Multiphonon Approaches to Complex Spectroscopy

Lo Iudice N, Bianco D, Knapp F† Andreozzi F, Porrino A, P. Vesely
Dipartimento di Scienze Fisiche, Universitá di Napoli Federico II and INFN, Sezione di Napoli, Napoli, Italy
E-mail: loiudice@na.infn.it

Abstract. An equation of motion method for solving the nuclear eigenvalue problem in a basis of microscopic multiphonon states has been upgraded so as to generate states solely composed of quasiparticle Tamm-Dancoff phonons. The method is applied to the neutron rich $^{20}$O. A space covering up to three-phonon states is adopted to compute the electric dipole response.

1. Introduction

The most common extension adopted to describe the anharmonic features of collective modes and giant resonances is known as second random-phase approximation (SRPA). It consists in coupling the RPA modes to two-particle two-hole or four quasiparticles configurations [1]. Recent implementations were based on density functional non relativistic [2] and relativistic [3] theories.

Another popular generalization is the quasiparticle phonon model (QPM) [4]. This approach, being based on a Hamiltonian of separable form, is able to extend further the RPA approach by including also a fraction of three-phonon states.

SRPA [3, 5] and QPM [6, 7] were applied to neutron rich nuclei to study the properties of the so called pygmy resonance induced by the oscillation of the valence neutron skin against the core. Such a resonance, whose first evidence was provided about a decade ago [8], is now under intense experimental investigation. A list of references can be found in [9].

We proposed, recently, an equation of motion phonon method (EMPM) [10, 11] for solving the eigenvalue problem in a basis of multiphonon states generated iteratively starting from Tamm-Dancoff phonons. These multiphonon states, however, had a hybrid structure. They were composed of particle-hole ($ph$) operators coupled to $(n-1)$-phonon states.

This inconsistency is removed in the new formulation, where all quantities are expressed solely in terms of phonons. Moreover, these phonons are constructed in the quasiparticle ($qp$) scheme so as to allow the study of open shell nuclei.

† Present address: Faculty of Mathematics and Physics Charles University, Prague, Czech Republic
Simple and physically transparent equations generate iteratively a multiphonon basis which can be used to solve the eigenvalue problem for a Hamiltonian of general form. No approximations, apart from the one inherent the \(qp\) formalism, are involved. 

The method was already applied to the neutron rich \(^{20}\)O, chosen as test-ground, to compute the \(E1\) response in a space including up to three phonons \([12]\). A Nilsson+BCS basis in a restricted space was adopted. Here, we use a HF+ BCS basis in the same configuration space.

2. Brief outline of the method

We first solve the \(qp\) TDA eigenvalue equation

\[
\sum_{rs,qt} A_{rs,qt}^\lambda c_{qt}^\lambda = \left\{ \left[ (\epsilon_r + \epsilon_s) - E_\lambda \right] \delta_{rq} \delta_{st} + \mathcal{V}_{rs,qt} \right\} c_{qt}^\lambda = 0,
\]

where \(\epsilon_r\) are quasiparticle energies and \(\mathcal{V}_{rs,qt}\) are the matrix elements of the two-body potential between two-quasiparticle states.

The eigenstates \(|\lambda\rangle\) of energy \(E_\lambda\) define the phonon operators

\[
O_\lambda^\dagger = \sum_{r<s} c_{rs}^\lambda \alpha_r^\dagger \alpha_s^\dagger,
\]

where \(\alpha_r^\dagger (\alpha_r)\) creates (annihilates) a quasiparticle with respect to the BCS vacuum \(|0\rangle\).

The main objective of the method is to generate a set of multiphonon states \(|n;\alpha\rangle\) of the form

\[
|n;\beta > = \sum_{\lambda\alpha} C^{(\beta)}_{\lambda\alpha} O_{\lambda}^\dagger |n-1;\alpha > .
\]

The procedure developed to generate these states starts with introducing the equations of motion

\[
< n, \beta | [H, O_{\lambda}^\dagger] | n-1, \alpha > = \left( E^{(n)}_{\beta} - E^{(n-1)}_{\alpha} \right) < n, \beta | O_{\lambda}^\dagger | n-1, \alpha > .
\]

We have to expand the commutator on the left-hand side and use closure relations, just as in Refs. \([10, 11]\). After making additional manipulations, we obtain

\[
\sum_{\lambda'\alpha'} A_{\lambda\alpha,\lambda'\alpha'}^{\beta} X_{\lambda\alpha}^{\beta} = E_{\beta} X_{\lambda\alpha}^{\beta},
\]

where

\[
X_{\lambda\alpha}^{(\beta)} = < n, \beta | O_{\lambda}^\dagger | n-1, \alpha >
\]

and

\[
A_{\lambda\alpha,\lambda'\alpha'} = (E_\lambda + E_\alpha) \delta_{\lambda\lambda'} \delta_{\alpha\alpha'} + \mathcal{V}_{\lambda\alpha,\lambda'\alpha'}.
\]

As shown by the above formula, the matrix \(A_{\lambda\alpha,\lambda'\alpha'}\) is expressed solely in terms of phonons and is in close correspondence with the \(qp\) TDA matrix \(A_{rs,qt}\) defined by the eigenvalue equation (1). Indeed, \(A_{\lambda\alpha,\lambda'\alpha'}\) is formally deduced from \(A_{rs,qt}\) by replacing the quasiparticle energies \(\epsilon\) with the phonon energies \(E_\lambda\) and the quasiparticle two-body potential \(\mathcal{V}_{rs,qt}\) with the phonon-phonon potential \(\mathcal{V}_{\lambda\alpha,\lambda'\alpha'}\).
Equation (5) is not yet an eigenvalue equation. This is obtained once we express the $X$ amplitudes (6) in terms of the $C$ coefficients by making use of the expansion (3). Once this is done, we obtain from (5) the generalized eigenvalue equation within the $n$-phonon subspace

$$\sum_{\lambda' \alpha'} (A \mathcal{D})(\lambda \alpha, \lambda' \alpha') C^{\beta}_{\lambda' \alpha'} = E^\beta \sum_{\lambda' \alpha'} \mathcal{D}_{\lambda \alpha, \lambda' \alpha'} C^{\beta}_{\lambda' \alpha'},$$

where

$$\mathcal{D}_{\lambda \alpha, \lambda' \alpha'} = < n - 1, \alpha' | O_{\lambda'} O_\lambda^\dagger | n - 1, \alpha >$$

is the metric matrix. $A$ and $\mathcal{D}$ are expressed in terms of quantities defined in the $(n - 1)$-phonon subspace. The recursive character of these formulas allows to solve Eqs. (8) iteratively. The redundant states are eliminated by a procedure [10, 11] based on the Cholesky decomposition method. A set of orthonormal multiphonon states $\{|0>, |1>, \ldots |n, \alpha > \ldots\}$ is generated. The Hamiltonian is diagonal in each $n$-phonon block. The terms coupling different blocks are also expressed solely in terms of phonons and have a simple structure. The Hamiltonian matrix can therefore be brought easily to diagonal form yielding the eigenfunctions

$$| \Psi_\nu > = \sum_{n, \alpha} C^{(\nu)}_{\alpha}(n; \alpha).$$

As shown by the formula, the ground state is treated at the same footing of the other states and is explicitly correlated. Indeed, our approach does not rely on any approximation, except for the one inherent the Bogoliubov $qp$ transformation.

3. Application of the method to $^{20}$O

A test calculation of the $E1$ response in $^{20}$O was already performed [12] using a Nilsson basis. The lowest three major shells, up to $(s, d)$, were used to generate the constituent $qp$ TDA phonons. All multiphonon states spanning up to the entire three-phonon subspace were taken into account. The space considered is slightly more restricted than the one used in a shell model calculation [13].

Here, we replace the Nilsson with a HF basis. The HF states incorporate high energy configurations even if the calculation is performed in a restricted space. They are used to generate the quasiparticle states in BCS. As in [12], a Brueckner $G$-matrix derived from the CD-Bonn nucleon-nucleon potential are adopted.

The spurious admixtures induced by the center of mass motion and by the particle number violation, caused by the Bogoliubov transformation, are removed in the preliminary TDA stage. We have just to adopt the Gramm-Schmidt procedure to orthogonalize the $2qp$ states to $\propto \vec{R}_{CM} |0 >$ and $\propto \hat{n} |0 >$ and use this spurious free basis to generate the TDA phonons. The multiphonon states resulted to be spurious free as well, since their phonon structure remains unchanged all along the procedure.

As shown in Figure 1, the $E1$ reduced strength gets dramatically damped and spread as the two-phonon space is included. The three-phonons induce further quenching and fragmentation.
Figure 1. $E1$ reduced transition strengths using a HF basis in different multiphonon spaces

Figure 2. $E1$ reduced transition strengths obtained from using a Nilsson basis

Figure 3. $E1$ cross sections computed in a HF+BCS basis

The phonon coupling induces damping and quenching also in the Nilsson basis (Figure 2). Though HF and Nilsson spectra are seen to differ in several details, in both cases, few low-lying peaks appear below 16 MeV, consistently with early experiments [8], especially when three-phonon states are included.

The effects of the phonon coupling are visible also in the cross sections, shown in Figure 3. Due to the smoothing action of the width ($\Gamma = 1 MeV$), however, the cross section emphasizes the damping and spreading induced by the two phonons, while the role of the three phonons is attenuated.

The transition densities shown in Figures 4 disclose the nature of two typical states. The plot in the left panel clearly indicates that the peak in the GDR region describes an out of phase oscillation of protons against neutrons. On the other hand,
the transition density pertaining to a low-lying state plotted in the right panel is typical of an oscillation of neutron skin versus the core indicating the occurrence of a pygmy resonance.

No evidence of such a collective mode emerged from the calculation using the Nilsson basis [12]. The low-lying states obtained in this basis were simply non collective. Apparently, the high energy configurations incorporated in the HF basis play an important role in determining the relative weight of proton and neutron components.

The discrepant results obtained in the HF and Nilsson basis strongly suggests that a larger configuration space must be used in generating the constituent TDA phonons.

On the other hand, the number of multiphonon states becomes prohibitively large as the number of major shells increases. A severe truncation of the multiphonon space is necessary. The phonon formalism is naturally suited to such a task. It allows, in fact, to select the TDA phonons according to their energy and collectivity. A work along these line is being completed.

References

[1] Wambach J 1988 Rep. Prog. Phys. 51 989
[2] Gambacurta D, Grasso M and Catara F 2010 Phys. Rev. C 81 054312
[3] Litvinova E, Ring P and Tselyaev V 2010 Phys. Rev. Lett. 105 022502
[4] Soloviev V G 1992 Theory of atomic nuclei : Quasiparticles and Phonons (Bristol: Institute of Physics)
[5] Paar N, Vretenar D, Khan E and Colò G 2007 Rep. Prog. Phys. 70 691 for review and references
[6] Tsoneva N, Lenske H and Ch. Stoyanov 2004 Phys. Lett. B 586 213
[7] Tsoneva N and Lenske H 2008 Phys. Rev. C 77 024321
[8] Leistenschneider A et al. 2001 Phys. Rev. Lett. 86 5442
[9] Tami A et al. 2011 Phys. Rev. Lett. 107 062502
[10] Andreozzi F, Knapp F, Lo Indice N, Porrino A and Kvasil J 2007 Phys. Rev. C 75 044312
[11] Andreozzi F, Knapp F, Lo Indice N, Porrino A and Kvasil J 2008 Phys. Rev. C 78 054308
[12] Lo Indice N, Bianco D, Knapp F, Andreozzi F, Porrino A and Vesely P 2011 Proceedings 14th International Symposium on Capture Gamma-Ray Spectroscopy and Related Topics (CGS14)/(Guelph) (Singapore: World Scientific) to be published
[13] Sagawa H and Suzuki T 1999 Phys. Rev. C 59 3116