The potential barrier of the closed universe expansion has been investigated and its overcoming condition has been obtained. The restrictions on the Friedmann integrals, cosmological constant and medium components energy densities have been analyzed. The phase-space has been considered and the phase curves of eternally expanding closed universes have been plotted. A questionable coincidence of the our Universe Friedmann integrals has been discussed.

1 Introduction

As it is known, data on distance modulus versus redshift reveal the presence of an acceleration of the cosmological expansion [123]. This is possible in existence of a cosmological repulsion which is described usually by Λ-term or equivalently by the presence of an vacuum-like medium with a negative pressure. An open universe will expand forever even if Λ = 0. A closed universe at Λ = 0 will stop expanding in future and begin to recollapse without fail. If Λ > 0, the expansion becomes accelerated when the doubled vacuum density exceeds the decreasing matter density. The open universe is sure to pass this condition and its eternal expansion has not an alternative.

Reaching of the closed universe the acceleration state and its consequent eternal expansion requires a potential barrier overcoming which is possible under certain conditions. In this work a restriction on the closed universe parameters being necessary to getting over the barrier and the universe dynamics in this case are investigated.

2 Condition of eternal expansion

The closed universe dynamics is described by the Einstein field equation for curvature radius $a$ of 3-space in comoving with cosmological expansion nonrotating (synchronous) frame:

$$\dot{a}^2 = \frac{A_m}{a} + \frac{a^2}{A_v^2} - 1.$$  (1)

Here and below convention $c = 1$ is used. The evolution constants $A_m$ and $A_v$ following [56] are named the Friedmann integrals. The matter integral

$$A_m = \frac{8\pi}{3} G \rho_m a^3,$$  (2)

where $\rho_m$ is the dust-like matter density including a dark matter, $G$ is the gravitational constant. The vacuum integral $A_v$

$$A_v^2 = \frac{8\pi}{3} G \rho_v = \frac{\Lambda}{3},$$  (3)

where $\rho_v$ is the vacuum energy density, $\Lambda$ is the cosmological constant. The equation (1) has been written without taking into account a radiation which contribution dominates at an early stage of the universe evolution and is negligible at the
are plotted in Fig. 1. At the point \( a \),

\[
A_m = -\frac{A_m}{a}, \quad U_v = -\frac{a^2}{A_v^2}, \quad U = U_m + U_v
\]

and write the equation \( (1) \) in form of an "energy conservation law"

\[
\dot{a}^2 + U(a) = -1.
\]

Graphs of the functions \( U_m(a), U_v(a), U(a) \) are plotted in Fig. 1. At the point \( a = \tilde{a} \),

\[ U \begin{cases} 
\tilde{U}_v \\
U_m \\
\tilde{U}_m \\
\tilde{U}
\end{cases} \]

\[ A_v \tilde{a} \tilde{a}' A_m \]

Figure 1: Potential barrier

\[
\tilde{a}^3 = \frac{1}{2} A_m A_v^2 (4) \]

the maximum \( \tilde{U} \) of the potential barrier \( U(a) \) is disposed,

\[
\tilde{U} = U(\tilde{a}) = -\left( \frac{\alpha A_m}{A_v} \right)^{2/3}, \quad (5)
\]

where \( \alpha = 3\sqrt{3}/2 \). At this point the balance of matter gravitation and vacuum antigravitation is achieved, \( \rho_m = 2\rho_v \). The levels \( \tilde{U}, \tilde{U}_m, \tilde{U}_v, 0 \) are equidistant:

\[
\tilde{U} - \tilde{U}_m = \tilde{U}_m - \tilde{U}_v = \tilde{U}_v = \frac{1}{3} \tilde{U}.
\]

At the point \( a = \tilde{a} \) the deceleration parameter changes the sign:

\[
q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\tilde{a}^3}{a^3 + aA_v^2 + 2a^3} (6)
\]

At \( a < \tilde{a} \) the matter gravitation dominates and \( q > 0 \), at \( a > \tilde{a} \) the vacuum antigravitation dominate and \( q < 0 \). At the point \( a = a' \),

\[
a'^3 = 2\tilde{a}^3
\]

\[
U'' = U'_v \text{ and } \rho_m = \rho_v. \text{ If the level } U = -1 > U''_m = U'_v \text{ then } A_m > A_v \text{ (as represented in Fig. 1); } A_m \leq A_v \text{ otherwise. The Friedmann integral } A_m \text{ is a maximum value of the curvature radius in the standard model } \Lambda = 0, \text{ the integral } A_v \text{ equals an initial value } a \text{ in the de Sitter universe } \rho_m = 0:
\]

\[
a(t) = A_v \cosh(t/A_v). \quad (7)
\]

A closed universe will expand eternally if the level \( U = -1 \) is above the potential barrier summit \( U = \tilde{U} \), \( |\tilde{U}| \geq 1 \) (the equality sign corresponds to the asymptotic approaching \( a \) to \( \tilde{a} \)):

\[
\frac{\alpha A_m}{A_v} \geq 1. \quad (8)
\]

The condition of potential barrier overcoming corresponds to the strict inequality. Thus, universe fate is determined by the Friedmann integrals ratio. The condition \( (8) \) means the limitation on the cosmological constant

\[
\Lambda \geq \frac{4}{3} A_m^{-2}. \quad (8')
\]

The \( A_m \) value \( (2) \) can be estimated very rough due to a considerable uncertainty of the scale factor \( a \). According to \( [5, 6] \) \( A_v \sim A_m \) and even some more, and then a fulfilment of the condition \( (8) \) turns out to be problematic.

Find out the condition \( (8) \) for the density parameters \( \Omega_m \) and \( \Omega_v \),

\[
\Omega_m = \frac{\rho_m}{\rho_c} = \left( 1 + \left( \frac{a}{a'} \right)^3 - \frac{a}{a'} \left( \frac{A_v}{A_m} \right)^{2/3} \right)^{-1} \quad (9)
\]

\[
\Omega_v = \frac{\rho_v}{\rho_c} = \left( \frac{a}{a'} \right)^3 \Omega_m = (HA_v)^{-2} \quad (10)
\]

where \( \rho_c \) is the critical density, \( \rho_c = 3H^2/8\pi G \), \( H = \dot{a}/a \) is the Hubble constant. The density parameter \( \Omega_m \) achieves a maximum at \( a = a_1 \)

\[
a_1 = \frac{A_v}{\sqrt{3}}
\]

\[ \Omega_m = 1 \text{ at } a = 0 \text{ and } a = A_v. \text{ The parameter } \Omega_v \text{ achieves a maximum (and the Hubble constant } H - \text{ a minimum) at } a = a_2
\]

\[
a_2 = \frac{3}{2} A_m;
\]
is described by a the curve in $(\Omega_m, \Omega_v)$ plane. One and only one curve passes through each of the point $(\Omega_m, \Omega_v)$, excepting (0, 1) and (1, 0). It means an observational determination of the values $\Omega_m, \Omega_v$ fixes the universe evolution in the model circumscribed (11, 12). The phase-space area $(\Omega_m, \Omega_v)$ satisfying the condition (8), (13) is disposed between the curves 1 and 1′ which correspond to the equality $A_v = \alpha A_m$. All the curves in this area corresponding the universes expansion begin at the point (1, 0); the evolutions are completed by the de Sitter universe (0, 1). On the curves 2 and 3 the correlations $A_v = 2A_m$ and $A_v = A_m$ are fulfilled respectively. Below the curve 1 universes expand from the Big Bang (1, 0) till a maximum value $a$ defined left point of intersection of the straight line $U = -1$ and the curve $U(a)$ when $\Omega_m$ and $\Omega_v$ go to infinity, afterwards the universes begin to retract and recollapse (1, 0). Above the curve 1′ expansion goes on the other (large $a$) side of the potential barrier without the Big Bang from a minimum value $a$ defined the right intersection point of the lines $U = -1$ and $U(a)$ to the de Sitter metric (0, 1). A passage from (1, 0) to (0, 1) on a straight line corresponds a flat universe $\Omega_m + \Omega_v = 1$. The dependence $a(t)$ at $t \to \infty$ becomes exponential

$$a(t) = A_v \exp(t/A_v),$$

which describes the final fate of any expanding universe with the $(\Omega_m, \Omega_v)$ values above the curve 1.

### Figure 2: Phase curves

Curves in Fig. 2 represents the dynamics of universes according to (12). The point (1, 0) corresponds to a Big Bang, the point (0, 1) signifies a de Sitter universe. The evolution of an universe with the fixed Friedmann integrals ratio $A_m/A_v$
Plots of the parameters $\Omega_m$ (9), $\Omega_v$ (10), $q$ (11) versus $a/\tilde{a}$ for the universe $A_v = 2A_m$ (the curve 2 in Fig. 2) and for the almost flat universe $A_v = 10^{-2}A_m$ are given in Fig. 3(a) and 3(b) respectively.

It should be mentioned that at an early stage of an universe evolution a radiation density $\rho_r$ dominates, then phase-space is three-dimensional ($\Omega_m, \Omega_v, \Omega_r$). The phase curves begin at the point $(0,0,1)$; they are directed to the $\Omega_r$ axis for a closed universe, and when the matter grows predominant over the radiation, the curves pass on ($\Omega_m, \Omega_v$) plane as it is shown in Fig. 2. The vacuum energy density $\rho_v$ is to prevail at a still more early stage preceding radiation-dominated, and then the universe develops from de Sitter metric to de Sitter metric along a closed phase curve. This curve simplified has been plotted here for a flat universe $\Omega_m + \Omega_v + \Omega_r = 1$.

4 Summary

The measured cosmological expansion acceleration means that our Universe will expand eternally. The condition of the closed universe eternal expansion restricts the Friedmann integrals ratio by the correlation (8). Written down for the density parameters $\Omega_m, \Omega_v$ this correlation (13) separate the area in the phase-space (Fig. 2). The measured parameters $\Omega_m, \Omega_v$ of our Universe are found in this area. The Friedmann integrals of our Universe do not coincide. The phase curves of the eternally expanding closed universes have been plotted. The dependence of the cosmological parameters $\Omega_m, \Omega_v, q, H$ on the universe curvature radius $a$ has been investigated.

References

[1] A.G. Riess et al., Astron. J. 116 (1998) 1009, astro-ph/9805201
[2] S. Perlmutter et al., Astrophys. J. 517 (1999) 565, astro-ph/9812133
[3] S.M. Carrol, Living Rev. Rel. 4 (2001) 1, astro-ph/0004075
[4] A.A. Starobinski, Grav. Cosmol. 6 (2000) 157, astro-ph/9912054
[5] A.D. Chernin, Uspekhi Fisicheskikh Nauk 171 (2001) 153.
[6] A.D. Chernin, New Astron. 7 (2002) 113, astro-ph/0107071
[7] A. Benoit et al., astro-ph/0210306
[8] A. Linde, hep-th/0211048