Abstract

Motivated by recent CELO measurements and the progress of the theory of B decays, we investigate $B \to PV(P = \pi, K, V = K^*, \rho, \omega)$ decay modes in the framework of QCD improved factorization. We find that all the measured branching ratios are well accommodated in the reasonable parameter space and predictions for the other decay modes are well below the experimental upper limits. We also have calculated $CP$ asymmetries in these decay modes.

PACS Numbers: 13.25Hw, 12.38Bx, 12.15Hh
1 Introduction

It is of great interest and importance to investigate the decays of B mesons to charmless final states to study the weak interactions and CP violation. In past years, we have witnessed many experimental and theoretical progresses in the study of B physics with the observations of many B charmless decay processes and improvements of the theory of B decays.

Most of the theoretical studies of B decays to pseudocalar and vector final states are based on the popular *Naive Factorization approach*. As it was pointed out years ago in Ref.[2], the dominant contribution in B decays comes from the so-called *Feynman mechanism*, where the energetic quark created in the weak decay picks up the soft spectator softly and carries nearly all of the final-state meson’s momentum. It is also shown that Pion form factor in QCD at intermediate energy scale is dominated by Feynman mechanism[3, 4, 5]. From this point, we can understand why the naive factorization approach have worked well for B and D decays, and the many existing predictions for B decays based on naive factorization and spectator ansatz do have taken in the dominant physics effects although there are shortcomings. However, with the many new data available from CLEO and an abundance of data to arrive within few years from the B factories BaBar and Belle, it is demanded highly to go beyond the naive factorization approach.

Recently, Beneke et al., have formed an interesting QCD factorization formula for B exclusive nonleptonic decays[6, 7]. The factorization formula incorporates elements of the naive factorization approach (as leading contribution) and the hard-scattering approach (as subleading corrections), which allows us to calculate systematically radiative (subleading nonfactorizable) corrections to naive factorization for B exclusive nonleptonic decays. An important product of the formula is that the strong final-state interaction phases are calculable from the first principle which arise from the hard-scattering kernel and hence process dependent. The strong phases are very important for studying CP violation in B decays. Detail proofs and arguments could be found in[7]. Here we recall briefly the essence of the QCD factorization formula as follows.

The amplitude of $B$ decays to two light mesons, say $M_1$ and $M_2$, is obtained through the hadronic matrix element $\langle M_1(p_1)M_2(p_2)\mid \mathcal{O}_i \mid B(p) \rangle$, here $M_1$ denotes the final meson that picks up the light spectator quark in the $B$ meson, and $M_2$ is the another meson which is composed of
the quarks produced from the weak decay point of $b$ quark. Since the quark pair, forming $M_2$, is ejected from the decay point of $b$ quark carrying the large energy of order of $m_b$, soft gluons with the momentum of order of $\Lambda_{QCD}$ decouple from it at leading order of $\Lambda_{QCD}/m_b$ in the heavy quark limit. As a consequence any interaction between the quarks of $M_2$ and the quarks out of $M_2$ is hard at leading power in the heavy quark expansion. On the other hand, the light spectator quark carries the momentum of the order of $\Lambda_{QCD}$, and is softly transferred into $M_1$ unless it undergoes a hard interaction. Any soft interaction between the spectator quark and other constituents in $B$ and $M_1$ can be absorbed into the transition formfactor of $B \rightarrow M_1$. The non-factorizable contribution to $B \rightarrow M_1M_2$ can be calculated through the diagrams in Fig.1.

In this paper we study $B \rightarrow PV$ decays within the framework of the QCD improved factorization approach \[6, 7\]. In Sec.II We present notations and calculations. In Sec.III we compare our numerical results with the experimental data presented by CLEO collaboration \[8\]. We find that all the measured branching ratios are well accommodated in the reasonable parameter space and predictions for the other decay modes are well below their upper limits. We also give our predictions of direct CP asymmetries and time integrated CP asymmetries in the decay modes. Large direct CP violation asymmetries are predicted for the decay modes $B^0 \rightarrow \pi^0\omega, K^-\rho^+, K^0\omega, B^- \rightarrow K^-K^{*0}, K^-\rho^0, \pi^0K^{*-}, K^-\omega$. The direct CP asymmetries in the observed decay modes $B^- \rightarrow \pi^-\rho^0, \pi^-\omega$ and $B^0 \rightarrow \pi^\pm\rho^\pm$ are predicted to be around few percentages.

2 Calculations

The effective Hamiltonian for $B$ decays is given by \[9\],

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{cq}^* \left( \sum_{i=1}^{10} C_i O_i^{cu} + \sum_{i=3}^{10} C_i O_i + C_g O_g \right) + V_{cb}V_{eq}^* \left( \sum_{i=1}^{2} C_i O_i^{cc} + \sum_{i=3}^{10} C_i O_i + C_g O_g \right) \right], \quad (1)$$
with the effective operators given by

\begin{align*}
O_1^u &= (\bar{q}_a u_a)_{V-A} \cdot (\bar{u}_\beta b_\beta)_{V-A}, & O_2^u &= (\bar{q}_a u_\beta)_{V-A} \cdot (\bar{u}_\beta b_\alpha)_{V-A}, \\
O_1^c &= (\bar{q}_a c_\alpha)_{V-A} \cdot (\bar{c}_\beta b_\beta)_{V-A}, & O_2^c &= (\bar{q}_a c_\beta)_{V-A} \cdot (\bar{c}_\beta b_\alpha)_{V-A}, \\
O_3 &= (\bar{q}_a b_\alpha)_{V-A} \cdot \sum_{q'} (\bar{q}_q' q_\beta)_{V-A}, & O_4 &= (\bar{q}_a b_\beta)_{V-A} \cdot \sum_{q'} (\bar{q}_q' q_\alpha)_{V-A}, \\
O_5 &= (\bar{q}_a b_\alpha)_{V-A} \cdot \sum_{q'} (\bar{q}_q' q_\beta)_{V-A}, & O_6 &= (\bar{q}_a b_\beta)_{V-A} \cdot \sum_{q'} (\bar{q}_q' q_\alpha)_{V-A}, \\
O_7 &= \frac{3}{2} (\bar{q}_a b_\alpha)_{V-A} \cdot \sum_{q'} e_q' (\bar{q}_q' q_\beta)_{V-A}, & O_8 &= \frac{3}{2} (\bar{q}_a b_\beta)_{V-A} \cdot \sum_{q'} e_q' (\bar{q}_q' q_\alpha)_{V-A}, \\
O_9 &= \frac{3}{2} (\bar{q}_a b_\alpha)_{V-A} \cdot \sum_{q'} e_q' (\bar{q}_q' q_\beta)_{V-A}, & O_{10} &= \frac{3}{2} (\bar{q}_a b_\beta)_{V-A} \cdot \sum_{q'} e_q' (\bar{q}_q' q_\alpha)_{V-A}, \\
O_9 &= (g_s / 8\pi^2) m_b \bar{d}_\alpha \sigma^{\mu\nu} R \frac{\lambda_{\alpha\beta}}{2} b_\beta G^A_{\mu\nu}.
\end{align*}

Here \( q = d, s \) and \((q' \epsilon \{u, d, s, c, b\})\), \( \alpha \) and \( \beta \) are the SU(3) color indices and \( \lambda^A_{\alpha\beta} \), \( A = 1, \ldots, 8 \) are the Gell-Mann matrices, and \( G^A_{\mu\nu} \) denotes the gluonic field strength tensor. The Wilson coefficients evaluated at \( \mu = m_b \) scale are

\begin{align*}
C_1 &= 1.082, & C_2 &= -0.185, \\
C_3 &= 0.014, & C_4 &= -0.035, \\
C_5 &= 0.009, & C_6 &= -0.041, \\
C_7 &= -0.002/137, & C_8 &= 0.054/137, \\
C_9 &= -1.292/137, & C_{10} &= -0.262/137, \\
C_g &= -0.143.
\end{align*}

The non-factorizable contributions to \( B \to M_1 M_2 \) can be calculated through the diagrams in Fig.1. The results of our calculations are summarized compactly by the following equations

\begin{align*}
\mathcal{T}_p &= \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \text{Vev} \left\{ a_1^p(BM_1, M_2) (\bar{q} u)_{V-A} \otimes (\bar{u} b)_{V-A} + a_2^p(BM_1, M_2) (\bar{u} u)_{V-A} \otimes (\bar{q} b)_{V-A} \\
&\quad + a_3^p(BM_1, M_2) (\bar{q} q')_{V-A} \otimes (\bar{q} b)_{V-A} + a_4^p(BM_1, M_2) (\bar{q} q')_{V-A} \otimes (\bar{q} b)_{V-A} \\
&\quad + a_5^p(BM_1, M_2) (\bar{q} q')_{V+A} \otimes (\bar{q} b)_{V-A} + a_6^p(BM_1, M_2) (-2) (\bar{q} q')_{S+P} \otimes (\bar{q} b)_{S-P} \\
&\quad + a_7^p(BM_1, M_2) \frac{3}{2} e_q (\bar{q} q')_{V+A} \otimes (\bar{q} b)_{V-A} \\
&\quad + (-2) \left( a_8^p(BM_1, M_2) \frac{3}{2} e_q + a_{8a}(BM_1, M_2) \right) \frac{3}{2} e_q (\bar{q} q')_{S+P} \otimes (\bar{q} b)_{S-P} \\
&\quad + a_9^p(BM_1, M_2) \frac{3}{2} e_q (\bar{q} q')_{V-A} \otimes (\bar{q} b)_{V-A} \\
&\quad + \left( a_{10}^p(BM_1, M_2) \frac{3}{2} e_q + a_{10a}(BM_1, M_2) \right) (\bar{q} q')_{V-A} \otimes (\bar{q} b)_{V-A} \right\},
\end{align*}

\( 2 \).
where the symbol $\otimes$ denotes $\langle M_1 M_2 | j_2 \otimes j_1 | B \rangle \equiv \langle M_2 | j_2 | 0 \rangle \langle M_1 | j_1 | B \rangle$. $M_1$ represents the meson which picks up the spectator quark through this paper. For $M_1$ is light vector meson and $M_2$ is light pseudoscalar meson, the effective $a_i^q$'s which contain next-to-leading order(NLO) coefficients and $O(\alpha_s)$ hard scattering corrections are found to be

\[
\begin{align*}
\tilde{a}_{1,2}(BV, P) &= 0, \quad \tilde{a}_i^u(BV, P) = a_i^u(BV, P), \quad i = 3, 5, 7, 8, 9, 10, 8a, 10a. \\
\tilde{a}_1^u(BV, P) &= C_1 + \frac{C_2}{N} + \frac{\alpha_s C_F}{4\pi N} C_2 F_P, \\
\tilde{a}_2^u(BV, P) &= C_2 + \frac{C_3}{N} + \frac{\alpha_s C_F}{4\pi N} C_1 F_P, \\
\tilde{a}_3^u(BV, P) &= C_3 + \frac{C_4}{N} + \frac{\alpha_s C_F}{4\pi N} C_4 F_P, \\
\tilde{a}_4^u(BV, P) &= C_4 + \frac{C_5}{N} + \frac{\alpha_s C_F}{4\pi N} \left[ C_3 (F_P + G_P(s_q) + G_P(s_b)) + C_1 G_P(s_p) \right] \\
&\quad + (C_4 + C_6) \sum_{f=u}^b G_P(s_f) + G_{V,g} \right], \\
\tilde{a}_5^u(BV, P) &= C_5 + \frac{C_6}{N} + \frac{\alpha_s C_F}{4\pi N} C_6 (-F_P - 12), \\
\tilde{a}_6^u(BV, P) &= C_6 + \frac{C_7}{N} + \frac{\alpha_s C_F}{4\pi N} \left[ C_1 G_P'(s_p) + C_3 (G_P'(s_q) + G_P'(s_b)) \right] + (C_4 + C_6) \sum_{f=u}^b G_P'(s_f) + G_{P,g}' \right], \\
\tilde{a}_7^u(BV, P) &= C_7 + \frac{C_8}{N} + \frac{\alpha_s C_F}{4\pi N} C_8 (-F_P - 12), \\
\tilde{a}_8^u(BV, P) &= C_8 + \frac{C_9}{N}, \\
\tilde{a}_{8a}^u(BV, P) &= \frac{\alpha_s C_F}{4\pi N} \left[ \left( C_8 + C_{10} \right) \sum_{f=u}^b \frac{3}{2} e_f G_P'(s_f) + C_9 \frac{3}{2} (e_q G_P'(s_q) + e_b G_P'(s_b)) \right], \\
\tilde{a}_9^u(BV, P) &= C_9 + \frac{C_{10}}{N} + \frac{\alpha_s C_F}{4\pi N} C_{10} F_P, \\
\tilde{a}_{10}^u(BV, P) &= C_{10} + \frac{C_9}{N} + \frac{\alpha_s C_F}{4\pi N} C_9 F_P, \\
\tilde{a}_{10a}^u(BV, P) &= \frac{\alpha_s C_F}{4\pi N} \left[ \left( C_8 + C_{10} \right) \frac{3}{2} \sum_{f=u}^b e_f G_F(s_f) + C_9 \frac{3}{2} (e_q G_F(s_q) + e_b G_F(s_b)) \right],
\end{align*}
\]

where $q = d, s$. \quad $q' = u, d, s$ and $f = u, d, s, c, b$. $C_P = (N^2 - 1)/(2N)$ and $N = 3$ is the number of colors. The internal quark mass in the penguin diagrams enters as $s_f = m_f^2/m_b^2$. $\bar{x} = 1 - x$ and $\bar{u} = 1 - u$.

\[
F_P = -12 \ln \frac{\mu}{m_b} - 18 + f_P^I + f_P^{II},
\]
\[
f_P^f = \int_0^1 dx g(x) \phi_P(x), \quad g(x) = 3 \frac{1-2x}{1-x} \ln x - 3i\pi, \\
f_P^{II} = \frac{4\pi^2}{N} \frac{f_V f_B}{A_0(0) M_B^4} \int_0^1 dz \frac{\phi_B(z)}{z} \int_0^1 dx \frac{\phi_V(x)}{x} \int_0^1 dy \frac{\phi_P(y)}{y}, \quad (7)
\]

\[
G_{P,g} = -\int_0^1 dx \frac{2}{x} \phi_P(x), \quad (8)
\]

\[
G_P(s_q) = \frac{2}{3} - 4 \frac{3}{3 \ln \frac{\mu}{m_b}} + 4 \int_0^1 dx \phi_P(x) \int_0^1 du \ u\bar{u} \ln [s_q - u\bar{u}x - i\epsilon], \quad (9)
\]

\[
G'_{P,g} = -\int_0^1 dx \frac{3}{2} \phi_0(x) = -\frac{3}{2}, \quad (10)
\]

\[
G'_{P}(s_q) = \frac{1}{3} - \ln \frac{\mu}{m_b} + 3 \int_0^1 dx \phi_0^p(x) \int_0^1 du \ u\bar{u} \ln [s_q - u\bar{u}x - i\epsilon], \quad (11)
\]

For \(M_1\) is pseudoscalar and \(M_2\) is vector, the coefficients are

\[
a_{i,2}^a(BP,V) = 0, \quad a_i^a(BP,V) = a_i^a(BP,V), \quad i = 3, 5, 7, 8, 9, 10, 8a, 10a.
\]

\[
a_1^a(BP,V) = C_1 + \frac{C_2}{N} + \frac{\alpha_s C_F}{4\pi N} C_2 F_V,
\]

\[
a_2^a(BP,V) = C_2 + \frac{C_1}{N} + \frac{\alpha_s C_F}{4\pi N} C_1 F_V,
\]

\[
a_3^a(BP,V) = C_3 + \frac{C_4}{N} + \frac{\alpha_s C_F}{4\pi N} C_4 F_V,
\]

\[
a_4^a(BP,V) = C_4 + \frac{C_3}{N} + \frac{\alpha_s C_F}{4\pi N} \left[ C_3 (F_V + G_V(s_q) + G_V(s_b)) + C_1 G_V(s_p) \right]
+ (C_4 + C_6) \sum_{f=u}^b G_V(s_f) + G_{V,g},
\]

\[
a_5^a(BP,V) = C_5 + \frac{C_6}{N} + \frac{\alpha_s C_F}{4\pi N} C_6 (-F_V - 12),
\]

\[
a_6^a(BP,V) = C_6 + \frac{C_5}{N},
\]

\[
a_7^a(BP,V) = C_7 + \frac{C_8}{N} + \frac{\alpha_s C_F}{4\pi N} C_8 (-F_V - 12),
\]

\[
a_8^a(BP,V) = C_8 + \frac{C_7}{N},
\]

\[
a_9^a(BP,V) = C_9 + \frac{C_{10}}{N} + \frac{\alpha_s C_F}{4\pi N} C_{10} F_V,
\]

\[
a_{10}^a(BP,V) = C_{10} + \frac{C_9}{N} + \frac{\alpha_s C_F}{4\pi N} C_9 F_V,
\]

\[
a_{10a}^a(BP,V) = \frac{\alpha_s C_F}{4\pi N} \left[ (C_8 + C_{10}) \sum_{f=u}^b e_f G_V(s_f) + C_9 \frac{3}{2} (e_q G_V(s_q) + e_b G_V(s_b)) \right], \quad (12)
\]

where

\[
F_V = -12 \ln \frac{\mu}{m_b} - 18 + f_V^f + f_V^{II}, \quad (13)
\]
Where $\phi_P(x)$ and $\phi_P^0(x)$ are the pseudoscalar meson’s twist-2 and twist-3 distribution amplitudes (DA) respectively. $\phi_V(x) = \phi_{V,||}$ is the leading twist DA for the longitudinally polarized vector meson states. We have used the fact that light vector meson is mainly longitudinally polarized in B decays and the transversely polarized state is suppressed by the factor $M_V/M_B$. Furthermore in $B \to PV$ decays the vector meson should be completely polarized in longitudinal direction due to the angular momentum conservation requirement. In the derivation of the effective coefficients $a_i$’s we have used NDR scheme and assumption of asymptotic DAs. The infrared divergences in Fig.1. a – d are cancelled in their sum.

With the effective coefficients in Eqs. (5) and (12), we can write down the decay amplitudes of the decay modes (we only list four decay modes which have been detected in experiment here. The other decay modes are given in appendix A)

$$
\mathcal{M}(B^0 \to \pi^+ \rho^-) = \frac{G_F}{\sqrt{2}} f_{\rho} m_B^2 F_0^{B \to \pi} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_1(B^0\rho^+, \pi^-) + a_4(B^0\rho^+, \pi^-) \right. \right.
+ a_{10}(\bar{B}^0\pi^+, \rho^-) + a_{10a}(\bar{B}^0\pi^+, \rho^-) \left. \left. \right] + a_4^*(\bar{B}^0\pi^+, \rho^-) + a_{10}(\bar{B}^0\pi^+, \rho^-) + a_{10a}(\bar{B}^0\pi^+, \rho^-) \right\};
$$

$$
\mathcal{M}(\bar{B}^0 \to \pi^- \rho^+) = \frac{G_F}{\sqrt{2}} f_{\pi} M_B^2 A_0^{B \to \rho} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_1(\bar{B}^0\rho^+, \pi^-) + a_4(\bar{B}^0\rho^+, \pi^-) \right.$$ 
$$+ \left( a_6^u(B^0\rho^+, \pi^-) + a_8(B^0\rho^+, \pi^-) + a_{8a}(B^0\rho^+, \pi^-) \right) R_{\pi^-} + a_{10}(B^0\rho^+, \pi^-) + a_{10a}(B^0\rho^+, \pi^-) \right.$$ 
$$- \left[ a_4^*(\bar{B}^0\rho^+, \pi^-) + \left( a_6^u(\bar{B}^0\rho^+, \pi^-) + a_8(\bar{B}^0\rho^+, \pi^-) + a_{8a}(\bar{B}^0\rho^+, \pi^-) \right) R_{\pi^-} \right.$$ 
$$+ a_{10}(\bar{B}^0\rho^+, \pi^-) + a_{10a}(\bar{B}^0\rho^+, \pi^-) \right\};
$$

$$
\mathcal{M}(B^- \to \pi^- \rho^0) = \frac{G_F}{2} f_{\rho} M_B^2 F_0^{B \to \pi} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_2(B^-\pi, \rho^0) - a_4^u(B^-\pi, \rho^0) \right.$$ 
$$+ a_2(B^-\pi, \rho^0) + \frac{3}{2} \left( a_7(B^-\pi, \rho^0) \right.$$
$$+ a_9(B^-\pi, \rho^0) \right) + \frac{1}{2} a_{10}(B^-\pi, \rho^0) - a_{10a} \right\}.
$$
is the phase of

For the chiral enhancement factors for the pseudoscalar mesons, we take

\[ G_R (\pi^-) = \frac{G_F}{2} f_\pi M_B^2 A_{B_0}^{B \to \rho} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_1 (B^- \rho^0, \pi^-) + a_4^0 (B^- \rho^0, \pi^-) + a_8 (B^- \rho^0, \pi^-) + a_{10a} (B^- \rho^0, \pi^-) \right] \right\} \]

\[ + \left[ a_4^0 (B^- \rho^0, \pi^-) + a_6^0 (B^- \rho^0, \pi^-) + a_8 (B^- \rho^0, \pi^-) + a_{10a} (B^- \rho^0, \pi^-) \right] \right\} \quad \text{(19)} \]

\[ M(B^- \to \pi^- \omega) = \frac{G_F}{2} f_\pi M_B^2 A_{B_0}^{B \to \omega} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_1 (B^- \omega, \pi^-) + a_4^0 (B^- \omega, \pi^-) + a_8 (B^- \omega, \pi^-) + a_{10a} (B^- \omega, \pi^-) \right] \right\} \]

\[ + \left[ a_4^0 (B^- \omega, \pi^-) + a_6^0 (B^- \omega, \pi^-) + a_8 (B^- \omega, \pi^-) + a_{10a} (B^- \omega, \pi^-) \right] \right\} \quad \text{(20)} \]

Where \( R_b = \frac{1-\lambda^2/2}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| \) and \( R'_b = \frac{\lambda}{1-\lambda^2/2} \left| \frac{V_{ub}}{V_{cb}} \right| \). \( V_{cb}, V_{ud} \) and \( V_{us} \) are chosen to be real and \( \gamma \) is the phase of \( V_{ub}^* \). \( \lambda = |V_{us}| = 0.2196, \) \( R_{s\pi} = -2M_{s\pi}^2/(m_b (m_u + m_d)) \).

### 3 Numerical calculations and discussions of results

In the numerical calculations we use \([10]\)

\[ \tau(B^+) = 1.65 \times 10^{-12} \text{s}, \quad \tau(B^0) = 1.56 \times 10^{-12} \text{s}, \]

\[ M_B = 5.2792 \text{GeV}, \quad m_b = 4.8 \text{GeV}, \quad m_c = 1.4 \text{GeV}, \]

\[ f_B = 0.180 \text{GeV}, \quad f_\pi = 0.133 \text{GeV}, \quad f_K = 0.158 \text{GeV}, \]

\[ f_{K^*} = 0.214 \text{GeV}, \quad f_\rho = 0.21 \text{GeV}, \quad f_\omega = 0.195 \text{GeV}. \]

For the chiral enhancement factors for the pseudoscalar mesons, we take

\[ R_{s\pi} = R_{sK} = -1.2, \]
which are consistent with the values used in [6, 11, 12]. We should take care for \( R_{\pi^0} \). As pointed out in Ref.[7], \( R_{\pi^0} \) for \( \pi^0 \) should be \(-2M_{\pi}^2/(m_u(m_u + m_d))\) and equal to \( R_{\pi^0} \) due to inclusion isospin breaking effects correctly.

For the form factors, we take the results of light-cone sum rule [13, 14]

\[
F_{B \to \pi}(0) = 0.3, \quad F_{B \to K}(0) = 1.13F_{B \to \pi}(0),
\]

\[
A_{0 \to \rho} = 0.372, \quad A_{0 \to K^*} = 0.470,
\]

and assume \( A_{0 \to \omega}(0) = 1.2 \times A_{0 \to \rho}(0) \) since we find larger \( A_{0 \to \omega}(0) \) is preferred by experimental data.

We take the leading-twist DA \( \phi(x) \) and the twist-3 DA \( \phi_0(x) \) of light pseudoscalar and vector mesons as the asymptotic form [15]

\[
\phi_{P,V}(x) = 6x(1-x), \quad \phi_0(x) = 1.
\]

For the \( B \) meson, the wave function is chosen as [16, 17],

\[
\phi_B(x) = N_B x^2 (1-x)^2 \exp\left[-\frac{M_B^2}{2\omega_B^2} x^2\right],
\]

with \( \omega_B = 0.4 \) GeV, and \( N_B \) is the normalization constant to make \( \int_0^1 dx \phi_B(x) = 1 \). \( \phi_B(x) \) is strongly peaked around \( x = 0.1 \), which is consistent with the observation of Heavy Quark Effective Theory that the wave function should be peaked around \( \Lambda_{QCD}/M_B \).

We have used the unitarity of the CKM matrix \( V_{uq}V_{ub} + V_{cq}V_{cb} + V_{tq}V_{tb} = 0 \) to decompose the amplitudes into terms containing \( V_{uq}V_{ub} \) and \( V_{cq}V_{cb} \), and

\[
|V_{us}| = \lambda = 0.2196, \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.02,
\]

\[
|V_{cb}| = 0.0395 \pm 0.0017, \quad |V_{ud}| = 1 - \lambda^2/2.
\]

We leave the CKM angle \( \gamma \) as a free parameter.

The branching ratios of two body \( B \) decays is given by

\[
Br(B \to M_1 M_2) = \frac{\tau_B}{16\pi M_B} |\mathcal{M}(B \to M_1 M_2)|^2.
\]

For the case that the final state \( f \) is non-\( CP \)-eigenstate, the direct \( CP \) asymmetry parameter is defined as

\[
A_{CP}^{dir} = \frac{|\mathcal{M}(B^+ \to f)|^2 - |\mathcal{M}(B^- \to \bar{f})|^2}{|\mathcal{M}(B^+ \to f)|^2 + |\mathcal{M}(B^- \to \bar{f})|^2},
\]
and
\[
A_{CP}^{\text{dir}} = \frac{|\mathcal{M}(B^0 \to f)|^2 - |\mathcal{M}(\bar{B}^0 \to \bar{f})|^2}{|\mathcal{M}(B^0 \to f)|^2 + |\mathcal{M}(\bar{B}^0 \to \bar{f})|^2}.
\] (26)

For the neutral B decaying into CP eigenstate \(f\), i.e., \(f = \bar{f}\), the effects of \(B^0 - \bar{B}^0\) mixing should be taken into account. Thus the CP asymmetry is time dependent, which is given by
\[
A_{CP}(t) = A_{CP}^{\text{dir}} \cos(\Delta m t) - \frac{2 \text{Im}(\xi)}{1 + |\xi|^2} \sin(\Delta m t),
\] (27)
and the time-integrated CP asymmetry is obtained through
\[
A_{cp} = 1 - \frac{|\xi|^2 - 2 \text{Im}(\xi) (\Delta m/\Gamma)}{(1 + |\xi|^2)[1 + (\Delta m/\Gamma)^2]},
\] (28)
where \(\Delta m\) is the mass difference of the two mass eigenstates of neutral B mesons, and \(A_{CP}^{\text{dir}}\) is the direct CP asymmetry defined in eq.(26). The parameter \(\xi\) is given by
\[
\xi = \frac{V_{tb}^* V_{td}^* \langle f | H_{\text{eff}} | \bar{B}^0 \rangle}{V_{tb} V_{td}^* \langle f | H_{\text{eff}} | B^0 \rangle}.
\] (29)

The numerical results of the branching ratios \(B \to PV\) are shown in Fig.2 as the function of CKM angle \(\gamma\). We can see from Fig.2.1, 2.2, and 2.3 that for the three detected channels the predicted branching ratios agree well with the CLEO experiment data \([8]\). Our predictions for other decay modes are well below their 90\% C.L upper limits.

There are several works available with detail analyses of the CLEO new data of the decays of B to charmless PV states\([11, 12, 19]\). Cheng and Yang have renewed their predictions with the “generalized factorization” framework\([11]\). It is worth to note that the shortcomings in the “generalized factorization” are resolved in the framework employed in this paper. Non-factorizable effects are calculated in a rigorous way here instead of being parameterized by effective color number. Since the hard scattering kernels are convoluted with the light cone DAs of the mesons, gluon virtuality \(k^2 = \bar{x} m_b^2\) in the penguin diagram Fig.1.e has well defined meaning and leaves no ambiguity as to the value of \(k^2\), which has usually been treated as a free phenomenological parameter in the estimations of the strong phase generated though the BSS mechanism\([20]\). So that CP asymmetries are predicted soundly in this paper. We present the numerical result of the branching ratios of \(B \to PV\) decays in Table. 1 with the relevant strong phases shown explicitly. It shows that the strong phases are generally mode dependent.

Hou, Smith and Würtzwein have performed a model dependent fit using the recent CLEO data and found \(\gamma = 114^{+25}_{-21}\) degree. Using SU(3) flavor symmetry, Gronau and Rosner have
analysed the decays of B to charmless PV final states extensively and found several processes are consistent with cos γ < 0. In this paper we find cos γ < 0 is favored by the $B^+ \rightarrow \pi^- \rho^0$ and $\bar{B} \rightarrow \pi^- \bar{\rho} + \pi^+ \rho^-$ if their experimental center values are taken seriously. To meet its center value with cos γ < 0, $B^- \rightarrow \pi^- \omega$ would indicate larger form factor i.e. $A_{B^- \rightarrow \pi^- \omega}(0) > A_{B^- \rightarrow \pi^- \rho}(0)$.

In our numerical calculation, we have taken $A_{B^- \rightarrow \pi^- \omega}(0) = 0.446$ which is still consistent with the LCSR results $0.372 \pm 0.074^{[13]}$. It is also interesting to note that $\bar{B} \rightarrow \pi^+ \rho^-$ is suppressed by cos γ < 0 while $\bar{B} \rightarrow \pi^- \rho^+$ is enhanced. The difference between $Br(\bar{B} \rightarrow \pi^+ \rho^-)$ and $Br(\bar{B} \rightarrow \pi^- \rho^+)$ is much more sensitive to γ than their sum.

For comparison with the results in the literature, we table our predictions made for γ = 100° in Table 2. We find that most of our predictions agree with Ref. [17]. For $Br(\bar{B} \rightarrow \pi^0 K^{*0})$, our prediction is much smaller than the prediction of Ref. [11] which exceeds the upper limit slightly.

The direct and time-integrated CP asymmetries are shown in Fig 3. and Fig. 4 respectively. For γ around 100°, the direct CP asymmetries in the decay modes $B^- \rightarrow K^- K^{*0}$, $K^- \rho^0$, $\pi^0 K^{*-}$, $K^- \omega$ and $\bar{B} \rightarrow K^- \rho^+$, $K^0 \omega$ are as large as $(\pm)10% \sim (\pm)20%$. Unfortunatly the decay rates of these decay modes are quite small. From Fig. 4, we can see that the time integrated CP asymmetries in $\bar{B} \rightarrow \rho^+ \pi^-$, $\rho^0 \pi^0$, $\omega \pi^0$ are about $\pm 30% \sim \pm 60%$ for γ around 100°.
Table 2: Branching ratios (in units of $10^{-6}$) for the charmless decays modes studied by CLEO. Experimental results are taken from [8]. Our results are made for $\gamma = 100^\circ$. Cheng and Yang’s preferred predictions[11] (the case $N_{eff}^{c}(LL) = 2$ and $N_{eff}^{c}(LR) = 6$) are taken for comparison. The form factors used in Ref.[11] are very similar to ours.

| Decay modes | Our results | Ref[11] | CLEO $B_{fit}$ | $B$ or 90% $B$ UL |
|-------------|-------------|---------|----------------|-----------------|
| $B^{-} \rightarrow \pi^{-}\rho^{0}$ | 8.96 | 13.0 | $10.4^{+3.3}_{-3.4} \pm 2.1$ | $10.4^{+3.3}_{-3.4} \pm 2.1$ |
| $\bar{B}^{0} \rightarrow \pi^{+}\rho^{-}$ | 18.6 | 18.2 | Sum up | Sum up |
| $\bar{B}^{0} \rightarrow \pi^{-}\rho^{+}$ | 13.5 | 14.2 | $27.6^{+8.4}_{-7.4} \pm 4.2$ | $27.6^{+8.4}_{-7.4} \pm 4.2$ |
| $B^{-} \rightarrow \pi^{-}\omega$ | 9.82 | 10.7 | $11.3^{+3.3}_{-2.9} \pm 1.4$ | $11.3^{+3.3}_{-2.9} \pm 1.4$ |
| $\bar{B}^{0} \rightarrow \pi^{0}\rho^{0}$ | 0.11 | 0.75 | $1.6^{+2.0}_{-1.4} \pm 0.8$ | <5.5 |
| $B^{-} \rightarrow \pi^{0}\rho^{-}$ | 10.0 | 13.1 | | <43 |
| $\bar{B}^{0} \rightarrow \pi^{0}\omega$ | 0.004 | 0.02 | $0.8^{+1.9+1.0}_{-0.8-0.8}$ | <5.5 |
| $B^{-} \rightarrow K^{-}\rho^{0}$ | 0.71 | 1.10 | $8.4^{+4.0}_{-3.4} \pm 1.8$ | <17 |
| $B^{-} \rightarrow \pi^{-}\bar{K}^{*0}$ | 3.85 | 3.64 | $7.6^{+3.5}_{-3.0} \pm 1.6$ | <16 |
| $B^{-} \rightarrow K^{-}\bar{K}^{*0}$ | 0.26 | 0.39 | $0.0^{+1.3+0.6}_{-0.0-0.0}$ | <5.3 |
| $B^{-} \rightarrow \pi^{0}\bar{K}^{*-}$ | 6.42 | 4.34 | $7.1^{+11.4}_{-7.1} \pm 1.0$ | <31 |
| $B^{-} \rightarrow K^{-}\omega$ | 0.97 | 2.24 | $3.2^{+2.4}_{-1.9} \pm 0.8$ | <7.9 |
| $\bar{B}^{0} \rightarrow \bar{K}^{0}\omega$ | 0.40 | 1.89 | $10.0^{+5.4}_{-4.2} \pm 1.4$ | <21 |
| $\bar{B}^{0} \rightarrow K^{-}\rho^{+}$ | 1.29 | 3.49 | $16.0^{+7.6}_{-6.4} \pm 2.8$ | <32 |
| $\bar{B}^{0} \rightarrow \pi^{0}\bar{K}^{*0}$ | 0.58 | 3.92 | $0.0^{+1.3+0.5}_{-0.0-0.0}$ | <3.6 |
4 Summary

In this paper we have calculated the branching ratios and CP asymmetries of the charmless decays $B \to PV(P = (\pi, K), V = (\rho, \omega, K^*))$ in the QCD improved factorization approach which have been formed recently by Beneke et. al. [6, 7].

We have used LCSR form factors $F_{B\to\pi,K}(0)$ and $A_{0^{\rho,K^*}}(0)$ as inputs. The results of $\text{Br}(B^- \to \pi^- \rho^0)$ and $\text{Br}(B^0 \to \pi^{\pm} \rho^{\mp})$ agree with CLEO data[8] very well and favor $\cos \gamma < 0$ if their experimental center values are taken seriously. To meet its experimental center value and $\cos \gamma < 0$, the decay $B^- \to \pi^- \omega$ will prefer larger form factor $A_{0^{B\to\omega}}(0)$. For the other decay modes, the branching ratios are predicted well below their 90% C.L upper limits given in Ref[8]. We have also compared our results with Cheng and Yang’s renewed results of the branching ratios. For many decay modes, our results agree with theirs.

Working in the QCD improved factorization approach, we are allowed to calculate the strong phases to make predictions of CP asymmetries for the decay modes more sound than before. We find direct CP asymmetries in the observed decay modes are around few percentages level. Direct and time-integrated CP asymmetries in those decay modes have been tabled in Fig.3 and Fig.4 respectively.

Acknowledgments

We acknowledge the Grant-in-Aid for Scientific Research on Priority Areas (Physics of CP violation with contract number 09246105 and 1014028) and the Monbusho Found 10098178-00. We thanks JSPS(Japan Society for the Promotion of Science) for support.
Appendix A

The decay amplitudes of some of the $B \to PV$ decays in terms of the effective coefficients $a_i$'s:

\[
\mathcal{M}(\bar{B}^0 \to \pi^0 \rho^0) = -\frac{G_F}{2\sqrt{2}} f_\rho M_B^2 F_0^{B \to \pi} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_2(\bar{B}^0 \pi^0, \rho^0) - a_4^u(\bar{B}^0 \pi^0, \rho^0) \right] \\
+ \frac{3}{2} \left[ a_7(\bar{B}^0 \pi^0, \rho^0) + a_9(\bar{B}^0 \pi^0, \rho^0) \right] + \frac{1}{2} a_{10}(\bar{B}^0 \pi^0, \rho^0) - a_{10a}(\bar{B}^0 \pi^0, \rho^0) \right\} \\
- \left[ -a_4^e(\bar{B}^0 \pi^0, \rho^0) + \frac{3}{2} \left( a_7(\bar{B}^0 \pi^0, \rho^0) + a_9(\bar{B}^0 \pi^0, \rho^0) \right) + \frac{1}{2} a_{10}(\bar{B}^0 \pi^0, \rho^0) - a_{10a}(\bar{B}^0 \pi^0, \rho^0) \right] \right\} \\
- \frac{G_F}{2\sqrt{2}} f_\pi M_B^2 A_0^{B \to \rho} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_2(\bar{B}^0 \rho^0, \pi^0) - a_4^u(\bar{B}^0 \rho^0, \pi^0) \right] \\
- \left[ -a_4^e(\bar{B}^0 \rho^0, \pi^0) - \left( a_6^e(\bar{B}^0 \rho^0, \pi^0) - \frac{1}{2} a_8(\bar{B}^0 \rho^0, \pi^0) + a_{8a}(\bar{B}^0 \rho^0, \pi^0) \right) R_{\pi^0} \right] \\
- \frac{3}{2} \left( a_7(\bar{B}^0 \rho^0, \pi^0) - a_9(\bar{B}^0 \rho^0, \pi^0) \right) + \frac{1}{2} a_{10}(\bar{B}^0 \rho^0, \pi^0) - a_{10a}(\bar{B}^0 \rho^0, \pi^0) \right\} \right\}; (30)
\]

\[
\mathcal{M}(B^- \to \pi^0 \rho^-) = \frac{G_F}{2} f_\rho M_B^2 F_0^{B \to \pi} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_1(B^- \pi^0, \rho^-) + a_4^u(B^- \pi^0, \rho^-) \right] \\
+ a_1(B^- \pi^0, \rho^-) + a_{10a}(B^- \pi^0, \rho^-) \right\} \\
- \left[ a_4^e(B^- \pi^0, \rho^-) + a_{10}(B^- \pi^0, \rho^-) + a_{10a}(B^- \pi^0, \rho^-) \right] \right\} \\
+ \frac{G_F}{2} f_\pi M_B^2 A_0^{B \to \rho} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_2(\bar{B}^0 \rho^0, \pi^0) - a_4^u(\bar{B}^0 \rho^0, \pi^0) \right] \\
- \left[ -a_4^e(\bar{B}^0 \rho^0, \pi^0) - \left( a_6^e(\bar{B}^0 \rho^0, \pi^0) - \frac{1}{2} a_8(\bar{B}^0 \rho^0, \pi^0) + a_{8a}(\bar{B}^0 \rho^0, \pi^0) \right) R_{\pi^0} \right] \\
- \frac{3}{2} \left( a_7(\bar{B}^0 \rho^0, \pi^0) - a_9(\bar{B}^0 \rho^0, \pi^0) \right) + \frac{1}{2} a_{10}(\bar{B}^0 \rho^0, \pi^0) - a_{10a}(\bar{B}^0 \rho^0, \pi^0) \right\} \right\}; (31)
\]

\[
\mathcal{M}(\bar{B}^0 \to \pi^0 \omega) = \frac{G_F}{2\sqrt{2}} f_\omega M_B^2 F_0^{B \to \pi} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_2(\bar{B}^0 \pi^0, \omega) + a_4^u(\bar{B}^0 \pi^0, \omega) \right] \\
+ a_2(\bar{B}^0 \pi^0, \omega) + a_{10a}(\bar{B}^0 \pi^0, \omega) \right\} \\
- \left[ a_4^e(\bar{B}^0 \pi^0, \omega) + a_{10}(\bar{B}^0 \pi^0, \omega) + a_{10a}(\bar{B}^0 \pi^0, \omega) \right] \right\} \\
+ \frac{G_F}{2} f_\pi M_B^2 A_0^{B \to \rho} \lambda V_{cb} \left\{ R_u e^{-i\gamma} \left[ a_2(\bar{B}^0 \rho^0, \pi^0) - a_4^u(\bar{B}^0 \rho^0, \pi^0) \right] \\
- \left[ -a_4^e(\bar{B}^0 \rho^0, \pi^0) - \left( a_6^e(\bar{B}^0 \rho^0, \pi^0) - \frac{1}{2} a_8(\bar{B}^0 \rho^0, \pi^0) + a_{8a}(\bar{B}^0 \rho^0, \pi^0) \right) R_{\pi^0} \right] \\
- \frac{3}{2} \left( a_7(\bar{B}^0 \rho^0, \pi^0) - a_9(\bar{B}^0 \rho^0, \pi^0) \right) + \frac{1}{2} a_{10}(\bar{B}^0 \rho^0, \pi^0) - a_{10a}(\bar{B}^0 \rho^0, \pi^0) \right\} \right\}; (32)
\]
\[ +2 \left( a_3(B^0\pi^0, \omega) + a_5(B^0\pi^0, \omega) \right) + \frac{1}{2} \left( a_7(B^0\pi^0, \omega) + a_9(B^0\pi^0, \omega) \right) \\
- a_{10}(B^0\pi^0, \omega) + 2a_{10a}(B^0\pi^0, \omega) \right] \\
- \left[ a_6(B^0\pi^0, \omega) + 2 \left( a_3(B^0\pi^0, \omega) + a_5(B^0\pi^0, \omega) \right) \\
+ \frac{1}{2} \left( a_7(B^0\pi^0, \omega) + a_9(B^0\pi^0, \omega) - a_{10}(B^0\pi^0, \omega) + 2a_{10a} \right) \right] \\
+ \frac{G_F}{2\sqrt{2}} f_\pi M_B^2 A_{B^+\to \omega} \lambda V_{cb} \left\{ R_\alpha e^{-i\gamma} \left[ a_2(B^0\omega, \pi^0) - a_4(B^0\omega, \pi^0) \right] \\
- \left( a_6(B^0\omega, \pi^0) - \frac{1}{2} a_8(B^0\omega, \pi^0) + a_{8a}(B^0\omega, \pi^0) \right) R_\pi^0 \\
+ \frac{3}{2} \left( a_9(B^0\omega, \pi^0) - a_7(B^0\omega, \pi^0) + \frac{1}{2} a_{10}(B^0\omega, \pi^0) - a_{10a}(B^0\omega, \pi^0) \right) \right\}; \quad (32) \]

\[ \mathcal{M}(B^- \to K^- \rho^0) = \frac{G_F}{2} f_K M_B^2 A_{B^+\to \rho} \lambda V_{cb} (1 - 0.5\lambda^2) \left\{ R_\alpha e^{-i\gamma} \left[ a_1(B^-\rho^0, K^-) + a_4(B^-\rho^0, K^-) \right] \\
+ rki \left( a_6(B^-\rho^0, K^-) + a_8(B^-\rho^0, K^-) + a_{8a}(B^-\rho^0, K^-) \right) + a_{10}(B^-\rho^0, K^-) + a_{10a}(B^-\rho^0, K^-) \right\} \\
+ \frac{G_F}{2} f_\rho M_B^2 F_{B^+\to K} \lambda V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_\alpha e^{-i\gamma} \left[ a_2(B^-K^-, \rho^0) + \frac{3}{2} a_7(B^-K^-, \rho^0) \right] \\
+ \frac{3}{2} a_9(B^-K^-, \rho^0) \right\} + \frac{3}{2} a_7(B^-K^-, \rho^0) + a_9(B^-K^-, \rho^0) \right\}; \quad (33) \]

\[ \mathcal{M}(B^- \to \pi^-K^*) = \frac{G_F}{\sqrt{2}} f_K M_B^2 F_{B^+\to \pi} \lambda V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_\alpha e^{-i\gamma} \left[ a_1^*(B^-\pi^-, K^*) - \frac{1}{2} a_{10}(B^-\pi^-, K^*) \right] \\
+ a_{10a}(B^-\pi^-, K^*) \right\} + a_4^*(B^-\pi^-, K^*) - \frac{1}{2} a_{10}(B^-\pi^-, K^*) + a_{10a}(B^-\pi^-, K^*) \right\}; \quad (34) \]

\[ \mathcal{M}(B^- \to K^-K^*) = \frac{G_F}{\sqrt{2}} f_K M_B^2 F_{B^+\to K} \lambda V_{cb} \left\{ R_\alpha e^{-i\gamma} \left[ a_2^*(B^-K^-, K^*) - \frac{1}{2} a_{10}(B^-K^-, K^*) \right] \\
+ a_{10a}(B^-K^-, K^*) \right\} - a_4^*(B^-K^-, K^*) - \frac{1}{2} a_{10}(B^-K^-, K^*) + a_{10a}(B^-K^-, K^*) \right\}; \quad (35) \]
\[
\mathcal{M}(B^+ \to \pi^0 K^{*-}) = \frac{G_F}{2} f_K M_B^2 F_0^{B \to \pi} V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_1(B^- \pi^0, K^{*-}) + a_4^u(B^- \pi^0, K^{*-}) + a_{10}(B^- \pi^0, K^{*-}) + a_{10a}(B^- \pi^0, K^{*-}) \right] \\
+ \left[ a_5^u(B^- \pi^0, K^{*-}) + a_{10}(B^- \pi^0, K^{*-}) + a_{10a}(B^- \pi^0, K^{*-}) \right] \right\}
+ \frac{G_F}{2} f_\pi M_B^2 A_0^{B \to K^{*-}} V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_2(B^- K^{*-}, \pi^0) - \frac{3}{2} (a_7(B^- K^{*-}, \pi^0) - a_9(B^- K^{*-}, \pi^0) \right) \right\} \\
\mathcal{M}(B^+ \to K^- \omega) = \frac{G_F}{2} f_\omega M_B^2 F_0^{B \to K} V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_2(B^- K^-, \omega) + 2 (a_3(B^- K^-, \omega) \right. \right. \\
+ a_5(B^- K^-, \omega)) + \frac{1}{2} \left( a_7(B^- K^-, \omega) + a_9(B^- K^-, \omega) \right) \\
+ \left[ a_3(B^- K^-, \omega) + a_5(B^- K^-, \omega) \right) + \frac{1}{2} \left( a_7(B^- K^-, \omega) + a_9(B^- K^-, \omega) \right) \right\}
+ \frac{G_F}{2} f_K M_B^2 A_0^{B \to \omega} V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_1(B^- \omega, K^-) + a_4^u(B^- \omega, K^-) \\
+ R_{K^-} \left( a_6^u(B^- \omega, K^-) + a_8(B^- \omega, K^-) + a_{10}(B^- \omega, K^-) + a_{10a}(B^- \omega, K^-) \right) \\
+ \left[ a_4^u(B^- \omega, K^-) + R_{K^-} \left( a_6^u(B^- \omega, K^-) + a_8(B^- \omega, K^-) + a_{10}(B^- \omega, K^-) \right) + a_{10a}(B^- \omega, K^-) \right) \\
+ a_{10}(B^- \omega, K^-) + a_{10a}(B^- \omega, K^-) \right\} \\
\mathcal{M}(\bar{B}^0 \to \bar{K}^0 \omega) = \frac{G_F}{2} f_\omega M_B^2 F_0^{\bar{B} \to K} V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_2(\bar{B}^0 \bar{K}^0, \omega) + 2 (a_3(\bar{B}^0 \bar{K}^0, \omega) \right. \right. \\
+ a_5(\bar{B}^0 \bar{K}^0, \omega)) + \frac{1}{2} \left( a_7(\bar{B}^0 \bar{K}^0, \omega) + a_9(\bar{B}^0 \bar{K}^0, \omega) \right) \\
+ \left[ a_3(\bar{B}^0 \bar{K}^0, \omega) + a_5(\bar{B}^0 \bar{K}^0, \omega) \right) + \frac{1}{2} \left( a_7(\bar{B}^0 \bar{K}^0, \omega) + a_9(\bar{B}^0 \bar{K}^0, \omega) \right) \right\}
+ \frac{G_F}{2} f_K M_B^2 A_0^{\bar{B} \to \omega} V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_1(\bar{B}^0 \omega, \bar{K}^0) + R_{K^0} \left( a_6^u(\bar{B}^0 \omega, \bar{K}^0) \\
- \frac{1}{2} a_8(\bar{B}^0 \omega, \bar{K}^0) + a_{10a}(\bar{B}^0 \omega, \bar{K}^0) \right) \\
+ R_{K^0} \left( a_6^u(\bar{B}^0 \omega, \bar{K}^0) - \frac{1}{2} a_8(\bar{B}^0 \omega, \bar{K}^0) + a_{10a}(\bar{B}^0 \omega, \bar{K}^0) \right) + \frac{1}{2} a_{10}(\bar{B}^0 \omega, \bar{K}^0) + a_{10a}(\bar{B}^0 \omega, \bar{K}^0) \right) \right\} \\
\mathcal{M}(\bar{B}^0 \to K^- \rho^+) = \frac{G_F}{\sqrt{2}} f_K M_B^2 A_0^{\bar{B} \to \rho} V_{cb} (1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_1(\bar{B}^0 \rho^+, K^-) + a_4(\bar{B}^0 \rho^+, K^-) \right) + a_{10}(\bar{B}^0 \rho^+, K^-) + a_{10a}(\bar{B}^0 \rho^+, K^-) \right) \right\} \\
\]
\[ + R_{K^+} \left( a_6^0(B^0\rho^+, K^-) + a_8(B^0\rho^+, K^-) + a_{8a}(B^0\rho^+, K^-) \right) \\
+ a_{10}(B^0\rho^+, K^-) + a_{10a}(B^0\rho^+, K^-) \right] + \left[ a_8^c(B^0\rho^+, K^-) \right] \right) \}; (39) \\

\[ M(B^0 \rightarrow \pi^0 K^{*0}) = \frac{G_F}{2} f_{\pi} M_B^2 A_0^{B \rightarrow K^*} V_{cb}(1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_2(B^0 K^{*0}, \pi^0) - \frac{3}{2} a_7(B^0 K^{*0}, \pi^0) \right] \\
- a_9(B^0 K^{*0}, \pi^0) \right\} - \frac{3}{2} \left[ a_7(B^0 K^{*0}, \pi^0) - a_9(B^0 K^{*0}, \pi^0) \right] \\
- \frac{G_F}{2} f_K M_B^2 F_0^{B \rightarrow \pi} V_{cb}(1 - \frac{\lambda^2}{2}) \left\{ R_c e^{-i\gamma} \left[ a_4(B^0 K^{*0}, \pi^0) - \frac{1}{2} a_{10}(B^0 \pi^0, K^{*0}) \right] \\
+ a_{10a}(B^0 \pi^0, K^{*0}) \right\} + \left[ a_8^c(B^0 \pi^0, K^{*0}) - \frac{1}{2} a_10a(B^0 \pi^0, K^{*0}) \right]; (40) \]
References

[1] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C29, 637(1985), Z. Phys. C34, 103(1987).

[2] V.L. Chernyak and I.R. Zhitnitsky, Nucl. Phys. B345, 137(1990).

[3] N. Isgur and C.H Llewelyn-Smith, Phys. Rev. Lett. 52, 1080(1984); Nucl. Phys. B317, 526(1989).

[4] A. V. Radyushkin, Acta Phys. Pol. 15, 403(1984).

[5] N. G. Stefanis, hep-ph/9911373.

[6] M. Beneke, G. Buchalla, M. Neubert, and C.T. Schrajda, Phys. Rev. Lett. 83, 1914 (1999).

[7] M. Beneke, G. Buchalla, M. Neubert, and C.T. Schrajda, hep-ph/0006124.

[8] CLEO Collaboration, CLEO CONF 99-13; CLEO Collaboration, CLNS 99/1652 and CLEO 99-19.

[9] For a review, see G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).

[10] Particle Data Group, Eur. Phys. J. C3, 1 (1998).

[11] H. Y. Cheng and K. C. Yang, hep-ph/9910291.

[12] W.S. Hou, J.G. Smith and F. Würthwein, hep-ex/9910014.

[13] P. Ball, and V. M. Braun, Phys. Rev. D 58, 094016 (1998).

[14] P. Ball, JHEP09,005(1998).

[15] G.P. Lepage, and S.J. Brodsky, Phys. Lett. B 87, 359 (1979); V.L. Chernyak and A.R. Zhitinissky, Phys. Rep. 112, 173(1983); V. M. Braun and I. E. Filyanov, Z. Phys. C48, 239(1990).

[16] Y.Y. Keum, H.-n. Li, A.I. Sanda, preprint KEK-TH-642, NCKU-HEP-00-01, hep-ph/0004004, preprint NCKU-HEP-00-02, DPNU-00-14, hep-ph/0004173.

[17] C.D. Lü, K. Ukai, M. Z. Yang, preprint HUPD-9924, DPNU-00-15, hep-ph/004213.
[18] M. Gronau, Phys. Rev. Lett 63, 1451, (1989).

[19] M. Gronau and J.L. Rosner, Phys. Rev. D61, 073008(2000).

[20] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett, 43, 242 (1979).
Figure captions

Fig.1 Order $\alpha_s$ non-factorizable contributions in $B \to M_1M_2$ decays

Fig.2 $BR(B \to PV)$ as a function of $\gamma$ are shown as curves in units of $10^{-6}$. The BR measured by CLEO Collaboration are shown by horizontal solid lines. The thicker solid lines are its center values, thin lines are its error bars or the upper limit.

Fig.3 Direct $CP$ asymmetry of $B \to PV$ as a function of $\gamma$

Fig.4 Time-integrated $CP$ asymmetry as a function of $\gamma$ with the mixing parameter $\Delta m/\Gamma = 0.723$ in the SM
Figure 1:
Fig. 2.1, $\text{Br}(\bar{B}^0 \rightarrow \pi^- \rho^+ + \pi^+ \rho^-)$ vs $\gamma$

Fig. 2.2, $\text{Br}(\bar{B}^- \rightarrow \pi^- \rho^0)$ vs $\gamma$

Fig. 2.3, $\text{Br}(B^- \rightarrow \pi^- \omega)$ vs $\gamma$

Fig. 2.4, $\text{Br}(\bar{B}^0 \rightarrow \pi^0 \rho^0)$ vs $\gamma$

Fig. 2.5, $\text{Br}(B^- \rightarrow \pi^0 \rho^-)$ vs $\gamma$

Fig. 2.6, $\text{Br}(\bar{B}^0 \rightarrow \pi^0 \omega)$ vs $\gamma$

Fig. 2.7, $\text{Br}(B^- \rightarrow \pi^- K^{*0})$ vs $\gamma$

Fig. 2.8, $\text{Br}(B^- \rightarrow K^- K^{*0})$ vs $\gamma$

Fig. 2.9, $\text{Br}(B^- \rightarrow K^- \rho^0)$ vs $\gamma$

Fig. 2.10, $\text{Br}(\bar{B}^0 \rightarrow K^- \rho^+)$ vs $\gamma$

Fig. 2.11, $\text{Br}(\bar{B}^0 \rightarrow \pi^0 K^{*0})$ vs $\gamma$

Fig. 2.12, $\text{Br}(B^- \rightarrow \pi^0 K^{*-})$ vs $\gamma$

Fig. 2.13, $\text{Br}(B^- \rightarrow K^- \omega)$ vs $\gamma$

Fig. 2.14, $\text{Br}(\bar{B}^0 \rightarrow K^0 \omega)$ vs $\gamma$

Figure 2:
Figure 3:

Fig. 3.1a, $A_{cp}^{dir} (\bar{B}^0 \rightarrow \pi^- \rho^+) \text{ vs } \gamma$

Fig. 3.1b, $A_{cp}^{dir} (\bar{B}^0 \rightarrow \pi^+ \rho^-) \text{ vs } \gamma$

Fig. 3.2, $A_{cp}^{dir} (\bar{B}^0 \rightarrow \pi^0 \rho^0) \text{ vs } \gamma$

Fig. 3.3, $A_{cp}^{dir} (B^- \rightarrow \pi^- \rho^-) \text{ vs } \gamma$

Fig. 3.4, $A_{cp}^{dir} (B^- \rightarrow \pi^0 \rho^0) \text{ vs } \gamma$

Fig. 3.5, $A_{cp}^{dir} (B^- \rightarrow \pi^- \omega) \text{ vs } \gamma$

Fig. 3.6, $A_{cp}^{dir} (\bar{B}^0 \rightarrow \pi^0 \omega) \text{ vs } \gamma$

Fig. 3.7, $A_{cp}^{dir} (B^- \rightarrow \pi^- K^*0) \text{ vs } \gamma$

Fig. 3.8, $A_{cp}^{dir} (B^- \rightarrow K^- K^*0) \text{ vs } \gamma$

Fig. 3.9, $A_{cp}^{dir} (B^- \rightarrow K^- \rho^0) \text{ vs } \gamma$

Fig. 3.10, $A_{cp}^{dir} (\bar{B}^0 \rightarrow K^- \rho^+) \text{ vs } \gamma$

Fig. 3.11, $A_{cp}^{dir} (\bar{B}^0 \rightarrow \pi^0 K^-) \text{ vs } \gamma$

Fig. 3.12, $A_{cp}^{dir} (B^- \rightarrow \pi^0 K^-) \text{ vs } \gamma$

Fig. 3.13, $A_{cp}^{dir} (B^- \rightarrow K^- \omega) \text{ vs } \gamma$

Fig. 3.14, $A_{cp}^{dir} (\bar{B}^0 \rightarrow K^0 \omega) \text{ vs } \gamma$
Fig. 4.1, $A_{\text{cp}}(B^0 \rightarrow \rho^+\pi^-)$ vs $\gamma$

Fig. 4.2, $A_{\text{cp}}(B^0 \rightarrow \rho^-\pi^+)$ vs $\gamma$

Fig. 4.3, $A_{\text{cp}}(B^0 \rightarrow \rho^0\pi^0)$ vs $\gamma$

Fig. 4.4, $A_{\text{cp}}(B^0 \rightarrow \omega^0\pi^0)$ vs $\gamma$

Figure 4: