Numerical analysis of beam-to-column connection of pallet racks

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Abstract. The aim of this paper is a numerical study of the connections that are established between main elements of the steel structure of pallet racks, frames and beams. Frames that lie in the vertical plane consist of two perforated columns linked together by a system of diagonal and/or horizontal bracing welded or bolted to the columns. Beams connect adjacent frames and lie in the horizontal direction. Beam-end connectors are welded to or otherwise formed as an integral part of the beams, which has special devices which engage in holes or slots in the column. There are different types and designs of beam-end connectors which characterize the different racks manufacturers. Currently the only way to determine moment-rotation curve with structural properties of such connections is an experiment. In order to avoid a large number of expensive tests with aim to determine the main characteristics for different types of connection which in practice may be very much, this paper shows developed numerical model to simulate the experiment using the finite element method. Using the developed model can be made a global analysis of the structural behaviour and the calculation of elements according to the procedures defined in the relevant standards and recommendations.

1. Introduction
Pallet racks are free standing structures, usually made of cold-formed steel with ability to withstand heavy vertical loads and very less lateral loads and are designed to be as light as possible. Rack structures are constructed easy to be assembled. Depending on the purpose of the rack construction, the connections between the elements are usually realized by special connectors, sometimes with bolts and really rare by welding. In this paper connections that are established between two main elements of the steel structure of selective pallet racks, beams and columns, were analysed. In general classification and modelling of connections have been treated according to European standard Eurocode 3 [1]. As it is impossible to develop a general analytical model for calculating these connections, currently the only way to determine the properties of such connections is an experiment. In this paper is analysed the test procedure and results of beam-to-column connections according to the procedure defined in the FEM (European Materials Handling Federation) codes in order to determine the moment-rotation curve ($M$-$\Phi$ curve) [2]. In order to avoid a large number of expensive tests with aim to determine the $M$-$\Phi$ curves for different types of connection which in practice may be very much, this paper presented developed numerical model which simulate the experiment using the finite element method. After verification of the model with available experimental results, it can be applied to the various combinations of beam-to-column connections. Reliable determination of structural properties of the connection using the developed model can be used for a global analysis of
the structural behaviour and the calculation of elements according to the procedures defined in the code [2] that was used as the base document for the development of standard EN 15512 [3].

2. Connections of the pallet racks

The vertical frames and horizontal beams, usually made of thin-walled cold-formed profiles form a spatial frame structure of pallet racking system [4]. Upright frames lie in the vertical plane, in the cross aisle direction, normal to the main aisle of the rack. They consist of two perforated columns linked together by a system of diagonal and/or horizontal bracing usually bolted to the columns, Figure 1 a). This bracing system provide rack stability in cross-aisle direction. Connection with a floor through anchors is provided by baseplate, Figure 1 b). If it is necessary to increase the height of the racking system, it is achieved by column splice, Figure 1 c). Beams connecting adjacent frames and lying in the horizontal direction parallel to the main aisle, Figure 1 d). Beam end connectors are welded to or otherwise formed as an integral part of the beams, which has special devices which engage in holes or slots in the column. The down-aisle stability primary is provided by the stiffness of the joints between columns and beams. In practice, there are different types and designs of these connections, which characterize the different racks manufacturers [5]. The overall performance of a rack system depends on the efficiency of beam end connector.

![Figure 1. Connections between main elements of the steel structure of pallet racks: (a) frame, (b) baseplate, (c) upright splice and (d) beam to column.](image-url)

3. Analysis of beam-to-column connection

3.1. Structural properties of the connection

In the current practice connections are characterized as rigid or elastic, i.e. traditional method of calculation based on the assumption of an ideal relationship between the structural elements generated the maximum angle of rotation and bending moments as shown on Figure 2. However, the practice and laboratory tests have shown that there are connections with characteristics between the elastic and rigid. Therefore, the classification of connection was made as: simple or elastic, continuous or rigid and semi-rigid [1]. For semi-rigid joints it will be determined that $0 < \Phi_c < \frac{ql^2}{24EI}$ and bending moment $0 < M < \frac{ql^2}{12}$, Figure 2. Behaviour of connection is defined by the curve which shows the correlation between the bending moment in the connecting point $M_{j,Ed}$ and relative rotation of the joint $\Phi_{Ed}$. This curve is known in literature as $M-\Phi$ curve or $M-\Phi$ characteristic and can be determined experimentally, based on semi-empirical expressions given for various types of connection, applying some numeric methods, or based on the recommendations in the regulations that deal with this issue (Eurocodes, FEM codes). In some cases, the real $M-\Phi$ characteristic includes some rotation due to effects such as screw slippage, mistakes in execution, etc. This can result in significant initial rotations that should be included in the calculation of $M-\Phi$ curve. Using $M-\Phi$ curve it can be determined three main structural properties of the connection:

- bending strength $M_{j,Rd}$,
- rotational stiffness $S_j$ and
- rotational capacity $\Phi_{C,Ed}$.
3.2. Bending tests on beam end connectors

The beam end connector provides a semi rigid connection between the beam and the column. The semi-rigid behaviour is due to the distortion of column walls, tearing of column perforation and distortion of beam end connector. The structural behaviour of the column and beam end connector assembly is critical to the behaviour of the complete structure. It is influenced by a large number of factors: type of the column, thickness of the column, type of the beam, position of the beam on the connector, method of connecting the beam to the connector, bracket type and properties of the materials used. All combinations of these factors, which occur in the design of the structural system, shall be tested separately, unless it can be reasonably demonstrated that interpolation of results provides a conservative estimate of performance. The purpose of the test is to determine the stiffness and the bending strength of the beam end connector \([4, 6]\). For each column and connector assembly, a number of nominally identical tests shall be made so that the results may be interpreted statistically in accordance with \([1]\) and \([2]\).

3.2.1. Test arrangements

A short length of column shall be connected to a relatively very stiff testing frame at two points with a clear distance, \(h\), between them where:

\[
h < beam\ connector\ length + 2 \times column\ face\ width
\]  

Over this distance there shall be no contact during the test between the column and the testing frame. A short length of beam shall be connected to the column by means of the connector to be tested, and beam locks shall be in place. Typical examples of suitable test arrangements are shown in Figure 3. Sideways movement and twisting of the beam end shall be prevented by a lateral restraint which, however, allows the beam component to move freely in the direction of the load. Alternatively, a pair of connectors may be tested in parallel. The load shall be applied at 400 mm from the face of the column by an actuator at least 750 mm long between pinned ends, as shown in Figure 3. The rotation may be measured by:

- displacement transducers bearing onto a plate tack-welded to the beam close to the connector, but with enough clearance to allow for connector distortion (Gauges C1 and C2 in Figure 3), or
- by an inclinometer connected to the beam close to the connector.

Figure 4 shows a photograph of the realized tests, fully in accordance with the above described scheme. A bending test on beam end connectors is carried out in accordance with \([2]\) and \([3]\).

In this paper was analysed part of very large family of columns marked alphanumeric as S80L and beams marked as R140L both made by the same producer. The symbols L, ML, M, MH, and H in the columns respectively indicate the wall thickness of 1.25, 1.5, 2, 2.5 and 3 mm. The symbols L, M and H in the beams respectively denote the thickness of the beam wall of 1, 1.25 and 1.5 mm. Specified steels according to Table 1 as well as the properties are given for both elements in the connection.
Referring to the experiments carried out on various connections, a bending test on beam end connectors are determined by measuring the size of all related to the dimensions and position of the elements of the sample in the testing equipment, and it is shown on Figure 5 and given in Table 2 for connection S80M-R140L. Dimensions which determine the position of the device for measuring displacement are shown on Figure 6 as well as given in Table 3 for the same connection. Bending tests have been carried out on five samples with the combination of beam and column as shown in tables. Each sample consists of a part of the column, beam which is connected to the column through beam end connector and secured from falling out by a safety pin. The first four samples were subjected to the load caused by normal operating conditions (bending moment is conventionally defined as positive), while the fifth pattern is subjected to a stress in such a way that the load tends to separate the connection (bending moment is a negative value).

### Table 1. Material properties.

| Element   | Steel                  | $E$, N/mm$^2$ | $\rho$, kg/mm$^3$ | $\nu$,  | $f_{yb}$, N/mm$^2$ | $f_{u}$, N/mm$^2$ |
|-----------|------------------------|---------------|-------------------|--------|-------------------|------------------|
| Column    | S350 GD Z 200 UNI EN 10326 | 2.1 $\cdot$ 10$^5$ | 7.85 $\cdot$ 10$^{-6}$ | 0.3    | 350               | 420              |
| Beam      | S320 GD Z 200 UNI EN 10326 | 2.1 $\cdot$ 10$^5$ | 7.85 $\cdot$ 10$^{-6}$ | 0.3    | 320               | 390              |
Table 2. Dimensions which determine the position of a samples.

| Joint  | Sample  | $A$, mm | $B$, mm | $C$, mm | $D$, mm | $E$, mm | $F$, mm |
|--------|---------|--------|--------|--------|--------|--------|--------|
| S80M-R140L | B-5     | 400    | 119    | 290    | 386    | 530    | 24.1   |
|         | B-6     | 400    | 119    | 290    | 385    | 530    | 23.7   |
|         | B-7     | 400    | 119    | 290    | 386    | 530    | 23.5   |
|         | B-8     | 400    | 119    | 289    | 386    | 530    | 23.6   |
|         | B-10    | 400    | 119    | 290    | 142    | 530    | 23.5   |

Table 3. Dimensions which determine the position of device for measuring displacement under tests.

| Joint  | Sample  | $G$, mm | $H$, mm | $L$, mm | $M$, mm |
|--------|---------|--------|--------|--------|--------|
| S80M-R140L | B-5     | 139.8  | 69.1   | 43     | 17     |
|         | B-6     | 139.5  | 69.1   | 43     | 16     |
|         | B-7     | 139.1  | 69.3   | 42     | 17     |
|         | B-8     | 140.1  | 69.2   | 42     | 15     |
|         | B-10    | 139.00 | 69.2   | 42     | 29     |

3.2.2. Test procedure

The tests described here load the connector vertically downwards in shear. If tests in the reverse direction show results for stiffness and strength which are less than 50% of the values measured in these tests, then the actual figures shall be measured for use in design. Separate values for the stiffness and strength shall be obtained for both right and left hand connectors and the mean value used in design. An initial load, $F$, equal to 10% of the anticipated failure load may be applied to the assembly and then removed as a preload in order to bed in the components. The gauges should then be reset. The load, $F$, shall then be increased gradually until the maximum load is reached and the connection fails. The rotation of the connection shall be observed and, for each test, a plot of the moment $M$ and the rotation $\Phi$ shall be made, in which:

$$ M = a \cdot F $$  \hspace{1cm} (2)

and

$$ \Phi = \frac{\delta_2 - \delta_1}{d} $$  \hspace{1cm} (3)

where: $a$ - lever arm for the load $F$, $d$ - distance between the gauges C1 and C2, $\delta_1$ - deflection measured by gauge C1 and $\delta_2$ - deflection measured by gauge C2.

Based on tests realization connection rotation $\Phi$ is determined from the equation:

$$ \Phi = \frac{\delta_2 + \delta_3 - \delta_1}{L + G + M} $$  \hspace{1cm} (4)

where: $\delta_3$ - deflection measured by gauge C3, and $L$, $G$ and $M$ – dimensions shown in Figure 6.

Bending tests on beam end connectors up to collapse under normal operating conditions (bending moment is conventionally defined positive) were performed on this rack structure. An initial loading-unloading cycles up to the maximum loading level $F_0$ was performed, which is provided assembling and fitting of the elements in the connection, after which the load is increased incrementally until it reaches the value of the failure $F_{\text{ti}}$. In the Table 4 are given the maximum measured values of the achieved load for each sample which corresponding to the moment calculated according to formula 2. In the table are also given duration of each probe as well as each sample.
Table 4. The measured values of the achieved load.

| Joint   | Sample | $F_0$, kN | $F_r$, kN | $t_i$, s | $M_{ti}$, kNm |
|---------|--------|-----------|-----------|----------|---------------|
| S80M - R140L | B - 5  | 0.632     | 5.741     | 324      | 2.296         |
|         | B - 6  | 0.632     | 6.162     | 341      | 2.465         |
|         | B - 7  | 0.620     | 5.771     | 320      | 2.308         |
|         | B - 8  | 0.635     | 5.930     | 274      | 2.372         |
|         | B - 10 | -0.590    | -4.119    | 316      | -1.647        |

3.2.3. Derivation of the results

The failure moment, $M_{fr}$, shall be taken to be the maximum observed moment, as indicated in figure 7. For each column and connector assembly, the characteristic failure moment $M_k$ shall be calculated in accordance with section 3.2.2. The design moment for the connection is then $M_{Rd}$, where:

$$M_{Rd} = \frac{M_k}{\gamma_M} \tag{5}$$

in which: $\gamma_M$ - partial safety factor for connections, defined in [1] and [2].

Figure 7. Derivation of connector stiffness.

The rotational stiffness of the connector shall be obtained as the slope $S_y$ of a line through the origin which isolates equal areas between it and the experimental curve, below the design moment, $M_{Rd}$, as shown in Figure 7, provided that:

$$S_y \leq 1.15 \times \frac{M_k}{\Phi_{ki} \cdot \gamma_M} \tag{6}$$

The design value, $S_d$, of the connector stiffness shall be taken as the average value, $S_m$, as shown in Table 5, where:

$$S_m = \frac{1}{n} \sum_{i=1}^{n} S_i \tag{7}$$

Table 5. Test results.

| Joint   | Sample | $M_{te}$, kNm | $M_{tm}$, kNm | $M_k$, kNm | $M_{Rd}$, kNm | $S_{ti}$, kNm/rad | $S_{tm}$, kNm/rad |
|---------|--------|---------------|---------------|------------|----------------|--------------------|--------------------|
| S80M-140L | B-5    | 2.296         |               |            |                | 92.06              |                    |
|         | B-6    | 2.465         |               |            |                | 99.35              | 94.54              |
|         | B-7    | 2.308         | 2.360         | 1.957      |                | 87.06              |                    |
|         | B-8    | 2.372         |               |            |                | 99.68              |                    |
3.3. Numerical analysis

Numerical models for all tested joints were developed in the software Femap [7] based on the technical documentation of the producer and data given in Tables 1, 2 and 3. LS-Dyna software [8] was used for numerical analysis. The parts in the set are modelled with 3D 8-nodal finite elements and contain 263204 elements and 451489 nodes. The load transfer plate is modelled with the shell elements. In the Figure 8, a mesh of finite elements model of cantilever test for analysis of the beam-to-column connection is shown.

The system for transfer loading is modelled with 1D finite elements, i.e. rods. The connection between the parts in the assembly are modelled by the surface-to-surface contact elements. Contact finite elements are used for modelling the following connections: column-beam end connector, beam end connector-beam, beam-parts for blocking lateral movement. The analysis used an elasto-plastic material model with kinematic reinforcement [8].

![Figure 8. Model of cantilever test for beam to column connection of the joint S80M-R140L.](image)

![Figure 9. Boundary conditions and load.](image)

The column and side plates are fixed at the ends. Figure 9 shows the boundary conditions and applied load. At the end of the rod, the movement is set in the negative direction of the y axis. The value of the given displacement at the point of effect of the force is calculated according to the experimental data for the relation between the angle of rotation and the corresponding force value, i.e. moment of bending. Figure 10 shows the curves of the moment-rotation of the four samples of the joint S80M-R140L obtained experimentally, analytical and curve obtained by the numerical simulation of the experiment [6]. Using developed numerical model as well as the analogy with the procedure for determining the rotational stiffness of the connection according to the 3.2.3 numerical results of 73.55 kNm/rad was obtained for the joint S80M-R140M according to the diagram shown in Figure 11. The limitation that exists in the case of using only a numerical model is the application of the maximum moment resistance as the mean value in determining the rotational stiffness. Based this,

![Figure 10. Moment-rotation curves for joint S80M-R140L.](image)

![Figure 11. Moment-rotation curves for joint S80M-R140M.](image)
it can be concluded that the developed model of finite elements can be reliably applied to determine the construction properties of the considered connection.

Figures 12 and 13 show a comparative overview for the field of the displacement in the \( x \) direction with deformed configuration for the numerical model of the tested sample S80M-R140L and the numerical model of the joint S80M-R140M for which the test was not performed.

**Figure 12.** Filed of displacement in the \( x \) direction of the joint S80M-R140L.

**Figure 13.** Filed of displacement in the \( x \) direction of the joint S80M-R140M.

In Figures 14 and 15 the field of equivalent stress when reaching the maximum moment and the field of equivalent plastic deformations for the joint S80M-R140M is shown.

**Figure 14.** Field of equivalent stress for joint S80M-R140M.

**Figure 15.** Field of equivalent plastic deformation for joint S80M-R140M.

In the Table 6 is given a comparative view of the traditional analytical calculation of the bending moment and the deflection in the characteristic points for the different boundary conditions of the beam as well as the numerical derivation of the results [4].

The application of programs based on the finite element method in the analysis of structures gives the following advantages [4]:

- Simply creation and modification of numerical models necessary for a global analysis of the construction as well as analysis of each element of the structure,
- Field of stress, i.e. the checking of the capacity of all structural elements that cannot be reliably established experimentally,
- Field of displacement, i.e. displacement of all characteristic points of certain elements of the structure as well as of the construction as a whole,
- Analysis of the influence of initial inaccuracies in the construction on the stress, displacement and stability, generated in the fabrication and during installation.
Table 6. Calculation of the bending moment and deflection of the beam with different boundary conditions of the supports.

| Joint          | Rigid     | Simple    | Semi-rigid |
|----------------|-----------|-----------|------------|
| Model          | Beam R140L| Beam R140L| Beam R140L |
| Span, L, mm    | 2700      | 2700      | 2700       |
| Load, $F_q$, N | 29430     | 29430     | 29430      |
| Moment of area of beam, $I_b$, mm$^4$ | 1581600 | 1581600 | 1581600 |
| Moment of area of column, $I_c$, mm$^4$ | 384200 | 384200 | 384200 |
| Stiffness of beam-to-column connector, $S_b$, kNm/rad | $\infty$ | 0 | 94.54 |
| Equivalent rotational stiffness, $S_e$, kNm/rad | $\infty$ | 0 | Analytical [4] =61.90 |
| Bending moment in the supports, $M_A$, kNm | Analytical [4] =6.62 | =0 | Analytical [4] |
| Bending moment in the mid of span, $M_c$, kNm | Analytical [4] =3.31 | Numerical [4] =3.37 | Analytical [4] =9.93 | Numerical [4] =7.08 | Analytical [4] =8.60 | Numerical [4] =6.23 |
| Deflection, $f_c$, mm | Analytical [4] =4.542 | Numerical [4] =7.00 | Analytical [4] =22.71 | Numerical [4] =17.86 | Analytical [4] =19.06 | Numerical [4] =15.26 |
| Numerical results for deflection, $f_c$, mm | 4.529 | 22.67 | 19.02 |

4. Conclusion
Constructive design of the connection between the elements of the spatial steel structure of pallet racks is of great importance both for the load capacity and for the economy of this type of construction. In previous practice, the connections were generally treated as rigid or simple. If joints in the structural analysis are viewed only as simple or rigid, this negatively affects the cost-effectiveness of the construction. Determination of the structural properties of the beam-to-column connection based on the moment-rotation curve is the basis for further analysis of the connection behaviour as well as its influence on global structure. In order to avoid expensive experimental analysis and determination of this characteristic, it is resorted to the development of numerical models based on finite element method. In the simulation of the test conditions numerical models gave very good results. Numerical analysis enables rapid and at the same time reliable optimization without the need of experimental testing. But simple changing of the numerical model enable global analysis of the complete structure, too. In this way, it is possible to automatically generate all the necessary models for the global analysis.
of the rack structure using the finite element method, which allows very quick and simple optimization of the construction from the aspect of ultimate limit state and serviceability limit state, taking into account all relevant actions and their combinations in accordance with the valid regulations and standards. In order to accurately implement the calculation of the steel structure of pallet racks with semi-rigid connections and simplify it in practice, it is necessary to develop specialized software packages for calculating the rack structure or use already existing commercial software packages where is necessary the introduction of the stiffness matrix for elements with semi-rigid connections.

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