Chromatic transitions in the emergence of syntax networks

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The emergence of syntax during childhood is a remarkable example of how complex correlations unfold in nonlinear ways through development. In particular, rapid transitions seem to occur as children reach the age of two, which seems to separate a two-word, tree-like network of syntactic relations among words from a scale-free graphs associated to the adult, complex grammar. Here we explore the evolution of syntax networks through language acquisition using the chromatic number, which captures the transition and provides a natural link to standard theories on syntactic structures. The data analysis is compared to a null model of network growth dynamics which is shown to explore the evolution of syntax networks through language acquisition using the chromatic number, and thus, can be interpreted as the footprints of incompatibility relations, somewhat as opposed to modularity considerations.

Keywords: Complex Networks, Graph Colouring, Modularity, Syntax

I. INTRODUCTION

The origins of human language have been a matter of intense debate. Language is a milestone in our evolution as a dominant species and is likely to pervade the emergence of cooperation and symbolic reasoning $^{1-4}$. Maybe the most defining and defeating trait is its virtually infinite generative potential: words and sentences can be constructed in recursive ways to generate nested structures of arbitrary length $^{3,5}$. Such structures are the product of a set of rules defining syntax, which are extracted by human brains through language acquisition during childhood after a small sample of the whole combinatorial universe of sentences has been learned. And yet, in spite of its complexity, syntax is accurately acquired by children, who master their mother tongue in a few years of learning. Indeed, around the age of two, linguistic structures produced by children display a qualitative shift on their complexity, indicating a deep change on the rules underlying them $^{5,7}$. This sudden increase of grammar complexity is known as the syntactic spurt, and defines the edge between the two words stage, where only isolated words or combinations of two words occur, to a stage where the grammar rules governing this syntax are close to the one we can find in adult speech -although the cognitive maturation of kids makes the semantic content or the pronunciation different from the adult one. How can we explain or interpret such nonlinear pattern?

Statistical physicists have approached the problem of language evolution showing for example that nontrivial patterns are shared between language inventories (collections of words) and some genetic and ecological neutral models $^8$ -see $^9$ and references therein. However, most of these models do not make any assumption about the role played by actual interactions among words, or, more generally, linguistic units, which largely define the nature of linguistic structures. In this context, a promising approach to its structure and evolution involves considering language in terms of networks of interconnected units instead of unstructured collections of elements (e.g., words or syllables) $^{10}$. In this context, syntactic networks, in which nodes are words and links the projection of actual syntactic relations, have been shown to be an interesting abstraction to grasp general patterns of language production $^{10,12}$. Specially valuable has been the quantitative data obtained from syntax networks obtained along the process of syntax acquisition $^{7,13,14}$.

At the fundamental level, syntax can be understood as a set of symbols associated under a restricted universe of combinations somewhat similar to chemistry. Atoms and words would then be linked through compatibility relations defining what can be combined and what is forbidden. The power of this picture is supported by the use of linguistic methods in the systematic characterization of chemical structures $^{13}$. Chemical structure diagrams can thus be seen as some sort of language, with chemical species and bonds as key ingredients. In a more abstract fashion, we can say that general rules of combining elements within a given set of interacting pieces with well defined functional meaning is at work in both language and chemistry.

Following the chemical analogy, where abstract classes of “nodes” can be defined, we will take advantage of graph colorability theory as a general framework to detect transitions based on qualitative changes of compatibilities. Specifically, we suggest that a new combinatorial approach grounded on a graph colouring may enable a better understanding of the evolution of networks having internal relations of compatibility (e.g., some kind of syntactic rules). In this context, we propose the chromatic number -and associated measures- of the graph $^{16,18}$ as
FIG. 1: Optimal colourings of syntactic networks before and after the syntactic spur. (a) A syntactic network before the transition (3th corpus) is largely bipartite (this network accepts a 2-coloring). (b) Post-transition network (7th corpus) is remarkably more complex, which corresponds to high chromatic number $\chi(G) = 6$. All networks coming from Peter dataset. Time spent between these two corpora is about two months and a half (see text).

an indicator of network complexity. Within our context, it will be a surrogate of syntactic complexity. The chromatic number is defined as the minimal number of colors needed to paint all nodes of the graph in a way that no adjacent nodes have the same color \[17\]. The $q$-coloring problem, i.e., to know whether a graph can be colored with $q$ different colors is one of the most important NP-complete problems. From the statistical physics point of view, an analogous problem is defined within the context of the Potts model \[19\].

It is worth to emphasize that transitions in the evolution of the chromatic number have been widely studied in models of random graphs \[18, 20–22\]. Our exploration over sequences of syntactic graphs mapping child language acquisition also displayed transitions in the chromatic number (see below). This is, to the best of our knowledge, the first time that such transitions have been reported in a real system. Classes of nodes would be defined precisely by the fact that there are no connections among them, a measure conceptually opposite to graph modularity.

The remaining of the paper is organized as follows. Section II is devoted to a brief revision of the so-called Potts model as the way to introduce the chromatic number. In section III we apply these theoretical constructs to our problem and we analyze the obtained data by using different estimators of relevance, the most prominent of them being a null model of random sentence generation. In section IV we discuss the obtained results and we highlight a number of potential impacts of this kind of complexity estimators for complex networks.

II. GRAPHS AND COLORING: BASICS

We will work over undirected graphs. An undirected graph $G(V, E)$ -hereafter, $G$- is composed by the set of $V = \{v_1, ..., v_n\}$ nodes and a set $E = \{e_j\mid 1 \leq j \leq m\} \subseteq V \times V$ of edges. Each (unordered) pair $e_j = \{v_i, v_k\}$ depicts a link between nodes $v_i$ and $v_j$. The number of links $k(v_i)$ attaching node $v_i$ is the degree of the node and $\langle k \rangle$ is the average degree of the graph $G$. The degree distribution $P(k)$ accounts for the probability to select a node at random having degree $k$. The identity card of a graph is the so-called Adjacency matrix, $a(G)$, which is defined as follows:

$$a_{ij} = \begin{cases} 1, & \text{iff } (\exists e_k \in E) : (e_k = \{v_i, v_j\}) \\ 0, & \text{Otherwise} \end{cases}.$$

We observe that the adjacency matrix of undirected graphs is symmetrical, i.e., $a_{ij} = a_{ji}$.

The computation of the chromatic number can be formulated as the following combinatorial problem: What is the minimal number of ‘colours’ needed to paint all nodes
of the graph in such a way that no single node is connected to neighbors having the same color? We can map this problem into the antiferromagnetic q-dimensional Potts model at $T = 0$ \cite{13}. This model is a generalization of the classical Ising model for lattices: at every node of this lattice we place a particle having a spin which energetically constrains the state of its neighbors. Traditionally, spins can have only two states, namely $|↑⟩$ and $|↓⟩$. In the Potts model, compatibility relations take into account an arbitrary number $q > 2$ of different states.

Let us consider a partition of nodes $V$ containing $q$ different classes, namely, $G_q(V) = \{g_1, ..., g_q\}$ of $V$, i.e.:

$$\bigcap G_q = \emptyset \text{ and } \bigcup G_q = V \ , \quad (1)$$

The state $\sigma_i$ of node $v_i$ indicates the class of $G_q(V)$ to which the node belongs to, i.e., $\sigma_i \in g_j$. Let $\mathcal{F}_q(V)$ be the ensemble of all partitions of $V$ containing $q$ different classes. Every element in $\mathcal{F}_q(V)$ has the following energy penalty\footnote{In our approach, the energy units of this Hamiltonian are arbitrary.}:

$$\mathcal{H}(G_q) = J \sum_{i < j} a_{ij} \delta(\sigma_i, \sigma_j) \ , \quad (2)$$

where $J = 1$ is the coupling constant and $\delta$ is the Kronecker symbol:

$$\delta(\sigma_i, \sigma_j) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{Otherwise} \end{cases}$$

Intuitively, the higher the presence of pairs of connected nodes belonging to the same state, the higher will be the energy of the global state of the graph. Given a fixed $q$, the configurations displaying minimal energy may have an amount of non-solvable situations, leading to the unavoidable presence of connected nodes at the same state. This phenomenon is called frustration, and for these configurations, the ground state of the Hamiltonian defined in (2) displays positive energy. If there is no frustration, i.e., $\exists G_q \in \mathcal{F}_q(V)$, we can find a partition that satisfies:

$$\mathcal{H}(G_q) = 0 \ , \quad (3)$$

and we say that the graph is $q$-colorable, being the $q$ different colors the $q$ different classes or members of $G_q$. When the graph is $q$-colorable, there is at least one partition $G_q \in \mathcal{F}_q(V)$ such that, if $v_i, v_j \in V$ belong to the same class or color of the partition, namely $g_i \in G_q$. We deduce that:

$$(v_i, v_j \in g_i) \Rightarrow a_{ij} = 0 \ . \quad (4)$$

Relation (4) maps color classes onto disjoint sets of graph elements (adjacent nodes have a different color). Now, the coloring problem consists in finding the minimal number of classes (or colors) required to properly paint the graph. This is the so-called Chromatic Number of the graph $G$:

$$\chi(G) = \min \{q : (\exists G_q \in \mathcal{F}_q(V)) : \mathcal{H}(G_q) = 0\} \ . \quad (5)$$

Now suppose network partition(s) $G_q^* \in \mathcal{F}_q(V)$ having minimal energy, see equation (2), given a number of colours $q$:

$$G_q^* = \min_{G_q \in \mathcal{F}_q(V)} \{\mathcal{H}(G_q)\} \ . \quad (6)$$

In general, the process of search for the chromatic number yields a decreasing sequence of energies ending at $\mathcal{H}(G_q^*) = 0$:

$$\mathcal{H}(G_q^*) \leq ... \leq \mathcal{H}(G_q^*) = 0 \ , \quad (7)$$

In order to assess the statistical significance of chromatic numbers, we define the relative energy of any $q$-coloring as follows:

$$f_q(\chi) = \frac{\mathcal{H}(G_q^*)}{|E|} \ , \quad (8)$$

where $|E|$ is the number of edges in the graph $G$. This quantity $0 \leq f_q(\chi) \leq 1$ corresponds to the minimal (relative) number of frustrated links or violations (i.e., when adjacent nodes have the same color).

Despite the high complexity of this problem (computing the chromatic number in an arbitrary graph is a NP-hard problem) several bounds can be defined. A lower bound can be defined from the so-called Clique number. A clique is a subgraph in which every node is connected to all other nodes in the subgraph. The Clique number $\omega(G)$ is the size of the largest clique in the graph, which is a natural lower bound for $\chi(G)$ \cite{17}:

$$\omega(G) \leq \chi(G) \ . \quad (9)$$

Alternatively, an upper bound on $\chi(G)$ can be defined by looking at the $K$-core structure of $G$. The $K$ (core) is the largest subgraph whose nodes display degree higher or equal to $K$. Now, lets $K^*(G)$ be the $K$-core with largest connectivity that can be found in $G$:

$$K^* = \max \{K : K(G) \neq \emptyset\} \ . \quad (10)$$

Then, it can be shown that $K^*$ sets an upper bound to the chromatic number \cite{17}:

$$\chi(G) \leq K^* + 1 \ . \quad (11)$$

Finally, let us mention that, for some families of random graphs the chromatic number has an asymptotic behavior depending on the average connectivity \cite{15}, $\chi(G) \sim \frac{k}{\log k}$. However, the above relationship does not hold for scale-free networks with exponent $2 < \gamma < 3$. These heterogeneous networks cannot have a stable value of the chromatic number because their clique number \cite{9} diverges with the graph size, even at constant $\langle k \rangle$ \cite{23}. 

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1 In our approach, the energy units of this Hamiltonian are arbitrary.
III. THE EVOLUTION OF $\chi$ ALONG SYNTAX ACQUISITION

Here we study the evolution of the chromatic number through language development as captured by syntactic graphs. We compare the chromatic number with the lower and upper bounds provided by the clique number and the maximal $K$-core, respectively. We assess the relevance of computed chromatic numbers with the corresponding minimal energy. The combination of these two measurements enable us to interpret the nature of the chromatic number. Specifically, we can check whether changes in this number reflect a global pattern or instead some anomalous behaviour of a small, localized subgraph. Finally, we provide further validation of our analysis by comparing chromatic numbers in empirical and synthetic networks obtained through a random sentence generator.

A. Building the Networks of Early Syntax

Through the process, networks built upon the aggregation of syntactic structures from child’s productions grow and change in a smooth fashion until a rapid transition occurs \cite{14, 24} –see also \cite{10}. We reconstruct syntactic networks by projecting the raw constituent structure, i.e., phrase structure of children’s utterances, into linear relations among lexical items \cite{24}. Then, we aggregate all these productions in a single graph where nodes are lexical items and links represent syntactic relations between them \cite{14, 24}. These networks provide a unique window into the patterns of change occurring in the language acquisition process.

The two cases studied here are obtained from the CHILDES Database \cite{26, 27} which includes conversations between children and parents. Specifically, we choose Peter and Carl’s corpora, whose structure has been accurately extracted and curated. For both Peter and Carl’s corpora, we choose 11 different recorded conversations distributed in approximately uniform time intervals ranging from the age of $\sim$ 20 months to the age of $\sim$ 28 months. The chosen interval corresponds to the period in which the syntactic spur takes place. From every recorded conversation, we extract the syntactic network of child’s utterances obtaining a sequence of 11 syntactic graphs corresponding to the sequence of Peter conversations $\mathcal{G}_P, ..., \mathcal{G}_{P11}$ and Carl’s conversations $\mathcal{G}_C, ..., \mathcal{G}_{C11}$.

B. Chromatic transition from bipartite to multicoloured networks

From our graph collection –see section III A-, we obtain two sequences of chromatic numbers $s_P(\chi)$ and $s_C(\chi)$ corresponding to the evolution of the chromatic number in Peter and Carl datasets, respectively:

$$s_P(\chi) = \chi(\mathcal{G}_P1), ..., \chi(\mathcal{G}_{P11})$$
$$s_C(\chi) = \chi(\mathcal{G}_C1), ..., \chi(\mathcal{G}_{C11}) .$$

The above sequences display similar patterns with some interesting differences, see figure 2 and figure 4. For example, the middle stages of both datasets show an increase in the chromatic number. At the stage when the syntactic spur takes place, Peter’s dataset $s_P$ displays a sharp transition from a nearly constant, low chromatic number to a high chromatic number, which is fully consistent with the emergence of complex syntax. First tree networks in $s_P$ accept 2-colorings, i.e., they are bipartite, see figure 1. The grammar at this stage mainly generates pairs of complementary words, like:

$$\langle\text{verb, noun}\rangle \text{ or } \langle\text{adj., noun}\rangle .$$

Typical productions of this stage are, for example, ”car red” or “horsie run”. This pre-transition pattern, also called 2-stage grammar, corresponds to a highly restrictive grammar, e.g., syntactic structures like $\langle\text{verb, verb}\rangle$ do not exist. Instead, relations between lexical items are strongly constrained by their semantic content. On the other hand, Carl’s sequence $s_C$ shows $\chi \geq 3$ from the very beginning – i.e. these networks are not bipartite. A detailed inspection of Carl’s productions at this stage shows the presence of functional particles from the very beginning. This suggests that, in general, high chromatic numbers relates to high grammar flexibility, being this flexibility provided by the hinge role that have these particles in the global functioning of grammar.

| $f_1(\chi)$ | $f_2(\chi)$ | $f_3(\chi)$ | $f_4(\chi)$ | $f_5(\chi)$ | $f_6(\chi)$ | $f_7(\chi)$ | $f_8(\chi)$ | $f_9(\chi)$ | $f_{10}(\chi)$ | $f_{11}(\chi)$ |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $\mathcal{G}_P1$ | $\mathcal{G}_P2$ | $\mathcal{G}_P3$ | $\mathcal{G}_P4$ | $\mathcal{G}_P5$ | $\mathcal{G}_P6$ | $\mathcal{G}_P7$ | $\mathcal{G}_P8$ | $\mathcal{G}_P9$ | $\mathcal{G}_{P10}$ | $\mathcal{G}_{P11}$ |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1/49 | 5/105 | 66/434 | 131/644 | 87/589 | 157/903 | 104/659 | 95/717 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathcal{G}_C1$ | $\mathcal{G}_C2$ | $\mathcal{G}_C3$ | $\mathcal{G}_C4$ | $\mathcal{G}_C5$ | $\mathcal{G}_C6$ | $\mathcal{G}_C7$ | $\mathcal{G}_C8$ | $\mathcal{G}_C9$ | $\mathcal{G}_{C10}$ | $\mathcal{G}_{C11}$ |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6/140 | 5/119 | 11/156 | 6/128 | 10/152 | 14/199 | 61/361 | 65/442 | 71/439 | 93/592 | 131/687 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE I: Relative energy values of $q$-colorings in the Peter (top) and Carl (bottom) datasets (see text).
Still, the behaviour of $\chi(G)$ can be quite sensitive to the anomalous behaviour of small subgraphs. For example, the transition of $\chi(G_2) = 2$ to $\chi(G_3) = 3$, when Peter is about 23 months old, is due to a single triangle in a (largely) bipartite network — see figure 2 left. A combination of measurements enables us to assess whether the chromatic number represents the behaviour of a small number of nodes or it is the natural outcome of global network features. For example, we can compare $\chi(G)$ with the lower bound given by the clique number $\omega(G)$, and the upper bound provided by the maximal $K^*$-core connectivity $K^*(G)$. Therefore, each sequence $s_P(\chi), s_C(\chi)$ will be accompanied by two sequences, namely $\Omega, \kappa$:

$$
\Omega_{PC} = \omega(G_{P1,C1}), \ldots, \omega(G_{P11,C11})
$$

$$
\kappa_{PC} = K^*(G_{P1,C1}), \ldots, K^*(G_{P11,C11})
$$

Since every graph can be associated to a measure of $\chi$ relevance — see eq. (5) —, we will have a third sequence of $f$’s for every Peter’s graph and another associated to every Carl’s graph — see table I. For example, figure 2 - top — shows a clear trend towards increasing maximum clique and maximum $K^*$-core combined with increased relevance — see table I —, which indicates that the final chromatic number cannot be longer associated to any trivial clique. In any case, the relevance of the chromatic number as a
global complexity estimator is much more feasible after the transition. Both Peter and Carl sequences show that the chromatic number is often close to the clique number -see figure 2 (top). Maximum $K^*$-core size is generally more than twice the maximum clique size (see figure 2 (inset)). Then, the whole network structure (or a large part of it) has enough connectivity to enable the emergence of a non-trivial $K$-core structure. This is consistent with a manual inspection of grammars that generate a great amount of combinatorial complexity, i.e., a rich collection of compatibility relations.

C. Real syntax versus null model

Here, we compare the evolution of the chromatic number in real and simulated networks. A data-driven, syntax-free model that generates random child’s utterances having the same statistics of word production as Peter and Carl datasets is used as a null model [7]. This model definition enables us to assess if the high combinatorics displayed by post-transition networks emerge directly from an increasingly rich vocabulary. We build our model by extracting the following statistical parameters from the 11 recorded conversations in Peter and Carl corpora:

1. The number of sentences $S_P(i)$, $S_C(i)$ in the Peter and Carl datasets.

2. The probability distribution of structure lengths or the probability $P(s)$ that any syntactic structure has $s$ words. We obtain two different distributions, one for each dataset.

3. We assume that the probability of the $i$-th most frequent word is a scaling law:

$$p(i) = \frac{1}{Z} i^{-\beta},$$

with $1 \leq i \leq N_w(T)$, $\beta \approx 1$ -i.e., Zipf’s law- and $Z$ is the normalization constant:

$$Z = \sum_{i=1}^{N_w(T)} \left( \frac{1}{i} \right)^{\beta}.$$  \hspace{1cm} (13)

Notice that $Z$ depends on lexicon size, $N_w(T)$, which grows slowly at this stage.

We run the above model in the two datasets by generating $|S_{P,C}(i)|$ random sentences, each experiment is repeated 20 times. From the collection of randomly generated syntactic structures we construct a comparable sequence of syntax networks following the same method as in the real datasets -see section [IIA]. Figure 3 shows that our model generates random syntax networks with size and connectivity comparable to the ones measured in real networks. These statistical indicators display a huge increase during the studied period, being this increase sharper around the age of two, i.e., during the syntactic spurt [7]. As discussed in section II both the mean connectivity and network size play an important role when determining the values of $\omega$, $\chi$ and $K^*$.

Now, we compute the sequence of averaged chromatic numbers, $\bar{\chi}_{P,C}(\chi)$, for the simulated Peter and Carl syntax networks. Similarly, we generate the sequences of average clique number $\bar{\Omega}_{P,C}$ and the average maximum $K$-core $\bar{\kappa}_{P,C}$. The most salient property we find when comparing real networks obtained from both Peter and Carl’s corpora with their randomized counterparts is a huge increase of $\chi$ (and $\kappa$) in the simulated networks. That is, the ensemble of random strings displays higher complexity parameters than the real corpora. For example, at the end of the studied period, the three complexity estimators are close to 10 in Peter simulations and close to 9 Carl simulations -see figure 4 (bottom).

A very interesting feature is found at the first stages of the simulated Peter sequence: the random networks
are no longer bipartite—see section III B. In particular, the third corpus has an average chromatic number of 4, which is significantly higher than the observed chromatic number. In this case, the two-stage grammar imposes severe constraints on what is actually plausible in any pre-transition syntactic structure. This trend is also observed at later stages of language acquisition. In general, simulated networks have higher chromatic numbers than empirical networks, although both two types of networks have similar connectivities—by definition. In some cases, the average chromatic number of the graphs belonging to the random ensemble is twice the real one, see figure (2). To better understand the nature of these deviations, we have compared the behaviour of chromatic numbers against mean connectivity and the size of the largest connected component. Figure 4 shows a well-defined, non-trivial deviation between real networks and random networks. For example, the plot of chromatic number and network size display a quasi linear relationship in simulated networks—see figure (4) bottom. These plots suggest that the chromatic number is capturing essential combinatorial properties of the underlying system, which cannot be reproduced with a simple, syntax-free random generation model.

FIG. 4: Relationship between (right) the average degree and the chromatic number and (right) the size of the largest connected component and the chromatic number in the Peter (solid circles) and Carl (open squares). Simulation results are at the bottom while real data is shown at the top.

IV. DISCUSSION

Syntax is a characteristic, complex and defining feature of language organization. It pervades its capacity for unbounded generative power of the linguistic system [5], allows sentences to be organized in highly structured ways and is acquired in almost full power by children after being exposed to a limited repertoire of examples. Syntax is also one aspect of the whole: semantic and phonological aspects need to be taken into account, and they are all embedded in (and run by) a cognitive, brain-embodied framework [28]. Because of the dominant role played by how words actually interact with each other, computational and theoretical approaches dealing with word inventories or other statistical trends ignoring interactions are likely to be limited. Following previous work that takes advantage of complex networks approaches to language organization [10] we have made a step further in studying the organization of syntax graphs using graph coloring. The motivation of this approximation is twofold. On the one hand, graph colorability allows to properly detect correlations that are not captured by topological approaches. On the other hand, it seems a natural way to substantiate previous claims connecting syntax with compatibility relations common with other types of systems, such as chemical structures. Since graph coloring naturally defines compatibility through the presence or absence of a common label to every pair of nodes, it
seems the right framework to study the process of network growth in child language.

The behavior of the chromatic number accurately marks the syntactic spurt in language acquisition, i.e., it is a footprint of the generative power of the underlying grammar. There are limitations associated to the network definition. Syntactic relations are structure-dependent, not sequence dependent. Because the network is an aggregation of text sequences, it cannot fully grasp the hierarchical nature associated to syntactic constructs. Still, the chromatic number is a global measurement that can detect grammar constraints by analyzing the pattern of network interaction at different scales. That is, the network representation is an indicator of global linguistic performance and includes some combinatorial signal which can be properly detected with the chromatic number. In this context, standard network measurements like average degree, clustering or degree distribution are much more limited.

There are other, broader implications of our work. The chromatic number can be viewed as a reciprocal measure of standard community detection. Here, the chromatic number defines a partition of the network in classes of unlinked nodes. This definition is particularly relevant in networks where some kind of compatibility relation is at work in the wiring process. In this case, the standard community structure can be misleading, because elements of the same class cannot be connected. The case for syntactic graphs is paradigmatic but the partition induced by the chromatic number could shed light into the behaviour of many other systems. Additionally, we have proposed to assess the statistical significance of these partitions with the sequence of minimal violations -see equation (8). Future work should explore how the chromatic number (and related measures) can be exploited to detect forbidden links in the network. Deviations of the chromatic number (as the ones observed in this paper) suggest the presence of combinatorial constraints that must be taken into account, for example, when defining proper null-models.

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