A High-Precision Method for Dynamically Measuring Train Wheel Diameter Using Three Laser Displacement Transducers

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Abstract: Wheel diameter is a significant geometric parameter related to the safe operation of trains, and needs to be measured dynamically. To the best of the authors’ knowledge, most existing dynamic measurement methods and systems do not meet the requirement that the wheel diameter measurement error for the high-speed vehicle is less than 0.3 mm. In this paper, a simple method for dynamically and precisely measuring train wheel diameter using three one-dimensional laser displacement transducers (1D-LDTs) is proposed for the first time, and a corresponding measurement system which was developed is described. The factors that affect the measurement accuracy were analyzed. As a main factor, rail deformation caused by the wheel-rail interaction force at low (20 km/h) and high (300 km/h) speeds was determined based on the combination of multi-body dynamics and finite element methods, and the effect of rail deformation on measurement accuracy is greatly reduced by a comparative measurement. Field experiments were performed to verify the performance of the developed measurement system, and the results of the repeatability error and measurement error of the system were both less than 0.3 mm, which meets the requirement of wheel diameter measurements for high-speed vehicles.

Keywords: train wheel diameter measurement; dynamic measurement; laser displacement transducer; structural light vision transducer; multi-body dynamics and finite element methods

1. Introduction

Wheelsets are a key component of trains. The wear of wheel treads is becoming more serious, due to continuous increases in the speed and load of trains [1]. As wheel diameter is a significant geometric parameter related to the safe operation of trains, the diameter differences between the wheels of one wheelset, one bogie, one car, and one train must be controlled under a safe limitation. For example, the diameter difference within one wheelset of the high-speed vehicle in operation must be less than 0.3 mm, according to European Norm EN 15313 [2]. The wheel diameter changes every day because of wheel wear; thus, the dynamic measurement of diameter to detect overrun wheels in time is critically important.

Currently, there are many kinds of wheel diameter measurement methods and devices, which can be classified into two categories: shop measurements and field measurements. Shop measurements are usually performed periodically in the train garage. Traditionally, a measuring gauge [3] is utilized to manually measure the diameter based on the three-point measurement method. It is unreliable, as the skill of the user can significantly influence the result. To improve the measuring reliability, Torabi et al. proposed a method based on image processing with a measurement uncertainty of 0.4 mm [4]. Feng et al. proposed a method which uses parallelogram mechanisms together with...
one-dimensional laser displacement transducers (1D-LDTs) [5]. A non-contact measurement method using the LDT and CCD was put forward by Wu et al. [6] with a measuring accuracy of 0.2 mm. The underfloor wheelset lathe can be used in geometric parameter measurements of the wheel tread with the wheel remaining assembled to the bogie [7]; the accuracy of measuring the diameter is within 0.1 mm. The main disadvantage of shop measurement methods is that a potential safety hazard exists, because these methods cannot identify overrun wheelsets in time. Filed measurement is consequently the only choice for the timely discovery of overrun wheelsets. 1D-LDT and structural light vision transducers (SLVTs) are the two main methods for field diameter measurements. Chen et al. used commercial SLVTs to measure the geometric parameters of wheel tread, including wheel diameter [8,9]. Similar measurement systems were developed by corporations [10–12]. These methods used multiple sets of transducers which required global calibration on-site to unify the coordinated system; this approach is occasionally difficult, and the diameter measurement error is about 0.5 mm, which cannot meet the requirement of being less than 0.3 mm for the wheels of a high-speed vehicle, according to Chinese Standards [13]. The measurement accuracy of diameter can be improved by 1D-LDT, as the 1D-LDT has higher measurement accuracy. Naumann et al. proposed a measurement system using at least 26 1D-LDTs [14], which is difficult to apply on-site. We propose a series of methods using 1D-LDTs [15,16]. One 1D-LDT was used to dynamically measure the diameter and an eddy current sensor was used to locate the position of the wheel [15]. In order to improve the accuracy of the positioning, two eddy current sensors were used [16]. However, the measurement accuracy of these methods will be significantly reduced due to wear of the wheel tread and rail top. To solve this problem, we further proposed a simple method using three 1D-LDTs. The corresponding measurement system was developed, and field experiments were carried out. The results show that the repeatability error and measurement error are less than 0.3 mm, which meet the requirement of Chinese high-speed railway operations [13], and pave a new way for dynamically measuring wheel diameters with greater precision. The rest of this paper is organized as follows. The proposed method is described in Section 2. Section 3 analyses the factors that affect the measuring accuracy. In Section 4, the results of field experiments are presented. Finally, the conclusions of this study are given in Section 5.

2. Measurement Method

2.1. Measurement Method Based on One 1D-LDT

Figure 1a shows the method for dynamically measuring the wheel diameter using one 1D-LDT and eddy current sensor [15]. As shown in Figure 1a, the 1D-LDT is fixed on the rail at point A with an angle $\alpha$. When the train wheel runs on the rail along the X axis, the 1D-LDT measures the distance $l_1$ between the transducer and the wheel tread at the point of B. The coordinate of B is $(l_1 \cos \alpha, l_1 \sin \alpha)$. An eddy current sensor is mounted at point C to measure the wheel position, which is $L_1$ away from point A. In fact, the eddy current sensor can be replaced by 1D-LDT. Thus, the equation of the circle is expressed as

$$(x - L_1)^2 + (y - R)^2 = R^2. \quad (1)$$

where, $R$ is the radius of the wheel. According to the measured distance $l_1$ of the 1D-LDT when the wheel is in the position of point C, the diameter is calculated by

$$D = \frac{L_1^2 + l_1^2}{l_1 \sin \alpha} - \frac{2L_1}{\tan \alpha}. \quad (2)$$

The eddy current sensor is used to measure the wheel position; however, the output of the eddy current sensor changes slowly when it is closer to the lowest point of the wheel, as shown in Figure 1b. As a result, the positioning accuracy is low, which greatly influences the accuracy of diameter measurements. To improve the accuracy of the positioning, two eddy current sensors [16] are mounted symmetrically at appropriate positions on both sides of the point C, as shown in Figure 1c, and the two
outputs of the eddy current sensors change quickly at the point C, as shown in Figure 1d. There is a zero position of the difference between the two outputs, and this point is used to determine the position of the wheel, thus improving the positioning accuracy and the diameter measurement accuracy.

2.2. Measurement Method Based on Two 1D-LDTs

To reduce the influence of roundness on the diameter measurements, two 1D-LDTs were used, as shown in Figure 2a. The second transducer is fixed on the rail at the point E with an angle $\beta$, which is $L_2$ away from the point C. When the train wheel runs on the rail along the X axis, two transducers can measure the distances $l_1$ and $l_2$ synchronously, and the outputs of the two 1D-LDTs are shown in Figure 2b. Figure 2c shows the sum of the two outputs of 1D-LDTs. From Figure 2c, there is a minimum value which is used to determine the wheel position with a high level of accuracy. Thus, the measured distances $l_1$ and $l_2$ of the two 1D-LDTs are determined when the wheel is in the position of point C.
According to the Equation (2), the diameter of wheel is expressed as

\[ D = \frac{L_1^2 + L_2^2}{2l_1 \sin \alpha} - \frac{L_1}{\tan \alpha} + \frac{L_2^2 + l_2^2}{2l_2 \sin \beta} - \frac{L_2}{\tan \beta}. \] (3)

### 2.3. Measurement Method Based on Three 1D-LDTs

The above three methods (in Figure 1a,c, and Figure 2a) are based on the assumption that the point C between the wheel and the rail is fixed. In fact, it will change, due to wear of the wheel tread and rail top. If the point C is changed to C’, as shown in Figure 2d, it will yield a measurement error for the diamet er of the circle. From Equation (4), the diameter of the measured circle can be obtained. To reduce the measurement error, a simple method using three 1D-LDTs is proposed.

Figure 3a shows the setup for dynamically measuring the diameter using three 1D-LDTs, which are called S1, S2, and S3, respectively. Three transducers are mounted on the standard rail. The distances between S2 to S1 and S3 are L1 and L2, respectively. In addition, the emitting spots of S1 and S3 are projected to the wheel tread at the angles of \( \alpha \) and \( \beta \), respectively.

\[ (x - L_1)^2 + (y - R - h)^2 = R^2. \] (4)

where \( R \) is the radius of the measured circle. From Equation (4), the diameter of the measured circle \( D_1 \) is calculated by

\[ D_1 = \frac{L_1^2 + L_2^2 + h^2 - 2L_1l_1 \cos \alpha - 2l_1h \sin \alpha}{l_1 \sin \alpha - h}. \] (5)
Similarly, the diameter of the circle $D_2$ measured by $S_2$ and $S_3$ can be obtained. To reduce the influence of roundness, the average of the two diameters $D_1$ and $D_2$ is the optimal diameter $D_m$ of the measured circle. With $L_1 = L_2 = L$ and $\alpha = \beta$, it is expressed by

$$D_m = \frac{L^2 + l_1^2 + h^2 - 2Ll_1 \cos \alpha - 2l_1h \sin \alpha}{2l_1 \sin \alpha - 2h} + \frac{L^2 + l_2^2 + h^2 - 2Ll_2 \cos \alpha - 2l_2h \sin \alpha}{2l_2 \sin \alpha - 2h}. \quad (6)$$

When the train wheel passes through the measurement system, the three transducers synchronously measure the distance between the transducer and the wheel tread. Figure 4 shows the typical outputs of the transducers. The closest distance $h$ between $S_2$, and the measured circle can be obtained by fitting using the sampling points of $S_2$. Similar to the method using two 1D-LDTs, the sum of the outputs of $S_1$ and $S_3$ is used to determine the wheel position. Then, the measured distances $l_1$ and $l_2$ of the other two transducers $S_1$ and $S_3$ are obtained, respectively. Finally, the diameter of the measured circle can be determined according to Equation (6). Different from the methods shown in Figure 1a,c, and Figure 2a, $S_2$ in Figure 3b is used to measure distance $h$, rather than for determining the wheel position. If there is the wear on the wheel tread and rail top, the distance $h$, $l_1$, and $l_2$ will be changed correspondingly, and Equation (6) is still stable; thus, the diameter measurement accuracy will not be affected by the wear of the wheel tread and rail top.

![Figure 4. Typical measuring distance of the three transducers.](image)

Figure 5 shows the wheel tread profile. Point C is defined as the contact point between the wheel and the rail, which is 70 mm to the straight line $AB$ in the wheel rim segment. The rolling circle consists of all the contact points around the wheel, the diameter of which is defined as the wheel diameter. Point $E$ is the measured point of the 1D-LDT, which is in the straight line $DF$. The distance $d_{EC}$ between the measured point and the contact point is known by the layout of the three 1D-LDTs, the typical value of which is 47 mm. It should be noted that $D_m$ in Equation (6) is not the wheel diameter, but the diameter of the measured circle. To measure the wheel diameter, the distance $h_{EC}$ between the measured point and contact point should be determined. Thus, the SLVT is used to measure the wheel tread profile, the measurement principle of which is described in the reference [17].

The two-dimensional coordinates of the wheel tread profile can be obtained by the SLVT. After that, the straight line $AB$ can be obtained by fitting; then, the coordinate of contact point $C$ is determined by the distance to the line $AB$. As the distance $d_{EC}$ is known, the coordinate of the measured point can be accurately determined. The difference between the $y$-coordinates of the two points is $h_{EC}$, and the wheel diameter $D$ can be obtained by

$$D = D_m + 2h_{EC}. \quad (7)$$
3. Measurement Error Analysis

From Equations (6) and (7), the diameter can be calculated simply by measuring the special distances of \( l_1, l_2, h, \) and \( h_{EC} \). \( h_{EC} \) is measured by the SLVT, and the measurement error can be less than 0.1 mm [18], which results in an error for measuring the diameter of less than 0.2 mm. There are four other primary factors that can influence the diameter measurement accuracy: The S-shaped motion of the wheel, the measurement error of 1D-LDT, the positioning error of the wheel, and the rail deformation caused by the wheel-rail interaction force. The measurement errors caused by the S-shaped motion of the wheel and the rail deformation are systematic errors for which it is possible to compensate. The measurement errors caused by the measurement error of 1D-LDT and the positioning error of the wheel are random errors, for which it is not possible to compensate. In the following section, these factors are analyzed and simulated in theory, where \( L \) is set to 480 mm, \( h \) is set to 80 mm, \( \alpha \) is set to 60°, and \( l \) is set to 285 mm.

3.1. Measurement Error of the Transducer

Figure 6 shows the theoretical stimulation results of the diameter measurement error caused by the measurement errors of \( S_1 \) and \( S_3 \), as well as \( S_2 \). In Figure 6, the measurement error of the transducer is set between −0.2 and 0.2 mm at intervals of 0.05 mm. In fact, the measurement error of the transducer can be less than 0.05 mm [19]; thus, the diameter measurement error is less than 0.24 mm caused by the measurement errors of \( S_1 \) and \( S_3 \). For \( S_2 \), it is less than 0.15 mm.

3.2. Positioning Error of the Wheel

Figure 7 shows the theoretical stimulation result of the diameter measurement error produced by the wheel positioning error, where the positioning error is zero when \( S_2 \) is below the lowest point of
the wheel. According to Equation (6), if the positioning error is within 5.0 mm, the diameter measuring error is less than 0.13 mm. Actually, it is easy for the wheel positioning error being controlled to be less than 1 mm based on the sum of the outputs of \(S_1\) and \(S_2\); thus, the diameter measurement error caused by the wheel positioning error is negligible.

![Figure 7](image1.png)

**Figure 7.** Diameter measurement error caused by the positioning error of the wheel.

### 3.3. S-Shaped Motion of the Wheel

Figure 8 shows the tread profile with the S-shaped motion of the wheel, wherein the standard tread profile is obtained by measuring the standard wheel using the SLVT. Owing to the S-shaped motion of the wheel, the contact point \(C\) has changed to \(C'\), and the measured point \(E\) has changed to \(E'\) in the Y-axis direction. In Figure 4, the S-shaped motion displacement \(\Delta d_{EC}\) can be measured by the difference between the two straight lines in the wheel rim segment. Thus, the corrected distance \(d_{EC}'\) between the contact point and the measurement point is expressed by

\[
d_{EC}' = d_{EC} - \Delta d_{EC}.
\]

![Figure 8](image2.png)

**Figure 8.** Tread profile with the S-shaped motion of wheel.

Here, \(d_{EC}'\) is used to search the measured point in the tread profile with the S-shaped motion of the wheel, and the corrected distance \(h_{EC}'\) can be determined. In this way, the influence of the S-shaped motion of the wheel on the diameter measurement can be eliminated in theory.

### 3.4. Rail Deformation

Rail deformation caused by the wheel-rail interaction force is inevitable. As shown in Figure 9a, there is a distance \(\Delta d_S\) in the vertical direction of \(S_1\) to \(S_2\) and \(S_3\) to \(S_2\) because of rail deformation when
the train passes through the transducers; thus, the measured distance error $\Delta l$ of the transducer and the direction error $\Delta \alpha$ of the laser beam are produced. From Figure 9b, $\Delta l$ can be expressed by

$$\Delta l = \Delta d_S \sin \alpha.$$  

(9)

From Figure 9c, $\Delta \alpha$ is obtained by

$$\Delta \alpha = \arctan \left( \frac{\Delta d_S}{L} \right).$$  

(10)

To analyze the effect of rail deformation on diameter measurement, $\Delta d_S$ is a key parameter and should be determined. A vehicle-track, rigid-flexible coupling model was established using the combination of multi-body dynamics and finite element methods, and rail deformation was analyzed at low and high train speeds.

Figure 10 shows a dynamic model of the vehicle system, wherein the vehicle and the track constitute the vehicle-track coupling dynamic system. The vehicle model is a multi-body system composed of one body, two frames, four wheels, and the primary and secondary suspension systems. The finite element analysis software Abaqus CAE [20], which is developed by Dassault Systems and it is a complete solution for finite element modeling, visualization, and process automation, was used to produce the flexible rail at first, and the elastic part was then generated using the software Simpack [21], which is a general multi-body simulation software enabling analysts and engineers to simulate the non-linear motion of any mechanical or mechatronic system. The values of the parameters used in the simulation are listed in Table 1. Using a combination of the structural parameters and suspension parameters of the vehicle system, the vehicle-track rigid-flexible coupling model was established.
Thus, the diameter measurement errors are ±0.19 and ±0.17 mm.

Figure 12. Simulation results: displacements of the transducers at the speeds of (a) 20 km/h and (b) 300 km/h.

Figure 10. Dynamic model of the vehicle system.

Table 1. Parameters used in simulation.

| Parameters                          | Values                  | Parameters                          | Values                  |
|-------------------------------------|-------------------------|-------------------------------------|-------------------------|
| Mass of car body                    | 3.43 \times 10^4 kg    | Pitch moment of inertia of bogie    | 1205 kg·m^2            |
| Mass of bogie                       | 2235 kg                 | Yaw moment of inertia of bogie      | 2792 kg·m^2            |
| Mass of wheel                       | 1200 kg                 | Stiffness of primary suspension system | 9.198 \times 10^5 N/m |
| Roll moment of inertia of car body  | 9.25 \times 10^4 kg·m^2| Stiffness of secondary suspension system | 1.33 \times 10^5 N/m |
| Pitch moment of inertia of car body | 1.756 \times 10^5 kg·m^2| Damping of primary suspension system | 1 \times 10^8 N/s/m   |
| Yaw moment of inertia of car body   | 1.728 \times 10^5 kg·m^2| Damping of secondary suspension system | 8400 N/s/m            |
| Roll moment of inertia of bogie     | 1846 kg·m^2             |                                      |                         |

Displacement-time curves of S_1, S_2, and S_3 at the different speeds of the train using the vehicle-track rigid-flexible coupling model are shown in Figure 11. The displacement-time curve of the transducer can represent rail deformation, as it is fixed on the rail. From Figure 11a,b, rail deformation was different at low and high speeds. At a low speed of 20 km/h, the distance Δd_5 was 0.18 mm. At a high speed to 300 km/h, it was 0.15 mm. From Equation (9), the measured distance errors Δl were 0.16 and 0.14 mm at speeds of 20 and 300 km/h, respectively. Thus, the diameter measurement errors are ±0.75 and ±0.66 mm according to Figure 6.
For wheel diameter being in a range of 760–920 mm. Thus, the influence of rail deformation on diameter measurement errors are 0.18 mm. At a high speed to 300 km/h, it was 0.15 mm. From Equation (9), the measured distance is compensated by the comparative measurement with a standard wheel whose diameter is known. Thus, Equation (6) should be expressed as

\[
D_m = D_m' + \frac{(l_1' - l_1)(L^2 \sin \alpha - h^2 \sin \alpha - 2Lh \cos \alpha + hl_1' + hl_1 - l_1' l_1 \sin \alpha)}{2(l_1' \sin \alpha - h)(l_1 \sin \alpha - h)} + \frac{(l_2' - l_2)(L^2 \sin \alpha - h^2 \sin \alpha - 2Lh \cos \alpha + hl_2' + hl_2 - l_2' l_2 \sin \alpha)}{2(l_2' \sin \alpha - h)(l_2 \sin \alpha - h)}
\]

(11)

where \(D_m'\) is the standard diameter of the measured circle in the standard wheel, \(l_1'\) and \(l_2'\) are the output distances of \(S_1\) and \(S_3\) to measure the standard wheel, respectively. Figure 13 shows the simulation results based on the comparative measurement, where the diameter of the standard wheel is 840 mm. For both low and high speeds, the diameter measurement error is less than 0.1 mm, with the wheel diameter being in a range of 760–920 mm. Thus, the influence of rail deformation on diameter measurement can be greatly reduced.

**Figure 12.** Diameter measurement error caused by the angle change of the laser emission.

The diameter measurement errors caused by the rail deformation are systematic and can be compensated by the comparative measurement with a standard wheel whose diameter is known. Thus, Equation (6) should be expressed as

\[
D_m = D_m' + \frac{(l_1' - l_1)(L^2 \sin \alpha - h^2 \sin \alpha - 2Lh \cos \alpha + hl_1' + hl_1 - l_1' l_1 \sin \alpha)}{2(l_1' \sin \alpha - h)(l_1 \sin \alpha - h)} + \frac{(l_2' - l_2)(L^2 \sin \alpha - h^2 \sin \alpha - 2Lh \cos \alpha + hl_2' + hl_2 - l_2' l_2 \sin \alpha)}{2(l_2' \sin \alpha - h)(l_2 \sin \alpha - h)}
\]

(11)

where \(D_m'\) is the standard diameter of the measured circle in the standard wheel, \(l_1'\) and \(l_2'\) are the output distances of \(S_1\) and \(S_3\) to measure the standard wheel, respectively. Figure 13 shows the simulation results based on the comparative measurement, where the diameter of the standard wheel is 840 mm. For both low and high speeds, the diameter measurement error is less than 0.1 mm, with the wheel diameter being in a range of 760–920 mm. Thus, the influence of rail deformation on diameter measurement can be greatly reduced.

**Figure 13.** Diameter measurement error caused by an angle change of the laser emission.

4. Field Experiments

The corresponding system for dynamically measuring the train wheel diameter based on the aforementioned method was developed, and was installed in the storage line of Yinchuan railway station vehicle depot, China, as shown in Figure 14a. Figure 14b,c show images of the 1D-LDTs and SLVTs, respectively. Here, three developed 1D-LDTs and two developed SLVTs were used. In Figure 14b, the three 1D-LDTs were installed in a box, which was mounted on the standard rail. In the box, \(S_1\) and \(S_3\) were mounted symmetrically, 480 mm away from \(S_2\). The lasers of \(S_1\) and \(S_3\) were at an angle of 60°. In addition, \(S_2\) had a measurement range of 40 mm, and the start measurement distance was 90 mm. For \(S_1\) and \(S_3\), the measurement range was 80 mm, the start measurement distance was 280 mm. A grating ruler (LG-50, accuracy: 0.1 μm, resolution: 50 nm) was used to calibrate the
1D-LDTs. The results showed that the measurement accuracy was 20 μm for \( S_2 \), and that was 15 μm for \( S_1 \) and \( S_3 \). In Figure 14c, the SLVT was mainly composed by a camera (acA1600-20gm, Basler, pixel: 1626 × 1236), a lens (M1620-MPW2, Computer, focal length: 16 mm), and a line laser (EL65L200IG9, Elite, power: 200 mW), which was installed in an individual box. Two SLVTs, mounted on both sides of the standard rail, were used to produce a complete tread profile. When the train wheel passed through the measurement system, the three 1D-LDTs sampled synchronously with a frequency of 5 kHz, and the two SLVTs were triggered by a photoelectric switch. The real outputs of the three 1D-LDTs and two SLVTs are shown in Figure 14d,e respectively.

![Image of the field experiment and real outputs of the transducers](image_url)

**Figure 14.** Images of the field experiment and real outputs of the transducers: images of (a) developed measurement system, (b) one-dimensional laser displacement transducers (1D-LDTs), and (c) structural light vision transducers (SLVTs); outputs of (d) three 1D-LDTs and (e) two SLVTs.

To verify the performance of the developed measurement system, the diameters of four wheels with a small roundness error in a train car (ID: 350122), which passed through the measuring system three times in a month, were dynamically measured. The experimental information and results are shown in Table 2.
Table 2. Experimental information and results.

| Number | Data and Time      | Train Speed (km/h) | Temperature (°) | Wheel No. | 1 (mm)   | 2 (mm)   | 3 (mm)   | 4 (mm)   |
|--------|--------------------|--------------------|----------------|-----------|----------|----------|----------|----------|
| 1      | 18:21, 4 June, 2019| 3.1                | 26.6           |           | 891.10   | 881.64   | 887.67   | 889.12   |
| 2      | 18:22, 5 June, 2019| 3.5                | 28.4           |           | 890.68   | 881.45   | 887.59   | 889.09   |
| 3      | 23:08, 22 June, 2019| 3.9                | 20.3           |           | 891.22   | 881.45   | 887.32   | 889.27   |

Average value (mm) 891.00 881.51 887.53 889.16
Repeatability error (mm) 0.27 0.09 0.17 0.09
Result of the lathe (mm) 889.38
Deviation (mm) 0.22

Table 2 shows that our system has a good performance in repeatability error. The repeatability error of wheels 2 and 4 is 0.09 mm, and the maximum repeatability error is 0.27 mm. In addition, the diameter of wheel 4 was measured by an underfloor wheelset lathe (U2000-150, hegenscheidt, diameter measurement accuracy: 0.1 mm); the result was 889.38 mm. Thus, the deviation between our system and the lathe is 0.22 mm. Consequently, the repeatability and measurement errors of the proposed system are less than 0.3 mm, which meets the requirement of Chinese high-speed railway operations.

5. Conclusions

In this paper, a high-precision method using three 1D-LDTs for dynamically measuring wheel diameters was first proposed. The factors that influenced measurement accuracy were analyzed. As a main factor, the rail deformation caused by the wheel-rail interaction force at low (20 km/h) and high (300 km/h) speeds was determined based on the combination of the multi-body dynamics and finite element methods, and the effect of rail deformation on measurement accuracy was greatly reduced by a comparative measurement. A corresponding measurement system was developed. The performance of the proposed measurement system was verified by field experiments. The experimental results demonstrate that the repeatability and measurement errors of the system are less than 0.3 mm. Future work will focus on high-speed dynamic diameter measurements for high-speed trains.

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