\textbf{\gamma-Reward: A Novel Multi-Agent Reinforcement Learning Method for Traffic Signal Control}

Junjia Liu\textsuperscript{1}, Huimin Zhang\textsuperscript{1}, Zhuang Fu\textsuperscript{†,*}, Yao Wang\textsuperscript{†}
\textsuperscript{1}State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University
\textsuperscript{†}National Engineering Laboratory for Automotive Electronic Control Technology, Shanghai Jiao Tong University
Shanghai, 200240, China
junjialiu@sjtu.edu.cn, zhanghm1819@sjtu.edu.cn, zhf@sjtu.edu.cn, sjtuyao@sjtu.edu.cn

\textbf{Abstract}—The intelligent control of traffic signal is critical to the optimization of transportation systems. To solve the problem in large-scale road networks, recent research has focused on interactions among intersections, which have shown promising results. However, existing studies pay more attention to the sensation sharing among agents and do not care about the results after taking each action. In this paper, we propose a novel multi-agent interaction mechanism, defined as \textit{\gamma-Reward} that includes both original \textit{\gamma-Reward} and \textit{\gamma-Attention-Reward}, which use the space-time information in the replay buffer to amend the reward of each action, for traffic signal control based on deep reinforcement learning method. We give a detailed theoretical foundation and prove the proposed method can converge to Nash Equilibrium. By extending the idea of Markov Chain to the road network, this interaction mechanism replaces the graph attention method and realizes the decoupling of the road network, which is more in line with practical applications. Simulation and experiment results demonstrate that the proposed model can get better performance than previous studies, by amending the reward. To our best knowledge, our work appears to be the first to treat the road network itself as a Markov Chain.

\textbf{Index Terms}—Multi-agent, interaction mechanism, \textit{\gamma-Reward}, deep reinforcement learning, traffic signal control

I. INTRODUCTION

Traffic congestion has been an increasingly critical matter. It not only leads to increased traffic time and reduces the efficiency of travel, but also exacerbates noise and environmental pollution due to frequent acceleration and deceleration. According to relevant research, almost all collisions and delays in urban traffic are concentrated at intersections \textsuperscript{1}. Unreasonable signal control significantly leads to waste of traffic resource and traffic delays. Therefore, the key to solving urban congestion is to dredge the intersection.

Deep Reinforcement Learning (DRL) methods \textsuperscript{2} \textsuperscript{3} \textsuperscript{4} have been well applied in the traffic signal regulation of single-intersection and shown a better performance than traditional methods, such as Max-pressure \textsuperscript{5}. Recent works began to try to apply RL algorithms, especially multi-agent Reinforcement Learning (MARL), to multi-intersection and even large-scale road networks. Different from the single-intersection problem, the intelligent regulation of large-scale road networks needs to achieve synergistic control between various intersections. In other words, multiple agents need to interact with each other. They need to ensure their own intersection unobstructed, and at the same time, pay attention to the traffic flow status of surrounding or even remote intersections, so that they can ultimately improve the efficiency of overall road network. Latest research introduced the graph attention network \textsuperscript{6} to share sensation with each other, and get a inspired result.

A. Related work and Motivation

The existing traffic signal control (TSC) methods can be divided into two categories: rule-based methods and learning-based methods. The former transforms the problem into a rule-based optimization problem, the later one seeks control strategy from the traffic flow data.

For the rule-based method, such as Webster \textsuperscript{7}, GreenWave \textsuperscript{8} and Max-pressure, a traffic signal plan usually consists of a pre-timed cycle length, fixed cycle-based phase sequence, and phase split.

Webster is used for an isolated intersection which is a widely-used method in traffic signal control. It calculates the cycle length and phase split for a single intersection that minimizes the travel time of all vehicles passing the intersection. GreenWave is a classical method in transportation field to implement coordination, which aims to optimize the offsets to reduce the number of stops for vehicles traveling along one certain direction. Max-pressure aims to reduce the risk of over-saturation by balancing queue length between neighboring intersections by minimizing the pressure of the phases for an intersection.

Recently, reinforcement learning technique, as a popular learning-based method, has been proposed to control traffic signals for their capability of online optimization without prior knowledge about the given environment.

At present, DRL has been successfully applied to the single-intersection regulation of traffic signal \textsuperscript{3}. The results of various state-of-the-art DRL algorithms are compared in Ref. \textsuperscript{9}, showing that Deep Q-learning algorithm is more suitable for the solution of traffic signal control tasks. However, the problem with multiple intersections is still a current frontier. Existing MARL algorithms focus on collaboration between

The first two authors Junjia Liu and Huimin Zhang contributed equally to this paper. * Correspondence: zhf@sjtu.edu.cn (Z. F.), Tel.: +86-138 1649 6926 (Z. F.)
agents and can be divided into centralized and distributed methods according to different way of computing. Independent Q-learning (IQL) directly perform a Q-learning algorithm for each agent. However, the environment is shared in MARL and it will change with policy and state of each agent [10]. For one of the agents, the environment is dynamic and non-stationary, so this algorithm can’t converge, but it may also have better results in some applications. Ming Tan [11] compared IQL with Value Decomposition Networks (VDN). VDN algorithm integrates the value function of each agent to obtain a joint action value function. The integration method is to directly add the summation method. Moreover, QMIX [12], as a extend of VDN, uses state information and integrates them in a nonlinear way and gets a stronger approximation ability. VDN Both QMIX and VDN are typical centralized MARL algorithm.

On the basis of these centralized methods, Colight [6] introduced the concept of attention mechanism, and realized the communication and cooperation between agents with more interpretable. However, as mentioned in Ref. [6], the neighborhood scope is a constant, so the traffic flow information among intersections can’t be utilized to determine the range of the neighborhood. Except for sharing sensation like Colight, some other studies also try to share actions for jointly modeling [13]. Although Colight achieved immense success, we have some thoughts on this: Is graph attention network which calculates the whole road network together the best method to implement interaction? Is sensation sharing the only way to build cooperative agents?

B. Main contributions

In this paper, we use the DRL algorithm to build agents that control the intersections, and also regard the road network itself as a Markov Chain. We multiply the state of distant intersections by the discount rate \( \gamma \) (Note that it represents the space-time discount among multi-agents information, not the common meaning used in RL, and it will be distinguished in detail) and add it to the consideration of the current intersection. These information is used as a penalty to correct the calculation of current rewards, so that the agents have the ability of communication and collaboration. Due to the various traffic volume of each roads, the influence of the surrounding intersections may be different. Therefore, attention mechanism is introduced in this paper to correct the influence weight of the surrounding intersections on the current intersection.

To summarize our main contributions as follows:

- We propose a interaction mechanism, called \( \gamma \)-Reward, which collects the space-time information to amend the current reward, and has the ability to connect with neighbor agents or even further.
- We also use attention just in neighborhood for distinguishing various significance, imitate the idea of off-policy learning to update the attention score parameters in \( \gamma \)-Reward formula.
- It is found in the test results of various road networks that the \( \gamma \)-Reward series, including original \( \gamma \)-Reward and \( \gamma \)-Attention-Reward, are superior to either the traditional methods or the current multi-agent control methods.

II. PROBLEM FORMULATION & PROPOSED MARL METHOD

In this section, we introduce the basic knowledge of TSC problem and propose our MARL method.

A. Preliminary & Formulation

- **Lane**: Lane is part of a roadway that is designated to be used by a single line of vehicles [14]. There are two kinds of lanes: entering lane and exiting lane [15]. Each intersection consists of multiple lanes.
- **Phase**: A phase is a combination of movement signals [4]. Figure 1(a) shows eight main directions of vehicles at the intersection. Note that the direction of turning right is usually ignored in these problem, since it can execute every time without caring the traffic signal. The directions in the same phase need not to be conflict which is shown in Figure 1(b). Phase is the unit of traffic signal control, only one phase can turn green at a time.
- **Neighbor intersection**: The intersections which directly connect to the current intersection. In a common road network, each intersection usually have at most four neighbors.
- **Waiting vehicle**: If a vehicle on an entering lane have a speed lower than a threshold, then we define it as a waiting vehicle, which means it is slowing down to wait for the red light.

In order to use RL, we regard the traffic signal control problem as Markov Decision Process (MDP). Each intersection in the system is controlled by an unique RL agent. They

![Diagram](a) Eight main directions in a single intersection  
(b) Eight kinds of basic phases
need to observe part of the environmental state \( O \), and get actions \( A \) according to these states to determine which phase in the intersection needs to turn green. The effect of control is fed back from the environment in the form of reward \( R \). The goal of RL agent is to maximize the reward function by continuously exploring and utilizing on the basis of constant interaction with the environment. In this paper, the problem is to reduce the length of the queue \( q \) or the travel time \( T_w \) in the road network. To make this problem more suitable for RL, we can first abstract it into these parts \( \langle S, O, A, P, R, \pi, \gamma \rangle \):

Agent 1: \( s_0 \rightarrow a_1 \rightarrow s_1 \rightarrow a_2 \rightarrow s_2 \rightarrow \cdots \rightarrow s_n \rightarrow a_n \)

Agent 2: \( s_0 \rightarrow a_1 \rightarrow s_1 \rightarrow a_2 \rightarrow s_2 \rightarrow \cdots \rightarrow s_n \rightarrow a_n \)

Agent m: \( s_0 \rightarrow a_1 \rightarrow s_1 \rightarrow a_2 \rightarrow s_2 \rightarrow \cdots \rightarrow s_n \rightarrow a_n \)

Fig. 2. The traffic signal control problem regarding as MDP

- **Observation** \( o_t^i : o_t^i = (o_t^i, \ldots, o_t^i) \), where \( o_t^i \in O_t^i \). Every agents observe the length of the vehicle queue on entering lanes of their own intersection. Moreover, in order to cater to the design of the proposed \( \gamma \)-Reward algorithm, we also need to observe the number of vehicle on the exiting lanes which connect to neighbor intersections.

- **Action** \( a_t^i : a_t^i = (a_t^i, \ldots, a_t^i) \), where \( a_t^i \in A_t^i \). Action can be easily set as the number of phase which is chosen to be green.

- **Transition probability** \( P: P(o_{t+1}^i | o_t^i, a_t^i) \) describes the probability from state \( o_t^i \) to the next potential state \( o_{t+1}^i \).

- **Reward** \( r_t^i : \) After executing each action \( a_t^i \), we can get a return information to judge whether \( a_t^i \) is good enough for \( o_t^i \). We use the number of waiting vehicle on the entering lanes as a raw reward. For amendingary reward, we use \( R_{E} \) as a representation.

- **Policy** \( \pi: \) Policy is what agents need to learn in RL. It represents the goal of reducing travel time and increasing average speed. For a single agent, \( \pi^i : O_t^i \rightarrow A_t^i \).

- **Discount rate** \( \gamma \): This factor is the common meaning used in RL. To avoid confusion with \( \gamma \)-Reward, we use \( \gamma \) here to replace the original symbol \( \gamma \).

**B. Proposed \( \gamma \)-Reward Function**

Interaction play a critical role in MARL algorithms, either centralized or decentralized. In this paper, we propose an interaction mechanism among distributed agents, each agent is based on Double-Dueling-Deep Q Network (3DQN) \[16\] \[17\] \[18\], which is one of the best Q value based model until now. A detailed description of it can be found in Appendix A. We found that some studies already directly use 3DQN on TSC problem recently \[19\], but it can only be used as an IQL method in multi-intersection problem. For these problems, which we really need is an interaction mechanism. Therefore, we propose \( \gamma \)-Reward for interaction.

The idea was inspired by the basic theory of reinforcement learning. For the traffic signal problem, not only the timing decision should be regarded as MDP, road network itself is more like a Markov Chain. Since the decision of current intersection will have an effect on other intersections in future. So it should consider not only the TD-error (see Appendix A) in time, but also the 'TD-error' in space.

Figure 4 shows a diagram of multi-intersection road network with \( 3 \times 3 \) intersections. Since the decision of intersection \( E \) at time \( t \) will affect the next intersection \( F \) at time \( t+n \), the result of intersection \( E \) at time \( t \) should be affected not only by the current intersection reward \( r_{E,t} \), but also by the reward \( r_{j,t+n} \) of the surrounding intersections at time \( t+n \), where \( j \in B, D, F, H \). The formula of basic \( \gamma \)-Reward is as follows:

\[
R_{E}(t) = r_{E}(t) + \gamma \cdot \left[ \text{Sigmoid} \left( \sum_{j \in S} \left( \frac{R_{j}(t+n)}{r_{j}(t)} - 1 \right) \right) - 0.5 \right]
\]

\[
\text{Sigmoid}(x) = \frac{1}{1 + e^{-x}}
\]

Fig. 3. An interaction mechanism proposed based on 3DQN

Fig. 4. Diagram for \( \gamma \)-Reward
Where $F = \{B, D, F, H\}$ represents the four intersections around intersection $E$. The parameter $n$ is called delay span which represents the time span of most vehicles reach the next intersection after current action $a_t$. It is related to the length of the road and the average velocity on the exiting lane. Meanwhile, we also need to pay attention to the sampling interval of the algorithm at the time of sampling in simulator.

In Equation 1 \( \frac{R_j(t+n)}{r_j(t)} \) shows the change of traffic capacity at intersection $j$ between time $t$ and $t + n$. If \( \frac{R_j(t+n)}{r_j(t)} \) is greater than 1, it indicates that the traffic capacity of the intersection $j$ is deteriorated, that is, the decision of the intersection $E$ at the time of $t$ will cause the intersection $j$ connected to it to be more congested; conversely, \( \frac{R_j(t+n)}{r_j(t)} \) less than 1 indicates that the capacity of intersection $j$ is improved, and the decision of the intersection $E$ at time $t$ is a good decision for intersection $j$. Subtracting this ratio by 1 and summing up, gets the total impact of the intersection of $E$ at the time of $t$ on its surrounding intersections. In order to make this value as a penalty, the sigmoid function need to be used to limit its range to $(0, 1)$, and then subtracting 0.5, which is finally modified to the interval of $(-0.5, 0.5)$. After these transformations, if the penalty is greater than 0, it means that the surrounding traffic capacity will deteriorate in the future. In contrast, it means the traffic capacity is improved. The penalty finally multiplies by a discount factor $\gamma$ as an amendment to reward. Since the training goal of RL is to maximize the reward function, the existence of the penalty item forces the agent to pay attention to the situation around the intersection while improving its own policy.

Although the future reward value is used in Equation 1 it is achievable in the program. Since we use Q-learning as a basic model which is off-policy and will save the trajectory of state-action pair and the obtained reward into a replay buffer, and then sample to train. In this way, the raw reward $r_j(t)$ is saved first, and then corrected after $n$ steps, therefore the calculation process can be realized (see Appendix B for pseudocode).

C. Attention Mechanism for $\gamma$-Reward

The formula of $\gamma$-Reward proposed in the previous section is based on that each intersection in the road network have same situation, or in other words, the level of them is equal. But in reality, the levels of the intersections in the road network are different, some have only one or two lanes, and others may have four to six. Imagine if the intersection $E$ in Figure 4 is a two-way road which has eight lanes, the intersection $F$ is same as $E$, while on the other side, the intersection $G$ is a two-way which only has two lanes. The decision of intersection $E$ in $E \Rightarrow F$ and $E \Rightarrow G$ is inevitably different. The discharge of intersection $E$ can easily lead to excessive congestion at intersection $G$, but it may not be very serious for intersection $F$. In addition to the number of lanes at the intersection, the length between each intersection is different, which means that the maximum congestion length each intersection can accept is also different. In summary, when using a $\gamma$-Reward formula correction at an intersection, there are different influence weights for different intersections.

It is true that the above mentioned problems can be solved by setting different thresholds to get weights, like Ref. [20]. However, the actual road network situation is very complicated, there is no way to include all parameters into consideration. Therefore, we can learn the rules from the traffic data. In this respect, attention mechanism gives us a good solution.

Attention mechanism [21]–[23] is an algorithm first proposed to solve 'seq2seq' translation problem in NLP. Attention can be interpreted broadly as a vector of importance weights: To predict an element, such as a word in a sentence, attention vector can be used to estimate how strongly it is related to other elements, and the sum of its values can be used as an approximation of the target. It breaks the limits of Euclidean distance between data, captures long-range dependencies in the sentences, and provides smoother translations.

In addition to sequence data, Attention can also be used for other types of problem. In the graphics world, the Graph Convolution Network (GCN) [24] tells us that combining local graph structures with node features can achieve good performances in node classification tasks. However, the way GCN combines the characteristics of neighboring nodes is closely related to the structure of the graph, which limits the generalization ability of the trained model on other graph structures. The Graph Attention Network (GAT) [25] proposes a weighted summation of neighboring node features using the attention mechanism. The weights of the neighboring node features completely depend on the node characteristics and is independent of the graph structure.

Recent research has begun to introduce Attention’s ideas into MARL algorithms. Ref. [25] proposed a Multi-Agent Actor-Critic (MAAC) algorithm that combines attention mechanism. MAAC encodes the state of the surrounding agents, and obtains the contribution value of the surrounding agents to the current agent through the attention network, together with $(s, a)$ of the current agent as an input, the Q value is obtained through an MLP. While the Q network is updated in reverse, the attention network is updated and the attention scores of the surrounding agents for the current agent are also corrected. Colight applied attention mechanism to the traffic signal control problem of large-scale road network, it encodes the state and directly obtains the Q value through the Multi-heads Attention network.

1) Attention: Every agents can interact with their environment and get the observation on time. In this paper, the raw observation contains the number of car on the each lane at the current intersection. We first need to embed the observation from environment by applying a layer of Multi-layer Perception (MLP):

$$z_i = W_j o_i + b \quad (2)$$

In order to get the weight of the intersection $i$ to the adjacent intersection $j$, we need to combine their hidden variables $z_i, z_j$ by following dot product:
$e_{ij} = z_j^T W_k^T W_q z_i \quad (3)$

$e_{ij}$ represents the influence of the adjacent intersection $j$ on the current intersection $i$. It should be noted that the influence between them may not be necessarily equal. Then we normalize them by softmax function:

$\alpha_{ij} = \frac{\exp(e_{ij})}{\sum_{k \in N_i} \exp(e_{ik})} \quad (4)$

Impact value $v_{ij}$ can be calculate as $v_{ij} = \alpha_{ij} z_j$, which means the value $i$ needs to consider from $j$. Finally, adding them together and passing the RELU activation function, the final characterization of the intersection $i$ is obtained:

$h_i = \sigma \left( \sum_{j \in N_i} \alpha_{ij} z_j \right) \quad (5)$

2) $\gamma$-Attention-Reward: We use attention mechanism to consummate $\gamma$-Reward function by adding an attention score before the sum operation.

$R_i(t) = r_i(t) \cdot \left\{ 1 + \gamma \cdot \text{Sigmoid} \left( \sum_{j \in \mathcal{F}} \hat{\alpha}_{ji} \left( \frac{r_j(t + n)}{r_j(t)} - 1 \right) - 0.5 \right) \right\} \quad (6)$

Attention score is updated with the policies:

$J(\pi_\theta) = (R_i + \gamma' \max Q(o_i, a_i; \hat{\theta}, \hat{\alpha}_i) - Q(o_i, a_i; \theta, \alpha_i))^2 \quad (7)$

It is worth emphasizing that $\alpha_{ij}$ represents the importance from $j$ to $i$, but for $\gamma$-Reward we seek the influence from $i$ to $j$. So in Equation 6 and 7 we need to use $\alpha_{ji}$ for amending rewards of agent $i$.

Since the attention score $\alpha_{ji}$ is a real-time updated value, this paper uses it as an evaluation metric to dynamically assess the impact of surrounding intersections based on dynamic traffic data. While reward is also an evaluation indicator, which is a timely feedback obtained after performing an action at a certain state, used to evaluate the quality of the state-action pair. If we introduce $\alpha_{ij}$ into the calculation of reward and update it in real time, there is bound to be a problem. This causes the Attention layer to intentionally update the direction which reduces the impact of important neighbor intersections and increases those with less traffic flow in order to increase reward. If we do like this, the introduction of the attention score will be even worse than the original $\gamma$-Reward. To solve this problem, we can follow the Q-learning algorithm and use the off-policy idea to get target attention scores $\hat{\alpha}_{ij}$ in the reward calculation using target Attention layers. Its network parameters are updated with target Q parameters $\hat{\theta}$.

The whole framework of the proposed $\gamma$-Attention-Reward model is shown in Figure 5.

In Colight, there is a section devoted to the selection of the hyper-parameter Neighbor Number. It is found through experiments that the larger the $|N_i|$ is, the better the performance is. But when it is greater than 5, it takes more time to learn. This is because intersections within the scope of Colight’s $|N_i|$ needs to aggregate all the observations into one agent, and
the total number of agents is still equal to the intersection, which will inevitably lead to an increase in the amount of calculation. However, unlike Colignt, \( \gamma \)-reward does not need to consider the size setting of the neighbor number. We can just consider the neighbor number as a constant 5, which contains the current intersection and the four intersections directly connected to it. Note that, we do not need to employ Multi-heads Attention which leads to centralization. For the information of further intersections, recursiveness in the \( \gamma \)-Reward formula can work.

From the original \( \gamma \)-Reward part in Figure[3] we can figure the principle of it. The row in the figure represents the state information collected in time series. The column represents each agent. In order to prevent too much confusion, only the \( \gamma \)-Reward process of the red and blue agents is shown as a demonstration, and the interval of the scale variable \( n \) is also not shown here. Take the red agent at the bottom as an example, the orange-red block on the upper side of the diagonal represents the future observation of the agent. The orange block above the orange-red block affects the red block with \( \gamma \)2. By analogy, the further away from the current intersection, the smaller the effect of correction is. It is worth noting that only two-dimensional Replay Buffer is drawn here, in fact the real intersection has four or more surrounding intersections, so it should be a three-dimensional gradient figure.

III. THEORETICAL RESULTS

In this section, we establish theoretical results for the proposed algorithms. The key challenge of MARL algorithm is that the decision process may be non-stationary [27][28]. Since an agent is not aware actions of the other agents or limited to communicate, the transition probabilities \( P(o_{t+1}^i | o_t^i, a_t^i) \) are not stationary and change as the other agents change. So we need to demonstrate the decision process is stationary by using proposed algorithms. Specifically, we prove that they can converge to a Nash equilibrium. The argument is started with the following definitions.

**Definition 3.1:** A decentralized MARL decision process is stationary (or homogeneous), iff, for each agent \( i \) and all \( p, q \in \mathbb{N} \), \( o^i, a^i \in A^i \) [27]

\[
\sum_{a^{-i}_{t'} \in A^{-i}_{t'}} P_i (o_{t+1}^i | o_t^i, a_t^i) = \sum_{a^{-i}_{t'} \in A^{-i}_{t'}} P_i (o_{t+1}^i | o_t^i, a_t^i)
\]

and \( P \) for global state \( s \in S \) must be stationary either.

\[
\sum_{a^{-i}_{t'} \in A^{-i}_{t'}} P (s_{t+1}^i | s_t^i, a_t^i) = \sum_{a^{-i}_{t'} \in A^{-i}_{t'}} P (s_{t+1}^i | s_t^i, a_t^i)
\]

where \( a^{-i} = a \setminus \{ a^i \} \).

Based on the stationary MDP, we can give a definition of global optimal reward for proving the astringency of proposed methods by extending the definition by Ref. [29].

**Definition 3.2:** For a stationary MDP, the global optimal reward can be decomposed into a sum of local optimal reward for each reward function \( f_i \)

\[
\rho^* = \sum_{i=1}^{m} \rho^*_i \quad (10)
\]

**Proof Sketch** For a given stationary MDP, there must exists a stationary \( P^*_i \), where \( \pi^*_i \) is the optimal policy of agent \( i \)

\[
\rho^*_i = \sum_{o^i \in O^i} P^*_i(o^i)f_i(o^i, a^i | a^i = \pi^*_i(o^i)) \quad (11)
\]

**Definition 3.3:** In stochastic game, a Nash equilibrium point is a tuple of \( m \) strategies \( (\pi^1, \ldots, \pi^m) \) such that for all \( s \in S \) and \( i = 1, \ldots, m \)

\[
\nu_i(s | \pi^1, \ldots, \pi^m) \leq \nu_i(s | \pi^1, \ldots, \pi^i, \pi^{i+1}, \ldots, \pi^m) \quad (12)
\]

for all \( \pi^i \in \Pi^i \), where \( \Pi^i \) is the set of policies of total \( m \) agents.

**A. Stationarity of \( \gamma \)-Reward series**

First we give the proof of stationarity. Unless the MDP is stationary, the model cannot guarantee convergence to the optimal.

**Assumption 3.1:** The original reward function can be represented as

\[
r^i_t = f(o^i_t, a^i_t) \quad (13)
\]

where \( o^i, a^i \) include state-action pair from time step 1 to \( t \). Assume that \( \gamma \)-Reward series are special reward function \( f(o, a) \).

\[
R^i_t = f((o^i_t, o^{-i}_k), (a^i_t, a^{-i}_k)) \quad (14)
\]

\( k \in [1, N] \).

**Assumption 3.2:** As a continuing task without definite ending, the expected reward \( G_t \) of TSC problem is defined as follow with a discount rate \( \gamma \)

\[
G_t \doteq r_{t+1} + \gamma' r_{t+2} + \gamma'^2 r_{t+3} + \cdots = \sum_{k=0}^{N-t} (\gamma')^k r_{t+k+1} \quad (15)
\]

**Assumption 3.3:** The Q function is based on trajectory of expected return.

\[
Q(o, a | \pi) = \sum P(path_{o,a} | \pi) \cdot G(path_{o,a}) \quad (16)
\]

\[
a_t = \arg \max Q_{\text{max}}(o, a | \pi) \quad (17)
\]

**Theorem 3.1:** With \( \gamma \)-Reward as an interaction mechanism, the decision process of distributed DQN algorithm (R\mathcal{P}) is stationary.
Proof Sketch According to Assumption 3.2 and Assumption 3.3, the value function with \( \gamma \)-Reward (\( RQ \)) can be written as follow

\[
(RQ)_{i}^{*}(o, a|\pi) = \sum_{t=0}^{N} (RP)_{i}^{t}(path_{o_{t}, a_{t}}|\pi) \ast (RG)_{i}^{t}(path_{o_{t}, a_{t}}) = Q(\langle o_{t}^{i}, a_{t}^{i} \rangle, \langle a_{t}^{i}, a_{k}^{i} \rangle | \pi)
\]

The calculation of (\( RQ \)) is related to the except reward path. (\( RQ \)) is a discounted sum of amendatory reward \( R_{i}^{t} \), which is bound up with not only \( (o_{t}^{i}, a_{t}^{i}) \), but also \( (o_{t-1}^{i}, a_{t-1}^{i}) \) from the other agents. Since (\( RQ \)) records the future trajectory of amendatory reward from time step \( t \) to the end of episode, it must contain the previous and posterior state-action pair as a vector, like Equation (14) shown in Assumption 3.1. According to the above two properties, we can decompose (\( RP \)) by using Equation (7)

\[
(RP)_{i}^{t}(o_{t+1}^{i}, o_{t}^{i}, a_{t}^{i}) = P_{i}^{t}(o_{t+1}^{i}, o_{t}^{i}, \text{argmax}(RQ)_{i}^{t}(o, a)) = P_{i}^{t}(o_{t+1}^{i}, o_{t}^{i}, a_{k}^{i})
\]

where \( k \in [1, N] \). Obviously, (\( RP \)) satisfy the property in Definition 3.1. Thus, the process with \( \gamma \)-Reward series is a stationary process.

\[\square\]

B. Convergence of \( \gamma \)-Reward series

**Assumption 3.4**: Let the local optimal except reward \( G_{i}^{*} = \rho_{i}^{*} \), then the limitation of original reward is also equal to \( \rho_{i}^{*} \) due to the negative reward we set.

\[
\lim_{t \to N} R_{i}^{*} = \rho_{i}^{*}
\]

**Theorem 3.2**: \( \gamma \)-Reward formula can converge to local optimal reward.

**Proof Sketch** On the basis of Assumption 3.4, both \( R_{i}^{*} \) and \( R_{i}^{t+n} \) approach to \( \rho_{i}^{*} \), and the penalty term will disappear.

\[
\lim_{t \to N} \rho_{i}^{*} = \rho_{i}^{*} \rightarrow \rho_{i}^{*}
\]

Thus, \( \gamma \)-Reward formula converges to local optimal reward. Obviously, \( \gamma \)-Attention-Reward has the same property. \[\square\]

C. Optimality of \( \gamma \)-Reward model

For a stochastic game with multi-agent, the optimal point of the whole system is actually a Nash equilibrium point which is declared in Definition 3.3. \( \nu^*(s|\pi_1^*, \ldots, \pi_m^*) \) can be interpreted as the discounted except reward \( G \). According to previous two theorems, we can draw the following conclusions.

**Theorem 3.3**: With \( \gamma \)-Reward model, the multi-agent system can converge to a Nash equilibrium point.

### IV. Experiment

We use a simulator called Cityflow [31] rather than common SUMO [32], since it is more than twenty times faster than SUMO. Moreover, we use Ray [33] framework for the RL algorithm, the algorithm is based on Double-Dueling-Deep Q Network (3DQN).

#### A. Datasets

In the experiment, we use both synthetic data and real world data. We share the same real world dataset with Colight for convenience. The datasets mainly include two parts, roadnet and flow. Roadnet describes the number of intersections in the road network, the coordinates, and the number of lanes owned by each intersection. Flow is based on vehicles and lists thousands of vehicles, each vehicle has its own property, such as length, width, max of accuracy, max of speed and, the most importantly, trajectory. The experiment used the real world data of Hangzhou, Nanjing in China and also New York in USA. Meanwhile, we used two kinds of synthetic data, arterial and grid type. We counted their characteristics and presented them in Table I. \( Grid_{3 \times 3uni} \) is a one-way traffic and \( Grid_{3 \times 3bi} \) is two-way with same road network.

#### B. Baseline Methods

In Chapter II we have already introduced methods for traffic signal control, including traditional rule-based methods and learning-based methods. The most primitive rule-based methods are still the most common methods nowadays. As a mature rule-based method, Max-Pressure can be used as a representative.

#### Table I

| DataSet      | Intersections | Mean | Std  | Max  | Min  |
|--------------|---------------|------|------|------|------|
| Arterial1x6† | 6             | 300  | -    | -    | -    |
| Grid3x3uni† | 9             | 300  | -    | -    | -    |
| Grid3x3bi†  | 9             | 300  | -    | -    | -    |
| NewYork16x3  | 48            | 240.79 | 10.08 | 274 | 216 |
| Jinan3x4     | 12            | 526.63 | 86.70 | 676 | 256 |
| Hangzhou4x4  | 16            | 250.79 | 38.21 | 335 | 208 |

† Traffic flow from synthetic data are uniform, so there is no need to count another three values.

**Proof Sketch** First we give the definition of optimal \( \nu^* \) with Definition 3.2.

\[
\nu^*(s|\pi_1^*, \ldots, \pi_m^*) = \rho^* = \sum_{i=1}^{m} \rho_i^*
\]

From Equation (21) we can figure that the local optimal reward of \( R^i \) is depend on local optimal of the other agents \( \rho_i^* \). In other words, if there is an agent which does not converge to local optimal reward, the other will also not be optimal. From this we can make a conclusion that \( \gamma \)-Reward forces agents to care about the others and let the whole system finally converge to a Nash equilibrium point. \[\square\]
Learning-based methods have been prosperous under the development of deep learning and data science in recent years. They are characterized by the use of large-scale data to approximate optimal strategies through iterative learning. We have chosen several methods as baseline:

- **IQL**: Since $\gamma$-Reward is based on the decoupling idea of IQL, and introduces an interaction mechanism, so it can demonstrate the impact of interaction mechanism compared to IQL. Here we use original 3DQN, like Ref. [19], for comparison.

- **QMIX**: This is a mature MARL algorithm, which integrates all agents into the same model and concentrates on the learning of joint action reward functions. As a typical one-model method, comparing it with $\gamma$-Reward can effectively observe the advantages and disadvantages of joint learning and independent learning for interaction.

- **Colight**: Unlike $\gamma$-Reward, Colight is more like QMIX, but not learns a joint action. It uses Attention layers to train the surrounding observation code to replace the current intersection’s observation. Due to its fully collection of observation, it can apply Multi-heads Attention. By this way, the interaction between agents is realized. $\gamma$-Attention-Reward has made some improvements on this basis. The method of Replay Buffer Amendment is employed to introduce the effect of the current intersection on the surrounding intersection, replacing the hyperparameter $|N_i|$ in Colight and adding consideration of the impact of time and space on the action.

### C. Evaluation Performance

Figure 6 and Table II shows the performance comparison between $\gamma$-Reward and the more powerful $\gamma$-Attention-Reward and baselines. Each model has trained 100 iterations, and each iteration run 3600 time steps in the simulator. Each action in
them lasts at least 10 seconds for avoiding rapid switching phase impracticably.

We use the average transit time of the vehicle to evaluate the performance of the model, which is the standard evaluation method in the field of traffic signal regulation.

The performance of Learning-based methods is significantly better than rule-based Max-Pressure (Table II), which is widely proved in many researches.

Among the independent learning RL model, the performance of $\gamma$-Reward series far exceeds the IQL model. This demonstrates the importance of interaction between agents for global performance improvement.

It is worth noting that, in all road network, the independent learning RL model shows a stronger astringency than one-model QMIX. That probably because for a single model excessive dimensions can make the policy more difficult to learn. However, Colight does not show divergence, while achieved good results. This may benefits from that it does not make joint decisions through the joint action function, but by sharing network parameters, so that all agents generate independent actions. Since the agents share the model parameters, they will undoubtedly ignore individual differences and sacrifice some performance. Compared to the proposed model, intense oscillations sometimes occur in Colight during training, this is also a result of sharing parameters. Once the model iterating into a wrong direction, it will misleading all agents and lead to a horrible congestion in the whole road network. The performance of proposed methods are even better than Colight, which means sharing sensation is not the only way to realize interaction. Sharing results can also help to focus on the whole road network for a single intersection.

In real world road networks, $\gamma$-Reward and $\gamma$-Attention-Reward do not show gigantic difference. The reason is that all real world road networks we used are two-way road. We will detailed introduce the study about the attention score later and can be figured in Figure 9. Attention does not play an important role. It shown its effect in specifically synthetic road networks. Compared Grid$_{6\times6}$bi and Grid$_{6\times6}$uni in Figure 9, Attention layers distinguish one-way and two-way, and assist agents to achieve a better performance. This will also describe later by revealing the detail of Attention layers.

**D. Study of hyperparameter $\gamma$**

We use Arterial$_{1\times6}$ road network to study the impact of different $\gamma$ value. We have chosen $[0.3, 0.5, 0.7, 0.9]$ four $\gamma$ values and compared the results. As shown in Figure 7, 0.5 may be a balance point of the penalty item. So for the hyperparameter $\gamma$, we all set it to 0.5.

**E. Visualization of Proposed method**

The core idea of the $\gamma$-Reward algorithm is to correct the reward in the Replay buffer. In this section, we use the Arterial$_{1\times6}$ road network as an example to show how the reward values between different intersections affect each other. Compare the Grid$_{3\times3}$uni and Grid$_{3\times3}$bi road networks to demonstrate the role of the attention mechanism in the $\gamma$-Reward algorithm improvement.

1) **Visualization of $\gamma$-Reward Function:** In Figure 8 dashed lines represent the original reward, and solid lines represent the corrected Reward. Since the linear road network is selected, there are no more than two adjacent intersections, so it is easier to observe the influence between the intersections.

Observe the red line in the light yellow area, which represents the reward for the second intersection. It can be found that the solid line in this area is almost above the dashed line, meaning that after amendment, rewards become better. The reason is shown in the light blue area, dark blue and orange lines are getting rise, which means the traffic situation is getting better both in intersection 1 and 3. We believe that in the process of getting better at these two intersections, some of efforts are contributed by intersection 2. The lag between light yellow and blue area is up to the delay span $n$.

From Figure 8 we can see that in the actual training, the $\gamma$-Reward algorithm, like what we expected, introduces future changes in nearby intersections.

2) **Effect of Attention Mechanism:** Figure 9 shows attention scores from Hangzhou road network. We can find that except current intersection 6, others are declined and tending to the
TABLE II
PERFORMANCE COMPARISON

| Model          | Grid3×3bi | Grid3×3uni | NewYork16×3 | Jinan3×4 | Hangzhou4×4 |
|----------------|-----------|------------|-------------|----------|-------------|
| Max-Pressure   | 204.72    | 186.06     | 405.69      | 359.81   | 431.53      |
| IQL            | 191.05    | 157.51     | 248.46      | 371.74   | 406.27      |
| QMIX           | 565.70    | 619.32     | 216.56      | 571.78   | 587.46      |
| Colight        | 104.89    | 100.96     | 169.66      | 301.78   | 311.15      |
| γ-Reward       | 135.38    | 96.44      | 162.18      | 303.97   | 304.90      |
| γ-Attention-Reward | 96.14 | 93.93      | 141.16      | 286.27   | 284.24      |

Fig. 9. Attention score of intersection 6 selected from Hangzhou4×4 road network

Fig. 10. Attention layers learn the different of surrounding from traffic flow; Grid3×3bi and Grid3×3uni road network are used for comparison to show the effect of attention layers.

same value. Therefore, Attention does not play an important role in real world datasets. The reason could be that all of them are two-way road and have same number of lanes. In order to highlight its effect, we need to compare the situation between one-way and two-way traffic in a same road network. Thus, we use Grid3×3uni and Grid3×3bi for visualization. Figure 10 shows the comparison between one-way and two-way 3×3 network. Comparing Fig. 10(a) with Fig. 10(b), the score change of intersection 5 in Fig. 10(a) is the equalization of surrounding intersection 2, 4, 6 and 8. While in Fig. 10(b), the intersection from the direction of exiting is obviously holding a commanding edge. This means that attention mechanism does have a significant effect in understanding the structure of the road network. With the attention mechanism, the reward amendment is more concerned with the results of its actions, which is crucial in the revision of reward.

V. DISCUSSION FOR REAL WORLD APPLICATION

Traffic signal control is a practical problem, and the proposed control plan should aim at solving practical problems. Therefore, it is necessary to take into account the limitations that may exist in practice.

Real-time traffic communication may cause communication delay, information security problem and risk of packet loss [34]. At this time, it is necessary to decouple the road
network calculation. It is worth noting that decentralized is
the inclination in TSC problem [20] [35].

Colight, while using Multi-heads Attention technology and
parameter sharing, summarizes the relationship of the global
road network and reduces the time complexity and space
complexity of training. However, if you want to apply it to the
actual application, you need to obtain the global road network
information in real time to the central server for calculation,
and the resulting joint action is sent to each intersection signal
light. Note that it is not just transmitting actions of each traffic
signal, but also their sensor observations!

Another disadvantage of this is that if there is a new
intersection adding to the road network, or if the control center
want to extend the coverage of the road network, retrain is
unavoidably. Decoupling the road network like the \(\gamma\)-Reward
series algorithm allows each agent to focus on itself and up
to four intersections. From Figure [11] we can compare the
complexity. The computational complexity of a single agent
is much smaller than the global one, and can even run directly
on intelligent traffic which has embedded edge computing
devices. In addition, if the road network structure is simple,
original \(\gamma\)-Reward can be used directly for convenience.

If the road network structure changes, you only need to
train the newly added intersections separately. Thus, the idea
of road network decoupling provides a scalability solution for
traffic signal control problems.

VI. CONCLUSION

In this paper, we propose the \(\gamma\)-Reward method and its
variant \(\gamma\)-Attention-Reward that introduces the attention mech-
anism to solve the problem of intelligent control of traffic
signal. Specially, we give a detailed proof of them and show
that they can converge to Nash equilibrium. We conduct com-
prehensive experiments using both real-world and synthetic
data. They confirm that our proposed methods have a superior
performance over state-of-the-art methods. In asymmetry road
network, \(\gamma\)-Attention-Reward show inspired results than \(\gamma\)-
Reward by adding attention mechanism. Moreover, we in-
vestigate thoroughly the effect of reward amendment and
attention mechanism in achieving interaction. Compared to
the recently proposed Colight, \(\gamma\)-Reward series replaces the graph
convolution with recursion, decoupling the road network, and
is more suitable for practical applications.

ACKNOWLEDGMENT

This project is derived from the 46th project of Deecamp
2019, which won the Best Technology Award, and we thanks
for the effort of our teammates in "Multi-Commander".

Thanks to the practice platform provided by Dr. Kai-Fu
Lee and the support of APEX Lab of Shanghai Jiaotong
University. We also gratefully acknowledge the support from
the National Natural Science Foundation of China (Grant No.
61973210), the projects of SJTU (Grant Nos. YG2019ZDA17;
ZH2018QNB23).

REFERENCES

[1] H. R. TO and M. M. Barker, “White paper european transport policy
for 2010: time to decide,” 2001.
[2] X. Liang, X. Du, G. Wang, and Z. Han, “Deep reinforcement learn-
ing for traffic light control in vehicular networks,” arXiv preprint arXiv:1803.11115
[3] 2018.
[3] A. Wei, G. Zheng, H. Yao, and Z. Li, “Intelligent: A reinforcement
learning approach for intelligent traffic light control,” in Proceedings
of the 24th ACM SIGKDD International Conference on Knowledge
Discovery & Data Mining, pp. 2496–2505, ACM, 2018.
[4] H. Wei, G. Zheng, V. V. Gayah, and Z. Li, “A survey on traffic signal
control methods,” CoRR, vol. abs/1904.08117, 2019.
[5] P. Varaiya, “The max-pressure controller for arbitrary networks of
signalized intersections,” in Advances in Dynamic Network Modeling
in Complex Transportation Systems, pp. 27–66, Springer, 2013.
[6] H. Wei, N. Xu, H. Zhang, G. Zheng, X. Zang, C. Chen, W. Zhang,
Y. Zhu, K. Xu, and Z. Li, “Colight: Learning network-level cooperation
for traffic signal control,” arXiv preprint arXiv:1905.05717, 2019.
[7] P. Koonce and L. Rodegards, “Traffic signal timing manual,” tech. rep.,
United States. Federal Highway Administration, 2008.
[8] J. Török and J. Kertész, “The green wave model of two-dimensional
traffic: Transitions in the flow properties and in the geometry of the traffic jam,”
Physica A: Statistical Mechanics and its Applications, vol. 231, no. 4, pp. 515–533, 1996.
[9] S. S. Mousavi, M. Schukat, and E. Howley, “Traffic light control using
deep policy-gradient and value-function-based reinforcement learning,”
IET Intelligent Transport Systems, vol. 11, no. 7, pp. 417–423, 2017.
[10] J. Foerster, I. A. Assael, N. de Freitas, and S. Whiteson, “Learning
to communicate with deep multi-agent reinforcement learning,” in Advances
in Neural Information Processing Systems, pp. 2137–2145, 2018.
[11] M. Tan, “Multi-agent reinforcement learning: Independent vs. coopera-
tive agents,” in Proceedings of the tenth international conference on
machine learning, pp. 330–337, 1993.
[12] T. Rashid, M. Samvelyan, C. S. De Witt, G. Farquhar, J. Foerster, and
S. Whiteson, “Qmix: monotonic value function factorisation for deep
multi-agent reinforcement learning,” arXiv preprint arXiv:1803.11485,
2018.
[13] Y. Yang, R. Luo, M. Li, M. Zhou, W. Zhang, and J. Wang, “Mean field
multi-agent reinforcement learning,” arXiv preprint arXiv:1802.05438,
2018.
[14] Wikipedia contributors, “Lane — Wikipedia, the free encyclopedia,”
2019. [Online; accessed 1-November-2019].
[15] A. Stevanovic, Adaptive traffic control systems: domestic and foreign
state of practice. No. Project 20-5 Topic 40-03, 2010.
[16] V. Mnih, K. Kavukcuoglu, D. Silver, A. A. Rusu, J. Veness, M. G.
Bellemare, A. Graves, M. Riedmiller, A. K. Fidjeland, G. Ostrovski,
et al., “Human-level control through deep reinforcement learning,”
Nature, vol. 518, no. 7540, p. 529, 2015.

1Our project of Deecamp 2019 is open-source in Github: https://github.com/multi-commander/Multi-Commander
A. Fundamental RL algorithms

1) Temporal-Difference Learning: Temporal-Difference Learning (TD learning), proposed by Sutton [36], combining with Dynamic Programming (DP) and Monte Carlo (MC) methods, becomes the core idea of reinforcement learning. Like the Monte Carlo algorithm, it does not need to know the specific environment model, and can learn directly from experience. On the other hand, it also inherits bootstrapping from DP algorithm, which is the unique feature of TD learning: predictions are used as targets during the course of learning [37]. Monte Carlo simulates (or experiences) an episode until it ends, then estimates the state value based on the value of each state. In contrast, TD learning simulates an episode with one step (or several steps) per action which based on the reward of the new state, and then estimate the state value before execution.

The Q-learning algorithm [38] is a groundbreaking algorithm. TD learning is used here for off-policy learning.

\[ \delta_t = r_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \]

where \( \delta_t \) represents TD-error.

2) Deep Q-network: Deep Q-network (DQN) is a powerful off-policy algorithm which has achieved good results in many fields since 2015 [15]. It uses neural network to approximate the Q-value function instead of tabling.

\[ Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[ y_j - Q(S_t, A_t) \right] \]

\[ y_j = r_{t+1} + \gamma \max_a Q(S_{t+1}, a) \]

3) Double Deep Q-network: Q-learning uses \( \max \) to select the best action, which causes a Maximization Bias problem. So [17] solved this by designing a Double Q-learning, it only differs in the calculation of the target Q-value:

\[ y_j = R_j + \gamma' \max_{a'} \left( \phi(S_j'), \arg\max_{a,w} Q(\phi(S_j'), a, w) \right) \]

4) Dueling Deep Q-network: Another improvement for DQN is Dueling DQN [15], which decomposes the Q network into two separate control streams, a value function \( V(s) \), and a state-based action advantage function \( A(s, a) \). These two control flows obtain an estimate value of the Q function through a special aggregating layer.

\[ Q(s, a; \theta, \alpha, \beta) = V(s; \theta, \beta) + \left( A(s, a; \theta, \alpha) - \frac{1}{|A|} \sum_{a'} A(s, a'; \theta, \alpha) \right) \]
B. Pseudocode for $\gamma$-Reward series

**Algorithm 1** $\gamma$-Attention-Reward Algorithm for MARL Traffic Lights Control

1: Initialize $E$ parallel environments with $N$ agents
2: Initialize replay memory $D$ to capacity $N_D$
3: Initialize raw replay memory $D_r$ to capacity $N_D + n$
4: Initialize action-value function $Q$ with random weights $\theta$
5: Initialize target action-value function $\hat{Q}$ with weights $\theta^- = \theta$
6: Initialize attention scores $\alpha_{i,j}$
7: $T_{update} \leftarrow 0$
8: for episode = 1, $M$ do
9:   Reset environments, and get initial $o_i$ for each agent $i$
10:   for $t = 1, T$ do
11:      Select actions $a_i \sim \pi_i(o_i)$ for each agent $i$ in each environment $e$
12:      Send actions $a_i$ to all parallel environments and get $o'_i, r_i$ for all agents
13:      Store $(a_i, o_i, r_i, o'_i)$ in $D$
14:      $T_{update} = T_{update} + N$
15:      if $T_{update} \leq$ min steps per update + $n$ then
16:         Replay Buffer Amendment($D_r, \alpha_{i,j}$)
17:         Update Policies:
18:            Calculate $a_{i}^{B} \sim \pi_i^B(o_i^B), i = 1 \ldots N$
19:            Calculate $Q_i^\psi(a_{1 \ldots N}^B, a_{1 \ldots N}^B)$ for all $i$ in parallel
20:            Update policies using $\nabla_{\theta,i} J(\pi_\theta)$ and Adam
21:      Update target $Q$ parameters: $\hat{\theta} \leftarrow \theta$
22:      Update target attention parameters: $\hat{\alpha} \leftarrow \alpha$
23:      $T_{update} \leftarrow 0$
24:   end if
25:   end for
26: end for

**Algorithm 2** Replay Buffer Amendment

1: function REPLYBUFFERAMENDMENT($D_r, \alpha_{i,j}$)
2:   for $i = 1, N$ do
3:      index = len($D_r$) - $n$
4:      while index > ex_index do
5:         $(o_i, a_i, r_i, o'_i) = D_{r,i}(j)$
6:         $R_i = \gamma$-Attention-Reward function($r_i, \alpha_{i,j}$)
7:         Store $(o_i, a_i, R_i, o'_i)$ in $D$
8:         $j = j - 1$
9:      end while
10:     ex_index = len($D_r$) - $n$
11: end for
12: end function