Confinement- Deconfinement Phase Transition and Fractional Instanton Quarks in Dense Matter

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We present arguments suggesting that large size overlapping instantons are the driving mechanism of the confinement-deconfinement phase transition at nonzero chemical potential \( \mu \). The arguments are based on the picture that instantons at very large chemical potential in the weak coupling regime are localized configurations with finite size \( \rho \sim \mu^{-1} \). At the same time, the same instantons at smaller chemical potential in the strong coupling regime are well represented by the so-called instanton-quarks with fractional topological charge \( 1/N_c \). We estimate the critical chemical potential \( \mu_c(T) \) where transition between these two regimes takes place. We identify this transition with confinement-deconfinement phase transition. We also argue that the instanton quarks carry magnetic charges. As a consequence of it, there is a relation between our picture and the standard t’Hooft and Mandelstam picture of the confinement. We also comment on possible relations of instanton-quarks with “periodic instantons”, “center vortices”, and “fractional instantons” in the brane construction. We also argue that the variation of the external parameter \( \mu \), which plays the role of the vacuum expectation value of a “Higgs” field at \( \mu \gg \Lambda_{QCD} \), allows to study the transition from a “Higgs-like” gauge theory (weak coupling regime, \( \mu \gg \Lambda_{QCD} \)) to ordinary QCD (strong coupling regime, \( \mu \ll \Lambda_{QCD} \)). We also comment on some recent lattice results on topological charge density distribution which support our picture.

PACS numbers:

I. INTRODUCTION

This talk is based on a number of original results\(^2\),\(^3\) obtained with different collaborators at different times.

Color confinement, spontaneous breaking of chiral symmetry, the \( U(1) \) problem and the \( \theta \) dependence are some of the most interesting questions in QCD. Unfortunately, progress in the understanding of these problems has been extremely slow. At the end of the 1970’s A. M. Polyakov\(^4\) demonstrated charge confinement in QED\(_3\). This was the first example where nontrivial dynamics was shown to be a key ingredient for confinement. The instantons (the monopoles in 3d) play a crucial role in the dynamics of confinement in QED\(_3\). Instantons in four dimensional QCD were discovered 30 (!) years ago\(^5\). However, their role in QCD\(_4\) remains unclear even today due to the divergence of the instanton density for large size instantons.

Approximately at the same time instanton dynamics was developed in two dimensional, classically conformal, asymptotically free models (which may have some analogies with QCD\(_4\)). Namely, using an exact accounting and resummation of the \( n \)-instanton solutions in \( 2d \ CP^{N_c-1} \) models, the original problem of a statistical instanton ensemble was mapped unto a \( 2d \)-Coulomb Gas (CG) system of pseudo-particles with fractional topological charges \( \sim 1/N_c \) (the so-called instanton-quarks)\(^6\). The instanton-quarks do not exist separately as individual objects. Rather, they appear in the system all together as a set of \( \sim N_c \) instanton-quarks so that the total topological charge of each configuration is always an integer. This means that a charge for an individual instanton-quark cannot be created and measured. Instead, only the total topological charge for the whole configuration is forced to be integer and has a physical meaning. This picture leads to the elegant explanation of confinement and other important properties of the \( 2d \ CP^{N_c-1} \) models\(^6\). Unfortunately, despite some attempts\(^7\), there is no demonstration that a similar picture occurs in \( 4d \) gauge theories, where the instanton-quarks would become the relevant quasiparticles. Nevertheless, there remains a strong suspicion that this picture, which assumes that instanton-quarks with fractional topological charges \( \sim 1/N_c \) become the relevant degrees of freedom in the confined phase, may be correct in QCD\(_4\).

On the phenomenological side, the development of the instanton liquid model (ILM)\(^8\),\(^9\) has encountered successes (chiral symmetry breaking, resolution of the \( U(1) \) problem, etc) and failures (confinement could not be described by well separated and localized lumps with integer topological charges). Therefore, it is fair to say that at present, the widely accepted viewpoint is that the ILM can explain many experimental data (such as hadron masses, widths, correlation functions, decay couplings, etc), with one, but crucial exception: confinement. There are many arguments against the ILM approach, see e.g.\(^10\), there are many arguments supporting it\(^4\).

In this talk we present new arguments supporting the idea that the instanton-quarks along with instantons are the relevant quasiparticles in the strong coupling regime. In this case, many problems formulated in\(^10\) are naturally resolved as both phenomena, confinement and chiral symmetry breaking are originated from the same vac-

*Invited talk delivered at the Light Cone Workshop, July 7-15, 2005, Cairns, Australia.
uum configurations, instantons, which may have arbitrary scales: the finite size localized lumps with integer topological charges, as well as set of $N_c$ fractionally $1/N_c$-charged correlated objects sitting at arbitrary large distances from each other. In this picture when fractionally charged $1/N_c$ constituents propagate far away from each other, the confinement could be a natural consequence of a dynamics of these well correlated objects. We emphasize that along with instanton quarks there are ordinary instantons with integer topological charges. Indeed, if the instanton-quarks are close to each other they bound together and likely to form an ordinary instanton. If the instanton quarks far away from each other, the description in terms of fundamental instanton quarks is more appropriate. The precise probability for each configuration depends on the interplay between action and entropy. Such a feature when the well-localized instantons and de-localized instanton-quarks coexist may lead to the understanding why the chiral symmetry breaking phenomenon (which, as ILM suggests, is due to the well-localized instantons) and the confinement-deconfinement phase transitions (which is due to the de-localized instanton quarks, according to the present proposal) are so close to each other. In our picture such a “conspiracy” is a simple reflection of the fact that both phenomena are due to the same configurations, instantons, which however can be in different configurations.

More importantly, we make some very specific predictions which can be tested with traditional Monte Carlo techniques, by studying QCD at nonzero isospin chemical potential. Our conjecture can be explicitly and readily tested in numerical simulations which can be made with traditional Monte Carlo techniques, by studying QCD at nonzero isospin (rather than baryon) chemical potential. We also comment on relations with different works. Finally, we make some comments on recent lattice results on topological density distribution.

II. Instantons at Large $\mu$

At low energy and large chemical potential, the $\eta'$ is light and described by the Lagrangian derived in 2:

$$L_{\varphi} = f^2(\mu)\left[\left(\partial_\mu \varphi\right)^2 - u^2(\partial_\mu \varphi)^2\right] - V_{\text{inst}}(\varphi).$$

where the $\varphi$ decay constant, $f^2(\mu_B) = \mu_B^2/8\pi^2$ and $f^2(\mu_I) = 3\mu_I^2/16\pi^2$, and its velocity, $u^2 = 1/3$. We define baryon and isospin chemical potentials as $\mu_B = (\mu_u + \mu_d)/2$. The nonperturbative potential $V_{\text{inst}} \sim \cos(\varphi - \theta)$ is due to instantons, which are suppressed at large chemical potential.

The instanton-induced effective four-fermion interaction for 2 flavors, $u, d$, is given by 14, 15:

$$L_{\text{inst}} = \int d\rho n(\rho) \left\{ \left( \bar{u}_R u_L \right)(\bar{d}_R d_L) + \frac{1}{3} \left[ (\bar{u}_R \lambda^\mu u_L)(\bar{d}_R \lambda^\nu d_L) \right] - \frac{3}{4} (\bar{u}_R \sigma_{\mu\nu} \lambda^\mu u_L)(\bar{d}_R \sigma_{\mu\nu} \lambda^\nu d_L) \right\} + \text{H.c.}$$

We study this problem at nonzero temperature and chemical potential for $T \ll \mu$, and we use the standard formula for the instanton density at two-loop order 2:

$$n(\rho) = C_N(\beta_I(\rho))^2 \rho^{-5} \exp[-\beta_{II}(\rho)] \times \exp[-(N_f \mu^2 + 1/3(2N_c + N_f)\pi^2T^2\rho^2)],$$

where

$$C_N = 0.466e^{-1.679N_c1.34N_f}/(N_c - 1)(N_c - 2)^{1/2},$$

$$\beta_I(\rho) = -b \log(\rhoA),$$

$$\beta_{II}(\rho) = \beta_I(\rho) + \frac{b'}{2b} \log \left( \frac{2\beta_I(\rho)}{b} \right),$$

$$b = \frac{11}{3} N_c - \frac{2}{3} N_f,$$

$$b' = \frac{34}{3} N_c^2 - \frac{13}{3} N_f N_c + \frac{N_f}{N_c}.$$
By taking the average of Eq. (3) over the state with nonzero vacuum expectation value for the condensate, one finds

\[ V_{\text{inst}}(\varphi) = - \int d\rho n(\rho) \left( \frac{4}{3} \pi^2 \rho^3 \right)^2 \times 12|X(\mu)|^2 \cos(\varphi - \theta) \]

\[ = -a(\mu, T) \mu^2 \Delta^2 \cos(\varphi - \theta), \]

where \(|X(\mu)| = 3\sqrt{3} \Delta \sqrt{\rho} \) and \(|X(\mu)| = 3\sqrt{3} \mu^2 \Delta \sqrt{\rho} \), and \( \Delta \) is the gap \[2, 13\]. Therefore the mass of the \( \varphi \) field is given by

\[ m = \sqrt{\frac{a(\mu, T)}{2f(\mu)}} \mu \Delta. \]

The approach presented above is valid as long as the \( \varphi \) field is lighter than \(~2\Delta\), the mass of the other mesons in the system \[2\], that is if

\[ a(\mu, T) \leq 8f^2(\mu)/\mu^2. \]

This is exactly the vicinity where the Debye screening scale and the inverse gap become of the same order of magnitude \[2\], and therefore, where the instanton expansion breaks down.

For reasons which will be clear soon, we want to represent the Sine-Gordon (SG) partition function \[1, 4\] in the equivalent dual Coulomb Gas (CG) representation \[2\],

\[ Z = \sum_{M=1}^{\infty} \frac{(\lambda/2)^M}{M!} \int d^4x_1 \ldots d^4x_M \exp\left[-i\theta \sum_{a=1}^{N_f} Q_a \cdot e^{-\frac{1}{2} \sum_{a>b=0}^{M} Q_a Q_b G(x_a - x_b)} \right]

\[ G(x_a - x_b) = \frac{1}{4\pi^2(x_a - x_b)^2}, \lambda \equiv \frac{a\mu^2 \Delta^2}{u}. \]

Physical interpretation of the dual CG representation \[6\]:

\( a) \) Since \( Q_{\text{net}} \equiv \sum_a Q_a \) is the total charge and it appears in the action multiplied by the parameter \( \theta \), one concludes that \( Q_{\text{net}} \) is the total topological charge of a given configuration.

\( b) \) Each charge \( Q_a \) in a given configuration should be identified with an integer topological charge well localized at the point \( x_a \). This, by definition, corresponds to a small instanton positioned at \( x_a \).

\( c) \) While the starting low-energy effective Lagrangian contains only a colorless field \( \varphi \) we have ended up with a representation of the partition function in which objects carrying color (the instantons) can be studied.

\( d) \) In particular, \( II \) and \( \bar{II} \) interactions (at very large distances) are exactly the same up to a sign, order \( g^6 \), and are Coulomb-like. This is in contrast with semiclassical expressions when \( II \) interaction is zero and \( \bar{II} \) interaction is order \( 1/g^2 \).

\( e) \) The very complicated picture of the bare \( II \) and \( \bar{II} \) interactions becomes very simple for dressed instantons/anti-instantons when all integrations over all possible sizes, color orientations and interactions with background fields are properly accounted for.

\( f) \) As expected, the ensemble of small \( \rho \sim 1/\mu \) instantons can not produce confinement. This is in accordance with the fact that there is no confinement at large \( \mu \).

### III. INSTANTONS AT SMALL \( \mu \)

We want to repeat the same procedure that led to the CG representation in the confined phase at small \( \mu \) to see if any traces from the instantons can be recovered. We start from the chiral Lagrangian and keep only the diagonal elements of the chiral matrix \( U = \exp\{\text{diag}(\phi_1, \ldots, \phi_{N_f})\} \) which are relevant in the description of the ground state. Singlet combination is defined as \( \phi = \text{Tr} U \). The effective Lagrangian for the \( \phi \) is

\[ L_\phi = f^2(\partial_\mu \phi)^2 + E \cos\left(\frac{\phi - \theta}{N_c}\right) + \sum_{a=1}^{N_f} m_a \cos \phi_a \]

A Sine-Gordon structure for the singlet combination corresponds to the following behavior of the \( (2k)^{th} \) derivative of the vacuum energy in pure gluodynamics \[11\],

\[ \frac{\partial^{2k} E_{\text{vac}}(\theta)}{\partial \theta^{2k}} \bigg|_{\theta=0} \sim \int \prod_{i=1}^{2k} \left\langle Q(x_1) \ldots Q(x_{2k}) \right\rangle \]

\[ \sim \left( \frac{i}{N_c} \right)^{2k}, \quad \text{where} \quad Q \equiv \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}_{\mu\nu}. \]

The same structure was also advocated in \[17\] from a different perspective. As in \[7\] the Sine-Gordon effective field theory \[8\] can be represented in terms of a classical statistical ensemble (CG representation) similar to \[7\] with the replacements \( \lambda \rightarrow E, \ u \rightarrow 1 \), more precisely,

\[ Z = \sum_{M=0}^{\infty} \frac{(E/2)^M}{M!} \int d^4x_1 \ldots d^4x_M \times \sum_{Q_a = \pm 1/N_c} \int D\phi \frac{1}{E}\int d^4x(\partial_\mu \phi)^2 \times \left( e^{i \sum_{a=1}^{N_f} Q_a(\phi(x_a) - \theta)} \right). \]

The functional integral is trivial to perform and one arrives at the dual CG action,

\[ Z = \sum_{M=0}^{\infty} \frac{(E/2)^M}{M!} \int d^4x_1 \ldots d^4x_M \times \left( e^{-i\theta \sum_{a=1}^{N_f} Q_a} \right) \cdot \frac{1}{4\pi^2(x_a - x_b)^2}, \]

\[ G(x_a - x_b) = \frac{1}{4\pi^2(x_a - x_b)^2}. \]
The fundamental difference in comparison with the previous case is that while the total charge is integer, the individual charges are fractional \( \pm 1/N_c \). This is a direct consequence of the \( \theta/N_c \) dependence in the underlying effective Lagrangian before integrating out \( \phi \) fields, see eq. (10).

Physical Interpretation of the CG representation of theory (11):

(a) As before, one can identify \( Q_{\text{net}} \equiv \sum_a Q_a \) with the total topological charge of the given configuration.

(b) Due to the \( 2\pi \) periodicity of the theory, only configurations which contain an integer topological number contribute to the partition function. Therefore, the number of particles for each given configuration \( Q_i \) with charges \( \sim 1/N_c \) must be proportional to \( N_c \).

(c) Therefore, the number of integrations over \( d^4 x_i \) in CS representation exactly equals \( 4N_c k \), where \( k \) is integer. This number \( 4N_c k \) exactly corresponds to the number of zero modes in the \( k \)-instanton background. This is basis for the conjecture that at low energies (large distances) the fractionally charged species, \( Q_i = \pm 1/N_c \), are the instanton-quarks suspected long ago.

(d) For the gauge group, \( G \), the number of integrations would be equal to \( 4kC_2(G) \) where \( C_2(G) \) is the quadratic Casimir of the gauge group (\( \theta \) dependence in physical observables comes in the combination \( \frac{\theta}{C_2(G)} \)). This number \( 4kC_2(G) \) exactly corresponds to the number of zero modes in the \( k \)-instanton background for gauge group \( G \).

(e) The CG representation corresponding to eq. (5) describes the confinement phase of the theory.

One obvious objection for such an identification of \( Q_a \) with the topological charge immediately comes in mind: it has long been known that instantons can explain most low energy QCD phenomenology with the exception confinement; and we claim that confinement also arises in this picture: how can this be consistent? We note that quark confinement cannot be described in the dilute gas approximation, when the instantons and anti-instantons are well separated and maintain their individual properties (sizes, positions, orientations), as it happens at large \( \mu \). However, in strongly coupled theories the instantons and anti-instantons lose their individual properties (instantons will “melt”) their sizes become very large and they overlap. The relevant description is that of instanton-quarks which can be far away from each other, but still strongly correlated. For such configurations the confinement is a possible outcome of the dynamics.

We should remark here that a precise form of the potential in the form of a single function \( \sim \cos(\theta/N_c) \) is not a crucial issue for discussions below. A combination of a number of terms, \( a_k \cos(k\theta/N_c) \) may change the interactions of the instanton quarks. However, the most important element here remains the same: the \( \theta/N_c \) behavior is well established result and remains untouched even when more complicated terms are introduced. It will lead to the fractional charges \( Q_a = \pm 1/N_c \) in Coulomb Gas representation in big contrast with weakly interacting phase at large \( \mu \) where only integer topological charges appear.

We should also comment at this point that our numerical estimates below are based exclusively on the instanton density at large \( \mu \) while we approaching the critical value. In this region the potential is well established and unique. Therefore, our results below are not sensitive to the specific details of the potential at small \( \mu \) when some additional terms might be present.

IV. CONJECTURE.

We thus conjecture that the confinement-deconfinement phase transition takes place at precisely the value where the dilute instanton calculation breaks down. At large \( \mu \) the weakly interacting phase (CS) is realized. Instantons are well localized configurations with a typical size \( \mu^{-1} \). Color in CS phase is not confined. At low \( \mu \) the strong interacting regime is realized and color is confined. Instantons are not well localized configurations, but rather are represented by \( N_c \) instanton quarks which can propagate far away from each other. The value of the chemical potential as a function of temperature, \( \mu_c(T) \) is given by saturating the inequality (10).

Few remarks are in order. First, we can estimate the critical \( \mu_c \) not only at \( T = 0 \), but also at \( T \neq 0 \) as long as the temperature is relatively small such that our approach is justified. Indeed, in the weak coupling regime the \( T \) dependence of the instanton density is determined by a simple insertion \( \sim \exp[-(2N_c + N_f)\pi^2 T^2/\rho^2] \) into the expression for the density. The temperature dependence also enters the expression for \( \Delta(T) \). As long as \( \Delta(T) \) does not vanish and we are in CS phase, our calculations are justified, and the critical \( \mu_c(T) \) can be estimated as a function of \( T \) at relatively small \( T \) as shown in FIG.1.

We should emphasize that in our picture the nature of the phase transition is universal and it is not sensitive to the specific values of \( N_c \) and \( N_f \), in spite of the fact that the ground state of the superconducting phase is very sensitive to the values of \( N_c \), \( N_f \) and quark mass (CFL, 2SC, crystalline or even more complicated phases).

V. NUMERICAL RESULTS.

The critical chemical potential as a function of temperature is implicitly given by \( a(\mu_c(T), T) = 8f^2(\mu_c(T))/\mu_c^2(T) \). We can calculate \( a(\mu_c(T), T) \) from (11). We are however limited to temperatures where Cooper pairing takes place, i.e. for \( T \leq 0.567 \Delta \). We

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1 We refer to ref. where it is argued, based on analysis of two dimensional \( CP^{N-1} \) model, that much more complicated structure for the instanton quark interactions could result.
TABLE I: Results

|                | $N_c = 3, N_f = 2$ | $N_c = 3, N_f = 3$ | $N_c = 2, N_f = 2$ |
|----------------|--------------------|--------------------|--------------------|
| $\mu_{BC}/\Lambda$ | 2.3                | 1.4                | 3.5                |
| $\mu_{IC}/\Lambda$ | 2.6                | 1.5                | 3.5                |

![Graph](image)

FIG. 1: Critical isospin chemical potential for the confinement-deconfinement phase transition as a function of temperature (solid curve). The dashed curve represents the critical value.

VI. INSTANTON QUARKS AS MONOPOLES.

Having formulated our conjecture and the results which follow from it, the question about the relation between the standard ‘t Hooft and Mandelstam picture of the confinement and our proposal (when confinement is due to the instanton-quarks) can be formulated. The key point of the ‘t Hooft - Mandelstam approach is the assumption that dynamical monopoles in QCD exist and Bose condense. The goal of this section is to argue that the instanton-quarks carry the magnetic charges. Therefore, in principle, they may play the role of the dynamical monopoles which are the key players in the ‘t Hooft and Mandelstam framework. In this case both pictures could be the two sides of the same coin.

Expression clearly shows that the statistical ensemble of particles interact according to the Coulomb law. An immediate suspicion following from this observation is that these particles carry a magnetic and/or electric charge, since charges of that type interact precisely in the above manner. This suspicion will be corroborated in a moment. The charges $Q_a$ were originally introduced in a very formal manner so that the QCD effective low energy Lagrangian can be written in the dual CG form. In the previous sections we presented arguments that the particles $Q_a$ carry fractional topological charges and can be identified with instanton quarks. Now we shall argue that these particles also carry the magnetic charges.

As a short detour, let us remind few important results regarding the SU(2) Georgi-Glashow model in the weak coupling regime, with a $\theta$-term when the scalar $\Phi^a$ have a large VEV. The monopole solution can be constructed explicitly and the well-known Witten’s effect, where the monopole acquires an electrical charge, takes place. Let $N$ denote the generator of large gauge transformations corresponding to rotations in the $U(1)$ subgroup of SU(2) picked out by the gauge field, i.e. rotations in SU(2) about the axis $n^a = \frac{1}{\sqrt{3}}v^\alpha$. Rotations by an angle of $2\pi$ about this axis must yield the identity for arbitrary configurations, which implies that the magnetic monopoles carry an electric charge proportional to $\theta$. Indeed,

$$1 = e^{i2\pi N} = e^{i2\pi \frac{Q}{b} - i\theta \frac{Q}{2b}}, \quad (12)$$

where,

$$M = \frac{1}{v} \int d^3x D_i\Phi^a B^a_i, \quad Q = \frac{1}{v} \int d^3x D_i\Phi^a E^a_i,$$

are the magnetic and electric charge operators respectively, expressed in terms of the original fields, and $v$ is the vacuum expectation value $\langle \Phi^a \rangle$ at infinity. The combination $\frac{Q}{b} = n_m$ in Eq. (12) is an integer and determines the magnetic charge of the configuration. As usual, it is assumed that remains correct in the strong coupling regime when $v$ is not large and/or in the...
more radical case when \( \Phi^a \) is not present in the original formulation. Indeed, as explained in \[21\] the existence of \( \Phi^a \) is not essential and some effective fields may play its role. One finds that monopoles do exist and the Witten effect expressed by formula \[12\] remains unaltered even when monopoles appear as singularities in the course of the gauge fixing procedure as described in \[21\].

Restricting attention to terms which are proportional to the \( \theta \)-parameter, a comparison between the CG representation, Eq. \[11\], and Eq. \[12\] will now be carried out. From the CG representation, Eq. \[11\] the relevant term is the total charge, \( Q_{\text{net}} \), of the configuration, while in Eq. \[12\] the relevant factor is the total magnetic charge \( \frac{4\pi e M}{4\pi} \) for each time slice. The following identification is then made\(^2\):

\[
Q_{\text{net}} = \frac{eM}{4\pi} = n_m \in \mathbb{Z}.
\]  

(13)

From these simple observations one can immediately deduce that our fractional magnetic charges \( Q_a \) cannot be related to any semi-classical solutions, which can carry only integer charges; rather, configurations with fractional magnetic charges should have pure quantum origin.

One should notice here that the connection between monopoles and instantons on the classical level is not a very new idea \[22\]. Indeed, for example, quite recently, such a relation was established for the periodic instantons (also called calorons) defined on \( R^4 \times S^1 \) \[26\], see also \[27\] and \[28\] where monopoles and instantons are intimately related objects in semiclassical construction.

Furthermore, a similar relation was seen in the study of Abelian projection for instantons \[26, 27\], albeit at the classical level. In particular in ref. \[27\] it was demonstrated that the instanton’s topological charge, \( Q \), is given in terms of the monopole charge \( M \) forming the loop as follows \( Q = \frac{eM}{4\pi} \). This formula is very similar to our relation \[13\], where the total topological charge, \( Q_{\text{net}} \), for a configuration containing a number of particles, described by the system \[11\] was identified with the total magnetic charge for each Euclidean time slice for the same configuration. Further to this point, lattice simulations do not contradict this picture where large instantons induce the magnetic monopole loops forming large clusters, see e.g. \[28\] and references therein.

We conclude this section with few following remarks. 1). The relation between topological charge in 4d and magnetic charge in 3d is understood only on the level of classical equations of motion, \[22-27\]. However, this knowledge does not provide us with answers on the crucial questions such as: “what is dynamical properties of these monopoles?”, “do they condense or, rather, they propagate only for short distances for a short period of time?”

2). A similar to eq. \[13\] identification could be made for a different system with large \( \mu \) \[7\] when only small size \( \rho \sim \mu^{-1} \) instantons are present. However, in this case it is quite obvious that the description in terms of the monopole loops makes no sense because the typical size of the loops is very small, of order \( \sim \mu^{-1} \), and the magnetic charge is obviously screened on large distances. Therefore, monopole charge of constituents play no role for such ensembles.

3). In contrast with the small instantons, the constituents of large size instantons (instanton quarks with charge \( 1/N_c \) ) may propagate far away from each other. In this case the description in terms of the instanton quarks which carry the monopole charges could be appropriate. For such configurations the magnetic charges of the instanton quarks should manifest themselves in some way. In particular, if the magnetic charges Bose-condense, this indicates the onset of quark confinement. To investigate the possibility for such a condensation an expression for the magnetic charge creation operator, \( \mathcal{M} \), must be found and its VEV (magnetization) calculated. Such a program is very ambitious, and obviously beyond the scope of the present work.

VII. CONCLUDING COMMENTS

A. Main Results

The main leitmotiv of this talk is based on the conjecture that the confinement-deconfinement phase transition at nonzero chemical potential and small temperature is driven by instantons. The instantons qualitatively change the shapes at the transition: they small well-localized objects at large \( \mu \gg \mu_c \); they become arbitrary large, strongly overlapped configurations at small \( \mu \ll \mu_c \) in which case description in terms of the instanton quarks become appropriate. Let us emphasize again: the instanton quarks are point like defects which have pure quantum origin and can not be described as semiclassical configurations. They are characterized by 4 translational collective variables, such that \( k \) units of the topological charge are represented by coherent superposition of \( N_c \) instanton quarks (per unit charge) to make together \( 4N_c k \) collective variables. This number precisely matches the number of the instanton parameters with topological charge \( k \). While the instanton quarks can be arbitrary far away from each other, they keep the information about their origin; they are correlated. Therefore, instanton quarks form not a random, but rather, the coherent large size configurations.

Furthermore we make a quantitative prediction for the

\(^2\) Of course we assume here that a configuration is static, or slowly depending on time. Therefore, the identification \[13\] should be considered as a relation if the instanton quarks \( Q_a \) were treated as classical sources. It is definitely not the case for the dynamical system under study. Nevertheless, relation \[13\] serves as a good argument suggesting that instanton quarks carry the magnetic charges. The crucial questions are: can these monopoles propagate far away from each other? do these monopoles condense?
critical value of the chemical potential where this transition between two descriptions takes place: \( \mu_c \sim 3\Lambda_{\text{QCD}} \) at \( T = 0 \). This prediction can be readily tested on the lattice at nonzero isospin chemical potential.

B. Future Directions

There are well established lattice method which allow to introduce isospin chemical potential into the system, see e.g. \[10, 30, 31\]. Independently, there are well-established lattice methods which allow to measure the topological charge density distribution, see e.g. \[29\]. We claim that the topological charge density distribution measured as a function of \( \mu_I \) will experience sharp changes at the same critical value \( \mu_I = \mu_c(T) \) where the phase transition (or rapid crossover) occurs. Indeed, the changes in the topological charge density distribution are expected due to the fundamental differences in \( \theta \) dependence in two different regimes. We identify these changes with confinement-deconfinement transition based on the arguments presented above. We strongly advocate the lattice community to perform such an analysis to see whether corresponding “accidental coincidence” indeed takes place. Such an analysis would provide an unique opportunity to study a transition from “Higgs-like” gauge theory by varying the external parameter \( \mu_I \) which plays the role of the vacuum expectation value of a Higgs field\(^3\). In such an analysis one could explicitly study what is happening with finite size instantons (which are under complete theoretical control at large \( \mu_I > \mu_c \)) when transition from weak coupling regime to strong coupling regime occurs.

C. Relation to Other Studies

Here we would like to make few comments on relation to other works.

\( \text{i).} \) As we already mentioned, at the intuitive level there seems to be a close relation between instanton quarks and the “periodic instanton” \[23, 32\]. Indeed, in these papers it has been shown that the large size instantons and monopoles are intimately connected and instantons have the internal structure resembling the instanton-quarks. Also, it has been shown that the constituents carry the magnetic charges. More than that, it has been also argued that large size instantons likely were missing in the lattice simulations, which is consistent with the picture advocated in the present work. Unfortunately, one should not expect to be able to account for large instantons using semiclassical technique to bring this intuitive correspondence onto the quantitative level. However, such a mapping may help us to understand the relation between pictures advocated by ’t Hooft and Mandelstam \[12\] on one hand and picture where instanton-quarks are the key players, on the other hand.

\( \text{ii).} \) There seems to be another close relation (albeit at the intuitive level) between the instanton quarks and configurations with center vortices and nexuses with fractional fluxes \( 1/N_c \), see recent papers on the subject and earlier references therein \[32\]. In particular, the total topological charge for entire configuration in both cases is always integer. Locally, however, essentially independent units carry fractional charges \( 1/N_c \). While the geometrical and topological properties are very similar in both cases, there is, however, a fundamental difference between the two: center vortices/nexuses are classical configurations, while the instanton quarks (and everything which accompanying them) have pure quantum origin. This remark is also applied to the “periodic instanton” mentioned above. This difference, in particular, manifests itself for the gauge group \( G \) different from \( SU(N_c) \). In this case the fractional topological charge carried by instanton quarks is \( 1/C_2(G) \), see Section III. At the same time, in general, \( C_2(G) \) is not related to the center of the group playing a crucial role in construction of center vortices \[32\].

\( \text{iii).} \) Using the overlap formalism for chiral fermions \[33\], it has been demonstrated \[34\] that there is a strong evidence for there existence of gauge field configurations with fractional topological charge \( Q = 1/2 \) for \( SU(2) \) gauge theory.

\( \text{iv).} \) There is an interesting recent development in lattice computations which in principle would allow to study the topological charge fluctuations in QCD vacuum without any assumptions or guidance based on some specific models for QCD vacuum configurations \[31\]. Our remark here that the picture based on the instanton quarks advocated here is consistent with these recent lattice results \[31\]. Indeed, the most profound finding of ref. \[31\] is demonstration that the topological density distribution in QCD has “inherently global” structure. It is definitely consistent with our picture when the point like instanton quarks can be far away from each other, but still keep the correlation at arbitrary large distances.

Another interesting observation by ref. \[31\] can be explained as follows. If 4D structures of finite size (such as instantons with finite size \( \sim \Lambda_{\text{QCD}} \)) dominate the continuum limit, than these coherent regions of size \( \sim \Lambda_{\text{QCD}} \) should exhibit scaling behavior when the lattice spacing is changed. This feature has not been observed in ref. \[31\]. Therefore, it has been suggested that, in physical units, the corresponding 4D structures should shrink to mere points in the continuum limit.

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\( ^3 \) “Higgs-like” gauge theories characterized by some finite expectation value of the Higgs field, when topological defects have finite size, \( \theta \) dependence is trivial, \( \sim \cos \theta \), and weak coupling regime is realized. This is in contrast with “Non-Higgs” gauge theories, like QCD at zero temperature and chemical potential when no fundamental scalar fields exist, \( \theta \) dependence appears in form of \( \theta/N_c \), and weak coupling regime can not be achieved in description of the large distance physics.
ation is certainly not in contradiction with our picture where instanton quarks are indeed, the effective 4D point like constituents classified by 4 translational zero modes.

As we discussed earlier in the text, the instanton quarks in static limit carry the magnetic charges. At the same time, the magnetic charge of the entire large-size instanton (with all its constituents with fractional charges $1/N_c$) must be zero. Therefore instanton quarks are attached to each other by magnetic strings such that total magnetic flux of whole system is zero. While the fluxes are $1/N_c$, they can be probed by quarks in fundamental representation. This picture, again, is consistent with feature of the “skeleton” (minimal hard-core substructure exhibiting the global behavior) from ref. [31] which is viewed as a network of world lines for point-like objects.

Finally, the dual picture of our CG representation (describing the instanton quarks) is nothing but the effective chiral lagrangian for Goldstone fields, see eq. [3]. This "obvious" connection between confinement and chiral symmetry breaking phenomenon in our framework is consistent with speculation of ref. [31] that the corresponding long distance correlations might be associated with long range propagation of Goldstone fields.

It is too early to say whether ref. [31] finds precisely the features we have been advocating to exist for quite a range propagation of Goldstone fields.

vi). As the final remark: the $\theta$ parameter played a key role in all discussions presented above. However, the region of $\mu_c(T)$ where transition is expected to occur (see Table 1) is not very sensitive to value of $\theta$. Indeed, the $\theta$ dependence in physical observable comes with extra suppression $\sim m_q$ which is very small factor. This is exactly the reason why all results for $\mu_c(T)$ are quoted for $\theta = 0$. This is definitely not the case when transition from normal to superfluid phase is considered as a function of baryon chemical potential at $N_c = 2$, or as a function of isotopical chemical potential at $N_c \geq 3$. In these cases the transitions are happening at $\mu \sim m_\pi(\theta)$ where very nontrivial dependence $\mu_c(T)$ on $\theta$ is expected [37].

Acknowledgements

Author thanks Pierre van Baal for numerous, very insightful and never ending discussions on the subject. Author also thanks all collaborators of the papers [1]-[8] which constitute the main bulk of this talk. Author also thanks Rajamani Narayanan for the correspondence regarding papers [11] and [12]. Alex Buchel for correspondence regarding the papers [19] and [20], and Michael Creutz for correspondence regarding the papers [37]. I am also thankful to the organizers of the Light Cone Meeting, Cairns, Australia, 2005, for inviting me to speak on this subject. The work was supported, in part, by the Natural Sciences and Engineering Research Council of Canada.

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