CALLING A SPADE A SPADE:
MATHEMATICS IN THE NEW PATTERN
OF DIVISION OF LABOUR

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The man who could call a spade a spade
should be compelled to use one.
It is the only thing he is fit for.
—Oscar Wilde

INTRODUCTION

The growing disconnection of the majority of population from mathematics is becoming
a phenomenon that is increasingly difficult to ignore. It is reflected in the mass culture
and has become part of Zeitgeist—and evidenced, in detail, in talks [17, 22, 23] at the
Mathematical Cultures meetings.

This paper attempts to point to deeper roots of this cultural and social phenomenon. It
concentrates on mathematics education, as the most important and better documented area
of interaction of mathematics with the rest of human culture.

I argue that new patterns of division of labour have dramatically changed the nature and
role of mathematical skills needed for the labour force and correspondingly changed the
place of mathematics in popular culture and in the mainstream education. The forces that
drive these changes come from the tension between the ever deepening specialisation of
labour and ever increasing length of specialised training required for jobs at the increas-
ingly sharp cutting edge of technology.

Unfortunately these deeper socio-economic origins of the current systemic crisis of
mathematics education are not clearly spelt out, neither in cultural studies nor, even more
worryingly, in the education policy discourse; at the best, they are only euphemistically
hinted at.

This paper is an attempt to describe the socio-economic landscape of mathematics ed-
ucation without resorting to euphemisms. This task imposes on the author certain restric-
tions: he cannot take sides in the debate and therefore has to refrain from giving any
practical recommendations. Also it makes necessary a very clear disclaimer:

The author writes in his personal capacity. The views expressed do not neces-
sarily represent position of his employer or any other person, organisation, or
institution.

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THE NEW DIVISION OF LABOUR

It’s the economy, stupid.
James Carville

It is time to recognise that discussion of mathematics education takes place in a socio-economic landscape which has never before existed in the history of humanity.

This, largely unacknowledged, change, can be best explained by invoking Adam Smith’s famous words displayed on the British £20 banknote, Figure 1:

The words on the banknote:

“The division of labour in pin manufacturing (and the great increase in the quantity of work that results)”

are, of course, a quote from Adam Smith’s The Wealth of Nations. They are found on the very first page of Chapter I of Book I with the now famous title Of The Division of Labour:

“One man draws out the wire; another straights it; a third cuts it; a fourth points it; a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on is a peculiar business; to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations.”

And Adam Smith comes to the conclusion:

Separation of the pin production process into 18 operations increases the productivity by factor of 240.

By the end of the 20th century, the ever deepening division of labour has reached a unique point in history of humankind when 99% of people have not even the vaguest

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1It’s the economy, stupid. According to Wikipedia, this phrase, frequently attributed to Bill Clinton, was made popular by James Carville, the strategist of Clinton’s successful 1992 presidential campaign against George H. W. Bush.
idea about the workings of 99% of technology in their immediate surrounding. This is especially true in respect of mathematics.

Every time you listen to iPod or download a compressed graphic file from the Internet, extremely sophisticated mathematical algorithms come into play. A smartphone user never notices this because these algorithms are encoded deep inside the executable files of smartphone apps. Nowadays mathematics (including many traditional areas of abstract pure mathematics, such as number theory, abstract algebra, combinatorics, spectral analysis, to name a few) is used in our everyday life thousands, may be millions, times more intensively than 50 or even 10 years ago. Mathematical results and concepts involved in practical applications are much deeper and more abstract and difficult than ever before. One of the paradoxes of modern times is that this makes mathematics invisible because it is carefully hidden behind a user friendly smartphone interface.

There are more mobile phones in the world now than toothbrushes. But the mathematics built into in a mobile phone or a MP3 player is beyond understanding of most graduates from mathematics departments of British universities. However, practical necessity forces us to teach a rudimentary MP3 technology, in cookbook form, to electronic engineering students; its mathematical content is diluted or even completely erased. This is even more true of MP4.

In the emerging division of intellectual labour, mathematics is a 21st century equivalent of sharpening a pin.

The only catch is that a pin-sharpen of Adam Smith’s times could be trained on the job in a day. Training of a mathematically competent worker for high tech industries requires at least 15 years of education from ages 5 to 20.

*It is this tension between the ever increasing degree of specialisation and the ever increasing length of specialised training that lies at the heart of the matter.*

**Politics and economics**

The issue of new patterns of division of labour starts to emerge in political discourse. I give here some examples.

The book by Frank Levy and Richard Murnane *The New Division of Labor* [[19]], published in 2004 and based on the material from the USA, focuses on economic issues viewed from a business-centered viewpoint. Here is a characteristic quote:

In economic terms, improved education is required to restore the labor market to balance. […] the falling wages of lower skilled jobs reflect the fact that demand was not keeping with supply. If our predictions are right, this trend will continue as blue-collar and clerical jobs continue to disappear.

Better education is an imperfect tool for this problem. The job market is changing fast and improving education is a slow and difficult process. [[19] p. 155].

Elizabeth Truss, Member of Parliament for Conservative Party (who is now an Under-secretary of State in the Department for Education), not long ago published a report [[25]] where she addressed the issue of “hourglass economy” in the context of education policy (see also [[26]])

The evidence suggests increased polarisation between high skilled and unskilled jobs, with skilled trades and clerical roles diminishing. Long standing industries are becoming automated, while newly emerging industries demand high skills. Formal and general qualifications are the main route into these jobs. At the top level MBAs and international experience is the new benchmark. Despite popular
perception, the middle is gradually disappearing to create an ‘hourglass economy’.

In the next section, we shall return to “hourglass economy” and the “hourglass” shape of demand for mathematics education of different levels of students’ attainment. Meanwhile, I refer the reader to the views of numerous economists concerning “job polarisation” (Autor [5], Goos et al. [15]), “shrinking middle” (Abel and Deitz [1]), “intermediate occupations” and “hourglass economy” (Anderson [4]). The same sentiments about the “disappearing middle” are repeated in more recent books under catchy titles such as Tyler Cowen’s The Average is Over [12]; they are obviously becoming part of Zeitgeist. Although their book is optimistic, Brynjolfsson and McAfee [9] emphasise the way in which the application of the knowhow in the upper half of the hourglass causes the hollowing out of the need for mathematical capability in the “neck”.

It is instructive to compare opinions on job polarisation and its impact on education coming from the opposite ends of political spectrum.

Judging by his recent book [16], Alan Greenspan appears to care only about the top part of the hourglass:

> The skill structure of the workforce overall does not match the needs implied by the complexity of our capital infrastructure, most specially in areas of high-tech industry.

A voice from the left (Elliot [13]), on the contrary, accuses the masters of the universe of intentionally dumbing down education:

> We need, I should say, to look for an analysis in the direction of global developments in the capitalist labour process—especially the fragmentation of tasks, the externalization of knowledge (out of human heads, into computer systems, administrative systems and the like)—and the consequent declining need, among most of the population, regarded as employees or workers, for the kinds of skills (language skills, mathematical skills, problem-solving skills etc.) which used to be common in the working class, let alone the middle classes. This analysis applies to universities and their students. Dumbing-down is a rational—from the capitalist point of view—reaction to these labour-process developments. No executive committee of the ruling class spends cash on a production process (the production of students-with-a-diploma) that, from its point of view, is providing luxury quality. It will continuously cut that quality to the necessary bone. It is doing so. This, to repeat the point, is a global tendency rooted in the reality of capitalist production relations.

But Greenspan appears to take a relaxed, some may say “devil-take-the-hindmost” view of “dumbing-down” in education; his words deserve to be put in boldface:

> While there is an upside limit to the average intellectual capabilities of population, there is no upper limit to the complexity of technology.

Many (including me) may disagree with this extreme claim—but it has a chance to influence political and business decisions.

**Implications for mathematics education**

We have to realise that it is no longer an issue whether the role of mathematics in society is changing: the change is being ruthlessly forced on us by Adam Smith’s ‘invisible hand’.

In particular, changing economic imperatives lead to the collapse of the traditional pyramid of mathematics education. Let us look at the diagram in Figure [2]
The diagram is not made to any scale and should be treated qualitatively, not quantitatively. The left hand side of the pyramid suggests how the distribution of mathematical attainment looked in the mid 20th century, with pupils / students / graduate students at every level of education being selected from a larger pool of students at the previous level. In the not so distant past, every stage in mathematics education matched the economic demand for workers with a corresponding level of skills. From students’ point of view, every year invested in mathematics education was bringing them a potential (and immediately cashable) financial return.

The traditional pyramid of mathematics education was stable because every level had its own economic justification and employment opportunities. I have included as Appendix the examination paper Post Office Entrance Examination from 1897 which is being circulated among British mathematicians as a kind of subversive leaflet. A century ago, good skills in practical arithmetic were opening up employment opportunities for those in the reasonably wide band of the diagram on the left, the one which has now become the bottleneck of the ‘hourglass’ on the right. Nowadays this level of skills is economically redundant; its only purpose is to serve as an indication of, and a basis for, a person’s progress to higher, more economically viable, levels of mathematics education.

The right hand side of the pyramid suggests what we should expect in the future: an hourglass shape, with intermediate levels eroded. Certain levels of mathematics education are not supported by immediate economic demand and serve only as an intermediate or preparatory step for further study. From an individual’s point of view, the economic return on investment in mathematical competence is both delayed and less certain. Once this is realised, it seems likely to weaken the economic motivation for further study.

The cumulative nature of learning mathematics makes a “top-heavy” model of education unsustainable: what is the motivation for students to struggle through the neck of the hourglass? Whether they realise it or not children and their families subconsciously apply a discounted cash flow analysis to the required intellectual effort and investment of time as compared to the subsequent reward. Elizabeth Truss [27] proposes a “supply-side reform” of education and skills training as a solution to the hourglass crisis. But supply-side stimuli work best for large scale manufacturers and suppliers. In mathematics education, the key links in the supply chain are children themselves and their families; in the global “knowledge economy” too many of them occupy a niche at best similar to that of subsistence farmers in global food production, at worst similar to that of refugees living on food donations. And supply-side economics does not work for subsistence farmers, who need demand for their work and their products, and demand with payment in advance, not
in 20 years. Mathematics education has a 20 years long production cycle, which makes supply-side stimuli meaningless.

An additional pressure on mathematics education in the West is created by division of labour at an international level: in low wage economies of countries like India, learning mathematics still produces economic returns for learners that are sufficiently high in relation to meagre background wages and therefore stimulate ardent learning. As a result, the West is loosing ability to produce competitively trained workers for mathematically intensive industries.

Should we be surprised that the pyramid of mathematics education is no longer a pyramid and collapses?

**The neck of the hourglass**

The mathematical content of the neck can be described in educationalist terminology used in England as Key Stage 3 and Key Stage 4 mathematics:

Key Stage 3 mathematics teaching [...] marks a transition from the more informal approach in primary schools to the formal, more abstract mathematics of Key Stage 4 and beyond. [14, p. 6]

It is informal concrete mathematics and more abstract formal mathematics that make the two bulbs of the hourglass.

Why do we need abstract mathematics? A highly simplified explanation might begin with the fact that money, as functions in the modern electronic world, is a mathematical abstraction, and this abstraction rules the world.

Of course, this always was the case. However, in 1897 competent handling of money required little beyond arithmetic and use of tables of compound interest, and clerks at the Post Office were supposed to be mathematically competent for everyday retail finance, see Questions 7 and 8 in the Appendix. Nowadays, the mathematical machinery of finance includes stochastic analysis, among other things. Worse, the mathematics behind information technology that supports financial transactions is also very abstract.

Let us slightly scratch the touchscreen of a smartphone or tablet and look at what is hiding below the ordinary spreadsheet.

I prepared the following example for my response to a report from ACME Mathematical Needs: Mathematics in the workplace and in Higher Education [3]. The report provides the following case study as an important example of use of mathematics.

**6.1.4 Case study: Modelling the cost of a sandwich**

The food operations controller of a catering company that supplies sandwiches and lunches both through mobile vans and as special orders for external customers has developed a spreadsheet that enables the cost of sandwiches and similar items to be calculated. [...] This task would not be too challenging to “women and girl clerks” of 1897, and dealt with by ordinary arithmetic—with the important exception of the “development of a spreadsheet”. Let us look at it in more detail.

Anyone who ever worked with a spreadsheet of the complexity that required for the steps involved in producing sandwiches should know that the key mathematical skill needed is an awareness of the role of brackets in arithmetical expressions and an intuitive feel of how the brackets are manipulated, something that is sometimes called “structural arithmetic” [14] or “pre-algebra”. At a slightly more advanced level working with spreadsheets

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2I used this example in my paper [7].
To illustrate this point, I prepared a very simple spreadsheet in OpenOffice.org Calc (it uses essentially the same interface as Microsoft Excel).

### Figure 3.

|   | A         | B         | C         | D         | E         |
|---|-----------|-----------|-----------|-----------|-----------|
| 1 | “Modelling” the cost of a sandwich |                                           |           |           |           |
| 2 | Costs are in pence                  |           |           |           |           |
| 6 | Ingredient                          | Single Decker | Double Decker |
| 8 | Bun                                 | 20         | 20        |           |           |
| 9 | Burger                              | 35         | 70        |           |           |
|10 | Cheese                              | 15         | 15        |           |           |
|11 | Onion                               | 5          | 10        |           |           |
|12 | Mustard                             | 5          | 10        |           |           |
|13 | Lettice                             | 5          | 5         |           |           |
|14 | Total Cost                          | **85**     |           |           |           |

### Figure 4.

|   | A         | B         | C         | D         | E         |
|---|-----------|-----------|-----------|-----------|-----------|
| 1 | “Modelling” the cost of a sandwich |                                           |           |           |           |
| 2 | Costs are in pence                  |           |           |           |           |
| 6 | Ingredient                          | Single Decker | Double Decker |
| 8 | Bun                                 | 20         | 20        |           |           |
| 9 | Burger                              | 35         | 70        |           |           |
|10 | Cheese                              | 15         | 15        |           |           |
|11 | Onion                               | 5          | 10        |           |           |
|12 | Mustard                             | 5          | 10        |           |           |
|13 | Lettice                             | 5          | 5         |           |           |
|14 | Total Cost                          | **85**     | **130**   |           |           |

requires understanding of the concept of functional dependency in its algebraic aspects (frequently ignored in pre-calculus).
Look at Figure 3 if the content of cell C14 is \( \text{SUM}(C8:C13) \) and you copy cell C14 into cell D14 (see Figure 4), the content of cell D14 becomes \( \text{SUM}(D8:D13) \) and thus involves change of variables. What is copied is a structure of an algebraic expression, not even an algebraic expression itself. And of course this is not copying of the value of this expression: please notice that the value 85 becomes 130 when moved from cell C14 to cell D14!

Intuitive understanding that \( \text{SUM}(C8:C13) \) is in a sense the same as \( \text{SUM}(D8:D13) \) is best achieved by exposing a student to a variety of algebraic problems which convince him/her that a polynomial of kind \( x^2 + 2x + 1 \) is, from an algebraic point of view, the same as \( z^2 + 2z + 1 \), and that in a similar vein, the sum

\[
C8 + C9 + C10 + C11 + C12 + C13
\]

is in some sense the same as

\[
D8 + D9 + D10 + D11 + D12 + D13.
\]

However the computer programmer (the one who does not merely use spreadsheets, but who writes background code for them), needs a understanding of what it means for two expressions to be “the same”. Experience suggests rather clearly that majority of graduates from mathematics departments of British universities, as well as majority of British school mathematics teachers do not possess language that allows them to define what it means that two expressions in a computer code involving different symbols (and, frequently, different operations) are “actually the same”.

This is a general rule: when a certain previously “manual” mathematical procedure is replaced by software, design and coding of this software requires a much higher level of mathematical skills than those needed for the procedure which has been replaced—but from a much smaller group of workers.

LONG DIVISION

For simplistic discussions in the media, the neck of the hourglass can be summarised in just two words: long division.

One of my colleagues who read an early draft of this paper wrote to me:

“I would not touch long division, as an example, with a ten-foot pole, because it leads to wars.”

But I am touching it exactly because it leads to wars—to the degree that the words “long division” are used as a symbol for the socio-economic split in English education.

Why is long division so divisive? Because it is remarkably useless in the everyday life for 99% of people. We have to accept that the majority of the population do not need “practical” mathematics beyond the use of a calculator, and from the “practical” point of view long division can follow slide rules and logarithm tables into the dustbin of history. But why are long multiplication and long division so critical for squeezing the learners through the hourglass neck? Because many mathematicians and mathematics educators believe that these “formal written methods” should be introduced at a relatively early stage not because of their “real life relevance'' but with the aim to facilitate children’s deep interiorisation of the crucially important class of recursive algorithms which will make the

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3I heard claims that fractions have to be excluded from school curriculum for the same reason: only a small minority of school students will ever need them in real life.

“Who of the colleagues present here have lately had to add \( \frac{2}{3} \) and \( \frac{3}{7} \)?”

—this question was asked at one of the recent meetings of experts in mathematics education.
basis of children’s later understanding of polynomial algebra—and, at later stages, the “semi-numerical” algorithms, in terminology of the great Donald Knuth [18]. However there is nothing exceptional in long division, many other algorithms can play in mathematics education the same propaedeutic role, and all of them could be similarly dismissed as not having any “real life relevance” because they are needed only by a relatively narrow band of students, those who are expected to continue to learn mathematics up to more advanced stages and work in mathematics-intensive industries. In short, “long division” is an exemplification of what I later in this paper call “deep mathematics education”.

The left-wing camp in education draw a natural conclusion: long division is hard, its teaching is time- and labour-consuming and therefore expensive, and it will eventually be useful only for a small group of high-flyers—so why bother to teach it?

This is indeed the core question:

**Does the nation have to invest human and financial resources into pushing everyone through the hourglass neck? Or should it take a conscious effort to improve the quality of deep mathematics teaching for a limited number of students?**

This is the old conundrum of the British system of education. A recent BBC programme [6] has revealed that Prince Charles in the past lobbied for more (academically selective) grammar schools. Former Education Secretary (Labour) David Blunkett told about his exchanges with Prince Charles:

I would explain that our policy was not to expand grammar schools, and he didn’t like that.

He was very keen that we should go back to a different era where youngsters had what he would have seen as the opportunity to escape from their background, whereas I wanted to change their background. (The emphasis is mine—AB.)

This is a brilliant formulation of the dilemma, and it is especially good in the case of mathematics education because the hourglass shape of economic demand for different levels of mathematics education puts the emotive word “escape” on a solid economic foundation: it is the escape through the hourglass neck.

While I would be delighted, and relieved, to be convinced by arguments to the contrary, at this point I can see the solutions offered by the Left and the Right of British education politics as deficient in ways that mirror each other:

- The Left appear to claim that it is possible to have quality mathematics education for everyone. While their position is sincerely held, still, as I see it, it leads to inconsistencies which can be avoided only by lowering the benchmark of “quality” and ignoring the simple economic fact that what they call “quality education” is needed neither by learners, nor by their future employers.

- The Right appear to claim that administrative enforcement of standards will automatically raise the quality of education for everyone. It is also a sincerely held position, but, as I see it, it leads to inconsistencies which can be avoided only by preparing escape routes for their potential voters’ children in the form of “free schools”.

My previous analysis has not made any distinction between “state” and “private” schools; this reflects my position—I do not believe that mainstream private schools, or “free schools” (even if they are privatised in the future) make any difference in the systemic crisis of mathematics education.
Back to Zunft?

At this point we need to take a closer look at division of labour. Braverman [8] emphasises distinction between social division of labour between different occupational strata of society and detailed division of labour between individual workers in the workplace.

The division of labor in society is characteristic of all known societies; the division of labor in the workshop is the special product of capitalist society. The social division of labor divides society among occupations, each adequate to a branch of production; the detailed division of labor destroys occupations considered in this sense, and renders the worker inadequate to carry through any complete production process. In capitalism, the social division of labor is enforced chaotically and anarchically by the market, while the workshop division of labor is imposed by planning and control. [8, pp. 50–51]

It is the emerging new workplace division of labour that makes mathematics “on average” redundant, while “chaotic and anarchic” social division exacerbate singularities in supply of mathematically skilled workers and paralyses policy decisions in education. Let us return for a second to my Adam Smith simile:

A pin-sharpener of Adam Smith’s times could be trained on the job in a day.
Training of a mathematically competent worker for high tech industries requires at least 15 years of education from ages 5 to 20.

In relation to mathematics, social factors and, consequently, social division of labour attain increasing importance for a simple reason: who but families are prepared to invest 15 years into something so increasingly specialised as mathematics education?

And, by the way, what instructional system was in place before the division-of-labor sweatshops glorified by Adam Smith? The Zunft system. In German, Zunft is a historic term for a guild of master craftsmen (as opposed to trade guilds). The high level of specialisation of Zunft could be sustained only by hereditary membership and training of craftsmen, from an early age, often in a family setting. It is hard not to notice a certain historical irony…

The changing patterns of division of labour affect mathematics education in every country in the world. But reactions of the government, of the education community, of parents from different social strata depend on political and economic environment of every specific country. So far I analysed consequences for education policy in England; when looking overseas and beyond the anglophone world, one of more interesting trends is mathematics homeschooling and “math circles” movements in two so different countries as the USA and Russia. In both countries mathematically educated sections of middle class are losing the confidence in their governments’ education policies and in the competence of the mathematics education establishment.

Statistics of mathematics homeschooling are elusive, but what is obvious is the highest quality of intellectual effort invested in the movement by its leading activists—just have a look at books [10, 21, 29]. At the didactical level, many inventions of mathematics homeschoolers are wonderful but intrinsically unscalable and cannot be transported into the existing system of mass education. I would say that their approach is not a remedy for the maladies of mainstream education; on the contrary, the very existence of mathematics homeschoolers is a symptom of, and a basis for a not very optimistic prognosis for, the state of mass mathematics education.

Still, in my opinion, no-one in the West has captured the essence of deep mathematics education better then they have.
At the didactic level, bypassing the hourglass neck of economic demand for mathematics means development of deep mathematics education. I would define it as Mathematics education in which every stage, starting from pre-school, is designed to fit the individual cognitive profile of the child and to serve as propaedeutics of his/her deep study of mathematics at later stages of education.

To meet these aims, “deep” mathematics education should should unavoidably be joint-up and cohesive.4

To give a small example in addition to the already discussed long division, I use another stumbling block of the English National Curriculum: times tables. The following is a statutory requirement:

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work. [20]

This requirement is much criticised for being archaic (indeed, why 12?), cruel and unnecessary. But to pass through the neck of the hourglass, children should know by heart times tables up to 9 by 9; even more, it is very desirable that they know by heart square numbers up to $20^2 = 400$, because understanding and “intuitive feel” of behavior of quadratic functions is critically important for learning algebra and elementary calculus.

The concept of “deep mathematics education” is not my invention. I borrowed the words from Maria Droujkova, one of the leaders of mathematics homeschooling. Her understanding of this term is, first of all, deeply human and holistic.

In her own words5

The math we do is defined by freedom and making. We value mastery - with the understanding that different people will choose to reach different levels of it. The stances of freedom and making are in the company’s motto:

Make math your own, to make your own math.

When I use the word “deep” as applied to mathematics education, I approach it from that natural math angle. It means deep agency and autonomy of all participants, leading to deep personal and communal meaning and significance; as a corollary, deep individualization of every person’s path; and deep psychological and technological tools to support these paths.

Droujkova uses, as an example, iterative algorithms, and her approach to this concept is highly relevant for the discussion of the propaedeutic role of “long division”:

From the time they are toddlers, children play with recursion and iteration, in the contexts where they can define their own iterating actions. For example, children design input-output “function machines” and connect the output back to the input. Or experiment with iterative splitting, folding, doubling, cutting with paper, modeling clay, or virtual models. Or come up with substitution and tree fractals, building several levels of the structure by iterating an element and a transformation. Grown-ups help children notice the commonalities between these different activities, help children develop the vocabulary of recursive and iterative algorithms, and support noticing, tweaking, remixing, and formulating of particular

4The Moscow Center for Continuous Mathematics Education, http://www.mccme.ru/index-el.html emphasise this aspect by putting the word “continuous” into their name; they have obvious focus on bridging the gap between school and university level mathematics, while homeschoolers tend to start at the pre-school stage.

5Private communication.
properties and patterns. As children mature, their focus shifts from making and remixing individual algorithms to purposeful creation and meta-analysis of patterns. For example, at that level children can compare and contrast recursion and iteration, or analyze information-processing aspects of why people find recursive structures beautiful, or research optimization of a class of recursive algorithms.

Maria Droujkova describes a rich and exciting learning activity. But it would be impossible without full and informed support from children’s families. To bring this education programme to life, you need a community of likely-minded and well-educated parents. It could form around their children’s school (and would almost inevitably attempt to control the school), or around a “mathematical circle”, informal and invisible to educational establishment and therefore free from administrative interference, or, which is much more likely in our information technology age, grow as an Internet-based network of local circles connected by efficient communications tools—and perhaps helped by parents’ networking in their professional spheres. These “communities of practice”, as Droujkova calls them using a term coined by Wenger [28], are Zünfte at the new turn of history’s spiral. I see nothing that makes it unfeasible.

I wish mathematics homeschoolers all the luck. But their work is not a recipe for mainstream education.

**Conclusion**

*I came here knowing we have some sickness in our system of education; what I have learned is that we have a cancer!*  
Richard Feynman, *Surely You’re Joking, Mr. Feynman!*

In this paper, I have attempted to describe how deepening specialisation and division of labour in the economy affects the mathematics education system, changes its shape, undermines its stability, and (at least in England) provokes political infighting.

I wish to reiterate that I am not taking sides in these fights. I do not wish to lay blame on anyone, or criticise anyone’s views. My paper is a call for a sober, calm, and apolitical discussion of the socio-economic roots of the current crisis in mathematics education.

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1. Simplify
\[ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} \]
\[ \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} \]
\[ \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} \]
\[ \frac{1}{4} - \frac{1}{5} - \frac{1}{6} - \frac{1}{7} \]
\[ \frac{1}{4} + \frac{1}{5} \]
\[ \frac{1}{4} - \frac{1}{5} \]
\[ \frac{1}{6} + \frac{1}{7} \]
\[ \frac{1}{6} - \frac{1}{7} \]
\[ \frac{1}{102} \]
\[ \frac{1}{135} \]

2. If 725 tons 11 cwt. 3 qrs. 17 lbs. of potatoes cost £3386, 2s. 2½d. how much will 25 tons 11 cwt. 3 qrs. 17 lbs. costs (sic)?

3. Extract the square root of 331930385956.

4. A purse contains 43 foreign coins, the value of each of which either exceeds or falls short of one crown by the same integral number of pence. If the whole contents of the purse are worth £10, 14s. 7d., find the value and number of each kind of coin. Show that there are two solutions.

5. Explain on what principle you determine the order of the operations in
\[ \frac{1}{2} + \frac{3}{4} - \frac{5}{6} - \frac{7}{8} \times \frac{9}{10} \]
and express the value as a decimal fraction. Insert the brackets necessary to make the expression mean :-
Add \( \frac{1}{2} \) to \( \frac{3}{4} \), divide the sum by \( \frac{5}{6} \), from the quotient subtract \( \frac{7}{8} \), and multiply this difference by \( \frac{9}{10} \).

6. Show that the more figures 2 there are in the fraction 0.222...2, the nearer its value is to \( \frac{2}{5} \). Find the difference in value when there are ten 2s.

7. I purchased £600 worth of Indian 3 per cent. stock at 120. How much Canadian 5 per cent. stock at 150 must I purchase in order to gain an average interest of 3 per cent. on the two investments?

8. If five men complete all but 156 yards of a certain railway embankment, and seven men could complete all but 50 yards of the same embankment at the same time, find the length of the embankment.

9. Find, to the nearest day, how long £390, 17s. 1d. will take to amount to £405, 14s. 3d. at 3 \( \frac{1}{4} \) per cent. per annum (365 days) simple interest.

10. A certain Irish village which once contained 230 inhabitants, has since lost by emigration three-fourths of its agricultural population and also five other inhabitants. If the agricultural population is now as numerous as the rest, find how the population was originally divided.