Magnetically induced transparency of circularly polarized laser beam in plasmas

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Abstract. For the intensities greater than $10^{18}$ W/cm², circularly polarized radiation can propagate in electron plasma whose density is greater than the critical density. A strong flow of relativistic electrons, axially co-moving with the pulse arises. At this point the magnetic field of the electromagnetic wave becomes important. In the present paper, three regimes of propagation of circularly polarized laser beam in magnetized plasma are identified. An appropriate expression for the non-linear dielectric tensor has been used in the analysis under paraxial approximation. Two modes of propagation, viz, extraordinary mode and ordinary mode exist; because of the relativistic mechanism the induced magnetic field significantly affect the propagation of laser beam in plasma. Further studied for inhomogeneous plasma and penetration in overdense plasma is indicated. The induced magnetic field leads to the magnetically induced transparency due to extraordinary mode.

1. Introduction

The interaction of intense short laser pulse with plasma involves a number of interesting nonlinear physical phenomenons, including the generation of quasi-static intense magnetic fields. The recent achievement of powerful short laser pulse has lead to renewal interest in interaction of radiation with matter into relativistic regime. The generation of magnetic field in high intensity laser-plasma interaction has received considerable attention these include fast electron and ion generation, indicating that ultra strong electric field and magnetic field are generated in the plasma [1].

The development of multi terawatt femtosecond lasers makes it possible to achieve a regime of laser matter interaction never reached before. Now interaction mechanisms are being proposed theoretically to model the interaction, including the anomalous skin effect, hole boring and self induced transparency [2, 3, 4]. During the interaction of an intense laser with plasma, a magnetic field is generated and betatron resonance occurs between the electron and the electric field of the laser, and the electrons are accelerated to high energies. The possibility of light propagation through overdense plasmas was studied in the relativistic regime for first time by Akhiezer et al and Dawson, et al [5, 6]. In this new regime of high laser intensity the quiver velocity of electrons is relativistic; one of he main
effects in laser plasma interaction is associated with the relativistic increase of the inertial electron mass and consequent lowering of the natural plasma frequency that may crucially modify the optical properties of plasma. The superintense electromagnetic radiation would be able to propagate through a classically overdense plasma due to the relativistic correction to the electron mass, the so-called induced transparency effect.

In the present paper we describe propagation of circularly polarized high-intensity laser radiation in plasmas. The propagation of an electromagnetic wave in a magnetized plasma, in the so-called extraordinary mode has been considered. Such a wave field causes the plasma electrons to gyrate in the orbit whose radii depend on the radiation intensity, on the wave frequency and on the electron density. This gyration of the plasma electrons induces a magnetic field, which for the left circular polarization is parallel and for right circular polarization is antiparallel to the direction of wave propagation. In section 2, analytical formulation is presented with dielectric tensor in relativistic magnetoplasma, wave propagation equation for extraordinary mode and critical-divider curve for circularly polarized laser beam. Numerical results and discussion is made in section 3, followed by conclusion in section 4.

2. Analytical formulation

2.1. Dielectric tensor in relativistic magnetoplasma

Consider the propagation of circularly polarized electromagnetic beam in the self generated or externally applied static magnetic field $B_0$ (assume to be along the z-axis) in either of the two counter-rotating circularly polarized modes known as extraordinary or ordinary mode of propagation. The electric vector can be written as

$$ E_\pm = A_{1,2} \exp\left[i(\omega t - k_{0z} z)\right] $$

where

$$ E_\pm = A_{1,2} = E_z \pm iE_y, \quad k_{0z} = \left(\frac{\omega}{c} \sqrt{\varepsilon_0} \right)^{1/2} $$

is the propagation constant. The Gaussian intensity distribution of these modes at $z = 0$ is given by

$$ A_{1,2} A_{1,2}^* = E_{0z}^2 \exp\left(-\frac{r^2}{r_0^2}\right) $$

where ‘r’ is the radial coordinate of the cylindrical coordinate system and ‘$r_0$’ is the initial beam width.

Following Sodha et al. [7] and analyzing on the similar lines of Asthana et al [8] the dielectric tensor in a magnetoactive plasma due to relativistic variation of mass for both modes of propagation can be written as [9]

$$ \varepsilon_z = \varepsilon_{zz} + i\varepsilon_{zz} (E_z \cdot E_z^*) $$  \hspace{1cm} (2)

$$ \varepsilon_{zz} = \varepsilon_{zz} + \varepsilon_{zz} (E_z \cdot E_z^*) $$  \hspace{1cm} (3)

where,

$$ \varepsilon_{zz} = 1 - \frac{\omega_p^2}{\omega^2 \left(1 + \frac{\omega_p^2}{\omega^2}\right)} $$

and

$$ \varepsilon_{zz} = \left(1 - \frac{\omega_p^2}{\omega^2}\right) $$
are the linear terms and $\omega_{2s}$, $\omega_{2}$ are nonlinear terms defined later for relativistic magnetoplasma. Here

$$\omega_p^2 = \left(\frac{4\pi ne^2}{m_r}\right)$$

is the relativistic plasma frequency and

$$\omega_c = \left(\frac{eB}{mc}\right)$$

the cyclotron frequency. At relativistic intensity correction in electron mass for circularly polarized wave is

$$m_r = m\left[1 + \frac{e^2 E E^*}{m^2 \omega^2 c^2}\right]^{1/2} \quad (4)$$

It is seen that the relativistic dependence of the mass of electron on its speed leads to the nonlinear dielectric function, which depends on electric vector of the wave. Since the relativistic nonlinearity of the dielectric function occur through plasma frequency, it should manifest itself in a time of the order of period of plasma oscillations viz., $\omega_p^{-1}$. Hence effective dielectric constant in magnetized plasma is

$$\varepsilon'_z = 1 - \frac{\omega_p^2}{\omega^2} \left[1 + \frac{e^2 E E^*}{m^2 \omega^2 c^2}\right]^{-1/2} \quad (5)$$

For arbitrarily large nonlinearity the dielectric constant is

$$\varepsilon'_z = \varepsilon'_{zs} - r^2 \varepsilon'_{2s}(f) \quad (6)$$

where

$$\varepsilon'_{zs} = \varepsilon'_{z} + \frac{\Omega_p^2}{(1 \mp \Omega_c)} \left[1 - \frac{\alpha E^2}{f^2} \left(\frac{\varepsilon'_z(1)}{\varepsilon'_z(z)}\right)^{1/2}\right]^{-1/2} \quad (7)$$

$$\varepsilon'_{2s} = \frac{\Omega_p^2}{(1 \pm \Omega_c)} \alpha \frac{E^2}{r^2 f^2} \left[1 + \frac{\alpha E^2}{f^2} \left(\frac{\varepsilon'_z(1)}{\varepsilon'_z(z)}\right)^{1/2}\right]^{-3/2} \quad (8)$$

with

$$\Omega_p = \left(\frac{\omega_p}{\omega}\right)$$

$$\Omega_c = \left(\frac{\omega_c}{\omega}\right)$$

and

$$\alpha = \left(\frac{\varepsilon^2}{m^2 \omega^2 c^2}\right)$$
2.1. Wave propagation for extraordinary mode

The wave equation in its general form for a varying field in magnetized plasma is written as

\[ \nabla^2 \mathbf{E} - \nabla (\nabla \cdot \mathbf{E}) + \left( \frac{\omega^2}{c^2} \right) \varepsilon \varepsilon_0 \mathbf{E} = 0 \]  

which is valid when

\[ \left( \frac{\omega^2}{c^2} \right) \left( \frac{1}{\varepsilon_0} \right) \nabla^2 \left( \ln \varepsilon \right) \ll 1 \]  

(10)

This condition is valid for most situations of interest. Since the variation of the field in \( x \) and \( y \) direction are very slow as compared to those in \( z \) direction, the two modes corresponding to \( A_1 \) and \( A_2 \) are loosely coupled. Hence we consider only one mode at a time. So one of the modes can be taken to be zero, and the behavior of remaining one in the magnetic field of plasma is studied.

A first order JWKB solution of the one-dimensional wave equation for extraordinary mode

\[ \frac{\partial^2 A_1}{\partial z^2} + \frac{1}{2} \left( 1 + \varepsilon \varepsilon_0 \right) \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) A_1 + \frac{\omega^2}{c^2} \varepsilon \varepsilon_0 A_1 = 0 \]  

(11)

is

\[ A_i(r, z) \propto \left[ \frac{\varepsilon \varepsilon_0}{\varepsilon \varepsilon_0(f)} \right]^{1/4} \exp \left\{ i \left[ \omega t - \frac{\omega}{c} \int (\varepsilon \varepsilon_0)^{1/2} dz \right] \right\} \]  

(12)

The solution is valid if both the first- and second order derivative of \( \varepsilon \varepsilon_0 \) is negligibly small. The above solution suggests a solution of (9) of the form

\[ A_i(r, z) = A(r, z) \left[ \frac{\varepsilon \varepsilon_0}{\varepsilon \varepsilon_0(f)} \right]^{1/4} \exp \left\{ i \left[ \omega t - \frac{\omega}{c} \int (\varepsilon \varepsilon_0)^{1/2} dz \right] \right\} \]  

(13)

where

\[ k_s(f) = \frac{\omega}{c} \left[ \varepsilon \varepsilon_0(f) \right]^{1/2} \]

\[ k_s(0) = \frac{\omega}{c} \left[ \varepsilon \varepsilon_0(f = 1) \right]^{1/2} \]

and \( A(r, z) \) may be expressed as

\[ A(r, z) = A_s(r, z) \exp \left[ -i \frac{\omega}{c} (\varepsilon \varepsilon_0)^{1/2} S(r, z) \right] \]  

(14)

\( S(r, z) \) is called the eikonal and is related to the curvature of the wave front. Substituting for \( A_i(r, z) \) and \( A(r, z) \) from (13) and (14) into (9), neglecting \( (\partial^2 A / \partial z^2) \) and derivatives of \( \varepsilon \varepsilon_0 \) which is justifiable for slowly converging and diverging beams and separating real and imaginary parts of the resulting equation, one obtains

\[ \frac{2}{c} \frac{\partial S}{\partial z} + \frac{1}{2} \left( \frac{\partial}{\partial z} + \frac{\partial}{\partial r} \right) \left( \frac{\partial S}{\partial r} \right)^2 + r^2 \frac{\varepsilon \varepsilon_0}{2 \varepsilon \varepsilon_0 A_0} \left( 1 + \varepsilon \varepsilon_0 \right) \left( \frac{\partial^2 A}{\partial r^2} + \frac{\partial A}{\partial r} \right) = 0 \]  

(15)
The solution of equation (15) and (16) for an initially Gaussian beam can be written as

\[ A_i^2 = \left( \frac{E_{i}}{T^2} \right) \left[ \frac{E_{i}(0)}{E_i(f)} \right] \exp \left( \frac{-r^2}{2f^2} \right) \] (17)

\[ S = \frac{r^2}{2} \beta(z) + \eta(z) \] (18)

\[ \beta(z) = 2 \frac{1}{f(z)} \left( 1 + \frac{E_{i}}{E_{i0}} \right) \frac{df}{dz} \] (19)

where, \( \beta(z) \) is the curvature of the wave front. The parameter \( f(z) \) is obviously the beamwidth parameter, which in view of equation (1) must be equal to 1 at \( z = 0 \), further, an initially parallel beam at \( z = 0 \) requires \( (df/dz)_{z=0} = 0 \). Thus the two boundary conditions on \( f \) are

\[ f = 1 \quad \text{and} \quad \left( \frac{df}{dz} \right) = 0 \quad \text{at} \quad z = 0 \] (20)

From equation (13), (14), and (17) one obtains

\[ |A_i|^2 = \left( \frac{E_{i}(0)}{E_i(z)} \right) \left[ \frac{E_{i}(0)}{E_i(f)} \right] \exp \left( \frac{-r^2}{2f^2} \right) \] (21)

If we now substitute the expression for \( S \), \( A_i^2 \) and \( E_i \) from equation (17), (18) and (6) respectively in equation (15) and collect the coefficient of \( r^2 \) (paraxial approximation), we obtain the equation governing the beam width parameter for extraordinary mode as

\[ \frac{d^2 f}{dz^2} = -\frac{1}{2} \left( 1 + \frac{E_{i}}{E_{i0}} \right) \frac{\omega^2}{\alpha E_{i0}} \frac{\alpha E_{i0}^2}{2f^2} \left[ 1 + \frac{\alpha E_{i0}^2}{2f^2} \right] \left[ \frac{E_{i}(0)}{E_i(f)} \right]^{-3/2} \]

\[ + \frac{c^2}{4\omega^2 E_{i0} r_i f^3} \left( 1 + \frac{E_{i}}{E_{i0}} \right)^2 \] (22)

Transforming the coordinate \( z \) and initial beam width \( r_i \) to dimensionless forms:

\[ \xi = \left( \frac{z c}{r_i \omega} \right) \quad ; \quad \rho_\circ = \left( \frac{r_i \omega}{c} \right) \]

and defining

\[ p = \alpha E_{i0}^2 \]

\[ \phi(\xi) = (1 + F\xi) \]

We get the characteristic beam propagation equation for extraordinary mode

\[ E_{i0} \frac{d^2 f}{dz^2} = \frac{1}{4} \left( 1 + \frac{E_{i0}}{E_{i0}} \right) \left[ 1 + \frac{E_{i0}}{E_{i0}} \right] \frac{\Omega^2 \rho_\circ^2}{2(1-\Omega)} \left[ \frac{E_{i}(0)}{E_i(f)} \right] \left[ 1 + p \frac{E_{i}(0)}{E_i(z)} \right]^{3/2} \] (23)
For an initial plane wave front of the beam, the initial conditions on \( f' \) are \( (f = 1) \) and \( (d/d\xi) = 0 \) at \( (\zeta = 0) \). When two term on right hand side of equation (23) cancel each other at \( (\zeta = 0) \), \( (d^2f/d\xi^2) = 0; \) since \( (d/d\xi) = 0 \) at \( (\zeta = 0) \) and \( (f = 1) \) for all values of \( \zeta \); in other words, the beam propagate without convergence or divergence.

2.3. Critical-Divider curve

For \( (d^2f/d\xi^2) \) to vanish at some value of \( f \) (or \( \xi \)) the beam power \( p \) must have a value \( p_c \) such that

\[
\rho^2 = \frac{1}{\Omega^2_p} \left[ 1 + \frac{\hat{e}_{es}}{\hat{e}_{ei}} \right] \left( 1 - \Omega \right) \left[ 1 + p \frac{[\hat{e}_{e}(1)]^{3/2}}{[\hat{e}_{e}(z)]^{3/2}} \right]
\]

(24)

At \( \zeta = 0, f = 1, \hat{e}'(z) = \hat{e}'(1), \rho = \rho_o \) and \( p_c = p_{oc} \) so that we have

\[
\rho^2_o = \frac{1}{\Omega^2_{oc}} \left[ 1 + \frac{\hat{e}_{es}}{\hat{e}_{ei}} \right] \left( 1 - \Omega \right) \frac{1 + p_{oc}}{p_{oc}}^{3/2}
\]

(25)

If the initial beam power \( p_o \) and the beam width \( \rho_o \) satisfy (25) we shall have \( (d^2f/d\xi^2) = 0 \) at \( (\zeta = 0) \) ensuring that \( (d/d\xi) \) and \( f' \) retain their initial boundary values throughout the path of propagation; such a propagation without change of beam width is called uniform wave-guide propagation. One can observe that \( p_{oc} \to \infty \) in both the limits \( p_{oc} \to 0 \) and \( p_{oc} \to \infty \) and it has a minimum of \( [{\text{eq} (1)}]^{3/2}/\Omega \) at \( p_{oc} = 1 \).

3. Numerical results and discussion

As stated before the uniform wave guide propagation requires the initial beam power (denoted by \( p_{oc} \)) and beam width \( \rho_o \) to satisfy (25). The \( p_{oc} \) versus \( p_{oc} \) relation according to this equation has been shown by the graph \( p_c \) in Figure 1. If the initial point \( (p_{oc}, \rho_o) \) does not lie on this critical power graph (or simply the critical curve) \( (d^2f/d\xi^2) \) will not vanish at \( (\zeta = 0) \), having a positive value if this point lies on the same side of the critical curve as the origin and negative otherwise. If the point \( (p_{oc}, \rho_o) \) lies on the other side of the curve as the origin so that \( (d^2f/d\xi^2) < 0 \) at \( (\zeta = 0) \), the propagation of the beam through the plasma will result in \( (d/d\xi) \) falling below zero and consequently \( f \) falling below unity. As \( f \) falls below unity, \( (p_{oc}/f^2) \) increases and \( \rho_o f \) decreases. The operative point \( (p_{oc}/f^2, \rho_o f) \) therefore moves towards a downward right direction necessarily meeting the critical curve at some value of \( f \) causing \( (d^2f/d\xi^2) \) to vanish. This point will correspond to a point of inflexion in the \( f \) versus \( \xi \) curve beyond which \( (d^2f/d\xi^2) \) will become positive and \( (d/d\xi) \) will start increasing (decreasing in magnitude) subsequently reaching a value zero and \( f \) reaching a positive minimum. One should however remember that the curve for \( (d/d\xi) \) to vanish for \( f \neq 1 \) equation (24) will be somewhat different from the curve shown in Figure 1 (referring to \( f = 1 \)) equation (25) although the similarity of these two equations suggests that they will have the same general appearance. One can easily conclude that the beam in this case converges with \( f \) oscillating between unity and a positive minimum, producing multiple foci. Against this, if the point \( (p_{oc}, \rho_o) \) falls on the same side of the critical curve as the origin \( (d^2f/d\xi^2) > 0 \) at \( (\zeta = 0) \) and the beam propagation will result in \( f \) increasing above unity, causing the operative point \( (p_{oc}/f^2, \rho_o f) \) to shift in an upward left direction. In the case where the operative point meets the critical curve at some value of \( f \) and crosses towards the other side, the beam-width parameter \( f \) will reach a finite maximum and will oscillate between this maximum and unity through the path of the beam. Such propagation is known as self-guiding propagation.

In the case of the self-focusing or self-guiding the necessary condition is the vanishing of \( (d^2f/d\xi^2) \) at some \( \xi \) or \( f \) value, which requires from equation (24)
\[
\frac{(1 - \varepsilon_s'(z))^2}{\varepsilon_s'(z)} = \left(1 + \frac{\varepsilon_s'}{\varepsilon_{\text{real}}}ight)^2 (1 - \Omega_p) \frac{\Omega_p^2}{p_s^2 p_s' \varepsilon_s'(1)}
\]  
(26)

while the beam-width parameter \( f \) can be expressed as

\[
f^2 = \frac{p_s}{\left\{ \Omega_p^2 \left(1 - \Omega_p \right) \left(1 - \varepsilon_s'(z) \right) \right\} - 1}
\]  
(27)

**Figure 1:** The critical curve (\( p_c \)) and divider curve (\( p_d \)) in the dimensionless initial beamwidth (\( r_o \omega/c \)) versus dimensionless initial beam power \( p_o = \alpha E_0^2 \) plane. Regions I, II and III have been illustrated. The curves correspond to \( \Omega_p^2 = 0.8 \). The points \( a \) (8, 0.3), \( b \) (30, 2), \( c \) (80, 3) and \( d \) (90, 4) refer to typical points for which \( f, \xi \) variation has been given in Figure 2.

Equation (26) gives a positive value for \( \varepsilon_s'(z) \) in the range \( 0 < \varepsilon_s'(z) < 1 \) for any chosen (positive) values of \( \Omega_p, p_o \) and \( p_s \), but (27) yields a real value for \( f \) only for

\[
1 - \varepsilon_s'(z) < \frac{\Omega_p^2}{1 - \Omega_p}
\]

implying that

\[
\frac{1}{\varepsilon_s'(z)} < \frac{1}{1 - \Omega_p^2 \left(1 - \Omega_p \right)}
\]
In view of this inequality, (26) leads to

\[ \rho_c^4 \rho_o^2 > \left(1 + \frac{e_{oc}}{e_{oc}}\right)^2 (1 - \Omega_p^2)^2 \]
\[ \frac{\Omega_p^4}{\Omega_o^4} \left[1 - \frac{\Omega_o^2}{(1 - \Omega_o^2)}\right] \]

Replacing the inequality sign by an equality we obtain a relation between \(\rho_o\) and \(\rho_o\) depicted in Figure 1 by the curve \(p_d\). It is obvious from inequality (25) that the curve \(p_d\) is meaningful only for \(\Omega_p^2 / (1 - \Omega_o^2) < 1\). The curves \(p_c\) and \(p_d\) divide the \((\rho_o, \rho_o)\) plane in three regions [10] shown as I, II and III in the figure. If the initial point \((p_o, \rho_o)\) lies in region I the propagation of the beam can not shift the operative point \((p, \rho)\) to the critical curve and the beam-width parameter will steadily increase with \(\xi\), i.e. the beam will diverge; in case the initial point \((p_o, \rho_o)\) is in region II the beam will be self-guided and if it is in region III the beam will be self-focused. The \(f\) versus \(\xi\) graphs for four representative points \(a\) in region I, \(b\) and \(c\) in region II and \(d\) in region III have been obtained by numerically integrating (23) and are illustrated in Figure 2. The \((p_o, \rho_o)\) coordinates for these four curves are respectively \(a\) (8, 0.3), \(b\) (30, 2), \(c\) (80, 3) and \(d\) (90, 4).

**Figure 3.** Variation of dimensionless axial intensity \(p_o/f^2\) of the beam with distance of propagation \(\xi\) for linear profile; \(p_o = 30\) and \(r_o / c = 4\), for extraordinary mode. The cases refer to underdense and overdense plasma both at \(\xi = 0\).

**Figure 4.** The variation of the beamwidth parameter against the normalized propagation distance of the laser beam with magnetic field (extraordinary mode) and without magnetic field in inhomogeneous plasma.

Figure 3 shows the dimensionless axial beam intensity \(p_o/f^2\) against distance of propagation \(\xi\) for inhomogeneous plasma. For a linear density profile corresponding to critical density \(\Omega_p^2 = 0.8\) (underdense) and \(\Omega_p^2 = 2.0\) (overdense) graphs are illustrated for an extraordinary mode. The figure shows that the successive intensity peaks are higher and grow faster for overdense plasma, which indicates significant penetration of intense laser beam in the overdense plasma. Figure 4 shows the effect of magnetic field in inhomogeneous plasma for extraordinary wave. Graphical plot illustrates the variation of the beamwidth parameter against the normalized propagation distance in presence and in absence of magnetic field.
4. Conclusion
The analytical work is based on the well-known extraordinary wave propagation in relativistic magnetoactive plasma. From calculations we conclude that wave propagation and focusing enhances in presence of magnetic field. The laser light can propagate through overdense-magnetized plasma as an extraordinary wave in the relativistic regime, which is known as magnetically induced transparency.

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