Large deflections of a structurally damped panel in a subsonic flow

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Received: 20 March 2020 / Accepted: 4 July 2020 / Published online: 18 July 2020 © Springer Nature B.V. 2020

Abstract The large deflections of panels in subsonic flow are considered, specifically a fully clamped von Karman plate accounting for both rotational inertia in plate filaments and (mild) structural damping. The panel is taken to be embedded in the boundary of the positive half-space in \( \mathbb{R}^3 \) containing a linear, subsonic potential flow. Solutions are constructed via a semigroup approach despite the lack of natural dissipativity associated with the generator of the linear dynamics. The flow–plate dynamics are then reduced—via an explicit Neumann-to-Dirichlet (downwash-to-pressure) solver for the flow—to a memory-type dynamical system for the plate. For the non-conservative plate dynamics, a global attractor is explicitly constructed via Lyapunov and recent quasi-stability methods. Finally, it is shown that, via the compactness of the attractor and finiteness of the dissipation integral, all trajectories converge strongly to the set of stationary states.

Keywords Mathematical aeroelasticity · Von Karman plate · Flutter · Semigroup · Quasi-stability · Attractors · Stabilization

Mathematics Subject Classification 74F10 · 74K20 · 76G25 · 35B40 · 35G25 · 37L15

1 Introduction

In this treatment, we present and discuss rigorous results for a panel flutter model appearing in the classic aeroelasticity literature [19–21], namely a subsonic, inviscid potential flow interacting with the large deflections of a fully clamped plate. We specifically take a von Karman-type plate, allowing for rotational inertia effects in plate filaments as well as some (small) amount of structural damping. We are concerned with the Hadamard well-posedness of solutions (existence, uniqueness, and continuous dependence upon data), as well as qualitative properties of solutions beyond the transient regime. Specifically, we are interested in aeroelastic instabilities such as flutter.

Aeroelastic flutter, in a flow–plate system, is a particular type of feedback instability where the flow’s aerodynamical loading at the fluid–structure interface destabilizes the otherwise stable damped plate [21,26]. The flutter instability occurs as a bifurcation in the flow parameters, typically that of the unperturbed flow velocity \( U \). Such an instability can manifest itself via chaotic plate oscillations [23], but often occurs in the form of limit cycle oscillations. Stationary instability—elastic bucking—is also possible, depending on the mechanical forcing and flow quantities in the system. Hence, a variety of interesting questions present
themselves: (i) Can one detect, from the parameters in the problem, if flutter will occur? (ii) Is flutter, so to speak, suppressed by the inclusion of some form of mechanical damping? (iii) In what ways does the long-time behavior of the flow–plate system depend on the initial configuration? Engineers, for instance, often observe/remark that panel flutter only occurs supersonically \[20,22\]. In this paper, we attempt to address these points in a mathematically rigorous way, in the fully infinite-dimensional setting, taken from the strongly coupled flow–plate PDE system presented below.

Here, we will present the construction of both strong and weak PDE solutions through the semigroup approach, in particular characterizing the domain of the generator for the flow–plate dynamics. We utilize the structure of the potential flow equation to explicitly construct a particular Neumann-to-Dirichlet map for the flow, which permits a closed representation for the plate with memory effects scaled by the characteristic flow velocity. For the resulting non-conservative plate with memory, we construct a smooth, compact global attractor of finite dimension, so long as some structural damping is present in the plate. This attractor is global in the sense that it attracts bounded sets in the state space with uniform rate, i.e., does not depend on specific initial data. With the attractor in hand, we utilize the finiteness of the dissipation integral, along with the compactness of the plate-to-flow (Neumann) mapping, to show that subsonic trajectories always converge to the stationary set; this is to say, we confirm the engineering assertion that subsonic panel systems have stationary end behavior.

Many of the relevant and related results presented in this paper come from a variety of places, e.g., \[4,5,13,18,26,29,34\]. These references span books, book chapters, surveys, older papers, and newer papers, in some cases with sparse details. To our knowledge, a full rigorous discussion of this subsonic flow-plate system—from PDE model to well-posedness to attractors to stabilization—has not appeared. We do note the paper \[34\] of this form, as well as \[35\] where inertia is neglected, which considers a thermoelastic panel. In our case, we choose a particular structurally damped panel, where a full exposition is possible. Indeed, we consider the case of a clamped von Karman plate with rotational inertia effects included, as well as appropriately scaled mild structural damping, precisely because the model permits a clean sequence of rigorous results and a linear discussion. The style of the paper is to provide all of the main results (and the lemmata on which they are built) in a formal mathematical way, without necessarily proving each of these supporting facts. In the relevant cases below, we clearly provide the reference for the proofs and further discussion. In what follows below, we provide new approaches and/or proofs for the system (e.g., making use of the recent quasi-stability approach \[7\]) which have not been applied to this model in the literature. And, in the case of our main stabilization result, we provide a detailed proof.

1.1 Panel flutter model

The large deflections of an aeroelastic panel are typically modeled via the plate theory of von Karman \[13,17\], going back to the early aeroelasticity literature \[3,19\]. Here, we also choose this cubic-type model based upon the quadratic strain–displacement law \[17,27\]. At equilibrium, we model the centerline of the plate by a bounded domain \( \Omega \subset \{ x_3 = 0 \} \) with smooth boundary \( \Gamma \) having unit outward normal \( \nu \).

The inviscid potential flow corresponds to the linearization of compressible Navier–Stokes about the stationary state \( U e_1 \), i.e., constant flow of velocity \( U \) in the \( x \)-direction; we normalize the flow parameters so that \( U = 1 \) corresponds to the speed of sound, i.e., Mach 1. The flow environment we consider as \( \mathbb{R}^3_+ = \{ \mathbf{x} \in \mathbb{R}^3 : x_3 > 0 \} \) so that the plate’s centerline \( \Omega \subset \partial \mathbb{R}^3_+ \).

In this situation, then, \( u : \Omega \times [0, \infty) \to \mathbb{R}^3 \) corresponds to the transverse plate deflections; \( \phi : \mathbb{R}^3_+ \times [0, \infty) \to \mathbb{R}^3 \) is the perturbation velocity potential, such that \( v = U e_1 + \nabla \phi \) is the perturbed flow field. Then, the evolution flow–plate system of interest here is given by:

\[
\begin{align*}
&((1 - \alpha \Delta) u_t + \Delta^2 u + k(1 - \alpha \Delta) u_t + f_1(u) \quad \text{in} \quad \Omega \times (0, T), \quad u(0) = u_0; \quad u_t(0) = u_1 \\
&u = \partial_t u = 0 \quad \text{on} \quad \partial \Omega \times (0, T), \\
&(\partial_t + U \partial_{x_3})^2 \phi = \Delta \phi \quad \text{in} \quad \mathbb{R}^3_+ \times (0, T), \quad \phi(0) = \phi_0; \quad \phi_t(0) = \phi_1 \quad \text{in} \quad \mathbb{R}^3_+, \\
&\partial_{x_3} \phi = [(\partial_t + U \partial_{x_3}) u]_{\text{ext}} \quad \text{on} \quad \{ x_3 = 0 \} \times (0, T).
\end{align*}
\]

(1.1)

Above, parameters such as mass, density, thickness, and stiffness have been scaled out. The remaining parameters are those relevant to this mathematical analysis: \( U, \alpha, k \). Here, \( \alpha > 0 \) corresponds to the accommodation of rotational inertia of plate filaments \[27\].
whereas \( k > 0 \) corresponds to the presence of mild structural damping. The function \( p_0(\mathbf{x}) \) corresponds to a stationary pressure on the top surface of the plate. The notation \([\cdot]_{\text{ext}}\) above means extension by zero from \( \Omega \to \mathbb{R}^2 \) with corresponding restriction \( r_{\Omega}[\cdot] \), and the standard trace operator denoted by \( \gamma[\cdot] \) onto \( \partial \Omega \) or \( \{x_3 = 0\} \) is utilized (of course in the appropriate functional senses).

The (scalar) von Karman nonlinearity \([13,17,27]\) is given through the von Karman bracket and the Airy stress function. The bracket is

\[
[u, w] = (\partial_{x_1}^2 u)(\partial_{x_2}^2 w) + (\partial_{x_2}^2 u)(\partial_{x_1}^2 w) - 2(\partial_{x_1} \partial_{x_2} u)(\partial_{x_1} \partial_{x_2} w),
\]

while the Airy function is defined as an elliptic solver,

\[
u = v(u) \quad \text{is the solution to} \quad \Delta^2 v = -[u, u] \quad \text{in} \quad \Omega, \quad v = \partial_\nu v = 0 \quad \text{on} \quad \Gamma. \quad (1.2)
\]

Finally, letting \( F_0(\mathbf{x}) \) represent a stationary planar force on \( \Omega \) corresponding to in-plane plate loading (pre-stressing), we have the von Karman nonlinearity

\[f_0(u) = -[u, v(u)] + F_0.\]  

Remark 1.1 The damping in (1.1) would be delineated as “square-root” type in the case \( \alpha = 0 \), as it is in some sense an interpolation between weak damping of the form \( +k_0 u_t \) and Kelvin–Voigt damping of the form \( +k_2 \Delta^2 u_t \). This type of damping is popular in engineering since it most accurately reproduces physical (low) modal damping decay rates. See \([24, \text{Section 2.1}]\) for detailed discussion and further references. In our treatment, we refer to it as mild structural damping.

1.2 Notation and conventions

In this paper, we utilize the standard notation and conventions for \( L^p(\Omega) \) spaces and Sobolev spaces of order \( s \in \mathbb{R} \), \( H^s(\Omega) \) where \( \partial \Omega \) is some domain. The space \( H^0(\Omega) \) denotes the completion of the test functions \( C_0^\infty(\Omega) \) in the \( H^s(\Omega) \) norm with dual \( H^{-s}(\Omega) \). For our norm notation, we will denote \( \| \cdot \|_{H^s(\partial \Omega)} = \| \cdot \|_s \), where the spatial domain will be clear from context; we will identify \( \| \cdot \|_{L^2(\partial \Omega)} = \| \cdot \|_0 \), omitting \( s = 0 \). Inner products on \( \mathbb{R}^3_+ \) will be denoted by \( \langle \cdot, \cdot \rangle := \langle \cdot, \cdot \rangle_{L^2(\mathbb{R}^3_+)} \), and on \( \partial \mathbb{R}^3_+ \) we utilize the notation \( \langle \cdot, \cdot \rangle := \langle \cdot, \cdot \rangle_{L^2(\partial \mathbb{R}^3_+)} \). The trace operator on \( H^1(\partial \Omega) \) spaces will be denoted by \( \gamma[\cdot] \) with range in \( H^{1/2}(\partial \Omega) \). We denote an open ball of radius \( R \) in a Banach space \( X \) by \( B_R(X) \).

Throughout the entirety of this paper, unless otherwise explicitly stated, we consider \( U \in [0, 1) \).

1.3 Energies and solutions

The energetic constraints for solutions manifest themselves through natural topological requirements, namely for \( L^2(\Omega) \) given by

\[
\| \cdot \|^2_{L^2(\Omega)} := \alpha \| \nabla \cdot \|_{L^2(\Omega)}^2 + \| \cdot \|^2_{L^2(\Omega)}^2,
\]

finite energy solutions should have the properties:

\[
u \in C(0, T; H^0(\Omega)) \cap C^1(0, T; L^2(\Omega)); \quad \phi \in C(0, T; W_1(\mathbb{R}^3_+)) \cap C^1(0, T; L^2(\mathbb{R}^3_+)), \quad (1.4)
\]

where \( W_1(\mathbb{R}^3_+) \) denotes the homogeneous Sobolev space of order 1. Here,

\[
W_1(\mathbb{R}^3_+) = \left\{ \phi \in L^2_{loc}(\mathbb{R}^3_+) : \nabla \phi \in L^2(\mathbb{R}^3_+) \right\},
\]

which is to say the space topologized by the gradient norm \( \| \nabla \phi \|_{L^2(\mathbb{R}^3_+)} \) without \( L^2(\mathbb{R}^3_+) \) norm control.

To set provide a dynamical systems framework, the principal state space is taken to be

\[
Y = Y_{fl} \times Y_{pl} \equiv (W_1(\mathbb{R}^3_+) \times L^2(\mathbb{R}^3_+)) \times \left( H^0(\Omega) \times L^2(\Omega) \right). \quad (1.5)
\]

We will also consider a stronger space on finite time intervals:

\[
Y_s \equiv H^1(\mathbb{R}^3_+) \times L^2(\mathbb{R}^3_+) \times H^0(\Omega) \times L^2(\Omega). \quad (1.6)
\]

The energies corresponding to finite energy solutions of (1.1), and the above space \( Y_s \), are given below.

\[
E_{pl} = \frac{1}{2} \left[ \| u_t \|^2_{L^2(\Omega)} + \| \Delta u \|^2 + \frac{1}{2} \| \Delta v(u) \|^2 \right] - \langle F_0, [u, u] \rangle + \langle p_0, u \rangle, \quad (1.7)
\]

\[
E_{fl} = \frac{1}{2} \| \phi_t \|^2 + \| \nabla \phi \|^2 - U^2 \| \partial_{x_3} \phi \|^2 \quad (1.8)
\]

\[
E_{int} = 2U \langle \gamma(\phi, \partial_{x_3} u) \rangle, \quad (1.9)
\]

\[
\mathcal{E} = E_{pl} + E_{fl} + E_{int}. \quad (1.10)
\]

The pair \((\phi, u)\) as in (1.4) is said to be a strong solution to (1.1) on \([0, T]\) if:

- \((\phi_t, u_t) \in L^1 \left( a, b; H^1(\mathbb{R}^3_+) \times H^2(\Omega) \cap H_0^1(\Omega) \right)\)

for any \((a, b) \subset [0, T]\).
Strong solutions are point-wise solutions.

The pair \((\phi, u)\) is said to be a generalized solution to problem (1.1) on the interval \([0, T]\) if there exists a sequence of strong solutions \((\phi^n(t); u^n(t))\) with some initial data \((\phi_0^n, \phi_1^n; u_0^n; u_1^n)\) such that \((\phi^n, u^n)\) converge to \((\phi, u)\) in the sense of \(C([0, T]; Y)\) as \(n \to \infty\). Such solutions correspond to semigroup solutions for initial data in \(Y\) rather than the domain of the generator.

Lastly, the pair \((u, \phi)\), with

\[
\begin{align*}
\mathcal{W}_T &\equiv \left\{ u \in L^\infty(0, T; H_0^2(\Omega)), \quad \partial_t u(x, t) \in L^\infty(0, T; H_0^1(\Omega)) \right\}, \\
\mathcal{Y}_T &\equiv \left\{ \phi \in L^\infty(0, T; H^1(\mathbb{R}^d)), \quad \partial_t \phi(x, t) \in L^\infty(0, T; L^2(\mathbb{R}^d)) \right\},
\end{align*}
\]

is said to be a weak solution to (1.1) on \([0, T]\) if

\[
\begin{align*}
&u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x) \quad \text{and} \quad \phi(x, 0) = \phi_0(x), \quad \phi_t(x, 0) = \phi_1(x), \\
\int_0^T &\left( (1-\alpha \Delta) \partial_t u(t), \partial_t w(t) \right) - k(1-\alpha \Delta) (\partial_t u(t), w(t)) - \left( r_\Omega \gamma \phi(t), \partial_t w(t) + U \partial_{x_i} w(t) \right) \, dt = \langle u_1 - r_\Omega \gamma \phi_0, w(0) \rangle_{L^2(\Omega)} \quad \text{for all test functions} \ w \in \mathcal{W}_T \\
\int_0^T &\left( (\partial_t + U \partial_{x_i}) \psi(t), (\partial_t + U \partial_{x_i}) \phi(t) \right) - \langle \nabla \phi(t), \nabla \psi(t) \rangle + \left( (\partial_t + U \partial_{x_i}) u(t), r_\Omega \gamma \psi(t) \right) \, dt = \langle \phi_1 + U \partial_{x_i} \phi_0, \psi(0) \rangle \\
&\quad \text{for all test functions} \ \psi \in \mathcal{Y}_T \quad \text{such that} \ \psi(T) = 0.
\end{align*}
\]

It is clear that strong solutions are generalized, and we state without proof that generalized solutions are in fact weak—see the discussion of abstract second-order equations in [13].

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pact global attractor), and quasi-stability on the absorbing ball provides finite dimensionality of the attractor in the state space as well as additional smoothness.

Finally, in Sect. 7, we present the main result here on stabilization of the dynamics to the equilibria set. Obtaining the result depends critically on all that has been established in the previous sections. After showing that plate trajectories in fact converge to stationary points, we then lift this convergence to the flow. This result utilizes: compactness of the attractor, finiteness of the dissipation integral, and compactness of the specific Neumann-to-Dirichlet map provided through the explicit solution representation discussed above.

The main results in each of the above sections are stated precisely in their respective sections. Proofs are provided in most cases, but where they are not, precise references are given.

1.5 Discussion of results herein and relationship to the literature

Let us discuss the previous mathematical work on this and closely related models. Early engineering references address panel flutter (in the comparable formulation to (1.1)) as motivated by the paneling and external layers on aircraft and projectiles [19,20]. We make special note of the work of Bolotin [3], whose early work has the mathematical formulation of the flow–plate system here, as well as good mathematical insight into a variety of qualitative features of the dynamics. Later, the work of Chueshov et al. began to address flow–plate models in a modern PDE and dynamical systems sense [4,5,18]—indeed, Chueshov should be given credit as a driving force for the analysis of this and other models of mathematical aeroelasticity. More broadly, we mention other seminal works in mathematical aeroelasticity [2,25], as well as the surveys [10,11] and the book chapter [26] which provide an overview of mathematical aeroelasticity, including some modeling discussions for configurations other than that of a panel.

Specific to the models described in this treatment, we point to early work on the delay dynamical system as it appears here can be found in [6,8]. Later, the works [4,5,18] consider the system presented here as (1.1). Well-posedness is addressed through Galerkin constructions with good microlocal estimates [32] applied by decoupling the flow–plate system. Later, stabilization-type results appeared for the flow–plate system when beneficial thermal effects accounted for in the plate [34,35], in particular parabolic dissipation. In [34], the thermoelastic plate takes \( \alpha > 0 \) and the arguments utilize analyticity of the underlying semigroup for the linearization; in [35], \( \alpha = 0 \) and only partial parabolic smoothing is available. In fact, these papers provide a motivation for our stabilization work in Sect. 7, and these papers are the closest in the literature to a complete description—as we provide here—of long-time behavior, albeit in the thermoelastic case. In this work, we trade thermal effects for engineering-type mechanical damping in the plate, not necessary in [34,35]. In the monograph [13], many results appeared for attractors for the plate system, though not explicitly using the more recently developed quasi-stability theory. Convergence to equilibrium for the model discussed here was outlined in both [9,13], without details, largely based on the foundation laid in [34,35]. In the case without rotational inertia—namely \( \alpha = 0 \)—well-posedness was obtained for the first time in [38] using a semigroup approach, and later again with boundary dissipation in [29]. The work [14] provides a well-posedness construction for \( \alpha = 0 \) through the clever use of an absorbing boundary condition for the flow. Attractors and determining modes for the associated delay dynamical system when \( \alpha = 0 \) are considered in [15] and the more recent [39]. Stabilization to equilibria was considered in the \( \alpha = 0 \) case (with only weak damping) in the sequence [30] and [31]; these results are much more complicated, the nonlinearity is not principally that of von Karman, and the results are in some sense partial. Specifically, convergence to equilibrium (as presented here) is obtained for \( \alpha = 0 \) in the following cases: (i) smooth initial data, with any imposed damping \( k > 0 \), for von Karman or Berger-type plate nonlinearities; (ii) finite energy data, with damping of the form \( k(w_t + w) \), \( k > k_{\min} \), again for von Karman or Berger nonlinearities; (iii) and finally, for finite energy data, with damping of the form \( k w_t \), \( k > k_{\min} \), for the Berger plate.

We emphasize that stabilization to equilibria results of the sort presented here become much more complicated via the inclusion of thermal effects, or when \( \alpha = 0 \); hence, the treatment here gives the simplest picture of the underlying challenges and mechanisms for stabilizing the flow–plate system, despite the fact that the aforementioned modifications are certainly more complex physical systems.
Thus, for the treatment at hand, we choose a physically relevant scenario where many results can be presented in a clean and clear manner, utilizing the state-of-the-art tools for modern dynamical systems theory. The results here, in some cases, are not sharp (with respect to parameters, for instance), and not every result is novel. In fact, many of the results presented here have appeared in disjointed publications listed above over the past 30 years (with varying degrees of detail). Of the results here, those which have appeared before are given here largely with novel proofs using recent dynamical systems technology.

The highlights of this treatment are:

- A semigroup approach to well-posedness of strong and generalized solutions to the flow–plate system. This is a recent treatment for these strongly coupled, non-dissipative dynamics, and takes the system as a whole, exploiting cancellations through calculations performed on strong solutions rather than relying on microlocal trace results for decoupled dynamics; we believe this to be an elegant and instructive manner of obtaining solutions;
- A direct construction—without appealing to abstract theorems for second-order evolutions—of an absorbing ball for the plate dynamics (appropriately modified from [15]), required because of the non-gradient structure of the reduced dynamics, making use of a non-standard Lyapunov approach on the reduced plate dynamical system with memory;
- The use of modern quasi-stability theory [7] directly on the absorbing ball, yielding, all at once from that powerful theory, a compact global attractor for plate dynamics that is smoother than the finite energy space as well as having finite fractal dimensional;
- A complete proof of subsonic convergence to equilibrium, utilizing the compactness of the plate attractor, finiteness of the mechanical dissipation integral, and the compactness of the plate-to-flow lifting, without invoking any sort of thermoelastic smoothing or dissipation as in [34,35].

2 Well-posedness and boundedness of solutions

In this section, we will construct strong and generalized (and hence weak) solutions via a semigroup approach. This approach was first utilized in the case $\alpha = 0$ in [38] and some subsequent references [16,26,29] based on it (also for $\alpha = 0$). We provide here the abstract setup and semigroup generation result for the model at hand, (1.1) when $\alpha > 0$. We will rewrite the linear problem abstractly on the finite energy space, using appropriate constituent operators. The generator, then, for the underlying linear flow–plate dynamics is $\omega$-dissipative [33] and maximal in the appropriate sense, yielding generation. Then, exploiting the local Lipschitz property of the von Karman nonlinearity, we will obtain local-in-time solutions for the nonlinear problem. Lastly, a priori estimates on solutions (which exploit the superlinear nature of the nonlinearity) provide global solutions in $Y_s$ (as in (1.6)) on any $[0, T]$. Further energy estimates ensure that the solution is uniformly bounded in time in the extended space $\mathcal{Y}$ as in (1.5).

2.1 Operators and abstract restatement of the linear problem

Let $A : \mathcal{D}(A) \subset L^2(\mathbb{R}^3_+) \to L^2(\mathbb{R}^3_+)$ be the positive (recall: $0 \leq U < 1$), self-adjoint operator

$$Af = -\Delta f + U^2 \partial_{x_3}f + \varphi f,$$

$$\mathcal{D}(A) = \{ f \in H^2(\mathbb{R}^3_+) : \partial_{x_3}f|_{x_3=0} = 0 \},$$

where $\mu > 0$.\footnote{The perturbation $\mu > 0$ is introduced to dispense with the zero eigenvalue; later that this will be taken as a bounded perturbation on $\mathcal{Y}$ and removed to obtain the problem as originally stated.} It is clear, then, that $\mathcal{D}(A^{1/2}) = H^1(\mathbb{R}^3_+)$ (in the sense of topological equivalence). The corresponding Neumann map $N_0 : L^2(\Omega) \to L^2(\mathbb{R}^3_+)$ obtained through Green’s formula [28] is given by

$$\psi = N_0 w \iff (-\Delta + U^2 \partial_{x_1}^2 + \mu) \psi = 0 \text{ in } \mathbb{R}^3_+ \text{ with } \partial_{x_3} \psi|_\Omega = [w]_{\text{ext}},$$

(2.1)

We have from [13,28] that $A^{3/4-\epsilon} N_0 : L^2(\mathbb{R}^2) \to L^2(\mathbb{R}^3_+)$ is continuous. Moreover, we have the adjoint identification of the Dirichlet trace

$$N_0^* A f = \gamma \{ f \}, \ f \in H^1(\mathbb{R}^3_+).$$

Introduce now the differential operator $D(\varphi) \equiv 2U \varphi$, defined on $\mathcal{D}(\varphi^{1/2})$ and the biharmonic operator $\mathcal{A} u = \Delta^2 u$, defined on $\mathcal{D}(\mathcal{A}) = (H^4 \cap H^2_0)(\Omega)$. In this case, $\mathcal{A}$ is a positive, self-adjoint operator on $L^2(\Omega)$ with $D(\mathcal{A}^{1/2}) = H^2_0(\Omega)$. Lastly, we define the operator $M_\alpha = (1 - \alpha \Delta)$ with domain $\mathcal{D}(M_\alpha) = (H^2 \cap H^1_0)(\Omega)$ and $\mathcal{D}(M_\alpha^{1/2}) = H^1_0(\Omega) = L^2_0(\Omega)$.\footnote{The perturbation $\mu > 0$ is introduced to dispense with the zero eigenvalue; later that this will be taken as a bounded perturbation on $\mathcal{Y}$ and removed to obtain the problem as originally stated.}
Considering the above, we have the abstract formulation of the homogeneous linear version of (1.1):

\[
\begin{aligned}
\phi_{tt} + A(\phi + N_0(u_t + U u_x)) \\
-\mu \phi + D(\phi_t) = 0, \quad \text{in } [\mathcal{D}(A^{1/2})]'
\end{aligned}
\]

\[
\begin{aligned}
M_\alpha u_{tt} + k M_\alpha u_t + \mathcal{A} u \\
- N_0^* A (\phi_t + U \phi_x) = 0, \quad \text{in } [\mathcal{D}(\mathcal{A}^{1/2})]'
\end{aligned}
\]

(2.2)

These natural state space then becomes

\[Y_\alpha = Y_1 \times Y_{2,\alpha} = \mathcal{D}(A^{1/2}) \times L^2(\mathbb{R}^2_+) \times \mathcal{D}(\mathcal{A}^{1/2}) \times \mathcal{D}(M_\alpha^{1/2}),\]

with natural inner product: \[y = (\phi_1, \phi_2; u_1, u_2)\] and \[y' = (\phi_1', \phi_2'; u_1', u_2')\]

\[(y, y')_Y_\alpha = (\phi_1, \phi_1')_{\mathcal{D}(A^{1/2})} + (\phi_2, \phi_2')_{L^2(\mathbb{R}^2_+)} + (u_1, u_1')_{\mathcal{D}(\mathcal{A}^{1/2})} + (u_2, u_2')_{\mathcal{D}(M_\alpha^{1/2})}.\]

(2.3)

For \(\alpha > 0\) fixed and any \(\mu > 0\), we have immediately that \(Y_\alpha = Y_\gamma\) in the sense of topological equivalence.

Writing (2.2) as a first-order system for \[y = (\phi_1, \phi_2; u_1, u_2)^T\] leads to the overall dynamics operator \([T_\alpha : \mathcal{D}(T_\alpha) \subset Y_\alpha \rightarrow Y_\alpha\]

expressed (in a distributional sense) by:

\[
T_\alpha y = \begin{bmatrix}
0 & -I & 0 & 0 \\
A & D & U A N_0 \partial \gamma & A N_0 \\
0 & 0 & 0 & -I \\
-M_\alpha^{-1} U N_0^* A \partial \gamma & -M_\alpha^{-1} N_0^* A & M_\alpha^{-1} \mathcal{A} & k I
\end{bmatrix} \begin{bmatrix}
\phi_1 \\
\phi_2 \\
u_1 \\
u_2
\end{bmatrix},
\]

(2.4)

with

\[
\mathcal{D}(T_\alpha) = \left\{ y \in [\mathcal{D}(A^{1/2})]^2 \times [\mathcal{D}(\mathcal{A}^{1/2})]^2 : \right. \]

\[
\left. \begin{array}{l}
\phi_1 + N_0(u_2 + U \partial_x u_1) \in \mathcal{D}(A); \\
M_\alpha^{-1/2}[\mathcal{A} u_1 - N_0^* A (\phi_2 + U \partial_x \phi_1)] \in L^2(\Omega)
\end{array} \right\}.
\]

(2.5)

In this case, the linearized, homogeneous version of (1.1), perturbed by \(\mu\), is represented by the abstract ODE

\[
\frac{dy}{dt} + T_\alpha y = 0; \quad y(0) = y_0 \in Y_\alpha.
\]

(2.6)

### 2.2 Generation

In this section, we use the Lumer–Philips theorem \([33]\) to obtain:

**Theorem 2.1** The operator \(T_\alpha : \mathcal{D}(T_\alpha) : Y_\alpha \rightarrow Y_\alpha\) is \(\omega\)-accretive and maximal; hence, \(-T_\alpha\) is the generator of a strongly continuous semigroup of bounded linear operators on \(Y_\alpha\) (and hence also on \(Y_\gamma\)).

**Proof Outline** To show that \(T_\alpha\) is \(\omega\)-accretive, we first consider a modified inner product on \(Y_\gamma\): For \(y = (\phi_1, \phi_2; u_1, u_2)^T\) and \(y' = (\phi_1', \phi_2'; u_1', u_2')^T\)

\[(y, y') = (y, y')_Y_\gamma + U \langle \partial \gamma u_1, r \partial \gamma \phi_1 \rangle + \lambda (\nabla u_1, \nabla u_1'),\]

for \(\lambda = \lambda(U)\) a parameter chosen to ensure positivity of the inner product \(((\cdot, \cdot))\). It is a straightforward exercise using the Sobolev embedding theorems and the Hardy inequality to check that \(((\cdot, \cdot))\) is in fact an inner product on \(Y_\gamma\) whose topology is equivalent to that given by the original inner product \(((\cdot, \cdot))_Y_\gamma\).
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Eliminating $\phi_1$ and $u_1$, and applying $M_\alpha$ to the last equation in (2.7), we obtain the operator

$$G = \left( \begin{array}{c}
1 A + D + \eta I \\
- N_0^* A (I + U/\eta \partial_\xi) \frac{1}{\eta} \eta \phi + (\eta + k) M_\alpha \\
\end{array} \right),$$

and the equation

$$G (\phi_2, u_2) = \left( \begin{array}{c}
\psi_2 - \frac{1}{\eta} A \psi_1 - \frac{U}{\eta} A N_0 \partial_\xi \psi_1 \\
M_\alpha w_2 - \eta \phi w_1 + \frac{1}{\eta} N_0^* A \partial_\xi \psi_1.
\end{array} \right) \in [P(A^{1/2})]' \times [P(\omega^{1/2})]' \tag{2.9}$$

Taking $V = P(A^{1/2}) \times P(\omega^{1/2})$, and considering $G : V \rightarrow V'$, we obtain that $G$ is $m$-monotone and coercive for appropriately chosen $\eta(U)$, and hence a corollary to Minty’s theorem [13, Proposition 1.2.5] ensures that $G$ is surjective. Elliptic regularity for $A$ and $\omega$ then provides that $y = (\phi_1, \phi_2; u_1, u_2)^T \in P(T_\alpha)$ (with appropriate estimates), giving the solution to (2.7).

With generation accomplished on $Y_\alpha$ taken with the topology induced by $(\cdot, \cdot)$, we obtain immediate semigroup generation on $Y_\alpha$ in the natural norm induced by (2.3), and again, via topological equivalence, semigroup generation on $Y_\alpha$. □

2.3 Locally Lipschitz perturbation on finite time intervals

Consider the perturbation operator $F : Y_\alpha \rightarrow Y_\alpha$ given by $F(y) = (0, \mu \phi; 0, M_\alpha^{-1} [p_0 - f_\alpha(u)])^T$. Then the abstract system

$$\frac{dy}{d\tau} + T_\alpha y = F(y), \quad y(0) = y_0 \in Y_\alpha$$

is equivalent to the main flow–plate system (1.1). As is shown in [13], the sharp regularity of the Airy stress function provides the a local Lipschitz property for the von Karman nonlinearity:

Applying the standard perturbation semigroup argument [33], we obtain:

**Lemma 1** With $T_\alpha$ and $F$ as above, the equation

$$\frac{dy}{d\tau} + T_\alpha y = F(y), \quad y(0) = y_0 \in P(T_\alpha)$$

has a unique local-in-time strong solution on $[0, t_{\text{max}})$. When $y_0 \in Y_\alpha$, we have a unique local-in-time mild solution. In both cases, when $t_{\text{max}}(y_0) < \infty$, we have that $||y(t)||_{Y_\alpha} \rightarrow \infty$ as $t \nearrow t_{\text{max}}(y_0)$.

Identifying the abstract ODE in (1) with the flow–plate system in (1.1), we obtain a local-in-time existence and uniqueness result.

**Corollary 1** Consider the system in (1.1) with $U \in L^2(\Omega)$ and $F_0 \in H^3(\Omega).$ Then, for $y_\alpha = (\phi_0, \phi_\alpha; u_0, u_1) \in P(T_\alpha)$ (resp. $Y_\alpha$) there exists a unique, local-in-time strong (resp. generalized) solution $(\phi(t), \phi(t); u(t), u_1(t))$ as defined in Sect. 1.3.

**Remark 2.1** Energy methods and the direct estimate

$$||\phi(t)||_{L^2(\mathbb{R}_+^3)} \leq ||\phi(0)||_{L^2(\mathbb{R}_+^3)} + \int_0^t ||\phi(\tau)||_{L^2(\mathbb{R}_+^3)} d\tau, \tag{2.11}$$
yields that the solutions in Corollary 1 are valid for \( t \in [0, T] \) for any \( T > 0 \). This point will be superseded by the following section.

**Remark 2.2** The abstract setup above will obtain, mutatis mutandis, for \( \alpha = 0 \)—see [38], or see [14] for an alternative viscosity approach.

### 2.4 Bounds in energy norm \( Y \) and global solutions

In this section, we remark that in the norm of \( Y \) as defined in (1.5), solutions are global-in-time bounded. This allows us to extend our result in Corollary 1 to be global in the sense of a solution for \( t \in [0, \infty) \). Such extension permits the analysis of long-time behavior of solutions.

The bounds in the following proposition are critical to obtaining the global-in-time boundedness mentioned above.

**Proposition 1** First, for \( \phi \in W_1(\mathbb{R}^3_+) \):

\[
\|r_\Omega Y[\phi]\|_{L^2(\Omega)} \leq C_\Omega \|\nabla \phi\|_{L^2(\mathbb{R}^3_+)}.
\]

Next, the interactive energy \( E_{int} \), as defined (1.7), is controlled in the following way:

\[
|E_{int}(t)| \leq \delta \|\nabla \phi(t)\|_{\mathbb{R}^3_+}^2 + C(U, \delta)\|u_x(t)\|_{\Omega}^2, \quad \delta > 0.
\]

Lastly, the nonlinear potential energy provides control of low frequencies: For any \( \eta, \epsilon > 0 \), there exists \( M_{\eta, \epsilon} \) such that

\[
\|u\|_{L^\infty_\eta}^2 \leq \epsilon \|\Delta u\|^2 + \|\Delta v(u)\|^2 + M_{\eta, \epsilon}, \quad \forall u \in (H^2 \cap H^1_\Omega)(\Omega).
\]

See [13, 38] for detailed proofs of the above facts; here, we suffice to say that (2.12) follows from the Hardy inequality, and from (2.12) the estimate (2.13) follows via Young’s; (2.14) is obtained through a compactness uniqueness argument that exploits superlinearity of \( f_v \).

Now, let us define the positive part of the energy \( E \) as:

\[
E_+(t) = \frac{1}{2} \|u_t\|_{L^2_0(\Omega)}^2 + \|\Delta u\|^2 + \frac{1}{2} \|\Delta v(u)\|^2 + \|\phi_t\|^2 + \|\nabla \phi\|^2.
\]

Then, via Proposition 1, we obtain control of the unsigned energy by the positive part:

**Lemma 2** For generalized solutions to (1.1), there exist positive constants \( c, C, M \) that are positive, and do not depend on the individual trajectory, such that:

\[
cE_+(t) - M_{p_0, F_0} \leq E(t) \leq CE_+(t) + M_{p_0, F_0}.
\]

The next lemma is the energy identity, as defined through (1.7):

**Lemma 3** Weak (and hence generalized and strong) solutions to (1.1) satisfy the energy identity

\[
E(t) + k \int_0^t \|u_t\|_{L^2_0(\Omega)}^2 \, d\tau = E(0).
\]

Synthesizing all of the above lemmata, we obtain:

**Lemma 4** Any weak (and hence generalized or strong) solution to (1.1) will satisfy the bound

\[
\sup_{t \geq 0} \left\{ \|u_t\|_{L^2_0(\Omega)}^2 + \|\Delta u\|_{\Omega}^2 + \|\phi_t\|_{\mathbb{R}^3_+}^2 + \|\nabla \phi\|_{\mathbb{R}^3_+}^2 \right\} \leq C(\|y_0\|_{Y}) < +\infty.
\]

Thus, solutions are (Lyapunov) stable in time in the norm \( Y \).

An immediate corollary from the energy identity (3) and the above boundedness is the finiteness of the dissipation integral, which is used critically below.

**Corollary 2** Any solution (1.1) satisfying the energy identity with \( k > 0 \) has the property

\[
\int_0^\infty \|u_t(t)\|_{L^2_0(\Omega)}^2 \, d\tau \leq K(\|y_0\|_{Y}) < \infty.
\]

**Remark 2.3** We note that global-in-time boundedness of solutions cannot be obtained without accounting for nonlinear effects. Also, we note that any generalized solution has the properties: \( y(t) \in C([0, T]; Y_s) \) and \( y(t) \in C([0, \infty); Y) \). This is to say, on infinite time intervals we lose control of the quantity \( \|\phi(t)\|_0 \).

**Remark 2.4** In the case of \( \alpha = 0 \), as examined in [35] for the thermoelastic plate, as well as in [30,31], the dissipation integral is bounded, but its form is modified to the dissipation structure specific to those cases.

### 3 Equilibria set

Before moving on to discuss the long-time behavior of trajectories, it is worthwhile to discuss the equilibrium/stationary solutions. Thus, in this section, we consider the stationary points of the dynamics \((S_t, Y)\), i.e.,
stationary solutions for (1.1) corresponding to:
\[
\begin{align*}
\Delta^2 u + f_t(u) &= p_0(x) + U r_{\Omega} \gamma [\partial_{x_3} \phi] \\
 u &= \partial_x u = 0 \\
\Delta \phi - U^2 \partial^2_{x_3} \phi &= 0 \\
\partial_{x_3} \phi &= U [\partial_{x_3} u]_{\text{ext}} \\
& \quad x \in \Omega \quad x \in \Gamma' \quad x \in \mathbb{R}^3_+ \quad x \in \partial \mathbb{R}^3_+ \\
\end{align*}
\]
(3.1)

Below, \(W_2(\mathbb{R}^3_+) = \{ \phi \in L^2_{\text{loc}}(\mathbb{R}^3_+) : D^2 \phi \in L^2(\mathbb{R}^3_+), |\alpha| = 1, 2 \},\) and a weak solution to (3.1) is defined as a pair \((u, \phi) \in H^2(\Omega) \times W_1(\mathbb{R}^3_+),\) such that
\[
\begin{align*}
\langle \Delta u, \Delta w \rangle - \langle |u|, v(u) + F_0 \rangle, w \rangle \\
+ U \langle \gamma [\phi], \partial_{x_3} w \rangle = & \langle p_0, w \rangle
\end{align*}
\]
and
\[
\begin{align*}
\langle \nabla \phi, \nabla \psi \rangle_{\mathbb{R}^3_+} - U^2 \partial_{x_3} \phi, \partial_{x_3} \psi \rangle_{\mathbb{R}^3_+} \\
+ U \langle \partial_{x_3} u, \gamma [\psi] \rangle = & 0
\end{align*}
\]

We have the following theorem for the stationary problem as given in [13, Theorem 6.5.10]:

**Theorem 3.1** Suppose \(0 \leq U < 1\) with \(p_0 \in L^2(\Omega)\) and \(F_0 \in H^3(\Omega).\) Then, a weak solution \((u(x), \phi(x))\) to (3.1) exists and satisfies the additional regularity property:

\((u, \phi) \in (H^4 \cap H^2_0)(\Omega) \times W_2(\mathbb{R}^3_+).\)

Such solutions correspond to the extremal points of the potential energy functional

\[
P(u, \phi) = \frac{1}{2} \| \Delta u \|_{\mathbb{R}^3_+}^2 + \Pi(u) + \frac{1}{2} \| \nabla \phi \|_{\mathbb{R}^3_+}^2
\]
\[
- U^2 \| \partial_{x_3} \phi \|_{\mathbb{R}^3_+}^2 + U \langle \partial_{x_3} u, \gamma [\phi] \rangle_{\Omega},
\]

considered for \((u, \phi) \in H^2(\Omega) \times W_1(\mathbb{R}^3_+).\)

We denote by \(N\) the set of stationary solutions from Theorem 3.1. In general, \(N\) has multiple elements. The reference [13] provides an example (a choice of \(p_0\) and \(F_0\)) where there are multiple stationary points: Let \(p_0(x) \equiv 0\) and \(F_0 = -\beta \left(x_1^2 + x_2^2\right).\) Then, there exists \(\beta > 0\) such that for \(\beta > \beta_0\) we have at least three solutions: \((0, 0), (u_\beta, \phi_\beta)\) and \((-u_\beta, -\phi_\beta)\)—see also [17].

For given loads \(F_0\) and \(p_0,\) the set of stationary solutions is generically finite. This is to say that there is an open dense set \(\mathcal{R} \subset L^2(\Omega) \times H^4(\Omega)\) such that if \((p_0, F_0) \in \mathcal{R}\) then the corresponding set of stationary solutions \(N\) is finite. This follows from the Sard–Smale theorem, as shown in [13, Theorem 1.5.7 and Remark 6.5.11].

### 4 Flow with given Neumann plate data

To perform the qualitative analysis below, it will be necessary to consider the flow equation with *prescribed* Neumann data. This will allow us to explicitly compute the relevant Neumann-to-Dirichlet-type map in terms of the plate dynamics, among other quantities.

Consider the problem:

\[
\begin{align*}
(\partial_t + U \partial_{x_3})^2 \phi &= \Delta \phi & & \text{in } \mathbb{R}^3_+ \times (t_0, T) \\
\partial_{x_3} \phi \bigg|_{x_3 = 0} &= h(x, t) & & \text{in } \mathbb{R}^2 \times (t_0, T) \\
\phi(t_0) &= \phi_0; \quad \phi_t(t_0) &= \phi_1 & & \text{in } \mathbb{R}^3_+
\end{align*}
\]
(4.1)

We have the following theorem from [5, 13, 32]:

**Theorem 4.1** Assume \(U \geq 0, U \neq 1;\) take \((\phi_0, \phi_1) \in H^1(\mathbb{R}^3_+) \times L^2(\mathbb{R}^3_+).\) If \(h \in C([t_0, \infty); H^1(\mathbb{R}^2))\) then (4.1) is well posed (in the weak sense) with

\(\phi \in C\left([t_0, \infty); H^1(\mathbb{R}^3_+)\right), \phi_t \in C\left([t_0, \infty); L^2(\mathbb{R}^3_+)\right).\)

**Remark 4.1** In fact, a stronger regularity result is available. Finite energy \(H^1(\Omega) \times L^2(\Omega)\) solutions are obtained with \(h \in H^{1/3}(0, T) \times L^2\) [28, 36].

#### 4.1 Flow decomposition and properties

We may decompose the flow problem from (4.1) into two pieces corresponding to zero Neumann data and zero initial data, respectively: \(\phi^\ast\) solves (4.1) with \(h \equiv 0,\) and \(\phi^{**}\) solves (4.1) with \(\phi_0 = \phi_1 \equiv 0.\)

In line with our well-posedness result, Corollary 1, we will consider:

\(h(x, t) \equiv [u_t + U u_{x_1}]_{\text{ext}} \in C([0, T]; H^1(\mathbb{R}^2)).\) (4.2)

#### 4.2 Point-wise formulae

In this section, we will look at \(\phi^\ast\) and \(\phi^{**}\) separately and establish results regarding each of them. These results will be used to obtain useful estimates in the next section.

For the analysis of \(\phi^\ast\), we use the tools developed in [5, 18], namely the Kirchhoff-type representation for the solution \(\phi^\ast(x, t) \in \mathbb{R}^3_+\) (see, e.g., [13, Theorem 6.6.12]). We conclude that if the initial data \(\phi_0\) and \(\phi_1\) are localized in the ball \(K_{\rho} \equiv \mathbb{R}^3_+ \cap B_{\rho}(\mathbb{R}^3),\) then by finite dependence on the domain of the signal
in 3-D (Huygens’ principle), one obtains for any \( \tilde{\rho} \) that
\[
\phi^*(\mathbf{x}, t) \equiv 0 \text{ for all } \mathbf{x} \in K_{\tilde{\rho}} \text{ and } t \geq t_{\tilde{\rho}}.
\]
Thus, \( \phi^* \) tends to zero in the sense of the local flow energy, i.e.,
\[
\| \nabla \phi^* (t) \|_{L^2(K_{\tilde{\rho}})}^2 + \| \phi^*_t (t) \|_{L^2(K_{\tilde{\rho}})} \to 0, \quad t \to \infty,
\]
for all fixed \( \tilde{\rho} > 0 \). Also, in this case,
\[
(\partial_t + U \partial_{x_1}) \gamma(\phi^*) \equiv 0, \quad \mathbf{x} \in \Omega, \quad t \geq t_{\tilde{\rho}}.
\]

On the other hand, for \( \phi^{**} \) as above, we have the following result taken from, for instance, [5, Theorem 3.3]:

**Theorem 4.2** Let
\[
h(\mathbf{x}, t) = [u_t(x_1, x_2, t) + U u_{x_1}(x_1, x_2, t)]_{\text{ext}},
\]
there exists a time \( t^*(\Omega, U) \) such that, for all \( t > t^* \), we have the following representation for the weak solution:
\[
\phi^{**}(\mathbf{x}, t) = -\frac{\chi(t - x_3)}{2\pi} \int_{x_3}^{t^*} \int_{0}^{2\pi} (u_1^+ (\mathbf{x}, t, s, \theta)) \, d\theta ds,
\]
where \( \chi(s) \) is the Heaviside function. The time \( t^* \) is given by:
\[
t^* = \inf \{ t : \mathbf{x}(U, \theta, s) \notin \Omega \text{ for all } (x_1, x_2) \in \Omega, \quad \theta \in [0, 2\pi], \quad \text{and } s > t \},
\]
with
\[
\mathbf{x}(U, \theta, s) = (x_1 - (U + \sin \theta) s, x_2 - s \cos \theta) \subset \mathbb{R}^2
\]
(not to be confused with \( \mathbf{x} = (x_1, x_2) \)).

The elementary formula below is of critical importance to the stabilization arguments here; it is shown [4,5] and also appears critically in [34,35]:
\[
\partial_{x_3} \phi^{**}(\mathbf{x}, t, s, \theta) = -\frac{d}{ds} h^\dagger (\mathbf{x}, t, s, \theta) - U \partial_{x_1} h^\dagger (\mathbf{x}, t, s, \theta)
\]
\[
- \frac{s}{\sqrt{s^2 - x_3^2}} [M_\theta h^\dagger](\mathbf{x}, t, s, \theta)
\]
where \( M_\theta = \sin \theta \partial_{x_1} + \cos \theta \partial_{x_2} \) and
\[
u^+(\mathbf{x}, t, s, \theta) = [u]_{\text{ext}} (x_1 - Us + \sqrt{s^2 - x_3^2} \sin \theta,
\]
\[
x_2 - \sqrt{s^2 - x_3^2} \cos \theta, t - s), \quad t > t_{\tilde{\rho}}.
\]
in computing partials of \( \phi^{**} \). For \( \phi_t^{**} \), we have:
\[
\phi_t^{**}(\mathbf{x}, t) = \frac{1}{2\pi} \int_0^{2\pi} d\theta u_1^+(\mathbf{x}, t, t^*, \theta)
\]
\[
- \int_0^{2\pi} d\theta u_1^+(\mathbf{x}, t, x_3, \theta)
\]
\[
+ \int_{x_3}^{t^*} ds \frac{s}{\sqrt{s^2 - x_3^2}}
\]
\[
\int_0^{2\pi} d\theta [M_\theta u_1^+(\mathbf{x}, t, s, \theta)]
\]
\[
(4.10)
\]
\[
\text{Remark 4.2} \text{ We note that to use these formulae in a classical sense (i.e., to produce direct bounds), one must have point-wise in time that } u_t \in H^1(\Omega), \text{ which critically depends on the presence of } \alpha > 0.
\]
Similarly, the spatial partials for \( i = 1, 2 \) are:
\[
\phi_{x_i}^{**}(\mathbf{x}, t) = \frac{1}{2\pi} \int_{x_3}^{t^*} \int_0^{2\pi} \partial_{x_i} u_1^+(\mathbf{x}, t, s, \theta) \, d\theta ds
\]
\[
= \frac{1}{2\pi} \int_{x_3}^{t^*} \int_0^{2\pi} U \partial_{x_1} u_1^+(\mathbf{x}, t, s, \theta) \, d\theta ds
\]
\[
+ \frac{1}{2\pi} \int_{x_3}^{t^*} \int_0^{2\pi} \partial_{x_i} u_1^+(\mathbf{x}, t, s, \theta) \, d\theta ds
\]
(4.11)
Differentiation in \( x_3 \) is direct, and so with cancellation, it yields:
\[
\partial_{x_3} \phi^{**}(\mathbf{x}, t)
\]
\[
= \partial_{x_3} u_1^+(\mathbf{x}, t - x_3, x_2, t - x_3)
\]
\[
+ \frac{1}{2\pi} \int_{x_3}^{t^*} \frac{x_3}{\sqrt{s^2 - x_3^2}} \int_0^{2\pi} d\theta
\]
\[
\left[ (\partial_t + U \partial_{x_1})(M_\theta u_1^+) \right](\mathbf{x}, t, s, \theta).
\]
(4.13)
From these direct calculations, bounds on solutions can be obtained directly [34, Lemma 8] using interpolation:

**Lemma 5** For (4.1), taken with \( h(\mathbf{x}, t) = (u_t + U u_{x_1})_{\text{ext}} \), we have
\[
\| \nabla \phi^{**}(t) \|_{K_{\tilde{\rho}}} + \| \phi^{**}(t) \|_{K_{\tilde{\rho}}}
\]
\[
\leq C(\rho) \left\{ \| u(t) \|_{H_0^{2+n}(\Omega)} + \| u_t(t) \|_{H_0^{1+n}(\Omega)} \right\}
\]
(4.14)
for \( s, \eta \geq 0, 0 < s + \eta < 1/2 \) and \( t > t^* (U, \Omega) \).

From here, for smooth solutions, we can explicitly solve for the needed Dirichlet trace of the material derivative appearing on the RHS of (1.1) in the plate equation in terms of the Neumann data \( h = [u_t + U u_{x_1}] \). Considering the term

\[
 r_\Omega r \gamma \left( \partial_t - U \partial_{x_1} \right) \phi = r_\Omega r \gamma \left( \partial_t + U \partial_{x_1} \right) \phi^* 
\]

for \( t > t_\rho \) by (4.4), where again \( \rho \) corresponds to the \( \text{supp}(\phi_0), \text{supp}(\phi_1) \subset K_\rho \). Using the above expressions for \( \partial_t \phi^* \) and \( \partial_{x_1} \phi^* \), we obtain

\[
 r_\Omega r \gamma \left( \partial_t - U \partial_{x_1} \right) \phi \left[ \phi^* \right] 
= - ( \partial_t + U \partial_{x_1} ) u - q(u'),
\]

for \( t \geq \max\{t^*, \ t_\rho \} \) \( (t^* \text{ as defined above in (4.6)}) \) with

\[
 q(u') = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{t^*} ds \int_0^{2\pi} d\theta 
\]

\[
 \left[ M_\theta^2 [ u ]_{ext} \left( x(u, \theta, s) \right) \right],
\]

and \( x(u, \theta, s) \) as in (4.7).

The notation above for \( u' \) indicates the entire set \( \{ u(t + s) : s \in (-t^*, 0) \} \), where \( t^* \) is the fixed delay time given in (4.6) depending only on \( \Omega \) and \( U \); this notation is used in considerations with dynamical systems with delay/memory [6, 8, 13].

\section{5 Reduced plate dynamics}

5.1 Plate reduction theorem

From all of the point-wise formulae of the previous section—including the calculation of the Neumann-to-material-derivative trace in (4.16)—we obtain the theorem below by waiting a time \( t^* = \max\{t^*, \ t_\rho \} \). The result is as stated in [5, Theorem 2.2].

\begin{theorem}
 Let \( k \geq 0 \), and \( (\phi_0, \phi_1; u_0, u_1) \in H^k_0(\Omega) \times L^2(\Omega) \times H^1(\mathbb{R}^2_+) \times L^2(\mathbb{R}^2_+) \). Assume that there exists an \( \rho \) such that \( \phi_0(x) = \phi_1(x) = 0 \) for \( x \) outside \( K_\rho \). Then, there exists a time \( t^*(\rho, U, \Omega) > 0 \) such that for all \( t > t^* \) the plate solution \( u(t) \) to (1.1) satisfies the following equation (in a weak sense):

\[
 M_\alpha u_{tt} + \Delta^2 u + k \Delta u + f(u) 
= p_0 - ( \partial_t + U \partial_{x_1} ) u - q(u')
\]

(5.1)

with

\[
 q(u') \equiv \frac{1}{2\pi} \int_0^{t^*} \int_0^{2\pi} ds \int_0^{2\pi} d\theta [ M_\theta^2 u_{ext} ](x_1) 
- (U + \sin \theta) s, \quad x_2 = - s \cos \theta, \quad t - s,
\]

(5.2)

with \( M_\theta \) and \( t^* \) as in the previous section.

We then have the following direct estimates on the delay potential \( q(u') \) [5, 13, 15, 18]:

\begin{proposition}
 Let \( q(u') \) be given by (5.2). Then,

\[
 ||q(u')||_{\pi}^2 \leq c t^* \int_{t-t^*}^t ||u(\tau)||^2 d\tau
\]

(5.3)

for any \( u \in L^2(t-t^*, t; H^1_0(\Omega)) \). If \( u \in L^2_{loc}(-t^*, +\infty; H^1(\Omega)) \) we also have

\[
 ||q(u')||^2 \leq c t^* \int_{t-t^*}^t ||u(\tau)||^2 d\tau,
\]

(5.4)

\end{proposition}

5.2 Reduced dynamical system \((T_t, H)\)

With these estimates, the system given in (5.1)–(5.2) is independently well posed [15] as plate equation with memory [13, 15]. Specifically, on the space for initial data

\[
 u(0) = u_0 \in H^k_0(\Omega), \quad u_t(0) = u_1 \in H^1_0(\Omega),
\]

\[
 u|_{t=t^*, 0} = \eta \in L^2(-t^*, 0; H^1_0(\Omega)).
\]

In application, we will consider an initial datum \( y_0 \in Y \) corresponding to the full flow–plate dynamics \( S_t(y_0) \) in (1.1). We want a sufficiently long time \( t^*(\rho, U, \Omega) \) and employ the reduction result Theorem 5.1, and we may consider the “initial time” \( (t = t_0 > t^*) \) for the delay dynamics. At such a time, the data which are fed into (5.1) are \( x_0 = (u(t_0), u_t(t_0), u_{tt}(t_0)) \), where these data are determined by the full dynamics of (1.1) on \( (t_0 - t^*, t_0) \). Thus, given a trajectory \( S_t(y_0) = y(t) = (\phi(t), \phi_t(t); u(t), u_t(t)) \in Y \), we may analyze the corresponding delay evolution \((T_t, H)\), with \( H \equiv L^2_{loc}(\Omega) \times L^2(\Omega) \times L^2(-t^*, 0; H^1_0(\Omega)) \), with given data \( x_0 \in H \). We then have that \( T_t(x_0) = (u(t), u_t(t); u') \) with \( x_0 = (u_0, u_1, \eta) \). The natural norm is taken to be

\[
 ||(u, v; \eta)||^2_H \equiv ||\Delta u||^2 + ||v||^2 + \int_{-t^*}^0 ||\Delta \eta(t + s)||^2 ds.
\]

Using standard multiplier methods, along with the a priori boundedness in Lemma 4, we obtain via Grönwall’s inequality the Lipschitz estimate below.
Lemma 6 Suppose $u^i(t)$ for $i = 1, 2$ are solutions to (5.1) with different initial data and $z = u^1 - u^2$. Additionally, assume that
\[
||u^i_t(t)||_{L^2_0(\Omega)}^2 + ||\Delta u^i(t)||^2 \leq R^2, \quad i = 1, 2
\]
for some $R > 0$ and all $t \in [0, T]$. Then, there exists $C > 0$ and $a_R \equiv a_R(t^*) > 0$ such that
\[
||z_t(t)||_{L^2_0(\Omega)}^2 + ||\Delta z(t)||^2 \\ \leq C e^{a_R t} \left( ||\Delta (u^1_0 - u^2_0)||^2 \\ + ||u^1_1 - u^2_1||^2_1 + \int_{-t^*}^0 ||\eta^1(\tau) - \eta^2(\tau)||^2_2 d\tau \right)
\]
for all $t \in [0, T]$.

6 Smooth global attractor for reduced plate dynamics

The main result in this section is that the plate dynamical system $(T_t, H)$ has a compact global attractor which has additional nice properties.

We recall that (see, e.g., [1, 13]) for a generic dynamical system $(S_t, H)$, a compact global attractor $A \subset H$ is an invariant set (i.e., $S_t A = A$ for all $t \in \mathbb{R}_+$) that uniformly attracts any bounded set $B \subset H$: $\lim_{t \to +\infty} d_H(S_t B \mid A) = 0$, where $d_H$ corresponds to the Hausdorff semidistance. As we will see, $A$ will have finite fractal dimension in this case: $\text{dim}_f A < \infty$. The fractal dimension of a set is defined in terms of minimal coverings, [7, 13], and a set with finite fractal dimension can be included as a subset of some higher-dimensional Euclidean space.

Theorem 6.1 (Smooth, Finite-Dimensional Global Attractor) Let $k > 0$, $U \neq 0$, $p_0 \in L^2(\Omega)$, and $F_0 \in H^3(\Omega)$ in (1.1). Also assume the flow data $\phi_0, \phi_1 \in Y$ are localized (with supports in $K_\rho$, as in Theorem 5.1). Then, the corresponding delay system $(T_t, H)$ has a compact global attractor $A$ of finite fractal dimension in $H$. Moreover, $A$ has additional regularity: any full trajectory $y(t) = (u(t), u_t(t), u^1(t)) \subset A$, $t \geq 0$, has the property that $u \in C_r(\mathbb{R}; H^3(\Omega) \cap H^2_0(\Omega))$, $u_t \in C_r(\mathbb{R}; H^2_0(\Omega))$, and $u_{tt} \in C_r(\mathbb{R}; H^1_0(\Omega))$.

This can be rephrased for the full system $(S_t, Y)$ by projecting on the first two components of $H$:

Corollary 3 With the same hypotheses as Theorem 6.1, there exists a compact set $\mathcal{U} \subset H^3(\Omega) \times H^2_0(\Omega)$ of finite fractal dimension such that for any weak solution $(u, u_t; \phi, \phi_t)$ to (1.1) has
\[
\lim_{t \to +\infty} dy_{pl}\left((u(t), u_t(t)), \mathcal{U}\right) = \lim_{t \to +\infty} \inf_{\mathcal{U}} (||u(t) - w_0||^2 + ||u_t(t) - w_1||^2) = 0.
\]
We also have the additional regularity $\mathcal{U} \subset (H^3(\Omega) \cap H^2_0(\Omega)) \times H^2_0(\Omega)$.

The proof of Theorem (6.1) proceeds in two steps: the construction of an absorbing ball by Lyapunov methods, followed by the attainment of the so-called quasi-stability property on the absorbing ball.

6.1 Construction of absorbing ball

For the non-conservative plate dynamics given by (5.1), we explicitly construct the absorbing ball via a Lyapunov approach which is a clear modification of that in [15]. Recalling the definition of $E_{pl}$ from (1.7) and defining the quantities
\[
\Pi_s(u) = \frac{1}{2} ||\Delta u||^2 + \frac{1}{2} ||\Delta v(u)||^2,
\]
\[
E_s(u, u_t) = \frac{1}{2} ||u_t||^2_{L^2_0(\Omega)} + \Pi_s(u),
\]
we consider the Lyapunov-type function (the relevant adaptation of that in [15])
\[
V(T_t(x)) = E_{pl}(u(t), u_t(t)) + v\left((M_{2}^{1/2}u_t, M_{2}^{1/2}u)\right) + k\left((M_{2}^{1/2}u, M_{2}^{1/2}u)\right) + \mu \int_0^t \int_{-s}^t \Pi_s(u(\tau)) d\tau ds,
\]
(6.1)
where $T_t(x) = x(t) = (u(t), u_t(t), u_{tt})$ for $t \geq 0$,

The elementary inequality
\[
\int_0^t \int_{-s}^t \Pi_s(u(\tau)) d\tau ds \leq t^n \int_{-t}^t \Pi_s(u(\tau)) d\tau,
\]
we establish the topological equivalence between $V(T_t(x))$ and $E_s$, which is given by the following lemma.

Without loss of generality, take $t_0 = 0$. 

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Lemma 7 With \((T_t, H)\) defined in Sect. 5.2, and \(V\) defined as in (6.1), we have that there exists \(v_0 > 0\) such that for all \(0 < \nu \leq v_0\) there are \(c_0(v_0), c_1, c(v_0), C > 0\)

\[
c_0 E_a - c \leq V(T_t(x)) \leq c_1 E_a + \mu C r^s \int_{-t^s}^0 \Pi_a(u(t + \tau)) d\tau + c. \tag{6.2}
\]

A careful but direct calculation of \(\frac{d}{dt} V(T_t(x))\), coupled with the estimates on the nonlinear potential energy Lemma 2.14 and the estimate on \(q(u^t)\) at the \(L^2\) level in Lemma 2, produces, for \(0 < \nu < \min\{v_0, 1\}\), and for \(\mu\) sufficiently small, the following lemma:

Lemma 8 For all \(k > 0\), there exist \(\mu, \nu > 0\) sufficiently small, and \(c(\mu, \nu, t^s, k), C(\mu, \nu, p_0, F_0) > 0\) such that

\[
\frac{d}{dt} V(T_t(x)) \leq C - c \left\{ E_a(u, u_t) \right\} + \int_{-t^s}^0 \Pi_a(u(t + \tau)) d\tau \right\}. \tag{6.3}
\]

From this lemma and the upper bound in (6.2), we have a \(\delta(k, \mu, \nu) > 0\) and a \(C(\mu, \nu):\)

\[
\frac{d}{dt} V(T_t(x)) + \delta V(T_t(x)) \leq C, \quad t > 0. \tag{6.4}
\]

The estimate above in (6.4) implies (via an integrating factor) that

\[
V(T_t(x)) \leq V(x) e^{-\delta t} + \frac{C}{\delta} (1 - e^{-\delta t}). \tag{6.5}
\]

Hence, the set

\[
\mathcal{B} = \left\{ x \in H : V(x) \leq 1 + \frac{C}{\delta} \right\}
\]

is a bounded forward invariant absorbing set for \((T_t, H)\). This, along with (6.2), gives that \((T_t, H)\) is ultimately dissipative in the sense of dynamical systems [1,37].

6.2 Tools from quasi-stability theory

We now proceed by discussing the specific tool we use in the construction of the attractor: quasi-stability [7, 13].

Condition 6.1 Consider second-order (in time) dynamics \((S_t, H)\) where \(H = X \times Y \times Z\) with \(X, Y, Z\) Hilbert, and \(X\) compactly embedded into \(Y\). Further, suppose \(y = (x_0, x_1, z_0) \in H\) with \(S_t y = (x(t), x_t(t), z^t)\) where the function \(x \in C([0, \infty), X) \cap C^1([0, \infty), Y)\), \(z \in L^2_{loc}(\mathbb{R}_+, Z)\), where again \(z^t = (z(t + \tau) : \tau \in (-t^s, 0)}\) for some characteristic time \(t^s\).

Condition 6.1 restricts our attention to second-order, hyperbolic-like evolutions.

Condition 6.2 Suppose the evolution operator \(S_t : H \rightarrow H\) is locally Lipschitz, with Lipschitz constant \(a(t) \in L^\infty([0, \infty))\):

\[
\|S_t y_1 - S_t y_2 \|^2_H \leq a(t) \|y_1 - y_2 \|^2_H. \tag{6.6}
\]

Definition 1 With Conditions 6.1 and 6.2 in force, suppose that the dynamics \((S_t, H)\) admit the following estimate for \(y_1, y_2 \in B \subset H:\)

\[
\|S_t y_1 - S_t y_2 \|^2_H \leq e^{-y^t} \|y_1 - y_2 \|^2_H + C_q \sup_{\tau \in [0, t]} \|x_1 - x_2 \|^2_{Y^\tau}, \quad \text{for some \(q, C_q > 0\),} \tag{6.7}
\]

where \(X \subseteq Y \subseteq Y^\tau\), and the last embedding is compact. Then, we say that \((S_t, H)\) is quasi-stable on \(B\).

We now run through a handful of consequences of the type of quasi-stability described by Definition 1 for dynamical systems \((S_t, H)\) satisfying Condition 6.1 [13, Proposition 7.9.4].

Theorem 6.2 If a dynamical system \((S_t, H)\) satisfying Conditions 6.1 and 6.2 is quasi-stable on an absorbing ball \(B \subset H\), then there exists a compact global attractor \(A \subset \subset H\),

\[
\sup_{t \in \mathbb{R}} \left\{ \|x_t(t) \|^2_X + \|x_t(t) \|^2_Y \right\} \leq C,
\]

where the constant \(C\) above depends on the “compactness constant” \(C_q\) in (6.7).

Elliptic regularity can then be applied to the equation itself generating the dynamics \((S_t, H)\) to recover regularity for \(x(t)\) in a norm higher than that of the state space \(X\).

6.3 Quasi-stability on the absorbing ball

For the discussion of quasi-stability, we begin with the standard observability and energy inequalities which follow from energy methods developed for the wave
The details presented for $\alpha = 0$ in [15] are unaltered different here.

Let us utilize the notation that $E_z(t) = ||z(t)||^2_{L^2(\Omega)} + ||\Delta z(t)||^2$. We state the following estimates without proof.

**Lemma 9** (Preliminary estimates) Let 
\[ u^i \in C(0, T; H_0^1(\Omega)) \cap C^1(0, T; L_2^2(\Omega)) \cap L_2^2(-t^*, T; H_0^1(\Omega)) \]

solve (5.1) with appropriate initial conditions on $[0, T]$ for $i = 1, 2, T \geq 2t^*$. Additionally, assume $(u^i(t), u^i_1(t)) \in B_R(Y_{pl})$ for all $t \in [0, T]$. Then, the following estimates on $z$ holds for some $\delta \in (0, 2]$: 

\[ E_z(t) + \int_s^t E_z(\tau) d\tau \leq a_0 \left( E_z(s) + \int_{s-t^*}^s ||z(\tau)||^2_{L_2^2} d\tau \right) + C(T, R, \delta) \sup_{\tau \in [s,t]} ||z(\tau)||^2_{L_2^2} - a_1 \int_s^t \langle f(u^1) - f(u^2), z_1 \rangle d\tau. \]  

(6.8)

\[ \frac{T}{2} \left[ E_z(T) + \int_{T-t^*}^T E_z(\tau) d\tau \right] \leq a_2 \left( E_z(0) + \int_{-t^*}^0 ||z(\tau)||^2_{L_2^2} d\tau \right) + C(T, R, \delta) \sup_{\tau \in [0,T]} ||z(\tau)||^2_{L_2^2} - a_3 \int_0^T \int_s^T \langle f(u^1) - f(u^2), z_1 \rangle d\tau d\sigma - a_4 \int_0^T \langle f(u^1) - f(u^2), z_1 \rangle d\tau, \]

(6.9)

with the positive $a_i$, independent of $T$ and $R$.

We note the elementary bound 
\[ |\langle f(u^1) - f(u^2), z_1 \rangle| \leq C_\varepsilon ||f(u^1) - f(u^2)||^2_{L_2^2} + \varepsilon ||z_1||^2_{L_2^2}. \]

Utilizing the above bound directly, and invoking the locally Lipschitz nature of the von Karman nonlinearity in (2.10), we see (rescaling constants) that 

\[ \int_s^t |\langle f(u^1) - f(u^2), z_1 \rangle| d\tau \leq \varepsilon \int_s^t ||z_1||^2 + C(\varepsilon, R, \delta, |t-s|) \sup_{[s,t]} ||z(\tau)||^2_{L_2^2}. \]

(6.10)

This yields the estimate from which the quasistability property of $(T, H)$ can be deduced.

**Lemma 10** Suppose $z = u^1 - u^2$ as before, with $y^i(t) = (u^i(t), u^i_1(t), u^i_2(t)), y^j(t) \in \mathcal{B}$ (i.e., the trajectories lie in the absorbing ball) for all $t \geq 0$. Also, let $\delta > 0$ and $E_z(t)$ be defined as above. Then, there exists a time $T$ such that the following estimate holds:

\[ E_z(T) + \int_{T-t^*}^T ||z(\tau)||^2_{L_2^2} d\tau \]

\[ \leq \beta \left( E_z(0) + \int_{-t^*}^0 ||z(\tau)||^2_{L_2^2} d\tau \right) + C(R, T, t^*, \delta) \sup_{\tau \in [0,T]} ||z(\tau)||^2_{L_2^2-\delta} \]

(6.11)

with $\beta < 1$.

**Proof of Lemma 10** Applying (6.10) to (6.8) and (6.9) with $s = 0$ and $t = T$, we obtain

\[ E_z(T) + \int_0^T E_z(\tau) d\tau \leq a_0 \left( E_z(0) + \int_{-t^*}^0 ||z(\tau)||^2_{L_2^2-\eta} d\tau \right) + C(T, R, \eta, \varepsilon) \sup_{\tau \in [0,T]} ||z_1||^2_{L_2^2-\eta} + \varepsilon a_1 \int_0^T ||z_1(\tau)||^2_{L_1} d\tau. \]

(6.12)

and

\[ \frac{T}{2} \left[ E_z(T) + \int_{T-t^*}^T E_z(\tau) d\tau \right] \leq a_2 \left( E_z(0) + \int_{-t^*}^0 ||z(\tau)||^2_{L_2^2-\eta} d\tau \right) + C(T, R, \eta, \varepsilon) \sup_{\tau \in [0,T]} ||z(\tau)||^2_{L_2^2-\eta} + C T \varepsilon \int_0^T ||z(\tau)||^2_{L_1} d\tau \]

(6.13)

for $T \geq \max\{t^*, 1\}$. After adding (6.12) and (6.13) and invoking the Sobolev embeddings, we can drop suitable
we have existence of the global attractor $A \subset \subset H$ for the system $(T, H)$ and the estimate:

$$\sup_{t \in \mathbb{R}} \left\{ \|u_t(t)\|^2_{H_0^1(\Omega)} + \|u_{tt}(t)\|^2_{H_0^1(\Omega)} \right\} \leq C. \quad (6.16)$$

Consequently, $u_t(t) \in H_0^2(\Omega)$ and $u_{tt}(t) \in H_0^1(\Omega)$ for each $t \in \mathbb{R}$. Applying elliptic regularity to the equation

$$\Delta^2 u = -M_\alpha u_t + kM_\alpha u_t + [u, v(u) + F_0] + p_0 - u_t - Uu_x - q(u') \in H^{-1}(\Omega)$$

point-wisely in time, we obtain that $u(t) \in H^3(\Omega) \cap H_0^2(\Omega)$. Therefore, $A$ possesses better regularity than $H$ and we obtain the statement of Theorem 6.1.

### 7 Stabilization to equilibria set

Finally, in this section, we show that in the subsonic case, when any amount of damping is present $k > 0$, that trajectories stabilize to the equilibrium set $\mathcal{N}$ as in Sect. 3. (This is in stark contrast to the case when $\alpha = 0$.) From an applied point of view, this means that for subsonic flows, physical panels do not experience aerodynamic instability. Said differently: there is no subsonic panel flutter. The main theorem proved in this section is presented below.

**Theorem 7.1** Let $\alpha > 0$, $0 \leq U < 1$, $k > 0$, and the assumptions of Theorem 5.1 be in force. Then for any weak solution $(u(t); \phi(t))$, with flow initial data $\phi_0$ and $\phi_1$ localized to $K_p$, we have

$$\lim_{t \to \infty} \inf_{\tilde{\phi} \in \mathcal{N}} \left\{ \|u(t)\|_{L^2(K_\beta)}^2 + \|u_t(t)\|_{H_0^1(K_\beta)}^2 + \|\phi(t) - \tilde{\phi}(t)\|_{H_0^1(K_\beta)}^2 \right\} = 0 \quad (7.1)$$

for any $\tilde{\beta} > 0$.

This theorem is discussed concisely in [9,13].

By the generic finiteness of the stationary set discussed in Sect. 3, for most loads $p_0$ and $F_0$, the set $\mathcal{N}$ is finite. In which case, the equilibria set corresponding to Theorem 3.1 is discrete and isolated. When this occurs, we can improve the result above.

**Corollary 4** Assume that $\mathcal{N}$ is an isolated set. Let the hypotheses of Theorem 7.1 be in force; then, for any...
generalized solution \((u, \phi)\) to (1.1) there exists a stationary point \((\tilde{u}, \tilde{\phi})\) satisfying (3.1) such that
\[
\lim_{t \to \infty} \left\{ \|u(t) - \tilde{u}\|_2^2 + \|u_t(t)\|^2 + \|\phi(t) - \tilde{\phi}\|_{H^1(K_\rho)} + \|\phi_t(t)\|^2_{L^2(K_\rho)} \right\} = 0,
\]
for any \(\rho > 0\).

The proof of this theorem proceeds in steps, utilizing first the finiteness of the dissipation integral in Lemma 2 (from uniform boundedness of trajectories in the norm \(Y\) in Lemma 4). These facts along with the compactness of the attractor for \((T_t, H)\) allow us to conclude strong convergences for the plate. We then “transfer” these strong convergences to the flow via the Neumann lift corresponding to the \(\phi^{**}\) in Theorem 4.2 and the resulting Neumann-to-Dirichlet expression for the material flow derivative (5.2).

7.1 Step 1: plate convergences

**Proposition 3** Let \(u\) be a generalized solution to (1.1) with \(\alpha > 0\). Then, \(M_\alpha u_t(t) \to 0\) in \(H^{-1}(\Omega)\) as \(t \to \infty\).

**Proof of Proposition 3** Multiplying the plate equation in (1.1) by a test function \(w \in C_0^\infty(\Omega)\), we obtain
\[
\langle M_\alpha u_{tt}, w \rangle = -\langle \Delta u, \Delta w \rangle + \langle p_0, w \rangle - k(M_\alpha u_t, w)
+ \langle [u(t), v + F_0], w \rangle + \langle r_\Omega \gamma(\phi_t + U\phi), w \rangle.
\]

The first and second terms on the RHS of the equality are uniformly bounded in time by Lemma 4.

For the third term, we have
\[
\|k(M_\alpha u_t, w)\| \leq k(\|u_t\|_1 \cdot \|w\|) + k\alpha(\|\nabla u_t\|_1 \cdot \|\nabla w\|)
\]
(7.3)

For the fourth term, Theorem 1.43 and Corollary 1.45 from [13] provide us with the following estimates for \(f_v:\)
\[
\|\langle [u, v + F_0], w \rangle\| \leq \|u\|_1 \cdot \|v\| + \|u_t\|_{-1, \Omega} \cdot \|w\|_1
\leq C_1 \|u\|^2_2 \|w\| + C_2 \|u_t\|_2 \|F_0\|_2 \|w\|_1.
\]

(7.4)

To estimate the fifth term above in (7.2), recall that
\[
\begin{align*}
&= -\langle \partial_t + U\partial_x \rangle \gamma(\phi(t))
\end{align*}
\]
for \(t \geq t^*\), where \(t^*\) and \(q(u')\) are defined as in Theorem 5.1. Also,
\[
\begin{align*}
\langle r_\Omega \gamma(\phi_t + U\phi), w \rangle &\leq \|r_\Omega \gamma(\phi_t + U\phi), w\| \cdot \|w\|
\leq (\|u_t\|_1 + U\|u\|_2 + \|q(u')\|) \|w\|
\leq C (\|u_t\|_1^2 + U\|u\|_2^2)
\end{align*}
\]
(7.5)

The right-hand sides of (7.3), (7.4), and (7.5) are then uniformly bounded in time by Lemma 4. Hence, by (7.2), \(\|M_\alpha u_t, w\| = \|\partial_t(M_\alpha u_t, w)\|\) is uniformly bounded in time for each \(w \in C_0^\infty(\Omega)\).

Next, we see that
\[
\begin{align*}
\int_0^\infty \|M_\alpha u_t, w\|^2 d\tau
\leq \int_0^\infty \|u_t, w\|^2 d\tau + \alpha \int_0^\infty \|\nabla u_t, \nabla w\|^2 d\tau
\leq C(\|w\|) \int_0^\infty \|u_t\|^2_{1, \Omega} < \infty
\end{align*}
\]
by the finiteness of the dissipation integral given in Corollary 2. Hence, by Barbalat’s lemma and density, \(^4\) we obtain
\[
\lim_{t \to \infty} \langle M_\alpha u_t(t), w \rangle = 0, \quad \forall w \in H_0^1(\Omega).
\]
\(\square\)

\(A\) being the global attractor is the \(\omega\)-limit set of the absorbing ball and includes \(\omega\)-limit sets of all the trajectories of the dynamical system \((T_t, H)\). Projecting onto the first two components of \(H\), we obtain the following proposition:

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Proposition 4 Let \( \mathcal{U} \) be as in Corollary 3 and \( u \) be a generalized solution to (1.1). Then, given any sequence of moments of time converging to infinity, there is a subsequence \( \{t_n\} \) and \( (\tilde{u}, \tilde{\bar{v}}) \in \mathcal{U} \subset H^1_0(\Omega) \times H^1_0(\Omega) \) (depending on \( \{t_n\} \)) such that

\[
\lim_{n \to \infty} ||u(t_n) - \tilde{u}||_2^2 = \lim_{n \to \infty} ||u(t_n) - \tilde{\bar{v}}||_1^2 = 0.
\]

Remark 7.1 From this point on till the end of Sect. 7.3, we will focus our attention on the subsequence \( \{t_n\} \), as in Proposition 4. In these sections, we establish results pertaining to our solution \( (u, \phi) \) evaluated on the time sequence \( \{t_n\} \). Finally, in Sect. 7.4 we will show (via a contradiction argument) how the existence of such a subsequence can be used to prove the statement of the theorem.

Combining the fact that \( M^{-1}_a: H^{-1}(\Omega) \to H^1_0(\Omega) \) is continuous with Proposition 3, which states that \( M_a u(t) \to 0 \) in \( H^{-1}(\Omega) \), we obtain immediately:

**Proposition 5** Let \( u \) be a generalized solution to (1.1). Then,

\[
\lim_{t \to \infty} ||u(t)||_1 = \lim_{t \to \infty} ||M^{1/2} a u(t)|| = 0.
\]

Remark 7.2 We note, of course, that this convergence is substantially weakened to \( ||u(t)|| \to 0 \) when \( \alpha = 0 \) which creates complications in the arguments that follow—see [30].

Now, given a sequence of moments in time that converges to infinity, from Proposition 4 we have that there exists a subsequence \( \{t_n\} \) and a point \( \tilde{u} \in H^2_0(\Omega) \) such that

\[
||u(t_n) - \tilde{u}||_2 \to 0, \text{ as } n \to \infty. \tag{7.8}
\]

Moreover, because

\[
||u(t_n + \tau) - u(t_n)|| \leq \int_{t_n}^{t_n + \tau} ||u(t)(s)||_1 ds
\]

and \( u(t) \in C(H^1_0(\Omega)) \) with \( u(t) \to 0 \) in \( H^1_0(\Omega) \), we conclude that

\[
\max_{\tau \in [-a,a]} ||u(t_n + \tau) - \tilde{u}||_1 \to 0 \text{ as } n \to 0 \tag{7.9}
\]

for every finite \( a > 0 \). By interpolation, for \( \delta > 0 \), we obtain

\[
\max_{\tau \in [-a,a]} ||u(t_n + \tau) - \tilde{u}||_2 \to 0 \text{ as } n \to \infty \tag{7.10}
\]

Since \( ||u(t_n) - \tilde{u}||_2 \) is bounded, from equations 7.9 and 7.10, we have

\[
\max_{\tau \in [-a,a]} ||u(t_n + \tau) - \tilde{u}||_{2-\delta,\Omega} \to 0 \text{ as } n \to \infty. \tag{7.11}
\]

Using a simple contradiction argument, we can push the Sobolev index to 2.

**Proposition 6** Let \( u \) be a generalized solution to (1.1). Then, for any sequence of times converging to infinity, there exists a subsequence \( \{t_n\} \) and a corresponding \( \tilde{u} \in H^2_0(\Omega) \) such that

\[
\max_{\tau \in [-a,a]} ||u(t_n + \tau) - \tilde{u}||_2 \to 0 \text{ as } n \to \infty. \tag{7.12}
\]

**Proof of Proposition 6** Given any sequence of times converging to infinity, we know there exists a subsequence \( \{t_n\} \) and a point \( \tilde{u} \) such that (7.11) holds. Now, for any fixed \( a \), consider a subsequence

\[
\max_{\tau \in [-a,a]} ||u(t_n + \tau)||_2 \to ||u(t_n + \tau_k)||_2
\]

for some \( \tau_k \in [-a,a] \). From Corollary 3, we know that the sequence \( \{u(t_n + \tau_k)\}_{k=1}^\infty \) has a convergent subsequence \( u(t_{n_k} + \tau_{k}) \to \tilde{u} \in H^2_0(\Omega) \), as well as in any lower Sobolev space; then,

\[
||u(t_{n_k} + \tau_k) - \tilde{u}||_2 \to 0. \tag{7.12}
\]

From (7.11), we know that

\[
||u(t_{n_k} + \tau_k) - \tilde{u}||_2 \to 0 \tag{7.13}
\]

Substituting (7.13) in (7.12), we see \( ||\tilde{u} - \bar{v}||_2 \to 0 \) and we identify the limits.

Hence, any subsequence of \( \max_{\tau \in [-a,a]} ||u(t_n + \tau) - \tilde{u}||_2 \) has a further convergent subsequence that converges to 0, yielding the result. \( \square \)

7.2 Step 2: lifting to flow convergences

We emphasize that this step where the assumption \( \alpha > 0 \) is critical to the arguments presented here. To bound
each of the following terms and show φ(t) ∈ H^1(K_ρ), we need ∥u(t)∥_1 → 0 and

\[ \max_{\tau \in [-a, a]} ∥u(t_n + \tau) - \bar{u}∥_2 \to 0 \text{ as } n \to \infty, \]

where the second convergence in turn also requires ∥u(t)∥_1 → 0. The manipulation of the classical flow formulae in Sect. 4.2 to obtain estimates is much more difficult in the case when \( \alpha \neq 0 \).

Let \( \bar{\phi}(x, t) = \frac{1}{2\pi} \int_{x_3}^t ds \int_0^{2\pi} d\theta U \partial x_1 \bar{u}^+(x, t, s, \theta) \).

For \( t > t_\rho \), \( \phi = \phi^{**} \); thus, we can replace \( \phi \) with \( \phi^{**} \) in (4.5); we then have the following estimates:

\[ \int_{K_\rho} dx |\partial x \phi(x, t_n) - \partial x \bar{\phi}(x, t_n)| \]

\[ = \frac{1}{2\pi} \int_{K_\rho} dx \left| \int_{t_n}^t ds \int_0^{2\pi} d\theta \partial x \right| \left( (\partial_t + U \partial x_1)(u - \bar{u}) \right) \nabla \parallel (x, t_n, s, \theta) \parallel \leq t^a \left( \max_{\tau > t - t^*} ∥u(\tau)∥_1 + U \max_{\tau \in [-t^*, t^*]} ∥u(t_n + \tau) - \bar{u}∥_2 \right) \int_{K_\rho} dx. \]

(7.14)

For \( j = 1, 2 \):

\[ \int_{K_\rho} dx |\partial x_j \phi(x, t_n) - \partial x_j \bar{\phi}(x, t_n)| \]

\[ = \frac{1}{2\pi} \int_{K_\rho} dx \left| \int_{t_n}^t ds \int_0^{2\pi} d\theta \partial x_j \right| \left( (\partial_t + U \partial x_1)(u - \bar{u}) \right) \nabla \parallel (x, t_n, s, \theta) \parallel \leq t^a \left( \max_{\tau > t - t^*} ∥u(\tau)∥_1 + U \max_{\tau \in [-t^*, t^*]} ∥u(t_n + \tau) - \bar{u}∥_2 \right) \int_{K_\rho} dx. \]

(7.15)

and

\[ \int_{K_\rho} dx |\partial x \phi(x, t_n) - \partial x \bar{\phi}(x, t_n)| \]

\[ \leq \int_{K_\rho} dx |(u - \bar{u})(x_1 - U x_3, x_2, t_n - x_3)| \]

\[ + \frac{1}{2\pi} \int_{K_\rho} dx \left| \int_{t_n}^t ds \int_0^{2\pi} d\theta [M_{\theta}] \right| \left( (\partial_t + U \partial x_1)(u - \bar{u}) \right) \nabla \parallel (x, t_n, s, \theta) \parallel \]

\[ \leq \left( ∥u - \bar{u}∥_1 + \max_{t > t - t^*} ∥u(t)∥_1 \right. \]

\[ + U \max_{\tau \in [-t^*, t^*]} ∥u(t_n + \tau) - \bar{u}∥_2 \int_{K_\rho} dx. \]

(7.16)

Also, from equation (4.10), we obtain

\[ ∥\phi(t, x)∥_{K_\rho} \leq \max_{t > t - t^*} ∥u(t)∥_1 \{ 2 + t^s \}. \]

(7.17)

By Propositions 5 and 6, all terms on the right-hand side of estimates (7.14)–(7.17) approach zero. Hence, we obtain the convergence

\[ ∥\phi(t_n) - \bar{\phi}(t_n)∥_{1,K_\rho} + ∥\phi(t_n)∥_{K_\rho} \to 0, \text{ as } n \to \infty. \]

7.3 Step 3: weak solution

In this step, we show that (\( \bar{\phi}; \bar{\phi} \)), as constructed in the previous steps, is a weak solution to (3.1). We multiply the plate equation in (1.1) by smooth function \( w \in C^\infty_0(\Omega) \) in \( L^2(\Omega) \), integrate from \( t_n \) to \( t_n + a \) for some \( a > 0 \), and integrate by parts to obtain

\[ \langle u(t, w) \rangle_{t_n}^{t_n+a} + \int_{t_n}^{t_n+a} \langle \Delta u, \Delta w \rangle + \langle \alpha \nabla u_t, \nabla w \rangle \int_{t_n} \]

\[ + k \int_{t_n}^{t_n+a} \langle u(t), w \rangle \int_{t_n} \]

\[ - k \alpha \int_{t_n}^{t_n+a} \langle \nabla u_t, \nabla w \rangle \int_{t_n} \]

\[ - \int_{t_n}^{t_n+a} \langle \lambda + F_0, w \rangle + \int_{t_n}^{t_n+a} \langle p_0, w \rangle \int_{t_n} \]

\[ \int_{r}\Omega \left[ \gamma(\phi_u + U \partial x_1) \right] \int_{t_n} \]

Each term may be estimated:

\[ \left| \int_{t_n}^{t_n+a} \langle \Delta u - \Delta \bar{u}, \Delta w \rangle \int_{t_n} \right| \]

\[ \leq a ∥\Delta w∥ \max_{t \in [0,a]} ∥u(t_n + \tau) - \bar{u}(\tau)∥_2. \]

\[ k \int_{t_n}^{t_n+a} \left| \langle u(t), w \rangle \int_{t_n} \right| \]

\[ \leq a ∥w∥ \max_{t \in [0,a]} ∥u(t_n + \tau)∥_1. \]

\[ \left| \langle u(t), w \rangle \right|_{t_n}^{t_n+a} \]
Similarly, multiplying the fluid part of equation 1.1 with \( \psi \)

\[
\langle \alpha \nabla u(t, t), \nabla w \rangle \bigg|_{t_n}^{t_n+a}
\]

\[
\leq 2\alpha \|w\| \max_{t > t_n} \|u(t)\|_1.
\]

\[
k\alpha \int_{t_n}^{t_n+a} |(\nabla u(t, t), \nabla w)\| dt
\]

\[
\leq k\alpha a \|w\| \max_{t \in [t_n, t_n+a]} \|u(t)\|_1. \tag{7.18}
\]

We substitute \( x_3 = 0 \) in (4.11) and (4.10) to obtain the point-wise expression for \( \|r_{\Omega}\| \gamma \langle \phi_t + U \phi_{x_1} \rangle, w \rangle \), which is then used to obtain the following estimate:

\[
\int_{t_n}^{t_n+a} |(r_{\Omega}(\gamma \phi_t + U \phi_{x_1}), w)dt - U(\gamma (\phi_{x_1}, w))| \leq (t^* + 2 + Ut^*) \max_{t > t_n} \|u(t)\|_1
\]

\[
+ U^2 t^* \max_{t \in [-t^*, a]} \|u(t_n + t) - \bar{u}\|_2. \tag{7.19}
\]

As we have noted, \( f_t(u) = [u, v(u) + F_0] \) is locally Lipschitz for each \( F_0 \in H^3(\Omega) \), yielding

\[
\|u(t, v(u) + F_0) - [\bar{u}, v(\bar{u}) + F_0], w]\| \leq \|u(t, v(u) + F_0) - [\bar{u}, v(\bar{u}) + F_0]\| \|w\|
\]

\[
\leq C(\|u\|, \|w\|_2, \|F_0\|) \|u - \bar{u}\|_2. \tag{7.20}
\]

Each term on the right-hand side of (7.18)–(7.20) goes to zero as \( n \to \infty \) by Propositions 5 and 6. Hence, by density, we obtain the following relation for all \( w \in H^3(\Omega) \):

\[
\langle \Delta \bar{u}, \Delta w \rangle - \langle [\bar{u}, v(\bar{u}) + F_0], w \rangle
\]

\[
+ U \gamma (\bar{\phi}, \phi_{x_1}, w) = (p_0, w). \tag{7.21}
\]

Similarly, multiplying the fluid part of equation 1.1 with \( \psi \in C_0^\infty (\mathbb{R}^3_+) \) and integrating from \( t_n \) to \( t_n + a \), we get

\[
\int_{t_n}^{t_n+a} (\dot{\phi}_t, \psi) dt + \int_{t_n}^{t_n+a} U^2 (\phi_{x_1}^2, \psi) dt
\]

\[
+ \int_{t_n}^{t_n+a} (2U \phi_{x_1} \phi_t, \psi) dt
\]

\[
= \int_{t_n}^{t_n+a} (\Delta \phi, \psi) dt. \tag{7.22}
\]

This implies

\[
(\phi_t, \psi) \bigg|_{t_n}^{t_n+a} - \int_{t_n}^{t_n+a} U^2 (\phi_{x_1} \phi_t, \psi) dt
\]

\[
- 2U \int_{t_n}^{t_n+a} (\phi_t, \psi) dt + \int_{t_n}^{t_n+a} (\nabla \phi, \nabla \psi) dt
\]

\[
+ \int_{t_n}^{t_n+a} ((\partial_t + \partial_{x_1} u)(\gamma \psi) = 0. \tag{7.23}
\]

We now estimate each term. Recall that \( K_\rho \subset \subset \mathbb{R}^3_+ \) contain the support of \( \psi \). Then:

\[
U^2 \int_{t_n}^{t_n+a} \mid (\phi_{x_1} \phi - \phi_{x_1} \bar{\phi}, \phi_{x_1} \phi) \mid dt
\]

\[
\leq aU^2 \|\phi_{x_1} \phi \| \max_{t \in [0,a]} \|\phi(t_n + \tau) - \bar{\phi}\|_{1,K_\rho}. \tag{7.24}
\]

\[
2U \int_{t_n}^{t_n+a} \mid (\phi_t, \phi_{x_1} \phi) \mid dt
\]

\[
\leq 2aU \|\phi_{x_1} \phi \| \max_{t \in [0,a]} \|\phi(t_n + \tau)\|_{1,K_\rho}. \tag{7.25}
\]

\[
\int_{t_n}^{t_n+a} \mid (\nabla \phi - \nabla \bar{\phi}, \nabla \psi) \mid dt
\]

\[
\leq a \|\nabla \psi\| \max_{t \in [0,a]} \|\phi(t_n + \tau) - \bar{\phi}\|_{1,K_\rho}. \tag{7.26}
\]

\[
\int_{t_n}^{t_n+a} \mid ((\partial_t + U \partial_{x_1} u - U \partial_{x_1} \bar{u}, \gamma \psi) \mid dt
\]

\[
\leq a \max_{t \in [0,a]} \|u(t)\|_{1} + a \max_{t \in [0,a]} \|u(t_n + \tau) - \bar{u}\|_2. \tag{7.27}
\]

Applying (7.14)–(7.17) to (7.24), and again straightforwardly invoking Propositions 5 and 6, we see that each term on the right-hand side of (7.24) approaches zero. Whence we obtain

\[
(\nabla \bar{\phi}, \nabla \psi) - U^2 (\phi_{x_1} \bar{\phi}, \phi_{x_1} \psi)
\]

\[
+ U (\partial_{x_1} \bar{u}, \gamma \psi) = 0. \tag{7.28}
\]

for any \( \psi \in H^1(\mathbb{R}^3_+) \). Thus, \( (\bar{u}, \bar{\phi}) \) satisfies (7.21) and (7.25) and is hence a weak solution.

7.4 Step 4: final result

We therefore have shown that any sequence of time converging to infinity contains a subsequence \( \{t_n\} \) for which \( (u(t_n); \phi(t_n)) \) converges to some stationary solution. We conclude by improving this convergence to the set \( N \).
Proposition 7 For \((u, \phi)\), a generalized solution to (1.1) where \(\phi_0, \phi_1\) have localized support in \(K_\rho\), we have:
\[
\lim_{t \to \infty} \inf_{[\bar{u}, \bar{\phi}] \in \mathcal{N}} \left\{ ||u(t) - \bar{u}(t)||^2_{L^2(\Omega)} + ||u_t(t)||^2_{L^2(\Omega)} + ||\phi(t) - \bar{\phi}(t)||^2_{H^1(K_\rho)} + ||\phi_t(t)||^2_{L^2(K_\rho)} \right\} = 0,
\]
for any \(\bar{\rho} > 0\).

Proof of Proposition 7 Assume the statement is not true. Then, there is a sequence \(t_m \to \infty\) and some \(\epsilon > 0\) so that for all \(m\) sufficiently large
\[
\inf_{[\bar{u}, \bar{\phi}] \in \mathcal{N}} \left\{ ||u(t_m) - \bar{u}(t_m)||^2_{L^2(\Omega)} + ||u_t(t_m)||^2_{L^2(\Omega)} + ||\phi(t_m) - \bar{\phi}(t_m)||^2_{H^1(K_\rho)} + ||\phi_t(t_m)||^2_{L^2(K_\rho)} \right\} > \epsilon.
\]
But for any such sequence \(\{t_m\}\), we have shown that there exists a subsequence \(\{t_{m_k}\}\) such that
\[
\lim_{k \to \infty} \inf_{[\bar{u}, \bar{\phi}] \in \mathcal{N}} \left\{ ||u(t_{m_k}) - \bar{u}(t_{m_k})||^2_{L^2(\Omega)} + ||u_t(t_{m_k})||^2_{L^2(\Omega)} + ||\phi(t_{m_k}) - \bar{\phi}(t_{m_k})||^2_{H^1(K_\rho)} + ||\phi_t(t_{m_k})||^2_{L^2(K_\rho)} \right\} \to 0,
\]
which is a contradiction. Hence,
\[
\lim_{t \to \infty} \inf_{[\bar{u}, \bar{\phi}] \in \mathcal{N}} \left\{ ||u(t) - \bar{u}(t)||^2_{L^2(\Omega)} + ||u_t(t)||^2_{L^2(\Omega)} + ||\phi(t) - \bar{\phi}(t)||^2_{H^1(K_\rho)} + ||\phi_t(t)||^2_{L^2(K_\rho)} \right\} = 0.
\]

With the above claim, we conclude the proof of Theorem 7.1. In the case that \(\mathcal{N}\) is isolated (e.g., finite), (7.26) collapses to the result of Corollary 4.

Acknowledgements The authors would like thank the referees for their critical reading and thoughtful criticisms and suggestions which greatly helped improve the quality of the manuscript. The second author would like to thank the National Science Foundation for its generous support.

Funding Justin T. Webster would like to thank the National Science Foundation and acknowledge his partial funding from NSF Grant DMS-1907620.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Availability of data and material Not applicable.

Code availability Not applicable.

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