Distributed Uplink Resource Allocation in Cognitive Radio Networks – Part II: Equilibria and Algorithms for Joint Access Point Selection and Power Allocation

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Abstract

The main objective of this two part paper is to formulate and address the problem of distributed uplink resource allocation in multi-carrier cognitive radio networks (CRN) with multiple Access Points (APs). When the APs operate on non-overlapping spectrum bands, such problem is essentially a joint spectrum decision and spectrum sharing problem. In this network, the cognitive users (CUs) are endowed with greater flexibility than the single AP network we considered in the first part of the paper [2]: they can optimize their uplink transmission rates by means of: 1) association to a suitable AP and 2) sharing the set of channels that belong to this AP with other CUs associated with this AP. Clearly these two steps are interdependent, and our objective is to devise suitable algorithms by which the CUs can perform these two steps in a distributed and efficient fashion.

In the first part of this paper, we have studied solely the spectrum sharing aspect of the above problem, and proposed algorithms for the CUs in the single AP network to efficiently share the spectrum. In this second part of the paper, we build upon our previous understanding of the single AP network, and formulate the joint spectrum decision and spectrum sharing problem in a multiple AP network into a non-cooperative game, in which the feasible strategy of a player contains a discrete variable (the AP/spectrum decision) and a continuous vector (the power allocation among multiple channels). The structure of the game is hence very different from most non-cooperative spectrum management game proposed in the literature. We provide characterization of the Nash Equilibrium (NE) of this game, and

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present a set of novel algorithms that allow the CUs to distributively and efficiently select the suitable AP and share the channels with other CUs. Finally, we study the properties of the proposed algorithms as well as their performance via extensive simulations.

I. INTRODUCTION

A. Motivation and Related Work

The objective of this two part paper is to provide the analytical framework as well as the solutions to the joint AP selection and power allocation problem in a CRN in the presence of multiple APs. As mentioned in the first part of the paper, the need for such joint optimization may arise in a CRN with multiple CUs and multiple APs, for example, the IEEE 802.22 cognitive radio Wireless Regional Area Network (WRAN) [3]. In such network, a particular geographical region may be served by multiple service providers (SPs), or by multiple APs installed by a single SP [4]. Consequently, the CUs, on top of being able to share the spectrum offered by a particular SP/AP, also have the flexibility of deciding on their SP/AP association. As suggested in [5] and [6], it would be generally beneficial (in terms of either system-wide or individual performance), compared with traditional closest AP assignment strategy, to allow the users in multiple AP networks to include the AP association as an additional decision variable.

In this part of the paper, we consider a CRN with multiple CUs and APs. The available spectrum is partitioned by the APs, and they operate on non-overlapping spectrum bands. Each CU’s objective is to connect to a single AP for communication. The CUs can concurrently use all the channels that belong to its associated AP for transmission if desired, but different CUs interfere with each other if they use the same channel. In the considered network, the CUs first need to select an appropriate AP for communication, a task that can be viewed as a spectrum decision task because the CUs are essentially choosing a “best” spectrum band in terms of transmission rate. Then they face the spectrum sharing problem when they try to dynamically allocate their communication power across the channels that belong to the selected AP. Clearly for a fixed system wide CU-AP association, the entire network is reduced to multiple single AP sub-networks, and the (near-) optimal spectrum sharing solution for each of the sub-network is studied in the first part of this paper [2]. Although as we have demonstrated that for each sub-network, our proposed spectrum sharing scheme has the potential of maximizing the achievable sum rate, in a multiple AP network, the system performance is inevitably tied to the quality of the system wide CU-AP association decision as well. A bad CU-AP association decision will result in unsatisfactory system performance regardless of the underlying sharing scheme. Consequently, the association problem and the sharing problem are strongly interdependent, and in this part of the paper, we intend to propose
distributed and efficient algorithm for the CUs in the network to carry out both of the tasks of AP association (spectrum decision) and power allocation (spectrum sharing).

A related problem of joint cell selection/base station (BS) association and power control has been addressed in infrastructure-based cellular networks. [7] and [8] are early works trying to tackle this problem in an uplink spread spectrum cellular network. The objectives are to let the users find a best site selection and power allocation tuple such that all users’ target signal to interference ratio (SIR) are met, and each user’s transmission power is minimized. The authors of [5] and [6] cast a similar problem (with an objective to maximize individual power efficiency or minimizing individual cost) into game theoretical frameworks, and propose algorithms to find the Nash Equilibrium (NE) of the proposed games. One of the most important differences between our work and the above cited works is that the power allocation problems in these works are essentially scalar value optimization problem: each user only needs to decide on its power level once a BS is selected, while in our work, individual power allocation is a vector optimization problem as the CUs have the flexibility to use all the channels that belong to a particular AP concurrently. This fundamental difference makes the considered problem more complex, hence the analytical frameworks provided by the above cited works are not suitable for our problem. [9] is a recent work using non-cooperative game theory to address the problem of distributed energy-efficient power control in uplink multi-carrier CDMA system. Similarly as in the above cited works, the solution proposed by the authors mandates that the users choose a single optimum channel as well as a scalar power level to transmit on the selected channel. [4] is a recent work considering the uplink dynamic spectrum sharing problem in a multi-carrier multiple service provider CRN. The authors propose algorithms for the users to select the size of the spectrum and the amount of power for transmission. One important assumption of this work is that the users can connect to multiple APs at the same time (we refer to such network as multiple-connectivity network), an assumption that simplifies the analysis significantly but may induce considerable signaling overhead on the network side as well as hardware implementation complexity on the cognitive device. Even such issues may be resolved in the future, our work, which analyzes the single-connectivity network, can serve as a benchmark for comparison between single-connectivity and multiple-connectivity networks.

We also argue that the problem under consideration is in many aspects more complicated than the traditional AP association problems arise in the 802.11 WLAN network (for example, [11], [12] and [13] and the reference therein). Typically, AP association is aiming to optimize different system performance

\footnote{In WLAN literature, such network is also referred to as “multi-homing” network, see [10] and the reference therein.}
metrics (throughput, fairness, etc), and only simple individual throughput estimates within each AP are used to update the current association profile. Indeed, in 802.11 WLAN network, the throughput of an individual AP with fixed number of users and fixed physical bit rate can be approximated using simple analytical formulae \cite{14}, and this result has greatly simplified the analysis of many work dealing with dynamic AP association in WLAN, e.g., \cite{13} and \cite{10}.

We note here that the problem of how to dynamically perform the task of both spectrum decision and spectrum sharing may arise in different CRN configuration as well. Many of the current works addressing the spectrum management problem in multi-channel multi-user CRN focus only on the spectrum sharing aspect of the problem. For example, in \cite{15}, \cite{16}, \cite{17}, a set of iterative water-filling (IWF) based algorithms are proposed to find a distributed solution of power allocation in multi-channel, multi-user CRN. One important assumption underlying these works is that the CUs are able to use all the channels simultaneously. This assumption might not be valid in the situation where the available spectrum is fragmented due to licensed user activities and the CUs are equipped with 1-agile radio which can only use a single chunk of continuously aligned channels at a time \cite{23}. In this scenario, the CUs need to first decide on which chunk of channels to use, and then make subsequent power allocation decisions on the selected set of channels, i.e., the CUs are required to perform the task of joint spectrum decision and spectrum sharing. It is our belief that our work can also serve to shed some lights on providing solutions to the above problem, as the network configuration considered in our work is sufficiently similar to the configuration mentioned above.

B. Contributions and Organization of This Work

To the best of our knowledge, this is the first work that proposes distributed algorithms to deal with joint AP selection and power allocation problem in a multi-channel multi-AP CRN. We cast the problem into a non-cooperative game framework, in which each CU’s objective is to maximize its own transmission rate, and its strategy space is the union of a discrete set (the set of possible APs) and a multi-dimensional continuous set (the set of feasible power vectors). Although non-cooperative game theory has recently been extensively applied to solve the resource allocation problem in CRN (e.g., \cite{15}, \cite{17}, \cite{24} and the reference therein), our formulation is considerably different and more involved because of such “hybrid”

\textsuperscript{2}IWF is originally proposed in \cite{18} in the context of DSL network, and subsequently applied to wireless network with vector multiple-access channel \cite{19} and with gaussian interference channel in \cite{20}, \cite{21}, \cite{22}.

\textsuperscript{3}See \cite{23} for discussion of agile radios and the possibility of this scenario in actual CRN implementations.
nature of the strategy space of the game. We analyze in detail the equilibrium solution of the game, and develop a suite of algorithms with provable convergence guarantees that enable the CUs to distributedly compute the equilibrium solution.

We organize our paper as follows. In section II, we present the network model under consideration and formulate the problem into a non-cooperative game. In section III, we analyze the properties of the equilibrium solution. In section IV, we provide our main algorithm and its convergence results. In section V and VI, we provide important extensions of the JASPA. We present simulation results in section VII and conclude the paper in section VIII.

II. PROBLEM FORMULATION

A. Considered Network and Some Assumptions

We consider the following cognitive network configuration. Suppose there are a set \( \{1, 2, \cdots, N\} \equiv N \) of CUs, a set \( \{1, 2, \cdots, K\} \equiv K \) of channels and a set \( \{1, 2, \cdots, W\} \equiv W \) of APs in the network, and we normalize the total available bandwidth to 1. Each AP \( w \in W \) is assigned with a subset of channels \( K_w \subset K \). We focus on the uplink scenario where each CU wants to connect to one of the APs for transmission. The followings are our main assumptions of the network.

A-1) Each CU \( i \) is able to associate to all the APs, and each AP covers entire area of the network.

A-2) The APs covering the same area operate on non-overlapping portions of the available spectrum.

A-3) The set of spectrum can be used exclusively by the CRN for a relative long period of time.

A-4) Each CU can associate to a single AP at a time; it can concurrently use all the channels of the associated AP, if desired.

A-5) Each AP is equipped with single-user receivers. Different APs in the network do not compete with each other for revenue.

Assumption A-1) is made merely for ease of presentation, and our work can be extended to the scenarios where different APs cover different areas of the network, and where the CUs can only connect to the subset of APs that cover them.

Assumption A-2) is commonly used when considering AP association problems in WLAN (e.g., [10]), or the spectrum sharing problem in cognitive network with multiple service providers (e.g., [4]). It is made to mitigate interference between neighboring APs. It can be achieved either by 1) the APs agree offline the partition of the spectrum \(^4\) or 2) the APs jointly run a distributed online spectrum assignment algorithm.

\(^4\)In the presence of multiple SPs, such offline negotiation can be made possible by the coordination of a spectrum clearing house, as suggested in [4].
similar to the ones proposed in [11] to determine the best spectrum assignment. How to determine the “optimum” partition of the spectrum is out of the scope of this paper. Assumption A-1) and A-2) imply that $K_w \cap K_q = \emptyset$, $\forall w \neq q, k, q \in \mathcal{W}$.

B. System Model

Let \{\{|h_{i,w}(k)|^2\}_k \in K_w\} be the set of power gains from CU $i$ to AP $w$ on all its channels; Let \{\{n_w(k)\}_k \in K_w\} be the set of environmental noise powers on all channels for AP $w$; Let the $N \times 1$ vector $\mathbf{a}$ denote the association profile in the network, with its $i^{th}$ element $a(i) = w$ indicating that CU $i$ is associated to AP $w$. Each CU $i$ is able to obtain its own channel gains to all the APs, \{|h_{i,w}(k)|^2\}_k \in K_w, w \in \mathcal{W}$, via feedback from the APs, but it does not need to have the knowledge of other CUs’ channel gains in the network.

Let $p_{i,w}(k)$ represent the amount of power CU $i$ transmits on channel $k$ when it is associated with AP $w$; Let $\mathbf{p}_{i,w} = \{p_{i,w}(k)\}_k \in K_w$ be the power profile of CU $i$ when it is associated with AP $w$; let $\mathbf{p}_{-i,w}$ be the joint power profiles of all the CUs other than $i$ that is associated with AP $w$: $\mathbf{p}_{-i,w} \triangleq \{p_{j,w}\}_j:j \neq i, a(j) = w$.

By construction, for all $w \in \mathcal{W}$, if $w \neq a(i)$, then $p_{i,w} = 0$. The power profiles of the CUs must satisfy the following two constraints (as in [2]): 1) Total power constraints; 2) Positivity constraints. As such, each CU’s feasible power allocation when it is associated with AP $w$ can be expressed as:

$$\mathcal{F}_{i,w} = \left\{\mathbf{p}_{i,w} : \sum_{k \in K_w} p_{i,w}(k) \leq \bar{p}_i, \; p_{i,w}(k) \geq 0, \; \forall k \in K_w\right\}.$$  

Again assume that there is no interference cancelation performed at the AP, then for a fixed AP association and power allocation configuration, CU $i$’s uplink transmission rate (when it is associated with AP $w$) can be expressed as follows:

$$R_i(\mathbf{p}_{i,w}, \mathbf{p}_{-i,w}, w) = \frac{1}{K} \sum_{k \in K_w} \log \left(1 + \frac{|h_{i,w}(k)|^2 p_{i,w}(k) + \sum_{j:a(j) = w, j \neq i} |h_{j,w}(k)|^2 p_{j,w}(k) + \sum_{j:a(j) = w, j \neq i} |h_{j,w}(k)|^2 p_{j,w}(k)}{n_w(k) + \sum_{j:a(j) = w, j \neq i} |h_{j,w}(k)|^2 p_{j,w}(k)}\right)$$ (1)

$$= \frac{1}{K} \sum_{k \in K_w} \log \left(1 + \frac{|h_{i,w}(k)|^2 p_{i,w}(k)}{n_w(k) + I_i(k)}\right) \triangleq R_i(\mathbf{p}_{i,w}, \mathbf{I}_{i,w}, w)$$ (2)

where $I_i(k)$ denotes the aggregated received transmission power level on channel $k$ except CU $i$, i.e.,

$$I_i(k) \triangleq \sum_{j:a(j) = w, j \neq i} |h_{j,w}(k)|^2 p_{j,w}(k), \quad \mathbf{I}_{i,w} \triangleq \{I_i(k)\}_{k \in K_w}.$$ (3)

We note that, if $w = a(i)$, then $\mathbf{I}_{i,w}$ can be viewed as the set of interference currently experienced by CU $i$; if $w \neq a(i)$, $\mathbf{I}_{i,w}$ can be viewed as the set of interference that CU $i$ would experience if it were to switch to AP $w$. 

We see that (1) and (2) are equivalent definitions of the CU $i$'s transmission rate. We will use either definition in the following paragraph depending on the context.

C. A Non-Cooperative Game Formulation

We model each CU $i$ as a selfish agent with the objective to find strategy $(w^*, p^*_i, w^*)$ that maximizes its transmission rate, based on the current state of the network:

\[
(w^*, p^*_i, w^*) \in \arg \max_{w \in W} \max_{p_i \in \mathcal{F}_{i,w}} R_i(p_i, w^*; w).
\]

We are now ready to define a non-cooperative game $\mathcal{G}$:

\[
\mathcal{G} \triangleq \{\mathcal{N}, \{\chi_i\}_{i \in \mathcal{N}}, \{R_i\}_{i \in \mathcal{N}}\}
\]

where the CUs $i \in \mathcal{N}$ are the players in the game; each CU’s strategy space can be expressed as $\chi_i \triangleq \bigcup_{w \in W} \{w, \mathcal{F}_{i,w}\}$; each CU’s utility function is its transmission rate $R_i(p_i, w^*; w)$ as defined in (1). We emphasize that each feasible strategy of a player contains a discrete variable and a continuous vector, which makes the game $\mathcal{G}$ different from (and thus more complicated than) most of the games considered in the context of network resource allocation. We refer to the strategy space $\{\chi_i\}_{i \in \mathcal{N}}$ of this game as hybrid strategy space.

The NE of this game is defined as the tuple $\{a^*(i), p^*_i, a^*(i)\}_{i \in \mathcal{N}}$ such that the following set of equations are satisfied:

\[
(a^*(i), p^*_i, a^*(i)) \in \arg \max_{w \in W} \max_{p_i \in \mathcal{F}_{i,w}} R_i(p_i, w^*; w), \quad i \in \mathcal{N}
\]

or equivalently, $\forall i \in \mathcal{N}, \ w \in \mathcal{W}, \ p_i \in \mathcal{F}_{i,w},$

\[
R_i(p^*_i, a^*(i); p^*_{-i,a^*(i)}; a^*(i)) \geq R_i(p_i, w^*_{-i,a^*(i)}; w).
\]

Note that $p^*_{-i,w}$ is defined as the power profiles of all the CUs other than CU $i$ that is associated with AP $w$ in the NE: $p^*_i, w \triangleq \{p^*_j, a^*(j)\}_{j \neq i, a^*(j) = w}$. We call the equilibrium profile $a^*$ a NE association profile, and $p^*_{a^*} \triangleq \{p^*_i, a^*(i)\}_{i \in \mathcal{N}}$ a NE power allocation profile. In order to avoid duplicated definitions, we call the tuple $(a^*, p^*_{a^*})$ a joint equilibrium profile (JEP) of the game $\mathcal{G}$ (instead of a NE). It is clear from either of the above definition that in a JEP, the system is stable in the sense that no CU has the incentive to deviate from either its AP association or its power allocation.

III. PROPERTIES OF THE JEP

In this section, we introduce the notion of the potential function for the game $\mathcal{G}$, and its relationship with the JEP. This function plays an important role in our following analysis of the existence of JEP and the proof of convergence of the algorithm. We then prove that the JEP always exists for the game $\mathcal{G}$. 
A. The Potential Function

Consider a simpler problem in which the association vector $a$ is predetermined and fixed. In this case, the CUs do not need to choose their AP associations, thus the problem of finding the JEP defined in (6) reduces to the one of finding the NE power allocation profile $p^*_a$ that satisfies:

$$p^*_{i,a(i)} \in \arg \max_{p_i \in F_{i,a(i)}} R_i(p_i, p^*_a, a(i)), \quad \forall \, i \in \mathcal{N}. \quad (7)$$

For a specific AP $w$, denote the set of CUs associated with it to be $\mathcal{N}_w$: $\mathcal{N}_w \triangleq \{i : a(i) = w\}$. It is clear that $\{\mathcal{N}_w\}_{w \in \mathcal{W}}$ is a partition of $\mathcal{N}$. We use $p_w \triangleq \{p_{i,w}\}_{i \in \mathcal{N}_w}$ to denote the long vector containing the power profiles of all CUs associated with AP $w$. When $a$ is fixed, the activity of the set of CUs $\mathcal{N}_w$, $w \in \mathcal{W}$ does not affect the activity of the set of CUs $\mathcal{N}_q$, $q \in \mathcal{W}$, $q \neq w$, because of the fact that AP $w$ and $q$ operate on different sets of channels. Consequently, the original game $G$ introduced in (5) can be decomposed into $\mathcal{W}$ independent small games, with each small game $G^a_w$ defined as:

$$G^a_w = \{\mathcal{N}_w, \{F_{i,w}\}_{i \in \mathcal{N}_w}, \{R_i\}_{i \in \mathcal{N}_w}\}. \quad (8)$$

Clearly, each of such small game has the same form as the spectrum sharing game $G$ analyzed in Section III of the first part of the paper. We define the potential function for the small game $G^a_w$ as well as for the original game $G$ as follows.

**Definition 1:** The potential function of the game $G^a_w$ under a feasible power profile $p_w$ is defined as:

$$P_w(p_w; a) = \frac{1}{K} \sum_{k \in \mathcal{K}_w} \left( \log \left( n_w(k) + \sum_{i \in \mathcal{N}_w} |h_{i,w}(k)|^2 p_{i,w}(k) \right) - \log n_w(k) \right).$$

The system potential function under a specific $a$ and a feasible $p$ is defined as the sum of the potential functions associated to all games $\{G^a_w\}_{w \in \mathcal{W}}$:

$$P(p; a) = \sum_{w \in \mathcal{W}} P_w(p_w; a). \quad (9)$$

Define $F^a_w \triangleq \prod_{i \in \mathcal{N}_w} F_{i,w}$ as the joint feasible set for the CUs that are associated with AP $w$ under the association profile $a$, and let $\mathcal{F}^a \triangleq \prod_{w \in \mathcal{W}} F^a_w$. Let $\mathcal{E}(a)$ denote the set of all NE power profiles for the game $G^a_w$, then $\mathcal{E}(a) \triangleq \prod_{w \in \mathcal{W}} \mathcal{E}_w(a)$ is the set of all NE power profiles for the game $G$ under fixed association profile $a$. Let $p^*_w(a)$ be any one of such NE power profiles for game $G^a_w$, i.e., $p^*_w(a) \in \mathcal{E}_w(a)$; let $p^*(a)$ be any one of the NE power profiles of the network, $p^*(a) \in \mathcal{E}(a)$. The

\[\text{5} \text{Indeed, as argued in Section III and IV-A of [2], the spectrum sharing game } G \text{ (hence the small game } G^a_w) \text{ may have a connected set of NE power profiles.}\]
following corollary regarding to the relationship between \( p^*_w(a) \) and the potential functions is a straightforward consequence of Theorem 1 and Corollary 1 of [2].

**Corollary 1:** For fixed \( a \), a feasible \( p_w \in \mathcal{F}_w \) maximizes the potential function \( P_w(p_w; a) \) if and only if it is in the set \( \mathcal{E}_w(a) \). We define the maximum value of the potential function:

\[
P_w(a) \triangleq \max_{p_w \in \mathcal{F}_w} P_w(p_w; a) = P_w(p^*_w(a); a)
\]  

(10)

as an equilibrium potential (EP) for AP \( w \) under association profile \( a \).

For a fixed \( a \), a feasible \( p \in \mathcal{F}^a \) that maximizes the system potential function \( P(p; a) \) if and only if it is in the set \( \mathcal{E}(a) \). Similarly as above, we refer to the maximum value of the system potential function as the system equilibrium potential (SEP) under association profile \( a \), and denoted it by \( \bar{P}(a) \):

\[
\bar{P}(a) \triangleq \sum_{w \in \mathcal{W}} \bar{P}_w(a).
\]  

(11)

**B. Existence of JEP**

In this section, we discuss the existence of the JEP as defined in (6). We emphasize here that determining the existence of the JEP (which is a pure NE) for the game \( G \) is by no means a trivial proposition. Due to the hybrid structure of the game \( G \), the standard results on the existence of pure NE of either continuous or discrete games can not be applied. Consequently, we have to explore the structure of the problem in proving the existence of JEP for the game \( G \).

From Corollary 1 we see that a specific \( a \) can be mapped to a SEP, denoted by \( \bar{P}(a) \). We claim that any one of the AP association profiles \( \tilde{a} \) that maximizes the SEP, along with any one of its corresponding system power profile \( p^*(\tilde{a}) \in \mathcal{E}(\tilde{a}) \), constitute a JEP as defined in (6). We state this observation in the following theorem.

**Theorem 1:** The game \( G \) as defined in (5) always admits a JEP. An association profile \( \tilde{a} \in \arg \max_a \bar{P}(a) \), along with any one of its corresponding NE power allocation profile \( p^*(\tilde{a})(j) = \left\{ p^*_i(\tilde{a}(i)) \right\}_{i \in \mathcal{N}} \in \mathcal{E}(\tilde{a}) \), constitute a JEP of the game \( G \) that satisfies (6).

**Proof:** We prove this theorem by contradiction. Suppose \( \tilde{a} \) maximizes the system potential, but \( \tilde{a} \) is not a NE association profile. Then there must exist a CU \( i \) who wants to switch from \( \tilde{a}(i) = \tilde{w} \) to a different AP \( \tilde{w} \neq \tilde{w} \). Define a new association profile \( \tilde{\tilde{a}} \) as:

\[
\tilde{\tilde{a}}(j) = \begin{cases} 
\tilde{a}(j) & \text{for } j \neq i \\
\tilde{w} & \text{for } j = i.
\end{cases}
\]  

(12)
Let $\mathbf{p}^*(\tilde{a}) \in \mathcal{E}(<\tilde{a})$ and $\mathbf{p}^*(\tilde{a}) \in \mathcal{E}(<\tilde{a})$. The maximum rate that CU $i$ can get after switching to $\tilde{w}$ if all other CUs do not change their actions:

$$
\hat{R}_i(\mathbf{p}_{i,\tilde{w}}, \mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{w}) = \sum_{k \in \mathcal{K}_w} \log \left( 1 + \frac{|h_{i,\tilde{w}}(k)|^2 \tilde{p}_{i,\tilde{w}}(k)}{n_{\tilde{w}}(k) + \sum_{j: \tilde{a}(j) = \tilde{w}} |h_{j,\tilde{w}}(k)|^2 \tilde{p}_{j,\tilde{w}}^*(k)} \right)
$$

where $I_i^*(k)$ is defined similarly as in (3), and the vector $\mathbf{p}_{i,\tilde{w}}$ is determined by:

$$
\mathbf{p}_{i,\tilde{w}} = \arg \max_{p_i \in \mathcal{F}_i, \tilde{w}} \hat{R}_i(\mathbf{p}_i, \mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{w}).
$$

We can view the rate $\hat{R}_i(\mathbf{p}_{i,\tilde{w}}, \mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{w})$ as CU $i$’s estimate of the maximum rate it can get if it were to switch to AP $\tilde{w}$.

Because CU $i$ prefers $\tilde{w}$, from the definition of the JEP (6) we see that its current communication rate must be strictly less than its estimated maximum rate, i.e., the following must be true:

$$
R_i(\mathbf{p}_{i,\tilde{w}}^*(\tilde{a}), \mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{w}) < \hat{R}_i(\mathbf{p}_{i,\tilde{w}}, \mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{w})
$$

where $R_i(\mathbf{p}_{i,\tilde{w}}^*(\tilde{a}), \mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{w})$ is the actual transmission rate for CU $i$ in the association profile $\tilde{a}$, and it can be expressed as follows:

$$
R_i(\mathbf{p}_{i,\tilde{w}}^*(\tilde{a}), \mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{w}) = \sum_{k \in \mathcal{K}_w} \log \left( 1 + \frac{|h_{i,\tilde{w}}(k)|^2 \tilde{p}_{i,\tilde{w}}^*(k)}{n_{\tilde{w}}(k) + I_i^*(k)} \right)
$$

Combining (13), (15) and (16) we must have that:

$$
P_{\tilde{w}}(\mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{a}) - P_{\tilde{w}}(\mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{a}) < P_{\tilde{w}}(\mathbf{p}_{i,\tilde{w}}^*(\tilde{a}); \tilde{a}) - P_{\tilde{w}}(\mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{a}).
$$

We notice that the term $P_{\tilde{w}}(\mathbf{p}_{\tilde{w}}^*(\tilde{a}); \tilde{a})$ is equivalent to $P_{\tilde{w}}(\mathbf{p}_{i,\tilde{w}}^*(\tilde{a}); \tilde{a})$ due to the equivalence of the following sets:

$$
\{ j : j \neq i, \tilde{a}(j) = \tilde{w} \} = \{ j : j \neq i, \tilde{a}(j) = \tilde{w} \}.
$$

Recall that from Corollary 1 we have that the NE power allocation profile maximizes the potential function: $\mathbf{p}_{\tilde{w}}^*(\tilde{a}) \in \arg \max_{\mathbf{p}_{\tilde{w}} \in \mathcal{F}_{\tilde{w}}} P_{\tilde{w}}(\mathbf{p}_{\tilde{w}}; \tilde{a})$. Observe that the set of CUs associated with AP $\tilde{w}$ under
profile \( \tilde{\alpha} \) is the same as the set of CUs associated with AP \( \tilde{\alpha} \) under profile \( \tilde{\alpha} \) excluding CU \( i \), we must have \( p_{-i,\tilde{w}}^{*}(\tilde{\alpha}) \in \mathcal{P}_{\tilde{w}}^{\tilde{\alpha}} \). Consequently, the following is true:

\[
P_{\tilde{w}}(p_{\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha}) \geq P_{\tilde{w}}(p_{-i,\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha}) = P_{\tilde{w}}(p_{-i,\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha})
\]

(19)

where \((a)\) is from (18). Similarly, we have that:

\[
P_{\tilde{w}}(p_{\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha}) \geq P_{\tilde{w}}(\tilde{p}_{i,\tilde{w}}, p_{\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha}).
\]

(20)

Combining (19), (20) and (17), we have that:

\[
P_{\tilde{w}}(p_{\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha}) - P_{\tilde{w}}(p_{\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha}) < P_{\tilde{w}}(\tilde{p}_{i,\tilde{w}}, p_{\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha}) - P_{\tilde{w}}(p_{\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha})
\]

which essentially says that after \( i \) switched to AP \( \tilde{w} \), the decrease of EP of AP \( \tilde{w} \) is less than the increase of the EP of AP \( w \). In other words, we have that:

\[
P_{\tilde{w}}(p_{\tilde{w}}^{*}(\tilde{\alpha}); \tilde{\alpha}) + P_{\tilde{w}}(p_{\til{w}}^{*}(\til{\alpha}); \til{\alpha}) < P_{\til{w}}(p_{\til{w}}^{*}(\til{\alpha}); \til{\alpha}) + P_{\til{w}}(p_{\til{w}}^{*}(\til{\alpha}); \til{\alpha}).
\]

(21)

Noticing that the equilibrium potentials of all the APs other than \( \tilde{w} \) and \( \tilde{w} \) are the same between the profile \( \tilde{\alpha} \) and \( \tilde{\alpha} \), thus adding them to both sides of (21) we have that:

\[
\sum_{w \in \mathcal{W}} P_{\til{w}}(p_{\til{w}}^{*}(\til{\alpha}); \til{\alpha}) < \sum_{w \in \mathcal{W}} P_{\til{w}}(p_{\til{w}}^{*}(\til{\alpha}); \til{\alpha})
\]

(22)

which is equivalent to:

\[
\bar{P}(\til{\alpha}) < \bar{P}(\til{\alpha}).
\]

(23)

This is a contradiction to the assumption that \( \bar{P}(\til{\alpha}) \) maximizes the system potential. We conclude that \( \til{\alpha} \) must be a NE association profile. Clearly, \( p^{*}(\til{\alpha}) \) is a NE power allocation profile. Consequently, we have that \((\til{\alpha}, p^{*}(\til{\alpha}))\) is a JEP.

**IV. The Proposed Algorithm**

In this section, we introduce our main algorithm, referred to as the Joint Access point Selection and Power Allocation (JASPA) algorithm, that allows the CUs in the network to distributely compute the JEP. To this end, we first introduce a simple scheme that assigns the CUs to their closest AP, a scheme which essentially separates the process of AP association and power allocation. This scheme, although relatively simple, offers valuable insights upon which we build the JASPA algorithm, in subsection IV-B.
A. Closest AP Association Algorithms

Consider a fixed AP association profile $\mathbf{a}$ in which each CU is assigned to its closest AP. The “closeness”, or “distance” from a CU to the APs can be measured either by the physical distance between them, or by the strength of pilot/control signal received by the CU from the AP. Assuming that each CU has a single closest AP, then the AP association profile is unique and the computation of JEP reduces to the problem of finding the NE power allocation profile. Moreover, as mentioned before, the CUs are partitioned into independent sets $\mathcal{N}_w \triangleq \{i : a(i) = w\}$, and the CUs in each set $\mathcal{N}_w$ can compute their NE power allocation profile without taking into consideration the behaviors of the CUs in other sets.

Clearly, this scheme separates the process of spectrum decision and spectrum sharing, and the CUs only need to carry out the task of sharing the spectrum available to the designated AP with other CUs. However, as we probably can speculate, no matter how efficient such sharing scheme is, the overall system performance might suffer because of the fixed and inefficient AP assignment. We will see such performance degradation later in the simulation section.

We note that from Proposition 2 and 3 in [2], for a specific association profile $\mathbf{a}$, the set of CUs $\mathcal{N}_w$ that is associates to the same AP $w$ are able to distributedly decide on their NE power allocation profiles by running either the A-IWF or the S-IWF algorithm (cf. Algorithm 1 and Algorithm 2 in [2]).

B. The Joint AP Selection and Power Allocation Algorithm

We name the proposed algorithm Joint Access Point Selection and Power Allocation (JASPA) algorithm. Intuitively, the proposed algorithm works as follows. For a fixed AP association profile, all CUs calculate iteratively their NE power allocations. After convergence, they individually try to see if they can strictly increase their communication rates by switching to another AP, assuming that all other CUs keep their current AP associations and power profiles. When CU $i$ decides that its next best AP association should be $w_i^*$, we record his decision by a $W \times 1$ best reply vector $\mathbf{b}_i : \mathbf{b}_i = \mathbf{e}_{w_i^*}$, where $\mathbf{e}_j$ denotes a $W \times 1$ elementary vector with all entries 0 except for the $j^{th}$ entry, which takes the value 1. In the next iteration, CU $i$’s actual AP association decision is made according to a $W \times 1$ probability vector $\beta_i^t$, which is properly updated in each iteration according to $\mathbf{b}_i$. We also suppose that each CU has a length $M$ memory, operated in a first in first out (FIFO) fashion, that records its last $M$ best reply vectors.

The proposed algorithm is detailed as follows.

---

6This assumption is without loss of generality because if two APs have the same “distance” to a CU, they can be further ranked by other closeness criterion.
1) Initialization: Let \( t=0 \), CUs randomly choose their APs.

2) Calculation of the NE Power Allocation Profile: Based on the current association \( a^t \), all the CUs calculate their NE power allocations \( p^*_i(a^t) \), either by A-IWF or S-IWF algorithm. We call the process of reaching such intermediate equilibrium an “inner loop”.

3) Selection of the Best AP Association: Each CU \( i \) talks to all the APs in the network, obtains necessary information in order to find a set of APs \( W^t_i \) such that all \( w \in W^t_i \) satisfies \( w \neq a^t(i) \) and:

\[
\max_{p_i,w \in F_i,w} R_i(p_{i,w}, p^*_w(a^t); w) > R_i(p_i^*(a^t), p_{-i}^*(a^t); a^t(i)). \tag{24}
\]

If \( W^t_i \neq \emptyset \), obtain the \( w^*_i \in W^t_i \) that can offer the maximum rate (ties are randomly broken); otherwise, let \( w^*_i = a^t(i) \). Set the best reply vector \( b^t+1_i = e_{w^*_i} \).

4) Update Probability Vector: For each CU \( i \), update the \( W \times 1 \) probability vector \( \beta^t_i \) according to:

\[
\beta^{t+1}_i = \begin{cases} 
\beta^t_i + \frac{1}{M}(b^t+1_i - b^t-M_i) & \text{if } M \leq t \\
\beta^t_i + \frac{1}{M}(b^t+1_i - b^1_i) & \text{if } M > t > 0 \\
b^1_i & \text{if } t = 0.
\end{cases} \tag{25}
\]

Shift \( b^t+1_i \) into the end of the memory; shift \( b^t-M_i \) out from the front of the memory if \( t \geq M \).

5) Determine the Next AP Association: Each CU \( i \) samples the AP index for association at next iteration according to the probability \( \beta^t+1_i \), i.e.,

\[
a^{t+1}(i) \sim \text{multi}(\beta^t+1_i) \tag{26}
\]

where \( \text{multi(.)} \) represents a multinomial distribution.

6) Continue: Let \( t = t+1 \), and go to Step 2).

We make several comments regarding to the above JASPA algorithm.

Remark 1: It is crucial that each CU finally decides on choosing a single AP for transmission. Failing to do so will result in system instability, in which the CUs switch AP association indefinitely, and much of the system resource will be wasted for closing old connections and re-establishing new connections between the APs and CUs. In another word, it is preferable that for all \( i \in \mathcal{N} \), \( \lim_{t \to \infty} \beta^t_i = \beta^*_i \) and:

\[
\beta^*_i(w) = \begin{cases} 
1 & \text{for a single } w \in \mathcal{W} \\
0 & \text{otherwise.}
\end{cases} \tag{27}
\]

Remark 2: The best reply vectors \( \{b^t+1_i\}_{i \in \mathcal{N}} \) are decided in each iteration based on the other CUs’ AP associations and power profiles in the previous iteration. It is straightforward to show that in order to calculate \( \max_{p_i,w \in F_i,w} R_i(p_{i,w}, p^*_w(a^t); w) \) for different \( w \in \mathcal{W} \), individual CU \( i \) does not need to know the strategies of all other CUs in the network, nor does it need to know the system association profile
Instead, it only requires the information of aggregated interference plus noise on each channel from each AP of the last iteration. This is precisely the necessary information needed for finding the set \( W_t \) in Step 3) of the JASPA. This property of the algorithm contributes to the reduction of the amount of messages exchanged between APs and each CU when making association decisions.

Remark 3: Considering the overhead regarding to end an old connection and re-establish a new connection, it is reasonable to assume that a selfish CU is unwilling to abandon its current AP if the new one cannot offer significant improvement of the data rate. We can model such unwillingness of the CUs by introducing a connection cost \( c_i \geq 0 \), which is a private parameter for each CU \( i \). A CU \( i \) will only switch to a new AP if the new one can offer rate improvement of at least \( c_i \), i.e., it will only switch to those APs \( w \in W_t \) that satisfies:

\[
\max_{p_{i,w} \in F_{i,w}} R_i(p_{i,w}, p_w^*(a^t); w) \geq R_i(p_i^*(a^t), p_{\ast-1}(a^t); a^t(i)) + c_i.
\]

From a system point of view, such unwillingness to switch by the CUs might contribute to improved convergence speed of the algorithm, but might also result in reduced system throughput. These two phenomena are indeed observed in our simulations, please see section VII for examples. We note that the equilibrium solution resulted from using the costs \( \{c_i\}_{i \in \mathcal{N}} \) is closely related to the notion of “\( \epsilon \)-equilibrium” in the game theory. See chapter 4 of [25] for details.

C. Proof of Convergence

In this section, we prove that the JASPA algorithm converges to a JEP globally, i.e., the algorithm converges regardless of the initial starting points of the algorithm, or the realizations of the channel gains.

We first introduce some notations. Let \( c_i^t \) be a vector denoting the best reply association profile at time \( t \), i.e., \( c_i^t(i) = w \) if and only if \( b_i^t(w) = 1 \). Define a set \( C \) and \( A \) as follows: \( c \in C \iff c \) infinitely often in \( \{c_i^t\}_{t=1}^\infty \), and \( a \in A \iff a \) infinitely often in \( \{a_i^t\}_{t=1}^\infty \). We first provide a proposition stating that there must exist a NE association profile \( a^* \) that satisfies \( a^* \in A \). The proof of this proposition can be found in Appendix A.

Proposition 1: Choose \( M \geq N \). Then at least one element in the set \( A \), say \( a^* \), is a NE association profile. Moreover, \((a^*, p^*(a^*)) \) is a JEP (satisfy equation (6)).

Using the result in Proposition 1 we obtain the following convergence results.

Theorem 2: When choosing \( M \geq N \), the JASPA algorithm produces a sequence \( \{(a_i^t, p_i^*(a_i^t))\}_{t=1}^\infty \) that converges to a JEP \((a^*, p^*(a^*))\) with probability 1.
Proof: We first show that the sequence \( \{a^t\}_{t=1}^{\infty} \) converges to an equilibrium profile \( a^* \). Notice that if at time \( T \), \( a^T = a^* \), and in the next \( M \) iterations, we always have \( a^{T+t} = a^* \), \( t = 1, \ldots, M \), then the algorithm converges.

Let \( A^* \subset A \) contains all the NE association profiles in \( A \). Let \( \{a^{t(k)}: k \geq 1\} \) be the infinite subsequence satisfying \( a^{t(k)} \in A^* \). Without loss of generality, assume \( t(k) - t(k-1) \geq M \). Let us denote by \( C_k \) the event in which the process converges to an \( a^* \) \( \in \) \( A^* \), after a sequence of best replies equals to \( a^* \) of length \( M \) occurs, starting at time \( t(k) \): \( C_k = \bigcap_{l=1}^{M} \{a^{t(k)+l} = a^*\} \). Note, \( Pr\left(\bigcap_{k \geq 1} C^c_k \bigg| C^c \right) \geq \left(\frac{1}{M}\right)^N \times M \), because whenever \( a^* \) appears, each CU \( i \)'s best reply should be \( a^*(i) \), hence \( a^*(i) \) will be inserted into the last slot of CU \( i \)'s memory. Then with probability \( \left(\frac{1}{M}\right)^N \), all CUs sample the last memory and \( a^* \) will appear in the next iteration. Thus,

\[
Pr\left(\bigcap_{k \geq 1} C^c_k \right) = \lim_{T \to \infty} Pr\left(\bigcap_{k=1}^{T} C^c_k \right) = \lim_{T \to \infty} \prod_{k=1}^{T-1} (1 - Pr(C_{k+1}|C_k)) 
\leq \lim_{T \to \infty} \left(1 - \left(\frac{1}{M}\right)^N \times M \right)^{T-1} = 0.
\]

(28)

This says \( Pr(a^t \text{ converges to a } a^* \in A^* \text{ eventually}) = 1 \). Finally, because \( p^*(a^*) \in E(a^*) \) is a NE power allocation profile, we conclude that \( (a^*, p^*(a^*)) \) is a JEP.

We mention that the requirement on the length of the memory is technical in order to facilitate the proof. In simulations, we observe that such requirement is not necessary for ensuring convergence.

Now that we have shown the convergence of the JASAP to the JEP, it is of interest to evaluate the “quality” of such network equilibrium. In this work, we use the system throughput to measure the quality of the JEP, and our simulation results (to be shown in section VII) are very encouraging.

V. EXTENSIONS TO THE JASPA ALGORITHM

The JASPA algorithm presented in the previous section is “distributed” in the sense that the computation that each CU needs to carry out in each iteration only requires some local/summary information, i.e., the aggregated interference plus noise at different APs in different channels, and the CU’s own channel gain. However, this algorithm requires that for each AP association profile \( a^t \), an intermediate equilibrium \( p^*(a^t) \) should be reached, and at each iteration \( t \) the CUs cannot choose their next AP association profile until the system reaches such equilibrium. This requirement poses a relatively strong level of coordination among the CUs (although this issue can be alleviated by letting the APs orchestrate the updating instances), which is not entirely desirable for a distributed algorithm.

In this section, we propose two algorithms that do not require that the CUs reach any intermediate equilibria. Specifically, we propose 1) a sequential version of the JASPA algorithm (Se-JASPA) in which
CUs act one by one in each step, and 2) a simultaneous/parallel version of the JASPA algorithm (Si-JASPA) in which CUs act at the same time.

The Se-JASPA algorithm is detailed in Table I.

1) Initialization (t=0): Each CU randomly chooses $a^0(i)$ and $p^0_{i,a^0(i)}$
2) Determine the Next AP Association:
   If it is CU $i$’s turn to act, (e.g., $\{(t+1)\mod N\} + 1 = i$), then CU $i$
   finds a set $W^*_i$ s.t.:
   
   $W^*_i = \arg\max_w \max_{p_{i,w} \in F_{i,w}} R(p_{i,w}, p^*_i; w)$
   Then it selects an AP by randomly picking $w^* \in W^*_i$ and setting
   $a^{t+1}(i) = w^*$.
   For other CUs $j \neq i$, $a^{t+1}(j) = a^t(j)$
3) Update the Power Allocation:
   Denote $w^* = a^{t+1}(i)$, Then CU $i$ calculates $p^{t+1}_i$ as
   
   $p^{t+1}_i = \begin{cases} 
   \arg\max_{p_{i,w^*} \in F_{i,w^*}} R_i(p_{i,w^*}, p^*_i; w^*), & \text{if } w^* \neq a^t(i) \\
   \arg\max_{p_{i,w^*} \in F_{i,w^*}} R_i(p_{i,w^*}, p^*_i; w^*), & \text{otherwise}
   \end{cases}$
   For other CUs $j \neq i$, $p^{t+1}_j = p^t_j$
4) Continue: Let $t\to t+1$, and go to Step 2)

**TABLE I**

**THE SE-JASPA ALGORITHM**

We partially characterize the convergence behavior of Se-JASPA algorithm in the following theorem, the proof of which can be found in Appendix B.

**Theorem 3:** The sequence of system potential $\{P(p^t, a^t)\}_{t=1}^\infty$ produced by the Se-JASPA algorithm is non-decreasing and converging.

Some brief comments regarding to the Se-JASPA algorithm is in order. We see that the Se-JASPA algorithm differs from the JASPA algorithm in several important ways. Firstly, a CU $i$ does not need to keep its best reply vector $b^t$ as it does in JASPA. It decides on its AP association greedily in step 2).

Secondly, a CU $i$, after deciding a new AP $a^{t+1}(i) = w^*$, does not need to go through the process of reaching an intermediate equilibrium with all other CUs to obtain $p^{t+1}_{i,w^*}$. However, the CUs still need to be coordinated for the exact sequence of their update, because in each iteration only a single CU is allowed to act. Such order of update can be agreed upon and enforced by the APs in the network. As might be inferred by the sequential nature of this algorithm, when the number of CUs is large, the convergence becomes slow.

The Si-JASPA algorithm, as detailed in Table III, overcomes the above difficulties encountered in Se-JASPA. We note that in the algorithm, the variable $T^i_t$ represents the duration that CU $i$ has stayed in the
1) Initialization (t=0): Each CU \( i \) randomly chooses \( a_i^0 \) and \( p_i^0 \).

2) Selection of the Best Reply Association:
Each CU obtains the AP \( w_i^* \) and set \( b_i^{t+1} \) following Step 3) of JASPA.

3) Update Probability Vector:
Each CU \( i \) updates the probability vector \( \beta_i^t \) according to (25).
Shift \( b_i^{t+1} \) into the memory; shift \( b_i^{t-M} \) out of memory if \( t \geq M \).

4) Determine the Next AP Association:
Each CU \( i \) samples the AP index for association as in (26).

5) Compute the Best Reply Power Allocation:
Let \( w_i^{t+1} = a_i^{t+1} \). Each CU \( i \) calculates \( p_i^* \) as
\[
p_i^* = \max_{p_i, w_i^{t+1}} R_i(p_i, w_i^{t+1}, p_i^{t-M}, w_i^{t-M}); w_i^{t+1}\]

6) Update the Duration of Stay:
Each CU \( i \) maintains and updates a variable \( T_i \):
\[
T_i = \begin{cases} 
1 & \text{if } a_i^{t+1} \neq a_i^t \\
T_i + 1 & \text{if } a_i^{t+1} = a_i^t
\end{cases}
\]

7) Update the Power Allocation:
Each CU \( i \) calculates \( p_i^{t+1} \) as follows:
\[
p_i^{t+1} = \begin{cases} 
p_i^* & \text{if } a_i^{t+1} \neq a_i^t \\
(1 - \alpha_{T_i})p_i^t + \alpha_{T_i}p_i^* & \text{if } a_i^{t+1} = a_i^t
\end{cases}
\]

8) Continue: Let \( t = t+1 \), and go to Step 2)

---

**TABLE II**

| **THE SI-JASPA ALGORITHM** |

The Si-JASPA is almost the same as the JASPA except that each CU, after switching to a new AP, does not need to go through the process of joint computation of the intermediate equilibrium solution. Instead, the CUs can make their AP decision “continuously”. The level of coordination among the CUs required for this algorithm is minimum among all the three algorithms introduced so far. The simultaneous update required by this algorithm can be realized by either one of the following approaches:

- The APs agree upon the update interval off-line. Each CU is equipped with a timer. The first time a CU comes into the system, it is informed by its initial associated AP the update interval and the next update instance. After that, this CU can perform update on its own.
- The APs agree upon the update interval off-line. When the time comes for the update, the APs...
individually alert the CUs associated with them by broadcasting.

Extensive simulations suggest that this algorithm converges faster than the Se-JASPA.

VI. JASPA BASED ON NETWORK-WIDE JOINT-STRATEGY

The Se/Si-JASPA algorithms introduced in the previous section relieves the CUs from the burden of reaching intermediate equilibrium. However, the lack of general proof of convergence for them might be a concern to us (although they appear to be always convergent in practice). In this section, an alternative algorithm with convergence guarantee is proposed. This algorithm allows the CUs, as in the Se/Si-JSPA, to jointly select their power profiles and AP association without the need to reach the intermediate equilibria. We will see later that compared with all the algorithms introduced previously, the algorithm studied in this section requires considerably different information/memory structure for both the CUs and the APs. Among others, it requires that the CUs maintain in their memory some history of the network-wide joint strategy of all CUs. We henceforth name this algorithm Joint-strategy JASPA (J-JASPA).

A. The J-JASPA Algorithm

We first give some definitions. As all previously mentioned algorithms, the J-JASPA algorithm is iterative in nature, thus in the following we use \( t \) to denote the \( t^{th} \) iteration of the algorithm, if needed.

- Let \( N^t_w \triangleq \{ i : a^t(i) = w \} \) be the set of CUs that are associated with AP \( w \) in iteration \( t \).
- Let \( Q \) be any subset of \( N \). Define the last time that the subset \( Q \) of CUs is associated with a particular AP \( w \) as \( \widehat{t}^w_w(\mathcal{Q}) \), i.e.,

\[
\widehat{t}^w_w(\mathcal{Q}) = \begin{cases} 
\arg \max_i \{ N^t_w = \mathcal{Q} \} & \text{if } \bigcup_{t \geq 0} \{ N^t_w = \mathcal{Q} \} \neq \emptyset \\
\infty & \text{otherwise.} 
\end{cases} 
\tag{30}
\]

- Let \( I(N^t_w) \) be the joint interference profile by the subset of CUs \( N^t_w \) that is associated with AP \( w \):

\[
I(N^t_w) \triangleq \{ I^t_{i,w} \}_{i \in N^t_w}, \quad \text{where } I^t_{i,w} \text{ is defined in (3)}.
\]

As we mentioned before, one of the distinct feature of J-JASPA algorithm is the information/memory structure required for carrying out the computation. Specifically, each CU \( i \) keeps three different memories, each of which is of length \( M \) and operates in a FIFO fashion. The first memory, referred to as association memory (AM), records CU \( i \)'s last \( M \) associated AP \( \{ a^t(i) \}^T_{t=T-M+1} \), i.e., \( AM_i(m) = a^{T-M+m}(i) \). Here we use \( AM_i(m) \) to denote \( m^{th} \) element in CU \( i \)'s AM. The second memory, referred to as interference memory (IM), records the last \( M \) system interference levels for CU \( i \), \( \{ I^t_{i,w} \}_{t-T-M+1} \), where \( I^t_{i,w} \triangleq \{ I^t_{i,w} \}_{w \in \mathcal{V}} \). The third memory, referred to as rate memory, records the last \( M \) CU \( i \)'s sum rate, \( \{ R^T_i \left( p^{a^t(i)}_{a^t(i)} ; a^t(i) \right) \}_{t=T-M+1} \).
Each AP $w$ is also required to keep track of some local quantities regarding to the history of the CU behaviors. Specifically, AP $w$ keeps track of the following variables for each subset $Q \subseteq \mathcal{N}$ that has been associated with AP $w$ during time $[0, T]$ at least once:

- The local power profile $p(Q) = \left\{ p_i^w(Q) \right\}_{i \in \mathcal{N}_w}$.
- The local interference profile $I(Q) = \left\{ I_{i,w}^w(Q) \right\}_{i \in \mathcal{N}_w}$.
- The total number of times that $Q$ has been played: $\hat{T}(Q) = \sum_{t \leq T} 1\{ N_w^t = Q \}$, where $1\{.\}$ is the indicator function.

Then the J-JASPA algorithm can be detailed as follows:

1) **Initialization:** Let $t = 0$, each CU $i$ randomly chooses the $a_0(i) \in \mathcal{W}$ and $p_0^i \in \mathcal{P}_i, a_0(i)$.

2) **Update CU Memory:** For each $i \in \mathcal{N}$, talk to all AP in the system and obtain $I_i^t$. Shift $a'_t(i)$, $I_i^t$, and $R_i^t \left( p_i^{t, a'_t(i)}, p_{-i, a'_t(i)}; a'^t(i) \right)$ into the end of the AM, IM and RM, respectively. If $t > M$, shift the first element of the AM, IM and RM out of the memory.

3) **Update AP Memory:** For each $w \in \mathcal{W}$, update the vectors: $p(\mathcal{N}_w^t) = \left\{ p_{i,w}^t \right\}_{i \in \mathcal{N}_w^t} : I(\mathcal{N}_w^t) = \left\{ I_{i,w}^t \right\}_{i \in \mathcal{N}_w^t}$.

4) **Sample Memory:** Let $\hat{M} = \min\{M, t\}$, each CU $i$ uniformly samples its association memory:

$$\text{Sample index}^t_i = \text{multi}\left( \frac{1}{\hat{M}}, \ldots, \frac{1}{\hat{M}} \right);$$

Let $\tilde{a}^t_i = AM_i(\text{index}^t_i)$, $\hat{I}_i^t = IM_i(\text{index}^t_i)$, $\hat{R}_i^t = RM_i(\text{index}^t_i)$. \hspace{1cm} (31)

5) **Calculate Best AP Association:** Each CU $i$ finds association according to $\tilde{a}^t_i$, $\hat{I}_i^t$ and $\hat{R}_i^t$, i.e., find the set of APs $\mathcal{W}_i^*$ such that:

$$\mathcal{W}_i^* = \left\{ w : \max_{p_{i,w} \in \mathcal{P}_i,w} R_i^t \left( p_{i,w}, \hat{I}_{i,w}^t ; w \right) > \hat{R}_i^t \right\} \cup \tilde{a}_i^t. \hspace{1cm} (32)$$

Then randomly pick $w^* \in \mathcal{W}_i^*$, and set $a^{t+1}_i(i) = w^*$.

6) **Calculate Power Allocation:** Each CU $i$ switches to AP $a^{t+1}_i(i)$. Let $w = a^{t+1}_i(i)$, then CU $i$ obtain the following quantities from this AP $w$: $\hat{T}(\mathcal{N}_w^{t+1})$, $I_i(\mathcal{N}_w^{t+1})$, and $p_i(\mathcal{N}_w^{t+1})$. If $\hat{T}(\mathcal{N}_w^{t+1}) \geq 1$ (the set of CUs $\mathcal{N}_w$ has been associated with AP $w$ at the same time before), let $\alpha = \alpha \hat{T}(\mathcal{N}_w^{t+1})$, and:

$$p_i^{t+1} = (1 - \hat{\alpha}) p_i(\mathcal{N}_w^{t+1}) + \hat{\alpha} \Phi_i \left( I_i(\mathcal{N}_w^{t+1}) \right). \hspace{1cm} (33)$$

\text{Here, we use “local” to signify the fact that individual AP can gather these information without the need to communicate with other APs.}
If \( \hat{T}(N^t_w + 1) = \infty \) (the set of CUs \( N_w \) has not been associated with AP \( w \) at the same time before), randomly pick \( p^{t+1}_i \in F_{i,w^{t+1}(i)} \).

7) **Continue:** Let \( t = t + 1 \), go to step 2).

We see that although algorithmically the J-JASPA is similar to the previously introduced algorithms in the sense that the AP associations are decided probabilistically, and the power profiles are computed based on historical profiles and newly computed components \( \Phi(.) \), there are several significant differences between the J-JASPA and the previously introduced algorithms.

1) In J-JASPA, each CU calculates its best AP association according to a *sampled historical* network state, while in the JASPA and Si-JASPA, it calculates this quantity according to the *current* network state.

2) In J-JASPA, CUs’ AP association is the same as their *best AP association*, while in Si-JASPA and JASPA, their AP association is *sampled from the memory*.

3) This algorithm requires APs to have memory. Each AP needs to record the local power allocation and interference profiles for *all* the different sets of CUs that have been associated with it in the previous iterations, while in the previously introduced algorithms, the APs do not need to have memory.

4) The J-JASPA requires larger memory for the CUs for constructing AM, IM and RM.

5) The J-JASPA requires extra communications between the CUs and the APs (mainly in step 6).

We will see in the next subsection that it is exactly these changes in the algorithm and the extra requirements in terms of memory and communication that enables the J-JASPA to have provable convergence guarantees without the need to reach the intermediate equilibria. This is a significant improvement compared with the original JASPA, which does need the CUs to reach the intermediate equilibria, and the Se/Si-JASPA, for which we are not able to provide complete convergence proofs. We have also observed in simulation that J-JASPA converges faster than Se/Si-JASPA.

**B. The convergence of the J-JASPA algorithm**

In this subsection, we show that the J-JASPA algorithm converges to a JEP.

Define the set \( \mathcal{A} \) as follows: \( \mathbf{a} \in \mathcal{A} \iff \mathbf{a} \) infinitely often in \( \{ \mathbf{a}^t \}_{t=1}^{\infty} \). We first provide a proposition characterizing the power profiles of the CUs in the network every time a profile \( \mathbf{a} \in \mathcal{A} \) appears.

**Proposition 2:** Choose \( \mathbf{a} \in \mathcal{A} \). Let \( \{t(n)\}_{n=1}^{\infty} \) be the subsequence of \( \{t\}_{t=1}^{\infty} \) such that \( \mathbf{a} \) is played, i.e., \( \{t(n) : \mathbf{a}^{t(n)} = \mathbf{a}\} \). Then we have for all \( w \in \mathcal{W} \), \( \lim_{n \to \infty} p^{t(n)}_w = p^*_w(\mathbf{a}) \), where \( p^*_w(\mathbf{a}) \) is a NE power allocation profile for AP \( w \) under \( \mathbf{a} \), i.e., \( p^*_w(\mathbf{a}) \in \mathcal{E}_w(\mathbf{a}) \). Furthermore, we have that \( \lim_{n \to \infty} \mathcal{P}_w(\mathbf{p}^{t(n)}_w, \mathbf{a}^{t(n)}) = \tilde{P}_w(\mathbf{a}) \) and \( \lim_{n \to \infty} P(\mathbf{p}^{t(n)}, \mathbf{a}^{t(n)}) = \tilde{P}(\mathbf{a}) \).
Proof: For a \( w \in W \), let \( \mathcal{N}_w = \{ i : a(i) = w \} \). Define another subsequence \( \{ \hat{t}(n) \}_{n=1}^{\infty} \) in which the subset of CUs \( \mathcal{N}_w \) is associated with AP \( w \). Clearly, \( \{ t(n) \}_{n=1}^{\infty} \) is a subsequence of \( \{ \hat{t}(n) \}_{n=1}^{\infty} \). From the J-JASPA algorithm, we see that at each \( \hat{t}(n) \), (33) implements the single AP A-WF (cf. Algorithm 1 in [2]) with the fixed set of CUs \( \mathcal{N}_w \). Thus, from Proposition 2 in [2] we have that the subsequence \( \{ p^{\hat{t}(n)}_w \}_{n=1}^{\infty} \) converges to \( p^*_w(a) \in \mathcal{E}_w(a) \), which is a NE power allocation profile under fixed system association profile \( a \). Consequently, the infinite sub-subsequence \( \{ p^{\hat{t}(n)}_w \}_{n=0}^{\infty} \) also converges to the same \( p^*_w(a) \). From Corollary [2], we have that \( \lim_{n \to \infty} P_w(p^{\hat{t}(n)}_w; a^{\hat{t}(n)}) = P_w(a) \) and \( \lim_{n \to \infty} P(p^{\hat{t}(n)}_w, a^{\hat{t}(n)}) = P(a) \). \( \blacksquare \)

We need the following set of definitions to proceed. Let \( \hat{a}^t \) be the sampled system profile at time \( t \): \( \hat{a}^t(i) \triangleq \tilde{a}^t_i \), \( \forall i \). For a specific \( a \), define the subsequence \( \{ t(n, a) \} \) to be the time instances that \( a \) appears and is immediately sampled by all the CUs, i.e., \( \{ t(n, a) : a^{t(n,a)} = a \text{ and } \tilde{a}^{t(n,a)} = a \} \). Note that if \( a^t = a \), then according to step 5) of the J-JASPA algorithm, with positive probability \( \hat{a}^t = a \). Thus, if \( a \in A \), then \( \{ t(n, a) \} \) is a infinite sequence. Define \( R^*_i \left( \tilde{I}^{t(i)}_i; w \right) \triangleq \max_{p_{i,w} \in \mathcal{F}_{i,w}} R_i(p_{i,w}, \tilde{I}^{t(i)}_i; w) \) to be the maximum rate CU \( i \) can achieve in AP \( w \) based on sampled interference \( \tilde{I}^{t(i)}_i \). Define a set:

\[
B_i \left( \tilde{I}^{i}_i, \hat{a}^t(i) \right) \triangleq \left\{ w : \bar{R}^*_i \left( \tilde{I}^{t(i)}_i; w \right) > \tilde{R}^*_i \right\} \cup \hat{a}^t(i)
\]  

(34)

where \( \tilde{R}^*_i \) is defined in (31). We call the set \( B_i \left( \tilde{I}^{i}_i, \hat{a}^t(i) \right) \) CU \( i \)'s best association set at time \( t \). From step 5) of the J-JASPA algorithm, all \( w \in B_i \left( \tilde{I}^{i}_i, \hat{a}^t(i) \right) \) has positive probability to be picked by CU \( i \) in iteration \( t + 1 \). Let \( B_i \left( \tilde{I}^{i}_i(a), a(i) \right) \triangleq \lim_{n \to \infty} B_i \left( \tilde{I}^{t(n,a)}_i, \hat{a}^{t(n,a)}(i) \right) \). We provide a technical characterization of the best association set. See Appendix C for proof.

Proposition 3: For a specific CU \( i \) and a system association profile \( a \in A \), suppose there exists a \( w \neq a(i) \) such that \( w \in B_i \left( \tilde{I}^{i}_i(a), a(i) \right) \), i.e., CU \( i \) has the incentive to move to a different AP in the limit. Then there exists a large enough constant \( N^*_i(a) \) such that for all \( n > N^*_i(a) \), we have:

\[
B_i \left( \tilde{I}^{i}_i(a), a(i) \right) \subseteq B_i \left( \tilde{I}^{t(n,a)}_i, \hat{a}^{t(n,a)}(i) \right).
\]  

(35)

In words, this propositions says that suppose a specific association profile \( a \) happens infinitely often (hence is sampled infinitely often), and suppose in the limit, when \( a \) is sampled, a CU \( i \) prefers a \( w \neq a(i) \), then after iteration \( t(N^*_i(a), a) \), it must prefer \( w \) in every time instance \( t(n, a) \), where \( n \geq N^*_i(a) \). Now we are ready to provide the main result for the J-JASPA algorithm.

Theorem 4: The J-JASPA algorithm converges to a JEP with probability 1.

Proof: Consider the sequence \( \{(a^t, p^t)\}_{t=1}^{\infty} \). Choose \( \tilde{a} \) to be any system association profile that satisfies the following: \( \tilde{a} \in \arg \max_{a \in A} \{ P(a) \} \). We first show that \((\tilde{a}, p^*(\tilde{a}))\) is a JEP.

\( ^8 \)Note that such limit exist because of Proposition [2]
Suppose \((\overline{a}, p^*(\overline{a}))\) is not a JEP, then there exists a CU \(i\), and a \(\overline{w} \neq \overline{a}(\overline{i})\) such that \(\overline{w} \in B^*_i(\overline{I}^*_{\overline{i}, \overline{a}(\overline{i})}; \overline{I}^*_{\overline{i}, \overline{a}(\overline{i})})\).

This implies that there exists an \(\varepsilon > 0\) such that:

\[
R^*_{\overline{i}}(\overline{I}^*_{\overline{i}, \overline{w}}; \overline{w}) - R^*_1(\overline{I}^*_1, \overline{a}(\overline{i}); \overline{a}(\overline{i})) \geq \varepsilon. \tag{36}
\]

Define a new association profile \(\overline{a}\) as:

\[
\overline{a} = \begin{cases} 
\overline{a}(j) & \text{for } j \neq \overline{i} \\
\overline{w} & \text{for } j = \overline{i}.
\end{cases} \tag{37}
\]

Following the steps we already went through in Theorem 1 from (12) to (23), we can show that:

\[
\hat{P}(\overline{a}) < \hat{P}(\overline{a}). \tag{38}
\]

It is clear that if \(\overline{a} \in \mathcal{A}\), then the above is a contradiction to the assumption that \(\overline{a} \in \text{arg max}_{a \in \mathcal{A}} \{\hat{P}(a)\}\). In the following, we show that \(\overline{a} \in \mathcal{A}\), thus completing the proof.

From Proposition 3 there exists a \(N^*_1(\overline{a})\) large enough that for all \(n > N^*_1(\overline{a}), \overline{w} \in B^*_1(\overline{I}^{t(n,\overline{a}), \overline{a}(t(n,\overline{a})}(i))\).

Take any \(n > N^*_1(\overline{a})\). We know that from the definition, in iteration \(t(n, \overline{a}), \overline{a}^{t(n,\overline{a})}(i) = \overline{a}, \forall \ i \in \mathcal{N}\). From Step 5 in the J-JASPA algorithm, we see that with positive probability, in iteration \(t(n, \overline{a}) + 1\), CU \(j \neq \overline{i}\) chooses to stay in \(\overline{a}(j)\), and CU \(\overline{i}\) chooses to switch to \(\overline{w}\). This implies that the association profile \(\overline{a}\) happens with positive probability in every instance \(t(n, \overline{a}) + 1\). Because \(\{t(n, \overline{a})\}\) is a infinite sequence, \(\overline{a}\) happens infinitely often, i.e., \(\overline{a} \in \mathcal{A}\).

In summary, we conclude that \(\overline{a}\) must be a NE association profile, and thus, \((\overline{a}, p^*(\overline{a}))\) is a JEP.

Finally, following the proofs of Theorem 2 we can show similarly that the sequence \(%(a^t, p(a^t))\%_{t=1}^\infty\) that produced by J-JASPA converges to a JEP with probability 1.

\[\blacksquare\]

VII. Simulation Results

In this section, we present various simulation results to validate the proposed algorithms. We first show the results regarding to the convergence properties, and then present the results regarding to the system throughput performance. Due to the space limit, for each experiment we show the results obtained by running either Si/Se-JASPA and J-JASPA, or the results obtained by the original JASPA.

We have the following general settings for the simulation. We place multiple CUs and APs randomly in a \(10m \times 10m\) area; we let \(d_{i,w}\) denote the distance between CU \(i\) and AP \(w\), then the channel gains between CU \(i\) and AP \(w\), \(%h_{i,w}(k)\%_{k \in \mathcal{K}_w}\), are independently drawn from an exponential distribution with mean \(\frac{1}{d_{i,w}^2}\) (i.e., \(|h_{i,w}(k)|\) is assumed to have Rayleigh distribution). We let the available channels to be evenly pre-assigned to different APs. When we say a “snapshot” of the network, we refer to the...
network with fixed (but randomly generated as above) AP, CU locations and channel gains. We set the length of the individual memory as $M = 10$. For ease of presentation and comparison, when we use the JASPA algorithm with connection cost, we let all the CUs’ connection cost $\{c_i\}_{i \in \mathcal{N}}$ be identical.

A. Convergence

We only show the results for Si/Se-JASPA and J-JASPA in this subsection. We first consider a network with 20 CUs, 64 channels, and 4 APs. Fig. 1 shows the evolution of the system throughput as well as the values of the system potential function generated by a typical run of the Se-JASPA, Si-JASPA, J-JASPA and Si-JASPA with connection cost $c_i = 3$ bit/sec $\forall i \in \mathcal{N}$. We observe that the Si-JASPA with connection cost converges faster than Si-JASPA and Se-JASPA, while Se-JASPA converges very slowly. After convergence, the system throughput achieved by Si-JASPA with connection cost is smaller than that of the other three algorithms. Notice that in the right part of Fig. 1, the system potential generated by the Se-JASPA is non-decreasing along iterations. This property has been identified in Theorem 3.

Fig. 2 shows the evolution of the AP selections made by the CUs in the network during a typical run of the Si-JASPA algorithm. We only show 3 out of 20 CUs (we refer the selected CUs as CU 1, 2, 3 for easy reference) in order not to make the figure overly crowded. Fig. 2 shows the corresponding evolution of the probability vectors $\{\beta^t_i\}_{i=1}^{200}$ for the three of the CUs selected in Fig. 3. It is clear that upon convergence, all the probability vector converges to a 0-1 vector. We then evaluate how the number of CUs in the network affects the speed of convergence of different algorithms. In order to do so, we compare the average iterations to achieve convergence in the network with 4 APs, 64 channels and different number of CUs, for the following three algorithms 1) Si-JASPA, 2) Se-JASPA, 3) J-JASPA, 4) Si-JASPA with connection cost.
The Value of $\beta_i$:

- $\beta_1^{(1)}$
- $\beta_1^{(2)}$
- $\beta_1^{(3)}$
- $\beta_1^{(4)}$
- $\beta_2^{(1)}$
- $\beta_2^{(2)}$
- $\beta_2^{(3)}$
- $\beta_2^{(4)}$
- $\beta_3^{(1)}$
- $\beta_3^{(2)}$
- $\beta_3^{(3)}$
- $\beta_3^{(4)}$

Fig. 2. Convergence of the probability vector $\beta_t^1, \beta_t^2$ and $\beta_t^3$ of CUs 1, 2, 3.

connection cost $c_i = 3$ bit/sec for all CUs. From Fig. 4, we see that when the number of CUs in the system becomes large, the sequential version of the JASPA takes significantly longer time to converge than the other three simultaneous versions of the JASPA algorithm. Moreover, the J-JASPA shows faster convergence than the Si/Se-JASPA. We can also see that the connection costs adopted by individual CUs indeed have positive effects on the convergence speed of the system.

Note that each point in this figure represents the average of 100 independent runs of each algorithm on randomly generated network snapshots.

B. System Throughput Performance

We then evaluate the network throughput performance achievable by the JEP computed by the JASPA. We first investigate a small networks with 8 CUs, 64 channels and 1, 2, 3, 4 APs, and compare the performance of JASPA related algorithms to the maximum network throughput that can be achieved for the same network. The maximum network throughput for a snapshot of the network is calculated by the following two steps: 1) for a specific AP-CU association profile, say $\mathbf{a}$, calculate the maximum
network throughput (denoted by $T(a)$) by summing up the maximum capacity of individual APs in the network; 2) enumerate all possible AP-CU association profiles, and find $T^* = \max_a T(a)$. It is clear now that the reason we choose to focus on such relatively small networks in this experiment is that for a large network, the time it takes for the above exhaustive search procedure to find the maximum network throughput becomes prohibitive.

The result is shown in Fig. 5, where each point on the figure is obtained by running the algorithms on 100 independent snapshots of the network. We see that the JASPA algorithm performs very well with little throughput loss, while the closest AP algorithm, which separates the tasks of spectrum decision and spectrum sharing, performs poorly.

We then start to look at the performance of larger networks with 30 CUs, up to 16 APs and up to 128 channels. Fig. 6 shows the comparison of the performance of JASPA, JASPA with individual cost $c_i = 3$ bit/sec and $c_i = 5$ bit/sec, and the closest AP algorithm mentioned in section IV-A. We adopt the actual distance as the measure of “closeness” in the closest AP algorithm. Each point in this figure is the average of 100 independent runs of the algorithms.

Due to the prohibitive computation time required, we are unable to obtain the maximum system throughput for these relatively large networks. We instead compute the equilibrium system throughput that can be achieved in a game if all CUs are able to connect to multiple APs at the same time. We refer to this as the multiple-connectivity network. It is clear that in such network, there is no need for the CUs

\[ \text{Multiple access channel sum capacity.} \]

For a single AP with fixed number of users and channel gains, the maximum capacity is the well-known multiple access channel sum capacity.
to perform the AP selection, and the CUs in this network enjoy the flexibility of being able to connect to multiple APs at the same time. However, we observe that the performance of JASPA is close to that of the “multiple-connectivity” network.

From Fig. 6 we see that when the number of APs increases, the throughput of the JASPA algorithm becomes much better than the closest AP algorithm, a phenomenon that is partly due to the fact that for the closest AP algorithm, the separation of the AP selection and power allocation process results in the insufficient use of the spectrum: when the number of AP increases, it becomes increasingly more probable that several APs are idle because no CUs are close to them. Fig. 6 along with Fig. 4 also serve to confirm our early speculation that algorithms with connection cost can indeed improve the convergence speed while reducing the system throughput.

We also observe from Fig. 6 that generally the system throughput increases as the number of APs increases, which suggests that the scheme that partitions the available spectrum and assigns them to the APs with different geographical locations is indeed more favorable than the scheme which uses a single AP to manage all the spectrum. This phenomenon can be explained partly by reasoning as follows: when using a single AP, it is likely that many of the CUs are located far away from the AP, and thus none of their channels have good quality; on the other hand, when using multiple APs, although each CU can only use part of the available channels, it is more probable that there is one or more APs that are located in its vicinity, thus is able to provide good channel quality. We have to mention here that, although not shown in Fig. 6 placing too many APs in the network may also result in reduced network throughput as the chance of idle APs increases as the number of AP increases (an extreme case is that the number of APs is larger than the number of CUs).

VIII. CONCLUSION

In this paper, we addressed the joint AP association and power allocation problem in a CRN. We formulate the problem into a non-cooperative game with hybrid strategy space. We characterized the NE of this game, and provided distributed algorithms to reach such equilibrium. Empirical evidence gathered from simulation experiments suggests that the equilibrium has very promising quality in term of the system throughput.

There can be many future extensions to this work. First of all, the non-cooperative game with hybrid strategy space analyzed in this paper can be applied to many other problems as well, for example, the CRN with interference channel and segmented spectrum mentioned at the end of section I-A. Secondly, for the problem considered in this work, it is beneficial to characterize quantitatively the efficiency of
the JEP, and to provide solutions for efficiency improvement. Thirdly, it will be interesting to analyze
the effect of time-varying channel gains and the arrival and departure of the CUs on the performance of
the algorithm, and to propose suitable heuristic dealing with these situations.

APPENDIX A

PROOF OF PROPOSITION[1]

Proof: Choose \( c \in C \) and suppose that at time \( t \), \( c^t = c \). If \( c \) is an equilibrium association profile,
then with probability at least \((\frac{1}{M})^N\) (all CUs chooses \( a^t(i) = c^t(i) \)), we have that \( a^t \) is an equilibrium
association profile. Because \( c^t = c \) happens infinitely often, we must have \( a^t = c \) happens infinitely
often, i.e., there exists an equilibrium association in the set \( A \).

Suppose \( c \) is not an equilibrium association. Then consider the following steps of operation.

Step 1): With probability at least \((\frac{1}{M})^N\), \( a^t = c \), and \( c \in A \). Because \( c \) is not a NE, then without loss
of generality, assume that CU \( i \) is better off by switching to \( \hat{w}_i \): \( c^{t+1}(i) = \hat{w}_i \neq c^t(i) \). Then we must have
that \( \hat{P}(a^t) < \hat{P}((\hat{w}_i, a^t_{-i})) \), a fact from (23). Then with probability at least \((\frac{1}{M})^N\) (all player \( j \) except
player \( i \) choose \( a^{t+1}(j) = c^t(j) = a^t(j) \) that \( a^{t+1} = (\hat{w}_i, a^t_{-i}) \), and we have \( \hat{P}(a^{t+1}) > \hat{P}(a^t) \). Put
index \( i \) in the the set \( U : U = \{i\} \). We note in this stage, we have: \( a^{t+1}(i) = c^{t+1}(i) \). Similarly, at \( t + 2 \),
if we are able to find a CU \( j \neq i \) with \( c^{t+2}(j) = \hat{w}_j \neq a^{t+1}(j) \) (i.e. CU \( j \) is better off if switching to AP \( \hat{w}_j \), we let \( U = \{i, j\} \). Then again with positive probability, we have \( a^{t+2} = (\hat{w}_j, a^t_{-j}) \). Consequently,
\( \hat{P}(a^{t+2}) > \hat{P}(a^{t+1}) \). We note in this stage, the following is true: \( a^{t+2}(j) = c^{t+2}(j) \). Continue this
process, until we reach a time \( t + n <= t + N \) such that only CUs in the set \( U \) are willing to switch.
Let \( E \) be the complement set of \( U \).

Step 2): We must have that for \( j \in E \), \( a^{t+n-1}(j) = c^{t+n}(j) \). On the other hand, for all \( i \in U \), from
the argument in Step 1), we see that there must exist a \( 0 < k_i < n \) such that \( c^{t+k_i}(i) = a^{t+n-1}(i) \). Pick
\( q \in U \) such that \( q = \arg \min_{i \in U \cap E^{t+n}(i) \neq a^{t+n-1}(i)} k_i \). Consequently, we can shift \( c^t \) out of the memory
and still be able to construct \( a^{t+n} = (\hat{w}_q, a^{t+n-1}_q) \) with positive probability, because all the elements in \( a^{t+n-1} \)
must have been appeared once in \( \{c^t\}_{t=t+1} \). Move \( q \) out of \( U \) and into \( E \), and continue Step 2)
until only CUs in the set \( E \) are willing to switch. Switch the role of \( U \) and \( E \), and continue Step 2).

By continuously performing the above operations, \( \{\hat{P}(a^t)\}_{t=1}^\infty \) is a strictly increasing sequence, and
there must exist a finite time instance \( T < \infty \) such that it is not possible to find \( a^{t+T+1} \) that differs
with \( a^{t+T} \) with a single element and has the property \( \hat{P}(a^{t+T+1}) > \hat{P}(a^{t+T}) \). Consequently, \( a^{t+T} \) is an
equilibrium profile. We let \( a^* = a^{t+T} \).The finiteness of \( T \) comes from the finiteness of the number of values of \( \hat{P}(a) \) (due to the finiteness of the choice of \( a \)). Such finiteness combined with the strict positivity
of the probability of performing each operation in Step 1) and Step 2) implies that the probability of reaching \(a^*\) from \(c\) is non-zero.

We conclude from the above analysis that with positive probability, a NE profile \(a^*\) will appear after \(a^t\) in finite steps. Because \(a^t = c\) happens infinitely often, we must also have that \(a^*\) happens infinitely often, i.e., \(a^* \in A\).

Finally, it is straightforward to see that the fact that \(a^*\) is an equilibrium association profile suggests that the tuple \((a^*, p^*(a^*))\) is a JEP.

**APPENDIX B**

**PROOF OF THEOREM**

Proof: Suppose that at time \(t+1\), it is CU \(i\)'s turn to move. Let \(w = a^t(i)\) be the CU \(i\)'s associated AP at time \(t\). We have the following two situations.

1) **At time \(t+1\), CU \(i\) is best off switching to \(\hat{w} \neq a^t(i)\).** In this situation, CU \(i\)'s communication rate at time \(t\) under association profile \(a^t\) is as follows:

\[
R_i(p_{i,w}^t, p_{i,-w}^t; w) = \sum_{k \in K_w} \log \left( 1 + \frac{|h_{i,w}(k)|^2 p_{i,w}(k)}{n_w(k) + \sum_{j : a^t(j) = w} |h_{i,w}(k)|^2 p_{j,w}(k)} \right)
\]

\[
\overset{(a)}{=} P_w(p_i^t; a^t) - P_w(p_i^t; a^{t+1}) \overset{(b)}{=} P_w(p_i^{t+1}; a^t) - P_w(p_i^{t+1}; a^{t+1})
\]

where (a) and (b) are true because of the fact that at time \(t+1\), CU \(i\) no longer associates with AP \(w\), and all other CUs keep their power and association profile the same as in time \(t\).

CU \(i\)'s communication rate after it finishes switching to \(\hat{w}\) is:

\[
R_i(p_{i,\hat{w}}^{t+1}, p_{\hat{w}; i,-w}^t; \hat{w}) = \sum_{k \in K_{\hat{w}}} \log \left( 1 + \frac{|h_{i,\hat{w}}(k)|^2 p_{i,\hat{w}}^{t+1}(k)}{n_{\hat{w}}(k) + \sum_{j : a^\hat{w}(j) = \hat{w}} |h_{i,\hat{w}}(k)|^2 p_{j,\hat{w}}^{t+1}(k)} \right)
\]

\[
= P_{\hat{w}}(p_{i}^{t+1}, p_{\hat{w}; i,-w}^t; \hat{w}) - P_{\hat{w}}(p_{i}^{t}, a^t) = P_{\hat{w}}(p_{i}^{t+1}, a^t) - P_{\hat{w}}(p_{i}^{t}, a^t).
\]

Because \(R_i(p_{i,\hat{w}}^{t+1}, p_{\hat{w}; i,-w}^t; \hat{w}) > R_i(p_{i,w}^t, p_{i,-w}^t; w)\), we have:

\[
P_{\hat{w}}(p_{i}^{t+1}, a^t) + P_{\hat{w}}(p_{i}^{t+1}, a^{t+1}) > P_{\hat{w}}(p_{i}^{t}, a^t) + P_{\hat{w}}(p_{i}^{t}, a^{t+1}).
\]

Arguing similarly as in (23), we have that SEP must satisfy: \(P(p_i^{t+1}; a^{t+1}) > P(p_i^t; a^t)\).

2) **At Time \(t+1\), CU \(i\) stays in \(w\).** Notice that in this case, we have \(a^{t+1} = a^t\). From Proposition 3 of \([2]\), we have the following inequality:

\[
P_w(p_{i,w}^{t+1}, p_{i,-w}^{t+1}; a^{t+1}) = P_w(p_{i,w}^{t+1}, p_{i,-w}^{t}; a^t) \geq P_w(p_{i,w}^{t}, p_{i,-w}^{t}; a^t),
\]

thus, \(P(p_i^{t+1}; a^{t+1}) \geq P(p_i^t; a^t)\). We conclude that in both cases, the system potential is non-decreasing.

Because \(P(p_i^t; a^t)\) is upper bounded, \(\{P(p_i^t; a^t)\}_{t=1}^\infty\) is a converging sequence.
APPENDIX C

PROOF OF PROPOSITION\textsuperscript{3}

Proof: From Proposition\textsuperscript{2} if \textbf{a} \in \mathcal{A}, we have that \( \lim_{n \to \infty} \mathbf{P}_w^{(n, \mathbf{a})} = \mathbf{p}_w^{\ast}(\mathbf{a}), \forall \ i \in \mathcal{N} \), which implies that \( \lim_{n \to \infty} \mathbf{I}_{i,w}^{(n, \mathbf{a})} = \mathbf{I}_{i,w}^{\ast}(\mathbf{a}), \forall \ i \in \mathcal{N} \). This resulted combined with the continuity of the function \( R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) \) with respect to \( \mathbf{I}_{i,w}^{(n, \mathbf{a})} \), and the continuity of the function \( \widehat{R}_i(n, \mathbf{a}) = R_i \left( \mathbf{p}_w^{(n, \mathbf{a})}, \mathbf{a}^{(n, \mathbf{a})}(i) \right) \) with respect to \( \mathbf{p}_w^{(n, \mathbf{a})} \), further implies that, for any \( \delta > 0 \), there must be a \( N(\delta) \) such that for all \( n > N(\delta) \), the followings are true:

\[
\max_{w \in \mathcal{W}} \left| R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) - R_i^* \left( \mathbf{I}_{i,w}^{\ast}(\mathbf{a}); w \right) \right| < \delta, \\
\left| \widehat{R}_i(n, \mathbf{a}) - R_i \left( \mathbf{p}_w^{\ast}(\mathbf{a}); \mathbf{a}(i) \right) \right| < \delta. \tag{41}
\]

For any \( w \neq \mathbf{a}(i) \) such that \( w \in B_i \left( \mathbf{I}_{i,w}^{\ast}(\mathbf{a}), \mathbf{a}(i) \right) \), there must exist a \( \epsilon_w > 0 \) such that:

\[
R_i^* \left( \mathbf{I}_{i,w}^{\ast}(\mathbf{a}); w \right) - R_i \left( \mathbf{I}_{i,a(i)}^{\ast}(\mathbf{a}); \mathbf{a}(i) \right) \geq \epsilon_w. \tag{42}
\]

Take \( \epsilon > 0 \) such that \( \epsilon = \min_{w \in B_i \left( \mathbf{I}_{i,w}^{\ast}(\mathbf{a}), \mathbf{a}(i) \right)} \epsilon_w \), and choose a \( \hat{\delta} \) small enough such that \( 0 < 2\hat{\delta} < \epsilon \), and let \( N_i^\ast(\mathbf{a}) \triangleq N(\hat{\delta}) \). We have that for all \( n > N_i^\ast(\mathbf{a}) \), the following is true:

\[
R_i^* \left( \mathbf{I}_{i,w}^{\ast}(\mathbf{a}); w \right) - R_i \left( \mathbf{I}_{i,a(i)}^{\ast}(\mathbf{a}); \mathbf{a}(i) \right) \\
= R_i \left( \mathbf{I}_{i,w}^{\ast}(\mathbf{a}); w \right) + R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) - R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) - \widehat{R}_i(n, \mathbf{a}) - R_i \left( \mathbf{I}_{i,a(i)}^{\ast}(\mathbf{a}); \mathbf{a}(i) \right) \\
\leq R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) - \widehat{R}_i(n, \mathbf{a}) + \left| R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) - R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) \right| + \left| \widehat{R}_i(n, \mathbf{a}) - R_i \left( \mathbf{I}_{i,a(i)}^{\ast}(\mathbf{a}); \mathbf{a}(i) \right) \right| \\
\leq R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) - \widehat{R}_i(n, \mathbf{a}) + \delta + \hat{\delta}. \tag{43}
\]

Consequently, we have that for all \( n > N_i^\ast(\mathbf{a}) \), \( R_i^* \left( \mathbf{I}_{i,w}^{(n, \mathbf{a})}; w \right) - \widehat{R}_i(n, \mathbf{a}) \geq \epsilon - 2\hat{\delta} > 0 \), which implies that \( w \) must be in the set \( B_i \left( \mathbf{I}_{i,w}^{\ast}(\mathbf{a}), \widehat{\mathbf{a}}^{(n, \mathbf{a})}(i) \right) \). The claim is proved.

\[\blacksquare\]

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