A REVIEW OF THE $1/N$ EXPANSION IN RANDOM TENSOR MODELS

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Matrix models are a highly successful framework for the analytic study of random two-dimensional surfaces with applications to quantum gravity in two dimensions, string theory, conformal field theory, statistical physics in random geometry, etc. Their success relies crucially on the so called $1/N$ expansion introduced by ’t Hooft.

In higher dimensions matrix models generalize to tensor models. In the absence of a viable $1/N$ expansion tensor models have for a long time been less successful in providing an analytically controlled theory of random higher dimensional topological spaces. This situation has drastically changed recently. Models for a generic complex tensor have been shown to admit a $1/N$ expansion dominated by graphs of spherical topology in arbitrary dimensions and to undergo a phase transition to a continuum theory.

Keywords: Random Tensors; $1/N$ expansion; Critical behavior.

1. Introduction

Random matrices were introduced by Wishart\textsuperscript{1} for the statistical analysis of large samples and first used in physics by Wigner\textsuperscript{2} for the study of the spectra of heavy atoms. An invariant matrix ensemble is a probability distribution for a random $N \times N$ matrix $M$ with probability density $\frac{1}{Z} \frac{1}{2} e^{-N \text{Tr} S(M M^\dagger)}$ for some polynomial $S$ (or $\frac{1}{2} e^{-N \text{Tr} S(M)}$ for hermitian matrices $M = M^\dagger$).

The moments and partition function of such a probability distribution can be evaluated in perturbations as sums over ribbon Feynman graphs. The weights of the graphs are fixed by the Feynman rules. In his seminal work\textsuperscript{3} ’t Hooft realized that the size of the matrix $N$ endows an invariant matrix ensemble with a small parameter, $1/N$, and the perturbative expansion can be reorganized as a series in $1/N$ indexed by the genus. At leading order in the large $N$ limit only planar graphs\textsuperscript{4} contribute.\textsuperscript{5}

As the planar graphs form a summable family, invariant matrix models undergo a phase transition to a continuum theory of random infinitely refined surfaces when the coupling constant is tuned to some critical value.\textsuperscript{6,7}

\textsuperscript{1}Where $M^\dagger$ denotes the hermitian conjugate
\textsuperscript{2}Subsequent terms in the $1/N$ series can be accessed through double scaling limit\textsuperscript{8,9}
framework for the analytic study of two-dimensional random geometries coupled
with conformal matter\cite{10,13} and through the KPZ correspondence\cite{19,23} they relate to
conformal field theory on fixed geometries.

The success of matrix models inspired their generalization in higher dimensions
to random tensor models\cite{23,29} in the hope to gain insights into conformal field
theory, statistical models in random geometry and quantum gravity in three and four
dimensions. In spite of these initial high hopes tensor models have, until recently,
failed to provide an analytically controlled theory of random geometries: no progress
could be made because for a long time no generalization of the \(1/N\) expansion to
tensors has been found.

The situation has drastically changed with the discovery of the colored\cite{30,32} ran-
dom tensor models. The perturbative series of the colored models supports a \(1/N\) expansion\cite{33,35} indexed by the \textit{degree}, a positive integer which plays in higher dimen-
sions the role of the genus, but is \textit{not} a topological invariant. Leading order graphs,
called \textit{melonic}\cite{36} triangulate the \(D\)-dimensional sphere in any dimension\cite{33,35} and
are a summable family\cite{36}. Like their two dimensional counterparts, tensor models un-
dergo a phase transition to a theory of continuous infinitely refined random spaces
when tuning to criticality. Colored random tensors\cite{37} give the first analytically
accessible theory of random geometries in higher dimensions\cite{38,46}.

The results obtained for the colored models extend to all invariant models for
a complex tensor\cite{47}. The colors arise naturally as a canonical bookkeeping device
tracking the indices of the tensor and provide the key to the \(1/N\) expansion. We
present in this paper an overview of these results. The ensuing theory of random
tensors generalizing invariant matrix models to higher dimensions is universal\cite{48,51}
Tensor models have been generalized to tensor field theories\cite{52,57}, which have been
shown to be (super) renormalizable and, at least in two instances\cite{53,54}, asymptoti-
cally free.

2. Tensor Models

Invariant tensor models for a random tensor of rank \(D\) are, loosely speaking, prob-
ability measures of the form

\[
d\nu = \frac{1}{Z} e^{-N^{D-1} S(T, \bar{T})} \left( \prod dT_{n_1...n_D} d\bar{T}_{\bar{n}_1...\bar{n}_D} \right), \tag{1}
\]

where the “action” \(S(T, \bar{T})\) is some invariant polynomial.

Let \(T\) be a covariant complex tensor of rank \(D\) transforming under the external
tensor product of \(D\) fundamental representations of the unitary group \(\otimes_{i=1}^{D} U(N_i)\),
for some fixed dimensions \(N_1, \ldots, N_D\). The tensor \(T\) can be seen as a collection of
\(D\)-dimensional vectors \(T_{n_1...n_D}, \ (n_i = 1, \ldots, N_i)\) transforming as

\[
T'_{a_1...a_D} = \sum_{n_1,...,n_D} U_{a_1n_1} \cdots U_{a_Dn_D} T_{n_1...n_D},
\]

\[
T'_{\bar{a}_1...\bar{a}_D} = \sum_{\bar{n}_1,...,\bar{n}_D} \bar{U}_{\bar{a}_D\bar{n}_D} \cdots \bar{U}_{\bar{a}_1\bar{n}_1} \bar{T}_{\bar{n}_1...\bar{n}_D}. \tag{2}
\]
Each unitary group $U(N_i)$ acts independently on its corresponding index. In particular, as the dimensions $N_1, \ldots, N_D$ might very well be different, $T$ has no symmetry properties under permutation of its indices. For simplicity we restrict to $N_i = N$, for all $i$. The complex conjugate tensor $\bar{T}_{n_1, \ldots, n_D}$ is a contravariant tensor of rank $D$. We will denote $\bar{n}_i$ the indices of $\bar{T}$ and $n$ the $D$-uple of integers $(n_1, \ldots, n_D)$. We call the position of an index its color: $n_1$ has color 1, $n_2$ has color 2 and so on.

### 2.1. Invariants

By the fundamental theorem of classical invariants of the unitary group any invariant function of the tensor $T$ can be expressed as a linear combination of *trance invariants*\(^{155}\) built by contracting pairs of covariant and contravariant indices in a product of tensor entries

$$\text{Tr}(T, \bar{T}) = \sum_{\bar{n}_1, \ldots} \prod_{i=1}^{n_1} \delta_{n_1, \bar{n}_1} T_{n_1, \ldots, \bar{T}_{n_1, \ldots}}$$

where all the indices are saturated. Because the unitary group acts independently on each index, the contractions must preserve the color: the first index $n_1$ of a $T$ must always contract with the first index $\bar{n}_1$ on some $\bar{T}$, the second index $n_2$ of $T$ with the second index $\bar{n}_2$ on some $\bar{T}$ and so on.

The trace invariants can be represented as (bipartite, closed, $D$-colored) graphs. To draw the graph associated to a trace invariant we represent every $T_{n_1, \ldots}$ by a white vertex $v$ and every $\bar{T}_{\bar{n}_1, \ldots}$ by a black vertex $\bar{v}$. The contraction of two indices $\delta_{n_i, \bar{n}_i}$ is represented by a line $l^i = (v, \bar{v})$ connecting the two corresponding vertices. The lines inherit the color $i$ and always connect a black and a white vertex.

Some examples of trace invariants for rank 3 tensors are represented in figure 1. For instance the leftmost graph corresponds to the invariant

$$\sum \delta_{a_1, p_1} \delta_{a_2, q_2} \delta_{a_3, r_3} \delta_{b_1, r_1} \delta_{b_2, p_2} \delta_{b_3, q_3} \delta_{c_1, q_1} \delta_{c_2, r_2} \delta_{c_3, p_3} T_{a_1, a_2, a_3} T_{b_1, b_2, b_3} T_{c_1, c_2, c_3} T_{p_1, p_2, p_3} T_{q_1, q_2, q_3} T_{r_1, r_2, r_3},$$

where $T_{a_1, a_2, a_3}$ is represented by the vertex $a$ in the drawing and so on. The trace invariant associated to a graph $B$ is

$$\text{Tr}_B(T, \bar{T}) = \sum \delta^B_{\{n^r, \bar{n}^\ell\}} \prod_{v, \bar{v}} T_{n^v, \bar{n}^\ell} \bar{T}_{\bar{n}^v, n^\ell}, \quad \text{with} \quad \delta^B_{\{\bar{n}^r, n^\ell\}} = \prod_{i=1}^{D} \prod_{\nu = (v, \bar{v})} \delta_{n_i^\nu, \bar{n}_i^\nu},$$

where $v$ (resp. $\bar{v}$) run over all the white (resp. black) vertices of $B$, $l^i$ runs over the lines of color $i$ of $B$ and all the indices are summed. There exists a unique graph
with two vertices (all its $D$ lines connect the two vertices). We call it the $D$-dipole and denote it $B_1$. The associated invariant is the unique quadratic invariant

$$\text{Tr}_{B_1}(T,\bar{T}) = \sum_{\vec{n}, \bar{\vec{n}}} T_{\vec{n}}\bar{T}_{\bar{\vec{n}}} \prod_{i=1}^{D} \delta_{n_i,\bar{n}_i}. \tag{6}$$

We consider in the sequel the most general “single trace” tensor model, that is the action is a sum over invariants corresponding to connected graphs $B$

$$S(T,\bar{T}) = \text{Tr}_{B_1}(T,\bar{T}) + \sum_B t_B N^{-\frac{2}{(D-2)}} \omega(B) \text{Tr}_{B}(T,\bar{T}), \tag{7}$$

with $t_B$ some coupling constants, and we singled out the quadratic part corresponding to $B_1$. In equation (7) we have added a scaling in $N^{-\frac{2}{(D-2)}} \omega(B)$ for every invariant which we will fix later.

The partition function of an invariant tensor model writes then

$$Z(t_B) = \int \left( \prod_{\vec{n} = \bar{\vec{n}}} dT_{\vec{n}} d\bar{T}_{\bar{\vec{n}}} \right) e^{-N^{D-1} S(T,\bar{T})}, \tag{8}$$

where $\vec{n} = \bar{\vec{n}}$ runs over all the $D$-uples of integers $(n_1, \ldots n_D), \ n_i = 1 \ldots N$. The scaling $N^{D-1}$ is canonical: it is the only scaling which leads to a well defined large $N$ limit as it can be seen already for the Gaussian distribution. The observables are (again) the trace invariants represented by $D$-colored graphs.

The partition function is evaluated by Taylor expanding with respect to $t_B$ and evaluating the Gaussian integral in terms of Wick contractions. This leads to a representation in Feynman graphs. The Feynman graphs are made of effective vertices coming from the invariants $\text{Tr}_{B}(T,\bar{T})$ (which are graphs $B$ with colors 1, . . . , $D$) connected by effective propagators (Wick contractions, pairings of T’s and $\bar{T}$’s).

A Wick contraction of two tensor entries $T_{a_{\vec{n}}} \cdot \bar{T}_{\bar{a}_{\bar{\vec{n}}}}$ and $\bar{T}_{\bar{a}_{\bar{\vec{n}}}}$ with the quadratic part (8) consists in replacing them by $\frac{1}{N^{D-1}} \prod_{i=1}^{D} \delta_{a_i,\bar{a}_i}$. We represent the Wick contractions by dashed lines to which we assign a new color, 0. The Feynman graphs (henceforth denoted $G$) are then $D + 1$ colored graphs, see figure 2. We denote $B_{(\varrho)}$, $\varrho = 1, \ldots |\varrho|$ the effective vertices (subgraphs with colors 1, . . . , $D$) of a Feynman graph $G$. The free energy is a sum over closed, connected $(D + 1)$-colored graphs

$$F(t_B) = -\ln Z(t_B) = \sum_{G} \frac{(-1)^{|\varrho|}}{s(G)} \left( \prod_{\varrho=1}^{|\varrho|} t_{B_{(\varrho)}} \right) A(G), \tag{9}$$

where $s(G)$ is a symmetry factor and $A(G)$ is the amplitude of $G$

$$A(G) = \sum_{\{\vec{n}, \bar{\vec{n}}\}} \left[ \prod_{\varrho} N^{D-1-\frac{2}{(D-2)} \omega(B_{(\varrho)})} \delta_{B_{(\varrho)}, \{\vec{n}, \bar{\vec{n}}\}} \right] \left[ \prod_{\varrho \neq (v, \bar{v})} \frac{1}{N^{D-1-1}} \prod_{i} \delta_{n_i,\bar{n}_i} \right]. \tag{10}$$
2.2. Colored Graphs and topological spaces

The ribbon graphs of matrix models represent triangulated surfaces. Similarly the colored graphs of tensor models represent triangulated spaces. This is encoded in the following theorem.

**Theorem 2.1.** Any closed connected $D+1$ colored graph is a $D$ dimensional normal simplicial pseudo manifold.

Loosely speaking a pseudomanifold is a generalization of a manifolds having a finite number of conical singularities. One can visualize this pseudomanifold by gluing simplices. We restrict our discussion to the case $D = 3$. The $D+1$ colored graphs have lines of four colors 0, 1, 2 and 3.

![Fig. 3. Gluing of tetrahedra associated to a graph. (a) Vertex. (b) Line. (c) Face.](image)

We associate a positive (resp. negative) oriented tetrahedron to every four valent white (resp. black) vertex. The triangles bounding the tetrahedron are dual to the lines emanating from the vertex (see figure 3(a)) and inherit their color. Thus a tetrahedron is bounded by four triangles of colors 0, 1, 2 and 3. The coloring of the triangles induces colorings on all the elements of the tetrahedron: the edge of the tetrahedron common to the triangles 1 and 2 is colored by the couple of colors 12, and so on. Similarly the apex of the tetrahedron common to the triangles 1, 2 and 3 is colored with the triple of colors 123. A line in the graph represents the unique gluing of two tetrahedra which respects all the colorings. Thus (see figure 3(b)) we glue the triangle of color 3 of one tetrahedron on the triangle of color 3 of the other such that the edge 13 (23 resp. 03) is glued on the edge 13 (23 resp. 03), and the apex 123 (023 resp. 013) is glued on the apex 123 (023 resp. 013). The full cellular structure of the resulting gluing of tetrahedra is encoded in the colors. For instance
an edge 13 is represented by a subgraph with colors 13 in the graph $G$ (see figure 3(c)). We call the subgraphs with two colors of $G$ its faces.

Two remarks are in order. First a classical result by Pezzana\cite{59} ensures that the perturbative expansion of tensor models generates all possible manifolds.

**Theorem 2.2 (Pezzana’s Existence Theorem).** *Any piecewise linear $D$ dimensional manifold admits a representation as an edge colored graph.*

In fact for every manifold one can build an infinity of graphs representing it.

Second, one can ask what is the topological interpretation of an initial trace invariants. As an invariant is a graph with 3 colors (see figure 4), it represents a surface. Adding the lines of color 0 comes to taking the topological cone over this surface (and if the surface has non zero genus it leads to a conical singularity). The invariant represents a “chunk” of space, and the $3+1$ colored graph can be seen as the gluing of such chunks together. A chunk can alternatively be seen by associating tetrahedra to the vertices of the invariant (decorated by external halflines of color 0) and observing that the invariant represents the gluing of these tetrahedra along triangles of colors 1, 2 and 3, but not along the triangles of color 0. As in every tetrahedron the triangle of color zero is opposed to an unique vertex, this chunk is a gluing of tetrahedra (along triangles) around a vertex\cite{57}.

### 2.3. The $1/N$ expansion

We start by a technical prelude. The graphs of matrix models are ribbon graphs made of vertices lines and faces. Consider a (closed connected) ribbon graph with an even number of trivalent vertices, $V = 2p$, thus having $L = 3p$ lines. The number of faces of the graph can be expressed as a function of only the number of vertices and the genus $g$ of the graph

$$F = p + 2 - 2g .$$

A similar relation holds in higher dimensions for $D$ colored graphs. To every graph $B$ with $D$ colors we associate a non negative integer $\omega(B)$ (which we call its degree) such that the number of faces (subgraph with two colors) of $B$ writes\cite{34,35}

$$F = \frac{(D-1)(D-2)}{2} p + (D-1) - \frac{2}{(D-2)!} \omega(B) ,$$

**Fig. 4.** A trace invariant and its associated surface.
with \( p \) is the half number of vertices of 8. Naturally a similar relation holds for graphs with \( D + 1 \) colors by shifting \( D \) to \( D + 1 \).

The crucial property of the degree is that, like the genus, it is non-negative \( \omega(B) \geq 0 \). It is an intrinsic integer number one can compute starting from the graph. However, contrary to the genus, the degree is not a topological invariant but it mixes information about the topology and cellular structure. The idea is that when counting faces one can identify ribbon graphs \( J \) (called jackets \(^{33–35,39,61}\)) embedded in the colored graph. One can separately count the number of faces of each jacket in terms of its genus \( g(J) \). Summing over all jackets one gets a counting of the total number of faces of the colored graph in terms of the sum of these genera \( \omega(B) \equiv \sum_J g(J) \) which is the degree of 8. Some examples (for graphs with \( 3 + 1 \) colors) are presented in figure 5.

![Fig. 5. 3+1 colored graphs of degree (a) \( \omega(G) = 0 \). (b) \( \omega(G) = 4 \). (c) \( \omega(G) = 10 \).](image)

We now compute the amplitude of a graph. We fix the scaling of the invariants in the action eq. (7), \( \omega(B) \), to be exactly there degree and evaluate eq. (10). The non-trivial part comes from counting the number of independent sums. Recall that each solid line of colors \( 1, \ldots, D \) represents the identification of one index, while the dashed lines of color 0 represent the identifications of \( D \) indices. It follows that an index, say \( n_1 \), is identified once along a solid line of color 1, then along a dashed line 0, then along a solid line 1, the along a dashed line, and so on until the cycle of colors 0 and 1 closes. We thus get a free sum over an index, hence a factor \( N \) for every cycle (i.e. face) with colors 0i. The total number of faces expresses in terms of the degree of \( G \) and a short computation yields\(^{10}\)

**Theorem 2.3.** The amplitude of a closed connected \( D + 1 \) colored graph, \( 10 \) is

\[
A(G) = N^{D-\omega(G)}. \tag{13}
\]

This is the \( 1/N \) expansion in random tensor models generalizing in arbitrary dimension the familiar \( 1/N \) expansion of matrix models, \( A(G) = N^{2-2g(G)} \).

2.4. **The leading order graphs**

All the results presented so far particularize when \( D = 2 \) to the classical matrix models results. In particular \( 2+1 \) colored graphs are ribbon graphs and the degree is
Fig. 6. Melons with $p = 2$ and $3 + 1$ colors.

the genus. At leading order only the $D + 1$ colored graphs of degree zero contribute. The structure of the $D + 1$ colored graphs of degree zero is very different for $D = 2$ (matrices) and $D \geq 3$ (tensors). The $D + 1$ dipole has 2 vertices and $\frac{D(D-1)}{2}$ faces (one for each couple of colors $ij$) hence degree 0 (figure 5(a)). For $D \geq 3$ all the graphs of degree zero with $2p + 2$ vertices can be obtained by inserting two vertices connected by $D$ lines arbitrarily on any line of a $D + 1$ colored graph of degree zero with $2p$ vertices. This of course does not hold for $D = 2$. We call these graphs melons (see figure 6).

The graph with two vertices and $D + 1$ lines represents the coherent identification of two $D$ simplices along their boundary, hence it represents a sphere in $D$ dimensions. Two vertices connected by $D$-lines represent a ball in $D$ dimensions. The iterative insertion of balls into spheres preserves the topology, hence

**Theorem 2.4.** For any $D$, the graphs of degree 0 have spherical topology.

As the melonic graphs are generated by an iterative insertion procedure, they can be mapped onto abstract (colored, $D + 1$-ary) trees. The trees are a summable family, hence the melonic graphs are a summable family with a finite radius of convergence. The weight of a melonic graph depends on the coupling constants of the model and tuning to criticality tensor models undergo, like matrix models, a phase transition to continuous infinitely refined random spaces.

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