Effect of large-angle scattering, ion flow speed and ion-neutral collisions on dust transport under microgravity conditions

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Abstract. The transport of dust particles through a plasma depends mostly on the ion drag force, the neutral drag force and the electrostatic force. The standard expressions for these forces were originally derived for a single dust particle placed in a collisionless plasma, with negligible flow speeds of the ions. Recent theories show deviations from the standard expressions for the charging and the ion drag force acting on dust particles in a plasma, when there are collisions and a significant ion flow. Experiments show only a small deviation from the standard expressions for the ion drag. We have extended a self-consistent dusty plasma model for a radio-frequency discharge with recent theories regarding the calculation of the ion drag force, including the effect of ion scattering beyond the screening length, ion flow and ion-neutral collisions. A change in the dust charge due to these collisions is also considered. Inside the dust-free void, that is generated by the ion drag force, scattering beyond the screening length is very important. Inside the dust cloud however, the effect is only moderate. Ion flow speeds under typical discharge parameters are low, except near the electrodes. Therefore, the effect of the ion flow speed on the ion drag force is very small. Collisions only increase the ion drag force near the outer walls. Only there does the screening length become much larger than the ion mean-free path. The dust charge however, is strongly reduced inside the void, and near the edge of the dust cloud, which is due to the low ion flow in both regions.

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When we compare our model with experiments, we conclude that in the bulk of the discharge and at the void edge, large angle scattering is important and the velocity-dependent linearized Debye length is the appropriate screening length. Using small angle scattering with the electron Debye length actually overestimates the ion drag, resulting in inconsistent values of the electric field and the ion drift speed.

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1. Introduction

Small particles introduced in a plasma will be accelerated under the influence of a range of forces, including: gravity, thermophoresis, neutral drag, ion drag and the electrostatic force. In this paper, we are only concerned with the transport of dust under microgravity conditions. Furthermore, it has been shown that the effect of thermophoresis is usually much smaller than the effect of the other forces [1, 2], as long as one of the electrodes is not heated or cooled. Thus, the three most important forces in a complex plasma under microgravity conditions, are the neutral drag, the electrostatic force and the ion drag (and the mutual interaction between the charged dust particles). It is intriguing to realize that none of these forces is completely understood either theoretically or through experiments in a flowing, collisional plasma.

For the neutral drag force, the Epstein formulation is generally used [3]. It describes the interaction of neutral particles with hard spheres. The momentum transfer by the neutral particles to the spheres during collisions depends on the way they reflect from the spheres. Usually, specular reflection is assumed, which is similar to the way light reflects from a smooth surface, such as a mirror. In [4], it was found that the neutral drag force determined from experiment corresponded well with this standard formulation. However, in two very recent experiments [5, 6] it was shown...
that the neutral drag force might very well be much smaller than the value given by the Epstein formulation for specular reflection. It should be noted that in [5], the mass of the dust particles was underestimated by almost an order of magnitude (for aluminum dust particles, with a mass density of $2.7 \text{ g cm}^{-3}$ and a radius of $0.6 \mu\text{m}$, we find a mass of $\approx 2.5 \times 10^{-15} \text{ kg}$ in stead of the reported $2.5 \times 10^{-16} \text{ kg}$), which results in errors in the analysis.

The electrostatic acceleration depends on the charge of the dust particles. Normally, the charge on a dust particle in a plasma is calculated using Orbital Motion Limited (OML) theory [7]. It describes how ions and electrons are captured by a charged spherical particle, using conservation of energy and angular momentum. This theory then gives the electron and ion current to a charged dust particle. From the balance of these currents, the potential at the dust particle surface is calculated and from this potential the dust particle charge. Even though this theory depends on many assumptions, a Particle-In-Cell (PIC) model used to calculate the ion drag force on a single dust particle [8], shows remarkable good agreement with OML theory, again for a collisionless flowing plasma. However, it was only used in the limiting cases of dust particle radii much larger than the ion Debye length, or in the case of a plasma with equal ion and electron temperature. In a recent paper, Khrapak et al [9] have experimentally shown that collisions can dramatically reduce the charge on dust particles in a plasma. The reduction in charge is caused by an increase in the ion current reaching the dust particle due to charge-exchange collisions. In these collisions, a fast moving ion collides with a neutral, resulting in a fast moving neutral and a slow moving ion. This slow moving ion can then be captured by the dust particle. They have also given a theoretical treatment for the calculation of the ion current in the presence of collisions based on derivations presented in [10].

The ion drag force strongly depends on the shielding of the dust particle by plasma particles and thus on the potential distribution around the dust particle. In a simple plasma, the heavy ions are surrounded by electrons, which causes the potential around the positively charged ions to vanish faster than the $1/r$ dependency of a simple point charge. This faster drop of the potential is referred to as ‘shielding’ by the electrons and the typical scale length over which the potential drops is called the ‘shielding length’, which in a simple plasma corresponds to the linearized Debye length. In a dusty, or complex plasma, the dust particle is the heaviest species. Therefore, the shielding in such a plasma is more difficult. The scale length over which the potential around a dust particle vanishes is now some function of both the electron as well as the ion Debye length, especially since dust particles typically obtain a negative charge.

The ion drag also depends on the speed with which the ions flow through the complex plasma [11]. Large ion flow speeds cause a strong anisotropy in the ion distribution around the dust particle, which causes a strong anisotropy in the shielding of the dust particle. Furthermore, the ion speed determines the size of the smallest closed orbit, or Coulomb radius, around a charged dust particle. This length scale plays an important role in the scattering of ions in the potential around the dust particle, as will be shown.

Finally, collisions of ions with neutrals change the ion drag force [12, 13] in a similar way as collisions change the dust charge; particles which were outside the solid angle of particle-scattering or particle-capture enter the solid angle after a collision, which increases the ion drag force acting on the dust particle.

Different experimental determinations of the ion drag force in complex plasmas [14]–[16] give ambiguous answers and usually can be explained by both the original approach of Barnes et al [17] as well as the more recent theoretical description [11], which has led to many discussions e.g. [18, 19].

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We have included most of the more recent theoretical considerations regarding the ion drag force and the charging of dust particles in a flowing, collisional plasma in a previous model [1, 2]. In this paper, we compare the effects of these additions on the dust transport and on the dust and plasma parameters in a complex radio-frequency (RF) plasma under microgravity conditions. Section 2 will discuss the model in some more detail and shows the different additions to the model. Section 3 will show the dust transport and dust and plasma parameters for the different models used and discusses the importance of ion flow, collisions and scattering beyond the screening length by comparing the different parameters. In section 4, we will compare our modelling results with recent experimental results and in section 5, we will present our conclusions.

2. Model

2.1. Plasma equations

The model solves the particle balance equations for the plasma species \( i \) using the drift-diffusion approximation:

\[
\frac{\partial n_i}{\partial t} + \nabla \cdot \Gamma_i = S_i, \tag{1}
\]

\[
\Gamma_i = n_i \mu_i E - D_i \nabla n_i. \tag{2}
\]

\( \mu_i \) is the mobility of the particle \( i \) and \( D_i \) is the diffusion coefficient. The source terms include ionization, excitation and recombination on the dust particle surface. The electric field is found using the Poisson equation including the dust particle charge \( Q_D = -Z_D e \)

\[
\nabla^2 V = -\frac{e}{\epsilon_0} (n_+ - n_e - n_- - Z_D n_D), \tag{3}
\]

\[
E = -\nabla V. \tag{4}
\]

The ions are too heavy to follow the instantaneous electric field \( E \), and therefore an effective electric field \( E_{\text{eff}} \) is calculated [1, 2]. The electron energy balance is solved using a similar drift-diffusion approach

\[
\frac{\partial w}{\partial t} + \nabla \cdot \Gamma_w = -e \Gamma_e \cdot E + S_w, \tag{5}
\]

\[
\Gamma_w = \frac{5}{3} \mu_e w E - \frac{5}{3} D_e \nabla w, \tag{6}
\]

where \( w \) is the average electron energy density \( w = n_e \epsilon \). The source terms are calculated using a two-term Boltzmann solver for the electron energy distribution function, linking them to the average energy [1, 2]. The ions are assumed to locally dissipate their energy in collisions with the background neutrals. This and the heat originating from the recombination of ions and electrons on the dust particle surface, results in the heating of the background gas.
2.2. Dust charging

Dust particles in a plasma act as small probes and collect ions and electrons. Due to the high mobility of the electrons compared to the ions, the charge is usually negative. The corresponding potential, called the ‘floating potential’, is reached when the current of electrons to the dust particle surface equals the current of ions. These currents are calculated using OML theory. For a negatively charged dust particle, the currents are given by

\[ I_+ = \pi r_D^2 n_e \sqrt{\frac{2E_+}{m_+}} \left( 1 - \frac{eV}{E_+} \right), \tag{7} \]

\[ I_e = -\pi r_D^2 n_e \sqrt{\frac{8kT_e}{\pi m_e}} \exp \left( \frac{eV}{kT_e} \right). \]

Here \( r_D \) is the radius of the dust particle, \( n_+ \) and \( n_e \) are the ion and electron density respectively (note that we do not consider electronegative discharges here), \( m_+ \) and \( m_e \) are the ion mass and electron mass, \( E_+ = (4kT_{\text{gas}}/\pi + m_+ u^2_+/2) \) is the ion energy consisting of a thermal part and a kinetic part from the drift velocity of the ions in the discharge (calculated from \( u_+ = \mu_+ E_{\text{eff}} \)). \( V \) is the potential at the surface of the dust particle with respect to the potential of the surrounding plasma, and \( T_e \) is the electron temperature. Putting \( I_+ + I_e = 0 \) and using a Newton iteration method, the floating potential \( V \) can be found. The charge is then calculated using \( eZ_D = 4\pi \epsilon_0 r_D V \). It should be noted that the above equations are correct to a numerical factor of the order of unity only for the limit of \( u_+ \downarrow 0 \).

Near the walls \( n_+/n_e \) can be so large that the dust charge can become positive. In that case, the role of the ion current and electron current is reversed, meaning that the ion current is exponentially decreased and the electron current is increased for increasing positive potential.

Ions in their orbit around dust particles can lose angular momentum when they collide with a background neutral. Assuming that the ion was close enough, this might cause the ion to be collected by the dust particle, increasing the ion current to the dust particle and thereby reducing the negative floating potential. This depends on the mean-free path of the ions and on the shielding length around the charged dust particle. For typical discharge parameters Khrapak et al [9] derived an alternative equation for the ion current to a negatively charged dust particle in the presence of these collisions

\[ I_+ = -\pi r_D^2 n_+ \sqrt{\frac{2E_+ eV}{m_+}} \left( 1 - 0.1 \frac{eV}{E_+} \frac{\lambda_D}{l_{\text{mf,p}}} \right), \tag{8} \]

where \( \lambda_D^{-1} = \sqrt{1/\lambda_e^2 + 1/\lambda_+^2(u_+)} \) is the linearized Debye length, with \( \lambda_+^2(u_+) = \lambda_e^2(v_T) [1 + (u_+^2/v_T^2)] \) and \( l_{\text{mf,p}} \) the ion mean-free path for collisions with the neutrals, which we have defined by \( l_{\text{mf,p}} = v/\nu \), with \( \nu \) as the collision frequency. After collisions with neutrals the ions have a negligible drift speed and therefore we assume that they cannot reach the surface of positively charged dust particles. Thus, for positively charged dust particles we use the OML theory; the equivalent of equation (7) for positively charged dust particles.
An important effect of the change in current towards the dust particles is the corresponding change in recombination of plasma particles on the dust. The recombination rate is calculated from the currents reaching the dust particles. Since these currents are increased, the recombination rate is also increased. This means that the change in current has an effect on the plasma parameters. Furthermore, the heating of the dust particles by this recombination is also reduced and this results in a lower heating of the background gas. In turn, this has an effect on the thermophoretic force acting on the dust particles. The thermophoretic force plays only a minor role however.

2.3. Forces and dust transport

The thermophoretic force and the neutral drag force are estimated using the standard equations [1, 2]. We did not change the neutral drag, despite the conclusions in [5, 6], since the code used, computes towards the final equilibrium solution. The neutral drag does not play a role in this final equilibrium position, assuming that it only acts as a damping force. When considering the exact dynamics of the dust transport, it would strongly influence the timescales of dust transport. In [6], the deviation from the standard Epstein formulation of the neutral drag was determined from the dynamics of dust transport, after a perturbation of dust clouds from their equilibrium position.

The electrostatic force is calculated using

\[ F_E = Z_D e \overline{E}, \]  

where \( Z_D \) is either solved using equations (7) or equation (8) and \( \overline{E} \) is the time-averaged electric field.

The ion drag consists of two parts: the transfer of momentum by ions collected by the dust particle and the transfer of momentum by ions deflected by the dust particle. The force is given by:

\[ F_{id} = m_+ \int v f_+ (v) [\sigma_c (v) + \sigma_s (v)] \, dv, \]  

where \( v \) is the total ion velocity and \( f_+ (v) \) is the ion velocity distribution function. The cross-section for ion capture \( \sigma_c (v) \) is found from OML theory

\[ \sigma_c (v) = \pi r_D^2 \left( 1 + \frac{2 \rho_0 (v)}{r_D} \right), \]  

with \( \rho_0 (v) = Z_D e^2 / 4 \pi e_0 m_+ v^2 \) the Coulomb radius, which is the closed orbit of an ion with velocity \( v \) in the potential of a dust particle with charge \( Z_D \). The cross-section for deflection, or scattering, depends on the specific form of the potential around the dust particle. Assuming a pure Coulomb potential with certain cut-off radii for the impact parameters, \( b_{min} \) and \( b_{max} \), it is found from:

\[ \sigma_s (v) = 4 \pi \int_{b_{min}}^{b_{max}} \frac{b \, db}{1 + (b/\rho_0 (v))^2} = 4 \pi \rho_0^2 (v) \Lambda (v), \]
where $b$ is the impact parameter of the ion approaching the dust particle and $\Lambda(v)$ is called the Coulomb logarithm. The original approach [17] uses

$$
\Lambda(v) = \frac{1}{2} \ln \left[ \frac{\rho_0^2(v) + b_{\text{max}}^2}{\rho_0^2(v) + b_{\text{min}}^2} \right],
$$

where $b_{\text{min}}$ is the collection radius given by equation (11), and we will show both the results for $b_{\text{max}} = \lambda_D(v)$ and $b_{\text{max}} = \lambda_e$. The approach by [11] is to include ions scattered beyond the screening length, by choosing $b_{\text{max}} = \lambda_D(v)(1 + 2\rho_0(v)/\lambda_D(v))^{1/2}$, so that the Coulomb logarithm becomes

$$
\Lambda(v) = \ln \left[ \frac{\rho_0(v) + \lambda_D(v)}{\rho_0(v) + r_D} \right],
$$

which results in an increased ion drag force when the linearized Debye length is used. The above form of the Coulomb integral is valid for $\beta(v) = \rho_0(v)/\lambda_D(v) \lesssim 5$ which is the case for the dust cloud and near the void edge (see section 3). Inside the void $\beta(v) \gg 1$. In this case, a better form of the scattering cross-section is given by $\sigma_s(v) = 8\pi\lambda_D^2\beta^2(v)/(1 + 1.5\beta^{1.65}(v))$ [20]. We did not use this form of the scattering cross-section here.

Equation (14) is not accurate for larger ion flow speed (with thermal Mach numbers $M_T = u_e/v_T$ larger than $\approx 5$ for our discharge parameters [8]). Hutchinson [8] used a PIC approach to calculate the ion drag for higher ion flow speed and he concluded that good agreement can be found when in the calculation of the Coulomb integral, the velocity is as given in the total ion energy

$$
m_+v^2/2 = 4kT_{\text{gas}}/\pi + m_+u_e^2/2
$$

is replaced by

$$
m_+v^2/2 = 4kT_{\text{gas}}/\pi + m_+u_e^2/2 \left[ 1 + \left( \frac{u_e}{u_b} \right)/(0.5 + 0.05 \ln \left( \frac{m_+}{Z_e} \right) + \sqrt{\frac{T_e}{T}}) \right],
$$

where $u_b = (kT_e/m_+)^{1/2}$ is the Bohm speed and $m_+/Z_e = 40$ in the case of argon (here) and 1 in the case of atomic hydrogen ions.

When ions collide with background neutrals and the mean-free path becomes comparable to or smaller than the screening length, more ions can be effectively deflected by the dust particles, since they lose angular momentum in these collisions. In [12, 13], a theoretical description was constructed for the calculation of the ion drag force, including these collisions by adding a ‘collisional function’, $K$, to the Coulomb integral. The ion drag force is now calculated as

$$
F_{\text{id}} = F_{\text{id}}^c + F_{\text{id}}^o,
$$

$$
= n_+m_+v u_e \left( \sigma_s(v) + \pi\rho_0^2(v) \left[ \Lambda(v) + K(\frac{\lambda_D(v)}{l_{\text{mfp}}} \right) \right]
$$

with

$$
K(x) = x \arctan(x) + \left( \frac{\sqrt{\pi}}{2} - 1 \right) \frac{x^2}{1 + x^2} - \sqrt{\frac{\pi}{2}} \ln (1 + x^2).
$$
Two comments are in order here; firstly, the above form of the collision function, even though it is independent of $\beta(v)$, was derived for $\beta(v) < 1$ assuming a shifted Maxwellian distribution function. The effect of the collisions on the velocity distribution function of the ions is not taken into account. Secondly, the collision integral is not derived from the binary collision approach, but using the linear dielectric response formalism [12, 13]. The two limiting cases, $M_T \ll 1$ and $M_T \gg 1$ were both analytically derived, but the exact analytical form for $M_T > 1$ (and not $M_T \gg 1$) has not yet been found.

In our model, the dust transport is solved assuming that the neutral drag is always in balance with the sum of the other forces, i.e. neglecting the dust inertia. Even though this is not always true, it gives the (final) equilibrium solution and provides a way to write the dust flux as

$$\Gamma_D = -\mu_D n_D \mathbf{E}_{\text{eff}} - D_D \nabla n_D - \frac{32}{15} \frac{n_D \sigma_D}{m_D v_{m,D} \pi v_{th}} \kappa_T \nabla T_{\text{gas}} + \frac{n_D \mathbf{F}_{\text{id}}}{m_D v_{m,D}},$$

(18)

where $\mu_D = eZ_D/m_D v_{m,D}$, $\sigma_D = \pi r_D^2$, $\kappa_T$ is the translational part of the gas thermal conductivity, $v_{th}$ is the thermal velocity of the background gas and $v_{m,D}$ is the momentum transfer frequency of the dust. We also see that a different form of the Epstein drag will only result in an overall constant. The dust diffusion coefficient is taken from [21], which includes the effect of the mutual Coulomb interaction and the possibility of the formation of crystalline incompressible dust structures:

$$D_D = \frac{1}{v_{m,D} d n_D},$$

(19)

$$P_{cr} = \frac{1 + \alpha \kappa}{3\alpha} N_{nn} \Gamma P_D \exp(-\alpha \kappa).$$

(20)

Here, $\Gamma$ is the ratio of the Coulomb energy over the thermal energy, $\Gamma = (eZ_D)^2/4\pi \varepsilon_0 \Delta k T_D$, $\Delta = n_D^{-1/3}$ is the mean distance between dust particles, $\kappa = \Delta / \lambda_D(v)$, $N_{nn}$ and $\alpha$ are constants depending on the lattice structure of the crystalline dust [22]. $P_D = n_D k_b T_D$, with $T_D$ the kinetic dust particle temperature. This temperature is coupled to the background gas temperature, which depends on the recombination of plasma particles on the dust particle surface. Because of this and because of the fact that the above equation depends on the dust charge directly, the dust diffusion depends on the charging theory used.

This is a closed set of equations which allows us to self-consistently solve the plasma and dust parameters. For the solution, we regularly recalculate the average plasma profiles in agreement with the actual dust density and charge profiles.

3. Results for different models

In this section, we compare the dust transport for different models used. We begin by briefly showing the results with the 'classical' approach by Barnes et al (small angle scattering) and then compare them with the results obtained by using the approach of Khrapak et al (large angle scattering). In all the following results, we modelled an argon discharge at 40 Pa background pressure and at a 100 volt peak-to-peak applied potential. The geometry is similar to the geometry of the PKE chamber [23]. We introduced 1 million dust particles with a radius of 6.8 $\mu$m. In the
experiments, dust particles were introduced through two shakers in the electrodes. In our model, we add source terms for the dust particles at the same positions, simulating the ejection of the dust particles into the plasma from the electrodes.

3.1. Small-angle scattering and large-angle scattering

Figures 1 and 2 show the final dust densities for the Barnes approach with the linearized Debye length as the cut-off radius and the Barnes approach with the electron Debye length. Similar results were also shown in [1, 2], where the ion drag force was enhanced by a factor of 5, instead of using the electron Debye length as the cut-off length. Even though these results are not new, we learn one important thing; apparently, the fact that the void does not appear, implies that the momentum transfer cross-section for ion-dust collisions should be larger than that calculated using the Coulomb scattering theory with the linearized Debye radius as the upper limit for the impact parameter. The ion drift itself is not sufficient to produce a void. Apparently, the linearized Debye length does not approach the electron Debye length near the edge of the bulk of the discharge.

Since it is assumed that negative dust particles are screened by positive ions, the electron Debye length is considered to be an unphysical cut-off length for the potential around negatively charged dust particles. Therefore Khrapak et al introduced the idea of having ions scattered beyond the Debye length. They introduced a parameter, $\beta(v)$, which depends on the dust charge, the ion flow and the linearized Debye length, $\beta(v) = \rho_0(v)/\lambda_D(v)$. When this parameter is large,
Figure 2. The final dust density profile when the ion drag force is calculated with the Barnes logarithm and the electron Debye length as the cut-off radius. Clearly, this increases the ion drag force in the bulk of the discharge enough and a dust-free void is formed.

closed ion orbits exist beyond the linearized Debye length, which means that ions can be deflected beyond the screening length. Figure 3 shows this parameter calculated self-consistently in our model for the final dust density profile of figure 2. We see that inside and near the edge of the void, $\beta(v) > 1$, which means that the Barnes form of the logarithm is no longer valid. Of course, inside the void no dust particles are present, so the effect of scattering of ions beyond $\lambda_D(v)$ is only important near the edge of the void for the dust cloud observed in our simulations. However, for a dust particle introduced in the void the ion drag force is such that it will still be accelerated out of the void and in that case scattering beyond the screening length is important.

When we look at the final dust density profile calculated with the Khrapak form of the ion drag, shown in figure 4, we indeed see a change in the shape of the void around the quasi-neutral bulk of the discharge. The final size of the void is somewhat smaller and the void has a more rectangular shape. The density gradients near the edges of the void are steeper than in the case of the calculation with the Barnes approach, which indicates a narrower potential well around the point of force balance. The importance of large angle scattering is shown in figure 5. Inside the dust cloud the value of $\beta(v)$ is moderate. Inside the void however $\beta(v) \gg 1$.

The total force (ion drag + thermophoretic + electrostatic) in the radial direction, at the axis of symmetry ($Z = 0.027$ m) after 2 s, is shown in figure 6. It shows the force for the Barnes approach with the linearized Debye length, where no void is formed (red, marked B). Inside the bulk, $\beta$ is much larger than 1, so the ion drag force calculated with the approach by Khrapak (blue, marked C) is larger than the ion drag force calculated with the approach by
Figure 3. The parameter measuring the importance of scattering beyond the linearized Debye length (large angle scattering). For $\beta > 1$, this scattering is important. The dashed lines correspond to the dust density contours from figure 2. Scattering beyond $\lambda_D$ seems to be important only inside the void or near the void edge.

Barnes with the electron Debye length (black, marked A). However, $\beta$ drops to smaller values rapidly for increasing radial position, and in that case the approach by Barnes with the electron Debye length can overestimate the ion drag force, especially for larger particles, as was also mentioned and shown in [16, 18]. Another important observation is that the void edge does not correspond to the point where $F_r = 0$. However, figure 6 does not include the mutual interaction force between the charged dust particles, which prevents them from reaching the force balance point exactly.

3.2. Ion flow speed

We have already seen that the ion flow in this discharge is not large enough to make the linearized Debye length approach the electron Debye length near the edge of the bulk of the discharge. However, ion flow can have an important effect on the ion drag force. Figure 7 shows the ion drift velocity normalized to the thermal ion velocity, where we assume that $T_i \approx T_n = 293$ K, for the solution with the Khrapak logarithm.

We see that the ions move close to the thermal velocity in the bulk of the discharge. Axially, they are accelerated by the dust, but mainly by the global electric field induced by the applied potential on the electrodes. Radially, this electric field plays only a minor role. The main electric field in the radial direction is caused by the charged dust cloud and by the electric field due to the
Figure 4. The final dust density profile for the calculation with the Khrapak form of the Coulomb logarithm. Notice the change in the shape and size of the void. Also note that the radial dust transport is smaller here.

The difference in mobility of the ions and electrons moving to the outer walls. It is interesting to note that the scattering of ions on the charged dust particles decelerates the ions inside the dust cloud back to thermal drift velocities. Once they reach the edge of the dust cloud, they escape and are accelerated towards the outer walls again. This behaviour of the ion drift speed was also assumed in [24], where an analysis of dust waves lead to the conclusion that the ions were decelerated in the dust to subthermal drift speeds. We see that the thermal ion Mach number, $M_T$, never exceeds 4 inside the dust cloud, except close to the electrodes in the (pre-)sheaths. We therefore mainly expect an effect of the ion flow speed on the ion drag force close to the electrodes. Near the void edge the Mach number, $M_T \sim 2-4$, which is in good correspondence with [24, 25]. This value of the Mach number near the void edge is to be expected since the ratio of the ion drag force and the electrostatic force decreases rapidly for $M_T > 1$ [25] and force balance is reached within this range of Mach numbers.

Figure 8 shows the final dust density profile when we use the Khrapak form of the Coulomb logarithm together with the empirical adjustment of the ion drift velocity as suggested by Hutchinson (the filled contours), as well as the final dust density profile when we only use the Khrapak form of the Coulomb logarithm (solid lines).

In our discharge, the electron energies range from 3–7 eV, so on average the electron energy is approximately 5 eV. This means that the Bohm velocity corresponds to a thermal Mach number of $M_B \approx 13$. According to [8], the effect of ion drift becomes important for $v_{\text{eff}} \approx 0.55 v_B$, which thus corresponds to a thermal Mach number of $M_T \approx 7$. The collection force however
Figure 5. $\beta(v)$ for the solution with the Khrapak form of the Coulomb logarithm. Inside the dust cloud $\beta(v)$ stays below 7, which means that equations (8) and (14) are reasonable. Inside the void again scattering beyond the screening length is very important. Dashed lines correspond to the final dust density profile shown in figure 4.

becomes dominant when $M_T \approx \sqrt{(\lambda_D/a)\beta_T}$, where $\beta_T = e^2 Z_D/4\pi\epsilon_0 kT \lambda_D$ [26]. In the sheath, this corresponds to a thermal Mach number of $M_T \approx 8.5$.

The shape of the dust cloud edge in front of the electrodes is therefore determined by the collection force and is approximately the same for both solutions. The shape of the inner edge of the void is mainly determined by ions flowing below the Bohm speed. For our solution of the ion flow, the increase in the scattering ion drag force is approximately 15%. Inside the dust cloud the ions are first accelerated, leading to a slight increase in the ion drag force, but then decelerated again by (recombination on) the negatively charged dust particles. The dust contours on the outside of the dust cloud are again the same, since ions are showing almost no drift there, as is shown in figure 7.

3.3. Collisions

We consider two effects of collisions between flowing ions and neutrals (charge-exchange collisions), namely the effect of these collisions on the ion flow towards the dust particles and thus the charging of dust particles, and on the increase in momentum transfer caused by these collisions and the corresponding increase in the ion drag force. One of the important parameters here is the ratio of the linearized Debye length and the ion-neutral mean-free path. Figure 9 shows this parameter for the calculation with the Khrapak form of the logarithm.
In figure 9, we see that inside the void the ratio is very small. This has to do with the small size of the linearized Debye length, which in the bulk equals the ion Debye length. The mean-free path is calculated for constant momentum transfer frequency and therefore only depends on the drift velocity of the ions. We expect no effect of the collisions on the ion drag force in the bulk of the discharge. Towards the outer edge of the dust cloud however, the ‘Collision function’, $K$, becomes comparable to the Coulomb logarithm $\Lambda$, which happens when $x \approx 3$ in equation (17).

When we compare the dust density distribution for the solution with the Khrapak form including the adjustment for ion flow and a similar solution, but now including the effect of collisions on the ion drag force, we indeed see the largest shift near the outer edge of the dust cloud, which is shown in figure 10.

Finally, we included the effect of collisions on the charging of dust particles. It is important to realize that even though $x$ might be smaller than 1, the charging can still be affected when $eV/E_+$ is large enough, and this is exactly the case in the bulk of the discharge.

Figure 11 shows the charge per dust particle for the calculation when we use the Khrapak form of the Coulomb logarithm together with the standard OML expressions for the currents towards the dust particles. Figure 12 shows the same when the effect of ion-neutral collisions is included in the current balance towards the dust particles. We see that the charge a dust particle in the bulk would have is reduced by a factor of 2 or more. Near the void edge, the effect of the collisions is very small, since both $\lambda_D/l_{mf\beta}$ and $eV/E_+$ are small. Near the outer edge of the

Figure 6. The total radial force at the axis of symmetry for both the approach by Barnes et al with the electron Debye length (black, marked A), the linearized Debye length (red, marked B) as well as the approach by Khrapak et al (blue, marked C). The force balance point shifts inwards for the latter approach, resulting in a smaller void and less transport outwards. The green line indicates $F_r = 0$. 

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dust cloud the ions are decelerated and $eV/E_+ \rightarrow 0$ becomes large again, reducing the charge on the dust particles. Near the walls the linearized Debye length becomes very large, as is shown in figure 9, and the charge is reduced here as well.

Finally in figure 13, we show the total radial force along the axis of symmetry for the simulations above. We see that the total force with the reduced charge deviates the most from the simple Khrapak solution. Inside the void, the force is much more repelling than in the other cases, since the electrostatic force pulling the particles in is reduced. The force balance point (where $F_r = 0$) also lies slightly more inward. It is interesting to see that only for the reduced charge calculation, the total force actually changes sign at the force balance point. The other forces stay very close to 0 for a large radial interval before becoming negative near the outer wall.

It is also here that the effect of the collisions on the ion drag force becomes important. Towards the outer wall (at the outer edge of the dust cloud), the force is clearly less negative, due to the increase in the outward ion drag force. Near the edge of the void the force is slightly increased due to the flow speed. This effect is very small however.

4. Comparison with experiments

Here, we will discuss the experimental results as found and discussed in [14, 18, 19] and [15, 16]. The first papers discuss measurements of trajectories of particles falling through the bulk of a
RF discharge. The last two discuss the formation of voids around (Langmuir) probes introduced into a complex plasma.

4.1. Particles falling through a RF discharge

In this experiment, dust particles were released from the topside of a vertical tube at relatively high pressures. The dust particles fell through a plasma volume powered by a RF generator. Using a laser sheet and a charge-coupled device (CCD) camera, the trajectories of the dust particles were captured. From the trajectories perpendicular to the tube, the ratio of the electrostatic force and ion drag force was determined. This ratio (typically between 0.5–5) was then plotted against the Debye length over the dust particle radius and the Debye length over the ion mean-free path. One of the first observations was that $MT \leq 2$, which seems reasonable when we look at the ion flow in the bulk of our simulations. Secondly, it was stated that for the higher pressures, the assumption of the collisionless theory might fail. From our calculations, the effect of the collisions on the charge of the dust particle might be even more important. This would mean that for a given ratio, the ion drag force would actually have to be smaller than in the case where the charge of the dust particle is calculated using OML theory. One statement made in [14] is that in the case of the ion drag force calculated with the approach by Khrapak, the ion drag force shows

Figure 8. A zoom in on the dust density profiles for the calculation using the Khrapak form (black, solid lines) and the case where the effect of flow has been included (coloured contours). Note that the values and spacings are equal for the lines and the coloured contours. The effect of flow is mainly visible near the void edge and the outside of the cloud.
Figure 9. The ratio of the linearized Debye length to the mean-free path for ion-neutral collisions. When this ratio is large, collisions occur within the screening length of the dust particles and will affect both the charging of the dust particles as well as the ion drag force acting on the dust particles. Dashed lines correspond to the final dust density profile as shown in figure 8 (filled colours).

a dramatic increase up to a factor of 40. Figure 14 shows the ratio of the radial ion drag force to the radial component of the electrostatic force. (Similar to the ion drag force and electrostatic force considered in the experiment.) Clearly, even for the approach with the Khrapak logarithm and the linearized Debye length, the ratio of the two forces lies between 2 and 4 in the bulk of the discharge, which seems to be in reasonable agreement with figure 3 in [14]. Of course, we did not include the ratio for the calculation using the Barnes form of the Coulomb logarithm together with the linearized Debye length as the cut-off radius. From figure 6, and the fact that no void is present for this calculation, we can see that the ratio would be very small. So comparing the Khrapak form of the Coulomb logarithm with the linearized Debye length as the cut-off radius with the Barnes form and the linearized Debye length, would indeed show a large difference. But in [14], the electron Debye length was used as the maximum impact parameter.

4.2. Void formation around probe

In these experiments probes were introduced in a dusty plasma, biased negatively with respect to the surrounding plasma. The electric field accelerates positive ions towards the probe tip and negatively charged dust particles away from the probe tip. The ions streaming towards the probe tip are collected and deflected by the dust particles, and the corresponding ion drag force acts...
Figure 10. A zoom in on the dust density profiles for the calculation using the Khrapak form with the adjustment for ion flow (black, solid lines) and the case where the effect of collisions has been included (coloured contours). The largest shift in dust density is observed near the outside of the dust cloud.

towards the probe. The equilibrium position of the dust around the probe comes from the balance between the electrostatic force away from the probe and the ion drag force towards the probe. This results in the formation of a void around the tip of the probe.

In [15] dust particles were suspended in the anode spot of a dc discharge, after which a probe was introduced into the plasma. The reported electric field around the probe is in the range of $1200 \sim 3400 \text{ V m}^{-1}$, which is much larger than the vertical electric field which balanced the force of gravity, resulting in the original vertical equilibrium position of the dust particles in the anode spot. The average ion drift velocity reported was $1000 \text{ m s}^{-1}$, which, for room temperature, means a thermal Mach number of $M_T \sim 3$, which is in agreement with the Mach number at the void edge found in our simulations.

The ion drag in [15] was calculated using the Barnes equation with the electron Debye length as the upper value for the impact parameter. This means that for $M_T \sim 3$ the ion drag might be somewhat overestimated for the radius of the dust particles ($2.9 \mu \text{m}$) and the plasma density ($\sim 10^{15} \text{ m}^{-3}$). Using numbers reported in the paper, we find $\beta(v) \approx 5$. This means that it is justified to use equation (14) to calculate the ion drag. This way, we find a total ion drag force of $F_{id} \approx 3.1 \times 10^{-12} \text{ N}$. This is about an order of magnitude smaller than the value found in [15] ($\approx 2 \times 10^{-11} \text{ N}$).

The electric field required for balance with the ion drag force is then given by $E = F_{id} / Q_D$. The charge reported in the paper was $Q_D \sim 5200e$. Using this we then find an electric field of...
The number of electrons on a dust particle, $Z_D$, calculated for the Khrapak logarithm. The charge is given by $Q_D = -eZ_D$, so that a positive number in the graph corresponds to a negative dust charge, and a negative charge number to positively charged dust. Note that the charge is calculated throughout the chamber, even where no dust is present.

approximately 3725 V m$^{-1}$. The value calculated in [15] seems to be an order of magnitude too low, since $2 \times 10^{-11} N/(5200e) \approx 24\,000$ V m$^{-1}$, where (3400–1200) V m$^{-1}$ is reported.

Using an ion-neutral collision cross-section of $65 \times 10^{-20}$ m$^2$ [16] and a neutral density $N \sim 10^{21}$ m$^{-3}$, we find the argon ion mobility as $\mu_i \approx 1$. Which means that the drift velocity $u_i = \mu_i E$ would give $u_i \approx 4 \times 10^3$ m s$^{-1}$. This value is of the same order as the drift velocity assumed (500–1500 m s$^{-1}$), even though it is somewhat too high. It is clear that for the formation of a void around a biased probe in a dusty plasma, the ion drag force calculated assuming deflection of ions beyond the screening length gives the best results.

In [16], a probe was introduced inside the void of a dusty plasma under microgravity. We have seen that inside the void $\beta(v) \gg 1$ and that the charge of dust particles is reduced with respect to the OML charge. However, the ratio of $F_{id}/F_E$ remains approximately the same.

The parameters in [16] are almost the same as used in our simulations, except for the dust particle size. The forces in balance are also in the $10^{-12}$ N range, however, the charge is calculated using OML theory. Again, estimating the ion drift velocity from the balance with the electric field (and using the OML value of the dust charge $Q_D = 10\,000e$), we find for the solution with Barnes an electric field of $E = 850$ V m$^{-1}$ while for the solution with Khrapak and the ion Debye length $E = 331$ V m$^{-1}$.

The first results in an ion drift velocity of approximately $u_i = 1000$ m s$^{-1}$, while the latter gives approximately $u_i = 300$ m s$^{-1}$. Even though both solutions are in the right order, in the
first approach it was assumed that the ions were flowing with the Bohm speed, \( u_+ \equiv u_B = 2800 \text{ m s}^{-1} \), where in the latter approach ions were assumed to flow at the thermal speed, \( u_+ = v_T = 300 \text{ m s}^{-1} \). So the approach with Khrapak and the ion Debye length is more self-consistent, as far as the electric field and the ion flow calculated from the force balance is concerned.

As was mentioned, using thermal ions and the ion Debye length gives the lower limit for the ion drag force. The real ion drag force lies somewhere in between. This seems to be consistent with the experiment mentioned in [15] and discussed above. It is likely that at the void edge, ions are flowing with \( M_T \approx 3 \) and scattering beyond the screening length given by the linearized Debye length is important. Using the approach by Barnes with the electron Debye length strongly overestimates the ion drag force, which results in inconsistent ion drift velocities.

5. Conclusions

In a complex plasma, the interaction between the plasma particles and the dust particles plays the dominant role in the transport of the dust. This interaction is mainly determined by the distance over which charged particles are affected by the potential around the dust particles. This distance is a complex function of both the electron and the ion Debye length (which depends on the ion flow), but also depends on the presence of ion-neutral collisions. Our modelling and the comparison between experiments shows that scattering beyond the linearized Debye length is
very important for the interaction between ions and dust particles, and therefore the solution with
the modified Coulomb logarithm proposed by Khrapak et al should be used. Still, the formation
of a void is observed for all the simulations, except for the Barnes approach with the linearized
Debye length. The flow speed of the ions is not sufficient to make the linearized Debye length
approach the electron Debye length in this case, and the Barnes form of the Coulomb logarithm
results in an insufficient ion scattering cross-section.

When applying the theory to experiments performed in the recent past, it seems that the
ion drag force has been overestimated in most cases, by using the Barnes approach with the
electron Debye length. This is also confirmed by our simulations, where the void is much larger
for the calculation with this approach than with the approach by Khrapak. This smaller size of
the void also corresponds better with the size of the void usually observed in experiments under
microgravity conditions done in the PKE chamber.

The shape of the void observed in these experiments depends on the pressure and input
power and on the size of the dust particles used. In [27], small in situ grown particles form a
clear lentil-shaped void. Small particle size means small charge, which means small \( \beta(v) \). Both
the solution of Barnes et al and the solution of Khrapak give reasonable and similar solutions in
this limit, which, from our simulations, result in more round-shaped voids. The pressure in these
experiments was also relatively high, resulting in slow ion drift speeds and many ion-neutral
collisions.

In other typical experiments, larger particles were used, for instance with a 7.5 \( \mu m \) radius
in [23]. The shape of the void in this experiment is less lentil shaped and more rectangular,
especially for higher input powers. Since near the void edge, the ion drift speed is always similar
\( (M_F \approx 2–3) \), the higher dust particle radius will result in a higher \( \beta(v) \), so that the solution with

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**Figure 13.** The net radial force along the axis of symmetry.
the modified Coulomb integral differs greatly from the solution with the Barnes form of the Coulomb logarithm, which in our simulation indeed produces more rectangular shaped voids.

Ion-neutral collisions in a dusty plasma under microgravity do not have an important effect on the ion drag force for the range of parameters considered here, except at the outer edge of the dust cloud, near the outer walls of the discharge. They do change the charge however, especially when the ion flow is low and the dust charge high, which is the case in the bulk of the discharge.

Ions do not flow faster than $M_{T} \sim 4$ in dust clouds in the dusty plasmas modelled, except in front of the electrodes. Inside the dust cloud they are decelerated again, in some simulations even below the thermal velocity. This means that the correction proposed by Hutchinson plays only a minor role in the transport of dust in dusty RF discharges like the one modelled here. In other types of discharges, such as dc discharges, this might be different though. The value of the Mach number near the void edge always lies between 2 and 4. Large angle scattering plays a very important role inside the void of a dusty discharge under microgravity. Inside the dust cloud, however, $\beta(\nu)$ stays below a value of 7. This means that using the Barnes approach and the electron Debye length actually overestimates the ion drag force.

As a final remark, it is worthwhile to mention that we do not observe a rapid dust density increase on the inside of the void. This can be due to the limited resolution in our simulations, since this high-density layer on the inside is typically a few particle layers thick. It is still a question whether or not this sharp void-dusty plasma boundary results from the matching between the two regions and the necessary space charge layers [28], or is a shock-like structure [29].
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