Kernel Function Based Mean Matrix Estimation and Its Application to Radar Target Detection

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Abstract. The estimation of mean matrix for a set of Hermitian positive definite (HPD) matrices is of fundamental importance to radar target CFAR detection, which can be considered based on a geometric framework. However, false targets can cause large errors in the estimation of the mean clutter matrix. In this paper, we consider the nonlinearity of kernel function and the mean matrix estimation method based on kernel function is designed. In order to reduce the influence of false targets in the estimation of mean matrix, we apply the kernel function to the nonlinear adjustment of the geometric distance. Then, the kernel function based constant false alarm rate (CFAR) detector is designed. Finally, numerical experiments based on simulated clutter data and measured sea clutter data are performed. We come to a conclusion that the method of matrix mean value estimation based on kernel function can restrain the false target in the reference cells effectively.

1. Introduction

Target detection in the background of sea clutter is always a difficult problem for researchers [1-4]. In the field of moving targets detection and recognition, researchers distinguish target from clutter by extracting the characteristics of the signal. In the early days, the typical method of target detection was constant false alarm rate (CFAR) and its improved algorithm based on background clutter distribution model [2-4]. With the background clutter distribution, we can quantitatively analyse the detection rate and false alarm rate of target detection. However, there is still no accurate model to characterize sea clutter. K distribution model is the most widely used, which is a good fit to the amplitude statistical characteristic of the real observed data.

However, the sea clutter is difficult to be represented by a unique K distribution model due to its non-stationary characteristics, and there is not a unique detector to detect targets. So, the researchers considered the correlation characteristics of the signal and designed matrix constant false alarm rate (CFAR) detector based on the basic concept of information geometry theory. In the field of radar target detection, the representative research achievement is the research of pulse Doppler radar matrix CFAR detection based on positive determinate matrix manifolds. It’s researched by Barbaresco et al. researchers of Thales Air Systems, France [5-8]. Mean matrix estimation is an important step in matrix CFAR detection [9-11]. In manifolds, the sum of the distances from the mean matrix to sampled matrices is minimized. However, false targets can cause large errors in the estimation of the mean clutter matrix. For this reason, we consider the nonlinearity of kernel function and design the kernel function based mean matrix estimation method. Based on the theory of information geometry, we design the CFAR detector based on kernel function. Then, numerical tests of the simulation data and measured data are performed.
carried out to verify that kernel function based mean matrix estimation method can effectively suppress false targets in reference cells.

2. Kernel function based mean matrix estimation and CFAR detector

2.1. Signal model
The target model can be expressed as

$$s = \alpha p$$

(1)

where $\alpha$ is expressed as the power of the target signal and $p$ is expressed as the doppler steering vector, which can be denoted as

$$p = \frac{1}{\sqrt{N}}[1, \exp(j2\pi f_d), \ldots, \exp(j2\pi(N-1)f_d)]^T$$

(2)

where $f_d$ stand for the normalized doppler frequency.

In this paper, sea clutter is taken as the background noise. The results show that most of the sea clutter amplitudes obey K-distribution and the power spectrum belongs to Gaussian spectrum. As a newly constructed hybrid model, K-distribution is one of the most widely recognized models that can accurately model sea clutter. The probability density function of the amplitude is

$$p(x) = \frac{2\mu}{\Gamma(\nu)} \left( \frac{\mu x}{2} \right)^{\nu} K_{\nu-1}(\mu x),$$

(3)

where $\nu$ and $\mu$ represent the shape parameter and scale parameter, respectively. $\Gamma(\cdot)$ is the Gamma function, $K_{\nu-1}(\cdot)$ is the $(\nu-1)$-order of modified Bessel functions of the second kind.

2.2. Kernel function based mean matrix estimation
Estimating the mean matrix of clutter is very important in the work of matrix CFAR detector. However, false targets can cause large errors in the estimation of the mean clutter matrix. In order to reduce the influence of false object on the estimation of clutter mean, we use kernel function to adjust the geometric metric nonlinearly. We use the Gaussian kernel function to adjust the symmetrized Kullback-Leibler (SKL) divergence and calculate the mean value matrix by using the SKL divergence after nonlinear processing. SKL divergence can be expressed as

$$D_{SKL}(R_1, R_2) = \frac{1}{2} \left[ tr \left( R_1^{-1}R_2 + R_2^{-1}R_1 \right) - 2N \right]$$

(4)

where $R_1, R_2$ are symmetric positive definite matrices. Gaussian kernel is the most commonly used kernel function. We use Gaussian kernel for nonlinear transformation of SKL divergence, as follow

$$K_{GSKL}(R_1, R_2) = \exp \left( -\frac{D_{SKL}(R_1, R_2)}{2\sigma^2} \right)$$

(5)

where $\sigma$ represents the width parameter. Mean matrices $\bar{R}$ can be given by the solution of the following maximum problem

$$\bar{R} = \arg \max_R \frac{1}{N} \sum_{i=1}^{N} \exp \left( -\frac{1}{4\sigma^2} tr \left( R_1^{-1}R + R^{-1}R_1 - 2I \right) \right)$$

(6)

We define the objective function $F(R)$, as follow

$$F(R) = \frac{1}{N} \sum_{i=1}^{N} \exp \left( -\frac{1}{4\sigma^2} tr \left( R_1^{-1}R + R^{-1}R_1 - 2I \right) \right)$$

(7)

The gradient is
$$\nabla F(R) = -\frac{1}{4N\sigma^2} \sum_{i=1}^{N} \left\{ -R^{-2}R_i + R_i^{-1} \right\} \exp\left( -\frac{1}{4\sigma^2} \text{tr}\left( R_i^{-1}R + R_iR_i^{-1} - 2I \right) \right)$$  \hspace{1cm} (8)

Let \( \nabla F(R) = 0 \), that is,

$$\sum_{i=1}^{N} \left\{ -R^{-2}R_i + R_i^{-1} \right\} \exp\left( -\frac{1}{4\sigma^2} \text{tr}\left( R_i^{-1}R + R_iR_i^{-1} \right) \right) = 0$$  \hspace{1cm} (9)

A pair of upper transformation can be obtained:

$$R = \left( \frac{\sum_{i=1}^{N} R_i^{-1}E_i}{\sum_{i=1}^{N} R_iE_i} \right)^{-\frac{1}{2}}$$  \hspace{1cm} (10)

where

$$E_i = \exp\left( -\frac{1}{4\sigma^2} \text{tr}\left( R_i^{-1}R + R_iR_i^{-1} \right) \right)$$  \hspace{1cm} (11)

Using Weiszfeld algorithm, the iterative solution of the mean matrix \( \bar{R} \) can be obtained

$$\bar{R}_{i+1} = \left( \frac{\sum_{i=1}^{N} R_i^{-1}E_i}{\sum_{i=1}^{N} R_iE_i} \right)^{-\frac{1}{2}}$$  \hspace{1cm} (12)

where

$$E_i = \exp\left( -\frac{1}{4\sigma^2} \text{tr}\left( R_i^{-1}R + R_iR_i^{-1} \right) \right)$$  \hspace{1cm} (13)

2.3. Kernel function based SKL divergence CFAR detector

Comparing with the traditional matrix CFAR detector based on SKL divergence (SKL-CFAR), we design the kernel function-based CFAR detector (KSKL-CFAR). The KSKL-CFAR detector uses the kernel function based mean matrix estimation method to find the mean value matrix of the reference cell correlation matrices. According to the mean matrix obtained, we compare \( \text{SKL}(R_{\theta}, \bar{R}) \) to the threshold to determine if the target exists. Because there is not a numerical solution of the threshold \( \eta \), we calculate the threshold estimate \( \hat{\eta} \) using Monte Carlo simulation. Finally, the detection statistic is described below:

$$\begin{align*}
H_1 & : \\
\text{SKL}(R_{\theta}, \bar{R}) & \geq \hat{\eta} \\
H_0 & :
\end{align*}$$  \hspace{1cm} (14)

As described in Table 1, the provided algorithm consists of five steps to obtain the detection statistic and set the adaptive threshold.

| Algorithm steps |
|-----------------|
| **Step 1:** Take \( m \) pulse from each cell, and calculate their correlation matrices \( \{R_1, R_2, \cdots, R_{m}, R_{d}\} \) . |
| **Step 2:** Calculate the mean matrix \( \bar{R} \) of the reference cell using the kernel function based mean matrix estimation method. |
| **Step 3:** Calculate the threshold estimate using Monte Carlo simulation \( \hat{\eta} \) . |
| **Step 4:** Calculate the SKL divergence of the detection matrix \( R_{d} \) with \( \bar{R} \) . |
Step 5: Judgements are rendered in accordance with $SKL(\mathbf{R}_D, \bar{\mathbf{R}}) \triangleq \bar{\eta}$.

3. Simulation and analysis

In order to verify the detection performance, numerical experiments under simulation data and sea clutter data are carried out respectively in this chapter. KSKL method is compared with SKL method.

3.1. Numerical experiments under simulated data

We experimented with simulated signals. Clutter is simulated by K distribution with shape parameter 1 and scale parameter 0.5. The simulation target uses the \( (2) \) as the driving vector, and the normalized doppler frequency \( d_f \) is set to 0.15. Sample frequency is 1000 Hz.

Firstly, the comparison experiment of normalized statistics is carried out under the actual measurement data. We randomly generate a set of clutter data consisting of 13 range cells, each with 8 pulses of data. The simulation target of 5dB is added to the 9th range cell, and the remaining 16 range cells are used as reference cells. The simulation false target of 5dB is added to the 4th range cell. We calculate the geometric mean of reference cells and normalized statistic of each cell. Then, we can get the result of Figure 1. As a result, the normalized statistic of false target in SKL method are higher than the real target and the KSKL method produces the opposite result. It is concluded that the KSKL method has an inhibition effect on false targets in reference cells.

Next, Monte Carlo method is used to verify the detection performance of KSKL-CFAR. 12,500 sets of clutter data are generated randomly. Each set of data contains 13 range cells and each range cell contains 8 pulses. Considering normal circumstances, we desire the false alarm rate $P_{fa}$ to be $10^{-3}$. Since there is no analytic solution to the threshold, the estimated threshold is obtained by Monte Carlo experiment using the first 12,000 sets of data. The latter 500 data sets are used to test the detection performance. The target signal with SCR $= -10 - 20dB$ is added into the 9th cell and the false target with the same SCR is added into the 4th cell. Figure 2 shows the detection performance of KSKL-CFAR and SKL-CFAR detectors on simulation data. As a result, KSKL-CFAR detector have better performance than SKL-CFAR detector. It is concluded that the KSKL method has an inhibition effect on false targets in reference cells too.
3.2. Numerical experiments under measured data
We use the actual measurement data of IPIX radar collected by McMaster University of Canada. The parameters are listed in Table 2.

Firstly, we removed unit 9 because it contained targets. Then, we divide the first 100,000 pulses per cell into 90,000 pulses for the estimation of threshold values and 10,000 pulses for the estimation of detection probabilities. We experiment with 8 pulses. Since there is no target signal in the rest of measured data, the simulation target with SCR = −10 − 20dB is added to the 9th range cell and the false target with the same SCR is added into the 4th cell. Figure 3 shows the detection performance of different detectors when $P_{fa} = 10^{-3}$. As a result, KSKL-CFAR detector have better performance than SKL-CFAR detector. It is concluded that KSKL-CFAR had significant inhibition effect on false targets under the measured data.

Table 2. Realized IPIX radar data 17#19931107_135603_starea. CDF parameters

| Parameter                  | Value      |
|----------------------------|------------|
| Radar frequency/GHz        | 9.39GHz    |
| Pulse repeat frequency/Hz  | 1000Hz     |
| Polarization mode          | HH         |
| Number of distance cells   | 14         |
| Number of pulses           | 131,072    |

Figure 3. Detection performance of KSKL-CFAR and SKL-CFAR detectors on measured data

4. Conclusion
Target detection in the background of sea clutter is always a difficult problem for researchers. the researchers considered the correlation characteristics of the signal and designed matrix constant false alarm rate (CFAR) detector based on the basic concept of information geometry theory. However, false targets can cause large errors in the estimation of the mean clutter matrix. In this paper, we consider the influence of false targets on the traditional mean matrix estimation method. Considering the nonlinearity of kernel function, a mean matrix estimator based on kernel function is designed. We design numerical experiments based on simulation data and measured data. The results show that the method of matrix mean value estimation based on kernel function can restrain the false target in the reference cells effectively. In future research, we can consider studying the effect of kernel function parameters on mean estimation.
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