Cluster correlations in multifragmentation

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Abstract. The possibility of strong cluster correlations in multifragmentation reactions is explored based on the antisymmetrized molecular dynamics calculations. The number of emitted protons, which is smaller than predicted by usual transport models, can be explained by introducing the two-nucleon collision process to the final states in which one or both of the scattered nucleons may form clusters (such as deuterons, tritons and \(\alpha\) particles) with other nucleons in the system. The cluster correlation has strong impacts on the whole collision dynamics. The multifragmentation data at 50 MeV/nucleon and above are well reproduced with this strong cluster correlation if pairs of clusters with small relative velocities are assumed to form light nuclei such as Li and Be isotopes. An analysis is shown on symmetry energy effects in emission of nucleons and clusters.

1. Introduction
In medium-energy heavy-ion collisions, the system is compressed at an early stage and then expands so that the whole system disintegrates into many intermediate mass fragments and light particles. It is a highly challenging problem to explain the time evolution of such reactions, which requires a fully many-body approach taking into account some quantum features. The reaction dynamics can also be regarded as a manifestation of nuclear matter properties, such as the equation of state and the liquid-gas phase transition. In addition to the mean-field aspect, many-body correlations are also an important aspect of matter properties.

The correlations are not only microscopic effects on top of the bulk nuclear matter property but also can have strong impacts on the bulk property and the global collision dynamics. For example, let us consider four nucleons in matter at a temperature of \(T = 10\) MeV. If they are not correlated, the average energy is \(\langle E \rangle = \frac{3}{2}T \times 4 = 60\) MeV, while it is \(\langle E \rangle = -28.3\) MeV + \(\frac{3}{2}T \times 1 = -13.3\) MeV if the four particles always form an \(\alpha\) cluster. The energy difference seems large enough to change the global collision dynamics if such correlations appear during heavy-ion collisions. Thus it is important to take into account many-body correlations together with the bulk nuclear matter properties.

The experimental data shows that the clusters are actually an important part of multifragmenting systems. In central Xe + Sn collisions at 50 MeV/nucleon [1], only about 10 \% of the total protons in the system are emitted as free protons, while the other protons are bound in light particles (\(d, t, ^3\text{He}, \alpha\)) and heavier fragments in the final state. The number of protons in \(\alpha\) particles (20 \%) is more than the number of free protons (10 \%). Even at 1 GeV/nucleon, it is known that about half of protons are still bind in clusters and fragments in Au+Au collisions [2]. In theoretical studies, there are large discrepancies in the number of emitted nucleons predicted by different transport models (see Ref. [3] for example), and most transport models tend to overestimate the experimental data of the proton multiplicity.
Currently there is a lot of interest in the equation of state of asymmetric nuclear matter at various densities \[4, 5\]. Most of the effects of the nuclear symmetry energy are probably understood as phenomena occurring on top of the collision dynamics governed mainly by the properties of symmetric nuclear matter. For the study of symmetry energy, cluster correlations are important firstly because correlations may influence the global collision dynamics as mentioned above, and secondly because many experimental probes are obtained from clusters such as the \(t^3\)He ratios. It has been also shown by Natowitz et al. that the existence of clusters enhances the symmetry energy at low densities \[6\].

The aim of this paper is to demonstrate the strong effects of cluster correlations in multifragmentation reactions based on the antisymmetrized molecular dynamics (AMD) calculations. In particular, the stochastic process of the transition induced by the two-body residual interaction is improved by taking into account the cluster correlations in the final states of the transition.

2. AMD with usual two-nucleon collisions
As one of the transport models for nuclear reaction, AMD solves the time evolution of many-nucleon system starting with two boosted nuclei with a given impact parameter and a suitable distance between them. The very early version of AMD \[7\] is reviewed here because it is the basis of the extension to incorporate cluster correlations in the following sections.

2.1. AMD wave function and the equation of motion

To describe the state at each time, AMD employs a single Slater determinant of Gaussian wave packets for \(A\) nucleons

\[
\langle \mathbf{r}_1 \ldots \mathbf{r}_A | \Phi(Z) \rangle \propto \det_{ij} \left[ \exp \left\{ \nu (\mathbf{r}_i - \mathbf{Z}_j / \sqrt{\nu})^2 \right\} \chi_{\alpha_i}(i) \right],
\]

(1)

where \(\chi_{\alpha_i}\) are the spin-isospin states with \(\alpha_i = p \uparrow, p \downarrow, n \uparrow, \) or \(n \downarrow\). Thus the many-body state \(\langle \Phi(Z) \rangle\) is parametrized by a set of complex variables \(Z = \{Z_i\}_{i=1, \ldots, A}\), where \(A\) is the number of nucleons in the system. The width parameter \(\nu = (2.5 \text{ fm})^{-2}\) is treated as a constant parameter common to all the wave packets. Up to the antisymmetrization effect, the real and imaginary parts of \(Z_i\) correspond to the position and momentum centroids, respectively. This choice of the wave function is suitable for the study of fragmentation reactions because it ensures that each single-particle wave function does not belong to different fragments simultaneously.

The time evolution of the wave packet parameters \(Z\) is determined, up to the extensions described later, by applying the time-dependent variational principle

\[
\delta \int_{t_1}^{t_2} \frac{\langle \Phi(Z) | (i\hbar \frac{d}{dt} - H) | \Phi(Z) \rangle}{\langle \Phi(Z) | \Phi(Z) \rangle} dt = 0 \quad \text{with} \quad \delta Z(t_1) = \delta Z(t_2) = 0,
\]

(2)

from which the equation of motion for \(Z\) is obtained,

\[
i \hbar \sum_{j\tau} C_{\sigma j\tau} \frac{dZ_{j\tau}}{dt} = \frac{\partial \mathcal{H}}{\partial Z_{i\sigma}} \quad \text{or} \quad \dot{Z}_i = \{Z_i, \mathcal{H}\}_\text{PB},
\]

(3)

where \(\sigma, \tau = x, y, z\) are the labels for the components of \(Z_i\) \((i = 1, 2, \ldots, A)\). The Hamiltonian \(\mathcal{H}\) in the equation of motion is the expectation value of the Hamiltonian operator \(H\) with an additional correction for the spurious zero-point energies of the center-of-mass motions of fragments. A positive definite Hermitian matrix \(C_{\sigma j\tau}\) appears in the equation of motion, suggesting that the variables \(Z\) are not canonical coordinates. By defining the Poisson brackets
employing the matrix $C_{i\sigma,j\tau}$, the equation of motion can be written in a brief form. The effective interactions for the mean-field calculations, such as the Gogny force and the Skyrme force, have been usually employed in the Hamiltonian $H$. The calculated results presented in this paper were obtained with the Skyrme SLy4 force [8] unless otherwise mentioned. The equation of motion can be interpreted intuitively as representing the motion of individual wave packets in the mean-field potential.

### 2.2. Two-nucleon collisions

In addition to the mean-field effect in the equation of motion, the residual interaction plays important roles in collisions at energies of more than 10 MeV/nucleon. This effect has been incorporated into AMD as two-nucleon collisions in a similar way to other transport models. The two-nucleon collision is an effect beyond the mean-field approximation with a single Slater determinant, it cannot be described by a deterministic term in the equation of motion derived from the time-dependent variational principle. In AMD, a two-nucleon collision is treated as a stochastic transition form an AMD state to one of the possible other AMD states. The total probability of a collision and the partial transition probabilities to specific states are decided based on the differential cross section $(d\sigma/d\Omega)^{NN}$ of the two-nucleon scattering, which depends on the scattering energy and may be modified in nuclear medium. In the calculations presented in this paper, we employ the in-medium density-dependent total cross sections by Li and Machleidt [9] for the kinetic energies $E > 35$ MeV in the nucleon-nucleon center-of-mass system, while the cross sections at $E = 35$ MeV are employed at lower energies. Angular distributions are assumed to be similar to the experimental data in the free space.

As the AMD variables $Z$ do not have an intuitive meaning of nucleon coordinates due to the effect of antisymmetrization, we have introduced physical coordinates $W = \{W_1, \ldots, W_A\}$ which are nonlinear functions of $Z$. Let us consider a two-nucleon collision between the 1st and the 2nd nucleons. The assumption, which is usually taken for granted but is essential for the discussion in this paper, is that only the two coordinates are changed by the collision with a randomly generated scattering angle, so that the physical coordinates after the collision are written as $W' = \{W'_1, W'_2, W'_3, \ldots, W'_A\}$. The Pauli-blocking for the final state of the collision is considered with the physical coordinates. If the transformation form $W'$ to AMD variables $Z'$ exists, the transition from $|\Phi(Z)\rangle$ to $|\Phi(Z')\rangle$ is allowed.

### 2.3. A typical result

It is known that the early version of AMD with usual two-nucleon collisions does not reproduce multifragmentation data except for collisions of light nuclei. Figure 1 shows the time evolution of the density in an event of central Xe + Sn collision at 50 MeV/nucleon. The initial two nuclei essentially pass through each other, so that two relatively large fragments are typically observed together with some small fragments and many emitted nucleons. The charge distribution of fragments is compared with the INDRA experimental data [1] in Fig. 2. The number of fragments with $Z > 20$ is seriously overestimated while the number of intermediate mass fragments is underestimated. The proton multiplicity $M_p = 40.2$ is too large while the $\alpha$-particle multiplicity $M_\alpha = 2.5$ is too small compared to the experimental multiplicities detected by the INDRA detectors ($M_p = 8.4$ and $M_\alpha = 10.1$) [1]. The situation is essentially the same as what was found in Ref. [10] for Ca + Ca collisions at 35 MeV/nucleon.

The origin of this failure in multifragmentation was interpreted as due to the restriction of the fixed shape of Gaussian wave packets. In Refs. [10, 11, 12], the effect of the change of the wave packet shape was introduced into AMD as stochastic fluctuations to the wave packet centroids. The change of the shape of each wave packet in the mean field is calculated based on the Vlasov equation. This effect can be considered as wave packet splitting or quantum branching in which different branches evolve towards different reaction channels. By considering the wave packet
splitting, the reproduction of the change distribution of intermediate mass fragments improves very much as shown in Ref. [12]. Nevertheless, discrepancies still exist between theory and experiment for the proton and α-particle multiplicities.

In this paper, however, we do not consider the wave packet splitting. In the following sections, we explore an alternative improvement of the model to allow strong cluster correlations, aiming at an explanation of the proton and α-particle multiplicities together with the fragment yields. The results of AMD with both options of wave packet splitting and cluster correlations were reported in Ref. [13], while all the results presented in this paper are obtained without wave packet splitting.

3. Transition with cluster correlations

3.1. Outline of formulation

The two-nucleon collision process in AMD is a transition from an AMD wave function $|\Psi_i\rangle$ to one of possible final states $|\Psi_f\rangle$ which is assumed to occur instantaneously and to conserve the energy expectation value $\langle \Psi_i | H | \Psi_i \rangle = \langle \Psi_f | H | \Psi_f \rangle$. As the transition is induced by the residual interaction between the two colliding nucleons, the transition rate may be expressed as

$$ W_{i\rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i) $$

which is similar to Fermi’s golden rule. However, some care should be taken since the transition is considered here between states which are not eigenstates of the unperturbed Hamiltonian.

An important question here is how to choose the complete basis $\{ |\Psi_f \rangle \}$ of final states. In the usual implementation of two-nucleon collisions, only the states of the two nucleons are changed
under the assumption that these two nucleons are not correlated with the other nucleons in the system. On the other hand, if the correlations exist in the final states between the scattered nucleons and other nucleons, it is more reasonable to construct the final states \(|\Psi_f\rangle\) taking account of the correlations.

Cluster correlations can exist in nuclear medium under various conditions. For example, a calculated result is shown in Fig. 2 of Ref. [14] by Danielewicz et al., indicating that a deuteron-like pair will propagate in nuclear medium if the momentum of the pair is more than twice the Fermi momentum of the medium. We assume here that such clusters with \(A = 2, 3\) and \(4\) can propagate in the medium if it is allowed by the Pauli principle. In AMD, if several wave packets (with different spins and isospins) are placed at the same phase space point, these wave packets will tend to move together as a cluster by the equation of motion. On the other hand, if the wave packets are placed randomly in the phase space, the chance for these nucleons to form a cluster after propagation is small. Therefore, in order to respect the possibility of forming a cluster in the final state of a two-nucleon collision, the set of final states \(|\Psi_f\rangle\) should be suitably constructed.

To obtain the transition probabilities to clusterized states, we employ an approximation similar to Ref. [14]. As an example of possible final states \(|\Psi_f\rangle\) for a collision of two nucleons \(N_1\) and \(N_2\) with the initial relative velocity \(v_{NN}\), let us consider a case that \(N_1\) (\(N_2\)) forms a cluster \(C_1\) (\(C_2\)) with another nucleon \(B_1\) (\(B_2\)) in the final state. The partial differential cross section to this final channel is given by

\[
v_{NN}d\sigma(N_1B_1N_2B_2 \rightarrow C_1C_2) = \frac{2\pi}{h}|\langle \phi_1^\prime|\phi_1^+q\rangle|^2 |\langle \phi_2^\prime|\phi_2^*-q\rangle|^2 |M|^2 \delta(H - E) \frac{P_{rel}^2dp_{rel}d\Omega}{(2\pi h)^3},
\]

where \(M\) is the matrix element for the two-nucleon scattering to the final state with the relative momentum \(p_{rel}\) and the scattering angle \(\Omega\) in the two-nucleon center-of-mass system. The overlap matrix \(\langle \phi_1^\prime|\phi_1^+q\rangle\) is taken between \(|\phi_1^+q\rangle = e^{iqr_1}|\phi_1\rangle\) and \(|\phi_1^\prime\rangle\), where \(|\phi_1\rangle\) and \(|\phi_1^\prime\rangle\) are the initial and final states of the \(N_1 + B_1\) system, respectively, and the operator \(e^{iqr_1}\) gives the momentum transfer to the nucleon \(N_1\). The clusterized states \(|\phi_1\rangle\) and \(|\phi_2\rangle\) are approximated by the simple harmonic oscillator \((0s)^n\) configuration with the oscillator constant associated with the wave packet width \(\nu\) in AMD, so that any final state of the collision is represented by an AMD wave function. If the two-nucleon matrix element \(M\) is identified with that in the usual two-nucleon scattering without special consideration on cluster formation

\[
v_{NN}d\sigma_{NN} = \frac{2\pi}{h}|M|^2 \delta(H - E) \frac{P_{rel}^2dp_{rel}d\Omega}{(2\pi h)^3},
\]

the cluster-forming cross section can be obtained from the two-nucleon collision cross section by

\[
\left(\frac{d\sigma}{d\Omega}\right)_{N_1B_1N_2B_2 \rightarrow C_1C_2} = \frac{(P_{rel}^2/|\partial H/\partial p_{rel}|)_{C_1C_2}}{(P_{rel}^2/|\partial H/\partial p_{rel}|)_{NN}} |\langle \phi_1^\prime|\phi_1^+q\rangle|^2 |\langle \phi_2^\prime|\phi_2^*-q\rangle|^2 \left(\frac{d\sigma}{d\Omega}\right)_{NN}.
\]

The actual situation of a two-nucleon collision requires more considerations because there are many possible ways of forming a cluster for each of the scattered nucleons \(N_1\) and \(N_2\). Figure 3 illustrates an example in which one of the scattered nucleons (assumed to be a proton with up-spin) may form a deuteron cluster with any one of \(B_1\), \(B_2\) or \(B_3\) (neutrons with up-spin). When \(N + B_i\) (for each \(i = 1, 2, 3\)) forms a deuteron cluster after the momentum transfer \(q\) to \(N\), the centroids of these wave packets are moved to the same phase space point in the final state \(|\Phi_i^\prime\rangle\) without changing the center-of-mass state of the \(N + B_i\) system. Since the different final states \{|\Phi_i^\prime\rangle\} are not orthogonal to one another, it is wrong to simply add the probabilities
Figure 3. An example of deuteron formation in a two-nucleon collision, in which one of the scattered nucleon $N$ (proton with up-spin) may form a deuteron cluster with any one of $B_1$, $B_2$ or $B_3$ (neutrons with up-spin). The filled circles depict the wave packet centroids.

Then the probability of deuteron formation is calculated by

$$P = \langle \Phi^q | \hat{P} | \Phi^q \rangle = \sum_{ij} \langle \Phi^q | \Phi^q_i \rangle N_{ij}^{-1} \langle \Phi^q_j | \Phi^q \rangle,$$  

which should replace the overlap probability $|\langle \phi^q_1 | \phi^{+q}_{1}\rangle|^2$ or $|\langle \phi^q_2 | \phi^{-q}_{2}\rangle|^2$ in Eq. (7). When a deuteron is formed with the probability $P$, one of the final states $\{ \Phi^q_j \}$ is chosen in a suitable branching ratio. The energy-conserving nucleon-nucleon relative momentum $p_{\text{rel}}$ and the phase space factor in Eq. (7) should be calculated for each final state $\Phi^q_j$. With the rest of the probability $1 - P$, this up-spin proton $N$ does not form a deuteron with any up-spin neutron.

The procedure is repeated for the cluster formation with nucleons ($B$’s) with other spin and isospin directions until formation of an $\alpha$-cluster is considered. The particle $N$ should be regarded as a cluster, instead of a nucleon, if a (sub)cluster has been already formed in previous steps of the repetition.

A correction for cluster correlations in AMD was tested in a different approach [15] by introducing stochastic coalescence of wave packets in order to keep the probability of finding the subsystem in the cluster ground state. The two approaches are conceptually similar in spite of the different implementations. However, an advantage of the new approach here is that the energy and momentum are naturally conserved in the two-nucleon collision process that is assumed to occur instantaneously.

3.2. Results

When cluster formation is turned on in the final states of two-nucleon collisions, many clusters are created in heavy-ion collisions as soon as two nuclei overlap and hard two-nucleon collisions occur, as shown in Fig. 4. Clusters are formed by two-nucleon collisions and propagated by the AMD equation of motion [Eq. (3)]. A cluster is broken by the propagation in some cases by different forces acting on the nucleons in the cluster, but a cluster is more frequently broken when one of the nucleons in the cluster collide with another nucleon in the system. It should be noted that, even though the information of existing clusters is useful to understand the situation, the time evolution of the system does not depend on whether a cluster of wave packets is formed by a two-nucleon collision or these wave packets are at the same point accidentally or for some other reason (except for the extension in the next section).
Figure 4. Time evolution of the numbers of nucleons in clusters in Xe + Sn central collisions at 50 MeV/nucleon. The parts, from bottom to top, represent the number of nucleons in 4-, 3- and 2-nucleon clusters, respectively.

Figure 5. The time evolution of the density projected onto the reaction plane in an event of central Xe + Sn collision at 50 MeV/nucleon, calculated by the AMD with cluster formation in the final states of two-nucleon collisions. The density is shown at every 40 fm/c from left to right. The size of the shown area is 40 fm × 40 fm.

Figure 6. The charge distribution of fragments produced in central Xe + Sn collisions at 50 MeV/nucleon. The red histogram was calculated with cluster formation in the final states of two-nucleon collisions. The statistical decays of hot fragments at \( t = 600 \text{ fm/c} \) have been calculated. The blue points show the experimental data by INDRA [1].
Figure 7. The relation between the gas fraction $Z_{\text{gas}}/Z_{\text{tot}}$ and the proton multiplicity $M_p$ in Xe + Sn central collisions at 50 MeV/nucleon. The arrow connecting two calculated points shows the effect of cluster correlations in the AMD calculation. The larger filled circle shows an area which is roughly consistent with the experimental data. The dashed arrow represents the effect of inter-cluster correlations.

Strong effect of cluster correlations can be easily seen in the time evolution of the density in Xe + Sn central collisions at 50 MeV/nucleon as shown in Fig. 5. Compared to Fig. 1 without cluster correlations, the expansion to the transverse directions is now very strong, and the system is disintegrating into small pieces. This strong expansion to the transverse directions, or strong stopping, can be understood from the large phase space factor in Eq. (7) when cluster formation is allowed, even though the same two-nucleon cross section $(d\sigma/d\Omega)_{\text{NN}}$ is employed for both calculations.

The fragment charge distribution has been drastically changed by cluster correlations as shown in Fig. 6 compared with Fig. 2. The proton multiplicity has decreased from $M_p = 40.2$ to 10.9 and the $\alpha$-particle multiplicity has increased from $M_{\alpha} = 2.5$ to 23.2 by the incorporation of cluster correlations. Although the proton multiplicity is now consistent with the experimental data [1], the $\alpha$ particles are produced too much and heavier fragments are not sufficient. In short, the tendency of turning into cluster gas is too strong compared to experimental data.

To characterize the situation of a multi-fragmenting system, it is useful to pay attention to the liquid-gas separation, where the liquid is defined as the part of the system consisting of fragments with $A > 4$ and the gas part consists of emitted light particles with $A \leq 4$. The charge in the gas part relative to the total charge of the system changes from $Z_{\text{gas}}/Z_{\text{tot}} = 0.55$ to 0.78 by cluster correlations, while the INDRA data suggest that the gas fraction $Z_{\text{gas}}/Z_{\text{tot}}$ should be between 0.4 and 0.5. Figure 7 shows the relation between the gas fraction and the proton multiplicity. As the experimental point exists out of the line connecting the calculated results with and without cluster correlations, it is not likely that the data can be reproduced by a minor tuning such as adjusting the probability of cluster formation.

4. Effects of inter-cluster correlations
The reason of the unphysically strong tendency of cluster gas may be due to the fact that the motion of clusters is almost classical in AMD calculations when the equation of motion for the wave packet centroids is solved. In reality, a few clusters can form a bound state corresponding to the ground state of a light nucleus such as the Li and Be isotopes. The binding of clusters in many of these nuclei is weak so that there is only one bound intrinsic state. Even though the AMD wave function can describe the ground state, the phase space of classically bound configurations does not necessarily have a correct volume corresponding to a single quantum state. When the binding is weak, the classical phase space is much smaller than the quantum phase space, which suppresses the binding probability of clusters in AMD.

To test this idea, some inter-cluster correlation is introduced as a stochastic process of bonding clusters. The relative momentum between clusters is replaced by zero if moderately separated clusters ($R_{\text{rel}} < 5 \text{ fm}$) are moving away from each other with a small relative momentum ($R_{\text{rel}} \cdot P_{\text{rel}} < 0$ and $P_{\text{rel}}^2/2\mu < 8 \text{ MeV}$). Thus this cluster bonding process works to select...
bound states of two or several clusters during the time evolution of reactions.

This inter-cluster correlation certainly reduces the tendency of cluster gas. For central collisions of Xe + Sn at 50 MeV/nucleon, the α-particle multiplicity is reduced from $M_\alpha = 23.2$ to 10.7 by the inter-cluster correlation, while the the proton multiplicity changes only slightly from $M_p = 10.9$ to 10.8. The gas fraction $Z_{\text{gas}}/Z_{\text{tot}} = 0.43$ is also consistent with the INDRA data [1] when the inter-cluster correlation is taken into account with the parameters mentioned above.

The degree of stopping, namely how much the kinetic energies of the initial nuclei are converted to the transverse directions, is an important quantity to characterize the global dynamics. It can be quantified by using the kinetic energy components of emitted particles

$$R_E = \frac{\sum (E_x + E_y)}{2 \sum E_z},$$

(10)

where the sums are for all charged products ($Z \geq 1$). The INDRA data show $R_E \approx 0.6$ at this energy [16]. As shown in Fig. 1, the stopping is too small ($R_E = 0.50$) with usual two-nucleon collisions without cluster formation, while the stopping increases ($R_E = 0.61$) and consistent with data when cluster formation is taken into account. The inter-cluster correlation did not influence the stopping in this case. It should be noted that the degree of stopping also depends on the choice of the two-nucleon collision cross sections which is not very well known in nuclear medium, and therefore it is always important to carefully check the calculated stopping in order to describe the global collision dynamics reliably.

Higher energy collisions are studied for the Au + Au central collisions. As shown in Fig. 9, the calculation with the same options of the cluster and inter-cluster correlations reproduces the experimental data [17] of fragment charge distribution at both 150 and 250 MeV/nucleon. The hydrogen multiplicity is also well reproduced. However, the proton multiplicity is overestimated (and therefore the deuteron multiplicity is underestimated) by the calculation, which may be indicating that the high momentum components in the cluster internal wave functions should be taken into account more precisely than assuming the harmonic oscillator (0s)$^n$ configurations.

5. Symmetry energy effects on cluster emission
The results in the previous section demonstrate that the global dynamics and fragmentation are reasonably described by AMD with cluster and inter-cluster correlations, at least for central
Figure 9. The charge distribution of fragments produced in central Au + Au collisions at 150 (left) and 250 (right) MeV/nucleon. The red histogram was calculated by AMD with cluster and inter-cluster correlations. The blue points show the experimental data of Reisdorf et al. [17].

Figure 10. Multiplicities of light particles emitted to $60^\circ < \theta_{CM} < 120^\circ$ in $^{112}\text{Sn} + ^{112}\text{Sn}$ and $^{124}\text{Sn} + ^{124}\text{Sn}$ central collisions at 50 MeV/nucleon, calculated by AMD with cluster and inter-cluster correlations. Results with the Skyrme SLy4 interaction ($L = 46$ MeV) and its modified version for a stiffer density-dependence of the symmetry energy ($L = 108$ MeV) are shown.

|       | $^{124}+^{124}$ | $^{112}+^{112}$ | DRatio |
|-------|----------------|----------------|--------|
| $Y(t)/Y(^3\text{He})$ | 4.1             | 2.0             | 2.0    |
| $Y(n)/Y(p)$            | 4.5             | 2.1             | 2.2    |

Table 1. The $t/^3\text{He}$ and $n/p$ yield ratios for the particles emitted to $60^\circ < \theta_{CM} < 120^\circ$ in $^{112}\text{Sn} + ^{112}\text{Sn}$ and $^{124}\text{Sn} + ^{124}\text{Sn}$ central collisions at 50 MeV/nucleon calculated by AMD with cluster and inter-cluster correlations. The double ratios (DRatio) are taken between these two reaction systems. Results with the Skyrme SLy4 interaction ($L = 46$ MeV) and its modified version for a stiffer density-dependence of the symmetry energy ($L = 108$ MeV) are shown in the left and right sides, respectively.
collisions of $A \approx 120$ nuclei at 50 MeV/nucleon. Therefore we expect that the cluster emission and the symmetry energy effects in it are reliably predicted by AMD for similar systems.

Figure 10 shows the multiplicities of light particles in central $^{112}$Sn + $^{112}$Sn and $^{124}$Sn + $^{124}$Sn collisions at 50 MeV/nucleon. Only the particles emitted to the transverse directions $60^\circ < \theta_{\text{CM}} < 120^\circ$ in the center-of-mass system are employed for the analysis here to see the effects of the symmetry energy. The calculations are performed with two different effective interactions. The one labeled by $L = 46$ MeV is the Skyrme SLy4 force and the other one labeled by $L = 108$ MeV is a force modified based on the SLy4 force to have a stiffer density-dependence of the symmetry energy without changing the equation of state for isospin-symmetric and spin-saturated systems. In all the cases, neutrons and tritons are emitted more than protons and $^3$He, respectively. Furthermore the relations in the neutron and proton yields look similar to those in the triton and $^3$He yields. However, as shown in Table 1, the detailed number of the $t/\alpha$He ratio is not always the same as the $n/p$ yield ratios, in particular for $L = 108$ MeV. When further ratios are taken between two reactions, the double ratios for $t/\alpha$He and $n/p$ are close. Note that the double ratios for both $t/\alpha$He and $n/p$ are expected to be $e^{\alpha-\beta}$ if the yields satisfy isoscaling [18] with the parameters $\alpha$ and $\beta$. It can be also seen that the double ratio depends on the stiffness of the symmetry energy.

The analysis here was done by using particles at all energies. Similar study for the energy spectra to compare with the data [20] and other calculations [19] is an interesting subject to do in near future.

6. Summary

In multifragmentation reactions in wide range of incident energies, it has been well known that $\alpha$ particles are copiously produced in addition to intermediate mass fragments. In head-on collisions of $A \approx 120$ nuclei at 50 MeV/nucleon, for example, more than 20% of protons in the system are bound in $\alpha$ particles, while only about 10% of protons are emitted as free protons. Microscopic transport models, including the standard version of AMD, have not been able to consistently describe the formation of clusters and fragments. Nucleon multiplicity is seriously overestimated in most cases. This suggests that cluster correlations should be more explicitly considered in the dynamics of heavy-ion collisions.

The AMD approach was extended by taking into account the formation of correlated clusters in the final states of every two-nucleon collision. The probabilities of transition to different clustering configurations are evaluated based on the usual two-nucleon collision cross sections assuming that the transition is induced by the two-body residual interaction. The non-orthogonality of the final states was carefully considered.

Calculations show that the cluster correlations have a drastic impact on the whole collision dynamics. Not only the increase of the $\alpha$-particle multiplicity and the decrease of the nucleon multiplicity, but also the separation ratio into liquid ($A > 5$) and gas ($A < 4$) is affected. The tendency of turning into cluster gas is too strong compared to experimental data. For observables such as the fragment and cluster yields, the kinetic energies and the degree of stopping, much better agreement with data is obtained by further considering the correlations between clusters.

With this new version of AMD with cluster and inter-cluster correlations, the global dynamics and fragmentation in heavy-ion collisions are reproduced reasonably well. This is an important step to reliably extract the information of nuclear matter equation of state from the analysis of emission of nucleons, clusters and heavier fragments.

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