Is quantum mechanics compatible with a deterministic universe? Two interpretations of quantum probabilities

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Abstract
Two problems will be considered: the question of hidden parameters and the problem of Kolmogorovity of quantum probabilities. Both of them will be analyzed from the point of view of two distinct understandings of quantum mechanical probabilities. Our analysis will be focused, as a particular example, on the Aspect-type EPR experiment. It will be shown that the quantum mechanical probabilities appearing in this experiment can be consistently understood as conditional probabilities without any paradoxical consequences. Therefore, nothing implies in the Aspect experiment that quantum theory is incompatible with a deterministic universe.

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1. Introduction

In this paper I propose to analyze the theoretical consequences of two distinct understandings of quantum mechanical probabilities. Assume that someone perform a measurement \( a \) on an entity of state \( W \). Denote \( A \) one of the possible outcomes of the measurement \( a \). In other words, the preparation part of the measurement, event \( a \), has happened, within circumstances identified as \( W \), and one of the outcome events occurs, denoted by \( A \). Repeating many times this measurement we can count the probability (relative frequency) of occurrence of \( A \). Assume that quantum mechanics describes the above situation. One can somehow figure out a Hermitian operator \( \hat{a} \) corresponding to \( a \), a density operator \( \hat{W} \) corresponding to the state \( W \), and the outcome \( A \) can be identified with a suitable projector \( \hat{A} \) from the spectral decomposition of \( \hat{a} \). The relative frequency of the outcome \( A \) is equal to \( \text{tr}(\hat{W}\hat{A}) \).

Now, the question is this: How can we interpret the probability \( \text{tr}(\hat{W}\hat{A}) \)? I suggest to analyze the following two different interpretations:

a) Property Interpretation: \( \text{tr}(\hat{W}\hat{A}) = p(\hat{A}) \), that is the probability that the entity has the property \( \hat{A} \), which property consists in that “the outcome \( A \) occurs whenever measurement \( a \) is performed”.

b) Minimal Interpretation: \( \text{tr}(\hat{W}\hat{A}) = p(A|a) \), that is the conditional probability of the occurrence of the outcome \( A \), given that the measurement \( a \) is performed.

At first sight one may think that there is no principal difference between the two above interpretations. For example it is obviously true that a) entails b). Indeed, assume that there is a property \( \hat{A} \) which already guarantees the occurrence of the outcome \( A \). Then, whenever we perform the measurement preparation \( a \), the occurrence of the outcome \( A \) depends on whether the entity has the property \( \hat{A} \) or not. Therefore \( p(A|a) = p(\hat{A}) \).

But the converse is not true: b) does not entail a), simply because interpretation b) does not assume at all the existence of a property which would guarantee the outcome \( A \). And this makes a big difference. As we will see in the following sections, these two distinct interpretations of the quantum mechanical probabilities lead to extremely different conclusions. We will consider two problems: the question of hidden parameters and the problem of Kolmogorovity of quantum probabilities. Both of them will be analyzed from the point of view of the Property Interpretation and the Minimal Interpretation. Our analysis will be focused on the Aspect-type EPR experiment, as a particular example.

2. The EPR experiment

An important historical step was Bell’s analysis of the EPR experiment. The great advantage of Bell’s approach to the problem of hidden variables was that even though he used part of the machinery of quantum mechanics, one does not need to use it, but only elementary probability calculus and the experimental results. And that is why his proof of the non-existence of (local) hidden variables has been regarded as the most serious.

Consider an experiment corresponding to the Clauser and Horne derivation of a Bell-type inequality. It is like Aspect’s experiment with spin-1/2 particles (Figure 1). The four directions in which the spin components are measured are coplanar with angles \( \angle(a, a') = \angle(a', b) = \angle(a, b') = 120^\circ \) and \( \angle(b, a') = 0 \). Denote \( N \) the number of particle pairs emitted by the source. Let \( N(A) \) be the detected number of the outcomes “spin up at the left wing in \( a \) direction”. We will consider the relative frequency \( p(A) = \frac{N(A)}{N} \).

Denote \( p(A'), p(B), p(B') \) the same relative frequencies for the outcomes \( A', B \) and \( B' \). It is also registered how many times the switches choose this or that direction. The corresponding relative frequencies
are denoted by $p(a), p(a'), p(b), p(b')$. Assume that the left and the right measurements as well as the choices of the left and the right switches are spatially separated.

Performing the experiments we observe the following facts:

$$
\begin{align*}
    p(A) &= p(A') = p(B) = p(B') = \frac{1}{4} \\
    p(a) &= p(a') = p(b) = p(b') = \frac{1}{2} \\
    p(A \land a) &= p(A) = \frac{1}{4} \\
    p(A' \land a') &= p(A') = \frac{1}{4} \\
    p(B \land b) &= p(B) = \frac{1}{4} \\
    p(B' \land b') &= p(B') = \frac{1}{4} \\
    p(A \land a') &= p(A' \land a) = p(B \land b') = p(B' \land b) = 0 \\
    p(A \land B) &= p(A \land B') = p(A' \land B') = \frac{3}{32} \\
    p(A' \land B) &= 0 \\
    p(a \land a') &= p(b \land b') = 0 \\
    p(a \land b) &= p(a' \land b') = p(a' \land b) = p(a' \land b') = \frac{1}{4} \\
    p(A \land b) &= p(A \land b') = p(A' \land b) = p(A' \land b') = \\
    &= p(B \land a) = p(B \land a') = p(B' \land a) = p(B' \land a') = \frac{1}{8}
\end{align*}
$$

The quantum mechanical description of the experiment is as follows:

Let $H^2$ be a two-dimensional Hilbert space of a spin-1/2 particle spin-states. Denote $\psi_{+a}$ and $\psi_{-a}$ the normalized eigenstates corresponding to “spin up” and “spin down” in the $a$ direction respectively. Given two particles the space of their spin-states is $H^2 \otimes H^2$. The state of the two-particle system is assumed to be the 0-spin state, represented as $\hat{W} = \hat{P}_{\psi_s}$, where $\psi_s = \frac{1}{\sqrt{2}} (\psi_{+a} \otimes \psi_{-a} - \psi_{-a} \otimes \psi_{+a})$ for an arbitrary direction $a$. The outcomes are identified with the following projectors:

$$
\begin{align*}
    \hat{A} &= \hat{P}_{\text{span}\{\psi_{+a} \otimes \psi_{+a}, \psi_{-a} \otimes \psi_{-a}\}} \\
    \hat{A}' &= \hat{P}_{\text{span}\{\psi_{+a} \otimes \psi_{+a}, \psi_{-a} \otimes \psi_{+a}\}} \\
    \hat{B} &= \hat{P}_{\text{span}\{\psi_{-b} \otimes \psi_{+b}, \psi_{+b} \otimes \psi_{+b}\}} \\
    \hat{B}' &= \hat{P}_{\text{span}\{\psi_{-b} \otimes \psi_{+b}, \psi_{+b} \otimes \psi_{+b}\}}
\end{align*}
$$

Figure 1: The Aspect experiment with spin-$\frac{1}{2}$ particles
Quantum mechanics is in agreement with the observations in the following sense:

\[
\frac{p(A \land a)}{p(a)} = \text{tr}(\hat{W}A) = \frac{p(A' \land a')}{p(a')} = \text{tr}(\hat{W}A') = \frac{p(B \land b)}{p(b)} = \text{tr}(\hat{W}B) = \frac{p(B' \land b')}{p(b')} = \text{tr}(\hat{W}B') = \frac{1}{2} \]

\[
\frac{p(A \land B \land a \land b)}{p(a \land b)} = \frac{p(A \land B)}{p(a \land b)} = \text{tr}(\hat{W}AB) = \frac{1}{2} \sin^2 \angle(a, b) = \frac{3}{8}
\]

\[
\frac{p(A' \land B \land a' \land b)}{p(a' \land b')} = \frac{p(A' \land B)}{p(a' \land b')} = \text{tr}(\hat{W}A'B) = \frac{1}{2} \sin^2 \angle(a', b) = 0
\]

\[
\frac{p(A' \land B' \land a' \land b')}{p(a' \land b')} = \frac{p(A' \land B')}{p(a' \land b')} = \text{tr}(\hat{W}A'B') = \frac{1}{2} \sin^2 \angle(a', b') = \frac{3}{8}
\]

Now we analyze the above experiment from the point of view of the two distinct interpretations of quantum probabilities.

3. The Property Interpretation

3.1. The hidden parameter problem

According to the Property Interpretation, the occurrence of an outcome $A$, for example, reflects the existence of a property $\sim A$. Therefore, for the probability that the corresponding property exists we have

\[
p(E \land e) = \frac{p(E \land e)}{p(e)} \quad E, F = A, A', B, B'
\]

\[
p(E \land F \land e \land f) = \frac{p(E \land F \land e \land f)}{p(e \land f)} \quad e, f = a, a', b, b'
\]

As it turns out from (1) we encounter a correlation among the outcomes of spatially separated measurements. If local hidden variable theories could exist, one would be able to try to explain this correlation via a common cause mechanism. Briefly recall the usual assumptions describing the notion of a local hidden variable. Assume that there is a parameter $\lambda$, taken as an element of a probability space $\langle \Lambda, \Sigma(\Lambda), \rho \rangle$, such that the quantum mechanical probabilities can be represented as follows:

\[
p(\lambda, \sim A \land B) = p(\lambda, \sim A) p(\lambda, B)
\]

\[
p(\sim A) = \int_{\Lambda} p(\lambda, \sim A) d\rho
\]

\[
p(B) = \int_{\Lambda} p(\lambda, B) d\rho
\]

\[
p(\sim A \land B) = \int_{\Lambda} p(\lambda, \sim A) p(\lambda, B) d\rho
\]

Now, for real numbers such that

\[
0 \leq x, x', y, y' \leq 1
\]

the following elementary inequality holds

\[
-1 \leq xy - x'y' + x'y - x - y \leq 0
\]

Applying this inequality, we have
\[-1 \leq p(\lambda, \vec{A} \land \vec{B}) - p(\lambda, \vec{A} \land \vec{B}) + p(\lambda, \vec{A} \land \vec{B}) + p(\lambda, \vec{A} \land \vec{B}) \leq 0\]

Integrating this inequality we have
\[-1 \leq p(\vec{A} \land \vec{B}) - p(\vec{A} \land \vec{B}) + p(\vec{A} \land \vec{B}) + p(\vec{A} \land \vec{B}) - p(\vec{A} \land \vec{B}) \leq 0\]  \hspace{1cm} (5)

This is one of the well-known Clauser-Horne inequalities (one can get all the others by varying the roles of $\vec{A}$, $\vec{A}'$ and $\vec{B}$, $\vec{B}'$). Returning to the Aspect experiment, from (3) we have
\[
\begin{align*}
    p(\vec{A}) &= p(\vec{A}) = p(\vec{B}) = p(\vec{B}) = \frac{1}{2} \\
p(\vec{A} \land \vec{B}) &= p(\vec{A} \land \vec{B}) = p(\vec{A} \land \vec{B}) = \frac{3}{8} \\
p(\vec{A} \land \vec{B}) &= 0
\end{align*}\]  \hspace{1cm} (6)

These probabilities violate the Clauser-Horne inequality. Thus, according to the usual conclusion, there is no local hidden variable theory reproducing the quantum mechanical probabilities. In other words, no common cause explanation is possible for the correlations between space-like separated measurement outcomes in the EPR experiment. That is, quantum mechanics seems to violate Einstein causality.

### 3.2. The question of Kolmogorovity

Recall Pitowsky’s important theorem about the conditions under which a probability theory is Kolmogorovian. Let $S$ be a set of pairs of integers $S \subseteq \{\{i, j\} \mid 1 \leq i \leq j \leq n\}$. Denote by $R(n, S)$ the linear space of real vectors having a form like $(f_1, f_2, \ldots, f_n, \ldots, f_{ij}, \ldots)$. For each $\varepsilon \in \{0, 1\}^n$, let $u^\varepsilon$ be the following vector in $R(n, S)$:
\[
\begin{align*}
    u^\varepsilon_i &= \varepsilon_i \quad 1 \leq i \leq n \\
u^\varepsilon_{ij} &= \varepsilon_i \varepsilon_j \quad \{i, j\} \in S
\end{align*}\]  \hspace{1cm} (7)

The classical correlation polytope $C(n, S)$ is the closed convex hull in $R(n, S)$ of vectors $\{u^\varepsilon\}_{\varepsilon \in \{0, 1\}^n}$:
\[C(n, S) = \left\{ a \in R(n, S) \mid a = \sum_{\varepsilon \in \{0, 1\}^n} \lambda_\varepsilon u^\varepsilon \text{ such that } \lambda_\varepsilon \geq 0 \text{ and } \sum \lambda_\varepsilon = 1 \right\} \]  \hspace{1cm} (8)

Consider now events $A_1, A_2, A_3, \ldots, A_n$ and some of their conjunctions $A_i \land A_j \ (\{i, j\} \in S)$. Assume that we know their probabilities, from which we can form a so called correlation vector
\[p = (p_1, p_2, \ldots, p_n, \ldots, p_{ij}, \ldots) = (p(A_1), p(A_2), \ldots, p(A_n), \ldots, p(A_i \land A_j), \ldots) \in R(n, S)\]

We will then say that $p$ has a Kolmogorovian representation if there exist a Kolmogorovian probability space $(\Omega, \Sigma, \mu)$ and measurable subsets $X_{A_1}, X_{A_2}, X_{A_3}, \ldots, X_{A_n} \in \Sigma$ such that
\[
\begin{align*}
p_i &= \mu(X_{A_i}) \quad 1 \leq i \leq n \\
p_{ij} &= \mu(X_{A_i \land A_j}) \quad \{i, j\} \in S
\end{align*}\]

**Theorem 1** (Pitowsky, 1989) A correlation vector $p = (p_1, p_2, \ldots, p_n, \ldots, p_{ij}, \ldots)$ has a Kolmogorovian representation if and only if $p \in C(n, S)$.  

5
From the definition of the polytope, equations (7) and (8), it is obvious that the condition $p \in C(n, S)$ has the following meaning: the probabilities can be represented as weighted averages of the classical truth values defined on the corresponding propositional logic.

In case $n = 4$ and $S = S_4 = \{\{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}\}$, the condition $p \in C(n, S)$ is equivalent with the following inequalities:

$$0 \leq p_{ij} \leq p_j \leq 1$$
$$0 \leq p_{ij} \leq p_j \leq 1 \quad i = 1, 2 \quad j = 3, 4$$
$$p_i + p_j - p_{ij} \leq 1$$

\[\begin{align*}
-1 \leq p_{13} + p_{14} + p_{24} - p_{23} - p_1 - p_4 & \leq 0 \\
-1 \leq p_{23} + p_{24} + p_{14} - p_{13} - p_2 - p_4 & \leq 0 \\
-1 \leq p_{14} + p_{13} + p_{23} - p_{24} - p_1 - p_3 & \leq 0 \\
-1 \leq p_{24} + p_{23} + p_{13} - p_{14} - p_2 - p_3 & \leq 0
\end{align*}\] \hspace{1cm} (9)

The last inequality of (10) is nothing else but inequality (5) if $p_1 = p(\bar{A}), p_2 = p(\bar{A'}), p_3 = p(\bar{B}), p_4 = p(\bar{B'})$ and $p_{13} = p(\bar{A} \land \bar{B}), p_{14} = p(\bar{A'} \land \bar{B}), p_{23} = p(\bar{A} \land \bar{B'}), p_{24} = p(\bar{A'} \land \bar{B'})$. Therefore, substituting for the probabilities in the last inequality of (10) the values as were calculated from quantum mechanics in (6), we have

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} - 0 - \frac{1}{2} - \frac{1}{2} = \frac{1}{8} > 0$$ \hspace{1cm} (11)

Consequently,

$$p = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{3}{8}, 0, \frac{3}{8}\right) \notin C(4, S)$$

Now, Pitowsky concludes: “We have demonstrated that $p \notin C(n, S)$ and therefore we cannot explain the statistical outcome by assuming that the source is an “urn”, containing electron pairs in the singlet state, such that the distribution of the properties $\bar{A}, \bar{A'}, \bar{B}, \bar{B'}$ in this “urn” is fixed before the measurement.” (Pitowsky, p. 82; notation changed for sake of uniformity). “The violations of these constraints on correlations by quantum frequencies thus poses a major problem for all schools of classical probability. I take this fact to be the major source of difficulty which underlies the interpretation of quantum theory.” (Ibid., p. 87).

4. The Minimal Interpretation

As we can see, the Property Interpretation leads to at least three paradoxical difficulties, which seem to be unresolved: 1) There is no local hidden parameter theory possible which could provide a common cause explanation for the EPR correlations, therefore 2) quantum mechanics violates the Einstein causality. 3) Quantum mechanics violates the axioms of a Kolmogorovian probability theory, and this violation makes it very difficult to provide a consistent interpretation for the quantum mechanical probabilities. These difficulties lead to conclusion that quantum mechanics is not compatible with a deterministic universe (see Szabó 1994).

These are extremely grave, but widely accepted, conclusions. However, as we are going to show in this section, none of the above paradoxical difficulties appear if we accept the Minimal Interpretation of quantum probabilities.
4.1. The question of Kolmogorovity

Now the quantum mechanical probabilities mean conditional probabilities only:

\[
\text{tr}(\hat{W}E) = p(E|e) = \frac{p(E \wedge e)}{p(e)} \quad E = A, A', B, B' \quad e = a, a', b, b'
\]

\[
\text{tr}(\hat{W}F) = p(E \wedge F|e \wedge f) = \frac{p(E \wedge F \wedge e \wedge f)}{p(e \wedge f)} \quad E = A, A' \quad e = a, a'
\]

\[
F = B, B' \quad f = b, b'
\]  

(13)

where the events \(a, a', b, b'\) play an explicit role. Therefore, the question of Kolmogorovity has to be phrased in another way: We have 8 events, \(A, A', B, B', a, a', b, b'\) and all of their 28 possible conjunctions. The probabilities of these 8+28 events are empirically given by (1), and these probabilities are in harmony with quantum mechanics in the sense of (13). Now, the question is whether a Kolmogorovian probability space \((\Omega, \Sigma, \mu)\) and measurable subsets

\[X_A, X_{A'}, X_B, X_{B'}, X_a, X_{a'}, X_b, X_{b'} \in \Sigma\]

exist, such that

\[
p(E) = \mu(X_E) \quad p(E \wedge F) = \mu(X_E \cap X_F) \quad E, F = A, A', B, B', a, a', b, b'
\]

(14)

in order to reproduce the empirical observations (1), and

\[
p(E \wedge F \wedge e \wedge f) = \frac{p(E \wedge F \wedge e \wedge f)}{p(e \wedge f)} = \frac{\mu(X_E \cap X_F \cap X_e \cap X_f)}{\mu(X_e \cap X_f)} \quad E = A, A' \quad e = a, a'
\]

\[
F = B, B' \quad f = b, b'
\]

(15)

in order to reproduce the quantum mechanical probabilities, consistently, as conditional probabilities \(*^{\dagger}\). We are going to prove now that such a Kolmogorovian representation does exist. It follows from the Pitowsky theorem that there exists a Kolmogorov probability space satisfying (14) iff \(p \in C(\mathcal{S}_{28}^\text{max})\), where the correlation vector \(p\) consists of probabilities described in (1). If we assume that this condition is satisfied and a suitable probability space exists, then, as a consequence of the properties described in (1), (15) is also satisfied. Indeed, as a consequence of \(p(A \wedge a) = p(A)\) we have \(\mu(X_A \wedge X_a) = \mu(X_A)\) which implies that \(\mu(X_A \wedge X_a) = 0\). In the same way we have \(\mu(X_B \wedge X_b) = 0\). \(X_A \wedge X_B\) can be written as a union of two disjunct subsets

\[\]

* We don’t need a new formal definition of a Kolmogorovian representation in the Minimal Interpretation. Only the set of “empirical probabilities” is different. Instead of numbers like \(\frac{p(E \wedge e)}{p(e)}\), the numbers \(p(E)\), \(p(e)\) and \(p(E \wedge e)\) are the “given probabilities” for which we are seeking a Kolmogorovian representation. If such representation exists, the “quantum probabilities” \(\frac{p(E \wedge e)}{p(e)}\) are also represented as traditional conditional probabilities

\[
\frac{\mu(X_E \cap X_e)}{\mu(X_e)}\]

according to the Bayes-law.

† We need (15) only to guarantee the correct value of \(\mu(X_E \cap X_F \cap X_e \cap X_f)\), which is normally outside of the scope of the Pitowsky theorem.
\[ X_A \cap X_B = \left( \frac{(X_A \cap X_B) \setminus (X_A \setminus X_a) \cup (X_B \setminus X_b)}{X_A \cap X_B \cap X_a \cap X_b} \right) \cup \left( \left( X_A \setminus X_a \right) \cup \left( X_B \setminus X_b \right) \right) \]

Therefore

\[ \mu(X_A \cap X_B) = \mu(X_A \cap X_B \cap X_a \cap X_b) + \mu(X_A \setminus X_a) \cup (X_B \setminus X_b) \]

The same holds for \( A \) and \( B' \), \( A' \) and \( B \) as well as for \( A' \) and \( B' \). Equation (15) therefore obeys.

Now, the question is whether the probability vector

\[ p = \begin{pmatrix} p(A) p(A') \ldots p(b) p(b') p(A \wedge A') p(A \wedge B) \ldots p(b \wedge b') \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \]

satisfies condition

\[ p \in C_{\text{max}} \]

or not. In case \( n > 4 \) there are no derived inequalities which would be equivalent to the condition \( p \in C(n, S) \) (see Pitowsky 1989 for the details). We thus have to directly examine the geometric condition (17). I testified condition (17) by computer (see Appendix) and the result is affirmative:

\[ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 32 & 32 & 32 & 32 & 32 & 32 & 32 \end{pmatrix} \in C_{\text{max}} \]

therefore the probabilities measured in the Aspect experiment do have Kolmogorovian representation as well as the quantum mechanical probabilities, in this experiment, can be represented as conventional (defined by the Bayes law) conditional probabilities in a Kolmogorovian probability theory.

4.2. The hidden parameter problem

According to the Minimal Interpretation there are no “properties” corresponding to the outcomes of the measurements we can perform on the system (at least quantum mechanics has nothing to do with such properties). We do not need, therefore, to explain the correlation between spatially separated occurrences of such (non-existing) properties. But, there do exist (observable physical) events corresponding to the performance-preparations of various measurements and other events which correspond to the outcomes.

Each such event occurs with certain probability shown in (1). What we observe in the EPR experiment is a correlation between spatially separated outcomes. And the question is whether a local hidden variable ex-
planation for such a correlation is possible or not. Thus, the whole formulation of the common cause problem has to be reconsidered.

A local hidden variable theory (regarded as a mathematically well formulated representation of a deterministic and non-violating Einstein causality universe) has to reproduce the probability of each event, i.e., the probabilities of the outcomes and that of the performance-preparation. The assumed “parameter” \( \lambda \in \Lambda \) should represent the state of the part of the universe which belongs to the common past of the two separated measurements (Figure 2) such that not only for the outcomes but also for the performance-preparations we have

\[
p(E) = \int_{\Lambda} p(\lambda, E) \, d\lambda \quad E = A, A', B, B', a, a', b, b'
\]  

(18a)

The same holds for the conjunctions

\[
p(E \land F) = \int_{\Lambda} p(\lambda, E \land F) \, d\lambda \quad E = A, A', a, a' \quad F = B, B', b, b'
\]  

(18b)

We also assume that the underlying hidden variable theory is Einstein-local. This means that if there is a correlation between any two spatially separated events, it should be the consequence of the \( \lambda \)-dependence of the corresponding probabilities, but not the consequence of a direct physical interaction. This assumption is, as usual, formulated via the following relations:

\[
p(\lambda, E \land F) = p(\lambda, E) p(\lambda, F) \quad E = A, A' \quad F = B, B'
\]  

(19a)

\[
p(\lambda, e \land f) = p(\lambda, e) p(\lambda, f) \quad e = a, a' \quad f = b, b'
\]  

(19b)

\[
p(\lambda, E \land f) = p(\lambda, E) p(\lambda, f) \quad E = A, A' \quad f = b, b'
\]  

(19c)

\[
p(\lambda, e \land F) = p(\lambda, e) p(\lambda, F) \quad e = a, a' \quad F = B, B'
\]

Relations (19a) express the \( \lambda \)-level independence of the outcomes (in other words, the screening off the outcomes by the hidden parameter), relations (19b) express the \( \lambda \)-level independence of the choices which measurement will be performed. Finally, equations (19c) represent the \( \lambda \)-level independence of the outcomes from the spatially separated choices (parameter independence).

Now, the question is whether there exists such a parameter satisfying conditions (18) and (19). One can apply the following theorem:

**Theorem 2** With the notation of the section 3.2 consider events \( A_i \) \( i \in S \) and a set of indexes \( S \). Assume that a correlation vector \( p = \{ p(A_i), p(A_i \land A_j), \ldots \} \) can be represented as convex combination of parameter-depending correlation vectors \( \{ \pi(\lambda, A_i), \ldots, \pi(\lambda, A_i \land A_j), \ldots \} \).

\[
p(A_i) = \int_{\Lambda} \pi(\lambda, A_i) \, d\lambda \quad \text{for } 1 \leq i \leq n \quad \text{(20)}
\]

such that

\[
p(A_i \land A_j) = \int_{\Lambda} \pi(\lambda, A_i \land A_j) \, d\lambda \quad \text{for } \{i, j\} \in S
\]

* We cannot use the formal reproduction of the Clauser-Horne derivation described in section 3.1 because the number of events and the number of investigated conjunctions are larger than four.
\[ \pi(\lambda, A_i \wedge A_j) = \pi(\lambda, A_i) \cdot \pi(\lambda, A_j) \quad \text{for each } \{i, j\} \in S \quad (21) \]

Then \( p \in C(n, s) \).

(See Szabó 1993 for the proof.)

Equations (18) correspond to (20) and (19) to (21) in case

\[
\begin{align*}
A_1 &= A \\
A_2 &= A' \\
A_3 &= a \\
A_4 &= a' \\
A_5 &= B \\
A_6 &= b \\
A_7 &= B' \\
A_8 &= b'
\end{align*}
\]

\[ S = S_{16} = \{\{1, 3\}, \{1, 4\}, \{1, 7\}, \{1, 8\}, \{2, 3\}, \{2, 4\}, \{2, 7\}, \{2, 8\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}\} \]

According to Theorem 2, if there exists a hidden parameter theory satisfying conditions (18) and (19) then the observed probabilities should satisfy condition \( p \in C(8, S_{16}) \).

In (1) we gathered all the information known about the observed probabilities in the Aspect-type spin-correlation experiment. We can collect these data in a correlation vector. The question is whether this correlation vector is contained in the classical correlation polytope or not:

\[ p = (1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1. There can exist no hidden parameter theories reproducing all the empirically testable part of quantum mechanics. At least, such a hidden variable theory should violate Einstein-causality.

2. Quantum mechanics is not a Kolmogorovian probability theory.

The latter is related to the problem of determinism in the following way. The probabilities of future events in a deterministic world should be interpreted as weighted averages of the possible (classical) truth value assignments, since only in this case we can interpret probabilities epistemically, relating them to our lack of knowledge. In other words, only in this case one can assume that the state of affairs is settled in advance, but we do not have enough information about this settlement. But we also know that this is true only if the corresponding probabilities admit a Kolmogorovian representation (Pitowsky 1989).

In this paper, I have challenged the above far reaching conclusions by showing that these are all rooted in a particular interpretation of quantum probabilities, namely in the Property Interpretation.

Moreover, the Property Interpretation is contradictory in itself. According to it we assume the existence of properties which surely predetermines the outcomes of the possible measurements. These properties appear before the measurements are performed. (Otherwise we play another ball-game, namely, the Minimal Interpretation.) Therefore we have to assume that the distribution of these properties in a statistical ensemble is fixed before the measurements. On the basis of the Property Interpretation we derive then various “No Go” theorems like the Bell theorem, the Kochen-Specker theorem (Pitowsky 1989) or the GHSZ-Mermin theorem (Mermin 1990), which assert that such properties cannot exist. From this contradiction it does not at all follow either that quantum mechanics is a non-Kolmogorovian probability theory or that there are no hidden parameter theories without violation of Einstein-causality. But it does follow that the Property Interpretation is an untenable interpretation of quantum probabilities.

In opposition to the Property Interpretation, we have seen, at least in case of the crucial Aspect experiment, that the quantum mechanical probabilities can be consistently understood as conditional probabilities without any paradoxical consequences. The Minimal Interpretation provides that the quantum mechanical model of the Aspect experiment remains within the framework of the Kolmogorov probability theory, and nothing excludes the existence of local hidden parameters. Therefore, this experiment does not imply that quantum theory is incompatible with a deterministic universe.

One may argue that we had to pay dearly for that. We had to give up the existence of properties pre-determining the outcomes, and we had to include the performance-preparations of measurements as events in the probability-theoretic description of quantum systems. However, this price was only a fictitious one. The existence of properties is excluded by quantum mechanics itself. The Minimal Interpretation only says that we mustn’t do calculation as if they were existing. The other problem is a little more serious. Still, if we take the question “Is quantum mechanics compatible with a deterministic universe?” seriously, we cannot ignore that a “deterministic universe” includes not only the causal determination of the measurement outcomes, but it also includes causally deterministic “decision” processes of whether this or that measurement is being performed. No matter whether these decisions are made by machines like the switches in the Aspect experiment or by human beings (the problem of the free will, however, is beyond the scope of this paper), such processes must be deterministic in a deterministic world (Cf. Brans 1988). The unpredictability of such decisions, in the same way as that of the measurement outcomes, appears epistemically, in connection of the lack of knowledge.

Finally, I would like to emphasize that all of my analysis in this paper referred only to a particular situation of the Aspect experiment. I did challenge, however, the following two widely accepted views: The Aspect experiment provides empirical data (with full agreement to the results of the quantum mechanical calculations), such that 1) there cannot exist a Kolmogorovian probability model, in which these data could be represented, and 2) there cannot exist a local hidden variable model capable of reproducing these empirical data. It has been shown that there is a possible understanding of these data (the “minimal interpretation”), which provides 1) a consistent representation of the measured relative frequencies by a Kolmogorovian probability model, and 2) a local hidden parameter model reproducing the empirical frequencies.

Taking into account, however, that the analyzed Aspect experiment is usually regarded as a crucial particular example, these results can provide a good basis for further analysis.
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Appendix

The non-zero weights of vertices for correlation vector

\[ p = \left( p(A) p(A') p(B) \ldots p(b) p(b') p(A \wedge A') p(A \wedge B) p(A \wedge B') \ldots p(b \wedge b') \right) = \]

\[ = \left( \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{8} \frac{1}{8} \frac{1}{8} \frac{1}{8} \ldots \frac{1}{32} \frac{1}{32} \frac{1}{32} \frac{1}{32} \frac{1}{8} \frac{1}{8} \frac{1}{8} \frac{1}{8} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \right) \]

in convex linear combination: \[ p = \sum_{\varepsilon \in \{0,1\}^n} \lambda_{\varepsilon} u^{\varepsilon} \]

| \( \varepsilon \) | \( \lambda_{\varepsilon} \) |
|---|---|
| (0, 0, 0, 0, 0, 0, 0, 0) | 1.331953826593235 \( \times 10^{-004} \) |
| (0, 0, 0, 0, 0, 0, 0, 1) | 3.981811842052041 \( \times 10^{-012} \) |
| (0, 0, 0, 0, 0, 0, 1, 0) | 1.5858103251792 \( \times 10^{-011} \) |
| (0, 0, 0, 0, 1, 0, 0, 0) | 1.719473348427147 \( \times 10^{-011} \) |
| (0, 0, 0, 0, 1, 0, 0, 1) | 9.3749128207016 \( \times 10^{-002} \) |
| (0, 0, 0, 1, 0, 0, 0, 0) | 1.659025110711809 \( \times 10^{-003} \) |
| (0, 0, 1, 0, 0, 0, 0, 0) | 2.302471218278586 \( \times 10^{-009} \) |
| (0, 0, 1, 0, 0, 0, 0, 1) | 9.37536117362976 \( \times 10^{-002} \) |
| (0, 0, 1, 0, 0, 0, 1, 0) | 9.374772757291794 \( \times 10^{-002} \) |
| (0, 0, 1, 0, 0, 1, 0, 0) | 4.57967041025964 \( \times 10^{-012} \) |
| (0, 0, 1, 0, 0, 1, 0, 1) | 1.984600201190845 \( \times 10^{-012} \) |
| (0, 0, 1, 0, 1, 0, 0, 0) | 7.13608848838531 \( \times 10^{-015} \) |
| (0, 0, 1, 0, 1, 0, 0, 1) | 3.12548503279686 \( \times 10^{-002} \) |
| (0, 0, 1, 1, 0, 0, 0, 0) | 6.30150407232577 \( \times 10^{-006} \) |
| (0, 0, 1, 1, 0, 0, 0, 1) | 3.12410371534824 \( \times 10^{-002} \) |
| (0, 0, 1, 1, 0, 0, 1, 0) | 3.07732580040465 \( \times 10^{-006} \) |
| (0, 0, 1, 1, 0, 1, 0, 0) | 2.11055461302178 \( \times 10^{-011} \) |
| (0, 0, 1, 1, 0, 1, 0, 1) | 1.05007003980725 \( \times 10^{-007} \) |
| (0, 0, 1, 1, 1, 0, 0, 0) | .1249948143959045 |
| (0, 0, 1, 1, 1, 0, 0, 1) | 1.051358132497973 \( \times 10^{-011} \) |
| (0, 0, 1, 1, 1, 0, 1, 0) | 3.12484651803970 \( \times 10^{-002} \) |
| (0, 1, 0, 0, 0, 0, 0, 0) | 4.604883748621225 \( \times 10^{-012} \) |
| (0, 1, 0, 0, 0, 0, 0, 1) | 1.379018774172458 \( \times 10^{-011} \) |
| (0, 1, 0, 0, 0, 0, 1, 0) | 1.07745362086089 \( \times 10^{-005} \) |
| (0, 1, 0, 0, 0, 1, 0, 0) | 3.125741332769394 \( \times 10^{-002} \) |
| (0, 1, 0, 0, 0, 1, 0, 1) | .1249817684292793 |
| (0, 1, 0, 0, 0, 1, 1, 0) | 2.092178116623722 \( \times 10^{-011} \) |
| (0, 1, 0, 0, 0, 1, 1, 1) | 4.593699486576952 \( \times 10^{-006} \) |
| (0, 1, 0, 0, 1, 0, 0, 0) | 6.67953545117687 \( \times 10^{-006} \) |
| (0, 1, 0, 0, 1, 0, 0, 1) | 9.37367007136344 \( \times 10^{-002} \) |
The reader can easily imagine similar explicit results for vector (22).

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