Mathematical model of parking space unit for triangular parking area

Intan Syahrini¹, Teti Sundari¹, Taufiq Iskandar¹,², Vera Halfiani², Said Munzir¹,²,³ and Marwan Ramli¹,²,³

¹Department of Mathematics, Syiah Kuala University, Banda Aceh 23111, Indonesia
²Dynamic Application and Optimization Research Group, Syiah Kuala University, Banda Aceh 23111, Indonesia
³Applied Mathematics Laboratory Syiah Kuala University, Banda Aceh 23111, Indonesia

E-mail: intansyah_89@yahoo.com, tetisundari27@gmail.com, tauis2005@gmail.com, verahalfiani@yahoo.com, smunzir2001@yahoo.com, corresponding author:ramlimarvan@gmail.com or marwan.math@unsyiah.ac.id

Abstract. Parking space unit (PSU) is an effective measure for the area size of a vehicle, including the free space and the width of the door opening of the vehicle (car). This article discusses a mathematical model for parking space of vehicles in triangular shape area. An optimization model for triangular parking lot is developed. Integer Linear Programming (ILP) method is used to determine the maximum number of the PSU. The triangular parking lot is in isosceles and equilateral triangles shape and implements four possible rows and five possible angles for each field. The vehicles which are considered are cars and motorcycles. The results show that the isosceles triangular parking area has 218 units of optimal PSU, which are 84 units of PSU for cars and 134 units of PSU for motorcycles. Equilateral triangular parking area has 688 units of optimal PSU, which are 175 units of PSU for cars and 513 units of PSU for motorcycles.

1. Introduction

Parking Space Unit (PSU) is an effective measure for the area size of a vehicle, including the free space and the width of the door opening of the vehicle. Parking facilities can be on-street or off-street parking [1]. A parking lot design should have a goal to maximize the number of parking space units while considering several things such as parking layout and pedestrian space around the parking area [2]. Parking facilities in public spaces that are easily accessible can also be considered [3]. A wireless-based smart parking system can be one of the solutions to facilitate a parking lot that can allow parking on separate floors, with the result that it will reduce the required land or area for parking space [4]. In other studies, an automated parallel parking system in small mobile vehicles [5] and a simple parking path programming strategy for automatic parking system (APS) [6] were proposed to facilitate the parking system.

AbdelFatah and Taha [7] suggested a design of a parking lot with three possible parking rows and five possible parking angles. Ramli et al. [8] developed the model of AbdelFatah and Taha by establishing four possible parking rows to be applied to the parking lot in Hermes Palace Mall of Banda Aceh. The result shows that by the selecting of efficient parking angles, the parking lots can accommodate the optimal number of vehicles so that it can increase the
revenue of parking facility manager. A study on on-street parking at Jl. T. Panglima Polem B.Aceh which is located in the city center was also presented in [9]. The result shows that the parking arrangement on the road is different from the arrangement obtained from the model developed based on the parking demand and requirement. The requirement for a parking area in a public place at the main trading center of Pasar Aceh was studied by Erliana [10]. The result shows that the available parking capacity is unable to meet the actual required parking capacity. Anggraini [11] examined the performance of the street by Cempaka Lima Clinic in Banda Aceh. The result shows that the on-street parking facilities affect the decrease of the street performance level.

Along with the increasing number of vehicles, the provision of parking lots especially outside the road body which can accommodate more vehicles has become indispensable. One solution that can be carried out is to design the parking space unit (PSU) in accordance with the shape of the land so that it can accommodate vehicles in larger quantities. This idea underlies the conduct of this research. Here, an optimization model of PSU will be developed by considering a parking area in triangular shape.

2. The Mathematical Model

The optimization model of parking space unit (PSU) that are discussed in this article can be seen in figure 1. This model is a modification of the model presented by Ramli et al. [6] to the parking lot in square or rectangular shape. The similar approach is applied for the triangular parking lot. The PSU optimization model can be described as four possible parking line. The four possible parking rows are the full $X_p$ exterior row, the $X_e$ exterior, the full $X_{ip}$ interior and the $X_i$ interior. Each possible parking row has possible parking angles of $30^\circ$, $45^\circ$, $60^\circ$, $75^\circ$ and $90^\circ$.

![Figure 1. Establishment of triangular parking lot and Parking Space Unit (PSU)](image)

In figure 1, $A_1$ is the width of the vehicle, $A_2$ is the width of the vehicle in the row, $B_1$ is the length of the vehicle, $B_2$ is the vehicle length in the row, $C_1$ is the parking row width from the wall to the parking line boundary, $C_1$ is the width of the parking row in the middle, $D$ is the vehicle alley, $E_1$ is the full exterior line width, $E_2$ is the exterior line width, $E_3$ is the width of the full interior row, $E_4$ is the width of the interior row, $W$ is the whole parking lot width, $L$
is the entire parking lot length and \( L' \) is the length of the parking lot for the row in the middle without an alley.

Based on figure 1, we can develop the objective function maximizing the PSU based on parking angle with decision variable \( X(ep, \theta), X(e, \theta), X(ip, \theta), X(i, \theta) \) and the number of PSU units based on the parking angle \( n(ep, \theta), n(e, \theta), n(ip, \theta), n(i, \theta) \) with possible parking corner of the vehicle 30°, 45°, 60°, 75° and 90°. Mathematically, the objective function for the maximum PSU problem can be written as follows,

\[
maksZ = \sum_{\theta=30^0,45^0,60^0,75^0,90^0} [n(ep, \theta) + n(e, \theta) + n(ip, \theta) + n(i, \theta)].
\]

The constraints of this problem are the width of the parking lot \( W \), the length of the parking lot \( L \), the length of the parking lot in the middle of \( L' \) and the number of vehicles in the form of positive integers.

\[
\theta = 30^0, 45^0, 60^0, 75^0, 90^0\left[E_{1\theta}X_{(ep, \theta)} + E_{2\theta}X_{(e, \theta)} + E_{3\theta}X_{(ip, \theta)} + E_{4\theta}X_{(i, \theta)}\right],
\]

\[
A2n(ep, \theta) - (L + L')X_{(ep, \theta)} \leq 0,
\]

\[
A2n(e, \theta) - X_{(e, \theta)}L \leq 0,
\]

\[
A2n(ip, \theta) - (L' + L_2)L_{(ip, \theta)} \leq 0,
\]

\[
A2n(i, \theta) - X_{(i, \theta)}L' \leq 0.
\]

The optimization coefficients of PSU for the vehicles are given in table 1.

In this article, the triangular parking lot that will be optimized is consists of two types, the isosceles triangular parking area which has two 45° angles and one 90° angle and equilateral triangular parking area. The length and width of each triangular parking area is assumed based on the side comparison of the two types of the triangular shape. The PSU optimization model in the triangular parking area follows the equations (1) to (6). The method applied here is linear integer programming which is calculated by using Lindo software to help find the most optimal results.

3. Results and Discussion

Table 2 shows the simulation results of the PSU optimization model for the isosceles triangular parking lot for cars and motorcycles. It can be seen that the optimal PSU for cars is 82 units consisting of one full exterior row \( (X_{ep}) \) with parking angle 75° taking in capacity of 56° PSUs and one full interior row \( (X_{ip}) \) with parking angle 60 taking in capacity of 28 PSUs. Meanwhile, the parking lot of the row with the minimum number of PSU will be used as parking space to optimize the number of PSU for motorcycles. For optimum motorcycle PSU, there are 134 PSUs consisting of one full exterior row \( (X_{ep}) \) with parking angle 75 accommodating 86 PSUs and two interior lines \( (X_i) \) with parking angle 45° and 60° accommodating 23 and 25 PSUs respectively. Based on the results, vehicle in the isosceles triangular parking lot can be illustrated as shown in figure 2.

Table 3 shows the simulation results of the PSU optimization model for equilateral triangular parking area for cars and motorcycles. It can be seen that the optimal PSU for cars is 175 units consisting of one full interior line \( (X_{ip}) \) with parking angle 45° accommodating capacity of 175 PSUs. Meanwhile, the parking lot of the row with the minimum number of PSU will be used as parking space for motorcycles. For optimum motorcycle PSU, there are 513 PSUs consisting of a full exterior row \( (X_{ep}) \) with parking angle 90° taking in 231 PSUs, a full interior row \( (X_{ip}) \) with parking angle 90° taking in 192 PSUs, and one interior line \( (X_i) \) with parking angle 90°
Table 1. Coefficient of parking space optimization for both vehicles

| Dimension (meter)                                      | Symbol | Angle 30° | 45° | 60° | 75° | 90° |
|-------------------------------------------------------|--------|-----------|-----|-----|-----|-----|
| Car (5 m x 2.5 m)                                      |        |           |     |     |     |     |
| Width                                                 | A1     | 2.50      | 2.50| 2.50| 2.50| 2.50|
| Width of parallel line                                 | A2     | 5.00      | 3.54| 2.89| 2.59| 2.50|
| Length                                                | B1     | 5.00      | 5.00| 5.00| 5.00| 5.00|
| Length of parallel line                                | B2     | 9.33      | 7.50| 6.44| 5.67| 5.00|
| Width of the parking line from wall to border of parking line | C1     | 4.67      | 5.30| 5.58| 5.48| 5.00|
| Width of the middle parking line                       | C2     | 3.58      | 4.42| 4.96| 5.15| 5.00|
| Circulation alley                                      | D      | 3.50      | 3.75| 4.50| 6.00| 7.00|
| Width of full exterior                                 | E1     | 11.75     | 13.47|15.04|16.63|17.00|
| Width of exterior                                      | E2     | 8.17      | 9.05|10.08|11.48|12.00|
| Width of full interior                                 | E3     | 10.66     | 12.59|14.42|16.30|17.00|
| Width of interior                                      | E4     | 7.08      | 8.17| 9.46|11.15|12.00|

Motorcycles (2 m x 0.7 m)

| Dimension (meter)                                      | Symbol | Angle 30° | 45° | 60° | 75° | 90° |
|-------------------------------------------------------|--------|-----------|-----|-----|-----|-----|
| Width                                                 | A1     | 0.70      | 0.70| 0.70| 0.70| 0.70|
| Width of parallel line                                 | A2     | 1.40      | 0.99| 0.81| 0.72| 0.70|
| Length                                                | B1     | 2.00      | 2.00| 2.00| 2.00| 2.00|
| Length of parallel line                                | B2     | 3.21      | 2.70| 2.40| 2.19| 2.00|
| Width of the parking line from wall to border of parking line | C1     | 1.61      | 1.91| 2.08| 2.11| 2.00|
| Width of the middle parking line                       | C2     | 1.31      | 1.66| 1.91| 2.02| 2.00|
| Circulation alley                                      | D      | 1.60      | 1.60| 1.60| 1.60| 1.60|
| Width of full exterior                                 | E1     | 4.52      | 5.17| 5.59| 5.73| 5.60|
| Width of exterior                                      | E2     | 3.21      | 3.51| 3.68| 3.71| 3.60|
| Width of full interior                                 | E3     | 4.22      | 4.92| 5.42| 5.64| 5.60|
| Width of interior                                      | E4     | 2.91      | 3.26| 3.51| 3.62| 3.60|

Taking in 90 PSUs. Based on the results, vehicles in the parking lot in the equilateral triangular shape can be illustrated as shown in figure 3.

Table 2. The simulation results of PSU models for cars and motorcycles in isosceles triangular parking

| Angle   | Car   | 30° | 45° | 60° | 75° | 90° |
|---------|-------|-----|-----|-----|-----|-----|
| Number of rows X(ep, θ) / Number of PSU n(ep, θ) | 0    | 0   | 0   | 1/56| 0   |
| Number of rows X(e, θ) / Number of PSU n(e, θ) | 0    | 0   | 0   | 0   | 0   |
| Number of rows X(ip, θ) / Number of PSU n(ip, θ) | 1/26 | 1/28| 0   | 0   | 0   |
| Number of rows X(i, θ) / Number of PSU n(i, θ) | 0    | 0   | 0   | 0   | 0   |

Motorcycles

| Angle   | Motorcycle | 30° | 45° | 60° | 75° | 90° |
|---------|------------|-----|-----|-----|-----|-----|
| Number of rows X(ep, θ) / Number of PSU n(ep, θ) | 0    | 0   | 0   | 1/86| 0   |
| Number of rows X(e, θ) / Number of PSU n(e, θ) | 0    | 0   | 0   | 0   | 0   |
| Number of rows X(ip, θ) / Number of PSU n(ip, θ) | 0    | 0   | 0   | 0   | 0   |
| Number of rows X(i, θ) / Number of PSU n(i, θ) | 1/23 | 1/25| 0   | 0   | 0   |
Figure 2. Optimal PSU setting for cars (a) and motorcycles (b) in isosceles triangular parking area

Table 3. Simulation results of PSU model of cars and motorcycles in equilateral triangular parking

| Angle | 30° | 45° | 60° | 75° | 90° |
|-------|-----|-----|-----|-----|-----|
| Car   |     |     |     |     |     |
| Number of rows $X(ep, \theta) / \text{Number of PSU } n(ep, \theta)$ | 0   | 0   | 1/55| 0   | 0   |
| Number of rows $X(e, \theta) / \text{Number of PSU } n(e, \theta)$   | 0   | 0   | 0   | 0   | 0   |
| Number of rows $X(ip, \theta) / \text{Number of PSU } n(ip, \theta)$ | 0   | 0   | 5/175| 0   | 0   |
| Number of rows $X(i, \theta) / \text{Number of PSU } n(i, \theta)$   | 0   | 0   | 0   | 0   | 0   |
| Motorcycles |     |     |     |     |     |
| Number of rows $X(ep, \theta) / \text{Number of PSU } n(ep, \theta)$ | 0   | 0   | 0   | 0   | 1/231|
| Number of rows $X(e, \theta) / \text{Number of PSU } n(e, \theta)$   | 0   | 0   | 0   | 0   | 0   |
| Number of rows $X(ip, \theta) / \text{Number of PSU } n(ip, \theta)$ | 0   | 0   | 0   | 0   | 1/192|
| Number of rows $X(i, \theta) / \text{Number of PSU } n(i, \theta)$   | 0   | 0   | 0   | 0   | 1/90 |

Figure 3. Optimal PSU setting for cars (a) and motorcycles (b) PSU in equilateral triangular parking area
4. Conclusion
The optimization model of parking space unit (PSU) in triangular parking area in this research is examined in two types of shape, namely isosceles and equilateral triangular shapes. The model can be described as four possible lines of parking, which is full exterior line Xep, exterior line Xe, full interior line Xip, and interior line Xi. Each possible parking line has a possible parking angle which are $30^\circ$, $45^\circ$, $60^\circ$, $75^\circ$ and $90^\circ$. The results show that the isosceles triangular parking area has 218 units of optimal PSU, which are 84 units for cars and 134 units for motorcycles. Equilateral triangular parking area has 688 units of optimal PSU, which are 175 units for cars and 513 units for motorcycles.

Acknowledgments
The authors thank the anonymous referees for their valuable suggestions which led to the improvement of the article. This research is funded by Laboratory Grant, Syiah Kuala University, 2017.

References
[1] Directorate General of Land Transportation of Indonesia 1996 Guide of Technical Organization of Parking Facility (Jakarta : Directorate General of Land Transportation)
[2] University of Houston 2012 Parking Lot Design Standards Section 9.0
[3] Ibrahim D M, Wahjudijati E and Ismail A 2013 Study of public space utilization for parking area at Cikuray Garut street J. Konstruksi STTGarut
[4] Aggarwal M, Aggarwal S and Uppal R S 2012 Int. J. Sci. Res. Pub. 2(10) 1-8
[5] Wahab M F, Moe1 A L, Abu A, Yaacob Z and Legowo A J. 2015 Eng. Appl. Sci. 10(16) 7107-12
[6] Wang J M, Wu S T, Ke C W and Tzeng B K 2013 Vehicle Engineering 1(3) 57-63
[7] Abdelfatah A S and Taha M A 2014 J. Traff. Logis. Engin. 2(3) 176-81
[8] Ramli M, Sary D P and Hafidzani V 2016 Optimization model of parking charge and income using Lagrange multiplier method Proc. AIP Conference Series pp. 030034-1 – 030034-8
[9] Munzir S, Ihsan M and Amin Z 2010 Linear Programming for Parking Slot Optimization Proceedings of the 6th IMT-GT Conference on Mathematics, Statistics and Its Applications (Kuala Lumpur) pp. 462-72
[10] Erliana H, Saleh M S and Anggraini 2014 J. Teknik Sipil 3(3) 188-96
[11] Anggraini R, Burhanuddin and Ilyas M I 2013 Study of public service space availability on the parking facility need and road current performance (case study of Cempaka Lima clinic in Banda Aceh) National Conference of Civil Engineering 7 (Surakarta) T53-T60