Decelerating Non-Static Plane Symmetric cosmological Model with Varying $\Lambda$

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Abstract: we consider the non-static plane symmetric space time with a stiff fluid and a variable cosmological constant $\Lambda$. We have obtained the solutions of field equations. Some physical and geometrical properties of the models are also discussed.

Keywords: non-static plane symmetric metric, cosmological model, Hubble’s parameter.

I. INTRODUCTION

In recent years there have been considerable interests in cosmological constant problem in cosmology. The simplest way out of this problem is to consider a varying cosmological term, which decays from huge to small in an expanding universe [1-4]. Several phenomenological model have been suggested by [5-17]. The cosmological constant $\Lambda$ has been one of the most mysterious and fascinating objects for cosmologist and theoretical physicists since its introduction in cosmology by Albert Einstein. The explanation of its origin is one of the most fundamental issue for our comprehension of general relativity and quantum field theory. In recent years there has been a lot of interest in the study the role of the cosmological constant $\Lambda$ at every early and later stages of the evolution of the universe. A wide range of observations mostly suggest that the universe possesses a non-zero cosmological constant. The $\Lambda$ term has been interpreted in terms of Higgs scalar field by Bergmann [18]. Drietlein [19] suggested that the mass of Higgs boson is connected with $\Lambda$ being a function of temperature and is related to the process of broken symmetries, and therefore it could be a function of time in an expanding universe. In quantum field theory, the cosmological constant is considered as the vacuum energy density. The general speculation is that the universe might have been created from an excited vacuum fluctuation (absence of inflationary scenario) followed by super cooling and reheating subsequently due to vacuum energy. Bianchi type $I-IX$ cosmological models are important in the sense of strings, isotropic, homogeneous etc. In the past five decades relativists has been interested in constructing string cosmological model. The model Bianchi type $VI_0$ of universe is initiated and explained solution of cosmological problem by Borrow [20]. Solution of Einstein field equation of $VI_0$ space time in stiff fluid has been investigated by Ellis and McCollum [21]. The class of Bianchi type $VI_0$ perfect fluid cosmological model associated with electromagnetic field has been obtained by Dunn and Tupper [22]. Some Bianchi type cosmological model in Biometric theory of gravitation presented by Reddy and Rao [23]. An algorithm for generating exact perfect fluid solution Einstein field equation, not satisfying the equation of state, for spatially homogeneous cosmological model of Bianchi type $VI_0$ has been presented by Shri Ram [24]. The solution of string cosmological models with magnetic field in general relativity have been obtained by Singh and Singh [25]. Solution of Bianchi string cosmological model with Bulk viscosity and magnetic field have been obtained Xing-Xiang [26-28]. Bianchi type $III$ for cloud string cosmological model described by Tikkar and Patil [29]. Tikkar and Patil [30] obtained some exact solutions. Bianchi type $I$ and Bianchi type $III$ discussed by Bali et al. [31-34] Recently, Bali and Poonia [35] investigated Bianchi type $VI_0$ Inflationary Cosmological Model in General Relativity. Tyagi et. al [36-38] obtained Bianchi Type $VI_0$ homogeneous cosmological model for anti-stiff perfect fluid for time dependent $\Lambda$ in general relativity. Inhomogeneous cosmological model for stiff perfect fluid distribution in general relativity and Barotropic perfect fluid in creation field theory with time dependent cosmological model. Bali et al [39] has investigated Bianchi type $VI_0$ in general relativity. Bianchi type $VI_0$ Universe model with time dependent $\Lambda$ term investigated By Shrimali et al. [40]. In this paper we obtain non static plane symmetric cosmological models with time-dependent cosmological constant. The plan of the paper is as follows. We present the metric and field equations in Section 2. In Section 3, we obtain exact solutions of the field equations. The physical and kinematical behaviors of the cosmological models are discussed in section 4. Finally, conclusions are summarized in the Section 5.
II. METRIC AND FIELD EQUATIONS

We consider the Riemannian space-time described by the line element

\[ ds^2 = e^{2h} \left( dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2 \right) \]  

(1)

Where \( r, \theta, z \) are the usual cylindrical polar coordinates and \( h \) & \( s \) are functions of \( t \) alone. It is well known that this line element is plane symmetric.

We consider the energy momentum tensor in presence of perfect fluid has the form

\[ T_{ij} = (\rho + p)v_i v_j + p g_{ij} \]  

(2)

where \( p \) is the pressure in the fluid and \( \rho \) is the energy density of the fluid and \( v^i \) is the four velocity vector defined by \( v^i = \delta^i_j \)

where \( i = 1,2,3,4 \) satisfying

\( v^iv_i = 1 \)

(3)

and perfect fluid obeys the equation of state

\[ p = \lambda \rho \]  

(4)

Where \( \lambda \) (0 \( \leq \) \( \lambda \) \( \leq \) 1) is constant.

The Einstein equation with time dependent \( \Lambda \) and \( 8\pi G = C = 1 \) given by

\[ R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij} - \Lambda g_{ij} \]  

(5)

For line element (5) and energy momentum tensor (2) commoving system of coordinates, field equations are

\[ e^{-2h} \left( 2\dot{h} + \ddot{h} + h^2 + 2\dot{h} \dot{s} \right) = -p - \Lambda \]  

(6)

\[ e^{-2h} \left( 2\dot{h} + h^2 \right) = -p - \Lambda \]  

(7)

\[ e^{-2h} \left\{ 2\dot{h} \dot{s} + 3\dot{h}^2 \right\} = \rho - \Lambda \]  

(8)

Here the over dot denotes differentiation with respect to \( t \).

The average scale factor (\( a \)), Volume scale factor (\( V \)), expansion scalar (\( \theta \)) and shear scalar (\( \sigma \)) are

\[ a = \left( rse^{4h} \right)^{\frac{1}{3}} \]  

(9)

\[ V = a^3 = \left( rse^{4h} \right) \]  

(10)

\[ \theta = V_j^i = 3\dot{h} + \dot{s} \]  

(11)

\[ \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} \]  

(12)

The generalized mean Hubble parameter (\( H \)) is given by

\[ H = \frac{\dot{a}}{a} \]  

(13)

An important observational quantity is the deceleration parameter \( q \) which is defined as

\[ q = -\frac{a\ddot{a}}{\dot{a}^2} \]  

(14)

The sign of \( q \) indicates whether the model inflates or not. The positive sign corresponds to the standard decelerating model whereas the negative sign indicates inflation.
III. SOLUTIONS OF THE FIELD EQUATIONS

we assume that the relation between the metric potentials given by

\[ e^n = \beta s^n \]  

(15)

Where \( \beta \) is and constant \( n > 1 \)

From equations (6) and (7), we obtained

\[ \frac{\ddot{s}}{s} + 2\dot{h} = 0 \]  

(16)

From equations (15) and (16), we get

\[ e^n = \beta \left[ (2n+1)t \right]^{\frac{n}{2n+1}} \]  

(17)

\[ s = \left[ (2n+1)t \right]^{\frac{n}{2n+1}} \]  

(18)

The metric (1), in this case, can be written as

\[ ds^2 = \left[ (2n+1)t \right]^{\frac{2n}{2n+1}} \left( dt^2 - dr^2 - r^2 d\theta^2 - \left[ (2n+1)t \right]^{\frac{2}{2n+1}} dz^2 \right) \]  

(19)

IV. PHYSICAL AND GEOMETRICAL PROPERTIES OF THE MODEL

The physical and geometrical quantities that are important in cosmology are average scale factor \( a \), spatial volume \( V \), mean Hubble parameter \( H \), the expansion scalar \( \theta \), shear scalar \( \sigma^2 \) which have the following expressions for the model (1) as given below

The average scale factor

\[ a = \left\{ r \beta^4 \left[ (2n+1)t \right]^{\frac{4n+1}{2n+1}} \right\}^{\frac{1}{3}} \]  

(20)

Spatial volume

\[ V = r \beta^4 \left[ (2n+1)t \right]^{\frac{4n+1}{2n+1}} \]  

(21)

The mean Hubble parameter

\[ H = \frac{(4n+1)}{3(2n+1)t} \]  

(22)

Scalar of expansion

\[ \theta = \frac{(4n+1)}{(2n+1)t} \]  

(23)
Shear scalar,
\[ \sigma^2 = \frac{7}{18} \frac{(4n+1)^2}{(2n+1)^2 t^2} \]  \hspace{1cm} (24)

The anisotropy parameter is given by
\[ A_m = \frac{7}{9} \]  \hspace{1cm} (25)

The deceleration parameter
\[ q = \frac{2(n+1)}{(4n+1)} \]  \hspace{1cm} (26)

The value of decelerating parameter is positive, hence, the model (3.5) represents a decelerating universe.

By taking \( p = \rho \) from equation (7) and (8), we get
\[ \Lambda = -\frac{n}{(2n+1)^2 t^2} \]  \hspace{1cm} (27)

From equation (8), we get
\[ p = \rho = \frac{(3n+1)n}{(2n+1)^2 t^2} \]  \hspace{1cm} (28)
V. CONCLUSIONS

In this paper, we have studied anisotropic non-static space-time with variable cosmological constant $\Lambda$. The cosmological constant inversely varies as square of time $t$, which corresponds to the model of Kalligas et al. [17]. We observed that at $t = 0$, the parameters $H, \theta, \sigma^2, p, \rho$ and $\Lambda$ are all infinite, as $t$ increases all parameters decreases. As $t$ leads to infinite value, the volume become infinitely large whereas the parameters converges to zero. Also model is anisotropic, as $\frac{\sigma}{\theta} = \text{constant}$. It is also observed that the deceleration parameter in the model is positive which indicates that the universe in the model is decelerating.

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