Perturbations such as rotation and polar-cap current (PC-current) have been believed to greatly affect the pulsar radio emission and polarization. The two effects have not been considered simultaneously in the literature; each one of these has been considered separately, and a picture has been deduced by simply superposing them, but such an approach can lead to spurious results. Hence, by considering pulsar rotation and PC-current perturbations together instead of one at a time, we have developed a single particle curvature radiation model, which is expected to be much more realistic. By simulating a set of typical pulse profiles, we have made an attempt to explain most of the observational results of pulsar radio emission and polarization. The model predicts that due to the perturbations the leading side component can become either stronger or weaker than the corresponding trailing one in any given cone, depending on the passage of the sight line and modulation (nonuniform source distribution). Further, we find that the phase delay of the polarization angle inflection point with respect to the core component greatly depends on the viewing geometry. The correlation between the sign reversal of circular polarization and the polarization angle swing in the case of core-dominated pulsars becomes obscure once the perturbations and modulation become significant. However, the correlation between the negative circular polarization and the increasing polarization angle and vice versa is very clear in the case of conal-double pulsars. The “kinky”-type distortions in polarization angle swing could be due to the incoherent superposition of modulated emission in the presence of strong perturbations.

Key words: polarization – pulsars: general – radiation mechanisms: non-thermal

Online-only material: figure set

1. INTRODUCTION

Pulsars that are famed by their highly periodic signals are now universally accepted as fast rotating and highly magnetized (mainly dipolar) neutron stars. Coherent curvature radiation caused by ultra-relativistic plasma streaming out along the open dipolar magnetic field lines is believed to be responsible for pulsar radio emission (e.g., Sturrock 1971; Ruderman & Sutherland 1975; Melikidze et al. 2000; Gil et al. 2004). The individual pulses from pulsars in general are highly random in strength as well as in appearance in longitude within the pulse window. However, the average profiles resulting from the summation of several hundreds of individual pulses have well-defined shapes, and they are unique in most cases. Further pulsars in general show an “S”-shaped characteristic polarization position angle (PPA) swing, which is attributed to the underlying dipole field geometry of the emission region (Radhakrishnan & Cooke 1969).

In general, the average profiles are made up of many components. Rankin (1983, 1990, 1993), Mitra & Deshpande (1999), and Mitra & Rankin (2002) recognized that the pulsar emission beams have a nested core–cone structure. However, since the conal components generally show asymmetry in their location with respect to the central core component, Lyne & Manchester (1988) suggested that the emission is “patchy.” The components often show asymmetry in their strengths between the leading and trailing sides of the profiles. Furthermore, some pulsars show the polarization angle that deviates from the standard “S” curve (Xilouris et al. 1998).

Among the several relativistic effects that have been proposed to understand pulsar emission and polarization, the effects of rotation such as aberration and retardation (A/R) and polar-cap current (PC-current) perturbation are found to be quite important. Due to pulsar rotation, the relativistic plasma has a corotation velocity component in addition to the intrinsic velocity along the dipole field lines. Hence, the net velocity of plasma will be aberrated in the direction of the pulsar rotation. Therefore, an inertial observer tends to see the plasma trajectory that differs significantly from the associated dipole field lines, and hence affects the pulsar emission and polarization (Blaskiewicz et al. 1991, hereafter BCW1991; Dyks 2008; Thomas & Gangadhara 2007, 2010; Dyks et al. 2010; Thomas et al. 2010; Kumar & Gangadhara 2012a, hereafter KG2012a; Wang et al. 2012). On the other hand, in other artificial models, the corotation of the pulsar magnetosphere is ignored and considered to be only the effect of PC-current on the underlying dipole field. The field lines will get curvature in the azimuthal direction due to the PC-current-induced toroidal magnetic field in addition to their intrinsic curvature in the polar direction. Therefore, the trajectory of field-line-constrained plasma becomes significantly different from the unperturbed case and hence affects the pulsar emission and polarization (Hibschman & Arons 2001; Gangadhara 2005; Kumar & Gangadhara 2012b, hereafter KG2012b).

By taking rotation into account, BCW1991 have predicted that the PPA inflection point lags behind the midpoint of the intensity profile by \( \sim 4r / r_{LC} \), where \( r_{LC} = cP / 2\pi \) is the light cylinder radius, \( c \) is the speed of light, and \( P \) is the pulsar rotation period. Later, Dyks (2008) confirmed this behavior. However, KG2012a showed that due to the combined effect of rotation, modulation, and viewing geometry, the phase lag of the PPA inflection point with respect to the central core will differ significantly from \( \sim 4r / r_{LC} \). On the other hand, KG2012b predicted that the PPA inflection point can even lead the central
core due to the PC-current-induced perturbation. Note that an asymmetry in the phase location of components is believed to arise from aberration and retardation effects (Gangadhara & Gupta 2001; Gupta & Gangadhara 2003; Dyks et al. 2004; Gangadhara 2005).

Further, by considering the rotation, BCW1991 predicted that the leading side intensity components dominate over the trailing ones. This is due to the fact that the curvature of source trajectory on the leading side becomes larger than that on the trailing side. Later, Thomas & Gangadhara (2007) confirmed this effect and Dyks et al. (2010), Thomas et al. (2010), KG2012a, and Wang et al. (2012) reconfirmed it. Although statistically cases with a stronger leading component are more common, converse cases are also quite significant (Lyne & Manchester 1988). By considering the PC-current-induced perturbation on the underlying dipole field, KG2012b showed that the leading side components can either become stronger or weaker than the corresponding ones on the trailing side depending upon the viewing geometry and modulation. However, the aforementioned prediction by KG2012b was in the absence of strong rotation effect.

In the literature, two types of circular polarization have been recognized: “antisymmetric” and “symmetric.” If the circular polarization changes its sense near the center of the pulse profile, then it is recognized as antisymmetric-type circular polarization; meanwhile, if the polarity of the circular polarization does not change throughout the pulse profile, then it is recognized as the symmetric type (Radhakrishnan & Rankin 1990). However, either antisymmetric or symmetric circular polarization can be associated with individual components (Han et al. 1998; You & Han 2006; KG2012a, KG2012b). In the past, only the antisymmetric-type circular polarization was thought to be an intrinsic property of curvature radiation (e.g., Michel 1987; Gil & Snikowski 1990a, 1990b; Radhakrishnan & Rankin 1990; Gil et al. 1993; Gangadhara 1997, 2010), and “symmetric”-type circular polarization was speculated to be a result of propagation effects. Recently, however, KG2012a demonstrated that in addition to antisymmetric, symmetric circular polarization can also be produced within the framework of curvature radiation if the effects of pulsar rotation, nonuniform plasma distribution, and viewing geometry are taken into account. Wang et al. (2012) reconfirmed these findings. KG2012b deduced similar results by considering PC-current-induced perturbation instead of pulsar rotation.

Pulsars show diverse behavior in circular polarization, and its association with the PPA swing is quite important for understanding the underlying geometry of the emission region. In the case of pulsars with antisymmetric circular polarization, Radhakrishnan & Rankin (1990) found a strong correlation between the sense reversal of circular polarization and the PPA swing: the sign reversal of circular polarization from negative to positive is associated with increasing PPA sweep and vice versa. Gangadhara (2010) confirmed the correlation and proposed that it is a geometric phenomenon. Further, KG2012a, KG2012b, from their simulation of polarization profiles, showed that the correlation exists when the rotation and PC-current perturbations are less significant. But Han et al. (1998) and You & Han (2006) noted that the sense reversal of circular polarization near the center of pulse profiles is not correlated with the PPA swing. However, they did find a strong correlation between the sense of the circular polarization and the PPA swing in double-conal pulsars. KG2012a speculated that such a correlation in the case of double-conal pulsars can arise if the modulations are asymmetrically located in the conal rings centered on the magnetic axis.

Among the several relativistic models proposed by taking into account pulsar rotation and the PC-current perturbations, only the models proposed by KG2012a, KG2012b, and Wang et al. (2012) can explain the full polarization state of the radiation field. Furthermore, they are much more realistic in the sense that in addition to strong perturbations, the effects of nonuniform source distribution (modulation) and viewing geometry are incorporated as essential ingredients. However, in the models proposed by KG2012a and Wang et al. (2012), the authors considered the effect of pulsar rotation and ignored PC-current perturbation as a special case. On the other hand, in yet another artificial case, KG2012b considered the effect of PC-current on the underlying dipole field by ignoring the corotation of the pulsar magnetosphere. Since both the effects are found to be quite dominant in affecting pulsar radio emission and polarization, they must be combined together.

In this paper, both rotation and PC-current perturbations are considered simultaneously and analyzed for their combined effect on pulsar emission. If we separately consider the rotation and PC-current perturbations and combine the results, the resulting conclusions can be erroneous (as we show in the next section), and this shows our work’s importance. Although pulsar radiation is generated via some coherent processes, we perform the modeling of pulsar radio emission in terms of single particle curvature radiation. Note that although to the first order the single particle approximation is not a bad assumption, in reality some factors influencing the coherence processes may favor or oppose the effects that we consider in this work. We present the theory of single particle curvature radiation in a rotating PC-current perturbed magnetic field and analyze the polarization state in Section 2. In Section 3, we present a set of simulated pulse profiles and speculate on the polarization properties by comparing the observed pulses. In Section 4 we discuss our findings, and present conclusions in Section 5.

2. CURVATURE RADIATION FROM A ROTATING PC-CURRENT PERTURBED MAGNETOSPHERE

Let us consider a stationary Cartesian coordinate system \(-XYZ\) with the origin \(O\) located at the center of a neutron star as an inertial observer frame (see Figure 1). Consider an inclined and rotating PC-current perturbed dipole with an inclination angle \(\alpha\) with respect to the rotation axis \(\Omega\), which is taken to be parallel to the \(Z\)-axis. The velocity \(\mathbf{v}\) of the relativistic source \(\mathbf{S}\), which is constrained to move along the rotating PC-current perturbed dipole field line \(f\), is given by

\[
\mathbf{v} = \kappa c \mathbf{b} + \Omega \times \mathbf{r},
\]

where \(\mathbf{b} = B / |B|\) and \(B = B_{0} + B_{1}\). \(B_{0}\) is the unperturbed dipole field, \(B_{1}\) is the PC-current-induced field, and \(\mathbf{r}\) is the position vector of the source (KG2012b). The parameter \(\Omega = \Omega_{0}\) is the pulsar angular velocity and \(\kappa\) specifies the normalized speed of the source with respect to the speed of light \(c\) along the associated perturbed field line.

The first term on the right-hand side of Equation (1) is the source velocity along the perturbed field lines. The second term is the induced velocity due to corotation of the pulsar magnetosphere. Note that due to the PC-current perturbation, the field lines that lie above the magnetic axis tend to azimuthally twist toward the pulsar rotation, whereas those that lie below
the magnetic axis twist in the opposite direction (see Figure 1 in KG2012b). Therefore, the contributions to the aberration of the source velocity $v$ by the above two terms add up to the negative sight line impact angle $\sigma$, but they try to cancel each other for the positive $\sigma$. However, since the aberration of the source velocity due to the effect of rotation is larger than that due to the PC-current perturbation for the current, which is on the order of Goldreich–Julian current, the net aberration will always be in the direction of pulsar rotation, and it is greater for negative $\sigma$.

Since the relativistic emission is beamed in the direction of the velocity $v$ with a half-opening angle $1/\gamma$, an observer can receive the emission only from a selected emission region whose boundary satisfies $\hat{n} \cdot \hat{v} = \cos(1/\gamma)$, where $\hat{v} = v/|v|$ and the sight line $\hat{n} = \{\sin \zeta, 0, \cos \zeta\}$ with $\zeta = \alpha + \sigma$. But the exact analytical solutions for the emission point coordinates ($\theta_0, \phi_0$) and ($\theta_{\text{pcc}}, \phi_{\text{pcc}}$) (see KG2012a, KG2012b for their definition) of the beaming region are difficult to find once the effects of rotation and PC-current perturbation are considered, and hence we seek numerical solutions. Note that in finding exact values for coordinates $\theta_0$ and $\phi_0$, the values obtained from KG2012b must be used as initial guess values for fast convergence, whereas KG2012a used those derived from Gangadhara (2010).

By using the parameters $r_n = 0.1$, $P = 1$ s, source Lorentz factor $\gamma = 400$, the current scale factor $\sigma = 1$, $\alpha = 10^5$, and $\sigma = \pm 5^5$, we computed $\theta_0$ and $\phi_0$ and presented them in Figure 2 as functions of $\phi'$. Due to the PC-current perturbation alone (represented by dotted line curves), the emission points in $\phi_0$ shift to later phases for positive $\sigma$ and to earlier phases for negative $\sigma$, whereas they are mostly unaffected in $\theta$. On the other hand, due to the effect of rotation (aberration) alone (represented by dashed line curves), the emission shifts to earlier phases in both $\theta$ and $\phi$. However, the phase shift of emission points in $\phi$ caused by rotation alone is larger than that caused by PC-current perturbation alone. This can be clearly seen in the phase shifts of the antisymmetric point of $\phi_0$ (phase at which $\phi_0$ is equal to 0° or 180°) indicated by arrows (styled same as $\phi_0$).

As a result, in the more realistic case of a rotating PC-current-perturbed dipole (represented by thick solid line curves), the emission points in $\theta$ shift to earlier phases in both cases of $\sigma$ by the same amount as in the case of the rotating dipole, whereas in $\phi$ they shift to the earlier phases by a smaller amount in the case of positive $\sigma$ and by a larger amount in the case of negative $\sigma$.

Note that if the emission region is modulated (nonuniform source distribution) in the azimuthal direction, which we show later, then the resulting intensity components will also show the aforementioned asymmetric phase shift between the positive and negative $\sigma$ cases.

We also computed $\theta_0$ and $\phi_0$ for the case of a rotating perturbed dipole by assuming $\theta_0 = \theta_0' + \delta\theta_{\text{rot}} + \delta\theta_{\text{pcc}}$ and $\phi_0 = \phi_0' + \delta\phi_{\text{rot}} + \delta\phi_{\text{pcc}}$, where $\theta_0'$ and $\phi_0'$ are the coordinates of emission points in the nonrotating unperturbed dipole. The changes in the colatitude $\delta\theta_{\text{rot}} = \theta_{0,\text{rot}} - \theta_0'$ and $\delta\theta_{\text{pcc}} = \theta_{0,\text{pcc}} - \theta_0'$ and those in the azimuth $\delta\phi_{\text{rot}} = \phi_{0,\text{rot}} - \phi_0'$ and $\delta\phi_{\text{pcc}} = \phi_{0,\text{pcc}} - \phi_0'$ are due to the aberration and PC-current perturbation, respectively. The parameters $\theta_{0,\text{rot}}$ and $\theta_{0,\text{pcc}}$ are the coordinates after separately considering the perturbations, rotation and PC-current, respectively, and similarly $\phi_{0,\text{rot}}$ and $\phi_{0,\text{pcc}}$. We find that, thus obtained, $\theta_0$ and $\phi_0$ more or less match the thick solid line curves (not shown in the figure). Therefore, the phase shifts as well as the changes in the magnitude of emission point coordinates $\theta_0$ and $\phi_0$ due to the two separate perturbations (rotation and PC-current) simply add up when the two effects are combined together.

The acceleration of source is given by

$$a = \frac{(\kappa\epsilon)^2}{|b|} \frac{d\hat{b}}{d\theta} + \frac{\kappa\epsilon^2}{|b|} \frac{d}{d\theta} \frac{d\hat{b}}{d\theta} + 2\kappa\epsilon(\Omega \times \hat{b}) + \Omega \times (\Omega \times \hat{b}),$$

(2)
where we used the expression of the arc length of the field line \( ds = |b| d\theta = k c dt \) and the total derivative \( dF/d\theta = \partial F/\partial \theta + (\partial F/\partial \phi)(d\phi/d\theta) \), where \( F \) stands for \( b,k \), etc. The first term on the right-hand side of Equation (2) is the acceleration of bunch due to curvature of the PC-current-perturbed field lines. Note that it includes both the intrinsic curvature due to the dipolar field and the induced curvature due to the PC-current perturbation on the dipole field. The small change in the source speed due to motion along the perturbed field line is represented by the second term, and it is very small among the other terms. The third term is the rotationally induced acceleration resulting from the Coriolis force and is in the direction of pulsar rotation. The last term is the induced acceleration due to the Centrifugal force that is acting away from the rotation axis. The PC-current-induced acceleration included in the first term becomes much more important at higher emission altitude, larger pulsar angular velocity, and smaller \( \alpha \). On the other hand, the Coriolis and the centrifugal accelerations, with the Coriolis term being the dominant one, become much more important at higher emission altitude, larger pulsar angular velocity, and larger \( \alpha \).

By using the same parameters as in Figure 2, we computed the radius of the curvature \( \rho \approx v^2/|\mathbf{a}| \) as a function of \( \phi' \) and plotted it in the upper panels of Figure 3. The PC-current perturbation (represented by the dotted line curves) leads to larger curvature with respect to the unperturbed ones (thin solid line curve), leaving the symmetric point of \( \rho \) mostly unaffected (KG2012b). On the other hand, rotation induces an asymmetry into the curvature between the leading and trailing sides of \( \phi' = 0^\circ \), since the leading side trajectory becomes more curved and \( \rho \) maximum shifts to the trailing side (e.g., KG2012a). Since the perturbation due to the PC-current in the case of positive \( \sigma \) opposes that due to the corotation, the net \( \rho \) in the rotating PC-current-perturbed dipole (thick solid line curve) becomes larger than that resulting from rotation alone, and vice versa in the case of negative \( \sigma \).

To assess the possibility of deriving \( \rho \) for the rotating PC-current-perturbed dipole by separately considering the perturbations due to PC-current and rotation, analogous to \( b_0 \) and \( \phi_0 \) in Figure 2, we considered the net curvature as \( k = k' + \delta k_{\text{rot}} + \delta k_{\text{pcc}} \), where \( k' \) is the curvature of the unperturbed dipolar field lines, \( \delta k_{\text{rot}} = k_{\text{rot}} - k' \), and \( \delta k_{\text{pcc}} = k_{\text{pcc}} - k' \). \( k_{\text{rot}} \) and \( k_{\text{pcc}} \) are the curvature vectors in the cases of the rotating dipole and PC-current-perturbed dipole, respectively. The resultant \( \rho \) can be derived as

\[
\rho = \left[ \frac{1}{\rho'^2} + \frac{1}{\rho_{\text{rot}}^2} + \frac{1}{\rho_{\text{pcc}}^2} \left( -2 \left( \frac{\dot{a}' \cdot \dot{a}_{\text{rot}}}{\rho' \rho_{\text{rot}}} + \frac{\dot{a}' \cdot \dot{a}_{\text{pcc}}}{\rho' \rho_{\text{pcc}}} - \frac{\dot{a}_{\text{rot}} \cdot \dot{a}_{\text{pcc}}}{\rho_{\text{rot}} \rho_{\text{pcc}}} \right) \right) \right]^{-1/2},
\]

where \( \rho', \rho_{\text{rot}}, \) and \( \rho_{\text{pcc}} \) are the radii of the curvatures and \( \dot{a}' \), \( \dot{a}_{\text{rot}} \), and \( \dot{a}_{\text{pcc}} \) are the unit acceleration vectors in the cases of the nonrotating dipole, rotating dipole, and PC-current-perturbed dipole, respectively. Thus obtained, \( \rho \), by adding the two separate perturbations computed independently, is superposed in the \( \rho \) panels of Figure 3 (see the dot-dashed line curves). We can see that it is significantly different from the actual \( \rho \) of the rotating PC-current-perturbed dipole.

The PPA \( \psi \) of the electric field of radiation due to relativistic sources, defined as the angle between the radiation electric field and the projected spin axis on the plane of the sky, can be computed by knowing the acceleration of the radiation source: \( \tan \psi = \hat{\mathbf{e}}_{\perp} \cdot \hat{a} / (\hat{\mathbf{e}}_{\parallel} \cdot \hat{a}) \), where \( \hat{\mathbf{e}}_{\perp} \) (projected spin vector) and \( \hat{\mathbf{e}}_{\parallel} \) are unit vectors in the directions perpendicular to \( \hat{a} \) (Gangadhara 2010). Using the same parameters as in the upper panels of Figure 3, we computed the position angle \( \psi \) as a function of \( \phi' \) and plotted in the lower panels of Figure 3. The perturbation due to the PC-current (represented by dotted line curves) causes the \( \psi \) curve to shift upward with respect to the standard rotating vector model (RVM) curve (nonrotating dipole, thin solid line curve).
curve), while its inflection point remains unaffected (Hibschman & Arons 2001; KG2012b). On the other hand, the rotation (aberration) causes the $\psi$ curve to shift upward or downward, depending on the sign of $\sigma$, and always shifts the inflection point to the trailing side. Therefore, in a more realistic case of rotating PC-current-perturbed dipole, the net $\psi$ will be shifted upward with respect to the one in the rotating case, while the inflection point lies mostly at the same phase as in the rotating dipole.

We also computed $\psi$ for the rotating perturbed dipole by assuming $\psi = \psi' + \delta \psi_{\text{rot}} + \delta \psi_{\text{pcc}}$, where $\psi'$ is due to the nonrotating dipole, $\delta \psi_{\text{rot}} = \psi_{\text{rot}} - \psi'$ and $\delta \psi_{\text{pcc}} = \psi_{\text{pcc}} - \psi'$ with $\psi_{\text{rot}}$ and $\psi_{\text{pcc}}$ are the position angles after considering the perturbation separately due to rotation and PC-current, respectively. We find that $\psi$ thus obtained in both the numerical (represented by thick dot-dashed line curves) and analytical perturbation theories (represented by thin dot-dashed line curves) and analytical perturbation theories (represented by thin dot-dashed line curves) and Hibschman & Arons 2001) significantly differs from the actual $\psi$ of the rotating PC-current dipole. Therefore, we believe that the two effects must be combined together to deduce the polarization state of the radiation field.

3. POLARIZATION STATE OF THE RADIATION FIELD

Radiation emitted by the relativistic accelerating sources will have a broad spectrum, and it is given by (Jackson 1998)

$$E(\mathbf{r}, \omega) = \frac{1}{\sqrt{2\pi}} \frac{q}{R_0 c} \frac{e^{i\omega_R/c}}{\kappa c} \int_{-\infty}^{+\infty} \frac{d\theta}{\mathbf{b}} \times \left[ (\mathbf{n} - \mathbf{b}) \times \mathbf{b} \right] e^{i(\omega - \Omega t)\mathbf{r}/c} d\theta.$$  \hspace{1cm} (4)

Note that time $t$ in the above equation must be replaced by the expression given in Equation (6) of KG2012a, and the parameters $\phi'$ must be replaced by $\Omega t$ and $\phi$ according to the expression given in Equation (11) of KG2012b. We solve the integral using the method given in KG2012a and find the polarization state of the radiation field in terms of the Stokes’ parameters $I, Q, U,$ and $V$.

3.1. Emission from Uniform Distribution of Sources

By assuming a uniform distribution of sources throughout the emission region, we computed the radiation field from the beaming region and its polarization state, as presented in Figure 4. Since the magnitude of rotation of the contour patterns of the total intensity and the linear polarization in ($\theta, \phi$) are more or less the same as that of the circular polarization, we present only the contour patterns of circular polarization. However, the total intensity and linear polarization will be at its maximum at the beaming region center and fall considerably toward the boundary, with a slightly larger emission toward the larger curvature region (see Figure 3 in KG2012a, KG2012b).

Due to pulsar rotation, the contour patterns of the circular polarization (panels (b) and (b')) rotate in the ($\theta, \phi$) plane with respect to those in the nonrotating dipole (panels (a) and (a')), and rotates from the trailing side to the leading side for both signs of $\sigma$. Furthermore, there arises an asymmetry in the strength of the positive and negative polarities of the circular polarization in such a way that the negative circular becomes much stronger. On the other hand, due to the PC-current perturbation, the rotation direction of the contour patterns of the circular polarization (panels (c) and (c')) is the opposite between the positive and negative $\sigma$ cases: for positive $\sigma$, the rotation direction is opposite to that due to the effect of rotation, whereas it is in the same direction for negative $\sigma$. However, the magnitude of rotation of the contour patterns due to the PC-current is less than that due to pulsar rotation. Also, due to the PC-current, the positive polarity of the circular polarization becomes stronger than the negative polarity for both $\pm \sigma$. Therefore, in the case of rotating PC-current perturbation (panels (d) and (d')), the rotation direction of the contour pattern...
Figure 4. Pattern of the circular polarization for the emission from the beaming region with uniform distribution of sources in the following cases: the nonrotating dipole—panels (a) and (a’); rotating dipole—panels (b) and (b’); nonrotating PC-current-perturbed dipole—panels (c) and (c’); and rotating perturbed dipole—panels (d) and (d’). In each panel, the emission is normalized with the corresponding maximum of the total intensity. Here, we used the parameters $\phi' = 0^\circ$ and $\nu = 600$ MHz, and the rest of the parameters are the same as in Figure 2.

The emission from the beaming region due to different plasma bunches will be incoherently added at the observation point if they are separated by a space larger than the radiation wavelength. The resultant emission that the observer receives of the circular polarization will be in the same direction as in panels (b) and (b’) but with a lower magnitude of rotation for positive $\sigma$ than for negative $\sigma$. Furthermore, due to the opposite selective enhancement of either the positive or negative polarity of the circular polarization by the two perturbations (rotation and PC-current), the circular polarization has more or less the same strength between the positive and negative polarities, which is similar to the nonrotating case.
will be the sum of the intensities from the different plasma bunches. Using the expressions given in Gangadhara (2010, Equations (33)–(36)), we computed the polarization state of the emitted radiation due to uniform distribution of sources, and they are presented in Figure 5. With the uniform distribution of sources, the PC-current perturbation alone (represented by the dotted line curves) does not affect the symmetry of the total intensity $I$ between the leading and trailing sides of $\phi' = 0^\circ$, which is similar to the nonrotating dipole, whereas the rotation introduces an asymmetry (represented by the dashed line curves) where the leading side becomes stronger due to an induced larger curvature of the source trajectory on that side (see Figure 3). In the more realistic rotating perturbed dipole, there remains an asymmetry similar to the case of the rotating dipole, wherein the emission significantly differs from that in the cases of the rotating dipole and the PC-current-perturbed dipole when considered separately.

Due to an incoherent addition of emission from the different bunches within the beaming region, the magnitude of linear polarization $L$ becomes slightly smaller than that of the total intensity $I$. However, the profile of $L$ more or less matches the corresponding $I$. The net survived circular polarization $V$ in the case of the rotating perturbed dipole (represented by the thick solid line curves) becomes very small because of the
two perturbations (rotation and PC-current), which selectively enhance the opposite polarities of $V$ as shown in Figure 4. The position angle increases in the case of positive $\sigma$, whereas it decreases in the case of negative $\sigma$ with the inflection point always shifted to the trailing side.

3.2. Emission Due to Nonuniform Distribution of Sources

We considered a Gaussian modulation function which is given in KG2012a, KG2012b to model the nonuniform distribution of sources in the emission region. We find the polarization state of the radiation field from the nonuniform distribution of sources using the expressions given in Gangadhara (2010, see Equation (38)).

3.2.1. Emission with Azimuthal Modulation

By considering a modulation in the magnetic azimuth with the peak at the meridional plane, we simulated the polarization profiles affected by the rotation and PC-current perturbations and plotted them in Figure 6 along with the profiles of unperturbed emission. We chose $r_0 = 0.05$, $f_0 = 1$, $\phi_P = 0^\circ$, $\sigma_\phi = 0.1$, and the rest of the parameters are the same as in Figure 5. We can see that due to perturbations both the emission and polarization are significantly affected in phase and magnitude. However, the maximum $L$ mostly remains unaffected. Pulsar rotation causes the intensity components to be shifted to earlier phases with respect to the fiducial plane, whereas the PPA inflection points to later phases for both signs of $\sigma$. On the other hand, PC-current causes the intensity components to shift to later phases and the PPA inflection point to earlier phases for positive $\sigma$ and vice versa for negative $\sigma$.

The net phase shift of the intensity component and PPA inflection point after combining the two perturbations are found to be $-2:16$ and $7:95$, respectively, in the case of $\sigma = 5^\circ$. They are found to be $-6:69$ and $8:90$ in the case of $\sigma = -5^\circ$. On the other hand, the sum of the phase shifts of the intensity components caused separately by the rotation and PC-current is found to be $-2:10$ and $-6:87$, respectively, in the cases of $\sigma = \pm 5^\circ$, whereas that of the PPA inflection point are found to be $6:44$ and $9:59$, respectively. Therefore, the absolute relative difference between the phase shift of the intensity component that resulted when the two perturbations are taken together and that due to the sum of two separate perturbations is about $<3\%$ in both cases of $\sigma$. For the PPA inflection point, this difference is found to be about $19\%$ in the case of $\sigma = 5^\circ$, and $8\%$ in the case of $\sigma = -5^\circ$. Note that the net phase shift of the intensity component becomes slightly larger than that of the net modulation $f$ (which is about $-1:92$ and $-5:71$, respectively, in the cases of $\sigma = \pm 5^\circ$) due to an induced asymmetry in the net radius of curvature at about the peak location of modulation. Also note that the above absolute relative differences of the phase shifts of the intensity components and the PPA inflection point become even more significant at higher altitudes ($r_n \sim 0.1$; see Figure 6.2 in the online journal).

Although circular polarization of opposite polarities from the background unmodulated emission roughly cancels out, a net circular with an asymmetry between the opposite polarities survives in the presence of modulation. This is because of an asymmetry in the magnitude of rotation of the emission pattern with respect to rotation phase in such a way that higher rotation magnitude toward the inner rotation phases as compared to that on outer phases (KG2012a). Hence, it results in the selective enhancement of the leading side circular over the trailing side circular. However, the asymmetry between the opposite polarities of the circular polarization becomes smaller for positive $\sigma$ than that for negative $\sigma$. This is due to the opposite behavior of the PC-current when introducing an asymmetry of the opposite polarities of $V$ between $\pm \sigma$ wherein it selectively enhances the trailing positive circular for $+\sigma$ and the leading positive circular for $-\sigma$. On the other hand, pulsar rotation causes the selective enhancement of the leading polarity of $V$, i.e., negative circular for $+\sigma$ and positive circular for $-\sigma$. Note that the circular polarization becomes the “symmetric” type for much broader modulation (and hence broader pulse width) due to increased asymmetry in the magnitude of rotation of the beaming region emission pattern between the inner and outer phases (see Figure 6.2 in the online journal).

3.2.2. Emission with Polar Modulation

By considering a modulation with plasma density gradient in the polar direction, we present the effects of rotation and PC-current perturbation on pulsar emission and polarization. In Figure 7, we present the simulation of hallow cone emission surrounding the magnetic axis with a modulation peak at $\theta_P = 2^\circ$. Since the minimum $\theta_0$ is about $\sim(2/3)\sigma$, which is about $3:3$ for $\sigma = \pm 5^\circ$, the observer’s sight line cuts the hollow cone emission only once in each pulsar rotation, and hence it results in a single component profile. For the sake of comparison, the cases before and after consideration of the perturbations are all shown in Figure 7, and the parameters’ normalization and the line representations are the same as in Figure 6. Note that, unlike the case of azimuthally modulated emission (see Figure 6), the maximum modulation strength that the observer encounters in the lab frame is not same in all the cases, as the minimum $\theta_0$ is slightly affected due to the perturbations. But, similar to the case of Figure 6, the emission is affected significantly before and after considering the perturbations due to the induced differences in $\rho$ at the peak locations of $f$. The net phase shifts of a component after combining the two perturbations in the cases $\sigma = \pm 5^\circ$ are found to be $-5:79$ and $-6:29$, respectively. On the other hand, the net phase shift obtained by adding the phase shifts due to the rotation and PC-current when considered separately in the cases $\sigma = \pm 5^\circ$ are found to be $-5:32$ and $-6:83$, respectively. Hence, the relative differences are about $8\%$ and $8.5\%$, respectively, with respect to the combined case of the perturbations. Note that these relative differences in the intensity phase shifts become larger at higher altitude ($\sim r_n = 0.1$).

The maximum normalized linear polarization $L$ is more or less the same in all the cases, similar to Figure 6. Although the position angle is significantly affected after combining the two perturbations, its inflection point lies at roughly the same phase as if due to rotation alone. The circular polarization $V$ becomes symmetric. In the case of positive $\sigma$, it is positive (dashed line) when only rotation is present on the other hand it is purely negative (dotted line) when PC-current alone present. This is due to emission patterns rotating in the opposite direction in the beaming region (see Figure 4). Since rotation dominates over PC-current, $V$ is positive in the combined case (continuous line). In the case of negative $\sigma$, $V$ becomes negative in all the cases, as the emission patterns rotate in the same direction. Hence, the net circular polarization after combining the two perturbations will also be symmetric with the survival of polarity similar to that due to the pulsar rotation alone.

By considering a hallow cone modulation with a peak at $\theta_P = 3^\circ/6$, we analyzed a case where the sight line cuts the hallow cone emission twice and it is presented in Figure 8.
Figure 6. Simulated pulse profiles with modulation in the azimuthal direction (the line representation is the same as in Figure 2). The total intensity $I$ is normalized with the maximum of $I$ of the combined case of rotation and PC-current, whereas $L$ and $V$ are normalized with the corresponding maximum of $I$ in their respective cases. We used the parameters $r_n = 0.05$, $f_\rho = 1$, $\phi_P = 0^\circ$, and $\sigma_\phi = 0.1$, and the rest of the parameters are the same as in Figure 5. (See the online version of the journal for Figures 6.1–6.3.)

(The complete figure set (3 images) is available in the online journal.)

The trailing side intensity component becomes weaker as well as narrower than that on the leading side due to the induced asymmetry in the curvature of source trajectories between the two sides. Since the sight line crosses the central maximum of the hollow cone on both the leading and trailing sides, there is a changeover of selective enhancement of emission over the part of the beaming region with smaller values of $\theta$ to that over the larger values of $\theta$ on the leading side, and vice versa on
the trailing side. Hence, there results an antisymmetric circular polarization with the sign reversal from positive to negative over the leading side component and vice versa on the trailing side for the case of positive $\sigma$. On the other hand, it is opposite for the case of negative $\sigma$ due to the opposite direction of rotation of the emission pattern of the beaming region.

3.2.3. Emission with Modulation in Both Azimuthal and Polar Directions

The radiation sources may be nonuniformly distributed in both the polar and azimuthal directions in the pulsar magnetosphere. The extreme cases, wherein modulation is
In Sections 3.2.1 and 3.2.2, respectively. In this section, we present a few more cases where modulation exists in both the polar and azimuthal directions. In Figure 6, we considered the cases \( \sigma_0 = 0.01 \) and \( \sigma_0 = 0.1 \), wherein the modulation effectively dominates in the azimuthal direction over that in the polar direction, and \( \sigma_0 = 0.001 \) and \( \sigma_0 = 0.5 \), wherein the modulation becomes effective in \( \theta \) due to the comparatively much larger coverage of \( \phi \); see, e.g., Figures 2 and 4. In the case of \( \sigma_0 = 0.01 \) and \( \sigma_0 = 0.1 \), the emission and polarization properties are similar to the case of \( \alpha = 10^\circ \) and \( \sigma = 5^\circ \) of Figure 6 with an antisymmetric circular polarization, wherein the sign reversal is from the negative polarity to the positive. On the other hand, even though the single modulation is considered in the case of \( \sigma_0 = 0.001 \) and \( \sigma_0 = 0.5 \), due to the viewing geometry, much elongation of the modulation in the azimuthal direction, and squeezing into a narrow cone, the modulation encountered by the sight line results in a blended two-component-like structure. It further results in a two-component-like structure in the intensity profile with, however, a much weaker trailing side due to the larger \( \rho \). Although circular polarization is still antisymmetric, the sign reversal becomes opposite to the case of \( \sigma_0 = 0.01 \) and \( \sigma_0 = 0.1 \), i.e., from positive to negative. Hence, in the presence of perturbations, it is difficult to see the correlation between the sign reversal of circular polarization and the PPA swing as both the types of circular sign reversal seem to be associated with increasing PPA swing. Note that the inflection point of the PPA swing is not derived in the case of \( \sigma_0 = 0.001 \) and \( \sigma_0 = 0.5 \) due to the difficulty in finding it because of the kinky nature in the PPA swing. Furthermore, note that similar behavior would be observed for the negative sight line (say \( \sigma = -5^\circ \), \( \phi_P = 180^\circ \)) except with the opposite sign reversal of the circular polarization in the respective cases and decreasing PPA swing (see Figure 10).

In Figure 11, we presented the simulations for different cases of two Gaussian modulations symmetrically located in a given ring centered on the magnetic axis. Although the asymmetry in \( \rho \) between the leading and trailing sides is the same in all the cases with much larger curvature on the leading side, there is a diverse asymmetry in the strength of the intensity component between the leading and trailing sides. This is due to an induced asymmetry in the strength of modulation that the inertial observer encounters between the leading and trailing sides. In the case of \( \theta_P = 4^\circ \) and \( \phi_P = \pm 15^\circ \), the observer encounters a much weaker modulation on the leading side than on the trailing side, which overcomes the influence of asymmetric \( \rho \). Hence, it results in a stronger trailing side component. Whereas in the case of \( \theta_P = 4^\circ \) and \( \phi_P = \pm 25^\circ \), the observer encountered asymmetry in the modulation strength between the leading and trailing sides weakens as compared to that in the cases of \( \theta_P = 4^\circ \) and \( \phi_P = \pm 15^\circ \). Hence, the leading side intensity component becomes stronger than the trailing one as the influence of asymmetry in \( \rho \) becomes much larger.
more important than that in $f$. In the case of $\theta_p = 4^\circ$ and $\phi_p = \pm 35^\circ$, the asymmetry in the modulation strength between the leading and trailing sides becomes even smaller, resulting mostly in the leading side component. In the case of $\theta_p = 4^\circ$ and $\phi_p = \pm 45^\circ$, the observer encounters a higher modulation strength on the leading side than on the trailing side, resulting in a single leading side component or a partial cone.

In all the cases, circular polarization is found to be symmetric over both the leading and trailing side components due to the selective enhancement of negative polarity caused by the rotation and PC-current perturbation. Note that in the case of $\theta_p = 4^\circ$ and $\phi_p = \pm 15^\circ$, for example, the emission from the region closer to the magnetic axis is selectively enhanced, and hence a selective enhancement of the inner circular occurs. Note that the polarity of symmetric-type circular polarization becomes positive over both the leading and trailing side components for the cases of negative $\sigma$ with the corresponding modulation parameters (see Figure 12). The kinks are introduced into the PPA swing due to the combined effect of modulation and perturbation caused by the rotation and PC-current.

4. DISCUSSION

The poloidal PC-current perturbs the underlying dipole field by inducing a toroidal magnetic field (Hibschman & Arons 2001; KG2012b), whereas the pulsar rotation causes the bending of the trajectory of the field line constrained plasma in the direction of pulsar rotation (e.g., BCW1991; Gangadhara 2005; Thomas & Gangadhara 2007; Dyks 2008; Dyks et al. 2010; Thomas et al. 2010; KG2012a; Wang et al. 2012). They significantly affect the phase locations of the intensity components as well as the PPA inflection point by introducing asymmetric phase shifts between them. Rotation (aberration) shifts intensity to the leading side and the PPA inflection point to the trailing side irrespective of the sign of $\sigma$ (e.g., BCW1991; Gangadhara 2005; Thomas & Gangadhara 2007; Dyks 2008; Dyks et al. 2010; Thomas et al. 2010; KG2012a; Wang et al. 2012). On the other hand, PC-current along with modulation can shift the intensity to the trailing side and the PPA inflection point to the leading side for positive $\sigma$, and vice versa for negative $\sigma$ (KG2012b). The influences of the rotation and PC-current add up for negative $\sigma$ but cancel each other when $\sigma$ is positive. Note that the opposite behavior of rotation and PC-current in the case of positive $\sigma$ is due to the opposite directions of induced curvature caused by the two perturbations.

Although the effects of rotation and PC-current perturbation can be understood qualitatively when they are combined together, their quantitative estimate becomes impossible when the two effects are considered separately, as shown in Sections 2 and 3. However, since the influence of the rotation is mostly larger than that of the PC-current, in the combined cases the effects of rotation mostly prevail with quite different magnitudes. For example, the phase delay between the PPA inflection
Figure 10. Same as Figure 9 except with $\sigma = -5^\circ$ and $\phi_P = 180^\circ$.

Figure 11. Same as Figure 9 but with two Gaussian modulations symmetrically located in a given conal ring at various azimuth. For simulation, we used the parameters $\sigma_\theta = 0.003$ and $\sigma_\phi = 0.1$, and the rest of the parameters are the same as in Figure 9. The dashed line curves in the first two columns of the upper panels represent an amplified $f$ to unity to identify with the corresponding intensity component.
point and the central component of pulse profile becomes much smaller for positive $\sigma$ as compared to that for negative $\sigma$ (see Figure 6). Hence, the above phase delays greatly depend on geometric parameters that have not been reported in the literature.

Even though the curvature of source trajectory on the leading side becomes larger than that on the trailing side due to the perturbation caused by the rotation and PC-current considered together, the net modulated intensity can become stronger either on the leading side or on the trailing side (see Figure 11) depending on the modulation location and the viewing geometry. Statistically, although there is observational support for the leading side component dominating over the trailing one, other contrary cases are also reported (Lyne & Manchester 1988). Hence, our model provides a more plausible explanation for the usual asymmetry between the leading and trailing side components. KG2012b, by considering the PC-current alone, also predicted the above behavior in the absence of a strong rotation effect. However, by considering the effect of pulsar rotation on the field line constrained plasma alone, BCW1991 predicted that the leading side intensity dominated over the trailing side, and later Thomas & Gangadhara (2007), Dyks (2008), Dyks et al. (2010), Thomas et al. (2010), KG2012a, and Wang et al. (2012) arrived at the same conclusion. Note that the "partial cone" pulsars, which show either missing or a much suppressed side in their conal-double component profiles (Lyne & Manchester 1988), can result due to the rotation and PC-current perturbation (see Figure 11).

Furthermore, the rotation and PC-current perturbation introduce an asymmetry into the strengths of opposite polarities of the circular polarization. When they are considered separately, they selectively enhance either the negative or positive polarity. Furthermore, the two effects cause the emission pattern to rotate in the $(\theta, \phi)$ plane in opposite directions for positive $\sigma$ and in the same direction for negative $\sigma$. Hence, it results in a smaller rotation of the emission pattern in the $(\theta, \phi)$ plane for the positive $\sigma$ case than that for negative $\sigma$. However, due to the rotation of the emission pattern, the symmetric-type circular polarization becomes evident in addition to the more common antisymmetric type with the usual asymmetry between the opposite polarities in the presence of nonuniform distribution of sources (see also KG2012a, KG2012b). Due to the perturbations, we find the sign reversal of circular polarization from negative to positive near the center of the pulse profile can be associated with either the increasing or decreasing PPA swing and vice versa (see Figures 9 and 10). These deductions are found to be in accordance with the Han et al. (1998) and You & Han (2006) observational findings.

Our simulations (see Figures 11 and 12) also confirm the Han et al. (1998) and You & Han (2006) findings that the negative polarity of the circular polarization is associated with increasing PPA, and vice versa for positive polarity, on both the sides of conal-double pulsar profiles. This is possible due to the combined rotation and PC-current perturbation under specific conditions when the modulation is more effective in the polar direction than in the azimuthal direction. However, KG2012a claimed that such correlation can exist provided that, in addition to the effective modulation in the polar direction, the modulations’ location must be asymmetric in a given conal ring centered on the magnetic axis.

Some pulsars, particularly millisecond pulsars, show the polarization angle behavior that deviates from the standard...
“S” curve (Xilouris et al. 1998). We find that the “kinky”-type distortions in PPA profile are due to the superposition of modulated incoherent profile over the regions that are larger than the wavelength of radiation in the presence of strong rotation and PC-current perturbations. Note that Gangadhara (2010) and KG2012a also obtained similar results. However, the results of Gangadhara (2010) were in the absence of any perturbation, and hence the PPA distortions from the standard “S” curve are smaller. Mitra & Seiradakis (2004) speculated that if the radio emission has a varying emission height across the pulse profile then the PPA swing will be nonuniform. Also, in the literature the kinky-type origin has been attributed to the multipolar magnetic field in the radio emission region (Mitra et al. 2000) and to the return currents in the pulsar magnetosphere (Ramachandran & Kramer 2003).

Note that we performed modeling of coherent pulsar radio emissions in terms of single particle curvature radiation. Although to the first order it is not a bad assumption, in reality the efficiency of the coherence process may depend on many factors, and hence there may be induced asymmetries in the pulsar emission and polarization between the leading and trailing sides. For example, the coherent emitters may be brighter on the one side of the profile than on the other side depending on the factors influencing the coherence process, and this can act in the same direction or in the opposite direction of the processes that we have considered in this paper.

Also note that the retardation simply makes the entire profile shift to an earlier phase without affecting the shape of the pulse profile, unless the received emission originated from a varying altitude with respect to the rotation phase (KG2012a). For simplicity, we assumed a constant emission altitude across the whole pulse and did not present the pulse simulations, including retardation effect, in this work. Furthermore, the current scale factor $\xi$ is taken to be unity, but it can be a bit different and may even have altitude dependency, which can slightly modify the results. We believe that our model is much closer to the realistic one, and it can explain most of the diverse behaviors of pulsar emission and polarization.

5. CONCLUSION

We believe that our model is much more realistic for pulsar emission and polarization than those proposed previously, as for the first time it simultaneously takes into account the perturbation caused jointly by the rotation and the PC-current. In addition, it considers the detailed study on nonuniform source distribution (modulation) and the influence of viewing geometry on pulsar emission. Hence, we believe that one can explain most of the observed emission and polarization properties of pulsars within the framework of curvature radiation. On the basis of our simulation of pulse profiles, we draw the following conclusions:

1. The effect of rotation and PC-current perturbation in the presence of nonuniform source distribution (modulation), along with viewing geometry, might be responsible for most of the observed diverse behavior of the polarization properties of pulsar radio emission.

2. Because of the perturbations, there arises an asymmetry in the phase shift of the intensity components and PPA inflection point for $\pm \sigma$. Furthermore, we note that the phase delay of the PPA inflection point with respect to that of the central component (core) becomes larger for negative $\sigma$ than for positive $\sigma$.

3. The leading side components can become either stronger or weaker than the corresponding trailing side components of a given cone. This is due to the induced asymmetry in the curvature of the source trajectory and the sight line encountered asymmetry in the modulation strength between the two sides.

4. Both the “antisymmetric” and “symmetric” types of circular polarization are possible within the framework of curvature radiation when the perturbation and modulation are operative.

5. In the presence of perturbation, the sign reversal of the “antisymmetric”-type circular polarization as well as the sign of the “symmetric”-type circular polarization do not show correlation with the sign of the PPA swing in the case of central core-dominated pulsars.

6. The “kinky”-type distortions in the PPA swing could be due to the incoherent addition of modulated emission in the presence of strong perturbations.

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