Influence of heat and mass transfer on inclined (MHD) peristaltic of pseudoplastic nanofluid through the porous medium with couple stress in an inclined asymmetric channel

Mohammed R. Salman¹, Ahmed M. Abdulhadi²
University of Baghdad-College of Science-Department of Math-Baghdad-Iraq
Email address: mawb1967@gmail.com (Mohammed R. Salman), ahm6161@yahoo.com (Ahmed M. Abdulhadi)

Abstract. The paper is intended for studying the Influence of heat and mass transfer on inclined magnetohydrodynamics peristaltic of pseudoplastic nanofluid through the porous medium with couple stress in an inclined tapered asymmetric channel. The governing equations of two-dimensional are the motion and energy equations are resolved and simplified under acting the low “Reynolds number” long wavelength by using perturbation technique series solutions. And have been obtained the effects of different parameters which are “Hartmann number”, “Darcy number”, “Brinkman number”, “Reynolds number”, “Froude number”, “Prandl number” and “Eckert number” and as well the “couple stress”, inclination of magnetic field, mean flow rate and incline angle of the channel on the “pressure gradient”, velocity, “pressure rise”, “Nusselt number”, “temperature”, “heat transfer coefficient” and survey the “trapping” phenomena inspection” through different graphs. And in order to illustrate the validity of the theoretical study, numerical results have been computed by using MATHEMATICA software.

Keywords: Peristaltic transport, Reynolds number, Hartmann number, Froude number, Porous Medium.

1. Introduction

The study a non-Newtonian fluid flows has a great interest among researchers Including medical, biological sciences, physiology, blood. Also on the industrial field where has encouraged the complex nature of fluids of which, oils, chemicals, petroleum and other fluids, has encouraged extensive studies and research into the properties of these fluids. where many researchers presented basic equations for various non-Newtonian liquids [1-5].

The couple stress fluid theory is one of the most important topics in the study of Newtonian fluids and represents the excellent study presented written by Stokes “Theories of fluids with micro-structure an introduction” [6,7]. And also there are many important studies of the couple stress fluid that explain the behavior of complex fluids such as blood, thrombosis due to blood clotting, liquid crystals, colloidal liquids and synovial liquids [8-10].

The peristaltic flows, due to extensive applications in industry, physiology, and agriculture in modern irrigation methods, The movement peristaltic flows of the Newtonian and non-Newtonian fluids, which results from contraction and expansion along the tube (channel), for example, the movement of food from the stomach to the small intestine and large intestine as well as the transfer of the ovum in the fallopian tube and the flow of urine flowing from the kidney to the bladder, the passage of the yellow matter from the gallbladder to the twelve, etc. They are the most common examples in the body. Through these and other applications, many several researchers have studied the of peristaltic flow in different cases[11-17].

The magnetohydrodynamics is currently involved in the detection and treatment of many diseases, the most important of which is the treatment of cancer tumors and control of bleeding caused by surgical
currencies, as well as MRI to diagnose the disease.magnetic field effect can also be used as a blood pump during operations of cardiovascular surgery (stenosis and arteriosclerosis). From these applications, many researchers studied transport problems in asymmetric channels and studied the effect of the magnetic field on fluid flow behavior. Nadeem and Akram [18,19] have studied the peristaltic flow of a couple stress fluid under the influence of an induced magnetic field in an asymmetric channel and effect of inclined magnetic field on a Williamson fluid model in an inclined asymmetric channel.

The concept of mass transfer is one of the most important topics that have wide applications in biological processes in the body of living things, including humans, such as transferring nutrients in the blood to nearby tissues, and kidney dialysis processes to purify blood from toxins and high urea. As well as in many industrial processes such as separation of the membrane, separation of petroleum components, distillation systems, water purification and the process of separating or adding chemical impurities. For example, Saleem and Haider [20] studied theoretical analysis of heat and mass transfer of the non-Newtonian fluid (Maxwell’s fluid) in peristaltic channels with creeping flow. Hayat et al. [21] examined and studied the simultaneous influences of convective conditions and nanoparticles on peristaltic motion. M.R. Salman and A.M. Abdulhadi [22] have analyzed theoretically Effects of MHD on Peristalsis Transport and Heat Transfer with Variables Viscosity in Porous Medium. Also many researchers worked in this field [23,24]. K. Ramesh [25] he studied the effect of heat and mass transfer on peristaltic flow of a couple stress fluid in an inclined asymmetric channel with there exist porous medium under the influence of an inclined magnetic field.

In this paper, we will study, discuss and interpret Influence of heat and mass transfer on inclined (MHD) peristaltic of pseudoplastic nanofluid through the porous medium with couple stress in an inclined asymmetric channel. This is done by simplifying and solving the governing equations under the influence of the very small Reynolds number and a large wavelength by use numerical results have been computed by using MATHEMATICA software and then we analyze these results through shapes and graphs for pressure gradient and temperature and pressure rise and stream function.

2. Problem description

The flow of an incompressible magnetohydrodynamic peristaltic couple stress fluid in an inclined asymmetric channel in a two-dimensional tapered width \(2d\) through a porous medium. The channel is inclined at an angle \(\alpha\) to the X-axis. The lower and upper walls \(Y = \tilde{H}_1\) and \(Y = \tilde{H}_2\) are at the distance \(d\) from the central line of the channel. Wave speed of peristaltic waves is denoted by \(c\) . We choose a Cartesian coordinate system in such a way that X-axis is taken along the axial direction and Y-axis is normal to the flow direction. to it (see Fig. (1)). The fluid is electrically conducting in uniform magnetic \((T_o)\) field \((B_o)\) and is inclined at an angle \((\Theta)\) to the Y-axis, where a uniform magnetic field \(B = (0,B_o,0)\). The induced magnetic field is ignored here by making a very small magnetic Reynolds number. Heat and mass transfer is examined through convective conditions. The flow is generated by sinusoidal waves propagating along the compliant walls of the channel. The upper and lower walls are maintained at the temperatures \((T_0)\) and \((T_1)\) . The geometry of the wall surfaces is given as follows:

\[
Y = \tilde{H}_1(\tilde{X}, t') = d + m'\tilde{X} + b_1\sin\left(\frac{2\pi}{\lambda}(\tilde{X} - ct')\right) \quad (1)
\]

\[
Y = \tilde{H}_2(\tilde{X}, t') = -d - m'\tilde{X} - b_2\sin\left(\frac{2\pi}{\lambda}(\tilde{X} - ct') + \phi\right) \quad (2)
\]

Where \(b_1\), \(b_2\) are the amplitudes of upper and lower walls, \(c\) is the "velocity of the wave", \(\lambda\) is the wavelength, is the non-uniform parameter, the phase variance \((\phi)\) varies in the range \((0 \leq \phi \leq \pi)\)
, where \((\phi = 0)\) consider to symmetric channel with waves out of phase i.e. both walls move towards outward or inward the same time and further \(b_1,b_2,d\) and \((\phi)\) satisfy the following condition at the inlet of divergent channel

\[
b_1^2 + b_2^2 + 2b_1b_2 \cos \phi \leq (2d)^2
\]

(3)

Here we assume the fluid to be electrically conducting in the presence of a uniform inclined magnetic field \(B=(0,B_o,0)\). To compute the "Lorentz force" we will put on a magnetic field just in \(Y\) direction and then we study the influence of it on the fluid flux.

Expression of an extra stress tensor in pseudoplastic fluid is [26] :

\[
\dot{S} + \lambda_1 \frac{D\dot{S}}{D\tau} + \frac{1}{2} (\lambda_1 - \mu_1) \left( A_1 \dot{S} + \dot{S} A_1 \right) = \mu A_1
\]

In which \(\lambda_1\) and \(\mu_1\) are the relaxation times. Also \(A_1 = \left[ \nabla \nabla + \left( \nabla \nabla \right)^T \right] \), \(A_1\) is the first Rivlin-Ericksen tensor with the velocity gradient, and

\[
d\dot{S} / d\tau = \partial \dot{S} / \partial \tau + \nabla \nabla \dot{S} \quad \text{and} \quad \frac{D\dot{S}}{D\tau} = d\dot{S} / d\tau - \left( \nabla \nabla \right) \dot{S} - \dot{S} \left( \nabla \nabla \right)^T
\]

Then

\[
\frac{D\dot{S}}{D\tau} = \partial \dot{S} / \partial \tau + \nabla \nabla \dot{S} - \left( \nabla \nabla \right) \dot{S} - \dot{S} \left( \nabla \nabla \right)^T
\]

Figure 1. Channel diagram asymmetric and tapered.

3. The governing equations

In the laboratory frame \((\tilde{x}, \tilde{y})\), the "energy equation" is written as follows

\[
\frac{\partial U}{\partial \tilde{x}} + \frac{\partial V}{\partial \tilde{y}} = 0
\]

(4)

\[
\rho \left[ \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial \tilde{x}} + V \frac{\partial U}{\partial \tilde{y}} \right] = -\frac{\partial \mu}{\partial \tilde{x}} \left( \dot{S}_{xx} \right) + \frac{\partial \mu}{\partial \tilde{y}} \left( \dot{S}_{xy} \right) - \sigma B_o \cos \Theta \left[ \tilde{U} \cos \Theta - \tilde{V} \sin \Theta \right] - \frac{\mu}{K_0} \tilde{U} + \rho g \sin \alpha
\]

(5)
The extra stress components \( \tilde{S}_{xx} \), \( \tilde{S}_{xy} \) and \( \tilde{S}_{yx} \) satisfy the following relations:

\[
\begin{align*}
\tilde{S}_{xx} &= -\alpha_0 B_0^2 \sin \theta \left[ U \cos \Theta - V \sin \Theta \right] - \frac{\mu}{K_0} \tilde{U} - \rho g \sin \alpha \\
\rho c_p \left[ \frac{\partial \tilde{T}}{\partial t} + \tilde{U} \frac{\partial \tilde{T}}{\partial X} + V \frac{\partial \tilde{T}}{\partial Y} \right] &= K \left[ \frac{\partial^2 \tilde{T}}{\partial X^2} + \frac{\partial^2 \tilde{T}}{\partial Y^2} \right] + \mu \left[ 2 \left( \frac{\partial \tilde{U}}{\partial X} \right)^2 + \left( \frac{\partial \tilde{V}}{\partial Y} \right)^2 \right] + \eta \left[ \frac{\partial^2 \tilde{U}}{\partial X^2} + \frac{\partial^2 \tilde{V}}{\partial Y^2} \right] + Q_0
\end{align*}
\]

The extra stress components \( \tilde{S}_{xx} \), \( \tilde{S}_{xy} \) and \( \tilde{S}_{yx} \) satisfy the following relations:

\[
\begin{align*}
\tilde{S}_{xx} &= -\alpha_0 B_0^2 \sin \theta \left[ U \cos \Theta - V \sin \Theta \right] - \frac{\mu}{K_0} \tilde{U} - \rho g \sin \alpha \\
\rho c_p \left[ \frac{\partial \tilde{T}}{\partial t} + \tilde{U} \frac{\partial \tilde{T}}{\partial X} + V \frac{\partial \tilde{T}}{\partial Y} \right] &= K \left[ \frac{\partial^2 \tilde{T}}{\partial X^2} + \frac{\partial^2 \tilde{T}}{\partial Y^2} \right] + \mu \left[ 2 \left( \frac{\partial \tilde{U}}{\partial X} \right)^2 + \left( \frac{\partial \tilde{V}}{\partial Y} \right)^2 \right] + \eta \left[ \frac{\partial^2 \tilde{U}}{\partial X^2} + \frac{\partial^2 \tilde{V}}{\partial Y^2} \right] + Q_0
\end{align*}
\]

In which \( \rho \) is the fluid density, \( \tilde{U} \) and \( \tilde{V} \) are the respective velocity components in \( \tilde{X} \) and \( \tilde{Y} \) directions, \( \tilde{P} \) is the pressure, \( \sigma' \) is the electrical conductivity of the fluid, \( g \) is the acceleration due to the gravity, \( \mu \) is the "viscosity", \( \eta \) is the "couple stress" viscosity parameter, \( K_0 \) is the permeability parameter, \( B_0 \) is the magnetic field, \( \kappa \) the "thermal conductivity" of fluid, \( \alpha \) is the inclination angle of the channel, \( \Theta \) is the inclination of the magnetic field, \( \tilde{T} \) is the temperature, \( c_p \) is the specific heat at a constant pressure, and \( Q_0 \) is the heat generation parameter. And \( \tilde{S}_{xx}, \tilde{S}_{xy} \) and \( \tilde{S}_{yx} \) are the components of the additional stress tensor. Respectively.

And in the laboratory frame \((\tilde{x}, \tilde{y})\), the flow is steady.

Now, for \( \tilde{U} \) and \( \tilde{V} \) be the respective velocity components along \( \tilde{X} \) and \( \tilde{Y} \) directions in the fixed frame, respectively. For the unsteady two-dimensional flow, the velocity components may be written \( \tilde{V} = (\tilde{U}, \tilde{V}, \tilde{W}) \) and the temperature function may be written \( T = T(\tilde{X}, \tilde{Y}) \).
The suitable boundary conditions comprising wall no-slip and convective boundary conditions are approaching as follows:

\[
U = 0, \quad T = T_0 \quad \text{at} \quad Y = \tilde{H}_1 \quad (11)
\]

\[
U = 0, \quad T = T_1 \quad \text{at} \quad Y = \tilde{H}_2 \quad (12)
\]

If we introduce the wave frame having coordinates \((\tilde{x}, \tilde{y})\) which march in the horizontal direction with the same wave speed \((c)\). Then in the laboratory frame \((X, Y)\), the unsteady flow can be considered as steady. The coordinates and velocities in the two frames by following relations

\[
\tilde{x} = \tilde{X} - ct, \quad \tilde{y} = \tilde{Y}, \quad \tilde{u} = \tilde{U} - c, \quad \tilde{v} = \tilde{V}, \quad \tilde{p} = \tilde{P}
\]

(13)

Where \(\tilde{u}, \tilde{v}\) are the velocity components in the wave frame \((\tilde{x}, \tilde{y})\).

4. Dimensionless parameter

For a dimensional analysis process we set up the following non-dimensional quantities:

\[
x = \frac{\tilde{x}}{\lambda}, \quad y = \frac{\tilde{y}}{d}, \quad t = \frac{ct}{\lambda}, \quad u = \frac{\tilde{u}}{c}, \quad v = \frac{\tilde{v}}{c}, \quad \delta = \frac{d}{\lambda}, \quad h_1 = \frac{\tilde{H}_1}{d}, \quad h_2 = \frac{\tilde{H}_2}{d}, \quad \theta = \frac{T - T_1}{T_0 - T_1}, \quad p = \frac{\tilde{p}}{\delta \mu c},
\]

\[
S_y = \frac{d}{c \mu} \frac{\tilde{S}_y}{\delta}, \quad K = \frac{K_0}{d}, \quad a = \frac{b_1}{d}, \quad b = \frac{b_2}{d}, \quad Re = \frac{p c d}{\mu}, \quad m = \frac{m^2}{d}, \quad Pr = \frac{\mu c}{\kappa}, \quad M = \frac{\sigma}{\mu d B_0}, \quad (14)
\]

\[
\gamma = \frac{\mu c}{d}, \quad Fr = \frac{c^2}{g d}, \quad Ec = \frac{c^2}{\mu}, \quad Br = Ec Pr, \quad \beta = \frac{Q_d^2}{K(T_1-T_0)}, \quad \mu = \frac{\partial \psi}{\partial x}, \quad \nu = -\frac{\partial \psi}{\partial x}
\]

Where \((\delta)\) a "wave number", \((Re)\) a "Renold number", \((\gamma)\) represent "couple stress" parameter, \((K)\) a "Darcy number", \((M)\) a "Hartman number", \((Fr)\) a "Froude number", \((\theta)\) is a "temperature distribution", \((Ec)\) a "Eckert number" and \((Pr)\) is a "Prandle number".

By using the transformations in Eqs.(13) and (14) into a governing Eqs.(4)-(7) we have the following form.

\[
\delta \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(15)

\[
\text{Re} \left[ \left( \frac{\partial \psi}{\partial y} + 1 \right) \frac{\partial \psi}{\partial x} \right] = \frac{d \psi}{dx} + \delta \frac{\partial}{\partial x} (S_{x\alpha}) + \delta \frac{\partial}{\partial y} (S_{y\alpha})
\]

(16)

\[
-M^2 \cos \Theta \left[ \left( \frac{\partial \psi}{\partial y} + 1 \right) \cos \Theta + \delta \frac{\partial \psi}{\partial x} \sin \Theta \right] - \frac{1}{K} \left( \frac{\partial \psi}{\partial y} + 1 \right) + \text{Re} \left( \frac{\partial \psi}{\partial x} \right)
\]

(17)

\[
\text{Re} \left[ \frac{\partial v}{\partial t} + (u + 1) \frac{\partial v}{\partial x} + \frac{1}{\delta} \frac{\partial v}{\partial y} \right] = -\frac{dp}{dy} + \delta \frac{\partial}{\partial x} (S_{x\alpha}) + \delta \frac{\partial}{\partial y} (S_{y\alpha})
\]

where \(\sigma^2 B_0^2 d_i^2 \frac{\delta}{\mu} \sin \Theta \left[ (u + 1) \cos \Theta + v \sin \Theta \right] - \frac{\delta v}{K} + \frac{\rho g d_i^2 \delta}{\mu} \delta \cos \alpha \)
\[ Re\ Pr \left[ \delta(u + 1) \frac{\partial \theta}{\partial x} + \nu \frac{\partial \theta}{\partial y} \right] = \left[ \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right] \]
\[ + Ec\ Pr \left[ 2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \left( \frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x} \right)^2 \right] \]
\[ + Ec\ Pr \left[ \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)^2 + \left( \frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 v}{\partial y^3} \right)^2 \right] + \beta \]

(18)

Similarly Eqs.(11) and (12) become

\[ \frac{\partial \psi}{\partial y} = 0 , \quad \theta = 0 \quad \text{at} \quad y = h_1 = -1 + m (x + t) - a \sin [2\pi x + \phi] \]

(19)

\[ \frac{\partial \psi}{\partial y} = 0 , \quad \theta = 1 \quad \text{at} \quad y = h_2 = 1 + m (x + t) + b \sin [2\pi x] \]

(20)

Eqs.(16)-(18), under long wavelength (\( \delta \ll 1 \)) and low "Reynolds number" approximation is given a following equations:

\[ \frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left( S \psi \right) - M^2 (\cos \Theta)^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) - \frac{1}{K} \left( \frac{\partial \psi}{\partial y} + 1 \right) + \frac{Re}{Fr} \sin \alpha \]

(21)

\[ \frac{\partial p}{\partial y} = 0 \]

(22)

\[ \frac{\partial^2 \theta}{\partial y^2} + Br \left[ \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \frac{1}{\gamma^2} \left( \frac{\partial^3 \psi}{\partial y^3} \right)^2 \right] + \beta = 0 \]

(23)

After removing the dimensions (by Eq.(14)), the boundary conditions are as follows:

\[ \psi = -\frac{F}{2} , \quad \frac{\partial \psi}{\partial y} = 0 , \quad \theta = 0 \quad \text{at} \quad y = h_1 \]

(24)

\[ \psi = \frac{F}{2} , \quad \frac{\partial \psi}{\partial y} = 0 , \quad \theta = 1 \quad \text{at} \quad y = h_2 \]

(25)

Where \((F)\) represents the dimensionless mean flows. Here

\[ F (x, t) = Q + a \sin [2\pi x + \phi] + b \sin [2\pi x] \]

(26)

In which \((Q)\) is the dimensionless mean flows in the wave frame, Here

\[ F = \int_{h_1(x)}^{h_2(x)} \frac{\partial \psi}{\partial y} dy = \psi(h_2) - \psi(h_1) , \quad Q = \frac{\tilde{Q}}{cd} \]

(27)

\[ \tilde{\psi} = \int \tilde{u}(\tilde{x}, \tilde{y}) d\tilde{y} \]

(28)

Here it is referred that the conditions of \((\psi)\) which is satisfy Eq.(27) and the conditions on \(\partial \psi / \partial y\)

means that the flow is no-slip.
Now we have \[ S_{xy} = \left( \frac{\partial^2 \psi}{\partial y^2} \right) \left( 1 - \frac{\xi}{\left( \frac{\partial^2 \psi}{\partial y^2} \right)^2} \right) \] where \( \xi = \left( \mu_i^2 - \lambda_i^2 \right) \) is the pseudoplastic fluid parameter [27].

By recompense this equation in Eq.(21)

\[
\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left( \frac{\partial^2 \psi}{\partial y^2} \right) \left( \frac{\partial^2 \psi}{\partial y^2} \right) - M^2 (\cos \Theta)^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) - \frac{1}{K} \left( \frac{\partial \psi}{\partial y} + 1 \right) + \frac{Re}{Fr} \sin \alpha
\]

\[
= \frac{\partial}{\partial y} \left[ \frac{\partial^2 \psi}{\partial y^2} \left( 1 + \frac{\xi}{\left( \frac{\partial^2 \psi}{\partial y^2} \right)^2} + \ldots \right) \right] - M^2 (\cos \Theta)^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) - \frac{1}{K} \left( \frac{\partial \psi}{\partial y} + 1 \right) + \frac{Re}{Fr} \sin \alpha
\]

\[
\Rightarrow \frac{\partial p}{\partial x} = \frac{\partial^3 \psi}{\partial y^3} + 3\frac{\partial^2 \psi}{\partial y^2} \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial y} - M^2 (\cos \Theta)^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) - \frac{1}{K} \left( \frac{\partial \psi}{\partial y} + 1 \right) + \frac{Re}{Fr} \sin \alpha
\]  

By Eq.(22) and Eq.(29) we get:

\[
\frac{\partial^4 \psi}{\partial y^4} + 3\frac{\partial^3 \psi}{\partial y^3} \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial y} - M^2 (\cos \Theta)^2 + \frac{1}{K} \frac{\partial^2 \psi}{\partial y^2} = 0
\]  

5. Coefficient of heat transfer

The coefficient of heat transfer at the upper wall is offered by the following law

\[
Z(x) = \left. \frac{\partial h_1}{\partial x} \left( \frac{\partial \theta}{\partial y} \right) \right|_{y = h_1}
\]  

And the coefficient of heat transfer at the lower wall is given by

\[
Z(x) = \left. \frac{\partial h_2}{\partial x} \left( \frac{\partial \theta}{\partial y} \right) \right|_{y = h_2}
\]  

6. Average pressure rise

The non-dimensional average rise in pressure \( \Delta p \) per wavelength is given by

\[
\Delta p = \int_0^1 \frac{\partial p}{\partial x} \, dx
\]  

7. Solution technique

In order to solve the equations (23), (29) and (30), we expand the flow quantities in a power series of \( \xi \) as follows

\[
\psi = \psi_0 + \xi \psi_1 + \xi^2 \psi_2 + \ldots,
\]

\[
P = P_0 + \xi P_1 + \xi^2 P_2 + \ldots,
\]

\[
\theta = \theta_0 + \xi \theta_1 + \xi^2 \theta_2 + \ldots,
\]

\[
Z = Z_0 + \xi Z_1 + \xi^2 Z_2 + \ldots.
\]

Substituting Eq.(34) into Eqs.(23),(29) and (30) we get:
7.1. Zero order
The corresponding systems at zero order are
\[ \frac{\partial^2 \theta_0}{\partial y^2} + Br \left[ \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 + \frac{1}{\gamma^2} \left( \frac{\partial^3 \psi_0}{\partial y^3} \right)^2 \right] + \beta = 0 \]
\[ \frac{dP_0}{dx} = \frac{\partial^3 \psi_0}{\partial y^3} - N_1^2 \left( \frac{\partial \psi_0}{\partial y} + 1 \right) + \frac{Re}{Fr} \sin \alpha \]
\[ \frac{\partial^4 \psi_0}{\partial y^4} - N_1^2 \frac{\partial^2 \psi_0}{\partial y^2} = 0 \]

Where \( N_1^2 = M^2 (\cos \Theta)^2 + \frac{1}{K} \) with corresponding boundary conditions
\[ \psi_0 = -\frac{F_0}{2} \frac{\partial \psi_0}{\partial y} = 0, \theta_0 = 0 \text{ at } y = h_1 \]
\[ \text{and } \psi_0 = \frac{F_0}{2} \frac{\partial \psi_0}{\partial y} = 0, \theta_0 = 1 \text{ at } y = h_2 \]

7.2. First order
The corresponding systems at first order are
\[ \frac{\partial^2 \theta_1}{\partial y^2} + 2Br \left[ \left( \frac{\partial^2 \psi_1}{\partial y^2} \right)^2 + \frac{1}{\gamma^2} \left( \frac{\partial^3 \psi_1}{\partial y^3} \right)^2 \right] = 0 \]
\[ \frac{dP_1}{dx} = \frac{\partial^3 \psi_1}{\partial y^3} + 3 \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 \frac{\partial \psi_0}{\partial y} - N_2^2 \frac{\partial \psi_1}{\partial y} \]
\[ \frac{\partial^4 \psi_1}{\partial y^4} + 3 \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 \frac{\partial^2 \psi_0}{\partial y^2} + 6 \frac{\partial^3 \psi_0}{\partial y^3} \frac{\partial^2 \psi_0}{\partial y^2} - N_1^2 \frac{\partial^2 \psi_1}{\partial y^2} = 0 \]

Where \( N_2^2 = M^2 + \frac{1}{K} \) with corresponding boundary condition
\[ \psi_1 = -\frac{F_1}{2} \frac{\partial \psi_1}{\partial y} = 0, \theta_1 = 0 \text{ at } y = h_1 \]
\[ \text{and } \psi_1 = \frac{F_1}{2} \frac{\partial \psi_1}{\partial y} = 0, \theta_1 = 0 \text{ at } y = h_2 \]

8. Perturbation Solutions
By using the series solution (regular perturbation technique), it is found that the solution of the zeroth and first Solutions system is given:

8.1. Zeroth- Order Solution
The solutions of Eqs.(35),(36),(37) are given by
\[ \psi_0 = C_3 + C_4 y + \frac{C_1 e^{-N_1}}{N_1^2} + \frac{C_2 e^{N_1}}{N_1} \]
\[ u_0 = C_4 + \frac{-C_1 e^{-N_1} N_1 + C_2 e^{N_1} N_1}{N_1^2} \]
\[
\frac{\partial P_0}{\partial x} = \frac{Re \sin \alpha}{Fr} + \frac{-C_1 e^{-\gamma N_i} N_1^3 + C_2 e^{\gamma N_i} N_1^3}{N_1^2} - \frac{N_1^2}{N_1^2} \left(1 + C_4 + \frac{-C_1 e^{-\gamma N_i} N_1 + C_2 e^{\gamma N_i} N_1}{N_1^2}\right)
\]

\[
\theta_0 = d_1 + d_2 y + \frac{2\gamma^2 N_1}{N_1^2}
\]

\[
Z_{0h} = \frac{\partial h_i}{\partial x} \left(\frac{\partial \theta_0}{\partial y}\right)_{y=h_i} = \left(-m - 2a \pi \cos \left(2\pi x + \phi \right)\right)(d_2 + \frac{1}{2\gamma^2 N_1} \left(Br C_2^2 e^{-2\gamma N_i} \left(\gamma^2 + N_1^2\right)\right)
\]

\[
\left(-Br C_2^2 e^{2\gamma N_i} \left(\gamma^2 + N_1^2\right) - 2\gamma N_1 \left(\gamma^2 \left(2Br C_2 C_2 + \beta\right) - 2Br C_2 C_2 N_1^2\right)\right)
\]

Moreover, the non-dimensionless average rise in \((\Delta p_0)\) per wavelength at this order is

\[
\Delta p_0 = \int_0^1 \left(\frac{\partial p_0}{\partial y}\right)_{y=0} dx
\]

### 8.2. First-Order Solution

Substituting the zeroth-order solution into Eqs.(38), (39),(40) and then we solve the resulting system with the existence of "boundary conditions", we get

\[
\psi_1 = C_7 + y C_8 + \frac{1}{8 N_1} \left[-2 e^{-\gamma N_i} \left(-6 C_1^2 C_2 y - \frac{15 C_1^2 C_2 + 4 C_5}{N_1}\right) + 2 e^{\gamma N_i} \left(-6 C_1^2 C_2 y + \frac{15 C_1^2 C_2 + 4 C_6}{N_1}\right) - C_1^3 e^{-3\gamma N_i} - C_2^3 e^{3\gamma N_i}\right]
\]

\[
u_1 = \frac{\partial \psi_1}{\partial y} = C_8 + \frac{1}{8 N_1} \left[3 C_1^3 e^{-3\gamma N_i} + 12 C_1^2 C_2 e^{3\gamma N_i} - 2C_2^3 e^{-3\gamma N_i} + 3 C_1^3 e^{3\gamma N_i} + 2 e^{-\gamma N_i} \left(-6 C_1^2 C_2 y - \frac{15 C_1^2 C_2 + 4 C_5}{N_1}\right) N_1\right.
\]

\[
+2 e^{\gamma N_i} \left(-6 C_1^2 C_2 y + \frac{15 C_1^2 C_2 + 4 C_6}{N_1}\right) N_1
\]

\[
+2 e^{\gamma N_i} \left(-6 C_1^2 C_2 y + \frac{15 C_1^2 C_2 + 4 C_6}{N_1}\right) N_1
\]
\[
\frac{dp_1}{dx} = \frac{3(C_1 e^{-\gamma y_i} N_i^2 + C_2 e^{\gamma y_i} N_i^2)^2}{N_i^6} \left(-C_1 e^{-\gamma y_i} N_i^3 + C_2 e^{\gamma y_i} N_i^3\right) \\
+ \frac{1}{8N_i} (27C_1 e^{-\gamma y_i} N_i^2 + 36C_1 C_2 e^{-\gamma y_i} N_i^2 - 36C_1 C_2 e^{\gamma y_i} N_i^2 - 27C_2^3 e^{3\gamma y_i} N_i^2) \\
+ 2C_1 e^{-\gamma y_i} \left(-2C_1 C_2 y + \frac{15C_1^2 C_2 + 14C_1}{N_i}\right) N_i^3 + 2C_2 e^{\gamma y_i} \left(-6C_1 C_2 y + \frac{15C_1^2 C_2 + 4C_6}{N_i}\right) N_i^3 \\
- (C_8 + \frac{1}{8N_i} (3C_1 e^{-\gamma y_i} + 12C_1 C_2 e^{-\gamma y_i} - 12C_1 C_2 e^{\gamma y_i} - 3C_2^3 e^{3\gamma y_i}) \\
+ 2C_1 e^{-\gamma y_i} \left(-6C_1 C_2 y - \frac{15C_1^2 C_2 + 14C_1}{N_i}\right) N_i^3 + 2C_2 e^{\gamma y_i} \left(-6C_1 C_2 y + \frac{15C_1^2 C_2 + 4C_6}{N_i}\right) N_i^3) N_i^2 \\
\theta_i = d_3 + y d_4 + \frac{1}{16\gamma^2 N_i} Br(9C_1 e^{-\gamma y_i} (\gamma^2 + 3N_i^2) + \frac{9C_2 e^{\gamma y_i} (\gamma^2 + 3N_i^2)}{N_i^4}) \\
- 8y N_i ((3C_1 C_2^2 + 2C_2 C_5 + 2C_2 C_6) y^2 + (3C_1 C_2^2 - 2C_2 C_4 - 2C_1 C_6) N_i^2) \\
+ 2C_1 e^{-\gamma y_i} (-6C_1 C_2 y (\gamma^2 + N_i^2) - \frac{9C_1^2 C_2 y^2 + 8C_5 y^2 + 33C_1 C_2 N_i^2 + 8C_5 N_i^2}{N_i}) \\
+ 2C_2 e^{\gamma y_i} \left(6C_1 C_2^3 y (\gamma^2 + N_i^2) - \frac{C_2 (9C_1 C_2^2 y^2 + 8C_5 y^2 + 33C_1 C_2 N_i^2 + 8C_5 N_i^2)}{2N_i}\right) \\
\text{The heat transfer coefficient at this order is given by} \\
Z_{th} = \frac{\partial h_i}{\partial x} \frac{\partial \theta_i}{\partial y} (y = 0) = \left[-m - 2a \pi \cos (2\pi x + \phi)\right] (d_4 + \frac{1}{16\gamma^2 N_i} Br(-12C_1 C_2 e^{-\gamma y_i}) \\
\left(\gamma^2 + N_i^2\right) + 12C_1 C_2 e^{\gamma y_i} (\gamma^2 + N_i^2) - 9C_1 e^{-\gamma y_i} (\gamma^2 + 3N_i^2) + 9C_2 e^{\gamma y_i} (\gamma^2 + 3N_i^2) \\
- 16y N_i ((3C_1 C_2^2 + 2C_2 C_5 + 2C_2 C_6) y^2 + (3C_1 C_2^2 - 2C_2 C_4 - 2C_1 C_6) N_i^2) \\
- 4C_1 e^{-\gamma y_i} \left(-6C_1 C_2 y (\gamma^2 + N_i^2) - \frac{9C_1^2 C_2 y^2 + 8C_5 y^2 + 33C_1 C_2 N_i^2 + 8C_5 N_i^2}{N_i}\right) \\
+ 4C_2 e^{\gamma y_i} N_i \left(6C_1 C_2^3 y (\gamma^2 + N_i^2) - \frac{C_2 (9C_1 C_2^2 y^2 + 8C_5 y^2 + 33C_1 C_2 N_i^2 + 8C_5 N_i^2)}{2N_i}\right) \\
The non-dimensional pressure rise \((\Delta p_1)\) a per wavelength is given as follows:
\[
\Delta p_1 = \int_0^1 \frac{\partial p_1}{\partial x} \bigg|_{y=0} \, dx \\
\text{9. Nusselt number}
\text{The non-dimensionless Nusselt number is obtained by the relation } Nu = -\frac{\partial \theta}{\partial y}, \text{this leads}
\]
$\frac{Nu}{d^2} = -d^2 - \frac{1}{2\gamma N_i^2} (BrC_1^2 e^{2y N_i} (\gamma^2 + N_i^2) - BrC_2^2 e^{2y N_i} (\gamma^2 + N_i^2))$

$-2\gamma N_i (\gamma^2 (2BrC_1 + \beta) - 2BrC_2 N_i^2)) - \frac{1}{16\gamma N_i} Br (-12C_1^4 e^{2y N_i})$ 

$(\gamma^2 + N_i^2) + 12C_1 e^{2y N_i} (\gamma^2 + N_i^2) - 9C_1^4 e^{-4y N_i} (\gamma^2 + 3N_i^2) + 9C_2^2 e^{-2y N_i}$

$(\gamma^2 + 3N_i^2) - 16\gamma N_i ((3C_1^2 C_2^2 + 2C_2 C_5 + 2C_6) \gamma^2 + (3C_1^2 C_2^2 - 2C_2 C_5 + 2C_1 C_6) N_i^2)$

$-4C_1 e^{-2y N_i} N_i \left( -6C_1 C_2 y (\gamma^2 + N_i^2) - \frac{9C_1^2 C_2^2 \gamma^2 + 8C_2 \gamma^2 + 33C_1^2 C_2 N_i^2 + 8C_2 N_i^2}{2N_i} \right)$

$+ 4 e^{2y N_i} N_i \left( 6C_1 C_2^3 y (\gamma^2 + N_i^2) - \frac{C_2 (9C_1 C_2^2 \gamma^2 + 8C_6 \gamma^2 + 33C_1^2 C_2 N_i^2 + 8C_2 N_i^2)}{2N_i} \right)$

10. Numerical results and discussion

In this section, we will be discussing the behavior and graphical description of various parameters like "Hartmann number" ($M$), "Darcy number" ($K$), "couple stress" parameter ($\gamma$), "heat generation parameter" ($\beta$), "inclination of magnetic field" ($\Theta$), "Brinkman number" ($Br$), "Reynolds number" ($Re$), "Froude number" ($Fr$), "Prandle number" ($Pr$), "Eckert number" ($Ec$), "mean flow rate" ($Q$) and "incline angle" of the channel ($\alpha$) and their effect on pressure gradient, temperature, "Nusselt number", heat transfer coefficient, pressure rise and the behavior of variously wave forms, and discussed in the form of streamlines. And has been using Mathematica program to find the results numerically and graphically.

10.1. The Pressure gradient distribution

In this section, the Figs.(2),(3),(6) and (7) Turns out that the pressure gradient ($dy/dx$) increases with the increase in the ($Re$), ($Q$) ($K$) and ($\phi$). While the behavior is reflected where the gradient pressure ($dy/dx$) is decreasing with an increase in ($Fr$), ($M$) As shown in the Figs.(4) and (5).

![Figure 2](image-url)  
**Figure 2.** Different of pressure gradient with variation of $Re$  

![Figure 3](image-url)  
**Figure 3.** Different of pressure gradient with variation of $Q$
10.2. Pumping Characteristics

The Figs.(8)-(13) are observed the behavior of average pressure rise ($\Delta p$) against mean flow rate ($Q$) for different flow parameters. We partition the region into five parts as follows: (I) Peristaltic pumping area ($\Delta p > 0, Q > 0$). In this area, the positive value of ($Q$) is wholly due to the peristalsis, and overcoming the pressure rise. (II) Augmented pumping region when ($\Delta p < 0, Q > 0$). In this region, a negative pressure difference assists the flow due to the peristalsis of the walls. (III) ($\Delta p < 0, Q < 0$) is the region where there is no flow. (IV) The region ($\Delta p > 0, Q < 0$) is a backward pumping region. (V) If $\Delta p = 0$ then this region is corresponds to the free pumping region. This flow occurs due to peristalsis in the walls. It is seen in Fig.(8), the increase of ($K$) leads to that ($\Delta p$) increasing in a co-pumping region ($\Delta p < 0, Q > 0$), but the case is reverse in the peristaltic pumping region($\Delta p > 0, Q > 0$), which ($\Delta p$) is a decreased with an increase ($K$). Fig. (9) shows that is inverse to what happened in Fig. (8) completely, where ($\Delta p$) decreases in a co-pumping region ($\Delta p < 0, Q > 0$) and ($\Delta p$) is an enhance in the peristaltic pumping region ($\Delta p > 0, Q > 0$) with increase of ($M$). The Figs.(10), (12) and (13) demonstrates the effect of ($Re, \alpha, \xi$) on ($\Delta p$). It is noted that ($\Delta p$) is increasing in all the regions mentioned above respectively. And we note the opposite in Fig. (11) shows an influence of ($Fr$) on average "pressure rise" ($\Delta p$). We notice from the graph the pumping rate ($\Delta p$) is a decreasing with increasing of ($Fr$) in all the regions mentioned.
10.3. Velocity Profile
The graphs are sketched in Figs.(14)-(19) at the fixed values of $x = 0.3$ and $t = 5$. The aim of this part is to discuss the influence of the non-uniform parameter ($m$), Hartmann number ($M$), the inclination angle of the magnetic field ($\Theta$), phase angle ($\phi$), the pseudoplastic fluid parameter ($\xi$) and mean flow rate ($Q$) on the velocity ($u$). It is noted that from figures, the velocity profile traces a parabola form trajectory with maximum value occurring in mid of the channel. From the Fig.(14), the velocity
increases with increase in \((m)\). Fig. (15) shows that the velocity decreases near the mid of the channel with an increase in \((M)\). When observing Fig. (16) it is seen that the behavior of \((\Theta)\) on the velocity profile is completely reverse compared to the case in \((M)\). Furthermore, it can be observed that the velocity decreases with enhance in \((\Phi)\). The velocity drop in the middle of the channel and increases near the walls with an increase of parameter \((\zeta)\) (see Fig. (18)). This confirms that the velocity decreases in the middle of the channel with a porous medium in it, so the more porous in a porous medium will be less resistance to the flow of fluid, this makes a velocity is the greater extent. We observed from Fig. (19) the velocity profile for the parameter \((Q)\) which depicts that the velocity \((u)\) enhances with an increase in \((Q)\).

Figure 14. Velocity \(u\) versus \(y\)-direction for different values \(m\)

Figure 15. Velocity \(u\) versus \(y\)-direction for different values \(M\)

Figure 16. Velocity \(u\) versus \(y\)-direction for different values \(\Theta\)

Figure 17. Velocity \(u\) versus \(y\)-direction for different values \(\Phi\)
10.4. Temperature distribution

This subsection devotes the effect of Hartmann number \( (M) \), inclination of the magnetic field \( (\Theta) \), Darcy number \( (K) \), heat generation parameter \( (\beta) \), Brinkman number \( (Br) \) and couple stress fluid parameter \( (\gamma) \) on the temperature of the fluid under the influence of peristalsis is appears in Figs.(20)-(25) for the fixed values of \( x = 0.3 \) and \( t = 5 \), which are depicts in the middle part of the channel. It is noticed from Fig.(20), the temperature increases with an increase of \( (M) \). Also, the temperature of the fluid is increased with an increase of \( (\beta) \) (see Fig.(21)). We see for Fig.(22), the temperature decreasing with increase in \( (K) \) in the center part of the channel and, while opposite behavior is observed near channel boundaries. The temperature can be higher when there is a porous medium. This applies to the rest of the parameters \( (\Theta) \) and \( (\gamma) \), where it is noticed from Figs.(23) and (24), the temperature decreases with increases of the inclination of the magnetic field \( (\Theta) \) and couple stress parameter \( (\gamma) \). It is observed from Fig.(25) that, the temperature increases with increasing of \( (Br) \) in core (center) channel. This case came because an increase in values of \( (Br) \) rises the resistance offered by shear inflow which in turn increases the heat generation due to the viscous dissipation effects and hence the temperature of the fluid increases, but the opposite near the boundary of the channel.
10.5. Heat transfer coefficient
This section devotes the effect of "Hartmann number" ($M$), heat generation parameter ($\beta$), "Brinkman number" ($Br$) and "couple stress" fluid parameter ($\gamma$) on the "heat transfer coefficient" $Z_{h1}$ at the wall. The visualization of heat transfer coefficient at the upper wall under the influence of the peristalsis in a channel. It is shown from the figures (26),(27),(28) that, heat transfer coefficient $Z_{h1}$ is wobbling. This is due to spread of sinusoidal waves along the channel walls, the heat transfer coefficient $Z_{h1}$ increases with the increase of ($Br$), ($M$), ($\beta$) respectively. It is noticed from Fig.(29), the absolute value of heat transfer coefficient $Z_{h1}$ decreases with an increase of ($\gamma$).

Figure 24. Temperature distribution $\theta$ for different values $\gamma$

Figure 25. Temperature distribution $\theta$ for different values $Br$

Figure 26. Heat transfer coefficient $Z_{h1}$ for different values of $M$

Figure 27. Heat transfer coefficient $Z_{h1}$ for different values of $M$
10.6. Nusselt number

In this paragraph the Figs. (30)-(35) denotes the effect of Hartmann number ($M$), the inclination of the magnetic field ($\Theta$), Darcy number ($K$), mean flow rate ($Q$), Brinkman number ($Br$) and couple stress fluid parameter ($\gamma$) on the "Nusselt number". It is noticed of a Figs.(30)-(33), the "Nusselt number" grow with enhance of ($Br$), ($K$), ($\Theta$) and ($Q$) respectively. While in Figs.(34) and (35) the case is inverse, with an enhancing of ($\gamma$) and ($M$) the "Nusselt number" ($Nu$) is decreasing.
11. Trapping phenomena

The trapping is a phenomenon interesting in peristaltic motion, where they are considered from a physical phenomenon which may be responsible for thrombus formation in blood and the movement of food bolus in the gastrointestinal tract. In this paper, we study trapping phenomenon for various values parameters, and a walls is peristaltic and the flow is no-slip, this trapped bolus is reinforcing along with the peristaltic wave. we observed that a bolus also in the fixed frame which may likely to be due the impression of non-zero time-average of the flow over one period of the wave. The trapping for different values of \((m, Q, \xi, M\) and \(K\)) are shown in Figs.(36)-(40) at fixed value of \(t = 0.5\). In this figures, we note that the significant variation and the strong impact of occurs bolus near on walls with high deformations. The Fig.(36) shows the effect non-uniform parameter \((m)\) on the trapping, when is examined we found the size of the trapped bolus is decreasing with an enhance in \((m)\). And the effect of the "mean flow rate" \((Q)\) on the trapping is observed of streamlines (see Fig.(37)). We found that the trapping bolus increase in size with enhancing of values of \((Q)\). The Fig.(38) we observe impact pseudoplastic fluid parameter \((\xi)\) on the trapping (As shown in the drawing streamlines) with increasing value of \((\xi)\) the size of the trapped is increasing. As for the variance values of the "Hartmann number" \((M)\) in the streamlines, The increase in the values of \((M)\), leads to an increase in the size of the trapping bolus (see Fig.(39)). It is noticed from Fig.(40) that, the trapping bolus decreases with an increase of the Darcy number \((K)\) in asymmetric channels.
Figure 37. Streamlines for $b = 0.3, \phi = \pi / 2, \theta = \pi / 3, a = 0.1, \xi = 0.05, m = 0.3, M = 1, K = 0.2, t = 0.5$ and for different $Q$:
(a) $Q = 0.5$ (b) $Q = 1$

Figure 38. Streamlines for $b = 0.3, \phi = \pi / 2, \theta = \pi / 3, a = 0.1, m = 0.3, Q = 0.5, M = 1, K = 0.2, t = 0.5$ and for different $\xi$
(a) $\xi = 0.02$ (b) $\xi = 0.04$.

Figure 39. Streamlines for $b = 0.3, \phi = \pi / 2, \theta = \pi / 3, a = 0.1, \xi = 0.05, m = 0.3, Q = 0.5, K = 0.2, t = 0.5$ and for different $M$:
(a) $M = 0.5$, (b) $M = 2$.

Figure 40. Streamlines for $b = 0.3, \phi = \pi / 4, \theta = \pi / 3, a = 0.1, \xi = 0.05, m = 0.2, Q = 0.5, M = 1, t = 0.5$ and for different $K$:
(a) $K = 0.1$ (b) $K = 0.3$. 
12. Conclusions
In this paper, we studied a mathematical model of the peristaltic transport of a pseudoplastic nanofluid in the tapered asymmetric channel with the effect of magnetic field and porous medium, and for heat effect with peristaltic transport through slanted tubes for the non-uniform walls. The effects of Hartmann number \((M)\), Darcy number \((K)\), couple stress parameter \((\gamma)\), heat generation parameter \((\beta)\), inclination of magnetic field \((\Theta)\), Brinkman number \((Br)\), Reynolds number \((Re)\), Froude number \((Fr)\), Prandtl number \((Pr)\), Eckert number \((Ec)\), mean flow rate \((Q)\) and incline angle of the channel \((\alpha)\).

We discussed the main findings as follows:

- The pressure gradient increases with an increase in \((Re)\), \((Q)\), \((K)\) and \((\phi)\) while it decreases for increasing of \((Fr)\), \((M)\).
- It has been found that the average rise in pressure \((\Delta p)\) increasing with the increase of \((K)\), \((M)\), \((Re)\) and \((\tilde{\zeta})\) while it decreases by increasing \((Fr)\) and \((b)\).
- The velocity profile increases with an increase \((m)\), \((\Theta)\) and \((Q)\) while it decreases with increasing \((M)\), and \((\phi)\).
- The axial velocity decreases with an increase in \((\tilde{\zeta})\) at the core part of the channel. However, opposite behavior on near channel boundaries.
- The temperature distribution is increased with an increase \((\beta)\) and \((M)\).
- The temperature decreases with an increase in \((K)\), \((\Theta)\) and \((\gamma)\) at the core part of the channel. However, opposite behavior on near channel boundaries.
- Heat transfer coefficient is a decreasing function with an increase in \((Br),(M)\) and \((\beta)\) while it enhances by increasing \((\gamma)\).
- The Nusselt number increases with increasing of \((Br)\), \((K)\), \((\Theta)\) and \((Q)\). While \((Nu)\) is decreasing with the increase of \((\gamma)\) and \((M)\).

If there is no-slip flow then the trapping bolus decreases with increasing of parameters \((m)\) and \((K)\) while it has a reverse behavior with increasing of \((Q)\), \((\tilde{\zeta})\) and \((M)\).

References
[1] S. Rashidi, M. Dehghan, R. Ellahi, M. Riaz, M.T. Jamal-Abad, Study of stream wise transverse magnetic fluid flow with heat transfer around an obstacle embedded in a porous medium, J. Magn. Magn. Mater. 378 (2015) 128–137.
[2] A. Zeeshan, R. Ellahi, M. Hassan, Magnetohydrodynam-ic flow of water/ethylene glycol based nano fluids with natural convection through a porous medium, Eur. Phys. J. Plus 129 (12) (2014) 1–10.
[3] M. Sheikholeslami, R. Ellahi, H.R. Ashorynejad, G. Domairry, T. Hayat, Effects of heat transfer in flow of nano fluids over a permeable stretching wall in a porous medium, J. Comput. Theor. Nanosci. 11 (2) (2014) 486 –496.
[4] R. Ellahi, X. Wang, M. Hameed, Effects of heat transfer and nonlinear slip on the steady flow of Couette fluid by means of Chebyshev spectral method, Z.Naturforsch. A 69(1–2) (2014) 1–8.
[5] R. Ellahi, E. Shivanian, S. Abbasbandy, T. Hayat, Analysis of some magnetohydrodynamic flows of third order fluid saturating porous space, J. Porous Med. 18 (2) (2015) 89 –98.
[6] V.K. Stokes, Couple stresses in fluids, Phys.Fluids 9 (9) (1966) 1709–1715.
[7] V.K. Stokes, Theories of Fluids with Microstructure, Springer, New York, 1984.
[8] T. Hayat, M. Mustafa, Z. Iqbal, A. Alsaedi, Stagnation point flow of couple stress fluid with melting heat transfer, Appl. Math. Mech. 34 (2) (2013) 167–176.
[9] N.S.Akbar, S.Nadeem, Intestinal flow of a couple stress nano fluid in arteries, IEEE Trans. Nanobioscience 12 (4) (2013) 332–339.

[10] D. Srinivasacharya, N. Srinivasacharyulu, O. Odelu, Flow and heat transfer of couple stress fluid in a porous channel with expanding and contracting walls, Int. Commun. Heat Mass Transfer 36 (2009)180–185.

[11] S. Nadeem, A. Riaz, R. Ellahi, Peristaltic flow of viscous fluid in a rectangular duct with compliant walls, Comput. Math. Model. 25 (3) (2014) 404 – 415.270 K. Ramesh / Journal .

[12] A. Riaz, R. Ellahi, S. Nadeem, Peristaltic transport of a Carreau fluid in a compliant rectangular duct, Alex. Eng. J. 53 (2) (2014) 475–484.

[13] M. Mishra, A.R. Rao, Peristaltic transport of a Newt-onian fluid in an asymmetric channel, J. Appl. Math. Phys. 54 (2003) 532–550.

[14] N. Ali, T. Hayat, Peristaltic flow of a micropolar fluid in an asymmetric channel, Comput. Math. Appl. 55 (2008) 589–608.

[15] P. Naga Rani, G. Sarojamma, Peristaltic transport of a Casson fluid in an asymmetric channel, Australas. Phys. Eng. Sci. Med. 27 (2) (2004) 49–59.

[16] T. Hayat, A. Alfsar, N. Ali, Peristaltic transport of a Johnson-Segalman fluid in an asymmetric channel, Math. Comput. Model. 47(2008) 380–400.

[17] T. Hayat, M. Javed, Exact solution to peristaltic transport of power-law fluid in asymmetric channel with compliant walls, Appl. Math. Mech. 31(10) (2010) 1231–1240.

[18] S. Nadeem, S. Akram, Peristaltic flow of a couple stress fluid under the effect of induced magnetic field in an asymmetric channel, Arch. Appl. Mech. 81 (2011) 97–109.

[19] S. Nadeem, S. Akram, Influence of inclined magnetic field on peristaltic flow of a Williamson fluid model in an inclined symmetric or asymmetric channel, Math. Comput. Model. 52 (2010) 107–119.

[20] M. Saleem, A. Haider, Heat and mass transfer on the peristaltic transport of non-Newtonian fluid with creeping flow, Int. J. Heat Mass Transf. 68 (2014) 514 –526.

[21] T. Hayat, H. Yasmin, B. Ahmad, B. Chen, Simultaneous effects of convective conditions and nanoparticles on peristaltic motion, J. Mol. Liq. 193 (2014) 74–82.

[22] M.R.Salman, A.M.Abdulhadi, Effects of MHD on Peristalsis Transport and Heat Transfer with Variables Viscosity in Porous Medium, Int. J. of Sci. and Res. (IJSR) Volume 7 Issue 2, February (2018) pages 612-623.

[23] S. Nadeem, N.S. Akbar, Effects of heat and chemical reactions on peristaltic flow of Newtonian fluid in a diverging tube with inclined MHD, Asia Pac. J. Chem. Eng. 6 (2011) 659 –668.

[24] S. Nadeem, S. Akram, Influence of inclined magnetic field on peristaltic flow of a Jeffrey fluid with heat and mass transfer in an inclined symmetric or asymmetric channel, Asia Pac. J. Chem. Eng. 7 (2012) 33 – 44.

[25] K.Ramesh, Influence of heat and mass transfer on peristaltic flow of a couple stress fluid through porous medium in the presence of inclined magnetic field in an inclined asymmetric channel, J. Mol. Liq. 219 (2016) 256-271

[26] T. Hayat, S. Noreen, M.S. Alhothuali, S. Asghar, A. Alhomaidan, Peristaltic flow under the effects of an induced magnetic field and heat and mass transfer, Int. J. Heat Mass Transf. 55 (2012) 443–452.

[27] N.S. Akbar, T. Hayat, S. Nadeem, S. Obaidat, Peristaltic flow of a Williamson fluid in an inclined asymmetric channel with partial slip and heat transfer, Int. J. Heat Mass Transf. 55 (2012) 1855 – 1862.