Investigation of phase objects using off-axis digital holography with a-priori known information on the reference wave

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Abstract. Off-axis digital holography with a-priori known information on the reference wave allows phase reconstruction from a single interference pattern, thus, being a promising tool for investigations of dynamic processes. The paper presents the method aimed for estimation of angles between the object and reference wave from a single digital hologram. This information is essential for hologram reconstruction using a-priori known data on the reference wave and may also be beneficial in other reconstruction methods. The technique was validated using a specially prepared phase object; the recording was performed at various angles between the object and reference beams. Main factors affecting the reconstructed phase quality were analyzed.

1. Introduction
Nowadays a wide variety of physical processes is studied using holographic techniques. Holography provides nondestructive recording of spatial and temporal variations of object properties. Analyzing the reconstructed phase one can study particle flows [1], elastic waves in solids [2], thin film vibrations and many other physical phenomena. At present time a wave front reconstruction and phase image processing are performed faster and easier by means of the computer techniques.

Digital photo detector arrays are more sensitive than holographic materials. Besides that, using phase and amplitude digital images one can numerically propagate the wave front backwards and thus get the exact coordinates of all registered amplitude objects. The most common methods of digital holography operate by analogy to that of classical holography: the reconstructed wave front is numerically propagated back to the object plane [3] which results in the formation of diffraction orders containing real and virtual images of the object. Technical characteristics of modern digital sensors used for hologram recording (pixel size and matrix dimensions) impose restrictions on the angle between the object and reference beams and cause reduction of the reconstructed image size. Another group of digital holographic techniques [4] allows one to reconstruct the object wave phase from a set of holograms recorded with a reference wave gaining a particular phase shift from one hologram to another. Such techniques do not provide an instantaneous phase recording and thus are not applicable for studies of fast processes.
2. Off-axis digital holography technique

The method applied in our research is free of these disadvantages and allows one to reconstruct full-size images of the real and virtual parts of the wave front from a single digital hologram [5]. We note however that angles between the object and reference beams are rather small and are difficult to measure. In our experiments a method of angle estimation from the recorded interference pattern had been developed and applied. The algorithm of the object wave amplitude and phase reconstruction is as follows. Assume the reference wave amplitude \( B \), the object wave amplitude \( A \) and the phase \( \phi \) to be constant within a small square area around each pixel. This approximation is feasible for large enough angles between the object and reference beams when the reference wave phase and the interference pattern intensity oscillate sufficiently rapidly. Then the interference of the object and reference waves can be described by the following equation:

\[
I = \left| B \exp(i\phi) + A \exp(i\alpha) \right|^2, \tag{1}
\]

Applying this equation to each pixel of the square area we obtain an overdetermined system of equations. Unknown parameters can be found then by solving this set of equations using the least-square method, i.e.:

\[
\sum w_m \left| I_m - |A \exp(i\alpha) + B \exp(i\phi)|^2 \right|^2, \tag{2}
\]

Weight factors \( w_m \) adjust an impact of each pixel to the minimized sum; their magnitudes decrease with the distance from the central pixel. That ensures that even if the hypothesis of \( B, A \) and \( \phi \) constancy near the edge of the area under consideration is not fulfilled, it almost does not affect the solution. This sum minimization leads to the system of three linear equations containing three unknown parameters constant within the area [5]. The system solution gives the object wave amplitude and phase at the central pixel. Note that the reference wave phase is supposed to be known. The phase of the object wave cannot be reconstructed without this information.

In most cases it is convenient to use a plane reference wave. So it can be defined precisely from the measured value of an angle between the reference and object waves. Instead of direct measurements of the angle, which is usually difficult due to its smallness, one can simulate the interference of two plane waves and compare the result obtained with the recorded interference pattern. Alternatively the angle can be calculated from a number of interference fringes recorded on the hologram per unit length using the equation:

\[
\beta = \arcsin \left( \frac{\lambda}{d} \right), \tag{3}
\]

where \( d \) is the interference fringe width.

Knowing the angle between the object and reference waves one can model the object wave numerically using the equation:

\[
B(x) = \exp \left[ \frac{2\pi i}{\lambda} x \cos(\beta) \right], \tag{4}
\]

3. Experiments and discussion

The described technique was validated in experiments with a specially prepared test object. First the required phase image was modeled on the computer in a form of a grayscale image. This image was projected on the high-resolution photographic negative film. The film was tightly fixed to the dichromated gelatin layer of a holographic plate. Being then exposed by laser radiation from the film side, dichromated gelatin undergoes photolytical dissociation which is stronger in those areas, where the emulsion absorbs more light i.e. where the film transmission is higher. Due to thus induced variations
of refractive index in the dichromated gelatin layer, the phase relief copying the grayscale image is formed. To ensure that the test object does not affect the wave front amplitude, intensities of the transmitted laser radiation in several planes were registered, as shown in Figure 1. Since the contour of the test object is visible only in the defocused image (due to light diffraction on the phase relief), the sample indeed does not affect the amplitude component of the light.

The experimental validation of the technique was performed using thus obtained test object. Several digital holograms were recorded with different angles between the object and reference waves. The reconstructed phase images showed good quality for a wide range of angles which indicates that the method applied has less angular restrictions then the Fourier-transformation technique. The major reason of the quality decrease of the reconstructed wave front in our experiments is a low contrast of the fringe pattern. Nevertheless it was shown that even a low-contrast digital hologram can be reconstructed and both the phase information and amplitude image can be obtained.

![Figure 1. Transmitted light intensities registered in the test object plane (a) and in an out-of-focus plane (b).](image1)

![Figure 2. Reconstructed amplitude (a) and phase (b) of the test object.](image2)

Thus, the technique of off-axis digital holography with a-priori known information on the reference wave has been tested with the specially made phase object. The main factors affecting the reconstructed phase quality were analyzed. The method of angles estimation had been suggested allowing to render automatic reconstruction of holograms. The applied technique of complex wave retrieval from a single off-axis hologram possesses several important advantages over more commonly used Fourier transformation and phase shifting holography techniques and, in particular, can provide reliable phase reconstruction in studies of dynamic processes.

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