Testing CPT and Lorentz Symmetry with Electrons and Positrons in Penning Traps

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Abstract. We present a theoretical analysis of signals for CPT and Lorentz violation in $g - 2$ and charge-to-mass-ratio experiments on electrons and positrons in Penning traps. Experiments measuring anomaly frequencies are found to be the most sensitive to CPT violation. We find that the conventional figure of merit for CPT breaking, involving the difference of the electron and positron $g$ factors, is inappropriate in this context, and an alternative is introduced. Bounds of approximately $10^{-20}$ are attainable.

INTRODUCTION

The CPT theorem \cite{1} is a general and powerful result that holds for local relativistic quantum field theories of point particles in flat spacetime. Any field theory of this kind must be invariant under the combined operations of charge conjugation C, parity reversal P, and time reversal T. As a consequence of this invariance, particles and antiparticles have equal masses, lifetimes, charge-to-mass ratios, and gyromagnetic ratios. The CPT theorem has been tested to great accuracy in a variety of experiments \cite{2}. The sharpest bound is obtained in experiments with neutral kaons, where the CPT figure of merit is

$$r_K \equiv \left| \frac{m_K - m_{\bar{K}}}{m_K} \right| \lesssim 2 \times 10^{-18}. \quad (1)$$

Experiments on electrons and positrons confined in Penning traps also yield sharp bounds on CPT violation. Indeed, these experiments provide the tightest bounds on CPT in the lepton system. Two types of experimental comparisons of electrons and positrons are possible in Penning traps. They involve making accurate measurements of cyclotron frequencies $\omega_c$ and anomaly frequencies $\omega_a$ of single isolated particles confined in the trap. The first compares the ratio $2\omega_a/\omega_c$ for particles and antiparticles. In the context of conventional quantum electrodynamics, this ratio equals $g - 2$ for the particle or antiparticle. A second experiment compares values of $\omega_c \sim q/m$, where $q > 0$ is the magnitude of the charge and $m$ is the mass, and is therefore a comparison of charge-to-mass ratios.

The conventional figure of merit adopted in $g - 2$ experiments on electrons and positrons is given as the relative difference in their $g$ factors \cite{3,4},

$$r_g^e \equiv \left| \frac{g_{e} - g_{e^+}}{g_{avg}} \right|, \quad (2)$$

which is known to be less than $2 \times 10^{-12}$. The bound obtained in charge-to-mass-ratio experiments \cite{5} is expressed as the ratio

$$r_{q/m}^e \equiv \left| \frac{(q_{e} - m_{e}) - (q_{e^+} + m_{e^+})}{(q/m)_{avg}} \right|, \quad (3)$$

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which is less than or equal to $1.3 \times 10^{-7}$.

Measurements of frequencies in atomic systems typically have experimental uncertainties four or five orders of magnitude better than the measurements made in kaon experiments. However, the figure of merit $r_K$ is poorer than $r_K^*$ by about six orders of magnitude. This raises some interesting questions about the Penning-trap experiments as to why they do not provide better tests of CPT despite having better experimental precision. However, it is impossible to pursue these types of questions in the context of conventional quantum electrodynamics, since CPT breaking is strictly forbidden. Instead, one would need to work in the context of a theoretical framework that incorporates CPT-violating interactions, making possible an investigation of possible experimental signatures. Only recently has such a theoretical framework in the context of the standard model been developed [6].

In this paper, we summarize the results of our analysis on CPT and Lorentz tests performed with electrons and positrons in Penning traps. A more complete description of this analysis can be found in Refs. [7,8].

THEORETICAL FRAMEWORK

The theoretical framework we use [6] is based on a general extension of the SU(3) $\times$ SU(2) $\times$ U(1) standard model in particle physics. It includes all possible leading-order CPT- and Lorentz-violating interactions that could arise from spontaneous symmetry breaking at a more fundamental level, such as in string theory. This type of CPT violation is a possibility in string theory because the usual axioms of the CPT theorem do not apply to extended objects like strings. In spontaneous symmetry breaking, the dynamics of the action remains CPT invariant, which means the framework can preserve desirable features of quantum field theory such as gauge invariance, power-counting renormalizability, and microcausality. The CPT and Lorentz violation occurs only in the solutions of the equations of motion and is similar to the spontaneous breaking of the electroweak theory in the standard model.

To analyze interactions involving electrons and positrons in a Penning trap, we use a restriction of the full particle-physics framework to quantum electrodynamics. The resulting model divides into two sectors, one that breaks CPT and one that preserves CPT, while both break Lorentz symmetry. Possible violations of CPT and Lorentz symmetry are parametrized by quantities that can be bounded by experiments. Within this framework, the modified Dirac equation describing a fermion with charge $q$ and mass $m$ in an electromagnetic field is given by

$$(i\gamma^\mu D_\mu - m - a_\mu \gamma^\mu - b_\mu \gamma_5 \sigma^\mu + \frac{i}{2} H_{\mu\nu} \sigma^{\mu\nu} + ic_\mu \gamma^\mu D^\nu + id_{\mu\nu} \gamma_5 \gamma^\mu D^\nu) \psi = 0$$

(4)

Here, $\psi$ is a four-component spinor, $iD_\mu = i\partial_\mu - qA_\mu$ is the covariant derivative, $A^\mu$ is the electromagnetic potential in the trap, and $a_\mu$, $b_\mu$, $H_{\mu\nu}$, $c_\mu$, $d_{\mu\nu}$ are the parameters describing possible violations of CPT and Lorentz symmetry. The properties of $\psi$ under transformations imply that the terms involving $a_\mu$, $b_\mu$ break CPT while those involving $H_{\mu\nu}$, $c_\mu$, $d_{\mu\nu}$ preserve it, and that Lorentz symmetry is broken by all five terms.

Since there have been no experimental observations to date of CPT or Lorentz breaking, the quantities $a_\mu$, $b_\mu$, $H_{\mu\nu}$, $c_\mu$, $d_{\mu\nu}$ must all be small. We can estimate the suppression scale for these parameters by taking the scale governing the fundamental theory as the Planck mass $m_{Pl}$ and the low-energy scale as the electroweak mass scale $m_{ew}$. The natural suppression scale for Planck-scale effects in the standard model would then be of order $m_{ew}/m_{Pl} \approx 10^{-17}$.

EXPERIMENTS IN PENNING TRAPS

We use this theoretical framework to analyze comparative tests of CPT and Lorentz symmetry on electrons and positrons in Penning traps. First, we note that the time-derivative couplings in (4) alter the standard procedure for obtaining a hermitian quantum-mechanical hamiltonian operator. To overcome this, we first perform a field redefinition at the lagrangian level that eliminates the additional time derivatives. We then use charge conjugation to obtain a Dirac equation and hamiltonian for the antiparticle.

In tests of CPT, experiments compare the cyclotron and anomaly frequencies of particles and antiparticles. According to the CPT theorem, electrons and positrons of opposite spin in a Penning trap with the same magnetic fields but opposite electric fields should have equal energies. The experimental relations $g - 2 = 2\omega_a/\omega_c$ and $\omega_c = qB/m$ provide connections to the quantities $g$ and $q/m$ used in defining the figures of merit $r_K^*$ and $r_K^*$.

We perform calculations using Eq. (4) to obtain possible shifts in the energy levels due to either CPT-breaking or CPT-preserving Lorentz violation. In this way, the effectiveness of Penning-trap experiments on electrons and positrons as tests of both CPT-breaking and CPT-preserving Lorentz violation can be analyzed. From the computed energy shifts we determine how the frequencies $\omega_c$ and $\omega_a$ are affected and whether the conventional figures of merit are appropriate.

In experiments performed in Penning traps, the dominant contributions to the energy come from interactions of the electron or positron with the constant magnetic field of the trap, while the quadrupole electric fields generate smaller effects. In a perturbative calculation, the dominant CPT- and Lorentz-breaking effects can therefore be
RESULTS

The results of our calculations for electrons and positrons in Penning traps [7,8] show that the leading-order effects due to CPT and Lorentz breaking cause corrections to the cyclotron and anomaly frequencies:

$$\omega_c^\pm \approx \omega_a^\pm \approx (1 - c_{10}^0 - c_{11}^e - c_{22}^e)\omega_c$$  \hspace{1cm} (5)

$$\omega_a^\mp \approx \omega_a \mp 2b_3^e + 2d_{50}^e m_e + 2H_{12}^e$$  \hspace{1cm} (6)

In our notation, $\omega_c$ and $\omega_a$ represent the unperturbed frequencies for the electron ($e^-$) and the positron ($e^+$), while $\omega_c^\pm$ and $\omega_a^\mp$ denote the frequencies including the corrections. Superscripts have also been added on the coefficients $b_3$, etc., to denote that these parameters describe the electron-positron system. From these relations we find the differences in the electron and positron cyclotron and anomaly frequencies to be

$$\Delta\omega_c^e \equiv \omega_c^e - \omega_c^+ \approx 0$$  \hspace{1cm} (7)

$$\Delta\omega_a^e \equiv \omega_a^e - \omega_a^+ \approx -4b_3^e$$  \hspace{1cm} (8)

We find that in the context of this framework, comparisons of cyclotron frequencies to leading order do not provide a signal for CPT or Lorentz breaking, since the corrections to $\omega_c$ for electrons and positrons are equal. However, comparisons of anomaly frequencies provide unambiguous tests of CPT since the CPT-violating term with $b_3$ results in a nonzero value for the difference $\Delta\omega_a^e$, while the CPT-preserving coefficients do not appear.

We also find that to leading order there are no corrections due to CPT or Lorentz violation to the $g$ factors for either electrons or positrons. This has important and unexpected results concerning the figure of merit $r_g$ in Eq. (2). With $g_{e^-} \approx g_{e^+}$ to leading order, we find that $r_g$ vanishes, which would seem to indicate the absence of CPT breaking. However, this conclusion would be incorrect because the model contains explicit CPT violation. In addition, our calculations show that with $\bar{b} \neq 0$ the experimental ratio $2\omega_a/\omega_c$ depends on the magnetic field and is undefined in the limit of a vanishing $B$ field. Therefore, the usual relation $g - 2 = 2\omega_a/\omega_c$ does not hold in the presence of CPT violation. For these reasons, we conclude that the figure of merit $r_g$ in Eq. (2) is inappropriate in the context of our framework. An alternative is suggested next.

Since a prediction of the CPT theorem is that electron and positron states of opposite spin in the same magnetic field have equal energies, we propose a model-independent figure of merit

$$r_{\omega_a}^e \equiv \frac{|E_{n,s}^{e-} - E_{n,-s}^{e+}|}{E_{n,s}^{e-}}$$  \hspace{1cm} (9)

where $E_{n,s}^{e\pm}$ are the energies of the relativistic states labeled by their Landau-level numbers $n$ and spin $s$. Our calculations show $r_{\omega_a}^e \approx |\Delta\omega_a^e|/2m_e \approx |2b_3^e|/m_e$. Assuming frequency resolutions on the order of 1 Hz, we estimate as a bound on this figure of merit,

$$r_{\omega_a}^e \lesssim 10^{-20}$$  \hspace{1cm} (10)

This definition of the figure of merit $r_{\omega_a}^e$ is compatible with the corresponding figure of merit $r_K$ arising from experiments with the neutral-kaon system. This is because both figures of merit involve ratios of energy scales, and therefore comparisons across experiments are more meaningful. This is not the case for the figures of merit $r_{\omega_a}^e$ and $r_K$, since each involves different physical quantities. Our estimate suggests that a somewhat tighter bound for $r_{\omega_a}^e$ is attainable in Penning-trap experiments than for the corresponding figure of merit $r_K$ arising from experiments with the neutral-kaon system. This result is more in line with the greater precision that is experimentally accessible in frequency measurements in a Penning trap. However, performing the CPT tests in the kaon system remains essential because neutral-meson CPT violation is controlled by distinct CPT-violating parameters that appear only in the quark sector [10].
In Ref. [8], we describe additional possible signatures of CPT and Lorentz violation. These include possible diurnal variations in the anomaly and cyclotron frequencies. Tests for these effects would provide bounds on some of the components of the parameters \( c_{\mu\nu}, d_{\mu\nu}, \text{ and } H_{\mu\nu} \).

One type of experiment looking for diurnal variations involves the electron alone or the positron alone. In the standard-model extension, these variations would occur because the components of the couplings in Eq. (6) would change as the Earth rotates. Consider the following quantities for the electron and positron:

\[
\Delta^e_{\omega^-} = \frac{|\mathcal{E}_{0,+1}^e - \mathcal{E}_{1,-1}^e|}{\mathcal{E}_{0,-1}^e}, \quad \Delta^e_{\omega^+} = \frac{|\mathcal{E}_{0,-1}^e - \mathcal{E}_{1,+1}^e|}{\mathcal{E}_{0,+1}^e}.
\]

Suitable figures of merit \( r_{\omega^\pm, \text{diurnal}}^e \) can be defined as the amplitude of the diurnal variations in \( \Delta^e_{\omega^-} \) and \( \Delta^e_{\omega^+} \), respectively. In the context of our framework, we find

\[
r_{\omega^\pm, \text{diurnal}}^e \approx \frac{2|\mp b^e_3 + d_{30}^e m_e + H_{12}^e|}{m_e}.
\]

The experimental issues involved in obtaining a bound on \( r_{\omega^\pm, \text{diurnal}}^e \) include maintaining stability in the magnetic field. For example, limiting variations in the magnetic field to a level of about 5 parts in \( 10^9 \) over the duration of the experiment would keep any drift in the 200 MHz anomaly frequency within a 1 Hz margin. The data would also need to be suitably binned according to the orientation of the magnetic field as a function of star time. A more elaborate approach to such diurnal experiments would be to mount the apparatus on a suitable rotating platform and thereby to investigate any geometrical dependence more directly.

An experiment of this nature on electrons alone or positrons alone would bound the combination \( \mp b^e_3 + d_{30}^e m_e + H_{12}^e \) of couplings in the standard-model extension. It would involve searching for leading-order corrections to the anomaly and cyclotron frequencies which exhibit periodicities of approximately 24 hours. Subleading order corrections involving tensor couplings might exhibit 12-hour periodicities. However, these effects would be suppressed relative to the leading-order effects in Eq. (11). All three of these quantities in Eq. (11) break Lorentz symmetry, but only the coupling \( b^e_3 \) breaks CPT. If a signal were detected, it would indicate Lorentz violation but not necessarily CPT violation. It would provide strong motivation for a subsequent experiment comparing anomaly frequencies of electrons and positrons, which would bound the CPT-breaking parameter \( b^e_3 \) in isolation.

Data for this type of experiment on electrons alone already exist, and a preliminary analysis has been performed [11]. Assuming a precision of approximately 1 Hz in detecting diurnal variations, we estimate a bound on Lorentz breaking of

\[
r_{\omega^\pm, \text{diurnal}}^e \lesssim 10^{-20}.
\]

CONCLUSIONS

We find that the use of a general theoretical framework incorporating CPT and Lorentz breaking allows a detailed investigation of possible experimental signatures in Penning-trap experiments on electrons and positrons. Our results indicate that the best tests of CPT symmetry in Penning traps emerge from comparisons of anomaly frequencies in \( g - 2 \) experiments. Our estimated bound on CPT from a variety of signals is approximately \( 10^{-20} \) in electron-positron experiments. A table showing these estimated bounds is presented in Ref. [8]. We also find that experiments searching for diurnal variations in electrons alone can provide bounds on Lorentz breaking at a level of approximately \( 10^{-20} \).

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