Cosmology of Brane-worlds

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Abstract

This talk presents an overview of the brane cosmology scenario, based on the idea that our Universe is a 3-brane embedded in a five-dimensional anti-de Sitter bulk space-time. Special emphasis is put on the novel features of this scenario: an unconventional cosmological evolution at high energy densities, i.e. in the early universe, and dark radiation, that embodies the gravitational effects of the bulk onto the brane, and which is shown to be generated during the high energy era by the production of bulk gravitons.

1. Introduction

In this contribution, I review the basic ingredients of the so-called Randall-Sundrum brane cosmology. This new cosmological scenario is based on the assumption that our universe is a brane: a sub-space embedded in a bulk spacetime, with a single extra dimension. In contrast with other braneworld models, the self-gravity of the brane is taken into account.

As will be recalled in this contribution, the cosmology for such a brane-universe is modified in two respects:

- the Friedmann equation is modified at high energy;
- the bulk influences the cosmological evolution via an additional term, usually called dark radiation or Weyl radiation.

Whereas the first modification has an impact only during the very early universe, since it is significant only at high energy, the second effect could have observable consequences today, as discussed below.

In this contribution, I do not discuss other important topics in the context of brane cosmology, such as the issue of cosmological perturbations, since this topic will be discussed by Roy Maartens. Finally, for the reader who wants to learn more on this subject, he/she can find in the literature several detailed reviews [1] that cover much more than the present contribution.
2. Homogeneous brane cosmology

2.1. The model

As in standard cosmology, the starting point is to assume homogeneity and isotropy in the three ordinary spatial dimensions. However, in the context of brane cosmology, these symmetries cannot be extended to the extra dimension, since the presence of the brane itself breaks homogeneity along the extra dimension. As a consequence, all physical quantities depend on time and on the extra dimension.

In a suitable coordinate system (Gaussian Normal), the five-dimensional metric can be written in the form

$$ds^2 = -n^2(t, y)dt^2 + a^2(t, y)d\vec{x}^2 + dy^2,$$

(1)

with the brane located at $y = 0$. To obtain the equations governing the cosmological evolution, one substitutes this ansatz into the five-dimensional Einstein equations (with the bulk cosmological constant $\Lambda$)

$$G_{AB} + \Lambda g_{AB} = \kappa^2 T_{AB}$$

(2)

where the energy-momentum tensor, assuming a bulk otherwise empty, is due to the brane matter and thus given by

$$T^{B}_{A} = \text{Diag}(-\rho_b(t), P_b(t), P_b(t), P_b(t), 0)\delta(y).$$

(3)

2.2. The cosmological evolution

It turns out that it possible to solve explicitly the five-dimensional Einstein’s equations (see [2]). Specializing the obtained solution at the brane location (denoted by the subscript 'b'), one finds the modified Friedmann equation [3, 4, 2]:

$$H_b^2 \equiv \frac{\dot{a}_b^2}{a_b^2} = \frac{\Lambda}{6} + \frac{\kappa^2 \rho_b^2}{36} + \frac{C}{a_b^4}$$

(4)

where $C$ is an integration constant. It can also be shown that, for an empty bulk, the usual conservation equation holds, which implies

$$\dot{\rho}_b + 3H_b(\rho_b + P_b) = 0.$$

(5)

For $\Lambda = 0$ and $C = 0$, the bulk is 5-D Minkowski and the cosmology is highly unconventional since the Hubble parameter is proportional to the brane energy density [3]. This has the unfortunate consequence to ruin the standard nucleosynthesis scenario that relies on the evolution of the expansion rate with respect to the relevant microphysical interaction rates.

To obtain a viable brane cosmology scenario, the simplest way is to use the idea of Randall and Sundrum [5], i.e. to make the following assumptions:
Consider a bulk with a negative cosmological constant $\Lambda < 0$

Assume the brane is endowed with an intrinsic tension $\sigma$, so that $\rho_b(t) = \sigma + \rho(t)$, where $\rho(t)$ is the energy density of usual cosmological matter.

With these assumptions, the Friedmann equation (4) yields

$$H_b^2 = \left(\frac{\Lambda}{6} + \frac{\kappa^4}{36}\sigma^2\right) + \frac{\kappa^4}{18}\sigma \rho + \frac{\kappa^4}{36}\rho^2 + \frac{C}{a_b^4}. \quad (6)$$

One recovers approximatively the usual Friedmann equation if

$$\frac{\Lambda}{6} + \frac{\kappa^4}{36}\sigma^2 = 0, \quad (7)$$

which is the condition imposed by Randall and Sundrum in their (non-cosmological) model to recover standard gravity, and which also implies

$$8\pi G \equiv \frac{\kappa^4}{6}\sigma. \quad (8)$$

However, this Friedmann equation is characterized by two new features:

- A $\rho^2$ term, which dominates at high energy;
- A radiation-like term, $C/a_b^4$, usually called dark radiation.

The cosmological evolution undergoes a transition from a high energy regime, $\rho \gg \sigma$, characterized by an unconventional behaviour of the scale factor, into a low energy regime which reproduces our standard cosmology. For $C = 0$ and an equation of state $w = P/\rho = \text{const}$, one can solve analytically the evolution equations and one finds

$$a(t) \propto t^{1/q} \left(1 + \frac{q t}{2\ell}\right)^{1/q}, \quad q = 3(1 + w). \quad (9)$$

One clearly sees the transition, at the epoch $t \sim \ell$, between the early, unconventional, evolution $a \sim t^{1/q}$ and the standard evolution $a \sim t^{2/q}$.

2.3. The bulk point of view

The above cosmological evolution can be obtained from a very different perspective [6], by starting from a static bulk metric, which, because of the cosmological symmetries and the (negative) cosmological constant, must be AdS-Schwarzschild in five dimensions:

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2d\Sigma^2, \quad f(R) = k + \frac{R^2}{\ell^2} - \frac{C}{R^2}, \quad k = 0, \pm 1. \quad (10)$$

In this coordinate system, the brane is moving and the so-called junction conditions $[K_{\mu\nu}] = -\kappa^2(S_{\mu\nu} - (S/3)g_{\mu\nu})$ give the modified Friedmann equation obtained above.
2.4. Constraints on the parameters

This cosmological scenario is essentially characterized by the value of the “fundamental mass scale” \( M_5 \) defined as \( \kappa^2 = M_5^{-3} \), since the other parameter, the AdS lengthscale \( \ell \equiv \sqrt{-\Lambda/6} \), is related to \( M_5 \) via the relation \[^3\] \[ M_{Pl}^2 = M_5^3 \ell, \]
which defines the four-dimensional Planck mass in this set-up. The scenario must satisfy two constraints:

- be compatible with the nucleosynthesis scenario, which means that the high energy regime, mentioned above, must take place before nucleosynthesis. This requires \( \sigma^{1/4} > 1 \) MeV, and since \( \sigma = 6/(\kappa^2 \ell) = 6M_5^3/M_{Pl}^2 \), this gives the constraint \( M_5 > 10^4 \) GeV.

- be compatible with the gravity experiments on small scales, which presently require \( \ell < 0.1 \) mm. This implies \( M_5 > 10^8 \) GeV.

As will be detailed in the next section, another observational constraint applies to the dark radiation constant \( C \).

3. Dark radiation and the production of bulk gravitons

So far, the bulk has been assumed to be strictly empty, apart from the presence of the brane. However, the fluctuations of brane matter generate bulk gravitational waves. Equivalently, at smaller scales, the scattering of brane particles can produce bulk gravitons:

\[ \psi + \bar{\psi} \rightarrow G \]

Therefore, the fact that the homogeneity hypothesis in the brane is not exactly satisfied necessarily leads to the presence of a flow of gravitons in the bulk.

3.1. Emission by the brane

The production of gravitons results into an energy loss for ordinary matter, which can be expressed as

\[ \frac{d\rho}{dt} + 3H(\rho + P) = - \int \frac{d^3p}{(2\pi)^3} C[f], \]

with the collision term

\[ C[f] = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_2}{2E_1} \frac{d^3p_2}{2E_2} \sum |\mathcal{M}|^2 f_1 f_2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p), \]
where $\mathcal{M}$ is the scattering amplitude for the process in consideration (the indices 1 and 2 correspond to the scattering particles $\psi$ and $\bar{\psi}$). The summed squared amplitude is given by
\[
\sum |\mathcal{M}|^2 \equiv \hat{g} \frac{\kappa^2}{8\pi} s^2, \quad \hat{g} = \frac{2}{3}g_s + 4g_v + g_f
\] (14)
with $s = (p_1 + p_2)^2$. When the brane matter is in thermal equilibrium with a temperature $T$, one finds
\[
\dot{\rho} + 4H\rho = -\frac{315}{512\pi^3} \hat{g} \kappa^2 T^8.
\] (15)

3.2. Bulk description of the radiation brane

It is much more difficult to describe what happens in the bulk. A possibility is to model the system Bulk + Radiating Brane by a generalized five-dimensional Vaidya metric \[7\]
\[
ds^2 = -f(R, v)dv^2 + 2dRdv + R^2dx^2, \quad f(R, v) = \mu^2 R^2 - \mathcal{C}(v),
\] (16)
describing an *ingoing* radiation flow. Here, $v$ is a null coordinate. If $\mathcal{C}(v)$ is constant, one recovers the AdS-Schwarzschild metric \[10\]. The generalized Vaidya’s metric is a solution of the five-dimensional Einstein’s equations with
\[
T_{AB} = \mathcal{F} k_A k_B, \quad k_A k^A = 0,
\] (17)
which means that the gravitons must be radial.

This description is in general too restrictive. More generally, the bulk must be seen as filled with a gas of gravitons with a distribution function $f$. Their energy-momentum tensor is given by
\[
\mathcal{T}_{AB} = \int d^5p \, \delta \left( p_M p^M \right) \sqrt{-g} f p_A p_B,
\] (18)
From the 5D Einstein equations, one can derive effective 4D Einstein equations \[8\], which in the homogeneous case yield
- the Friedmann equation
\[
H^2 = \frac{\kappa^2}{3} \left[ \left( 1 + \frac{\rho}{2\sigma} \right) \rho + \rho_D \right],
\] (19)
- the non-conservation equation for brane matter
\[
\dot{\rho} + 3H (\rho + p) = 2 \mathcal{T}_{RS} n^R u^S
\] (20)
which must be identified with \[13\];
The asymmetry $A_2$ as a function of $x$.

- the non-conservation equation for the “dark radiation” energy density $\rho_D$ (which includes all effective contributions from the bulk):

$$\dot{\rho}_D + 4H\rho_D = -2 \left( 1 + \frac{\rho}{\sigma} \right) T_{AB} u^A n^B - 2H \ell T_{AB} n^A n^B. \quad (21)$$

On the right hand side of this last equation, we find two terms with opposite signs: the first term, due to the energy flux from the brane into the bulk, contributes positively and thus increases the amount of dark radiation whereas the second term, due to the pressure along the fifth dimension, decreases the amount of dark radiation. To estimate quantitatively these terms, one needs to determine the distribution of the bulk gravitons.

This task was undertaken recently [9] by assuming that the bulk background stays approximately AdS during the whole evolution. For this background, one can compute analytically the trajectories of the bulk gravitons. Figure 1 shows such a few trajectories, together with the trajectory of the brane endowed with relativistic matter. A peculiar feature is that many (non-radial) gravitons tend to come back onto the brane and bounce off it. This gives a significant contribution to the transverse pressure effect, which almost, although not quite, compensates the flux effect.

The amount of dark radiation produced, in terms of the ratio $\epsilon_D \equiv \rho_D / \rho$ is plotted on Fig. 2 as a function of the initial radiation energy density (and compared with the analytical results of [10] and [7]).

3.3. Observational constraints

The computed amount of dark radiation can be confronted to observations (see e.g. [11]). Indeed, since dark radiation behaves as radiation, it must satisfy
Fig. 2. Amount of dark radiation $\epsilon_D = \rho_D / \rho$ as a function of the initial energy density on the brane $\rho_i$ (in units of $\sigma^4$), computed numerically in [9] and compared with HM [10] and LSR [7].

the nucleosynthesis constraint on the number of additional relativistic degrees of freedom, usually expressed in terms of the extra number of light neutrinos $\Delta N_{\nu}$. The relation between $\Delta N_{\nu}$ and $\epsilon_D$ is given by

$$\epsilon_D = \frac{7}{43} \left( \frac{g_*}{g_*^{\text{nucl}}} \right)^{1/3} \Delta N_{\nu}, \quad (22)$$

where $g_*^{\text{nucl}} = 10.75$ is the number of degrees of freedom at nucleosynthesis (in fact before the electron-positron annihilation). Assuming $g_* = 106.75$ (standard model), this gives $\epsilon_D \approx 0.35 \Delta N_{\nu}$. The typical constraint from nucleosynthesis

$$\Delta N_{\nu} < 0.2 \quad (23)$$

implies

$$\epsilon_D \equiv \frac{\rho_D}{\rho_r} < 0.03 \left( \frac{g_*}{g_*^{\text{nucl}}} \right)^{1/3} \quad (24)$$

which gives $\epsilon_D < 0.07$ with the degrees of freedom of the standard model.

4. Conclusions

The various models of extra dimensions with branes have raised a considerable interest in the last few years, motivated by their more or less direct connections with the recent developments in string/M theory. To make further progress in this direction, it is important to see how these models can be tested by experiments.
Roughly speaking, the tests can be classified into three broad categories: modification of Newton’s law; signatures in colliders; cosmology. As usual in high energy physics, if the scale characterizing new physics is too high then it cannot be reached directly in collider experiments. In this case cosmology is the only place where the effects of new physics can be, indirectly, observed.

As illustrated by numerous works in the last few years, Randall-Sundrum type cosmology is a very rich playground to study the very peculiar consequences of the braneworld idea in cosmology. As recalled in this contribution, its essential new features are: a $\rho^2$ term in the generalized Friedmann equation, which dominates at high energies; “dark radiation”, produced during the high energy phase, and of potential relevance for observations, via the nucleosynthesis constraints on the number of extra relativistic degrees of freedom.

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