Computation of topological charges of optical vortices via non-degenerate four-wave-mixing

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In this paper, we report an experiment, which demonstrates computation of topological charges of two optical vortices via non-degenerate four-wave-mixing process. We show that the output signal beam carries orbital angular momentum which equals to the subtraction of the orbital angular momenta of the probe light and the backward pump light. The $^{85}\text{Rb}$ atoms are used as the nonlinear medium, which transfer the orbital angular momenta of lights.

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Although polarization of light, which depends on spin angular momentum of photon, is familiar to people, its closely related counterpart, orbital angular momentum (OAM) of light, was quite unfamiliar to the researchers until very recently. The most common light beam that carries OAM is the so-called Laguerre-Gaussian (LG) mode beam. Optical beams with OAMs include screw topological wave front dislocation or vortices. A spiral phase ramp around a singularity where the phase of the wave is undefined. The order of the singularity multiplied by its sign is referred as the topological charge of the dislocation. This kind of light field is also called optical vortex (OV) due to its helicoidal wavefront.

There is a great deal of interest in the property of light with OAM and how it interacts with materials. Transfer of OAM of light to macroscopic particles has been reported. Using a light beam with OV to excite vortex state in a Bose-Einstein condensation (BEC) was proposed. Experiments of transfer of OAM between light and atoms were reported. Another interesting direction is studying the change of OAM of light during a nonlinear optical process. Conversion of topological charge of optical vortices in a Second-Harmonic Generation (SHG) has been reported. Studies of OAM of light in parametric down-conversion processes have also been explored.

In this paper we report an experiment, which demonstrates computation of topological charges of two optical vortices via non-degenerate four-wave-mixing. $^{85}\text{Rb}$ atoms are used to mediate the interaction among lights with optical vortices. Fig. 1 shows the experimental setup. The 3cm vapor cell contains isotopically pure $^{85}\text{Rb}$. The temperature of the cell is kept around 60°C, corresponding to a atomic density about $3.5\times10^{11}$cm$^{-3}$. A three layer magnetic shield is used to screen out the magnetic field of earth. The remaining magnetic field inside the shield is less than 1mGs. Three external cavity diode lasers (ECDL) are used in our experiment. Two ECDLs working at 795nm are used as the forward pump field and the probe field. A 780nm ECDL serves as the backward pump field. The frequency of the forward pump field is tuned to be resonant with $|5S_{1/2}, F = 2\rangle \rightarrow |5P_{1/2}, F = 2\rangle$ transition. The frequency of the backward pump field is tuned to be resonant with $|5S_{1/2}, F = 3\rangle \rightarrow |5P_{3/2}, F = 2\rangle$ transition. And the frequency of the probe field is tuned to be resonant with $|5S_{1/2}, F = 3\rangle \rightarrow |5P_{1/2}, F = 2\rangle$ transition. The forward pump field and the probe field are vertically polarized. The backward pump field and the probe field are focused inside the cell. The diameters of them are about 0.5mm. The forward pump beam is collimated and has a diameter about 3mm. The forward pump field and the backward pump field are made to be counter-propagating. There is a small angle (about 10mrad) between the probe field and the two pump fields. The three beams are made to be coincided inside the vapor cell. This kind of configuration forms a closed non-degenerate four-wave-mixing (FWM) process, which produces a 780nm signal. And the configuration of the polarizations ensures a maximum output signal with horizontal polarization. The output FWM signal is picked up by a PBS and then sent into a Mach-Zender (M-Z) interferometer. This interferometer serves as an analyzer of the OAM of the output light field. We will explain it in detail later. The output of the M-Z interferometer is monitored by a CCD camera.

We use two computer generated holograms to make the probe field and the backward field to be LG mode beams, which carry OAMs (or topological charges). These holograms have fork-like patterns in their center (see Fig. 1). When the hologram is illuminated by a normal Gaussian laser beam, the first-order diffracted beams will carry OAMs of $+\hbar$ or $-\hbar$ depending on the sign of the diffraction order. The field amplitude of a LG mode laser is given by

$$E_{lg}^{P} = E_0 \left( \frac{\sqrt{2}}{w} \right)^l e^{-u_0} e^{-\frac{r^2}{w}} L_p^l \left( \frac{2r^2}{w} \right),$$

(1)

where, $E_0$ is the amplitude. $w$ is the half-beam width. $L_p^l$ is the Laguerre polynomial. $r$ and $\phi$ are, respectively, the radial and angular coordinates in cylindrical polar coordinate system with its $z$ axis being along the beam prop-
agitation direction. In our experiment all three light fields are output from single mode fibers and are in TEM\(_{00}\) modes. We put two holograms after the output couplers of the backward pump beam and the probe beam respectively. The first-order diffraction is used. We use two translation stages to control the transverse position of the holograms. If the fork-shape structure is illuminated, the diffracted beam is a LG\(_l^0\) beam, which carries \(\pm h\) of OAM. The sign of the OAM depends on the order of the diffraction (+1 or −1). If we move the fork-shape structure outside the beam, then the diffracted beam is an ordinary Gaussian beam, which carries no OAM. The diffraction efficiencies of the two holograms are about 10\%, which is quite low. The power of forward pump field, backward pump field and probe field are all about 700\(\mu\)W.

Before presenting the experimental results, let’s first review some basic properties of the FWM process. This process utilizes the third-order nonlinearity \(\chi^{(3)}\) of the nonlinear medium. The output FWM signal is related with the input fields with relation [21],

\[
E_S = \chi^{(3)} E_F E_B E_P, \tag{2}
\]

where \(E_S, E_F, E_B\) and \(E_P\) are electrical amplitudes of the signal field, the forward pump field, the backward pump field and the probe field respectively. Here we assume scalar fields for simplicity.

The energy conservation law and the momentum conservation law must be obeyed. These conditions suggest that \(\omega_F + \omega_B - \omega_P = \omega_s\) and the phase-matching condition \(\vec{k}_F + \vec{k}_B - \vec{k}_P = \vec{k}_S\). In the traditional FWM experiment all beams carry no OAM. So the conservation of OAM is automatically fulfilled. But in our experiment, since we introduce OAMs into the backward pump field and the probe field, the OAM conservation law must be considered too. To write it explicitly,

\[
l_F + l_B - l_P = l_S. \tag{3}
\]

First we only make the backward pump field to carry OAM (i.e. \(l_B = 1\)). Since \(l_F = l_P = 0\), the output FWM signal should also carries \(+h\) of OAM (i.e. \(l_S = 1\)). To analyze the OAM of the output beam, A M-Z interferometer is used. This M-Z interferometer is different from the ordinary one. The beam is reflected odd times in one arm and is reflected even times in the other. The reason we use this kind of interferometer is that reflection can flip the sign of the LG beam [22]. A LG beam which carries \(+lh\) of OAM is changed to a LG beam which carries \(−lh\) of OAM after reflection. So the beams from the two arms of the M-Z interferometer have OAMs with opposite signs. When they interfere, they will produce an interference pattern that can be used to reveal the OAM carried by the input field. Fig. 2 (a) and (b) are the images recorded by CCD camera. (a) is the recorded signal when one arm of the M-Z interferometer is blocked. The field distribution is a doughnut-shape spot. Fig. 2(b) is recorded without the block. This time the signal fields from the two arms form an two-part spiral interference pattern.

After this first step we carry out an experiment, in which two beams with topological charge are used. The topological charge of the backward pump beam is \(+1(l_B = +1)\). The probe beam has the same topological charge as the backward pump beam. But it is reflected by a mirror before entering the vapor cell, so as mentioned above the reflection flips the sign of the topological charge. Therefore it carries \(−h\) of OAM (\(l_P = −1\)). From the angular momentum conservation law we know \(l_S = l_F + l_B - l_P = 0 + 1 - (-1) = 2\). We also analyze the output FWM signal with the M-Z interferometer. The results are shown in Fig. 3. Fig.3 (a) is the recorded signal when one arm of the M-Z interferometer is blocked. The field distribution is also a doughnut-shape spot. One thing worth noting is that the internal circle is actually not a circle but an irregular shape. This phenomena is caused by the geometry of the experimental setup. Because in our setup the probe beam and the two pump beams are not collinear exactly. There is a small angle (10mrad) between them. So they only coincide partly inside the cell. Consequently the output signal is not a perfect doughnut shape. Fig. 2(b) is recorded without the block. The interference pattern is a four-part structure, which has a windmill shape. These two images manifest the OAM of the output beam is \(2h\). In this sense this experiment can be viewed as an computation of the topological charges of backward pump field and the probe field. In our case, it is a subtraction.

This experiment demonstrates that OAM is an intrinsic property of light, just as polarization. In a nonlinear process which involves lights with non-zero OAM, the OAM conservation law must be obeyed also.

One can easily extend this setup to compute three topological charges by introducing OAM in the forward pump field. In principle, N-1 topological charges can be mixed using a N-wave-mixing process. However higher-order nonlinear process has a much smaller cross section. And cascading of many FWM processes seems to
be a more feasible way to do computation of OAMs of lights. Compared with other experimental schemes, such as SHG, the FWM configuration provides a larger flexibility.

In conclusion, we have reported an experiment which realized an computation of topological charges of two optical vortices. This experiment utilize the OAM conservation nature of the FWM process. The $^{85}$Rb atoms are used as the nonlinear medium, which transports the OAMs of lights. We show that OAM can be transferred from one beam to another. Then we demonstrate the subtraction of OAMs of two light beams. This experiment may find applications in optical computing and processing.

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Figure Captions

Fig. 1 (a) Experimental setup. P: Polarizer; BS: Beam splitter; PBS: Polarized beam splitter. The 3cm vapor cell contains isotopically pure $^{85}$Rb. A three layer magnetic shield is used to screen out the magnetic field of earth. Three external cavity diode lasers (ECDL) are used in our experiment. The backward pump field and the probe filed are focused inside the cell. The diameters of them are about 0.5mm. The forward pump beam is collimated and has a diameter about 3mm. The forward pump field and backward filed are made to be counter-propagating. There is a small angle (about 10mrad) between the probe field and the two pump fields. The three of them are made to be coincided inside the vapor cell. The output FWM signal is picked up by the PBS and then sent to a Mach-Zender (M-Z) interferometer. One output of the M-Z interferometer is directed to a CCD camera. This interferometer serves as an analyzer of the OAM of the incident beam. (b) Energy diagram of $^{85}$Rb. The frequency of the forward pump field is tuned to be resonant with $|5S_{1/2}, F = 2⟩ \rightarrow |5P_{1/2}, F = 2⟩$ transition. The frequency of the backward pump field is tuned to be resonant with $|5S_{1/2}, F = 3⟩ \rightarrow |5P_{1/2}, F = 2⟩$ transition. And the frequency of the probe field is tuned to be resonant with $|5S_{1/2}, F = 3⟩ \rightarrow |5P_{1/2}, F = 2⟩$ transition. The forward pump field is horizontally polarized. The backward pump field and the probe field are vertically polarized.

Fig. 2 (a) Recorded signal when one arm of the M-Z interferometer is blocked. The field distribution is a doughnut-shape spot. (b) Recorded signal without the block. This time the signal fields from the two arms forms an two-part spiral interference pattern. (c) and (d) is the result of theoretical calculation using Eq. (4).

Fig. 3 The same as Fig. 2, except that two input beams (the backward pump beam and the probe beam) contain non-zero OAM. The OAM of the output signal equals to the subtraction of the OAM of the backward pump field and the OAM of the probe field.

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(a) Experimental setup.

(b) Energy diagram of $^{85}\text{Rb}$.
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