D\bar{D} production and their interactions

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Abstract. S and P wave D\bar{D} scatterings are studied in a meson exchange model. When exploring their production in the e^+e^- annihilation process, we have considered the rescattering effect in a nonrelativistic method. We find that this effect may result in an anomalous line shape of the cross section provided that the final state interaction is unrealistically strong. However, it is difficult to understand the anomalous line shape observed by the BES collaboration with this mechanism. From the normal line shape for the B\bar{B} case, we set a constraint for the model parameters and further get an upper limit for the S wave binding solutions.

1. Introduction

Observation of new charmonium-like mesons, which are above the D\bar{D} threshold, motivates heated discussions on their possible structures. Due to the near-threshold feature, the exploration of heavy meson-antimeson bound states is very popular. The most famous example is X(3872), which was widely discussed by assuming it is a D\bar{D} molecule [1, 2, 3, 4, 5].

Interactions of heavy quark hadrons may be studied with non-relativistic formalisms. Because of the relatively small kinetic term in the Hamiltonian, existence of heavy quark molecules is possible. Here, we focus on the simplest D\bar{D} system. A D\bar{D} bound state was obtained in Ref. [6] with a quark-based model. A quasibound state was also found with the unitarized method [7, 8]. The vector meson exchange may result in a possible binding solution [9]. Our previous results cannot exclude its existence within the meson exchange framework [10] nor chiral quark model [11]. However, a very recent analysis indicates that the existence of a D\bar{D} bound state is difficult to understand [12]. Therefore, the question about the existence of the D\bar{D} bound state is still open. In Ref. [13], we reanalyzed the bound state problem with a meson exchange model at the hadron level, where \sigma, \rho, and \omega exchanges were considered.

To better understand their interactions, we also investigated the scattering problem [13]. We calculated, with various parameters, the S and P wave phase shifts and total cross sections, and derive the scattering length and scattering volume. The information from the scattering may complement our understanding of the bound state problem.

Recently, the BES Collaboration announced an anomalous line shape of the e^+e^- \rightarrow hadrons total cross section [14]. A similar line shape was also observed in the D\bar{D} production [15]. A possible contribution of two closely spaced narrow resonances was suggested in Ref. [16] to understand the observation. Since the anomalous structure is just above the D\bar{D} threshold and below \psi(3770), final state interactions (FSI) between the D and \bar{D} may contribute. FSI gives
non-negligible contributions to the $\psi(3770)$ decay according to Refs. [17, 18]. A part of the FSI effect has been considered in Ref. [19], but no anomalous line shape appears there. We explored whether this structure is due to the multiple $D\bar{D}$ rescattering effect in their production based on the calculated phase shifts, using the Yamaguchi separable potential approximation [20].

2. The model and $D\bar{D}$ interactions
Only the isoscalar $D\bar{D}$ system is considered. We use a one-meson-exchange model to study the interaction. Because of the parity conservation, the pion exchange is forbidden. The $\sigma$, $\rho$, and $\omega$ exchange potentials read

\begin{align}
V_\sigma &= -\frac{g_\sigma^2}{4\pi} r \left[ e^{-m_\sigma r} - e^{-\Lambda r} \right] - \frac{\Lambda^2 - m_\sigma^2}{2\Lambda} e^{-\Lambda r}, \\
V_\rho &= -3 \frac{G_V^2}{16\pi} r \left[ e^{-m_\rho r} - e^{-\Lambda r} \right] - \frac{\Lambda^2 - m_\rho^2}{2\Lambda} e^{-\Lambda r}, \\
V_\omega &= -\frac{G_\omega^2}{16\pi} r \left[ e^{-m_\omega r} - e^{-\Lambda r} \right] - \frac{\Lambda^2 - m_\omega^2}{2\Lambda} e^{-\Lambda r},
\end{align}

where we have introduced a cutoff through the monopole type form factor $F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}$ at each interacting vertex. For the coupling constants in the potentials, we use the values derived from the chiral multiplet assumptions and the vector meson dominance: $G_V = 5.22$ and $g_\sigma = 0.76$.

For the undetermined mass of $\sigma$, we choose two extreme cases, $m_\sigma = 400$ MeV and 600 MeV.

Once the potentials are given, one may solve the bound state and scattering problem. We present the binding energies of various cases in Table 1, where we also give the solutions for the $B\bar{B}$ case. It is observed that the binding energy is sensitive to the cutoff and is very large in the limit $\Lambda \to \infty$, which is unreasonable for a physical system. To see the cutoff dependence, we use several finite values $\Lambda = 0.8, 1.0, 1.2, 1.5, \text{ and } 2.0$ GeV in the calculation of scattering phase shifts.

| Systems | $m_\sigma$ (MeV) | $\Lambda$ (GeV) |
|---------|------------------|-----------------|
| $D\bar{D}$ (S wave) | 400 | 1.2 | 1.5 | 2.0 | $\infty$ |
| | 600 | $\times$ | -0.8 | -29.4 | -974.5 |
| | 400 | $\times$ | -1.4 | -31.8 | -980.9 |
| $B\bar{B}$ (S wave) | 600 | 1.2 | 1.5 | 2.0 | $\infty$ |
| | 600 | -8.3 | -57.4 | -186.0 | -4916.7 (n=1) |
| | 400 | -10.1 | -60.7 | -190.5 | -4924.8 (n=1) |
| $B\bar{B}$ (P wave) | 600 | 1.2 | 1.5 | 2.0 | $\infty$ |
| | 600 | $\times$ | $\times$ | $\times$ | -377.4 |
| | 400 | $\times$ | $\times$ | -0.6 | -383.1 |

We have calculated the S and P wave phase shifts in various cases in Ref. [13]. Here, we present only one example for each partial wave. Figure 1 (2) gives the S (P) wave phase shift and total cross section with $m_\sigma = 600$ (400) MeV. An S wave $D\bar{D}$ bound state is possible if the short range attraction is strong, e.g. $\Lambda \geq 1.5$ GeV. There is no P wave binding solutions even in the point particle limit ($\Lambda \to \infty$), but there is a bump in the P-wave total cross section.
Figure 1. The phase shift (a) and the total cross section (b) for the S wave $D\bar{D}$ scattering with $m_\sigma = 600$ MeV and different cutoff.

Figure 2. The phase shift (a) and the total cross section (b) for the P wave $D\bar{D}$ scattering with $m_\sigma = 400$ MeV and different cutoff.

With the obtained phase shifts, it is easy to derive the threshold parameters: the S wave scattering length and the P wave scattering volume. We list them in Table 2 and 3, respectively.

Table 2. S- wave $D\bar{D}$ scattering lengths in units of fm. The number of * in the table indicates existence of a bound state.

| $m_\sigma$ (MeV) | 0.8 | 1.0 | 1.2 | 1.5 | 2.0 | $\infty$ |
|------------------|-----|-----|-----|-----|-----|---------|
| 600              | 0.014 | 0.31 | 1.23 | -5.46(*) | -1.14(*) | 0.60(*) |
| 400              | 0.068 | 0.40 | 1.56 | -4.23(*) | -1.10(*) | 0.74(*) |

Table 3. The P- wave $D\bar{D}$ scattering volumes in units of fm$^3$.

| $m_\sigma$ (MeV) | 0.8 | 1.0 | 1.2 | 1.5 | 2.0 | $\infty$ |
|------------------|-----|-----|-----|-----|-----|---------|
| 600              | 0.0019 | 0.023 | 0.041 | 0.058 | 0.070 | 0.085 |
| 400              | 0.015 | 0.037 | 0.055 | 0.072 | 0.087 | 0.10 |
3. $D\bar{D}$ production and the rescattering effect

To include the P wave $D\bar{D}$ FSI in the production process, we adopt the Yamaguchi separable potential approximation [20]. The introduced parameters are determined so as to reproduce the calculated phase shifts. In this method, the total production cross section has two parts: $\sigma_{\text{prod}} = \sigma_1 + \sigma_2$. The first part gives the cross section without FSI, while the second one reflects the rescattering effect.

In the numerical evaluation, we explored several cases. With the phase shifts calculated in the meson exchange model, we do not find any anomalous line shape. However, if one considers an extreme case of very strong attraction (the scattering volume $a_1 = 2\text{ fm}^3$), we get an anomalous line shape. The comparison between this case and the one with $a_1 = 0.087\text{ fm}^3$ is shown in Fig. 3. The latter case corresponds to $m_\sigma = 400\text{ MeV}$, and $\Lambda = 2.0\text{ GeV}$. Because the scattering volume is much larger than the maximum number in Table 3, the extreme case may not be realistic. Therefore, although the strong FSI may lead to anomalous line shape in the production processes, one cannot interpret those observed by the BES Collaboration with this mechanism.

![Image of cross sections](a) (b)

**Figure 3.** The production cross section obtained for the potentials with the scattering volume $a_1 = 2.0\text{ fm}^3$ (a) and that with $a_1 = 0.087\text{ fm}^3$ (b). The dashed line corresponds to the $\sigma_1$ part of $\sigma_{\text{prod}}$. The experimental data are taken from Ref. [15].

For the heavier $B\bar{B}$ mesons, the S wave bound state is more likely to exist than the $D\bar{D}$ case. Because of the heavy quark symmetry and the smaller size of the $B$ meson, the cutoff in the potentials should be slightly larger than that in the $D\bar{D}$ case. We have performed a similar calculation [13]. The binding energies are given in Table 1. The values indicate the cutoff should not be very large. Now the P wave binding solution also exists with $\Lambda \geq 2.0\text{ GeV}$. On the other hand, if the P wave binding solution does not exist, we get an upper limit of the cutoff. As a result, the S wave $B\bar{B}$ and $D\bar{D}$ interactions are better understood. In fact, the P wave $B\bar{B}$ production provides some information. When there is a shallow bound state or a sharp resonance, the strong rescattering effect may affect the line shape of the total cross section. From the data of the BABAR Collaboration [21], the line shape of the P wave $B\bar{B}$ production cross section is normal, which gives the constraint $\Lambda < 2.0\text{ GeV}$. In fact, the investigation in detail reduces the upper limit to 1.7 GeV where there is a shallow P wave resonance.

4. Discussions

In our model calculation, the results for the bound state problem, the scattering problem, and the production problem are sensitive to the nearly universal cutoff. From the line shape of the $B\bar{B}$ production cross section, we have extracted the upper limit $\Lambda < 1.7\text{ GeV}$. We may further
set the constraint for the binding energy. For the $D\bar{D}$ S-wave bound state, the binding energy should be less than 10 MeV. For the S-wave $B\bar{B}$ pair, the binding energy is expected to be below 100 MeV. To see the effect of the form factor, we also investigated the dipole type one. The upper limit of the cutoff increases, but the constraint on the S-wave binding energies has little change.

If the $D\bar{D}$ S-wave bound state exists, its strong decay channel would be mainly $\eta_\epsilon\eta$ and $\chi_0\sigma\pi$. To look for this scalar state, one may investigate the following processes: $B$ decay, $\gamma\gamma$ fusion, $p\bar{p}$ collision, or $e^+e^- \rightarrow J/\psi + (D\bar{D})_s$. The $B\bar{B}$ bound state would mainly decay into $\Upsilon(1S)\omega$ and $\chi_{b0}(1P)\pi\pi$. One also has a chance to obtain this scalar state by $p\bar{p}$ collision.

5. Conclusions
We have studied the bound state problem, the scattering problem, and the production problem for the isoscalar $D\bar{D}$ pair in a meson exchange model. The existence of an S wave bound state is not excluded. For their production, we have considered the multiple rescattering effect using the calculated P wave phase shifts. In an extreme case of very strong attraction, we got an anomalous line shape. But we could not explain the BES observation with this mechanism.

We also studied the $B\bar{B}$ system within the same framework. Now the P wave bound state is also possible. Since the strong P wave interaction may affect the line shape of the cross section, we estimated the upper limit of the cutoff from their production. Assuming the same cutoff, we obtained the upper limit for the binding energy of the S wave $D\bar{D}$ ($B\bar{B}$) bound states to be 10 (100) MeV.

Acknowledgments
We enjoy the collaboration with Prof. Shi-Lin Zhu and Wei-Zhen Deng. This work was supported partly by the Japan Society for the Promotion of Science under Contract No. P09027; KAKENHI under Contract Nos. 17070002 (Priority area), 19540275, 20540281, 22105503, and 21-09027; National Natural Science Foundation of China under Grant Nos. 10625521, 10675008, 10705001, 10775146, 10721063, and 10805048; the Foundation for the Author of National Excellent Doctoral Dissertation of P.R. China (FANEDD) under Contract No. 200924; the Doctoral Program Foundation of Institutions of Higher Education of P.R. China under Grant No. 20090211120029; and the Program for New Century Excellent Talents in University (NCET) by the Ministry of Education of P.R. China under Grant No. NCET-10-0442.

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