Direct Measurement of intermediate-range Casimir-Polder potentials

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We present the first direct measurements of Casimir-Polder forces between solid surfaces and atomic gases in the transition regime between the electrostatic short-distance and the retarded long-distance limit. The experimental method is based on ultracold ground-state Rb atoms that are reflected from evanescent wave barriers at the surface of a dielectric glass prism. Our novel approach does not require assumptions about the potential shape. The experimental data confirm the theoretical prediction in the transition regime.

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Although being the most precisely tested theory in physics, quantum electrodynamics (QED) leads only in very special cases to measurable forces. One example are the well-known Casimir and van der Waals forces [1, 2]. In addition to their fundamental importance, a detailed understanding of these forces is crucial for testing new fundamental physics at short distances such as non-Newtonian gravitational forces [3, 4]. Furthermore, they have important technological implications for the development of micromachines with nanoscale moving parts [5, 6]. Today, Casimir forces between solids can be measured with high precision [7, 8, 9]. These measurements are all done with objects that are large compared to the relevant distances where Casimir forces become dominant. Therefore, the underlying theory contains the macroscopic properties of the objects, i.e. the dielectric functions. Moreover, also the geometry of the macroscopic bodies plays an important role due to the non-additivity of Casimir forces. A much cleaner situation is given when the test object is microscopic. This is in good approximation true for a single atom. In this case the force by which the atom is attracted towards a surface is often referred to as Casimir-Polder (CP) force. The shape of the CP potential depends on the distance from the surface. While in the limits of short and long distances CP-forces can be approximated by different power laws [1, 10], in the transition regime simple analytical expressions do not exist. Here, the potential is given by an integral over the frequency-dependent dielectric function of the substrate and the polarizability of the atom [11].

In the last two decades many efforts have been made to measure CP forces including sophisticated approaches such as diffraction and interferometry of cold atom beams at thin transmission gratings [12, 13] and quantum reflection of atoms from solid surfaces [14, 15, 16, 17]. In these experiments CP forces were studied indirectly by fitting the coefficients of the theoretical surface potentials to the measured data. Direct methods that have been developed up to now can be divided into spectroscopic [18, 19, 20, 21] and kinetic measurements. The latter include reflection of cold atoms from an atomic mirror [22] and surface-induced frequency-shifts of oscillating Bose-Einstein condensates in magnetic traps [23]. In this article the transition regime is probed for the first time by a direct model-free measurement. This is done by reflecting ultracold atoms from evanescent wave barriers similar to previous work [22]. However, here we systematically vary the mirror potential and introduce a new data analysis which allows for the direct investigation of surface potentials at sub-wavelength distances from the surface. The new method makes use of a known repulsive potential $V_{EW}(z)$ that is added to the unknown attractive surface potential $V_{SF}(z)$ with the goal to generate a potential barrier. By varying the strength of $V_{EW}(z)$, the height and the position of the barrier can be adjusted. The height is measured by reflecting cold atoms of a given energy. The position can be determined from the derivative of the barrier height with respect to the strength of $V_{EW}(z)$. This last step is the key feature for reconstructing the unknown surface potential.

The experimental situation is shown in Fig. 1. An evanescent wave leaking out from the surface of a trans-
parent substrate generates a repulsive dipole potential of the form

\[ V_{\text{EW}} = C_0 \cdot P \cdot \exp \left\{ -2 \frac{z}{z_0} \right\}, \]  

(1)

with a constant \( C_0 \) and the field decay length \( z_0 \) \( [24] \). The total potential is now the sum of the (attractive) unknown surface potential \( V_{\text{SF}} \) and the EW Potential

\[ V_{\text{tot}} = V_{\text{SF}} + V_{\text{EW}}. \]  

(2)

If the repulsive potential is strong enough, a potential barrier is formed at a distance \( z_B(P) \) from the surface. At the maximum \( V'_{\text{tot}} = 0 \), which means that

\[ V'_{\text{SF}}(z = z_B) = 2C_0 \cdot \frac{P}{z_0} \cdot \exp \left\{ -2 \frac{z_B}{z_0} \right\}. \]  

(3)

Furthermore, the height of the barrier is given by

\[ E_B = V_{\text{SF}}(z_B) + C_0 \cdot P \cdot \exp \left\{ -2 \frac{z_B}{z_0} \right\}. \]  

(4)

In the experiment this barrier height \( E_B \) is measured as a function of the laser power \( P \). From the data the derivative \( \frac{dE_B}{dP} \) is taken and compared to theory as follows. We differentiate \( 1 \) with respect to the laser power taking the inner derivatives into account and substitute eq. \( 3 \) into the result. Note that \( z_B(P) \) is a function of \( P \). This delivers the following equation:

\[ \frac{dE_B}{dP} = C_0 \cdot \exp \left\{ -2 \frac{z_B}{z_0} \right\}. \]  

(5)

The crucial point here is that this derivative depends only on the evanescent wave potential and not on the surface potential. From this expression the position of the barrier can be calculated to be

\[ z_B(P) = -\frac{z_0}{2} \cdot \ln \left( \frac{1}{C_0} \cdot \frac{dE_B}{dP} \right). \]  

(6)

With this knowledge of the barrier height \( E_B \) and the barrier position \( z_B \) the unknown surface potential \( V_{\text{SF}}(z_B) \) can be determined by solving \( 4 \).

The experimental task is to measure the height of the potential barrier as a function of the laser power. For a given laser power this is done by classical reflection of an ultracold atom cloud with variable kinetic energy from the barrier. The experiment is carried out with a setup explained in detail in \( [23] \). Here only a short summary is given. An ultracold atomic cloud is prepared in a Joffe-Pritchard type trap some hundred micrometers below the superpolished surface of a dielectric glass prism which is mounted upside down in a vacuum chamber. Almost pure condensates can be generated with some \( 10^5 \) atoms. For this experiment however, only very cold thermal clouds are prepared in order to avoid effects due to the interaction between the atoms at high density. The ultracold \( ^{87}\text{Rb} \) cloud is held in the magnetic trap at a fixed distance \( z_1 \) below the prism surface. Then a vertical laser beam (wavelength \( \lambda = 830 \text{ nm} \)) that propagates perpendicularly through the prism from the top is switched on adiabatically in order to generate a dipole trap with radial and axial trapping frequencies of \( \omega_{\text{d,r}} = 2\pi \times 50 \text{ Hz} \) and \( \omega_{\text{d,x}} = 2\pi \times 0.1 \text{ Hz} \). In this combined magnetic / dipole trap a cloud temperature of \( T \sim 100 \text{ nK} \) is measured. Now the atoms are accelerated towards the surface by a sudden shift of the magnetic trapping minimum to a new variable position \( z_2 \). After a waiting time of a quarter of an oscillation period during which the atoms accelerate in the shifted trap, the magnetic field is quickly ramped to a constant gradient which compensates for the gravitational force. The atoms now move towards the surface with a nearly constant velocity \( v_0 \). It is determined by absorption imaging of the position of the cloud in the first few milliseconds of its motion. While the atoms move to the surface they are slightly accelerated due to residual curvature of the levitation potential. This effect is taken into account as a correction of the measured velocity. The velocity at the surface is then given by \( v = \sqrt{v_0^2 + \omega_{\text{lev}}^2} \) with \( \omega_{\text{lev}} = 2\pi \times 4 \text{ Hz} \). During the reflection of the atoms the dipole trap guarantees radial confinement. The measured radial FWHM width of the atomic cloud is \( 40 \mu \text{m} \) such that the atoms are reflected only from the center of the evanescent wave. There, the potential barrier reaches its maximum due to the Gaussian intensity distribution of the EW laserspot. The EW laser is centered around \( \lambda = 765 \text{ nm} \) with a spectral width of \( \Delta \lambda = \pm 1 \text{ nm} \). The vertical potential generated by the dipole trap is very weak and can be neglected. After contact with the surface the number of reflected atoms is determined by absorption imaging. The measurement is repeated for various trap displacements \( \Delta z = z_2 - z_1 \) which correspond to different velocities \( v \). Typical results are shown in the inset of Fig. \( [2] \). The data points show a gradual decrease from the situation where all atoms are reflected (\( R = 1 \)) to full transmission \( R = 0 \). The width of this decrease is mainly dominated by the Gaussian velocity distribution of the atoms corresponding to the temperature of the cloud. It is also slightly effected by the inhomogeneous barrier height due to the Gaussian transversal intensity profile of the EW laser beam. Nonclassical broadening effects like quantum reflection and tunneling may play a role for the higher laser powers used, where the surface potential is steepened by the evanescent wave \( [26] \). However, its influence on the center of the decrease and by that on the measured barrier height is negligible. Thus the data points in Fig. \( [2] \) are fit with a model which implements only the two classical broadening mechanisms mentioned above. The result of the fits for various laser powers is plotted in the main part of Fig. \( [2] \) (open circles). For comparison, barrier heights are plotted as derived from the
Theoretical CP potentials valid in the different regimes:

\[
V_{\text{CP}} = \begin{cases} 
V_{\text{vdW}} = -\frac{C_3}{z^4} & (z \gg l), \\
V_{\text{ret}} = -\frac{C_3}{z^4} & (z \ll l), 
\end{cases}
\]

with a typical distance \(l\) separating both regimes. For calculating the potential coefficients \(C_j\) the values for an ideally conducting surface \(C_{j,n}^{\text{ic}}\) are taken from [27] and corrected for the dielectric surface. In the case of the \(C_4\) coefficient the correction is given by a factor \(\Phi(n)\) with refractive index \(n = 1.512\) [2]. In the case of the \(C_3\) coefficient an often made approximation leads to a correction factor of \(n^2\). Our corrected coefficients are \(C_3 = 5.8 \times 10^{-49} \text{ Jm}^3\) and \(C_4 = 5.4 \times 10^{-56} \text{ Jm}^4\). The transition length can be estimated from the intersection between the retarded and the nonretarded curve to be \(l \approx \frac{C_3}{C_4} = 92 \text{ nm}\). In this range we calculate the Casimir-Polder potential correctly by solving the full QED formula (5.39) in [11]. For this purpose the magnetic permeability of glass is \(\mu(\omega) = 1\) and the atomic polarizability \(\alpha(\omega)\) follows from [28]. The dielectric function \(\varepsilon(\omega)\) is determined from optical data which are available for glass in a wide range [28]. The barrier height which is derived from this correct CP potential fits best to the experimental data, although also here a deviation is observed. This deviation is particularly large for high laser powers, where the data exceed the theoretical values. A model-free comparison between experiment and theory is possible, if the surface potential is extracted from the measurement as explained above. The required derivative \(\frac{dE_B}{dt} \approx \frac{E_B(n+1) - E_B(n)}{P(n+1) - P(n)}\) is taken from a smoothed curve that can be obtained by fitting the measured barrier heights. The fit function must be chosen very carefully in order to stay model-free. A polynomial fit function e.g. would implicitly assume a surface potential of a certain shape. To maintain generality the values of the fit function at the measured laser powers are the parameters of the fit. Furthermore, the fit function fulfills the following three constraints: (1) the first derivative is positive at each point, (2) the second derivative is also positive at each point, and (3) the third derivative is negative at each point. The physical reason for (1) is that an increasing laser power leads to a growing barrier height. Constraint (2) is equivalent to the fact that for increasing laser power the barrier gets closer to the surface and constraint (3) means that the rate at which the barrier gets closer to the surface decreases. These assumptions are valid for any attractive surface potential whose attraction is growing with decreasing distance from the surface. This is the only assumption on the potential shape we make. The result of the fit is plotted as dots in Fig. 2 [30]. To guide the eye the fit points are linearly interpolated. From the fit points the surface potential is determined by equations [11] and [30]. The only parameters in this calculation are the ones describing the evanescent wave: the field decay length \(z_0\) is calculated from a measured incidence angle of \(\theta = 43.4^\circ \pm 0.1^\circ\), and a laser wavelength of \(\lambda = 765 \text{ nm}\) to \(z_0 = (430 \pm 10) \text{ nm}\). The proportionality factor \(C_0\) is calculated from standard dipole trap theory [24] with a measured beam waist of \(w_{0,x} = (170 \pm 5) \mu\text{m}\) and \(w_{0,y} = (227 \pm 5) \mu\text{m}\). The evanescent wave intensity is given (for p-polarized light) by \(\frac{E_{\text{EU}}}{E_{\text{in}}} = \frac{1}{n^2} \cdot \frac{4n^2 \cos^2(\theta)}{\cos^2(\theta) + n^2 (n^2 \sin^2(\theta) - 1)}\), with the refractive index \(n\) of the prism. With these parameters the proportionality constant is \(C_0 = (1.35 \pm 0.05) \times 10^{-27} \text{ J/W}\).

Figure 3 shows the surface potential determined from the measured barrier height (dots). Statistical errors are due to the spread of the data points in Fig. 4 around the fit curve. For the vertical axis they are given by a shift of the fit curve to the 95% confidence interval boundaries. For the horizontal axis the evaluation of the statistical error depends on the error of the gradient of the fit curve. An estimation of this error can be given by calculating the mean gradient and assuming a maximum gradient by a linear interpolation between the lower left border of the confidence interval with the upper right border. For the minimum gradient the upper left border is connected with the lower right border. This gives an uncertainty in the gradient \(\delta (\frac{dE_B}{dt})\). The uncertainty of the barrier position \(\delta z_B\) is then given by means of (6). Systematic errors are due to the uncertainty for \(z_0\), \(C_0\), \(P\) and \(\omega\). They turn out to be smaller than the systematic errors. Taking into account these uncertainties the measurements agree best with the full QED calculation. The retarded and the static potential can be excluded. Other experiments show that patch potentials from adsorbed Rubidium atoms can play a role in the surface poten-
The full theoretical curve also valid in the transition regime (trans) approaches the retarded Casimir-Polder potential (ret). The full theoretical, i.e., the nonretarded van der Waals potential (vdW) and in the large figure the theoretical surface potentials are plotted. This might be due to a permanent exposure of the prism surface to the evanescent wave which can either lead to laser induced desorption of atoms [32] or to an increased diffusion of atoms on the surface.

In this article direct measurements of the Casimir-Polder force between ground-state Rb atoms and the surface of a dielectric glass prism at distances between 160 nm and 230 nm are presented. A novel method has been introduced which is based on a test potential generated with an optical evanescent wave at the glass surface. The measurements do not coincide with the limiting formulas valid in the static and in the retarded regime. However, they do agree with a correct QED calculation and thus confirm the present theory. In addition to the mentioned measurement errors the observed small deviation might be caused by the incomplete knowledge of the dielectric function of the used borosilikate glass prism. For calculating the theoretical curve the well-known dielectric function of SiO$_2$ glass has been used. However, the optical properties of glasses vary depending on the exact type of glass [33]. It is therefore possible that the theoretical curve slightly deviates from the real situation in the experiment. Already a moderate increase in the experimental resolution will make it possible to discern between such theoretical and experimental errors and might reveal new physics that could be hidden in the observed deviation.

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Simpler fit functions have also been tested. It has turned out that with the given statistical errors of the data points e.g. a quadratic fit function cannot be discerned from the general fit function, although it does not fulfil condition (3).

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