PyTSK: A Python Toolbox for TSK Fuzzy Systems

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Abstract

This paper presents PyTSK, a Python toolbox for developing Takagi-Sugeno-Kang (TSK) fuzzy systems. Based on scikit-learn and PyTorch, PyTSK allows users to optimize TSK fuzzy systems using fuzzy clustering or mini-batch gradient descent (MBGD) based algorithms. Several state-of-the-art MBGD-based optimization algorithms are implemented in the toolbox, which can improve the generalization performance of TSK fuzzy systems, especially for big data applications. PyTSK can also be easily extended and customized for more complicated algorithms, such as modifying the structure of TSK fuzzy systems, developing more sophisticated training algorithms, and combining TSK fuzzy systems with neural networks. The code of PyTSK can be found at https://github.com/YuqiCui/pytsk.

Keywords: PyTSK, TSK fuzzy systems, fuzzy clustering, mini-batch gradient descent

1. Introduction

Takagi-Sugeno-Kang (TSK) fuzzy systems Mendel (2017) have been widely used in many classification and regression problems (Kar et al., 2014; Wu and Mendel, 2020). Traditional TSK fuzzy system optimization algorithms based on fuzzy clustering or evolutionary algorithms are usually time-consuming in big data applications. Mini-batch gradient descent (MBGD) based algorithms (Goodfellow et al., 2016), which have been widely used in neural network optimization, have also been adapted for TSK fuzzy system optimization in recent years (Wu et al., 2019; Cui et al., 2020; Du et al., 2020; Matsumura and Nakashima, 2017; Shi et al., 2021). They have demonstrated better generalization, lower training cost, and better scalability to large datasets. However, few libraries provide convenient and complete Python application programming interfaces (APIs) for developing TSK fuzzy systems, and even fewer based on deep learning frameworks such as PyTorch, TensorFlow and Keras. This may hinder future development and applications of TSK fuzzy systems.

This paper presents PyTSK, a Python toolbox that allows users to develop TSK fuzzy systems using fuzzy clustering or MBGD based optimization algorithms. For fuzzy clustering based optimization, we separate the antecedent and consequent of the TSK fuzzy system so that each part can be easily replaced by a more sophisticated algorithm, and hyperparameters of both modules can be separately or jointly tuned using APIs in scikit-learn, such as GridSearchCV(). For MBGD based optimization, we provide PyTorch-based APIs
Algorithm 1: Code snippet of optimizing a TSK fuzzy classifier with fuzzy c-means and ridge regression using PyTSK and scikit-learn APIs.

```python
from pytsk.cluster import FuzzyCMeans
from sklearn.pipeline import Pipeline
from sklearn.linear_model import RidgeClassifier

model = Pipeline(steps=[
    ('GaussianAntecedent', FuzzyCMeans(n_cluster=n_rule)),
    ('Consequent', RidgeClassifier())
])
model.fit(x_train, y_train) # Model training
y_pred = model.predict(x_test) # Model prediction
```

for developing TSK fuzzy models. Several state-of-the-art MBGD-based optimization tech-
niques, e.g., uniform regularization (Cui et al., 2020), DropRule (Wu et al., 2019), and high-
dimensional defuzzification (Cui et al., 2021), are implemented in PyTSK to improve the
generalization performance. The API design of PyTSK is similar to that of scikit-learn, i.e., training and prediction of TSK fuzzy models can be performed by calling `fit()` and `predict()` functions, respectively.

2. Design of PyTSK

This section introduces fuzzy clustering based TSK fuzzy system optimization algorithms, MBGD-based TSK fuzzy system optimization algorithms, and several advanced functionalities in PyTSK.

2.1 Fuzzy Clustering based TSK Fuzzy System Optimization

Fuzzy clustering algorithms, e.g., fuzzy c-means (Bezdek et al., 1984), can be used to de-
termine the antecedent parameters of a TSK fuzzy system, especially when Gaussian mem-
bersonship functions are used (Deng et al., 2010). The input of the consequent part can be
transformed into a linear vector, and the consequent parameters can be determined by linear regression algorithms (Xu et al., 2019).

PyTSK implements the fuzzy c-means algorithm as a subclass of `TransformerMixin()` in
scikit-learn. The `predict()` method outputs the corresponding membership degree of
each cluster, and the `transform()` method generates the linear consequent input. A com-
plete TSK fuzzy system can be built by using the `Pipeline()` class of scikit-learn to com-
bine the obtained antecedents with any linear regression algorithm, following scikit-learn
API design. A simple example of optimizing a TSK fuzzy classifier using PyTSK is shown
in Algorithm 1

2.2 MBGD-based TSK Fuzzy System Optimization

MBGD-based optimization algorithms are widely used in neural networks, particularly deep
learning (Goodfellow et al., 2016). They have also been adapted for TSK fuzzy system
optimization recently (Wu et al., 2019; Cui et al., 2020; Shi et al., 2021), for better generalization, lower training cost, and better scalability. MBGD-based optimization of fuzzy systems also makes the development of deep fuzzy neural networks easier, as both the fuzzy part and the neural network part can be trained by a single MBGD algorithm.

In PyTSK, MBGD-based TSK fuzzy system optimization is implemented in PyTorch. Two middleware, Firing Level (FL) Transformer and Input Transformer, are reserved for conveniently modifying the model structure. The model structure of a TSK fuzzy system and the reserved middleware are shown Fig. 1.

![Figure 1: The model structure of a TSK fuzzy system and the reserved middleware in PyTSK.](image)

More specifically, the antecedent module computes the normalized FLs of the rules (PyTSK supports both Gaussian and triangular membership functions), which are then transformed by the FL transformer as needed to impose additional rule weights, normalization, or more transformations. The input transformer transforms the input data as needed and then sends them to the consequent module, which can be used for additional feature selection or normalization.

A simple example of MBGD-based TSK fuzzy classifier optimization using PyTSK is shown in Algorithm 2. Note that the defined TSK fuzzy model inherits the Module() class of PyTorch, which means it can be optimized by either our provided API Wrapper() or any other PyTorch-derived API.

2.3 Advanced Techniques for MBGD-based TSK Fuzzy System Optimization

Studies have pointed out that MBGD-based TSK fuzzy system optimization may have some limitations. For example, MBGD-based training may generate dead rules and degrade the resulting TSK fuzzy model’s fitting ability (Cui et al., 2020); the saturation of the softmax function may limit the TSK fuzzy model’s performance on high-dimensional datasets (Cui et al., 2021); the gradient vanishing problem may make TSK fuzzy models sensitive to the choice of the optimizer (Cui et al., 2022). Therefore, PyTSK supports the following advanced techniques for improving the generalization performance of TSK fuzzy systems:

1. **Uniform regularization** (Cui et al., 2020), which forces each rule to have similar contributions to the prediction, so as to avoid dead rules and improve the generalization performance. One can train a TSK fuzzy model with uniform regularization by simply setting the parameter ur of the Wrapper() class in PyTSK.

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1. More examples and details can be found at [https://pytskdocs.readthedocs.io/en/latest/models.html](https://pytskdocs.readthedocs.io/en/latest/models.html)
Algorithm 2: Code snippet of MBGD-based TSK fuzzy classifier optimization using PyTSK and PyTorch APIs.

```python
import torch.nn as nn
from pytsk.gradient_descent.antecedent import AntecedentGMF
from pytsk.gradient_descent.training import Wrapper
from pytsk.gradient_descent.tsk import TSK
from torch.optim import Adam

gmf = nn.Sequential(
    AntecedentGMF(in_dim=in_dim, n_rule=n_rule),
    FLTransformer(),
)  # Defining antecedent and FL transformer
model = TSK(in_dim=in_dim, out_dim=n_class, n_rule=n_rule, antecedent=gmf,
            precons=InputTransformer)  # Define consequent and input transformer
wrapper = Wrapper(model, optimizer=Adam(model.parameters(), lr=lr),
                   criterion=nn.CrossEntropyLoss())  # Set wrapper
wrapper.fit(x_train, y_train)  # Model training
y_pred = wrapper.predict(x_test).argmax(axis=1)  # Model prediction
```

2. **High-dimensional TSK (HTSK)** (Cui et al., 2021), which uses the reciprocal of the input dimensionality as the exponent term of the rule firing levels in defuzzification to improve the generalization performance on high-dimensional datasets. HTSK can be implemented by simply setting the parameter `high_dim` to True when defining the Gaussian antecedent classes `AntecedentGMF()` and `AntecedentShareGM()`.

3. **DropRule** (Wu et al., 2019), which randomly drops some of the rules during MBGD-based optimization to improve the generalization performance. DropRule can be implemented by setting the FL Transformer as a PyTorch Dropout (Srivastava et al., 2014) layer.

4. **Deep/stacked fuzzy systems**, which stacks TSK fuzzy systems or deep neural networks to automatically extract more nonlinear features to enhance the model’s fitting ability. Since PyTSK implements a TSK fuzzy system as a `Module()` class of PyTorch, the way to build a deep/stacked TSK fuzzy system is the same as that of building a deep neural network.

3. **Conclusion**

This paper has introduced PyTSK, a Python toolbox for conveniently developing TSK fuzzy systems using fuzzy clustering or MBGD-based optimization algorithms. Models built by PyTSK can be easily customized, and are compatible with other machine learning models derived from scikit-learn and PyTorch. Several state-of-the-art MBGD-based techniques for improving a TSK fuzzy model’s generalization performance, e.g., uniform regularization, HTSK and DropRule, have also been implemented in PyTSK.

In the future, we will provide support of more advanced fuzzy clustering algorithms, more types of membership functions, and more regularization approaches for improving the
generalization or interpretability of TSK fuzzy systems. PyTSK also welcomes contributions or suggestions from the community.

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