New data on $0^+$ states in $^{158}$Gd

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Excited states in the deformed nucleus $^{158}$Gd have been studied in the \((p,t)\) reaction by using the Munich Tandem and Q3D spectrograph. 30 new excited $0^+$ states (three tentative) have been assigned up to the 4.3 MeV excitation energy. The total number of 34 excited $0^+$ states (four tentatively assigned) in a deformed nucleus, close to a complete level scheme, offers a new opportunity to test nuclear models and obtain more information on the structure of these special states.

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I. INTRODUCTION

Excited $0^+$ states in nuclei are specific modes of nuclear excitations which were intensively studied, especially in the last few decades. They have a different structure associated, e.g., with pair vibrations, beta vibrations, vibrations caused by spin-quadrupole forces, one- and two-phonon states, and so on. These states occupy a special place in nuclear physics. Theoreticians point out that many difficulties of the nuclear theory are concentrated just on excited $0^+$ states \[1\]. This conclusion was achieved already when spectra of only a few additional $0^+$ states at low energies above the $\beta$ vibrational state in deformed nuclei were known: the interacting boson model (IBM) \[2\] and the quasiparticle phonon model (QPM) \[3\] have been used extensively for comparison with experimental data. Theories met difficulties already for consideration of the properties of the first excited states, for example, at explaining the strong excitation of the first excited $0^+$ states in actinide nuclei. They represent the collective excitations different in character from the $\beta$ vibrations. Importance of the monopole and the quadrupole pairing field was realized \[4\] while trying to explain this observation. Garrett \[5\] reviewed the properties of the first excited $0^+$ states in deformed nuclei and showed that only in a few nuclei the states, considered as $\beta$ vibrational, met the original definition \[6\]. In all other nuclei they have a more complex structure.

The multiple $0^+$ states in deformed nuclei are so far a new challenge for all theoretical models. After the first observation of multiple $0^+$ states in $^{158}$Gd \[9\], the attempts were concentrated to understand the nature of these states in deformed nuclei in general, and, in particular, the observed 13 excited $0^+$ states in $^{158}$Gd. Simple calculations were performed by using the projected shell model \[10\] and geometric collective model \[11\]. The most popular approaches were applied in the framework of the IBM \[11\] and the QPM \[12\]. Some other theoretical approaches have been used to describe $0^+$ states in different nuclei, for example, within a model based on the Hamiltonian including the monopole pairing, quadrupole-quadrupole, and spin-quadrupole interactions, all diagonalized in the random phase approximation \[13\]. In fact, the nature of the $0^+$ states in these approaches are different. For example, the role of octupole components in the formation of $0^+$ states in actinides is radically different in the IBM and QPM \[14\]–\[16\]. Nevertheless, the models describe some properties of the energy spectra of $0^+$ states and the excitation cross sections. In comprehensive theoretical efforts to understand the nature of these $0^+$ excitations, an extensive mapping of excited $0^+$ states, information on the evolution of the abundance of $0^+$ states in the entire region of deformed nuclei and on the dependence of the abundance of $0^+$ states in the excitation energy spectrum are important.

Excited $0^+$ states are usually identified via \((p,t)\) reactions even in the compound and dense excitation spectra: they have a very distinct angular distribution. Early studies, for example in Ref. \[17\], were limited relative to the low excitation energies, and a limited number of excited $0^+$ states were observed. Intensive studies of the multiple $0^+$ states were triggered by the observation of 12 excitations with zero angular-momentum transfer via the \((p,t)\) reaction in the odd nucleus $^{229}$Pa \[18\] and 13 excitations in the even-even nucleus $^{158}$Gd \[9\]. Then, many experiments were carried out through the \((p,t)\) transfer campaign in the region of actinides \[19\]–\[24\] and rare earths \[25\]–\[29\]. A feature of some of these studies is that, simultaneously with $0^+$ states, many states with other spins of both parities have also been identified. The total spectra of $0^+$ states and, presumably, the total spectra of $2^+$ and $4^+$ states of a collective nature were accumulated. In deformed nuclei an excitation mode with angular momentum $J^\pi$ splits into states distinguished by its projection $K$ quantum number, which ranges from zero to $J$.

In addition to the states with structure $J^\pi K^\pi = 0^+ 0^+$ the states with a more general structure $J^\pi K^\pi = J^\pi 0^+$ are expected. States related to specific two-quasiparticle
modes are expected above twice the pairing gap energy. In addition, there are two-phonon excitation of collective modes, the quadrupole, octupole and hexadecapole phonons. The collective monopole pairing vibration have also to be considered. All these configurations will mix to some extent. Thus, a significant number of $0^+$ states with different structures should be observed in deformed nuclei.

So far, almost all the studies of the $0^+$ states have been performed for an excitation energy below 3 MeV. One attempt to expand this range was undertaken for $^{230}$Th [20]. In the region up to 4.5 MeV, the (p,t) spectra for two angles, 12.5° and 26° were measured. The ratio of cross sections for 8 states was corresponding to the ratio for $0^+$ states. However, this ratio does not exclude the identification of $3^+$ and $6^+$ states. The answer to the assumption that the spectrum of $0^+$ states is terminated in energy is still open. The angular distributions up to approximately 4 MeV were also measured for $^{165}$Er [20]. In the region from 3 to 4 MeV a steeply rising cross section was observed at small reaction angles for 9 excitation energies. However, a sharp minimum at about 17.5°, which is also a distinguishing feature of $0^+$ excitations, was absent in these angular distributions. Since such angular distributions correspond also to $2^+$ excitations in the DWBA calculations with taking into account an indirect transfer in the (p,t) reaction [22], these $0^+$ states can be assigned only tentatively.

Our initial aim was to carry out the $^{160}$Gd(p,t)$^{158}$Gd experiment for observation of the $0^+$ excitations in the region from 3 to 4.2 MeV, in addition to the already observed $0^+$ states below 3 MeV by Lesher et al. [8]. However, some of the circumstances discussed below led to the need to perform the experiment also for lower energies. Thus, we identified 230 states with different spins. The purpose of this paper is to present the results for $0^+$ states: we report the existence of 34 $0^+$ states in one nucleus below the excitation energy of 4.3 MeV. For four of them, including the 1952.4 keV state, the assignment is tentative. This number is the largest observed in any nucleus and provides a unique opportunity for testing new models on the nature of $0^+$ excitations in nuclei. Results on new $2^+$, $4^+$, and $6^+$ states will be presented in a forthcoming work.

II. EXPERIMENT DETAILS AND RESULTS

The first experiment in the high energy region has been performed at the Tandem accelerator of the Maier-Leibnitz-Laboratory of the Ludwig-Maximilians-University and Technical University of Munich using a 22 MeV proton beam on a $110 \mu g/cm^2$ target of isotopically enriched $^{160}$Gd (98.10%) with a 14 $\mu g/cm^2$ carbon backing. Known impurities in the target material consist of $^{158}$Gd (0.99%), $^{156}$Gd (0.33%), and $^{157}$Gd (0.44%). A 1.4 m long focal plane detector provides the particle identification of the ejectiles of masses 1 - 4 in the high-precision Q3D spectrometer. The resulting triton spectra have a resolution of 4 - 7 keV (FWHM) and are background-free. The acceptance of the spectograph was 14.43 msr for all angles, except for the most forward angle 5°, where it was 7.50 msr. Typical beam currents was about 1.0 $\mu$A.

The angular distributions of the cross sections were obtained from the triton spectra at eight laboratory angles from 5° to 40°. The low energy spectra in the interval from 0 to 3.4 MeV have been also measured at angle 5° for three magnetic setting, which are all overlapping with the neighboring regions.

For the calibration of the energy scale, the triton spectra from the reactions $^{154}$Gd(p,t)$^{152}$Gd have been measured at the same magnetic setting. The high energy spectrum of $^{158}$Gd was calibrated with the known energies from $^{152}$Gd, while the lower energy part was calibrated, at first, using the energies of the $0^+$ states assigned by Lesher et al. [9]. When the high-energy spectrum was shifted to the overlapping region with the low-energy spectrum it occurs to be impossible to combine the high and low energy spectra. The energy scales were different, and a necessary shift was found considerable different from the Q-value obtained using the Atomic Mass Tables [30]. To solve this problem, a second experiment was performed in the low-energy region on the 125 $\mu g/cm^2$ target of $^{160}$Gd. The acceptance of the spectograph was 9.16 msr for 6° and 15.94 msr for other angles. The resulting triton spectra have a slightly lower resolution of 8 - 9 keV (FWHM). For the calibration of the energy scale, the triton spectra from the reactions $^{172}$Yb(p,t)$^{170}$Yb are measured at the same magnetic settings. The well-known levels of $^{158}$Gd have been also used for calibration in this energy interval. As shown from Fig. 1 the spectra in both energy intervals calibrated by the reactions $^{154}$Gd(p,t)$^{152}$Gd and $^{172}$Yb(p,t)$^{170}$Yb coincide perfectly in the overlapping region, which is an evidence for the accuracy of the calibration. Fig. 2(a-c) shows the triton spectrum over the whole measured energy interval from 1.0 to 4.3 MeV, taken at the de-
FIG. 2. The triton spectrum from the $^{160}\text{Gd}(p,t)^{158}\text{Gd}$ reaction measured at angle 5°. The states assigned as $0^+$ states are labeled by their energies. Arrows indicate the states for which $0^+$ assignments in Ref. [9] are not confirmed in this study.

TABLE I.

Reaction Q-values obtained from the energy shifts of the $^{152}\text{Gd}$ and $^{170}\text{Yb}$ spectra, relative to the $^{158}\text{Gd}$ spectrum in the calibration procedure are compared with the Q-values calculated from the mass excesses. All data are given in keV.

|        | $^{152}\text{Gd}$ | $^{158}\text{Gd}$ | $^{170}\text{Yb}$ |
|--------|-------------------|-------------------|-------------------|
| $\triangle E$ | 1749.0 6         | 1238.1 6         |                   |
| $Q[\triangle E]$ | 6671.5 9        | 4912.9 7         | 6161.0 9         |
| $Q[\text{AME}]$ | 6659.9 3        | 4912.9 7         | 6152.3 6         |
| $\triangle Q$ | 11.6 10          |                   | 8.7 9             |

In the course of analysis of the measured spectra we found that the Q values for the (p,t) reactions on the $^{160}\text{Gd}$, $^{154}\text{Gd}$ and $^{172}\text{Yb}$ targets are in disagreement with the ones calculated using the Atomic Mass Tables [30]. They are given in Table I. The reaction Q-value for the $^{160}\text{Gd}$ target is used as the reference value. This means that for this nucleus the Q value which was determined from the data in the AME2016 was taken as a starting point in the calculations for other nuclei. As seen from the table, the Q value for the $^{154}\text{Gd}$ and $^{172}\text{Yb}$ targets, which is determined by the energy shift necessary to fit the peaks in the overlapping region, differs from the value obtained by using the atomic mass excesses. Since the differences for $^{154}\text{Gd}$ and $^{172}\text{Yb}$ targets are positive and close in values, the inaccuracy in the mass excess refers most likely to $^{160}\text{Gd}$ and/or $^{158}\text{Gd}$.

The analysis of triton spectra was performed using the program GASPAN [31]. The peaks in the spectra which are measured at 5° degree have been identified for 230 levels, though the peaks for all eight angles were identified only for 162 levels. The resulting angular distributions are shown for $0^+$ states in Fig. 2. Efficiency corrections for the target thickness at different angles have been taken into account.

The observed angular distributions are compared with calculations using the distorted wave Born approximation (DWBA). The coupled-channel approximation (CHUCK3 code of Kunz [32]) and the optical potential parameters suggested by Becchetti and Greenlees [33] for protons and by Flynn et al. [34] for tritons have been used in the calculations. Angular distributions of the $0^+$
FIG. 3. Angular distributions of assigned $0^+$ states in $^{158}$Gd and their fit with the CHUCK3 one-step calculations. The transfer configurations used in the calculations for the best fit are shown for every state (see text for details).
states are reproduced very well by a one-step process, which simplifies the calculations. The orbitals close to the Fermi surface have been used as the transfer configurations. For $^{158}\text{Gd}$ and $^{160}\text{Gd}$ such configurations include the orbitals, which correspond to those in the spherical potential, namely, $1h_{9/2}$, $2f_{5/2}$, $1i_{13/2}$, and $1h_{11/2}$. The DWBA angular distributions depend to some extent on the transferred configurations. The most noticeable difference is obtained for the angular distribution at the $(1i_{13/2})^2$ transfer configuration. For other configurations the difference consists in a different height of the maximum at about 20° and minor displacements of minimum. In addition, since the excited $0^+$ state must consist of many terms in the wave function with a coherent summing of individual amplitudes, this difference allows to obtain a better fit to the experimental angular distributions using mixed configurations. They are shown in Fig. 3. Only two transfer components are shown: the first one is the main constituent, while the second one improves the fit to the peak at 20° and to the minimum at about 15 - 18°, its admixture does not exceed 10%.

In Fig. 3 the experimental data are given in $\mu$b/sr and their values are plotted with the error bars while the Q-corrected CHUCK3 calculations are shown with full lines. The solid (red) lines present the firm assignments and the dashed (blue) lines show tentative assignments. The results of this study concerning $0^+$ states as compared with previous studies are collected in Table I. The high-precision study of the ($p$,t) reaction on $^{160}\text{Gd}$ was performed by Lesher et al. [4]. They confirmed the definite assignment of three excited $0^+$ states at 1196.1, 1452.3, and 1743.1 keV as well as four tentative $0^+$ assignments for 1952.4, 1957.4, 1972.2, and 2688.8 keV from the ($n$,γ) reaction [35]. Additionally, they found seven new $0^+$ states at 1577, 2277, 2338, 2643, 2911, 3077, and 3110 keV. The $0^+$ assignment at 1577 and 2277 keV was strengthened by the analysis of gamma rays from the ($n$,γ) reaction [35]. However, later in the next paper by Lesher et al. [36] for study of $0^+$ states in the ($n$,n'γ) reaction, no γ-rays were detected that decay level the 1577 keV. Therefore, the corresponding peak in the observed triton spectrum is interpreted as an existing excitation through the $^{156}\text{Gd}(p,t)^{154}\text{Gd}$ reaction on the $^{156}\text{Gd}$ impurity in the target [34]. The observed cross section 3.8 $\mu$b/sr is only slightly larger than the calculated 2.7 $\mu$b/sr under the assumption of identical cross sections for the ground state excitations in $^{158}\text{Gd}$ and $^{156}\text{Gd}$ nuclei.

For the states below 1743.2 keV, only the absolute cross section is shown in Table I as the result of our analysis, since their angular distributions were not measured. The angular distributions confirm the $0^+$ assignment for the state 1743.2 keV and, for the first time, for the 1957.3 keV state, although with a very small cross section in the latter case. The strong 1953.5 keV peak was attributed by Lesher et al. [4] to the excitation of the 1952.4 keV state and the 1960.1 keV peak to the state 1957.4 keV known from the previous publications, e.g. Ref. [35]. According to our study with correct calibration, the strong peak is observed at the 1957.3 keV and it should be attributed to the excitation of the known state 1954 keV. Lesher et al. in another publication [36] using an analysis of the γ-rays from the ($n$,n'γ) reaction confirm their private communication with Bucurescu and Meyer and concluded that the putative 1953.5 keV level actually has an energy of 1950.0 keV. Moreover, Bucurescu and Meyer find no evidence of the suggested 1960.1 keV level. In fact, when the energy of a strong peak is shifted in Ref. [36], the same shift should be applied also for this weak peak, giving an energy of about 1964 keV. Our calibration procedure produces an energy of 1966.5 keV for this peak. Nevertheless an additional test confirms the conclusion in Ref. [36]. Fig. 4 shows an overlapping of the 1957.3 keV peak with another single peak from the spectrum, such that their tails practically coincide. Thus, the small peak distinguished at 1960.1 keV by Lesher et al. (1966.5 keV in our case) is most likely a result of the tail from the 1957.3 keV strong peak. The same angular distribution for the peak at 1957.3 keV and for the doubtful peak at 1966.5 keV (see Fig. 3) is an additional argument.

A $0^+$ level at 1952.34±0.05 keV had been tentatively proposed from the neutron capture data [35]. A confirmation of this would be the observation of the $0^+$ state in the ($p$,t) reaction. However, the excitation of the 1952.4 keV state is very weak and, for that, measurements of the angular distribution, as it turned out, are not possible. Therefore, our data cannot confirm a $0^+$ assignment for this state and only a tentative spin can be inferred from the gamma ray data. The $0^+$ assignment is not supported for the 1972.2 keV state in Ref. [4]. Instead we found a weak and spread peak with the energy determined as 1977.6 keV, and its angular distribution supports the $0^+$ assignment.

In the part of the higher energy spectrum we support the $0^+$ assignment for two states out of the seven $0^+$ levels, all seven assigned in Ref. [4]. The energies of all these states differ in our study from those given in Ref. [4]. The reason resides in the different calibrations we used, whose details are given above. The details of calibra-
TABLE II.

Results of the present (p,t) experiment are compared with previous studies. The first column shows the energies measured by different methods and compiled in Ref. [37]. Next three columns show energies, relative (p,t) cross sections at 6°, and spins from Ref. [9]. The last three columns show the present results: energies, absolute (p,t) cross sections at 5°, and spin assignments. The errors of the differential cross sections are statistical, and an additional error of 10% should be taken into account due to the uncertainty in the thickness of the targets used.

| NDS Ref. [37] | Results of Lesher et al. Ref. [9] | Results of present study |
|---------------|----------------------------------|--------------------------|
| $E_{exp}(keV)$ | $E_{exp}(keV)$ | $d\sigma/d\Omega(\text{rel})$ | $I^*$ | $E_{exp}(keV)$ | $d\sigma/d\Omega(\mu b)$ | $I^*$ |
| 0.0 | 0.0 6 | 1000 8 | 0+ | 0.0 3 | 1320 12 | 0+ |
| 1196.165 8 | 1194.8 13 | 3.7 6 | 0+ | 1196.1 8 | 3.1 4 | 0+ |
| 1452.352 6 | 1452.4 6 | 305 6 | 0+ | 1452.3 3 | 389 6 | 0+ |
| 1576.930 16 | 1577.0 12 | 5.4 7 | 0+ | 1577.0 4 | 5.6 6 | 0+ |
| 1743.145 14 | 1742.7 9 | 0.6 3 | 0+ | 1743.2 5 | 1.8 2 | 0+ |
| 1935.5 6 | 1936.5 15 | 0.9 2 | 0+ | 1952.4 9 | 0.4 2 | 0+ |
| 1952.424 25 | 1953.5 6 | 30.8 14 | 0+ | 1952.4 9 | 35.9 9 | 0+ |
| 1957.424 25 | 1960.1 38 | 3.2 5 | 0+ | 1957.3 3 | 3.2 5 | 0+ |
| 1972 3 | 1972.2 31 | 0.4 2 | 0+ | 1977.6 8 | 1.2 2 | 0+ |
| 2276.02 3 | 2277.3 22 | 39.6 22 | 0+ | 2276.7 4 | 48.2 12 | 0+ |
| 2338.0 8 | 2339.8 13 | 10.7 7 | 0+ | 2333.4 5 | 6.7 4 | 0+ |
| 2437.2 4 | 2442.2 4 | 11.0 4 | 0+ | 2463.7 4 | 20.0 8 | 0+ |
| 2643.4 8 | 2649.5 8 | 18.1 10 | 0+ | 2632.7 4 | 2.3 7 | 2+ |
| 2688.8 8 | 2695.5 8 | 1.7 10 | 0+ | 2643.1 5 | 0.8 3 | 0+ |
| 2888.2 4 | 2914.5 5 | 8.7 13 | 0+ | 2972.6 4 | 14.6 9 | 0+ |
| 3013.4 7 | 3076.7 16 | 2.9 49 | 0+ | 3041.7 8 | 1.5 2 | (2+) |
| 3080.0 6 | 3109.9 11 | 1.2 5 | 0+ | 3072.2 5 | 10.0 5 | (2+) |
| 3234.5 5 | 3293.7 4 | 4.8 3 | 0+ | 3223.3 3 | 2.1 3 | (2+) |
| 3262.9 5 | 3344.5 5 | 18.0 6 | 0+ | 3293.7 4 | 10.0 5 | (2+) |
| 3388.6 9 | 3400.2 9 | 7.7 4 | (0+) | 3388.6 9 | 7.7 4 | (0+) |
| 3431.8 8 | 3546.2 7 | 2.5 3 | 0+ | 3431.8 8 | 10.3 5 | 0+ |
| 3569.6 7 | 3616.8 5 | 2.8 3 | 0+ | 3569.6 7 | 9.9 5 | 0+ |
| 3626.9 6 | 3626.4 8 | 9.9 5 | 0+ | 3626.4 8 | 22.7 6 | 0+ |
| 3641.7 8 | 3691.7 8 | 4.1 4 | 0+ | 3641.7 8 | 20.4 6 | 0+ |
| 3737.7 11 | 3819.2 7 | 2.7 6 | 0+ | 3737.7 11 | 2.7 6 | 0+ |
| 3819.8 | 3819.8 | 2.1 3 | 0+ | 3819.2 7 | 2.1 3 | (0+) |
| 3829.1 6 | 3848.2 8 | 5.0 4 | 0+ | 3829.1 6 | 5.0 4 | 0+ |
| 3876.1 6 | 3984.9 6 | 2.5 4 | 0+ | 3876.1 6 | 2.5 4 | 0+ |
| 4220.4 6 | 4258.1 6 | 3.3 4 | 0+ | 4220.4 6 | 3.3 4 | 0+ |

* The peak at 1952.4 keV is hidden by much more strong peak at 1957.3 keV. Its strength was estimated by fixing energy of 1952.4 keV in the process of fitting.
FIG. 5. The angular distributions for the states assigned as $0^+$ excitations in Ref. [9]. Our results do not confirm these assignments.

The disagreements between our data and the results by Lesher et al. [9] are not available in their publication. Therefore, in the following discussion, we will refer to the energies determined in our study. Using the present calibration, we support the $0^+$ assignment for the states 2276.4 and 2888.2 keV. The angular distributions obtained in our study for the rest of the five states reported as $0^+$ states in Ref. [9] are presented in Fig. 5 together with DWBA calculations. They allowed assignments of spin $4^+$ for the states 2333.4 and 2632.7 keV, $2^+$ for the state 2695.5 and 3041.7 keV. The angular distribution for the state 3079.2 keV does not allow the definite assignment. At the same time we found four new $0^+$ states at 2437.2, 2726.4, 2757.2, and 2914.5 keV. As seen from Fig. 5 their angular distributions indicate clear $0^+$ assignments.

There are, however, disagreements between some of our data and the results by Lesher et al. [9] from study of the $0^+$ states in the $(n,n'\gamma)$ reaction. The aim of this study was to define the collective nature of $0^+$ excitations assigned in their previous work using the $(p,t)$ reaction. The main way of decay of the low-lying $0^+$ states is to the first excited state $2^+$ at energy of 79.5 keV. Therefore, $\gamma$ rays of the corresponding energies were found in the $\gamma$ spectrum and their properties were investigated. This study confirms the data for the 1743.2, 1957.3 (with the energy correction discussed above) and 2276.7 keV. Study of the coincidences of the $\gamma$ rays feeding and de-exciting $0^+$ states confirms these assignments.

The disagreements start from the state with an energy of 2333.4 keV, as determined in the present study, and with 2338.0 keV in Ref. [9]. From the observation of the de-exciting $\gamma$ rays Lesher et al. have identified a $0^+$ state at energy 2340.0 keV although the gamma ray decaying to the first $2^+$ state was not observed. They concluded that the state of 2340.0 keV and that of 2338.0 keV observed in the $(p,t)$ reaction is the same state. However, the energy of corresponding peak with our calibration is 2333.4 keV and as seen from Fig. 5 the angular distribution for this level corresponds to the $4^+$ spin assignment. The energy 2338.0 keV and, especially, 2340.0 keV are excluded additionally by the triplet of peaks in the triton spectrum with the two known energies of 2355.0, 2344.7 keV [37] and with a third energy level in question. The triplet looks almost equidistant, which allows one to obtain the energy of the peak in question from the interval between the peaks of 2355.0 and 2344.7 keV. The obtained energy is 2334.4 keV that is close to the value found in this study. It can be assumed that the energy of 2340.0 keV refers to the $2^+$ state known from the $(d,p)$ reaction [57], but not to the $0^+$ state.

The $0^+$ state at energy 2644.2 keV was identified using the observation of the 2564.7 keV $\gamma$ ray and the excitation
function. This $0^+$ state was assigned in the (p,t) reaction in Ref. [9]. The 2564.7 keV $\gamma$ ray was attributed to the transition from the $0^+$ state to the $2^+_1$ state at the 79.4 keV energy [30]. However, the angular distribution of the $\gamma$ ray of 2564.7 keV is not completely isotropic which already excludes a definite $0^+$ assignment. Finally, the real energy of the corresponding peak as follows from our calibration is 2632.7 keV and as seen from Fig. 5, the angular distribution for this level corresponds to the $4^+$ spin assignment. Since the coincidences were not measured, it is not obvious that the $\gamma$ rays of 2564.7 keV refer to the de-excitation of the $0^+$ state into the $2^+_1$ state. Therefore, the situation with the identification of this state is not clear. Perhaps, the 2564.7 keV $\gamma$ ray refers to the de-excitation of the 2643.1 keV level, seen in the present study with close energy to the putative 2644.2 keV state, which is identified, however, as the $2^+_2$ level (see, Fig. 5). In a similar way, the 2832.0 keV $\gamma$ transition was used to identify the $0^+$ state at an energy of 2911.5 keV, also assigned as $0^+$ state in the (p,t) reaction. However, again the real energy of the corresponding peak is 2888.2 keV. In addition, we found 20 new $0^+$ states in the energy interval from 3200 to 4300 keV. This energy region was not investigated so far in the (p,t) reaction. The total number of $0^+$ states detected in one nucleus equals now 34, which is the largest of such states observed so far. For three of them the $0^+$ assignments are tentative. For some of these states, their energies observed in the (n,$\gamma$) reaction were found to be close within the error limits. Apart from the energies, there is no other information about these states. Therefore, one can not be sure that these states and the ones observed in the (p,t) reaction are the same, although the close proximity of the energies obtained in the two independent experiments are supporting the validity of our calibration.

As already mentioned, theoretical models have relatively modest results describing the spectra of multiple $0^+$ excitations. No attempt was made to fit individual $0^+$ states and, therefore, no predictions of the $0^+$ states having a correspondence with the specific experimental states. The point of the calculations was rather to see a number of $0^+$ excitations in the energy range up to about 3 MeV, and a general trend in the cumulative cross section with increasing energy. Such calculations were performed both within the framework of the QPM and the spdf-IBM, in particular, for $^{158}$Gd [11, 12]. The IBM calculations yields a number of $0^+$ states close to the experimental ones below 3 MeV, and many of the $0^+$ states of two-phonon octupole character, as shown in Fig. 7. The spdf-IBM failed to reproduce the increasing density of $0^+$ states above 3 MeV. In addition, several other $0^+$ states at higher excitation energy are calculated in Ref. [11], amounting to 23 excited $0^+$ states below 4 MeV. Therefore, spdf-IBM reproduces at least partially. The cross sections were not calculated in this publication since only the use of an extended Hamiltonian allows to perform such calculations [38].

The cross sections were calculated in the framework of the QPM. The experimental spectra of $0^+$ states, as compared to the calculated ones, are shown in Fig. 7. The QPM predicts a number of $0^+$ states which are close to the one observed below 3 MeV. However, this model fails in the cross section calculation for the first excited state. This state is excited very weakly, that may indicate its $\beta$-vibrational nature. A large cross section (33% of the cross section for g.s.) is observed for the second excited $0^+$ state, which is evidence of the similarity of its structure to the structure of the ground state. In contrast to this, the QPM predicts strong excitation just for the first excited $0^+$ state, that shows its resemblance with the ground state, and very weak excitations for all other $0^+$ states. Six of the QPM $0^+$ states (mostly the lowest) have a one-phonon character. Other states at higher excitation energy contain large, and, in many cases, the dominant two-phonon components. They are built on the collective octupole phonons almost in all cases, in agreement with the IBM calculation [11, 12].

New experimental data in the extended energy region represent an excellent opportunity to test these and other nuclear models. There is one additional aspect of such

![FIG. 7. The (p,t) cross sections at the angle 5° for $0^+$ states in $^{158}$Gd: experimental data (a) and calculated in the framework of the QPM (b).](image-url)
The identification of 30 excited $0^+$ states and a decrease of their excitation cross sections in the $(p,t)$ reaction with increasing the excitation energy \[16\]. Their structure becomes more complicated and octupole components in the wave function play an increasing role. The experimental spectrum of $0^+$ states presented in Fig. \[7\] demonstrates a somewhat different picture. A bump of states is observed in the region between 3.2 and 4.0 MeV and, if there is no termination of spectrum, a drop in the magnitude of the density of $0^+$ states is then seen in experimental data.

### III. CONCLUSION

We carried out a new high-precision $(p,t)$ reaction on an isotopically enriched target of $^{160}$Gd which allowed the identification of 30 excited $0^+$ states below 4.3 MeV in the spectrum of $^{158}$Gd. Thus, the total number of $0^+$ states in this nucleus is increased now up to 34. Such abundance of $0^+$ states have not previously been observed in any nucleus investigated so far. The $^{158}$Gd was the nucleus for which information on the multiple $0^+$ excitation was published for the first time in Ref. \[9\]. The new information can be interesting, especially among theoreticians, because several models were applied in an attempt to understand the nature of these states. Much richer new information will be of no less interest for theoreticians since the observation of thirty four $0^+$ states in one nucleus is the strongest challenge to our understanding of these excitations. In a forthcoming analysis of the obtained experimental data the $2^+$ and $4^+$ states and possible other states of the negative parity will be assigned. As in our previous publications this can allow to build collective bands with the $0^+$ states as band-heads which will bring further support for the collectivity of these states. The data from the $(p,t)$ reaction are interesting in one more aspect. As noted above, complete or almost complete sequences of states of the collective nature with a definite $J^+$ are available from this reaction. This allows to carry out a statistical analysis of these spectra with the aim of clarifying the measure of order and chaos in collective spectra \[39, 40\]. Moreover, such studies are helpful in the formation of sequences of states that can be interpreted as collective bands based on $0^+$ and other states. Collective bands with different $K$ for the $2^+$ and $4^+$ band heads can be formed, and this opens a new possibility to investigate the $K$-symmetry breaking \[40\].

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