Sliding Network Coding for URLLC

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Abstract—In this paper, we propose a network coding (NC) based approach to ultra-reliable low-latency communication (URLLC) over erasure channels. In transmitting multiple data packets, we demonstrate that the use of random NC can improve the reliability in terms of decoding error probability, while it incurs a longer decoding delay than a well-known $K$-repetition. To avoid a long decoding delay, we consider a sliding NC (SNC) design that allows a highly reliable transmission of each data packet with a guaranteed decoding delay. A few design examples are derived and their decoding error rates are analyzed. Through the analysis, we can show that the decoding error rate of SNC is much lower than that of $K$-repetition at the same spectral efficiency, which means a more reliable transmission can be achieved using SNC than $K$-repetition.

Index Terms—URLLC; Network coding; Sliding window

I. INTRODUCTION

In the fifth generation (5G) technology standard for cellular systems [1] [2], a number of new services and applications are to be supported. Among them, there are mission-critical applications (including industrial automation and autonomous vehicles) that are to be supported by ultra-reliable low-latency communication (URLLC) [3] [4] [5]. In general, URLLC is considered to be the most challenging task in 5G and future wireless networks due to two ambitious requirements, namely, high-reliability and low-latency, to be satisfied simultaneously for short-packet transmissions [6] [7] [8]. According to [9], 5G has identified several scenarios of URLLC with performance requirements. For example, for factory automation, the actuation of industrial devices has stringent performance requirements such as a latency of 1 millisecond (ms) and reliability of 99.9999%.

In general, in order to achieve a high reliability, hybrid automatic repeat request (HARQ) protocols can be used [10] [11]. In HARQ as a link-layer protocol for a peer-to-peer communication, coded packets are transmitted by a transmitter, and some of them are re-transmitted if a receiver is unable to decode them due to channel fading, interference, or any other reasons. To enable re-transmissions, the feedback from the receiver is sent to the transmitter. In general, HARQ protocols can achieve a high reliability. However, if there are frequent re-transmissions, the decoding delay of packets can be long as the transmitter needs to wait until it receives a feedback signal. There are variants to reduce decoding delay [12] [13] based on decoding error prediction.

An effective means to lower the delay in HARQ is to exploit transmit diversity, e.g., the same packet can be transmitted a number of times, called $K$-repetition [14] [15]. Since the probability of successful transmission increases with the number of repetitions, $K$, the number of re-transmissions can decrease, which leads to a short decoding delay at the cost of the spectral efficiency by a factor of $K$. This is often acceptable to meet a stringent delay constraint when the bandwidth is plentiful [16].

The notion of network coding (NC) has been introduced for efficient routing of multicast traffic [17] [18] and extended to various applications [19] [20] [21]. Among them, it is shown in [16] that NC can be used to meet low delay requirements in 5G.

As in [16], in this paper, we propose an approach based on NC to URLLC (the relationship between the proposed approach and the approach in [16] will be explained in Subsection I-A). In this approach, NC packets, which are linear combinations of original data packets, are transmitted together with original data packets. In particular, NC packets are generated using a sliding window of original data packets, and for this reason, the proposed approach is called sliding NC (SNC). This approach allows a receiver to decode a sequence of coded packets on-the-fly with a certain specific decoding delay. As a result, when a transmitter generates a sequence of packets at a certain rate and wishes to deliver each packet with a guaranteed delay in URLLC applications, the proposed approach can be used while providing a high transmission reliability.

The main contributions of the paper can be summarized as follows: i) the notion of SNC is proposed to transmit packets in on-the-fly mode to meet URLLC requirements in terms of decoding delay and packet decoding error rate; ii) various SNC designs are derived with a delay constraint; iii) the decoding error rate of SNC designs is analyzed, which shows that the decoding error rate can be significantly low compared to that of $K$-repetition at the same level of spectral efficiency.

A. Related Works

For URLLC, coded short packets are considered as in [22] [23], where a low decoding error rate is to be achieved for each short packet transmission. However, if a transmitter has a long message or a sequence of packets that are generated at a certain rate, it is necessary to consider streaming codes. In [16], using NC, an approach to generate a sequence of coded packets, as a streaming code, is proposed to exploit the rate-delay tradeoff. In particular, a linear combination of (past) data packets is inserted after a certain number of original data packets, say $l - 1$ packets, where $l \geq 2$, and transmitted together with original packets. As a result, the effective code rate becomes $\frac{l - 1}{l}$. In general, this approach works well when the channel condition is not severe. If the channel is not reliable for each packet transmission (as in random access channel [24] [25]), the approach in [16] cannot provide a sufficiently low decoding
error rate. In particular, for an erasure channel of an erasure probability of $\epsilon$, the channel capacity is $1 - \epsilon$ [26]. Since the code rate is $\frac{l-1}{l} = 1 - \frac{1}{l}$ for a positive integer $l$ in [16], it is required that $\frac{1}{l} > \epsilon$ for a highly reliable transmission. If $\epsilon$ is not sufficiently small due to hostile channel conditions, $l = 2$ (i.e., one NC packet after every one original data packet) may not ensure a highly reliable transmission (or a very low decoding error rate). Thus, the effective code rate needs to be low as that of $K$-repetition [14] [15], which is $\frac{1}{k}$. The proposed approach in this paper has a low effective code rate so that a very low decoding error rate can be achieved by inserting multiple NC packets after every one original data packet. In this sense, the proposed approach can be seen as a generalization of the approach in [16].

B. Organization of the Paper

The rest of the paper is organized as follows. In Section II, we present two erasure channel models. Two different approaches to reliable transmissions over erasure channels are discussed in Section III. In Section IV, we present the proposed approach, namely SNC, with some design examples. The decoding error probability of the SNC designs introduced in Section IV is analyzed in Section V. Simulation results are presented in Section VI and the paper is concluded in Section VII with a few remarks.

Notation: Matrices and vectors are denoted by upper- and lower-case boldface letters, respectively. The superscript $T$ denotes the transpose. The identity matrix is represented by $I$. $\mathbb{E}[\cdot]$ and $\text{Var}(\cdot)$ denote the statistical expectation and variance, respectively. $Q(x)$ represents the Q-function, which is defined as $Q(x) = \int_{x}^{\infty} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dz$.

II. ERASURE CHANNEL MODELS

In this section, we consider two erasure channel models. For convenience, assume that time is divided into discrete slots and a packet can be transmitted within a slot.

A. An Erasure Channel Model for Coded Packets

Consider a point-to-point channel and assume that each packet is a codeword. From [27] [23], the achievable rate of $n$-length code is given by

$$R(\rho, n, \epsilon) = \log_2(1 + \rho) - \sqrt{\frac{V(\rho)}{n}} Q^{-1}(\epsilon) + O\left(\frac{\log_2 n}{n}\right), \tag{1}$$

where $\rho$ is the signal-to-noise ratio (SNR), $\epsilon$ is the (codeword or packet) error probability, $V(\rho)$ is the channel dispersion that is given by

$$V(\rho) = \frac{\rho(2 + \rho)}{(1 + \rho)^2} (\log_2 e)^2. \tag{2}$$

Alternatively, the error probability becomes

$$\epsilon \approx Q\left(\sqrt{\frac{n}{V(\rho)}} \left(\log_2(1 + \rho) - \frac{N_{\text{bit}}}{n}\right)\right), \tag{3}$$

where $N_{\text{bit}} = Rn$ represents the number of message bits per packet and $n$ can be seen as the number of channel uses.

Here, the code rate, $\frac{N_{\text{bit}}}{n}$, should be lower than the capacity, $\log_2(1 + \rho)$, for a low error probability. As a result, the channel can be seen as an erasure channel with the erasure probability $\epsilon$ for each packet transmission.

Note that for fading channels, the right-hand side (RHS) in (3) is to be averaged over the SNR, $\rho$, to find the average erasure probability [28] [23], where the SNR is the received SNR that includes the random channel coefficient.

B. An Erasure Channel Model with Two-Step Random Access

For machine-type communication (MTC) in 5G, 2-step random access has been considered [29] in order to lower signaling overhead compared to conventional 4-step random access. In this subsection, we will show that the resulting channel of 2-step random access can also be seen as an erasure channel.

Like the conventional 4-step random access, suppose that a pool of $L$ preambles is used in 2-step random access. For data packet transmissions, a slot is divided into two sub-slots in a time division multiplexing (TDM) manner. In the first sub-slot, each active device transmits a preamble that is chosen from the preamble pool uniformly at random and then transmits a data packet, which forms the first step, as illustrated in Fig 1. In the second step, the receiver (which is a base station (BS)) sends the feedback signal to inform the decoding outcomes.

Suppose that there are $M$ active devices and consider an active device of interest that chooses a certain preamble. This device can successfully transmit its packet if the other devices choose the other preambles, and the corresponding probability is given by

$$p_s(M) = \left(1 - \frac{1}{L}\right)^{M-1}. \tag{4}$$
Thus, the probability of unsuccessful packet transmission of the active device becomes
\[
\epsilon = E[1 - p_e(M) | M \geq 1]
= 1 - E \left( \left(1 - \frac{1}{L} \right)^{M-1} | M \geq 1 \right)
= 1 - \sum_{m=1}^{\infty} \left(1 - \frac{1}{L} \right)^{m-1} \Pr(M = m | M \geq 1),
\]
where the expectation is carried out over \( M \) and \( \Pr(M = m | M \geq 1) \) is the conditional probability of \( M = m \) provided that \( M \geq 1 \). That is, from an active device’s point of view, the channel can be seen as an erasure channel with the erasure probability \( \epsilon \) [25]. If \( M \) follows a Poisson distribution with mean \( \lambda \), the erasure probability becomes
\[
\epsilon = 1 - \frac{e^{-\frac{\lambda}{L}} - e^{-\lambda}}{(1 - e^{-\lambda})(1 - \frac{1}{L})}, \quad L \geq 2,
\]
which can be approximated by \( \frac{1}{L} \), i.e., \( \epsilon \approx \frac{1}{L} \), if \( \lambda \ll L \). In other words, in order to have a sufficiently low \( \epsilon \), say \( 10^{-5} \), \( L \) has to be very large (i.e., \( 10^{5} \)).

As shown in (3) and (5), the channel erasure probability, \( \epsilon \), depends on a number of factors, and it would be difficult to achieve a sufficiently low \( \epsilon \) for ultra-reliable communications. For example, as in (3), the increase of the SNR, \( \rho \), may not lead to the decrease of \( \epsilon \) if the difference between the channel capacity, \( \log_2(1 + \rho) \), and the code rate, \( \frac{N_{hk}}{n} \), is fixed regardless of \( \rho \). To see this, let \( \delta = \log_2(1 + \rho) - \frac{N_{hk}}{n} > 0 \), which is assumed to be independent of \( \rho \) and \( n \). As \( \rho \to \infty \), \( V(\rho) \to V = \left( \log_2 e \right)^2 \). Thus, \( \epsilon \to Q(\sqrt{\frac{\delta}{\rho}}) \) as \( \rho \to \infty \). In other words, although the SNR, \( \rho \), approaches infinity, \( \epsilon \) cannot approach 0, but a non-zero constant. In addition, from (5), we can also see that \( L \) should be sufficiently large for a low erasure probability. However, since the radio resource is limited, it is difficult to increase \( L \). As a result, for ultra-reliable communications, there should be diversity techniques, since the channel erasure probability, \( \epsilon \), may not be sufficiently low.

III. RELIABLE TRANSMISSIONS

In this section, we discuss reliable transmission of a message consisting of \( M \) packets with a certain delay constraint for URLLC.

Throughout the paper, we have the following assumptions.

**A1** As discussed in Section II, each packet is independently transmitted over an erasure channel. Since each packet is encoded with parity bits, the receiver can decode it and declare its successful decoding or failure (then this packet is regarded as an erased one). As mentioned earlier, each packet is erased with a probability of \( \epsilon \).

**A. Repetition Diversity**

Suppose that each coded packet is transmitted \( K \) times according to \( K \)-repetition [14]. This results in an improvement of the reliability at the cost of the spectral efficiency by a factor of \( K \). For convenience, \( \frac{1}{K} \) is referred to as the effective spectral efficiency or code rate as \( K \)-repetition can be seen as a repetition code.

For convenience, assume that a block consists of \( K \) slots. Denote by \( V_{k,m} \) the \( k \)th slot of block \( m \). Then, with slight abuse of notation, let \( V_{k,m} = X_m \) to represent that the packet transmitting in the \( k \)th slot of block \( m \), \( k = 1, \ldots, K \), where \( X_m \) represents original data packet \( m \). That is, \( K \) copies of \( X_m \) is transmitted in a block for \( K \)-repetition.

The receiver is able to decode the packet if at least one of \( K \) copies can be correctly decoded. Thus, according to Assumption of A1, the decoding error probability becomes
\[
\epsilon_K = (K^{1-\epsilon} e)^{K} = \epsilon^K.
\]

For example, if \( \epsilon = 10^{-2} \), in order to achieve a target error rate of \( 10^{-5} \), \( K \) should be greater than or equal to 3. The associated delay for each packet is the time duration of \( K \) slots or one block. In fact, this delay can be seen as an upper-bound, because one of the copies can be decoded before all \( K \) copies are received.

In this paper, we consider the case that a transmitter has a message of \( M \) packets that are generated at a certain rate. When \( K \)-repetition is employed, each packet can be successfully decoded within a delay of \( K \) slots with a probability of \( \epsilon_K \). For a mission-critical application, the receiver may have to decode every packet within a certain delay, and decoding failures causing re-transmissions in HARQ could result in significant performance losses for the application. Thus, when transmitting \( M \) packets, it is desirable to have a sufficiently low decoding error rate to minimize the number of re-transmissions.

**B. Random Linear Network Coding**

In this subsection, we consider an approach that can effectively reduce the decoding error rate using NC [30].

As in [31] [32] [16], NC can be used for peer-to-peer communications. In order to deliver \( M \) packets, NC can be used. Suppose that the transmitter uses random linear NC (RLNC) and the \( n \)th encoded packet is given by
\[
Y_n = f_n(X_1, \ldots, X_M) = c_{n,1}X_1 \oplus \cdots \oplus c_{n,M}X_M, \quad n = 1, \ldots, \quad (8)
\]
where \( f_n(\cdot) \) is a random linear combination of \( M \) packets and the \( c_{n,m} \)'s are the encoding coefficients that are taken from the Galois field, \( GF(q) \). Here, \( q \) represents the size of the Galois field and \( \oplus \) represents the addition in \( GF(q) \), which is the XOR operation when \( q = 2 \). In (8), the packet is also a vector over \( GF(q) \). Throughout the paper, a linear combination of data packets, i.e., \( Y_n \) in (8), is referred to as an NC packet, while \( X_m \) is simply referred to as a (data) packet.

Note that each NC packet is to be encoded as an original data packet in \( K \)-repetition so that each encoded packet (whether it is an original packet, i.e., \( X_m \), or an NC packet, i.e., \( Y_n \)) is successfully received with a probability of \( 1 - \epsilon \) (or erased with a probability of \( \epsilon \)) according to Assumption of A1.
Let \( P(S, M) \) be the decoding probability with \( S (\geq M) \) successfully decoded NC packets. In [33] [34], with random encoding coefficients, it is shown that
\[
P(S, M) = \prod_{n=0}^{M-1} \left(1 - \frac{1}{q^{s-n}}\right).
\]
(9)

If all zero encoding coefficients are removed, \( P_{nc} \) is given by
\[
P(S, M) = \prod_{n=0}^{S-M} (-1)^n \binom{S}{n} U_q(M, S-n) \left(q^M - 1\right)^S,
\]
where \( U_q(m, n) = \prod_{j=0}^{n-1} (q^n - q^j) \). Note that since (9) is a lower-bound on (10), we will use (9) to see the performance of NC in this paper.

For a fair comparison with \( K \)-repetition in terms of the spectral efficiency, suppose that \( KM \) NC packets are be transmitted. With erasure probability \( \epsilon \), the probability of successful decoding of all \( M \) packets is given by
\[
P_{nc}(N, M; \epsilon) = \sum_{n=M}^{N} P(s, M) \binom{N}{s} (1 - \epsilon)^s \epsilon^{N-s},
\]
where \( N = MK \). For \( K \)-repetition, the probability of successful decoding of all \( M \) packets becomes
\[
P_K(N, M; \epsilon) = (1 - \epsilon_K)^M = (1 - \epsilon^K)^M.
\]
(12)

In Fig. 2, the performance of NC and \( K \)-repetition is shown when \( q = 4 \), \( K = 3 \), and \( M \in \{5, 10\} \). Note that both NC and \( K \)-repetition transmit a total of \( N = MK \) (coded) packets to transmit \( M \) original data packets. As \( M \) increases, NC performs better than \( K \)-repetition. Note that the performance of \( K \)-repetition is insensitive with respect to a finite \( M \) if \( \epsilon \) is sufficiently low.

We have a few important remarks as follows.

- The use of NC can provide a significant improvement in reliability compared to \( K \)-repetition with the same spectral efficiency [30]. However, since all \( M \) packets are to be delivered, no specific priority for the early packets is given. For example, in order to decode the first packet, the receiver needs to wait to receive at least \( M \) NC packets. This means that the lower bound on the delay of the first packet is \( M \). Thus, if \( M > K \), the decoding delay of packet of NC is longer than that of \( K \)-repetition. Note that since \( M \) is the lower bound on the delay in NC and \( K \) is the upper bound in \( K \)-repetition, even if \( M = K \), the packet transmission delay in NC is expected to be longer than that in \( K \)-repetition. Consequently, we can see that NC has a better reliability than \( K \)-repetition at the cost of packet transmission delay, which means that NC may not be suitable for URLLC.

- As mentioned earlier, when the packets are generated at a constant rate, the transmitter needs to wait till it has \( M \) original packets for NC. As a result, there would be an additional delay at the transmitter side.

- To reduce the decoding delay in NC, a small number of packets can be considered. For example, if there are 20 packets to be delivered, we can divide them into 4 groups so that each group has 5 packets. In this case, \( M \) becomes 5 (not 20). However, as shown in Fig. 2 (a), if \( \epsilon \) is sufficiently low (say \( \epsilon = 10^{-3} \)), the decoding error probability of NC can be higher than that of \( K \)-repetition (in addition to this, the decoding delay, which is \( M = 5 \), is longer than that of \( K \)-repetition, which is \( K = 3 \)). This (i.e., the use of small number of packets for NC) offsets the performance gain of NC.

### IV. SLIDING NETWORK CODING

In this section, we introduce SNC that can provide a relatively low transmission delay for each packet with improved reliability compared to \( K \)-repetition. In particular, SNC can take advantage of both \( K \)-repetition and NC using on-the-fly mode.

#### A. Examples

In order to illustrate the idea of SNC, consider an example with \( K = 2 \). Suppose that a block consists of two consecutive slots, which is denoted by \( (V_{1,m}, V_{2,m}) \). Here, \( V_k,m \) is the NC packet transmitted in the \( k \)th slot of block \( m \). As shown in Table I, for example, the NC packets are given. For \( m \geq 2 \), we have
\[
(V_{1,m}, V_{2,m}) = (X_m, X_{m-1} \oplus X_m).
\]
(13)

| \( m \) | \( V_{1,m} \) | \( V_{2,m} \) |
|---|---|---|
| 1 | \( X_1 \) | \( X_1 \) |
| 2 | \( X_2 \) | \( X_1 \oplus X_2 \) |
| 3 | \( X_3 \) | \( X_2 \oplus X_3 \) |
| 4 | \( X_4 \) | \( X_3 \oplus X_4 \) |
| ... | ... | ... |

At the end of block \( m \), suppose that the receiver is to decode \( X_{m-1} \). For example, consider \( m = 4 \) and the receiver is to decode \( X_3 \). Then, we have \( V_{1,4} = X_4 \) and \( V_{2,4} = X_3 \oplus X_4 \). In
addition, the receiver also has $V_{1,3} = X_3$ and $V_{2,3} = X_2 \oplus X_3$. Suppose that all the previous packets are successfully decoded. This means that $X_1$ and $X_2$ are decoded. Then, according to Assumption of A1, the decoding error probability of $X_3$ is as follows:

$$P_3 = \epsilon^2 (1 - (1 - \epsilon)^2)$$

The part (a) is due to the fact that the decoding error probability when the receiver receives $V_{1,3} = X_3$ and that when $V_{2,3} = X_2 \oplus X_3$ (since $X_2$ is assumed to be correctly decoded, $X_3$ can be decoded if $X_2 \oplus X_3$ is correctly decoded, which means that the decoding error probability with $X_2 \oplus X_3$ is $\epsilon$). The part (b) is due to the decoding error when $X_3$ is to decoded with $V_{2,4} = X_3 \oplus X_4$ and $V_{1,4} = X_4$. For successful decoding of $X_3$, both $V_{2,4}$ and $V_{1,4}$ should be correctly decoded. Thus, the associated error probability of decoding is $1 - (1 - \epsilon)^2$. It can also be shown that $X_1$ is decoded at $m = 2$ with the following decoding error probability:

$$P_1 = \epsilon^2 (1 - (1 - \epsilon)^2) = 2 \epsilon^3 + O(\epsilon^4), \quad (15)$$

for $m \geq 2$.

In order to see the advantage of SNC over $K$-repetition, let consider an example with $\epsilon = 0.01$ and a target decoding error probability of $p_{err} = 10^{-5}$. When $K$-repetition is used, we need to have $K \geq 3$. On the other hand, with SNC, as shown above, $K = 2$ is sufficient as the decoding error probability becomes $2 \epsilon^3 = 2 \times 10^{-6}$. That is, with a small number of slots per block (or repetitions), a lower decoding error probability can be achieved using SNC.

For decoding delay, compared to $K$-repetition, SNC has an additional delay of one block as $X_{m-1}$ is to be decoded at the end of block $m$. Note that the resulting approach is referred to as SNC, because $V_{2,m}$, which is an XOR of two packets, $X_{m-1}$ and $X_m$, is a linear combination of the packets within a sliding window of two consecutive packets (this becomes clear with $K \geq 2$, which will be discussed later).

Note that any incorrect decoding of the data packets will result in subsequent decoding errors, i.e., there is error propagation in SNC. Thus, it may be necessary to lower the decoding error probability of the first data packet, $X_1$. To this point, consider the example in Table II. It can be readily shown that $X_1$ can be decoded at $m = 3$ with the following decoding error probability:

$$P_1 = \epsilon^2 (1 - (1 - \epsilon)^2) = 4 \epsilon^4 + O(\epsilon^5). \quad (16)$$

Clearly, we have this decrease of the decoding error probability of $X_1$ at the cost of delay. That is, $X_{m-2}$ can be decoded at block $m$.

**Table II**

| $m$ | $V_{1,m}$ | $V_{2,m}$ |
|-----|-----------|-----------|
| 1   | $X_1$     | $X_1$     |
| 2   | $X_2$     | $X_1 \oplus X_2$ |
| 3   | $X_3$     | $X_1 \oplus X_3$ |
| 4   | $X_4$     | $X_2 \oplus X_4$ |

We can see that the two designs in Tables I and II have a spectral efficiency of $\frac{1}{K} = \frac{1}{2}$ and a decoding error rate of $O(\epsilon^3)$. While the resulting decoding error can be sufficiently low for some applications, it is also possible to have further lower error rates with $K \geq 2$ via a generalization of SNC, which will be discussed in the next subsection.

**B. A Design with More than Two Slots per Block**

In this subsection, we consider SNC with $K \geq 2$ through a generalization.

As mentioned earlier, we assume that a block consists of $K$ slots. Denote by $D$ the delay parameter such that $X_{m-D}$ is to be decoded at the end of block $m$. The encoded packets of block $m$ are now given by

$$V_{1,m} = X_m$$

$$V_{k,m} = X_{m-D} \oplus f_k(X_m, \ldots, X_{m-D+1}), \quad k = 2, \ldots, K \quad (17)$$

where $f_k(.)$ are different linear combinations of the data packets, $X_m, \ldots, X_{m-D+1}$, at block $m$. That is, the $(K-1)$ NC packets are

$$f_k(X_m, \ldots, X_{m-D+1}) = c_{k,1}X_m \oplus \cdots \oplus c_{k,D}X_{m-D+1}, \quad (18)$$

where $c_{k,d} \in GF(q)$ is the encoding coefficient for the $m$th packet, $V_{k,m}$, or

$$\begin{bmatrix} f_1 \\ \vdots \\ f_{K-1} \end{bmatrix} = C \begin{bmatrix} X_m \\ \vdots \\ X_{m-D+1} \end{bmatrix}, \quad (19)$$

where $[C]_{k,d} = c_{k,d}$ and the size of $C$ is $(K-1) \times D$. For convenience, the SNC in (17) is referred to the $(K, D, q)$-SNC design, where $K$ represents the inverse of the effective spectral efficiency, $D$ represents the delay in block, and $q$ is the size of Galois field. Consequently, we can see that each block $m$ consists of the current original data packet, $X_m$, and $K-1$ NC packets that are linear combinations of the current and past packets in SNC.

Note that unlike the approach of NC in Subsection III-B, since the packets in block $m$ are linear combinations of current and past packets, the transmitter has no encoding delay to form NC packets, and is able to send a new original packet per block in on-the-fly mode.

For example, consider the following simple design:

$$V_{1,m} = X_m$$

$$V_{k,m} = X_{m-D} \oplus X_{m-k+2}, \quad k = 2, \ldots, K \quad (20)$$

where linear combinations of two packets are considered for the NC packets, $V_{k,m}, \quad k = 2, \ldots, K$. In this design, the delay parameter, $D$, becomes $K-1$ and $C = I$. In Fig. 3, a sliding window of $K-1$ packets is shown to form $K-1$ NC packets in block $m$. From this, it is clear that $D$ becomes $K-1$. That is, (20) is a $(K, K-1, 2)$-SNC.

While the $(K, K-1, 2)$-SNC design in (20) is simple (as only two packets are combined for NC packets), the decoding delay can be long for a large $K$. In particular, the decoding delay becomes $K^2$ (in slots), which is $K$-time longer than that.
of $K$-repetition. This shows that SNC can provide a higher reliability at the cost of delay. Since it is desirable to have a short decoding delay, we now find the minimum decoding delay, $D$, for $(K, D, q)$-SNC.

Lemma 1: For $(K, D, q)$-SNC, the delay parameter, $D$, has to satisfy the following inequality:

$$ D \geq \log_q K. $$

Proof: For a given $q$, from (18), there can be $q^D - 1$ non-zero different NC packets that are linear combinations of $X_m, \ldots, X_{m-D+1}$. Thus, we have $K - 1 \leq q^D - 1$ or $K \leq q^D$, which leads to (21).

According to (21), we can see that the minimum decoding delay increases logarithmically with $K$. Any $(K, D, q)$-SNC design with $D = \lceil \log_q K \rceil$ is referred to as a minimum delay $(K, D, q)$-SNC. Clearly, the SNC in Table I is an example of a minimum delay $(K, D, q)$-SNC design, where $(K, D, q) = (2, 1, 2)$, while the SNC in Table II with $(K, D, q) = (2, 2, 2)$ does not have the minimum delay. Another example of a minimum delay $(K, D, q)$-SNC design with $(K, D, q) = (4, 2, 2)$ can also be found in Table III.

C. A Decoding Rule

At the receiver, noisy versions of $V_{k,m}$, which are denoted by $\hat{V}_{k,m}$, are received through an erasure channel. Under the assumption of A1, we have

$$ \hat{V}_{k,m} = \left\{ \begin{array}{ll} V_{k,m} & \text{w.p. } 1 - \epsilon \\ ? & \text{w.p. } \epsilon, \end{array} \right. $$

where ? represents the erasure. For decoding, we consider the following two stages:

S1 At the end of block $m$, the received NC packets, $\hat{V}_{k,m}$, $k = 1, \ldots, K$, are individually decoded. They are referred to as not-fully-decoded (NFD) packets. Since the channel is an erasure channel, each NFD packet is either successfully decoded (with probability $1 - \epsilon$) or unknown (with probability of $\epsilon$).

S2 Then, $X_{m-D}$ is to be decoded using the NFD packets and the fully-decoded (FD) packets of $X_{m-D-k}$, $k \geq 1$. In the second step, since $X_{m-D}$ is to be decoded at the end of slot $m$, $X_{m-D-k}$, $k \geq 1$, should be decoded in the previous blocks. Thus, they are available as FD packets. That is, at the end of block $m$, we have

$$ \ldots, X_{m-D-1}, X_{m-D}, X_{m-D+1}, \ldots, X_m. $$

(23)

To illustrate the decoding rule, consider an example with the SNC design in Table III. At the end of block $m$, suppose that the receiver finds that the following NFC packets are erased after the first step:

$$ \hat{V}_{1,m} = \hat{V}_{3,m} = \hat{V}_{4,m-1} = \hat{V}_{1,m-2} = \hat{V}_{2,m-2} = ?,$$

while the other NFD packets and all the FD packets are correctly decoded. Then, in the second step, the receiver has the following NC packets directly related to $X_{m-2}$:

$$ V_{2,m} = X_{m-2} \oplus X_m $$
$$ V_{4,m} = X_{m-2} \oplus X_m \oplus X_{m-1} $$
$$ V_{3,m-1} = X_{m-3} \oplus X_{m-2} $$
$$ V_{4,m-2} = X_{m-4} \oplus X_{m-3} \oplus X_{m-2}, $$

(25)

where the boxed variables are FD packets, which are assumed to be correct. Then, we can see that $X_{m-2}$ can be decoded from $X_{3,m-1}$ or $X_{4,m-2}$. Note that once $X_{m-2}$ is successfully decoded, it becomes an FD packet, which can help decode $X_m$ from $V_{2,m}$.

V. PERFORMANCE ANALYSIS

In this section, we present the performance analysis of SNC in terms of decoding error rate.

First, we consider the simple design in (20), i.e., $(K, K-1, q)$-SNC, which allows a tractable analysis to find the decoding error rate. As shown below, it seems that the error exponent of SNC can be about two times higher than that of $K$-repetition.

Lemma 2: Suppose that the receiver is to decode $X_{m-D}$ at the end of block $m$ when the SNC in (20) is used with $D = K - 1$. Provided that all the FD packets are correctly decoded, the decoding error probability of $X_{m-D}$ is given by

$$ p_{\text{snc}} = \epsilon^K (1 - (1 - \epsilon)^2)^{K-1} $$
$$ = 2^{K-1} \epsilon 2^{K-1} + O(\epsilon^{2K}). $$

(26)

Proof: To find the decoding error probability of $X_{m-D}$, two different sets of received signals are considered.

1) With the current block $m$, there are $(K-1)$ (XORed) copies of $X_{m-D}$ with the NFD packets, i.e., $V_{k,m}$, $k = 2, \ldots, D$. The decoding error becomes $(1 - (1 - \epsilon)^2)^{K-1}$.

2) There are $K$ past blocks that contain copies of $X_{m-D} = X_{m-K+1}$. For example, at block $m-K+1$, $V_{1,m-K+1}$ was transmitted according to (20). In addition, at block $m - d$, $V_{K-d+1,m-d} = X_{m-d-K+1} \oplus X_{m-K+1}$, $d = 1, \ldots, K - 1$, is transmitted. Since we assumed that all the FD packets are correctly decoded, in $V_{K-d+1,m-d} = X_{m-d-K+1} \oplus X_{m-K+1}$, $X_{m-d-K+1}$ is known if $V_{k-d+1,m-d}$ is correctly decoded. Thus, the associated error probability to decode $X_{m-K+1}$ for given $V_{K-d+1,m-d}$ is $\epsilon$. As a result, the error probability with
the received signals in the past blocks, i.e., $V_{1,m-K+1}$ and $V_{K-d+1,m-d}$, $d \in \{1, \ldots, K-1\}$, becomes $e^K$.

Consequently, the decoding error probability becomes the product of $e^K$ and $(1 - (1 - \epsilon)^2)^{K-1}$, which is given in (26).

There are some remarks.

- The SNC in Table I is a $(K, K - 1, 2)$-SNC design with $K = 2$. As shown in (26), the decoding error rate is $p_{snc} = 2e^3 + O(e^4)$, which agrees with that in (14).
- The decoding error rate in (26) can be regarded as an upper-bound as it is assumed that the erasure probability of any NFD packet in block $m$, $X_m$, is set to $\epsilon$. In practice, some of the NFD packets can be decoded in the previous decoding rounds. Thus, the effective erasure probability can be lower than $\epsilon$, which results in the actual decoding error rate that is lower than that in (26).

As shown in (26), an approximate decoding error rate of the $(K, K - 1, 2)$-SNC design in (20) is available. However, in a general $(K, D, q)$-SNC design, it is not straightforward to find such an expression and we need to define a few more parameters.

Define $\mu$ as the number of NCs in blocks $m-D, \ldots, m-1$ that are $X_{m-D}$ itself or linear combinations of $X_{m-D}$ and FD packets, $X_{m-D-k}$, $k \geq 1$. As an example, consider the SNC design in Table III. Since $X_{m-3}$ and $X_{m-4}$ are FC packets, it can be shown that

\begin{align*}
V_{1,m-2} &= X_{m-2} \\
V_{2,m-2} &= X_{m-4} \oplus X_{m-2} \\
V_{4,m-2} &= X_{m-4} \oplus X_{m-3} \oplus X_{m-2} \\
V_{3,m-1} &= X_{m-3} \oplus X_{m-2}.
\end{align*}

Thus, we have $\mu = 4$ in this example.

**Lemma 3**: Suppose that $D \leq K - 1$ and the coefficient matrix $C$ in (19) can be expressed (possibly after permutation of rows) as

$$C = \begin{bmatrix} \diag(c_{1,1}, \ldots, c_{D,D}) \end{bmatrix},$$

where $c_{d,d} \neq 0$, $d = 1, \ldots, D$. The decoding error rate of a $(K, D, q)$-SNC design is given by

$$p_{snc} = 2^D e^h + O(e^{h+1})$$

where $\mu \geq D$.

**Proof**: Eq. (27) implies that there exist $D$ $f_k$’s such that

$$f_k = c_{k,d}X_{m-d}, \quad c_{k,d} \neq 0, \quad k \in \{k_1, \ldots, k_D\}.$$  

At the end of block $m$, the receiver has $\hat{V}_{1,m-d}$, which are noisy versions of NFD packets, $X_{m-d}$, $d = 0, \ldots, D - 1$, as shown in (17). Thus, to decode $X_{m-d}$, $d = 0, \ldots, D - 1$, we need to define a few more

As shown in (28), the decoding error rate of SNC can decrease with the delay parameter $D$. That is, there is a trade-off between the delay and reliability.

As shown in Lemma 3, if $C$ is designed as in (27), we expect that $\mu$ increases with $D$ (since $\mu$ is upper-bounded by $D$). However, there might be a better design to maximize $\mu$ or minimize the decoding error rate for given $K$ and $D$. Finding an optimal SNC design is a further research topic to be studied in the future.

**VI. SIMULATION RESULTS**

In this section, we present simulation results with two different SNC designs, $(3, 2, 2)$-SNC and $(4, 2, 2)$-SNC, unless stated otherwise, and compare them with those of $K$-repetition. For simplicity, we only consider the case of $q = 2$ (i.e., binary NC).

In Fig. 4, the decoding error rate is shown over time (in blocks). As shown in Fig. 4 (a) for the performance when $K = 3$ and $\epsilon = 0.1$, $K$-repetition has a decoding error rate of $e^{3.3} = 10^{-3}$, while SNC provides a much lower error rate, which is about $4e^{5} = 4 \times 10^{-5}$ (according to (26)). That is, at the same spectral efficiency, SNC can provide a much lower decoding error rate than $K$-repetition at the cost of additional decoding delay. Fig. 4 (b) shows the performance when $K = 4$ and $\epsilon = 0.2$. We can see that the $(4, 2, 2)$-SNC design can perform better than $K$-repetition as expected. From Lemma 3, we can find an upper-bound on the decoding error probability of $(4, 2, 2)$-SNC, which is $4e^{5} \approx 2.56 \times 10^{-4}$. As
shown in Fig. 4 (b), indeed, it is an upper-bound. Note that the decoding error rate of \((4, 3, 2)\)-SNC can be \(2^{10^2} \epsilon^7 \approx 1.02 \times 10^{-4}\) according to (26), which is close to the actual decoding error rate of \((4, 2, 2)\)-SNC. Thus, the decoding delay can be less than \(D = K - 1\) in blocks without significant performance degradation in terms of decoding error rate, if SNC can be carefully designed.

Although \(K\)-repetition or SNC can lower the decoding error rate, it is impossible to completely avoid decoding failures. Thus, we may need to re-transmit packets if necessary in some applications. In Fig. 5, the probability of a certain number of re-transmissions is shown when a user transmits a message consisting of \(M \in \{50, 100\}\) packets. That is, assuming that a user is to transmit a message of \(M\) packets over a session, we obtain the probability that the total number of re-transmissions of packets is \(i \in \{0, \ldots\}\) within a session. As shown in Fig. 5 (a), the probability that one of \(M = 100\) packets is to be re-transmitted is about \(10^{-3}\) with \(K\)-repetition, while this probability becomes less than \(4 \times 10^{-5}\) with SNC. Clearly, SNC can significantly reduce the number of re-transmissions, which is important for URLLC design as each re-transmission results in additional packet transmission delay. In some mission-critical applications, additional unexpected delays can result in significant performance losses. Thus, for such mission-critical applications, SNC can be a good candidate as it can provide a high reliability with a guaranteed delay, i.e., a very low decoding error rate, say \(O(\epsilon^{M+D})\), with a delay of \(D\) blocks. It is also noteworthy that the probability of re-transmissions increases with \(M\), i.e., the length of message.

We also have similar results in Fig. 5 (b), where SNC provides a lower probability of re-transmissions than \(K\)-repetition. In particular, we see that there is no event of more than 2 re-transmissions with \(M \in \{50, 100\}\) when SNC is used. On the other hand, there are cases that require more than 2 re-transmissions in \(K\)-repetition.

In order to see the performance for different values of erasure probability, simulations are carried out for \(\epsilon \in [10^{-2}, 10^{-3}]\) and the results are shown in Fig. 6. If \(\epsilon\) is too low, we are unable to see any decoding error events. In this case, the theoretical prediction from (26) and (28) can be used.

In Fig. 6, we can see that the decoding error rate increases with the erasure probability, \(\epsilon\). In addition, as demonstrated earlier, (26) provides a good prediction of decoding error rate for \((4, 3, 2)\)-SNC as shown in Fig. 6 (a). For \((4, 2, 2)\)-SNC, (28) can be used as an upper-bound, which can be confirmed by Fig. 6 (b). By comparing Figs. 6 (a) and (b), it can be confirmed that the increase of \(K\) results in a lower decoding error rate, and (26) and (28) are useful to decide \(K\) so that a required decoding error rate can be met for a given \(\epsilon\).

In Fig. 6, we also include the performance of NC with a fixed \(M\). Note that NC is not an on-the-fly scheme, and its total delay (for both encoding and decoding) is \(2N = 2MK\) in packets or \(2M\) in blocks\(^1\) (each encoding or decoding delay is \(M\) in blocks). For a short delay, we can consider the case of \(M = D = 2\). From Fig. 6, we see that NC has a high decoding error rate than both \(K\)-repetition and SNC. To lower the decoding error rate, a larger \(M\), say \(M = 6\), can be used for NC. In this case, its performance is comparable to that of \(K\)-repetition or SNC. To lower the decoding error rate, a larger \(M\), say \(M = 6\), can be used for NC. In this case, its performance is comparable to that of

\(^{1}\)Recall that a block consists of \(K\) packets.
SNC. However, the decoding delay of NC becomes $2MK = 36$ (for $K = 3$) or 48 (for $K = 4$) in packets, while that of SNC is $K(D + 1) = 9$ (for $K = 3$) or 12 (for $K = 4$) in packets.

We can have SNC for a different value of $K$ as in (20), i.e., $(K, K - 1, 2)$-SNC. Fig. 7 shows the decoding error rates of SNC and $K$-repetition as functions of $K$ for a given erasure probability $\epsilon \in \{0.1, 0.3\}$. Clearly, SNC can reduce the number of repetitions or improve the effective spectral efficiency, compared to $K$-repetition. For example, with $\epsilon = 0.1$, in order to achieve a target decoding error rate of $10^{-6}$, $K$-repetition requires $K = 6$ repetitions. On the other hand, SNC requires $K = 4$ repetitions. Noting that this particular SNC design has a decoding delay of $K^2 = 16$ slots, while $K$-repetition has a decoding delay of $K = 6$ slots, we can see that SNC has about 2.66 times longer decoding delay than $K$-repetition, while the spectral efficiency is improved by a factor of $\frac{2}{3} = 1.5$ in this example. We can also have similar observations with $\epsilon = 0.3$. Note that the performance gap in terms of decoding error probability increases with $K$. Thus, with a lower target decoding error rate, the performance gap between SNC and $K$-repetition will be widened.

Fig. 7. Decoding error rates of $(K, K - 1, 2)$-SNC and $K$-repetition as functions of $K$ for a given erasure probability $\epsilon \in \{0.1, 0.3\}$.

VII. CONCLUDING REMARKS

In this paper, we proposed SNC to effectively exploit the performance gain of NC without a significant increase of decoding delay for URLLC. Since a sliding window of current and past packets was used to generate NC packets that are transmitted together with original packets in SNC as on-the-fly mode, SNC can be seen as a streaming code. As a result, SNC is well-suited to the case of URLLC where a transmitter needs to transmit packets generated at a constant rate with a high reliability and a guaranteed delay for each packet delivery. A few design examples of SNC were also derived and analyzed. It has been shown that the SNC’s minimum decoding delay can increase logarithmically with $K$ while its error exponent can be about 2-time larger than that of $K$-repetition.

While we mainly focused on introducing SNC in this paper with some design examples, there are a number of issues to be addressed in the future. Some of them are as follows.

- An optimal design of SNC is necessary. In Lemma 3, we showed that the error exponent can be greater than or equal to 2$D$. For a given pair of $(K, D)$, there might be an optimal design that maximizes the error exponent, which needs to be investigated in the future.
- We mainly considered SNC design examples with $q = 2$. As shown in Lemma 1 or (21), a large $q$ can help decrease the decoding delay. Thus, it will be necessary to study SNC with a large $q$.

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