The effectivity of APOS model based worksheets on the improper integral

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Abstract. The APOS model is a mathematical learning model based on APOS theory, with syntax consisting of phases: Orientation, Practicum, Small Group Discussion, Classroom Discussion, Exercise and Evaluation. The APOS model has been implemented in Integral Calculus courses by Class A Semester 3 Students in Mathematics Education Study Program FKIP UNIB 2017/2018 TA. Specifically for the improper integral subject, there are five students who choose to discuss improper integral independently in group. The purpose of this study is to see the effectivity of APOS Model Based Worksheets on the Improper Integral subject matter. The effectivity was analyzed from the mastery of the improper integral by giving a test, if the five students were still master the material, after the launch time of up to two months. The instrument used were a test, and an open questionnaire. From the results of this study, we conclude that mastery of the material was in category “very effective”. Based on the polling from the questionnaire, it can be concluded that the worksheet based on the APOS Model was effective to help the development of independent, diligent, thorough, resilient, responsible, jointly active, helpful, critical and creative characters of students. So, the Apos Based Worksheets was effective to use in calculus class on the Improper Integral material.

1. Introduction
One of a compulsory course in Mathematics Education Study Program S1 FKIP UNIB is Integral Calculus which has 4 (3-1) credits. The material of this course are: definite integral, indefinite integral, basic theorem of calculus, application of integral to calculate area, calculating volume of rotary object, transcendental function, and techniques of integration. Improper integrals subjects is an advanced material of calculus integral learning.

In Indonesian Education system, teacher students surely need to practice learning that support the curriculum 2013 (K13) context in their lecture. In curriculum 2013, learning process must be focus on how students can develop their knowledge, skills, and good attitudes. In line with this orientation, learning in 2013 curriculum should be done through active and creative learning so that students can develop critical thinking skills and communication skills and develop creativity as well. There are at least five points that teachers must develop in teaching in order to get the realization of this learning. Those five points are: to observe with the approach of science, developing the ability to ask or intellectual curiosity, the ability to think, experiment, and communication [1].

To support the practice of applying K13 for teacher students, the Integral Calculus courses are implemented by applying the APOS Model which is a refinement of the Learning Calculus Model Based on APOS Theory (MPK-APOS) [2] [3]. APOS is a learning theory devoted to mathematics learning at
the college level, which integrates computer use, learning in small groups, and takes account of mental constructions by students in understanding a mathematical concept. These mental constructions are: action, process, object, and schema which is abbreviated as APOS [4] [5] [6]. In general the construction of APOS Model is in Figure 1.

**Figure 1. Mathematics Learning Model Based on APOS Theory (APOS Model) [3]**

Figure 1 shows the syntax of APOS model consists of phases: Orientation, Practicum, Small Group Discussion, Classroom Discussion, Exercise and Evaluation. The time division for each phase depends on the weight of the course sks, for Integral Calculus with 4 (3-1) credits, the 15 'Orientation phase, Practicum 50', Small Group Discussion 50', Break 15', 50th Class Discussion ' Exercise / Evaluation 20 '. The Figure also shows the the Student-centered learning in APOS Model, supported by worksheet and computer applications can have an effect on students' mastery and long term memory of learning materials. Some characters such as active in learning, determined, self-confidence and caring can also arise within the student as the effect of accompaniment from learning by APOS Model.
The implementation of APOS model was supported by a Worksheet that contains activities for each phase, and the practicum phase was supported using maple computer applications. Yu Chii-Huei examines that Maple also plays a vital assistive role in problem-solving. She wants to extend the research topic by calculating the connotations of calculus and engineering mathematics [7]. As well as Yu, after implementing the APOS Model in integral indeterminate, integral of course, the fundamental theorem of calculus, the use of integral to calculate the area, calculate the volume of the rotary object, the transcendent function, and the integrating technique, we also want to extend the research in improper integral.

In line with the refinement of the APOS Model from MPK-APOS, then improvements were also made to the APOS Model Worksheet. The Worksheet originally designed consisted of Practicum Worksheets, Manual Worksheets, Class Discussion Sheets, Exercise or Evaluation sheets improved into Worksheets consisting of phases: Orientation, Practicum, Small Group Discussion, Class Discussion, Exercise or Evaluation. Worksheets for Integral Unreasonable in the practical phase include the Maple version 11 command. The material of improper integrals includes difficult material and students must thoroughly master the material of Differential and integral calculus. Here’s some explanations about Improper Integral [8][9][10][11].

For a function \(f\) the Riemann integral to the interval \([a, b]\), the definite integral definitions is

\[
\int_a^b f(x) \, dx = \lim_{|P| \to 0} \sum_{i=1}^n f(c_i) \Delta x_i
\]

means only when a and b are finite. This concept will be expanded to the case of the origin region of function \(f\) in the form:

Finite interval \((a, b] , [a, b), \) and \([a, c)\) or a combination of cases;

Infinite interval \([a, \infty), (-\infty, b], \) and \((-\infty, \infty)\) which can also load the hose case up.

\[1.1. \text{Improper integrals in the finite interval}\]

For every \( \varepsilon > 0 \) denegn \(0 < \varepsilon < b - a\), the function \(f\) integrated in \([a + \varepsilon, b]\), \(\lim_{x \to a^+} f(x) = \pm \infty\), and provided \( \lim_{x \to a^+} \int_a^{a+\varepsilon} f(x) \, dx = \lim_{c \to 0^+} \int_c^b f(x) \, dx = L \).

In this situation, the improper integrals in interval \([a, b]\) defined as

\[
\int_a^b f(x) \, dx = \lim_{x \to a^+} \int_a^{a+\varepsilon} f(x) \, dx = \lim_{c \to 0^+} \int_c^b f(x) \, dx = L
\]

It says the improper integral \(\int_a^b f(x) \, dx\) converges to \(L\). Then if

\[
\int_a^b f(x) \, dx = \lim_{x \to 0^+} \int_a^{a+\varepsilon} f(x) \, dx = \lim_{c \to 0^+} \int_c^b f(x) \, dx = \pm \infty \) doesn’t exist, the improper integral \(\int_a^b f(x) \, dx\) said diverges.

For every \( \varepsilon > 0 \) denegn \(0 < \varepsilon < b - a\), the function \(f\) integrated in \([a, b - \varepsilon]\), \(\lim_{x \to b^-} f(x) = \pm \infty\), and if \( \lim_{x \to b^-} \int_a^{b-\varepsilon} f(x) \, dx = \lim_{c \to 0^+} \int_c^b f(x) \, dx = L \) exist.

So the improper the improper integrals in interval \([a,b]\) defined as \(\int_a^b f(x) \, dx = \lim_{x \to b^-} \int_a^{b-\varepsilon} f(x) \, dx = \lim_{c \to 0^+} \int_c^b f(x) \, dx = L\).

It says the improper integral \(\int_a^b f(x) \, dx\) converges to \(L\). Then if

\[
\lim_{x \to b^-} \int_a^{b-\varepsilon} f(x) \, dx = \lim_{c \to 0^+} \int_c^b f(x) \, dx = \pm \infty \) doesn’t exist, the improper integral \(\int_a^b f(x) \, dx\) said diverges.

\[1.2. \text{Improper integral in the infinite interval}\]
For the improper integral in the infinite interval, we have 3 definitions

Let the function \( f \) integrated for every \([a, b]\) dan let \( \lim_{b \to \infty} \int_{a}^{b} f(x) \, dx = L \) the improper the improper integrals in interval \([a, \infty)\) defined as \( \int_{a}^{\infty} f(x) \, dx = \lim_{b \to \infty} \int_{a}^{b} f(x) \, dx = L \)

It says the improper integral \( \int_{a}^{\infty} f(x) \, dx \) convergent to \( L \). Then if

\[ \lim_{b \to \infty} \int_{a}^{b} f(x) \, dx = \pm \infty \text{ or does not exist, the improper integral } \int_{a}^{\infty} f(x) \, dx \text{ said to be divergent.} \]

Let the function \( f \) integrated for every \([a, b]\) dan let \( \lim_{a \to -\infty} \int_{a}^{b} f(x) \, dx = L \) the improper the improper integrals in interval \([ -\infty, b)\) defined as \( \int_{a}^{b} f(x) \, dx = \lim_{a \to -\infty} \int_{a}^{b} f(x) \, dx = L \)

It says the improper integral \( \int_{-\infty}^{b} f(x) \, dx \) convergence to \( L \). Then if \( \lim_{a \to -\infty} \int_{a}^{b} f(x) \, dx = \pm \infty \) or

do not exist, improper integral \( \int_{-\infty}^{b} f(x) \, dx \) said to be divergent.

Let the function \( f \) integrated for every finite intervals, with the improper integral \( \int_{-\infty}^{\infty} f(x) \, dx \) to be convergence to \( L \) and \( \int_{-\infty}^{\infty} f(x) \, dx \) convergence to \( M \). In this matter, the improper integral of function \( f \) in \((-\infty, \infty)\) defined as \( \int_{-\infty}^{\infty} f(x) \, dx = \int_{-\infty}^{c} f(x) \, dx + \int_{c}^{\infty} f(x) \, dx = L + M \)

It says the improper integral \( \int_{-\infty}^{\infty} f(x) \, dx \) convergence to \( L + M \). If one of that integrals divergence, so \( \int_{-\infty}^{\infty} f(x) \, dx \) said to be divergence.

During our Integral Calculus lectures carried out with the APOS Model, the positive effects had shown in Figure 1 are already appeared. Students are more mastering the material, more active in learning and cooperate in groups [12]. Therefore we would like to see whether after studying integral calculus based on APOS Model can also have a positive effect on the mastery of improper integral matter.

2. Research Procedure
We chose five students of 3rd semester FKIP Mathematics Education Study Program University of Bengkulu whose always active in every phase of APOS Learning Model as the subject of research as this study subjects. This is in purpose to find the effect of learning calculus-based APOS that has been implemented. The selection was done with the assumption that the positive effect of APOS Model will appear if each learning phase is implemented properly.

We asked the five students to work in a group to complete the Model APOS worksheet for improper integral material independently and collect it one week later. The APOS model worksheet includes the orientation phase, the practicum phase, the group discussion phase and the exercise phase. Learning on this improper integral material was done without the guidance of the lecturers and without the phase of class discussion due to time constraints.

Two months later, after the semester holiday, the five students were given a test and an open questionnaire to look at the instructional effect and the addition effect of the APOS Model and to know their own independent learning process. The tests and questionnaires conducted without prior notice, so that the memory and understanding of students about improper integrals can be detected accurately.

Analysis of the effectiveness of APOS Model Worksheets is supported by the results of data analysis from: student learning outcomes, and open questionnaires. To analyze student learning outcomes data descriptive analysis was used. Percentage of completeness of student learning outcomes for the cognitive domain using the following formula:

\[
\text{Completeness Score} = \frac{\text{Correct score obtained by students}}{\text{Maximum score}} \times 100 \% \quad (1)
\]

The completeness category of student learning outcomes is used classification according to Table 1.
Table 1. The Category of The Completeness of Learning Outcomes

| No | Ketuntasan (%) | Mutu | Kategori | Keterangan |
|----|----------------|------|----------|------------|
| 1  | [85, 100]      | A    | Very Good| Very Effective |
| 2  | [80, 85)       | A+   | Good     | Effective   |
| 3  | [76, 80)       | B    | Good Enough| Effective Enough |
| 4  | [70,75)        | B+   | Less Good| Less Effective |
| 5  | [65,70)        | C    | Less Good| Less Effective |
| 6  | [60,65)        | C+   | Less Good| Less Effective |
| 7  | [55, 60)       | D    | Not Effective |
| 8  | [45,55)        | D+   | Not Effective |
| 9  | [0,45)         | E    | Not Effective |

The level of achievement of the effectiveness of Model-APOS Worksheet based categories is described in Table 2.

Table 2. Criteria for the Effectiveness of Effectiveness on LK Effectiveness Based on the APOS Model

| No | Achievement (%) | Category |
|----|-----------------|----------|
| 1  | 0 – 20          | Not Effective |
| 2  | 21 – 40         | Less Effective |
| 3  | 41 – 60         | Effective |
| 4  | 61 – 80         | Enough |
| 5  | 81 - 100        | Very Effective |

Source: [2]

For data derived from open questionnaires, what was done was to collect all opinions of students totaling 5 people. Then general conclusions are drawn about their opinions on the APOS Model-Based Worksheet, and how they are in completing the Worksheet for these unnatural integral topics. And the answers about the characters that are built as a result of integral calculus learning by applying APOS Model worksheets.

3. Results and Discussions

The introduction of an unnatural integral is given in the practical phase through the Maple syntax. Examples begin with the introduction of infinite limits and limits in infinity. In the Class Discussion phase, a selected group was asked to explain how to solve one of the questions that was solved manually in the small group discussion phase. Furthermore, in the training phase, questions were solved manually. In the Evaluation phase, the lecturer can provide queries whenever possible. The evaluation can also be done by the lecturer by examining the Worksheet or observing student activities during the learning process. It could also be by giving a questionnaire to students. Although all phases are listed in the worksheet, for this integral improper material, the class discussion phase is not implemented. The evaluation phase is done after the student has been off for several months, so that the positive effect of this worksheet is visible. Table 3 below is an example of Maple's command in the Practicum phase.

Table 3. Improper Integral

| NO | MAPLE code | MAPLE answer |
|----|------------|--------------|
| Unlimited Limit and Limit on Unlimited |
| 1  | Limit((x^2+3*x-10)/(x^2-4*x+4),x=2)=limit((x^2+3*x-10)/(x^2-4*x+4),x=2); | limit((x^2+3*x-10)/(x^2-4*x+4),x=2); |
| 2  | Limit(exp(x), x=infinity);limit(exp(x), x=infinity); | limit(exp(x), x=infinity); |


3  ➢ Limit(exp(x), x= -infinity); limit(exp(x), x= -infinity);

Improper Integral
4  ➢ \( f:=x->1/((x-1)^{1/2}); \)
   ➢ \( \text{Int}(f(x), x=1..2) = \text{int}(f(x), x=1..2); \)

5  ➢ \( g:=x->1/((5 - x)^{1/2}); \)
   ➢ \( \text{Int}(g(x), x=1..5) = \text{int}(g(x), x=1..5); \)

6  ➢ \( h:=x->1/((x - 2)^{1/2}); \)
   ➢ \( \text{Int}(h(x), x=1..3) = \text{int}(h(x), x=1..3); \)

7  ➢ \( f:=x-> x/(1+x^2)^{1/2}; \)
   ➢ \( \text{Int}(f(x), x=0..\text{infinity}) = \text{int}(f(x), x=0..\text{infinity}); \)

8  ➢ \( f:=x-> x/(1+x^2)^{1/2}; \)
   ➢ \( \text{Int}(f(x), x= -\text{infinity}..0) = \text{int}(f(x), x=\text{infinity}..0); \)

9  ➢ \( f:=x-> x/(1+x^2)^{1/2}; \)
   ➢ \( \text{Int}(f(x), x= -\text{infinity}..\text{infinity}) = \text{int}(f(x), x= -\text{infinity}..\text{infinity}); \)

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It can be seen in Table 3 above that before entering into unnatural integral material, it is introduced first about infinite limits and limits in infinity. Maple's very simple command will certainly be difficult to digest by students who immediately learn the unnatural integral subject matter, without ever learning for the previous subject using APOS-based LK models.

In the small group discussion phase there are two parts discussed by students, the first is that students are directed through questions to understand Maple's answer, the second part of the student is directed to solve unnatural integral problems manually, ie without the help of maple. Examples of the questions are as follows.

Check the limit function value on the MAPLE answer for command no 1 - 3. What is the numerator value and how much the value of the denominator is from each function

Read the Calculus book to get answers to when a limit can be solved by lowering the function in the numerator and decreasing the function in the denominator.

Explain briefly how you complete the no 4 and 5 commands without computer help.

Calculate this improper integral

\[
\int_0^2 \frac{5}{x^2} \, dx
\]

(2)

All of the five subject were learning very well APOS Model Worksheets for improper integral materials. The result of the worksheets that they collect, those are their maple work and their interpretations in the practicum phase, their problem solving results in the discussion phase, and their problem solving in the excercise phase showed 82,125% mastery of improper integral. This result belongs to “very effective” category. Figure 2 below shows the results of the worksheet in the practicum phase and the class discussion phase for command no.4
Figure 2 shows that students are able to analyze maple execution results by learning from books and other sources independently. Maple execution results that automatically show the results they can describe in detailed and precise steps. This shows that the APOS worksheet has a positive effect on student independence.

A test was conducted on 2018 February 6th by taking three questions in the Model APOS worksheet exercise phase as a test question. This test was in purpose to find out whether students who are working on an improper integral Worksheet Based on APOS Model still remember about improper integral. Here are the questions given to the five subjects.

Find the integral of:

1. \( \int_{0}^{4} \frac{2}{x - 4} \, dx \)

2. \( \int_{0}^{3} \frac{x}{x^2 - 4} \, dx \)

3. \( \int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2 + 1}} \, dx \)

Students were given 90 minutes to solve the problems. After the answers of the students examined the results obtained 80.6% which is belongs to “very effective” category. AB and ET got 100% score, FR got 62% score, MK got 54% score, and RD got 87% score.

The goal to be achieved after a student learns Calculus well is to acquire basic knowledge and mathematical mindset, in the form of: (1) the critical, logical, and systematic thinking of scientific thinking; (2) the trained of reason and creativity after studying the various strategies and tactics in solving the calculus problem; (3) trained in designing simple mathematical models; (4) skilled in standard technical math supported by correct concepts, reasoning, formulas, and methods. [8] The test
result shows that there are 3 students who well-master about the improper integrals. Although after 2 months of semester off, and they do the test without learning first, AB, ET and RD still remember the improper integrals matter. So the goal of learning calculus well has been achieved. Nevertheless, two other students still lack the mastery of this materials.

FR and MK forgot about the concept of improper integral, although the integrating technique is correct. They still assume that the given problem is a definite integral, so that they resolve like resolving the definite integral. Students often think that indefinite, definite and improper integrals are mathematical objects of the same nature. Actually, the improper integral has a unique simple general solution, without any exception [13]. Based on the questionnaires students filled in, they found it was difficult to learn unnatural integral materials independently in a short time. In addition, since it has been two months not reading the material, some concepts and formulas were forgotten.

Figure 3. An example of MK’s answer of item test no.2

The results of this study indicate that the theory about the cone of learning Dale in Figure 4 below proved true. If students are involved in hands-on workshops they are able to recall 70% [14] [15]. The five students worked on Model APOS worksheet of improper integrals by continuing to discuss and explain each other the material that is understood and not understood. Although still minus presentations and class discussions seen from 2 students who quickly forget about the concept of improper integrals. They generally remember about integration techniques because the whole matter of integral calculus is only up to the integration technique all presented to the class. Thus they explain in the open questionnaire the independent group learning process they have done.

"FR: When working on a sheet about improper integrals, the five of us do it by dividing the tasks of each exercise. After that we discussed the answers together before collecting to the lecturer."

"ET: we explain by the same division, so everyone explains the shared material. So everyone explains with the same part and is heard by others."

Because mathematics is activity (doing mathematics), math activities not only focus on the final solution sought, but in the process which includes, among others, the search for patterns and relationships, conjectural tests, and estimation of results. In such activities, children are required to use and adapt existing knowledge leading to the development of new understandings (Riedesel, Schwartz, dan Clements in [16]).
Positive effectivity on the mastery of student materials can not be separated from the role of APOS Model Worksheet which contains 85% of the learning phases with APOS Model. Therefore, although APOS worksheet is not done in the classroom, the mastery of student material remains facilitated. It can even increase students' creativity in managing learning patterns and discussion patterns according to the circumstances they want. In this study, the five subjects formed a group chat to add their discussion facilities. While at home and having difficulty mastering the material, they can easily discuss it in group chat.

The practicum phase The APOS model includes instructions and maple application commands in resolving improper integrals problems. In this phase the student is asked to execute maple command and analyze it. Studying with computer applications them to spend more time on exploring the problem solving process, interpreting results, and focus on understanding [7] [17].

"MK: The APOS worksheet helps me understand the material, because by using maple, something abstract can be drawn clearly."

The group discussion phase guides students to analyze the process of improper integrals problem solving on maple and then linked with the existing theories in textbooks. Even according to FR in the open questionnaire answer, "if the answers to computer and manual applications are different, it becomes a challenge to investigate which answers are right and which are wrong". In this phase students are also asked to discuss some new problems and solve them without maple help based on their analysis of maple steps. The training phase is aimed at testing students' understanding of the material. FR states that she is reading and trying to do the exercises constantly to keep in mind.

Completing the group discussion phase and the training phase, it can develop High Order Thinking Skill students because of the problem solving emphasis on the process [17] [18] [16] [19] [20]. The five students have already felt this benefit, as stated by maestika in the open questionnaire, "When processing material from books, worksheet, and discussions often cause confusion that makes me think harder, creative, analytical and critical".

![Dale's Cone of Experiences](https://example.com/dale-cone.png)

**Figure 4.** Dale’s Cone of Experiences [16]
One of the effects of companions arising from calculus learning with the APOS Model is the building of positive characters in students (see Figure 1). Questions on the open questionnaire given to the five subjects on this subject received the following responses.

"Explain, what characters you get with learning using an APOS Model based worksheet "

AB: independent, diligent, raising curiosity, being responsible and cooperating with groups or friends.

ET: responsible for solving problems and exercises, teamwork, helping each other in working worksheet

FR: this lesson can make me an unyielding, conscientious, independent, and critical thinking person through group discussions and class discussions.

MK: 1. Diligent to read books. I became more motivated and wanted to know the details of the subject matter after seeing the results on maple. 2) meticulous. Because maple is very sensitive. 3) Be patient when I'm wrong in maple input. 4) creative thinking, analysis and critical. While reading material from books, worksheets, and discussions often leads to confusion that makes me think harder, more creative, analytical and critical.

RD: independence, diligent reading, thinking power increases.

The five students felt a positive effect after obtaining integral calculus learning with the APOS Model as discussed earlier. Here's the response of the five students about the benefits.

"Briefly explain the advantages of using the APOS Model in Integral Calculus courses"

AB: I find it easier to understand because of a lot of practice and longer to remember it.

ET: I used to get used to understanding myself rather than waiting for the lecturer to explain.

FR: The APOS model is very profitable because until now Integral matter last semester still stuck in my mind.

MK: advantages: increased interest in reading, training of critical thinking power.

RD: I remember more about the materials I have done.

After the five subjects entered semester 5, subject AB stated that until now, they still felt the benefits of learning with the APOS Model-based worksheet. AB said:

“what feels most is the formation of resilient and critical characters and of course confidence follows when we have reached a tenacious and critical stage in this integral calculus. When given the learning model of APOS last year, Ahbi used to explore questions independently and find out how to work integrally that has questions in various forms. because it's already familiar with the phases of group discussion, class discussion, and the training phase so doing the questions so it's not too heavy and already enjoying”

AB also felt that there was an advantage of understanding compared to friends in other classes who studied integral calculus conventionally. In semester 4, there was a change in class division in their generation. So that AB went to college with friends who were mostly in the third semester of lectures in other classes that studied conventionally. AB felt that students who previously studied with the APOS Model were more confident in working on questions that used integral concepts in semester 4 subjects. If learning conventionally, craft and effort to understand the material independently (learning at home) was lower than the APOS model that could be said obliged to study regularly independently. So that it affects the low level of understanding and confidence in solving problems. Furthermore, AB stated that students who study with APOS longer remember the concept because they are looking for themselves. AB said,

“whose name is human, there must can be forgetting something, but if we construct the concept by ourselves, let alone we write the plot with our own language, when we forget there are at least important points that are still left in memory and when we open the note again, immediately remember and understand again”

Positive effect perceived by students due to APOS Model that requires students to learn actively in groups with the help of Worksheets and computer applications. Tang & Titus said that utilizing activity worksheets in class and supplementing lecture contents with media materials have promoted active learning among students; generated appropriate learning activities such as interactive and cooperative
learning; focused students' attention on the application of principles and practice of problem solving skills; increased interaction between instructors and students; prepared students to apply the same techniques and concepts on homework problems that they were applied to in-class exercises. They found that (83%) indicated that the use of worksheets in fact facilitated cooperative learning. 93% of surveyed students reported that practicing activity worksheet problems in class enhanced their learning of course content. Between the interactive lecture and traditional expository lecture formats, students overwhelmingly preferred the former to the latter. Students seem more engaged, asked more questions, and were more focused than during traditional lectures [21].

The effect of learning This APOS model is also in line with Vigotsky’s social constructivism. As Vygotsky famously stated, “Every function in the child’s cultural development appears twice: first, on the social level, and later, on the individual level; first between people …, then inside the child[22] According to Vygotsky, students develop higher mental functions through mediated, social and collaborative activity. Thinking and reasoning emerge through practical activity in the social environment and in relation to the cultural, historical, and material reality of the activity. [23]. Vygotsky defines ZPD (Zone of Proximal Development) as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers”. So we need learning materials that allow learners and teachers to shift within the zone of proximal development gaining relational understanding of the content studied in that particular topic [24] in this research, we had used a worksheet and computer application and its works in our students learning.

4. Conclusion

From the results of this study, we conclude that mastery of the material was in category “very effective”. The responses from the questionnaire, showed that the worksheet based on the APOS Model was effective to help the development of independent, diligent, thorough, resilient, responsible, jointly active, helpful, critical and creative characters of students. So, the Apos Based Worksheets was effective to use in calculus class on the Improper Integral material.

The effectivity could be seen from the good mastery of a improper integral, that is the results of a test shows that 3 out of 5 students are not easy to forget the material. Even 2 college students show 100% mastery of integral matter. The worksheet effectivity also seen from the formation of independent , diligent, thorough, resilient, responsible, active cooperate, help each other, critical and creative characters of calculus students. However in this study we didn’t conduct the Classroom discussion Phase of Model APOS due to the time availability. So for further study, we need to see the effectivity of classroom discussion phase of APOS Model in improper integrals or any other mathematics topics.

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