A study of QCD coupling constant from fixed target deep inelastic measurements

V.G. Krivokhijine and A.V. Kotikov

Joint Institute for Nuclear Research, 141980 Dubna, Russia

Abstract

We reanalyze deep inelastic scattering data of BCDMS Collaboration by including proper cuts of ranges with large systematic errors. We perform also the fits of high statistic deep inelastic scattering data of BCDMS, SLAC, NM and BFP Collaborations taking the data separately and in combined way and find good agreement between these analyses. We extract the values of the QCD coupling constant \( \alpha_s(M_Z^2) \) up to NLO level. The fits of the combined data for the nonsinglet part of the structure function \( F_2 \) predict the coupling constant value \( \alpha_s(M_Z^2) = 0.1174 \pm 0.0007 \) (stat) \( \pm 0.0019 \) (syst) \( \pm 0.0010 \) (normalization). The fits of the combined data for both: the nonsinglet part of \( F_2 \) and the singlet one, lead to the values \( \alpha_s(M_Z^2) = 0.1177 \pm 0.0007 \) (stat) \( \pm 0.0021 \) (syst) \( \pm 0.0009 \) (normalization). Both above values are in very good agreement with each other.

1 Introduction

The deep inelastic scattering (DIS) leptons on hadrons is the basical process to study the values of the parton distribution functions (PDF) which are universal (after choosing of factorization and renormalization schemes) and can be used in other processes. The accuracy of the present data for deep inelastic structure functions (SF) reached the level at which the \( Q^2 \)-dependence of logarithmic QCD-motivated and power-like ones may be studied separately (for a review, see the recent papers [1] and references therein).

In the present paper we review the results of our analysis [2] at the next-to-leading (NLO) order of perturbative QCD for the most known DIS SF \( F_2(x, Q^2) \) taking into account SLAC, NMC, BCDMS and BFP experimental data [4]-[7]. We stress the power-like effects, so-called twist-4 (i.e. \( \sim 1/Q^2 \)) contributions. To our purposes we represent the SF \( F_2(x, Q^2) \) as the contribution of the leading twist part \( F_2^{pQCD}(x, Q^2) \) described by perturbative QCD and the nonperturbative part (i.e. twist-four terms \( \sim 1/Q^2 \)):

\[
F_2(x, Q^2) \equiv F_2^{full}(x, Q^2) = F_2^{pQCD}(x, Q^2) \left( 1 + \frac{\tilde{h}_4(x)}{Q^2} \right) \quad (1)
\]

The SF \( F_2^{pQCD}(x, Q^2) \) obeys the (leading twist) perturbative QCD dynamics including the target mass corrections (and coincides with \( F_2^{tw2}(x, Q^2) \) when the target mass corrections are withdrawn).

The Eq.(1) allows us to separate pure kinematical power corrections, i.e. so-called target mass corrections, so that the function \( \tilde{h}_4(x) \) corresponds to “dynamical” contribution.

\footnote{The evaluation of \( \alpha_s^3(Q^2) \) corrections to anomalous dimensions of Wilson operators, that will be done in nearest future by Vermaseren and his coauthors (see discussions in [3]), gives a possibility to apply many modern programs to perform fits of data at next-next-to-leading order (NNLO) of perturbative theory (see detail discussions in [3]).}
of the twist-four operators. The parameterization (1) implies that the anomalous dimensions of the twist-two and twist-four operators are equal to each other, that is not correct in principle. Meanwhile, in view of limited precision of the data, the approximation (1) and one in the footnote 2 give rather good predictions (see discussions in [8]).

Contrary to standard fits (see, for example, [8]-[10]) when the direct numerical calculations based on Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [11] are used to evaluate structure functions, we use the exact solution of DGLAP equation for the Mellin moments $M_{n}^{tw}(Q^{2})$ of SF $F_{2}^{tw}(x, Q^{2})$:

$$M_{n}^{k}(Q^{2}) = \int_{0}^{1} x^{n-2} F_{2}^{k}(x, Q^{2}) \, dx \quad \text{(hereafter } k = \text{full, } pQCD, \text{tw2}, \ldots)$$

and the subsequent reproduction of $F_{2}^{full}(x, Q^{2})$, $F_{2}^{pQCD}(x, Q^{2})$ and/or $F_{2}^{tw2}(x, Q^{2})$ at every needed $Q^{2}$-value with help of the Jacobi Polynomial expansion method [12, 13] (see also similar analyses at the NLO level [14] and at the NNLO level and above [15]).

In this paper we consider in detail only fits of the nonsinglet part (NS) of SF $F_{2}$ and review only final results in general case, that can be found in [2]. Moreover, we do not present exact formulae of $Q^{2}$-dependence of SF $F_{2}$ which are also given in [2]. We note only that the moments $M_{NS}(n, Q^{2})$ at some $Q_{0}^{2}$ is theoretical input of our analysis and the twist-four term $\tilde{h}_{4}(x)$ is considered as a set free parameters at each $x_{i}$ bin. The set has the form $\tilde{h}_{4}^{\text{free}}(x) = \sum_{i=1}^{I} \tilde{h}_{4}(x_{i})$, where $I$ is the number of bins. The constants $\tilde{h}_{4}(x_{i})$ (one per $x$-bin) parameterize $x$-dependence of $\tilde{h}_{4}^{\text{free}}(x)$.

\section{Fits of $F_{2}$: procedure}

Having the QCD expressions for the Mellin moments $M_{n}^{k}(Q^{2})$ we can reconstruct the SF $F_{2}^{k}(x, Q^{2})$ as

$$F_{2}^{k,N_{\text{max}}}(x, Q^{2}) = x^{a}(1-x)^{b} \sum_{n=0}^{N_{\text{max}}} \Theta_{n}^{a,b}(x) \sum_{j=0}^{n} c_{j}^{(n)}(\alpha, \beta) M_{j+2}^{k} (Q^{2}) ,$$

where $\Theta_{n}^{a,b}$ are the Jacobi polynomials and $a, b$ are the parameters, fitted by the condition of the requirement of the minimization of the error of the reconstruction of the structure functions (see Ref.[13] for details).

First of all, we choose the cut $Q^{2} \geq 1 \text{ GeV}^{2}$ in all our studies. For $Q^{2} < 1 \text{ GeV}^{2}$, the applicability of twist expansion is very questionable.

Secondly, we choose quite large values of the normalization point $Q_{0}^{2}$. There are several reasons of this choice:

\begin{itemize}
  \item Our perturbative formulae should be applicable at the value of $Q_{0}^{2}$. Moreover, the higher order corrections $\sim \alpha_{s}^{n}(Q_{0}^{2}) \,(n \geq 2)$, coming from normalization conditions of PDF, are less important at higher $Q_{0}^{2}$ values.
\end{itemize}

\footnote{The r.h.s. of the Eq.(1) is represented sometimes as $F_{2}^{pQCD}(x, Q^{2}) + \tilde{h}_{4}(x)/Q^{2}$. It implies that the anomalous dimensions of the twist-four operators are equal to zero.}

\footnote{We would like to note here that there is similar method to reproduce of structure functions, based on Bernstein polynomials. The method has been used in several analyses at the NLO level in [14] and at the NNLO level in [18].}
• It is necessary to cross heavy quark thresholds less number of time to reach $Q^2 = M_Z^2$, the point of QCD coupling constant normalization.

• It is better to have the value of $Q_0^2$ around the middle point of logarithmical range of considered $Q^2$ values. Then at the case the higher order corrections $\sim (\alpha_s(Q^2) - \alpha_s(Q_0^2))^n$ ($n \geq 2$) are less important.

We use MINUIT program [19] for minimization of two $\chi^2$ values:

\[
\chi^2(F_2) = \left| \frac{F_2^{exp} - F_2^{teor}}{\Delta F_2^{exp}} \right|^2
\]

We would like to apply the following procedure: we study the dependence of $\chi^2/DOF$ value on value of $Q^2$ cuts for various sets of experimental data. The study will be done for the both cases: including higher twists corrections and without them.

We use free normalizations of data for different experiments. For the reference, we use the most stable deuterium BCDMS data at the value of energy $E_0 = 200$ GeV \(^4\). Using other types of data as reference gives negligible changes in our results. The usage of fixed normalization for all data leads to fits with a bit worsen $\chi^2$.

3 Results of fits

Hereafter we choose $Q_0^2 = 90$ GeV\(^2\) that is in good agreement with above conditions. We use also $N_{max} = 8$, the cut $0.25 \leq x \leq 0.8$, where the nonsinglet evolution is dominant.

3.1 BCDMS $C^{12} + H_2 + D_2$ data

We start our analysis with the most precise experimental data \([3]\) obtained by BCDMS muon scattering experiment at the high $Q^2$ values. The full set of data is 607 points.

It is well known that the original analyses given by BCDMS Collaboration itself (see also Ref. \([3]\)) lead to quite small values $\alpha_s(M_Z^2) = 0.113$. Although in some recent papers (see, for example, \([8, 20]\)) more higher values of $\alpha_s(M_Z^2)$ have been observed, we think that an additional reanalysis of BCDMS data should be very useful.

Based on study \([21]\) (see also \([24]\)) we proposed in \([4]\) that the reason for small values of $\alpha_s(M_Z^2)$ coming from BCDMS data was the existence of the subset of the data having large systematic errors. We studied this subject by introducing several so-called $Y$-cuts (see \([21]\)). Excluding this set of data with large systematic errors leads to essentially larger values of $\alpha_s(M_Z^2)$ and very slow dependence of the values on the concrete choice of the $Y$-cut (see below).

We studied influence of the experimental systematic errors on the results of the QCD analysis as a function of $Y_{cut3}$, $Y_{cut4}$ and $Y_{cut5}$ applied to the data. We use the following $x$-dependent $y$-cuts:

\[
y \geq 0.14 \quad \text{when} \quad 0.3 \leq x \leq 0.4 \\
y \geq 0.16 \quad \text{when} \quad 0.4 \leq x \leq 0.5
\]

\(^4E_0\) is the initial energy lepton beam.

\(^5\)Hereafter we use the kinematical variable $Y = (E_0 - E)/E_0$, where $E_0$ and $E$ are initial and scattering energies of lepton, respectively.
Figure 1: The study of systimatics at different $Y_{\text{cut}}$ values. The QCD analysis of BCDMS $C^{12}, H_2, D_2$ data: the inner (outer) error-bars show statistical (systematic) errors.

$$y \geq Y_{\text{cut}}^3 \text{ when } 0.5 < x \leq 0.6$$

$$y \geq Y_{\text{cut}}^4 \text{ when } 0.6 < x \leq 0.7$$

$$y \geq Y_{\text{cut}}^5 \text{ when } 0.7 < x \leq 0.8$$

(4)

and several sets $N$ of the values for the cuts at $0.5 < x \leq 0.8$ given in the Table.

| $N$ | 0  | 1  | 2  | 3  | 4  | 5  | 6  |
|-----|----|----|----|----|----|----|----|
| $Y_{\text{cut}}^3$ | 0.14 | 0.16 | 0.16 | 0.18 | 0.22 | 0.23 |
| $Y_{\text{cut}}^4$ | 0.16 | 0.18 | 0.20 | 0.20 | 0.23 | 0.24 |
| $Y_{\text{cut}}^5$ | 0.20 | 0.20 | 0.22 | 0.22 | 0.24 | 0.25 |

Table. The values of $Y_{\text{cut}}^3$, $Y_{\text{cut}}^4$ and $Y_{\text{cut}}^5$.

The systematic errors for BCDMS data are given [6] as multiplicative factors to be applied to $F_2(x, Q^2)$: $f_r, f_b, f_s, f_d$ and $f_h$ are the uncertainties due to spectrometer resolution, beam momentum, calibration, spectrometer magnetic field calibration, detector inefficiencies and energy normalization, respectively.

For this study each experimental point of the undistorted set was multiplied by a factor characterizing a given type of uncertainties and a new (distorted) data set was fitted again in agreement with our procedure considered in the previous section. The factors ($f_r, f_b, f_s, f_d, f_h$) were taken from papers [6] (see CERN preprint versions in [6]).

The absolute differences between the values of $\alpha_s$ for the distorted and undistorted sets of data are given in the Fig. 1 as the total systematic error of $\alpha_s$ estimated in quadratures. The number of the experimental points and the value of $\alpha_s$ for the undistorted set of $F_2$ are also presented in the Fig. 1.

From the Fig. 1 we can see that the $\alpha_s$ values are obtained for $N = 1 \div 6$ of $Y_{\text{cut}}^3$, $Y_{\text{cut}}^4$ and $Y_{\text{cut}}^5$ are very stable and statistically consistent. The case $N = 6$ reduces the
systematic error in $\alpha_s$ by factor 1.8 and increases the value of $\alpha_s$, while increasing the statistical error on the 30%.

After the cuts have been implemented (hereafter we use the set $N = 6$), we have 452 points in the analysis. Fitting them in agreement with the same procedure considered in the previous Section, we obtain the following results:

$$\alpha_s(M_Z^2) = 0.1153 \pm 0.0013 \text{ (stat) } \pm 0.0022 \text{ (syst) } \pm 0.0012 \text{ (norm)},$$

$$= 0.1153 \pm 0.0028 \text{ (total experimental error)}$$

where hereafter the symbol “norm” marks the error of normalization of experimental data. The total experimental error is squared root of sum of squares of statistical error, systematic one and error of normalization.

### 3.2 SLAC, BCDMS, NMC and BFP data

After these cuts have been incorporated (with $N = 6$) for BCDMS data, the full set of combine data is 797 points.

To verify the range of applicability of perturbative QCD, we analyze firstly the data without a contribution of twist-four terms, i.e. when $F_2 = F_{2}^{pQCD}$. We do several fits using the cut $Q^2 \geq Q_{cut}^2$ and increase the value $Q_{cut}^2$ step by step. We observe good agreement of the fits with the data when $Q_{cut}^2 \geq 10 \text{ GeV}^2$ (see the Fig. 2).

Later we add the twist-four corrections and fit the data with the usual cut $Q^2 \geq 1 \text{ GeV}^2$. We have find very good agreement with the data. Moreover the predictions for $\alpha_s(M_Z^2)$ in both above procedures are very similar (see the Fig. 2).

So, the analysis of combine SLAC, NMC, BCDMS and BFP data are given the following results:

- When twist-four corrections are not included and the cut of $Q^2$ is 10 GeV$^2$ at the free normalization

$$\chi^2/DOF = 0.98 \quad \text{and} \quad \alpha_s(M_Z^2) = 0.1170 \pm 0.0009 \text{ (stat)} \quad (5)$$

- When twist-four corrections are included and the cut of $Q^2$ is 1 GeV$^2$

$$\chi^2/DOF = 0.97 \quad \text{and} \quad \alpha_s(M_Z^2) = 0.1174 \pm 0.0010 \text{ (stat)} \quad (6)$$

Thus, as it follows from nonsinglet fits of experimental data, perturbative QCD works rather well at $Q^2 \geq 10 \text{ GeV}^2$.

### 4 Summary

We have demonstrated several steps of our study \cite{2} of the $Q^2$-evolution of DIS structure function $F_2$ fitting all modern experimental data at Bjorken variable $x$ values: $x \geq 10^{-2}$.

From the fits we have obtained the value of the normalization $\alpha_s(M_Z^2)$ of QCD coupling constant. First of all, we have reanalyzed the BCDMS data cutting the range with large systematic errors. As it is possible to see in the Fig. 1, the value of $\alpha_s(M_Z^2)$ rises strongly when the cuts of systematics were incorporated. In another side, the value of $\alpha_s(M_Z^2)$ does not dependent on the concrete type of the cut within modern statistical errors.

We have found that at $Q^2 \geq 10 \div 15 \text{ GeV}^2$ the formulae of pure perturbative QCD (i.e. twist-two approximation together with target mass corrections) are in good agreement
Figure 2: The values of $\alpha_s(M_Z^2)$ and $\chi^2$ at different $Q^2$-values of data cut. The black points show the analyses of data without twist-four contributions. The white point corresponds to the case where twist-four contributions were added. Only statistical errors are shown.

The results for $\alpha_s(M_Z^2)$ are very similar (see [2]) for the both types of analyses: ones, based on nonsinglet evolution, and ones, based on combined singlet and nonsinglet evolution. They have the following form:

- from fits, based on nonsinglet evolution:
  
  \[
  \alpha_s(M_Z^2) = 0.1170 \pm 0.0009 \text{ (stat)} \pm 0.0019 \text{ (syst)} \pm 0.0010 \text{ (norm)}, \quad (7)
  \]

- from fits, based on combined singlet and nonsinglet evolution:
  
  \[
  \alpha_s(M_Z^2) = 0.1180 \pm 0.0013 \text{ (stat)} \pm 0.0021 \text{ (syst)} \pm 0.0009 \text{ (norm)}, \quad (8)
  \]

\( ^6 \)We note that at small $x$ values, the perturbative QCD works well starting with $Q^2 = 1.5 \div 2$ GeV$^2$ and higher twist corrections are important only at very low $Q^2$: $Q^2 \sim 0.5$ GeV$^2$ (see [22, 23] and references therein). As it is was observed in [24, 25] (see also discussions in [22, 23, 26]) the good agreement between perturbative QCD and experiment seems connect with large effective argument of coupling constant at low $x$ range.
When we have added twist-four corrections, we have very good agreement between QCD (i.e., first two coefficients of Wilson expansion) and data starting already with $Q^2 = 1$ GeV$^2$, where the Wilson expansion should begin to be applicable. The results for $\alpha_s(M_Z^2)$ coincide for the both types of analyses: ones, based on nonsinglet evolution, and ones, based on combined singlet and nonsinglet evolution. They have the following form:

- from fits, based on nonsinglet evolution:
  \[
  \alpha_s(M_Z^2) = 0.1174 \pm 0.0007 \text{ (stat)} \pm 0.0019 \text{ (syst)} \pm 0.0010 \text{ (norm)}, \quad (9)
  \]

- from fits, based on combined singlet and nonsinglet evolution:
  \[
  \alpha_s(M_Z^2) = 0.1177 \pm 0.0007 \text{ (stat)} \pm 0.0021 \text{ (syst)} \pm 0.0009 \text{ (norm)}, \quad (10)
  \]

Thus, there is very good agreement (see Eqs. (7), (8), (9) and (10)) between results based on pure perturbative QCD at quite large $Q^2$ values (i.e., at $Q^2 \geq 10 \div 15$ GeV$^2$) and the results based on first two twist terms of Wilson expansion (at $Q^2 \geq 1$ GeV$^2$, where the Wilson expansion should be applicable).

We would like to note that we have good agreement also with the analysis [20] of combined H1 and BCDMS data, which has been given by H1 Collaboration very recently. Our results for $\alpha_s(M_Z^2)$ are in good agreement also with the average value for coupling constant, presented in the recent studies (see [8, 27, 28] and references therein) and in famous Altarelli and Bethke reviews [29].

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