From $b \to s\gamma$ to the LSP Detection Rates in Minimal String Unification Models

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ABSTRACT

We exploit the measured branching ratio for $b \to s\gamma$ to derive lower limits on the sparticle and Higgs masses in the minimal string unification models. For the LSP('bino'), chargino and the lightest Higgs, these turn out to be 50, 90 and 75 GeV respectively. Taking account of the upper bounds on the mass spectrum from the LSP relic abundance, we estimate the direct detection rate for the latter to vary from $10^{-1}$ to $10^{-4}$ events/kg/day. The muon flux, produced by neutrinos from the annihilating LSP’s, varies in the range $10^{-2} - 10^{-9}$ muons/m$^2$/day.
1 Introduction

The supersymmetric theories provide the most promising framework to extend the standard model (SM) [1]. Local supersymmetry (SUSY) breaking induces soft SUSY breaking terms such as gaugino masses, scalar masses and trilinear and bilinear couplings of scalar fields in global SUSY models. The values of these soft terms determine the phenomenological properties of the models. Four-dimensional superstrings is a promising candidate for the unification of all interactions, and so far the only candidate theory for a consistent treatment of gravity on the quantum level. In four-dimensional string approaches such as the Calabi-Yau [2] or orbifold models [3], the expressions for the Kähler and the gauge kinetic functions of supergravity are known. In recent papers [4],[5],[6] the soft SUSY breaking terms have been derived from superstring theories under the assumption that SUSY is broken by the vacuum expectation value (vev) of the F-terms of the dilaton field $S$ and/or the moduli fields $T_i$. These gauge singlet fields are generically present in a large class of four-dimensional models, and their coupling to the gauge non-singlet matter are suppressed by powers of the Planck mass. This makes them ‘natural’ candidates to constitute the SUSY-breaking “hidden sector” which is needed in many phenomenological models of low-energy SUSY. The vev of the real part of $S$ gives the inverse square of the tree-level gauge coupling constant, and the vevs of the moduli fields $T_i$ parameterize the size and shape of the compactified space. In Ref. [7] it was assumed that only the dilaton field and an overall modulus field $T$ contribute to the SUSY breaking, and the soft SUSY breaking terms were parameterized by a goldstino angle $θ$, the gravitino mass $m_{3/2}$ and a set of parameters known as modular weights. The case with multi-moduli fields is discussed in Ref. [8].

We choose the modular weights such that one can have the appropriate large string threshold corrections to ensure the ‘merging’ of the three ‘low energy’ gauge couplings at the GUT scale. This is the so-called minimal string unification model [9]. It is worth repeating here that the natural unification scale in superstring models is $M_{\text{string}} \sim 0.5 \times g_{\text{string}} \times 10^{18}$ GeV, where $g_{\text{string}} = (ReS)^{-1/2} \sim 0.7$. However, the merging of the gauge coupling constants with the particle content of the MSSM (minimal SUSY SM) takes place at a scale $M_X \sim 3 \times 10^{16}$ GeV. Several mechanisms have been proposed in
order to explain this $M_X - M_{\text{string}}$ discrepancy. The string threshold corrections is an
elegant possibility since it does not require any new particles beyond the MSSM ones.

The investigation of the phenomenology of this approach leads to a number of ‘low
energy’ predictions which can be tested at LEPII, Tevatron and LHC, as we have empha-
sized in Refs.[10] and [11]. We have shown that the lightest neutralino in this model also
happens to be the lightest supersymmetric particle (LSP), and it is almost a ‘pure’ bino
where the gaugino function is larger than 0.97. Moreover, the cosmic abundance of the
LSP puts important constraints on the underlying supersymmetry breaking parameters,
namely one obtains an upper bound on the gravitino mass of about 600 GeV, which leads
to upper bounds on the sparticle spectrum of the model.

In this paper we wish to focus on the reduction of the allowed parameter space that
follows from the $b \to s\gamma$ physics. (It is well known that the constraints from $b \to s\gamma$
cause a dramatic reduction of the allowed MSSM parameter space) In models with dila-
ton/moduli SUSY breaking, we find that the $b \to s\gamma$ constraint imposes a lower bound on
the gravitino mass which implies stringent lower bounds on the entire SUSY spectrum (for
instance, the lower bound that we obtain on the chargino mass exceeds by a small amount
its present experimental bound of 85 GeV). We provide estimates for the detection rates
in direct and indirect neutralino searches within the range of the parameter space which
satisfies all the particle accelerator and relic abundance constraints.

This paper is organized as follows. In section 2 we review the soft SUSY breaking
terms obtained in Ref.[5] and their parameterization following Ref. [7]. Also, we study
the effect of leaving free the $B$-parameter. Section 3 deals with the constraints on SUSY
parameter space from the $b \to s\gamma$ decay. In section 4, we study the direct and indirect
LSP detection rates in the allowed range of the parameter space. We show that the
event rate $R$ lies between 0.1 and $10^{-4}$ events/kg/day while the muonic flux $\Gamma$ satisfies
$10^{-9} \leq \Gamma \leq 10^{-2}$ muon/m$^2$/day. We give our conclusions in section 5.
2 Minimal String Unification

In this section we give a brief review of the construction of the soft SUSY breaking terms in the minimal string unification scheme \[9\] and their low energy implications \[10\], \[11\]. A supergravity lagrangian is characterized by the Kähler potential $K$, a superpotential $W$ and the gauge function $f_a$, where $a$ refers to the gauge group. In the case of orbifold four-dimensional superstrings, the Kähler potential has the form \[12\]

\[
K = -\log(S + S^*) - 3\log(T + T^*) + \sum_i (T + T^*)^{n_i}\phi_i\phi_i^*.
\]

Here $S$ is the dilaton field which couples universally in all string models, $T$ is the overall modulus whose real part gives the volume of the compactified space, and $\phi_i$ are the chiral superfields. The $n_i$ are integers, called the modular weights of matter fields. At tree level the gauge kinetic function is given by $f_a = k_a S$, where $k_a$ is the Kac-Moody level of the gauge factor. In the phenomenological analysis that follows, $k_3 = k_2 = \frac{5}{3}k_1 = 1$. Assume that the fields which contribute to SUSY breaking are $S$ and $T$ through a non vanishing vevs of their auxiliary fields $F_S$ and $F_T$ respectively. We can take the following parameterization for the vevs:

\[
(G_{SS})^{1/2}F_S = \sqrt{3}m_{3/2}\sin\theta,
\]

\[
(G_{TT})^{1/2}F_T = \sqrt{3}m_{3/2}\cos\theta.
\]

Here $G_{ij} = \frac{\partial^2 G}{\partial \phi_i \partial \phi_j}$, $G = -3\ln\left(\frac{K}{3}\right) + \ln|W|^2$ and $e^{G/2} = m_{3/2}^2$ is the gravitino mass. The angle $\theta$ parameterizes the direction of the goldstino field $\tilde{\eta}$ (the goldstino fermion is eaten by the gravitino in the process of SUSY breaking) in the $S, T$ field space:

\[
\tilde{\eta} = \sin\theta \hat{S} + \cos\theta \hat{T},
\]

where $\hat{S}$ and $\hat{T}$ are the canonically normalized fermionic partners of the scalar fields $S$ and $T$. Thus this angle is called the goldstino angle.

The soft breaking terms have the form

\[
m_i^2 = m_{3/2}^2(1 + n_i \cos^2\theta),
\]

\[
M_a = \sqrt{3}m_{3/2}\frac{k_a \text{Re}S}{\text{Re}f_a}\sin\theta + m_{3/2}\cos\theta \frac{B_a'(T + T^*)\hat{G}_2(T, T^*)}{32\pi^3 \text{Re}f_a},
\]

\[
B_a'(T + T^*) = \frac{\text{Re}S}{\text{Re}f_a} + \frac{3\text{Re}f_a}{2k_a}(T + T^*) + O(T^2/T^*),
\]

\[
\hat{G}_2(T, T^*) = \sum_{i, j} (T + T^*)^{n_{ij}}\phi_i^*\phi_j^*.\]
\[ A_{ijk} = -\sqrt{3}m_{3/2}\sin \theta - m_{3/2}\cos \theta(3 + n_i + n_j + n_k), \]  
\[ B_\mu = m_{3/2}\left[-1 - \sqrt{3}\sin \theta - \cos \theta(3 + n_{H_1} + n_{H_2})\right], \]

where the definitions of the \( B'_a, \hat{G}_2 \) functions are given in Ref.[4]. It is clear that if \( S \) contributes dominantly to SUSY breaking (\( \theta = \pi/2 \)), we obtain the well known universal soft scalar and gaugino masses. Otherwise, the soft scalar masses depend on their modular weights and \( T \)-dependent threshold corrections that lead to non universal gaugino masses.

In Ref.[7] two sources for the \( B \)-parameter were considered, labeled by \( B_Z \) and \( B_\mu \). Here \( B_\mu \) is the coefficient of the \( H_1 - H_2 \) mixing term in the scalar Higgs sector potential. The source of \( B_Z \) is the mechanism of Ref.[13], where \( \mu \) arises only from couplings in the Kähler potential. The source of \( B_\mu \) is considered in Ref.[7], with \( \mu \) coming solely from the \( S \) and \( T \) sector. In general, there could be an admixture of these two cases. In our previous analysis we have focused on just \( B_\mu \), since this option for \( B \) allows a larger region of SUSY parameter space for electroweak radiative breaking. Here we will study the effect of leaving \( B \) as a free parameter to be determined from the electroweak breaking conditions.

An interesting input in order to further constrain the form of the soft terms is the requirement of the gauge couplings unification. As mentioned in the introduction, in string unification scenarios, the gauge coupling constants \( g_a \) of the three SM interactions are related at the string scale \( M_{String} \) by

\[ k_3 g_3^2 = k_2 g_2^2 = k_1 g_1^2 = \frac{8\pi}{\alpha'} G_{Newton} = g_{String}^2. \]  

In fact, it has been shown [14] that the string scale is of order \( g_{String} \times 0.5 \times 10^{17} \) GeV. Thus one finds that there is an order of magnitude discrepancy between the SUSY GUT and the string unification scales. Here we assume the so-called minimal string unification [9], namely that the only ‘light’ particles are the standard MSSM ones. We rely on the string threshold contributions to ‘cover’ the gap between the unification and the string scales.

It has been shown [1] that the following values of the modular weights lead to good agreement for \( \sin^2 \theta_W \) and \( \alpha_3 \).

\[ n_{Q_L} = n_{D_R} = -1, \quad n_{u_R} = n_{H_1} = -2, \quad n_{L_L} = n_{E_R} = n_{H_2} = -3. \]  

(9)
The above values of the modular weights are interesting because they provide an explicit model with non-universality which has phenomenological implications which differ from the case of the universal boundary conditions. From eq. (3) and above set (4) of values of the modular weights, we readily obtain the value of the scalar masses. Notice that if one require the absence of tachyonic mass at the compactification scale then we get
\[
\cos^2 \theta \leq \frac{1}{3}.
\]
The asymptotic gaugino masses read:
\[
M_3 = \sqrt{3} m_{3/2}(\sin \theta + 0.12 \cos \theta),
\]
\[
M_2 = \sqrt{3} m_{3/2}(\sin \theta + 0.06 \cos \theta),
\]
\[
M_1 = \sqrt{3} m_{3/2}(\sin \theta - 0.02 \cos \theta).
\]

The trilinear scalar coupling \( A_t \) which is related to the top quark Yukawa coupling is given by
\[
A_t = -m_{3/2}(\sqrt{3} \sin \theta - 3 \cos \theta).
\]
Finally \( B_\mu \) has the form
\[
B_\mu = m_{3/2}(-1 - \sqrt{3} \sin \theta + 2 \cos \theta).
\]

Given the boundary conditions in eqs. (10-12) at the compactification scale, we determine the evolution of the various couplings according to their renormalization group equation (RGE) to finally compute the mass spectrum of the SUSY particles at the weak scale. The electroweak symmetry breaking requires the following conditions among the renormalized quantities \( \mu_1^2, \mu_2^2 \) and \( \mu_3^2 \):
\[
\mu_1^2 + \mu_2^2 > 2\mu_3^2, \quad |\mu_3|^4 > \mu_1^2 \mu_2^2,
\]
and
\[
\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}, \quad \sin 2\beta = \frac{-2B_\mu}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2}.
\]
Here \( \tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle \) is the ratio of the two Higgs vevs that give masses to the up and down type quarks, and \( m_{H_1}, m_{H_2} \) are the Higgs masses at the electroweak scale. \( \mu_1^2, \mu_2^2 \) and \( \mu_3^2 \) satisfying the boundary conditions at \( M_{\text{string}} \):
\[
\mu_i^2 = m_{H_i}^2 + \mu^2, \quad i = 1, 2
\]
\[
\mu_3^2 = -B_\mu.
\]
Using equations (14) we find that $\mu$ and $\tan \beta$ are specified in terms of the goldstino angle $\theta$ and the gravitino mass $m_{3/2}$. It turns out that if only the top Yukawa coupling is of order unity, $\tan \beta$ is close to 2 and $|\mu|$ is quite large. For instance, $\mu$ is about 350 GeV for the lower chargino bound of 84 GeV.

As explained in Ref.[10] a further constraint on the parameter space is entailed by the demand of color and electric charge conservations. In particular, with the latter constraint one finds that $0.98 \, \text{rad} \leq \theta \leq 2 \, \text{rad}$. In the case with $B$ as a free parameter, we can determine it from equation (14) in terms of $\tan \beta$. Fig.1 shows the ratio of $B$ (at compactification scale) to $m_{3/2}$ versus the gravitino mass, for $\tan \beta = 6$. We note that the sign of $B$ in general is opposite that of $\mu$ for the correct electroweak symmetry breaking. For the value of $m_{3/2}$ fixed different values of $B/m_{3/2}$ in this figure corresponds to different values of $\theta$. In the same way, all the figures are plotted corresponding to different values of $\theta$.

![Figure 1](image_url)

Figure 1: The ratio of $B$ (at compactification scale) to $m_{3/2}$ versus the gravitino mass, with $\tan \beta = 6$.

We have previously shown [11] that the lightest neutralino is almost a ‘pure’ bino and is predicted to be the lightest sparticle (LSP). The LSP is a leading cold dark matter candidate and expected to play an important role in the large scale structure formation. Assuming a relic density parameter $0.1 \leq \Omega_{LSP} \leq 0.9$, with the Hubble parameter $h$ in the
range \(0.4 \leq h \leq 0.8\), we found that the maximum value of neutralino relic density \(\Omega_{LSP} h^2\) imposes an upper bound on \(m_{3/2}\) which is very sensitive to \(\theta\). In turn, this leads to a stringent upper bound on the LSP mass of about 160 GeV in the case of pure dilaton SUSY breaking, while this bound reaches 300 GeV in the case of \(\theta = 0.98\) rad which represents the maximum moduli contribution to SUSY breaking in this model. There is no point in the parameter space \((m_{3/2}, \theta)\), after imposing the experimental constraints on the sparticles, that leads to \(\Omega_{LSP} h^2\) less than the minimal value (0.014). Hence requiring the LSP to provide the correct amount of the cold dark matter does not lead to a lower bound on \(m_{3/2}\). However, in the next section we will show that the recent observational bounds on \(b \to s\gamma\) impose lower bounds on the sparticle masses of this model. Consequently, we are able to provide both the lower and upper bounds on the sparticle spectrum.

### 3 Constraints from \(b \to s\gamma\)

In this section we focus on the constraints on the parameter space \((m_{3/2}, \theta)\) which come from \(b \to s\gamma\) decay. The recent observation of this process by the CLEO collaboration \([17]\), \(1 \times 10^{-4} < BR(b \to s\gamma) < 4 \times 10^{-4}\) (at 95\% c.l.), has stirred interest in the possible bounds obtainable for supersymmetric models because of new contribution to this process \([18],[19]\). For a recent discussion in the context of the effective supergravities from string theories see Ref.\([20]\).

In MSSM there are additional contributions to the decay besides the SM diagrams with a W-gauge boson and an up-quark in the loop. The new particles running in the loop are: charged Higgs \((H^{\pm})\) and up-quark, charginos \((\chi^-)\) and up-squarks, gluino and down-squarks, neutralinos and down-squarks \([18]\). The total amplitude for the decay is the sum of all these contributions. The inclusive branching ratio for \(b \to s\gamma\) normalized to \(BR(b \to c\bar{e}\nu)\) is given by \([18]\)

\[
\frac{BR(b \to s\gamma)}{BR(b \to c\bar{e}\nu)} = \frac{|V_{ts} V_{tb}|^2 6\alpha_{em} \eta^{16/23} A_\gamma + \frac{8}{3} (\eta^{14/23} - \eta^{16/23}) A_g + C|^2}{\pi I(x_{cb})[1 - \frac{2}{3\pi} \alpha_S(m_b) f(x_{cb})]}.
\]

Here \(\eta = \frac{\alpha_S(m_W)}{\alpha_S(m_b)}\), and \(C\) represents the leading-order QCD corrections to \(b \to s\gamma\) amplitude at the scale \(Q = m_b\) \([21]\). The function \(I(x)\) is given by

\[
I(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x,
\]
and $x_{cb} = \frac{m_c}{m_b}$, while $f(x_{cb}) = 2.41$ is a QCD correction factor. The amplitude $A_\gamma$ is from the photon penguin vertex, and the $A_g$ from the gluon penguin vertex. The relevant contributions which we will consider come from the SM diagram plus those with the top quark and charged Higgs, and up-squarks and charginos running in the loop. Following the notation of Ref. [22] these contributions read:

$$A_{\gamma,g}^{SM} = \frac{3}{2} \frac{m_t^2}{M_W^2} f^{(1)}_{\gamma,g} \left( \frac{m_t^2}{M_W^2} \right),$$

$$A_{\gamma,g}^{H^-} = \frac{1}{2} \frac{m_t^2}{m_H^2} \left[ \frac{1}{\tan^2 \beta} f^{(1)}_{\gamma,g} \left( \frac{m_t^2}{m_H^2} \right) + f^{(2)}_{\gamma,g} \left( \frac{m_t^2}{m_H^2} \right) \right],$$

$$A_{\gamma,g}^{-} = A_{\gamma,g}^{\chi^-} + A_{\gamma,g}^{\chi^-}_2 + A_{\gamma,g}^{\chi^-}_3 + A_{\gamma,g}^{\chi^-}_4,$$

where $A_{\gamma,g}^{\chi^-}_i$ are given by

$$A_{\gamma,g}^{\chi^-}_1 = \sum_{j=1}^2 \frac{M_W^2}{M_{\chi_j}^2} |V_{tj}|^2 f^{(1)}_{\gamma,g} \left( \frac{m_{\tilde{t}}^2}{M_{\chi_j}^2} \right),$$

$$A_{\gamma,g}^{\chi^-}_2 = - \sum_{j,k=1}^2 \frac{M_W^2}{M_{\chi_j}^2} \left[ V_{tj} T_{k1} - \frac{V_{t2} m_{\chi_j} T_{k2}}{\sqrt{2} M_W \sin \beta} \right]^2 \frac{f^{(1)}_{\gamma,g} \left( \frac{m_{\tilde{t}}^2}{M_{\chi_j}^2} \right)}{f^{(2)}_{\gamma,g} \left( \frac{m_{\tilde{t}}^2}{M_{\chi_j}^2} \right)},$$

$$A_{\gamma,g}^{\chi^-}_3 = - \sum_{j=1}^2 \frac{M_W}{M_{\chi_j}} \frac{U_{tj} V_{tj}}{\cos \beta} f^{(3)}_{\gamma,g} \left( \frac{m_{\tilde{t}}^2}{M_{\chi_j}^2} \right),$$

$$A_{\gamma,g}^{\chi^-}_4 = \sum_{j,k=1}^2 \frac{M_W}{M_{\chi_j}} \frac{U_{tj} V_{tj}}{\cos \beta} \left( V_{tj} T_{k1} - \frac{V_{t2} m_{\chi_j} T_{k2}}{\sqrt{2} M_W \sin \beta} \right) T_{k1} f^{(3)}_{\gamma,g} \left( \frac{m_{\tilde{t}}^2}{M_{\chi_j}^2} \right).$$

The functions $f^{(i)}_{\gamma,g}, i = 1, 2, 3$ are given in [18], $V$ and $U$ are the unitary matrices which diagonalise the chargino mass matrix, while $T$ diagonalises the stop mass matrix [1].

As it is known, the charged Higgs contribution always interferes constructively with the SM contribution. The chargino contribution could give rise to a substantial destructive interference with the SM and Higgs amplitudes, depending on the sign of $\mu$, the value of $\tan \beta$ and the mass splitting between the stop masses. This way of presenting the chargino amplitude in equation (17) by splitting it into four pieces can help us to show when the chargino contribution can significantly reduce $b \rightarrow s\gamma$ [23]. It turns out numerically that the magnitudes of $A_{\gamma,g}^{\chi^-}_1$ and $A_{\gamma,g}^{\chi^-}_2$ are less than those of $A_{\gamma,g}^{\chi^-}_3$ and $A_{\gamma,g}^{\chi^-}_4$, especially with the LEPII lower bound on the chargino mass $m_\chi > 84$ GeV. Moreover, we

\[\text{We use the sign convention of } \mu \text{ in the chargino mass matrix opposite to that adopted in the Haber and Kane report [1].}\]
can observe that the sum of $A_{\gamma,g}^\chi_3 + A_{\gamma,g}^\chi_4$ is identically zero in the limit of degenerate up-squark masses. On the other hand, $A_{\gamma,g}^\chi_3$ and $A_{\gamma,g}^\chi_4$ each can be quite large because they are enhanced by large tan $\beta$.

First we make our analysis in the context of the choice $B_\mu$ (eq.(12)) for the $B$ parameter. Hence tan $\beta$ is fixed to be $\simeq 2$. In Figs.2 and 3 we show the values of the $BR(b \rightarrow s\gamma)$ corresponding to the gravitino mass $m_{3/2}$ in the allowed range we have determined in Ref [11] for $\mu < 0$ and $\mu > 0$ respectively.

Figure 2: The branching ratio $BR(b \rightarrow s\gamma)$ versus $m_{3/2}$ with $\mu < 0$, while tan $\beta \simeq 2$ from electroweak breaking.
Figure 3: The branching ratio $BR(b \to s\gamma)$ versus $m_{3/2}$, with $\mu > 0$ and $\tan \beta \simeq 2$.

It is remarkable that for $\mu < 0$, even taking the experimental upper bound $BR(b \to s\gamma) < 4 \times 10^{-4}$ we obtain a lower bound of $\simeq 90$ GeV on the gravitino mass while, for $\mu > 0$ the lower bound is 150 GeV. This can be explained as follows. For $\mu < 0$, the chargino contribution gives a destructive interference with the SM and $H^+$ contributions, but it is smaller in magnitude. This is due to the fact that $\tan \beta$ is of order 2, and the splitting of the two stop mass eigenstates $m^2_{\tilde{t}_1, \tilde{t}_2}$ is small since the L-R entry in the stop mass matrix is quite small with respect to the value of the diagonal elements which get a large gluino contribution in the renormalization group evolution. For $\mu > 0$ the chargino gives a constructive interference with the SM and $H^+$, and this makes the branching ratio larger than the experimental upper bound, unless the Higgs and chargino masses are sufficiently large.

Now we relax the assumption $B = B_\mu$ which forces $\tan \beta \simeq 2$ and let $B$ be a free parameter. For larger values of $\tan \beta$ and $\mu < 0$ we expect the chargino contribution to give rise to substantial destructive interference and the branching ratio of $b \to s\gamma$ can be less than the standard model value as shown in Fig.4.
Figure 4: The $b \to s\gamma$ branching ratio versus $m_{3/2}$, with $\mu < 0$ and $\tan \beta = 20$.

The new constraints from $b \to s\gamma$ shrink the allowed parameter space of the model as shown in Fig. 5 for the case $\mu < 0$. The model predicts that the mass of the chargino is greater than 90 GeV. The lower bound in the right selectron mass turns out to be 65 (110) GeV if $\theta = 0.98(\pi/2)$. Actually, the discovery of a right selectron with mass less than the chargino mass would be signal for a departure from the (pure dilaton dominated) universal soft SUSY breaking scenario. Also of much interest are the lower bounds on the lightest neutralino and lightest Higgs: they are 50 GeV and 75 GeV respectively. As we said, all these bounds apply in the case $B = B_\mu$. On the other hand, if we let $B$ free then for $\mu < 0$ no lower bound on $m_{3/2}$ is obtained.
The Allowed Range

Figure 5: The allowed parameter space with $\mu < 0$. The solid line corresponds to the constraints from $b \to s\gamma$ while the dashed line corresponds to the upper bounds on $m_{3/2}$ from the neutralino relic abundance.

4 The LSP detection rates

In models with conserved R-parity the lightest supersymmetric particle (LSP) is considered the favorite candidate for cold dark matter (CDM). As mentioned in section 1, in a previous analysis[11] we have shown that the lightest neutralino in the minimal string unification turns out to be the LSP and it is almost a pure bino. Moreover, requiring $0.1 \leq \Omega_{LSP} \leq 0.9$, with $0.4 \leq h \leq 0.8$, leads to relevant constraints on the parameter space ($m_{3/2}, \theta$). This leads to a stringent upper bound on the LSP mass of about 160 GeV in the case of pure dilaton supersymmetry breaking. In addition, severe limits on the parameter space were obtained in the last section by imposing the constraints that derive from $b \to s\gamma$. In this section we are interested in the detectability of the LSP of this model taking account of all relevant constraints.

It was shown in Ref.[24] that the detectability of neutralino dark matter is linked to the amplitude for $b \to s\gamma$, and the experimental bounds on the branching ratio for the inclusive $b \to s\gamma$ decay impose strong constraints on the region of the parameter space where sizable counting rates for relic neutralinos are expected. The main reason for this
is that both the counting rate and the branching ratio increase with decreasing mass of the Higgs bosons. In Ref. [25] it was shown that there are sizable regions of the parameter space with $R > 0.01$ including this constraint. Other authors [26] have recently claimed that there exists some possibility for further enhancement of the detectability of neutralino scattering with nuclei. It is certainly relevant to investigate the impact of the various restrictions (including those coming from $b \to s\gamma$ in section 3) on the neutralino-nuclei scattering in the class of superstring models under discussion.

Perhaps the most natural way of searching for the neutralino dark matter is provided by direct experiments, where the effects induced in appropriate detectors by neutralino-nucleus elastic scattering may be measured. The differential detection rate is given by

$$\frac{dR}{dQ} = \frac{\sigma \rho_\chi}{2m_\chi m_r^2} F^2(Q) \int_{v_{min}}^{\infty} \frac{f_1(v)}{v} dv, \quad (19)$$

where $f_1(v)$ is the distribution of speeds relative to the detector. The reduced mass is $m_r = \left(\frac{Q m_N}{2m_r^2}\right)^{\frac{1}{2}}$, where $m_N$ is the mass of the nucleus, $v_{min} = \left(\frac{Q m_N}{2m_r^2}\right)^{\frac{1}{2}}$, $Q$ is the energy deposited in the detector and $\rho_\chi$ is the density of the neutralino near the Earth. It is common to fix $\rho_\chi$ to be $\rho_\chi = 0.3 \text{GeV/cm}^3$. Instead, we will determine it from the relation

$$\rho_\chi = \Omega_\chi h^2 \times \rho_{\text{critical}}, \quad (20)$$

where $\rho_{\text{critical}} \sim 1.8 \times 10^{-29} \text{g/cm}^3$ and $\Omega_\chi h^2$ is the neutralino relic density, so we are treating $\rho_\chi$ as a function of the neutralino mass. We will compare the result with the one we would obtain if $\rho_\chi = 0.3 \text{GeV/cm}^3$. The quantity $\sigma$ is the elastic-scattering cross section of the LSP with a given nucleus. In general $\sigma$ has two contributions: spin-dependent contribution arising from $Z$ and $\tilde{q}$ exchange diagrams, and spin-independent (scalar) contribution due to the Higgs and squark exchange diagrams. For $^{76}\text{Ge}$ detector, where the total spin of $^{76}\text{Ge}$ is equal to zero, we have contributions only from the scalar part. The form factor in this case is given by [27]

$$F(Q) = \frac{3j_1(qR_1)}{qR_1} e^{-\frac{1}{2}q^2 s^2}, \quad (21)$$

where the momentum transferred is $q = \sqrt{2m_N Q}$, $R_1 = (R^2 - 5s^2)^{1/2}$ with $R = 1.2 fm A^{1/2}$ and $A$ is the mass number of $^{76}\text{Ge}$. $j_1$ is the spherical Bessel function and $s \simeq 1 fm$. 

13
The event rate $R$ is presented in Figs. 6 and 7 for the two cases, $\rho_\chi$ as function of $m_\chi$, and $\rho_\chi = 0.3 GeV/cm^3$. The detection rates are of order $10^{-1} - 10^{-4}$ events/kg/day. Also, we can see that the result significantly changes when we treat $\rho_\chi$ as a function of the neutralino mass.

Figure 6: The event rate $R$ versus $m_\chi$ with $\rho_\chi$ treated as function of $m_\chi$, and $\tan \beta \simeq 2$. The horizontal line denotes the present experimental sensitivity.

Figure 7: The event rate $R$ versus $m_\chi$ for $\rho_\chi = 0.3 GeV/cm^3$ and $\tan \beta \simeq 2$. The horizontal line as in Fig. 6.
A promising method for indirect detection of neutralinos in the halo is the observation of the energetic neutrinos from the annihilation of neutralinos that accumulate in the sun or in the earth. Among the annihilation products are ordinary neutrinos which may be observable in suitable detectors. The energies of the neutrinos are about a third of the LSP mass so they are easily distinguished from solar neutrinos or any other known background. The technique for the detection of such energetic neutrinos is through observation of upward muons produced by the charged current interactions of the neutrinos in the rock below the detector. Concentrating on the neutralino annihilation on the sun, the flux of such muons from neutralino annihilation can be written as

\[
\Gamma = 2.9 \times 10^8 m^{-2} yr^{-1} \tanh^2(t/\tau) \rho_\chi^{0.3} f(m_\chi) \zeta(m_\chi) \left(\frac{m_\chi^2}{GeV}\right)^2 \left(\frac{f_P}{GeV^{-2}}\right)^2.
\]  

(22)

The neutralino-mass dependence of the capture rates is described by [28]

\[
f(m_\chi) = \sum_i f_i \phi_i S_i(m_\chi) F_i(m_\chi) \frac{m_i^3 m_\chi}{(m_\chi + m_i)^2},
\]  

(23)

where the quantities \(\phi_i\) and \(f_i\) describe the distribution of element \(i\) in the sun and they are listed in Ref. [28], the quantity \(S_i(m_\chi) = S(\frac{m_\chi}{m_Ni})\) is the kinematic suppression factor for capture of neutralino of mass \(m_\chi\) from a nucleus of mass \(m_Ni\) [28] and \(F_i(m_\chi)\) The form factor suppression for the capture of a neutralino of mass \(m_\chi\) by a nucleus \(i\). Finally, the function \(\zeta(m_\chi)\) describes the energy spectrum from neutralino annihilation for a given mass.

In Figs.8 and 9 we present the results for muonic fluxes resulting from captured neutralinos in the sun for \(\rho\) as a function of the neutralino mass, and for \(\rho_\chi = 0.3 GeV/cm^3\). We see that the predicted muonic flux lies between \(10^{-2}\) and \(10^{-9}\) muon/\(m^2/\text{day}\). Clearly, large scale detectors are best suited for neutralino detection.
Figure 8: The muonic flux $\Gamma$ versus $m_\chi$ with $\rho_\chi$ considered a function of $m_\chi$, and $\tan \beta \simeq 2$.

Figure 9: The muonic flux $\Gamma$ versus $m_\chi$ for $\rho_\chi = 0.3$ and $\tan \beta \simeq 2$. 
5 Conclusions

The decay $b \to s\gamma$ has been employed to derive the most stringent lower bound on the gravitino mass in the ‘minimal’ string unification models. This leads to lower bounds on the sparticles and Higgs mass spectra. By combining this information with the upper bounds available from considerations of the LSP (‘bino’) relic abundance, we are able to estimate the direct and indirect detection rates for the latter. Large scale detectors are needed to discover the LSP of our scheme.

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