Sensorless Estimation and Nonlinear Control of a Rotational Energy Harvester

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Abstract. It is important to perform sensorless monitoring of parameters in energy harvesting devices in order to determine the operating states of the system. However, physical measurements of these parameters is often a challenging task due to the unavailability of access points. This paper presents, as an example application, the design of a nonlinear observer and a nonlinear feedback controller for a rotational energy harvester. A dynamic model of a rotational energy harvester with its power electronic interface is derived and validated. This model is then used to design a nonlinear observer and a nonlinear feedback controller which yield a sensorless closed-loop system. The observer estimates the mechanical quantities from the measured electrical quantities while the control law sustains power generation across a range of source rotation speeds. The proposed scheme is assessed through simulations and experiments.

1. Introduction

Energy harvesting is a promising solution to extend the operational lifetime of low-power sensor networks, which over the past few decades have relied on finite energy sources such as batteries. Examples of ambient energy sources that can be converted into useable electrical power include solar, radio frequency waves and vibrations [1]. Elimination of sensors and transducers, i.e. sensorless operation, in these harvesters reduces the cost and increases the system ruggedness and reliability. The main approaches to sensorless operation are based on saliency, Kalman filter and model reference techniques [2]. However, the estimation of mechanical quantities from measured electrical quantities still remains challenging because the existing techniques are computationally intensive, require special construction for estimation or require proper initialisation [3].

A rotational energy harvester which is based on balancing gravitational and motor torques has been reported in [4]. The control scheme discussed in [4] focuses on the impedance matching for optimal power transfer between the load and includes a current limit to prevent the mass from flipping around. The nonlinear controller presented here is based on Lyapunov stability criterion and aims to improve the harvester’s efficiency by using the estimated mechanical states of the harvester thereby providing sensorless operation.
2. Development and validation of the harvester’s mathematical model

The rotational energy harvester consists of a DC generator with its stator coupled to a continuously rotating source and a semicircular mass \( m \) attached to the rotor at a distance \( l \) from the axis of rotation (see Figure 1a). When power is drawn from the generator the torque between the stator and rotor (motor torque) is counteracted by the torque generated by the gravitational force acting on the offset mass (gravitational torque). The difference between these two torques creates a difference in the angular speeds of the stator and the rotor that can be tapped off as power. The generated power is collected in a storage supercapacitor for future use [4]. To ensure optimal power transfer from the harvester to the load, the load resistance, \( R_L \), should be closely matched to the harvester’s armature resistance, \( R_a \). The input impedance of a boost converter \( R_{in} \) can be controlled by varying its duty cycle \( \delta \) as \( R_{in} = R_L(1 - \delta)^2 \) [5]. Therefore, it is used as a power electronic interface circuit between the harvester and the load.

The experimental set-up for the harvester and the interface electronics is illustrated in Figure 1b. For a more detailed explanation of the construction and the choice of the circuit components, please refer to [4] and references therein.

From the free body diagram of the offset mass in Figure 1a, the torque balance on the mass attached to the rotor of the harvester is given by \( J \dot{\omega} = \Gamma_M - mgl \sin \theta \), where \( J \) is the moment of inertia of the mass, \( \omega \) is the angular velocity, and \( \theta \) is the deflection angle of the mass measured from the vertical axis. The motor torque \( \Gamma_M \) is calculated as \( \Gamma_M = -k_T I_{in} \), where \( k_T \) is the torque constant of the motor, and \( I_{in} \) is the current drawn by the DC generator. The negative sign indicates that the current flows out of the generator and into the boost converter. The voltage generated by the harvester, \( E_g \), when it is attached to a continuously rotating source at a speed \( \omega_s \) is calculated as \( k_E(\omega_s - \omega) \), where \( k_E \) is the “motor constant” of the DC generator.

Application of Kirchhoff’s voltage law to the circuit in Figure 1c and use of the relations \( I_{in} = \frac{V_{in}}{R_{in}} \) and \( R_{in} = (1 - \delta)^2 R_L \) yields

\[
V_{in} = \frac{k_E(\omega_s - \omega)(1 - \delta)^2 R_L}{R_a + (1 - \delta)^2 R_L} .
\]  

The standard averaged model that describes the dynamics of the boost converter composed of an inductor and a supercapacitor is (see [5] and references therein)

\[
\dot{I}_L = \frac{(1 - \delta)V_C}{L} + \frac{1}{L} \frac{k_E(\omega_s - \omega)(1 - \delta)^2 R_L}{R_a + (1 - \delta)^2 R_L} , \quad \dot{V}_C = \frac{I_L}{C} - \frac{V_C}{R_L C} .
\]  

Figure 1: Rotational energy harvester.
where \( V_C \) is the voltage across the supercapacitor of capacitance \( C \), \( I_L \) is the current flowing through the inductor of inductance \( L \), and \( V_{in} \) is as in (1).

From (1) and (2), the mathematical model of the harvester with the interface circuit is given by the equations

\[
\dot{x}_1 = x_2, \\
\dot{x}_3 = -\frac{(1-\delta)x_4}{L} + \frac{k_E(\omega_s - x_2)(1-\delta)^2R_L}{J(R_a + (1-\delta)^2R_L)} + \frac{k_Ek_T(\omega_s - x_2)}{R_a + (1-\delta)^2R_L}, \\
\dot{x}_4 = \frac{(1-\delta)x_3}{C} - \frac{x_4}{R_LC},
\]

(3)

where \( x_1, x_2 \) describe the angular position \( \theta \) and the angular velocity \( \omega \) of the mass, respectively, and \( x_3, x_4 \) describe the inductor current \( I_L \) and the output capacitor voltage \( V_C \) of the boost converter, respectively.

It can be inferred from the \( \dot{x}_2 \) equation that the operating condition: \( x_1 = \pm \pi/2 \) and \( x_2 = 0 \) maximises the amount of gravitational torque, \( \frac{mgl\sin x_1}{J} \), generated by the mass and ensures the largest difference in rotational speeds between the stator and rotor, \( \omega_s - x_2 \). As the source rotation increases, maintaining the angular position of the mass at \( \pi/2 \) prevents it from flipping over and synchronising with the source rotation.

To validate the model (3) the experimental set-up of the harvester and the simulated model are driven by the same input and the output data are collected. The relative errors of \( I_L \) and \( V_C \) for two selections of source rotations and fixed duty cycles are plotted in Figure 2. These errors, calculated as \( e_{i,rel} = \frac{|x_{i,meas} - x_{i,sim}|}{x_{i,meas}} \), for \( i = 3, 4 \), are below 0.2% which suggests that the mathematical model gives an accurate description of the system.

![Figure 2](https://via.placeholder.com/150)

(a) Duty cycle = 70%

(b) Duty cycle = 80%

Figure 2: Time histories of the source rotations (top), relative errors of the inductor current (middle), and relative errors of the output capacitor voltage (bottom).

### 3. Sensorless estimation

The observer design is based on the method described in [6]. We aim to accurately estimate the mechanical variables \( \text{i.e.} \) angular position and velocity, and the measured electrical variables \( \text{i.e.} \) inductor current and capacitor voltage.
Proposition 1: Consider the rotational energy harvester system as described in (3). The dynamical system
\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + \beta_2(x_3) - \frac{\partial \beta_1}{x_3} x_{30}, \\
\dot{x}_2 &= -\frac{mgl \sin(\dot{x}_1 + \beta_1(x_3))}{J} - \frac{\partial \beta_2}{x_3} x_{30} - \left(\frac{k_E k_T (\omega_s - \dot{x}_2 - \beta_2)}{R_a + (1 - \delta)^2 R_L}\right),
\end{align*}
\]
with states \([\dot{x}_1(t), \dot{x}_2(t)]^T \in \mathbb{R}^2\), \(b_1 > 0, b_2 < 0\), inputs: \(x_3(t) \in \mathbb{R}, x_4(t) \in \mathbb{R}, \delta(t) \in [0, 1] \) and \(\omega_s(t) \in \mathbb{R}\),
\[
\begin{align*}
\beta_1(x_3) &= \frac{b_1 - 1}{\rho} x_3, \\
\beta_2(x_3) &= \left(\frac{b_2}{\rho} \frac{k_E k_T}{\rho (R_a + (1 - \delta)^2 R_L)} x_3\right), \\
x_{30} &= \frac{(1 - \delta) x_4}{L} + \rho (\omega_s - \dot{x}_2 - \beta_2),
\end{align*}
\]
is such that \(\lim_{t \to \infty} (x_1(t) - \dot{x}_1(t)) = 0\) and \(\lim_{t \to \infty} (x_2(t) - \dot{x}_2(t)) = 0\), i.e. \(\dot{x}_1\) and \(\dot{x}_2\) are asymptotically converging estimates of \(x_1\) and \(x_2\) respectively.

Proof. Define \(z = [\beta_1(x_3) + \dot{x}_1, \beta_2(x_3) + \dot{x}_2]^T - [x_1, x_2]^T\), and note that
\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} = \begin{bmatrix}
0 \\
-\frac{mgl}{J} \frac{1 - \rho}{R_a + (1 - \delta)^2 R_L} - \frac{\rho}{x_3} \frac{\partial \beta_2}{x_3}
\end{bmatrix} \begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} - \frac{mgl}{J} \begin{bmatrix}
0 \\
\Gamma(x_1, z_1)
\end{bmatrix} z_1,
\]
where \(\rho = \frac{k_E (1 - \delta)^2 R_L}{L(R_a + (1 - \delta)^2 R_L)}\) and \(\Gamma = \sin(x_1 + z_1) - \sin(x_1) - z_1\). The variables \(b_1\) and \(b_2\) are chosen such that \(b_2 < 0\) and \(\frac{J b_2}{mgl b_1} \leq \frac{1}{\|\Gamma\|_2}\). The claim then follows from standard arguments on stability of passive systems, see [7].

4. Controller design

The equations of the model in (3) suggest that the angular position of the mass \(x_1\) can be adjusted by changing the amount of torque acting on it. This can be changed using the control variable \(\delta\).

Proposition 2: Consider the rotational energy harvester system (3). Assume \(\omega_s\) is constant.
The control law \(\delta = 1 - \sqrt{\frac{k_E k_T}{\mu R_L} - \frac{R_a}{R_L}}\), where \(\mu = -\frac{mgl (1 - \sin x_1)}{(\omega_s - x_2)^2} + \eta (\omega_s - x_2) \cos x_1 + \frac{mgl}{\omega_s}\) for \(0 < \eta << 1\) globally asymptotically stabilizes the equilibrium \((\pi/2, 0)\) of the \((x_1, x_2)\) subsystem. In addition \(\delta(t) \in [0, 1]\) and \(x_2(t) < \omega_s\) for all \(t \geq 0\).

Proof. Consider the candidate Lyapunov function \(V(x_1, x_2) = (1 - \sin x_1) + \frac{J}{\eta} \ln |\omega_s - x_2| - \frac{\omega_s}{\omega_s - x_2}\). Note that \(V \geq 0\) for \(0 < \eta << 1\), and \(V = 0\) for \(x_1 = \pi/2 + k\pi\) and \(x_2 = 0\). In addition \(\lim_{x_2 \to \omega_s} V = \infty\) The time derivative of \(V\) along the closed-loop trajectories is
\[
\dot{V} = -\frac{mgl x_2^2}{(\omega_s - x_2)^2} \leq 0.
\]
The claim then follows from standard LaSalle arguments, see [7].

Remark 1: The physical characteristics of the harvester along with the choice of \(\delta\) as the control variable restrict the operational range of the harvester. This limitation can be overcome if a saturation control scheme such as \(\min \left(\max \left(\mu, \frac{k_E k_T}{(R_L + R_a)} - \frac{mgl}{\omega_s}, \frac{k_E k_T}{R_a} - \frac{mgl}{\omega_s}\right), \frac{k_E k_T}{R_a} - \frac{mgl}{\omega_s}\right)\) is used.
5. Experimental results of the nonlinear observer and controller

Figure 3a demonstrates the performance of the observer described in Proposition 1 with measured electrical quantities, \( \omega_s = 75 \text{ rad/s}, \delta = 0.8, b_1 = 50, \) and \( b_2 = -70. \) The observer is used to implement the control law in Proposition 2. Figure 3b shows the angular position of the mass being held at approximately \( \pi/2 \text{ rad} \) for a source rotation \( \omega_s = 75 \text{ rad/s}. \) The error in angular position of the mass in the experimental implementation is due to rounding errors in the microcontroller. Figure 3c illustrates the closed-loop currents and voltages from the experiments and simulations.

![Figure 3](image)

(a) Estimated angular position (top) and angular velocity (bottom).
(b) Mass being held at an angular position of approximately \( \pi/2. \)
(c) Simulated/measured currents (top) and voltages (bottom).

Figure 3: Performance of the proposed observer-controller scheme.

6. Conclusions

A model of a rotational energy harvester with its power electronic interface has been derived and validated. This model has been used for the design of an observer and a controller, the performances of which have been demonstrated via experiments. The proposed observer method can be adapted for other types of harvesters.

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