A mechanism to generate mass: the case of fermions

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In a recent paper we have presented a mechanism to generate mass from gravitational interaction, based on the Mach principle, according to which the inertia of a body is a property of matter as well as of the background provided by the rest-of-the-universe. In we realized such an idea for a scalar field treating the rest-of-the-universe in its vacuum state. In the present paper we describe a similar mechanism for fermions.

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I. INTRODUCTION

In the realm of high-energy physics, the Higgs model produced an efficient scenario for generating mass to the vector bosons.

At its origin appears a process relating the transformation of a global symmetry into a local one and the corresponding presence of vector gauge fields.

This mechanism appeals to the intervention of a scalar field that appears as the vehicle which provides mass to the gauge vector field $W_{\mu}$. For the mass to be a real and constant value (a different value for each field) the scalar field $\varphi$ must be in a minimum state of its potential. This fundamental state of the self-interacting scalar field has an energy distribution given by $T_{\mu\nu} = V(\varphi_0) g_{\mu\nu}$. A particular form of self-interaction of the scalar field $\varphi$ allows the existence of a constant value $V(\varphi_0)$ that is directly related to the mass of $W_{\mu}$. This scalar field has its own mass, the origin of which rests unclear. In we presented a new mechanism depending on the gravitational interaction, that can provides mass to the scalar field. In the present paper we extend this strategy to the case of fermions.

Although the concept of mass pervades most of all analysis involving gravitational interaction, it is an uncomfortable situation that still to this day there has been no successful attempt to derive a mechanism by means of which mass appears as a direct consequence of a dynamical process depending on gravity.

The main idea concerning inertia in the realm of gravity according to the origins of General Relativity, goes in the opposite direction: while the Higgs mechanism explores the reduction of a global symmetry into a local one, the Mach principle suggests a cosmical dependence of local properties, making the origin of the mass of a given body to depend on the structure of the whole universe. In this way, there ought to exist a mechanism by means of which this quantity - the mass - depends on the state of the universe. The purpose of this paper is to exhibit an example of such mechanism.

We start by considering Mach principle as the statement according to which the inertia of a body $A$ are determined by the energy-momentum throughout all space. How could we describe such universal state that takes into account the whole contribution of the rest-of-the-universe onto $A$? There is no simpler way than to consider this state as the most homogeneous one and relate it to what Einstein attributed to the cosmological constant or, in modern language, the vacuum of all remaining bodies. This means to describe the energy-momentum distribution of all complementary bodies of $A$ in the Universe as

$$T_{\mu\nu}^U = \Lambda g_{\mu\nu}$$

This article is written in the following stages: in the next section I will do a revision of the mechanism in the case of matter being represented by a scalar field. We shall see how the existence of an action by the rest-of-the-universe on a massless field may provide it with a mass. In the following section we will turn our attention to the case of a massless spinor field. We end with some comments on this strategy and further consequences.

II. THE CASE OF SCALAR FIELD

Let $\varphi$ be a massless field the dynamics of which is given by the Lagrangian

$$L_\varphi = \frac{1}{2} \partial_\alpha \varphi \partial^\alpha \varphi$$

In the framework of General Relativity its gravitational interaction will be given by the Lagrangian

$$L = \frac{1}{\kappa} R + \frac{1}{2} W(\varphi) \partial_\alpha \varphi \partial^\alpha \varphi + B(\varphi) R - \frac{1}{\kappa} \Lambda$$ (1)

where for the time being the dependence of $B$ and $W$ on the scalar field is not fixed. This dynamics represents a scalar field coupled non-minimally with gravity. The cosmological constant is added by the reasons presented above and as we shall see it represents the influence of the

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rest-of-the-universe on $\varphi$. In the present proposed mechanism, such $\Lambda$ is the real responsible to provide mass for the scalar field. This means that if we set $\Lambda = 0$ the mass of the scalar field will vanish, as we shall see. The factor $W(\varphi)$ has been introduced to eliminate terms involving $(\partial_{\alpha} \varphi)^2$ that have their origin in the non-minimal coupling with gravity.

Independent variation of $\varphi$ and $g_{\mu\nu}$ yields

$$ W \Box \varphi + \frac{1}{2} W' \partial_{\alpha} \varphi \partial^{\alpha} \varphi - B' R = 0 \quad (2) $$

where for graphical simplicity we set $\alpha_0 \equiv 2/\kappa$ and $B' \equiv \partial B / \partial \varphi$. The energy-momentum tensor defined by

$$ T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L)}{\delta g^{\mu\nu}} $$

is given by

$$ T_{\mu\nu} = W \partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{1}{2} W \partial_{\alpha} \varphi \partial^{\alpha} \varphi g_{\mu\nu} + 2B (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) + 2\nabla_{\mu} \nabla_{\nu} B - 2B' g_{\mu\nu} + \frac{1}{\kappa} \Lambda g_{\mu\nu} \quad (3) $$

Taking the trace of equation (3) we obtain

$$ (\alpha_0 + 2B) \Box \varphi = - \partial_{\alpha} \varphi \partial^{\alpha} \varphi (W + 6B'') - 6B' \Box \varphi + 4 \frac{\Lambda}{\kappa} \quad (5) $$

where we use $\Box B = B' \Box \varphi + B'' \partial_{\alpha} \varphi \partial^{\alpha} \varphi$.

Inserting this result on the equation (2) yields

$$ \mathcal{M} \Box \varphi + \mathcal{N} \partial_{\alpha} \varphi \partial^{\alpha} \varphi - \mathcal{Q} = 0 \quad (6) $$

where

$$ \mathcal{M} \equiv W + \frac{6(B')^2}{\alpha_0 + 2B} $$

$$ \mathcal{N} \equiv \frac{1}{2} W' + \frac{B' (W + 6B'')}{\alpha_0 + 2B} $$

$$ \mathcal{Q} = \frac{4 \Lambda B'}{\kappa (\alpha_0 + 2B)} $$

Thus, through the non-minimal coupling with the gravitational field it follows that the scalar field acquires an effective self-interaction.

At this point it is worth to select among all possible candidates of $B$ and $W$ a particular one that makes the factor on the gradient of the field to disappear on the equation of motion by setting $\mathcal{N} = 0$. This condition will give $W$ as a function of $B$:

$$ W = \frac{2q - 6(B')^2}{\alpha_0 + 2B} \quad (7) $$

where $q$ is a constant. Inserting this result into the equation (6) yields

$$ \Box \varphi - \frac{2\Lambda}{q\kappa} B' = 0 \quad (8) $$

At this point one is led to set

$$ B = - \frac{\beta}{4} \varphi^2 $$

to obtain

$$ \Box \varphi + \mu^2 \varphi = 0 \quad (9) $$

where

$$ \mu^2 \equiv \frac{\beta \Lambda}{q \kappa} \quad (10) $$

For the function $W$ it follows

$$ W = \frac{2 \alpha_0 - 3 \beta^2 \varphi^2}{2 \alpha_0 - \beta \varphi^2} $$

in which we have choose $2q = \alpha_0$ in order to obtain the standard value for the dynamics in case $\beta$ is zero.

Thus as a result of the above process we have succeeded to provide to the scalar field a mass $\mu$ that depends on the constant $\beta$ and on the existence of $\Lambda$:

$$ \mu^2 = \beta \Lambda \quad (11) $$

If $\Lambda$ vanishes then the mass of the field vanishes. This is precisely what we envisaged to obtain: the net effect of the non-minimal coupling of gravity with the scalar field corresponds to a specific self-interaction of the scalar field. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe — represented by the cosmological constant — in the state in which all existing matter is on the corresponding vacuum. The values of different masses for different fields is contemplated in the parameter $\beta$.

A. Renormalization of the mass

The effect of the rest-of-the-universe on a massive scalar field can be analyzed through the same lines as above. Indeed, let us consider the case in which the free field dynamics is given by

$$ L = \frac{1}{\kappa} R + \frac{1}{2} \partial_{\alpha} \varphi \partial^{\alpha} \varphi + B(\varphi) R - V(\varphi) - \Lambda \quad (12) $$
where one is led to choose $B = a + b \varphi - 1/12 \varphi^2$ to obtain after some algebraic steps

$$\Box \varphi + V'_{\text{eff}} = 0$$

where the effective potential $V_{\text{eff}}$ is described in terms of $\Lambda$, the pre-existing potential $V$ and its derivatives. The net effect of the action of the rest-of-the-universe is in this case to modify the potential $V$ and to re-normalize the mass.

### III. THE CASE OF FERMIONS

Let us now turn our attention to the case of fermions. The massless theory for a spinor field is given by Dirac equation:

$$i\gamma^\mu \partial_\mu \Psi = 0 \quad (13)$$

In the framework of General Relativity its gravitational interaction is driven by the Lagrangian (we are using units were $\hbar = c = 1$)

$$L = \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{i}{2} \nabla_\mu \bar{\Psi} \gamma^\mu \Psi + \frac{1}{\kappa} R + V(\Phi) R - \frac{1}{\kappa} \Lambda + L_{\text{CT}} \quad (14)$$

where the non-minimal coupling of the spinor field with gravity is contained in the term $V(\Phi)$ that depends on the scalar

$$\Phi \equiv \bar{\Psi} \Psi$$

that preserves the gauge invariance of the theory under the map $\Psi \rightarrow \exp(i \theta \bar{\Psi})$. Note that the presence of the factor on $\Phi$ on the dynamics of $\Psi$ breaks the chiral invariance of the mass-less fermion, a condition that is necessary for a mass to appear.

For the time being the dependence of $V$ on $\Phi$ is not fixed. We have added a counter-term $L_{\text{CT}}$ for reasons that will be clear later on. On the other hand, the form of the counter-term should be guessed, from the previous analysis that we made for the scalar case, that is we set

$$L_{\text{CT}} = H(\Phi) \partial_\mu \Phi \partial^\mu \Phi \quad (15)$$

This dynamics represents a massless spinor field coupled non-minimally with gravity. The cosmological constant is added by the reasons presented above and as we shall see it represents the influence of the rest-of-the-universe on $\Psi$.

Independent variation of $\Psi$ and $g_{\mu\nu}$ yields

$$i\gamma^\mu \nabla_\mu \Psi + (R V' - H' \partial_\mu \Phi \partial^\mu \Phi - 2H \Box \Phi) \Psi = 0 \quad (16)$$

where $V' \equiv \partial V/\partial \Phi$. The energy-momentum tensor is given by

$$T_{\mu\nu} = \frac{i}{4} \bar{\Psi} \gamma(\mu \nabla \nu) \Psi - \frac{i}{4} \nabla(\mu \bar{\Psi} \gamma \nu) \Psi + 2V(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) + 2\nabla_\mu \nabla_\nu V - 2\Box V g_{\mu\nu} + 2H \partial_\mu \Phi \partial_\nu \Phi - H \partial_\mu \Phi \partial^\nu \Phi g_{\mu\nu} + \frac{\alpha_0}{2} \Lambda g_{\mu\nu} \quad (18)$$

Taking the trace of equation (17) we obtain after some algebraic manipulation:

$$(\alpha_0 + 2V + V') R = H' \Phi \partial_\mu \Phi \partial^\mu \Phi + 2H \Box \Phi - 6\Box V + 2\alpha_0 \Lambda \quad (19)$$

Inserting this result back on the equation (16) yields

$$i\gamma^\mu \nabla_\mu \Psi + (\chi \partial_\lambda \Phi \partial^\lambda \Phi + \chi \Box \Phi) \Psi + Z \Psi = 0 \quad (20)$$

where

$$Z \equiv \frac{2 \alpha_0 \Lambda V'}{\alpha_0 + 2V + \Phi \Box \Phi} \quad (21)$$

$$\chi \equiv \frac{V'(\Phi H' - 2H - 6V' \Box \Phi) \alpha_0 + 2V + \Phi \Box \Phi - 2H}{\alpha_0 + 2V + \Phi \Box \Phi}$$

At this stage it is worth selecting among all possible candidates of $V$ and $H$ particular ones that makes the factor on the gradient and on $\Box$ of the field to disappear from equation (20). The simplest way is to set $X = Y = 0$ which imply only one condition, that is

$$H = \frac{-3(V')^2}{\alpha_0 + 2V} \quad (21)$$

The non-minimal term $V$ is such that $Z$ reduces to a constant, that is

$$V = \frac{\alpha_0}{2} \left[(1 + \sigma \Phi)^{-2} - 1\right] \quad (22)$$

Then it follows immediately

$$H = -3\alpha_0 \sigma^2 (1 + \sigma \Phi)^{-4} \quad (23)$$

where $\sigma$ is a constant.

The equation for the spinor becomes

$$i\gamma^\mu \nabla_\mu \Psi - m \Psi = 0 \quad (24)$$
where
\[ m = \frac{4 \sigma \Lambda}{\kappa}. \] (25)

Thus as a result of the above process we have succeeded to provide to the spinor field a mass \( m \) that depends crucially on the existence of \( \Lambda \). If \( \Lambda \) vanishes then the mass of the field vanishes. This is precisely what we envisaged to obtain: the net effect of the non-minimal coupling of gravity with the spinor field corresponds to a specific self-interaction. The mass of the field appears only if we take into account the existence of all remaining bodies in the universe — represented by the cosmological constant — in the state in which all existing matter is on the corresponding vacuum. The values of different masses for different fields are contemplated in the parameter \( \sigma \).

The various steps of our mechanism can be synthesized as follows:

- The dynamics of a massless spinor field \( \Psi \) is described by the Lagrangian
  \[ L_D = \frac{i}{2} \bar{\Psi} \gamma^\mu \nabla_\mu \Psi - \frac{i}{2} \nabla_\mu \bar{\Psi} \gamma^\mu \Psi; \]

- Gravity is described in General Relativity by the scalar of curvature
  \[ L_E = R; \]

- The field interacts with gravity in a non-minimal way described by the term
  \[ L_{\text{int}} = V(\Phi) R \]

where \( \Phi = \bar{\Psi} \Psi; \)

- The action of the rest-of-the-universe on the spinor field, through the gravitational intermediary, is contained in the form of an additional constant term on the Lagrangian noted as \( \Lambda \);

- A counter-term depending on the invariant \( \Phi \) is introduced to kill extra terms coming from gravitational interaction;

- As a result of this process, after choosing \( V \) and \( H \) the field acquires a mass being described as
  \[ i \gamma^\mu \nabla_\mu \Psi - m \Psi = 0 \]

where \( m \) is given by equation (25) and is zero only if the cosmological constant vanishes.

This procedure allows us to state that the mechanism proposed here is to be understood as a form of realization of Mach principle according to which the inertia of a body depends on the background of the rest-of-the-universe. Besides, our strategy can be applied in a more general context in support of the idea that (local) properties of microphysics may depend on the (global) properties of the universe. We will come back to this in a future paper.

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