Diffusive topological transport in spatiotemporal thermal lattices

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Topological phases have been studied in photonic, acoustic and phononic metamaterials, promising a range of applications. Such topological modes usually stem from collective resonant effects in periodic lattices. One may, therefore, expect similar features to be forbidden for thermal diffusion that is purely dissipative and mostly incoherent, prohibiting collective resonances. Here we report the discovery of diffusion-based topological states supported by spatiotemporally modulated advections stacked over a fluidic surface. This arrangement imitates a periodic propagating potential in an effective thermal lattice. We observe edge states in topologically non-trivial and bulk states in topologically trivial lattices. Interface states form at boundaries between these two types of lattice, manifesting inhomogeneous thermal properties on the fluidic surface. Our findings establish a framework for topological diffusion and thermal edge or bulk states, and it may allow a distinct mechanism for the flexible manipulation of diffusive phenomena for robust heat and mass transfer.

Topological insulating phases1–5 have been unveiling a variety of new wave phenomena in metamaterials. A particularly interesting class of topological phenomena arises in open systems6–9, enabling novel phenomena across a variety of fields10–13, such as anomalous edge states1, mode switching14–16, unidirectional wave propagation17,18, single-mode laser19,20, pump-dependent lasing21–23 and so on. Two main recipes have been proposed to realize these classes of non-Hermitian topological responses. The first option is to drive the topological responses by magneto-optic effects24 or tailored lattice coupling25 in cooperation with the introduced non-Hermitian elements; the second is to induce non-trivial topological phases by pumping non-Hermitian elements like gain and loss26–30 into otherwise Hermitian systems.

Recently, the Hamiltonian associated with classical thermal exchange was discovered to follow a skew-Hermitian relation31, which may offer a hint to realize a topological insulating phase for thermal diffusion. On the other hand, the absence of a periodic potential and dynamic coherence for thermal transport fundamentally hinders this pathway. To synthesize a topological insulating phase in purely dissipative diffusion mechanism, it is of utmost importance to imitate the periodic potential and coherent interference in thermal transport, such that Bloch theory could be revived and diffusive topological transport could be thereby expected as the photonic and acoustic counterparts32–38.

Here we resort to judicious time-modulated Hermiticity features in both space and time to introduce an advective paradigm for the demonstration of thermal topological transport in periodically stacked fluid surfaces. We incorporate spatiotemporally modulated advections to enable an additional real dimension to thermal diffusion, whereas alternating advection arrangements offer imitated coherences and periodic potentials. Depending on the advective configuration, the effective bandgap can be either topologically non-trivial or trivial. We experimentally discover thermal topological modes with edge and interface states as well as conventional bulk states, manifested as stationary and deviated temperature distributions, respectively. The findings provide a feasible way of creating an effective periodic lattice on an advective fluidic surface, unlocking more opportunities and applications for topological thermal diffusion. Our proposed recipe for diffusive topological modes can further enlighten the manipulation of general diffusive fields39–41.

We consider a planar fluid surface possessing periodic advections in a virtual space $(x_p, y_p, z)$ (Fig. 1a) to demonstrate non-Hermitian topological modes for thermal diffusion. Owing to the dissipative nature of thermal diffusion, non-Hermiticity inherently exists in the proposed system. To create an effective coherent resonance for thermal fields over the fluidic surface, we apply spatiotemporal modulation onto the corresponding advective regions42 marked by specific colours (Fig. 1a). These advective blocks could be regarded as four modulated units, forming one effective lattice. The four-unit lattice is used to form a pair of effective dimers and a four-band system by effective decoupling between the units with the imposed advections. Such implementation could reveal multifarious band properties, topological transitions and midgap edge states for a wide range of parameters, as well as simplify practical advective arrangements, in contrast to lattices with two or three units43–34. Further periodic configuration of multiple effective lattices can yield an effective one-dimensional chain that can be induced on the fluidic surface. Here the heat transfer process between two neighbouring super-cells $n$ and $(n+1)$ via their joint interface in the virtual space $(x_p, y_p, z)$ can be written as

$$
\rho c \frac{\partial T_n}{\partial t} = \kappa \frac{\partial^2 T_n}{\partial x_p^2} + \rho c v_n \frac{\partial T_n}{\partial x_p} + \frac{h_n}{a_n} (T_{n+1} - T_n).
$$

(1)
In equation (1), \( \rho, c \) and \( \kappa \) denote the density, specific heat and thermal conductivity of the planar fluid surface, respectively; \( T_n \) is the temperature of a specific unit of one lattice (Fig. 1a), where subscript \( n \) indicates the unit index counting from 1. Further, \( a_n \) and \( h_n \) are the width and convective heat transfer coefficient of each unit. Equation (1) is the general thermal energy equation for describing heat transfer and exchange between any two neighbouring units with a joint interface. It consists of three terms: a conductive component \( (\kappa \frac{\partial T_n}{\partial z}) \), a convective component \( \left( \pm \rho c v_n \frac{\partial T_n}{\partial x} \right) \) and a heat exchange term via their joint interface \( \frac{h_n}{a_n} (T_{n+1} - T_n) \) derived from Newton’s law of cooling. It implies that heat exchange via the interface is proportional to their difference in temperatures. Hence, only temperature is involved here to describe the heat exchange between the two neighbouring layers \( n \) and \( (n+1) \) via their joint interface. It is worth noting that the exchange between layers \( (n-1) \) and \( n \) should be involved when we simultaneously take the three units \( (n-1), n \) and \( (n+1) \)—possessing two joint interfaces—as an entirety instead of the two units in equation (1). In that case, the total exchange via the two interfaces between units \( (n-1, n) \) and \( (n, n+1) \) becomes \( \frac{h_n}{a_n} (T_{n+1} + T_{n-1} - 2T_n) \), which is the superposition of the two exchanges \( \frac{h_n}{a_n} (T_{n+1} - T_n) \) and \( \frac{h_n}{a_n} (T_{n} - T_{n-1}) \) at the corresponding interfaces. This is equivalent to equation (1) since heat exchanges via each joint interface between any two neighbouring layers are the core of thermal coupling and they should be counted only once in the effective lattice. For simplification, we adopt the same width for each unit, thus leading to an effective lattice constant of 4a. We further assign a series of spatiotemporal advections \( (v_0, v_2, v_4) \) to each unit advection \( (v_n, v_{n+1}, v_{n+2}, v_{n+3}) \) of one lattice to create an effective ‘oscillation’ in such diffusive systems, that is, \( \pm \rho c v_n \frac{\partial T_n}{\partial x} \) in equation (1). Here we make \( v_i \geq 0 \) and \( v_i \geq 0 \) to indicate the scalar amplitudes of the imposed advective velocities on each unit, whereas ‘\( \pm \)’ indicates the advective direction in each unit. That is, \( \pm v_{i0} \) denotes the imposed advections

Fig. 1 | Topological transport in thermal diffusion. a, Schematic for realizing the topology in thermal diffusion by introducing periodic advections onto a planar fluid surface (coloured curves) in a virtual space \((x, y, z)\). These periodic advections independently propagate along the \( x \) direction and are arranged along the \( z \) direction, thus contributing to the effective ‘lattice’ with four units marked by \( n-(n+3) \). The lattice constant is 4a, whereas the field coupling between two adjacent units is \( m = \frac{m}{a_n} = \frac{m}{a_n} \). b, Phase diagram by modulating advections \((v, v_i)\). Among them, Phase I exists only when \( v = 0 \) or \( v_i = 0 \); Phases II and III are gapped, whereas Phase IV is gapless. c, Practical model for implementations in a transformed space \((x, y, z)\). It can be created via a conformal mapping with the model in the schematic, where the azimuthal and radial directions in c correspond to \( x \) and \( y \) in a, respectively. The purple dash-dot lines in a and c indicate their counterpart boundaries of a specific unit. Each effective lattice consists of four units on the fluid surface. They are modulated by corresponding dynamic components with consistent colours inserted under the fluid surface. The entire thermal process is modulated by ten effective lattices (bottom inset). d, Angular velocity (eigenfrequencies) sorted in a gapped phase as a function of \( v_i = m/k \) and \( v_i = 2m/k \). The top and bottom insets indicate non-trivial and trivial lattices, respectively.
proceeding along opposite directions with amplitudes $v_{xy}$. Owing to the spatiotemporal properties, these advective velocities imitate the periodic potential fields, yielding $v_{xy} = v_{n+1}^{n+1}$ between two adjacent lattices along the $z$ direction (Supplementary Equation (2)). On the interaction of imposed advectons and conductions, their thermal exchange $\frac{\kappa}{\rho} \left( T_{n+1} - T_n \right)$ effectively provides the coupling within two neighbouring units via the joint interface. Under the hypothesis of small fluid thermal conductivity $\kappa$, the first-order wave-like solution $T_n = Ae^{i(\Delta z_{xy} - \omega t)}$ hints at the possibility of switching equation (1) to a similar form of the Schrödinger equation, where $k_n = \frac{2\pi}{\Delta z_{xy}} = R \Delta z_{xy}$ and $\omega_n = -\frac{\kappa}{\rho c} \Delta z_{xy} - k_n v_n$ denote the effective wave-numbers and angular frequencies of the units for each lattice, respectively. The phase diagram for the effective band structure under specific advective configurations is presented in Fig. 1b, indicating four insulating Phases I–IV (Supplementary Note 2). To realize such an effective lattice in an available thermal system, we need to mimic the oscillations and periodic propagations with the imposed advection in a finite system. We adopt a strategy by connecting the two terminals along the $x$ direction (Fig. 1a), thus forming an annular fluid surface in the transformed space ($x,y,z$). The $z$ direction is unchanged in this process. Such a transition (Fig. 1c) follows a general conformal mapping (Methods). The thicknesses of the advective components and annular fluid surface in the transformed space are $b$ and $d$, respectively. Owing to the same radii of each unit, the effective wavenumbers are the same for the entire system, that is, $k_n = k = R^1$. The changing angular velocity corresponds to a real angular frequency ($\omega_n$) in the actual thermal system. Based on the Bloch theorem and the imposed periodic potential (velocity) field along the $z$ direction, the effective Hamiltonian can be expressed as

$$
H = i \begin{bmatrix}
\frac{\kappa}{\rho c} & -\frac{\kappa}{\rho c} & 0 & -\frac{\kappa}{\rho c} e^{i k_x a} \\
\frac{\kappa}{\rho c} & \frac{\kappa}{\rho c} & 0 & \frac{\kappa}{\rho c} e^{i k_x a} \\
0 & -\frac{\kappa}{\rho c} & \frac{\kappa}{\rho c} & 0 \\
\frac{\kappa}{\rho c} e^{-i k_x a} & 0 & \frac{\kappa}{\rho c} & \frac{\kappa}{\rho c}
\end{bmatrix}
$$

(2)

where $k$, denotes the effective Bloch wavenumber (Supplementary Note 1), and $I$ is the indentity matrix. The Bloch theorem is applied along the $z$ direction, that is, the direction of the one-dimensional lattice arrangement. Here $D = \frac{\rho c}{\kappa}$ denotes the diffusivity, whereas we further assume $m = \frac{\rho c}{\kappa} = \frac{\rho c}{\kappa}$ for simplification under the hypothesis of small fluid surface thickness $\Delta z$ (Fig. 1c). It is worth noting that equation (2) is named as the effective Hamiltonian, since it is obtained by solving the eigenvalue problem with the effective oscillatory parameters of wavenumbers $k$, and angular frequencies $\omega_n$, which do not naturally exist in thermal diffusion. The mathematical presence of ‘i’ in the entire Hamiltonian (equation (2)) indicates that the decoupled form of the matrix of heat exchanges (equation (2)) follows the skew Hermitian, which is in sharp contrast to the coupled terms of photonic/acoustic Hamiltonians also with a four-unit lattice[29-31]. Thus, the imposed advectons act as a Hermitian modulation in this diffusion scenario, and it is essentially equivalent to the non-Hermitian role played by gain and loss in the photonic counterpart[29-31]. Then, we can define the eigenvalues for the periodic lattice as

$$
E_{\pm} = -i (Dk^2 + m) \pm \sqrt{\frac{\eta}{2} \sqrt{\left| p \right| + \sqrt{p^2 - q^2 - l^2}}},
$$

$$
p = (k_n v_n)^2 + (k_n v_n)^2 - 4m^2, q = 2 (k_n v_n) (k_n v_n),
$$

$$
s = m^2 \left( 1 + e^{i \pi k} \right), l = 4m^2 \sin \left( \frac{\pi k_n}{2} \right).
$$

(3)

This system can support topological modes, since equation (2) obeys pseudo-anti-Hermiticity, that is, $H = -\eta H \eta$, where $\eta = \text{diag}(1,-1,1,-1)$. Considering one four-unit lattice, two types of advective coupling can be expected in equation (3) with imposed advectons, thus leading to effective couplings

$$
\sqrt{\left( \frac{k}{\rho c} \right)^2 - (k_n v_n \pm k_n v_{n+1})^2}
$$

between any two adjacent units. These advective couplings within the four-unit lattice support an effective bandgap $\Delta = \sqrt{p - \sqrt{p^2 - q^2}}$ and the four insulating phases shown in Fig. 1b. Here we focus on the response in gapped Phase III with $p - q > 0$ and $p < q$. In this phase, the integer winding number is zero or zero when the advection arrangement is $(v_n - v_{n+1} - v_{n+2})$ or $(-v_n - v_{n+1} v_{n+2})$. The other insulating phases and corresponding properties are discussed in Supplementary Note 2.

Non-zero and zero winding numbers correspond to the presence or absence of edge states at the boundaries, respectively. To verify their presence, we consider 40 units forming 10 effective lattices in the schematic shown in Fig. 1c (lower inset) and Extended Data Fig. 1, creating a finite fluidic system. The inherent conduction between each pair of adjacent lattices acts as the hopping term to generate the underlying edge and bulk states. When we consider the advective arrangement $(v_n - v_{n+1} - v_{n+2})$ for each lattice in the selected phase, the sorted angular velocities (eigenfrequencies) indicate the emergence of a pair of topological edge states within the bandgap (Fig. 1d, top inset). If the advective arrangement becomes $(-v_n - v_{n+1} v_{n+2})$ in the same phase, only the bulk state emerges (Fig. 1d, bottom inset). The thermal distribution of the edge and bulk states in the selected phase is illustrated in Fig. 2. The captured temperature profiles at specific moments of the non-trivial lattice scheme are presented in Fig. 2a,c. The thermal behaviours are robust, with stationary temperature profiles (Fig. 2c, right) during the entire thermal process, whereas some deviations relative to initial locations can be observed at the two boundaries of the system. By further tracing the experimental azimuths and spatial locations of the maximum temperature ($T_{\text{max}}$) of each unit on the fluidic surface, we find that they correspond to the first and last lattices at the two system boundaries, respectively, and slightly move to the two sides with respect to their initial azimuths (Fig. 2a) and spatial locations (Fig. 2c), revealing the non-zero winding number ($\pm 1$) (Supplementary Note 2). The spatial information of $T_{\text{max}}$ of the other units remains almost unchanged (Fig. 2a,c). Then, we study the state intensities of each unit, which could be described as the effective thermal resistance in analogy to topological electric circuits along the $z$ direction between two adjacent units and in the $x-y$ plane (azimuth direction) within one unit, that is, $R_{\text{eff},z} = \sum_{n} \frac{\eta}{\Delta z_{xy}}$ and $R_{\text{eff},xy} = \sum_{n} \frac{\Delta z_{xy}}{\eta}$, respectively, where $T_{\text{max},z}$ and $T_{\text{max},xy}$ are the temperature components along the $z$ direction of the adjacent lattices and $\Delta T_{\text{max},z}$ is the temperature-difference component between the highest and lowest temperatures in the $x-y$ plane of a specific unit. $P$ denotes the power of corresponding area. Since the eigenvalue reveals the energy per unit of time corresponding to the effective heat flux along the direction of lattice arrangements, each pole of $R_{\text{eff},z}$ and $R_{\text{eff},xy}$ can be used to indicate the corresponding modes on the fluidic surface. Thus, the larger $R_{\text{eff},z}$ and smaller $R_{\text{eff},xy}$ at the system boundaries compared with the other regions (Fig. 2e,g) hint at the existence of edge states (deviations observed in Fig. 2a,c), which suppress and contribute to the heat-flux propagation towards the corresponding directions, respectively. It is worth noting that stationary thermal distribution can be observed over the entire system, owing to the thermal equilibrium under such advective configurations. The smaller $R_{\text{eff},z}$ and larger $R_{\text{eff},xy}$ (not at the boundaries) lead to expedited/tough heat transfer towards these directions.

In contrast, the thermal features of the scheme with trivial lattices (Fig. 1d, bottom inset) are presented in Fig. 2b,d,f,h.
Fig. 2 | Measured results for topologically non-trivial and trivial responses in the gapped phase. 

**a, b.** Azimuths of \( T_{\text{max}} \) for each unit on the fluid surface at specific moments for the non-trivial (a) and trivial (b) lattice. 

**c, d.** Visualizations of experimental \( T_{\text{max}} \) locations (left) and thermal images (right) with respect to a (c) and b (d). The background colours on the projective planes correspond to the coordinates shown in Fig. 1c. The temperature profile on the fluid surface is effectively confined along the x and y directions in the non-trivial lattice, but it significantly expands along the x and y directions in the trivial lattice. These results agree well with the azimuths in a and b. 

The colour bars in c and d indicate the temperature ranges of the IR images. 

**e, f.** Sums of the amplitudes of the effective thermal resistance along the z direction for the system with the lattices in c (e) and d (f). 

**g, h.** Effective thermal resistances in the x–y plane for the two lattices in c (g) and d (h). In e–h, the orange columns with green dots denote the experimental values and ‘+’ denotes the theoretical values of the effective thermal resistances of each unit.
The temperature profiles and spatial information of $T_{\text{max}}$ (Fig. 2b,d) indicate that the thermal distributions exhibit distinctive differences between the initial and final moments. The characteristic thermal distributions showcase significant deviations towards both sides with respect to their initial positions in the units near both system boundaries and reveal relatively stable distributions in the intermediate units of the system over time. Such behaviours are robustly maintained till the steady state. By further evaluating the state intensities (effective thermal resistances along the $z$ direction and $x$-$y$ plane) of each unit for the trivial lattices, there are no significant differences between the effective thermal resistances of each unit in these directions, revealing the bulk state on the fluidic surface with zero winding number (Supplementary Note 2). Except the different characteristics of the temperature distributions, the thermal process of the scheme with trivial lattices is enhanced compared with the non-trivial one, since the temperature amplitudes shown in Fig. 2d indicate a faster relaxation process than those shown in Fig. 2c.

The topological edge and bulk states suggest the possibility of flexibly manipulating the thermal properties of a fluidic surface.

Here we further explore two cases with the tunable butting of topologically non-trivial and trivial lattices, possessing non-zero ($\pm 1$) and zero winding numbers, respectively, demonstrating controllable edge, interface and bulk states, whereas the advections are also modulated in the same gapped phase (Supplementary Note 2). For Case I, we configure five non-trivial (20 units) and five trivial (20 units) lattices to the fluidic surface (Fig. 3a). The characteristic thermal distributions and experimental temperature profiles are illustrated in Fig. 3b–e. The spatial information of $T_{\text{max}}$ at specific moments (Fig. 3b,c) indicate that the characteristic distributions of non-trivial lattices could maintain stationary locations in most of the units, whereas some deviations to the initial states occur in the units approaching the boundary of non-trivial and trivial lattices. For trivial lattices, the characteristic distributions showcase large deviations to the initial states in most of the intermediary units, whereas the $T_{\text{max}}$ locations of the last few units approaching the system boundary further indicate near-$\pi$ deviations. Such distributions in the non-trivial and trivial lattices reveal one edge state and one interface state at the system boundary in the non-trivial lattice and butting of the two
For Case II, we adjust the configuration strategy by inserting one non-trivial lattice (4 units) at the two system boundaries, whereas eight trivial lattices (32 units) are further filled in the regions between the two non-trivial lattices (4 units) at the two system boundaries, and more modulations can be anticipated with different lattice configurations and advections.

This work introduces an advective route to reveal the thermal topological modes by stacking periodic advections on a fluid surface. These spatiotemporally modulated advections introduce manipulation in momentum space for a purely dissipative phenomenon like thermal diffusion, enabling effectively non-trivial and trivial lattices. Thermal topological edge, interface and bulk states were observed on modulating the advections within these effective lattices in a gapped phase. Our findings point to a direction beyond conventional topological physics based on oscillatory fields, respectively, implying larger effective thermal resistance (Fig. 3d,e). The captured temperature profiles (Fig. 3b, right) further validate the above characteristic $T_{\text{max}}$ locations and demonstrate the distinctive thermal distributions of the two lattices. Besides, faster relaxation can also be observed in trivial lattices with significant homogenized temperature amplitudes at different moments.

By further evaluating the state intensities with the effective thermal resistance (Fig. 4d,e), two edge states at both system boundaries and two interface states at the butting of the non-trivial and trivial lattices are observed with larger (smaller) effective thermal resistances along the $z$ direction (in the $x$–$y$ plane) compared with the units of trivial lattices. The above two cases reveal the possibility of flexibly manipulating the thermal properties on a fluidic surface with non-trivial and trivial lattices, and more modulations can be anticipated with different lattice configurations and advections.
offering a unique platform to study unexpected non-Hermitian topological modes in purely diffusive systems. The critical strategy is to create the conduction–advection form in different diffusive fields with actual or effective advective terms. If the actual advection does not naturally exist such as electric conductivity, one feasible way is to adopt spatiotemporal parameters to derive an effective advective term from the intrinsic conduction. Besides, this work also suggests a distinctive mechanism of arbitrary diffusive manipulations with combinations of non-trivial and trivial lattices.

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Experimental demonstrations for insulating phases. The motors are started after the heating process. Based on the fabricated parameters of the system, the critical values on the exceptional points (EPs) should follow $k^2 T_{\text{cr}} = (m_0^2 + (e/pcd)^2) = 0.0225 s^2$. Four independent motors for spinning motions are employed to demonstrate the selected insulating phases and non-trivial/trivial behaviours (Fig. 3). The spatiotemporal advects for the four axes are $2[R m_{\text{non-threshold}}(2\pi t/ 10)], -2[R m_{\text{non-threshold}}(2\pi t/ 10)], -2[R m_{\text{threshold}}(2\pi t/ 10)]$ and $R m_{\text{threshold}}(2\pi t/ 10)$, where $t$ denotes time. The spatial information of $T_{\text{ann}}$ is measured by a thermocouple and dynamic components, whose data format is in the form of Cartesian coordinates. Thus, the experimental $T_{\text{ann}}$ azimuths (Figs. 2) can be calculated as $\theta = \arctan (\frac{y}{x})$. 

Experimental systems for topological manipulations. Since the demonstrations of manipulated Cases I and II require different lattice configurations on the fluidic surface under corresponding advects, additional transmission shafts and motors are employed to implement the experiments. For Case I, the fluid surface is actuated by five non-trivial (20 units) and five trivial (20 units) lattices. The velocities of the non-trivial/trivial components are independently modulated by four transmission shafts corresponding to the four advections of each lattice. The spinning velocities for each of these units non-trivial lattice are $3[R m_{\text{non-threshold}}(2\pi t/10)], -2[R m_{\text{non-threshold}}(2\pi t/10)], -2[R m_{\text{threshold}}(2\pi t/10)]$ and $2[R m_{\text{threshold}}(2\pi t/10)]$, whereas the velocities for each of the trivial lattices are $3[R m_{\text{threshold}}(2\pi t/10)], 2[R m_{\text{threshold}}(2\pi t/10)], -3[R m_{\text{threshold}}(2\pi t/10)]$ and $3[R m_{\text{threshold}}(2\pi t/10)]$. The same modified method is also implemented in Case II. Considering the advective configurations and manipulated behaviours, two non-trivial lattices (4 units each) at both system boundaries and eight trivial lattices (32 units) between them are adopted. The advections velocities for each of the units on the non-trivial lattices are $3[R m_{\text{non-threshold}}(2\pi t/10)], -2[R m_{\text{non-threshold}}(2\pi t/10)], -3[R m_{\text{threshold}}(2\pi t/10)]$ and $2[R m_{\text{threshold}}(2\pi t/10)]$, whereas the velocities for each of the internal trivial lattices are $3[R m_{\text{threshold}}(2\pi t/10)], 2[R m_{\text{threshold}}(2\pi t/10)], -3[R m_{\text{threshold}}(2\pi t/10)]$ and $3[R m_{\text{threshold}}(2\pi t/10)]$.
Extended Data Fig. 1 | The entire structure and experimental setups. **a** indicates the entire structure for implementing advections (effective oscillations) on the fluid surface. A constraint shell is adopted outside the fluid surface to keep the concentric rotations for each unit and the internal gear. During the measurements, this constraint shell is removed to directly capture the temperature-related data of the fluid surface. **b** and **c** present the fabricated dynamic components and their combinations consisting of 40 internal gear sets for modulating the fluid surface. The azimuthal direction of the units is marked in (**b**). The black dashed border in (**c**) indicates the region for IR images when fluid is coated on these dynamic components, while the interval of two neighboring deriving gears on a transmission shaft is 4a. The black dot marked with ‘1’ indicates the first unit of the entire system at the initial location of \( z = 0 \). **d** provides the real experiment setups, and the left upper inset presents the general engagement of the internal gear set for one unit cell. The colored dots indicate the corresponding motors.