The EFT likelihood for large-scale structure in redshift space

Giovanni Cabass
Max-Planck-Institut für Astrophysik,
Karl-Schwarzschild-Str. 1, 85741 Garching, Germany
E-mail: gcabass@mpa-garching.mpg.de

Received August 5, 2020
Revised October 3, 2020
Accepted December 12, 2020
Published January 29, 2021

Abstract. We study the EFT likelihood for biased tracers in redshift space, for which the bias expansion of the galaxy velocity field $v_g$ plays a fundamental role. The equivalence principle forbids stochastic contributions to $v_g$ to survive at small $k$. Therefore, at leading order in derivatives the form of the likelihood $P[\tilde{\delta}_g|\delta,v]$ to observe a redshift-space galaxy overdensity $\tilde{\delta}_g(\tilde{x})$ given a rest-frame matter and velocity fields $\delta(x), v(x)$ is fixed by the rest-frame noise. If this noise is Gaussian with constant power spectrum, $P[\tilde{\delta}_g|\delta,v]$ is also a Gaussian in the difference between $\tilde{\delta}_g(\tilde{x})$ and its bias expansion: redshift-space distortions only make the covariance depend on $\delta(x)$ and $v(x)$. We then show how to match this result to perturbation theory, and that one can consistently neglect the field-dependent covariance if the bias expansion is stopped at second order in perturbations. We discuss qualitatively how this affects numerical implementations of the EFT-based forward modeling, and how the picture changes when the survey window function is taken into account.

Keywords: cosmic web, cosmological parameters from LSS, redshift surveys

ArXiv ePrint: 2007.14988
1 Introduction and summary of main results

The effective field theory (EFT) of large-scale structure (LSS), EFTofLSS hereafter, allows for a controlled incorporation of the effects of fully nonlinear structure formation on small scales in the framework of cosmological perturbation theory [1, 2]. This is especially important when attempting to infer cosmological information from observed biased tracers such as galaxies, quasars, galaxy clusters, the Lyman-α forest, and others (see [3] for a review; in the following we will always refer to the tracers as “galaxies” for simplicity).

The prediction for the galaxy density field $\delta_g(x, \tau) = n_g(x, \tau)/n_g(\tau) - 1$ can be broken into two parts: a “deterministic” part $\delta_{g,\text{det}}$ which captures the modulation of the galaxy density by long-wavelength perturbations, and a stochastic residual which fluctuates due to the small-scale initial conditions. When integrating out small-scale modes, this effectively leads to a noise in the galaxy density.
Recently, refs. [4, 5] presented a derivation of the likelihood of the entire galaxy density field $\delta_g(x, \tau)$ given the nonlinear matter density field (i.e. the matter field evolved via gravity), in the context of the EFT. This result offers several advantages over approaches that aim at constraining a finite number of correlation functions:

- It puts the stochasticity of galaxies and the deterministic bias expansion on the same footing, showing that the form of the conditional likelihood is determined by the properties of the noise (such as the fact that, in first approximation, it is Gaussian with constant power spectrum on scales where the EFTofLSS can be applied).
- The likelihood is given in terms of the fully nonlinear density field, which can be predicted for example using N-body simulations, and thus isolates the truly uncertain aspects of the observed galaxy density.
- The likelihood is given by the functional Fourier transform of the generating functional. Since the latter contains all correlation functions, the derivation of [5] provides a correspondence between different terms in the likelihood and correlation functions.
- The conditional likelihood of the galaxy density field given the evolved matter density field is precisely the key ingredient required in full Bayesian (“forward-modeling”) inference approaches [6–11], and can be employed there directly [4, 12, 13] (see [14–17] for related approaches).

So far the EFT likelihood has been used with rest-frame halo catalogs only [12, 13]. However, we need to account for the mapping from rest-frame quantities to observations (for which we loosely use the term “projection effects”) if we want to apply the EFT-based Bayesian forward modeling to real data.

Let us consider what happens on sub-horizon scales (that will be the focus of this work). There, projection effects reduce to redshift-space distortions (RSDs), so what is necessary is an expression for the likelihood in redshift space. Luckily, the EFTofLSS in redshift space has been thoroughly investigated already, see e.g. [18–22]. The main ingredient we need to move from the rest-frame coordinates to redshift space is the galaxy velocity field $v_g$, and the equivalence principle strongly constrains the form of the EFT counterterms. Consider for example the well-known Kaiser formula for the redshift-space power spectrum [23]: the fact that we can use its quadrupole to test the growth rate without complications of bias is because on large scales the deterministic part of the galaxy velocity follows the matter velocity, $v_{g,\text{det}} = v$. The first EFT corrections enter at the derivative level, via operators like $v_{g,\text{det}} \supset \beta \nabla^2 v \nabla^2 v$.

Something similar happens for the stochasticity in the galaxy velocity, $\varepsilon_v$ in the notation of [3]. The equivalence principle forbids its power spectrum to have a constant part in the large-scale limit. For our purposes, the most important consequence is that the form of the conditional likelihood is fixed fully by the properties of the noise for the galaxy density in the rest frame, at leading order in a derivative expansion: no additional stochasticity is involved when we move to redshift space.

The main goal of the paper is to use this fact to compute the conditional EFT likelihood $P[\tilde{\delta}_g | \delta, v]$ to observe a redshift-space galaxy overdensity $\tilde{\delta}_g(\tilde{x})$ given a rest-frame matter and velocity fields $\delta(x)$, $v(x)$ at all orders in the deterministic bias expansion for $\delta_g$. Then, we study how our result connects to the perturbative treatment of the EFTofLSS, and discuss the difficulties (and ways around them) of implementing a cutoff on the galaxy and matter
fields (central to the EFT-based forward modeling) while retaining the full, nonperturbative form of the conditional likelihood. Finally, we look into what are the corrections coming from the noise in the galaxy velocity field and briefly touch on the survey window function. The short section below contains a more detailed summary.

Outline and summary of main results. The outline of the paper, and a summary of the main results, is as follows.

Section 2 contains a quick review of RSDs following section 9.3 of [3]. Then we summarize our notation and conventions and review the derivation of the rest-frame likelihood following [4, 5].

Section 3 derives the main result of the paper. We obtain the conditional likelihood for the redshift-space galaxy density at leading order in a derivative expansion but at all orders in perturbations (Sections 3.1 and 3.2). The result is

\[
P[\tilde{\delta}_g|\delta, v] = \left( \prod_{\tilde{x}} \sqrt{\frac{\tilde{J}[\delta, v](\tilde{x})}{2\pi P_{\epsilon g}^{(0)}}} \right) \exp \left( -\frac{1}{2} \int d^3\tilde{x} \frac{(\tilde{\delta}_g(\tilde{x}) - \tilde{\delta}_{g,\text{det}}[\delta, v](\tilde{x}))^2}{P_{\epsilon g}^{(0)} / \tilde{J}[\delta, v](\tilde{x})} \right),
\]

(1.1)

where:

- following the notation of [3] we denote the redshift-space coordinates by \( \tilde{x} \);
- \( \tilde{\delta}_g(\tilde{x}) \) are the data in redshift space;
- \( P_{\epsilon g}^{(0)} \) is the noise power spectrum, usually parameterized as \( \alpha / \pi \) for a dimensionless \( \alpha \);
- \( \tilde{\delta}_{g,\text{det}} \) is the deterministic bias expansion for the redshift-space galaxy overdensity. It is obtained by transforming to redshift space its rest-frame analog \( \delta_{g,\text{det}} \). For this reason it depends both on \( \delta \) and on the matter velocity \( v \);
- finally, we denote by \( \tilde{J} \) the Jacobian of the transformation from the rest frame to redshift space. In the distant-observer approximation \( \hat{n} = \text{const.} \) (\( \hat{n} \) being the line of sight) it is equal to \( 1 + \hat{n} \cdot \partial ||v_g(x)/H \) evaluated at \( x = x(\hat{x}) \), and with \( v_g = v_{g,\text{det}}[\delta, v] \). It depends on the matter overdensity and the matter velocity field because the deterministic bias expansion for the galaxy velocity and the coordinate change \( x = x(\hat{x}) \) depend on them.

Writing explicitly the Jacobian in the distant-observer approximation, eq. (1.1) then becomes

\[
P[\tilde{\delta}_g|\delta, v] = \left( \prod_{\tilde{x}} \sqrt{\frac{1 + \hat{n} \cdot \partial ||v_{g,\text{det}}[\delta, v](x(\hat{x}))/H}{2\pi P_{\epsilon g}^{(0)}}} \right) \times \exp \left( -\frac{1}{2} \int d^3\tilde{x} \frac{(\tilde{\delta}_g(\tilde{x}) - \tilde{\delta}_{g,\text{det}}[\delta, v](\tilde{x}))^2}{P_{\epsilon g}^{(0)}} \right) \left( 1 + \frac{\hat{n} \cdot \partial ||v_{g,\text{det}}[\delta, v](x(\hat{x}))}{H} \right).
\]

(1.2)

Section 3.3 contains multiple checks of eq. (1.1), such as the fact that we must obtain a Dirac delta functional setting \( \tilde{\delta}_g \) equal to \( \tilde{\delta}_{g,\text{det}} \) when the noise goes to zero (which, given that the likelihood is Gaussian in the data \( \delta_g \), is equivalent to requiring that its integral over \( \tilde{\delta}_g \) equals 1). This is explicit in eq. (1.1), thanks to the fact that the overall factor in front of the exponential is written as a product over redshift-space coordinates \( \tilde{x} \). Finally, we show
that it is possible to augment eq. (1.1) to include the modulation of the rest-frame noise by long-wavelength operators (effectively equivalent to a stochasticity of the bias coefficients in $\delta_{g,\text{det}}$). This amounts to replacing $P_0$ with a nonnegative “field-dependent covariance”.

**Section 4.** The conditional likelihood of eq. (1.1) above is written in real space. This is necessary if we want an expression that is valid at all orders in perturbations, since stopping at leading order in derivatives means that we always deal with local functionals. This can, however, obscure the link with perturbative approaches. In sections 4.1 and 4.2 we show that once we cutoff the Fourier modes of the fields at a scale $\Lambda$, it is straightforward to recover the expansion parameters of the EFTofLSS. This procedure allows us also to address the positivity of the Jacobian $\tilde{J}$ in eq. (1.1). The use of filtered fields (both galaxy and matter ones) is central to current applications of the EFT likelihood [12, 13]: in section 4.3 we discuss how the presence of the field-dependent Jacobian makes it complicated to retain a closed form of the likelihood and have it normalized with respect to the data, i.e. the galaxy field, once this is cut at $\Lambda$. Fortunately we also show that, as long as we stop at second order in the bias expansion, it is consistent to neglect the dependence of $\tilde{J}$ on $\delta$ and $\nu$, hence allowing an analytical normalization of the likelihood. We also briefly investigate how it is possible to bypass this problem while keeping the full dependence of the Jacobian on the matter density and velocity fields.

**Section 5** studies the impact of the stochasticity in the galaxy velocity field, $v_g = v_{g,\text{det}} + \varepsilon_v$. As discussed in the introduction, this is expected to be subleading on large scales. After reviewing the power spectrum of $\varepsilon_v$, we confirm this via explicit computation in a way similar to section 4.2. Figure 1 summarizes the relative importance of these contributions with respect to the ones studied in sections 4.1 and 4.2.

**Section 6** concludes the paper with a brief discussion of astrophysical selection effects and, especially, of the survey window function.

**Appendices A, B and C** contain some details on the calculations of sections 3, 4 and 5, respectively.

### 2 RSDs, notation and review of the rest-frame likelihood

#### 2.1 Redshift-space distortions

We start by a quick review of redshift-space distortions (we refer e.g. to [24–26] and references therein for additional details). In this section we follow closely the notation of [3] (see their section 9.3), while from the next section to the end of the paper we will use a modified version that better highlights how the transformation to redshift space depends on the matter density and velocity fields.

All tracers of large-scale structure are effectively observed via photon arrival directions ($\hat{n}$) and redshifts ($z$), inferred from the shift in frequency of the observed spectral energy distribution of the galaxy relative to the rest-frame frequency. Hence, an essential ingredient in the interpretation of large-scale structure is the mapping from rest-frame quantities to observations (see [27] for a concise recent review of the subject).

We can relate the observed position ($z, \hat{n}$) to the position of the galaxy in a global coordinate system $x^\mu = (\eta, \mathbf{x})$ by solving the geodesic equation from the observer’s location to the source, given the photon momentum at the observer specified by ($z, \hat{n}$). We can
also associate a “fiducial” position \( \tilde{x}^\mu = (\tilde{\eta}, \tilde{x}) \) to the galaxy by solving the same geodesic equation in a purely FLRW spacetime.\(^1\) The difference \( \Delta x \) between \( x \) and \( \tilde{x} \) effectively defines a coordinate transformation from observed to true galaxy positions, and we can find the expression for the observed galaxy density by computing how the zeroth component of the galaxy number current transforms under it.

On subhorizon scales \( k \gg \mathcal{H} \), which will be main focus of this work, the gravitational redshift terms (which involve the Newtonian potential directly) are negligible, as is the component of \( \Delta x \) orthogonal to the line of sight. This implies that the coordinate shift from the rest frame to redshift space is purely spatial and parallel to the line of sight. It is given by

\[
\tilde{x} = x + u_|| (x) \hat{n}(x) ,
\]  

where

\[
u_g = \frac{v_g(x)}{H} , \quad u_|| (x) = \hat{n}(x) \cdot u(x) , \quad \hat{n}(x) = \frac{x}{|x|} .
\]

Here \( v_g \) is the galaxy velocity, and for simplicity of notation we will not add a subscript to \( u_|| \).

If we make the assumption that the transformation of eq. (2.1) is one-to-one, which is true on perturbative scales (we will come back to this in section 4.2), we can obtain a fully nonlinear expression for the observed galaxy density perturbation as

\[
\tilde{\delta}_g (\tilde{x}) = \frac{1 + \delta_g (x)}{1 + \partial_|| u_|| (x)} - 1 .
\]

On the right-hand side we intend everything evaluated at \( x = x(\tilde{x}) \), and we have used the relation

\[
\left| \frac{\partial x^i}{\partial \tilde{x}^j} \right| = \left| \delta_j^i + \hat{n}^j \frac{\partial u_|| (x)}{\partial x^i} \right|^{-1} = \frac{1}{1 + \partial_|| u_|| (x)} ,
\]

where again on the right-hand side everything is evaluated at \( x = x(\tilde{x}) \), and \( \partial_i \) is defined as \( \hat{n} \cdot \nabla \) in eqs. (2.3), (2.4). This relation holds in the distant-observer approximation, where we approximate \( \hat{n} \) as slowly-varying over the survey volume. The final result of this paper, eq. (1.1), is not dependent on this approximation: as we will see in the next sections it will hold whatever the form of the Jacobian \( |\partial x^i/\partial \tilde{x}^j| \) is.

### 2.2 Notation and conventions

For the purposes of this paper it is fundamental to keep in mind the fact that the transformation to redshift space depends on galaxy velocity field. Let us then write eq. (2.1) as

\[
x = \mathcal{R}[v_g] (\tilde{x}) , \quad \tilde{x} = \mathcal{R}^{-1}[v_g] (x) .
\]

The Jacobian of the transformation from redshift space to the rest-frame coordinates is given by

\[
\left| \frac{\partial \mathcal{R}^{-1}[v_g] (x)}{\partial x} \right| = 1 + \partial_|| u_|| (x) \equiv J[v_g] (x) ,
\]

while the Jacobian of the inverse transformation is given by

\[
\left| \frac{\partial \mathcal{R}[v_g] (\tilde{x})}{\partial \tilde{x}} \right| = \frac{1}{J[v_g] (\tilde{x})} , \quad \text{with} \quad J[v_g] (\tilde{x}) = J[v_g] (\mathcal{R}[v_g] (\tilde{x})) .
\]

\(^1\)When also fixing a fiducial cosmology we obtain the additional Alcock-Paczyński distortions [28].
Consequently, eq. (2.3) becomes

\[ \tilde{\delta}_g(\tilde{x}) = \frac{1 + \delta_g(\mathcal{R}[v_g](\tilde{x}))}{f[v_g](\tilde{x})} - 1. \]  

(2.8)

Let us now move to the bias expansion, that will allow us to streamline the notation of eq. (2.8) a bit. First, we recap the bias expansion for the galaxy density field in the rest frame. If we define the nonlinear matter field as \( \delta \), we can write the deterministic bias relation as

\[ \delta_g(x) = \delta_{g,\text{det}}[\delta](x), \]  

(2.9)

where the functional \( \delta_{g,\text{det}}[\delta] \) contains all the operators constructed from the nonlinear matter field. Let us write it as

\[ \delta_{g,\text{det}}[\delta] = \sum_O b_O O[\delta]. \]  

(2.10)

Here we use the basis of [29] (see also sections 2.2–2.5 of [3], and see [30] for an alternative basis) to write the bias expansion at a fixed time. Then, in the rest frame and up to second order in perturbations (and leading order in derivatives) we have

\[ \delta_{g,\text{det}}[\delta](x) = b_1 \delta(x) + \frac{b_2}{2} \delta^2(x) + b_K K^2[\delta], \]  

(2.11)

where \( K^2 = K_{ij} K^{ij} \) and the tidal field \( K_{ij}[\delta] \) is equal to \( (\partial_i \partial_j / \nabla^2 - \delta_{ij}/3) \delta \).

We can then look at the galaxy velocity \( v_g \). The matter velocity field \( v \) plays a fundamental role in its deterministic bias expansion. More precisely, the equivalence principle ensures that at leading order in derivatives we have

\[ v_{g,\text{det}}[\delta,v](x) = v(x), \]  

(2.12)

i.e. we have \( \beta_v = 1 \) (we use the same notation as [3] for velocity-bias parameters). Terms schematically the form \( (\delta \cdots \delta)v \), or generically \( O[\delta]v \), are likewise forbidden. The leading correction takes the form [31, 32]

\[ v_{g,\text{det}}[\delta,v](x) \supset \beta \nabla^2 v \nabla^2 v(x), \]  

(2.13)

which is degenerate with \( \nabla \delta \) at linear order in perturbations. Notice that on the right-hand side we have allowed for a generic functional dependence on both the nonlinear matter field \( \delta \) and the nonlinear velocity field \( v \): a complete enumeration of all the operators contained in \( v_{g,\text{det}}[\delta,v] \) at leading order in derivatives, together with the proof that also for the velocity field it is possible to write a bias expansion at a fixed time, can be found in [29] (see also appendix B.5 of [3]).

As the reader might have noticed, in this section (and in section 2.1 as well) we have focused on the deterministic bias expansion for the galaxy density and velocity fields. A more detailed discussion of the noise \( v_g - v_{g,\text{det}}[\delta,v] \) is left to section 5: for the moment we emphasize that, in a derivative expansion, the leading source of noise is only the rest-frame noise \( \epsilon_g \), whose power spectrum (~ \( k^0 \) on large scales) is usually parameterized as \( \alpha / \pi_g \) for some dimensionless \( \alpha \) expected to be of order 1. This is also reviewed in the next section.

The fact that we focus on the deterministic bias expansion for \( v_g \) allows us to simplify the notation greatly. Until section 5 we will always understand \( v_g = v_{g,\text{det}}[\delta,v] \) in the coordinate transformation \( \mathcal{R} \): therefore we will drop the functional dependence of the latter on \( v_g \), i.e.

\[ \mathcal{R}[v_{g,\text{det}}[\delta,v]] \equiv \mathcal{R}. \]  

(2.14)
Similarly, since the Jacobian of the coordinate change also depends on $\delta$ and $v$ via $v_{\text{g,det}}$, we write
\begin{equation}
J[v_{\text{g,det}}[\delta, v]](x) \equiv J[\delta, v](x),
\end{equation}
and
\begin{equation}
\tilde{J}[v_{\text{g,det}}[\delta, v]](\tilde{x}) = J[v_{\text{g,det}}[\delta, v]](\mathcal{A}(\tilde{x})) \equiv \tilde{J}[\delta, v](\tilde{x}),
\end{equation}
which is consistent with the notation in eq. (1.1). Finally, we also define
\begin{equation}
\tilde{\delta}_{\text{g,det}}[\delta, v](\tilde{x}) = \frac{1 + \delta_{\text{g,det}}[\delta](\mathcal{A}(\tilde{x}))}{J[\delta, v](\tilde{x})} - 1.
\end{equation}
This functional is what appears in eq. (1.1).

We conclude this section with a recap of our notation for functional derivatives (importantly, both $x$ and $\tilde{x}$ are cartesian coordinates, so the following relations work equally well in the rest-frame coordinates and redshift space). Given a field $\chi(x)$, we have
\begin{equation}
\frac{\partial \chi(x)}{\partial \chi(y)} = \delta_D^{(3)}(x - y).
\end{equation}
This is the generalization of $\partial x^i / \partial x^j = \delta^i_j$. The right-hand side is dimensionful since we need to satisfy the relation
\begin{equation}
\frac{\partial}{\partial \chi(y)} \int d^3 x \chi(x) = 1,
\end{equation}
i.e. the equivalent of $\partial(\sum_i x^i) / \partial x^j = 1$. We define the functional Dirac delta by
\begin{equation}
\int D\chi F[\chi] \delta_D^{(\infty)}(\chi - \varphi) = F[\varphi]
\end{equation}
for any functional $F[\chi]$. Given the functional measure $D\chi = \prod_x d\chi(x)$, we can see that for practical purposes $\delta_D^{(\infty)}(\chi - \varphi)$ is a product of one-dimensional Dirac delta functions of $\chi(x) - \varphi(x)$ at each $x$. Analogous definitions (with $\delta_D^{(3)}(x - x') \rightarrow (2\pi)^3 \delta_D^{(3)}(k + k')$, etc.) hold for functionals of momentum-space fields.

### 2.3 Review of the rest-frame EFT likelihood

In this section we review the results of [5], where the rest-frame EFT likelihood was derived under the assumption of Gaussian noise and no stochasticity of the bias coefficients.

The difference between $\delta_\varphi$ and $\delta_{\text{g,det}}[\delta]$ that arises from integrating out short-scale modes that cannot be described within the EFT is captured by a noise $\varepsilon_\varphi(x)$. Let us assume that the noise is Gaussian with power spectrum $P_{\varepsilon_\varphi}(k)$. Locality (i.e. the fact that the error we make in describing galaxy clustering via eq. (2.10) at $x_1$ and $x_2$ is uncorrelated in the limit of large $|x_1 - x_2|$) and the absence of preferred directions impose that the noise power spectrum is analytic in $k^2 = |k|^2$, i.e.
\begin{equation}
P_{\varepsilon_\varphi}(k) = P_{\varepsilon_\varphi}^{(0)} + P_{\varepsilon_\varphi}^{(2)} k^2 + \ldots.
\end{equation}
The coefficients $P_{\varepsilon_\varphi}^{(n)}$ have dimensions of a length to the power $n + 3$: $P_{\varepsilon_\varphi}^{(0)}$ fixes the size of the noise (this is what is typically taken to be $\alpha / \bar{n}_g$), while we expect that for $n \geq 2$ we have
\begin{equation}
\frac{P_{\varepsilon_\varphi}^{(n)}}{P_{\varepsilon_\varphi}^{(0)}} \sim R^n,
\end{equation}
where
where $R_*$ is the typical nonlocality scale of galaxy formation. For dark matter halos, $R_*$ is expected to be of order of the halo Lagrangian radius $R(M_h)$ or of order of the nonlocality scale for matter $\sim 1/k_{NL}$ (that is the scale at which the dimensionless linear matter power spectrum becomes of order one), whichever is larger (see e.g. [33–36] for a discussion of deviations of $\alpha$ from 1 and for the scale dependence of $P_{\epsilon_g}(k)$).

Let us then take a wavenumber $\Lambda$ smaller than $1/R_*$. We can split the noise field in a short-wavelength part and a long-wavelength part. More precisely, the short-wavelength one is obtained by subtracting

$$\epsilon_{g,\Lambda}(k) = \epsilon_g(k) \Theta_H(\Lambda^2 - k^2)$$

(2.23)

from $\epsilon_g(k)$, where $\Theta_H$ is the Heaviside theta function. Since we are assuming the noise to be Gaussian the likelihood for the short modes and the long modes factorizes, as does the functional measure $D\epsilon_g$. Given that we cannot reliably describe short-wavelength modes, we can just integrate out the short-wavelength component of the noise, and remain with a likelihood for $\epsilon_{g,\Lambda}(k)$ only.

What is this likelihood? Since we have chosen $\Lambda$ such that the higher-derivative terms of eq. (2.21) are negligible, we can write it as

$$\mathcal{P}[\epsilon_g] = \left( \prod_{|k| \leq \Lambda} \sqrt{\frac{1}{2\pi P_{\epsilon_g}^{(0)}}} \right) \exp \left( -\frac{1}{2} \int_{|k| \leq \Lambda} \frac{|\epsilon_{g,\Lambda}(k)|^2}{P_{\epsilon_g}^{(0)}} \right).$$

(2.24)

The normalization of eq. (2.24) is such that, if $P_{\epsilon_g}^{(0)} \rightarrow 0$, we recover a Dirac delta functional that sets $\epsilon_{g,\Lambda}$ to zero. Before proceeding, let us discuss the assumption of Gaussian likelihood (we go back to this in section 4.2, see also figure 1). Is there a small parameter that allows us to expand around a Gaussian? The fact that we are restricting ourselves to long wavelengths ensures that higher-order $n$-point functions of the noise are suppressed. For example, consider the noise bispectrum. Via the same arguments that lead to eq. (2.21), we can take it to be a constant on large scales. By dimensional analysis we can take this constant to be a dimensionless $S_{\epsilon_g}$ times $P_{\epsilon_g}^{(0)}$ squared ($S_{\epsilon_g}$ is 1 for a Poisson likelihood, for example). The non-Gaussianity of the noise likelihood, then, is controlled by $S_{\epsilon_g}$ times the typical size of a noise fluctuation on a scale $\Lambda$, which scales as

$$\sqrt{P_{\epsilon_g}^{(0)}} \Lambda^3.$$

(2.25)

Hence, as long as $\Lambda$ is smaller than $S_{\epsilon_g}^{-2/3}(\bar{n}_g/\alpha)^{1/3}$, we are justified in expanding around a Gaussian likelihood.

Let us then multiply this likelihood by a (“Fourier-space”) Dirac delta functional

$$\delta_D^{(\infty)}(\delta_{g,\Lambda}(k) - \delta_{g,\text{det},\Lambda}[\delta_{\Lambda}](k) - \epsilon_{g,\Lambda}(k)).$$

(2.26)

Here we have cut both $\delta_{g}$ and $\delta_{g,\text{det}}$ at $\Lambda$, and we have constructed the deterministic galaxy field from the matter field cut also at $\Lambda$. This is the same procedure that was originally described in [4]. We will come back to it in section 4.1.

If we now functionally integrate over $\epsilon_{g,\Lambda}$, we obtain the conditional likelihood for the galaxy field given the matter field, i.e.

$$\mathcal{P}[\delta_{g,\Lambda}|\delta_{\Lambda}] = \left( \prod_{|k| \leq \Lambda} \sqrt{\frac{1}{2\pi P_{\epsilon_g}^{(0)}}} \right) \exp \left( -\frac{1}{2} \int_{|k| \leq \Lambda} \frac{|\delta_{g}(k) - \delta_{g,\text{det}}[\delta_{\Lambda}](k)|^2}{P_{\epsilon_g}^{(0)}} \right).$$

(2.27)
Here we have used the fact that the data and the deterministic galaxy density field have both support for $|k| \leq \Lambda$ to remove the cutoff from the fields themselves and replace it by a cutoff in the integral $\int k$, using the fact that these two fields appear quadratically in the likelihood.

Thanks to the fact that both the galaxy field and its deterministic expression in terms of $\delta_{\Lambda}$ appear quadratically in the exponent of eq. (2.27), we can switch to real space. More precisely, we can write

$$P[\delta_g, \delta_{\Lambda} | \delta_{\Lambda}] = \left( \prod_x \sqrt{\frac{1}{2\pi P_{\varepsilon_g}^{(0)}}} \right) \exp \left( -\frac{1}{2} \int d^3x \frac{\left( \delta_{g,\Lambda}(x) - \delta_{g,\text{det},\Lambda}[\delta_{\Lambda}](x) \right)^2}{P_{\varepsilon_g}^{(0)}} \right) ,$$

(2.28)

where the “$\Lambda$” subscripts stand for the fact that: (i) we cut the field $\delta_g$ in Fourier space and transform it back to real space; (ii) we construct $\delta_{g,\text{det}}$ from $\delta_{\Lambda}$, we cut it in Fourier space, and then transform it to real space. We then take the difference between $\delta_{g,\Lambda}(x)$ and $\delta_{g,\text{det},\Lambda}[\delta_{\Lambda}](x)$, square it, and integrate it over all $x$. Effectively, this tells us that it makes sense to write

$$P[\delta_g | \delta] = \left( \prod_x \sqrt{\frac{1}{2\pi P_{\varepsilon_g}^{(0)}}} \right) \exp \left( -\frac{1}{2} \int d^3x \frac{\left( \delta_g(x) - \delta_{g,\text{det}}[\delta](x) \right)^2}{P_{\varepsilon_g}^{(0)}} \right) ,$$

(2.29)

if we assume that the fields $\delta$ and $\delta_{g,\text{det}}[\delta]$ appearing in the integral above are cut at a scale longer than $1/R_*$, defined by eq. (2.22). In eqs. (2.28), (2.29) we have written the overall normalization as a real-space product for simplicity. It must also be intended as filtered: we will investigate this point in more detail in section 4.3. The higher-derivative stochasticities, i.e. the higher orders in an expansion of the noise power spectrum in $R_*^2 k^2$ are under perturbative control in real space as long as $\Lambda < 1/R_*$ (for more details we refer to section 4.3 of [37]).

In the next section we will start from eq. (2.29) to derive an expression for the conditional likelihood in redshift space. It is important to emphasize that we will work in the infinite-$\Lambda$ limit. This is necessary if we want to achieve a nonperturbative expression for the redshift-space likelihood: indeed, all the manipulations we will make rely strongly on the transformation from the rest frame to redshift space being a local functional of the galaxy density in real space, cf. eq. (2.17). We discuss how to connect to the results above (and by extension to [4, 12, 13], where $\Lambda$ is kept finite), in section 4.1. Let us elaborate more on the infinite-$\Lambda$ limit. This must be intended only as a “trick” that allows us to compute exactly the contribution to the likelihood of the coordinate change to redshift space, eq. (2.1). In this limit, there are many different contributions to the likelihood that also become important (the non-Gaussianity of the noise, the modulation of the noise by long-wavelength modes, etc.). By focusing only on the coordinate change in eq. (2.1), however, we are isolating at all orders in perturbations the terms that will not be affected by renormalization once we reintroduce a finite $\Lambda$ in section 4.1: even after short-scale modes are integrated out, the form of the “displacement” of the galaxy field to redshift space is unaffected (this is similar to what happens to the displacement from Lagrangian to Eulerian coordinates for the rest-frame likelihood, as discussed e.g. in [4, 5, 12]).

Before proceeding, we notice that in [37] we derived the impact of the stochasticities of bias coefficients on the conditional likelihood in the assumption that they follow a Gaussian probability distribution (on large scales this is the more relevant correction to a Gaussian likelihood with constant covariance). This effectively results in the replacement $P_{\varepsilon_g}^{(0)} \rightarrow P_{\varepsilon}^{[\delta]}$, where $P_{\varepsilon}^{[\delta]}$ is a positive-definite field-dependent covariance. We investigate how to include this in section 3.3.
3 Main result

First, let us define what we are after. For the purposes of Bayesian forward modeling, we still need the conditional likelihood for the data given the matter density and velocity in the rest frame. Indeed, this is what gravity-only N-body simulations most naturally output. The only difference with the conditional likelihood of section 2.3 is that we now want to have the data in redshift space. Hence we need the likelihood

$$P_{\tilde{\delta}_g | \delta, v} ,$$

which in terms of the joint likelihood $$P_{\tilde{\delta}_g, \delta, v}$$ and the likelihood for the matter density and velocity fields is given by

$$P_{\tilde{\delta}_g | \delta, v} = \frac{P_{\tilde{\delta}_g, \delta, v}}{P_{\delta, v}} .$$

Given that the relation between $$v_g$$ and $$\delta, v$$ is fully deterministic (its noise is discussed in section 5), we can compute the conditional likelihood of eq. (3.1) via the functional coordinate change summarized by eqs. (2.8), (2.14), (2.15), (2.16). We do this in section 3.1. Another way to arrive at $$P_{\tilde{\delta}_g | \delta, v}$$ is by integrating out the noise $$\varepsilon_g$$, similarly to what we do with the rest-frame likelihood. This provides an additional check of our end result. We do this in section 3.2.

Finally, in section 3.3 we perform additional checks of the soundness of our result, and discuss how to include the modulation of the noise power spectrum by the matter field.

3.1 Calculation via functional coordinate change

From eqs. (2.8), (2.14), (2.15), (2.16), we see that the coordinate change we need to do is

$$\tilde{\delta}_g(x) = \frac{1 + \delta_g(\mathcal{R}(x))}{\mathcal{J}[\delta, v](x)} - 1 ,$$

whose inverse is

$$\delta_g(x) = \mathcal{J}[\delta, v](x) \left( 1 + \tilde{\delta}_g(\mathcal{R}^{-1}(x)) \right) - 1 .$$

Importantly, $$\delta, v$$ do not change.

The fact that $$\delta, v$$ are not touched by the coordinate change has two consequences. First, thanks to eq. (3.2), it tells us that the transformation of the conditional likelihood is the same as that of the joint likelihood. That is, $$P_{\tilde{\delta}_g | \delta, v}$$ is given by

$$P_{\tilde{\delta}_g | \delta, v} = \frac{1}{\mathcal{J}[\delta, v](x)} \left( \frac{1}{2\pi P_{\varepsilon_g}^{(0)}} \right) \exp \left( -\frac{1}{2} \int d^3x \frac{(\delta_g(x) - \delta_g, \text{det}[\delta](x))^2}{P_{\delta_g}^{(0)}} \right)$$

with $$\delta_g(x) = \mathcal{J}[\delta, v](x) \left( 1 + \tilde{\delta}_g(\mathcal{R}^{-1}(x)) \right) - 1$$ in the exponent.

Here the expression for the rest-frame likelihood is the one of eq. (2.29), with $$\delta_g$$ in the exponent replaced with its expression in terms of $$\tilde{\delta}_g$$ as per eq. (3.4), and the overall factor is the functional Jacobian of the coordinate change (for which we have used a notation that
emphasizes how $\delta, v$ are not changed). Second, the continuum generalization of Leibniz’s rule for determinants\(^2\) ensures that

\[
\frac{1}{\left| \frac{\partial (\delta, \delta, v)}{\partial (\delta, \delta, v)} \right|} = \left| \frac{\partial \delta_g}{\partial \delta_g} \right| \left| \frac{\partial (\delta, v)}{\partial (\delta, v)} \right| = \left| \frac{\partial \delta_g}{\partial \delta_g} \right| .
\]

(3.7)

This is because the Jacobian matrix is triangular in field space, thanks to the fact that $\delta, v$ do not depend on the galaxy field. For such matrices the determinant is the product of the determinants on the diagonal.

Let us then compute the functional Jacobian of eq. (3.7). We only need the functional derivative

\[
\frac{\partial \delta_g(x)}{\partial \tilde{\delta}_g(x')} .
\]

(3.8)

From eq. (3.4), together with eq. (2.18), we see that it is equal to

\[
J_\delta v(x) \delta_D^{(3)}(\mathcal{R}^{-1}(x) - \tilde{x}') .
\]

(3.9)

We can then use the property of the Dirac delta function (valid for any invertible function $f$)

\[
\delta_D^{(3)}(f^{-1}(x) - \tilde{x}') = \delta_D^{(3)}(x - x') \left| \frac{\partial f^{-1}(x)}{\partial x} \right|
\]

(3.10)

together with eq. (2.6), i.e.

\[
\left| \frac{\partial \mathcal{R}^{-1}(x)}{\partial x} \right| = J_\delta v(x) ,
\]

(3.11)

to find

\[
\frac{\partial \delta_g(x)}{\partial \tilde{\delta}_g(x')} = \delta_D^{(3)}(x - x') ,
\]

(3.12)

which implies

\[
\left| \frac{\partial \delta_g}{\partial \delta_g} \right| = 1 .
\]

(3.13)

Switching to $\tilde{x}$ in the exponent of eq. (3.5), we then find

\[
P[\tilde{\delta}_g|\delta, v] = \left( \prod_x \sqrt{\frac{1}{2\pi \sigma_g^2}} \right) \exp \left( -\frac{1}{2} \int d^3 \tilde{x} \left( \frac{\tilde{\delta}_g(\tilde{x}) - \delta_g, \text{det} [\delta, v](\tilde{x})^2}{P_g^{(10)}} / J_\delta v(\tilde{x}) \right) \right) ,
\]

(3.14)

where the deterministic galaxy overdensity in redshift space is given by eq. (2.17), and the determinant $J_\delta v(\tilde{x})$ is the one of eq. (2.16).

\(^2\)I.e.

\[
\det \begin{pmatrix} A & B \\ 0 & D \end{pmatrix} = \det A \det D = \det \begin{pmatrix} A & 0 \\ C & D \end{pmatrix}
\]

(3.6)

for any $n \times n$ matrix $A$ and $m \times m$ matrix $D$. 


3.2 Integrating out the noise

In this section we arrive at the same result of eq. (3.14) via another route, i.e. by integrating out the noise $\varepsilon_g(x)$ similarly to what we do to arrive at the rest-frame likelihood.

Once we account for the noise $\varepsilon_g(x)$, eqs. (2.8), (2.17) tell us that the relation between $\tilde{\delta}_g(\tilde{x})$ and $\delta_g$ is

$$\tilde{\delta}_g(\tilde{x}) = \delta_g,_{\text{det}}[\delta](x) + \frac{\varepsilon_g(\mathcal{R}(\tilde{x}))}{J[\delta, v](\tilde{x})}. \quad (3.15)$$

Let us then multiply the Gaussian likelihood for $\varepsilon_g(x)$, that for a constant noise power spectrum can be written as

$$P[\varepsilon_g] = \left( \prod_x \sqrt{\frac{1}{2\pi P_{\varepsilon_g}(x)}} \right) \exp \left( -\frac{1}{2} \int d^3x \frac{\varepsilon_g^2(x)}{P_{\varepsilon_g}(x)} \right), \quad (3.16)$$

by a Dirac delta functional

$$\delta_D^{(\infty)} \left( \tilde{\delta}_g(\tilde{x}) - \tilde{\delta}_g,_{\text{det}}[\delta](\tilde{x}) - \frac{\varepsilon_g(\mathcal{R}(\tilde{x}))}{J[\delta, v](\tilde{x})} \right) \quad (3.17)$$

and functionally integrate over $D\varepsilon_g = \prod_x d\varepsilon_g(x)$. To do this, let us define

$$\tilde{\varepsilon}_g(\tilde{x}) = \frac{\varepsilon_g(\mathcal{R}(\tilde{x}))}{J[\delta, v](\tilde{x})}. \quad (3.18)$$

The functional generalization of the change-of-coordinates rule for the Dirac delta functional makes eq. (3.17) equal to

$$\frac{1}{\partial \tilde{\varepsilon}_g(\tilde{x}) / \partial \varepsilon_g(x')} \delta_D^{(\infty)} \left( \varepsilon_g(x) + \left\{ 1 + \delta_g,_{\text{det}}[\delta](x) \right\} - J[\delta, v](x) \left\{ 1 + \dot{\delta}_g(\mathcal{R}^{-1}(x)) \right\} \right). \quad (3.19)$$

The overall Jacobian is straightforward to evaluate. From eq. (3.18) we need to evaluate the functional determinant of

$$\frac{\delta_D^{(3)}(\mathcal{R}(\tilde{x}) - x')}{J[\delta, v](\tilde{x})}, \quad (3.20)$$

which is equal to $\delta_D^{(3)}(\tilde{x} - \tilde{x}')$ thanks to eq. (2.7) and the properties of the three-dimensional Dirac delta function. Hence we have

$$\left| \frac{\partial \tilde{\varepsilon}_g(\tilde{x})}{\partial \varepsilon_g(x')} \right| = 1. \quad (3.21)$$

Integrating $\varepsilon_g(x)$ out via eqs. (3.19), (3.21) then gives

$$P[\delta_g, \delta, v] = \left( \prod_x \sqrt{\frac{1}{2\pi P_{\varepsilon_g}(x)}} \right) \times \exp \left( -\frac{1}{2} \int d^3x \left\{ 1 + \delta_g(\mathcal{R}^{-1}(x)) - \left( 1 + \delta_g,_{\text{det}}[\delta](x) \right) / J[\delta, v](x) \right\}^2 \right) \quad (3.22)$$

Finally, switching to redshift space in the integral in the exponent we obtain the same result as in eq. (3.14). In appendix A we confirm this result a final time via manipulations very similar to the above.
3.3 Limit of zero noise, normalization and stochasticity of bias coefficients

In this section we look in more detail at the result of eqs. (1.1), (3.14).

- First, we confirm that in the limit $P_{\varepsilon_g}^{[0]} \rightarrow 0$ we obtain a Dirac delta functional that sets $\tilde{\delta}_g(\tilde{x})$ equal to its deterministic expression of eq. (2.17).

- Related to this, we check that our likelihood has the correct normalization with respect to $\tilde{\delta}_g(\tilde{x})$. That is, we check that integrating it over $\tilde{\delta}_g(\tilde{x})$ gives 1. As a consequence, we prove that indeed eq. (3.14) is equal to eq. (1.1). There is also an important advantage in moving from eq. (3.14) to eq. (1.1), since in the latter the likelihood is written exclusively in redshift-space coordinates $\tilde{x}$, that are the coordinates used in observations.

- Finally, we discuss how to include the impact of the stochasticity of bias coefficients following [37], that gives the leading correction to the rest-frame likelihood with constant noise power spectrum.

Limit of zero noise. Consider the probability distribution of $\varepsilon_g(x)$, i.e. eq. (3.16). In the limit $P_{\varepsilon_g}^{[0]} \rightarrow 0$ it is equal to a Dirac delta functional that sets $\varepsilon_g(x)$ identically equal to zero. Hence, if we multiply it by eq. (3.17) and integrate over the noise, we must obtain

$$\delta_D^{(\infty)}(\tilde{\delta}_g(x) - \tilde{\delta}_g, \text{det}[\delta,\nu](x)),$$

where the deterministic galaxy overdensity in redshift space is defined by eq. (2.17).

To see how this works out we take the limit $P_{\varepsilon_g}^{[0]} \rightarrow 0$ in eq. (3.22). We obtain

$$\left(\prod_x \frac{1}{J[\delta,\nu](x)}\right) \delta_D^{(\infty)}(\tilde{\delta}_g(\mathcal{R}^{-1}(x)) - \tilde{\delta}_g, \text{det}[\delta,\nu](\mathcal{R}^{-1}(x))).$$

We can then use the properties of the Dirac delta functional to change the argument of the two fields. The overall factor we get from the change of variables is

$$\frac{1}{|\delta_D^{(3)}(\mathcal{R}^{-1}(x) - \tilde{x}'|)}$$

where, as in all the equations above, we denote the determinant by $|\cdot|$ (here the determinant of a matrix in $x, x'$). Eq. (3.25) is equal to

$$\left(\prod_x \frac{1}{J[\delta,\nu](x)}\right)^{-1}$$

thanks to eq. (2.6) and the properties of the three-dimensional Dirac delta function, hence confirming eq. (3.23).

Normalization of the likelihood. With similar manipulations we can show that the redshift-space conditional likelihood is normalized if we integrate over the data, i.e. in $\mathcal{D}\tilde{\delta}_g = \prod_x \text{d}\tilde{\delta}_g(\tilde{x})$. In order to do this we switch the argument of the fields in the likelihood back to $x$, that is we take eq. (3.22), and perform the corresponding change of variables in the functional measure. We define

$$\tilde{\delta}_g(\mathcal{R}^{-1}(x)) = \Delta(x).$$
Thanks to the Jacobian of the inverse being the inverse of the Jacobian, the functional measure changes into

$$D \tilde{\delta}_g = D \Delta \frac{1}{|\partial \Delta(x)|} , \quad (3.28)$$

where $D \Delta = \prod_x d\Delta(x)$ and

$$|\frac{\partial \Delta(x)}{\partial \tilde{\delta}_g(x')}| = |\delta^{(3)}_{\Delta} (\tilde{x}^{-1}(x) - \tilde{x}')| = \prod_x \frac{1}{J[\delta,v](x)} , \quad (3.29)$$

as in eqs. (3.25), (3.26).

Hence, combining eqs. (3.28), (3.29) with eq. (3.22), more precisely the fact that the overall factor multiplying the exponential is now

$$\prod_x \sqrt{\frac{J^2[\delta,v](x)}{2\pi P_{\varepsilon g}^{(0)}}} , \quad (3.30)$$

after a simple shift of integration variables we see that the integral in $D \Delta$ is equal to 1.

It is then useful to recast our result in a form that makes more clear what is the limit of zero noise and the normalization of the likelihood. This just amounts to writing the overall normalization as a product over redshift-space coordinates as in eq. (1.1), i.e.

$$P[\tilde{\delta}_g|\delta,v] = \left( \prod_x \sqrt{\frac{J[\delta,v](\tilde{x})}{2\pi P_{\varepsilon g}^{(0)}}} \right) \exp \left( -\frac{1}{2} \int d^3 \tilde{x} \frac{\left( \tilde{\delta}_g(\tilde{x}) - \tilde{\delta}_g,\text{det}[\delta,v](\tilde{x}) \right)^2}{P_{\varepsilon g}^{(0)}} \right) . \quad (3.31)$$

**Stochasticity of bias coefficients.** All of the manipulations of this section go through in the same way if the constant noise power spectrum is replaced by a field-dependent one, i.e.

$$P_{\varepsilon g}^{(0)} \rightarrow P_{\varepsilon}[\delta] . \quad (3.32)$$

This is (at leading order in derivatives) the effect that the stochasticity of bias coefficients, which describes the fact that the noise generated by integrating short-scale modes can be modulated by long-wavelength perturbations, has on the rest-frame likelihood [37]. More precisely we have

$$P_{\varepsilon}[\delta](x) = P_{\varepsilon g}^{(0)} + 2 \sum_O P_{\varepsilon g,\varepsilon g,\delta}^{(0)} O[\delta](x) + \sum_{O,O'} P_{\varepsilon g,\varepsilon g,\delta}^{(0)} O[\delta](x) O'[\delta](x) , \quad (3.33)$$

where the bias coefficients $b_O$ and the operators $O[\delta]$ are defined in eq. (2.10), and $P_{\varepsilon}[\delta] \geq 0$ as long as the power spectra of the noises are such that the matrix

$$\begin{pmatrix}
  P_{\varepsilon g}^{(0)} & P_{\varepsilon g,\varepsilon g,\delta}^{(0)} & \cdots & P_{\varepsilon g,\varepsilon g,\delta}^{(0)} & \cdots \\
  P_{\varepsilon g,\delta}^{(0)} & P_{\varepsilon g}^{(0)} & \cdots & P_{\varepsilon g,\delta}^{(0)} & \cdots \\
  \vdots & \vdots & \ddots & \vdots & \ddots \\
  P_{\varepsilon g,\delta}^{(0)} & P_{\varepsilon g,\delta}^{(0)} & \cdots & P_{\varepsilon g}^{(0)} & \cdots \\
  \vdots & \vdots & \cdots & \vdots & \cdots 
\end{pmatrix} \quad (3.34)$$
is positive-definite (the fields $\varepsilon_{g,O}$ describe the stochasticity of the bias coefficients via the shift $b_O \rightarrow b_O + \varepsilon_{g,O}$ in the deterministic bias expansion. For simplicity of notation, and following [3], we have used the shorthand $P^{(0)}_{\varepsilon_{g,O}^*} = P_{\varepsilon_{g,O}^*}$).

The replacement of eq. (3.32) leads, then, to the final expression for the conditional likelihood in redshift space, i.e.

$$
P[\tilde{\delta}_g|\delta,v] = \left( \prod_{\tilde{x}} \frac{\sqrt{J[\delta,v](\tilde{x})}}{2\pi P_\varepsilon[\delta,v](\tilde{x})} \right) \exp \left( -\frac{1}{2} \int d^3\tilde{x} \frac{\left( \tilde{\delta}_g(\tilde{x}) - \tilde{\delta}_{g,det}(\tilde{x}) \right)^2}{P_\varepsilon[\delta,v](\tilde{x})/J[\delta,v](\tilde{x})} \right),$$

where we have defined

$$
P_\varepsilon[\delta,v](\tilde{x}) = P_\varepsilon[\delta](\tilde{x}),$$

with the additional dependence on $v$ coming from the change of the argument, cf. eq. (2.14).

4 Cutoffs and perturbativity

So far we have worked in the infinite-$\Lambda$ limit. As discussed at the end of section 2.3, this was necessary to arrive at an expression for the likelihood valid at all orders in perturbation theory. In turn, this is necessary if one wants to sample the likelihood numerically: perturbative (Edgeworth-like) expansions of the likelihood are not suitable to numerical manipulations. Let us now discuss how a finite $\Lambda$ can be reinstated and, most importantly, what are the consequences.

4.1 Reintroducing the cutoff

In section 2.3 we have seen that, for the Gaussian likelihood with constant noise power spectrum of [4, 5, 12], the cutoff was implemented in the following way:

- we construct the deterministic bias expansion $\delta_{g,det}$ from a matter field $\delta$ cut at a scale $\Lambda$. After getting $\delta$ from, e.g., an N-body simulation, one can transform it to Fourier space, cut it at $\Lambda$, transform it back to real space and construct the (local in real space) deterministic bias expansion out of it (refs. [4, 12, 13] follow this procedure);
- once we have the real-space $\delta_{g,det}[\delta_\Lambda]$, we can transform it to Fourier space, cut it, and then transform it back to real space;
- the data $\delta_g$ are also cut at $\Lambda$ following the same procedure.

The exponent in the likelihood, then, is constructed by taking the difference between the real-space $\delta_{g,\Lambda}$ and $\delta_{g,det,\Lambda}[\delta_\Lambda]$ constructed as above, squaring it, dividing it by $-2P^{(0)}_{\varepsilon_g}$ and integrating in $d^3x$.

To understand how this generalizes to our redshift-space likelihood we can first recall how the cutoff $\Lambda$ arises when we connect the likelihood to correlation functions. Roughly speaking, when we want to describe correlation functions via effective field theory techniques we deal with two types of momenta: loop momenta and external momenta. Loop momenta are integrated over. They run to infinity and the resulting UV-sensitivity of diagrams is absorbed by counterterms. Even after we have renormalized the theory, however, we still do not want to take external momenta to be arbitrarily hard. We know that our effective field theory fails around some physical scale (the nonlinear scale $k_{NL}$, or the galaxy nonlocality scale $1/R_*$), so we cannot trust our predictions on scales shorter than that. Hence, we take
external momenta to be smaller than some \( \Lambda \): the larger the \( \Lambda \), the more important the contribution of higher-order operators will be.

The presence of \( \Lambda \) can be accounted for straightforwardly once we collect all correlation functions in the so-called generating functional \( Z \). This is a functional of an “external current” \( J \) (not to be confused with eq. (2.6), i.e. the Jacobian of the transformation from redshift space to the rest-frame coordinates), such that functional derivatives of \( \ln Z \) evaluated at \( J = 0 \) yield the connected correlation functions of the theory. If we take \( J \) such that its Fourier modes are zero above \( \Lambda \), we are then sure that we only compute correlation functions with external momenta softer than \( \Lambda \).

This has a direct consequence on the likelihood. Indeed, the likelihood is the Fourier transform of the generating functional, and the fields \( \delta_g \) and \( \delta \) are the “duals” to the currents \( J_g \) and \( J \) for the correlation functions of galaxies and matter, respectively. Since these currents are cut at \( \Lambda \), \( \delta_g \) and \( \delta \) are cut as well. Finally, the deterministic bias expansion \( \delta_g,\text{det} \) is also cut because it is linearly coupled to the current \( J_g \) in the functional integral that defines the generating functional [5, 37, 38].

The exact same reasoning can then be applied to our redshift-space conditional likelihood. The Fourier modes, which are defined by the Fourier transform over the cartesian coordinates \( \tilde{x} \), of the data \( \delta_g \) and of the deterministic bias expansion in redshift space \( \delta_g,\text{det} \) are set to zero above a cutoff \( \Lambda \). The same happens for the Fourier modes (defined via Fourier transform over \( x \)) of the matter field. Importantly, the covariance \( P^{(0)}_g / \tilde{J}[\delta, \tilde{v}(\tilde{x})] \) in the exponent of eqs. (1.1), (3.31) (or their analog in eq. (3.35) when we want to include the stochasticity of bias coefficients), is constructed from the filtered matter field. The integral is then carried out over \( d^3\tilde{x} \).

- The fact that the integral is constructed from filtered fields is what makes the connection with a perturbative treatment possible. This is discussed in section 4.2 below.

- The numerator in eqs. (1.1), (3.31) or eq. (3.35) is the square of a difference, hence it is manifestly positive. As long as the denominator is positive, then, the whole exponent is positive. Since we are dealing with multi-dimensional integrals this does not guarantee, however, that the likelihood is well-behaved. We come back to this in section 4.3.

- As a “corollary” of this discussion, we see how the possibility of working with filters different from an isotropic hard cut in Fourier space arises. Let us go back to correlation functions, and work in the distant-observer/flat-sky approximation (as it is usually done when discussing the EFT of LSS in redshift space, see e.g. [19, 22]). For the sake of this discussion let us also focus on the power spectrum. We then have at least two possibilities. Either we work by taking multipoles of the power spectrum (in some

---

One can schematically picture this in the following way. The generating functional for galaxies can be defined as a functional integral over the matter field \( \delta \), i.e.

\[
Z[J_g] = \int D\delta \mathcal{P}[\delta] \exp \left( \int d^3x J_g(\mathbf{x})\delta_g,\text{det}(\mathbf{x}) + \cdots \right),
\]

where \( \mathcal{P}[\delta] \) is the matter probability distribution and \( \cdots \) denotes terms of higher order in \( J_g \) that describe the noise (its power spectrum, its higher-order correlation functions, and the stochasticity of bias coefficients), entering as counterterms to absorb the UV dependencies that arise once we integrate out the short modes of \( \delta \). Importantly, the deterministic bias expansion is coupled linearly to the current. Hence, if the current is cut at \( \Lambda \), modes of the field \( \delta_g,\text{det}(\mathbf{x}) \) above \( \Lambda \) do not appear in the path integral above. For more details we refer to [5, 37]. See also [38] for a discussion in the context of the EFT of LSS for matter.
orthonormal basis), or we bin it in both \(k = |k|\) and \(\mu = k \cdot \hat{n}\). The second one is the “wedges” approach proposed in [39]. When translated in terms of cutoffs, it means that we can implement two different cuts: one on \(k\) and one on \(k_\parallel = k \cdot \hat{n}\). The advantage of implementing a cut on \(k_\parallel\) softer than the one on \(k\) is that we can have more control on higher-order corrections from \(u_\parallel\) (as we will see in a moment, any occurrence of \(u_\parallel\) comes with a \(\partial_\parallel\)). Since the cutoffs that we implement in the analysis of correlation functions are exactly those on the fields appearing in the likelihood, we see that it is possible to cut \(\tilde{\delta}_g\) and \(\tilde{\delta}_{g,\text{det}}\) via an anisotropic filter. We leave a more detailed investigation to future work.

So far we have not discussed how to deal with the overall factor in front of the exponential. In the real-space expression for the rest-frame likelihood with constant noise power spectrum, eq. (2.29), it was intended as filtered at a scale \(\Lambda\) as well. However, it was very simple to implement such filter there even if we worked in real space, thanks to the fact that \(P_e^{(0)}\) is constant. How do we proceed now that the covariance depends on \(x\) (or, better, the redshift-space position \(\tilde{x}\))? We come back to this important point in section 4.3 below.

4.2 Connection with the perturbative treatment

We now have the tools to study the connection with the perturbative treatment of the EFTofLSS. The presence of the cutoffs is fundamental for this purpose.

- The matter field is cut at \(\Lambda\). This means that the real-space matter field is a collection of modes with wavenumbers up to \(|k| = \Lambda\). Since we take \(\Lambda\) below \(k_{\text{NL}}\), we know it is the linear power spectrum that controls the amplitude of these modes. For a power-law dimensionless linear power spectrum we have \(\sqrt{\Delta^2(k)} = (k/k_{\text{NL}})^{3+n_\delta}/2\), where \(\Delta^2(k) = k^3 P_L(k)/2\pi^2\). Then, the typical size of a filtered perturbation \(\delta(x)\) (in this section we drop the subscript “\(\Lambda\)" for simplicity of notation) scales as
  \[
  \delta(x) \sim \left(\frac{\Lambda}{k_{\text{NL}}}\right)^{3+n_\delta/2}.
  \]  
  Taking \(n\) derivatives of this field increases the scaling by \(n\) powers of \(\Lambda\). The index \(n_\delta\) is between \(-2\) and \(-1.5\) on the scales where loop corrections in the EFTofLSS become important, see e.g. [40–42] and section 4.1 of [3]. Hence, as long as \(\Lambda\) is sufficiently lower than \(k_{\text{NL}}\), eq. (4.2) ensures that \(|\delta(x)| < 1\) and that \(|\nabla^n\delta(x)/k_{\text{NL}}^n| < 1\).

- Consequently, in the deterministic bias expansion \(\delta_{g,\text{det}}\) for the galaxy overdensity in redshift space the same perturbative expansion that we are used to when dealing with correlation functions applies. Let us see how this works. As shown e.g. in [43] (see also eq. (9.44) of [3]), after taking into account the change of argument from \(x\) to \(\tilde{x}\) we obtain
  \[
  \tilde{\delta}_{g,\text{det}} = \delta_{g,\text{det}} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \delta^{(n)}_\parallel \left(u_\parallel^n(1 + \delta_{g,\text{det}})\right),
  \]  
  where all fields are evaluated at the redshift-space coordinate \(\tilde{x}\) and \(\delta_{g,\text{det}}\) is given by the bias expansion of eq. (2.10), constructed from a filtered matter field. The galaxy velocity \(u_\parallel\) is given by its deterministic bias expansion, e.g. eqs. (2.12), (2.13), constructed from a filtered \(\delta\) and a filtered matter velocity field. Eq. (4.2) ensures that a perturbative expansion of \(\delta_{g,\text{det}}[\delta\Lambda]\) is under control. What about the new terms,
i.e. those involving $u_\parallel$? These are also under perturbative control. In fact, we see that $u_\parallel$ never appears by itself, but it always comes with a derivative $\partial_\parallel$ (be it acting on $u_\parallel$ itself or on other fields). Given that on large scales the matter velocity scales like $(\nabla/\nabla^2)\delta$, the additional derivative $\partial_\parallel$ ensures that any occurrence of $u_\parallel$ in eq. (4.3) scales with $\Lambda$ in the same way as in eq. (4.2). Most importantly, recall that $v$ is actually proportional via a dimensionless coefficient to $(H\nabla/\nabla^2)\delta$. The factors of $H$ then simplify in eq. (2.2), and we get
\begin{equation}
\partial_\parallel u_\parallel \sim u_\parallel \partial_\parallel \sim \left(\frac{\Lambda}{k_{NL}}\right)^{\frac{3+n_4}{2}}.
\end{equation}
That is, the scaling with $\Lambda$ is still controlled by the nonlinear scale $k_{NL}$ only.\footnote{Notice that here we are not considering the overall growth rate $f = d\ln D_1/d\ln a$ that $u_\parallel$ is proportional to at linear order in perturbations (equal to 1 in an Einstein-de Sitter universe). Importantly, its presence does not affect the scaling of eq. (4.4) with $\Lambda$.

- The difference between the filtered $\tilde{\delta}_g$ and $\tilde{\delta}_{g,\text{det}}$ is controlled by the rest-frame noise cut at $\Lambda$. We are justified in neglecting the transformation of the noise field to redshift space (given by eq. (3.18), effectively) because we have shown above that such transformation is under perturbative control. The same argument applies to the modulation of the noise power spectrum by the matter field, eq. (3.33). Hence, the size of a noise fluctuation in real space scales as
\begin{equation}
\sqrt{P_{\tilde{g}}^{(0)}(\Lambda)^3}.
\end{equation}
That is, it is controlled by the mean separation between tracers for $P_{\tilde{g}}^{(0)} = \alpha/\bar{n}_g$. This scaling is especially important because, at variance with the deterministic bias expansion, our likelihood does not include the non-Gaussianity of the noise at all orders. However, since the corrections from the noise non-Gaussianity come in the form of higher powers in $\tilde{\delta}_g - \tilde{\delta}_{g,\text{det}}$ [5], we know that they are under control as long as $\Lambda$ is softer than $(1/\bar{n}_g)^{1/3}$.

- Finally, we can estimate the relative importance of the different ingredients in the likelihood of eqs. (1.1), (3.31). More precisely, we can compare the relative importance of higher-order terms in the deterministic bias expansion with higher-order terms in the field-dependent covariance $P_{\tilde{g}}^{(0)}/\tilde{J}[\delta,v](\tilde{x})$. Expanding both the numerator and the denominator of the exponential around linear theory, we see that going up to $n$th order in perturbations in $\tilde{\delta}_{g,\text{det}}$ is always more important than going up to the same order in the covariance. The former is enhanced with respect to the latter by the ratio (recall that $-2 \lesssim n_4 \lesssim -1.5$)
\begin{equation}
\left(\frac{\Lambda}{k_{NL}}\right)^{\frac{3+n_4}{2}} = \left(\frac{1}{k_{NL} P_{\tilde{g}}^{(0)}}\right)^\frac{3}{2} \left(\frac{\Lambda}{k_{NL}}\right)^{\frac{n_4}{2}}.
\end{equation}
The same counting goes through when including the stochasticity of bias coefficients (eqs. (3.35), (3.36) being the resulting likelihood), as discussed in [37].
Before proceeding, it is important to emphasize that these conclusions rely on the assumption of negative $n_\delta$. This assumption holds on mildly-nonlinear scales, where the EFTofLSS is usually employed to push the reach of perturbation theory. The linear power spectrum, however, turns around near $0.02\ h\text{Mpc}^{-1}$. On these large, linear scales, the relative importance of the various terms in the likelihood is changed: see e.g. figure 1 (this has been discussed also in [5, 13]).

The last of the bullet points above also allows us to address the positive-definiteness of the Jacobian $\tilde{J}$ of the coordinate change to redshift space. As long as the cutoff $\Lambda$ is soft enough, eq. (2.6) tells us that the Jacobian is equal to 1 plus small corrections. It is important to emphasize, however, that there is no physical reason why this should hold as we take $\Lambda$ to be larger and larger. Indeed, it is possible that a simply connected volume in observed coordinates does not correspond to a simply connected region in rest-frame coordinates, i.e. that the coordinate change $\mathcal{R}$ is multi-valued: this can happen for galaxies that have large peculiar velocities. Restricting $\Lambda$ to lie in the perturbative regime is what allows to get around this issue (see also section 9.3.2 of [3] for a discussion).

We can compare and contrast this with the positive-definiteness of $P_{\epsilon_0}^{\{0\}}$, or in general of $P_{\epsilon}\{\delta\}$ in eqs. (3.32), (3.33). Independently of whether we construct $P_{\epsilon}\{\delta\}$ from a filtered matter field or not, we know that it cannot be negative if the (Gaussian) noise likelihood has a positive-definite covariance. Consider the manifestly nonnegative combination

$$\left(\epsilon_g(x) + \sum_O \epsilon_{g,O}(x) O[\delta](x)\right)^2,$$

and functionally integrate it over the noise fields. As long as the noise covariance of eq. (3.34) is a positive-definite matrix, this integral is well defined and gives exactly eq. (3.33) times $\delta_D^{(3)}(0)$. Hence we know that also the combination in eq. (3.33) cannot be negative.\(^6\) If we take a soft $\Lambda$, such that the higher-order terms in eq. (3.33) are small corrections to $P_{\epsilon_0}^{\{0\}}$, we conclude that $P_{\epsilon}\{\delta\}$ is manifestly positive.

4.3 Covariance and normalization\(^7\)

We have seen in the previous section that the presence of a cutoff allows to make the connection with the perturbative treatment manifest. In this section we ask: how does it affect the overall normalization of the likelihood? This is an inherently nonperturbative question.

Since the galaxy field $\tilde{\delta}_g$ is now filtered at the scale $\Lambda$, it is clear that the integral of the likelihood over it is not equal to 1 unless the factor in front of the exponential is modified from its infinite-$\Lambda$ expression of section 3. This is a problem because, as we discussed in section 3.3, having the correct normalization ensures that we recover the correct limit of a Dirac delta functional when $P_{\epsilon_0}^{\{0\}}$ goes to zero. Moreover, an incorrect normalization would lead to the wrong maximum-likelihood point for the parameters in the covariance (e.g. $P_{\epsilon_0}^{\{0\}}$ itself).

\(^5\)Let us consider for example the relative importance of the noise non-Gaussianity and the contributions from the clustering bispectrum. In the deeply linear regime, it is not important which one of the two dominates: both are subleading with respect to the simple linear contribution $\delta_g - b_1\delta$ in the conditional likelihood. The non-Gaussianity of the noise is still suppressed by eq. (2.25), while the effect of clustering is suppressed as $\Lambda^{3/2+n_\delta}$, for positive $n_\delta$.

\(^6\)Notice that the proof goes through in the same way even if we cut the noise fields at some finite scale $\Lambda$. The only difference would be that $\delta_D^{(3)}(0)$ is replaced by its finite-$\Lambda$ counterpart evaluated at zero spatial separation, which is a positive number $\propto \Lambda^3$.

\(^7\)It is a pleasure to thank Fabian Schmidt for very useful discussions regarding the subject of this section.
We will now show that, if we include the full dependence of the covariance on the matter fields, the presence of a cutoff in Fourier space makes it difficult to normalize the likelihood with respect to the data while retaining a closed analytical form for it. Then, we discuss some ways in which one can get around this problem.

Let us consider the exponential in eqs. (1.1), (3.31), or in eq. (3.35) when we want to include the modulation of the noise by the matter field. As we discussed in section 4.1 the galaxy field and its deterministic bias expansion are cut, and both the covariance and the deterministic bias expansion are constructed from the filtered matter field. However, as far as the integral over the galaxy field is concerned, the only important cuts are those on $\tilde{\delta}_g$ and $\tilde{\delta}_g^{\text{det}}$. More precisely, in order to check the normalization we only need to check if the integral of the functional

$$
\exp \left( -\frac{1}{2} \int d^3\tilde{x} \frac{\left(\chi_{\Lambda}(\tilde{x}) - \varphi_{\Lambda}(\tilde{x})\right)^2}{P(\tilde{x})} \right)
$$

(4.8)

over $\chi_{\Lambda}(\tilde{x})$ is well defined.

First, we can perform a field redefinition $\chi_{\Lambda}'(\tilde{x}) = \chi_{\Lambda}(\tilde{x}) - \varphi_{\Lambda}(\tilde{x})$. Given that both fields are cut at $\Lambda$, the new field $\chi'$ also has no support for momenta below $\Lambda$, and the functional measure does not change. Dropping the subscript for simplicity of notation, we now have to integrate

$$
\exp \left( -\frac{1}{2} \int d^3\tilde{x} \frac{\chi_{\Lambda}^2(\tilde{x})}{P(\tilde{x})} \right)
$$

(4.9)

over $\chi_{\Lambda}(\tilde{x})$. Let us define

$$
\frac{\chi_{\Lambda}(\tilde{x})}{\sqrt{P(\tilde{x})}} = \xi(\tilde{x}),
$$

(4.10)

where we used the fact that $P(\tilde{x})$ is positive, cf. section 4.2. While a simple rescaling at the field level, this is a fully nonlinear redefinition in $\tilde{x}$. Therefore $\xi$ will contain all wavelengths even if the field $\chi_{\Lambda}$ is filtered. The Jacobian of the field redefinition does not depend on $\xi$, so the integral we are after is equal to

$$
\left| \frac{\partial \chi_{\Lambda}(\tilde{x})}{\partial \xi(\tilde{x}')} \right| \int D\xi \exp \left( -\frac{1}{2} \int d^3\tilde{x} \xi(\tilde{x}) \right) = \prod_{\tilde{x}} \sqrt{2\pi},
$$

(4.11)

where $D\xi = \prod_{\tilde{x}} d\xi(\tilde{x})$.

What about the Jacobian of the field redefinition? In appendix B we show that

$$
\frac{\partial \chi_{\Lambda}(\tilde{x})}{\partial \xi(\tilde{x}')} = \sqrt{P(\tilde{x}')} \hat{W}_{\Lambda}(|\tilde{x} - \tilde{x}'|),
$$

(4.12)

where $\hat{W}_{\Lambda}$ is the Fourier transform of the filter $W_{\Lambda}(k)$ defined by

$$
\chi_{\Lambda}(\tilde{x}) = \int k W_{\Lambda}(k) \chi(k) e^{ik \cdot \tilde{x}}.
$$

(4.13)

The field redefinition, and consequently the functional integral itself, is well defined only if this “matrix” is positive-definite (technically, if this is the kernel of a positive-definite linear operator in the space of square-integrable functions). Let us consider for example a
filter $W_{\Lambda}(k) = W(k^2/\Lambda^2)$. This encompasses the most common filters like a hard cut, as in eq. (2.23), or a Gaussian filter. It is then easy to convince oneself that, for a generic $P(\tilde{x}')$, eq. (4.12) is not a positive-definite matrix. A quick way to see this is the following. In eq. (4.12) we know that, for $W_{\Lambda}(k) = W(k^2/\Lambda^2)$, the Fourier transform of the filter can be written as a series expansion in $\nabla^2/\Lambda^2$, that is (see also appendix B)
\begin{equation}
\hat{W}_{\Lambda}(|\tilde{x} - \tilde{x}'|) = \delta_D^{(3)}(\tilde{x} - \tilde{x}') + \sum_{n=1}^{+\infty} c_n \frac{(-1)^n}{\Lambda^{2n}} \nabla^{2n} \delta_D^{(3)}(\tilde{x} - \tilde{x}'),
\end{equation}
where the $c_n$ are defined via
\begin{equation}
W\left(\frac{k^2}{\Lambda^2}\right) = 1 + \sum_{n=1}^{+\infty} c_n \frac{k^{2n}}{\Lambda^{2n}}.
\end{equation}
If we use the properties of the Dirac delta to move the derivatives on $\sqrt{P(\tilde{x}')}$, we see that we end up with a diagonal matrix (which also implies that we can easily compute the Jacobian of the field redefinition using the property $\ln \det = \text{Tr} \ln$). However, while $\sqrt{P(\tilde{x}')} = 0$ for all $\tilde{x}'$, there is no guarantee that this is true also after the infinite series of derivatives of eq. (4.14) acts on it.

While we will investigate this in more detail in future work as well, it is important to emphasize that it is not a showstopper. Indeed, we can already identify two ways in which this problem can be addressed.

First, it is what we learned about the perturbative scalings in the previous section (that mirrors what refs. [5, 37] also discuss) that comes to our rescue. We have seen that it is always more important to include nonlinearities in the forward model than nonlinearities in the field-dependent covariance. More precisely, eq. (4.6) tells us that second-order terms in $\delta_{\sigma, \text{det}}[\delta, \nu](\tilde{x})$ are enhanced by
\begin{equation}
\left(\frac{1}{k_{\text{NL}} P_{\epsilon y}^{(0)}}\right)^{3} \left(\frac{\Lambda}{k_{\text{NL}}}\right)^{n_{\delta}}
\end{equation}
with respect to linear terms in $\tilde{J}[\delta, \nu](\tilde{x})$. Comparing with cubic terms in $\delta_{\sigma, \text{det}}[\delta, \nu](\tilde{x})$, instead, would give an additional $\sim (\Lambda/k_{\text{NL}})^{(3+n_{\delta})/2}$, leading to
\begin{equation}
\left(\frac{1}{k_{\text{NL}} P_{\epsilon y}^{(0)}}\right)^{3} \left(\frac{\Lambda}{k_{\text{NL}}}\right)^{3+n_{\delta}}.
\end{equation}
This is close to being $\Lambda$-independent for $-2 \lesssim n_{\delta} \lesssim -1.5$.

Therefore we conclude that, as long as we stop at second order in perturbations in the deterministic bias expansion, we take $\tilde{J}[\delta, \nu](\tilde{x}) = 1$ in eqs. (1.1), (3.31),\(^8\) and we take $\Lambda$ such that the dimensionless number in eq. (4.16) is small, we are sure that we are not neglecting terms in the likelihood that are as (or more) important than the ones we are keeping. Since we are now effectively in the situation where $\sqrt{P(\tilde{x}')}$ is independent of $\tilde{x}'$, there are no problems with the normalization of the likelihood with respect to the data: it goes through straightforwardly as discussed in section 2.3.

A second solution, that does not require stopping at a finite order in perturbations, is the following. Let us consider a cubic filter $W_{\Lambda}(k)$, defined as
\begin{equation}
W_{\Lambda}(k) = \prod_{i=1}^{3} \Theta_H(\Lambda - |k^i|).
\end{equation}
\(^8\)Equivalently, in eq. (3.35) we take $\tilde{P}_y[\delta, \nu](\tilde{x})/\tilde{J}[\delta, \nu](\tilde{x})$ equal to $P_{\epsilon y}^{(0)}$. 

– 21 –
The calculations in appendix B go through in the same way, so that eq. (4.12) becomes

\[ \frac{\partial \Lambda(\tilde{x})}{\partial \xi(\tilde{x}')} = \sqrt{P(\tilde{x}')} \widehat{W}_\Lambda(\tilde{x} - \tilde{x}') , \]

(4.19)

with

\[ \widehat{W}_\Lambda(\tilde{x} - \tilde{x}') = \frac{1}{\pi^3} \prod_{i=1}^{3} \sin \left( \Lambda(\tilde{x}_i - \tilde{x}'_i) \right) . \]

(4.20)

Let us then put the fields on a lattice, \( \tilde{x} = al \) and \( \tilde{x}' = al' \) (\( l, l' \in \mathbb{N}^3 \)), with lattice spacing \( a = 2\pi/\Lambda \). It is straightforward to see that the matrix of eq. (4.19) is now diagonal in \( l, l' \): the filter is a positive number proportional to \( \Lambda^3 \) for \( l = l' \), and it is equal to zero if \( l \neq l' \). Since \( \sqrt{P(\tilde{x}')} \) remains positive also when evaluated on a lattice, eq. (4.19) is a positive-definite matrix and the functional integral giving the normalization of the likelihood is well defined.

5 Stochasticity in the galaxy velocity field

In this section we discuss what is the effect of the stochasticity in \( v_g \). As we discussed in section 2.2, this stochasticity is guaranteed to be subleading in derivatives by the equivalence principle (for more details we refer to [19] and section 2.8 of [3]). Following the notation of [3], we write

\[ v_g(x) = v_g,\text{det}[\delta, v](x) + \varepsilon v(x) . \]

(5.1)

Then, the fact that the source of relative acceleration between galaxies and matter can only be a functional of \( \nabla \delta(x) \) (and of derivatives of the tidal field at higher order in perturbations) guarantees that the power spectrum of \( \varepsilon v \) is of the form

\[ P_{\varepsilon v\varepsilon v}(k) = \mathcal{P}_{\varepsilon v\varepsilon v}(k) = P_{\varepsilon v\varepsilon v}(k^2 \delta^{ij} - k^i k^j) , \]

(5.2)

where the constants \( P_{\varepsilon v\varepsilon v}^{(2)} \) and \( P_{\varepsilon v\varepsilon v}^{(2)} \) have dimensions of a length to the 5th power. Similarly, the cross-correlation with \( \varepsilon g \) satisfies

\[ P_{\varepsilon v\varepsilon g}(k) = \mathcal{P}_{\varepsilon v\varepsilon g}(k) = P_{\varepsilon v\varepsilon g}(k^i \delta_j^i) , \]

(5.3)

where \( P_{\varepsilon v\varepsilon g}^{(1)} \) has dimensions of a length to the 4th power.

Before proceeding, let us explain the choice of notation in eq. (5.2). First, we emphasize that the absence of preferred directions allows both a term \( \propto k^2 \delta^{ij} \) and one \( \propto k^i k^j \).\footnote{Notice that neither [19] nor [3] included the term \( \propto k^2 \delta^{ij} \).} Let us decompose \( \varepsilon v(x) \) in a longitudinal part and a transverse part, i.e.

\[ \varepsilon v(x) = \nabla \varepsilon v_0(x) + \nabla \times \varepsilon_{\pm 1}(x) . \]

(5.4)

Then, eqs. (5.2), (5.3) are equivalent to having

\[ P_{\varepsilon v\varepsilon v_0}(k) = -P_{\varepsilon v\varepsilon v_0}^{(2)} , \]

(5.5a)

\[ P_{\varepsilon_{\pm 1}\varepsilon_{\pm 1}}(k) = -P_{\varepsilon_{\pm 1}\varepsilon_{\pm 1}}^{(2)} \delta^{ij} , \]

(5.5b)

\[ P_{\varepsilon_{\pm 1}\varepsilon_{\pm 0}}(k) = 0 , \]

(5.5c)

\[ P_{\varepsilon v\varepsilon g}(k) = P_{\varepsilon v\varepsilon g}^{(1)} . \]

(5.5d)
That is, the term $\propto k^2 \delta^{ij}$ comes from the transverse part of the noise field $\varepsilon_v(x)$, where we used the fact that its constant power spectrum must be proportional to $\delta^{ij}$ and its correlation with $\varepsilon_0(x)$ must vanish because of the absence of preferred directions.

We will see why this term is important in section 5.1 below. Before doing that, however, let us build some intuition for what the leading corrections to the EFT likelihood from eqs. (5.2), (5.3) are. At the linear level in perturbations, from eq. (4.3) we have

$$
\tilde{\delta}_g, \det(\tilde{x}) = \delta_g, \det[\delta](\tilde{x}) + \varepsilon_g(\tilde{x}) - \hat{n} \cdot \partial_\parallel \varepsilon_v(\tilde{x}) + \frac{\varepsilon_g(\tilde{x})}{H}.
$$

From this, we see that at this order the noise in $\tilde{\delta}_g$ is

$$
\varepsilon_g(\tilde{x}) - \hat{n} \cdot \partial_\parallel \varepsilon_v(\tilde{x}) \quad (5.7)
$$

Its power spectrum is a sum of three terms. The first is the rest-frame noise power spectrum. Then, we have the power spectrum of $\varepsilon_v$, and finally its correlation with $\varepsilon_g$. Using eqs. (5.2), (5.3), (5.5) we see that the contribution from the correlation of the rest-frame noise with the noise in $v_g$ is less suppressed in derivatives with respect to the contribution from the power spectrum of $\varepsilon_v$. More precisely, its scaling with $k^2$ is the same as the leading higher-derivative corrections to the rest-frame noise power spectrum, cf. eqs. (2.21), (2.22) (notice that we have a power of $k \cdot \hat{n} = k \mu$ from $\partial_\parallel$, and another power of $k \mu$ from $\sum_i n^i P_{\varepsilon_i, \varepsilon_g}(k)$: hence the dependence is on $k^2 \mu^2$). The terms coming from $P_{\varepsilon_v, \varepsilon_v}(k)$ carry an additional $k^2$ suppression (and give a $\mu^2$ or $\mu^4$ angular dependence).

In the next three sections we show how to derive these higher-derivative corrections to the likelihood. Moreover, we discuss what are the physical scales that suppress them.

### 5.1 Impact on the EFT likelihood

Let us now study the impact of $\varepsilon_v(x)$ on the EFT likelihood in more detail. The manipulations in section 3 relied strongly on the fact that the noise was local: since we are now dealing with higher-derivative corrections we cannot resum them at all orders in perturbations even if we work in the same infinite-$\Lambda$ limit of section 3. For this reason we will only sketch how to derive the more straightforward of these corrections and, most importantly, we will show in the next section that they are under perturbative control. Moreover, we will mostly work in Fourier space throughout this section since this makes manipulating higher-derivative terms a much easier task.

First, let us consider the noise correlators of eqs. (5.2), (5.3), together with the constant part of $P_{\varepsilon_g}(k)$ (we refer to [37] for a discussion on how to perturbatively include its higher-derivative corrections). If we construct the covariance matrix in Fourier space for $\varepsilon_g$ and $\varepsilon_v$ it is straightforward to see that its determinant is equal to

$$
\left( P_{\varepsilon_g}^{(2)} \right) P_{\varepsilon_g}^{(0)} P_{\varepsilon_v}^{(2)} k^6 + \left( P_{\varepsilon_v}^{(2)} P_{\varepsilon_g}^{(1)} \right)^2 k^6.
$$

That is, if $P_{\varepsilon_g}^{(2)} = 0$ we have gauge issues if we work with $\varepsilon_v$ as noise variable, and we must switch to a likelihood for $\varepsilon_0$ only.

A simplification arises if we consider the case where the velocity noise is uncorrelated with $\varepsilon_g$ and we take (introducing $P_v$ to simplify the otherwise very heavy notation)

$$
P_{\varepsilon_g, \varepsilon_v}^{(2)} = P_v^{(2)} \equiv P_v.
$$

- 23 -
The likelihood $P[\varepsilon_g, \varepsilon_v]$ then factorizes in
\[ P[\varepsilon_g, \varepsilon_v] = P[\varepsilon_g]P[\varepsilon_v]. \tag{5.10} \]

This leads to a great simplification. Indeed, we can work separately with $\varepsilon_g$ (whose likelihood we will still keep as in eq. (3.16), i.e. in real space) and with $\varepsilon_v$. The likelihood for the latter is given by
\[ P[\varepsilon_v] = \left( \prod_k \frac{1}{(2\pi)^{3/2} P_v k^3} \right) \exp \left( -\frac{1}{2} \int k \frac{\varepsilon_v(k)^2}{P_v k^2} \right). \tag{5.11} \]

Let us see how this leads to a derivative expansion for the EFT likelihood. First, we have the equivalent of section 3.2. Very schematically, we write this as
\[ P[\tilde{\delta}_g|\delta,v] = \int D\varepsilon_g D\varepsilon_v P[\varepsilon_g, \varepsilon_v] \delta(\infty) \det - \text{noise}. \tag{5.12} \]

What is important in this equation is that the we take the argument of the Dirac delta functional to be exactly as in section 3.2: the only difference is that now the galaxy velocity field is not equal to its deterministic bias expansion, but is instead given by eq. (5.1), i.e.
\[ v_g(x) = v_{g,\text{det}}[\delta,v](x) + \varepsilon_v(x). \tag{5.13} \]

The integral over the noise $\varepsilon_g$ goes through in the same way. We then arrive at the equivalent of eq. (3.22), that is\(^{10}\)
\[ \left( \prod_x \sqrt{\frac{1}{2\pi \mathcal{P}_{\varepsilon_g}^{(0)}}} \right) \exp \left( -\frac{1}{2} \int d^3 x \frac{\mathcal{R}[v_g](x) - (1 + \delta_{g,\text{det}}[\delta,v]) / J[v_g](x)^2}{\mathcal{P}_{\varepsilon_g}^{(0)} / J^2[v_g](x)} \right). \tag{5.14} \]

Here we emphasized the dependence of the Jacobian $J$ of the coordinate change on the full galaxy velocity field of eq. (5.13) by changing its argument from $J[\delta,v]$ to $J[v_g]$. Also, at variance with eq. (2.14), in the above equation we have
\[ \mathcal{R}[v_g] \equiv \mathcal{R}. \tag{5.15} \]

It is then only a matter of carrying out the integral over $\varepsilon_v$. We can do it perturbatively in derivatives by using a functional generalization of eq. (4.14). In appendix C.1 we show that
\[ P[\varepsilon_v] = \exp \left( -\frac{1}{2} \int_k P_v k^2 \frac{\partial}{\partial \varepsilon_v(k)} \cdot \frac{\partial}{\partial \varepsilon_v(-k)} \right) \delta_{D}^{(\infty)}(\varepsilon_v(k)). \tag{5.16} \]

When we integrate eq. (5.14) against $P[\varepsilon_v]$ we can expand the exponential and use the properties of the Dirac delta functional to move the derivatives from $\delta_{D}^{(\infty)}(\varepsilon_v(k))$ to eq. (5.14), arriving then at a series expansion for the conditional likelihood. Appendix C.2 shows how this works in practice. However, the results found so far are enough to discuss what are the parameters we are expanding the likelihood in: this is the subject of the next section.

\(^{10}\)For simplicity we are not going to include the stochasticity of bias coefficients. Our conclusions can be straightforwardly extended to account for a field-dependent noise covariance in the rest frame.
5.2 Checking the perturbative expansion

As in section 4.1 we reintroduce a filter at the scale Λ, applied to all the fields in eq. (5.14) (including the noise ε_v). It is then sufficient to study the scaling dimensions of the objects in the exponential of eq. (5.16) under a rescaling k → bk under a rescaling

\[ \frac{\partial \chi(k)}{\partial \chi(k')} = (2\pi)^3 \delta^{(3)}(k + k') . \]  

(5.17)

Given that both the Fourier modes \( \chi(k) \) of the dimensionless field \( \chi(x) \) and \( \delta^{(3)}(k + k') \) have dimension of a length to 3rd power, we see that \( \partial/\partial \chi(k') \) is dimensionless. Then, the only question is what is the “typical scale of variation” of eq. (5.14) with respect to the noise in the galaxy velocity field. From section 2.1, more precisely eqs. (2.1), (2.2), we see that every occurrence of the galaxy velocity field comes with an additional \( \partial/\partial v \).

Since we have two functional derivatives, we have an additional \( b^2 \) scaling, controlled by \( \mu^2/H^2 \). It is important to emphasize that derivatives with respect to \( \epsilon_v \) can lead to additional insertions of the fields \( \delta, \nabla v \) or \( \delta_g - \delta_g^{det} \). These contributions are less relevant in the infrared because they add powers of \( \Lambda \) according to eqs. (4.2), (4.4), (4.5): we will discuss them briefly in the next section.

What if we had not considered only the isotropic part (\( \propto k^2 \delta^{ij} \)) of \( P_{v_g,v_g}(k) \)? We would have obtained an additional overall \( \mu^2 \). From this we conclude that the corrections to the likelihood from the power spectrum of the noise in the galaxy velocity field scale at least as

\[ \frac{\mu^2 A^2}{h^2} P_v A^5, \quad \frac{\mu^2 A^2}{h^2} P_v \mu^2 A^5, \]  

(5.18)
as we had estimated via eq. (5.7). We confirm this by explicit calculation in appendix C.2.

The next question we have to ask, then, is what is the value we expect for \( P_v \). A rough upper limit comes with the following argument (see sections 2.7 and 2.8 of [3] for more details). In the deterministic bias expansion we have \( v_g = v \) at linear order in perturbations and derivatives. This, in turn, is proportional to \( (H \nabla v/\nabla^2) \delta \). \( \delta \) is proportional, via \( b_1 \), to \( \delta_g \) at this order. Let us consider then the stochasticity of \( \delta_g \). At second order in derivatives it is \( R_g^2 \nabla^2 \varepsilon_g \), so plugging this into \( (H \nabla v/\nabla^2) \delta \sim (H \nabla v/\nabla^2) \delta_g \) we get \( \varepsilon_v \sim H R_g^2 \nabla \varepsilon_g \), which means

\[ P_v \sim H^2 R_g^4 P \varepsilon_g^{(0)} . \]  

(5.19)

In the next section we confirm that the corrections from the correlation of the velocity noise with \( \varepsilon_g \) are more relevant on large scales than the ones of eq. (5.18). Moreover, we quickly discuss how to estimate the size of subleading terms coming from higher orders in \( \varepsilon_v \).

\(^{11}\)Since we are working perturbatively, there is no need to consider how loop corrections could affect the scaling dimensions (see also [40] for details).

\(^{12}\)A sharper estimate can be found by assuming a concrete physical model, e.g. that the stochastic velocity contribution of a given galaxy sample is due to the virial velocities within the host halos of mass \( M_h \) and Eulerian radius \( R_h \) of these galaxies. This gives \( P_v \) of order \( R_h^4 \) (that scales as \( M_h^{1/3} \)). See e.g. [44], section 2.8 of [3] and section 6 of [22] for details.
5.3 Leading correction from the noise in $v_g$

Let us discuss the impact of $\varepsilon_v$ in the field-level relation between $\delta_g$ and $\delta_g$ of eq. (2.8), i.e.

$$\delta_g(\hat{x}) = \frac{1 + \delta_g(\mathcal{F}[v_g](\hat{x}))}{J[v_g](\hat{x})} - 1.$$  \hfill (5.20)

Writing the rest-frame galaxy density field as the sum of the deterministic bias expansion and the stochasticity $\varepsilon_g$, we see that the impact of $\varepsilon_v$ is essentially twofold. Expanding $v_g$ around its deterministic bias expansion, we see that $\varepsilon_v$ can either go to multiply $\delta_g$, det $\delta_g$, or can appear by itself (The Jacobian of the coordinate change in eq. (5.20) is the most straightforward example).

- If it multiplies the deterministic bias expansion for $\delta_g$, its effect is basically the same as that of the stochasticity of the bias coefficients: it gives a modulation of the covariance suppressed by derivatives.

- If, on the other hand, $\varepsilon_v$ multiplies $\varepsilon_g$, we have a contribution that is similar to that coming from the non-Gaussianity of $\varepsilon_g$ (again suppressed by derivatives).

- Then, the leading contribution is when $\varepsilon_v$ appears by itself, e.g. thanks to the fact that in eq. (5.20) the Jacobian of the coordinate change to redshift space multiplies $1 + \delta_g$, and not only $\delta_g$. This is exactly what gives the contribution shown in eqs. (5.6), (5.7) (see also appendix C.2 for more details).

Let us now see how to account for the fact that the rest-frame noise $\varepsilon_v$ and $\varepsilon_g$ can be correlated, with

$$P_{\varepsilon_v \varepsilon_g}^{[1]} \sim \mathcal{H} R_{\varepsilon_g}^2 P_{\varepsilon_g}^{[0]}$$  \hfill (5.21)

via the same estimates that lead to eq. (5.19). The manipulations with functional integrals of section 5.2 continue to hold, with the only differences being that in eq. (5.16) $P_{v_v} k^2$ is replaced by $P_{\varepsilon_v \varepsilon_g} k \cdot \hat{n} = P_{\varepsilon_v \varepsilon_g} k \mu$, and one derivative with respect to $\varepsilon_v$ is replaced by a derivative with respect to $\varepsilon_g$. Consequently, we have one less power of $\mu \Lambda / \mathcal{H}$. As far as the scaling with $\Lambda$ is concerned, then, the scalings of eq. (5.18) become

$$\frac{\mu \Lambda}{\mathcal{H}} P_{\varepsilon_v \varepsilon_g}^{[1]} \mu \Lambda^4.$$  \hfill (5.22)

An interesting point is comparing the scalings of eqs. (5.18), (5.22) to the one from the higher-derivative corrections to the rest-frame noise power spectrum. The leading corrections there scale as $R_{\varepsilon_g}^2 \Lambda^2 P_{\varepsilon_g}^{[0]} \Lambda^3$. Hence, we see from eq. (5.19) that the ratio with the contribution from the power spectrum of $\varepsilon_v$ scales as

$$\frac{P_{v_v} \mu^2 \Lambda^7}{\mathcal{H}^2 R_{\varepsilon_g}^2 \Lambda^2 P_{\varepsilon_g}^{[0]} \Lambda^3} \sim R_{\varepsilon_g}^2 \mu^2 \Lambda^2,$$

$$\frac{P_{\varepsilon_v} \mu^4 \Lambda^7}{\mathcal{H}^2 R_{\varepsilon_g}^2 \Lambda^2 P_{\varepsilon_g}^{[0]} \Lambda^3} \sim R_{\varepsilon_g}^2 \mu^4 \Lambda^2,$$  \hfill (5.23)

which is always small for small $\Lambda$ thanks to it being softer than the nonlocality scale of galaxy dynamics. As far as the contribution from the correlation of $\varepsilon_v$ with $\varepsilon_g$ is concerned, instead, we have

$$\frac{P_{\varepsilon_v \varepsilon_g}^{[1]} \mu^2 \Lambda^5}{\mathcal{H}^2 R_{\varepsilon_g}^2 \Lambda^2 P_{\varepsilon_g}^{[0]} \Lambda^3} \sim \mu^2,$$  \hfill (5.24)

where we have used eq. (5.21).

We conclude this section with figure 1, that compares these scaling dimensions with those discussed in section 4.2.
6 Conclusions

In this paper we have studied how to implement redshift-space distortions in the EFT likelihood for large-scale structure. The equivalence principle forbids large-scale stochasticity in the galaxy velocity. Therefore, at leading order in a derivative expansion, the form of the likelihood is determined by the noise in the rest-frame galaxy density. The likelihood is still a Gaussian in the redshift-space galaxy density field $\tilde{\delta}_g(\tilde{x})$: the effect of redshift-space distortions is that of modifying its covariance. The galaxy noise power spectrum $P_{\epsilon_g}^{(0)}$ is replaced by the field-dependent $P_{\tilde{\epsilon}_g}^{(0)} / \tilde{J}[\delta, v](\tilde{x})$, where $\tilde{J}[\delta, v](\tilde{x})$ is the Jacobian of the coordinate change $\tilde{x} = x(\tilde{x})$ to redshift space (which depends on the matter density and velocity fields through the bias expansion for the galaxy velocity).

In the framework of the EFT-based forward modeling one wants the galaxy and matter fields to be cutoff at a scale $\Lambda$ (this is essentially what allows to make contact with the perturbative approach of the EFTofLSS), but also to sample the likelihood via, e.g., Hamiltonian Monte Carlo methods. This requires the likelihood to be normalized with respect to the data, i.e. the redshift-space galaxy density field $\tilde{\delta}_g(\tilde{x})$. As long as we restrict our bias expansion and the coordinate change to redshift space to second order in perturbations, we show that it is consistent to neglect the dependence of the covariance of the Gaussian likelihood on...
the matter fields. The normalization of the likelihood can then be obtained in a closed form even when dealing with a filtered galaxy field. Putting the theory on a lattice with spacing \( a = 2\pi/\Lambda \) is a second way to achieve this, its advantage being that it does not require to stop at a finite order in perturbation theory. This latter approach was implemented numerically (albeit for a rest-frame halo samples identified in N-body simulations) in the recent paper [45]. This paper checked the consistency between the real-space formulation of the likelihood and its Fourier-space counterpart of previous papers in this series, confirming that the two yield the same results for, e.g., the constraints on the amplitude of the primordial power spectrum (at fixed cosmological parameters and phases of the initial conditions).

Finally, we have explicitly computed the corrections from the noise \( \varepsilon_v \) in the galaxy velocity field, confirming they are indeed subleading on large scales. Interestingly, we have shown that they are as relevant as the contribution of higher-derivative terms in the rest-frame noise power spectrum if \( \varepsilon_v \) is correlated with the noise in the rest-frame galaxy density.

Two subjects that can be investigated in more detail in future work are selection effects and how to go beyond the flat-sky approximation (along with the interplay with the survey window function). However, we can already comment briefly about them.

- The presence of the line-of-sight vector can alter the bias expansion \( \delta_{g, \text{det}}[\delta] \) by adding a preferred direction to contract tensor indices with. For example, we can have the operator \( \hat{n}^i \hat{n}^j K_{ij}[\delta] \) appearing at linear order in perturbations. These terms arise when the selection function depends on the orientation of the galaxy [46–52], or when galaxies are identified through emission or absorption lines, whose observed strength depends on the line of sight due to radiative transfer effects [53–55]. This is not a problem, since the scaling dimensions for these operators are the same of those in section 4.2: it is then sufficient to identify all the operators at a given order in perturbations and include them into \( \delta_{g, \text{det}}[\delta] \) (see e.g. [22], and [56] for a recent study of their impact on constraints on cosmological parameters within the framework of the EFTofLSS).

- Finally, let us comment on the distant-observer approximation and the survey window function. First, none of our nonperturbative expressions for the likelihood of section 3 relied on the distant-observer approximation. Moreover, and most importantly, the scaling dimensions discussed in sections 4 and 5 are also unaffected by it. Hence, we can understand the line of sight vector to be position-dependent throughout all of our expressions. A survey will in general probe a finite part of our past light cone, i.e. a finite region in \((z, \hat{n})\) space (for example, future large-scale galaxy surveys such as SPHEREx [57], DESI [58], and Euclid [59, 60] will have footprints of order 10000 square degrees). Once we convert this region to its equivalent in \( \tilde{x} \) coordinates, we can then simply integrate our likelihood over this region only, instead of over all \( \tilde{x} \).

We can see, however, how this last point raises a very important issue, linked to the discussion of section 4.3. If we integrate our likelihood only over a subset of \( \tilde{x} \) values, this is essentially equivalent to putting the noise power spectrum to infinity away from such region. That is, we now have effectively ended up with a position-dependent noise power spectrum. This seems to lead to the same complications that we encountered when discussing the dependence of the covariance on the matter density and velocity fields. The difference is that there is no expansion parameter that allows us to expand around an \( \tilde{x} \)-independent survey window function. Interestingly, since the window function is local in real space, it would not affect the conclusions of section 4.3, where we showed that the covariance matrix is diagonal if we put the theory on a lattice with spacing equal to \( 2\pi/\Lambda \).
While we leave a more careful investigation to future work, it is important to emphasize that the complications discussed here arise because of the necessity of implementing the cutoff $\Lambda$ if one wants to make contact with EFT-based techniques. Bayesian forward modeling is already being applied to data from real surveys (e.g. the SDSS-III/BOSS galaxy sample), therefore accounting for selection effects and the survey window function. For example, we refer to [61–63], in which the forward model for redshift-space distortions is the same discussed here (see e.g. eq. (18) of [61]), and the differences with this work are essentially the likelihood for the galaxy distribution, the bias model, and especially the absence of the EFT cutoff $\Lambda$. The recent implementation of the EFT likelihood in real space of [45] suggests that it is feasible to introduce these ingredients in the analysis of [61–63].

We conclude with a comparison with the velocity reconstruction of [64, 65]. This reconstruction exploits the degree of anisotropy in a redshift-space galaxy catalog to measure the distortion from the expected real-space isotropy. One can reconstruct the matter field essentially by undoing the effect of the shift of eq. (2.1) if linear theory, the continuity equation for matter and linear local-in-matter-density bias are assumed. From there, it is possible to measure, for example, the parameter $f/b$ (where $f$ is the linear growth rate $f(a) = d \ln D_1(a)/d \ln a$). While [64] works in real space and [65] works in Fourier space (the latter having the advantage that, in linear theory, it is easy to restrict to long-wavelength, perturbative modes), both avoid the flat-sky approximation. In the language of section 2 this amounts to considering $\hat{n} = \hat{n}(x)$ (as discussed in the previous paragraph, none of the results in this paper depend on the flat-sky approximation). However, both works employ an expansion in spherical harmonics instead of using cartesian coordinates. While for a full-sky survey this is surely advantageous, it is less so once the partial sky coverage is taken into account. The formulation in cartesian coordinates of this paper and [45] should make the implementation of the window function more straightforward (see e.g. eq. (2.6) of [45] for more details).

Another difference between with [64, 65] and the approach discussed in this work lies in the fact that in the (EFT-based) forward modeling we do not move the data back to the rest-frame but we “forward-model” the rest-frame galaxy density to redshift space, and compare with data there. Finally, the EFT-based likelihood is constructed to go beyond linear perturbation theory, while still keeping nonlinear effects under control, thanks to the cutoff discussed in section 4.

Acknowledgments

It is a pleasure to thank Alex Barreira and Fabian Schmidt for very useful discussions, and Fabian Schmidt for collaboration on related topics. We acknowledge support from the Starting Grant (ERC-2015-STG 678652) “GrInflaGal” from the European Research Council.

A Integrating out the noise in redshift space

In this appendix we derive again eq. (3.14) by integrating out the noise, the difference with the calculation of section 3.2 being that we work directly in redshift space. Let us consider the relation

$$\delta_D^{(\infty)}(\hat{\delta}_g(\hat{x}) - \hat{\delta}_{g,\text{det}}[\delta, v](\hat{x}) - \hat{\varepsilon}_g(\hat{x})),$$

(A.1)

where

$$\hat{\varepsilon}_g(\hat{x}) = \frac{\varepsilon_g(\hat{\mathcal{R}}(\hat{x}))}{J[\delta, v](\hat{x})}.$$

(A.2)
We want to integrate eq. (A.1) against the likelihood for $\tilde{\varepsilon}_g(\tilde{x})$. Given eq. (A.2) and the transformation rules for probability density functionals, the likelihood for the noise in redshift space is given by

$$
\left| \frac{\partial \varepsilon_g(x)}{\partial \tilde{\varepsilon}_g(\tilde{x}')} \right| \left( \prod_x \sqrt{\frac{1}{2\pi P^{[0]}_g}} \right) \exp \left( -\frac{1}{2} \int d^3x \frac{\varepsilon^2_g(\mathcal{R}^{-1}(x))}{P^{[0]}_g / J^2[\delta, v](x)} \right),
$$

(A.3)

where we have used the likelihood for the rest-frame noise of eq. (3.16) and the inverse of eq. (A.2) which, thanks to the definitions of eqs. (2.6), (2.7), is given by

$$
\varepsilon_g(x) = J[\delta, v](x) \tilde{\varepsilon}_g(\mathcal{R}^{-1}(x)).
$$

(A.4)

Eq. (A.4) straightforwardly allows us to compute the Jacobian in eq. (A.3). Using the definition of functional derivative, eq. (2.18), we get

$$
J[\delta, v](x) \delta_D^{(3)}(\mathcal{R}^{-1}(x) - \tilde{x}') = \delta_D^{(3)}(x - x'),
$$

(A.5)

where we used eqs. (3.9), (3.10), (3.11). The overall Jacobian is then equal to one. Then, in the integral of eq. (A.3) we can change the argument to $\tilde{x}$, and integrate in $D\tilde{\varepsilon}_g = \prod_{\tilde{x}} d\tilde{\varepsilon}_g(\tilde{x})$ against the Dirac delta functional of eq. (A.1). This gives eq. (3.14) again.

## B Functional Jacobian matrix for a filtered field

In this short appendix we show how to arrive at eq. (4.12). The field redefinition we have performed is the one of eq. (4.10), which we can rewrite as

$$
\chi_\Lambda(\tilde{x}) = \left( \sqrt{P(\tilde{x})} \xi(\tilde{x}) \right)_\Lambda,
$$

(B.1)

i.e.

$$
\chi_\Lambda(\tilde{x}) = \int_k W_\Lambda(k) \left( \int_q \xi(q) \mathcal{P}(k - q) \right) e^{ik\cdot\tilde{x}},
$$

(B.2)

where we defined

$$
\sqrt{P(\tilde{x})} = \int_k \mathcal{P}(k) e^{ik\cdot\tilde{x}},
$$

(B.3)

and the filter $W_\Lambda(k)$ is defined as

$$
\chi_\Lambda(\tilde{x}) = \int_k W_\Lambda(k) \chi(k) e^{ik\cdot\tilde{x}}.
$$

(B.4)

Using the two relations

$$
\frac{\partial \xi(\tilde{x})}{\partial \xi(\tilde{x}')} = \delta_D^{(3)}(\tilde{x} - \tilde{x}'),
$$

(B.5a)

$$
\xi(q) = \int d^3\tilde{x} \xi(\tilde{x}) e^{-iq\cdot\tilde{x}},
$$

(B.5b)

we have

$$
\frac{\partial \xi(q)}{\partial \xi(\tilde{x}')} = e^{-iq\cdot\tilde{x}'}.
$$

(B.6)
Hence, from eq. (B.2) we find

$$
\frac{\partial \chi_\Lambda(\tilde{x})}{\partial \xi(\tilde{x}')} = \int_k W_\Lambda(k) e^{ik\cdot\tilde{x}} \int_q e^{-iq\cdot\tilde{x}'} \mathcal{P}(k-q) \int_p e^{ip\cdot\tilde{x}'}
$$

$$
= \int_k W_\Lambda(k) e^{ik(\tilde{x}-\tilde{x}')} \int_p e^{ip\cdot\tilde{x}'}
$$

$$
= \hat{W}_\Lambda(|\tilde{x} - \tilde{x}'|) \sqrt{P(\tilde{x}')}, \tag{B.7}
$$

where in the last step we have used eq. (B.3), and also that the filter is only a function of $k = |k|$.

As a byproduct of this derivation we can also obtain the series in eq. (4.14). Indeed, if we expand the filter $W_\Lambda(k) = W(k^2/\Lambda^2)$ as

$$
1 + \sum_{n=1}^{+\infty} c_n \frac{k^{2n}}{\Lambda^{2n}}, \tag{B.8}
$$

we can write the first integral in the second-to-last line of eq. (B.7) as

$$
\int_k W_\Lambda(k) e^{ik(\tilde{x}-\tilde{x}')} = \int_k \left(1 + \sum_{n=1}^{+\infty} c_n \frac{(-1)^n}{\Lambda^{2n}} \nabla^{2n}\right) e^{ik(\tilde{x}-\tilde{x}')} \tag{B.9}
$$

where the derivative can equivalently be taken with respect to $\tilde{x}$ or $\tilde{x} - \tilde{x}'$ (or, using the fact that the filter is only a function of $k = |k|$, with respect to $x'$ or $x' - x$). Formally pulling the sum out of the integral sign, we get

$$
\left(1 + \sum_{n=1}^{+\infty} c_n \frac{(-1)^n}{\Lambda^{2n}} \nabla^{2n}\right) \delta^3_D(\tilde{x} - \tilde{x}'). \tag{B.10}
$$

C Integrating out the velocity noise

In this appendix we derive eq. (5.16) and use it to compute one of the corrections to the conditional EFT likelihood $P[\delta_g|\delta,v]$ by integrating out the velocity noise (appendices C.1 and C.2, respectively).

C.1 Formal power series for the likelihood of the velocity noise

Let us consider eq. (5.11), i.e.

$$
P[\varepsilon_v] = \left(\prod_k \frac{1}{(2\pi)^3/2 P_v^3 k^3}\right) \exp \left(-\frac{1}{2} \int_k \frac{|\varepsilon_v(k)|^2}{P_v k^2}\right). \tag{C.1}
$$

We can define its functional Fourier transform by

$$
P[\varepsilon_v] = \int \mathcal{D}\mathbf{E} \left(\prod_k \frac{1}{(2\pi)^3}\right) P[\mathbf{E}] \exp \left(i \int_k \mathbf{E}(k) \cdot \varepsilon_v(-k)\right), \tag{C.2}
$$

where $\mathcal{D}\mathbf{E} = \prod_k d\mathbf{E}(k)$ and we notice that $\mathbf{E}(k)$ is dimensionless. Since the likelihood $P[\varepsilon_v]$
is a normalized Gaussian, we know that its functional Fourier transform must take the form
\[
P[E] = \exp \left( -\frac{1}{2} \int_k P_v k^2 |E(k)|^2 \right), \tag{C.3} \]
i.e. it is equal to 1 for vanishing \( E(k) \).

Once we have eqs. (C.2), (C.3) we can rewrite the likelihood as
\[
P[\varepsilon_v] = \int D E \left( \prod_k \frac{1}{(2\pi)^3} \right) \exp \left( -\frac{1}{2} \int_k P_v k^2 |E(k)|^2 \right) \exp \left( i \int_k E(k) \cdot \varepsilon_v(-k) \right), \tag{C.4} \]
which is also equal to
\[
\int D E \left( \prod_k \frac{1}{(2\pi)^3} \right) \exp \left( -\frac{1}{2} \int_k P_v k^2 \frac{\partial}{\partial \varepsilon_v(k)} \cdot \frac{\partial}{\partial \varepsilon_v(-k)} \right) \exp \left( i \int_k E(k) \cdot \varepsilon_v(-k) \right). \tag{C.5} \]

Then, bringing the \( E \)-independent exponential out of the functional integral gives eq. (5.16).

C.2 Perturbative corrections to the EFT likelihood

We now use eq. (5.16) to sketch how to compute the corrections to the vanishing-noise likelihood perturbatively. First, we are integrating eq. (5.14), i.e.
\[
\left( \prod_x \frac{1}{2\pi P_{\varepsilon_g}^{(0)}} \right) \exp \left( -\frac{1}{2} \int d^3 x \left\{ 1 + \frac{1}{1 + \delta_g / \delta_E^{-1}(x)} \right\} - \left( 1 + \frac{\partial^2}{\partial \varepsilon_v} \right) \right) \frac{J[v_g](x)}{J[v_g]^2}, \tag{C.6} \]
multiplied by eq. (5.16) over the velocity noise: we can then move the functional derivatives from the Dirac delta functional to eq. (5.14).

Expanding the exponential to first order in \( 1/P_{\varepsilon_g}^{(0)} \) we then need to compute, schematically,
\[
\left( -\frac{1}{2} \int_k P_v k^2 \frac{\partial}{\partial \varepsilon_v(k)} \cdot \frac{\partial}{\partial \varepsilon_v(-k)} \right) \bigg|_{\varepsilon_v(k) = 0}. \tag{C.7} \]
Using the relation \( \exp x \sim 1 + x \), this can be then resummed into the logarithm of the likelihood.

For simplicity, we consider only derivatives acting on the overall Jacobian \( J \) (whose square multiplies the difference between theory and data in eq. (C.6) above). When we expand the exponential in eq. (C.6) in \( 1/P_{\varepsilon_g}^{(0)} \), and expand the Jacobian of the coordinate change in \( \varepsilon_v \) around the deterministic bias expansion for the galaxy velocity, we see that we have both “disconnected” contributions (that e.g. modify the overall normalization) and “connected” contributions.

We can see an example of the former already at first order in \( 1/P_{\varepsilon_g}^{(0)} \). We switch to Fourier space inside the exponential, and rewrite eq. (C.6) as (we drop the overall factor for simplicity)
\[
\exp \left( -\frac{1}{2 P_{\varepsilon_g}^{(0)}} \int_{p,p'} \mathcal{J}^2(p) \mathcal{J}^2(-p-p') \right), \tag{C.8} \]
\[13\]Since the derivatives always appear squared there is no need to keep track of signs.
where we defined the Fourier transform of the Jacobian as $\mathcal{J}$, and the definition of $\mathcal{A}$ can
be seen by matching to eq. (C.6). The quantity to which we have to apply the differential
operator of eq. (C.7), then, is

$$
- \frac{1}{2P_{v_{\epsilon}}^{(0)}} \int_{\mathbf{p},\mathbf{p}'} \mathcal{J}(\mathbf{p}) \mathcal{J}(\mathbf{p}') \mathcal{A}(-\mathbf{p} - \mathbf{p}'), \quad (C.9)
$$

where $\partial / \partial \epsilon_v(\mathbf{k})$ acts on $\mathcal{J}(\mathbf{p})$ and $\partial / \partial \epsilon_v(-\mathbf{k})$ on $\mathcal{J}(\mathbf{p}')$. From eqs. (2.6), (5.13) we have that

$$
\frac{\partial \mathcal{J}(\mathbf{k})}{\partial \epsilon_v(\mathbf{k})} = \frac{i \mathbf{k} \cdot \mathbf{n}}{\mathcal{H}} \mathbf{n} (2\pi)^3 \delta_D^{(3)}(\mathbf{k} + \mathbf{k}') , \quad (C.10)
$$

so, using $\mathbf{n} \cdot \mathbf{n} = 1$, we find

$$
- \frac{1}{4P_{v_{\epsilon}}^{(0)}} \int_{k,p,p'} P_v k^2 \frac{(\mathbf{p} \cdot \mathbf{n})(\mathbf{p}' \cdot \mathbf{n})}{\mathcal{H}^2} (2\pi)^3 \delta_D^{(3)}(\mathbf{p} + \mathbf{k})(2\pi)^3 \delta_D^{(3)}(\mathbf{p}' - \mathbf{k}) \mathcal{A}(-\mathbf{p} - \mathbf{p}') . \quad (C.11)
$$

It is straightforward to see that this is proportional to $\mathcal{A}(\mathbf{0})$.

The connected contribution arises at second order in $1/P_{v_{\epsilon}}^{(0)}$ from expanding the Jacobi-
ian that multiplies $1 + \delta_{g,\text{det}}[\delta](\mathbf{x})$ in eq. (C.6). We have

$$
1 + \tilde{\delta}_{g}(\mathcal{R}^{-1}(\mathbf{x})) - \frac{1 + \delta_{g,\text{det}}[\delta](\mathbf{x})}{J[v_{g}(\mathbf{x})]} = 1 + \tilde{\delta}_{g}(\mathcal{R}^{-1}(\mathbf{x})) - \frac{1 + \delta_{g,\text{det}}[\delta](\mathbf{x})}{J[\delta_{g}v](\mathbf{x})} + \mathbf{n} \cdot \partial \epsilon_v(\mathbf{x}) / \mathcal{H} ,
$$

where it is useful to assign a symbol, $\mathcal{B}(\mathbf{x})$, to the square root of $\mathcal{A}(\mathbf{x})$. Hence, losing track
from now on of irrelevant overall factors and switching to Fourier space, we have to apply the
differential operator of eq. (C.7) to

$$
\int_{\mathcal{P}} \frac{(\mathbf{p} \cdot \mathbf{n}) \mathbf{n} \cdot \epsilon_v(\mathbf{p})}{\mathcal{H}} \mathcal{B}(-\mathbf{p}) \int_{\mathcal{Q}} \frac{(\mathbf{q} \cdot \mathbf{n}) \mathbf{n} \cdot \epsilon_v(\mathbf{q})}{\mathcal{H}} \mathcal{B}(-\mathbf{q}) . \quad (C.13)
$$

After taking the derivatives with respect to the velocity noise, we arrive at

$$
\int_{\mathcal{P}} P_v k^2 \frac{(\mathbf{k} \cdot \mathbf{n})^2}{\mathcal{H}^2} \mathcal{B}(\mathbf{k}) \mathcal{B}(-\mathbf{k}) \sim \frac{P_v}{\mathcal{H}^2} \int d^3x \frac{\partial \mid \nabla (\text{data} - \text{theory})}{P_v^{(0)}} \cdot \frac{\partial \mid \nabla (\text{data} - \text{theory})}{P_v^{(0)}} , \quad (C.14)
$$

where we have schematically called “data — theory” the Fourier transform back to real space
of $\mathcal{B}$ (it is straightforward to match with eq. (C.6), in the same way as when we defined $\mathcal{A}$
and $\mathcal{B}$ themselves). Moreover, the dimensions have been fixed by reinstating the overall
factor of $1/P_{v_{\epsilon}}^{(0)}$ squared that we had dropped throughout: $P_v / \mathcal{H}^2$ has dimensions of a length
to the 7th power, $\partial \mid$ and $\nabla$ have dimensions of an inverse length, $P_v^{(0)}$ has dimensions of a
length to the 3rd power, and finally both $d^3x/P_v^{(0)}$ and “data — theory” are dimensionless.

We can then compare this with the logarithm of the likelihood for vanishing velocity noise.
We immediately see that the presence of the square of “data — theory” gives a scaling $\Lambda^3$, cf.
eq (4.5). The two derivatives $\nabla$ then give a scaling $\Lambda^2$, and we recognize the overall
$P_v$. Importantly, we notice that there are two additional derivatives $\partial \mid$ controlled by the
Hubble scale $\mathcal{H}$. This agrees with the conclusions of section 5.2. To derive the scaling with $\Lambda$,
there we had used the fact that the “typical scale of variation” of functionals of $\epsilon_v$ was
$\partial \mid / \mathcal{H}$: eqs. (C.12), (C.13) provide a clear example of this.
References

[1] D. Baumann, A. Nicolis, L. Senatore and M. Zaldarriaga, Cosmological Non-Linearities as an Effective Fluid, *JCAP* 07 (2012) 051 [arXiv:1004.2488].

[2] J.J.M. Carrasco, M.P. Hertzberg and L. Senatore, The Effective Field Theory of Cosmological Large Scale Structures, *JHEP* 09 (2012) 082 [arXiv:1206.2926].

[3] V. Desjacques, D. Jeong and F. Schmidt, Large-Scale Galaxy Bias, *Phys. Rept.* 733 (2018) 1 [arXiv:1611.09787].

[4] F. Schmidt, F. Elsner, J. Jasche, N.M. Nguyen and G. Lavaux, A rigorous EFT-based forward model for large-scale structure, *JCAP* 01 (2019) 042 [arXiv:1808.02002].

[5] G. Cabass and F. Schmidt, The EFT Likelihood for Large-Scale Structure, *JCAP* 04 (2020) 042 [arXiv:1909.04022].

[6] K. Fisher, O. Lahav, Y. Hoffman, D. Lynden-Bell and S. Zaroubi, Wiener reconstruction of density, velocity, and potential fields from all-sky galaxy redshift surveys, astro-ph/9406009.

[7] J. Jasche, F.S. Kitaura, B.D. Wandelt and T.A. Ensslin, Bayesian power-spectrum inference for Large Scale Structure data, *Mon. Not. Roy. Astron. Soc.* 406 (2010) 60 [arXiv:0911.2493].

[8] J. Jasche, F.S. Kitaura, C. Li and T.A. Ensslin, Bayesian non-linear large scale structure inference of the Sloan Digital Sky Survey data release 7, *Mon. Not. Roy. Astron. Soc.* 409 (2010) 355 [arXiv:0911.2498].

[9] J. Jasche and B.D. Wandelt, Bayesian physical reconstruction of initial conditions from large scale structure surveys, *Mon. Not. Roy. Astron. Soc.* 432 (2013) 894 [arXiv:1203.3639].

[10] H. Wang, H.J. Mo, X. Yang, Y.P. Jing and W.P. Lin, ELUCID - Exploring the Local Universe with reConstructed Initial Density field I: Hamiltonian Markov Chain Monte Carlo Method with Particle Mesh Dynamics, *Astrophys. J.* 794 (2014) 94 [arXiv:1407.3451].

[11] M. Ata, F.-S. Kitaura and V. Müller, Bayesian inference of cosmic density fields from non-linear, scale-dependent, and stochastic biased tracers, *Mon. Not. Roy. Astron. Soc.* 446 (2015) 4250 [arXiv:1408.2566].

[12] F. Elsner, F. Schmidt, J. Jasche, G. Lavaux and N.-M. Nguyen, Cosmology inference from a biased density field using the EFT-based likelihood, *JCAP* 01 (2020) 029 [arXiv:1906.07143].

[13] F. Schmidt, G. Cabass, J. Jasche and G. Lavaux, Unbiased Cosmology Inference from Biased Tracers using the EFT Likelihood, *JCAP* 11 (2020) 008 [arXiv:2004.06707].

[14] E. Bertschinger and A. Dekel, Recovering the full velocity and density fields from large-scale redshift-distance samples, *Astrophys. J.* Lett. 336 (1989) L5.

[15] M. Schmittfull, T. Baldauf and M. Zaldarriaga, Iterative initial condition reconstruction, *Phys. Rev. D* 96 (2017) 023505 [arXiv:1704.06634].

[16] U. Seljak, G. Aslanyan, Y. Feng and C. Modi, Towards optimal extraction of cosmological information from nonlinear data, *JCAP* 12 (2017) 009 [arXiv:1706.06645].

[17] C. Modi, M. White, A. Slosar and E. Castorina, Reconstructing large-scale structure with neutral hydrogen surveys, *JCAP* 11 (2019) 023 [arXiv:1907.02330].

[18] M. Lewandowski, L. Senatore, F. Prada, C. Zhao and C.-H. Chuang, EFT of large scale structures in redshift space, *Phys. Rev. D* 97 (2018) 063526 [arXiv:1512.08831].

[19] A. Perko, L. Senatore, E. Jennings and R.H. Wechsler, Biased Tracers in Redshift Space in the EFT of Large-Scale Structure, *arXiv:1610.09321*.

[20] Z. Ding, H.-J. Seo, Z. Vlah, Y. Feng, M. Schmittfull and F. Beutler, Theoretical Systematics of Future Baryon Acoustic Oscillation Surveys, *Mon. Not. Roy. Astron. Soc.* 479 (2018) 1021 [arXiv:1708.01297].
[21] L. Fonseca de la Bella, D. Regan, D. Seery and D. Parkinson, *Impact of bias and redshift-space modelling for the halo power spectrum: Testing the effective field theory of large-scale structure*, *JCAP* **07** (2020) 011 [arXiv:1805.12394].

[22] V. Desjacques, D. Jeong and F. Schmidt, *The Galaxy Power Spectrum and Bispectrum in Redshift Space*, *JCAP* **12** (2018) 035 [arXiv:1806.04015].

[23] N. Kaiser, *Clustering in real space and in redshift space*, *Mon. Not. Roy. Astron. Soc.* **227** (1987) 1.

[24] F. Bernardeau, S. Colombi, E. Gaztanaga and R. Scoccimarro, *Large scale structure of the universe and cosmological perturbation theory*, *Phys. Rept.* **367** (2002) 1 [astro-ph/0112551].

[25] S. Saito, *Galaxy Clustering in Redshift Space*, https://wwwmpa.mpa-garching.mpg.de/komatsu/lectureseries/2016.html.

[26] Z. Vlah and M. White, *Exploring redshift-space distortions in large-scale structure*, *JCAP* **03** (2019) 007 [arXiv:1812.02775].

[27] D. Jeong and F. Schmidt, *Large-Scale Structure Observables in General Relativity*, *Class. Quant. Grav.* **32** (2015) 044001 [arXiv:1407.7979].

[28] C. Alcock and B. Paczynski, *An evolution free test for non-zero cosmological constant*, *Nature* **281** (1979) 358.

[29] M. Mirbabayi, F. Schmidt and M. Zaldarriaga, *Biased Tracers and Time Evolution*, *JCAP* **07** (2015) 030 [arXiv:1412.5169].

[30] L. Senatore, *Bias in the Effective Field Theory of Large Scale Structures*, *JCAP* **11** (2015) 007 [arXiv:1406.7843].

[31] V. Desjacques, M. Crocce, R. Scoccimarro and R.K. Sheth, *Modeling scale-dependent bias on the baryonic acoustic scale with the statistics of peaks of Gaussian random fields*, *Phys. Rev. D* **82** (2010) 103529 [arXiv:1009.3449].

[32] T. Baldauf, V. Desjacques and U. Seljak, *Velocity bias in the distribution of dark matter halos*, *Phys. Rev. D* **92** (2015) 123507 [arXiv:1405.5885].

[33] R.E. Smith, R. Scoccimarro and R.K. Sheth, *The Scale Dependence of Halo and Galaxy Bias: Effects in Real Space*, *Phys. Rev. D* **75** (2007) 063512 [astro-ph/0609547].

[34] N. Hamaus, U. Seljak, V. Desjacques, R.E. Smith and T. Baldauf, *Minimizing the Stochasticity of Halos in Large-Scale Structure Surveys*, *Phys. Rev. D* **82** (2010) 043515 [arXiv:1004.5377].

[35] T. Baldauf, U. Seljak, R.E. Smith, N. Hamaus and V. Desjacques, *Halo stochasticity from exclusion and nonlinear clustering*, *Phys. Rev. D* **88** (2013) 083507 [arXiv:1305.2917].

[36] K.C. Chan, N. Hamaus and V. Desjacques, *Large-Scale Clustering of Cosmic Voids*, *Phys. Rev. D* **90** (2014) 103521 [arXiv:1409.3849].

[37] G. Cabass and F. Schmidt, *The Likelihood for LSS: Stochasticity of Bias Coefficients at All Orders*, *JCAP* **07** (2020) 051 [arXiv:2004.00617].

[38] S.M. Carroll, S. Leichenauer and J. Pollack, *Consistent effective theory of long-wavelength cosmological perturbations*, *Phys. Rev. D* **90** (2014) 023518 [arXiv:1310.2920].

[39] E.A. Kazin, A.G. Sanchez and M.R. Blanton, *Improving measurements of H(z) and Da(z) by analyzing clustering anisotropies*, *Mon. Not. Roy. Astron. Soc.* **419** (2012) 3223 [arXiv:1105.2037].

[40] E. Pajer and M. Zaldarriaga, *On the Renormalization of the Effective Field Theory of Large Scale Structures*, *JCAP* **08** (2013) 037 [arXiv:1301.7182].

[41] A.A. Abolhasani, M. Mirbabayi and E. Pajer, *Systematic Renormalization of the Effective Theory of Large Scale Structure*, *JCAP* **05** (2016) 063 [arXiv:1509.07886].
[42] T. Baldauf, M. Mirbabayi, M. Simonović and M. Zaldarriaga, LSS constraints with controlled theoretical uncertainties, arXiv:1602.00674.

[43] R. Scoccimarro, Redshift-space distortions, pairwise velocities and nonlinearities, Phys. Rev. D 70 (2004) 083007 [astro-ph/0407214].

[44] F. Schmidt, Dynamical Masses in Modified Gravity, Phys. Rev. D 81 (2010) 103002 [arXiv:1003.0409].

[45] F. Schmidt, Sigma-Eight at the Percent Level: The EFT Likelihood in Real Space, arXiv:2009.14176.

[46] P. Catelan, M. Kamionkowski and R.D. Blandford, Intrinsic and extrinsic galaxy alignment, Mon. Not. Roy. Astron. Soc. 320 (2001) L7 [astro-ph/0005470].

[47] C.M. Hirata, R. Mandelbaum, M. Ishak, U. Seljak, R. Nichol, K.A. Pimbblet et al., Intrinsic galaxy alignments from the 2SLAQ and SDSS surveys: Luminosity and redshift scalings and implications for weak lensing surveys, Mon. Not. Roy. Astron. Soc. 381 (2007) 1197 [astro-ph/0701671].

[48] T. Okumura and Y.P. Jing, The Gravitational Shear — Intrinsic Ellipticity Correlation Functions of Luminous Red Galaxies in Observation and in ΛCDM model, Astrophys. J. Lett. 694 (2009) L83 [arXiv:0812.2935].

[49] C.M. Hirata, Tidal alignments as a contaminant of redshift space distortions, Mon. Not. Roy. Astron. Soc. 399 (2009) 1074 [arXiv:0903.4929].

[50] E. Krause and C. Hirata, Tidal alignments as a contaminant of the galaxy bispectrum, Mon. Not. Roy. Astron. Soc. 410 (2011) 2730 [arXiv:1004.3611].

[51] W. Fang, L. Hui, B. Menard, M. May and R. Scranton, Anisotropic Extinction Distortion of the Galaxy Correlation Function, Phys. Rev. D 84 (2011) 063012 [arXiv:1105.3421].

[52] S. Singh, R. Mandelbaum and S. More, Intrinsic alignments of SDSS-III BOSS LOWZ sample galaxies, Mon. Not. Roy. Astron. Soc. 450 (2015) 2195 [arXiv:1411.1755].

[53] Z. Zheng, R. Cen, H. Trac and J. Miralda-Escude, Radiative Transfer Modeling of Lyman Alpha Emitters. II. New Effects in Galaxy Clustering, Astrophys. J. 726 (2010) 38 [arXiv:1003.4990].

[54] S. Wyithe and M. Dijkstra, Non-Gravitational Contributions to the Clustering of Ly-alpha Selected Galaxies: Implications for Cosmological Surveys, Mon. Not. Roy. Astron. Soc. 415 (2011) 3929 [arXiv:1104.0712].

[55] C. Behrens, C. Byrohl, S. Saito and J.C. Niemeyer, The impact of Lyman-α radiative transfer on large-scale clustering in the Illustris simulation, Astron. Astrophys. 614 (2018) A31 [arXiv:1710.06171].

[56] N. Agarwal, V. Desjacques, D. Jeong and F. Schmidt, Information content in the redshift-space galaxy power spectrum and bispectrum, arXiv:2007.04340.

[57] O. Doré et al., Cosmology with the SPHEREX All-Sky Spectral Survey, arXiv:1412.4872.

[58] DESI collaboration, The DESI Experiment, a whitepaper for Snowmass 2013, arXiv:1308.0847.

[59] Euclid Theory Working Group, Cosmology and fundamental physics with the Euclid satellite, Living Rev. Rel. 16 (2013) 6 [arXiv:1206.1225].

[60] L. Amendola et al., Cosmology and fundamental physics with the Euclid satellite, Living Rev. Rel. 21 (2018) 2 [arXiv:1606.00180].

[61] J. Jasche and G. Lavaux, Physical Bayesian modelling of the non-linear matter distribution: new insights into the Nearby Universe, Astron. Astrophys. 625 (2019) A64 [arXiv:1806.11117].
[62] D.K. Ramanah, G. Lavaux, J. Jasche and B.D. Wandelt, Cosmological inference from Bayesian forward modelling of deep galaxy redshift surveys, Astron. Astrophys. 621 (2019) A69 [arXiv:1808.07496].

[63] G. Lavaux, J. Jasche and F. Leclercq, Systematic-free inference of the cosmic matter density field from SDSS3-BOSS data, arXiv:1909.06396.

[64] K.B. Fisher, O. Lahav, Y. Hoffman, D. Lynden-Bell and S. Zaroubi, Wiener reconstruction of density, velocity, and potential fields from all sky galaxy redshift surveys, Mon. Not. Roy. Astron. Soc. 272 (1995) 885.

[65] A.F. Heavens and A.N. Taylor, A Spherical Harmonic Analysis of Redshift Space, Mon. Not. Roy. Astron. Soc. 275 (1995) 483 [astro-ph/9409027].