Effect of the radial and azimuthal mode indices of a partially coherent vortex field upon a spatial correlation singularity

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Abstract. The existence of a spatial correlation singularity or a ring dislocation in the spatial coherence function when a vortex is present has been demonstrated recently. Here, we investigate how the spatial correlation singularity is affected by both the radial and azimuthal mode indices ($p$, $\ell$) in a partially coherent light field. Theoretically, we find that the spatial correlation singularity may exist even in a non-vortex beam ($\ell = 0$) due to the radial index. Numerical simulations show the number of ring dislocations in the far-field cross correlation function is equal to $2p + |\ell|$ for the low coherence cases. This is confirmed by our experimental results. This phenomenon may occur in any partially coherent vortex wave.

\[ S \] Online supplementary data available from stacks.iop.org/NJP/15/113053/mmedia

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1. Introduction

In the past few years, great interest has been focused on vortex fields. They are associated with wave phenomena and appear in a wide range of physical systems. Vortices denote singular (zero intensity) points in a given field that have indeterminate phase, and phase circulation a multiple of $2\pi$ in the plane transversal to the direction of wave propagation [1]. Optical vortices are characterized by an azimuthal phase term $\exp(-i\ell \varphi)$ that encircles the singular point, where $\ell$ is the azimuthal mode index (topological charge), and $\varphi$ denotes the azimuthal coordinate. Optical vortices find numerous applications in quantum information, astrophysics, microscopy and the manipulation of atoms or microscopic particles [2]. Some of these applications arise from the orbital angular momentum associated with the nature of propagation of the Poynting vector field [3].

The preservation of the vortex structure during propagation makes partially coherent vortex beams good candidates for various applications; accordingly, a better understanding of the spatial coherence properties is of fundamental importance. Coherence vortices and the phase singularities of a partially coherent field have been studied extensively within optical coherence theory [4–8]. In 2004, it was shown that a spatial correlation singularity will exist in the spatial coherence function when a vortex is present [6]. More recently, the effect of the topological charge $\ell$ on the cross-correlation function (CCF) of a partially coherent beam has been studied theoretically and experimentally [9–11]. It has been shown that spatial correlation functions have interesting topological properties associated with their phase singularities. However, it is noteworthy that all of these studies have assumed that the partially coherent optical field possesses a radial index $p = 0$, ignoring the effect of the radial mode index. To our knowledge, a generic understanding of how both the radial and azimuthal degrees of freedom influence the form of the CCF has not been presented yet. More importantly, the radial degree of freedom is a crucial facet for the correct description of the transversal state of these light fields and thus it is necessary to understand this in relation to the CCF.

In this study, we theoretically and experimentally delineate the form of the CCF for a partially coherent light field including both the azimuthal and radial mode indices. Our results show that there are $2p + |\ell|$ spatial correlation singularities or ring dislocations in the CCF for low coherence cases. This result has major significance for the measurement and use of partially coherent vortex light fields.
2. Cross-correlation function in the far-field

If we consider a cylindrical coordinate system, a fully coherent Laguerre–Gaussian (LG) optical vortex with topological charge \( \ell \) at the source plane \( z = 0 \) has a complex amplitude proportional to

\[
    u_{p, \ell}(\rho, \varphi, 0) \propto \rho^\ell L_p^{(|\ell|)} \left( \frac{2 \rho^2}{w_0^2} \right) \exp\left( -\frac{\rho^2}{w_0^2} \right) \exp\left( -i\ell \varphi \right) \exp\left( i\theta \right),
\]

where \( w_0 \) is the waist width, \( L_p^{(|\ell|)}(\cdot) \) is the associated Laguerre polynomial, \( p \) is the radial mode index, \( \ell \) is the azimuthal mode index, \( \rho \) and \( \varphi \) are the radial and azimuthal coordinates, respectively, and \( \theta \) is an arbitrary phase.

By propagating a fully coherent LG beam through a rotating optical diffuser, we can create a partially coherent LG beam that can be described using the Gaussian Schell-model correlator: 

\[
    \mu(\vec{\rho}_1, \vec{\rho}_2) = \exp\left( -|\vec{\rho}_1 - \vec{\rho}_2|^2 / L_c^2 \right),
\]

where \( L_c \) is the transverse coherence length. One can write the mutual coherence function (MCF) of a partially coherent vortex beam at the source plane \( z = 0 \) as [6]

\[
    \Gamma'(\vec{\rho}_1, \vec{\rho}_2, 0) \propto |\vec{\rho}_1 \cdot \vec{\rho}_2|^\ell \exp\left( -\frac{|\vec{\rho}_1 - \vec{\rho}_2|^2}{L_c^2} \right) L_p^{(|\ell|)} \left( \frac{2 \rho_1^2}{w_0^2} \right) L_p^{(|\ell|)} \left( \frac{2 \rho_2^2}{w_0^2} \right) \exp\left( -\left( \rho_1^2 + \rho_2^2 \right) / w_0^2 \right) \exp\left[ -i\ell \left( \phi_2 - \phi_1 \right) \right].
\]

In the far field, the MCF is given by [6]

\[
    \Gamma(\vec{\rho}, \vec{\rho}', z) = (1/\lambda z)^2 \int \int \Gamma'(\vec{\rho}_1, \vec{\rho}_2, 0) \exp\left[ -\frac{i2\pi}{\lambda z} (\vec{\rho}_1 \cdot \vec{\rho}_1 - \vec{\rho}_2 \cdot \vec{\rho}_2) \right] d\vec{\rho}_1 d\vec{\rho}_2,
\]

where \( \lambda \) is the wavelength.

3. Numerical simulations

From the far-field MCF in equation (3), one may determine the far-field CCF, \( \chi_c(\vec{\rho}) = \Gamma(\vec{\rho}, -\vec{\rho}) \), which is a more robust pattern than the intensity in a partially coherent beam [6]. The far-field CCF of partially coherent optical vortices with different radial and azimuthal mode indices is shown in figure 1, where the transverse coherence length \( L_c = w_0 \). The ring dislocations denote places where the two-point correlation vanishes, the physical meaning of which indicates an absence of coherence. One can also see the number of ring dislocations in the far-field CCF is dependent on both the radial and azimuthal mode indices of a partially coherent optical vortex beam. It is noted that Palacios et al [6] showed that a spatial correlation singularity exists when a vortex is present, whereas figure 1(c) shows in fact that a spatial correlation singularity may exist even in a non-vortex beam (i.e. \( \ell = 0 \)). A careful observation of the far-field CCFs of partially coherent vortex beams (figure 1) gives a definitive general relationship between the number of dislocation rings, \( N \), and the radial and azimuthal mode indices of a partially coherent beam as

\[
    N = 2p + |\ell|.
\]

It can be also noted that this relationship is consistent with the earlier experimental results for the limiting case of \( (p = 0, \ell = 1) \) [6], namely, there is only one dislocation ring in the far-field.
Figure 1. Far-field CCFs of partially coherent LG beams with different mode indices.

CCF. When $p = 0$ and $\ell \neq 0$, equation (4) is rewritten as $N = |\ell|$, which is consistent with the earlier theoretical study [9].

Equation (4) can be understood by considering the different steps involved in measuring the far-field CCF of a LG beam. Figure 2 and the supplementary video 1 (available at stacks.iop.org/NJP/15/113053/mmedia) show how the CCF cross-correlates two regions of the partially coherent vortex beam, diametrically opposite to each other (highlighted using the yellow circles in parts (a), (d) and (g)). The size of these regions is determined by the transverse coherence length, $L_c$. The product of the two fields in these two regions corresponds to the integrand of the far-field CCF and is represented in parts (c), (f) and (i) of figure 2. We observed that this product has multiple features. In the ‘horizontal’ direction, one sees that the $p$ ring dislocations originating from the $p \neq 0$ beam appear doubled up, one for each region. Further, in the ‘vertical’ direction, the phase changes due to the $\ell$th order vortex charge. As we increase the distance between the two regions, a number of destructive interferences appear, each corresponding to a ring dislocation in the CCF function. Altogether, the number of destructive interferences observed is given by $2p + |\ell|$, originating from the doubling up of rings due to the two regions and the $\ell$ component originating from the $2\pi \ell$ phase changes in the ‘vertical’ direction.

In order to understand the general relationship in equation (4), especially to highlight which ring dislocations are associated with $p$ and which are associated with $\ell$, we present in figure 3 the one dimensional far-field CCF and intensity distributions of a partially coherent vortex beam with $p = 1$ and $\ell = 2$, for various coherence lengths. In figure 3(a) one observes that for a high coherence length, the far-field CCF is the same as the intensity distribution. It is well known that the intensity profile of a coherent LG vortex beam with radial mode index $p$ has $p$ dark rings and a dark centre, therefore, for $p = 1$ in figure 3, we observe one dark ring in the intensity distribution at $\rho = 24.5w_0$ (indicated by the point $Q$ and the blue arrow in figure 3(a)). Accordingly, we can say that the zero of the far-field CCF at point $Q$ in figure 3(a) is associated with $p$. Furthermore, the former point $Q$ denotes zeros in both intensity and the far-field CCF. From figure 3(b), one observes that as the coherence length $L_c$ decreases to $10w_0$, the intensity at the former point $Q$ increases; while the far-field CCF at $Q$ decreases below zero, creating two new crossing points $Q_1$ and $Q_2$. When the coherence length decreases $L_c = 5w_0$, two zeros in the far-field CCF appear near the former dark centre of the intensity, as shown by points $Q_3$ and $Q_4$ in figure 3(c). Additionally, figures 3(d) and (e) indicate that the number of zeros in the far-field CCF is unchanged for the case of even lower coherence length. Recalling the recent studies [9, 11], that is, the number of ring dislocations in the far-field CCF is equal to
the azimuthal mode index $\ell$ when $p = 0$, we can draw a conclusion that there are $2p + |\ell|$ ring dislocations in the far-field CCF for low coherence length cases.

4. Experimental results

We measured the CCF of a partially coherent beam with the aid of a wavefront folding (WFF) interferometer [6]. A schematic overview of the experimental setup is shown in figure 4. A partially coherent optical field is generated by focusing a He–Ne laser beam ($\lambda = 633$ nm) onto a rotating holographic diffuser (Edmund Optics, 0.5° diffusing angle). The beam is recollimated onto a spatial light modulator (SLM, Holoeye HEO 1080P), where an intensity-coded phase function [13] is imprinted in order to generate LG beams with different radial and azimuthal mode indices. By selecting the first diffraction order with a pinhole ($P_1$), we realize a partially coherent LG beam at the focal plane of lens ($L_3$) with a designated radial and azimuthal mode index ($p$, $\ell$). This partially coherent LG beam is then imaged onto a CCD camera (Basler pilot piA640-210gm) by a telescope. With both optical arms in the WFF interferometer
Figure 3. Far-field CCF (—) and intensity (⋯⋯⋯⋯⋯⋯) distributions for a partially coherent vortex beam with $p = 1$ and $\ell = 2$, for various coherence lengths $L_c$.

(a) $L_c = 100w_0$; (b) $L_c = 10w_0$; (c) $L_c = 5w_0$; (d) $L_c = w_0$; (e) $L_c = w_0/5$.

Figure 4. Schematic of the experimental setup used for measuring the ring dislocations of the far-field CCFs in a partially coherent vortex field. $D_1 =$ rotating holographic diffuser, $L =$ lens, $SLM =$ spatial light modulator, $M =$ mirror, $P =$ pinhole, $CCD =$ charge coupled camera, $B =$ beam splitter cube and $DP =$ dove prism. Focal widths of lenses: $f_1 = f_2 = 400$ mm, $f_3 = 1000$ mm, $f_4 = 200$ mm and $f_5 = 400$ mm.

unblocked, the recorded interferograms can reveal the locations of ring dislocations in the CCF, $\chi_c(\vec{\rho}) = \Gamma(\vec{\rho}, -\vec{\rho})$ [6]. The relation between the $\chi_c(\vec{\rho})$ and the expected interferogram $F(x, y)$ is given by [6]

$$F(x, y) = I(-x, y) + I(x, -y) + 2\text{Re}[\chi_c(\vec{\rho})]\cos(\Delta\vec{k}_\perp \cdot \vec{\rho}),$$

where $I(-x, y)$ and $I(x, -y)$ are the intensities of the two beams emerging from the interferometer, respectively; and $\Delta\vec{k}_\perp$ is the difference between the wave vectors of the two beams.
Figure 5. Far-field CCFs of partially coherent LG beams ($\ell = 1, 3$ and $p = 1, 3$), where $w = 2.7$ mm is the beam waist on the CCD. The concentric dashed-line circles are an aid to the eye to visualize the ring dislocations.

In this study, we considered partially coherent LG beams with both radial and azimuthal mode indices each ranging from 0 to 5, in integer steps. As an example, figure 5 shows the interferograms of CCF when $\ell = 1, 3$ and $p = 1, 3$. As shown in figure 5, the ring dislocations are experimentally observed and visualized (dashed-line concentric circles indicate the ring dislocations).

One notes that there are many fringes in figure 5, due to the difference between the wave vectors of the two beams emerging from the interferometer. We remark that the fringe visibility vanishes at the boundary of the ring dislocation in CCF (see [6]). Moreover, there is a single-fringe shift at each ring dislocation because of the $\pi$ phase jump. Therefore, the fringes can help us recognize the ring dislocations more clearly. The spatial coherence of the optical field is controlled by changing the position of the holographic diffuser along the optical axis [12], and we observe that the coherence length influences the far-field CCF significantly.

Experimental results show that the ring dislocations in the CCF exhibit high visibility without any post processing, provided the coherence length is approximately equal to the beam waist, which is consistent with the numerical results in figure 3. Figure 6 shows the far-field CCF interferograms when $\ell = 2, 4$ and $p = 2, 4$. From figures 5 and 6, we clearly observe the relationship between the number of ring dislocations in the CCFs and the mode indices ($p$, $\ell$), which agrees with equation (4). Furthermore, as mode indices increase, the visibility of the ring dislocations in the far-field CCF is reduced. This can be seen as a fundamental limit due to the finite number of detectable optical degrees of freedom of the far-field CCFs [14]. In the present

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Figure 6. Far-field CCFs of partially coherent LG beams ($\ell = 2, 4$ and $p = 2, 4$), where $w = 3.2$ mm for radial index mode $p = 2$ (upper row) and $w = 2.7$ mm for radial index mode $p = 4$ (lower row). The concentric dashed-line circles are an aid to the eye to visualize the ring dislocations.

setup, we observe $2p + |\ell|$ ring dislocations for values of $\ell$ and $p$ up to 5, beyond which the visibility in the interferograms of CCF is too low to recognize the ring dislocations.

Additionally, for partially coherent LG beams with no vortices ($\ell = 0$), we performed two experiments for the cases of $p = 1$ and 2 (not shown here). The number of ring dislocations in the recorded far-field CCF is equal to $2p$, which obeys the relation in equation (4). Our experimental results verify that the number of ring dislocations in the CCF of such a partially coherent vortex beam is exactly equal to the absolute value of the azimuthal index $\ell$ when $p = 0$ and $\ell \neq 0$, which is in agreement with theoretical predictions [9].

5. Conclusion

The dependence of the spatial correlation singularity upon both the radial and azimuthal mode indices ($p, \ell$) in a partially coherent light field is studied. A relationship between the number of ring dislocations in the far-field CCF and both the radial and azimuthal mode indices is presented numerically and experimentally. We found that the number of ring dislocations in the far-field CCF is equal to $2p + |\ell|$ for the low coherence cases, and the visibility of the ring dislocations is clearest when the coherence length is approximately equal to the waist width of the beam. Though this phenomenon is observed in the optical field, it may occur in any partially coherent wave. This offers a powerful relationship useful for the characterization of
partially coherent fields with embedded vortices and higher order radial modes. Our results will be useful for applications using partially coherent vortex fields. As an example, the radial index adds a new degree of freedom that may be exploited for quantum communication or information processing.

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