QUANTUM SUPPRESSION OF ALIGNMENT IN ULTRASMALL GRAINS: MICROWAVE EMISSION FROM SPINNING DUST WILL BE NEGLIGIBLY POLARIZED

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ABSTRACT

The quantization of energy levels in small, cold, free-flying nanoparticles suppresses dissipative processes that convert grain rotational kinetic energy into heat. For interstellar grains small enough to have \(\sim GHz\) rotation rates, the suppression of dissipation can be extreme. As a result, alignment of such grains is suppressed. This applies both to alignment of the grain body with its angular momentum \(\mathbf{J}\), and to alignment of \(\mathbf{J}\) with the local magnetic field \(\mathbf{B}_0\). If the anomalous microwave emission is rotational emission from spinning grains, then it will be negligibly polarized at GHz frequencies, with \(P \lesssim 10^{-6}\) at \(\nu > 10 GHz\).

Key words: cosmic background radiation – dust, extinction – ISM: magnetic fields – molecular processes – polarization – solid state: refractory

1. INTRODUCTION

The emission from the interstellar medium (ISM) in the Milky Way and other star-forming galaxies includes strong mid-IR emission features at 3.3, 6.2, 7.7, 8.6, 11.3, 12.6, and 17 \(\mu m\) (see, e.g., Smith et al. 2007). The only viable explanation for this emission is a substantial interstellar population of nanoparticles with composition resembling polycyclic aromatic hydrocarbons (PAHs) and containing as few as \(\sim 40\) atoms (Tielens 2008). The PAHs have been identified by their characteristic IR emission features, but it is possible that nanoparticles with other compositions—such as silicates or metallic Fe—could also be abundant. For the densities and temperatures present in the ISM, nanoparticles containing fewer than \(\sim 10^7\) atoms will inevitably be spinning at \(\sim GHz\) frequencies.

The so-called anomalous microwave emission (AME) observed at 10–60 GHz was interpreted as rotational emission from rapidly rotating nanoparticles (Draine & Lazarian 1998a, 1999b). Given the PAH abundances and size distribution required to explain the observed mid-IR emission, it was natural to consider spinning PAHs as the source of the AME. However, a recent observational study (Hensley et al. 2016) failed to find the expected correlation of AME emission with PAH abundance. Hensley et al. (2016) therefore suggested that spinning non-PAH (e.g., silicate or iron) nanoparticles may also be present in the ISM. Possible emission from silicate and iron nanoparticles has been further discussed by Hoang et al. (2016), Hoang & Lazarian (2016b), and B. S. Hensley & B. T. Draine (2016, in preparation).

It is possible that some or all of the AME might be thermal emission from grain material with a pronounced opacity peak near \(\sim 30 GHz\), such as might occur for some magnetic materials (Draine & Lazarian 1999; Draine & Hensley 2013). However, the AME does not show the linear dependence on grain temperature that would be expected (Hensley et al. 2016), and the stringent observational upper limits on polarization are difficult to reconcile with the idea that the AME originates from the “large” grains that are generally aligned, with long axes tending to be perpendicular to the local magnetic field (see, e.g., Planck Collaboration et al. 2015a, 2015b). Accordingly, rotational emission from rapidly spinning nanoparticles appears to be most likely.

This paper examines the dynamics of dissipation in spinning interstellar nanoparticles, whether composed of hydrocarbons, silicates, or other materials. Two types of dissipation are discussed. One is the internal dissipation that allows a tumbling grain to minimize its rotational kinetic energy by aligning \(\hat{\mathbf{a}}_1\), the principal axis of largest moment of inertia, with its angular momentum \(\mathbf{J}\). The other is the dissipation that occurs in a static magnetic field \(\mathbf{B}_0\) when \(\mathbf{J}\) is not aligned with \(\mathbf{B}_0\). In both cases, rotational kinetic energy is converted to heat.

Dissipative processes in grains have usually been treated in the classical limit where the rotating body has many internal degrees of freedom. However, in very small grains, energy level quantization can suppress intramolecular vibration–rotation energy transfer (IVRET) and dissipation of rotational energy.

Here we examine the quantum suppression of dissipation in spinning grains. Vibration–rotation energy exchange must be suppressed when the vibrational energy level spacing \(\Delta E\) is larger than the intrinsic width \(\delta E\) of the energy levels. We estimate the suppression factor as a function of the grain’s size and vibrational energy content \(E_{vib}\).

We calculate the implications of this quantum suppression both for the alignment of \(\mathbf{J}\) with \(\mathbf{B}_0\) and for the alignment of the principal axis \(\hat{\mathbf{a}}_1\) with \(\mathbf{J}\). Quantum suppression effects are extreme for the smallest nanoparticles, leading to almost total suppression of alignment of \(\mathbf{J}\) with \(\mathbf{B}_0\) for the smallest (and therefore most rapidly rotating) grains. If the AME is rotational emission from nanoparticles, it will be essentially unpolarized.

The paper is organized as follows. Section 2 reviews the energy levels of spinning nanoparticles, and Section 3 examines the distribution of vibrational modes and energy levels. Section 4 estimates the factor \(\psi_0\left(E_{vib}\right)\) by which IVRET will be suppressed in a nanoparticle, as a function of the vibrational energy \(E_{vib}\) present in the nanoparticle. Section 5 concerns the quantum suppression of alignment of the grain body with \(\mathbf{J}\). In Section 6 we discuss the quantum suppression of magnetic dissipation in either paramagnetic or ferromagnetic grains. The rotation and alignment of spinning nanoparticles is calculated in Section 7. In Section 8 we calculate the
polarization of rotational emission from spinning nanoparticles as a function of frequency. For conditions characteristic of neutral diffuse clouds, we show that the rotational emission at frequencies >1 GHz should have very small polarization, <0.01%. In Section 9 we calculate the degree of polarization of thermal emission from spinning nanoparticles, and the dichroic extinction contributed by such particles. The results are discussed in Section 10, and summarized in Section 11.

2. ENERGY LEVELS OF A SPINNING NANOPARTICLE

2.1. Rotation

Consider grains that can be approximated by spheroids, with \( I_\parallel, I_\perp, I_z \) being the eigenvalues of the moment of inertia tensor. Consider the case \( I_\parallel > I_\perp \) (i.e., oblate spheroids). Let \( J \) be the total angular momentum quantum number. In the center-of-mass frame, the total energy of the grain is

\[
E_{v,J,K} = E_{v,0,0} + E_{\text{rot}},
\]

where the “rotation constants” \( A, B \) may depend on the vibration state \( v \), and the quantum number \( K \) is the projection of \( J \) along the symmetry axis \( \hat{a}_z \). For a spheroid with semiaxes \( a \) and \( b \) parallel and perpendicular to the symmetry axis, and volume-equivalent radius \( a_{\text{eff}} = (ab^2)^{1/3} \), the rotation constants are

\[
A = \frac{\hbar}{4\pi c I_\parallel} = \frac{5\hbar}{4\pi Mc 2b^2},
\]

\[
B = \frac{\hbar}{4\pi c I_\perp} = \frac{5\hbar}{4\pi Mc a^2 + b^2},
\]

where \( I_\parallel, I_\perp \) are the moments of inertia for rotation parallel or perpendicular to the symmetry axis, and \( M \) is the mass. An oblate spheroid \( (I_\parallel > I_\perp) \) has \( B > A \). For future reference, the number of atoms is

\[
N \approx 50 \left( \frac{a_{\text{eff}}}{5 \text{ Å}} \right)^3
\]

for density \( \rho = 3.4 \text{ g cm}^{-3} \) and mean atomic weight 21.6 amu, as might be appropriate for silicate material. Figure 1(a) shows the rotational states of an oblate spheroid with \( J \leq 25 \). For a given \( J \), there are \( J + 1 \) possible values of \( K \), with \( K = 0 \) giving the highest energy and \( K = J \) giving the lowest. Figure 1(b) shows the energy levels for \( J = 20 \).

2.2. Vibration

Suppose the grain has total internal energy \( E_{v,J,K} \) in vibration and rotation. The uncertainty \( \delta E \) of the internal energy is determined by the level lifetime. If \( A_{\text{rad}} \) is the probability per unit time of a spontaneous radiative transition, \( N_{\text{abs}} \) is the probability per unit time of absorbing a photon, and \( N_{\text{coll}} \) is the probability per unit time of an inelastic collision with a gas particle, then

\[
\delta E \approx \hbar (A_{\text{rad}} + N_{\text{abs}} + N_{\text{coll}}).
\]

Radiation can be either purely rotational or rovibrational; we write \( A_{\text{tot}} = A_{\text{rot}} + A_{\text{vib}} \). The Einstein A coefficient for purely rotational transitions is

\[
A_{\text{rot}} \approx \frac{2\omega_{\text{rot}}^3}{3\hbar^3} \nu_{\text{rot}}^2,
\]

where \( \omega_{\text{rot}} \approx 4\pi c B_v J \), and \( \mu_\perp \) is the rms electric dipole moment perpendicular to \( J \) for the spinning grain. If \( \nu_{\text{rot}} = \omega_{\text{rot}}/2\pi \), then

\[
A_{\text{rot}} \approx 4 \times 10^{-6} \left( \frac{\mu_\perp}{5 \text{ D}} \right)^2 \left( \frac{\nu_{\text{rot}}}{30 \text{ GHz}} \right)^3 \text{ s}^{-1}.
\]
Equation (8) is the semiclassical estimate for a sphere \((I_\parallel = I_\perp)\). For symmetric-top or asymmetric-top molecules, the rotational spectrum is more complex (e.g., Townes & Schawlow 1975) but the total photon emission rate for purely rotaional transitions can still be approximated by Equation (8).

Consider now the vibrational emission from a spherical particle of radius \(a_{\text{eff}}\). At long wavelengths, the absorption cross section for interstellar amorphous silicate grains is (B. T. Draine & B. Hensley 2016, in preparation)

\[
C_{\text{abs}}(\lambda) \approx 9 \times 10^{-19} \text{cm}^2 \left(\frac{100 \, \mu\text{m}}{\lambda}\right)^2 \left(\frac{a_{\text{eff}}}{10^{-7} \text{cm}}\right)^3; \tag{9}
\]

for this cross section, a grain with vibrational temperature \(T_{\text{vib}}\) radiates photons at a rate

\[
A_{\text{vib}} = 4\pi \int C_{\text{abs}}(\nu) \frac{B_\nu(T)}{h\nu} d\nu 
\approx 2.6 \left(\frac{T_{\text{vib}}}{10^2 \text{K}}\right)^5 \left(\frac{a_{\text{eff}}}{10^{-7} \text{cm}}\right)^3 \text{s}^{-1}. \tag{10}
\]

and power

\[
\dot{E}_{\text{vib}} = 4\pi \int C_{\text{abs}}(\nu) B_\nu(T) d\nu 
\approx 1.9 \times 10^{-13} \left(\frac{T_{\text{vib}}}{10^2 \text{K}}\right)^6 \left(\frac{a_{\text{eff}}}{10^{-7} \text{cm}}\right)^3 \text{erg s}^{-1}. \tag{11}
\]

In the interstellar radiation field, the nanoparticle absorbs starlight photons at a rate (see Figure 11 of Draine & Li 2001)

\[
\dot{N}_{\text{abs}} \approx 5 \times 10^{-7} U \left(\frac{a_{\text{eff}}}{10^{-7} \text{cm}}\right)^3 \text{s}^{-1}, \tag{12}
\]

where the dimensionless factor \(U\) is the intensity of the radiation field relative to the local interstellar radiation field (Mathis et al. 1983).

In gas of density \(n_\text{H}\) and temperature \(T_{\text{gas}}\), the gas–grain collision rate is

\[
\dot{N}_{\text{coll}} \approx n_\text{H} \left(\frac{8kT_{\text{gas}}}{\pi m_\text{H}}\right)^{1/2} \pi a_{\text{eff}}^2 
= 1.4 \times 10^{-7} \text{s}^{-1} \left(\frac{n_\text{H}}{30 \text{ cm}^{-3}}\right) \left(\frac{T_{\text{gas}}}{10^2 \text{K}}\right)^{1/2} \left(\frac{a_{\text{eff}}}{10^{-7} \text{cm}}\right)^2. \tag{13}
\]

\(\dot{N}_{\text{coll}}\) will be modified if the nanoparticle is charged because the induced-polarization interaction increases the collision rate with neutrals, and Coulomb focusing increases the rate of collision with electrons (for positive grain charge) or ions (for negative grain charge). Nevertheless, \(\dot{N}_{\text{coll}}\) will still be small compared to \(A_{\text{vib}}\), therefore the effects of grain charge are unimportant.

Thus for a silicate nanoparticle with \(a_{\text{eff}} \approx 5 \times 10^{-8} \text{ cm}\), we have a level lifetime

\[
\tau \approx 0.3 \left(\frac{T_{\text{vib}}}{10^2 \text{K}}\right)^5 + 4 \times 10^{-6} \left(\frac{\nu_{\text{tot}}}{30 \text{ GHz}}\right)^3 + 6 \times 10^{-8} U \nonumber \]

\[
+ 3 \times 10^{-8} \left(\frac{n_\text{H}}{30 \text{ cm}^{-3}}\right) \left(\frac{T_{\text{gas}}}{10^2 \text{K}}\right)^{1/2} \text{s}. \tag{14}
\]

corresponding to a level width

\[
\frac{\delta E}{\hbar c} \approx 1.6 \times 10^{-12} \left[\left(\frac{T_{\text{vib}}}{10^2 \text{K}}\right)^5 + 10^{-5} \left(\frac{\nu_{\text{tot}}}{30 \text{ GHz}}\right)^3 \right. \nonumber \]

\[
+ 2 \times 10^{-7} U + 10^{-7} \left(\frac{n_\text{H}}{30 \text{ cm}^{-3}}\right) \left(\frac{T_{\text{gas}}}{10^2 \text{K}}\right)^{1/2} \text{cm}^{-1}. \tag{15}
\]

The level lifetimes and widths are strongly dependent on the vibrational temperature \(T_{\text{vib}}\). Broadening due to collisions, microwave rotational emission, and starlight absorption are of secondary importance so long as the grain has vibrational energy content corresponding to \(T_{\text{vib}} \gtrsim 20 \text{ K}\).

3. VIBRATIONAL DENSITY OF STATES

A grain with \(N\) atoms, in its electronic ground state, has \(3N - 6\) vibrational degrees of freedom, each with vibrational quantum number \(\tilde{v}_i\). The vibrational state of the grain is specified by the list of vibrational quantum numbers \(v = \{\tilde{v}_1, \ldots, \tilde{v}_{3N-6}\}\) of the modes. Let \(N_v(E)\) be the number of distinct vibrational states \(v = \{\tilde{v}_1, \ldots, \tilde{v}_{3N-6}\}\) with total vibrational energy \(E_{\text{vib}} < E\). If the vibrational modes are approximated as a set of harmonic oscillators with frequencies \(\omega_i\), \(N_v(E)\) can be calculated using the Beyer–Swinehart algorithm (Beyer & Swinehart 1973; Stein & Rabinovitch 1973). We will consider silicate nanoparticles as an example, but our conclusions are insensitive to the detailed composition, and similar results would be obtained for PAHs or for Fe nanoparticles.

The spectrum of vibrational modes for silicates was discussed by Draine & Li (2001, hereafter DL01). DL01 found that the experimental specific heats (Leger et al. 1985) for basalt glass (50% SiO₂, 50% metal oxides by mass) and obsidian glass (75% SiO₂, 25% metal oxides by mass) could be reproduced if two-thirds of the modes were distributed according to a two-dimensional Debye model with Debye temperature \(\Theta_2 = 500 \text{ K}\), and one-third of the modes according to a three-dimensional Debye model with Debye temperature \(\Theta_3 = 1500 \text{ K}\). The lowest-frequency mode is estimated to have

\[
\hbar \omega_1 = \frac{k\Theta_2}{2^{2/3} (N - 2)^{2/3}}. \tag{16}
\]

The \((N - 2)^{-2/3}\) dependence arises from the assumption that some of the modes are distributed in the same way as for a two-dimensional Debye model. While surprising, this model does reproduce the measured specific heat for bulk basalt and obsidian down to 10 K (see Figure 2 of Draine & Li 2001). For \(\Theta_2 = 500 \text{ K}\) and \(N = 40\), Equation (16) gives \(\hbar \omega_1/\hbar c = 10 \text{ cm}^{-1}\). The lowest-frequency vibration will presumably be a bending or torsional mode of the nanocluster.

Using the mode spectrum prescription from DL01, \(N_v(E)\) is calculated using the Beyer–Swinehart algorithm. The result is shown in Figure 2(a) for five values of \(N\). Figure 2(b) shows \(E_{\text{GR}}\), where the vibrational density of states \(g_v = dN_v/dE\), and we have averaged over bins of width \(\Delta E/\hbar c = 1 \text{ cm}^{-1}\); the “noise” at the lowest energies arises from the stepwise character of the function \(N_v(E)\).

\[a\] For the three-dimensional Debye model, the lowest-frequency mode would scale as \(\omega_1 \propto (N - 2)^{-1/3}\).
For vibrational energies \( E_{\text{vib}} \), the number of states \( N_v(E) \) is huge (>10^{20}), and the vibrational states can be treated as a continuum, even for a nanoparticle with as few as 40 atoms. However, for low energies, the discreteness of the vibrational spectrum can have important consequences.

4. QUANTUM SUPPRESSION OF INTERNAL RELAXATION

From Figure 1 we see that an oblate grain with fixed \( J \) and initial \( K < J \) can reduce \( E_{\text{rot}} \) by increasing \( K \), e.g., \( K \to K+1 \). However, energy conservation requires that this energy be transferred to vibrational modes. For a transition \( K \to K+1 \) to be possible, one must have \( K < J \) and there must be another vibrational state \( v_2 \) such that

\[
|E_{v_2,J,K+1} - E_{v_1,J,K}| \lesssim \delta E, \tag{17}
\]

where \( \delta E \) is the width of the energy level due to radiative or collisional broadening. Figure 3 shows such a transition. If we approximate \( B_{v_2} \approx B_{v_1}, A_{v_2} \approx A_{v_1} \), then Equation (17) becomes

\[
E_{v_2,0,0} = E_{v_1,0,0} + \hbar c (B_v - A_v)(2K + 1) \pm \delta E. \tag{18}
\]

The probability of (18) being satisfied, i.e., for a state \( v_2 \) to be available at the required energy, is approximately

\[
\psi_q(E_{\text{vib}}) = 1 - \exp(-g_E \delta E), \tag{19}
\]

where \( g_E \) is the vibrational density of states. For a tumbling grain, we will take the rate of vibration–rotation energy exchange (due to viscoelastic dissipation or other processes) to be the “bulk” rate multiplied by \( \psi_q(E_{\text{vib}}) \), which we will refer to as the quantum suppression factor.

Condition (18) applies to viscoelastic dissipation, which exchanges energy between kinetic energy of rotation and vibrational energy while the lattice angular momentum \( J \) remains constant. The processes of “Barnett relaxation” (Purcell 1979)—where the rotational kinetic energy can be reduced if some of the angular momentum is taken up by the system of electron spins—and “nuclear spin relaxation” (Lazarian & Draine 1999a)—where angular momentum is transferred to the system of nuclear spins—are slightly different from viscoelastic damping, because some of the lattice angular momentum is transferred to the electron or nuclear spin systems, and the lattice angular momentum quantum number \( J \to J - 1 \). Such transitions are discussed in Section 6, but the same quantum suppression factor \( \psi_q \) applies.
The bottom panel of Figure 4 shows the quantum suppression factor $\psi(E_{\text{vib}})$ as a function of vibrational energy $E_{\text{vib}}$ for five selected sizes: $N = 40, 50, 100, 200,$ and 400 atoms. For each case, the dot shows the point where $g_E E = 1$; for energies $E_{\text{vib}}$ below this point, IVRET will be suppressed. The middle panel shows the vibrational temperature $T_{\text{vib}}$ as a function of $E_{\text{vib}}$, and the top panel shows the cooling time $\tau_{\text{cool}} \equiv E_{\text{vib}} / |dE_{\text{vib}} / dt|_{\text{rad}}$ where $|dE_{\text{vib}} / dt|_{\text{rad}}$ is the thermal power radiated by the grain. We define the critical temperature $T_{\text{crit}}$ to be the vibrational temperature at which $g_E E = 1$. Figure 5 shows the critical energy $E_{\text{crit}}$ and critical vibrational temperature $T_{\text{crit}}$ as a function of nanoparticle size.

5. ALIGNMENT OF THE GRAIN BODY WITH ANGULAR MOMENTUM $\mathbf{J}$

Consider a grain with angular momentum quantum number $J$. Alignment of the grain axis $\hat{a}_1$ with $\mathbf{J}$ is measured by

$$\langle \cos^2 \theta_{\hat{a}_1 J} \rangle = \frac{\langle K^2 \rangle}{J(J + 1)},$$

where $\theta_{\hat{a}_1 J}$ is the angle between $\hat{a}_1$ and $\mathbf{J}$. If vibration–rotation energy exchange is rapid, then the tumbling grain will have fluctuating $K$, with the probability of being in state $K$ given by

$$p_k(J, T_{\text{vib}}) = C^{-1} \exp \left[ \frac{\hbar c (B_v - A_v) K^2}{k T_{\text{vib}}(E)} \right] \quad K = -J, ..., J$$

where $C = \sum_{K=-J}^{J} \exp[\hbar c (B_v - A_v) K^2 / k T_{\text{vib}}]$. The tumbling grain will have $\langle K^2 \rangle = \sum K^2 p_K$.

Single-photon heating of a small nanoparticle will raise it to a high temperature, resulting in near-random orientation ($\langle \cos^2 \theta_{\hat{a}_1 J} \rangle \approx 1/3$). So long as IVRET is rapid, $\langle \cos^2 \theta_{\hat{a}_1 J} \rangle$ will gradually increase as the grain cools, and the states of lower rotational energy will be increasingly favored. When the grain temperature falls to $T_{\text{crit}}$, the rate of internal dissipation will be suppressed. As shown in Figure 4, the onset of suppression for a nanoparticle with $N = 100$ atoms is at $T_{\text{vib}} \approx 75$ K, and by the time the temperature has dropped to $\sim 65$ K the suppression factor is $\psi \approx 10^{-6}$.

We have (see Figure 4) $T_{\text{crit}} \approx 140$ K for $N = 40$, and $\approx 40$ K for $N = 400$. Because of the rapid drop in $\psi(E_{\text{vib}})$ when $T_{\text{vib}}$ drops below $T_{\text{crit}}$, we will approximate vibration–rotation energy exchange as rapid provided $T_{\text{vib}} > T_{\text{crit}}$; but as negligibly slow when $T_{\text{vib}} < T_{\text{crit}}$. Thus, after absorbing a starlight photon that heats it to $T_{\text{vib}} > T_{\text{crit}}$, the nanoparticle will have full internal relaxation, with $p_k$ given by Equation (21), as it cools down until reaching temperature $T_{\text{crit}}$, at which time the angle $\theta_{\hat{a}_1 J}$ is frozen until the next heating event caused by a starlight photon, or $\mathbf{J}$ is changed by collisions or radiation. For a grain undergoing stochastic heating by starlight photons, let
\( p_E \) be the probability of being in energy bin \( E \). Then, for grains with angular momentum quantum number \( J \)

\[
\langle \cos^2 \theta_{\alpha J} \rangle \approx \frac{1}{J(J+1)} \left[ \sum_{E>E_{\text{crit}}} p_E \sum_K K^2 \rho_K \langle J, T_E \rangle \right] + \left( \sum_{E=0}^{E_{\text{crit}}} p_E \sum_K K^2 \rho_K \langle J, T_{\text{crit}} \rangle \right) .
\]

(22)

We will evaluate \( \langle \cos^2 \theta_{\alpha J} \rangle \) below after discussion of the excitation of \( J \).

6. QUANTUM SUPPRESSION OF MAGNETIC DISSIPATION AND ALIGNMENT WITH \( B_0 \)

Now consider a nanoparticle spinning in the static interstellar magnetic field \( B_0 \). The unpaired electron spins in the nanoparticle couple to \( B_0 \). If \( J \) has a component perpendicular to \( B_0 \), then the (weak) magnetization of the grain will lag (in the grain frame, there is a rotating component of the magnetic field), and the coupling of \( B_0 \) to the unpaired spins will exert a torque on the spinning grain, acting to reduce the component of the total angular momentum \( J \) that is perpendicular to \( B_0 \). This is the mechanism of Davis & Greenstein (1951) for alignment of \( J \) by magnetic dissipation.

Let \( J_{\parallel B} \) and \( J_{\perp B} \) be the components of the angular momentum parallel and perpendicular to \( B_0 \). In a large paramagnetic grain at temperature \( T_{\text{vib}} \), magnetic dissipation will cause \( J_{\perp B} \) to change at a rate

\[
\left( \frac{dJ_{\perp B}}{dt} \right)_{\text{DG,d}} = \frac{J_{\perp B}}{\tau_{\text{DG,0}}} \left( \frac{T_0}{T_{\text{vib}}} \right)
\]

(23)

\[
\tau_{\text{DG,0}} = \frac{2\rho J^2}{5B_0^2 K_0},
\]

(24)

where

\[
K_0 \equiv \frac{\Im(\chi(\omega))}{\omega} \bigg|_{\omega=\delta} \approx 10^{-13} \left( \frac{18 \, \text{K}}{T_0} \right) \, \text{s}
\]

(25)

for normal paramagnetic dissipation (Jones & Spitzer 1967). For this estimate for \( K_0 \) to apply to a nanoparticle, there should be at least a few unpaired spins present in the system, so that (1) there are spins to respond to the magnetic field, and (2) there will be spin–spin coupling as per the estimate of Jones & Spitzer (1967) for \( K_0 \).

Early studies of grain alignment by paramagnetic dissipation (e.g., Davis & Greenstein 1951) assumed that a grain spinning in a weak static magnetic field \( H_1 \) would have the same dissipation as a stationary grain in a rotating magnetic field \( H_1 \). Because dissipation in a rotating magnetic field is suppressed when \( \omega > \tau_r^{-1} \), where \( \tau_r \) is the spin–spin relaxation time, this led to the expectation (Jones & Spitzer 1967) that Equations (23)–(25) would overestimate the dissipation when the rotation frequency \( \omega_{\text{rot}} \) is greater than \( \tau_r^{-1} \). However, Lazarian & Draine (2000, hereafter LD00) showed that the Barnett effect causes the dissipation rate in a weak static magnetic field \( H_1 \) to remain proportional to \( H_1^2/\omega_{\text{crit}}^3 \) even for \( \omega_{\text{crit}} > \tau_r^{-1} \) so that Equations (23)–(25) remain valid even for very rapid rotation.

LD00 termed this “resonance relaxation” because of the close analogy to electron paramagnetic resonance.

Thus, in a sufficiently large specimen of paramagnetic material, we expect Equations (23)–(25) to apply even for very rapid rotation rates. However, we will see below that quantum effects will reduce the value of \( K_0 \) in very small, cold, nanoparticles of paramagnetic material.

In a single-domain ferromagnetic or ferrimagnetic grain, the spins will be spontaneously aligned, and the magnetization dynamics are quite different from the paramagnetic case. Dissipation in ferromagnetic materials at high frequencies has been discussed by Drake & Hensley (2013), with attention to Davis–Greenstein alignment. Using the Gilbert equation (Gilbert 2004) for the dynamical magnetization with Gilbert parameter \( \alpha_G \approx 0.2 \), \( K_0 \) for a pure Fe grain is estimated to be (see Drake & Hensley 2013, Equation (97))

\[
K_0 \approx 3 \times 10^{-13}(1 + \cos^2 \Theta) \, \text{s},
\]

(26)

where \( \Theta \) is the angle between the spontaneous magnetization \( M \) and \( J \). For paramagnetism, \( K_0 \propto 1/T_0 \), but for ferromagnetism, \( K_0 \) does not depend directly on grain temperature (provided the temperature is well below the Curie temperature \( T_C \approx 1100 \, \text{K} \)). The estimate for \( K_0 \) for metallic Fe is (coincidently) only a factor of a few larger than the classical estimate for paramagnetism (25) at the typical grain temperature \( T_0 \approx 18 \, \text{K} \). The dependence on \( \Theta \) causes ferromagnetic dissipation to be faster if the spontaneous magnetization direction is close to \( J \), but \( (1 + \cos^2 \Theta) \) is at most a factor of two.

The time-dependent torques experienced by the unpaired spins are transferred to the lattice, which will both excite lattice vibrations (heat) and reduce the lattice angular momentum. Energy is conserved: the decrease in rotational kinetic energy must be accompanied by heating of the lattice. Figure 6 shows an example of such a transition \( J \to J-1 \). The example shown has \( K \to K-1 \), but other values of \( \Delta K \) can also apply.

Because energy must be conserved, magnetic dissipation can take place only if there is a suitable energy level \( v_2 \) such that (see Figure 6)

\[
E_{v_2,0,0} = E_{v_2,0,0} + \Delta E_{\text{heat}} \pm \delta E,
\]

(27)

where \( \delta E \) is the “level width,” and

\[
\Delta E_{\text{heat}} = 2B_vJ + (A_v - B_v)(K_0^2 - K_2^2).
\]

(28)

The likelihood of a vibrational state \( v_2 \) being available so that (27) can be satisfied is again given by the function \( v_q(E_{v_2}) \) defined in Equation (19).

For a nanoparticle heated by starlight photons, \( T_{\text{vib}} \) is a stochastic function of time, shown schematically in Figure 7. When the grain is vibrationally “hot” immediately following a photon absorption, the torques due to the static magnetic field act to disalign \( J \) and \( B_0 \) (by acting to increase \( J_{\perp B}^2 \)) but when \( T_{\text{vib}} \) drops below \( T_{\text{crit}} \), the dissipative torques have a net aligning effect (by acting to decrease \( J_{\perp B}^2 \)). When the temperature falls below \( T_{\text{crit}} \), the dissipation is strongly suppressed; the suppression factor \( v_q \) falls off so rapidly that the magnetic torques effectively cease, and we can think of the alignment as frozen until the next starlight photon is absorbed.

Averaging over the temperature fluctuations, the systematic aligning torque due to magnetic dissipation is

\[
\left( \frac{dJ_{\perp B}}{dt} \right)_{\text{DG,d}} = -\frac{J_{\perp B}}{\tau_{\text{DG,0}}} \langle \psi_{\text{DG,d}} \rangle
\]

(29)
be the components of the grain $\mathbf{E}$ with $\mathbf{B}$ that falls below $\Delta J$. Levels with $K = J$ are shown for vibrational states $v_1$ and $v_2$.

\begin{equation}
\frac{d J_f}{dt} = \frac{4 k T_0}{\tau_{DG,0}} \langle \psi_{DG,e} \rangle
\end{equation}

where again $n = -1$ for paramagnetism and $n = 0$ for ferromagnetism, and $I$ is the moment of inertia. It is this excitation that leads to disalignment of $J$ and $B_0$. The energy distribution functions $p_E$ for silicate nanoparticles with radius $a$ heated by the interstellar radiation field estimated for the solar neighborhood by Mathis et al. (1983) were calculated following Draine & Li (2001). Figure 8 shows the suppression factors $\langle \psi_{DG,d} \rangle$ and $\langle \psi_{DG,e} \rangle$.

\begin{equation}
\langle \psi_{DG,e} \rangle = \sum_E p_E \left( \frac{T_{vib}}{T_0} \right)^{n+1} \psi_q(E),
\end{equation}

where $n = -1$ for paramagnetism and $n = 0$ for ferromagnetism or ferrimagnetism.

The fluctuation–dissipation theorem implies that there must also be excitation if the lattice temperature $T_{vib} > T_c$ to increase $J_f$, acting to disalign $J$ with $B_0$. When $T_{vib} < T_c$, dissipation tends to reduce $J_f$, acting to align $J$ with $B_0$. When $T_{vib}$ falls below $T_c$, dissipation is suppressed, and the alignment is effectively frozen until the next heating event.

\begin{equation}
\langle \psi_{DG,d} \rangle \equiv \sum_E p_E \left( \frac{T_{vib}}{T_0} \right)^n \psi_q(E),
\end{equation}

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Papoular (2016) recently proposed that interstellar grains can be aligned with the magnetic field $B_0$ even if the grain material has zero magnetic susceptibility, because the ions and electrons in the grain will experience time-varying Lorentz forces unless the grain has angular velocity $\omega \parallel B_0$. The mechanism proposed by Papoular also relies on dissipation of rotational kinetic energy and transfer of energy between rotation and vibration, and would be subject to the same quantum suppression factor $\psi_q(E_{vib})$ that would apply to the paramagnetic dissipation envisaged by Davis & Greenstein (1951), or dissipation in superparamagnetic or ferromagnetic materials.

7. EXCITATION OF J, AND ALIGNMENT OF J WITH B

We now consider the balance between excitation and damping by the various torques acting on a spinning grain. Let $J_B$ and $L_B$ be the components of the grain’s angular momentum parallel and perpendicular to $B$. For an individual grain, $J_B(t)$ and $L_B(t)$ are stochastic variables. For an ensemble, we write

\begin{equation}
\frac{d}{dt} \frac{\langle J_B^2 \rangle}{2l} \approx \frac{G k T_{gas}}{\tau_H} - \frac{F \langle J_B^2 \rangle}{\tau_H} - \frac{4 \mu^2}{9 e^3} \langle \omega^4 \rangle \langle J_B^2 \rangle.
\end{equation}
\[
\frac{d}{dt} \left\langle J_{2,\text{B}}^2 \right\rangle \approx \frac{G}{\eta_1} kT_{\text{gas}} - \frac{F}{\eta_1} \left\langle J_{2,\text{B}}^2 \right\rangle - 4 \mu_\perp^2 \left\langle \omega^4 \right\rangle \left\langle J_{2,\text{B}}^2 \right\rangle - \frac{\left( \left\langle J_{2,\text{B}}^2 \right\rangle - \left\langle J_{2}^2 \right\rangle \right)}{I_{\text{DG,0}}} \left\langle \psi_{\text{DG,d}} \right\rangle,
\]

(34)

where \( J_{2}^2 \equiv 2kT_0 \left\langle \psi_{\text{DG,d}} \right\rangle / \left\langle \psi_{\text{DG,d}} \right\rangle \). Here we approximate the grains as spherical, so that we need not consider the orientation of the body of the grain relative to \( \mathbf{J} \). The characteristic timescale

\[
\tau_1 \equiv \frac{4 \mu_\perp^2}{n_{\text{eff}} n_m H} \left( \frac{\pi m_H}{8kT_{\text{gas}}} \right)^{1/2}
\]

\[
\approx 2 \times 10^{-6} \left( \frac{30 \text{ cm}^{-3}}{n_H} \right) \left( \frac{a_{\text{eff}}}{10^{-7} \text{ cm}} \right) \left( \frac{100 \text{ K}}{T_{\text{gas}}} \right)^{1/2} \text{ s} \quad \text{(35)}
\]

is the rotational damping time for a neutral grain in a gas of atomic H. \( F(a_{\text{eff}}) \) and \( G(a_{\text{eff}}) \) are dimensionless factors introduced by Draine & Lazarian (1998b) to allow for the actual rates for rotational damping and excitation arising from partial ionization of the gas, charging of the grain, and the effects of starlight and infrared emission from the grain. Note that \( N_{\text{abs}} \tau_1 \gg 1 \) (see Equation (12)): temperature fluctuations due to stochastic heating occur in a time much shorter than the time for angular momentum variations, alignment, etc.

Loss of rotational kinetic energy from purely rotational electric dipole radiation varies as \( \omega^2 \), but also depends on the orientation of the electric dipole moment with respect to the grain’s rotation axis. Here we take \( \mu_\perp \) to be an appropriately averaged mean square dipole moment perpendicular to the rotation axis. Following Draine & Lazarian (1998b), we suppose that

\[
\mu_\perp \approx \beta_0 \sqrt{N},
\]

(36)

where \( N \) is the number of atoms in the nanoparticle, and \( \beta_0 \) is a constant.

\( F \) and \( G \) are functions of the grain’s charge state and volume-equivalent radius \( a_{\text{eff}} \) (and the grain’s shape), as well as the ionization and temperature of the gas, and have been estimated for a variety of environments (Draine & Lazarian 1998b; Ali-Haimoud et al. 2009; Hoang et al. 2010). For the present illustration we take \( F(a_{\text{eff}}) \) and \( G(a_{\text{eff}}) \) calculated by B. S. Hensley & B. T. Draine (2016, in preparation) for spherical silicate nanoparticles with electric dipole moments corresponding to \( \beta_0 = 0.3 \) D and cold neutral medium (CNM) conditions (see Table 1).

\( I_{\text{DG,0}} \) is the “classical” Davis–Greenstein alignment time if the grain temperature is \( T_0 \). The factors \( \left\langle \psi_{\text{DG,d}} \right\rangle \) and \( \left\langle \psi_{\text{DG,e}} \right\rangle \) include the effects of variations in temperature away from the nominal temperature \( T_0 \) as well as the quantum suppression of fluctuations at temperatures below \( T_{\text{crit}} \). \( J_{2}^2 \) is the mean value of \( J_{2}^2 \) if the only torques are from paramagnetic dissipation and the associated thermal fluctuations:

\[
\frac{J_{2}^2}{2I} = \frac{\sum E P_E \psi_q(E) kT_E}{\sum E P_E \psi_q(E)} = kT_0 \left\langle \psi_{\text{DG,e}} \right\rangle / \left\langle \psi_{\text{DG,d}} \right\rangle.
\]

(37)

We average over \( E \) because the grain undergoes thermal fluctuations in a time short compared to the characteristic rotational damping time \( \tau_1/F \).

We introduce dimensionless parameters

\[
\beta \equiv \frac{8 \mu_\perp^2 \left\langle \omega^4 \right\rangle kT_{\text{gas}}}{9 \epsilon_0^3 \left\langle \omega^2 \right\rangle^2 I^2} \tau_1,
\]

(38)

\[
\gamma \equiv \frac{2 \left\langle \psi_{\text{DG,d}} \right\rangle \eta_1}{\tau_{\text{DG,0}}},
\]

(39)

\[
\zeta_0 = \frac{J_{2}^2}{2kT_{\text{gas}}} = \frac{T_0 \left\langle \psi_{\text{DG,e}} \right\rangle}{T_{\text{gas}} \left\langle \psi_{\text{DG,d}} \right\rangle},
\]

(40)

and dimensionless variables

\[
x \equiv \frac{\left\langle J_{2,\text{B}}^2 \right\rangle + \left\langle J_{2}^2 \right\rangle}{kT_{\text{gas}}},
\]

(41)

\[
y \equiv \frac{\left\langle J_{2,\text{B}}^2 \right\rangle - \left\langle J_{2}^2 \right\rangle}{kT_{\text{gas}}},
\]

(42)

Figure 9(a) shows \( F, G, \beta, \gamma, \) and \( \zeta_0 \) as functions of grain size \( a_{\text{eff}} \) for CNM conditions.

The total rotational kinetic energy of the grain is measured by the “thermality” variable \( x \)—the ratio of the average grain rotational kinetic energy to what it would be in LTE with the gas temperature—while \( y \) measures the disalignment of \( \mathbf{J} \) from \( B_0 \). In a statistical steady state, Equations (33) and (34) become

\[
\frac{\tau_1}{dt} x (1 - y) = 0 = G - Fx (1 - y) - \beta x^2 (1 - y),
\]

(43)

\[
\frac{\tau_1}{dt} y = 0 = 2G - Fxy - \beta x^2 y - \gamma (xy - \zeta_0).
\]

(44)

Equations (43) and (44) are solved to find the steady-state values of \( x \) and \( y \) for each grain size. For an assumed \( y \) we have

\[
x = \frac{(F + \gamma y)^2 + 4 \beta (3G + \gamma \zeta_0) x^1/2 - (F + \gamma)}{2 \beta},
\]

(45)

and for given \( x \) we have

\[
y = \frac{2}{3} - \gamma \left( \frac{2/3) x - \zeta_0}{(3F + \gamma) x + 3 \beta x^2} \right).
\]

(46)

It is straightforward to iterate to find self-consistent \( x \) and \( y \). Figure 9(a) shows \( x(\alpha) \) for CNM conditions. The rms rotation frequency for the ensemble is

\[
\nu_{\text{rms}} = \frac{1}{2 \pi} \left( \frac{x kT_{\text{gas}}}{I} \right)^{1/2} = \frac{1}{2 \pi} \left( \frac{15 x kT_{\text{gas}}}{8 \pi \rho a_{\text{eff}}^5} \right)^{1/2},
\]

(47)
and the characteristic emission frequency is\(^4\)

\[
\nu_{\text{char}} = \frac{1}{2\pi} \left( \frac{\omega^4}{1} \right)^{1/4} \approx \left( \frac{5}{3} \right)^{1/4} \nu_{\text{rms}}.
\tag{48}
\]

Figure 9(b) shows \(\nu_{\text{char}}\) for spinning grains as a function of radius \(a\), for CNM conditions and \(\beta_0 = 0.3\) D. We see that only the smallest grains \((a \lesssim 8\,\AA)\) have characteristic rotation frequencies above 10 GHz, and thus only the smallest particles contribute significantly to the AME, which typically peaks in the range \(\sim 20-30\,\text{GHz}\) (Planck Collaboration et al. 2011, 2016). Figure 9(c) shows the rms rotational quantum number as a function of \(a\). Even for the smallest size, we have \(\langle J^2 \rangle \gg 1\), justifying the classical treatment of the rotational dynamics in Equations (33) and (34).

7.1. Alignment of \(\mathbf{J}\) and \(\mathbf{B}_0\)

Let \(\theta_{\mathbf{JB}}\) be the angle between \(\mathbf{J}\) and \(\mathbf{B}_0\). The ensemble has

\[
\langle \cos^2 \theta_{\mathbf{JB}} \rangle = 1 - y = \frac{1}{3} + \gamma \left( \frac{2/3}{x} - \frac{z_0}{(3F + \gamma)x + 3\beta x^2} \right),
\tag{49}
\]

We define the alignment factor

\[
R_{\mathbf{JB}} \equiv \frac{3}{2} \left( \langle \cos^2 \theta_{\mathbf{JB}} \rangle - \frac{1}{3} \right) = \gamma \frac{x - (3/2)z_0}{(3F + \gamma)x + 3\beta x^2},
\tag{50}
\]

which varies from 0 to 1 as \(\langle \cos^2 \theta_{\mathbf{JB}} \rangle\) varies from 1/3 (random orientations) to 1 (perfect alignment of \(\mathbf{J}\) with \(\mathbf{B}_0\)).

For \(\gamma \ll 1\) we see that grain alignment is very small: \(R_{\mathbf{JB}} \propto \gamma \ll 1\). Note that \(R_{\mathbf{JB}}\) can be negative when \(z_0 > (2/3)x\). Quantum effects suppress magnetic dissipation when the grain is cold, but not during the brief intervals (following starlight heating) when it is hot. As a result, thermal fluctuations acting to increase \(\langle J^2 \rangle_{\mathbf{JB}}\) can under some circumstances be more important than dissipation, and can cause \(R_{\mathbf{JB}}\) to become negative. However, this occurs only under conditions where quantum suppression is so effective that \(\gamma \ll 1\) and \(|R_{\mathbf{JB}}| < 1\).

Figure 10 shows \(R_{\mathbf{JB}}\) as a function of grain size and as a function of the characteristic emission frequency \(\nu_{\text{char}}\). The solid lines show results where the alignment is calculated including the quantum suppression of paramagnetic dissipation in small grains when \(T < T_{\text{crit}}\).

For comparison, the alignment of \(\mathbf{J}\) with \(\mathbf{B}_0\) is also calculated assuming “classical” paramagnetic dissipation, as in the standard Davis–Greenstein treatment. These results are obtained by solving the same Equations (43) and (44) but setting \(\langle \psi_{\text{DG,a}} \rangle = \langle \psi_{\text{DG,a}} \rangle = 1\) when evaluating \(\gamma\) and \(z_0\). The classical Davis–Greenstein treatment predicts \(R_{\mathbf{JB}} \approx 15\%\) for grains spinning at \(\sim 30\,\text{GHz}\), whereas when quantum
suppression effects are included, the alignment factor $R_{JB}$ drops to $\lesssim 10^{-8}$.

Recent calculations of polarization of rotational emission from silicate nanoparticles (Hoang et al. 2016) and magnetic Fe nanoparticles (Hoang & Lazarian 2016b) concluded that magnetic dissipation processes would be effective at aligning the particles, with polarization at 30 GHz predicted to be as large as $\sim 30\%$ for $a = 4\,\text{Å}$ silicate nanoparticles with $\beta_0 \approx 0.1\,\text{D}$, and $\sim 40\%-50\%$ for $a \approx 5\,\text{Å}$ Fe nanoparticles, for assumed dust temperatures $T_D \approx 20\,\text{K}$. However, when the quantum suppression effects considered here are included, we predict minimal alignment of such particles, with extremely low polarization above $\sim 10\,\text{GHz}$.

For radii $a \gtrsim 50\,\text{Å}$, the quantum suppression effects become unimportant (i.e., $\psi_a \approx 1$) and the present treatment coincides with classical Davis–Greenstein alignment of nanoparticles, aside from the use of fluctuating grain temperatures rather than the assumption of a steady temperature. Figure 10 shows the alignment factor $R_{JB}$ to be decreasing with increasing grain size for $a_{\text{eff}} \gtrsim 50\,\text{Å}$ as the Davis–Greenstein alignment time $\tau_{DG,0} \propto a_{\text{eff}}^2$ (see Equation (24)) becomes long compared to the rotational damping time $\tau_{\text{damp}} \propto a_{\text{eff}}$ (see Equation (35)). The observed substantial alignment of the larger “classical” grains with $a_{\text{eff}} \gtrsim 0.1\,\mu\text{m}$ is thought to be due to the effects of systematic torques that drive suprathermal rotation (Purcell 1975, 1979; Lazarian & Draine 1997), including the important effects of starlight torques that can both drive $a_{\text{eff}} \gtrsim 0.1\,\mu\text{m}$ grains to suprathermal rotation (Draine & Weingartner 1996) and directly bring the grain’s angular momentum into alignment with $B_0$ (Draine & Weingartner 1997; Weingartner & Draine 2003; Hoang & Lazarian 2009a, 2009b, 2016a; Lazarian & Hoang 2011). It is also possible that the larger grains may contain superparamagnetic inclusions that enhance alignment (Jones & Spitzer 1967; Mathis 1986; Goodman & Whittet 1995). The radiative torques that are important for $a \gtrsim 0.1\,\mu\text{m}$ grains are negligible for the $a_{\text{eff}} \lesssim 0.01\,\mu\text{m}$ nanoparticles considered here; other possible systematic torques due, for example, to formation of $\text{H}_2$ and photoelectric emission are suppressed by the “thermal flipping” phenomenon (Lazarian & Draine 1999b) and can be neglected for the nanoparticles discussed here.

7.2. Alignment of $\hat{a}_1$ with $\mathbf{J}$

The polarization of microwave emission depends on the orientation of the grain’s angular velocity $\omega$, which will not be parallel to $\mathbf{J}$ unless $\mathbf{J}$ is parallel to the grain’s principal axis. The alignment of $\hat{a}_1$ with $\mathbf{J}$ is measured by

$$R_{\hat{a}_1,\mathbf{J}} \equiv \frac{3}{2} \left( \left( \cos^2 \theta_{\hat{a}_1,\mathbf{J}} \right) - \frac{1}{3} \right),$$

which again varies between 0 and 1 as the alignment of $\hat{a}_1$ with $\mathbf{J}$ goes from random to perfect.

Recall that alignment of $\hat{a}_1$ with $\mathbf{J}$ occurs because the spinning grain can reduce its kinetic energy by bringing the axis of largest moment of inertia into alignment with $\mathbf{J}$. Quantum suppression of dissipation will interfere with such alignment in very small grains. As an example, we consider oblate spheroids with axial ratio $b/a = 2$, i.e., $B_0/A_0 = 2/[1 + (a/b)^2] = 1.6$. The expectation value $\langle \cos^2 \theta_{\hat{a}_1,\mathbf{J}} \rangle$ is evaluated using Equation (22) and the rms value of $J$ in Figure 9(c). Figure 11 shows $R_{\hat{a}_1,\mathbf{J}}(a, J)$ as a function of effective radius $a_{\text{eff}}$ for CNM excitation conditions (see Table 1). For $a_{\text{eff}} \lesssim 5\,\text{Å}$, the alignment of $\hat{a}_1$ with $\mathbf{J}$ is minimal, although it becomes appreciable for $10\,\text{Å} \lesssim a_{\text{eff}} \lesssim 50\,\text{Å}$.

7.3. Alignment of $\omega$ with $\mathbf{J}$

The electric dipole rotational emission from a single grain is 100% polarized if viewed from a direction perpendicular to the
grain’s instantaneous angular velocity $\omega$. However, if $R_{aJ} < 1$, then $\omega$ and $J$ will not be parallel, and both $\hat{a}_1$ and $\omega$ will nutate around $J$ (see Figure 12). For an oblate spheroid, the angle $\theta_{aJ}$ between $\omega$ and $J$ is given by

$$\cos^2 \theta_{aJ} = \frac{q + (1 - q)\cos^2 \theta_{aJ}}{q^2 + (1 - q^2)\cos^2 \theta_{aJ}},$$

(52)

where $q \equiv l_i/l_e \geq 1$. In the limit of a sphere ($q \rightarrow 1$), $\omega||J$, and $\cos^2 \theta_{aJ} = 1$ independent of the alignment of $\hat{a}_1$ with $J$. For oblate particles with $q > 1$, misalignment of $\hat{a}_1$ with $J$ implies misalignment of $\omega$ with $J$, but this misalignment is only slight. For an ensemble of grains with angular momentum quantum number $J$,

$$\langle \cos^2 \theta_{aJ} \rangle = \sum_{E=E_{\text{crit}}} \sum_{E=0} \frac{p_E}{k} \frac{[q + (1 - q)K^2/J(J + 1)]}{[q^2 + (1 - q^2)K^2/J(J + 1)]} \times p_K(J, T_E) + \left[ \sum_{E=0} \frac{p_E}{k} \frac{[q + (1 - q)K^2/J(J + 1)]}{[q^2 + (1 - q^2)K^2/J(J + 1)]} \times p_K(J, T_{\text{crit}}) \right],$$

(53)

where $p_K(J, T)$ is given by Equation (21), and we define

$$R_{aJ} \equiv \frac{3}{2} \left( \langle \cos^2 \theta_{aJ} \rangle - \frac{1}{3} \right).$$

(54)

Figure 11 shows $R_{aJ}$ as a function of grain size $a_{\text{eff}}$, for $b/a = 1.5$ ($q = 1.385$) and $b/a = 2$ ($q = 1.6$). We see that $R_{aJ} > 0.95$: $\omega$ remains quite well-aligned with $J$, even when $R_{aJ}$ is small.

8. POLARIZATION OF ROTATIONAL EMISSION

If $R_{aJ} < 1$, a spinning grain will undergo nutation around $J$. A spinning grain will generally have a significant magnetic moment antiparallel to $\omega$ due to the Barnett effect (Dolginov & Mytrophanov 1976; Purcell 1979); this magnetic moment will cause $J$ to precess around $B_0$ with a period that is short enough that complete averaging over precession can be assumed. If we assume that the grain has an electric dipole moment $\mu_\perp$ perpendicular to $\omega$, then, after averaging over rotation, nutation, and precession, one can show that the rotational emission will have fractional polarization

$$P_{\text{rot}} = \frac{3R_{aJ}R_{\beta C}}{4 - R_{aJ}R_{\beta C}} \sin^2 \psi,$$

(55)

where $\psi$ is the angle between the viewing direction and $B_0$.

Figure 13 shows the predicted polarization as a function of frequency for viewing directions perpendicular to the static magnetic field ($\sin^2 \psi = 1$). For each frequency $\nu$ we assume the emission to be dominated by grains with $\nu_{\text{char}} = \nu$. At microwave frequencies $\nu > 5$ GHz, the predicted polarizations are extremely small. If the AME is rotational radiation from spinning dust grains, the polarization should be negligible.

9. POLARIZATION OF EXTINCTION OR THERMAL EMISSION

Nonspherical grains have absorption or scattering cross sections that depend on the orientation of the grain relative to the direction and polarization of the incident radiation. In the long-wavelength limit, the cross section depends on the direction of the polarization $E$ relative to the grain body, but not on the direction of propagation. We assume this to be the case in the following discussion.

We consider oblate spheroidal grains with symmetry axis $\hat{a}_1$. Let $\hat{e}$ be the direction of polarization, and let $C_\parallel$ and $C_\perp$ be cross sections for $\hat{e}||\hat{a}_1$ and $\hat{e}\perp\hat{a}_1$. If we view the ensemble of grains from a direction perpendicular to $B_0$, the ensemble of precessing and nutating grains will have mean cross sections per particle

$$\langle C_\parallel \rangle = R_{aJ}R_{\beta C}C_\parallel + (1 - R_{aJ}R_{\beta C}) \left( C_\parallel + 2C_\perp \right).$$

(56)
\[ \left\langle C \right\rangle_{\perp B} = R_{a/J}R_{JB}C_\perp + \left( 1 - R_{a/J}R_{JB} \right) \frac{C_\parallel + 2C_\perp}{3}. \]  

(57)

Thermal emission would then have polarization

\[ P_{\text{th,em}} = \frac{\left\langle C \right\rangle_{\perp B} - \left\langle C \right\rangle_{\parallel B}}{\left\langle C \right\rangle_{\perp B} + \left\langle C \right\rangle_{\parallel B}} \]

\[ = \frac{3R_{a/J}R_{JB}(C_\parallel - C_\perp)}{3R_{a/J}R_{JB}(C_\parallel + C_\perp) + 2(1 - R_{a/J}R_{JB})(C_\parallel + 2C_\perp)}. \]  

(58)

The polarization due to dichroic extinction by a column density \( N_g \) of dust grains is

\[ P_{\text{ext}} = \tanh(N_gC_{\text{pol}}). \]  

(59)

\[ C_{\text{pol}} = \frac{1}{2} \left( \left\langle C \right\rangle_{\perp B} - \left\langle C \right\rangle_{\parallel B} \right) = \frac{1}{2}R_{a/J}R_{JB}(C_\parallel - C_\perp). \]  

(60)

Thus for both emission and extinction the polarization is determined by the product \( R_{a/J}R_{JB} \) that determines the degree of grain alignment (see Figure 14).

10. DISCUSSION

Rouan et al. (1992) considered IVRET in spinning PAHs with \( N \approx 90 \), and concluded that IVRET was sufficiently rapid that Equation (21) should be a good approximation throughout the cooldown following absorption of a starlight photon.

Quantum suppression of dissipation in grains was reconsidered by LD00, who noted that the energy \( E_1 \) of the lowest vibrationally excited state would be appreciable in small grains, and argued that spin–lattice relaxation should be suppressed by a factor \( \propto \exp\left(-E_1/kT_{\text{vib}}\right) \) leading to suppression of IVRET when \( T_{\text{vib}} \) drops below \( \sim E_1/k \). LD00 estimated that this would reduce the polarization to \( \sim 2\% \) at 20 GHz, and to only \( \sim 0.5\% \) for particles small enough to spin at 30 GHz.

Sironi & Draine (2009) revisited IVRET in spinning PAHs. They argued that when the separation \( \Delta E \) of the vibrational energy levels becomes larger than \( \hbar \omega_{\text{rot}} \), vibrational–rotational energy exchange will be suppressed. For a PAH with \( \sim 200 \) carbon atoms, they estimated that IVRET would effectively cease when \( T_{\text{vib}} \) dropped below \( \sim 65 \) K, leaving the body axis only partially aligned with \( J \).

In the present paper we have argued for a different criterion: that \( \nu-R \) energy transfer is suppressed when \( g_k \delta E_{\text{vib}} < 1 \), where \( \delta E_{\text{vib}} \) is the width of the energy states. Equation (19) is proposed as an estimate for the quantum suppression factor \( \psi_q \). This criterion leads to \( \psi_q \ll 1 \) for very small grains—see Figure 8. However, Equation (19) probably overestimates the relaxation rate when \( g_k \delta E < 1 \): simply having a state \( v_2 \) available at the appropriate energy does not ensure that the coupling from \( v_1 \) to \( v_2 \) will be fast, because there may be other “selection rules” that must be satisfied to have the energy transfer proceed at the “classical” rate. Thus, the true suppression factor \( \psi_q \) may be smaller than estimated from Equation (19), and the actual degree of polarization of rotational emission may be even smaller than the already very small values estimated here and shown in Figure 10.

The analysis by LD00 implicitly assumed that the change in rotational energy first appeared as “heat” in the spin system, only later being transferred to the lattice by spin–lattice coupling. LD00 argued that when the spin–lattice coupling time \( \tau_1 \) became long, the spin system would become hot, and the paramagnetic dissipation rate would be reduced by a factor \( \left[ 1 + (g_k^2 \mu^2 / 8 \hbar^2 \gamma \tau_2)^2 \right]^{-1} \), where \( \tau_2 \) is the spin–spin coupling time.

In the present treatment, we did not include the possibility of temporarily accumulating heat in the spin system and later transferring it to the lattice. We consider this unlikely because the “heat capacity” of the spin system appears to be insufficient: if a nanoparticle containing \( N \) atoms has dimension \( a_{\text{eff}} \approx 5 \) \( \text{Å} \) \( (N/50)^{1/3} \) and contains \( N_s \) unpaired spins with magnetic moments \( \mu \), then in a ferromagnetic system with aligned spins, the internal magnetic field is

\[ H_{\text{int}} \approx N_i \mu / a_{\text{eff}}^{3/2} = 2100 \left( N_i / N \right) \left( \mu / 4 \mu_B \right) \text{G}, \]  

(61)

where we might consider silicate clusters with MgFeSiO₄ composition, containing \( N_i \approx (N/7) \) Fe²⁺ ions with \( \mu \approx 4 \mu_B \). The magnetic moment of one of the Fe²⁺ ions would have a precession frequency \( \omega_{\text{pre}} = 2 \pi \approx \mu_B H_{\text{int}} / h = 3 \) GHz, with energy levels split by \( \Delta E = \hbar \omega_{\text{pre}} \). For ferromagnesian olivines, such as FeMgSiO₄, Equation (61) for \( H_{\text{int}} \) is likely to be an overestimate, because they are expected to be antiferromagnetic (Duff 1968; Carmichael 1989). However, the local field within metallic Fe (with \( N_i / N = 1 \) and \( \mu \approx 2.2 \mu_B \)) is larger, \( H_{\text{int}} \approx 8000 \) G, yielding \( \omega_{\text{pre}} / 2 \pi \approx 11 \) GHz.

If the lattice is spinning with a rotation frequency \( \omega_{\text{rot}} \), a reduction in the lattice angular momentum by \( \hbar \) implies a reduction in rotational kinetic energy by \( \hbar \omega_{\text{rot}} \). For \( \hbar \omega_{\text{rot}} / 2 \pi \gtrsim 3 \text{GHz} \), the spin system cannot absorb the change in rotational energy \( \hbar \omega_{\text{rot}} > \hbar \omega_{\text{pre}} \) in a single-spin transition. However, it is conceivable that transitions involving multiple spins might be important, allowing the energy \( \hbar \omega_{\text{rot}} \) to be absorbed by the spin system. Further, it may be possible for the energy \( \hbar \omega_{\text{rot}} \) to be divided between the spin system and lattice vibrations; if this is possible, it could allow dissipation even when the vibrational density of states is low. These considerations merit further.
From nanoparticles spinning at kHz frequencies, we expect the AME to be negligibly polarized. However, other grain materials—in particular, PAHs or metallic Fe—would also have quantum suppression of alignment, qualitatively similar to the silicate example shown here. Thus, if the AME is dominated by rotational emission at kHz frequencies, we expect the AME to be negligibly polarized.

Our prediction of negligible polarization for rotational emission at microwave frequencies is consistent with most published observations. For the Perseus molecular complex, Battistelli et al. (2006) measured the total polarization at 11 GHz to be \( P = 3.4_{-1.9}^{+3.3} \% \) (95% confidence interval). At higher frequencies, the AME polarization for this region (also known as G159-18.5) is consistent with zero, as are observations of the AME from the Lynds 1622 dark cloud, the \( \rho \) Oph cloud, and the H II regions LPH96; see Figure 15. A polarization \( P = (2.2 \pm 0.5) \% \) was reported for the H II region RCW175 at 13.5 GHz, but Battistelli et al. (2015) comment that this could be due to synchrotron emission associated with RCW175. Most recently, Génova-Santos et al. (2016) have obtained very stringent upper limits of 0.39%, 0.52%, and 0.22% for the AME polarization at 16.7, 22.7, and 40.6 GHz toward the molecular complex W43r.

At this time, our prediction of negligible polarization for rotational emission at microwave frequencies is consistent with observations of the AME, with the exception of the Perseus molecular complex, where Battistelli et al. (2006) found \( P = 3.4_{-1.9}^{+3.3} \% \) at 11 GHz. If correct, this would be incompatible with our predictions for the polarization of rotational emission, and would imply another mechanism for the AME in the Perseus molecular complex. We urge further observations of the Perseus complex to confirm this result.

Our modeling of the grain dynamics (Equations 33 and 34) has implicitly assumed that the damping and excitation functions \( F \) and \( G \) do not depend on the orientation of the grain’s angular momentum \( J \)—the only dependence on orientation enters through the magnetic dissipation term in Equation 34. However, if the starlight illuminating the dust is anisotropic, and the grain is appreciably nonspherical, the photon absorption rate for a grain will depend on the orientation of the grain relative to the starlight anisotropy. If the grain axis \( \hat{\alpha}_1 \) is aligned with \( J \), then the time-averaged photon absorption rate will in principle depend on the orientation of \( J \) and on the angle \( \theta_{\hat{\alpha}_1 J} \). Recognizing that \( \hat{\alpha}_1 \) will be nutating around \( J \), and \( J \) will be precessing around \( B_0 \) (the grain will generally have a magnetic moment), we do not expect \( F \) and \( G \) to have an appreciable dependence on the orientation of \( J \), but in principle this dependence is not zero unless the starlight is isotropic. Because the grain’s angular momentum is changed by absorption of the starlight photon and the subsequent emission of infrared photons, the rotational distribution function for the grains may develop a small degree of anisotropy as a result of the starlight anisotropy. Anisotropic starlight can produce a small degree of polarization in the PAH emission features from spinning PAHs in photodissociation regions (Leger 1988; Sironi & Draine 2009). For rotational emission this effect is expected to be slight. If future observations find small but nonzero polarization for the AME, the possible contribution from this mechanism should be evaluated quantitatively.

If future observations find the AME to be polarized with \( P \gtrsim 0.01 \% \) at \( \nu \gtrsim 10 \) GHz, it will be evidence that the AME is not entirely rotational emission from nanoparticles, or that starlight anisotropy has generated a small degree of polarization in the rotational emission.

While quantum suppression of alignment in nanoparticles with radii \( a \lesssim 10 \) Å will lead to effectively zero polarization of any rotational emission at GHz frequencies, Davis–Greenstein alignment is expected to be able to significantly align grains in the size range \( a_{\text{eff}} \approx 30–60 \) Å. The minimal degree of polarization in the far ultraviolet (Martin et al. 1999; Whittet 2004) suggests that grains in this size range must either be nearly spherical or have low abundance.

11. SUMMARY

The principal results of this paper are as follows.

1. Dissipation due to viscoelasticity or Barnett dissipation in a spinning grain is suppressed when the vibrational energy \( E_{\text{vib}} \) falls below a critical value \( E_{\text{crit}} \); the grain’s rotational kinetic energy cannot be converted to vibrational energy because there are no suitable vibrational states. This suppresses alignment of the grain’s body axis \( \hat{\alpha}_1 \) with its angular momentum \( J \).

2. Paramagnetic or ferromagnetic dissipation in a nanoparticle spinning in a static magnetic field \( B_0 \) is also suppressed when \( E_{\text{vib}} < E_{\text{crit}} \). This suppresses alignment...
of the grain’s angular momentum $J$ with the galactic magnetic field $B_0$, with the greatest suppression for the smallest grains.

3. For conditions typical of the neutral ISM, the rotational emission from interstellar dust at frequencies $\nu > 1$ GHz is expected to be negligibly polarized, with $P < 10^{-6}$ for $\nu > 10$ GHz (see Figure 13). If the AME arises from spinning grains, it should be essentially unpolarized, consistent with observations to date except for AME polarization $P = 3.4^{+1.5}_{-1.0}$% at 11 GHz reported for the Perseus molecular complex (Battistelli et al. 2006). Further measurement of AME polarization in the Perseus complex would be of great interest.

4. Ordinary paramagnetic dissipation should be able to align interstellar grains in the size range 30–60 Å. The rapid fall-off in starlight polarization in the far ultraviolet suggests that grains in this size range either are nearly spherical or contribute only a small fraction of the far-ultraviolet extinction.

The above conclusions are not sensitive to the grain material, and apply to rotational emission from spinning PAHs, nanosilicates, or iron nanoparticles.

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