Periodic Orbit theory for Resonant Tunneling Diodes: comparison with quantum and experimental results

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We investigate whether the quantal and experimental amplitudes of current oscillations of Resonant Tunneling Diodes in tilted fields are obtainable from Periodic Orbit (PO) theories by considering recently proposed PO approaches. We show, for the first time, that accurate amplitudes and frequency shifts for the current oscillations (typically to within a few %) can be obtained from a simple analytical formula both in the stable (torus-quantization) limit and the unstable regimes of the experiments which are dominated by isolated PO’s. But we find that the PO approach does not describe quantitatively the dynamically interesting intermediate experimental regimes which appear to be dominated by contributions from complex orbits and multiple non-isolated PO’s. We conclude that these regimes will not easily be described by the usual PO approach, even with simple normal forms.

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Mesoscopic systems have been extensively investigated as probes of ‘quantum chaos’ in real systems. Among these, the Resonant Tunneling Diode (RTD) in tilted fields, first introduced in 1994, has attracted much attention because of the diversity of observed effects it exhibits which have been attributed to Periodic Orbits and ‘soft chaos’. For instance, it has been used as an experimental probe of spectral fluctuations due to unstable periodic orbits, quantum scarring, bifurcations, ‘ghosts’ and the torus quantization regime. All these effects manifest themselves through observed oscillations in the tunneling current of varying amplitude and frequency.

The well-known Gutzwiller Trace Formula (GTF) is a powerful tool in the quantization of chaotic systems. It relates the frequencies and amplitudes of oscillations in the density of states to the actions and stabilities of classical periodic orbits (PO’s) in a simple analytic formula. However to date we have no equivalent formula which describes the corresponding oscillations in the tunneling current. Hence some interpretations of the experiments, for instance whether bifurcating orbits are seen or not, remain controversial. In a semiclassical treatment of the current was presented, evaluating the Bardeen tunneling matrix element numerically within the Wigner phase-space representation. Thus they could reproduce some interesting experimental features such as regimes of period-doubling of the current.

We show here that a simple analytical formula developed by Bogomolny and Rouben can yield quantitatively the amplitudes of contributions from isolated periodic orbits - stable or unstable. This represents the first demonstration that the amplitudes of the current oscillations may be quantitatively described by a periodic orbit expansion. The formula is in good agreement with results obtained using the approach of. The latter approach provides more flexibility allowing, for instance, excited initial states (which are not required in our calculation). But the advantage of the new formula lies in its simplicity - it is as easy to evaluate as the GTF and in the physical insight which the analytical expression provides. For instance, it exposes a shift between the frequency of the Gutzwiller density of states oscillations and the current oscillations. This shift is small (<1%) but the resolution of the quantum scaled calculations easily exceeds this. The comparison carried out here confirms that this shift is indeed easily detectable and accurately predicted in terms of classical quantities. Also, the formula reproduces observed features due to torus-quantization, in agreement with quantal calculations and the simple model we proposed previously.

We have applied here a rigorous test to this formula, since we have compared it with a broad range of accurate amplitudes obtained from a scaled quantum spectrum. We also compare with the extensive set of experimental data obtained at Bell Labs.

Many of the most interesting experimental features, such as period-doublings occur in an intermediate regime characterized by contributions from multiple non-isolated orbits and complex PO’s (ghosts). We find here that agreement in this regime is qualitative: both the formula and the approach of yield the rough range of period-doubling regions, but the amplitudes are in poor agreement with quantal results and experiment. In particular, in the two ‘ghost’ regions we identify, we cannot account for the amplitude of the current oscillations, even with normal form corrections. The strength and persistence of these contributions remains one of the most puzzling features of these experiments since in general ‘ghosts’ are strongly damped away from the tangent bifurcation where they appear.

We recall briefly the RTD model. In essence, the physical picture is as follows: an electric field F (along ) and a magnetic field B in the x−z plane (at tilt angle θ to the x axis) are applied to a double barrier quantum well.
Electrons in a 2DEG accumulate at the first barrier and tunnel through both barriers giving rise to a tunneling current $I$. In the process they probe the classical trajectories -regular or chaotic- arising from specular reflection at the barrier walls. The current oscillates as a function of applied voltage $V$. After rescaling with respect to $B$, the dynamics at given $\theta$ and ratio of injection energy to voltage ($R = E/V \sim 0.15$ for the Bell Lab experiments) depends only on the parameter $\epsilon = V/LB^2$. Regular behaviour occurs at high $\epsilon \sim 20000$ (in atomic units), chaotic behaviour for low $\epsilon \sim 1000$. The well width is $L = 1200 \text{Å}$. As we study here small angles ($\theta \leq 27^\circ$), we neglect the shift $\delta z \sim d \tan \theta$ due to the mean distance $d$ between the 2DEG and the left inner barrier ($x = 0$). At $\theta = 0^\circ$ the current consists of pure period one oscillations, of amplitude independent of $\epsilon$, associated with a straight line PO $(t_0)$, bouncing alternately between walls. This is our reference current amplitude $I_0$ and we normalise all amplitudes (semiclassical, quantal and experimental) to $I_0$.

As $\theta$ increases $t_0$ is no longer a straight line but continues to dominate the period-one oscillations. Due to the relatively short coherence time $\tau \sim 0.1 \text{ps}$ just four of the shortest PO’s ($t_0$, its second traversal $2t_0$ and the period-two PO’s $S_1$ and $S'$) account for all experimental features studied here. Their shape is shown in Fig. 1. Nevertheless, their dynamical behaviour is far from simple. Already in [1] it was observed that these classical PO’s appear and disappear abruptly. $t_0$ undergoes a series of tangent bifurcations, where it ceases to exist as a real PO but leaves a complex ‘ghost’. Subsequently, below the tangent bifurcation, a new similar looking PO $t_0'$ re-appears from the opposite side of the Surface of Section (SOS) and re-stabilizes. $S'$ appears abruptly at the discontinuity in the potential between a barrier and the energy surface due to the mean distance $d$ between the 2DEG and the left inner barrier ($x = 0$). At $\theta = 0^\circ$ the current consists of pure period one oscillations, of amplitude independent of $\epsilon$, associated with a straight line PO $(t_0)$, bouncing alternately between walls. This is our reference current amplitude $I_0$ and we normalise all amplitudes (semiclassical, quantal and experimental) to $I_0$.

The theoretical scaled current (neglecting experimental broadening due to incoherent processes) is a density of states weighted by a tunneling matrix element: $I(N) = \sum_i W_i \delta(N - N_i)$.

- **FIG. 1.** Shape in $x-z$ plane of the main PO’s $t_0$ (period-one), $S'$ and $S_1$ (period-2). The SOS’s illustrate, for $\theta = 11^\circ$, three generic dynamical regimes typical of $\theta = 10-30^\circ$: (1) $\epsilon = 20000$. Large stable island. (2) $\epsilon = 7000$. Intermediate regime. $t_0$ about to undergo a tangent bifurcation near the edge of the SOS, which will remove the real PO. This regime is characterised by contributions from non-isolated PO’s or complex orbits. (3) $\epsilon = 3000$. Unstable regime. The new $t_0'$ PO has re-appeared on the far side of the SOS. A strong period-two signal from the isolated unstable PO $S_1$, which occupies the central region most accessible to the tunneling electrons, is seen in the experiments.

The experimental range ($V = 0.1-1.1 \text{ V}$) corresponds to $N \sim 12 - 43$, which gives an average $N \sim \hbar^{-1} \sim 28$ corresponding to $V \sim 0.5 \text{ V}$. In fact $N$ is a re-scaled magnetic field $\sqrt{2mLc(R + 1/2)}/\pi$. In our calculations we used the Bardeen matrix element [16] for the tunneling probability. Then as explained in [13], one can re-express the matrix element in terms of energy Green’s functions and use their semiclassical expansion over classical paths. We consider the initial state describing the electrons prior of tunneling to be the lowest Landau state: $\phi_0(z) = \sqrt{B \cos \theta / \pi} \exp(-B \cos \theta z^2/2)$. Then the tunneling current is given by:

$$I(B) \propto \text{Re} \int dz dz' \sum_{c'(z \rightarrow z')} m_{12}^{-1/2} e^{iB(z + z')e^{-B \cos \theta(z^2 + z'^2)/2}}$$

where $m_{12} = \frac{\partial z}{\partial p_{z0}}$.

The integrals were evaluated analytically by stationary phase with the condition $\partial S/\partial z = \partial S/\partial z' = 0$. This condition implies that only PO’s starting with null momentum $p_z = 0$ contribute. The resulting contribution to the normalized current $I_{norm}(B) = I(B)/I_0$ for a given periodic orbit is approximated by:

$$I_{norm}(B) = \text{Re} \frac{e^{iB(S + \Delta S) + \mu \pi/2 - B\xi}}{\sqrt{-\cos \theta m_{12} + m_{21}/\cos \theta + 2m_{11}}}$$

$$\Delta S = \cos \theta z_0^2/(1 + \gamma^2); \quad \xi = \cos \theta z_0^2/(1 + \gamma^2)$$

$$\gamma = \frac{m_{11} - 1}{\cos \theta m_{12}},$$

where $\mu$ is a Maslov index, $\Delta S$ and $m_{ij}$ are the scaled
action and element of the classical monodromy matrix of the PO, with starting position \((x = 0, z = z_0)\). The semiclassical theory predicts that the frequency of the current oscillations is shifted relative to the scaled action \(S\) by \(\Delta S\).

We show in Fig. 2 the scaled action of \(t_0\), the semiclassical frequency \(S + \Delta S\) and the quantum frequency obtained by Fourier transform. The shift of frequency is \(\sim 1\%\) at most, but clearly we can see that the shifted frequency is in excellent agreement with the quantum results.

For the damping we find that the amplitudes of the maximum period-doubling current would coincide if we chose \(\tau\) in the range \(0.10 - 0.12\) ps for a given angle. This is remarkably consistent with the expected \(\tau \sim 0.1\) ps suggested in Fig. 3. We chose a representative value \(\tau = 0.11\) ps for all the experimental amplitudes. In effect period-two amplitudes are damped by incoherent processes by about an order of magnitude relative to \(I_0\).

In Fig. 3 we explained how one may extract experimental PO amplitudes in the case where the current has just a pure period one or two oscillation, by removing the smooth non-oscillatory component. This is only possible in a restricted range of the experiments. Also, to compare with theory we must in addition consider two factors: \(i\) the experimental features are displaced to a lower voltage \(V\) in the range from \(0.93\) at most, but clearly we can see that the shifted frequency is in excellent agreement with the quantum results.

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born’ isolated real $t_0'$ orbit dominates the current for $\epsilon < 2500$ [regime(3)].

In regime (2) in contrast we have the non-isolated contributions of $2t_0$ and $S'$ which cannot be added straightforwardly. Hence we simply show the individual contributions. Here agreement between the formula and quantum calculations/experiment is poor. $S'$ appears abruptly at a cusp bifurcation ($\epsilon \sim 18000$) and disappears in a tangent bifurcation at lower $\epsilon$, below which the experiment shows a slowly decaying ‘plateau’ due to a complex orbit (CO).

![Diagram](image)

**FIG. 4.** Quantal (QM), semiclassical (SC) and experimental amplitudes for period two current at (a) $\theta = 11^\circ$, (b) $\theta = 20^\circ$ and (c) $\theta = 27^\circ$. As previously we have 3 regimes.

1. stable and (3) unstable isolated PO, where the semiclassical formula is good. In regime (2) we have the non-isolated contributions of $2t_0$ and $S'$ which cannot be added straightforwardly. Hence we simply show the individual contributions. Here agreement between the formula and quantum calculations/experiment is poor. $S'$ appears abruptly at a cusp bifurcation ($\epsilon \sim 18000$) and disappears in a tangent bifurcation at lower $\epsilon$, below which the experiment shows a slowly decaying ‘plateau’ due to a complex orbit (CO).

In Fig. 4 we show a comparison between experimental, quantal and semiclassical amplitudes of the period two current at $\theta = 11^\circ$, $20^\circ$ and $27^\circ$. We were unable to read reliable experimental period-two amplitudes for $\theta = 11^\circ$ as a strong period one beat is also present. At $\theta = 20^\circ$ and $27^\circ$, the quantal calculations and the experiments are in very good agreement. As expected in the large stable island regime $\epsilon > 25000$ [regime (1)] agreement between the semiclassics and quantum is excellent. This is also the case for $\epsilon < 3000$ [regime (3)]. Here, the isolated unstable PO $S_1$ describes the current very well.

However, in the intermediate regime (2), the quantum current requires a coherent superposition of the non-isolated PO’s $2t_0$ and $S'$. A straightforward sum (allowing for their phases) yields poor results. $2t_0$ and $S'$ have near identical actions, unresolvable in the quantum Fourier transform spectroscopy. The $2t_0$ contribution comes from Miller tori localized on the large stable island and, due to the moderate values of $\hbar$, substantially beyond the island boundary. A phase-space analysis with Wigner and Husimi functions shows that the $S'$ scars are mixed in with the outer tori of the $t_0$ island. Hence both their action and phase-space localization coincide. At $11^\circ$ however, the contribution of $S'$ is small and occupies a narrow range in $\epsilon$. In this case the semiclassical amplitudes are quite good. This is not the case at $27^\circ$.

Both the individual island ($2t_0$) and $S'$ contributions are significant between $\epsilon = 20000 - 12000$ and there is no agreement with the quantal results. For $\epsilon < 8000$ the $2t_0$ contribution is negligible and $S'$ has disappeared into the complex plane in a tangent bifurcation. Even including the $S'$-‘ghost’, we were unable to obtain quantitative agreement in this complex orbit (CO) region spanning $\epsilon = 8000 - 5000$. We note that the slowly declining ‘plateau’ seen quantally and in the experiment is a surprising and unexpected feature, since ‘ghosts’ should be exponentially suppressed as the imaginary component of the action grows.

We have also compared with the alternative semiclassical approach in [14]. All regimes considered here and in [14] involved PO’s with initial $p_z = 0$. In that case, and assuming the lowest Landau state for the initial state, we have found that the numerical integral (5) of [14] reduces to the analytical formula here. Both encounter the same difficulties in regime (2). We note that although [14] requires the $p_z = 0$ selection rule, [14] requires only $p_z$ to be small. As yet we have no unambiguous experimental detection of a PO with $p_z > 0$.

Finally, we note that in general, the approach of [14] would predict complex stationary phase points. Their complex part has been neglected in order that the theory may obtain PO’s. Our work [17] indicates that the consistent failure of the PO formalism in the intermediate regime, despite the usual normal form corrections, may require that the usual PO picture be partly abandoned since complex non-periodic contributions -as opposed to ‘ghosts’ which are complex PO’s- may be essential.

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