wheel. Considering that the motion of the robot can be divided into two types from the relative dimensions of the left and right wheels, one is the straight motion with equal wheel diameter, and the other is the turning motion with different wheel diameter, the parameter design of the straight-line walking wheel diameter should not only maximize the output torque transmission efficiency during straight-line walking, but also provide the most reasonable size and position conditions for the variable diameter motion of the side wheel diameter turning motion.

We define the ratio of the larger wheel diameter to the smaller wheel diameter of the robot scalable wheel in the turning state as the scaling ratio. The scaling ratio represents the wheel diameter changing ability and the angle adaptation ability of the wheeled robot. As shown in Figure 12, the zoom angle $\rho$ of the variable diameter wheel, that is, the included angle between the holding spoke and the tangent of the coupling driving wheel, represents the deformation ability of the robot.

![Fig. 12 Schematic diagram of variable diameter wheel switching wheel diameter](image)

The wheel diameter of the two mobile wheels of the robot is adjustable. The general walking radius is defined, that is, if the wheel diameter of linear walking is $R^2$, the size range is: $R_{\text{min}} < R^2 < R_{\text{max}}$.

The radius dimension of the coupling driving wheel is $n$, the length of the holding spoke rod is $m$, and the tangent included angle at the connection between the spoke and the driving wheel is $\rho$. According to the cosine theorem, the radius dimension of variable diameter wheel is:

$$R^2 = (n^2 + m^2 - 2mn \sin \rho)$$

Considering that the variable diameter wheeled rolling robot will not produce side slip during linear walking, but a pure rolling motion, the linear walking motion equation of the wheeled rolling robot is listed based on the nonholonomic system theory:

$$
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & -a \sin \phi \\
\sin \phi & a \cos \phi \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
v \\
a
\end{bmatrix}
$$

Let $K = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$, then we have:

$$
K = \begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & -a \sin \phi \\
\sin \phi & a \cos \phi
\end{bmatrix}
\begin{bmatrix}
v \\
a
\end{bmatrix} = A \begin{bmatrix}
v \\
a
\end{bmatrix}
$$

Where $A = \begin{bmatrix}
\cos \phi & -a \sin \phi \\
\sin \phi & a \cos \phi
\end{bmatrix}$.

and $A^{-1} = \begin{bmatrix}
\frac{\cos \phi}{a} & \sin \phi \\
\frac{-1}{a} \sin \phi & \frac{1}{a} \cos \phi
\end{bmatrix}$, the inverse kinematics matrix equation of the wheeled rolling robot can be obtained as follows:

$$
\begin{bmatrix}
v \\
a
\end{bmatrix} = A^{-1} \begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix}
$$

When the torque of the servo actuator of the rolling robot is set to be constant in ADAMS simulation, the torque of the center of gravity adjustment device input by the robot is constant, and the linear motion speed curve and displacement trajectory of the wheeled rolling robot are obtained as shown in Figure 13 and Figure 14 below.

![Fig. 13 speed curve of robot linear rolling](image)
Due to the constant torque of the servo actuator and the continuous increase of the eccentric torque input by the robot center of gravity adjustment device, the angular acceleration of the rolling robot will continue to increase, and the body motion speed, rolling wheel linear speed and robot displacement will continue to increase.

5.2 Turning motion analysis

The robot walks in a straight line with the general walking radius $R_L$, when it travels to the curve area (as shown in Figure 15), the left wheel of the robot immediately shrinks and the right wheel expands to complete the left turn. The roll angle of the robot turning state is defined as $\alpha$. The larger the roll angle is, the smaller the turning radius $R_H$ of the robot is; Smaller turning radius enable the robot greater motion flexibility, which is also an important index for the motion performance of the robot.

The robot roll angle according to the geometric relationship is:

$$\alpha = \arccos \frac{R_r - R_t}{L}$$

The turning radius is:

$$R_H = \frac{L}{2} \cdot \frac{R_t + R_r}{(R_r - R_t)} = \frac{R_r + R_t}{2 \tan \alpha}$$

Considering the limit rotation size of the robot, we get:

$$\alpha_{max} = 6.2^\circ, \quad R_{H \min} = 1.76m$$

If the overall turning linear speed of the robot is $v$ and the angular speed is $\omega$, the linear speeds of the left wheel and the right wheel are $v_l$ and $v_r$ respectively. According to the coaxial movement relationship of the left and right reverse installed power take-off shafts, the speeds of the two wheels are:

$$v_l = (1 - \lambda) \omega R_L^p; \quad v_r = (1 + \lambda) \omega R_L^p$$

5.3 Uphill motion analysis

In the process of climbing, due to the influence of gravity and slope friction, the wheeled rolling robot is easy to be in an unstable state, which has a great impact on the robot to maintain the balance of uphill motion. Two preconditions are defined before analyzing the slope motion of the robot: 1) In the climbing process, the extrusion deformation between the outer circle circular arc and the slope is ignored, and the variable diameter wheel is a rigid body; 2) In the process of climbing, the action wheel rolls purely without slipping.

The overall weight of the system is divided into two parts. The mass of the center of gravity adjustment device is specified as $m_1$ and the mass of other parts is specified as $m_2$.

The orthogonal coordinate system consolidated on the slope, the starting position of the robot is set at point $O$, the displacement is on the slope with inclination $\gamma$ is $x$, $\theta$ is the angle of the gravity center adjustment device relative to the vertical direction, and $\phi$ is the driving torque of the robot, as shown in Figure 16 above.

On the premise that the wheeled rolling robot can climb the slope, the eccentric driving torque $\tau$ generated by the robot center adjustment device moves clockwise, and the friction $F_f$ between the slope and the robot contact wheel
is greater than the slope component of the robot system, which is defined as $F_1$.

Therefore, the force conditions for the rolling robot to move uphill is:

$$\gamma \leq \arcsin\left(\frac{m}{m + M R}\right)$$

$$\gamma \leq \arctan\mu$$

The constraint equation of robot slope motion is analyzed according to the generalized coordinate method. Assuming that the left and right wheels of the wheeled rolling robot do not slide or expand radially during the uphill process, the constraint equation of the robot on the slope is:

$$\dot{x} = R\dot{\theta}$$

According to the integration of the above formula, the shape and position equation of the whole system can be expressed by the angle $\theta$ of variable diameter rotation and the angle $\varphi$ of the offset of the center of gravity adjustment device relative to the vertical direction. The generalized coordinates of the system are selected as:

$$q_1 = \theta, q_2 = \varphi$$

The Lagrange function [29] of the robot system is:

$$L = T - V$$

The Lagrange equation of the system can be expressed as:

$$\begin{cases}
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau \\
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = \gamma
\end{cases}$$

It can be seen from the above formula that there are two control variables $\theta$ and $\varphi$ of the robot system, while there is only one input variable of the system. This input coupled underactuated system can establish its nonholonomic constraint equation through the dynamic model of the robot system.

Besides, $\theta = \frac{x}{R}, \dot{\theta} = \frac{\dot{x}}{R}$, the constraint relationship of slope motion of wheeled rolling robot is:

$$\lambda_1 \dot{x} + \lambda_2 \dot{\varphi} + \lambda_3 \varphi^2 + \lambda_4 = 0$$

Where:

$$\lambda_1 = (m_1 + 4m_2)R - m_1 l\cos(\varphi - \gamma)$$

$$\lambda_2 = m_1 (RL - R^2 - l^2)$$

$$\lambda_3 = -m_1 R\sin(\varphi - \gamma)$$

$$\lambda_4 = (m_1 + 2m_2)gR\sin\gamma - m_1 gl\sin \varphi$$

The equilibrium state in the process of slope motion can be divided into slope static state and slope uniform motion. Starting from the slope static state, the slope motion performance of the robot is studied, and the critical state of the robot on the slope is analyzed, so as to solve the critical value of the robot climbing angle $\gamma$.

When the robot is stationary on the slope, there are:

$$\begin{cases}
\dot{x} = 0, \ddot{x} = 0, \varphi = 0, \varphi = 0
\end{cases}$$

Brought into the Lagrange equation of the system, we have:

$$\begin{cases}
(m_1 + 2m_2)gR\sin \gamma = \tau_0 \\
m_1 gl\sin \varphi_0 = \tau_0
\end{cases}$$

Where, $\varphi_0$ is the critical deflection angle of the center of gravity adjustment device relative to the vertical plane, and $\tau_0$ is the critical output torque.

Assuming that the power input of the robot is continuous and can provide strong output torque, the following requirements shall be met:

$$\tau \geq \tau_{0\text{max}} = m_1 gl$$

The climbing ability of rolling robot is limited. Whether the slope movement can be completed is directly related to the slope angle. It is assumed that the maximum slope angle that the robot can pass through, that is, the limit slope angle of the robot climbing, is $\gamma_{\text{max}}$.

From $$(m_1 + 2m_2)gR\sin \gamma = m_1 gl\sin \varphi_0$$

$$\varphi_0 = \arcsin \left(\frac{(m_1 + 2m_2)gR}{m_1 l}\right)$$

According to the properties of arcsine function, the definition domain is [-1, 1], therefore, the slope inclination angle $\gamma$ must be bounded, thus:

$$M_t = m_1 + 2m_2$$

$$\gamma \leq \arcsin \left(\frac{m_1 l}{M_t} R\right)$$

It can be seen that when the rolling friction coefficient and the length of parallel mechanism are certain, the greater the mass ratio, the better the climbing
performance. The second equilibrium state of the robot's slope motion, that is, the uniform linear motion of the slope, also meets the above relationship. Therefore, the maximum slope inclination of the wheeled robot when moving on the slope is:

$$\gamma_{\text{max}} = \arcsin \left( \frac{m_1}{M_t} \frac{l}{R} \right)$$

When the slope angle exceeds the maximum slope angle that the robot can move, the robot will be unstable. Therefore, in the actual operation process, reasonable path planning is particularly important for the robot movement. Experiments show that when the center of gravity adjustment device moves to the horizontal position, that is, the position parallel to the slope, the wheeled robot reaches the theoretical maximum climbing angle. Taking the structural parameters of the robot into account, the maximum climbing angle is $\gamma=10.2^\circ$. The relationship between the uphill ability of the robot during operation and the mass ratio and size ratio is shown in Figure 17.

![Fig. 17 variation of uphill capacity](image)

6 Basic walking performance test of prototype

According to the theoretical analysis of robot function positioning and schematic design, a conceptual prototype is processed and assembled, as shown in Figure 18. The splicing arc piece, collet, spoke, wheel frame and other parts are self-designed aluminum alloy processing products.

Three steering gears are evenly distributed in the middle of the frame. The motion program is input in the steering gear control board to give instructions to the robot for desired practical tasks. The preliminary experiments of equal wheel diameter straight-line walking and different wheel diameter around turning are carried out in the laboratory environment.

![Fig. 18 physical picture of robot conceptual prototype](image)

As shown in Fig. 19, the robot is placed in a simulated asphalt pavement environment with a friction coefficient of about 0.6, the robot rolls forward driven by the eccentric torque generated by the center of gravity adjustment device, and the motion speed is adjusted by controlling the offset of the center of gravity adjustment device of the robot based on the parallel mechanism relative to the axis center position of the system.

The experimental results show that the fastest running speed of the robot is 0.8m/s. If the robot walks on other roads with low friction resistance, such as ceramic tile ground or cement road, its walking speed will be greater than the experimental results, so the walking speed parameters of the robot basically meet the design requirements of 1m / s.
Test the eccentric torque driving angle of the rolling robot under different motion environment and motion speed, according to the actual environment, the running speeds of the robot are 0.45m/s, 0.90m/s and 1.35m/s respectively, the movement environment is selected on ceramic tile pavement, cement pavement and asphalt pavement with low friction coefficient, and the change of driving angle of robot gravity center adjustment device is observed respectively. The experimental results are shown in Table 2 below.

| Robot running speed | Rolling pavement environment | Driving angle /° |
|---------------------|-----------------------------|------------------|
| 0.45m/s             | Tile pavement               | 24.50            |
|                     | Smooth cement               | 35.00            |
|                     | asphalt pavement            | 45.50            |
| 0.90m/s             | Tile pavement               | 30.00            |
|                     | Smooth cement               | 38.50            |
|                     | asphalt pavement            | 50.80            |
| 1.35m/s             | Tile pavement               | 40.50            |
|                     | Smooth cement               | 45.55            |
|                     | asphalt pavement            | 61.50            |

Figure 20 shows the performance test experiment of different wheel diameter around turning. Figure (a) shows the path diagram of robot around motion silhouette synthesis. The robot zoom ratio is set to 10:9 and the theoretical surround radius \( R_H \) is 3.5m. The experimental results show that the actual \( R_H \) is 3.48m. Figure (b) shows the limit zoom ratio surround turning motion experiment carried out by the robot. According to the comprehensive experimental results, the surround motion of the robot basically meets the initial design parameters.

7 Conclusions

(1) By analyzing and comparing the motion modes and driving structures of various typical robot mechanisms at home and abroad, a new variable diameter wheel rolling robot driven by gravity center adjustment device is designed. The robot is driven by the center of gravity adjustment device and two scalable gear trains as the motion mechanism. The ground contact wheel composed of multiple groups of circular arc pieces is restrained by spokes to adjust the wheel diameter of the robot, achieve the linear walking of equal wheel diameter and the rotary movement of different wheel diameter, and can complete the walking tasks in different road
environments.

(2) Based on classical mechanics, the kinematics characteristics of linear rolling, turning and slope motion of rolling robot are analyzed. The energy method is used to establish a slope motion theory model, the relationship between the climbing angle and size ratio and mass ratio of the robot climbing movement is plotted. On this basis, the uphill ability of the wheeled robot is evaluated, and the maximum climbing angle of the robot is obtained.

(3) The structure of the robot prototype is introduced, and the robot linear walking experiment and plane steering experiment are carried out to verify the feasibility of the robot movement.

8 Declaration

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Authors’ contributions

The author’s contributions are as follows: Hui Bian was in charge of the whole trial; Chun Zhang and Shijie Wang wrote the manuscript; Other students are responsible for assisting the prototype experiment and result analysis.

Competing interests

This paper does not involve competing economic interests.

Consent for publication

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