Chaos Controlling Problems for Circuit Systems with Josephson Junction

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Abstract. The complex dynamical characters of the Josephson junction circuit system are studied and the tunnel effect is considered. The dynamical equation of the system is established. The route from periodic motion to chaos is illustrated using bifurcation diagram. An adscititious coupling controller is constructed to control the chaos.

1. Introduction
Before 1972, realizations of the volt were made by assigning values to carefully stabilized banks of Weston cells. These Weston cells served as a kind of flywheel to maintain the unit of voltage between the comparison experiments with the SI definition of the volt. Drifts and problems with the transportability of Weston cells limited the uniformity of voltage standards around the world to about 1 ppm. The voltage standard clearly needed an intrinsic realization, much as the meter, is defined in terms of the speed of light. Fortunately, such an intrinsic standard became feasible with the discovery of the Josephson effect in 1962 [1]. The dynamics of the Josephson junction circuit system have been studied widely[2-4]. The discrete Josephson junction circuit could be either in periodical state or chaotic state, depending on the parameters used. The question is whether the chaotic state can be controlled or driven to a stable state. Starting with the pioneering work of Ott et al.,[5] various chaos control methods have been proposed[6] and applied to physical and nonlinear dynamical systems[7].

In the paper, the complex dynamics characters of an equivalent circuit of Josephson junction circuit, RSJ model, is studied. And the chaos is controlled by two methods. The advantages of the two controlled methods are discussed.

2. The RSJ model of Josephson junction circuit and its route to chaos
An equivalent circuit of Josephson junction circuit, RSJ model, is depicted in Figure 1. $I(t)$ is current supplied by outer power source. $R$ is resistance of junction. $C$ is capacitance of junction and $I_c(t)$ is the maximum Josephson current when temperature is $T$ without noise. Because super-current and normal current pass through Josephson junction, resistance effect and capacity effect must be considered.

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Figure 1. RSJ model of equivalent circuit of Josephson junction circuit system.

\[ I = I_c \sin \varphi \]

\[ I_s = I_c + I_n + I_d \]  \hspace{1cm} (1)

Where \( I_s \) is current passing through ideal junction, \( I_n \) is normal current, \( I_d \) is displacement current and \( I_c \) is direct-current of current passing through detour. In which \( I_c = I_s \sin \varphi \), here \( \varphi \) is phase difference between both ends of wave function. If the tunnel effect of resistance is considered

\[ I_c = (1 + \mu \cos \varphi)V / R \]  \hspace{1cm} (2)

The equation (1) can be depicted

\[ I_{sc} = I_c \sin \varphi + (1 + \mu \cos \varphi)V / R + CdV / dt \]  \hspace{1cm} (3)

Relation of phase difference \( \varphi \) and voltage \( V \) is

\[ \frac{d\varphi}{dt} = 2eV(t) / h \]  \hspace{1cm} (4)

Where \( h \) is Plank constant. So equation (3) can be written as

\[ I_{sc} = (\frac{hC}{2e}) \frac{d^2\varphi}{dt^2} + (\frac{h}{2eR})(1 + \mu \cos \varphi) \frac{d\varphi}{dt} + I_c \sin \varphi \]  \hspace{1cm} (5)

If current contains direct-current and alternating current, the dynamics equation of Josephson junction circuit is

\[ I_{sc} + I_{sc} \sin \Omega t = (\frac{hC}{2e}) \frac{d^2\varphi}{dt^2} + (\frac{h}{2eR})(1 + \mu \cos \varphi) \frac{d\varphi}{dt} + I_c \sin \varphi \]  \hspace{1cm} (6)

Adopt non-dimensional constant

\[ \rho = I_{sc} / I_c, \quad \alpha = I_{sc} / I_c, \quad \beta = 2eI_cCR^2 / h \]

And denoting

\[ \omega = \Omega \sqrt{hc / 2eI_c}, \quad \tau = t \sqrt{2eI_c / hC} \]

Equation (6) can be depicted as

\[ \frac{d^2\varphi}{d\tau^2} + \frac{1}{\sqrt{\beta}}(1 + \mu \cos \varphi) \frac{d\varphi}{d\tau} + \sin \varphi = \rho + \alpha \sin \omega \tau \]  \hspace{1cm} (7)

Denoting \( \varphi = x, d\varphi / d\tau = y, \tau = 1 / \sqrt{\beta} \). Equation (7) can be rewritten in the standard form of the two-dimensional system

\[
\begin{cases}
\dot{x} = y \\
\dot{y} = \rho + \alpha \sin \omega \tau - \sin x - r(1 + \mu \cos x)y
\end{cases}
\]  \hspace{1cm} (8)
Denoting the right hand of equation (8) equal to 0, the equilibrium points are
\[
\begin{align*}
\begin{cases}
y = 0 \\
x = \arcsin(p + \alpha \sin \omega r)
\end{cases}
\end{align*}
\] (9)

The Jacobian Matrix of equation (8) is
\[
J = \begin{bmatrix}
0 & 1 \\
-\cos x + r \mu \sin x & -r(1 + \mu \cos x)
\end{bmatrix}
\] (10)

The eigenvalue of equation (10) is
\[
[\lambda E - J] = \begin{bmatrix}
\lambda & -1 \\
\cos x - r \mu \sin x & \lambda + r(1 + \mu \cos x)
\end{bmatrix}
\] (11)

Namely
\[
\lambda^2 + r(1 + \mu \cos x)\lambda + \cos x - r \mu \sin x = 0
\] (12)

Equation (12) is a two-dimensional equation, and it can be solved by numerical method. According to the value of \(\lambda\), the type and character of bifurcation can be divided.

When the values of parameters \(\rho, \omega, r, \mu\) are given as 0,0.5,1,0.2, and \(\alpha\) is regarded as a bifurcation parameter, the route to chaos of the system can be analyzed by numerical solution. Chaotic motion can be easily observed in the system. If changing the value of bifurcation parameter, \(\alpha\), the periodic behaviors will turn into chaotic motion. The motion of the system turns from periodic behaviours to chaotic motion, when changing the value of \(\alpha\) from 1.6 to 1.67. Its bifurcation diagram is depicted in Figure 3 and its Lypunov exponent map is depicted in Figure 4. When value of \(\alpha\) is given as 1.6 the motion of the system (8) is period-1 motion as depicted in Figure 2.(a). The periodic orbit loses its stability and became doubling bifurcation as in Figure 2 (b) to Figure 2 (d) with the value of \(\alpha\) increasing from 1.6 to 1.638. Chaotic motion will appear in system when the value of \(\alpha\) is more than 1.638, as depicted in Figure 2 (d). Its chaotic motion is proved by its Lypunov exponent map in Figure 4.
3. Two methods of chaos controlling

3.1. Controlling chaos based on adscititious coupling controller

Denoting the dynamic equation of a chaos system as

\[ \dot{x} = f(x, c) \]  

(13)

Where \( x \in \mathbb{R}^n \) is the state variable of the system, \( c \) is system’s parameter. And denoting the value of \( c \) can make system (13) chaotic.

Then couple a autonomous linear system onto the main system of the chaos. Supposing the adscititious coupling controller is a simple progressive stable system. It can be depicted as

\[ \dot{y} = g(y, e) \]  

(14)

Where \( y \in \mathbb{R}^m \), \( e \) is control parameter of the controller and it can be changed easily. If the dimension of the control system \( m = 2 \), the adscititious coupling controller is

\[ \dot{y} + e(y - x) = 0 \]  

(15)

The original chaotic system added a coupling matrix which couples with the controlled will be

\[ \dot{x} = f(x, c) + e(x - y) \]  

(16)

The adscititious coupling controller will add an additional signal to the original main system via coupling matrix. And the coefficients \( e, e \) are the characteristic parameters of the controller in equation (15) and (16). The different controlling result can be obtained by changing the value of the characteristic parameters \( e, e \).

This is a review that chaotic motion will appear in system when the value of \( \alpha \) is more than 1.65 as shown in Figure 2 (d). Now add a two-dimensional coupling controller to the system (8). The variables of the controlling system equation are used as \( u, v \). The controlled system is depicted as

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= \rho + \alpha \sin \omega t - \sin x - r(1 + \mu \cos x)y + e(u - y) \\
\dot{u} &= e(y - u) \\
\dot{v} &= u
\end{align*}
\]  

(17)

Where \( e, e \) are the controlling parameters. And the value of parameter \( e \) is 0.3 and keeps invariant. Then changing the value of parameter \( e \), the chaos of the system (8) can be controlled to periodic motion. The phase plane portraits and bifurcation diagram of the controlled system are shown in Figure 5 and Figure 6. From Figure 5 and Figure 6 the controlled effect can be seen. The stable period-4, period-2 and period-1 orbits can be obtained via increasing the controlled strength \( e \).
3.2. Controlling chaos via adding nonlinear state feedback variables

A nonlinear controlling function can be constructed by the state variable of the system which depicted as

\[ u = k f(x_1, x_2, \ldots, x_n) \]  

(18)

Here \( k \) is the controlling strength.

Adding the controlling function (18) to first equation of the system (8), the controlled system can be depicted as

\[
\begin{align*}
    \dot{x} &= y + u \\
    \dot{y} &= \rho + \alpha \sin \omega \tau - \sin x - r(1 + \mu \cos x)y
\end{align*}
\]

(19)

The different controller \( u = kf(x_1, x_2, \ldots, x_n) \) can be designed according to the different controlling goal. There is a constant \( k_0 \) to the control of limit cycle. It makes the eigenvalue of the linear matrix in equilibrium point satisfy with

\[
R_1(\lambda) \big|_{k=k_0} = 0, \quad I_1(\lambda) \big|_{k=k_0} = 0, \quad dR_1(\lambda) / dk \big|_{k=k_0} = 0
\]

(20)

Here \( k_0 \) is the critical controlling parameter. Supposing

\[
a_i = \frac{\partial f}{\partial x_i} \big|_{x=x_j}, \quad a_z = \frac{\partial f}{\partial y} \big|_{y=y_j}
\]

(21)

Where \( x_j, y_j \) are coordinates of the equilibrium point. And supposing

\[ Q = \rho + \alpha \sin \omega \tau - \sin x - r(1 + \mu \cos x)y, \quad P = y + kf(x, y) \]

(22)
The Jacobian Matrix of equation (19) is
\[
J = \begin{bmatrix}
\frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\
\frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y}
\end{bmatrix} = \begin{bmatrix}
ka_1 & 1+ka_2 \\
ry\mu\sin x - \cos x & -r(1+\mu\cos x)
\end{bmatrix}
\] (23)

The eigenvalue of equation (23) is
\[
\lambda^2 + [r(1+\mu\cos x) - ka_1]\lambda + [(1+ka_2)(\cos x - ry\mu\cos x) - ka_r(1+\mu\cos x)] = 0
\] (24)

Supposing \(b_0 = 1.0, b_1 = 1, b_2 = r(1+\mu\cos x) - ka_1, b_3 = (1+ka_2)(\cos x - ry\mu\cos x) - ka_r(1+\mu\cos x)\)

The array can be constructed as
\[
\Delta_1 = b_1, \quad \Delta_2 = \begin{bmatrix} b_1 & b_0 \\ b_3 & b_2 \end{bmatrix}
\] (25)

If the value of controlling strength \(k\) makes \(\Delta_1 > 0, \Delta_2 > 0\), the equilibrium point of system (19) is progressive stable and the value of controlling strength \(k\) equal to the critical controlling parameter \(k_c\), the system is stable.

Choosing a controller of the system (19) as
\[
u = k(x^2 - y^2)
\] (26)

The chaos of the system (8) shown in Figure 2 (d) can be controlled to the stable period-1 orbit. The phase plane portraits of the controlled system are shown in Figure 7.

4. Conclusion

The dynamic system of the Josephson junction circuit system exhibits a rich variety of nonlinear behaviours as certain parameters varied. Due to the effect of nonlinearity, regular or chaotic motions may occur. In this paper, both analytical and computational methods have been employed to study the dynamical behaviours of the nonlinear system.

The periodic and chaotic motions of the system are obtained by the numerical methods such as phase portrait and bifurcation diagram. The changes of parameters play a major role for the nonlinear system. Chaotic motion is the motion which has a sensitive dependence on initial condition in deterministic physical systems.
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