Local computer model emulating the results of the Pan et al. experiment

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Abstract

It is a widespread current belief that objective local models cannot explain the quantum optics experiment of Pan et al. By presenting a model that operates on independent computers, we show that this belief is unfounded. Three remote computers (Alice, Bob and Claire), that never communicate with each other, send measurement results to a fourth computer that is in charge of collecting the data and computing correlations. The result obtained by our local simulation is in better agreement with the ideal quantum result than the Pan et al. experiment. We also show that the local model presented by Pan et al. that cannot explain the quantum results contains inappropriate reasoning with profound consequences for the possible results of any local model that uses probability theory.

1 Introduction

The paper of Pan et al. presents experimental results using quantum optics and in addition a special version of an objective local theory (POLT). For the actual experiment we refer the reader to the original reference and describe here only POLT, their objective local model, in detail. Three photons were detected in coincidence with a fourth one (i.e. two entangled photon pairs are used) after either a linear or circular polarization measurement. For each photon \( i = 1, 2, 3 \) the authors have introduced “elements of reality \( X_i \) with values \( \pm 1 \) for \( H'(V') \)
polarizations and $Y_i$ with values $\pm 1$ for R(L)" polarizations. These elements of reality are treated as random variables because they in turn depend on another common random variable $\Lambda$. $\Lambda$ may, for example represent some hidden property of the two detected photon pairs and may have any mathematical form (e.g. that of a vector labelling all four photons). It is now common practice \[2\] to introduce a counterfactual argument of the following kind. Given that a measurement has been performed with one given setting e.g. the $xxy$ experiment resulting in the measurement $X_1, Y_2, Y_3$, one could have performed the measurement using another setting, say an $xyx$ experiment or a $yx$ experiment resulting in $Y_1, X_2, Y_3$ or $Y_1, Y_2, X_3$, respectively, with the same $\lambda$. Here $\lambda$ is the value that the random variable $\Lambda$ has assumed in the actual measurement. Therefore, extending this argument to all $X_i$ and $Y_i$, the authors claim that the elements of reality $X_i$ and $Y_i$ satisfy the relations

\[ Y_1 \cdot Y_2 \cdot X_3 = -1, \quad Y_1 \cdot X_2 \cdot Y_3 = -1, \quad X_1 \cdot Y_2 \cdot Y_3 = -1 \]  

(1)

and

\[ Y_i \cdot Y_i = +1 \]  

(2)

and therefore by Eq. (1) that

\[ X_1 \cdot X_2 \cdot X_3 = -1 \]  

(3)

whereas Quantum Mechanics predicts a $+1$ result for Eq. (3). This results represents a contradiction between POLT and Quantum Mechanics. As has been analyzed elsewhere in more detail \[3, 4\], this kind of reasoning can be seriously questioned. Key to the understanding of the critique is the following. The counterfactual argument that one could have measured with another setting and the same $\lambda$ does, in our opinion, not yet cause any harm and may be logically correct. However, to assume further that the results of the real experiment can be understood in terms of and actually do contain elements all with identical $\lambda$’s to obtain Eqs. (1-3) requires several additional mathematical assumptions. One of these additional assumptions, that permeates all arguments that start with counterfactual reasoning, is that all random variables involved can be defined on a single probability space. As outlined in greater detail in \[5\], in order to model an experiment one first needs to start with a one-to-one correspondence between the experiments and the elements $\omega$ of the probability space (see Feller \[6\]). A random variable $\chi$ assigns to each indecomposable event $\omega$ a measurement $\chi(\omega)$, i.e., a value that the random variable $\chi$ assumes at $\omega$. In order to algebraically manipulate the random variables, such as in the conclusion that Eqs. (1) and (2) imply Eq. (3) it is necessary that all the random variables are defined on the same probability space. Because in (1) the results of Eq. (1) originate from three different experiments, each of them having a different configuration of measuring equipment, and performed over three different time intervals, Eq. (1) needs to be rewritten in the form

\[ Y'_1 \cdot Y'_2 \cdot X'_3 = -1, \quad Y''_1 \cdot X''_2 \cdot Y'''_3 = -1, \quad X'''_1 \cdot Y'''_2 \cdot Y'''_3 = -1 \]  

(4)
It is only when we equate

\[ Y_1^\prime = Y_1^{\prime\prime}, \quad Y_2^\prime = Y_2^{\prime\prime\prime}, \quad Y_3^{\prime\prime} = Y_3^{\prime\prime\prime} \quad (5) \]

that Eq. (3) can be written in the form

\[ X_1^\prime \cdot X_2^{\prime\prime} \cdot X_3^{\prime\prime\prime} = -1 \quad (6) \]

Our model exploits the fact that a fourth experiment is performed, in a separate time interval. We do not identify the product of the results of three independent experiments with the result of this fourth experiment

\[ X_1^\ast \cdot X_2^\ast \cdot X_3^\ast = +1 \quad (7) \]

because equating:

\[ Y_1^\prime = Y_1^{\prime\prime}, \quad Y_2^\prime = Y_2^{\prime\prime\prime}, \quad Y_3^{\prime\prime} = Y_3^{\prime\prime\prime}, \quad X_1^{\prime\prime\prime} = X_1^\ast, \quad X_2^{\prime\prime} = X_2^\ast \quad \text{and} \quad X_3^\prime = X_3^\ast \quad (8) \]

means that the six random variables figuring in Eqs. (11) and (13) can all be defined on a single probability space. This clearly contradicts Vorobev’s theorem [7], which states that this may not be possible since the four threefold joint distributions form a closed loop. In particular, if in addition to the above mentioned parameter \( \Lambda \), time and setting dependent equipment parameters \( \Lambda_e(t_m) \) are in play, then it can be easily shown that Eq. (8) does not hold, yet one still has an objective local model (here \( e \) denotes equipment setting and \( t_m \) the measuring time). Thus in this case the six random variables figuring in Eqs. (11) and (13) can not be defined on a single probability space. The simple reason for this is that each measurement time \( t_m \) labels a different light cone in the sense of relativity theory and therefore a different set of causal influences. These in turn may then signify different experimental conditions and the need for different probabilistic setups as well.\(^1\)

Equations (11) and (13) also imply that the promised “all versus nothing” (a single experiment ruling out local realistic theories in favor of standard quantum mechanics) is not corresponding to the real experiment, because four distinct experiments need to be carried out before any conclusions can be drawn. Needless to say, that in order to obtain statistically meaningful results, a large number of these four distinct experiments will have to be performed. The “all versus nothing” is based on the inappropriate assumption that the results of the real experiment can be understood and actually do contain elements all with the identical \( \lambda \)'s to obtain Eqs. (11-13), as discussed above.

In the following we present a simple model that goes beyond POLT by introducing time related variables. Our model is objective local because it can be implemented on independent computers. We are aware of the fact that the

\(^1\)We remark in this connection that any detailed theory of the Pan et al. experiment that uses relativity will need to introduce four-vectors that in turn introduce time.
model is not the most general and does not prove the existence of $\Lambda_e(t_m)$. It also does not provide any finality to the discussions surrounding the theorem of Bell and other related experiments. However, we maintain that oversimplified models such as POLT, (that employ only one $\Lambda$ and one single probability space for four distinct experiments) are inadequate for any serious argumentation designed to prove non-locality in nature. Therefore, if one wants to make statements about objective locality out of certain experimental results, it will also be necessary to improve in the future the actual experiments so that models such as the one presented below can be ruled out.

2 Description of the model

We start our extended model involving independent computers by introducing the following functions [5]. Let

$$ r_k(t) = \text{sign} \left[ \sin(2^k \pi t) \right] \text{ for } t > 0 $$

(9)

denote the $k$-th Rademacher function. Note that $r_k$ has period $2^{-(k-1)}$. The following table is to be implemented in the network simulation.

|        | $yyx, t_0 < t < t_1$ | $xyy, t_2 < t < t_3$ | $xxy, t_4 < t < t_5$ | $xxx, t_6 < t < t_7$ |
|--------|---------------------|---------------------|---------------------|---------------------|
| Comp1  | $Y'_1 = -r_1$       | $Y''_1 = -r_1$      | $X'''_1 = r_2 \cdot r_3$ | $X^*_1 = r_2 \cdot r_3$ |
| Comp2  | $Y'_2 = r_2$        | $X''_2 = r_1 \cdot r_3$ | $Y''''_2 = r_2$      | $X^*_2 = r_1 \cdot r_3$ |
| Comp3  | $X'_3 = r_1 \cdot r_2$ | $Y''''_3 = r_3$      | $Y''''_3 = -r_3$     | $X^*_3 = r_1 \cdot r_2$ |

Here $t_1 - t_0$ is the length of time the $yyx$ experiment is running, $t_2 - t_1$ is the length of time it takes the experimenters to switch the experimental set-up from an $yyx$ experiment to an $xyy$ experiment. $t_3 - t_2$ is the length of time the $xyy$ experiment is running and $t_4 - t_3$ again the time to switch and so forth as described in Table 1. Each of the three equations in Eq. 4 holds on the entire time interval where they are defined. Moreover, we have

$$ X'^*_1 \cdot X'^*_2 \cdot X'^*_3 = +1 $$

(10)

instead of Eq. 3, if we mimic at a later time the $xxx$ experiment according to the last column in Table 1. Furthermore, each $X$ and each $Y$ equals $+1$ or $-1$ half of the time. The essential point here is, of course, that for a given time in which a given experiment e.g. $yyx$, is being performed, we can assume that equipment parameters are such that $Y$ may be described by a certain Rademacher function, e.g. $Y_3 = r_3$, while at a later time, for which the $xyy$ experiment is performed we may have $Y_3 = -r_3$. (Here we have made use of the fact that in the Pan et
al. experiment the settings e.g. $yyx$ are set and used for a longer period of time so that communication between the stations is possible by sub-light velocities.)

For a given setting, the outcomes of the various experiments are, of course, only “known” at a given detector, not at the others. This is so because, as will be further described in the implementation, each of the players have only the part of the table that corresponds to them, while the Host only determines when the experiments start, and he does not have any knowledge of the entries on Table 1. Only the choice of measurement time, which is random, determines the outcomes together with the Rademacher functions that are characteristic for a given setting. Of course the three Rademacher functions in Table 1 can be replaced by three other Rademacher functions with arbitrarily large but different subscripts if faster fluctuation between $+1$ and $-1$ is desired.

### 3 Implementation of the model

The networking program works as follows. We make use of a Visual Basic template available online, and engineer it such that three computers (Alice, Bob and Claire) are connected to a fourth computer (Host). The Host first listens from its ethernet port until the three computers are connected. The Host is able to determine whether or not all the three remaining computers are reachable with the given domain. This is done by:

- Pressing the *Port Settings* button (by default set to *First Local Port* = 600, *First Remote Port* = 700, and *Maximum Number of Connections* = 3).
- Pressing the *Host* button.

The three players connect to the remote Host whose IP number should be entered by each player in their *Connect To* window, with the settings

- *First Local Port* = 600.
- *First Remote Port* = 701 for Alice, 702 for Bob and 703 for Claire.

When the three players are connected, the Host can see a new button, which enables him to

- **Launch the Simulation.**

By pressing it, it tells Alice, Bob and Claire, to set a common time label, and to start sending data according to Table 1 at certain time intervals.

At a given time $t$, the player at computer $i$ will make either a $x$ experiment, with a measurement result denoted $X_{i,t}$, or a $y$ experiment, with a measurement result denoted $Y_{i,t}$, the result of each measurement being either $+1$ or $-1$, according to Table 1. The players perform successively four sequences of “experiments”: a sequence of $yyx$ experiments (Alice and Bob perform a $y$ measurement, while Claire performs a $x$ measurement), a sequence of $yxy$ experiments, a third sequence of $xyy$ experiments and finally a sequence of $xxx$
experiments. Each player sends his/her label (x or y) according to his/her performed experiment, together with a measurement result to the Host, in charge of collecting the data and processing the product of the three measurement results sent by the players. The code was written so that, at a given time, each player can find in the code only the relevant part of Table 1 while this table is unaccessible to the Host. Very importantly, during the process, the players never communicate with each other, only unidirectionally to the Host. This is to say, they are only sending data through their ethernet cards.

This networking simulation exhibits a perfect agreement with the ideal quantum results. The data sent to the Host by the three players Alice, Bob, and Claire successively yields the following:

\[ Y_1' \cdot Y_2' \cdot X_3' = -1, \quad Y_1'' \cdot X_2'' \cdot Y_3'' = -1, \quad X_1''' \cdot Y_2''' \cdot Y_3''' = -1 \]  \hspace{1cm} (11)

and finally,

\[ X_1^* \cdot X_2^* \cdot X_3^* = +1, \]  \hspace{1cm} (12)

in accordance with Eqs. 11 and 7.

The detailed program can be obtained from us by e-mail upon request at the following electronic address: guillaume.adenier@msi.vxu.se.

4 Conclusion

We have presented a game that obeys all the rules of locality that are obeyed in the actual experiment of [1] and that results in the quantum mechanical correlations. With respect to the latter, our simulation outperforms the experimental results of Pan, Bouwmeester, Daniell, Weinfurter and Zeilinger [1], and therefore clearly invalidates their claim that objective local parameters cannot explain their experimental results. Future experiments and models need to consider therefore the effects of time and setting dependent equipment parameters.

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