Comment on “The Notion of Locality in Relational Quantum Mechanics”

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Abstract
A recent paper (Martin-Dussaud et al. in Found Phys 49:96, 2019) has given a lucid treatment of Bell’s notion of local causality within the framework of the relational interpretation of quantum mechanics. However, the authors went on to conclude that the quantum violation of Bell’s notion of local causality is no more surprising than a common cause. Here, I argue that this conclusion is unwarranted by the authors’ own analysis. On the contrary, within the framework outlined by the authors, I argue that far from saving the notion of ‘locality’ from the grip of Bell’s theorem, the authors have deprived it of a meaningful definition.

1 Background
In [1], an analysis of Bell’s notion of local causality was given from the standpoint of the relational interpretation of quantum mechanics [2]. The authors made the following points, which we will accept for present argumentation’s sake. To paraphrase:

(1) Bell’s notion of local causality is a statement about the beables of the theory (i.e. the candidate elements of reality);
(2) The definition and properties of the beables, and hence the applicability of local causality, depends on one’s interpretation of quantum mechanics;
(3) According to the relational interpretation, all physically meaningful beables relative to an observer are located within that observer’s past light-cone;

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Fig. 1  A space–time diagram showing the approximate location of the beables \( \{A, B, N, M, \Lambda\}_O \) relative to an observer \( O \), according to [1]. The circles are the space–time regions where local measurements take place, and the thin lines delineate their past and future light-cones. Since \( O \) considers \( \{A, N\}_O \) to be space-like separated from \( \{B, M\}_O \), a violation of Bell’s inequalities for these beables implies a violation of local causality relative to this observer.

(4) In particular, Bell’s criterion of local causality can only be meaningfully defined (and violated) relative to an observer whose past light cone encompasses all relevant beables.

An observer in this context refers to a physical system that is localized to a time-like trajectory of finite or infinite extent in space–time (a world-line segment). It will be useful to restrict our attention to finite observers, whose world-lines stretch from an initial ‘starting event’ to a final ‘terminal event’. The ‘past light cone’ referenced in point (3) above is then understood to mean the past light-cone of the observer’s terminal event, which encompasses the past light-cones of all other events on the observer’s world-line (this elaboration was not made in [1] but I take it to be consistent with their analysis).

We now review the last point (4), and its analysis in [1]. A Bell measurement scenario is defined by a set of beables \( \{A, B, N, M, \Lambda\}_O \). The beables \( A, B \) refer to the outcomes of two measurements on different parts of a quantum system. \( N \) is a set of beables that are in the causal past of \( A \) (but not \( B \)), which includes the measurement setting relevant to \( A \). Similarly \( M \) is a set of beables that are in the causal past of \( B \) (but not \( A \)), which includes the measurement setting relevant to \( B \). \( \Lambda \) represents a set of beables in the common causal past of \( A, B \), including the preparation of the quantum system. All of these beables are interpreted as being ‘physically meaningful’ relative to the observer \( O \) (whose terminal event is designated \( O_T \)) and are therefore all located in this observer’s past light-cone. It remains to be specified exactly where in the past light-cone they are situated, which will be the main point of contention taken up later on; for the time being let us follow [1] by locating the beables as shown in Fig. 1.

The authors of [1] do not specify the entire world-line of \( O \) because their analysis only depends on the observer’s ‘past light cone’ and hence only on the terminal event of the world-line, \( O_T \). For any observer terminating at this event, the beables \( A, B, N, M, \Lambda \) will be in their past light cone and thus potentially physically meaningful. Any such observer can potentially witness the experimental violation of Bell’s...
inequalities and hence reject Bell’s condition of local causality. The latter condition may be expressed relative to the observer $O$ as the following condition on the observed probabilities:

$$P(a|\lambda, n, b)_O = P(a|\lambda, n)_O ,$$

(1)

where the lowercase letters represent the specific values of the associated beables. It is important to notice that the relativisation of the beables to the observer $O$ has apparently gotten us no further towards explaining or avoiding this violation of local causality—it has only made it a problem for a smaller class of observers than is usually considered, namely those terminating at an event in the future light-cones of both measurements. For these observers, however, the problem seems just as strong as it would be without the relational interpretation.

At this juncture, the authors of [1] choose to bite the bullet and give up on local causality. However, they argue that ‘causes and effects of events are no further away than permitted by the velocity of light’, and hence that there is no need to invoke ‘superluminal interaction’ to explain the correlations. According to the authors, ‘there are correlations between A and B because there is a common cause in their common past’. However, it must be noted that the mere existence of a common cause in the common past of $A$ and $B$ is not sufficient to claim an explanation of the correlations. Whether it is sufficient rather depends on the model that one uses for the purpose of ‘causal explanation’. If one were to use a classical model of causal explanation, then it is known that common causes are in fact not sufficient to explain correlations that violate Bell’s inequalities [3,4]. The authors seem to be aware of this fact, because they emphasize that they do not accept classical causal explanations, on the grounds that these are ‘strongly related’ to determinism, and quantum theory is fundamentally non-deterministic in the relational interpretation. We mention that although the authors themselves do not propose any alternative model of causal explanation that could explain quantum correlations as common causes, such models have been proposed by others [5–9].

While this strategy of rejecting classical common-cause explanations in favour of ‘quantum common causes’ is perfectly respectable, it is also quite independent of the relational interpretation. Indeed, if one does not regard local causality as being the proper notion of ‘locality’ for non-deterministic beables, as the authors suggest, then it hardly seems to matter whether these beables are relational or not: all of the heavy lifting has already been done by the appeal to ‘non-determinism’, and there is nothing that conceptually requires relationalism. By all appearances, the quantum causal models proposed in the citations at the end of the previous paragraph can explain quantum correlations without needing to appeal to the relational interpretation.

I contend that, by rejecting classical causal explanations, the authors of [1] have given up too easily. In the remainder of this comment, I show that a more careful examination of beables in the relational interpretation allows us to avoid violations of Bell’s notion of local causality without needing to simultaneously reject a classical model of causal explanation. However, as we will see in the following sections, this evasion of non-locality comes at a steep price: when carried to its logical limits, the
relational interpretation leaves the concept of ‘locality’ without a meaningful definition at all.

Our argument is organized as follows. In the next section, we point out that there is an inherent ambiguity about where in space–time the ‘physically meaningful’ beables are for a given observer. We argue that the only way to resolve the ambiguity is to confine beables to the observer’s own world-line. We then point out that this permits a single observer to give a classical causal explanation for Bell inequality violations, provided that we allow the direction of causal relations to also be relative to the observer. Going further, in the next section we point out that the notion of space–time intervals between the beables of different observers is actually ill-defined in the relational interpretation. This calls into question the meaning of concepts such as ‘world-line’, ‘light-cone’ and hence also ‘local causality’. Consequently, relational quantum mechanics cannot be said to be local in unambiguous terms.

2 When Does a Beable Begin to be?

Imagine that you, a keen astronomer, are the observer \( O \). You are the witness to a shocking event, let us say a murder, that occurs in front of your eyes as you are watching Mars through your telescope. The perpetrator and victim are both known to you: the troubled relationship between Alice and Bob is no secret among the close-knit quantum information community. But now Alice has taken things past the point of no return, plunging a knife deep into Bob’s heart. Let us call the violent act \( M \), and let \( W \) be the event that light from the murder lands upon your startled eyes. The terminal event \( O_T \) of your observer-world-line occurs shortly after \( W \) (we may suppose the shock of the event transforms you into a wholly different person after \( O_T \), thereby terminating your worldline). Thus \( M \) clearly satisfies the criterion of being within your past light-cone. The question is, where in your past light-cone did \( M \) actually happen?

Hurriedly, you take a map of Minkowski space–time from the wall, intending to mark with your pencil the exact space–time event of the murder \( M \). Logic compels you to say that it happened on Mars, and so you trace backwards the path of the photons from the event \( W \)—whose co-ordinates are obviously known to you—all the way back to their origin on Mars’ surface, to a location just outside a popular Mars disco.

Then doubt sets in—on what grounds can you say that \( M \) really happened? The only fact directly accessible to you is the fact that photons carrying the image of a murder impacted your retinas at event \( W \). Surely you are entitled to reason that this was caused by an actual murder that took place elsewhere, but this only seems to support the notion that \( M \) is not itself a beable, but rather it is something more of an analytic fact deduced from the actual beables. These latter would include \( W \), plus any relevant memories from your life history, but not anything resembling a direct experience of (i.e. physical interaction with) the actual murder \( M \). For you, there is really no \( M \) in the sense of ‘beable’, but only \( W \), which occurs right where you were standing at the telescope, and not 64 million kilometers away on Mars.

Well, what about Bob? \( M \) is surely a beable to him. Moreover, on being murdered, Bob might reasonably expect light from the event to reach your eyes informing you of
the betrayal—he might even be certain that you are in the habit of observing Mars at precisely the moment that light from the dreadful deed would reach Earth. But to him, it is $M$ that is the real beable and $W$ that is only an abstract rational deduction from it. We are only taking seriously what the relational interpretation says: beables arise in local interactions between systems. $W$ arises from the interaction between the light arriving at your telescope and you, and so it is physically meaningful to you (but not to Bob). $M$ arises from the deadly interaction between Alice and Bob, and is physically meaningful to them, but not to you.

Let me pause to examine a possible objection to this line of thought. Surely, there is an objective sense in which $M$ actually happened that goes beyond the matter of whether light from $M$ reached $O$. Suppose that at the critical moment the International Space Station happened to pass between you and Mars, obstructing the view through your telescope, and preventing any light rays from Mars from reaching you. Nevertheless, Bob is dead, and will be dead when you next meet him (in the morgue). As it stands, the assumption (3) is too vague to tell us whether in this case $M$ ought to be a beable or not; it tells us that beables must be somewhere in the past light cone, but not where, or when. If we are to fully understand how the relational interpretation deals with Bell’s theorem, we must lift this ambiguity.

Let us therefore begin by adopting the extreme position that there is nothing that happens within the past light-cone of an observer that is not a beable for that observer, and thus strengthen statement (3) to the following:

(3′): Light-cone version: All physically meaningful beables relative to an observer are located within that observer’s past light-cone, and all beables in the observer’s past light cone are physically meaningful to that observer.

This innovation allows us to maintain some measure of observer relativism but without going all the way. It allows us to say that a beable comes into existence for an observer merely when it is possible ‘in principle’ for information about it to reach the observer.

To reveal the problem with (3′) it is enough to probe into the deeply unsatisfactory concessions lurking within that phrase ‘in principle’. The past light-cone refers to light travelling in a vacuum, yet nowhere in the universe is a true vacuum to be found. Thus, the rule (3′) would have us admit that beables become physically meaningful to us a moment before light actually reaches us in any realistic circumstance, having been slowed down by an intervening medium. They are said to be physically meaningful only because the light would have reached us at the same time if only the intervening matter hadn’t been there.

If the absurdity of that counterfactual isn’t troubling enough, consider that this approach denies any possible connection between beables and local physical interactions. Suppose I were to seal up an observer in a tank that maintained the quantum state of their body at the highest possible purity, preventing nearly all their physical interactions with the external environment. Relative to that observer, is it really sensible to insist on the physical meaningfulness of external events that cannot possibly affect him in any way? Note that one cannot escape from this bind by saying that these events would be relevant to the observer’s experience in a possible future in which they are released from the tank. To talk meaningfully about an observer, we must designate a terminal event $O_T$ after which the observer effectively ceases to exist, for it is this
event that defines the past light-cone as a strict subset of space–time. If the observer is kept in a tank until this point, the argument carries through unchanged. One could try appealing to a counterfactual—that the beables are physically meaningful by virtue of the consequences they would have if the observer weren’t in the tank—but this leads us astray from the original relational interpretation, which assigns beables to physical interactions that actually occur, not to hypothesized local interactions that might occur.

There seems, in short, no way to square \((3')\) with the spirit of the relational interpretation. In fact, any proposal that treats beables as physically meaningful that are not located on the observer’s world-line is vulnerable to the same type of argument. Thus, if we hold fast to the relational interpretation’s maxim that beables arise in local physical interactions with the observer, then our only recourse is to conclude that these beables are located along the observer’s world-line and nowhere else. Thus we should rather strengthen \((3)\) to the following:

\[(3''): \text{World-line version: All physically meaningful beables relative to an observer are located along that observer’s past world-line.}\]

This rather simple manoeuvre completely transforms the nature of quantum theory’s apparent conflict with local causality. In effect, there is trivially no conflict, because for any real observer \(O\), the beables that correspond to that observer’s experience of the two measurement events cannot be space-like separated. Thus, in cases where correlations do not admit factorization by a purported common cause, the criterion of local causality can present no objection to an explanation in terms of a direct causal influence from one to the other. Let us spell this out in a little more detail.

In general, the observers for whom the key beables \(A, B, N, M, \Lambda\) are physically meaningful can be classified into three categories (see Fig. 2). In the first category are those such as the observer \(O_1\) in the figure, for whom the beables \(\{A, B, N, M, \Lambda\}_{O_1}\) occur in a time-like sequence in which \(\{A, N\}_{O_1}\) come before \(\{B, M\}_{O_1}\). Since the relevant beables are time-like (rather than space-like) separated events, it may be possible to explain the Bell-inequality-violating correlations by means of an underlying
direct causal influence (provided we can account for the fact of no-signaling; we will address this shortly). A symmetric argument can be made for observers like $O_2$ for whom $\{B, M\}_{O_2}$ are in the time-like past of $\{A, N\}_{O_2}$, whose explanation of the statistics may also appeal to an underlying direct cause, but in the opposite direction to that stipulated by $O_1$. The third class of observers are those like $O_3$, for whom the beables $\{N, M\}_{O_3}$ first become physically meaningful at a single space–time event, and subsequently $\{A, B\}_{O_3}$ become meaningful at a later event along the world-line. Again, $O_3$ is free to suppose a direct cause (in either direction, as they see fit) between $\{A, N\}_{O_3}$ and $\{B, M\}_{O_3}$ to help explain the observed violations of Bell inequalities, since the beables in question are again time-like separated.

An objection to this can be raised on the grounds that if, say, $O_1$ wishes to explain the violation of Bell inequalities by appealing to a causal influence from $N_{O_1}$ to $B_{O_1}$, then she is also obliged to explain why this supposed influence does not permit her to transmit a signal to $B_{O_1}$ by manipulating $N_{O_1}$. After all, a ‘causal relation’ would usually be understood to entail the ability to transmit information, whereas it is well-known that quantum mechanics forbids such signalling.

Curiously, there is a way that the relational interpretation could avoid this particular difficulty, by redefining ‘causal relation’ so as to encompass classes of causal relation that do not entail signalling. One way to motivate this would be to appeal to the relativity principle, extending it to include the direction of causal influences: Relativity of causal direction In cases where different observers dispute the causal ordering of beables, no physical experiment can be devised that would indicate a preferred causal order.

Adopting this principle would imply distinguishing two different classes of causal relations in nature: those for whom both the existence and direction of the relation is agreed upon by all observers, and those which may be said to exist but whose direction is disputed among observers at different space–time locations. According to the above principle, only the former type of causal relation could be used for signalling.

To see why non-signaling is implied for the latter type of causal relation, suppose that it were possible to perform a Bell experiment in which $O_1$ could signal information to $B_{O_1}$ by manipulating $N_{O_1}$. Then this would break the symmetry, forcing $O_2$ to agree that $N$ is indeed the physical cause of $B$. This clearly favours $O_1$’s ordering of the beables over the order in which the beables appear to $O_2$, since only $O_1$’s ordering is consistent with the axiom that causes precede their effects in time. The experiment therefore would indicate a preferred ordering of the beables that would vindicate $O_1$’s assignment of causal direction over that of $O_2$. According to the principle of ‘relativity of causal direction’, no such experiment should be possible, and hence the premise is false: it should not be possible to signal from $N$ to $B$. Similarly, a symmetric argument can be given to forbid signalling from $M$ to $A$.

The relativity of causal direction admittedly stretches the imagination, since we are not used to thinking of the directions of causal relations as things that could be observer-relative in the same way as concepts like simultaneity, or the time-ordering of space-like separated events. Yet some recent work has given us reasons to think of causality as a relative concept [10–13]. Bell-inequality violation can therefore be interpreted as the discovery of this second category of causal relations in nature. (In fact, a proposal along these lines was recently put forth by Argaman in Ref.
[14], although interestingly it was inspired by a retrocausal interpretation of quantum mechanics, and not the relational interpretation as we have considered here).

### 3 Some Holes in the Plot

The formulation of local causality, as we saw at the beginning, depends on explicit reference to the light-cone structure of space–time. We have so far taken it for granted that there is a unique and meaningful way to embed the beables in such a space–time. However, the world-line postulate (3″′) calls this assumption into question. On closer examination, it seems to commit us to a picture of reality that is fundamentally “solipsistic”: since each observer is associated with its own set of beables, then we are faced with many distinct sets of beables, which are physically meaningful only for their respective observers, and it is not clear how to unify these disjoint sets into a single space–time manifold with the required light-cone structure. The situation is not entirely hopeless, because the relational interpretation does permit a beable to be shared by many observers, for instance when it is the product of a single physical interaction involving many observers at once. However, this alone does not guarantee that a sensible notion of ‘local causality’, or even of space–time, can be constructed.

To elaborate, consider that from the point of view of any given observer all beables according to (3″′) occur in a single place, i.e. where the observer is. In what manner is the notion of spatial distance between beables supposed to arise from this picture? In more detail, let \{X\}_O_1 represent a beable that arises in a physical interaction between \mathcal{O}_1 and a system \mathcal{S}_1, and let \{Y\}_O_2 represent a beable that arises in a physical interaction between \mathcal{O}_2 and a system \mathcal{S}_2. Suppose further that the observers do not interact with each other or each others’ systems. By definition, \(X\) is physically meaningful only to \mathcal{O}_1, and \(Y\) is physically meaningful only to \mathcal{O}_2. We now ask: what is the space–time interval between these two beables? Evidently, the question is ill-posed, for there is no observer relative to which \(X\) and \(Y\) do not exist within the same universe: \(X\) is part of the manifold of \mathcal{O}_1’s beables, and \(Y\) is part of the manifold of \mathcal{O}_2’s beables.

One might hope to retreat to the more moderate position that ‘locality’ may be said to be satisfied just so long as there exists some possible embedding of the different observers’ beables into a single background space–time such that the causal relations among beables respect the light-cone structure, that is, long as a picture like Fig. 1 can be consistently drawn. Such a move would entail declaring a set of space–time intervals between the world-lines of the different observers. But what meaning can these intervals have? If, for instance, the ‘distance’ between \(X\) and \(Y\) is to have physical meaning, it must itself be represented by a beable, which by the relational interpretation’s account must be physically meaningful to some observer. But, as we have just noted, there may not be any observer for whom both \(X\) and \(Y\) actually occur. To declare a value for the space–time interval between them is to assert a value for a meaningless concept.

It is therefore not really the case that [1] has shown the relational interpretation to be ‘local’. What has been shown is that the standard definitions of locality do not apply to the relational interpretation, because the beables are only physically meaningful
relative to an observer. This means, as we have seen, that they do not necessarily exist within a single space–time manifold with a well-defined light-cone structure. Since the formulation of ‘local causality’ clearly depends on this feature, it is no longer well-defined in the relational interpretation.

Of course, the above difficulties are chiefly the result of insisting that ‘local causality’, whatever it might mean, ought to be a condition placed only on the beables that have direct physical meaning, namely those that arise in the observers’ direct physical interactions. But perhaps this is too restrictive?

One way to achieve a unified picture is to introduce the idea, hinted at earlier, of an ‘abstract beable’: an occurrence which is not necessarily directly witnessed by an observer but is inferred to have happened ‘elsewhere’, and which can then be assigned a space–time location. In the example of the Bell experiment, consider a meeting between the observers $O_1$ and $O_2$, who then compare their records of their different histories of beables. These observers may find it compelling to make an identification between the beable $\{A\}_{O_1}$ (seen by $O_1$) and the beable $\{A\}_{O_2}$ (seen by $O_2$), by choosing, as a matter of convention, to regard these as having been different points of view of one and the same abstract beable $A$. This $A$ really is an abstraction, because it is derived from a pair of beables that are not jointly ‘physically meaningful’ for any particular observer. Yet it arguably does have a claim to being physically meaningful for the collective of observers whose beables it relates, since it serves as a kind of book-keeping device that allows them to establish correspondences between their separate manifolds of beables. Then, by filling the universe with observers and allowing them to pool their separate experiences, one might hope to organize their separate sets of beables into a single manifold of abstract beables whose space–time intervals are agreed upon by all members of the community. The obvious way to define ‘local causality’ in this framework would be to refer it to the light-cone structure of this background manifold of ‘abstract beables’.

However, this turns out to be no solution at all. For then it must be admitted that the beables to which Bell’s theorem applies are in fact abstract beables. So, for instance, the outcome of a measurement by $O_1$ is a valid (abstract) beable for $O_2$, even though it is space-like separated from $O_2$. That is to say, we are back in the paradigm of beables as ‘objective’ observer-independent events, and thus cannot evade Bell’s theorem by appealing to the relational character of the beables, which was the authors’ original intention.

This exposes the core of the relational interpretation’s dilemma: it was precisely the observer-relationalism of the beables that allowed the relational interpretation to escape the premises of Bell’s theorem. Yet this very same feature renders the notion of space–time without clear meaning, and hence also the notion of ‘locality’ itself. Any naïve attempt to recover the space–time manifold by rendering the beables objective again would also make it conform again to the premises of Bell’s theorem.

There is still a possibility that the relational interpretation could articulate a more sophisticated concept of space–time that is able to retain the observer-relativism of the beables in it, and then show that quantum mechanics is ‘local’ with respect to this space–time. However it is far from obvious whether this is possible, and until it is demonstrated, one cannot justify any definitive statements about the ‘locality’ of the interpretation.

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4 Conclusions and Open Problems

While the relational interpretation of beables in [1] does not necessarily run into conflict with Bell’s notion of local causality, we have seen that this is mainly because it is unclear how to even define ‘local causality’ within the metaphysical picture suggested by the interpretation. As such, claims of the ‘locality’ of the interpretation should be treated with suspicion.

One can at least postpone the above problem by restricting attention to just a single observer and the beables that are physically meaningful to them. We have argued that the violation of Bell inequalities between a single observer’s beables would still demand an explanation, because it is in conflict with the common cause explanation supplied by classical causal modeling. The solution of [1] was to reject classical causal modeling; we have suggested instead that, since the beables in question are necessarily time-like separated from the point of view of a given observer, the violation can be explained simply by positing a direct causal relationship. Of course, to agree with experiment, this would have to be a special type of causal relationship that does not enable signalling; we proposed that this constraint can be justified using the principle of relativity of causal direction. This model has the unique feature of being able to accommodate both classical causal modelling and quantum phenomena without running afoul of Bell’s theorem, due to the relational character of the beables. This may make it an interesting topic for future research, provided that the aforementioned problem of defining space–time and locality for multiple observers can be addressed.

In conclusion, in pursuing the consequences of the relational interpretation’s definition of ‘physically meaningful beables’, we have uncovered an open problem, namely the challenge of articulating a unified picture of space–time that is compatible with the interpretation, and more generally of explaining how the disparate sets of observer-relative beables can give rise to the appearance of a single objective reality. It has been argued by M. Müller that in a general ‘solipsistic’ setting (i.e. where different observers have a priori different realities), it is possible for objective reality to emerge if one adopts the principle that Solomonoff induction correctly predicts each observer’s future observations. Whether this strategy could be adapted to the relational interpretation to give meaning to space–time structure, and whether the resulting picture can be classified as ‘local’ in a meaningful sense, remains to be seen.

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