Conductivity sum rule, implication for in-plane dynamics and $c$-axis response

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Abstract

Recently observed $c$-axis optical sum rule violations indicate non-Fermi liquid in-plane behavior. For coherent $c$-axis coupling, the observed flat, nearly frequency independent $c$-axis conductivity $\sigma_1(\omega)$ implies a large in-plane scattering rate $\Gamma$ around $(0, \pi)$ and therefore any pseudogap that might form at low frequency in the normal state will be smeared. On the other hand incoherent $c$-axis coupling places no restriction on the value of $\Gamma$ and gives a more consistent picture of the observed sum rule violation which, we find in some cases, can be less than half.

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C-axis electrodynamics is important in distinguishing high-$T_c$ cuprates from conventional superconductors and for understanding mechanism in the cuprates [1]. Since the cuprates have layered structures, interlayer coupling between two adjacent CuO$_2$ planes plays an essential role, and its exact nature impacts on the recent experimental observation [2] of a significant violation of the conventional optical sum rule of Ferrel, Glove and Tinkham [3]. The normalized missing spectral weight (NMSW) is different from one and of order of a half in several high-$T_c$ cuprates. NMSW is the difference between the area under the real part of the conductivity $\sigma^{n(s)}(\omega) = \sigma_1^{n(s)}(\omega) + i\sigma_2^{n(s)}(\omega)$ in the normal minus superconducting state, divided by the superfluid density which is obtained from $\sigma_2^s(\omega)$. We have pointed out in Ref. [4] that in order to explain NMSW of less than one, it is necessary to consider non-Fermi liquid models for the in-plane dynamics, regardless of the nature of the interlayer coupling. A possible such model introduces a pseudogap in the normal state above $T_c$ as is observed [2]. Another is the ”mode” coupling model determined from consideration of ARPES data by Norman et al [5]. This model has been used to describe kinetic, as opposed to potential energy driven superconductivity.

In this paper, we show that coherent c-axis coupling, even with a pseudogap, cannot easily explain recent experimental findings, but that incoherent coupling describes them including the observed low frequency behavior of the effective NMSW.

The interlayer coupling is represented by a Hamiltonian $H_c$ [7–9]:

$$H_c = \sum_{ij, \sigma} \left[ t_{ij} c_{i \sigma}^+ c_{j \sigma} + H.c. \right], \tag{1}$$

where the hopping matrix $t_{ij}$ describes weak interlayer tunneling and $c_{i \sigma}^+$ creates an electron with spin $\uparrow$ at the site $i$ in the plane 1. We classify interlayer couplings in two classes because the results are remarkably different depending on their nature: i) One is coherent coupling ($t_{ij} = t_\perp$), which originates from an overlap of electronic wave functions between the two planes. For an in-plane Fermi liquid, it was shown in Ref. [4] that the superfluid density ($\rho_s$) is equal to the missing spectral weight $(N_n(\omega_c) - N_s(\omega_c) = 8 \int_{0}^{\omega_c} d\omega' \left[ \sigma_{1n}^s(\omega') - \sigma_{1c}^s(\omega') \right])$, where $\omega_c$ is a cutoff frequency of the order of a bandwidth). Thus coherent coupling (unless the
density of states has a strong variation with energy) can explain the experimental results on optimally doped YBCO and over-doped Tl2201 \[10\]. ii) The other is incoherent coupling, for which \( t_{ij} = V_i \delta_{ij} \), where \( V_i \) is an impurity potential. In this case, we showed in Ref. \[3\] that NMSW \( \geq 1.58 \). The characteristic difference between coherent and incoherent coupling is whether or not electron momentum is conserved in the interlayer transfer.

In the presence of an external vector potential \( A_z \), \( H_c \) is modified to \( H_c(A_z) \) by the phase factor \( \exp(\pm i e A_z) \). It is sufficient to expand \( H_c(A_z) \) to second order in \( A_z \). The current \( j_c = -\delta H_c(A_z)/\delta A_z = j_p + j_d \), where \( j_p = -i e d \sum \sigma t_{\perp} [c^{+}_i \sigma c_{i2\sigma} - c^{+}_{i2\sigma} c_{i\sigma}] \) and \( j_d = e^2 d^2 H_c A_z \) with \( d \) the interlayer spacing. In linear response theory, \( \langle j_c \rangle = [-\Pi + e^2 d^2 \langle H_c \rangle] A_z \), where \( \Pi \) is the current-current correlation function associated with \( j_p \) and \( \langle H_c \rangle \) is the perturbation of \( j_d \) due to \( H_c \).

From the \( c \)-axis conductivity sum rule \[2,11\] the superfluid density \( \rho_s \) can be written as

\[
\rho_s = 8 \int_{0^+}^{\omega_k} d\omega [\sigma^0_{1c}(\omega) - \sigma^0_{1c}(\omega)] - 4\pi e^2 d^2 \left[ \langle H_c \rangle^s - \langle H_c \rangle^n \right],
\]

where \( \omega_k \) is the cutoff frequency for the interband transitions that \( H_c \) does not describe, and we use units such that \( \hbar = c = k_B = 1 \) and set the volume of the system to be unity.

The penetration depth \( \lambda_c \) can be calculated in two ways. Based on the Kramers-Kronig relation for the conductivity, we obtain \( \lambda_c \) as

\[
\lambda_c = \frac{1}{4\pi \rho_s} = \lim_{\omega \to 0} [\omega \text{Im} \sigma_c(0, \omega)].
\]

Alternatively, using Eq.\(3\) we can also calculate \( \lambda_c(= 1/\sqrt{\rho_s}) \). Combining these two equations for \( \lambda_c \), we obtain the formula:

\[
\frac{(N_n - N_s)}{\rho_s} = \frac{1}{2} + \frac{1}{2} \sum_{k,p} |t_{k-p}|^2 \frac{G(k,\omega)G(p,\omega) - G_0(k,\omega)G_0(p,\omega)}{\sum_{k,p} |t_{k-p}|^2 F(k,\omega)F^+(p,\omega)},
\]

where \( G(k,\omega) \) and \( F(k,\omega) \) are superconducting Green functions and \( G_0(k,\omega) \) is in the normal state. For coherent \( c \)-axis coupling \( (|t_{k-p}|^2 = t_{\perp}^2 \delta_{k-p}) \) electron momentum parallel to the plane is conserved and \( t_{\perp}^2 \) may depend on the in-plane momentum. For incoherent coupling, an impurity configuration average is implied over a potential \( V_i \) \( (|t_{k-p}|^2 = |V_{k-p}|^2) \), and electron momentum is not conserved, and \( k \) and \( p \) remain unconstraint.

**Coherent coupling.** As the simplest case, one can consider a normal state spectral function \( A_0(k,\omega) = \Gamma/[(\omega - \xi_k)^2 + \Gamma^2] \) based on Fermi-liquid theory. The in-plane scattering rate
Γ is expected to depend strongly on direction of the momentum \( k \) in the two dimensional Brillouin zone. Cold spot exists along \((π, π)\) and hot spot along \((0, π)\) \[6\]. The coherent \( c \)-axis matrix element \( t_⊥ \), however, is itself dependent on direction and \( t_⊥^2 \) varies as \( \cos^4(2φ) \), where \( φ \) is the angle defining the in-plane direction of \( k \). This factor makes the \( c \)-axis conductivity sensitive mainly to the hot spot, although it remains Drude-like. Here we can ignore both \( φ \) dependences and interpret Γ as the scattering rate coming from \((0, π)\) region (hot spot).

However, except for YBCO at optimum doping, \( \sigma_1(ω) \) vs \( ω \) is found to be almost flat over an energy range of a few 100 meV. This implies that \( Γ \) and \( Γ_s \) need to be several 100 meV.

Non-Fermi liquid models with large \( Γ \) have appeared in the literature based on ARPES data in Bi2212. Norman et al. \[6\] have derived a simple phenomenological model - called the mode model in which the normal state is described by a large frequency independent \( Γ \) and the superconducting state by \( Γ(ω) = Γ θ(ω − Δ(0) − ω_{mode}) \), i.e. a Fermi liquid is recovered in this state as \( Γ(ω) = 0 \) for \( ω < Δ(0) + ω_{mode} \), where \( Δ(0) \) is the gap and \( ω_{mode} \) is the frequency of some collective mode. In addition, a pseudogap \( \tilde{Δ} \) can be introduced in the normal state as in the preformed pair model \[12\] with no long range phase coherence above \( T_c \). We take the pseudogap to have the same \( d \)-wave symmetry as does the superconducting gap in agreement with experiment \[3\] but it may not have the same amplitude. A possible phenomenological form of spectral function \( \tilde{A}(k, ω) \) is \( \tilde{A}_+(k, ω) + \tilde{A}_-(k, ω) \), where \( \tilde{A}_±(k, ω) = Γ (1 ± \xi_k/\tilde{E}_k)/[(ω ± \tilde{E}_k)^2 + Γ^2] \). Here \( \tilde{E}_k = \sqrt{\xi_k^2 + \tilde{Δ}_k^2} \) and the factors \((1 ± \xi_k/\tilde{E}_k)\) do not imply the phase coherence of BCS theory.

If the frequency cut-off \( ω_c \) is much larger than any energy scale in our consideration, then under the assumption of a cylindrical Fermi surface, it can be shown that as \( T \to 0 \),

\[-T \sum_{ω,k} G(k, ω)^2 = 1/2 + K (iΔ(0)/Γ_s)/π \]

and

\[T \sum_{ω,k} F(k, ω)^2 = 1/2 - K (iΔ(0)/Γ_s)/π \]

for superconducting state with an in-plane scattering rate \( Γ_s \), which is assumed, for simplicity, to be frequency independent, and

\[-T \sum_{ω,k} G_0(k, ω)^2 = 1/2 + K (i\tilde{Δ}/Γ)/π \]

for normal pseudogap state. Here \( K \) is the complete elliptic integral of the first kind and we can set the density of states \( N(0) \) at the Fermi level equal to be unity because it does not appear in NMSW. The in-plane angle dependence of \( t_⊥^2 \) such as \( \cos^4(2φ) \) does not change the results.
as long as $\omega_c \gg \Delta(0)$ and $\hat{\Delta}$. Therefore, NMSW becomes

$$\frac{(N_n - N_s)}{\rho_s} = \frac{1}{2} + \frac{\text{K} \left( i\hat{\Delta}/\Gamma \right) - \text{K} \left( i\Delta(0)/\Gamma_s \right)}{\pi - 2 \text{K} \left( i\Delta(0)/\Gamma_s \right)}$$.

(4)

$\Gamma_s$ may be different from $\Gamma$ and much smaller as ARPES data imply [5]. Such data are more consistent with the existence of quasiparticles in the superconducting state than in the normal state.

It is worthwhile illustrating the implications of Eq. (4) in some detail. i) Suppose the in-plane scattering rate $\Gamma_s$ is negligible compared with $\Delta(0)$, then Eq. (4) reduces to

$$\frac{(N_n - N_s)}{\rho_s} = \frac{1}{2} + \frac{K \left( i\hat{\Delta}/\Gamma \right)}{\pi}$$.

In this case, the conventional sum rule is recovered when $\hat{\Delta}/\Gamma \ll 1$ because the spectral function $\tilde{A}(k, \omega)$ becomes insensitive to the pseudogap. ii) If $\Delta(0)/\Gamma_s \gg 1$ and $\hat{\Delta}/\Gamma_s \gg 1$, then NMSW $\simeq 1/2$. To get 1/2, it is not necessary that $\Delta(0) \simeq \hat{\Delta}$ and $\Gamma_s \simeq \Gamma$ simultaneously; in other words, a mismatch between $\hat{\Delta}$ and $\Delta(0)$ does not matter as long as $\Gamma$ and $\Gamma_s$ are both negligible. But a small value of $\Gamma$ gives a frequency dependence of $\sigma_n(\omega)$, which disagrees with the observation that it is flat for $\omega$ up to a few 100 meV. Thus, the large $\Gamma$ and $\Gamma_s$ limit must be applied, and consequently any pseudogap region at low $\omega \lesssim \hat{\Delta}$ will be filled in and the model cannot describe the observation made in underdoped YBCO. A cancelation between $\sum_{\omega, k} G(k, \omega)^2$ and $\sum_{\omega, k} G_0(k, \omega)^2$ in Eq. (3) can still arise for large $\Gamma$ if there is a match between $\Delta(0)$ and $\hat{\Delta}$ and between $\Gamma$ and $\Gamma_s$ as can be seen from Fig.1, where we plot $-T \sum_{\omega, k} G_0(k, \omega)^2$ (or $-T \sum_{\omega, k} G(k, \omega)^2$) for three values of the pseudogap (or gap) as a function of $\Gamma$ (or $\Gamma_s$). We also show $T \sum_{\omega, k} F(k, \omega)^2$. Clearly the large $\Gamma$ region can also give a value of the sum rule bigger or smaller than 1/2. Should the pseudogap be larger than the superconducting gap (point $b$ in Fig. 1) and $\Gamma_s$ not too much smaller than $\Gamma$ (point $a$), the sum rule will be less than 1/2 while if $\Gamma_s$ is much less than $\Gamma$ such as for point $d$, it will be larger than 1/2 but less than 1. Other combinations could also be used. To be more explicit, suppose that $\hat{\Delta}/\Gamma = \Delta(0)/\Gamma_s + \gamma$ with $\Delta(0)/\Gamma_s \gg \gamma$, then $\text{K}(i\hat{\Delta}/\Gamma)/\pi - \text{K}(i\Delta(0)/\Gamma_s)/\pi \simeq -(\gamma/4)\Delta(0)/\Gamma_s$. Since $1/2 - \text{K}(i\Delta(0)/\Gamma_s)/\pi \simeq (1/8)(\Delta(0)/\Gamma_s)^2$ for $\Delta(0)/\Gamma_s \ll 1$, NMSW $\simeq 1/2 - \gamma \Gamma_s/\Delta(0)$ for $\gamma \ll \Delta(0)/\Gamma_s \ll 1$. Based on ARPES data, we
may assume that $\tilde{\Delta} \simeq \Delta(0)$ and $\Gamma \gg \Gamma_s \to 0$. In this case NMSW $\simeq 1/2 + (a - d)/(2c)$, which is obviously greater than 1/2, but small $\Gamma_s$ will not give the observed flat response $\sigma_c^n(\omega)$ in the superconducting state.

So far we have assumed that both superconducting and normal state are at $T \to 0$. Because it is not possible to access the normal state at low $T$, Basov et al. have used $T = T_c$ instead. In the inset of Fig. 1, we plot $-T\sum_{\omega,k} G_0^2$ with $\tilde{\Delta} = 4T_c$ and $\Gamma = T_c$ as a function of $T$. (We emphasize that $-T\sum_{\omega,k} G_0^2$ is nothing but NMSW if $\Gamma_s \ll \Delta(0)$.) It is clear that including the $T$ dependence in the NMWS does not qualitatively change the physics of the conductivity sum rule.

**Incoherent coupling.** For incoherent $c$-axis coupling, we need a specific model for the impurity scattering potential $|V_{k-p}|^2$ and need to average over impurity configuration. Part of the disorder scattering can be due to a mismatch of overlap matrix elements $t_\perp$ between planes. We use a simple model for the scattering potential $|V_{k-p}|^2 = |V_0|^2 + |V_1|^2 \cos(2\phi_k) \cos(2\phi_p)$ \[8\] and assume $|V_0|^2 = |V_1|^2$ for simplicity. The NMSW is now

$$\frac{(N_n - N_s)}{\rho_s} = \frac{1}{2} + \frac{1}{2} \sum_\omega \frac{[\tilde{\kappa}'^2 K(\tilde{\kappa}) - \kappa'^2 K(\kappa)]}{[\kappa'^2/\kappa] K(\kappa) - E(\kappa)/\kappa]^2}, \tag{5}$$

where $\tilde{\kappa} = \tilde{\Delta}/\sqrt{\Delta^2 + (\omega + \Gamma_s \text{sgn}\omega)^2}$, $\tilde{\kappa}' = \sqrt{1 - \tilde{\kappa}^2}$, $\kappa = \Delta(T)/\sqrt{\Delta(T)^2 + (\omega + \Gamma_s \text{sgn}\omega)^2}$, $\kappa' = \sqrt{1 - \kappa^2}$, and $E$ is the complete elliptic integral of the second kind. As one can easily see, $\tilde{\kappa}'^2 K(\tilde{\kappa})$ becomes $(\pi/2)^2$ if $\tilde{\Delta} = 0$ regardless of $\Gamma$. It can also be inferred from Eq. (5) that i) as $\tilde{\Delta}$ increases or $\Gamma$ decreases, NMSW becomes smaller because $\tilde{\kappa}$ is closer to 1 for a given $\omega$, and ii) as $\Gamma_s$ increases NMSW becomes smaller because $\kappa$ is closer to 0. In the incoherent coupling case, the normal state response $\sigma_c^n \propto |V_0|^2$ is independent of $\omega$ and there is no restriction on $\Gamma$ \[4,8\]. In the inset of Fig. 2, we plot NMSW as a function of $\Gamma_s$ with fixed values of $\alpha = 0$ and 1, where $\alpha \equiv \tilde{\Delta}/\Delta(0)$, and $\Gamma = \Delta(0)$. As expected, NMSW with $\alpha = 1$ decreases with increasing $\Gamma_s$. Note that NMSW with $\alpha = 0$ is almost unchanged with increasing $\Gamma_s$ because the numerator and the denominator of the second term in Eq. (5) are both decreasing in similar manner. In the case $\tilde{\Delta} = \Delta(0)(1 + \delta)$ \[\delta \ll 1\] with $\Gamma = \Gamma_s = 0$, Eq. (5) reduces to NMSW $\simeq 1/2 - 1.083\delta$, which is less than 1/2. For $\Gamma = \Gamma_s \neq 0$, the
coefficient of $\delta$ will be changed. In the general case we need to evaluate the sums in Eq. 3 for given $\tilde{\Delta}$, $\Delta(0)$, $\Gamma$ and $\Gamma_s$. In the main frame of Fig. 2, we plot NMSW as a function of $\Gamma$ with a few values of $\alpha$ assuming $\Gamma_s = 0$ since we have seen that $\Gamma_s \neq 0$ decreases NMSW. As one sees, NMSW is greater than 1 and independent of $\Gamma$ if $\alpha = 0$. Note that NMSW with $\alpha = 0$ is an asymptote of NMSW with nonzero $\alpha$ because the effect of $\tilde{\Delta}$ on NMSW vanishes when $\Gamma \gg \tilde{\Delta}$. As we expect, NMSW decreases with increasing $\alpha$ and decreasing $\Gamma$. If $\alpha \geq 1.46$ and $\Gamma \ll \Delta(0)$, then NMSW < 0. Generally speaking, in the incoherent coupling model, we can explain NMSW $\approx 1/2$ with $\tilde{\Delta} \approx \Delta(0)$, $\Gamma > \Delta(0)$ and $\Gamma_s < \Delta(0)$.

Additional insight into the sum rule can be obtained from consideration of $\sigma_{1c}^{n,s}(\omega)$. In the top panel of Fig. 3, we compare $\sigma_{1c}^{n}(\omega)$ (solid curve), with pseudogap $\tilde{\Delta} = \Delta(0)$, with $\sigma_{1c}^{s}(\omega)$ (dashed curve). In both cases, $\Gamma$ is taken to be zero and therefore both curves go to zero at $\omega = 0$. At small $\omega \lesssim \Delta(0)$ the solid and dashed curves are almost the same. However, they start to differ at a higher $\omega$, with $\sigma_{1c}^{s}(\omega)$ falling below $\sigma_{1c}^{n}(\omega)$. We already know that for these parameter the sum rule is exactly 1/2. But in this example, the effective NMSW defined as $\mathcal{N}(\omega) = [N_n(\omega) - N_s(\omega)]/\rho_s$ is nearly zero up to $\omega \gtrsim \Delta(0)$ as shown in the dashed curve in the bottom panel of Fig. 3. This does not agree with the findings of Basov et al., where $\mathcal{N}(\omega)$ increases almost linearly out of zero and rapidly move toward its saturated value. It is clear that, even for the incoherent $c$-axis coupling case, the argument presented in Ref. [13] that the preformed pair model agrees with a sum rule of 1/2 due to a simple cancelation between the normal and superconducting state is deficient when its detailed approach toward its saturated value is considered, i.e. when we compare the low $\omega$ behavior of $\mathcal{N}(\omega)$ with experiment.

A more reasonable model is obtained for finite normal state $\Gamma$. Returning to the solid curve of the top panel and including a finite $\Gamma$ in the pseudogap state fills in the region below $2\tilde{\Delta}$ by transferring spectral weight out of the region above this. In the middle panel we show results (solid curve) for a pseudogap $\tilde{\Delta} = 1.4\Delta(0)$ and $\Gamma = 0.5\Delta(0)$ for illustration only. Other choices could also have been made showing that our explanations are robust. Comparison of the solid curve (normal state) with the superconducting (dashed) curve ($\Gamma_s =$
0) shows a very different low $\omega$ behavior. Now $\mathcal{N}(\omega)$ will grow rapidly as $\omega$ increases from $\omega = 0$. This behavior is shown as the solid curve of the bottom panel. $\mathcal{N}(\omega)$ grows linearly out of $\omega = 0$ and nearly reaches its saturated value ($\simeq 0.52$) by $\omega \approx 6 \sim 7\Delta(0)$ in much better agreement with experiment. One can also conceive that $\tilde{\Delta}$ and $\Gamma$ are larger than considered above and that the normal state may not exhibit much of a pseudogap because of a large $\Gamma$, but that the sum rule is still nevertheless less than 1. This could explain optimally doped Tl2201 with NMSW $\approx 0.6$ where, however, no in-plane pseudogap has been reported although in the data of Basov et al., $\sigma_{1c}^n(\omega)$ is dropping with decreasing $\omega$ at small $\omega$ consistent with a smeared pseudogap.

Conclusion. We conclude that incoherent $c$-axis coupling can more easily explain the observed violations of the optical sum rule than can coherent coupling, but that non-Fermi liquid in-plane behavior is required. It is possible that the normalized missing spectral weight is less than $1/2$. A sharp increase in accumulated normalized spectral weight as a function of cut-off frequency rising to a value close to its saturated value within a few times the gap as is observed can also be understood within our phenomenological model but not within a pure preformed pair model [13].

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FIGURES

FIG. 1. The quantity $-T \sum_{\omega,k} G^2_0$ (or $-T \sum_{\omega,k} G^2$) as a function of $\Gamma/T_c$ (or $\Gamma_s/T_c$) for different values of $\tilde{\Delta}$ and $\Delta(0)$: $\tilde{\Delta} = T_c$ (dashed curve), $\tilde{\Delta} = \Delta(0) = 2.5T_c$ (solid curve) and $\tilde{\Delta} = 4T_c$ (dotted curve). The solid curve is $T \sum_{\omega,k} F^2$. The inset shows the $T$ dependence of $-T \sum_{\omega,k} G^2_0$ with $\tilde{\Delta} = 4T_c$ and $\Gamma = T_c$.

FIG. 2. NMSW as a function of $\Gamma/\Delta(0)$ for different values of $\alpha \equiv \tilde{\Delta}/\Delta(0)$ with $\Gamma_s = 0$ and $T \ll \Delta(0)$. The inset gives NMSW as a function of $\Gamma_s/\Delta(0)$ for two specific cases: $\alpha = 0$ with any $\Gamma$ (dashed curve) and $\alpha = 1$ with $\Gamma = \Delta(0)$ (solid curve).

FIG. 3. Top panel shows $\sigma^{n}_{IC}(\omega)/\sigma_{cn}$ with $\tilde{\Delta} = \Delta(0)$ and $\Gamma = 0$ (solid curve), where $\sigma_{cn} = 4\pi n_i (edN(0)V_0)^2$ with an impurity density $n_i$. The dashed curve ($\sigma^{s}_{IC}(\omega)/\sigma_{cn}$) is for the superconducting state with $\Delta(0)$ and $\Gamma_s = 0$. Middle panel gives $\sigma^{n}_{IC}(\omega)/\sigma_{cn}$ but now with $\tilde{\Delta} = 1.4\Delta(0)$ and $\Gamma = 0.5\Delta(0)$. Note the transfer of spectral weight from higher to lower frequency as compared to case when $\Gamma = 0$ (solid curve in the top panel). Bottom panel shows the effective NMSW $\mathcal{N}(\omega)$. The dashed curve is for the preform pair model with $\Gamma = \Gamma_s = 0$ and $\tilde{\Delta} = \Delta(0)$. Its saturated value of $1/2$ is not reached at $\omega = 6\Delta(0)$ where it is still only about 0.3. The solid curve includes smearing into the normal state with $\tilde{\Delta} = 1.4\Delta(0)$ and $\Gamma = 0.5\Delta(0)$. Its saturated value is about 0.52 which is nearly reached by $\omega = 6\Delta(0)$. 
Fig. 1 (Kim & Carbotte)
Fig. 2 (Kim & Carbotte)

\[ \frac{\Gamma_s}{\Delta(0)} \]

\[ \alpha=0 \]

\[ \alpha=1 \]

\[ \alpha=1.46 \]
Fig. 3 (Kim & Carbotte)