The system of vertical movement stabilization of the geophysical rocket for the purpose of high-precision measurement

A S Verkner and E O Guryanova
MIREA-Russian Technological University, Institute of Cybernetics, automatic systems department, Moscow 119454 Russia
aleksverk@mail.ru

Abstract. The paper shows the system of vertical movement stabilization of the geophysical rocket (GR) for the purpose of high-precision measurement. The stabilization system is one of the main systems comprising GR’s vehicle information and control system (VICS). The model uses kinematic relations reflecting continuous changes of yaw, pitch and roll angles expressed with dynamic and kinematic equations. The computer model of GR vertical movement stabilization was developed, verified and simulated by means of SimInTech dynamic modeling.

1. Introduction.
The purpose of this paper is the research and simulation of the GR vertical movement. The subject of the research is the GR dynamic control algorithms являющиеся алгоритмы управления динамикой GR in the mode of stabilization at Euler angles. The significance of this paper relates to the improvement of GR dynamics and specifically the obstacle avoidance algorithms. The practical significance comes from the fact that most tests are in virtual mode or simulated, which requires computer models easy to scale and parametrize.

2. GR VICS stabilization system.
Autopilot (Figure 1) controls the GR mass center, as well as GR stabilization with the reference to the mass center of three connected yaw ($\psi$), pitch ($\theta$) and roll ($\Phi$) rocket axis. [1]

![Figure 1. Functional diagram of GR autopilot.](image)
3. Description of mathematical model of UAV dynamics for stabilization problem.

The mathematical model is described by the kinematic and dynamic equations for GR, which were derived using Euler’s equations, Newton’s second law and kinematic conversions.

The following allowances were taken in the model and represented by constant values: resistance force affecting the rocket dynamics, aerodynamic and traction moments, inertia moment, engine traction.

The specifications and parameters of MP-12 geophysical rocket were taken as basic data.

Table 1. Basic data.

| Acceleration of gravity (m/s²) | Mass (kg) | Aerodynamic and traction moments (N*m) | Moment of inertia (kg*m²) | Aerodynamic and propulsive forces (N) | Elements of the matrix guides of the cosines |
|-------------------------------|-----------|---------------------------------------|--------------------------|--------------------------------------|-----------------------------------------------|
| g=9.81                        | m=1400    | m₁=130                               | I₁=0.4127                | fₐ,p₁=193.4                          | t₁₃=0.000313                                  |
|                               |           | m₂=128                               | I₂=1.39                  | fₐ,p₂=201.7                          | t₂₃=0.0004                                    |
|                               |           | m₃=100                               | I₃=1.02                  | fₐ,p₃=188.6                          | t₃₃=0.00027                                   |

Euler’s equations:

\[
\begin{align*}
\frac{dp}{dt} &= I_1^{-1} \left[ (I_2 - I_3) qr + m_{B1} \right] \\
\frac{dq}{dt} &= I_2^{-1} \left[ (I_3 - I_1) pr + m_{B2} \right] \\
\frac{dr}{dt} &= I_3^{-1} \left[ (I_1 - I_2) pq + m_{B3} \right]
\end{align*}
\]  

(1)

, where p, q, r – angle velocity in axis X, Y, Z; I₁, I₂, I₃ – inertia moments; mₐ₁, mₐ₂, mₐ₃ – aerodynamic and traction moments.

Quaternion matrix:

\[
\begin{align*}
\dot{q}_0 &= 0 - p - q - r \\
\dot{q}_1 &= p 0  r - q \\
\dot{q}_2 &= 2 q - r 0 p \\
\dot{q}_3 &= r q - p 0
\end{align*}
\]  

(2)

, where q₀, q₁, q₂, q₃ – quaternion elements.

Euler’s angle equations:

\[
\begin{align*}
tg (\psi) &= \frac{2(q₁q₂ + q₀q₃)}{q₀^2 - q₁^2 - q₂^2 - q₃^2} \\
\sin (\theta) &= -2(q₁q₃ + q₀q₂) \\
tg (\phi) &= \frac{2(q₂q₃ + q₀q₁)}{q₀^2 - q₁^2 - q₂^2 - q₃^2}
\end{align*}
\]  

(3)

, where ψ, θ, φ – yaw, pitch and roll angle; q₀, q₁, q₂, q₃ – quaternion elements. [2]
4. Computer modelling of GR kinematic and dynamic equations.

Computer modelling of GR kinematic and dynamic equations is performed in SimInTech. While modelling, the programming language blocs were used, that create the blocs enable of performing complex operations.

![Figure 2. Computer modelling of GR kinematic and dynamic equations.](image)

**Figure 3.** Dependence of pitch angle on GR movement time.

Dependence of pitch angle on GR movement time (Figure 3) is a graphical representation of mathematical model equations. The computer model of GR kinematic and dynamic equations (Figure 2) is universal and can be used for the problem of yaw, pitch and roll stabilization. To control the pitch stabilization, the computer model (Figure 2) should be converted and simplified.

5. Computer modelling of GR stabilization system in vertical plane.

![Figure 4. Functional diagram of GR pitch stabilization system.](image)

The control action, represented by traction moment $m_{R2}$, was added to computer model of GR kinematic and dynamic equations (Figure 2). Since the computer model is multi-connected and linear, angle velocities $p$ and $r$ were taken as constant values, which significantly simplifies the computer model. The model feedback was also complemented with converter for connection of the pitch angle.
and traction moment. As a result of the conversions, the computer model of GR stabilization system in vertical plane was obtained (Figure 5).

**Figure 5.** Computer model of GR pitch stabilization system.

The figure of system poles (Figure 6) shows that the stabilization system is stable as all roots are in the negative half-plane. [4]

**Figure 6.** Poles of GR pitch stabilization system.

The efficiency of the system was evaluated by the system transient response (Figure 7). The time of transition process makes $t_{pp}=0.006 \tau$, the overshoot makes $\sigma=1.5\%$.

**Figure 7.** Transient response of GR pitch stabilization system.
6. Development of the continuous controller for the problems of GR pitch stabilization.

![Diagram](image)

**Figure 8.** Structural scheme of GR pitch stabilization with the use of continuous controller.

The proportional-integral differential (PID) controller was used to improve the system performance, as it is the most commonly-used type of continuous controllers. The SimInTech computer optimization was used for setting of PID-controller parameters.

PID-controller can implement various kinds of control laws, each makes different effect on the system. The analysis of the effect of P, PI and PID – control laws on GR vertical dynamics stabilization system was made to achieve the ultimate results. The system with PID-controller showed the best quality performance.

![Diagram](image)

**Figure 9.** Computer model of GR pitch stabilization system for implementation of the PID-control law.

The coefficients of PID control calculated by software optimisation make \( CP = 2.40069; \ CI = 12.8; \ CD = 0.01125. \)

The figure (Figure 10) depicts the poles of the PID-control law system and shows that the system is stable as all poles are in the left half-plane.

![Diagram](image)

**Figure 10.** The poles of GR pitch stabilization system with PID-controller.

The efficiency of the system was evaluated by the system transient response (Figure 11). The time of transition process makes \( \text{tpp}=0.0015 \tau. \), the overshoot makes \( \sigma=8.6\%. \) According to the results of computer modelling, in the system where continuous controller is used the time of the transient response has decreased 4 times.
Figure 11. Transient response of GR stabilization system shows the implementation of the PID-control law.

7. Development of digital GR stabilization system in vertical plane.

Figure 12. Block diagram of digital GR stabilization system in vertical plane.

Recently, to make the practical application of the control system more user-friendly, the digital control systems are used (DCS). To obtain the digital GR stabilization system in vertical plane, the analog control system should be converted to digital one. The model of analog system should be supplemented with such discrete elements as analog-to-digital converter (ADC), digital-to analog converter (DAC), and the continuous PID-controller should be replaced by the discrete one, thus the whole control system would be digital.

The coefficients of the discrete controller were obtained by converting the coefficients of the continuous controller to digital coefficients.

The sampled-data transfer function of the PID-controller for GR pitch stabilization system is shown in the formula (4):

$$W_{reg}(z) = \frac{11.25 - 24.90069z + 13.66349z^2}{-z + 2}$$

(4)

For computer model of the digital GR stabilization system in vertical plane (Figure 13), the following DAC blocks were used: the level quantizer, the amplifying element and the extrapolator. The ADC and discrete PID-controller were implemented with the use of prefabricated blocks. [5]
Figure 13. Computer model of the digital GR stabilization system in vertical plane.

The figure (Figure 14), depicting the poles of the digital system, shows that the system is stable as all roots are in the negative half-plane.

Figure 14. The poles of the digital GR stabilization system in vertical plane.

The figure (Figure 15) shows the transient response of the digital GR stabilization system in vertical plane. The digital system is almost equivalent to the continuous one, so its efficiency could be evaluated by testing methods used for analog systems. The time of transition process makes $t_m=0.0018\tau$, the overshoot makes $\sigma=8.6\%$.

Figure 15. The transient response of the digital GR stabilization system in vertical plane.

The time of transition process where continuous controller is used makes $0.0015\tau$, and the time of transition process in digital system makes $0.0018\tau$, which proves that performance deterioration in digital system is insignificant.
8. Summary.
This paper in theory studies the altitude control system of the geophysical rocket MP-12. It develops the mathematical model of the rocket’s dynamics as an object of control. The main data were taken from the open information sources using the example of the geophysical rocket MP-12. The computer model of the GR stabilization system in vertical plane was simulated by SimInTech dynamic modeling. To improve the system’s performance, the continuous controller and computer modeling was used. The digital controller was used for the purpose of GR pitch stabilization and its computer modeling.

References
[1] Tumanov A V, Zuev A G and Sukhanov E D 2008 Telecontrol and homing methods in cruise missile control systems, Publishing House of MSTU N.E. Bauman pp 13-26
[2] Peter H. Zipfel 2007 Modeling and Simulation of Aerospace Vehicle Dynamics. - Second Edition. - University of Florida Gainesville, Florida: Education series pp 367-75
[3] Krasnov N F, Koshevoi V N, Danilov A N and Zakharchenko V F 1968 Aerodynamics of rockets, Ed. prof. Krasnova N.F. M.: Higher School pp 9-82
[4] Awad Ahmed and Wang Haoping 2016 Roll-pitch-yaw autopilot design for nonlinear time varying missile using partial state observer based 6 global fast terminal sliding mode control, Chinese Journal of Aeronautics
[5] Besekersky V A 1976 Digital automatic systems, main edition of physical and mathematical literature, Publishing House "Science" pp 9-136