SUPERSYMMETRIC STRUCTURE OF SPACETIME AND MATTER
-SUPERON QUINTET HYPOTHESIS-

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Abstract

A unified description of spacetime and matter at the Planck scale is proposed by using the irreducible representation of N=10 extended Super-Poincaré algebra, where all matters and all forces except the graviton are the supersymmetric composites made of the fundamental objects with spin 1/2, superon quintet. All the local gauge interactions in GUTs are investigated systematically by using the superon diagrams. The proton is stable and the flavor changing neutral current process is suppressed in the superon pictures of GUTs. The fundamental action of the superon model is proposed. The characteristic predictions which can be tested in the (coming) high energy experiments are discussed briefly.
It seems generally accepted that the (local) supersymmetry (SUSY)\[1\] is the most promising notion for unifying all elementary particles including the graviton within the framework of the local field theory. However, as shown by Gell-Mann\[2\], SO(8) maximally extended supergravity theory (SUGRA) is too small to accommodate all observed particles as elementary fields.

On the contrary, at the risk of the local field theory at the moment, it is interesting from the viewpoints of simplicity and beauty of Nature to attempt the accommodation of all observed elementary particles in a single irreducible representation of a certain group(algebra). In the previous paper\[3\], by identifying the graviton as the Clifford vacuum state( not necessarily the lowest energy state) of SO(N) extended super-Poincaré algebra(SPA) we have studied the irreducible representations of SO(N) SPA for the massless case. And we have shown\[3\] that from only the group theoretical arguments SO(N) SPA with N=10,11 and 12 may be relevant to the unified description of matters and forces and N=10 stands out among them. Because the assignment of quantum numbers adopted in Ref.[3] to 10 supercharges $Q^N (N = 1, 2, \ldots 10)$ of SO(10) SPA is unique in order to realize all observed quarks, leptons and gauge bosons as the low energy massless states of the representations.

In this letter, after the brief review of Ref.[3] for the self-contained arguments, we try to interpret the results of Ref.[3] from the viewpoints of the internal structure of the quarks, leptons and gauge bosons except the graviton. We assume that at (above) the Planck energy scale Nature(spacetime and matter) possesses SO(10) super-Poincaré symmetric structure and that all of the massless irreducible representations of SO(10) SPA reveal the structure.

In Ref.[3], by noting that 10 generators $Q^N$ of SO(10) SPA are the fundamental representations of SO(10) internal symmetry and that $SO(10) \supset SU(5) \supset SU(3) \times SU(2) \times U(1)$ we have decomposed 10 generators $Q^N$ of SO(10) SPA as follows with respect to SO(10) internal symmetry

$$10 = \bar{5}(Q^N; N = 1, 2, \ldots 5) + \bar{5}^*(Q^N; N = 6, 7, \ldots 10)$$

$$= \{(\mathbf{2}, \mathbf{1}; -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}) + (\mathbf{1}, \mathbf{2}; 1, 0)\} + \{(\mathbf{3}^*, \mathbf{1}; \frac{1}{3}, \frac{1}{3}, \frac{1}{3}) + (\mathbf{1}, \mathbf{2}^*; -1, 0)\}, (1)$$

where we have written ( SU(3), SU(2); electric charges ). For massless case( $P_\mu P^\mu = 0$ ) in order to see easily the helicity contents of the irreducible representation we go to the little algebra, where we can always choose the light-like frame $P_\mu = \epsilon(1, 0, 0, 1)$. 

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In terms of two-component Weyl spinors, the little algebra for the supercharges in this frame now becomes after suitable rescaling

\[ \{ Q^M_\alpha, Q^N_\beta \} = \{ \overline{Q}^M_\dot{\alpha}, \overline{Q}^N_\dot{\beta} \} = 0, \quad \{ Q^M_\alpha, \overline{Q}^N_\dot{\beta} \} = \delta_\alpha^{\dot{\beta}} \delta^{MN}, \]  

(2)

where \( \alpha, \beta = 1, 2 \) and \( M, N = 1, 2, \ldots 10 \). As a consequence of (2) the spinor charges \( Q^M_1, \overline{Q}^M_1 \) satisfy the algebra of annihilation and creation operators respectively and can be used to construct a 4-dimensional Fock space with positive metric. For the massless case, the Clifford vacuum \( | \Omega(\lambda) \rangle \) is a representation of the little group \( E_2 \) of a light-like vector, i.e. a massless state of a given helicity \( \pm \lambda \), if space inversion is considered. We identify the graviton with the Clifford vacuum \( | \Omega(\lambda) \rangle \) (not necessarily the lowest energy state), which satisfies

\[ Q^M_\alpha | \Omega(\lambda) \rangle = 0 \]  

(3)

and build up a new state with helicity \( (2 - \frac{n}{2}) \) by

\[ \overline{Q}^{M_1}_{\dot{\alpha}} \overline{Q}^{M_2}_{\dot{\beta}} \ldots \overline{Q}^{M_n}_{\dot{\gamma}} | \Omega(\lambda) \rangle. \]  

(4)

(Note that the helicities of such states are determined by the SO(10) SPA.) These states given by the Clifford vacuum \( | \Omega \rangle \) and all states of (4) obtained from \( | \Omega \rangle \) by multiplying with every possible product of the creation operators \( Q^M_1 \) span an irreducible \( 2 \cdot 2^{10} \) dimensional representation of the little algebra (2) of SO(10) SPA. It contains helicities up to \( \pm 3 \), if parity is included. For a reference we show in the following explicitly all states of SO(10) SPA and specify them by SO(10) dimension \( d \) and the helicity \( \lambda \), as \( d(\lambda) \):

\[ [1(+2), 10(+\frac{3}{2}), 45(+1), 120(+\frac{1}{2}), 210(0), 252(-\frac{1}{2}), 210(-1), 120(-\frac{3}{2}), 45(-2), 10(-\frac{5}{2}), 1, 1(-3)] + \text{CPT-conjugate}. \]  

(5)

By noting that the helicity of every such state as (3) and (4) is automatically determined by SO(10) SPA and that \( Q^M_1 \) and \( \overline{Q}^M_1 \) satisfy the algebra of the annihilation and the creation operators for the spin \( \frac{1}{2} \) particle, we speculate that these states (3) and (4) are the relativistic (gravitational) massless composite states spanned upon the mathematical (not the physical vacuum with the lowest energy) Clifford vacuum
and are composed of the fundamental object $Q^N$ superon with spin $\frac{1}{2}$. Therefore we regard (1) as a *quintet of superons* and a *quintet of antisuperons*. The identification of the generators of SO(10) SPA with the fundamental objects (particles) is strange so far especially from the viewpoint of the familiar local gauge field theory. We will consider these problems later and show (a possibility of) a fundamental (local) field theory of the superons.

Now we envisage the Planck scale physics as follows: At (above) the Planck energy scale spacetime and matter have the structure described by SO(10) SPA, where the gravity dominates and creates the superon-quintet and the antisuperon-quintet pair from the vacuum in such a way as superon-composites massless states span the irreducible massless representations of SO(10) SPA.

From the viewpoints of the superon hypothesis we can reinvestigate more concretely the physical meaning of the results obtained in Ref.[3]. For simplicity we use the following notations for superons $Q^N$ ($N = 1, 2, ..10$).

For the superon quintet $\tilde{5}$: \[
\left[ (3, 1; \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}), (1, 2; 1, 0) \right],
\]
we use \[
\left( Q^a, Q^b, Q^c, Q^m, Q^n; a, b, c = 1, 2, 3; m, n = 4, 5 \right) \tag{6}
\]
and for the antisuperon-quintet $\tilde{5}^*$: \[
\left[ (3^*, 1; \mp \frac{1}{3}, \mp \frac{2}{3}, \mp \frac{1}{3}), (1, 2^*; -1, 0) \right],
\]
we use \[
\left( Q^*_a, Q^*_b, Q^*_c, Q^*_m, Q^*_n; a, b, c = 1, 2, 3; m, n = 4, 5 \right). \tag{7}
\]

Accordingly we can specify explicitly all the states corresponding to observed quarks, leptons and massless gauge bosons of the standard model (SM) [4] presented in Ref.[3] as follows. The multiplicity of the fermionic states identified with respect to (SU(3), SU(2); electric charges) is counted in the two-component Weyl spinor unit. (SO(10) normalization factor is neglected.)

**$\nu_e, e_L$ type leptons:** four generations from $\underline{120}$: $(Q_m Q^*_4 Q^*_5)$, $(Q_a Q^*_4 Q^*_m)$ and conjugate states.

*four generations from $\underline{252}$: $\varepsilon_{abc} Q^a Q^b Q^c, \varepsilon_{abc} Q^a Q^b Q^c, Q^a Q^b Q^c, Q^*_a Q^*_b Q^*_m$, and conjugate states.*

**$e_R$ type leptons:** two generations from $\underline{120}$: $\varepsilon_{abc} Q^a Q^b Q^c$ and conjugate state.

*two generations from $\underline{252}$: $\varepsilon_{abc} Q^a Q^b Q^c, Q^a Q^b Q^c, Q^*_m, Q^*_m$ and conjugate state.*
\( (u,d)_L \) type quarks: two generations from 120: \( \varepsilon_{abc}Q_b^*Q_c^*Q_m^* \) and conjugate state.
four generations from 252: \( \varepsilon_{abc}Q_b^*Q_c^*Q_d^*Q_m^*, \varepsilon_{abc}Q_b^*Q_c^*Q_d^*\varepsilon_{mn}Q_m^*Q_n^* \) and conjugate states.

\( (u)_R \) type quarks: two generations from 120: \( Q_a\varepsilon_{mn}Q_mQ_n \) and conjugate state.
two generations from 252: \( \varepsilon_{abc}Q_bQ_cQ_d^*Q_e^*Q_f^*Q_m \) and conjugate state.

\( (d)_R \) type quarks: four generations from 120: \( Q_a^*\varepsilon_{abc}Q_bQ_cQ_d^* \) and their conjugate states.
six generations from 252: \( \varepsilon_{abc}Q_bQ_cQ_d^*Q_e^*Q_f^*Q_g^*Q_m \) and conjugate states.

\( SU(2) \times U(1) \) gauge bosons: one singlet state from 45: \( \frac{1}{\sqrt{2}}(Q_4Q_1^*-Q_5Q_2^*) \) and one triplet states from 45: \( \{ Q_4Q_5^*, \sqrt{\frac{1}{2}}(Q_4Q_4^*+Q_5Q_5^*), Q_5Q_4^* \} \).

\( SU(3) \) gluons: one octet state from 45: \( \{ Q_1Q_2^*, Q_2Q_3^*, -Q_1Q_2, \frac{1}{\sqrt{2}}(Q_1Q_1^*-Q_2Q_2^*), Q_2Q_1^*, \frac{1}{\sqrt{6}}(2Q_3Q_3^*-Q_2Q_2^*-Q_1Q_1^*), -Q_3Q_2^*, Q_3Q_1^* \} \).

\( SU(2) \) Higgs Boson: one doublet state from 210: \( \varepsilon_{abc}Q_aQ_bQ_cQ_m \) and conjugate state.

\( (X,Y) \) leptoquark bosons in GUT(\( SU(5)[5] \) and \( SO(10)[6] \)): From 45 we obtain: \( Q_a^*Q_m \) and conjugate state.

The specifications of \( SU(5) \) leptoquark states are interesting concerning the proton decay, for the symmetry breaking via \( SU(5) \) invariance may be worthwhile to consider. For the gauge bosons we have considered only the adjoint representation of \( SO(10) \) SPA.

Although the mass generation mechanism, i.e. the mechanism of the symmetry breaking,

\[ [SO(10) \text{ SPA(massless)}] \]
\[ \rightarrow [ \ldots ] \]
\[ \rightarrow [ \text{Poincaré algebra (massless in part) with } SU(3) \times SU(2) \times U(1) ] \]
\[ \rightarrow [ \text{Poincaré algebra(massless in part) with } SU(3) \times U(1) ] \]
is unknown, we dare to perform \( SU(3) \times SU(2) \times U(1) \) invariant recombinations of the helicity states of the massless irreducible representation of the little algebra of \( SO(10) \) SPA in order to see the possible contents of \( SU(3) \times SU(2) \times U(1) \) invariant massive states of the irreducible representation of the little algebra of Poincaré algebra.
Through the recombination, many of the lower helicity \((\pm \frac{3}{2}, \pm 1, \pm \frac{1}{2}, 0)\) states of SO(10) SPA are converted to the longitudinal components of the higher spin massive states of Poincaré algebra and others remain massless.

In Ref.[3] we have carried out the recombination among \(2 \cdot 2^{10}\) helicity states and found surprisingly all massless states necessary and sufficient for the SM with three generations of quarks and leptons appear in the surviving massless states. A few characteristic (i.e. independent on the intermediate symmetry breaking pattern) predictions which can be tested by the high energy particle experiment are presented in Ref.[3].

Now superons are for quarks, leptons, gauge bosons except the graviton, and Higgs bosons what quarks are for baryons and mesons. Then it is interesting to investigate the the symmetry breaking (mass pattern) of SO(10) SPA by performing the similar analysis used in the quark model for the hadron physics[7]. It gives the phenomenological informations to solve the superon dynamics at the Planck scale.

While, towards the construction of the field theory of the superon and for surveying the physical (phenomenological) implications of the superons for the unified gauge models (SM and GUTs) it is very important to understand all the gauge couplings of the unified gauge models in terms of the superon pictures. For simplicity we neglect the mixing between the states and take by using the conjugate representations naively the following left-right symmetric assignment for quarks and leptons, i.e. \((\nu_l, l^-)_R = (\bar{\nu}_l, l^+_L, etc.

For \((\nu_e, e), (\nu_\mu, \mu), (\nu_\tau, \tau)\) we take \((Q_m Q_4^* Q_5^*), (Q_a Q_6^* Q_m), Q_a Q_b^* Q_b Q_m^* Q_m \) and the conjugate states respectively.

For \((u, d), (c, s), (t, b)\) we take \(\epsilon_{abc} Q_b^* Q_c^* Q_m^* , \epsilon_{abc} Q_b^* Q_c^* Q_l \epsilon_{nm} Q_m^* , \epsilon_{abc} Q_b^* Q_c^* Q_d Q_m^* \), and the conjugate states respectively.

The superon line Feynmann diagram is obtained by replacing the single line in the Feynmann diagram of the gauge models by the corresponding multiple superon lines. We discuss as a few examples the following processes, i.e.

\[(i) \ \beta \ \text{decay:} \ n^0 \rightarrow p^+ + e^- + \bar{\nu}_e, \ (ii) \ \pi^0 \rightarrow 2\gamma, \ (iii) \ \text{the proton decay:} \ p^+ \rightarrow e^+ + \pi^0, \ (iv) \ \text{a flavor changing neutral current process(FCNC):} \ K^+ \rightarrow \pi^+ + \nu_e + \bar{\nu}_e \text{ and (v) an advocated typical process of the (non-gauged) compositeness:} \ \mu \rightarrow e + \gamma. \]

To translate the vertex of the Feynmann diagram of the unified gauge model into that of the superons, we assume that the superon-antisuperon pair creations and pair
annihilations within a single state for a quark, a lepton and a (gauge) boson (i.e. within a single SO(10) SPA state) are forbidden. This rule seems natural because every state is the irreducible representation of SO(10) SPA and is prohibited from the decay without any remnants, i.e. without the interaction between the superons contained in the different states. Now it is straightforward to translate uniquely the Feynmann diagram of the unified gauge models into that of the superon model.

For the processes (i) and (ii) we can draw the corresponding similar tree-like superon line diagrams easily, where the triangle-like superon diagram does not appear. For the process (iii) we consider the Feynmann diagrams for the proton decay of GUTs and find that the corresponding superon line diagrams do not exist due to the mismatch of the superons contained in the quarks(u and d) and the gauge bosons(X and Y) at the gauge coupling vertices. This means that irrespective of the masses of the gauge bosons the proton is stable, at least against $p^+ \rightarrow e^+ + \pi^0$.

For FCNC process (iv) the penguin-type and the box-type superon line diagrams are to be studied corresponding to the penguin- and box-Feynmann diagrams for $K^+ \rightarrow \pi^+ + \nu_e + \bar{\nu}_e$ of GUTs. Remarkably the superon line diagrams which have only the up-quark for the internal quark line exist. This is the indication of the strong suppression of the FCNC process, at least for the process $K^+ \rightarrow \pi^+ + \nu_e + \bar{\nu}_e$. For the process (v) the corresponding tree-like superon line diagram does not exist, i.e. $\mu \rightarrow e + \gamma$ decay mode is absent in the superon (composite) model.

It should be noticed that among the abovementioned superon line diagrams, the superon line diagrams corresponding to the penguin Feynmann diagram have the different topological structures, i.e. the twist of the superon lines within the propagator occurs. Such topological structures remain to be studied in detail. As an example of the superon line diagram we show in Figure 1 one of the penguin Feynmann diagrams for the process (iv).

Finally we consider the possibility of the fundamental theory which describes the superon dynamics. In Ref.[8], we have investigated the nonlinear representation of N=1 SUSY[9] in two dimensional spacetime. We have shown that by using Noether procedure we can construct the supercharge $Q$ explicitly and carry out the canonical quantization for the fundamental (Goldstone) spinor field $\psi(x)$ so that the super-Poincaré algebra can be satisfied at the quantized level. The spinor supercurrent density giving the supercharge $Q_\alpha$ has been written as follows

$$J^\mu(x) = \frac{1}{i} \sigma^\mu \psi(x) - \kappa \{ \text{the higher orders of } \psi(x) \}, \quad (\mu = 1, 2) \quad (8)$$
where $\kappa$ is an arbitrary constant with the second power of length and $\sigma^\mu$ are Pauli matrices, $\sigma^0 = 1$. This is a nice indication for our assumption that the generator (supercharge) $Q^N$ of SO(10) SPA represents the fundamental object (particle) superon with spin $1/2$, which obeys the Fermionic quantum statistics. (8) is the field-current identity with spin $1/2$ including, as shown by the term proportional to $\kappa$, the composite fields corresponding to the states of the irreducible representation of SO(10) SPA generated by the currents. Therefore we speculate that the fundamental theory of the superon quintet model of spacetime and matter at (above) the Planck scale is SO(10) nonlinear supersymmetry (NL SUSY) in the curved spacetime, and all the helicity-states including the observed quarks, leptons and gauge bosons except the graviton are the massless relativistic (gravitational) composite states composed of Goldstone fermions, superons.

From these considerations we propose the following Lagrangian as the fundamental theory of SO(10) superon model of spacetime and matter.

$$L = -\frac{e^3}{16\pi G}eR - \frac{e^3}{8\pi G}e\Lambda - \frac{1}{\kappa}e|W|,$$

(9)

$$|W| = detW^\nu_{\mu} = det(\delta^\nu_{\mu} + \kappa T^\nu_{\mu}),$$

(10)

$$T^\nu_{\mu} = \frac{1}{2i} \sum_{i,j=1}^{10} (\bar{s}_i O_{ij} \gamma^\mu s^j - D^\mu \bar{s}_i s^j O_{ij}),$$

(11)

where $\kappa$ is yet an arbitrary constant with the dimension of the fourth power of length, $e = det e^a_{\mu}$, $D_\mu = \partial_\mu + \frac{1}{2} \omega^{ab}_{\mu} \sigma_{ab}$ and $R$ and $\Lambda$ are the scalar curvature and the yet arbitrary cosmological constant, respectively. $O_{ij}$ is a $10 \times 10$ unitary matrix representing the quantum mechanical mixing among the superon quintet states, which may be probable but unpleasant from the elementary nature of the superon. The first term of (9) is the gravity action and the third term is the action for the superon $s(x)^i$, ($i = 1, 2, \ldots, 10$), which is global SO(10) NL SUSY in the curved spacetime. The invariance of the total action (9) under global SO(10) NL SUSY will be proved explicitly by performing the 1.5 order formalism adopted for proving the invariance of the SUGRA action[10]. The states with helicity $\pm 3, \pm \frac{5}{2}$ and $\pm 2$ (except the graviton) necessary for completing the irreducible representation of SO(10) SPA appear after specifying the contorsion in the spin connection $\omega^{\mu}_{ab}(e^\mu_a, s^i)$, for these states are 10-, 9- and 8-superon states respectively and hidden in (9). We can anticipate the invariance of (9) under the global SO(10) NL SUSY, which may
be included in the scope of Ref[11]. The structure of the true vacuum of (9) should be studied in detail.

From the phenomenological viewpoints, the current algebra may be useful and practical. From the commutator of the supercharges,

\[ \{ Q^M_\alpha, Q^N_\beta \} = 2\delta^{MN} \sum_{\mu=0}^{3} (\sigma_{\mu})_{\alpha \beta} P^\mu, \]  

we can express the Hamiltonian density in the form of the products of the super-currents of SO(10) NL SUSY obtained by Noether’s procedure as demonstrated in Ref.[8]. For understanding the superon dynamics phenomenologically it is important to fit all the decay data of low lying hadrons in terms of the superon line diagram amplitudes, which may give interesting structures among the various amplitudes.

The unified gauge models (SM and GUT) may be for the yet hypothetical SO(10) superon model what the Landau-Ginzburg theory is for the BCS theory of the superconductivity, i.e. they may be the (effective) theories at the low energy.

Besides those nice aspects of SO(10) superon model mentioned above, much more open questions are left.

However we speculate that the beautiful complimentality between the gauge unified models(SM and GUT) and the superon model, i.e. the former is strengthened or revived by taking account of the topology of the latter superon diagram, while drawing the superon diagram of the latter is guided by the Feynmann diagram for the gauge interaction of the former, may be an evidence of SO(10) SPA structure of spacetime and matter behind the gauge models, i.e. an evidence of the superon quintet hypothesis . The experimental search of a predicted new spin $\frac{3}{2}$ lepton doublet ($\nu_\Gamma, \Gamma^-$) with the mass of the electroweak scale[3] is important. The clear signals of the new particles may be similar to the top-quark pair production event without the jet production.

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Figure captions: One of the penguin diagrams of $K^+$ decay. Superons are labeled by the indices used in Eqs.(6) and (7). Figure 1 $K^+ \rightarrow \pi^+ + \nu_e + \bar{\nu}_e$
Figure 1  One of the Penguin diagrams of $K^+$ decay

$K^+ \rightarrow \pi^+ + \nu_e + \overline{\nu}_e$