Abstract
The relatively warm climate found in the North-Western Europe is due to the Gulf Stream that circulates warm saline water from southern latitudes to Europe. In North Atlantic Ocean the stream gives out a large amount of heat, cools down and sinks to the bottom to complete the Thermo-Haline Circulation. There is considerable debate on the stability of the stream to inputs of fresh water from the melting ice in Greenland. The circulation, being switched off, will have massive impact on the climate of Europe. Intergovernmental Panel on Climate Change (IPCC) has warned of this danger in its recent report. Our aim is to model the Thermo-Haline Circulation at the point where it sinks in the North-Atlantic. We create a two-dimensional discrete map modeling the salinity gradient and vertical velocity of the stream. We look for how a perturbation in the form of fresh water release can destabilize the circulation by pushing the velocity below a certain threshold.

1 Introduction
In the ocean, water motion is not only generated by wind-forcing. The horizontal temperature and salinity differences caused by climatic influences at the sea surface, cause density differences which initiate circulation. Such circulation requires that expansion should take place at a higher pressure than contraction does. In terms of ocean and water temperature, this means that the heat source must lie at a lower level than the cold source. In the ocean, the heat and cold sources are located at the same level, at the sea surface. Water close to a heat source attains a lower density than water close to a cold surface. It becomes lighter and spreads at the surface in the direction of the cold source. For continuity, water below the heat source will ascend and water below the cold source will descend and while spreading below, it will become warmer through heat conduction and mixing. In the layers down below, cold water moves from the higher latitudes to the lower ones and in the upper layer, warm water flows in the opposite direction. This is not the entire story. At the sea surface globally, there are regions rich in salinity as well as those deficient of salinity. The former has more evaporation and ice formation (a cold source) and the latter has more precipitation, continental run-offs and ice-melting (a heat source). This causes a haline circulation. In places, the Thermal and the Haline circulations act in the same direction, reinforcing each other to form what is called the Global Thermohaline Circulation (THC). It is often referred to as the Great Conveyor Belt. In the North Atlantic, it manifests itself as the meridional overturning current (MOC). This current, through the Gulf Stream and the North Atlantic Current, transports large quantities of warm water to the northern latitudes. This has a strong effect on climatic conditions. Compared to the corresponding parts of North America, Northern Europe has a much milder climate due to this heat transport. It has been proposed by the Intercontinental Panel on Climate Change (IPCC) that the challenge of global warming is real and it is man-made. There are concerns that global warming can affect the stability of the MOC. During the last deglaciation (whose remnants are the Great Lakes in the North America), a large amount of melt water entered the North Atlantic causing a shallower THC cell. North Atlantic’s ice cover has significantly changed over the past 20,000 years resulting in drastic climatic changes. However the ice cover over Greenland is much more massive than the polar sea ice cover. Melt water from Greenland could trigger a switchover of the MOC.

2 The Model
The aim here is to create a simple 2-D discrete model of MOC. The amount of scaled fresh water intake is
taken as a parameter. The impact of its variation is studied. Our starting point is a modification of an MOC model by Timmerman, Lohmann and Manahan [2] [3] [4].

\[
\dot{x} = x(1-x) + \mu + \frac{y}{h} \quad (1)
\]

\[
\dot{y} = \frac{h}{T^2} x - g + \frac{\alpha}{T} y \quad (2)
\]

Discretization yields

\[
x_{n+1} = ax_n(1 - x_n) + \mu + \frac{y_n}{h} \quad (3)
\]

\[
y_{n+1} = \frac{h}{T^2} x_n - g + \frac{\alpha}{T} y_n \quad (4)
\]

Now we shall fix some values for the parameters of the map. As the depth to which the meridional overturning descends is approximately 3 km, we shall take \( h = 3000 \). We shall take the proportionality constant \( \alpha \) as 1, \( T \), we take as 20, \( g \), we keep as the acceleration due to gravity on Earth surface, 9.8. With these values, the map much simplifies to

\[
x_{n+1} = ax_n(1 - x_n) + \mu + \frac{y_n}{3000} \quad (5)
\]

\[
y_{n+1} = 7.5x_n - 9.8 + 0.05y_n \quad (6)
\]

### 2.1 Low Fresh Water Intake

The variation of the map with \( a \) is not of much interest to us. \( \mu \) is a scaled variable and we shall study its variation in the range (0, 1). We begin with \( \mu = 0.01 \) and \( a = 1.1 \). The reason for the choice of \( \mu \) is that we are talking of the North Atlantic, where freezing rather than melting predominates, and physically that means a small, if any, fresh water intake for THC. The fixed points of the system are

\[
x_1 = -0.0435779, y_1 = -10.6598
\]

\[
x_2 = 0.136879, y_2 = -9.23516
\]

The first fixed point is a saddle point, however, the second fixed point is stable. For the unstable fixed point, if we start with a smaller value than that fixed point, the orbit diverges to negative infinity for both variables. But starting with a value more than this fixed point, the orbit converges to the stable fixed point (See Figure 1).

### 2.2 High Fresh Water Intake

Now our intent is to find out what happens to the system if suddenly a large amount of melt water finds its way into the MOC. This would correspond to a large value of \( \mu \). \( \mu = 0.8 \) is chosen. This corresponds to a sudden inflow of melt water from the Greenland ice cover. This is a low probability situation, but, in any case accentuated by the effects of global warming. The fixed points of the system are found to be

\[
x_1 = -0.805595, y_1 = -16.6758
\]

\[
x_2 = 0.898897, y_2 = -3.21924
\]

This time also, the second point is a stable fixed point, however, the first fixed point is a saddle and therefore unstable. As can be seen, the stable solution is one with a high salinity gradient, which is to be expected on addition of fresh water to the system and a lower velocity, signifying the weakening of the MOC. Now suppose, we were to begin from the previous stable fixed point, it would be of interest to see how the system evolves. This time the system settles down into the new fixed point in a matter of fifty time steps (See Figure 2).

### 2.3 Bifurcation Plot

We are interested in how the stable fixed points of the map vary with change in \( \mu \). This can be visualised by the Bifurcation Diagram for the range of (0, 1). The bifurcation diagram shows a smooth slowing down of the stream velocity with respect to \( \mu \). Close to \( \mu \sim 0.9 \) we can see the first bifurcation of the map (See Figure 3). It is seen that the bifurcation parameter \( \mu_B \) remains almost fixed at \( \sim 0.9 \) with respect to variation in the parameter ‘\( T \)’. Thus, the system is robust with respect to changes in the parameter ‘\( T \)’.

### 3 Conclusion

The interpretation that we make of this is that a gradual change in would cause the stream to slow down only gradually and only a sudden deglaciation leading to release of a massive amount of fresh water can cause the Gulf Stream to slow down. Therefore, it can be said that this is a low probability but disastrous event. What we would like to further do with the map is to standardise the values of the variables using real values and to see how the discrete time step translates into physical time because this would give us some idea of the time in which the system can get destabilised.
Figure 1: Time Series and Phase Plot showing convergence for low fresh water intake

Figure 2: Time Series and Phase Plot showing convergence for high fresh water intake
Figure 3: Bifurcation Plot showing variation of vertical velocity with change in µ for different values of parameter T.
References

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