Numerical algorithm for the problem of the technological process of filtering low-concentrated suspensions

B Yu Palvanov¹, U Saidov² and M Sharipov³

¹ Urgench branch of Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, 110, Al - Khorezm str., Urgench city, Uzbekistan
² Samarkand branch of Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, 47 A, Shohruh Mirzo str., Samarkand city, Uzbekistan
³ Urgench state university, 14, Kh.Alimdjan str., Urgench city, Uzbekistan

⁴ E-mail: Bozorboy@yandex.ru

Abstract. The paper presents a mathematical model in the form of a second-order nonlinear differential equation with initial and boundary conditions and a numerical method for solving the problem based on the finite-difference method of the technological process of filtration of low-concentration suspensions to determine the ranges of variation of the ion-exchange filter parameters, as well as the results of computational experiments. When developing a mathematical model, the following main filter parameters and physico-mechanical properties of solutions (liquids) were taken into account: filtration rate, rate of sedimentation of gel particles on the pores of the filter, filter porosity, filter layer thickness, kinetic coefficient, dynamic viscosity coefficient and outlet concentration of the suspension. The developed mathematical model and a numerical solution method allow determining the deposition rate of gel particles in the pores of the filter, the outlet concentration of the solution and changes in the concentration of the driving solution of the ion-exchange filter.

1. Introduction

Wastewater from chemical, machine-building and other industries contains toxic ions of heavy metals, which, when released into water bodies, adversely affect the surrounding flora and fauna, and have a toxicological effect when they enter the human body.

To protect the wastewater and the groundwater intake from sources of pollution emitted by industrial facilities, it is necessary to develop effective methods, technical means and technologies based on a comprehensive study of the process as a whole. In the technology of purification of liquid ionized industrial emissions, filters with ion exchange filter partitions are usually used. This technology is widely used in drinking water production, food processing, raw materials treatment, etc.

As follows from the analysis of research and development study on this problem, one of the most effective methods for conducting a comprehensive study and making decisions on the process of filtering ionized liquids is a computer modeling and computational experiments (CE) under wide variation in filter operation modes, physicochemical properties of the solution and technical characteristics of the filter unit.
This article discusses the development of an adequate mathematical model, numerical algorithm and calculation methods. To analyze the functioning of the technological process of suspension filtering, a series of CE was carried out on a computer.

Since the filtration process is one of the main stages in the preparation of products and raw materials in food, pharmaceutical, mining and metallurgical, oil refining industries, the quality and quantity of output products depends on the correct choice of operating modes of filtering units.

In recent years, significant results of a practical and theoretical nature have been obtained on the technologies of separation, sorting, and filtration of loose and liquid materials, and of ionized solutions. In particular, in [1], a population balance model was formed for the transport of suspension particles in porous media. Equations for particles and pore size distributions were derived from the stochastic “Master” equation. The model took into account the reduction in particle flux by restricting large particles to be conveyed through small pores. The analytical solution for low concentration particles was obtained for the basic particles and the pore size distribution. The averaged equations differ significantly from the traditional ones by the deep filtration model.

In [2], a phenomenological model of deep-water infiltration was proposed. The proposed mathematical model was combined with the equation of adjectival dispersion and the nonlinear equation of the considered technological process kinetics. The model includes dispersions and composes spatial and temporal changes in a porous medium. It was assumed that at any place inside the column, the filter deposit is formed as an irreversible bed filling with the subsequent formation of a reversible deposit at the working stage. The latter continues until the deposit reaches locally its maximum value. Then a break-through occurs through the filter. The equations were solved numerically using an explicit finite difference scheme. The results obtained were compared with field experiments on “EPA” installations conducted by the Israeli Water Company “Mekorot”.

The study in [3] described filtration experiments for a carbon nanotube through a screen filter. The paper provides a mathematical model for simulating carbon nanotube filtration experiments and considers a universal nanoparticle analyzer, and results related to non-spherical aerosol particles were presented.

In [4], a two-dimensional transient mathematical model was presented, showing the behavior of deep-bed filtration for aluminum. The equations for the consumption and mass of the fraction were solved using the CFX software package. The model takes into account all the basic physical processes that occur during infiltration. The concentration of inclusions remaining in the fluid and settling in the pores of the filter partitions was calculated at each time step of the model. The developed mathematical model and its software were applied for various industrial geometries of filtering units. To prove the mathematical model, the results obtained were compared with the available experimental industrial data and the influence of various model parameters on the filtration process indices was investigated.

The authors of [5] investigated the complex mechanisms of particle deposition in a filter candle. In order to study the filtration process and its modeling, filtration experiments were carried out with a suitable oil suspension particle in an experimental filter.

In [6], the issues of considering the reverse influence of the technological characteristics of the process (contamination concentration of liquid and settlement) and the characteristics of the medium (coefficients of porosity, filtration, diffusion, mass transfer, etc.) were presented and solved by the example of liquid purification in magnetic and sorption filters. An algorithm was presented for the numerical-asymptotic approximate solution of the corresponding problems of the model, described by a system of nonlinear singular differential equations of the “Convection-diffusion-mass transfer” type.

In the work [7] investigated the adsorptive removal of cationic surfactants from water using a hydrophobic adsorbent polymer. The equilibrium and kinetics of benzalkonium chloride adsorption using Amberlite XAD-16 were studied in a cyclic absorber. Fixed bed catalyst adsorption experiments were conducted to determine the dynamic loading capacity of the adsorbent.

In [8], the flow of liquids containing suspended particles in porous media was considered. A mathematical model of the interaction of a monodisperse suspension with a pore structure was presented.
The problem of determining the unknown parameters of the ion-exchange filter process by means of mathematical modeling was considered in [9]. The mathematical model was compiled from the material balance equation describing the kinetics of the ion exchange process for the nonequilibrium case. First, a numerical solution to the direct problem was given - the calculation of the concentration of impurities at the filter outlet. Then the inverse problem was formulated - finding the parameters of the ion-exchange process in nonequilibrium conditions. A method was proposed for determining the approximate values of these parameters from the concentration of impurities measured at the filter output.

The analysis of the conducted studies showed that when the mixture is filtered, the pores of the filter partition become clogged, and the hydraulic pressure on the filter surface increases, hence, the sediment layer and the filter partitions are deformed.

To take into account the influence of these factors in the suspension filtration technology, it is necessary to simulate the process, carry out CE on a computer and determine the operating mode and time of filter switching, as well as the ranges of variation of the basic parameters of filtering unit.

2. Materials and methods

To study the above process, we assume that the mixture flow occurs under the influence of a constant pressure drop in the filtration section at a time-varying flow rate \( q = q(t) \) (colmation of canals, reservoirs, earth dams, as well as the phenomenon of suffusion in the mentioned structures, etc.) or under the influence of a time variable pressure drop in the filtration section at a constant mixture flow rate \( q = q_0 \). In this case, it is assumed that the porous medium and the suspension are such that during filtration, the last part of the solid matter of the suspension is retained by the porous medium; the part of the previously settled particles breaks off and enters the filtration flow, and the other part is carried by the filtration flow beyond the area under consideration (figure 1).

Consider a unit volume of pore space in the process of mixture filtration. Note that saturation is the amount of substance per unit volume of pore space.

![Figure 1. Design scheme for filtering suspensions.](image)

From the law of conservation of balance, motion and mass conservation we have [11]:

\[
\frac{\partial \theta}{\partial t} + \frac{U \partial \theta}{m \partial x} + \frac{\partial \alpha}{\partial t} + (1 - m_0) \frac{\partial \delta}{\partial t} = \mu_0 \frac{\partial^2 \theta}{\partial x^2},
\]

\[\theta - \theta_i = \frac{\alpha}{1 - \delta}, \tag{2}\]

\[\frac{\partial \delta}{\partial t} = \lambda (\theta - \gamma \delta) \tag{3}\]

Equation (2) is written as:
\[ \alpha = (\theta - \theta_1) (1 - \delta), \]

and then we get

\[
\frac{\partial \alpha}{\partial t} = - (\theta - \theta_3) \frac{\partial \delta}{\partial t} + (1 - \delta) \frac{\partial \theta}{\partial t} - (1 - \delta) \frac{\partial \theta_1}{\partial t}.
\]

Substituting equations (3) and (4) into (1), we obtain

\[
\frac{\partial \theta}{\partial t} + \frac{U}{m} \frac{\partial \theta}{\partial x} - \lambda (\theta - \theta_1) (\theta - \gamma \delta) + (1 - \delta) \frac{\partial \theta}{\partial t} - (1 - \delta) \frac{\partial \theta_1}{\partial t} + \lambda (1 - m_0 - \theta + \theta_3) (\theta - \gamma \delta) = \mu_0 \frac{\partial^2 \theta}{\partial x^2},
\]

or

\[
(2 - \delta) \frac{\partial \theta}{\partial t} + \frac{U}{m} \frac{\partial \theta}{\partial x} - \lambda (1 - m_0 - \gamma \delta + \theta_3) \theta + \lambda \gamma \delta (1 - m_0 + \theta_3) + \lambda \theta^2 -(1 - \delta) \frac{\partial \theta_1}{\partial t} = \mu_0 \frac{\partial^2 \theta}{\partial x^2}.
\]

For simplicity, introduce the notation

\[
\lambda_1 = \lambda (1 - m_0 - \gamma \delta + \theta_3), \quad \lambda_2 = \lambda \gamma \delta (1 - m_0 + \theta_3),
\]

and then we get

\[
(2 - \delta) \frac{\partial \theta}{\partial t} + \frac{U}{m} \frac{\partial \theta}{\partial x} - \lambda_1 \theta + \lambda_2 + \lambda \theta^2 -(1 - \delta) \frac{\partial \theta_1}{\partial t} = \mu_0 \frac{\partial^2 \theta}{\partial x^2}.
\]

In view of the above, the actual statement of the problem takes the following form:

\[
\begin{cases}
(2 - \delta) \frac{\partial \theta}{\partial t} + \frac{U}{m} \frac{\partial \theta}{\partial x} - \lambda_1 \theta + \lambda_2 + \lambda \theta^2 -(1 - \delta) \frac{\partial \theta_1}{\partial t} = \mu_0 \frac{\partial^2 \theta}{\partial x^2}, \\
\frac{\partial \delta}{\partial t} = \lambda (\theta - \gamma \delta), \\
\frac{d \theta_3}{dt} = \frac{1 - \theta}{2 - \delta} \frac{d \Theta}{dt} + \frac{1}{2 - \delta} \left[ \lambda \left( \Theta - \gamma \bar{\delta} \right) (1 - \bar{\Theta}) - \frac{\theta_3 U}{H_0 (1 - \theta_3)} \right] + \lambda \left( \Theta - \gamma \bar{\delta} \right) + \frac{U}{H_0 (1 - \theta_3)}, \\
+ \Theta_3 \left[ \Theta - \gamma \bar{\delta} \right] + \frac{U}{H_0 (1 - \theta_3)}, \quad \frac{1}{2 - \delta} (t),
\end{cases}
\]

where

\[
\Theta(t) = \frac{1}{0} \Theta(x,t) dx; \quad \bar{\delta}(t) = \frac{1}{0} \delta(x,t) dx.
\]

The initial and boundary conditions for system (5) have the following form [10]:
\[
\begin{aligned}
\theta(t, x) &= \varphi_1(x), \quad \delta = 0, \quad \Theta_1 = 0 \text{ at } t = 0, \\
\theta(t, 0) &= \Theta_0, \quad \text{ at } x = 0, \\
\theta(t, 1) &= \varphi_2(t) \quad \text{ at } x = 1,
\end{aligned}
\]

where

\[
\varphi_1(x) = e^{-iH_0x}, \quad \varphi_2(t) = \Theta_0 \left(1 - A_1 A_2 e^{-A_2 t} \int_0^1 \left(2\sqrt{\alpha t} \right) dx \right),
\]

\[
B = m_0(1 - m_2) / q_0, \quad \alpha = A_1 A_2 B x.
\]

Here \( \theta \) - is the volume concentration of suspended solids in the flowing mixture; \( \Theta_0 \) - initial concentration of suspensions; \( \delta \) - the rate of particles sedimentation in the pore space; \( U \) - filtration rate; \( m \) - filter porosity; \( m_0 \) - initial porosity of the filter; \( \Theta_1 \) - outlet concentration of the mixture; \( U_0 \) - initial filtration rate; \( H_0 \) - the thickness of the filter partition; \( m_2 \) - the porosity of the settled mass; \( \mu_0 \) - coefficient of artificial viscosity; \( \lambda \) - kinetic coefficient; \( A_1, A_2 \) - experimental parameters; \( q_0 \) - initial unit flow; \( \gamma \) - specific gravity of the substance; \( I_0 \) - zero-order Bessel function.

Since the problem posed is described by nonlinear second-order partial differential equations, it is difficult to obtain its solution in an analytical form. Therefore, the problem posed will be solved by a numerical method based on the finite-difference approximation of differential operators to incremental ones. As a result, we obtain a system of algebraic equations for the sought variables [10-11].

To numerically integrate the problem and provide a stable and conservative scheme, we use an implicit finite difference scheme and obtain the following:

\[
(2 - \delta) \frac{\Theta_{t+1} - \Theta_t}{\tau} + \frac{U}{m} \frac{\Theta_{t+1} - \Theta_{t-1}}{h^2} - \lambda_3 \Theta_{t+1} + \lambda_2 + \lambda \left(\Theta_{t+1}\right)^2 - (1 - \delta) \frac{\partial \Theta_t}{\partial t} = \mu_0 \frac{\Theta_{t+1} - 2\Theta_{t+1} + \Theta_{t-1}}{h^2}.
\]

The nonlinear terms in equation (7) are linearized as follows:

\[
\left(\Theta_{t+1}\right)^2 = 2\Theta_{t+1} \cdot \Theta_{t-1} - \left(\Theta_t\right)^2.
\]

We obtain

\[
(2 - \delta) \frac{\Theta_{t+1} - \Theta_t}{\tau} + \frac{U}{m} \frac{\Theta_{t+1} - \Theta_{t-1}}{h^2} - \lambda_3 \Theta_{t+1} + \lambda_2 + 2\lambda \Theta_{t+1} \cdot \Theta_{t-1} - \lambda \left(\Theta_t\right)^2 - (1 - \delta) \frac{\partial \Theta_t}{\partial t} = \mu_0 \frac{\Theta_{t+1} - 2\Theta_{t+1} + \Theta_{t-1}}{h^2},
\]

or

\[
(2 - \delta) \frac{\Theta_{t+1} - \Theta_t}{\tau} + \frac{U}{m} \frac{\Theta_{t+1} - \Theta_{t-1}}{h^2} - \left(\lambda_3 - 2\lambda \Theta_{t-1}\right) \Theta_{t+1} + \lambda_2 - \lambda_3 = \mu_0 \frac{\Theta_{t+1} - 2\Theta_{t+1} + \Theta_{t-1}}{h^2}.
\]

Here

\[
\lambda_3 = \lambda \left(1 - m_0 - \gamma \delta_1 + \Theta_{t+1}\right), \quad \lambda_2 = \lambda \gamma \delta_1 \left(1 - m_0 + \Theta_{t+1}\right), \quad \lambda_3 = \lambda \left(\Theta_t\right)^2 + (1 - \delta) \frac{\partial \Theta_t}{\partial t}.
\]
Further, grouping the terms of equation (7), we obtain

\[ a_i \theta_{i+1} - b_i \theta_i + c_i \theta_{i-1} = -d_i, \quad (8) \]

where

\[ a_i = \frac{\mu_0}{h_i^2} - \frac{U}{2mh_i}, \quad b_i = \frac{2\mu_0}{h_i^2} + \frac{2 - \delta_i}{\tau} + \lambda_i, \quad c_i = \frac{\mu_0}{h_i^2} + \frac{U}{2mh_i}, \quad d_i = \frac{2 - \delta_i}{\tau} \theta_i^e - \lambda_{2i} + \lambda_{3i}. \]

We seek a solution to problem (8) in the form

\[ \theta_i = A_i \theta_{i+1} + B_i \]

where the sweep coefficients are determined using

\[ A_i = \frac{a_i}{b_i - c_i A_{i-1}}, \quad B_i = \frac{a_i + c_i b_{i-1}}{b_i - c_i A_{i-1}}, \quad i = 1, 2, ..., N - 1. \]

The sweep coefficients \( A_0, B_0 \) are determined from condition (6). In this case - \( A_0 = 0, B_0 = 1 \). Solving equation (8), we determine the mixture concentration on the surface of the filtering partition of the filter. After determining the concentration at each time layer, we calculate the deposition rate of particles \( \delta \) in the pores of the filter and the output concentration \( \theta_3 \) of the solution.

Knowing the values of \( \theta_3 \) and \( \delta \) at each time layer, we calculate the changes in the pressure drop using the equation of motion of the liquid phase:

\[ \frac{c_p}{\rho H} = \frac{\mu H U}{\rho H k_o (1 - \delta)^2} - \frac{U}{\rho H k_o (1 - \delta)^2} \frac{d \theta_i}{dt}, \]

or

\[ \frac{c_p}{\rho H} = \frac{\mu H U}{k_o (1 - \delta)^2} + \frac{\rho H U}{k_o (1 - \delta)^2} \frac{d \theta_3}{dt}, \quad (9) \]

provided \( p(0, t) = p_0 \).

Here \( k_o \) is the permeability coefficient; \( \mu \) - fluid viscosity; \( \rho \) - fluid density; \( H \) the height of the filter column, \( p \) - the surface pressure inside the filter column.

When calculating the pressure in formula (9), the right-hand side of the third equation of system (5) is substituted by \( d \theta_3 / dt \).

3. Results and discussion

According to the given algorithm, a numerical calculation was made to determine the concentration of particles in the solution in the technological process of purifying the suspension. The calculations were carried out with the following initial data:

\[ \rho = 2200.0 \, \text{kg/m}^3; \quad \mu = 0.983 \, \text{kg/m/s}; \quad k_o = 0.0023 \, \text{m}^2; \quad \lambda = 0.0026 \, \text{s}^{-1}; \]
\[ H = 1.0 \, \text{m}; \quad \gamma = 0.0079; \quad \delta_0 = 1 \times 10^{-5}; \quad \theta_0 = 0.478 \times 10^{-5}. \]

According to the numerical calculations performed on a computer, one of the essential parameters for suspension filtering is the porosity of the filter. The porosity of the filter affects the flow rate of the suspension and the deposition of gel particles. Numerical calculations showed that the filling of the filter pores with gel particles occurs in the upper layers of the filter, and under the action of hydraulic surface pressure, they move along with the depth of the filter partition.
Concentration change in time and filter thickness.

Numerical experiments showed that the concentration increase in the upper layers of the filter is faster than in the lower layers (figure 2). Besides, the concentration in the filter pores at the initial time point $t = 1-3 \ h$ decreases linearly along with the depth of the filter pores, at $t > 4 \ h$, the decrease in concentration along with the depth of the filter pores gradually turns into a logarithmic law (figure 3).

Concentration change at a fixed time depending on the filter pore thickness.

Numerical results showed that with an increase in the filter, the change in concentration decreases, this is especially noticeable in the lower layer of the filter (figure 4).
Figure 4. Change in concentration at different values of filter porosity depending on time: a) at $x = 10$ cm, b) at $x = 20$ cm.

To calculate the input parameters response to the process and the operating mode of the filter, a number of numerical experiments were conducted for various values $H_0$ and $U_0$. For the first series of experiments, leaving $U_0 = 0.00021 \text{ m/s}$ unchanged, the numerical calculations were carried out at $H_0 = 0.2; 0.4; 0.5; 0.25; 0.35 \text{ m}$. According to the numerical calculations carried out on a computer, with an increase in the thickness of the filter partition, the concentration of gel particles on the filter surface grows exponentially. This is especially noticeable at $H_0 > 0.25 \text{ m}$.

For the second series of experiments, the values of $U_0 = 0.0002; 0.001; 0.0004; 0.0005 \text{ m/s}$ were changed. The results obtained showed that the thicker the filter, the better the filtrate; however, it takes more time to clean the same volume, and the pressure in the column grows with an increase in $H_0$.

4. Conclusions. With an increase in the suspension feed rate into the column of the unit, the containment of particles in the filter column decreases. This is because, on the one hand, an increase in rate brings a large number of suspended particles and promotes rapid filling of the filter pores, on the other hand, it creates an additional force to break off already settled particles, and thereby degrades the quality of the filtered fluid.

References

[1] Santos A and Bedrikovetsky P 2004 *Computational and Applied Mathematics* **23**(2-3) 259-84
[2] Gitisa Vitaly et al. 2010 *Chemical Engineering Journal* **163** 78-85
[3] Jing Wang and David Y.H. Pui. 2013 *Chem. Eng. Sci.* **85** 69-76
[4] Duygu Kocaefe, Rung Tien Buia and Peter Waite 2009 *Applied Mathematical Modelling* **33** 4013-30
[5] Fernandez X R, Rosenthal I, Anlauf H and Nirschl H 2001 *Chemical Engineering Research and Design* **89**(12) 2776-84
[6] Safonyk Andrii and Bomba Andrii. 2014 *International Journal of Applied Mathematical Research*. 4(1) 10.14419/ijamr.v4i1.3805
[7] Turku I and Sainio T 2009 *Separation and Purification Technology* **69** 185-94
[8] Kapranov Yu I 2000 *Applied Mechanics and Technical Physics* **41**(2) 113-21
[9] Mukanova Balgaisha and Glazyrina Natalya 2015 *Mathematical Problems in Engineering* 2015. 10.1155/2015/357829
[10] Palvanov B *et al*. 2020 *IOP Conf. Ser.: Mater. Sci. Eng.* **862** 062010
[11] Palvanov B Yu 2016 *Problems of Computational and Applied Mathematics* **1** 48-63