On rescattering effects in the reaction $\pi^-d \to \pi^-d$

V.V. Baru$^{a,b}$, A.E. Kudryavtsev$^b$, V.E. Tarasov$^b$

$^a$Institut für Kernphysik, Forschungszentrum Jülich GmbH, D–52425 Jülich, Germany

$^b$Institute of Theoretical and Experimental Physics, 117259, B. Cheremushkinskaya 25, Moscow, Russia

Abstract

We discuss rescattering corrections to the impulse approximation for the processes $\pi^-d \to \pi^-d$ and $\gamma d \to \pi^0d$. It is shown that the rescattering effects (RE) give non-negligible contribution to the real part of these amplitudes. At the same time the contributions from the imaginary parts of impulse and rescattering corrections drastically cancel each other. This cancellation means that the processes $\pi^-d \to \pi^0nn$ and $\gamma d \to \pi^+nn/\pi^-pp$ are strongly suppressed near threshold as required by the Pauli principle.

1 Introduction

The study of the reactions $\gamma d \to \pi^0d$ and $\pi^-d \to \pi^-d$ near threshold has attracted continuous attention in the past few decades. Moreover the new experimental data appeared due to recent success of the accelerator technologies stimulate increasing theoretical interest in this field. In this paper we would like to concentrate on the rescattering effects (RE) and their role for these reactions. Indeed, these effects are found to be important in many of theoretical investigations of the reaction $\gamma d \to \pi^0d$ (cf., e.g. Refs. [1, 2, 3, 4]). However, recently in Ref. [5] the discussion about the role of these effects was renewed. In particular it was emphasized in [5] that the contribution from the two-step process $\gamma d \to \pi^-pp \to \pi^0d$ (cf. Fig. 1a) is totally compensated by the loop corrections to the impulse approximation (LCIA) (cf. Fig. 1b). This was argued by the Pauli principle for the intermediate NN states. Thus, the rescattering effects in Ref. [5] do not contribute to the process of coherent $\pi^0$ photoproduction on deuteron near threshold. Obviously, this conclusion of Ref. [5] disagrees with the results of other calculations performed, e.g., in Refs. [1, 2, 3, 4].

Let us discuss the arguments of Ref. [5] in more detail:

i) The final $\pi^0d$ state has quantum numbers $J^P = 1^-$ at low energies where pion is in the $S$ wave with respect to the deuteron. However, the only possible state of the system $pp\pi^-$ with $l_1 = l_2 = 0$ is $0^-$ (here $l_1$ is the orbital angular momentum of the $pp$ system and $l_2$ is the orbital momentum of the pion relative to the $pp$ system). Therefore the $S$-wave intermediate state $pp\pi^-$ does not contribute to the process $\gamma d \to \pi^0d$.

ii) In other words, the contribution of the diagram in Fig. 1a has to be compensated by the loop corrections to the impulse approximation (Fig. 1b) because of antisymmetry of the wave function for the pair of the intermediate nucleons.
Note, that the process $\gamma d \rightarrow \pi^0 np \rightarrow \pi^0 d$ is allowed by quantum numbers. However the amplitude $\gamma n \rightarrow \pi^0 n$ which contributes to this reaction is about factor of 20 smaller than the corresponding amplitude for the charged pion production.

In this remark we are going to discuss the role of rescattering effects for the process of pion-deuteron elastic scattering at low energies. The diagrams corresponding to RE and LCIA for the $\pi d$-scaterring are very similar to the ones for the reaction $\gamma d \rightarrow \pi d$ (cf. Fig. 1 and Fig 2b and 2c). Therefore we will investigate the relevance of RE and the problem of the cancellation of RE and LCIA performing the calculation of the $\pi d$-scattering amplitude.

The $\pi d$-scattering length was measured with a high accuracy [6, 7] and its value coincides with the theoretical predictions (cf., e.g., Refs. [8, 9, 10, 11]). In all these theoretical calculations rescattering effects (including the two-step charge exchange process $\pi^- p \rightarrow \pi^0 n \rightarrow \pi^- p$) give significant contribution to the value of the pion-deuteron scattering length.

In what follows we will directly demonstrate that the real part of the rescattering diagram (cf. Fig 2c) gives non-negligible contribution to the pion-deuteron scattering length. It is not compensated by the real part of LCIA (cf. Fig 2b). However, the imaginary parts of RE and LCIA cancel each other. This cancellation means that there is no contribution to observables from the $\pi NN$ states forbidden by the Pauli principle.

## 2 Calculation of the $\pi d$ scattering amplitude

Below we use a simple potential approach for the calculation of the $\pi N$-scattering amplitude. This approach was already applied to the problem of the determination of the $\pi N$-scattering length in Ref. 9. The model utilizes a pion-nucleon potential $V_{\pi N}(p, q)$ which is required for solving the Lippman-Schwinger equation

$$ T = V + VGT. $$

The S-wave $\pi N$-lengths $b_0$ and $b_1$ are related to the scattering length $a_{\pi N}$ by the equation

$$ a_{\pi N} = b_0 + b_1 I \tau, $$

where $I$ and $\tau$ are isospin operators for pion and nucleon, $b_0$ and $b_1$ are isoscalar and isovector scattering lengths. The analyses [9, 10] of the experimental data [6, 7] show that the absolute values of $b_0$ and $b_1$ are small compared to the typical scale of the problem $\sim \mu^{-1}$ (where $\mu$ is the pion mass). Note also that $b_0 \ll b_1$. Thus, the amplitude $T$ in eq. (1) may be perturbatively expanded in terms of the potential $V_{\pi N}(p, q)$.

Following Ref. 9 we choose $V_{\pi N}$ in the S-wave in the separable form:

$$ V_{\pi N}(k, q) = -\frac{(\lambda_0 + \lambda_1 I \tau)}{2m_{\pi N}} g(k) g(q), $$

where $g(k) = (c^2 + k^2)^{-1}$, $m_{\pi N} = m\mu/(m + \mu)$ and $m$ is the nucleon mass. The cut off parameter $c$ characterizes the range of the $\pi N$-forces, and usually it is varied in the range $2.5\mu \leq c \leq 5\mu$ [9, 10]. The parameters $\lambda_0$ and $\lambda_1$ are chosen in such a way to reproduce the scattering lengths $b_0$ and $b_1$. In what follows we will calculate the pion-deuteron scattering amplitude up to the second order in terms of the potential $V_{\pi N}$. With this accuracy $\lambda_0$ and $\lambda_1$ are equal to
\[ \lambda_0 = \frac{c^4}{2\pi^2} \left( b_0 - \frac{c}{2} \left( b_0^2 + 2b_1^2 \right) \right) , \]
\[ \lambda_1 = \frac{c^4}{2\pi^2} b_1 \left( 1 - \frac{c}{2} (2b_0 - b_1) \right) . \]  

(4)

Corrections to these expressions are of the order of \( \sim O(b_0^3, b_1^3) \) which are negligible.

Let us calculate the pion-deuteron scattering length using the potential \( V_{\pi N} \) (cf. eq. (3)).

i) Single scattering amplitude in the Born approximation

The diagram corresponding to this amplitude is shown in Fig. 2a. The expression for the \( \pi d \) amplitude \( f_1^{(V)} \) corresponding to the sum of two diagrams with the scattering of pion on proton and neutron has the form:

\[ f_1^{(V)} = - \frac{\mu}{(2\pi)(1 + \mu/m_d)} \int dP \varphi_d^2(p) \left[ V_{\pi - p} + V_{\pi - n} \right] . \]  

(5)

Here \( \varphi_d(p) \) is the deuteron wave-function in the momentum space with the normalization condition \( \int dP \varphi_d^2(p) = (2\pi)^3 \). Neglecting by the small corrections of order of \( \sim \mu/m \), one may take out the potential \( V \) in Eq. (5) of the integral and then get:

\[ f_1^{(V)} = 2 \left[ b_0 - \frac{c}{2} \left( b_0^2 + 2b_1^2 \right) \right] . \]  

(6)

This contribution is real as it should be in the Born approximation. Note also that the value \( f_1^{(V)} \) depends on the value of the parameter \( c \).

ii) Single scattering in the one-loop approximation

The diagram for the one-loop correction to the Born approximation is shown in Fig. 2b. We have to calculate the sum of two diagrams with the scattering of pion on proton and neutron taking into account the sum over all intermediate states. The expression for the amplitude \( f_{\pi d}^{(1)VGV} \) corresponding to this sum has the form:

\[ f_{\pi d}^{(1)VGV} = \frac{2\mu}{1 + \mu/m_d} \left[ (\lambda_0^2 + \lambda_1^2)I(\Delta m = 0) + \lambda_1^2 I(\Delta m) \right] , \]

\[ I(\Delta m) = \int dP \frac{g^2 \left( \frac{m k - \mu P}{m + \mu} \right) \varphi_d^2(p)}{(2m_{\pi N})^2} \int ds g^2 \left( \frac{m k - \mu s}{m + \mu} \right) \frac{1}{2\mu} \right] . \]  

(7)

Here \( k \) is the 3-momentum of the initial and final pion, \( \Delta m = m_{\pi^-} + m_p - m_{\pi^0} - m_n = 3.3 \text{ MeV} \) is the excess energy for the charge exchange process \( \pi^- p \rightarrow \pi^0 n \) in the intermediate state. For the case of the elastic rescattering \( \Delta m = 0 \).

The integral in eq. (7) is calculated numerically for some values of the cut off parameter \( c \). In the limit of large \( c \), i.e. when \( c \gg \mu \) and for \( \mu/m \ll 1 \) this integral can be calculated analytically.
\[ f_{\pi d}^{(1)VGV} = c(b_0^2 + 2b_1^2) + 2i \left[ k_0(\Delta m = 0)(b_0^2 + b_1^2) + k_0(\Delta m)b_1^2 \right], \]  

(8)

where we introduced the notation \( k_0^2 = k^2 + 2\mu\Delta m - 2\mu\varepsilon_d \). Note that \( k \ll c \) near the threshold.

Thus, in the limit of large \( c \) the resulting contribution from the impulse approximation (cf. Fig. 2a and 2b) to the real part of the \( \pi d \)-scattering amplitude is

\[ \text{Re} f_{\pi d}^{1(V)} = \text{Re} f_{\pi d}^{(1)VGV} = 2b_0. \]  

(9)

This is a naive but expected result for the real part of the amplitude corresponding to the impulse approximation. The values of \( \text{Re} f_{\pi d}^{(1)VGV} \) for the charge exchange process \( \pi^-d \to \pi^0nn \to \pi^-d \) are presented in Table 1 for the different values of parameter \( c \). In contrary to the real part of the loop amplitude the imaginary part of \( f_{\pi d}^{(1)VGV} \) (cf. eq. (8)) does not depend on \( c \) as required by the unitarity.

Now let us discuss the contribution to the pion-deuteron scattering length from the double scattering process.

iii) Double scattering contribution

Double scattering diagram is shown in Fig. 2c. Performing the calculation we have the following integral for the doublescattering amplitude \( f_{\pi d}^{(2)} \) (cf. Ref. [9] for details):

\[ f_{\pi d}^{(2)} = \frac{4c^4}{(2\pi)^5} \left[ (b_0^2 - b_1^2) J(\Delta m = 0) - b_1^2 J(\Delta m) \right], \]

\[ J(\Delta m) = \int \frac{d\mathbf{q}_1 d\mathbf{q}_2 \varphi_d(\mathbf{q}_1) \varphi_d(\mathbf{q}_2) g^2(\mathbf{k} + \mathbf{q}_1 - \mathbf{q}_2)}{\left[ (\mathbf{k} + \mathbf{q}_1 - \mathbf{q}_2)^2 + (\mu/m) (q_1^2 + q_2^2) + 2\mu (\varepsilon_d - \Delta m) - k^2 - i\varepsilon \right]^3}. \]  

(10)

In the limit of large \( c \) and for \( \mu/m \ll 1 \) this integral is reduced to the following expression:

\[ f_{\pi d}^{(2)} = 2(b_0^2 - b_1^2) \int \Psi_d^2(r) \frac{e^{-ikr + ik_0(\Delta m = 0)r}}{r} d\mathbf{r} - 2b_1^2 \int \Psi_d^2(r) \frac{e^{-ikr + ik_0(\Delta m)r}}{r} d\mathbf{r}, \]  

(11)

where \( \Psi_d(r) \) is the deuteron wave function in the coordinate space.

In the limit of small \( k \) and \( k_0 \), i.e. near the threshold for the real part of \( f_{\pi d}^{(2)} \) we get

\[ \text{Re} f_{\pi d}^{(2)} = 2(b_0^2 - 2b_1^2) \left\langle \frac{1}{r} \right\rangle_{\pi d} \]  

(12)

This expression is well known as a static limit for the doublescattering amplitude, see, e.g. [12] and references therein.

The imaginary part of the amplitude \( f_{\pi d}^{(2)} \) in the same limit is

\[ \text{Im} f_{\pi d}^{(2)} = 2k_0(\Delta m = 0) (b_0^2 - b_1^2) - 2k_0(\Delta m) b_1^2. \]  

(13)

Note that this contribution is negative because \( b_1 \gg b_0 \).
iv) Total pion-deuteron amplitude

Let us discuss the value for total pion-deuteron scattering amplitude in the limit of large $c$ ($c \gg \mu$) and for $\mu/m \ll 1$. For the imaginary part of the resulting amplitude in this limit from eqs. (8) and (13) we get

$$\text{Im} f_{\pi d} \approx 4k_0(\Delta m = 0)b_0^2.$$  \hspace{1cm} (14)

Thus, we obtain that the contributions from LCIA and RE to the imaginary part of the pion-deuteron scattering amplitude cancel each other in the leading order (i.e. terms $\sim b_1^1$). The non-vanishing part of $\text{Im} f_{\pi d}$ is proportional to $b_0^2$ what corresponds to the elastic rescattering process $\pi^- d \rightarrow \pi^- pn \rightarrow \pi^- d$. Note, that the imaginary parts of both expressions (8) and (13) behave as two-particle phase space, i.e. proportional to $k_0 \sim Q^{1/2}$, where $Q$ is the kinetic energy of the intermediate $\pi NN$ system. However, three-particle $\pi NN$ phase space should behave as $Q^2$. This paradox can be resolved if we remind the reader that the approximation $\mu/m \ll 1$, which implies that the kinetic energies of the intermediate nucleons in Eqs. (7) and (10) are neglected, was used to obtain Eqs. (8) and (13). This approximation corresponds to the the rescattering of pion on the fixed centers. That is why the imaginary parts in Eqs. (8) and (13) behave as $Q^{1/2}$. Of course, one can avoid this unnecessary simplification and calculate Eqs. (7) and (10) with taking into account the terms of order of $O(\mu/m)$. But it should not change the main result that the contributions from LCIA and RE to the imaginary part of the $\pi d$-scattering amplitude with intermediate charge exchange cancel each other. The result (14) means that the only possible final state which can be formed in the S-wave in the process of deuteron desintegration is $pn\pi^-$ (with $S = 1$ and $I = 0$ for pair of nucleons). The virtual charge exchange does not contribute to the imaginary part of the pion-deuteron amplitude. This conclusion is in agreement with the remark of Ref. \cite{5}.

At the same time we would like to stress that there is no complete cancellation between real parts of the amplitudes $f^{(1)\text{VGV}}_{\pi d}$ and $f^{(2)}_{\pi d}$, i.e. the resulting contribution from LCIA and RE to the real part of the pion-deuteron scattering amplitude is not small. This conclusion, which is also correct for the process $\gamma d \rightarrow \pi^0 d$, is in contrary to the arguments of Ref. \cite{5}. As can be seen from Eq. (8), the expression for $\text{Re} f^{(1)\text{VGV}}_{\pi d}$ depends linearly on the cut-off parameter $c$ for large values of $c$ and $\mu/m \ll 1$, whereas $\text{Re} f^{(2)}_{\pi d}$ in the same limit is totally determined by the deuteron wave function, i.e. independent of $c$ (c.f. Eq. (12)). Therefore the cancellation of the real parts of the amplitudes $f^{(1)\text{VGV}}_{\pi d}$ and $f^{(2)}_{\pi d}$ can not be achieved in this limit (the value $c = 2 < |1/r| >_d \approx 1.2\mu$ is obviously not realistic).

In Ref. \cite{9} we have calculated the sum of the real parts of the diagrams presented in Fig. 2 varying the parameter $c$ in the limits $2.5\mu \leq c \leq 3.5\mu$. The results of the present numerical calculation are presented in Table 1 for the case when $c$ varies in a larger range and the terms $\sim O(\mu/m)$ are taken into account. In the calculation we use the purely hadronic values for $b_0$ and $b_1$ presented in Ref. \cite{12}, i.e. $b_0 = -2.2 \times 10^{-3} m^{-1}_\pi$; $b_1 = 90.5 \times 10^{-3} m^{-1}_\pi$. This Table clearly confirms the conclusion discussed above that the real parts of the diagrams of Fig. 2b and 2c do not cancel each other.
3 Summary

We developed a consequent potential approach to the problem of the calculation of the pion-deuteron scattering length. The $\pi d$ amplitude was calculated including terms of the second order with respect to the pion-nucleon potential $V_{\pi N}$. The proper symmetrization of the wave function for the intermediate nucleons is taken into account automatically in our approach.

We show that there is a significant cancellation of the contributions from the imaginary parts of LCIA (cf. Fig. 2b) and RE (cf. Fig. 2c). This cancellation is expected. It simply reflects the fact that the process $\pi^-d \to \pi^0 nn$ is strongly suppressed near threshold as required by the Pauli principle. However, no such cancellations take place between the real parts of these processes. The integrals for the real parts of the amplitudes (7) and (10) are quite different. In particular, they have different dependence on the cut off parameter $c$ in the formfactor. Therefore we see no reasons for the cancellation of $\text{Re} f_V^{(1)}$ and $\text{Re} f_V^{(2)}$.

The situation for the reaction $\gamma d \to \pi^0 d$ is quite analogous to that discussed for the reaction $\pi d \to \pi d$. There is no any reasons for the cancellation of the real parts of the diagrams shown in Fig. 1a and 1b. This conclusion is in agreement with the results of papers 1, 2, 3, 4 where rescattering effects are found to be important for the reaction $\gamma d \to \pi^0 d$.

We would like to thank A. M. Gasparyan and V. G. Ksenzov for useful discussions. This work was partly supported by RFBR grant N0 02-02-16465, DFG-RFBR grant 436 RUS 113/652/0-1.

References

[1] J.H. Koch and R.M. Woloshin, Phys. Rev. C 16, 1968 (1977).
[2] P. Bosted and J.M. Laget, Nucl. Phys. A 296, 413 (1978).
[3] G. Fälldt, Phys. Scripta 22, 5 (1980).
[4] S. R. Beane et al., Nucl. Phys. A618, 381 (1997)
[5] M. Rekalo and Egle Tomasi-Gustafsson, Phys. Rev. C66, 015203 (2002); e-print nucl-th/0112063 (2001).
[6] D. Chatellard et al., Phys. Rev. Lett., 74, 4157 (1995); Nucl. Phys. A 625, 855 (1997).
[7] H.-Ch. Schröder et al., Phys.Lett. B469, 25 (1995), Eur. Phys. J. C 21, 473 (2001).
[8] V.M. Kolybasov and A.E. Kudryavtsev, Nucl. Phys B 41, 510 (1972).
[9] V.V. Baru and A.E. Kudryavtsev, Phys. Atom. Nucl. 60, 1475 (1997), $\pi N$-Newsletter, 12, 64 (1997)
[10] T.E.O. Ericson, B. Loiseau and A.W. Thomas, e-print hep-ph/0009312 (2000), submitted to Phys. Rev. C.
[11] A. Deloff, Phys. Rev. C 64, 065205 (2001); nucl-th/0104067
[12] T. Ericson and W. Weise, Pions and Nuclei. Clarendon Press. Oxford, 1988, p. 126.
| $c$ | $Re f_{\pi d}^{(1)} [\text{fm}]$ | $Re f_{\pi d}^{(2)} [\text{fm}]$ |
|-----|---------------------------------|---------------------------------|
| $2\mu$ | 0.0280                          | -0.0084                         |
| $3\mu$ | 0.0443                          | -0.0098                         |
| $4\mu$ | 0.0608                          | -0.0104                         |
| $5\mu$ | 0.0774                          | -0.0107                         |

Table 1: The real parts of the contributions from the diagrams shown in Fig. 2b and 2c for the charge exchange process.

Figure 1: Diagrams with intermediate negative pion rescattering contributing to the process $\gamma d \rightarrow \pi^0 d$.

Figure 2: Feynman diagrams contributing to the $\pi d$-scattering amplitude: $a$ – diagram of the first order on the $\pi N$ potential; $b, c$ – diagrams of the second order on the $\pi N$ potential.