Design analysis of a Helium re-condenser

P K Muley\textsuperscript{1,2}, S L Bapat\textsuperscript{2} and M D Atrey\textsuperscript{2}

\textsuperscript{1} Mechanical Engineering Department, Sardar Patel College of Engineering, Mumbai 400058, India
\textsuperscript{2} Mechanical Engineering Department, Indian Institute of Technology Bombay, Mumbai 400076, India
E-mail: slbapat@iitb.ac.in

Abstract. Modern helium cryostats deploy a cryocooler with a re-condenser at its II stage for in-situ re-condensation of boil-off vapor. The present work is a vital step in the ongoing research work of design of cryocooler based 100 litre helium cryostat with in-situ re-condensation. The cryostat incorporates a two stage Gifford McMahon cryocooler having specified refrigerating capacity of 40 W at 43 K for I stage and 1 W at 4.2 K for II stage. Although design of cryostat ensures thermal load for cryocooler below its specified refrigerating capacity at the second stage, successful in-situ re-condensation depends on proper design of re-condenser which forms the objective of this work. The present work proposes design of helium re-condenser with straight rectangular fins. Fins are analyzed for optimization of thermal performance parameters such as condensation heat transfer coefficient, surface area for heat transfer, re-condensing capacity, efficiency and effectiveness. The present work provides design of re-condenser with 19 integral fins each of 10 mm height and 1.5 mm thickness with a gap of 1.5 mm between two fins, keeping in mind the manufacturing feasibility, having efficiency of 80.96 % and effectiveness of 10.34.

1. Introduction
Helium is a scarce cryogenic resource with lowest normal boiling point of 4.2 K. Various cryogenic applications require liquid helium resulting in increasing global consumption of liquid helium. Conventional cryostats need periodic refilling of liquid helium. The process of replenishment of a cryostat has inherent demerit of heavy loss of liquid helium from the Mother Dewar and causes the discontinuity of operation. Hence, operation of conventional cryostats need a helium liquefaction plant with helium recovery system, involving huge infrastructure and associated cost. A cryocooler based helium cryostat with provision for in-situ re-condensation of boil-off helium vapor is the latest option for conservation of precious cryogen by effectively minimizing its boil-off to practically zero. The modern cryostats use a two stage cryocooler with a re-condenser. The surface of re-condenser is maintained at a temperature slightly lower than boiling point of liquid cryogen at the prevailing working pressure by its mechanical contact with II stage of cryocooler. Boil-off helium vapor resulting due to heat in-leak to the cryostat condenses over the re-condenser surface thus, maintaining a constant liquid helium level over long duration [1]. This eliminates the necessity of periodic replenishment of cryostat with liquid helium as in case of conventional cryostats. The present day
cryocooler based re-condensing cryostat is proving to be a technological replacement for the helium liquefaction plant in the vicinity of the cryostat, without any loss of purity of liquid cryogen. Mechanical design of present re-condensing cryostat ensures thermal load for cryocooler below the specified refrigerating capacity of 1 W at II stage. However, effective design and precise fabrication of re-condenser is the most vital aspect in design of re-condensing cryostat to ensure complete in-situ re-condensation of boil-off helium vapor. The principle in design of re-condenser is to provide sufficient surface area for effective heat transfer during the process of re-condensation. The surface area requirement of the re-condenser primarily depends upon the capacity of cryocooler and the heat transfer coefficient. The fabrication of present cryocooler based helium cryostat for 100 liter liquid helium capacity with in-situ re-condensation is presently underway.

IUAC, India has developed a 330 litre cryocooler based helium cryostat for superconducting quadrupole double magnet for Hybrid Recoil Mass Analyzer (HYRA) beam line. Two SRDK - 415 cryocoolers (1.5 W at 4.2 K) are used in this cryostat. The re-condenser of this cryostat has a surface area of 0.2 m$^2$ having 24 fins of 1.3 mm thickness and 1.7 mm gap between two fins [2].

2. Re-condenser arrangement
A re-condensing cryostat comprises outer and inner vessel and a sock. Sock is a mechanical structure which consists of assembly of components mounted on the top of outer and inner vessel for insertion and resting of cryocooler in the cryostat. The present cryostat is to deploy one Sumitomo make RDK 408D2 two stage Gifford McMahon (GM) cryocooler with refrigerating capacity of 40 W at 43 K and 1 W at 4.2 K for its I and II stage respectively. The re-condenser is to be mechanically coupled to II stage of cryocooler. Thus, the surface of re-condenser is to be maintained at a temperature slightly lower than boiling point of liquid cryogen at prevailing working pressure in the cryostat. This will ensure in-situ re-condensation of boil-off helium vapor in cryostat. The re-condenser is provided with fins for enhancing surface area for heat transfer during re-condensation and is to be machined precisely from a blank of OFHC copper. Figure 1 shows schematic arrangement of helium re-condenser.

![Schematic of helium recondenser](image)

**Figure 1.** Schematic of helium recondenser

3. Design Analysis
Phenomenon of re-condensation of helium vapor is a laminar film condensation process. Condensate tends to wet the condensing surface of re-condenser and forms a thin liquid film on it. Helium vapor is at saturation temperature corresponding to working pressure as it moves through ullage space to condensing surface. Latent heat of the boil-off helium vapor is transferred to re-condensing surface through liquid helium film on its surface. Condensed liquid helium then flows down under gravity. The film thickness grows continuously due to newly condensing vapor in the downward direction. Continuous film offers thermal resistance and prevents further heat transfer between helium boil-off
vapor and re-condensing surface. In the present arrangement, drop-wise condensation is not considered as viable. Hence, design analysis of re-condensing surface is on the basis of film condensation. The assumptions made are listed below [3].

- Condensing gases are pure, free from contamination or residual non-condensable gases.
- Contact thermal resistance between mating surfaces of re-condenser and II stage cryocooler flange is negligible.
- Temperature of re-condensing surface \( T_s = 4.2 \) K is same as that of II stage cryocooler flange and is constant and uniform over the entire surface of the re-condenser.
- Temperature at liquid-vapor interface is the saturation temperature \( T_{sat} = 4.3192 \) K corresponding to working pressure of 1.1 bar in the cryostat.
- The liquid helium film has uniform mean film temperature of \( T_m = 4.2596 \) K.
- The liquid helium from the film formed flows under the action of gravity and the flow is laminar.
- The film of liquid helium is in good thermal contact with the re-condensing surface.
- Heat transfer across the liquid helium film layer is only by conduction.
- Properties \( \rho_L, \mu_L, c_p, \) and \( k \) are evaluated at \( T_m \) whereas \( \rho_v, v_g, v_f \) and \( h_{fg} \) at \( T_{sat} \) for helium and remain constant.
- Normal viscous forces, inertia forces and shear stress at liquid-vapor interface are negligible.

3.1. Surface area required for re-condensation

Present analysis is carried out considering re-condensation phenomenon occurring under stable operating conditions of the cryostat. The surface area required for effective re-condensation is a function of cryocooler capacity. The re-condenser for the present work is designed for condensation load of 1 W which includes entire heat in leak from outer and inner vessels, sock and neck to the cryostat including heat load due to connecting leads for sensors. The working pressure inside the cryostat is assumed to be 1.1 bar \( (T_{sat,1.1} = 4.3192 \) K). The re-condenser surface temperature is \( T_s = 4.2 \) K. Thus, \( \Delta T = T_{sat} - T_s = 0.1192 \) K. The mean film temperature of liquid helium film is the average of \( T_s \) and \( T_{sat} \) \( (T_m = 4.2596 \) K). The diameter of II stage cryocooler flange is 64 mm. For the purpose of analysis, the approach adapted is to consider a vertical cylinder at 4.2 K. Condensation heat transfer rate and condensation heat transfer coefficient, \( h \) are determined using non-dimensional numbers \( Nu, Ra, Pr \) and \( Ja \) using equations given below.

\[
Pr = \frac{\mu \ast c_p}{k} \tag{1}
\]

\[
Ja = \frac{(c_p \ast \Delta T) \ast [h_{fg} + 0.68 \ast (c_p \ast \Delta T)]}{(\rho_L \ast (\rho_L - \rho_v) \ast g \ast Pr \ast H^3) / (\mu_L^2)} \tag{2}
\]

\[
Ra = \frac{\rho_L \ast (\rho_L - \rho_v) \ast g \ast Pr \ast H^3}{(\mu_L^2)} \tag{3}
\]

\[
Nu = 0.9428 \times (Ra / Ja)^{1/4} \tag{4}
\]

Equation (4) is applicable for following condition [5]

\[
H / D < 0.007 \ast (Ra / Ja)^{1/4} \tag{5}
\]

\[
h = (Nu \ast k) / H \tag{6}
\]

\[
A_s = Q_{condensation} / (h \ast \Delta T) \tag{7}
\]

### Table 1. Properties of helium [4]

| Property | Liquid He Film at \( T_m = 4.2596 \) K | He vapor at \( T_{sat} = 4.3192 \) K |
|----------|----------------------------------------|----------------------------------|
| \( \rho_L \) (kg/m\(^3\)) | 124.55 | 18.253 |
| \( \mu_L \) (Pa.s) | 3.162 \times 10^{-6} | 0.054786 |
| \( c_p \) (J/kg.K) | 5.3406 \times 10^3 | 0.0081346 |
| \( k \) (W/m.K) | 0.018724 | 20.229 \times 10^3 |
| \( \rho_v \) (kg/m\(^3\)) | 18.253 | 20.229 \times 10^3 |
| \( v_g \) (m\(^3\)/kg) | 0.054786 | 20.229 \times 10^3 |
| \( v_f \) (m\(^3\)/kg) | 0.0081346 | 20.229 \times 10^3 |
| \( h_{fg} \) (J/kg) | 20.229 \times 10^3 | 20.229 \times 10^3 |

\( Pr \) and \( Ja \) are independent of dimensions of condensing surface. Using equation (1) and (2) and values in table1, \( Pr = 0.9018 \) and \( Ja = 0.0308 \). \( Ra \) depends on height of vertical cylinder at 4.2 K. Hence,
combining equation (3), (4), (6) and (7) above, the surface area required for condensation of 1 W heat load and $\Delta T = 0.1192$ K, is calculated as $A_s = 0.008645$ m$^2$. Hence, required height of vertical cylinder at 4.2 K, $H_{Cyl} = 0.043$ m.

3.2. Minimum fin gap

In actual practice, the space between two adjacent fins should easily permit the flow of condensate over the fins and the flow of boil-off helium vapor between two liquid films formed on the two adjacent fin surfaces. Figure 2 shows the phenomenon of film condensation over fins. At $y = H$ (tip of fin); $\delta = \delta_{max}$, the maximum film thickness [3] is given by equation (8). The lower limit of the fin gap is decided by thickness of film that develops on the surface.

$$\delta_{max} = \left(\frac{(4 * k * \mu_L * \Delta T * H)}{(\rho_L * (\rho_L - \rho_v) * g * h_f)g}\right)^{\frac{1}{4}} \quad (8)$$

![Figure 2. Film condensation phenomenon over fins](image)

Iterative calculations are carried out to estimate values of $T_{sat}$ and $p_{sat}$ in the cryostat and the maximum film thickness $\delta_{max}$ at the tip of fin using following equations.

$$Q_{boil-off} = \dot{m} * (h_f g + c_{pl} * \Delta T) \quad (9)$$

$$V_f = \dot{m} * v_f \quad (10)$$

$$\delta_{max} = \frac{V_f}{2 * L * H} \quad (11)$$

Hence, the minimum gap between two fins $G_{min}$ is obtained as given in equation (12).

$$G_{min} = 2 * \delta_{max} * \left(1 + \frac{h_f}{v_f}\right) \quad (12)$$

3.3. Fin Length

The assumptions made in the analysis of fin design are listed below [3].

- Fins are rectangular straight fins.
- Heat conduction by the fin is one dimensional and steady state.
- Fins are of finite length and adiabatic tip.

Conduction heat transfer through a fin and fin parameter $m$ [3] are given as

$$Q_{fin} = \sqrt{h * p * k * A_c * \Delta T * \tan h (m * H)} \quad (13)$$

$$m = \left(\frac{(h * p)}{(k * A_c)}\right)^{\frac{1}{2}} \quad (14)$$

For fins of finite length and adiabatic tip, the required condition is $m * H < 4.6$ [3] for applying equation (13). For fin effectiveness $\varepsilon > 2$, the required criterion is $(k * p)/(h * A_c) > 4$ [3] which justifies use of fins on a surface. Hence, thin and closely spaced fins of high thermal conductivity material are preferred. The material for re-condenser in the present work is OFHC copper having thermal conductivity $k = 250$ W/m.K at 4.2 K [6]. Considering manufacturing feasibility [2], [7], the
dimensions of fin selected for the present work are 1.5 mm thickness and 1.5 mm gap between two fins. These specifications satisfy the above criterion. Total fin length $L$ varies only as a function of fin thickness $t$ and fin gap $G$. Total fin length $L$ is 2053.54 mm. For the present work, the pressure in the cryostat is 1.1 bar.

3.4. Fin height
Analysis is carried out for above configuration to determine condensation heat transfer coefficient and condensation capacity of fins for fin height ranging from 1 to 50 mm. Variation of condensation heat transfer coefficient with respect to fin height is shown in figure 3. From figure 3, it is seen that condensation heat transfer coefficient decreases rapidly up to 10 mm fin height, following a steep curve. For fin height above 10 mm the trend becomes almost flat for the rest of values in the range. Height of fin is selected as 10 mm.

\[ h = \frac{\tan h (m \times H)}{(m \times H)} \]  
\[ \varepsilon = \frac{Q_{\text{fin}}}{Q_{\text{without fins}}} \]  
\[ Q_{\text{without fins}} = h \times A_c \times \Delta T \]  

From figure 4, it is seen that conduction heat transfer by fins increases with increase in fin thickness at constant fin height and also with increase in fin height at constant fin thickness. Conduction heat transfer increases rapidly up to 2 mm fin thickness following a steep curve. For thickness above 2 mm, the trend is almost flat for various constant fin heights of 6, 10 and 15 mm.

![Figure 3. Variation of condensation heat transfer coefficient with fin height](image3)

![Figure 4. Variation of conduction heat transfer rate with fin thickness](image4)
Variation of fin efficiency with respect to fin thickness is shown in figure 5. From figure 5, it is seen that fin efficiency increases with increase in fin thickness at constant fin height and also with increase in fin height at constant fin thickness. Fin efficiency increases rapidly up to 2 mm fin thickness following a steep curve. For thickness above 2 mm, the trend is almost flat for various constant fin heights of 6, 10 and 15 mm.

![Figure 5. Variation of fin efficiency with fin thickness](image)

**Figure 5.** Variation of fin efficiency with fin thickness

Variation of fin effectiveness with respect to fin thickness is plotted in figure 6. From figure 6, it is seen that fin effectiveness increases with increase in fin thickness at a constant fin height whereas it decreases with increase in fin height at a constant fin thickness. Fin effectiveness increases rapidly up to 2 mm fin thickness following a steep curve. For thickness above 2 mm, the trend is almost flat for various constant fin heights of 6, 10 and 15 mm.

![Figure 6. Variation of fin effectiveness with fin thickness](image)

**Figure 6.** Variation of fin effectiveness with fin thickness

4. Results and discussions

In the present work, analysis is carried out for design of finned re-condenser for 100 litre capacity liquid helium cryostat. The cryostat is to deploy one Sumitomo RDK408 D2 two stage GM cryocooler. Specified refrigeration capacity of the cryocooler for first stage is 40 W at 43 K and 1 W at 4.2 K for second stage. The finned re-condenser for in-situ re-condensation is designed for operating conditions of 1.1 bar working pressure in the cryostat and heat load of 1 W. Helium is a saturated vapor at 1.1 bar. Considering manufacturing feasibility, 1.5 mm thick straight rectangular fins are chosen with a gap of 1.5 mm. From the analysis, height of fin is selected as 10 mm. The total length of finned surface is 2053.54 mm. The maximum film thickness at the tip of fin is 0.0190 mm. The theoretical value of minimum gap between fins is 0.287 mm. Thus, the specification of 1.5 mm gap between two fins in the selected fin configuration ensures easy passage of helium vapor over the fins. The surface area of fins is 0.02 m$^2$ and condensation heat transfer coefficient is 1385 W/m$^2$.K.
Fin thickness of 1.5 mm is found optimum resulting in fin efficiency of above 80.96% and fin effectiveness of 10.34.

5. Conclusion
For working condition of 1.1 bar pressure and 1 W boil-off heat transfer rate, the chosen dimensions of finned re-condenser with fin thickness of 1.5 mm, gap of 1.5 mm and height of 10 mm with total length of 2.053 m will serve the purpose of in-situ re-condensation.

6. References
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Nomenclature

\( A_c \) Area of cross section \( m^2 \)
\( A_s \) Surface area \( m^2 \)
\( c_p \) Specific heat capacity of fluid \( J/kg.K \)
\( c_{pL} \) Specific heat capacity of liquid at \( T_m \) \( J/kg.K \)
\( g \) Acceleration due to gravity \( m/s^2 \)
\( G \) Space/ gap between two fins \( m \)
\( G_{min} \) Minimum gap between two fins \( m \)
\( h \) Condensation Heat transfer coefficient \( W/m^2.K \)
\( h_{fg} \) Latent heat of condensation \( J/kg \)
\( H \) Height of fin \( m \)
\( H_{cyl} \) Height of vertical cylinder at 4.2 K \( m \)
\( Ja \) Jakob number
\( k \) Thermal conductivity \( W/m.K \)
\( L \) Total length of finned surface \( m \)
\( m \) Fin parameter \( m^{-1} \)
\( \dot{m} \) Mass flow rate of boil-off helium vapor \( Kg/s \)
\( Nu \) Nusselt number
\( P \) Perimeter of fin \( m \)
\( Pr \) Prandtl number
\( p_{sat} \) Saturation pressure \( bar \)
\( Q_{condensation} \) Condensation heat transfer \( W \)
\( Q_{conduction} \) Conduction heat transfer \( W \)
\( Q_{boil-off} \) Heat load equivalent to boil-off \( W \)
\( Q_{fin} \) Heat conduction by fins \( W \)
\( Q_{without fins} \) Heat transfer without fins \( W \)
\( Ra \) Rayleigh number
\( T_{sat} \) Saturation temperature \( K \)
| Symbol   | Description                                                                 | Unit  |
|----------|-----------------------------------------------------------------------------|-------|
| $\Delta T$ | Surface temperature                                                        | K     |
| $T_m$    | Mean film temperature                                                      | K     |
| $\Delta T$ | Difference between saturation and surface temperature                      | m     |
| $t$      | Thickness of fin                                                            | m     |
| $v_f$    | Specific volume of liquid helium at saturation temperature                 | m$^3$/kg |
| $v_g$    | Specific volume of helium vapor at saturation temperature                  | m$^3$/kg |
| $V_f$    | Volume of liquid helium                                                     | m$^3$ |

**Symbols**

- $\delta$: Thickness of liquid helium film, m
- $\delta_{max}$: Maximum thickness of liquid helium film, m
- $\varepsilon$: Fin effectiveness
- $\eta$: Fin efficiency, %
- $\rho_L$: Density of liquid helium in the film, Kg/m$^3$
- $\rho_v$: Density of helium vapor, Kg/m$^3$
- $\mu_L$: Dynamic viscosity of liquid helium in the film, Pa.s

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