COMPACT AND “COMPACT” OPERATORS ON STANDARD HILBERT MODULES OVER $C^*$-ALGEBRAS

ZLATKO LAZOVIC

Communicated by E. Troitsky

ABSTRACT. We construct a topology on the standard Hilbert module $H_A$ over a unital $C^*$-algebra and topology on $H^*_A$ (the extension of the module $H_A$ by the algebra $A^*$) such that any “compact” operator (i.e. any operator in the norm closure of the linear span of the operators of the form $z \mapsto x \langle y, z \rangle$, $x, y \in H_A$ (or $x, y \in H^*_A$)) maps bounded sets into totally bounded sets.

REFERENCES

1. M. Frank, Self-duality and $C^*$-reflexivity of Hilbert $C^*$-modules, Z. Anal. Anwend. 9 (1990), no. 2, 165–176.
2. D. J. Kečkić and Z. Lazović, Compact and “compact” operators on standard Hilbert modules over $W^*$-algebras, Ann. Funct. Anal. 9 (2018), no. 2, 258–270.
3. E. C. Lance, Hilbert $C^*$-modules: A Toolkit for Operator algebraists, London Math. Soc. Lecture Note Ser. 210, Cambridge Univ. Press, Cambridge, 1995.
4. V. M. Manuilov, Diagonalization of compact operators in Hilbert modules over finite $W^*$-algebras, Ann. Global Anal. Geom. 13 (1995), no. 3, 207–226.
5. V. M. Manuilov, Diagonalization of compact operators on Hilbert modules over $C^*$-algebras of real rank zero (in Russian), Mat. Zametki 62 no. 6 (1997), 865–870; English translation in Math. Notes 62 (1997), 726–730.
6. V. M. Manuilov and E. V. Troitsky, Hilbert $C^*$-Modules, Transl. Math. Monogr. 226 Amer. Math. Soc., Providence, 2005.
7. W. L. Paschke, Inner product modules over $B^*$-algebras, Trans. Amer. Math. Soc. 182 (1973), 443–468.
8. W. L. Paschke, Inner product modules arising from compact automorphism groups of von Neumann algebras, Trans. Amer. Math. Soc. 224 (1976), 87–102.
Faculty of Mathematics, University of Belgrade, Studentski trg 16-18, 11000 Belgrade, Serbia.

E-mail address: zlatkol@matf.bg.ac.rs