FCC-he sensitivity estimates on the anomalous electromagnetic dipole moments of the top-quark

M. A. Hernández-Ruiz*, 1 A. Gutiérrez-Rodríguez†, 2 M. Köksal‡, 3 and A. A. Billur§ 4

1Unidad Académica de Ciencias Químicas, Universidad Autónoma de Zacatecas
Apartado Postal C-585, 98060 Zacatecas, México.
2Facultad de Física, Universidad Autónoma de Zacatecas
Apartado Postal C-580, 98060 Zacatecas, México.
3Department of Optical Engineering, Sivas Cumhuriyet University, 58140, Sivas, Turkey.
4Department of Physics, Sivas Cumhuriyet University, 58140, Sivas, Turkey.

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Abstract

In this paper, we study the production of a top-quark in association with a bottom-quark and an electron-neutrino at the Future Circular Collider Hadron Electron (FCC-he) to probe the sensitivity on its magnetic moment (\(\hat{a}_V\)) and its electromagnetic dipole moment (\(\hat{a}_A\)) through the process \(e^- p \rightarrow e^- \gamma p \rightarrow \bar{t} \nu_e b p\). Assuming a large amount of collisions, as well as of data with cleaner environments, the FCC-he is an excellent option to study new physics, such as the \(\hat{a}_V\) and \(\hat{a}_A\). For our sensitivity study on \(\hat{a}_V\) and \(\hat{a}_A\), we consider center-of-mass energies \(\sqrt{s} = 7, 10 \text{ TeV}\) and luminosities \(\mathcal{L} = 50, 100, 300, 500, 1000 \text{ fb}^{-1}\). In addition, we apply systematic uncertainties \(\delta_{\text{sys}} = 0\%, 3\%, 5\%\) and we consider unpolarized and polarized electron beam. Our results show that the FCC-he is a very good perspective to probe the \(\hat{a}_V\) and \(\hat{a}_A\) at high-energy and high-luminosity frontier.

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* mahernau@uaz.edu.mx
† alexgu@fisica.uaz.edu.mx
‡ mkoksal@cumhuriyet.edu.tr
§ abillur@cumhuriyet.edu.tr
Electron-proton ($e^-p$) colliders have been and they continue to be considered as ideal machine to probe physics beyond the Standard Model (BSM). These $e^-p$ colliders, such as the Future Circular Collider Hadron Electron (FCC-he) \cite{1-6} develops options for potential high-energy frontier circular colliders at the post Large Hadron Collider (LHC) era. Such colliders will open up new perspective in the field of fundamental physics, especially for the particle physics. Many potential features in favor of this type of electron-proton colliders are the following: 1) Provides a cleaner environment compared to the $pp$ colliders and higher center-of-mass energies that to the $e^+e^-$ ones. The center-of-mass energies are much higher than that of the future International Linear Collider (ILC) and the Compact Linear Collider (CLIC). 2) Connection $e^-p$ and $pp$ physics. 3) Provides a cleaner environment with suppressed backgrounds from strong interactions. 4) $e^-p$ when added to $pp$ turns the $pp$ colliders into high precision Higgs and top-quark facilities. Removes the Parton Distribution Function (PDF) and coupling constant uncertainties in $pp$, $gg$ fusion processes. 5) This collider could also provides additional and sometimes unique ways for studying the Higgs boson and top-quark physics, as well as the exploration of Electroweak Symmetry Breaking (EWSB) phenomena, with unmatchable precision and sensitivity. 6) Statistics enhanced by several orders of magnitude for BMS phenomena brought to light by the LHC. 7) Benefit from both direct (large $Q^2$) and indirect (precision) probes. 8) Provides solid answers to open questions of the Standard Model (SM) like: hierarchy problem, prevalence of matter over antimatter, the neutrino masses, dark matter and dark energy. In summary, $e^-p + pp$ deliver high precision of Higgs boson, top-quark and QCD and electroweak physics complementary to $e^+e^-$. Furthermore, $e^-p$ is an stimulating, realistic option for a next energy frontier collider for particle physics. For a comprehensive study on the physics and detector design concepts we refer the readers to Refs. \cite{1-6}.

Next the detection of the top-quark, there has been an enormous motivation to investigate the properties and the potential of top-quark in great detail both in production and in decay. Increasingly sophisticated experimental results of the current colliders are complemented by very precise theoretical predictions in the framework of the SM of particle physics and beyond. Specifically, the anomalous coupling $tt\gamma$, which is the subject of this paper, have been studied in detail in hadron colliders and at a future high-luminosity high-energetic linear
electron-positron colliders, for a reviews exhaustive see Table I of Ref. \cite{7} and references therein \cite{8,18}.

The SM prediction for the Magnetic Moment (MM) and the Electric Dipole Moment (EDM) of the top-quark, that is $a_t$ \cite{19} and $d_t$ \cite{20,22} reads:

$$a_t = 0.02, \quad d_t < 10^{-30}(ecm). \quad (1)$$

The $a_t$ can be tested in the current and future colliders such as the LHC, CLIC, the Large Hadron-Electron Collider (LHeC) and the FCC-he. In the case of the $d_t$, its value is strongly suppressed as shown in Eq. (1), and is much too hard to be observed. However, it is very attractive for probing new physics BSM. Furthermore, it is considered as a source of CP violation.

With everything already mentioned, the FCC-he potential of $e^-p$ collisions at $\sqrt{s} = 7.07 TeV$ and $10 TeV$ and high luminosities $\mathcal{L} = 50 - 1000 fb^{-1}$, offer one of the best opportunities to test and improve our understanding of the top-quark physics. In special their MM and EDM, and as we already mentioned with the latter considered as a source of CP violation. CP violation can explain why there is more matter than antimatter in the universe, which is a topic of great relevance between the scientific community of particles and fields.

The MM and EDM of the top-quark, that is $\hat{a}_V$ and $\hat{a}_A$ can be probed in high-energy electron-proton collisions through the neutral current top-quark production, there are mainly two modes, i) Deep Inelastic Scattering (DIS) and ii) Photoproduction. Single top-quark and top-quark pair production is possible by both mechanisms.

In this paper we study in a model-independent way the dipole moments of the top-quark through the process of single top-quark production $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e b p$. Fig. 1 shows the schematic diagram for the process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e b p$, while, the Feynman diagrams contributing to the reaction $e^-\gamma \rightarrow \bar{t}\nu_e b$ they are shown in Fig. 2.

The rest of the paper is organized as follows. In Section II, we introduce the top-quark effective electromagnetic interactions. In Section III, we sensitivity estimates on top-quark anomalous electromagnetic couplings through $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e b p$ collisions. Finally, we present our conclusions in Section IV.
II. SINGLE TOP-QUARK PRODUCTION VIA THE PROCESS $e^{-} p \rightarrow e^{-} \gamma p \rightarrow \bar{t} \nu_{e} b p$

A. Effective interaction of $t \bar{t} \gamma$

Due to the absence so far of any signal of new heavy particles decaying into top-quark, an attractive approach for describing possible new physics effects in a model independent way is based on effective Lagrangian. In this approach, all the heavy degrees of freedom are integrated out leading to obtain the effective interactions between the SM particles. This is justified because the related observables have not shown any significant deviation from the SM predictions so far. From the effective Lagrangian approach, potential deviations of its value from the SM for the anomalous $t \bar{t} \gamma$ coupling are described of the effective Lagrangian given by

$$L_{\text{eff}} = L_{\text{SM}} + \sum_{n} \frac{c_n}{\Lambda^2} \mathcal{O}_n^{(6)} + h.c.$$  \hspace{0.5cm} (2)

In Eq. (2), $L_{\text{eff}}$ is the effective Lagrangian gauge-invariant which contains a series of dimension-six operators built with the SM fields, $L_{\text{SM}}$ is the renormalizable SM Lagrangian, $\Lambda$ is the scale at which new physics expected to be observed, $c_n$ are dimensionless coefficients and $\mathcal{O}_n^{(6)}$ represents the dimension-six gauge-invariant operator.

We write the most general effective vertex of $t \bar{t} \gamma$ \cite{9, 12, 13, 23, 24} as:

$$L_{t \bar{t} \gamma} = -g_{e} Q_{t} \bar{t} \Gamma^{\mu}_{t \bar{t} \gamma} t A_{\mu},$$ \hspace{0.5cm} (3)

this equation includes the SM coupling and contributions from dimension-six effective operators. In addition, $g_{e}$ is the electromagnetic coupling constant, $Q_{t}$ is the top-quark electric charge and the Lorentz-invariant vertex function $\Gamma^{\mu}_{t \bar{t} \gamma}$ is given by

$$\Gamma^{\mu}_{t \bar{t} \gamma} = \gamma^{\mu} + \frac{i}{2m_{t}} (\hat{a}_{V} + i \hat{a}_{A} \gamma_{5}) \sigma^{\mu\nu} q_{\nu}.$$ \hspace{0.5cm} (4)

Here $m_{t}$ is the mass of the top-quark, $q$ is the momentum transfer to the photon and the couplings $\hat{a}_{V}$ and $\hat{a}_{A}$ are real and related to the anomalous magnetic moment ($a_{t}$) and the electric dipole moment ($d_{t}$) of the top-quark, respectively. The relations between $\hat{a}_{V}(\hat{a}_{A})$ and $a_{t}(d_{t})$ are given by
\[ \hat{a}_V = Q_t a_t, \]  
\[ \hat{a}_A = \frac{2 m_t}{e} d_t. \]  

The operators contribute to top-quark electromagnetic anomalous couplings [25–27] are

\[ \mathcal{O}_{uW}^{33} = \bar{q}_L \sigma^{\mu\nu} t_R \tilde{\phi} W^a_{\mu\nu} + h.c, \]  
\[ \mathcal{O}_{uB\phi}^{33} = \bar{q}_L \sigma^{\mu\nu} t_R \tilde{\phi} B_{\mu\nu} + h.c, \]  

where \( \bar{q}_L \) is the quark field, \( \sigma^{\mu\nu} \) are the Pauli matrices and \( \tilde{\phi} \) is the Higgs doublet, while \( W^a_{\mu\nu} \) and \( B_{\mu\nu} \) are the \( U(1)_Y \) and \( SU(2)_L \) gauge field strength tensors, respectively.

From the parametrization given by Eq. (3), and from the operators of dimension-six given in Eqs. (7) and (8) we obtain the corresponding CP even \( \hat{a}_V \) and CP odd \( \hat{a}_A \) observables:

\[ \hat{a}_V = \frac{2 m_t}{e} \sqrt{2} \frac{\bar{v}}{\Lambda^2} \text{Re}\left[ \cos \theta_W C_{u\phi}^{33} + \sin \theta_W C_{uW}^{33} \right], \]  
\[ \hat{a}_A = \frac{2 m_t}{e} \sqrt{2} \frac{\bar{v}}{\Lambda^2} \text{Im}\left[ \cos \theta_W C_{u\phi}^{33} + \sin \theta_W C_{uW}^{33} \right]. \]

These observables contain \( v = 246 \text{ GeV} \), the breaking scale of the electroweak symmetry and \( \sin \theta_W(\cos \theta_W) \), the sine(cosine) of the weak mixing angle.

**B. Cross-section of \( e^- p \to e^- \gamma p \to \bar{t} \nu_e b p \)**

The FCC-he will be designed to operate in \( e^- p \) collision mode, but it can also be operated as a \( e^- \gamma, \gamma p \) and \( \gamma \gamma \) collider. A promising mechanism to generate energetic photon beams in a FCC-he is Equivalent Photon Approximation (EPA) [28–30] using the Weizsacker-Williams Approximation. In EPA, photons emitted from incoming hadrons (leptons) which have very low virtuality are scattered at very small angles from the beam pipe. The emitted quasireal photons have a low \( Q^2 \) virtuality and are therefore almost real. In this paper our calculations are based on photon-electron fluxes through the subprocess \( e^- \gamma \to \bar{t} \nu_e b \) and the representative leading order Feynman diagrams are depicted in Fig. 2. Here we put in evidence the contribution of elastic process with an intact proton in the final state, as well
as the inelastic component for the leptonic final state. The spectrum of EPA photons which are emitted by proton is given by \[ f_\gamma(x) = \frac{\alpha}{\pi E_p} \{[1 - x][\varphi(\frac{Q_{\text{max}}^2}{Q_0^2}) - \varphi(\frac{Q_{\text{min}}^2}{Q_0^2})]\}, \quad (11) \]

where \( x = E_\gamma/E_p \) and \( Q_{\text{max}}^2 \) is the maximum virtuality of the photon. In our calculations, we use \( Q_{\text{max}}^2 = 2 \text{GeV}^2 \). The minimum value of the \( Q_{\text{min}}^2 \) is given by

\[ Q_{\text{min}}^2 = \frac{m_p^2 x^2}{1 - x}, \quad (12) \]

From Eq. (11), the function \( \varphi \) is the following

\[ \varphi(\theta) = (1 + ay) \left[ -\ln(1 + \frac{1}{\theta}) + \sum_{k=1}^{3} \frac{1}{k(1 + \theta)^k}\right] + \frac{y(1 - b)}{4\theta(1 + \theta)^3} \]

\[ + c(1 + \frac{y}{4}) \left[ \ln\left(\frac{1 - b + \theta}{1 + \theta}\right) + \sum_{k=1}^{3} \frac{b^k}{k(1 + \theta)^k}\right], \quad (13) \]

where explicitly \( y, a, b \) and \( c \) are as follows

\[ y = \frac{x_2^2}{(1 - x_2)}, \quad (14) \]

\[ a = 1 + \frac{\mu_p^2}{4} + 4m_p^2 \frac{Q_0^2}{Q_0^2} \approx 7.16, \quad (15) \]

\[ b = 1 - \frac{4m_p^2}{Q_0^2} \approx -3.96, \quad (16) \]

\[ c = \frac{\mu_p^2 - 1}{b^4} \approx 0.028. \quad (17) \]

Hence, the total cross-section of the scattering \( e^- p \to e^- \gamma p \to \bar{t}\nu_e p \) can be expressed as

\[ \sigma(e^- p \to e^- \gamma p \to \bar{t}\nu_e p) = \int f_\gamma(x) d\sigma(e^- \gamma \to \bar{t}\nu_e b) dx, \quad (18) \]
where $\sigma(e^-\gamma \rightarrow \bar{t}\nu_e b)$ is the cross-section of the scattering $e^-\gamma \rightarrow \bar{t}\nu_e b$ and $f_\gamma(x)$ is the spectrum of equivalent photons which is given in Eq. (11).

It is worth mentioning that, there are different ways to optimize the signal sensitivity $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e b p$ and reduce the background. This is possible if we apply cut-based optimization, in addition to considering polarized electron beam.

We base our results on the following kinematic acceptance cuts in order to optimize the significance of the signal over all the backgrounds:

\begin{align*}
\text{Cut-1: } p_T^b &> 20 \text{ GeV}, \\
\text{Cut-2: } |\eta^b| &< 2.5, \\
\text{Cut-3: } p_T^{\nu_e} &> 15 \text{ GeV}.
\end{align*}

(19)

In Eq. (19), $p_T^b$ is the transverse momentum of the final state bottom-quark, $\eta^b$ is the pseudorapidity and $p_T^{\nu_e}$ is the transverse momentum of the electron-neutrino. The outgoing particles are required to satisfy these isolation cuts.

An essential feature in the design of current and future colliders of high-energy physics, is the implementation of polarized particles beams. Most accelerators have been modified or are being designed with the possibility of using polarized particles sources, such as the FCC-he. The possibility of using polarized electron beams can constitute a strong advantage in searching for new physics \[32\]. Furthermore, the electron beam polarization may lead to a reduction of the measurement uncertainties, either by increasing the signal cross-section, therefore reducing the statistical uncertainty, or by suppressing important backgrounds. In summary, one another option at the FCC-he is to polarize the incoming beam, which could maximize the physics potential, both in the performance of precision tests and in revealing the properties of the new physics BSM.

The general formula for the total cross-section for an arbitrary degree of longitudinal $e^-$ and $e^+$ beams polarization is given by \[32\]

\begin{align*}
\sigma(P_{e^-}, P_{e^+}) = \frac{1}{4}[(1 + P_{e^-})(1 + P_{e^+})\sigma_{++} + (1 - P_{e^-})(1 - P_{e^+})\sigma_{--} \\
+ (1 + P_{e^-})(1 - P_{e^+})\sigma_{+-} + (1 - P_{e^-})(1 + P_{e^+})\sigma_{-+}],
\end{align*}

(20)

where $P_{e^-}(P_{e^+})$ is the polarization degree of the electron (positron) beam, while $\sigma_{-+}$ stands for the cross-section for completely left-handed polarized \(e^-\) beam $P_{e^-} = -1$ and completely
right-handed polarized $e^+$ beam $P_{e^+} = 1$, and other cross-sections $\sigma_{+-}$, $\sigma_{++}$ and $\sigma_{+-}$ are defined analogously.

The main anomalous electromagnetic couplings affecting top-quark physics that are of interest for our study are $\hat{a}_V$ and $\hat{a}_A$. We have calculated the dependencies of the $e^- p \rightarrow e^- \gamma p \rightarrow \bar{t} \nu_e b p$ production cross-sections for the FCC-he at 7.07 TeV and 10 TeV on $\hat{a}_V$ and $\hat{a}_A$ using CalcHEP [31]. Furthermore, for our study we consider unpolarized and polarized electron beam, as well as the basic acceptance cuts given in Eq. (19). Assuming only one anomalous coupling to be non-zero at at time, we obtain the following results for the total cross-section in terms of the dipole moments of the top-quark:

\begin{enumerate}
  \item Total cross-section for $\sqrt{s} = 7.07$ TeV and $P_{e^-} = 0\%$.
    \[ \sigma(\hat{a}_V) = \left[ (0.00199)\hat{a}_V^2 + (0.0000350)\hat{a}_V + 0.000522 \right] (pb), \]
    \[ \sigma(\hat{a}_A) = \left[ (0.00199)\hat{a}_A^2 + 0.000522 \right] (pb). \]

  \item Total cross-section for $\sqrt{s} = 10$ TeV and $P_{e^-} = 0\%$.
    \[ \sigma(\hat{a}_V) = \left[ (0.00499)\hat{a}_V^2 + (0.0000217)\hat{a}_V + 0.000777 \right] (pb), \]
    \[ \sigma(\hat{a}_A) = \left[ (0.00499)\hat{a}_A^2 + 0.000777 \right] (pb). \]

  \item Total cross-section for $\sqrt{s} = 7.07$ TeV and $P_{e^-} = -80\%$.
    \[ \sigma(\hat{a}_V) = \left[ (0.00358)\hat{a}_V^2 + (0.0000614)\hat{a}_V + 0.00094 \right] (pb), \]
    \[ \sigma(\hat{a}_A) = \left[ (0.00358)\hat{a}_A^2 + 0.00094 \right] (pb). \]

  \item Total cross-section for $\sqrt{s} = 10$ TeV and $P_{e^-} = -80\%$.
    \[ \sigma(\hat{a}_V) = \left[ (0.00898)\hat{a}_V^2 + (0.000037)\hat{a}_V + 0.0014 \right] (pb), \]
    \[ \sigma(\hat{a}_A) = \left[ (0.00898)\hat{a}_A^2 + 0.0014 \right] (pb). \]
\end{enumerate}

We see that the sensitivities on the total cross-section and on the coefficients of $\hat{a}_V$ and $\hat{a}_A$ increase with the centre-of-mass energy, as well as with the polarized electron beam,
confirming the expected competitive advantage of the high-energies attainable with the FCC-he.

We first present the total cross-section of the signal $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$ as a function of the $\hat{a}_V$ and $\hat{a}_A$ for the center-of-mass energies of the FCC-he, that is $\sqrt{s} = 7.07\, TeV$ and $\sqrt{s} = 10\, TeV$, as shown through Figs. 3-6. These results are obtained after applying the kinematic cuts given in Eq. (19) and with unpolarized electron beam $P_{e^-} = 0\%$. The results show a clear dependence of the total cross-section of the $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$ scattering with respect to $\hat{a}_V$ and $\hat{a}_A$, as well as with the center-of-mass energies of the FCC-he.

In the case of the cross-section of the photo-production process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$ after application of cuts given by Eq. (19) and with polarized electron beam $P_{e^-} = -80\%$, the total cross-section is about 1.8 times larger than that of the photo-production process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$ with unpolarized electron beam $P_{e^-} = 0\%$, as shown in Figs. 9-12.

Before continuing with our study, it is worth making a discussion about our results obtained in Figs. 3-6 and 9-12. While the theory predictions for $\hat{a}_V$ and $\hat{a}_A$ in Eqs. (5) and (6) as well as the total cross-section that contains the anomalous coupling $t\bar{t}\gamma$ have been made in different contexts (see Table I of Ref. [7]), the $\hat{a}_V$ and $\hat{a}_A$ have not been measured experimentally yet. Therefore, one only has the option of comparing the measured $\hat{a}_V$ and $\hat{a}_A$ and the total cross-section with the theoretical calculation of Refs. [11, 12]. The authors of Ref. [11] specifically measure $\sigma(\gamma e^- \rightarrow t\bar{t})$ with 10\% (18\%) error obtaining the following results for the MM and the EDM of the top-quark at the LHeC: $|\kappa| < 0.05(0.09)$ and $|\tilde{\kappa}| < 0.20(0.28)$. While in our case, with the process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$, we obtain: $\hat{a}_V = (-0.1480, 0.1438)$ and $\hat{a}_A = |0.1462|$ with $\sqrt{s} = 10\, TeV$, $\mathcal{L} = 1000\, fb^{-1}$, $\delta_{sys} = 5\%$, $P_{e^-} = 0\%$ and 95\% C.L.. With polarized electron beam $P_{e^-} = -80\%$, we obtain: $\hat{a}_V = (-0.1394, 0.1353)$ and $\hat{a}_A = |0.1374|$. Although the conditions for the study of both processes $\gamma e^- \rightarrow t\bar{t}$ and $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$ are different, our result are competitive with respect to the results reported in Ref. [11]. In addition, it should be noted that our results are for 95\% C.L., while those reported in Ref. [11] are for 90\% C.L.. On the other hand, from the comparison of our result using the process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$ at the FCC-he, with respect to the process $pp \rightarrow p\gamma^*\gamma^* p \rightarrow pt\bar{t}p$ at LHC, our results show a significant improvement. Furthermore, it is noteworthy that with our process the total cross-sections is a factor $\mathcal{O}(10^3)$ between $pp \rightarrow p\gamma^*\gamma^* p \rightarrow pt\bar{t}p$ and $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$, that is, our results project 3 orders of magnitude better than those reported in Ref. [12]. These projections
shows that the sensitivity on the anomalous couplings $\hat{a}_V$ and $\hat{a}_A$ can be improved at the FCC-he by a few orders of magnitude in comparison with the projections of the LHC.

III. MODEL-INDEPENDENT SENSITIVITY ESTIMATES ON THE $\hat{a}_V$ AND $\hat{a}_A$

To determine the sensitivity of the non-standard couplings, $\hat{a}_V$ and $\hat{a}_A$, Eqs. (9) and (10) we use the results from Section II, for the process $e^- p \rightarrow e^- \gamma p \rightarrow \bar{t}\nu_e bp$ at the FCC-he. We consider the center-of-mass energies $\sqrt{s} = 7.07, 10 TeV$ and luminosities $\mathcal{L} = 50, 100, 300, 500, 1000 fb^{-1}$ with unpolarized and polarized electron beam. Furthermore, we consider the kinematic acceptance cuts given by Eq. (19), take into account the systematic uncertainties $\delta_{sys} = 0\%, 3\%, 5\%$ and we follow three different confidence level (C.L.) 68\%, 90\% and 95\% and to make our study more effective we perform a $\chi^2$ test define as:

$$
\chi^2(\hat{a}_V, \hat{a}_A) = \left( \frac{\sigma_{SM} - \sigma_{BSM}(\sqrt{s}, \hat{a}_V, \hat{a}_A)}{\sigma_{SM} \sqrt{(\delta_{st})^2 + (\delta_{sys})^2}} \right)^2.
$$

Here $\sigma_{SM}$ is the cross-section from the SM, while $\sigma_{BSM}(\sqrt{s}, \hat{a}_V, \hat{a}_A)$ is the total cross-section which contains contributions from the SM, as well as non-standard contributions which come from the anomalous couplings $\hat{a}_V$ and $\hat{a}_A$. $\delta_{st} = \frac{1}{\sqrt{N_{SM}}}$ and $\delta_{sys}$ are the statistical and systematic uncertainties. In our study we consider $\delta_{sys} = 0\%, 3\%, 5\%$. The number of events for the process $e^- p \rightarrow e^- \gamma p \rightarrow \bar{t}\nu_e bp$ is given by $N_{SM} = \mathcal{L}_{int} \times \sigma_{SM} \times BR \times \epsilon_{b-tag}$, where $\mathcal{L}_{int}$ is the integrated FCC-he luminosity and $b$-jet tagging efficiency is $\epsilon_b = 0.8$ \cite{33}. The top-quark decay almost 100\% to $W$ boson and $b$ quark, specifically $\bar{t} \rightarrow \bar{b}W^-$, where the $W$ boson decays into leptons and hadrons.

The $\chi^2(\hat{a}_V, \hat{a}_A)$ analysis due systematic uncertainties is studied for three representative values of $\delta_{sys}$ at 0\%, 3\% and 5\%, respectively. And the sensitivity of $\hat{a}_V$ and $\hat{a}_A$ at 95\% C.L. is found to be of the order of $10^{-1}$ with $\sqrt{s} = 10 TeV$, $\mathcal{L} = 1000 fb^{-1}$ and we consider both cases, that is, unpolarized and polarized electron beam, as shown in Table VI (which includes the acceptance cuts, Eq. (19)). The order of the sensitivity on the anomalous couplings $\hat{a}_V$ and $\hat{a}_A$ for other values of $\sqrt{s}$ and $\mathcal{L}$ varies between $10^{-2} - 10^{-1}$ at 68\% C.L. and 90\% C.L., as shown in Tables I-V. Our study shows that the anomalous $t\bar{t}\gamma$ vertex at the FCC-he can be probed to a very good accuracy and is comparable with others existing limits, see Table I, Ref. \cite{7}. 

10
Figs. 7-8 (unpolarized electron beam) and 13-14 (polarized electron beam) show the 95% C.L. contours for the anomalous top-quark dipole couplings $\hat{a}_V$ and $\hat{a}_A$ with the assumed energies and luminosities of $\sqrt{s} = 7.07, 10$ TeV and $L = 50, 250, 1000$ fb$^{-1}$. With the uncertainty of 0%, the 95% C.L. sensitivity on the couplings are found to be $\hat{a}_V \in [-0.45, 0.05]$, $\hat{a}_A \in [-0.25, 0.25]$ with $P_{e^-} = 0\%$, and $\hat{a}_V \in [-0.120, 0.120]$, $\hat{a}_A \in [-0.120, 0.120]$ with $P_{e^-} = -80\%$. Of the relations given by Eqs. (5) and (6), the sensitivities on the anomalous dipole moments of the top-quark $\hat{a}_V$ and $\hat{a}_A$ are corresponding to the following sensitivities on the magnetic and electric dipole moments of the top-quark:

$$P_{e^-} = 0\%: \quad -0.675 \leq a_t \leq 0.675, \quad 95\% \text{ C.L.,}$$

$$-1.433 \leq d_t(10^{-17} ecm) \leq 1.433, \quad 95\% \text{ C.L.}$$

and

$$P_{e^-} = -80\%: \quad -0.180 \leq a_t \leq 0.180, \quad 95\% \text{ C.L.,}$$

$$-6.878 \leq d_t(10^{-18} ecm) \leq 6.878, \quad 95\% \text{ C.L.}$$

For the anomalous magnetic and electric dipole moments, an improvement is reachable in comparison with the constraints obtained from the $\gamma e^- \rightarrow t\bar{t}$ [11] and $pp \rightarrow p\gamma^*\gamma^*p \rightarrow p\bar{t}t\bar{p}$ [12] searches mentioned previously.

IV. CONCLUSIONS

In this paper, we have study feasibility of measuring the non-standard couplings $\hat{a}_V$ and $\hat{a}_A$ coming from the effective electromagnetic interaction $tt\gamma$ through the process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_ep\bar{p}$ at the FCC-he. Specifically, we assume energies from 7.07 and 10 TeV and integrated luminosities of at least 50, 100, 300, 5000 and 1000 fb$^{-1}$. Further our sensitivity study is cut-based, polarized electron beam and sources of systematic uncertainties such as leptons and $b$-jet identification, as well as in a $\chi^2(\hat{a}_V, \hat{a}_A)$ test to extract, enhance and optimize the expected signal cross-section and the sensitivity on $\hat{a}_V$ and $\hat{a}_A$. We find that the total cross-section $\sigma(e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_ep\bar{p})$ has a strong dependence on the anomalous couplings $\hat{a}_V$ and $\hat{a}_A$, as well as with the center-of-mass energies of the FCC-he and therefore strong sensitivity estimated are obtained on $\sigma(e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_ep\bar{p})$ (see Figs. 3-6 and 9-12) and $\hat{a}_V$ ($\hat{a}_A$) (see Tables I-VI). Therefore, our results show that with the process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_ep\bar{p}$ at the FCC-he, the sensitivity estimated on the MM and the EDM of the top-quark can be
significantly strengthened. Specifically, with $1000 \text{ fb}^{-1}$ of data, $\sqrt{s} = 10 \text{ TeV}$, $\delta_{\text{sys}} = 5\%$ and $P_{e^-} = -80\%$ we obtain: $\hat{a}_V = (-0.1394, 0.1353)$ and $\hat{a}_A = |0.1374|$. Our results are competitive with those results shown in Table I of Ref. [7]. At this time, the FCC-he is an excellent option for the electron-proton collider. It will be useful for any new physics study. Fortunately, future of $e^-p$ colliders remain promising as it is a natural option like a hybrid between the hadron pp and linear $e^+e^-$ colliders.

It is worth mentioning that, additional improvements could be achieved on the observables of the top-quark, especially in their electromagnetic properties to the extent that more sophisticated analysis methods are apply. In addition to the improvement in the technology of detection of the current and future high-energy physics colliders.

Acknowledgments

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TABLE I: Sensitivity on the $\hat{a}_V$ magnetic moment and the $\hat{a}_A$ electric dipole moment of the top-quark through the process $e^{-}p \rightarrow e^{-}\gamma p \rightarrow \bar{t}\nu_ebp$.

| $\sqrt{s} = 7.07$ TeV, 68% C.L. | $P_{e^-} = 0\%$ | $P_{e^-} = -80\%$ |
|-------------------------------|----------------|-----------------|
| $\mathcal{L} (fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
| 50 | 0% | [-0.2733, 0.2557] | 0.2644 | [-0.2370, 0.2199] | 0.2282 |
| 50 | 3% | [-0.2742, 0.2566] | 0.2652 | [-0.2383, 0.2212] | 0.2295 |
| 50 | 5% | [-0.2756, 0.2580] | 0.2667 | [-0.2406, 0.2234] | 0.2318 |
| 100 | 0% | [-0.2313, 0.2137] | 0.2223 | [-0.2007, 0.1836] | 0.1919 |
| 100 | 3% | [-0.2327, 0.2151] | 0.2237 | [-0.2029, 0.1857] | 0.1941 |
| 100 | 5% | [-0.2351, 0.2175] | 0.2261 | [-0.2066, 0.1894] | 0.1978 |
| 300 | 0% | [-0.1779, 0.1603] | 0.1689 | [-0.1547, 0.1375] | 0.1458 |
| 300 | 3% | [-0.1811, 0.1635] | 0.1720 | [-0.1594, 0.1423] | 0.1506 |
| 300 | 5% | [-0.1862, 0.1686] | 0.1772 | [-0.1669, 0.1498] | 0.1581 |
| 500 | 0% | [-0.1577, 0.1401] | 0.1487 | [-0.1372, 0.1201] | 0.1283 |
| 500 | 3% | [-0.1622, 0.1446] | 0.1532 | [-0.1440, 0.1268] | 0.1351 |
| 500 | 5% | [-0.1694, 0.1518] | 0.1603 | [-0.1540, 0.1368] | 0.1451 |
| 1000 | 0% | [-0.1341, 0.1165] | 0.1250 | [-0.1169, 0.0997] | 0.1079 |
| 1000 | 3% | [-0.1414, 0.1238] | 0.1323 | [-0.1275, 0.1103] | 0.1186 |
| 1000 | 5% | [-0.1519, 0.1343] | 0.1429 | [-0.1414, 0.1242] | 0.1325 |

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TABLE II: Sensitivity on the $\hat{a}_V$ magnetic moment and the $\hat{a}_A$ electric dipole moment of the top-quark through the process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_bp$.

| $\mathcal{L} (fb^{-1})$ | $\delta_{\text{sys}}$ | $P_{e^-} = 0\%$ | $P_{e^-} = -80\%$ |
|-------------------------|----------------------|----------------|-----------------|
|                         | $\hat{a}_V^{}$ | $|\hat{a}_A^{}|$ | $\hat{a}_V^{}$ | $|\hat{a}_A^{}|$ |
| 50                      | 0%       | [-0.3084, 0.2908] | 0.2995 | [-0.2673, 0.2502] | 0.2585 |
| 50                      | 3%       | [-0.3093, 0.2917] | 0.3004 | [-0.2688, 0.2516] | 0.2600 |
| 50                      | 5%       | [-0.3110, 0.2934] | 0.3021 | [-0.2713, 0.2542] | 0.2626 |
| 100                     | 0%       | [-0.2608, 0.2432] | 0.2518 | [-0.2262, 0.2090] | 0.2174 |
| 100                     | 3%       | [-0.2624, 0.2447] | 0.2534 | [-0.2286, 0.2115] | 0.2199 |
| 100                     | 5%       | [-0.2651, 0.2475] | 0.2562 | [-0.2328, 0.2156] | 0.2240 |
| 300                     | 0%       | [-0.2003, 0.1827] | 0.1913 | [-0.1740, 0.1569] | 0.1652 |
| 300                     | 3%       | [-0.2039, 0.1863] | 0.1949 | [-0.1794, 0.1623] | 0.1706 |
| 300                     | 5%       | [-0.2097, 0.1921] | 0.2008 | [-0.1879, 0.1707] | 0.1791 |
| 500                     | 0%       | [-0.1774, 0.1598] | 0.1684 | [-0.1542, 0.1371] | 0.1454 |
| 500                     | 3%       | [-0.1825, 0.1649] | 0.1735 | [-0.1619, 0.1448] | 0.1531 |
| 500                     | 5%       | [-0.1906, 0.1730] | 0.1816 | [-0.1732, 0.1561] | 0.1644 |
| 1000                    | 0%       | [-0.1507, 0.1331] | 0.1416 | [-0.1311, 0.1140] | 0.1222 |
| 1000                    | 3%       | [-0.1589, 0.1413] | 0.1498 | [-0.1432, 0.1260] | 0.1343 |
| 1000                    | 5%       | [-0.1708, 0.1532] | 0.1618 | [-0.1589, 0.1418] | 0.1501 |

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TABLE III: Sensitivity on the $\hat{a}_V$ magnetic moment and the $\hat{a}_A$ electric dipole moment of the top-quark through the process $e^- p \rightarrow e^- \gamma p \rightarrow \bar{t} \nu_ebp$.

$$\sqrt{s} = 7.07 \text{ TeV, 95\% C.L.}$$

| $L \,(fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
|-----------------|----------------|-------------|----------------|----------------|----------------|
| 50              | 0\%            | [-0.3790, 0.3614] | 0.3701 | [-0.3283, 0.3111] | 0.3195 |
| 50              | 3\%            | [-0.3802, 0.3626] | 0.3713 | [-0.3301, 0.3130] | 0.3213 |
| 50              | 5\%            | [-0.3822, 0.3646] | 0.3734 | [-0.3333, 0.3161] | 0.3245 |
| 100             | 0\%            | [-0.3201, 0.3025] | 0.3112 | [-0.2775, 0.2603] | 0.2687 |
| 100             | 3\%            | [-0.3221, 0.3045] | 0.3132 | [-0.2805, 0.2633] | 0.2717 |
| 100             | 5\%            | [-0.3255, 0.3079] | 0.3166 | [-0.2856, 0.2685] | 0.2768 |
| 300             | 0\%            | [-0.2454, 0.2278] | 0.2365 | [-0.2130, 0.1958] | 0.2041 |
| 300             | 3\%            | [-0.2498, 0.2322] | 0.2409 | [-0.2196, 0.2025] | 0.2108 |
| 300             | 5\%            | [-0.2570, 0.2394] | 0.2481 | [-0.2301, 0.2130] | 0.2213 |
| 500             | 0\%            | [-0.2171, 0.1995] | 0.2081 | [-0.1885, 0.1713] | 0.1797 |
| 500             | 3\%            | [-0.2234, 0.2058] | 0.2144 | [-0.1980, 0.1808] | 0.1892 |
| 500             | 5\%            | [-0.2334, 0.2158] | 0.2245 | [-0.2120, 0.1948] | 0.2032 |
| 1000            | 0\%            | [-0.1840, 0.1664] | 0.1750 | [-0.1599, 0.1428] | 0.1511 |
| 1000            | 3\%            | [-0.1942, 0.1766] | 0.1852 | [-0.1748, 0.1577] | 0.1660 |
| 1000            | 5\%            | [-0.2090, 0.1914] | 0.2000 | [-0.1943, 0.1771] | 0.1855 |

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TABLE IV: Sensitivity on the $\hat{a}_V$ magnetic moment and the $\hat{a}_A$ electric dipole moment of the top-quark through the process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t}\nu_e bp$.

| $\sqrt{s}$ = 10 TeV, 68% C.L. | $P_{e^-} = 0\%$ | $P_{e^-} = -80\%$ |
|-----------------------------|-----------------|-----------------|
| $\mathcal{L} (fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
| 50  | 0\%  | [-0.1863, 0.1821] | 0.1845 | [-0.1612, 0.1570] | 0.1591 |
| 50  | 3\%  | [-0.1872, 0.1830] | 0.1854 | [-0.1625, 0.1584] | 0.1605 |
| 50  | 5\%  | [-0.1887, 0.1845] | 0.1869 | [-0.1648, 0.1377] | 0.1628 |
| 100 | 0\%  | [-0.1570, 0.1528] | 0.1551 | [-0.1358, 0.1317] | 0.1338 |
| 100 | 3\%  | [-0.1585, 0.1542] | 0.1566 | [-0.1381, 0.1339] | 0.1360 |
| 100 | 5\%  | [-0.1609, 0.1567] | 0.1591 | [-0.1418, 0.1567] | 0.1397 |
| 300 | 0\%  | [-0.1198, 0.1156] | 0.1179 | [-0.1037, 0.0996] | 0.1017 |
| 300 | 3\%  | [-0.1230, 0.1188] | 0.1211 | [-0.1085, 0.1044] | 0.1065 |
| 300 | 5\%  | [-0.1282, 0.1239] | 0.1263 | [-0.1158, 0.1117] | 0.1138 |
| 500 | 0\%  | [-0.1057, 0.1015] | 0.1037 | [-0.0915, 0.0874] | 0.0895 |
| 500 | 3\%  | [-0.1103, 0.1061] | 0.1083 | [-0.0983, 0.0942] | 0.0963 |
| 500 | 5\%  | [-0.1172, 0.1130] | 0.1153 | [-0.1077, 0.1036] | 0.1057 |
| 1000| 0\%  | [-0.0892, 0.0850] | 0.0872 | [-0.0773, 0.0732] | 0.0752 |
| 1000| 3\%  | [-0.0965, 0.0923] | 0.0945 | [-0.0877, 0.0836] | 0.0857 |
| 1000| 5\%  | [-0.1063, 0.1021] | 0.1044 | [-0.1002, 0.0961] | 0.0981 |

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TABLE V: Sensitivity on the $\hat{a}_V$ magnetic moment and the $\hat{a}_A$ electric dipole moment of the top-quark through the process $e^- p \to e^- \gamma p \to \bar{t}\nu_e b p$.

| $\sqrt{s} = 10$ TeV, 90% C.L. | $P_{e^-} = 0\%$ | $P_{e^-} = -80\%$ |
|-------------------------------|-----------------|-----------------|
| $L (fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
| 50 | 0% | [-0.2108, 0.2066] | 0.2090 | [-0.1823, 0.1782] | 0.1803 |
| 50 | 3% | [-0.2118, 0.2075] | 0.2100 | [-0.1838, 0.1797] | 0.1818 |
| 50 | 5% | [-0.2135, 0.2093] | 0.2117 | [-0.1864, 0.1823] | 0.1844 |
| 100 | 0% | [-0.1776, 0.1734] | 0.1757 | [-0.1536, 0.1495] | 0.1516 |
| 100 | 3% | [-0.1792, 0.1750] | 0.1774 | [-0.1561, 0.1520] | 0.1541 |
| 100 | 5% | [-0.1820, 0.1778] | 0.1802 | [-0.1603, 0.1562] | 0.1583 |
| 300 | 0% | [-0.1354, 0.1312] | 0.1335 | [-0.1172, 0.1131] | 0.1152 |
| 300 | 3% | [-0.1391, 0.1348] | 0.1372 | [-0.1227, 0.1186] | 0.1206 |
| 300 | 5% | [-0.1449, 0.1407] | 0.1430 | [-0.1309, 0.1268] | 0.1289 |
| 500 | 0% | [-0.1195, 0.1152] | 0.1175 | [-0.1034, 0.0993] | 0.1013 |
| 500 | 3% | [-0.1246, 0.1204] | 0.1227 | [-0.1111, 0.1070] | 0.1091 |
| 500 | 5% | [-0.1325, 0.1283] | 0.1306 | [-0.1218, 0.1176] | 0.1197 |
| 1000 | 0% | [-0.1008, 0.0966] | 0.0988 | [-0.0873, 0.0832] | 0.0852 |
| 1000 | 3% | [-0.1090, 0.1048] | 0.1071 | [-0.0991, 0.0950] | 0.0970 |
| 1000 | 5% | [-0.1202, 0.1159] | 0.1183 | [-0.1132, 0.1091] | 0.1112 |
TABLE VI: Sensitivity on the $\hat{a}_V$ magnetic moment and the $\hat{a}_A$ electric dipole moment of the top-quark through the process $e^-p \rightarrow e^-\gamma p \rightarrow \bar{t} \nu_e bp$.

| $\sqrt{s}$ (TeV) | $P_{e^-} = 0\%$ | $P_{e^-} = -80\%$ |
|------------------|-----------------|-----------------|
| $\mathcal{L} (fb^{-1})$ | $\delta_{sys}$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ | $\hat{a}_V$ | $|\hat{a}_A|$ |
| 50 | 0% | [-0.2600, 0.2558] | 0.2583 | [-0.2248, 0.2207] | 0.2228 |
| 50 | 3% | [-0.2612, 0.2570] | 0.2595 | [-0.2267, 0.2225] | 0.2246 |
| 50 | 5% | [-0.2633, 0.2591] | 0.2616 | [-0.2299, 0.2258] | 0.2279 |
| 100 | 0% | [-0.2190, 0.2147] | 0.2172 | [-0.1893, 0.1852] | 0.1873 |
| 100 | 3% | [-0.2210, 0.2168] | 0.2192 | [-0.1925, 0.1883] | 0.1904 |
| 100 | 5% | [-0.2245, 0.2202] | 0.2227 | [-0.1976, 0.1935] | 0.1956 |
| 300 | 0% | [-0.1669, 0.1627] | 0.1650 | [-0.1444, 0.1402] | 0.1423 |
| 300 | 3% | [-0.1714, 0.1671] | 0.1695 | [-0.1511, 0.1470] | 0.1491 |
| 300 | 5% | [-0.1786, 0.1743] | 0.1768 | [-0.1613, 0.1572] | 0.1593 |
| 500 | 0% | [-0.1471, 0.1429] | 0.1452 | [-0.1273, 0.1232] | 0.1253 |
| 500 | 3% | [-0.1535, 0.1493] | 0.1517 | [-0.1368, 0.1327] | 0.1348 |
| 500 | 5% | [-0.1633, 0.1590] | 0.1614 | [-0.1500, 0.1459] | 0.1480 |
| 1000 | 0% | [-0.1241, 0.1198] | 0.1221 | [-0.1074, 0.1033] | 0.1053 |
| 1000 | 3% | [-0.1342, 0.1300] | 0.1323 | [-0.1220, 0.1179] | 0.1199 |
| 1000 | 5% | [-0.1480, 0.1438] | 0.1462 | [-0.1394, 0.1353] | 0.1374 |
FIG. 1: A schematic diagram for the process $e^- p \rightarrow e^- \gamma p \rightarrow \bar{t} \nu_e bp$.

FIG. 2: Feynman diagrams contributing to the subprocess $e^- \gamma \rightarrow \bar{t} \nu_e b$.
FIG. 3: The total cross sections of the process $e^- p \rightarrow e^- \gamma p \rightarrow \bar{t} \nu_e bp$ as a function of $\hat{a}_V$ for center-of-mass energies of $\sqrt{s} = 7.07, 10 \text{ TeV}$ at the FCC-he.

FIG. 4: Same as in Fig. 3, but for $\hat{a}_A$. 

20
FIG. 5: The total cross sections of the process $e^− p \rightarrow e^− \gamma p \rightarrow \bar{\nu}_e b p$ as a function of $\hat{a}_V$ and $\hat{a}_A$ for center-of-mass energy of $\sqrt{s} = 7.07 \ TeV$ at the FCC-he.

FIG. 6: Same as in Fig. 5, but for center-of-mass energy of $\sqrt{s} = 10 \ TeV$. 
FIG. 7: Sensitivity contours at the 95% C.L. in the $\hat{a}_V - \hat{a}_A$ plane through the process $e^- p \rightarrow e^- \gamma p \rightarrow \bar{t} \nu_b p$ for $\sqrt{s} = 7.07 \, \text{TeV}$ at the FCC-he.

FIG. 8: Same as in Fig. 7, but for $\sqrt{s} = 10 \, \text{TeV}$.
FIG. 9: Same as in Fig. 3, but with polarized beams $P_{e^-} = -80\%$.

FIG. 10: Same as in Fig. 4, but with polarized beams $P_{e^-} = -80\%$. 
FIG. 11: Same as in Fig. 5, but with polarized beams $P_{e^-} = -80\%$.

FIG. 12: Same as in Fig. 6, but with polarized beams $P_{e^-} = -80\%$. 
FIG. 13: Same as in Fig. 7, but with polarized beams $P_{e^-} = -80\%$.

FIG. 14: Same as in Fig. 8, but with polarized beams $P_{e^-} = -80\%$. 