Wireless Sensor Network Positioning Method Based on the Boundary of Sea Level in Three-Dimensional Space

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Abstract. All locating one unknown node requires at least four reference nodes in a three-dimensional space. If three reference nodes are used, then at least two places can satisfy the result of positioning in a wireless sensor network. Hence, on the basis of the boundary of sea level, the reference nodes are on the sea level below, whereas the unknown nodes are above. Here, we determine the solution of positioning both unknown and reference nodes with different spatial locations to find three reference nodes that are noncollinear and are the nearest to one unknown node. The experiment proves the accuracy of positioning with three reference nodes, which are furnished by the boundary of sea level. We conclude that the method of positioning that only uses three reference nodes can exactly realize the position and simultaneously increase the speed of positioning.

1. Introduction

Localization in wireless sensor network (WSN) is significant in practical application [1]. In recently years, various location algorithms have been proposed as references [2-4] but most of them are for two-dimensional (2D) space. In the present study, we present a new positioning technology based on a single side of two reference nodes in the literature [5]. For example, in monitoring the marine environment, nodes are placed at different depths below the sea level for the detection of water temperature, ocean currents, and biological events, or one network that floats in the sky for monitoring air pollution. These nodes are placed in three-dimensional (3D) space. Thus, the location algorithms are important for non-planar sensor network [6]. This study achieves positioning based on three points in a 3D space, while saving cost and time for positioning by using few reference nodes.

The rest of the paper is organized as follows. The concept of sea level reference node distribution is discussed in Section 2. Section 3 introduces the principle of sea level boundary positioning in 3D. The implementation of the positioning process is discussed in Section 4. We discuss the experimental settings and results and compare the proposed method and other localization techniques in Section 5. Finally, the results are discussed in Section 6.

2. Node Distribution

On the basis of the boundary of the sea level, reference nodes (also called known nodes) are placed above the sea level, whereas blind nodes (also called unknown nodes) are placed below (Figure. 1). The known and unknown nodes are divided by the ABCD curve, but every known node has its own unique identifier, and record as ID. If the reference direction from AB to DC curve is selected, then the IDs gradually increase along the direction and the unknown nodes belong to the left of the sea...
level boundary. If the reference direction from CD to BA curve is selected, then the IDs gradually decrease and the unknown nodes belong to the right of the sea level boundary. If the ABCD plane is linear, then it is called flat surface split.

![Figure 1. Two Node distribution based on the boundaries of sea level.](image)

### 3. Principle of Sea Level Boundary Positioning

Every reference node sends its location information to other nodes, and all nodes send the distance information to every node through WSN [7]. If we know the distance from anchor nodes to reference nodes and the coordinates of reference nodes, we can obtain the location information of unknown nodes.

Suppose the coordinates of three reference nodes are $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, and $P_3(x_3, y_3, z_3)$, the coordinate of any unknown node is $M(x, y, z)$ or $N(x, y, z)$ (Figure 2). If the distances from $M$ or $N$ to $P_1$, $P_2$ and $P_3$ are $d_1$, $d_2$, and $d_3$, respectively. Then the distances between two points can be deduced as follows:

$$
\begin{align*}
(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 &= d_1^2 \quad (1) \\
(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2 &= d_2^2 \quad (1) \\
(x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2 &= d_3^2 \quad (1)
\end{align*}
$$

Eq. (1) is rearranged as

$$
\begin{align*}
(Q + R)z &= d_1^2 - x_1^2 - y_1^2 - z_1^2 - d_2^2 + d_3^2 \\
(Q + R)x &= x_1^2 - x_2^2 + y_2^2 - y_1^2 + z_2^2 - z_1^2 \\
(Q + R)y &= y_1^2 - y_3^2 + z_3^2 - z_1^2
\end{align*}
$$

where $Q = (x_1^2 - x_2^2 + y_2^2 - y_1^2 + z_2^2 - z_1^2 + d_2^2 - d_1^2)/2$ and $R = (x_2^2 - x_3^2 + y_3^2 - y_2^2 + z_3^2 - z_2^2 + d_3^2 - d_1^2)/2$.

$M(x_M, y_M, z_M)$ or $N(x_N, y_N, z_M)$ can be obtained from Eq. (2).

#### 3.1. Selecting reference nodes

First, we must determine whether $P_1$, $P_2$, and $P_3$ are collinear. If $\exists \lambda$, then $\overrightarrow{P_1P_2} = \lambda \overrightarrow{P_2P_3}$ and $P_1$, $P_2$, and $P_3$ are collinear. Thus, the result of positioning has countless locations, and we need not select three new points unless $\overrightarrow{P_1P_2} \neq \lambda \overrightarrow{P_2P_3}$.

#### 3.2. Plane equation for $P_1$, $P_2$, and $P_3$

If $P_1$, $P_2$, and $P_3$ are non-collinear, then they can only form a plane. We propose the following plane equation for $P_1$, $P_2$, and $P_3$: $Ax + By + Cz + D = 0$ (3)

$P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, and $P_3(x_3, y_3, z_3)$ are combined in Eq. (3) and expressed in a matrix form as follows:
\[
\begin{bmatrix}
x_1 & y_1 & z_1 \\
x_2 & y_2 & z_2 \\
x_3 & y_3 & z_3 \\
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C \\
\end{bmatrix}
= 
\begin{bmatrix}
-D \\
-D \\
-D \\
\end{bmatrix}
\]  

We define \( AA = \begin{bmatrix}
x_1 & y_1 & z_1 \\
x_2 & y_2 & z_2 \\
x_3 & y_3 & z_3 \\
\end{bmatrix} \) because \( P_1, P_2, \) and \( P_3 \) are non-collinear, and the inverse of matrix \( AA \) exists. Coefficients \( A, B, \) and \( C \) are expressed as parameter \( D \), as shown in Eq. (4), and \( D \) is removed. We eventually obtain the plane equation for \( P_1, P_2, \) and \( P_3 \) as follows:

\[
A'x + B'y + C'z + 1 = 0
\]  

where \( A', B', \) and \( C' \) are the coefficients after the removal of \( D \).

3.3. Determining the solution of positioning

If \( M \) and \( N \) are coincident (Figure. 2(a)), then the location of unknown nodes are only on the plane for \( P_1, P_2, \) and \( P_3 \). Thus,

\[
\hat{A}(x_y - x_1) + B(y_y - y_1) + C(z_y - z_1) = 0
\]

The normal vector of the plane is \( \vec{F}_1 = (A', B', C') \), the normal vector of the XOY plane is \( \vec{F}_2 = (0,0,1) \), and the angle between normal vectors \( \vec{F}_1 \) and \( \vec{F}_2 \) is \( \theta \). Then,

\[
\cos \theta = \frac{C'}{\sqrt{A'^2 + B'^2 + C'^2}}
\]

- If \( \cos \theta = 1 \), then plane \( P_1P_2P_3 \) is parallel with plane XOY (Figure. 2(b)). Point \( M \) is above plane \( P_1P_2P_3 \), whereas point \( N \) is beneath this plane. For point \( M \),

\[
z_M > z_1
\]

For point \( N \),

\[
z_N < z_1
\]

- If \( \cos \theta = 0 \), then plane \( P_1P_2P_3 \) is perpendicular to plane XOY (Figure. 2(c)). Point \( M \) is on the right of the plane \( P_1P_2P_3 \), whereas point \( N \) is on the left. For point \( M \),

\[
x_M > x_1
\]

For point \( N \),

\[
x_N < x_1
\]

- If \( \cos \theta \neq 1 \) and \( \cos \theta \neq 0 \), then the angle between normal vectors \( \vec{F}_1 \) and \( \vec{F}_2 \) is \( \left(0, \frac{\pi}{2}\right)\) (Figure. 2(d) and 2(e)).

\[
f_1(x, y, z) = Ax + By + Cz + 1
\]

\[
f_2 = f_1(x_y, y_y, z_y) = Ax + By + Cz + 1
\]
Rearranging Eq. (12) and (13) yield

\[ A'x_M + B'y_M + C'(z_M - f_2/C') + 1 = 0 \] (14)

For point M,

\[ f_2/C' > 0 \] (15)

For point N,

\[ f_2/C' < 0 \] (16)

**Figure 2.** Decision on positioning node

4. Implementation of the Positioning Process

On the basis of the principle of positioning, the method of three reference nodes can be designed based on range. If the increasing direction of the ID numbers is selected as the reference direction and all positioned nodes belong to the left of the interface, which are placed on the boundaries of the sea level. **Figure. 3** shows the design of the localization algorithm. The distances are measured from the anchor nodes to the three reference nodes, and the positioned nodes are calculated using Eq. (3) to obtain solution \( M(x_M, y_M, z_M) \) or \( N(x_N, y_N, z_N) \). First, whether the two solutions are coincident is determined using Eq. (6). The angle between the plane of the three reference nodes and XOY plane is then calculated by Eq. (7). Finally, Eq. (8)–(16) and the two solutions are used to obtain the desired position solution.
5. Experimental Settings and Results

The experiment performs a simulation by using MATLAB 2010b, building one 3D space in a 100×100×100 area. A total of 200 blind nodes and 50 reference nodes are randomly placed in the 3D space for the positioning of the three reference nodes [8], as shown in Figure 4. The result of the simulation shows that the proposed algorithm can accurately perform positioning.

Figure 3. Flowchart of WSN localization algorithm based on the boundary of sea level in 3D space

Figure 4. Experimental results for positioning of the three reference nodes based on the boundary of sea level
However, an error still exists in the actual environment, thereby making the measured distance deviate from the real value [9]. A distance error is further added to deliberately simulate. Then, the result of positioning with four reference nodes is compared to that with three reference nodes. If the positioning error is within the allowable range, then the positioned node is accurate and the accuracy rate of which is expressed as follows:

$$\eta_p = \frac{n}{N} \times 100 \%$$ \hspace{1cm} (17)

where \( N \) is the total number unknown nodes and \( n \) denotes the number of accurately positioned unknown nodes.

The experiment environment is an area of 100×100×100, and 200 unknown nodes are randomly placed in the 3D space. The selected reference nodes are 20, 30, 40, 50, 60, and 70. The positioning accuracy is obtained by the average of 10 times repeated for five times. The results are shown in Table 1, which illustrates that the positioning accuracy of the three reference nodes is better than that of the four reference nodes in the algorithm. The proposed approach is also compared with other localization techniques in terms of the number of reference nodes in positioning, accuracy, and cost [10-12], as shown in Table 2.

**Table 1 Results of Positioning Accuracy**

| Number of Reference Nodes | Positioning Accuracy in algorithm (\( \eta_p \)) |
|---------------------------|---------------------------------------------|
|                           | Three nodes (Five times) | Four nodes (Five times) |
| 20                        | 89 89 97 94.5 97         | 80 95.5 86 87 86       |
| 30                        | 92 89 94 92.5 94         | 93.5 74 65.5 75       |
| 40                        | 93.5 95 93 84 93         | 97 78 94.5 90         |
| 50                        | 92.5 91 92.5 90.5 92.5   | 86 90.5 90.5 74.5     |
| 60                        | 94.5 91 91 88.5 91       | 91.5 85 92 76         |
| 70                        | 95 85 87 93 87           | 81 83 76.5 89.5       |

**Table 2 Comparison of the Proposed Positioning Algorithm with Other Methods**

| Index                                | Proposed Algorithm | Other Methods  |
|--------------------------------------|--------------------|---------------|
| Number of known nodes in positioning | Three nodes        | No less than four nodes |
| Accuracy                             | Accurate           | Accurate      |
| Cost                                 | Minimal            | Substantial   |

6. Conclusion

In this work, the proposed positioning algorithm based on the boundary of sea level can minimize the number of known nodes in 3D space. This study proves that the theory of three known nodes is feasible, and positioning accuracy is improved by experiment simulation. Although the proposed method only needs three reference nodes, whether the unknown node belongs to the right or left must be determined. Hence, the proposed positioning method is useful for a calm sea. However, unknown node positioning for rough sea must consider positioning by dividing the plane into different small areas because two areas might have positioning solutions for the plane of the reference nodes. Application of the proposed method is feasible and has obvious advantages compared to other
positioning technologies in three-dimensional space, because the position of unknown nodes can be obtained through three known nodes in a certain area of the plane.

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