Singlet pairing and superfluidity in \(t\)-\(J\) ladders with Mott insulating stripes

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We study the ground state of inhomogeneous four-leg \(t\)-\(J\) ladders with Mott insulating stripes in the inner two legs. We show that, for anisotropic exchange couplings, a novel singlet-pair superfluid phase occurs with the fermions in the pairs residing on different sides of the Mott insulating stripe. In this phase, the binding energy is negative and the inter-leg singlet-pair correlation function decays algebraically with distance. On the other hand, correlation functions associated with inter-leg triplet pairing, intra-leg singlet and triplet pairing, single particles and spins, decay exponentially.

Introduction. In spite of being one of the most studied phenomena in condensed matter physics over the past three decades, consensus has not yet been reached as to which is(are) the underlying mechanism(s) for high-temperature superconductivity [1, 2]. In high-temperature superconductors, pairing occurs between electrons in the presence of strongly repulsive interactions. A complex competition of different orders makes it difficult to separate which ones aid and which ones are inimical to superconductivity [3]. One of the orders that has been observed in a variety of experiments in the doped cuprates family is related to the formation of charge density waves (CDW’s), dubbed stripes [4–6].

The presence of strong interactions makes it difficult to solve even the simplified effective models, e.g., the Hubbard and \(t\)-\(J\) models, that have been argued to contain the essential ingredients needed to describe high-temperature superconductivity [1, 7, 8]. Within the last fifteen years, a new way to explore the physical phenomena described by those models has emerged in the field of ultracold gases in optical lattices [9, 10], in which artificial lattices are created using laser beams and are loaded with ultracold atoms. This allows experimentalists to engineer nearly ideal realizations of effective model Hamiltonians with remarkable control and tunability, and to potentially identify the phases that can be described by those Hamiltonians. Recent experiments with ultracold fermions in two-dimensional lattices have made great progress in the exploration of the phases described by the two-dimensional Hubbard model at and away from half-filling [11–20], for example.

In experiments with ultracold gases, inhomogeneous trapping potentials (usually generated by the same laser beams that create the artificial lattice) maintain the gas confined. This results in inhomogeneous density distributions, with the coexistence of space-separated metallic and Mott insulating domains [21–23]. While inhomogeneities are generally regarded as a nuance, because one usually wants to understand phases of translationally invariant models, in this work we are interested in properties that are unique to inhomogeneous systems. They could be of relevance to phenomena such as high-temperature superconductivity because of, e.g., the presence of stripes. More specifically, we are interested in the properties of the conducting regions that surround Mott insulating domains. They can develop uncommon correlations because of proximity effects to an antiferromagnet [24–27]. In addition, we focus on ladder geometries because Mott insulating stripes have the potential to generate (or single out) pairing correlations with desired symmetries between surrounding conducting legs.

Thus, we study the \(t\)-\(J\) model on ladders in which the legs have different on-site potentials. A large negative on-site potential in the inner legs allows us to create Mott insulating stripes with controllable widths. In such systems, we show that anisotropic exchange couplings produce a novel form of pairing in which the paired fermions reside on sites across the Mott insulating stripe [See Fig. 1]. To shed light on the occurrence of pairing and superfluidity, we probe binding energies as well as inter-leg pairing correlations, for both the singlet and triplet channels. We also study intra-leg pairing, one-particle, and spin-spin correlations. The inter-leg singlet-pair correlations are found to decay algebraically, while the others decay exponentially. These results are contrasted with those obtained in homogeneous two-leg ladders, which exhibit a singlet-pair superfluid phase at low hole doping [28, 29].

![FIG. 1. Numerically obtained site-occupation profile, depicted as color bars, in a four-leg ladder with \(L_x = 20, N_L = N_r = 30, V = -40\), for isotropic couplings \(J_x = J_y = 0.33\) [see Eq. (1)]. A robust Mott stripe \((\langle \hat{n}_i \rangle \approx 1)\) is present in the inner two legs, while the two outer legs exhibit an average site occupation \((\langle \hat{n}_i \rangle \approx 0.5)\). The blue circles, connected by the dashed lines, show the inter-leg pairing investigated. The arrows indicate the pairs’ motion in the superfluid state.](image)
Model and method. The t-J Hamiltonian has been extensively studied as a paradigmatic model to understand the effects of strong correlations for large values of the ratio between the on-site repulsion strength $U$ and the hopping parameter $t$ [1, 7, 8, 28–37]. It can be simulated in optical lattice experiments by trapping two species of fermions in deep lattices so that $U/t \gg 1$, or using ultracold polar molecules [38, 39]. The t-J Hamiltonian reads $H_{tJ} = -t \sum_{\langle ij \rangle, \sigma} (\hat{c}_{i\sigma}^{\dagger} \hat{c}_{j\sigma} + \text{H.c.}) + J \sum_{\langle ij \rangle} (\hat{S}_i^x \hat{S}_j^x - \frac{1}{4} \hat{n}_i \hat{n}_j)$, where the operator $\hat{c}_{i\sigma}^{\dagger}$ ($\hat{c}_{i\sigma}$) creates (annihilates) a fermion with spin $\sigma = \uparrow, \downarrow$ on site $i$, and $\langle ij \rangle$ denotes the constrained summation over pairs of nearest neighbor sites. $\hat{S}_i^x = \hat{c}_{i\uparrow}^{\dagger} \sigma_x \hat{c}_{i\downarrow}$, and $\hat{n}_i = \sum_\sigma \hat{c}_{i\sigma}^{\dagger} \hat{c}_{i\sigma}$ are the spin (\(\vec{\sigma}\) are the Pauli matrices) and site occupation operators at site $i$, respectively. Note that the fermionic degrees of freedom must be projected onto the Hilbert subspace without double occupancies.

We study the ground state of the t-J model in a ladder geometry, with $L_x$ sites in the $x$-direction and $L_y$ sites in the $y$-direction [see Fig. 1], for which the Hamiltonian can be written as:

$$\hat{H} = -t_x \sum_{\sigma} \sum_{i_x=1}^{L_x-1} \sum_{i_y=1}^{L_y-1} \left( \hat{c}_{i_x,i_y,\sigma}^{\dagger} \hat{c}_{i_x+1,i_y,\sigma} + \text{H.c.} \right)$$

$$-t_y \sum_{\sigma} \sum_{i_x=1}^{L_x-1} \sum_{i_y=1}^{L_y-1} \left( \hat{c}_{i_x,i_y,\sigma}^{\dagger} \hat{c}_{i_x,i_y+1,\sigma} + \text{H.c.} \right)$$

$$+ J_x \sum_{i_x=1}^{L_x-1} \sum_{i_y=1}^{L_y-1} \left( \hat{S}_{i_x,i_y}^x \hat{S}_{i_x+1,i_y}^x - \frac{1}{4} \hat{n}_{i_x,i_y} \hat{n}_{i_x+1,i_y} \right)$$

$$+ J_y \sum_{i_x=1}^{L_x-1} \sum_{i_y=1}^{L_y-1} \left( \hat{S}_{i_x,i_y}^y \hat{S}_{i_x,i_y+1}^y - \frac{1}{4} \hat{n}_{i_x,i_y} \hat{n}_{i_x,i_y+1} \right)$$

$$+ \sum_{i} V_i \hat{n}_i.$$ (1)

In the remainder of the paper, $t_x = t_y = 1$ sets the energy scale. We focus on four-leg ladders ($L_y = 4$) with an inhomogeneous potential in the $y$-direction. This allows us to create Mott insulating stripes in the inner legs (for which we take $V_{i_y=2} = V_{i_y=3} = V = -40$) while maintaining the filling below one in the outer legs (for which we take $V_{i_y=1} = V_{i_y=4} = 0$). Both isotropic and anisotropic exchange couplings are considered. In optical lattice experiments, different inter- and intra-leg exchange couplings can be engineered by having different inter- and intra-leg hopping amplitudes, or, in systems with ultracold polar molecules, by changing the direction of an applied electric field [39].

The ground state of Eq. (1) is obtained numerically via the density matrix renormalization group (DMRG) method [40, 41]. We dynamically use up to 8000 DMRG many-body states so that the truncation error is of the order of $10^{-7}$ [42]. The calculations are done for $J_y = 0.33$, a value commonly used in studies of the t-J model in two-dimensional lattices [30, 31, 34], and different values of $J_y$. We focus on four-leg ladders with $N_\uparrow = N_\downarrow$ and average site occupation $(N_\uparrow + N_\downarrow)/(2L_y L_x) = 0.75$, so that $(\langle \hat{n}_i \rangle \approx 1$ in the inner two legs and $(\langle \hat{n}_i \rangle \approx 0.5$ in the outer two legs. $N_\uparrow$ ($N_\downarrow$) stands for the number of fermions with spin up (down). For computational convenience, open boundary conditions are adopted in the $x$-direction.

Binding energy. We start the exploration of the pairing tendencies of the four-leg ladder geometry by examining the binding energy, $E_b = E_0(N_\uparrow + 1, N_\downarrow + 1) + E_0(N_\uparrow, N_\downarrow) - 2E_0(N_\uparrow + 1, N_\downarrow)$, where $E_0(N_\uparrow, N_\downarrow)$ stands for the ground state energy in a system with $N_\uparrow$ ($N_\downarrow$) spin up (down) fermions. $E_b < 0$ in the thermodynamic limit means that the energy of two interacting particles (or holes, depending on the filling) is lower than that of two noninteracting ones. As a result, the system exhibits a tendency toward pair formation.

Figure 2(a) shows results for $E_b$ vs $J_y$ in four-leg ladders with $L_x = 28$. One can see that the binding energy becomes increasingly negative as $J_y$ increases. Finite-size scaling analyses of $E_b$ vs $1/L_x$ are reported in Fig. 2(b). While for the isotropic ($J_y = J_x = 0.33$) case the binding energy appears to be positive (but very small) in the thermodynamic limit, for the anisotropic ones the extrapolations indicate that they exhibit large negative values.

At this point one could ask whether the Mott stripe in between the outer two legs with $(\langle \hat{n}_i \rangle \approx 0.5$ is necessary to observe such robust negative binding energies. This is a valid concern as connecting those two outer legs by removing the Mott stripe could lead to similar (or even larger) negative binding energies. Homogeneous two-leg ladders at lower hole doping $(\langle \hat{n}_i \rangle \approx 0.8$ were shown to
support singlet-pair superfluidity in Refs. [28, 29]. For \( L_x = 28 \) [Fig. 2(a)], \( E_b \) for two-leg ladders can be seen to depend weakly (compared to the results for the four-leg ladder) on the value of \( J_y \). Finite-size scaling analyses of \( E_b \) for two-leg ladders [Fig. 2(b)], for the four values of \( J_y \) studied, suggest that the binding energies are very small and positive, or vanish, in the thermodynamic limit.

**Pairing correlation functions.** A unique feature of our setup is that the Mott stripe at the center of a four-leg ladder can mediate pairing between fermions on the opposite outer legs. To identify the pairing tendency that is dominant, we compute the inter-leg and intra-leg singlet- and triplet-pair correlation function

\[
    P_{x_1,x_2} = \langle \hat{\Delta}^\dagger_{x_1} \hat{\Delta}_{x_2} \rangle, 
\]

where \( \Delta^\dagger_i = \frac{1}{\sqrt{2}} (c^\dagger_{i,1,\uparrow} c^\dagger_{i,L_y,\downarrow} - c^\dagger_{i,1,\downarrow} c^\dagger_{i,L_y,\uparrow}) \) for singlet \( (P^S) \), and \( \Delta^\dagger_i = \frac{1}{\sqrt{2}} c^\dagger_{i,1,\downarrow} c^\dagger_{i,L_y,\uparrow} \) for triplet \( (P^T) \), inter-leg pairing; and \( \Delta_{x_1} = \frac{1}{\sqrt{2}} (c^\dagger_{i_1,1,\uparrow} c^\dagger_{i_1+1,\downarrow} - c^\dagger_{i_1,1,\downarrow} c^\dagger_{i_1+1,\uparrow}) \) for singlet \( (P^S_{1D}) \), and \( \Delta^\dagger_{x_1,y} = c^\dagger_{i_1,y,\downarrow} c^\dagger_{i_1+1,y,\uparrow} \) for triplet \( (P^T_{1D}) \), intra-leg pairing. (We checked that the results for the triplet-pair correlations are identical if one considers spin-up fermions, and that intra-leg correlations are identical in the two outer legs.) Since we use open boundary conditions in our numerical calculations, in what follows we report the average over all correlations at the same distance

\[
    P(r) = \frac{1}{N} \sum_{|x_1-x_2|=r} P_{x_1,x_2},
\]

where \( N \) is the total number of pairs of sites \( \{x_1, x_2\} \) satisfying \( |x_1 - x_2| = r \).

In Fig. 3(a) we plot the inter-leg singlet-pair correlations vs \( r \) in four-leg ladders. For the isotropic case, their decay is approximately algebraic. Increasing the value of \( J_y \) results in an enhancement of those correlations. This is the opposite to what happens for the triplet inter-leg correlations, depicted in Fig. 3(b). Their decay is also close to algebraic in the isotropic case, but increasing the value of \( J_y \) results in a clear exponential decay. The insets in Figs. 3(a) and 3(b) show the intra-leg singlet- and triplet-pair correlations for the same values of \( J_y \) as in the main panels. They can also be seen to decay exponentially with \( r \). These results suggest that, for anisotropic exchange couplings, four-leg ladders with a Mott stripe in the inner two legs exhibit singlet-pair superfluidity with the fermions in the pair being on opposite legs about the Mott insulating stripe.

In Figs. 3(a) and 3(b), we also plot the inter-leg singlet- and triplet-pair correlations, respectively, in homogeneous two-leg ladders. Figure 3(a) shows that for isotropic exchange couplings the inter-leg singlet-pair correlations in the two-leg ladder decay slightly faster than those in four-leg ladder. An anisotropy in the exchange couplings results in a slower decay of the inter-leg singlet correlations. However, for \( J_y = 0.75 \) at the largest distance accessible to us, they are nearly an order of magnitude smaller than those in the four-leg ladder. The decay of the inter-leg triplet correlations [Fig. 3(b)], as well as of the intra-leg singlet and triplet-pair correlations (not shown), is exponential in two-leg ladders. The comparison between the binding energies, and between the inter-leg singlet-pair correlations in two- and four-leg ladders, highlights the importance of the presence of the Mott stripe in the inner two legs of the four-leg ladders for the occurrence of singlet-pair superfluidity between legs with \( \langle \hat{n}_i \rangle \approx 0.5 \).

**One-particle and spin-spin correlations.** The quantum many-body phase realized in four-leg ladders with a Mott stripe in the inner legs can be further differentiated from the one in two-leg ladders with \( \langle \hat{n}_i \rangle \approx 0.5 \) by studying the intra-leg one-particle and spin-spin correlations. In a Luttinger liquid, a relevant point of reference for our systems as we are dealing with quasi-one-dimensional geometries, those correlations exhibit an algebraic decay [43].

We compute the one-particle density matrix

\[
    \rho_{x_1,x_2} = \langle \hat{c}^\dagger_{x_1,1,\uparrow} \hat{c}_{x_2,1,\downarrow} \rangle,
\]

(the results for spin-up fermions are identical), and the spin-spin correlation function

\[
    S^z_{x_1,x_2} = \langle \hat{\sigma}^z_{x_1,1} \hat{\sigma}^z_{x_2,1} \rangle,
\]
and report averages over correlations at the same distance, which are calculated as in Eq. (3).

In Figs. 4(a) and 4(b), we show results obtained for $\rho(r)$ and $S(r)$, respectively, in four-leg ladders with a Mott stripe (for the same values of $J_y$ as in Fig. 3). For the isotropic case, both correlation functions exhibit a near algebraic decay with $r$. However, for $J_y = 0.75$, $\rho(r)$ and $S(r)$ can be seen to decay exponentially. This is the result of the single-particle and spin excitations being gapped in the singlet-pair superfluid phase.

The insets in Figs. 4(a) and 4(b), show results for the same correlation functions in homogeneous two-leg ladders with $\langle \hat{n}_i \rangle \approx 0.5$. In stark contrast to the results for four-leg ladders with a Mott stripe in the inner legs, and to the results in Ref. [28] in homogeneous two-leg ladders with $\langle \hat{n}_i \rangle \approx 0.8$, in the two-leg ladders with $\langle \hat{n}_i \rangle \approx 0.5$ one can see that $\rho(r)$ and $S(r)$ decay algebraically with $r$. Hence, in the anisotropic case in two-leg ladders (at $\langle \hat{n}_i \rangle \approx 0.5$) there is a competition between one-particle, spin-spin, and inter-leg singlet-pair correlations, all of which are found to decay algebraically. There is no such competition in four-leg ladders with a Mott stripe in the inner legs in which the one-particle and spin-spin correlations decay exponentially.

**Summary and discussion.** Using DMRG calculations, we explored the ground state properties of four-leg $t$-$J$ ladders with a Mott insulating stripe in the inner two legs, and mean occupation per site $\langle \hat{n}_i \rangle \approx 0.5$ in the outer two legs. The different average site occupation per site in the legs was enforced by means of a strong trapping potential across the legs. We showed that, for anisotropic exchange couplings (stronger between the legs than within the legs), an inter-leg singlet-pair superfluid phase occurs (the fermions that form each pair are in opposite outer legs). The singlet-pair superfluid phase is characterized by a negative binding energy, algebraically decaying inter-leg (between the outer legs) singlet-pair correlations; and exponentially decaying inter-leg triplet-pair correlations, intra-leg singlet- and triplet-pair correlations, as well as intra-leg one-particle and spin-spin correlations. We also studied (not shown) the intra-leg density-density correlations in the superfluid phase. As in the singlet-pair superfluid phase in homogeneous two-leg ladders at $\langle \hat{n}_i \rangle \approx 0.8$ [28, 29], in our four-leg ladders the density-density correlations decay algebraically but more rapidly than the inter-leg singlet-pair correlations.

We contrasted the results obtained for the inhomogeneous four-leg ladders with those for homogeneous two-leg ladders with a mean occupation per site $\langle \hat{n}_i \rangle \approx 0.5$, for which no singlet-pair superfluid phase was found. This contrast makes apparent the importance of having the Mott insulating stripe in the inner two legs of the four-leg ladders. Such a Mott insulating stripe serves a double purpose: (i) it has antiferromagnetic correlations that mediate the interaction between the fermions in outer two legs such that they form singlets, and (ii) because of the double-occupancy constraint, the Mott stripe constrains the motion of the fermions in the outer legs to be one dimensional, which makes the singlet-pair phase robust against delocalization of the fermions across the legs (no inter-leg kinetic energy can be gained in our constrained geometry). The latter is not the case in the homogeneous two-leg ladders in which the fermions can hop between the legs.

We expect the singlet-pair superfluid phase found in four-leg ladders with a Mott stripe in the two inner legs to occur in wider ladders with a larger even number of inner legs forming the Mott stripe. For a sufficiently strong anisotropy in the exchange couplings, a larger number of inner Mott insulating legs should only result in a decrease of the magnitude of the negative binding energy (the Mott-stripe mediated interaction between fermions in the outer legs will become weaker). Testing this, as well as finding accurate phase diagrams when changing the anisotropy in the exchange couplings, the average filling in the outer legs, and the number of inner legs belonging to the Mott insulating stripe, is beyond our current computational capabilities. This is something that could be explored using optical lattice experiments.

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