High gradient pulsed quadrupoles for novel accelerators and space charge limited beam transport

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Abstract. Novel acceleration schemes like plasma wake-field based accelerators demand for high gradient focusing elements to match the Twiss parameters in the plasma to the transport lattice of the conventional accelerator beamlines, with typically much higher beta-functions. There are multiple candidates for achieving high gradient focusing fields, each one having certain drawbacks. Permanent magnets are limited in tunability, plasma lenses might degrade the transverse beam quality significantly and conventional magnets cannot reach very high gradients and often cannot be placed in direct proximity of the plasma accelerator because of their size. In this paper we present design considerations and simulations on compact, high gradient, pulsed quadrupoles, that could be used e.g. for final focusing of space charge dominated bunches into a LWFA (Laser Wake-Field Accelerator) at SINBAD or other facilities with similar demands. The target design gradient is 200 T/m at a physical aperture on the order of 10 mm.

1. Introduction
Free electron lasers (FELs) and high energy applications demand for electron accelerators with high peak brightness. To supply such beams novel acceleration techniques are being investigated, two of the most promising being beam- and laser-driven plasma based wakefield acceleration \cite{1,2}. In contrast to other wakefield acceleration schemes plasma wakefields exhibit strong transverse (focusing/defocusing) forces, which can reach MT/m focusing gradients \cite{3}, which implies very small beta-functions of the bunches driving and witnessing the plasma wakefields. To avoid emittance growth due to beta-function mismatch \cite{4,5} a matched, i.e. slow transition from the extremely high transverse fields in the plasma of up to kT/m to conventional beam optics in the accelerator beamline before and/or after the plasma accelerator is needed. In modern plasma wakefield acceleration experiments this is pursued by using strong focusing magnets \cite{6,7}, often in combination with tapered plasma density up- and downramps \cite{8,9,10}, which ease the demands on the magnets.

Permanent magnet quadrupoles are being considered for this purpose due to their high field gradient capabilities \cite{11} and their compactness. The latter is especially important for beams which are affected by high space charge forces, that can degrade the beam quality in long drifts \cite{12}. Nevertheless, these quadrupoles are inherently lacking tunability of the magnetic field gradient, which is a drawback especially in experimental setups. Active plasma lenses \cite{13}, which can also achieve the needed focusing gradients, are still facing severe beam quality degradation due to nonlinearities in the focusing fields \cite{14,15}.
We therefore propose an air-coil, pulsed, normal-conducting quadrupole to be used in the matching section at entry and exit of plasma-based accelerators. A design gradient of 200 T/m at a magnet length of 20 mm is pursued to offer an alternative to permanent magnet based quadrupoles [6]. Design considerations on the magnet, a layout of the pulsed power supply and numerical studies of the field quality are presented.

2. Magnet design

The magnet design is based on 4 straight conductors with a \( \cos(2\theta) \)-shaped cross-section (as shown in Fig. 1) and a length of 20 mm, which are connected in series at the ends via 3 cranks, forming an air-coil [16]. A high voltage pulse is applied to the coil to drive a high current with \( \mu \)s duration through the conductor. To counteract the expulsion of the current from the center of the conductor via the skin-effect, laminated, drilled litz-wires are stacked to form the \( \cos(2\theta) \) conductor shape. To define conductor thickness (Fig. 1, \( d \)) and inner conductor radius (Fig. 1, \( R \)), which determine the needed current density and maximum beamline aperture, the current, that is needed to achieve the design gradient of 200 T/m at varying radius, is simulated in CST Microwave Studio. A simplified model consisting of four infinitely long \( \cos(2\theta) \) conductors is used for these simulations. As Fig. 2 shows, the necessary peak current can be reduced nearly linearly with the inner conductor radius.

At constant inner radius, the current and current density, necessary to achieve 200 T/m at varying conductor thickness \( d \), are calculated and a linear dependency is found for the total current, as shown in Fig. 3.

As discussed in the next section, the current density is the main figure of merit for pulse

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**Figure 1.** Half section view of the pulsed quadrupole. The four \( \cos(2\theta) \)-shaped straight sections are parallel to the beam direction. Current input/output (bottom left) and one of the three connection cranks between straight conductor sections (top right) can be seen.
Figure 2. Peak pulse current for quadrupole gradient of 200 T/m for varying quadrupole conductor aperture radii at 2.5 mm conductor thickness.

Figure 3. Peak pulse current and current density in the conductor for varying conductor thicknesses at 8 mm conductor aperture radius.
heating, while the absolute peak current defines the demand on the pulsed power circuit. Thus a trade-off between peak current and current density is sought. Also, mechanical tolerances and constraints for the production of the quadrupole are considered and a conductor thickness of 3 mm at an inner conductor radius of 8 mm is chosen, which results in a peak current of 28 kA for the 200 T/m quadrupole prototype. Further optimisation with respect to necessary peak current and field quality might be possible after assessing the field shape and mechanical handling.

3. Pulsed power considerations
The characteristics of the pulse current are mainly determined by the necessary maximum value of 28 kA for 200 T/m and the heating of the magnet during a current pulse. No active cooling is foreseen in the magnet design, which means that passive heat transfer out of the magnet and passive cooling have to compensate the pulse heating. Considering only the heat transfer through the (copper) conductor, an equilibrium between pulse heating and heat transfer/cooling would be achieved for

$$\rho \cdot l \cdot j^2 \approx \lambda \cdot \frac{\Delta T}{l}$$

where $\rho$ is the resistivity of the copper conductor, $l$ is the conductor length inside of the magnet, $j$ the (average) current density, $\lambda$ the thermal conductivity and $\Delta T$ the temperature difference between the quadrupole conductor and the outside connections, which are passively cooled. It is obvious that only the current density defines the heat load on the coil. The peak current defines the requirements on the electrical pulse circuit, as the maximum current has to be conducted by the circuits power switch and (together with the pulse length) it defines the current rise rate requirements of the switch and the pulse capacitors. Therefore a trade-off between current density and total peak current has to be found for the coil conductor layout. By e.g. setting a limit on the temperature rise in the conductor to less than 50 degree, corresponding to a power loss in the conductor of ca. 10 W and assuming the parameters shown above at a pulse repetition rate of 10 Hz, the maximum pulse length would be 25 $\mu$s (i.e. a resonance frequency of capacitor and quadrupole inductance of 20 kHz). A reduced pulse length is even more favourable for further reduction of the heat load on all components in the pulse circuit.

To limit the total power consumed by the circuit in continuous operation, recirculation of the current pulses is pursued, as described in [17]. Power reduction of up to 80 % is predicted for this, which significantly lowers the demands on the high voltage supply at the expense of two extra diodes, a parallel dummy inductance and an additional power switch. The requirements on this additional switch are nevertheless lower than on the main switch by correspondingly dimensioning the parallel inductance w.r.t. the inductance of the quadrupole coil.

4. Field quality
To evaluate the magnetic field quality a full model of the conductor, including the $\cos(2\theta)$-shaped straight parts, the cranks (e.g. Fig. 1, top right) and current connectors (Fig. 1, bottom left) was simulated in CST Microwave Studio. For a precise analysis the field gradients, transverse and longitudinal field quality, especially in the range of the good field region (GFR), and the effective length of the magnet are calculated.

Within the aperture radius Fig. 4 shows a steady gradient of approx. 180 T/m. This is about 10 % lower than the required 200 T/m. The discrepancy is attributed to loss of magnetic flux at the open ends of the conductors in the full quadrupole model. To achieve the target gradient a magnetic shielding around the central conductor straight section is foreseen. Decreasing the aperture, increasing the current or choosing a thinner conductor might be considered to increase the gradient further if necessary.

To meet the desired requirements on the field quality, the field error $\Delta G / G$ in the GFR should
Figure 4. Magnetic field gradient in transverse plane in the z-center of the straight conductor section. Pixels showing severely reduced gradient in the periphery of the aperture are caused by numerical artifacts at the conductor surfaces.

not exceed a value of $10^{-2}$ for a linear accelerator. Figure 5 shows the field error along the x-axis at the center of the straight conductor section. The $10^{-2}$ reference threshold is shown in red. Field errors are significantly lower than this value in the inner 8 mm of the aperture, which is sufficient for an application in novel accelerator experiments. The effective length of the quadrupole is calculated to be 31 mm, considerably longer than the straight conductor section length of 20 mm.

5. Conclusion
In summary we have shown scalings and field quality simulations as well as considerations on the pulsed power electronics layout for a high gradient, compact, pulsed $\cos(2\theta)$ air coil quadrupole, which is proposed as a tunable lens for applications in novel accelerator schemes and other space charge dominated beam accelerators. Field errors are shown to be significantly less than the targeted maximum allowed error in a linear accelerators of $10^{-2}$ within a good field region of 8 mm in diameter. The 10 % difference to the target gradient of 200 T/m is planned to be addressed by a magnetic shielding around the central part of the magnets conductors. The effective quadrupole length of 31 mm was found to be significantly longer than the 20 mm length of the straight conductor sections. Mechanical design of the magnet is being prepared for a final assessment of the magnetic field parameters.
Figure 5. Field error along x-axis at the center of the magnet (blue) and target field quality (red).

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