1 introduction

Frustrated magnets are particularly interesting as they often involve unusual low temperature magnetic behaviors. Depending on the nature of interactions, the local connectivity, spin dimensionality, these systems stabilize unconventional magnetic ground states such as non collinear Néel orders, topological spin glasses, classical or quantum spin liquids, spin ices \[ 1 \]. These observations are both experimental and theoretical even though some of them, like topological spin glasses, are still conjectures. Among these ground states, two families have a striking property. Spin liquids, as well as spin ices, possess a residual entropy at zero temperature. This anomaly and the paradox it raises when considering the third principle of thermodynamics, are major points that people have tried to answer.

Theoretical studies of frustrated antiferromagnets started half a century ago with the exact solution of the triangular Ising antiferromagnet by Wannier \[ 2 \]. In the following years, many lattices have been identified where spin models, from Ising symmetry (Z2) to Heisenberg symmetry (O(3) or SU(2)), have disordered ground states. Among them, two have attracted a lot of interest: the kagomé lattice and the pyrochlore lattice \[ 3 \]. The first one is a two dimensional arrangement of corner sharing triangles, while the latter is a three dimensional structure of corner sharing tetrahedra. When considering antiferromagnetic nearest neighbour interactions, they both display spin liquid like behaviors, well characterized theoretically. Spin correlations functions are exponentially decaying at finite temperatures, even sometimes at \( T = 0 \), in both classical and quantum cases.

Recently, another lattice has received attention as it can be described as the two dimensional analog of the pyrochlore lattice: the checkerboard lattice. One very interesting point in this lattice is that its geometry mimics the local environment of each site of the pyrochlore lattice, but is much more simpler as it is two dimensional, and based on a square lattice structure. This has allowed Lieb and Schupp \[ 4 \] to establish exact results in the quantum case (\( S = 1/2 \)) showing that the spectrum in this system is very peculiar as ground states are necessity singlets in finite size clusters. For \( S \geq 1 \), it has been shown numerically \[ 5 \] within an 1/S expansion that the ground state may have a local magnetization at zero temperature while for \( S = 1/2 \) it was proposed that the system orders in a valence bond solid ground state. A recent work \[ 6 \] derived a phase diagram of the ground states for varying \( S \) stating that this system is most likely ordered for all \( S \). Exact diagonalization of finite clusters \[ 7 \] for \( S = 1/2 \) reached the same conclusion and define the ground state as a 4-spin valence bond solid. A perturbative expansion also found a 4-spin valence bond solid \[ 8 \] but with a different unit cell. There are now strong evidences that for small \( S \) this system should have long range order in a valence bond solid order parameter.
In this paper, we look for an exact solution on this lattice in the opposite limit of high spin dimensionality. Therefore, we study a classical Hamiltonian that corresponds to the generalisation of the Heisenberg model with $D$-component spin vectors \[ \mathcal{H} = -\frac{1}{2} \sum_{rr'} J_{rr'} \mathbf{s}_r \cdot \mathbf{s}_{r'}, \quad \| \mathbf{s}_r \| = 1 \] (1) and taking the limit $D \to \infty$. In this limit the problem becomes exactly soluble for all lattice dimensionalities $d$, and the partition function of the system coincides with that of the spherical model. The $D = \infty$ model properly accounts for the profound role played, especially in low dimensions, by the Goldstone or would be Goldstone modes. At the same time, the less significant effects of the critical fluctuation coupling leading, e.g., to the quantitatively different nonclassical critical indices, die out in the limit $D \to \infty$. Thus this model is a relatively simple yet a powerful tool for classical spin systems. It should not be mixed up with the $N$-flavour generalization of the quantum $S = 1/2$ model in the limit $N \to \infty$, including its $1/N$ expansion. The $N$-component nonlinear sigma model (see, e.g., Refs. [13], as well as Ref. [14] for the $1/N$ expansion) is a quantum extension of Eq. (1) in the long-wavelength region at low temperatures. Effective free energies for the $N$-component order parameter appear, instead of Eq. (1), in conventional theories of critical phenomena. Using them for the $1/N$ expansion (see, e.g., Ref. [15]) is a matter of taste. While yielding the same results for the critical indices as the lattice-based $1/D$ expansion, it misses the absolute values of the nonuniversal quantities.

In the following, we give the solution of this model on the checkerboard lattice and show that its ground state is a classical spin liquid. Particularly, it is shown that many properties are similar to the pyrochlore lattice case [23] due to the generalisation of the Heisenberg model with $D$-component spin vectors, as well as Ref. [17,18] for the $1/D$ expansion) in the long-wavelength region at low temperatures. Effective free energies for the $N$-component order parameter appear, instead of Eq. (1), in conventional theories of critical phenomena. Using them for the $1/N$ expansion (see, e.g., Ref. [15]) is a matter of taste. While yielding the same results for the critical indices as the lattice-based $1/D$ expansion, it misses the absolute values of the nonuniversal quantities.

In the following, we give the solution of this model on the checkerboard lattice and show that its ground state is a classical spin liquid. Particularly, it is shown that many properties are similar to the pyrochlore lattice case [23] despite the difference of lattice dimensionality. This strongly suggests that these frustrated systems are mainly driven by their local environment, i.e. by their topological frustration, at least for $D \to \infty$. The rest of this article is organized as follows. In Sec. 2 the structure of the checkerboard lattice and its collective spin variables are described. In Sec. 3 the formalism of the $D = \infty$ model is tailored for the checkerboard lattice. The diagrams of the classical spin diagram technique that do not disappear in the limit $D \to \infty$ are summed up. The general analytical expressions for the thermodynamic functions and spin CFs for all temperatures are obtained. In Sec. 4 the thermodynamic quantities of the checkerboard antiferromagnet are calculated and compared with MC simulation results as well as exact and analytical results previously obtained on the pyrochlore lattice in the whole temperature range. In Sec. 5 the real space correlation functions are computed. We finally discuss our results in Sec. 6 and conclude.

### 2 Lattice structure and the Hamiltonian

Checkerboard lattice shown in Fig. 1 consists of corner-sharing tetrahedra. Each node of the corresponding Bra-

![Fig. 1. The checkerboard lattice. It can be pictured as a square lattice of tetrahedra (top). Locally, it reproduces the same environment as the three dimensional pyrochlore lattice. In our calculations, it has been described by a square lattice with a 2-spins unit cell (bottom).](image-url)
where the interaction matrix is given by
\[ \tilde{V}_q = 2J \begin{pmatrix} a & 2uv \\ 2uv & b \end{pmatrix} \] (6)
with \( a = \cos(q_x) \), \( b = \cos(q_y) \), \( u = \cos(q_x/2) \) and \( v = \cos(q_y/2) \).

At the second stage, the Hamiltonian \( \mathcal{H} \) is finally diagonalized to the form
\[ \mathcal{H} = \frac{1}{2N} \sum_{nq} \tilde{V}_q \sigma_q^n \sigma_{-q}^n \] (7)
where \( \tilde{V}_q = 2J \nu_n(q) \) are the eigenvalues of the matrix \( \tilde{V}_q^{ll'} \) taken with the negative sign,
\[ \nu_1 = 1, \quad \nu_2 = -(1 + \cos(q_x) + \cos(q_y)) \] (8)
The diagonalizing transformation has the explicit form
\[ U^{-1}_{nl}(q)V_{ll'}(q)U_{ln'}(q) = \tilde{V}_q \delta_{nn'} \] (9)
where the summation over the repeated indices is implied and \( U \) is the real unitary matrix, \( U^{-1} = U^T \), i.e., \( U^{-1}_{nl} = U_{ln} \). The columns of the matrix \( U \) are the two normalized eigenvectors \( U_n = (U_{1n}, U_{2n}) \) of the interaction matrix \( V \):
\[ U_1 = \sqrt{\frac{1 + a}{2 + a + b} \left( \frac{2uv}{1 + a} \right)}, \]
\[ U_2 = \sqrt{\frac{1 + b}{2 + a + b} \left( \frac{1 + a}{2uv} \right)} \] (10)
The normalized eigenvectors satisfy the requirements of orthogonality and completeness, respectively:
\[ U_{ln}(q)U_{ln'}(q) = \delta_{nn'}, \quad U_{ln}(q)U_{ln}(q) = \delta_{ll} \] (11)
The Fourier components of the spins \( s_q^l \) and the collective spin variables \( \sigma_q^n \) are related by
\[ s_q^l = U_{ln}(q)\sigma_q^n, \quad \sigma_q^n = s_q^l U_{ln}(q) \] (12)
The largest dispersionless eigenvalue \( \nu_1 \) of the interaction matrix [see Eq. (3)] manifests frustration in the system which precludes an extended short-range order even in the limit \( T \to 0 \). Independence of \( \nu_1 \) of \( q \) signals that 1/2 of all spin degrees of freedom are local and can rotate freely. The other eigenvalue satisfy
\[ \nu_2(q) \cong -3 + q^2/2 \] (13)
at small wave vectors, \( q^2 \equiv q_x^2 + q_y^2 \ll 1 \). Within our description of the checkerboard lattice, the would be usual Goldstone mode corresponds to \( \nu_2 \). At the corners of the Brillouin zone \( (\pm \pi, \pm \pi) \), this mode becomes degenerate with the flat one and defines low energy excitations.
\[ \nu_2(q) \cong 1 - q^2/2 \] (14)

3 Classical spin diagram technique and the large-\( D \) limit

The exact equations for spin correlation functions in the limit \( D \to \infty \), as well as the \( 1/D \) corrections, can be the most conveniently obtained with the help of the classical spin diagram technique. A complete description of the technique applied to a non bipartite lattice can be
found in Ref. [28]. We only give here an outline of the calculations.

Our goal is to compute spin-spin CF’s

\[ s_{ll}^{u}(q) = \langle \sigma_{n}^{+} \sigma_{n}^{-} \rangle \]  

(15)

which are related to CF’s of the \( \sigma \) variables

\[ \sigma_{n}^{\alpha}(q) = \frac{D}{N} \langle \sigma_{\alpha n} \sigma_{\alpha -n}^{\alpha} \rangle = \frac{1}{N} \langle \sigma_{n}^{\alpha} \sigma_{-n}^{\alpha} \rangle, \]

(16)

through the relation

\[ s_{ll}^{u}(q) = U_{ln}(q) U_{ln}(q) \sigma_{n}^{\alpha}(q) \]

(17)

following from Eq. (2).

The analytical expression for the \( \sigma \) CF in the SCGA has the Ornstein-Zernike form

\[ \sigma_{n}(q) = \frac{D \tilde{A}}{1 - A \beta V_{n}}. \]  

(18)

In the large-\( D \) limit the expression of \( \tilde{A} \) simplifies \[ \tilde{A} \] and is given by

\[ \tilde{A} = \frac{2}{D+1+\sqrt{1+8L/D}}. \]  

(19)

Here the dispersion \( L \) is given by the formula

\[ L = \frac{\tilde{A}}{2} \sum_{n} v_{0} \int \frac{dq}{(2\pi)^{d}} \frac{(\beta V_{n})(\beta V_{n})}{1 - A \beta V_{n}}. \]  

(20)

The summation \( 1/N \sum_{q} \ldots \) is replaced by the integration over the Brillouin zone, \( v_{0} \) is the unit cell volume, and \( d \) is the spatial dimensionality. For the square lattice we have \( v_{0} = 1 \). The expression for \( L \) can be simplified to

\[ L = \frac{\tilde{P} - 1}{2 \tilde{A}}, \quad \tilde{P} = \frac{1}{2} \sum_{n} P_{n}, \]  

(21)

where \( P_{n} \) is the lattice Green function associated with the eigenvalue \( n \):

\[ P_{n} = v_{0} \int \frac{dq}{(2\pi)^{d}} \frac{1}{1 - A \beta V_{n}}. \]  

(22)

Now one can eliminate \( L \) from Eqs. (13) and (21), which yields the basic equation of the large-\( D \) model,

\[ D \tilde{A} \tilde{P} = 1. \]  

(23)

This nonlinear equation determining \( \tilde{A} \) as a function of temperature differs from those considered earlier \[ \text{[21][22][23][27]} \] by a more complicated form of \( \tilde{P} \) reflecting the lattice structure. The form of this equation is similar to that appearing in the theory of the usual spherical model. \[ \text{[1][11]} \]

The meanings of both equations are, however, different. Whereas in the standard spherical model a similar equation account for the pretty unphysical global spin constraint, Eq. (23) here is, in fact, the normalization condition \( \langle s_{r}^{2} \rangle = 1 \) for the spin vectors on each of the lattice sites \( r \). Indeed, calculating the spin autocorrelation function in the form symmetrized over sublattices with the help of Eqs. (17), (11), and (18), one obtains

\[ \langle s_{r}^{2} \rangle = v_{0} \int \frac{dq}{(2\pi)^{d}} \frac{1}{2} \sum_{n} s_{ll}^{u}(q) \]

(24)

That is, the spin-normalization condition is automatically satisfied in our theory by virtue of Eq. (23). After \( \tilde{A} \) has been found from this equation, the spin CFs are readily given by Eqs. (13) and (17).

To avoid possible confusions, we should mention that in the paper of Reimers, Ref. [30], where Eq. (18) with the bare cumulant \( \Lambda = 1/D \) has been obtained, the theoretical approach has been called “Gaussian approximation (GA)”. This term taken from the conventional theory of phase transitions based on the Landau free-energy functional implies that the Gaussian fluctuations of the order parameter are considered. In the microscopic language, this merely means calculating correlation functions of fluctuating spins after applying the MFA. Such an approach is known to be inconsistent, since correlations are taken into account after they had been neglected. As a result, for the checkerboard lattice one obtains a phase transition at the temperature \( T_{c} = T_{c}^{MFA} = 2J/D \) but immediately finds that the approach breaks down below \( T_{c} \) because of the infinitely strong fluctuations. In contrast to this MFA-based approach, the self-consistent Gaussian approximation used here allows, additionally, to the Gaussian fluctuations of the molecular field, which renormalize \( \tilde{A} \) and lead to the absence of a phase transition for this class of systems. The SCGA is, in a sense, a “double-Gaussian” approximation; one concerning fluctuations of the order parameter and the other one describing fluctuations of the molecular field.

To close this section, let us work out the expressions for the energy and the susceptibility of the checkerboard antiferromagnet. For the energy of the whole system, using Eqs. (13) and (16), as well as the equivalence of all spin components, one obtains

\[ U_{\text{tot}} = \langle \mathcal{H} \rangle = -\frac{N}{2} \sum_{n} v_{0} \int \frac{dq}{(2\pi)^{d}} \tilde{V}_{n} \sigma_{n}^{\alpha}(q). \]  

(25)

To obtain the energy pro spin \( U \), one should divide this expression by \( 2N \). With the use of Eq. (15), the latter can be expressed through the lattice Green’s function \( \tilde{P} \) of Eq. (24); then with the help of Eq. (23) it can be put into the final form

\[ U = \frac{T}{2} \left( D - \frac{1}{\Lambda} \right). \]  

(26)

The susceptibility pro spin symmetrized over sublattices can be expressed through the spin CFs as

\[ \chi_{q} = \frac{1}{2DT} \sum_{ll} s_{ll}^{u}(q). \]  

(27)
With the use of Eq. (27) this can be rewritten in the form
\[ \chi_q = \frac{1}{2D\xi} \sum_n W_n^2(q)\sigma^n(q), \quad W_n(q) \equiv \sum_l U_{ln}(q), \]
where the diagonalized CFs are given by Eq. (18). From Eq. (10) it follows that in the limit \( q \to 0 \) one has \( W_1 = 0 \) and \( W_2 = \sqrt{2} \). Thus the homogeneous susceptibility \( \chi \) simplifies to
\[ \chi = \frac{1}{2D\xi} \sigma^2(0). \] (28)

As we shall see in the next section, disappearance of the \( 2 \) terms with \( n = 1 \) from this formula ensures the nondiverge of the homogeneous susceptibility of the checkerboard antiferromagnet in the limit \( T \to 0 \). The situation for \( q \neq 0 \) is much more intricate and it will be considered below in relation to the neutron scattering cross section.

**4 Thermodynamics of the checkerboard antiferromagnet**

To put the results obtained above into the form explicitly well behaved in the large-\( D \) limit and allowing a direct comparison with the results obtained by other methods for systems with finite values of \( D \), it is convenient to use the mean-field transition temperature \( T_c^{\text{MFA}} = 2J/D \) as the energy scale. With this choice, one can introduce the reduced temperature \( \theta \) and the so-called gap parameter \( G \) according to
\[ \theta \equiv \frac{T}{T_c^{\text{MFA}}}, \quad G \equiv \frac{D}{\theta} \chi. \] (30)

In these terms, Eq. (23) rewrites as
\[ \theta G \bar{P}(G) = 1 \] (31)
and determines \( G \) as function of \( \theta \). Here \( \bar{P}(G) \) is defined by Eq. (21), where
\[ P_1 = \frac{1}{1-G}, \quad P_2 = r_0 \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{1}{1-G
u_2(\mathbf{q})}. \] (32)
The \( \sigma \) CFs of Eq. (13), which are proportional to the integrands of \( P_n \), can be rewritten in the form
\[ \sigma^n(q) = \frac{\theta G}{1-G\nu_n(q)}. \] (33)

Further, it is convenient to consider the reduced energy pro spin defined by
\[ \bar{U} \equiv U/|U_0|, \quad U_0 = -J, \] (34)
where \( U_0 \) is the energy pro spin at zero temperature. With the help of Eq. (26) \( \bar{U} \) can be written as
\[ \bar{U} = \theta - 1/G. \] (35)

The homogeneous susceptibility \( \chi \) of Eq. (21) can be rewritten with the help of Eq. (13) in the reduced form
\[ \chi \equiv 2J\chi = \frac{G}{1+3G}. \] (36)

The sense of calling \( G \) the “gap parameter” is clear from Eq. (36). If \( G = 1 \), then the gap in correlation functions closes: \( \sigma^1 \) turns to infinity, and \( \sigma^2 \) diverges at \( q \rightarrow (\pm \pi, \pm \pi) \). For nonordering models, it happens only in the limit \( \theta \to 0 \), however. The solution of Eq. (36) satisfies \( G \leq 1 \) and goes to zero at high temperatures. If \( \theta \ll 1 \), the function \( \bar{P} \) is dominated by \( P_1 = 1/(1-G) \). The ensuing asymptotic form of the gap parameter at low temperatures reads
\[ G \simeq 1 - \frac{\theta}{2}, \quad \theta \ll 1. \] (37)

At high temperatures, Eq. (31) requires small values of \( G \). Here, the limiting form of \( \bar{P} \) can be shown to be \( \bar{P} \simeq 1 + (3/2)\theta^2 \). The corresponding asymptote of \( G \) has the form
\[ G \simeq \frac{1}{\theta} \left( 1 - \frac{3}{2\theta^2} \right), \quad \theta \gg 1. \] (38)

The numerically calculated temperature dependence of \( G \) is shown in Fig. 3. Note that in the MFA one has \( G = 1/\theta \) which attains the value 1 at \( \theta = 1 \).

The temperature dependence of the reduced energy of Eq. (15) is shown in Fig. 4. Its asymptotic forms following from Eqs. (37) and (38) are given by
\[ \bar{U} \simeq \begin{cases} \frac{3}{(2\theta)}, & \theta \gg 1 \\ \frac{1}{1 + \theta/2}, & \theta \ll 1. \end{cases} \] (39)

This implies the reduced heat capacity \( \bar{C} = d\bar{U}/d\theta \) equal to 1/2 at low temperatures. To compare with the MC simulation data of Ref. [31] for the heat capacity of the
Heisenberg model we will use, instead of $\tilde{C}$, the true heat capacity $C = d\tilde{U}/dT = (D/2)\tilde{C}$ [see Eqs. (30) and (34)], which in our approach tends to $D/4 \gg 3/4$ at low temperatures. The temperature dependence of the heat capacity, compared with previous MC results of the Heisenberg antiferromagnet on the pyrochlore lattice [22], as well as exact solution of the infinite component classical antiferromagnet [23], are shown in Fig. 5. We see that the behavior of these two different models on the pyrochlore lattice are very similar to our results on the checkerboard lattice, thus confirming the analogy of these two structures despite their different space dimensionality.

Using Eq. (39) we compute the low and high temperature asymptotic behavior of the reduced susceptibility $\tilde{\chi}$

$$
\tilde{\chi} \sim \begin{cases} 
\frac{1}{4} - \frac{\theta}{32}, & \theta \ll 1 \\
(1/\theta)(1 - 3/\theta), & \theta \gg 1
\end{cases}
$$

Its dependence at all temperatures is reported in Fig. 6. Here also, we recover a behavior very similar to the one of the pyrochlore lattice where MC simulations and analytical results have been obtained for the Heisenberg antiferromagnet [32] as well as exact results for the infinite component classical antiferromagnet [23]. The similarity between our results and the local approximation developed in Ref. [33] suggest that the checkerboard antiferromagnet has its thermodynamics governed by local correlations, as it is expected in a classical spin liquid.

5 Real-space correlation functions

The low-temperature behavior of the $\sigma$ correlation functions of Eq. (33) is dominated by their asymptotic form at small wave vectors, where the would be Goldstone mode is defined, i.e at $q = (\pm \pi, \pm \pi) + \tilde{q}, |\tilde{q}| << 1$. According to Eqs. (8), we find, (14), and (37), by

$$
\sigma^1 \approx 2, \quad \sigma^2 \approx \frac{2\kappa^2}{\kappa^2 + \tilde{q}^2}
$$

where the quantity $\kappa^2 = \theta$ in $\sigma^2$ defines the correlation length

$$
\xi_c = \frac{1}{\kappa} = \frac{1}{\sqrt{\theta}}
$$
Appearance of this length parameter implies that the real-space spin CFs that, according to Eqs. (17) and (18), by
\[ s_{ij}'' = v_0 \int \frac{dq}{(2\pi)^d} e^{i\mathbf{q} \cdot (r_i - r_j)} U_{ln}(\mathbf{q}) U_{l'n}(\mathbf{q}) \sigma^n(\mathbf{q}) \] (43)
decay exponentially at large distances at nonzero temperatures. In contrast to conventional lattices, divergence of \( \xi_c \) at \( \theta \to 0 \) does not lead here to an extended short-range order, i.e., to strong correlation at distances \( r < \xi_c \). The zero-temperature CFs are purely geometrical quantities which are dominated by \( \sigma^n \) and have the form
\[ s_{ij}'' = 2v_0 \int \frac{dq}{(2\pi)^d} e^{i\mathbf{q} \cdot (r_i - r_j)} U_{11}(\mathbf{q}) U_{11}(\mathbf{q}) \] (44)

It is convenient to enumerate CFs by the numbers \( n, n' \), defined by Eq. (18). Thus \( s_{n,n'}'' \) is the correlation function of the \( l \) sublattice spin of the “central” unit-cell \((0,0)\) with the \( l' \) sublattice spin of the unit-cell translated by \((n,n')\). Let us calculate the CFs with \( l = l' = 1 \) at large distances along the horizontal line in Fig. 2 at small but non zero temperature. We use the asymptotic form of the matrix \( U_{ln}(\mathbf{q}) \) for \( q = (\pm \pi, \pm \pi) + q_r, |q_r| \ll 1 \), as well as the asymptotic form of CFs in Eq. (11). This allows to write this correlation function as
\[ s_{ij}'' = v_0 \int \frac{dq}{(2\pi)^d} (-1)^n e^{i\mathbf{q} \cdot \mathbf{r}_i} (U_{11}^2(\mathbf{q}) \sigma^1 + U_{12}^2(\mathbf{q}) \sigma^2) \]
\[ = 2v_0 \int \frac{dq}{(2\pi)^d} (-1)^n e^{i\mathbf{q} \cdot \mathbf{r}_i} \left( \frac{\tilde{q}_r^2}{\tilde{q}_r^2} + \frac{\tilde{q}_r^2}{\tilde{q}_r^2} \kappa^2 + \frac{\tilde{q}_r^2}{\tilde{q}_r^2} \right) \] (45)

Simplifying this expression and taking into account that the integral of the cosine over the whole Brillouin zone is zero, one arrives at the asymptotic form
\[ s_{n,0}^{11} = \frac{(-1)^n \kappa}{\pi n} K_1(\kappa n) \approx \begin{cases} \frac{(-1)^n}{\pi n^2}, & \kappa n \ll 1 \\ \frac{(-1)^n \kappa}{\sqrt{2\pi}} e^{-\kappa n}, & \kappa n \gg 1 \end{cases} \] (46)

where \( K_\nu(x) \) is the Macdonald (modified Bessel) function. The strong decrease of this correlation function with distance even at \( T = 0, (\kappa = 0) \) is not surprising, since our solution spans the whole highly degenerate ground-state manifold and this degeneracy is not lifted in the limit \( D \to \infty \). At small but non zero temperature, spin-spin correlations are exponentially decaying as if interactions were renormalized to zero and drive this system to a paramagnetic fix point for finite temperatures. Let us calculate more generally the CFs with \( l = 1, 2 \) and \( l' = 1, 2 \) at large distances along the horizontal line in Fig. 2 at zero temperature. Following the same previous procedure, we obtain
\[ s_{n,0}^{11} = s_{n,0}^{22} = \frac{(-1)^n}{\pi n^2}, \quad s_{n,0}^{12} = \frac{(-1)^n}{\pi (n + 1/2)^2} \] (47)

where \( n \) is in unit of the inter-cell distance. We note that despite the divergence of the correlation length \( \xi_c \) when lowering the temperature, the model do not order, which clearly defines this system as a classical two dimensional spin liquid.

6 Discussion

In the main part of this article, we have presented in detail the exact solution for the \( D = \infty \) component classical antiferromagnet on the checkerboard lattice. The solution does not show ordering at any temperature due to the strong degeneracy of the ground state, and thermodynamic functions behave smoothly. In contrast to conventional two-dimensional magnets, there is no extended short-range order at low temperature, and \( T = 0 \) is not a critical point of the system. Although the correlation associated to the would be Goldstone mode diverges as \( \xi_c \propto T^{-1/2} \), the power law decay \( \langle s_n s_r \rangle \propto 1/r^2 \) at zero temperature of the spin correlation functions leads to the loss of correlations at the scale of the interatomic distance. All these properties characterize this model as a two-dimensional classical spin liquid. As these results are obtained within the \( D = \infty \) component classical antiferromagnet, it is expected that it mimics the large \( S \) value Heisenberg antiferromagnet on the checkerboard lattice. Surprisingly, there are results [10,11] suggesting that for different \( S \), the Heisenberg antiferromagnet should be ordered on this lattice. At this stage, the discrepancy between the different approaches is not clear. A possible explanation, as reported in Ref. [15], is that for large \( S \), there should be quantum order by disorder (as supported in [6]) but this order by disorder comes with an energy scale of \( J/S \) rather than \( J \). Therefore, in the classical limit, the temperature at which the correlation length grows exponentially goes to zero. In other words, the limits \( T \to 0 \) and \( S \to \infty \) (or...
predictions. Here again, finite temperature properties of the infinite component spin vector model have been obtained close to the $D = 3$ Heisenberg case. However, at $T \to 0$, the method misses the “order by disorder” phenomenon which selects the coplanar spin manifold. As a consequence, the $D = 3$ zero temperature specific heat is much closer to $C = 11/12k_B$ than, it would be if really one third of the degrees of freedom were still fluctuating (as suggested by a mean field analysis). The $1/D$ corrections could resolve this discrepancy is still a work to be done.

Despite the dimensionality of the checkerboard lattice ($d = 2$), there are many similarities with previously obtained results on the pyrochlore lattice ($d = 3$). The specific heat in particular, the same value at zero temperature $C = 11/12k_B$. This is not surprising as the spin liquid behavior implies that the lattice is mainly described by local fluctuations. In fact, it is true at least for finite temperature thermodynamics. Why the present approach can lead to correct results for the pyrochlore case and not for the checkerboard case at very low temperatures has not been addressed in the present work. We can only note that the common point between the two lattices is the local connectivity which is clearly well taken into account within the $D \to \infty$ formalism. Whether dimensionality is relevant is not clear. At this stage, it can be noted that for the kagomé case, the $D \to \infty$ formalism has missed the reduced specific heat mechanism (see Ref. [23,24]) while for the pyrochlore case, it has been reproduced (see Ref. [23]). Therefore, discrepancies with $D = 3$ descriptions could be ascribed to dimensionality although other subtle phenomena that are not taken into account when $D \to \infty$ cannot be ruled out without studying the same model when including $1/D$ corrections.

To conclude, this classical approach indicates that for infinite component spin value, the checkerboard antiferromagnet should be disordered and behave as a spin liquid. We give some arguments why the present approach is at variance with results of Ref. [23]. It is interesting to note that for small spin value, the checkerboard antiferromagnet is expected to order $\mathbb{Z}_2$. Therefore the checkerboard antiferromagnet cannot be compared to the pyrochlore antiferromagnet even if its geometry defines it as its two dimensional analog. Even if they have the same unit cell, it is probably the global geometry of the underlying Bravais lattice that drives the physics and not only the local connectivity.

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