Characteristics of scalar dispersion in turbulent-channel flow

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May 11, 2014

The dispersion of a passive scalar by wall turbulence, in the limit of infinite Péclet number, is analyzed using frozen velocity fields from the DNS by [del Álamo & Jiménez (2001)]. The Lagrangian trajectories of fluid particles in those fields are integrated and used to compute the first and second-order moments of the distribution of fluid-particle displacements. It is shown that the largest scales in the flow dominate turbulent diffusion, and the computed dispersions are in good agreement with measurements in the atmospheric boundary layer. This agreement can be understood noting that the life times of the large structures are much longer than the time scale of the transition from linear to Gaussian particle spreading in the cross-stream plane. Numerical experiments performed computing the Lagrangian trajectories in reference frames moving at different velocities suggest that this transition is controlled by the difference between the mean streamwise velocity and the phase speed of the large-scale structures of the cross-stream velocities. In the streamwise direction, the effect of the mean shear dominates and produces elongated scalar patches, with dispersion exponents which are different from the transverse ones.

1 Introduction

The prediction of the diffusion characteristics in turbulent shear flows, particularly in those near walls, is a notoriously difficult problem. While for example, the width of a contaminant plume follows relatively well a Gaussian spreading law in isotropic turbulence, or even in wall-bounded flows when measured far enough from the source, the same is not true when the spreading is measured closer to the source (Nokes & Wood, 1988), near the wall, or in atmospheric flows. This is an important consideration in many practical applications, such as in the prediction of dispersal of pollution from industrial plants, or of hazardous substances from either accidental or malicious releases. There are many other problems in which this subject is important, apart from the ones already mentioned. For instance, the diffusion of odors in the atmosphere is known to affect

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the migrational patterns of some insects, and it is not known whether similar
effects occur in other anisotropic flows, such as near-surface ocean turbulence,
where it could influence the rate of decay of the thermal wake of vehicles. The
solution to these problems is typically estimated using empirical laws (Brown
et al., 1997), or computed from semi-empirical models (Hanna et al., 1999).
Many of these models are used for regulatory purposes, and the fact that some
of them produce different results for the same input data is an indication of the
difficulty of the problem. This has led to the development of standardization
programs (Olesen, 1995) with the purpose of establishing systematic procedures
for the development and testing of dispersion models, based on compilations of
meteorological data from field experiments. However, due to the inherent dif-
ficulty of performing such experiments, the data sets are scarce, the number
of measured magnitudes is limited and some of them are of doubtful accuracy
(Olesen, 1994).

Since the atmospheric effects are observed over scales of hundreds of meters,
and there are sound theoretical arguments to expect small-scale turbulence to
produce Gaussian diffusion at such long distances, it is tempting to conclude
that the reason for the anomalous spreading is the presence of the very large
anisotropic scales (VLAS) in turbulent wall flows. Recently we have performed
a direct numerical simulation of turbulent channel flow at moderate Reynolds
number, which we believe to be the first one in which both the Reynolds number
is high enough to observe some scale separation, and in which the numerical box
is large enough not to interfere with the dynamics of the largest scales. The
present work, which used flow data from this simulation, is intended to be a first
step in using direct numerical simulation in the subject of atmospheric dispersal,
which might contribute to diminishing the current experimental uncertainties.

2 Computing dispersion from frozen fields

We will consider the release of a passive scalar into turbulent-channel flow in the
limit of infinite Péclet number $U_b h / D$ (here $U_b$ is the bulk mean velocity in the
channel, $h$ is the channel half-width, and $D$ is the kinematic diffusivity of the
scalar). In that case the dispersion of the scalar is controlled by the Lagrangian

| Case | $U_{adv}/U_b$ | Spatial Resolution | No. of Fields | No. of Particles per Field |
|------|---------------|--------------------|--------------|---------------------------|
| 1    | 0             | full               | 1            | $2 \times 10^5$           |
| 2    | 0.84          | full               | 1            | $2 \times 10^5$           |
| 3    | 0             | $\lambda_x, \lambda_z > 0.25 h$ | 3            | $2 \times 10^5$           |
| 4    | 0.84          | $\lambda_x, \lambda_z > 0.25 h$ | 3            | $2 \times 10^5$           |

Table 1: Summary of computed cases.
trajectories $\mathbf{x}$ of the fluid elements that transport it, given by

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x}(t), t).$$

(1)

The main difficulty of computing the Lagrangian trajectories of fluid particles lies in knowing the unsteady three-dimensional velocity field $\mathbf{u}(\mathbf{x}, t)$, which has to be computed from the continuity and the Navier-Stokes equations, leading to a problem much more expensive than the integration of (1) itself. Due to the preliminary nature of this work, and in order to avoid the computational expense of integrating in time the Lagrangian trajectories coupled with the velocity field, we have decided to calculate the former using frozen velocity fields that were already available from the DNS of turbulent-channel flow by [del Alamo & Jiménez (2001)]. This simulation was performed at a Reynolds number $\text{Re}_\tau = 550$ based on the friction velocity $u_\tau$ and on the channel half-width $h$, and its most important characteristic is that the computational domain is large enough not to interfere with the largest scales in the flow, which will allow us to study their effect on the scalar dispersion. The size of the numerical box is $L_x \times L_y \times L_z = 8\pi h \times 2h \times 4\pi h$ in the streamwise, wall-normal and spanwise directions, respectively. In isotropic turbulence the frozen-field approximation would be reasonable for times much shorter than the characteristic life time of the eddies, which is proportional to their turnover time. However, this might not be true in wall turbulence, where the flow features are known to travel in the streamwise direction with an advection velocity of the order of $U_b$ (Wills, 1964). This advection velocity, acting on scales of length $\lambda$, introduces a convective time scale $T_c \sim \lambda / U_b$ that is always shorter that the eddy-turnover time $T_L \sim \lambda / u_\tau$. We have tried to take into account the effect of the mean advection by integrating the Lagrangian trajectories from the frozen velocity fields in a moving reference frame,

$$\frac{d\mathbf{x}}{d\tau} = \mathbf{u}[\mathbf{x}(\tau) - \tau \mathbf{U}_{\text{adv}}, t_0].$$

(2)

Here $\mathbf{u}(\mathbf{x}, t_0)$ is the instantaneous frozen velocity field at $t = t_0$, and $\mathbf{U}_{\text{adv}} = (U_{\text{adv}}, 0, 0)$ is the velocity of the reference frame, which can be interpreted physically as a choice for the advection velocity of the frozen fields. This choice affects the paths of fluid particles by modifying their velocities relative to the turbulent structures. In order to evaluate the effect of the advection velocity of the frozen fields in scalar dispersion, we have integrated (2) for two different values of $U_{\text{adv}}$. In one case we have chosen $U_{\text{adv}} = 0$, while in the other one we have set it to be equal to the representative phase velocity of the large energetic scales in the flow, which have widths and lengths of the order of or larger than $h$ (del Álamo & Jiménez, 2001). There are several possible ways to compute the phase velocity of a flow variable (Wills, 1964; Hussain & Clark, 1981; del Álamo & Jiménez 2002). Here we have computed it from the frequency-wave number power spectrum $P(\omega, k_x, y)$ as in [Wills (1964)], where the phase velocity is defined as the velocity $U_c(y)$ of the moving reference frame for which the
Let $y/h$ be the non-dimensionalized spanwise coordinate. The phase velocity $U_c$ of the large-scale spanwise velocity component as a function of wall distance. Only structures such that $\lambda_x, \lambda_z \geq 0.25 h$ are taken into account. The advection velocity $U_{adv}$ of the frozen fields.

The frequency-wave number power spectrum has been computed using time histories of velocity fields that were available from the DNS, similarly to [Choi & Moin (1990)]. In the present case, the maximum and the minimum frequencies imposed by the temporal sampling are $\omega_{min} = 0.14 U_b/h$ and $\omega_{max} = 71 U_b/h$. Due to storage limitations, the time histories of the velocity field were spatially filtered by removing all the length scales either shorter or narrower than 0.25 h using a Fourier cut-off filter. Figure 1 displays the average phase speed of the low-pass filtered spanwise velocity fluctuations as a function of wall-distance, and non-dimensionalized with the bulk mean velocity $U_b$ (solid line), together with its average across the channel width (dashed line)

$$U_{adv} = \frac{1}{2h} \int_0^{2h} U_c(y)dy = 0.84 U_b.$$ 

We have chosen this value as the advection velocity of the frozen fields to be used in (2) for our second set of experiments. [Kim & Hussain (1993)] computed the propagation speeds of several turbulent magnitudes, including the velocity components, in a fully-resolved $Re_{\tau} = 180$ channel. They obtained a phase velocity of $\omega$ in the near-wall region approximately equal to $10 u_{\tau}$, which is the same that we have measured in the low-pass filtered $Re_{\tau} = 550$ channel. In the outer region, however, they obtain advection velocities approximately 10% higher than we do, and which are closer to the local mean velocity in their

Figure 1: ---, phase velocity $U_c$ of the large-scale spanwise velocity component as a function of wall distance. Only structures such that $\lambda_x, \lambda_z \geq 0.25 h$ are taken into account. ---, advection velocity $U_{adv}$ of the frozen fields.
Figure 2: Lagrangian time scale $U_b T_L / h$ of the spanwise velocity, low-pass filtered in $z$ ($\lambda_z > 0.25 h$), as a function of the streamwise wavelength $\lambda_x$ and wall distance $y$.

This is not surprising. If we think that turbulent structures propagate roughly at the average streamwise velocity that they feel, then the smaller scales should follow the local mean velocity better than the large ones.

The integral time scale $T_L$ in (3) measures the characteristic time associated to the turbulent fluctuations of a given magnitude with a certain length $\lambda_x = 2\pi/k_x$ at a given wall-distance, and in a reference frame moving with their local advection velocity. This magnitude can be interpreted as the Lagrangian time scale seen by an observer following the mean trajectories of the eddies, or in other words, as the typical lifetime of the structures of a given length. The Lagrangian time scale of the fluctuations of spanwise velocity has been represented in figure 2, low-pass filtered in $z$, as a function of streamwise wavelength $\lambda_x$ and wall distance $y$. The figure shows that the lifetimes of the large scales of $w$ can be very long, even comparable to a wash-out time $8\pi h / U_b$. The values of $T_L$ for the other two components of velocity, not shown here, are similar, and give an a-priori estimate of the longest intervals of time for which we can expect the integration of (2) to provide reasonably accurate results. Note that this prediction would only be true if the large scales controlled the characteristics of dispersion. In order to analyze their importance in this phenomenon, we have solved (2) using both fully-resolved and cut-off filtered fields. Overall, we have integrated (2) in four different cases, depending on the choice of $U_{adv}$ and of the spatial resolution. These cases have been summarized in table 1, indicating the number of different fields that have been used for each case, as well as the number of trajectories that have been computed per field. The time discretization is fourth-order Runge-Kutta, and third-order B-splines have been
$\langle x \rangle$ / $\sigma_z$ / $h$ $\sim$ $\langle x \rangle$ / $h$ $\sim$ $\langle x \rangle$ / $h$ $^{1/2}$

Figure 3: R.m.s. of the spanwise displacement $\sigma_z$ of fluid particles as a function of their mean streamwise displacement $\langle x \rangle$. The different curves correspond to $y_0/h = 0.1(0.1)$ from bottom to top. The dashed lines have logarithmic slopes 1 and 1/2. (a), using $h$ as the scale of $\sigma_z$ and $\langle x \rangle$; (b), using $w'(y_0)t$ as the scale of $\sigma_z$ and $U(y_0)t$ as the scale of $\langle x \rangle$.

used to interpolate the velocity field from the collocation points of the DNS.

3 Results. One-point statistics

The single-particle statistics $\langle x_i \rangle$ and $\sigma_i = \langle (x_i - \langle x_i \rangle)^2 \rangle^{1/2}$ are of great interest because they indicate respectively the mean displacement of the center of a typical scalar patch and its size in the three directions of space, and also because the latter is often measured as a function of the former in field experiments, which will allow us to test the approximation (2). These magnitudes are functions of the initial position $y_0$ of the fluid element, of its instantaneous position $y$, and of time. Operating on (1) it is possible to obtain (Hunt, 1985) that

$$\partial_t \sigma_i^2 = u'_i(y)u'_i(y_0) \int_0^t \rho_{ii}(r_x - \tau U_{adv}, r_z, y_0, y, t - \tau) d\tau,$$

where $\rho_{ii}$ is the two-point autocorrelation coefficient of the $i$th-component of the velocity vector. This magnitude is a function of the streamwise and spanwise separations $r_x$ and $r_z$, of the initial and instantaneous wall-distances, and of time. Note that the frozen-field approximation is equivalent to setting $t - \tau = 0$ in $\rho_{ii}$ in (4). For times and spatial separations short compared to the corresponding integral scales, the velocity field is almost fully correlated, $\rho_{ii} \approx 1$, and $y_0 \approx y$. We then have

$$\sigma_i \approx u'_i(y_0)t \sim \langle x \rangle \approx U(y_0)t.$$  

On the other hand, for very long temporal and spatial separations the velocity field is approximately decorrelated, $\rho_{ii} \approx 0$, and the integral in the right-hand side of (4) is roughly independent of its upper limit. We then obtain the
Figure 4: Fraction $F_i$ of the $i$th-component of turbulent kinetic energy contained in the cut-off filtered fields as a function of wall-distance. ---, $u$; -----, $v$; -------, $w$.  

A Gaussian-spreading law $\sigma_i \sim \langle x \rangle h^{1/2}$. Both asymptotic behaviors can be observed in figure 4 where the solid curves show $\sigma_z$ from case 2 as a function of the mean streamwise displacement for ten equispaced intervals of initial wall-distances, from the wall to the center of the channel. In figure 3(a) we have used the channel half-width as the length scale for $\sigma_z$ and $\langle x \rangle$, while in figure 3(b) we have scaled $\sigma_z$ with $u'(y_0)t$ and $\langle x \rangle$ with $U(y_0)t$. The figures show that in the short-range limit, the curves representing $\sigma_z$ are parallel to the dashed line with logarithmic slope 1, while far away from the release point the curves are roughly parallel to the dashed line with logarithmic slope $1/2$. It can be observed in figure 3(b) that the scaling (5) collapses well the plume width corresponding to different release points, at least at short distances from the source. As expected, the collapse worsens beyond the turning point in the curves, where their slope starts decreasing and (5) is no longer valid. The characteristic position of this turning point is a measure of the shortest integral scale involved in the dispersion process.

Equation (4) also suggests that the large scales may play an important role in turbulent dispersion. Coherent structures with $\lambda_x/h > 2$ and $\lambda_z/h \approx 1 - 2$ are known to be correlated all across the channel half-width and to contain a large fraction of the turbulent kinetic energy (del Álamo & Jiménez 2002), which suggests that they should contribute substantially to the right-hand side of (4). Figure 4 displays the fraction $F$ of the total streamwise (solid line), wall-normal (dashed line) and spanwise (dotted line) kinetic energy contained in the cut-off filtered fields as a function of wall-distance. The figure shows that the structures which are longer and wider than $0.25 h$, contain most of the
Figure 5: R.m.s. of the displacement of fluid particles in the $i$th-direction as a function of their mean streamwise displacement $\langle x \rangle$. Top, $\sigma_x$; middle, $\sigma_z$; bottom, $\sigma_y$. Full DNS fields (case 2); cut-off filtered DNS fields (case 4). In all the cases $U_{adv} = 0.84 U_b$. (a), $y_0^+ < 100$; (b), $0.2 < y_0/h < 1$.

The kinetic energy of $u$ and $w$ in the outer region of the flow, and hence could be expected to produce values of $\sigma_x$ and $\sigma_z$ similar to the ones generated by the full fields. On the other hand, the small scales of $v$ contain relatively more kinetic energy than those of $u$ and $w$, suggesting that the value of $\sigma_y$ computed from the filtered fields should be less approximate to the one obtained from the full fields. This is actually what is observed in figure 5 where we have plotted the three components of $\sigma$ (from top to bottom $\sigma_x$, $\sigma_z$ and $\sigma_y$) computed from the full (case 2, solid lines) and the filtered (case 4, dashed lines) moving frozen DNS fields. In figure 5(a) the patch size has been averaged for fluid particles released in the near-wall region ($y_0^+ < 100$), while in figure 5(b) the average has been performed for initial positions in the outer region ($0.2 < y_0/h < 1$). The results from the filtered fields compare fairly well to those from the fully resolved ones in the outer region, while they underestimate the different components of the r.m.s. in the near-wall region. Note that the agreement between the different sets of data is better wherever $F$ is higher and vice-versa, supporting the argument above. These observations agree with the previous work by [Armenio et al. (1999)], who performed a similar analysis using time-evolving velocity fields, with application to LES modeling. The results from the stationary frozen DNS fields (cases 1 and 3), not shown here, behave in the same way as the ones we have presented in figure 5.

Note that the standard deviations in figure 5 are always much lower than $\langle x \rangle$, implying that the basic motion of the particles is advection by the local mean velocity, $\langle r_x \rangle \approx U(y_0)\tau$, while the spreading around that position is slow.

Figure 5 also shows the relative magnitude of $\sigma$ along the different axes, which can help us have an idea about the evolution of the shape of a typical scalar patch with the distance to the source. In the short-range region the three standard deviations grow at the same rate, and a typical cloud of scalar
would initially conserve its original shape as it moves away from the release point. However, after the cloud has traveled a certain distance it would start elongating very rapidly, as we can deduce from the increase in the slope of $\sigma_x$ that takes place in the figure at long distances. The comparison of figures (a) and (b) indicates that this phenomenon occurs at a shorter distance to the source for lower values of $y_0$, which is more apparent in figure (a). This figure displays the logarithmic slope of $\sigma_x$ (from case 2) as a function of time for ten equispaced intervals of $y_0$. The slope of $\sigma_x$ increases and reaches a maximum value in times which are longer as the curves move from the left to the right, which correspond to increasing values of $y_0$. There are strong arguments to think that this effect is due to the mean shear. Scaling time with $\partial_y U$ in figure (b) makes the position of the maxima of the different curves collapse, indicating that the time scale associated to this phenomenon is the inverse of the mean shear. Besides, in a different experiment we integrated (2) using fields from which we had removed the mean velocity profile, and the resulting $\sigma_x$ behaved in the same way as $\sigma_y$ and $\sigma_z$, and did not show the transient increase in slope. Tennekes & Lumley (1972) show that in a flow subjected to a uniform shear $S$, the dispersion in the streamwise direction increases asymptotically with time as $(St)^{3/2}$. This value of the logarithmic slope lies roughly in the center of the set of different maximum values that we obtained from the DNS fields, and the scatter in the numerical values might be explained by the fact that $\partial_y U$ is not uniform in a turbulent channel.

3.1 Comparison with atmospheric data

In section 2 we discussed the a-priori validity of our study, obtaining a rough estimate of the longest time intervals for which we could expect reasonable results from the model problem (2). Here we analyze a-posteriori the frozen-turbulence approximation, by comparing the computed dispersion characteristics with those measured in the atmospheric boundary layer. Figure 7 displays the r.m.s. of the spanwise displacement of fluid particles as a function of their mean streamwise displacement. The symbols come from field experiments, most of which were compiled by Nielsen et al. (2002) and Olesen (1995). The atmospheric data sets are difficult to compare among themselves and to the numerical results. In general, the experiments consist of releasing a passive tracer from a smokestack and measuring its near-ground concentration along arcs situated at increasing distances from the release point. However, neither the releases nor the measurements were performed at the same ground distances in the different experiments, the monitoring procedures also differed, and so did the topological and meteorological conditions. Thus, any quantitative conclusion from the observation of figure 7 should be taken as orientative, as it is also suggested by the dispersion of the data in the figure. The solid line comes from our numerical results with $U_{adv} = 0.84 U_b$, while the dashed line corresponds to $U_{adv} = 0$. In both cases we have represented the average values over the interval of particle positions $y/h < 0.1$, in order to compare with the atmospheric near-ground measurements. It should be noted however, that the numerical results con-
tain the contributions from particles released at all the possible wall distances across the channel, while the particles were released at a single ground-distance in the experiments. Even so, the agreement between the numerical and the experimental results is reasonably good for the results from the moving fields. They underestimate somewhat $\sigma_z$ at long streamwise distances, but this could be due to the fact that the atmospheric data in the corresponding experiment (the Copenhagen data set, represented with squares) were taken under unstably stratified atmospheric conditions, with Monin-Obukhov lengths of the order of $-100$ m (Olesen, 1994). The results from the stationary fields look qualitatively correct, but they predict widths lower than the experimental values. This can be understood considering that, since the particles move approximately with the mean flow velocity, the mean streamwise separation that goes into the correlation function in (4) is \( \langle r_x \rangle \approx U(y_0) \tau \), so that the first argument in the autocorrelation coefficient of \( w \) is
\[
\langle r_x \rangle - \tau U_{adv} \approx (U(y_0) - U_{adv}) \tau.
\]
In the stationary fields, fluid elements separate faster from their initial positions with respect to the flow structures than in the moving ones, because the difference between the mean velocity and that of the reference frame is higher in the former than in the latter. This is true all across the channel, except in the near wall region where \( U(y_0) \) is small. The fluid particles in the stationary snapshots therefore feel a more uncorrelated velocity field than in the advecting cases, leading to lower values of $\sigma_z$. This argument is supported by figure S which shows the logarithmic slope of $\sigma_z$ as a function of time (figure S(a)) and as a function of \( |\langle r_x \rangle - \tau U_{adv}| \) (figure S(b)), for five equispaced intervals of \( y_0 \) from the wall to the center of the channel. In figure S(a) the logarithmic slope of $\sigma_z$ decreases faster with time for the stationary fields (case 1), plotted with dotted
Figure 7: R.m.s. of the spanwise displacement of fluid particles, $\sigma_z/h$, as a function of their mean streamwise displacement $(x)w'/(Uh)$. Lines, data from DNS fields; $\cdots$, $U_{adv} = 0.84U_h$; $\cdots$, $U_{adv} = 0$. Symbols, atmospheric experiments; $\circ$, Ecofin Project (Nielsen et al., 2002); $\triangle$, Prairie Grass (Olesen, 1995); $\ast$, Lillestrøm; $\Box$, Copenhagen (Olesen, 1995).
lines, than for the moving ones (case 2), represented with solid lines. On the
other hand, the curves in figure 8(b) collapse fairly well except for the curve on
the left-hand side of the plot, which corresponds to \( y_0/h = 0.2 \). This position
is near the critical layer at \( y_c = 0.23h \), where \( U(y_c) = U_{adv} \). In that region, the
mechanism that we have just described is weaker, and it is reasonable to think
that the dispersion is dominated by the turbulent velocity fluctuations and not
by the mean advection. Note that in the case of the stationary fields the critical
layer is located at the walls, which may explain why all the dotted lines collapse
well in figure 8(b). The behavior that we have observed in figure 8 is common
to the results from both the fully-resolved and the cut-off filtered fields.

As we have mentioned, the location of the point where the logarithmic slopes
of \( \sigma_{y,z} \) decay from 1 to their asymptotic value of \( 1/2 \), is a measure of the integral
scale most relevant in turbulent dispersion in the cross-stream plane. The fact
that the results from the moving frozen fields, which neglect the time evolution
of turbulent structures, are able to predict the position of the turning region in
the data from the field experiments, can help us identify that integral scale. The
large scales of \( w \) have lifetimes (see figure 2) which are approximately 4 times
longer than the time scale of to the decay of the slope of \( \sigma_z \) from the moving fields
(figure 5b). This suggests that the temporal decay of the turbulent structures
may not be important in the decorrelation that the particles feel as they move
in the flow. On the other hand, the streamwise separation corresponding to the
transition of \( \sigma_z \) shown in figure 8(b), is \( r_x \approx h \). This length is essentially equal
to the position of the peak of the premultiplied energy spectrum of \( w \) (del Álamo
& Jiménez, 2001), which is a measure of its streamwise integral scale. These
observations also apply to the wall-normal direction, for which the experimental
information is much more scarce than in the spanwise direction. They suggest
that the transition in \( \sigma_{y,z} \) may be caused by the difference between the mean
velocity of the flow, and the phase speed of the velocity components in the
cross-stream plane.

4 Conclusions

The results show that the large scales of turbulent channels play a very impor-
tant role in turbulent dispersion in the outer region of the flow, specially in the
streamwise and spanwise directions. These structures contain a large fraction
of the turbulent kinetic energy, and they are correlated across the full channel
(del Álamo & Jiménez 2002), so they are expected to contribute substantially to
the standard deviations in (1). Filtered velocity fields retaining only structures
with \( \lambda_x, \lambda_z > 0.25h \) produce more than 90% of \( \sigma_{x,z} \) and roughly 80% of \( \sigma_y \)
in the outer region. These results indicate that LES should be a valuable tool in
the study of scalar dispersion.

The transition from linear to Gaussian spreading is due to the decorrelation
of the velocity field along the Lagrangian trajectories of the particles. The life
times of the large scales of the spanwise velocity are roughly 4 times longer
than the time scale of the decay in the slope of the plume width from 1 to 1/2.
Figure 8: Logarithmic slope of $\sigma_z$ for $y_0/h = 0.2(0.2)1$. (a), as a function of time $U_b t/h$. (b), as a function of the separation $|\langle r_x \rangle - \tau U_{adv}|/h$. $U_{adv} = 0.84 U_b$, $U_{adv} = 0$. The solid line that does not collapse well in the left-hand side of (b) corresponds to $y_0 = 0.2 h$, near the critical layer at $y_c = 0.23 h$.

Thus, the time evolution of turbulent structures does not seem to be significant in the decorrelation process that leads to Gaussian spreading, and to a first approximation, it may be possible to study turbulent diffusion using frozen velocity fields. In fact, we have integrated the Lagrangian trajectories of fluid particles from frozen velocity fields, obtaining values of $\sigma_z$ that agree well with atmospheric measurements. The agreement is better when the trajectories are computed in a reference frame moving with the average phase velocity of the large scales. The stationary frozen fields, on the other hand, produce values of $\sigma_z$ lower than those from the field experiments. This is so because the decorrelation times experienced by the fluid elements in the stationary fields are shorter than those in the moving ones. In both cases, the decay in the slope of $\sigma_z$ takes place when the streamwise separation of the particles relative to the velocity fields, $|\langle r_x \rangle - \tau U_{adv}|$, is roughly equal to the streamwise integral scale of $w$. Since the particles move in the $x$ direction following approximately the mean velocity profile, their separation with respect to their initial positions is given by $(U(y_0) - U_{adv}) \tau$, suggesting that the main cause of the transition from linear to Gaussian spreading is the difference between the mean streamwise velocity and the phase speeds of the velocity components in the cross-stream plane.

The mean shear is the dominating mechanism in the streamwise direction. It generates values of $\sigma_x$ much greater than the ones in the other two directions, and leads to very elongated patch shapes. Although this consideration is not important in the case of the dispersion of contaminants from a continuous source, it may be fundamental in the case of discrete releases.

This work was supported by grant BFM 2000-1468 of CICYT. J.C.A. was supported by the CTR and by the Spanish Ministry of Education. The authors would like to thank Helge R. Olesen at NERI, Denmark for providing a digital version of the atmospheric data sets collected by the Harmonisation Group. We
are also indebted to Julian C. R. Hunt, with whom we had fruitful discussions related to the subject of this work.

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