Optimization and optimality of frames with equally stable parts (links)

A I Shein
Penza State University of Architecture and Construction, 28, Titova str., Penza, Russian Federation
shein-ai@yandex.ru

Abstract. The problem of optimal design of transport overpass supports has the form of the problem of optimization of multi-storey frame system from the condition of stability. Analysis of the work of individual elements of frame systems in existing design solutions shows that not all parts of the frame are effectively involved in ensuring the overall stability of the structure. In this regard, it is important to solve the problem of optimization of frame elements with equal stability of individual parts. It is important to know the ratio of optimal frames and optimal frames with equally stable links. In an apparatus of forming a mathematical model of the critical state is adopted the method of displacement with the trigonometric functions, the effects of longitudinal forces on the reaction. The condition of the critical state of multi-storey frames, the stiffness matrix of which is the Jacobian matrix, is written by the determinant equation. The General solution of the problem of optimization of multi-storey frames in an analytical, closed form was obtained by the author earlier. For a complete analytical solution of optimization problems for equally stable parts, it is necessary to find solutions to two optimization problems for free frame links and two problems for non-free frame links. By a decision established that, forming the frame of the optimal revostock parts, we obtain a frame equivalent to the optimal non-segmented frame. And, therefore, the position stated by A. F. Smirnov that the most rational of the stability condition of the system will be such, all links (parts) of which have equal stability is true. The frame of the minimum mass with equal parts is optimal from the stability condition.

1. Introduction
One of the important directions of development of structural mechanics is the development of methods of optimal design of load-bearing structures of buildings and constructions. The solution of problems in this area in the first place can provide cheaper designs and reduce their material consumption.

Recently in construction multi-storey designs in the form of multilevel frames began to be applied more often. At the same time, the supporting elements of the frame are designed from materials with high mechanical properties, which contributes to the creation of cheaper and at the same time more flexible elements. This leads to the fact that the bearing capacity of frame frames is determined by their stability in the elastic or elastic-plastic stages. Analysis of the work of individual elements of frame systems in existing design solutions shows that not all parts of the frame are effectively involved in ensuring the overall stability of the structure. In this regard, the problem of optimization of frame elements with ensuring equal stability of individual parts becomes urgent.
2. Basic relations of the computational model

When optimizing the cross sections of the elements of frame systems, the bearing capacity of which is determined by their stability, the known elements are the configuration, support anchorages, material and nodal loads. The optimized frame consists of $n$ rods of unknown linear bending stiffness ($i_j=x_j$), which we assume to be constant within each rod. We want to find the values of variables $x_1, x_2, ..., x_n$, in which the weight (volume) function $G = \sum_{i=1}^{m} \lambda_i \cdot x_i$ takes the minimum value. Here $\lambda_i$ is the structural flexibility of the $i$-th stem. It is necessary that the frame is in critical condition, and the necessary conditions of strength are met.

Design schemes of multi-storey overpasses are free and / or non-free frame systems, which are characterized by the exhaustion of the bearing capacity in the form of loss of stability. Thus, the problem of optimal design of the transport overpass takes the form of the problem of optimizing the multi-storey frame system from the stability condition. The most important are the closed solutions of optimization problems, allowing to obtain ready-made formulas for the design of individual classes of structures [1-5].

We consider the problem of optimization of the frame system from condition of stability. As a tool for formation of a mathematical model of the critical state, we take the method of displacement with trigonometric functions of the influence of longitudinal forces on the reaction. Condition of critical state of multi-storey frames (figure 1), the stiffness matrix which is a Jacobian matrix, usually determined by the determinantal equation of critical state

$$\det\begin{bmatrix} r_{11} & r_{12} & 0 & 0 & \ldots & 0 \\ r_{21} & r_{22} & r_{23} & 0 & \ldots & 0 \\ 0 & r_{32} & r_{33} & r_{34} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & r_{m-1} & r_{mm} \end{bmatrix} = 0. \quad (1)$$

Here $r_{ik}$ – response to additional connections from the unit displacements of nodes that depend on the unknown linear stiffness of the mullions and transoms and on parameters of longitudinal force

$$v_j = \sqrt{\frac{N_j \cdot h_j}{x_j}}, \quad (2)$$

where $N_j$ – is the longitudinal force in the rod; $h_j$ – is the length of the rod.

In solving the verification problem, the transcendental equation obtained in the disclosure of the determinant of the matrix (1) is used to find the critical load. At the same time, the critical state is achieved, as a rule, when some parts of the frames are in a state of constraint, and others in the forced types of loss of stability. This is due to some arbitrariness in the appointment of frames rigidity.

In solving the problem of frame stiffness optimization by the criterion of minimum weight from the stability condition, equation (1) is used to construct the boundary hypersurface of the stability region in the n-dimensional stiffness space. The solution to this problem is a point lying on the hypersurface with the coordinates of the required stiffness. At the same time, the ratio of stiffness is determined by the longitudinal forces in the racks of frames.

Let us consider the question of the optimality of frames with equidistant parts (links). At the same time we resolve the question of the ratio of optimal frames and optimal frames with equidistant links. General solution of the optimization problem is obtained in [1].
3. Solving optimization problems

Figure 1 and figure 2 show the individual links of free and non-free frames: figure 1b and figure 2b – links of the first tier, in figure 1a and figure 2a – links of the tier $k$. We denote the unknown stiffness of the struts of the links $x_k$ of the beams $x_{m+k-1}$ and $x_{m+k}$, for the lower and upper bolt, respectively. Let us determine the optimal stiffness of these parts from the condition of the critical state. The condition of critical state is written in the form of equality to zero of the determinants composed of coefficients of
the method of displacement of each part of the frame. For a complete analytical solution of the specified optimization problems, it is necessary to find solutions to two optimization problems for the free frame links and two problems for the non-free frame links. For links of the first tier, we obtain:

Design schemes of multi-storey overpasses are free and/or non-free frame systems, which are characterized by the exhaustion of the bearing capacity in the form of loss of stability. Thus, the problem of optimal design of the transport overpass takes the form of the problem of optimizing the multi-storey frame system from the stability condition. The most important are the closed solutions of optimization problems, allowing to obtain ready-made formulas for the design of individual classes of structures [1-5].

Find min

\[ \gamma_1(v_i) = \frac{24x_i^2}{\lambda_{m+1}^2}, \] (3)

at

\[ r_{i1} = 0, \] (4)

where

\[ r_{i1} = \frac{\xi_i}{g v_i} + 6x_{m+1}, \] (5)

and

\[ r_i = 4\xi_i \phi_2(v_i) + 6x_{m+1}, \] (6)

for free and non-free frame, respectively. Solving the optimization problem by the Lagrange method, we get

\[ \gamma_i(v_i) = \frac{24x_i^2}{\lambda_{m+1}^2}, \] (7)

\[ x_i = \frac{N_i h_i}{V_i}, \] (8)

\[ x_{m+1}^n = -\frac{1}{6} x_i \frac{v_i}{g v_i}, \] (9)

\[ 0(v_i) = \frac{x_i c}{\lambda_{m+1}}, \] (10)

\[ x_{m+1}^m = \frac{4x_i \phi_2(v_i)}{c}, \] (11)

For the 2, 3, ... \( k \) links, the optimization task will be:

Find min

\[ V = t^2 x_k + x_{m+k-1}^2 + \frac{\lambda_{m+k}^2 x_{m+k}^u}{\lambda_{m+k}^2}, \] (12)

at

\[ \det[r_k(x)] = \begin{vmatrix} r_{k-k-1} & r_{k-k} \\ r_{k-k} & r_{k} \end{vmatrix} = 0, \] (13)

where

\[ r_{k-k} = \frac{x_k v_k}{g v_k} + 6x_{m+k}, \] (14)

\[ r_{k-k-1} = -\frac{x_k v_k}{\sin v_k}, \] (15)

or

\[ r_{k-k} = 4\xi_k \phi_2(v_k) + 6x_{m+k}, \] (16)

\[ r_{k-k-1} = 2\xi_k \phi_3(v_k), \] (17)

Constraints of the tasks, according to the method described in [1], are converted to

\[ \psi_{k-1} = \lambda_{m+k-1} \cdot r_{k-1,k-1} - r_{k-1,k} - \lambda_{m+k} = 0, \] (18)
\[ \psi_k = \pm \overline{r}_{m+k-1} \cdot r_{k,k-1} + \overline{r}_{m+k} \cdot r_{k,k} = 0, \]  

moving from determinant to individual equations. The transformed optimization task takes the form:

\[
\begin{align*}
\min V \\
\psi_{k-1} &= 0, \\
\psi_k &= 0.
\end{align*}
\]  

Stationary conditions are:

\[
\begin{align*}
\frac{\partial \varphi}{\partial x_u} &= \lambda + \lambda \cdot = 0, \\
\psi_{k-1} &= 0, \\
\psi_k &= 0, \\
\psi_{n} &= 0, n = k, k - 1.
\end{align*}
\]  

Substituting expressions for \( \psi \) into this system, we finally get:

\[
\begin{align*}
\left( \lambda_k^2 + \lambda_{m+k}^2 \right) \cdot \gamma_1 (v_k) + 2 \overline{r}_{m+k-1} \cdot r_{m+k} \cdot \gamma_2 (v_k) = 24 \overline{r}_1^2, \\
X_k &= N_k h_k / v_k^2, \\
x_{m+k-1}^u &= \left( \frac{-x_k \cdot v_k \cdot \overline{r}_{m+k-1} + x_k \cdot v_k \cdot \overline{r}_{m+k}}{\sin v_k} \right) / (6 \overline{r}_{m+k-1}), \\
x_{m+k}^H &= \left( \frac{-x_k \cdot v_k \cdot \overline{r}_{m+k} + x_k \cdot v_k \cdot \overline{r}_{m+k-1}}{\sin v_k} \right) / (6 \overline{r}_{m+k}),
\end{align*}
\]  

for free frames and

\[
\begin{align*}
\left( \lambda_k^2 + \lambda_{m+k}^2 \right) \cdot \theta_1 (v_k) + 2 \overline{r}_{m+k-1} \cdot r_{m+k} \cdot \theta_2 (v_k) = \overline{r}_1^2, \\
x_k &= N_k h_k / v_k^2, \\
x_{m+k-1}^u &= \left( -4x_k \cdot \phi_2 (v_k) \overline{r}_{m+k-1} + 2x_k \cdot \phi_3 (v_k) \overline{r}_{m+k} \right) / (6 \overline{r}_{m+k-1}), \\
x_{m+k}^H &= \left( -4x_k \cdot \phi_2 (v_k) \overline{r}_{m+k} + 2x_k \cdot \phi_3 (v_k) \overline{r}_{m+k-1} \right) / (6 \overline{r}_{m+k});
\end{align*}
\]  

for parts of non-free frames.

Connect the individual links (figures 1, 2), following the condition:

\[
N_k = \sum P_n, n = k, k + 1, ..., m.
\]  

As a result, we get:

\[
\begin{align*}
\overline{x}_k &= x_k, \\
\overline{r}_{m+k} &= x_{m+k}^H + x_{m+k}^u.
\end{align*}
\]  

4. Conclusions

Thus, composing the frame of the optimal equidistant parts, we obtain a frame equivalent to the optimal undivided frame [1]. And, therefore, the position stated is true that the most rational of the stability condition of the system will be the one where all links (parts) have equal stability. The frame of the minimum mass with equal parts is optimal from the stability condition

For frames with girders of the same design flexibility \( \lambda \) condition of equal stability can be written in the form

\[
Z_k = \text{const}, \quad k = 1, 2, 3, 4, m.
\]  

where \( m \) is the number of floors of the system.
At the same time, the equality between the flexibility of the racks leads to the equality of the optimal parameters of the longitudinal forces

\[ v_i = \text{const}, \ k = 2, 3, 4 \ldots m, \]  

and, consequently, to the equality of stresses in the compressed elements of frames

\[ \sigma_k = \frac{N_k}{F_k} = \frac{\nu^2 E}{k^2 \lambda_k} = \text{const}, \ k = 2, 3 \ldots m. \]  

The analytical condition of equal stability of \( n \) parts of the structural system is the possibility of identical replacement of the General criterion of the critical state by \( n \) criteria and, conversely, if the system is described by \( n \) criteria of the critical state and, at the same time, the General criterion turns into an identity, then this system is a set of \( n \).

As shown above, to ensure equal stability of regular systems, it is sufficient to introduce into the optimization model a condition \( Z_k = \text{const} \).

In case of designing more complex equidistant structures to ensure the identity of solutions of \( n \)-criterion and one-criterion problems, additional determinant conditions are required to ensure the compatibility of the deformation of discrete and continuous models.

References

[1] Shein A I and Zemtsova O G 2018 Analytical Solution of Optimization Problem of Stability of Frame Systems International Multi-Conference on Industrial Engineering and Modern technologies IOP Conf. Series: Materials Science and Engineering 463 04206.

[2] Mukhanov A V 2013 Optimization of building structures on the basis of numerical andanalytical solution of problems of mechanics of heterogeneous bodies (Rostov-na-Donu: Rostov State University of Civil Engineering) p 24

[3] Mironenko I V 2013 Optimization of the structures of reinforced concrete beams and frames bythe method of evolutionary modeling (Bryansk: Bryansk State Engineering and Technology Academy) p 20

[4] Geniev G A 1979 On the principle of equi-gradientness and its application to optimization problems of stability of rod systems Construction mechanics and design of structures 6 8-13.

[5] Tsypinas I K 1968 On the question of the synthesis of optimal systems subject to loss ofstability Lithuanian mechanical collection 2(3) 22-3