ON THE BLACK HOLE MASS—X-RAY EXCESS VARIANCE SCALING RELATION
FOR ACTIVE GALACTIC NUCLEI IN THE LOW-MASS REGIME

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ABSTRACT

Recent studies of active galactic nuclei (AGNs) found a statistically inverse linear scaling between the X-ray normalized excess variance $\sigma_{\text{rms}}^2$ (variability amplitude) and the black hole (BH) mass spanning over $M_{\text{BH}} = 10^6-10^9 M_\odot$. Suggested as having a small scatter, this scaling relation may provide a novel method to estimate the BH mass of AGNs. However, a question arises as to whether this relation can be extended to the low-mass regime below $\sim 10^6 M_\odot$. If confirmed, it would provide an efficient tool to search for AGNs with low-mass BHs using X-ray variability. This paper presents a study of the X-ray excess variances for a sample of AGNs with BH masses in the range of $10^5-10^6 M_\odot$ observed with XMM-Newton and ROSAT, including data both from the archives and from newly preformed observations. It is found that the relation is no longer a simple extrapolation of the linear scaling; instead, the relation starts to flatten at $\sim 10^6 M_\odot$ toward lower masses. Our result is consistent with the recent finding of Ludlam et al. Such a flattening of the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation is actually expected from the shape of the power spectrum density of AGNs, for which the break frequency is inversely scaled with the mass of BHs.

Key words: galaxies: active – galaxies: nuclei – X-rays: galaxies

1. INTRODUCTION

Rapid X-ray variability is one of the basic observational characteristics of active galactic nuclei (AGNs; McHardy 1985; Grandi et al. 1992; Mushotzky et al. 1993). It is a useful tool for studying black holes (BHs) and the central engines of AGNs, since the X-ray emission is thought to originate from the innermost region of an accretion flow around the BH. One commonly used method for characterizing the variability is the power spectrum density (PSD) analysis, which quantifies the amount of variability power as a function of temporal frequency (Green et al. 1993; Lawrence & Papadakis 1993). The PSD of AGNs has been found to be well described by a broken power law (e.g., Papadakis & McHardy 1995; Edelson & Nandra 1999; Uttley et al. 2002; Markowitz et al. 2003; Vaughan et al. 2003b). The break frequency is found to be inversely scaled with the BH mass in a linear way with a possible dependence on the scaled accretion rate $\dot{m}$ (in units of the Eddington accretion rate; McHardy et al. 2006; González-Martín & Vaughan 2012). These results are remarkable in the sense that AGNs show similar X-ray variability properties to black hole X-ray binaries (BHXBs), indicating that AGNs are scaled-up versions of BHXBs (see also Zhou et al. 2015).

However, reliable PSD analyses require well-sampled, high-quality X-ray data of time series which are generally hard to obtain for large samples of AGNs for the current X-ray observatories. Instead, an easier-to-calculate quantity, the “normalized excess variance” $\sigma_{\text{rms}}^2$ (e.g., Nandra et al. 1997; Turner et al. 1999), is commonly used to quantify the X-ray variability amplitude. Early studies revealed correlations between the excess variance and various parameters of AGNs, such as X-ray luminosity, spectral index, and the FWHM of the H$\beta$ line (e.g., Nandra et al. 1997; Turner et al. 1999; George et al. 2000; Markowitz & Edelson 2001). Later work suggested that these correlations are in fact by-products of a more fundamental relation with BH mass: the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation (Lu & Yu 2001; Bian & Zhao 2003; Papadakis 2004). In fact, this relation conforms to the scaling relation for the PSD break frequency with BH mass (Papadakis 2004; O’Neill et al. 2005; Ponti et al. 2012). O’Neill et al. (2005) confirmed the anti-correlation between the excess variance and $M_{\text{BH}}$ with a large AGN sample observed with ASCA. Zhou et al. (2010) obtained a tight correlation, using high-quality XMM-Newton light curves of AGNs whose BH masses were measured with the reverberation mapping technique. The intrinsic dispersion of the relationship ($\sim 0.2$ dex) is comparable to that of the relation between $M_{\text{BH}}$ and stellar velocity dispersion for the galactic bulge (Tremaine et al. 2002). By making use of a large sample of 161 AGNs observed with XMM-Newton for at least 10 ks for each object, Ponti et al. (2012, henceforth P12) reaffirmed this relationship (using the excess variance calculated on various timescales of 10, 20, 40, and 80 ks), but found only a weak dependence on the accretion rate. The significant correlation with small scatters (0.4 dex for the reverberation mapping sample, 0.7 dex for the CAIXAvar, sample; see Ponti et al. 2012 for details) suggested that it may provide similarly or even more accurate BH mass estimates compared to the method based on the single epoch optical spectra (Kaspi et al. 2000; Vestergaard & Peterson 2006). Moreover, unlike the commonly used virial method which is susceptible to the orientation effect of AGNs (Collin & Kawaguchi 2004), X-ray variability can be considered as inclination independent.

However, the previous studies were based on AGN samples with mostly supermassive BHs of $M_{\text{BH}} > 10^6 M_\odot$, and little is known about the relation of AGNs with $M_{\text{BH}} < 10^6 M_\odot$. The $\sigma_{\text{rms}}^2$ of a few AGNs with $M_{\text{BH}} \sim 10^6 M_\odot$ were presented and
found to have the largest $\sigma_{\text{rms}}^2$ values among AGNs over a large $M_{\text{BH}}$ range (Minniti et al. 2009; Ai et al. 2011). Ponti et al. (2012) proposed that the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation may show a deviation from the linear relation in the low-mass regime; however, the data is too sparse to draw a firm conclusion. Thus, the question of whether this relation can be extended to $M_{\text{BH}} < 10^6 M_\odot$ remains unanswered.\footnote{While we were writing this paper, a new paper (Ludlam et al. 2015) appeared which carried out a similar study and achieved a result similar to ours in this work.}

The answer to this question is important in at least two aspects. First, if the answer is “yes,” then the relation would provide a valuable method for finding the so-called low-mass AGNs with $M_{\text{BH}} \lesssim 10^6 M_\odot$, sometimes referred to as AGNs with intermediate-mass black holes. This is of particular interest since in these AGNs, the commonly used virial method involving optical broad emission lines becomes difficult in practice due to the faint AGN (broad line) luminosities, which are outshone by the host galaxy starlight. In fact, there have been a few attempts in practice by adopting such an assumption. By extrapolating the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation to below $10^6 M_\odot$, Kamizasa et al. (2012) selected a sample of 15 candidate low-mass AGNs using the X-ray excess variance. Second, the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation must have a cutoff somewhere; otherwise, the variability would become unrealistically large for ultra-luminous X-ray sources and BHXBs, which is not seen in observations however (e.g., González-Martín et al. 2011; Zhou 2015). From a theoretical perspective, the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation is actually a manifestation of the inverse scaling of the break frequency of the AGN PSD with BH mass, since the excess variance is the integral of the PSD over the frequency domain (van der Klis 1989, 1997). In fact, a break of the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation at low BH masses had been predicted based on the current understanding of the PSD of AGNs (Papadakis 2004; O’Neill et al. 2005; Ponti et al. 2012), and the exact break mass (the mass at which the break occurs) depends on the shape of the PSD. Therefore, a study of the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation in the low-mass regime may provide a constraint to the break mass as well as the shape of the AGN PSD.

In this paper, we study the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation in the $M_{\text{BH}} = 10^5-10^6 M_\odot$ range, using an optically selected low-mass AGN sample from our previous work (Dong et al. 2012). We use both new observations and archival data obtained with XMM-Newton, as well as archival data from ROSAT. This paper is organized as follows. The introduction of our sample and data reduction are presented in Section 2. In Section 3, the excess variance is introduced, as are the PSD models of AGNs concerned in this study. The results and discussion are presented in Section 4. Throughout the paper, a cosmology with $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$ is adopted.

2. SAMPLE, OBSERVATION, AND DATA REDUCTION

There are over 300 known low-mass (type 1) AGNs with $M_{\text{BH}} \lesssim 10^6 M_\odot$ so far. The largest is from our work (Dong et al. 2012), which was selected homogeneously from the Sloan Digital Sky Survey (DR4), and comprises 309 objects with $M_{\text{BH}} < 2 \times 10^6 M_\odot$. The BH masses were estimated from the luminosity and the width of the broad H$\alpha$ line, using the virial mass formalism of Greene & Ho (2005, 2007). One feature of this sample is the accurate measurements of the AGN spectral parameters, and hence the BH masses and Eddington ratios. Compared to previous samples, this sample is more complete as it includes more objects with low Eddington ratios down to $L_{\text{bol}}/L_{\text{Edd}} \sim 0.01$ (see also Yuan et al. 2014). We compile a working sample of low-mass AGNs with usable X-ray data from this parent sample.

We search for X-ray observations from both the XMM-Newton and ROSAT PSPC data archives to maximize the sample size. For XMM-Newton observations the 3XMM-DR4 catalog is used. We also add new observations of three objects from our program to study low-mass AGNs with XMM-Newton (proposal ID: 074422, PI: W. Yuan). We consider only those observations with exposure times longer than 10 ks. There are 26 XMM-Newton observations for 16 objects and 6 ROSAT observations for 6 objects found in the archives. The new observations of J0914+0853, J1347+4743, and J1153+4612 were performed by XMM-Newton in faint imaging mode on 2014 November 1, 22, and December 4 with exposure times of 36, 31, and 14 ks, respectively.

The X-ray data are retrieved from the XMM-Newton and ROSAT data archives. We follow the standard procedure for data reduction and analysis. For the XMM-Newton observations, we use the data from the EPIC PN camera only, which have the highest signal to noise. Light curves are extracted from observation data files using the XMM-Newton Science Analysis System (SAS) version 12.0.1. Events in the periods of high flaring backgrounds are filtered out.\footnote{Except for the object J1153+4612, since it is bright enough (with a mean count rate = 2.1 counts s$^{-1}$) that the aforementioned influence on the timing analysis can be ignored.} Observations with cleaned exposure times shorter than 10 ks are also excluded. Typical source extraction regions are circles with a 40 arcsec radius. Only good events (single and double pixel events, i.e. PATTERN $\leq 4$) are used for the PN data. Background light curves are extracted from source-free circles with the same radius. Finally, the SAS task EPICLCCORR is applied to make corrections for each of the XMM-Newton light curves. The energy band 0.2–10 keV is used. The time bins of the light curves are chosen to be 250 s, which are the same as in P12 for easy comparison. As an example, Figure 1 shows some of the typical light curves (panels 1–4).

For ROSAT PSPC observations, the XSELECT package is used to extract source counts and light curves. Typical source extraction regions are circles of 50 arcsec radius, and the same aperture is used to extract background light curves. The energy band for ROSAT observations is 0.1–2.4 keV. The time bins are also set to be 250 s for the same reason. Examples of the ROSAT light curves are also shown in Figure 1 (panel 5).

All of the light curves exhibit significant variability on short timescales, as shown in the figure. For a few sources, due to short observational intervals and relatively low signal-to-noise ratios (S/Ns), the intrinsic variability is overwhelmed by random fluctuations because of the large statistical uncertainties (the statistical uncertainty is larger than the source variability; see Section 3.1 for the definition of the excess variance). In such cases, a meaningful excess variance cannot be obtained and its value is consistent with being zero (the same situation also happened in some objects or observations in previous studies, e.g., O’Neill et al. 2005; Ponti et al. 2012). We find
objects with 15 observations. Among these objects, 10 were observed with XMM-Newton for a total of 13 observations, and 2 objects were observed with ROSAT for 2 observations (J1223 +0726 was observed both in XMM-Newton and ROSAT observations). Table 1 summarizes the basic parameters of the sample sources and the information on the X-ray observations. The BH masses are taken from Dong et al. (2012), which are in the $10^5−10^6$ $M_\odot$ range and the accretion rates are in the 0.06−0.90 range (Figure 2). All of the objects are all at very low redshifts $z \lesssim 0.21$ with a median $z = 0.090$.

3. MEASUREMENT OF EXCESS VARIANCE AND THE PSD MODELS

3.1. Excess Variance and the Uncertainty

Following Nandra et al. (1997; see also Turner et al. 1999; Vaughan et al. 2003a; Ponti et al. 2004), the normalized excess variance is calculated using the following definition:

$$\sigma_{\text{rms}}^2 = \frac{1}{N\mu^2} \sum_{i=1}^{N} \left( \frac{X_i - \mu}{\sigma_i} \right)^2,$$

where $N$ is the number of good time bins of an X-ray light curve, $\mu$ is the unweighted arithmetic mean of the count rates, and $X_i$ and $\sigma_i$ are the count rates and their uncertainties, respectively, in each bin.

As shown by van der Klis (1989, 1997), the excess variance is the integral of the PSD of a light curve over a frequency interval given by Equation (2),

$$\sigma_{\text{rms}}^2 = \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} P(\nu) d\nu,$$

where $\nu_{\text{min}} = \frac{1}{T}$, $\nu_{\text{max}} = \frac{1}{2T}$, and $T$ and $\Delta T$ are the time length and the bin size of the light curve, respectively. For a given light curve, it is clear that the exact value of the excess variance is dependent on the length of the light curve (e.g., observational duration) as well as on the bin size $\Delta T$. In order to compare the excess variances of different objects or observations in our sample, the duration (timescale) and the bin size of the light curves should be set to be the same for all of the objects. For this purpose, the light curves are divided into one or more segments of 10 ks in length, and for each of the segments the excess variance is calculated. For observations having more than one segment, the mean of the segments is taken. The results are listed in Table 1.

The uncertainty of the excess variance comes from two sources, one of the measurement uncertainty and the other of the stochastic nature of the variability process, as shown by Vaughan et al. (2003a). The measurement uncertainties of the excess variance are estimated following Vaughan et al. (2003a):

$$\left( \Delta \sigma_{\text{rms}}^2 \right)_{\text{mea}} = \sqrt{\frac{\sigma^2}{N \mu^2}} \left( \frac{2}{N \mu^2} \right)^{1/2} + \sqrt{\frac{\langle \sigma_i^2 \rangle}{N \mu^2}} \left( \frac{2F_{\text{var}}}{N \mu^2} \right)^{1/2},$$

where $\langle \sigma_i^2 \rangle$ is the mean of the square of count rate uncertainties, $F_{\text{var}}$ is the fractional variability ($F_{\text{var}} = \sqrt{\sigma_{\text{rms}}^2}$), and the other quantities are defined in Equation (1).

As discussed in Vaughan et al. (2003a), the light curves of AGNs are simply stochastic, meaning that each observed light...
The curve is only one realization of the underlying random variability process, and each realization can exhibit a slightly different mean count rate and variance. The random fluctuations between different realizations lead to non-ignorable scatter in the excess variance. This phenomenon can be reflected in our result: for a source with more than one observation (such as J1023+0405, No. X1 in Table 1), the excess variance varies considerably. The scatter can be significantly reduced if the light curves are sufficiently long, or, equivalently, have a sufficient number of data points. We then compute the uncertainty due to the stochastic nature of the variability process using the method introduced in Vaughan et al. (2003a). We first build a PSD model\(^8\) to simulate a light curve using the method of Timmer & Koenig (1995) with a bin size of 250 s and sufficiently long duration. The light curve is divided into 1000 separate segments of 10 ks. The distribution of the calculated excess variances of all of the segments is obtained, from which the range of stochastic uncertainty is found. We take the 68% confidence range for the expected excess variance on each of the timescales to get the stochastic scatter. Thus, from our simulations, the scatter due to this random process is

\[
\log(\sigma_{\text{rms}}) = 0.29 \text{rms}^2 s^{-2} \text{D} = 0.24 + 0.04 \text{ for a 10 ks light curve with a 250 s bin size.}
\]

Finally, the total uncertainties of the excess variance are obtained by combining in quadrature the stochastic and measurement uncertainties, which are at the 68% confidence level. The obtained uncertainties of \(\sigma_{\text{rms}}^2\) are given in Table 1. For an object having more than one observation, the mean and its uncertainty are calculated and used as the final excess variance in the following analysis.

\[\text{Figure 2. Distributions of the black hole mass and scaled accretion rate for our sample, taken from Dong et al. (2012).}\]

\[\text{Table 1 Sample and Information on X-Ray Observations}\]

| Num | SDSS Name | \(z\) | \(\log(M_{BH})\) | \(L_{bol}/L_{Edd}\) | ObsID/SEQID | Count Rate | Expo. | \(\sigma_{\text{rms,10ks}}\) | Err\(10\)ks |
|-----|-----------|------|-----------------|-----------------|-------------|-----------|-------|-----------------|-----------|
| 0   | J102348.4+040553.7 | 0.099 | 5.44 | 0.32 | 0108670101 | 0.08 | 51.00 | 0.044 | 0.019/0.019 |
| 0   | J114008.7+030711.4 | 0.081 | 5.70 | 0.90 | 0305920201 | 0.13 | 106.50 | 0.057 | 0.012/0.012 |
| 0   | J122349.6+072657.9 | 0.075 | 5.63 | 0.55 | 0205090101 | 0.19 | 24.00 | 0.027 | 0.025/0.025 |
| 0   | J14350.6+033842.6 | 0.028 | 5.73 | 0.06 | 0305920401 | 0.12 | 22.00 | 0.101 | 0.048/0.048 |
| 0   | J010712.0+140845.0 | 0.077 | 6.09 | 0.34 | 0305920101 | 0.13 | 11.75 | 0.065 | 0.032/0.032 |
| 0   | J135724.53+652505.9 | 0.106 | 6.20 | 0.47 | 0305920301 | 0.13 | 18.75 | 0.040 | 0.017/0.017 |
| 0   | J082433.3+380013.2 | 0.103 | 6.11 | 0.41 | 0403760201 | 0.09 | 15.00 | 0.097 | 0.059/0.058 |
| 0   | J091449.0+085321.1 | 0.140 | 6.28 | 0.37 | 0744220701 | 0.76 | 31.75 | 0.040 | 0.015/0.011 |
| 0   | J134738.2+474301.9 | 0.064 | 5.63 | 0.67 | 0744220801 | 0.63 | 20.75 | 0.028 | 0.009/0.009 |
| 0   | J115341.78+461242.3 | 0.025 | 6.09 | 0.29 | 0744220301 | 2.16 | 13.75 | 0.019 | 0.007/0.007 |
| 0   | J122349.6+072657.9 | 0.075 | 6.43 | 0.29 | RP600009N00 | 0.10 | 16.00 | 0.186 | 0.077/0.076 |
| 0   | J11644.65+402635.6 | 0.202 | 5.83 | 0.45 | RP700855N00 | 0.08 | 18.50 | 0.032 | 0.030/0.029 |

Note. Column 1: X or R denotes the object observed with XMM-Newton or ROSAT, respectively, and X9, X10, and XI1 are new observations from our program (proposal: 074422, PI: W. Yuan); Column 2: object name; Column 3: redshift; Column 4: black hole mass in units of the solar mass \(M_\odot\), from Dong et al. (2012); Column 5: Eddington ratios; Column 6: observation ID for XMM-Newton or sequence ID for ROSAT observation; Column 7: mean count rate of each observation (counts s\(^{-1}\)); Column 8: cleaned exposure time (ks); Column 9 and 10: excess variances and errors calculated on a timescale of 10 ks.

\(^8\) The PSD models used in this paper will be introduced in the next subsection, and all of the models yield essentially the same results.
is the break frequency, which is the integral of the PSD over the frequency span \([v_{\text{min}}, v_{\text{max}}]\), where \(v_{\text{max}}\) is set by the bin size (250 s) and \(v_{\text{min}}\) is set by the length (10 ks) of the light curve. The PSD of three AGNs with different \(M_{\text{BH}}\) and thus different break frequency \(v_{\text{br}}\) are shown as examples corresponding to the three cases in Equation (5) of \(v_{\text{br}} \leq v_{\text{max}}\), \(v_{\text{min}} < v_{\text{br}} \leq v_{\text{max}}\), and \(v_{\text{br}} > v_{\text{max}}\), respectively.

### 3.2. PSD Models of AGNs

As mentioned above, the excess variance is the integral of the PSD of a light curve over a frequency interval. If the shape of the PSD is known, then it is possible to calculate the values of the excess variances, and thus to derive the model relation between the excess variance and BH mass, which can be compared to observations. For AGNs, it has been shown that the PSD function has a standard shape of a broken power-law:

\[
P(\nu) = \begin{cases} 
A \left(\nu/\nu_{\text{br}}\right)^{-2}, & \nu \leq \nu_{\text{br}} \\
A \left(\nu/\nu_{\text{br}}\right)^{-4}, & \nu > \nu_{\text{br}}
\end{cases}
\]

where \(\nu_{\text{br}}\) is the break frequency (McHardy et al. 2006; González-Martín & Vaughan 2012). For an AGN with given \(M_{\text{BH}}\) and \(m\), \(v_{\text{br}}\) can be determined, although the exact value differs somewhat in different models. From Equations (2) and (4), the excess variance can be expressed as

\[
\sigma_{\text{rms}}^2 = \begin{cases} 
C_1 v_{\text{br}} \left(\nu_{\text{max}}^{1/3} - \nu_{\text{min}}^{1/3}\right), & \nu_{\text{br}} \leq \nu_{\text{min}} \\
C_1 \ln \left(\nu_{\text{br}}/\nu_{\text{min}}\right) - \frac{\nu_{\text{br}}}{\nu_{\text{max}}} + 1, & \nu_{\text{min}} < \nu_{\text{br}} \leq \nu_{\text{max}} \\
C_1 \ln \left(\nu_{\text{max}}/\nu_{\text{min}}\right), & \nu_{\text{br}} > \nu_{\text{max}}
\end{cases}
\]

where \(C_1 = A \nu_{\text{br}}\) is defined as the PSD amplitude (Papadakis 2004; see also model A in González-Martín et al. 2011 for details). It has been suggested that \(v_{\text{br}}\) is inversely scaled with \(M_{\text{BH}}\) with a possible dependence on the scaled accretion rate in Eddington units. The relationship between PSD and \(\sigma_{\text{rms}}^2\) is illustrated in the sketch in Figure 3, in which the PSD of three objects in our sample or P12 are shown. The break frequencies derived from \(M_{\text{BH}}\) and \(m\) using the González-Martín & Vaughan (2012) scaling relation are also indicated for the three objects, which represent the three cases in Equation (5), respectively.

However, although the broken power-law model given in Equation (4) is widely accepted, the exact values of the amplitude \(C_1\) and the break frequency \(v_{\text{br}}\) differ somewhat in different studies. Papadakis (2004) first suggested a model with \(v_{\text{br}} = 17/(M_{\text{BH}}/10^6 M_\odot)\) (Hz) and \(C_1 = A \nu_{\text{br}} = 0.017\). Later studies with larger samples found a secondary dependence of \(v_{\text{br}}\) on the scaled accretion rate \(\dot{m}\) (McHardy et al. 2006; González-Martín & Vaughan 2012), in addition to the dependence on \(M_{\text{BH}}\). In recent studies, the three following PSD models of AGNs were used:

**Model A:** Adopted by González-Martín et al. (2011), this model assumes the scaling relation \(v_{\text{br}} = 0.003 \dot{m} (M_{\text{BH}}/10^6 M_\odot)^{-1}\) suggested by McHardy et al. (2006) and a constant PSD amplitude of \(C_1 = A \nu_{\text{br}} (=0.017 \pm 0.006)\; (\text{Papadakis 2004})\).

**Model B:** The scaling relation for \(v_{\text{br}}\) is the same as in model A. As argued by Ponti et al. (2012), the dispersion of the \(M_{\text{BH}}-\sigma_{\text{rms}}^2\) relation arising from different \(\dot{m}\) derived from model A is too large to account for the observational data, and a new \(\dot{m}\)-dependent PSD amplitude was suggested: \(C_1 = \alpha \dot{m}^{-\beta}\), where \(\alpha = 0.003^{+0.002}_{-0.001}\) and \(\beta = 0.8 \pm 0.15\) were fitted in their study.

**Model C:** Similar to model A (\(C_1 = 0.017\)), but an improved scaling relation \(\nu_{\text{br}} = 0.001 \alpha \beta^{-2} (M_{\text{BH}}/10^6 M_\odot)^{-1}\) suggested by González-Martín & Vaughan (2012), is adopted, which predicts a weaker dependence on \(\dot{m}\).

### 4. RESULTS AND DISCUSSION

#### 4.1. \(M_{\text{BH}}-\sigma_{\text{rms}}^2\) Relation in the Low-mass Regime

The measured excess variances of 11 low-mass AGNs from our sample are calculated based on the X-ray light curves obtained in Section 2 using Equation (1). The results are listed in Table 1. To enlarge the sample, we make use of both the *XMM-Newton* and *ROSAT* data, which have somewhat different energy bands. It has been shown that the excess variance is not sensitive to energy bands in the range concerned here (Ponti et al. 2012). To enlarge the working sample, we also include eight objects\(^9\) presented in Ludlam et al. (2015) which are not included in our sample. The excess variance values given in Ludlam et al. (2015) are used, which were calculated using a bin size of 200 s, which is different from ours (250 s). However, the effect of such a difference on the calculated excess variance is negligible (<2%), as found by performing simulations using the method of Timmer & Koenig (1995).

Here, we study the relation between \(\sigma_{\text{rms}}^2\) and \(M_{\text{BH}}\) by combining the low-mass sample and the P12 sample. The combined sample has \(M_{\text{BH}}\) spanning four orders of magnitude, \(10^5-10^9 M_\odot\). The results are shown in Figure 4. Also plotted is the relation \(\log(\sigma_{\text{rms}}^2) = -2.09 - 1.03 \log\left(M_{\text{BH}}/10^6 M_\odot\right)\) derived from the P12 sample by Ponti et al. (2012). It shows that the \(\sigma_{\text{rms}}^2\) values of the low-mass sample are comparable to the largest values in P12 with higher \(M_{\text{BH}}\). However, for all of the sources in the combined low-mass sample except one, the

\(^9\) These objects are GH18, GH49, GH78, GH112, GH116, GH138, GH142, and GH211.
excess variances fall systematically below the extrapolation of the $M_{\text{BH}} - \sigma_{\text{rms}}^2$ relation derived from the high-mass P12 sample. Our result indicates that in the low-mass regime, the $M_{\text{BH}} - \sigma_{\text{rms}}^2$ relation may deviate from the previously known linear relation, and is likely to flatten out around $\sim 10^6 M_\odot$ toward the low-mass end. In fact, the low-mass sample objects themselves do not show any correlation between $\sigma_{\text{rms}}^2$ and $M_{\text{BH}}$ (the Spearman’s correlation test gives a null probability of 0.42). To quantify the statistical significance of this deviation, we perform two statistical tests, assuming that the previous inverse linear relation is a good description of the $M_{\text{BH}} - \sigma_{\text{rms}}^2$ relation over the entire mass range. First, we use the two-sided binomial test to find out the probability of having 18 out of 19 low-mass AGNs fall below the relation as observed, whereas an equal probability (50%) of falling on either side of the relation is expected. This provides a probability of $7 \times 10^{-5}$. Moreover, we fit an inverse linear relation with the slope fixed at $-1$ in the log–log space to the data of both the low-mass and the P12 sample. The reduced $\chi^2$/degrees of freedom ( dof) of the fit is 836/53. Then, we assume that the inverse linear relation breaks to a constant at masses below a so-called break mass, which is fitted as a free parameter. The $\chi^2$/ dof value for this model is 705/52 (the break mass is fitted to be $1.7 \times 10^6 M_\odot$). This improves the fit dramatically by reducing $\chi^2$ by $\Delta \chi^2 = 131$ for one additional free parameter. Although the fitting is statistically not acceptable in terms of the large reduced $\chi^2$ (which may arise from some intrinsic scatter inherent to the relationship), as an approximation, we employ the F-test (Blevington & Robinson 1992) to test the significance of adding the flattening term, which does NOT improve the fit. This yields a small $p$-value 0.003. We thus conclude that the flattening of the inverse linear $M_{\text{BH}} - \sigma_{\text{rms}}^2$ relation toward low-mass AGNs is statistically significant. The fitted break mass is $\sim 1.7 \times 10^6 M_\odot$.

We also fit a simple linear relation with the slope as a free parameter in the log–log space using the data of both the low-mass and P12 samples, using the bisector method as in Ponti et al. (2012; see also Appendix A of Bianchi et al. 2009). The best-fit relations are over-plotted in Figure 4 as dotted lines. We find $\log(\sigma_{\text{rms}}^2) = (-2.09 \pm 0.044) + (-0.58 \pm 0.053) \log (M_{\text{BH}}/10^7 M_\odot)$. The slope $\sim -0.58 \pm 0.053$ deviates significantly from the expected value $-1$ based on previous studies of AGN samples with higher $M_{\text{BH}}$ (e.g., Zhou et al. 2010; Ponti et al. 2012).

For low-mass AGNs, there appears to be a large scatter in the $\sigma_{\text{rms}}^2$ values spanning almost one decade. The distribution of the excess variances (in logarithm) for our sample objects with $M_{\text{BH}} < 2 \times 10^6 M_\odot$ is plotted in Figure 5 (left panel), along with a fitted Gaussian distribution (dashed line). It would be interesting to examine the intrinsic dispersion of their $\sigma_{\text{rms}}^2$ distribution by taking into account the uncertainty of individual $\sigma_{\text{rms}}^2$. The maximum-likelihood method as introduced by Maccacaro et al. (1988) is used to quantify the intrinsic distribution (assumed to be Gaussian) that is disentangled from the uncertainty of each of the measured excess variance (also assumed to follow a Gaussian distribution). We find a mean $\log(\sigma_{\text{rms}}^2) = -1.41 \pm 0.091$ with a standard deviation of $\sigma = 0.22 \pm 0.008 \pm 0.008$. The confidence contours of the two parameters are shown in Figure 5 (right panel). This indicates that the apparently large scatter of $\sigma_{\text{rms}}^2$ can mostly be attributed to the uncertainty of each of the measurements (including both the measurement and the stochastic uncertainties), and the intrinsic dispersion is small but non-negligible (the standard deviation of $\log(\sigma_{\text{rms}}^2) \approx 0.15 - 0.3$ at the 68% confidence level). It may also suggest that the dependence of $\sigma_{\text{rms}}^2$ on any other parameters (e.g., accretion rate) is likely not strong.

In conclusion, the previously determined inverse scaling of $M_{\text{BH}} - \sigma_{\text{rms}}^2$ for AGNs with supermassive BHs cannot be extrapolated to the low-mass regime, but starts to flatten at around $M_{\text{BH}} \sim 10^6 M_\odot$. This implies that although the excess variance can still be used to search for AGNs with $M_{\text{BH}} \lesssim 10^6 M_\odot$, it fails to provide accurate BH mass estimates in this mass range. This result is consistent with that obtained in a recent paper by Ludlam et al. (2015). It is also suggested that for AGNs in the $10^3 - 10^6 M_\odot$ range, the intrinsic dispersion of the excess variances is likely to be small (standard deviation of $\log(\sigma_{\text{rms}}^2) \approx 0.15 - 0.3$).

4.2. Comparison with Model Predictions

Here, we compare the observed $M_{\text{BH}} - \sigma_{\text{rms}}^2$ relation with the predictions based on the above three PSD models. In theory, the $M_{\text{BH}} - \sigma_{\text{rms}}^2$ relation can be deduced from the PSD of AGNs using Equation (5) once the shape and parameters of the PSD model are known. We take the three PSD models from Section 3.2 and fix the $\nu_{\text{lo}}(M_{\text{BH}}, m_i)$ relation as their original forms. Since the PSD amplitude $C_1$ was determined by fitting the $M_{\text{BH}} - \sigma_{\text{rms}}^2$ relation in previous works (Papadakis 2004; Ponti et al. 2012), it is set to be a free parameter here (two parameters $\alpha$ and $\beta$ for model B since $C_1 = \alpha m^{-\beta}$). The bivariate models of $\sigma_{\text{rms}}^2(M_{\text{BH}}, m_i)$ from Equation (5) are fit to the data of both the low-mass and P12 samples using the simple $\chi^2$-minimization fitting. The results are given in Table 2 where the errors of the fitted normalization are at the 68% confidence level. The best-fit PSD amplitudes are very close to their original values for all of the models.

The model $M_{\text{BH}} - \sigma_{\text{rms}}^2$ relations using the best-fit PSD normalizations are plotted in Figure 6 for the three models,
Table 2
Fitting Results of the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ Relation with the Three PSD Models

| PSD model | PSD Amplitude | $\chi^2$/dof |
|-----------|---------------|--------------|
| A         | $C_1 = 0.032 \pm 0.001$ | 2048/53      |
| B         | $\alpha = 0.0074 \pm 0.0004, \beta = 0.80 \pm 0.02$ | 598/52       |
| C         | $C_1 = 0.022 \pm 0.001$ | 820/53       |

along with the data. The black, red, and blue solid lines represent the relations for three typical accretion rates ($\dot{m} = 0.02, 0.29, 0.97$), respectively, which are the mean (in logarithm) of $\dot{m}$ in three $\dot{m}$ bins (0.01–0.1, 0.1–0.5, and 0.5–4.0). The objects with $\dot{m}$ in the three $\dot{m}$ bins are plotted in corresponding colors. It can be seen that all three models can well reproduce the observed trend of the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation quantitatively. For a given accretion rate, an inverse proportion is predicted in the high $M_{\text{BH}}$ regime, whereas in the low-mass regime it flattens toward lower $M_{\text{BH}}$. The exact value of $M_{\text{BH}}$ at which the relation starts to flatten (the break mass) depends on $\dot{m}$: the higher $\dot{m}$, the larger the break mass. This trend generally holds for all three PSD models, although the break mass varies from model to model, in addition to its dependence on $\dot{m}$.

Besides this general trend, there are some noticeable differences between the three models. For model A, the strong dependence on $\dot{m}$ leads to a large scatter in the high-mass regime over a range of $\dot{m}$. Model B predicts a smaller scatter, but the scatter is relatively larger in the low-mass regime than in the high-mass regime. Compared to model A, model C also predicts a smaller scatter over the whole $M_{\text{BH}}$ range given its weak dependence of $\sigma_{\text{rms}}^2$ on $\dot{m}$.

The inverse proportion of the $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation and its flattening toward low $M_{\text{BH}}$ can easily be understood from Equation (5) and Figure 3. For $\nu_{\text{br}} \leq \nu_{\text{min}}$ (e.g., NGC 3227 in Figure 3), as $\nu_{\text{br}}$ increases (and $M_{\text{BH}}$ decreases, since $\nu_{\text{br}} \propto M_{\text{BH}}^{-1}$), the integral of the PSD within $[\nu_{\text{min}}, \nu_{\text{max}}]$ increases accordingly, resulting in a linearly increasing $\sigma_{\text{rms}}^2$. Thus, the previously known $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation (for $M_{\text{BH}} > 10^6 M_\odot$) is a manifestation of the more fundamental dependence of $\nu_{\text{br}}$ on $M_{\text{BH}}$ (see also Papadakis 2004; Ponti et al. 2012). It also shows that the secondary dependence of $\nu_{\text{br}}$ on $\dot{m}$ introduces a scatter, the extent of which depends on the models. For $\nu_{\text{min}} < \nu_{\text{br}} \leq \nu_{\text{max}}$ (e.g., MRK 766), the relation becomes non-linear. For $\nu_{\text{br}} > \nu_{\text{max}}$ (e.g. J1023+0405), $\sigma_{\text{rms}}^2$ becomes independent of $\nu_{\text{br}}$ (thus of $M_{\text{BH}}$) and remains a constant.

We have shown that the observed $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation over almost the whole $M_{\text{BH}}$ range for AGNs of $M_{\text{BH}} = 10^5-10^9 M_\odot$ can be explained qualitatively by the PSD shape of AGNs and the dependence of $\nu_{\text{br}}$ on $M_{\text{BH}}$, although the exact relation depends on the details of the models. It would be interesting to investigate which of the above models give a better description of the data, taking advantage of the extended $M_{\text{BH}}$ range provided by our sample. Here, we discuss this issue only briefly by comparing the deviations of the data from the models in terms of the fitted $\chi^2$, although they are too large for the fits to be acceptable nominally for all the three models (the large $\chi^2$ values are partly due to the somewhat large uncertainties in $M_{\text{BH}}$ and $\dot{m}$, which are not taken into account in the fitting).

We discuss the three models, respectively. (i) Model A has a much larger $\chi^2$ value than models B and C, and is likely not a good description of the data. The same suggestion was also argued by Ponti et al. (2012). (ii) For model B, the fitted parameter $\beta$ ($=0.80 \pm 0.02$) is consistent with that of Ponti et al. (2012). For the objects with $M_{\text{BH}} < 2 \times 10^6 M_\odot$, no correlation is found between $\sigma_{\text{rms}}^2$ and $\dot{m}$ (the Spearman correlation test gives a null probability $p = 0.42$), which seems to be inconsistent with the model prediction (see Figure 6, middle panel). However, this may be partly due to the uncertainties in determining $\dot{m}$. (iii) For model C, our result yields a PSD amplitude consistent with that of Papadakis (2004), which is only weakly dependent on $\dot{m}$. This leads the excess variances converging to a constant value toward low $M_{\text{BH}}$, which is roughly consistent with the small intrinsic scatter (not zero, however) found above. Overall, based on the fitted $\chi^2$ values (Table 2), we tend to consider model B, and perhaps model C, to be better descriptions of the PSD shape of AGNs. However, a rigorous comparison of the PSD models is beyond the scope of this paper and will be carried out in a future work with a larger sample and better quality data.

5. CONCLUSION

The $M_{\text{BH}}-\sigma_{\text{rms}}^2$ relation for AGNs in the low-mass regime ($M_{\text{BH}} \lesssim 10^6 M_\odot$) is investigated using a sample of 11 low-
same sample of objects. The sources with mass AGNs observed by XMM-Newton and ROSAT, including both new observations and archival data, as well as 8 sources from Ludlam et al. (2015). We find that the inverse linear \( M_{\text{BH}} - \sigma_{\text{mm}} \) relation established in the high-mass regime in previous studies (e.g., Zhou et al. 2010; Ponti et al. 2012) fails to extend to \( M_{\text{BH}} \) below \( 10^6 M_\odot \). The relation begins to flatten at \( \sim 10^6 M_\odot \), below which the excess variances seem to remain constant. Our result is in good agreement with that obtained from a recent similar study by Ludlam et al. (2015). This is in fact consistent with the model prediction from our current understanding of the PSD of AGNs and the dependence of the break frequency \( \nu_b \) on \( M_{\text{BH}} \). Our result suggests that while the X-ray excess variance may still be used to search for low-mass AGN candidates,\(^{10}\) it fails to provide reliable estimation of the BH mass for AGNs with \( M_{\text{BH}} \lesssim 10^6 M_\odot \). In this \( M_{\text{BH}} \) regime, it is also found that the excess variances show small intrinsic dispersion when their uncertainties (both measurement and stochastic) are taken into account.

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\(^{10}\) As those already carried out by, e.g., Kamizasa et al. (2012), by assuming that the reverse linear scaling relation can be extended to the low-mass regime.

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**Figure 6.** Same as Figure 4 except that the solid lines represent the theoretical relations derived from the best-fit PSD models assuming different normalization and the dependence of break frequency on black hole mass and accretion rate (models A, B, and C; see the text for details). The three colors correspond to three accretion rates of \( m = 0.02 \) (black), 0.29 (red), and 0.97 (blue), which are chosen to be the mean (in logarithm) of \( m \) in three \( m \) bins 0.01–0.1, 0.1–0.5, and 0.5–4.0 of the whole sample of objects. The sources with \( m \) in the three \( m \) bins are plotted in corresponding colors.
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