Isospin splitting of pion elliptic flow in relativistic heavy-ion collisions

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Based on the framework of an extended multiphase transport model with mean-field potentials in both the partonic phase and the hadronic phase, we explain the elliptic flow difference between $\pi^+$ and $\pi^-$ in the beam-energy scan program at the relativistic heavy-ion collider by incorporating the vector-isovector potential for quarks and antiquarks with different isospins. It is found that the isospin splitting of charged pion elliptic flow favors a strong vector-isovector interaction, and thus serves as a probe of the quark matter equation of state as well as the QCD phase structure at finite baryon and isospin chemical potentials.

Understanding the properties of the strongly interacting quark-gluon plasma (QGP) as well as the hadron-quark phase transition is one of the main goals of relativistic heavy-ion collision experiments. The elliptic flow ($v_2$) characterizing the different numbers of particles moving in the reaction plane and out of the reaction plane is a good probe of the QGP properties \cite{1, 2}. The number-of-constituent-quark (NCQ) scaling of the elliptic flow in ultrarelativistic heavy-ion collisions serves as an evidence of the existence of the QGP, indicating that the elliptic flow is mostly developed in the partonic phase. Recently, 'low-energy' relativistic heavy-ion collisions were carried out in the beam-energy scan (BES) program at the relativistic heavy-ion collider (RHIC), in order to search for the signal of the critical point of the hadron-quark phase transition. Among various interesting phenomena different from those observed in ultrarelativistic-heavy-ion collisions is the splitting of $v_2$ between protons and antiprotons, $K^+$ and $K^-$, and $\pi^+$ and $\pi^-$ \cite{3–8}, showing the breakdown of the NCQ scaling law. The observed $v_2$ splitting can be due to the larger $v_2$ for transported quarks than that for produced quarks, or similarly, their different rapidity dependencies \cite{11, 12}, hydrodynamic evolution of the QGP at finite baryon chemical potential \cite{13, 14}, and the smaller radial flow of particles than their antiparticles \cite{15}. Although the $v_2$ splitting between $\pi^-$ and $\pi^+$ as well as its linear dependence on the charge asymmetry was proposed to be attributed to the electric quadrupole moment in the produced QGP induced by the chiral magnetic wave \cite{11}, it is disfavored by transport simulations based on the chiral kinetic equations of motion \cite{17, 18}.

In our previous study \cite{19, 20}, we have shown that the $v_2$ splitting between protons and antiprotons as well as between $K^+$ and $K^-$ can be attributed to the different mean-field potentials for particles and their antiparticles. We will demonstrated in the present study that the $v_2$ splitting between $\pi^+$ and $\pi^-$ can be explained by the different mean-field potentials for particles with different isospins.

The elliptic flow in heavy-ion collisions at intermediate energies was regarded as a probe of the mean-field potential or the nuclear matter equation of state \cite{21}. In noncentral heavy-ion collisions at relativistic energies, the produced QGP is of an almond shape in the transverse plane. Particles with attractive potentials are more likely to be trapped in the system and move in the direction perpendicular to the reaction plane, while those with repulsive potentials are more likely to leave the system and move along the reaction plane, thus reducing and enhancing their respective elliptic flows. In the baryon-rich matter produced in low-energy relativistic heavy-ion collisions, light quarks and baryons are affected by a more repulsive potential compared to light antiquarks and antibaryons as a result of the vector coupling, leading to the $v_2$ splitting between protons and antiprotons as well as between $K^+$ and $K^-$ \cite{13, 20}. Since heavy-ions are neutron-rich nuclei, the produced matter is not only baryon-rich but also neutron-rich or $d$-quark-rich, where $d$ ($\bar{u}$) quarks are expected to have a more repulsive potential than $u$ ($\bar{d}$) in the presence of a vector-isovector interaction. This will enhance the $v_2$ of $\pi^-$ while reduce the $v_2$ of $\pi^+$, qualitatively consistent with that observed experimentally. Since the stopping power becomes larger with the decreasing collision energy, the medium at midrapidities is expected to be not only more baryon-rich but also more neutron-rich, again qualitatively consistent with the larger $v_2$ splitting between $\pi^+$ and $\pi^-$ at lower collision energies. The isospin splitting of charged pion $v_2$ can thus be used to constrain the strength of the vector-isovector interaction and to help determine the quark matter equation of state as well as the QCD phase structure at finite baryon and isospin chemical potentials \cite{22}.

The present study is based on the framework of an extended multiphase transport model (extended...
AMPT) \[11, 20, 23\]. In this model the momentum distribution of initial partons is from melting hadrons generated by the heavy-ion jet interaction generator (HI-JING) model \[24\]. The coordinates of these partons in the transverse plane are set to be the same as those of the colliding nucleons from which they are produced, while their coordinates in the longitudinal direction are uniformly distributed within the finite thickness of the participant matter by considering the Lorentz contraction. The evolution of the partonic phase is then described by a relativistic transport approach, with the mean-field potentials for pions, kaons, and the initial partonic medium and the hadronization criterion employed in the present study compared with those used in Ref. \[30\].

The mean-field potentials of partons are given by a 3-flavor NJL model with isovector interactions, and its Lagrangian can be written as \[11\]

\[
\mathcal{L}_\text{NJL} = \bar{q}(i\gamma\cdot\nabla - m)q + \frac{G_s}{2} \sum_{a=0}^{8} [(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2] \\
- \frac{G_V}{2} \sum_{a=0}^{8} [(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2] \\
- K \{ \det[\bar{q}(1 + \gamma_5)] + \det[\bar{q}(1 - \gamma_5)] \} \\
+ G_{IS} \sum_{a=1}^{3} [(\bar{q}\lambda_a q)^2 + (\bar{q}\gamma_5\lambda_a q)^2] \\
- G_{IV} \sum_{a=1}^{3} [(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2]. \tag{1}
\]

In the above, \( q = (u, d, s)^T \) and \( m = \text{diag}(m_u, m_d, m_s) \) are the quark fields and the current quark mass matrix for \( u, d, \) and \( s \) quarks, respectively; \( \lambda_a \) are the Gell-Mann matrices with \( \lambda_0 = \sqrt{2/3}I \) in the 3-flavor space with the SU(3) symmetry; \( G_S \) and \( G_V \) are respectively the scalar-isoscalar and the vector-isoscalar coupling constant; and the \( K \) term represents the six-point Kobayashi-Maskawa-T’Hooft interaction that breaks the axial \( U(1)_A \) symmetry \[35\]. The additional \( G_{IS} \) and \( G_{IV} \) terms represent the scalar-isovector and the vector-isovector interactions, with \( G_{IS} \) and \( G_{IV} \) the corresponding coupling constants, respectively. Since the Gell-Mann matrices with \( a = 1, 2, 3 \) are identical to the Pauli matrices in \( u \) and \( d \) space, the isovector couplings break the SU(3) symmetry while keeping the isospin symmetry. In the present study, we employ the parameters \( m_u = m_d = 3.6 \text{ MeV}, m_s = 87 \text{ MeV}, G_S A^2 = 3.6, \) \( K A^3 = 8.9, \) and the cutoff value in the momentum integral \( \Lambda = 750 \text{ MeV} \) given in Refs. \[36, 37\]. As is known, the position of the critical point for the chiral phase transition is sensitive to \( G_V \) \[37, 39\], which was later constrained within \( 0.5 G_S < G_V < 1.1 G_S \) from the relative \( v_2 \) splitting between protons and antiprotons as well as between \( K^+ \) and \( K^- \) in relativistic heavy-ion collisions \[9\]. In the present study, we choose \( G_V = 1.1 G_S \) throughout the calculation.

In the mean-field approximation and considering only the flavor-singlet state for the vector-isoscalar term, the single-particle Hamiltonian for partons with flavor \( i \) \((i = u, d, s)\) can be written as

\[
H_i = \sqrt{M_i^2 + p_i^2} \pm \frac{2}{3} G_V (\rho_i^u + \rho_i^d + \rho_i^s) \\
\pm G_{IV} \tau_3 (\rho_i^0 - \rho_i^3). \tag{2}
\]

Here we take the convention that the upper (lower) sign is for quarks (antiquarks). \( M_i \) is the constituent quark mass determined by the gap equation

\[
M_i = m_i - 2 G_s \sigma_1 + 2 K \sigma_3 (\sigma_u - \sigma_d). \tag{3}
\]
where $\sigma = \langle q_i\bar{q}_i \rangle$ is the quark condensate, $(i, j, k)$ is any permutation of $(u, d, s)$, and $\tau_{3u}$ is the isospin quantum number of quarks, i.e., $\tau_{3u} = 1$, $\tau_{3d} = -1$, and $\tau_{3s} = 0$. $\vec{p}_i^\tau = \vec{p}_i^\tau + \frac{2}{3} G_V (\vec{\rho}_u^\tau + \vec{\rho}_d^\tau + \vec{\rho}_s^\tau) + G_{1V} \tau_{3b}(\vec{\rho}_u^\tau - \vec{\rho}_d^\tau)$ is the real momentum of partons, with $\vec{p}_i^\tau$ being the canonical momentum and $\vec{p}_i^\tau$ being the space components of the net quark density. $\vec{p}_i^\tau \equiv \langle \vec{q}_i \gamma_q^\tau \rangle$ in Eq. (2) is the time component of the net quark density. The quark condensate and the 4-component net quark density can be calculated from the phase-space distributions of quarks $(f_i)$ and antiquarks $(\bar{f}_i)$ through the relations

$$\langle \bar{q}_i \gamma_q \rangle = -2N_c \int_0^\Lambda \frac{d^3 p}{(2\pi)^3} \frac{M_i}{\sqrt{M_i^2 + p_i^\tau}} (1 - f_i - \bar{f}_i),$$

$$\langle \bar{q}_i \gamma_q \rangle = 2N_c \int_0^\Lambda \frac{d^3 p}{(2\pi)^3} \frac{\rho_i^\mu}{\sqrt{M_i^2 + p_i^\tau}} (f_i - \bar{f}_i),$$

where $2N_c$ is the quark spin and color degeneracy. In the transport approach, the whole system is divided into grids with $f_i$ and $\bar{f}_i$ at each cell calculated from averaging over parallel events using the test-particle method. In this way, the spatial distributions of the above quantities can be obtained.

Given the single-particle Hamiltonian as Eq. (2), partons with flavor $i$ evolve according to the following canonical equations

$$\frac{d\vec{r}_i^\tau}{dt} = \frac{\partial H_i}{\partial \vec{p}_i^\tau} = \frac{\vec{p}_i^\tau}{\sqrt{M_i^2 + p_i^\tau}},$$

$$\frac{d\rho_i^\tau}{dt} = -\frac{\partial H_i}{\partial \vec{r}_i^\tau} + \frac{d}{dt} \left[ \frac{2}{3} G_V (\vec{\rho}_u^\tau + \vec{\rho}_d^\tau + \vec{\rho}_s^\tau) + G_{1V} \tau_{3b}(\vec{\rho}_u^\tau - \vec{\rho}_d^\tau) \right].$$

The time component of the vector-isovector potential $G_{1V} \tau_{3b}(\vec{\rho}_u^\tau - \vec{\rho}_d^\tau)$ is expected to give a more repulsive (attractive) potential for $d$ ($u$) quarks in the baryon-rich and $d$-quark-rich partonic phase. The space component of the vector-isovector potential $-G_{1V} \tau_{3b}(\vec{\rho}_u^\tau - \vec{\rho}_d^\tau)$ may have an opposite effect, but its effect is small at the early stage when the net quark flux has not been developed. A similar argument for the vector-isoscalar potential can be found in Refs. [40, 41]. For the ease of discussion, we define $R_{1V} = G_{1V}/G_S$ as the reduced strength of the vector-isovector coupling and will discuss its effects on the isospin splitting of the elliptic flow. The scalar-isovector interaction leads to the constituent mass splitting of $u$ and $d$ quarks, and the effect is only pronounced around the boundary of the chiral phase transition [22] at the later stage of the partonic phase when the density is rather low, and it has been found to have negligible effects on the isospin splitting of the elliptic flow.

We first fit the parton scattering cross section in order to reproduce the charged particle $v_2$ at RHIC-BES energies. As shown in Fig. 1 using the same event-plane method [1, 2] as in the experimental analysis, the extended AMPT model with an isotropic parton scattering cross section of $10$ mb can reproduce reasonably well the transverse-momentum ($p_T$) dependence of mid-pseudorapidity charged particle $v_2$ at all RHIC-BES energies. The cross section is larger than that in Ref. [20], as a result of using an improved coalescence approach. As discussed in Ref. [27], a larger cross section is needed to reproduce the same hadron $v_2$ once combinations of partons close in momentum space is favored, compared with the naive spatial coalescence scenario.

We choose minbias Au+Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV as a representative system to discuss the effect of the vector-isovector interaction on the isospin splitting of the elliptic flow. The transverse-momentum dependence of $v_2$ for light quarks and antiquarks with different isospins

![FIG. 1: (color online) Comparison of the transverse momentum dependence of charged particle elliptic flows at mid-pseudorapidities ($|\eta| < 1$) at RHIC-BES energies from the extended AMPT model with those measured by the STAR Collaboration [12].](image1)

![FIG. 2: (color online) Transverse momentum dependence of the elliptic flows for mid-pseudorapidity ($|\eta| < 1$) light quarks and antiquarks in minbias Au+Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV from the extended AMPT model with (right) and without (left) the vector-isovector interaction.](image2)
are displayed in Fig. 2. The $v_2$ splitting between quarks and antiquarks is due to the vector-isoscalar potential. Without the vector-isovector interaction ($R_{IV} = 0$), $u$ and $d$ quarks as well as $\bar{u}$ and $\bar{d}$ have the same elliptic flow. With the vector-isovector interaction ($R_{IV} = 2$), $d$ quarks have a slightly larger $v_2$ than $u$ quarks. This shows that the time component of the vector-isovector potential has the dominate effect over the space component, leading to a more repulsive (attractive) potential for $d$ ($u$) quarks, as discussed above. One expects that $\bar{u}$ ($\bar{d}$) should also be affected by a more repulsive (attractive) potential according to Eq. [2], but this is not clearly seen from their elliptic flows.

Since $\pi^+ - \pi^-$ is composed of $u\bar{d}$ ($u\bar{d}$), the isospin splitting of light quark $v_2$ is expected to be preserved during the hadronization and lead to the isospin splitting of charged pion $\nu_2$. For resonance states with different isospins, e.g., $\rho^0$ or $\Delta^{++}$, formed from the quark coalescence, they eventually decay into charged pions and carry information of the isospin-dependent potentials on their constituent light quarks. The final elliptic flows of mid-pseudorapidity $\pi^+$ and $\pi^-$ are displayed in Fig. 3 for the same collision system as in Fig. 2. It is seen that the resulting $v_2$ as a function of transverse momentum from the extended AMPT model has a similar overall magnitude compared with those measured by the STAR Collaboration. A slightly larger $v_2$ for $\pi^-$ than $\pi^+$ is observed with the vector-isovector interaction, while the $v_2$ for $\pi^-$ and $\pi^+$ are similar without the vector-isovector interaction.

To illustrate more clearly the effect of the vector-isovector interaction, although the mean value of the $v_2$ difference is negative due to the weak hadronic potential, it is consistent with 0 within the statistical error. The $v_2$ splitting increases with the increasing coupling strength of the vector-isovector interaction until it is about to saturate for $R_{IV} > 2$. To understand the saturation effect from the vector-isovector interaction, we further show in the lower panel of Fig. 4 the evolution of the time component of the vector-isoscalar potential $U_{IV} = G_{IV}(\rho_u^0 + \rho_d^0 + \rho_s^0)$ as well as the vector-isovector potential $U_{IV} = G_{IV}(\mu^0_u - \mu^0_d)$. For different coupling strengths in the central region of the partonic phase. Although the magnitude of $U_{IV}$ increases with increasing $R_{IV}$, it decreases more rapidly for $R_{IV} = 5$ compared with that for $R_{IV} = 2$. This is due to the quicker reduction of the isospin asymmetry in the high-density phase as a result of the stronger vector-isovector potential. For the vector-isoscalar potential, it has a much larger magnitude compared with the vector-isovector potential.

In Fig. 5 the collision energy dependence of the $p_T$-integrated $v_2$ difference between mid-pseudorapidity $\pi^+$
and $\pi^-$ from the extended AMPT model is compared with those measured by the STAR Collaboration. At each collision energy, tens of thousands events were generated from transport simulations. The decreasing trend of $v_2$ splitting with increasing collision energy is qualitatively reproduced by the extended AMPT model. Although the mean values of the $v_2$ difference between $\pi^+$ and $\pi^-$ have a smaller magnitude compared with the experimental data, they have overlaps within statistical errors. Our calculations show that the $v_2$ difference between $\pi^+$ and $\pi^-$ tends to saturate. Results from our transport approach reproduce the decreasing $v_2$ splitting with increasing collision energy seen in experiments, and the experimentally observed $v_2$ difference between $\pi^+$ and $\pi^-$ favors a strong vector-isovector coupling, with its coupling strength larger than twice the scalar-isoscalar coupling strength. Our finding is important for understanding the quark matter equation of state as well as the QCD phase structure at finite baryon and isospin chemical potentials.

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