Electrodynamic processes models in atmospheric surface layer

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Abstract. The paper outlines the methodological approaches to the problem of mathematical modeling of electrical processes taking place in the surface layer of the atmosphere. A comparative analysis of various problem statements was carried out in terms of the possibilities of qualitative and quantitative analysis, as well as a set of hypotheses taken into account in the models regarding the properties of the studied processes.

Introduction
The atmospheric surface layer is characterized by existence of turbulent exchange processes, surface ionization sources (radioactivity) and aerosol particle sources both of natural and artificial origin. The electrodynamic status of the surface layer is determined by the so-called electrode effect which identifies the population of processes taking place in the ionized medium (air) close to the electrode surface (ground) which lead to dependence of local electrical atmospheric characteristics on elevation [9]. Meteorological factors have a significant impact on electrical and hydrodynamic features of the surface layer. Under the “good weather” conditions the thickness of the electrode layer changes from several dozens of centimeters to dozens of meters [9].

Atmospheric-electrical observations which are usually taken close to the ground surface, solve a number of general and special problems of the atmospheric physics; herewith a special scientific and practical interest is generated by behavior of electrical field which is stipulated by superposition of local disturbances on its global variations. The main regulatory daily disturbance of the electric field is its unitary variation [1,7,8,11] which represents synchronized changes at any point of observation (with a minimum of (04 h -05 h UT) and a maximum of (19 h -20 h UT)) due to the changes of the ionosphere potential as a consequence of influence of the global generator of the atmospheric electrical field connected with thunderstorm activity of the ground equatorial zone. Discrimination of global disturbances of the electrical field against the background of the local factor activity, in the first instance of anthropogenic impact (aerosol and radioactive air pollution), on the atmosphere represents a topical problem of experimental research of electricity in the surface layer, which is only partially solved by a selection of globally representative points of observation, particularly in highland and polar regions of the earth [10,11]. The impact of the electrode effect and meteorological...
conditions should be taken into account even in the most favorable conditions of the atmosphere-electrical observations.

The problem of mathematic simulation of the electric state of the surface layer is formulated as a problem of finding distribution of polar light ions (air ions) concentrations, electric field and electric current density in the surface layer [11]. Depending on meteorological state of the atmosphere the two extreme cases will be considered: the classical (non-turbulent) [4,15] electrode effect and the turbulent [7,16,19] electrode effect. Presence in the atmosphere of aerosol particles which represent a drain for the air ions, have an impact on the electrode effect, and under sufficiently great concentration the electric state of the surface layer can be characterized by only heavy ions which appear due to interaction of aerosol with the air ions.

Theoretical Justification

For the horizontally homogeneous turbulent surface layer free from aerosol particles the electrodynamic equations of the model will have the following form [9]:

\[ \frac{\partial n_{1,2}}{\partial t} + \frac{\partial}{\partial z} \left( b_{1,2} n_{1,2} E \right) - \frac{\partial}{\partial z} \left( D_T (z,t) \frac{\partial n_{1,2}}{\partial z} \right) = q - \alpha n_1 n_2, \quad \frac{\partial E}{\partial z} = 4\pi e (n_1 - n_2), \]

where \( n_{1,2} \) are volumetric concentrations of polar air ions, \( b_{1,2} \) is their mobility, \( E \) is the electric field intensity, \( q(z) \) is the function of the ionization rate, \( \alpha \) is the recombination factor, \( D_T(z,t) \) is the turbulent diffusion factor, \( e \) is the elementary charge.

The boundary conditions are \( n_{1,2}(z_0) = 0, \quad n_{1,2}(\infty) = \left( q(\infty) / \alpha \right)^{1/2}, \quad E(z_0) = E_0 \) for turbulent electrode effect and only \( n_2(0) = 0 \) for classical one. In turbulent case introduction of the roughness parameter \( z_0 \) which depends on the Reynolds number is equivalent to determination of the nature of the dynamic interaction of the turbulent flow with underlying surface. In the event of an aerodynamically smooth surface the parameter \( z_0 = 0 \).

To analyze the system (1) we represent it in the dimensionless form: \( t' = t/T, \quad z' = z/l_1, \quad n_{1,2}' = n_{1,2}/n_\infty, \quad E' = E/E_\infty, \quad n_\infty = \sqrt{q_\infty/\alpha}, \quad l_1 = D_T \cdot \tau, \quad \tau = (q_\infty \cdot \alpha)^{-1/2} \). Thereat we get the following system:

\[ \frac{\tau}{T} \frac{\partial n_{1,2}'}{\partial t'} - \frac{\partial}{\partial z'} \left( \xi_{1,2} \frac{\partial n_{1,2}'}{\partial z'} \right) + \xi_{1,2} \frac{\partial}{\partial z'} \left( n_{1,2}' E' \right) = \frac{q}{q_\infty} - n_1', \quad \frac{\partial E}{\partial z'} = \gamma (n_1' - n_2'), \]

Characteristic lead time of the hydrodynamic processes (T) equals to several hours whereas the lead time of the electric processes (\( \tau \)) is 250 sec. under the \( q \) values equal to \( 10^7 \) m\(^{-3}\)s\(^{-1}\) and \( \alpha \) values equal to \( 1.6 \times 10^{-12} \) m\(^3\)s\(^{-1}\). Therefore, employment of stationary models of the electrode effect for simulation of the surface layer electric state is appropriate.

The system of equations (2) is characterized by the two dimensionless parameters:

\[ \xi_{1,2} = \frac{b_{1,2}}{l_1}, \quad \gamma = 4\pi e l_1 \frac{n_\infty}{E_\infty}. \]

As it is evident, in the case of \( |\gamma| = 1 \) the electric field generated by the density of the electric space charge near the ground surface may be neglected.

Electrode Effect Models: Theory and Discussions
The deduced parameters $\xi_{1,2}$ and $\gamma$ can serve as criteria for the use of one or another model of the electrode effect for investigation of the electric surface layer depending on physical and meteorological conditions.

1) Classical Electrode Effect. When the parameter $\xi_{1,2} \geq 1$, the spatially-temporal distribution (throughout the height of $z$) of air ions in the surface layer is stipulated by electric forces only. The system of equations of the model has the following form [4,16]:

$$\pm \frac{d}{dz}(b_{1,2}n_{1,2}E) = q(z) - \alpha n_n n_2, \quad \frac{dE}{dz} = 4\pi e(n_1 - n_2).$$ (4)

Here the $\infty$ sign means the upper limit of the area of the electrode effect action. The usual height of the classic electrode layer amounts to $3 - 5$ meters. In case of the electric field increase from 100 to 500 V/m the electrode effect value ($E(z)/E_1$) on the ground surface does not change practically, therefore the scope of distribution of the electric values increases. In the result parameters of the electrode layer change: the parameter $E(z)/E_1$ increases at the height of several meters with the electric field enhancement, the parameter $n_1/n_2$ does not change practically and the $n_2/n_1$ decreases.

Under the “good weather” conditions, the space electric charge close to the ground surface is positive and the scope of its distribution is determined by the thickness of the classical electrode layer and amounts to several meters. The values of the space charge density are determined both by the power of the source of ion formation and by the electric field amount.

A negative space charge appears in the presence of a thin layer (several dozens of centimeters) of increased ionization close to the surface surface and leads to reversing of the electrode effect. This effect appears under a slight degree of air ionization but in weak electric fields (about several dozens of Volts per meter). In case of electric field enhancement or increase of scope of the function of ion formation the space charge becomes positive.

2) Turbulent Electrode Effect. If parameter $\bar{\xi}_{1,2}<1$, the transfer of air ions in the atmosphere shall take place together with electric forces and turbulent air flows.

The system of equations [7, 15, 19] is:

$$-\frac{d}{dz}(D_{1,2}(z)\frac{dn_{1,2}}{dz}) + \frac{d}{dz}(b_{1,2}n_{1,2}E) = q(z) - \alpha n_n n_2, \quad \frac{dE}{dz} = 4\pi e(n_1 - n_2).$$ (5)

In turbulent atmosphere the thickness of the electrode layer increases together with increase of the wind velocity value. Under low wind velocities (less than 1 m/sec) the profile of concentration of positive ions $n_1$ increases very fast and at the height of one meter it reaches its asymptotic value. In this case the positive space charge close to the surface will be highest possible and at the height of 6 meters the difference between $n_1$ and $n_2$ values does not exceed 5%. All this makes the profile look like distribution of electric values close to the surface in case of the classic electrode effect. In case of wind velocity increase up to 5 -6 m/s the $n_1$ and $n_2$ profiles become similar and the difference between the values at the height of one meter does not exceed 10%. The thickness of the electrode layer and the scope of distributions of its electric specifications increase and reach several dozens of meters.

With increase of the wind velocity the electrode effect parameter $E(z)/E_1$ at the height of 1-2 meters (which is standard for the atmosphere-electrical observations) increases due to increase of the scope of distribution, but in the whole electrode layer parameter ($E(z)/E_1$) it remains constant. This may be due to the fact that the turbulence is not an electric field generator.

With increase of degree of the air ionization and a low wind velocity (one meter per second maximum) a negative space charge appears close to the ground surface as is in case with the classic electrode effect; herewith the scope of its distribution increases up to 10-15 meters and density of the electric charge decreases. With increase of the turbulent intermixing or increase of the electric field the space charge becomes positive. Electric field enhancement decreases the effect of turbulence — the positive space charge increases, $(E(z)/E_1)$ values close to the ground surface slightly increase and distributions of the electric values become similar to the classic case.
3) Strong Turbulent Mixture Approximation. In case of fulfilment of the condition where \( \square_{1,2} \times < 1 \), by way of using the small parameter expansion method \( (1,2) \) the initial system (1) splits into the system of linear (according to the electric field) equations and in the zero-order approximation the air ion concentration does not depend on the electric field intensity but is determined by the turbulent interchange, by ionization and recombination processes.

In approximation of the strong turbulent interchange the system of equations has the following form \([6,7,13,17,19]\):

\[
\begin{align*}
- \frac{d}{dz} \left( D_T(z) \frac{dn_{1,2}}{dz} \right) &= q - \alpha n_1 n_2, \\
- D_T(z) \frac{d^2 E}{dz^2} + 4 \pi \lambda(z) E &= 4 \pi j_0,
\end{align*}
\]

where is \( \lambda \) – electrical conductivity, \( j \) – electric current density. The new boundary conditions for electric field are:

\[
\frac{dE}{dz} \bigg|_{z=0} = 0, \quad E \bigg|_{z=\infty} = \frac{j_0}{\lambda_\infty}.
\]

Distribution of the air ions along \( z \) height is determined by the \( l_m = \left( D_m \tau \right)^{1/2} \), \( \tau = (q_\alpha \lambda)^{1/2} \) scale which represents the distance which an ion passes during its lifetime due to the turbulent diffusion. Distribution of the electric field with reference to the height is determined by the change of electric conductivity with reference to the height and by the characteristic scale \( L_m = \left( D_m \tau_{\infty} \right)^{1/2} \), \( \tau_{\infty} = (4 \pi \lambda_{\infty})^{-1} \). The \( L_m \) scale determines thickness of the turbulent electrode layer, \( m \) is the parameter of the surface layer stratification. Calculations for the events with stable \( (m = 0) \), neutral \( (m = 1) \) and thermally unstable \( (m = 4/3) \) surface layer stratification showed that with increase of the degree of the atmospheric instability the scope of the electric characteristic distribution would increase. Density of the electric charge drastically increases near the surface and then decreases, and the most dramatic changes of the electric charge density were deduced with the thermally instable stratification. For the neutral stratification of the surface layer the change of air ion concentration and of the electrical conductivity with reference to the height has a logarithmic nature.

4) The Electrode Effect under Conditions of Aerosol Pollution of Atmosphere. With presence of aerosol particles in atmosphere the right part of the ionization-recombination equations comes up with members which describe the interaction of air ions with aerosol particles. Besides, there are added equations that describe turbulent transfer of the newly formed heavy ions. Given that conditions of equilibrium between the aerosol particles and light ions are met and neglecting the current of charged heavy ions generated within the process of connection of the aerosol with the air ions, the system of equations for the turbulent surface layer has the following form \([9,12,13]\):

\[
\begin{align*}
\frac{d}{dz} \left( D_T(z) \frac{dn_{1,2}}{dz} \right) \pm \frac{d}{dz} \left( b_{1,2} n_{1,2} E \right) &= q - \alpha n_1 n_2 - \eta_1 n_{1,2} N_{1,2} - \eta_2 n_{1,2} N_0, \\
\frac{dE}{dz} &= 4 \pi e \left( n_1 - n_2 + N_1 - N_2 \right), \\
N_1 + N_2 + N_0 &= N = const, \\
\frac{d}{dz} \left( \tau(z) \frac{dN_{1,2}}{dz} \right) &= \eta_1 n_{1,2} N_0 - \eta_2 n_{1,2} N_{1,2},
\end{align*}
\]
where \( N_{1,2} \) are space concentrations of polar heavy ions, \( \Box(z) \) is the factor of turbulent change for heavy ions.

The boundary conditions for the light ions and the boundary field have the form which is similar to the conditions in the atmosphere free from aerosol (1) - (3), and for the heavy ions it will be as follows:

\[
\left( \frac{dN_{1,2}}{dz} \right)_{z=z_0} = 0, \quad N_1(\infty) = N_2(\infty) = N_0.
\]  

(9)

In the classical electrode layer appearance of aerosol particles in the surface air \((N/\Box)(10^8 - 10^9) \text{ m}^{-3}\) diminishes thickness of the electrode layer and the \(E_0/E_0\) value does not change practically. With concentration of aerosol particles exceeding the value of \(5 \times 10^9 \text{ m}^{-3}\) the electric condition of the surface layer is determined mostly by heavy ions. Enhancement of the electric field close to ground with aerosol presence same as in clear atmosphere leads to increase of the \(E(z)/E_0\) parameter at the height of several meters, but to a lesser extent. Herewith other electrode layer parameters change in the following way: \(n_2/n_1\) and \(N_2/N_1\) diminish, \(N_1/N_1\) increase, \(n_1/n_1\) remain practically constant.

In the turbulent electrode layer presence of the aerosol particles with concentration less than \(10^9 \text{ m}^{-3}\), as is in the previous case, does not practically affect its characteristics. With aerosol concentration values of the order of \(10^9 \text{ m}^{-3}\) in the layer several meters thick presents reversing of the electric field which shows predominance of the negative space charge generated by heavy ions over the positive space charge conditioned by air ions.

Further increase of the aerosol particle concentration results in the electric structure of the surface layer being determined by distribution of heavy ions while concentration of light ions may be neglected. In this case the allowed suppositions are not fulfilled and it is necessary to go over from the electrode effect model to the models of turbulent transfer of heavy ions in the surface layer.

It should be noted that all above electromechanical models are steady which is conditioned by interrelation of characteristic times of development of meteorological and electrical processes in atmosphere.

### Unsteady Electrodynamic Model of the Surface Layer.

1) **Classical Electrode Effect Approximation.** Equations of the model with initial conditions [2,5]:

\[
\frac{\partial n_{1,2}}{\partial t} \pm b_{1,2} \frac{\partial (E \cdot n_{1,2})}{\partial z} = q(z) - \alpha n_1 n_2, \quad \frac{\partial E}{\partial z} = \frac{e}{\varepsilon_0} (n_1 - n_2),
\]

\[
\left. n_{1,2}(z) \right|_{t=0} = \frac{q}{\alpha} \left( 1 - e^{-\frac{z}{b_1}} \right), \quad \left. E(z) \right|_{t=0} = E_0,
\]  

\(10\)

where \( l_0 \) is the classical electrode layer thickness. The boundary conditions are the same as in the stationary case.

The conducted numerical calculations show that with the course of time thickness of the electrode layer increases and herewith at the height of 0.5 to 1 meters the electrode effect values change insignificantly. Concentration of positive and negative air ions near the ground surface decreases within the first 5 minutes and afterwards it becomes practically constant. The space charge density changes significantly within the first 3 minutes and afterwards it is practically constant. With a low degree of air ionization but with low electric fields (up to 30 V/m) same as in the steady case, the space charge at the height of 0.5 – 2 meters has a negative sign. With the electric field enhancement the space charge sign becomes positive. Time of the steady mode setup amounted to about 5-7 minutes.

2) **Turbulent Electrode Effect Approximation.** Equations of the model with initial conditions are [3,5]:
\[
\frac{\partial n_{1,2}}{\partial t} + b_{1,2} \frac{\partial (E \cdot n_{1,2})}{\partial z} - \frac{\partial}{\partial z} \left( D_{1,2}(z) \frac{\partial n_{1,2}}{\partial z} \right) = q(z) - \alpha n_{1,2},
\]
\[E(z) \big|_{t=0} = E_0,
\]
where \( l_t \) is the turbulent electrode layer thickness. The boundary conditions are the same as in the stationary case.

Analysis of the calculation results shows that with increase of the turbulent diffusion thickness of the electrode layer increases. Time of setup of the steady mode in the turbulent electrode layer amounts to about 15 minutes.

3) Convective Transfer Accounting. Equations of the model are

\[
\frac{\partial n_{1,2}}{\partial t} + \frac{\partial}{\partial z} \left( \mathbf{u}(z) \cdot n_{1,2} \cdot E \right) - \frac{\partial}{\partial z} \left( D_{1,2}(z) \frac{\partial n_{1,2}}{\partial z} \right) = q(z) - \alpha n_{1,2},
\]
\[
\frac{\partial E}{\partial z} = \frac{e}{\varepsilon_0} (n_t - n_z),
\]
where \( \mathbf{u} \) is the velocity of convective transfer. It was estimated that the convective transfer could significantly affect structure of the electrode layer near the ground surface [12,18]. It was derived that the time of the steady mode setup amounted to about 10-15 minutes and that it did not depend on the convective transfer speed.

Modelling of Global Disturbances of Electric Field.

With the use of the Poisson equation after integration of the equation system (1) it is possible to get equation for density of the complete electric current (\( j(t) \)) in the surface layer of atmosphere [1,9]:

\[
\frac{\partial E}{\partial t} + 4\pi \lambda E - D_t(z,t) \frac{\partial^2 E}{\partial z^2} = 4\pi j(t).
\]

Boundary and initial conditions for the equation (4) with a relevant left part can be represented in the following form:

\[
E \big|_{z=0} = E_0, \quad E \big|_{t=0} = E(0,z) \bigg| \frac{\partial E}{\partial t} + 4\pi \lambda E \bigg|_{z=0} = 4\pi j(t).
\]

Variations of the electric current density \( j(t) \) near the ground surface are determined by unsteady of electric fields above the surface layer, i.e. they are conditioned by global changes of the ionosphere potential caused either by thunderstorm current generators or by generators working in the upper atmosphere.

With change of the electric current density in the left part (13) according to the harmonic law in the electric field in the turbulent surface layer there is phase displacement of the electric field oscillation with reference to oscillations of the electric current density [1,9]. This is conditioned by changes of electric characteristics of the electrode layer due to differing degrees of the turbulent interchange in the atmosphere during within days.

Summary

At present there are many mathematical models of aerodynamics of the atmosphere surface layer and ways of their solving including numerical modelling with the use of modern computing facilities. Anyhow, a large population of both electrical and meteorological processes and phenomena taking place in actual atmosphere demand a correct use of theoretical results. When analyzing information of regular atmosphere-electrical observations we can broadly employ the electrode effect models,
whereas to solve problems related to electric characteristic variations we need nonstationary models with due regard of the increasingly larger role of aerosol and radioactive atmospheric pollution.

When choosing points and arrangement of atmospheric-electrical observations with due regard to the tasks in hand it is also necessary to take into account theoretical representations. It is especially important for implementation of longstanding global or regional monitoring of the electric condition of atmosphere. It is highly recommended to collect relevant meteorological information for further correct interpretation of the obtained data.

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