Assessing free tropospheric quasi-equilibrium for different GCM resolutions using a cloud-resolving model simulation of tropical convection

Xu Wang · Guang J. Zhang · E. Suhas

Received: 1 May 2021 / Accepted: 27 February 2022 / Published online: 4 April 2022
© The Author(s) 2022

Abstract
This study examines the free-tropospheric quasi-equilibrium at different global climate model (GCM) resolutions using the simulation of tropical convection by a cloud-resolving model during the Tropical Western Pacific International Cloud Experiment. The simulated dynamic and thermodynamic fields within the model domain are averaged over subdomains of different sizes equivalent to different GCM resolutions. These coarse-grained fields are then used to compute CAPE and its change with time, and their relationships with simulated convection. Results show that CAPE change with time is controlled predominantly by variations of thermodynamic properties in the planetary boundary layer for all subdomain sizes ranging from 64 to 4 km. Lag correlation analysis shows that CAPE generation by the free-tropospheric dynamical advection ($d\text{CAPE}_l$) leads convective precipitation but is in phase with convective mass flux at 600 mb and 500 mb vertical velocity for all subdomain sizes. However, the correlation coefficients and regression slopes decrease as the subdomain size decreases for subdomain sizes smaller than 16 km. This is probably due to increased randomness of convection and more scale-dependence of the relationships when the subdomain size reaches the grey zone. By examining the sensitivity of the relationships of convection with $d\text{CAPE}_l$ to temporal scales in different subdomain size, it shows that the quasi-equilibrium between $d\text{CAPE}_l$ and convection holds well for timescales of 30 min or longer at all subdomain sizes. These results suggest that the free tropospheric quasi-equilibrium assumption may still be useable even for GCM resolutions in the grey zone.

1 Introduction
Convection occurs on spatial scales from sub-kilometer for shallow cumulus to tens of kilometers or larger for organized mesoscale convective systems. Despite its small scales, the role of deep convective towers in maintaining the atmospheric thermodynamic structure, atmospheric circulation and tropical energy balance was recognized more than half a century ago (Riehl and Malkus 1958; Manabe and Strickler 1964). Global climate models (GCMs) in the past and up till now typically have a horizontal grid spacing of ~ 100 km or larger. Within such a GCM grid box, the collective effect of convection is parameterized. In doing so, various assumptions were made to relate convection to grid-scale fields. Kuo (1965, 1974) proposed a moisture convergence closure, which related convection to column moisture convergence in the atmosphere. This closure was later modified by Tiedtke (1989) to determine cloud base updraft mass flux using moisture convergence in the subcloud layer. However, moisture convergence-based closure has been faulted for causing grid-point storms (Yano et al. 1998) and artificial CISK (conditional instability of the second kind) (Ooyama 1982). Arakawa and Schubert (1974) proposed a convective quasi-equilibrium, which assumed that the removal of convective instability by a population of convective clouds of different sizes within a GCM grid box equals the generation of convective instability by GCM-resolved processes. The convective quasi-equilibrium is probably the most fundamental assumption among many assumptions involved in convective parameterization schemes. From the tropical circulation point of view, Emanuel et al. (1994) showed that under the convective quasi-equilibrium framework the tropical atmospheric systems such as the Hadley circulation, tropical cyclones and Madden–Julian oscillation (MJO), are...
Other closures have also been proposed since then, including CAPE (convective available potential energy) relaxation-based closure (Fritsch and Chappell 1980; Kain and Fritsch 1993; Kain 2004; Zhang and McFarlane 1995), boundary layer turbulent kinetic energy (TKE) and convective inhibition (CIN)-based closure (Mapes 2000), boundary layer quasi-equilibrium closure (Raymond 1995), and free tropospheric quasi-equilibrium closure (Zhang 2002). Yano et al. (2013) provides a thorough review of different closure assumptions and their merits. CAPE is an approximation of cloud work function used in the Arakawa-Schubert (1974) convection scheme. It assumes that convective instability in the atmosphere is removed by convection within a relaxation time scale of a few hours. Based on the concept of activation control of convection by Mapes (1997), Mapes (2000) proposed a closure relating convection to boundary layer TKE and CIN. Fletcher and Bretherton (2010) and Hohenegger and Bretherton (2011) further tested it using a cloud-resolving model and single column model.

Zhang (2002) analyzed the observational data from the U.S. DOE Atmospheric Radiation Measurement (ARM) program and found that the net variation of convective instability as measured by CAPE is significant in both convective and non-convective situations. This variation is largely controlled by boundary layer temperature and moisture fluctuations. Further analyses of tropical and midlatitude observational data by Zhang (2003) and Donner and Phillips (2003) reached similar conclusions. This indicates that contributions to the net CAPE variation from temperature and moisture changes in the troposphere above the planetary boundary layer are insignificant. Based on this finding, Zhang (2002) proposed a free-tropospheric quasi-equilibrium (FTQE), in which CAPE generation by free-tropospheric large-scale processes is balanced by the removal of CAPE from convective heating. By design, a positive CAPE generation by the free tropospheric large-scale processes was also used as a trigger condition for convection onset. A similar trigger condition was used in Xie et al. (2004, 2019). An evaluation of trigger conditions in many convective parameterization schemes using observational data showed that this trigger condition is among the best performing trigger functions (Suhas and Zhang 2014). The application of the FTQE in the Zhang-McFarlane convection scheme (Zhang and McFarlane 1995) in the National Center for Atmospheric Research Community Atmosphere Model version 3 (NCAR CAM3), Community Climate System Model version 3 (CCSM3) and Community Earth System Model version 1 (CESM1) has shown improved simulations of some features of tropical convection, including MJO and intertropical convergence zone (ITCZ) (Zhang and Mu 2005a, b; Zhang and Wang 2006; Song and Zhang 2009, 2018; Zhang and Song 2010). Wilcox and Donner (2007) implemented it into the Donner (1993) scheme in the Geophysical Fluid Dynamics Laboratory (GFDL) Atmospheric Model AM2. They showed, among other improvements, that the free tropospheric quasi-equilibrium closure greatly improved the simulation of the frequency of extreme precipitation events in the AM2. Benedict et al. (2013) showed that a version of the GFDL AM3 with the FTQE closure produced a better simulation of MJO than in the standard AM3 model. The FTQE closure was also implemented into the Finite-volume Atmospheric Model of IAP/LASG (FAMIL) GCM (Zhou et al. 2015) and tested in single column models of the NCAR CAM4 and CAM5 (Wang and Zhang 2013). Bechtold et al. (2014) incorporated this free-tropospheric quasi-equilibrium, with modifications to include some non-equilibrium elements from the planetary boundary layer, into the European Centre for Medium-range Weather Forecasts (ECMWF) Integrated Forecasting System (IFS) and showed that it improved the forecast of the diurnal variation of convection.

Both observational and GCM evaluations of FTQE are for GCM grid-spacings of ~200 km or larger. As GCM resolutions increase, is FTQE still a good assumption? Developing a convection scheme that can adapt to different GCM resolutions, i.e., scale-aware, has been an active research topic in recent years since Arakawa et al. (2011) proposed it (Arakawa and Wu 2013; Grell and Freitas 2014). Kwon et al. (2017). However, all these studies are concerned with how to factor in the fact that convective cloud fraction will no longer be negligible as GCM resolution increases to the grey zone (~10 km or less). Not as much has been explored on whether the closures of convective schemes are still usable as the GCM resolution approaches the grey zone. Suhas and Zhang (2015) evaluated several closure assumptions including CAPE-based, moisture convergence-based, and boundary layer turbulent kinetic energy (TKE)-based closures using a cloud-resolving model (CRM) simulation for different GCM resolutions. They found that the moisture convergence closure of Tiedtke (1989) is well correlated to convective precipitation and convective cloud mass flux at the 600 hPa level for a large range of grid spacings from 128 to 4 km. To what extent is the FTQE assumption still applicable or appropriate for use in convective parameterization at high horizontal resolutions? We will address this issue in the paper.

At this point, it is necessary to clarify a nomenclature. In relating convection to large-scale fields through a closure,

---

1 Here and in the rest of this paper trigger condition means a set of conditions that the atmospheric state or processes must satisfy before the call to convective parameterization is triggered.
the large-scale fields are often referred to as “forcing”, by which it implies a causality, that is, the large-scale forcing causes convection. Since the FTQE involves atmospheric circulation, it can be equally valid to argue that the large-scale circulation is a result of convection. More discussion on this will be presented in Sect. 3. In the paper, we will refrain from using the word “forcing” to describe the circulation unless it is specified from observations to drive the CRM, in which case it is indeed used as forcing in the CRM setup.

The paper is organized as follows. In Sect. 2, the CRM simulation data used for the evaluation will be described, along with a brief description of the free-tropospheric quasi-equilibrium and the analysis method. Section 3 will present the results. Section 4 will conclude the paper with a summary and some discussions.

## 2 Data and analysis method

The use of the cloud-resolving model (CRM) output for evaluating or developing convective parameterization schemes has been explored in the past (Xu et al. 1992; Plant and Craig 2008; Jones and Randall 2011). Although there are deficiencies in simulating the detailed structure of convective systems and the intensity of convective updrafts (Bryan et al. 2003; Varble et al. 2011), the macroscopic behavior of convection under a given large-scale condition is realistically simulated. Therefore, CRM data are still suitable for evaluating convective parameterization schemes.

The data used in this study are the same as those used in Suhas and Zhang (2015). They are from a CRM simulation of convection (Zeng et al. 2011) during the intensive observation period (IOP) of Department of Energy (DOE) Atmospheric Radiation Measurement (ARM) program’s Tropical Warm Pool International Cloud Experiment (TWP-ICE) in Darwin Australia during the northern Australian summer monsoon season of 2006 (May et al. 2008). The model is the three-dimensional Goddard Cumulus Ensemble (GCE) model (Tao and Simpson 1993). It has 41 vertical levels from the surface to 21 km height, with a vertical resolution varying from 42.5 m at the bottom to 1 km at the top, and a horizontal resolution of 1 km. The simulation covers the monsoon break period from 2100 UTC 4 February 2006 to 2100 UTC 10 February 2006. The model domain covers an area of 256 km x 256 km over the ARM Darwin site. The model simulation is output at 6-min interval.

To evaluate the free tropospheric quasi-equilibrium assumption for different subdomain sizes equivalent to various GCM resolutions, the raw data is averaged over different spatial and temporal scales. For spatial averaging, subdomain sizes of 64 km, 32 km, 16 km, 8 km and 4 km are used. Within each subdomain, we first identify convective grid points at the native CRM resolution of 1 km. A grid point is considered convective if its vertical velocity is greater than 1 m/s or less than –1 m/s and the sum of the mixing ratios of cloud liquid water and ice water exceeds 1 × 10⁻⁵ kg/kg. The GCM-equivalent domain-averaged convective mass flux \( M_c = \sum_i \rho w / N \) is calculated to measure convective activity in each subdomain. Here \( w \) is vertical velocity at a CRM grid point, the summation is over all convective grid points within the subdomain, and \( N \) is the total number of CRM grid points for the subdomain. For convective precipitation, if a CRM column contains any convective grid point, then the surface precipitation is classified as convective, and the subdomain-mean convective precipitation is given by \( P_c = \sum_i P / N \). Here the summation is over all convective columns. For temporal averaging, 1, 2, and 3 h are chosen.

Before going into the details of methodology of evaluating the free tropospheric quasi-equilibrium, we first define CAPE and its time rate of change resulting from subdomain-scale processes. CAPE is defined by the vertical integral of buoyancy of a parcel lifted from the most unstable level in the planetary boundary layer (PBL) to the level of neutral buoyancy:

\[
\text{CAPE} = \int_{p_b}^{p_t} R_d (T_{pv} - \overline{T_v}) d\ln p
\]

where \( T_{pv} = (1 + 0.608 q_t - q_i) \) and \( \overline{T_v} = \overline{T}(1 + 0.608 q_i) \) are virtual temperatures of the air parcel and the domain average. \( p_b \) and \( p_t \) are pressure values at the parcel’s originating level and the neutral buoyancy level, respectively. \( R_d \) is gas constant for dry air and \( q_i \) is liquid water condensed following the reversible moist adiabat of the air parcel. Subscript \( p \) stands for parcel’s properties and overbar for domain average. Thus, CAPE change with time can be rewritten as:

\[
\frac{\partial \text{CAPE}}{\partial t} = \frac{\partial \text{CAPE}_p}{\partial t} + \frac{\partial \text{CAPE}_e}{\partial t}
\]

where \( \frac{\partial \text{CAPE}_p}{\partial t} = R_d \frac{\partial}{\partial t} \int_{p_t}^{p_b} T_p d\ln p \) and \( \frac{\partial \text{CAPE}_e}{\partial t} = -R_d \frac{\partial}{\partial t} \int_{p_t}^{p_b} \overline{T_v} d\ln p \)

represent contributions to CAPE change from changes of the parcel’s thermodynamic properties and its environment in the free troposphere, respectively. For an undiluted parcel, its virtual temperature is determined by the temperature and moisture at its originating level in the PBL. With simple manipulation it can be shown (Zhang et al. 1998) that \( \frac{\partial \text{CAPE}_e}{\partial t} \)

is related to surface entropy flux, and \( \frac{\partial \text{CAPE}_e}{\partial t} \) is the change of the thickness of the convective layer. Using ARM observations, Zhang (2002, 2003) and Donner and Phillips (2003) found that CAPE variation is largely controlled by boundary layer thermodynamic changes, that is, \( \frac{\partial \text{CAPE}_e}{\partial t} \approx 0 \). Based on this finding, Zhang (2002) further decomposed \( \frac{\partial \text{CAPE}_e}{\partial t} \) into contributions from large-scale and convective scale processes.
\[ \frac{\partial \text{CAPE}_e}{\partial t} = \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_c + \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_{ls} \approx 0 \]  \hspace{1cm} (3)

This leads to the free tropospheric quasi-equilibrium assumption, which states that CAPE generation by large-scale advection in the free troposphere is in equilibrium with the CAPE removal by convective heating:

\[ \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_c \approx - \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_{ls} \]  \hspace{1cm} (4)

This assumption was used to devise a closure to determine convective mass flux at the cloud base (Zhang, 2002; Zhang and Mu, 2005a, b). In fact, the left-hand side of Eq. (4) is proportional to the cloud-based convective mass flux \( M_b \):

\[ \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_c = M_b \int_{p_t}^{p_0} (1 + 0.608q) \left( -\eta \frac{\partial S}{\partial p} \right) + 0.608T \left[ -\frac{\partial q}{\partial p} + \delta (q_s - q) \right] dlnp \]  \hspace{1cm} (5)

where \( \eta \) is the net cloud mass flux normalized by the cloud base mass flux, i.e., \( \eta = M_c/M_b \). The vertical integral on the rhs of Eq. (5) represents the consumption rate of CAPE per unit cloud-base mass flux [see Eq. (8) of Zhang (2002)]. Thus,

\[ M_b = \max \left\{ \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_{ls}, 0 \right\} / F \]  \hspace{1cm} (6)

Here \( F \) denotes the vertical integration on the rhs of Eq. (5). Note that the diagnostic relationship in Eq. (4) is well-known in large-scale tropical dynamics through scale analysis (e.g. Yano and Bonazzola 2009) and is the basis of weak temperature gradient approximation of tropical circulation (Sobel et al. 2001). But it was not formally used in the context of convective parameterization until Zhang (2002).

To evaluate the free tropospheric quasi-equilibrium closure for a given subdomain size mimicking a corresponding GCM resolution, the following procedure is taken to calculate CAPE changes due to subdomain-scale advection. First, CAPE is computed using Eq. (1) and subdomain-mean fields, and we call it \( \text{CAPE}_{eq} \). Second, we compute advective tendencies of temperature \( -\mathbf{\nabla} \cdot \mathbf{\nabla} \bar{q} - \bar{\omega} \frac{\partial \bar{q}}{\partial p} \) and moisture \( -\mathbf{\nabla} \cdot \mathbf{\nabla} \bar{q} - \bar{\omega} \frac{\partial \bar{q}}{\partial p} \) using subdomain-mean temperature, moisture and velocity fields from neighboring subdomains and finite difference as if they were representing GCM grid point variables. Here \( s \) is dry static energy normalized by heat capacity of air, \( s = T + gz/c_p \). The forcing from observations used to drive the CRM is also added uniformly to each subdomain, as was done in the CRM simulation. These advective tendencies are then used to update the subdomain-mean temperature and moisture fields at all levels above the parcel’s originating level, so that the parcel’s properties are not affected by this update:

\[ \frac{\partial \text{CAPE}_e}{\partial t} = \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_c + \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_{ls} \approx 0 \]  \hspace{1cm} (7a)

\[ \overline{T}_1 = \overline{T}_0 + \left( -\mathbf{\nabla} \overline{\bar{q}} - \bar{\omega} \frac{\partial \overline{\bar{q}}}{\partial p} \right) \Delta t + F_{adv} \Delta t \]  \hspace{1cm} (7b)

where \( \Delta t \) is 6 min, the time interval for CRM model output, and \( \overline{T}_0 \) and \( \overline{\bar{q}}_0 \) are temperature and moisture used to compute \( \text{CAPE}_0 \). \( F_{adv} \) and \( F_{advq} \) are observed forcing of temperature and moisture that is used to drive the CRM. Next, CAPE is recomputed using Eq. (1) with the updated temperature \( \overline{T}_1 \) and moisture \( \overline{\bar{q}}_1 \), and we call it \( \text{CAPE}_1 \). Thus, \( \text{CAPE}_1 \) is equivalent to CAPE in the atmosphere after the dynamics core but before convection parameterization is called in GCMs such as the NCAR CAM5. Finally, we obtain the CAPE change due to subdomain-scale (or GCM grid-scale) advection:

\[ \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_{ls} = \frac{\text{CAPE}_1 - \text{CAPE}_0}{\Delta t} \]  \hspace{1cm} (8)

Hereafter we will use the shorthand \( \text{dCAPE}_{ls} \) for \( \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_{ls} \). Note that neither \( \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_c \) nor \( \left( \frac{\partial \text{CAPE}_e}{\partial t} \right)_{ls} \) has contributions from changes of thermodynamic properties at and below the parcel’s initiation level in boundary layer. Thus, this closure is decoupled from the surface and boundary layer processes that affect the convective parcel’s properties. While this may seem unconventional since it is well known that convection is rooted in the boundary layer, it should be pointed out that heating and moistening, such as those from surface turbulent fluxes, in the boundary layer often do not directly affect deep convection. They first deepen the boundary layer, which subsequently leads to the development of shallow convection. Many previous observational and modeling studies have pointed out that shallow convection can serve to precondition the lower troposphere, so that the parcel’s properties are not affected by this update.

\[ \text{dCAPE}_{eq} \]
Assessing free tropospheric quasi-equilibrium for different GCM resolutions using a…

controlled by the PBL properties. Therefore, deep convection is indirectly related to PBL properties in this closure.

3 Results

Figure 1 shows the time series of observed precipitation during the IOP and dCAPE_{ls} due to observed/background large-scale forcing applied to the CRM. Observationally, there is a close correspondence between variations of precipitation and dCAPE_{ls}, with a correlation coefficient of 0.90 for points with positive dCAPE_{ls}. Figure 2 shows the time evolution of model precipitation and dCAPE_{ls} for selected subdomain sizes (64 km and 32 km). Also shown are contributions to dCAPE_{ls} from the imposed observed forcing and advection from model generated circulations. For a 64 km subdomain, the total dCAPE_{ls} largely comes from the contribution of model-generated circulations, and the prescribed, observed forcing is relatively small. As a result, the temporal variation of dCAPE_{ls} is largely dominated by that of the contribution from model circulation, with a correlation coefficient of 0.92 between the two, and the correlation between the total dCAPE_{ls} and the observed large-scale contribution is only 0.59. The precipitation variation clearly follows that of the total dCAPE_{ls}. In a 32 km subdomain, dCAPE_{ls} variation also predominantly comes from the model-generated contribution, with a correlation coefficient of 0.93. The contribution from the background forcing becomes much less
64 km (1 hr avg.)
(a) slope: 0.938
  corr: 0.978

32 km (1 hr avg.)
(b) slope: 0.949
  corr: 0.983

16 km (1 hr avg.)
(c) slope: 0.958
  corr: 0.987

8 km (1 hr avg.)
(d) slope: 0.963
  corr: 0.989

4 km (1 hr avg.)
(e) slope: 0.966
  corr: 0.990
Fig. 3 Scatter plots of total CAPE change (dCAPE/dt) versus CAPE change due to thermodynamic changes in the planetary boundary layer (dCAPE_p/dt) for different subdomain sizes ranging from 64 to 4 km. Each point represents a 1-h average of model output at 6 min interval. The regression lines (black), slopes and correlation coefficients are also provided. For visual clarity, only a portion (chosen randomly) of data points is used in the scatterplots at 16, 8 and 4 km subdomain sizes, but all data are used in the calculation of statistics.

The linear relationship between dCAPE_p and mass flux begins to degrade, with the median values plateauing off from the linear regression line, when the subdomain size further decreases to 8 km and 4 km, indicating that a larger amount of CAPE generation is needed to produce a given amount of convection as the GCM grid size decreases. The scatter is also increased. There are two possible explanations for the degeneration of the linear relationship at small subdomain sizes. First, as the subdomain size decreases, for a given grid-scale circulation and thermodynamic state, the stochastic behavior of convection becomes more prominent, thereby making convection in individual subdomains deviate more from the convective quasi-equilibrium. Second, as the subdomain size decreases, some of the convection may become GCM grid-resolved, resulting in less grid-scale convective mass flux for given grid-scale state. This scale-aware issue should become more noticeable when the subdomain size becomes smaller.

To further evaluate the accuracy of free tropospheric quasi-equilibrium as GCM grid size decreases, Fig. 5 shows the lag correlation between dCAPE ls and convective precipitation, 600 mb convective mass flux, and 500 hPa vertical velocity, respectively, averaged over different subdomain sizes. The maximum correlation occurs when dCAPE ls leads convective precipitation. The lead time decreases from 18 min for 64 km subdomain size to 10 min for sub-10 km subdomain size. The maximum correlation coefficient remains about the same from 64 km subdomain sizes to 16 km subdomain sizes but decreases as the subdomain size further decreases. The variation of time lag with subdomain size can be explained by the strength of GCM grid-scale forcing and the time it takes for convection to adjust. Wang and Randall (1996) demonstrated that it takes longer for convection to adjust under weak large-scale forcing, and vice versa. Figure 4 shows that when the subdomain size is larger, the forcing is smaller. Thus, it takes longer for convective heating to adjust the atmosphere to an equilibrium state. The maximum correlation between convective mass flux and dCAPE ls occurs at zero lag for all subdomain sizes. This indicates that the development of convective precipitation lags convective updraft mass flux at 600 mb, the larger the subdomain, the longer the time lag. The fact that the highest correlation occurs between convective mass flux and dCAPE ls at zero lag suggests that using dCAPE ls as a closure to determine the amount of convection at a given GCM time step is reasonable. However, this should not be construed as a causal relationship, but rather a diagnostic one. In a balanced state it may be difficult or pointless to identify causality. In the past convection has been considered both as a cause and a result of large-scale circulation, depending on one’s viewpoint. In dynamic models of tropical circulation (e.g. Gill 1980), the large-scale circulation is often viewed as a response to convective heating. On the other hand, in

significant, particularly during heavy precipitation events. The time series for other 64 km and 32 km subdomains are similar to those shown in the figure. This clearly demonstrates that the model-generated circulation on subdomain scale becomes more closely related to convection and the local relationship between convection and circulation is less dependent on the imposed forcing for the CRM as the subdomain size decreases.

Zhang (2002, 2003) and Donner and Phillips (2003) showed using field observational data that on spatial scales of a few hundred kilometers, variability in CAPE is mainly controlled by planetary boundary layer variability in thermodynamic fields. In tropical atmosphere, this is at least partly due to the weak temperature gradients in the free troposphere. To examine if the CAPE variation is still controlled partly due to the weak temperature gradients in the free troposphere. To examine if the CAPE variation is still controlled by the PBL properties as the spatial scale decreases, Fig. 3 shows the scatter plots of net CAPE change (dCAPE) versus contributions from boundary layer changes (dCAPE_p), that is, the left-hand side vs. the first term on the right-hand side of Eq. (2), using 1-h average data. For all subdomain sizes, there is a strong correlation between dCAPE and dCAPE_p, with correlation coefficients greater than 0.97. The slopes show that boundary layer contribution accounts for up to 93% of the total dCAPE. Furthermore, the relative contribution from the boundary layer increases as the domain size decreases. For example, at the 64 km subdomain size the regression slope is 0.938; it increases to 0.966 at 4 km subdomain size. These results, together with earlier observational results (Zhang 2002, 2003; Donner and Phillips 2003), indicate that boundary layer control of CAPE applies to all scales from ~ 100 km to grey zone scales of sub-10 km. In other words, Eq. (4) is a very good approximation to relate convection to GCM grid-scale CAPE generation in the free troposphere for all GCM grid spacings down to 4 km.

To verify this, Fig. 4 shows the relationship between dCAPE ls and convective mass flux at 600 mb. The red, cyan and orange lines are median values, first and third quartiles in each dCAPE ls bin, respectively. As in Suhas and Zhang (2015), the 600 mb convective mass flux is used to measure the amount of convection because it is difficult to define cloud base accurately in the CRM. Convective mass flux increases linearly with dCAPE ls for 64 km, 32 km and 16 km, indicating GCM grid-scale advective CAPE generation, as represented by dCAPE ls, is closely related to convective mass flux at these subdomain sizes.
Fig. 4 Scatter plots of convective mass flux at 600 mb and dCAPE for different subdomain sizes. The blue line is linear regression, with correlation coefficient and slope shown at the top of each plot. Red solid curves are for median values and green and yellow solid curves are for first and third quartiles, respectively. For plotting the quartile curves, the data are divided into a number of bins and the bin sizes are larger for smaller subdomains. For visual clarity, only a portion (chosen randomly) of data points is used in the scatterplots, but all data are used in the calculation of statistics. Note that the horizontal and vertical scales are different across the plots.
cloud-resolving models (e.g., Grabowski 2001) or single-column models (Randall et al. 1996), large-scale forcing is prescribed to drive the models and convection in the models responds to the forcing. In the real world, large-scale circulation and convection interact with each other and neither view provides a complete picture of this interaction. In the context of convective parameterization, the amount of convection determined by a parameterization closure can be viewed as that needed to balance the model-predicted thermodynamic state or process, be it a cause or an effect. Therefore, it is
unnecessary to determine whether one is the cause or the effect of the other.

Previous work by Davies et al. (2013) and Kumar et al. (2015) using Doppler radar observations of convection and ECMWF reanalysis data in Darwin, Australia found that convection is highly correlated with 500 mb large-scale vertical velocity. Another study by Qiao and Liang (2016) used a regional climate model Climate-Weather Research and Forecasting (CWRF) to evaluate the effects of cumulus parameterization closures on summer precipitation simulation over the U.S. east coast and Gulf of Mexico. They showed that vertical velocity and moisture convergence-based closures reproduce the observed precipitation pattern and amount, and capture the frequency of heavy rainfall events better than CAPE-based closures. Since the large-scale generation of CAPE is closely linked to vertical velocity through vertical advection of dry static energy, this indirectly supports the free tropospheric quasi-equilibrium closure. As shown in Fig. 5c, the correlation coefficients reach a maximum at zero lag between \( \Delta \text{CAPE}_{ls} \) and 500 mb vertical velocity for all subdomain sizes, the same as that between \( \Delta \text{CAPE}_{ls} \) and convective updraft mass flux at 600 mb, indicating that \( \Delta \text{CAPE}_{ls} \), 500 mb vertical velocity and convection occur concurrently. This concerted action between convection, adiabatic cooling and upward motion was noticed 30 years ago by Fraedrich and McBride (1989) as a free ride. It is also the basis of the weak temperature gradient approximation (Sobel et al. 2001). In terms of the values of correlation coefficients, \( \Delta \text{CAPE}_{ls} \) and 500 mb vertical velocity are highly correlated, with coefficients over 0.8 for all subdomain sizes. Nonetheless, it should be noted that \( \Delta \text{CAPE}_{ls} \) does not necessarily have a one-to-one relationship with the 500 mb vertical velocity since vertical motion at other levels also contributes to \( \Delta \text{CAPE}_{ls} \).

Figure 6 shows the lag correlation between \( \Delta \text{CAPE}_{ls} \) and vertical velocity (color shading) at different levels for different subdomain sizes. At zero lag, there is a broad maximum correlation in the mid-troposphere between 600 and 300 hPa. Also, there is a clear tilted structure, showing a time lag between the lower and upper troposphere for all subdomain sizes, with the lower tropospheric vertical velocity leading \( \Delta \text{CAPE}_{ls} \) and the upper tropospheric vertical velocity lagging \( \Delta \text{CAPE}_{ls} \). The vertical velocity below 800 hPa leads \( \Delta \text{CAPE}_{ls} \) by as much as 2 h for 64 km subdomain size, reduced to less than half an hour for 4 km subdomain size. This indicates that the low-level grid-scale circulation acts first to generate \( \Delta \text{CAPE}_{ls} \). Also shown in Fig. 6 is the time-lag correlation between convective mass flux and \( \Delta \text{CAPE}_{ls} \) (contours). Similar to vertical velocity, convective mass flux in the lower troposphere also leads \( \Delta \text{CAPE}_{ls} \), although the vertical tilt in the time lag-height plot is not as much as that for vertical velocity. This suggests that vertical velocity below 800 hPa develops first, followed immediately by shallow convection, which reinforces the vertical motion. Then \( \Delta \text{CAPE}_{ls} \) peaks, accompanied by deep convection in the troposphere. This is consistent with previous observational and modeling studies that shallow convection often appears before deep convection (Wu 2003; Zhang and Klein 2010). The lead time decreases with subdomain size, reflecting that convection responds faster to stronger forcing as the subdomain size decreases.

From Figs. 5 and 6, convective mass flux and grid-scale vertical velocity \( w \) are closely related. Since \( \bar{w} = \frac{M_c}{\rho} + (1 - \sigma)\bar{\omega}, \) where \( \bar{w} \) is the subdomain-mean vertical velocity, \( M_c \) is convective mass flux, \( \rho \) is the air density, \( \sigma \) is the convective fraction and \( \bar{\omega} \) is vertical velocity in the convection-free environment within the subdomain, the difference between \( \bar{w} \) and \( M_c/\rho \) measures the compensating subsidence in the convection-free environment. Figure 7 shows the vertical profiles of \( \bar{w} \) and \( M_c/\rho \) for different subdomain sizes. Convective mass flux exceeds \( \bar{w} \) at most levels except in the boundary layer, meaning that there is systematic compensating subsidence in the convective environment. The strength of the subsidence relative to convective mass flux or subdomain-mean vertical velocity decreases with subdomain size. In other words, more compensating subsidence occurs non-locally outside the subdomain where convection occurs as the subdomain size decrease. This is in agreement with the conceptual model of scale-dependence of convective effects as envisioned by Arakawa et al. (2011).

Figure 8 shows the regression slope between 600 mb mass flux and \( \Delta \text{CAPE}_{ls} \) for different subdomain sizes. Similar to the correlation coefficients, for a given subdomain size the regression slope is the largest at zero lag. For different subdomain sizes, the regression slope first increases from 64 km subdomain sizes to 16 km subdomain sizes, but then decreases as the subdomain size becomes smaller, similar to that of correlation coefficient shown in Fig. 5b. However, the fluctuations are relatively smaller compared to the regression slopes themselves, in the range of 12–14 (kg m\(^{-2}\) day\(^{-1}\))/(J kg\(^{-1}\) h\(^{-1}\)). The slight decrease of the slopes for subdomain sizes < 32 km can be explained by the following reasoning. Convection, as measured by precipitation or updraft mass flux, is positive-definite (i.e. the amount of convection cannot be negative) whereas the subdomain-scale \( \Delta \text{CAPE}_{ls} \) can be either positive or negative. In neighboring subdomains, \( \Delta \text{CAPE}_{ls} \) can be either positive or negative. In strongly positive \( \Delta \text{CAPE}_{ls} \) subdomains, it is likely there is active convection and convective precipitation. In negative \( \Delta \text{CAPE}_{ls} \) subdomains, there is likely no convection. Assume that there is a linear relationship between convection and \( \Delta \text{CAPE}_{ls} \) in subdomains where \( \Delta \text{CAPE}_{ls} \) is positive. When averaging over a larger
Fig. 6 Lag correlation between vertical velocity and $d\text{CAPE}_{50}$ (color shading), and between convective mass flux and $d\text{CAPE}_{50}$ (contours) at (a) 64 km, (b) 32 km, (c) 16 km, (d) 8 km and (e) 4 km. Contour lines start at 0.2 with an interval of 0.2. Negative value in x-axis means $d\text{CAPE}_{50}$ lags vertical velocity and mass flux.
subdomain containing subdomains with both positive and negative \( \text{dCAPE}_{ls} \), the rate of reduction of convective updraft mass flux will be smaller than the rate of reduction of \( \text{dCAPE}_{ls} \) because the former contains positive or zero values only while the latter contains both positive and negative values. Thus, the ratio of convective mass flux to \( \text{dCAPE}_{ls} \) for larger subdomains is larger than that for smaller subdomains, as seen in Fig. 8. A similar GCM-resolution dependence was also recognized by Xiao et al. (2015) and Yun et al. (2017) when examining the scale-awareness of vertical transport in the Zhang-McFarlane convection scheme.

Finally, to examine the sensitivity of the relationships between \( \text{dCAPE}_{ls} \) and convective mass flux to temporal scales for different subdomain sizes, Fig. 9 shows the variation of correlation coefficients and regression slopes between \( \text{dCAPE}_{ls} \) and 600 mb convective mass flux with time averaging intervals from 6 min to 3 h for all subdomain sizes. The correlation coefficients for 32 km–4 km subdomain sizes have a maximum at 30 min averaging time and then decrease slightly as the averaging intervals increase to 3 h. For 64 km subdomain size, the correlation coefficient reaches a maximum at 1-h averaging interval. However, its variation with averaging time intervals is relatively small. For the regression slopes, their variations with averaging time intervals have a similar pattern to that for the correlation coefficients. Therefore, these indicate that free tropospheric quasi-equilibrium between the subdomain scale forcing (i.e. \( \text{dCAPE}_{ls} \)) and convection holds well for timescales of 30 min or longer, implying that for GCM applications with 30-min timestep, the free tropospheric quasi-equilibrium assumption is suitable.

4 Conclusions

This study uses the cloud-resolving model simulation of convection in Darwin, Australia to examine the validity of the free tropospheric quasi-equilibrium assumption for different spatial scales equivalent to GCM grid sizes from 64 to 4 km. The CRM output is averaged over different subdomains equivalent to different GCM resolutions to investigate the relationships between convection, as measured by convective precipitation and convective mass flux at the 600 mb level, and the generation of convective available potential energy, \( \text{dCAPE}_{ls} \), by subdomain-scale circulation. Results show that although observed advective forcing is used to drive the CRM simulation of convection, CAPE generation for convection on subdomain scales is dominated by model-generated circulation when subdomain sizes become smaller.
It is further shown that the total CAPE variation is controlled by contributions from the boundary layer properties on all spatial scales from 64 km down to 4 km subdomain sizes. The smaller the subdomain size, the larger the contribution from the boundary layer. For 64 km, the boundary layer contributes about 93% to the total CAPE variation. For sub-10 km (8–4 km) subdomain sizes, it contributes as much as 96% to the total CAPE variation. This implies that convective removal and large-scale generation of CAPE in the free troposphere are largely in balance, with their sum accounting for about 7–4% of the total CAPE change (Fig. 3).

Convective precipitation, mass flux at 600 mb and vertical velocity at 500 mb all correlate well with dCAPE_{ls} for subdomain sizes from 64 to 4 km, suggesting that the free tropospheric quasi-equilibrium assumption can be applied to GCMs with resolutions in the grey zone. However, the correlation coefficient decreases as sub-domain size decreases for subdomain sizes smaller than 16 km. This is likely due to increased randomness of convection and more noticeable scale-dependence of the relationships when the subdomain size reaches the grey zone. The regression slope also becomes smaller with decreasing subdomain sizes on these scales, although the decrease is relatively small. This can be explained by the fact that convection is positive-definite whereas the free tropospheric forcing can be either positive or negative. Thus, as the subdomain size becomes smaller, the same amount of CAPE generation by the grid-scale circulation will correspond to a smaller amount of convection in the averaging domain. This implies that although free tropospheric quasi-equilibrium can still be useable as the GCM resolution increases, it needs to be modified to account for the decreasing regression slope in order to make it scale-aware. Finally, the sensitivity of relationship between dCAPE_{ls} and convective mass flux at 600 mb to temporal scales for all subdomain sizes is examined. It shows that their correlations remain similar for all subdomain sizes for timescales of 30 min or longer. However, there is a
noticeable increase for 8 km and 4 km subdomain sizes from 6 to 30 min intervals. As for the regression slopes, there is also little change for different averaging intervals for all subdomain sizes, especially timescales longer than 30 min. These results indicate that the free tropospheric quasi-equilibrium between dCAPEls and convection holds well for 30 min or longer timescales at all subdomain sizes from 64 to 4 km.

As the computing power increases, GCMs have begun to increase the horizontal resolution to 25 km or higher. This requires convective schemes to be able to adapt to the higher resolutions. While some schemes have incorporated the fact that convective cloud fraction within a GCM grid box typically increases with the GCM resolution, not much has been done to modify convection parameterization closure. This study provides a diagnostic evaluation on a closure assumption, which can be used to guide future development of a scale-aware closure for convection parameterization in high-resolution GCMs.

Acknowledgements This material is based upon work supported by the Department of Energy, Office of Science, Biological and Environmental Research Program (BER) under Award Number DE-SC0022064 and by the NSF Grant AGS-2054697. Xu Wang is also supported by the National Natural Science Foundation of China (NSFC) under Award Number 41805069 and the Office of China Postdoctoral Council (OCPC) under Award Number 20190003. We would like to thank Dr. Xiping Zeng for making the cloud-resolving model simulation output available to us. The post-processed data used in this study and associated codes are provided through a public repository at http://doi.org/10.5281/zenodo.4542461.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

Arakawa A, Schubert WH (1974) Interaction of a cumulus cloud ensemble with the large-scale environment, Part 1. J Atmos Sci 31(3):674–701
Arakawa A, Wu CM (2013) A unified representation of deep moist convection in numerical modeling of the atmosphere. Part I J Atmos Sci 70:1977–1992. https://doi.org/10.1175/JAS-D-12-0303.1
Arakawa A, Jung JH, Wu CM (2011) Toward unification of the multi-scale modeling of the atmosphere. Atmos Chem Phys 11:3731–3742. https://doi.org/10.5194/acp-11-3731-2011
Bechtold P, Semane N, Lopez P, Chaboureau J-P, Beljaars A, Bormann N (2014): Representing equilibrium and non-equilibrium convection in large-scale models, J Atmos Sci DOI: http://dx.doi.org/https://doi.org/10.1175/JAS-D-13-0163.1
Benedict JJ, Sobel AH, Maloney ED, Frierson DM, Donner LJ (2013) Tropical intraseasonal variability in Version 3 of the GFDL Atmosphere Model. J Clim 26:426–499
Bryan GH, Wyngaard JC, Fritsch JM (2003) Resolution requirement for the simulation of deep moist convection. Mon Wea Rev 131:2727–2792
Davies L, Jakob C, May P, Kumar VV, Xie S (2013): Relationships between the large-scale atmosphere and the small-scale convective state for Darwin, Australia. J Geophys Res Atmos 118:11,534–11,545, https://doi.org/10.1002/jgrd.50645
Donner LJ (1993) A cumulus parameterization including mass fluxes, vertical momentum dynamics, and mesoscale effects. J Atmos Sci 50:889–906
Donner LJ, Phillips VT (2003) Boundary layer control on convective available potential energy: implications for cumulus parameterization. J Geophys Res 108:4701. https://doi.org/10.1029/2003JD003773
Emanuel KA, Neelin JD, Bretherton CS (1994) On large-scale circulation in convective atmospheres. Q J Roy Meteorol Soc 120:1111–1143
Fletcher JK, Bretherton CS (2010) Evaluating boundary-layer based mass flux closures using cloud-resolving model simulations of deep convection. J Atmos Sci 67:2212–2225
Fraedrich K, McBride JL (1989) The physical mechanism of CISK and the free-ride balance. J Atmos Sci 46:2642–2648. https://doi.org/10.1175/1520-0469(1989)046<3c2642:TPMOCA>3.0.CO;2
Fritsch JM, Chapell CF (1980) Numerical prediction of convectively driven mesoscale pressure systems, Part 1: Convective parameterization. J Atmos Sci 37:1722–1733
Gill AE (1980) Some simple solutions for heat-induced tropical circulation. Q J R Meteorol Soc 106(449):447–462
Grabowski WW (2001) Coupling cloud processes with the large-scale dynamics using the cloud-resolving convection parameterization (CRCP). J Atmos Sci 58:978–997
Grell GA, Freitas SR (2014) A scale and aerosol aware stochastic convective parameterization for weather and air quality modeling. Atmos Chem Phys 14:5233–5250. https://doi.org/10.5194/acp-14-5233-2014
Hohenegger C, Bretherton CS (2011) Simulating deep convection with a shallow convection scheme. Atmos Chem Phys 11:10389–10406
Jones TR, Randall DA (2011) Quantifying the limits of convective parameterizations. J Geophys Res 116:D08210. https://doi.org/10.1029/2010JD014913
Kain JS, Fritsch JM (1993) Convective parameterization for mesoscale models: The Kain-Fritsch scheme, The Representation of Cumulus Convection in Numerical Models, Meteorological Monographs, 24.
Kain JS (2004) The Kain-Fritsch convective parameterization: an update. J Appl Met 43:170–181
Kemball-Cook SR, Weare BC (2001) The onset of convection in the Madden-Julian Oscillation. J Clim 14:780–793
Kumar V, Jacob C, Protat A, Williams C, May P (2015) Mass-flux characteristics of tropical cumulus clouds from wind profiler observations at Darwin, Australia. J Atmos Sci 72:1837–1855
Kuo HL (1965) On the formation and intensification of tropical cyclones through latent heat release by cumulus convection. J Atmos Sci 32:1232–1240
Kuo HL (1974) Further studies of the parameterization of the influence of cumulus convection on large-scale flow. J Atmos Sci 31:1232–1240
Kuo HL (1974) On the parameterization of the influence of cumulus convection on large-scale flow. J Atmos Sci 31:1232–1240
Kuo HL (1974) On the parameterization of the influence of cumulus convection on large-scale flow. J Atmos Sci 31:1232–1240
community climate model, version 3. J Geophys Res 110:D09109. https://doi.org/10.1029/2004JD005617
Zhang GJ, Song X (2010) Convection parameterization, tropical Pacific double ITCZ, and upper ocean biases in the NCAR CCSM3. Part II: Coupled feedback and the role of ocean heat transport. J Climate 23:800–812
Zhang GJ, Wang H (2006) Toward mitigating the double ITCZ problem in NCAR CCSM3. Geophys Res Lett 33:L06709. https://doi.org/10.1029/2006GL025229
Zhang Y, Klein SA (2010) Mechanisms affecting the transition from shallow to deep convection over land: inferences from observations of the diurnal cycle collected at the ARM Southern Great Plains site. J Atmos Sci 67(9):2943–2959
Zhou L, Bao Q, Liu Y, Wu G, Wang W-C, Wang X, He B, Yu H, Li J (2015) Global energy and water balance: Characteristics from Finite- volume Atmospheric Model of the IAP/ LASG (FAMIL1). J Adv Model Earth Syst 7:1–20. https://doi.org/10.1002/2014MS000349

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.