Are strongly magnetized degenerate stars cooling by axion emission?

M. Kachelrieß

INFN, Lab. Naz. del Gran Sasso, I–67010 Assergi (AQ), Italy

Abstract. We considered recently as a new axion production mechanism the process $e^- \rightarrow e^- + a$ in a strong magnetic field $B$. Requiring that for a strongly magnetized neutron star the axion luminosity is smaller than the neutrino luminosity we obtained the bound $g_{ae} \lesssim 10^{-10}$ for the axion electron coupling constant. This limit is considerably weaker than the bound derived earlier by Borisov and Grishina using the same method. Applying a similar argument to magnetic white dwarf stars we obtained $g_{ae} \lesssim 9 \cdot 10^{-13}(T/10^7\text{K})^{5/4}(B/10^{10}\text{G})^{-2}$, where $T$ is the internal temperature of the white dwarf. Here we note that the observed lack of magnetized white dwarfs with low-temperature in the galactic disc could also be interpreted as a signature of axion emission. Moreover, we speculate that axion emission could explain why the putative galactic halo population of white dwarfs is so dim.

1. Introduction

The axion $a$ is a pseudoscalar boson introduced by Peccei and Quinn to solve the strong CP-problem [1]. Although the naturalness of this solution can be criticized from a more fundamental point of view, the axion is still generally considered as the best motivated cold dark matter particle apart from the neutralino. Moreover, considerable experimental efforts are dedicated to axion searches.

Here we report on the calculation of a new axion production process, namely the emission of axions by electrons in the magnetic field $B$ of a strongly magnetized neutron or white dwarf (WD) star, in Ref. [2]. We
derived a new limit for the axion electron coupling constant $g_{ae}$, but noted also that the observations of magnetized WDs in the galactic disc show at low-temperature a lack compared to non-magnetized WDs. This could be interpreted as a possible signature of axion emission. Furthermore, we speculate that a substantial fraction of the galactic dark matter halo seen by the MACHO and EROS experiments consists of magnetized WDs. Since the cooling time of a magnetized WD is drastically reduced by axion emission, this could explain the non-observation of halo WDs in optical searches.

2. Axion cyclotron emissivity from degenerated electrons

The solutions of the Dirac equation for electrons in an external homogeneous, magnetic field are given by Landau states. Using the gauge $A = (0, 0, Bx, 0)$ they can be characterized by the Landau quantum number $N = 0, 1, \ldots$, the $y$ and the $z$ components of the momentum $p$, and the eigenvalue $\tau = \pm 1$ of a suitable polarization operator. Axion cyclotron emission is the transition of an electron from an excited Landau level $n$ into a level $n' < n$ thereby emitting an axion $a$ with momentum $\hat{k} = (\omega, \vec{k})$.

The luminosity $L_a$ emitted by $N$ electrons occupying the volume $V$ due to this process is given by

$$L_a = \lim_{T \to \infty} \frac{1}{T} \sum_{\lambda, \lambda'} \sum_{\vec{k}} \omega |S_{\pm}|^2 S,$$

where $S$ is the $S$-matrix element of the process $e^- \rightarrow e^- + a$, the summation index $\lambda$ indicates the set of quantum numbers $\lambda = \{n, \tau, p_y, p_z\}$,

$$S = f(E)[1 - f(E')]$$

and $f(E)$ are Fermi-Dirac distributions functions. As it is characteristic for processes in strong magnetic fields, the $S$-matrix element consists essentially of Laguerre functions $I$,

$$I_{n', n}(\kappa) = \sqrt{\frac{n'!}{n!}} \kappa^{(n-n')/2} e^{-\kappa/2} L_{n'}^n(\kappa).$$

The argument of the $I$-functions is given by $\kappa = (k \sin \theta)^2/(2eB)$, $\theta$ is the angle between $\vec{B}$ and $\vec{k}$, and $L_{n'}^n(x)$ are Laguerre polynomials. The numerical evaluation of the terms in Eq. (1) becomes cumbersome already for moderate $n$. Therefore we take advantage of the degeneracy of the electron gas inside inside a neutron or white dwarf star and employ an approximation commonly used in calculations of neutrino emission rates: Transitions between different Landau states are only possible, if the states
are lying inside the shell \([E_F - T, E_F + T]\), where \(E_F\) is the Fermi energy of the electrons. In this approximation, the axion cyclotron emissivity \(\varepsilon_a = \mathcal{L}_a/V\) is given by

\[
\varepsilon_a = \frac{eB}{2\pi^2} \sum_{n=1}^{n_{\text{max}}} \sum_{n' < n} \sum_{\tau = \pm} \int_0^\pi d\theta \sin \theta \frac{E_0}{p_z} \left( \frac{E_0}{E - p_z \cos \theta} \right)^2 \frac{d\Gamma_{0n'\tau} (\vec{k})}{d\theta_0} \omega^2 e^{\beta\omega} - 1.
\]

(4)

Here, \(d\Gamma_0\) is the differential decay width of an electron in its rest frame as given in Ref. [2] and \(n_{\text{max}} = \text{int} \left( \frac{(E_F^2 - m_e^2)}{2eB} \right)\). Note that Eq. (4) is valid for arbitrary magnetic field strengths.

Although the two infinite sums over \(n\) and \(n'\) have now been replaced by finite sums, for low magnetic field strengths or high densities it is still necessary to compute Laguerre polynomials with high index. This can be avoided by the use of a Bessel function approximation in the semiclassical, ultrarelativistic case and by the use of the \(\kappa \to 0\) limit of the \(I\)-functions in the classical case.

Our numerical evaluation of Eq. (4) confirmed the semi-classical limit valid for \(E \gg m_e\) and \(B \ll B_{cr} = m_e^2/e \approx 4.41 \times 10^{13}\) Gauss, i.e. for neutron stars with not too high magnetic fields, derived in Ref. [3] by Borisov and Grishina. However, applying axion cyclotron emission as an additional cooling mechanism to neutron stars and requiring that the axion luminosity is smaller than the total neutrino luminosity, we could constrain the axion electron coupling constant only to \(g_{ae} \lesssim O(10^{-10})\). This bound is three orders of magnitude weaker than the bound derived in Ref. [3] considering the same process. The reason for this discrepancy is that the authors of Ref. [3] derived their bound by requiring that the emissivity due to the process \(e^- \to e^- + a\) is smaller than due to neutrino cyclotron emission \(e^- \to e^- + \nu + \bar{\nu}\) instead of comparing \(\varepsilon_a\) with the total emissivity of a neutron star.

We now apply the same line of arguments to magnetic WDs. In Fig. 1, \(\tilde{\varepsilon}_a = \varepsilon_a/\alpha_{1eV}\) is shown for the electron density \(n_e = 10^{23}\text{cm}^{-3}\) and the temperatures \(T = 10^6\text{K}, T = 10^7\text{K}\) and \(T = 10^8\text{K}\), respectively, together with the fit function

\[
\tilde{\varepsilon}^{\text{fit}}_a = 2.6 \frac{\text{erg}}{\text{cm}^3\text{s}} \left( \frac{B}{10^9\text{G}} \right)^4 \left( \frac{T}{10^7\text{K}} \right).
\]

(5)

The dependence \(\varepsilon \propto B^4T\) in the classical limit could be expected from dimensional considerations.

The photon luminosity of the surface of a WD can be written as an
effective emissivity $\varepsilon_\gamma$, i.e. as an energy-loss per volume and time,

$$\varepsilon_\gamma = 3.3 \times 10^3 \frac{\text{erg}}{\text{cm}^3 \text{s}} \left(\frac{T}{10^7 \text{K}}\right)^{7/2}.$$  \hspace{1cm} (6)

If we require that a magnetized WD do not emit more energy in axions than in photons, we obtain

$$g_{ae} \lesssim 9 \cdot 10^{-13} \left(\frac{T}{10^7 \text{K}}\right)^{5/4} \left(\frac{B}{10^{10} \text{G}}\right)^{-2}.$$  \hspace{1cm} (7)

Since this bound is quite sensitive to $B$ and the knowledge of the internal magnetic field strengths of WDs is poor, it is hard to derive a precise bound for $g_{ae}$.

3. Cooling times of magnetized white dwarfs

The thermal history of a WD can be viewed essentially as a cooling process

$$\frac{dt}{dT} = -\frac{c_V(T)}{\varepsilon(T)},$$  \hspace{1cm} (8)
Table 1. Fraction of all white dwarfs (WDs) and of magnetized WDs in three temperature bins.

| $T_{\text{eff}}$[10$^3$K] | Fraction of all WDs | Fraction of magnetized WDs |
|--------------------------|---------------------|-----------------------------|
| 40-80                    | 1%                  | 0%                          |
| 20-40                    | 23%                 | 50%                         |
| 12-20                    | 76%                 | 50%                         |

emissivity $\varepsilon_\nu$, which is only for the hottest WDs with $T \gtrsim 10^{7.8}$K important and decreases additionally the cooling times. Since $\varepsilon_a \propto T$, compared to $\varepsilon_\gamma \propto T^{7/2}$, axion emission becomes more and more important during the cooling history of magnetized WDs. Therefore the fraction of magnetized WDs among all WDs should diminish drastically for low enough luminosities.

In Table I, we compare the fraction of strongly magnetized WDs with the fraction of all WDs in three different temperature bins. The distribution of hot strongly magnetized WDs adapted from Ref. spans approximately three order of magnitudes in surface dipole field strength, $B_{\text{dipole}} = 3 \cdot 10^6 - 10^9$ G, and has a maximum at $B_{\text{dipole}} \approx 3 \cdot 10^7$ G. It consists of only 18 stars, so some caution in interpreting the data is appropriate. Nevertheless, the fraction of magnetized WDs in the last temperature bin is considerably diminished compared to the total WD population. One possible explanation for this could be the additional energy loss of magnetized WDs due to axion cyclotron emission.

An obvious test for the hypothesis that magnetized WDs emit axions is the comparison of the observed temperature distribution of disc magnetized WDs with the predicted one. In Fig. 2 we show the calculated fraction of magnetized WDs in two different temperature bins as function of the magnetic field strength. For comparison, the observed fractions of all disc WDs and of magnetized WDs are shown with error bars on the left and on the right side of the panel. For internal magnetic field strengths $B_{10} \lesssim 10(10\text{meV}/m_a)^2$, the observed temperature distribution of all disc WDs is well reproduced, while for higher magnetic field strengths the observed temperature distribution of the magnetized WDs agrees also reasonably well with the predicted one. ($B_{10}$ denotes $B/10^{10}$ G.)

The most stringent upper limit for the axion mass, $m_a \lesssim 10$ meV, was derived studying the evolution of red giant stars. Using this value for
Figure 2. Fraction $f_i$ of magnetic WDs in two temperature bins as function of the magnetic field strength $B_{10}$ for $m_a = 10$ meV and $n_{\text{ion}} = 4.5 \times 10^{28} \text{cm}^{-3}$; on the left side the observed values of all WD, on the right side the observed values of magnetized WD.

the axion mass results in an average internal field of $B_{10} \sim 30$ in order to reproduce the observed temperature distribution of magnetized WDs. Although the internal field strengths of magnetized white dwarfs are generally assumed to be in the range $B_{10} = 0.01 \ldots 1$, they could approach even $B_{10} = 100$. Nevertheless, if one concedes a factor 5 as error in the upper mass limit, the necessary internal magnetic strengths of the magnetized white dwarfs are in the general accepted range.

Finally, let us speculate about the nature of the galactic dark matter halo. A preferred interpretation of the microlensing events seen by MACHO and EROS is that a substantial fraction of the galactic dark matter halo consists of compact objects with the most probable mass around $0.5 M_\odot$. For this mass range, WDs which are known to exist in large numbers are the natural candidates, although exotic objects like primordial black holes or gravitino stars cannot be excluded.

Various optical searches did not detect a substantial halo population of WDs. Hence the supposed halo population has to be dim enough to have eluded detection. But then the estimated cooling times of 11 to 15.5 Gyrs make them as old as globular clusters and raise the question whether this allows reasonable life times for the progenitors of the WDs. Therefore, such a halo seems to be only possible if the halo WDs are distinguished by some physical property which shortens their cooling times. This different physical property could be a strong internal magnetic field of the
halo WDs. For magnetic fields $B_{10} \gtrsim 5 \times 10^5 \text{meV/m}^2$, their cooling times would drop below 1 Gyr and would make a halo consisting of magnetized WDs invisible.

4. Summary

We have derived the axion emissivity of a magnetized electron gas due to the process $e^- \rightarrow e^- + a$ for arbitrary magnetic field strengths $B$. Applying axion cyclotron emission as an additional cooling mechanism to neutron stars we could constrain the axion electron coupling constant to $g_{ae} \lesssim \mathcal{O}(10^{-10})$. In the case of white dwarfs we could derive the more stringent limit Eq. (7). We have noted that the lack of low-temperature magnetized white dwarfs could be interpreted as signature of an additional energy loss due to axion cyclotron emission.

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