Studies of light neutron-excess nuclei from bound to continuum

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Abstract. The generalized two-center cluster model (GTCM), which can handle various single particle configurations in general two center systems, is applied to the light neutron-rich system, $^{12}\text{Be} = \alpha + \alpha + 4N$. We discuss the change of the neutrons' configuration around two $\alpha$-cores as a variation of an excitation energy. The covalent, ionic and atomic configurations coexist with the degenerate feature above the $\alpha+^8\text{He}_{g.s.}$ particle-decay threshold. We find the strong enhancement in the monopole excitation from the ground state to the excited states. The GTCM calculation is also applied to even Be isotopes, and the systematics on the structural changes from bound region to continuum is discussed.

1. Introduction

In the last two decades, developments of experiments with secondary RI beam have extensively advanced the studies on light neutron-rich nuclei. In particular, much efforts have been devoted to the investigation of molecular structure in Be isotopes. The Be isotopes can be considered as typical examples of two-center superdeformed systems which build on an $\alpha+\alpha$ rotor of $^8\text{Be}$. Theoretically, molecular orbital (MO), such as the $\pi^-$ and $\sigma^+$ orbitals associated with the covalent bonding in atomic molecules, have been successful in understanding the low-lying states of these isotopes [1].

The MO model can describe many kinds of characteristic properties of these isotopes, but they are mainly limited to the analysis on low-lying bound states, and theoretical studies on the highly excited states above the particle-decay threshold is still open area. In contrast to the situation of theoretical studies, recent experiments on $^{12}\text{Be}$, for instance, revealed the existence of many resonant states [2, 3, 4], which strongly decay into $^6\text{He}_{g.s.} + ^6\text{He}_{g.s.}$ and $\alpha + ^8\text{He}_{g.s.}$. Similar resonances, decaying to He isotopes, have also been observed in other Be isotopes, such as $^{10}\text{Be}=\alpha + ^6\text{He}$ [5] and $^{14}\text{Be}=^6\text{He} + ^8\text{He}$ [6].

In the present study, we investigate the structural changes from the bound states to the continuum states in even Be isotopes. In order to investigate the continuum states above the particle decay threshold, the intrinsic structures and their coupling to the scattering states should be treated in a unified manner. For this purpose, we introduce the generalized two-center cluster model (GTCM) [7, 8, 9]. In this model, the covalent MO configuration can be smoothly connected to the atomic or ionic one, in which valence neutrons are localized around one of the $\alpha$ cores. Furthermore, it becomes possible to describe both the formation of the covalent MO structures and its decays into a continuum energy state. In $^{12}\text{Be}$ for instance,
this model can handle the formation of an atomic structure of $^6\text{He}+^6\text{He}$, ionic structures of $\alpha+^8\text{He}$ and $^5\text{He}+^7\text{He}$, and covalent MO structures in a consistent manner. In this report, we mainly investigate the intrinsic structure of $^{12}\text{Be}$ and enhancement phenomena in the monopole transitions of $0^+_1\text{g.s.} \rightarrow 0^+_\text{ex.}$, which is embedded in continuum. We also apply GTCM to even Be isotope of $^{8+10}\text{Be}$, and investigate the structural changes from a bound region to continuum region.

2. Framework

The detailed explanation of GTCM has already been published in Refs. [7, 8, 9], and we briefly show the formulation of GTCM in the following. In this model, the total wave function of $^{12}\text{Be}$ is given by the superposition of the basis $\{\Phi_{m,J}^\nu(K)(S)\}$, where,

$$\Phi_{m,J}^\nu(K)(S) = \hat{P}_K^{\nu} A \left\{ \psi_L(\alpha)\psi_R(\alpha) \prod_{j=1}^4 \phi_j(m_j) \right\}_S .$$

The $\alpha$-cluster $\psi_\alpha(n=L, R)$ is expressed by the $\{0s\}^4$ configuration of the harmonic oscillator (HO) centered at the left($L$)- or right($R$)- side with the relative distance $S$. The single-particle wave function for the four valence neutrons localized around one of the $\alpha$ clusters is given by an atomic orbital (AO) $\phi(p_k, i, \tau)$, and $0\nu$-orbitals $p_k (k = x, y, z)$ around $i$ (= L or R) with the spin $\tau (= \uparrow$ or $\downarrow$). Here, $\{m_j\}$ are indices of AO $\{p_k, i, \tau\}$ and $\mathbf{m}$ represents a set of AOs for the four neutrons, $\mathbf{m}=\{m_1,m_2,m_3,m_4\}$. The intrinsic basis functions with the full anti-symmetrization $A$ are projected to the eigenstate of the total spin $J$, its intrinsic angular projection $K$, and the total parity $\pi$ by the projection operator $\hat{P}_K^{\nu}$.

The total wave function is finally given by taking the superposition over $S$, $\mathbf{m}$ and $K$ as

$$\hat{\Psi}_\nu^J = \int dS \sum_{\mathbf{m},K} C_{\mathbf{m}K}^\nu(S) \Phi_{\mathbf{m},J}^\nu(K)(S) .$$

The coefficients for the $\nu$-th eigenstate, $C_{\mathbf{m}K}^\nu(S)$, are determined by solving a coupled channel GCM (Generator Coordinate Method) equation [10]. We include all the possible AO configurations for the four valence neutrons [7, 8]. The present calculation is restricted to the axially symmetric ($K = 0$) case; however, we include all the possible AO configurations for the four valence neutrons within this approximation. Therefore, the model space of MO, where each valence neutron rotates around two centers simultaneously, is also covered [7, 8]. As for the nucleon-nucleon interaction, we use the Volkov No.2 and the G3RS for the central and spin-orbit parts, respectively. The parameters in the interactions and the size parameter of HO are the same as those applied in Refs. [7, 8], which successfully reproduce the properties of $^{10,12}\text{Be}$ [7, 8] and the threshold energies of $\alpha+{^8}\text{He}_{g.s.}, {^6}\text{He}_{g.s.}+{^6}\text{He}_{g.s.}$, and $^5\text{He}_{g.s.}+{^7}\text{He}_{g.s.}$ [9].

3. Results

3.1. Energy spectra of $^{12}\text{Be}$

The application of GTCM to $^{12}\text{Be} = \alpha + \alpha + 4N$ has already been published in Refs. [8, 9]. In the present article, we mainly report the enhancement phenomena in the monopole transitions. In $^{12}\text{Be}$, various chemical-bonding-like states appears: the covalent MO states of $(\pi_{3/2}^-)^2(\sigma_{1/2}^+)^2$ and $(\pi_{3/2}^-)^2(\pi_{1/2}^+)^2\{0^+_1\}$, the ionic states of $\alpha+{^8}\text{He}_{g.s.}$ and $^5\text{He}_{g.s.}+{^7}\text{He}_{g.s.} (0^+_1)$, the atomic state of $^6\text{He}_{g.s.}+{^6}\text{He}_{g.s.} (0^+_1)$, and the covalent-atomic hybrid state of $(^6\text{He}+{^5}\text{He})(\sigma_{1/2}^+)^2(0^+_1)$. The energy spacing of these chemical bonding states is small, say about $\sim 1\sim 2$ MeV, and hence these states reveal a strong degenerate feature [8].
The covalent states, $0_2$ and $0_3^{+}$, correspond to the neutrons’ excitation mode from the ground $0_1^+$ state, in which the orbits of the excess neutrons are excited around two $\alpha$ cores, while the ionic states, $0_3^+$ and $0_4^+$, are realized as the cluster excitation mode from the ground and $0_2^+$ states, respectively. The cluster excitation means that the $\alpha-\alpha$ relative motion is excited, and its wave function has an additional node by comparison with the lower bound states. The $0_1^+$ state is the simultaneous excitation mode, corresponding to the double excitation of the $\alpha$ clusters’ and excess neutrons’ degrees of freedom. The detailed analysis of the intrinsic structures for the individual energy levels are shown in Refs. [8, 9]

### 3.2. Monopole transitions in $^{12}$Be

All the excited states have a large magnitude of the (isoscalar) monopole transition from the ground ($\nu=1$) state to the $\nu$-th $0^+$ states [11], which is defined by

$$M(IS) = <\Psi^\nu| \sum_{i=1}^{12} r_i^2 | \Psi^1 > \equiv <0^+_\nu| \sum_{i=1}^{12} r_i^2 | 0^+_1 > . \quad (3)$$

Here, $r_i$ denotes the radial coordinate for the $i$-th nucleon in $^{12}$Be. In general neutron-excess systems, an isovector part of monopole matrix elements does not vanish and contributes to the total strength of a monopole transition. However, the isovector excitation becomes zero if the target nucleus is an isospin-saturated system, such as the $\alpha$ particle. In recent experiments, nuclear reactions with an $\alpha$ target are often employed to probe excited states of neutron excess systems [4]. In this study, therefore, we consider only the isoscalar part of the monopole matrix element.

In order to discuss the magnitude of the monopole strength clearly, in Table 1, we show the ratio of the calculated strength to the respective single particle strength, $|M(IS)/M^{s.p.}|$. The single particle strength is defined by the simple HO wave function in the $p$-shell [12] as:

$$M^{s.p.} = <1p, b|r^2|0p, b > = 2b^2 \sqrt{3 \pi} . \quad (4)$$

Here, $b$ is a size parameter of HO ($b=1.46$ fm), and $0p$ and $1p$ denote the radial wave function of HO with the zero and one radial node, respectively. As can be seen in Table 1, all the strength is comparable to or a few times larger than the single particle strength, $M^{s.p.}$, and these values are almost the same magnitude as the transition to the $0_2^+$ state at $E_x=7.65$ MeV in $^{12}$C, having a $3\alpha$ cluster structure. In $^{12}$C, the ratio of the monopole strength of $0_1^+ \rightarrow 0_2^+$ is about 4.6 [13]. The result of $^{12}$Be is consistent to the analysis of the monopole strength in Ref. [12], where the enhancement of the low-lying monopole strength is discussed by cluster formations.

It should be noted that all the monopole strength, corresponding to the various cluster structures, appears at $E_x \leq 20$ MeV in the present system. In marked contrast to this result, in a naive mean-field picture, the $2\hbar\omega$ ($\sim 35$ MeV) jump is needed for monopole excitations. This means that it is quite difficult to explain the monopole strength shown in Table 1 by mean-field models and hence, the present result is considered to be abnormal in a naive single particle picture.

In a system with a considerable neutron excess, there are almost degenerate monopole states at much lower energies than the energy region expected in a naive shell-model picture, and the transition strengths to these states are comparable with the single-particle strength. Among the various $0^+$ states, strong enhancement occurs for the transition to the $0_3^+$ state ($\alpha+^8$He$_{g.s.}$), which is a direct cluster’s excitation mode from the ground state. An enhancement can also be seen for the $0_4^+$ state with the two neutrons’ excited configuration, $(\pi^+_{3/2})^2(\pi^-_{1/2})^2$. This enhancement is due to the mixing of $(\pi^-_{3/2})^2(\sigma^+_{1/2})^2$ and $(\pi^-_{3/2})^2(\pi^-_{1/2})^2$ as pointed out in Ref. [14].
Ratio

| State | Main configuration | Ratio |
|-------|--------------------|-------|
| $0^+_2$ | $(\pi_{3/2})^2(\pi_{1/2})^2$ | 2.59 |
| $0^+_3$ | $\alpha+^{8}\text{He}_{g.s.}$ | 3.53 |
| $0^+_4$ | $^{6}\text{He}_{g.s.}+^{8}\text{He}_{g.s.}$ | 0.92 |
| $0^+_5$ | $(^{8}\text{He}+^{4}\text{He})(\sigma_{1/2}^+)^2$ | 1.48 |
| $0^+_6$ | $^{5}\text{He}_{g.s.}+^{7}\text{He}_{g.s.}$ | 1.76 |

Table 1. The ratio of the total monopole strength and the single particle strength in the 0p-shell.

However, the strength for $0^+_3$ is about two times the magnitude of the strength for $0^+_1$, and the transition to the cluster excited state, $0^+_3$, is strongest in all the monopole strength. We decomposed the total monopole strength into each of degrees of freedom, such as the excess four neutrons part and the $\alpha-\alpha$ part. From the result of the decomposition, we found that, in the $0^+_3$ state, the strength arising from the $\alpha-\alpha$ core part dominantly contributes to the total strength of the monopole transition. The detailed analysis of the monopole strength has already been reported in Ref. [11].

3.3. Systematics of even Be isotopes

We discuss the systematics of even Be isotopes by applying the similar calculation of GTCM. The energy spectra of $^{10}\text{Be}$ with $J^p=0^+$ is shown in Fig. 1(a). The total reaction probability, $1-\text{R}$, which is defined by the scattering matrices of the $\alpha+^{6}\text{He}_{g.s.}$ elastic scattering ($S_{el}$), is plotted by the dotted curve at the upper part in Fig. 1(a).

Below the $\alpha$ decay threshold (zero energy point), two bound states appear. The intrinsic structure of the $0^+_1$ and $0^+_2$ states are nicely described by the MO configuration of $(\pi_{3/2})^2$ and $(\sigma_{1/2})^2$, respectively. Since the $\sigma_{1/2}^+$ orbit has an enlarged distribution along the $\alpha-\alpha$ axis, the $\alpha-\alpha$ clustering is well developed in the $0^+_1$ state [7].

A prominent peak appears around $E \sim 3$ MeV in the total reaction probability. This enhancement of the reaction probability is due to the formation of the resonance with the ionic structure of $\alpha+^{4}\text{He}(2^+_1)$, which is shown by the $0^+_3$ state in Fig. 1(a). At the same energy region, there is a broad continuum structure, shown by $0^+$ in parenthesis with a transparent box. This continuum state has a main component of an ionic structure of $\alpha+^{6}\text{He}_{g.s.}$. Two $0^+$ states embedded in continuum correspond to the cluster’s relative excitation mode from the bound states: The $0^+_5$ state is cluster’s excitation from the ground $0^+_1$ state, while the continuum $(0^+)$ can be considered as the similar excitation mode from the $0^+_2$ state. These states are just analog states to the ionic state, $\alpha+^{6}\text{He}_{g.s.}$ $(0^+_2)$ and $^{5}\text{He}_{g.s.}+^{7}\text{He}_{g.s.}$ $(0^+_6)$, in $^{12}\text{Be}$.

The reason why the cluster excitation mode from $0^+_2$ does not form the resonance can be explained in the following: Since a clusters’ excited state must be orthogonal to the low-lying bound state, its size must be extended by comparison to the low-lying state. The $0^+_5$ state has a spatially extended structures due to the shape of the $\sigma^+$ orbit configuration, and the cluster excited state orthogonal to $0^+_1$ cannot keep an appropriate distance to form a quasi bound state.

The energy spectra for $^{14}\text{Be}$ is shown in Fig. 1(b). In this figure, we can clearly confirm the similar level structure to $^{10}\text{Be}$. In the bound region, two $0^+$ states appears, while two resonant states are embedded in continuum energy. The ground $0^+_1$ state has a spatially compact structure, which can be nicely understood by the shell model picture, while the $0^+_2$ state has a well developed $\alpha-\alpha$ cluster structure. Above the $^{6}\text{He}_{g.s.}+^{8}\text{He}_{g.s.}$ threshold, two resonant states are identified by solving the $^{8}\text{He}+^{8}\text{He}$ collision, and they have the ionic structures, such as $^{6}\text{He}_{g.s.}+^{8}\text{He}_{g.s.}$ $(0^+_3)$ and $^{6}\text{He}(2^+_1)+^{8}\text{He}_{g.s.}$ $(0^+_4)$. The $0^+_3$ and $0^+_4$ states correspond to the
Figure 1. (a) Energy spectra of $^{10}$Be ($J^e=0^+$). The threshold energies of the $\alpha+^6$He$_{g.s.}$ channel is taken to be the origin. The dotted curve at the right part represents the reaction probability for the central collision of $\alpha+^6$He$_{g.s.}$. (b) The same figure as (a) but for $^{14}$Be. The $^6$He$_{g.s.}+^8$He$_{g.s.}$ threshold energy is set to be the origin.

cluster’s excitation modes from the two bound states, the $0^+_1$ and $0^+_2$ states, respectively. In $^{10}$Be, one of two $0^+$ states above the $\alpha$ threshold becomes a broad continuum state, but, in $^{14}$Be, both unbound $0^+$ states becomes the resonant states. This is because of the increase of excess neutrons. Due to the increase of excess neutrons, the attraction between $^6$He and $^8$He in $^{14}$Be is larger than that between $\alpha$ and $^6$He in $^{10}$Be. Thus, unbound $0^+$ states are stabilized as the resonant states in $^{14}$Be.

We have also analyzed the level structure of $^{16}$Be. There is a possibility of the resonance formation of a nuclear dimer such as $^8$He$_{g.s.}+^8$He$_{g.s.}$ although $^{16}$Be itself is an unstable nucleus with respect to two neutrons emission, $^{16}$Be = $^{14}$Be + 2N. The ratio of $N$/A in the $^8$He nucleus is 0.75, which is the highest value in bound neutron excess systems. Thus, the $^8$He+$^8$He atomic state corresponds to the nuclear dimers with extremely neutron excess, and it is very interesting to investigate the possibility of its formation. In Figs. 2 (a) and (b), the energy spectra for $^8$Be and $^{16}$Be are shown, respectively.

In the $^8$Be nucleus (Fig. 2(a)), there is no bound state, and the sharp resonance with the $\alpha+\alpha$ structure exists just above two $\alpha$ threshold. On the contrary, we have found that the ground state in $^{16}$Be appears below the $^8$He$_{g.s.}+^8$He$_{g.s.}$ threshold by about 4 MeV, as show in Fig. 2(b). This ground state corresponds to the unbound states with respect to the $^{14}$Be$_{g.s.}+2N$ threshold by about 3 MeV. The ground state in $^{16}$Be, which is predicted by the GTCM calculation, can be considered as an analog state of $\alpha+\alpha$ in $^8$Be. Due to the effect of excess neutrons, the unbound resonance of $\alpha+\alpha$ becomes a “bound state” below the $^8$He$_{g.s.}+^8$He$_{g.s.}$ above the threshold, we have confirmed a resonance like behaviors as shown in the scattering matrix of $^8$He+$^8$He (the solid curve at the right part). The analysis of the wave function in $^{16}$Be is now under way.

4. Summary
We have studied the exotic structures of $^{12}$Be by applying GTCM. Above the particle decay threshold, various chemical-bonding-like structures appear with the small energy interval. All
Figure 2. (a) Energy spectra of $^8$Be ($J^\pi=0^+$). The threshold energies of the $\alpha+\alpha$ channel is taken to be the origin. The dotted curve at the right part represents the reaction probability for the central collision of $\alpha+\alpha$. (b) The same figure as (a) but for $^{16}$Be. The $^8$He$_{g.s.}+^8$He$_{g.s.}$ threshold energy is set to be the origin.

the energy levels have the monopole strength comparable to or larger than the single particle strength. In particular, the largest strength is predicted for the $0_3^+$ state, which is the direct cluster excitation mode from the ground state. These enhancements in the monopole transition can be observed in the much lower energy region than the energy region in the shell model expectation.

Various excited states are obtained in even Be isotopes of $^8$-16Be. The cluster excitation from the MO states to the ionic He-states occurs in the similar manner in $^{10,14}$Be. According to the analysis on $^{12}$Be, the strong monopole enhancement can also be expected in $^{10,14}$Be. In $^{16}$Be, the $^8$He$_{g.s.}+^8$He$_{g.s.}$ configuration, which is analog to $\alpha+\alpha$ in $^8$Be, is predicted. The systematic analysis on the intrinsic wave function of even Be isotopes is now underway.

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