A micro dalton resolution mass sensor using optically cooled microdisks via short-time measurement

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We propose a mass sensor using optically trapped and cooled dielectric microdisks with "measuring after cooling" scheme. The center-of-mass motion of a trapped particle in vacuum can experience extremely low dissipation resulting in robust decoupling from the heat bath. Thus the measurement can be accomplished in a short time before a jump in the phonon number after the optomechanical cooling is finished. This method can lead to $10^{-6}$ dalton mass sensitivity which is a thousand times smaller than the mass of one electron.

I. INTRODUCTION

Mechanical resonators are widely used as inertial balances to detect small quantities of adsorbed mass due to the shifts in their vibrational frequencies. Nanoelectromechanical systems have been proposed for highly sensitive mass detection of neutral species. Significant progresses have been made by using nanofabricated resonators[1,2] and carbon nanotubes[3-5] for mass sensing. However, so far, the best mass resolution achieved with microfabricated resonators has been 200yg in the base pressure of $10^{-10}$Torr[3,4], whereas carbon nanotube resonators have achieved mass resolutions as low as 2yg in base pressure of $3 \times 10^{-11}$mbar[5]. Here we demonstrate an optically levitated microdisk based optomechanical cooling with $10^{-6}$Da resolution. The masses of neutral atoms, electron, neutrons and other chemical molecules and biological molecules can be measured in all-optical domain, which have applications in mass spectrometry[6], magnetometry[7] and surface science[8].

Quantum optomechanical techniques in both the optical and microwave regimes will provide motion and force detection near the fundament limit imposed by quantum mechanics[9,10]. Optomechanical quantum control requires the mechanical oscillator to be in or near its quantum ground state. Therefore, nowadays the investigations of optomechanical cooling become a research focus. The two main methods being pursued in optomechanics are “backaction cooling” which requires the stable operation of a high-finesse cavity[11-14], and “feedback cooling” which requires precision measurement and feedback[15-17]. Recently much effort has been directed toward optically levitating nano-and micro-mechanical oscillators in ultrahigh vacuum such as nanospheres[14,17], nanodiamonds[18,19], microdisk[20,21], and even the living organisms[22]. The laser trapped objects has no physical contact to the environment, leads to ultralow mechanical damping which makes it a promising system for ground state cooling even at room temperatures[14,17,20].

However, the optomechanical cooling will increase the mechanical damping rate and cause the extra damping, thus will dramatically increase the linewidth of the oscillator[23]. This effect can significantly decrease the precision which strongly depends on the linewidth on spectrum. Here we propose a unique approach toward this problem, wherein the detection can be accomplished by a short integration time after the ground state cooling. An optically trapped microdisk in vacuum is well isolated from the thermal environment and can have a mechanical damping $\sim 10^{-7} Hz$. In this case, the thermal decoherence rate reduces notably, thus the detection can be done before the mechanical oscillator are heating out of the ground state in the absence of cooling scheme. In addition, the decoherence and heating rates are fundamentally limited by the momentum recoil of scattered photons and can be reduced simply by using high finesse cavity and increasing the trapping frequency[14,24]. Based on the "measuring after cooling" technique, we theoretically show that the mass sensitivity can be improved by 6-7 orders than the traditional nanomechanical resonators[3-5], reaching $10^{-6}$Da resolution. Mass spectrometers actually measure mass-to-charge ratios. Here the true mass of the atoms or molecules can, in principle, be determined without any need to ionize, even the mass of the electrons can also be determined. This unprecedented level of sensitivity allows us to measure precisely the mass of neutral particle and to distinguish between isotopes in inertial mass spectrometry measurements.

II. METHOD

Our approach involves optically trapping and cooling a cleanroom fabricated microdisk without clamping to the substrate at room temperature. We choose a silicon nitride microdisk of radius $r = 1 \mu m$ and thickness $b = 50 nm$. The ultralight weight of the microdisk($\sim 10^{-16}kg$) may allow an ideal situation in which a freely levitated disk is trapped against the gravity in the anti-
Here of a strong optical standing wave. The light fields of wavelength $\lambda = 1\mu m$ is used for trapping. A second field is used to cool and to measure the center-of-mass motion of the disk as shown in the Fig.1. Following Ref.[17], we can cool the levitated microdisk parametrically by a feedback loop. Nanomechanical resonators can function as 1) cooling. The equation of motion for the microdisk’s motion in $x$ direction (polarization direction) is

$$\ddot{x}(t) + \gamma x(t) + \omega_m^2 x(t) = \frac{1}{m}[\xi(t) + F_{opt}(t)].$$  \hspace{1cm} (2)

Here $\xi(t)$ denoting fluctuating forces acting on the microdisk due to the impact of the air molecules. For white noise, one has that $\langle \xi(t)\xi(t') \rangle = 2k_B T m \gamma \delta(t-t')$ according to the fluctuation dissipation theorem, where $k_B$ is the Boltzmann constant and $T$ is the temperature of the chamber. The oscillator’s damping rate can be written as $\gamma = \gamma_m + \gamma_{fb}$, where $\gamma_m$ accounts for the interaction with the background gas and $\gamma_{fb}$ is the damping introduced by feedback cooling. $F_{opt}(t) = \Delta k_{cool}(t)x(t)$ is a time-varying, nonconservative optical force introduced by parametric feedback, here $\Delta k_{cool}(t)$ is the additional trapping stiffness. Therefore, feedback cooling leads to shifts $\gamma_{fb}$ and $\Delta \omega$ in the disk’s natural damping rate $\gamma_m$ and oscillation frequency $\omega_m$. Activation of the parametric feedback loop gives rise to the additional damping, and predicts that the CM temperature is reduced down to

$$T_{CM} = T \frac{\gamma_m}{\gamma_m + \gamma_{fb}}.$$  \hspace{1cm} (3)

2) measurement. To accurately measure the microdisk’s frequency, we need to decrease the CM temperature. This can only be accomplished by using the cooling laser and hence an additional optical feedback damping. On the other hand, high feedback damping gives rise to large oscillation linewidth and significantly decreases the sensitivity. Thus we will not apply the parametric feedback in the measurement. One can stop feedback loop by ceasing the modulation in the cooling laser intensity at a time $t'$, thus we have

$$F_{opt}(t) = 0 \quad \text{for} \quad t > t'.$$  \hspace{1cm} (4)

It leads to shifts $\gamma_{fb} = 0$, and the total damping reduces to $\gamma_m$. Thus we only analyze here the heat of the mechanical motion of the center-of-mass of a dielectric disk due to the gas pressure and the photon recoil.

The variance of the position can be computed by solving the differential stochastic equation. We suppose $F_{opt}(t) = 0$ in the Eq.(2) and consider $\omega_m \gg \gamma_m$ which is always fulfilled in the levitating resonators, then we have

$$\langle [x(t) - \langle x(t) \rangle]^2 \rangle \approx \frac{k_B T}{\hbar \omega_m^2} [1 - e^{-\gamma_m t}].$$  \hspace{1cm} (5)

By considering the equipartition principle, the variance allows us to compute the increase of energy by taking $\Delta E(t) = m \omega_m^2 \langle [x(t) - \langle x(t) \rangle]^2 \rangle$. Hence one can compute the time $\Delta t$ required to increase one quantum $\hbar \omega_m$ of energy in the quantum harmonic oscillator. In the case of $\hbar \omega_m \ll k_B T$, the time $\Delta t$ is given by solving $\Delta E(\Delta t) = \hbar \omega_m$ and reads

$$\Delta t = \frac{1}{\gamma_m} \log\left(1 + \frac{\hbar \omega_m}{k_B T} \right) \approx \frac{\hbar \omega_m}{k_B T \gamma_m} = \frac{1}{\Gamma_{th}}.$$  \hspace{1cm} (6)

Here $\Gamma_{th} = k_B T \gamma_m / \hbar \omega_m$ is often referred to as the thermal decoherence rate and given by the inverse time it takes for one quantum to enter from the environment. The recoil imparted on the particle by a scattered photon is a small effect since the photon momentum is small compared to the momentum of the mechanical oscillator. However, at very low temperatures and pressures,
collisions with air molecules become negligible and photon recoil takes over as the dominating decoherence process. Considering that the strong scattering introduces recoil heating, one can obtain the total decoherence rate $\Gamma' = \Gamma_{th} + \Gamma_{recoil}$, and the total heating time

$$\Delta t' = \frac{1}{\Gamma'}. \quad (7)$$

Hence, we can choose a measurement time $\tau$ that is sufficiently short to ensure that the measurement can be accomplished before a single phonon is exchanged with the thermal bath in the absence of cooling scheme.

We now turn to the evaluation of the minimum measurable frequency shift, $\delta \omega$, limited by thermomechanical fluctuations of a levitated resonator readout by a probe laser. In such a measurement, the resonator is driven at a constant mean square amplitude, $x_c$, by the harmonic trapping potential. An estimate for $\delta \omega$ can be obtained by integrating the weighted effective spectral density of the frequency fluctuations by the normalized transfer function of the measurement loop. Performing this integration for the case where $Q \gg 1$, we obtain[25]

$$\delta \omega = \sqrt{\gamma_m k_B T_{CM} \Delta f / k_{trap} x_c^2}. \quad (8)$$

Here, $\Delta f$ is the measurement bandwidth. Experimentally, the measurement time $\tau$ is defined by the detection bandwidth $\tau = 1/2 \pi \Delta f$.

If the detection can be completed in a short time, fulfilling that $\tau \leq \Delta t'$, the phonon number can remain unchanged after the feedback cooling is stopped. It allows the measurement in ground state and the ultralow CM temperature $T_{CM}$ can be achieved. To obtain the minimum detectable mass, we rewrite Eq.(8) using the expression for $\delta m$ given in Eq.(1) supposing $\tau = \Delta t'$.

$$\delta m = \frac{1}{\omega_m^2} \sqrt{2m \gamma_m k_B T_{CM} / \pi x_c^2 \Delta t'}. \quad (9)$$

III. SENSITIVITY

In equilibrium, the microdisk is situated at $z = 0$, in the anti-node of an optical standing wave $E(z) \propto \cos kz$ (see Fig.1). The optical field polarizes the dielectric disk, yielding a gradient force trap around the equilibrium position. In the absence of any internal forces, the optical field traps a thin disk ($d \ll \lambda$) with a restoring frequency given by

$$\omega_m = \left[ \frac{2k^2 I_t (\varepsilon_r - 1)}{\rho c} \right]^{1/2}. \quad (10)$$

Here $I_t$ is trapping beam intensity, $k = 2\pi/\lambda$ is the optical wavevector, $\rho$ is the mass density and $\varepsilon_r$ is the dielectric constant. For the silicon nitride microdisks with the dielectric constant $\varepsilon_r = 4$, $\rho = 27000 kg/m^3$ we obtain the trapping frequency along $x$-axis $\omega_m = 54 MHz$ with $I_t = 10 W/\mu m^2$. The $x_c$ is the maximum root mean square(rms) amplitude produced a predominantly linear response. For a Gaussian field distribution, the nonlinear coefficients are given by $\xi = -2/\lambda^2[26]$. Considering the beam waist radius $W = r = 1 \mu m$, for small displacements, $|x_c| \ll |\xi|^{-1/2} = 0.7 \mu m$, the nonlinearity is negligible. In our considerations, $x_c$ is taken to be about one orders of magnitude smaller, we choose $x_c = 50 \mu m$.

Consider the mass of the microdisk $m = 4.2 \times 10^{-16} kg$, one can obtain the trapping stiffness $k_{trap} = m \omega_m^2 = 1.22 N/m$.

A laser-trapped microdisk in ultrahigh vacuum, by contrast, has no physical contact to the environment. Thus, the mechanical damping of a levitated microdisk $\gamma_m$ is limited only by collisions with residual air molecules. From kinetic theory we find that[21]

$$\gamma_m = \frac{32 P}{\pi \rho \beta}, \quad (11)$$

where $P$ is the pressure, $v = \sqrt{3k_B T/m_{gas}}$ and $m_{gas}$ are the root-mean-square velocity and mass of gas molecules, respectively. In order to achieve ultrahigh damping rate, the measurement must be completed in ultrahigh vacuum. The measurement set-up in Ref.[5] was prepared by lowering the base pressure to $3 \times 10^{-11} mbar$ to minimize adsorption of unwanted molecules. Let us choose the almost same parameter in simulation, supposing $P = 10^{-11} mbar$, then we obtain $\gamma_m = 1.5 \times 10^{-7} Hz$ and the thermal decoherence rate $\Gamma_{th} = 0.12 Hz$ with room temperature.

We have assumed that the trapping beam has a Gaussian profile. For cavity optomechanics, it will be necessary to trap the microdisk within a Fabry–Perot cavity, as illustrated in Fig.1. We now consider the effect of photon recoil heating. The number of coherent oscillations before a jump in the phonon number can be written as[20]

$$N = \frac{1}{2\pi} \frac{V c \omega_m^2 \rho c}{k^2 I_t}, \quad (12)$$

Here, $c = ck$, $V$ is the volume of the disk and $V_c = \pi W^2 L/4$ is the cavity mode volume. We assume a high finesse cylinder cavity of length $L = 1 cm$, and finesse $F_c = 3 \times 10^5[14]$, leading to a cavity decay rate $\kappa = c \pi /2F_c L = 1.6 \times 10^9 Hz$. Thus we obtain $N = 2.3 \times 10^5$ for a trapped SiN microdisk with frequency $\omega = 54 MHz$. Then the photon recoil heating can be obtained by $\Gamma_{recoil} = \omega_m / 2\pi N = 37.6 Hz$. Plugging these values into the Eq.(7), then we get the total heating time $\Delta t' = 26.5 ms$.

According to Eq.(11), a low pressure implies a low damping rate and thus, we find that lower CM temperature can be achieved at higher vacuum. Recalling that the typical cooling rate is of $\gamma_{th} \approx 20 Hz$ in the Ref.[17], one can cool its motional degrees of freedom from room
temperature to $T_{CM} = 2.3\mu K$ with $P = 10^{-11}$mbar according to Eq.(3). Note that, in the photon dominated regime and in the absence of feedback cooling, the depth of the trapping potential can be estimated by $\Delta U \approx (1/2)k_{T_{eff}}\delta m^2 = 10^{-15} J$. The phonon energy $k_B T_{eff} \approx 10^{-29} J$, this energy is much smaller than the depth of the trapping potential in our scheme, and therefore the particle is unlikely to escape as it heats up without feedback control. We list all the fundamental parameters in the Table I. Following Eq.(9), one can obtain the sensitivity of the mass probing $\delta m = 1.5 \times 10^{-33} kg$, this value corresponds to $0.9 \times 10^{-6}$Da and $0.84$keV in the natural unit. We expect the ultrahigh resolution mass sensor can be achieved under these conditions.

The major determinants of the mass sensitivity is the gas pressure in the chamber. The lower pressure limit of sputter-ion pumps is in the range of $10^{-11}$mbar. Lower pressures in the range of $10^{-12}$mbar can only be achieved when the sputter-ion pump works in a combination with other pumping methods[27,28]. To investigate the impact of pressure, we depict the sensitivity as a function of the pressure in Fig.2 by the red line. According to the figure, higher mass sensitivity requires a lower pressure and we find that micro dalton resolution can be achieved at ultrahigh vacuum ($10^{-11}$mbar). Compared with the previous sensitive measurement with cooling[10], the distinct difference of our scheme is the ”measuring after cooling” method. We first radiate a cooling field on the optically levitated system. Once the trapping microdisk is cooled to the ground state, then we stop cooling and begin to measure, the masses can be detected with a short integration time before a jump in the phonon number, thus the CM temperature keeps the same in probing. Considering the optical damping $\gamma_{fb} \approx 20Hz$ in cooling, we also depict the sensitivity-pressure relation for the traditional cooling scheme with the sample time of 1s by the blue dash line. In contrast to this, our sensing displays notable advantage in precision under low pressure environment. The sensitivity can be improved about 3 orders of magnitude than ”measuring during cooling” as shown in Fig.2, and 6-7 orders than the traditional electrical method. Moveover, the resonance frequency can be detected at a short time($\sim 26ms$) defined by our detection bandwidth ($\sim 6Hz$). A long averaging time would lower the signal to noise ratio. A short sample time is required to extract the signal from the noise since the amount of energy in the burst is to be compared with the thermal fluctuations in the same time interval[29].

IV. CONCLUSION

In conclusion, we propose an ultrasensitive mass sensor using optically trapped microdisk via the method of the ”measuring after cooling”. We have demonstrated that the measurement can be finished at a short time before the mechanical oscillator are heating out of the ground state without cooling. In the absence of cooling damping, the sensitivity can be improved remarkably. The resolution of $10^{-6}$Da allows us to detect precisely the additional mass in the adsorption of an atom or molecule. Compared with traditional electrical mass detection there are a lot of incomparable advantages for this optical mass sensing. For example, the heat effect and energy loss caused by circuits can be removed during the optical mass measurement, the spectral width in optical sensing is narrower($10^{-7}$Hz) than in electrical technique and the ultrashort measuring time($26ms$) allows ultrahigh time resolution. The all-optical mass sensing provides a new platform in nanoscale measurement, and has a promising to enhance the mass sensitivity and accuracy in the future.

| Parameter                  | Units | Value |
|----------------------------|-------|-------|
| Radius of microdisk, $r$   | $\mu m$ | 1     |
| Thickness of microdisk, $b$| nm    | 50    |
| Beam waist radius, $W$     | $\mu m$ | 1     |
| Trapping beam intensity, $I_t$ | $W/\mu m^2$ | 10   |
| Trapping beam wavelength, $\lambda$ | $\mu m$ | 1     |
| Air pressure, $P$          | mbar  | $10^{-11}$ |
| Chamber temperature, $T$   | K     | 300   |
| Damping of feedback, $\gamma_{fb}$ | Hz | 20    |
| Cavity finesse, $F_c$       | null  | $3 \times 10^5$ |
| Cavity length, $L$         | cm    | 1     |
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