Superfield Quantization

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Abstract

We present a superfield formulation of the quantization program for theories with first class constraints. An exact operator formulation is given, and we show how to set up a phase-space path integral entirely in terms of superfields. BRST transformations and canonical transformations enter on equal footing, and they allow us to establish a superspace analog of the BFV theorem. We also present a formal derivation of the Lagrangian superfield analogue of the field-antifield formalism, by an integration over half of the phase-space variables.

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1 Introduction

When quantizing gauge theories of the most general kind, an absolutely fundamental rôle is played by a rigid symmetry that transforms bosons into fermions, and vice versa. Ever since its discovery, be it in the Hamiltonian or Lagrangian formulation, it has been a dream to make this symmetry manifest by means of a superfield formulation. Some of the first preliminary steps in this direction were taken in ref. [5], and there has since been an enormous amount of literature on the subject (see, e.g., ref. [6] for a more recent approach, and references therein). Almost all of these attempts have been restricted to the Lagrangian formalism, and it is probably fair to say that a completely general formulation has not been found yet.

In view of this, we find it appropriate to reconsider the problem from a completely new direction. As is usually the case, it is instructive to go back to the basic starting point, which is the operator formalism. There the essential ingredients are the equal-time canonical commutation relations and the Heisenberg equations of motion, which describe the time evolution of the system. The needed superspace will therefore be two-dimensional, consisting of just $t$, ordinary time, and a new Grassmann-odd direction, which we denote by $\theta$. All coordinates and momenta, and all operators involving them, must then be generalized to superfield operators. Because of the Grassmann-odd nature of the additional coordinate, any object $A(t, \theta)$ defined in terms of these basic superfields will have a formal Taylor expansion in $\theta$ that truncates after just one term:

$$A(t, \theta) = A_0(t) + \theta A_1(t). \quad (1.1)$$

Therefore $A(t, \theta)$ will have the same statistics as $A_0(t)$. While the superspace of this construction is inherently two-dimensional, it in an obvious manner becomes extended to a $(d+1)$-dimensional superspace of coordinates $(x^\mu, \theta)$ when considered in the context of a Lorentz covariant quantum field theory in $d$ dimensions.

Obtaining a superfield formulation of the quantization program has the obvious advantage that the fermion-boson symmetry between all required fields, ghosts, ghosts-for-ghosts, Lagrangian multipliers, etc., becomes manifest. All standard superspace techniques are then applicable to the analysis of the perturbative expansion. The BRST symmetry in particular will be kept in a manifest manner at all stages of the computation. At a purely conceptual level it is pleasing to see that, as has long been suspected, the BRST symmetry can be understood in terms of an extension of space-time to include an additional fermionic direction. This puts the quantization program for dynamical systems with first-class constraints into a new geometrical framework.

We shall not here be concerned with the application of the superfield formalism to specific (field) theories, but shall rather seek a completely general framework, independent of the underlying Hamiltonian dynamics. Our initial input shall thus be the already proven existence of suitable BRST operators for the dynamics governed by first-class constraints. This entails the extension of the symplectic phase space to a possibly huge set of additional ghost fields and Lagrangian multipliers, all of which has been completely and rigorously established [1, 2, 3].

Once the required set of fields has been introduced, the essential three ingredients of the quantization program in the Hamiltonian formalism are: 1) The bosonic Hamiltonian operator $\hat{H}$, 2) The fermionic BRST operator $\hat{\Omega}$, and 3) A fermionic gauge-fixing function $\hat{\Psi}_1$. In our sought-for

*From now on we for simplicity denote this “BRST symmetry”, even when referring to its Hamiltonian counterpart.
superfield formulation, these three objects must be grouped into suitable combinations. It turns out to be most natural to link together the Hamiltonian and the BRST operator into one single (fermionic) object \( \hat{Q} = \hat{\Omega} + \theta \hat{H} \), which is nilpotent. The fermionic object \( \hat{\Psi}_1 \) is thus left over, without a bosonic superfield partner. As might have been expected from the operator quantization program, this is due to the fact that the one other fundamental freedom of the theory, that of canonical transformations on the phase space, has not been taken into account. Just as the (bosonic) Hamiltonian is linked together with the (fermionic) generator of BRST transformations, so the (fermionic) gauge-fixing function \( \hat{\Psi}_1 \) is linked with a (bosonic) generator of canonical transformations. All four objects enter in a tightly-knit way, and it should then come as no surprise that also proofs of gauge independence in this superfield formulation involve the use of both types of transformations.

Our paper is organized as follows. In the next section we introduce the required superspace derivatives, and postulate the superfield operator equations of motion. Consistency conditions are found to be satisfied, and the resulting dynamics reduces, in the subsector of original variables to that of the original Heisenberg equations of motion. We proceed to derive, in superfield language, the gauge independence of physical matrix elements, using the exact operator formulation. In Section 4 we propose a superfield phase-space path integral, and demonstrate that it satisfies all the criteria one must require for consistency with original path integral formulation. In particular, as one of the most important steps of this section, we establish a superfield version of the BFV Theorem. In Section 5 we show how to formally derive an analogous superfield Lagrangian path integral by the formal integration over half of the phase-space variables. Introducing suitable sources for BRST-transforms in the Hamiltonian language, we are naturally led to a superfield formulation of the field-antifield formalism. In particular, we can formally show that the superfield action must satisfy a superfield quantum Master Equation based on a nilpotent superfield \( \Delta \)-operator. Section 6 contains our conclusions. In Appendix A we outline an alternative proof of the superfield BFV Theorem, here using a more symmetric set of transformations in the path integral. We give our conventions, and some useful superfield identities, in Appendix B.

2 The Operator Formalism

Let us begin by introducing some notation. We consider a dynamical system with first-class constraints (generators of gauge symmetries), and of phase-space dimension \( 2N \). The phase space variables are denoted collectively by \( z_0^A(t) \). They have general Grassmann parities \( \epsilon(z_0^A) \equiv \epsilon_A \). We distinguish usual \( c \)-numbers from their associated operator counterparts by means of hats. Thus, for example, \( \hat{z}_0^A(t) \) are the operators corresponding to \( z_0^A(t) \). Grassmann parities of operators are of course inherited from their \( c \)-number ancestors. The supercommutator of two operators is defined as usual:

\[
[\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - (-1)^{\epsilon(\hat{A})\epsilon(\hat{B})}\hat{B}\hat{A}.
\] (2.1)

We begin by making a superfield extension of the phase space variables. In operator form,

\[
\hat{z}^A(t, \theta) \equiv \hat{z}_0^A(t) + \theta \hat{z}_1^A(t).
\] (2.2)

Obviously, the superpartner \( \epsilon(z_1^A) = \epsilon_A + 1 \) has opposite statistic. The superfield \( \hat{z}^A(t, \theta) \) is uniquely determined by a first order equation of motion (see Section 2.2) and initial data at time \( t_i \) and \( \theta_i = 0 \).
Quantization is imposed by equal-$t$-equal-$\theta$ canonical commutator relations

\[ [\hat{z}^A(t, \theta), \hat{z}^B(t, \theta)] = i\hbar \dot{\omega}^{AB}(t, \theta) , \]  

(2.3)

In order to maintain this quantization relation for all pairs $(t, \theta)$, the $\dot{\omega}^{AB}(t, \theta)$ should commute with the evolution operator in the $t$ and the $\theta$ direction. We shall guarantee this by imposing the simplifying ansatz

\[ \dot{\omega}^{AB}(t, \theta) = \omega^{AB} \hat{1} , \]  

(2.4)

for some constant invertible symplectic metric $\omega^{AB}$. This commutes trivially with any evolution. As a result quantization (i.e. the process of reducing commutators with the help of eq. (2.3)) and translation in the $t$ and $\theta$ directions commute. For an arbitrary operator $\hat{A}(\hat{z}^A(t, \theta), t, \theta)$ we define $(t', \theta')$-translations $\tau(t', \theta')$ by

\[ (\tau(t', \theta')\hat{A})(\hat{z}^A(t, \theta), t, \theta) \equiv \hat{A}(\hat{z}^A(t-t', \theta-\theta'), t-t', \theta-\theta') . \]  

(2.5)

We therefore have

\[ \tau(t', \theta')[\hat{A}, \hat{B}] = [\tau(t', \theta')\hat{A}, \tau(t', \theta')\hat{B}] , \]  

(2.6)

where it is understood that all commutators are replaced with the l.h.s. of the canonical quantization relations (2.3). Using the principle that quantization and translation should commute we see that the quantization relation eq. (2.3) (with ansatz (2.4)) is equivalent to the ordinary equal-time relation:

\[ [\hat{z}^A_0(t), \hat{z}^B_0(t)] = i\hbar \omega^{AB} \hat{1} . \]  

(2.7)

We now define two superspace derivatives by

\[ D \equiv \frac{d}{d\theta} + \theta \frac{d}{dt} , \quad \bar{D} \equiv \frac{d}{d\theta} - \theta \frac{d}{dt} , \]  

(2.8)

and, since we shall also be considering explicit differentiation,

\[ D \equiv \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial t} , \quad \bar{D} \equiv \frac{\partial}{\partial \theta} - \theta \frac{\partial}{\partial t} . \]  

(2.9)

Note that, as expected, $D$ and $\bar{D}$ both act like “square roots” of the time derivative:

\[ D^2 = \frac{d}{dt} = -\bar{D}^2 , \quad [D, \bar{D}] = 0 . \]  

(2.10)

Although $D$ and $\bar{D}$ themselves are fermionic, their squares are thus bosonic operators without any trace left of their fermionic origin. As we shall see shortly, this property is crucial in setting up a correct superfield formulation.

Without considering the superfield extension, the existence of these two operators for an arbitrary admissible gauge algebra (possibly open, of any rank, and possibly reducible, at any finite stage) associated with the first-class constraints is known from the work of ref. [1]. They are defined on
the extended phase space of fields that includes the required number of symplectic ghost field pairs and Lagrange multipliers. Here we for the moment assume that the required superfield extension is possible, and we shall subsequently, in the next section, verify that this indeed is the case.

In our construction it turns out that a more fundamental rôle is played by two Grassmann-odd operators, which are combinations of the BRST operator and the Hamiltonian:

\[
\hat{Q} = \hat{Q}(\hat{z}(t, \theta), \theta) = \hat{\Omega}(\hat{z}(t, \theta)) + \theta \hat{H}(\hat{z}(t, \theta)) , \\
\hat{\bar{Q}} = \hat{\bar{Q}}(\hat{z}(t, \theta), \theta) = \hat{\Omega}(\hat{z}(t, \theta)) - \theta \hat{H}(\hat{z}(t, \theta)).
\]  

(2.12)

They are both nilpotent (anticommuting) by virtue of eq. (2.11):

\[
[\hat{Q}, \hat{Q}] = 0 , \\
[\hat{Q}, \hat{\bar{Q}}] = 0 , \\
[\hat{\bar{Q}}, \hat{\bar{Q}}] = 0 .
\]  

(2.13)

We demand that both \(\hat{H}\) and \(\hat{\Omega}\) be hermitian, a property which is then inherited by \(\hat{Q}\) and \(\hat{\Omega}\) if \(\theta\) is real:

\[
\hat{H}^\dagger = \hat{H} , \\
\hat{\Omega}^\dagger = \hat{\Omega} , \\
\hat{Q}^\dagger = \hat{Q} , \\
\hat{\bar{Q}}^\dagger = \hat{\bar{Q}}.
\]  

(2.14)

2.1 Gauge Fixing

All of these relations hold before any gauge fixing. As is well known, for theories with first-class constraints, the supercommutation relations (2.11) encode the gauge algebra of the theory, and we can view \(\hat{H}\) and \(\hat{\Omega}\) as generating operators of this gauge algebra. The phase space operators \(\hat{z}_A^0\) is of course extended to include the full required set of ghost fields, and, in Darboux coordinates, their canonically conjugate momenta. Their superfield generalizations follow from eq. (2.2). To obtain well-defined dynamics, gauge fixing must be correctly implemented. According to the BFV theorem [1], this is accomplished with the help of a gauge fermion \(\hat{\Psi}_1\), whose supercommutator with \(\hat{\Omega}\) is added to \(\hat{H}\):

\[
\hat{H}_{\hat{\Psi}_1} = \hat{H} + (i\hbar)^{-1}[\hat{\Psi}_1, \hat{\Omega}] .
\]  

(2.15)

The subscript 1 will be explained below.

Let us now consider the problem in the superfield formulation. We propose to achieve the required gauge fixing by means of the following construction. Introduce two bosonic Hermitian operators \(\hat{\Psi}'\) and \(\hat{\Psi}''\) (and the corresponding barred operators \(\hat{\bar{\Psi}}'\) and \(\hat{\bar{\Psi}}''\), where the explicit \(\theta\)-dependence comes with the sign changed, similarly to eq. (2.12).):

\[
\hat{\Psi}' = \hat{\Psi}'(\hat{z}(t, \theta), \theta) = \hat{\Psi}'_0(\hat{z}(t, \theta)) + \theta \hat{\Psi}'_1(\hat{z}(t, \theta)) , \\
\hat{\Psi}'' = \hat{\Psi}''(\hat{z}(t, \theta), \theta) = \hat{\Psi}''_0(\hat{z}(t, \theta)) + \theta \hat{\Psi}''_1(\hat{z}(t, \theta)).
\]  

(2.16)

They are composed of, in total, four components, \(\hat{\Psi}'_0, \hat{\Psi}'_1, \hat{\Psi}''_0\) and \(\hat{\Psi}'_1\), of the following statistics:

\[
\epsilon(\hat{\Psi}'_0) = 0 = \epsilon(\hat{\Psi}''_0) , \\
\epsilon(\hat{\Psi}'_1) = 1 = \epsilon(\hat{\Psi}''_1) .
\]  

(2.17)

We shall see below that a combination of the one-components \(\hat{\Psi}'_1\) and \(\hat{\Psi}''_1\) comprise the usual gauge fermion of the BFV construction when \(\hat{\Psi}_0 = 0\). This explains why we put the subindex 1 in eq. (2.15).

For notational convenience, let us denote \(\hat{\Psi}'_0\) by just \(\hat{\Psi}_0\). The relevant nilpotent BRST operator is

\[
\hat{\Omega}_{\hat{\Psi}_0} = e^{(i\hbar)^{-1}\text{ad}_{\hat{\Psi}_0}}\hat{\Omega} = e^{(i\hbar)^{-1}\hat{\Psi}_0} \hat{\Omega} e^{-(i\hbar)^{-1}\hat{\Psi}_0} ,
\]  

(2.18)
where we have let
\[ \text{ad} \hat{X} \equiv [\hat{X}, \cdot ] \]
denote the adjoint action of an operator \( \hat{X} \). Next, choose a \( \hat{\Psi} \)-dressed \( Q \)-operator be a sum of an “exponential” and a “linear” gauge-fixing part:
\[
\hat{Q}_{\hat{\Psi}} = \hat{Q}_{\hat{\Psi}}(\hat{\zeta}(t, \theta), \theta) = e^{(i\hbar)^{-1} \text{ad} \hat{\Psi}(\hat{\zeta}(t, \theta), \theta)} \hat{Q}(\hat{\zeta}(t, \theta), \theta) + (i\hbar)^{-1} [\hat{\Psi}''(\hat{\zeta}(t, \theta), \theta), \hat{Q}_{\hat{\Psi}}(\hat{\zeta}(t, \theta))] .
\]
(2.20)

We shall later, in Section 2.2, see that \( \hat{Q}_{\hat{\Psi}} \) plays the rôle of a “super-Hamiltonian” i.e., an evolution operator for the superfields. It can be written in terms of two components:
\[
\hat{Q}_{\hat{\Psi}}(\hat{\zeta}(t, \theta), \theta) = \hat{\Omega}_{\hat{\Psi}}(\hat{\zeta}(t, \theta)) + \theta \hat{H}_{\hat{\Psi}}(\hat{\zeta}(t, \theta)) ,
\]
(2.21)

where
\[
\hat{\Omega}_{\hat{\Psi}} = \hat{\Omega}_{\hat{\Psi}}(\hat{\zeta}(t, \theta)) \equiv \hat{Q}_{\hat{\Psi}}(\hat{\zeta}(t, \theta), 0) ,
\]
\[
\hat{H}_{\hat{\Psi}} = \hat{H}_{\hat{\Psi}}(\hat{\zeta}(t, \theta)) \equiv \frac{\partial}{\partial \theta} \hat{Q}_{\hat{\Psi}}(\hat{\zeta}(t, \theta), \theta) .
\]
(2.22)

Inserting the definition (2.12) into eq. (2.20), we see that going from the operator \( \hat{Q} \) to \( \hat{Q}_{\hat{\Psi}} \) corresponds to the following modifications of \( \hat{\Omega} \) and \( \hat{H} \):
\[
\hat{\Omega}_{\hat{\Psi}} = \hat{\Omega}_{\hat{\Psi}_0} + (i\hbar)^{-1} [\hat{\Psi}''_0, \hat{\Omega}_{\hat{\Psi}_0}] ,
\]
(2.23)

and
\[
\hat{H}_{\hat{\Psi}} = e^{(i\hbar)^{-1} \text{ad} \hat{\Psi}_0} \left( \hat{H} + (i\hbar)^{-1} \left[ \int_0^1 \text{d} \alpha \left( e^{-\alpha(i\hbar)^{-1} \text{ad} \hat{\Psi}_0} \hat{\Psi}_1' \right) + e^{-(i\hbar)^{-1} \text{ad} \hat{\Psi}_0} \hat{\Psi}_0' \right] \right) + (i\hbar)^{-1} [\hat{E}_0(\hat{\Psi}_1) + \hat{\Psi}_1'', \hat{\Omega}_{\hat{\Psi}_0}] .
\]
(2.24)

We have here used the shorthand notation of
\[
\hat{E}_0(\hat{X}) \equiv \int_0^1 \text{d} \alpha \ e^{\alpha(i\hbar)^{-1} \text{ad} \hat{\Psi}_0} \hat{X} = f \left( (i\hbar)^{-1} \text{ad} \hat{\Psi}_0 \right) \hat{X} ,
\]
(2.25)

where
\[
f(x) = \int_0^1 \text{d} \alpha \ e^{\alpha x} = \frac{e^x - 1}{x} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!} .
\]
(2.26)

It follows from eqs. (2.23) and (2.24) that the definition (2.20) has the following effects:

- \( \hat{\Psi}_0 \equiv \hat{\Psi}'_0 \) induces a canonical transformation on both \( \hat{\Omega} \) and \( \hat{H} \).

- Both \( \hat{\Psi}_1' \) and \( \hat{\Psi}_0'' \) induce gauge-fixing terms in the Hamiltonian. From now on we will therefore call \( \hat{H}_{\hat{\Psi}} \) of eq. (2.24) the superfield unitarizing Hamiltonian.

- \( \hat{\Psi}_0'' \) adds a term to the zero-component \( \hat{\Omega}_{\hat{\Psi}} \), so that it in general is different from the pertinent BRST operator \( \hat{\Omega}_{\hat{\Psi}_0} \).
All hermiticity properties (2.14) are inherited by the $\hat{\Psi}$-dressed operators. This follows from the fact that the superfield $\hat{\Psi}$ must be hermitian (consistent with the fact that the original “gauge fermion” $\hat{\Psi}_1$ must be chosen antihermitian):

$$\hat{\Psi}^\dagger = \hat{\Psi}, \quad \hat{\Psi}_1^\dagger = -\hat{\Psi}_1 .$$

(2.27)

The operator $\hat{E}_0$ is invertible:

$$\hat{E}_0^{-1}(\hat{X}) = \left(\frac{1}{f}\right) \left((i\hbar)^{-1}\text{ad}\hat{\Psi}_0\right) \hat{X} ,$$

(2.28)

where

$$\frac{1}{f(x)} = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!} = 1 - \frac{x}{2} - \sum_{k=1}^{\infty} B'_k \left(-x^2\right)^k (2k)! ,$$

(2.29)

is the generating function for the Bernoulli numbers. This shows that one can change the 1-component of the gauge-fixing from a “exponential” gauge to a “linear” gauge and vice-versa. This is an important property, and it shows that one of the four gauge-fixing components mentioned above in fact is redundant. However, depending on the application, different forms of the gauge-fixing can prove convenient.

The “super-Hamiltonian” $\hat{Q}_{\hat{\Psi}}$ is BRST invariant:

$$[\hat{Q}_{\hat{\Psi}}(\hat{z}(t, \theta), \hat{\Omega}_{\hat{\Psi}_0}(\hat{z}(t, \theta))) = 0 .$$

(2.30)

This leads to the fact that the BRST operator $\hat{\Omega}_{\hat{\Psi}_0}(\hat{z}(t, \theta))$ is a constant of motion; see also the discussion in Section 2.2-2.3.

For consistency (the equations of motion should be integrable, see Section 2.2) we require that the “super-Hamiltonian” is nilpotent:

$$[\hat{Q}_{\hat{\Psi}}(\hat{z}(t, \theta), \hat{\Omega}_{\hat{\Psi}_0}(\hat{z}(t, \theta))) = 0 .$$

(2.31)

Remarkably, this crucial condition yields both of the equations

$$[\hat{\Omega}_{\hat{\Psi}}(\hat{z}(t, \theta)), \hat{Q}_{\hat{\Psi}}(\hat{z}(t, \theta), \theta) = 0 .$$

(2.32)

Conversely, they of course imply the nilpotency condition (2.31). The conditions (2.32) are automatically satisfied for gauges with $\hat{\Psi}_0'' = 0$. Although there are non-trivial gauges with $\hat{\Psi}_0'' \neq 0$ fulfilling this consistency requirement (2.32), we shall for simplicity from now on limit ourselves to the situation where $\hat{\Psi}_0'' = 0$. Then $\hat{\Omega}_{\hat{\Psi}}$ coincides with the BRST operator $\hat{\Omega}_{\hat{\Psi}_0}$, and the unitarizing operators can, as explained earlier, be written entirely with the help of the “exponential” form of gauge fixing. However, most of what follows can easily be extended to include consistent $\hat{\Psi}_0'' \neq 0$ gauges.

† Note that there are two different conventions for the Bernoulli numbers in the literature.
2.2 Equations of Motion

Consider now the following superfield equation of motion:

\[ i\hbar D\dot{z}^A(t, \theta) = -[\hat{Q}_\psi(\dot{z}(t, \theta), \theta), \dot{z}^A(t, \theta)] \, . \]  
\[ (2.33) \]

By applying the \( D \)-operator again, and taking into account the explicit \( \theta \)-dependence on the r.h.s. of eq. (2.33) when performing the second differentiation, as well as using the Jacobi identity for the supercommutator, one finds, remarkably,

\[ i\hbar \frac{d}{dt}\dot{z}^A(t, \theta) = -[\hat{H}_\psi(\dot{z}(t, \theta)), \dot{z}^A(t, \theta)] \, . \]  
\[ (2.34) \]

In fact, this is just the anticipated form of the Heisenberg equation of motion for the superfield \( \dot{z}^A \) and the unitarizing Hamiltonian \( \hat{H}_\psi \). It is an exact superfield operator relation.

Multiplying both sides of eq. (2.34) by \( \theta \) from the left, and substituting into eq. (2.33), one finds that the other independent relation contained in (2.33) is

\[ i\hbar \frac{d}{d\theta}\dot{z}^A(t, \theta) = -[\hat{\Omega}_\psi(\dot{z}(t, \theta)), \dot{z}^A(t, \theta)] \, . \]  
\[ (2.35) \]

Thus, while the ordinary time evolution is generated by the Hamiltonian \( \hat{H}_\psi \), the evolution in the Grassmann-odd direction is generated by the operator \( \hat{\Omega}_\psi \). The condition \( (d/d\theta)^2\dot{z}^A(t, \theta) = 0 \) is consistently reproduced from the right hand side of eq. (2.35) on account of the nilpotency condition on \( \hat{\Omega}_\psi \) from eq. (2.11). Conversely, in a superspace formulation, the nilpotency condition on the BRST charge can be viewed as a trivial consequence of the fermionic nature of the \( \theta \)-direction. (Remarkably, due solely to the presence of the explicit \( \theta \)-dependence in the definition (2.12), the nilpotency condition on \( \hat{Q}_\psi \), eq. (2.13), does not lead to the analogous and would-be fatal conclusion that the right hand side of eq. (2.34) should vanish.)

By differentiating eq. (2.34) with respect to \( \theta \), and differentiating eq. (2.35) with respect to \( t \), we see that the integrability condition \( [d/dt, d/d\theta]\dot{z}^A(t, \theta) = 0 \) is satisfied on account of the second equation in (2.32), and because of the Jacobi identity for supercommutators.

2.3 Constants of Motion

Let us now analyze the two eqs. of motion (2.34) and (2.35) in more detail. First, although eq. (2.34) looks deceptively like the correct quantum mechanical equation for the original phase space operators \( \dot{z}_0^A(t) \), it should be recalled that both components of the superfields \( \dot{z}^A(t, \theta) \) are involved, and that they in principle could mix through \( \hat{H}_\psi(z(t, \theta)) \). Consider, however, the \( \theta \)-dependence of the unitarizing Hamiltonian. We find it by taking the derivative, and using eq. (2.35):

\[ \frac{d}{d\theta}\hat{H}_\psi(\dot{z}(t, \theta)) = -(i\hbar)^{-1}[\hat{\Omega}_\psi(\dot{z}(t, \theta)), \hat{H}_\psi(\dot{z}(t, \theta))] = 0 \, , \]  
\[ (2.36) \]

where the last equality follows from eq. (2.11). Thus, the operator \( \hat{H}_\psi(z(t, \theta)) \) is, despite appearances, \( \theta \)-independent. Similarly, one finds that

\[ \frac{d}{d\theta}\hat{\Omega}_\psi(\dot{z}(t, \theta)) = -(i\hbar)^{-1}[\hat{\Omega}_\psi(\dot{z}(t, \theta)), \hat{\Omega}_\psi(\dot{z}(t, \theta))] = 0 \, . \]  
\[ (2.37) \]
Also the operator $\hat{\Omega}_\Psi(z(t, \theta))$ is therefore $\theta$-independent. The corresponding argument applied to eq. (2.34) shows that, as expected, both the unitarizing Hamiltonian $\hat{H}_\Psi$ and the operator $\hat{\Omega}_\Psi$ are constants of motion:

$$
\frac{d}{dt} \hat{H}_\Psi(\hat{z}(t, \theta)) = -(i\hbar)^{-1}[\hat{H}_\Psi(\hat{z}(t, \theta)), \hat{H}_\Psi(\hat{z}(t, \theta))] = 0
$$

$$
\frac{d}{dt} \hat{\Omega}_\Psi(\hat{z}(t, \theta)) = -(i\hbar)^{-1}[\hat{H}_\Psi(\hat{z}(t, \theta)), \hat{\Omega}_\Psi(\hat{z}(t, \theta))] = 0 .
$$

One important consequence of the above results is the following. Since both $\hat{\Omega}_\Psi$ and $\hat{H}_\Psi$ are $\theta$-independent, and hence in fact only functions of the zero-components $\hat{z}_0^A$, the fundamental supercommutator relations between them can be satisfied by the conventional solutions [1], once $\Psi$-dressed in the manner described above. The existence of appropriate solutions to the fundamental supercommutation relation (2.31) is therefore guaranteed.

### 2.4 Component Formulation

Letting $\theta = 0$, the two superfield equations of motion (2.34) and (2.35) reduce to

$$
i\hbar \frac{d}{dt} \hat{z}^A(t) = - [\hat{H}_\Psi(\hat{z}_0(t)), \hat{z}^A_0(t)]
$$

and

$$
i\hbar \hat{z}^A(t) = - [\hat{\Omega}_\Psi(\hat{z}_0(t)), \hat{z}^A_0(t)] ,
$$

respectively. The former (2.39) of these relations is precisely the correct Heisenberg eq. of motion for the original phase space operators $\hat{z}_0^A(t)$ with respect to the unitarizing Hamiltonian $\hat{H}_\Psi$. The latter relation (2.40) shows that the corresponding superfield partners $\hat{z}_1^A(t)$ are given, through an exact operator identity, by the $\hat{\Omega}_\Psi$-transform of the original $\hat{z}_0^A$'s.

The two equations (2.39) and (2.40) are in fact equivalent to (2.34) and (2.35). This is trivial for the former equation (2.34) by performing a super translation of (2.39). The latter equation follows from two observations: First, by (2.40), the Jacobi identity and nilpotency of $\hat{\Omega}_\Psi(\hat{z}_0(t))$

$$
[\hat{z}_1^A(t), \hat{\Omega}_\Psi(\hat{z}_0(t))] = 0 .
$$

Second, $\hat{\Omega}_\Psi(\hat{z}(t, \theta))$ is independent of $\theta$:

$$
\hat{\Omega}_\Psi(\hat{z}(t, \theta)) = \hat{\Omega}_\Psi \left( \hat{U}^{-1}(t) \hat{z}_0(t) \hat{U}(t) \right) = \hat{U}^{-1}(t) \hat{\Omega}_\Psi(\hat{z}_0(t)) \hat{U}(t) = \hat{\Omega}_\Psi(\hat{z}_0(t)) ,
$$

for

$$
\hat{U}(t) \equiv \exp \left[ (i\hbar)^{-1} \theta \hat{\Omega}_\Psi(\hat{z}_0(t)) \right] .
$$

Now eq. (2.35) follows quite easily.
2.5 Ghost Number Assignments

At this point it is appropriate to briefly comment on the question of ghost number assignments. As is well known, the operator \( \hat{\Omega} \) (and hence also \( \hat{\Omega}_0 \)) is not only Grassmann-odd, but also carries ghost number of one unit: \( \text{gh}(\hat{\Omega}) = +1 \). Treating eq. (2.35) as a first principle, we define \( \text{gh}(\theta) = -1 \).

Such an assignment is totally compatible with the other eq. of motion, (2.34). However now the operators \( D, \hat{\Psi}, \hat{Q} \) and \( \hat{\bar{Q}} \hat{\Psi} \) carry no definite ghost number. In fact, the ghost number discrepancy between the two supercomponent parts is the same for all these operators, namely two units, cf. (2.44).

However, considering the way in which the defining equation (2.33) splits up into the two independent equations (2.34) and (2.35), it is immediately clear that this indefinite ghost number of \( D \) and, say, \( \hat{Q} \hat{\Psi} \), is of absolutely no consequence. The mismatch can be easily corrected, for example, by inserting a bosonic constant \( \eta \) carrying ghost number +2 in front of the one-component of the above mentioned operators:

\[
\begin{align*}
D &= \frac{d}{d\theta} + \eta \theta \frac{d}{dt}, \\
\hat{\Psi} &= \hat{\Psi}_0 + \eta \theta \hat{\Psi}_1, \\
\hat{Q} &= \hat{\Omega} + \eta \theta \hat{H}, \\
\hat{\bar{Q}} \hat{\Psi} &= \hat{\Omega} \hat{\Psi}_0 + \eta \theta \hat{H} \hat{\Psi}.
\end{align*}
\]

Note that \( D \) is still essentially the square root of the time derivative:

\[
D^2 = \eta \frac{d}{dt}.
\]

Let us finally mention that, for consistency, one must add to the usual ghost number operator \( \hat{\Omega} \hat{\Psi}_0 \) a term \( +2 \eta \hat{\partial}_\eta \). Since all these redefinitions are inessential, we prefer, however, to avoid them.

2.6 The Physical-State Condition and Gauge Independence

To define physical states, we require that \( |\text{phys}\rangle_0 \) be annihilated by \( \hat{\Omega}_0 \hat{\Psi}_0(t, \theta) \):

\[
\hat{\Omega}_0 \hat{\Psi}_0 |\text{phys}\rangle_0 = 0.
\]

By hermiticity of \( \hat{\Omega}_0 \hat{\Psi}_0(t, \theta) \), this is equivalent to

\[
\hat{\Psi}_0 \langle\text{phys}| \hat{\Omega}_0 \hat{\Psi}_0 = 0.
\]

These two conditions are identical to the usual requirements for physical states \( \hat{\Psi}_0 \). Note that we have labelled our physical states by the subscript \( \hat{\Psi}_0 \). This is because, as we have seen above, this part of the superfield gauge boson \( \hat{\Psi} \) is responsible for a canonical transformation, and the corresponding physical states must be rotated accordingly. Conventionally one chooses \( \hat{\Psi}_0 = 0 \) from the outset, but the above definition is the suitable one in our more general case.

For the reader who is puzzled by the fact that only the original \( \hat{\Omega}_0 \hat{\Psi}_0 \) BRST operator enters in the definition of physical states, we note that in the case of exponential gauge fixing \( \Psi = \Psi' = \Psi'_0 + \theta \Psi'_1 \) with \( \Psi'' = 0 \), it trivial to rewrite the physical-state condition in terms of \( \hat{Q}_\hat{\Psi} \) and \( \hat{\bar{Q}} \hat{\Psi} \), which one would have guessed should enter into the definition of physical BRST cohomology. Indeed,

\[
\left( \hat{Q}_\hat{\Psi} + \hat{\bar{Q}} \hat{\Psi} \right) |\text{phys}\rangle_0 = 0.
\]
Corresponding to the definition of physical states, we also define physical operators \( \hat{A}_{(\Psi)}(\hat{z}(t, \theta), \theta) \) as those that supercommute with \( \hat{\Omega}_{\Psi_0}(\hat{z}(t, \theta)) \):

\[
[\hat{A}_{(\Psi)}(\hat{z}(t, \theta), \theta), \hat{\Omega}_{\Psi_0}(\hat{z}(t, \theta))] = 0 .
\] (2.50)

We have here explicitly given the operator \( \hat{A}_{(\Psi)}(\hat{z}(t, \theta), \theta) \) a subscript \( \Psi \) in order to emphasize that the operator

\[
\hat{A}_{(\Psi)}(\hat{z}(t, \theta), \theta) = \hat{U}_{\Psi}(t, \theta) \hat{A}(\hat{z}(t_i, \theta_i), \theta_i) \hat{U}_{\Psi}^{-1}(t, \theta)
\] (2.51)

is evolving from initial conditions at \( t_i \) and \( \theta_i = 0 \) according to Heisenberg equation of motion by an evolution operator

\[
\hat{U}_{\Psi}(t, \theta) = T \exp \left[ I \left( -(ih)^{-1} \hat{Q}_{\Psi} + D \right) \right] (t, \theta)
\] (2.52)

depending on the gauge \( \hat{\Psi} \). Here \( T \) stands for time-ordering with respect to time \( t \) and we have introduced the operator \( I = D^{-1} \), inverse to \( D \). It can be written as an integral operator

\[
I[F](t, \theta) \equiv \int dt'd\theta' K(t, \theta; t', \theta') F(t', \theta')
\] (2.53)

with the kernel

\[
K(t, \theta; t', \theta') \equiv 1_{[t_i, t]}(t') - \theta \delta(t - t') \delta(\theta') .
\] (2.54)

Here \( 1_{[t_i, t]}(t') \) is just the characteristic function, \( i.e. \),

\[
1_{[t_i, t]}(t') = 1 \text{ if } t_i \leq t' \leq t ,
\] (2.55)

and zero otherwise.

The above physicality definitions (2.47) and (2.50) can now be used to provide a proof of gauge independence of physical matrix elements

\[ \hat{\Psi}_0(\text{phys}|\hat{A}_{(\Psi)}(\hat{z}(t, \theta), \theta)|\text{phys}'\hat{\Psi}_0) \]

in superfield language. First, since \( \hat{\Psi}_0 \) simply generates canonical transformations, the picture changes with the choice of \( \hat{\Psi}_0 \), and it does not make sense to try to change \( \hat{\Psi}_0 \) in a given fixed \( \hat{\Psi}_0 \)-picture. Indeed, the whole formalism is invariant under canonical transformations, provided we redefine the physical states accordingly, as explained above. Without any loss of generality we can therefore restrict ourselves to changes \( \hat{\Psi} \to \hat{\Psi} + \Delta \hat{\Psi} \) with

\[
\Delta \hat{\Psi} = \theta \Delta \hat{\Psi}_1
\] (2.56)

keeping zero component \( \hat{\Psi}_0 \) fixed. This induces a change in \( \hat{Q}_{\Psi} \) of the form

\[
\Delta \hat{Q}_{\Psi} = (ih)^{-1}[\hat{E}_0(\Delta \hat{\Psi}), \hat{\Omega}_{\Psi_0}] .
\] (2.57)

The change in \( \hat{Q}_{\Psi} \) is thus effectively that of a BRST supercommutator. This is the crucial property, shared with the conventional treatment \( [\hat{\Pi}] \), which allows us to demonstrate independence of the chosen gauge.

We now introduce a “gauge changing” operator \( [\hat{G}] \)

\[
\hat{G}(t, \theta) = \hat{U}_{\Psi + \Delta \Psi}(t, \theta) \hat{U}_{\Psi}^{-1}(t, \theta) = T \exp \left[ -(ih)^{-1} I \Delta \hat{Q}_{\Psi} \right] (t, \theta) .
\] (2.58)
It satisfies the following equation of motion,

\[ i\hbar \dot{G}(t, \theta) = -\Delta \hat{Q}_\psi(t, \theta) \hat{G}(t, \theta) , \] (2.59)

subject to the boundary condition

\[ \hat{G}(t=t_i, \theta=\theta_i) = 1 . \] (2.60)

Consider now the solutions of the equation of motion (2.33) with respect to the new \( \hat{Q} \)-operator \( \hat{Q}_\psi + \Delta \hat{\psi} \). They are related to the previous solutions through

\[ \hat{z}^A_{(\psi+\Delta \psi)} = \hat{G} \hat{z}^A_{(\psi)} \hat{G}^{-1} . \] (2.61)

In general, for arbitrary operators \( \hat{A} \),

\[ \hat{A}_{(\psi+\Delta \psi)} = \hat{G} \hat{A}_{(\psi)} \hat{G}^{-1} . \] (2.62)

Let us now restrict ourselves to an infinitesimal change of superfield gauge bosons, for which the above solution for \( \hat{G} \) simply reads

\[ \hat{G} = 1 + \delta \hat{G} , \quad \delta \hat{G} = - (i\hbar)^{-2} I \left[ \hat{E}_0(\delta \hat{\psi}), \hat{\Omega}_{\psi_0} \right] = - (i\hbar)^{-2} \left[ I \hat{E}_0(\delta \hat{\psi}), \hat{\Omega}_{\psi_0} \right] , \] (2.63)

due to the fact that \( \hat{\Omega}_{\psi_0} \) is a constant of motion in both the \( t \) and \( \theta \) directions. This fact also implies that the BRST charge \( \hat{\Omega}_{\psi_0}(t, \theta) \) away from the initial Cauchy surface at initial time \( t_i \) and \( \theta_i = 0 \) does not depend on the gauge \( \psi \). The relation (2.62) becomes

\[ \delta \hat{A}_{(\psi)} = \left[ \delta \hat{G}, \hat{A}_{(\psi)} \right] . \] (2.64)

As a nice check, we note explicitly, that, as it should,

\[ \delta \left( \hat{\Omega}_{\psi_0} \right)_{(\psi)} = \left[ \delta \hat{G}, \hat{\Omega}_{\psi_0} \right] = 0 . \] (2.65)

For an arbitrary operator \( \hat{A} \) we get, using equation (2.50),

\[ \delta \hat{A}_{(\psi)} = - \left[ \hat{A}_{(\psi)}, \delta \hat{G} \right] \]
\[ = (i\hbar)^{-2} \left[ \hat{A}_{(\psi)}, I \hat{E}_0(\delta \hat{\psi}), \hat{\Omega}_{\psi_0} \right] \]
\[ = (i\hbar)^{-2} \left[ \hat{A}_{(\psi)}, I \hat{E}_0(\delta \hat{\psi}), \hat{\Omega}_{\psi_0} \right] , \] (2.66)

which is BRST-exact. Thus,

\[ \psi_0(\text{phys}|\delta \hat{A}_{(\hat{\psi})}(\hat{z}(t, \theta), \theta)|\text{phys})_{\hat{\psi}_0} = 0 . \] (2.67)

This shows that physical matrix elements do not depend on the chosen gauge. Following the principles laid out in refs. [7] and [3], one can analyze the condition of physical unitarity along similar lines.
2.7 Ward Identities

The BRST symmetry gives rise to a superfield formulation of Ward identities derivable from this operator formulation. To see this, we proceed as in the usual case\[3\] by introducing for the superfield operators \( \hat{z}^{A} \) external c-number sources, now themselves also superfields: \( J_{A}(t, \theta) = J_{A}^{0}(t) + \theta J_{A}^{1}(t) \), of opposite statistics: \( \epsilon(J_{A}) = \epsilon_{A} + 1 \). We take the opportunity here to be slightly more general, and introduce as well c-number sources, again superfields, \( z_{A}^{*}(t, \theta) = z_{A}^{0*}(t) + \theta z_{A}^{1*}(t) \), with the same statistics: \( \epsilon(z_{A}^{*}) = \epsilon_{A} \). These additional superfields are to be sources of BRST-transforms of the superfield phase-space variables\[2\]. Let us first introduce, for solutions \( \hat{z}^{A} \) of eq. \( (2.33) \), a generating operator \( \hat{Z}(t, \theta) = \hat{Z}(J, z^{*}; t, \theta) \) satisfying the following equation:

\[
\frac{i\hbar D}{\partial t} \hat{Z}(t, \theta) = \Delta \hat{Q}(t, \theta) \hat{Z}(t, \theta) ,
\]

where

\[
- \Delta \hat{Q}(t, \theta) = ( -1 )^{\epsilon_{A}+1} J_{A}(t, \theta) \hat{z}^{A}(t, \theta) + ( -1 )^{\epsilon_{A}} (i\hbar)^{-1} z_{A}^{*}(t, \theta) \left[ \hat{z}^{A}(t, \theta), \hat{Q}(z^{A}(t, \theta)) \right] ,
\]

and subject to the initial boundary condition at time \( t_{i} \) and \( \theta_{i} = 0 \)

\[
\hat{Z}(t = t_{i}, \theta = \theta_{i}) = \hat{1} .
\]

We can write the solution as

\[
\hat{Z}(t, \theta) = T \exp \left[ (i\hbar)^{-1} I \Delta \hat{Q} \right](t, \theta) .
\]

Note that there is no explicit \( t \) or \( \theta \) dependence inside \( \hat{Z}(t, \theta) \).

The corresponding operator that interpolates between two events \( t_{1}, \theta_{1} \) and \( t_{2}, \theta_{2} \) is given as

\[
\hat{Z}_{21} \equiv \hat{Z}(t_{2}, \theta_{2}; t_{1}, \theta_{1}) = \hat{Z}(t_{2}, \theta_{2}) \hat{Z}^{-1}(t_{1}, \theta_{1}) .
\]

It satisfies

\[
\hat{Z}_{11} = \hat{1} , \quad \hat{Z}_{12} = \hat{Z}_{21}^{-1} , \quad \hat{Z}_{32} \hat{Z}_{21} = \hat{Z}_{31} .
\]

We shall especially be interested in the full evolution

\[
\hat{Z}_{fi} = \hat{Z}(t_{f}, \theta_{f}; t_{i}, \theta_{i})
\]

from the initial time \( t_{i} \) and \( \theta_{i} = 0 \) to the final time \( t_{f} \) and \( \theta_{f} = 0 \). Obviously we have \( \hat{Z}(t, \theta) = \hat{Z}(t, \theta; t_{i}, \theta_{i}) \).

We now transform all superfield operators \( \hat{A}(\hat{z}(t, \theta), \theta) \) into a new representation, which is determined by the external sources:

\[
\hat{A}'(t, \theta) \equiv \hat{Z}^{-1}(t, \theta) \hat{A}(t, \theta) \hat{Z}(t, \theta) .
\]

In particular, the original phase space operators turn into

\[
\hat{z}'^{A}(t, \theta) = \hat{Z}^{-1}(t, \theta) \hat{z}(t, \theta) \hat{Z}(t, \theta) .
\]

An important property is that, due to the boundary condition \( (2.70) \), all operators coincide in their two different representations at the initial point \( t_{i} \) and \( \theta_{i} = 0 \):

\[
\hat{A}'(t_{i}, \theta_{i}) = \hat{A}(t_{i}, \theta_{i}) .
\]
From the original Heisenberg equation of motion \( (2.33) \) we find, by substitution, that the new equation of motion reads slightly different:

\[
\text{i} h D\hat{z}^A(t, \theta) = - \left[ \hat{Q}^\prime_q(t, \theta) + \Delta \hat{Q}'(t, \theta), \hat{z}^A(t, \theta) \right] ,
\]

or for a general operator

\[
\text{i} h D\hat{A}'(t, \theta) = - \left[ \hat{Q}^\prime_q(t, \theta) + \Delta \hat{Q}'(t, \theta), \hat{A}'(t, \theta) \right] + \text{i} h D\hat{A}'(t, \theta) .
\]

This precisely describes the dynamics of a shifted \( \hat{Q}_q \)-operator \( \hat{Q}^\prime_q + \Delta \hat{Q}'(t, \theta) \) that is, one with the superfields \( J_A \) and \( \hat{z}^* \) playing just those rôles of sources as we described above.

Note that the fundamental supercommutation relations are unaltered by switching to the new representation:

\[
\begin{align*}
[\hat{Q}^\prime_q(t, \theta), \hat{Q}^\prime_q(t, \theta)] &= 0 \\
[\hat{Q}'(t, \theta), \hat{Q}'(t, \theta)] &= 0 \\
[\hat{H}'(t, \theta), \hat{Q}'(t, \theta)] &= 0 .
\end{align*}
\]

But because of the \( t \) and \( \theta \)-dependent sources, the transformed BRST operator \( \hat{Q}_0' \) is no longer a constant of motion. Indeed, from \( (2.79) \)

\[
\text{i} h D\hat{Q}_0'(t, \theta) = (-1)^{c_A+1} J_A(t, \theta) [\hat{z}^A(t, \theta), \hat{Q}_0'(t, \theta)] .
\]

Starting from eq. \( (2.71) \), we now obtain the following identity:

\[
\frac{\delta}{\delta z^*_A(t, \theta)} \hat{Z}(t_f, \theta_f; t_i, \theta_i) = \frac{\delta}{\delta z^*_A(t, \theta)} \text{ Tr } \left[ (\text{i} \hbar)^{-1} I \Delta \hat{Q}_0' \right] (t_f, \theta_f) = -(\text{i} \hbar)^{-2} \hat{Z}(t_f, \theta_f; t_i, \theta_i) K(t_f, \theta_f; t_i, \theta_i) [\hat{z}^A(t, \theta), \hat{Q}_0'(t, \theta)] \hat{Z}(t, \theta; t_i, \theta_i) = -(\text{i} \hbar)^{-2} \hat{Z}(t_f, \theta_f; t_i, \theta_i) [\hat{z}^A(t, \theta), \hat{Q}_0'(t, \theta)] ,
\]

where we used that \( K(t_f, \theta_f; t_i, \theta) = 1 \), because \( \theta_f = 0 \). This in turn allows us to rewrite the evolution equation \( (2.81) \) for \( \hat{Q}_0' \) in the form of

\[
(\text{i} \hbar)^{c_A+1} J_A(t, \theta) \frac{\delta}{\delta z^*_A(t, \theta)} \hat{Z}(t_f, \theta_f; t_i, \theta_i) = (\text{i} \hbar)^{-1} \hat{Z}(t_f, \theta_f; t_i, \theta_i) D\hat{Q}_0'(t, \theta) .
\]

We let \( |0\rangle_{\hat{Q}_0} \) denote the vacuum state; it also satisfies the physical-state condition \( (2.47) \). We can define the generator \( \hat{Z}(J, z^*) \) or action \( W_c \) for connected diagrams as

\[
\hat{Z}(J, z^*) = \exp \left[ \frac{\text{i}}{\hbar} W_c(J, z^*) \right] = \hat{Q}_0 |0\rangle \hat{Z}(J, z^*; t_f, \theta_f; t_i, \theta_i) |0\rangle_{\hat{Q}_0} .
\]

We can now finally derive the Ward identity in very compact form. From eq. \( (2.88) \) we get, upon integrating over \( t \) from \( t_i \) to \( t_f \), and over \( \theta \) as well:

\[
\int_{t_i}^{t_f} J_A(t, \theta) \, dt \, d\theta \frac{\delta}{\delta z^*_A(t, \theta)} W_c(J, z^*) = 0 .
\]

Although this relation appears to be derivable directly from the equations of motion only, it encodes in an essential way the gauge (BRST) symmetry of the system. This is clearly seen when one traces it back to the evolution equation \( (2.81) \). The equation \( (2.85) \) can also be used to derive a superfield analog of the Master Equation for the effective action. We shall return to this in section 4.
3 A Super Phase-Space Path Integral

We proceed from the operator formalism to the naive (quasiclassical) phase space path integral in the usual manner. First we introduce the graded super Poisson bracket

\[ \{ F(z(t, \theta)), G(z(t, \theta)) \} \equiv F^{\omega A B} \partial_{\omega A} G^{\partial B}. \]  

(3.1)

We next consider the classical counterparts of all the previous operator relations. For \( \Omega(z(t, \theta)) \) and \( H(z(t, \theta)) \),

\[ \{ \Omega(z(t, \theta)), \Omega(z(t, \theta)) \} = 0 \quad \text{and} \quad \{ H(z(t, \theta)), \Omega(z(t, \theta)) \} = 0. \]  

(3.2)

The Grassmann-odd \( Q(z) \) and \( \bar{Q}(z) \) are defined by

\[ Q(z(t, \theta), \theta) \equiv \Omega(z(t, \theta)) + \theta H(z(t, \theta)), \]
\[ \bar{Q}(z(t, \theta), \theta) \equiv \Omega(z(t, \theta)) - \theta H(z(t, \theta)). \]  

(3.3)

They are nilpotent in terms of the Poisson bracket, by virtue of eq. (3.2):

\[ \{ Q(z(t, \theta)), Q(z(t, \theta)) \} = \{ \bar{Q}(z(t, \theta)), \bar{Q}(z(t, \theta)) \} = \{ Q(z(t, \theta)), \bar{Q}(z(t, \theta)) \} = 0. \]  

(3.4)

The classical equation of motion is taken to be

\[ Dz^A(t, \theta) = -\{ Q(\Psi(z(t, \theta)), z^A(t, \theta)) \}. \]  

(3.5)

By the same mechanism as before, this is equivalent to

\[ \frac{d}{dt}z^A(t, \theta) = -\{ H(\Psi(z(t, \theta)), z^A(t, \theta)) \}. \]
\[ \frac{d}{d\theta}z^A(t, \theta) = -\{ \Omega(\Psi(z(t, \theta)), z^A(t, \theta)) \}. \]  

(3.6)

(3.7)

Going through the analogous manipulations as in the operator formulation, we find, as expected, the following relations:

\[ \frac{d}{d\theta}H(\Psi(z(t, \theta))) = -\{ \Omega(\Psi(z(t, \theta)), H(\Psi(z(t, \theta))) \} = 0 \]
\[ \frac{d}{d\theta}\Omega(\Psi(z(t, \theta))) = -\{ \Omega(\Psi(z(t, \theta)), \Omega(\Psi(z(t, \theta))) \} = 0 \]
\[ \frac{d}{dt}H(\Psi(z(t, \theta))) = -\{ H(\Psi(z(t, \theta)), H(\Psi(z(t, \theta))) \} = 0 \]
\[ \frac{d}{dt}\Omega(\Psi(z(t, \theta))) = -\{ H(\Psi(z(t, \theta)), \Omega(\Psi(z(t, \theta))) \} = 0. \]  

(3.8)

Thus \( H(\Psi(z(t, \theta))) \) and \( \Omega(\Psi(z(t, \theta))) \) are constants of motion in terms of evolution in both \( t \) and \( \theta \).

The two superfield equations of motion (2.34) and (2.35) are therefore equivalent to

\[ \dot{z}_0^A(t) = -\{ H(\Psi(z_0(t)), z_0^A(t)) \} \]
\[ z_1^A(t) = -\{ \Omega(\Psi(z_0(t)), z_0^A(t)) \}. \]  

(3.9)

(3.10)

completely in analogy with the operator relations.
3.1 The Action

The next step consists in proposing an action by means of which the above superfield equation of motion, eq. (3.5), will follow by a variational principle. Consider

\[ S(z) = \int dt \ d\theta \left[ \frac{1}{2} \dot{z}^A(t, \theta) \ \omega_{AB} \ Dz^B(t, \theta)(-1)^\epsilon_B - Q_\Psi(z(t, \theta), \theta) \right] \]

\[ = - \int dt \ d\theta \left[ \frac{1}{2} (Dz^A(t, \theta)) \ \omega_{AB} \ z^B(t, \theta) + Q_\Psi(z(t, \theta), \theta) \right], \tag{3.11} \]

where \( \omega_{AB} \) is the inverse of \( \omega^{AB} \). By variation we precisely obtain the equation of motion (3.5).

The above action \( S \) is therefore a good candidate to be exponentiated, and integrated over in the superfield path integral:

\[ Z = \int [dz] \ \exp \left[ \frac{i}{\hbar} S(z) \right]. \tag{3.12} \]

The functional superfield integration over \( z^A(t, \theta) \) contains no additional measure factor. In a proper treatment of the phase-space path integral, one should start with the multiplication algebra of “symbols” \[ \[ \], and a path integral defined through a precise limiting procedure from a suitable discretization, for which eq. (3.12) is just the formal counterpart. For simple regulators in quantum field theory, the above definition may suffice, at least as far as the perturbative expansion is concerned.

It is a remarkable fact, already hinted at in the operator formulation, that the BRST charge \( \Omega \) and the Hamiltonian \( H \) enter on an almost equal footing. In the superspace action (3.11) we again see that it is \( Q \), the “superfield combination” of these two fundamental objects, which plays the rôle of the superfield Hamiltonian.

For consistency we require that the above path integral reduces to the usual phase space path integral upon integration over \( \theta \) in the action, and upon functional integration over the superfield components \( z^A_1(t) \). The \( \theta \)-integration in the action is straightforward, as it simply projects out the \( \theta \)-term in the integrand. We get:

\[ S(z_0, z_1) = \int dt \left[ \frac{1}{2} z_0^A(t) \ \omega_{AB} \ z_0^B(t) + \frac{1}{2} (-1)^\epsilon_B \ z_1^A(t) \ \omega_{AB} \ z_1^B(t) - H_\Psi(z_0(t)) - z_1^A(t) \ \partial_A \Omega_\Psi(z_0(t)) \right]. \tag{3.13} \]

As an intermediate check, we note that the action in this form implies the equations of motion (3.9) and (3.10) for \( z^A_0(t) \) and \( z^A_1(t) \), respectively. Let us next perform the \( z_1 \)-integration. After completing the square, and making use of the nilpotency condition for \( \Omega \) in eq. (3.2), we find

\[ Z = \int [dz_0] \ \text{Pf}(\omega) \ \exp \left[ \frac{i}{\hbar} \int dt \left\{ \frac{1}{2} z_0^A(t) \ \omega_{AB} \ z_0^B(t) - H_\Psi(z_0(t)) \right\} \right], \tag{3.14} \]

which precisely is the required expression. Even the canonically invariant Liouville measure is correctly reproduced – as a result of the gaussian \( z_1 \)-integration. In fact, the fields \( z_1(t) \) play two rôles in the phase-space path integral. On-shell, at the classical level, they are identitied with the BRST-transformed \( z_0^A(t) \)-fields, while off-shell, after a shift, they can be viewed as Pfaffian ghosts.

\[ \dagger \] Here we make use of Berezin integration \( \int d\theta \equiv \frac{d}{d\eta} \). To obtain an action with ghost number 0 one could divide the left hand side of (3.11) by \( \eta \) and use the prescriptions given in Section 2.3. For simplicity we prefer the present formulation.
3.2 Gauge Independence in the Superfield Formulation

The BFV Theorem [1] provides the path integral analog of the operator proof of gauge independence. Gauge independence can be proven in both the reduced path integral (3.14) (where $z_1$ is integrated out), and in the full superfield path integral. We start with the superfield version because it is simpler than the component version. One might have expected that the natural object to replace the BRST operator $\Omega_{\Psi_0}$ in the required transformation of variables would be the nilpotent "super-Hamiltonian" $Q_\Psi$. This is, however, not the case, as we shall see below. While it is possible to prove gauge independence by such a replacement, a few other complications occur. For this reason we have decided to relegate this type of proof to an appendix, while we here prove the superfield analog of the BFV Theorem by means of transformations based on $\Omega_{\Psi_0}$. It is of course still entirely based on the superfields $z^A$, with no need to split them up into components.

Let us adapt the following natural conventions: For a general function $F = F(z(t, \theta), \theta)$ with explicit $\theta$-dependence define

$$F_0(z(t, \theta)) = F(z(t, \theta), 0),$$
$$F_1(z(t, \theta)) = \frac{\partial}{\partial \theta} F(z(t, \theta), \theta),$$

so that $F = F_0 + \theta F_1$.

Consider the superfield path integral (3.12) with exponential type of gauge fixing functions $\Psi = \Psi' = \Psi'_0 + \theta \Psi'_1$. Let us assume that the gauge fixing functions have been changed infinitesimally $\Psi' \rightarrow \Psi' + \delta \Psi'$ causing the Hamiltonian part $Q_\Psi = e^{ad \Psi} Q$ of the action to change with

$$\delta Q_\Psi = \{ E(\delta \Psi'), Q_\Psi \} = \{(E(\delta \Psi'))_0, Q_\Psi \} + \theta \{(E(\delta \Psi'))_1, \Omega_{\Psi_0} \},$$

where

$$E(F) \equiv \int_0^1 d\alpha \ e^{+\alpha ad \Psi'} F,$$

and

$$ad F \equiv \{ F, \cdot \}.$$  

We will now show that this change is cancelled through an internal rearrangement of the path integral, and we can hence conclude that the path integral does not depend on the chosen gauge fixing. The first term in (3.16) is cancelled by a canonical transformation

$$\delta z^A(t, \theta) = \{z^A(t, \theta), (E(\delta \Psi'))_0(z(t, \theta))\}.$$  

A canonical transformation produces no Jacobian, and the kinetic term is invariant (up to a boundary term) because there is – by construction – no explicit $\theta$-dependence inside $(E(\delta \Psi'))_0(z(t, \theta))$. The change in the Hamiltonian part yields precisely

$$\{ Q_\Psi, (E(\delta \Psi'))_0 \},$$

i.e. minus the first term in (3.16). The second term in (3.16) is cancelled by a BRST type of transformation

$$\delta z^A(t, \theta) = \{z^A(t, \theta), \Omega_{\Psi_0}(z(t, \theta))\} \mu,$$
where $\mu$ is an odd functional of the form

$$\mu = -i \hbar \int dt' \ d\theta' \ (E(\delta \Psi'))_1(z(t', \theta')). \quad (3.22)$$

Here both the Hamiltonian and the kinetic part of the action is invariant. The latter because there is no explicit $\theta$-dependence in $\Omega_{\Psi_0}$. Finally the Jacobian produces the second term:

$$J - 1 = (-1)^{e_A} \int \delta z^A (t, \theta) \frac{\delta}{\delta z^A(t', \theta')} \ dt' \ d\theta' \delta (t' - t) \ dt \ d\theta$$

$$= i \hbar \int dt \ d\theta \ \{(E(\delta \Psi'))_1(z(t, \theta)), \ \Omega_{\Psi_0}(z(t, \theta))\}. \quad (3.23)$$

We have thus shown that the change in gauge, $\Psi' \to \Psi' + \delta \Psi'$, can be reabsorbed by a combination of a superfield canonical transformation and a superfield BRST transformation. The path integral is hence formally independent of $\Psi'$.

### 3.3 Gauge Independence in the Original Sector

We now outline the proof in the formulation (3.14) where $z_1$ has been integrated out. It is interesting it is own right to see how the proof generalizes in the original sector. Because of the “exponential” gauge fixing $\Psi = \Psi' = \Psi'_0 + \theta \Psi'_1$, the proof differs in some details from the original proof [1].

We first introduce, in analogy with eq. (2.25),

$$E_0(F) \equiv \int_0^1 d\alpha \ e^{+\alpha \text{ad}_{\Psi_0} F}. \quad (3.24)$$

An infinitesimal change in the gauge fixing produces the following change in the Hamiltonian

$$\delta H_\Psi = \{\delta \Xi_0, H_\Psi\} + \{\delta \Xi_1, \Omega_{\Psi_0}\} \quad (3.25)$$

where

$$\begin{align*}
\delta \Xi_0 & \equiv E_0(\delta \Psi'_0), \\
\delta \Xi_1 & \equiv E_0(\delta \Psi'_1) + \{E_0(\Psi'_1), E_0(\delta \Psi_0)\} + \left\{ \int_0^1 d\alpha \int_0^1 d\beta \left(e^{+\alpha \text{ad}_{\Psi_0} \delta \Psi'_0}, e^{+\text{ad}_{\Psi_0} \Psi'_1} \right) \right\}. \ (3.26)
\end{align*}$$

Now the idea is the same as in the superfield approach. First, we perform a canonical transformation

$$\delta z^A_0(t) = \{z^A_0(t), \delta \Xi_0(z_0(t))\}. \quad (3.27)$$

This changes the Hamiltonian part with

$$\delta \int dt \ H_\Psi(z_0(t)) = \int dt \{H_\Psi(z_0(t)), \delta \Xi_0(z_0(t))\}. \quad (3.28)$$

Next, consider the BRST variation

$$\delta z^A_0(t) = \{z^A_0(t), \Omega_{\Psi_0}(z_0(t))\} \ \mu. \quad (3.29)$$

with

$$\mu = -i \hbar \int dt' \ \delta \Xi_1(z_0(t')). \quad (3.30)$$
The action is invariant, but the Jacobian equals
\[ J = (-1)^{c_A} \int \int \delta z_0^A(t) \frac{\delta}{\delta z_0^A(t')} dt' \delta(t' - t) \ dt = \frac{i}{\hbar} \int dt \ \{ \delta \Xi_1(z_0(t)), \Omega_{\Psi_0}(z_0(t)) \} . \quad (3.31) \]

The two new terms in eq. (3.24) are thus explicitly cancelled, and we conclude again that the path integral is independent of \( \Psi' \).

4 Super Antifield Formalism

So far our considerations have been restricted to the Hamiltonian counterpart of BRST quantization. It is natural to seek an extension of this to the Lagrangian framework, which, for example, with little effort provides a manifestly Lorentz invariant description. To encompass the complete set of all possible gauge theories, we know that the appropriate language should be that of the field-antifield formalism \[8\]. This leads us to consider the reformulation of this field-antifield construction in superfield language.

The essential ingredient in the field-antifield formalism is a Grassmann-odd and nilpotent operator \( \Delta \), whose failure to act like a differentiation defines a Grassmann-odd bracket, the antibracket, and an associated antisymplectic geometry. Before we proceed with the derivation of a superfield analog of the field-antifield formalism, it is therefore useful to first pause and consider an appropriate generalization of these concepts to superfields. Although we will not need it in the present preliminary stage, we choose do it in the more general covariant formulation, where the antisymplectic coordinates have not necessarily been specified in Darboux form. It will be useful for later developments, where one would like to build a more abstract and coordinate-independent field-antifield formalism in superfield language.

4.1 General Covariant Theory

In this subsection we therefore describe a general odd symplectic superspace with \( 4N \) variables
\[ \Gamma^A(t, \theta) = (\Phi^\alpha(t, \theta); \Phi^\ast_{\alpha}(t, \theta)) , \quad (4.1) \]
where \( \Phi^\alpha(t, \theta) \) and \( \Phi^\ast_{\alpha}(t, \theta) \) have the same statistics \( \epsilon_\alpha \). We shall later, in section 4.4, let our previous superfield phase-space variables \( z^A(t, \theta) \) play the roles of \( \Phi^\alpha(t, \theta) \). In that particular case \( N \) is thus even.

Given a volume density \( \rho \) and a Grassmann-odd symplectic metric \( E^{AB}(t, \theta; t', \theta') \) the covariant “odd superfield Laplacian” \( \Delta \) reads
\[ \Delta = \frac{1}{2} (-1)^{c_A} \rho^{-1} \int dt \ d\theta \ \frac{\delta}{\delta \Gamma^A(t, \theta)} \rho \ E^{AB}(t, \theta; t', \theta') \ dt' \ d\theta' \ \frac{\delta}{\delta \Gamma^B(t', \theta')} . \quad (4.2) \]

We assume that the measure density and the antisymplectic structure are compatible in the sense that \( \Delta \) is nilpotent: \( \Delta^2 = 0 \). This \( \Delta \)-operator gives rise to a superfield antibracket in the conventional manner, through its failure to act as a derivation:
\[ (F, G) = (-1)^{c_F} [[\Delta, F], G]_1 = -(-1)^{(c_F + 1)(c_G + 1)} (G, F) \]
\[
\int \int F \frac{\delta}{\delta \Gamma^A(t, \theta)} \ dt \ d\theta \ E^{AB}(t, \theta; t', \theta') \ dt' \ d\theta' \ \frac{\delta}{\delta \Gamma^B(t', \theta')} G .
\]

(4.3)

We have here also indicated its symmetry property.

The antisymplectic metric should be non-degenerate, it should have Grassmann parity

\[ \epsilon(E^{AB}(t, \theta; t', \theta')) = \epsilon_A + \epsilon_B + 1 , \]

(4.4)

and symmetry

\[ E^{BA}(t', \theta'; t, \theta) = -(-1)^{\epsilon_A+1} E^{AB}(t, \theta; t', \theta') . \]

(4.5)

It satisfies the Jacobi identity

\[ \sum_{\text{cycl. } A, B, C} (-1)^{\epsilon_A+1}(\epsilon_C+1) \int E^{AD}(t^1, \theta^1; t^4, \theta^4) \ dt^4 \ d\theta^4 \ \frac{\delta}{\delta \Gamma^D(t^4, \theta^4)} E^{BC}(t^2, \theta^2; t^3, \theta^3) = 0 . \]

(4.6)

For a general bosonic vector field

\[
X = \int X^A(t, \theta) \ dt \ d\theta \ \frac{\delta}{\delta \Gamma^A(t, \theta)} \\
= \int X^A_0(t) \ dt \ \frac{\delta}{\delta \Gamma_0^A(t)} + \int X^A_1(t) \ dt \ \frac{\delta}{\delta \Gamma_1^A(t)} ,
\]

(4.7)

with components \( X^A(t, \theta) = X^A_0(t) + \theta X^A_1(t) \) of Grassmann parity \( \epsilon(X^A) = \epsilon_A \), the divergence \( \text{div}_\rho X \) is defined as the proportionality factor between the Lie-derivative of the volume-form \( \mathcal{L}_X \Omega_{\text{vol}} \) and the volume-form \( \Omega_{\text{vol}} \) itself:

\[ \mathcal{L}_X \Omega_{\text{vol}} = \text{div}_\rho X \ \Omega_{\text{vol}} , \]

(4.8)

or

\[
\text{div}_\rho X = (-1)^{\epsilon_A} \rho^{-1} \int dt \ d\theta \ \frac{\delta}{\delta \Gamma^A(t, \theta)} \rho \ X^A(t, \theta) \\
= (-1)^{\epsilon_A} \rho^{-1} \int dt \ \frac{\delta}{\delta \Gamma_0^A(t)} \rho \ X_0^A(t) + (-1)^{\epsilon_A+1} \rho^{-1} \int dt \ \frac{\delta}{\delta \Gamma_1^A(t)} \rho \ X_1^A(t) .
\]

(4.9)

Note that if \( X^A(t, \theta) = X^A(\Gamma(t, \theta), t, \theta) \) is an “ultra-local vector field”, then

\[ \text{div}_\rho X = \rho^{-1} X[\rho] . \]

(4.10)

If we restrict ourselves to a class of coordinate patches that are pairwise mutually connected by ultra-local super transformations \( \Gamma^A = F^A(\Gamma(t, \theta), t, \theta) \), then it is consistent to choose \( \rho \) to be the same for all patches, i.e. a coordinate change within the above mentioned class will not change the value of \( \rho \). In other words, under this restricted class of reparametrizations \( \rho \) behaves not only as a scalar density, but as a scalar. From a geometrical point of view there is hence no reason to insert a non-trivial measure density \( \rho \) different from 1. When \( \rho = 1 \) the divergence of an ultra-local vector field \( X^A(t, \theta) = X^A(\Gamma(t, \theta), t, \theta) \) vanishes identically, so ultra-locally we have an analogue...
of the Liouville Theorem. For a general Hamiltonian (but not necessarily ultra-local) vectorfield
\( X_F = (F, \cdot) = - (\cdot, F) \), where \( F \) is Grassmann-odd function, the divergence
\[
\text{div}_\rho X_F = -2\Delta(F)
\]  
(4.11)
is given by the odd Laplacian, as in the normal case [9].

4.2 Darboux Coordinates

In Darboux coordinates the fundamental antibracket relations read in superfield form:
\[
E^{\alpha\beta}(t, \theta; t', \theta') = \langle \Phi^\alpha(t, \theta), \Phi^\beta(t', \theta') \rangle = \delta^\alpha_\beta \delta(t-t') \delta(\theta-\theta') .
\]  
(4.12)
In components the non-vanishing bracket relations become
\[
\begin{align*}
(\Phi^\alpha_0(t), \Phi^\beta_1(t')) &= (-1)^{\epsilon_\alpha} \delta^\alpha_\beta \delta(t-t') , \\
(\Phi^\alpha_1(t), \Phi^\beta_0(t')) &= \delta^\alpha_\beta \delta(t-t') .
\end{align*}
\]  
(4.13)
We see that the \((-1)^{\epsilon_\alpha} \Phi^\alpha_1(t)\) are the antisymplectic conjugate variables to the original variables \(\Phi^\alpha_0(t)\). In the same manner \(\Phi^\beta_1(t)\) are conjugate to the superpartners \(\Phi^\gamma_0(t)\).

4.3 Path Phase-Space Antibracket

As an aside, we note that one can lift [12] the equal-\(t\)--equal-\(\theta\) even symplectic structure
\[
\{F(z(t, \theta)), G(z(t, \theta))\} \equiv F \delta_A \omega^{AB}(z(t, \theta)) \delta_B G .
\]  
(4.16)
to an odd path-space symplectic structure
\[
(F, G) = \int \int F \frac{\delta}{\delta z^A(t, \theta)} dt \frac{d\theta}{\delta z^B(t', \theta')} E^{AB}(t, \theta; t', \theta') dt' \frac{d\theta'}{\delta z^B(t', \theta')} G .
\]  
(4.17)
by the ultralocal ansatz
\[
E^{AB}(t, \theta; t', \theta') = \omega^{AB}(z(t, \theta)) \delta(t-t') \delta(\theta-\theta')(-1)^{\epsilon_B} .
\]  
(4.18)
Note that the Jacobi identity for $\omega^{AB}$ carries over to the Jacobi identity for $E^{AB}$. With the measure density $\rho = 1$ we can form a nilpotent odd Laplacian

$$\Delta = \frac{1}{2}(-1)^{\epsilon A} \int dt \, d\theta \frac{\overrightarrow{\delta}}{\delta z^A(t, \theta)} E^{AB}(t, \theta; t', \theta') \, dt' \, d\theta' \frac{\overrightarrow{\delta}}{\delta z^B(t', \theta')} \int dt \left\{ (-1)^{\epsilon A} \omega^{AB}(z_0(t)) \frac{\overrightarrow{\delta}}{\delta z^A_1(t)} \frac{\overrightarrow{\delta}}{\delta z^A_0(t)} \right. \\
\left. - \frac{1}{2} (-1)^{\epsilon A} z^C(t) \partial_C \omega^{AB}(z_0(t)) \frac{\overrightarrow{\delta}}{\delta z^B_1(t)} \frac{\overrightarrow{\delta}}{\delta z^A_1(t)} \right\}.$$ (4.19)

In components the corresponding antibracket reads

$$(z_0^A(t), z_0^B(t')) = 0,$$  
$$(z_0^A(t), z_1^B(t')) = \omega^{AB}(z_0(t)) \delta(t - t'),$$ 
$$(z_1^A(t), z_1^B(t')) = z^C(t) \partial_C \omega^{AB}(z_0(t)) \delta(t - t').$$ (4.20)

Such an antisymplectic structure has previously, in a different context, been considered in [10].

Finally, a set of Darboux coordinates

$$z^A(t, \theta) = (q^\alpha(t, \theta); p^*_\alpha(t, \theta))$$ (4.21)

satisfies

$$(q^\alpha(t, \theta), p^*_\beta(t', \theta')) = \delta^\alpha_\beta \delta(t - t') \delta(\theta - \theta').$$ (4.22)

### 4.4 The Hamiltonian Master Equation

Once the heuristic phase-space path integral has been established in superspace form, it is a small step to formally derive from it a corresponding Lagrangian field-antifield path integral in superfield form. There have already previously been some suggestions for such a superfield formulation of the field-antifield formalism [11], but they have not started with the Hamiltonian phase-space path integral itself, and, as we shall see, this leads to some differences.

Before proceeding to the formal derivation of the field-antifield superfield path integral, let us first return briefly to the Hamiltonian operator formulation. We have already seen how the fundamental Ward identities for the generator $W_c$ in the phase space formulation could be compactly cast in the form of eq. (2.85). Let us now go one step further, and define classical variables in the usual way

$$z^A_{cl}(t, \theta) \equiv \frac{\overrightarrow{\delta}}{\delta J_A(t, \theta)} W_c(J, z^*) = - W_c(J, z^*) \frac{\overleftarrow{\delta}}{\delta J_A(t, \theta)}. \quad (4.23)$$

Furthermore, we assume as usual that this relation can be inverted so that we can express $J_A(t, \theta)$ in terms of $z^A_{cl}(t, \theta)$. We can then define the effective action $\Gamma_{eff}$ in the standard way of a Legendre transform:

$$\Gamma_{eff}(z_{cl}, z^*) = W_c(J, z^*) - \int J_A(t, \theta) \, dt \, d\theta \, z^A_{cl}(t, \theta), \quad (4.24)$$
where on the right hand side we insert the solution $J_A(t, \theta)$ as a function of $z^A_{cl}(t, \theta)$. It follows that

$$J_A(t, \theta) = \frac{\delta}{\delta z^A_{cl}(t, \theta)} \Gamma_{eff}(z_{cl}, z^*) = -\Gamma_{eff}(z_{cl}, z^*) \frac{\delta}{\delta z^A_{cl}(t, \theta)}. \tag{4.25}$$

So the Ward identity (2.85) turns into a generalized Zinn-Justin equation:

$$0 = \int \delta z^A_{cl}(t, \theta) dt \, d\theta \, \frac{\delta}{\delta z^A_{cl}(t, \theta)} \Gamma_{eff}(z_{cl}, z^*) = -\frac{1}{2} (\Gamma_{eff}(z_{cl}, z^*), \Gamma_{eff}(z_{cl}, z^*))_{cl}. \tag{4.26}$$

where we have introduced the antibracket, here with respect to the “classical” superfields $z_{cl}$ and BRST superfield sources $z^*$. This equation has an uncanny resemblance to only the classical part of a conventional Lagrangian Master Equation written in superfield language. Of course, no approximations are involved at this stage, so the above equation is exact, as is clear from the way in which it was derived directly from exact operator relations. There are no “quantum corrections” to the superfield Master Equation, when written as above in terms of the effective action.

One way to derive a more conventional type of Master Equation from the operator formalism, is to formally introduce a functional Fourier transform $e^{i \bar{h} W_{H}^{(gf)}(z, z^*)}$ of the generator of connected diagrams $Z(J, z^*)$ with respect to the variables $J \leftrightarrow z$:

$$\exp \left[ \frac{i}{\hbar} W_{H}^{(gf)}(z, z^*) \right] = \int [dJ] \exp \left[ \frac{i}{\hbar} \left[ W_{c}(J, z^*) - \int J_A(t, \theta) dt \, d\theta \, z^A(t, \theta) \right] \right]. \tag{4.27}$$

Here $W_{H}^{(gf)}(z, z^*)$ plays the rôle of an action for a path integral with field variables $z^A$. In fact, $W_{H}^{(gf)}(z, z^*)$ is a Hamiltonian counterpart of a gauge-fixed Lagrangian BV action. It satisfies a $8N$-dimensional phase space quantum Master Equation. To see this, let us write down the phase space odd Laplacian on the doubly extended space of $z^A$ and $z^A_{*}$:

$$\Delta_{z} = (-1)^{\varepsilon A} \int dt \, d\theta \frac{\delta}{\delta z^A_{cl}(t, \theta)} \frac{\delta}{\delta z^A_{*}(t, \theta)}. \tag{4.28}$$

The Ward identity (2.85) can now be rewritten as a more conventional-looking quantum Master Equation:

$$\Delta_{z} \exp \left[ \frac{i}{\hbar} W_{H}^{(gf)}(z, z^*) \right] = 0. \tag{4.29}$$

The $W_{H}^{(gf)}(z, z^*)$ defined in the above way does not depend on the gauge fermion $\hat{\Psi}$, and it should of course be identified with an action already gauge-fixed, i.e. no extra gauge fixing is needed at this point. That is why we added a superscript “(gf)”. In particular, one can take the external sources $z^A_{*}$ to vanish.

### 4.5 The Lagrangian Master Equation

Let us now show how analogous results can be reproduced from the path integral point of view. We shall also derive the counterpart where the some phase space variables $\Pi_{\alpha}(t, \theta)$ are integrated out.

$$z^A(t, \theta) = (\Phi^\alpha(t, \theta); \Pi_{\alpha}(t, \theta)). \tag{4.30}$$
In particular, if one integrates out precisely half of the phase space variables, taken to be momenta, one obtains the Lagrangian version of the Master Equation. Our derivation is essentially a superfield extension of the one presented in ref. [13].

We start with the Hamiltonian action mentioned in Section 3.1:

$$S^{(0)}(z) = \int dt \, d\theta \left[ \frac{1}{2} z^{A}(t, \theta) \omega_{AB} \dot{D}^{B}(t, \theta)(-1)^{\epsilon B} - Q_{\Psi^{(0)}}(z(t, \theta), \theta) \right]$$  \hspace{1cm} (4.31)

Here we have included an initial gauge fixing $\Psi^{(0)} = \Psi^{(0)}(z(t, \theta), \theta)$ to be as general as possible. (Specifically, we shall later need this initial gauge fixing in the $\Pi_{\alpha}(t, \theta)$ sector). The Hamiltonian master action $W_{H}(z, z^{*})$ is introduced as the above action, linearly extended with sources $z^{*}_{A}$ for the BRST transformation.

$$W_{H}(z, z^{*}) = S^{(0)}(z) + \int z^{*}_{A}(t, \theta) \, dt \, d\theta \left\{ z^{A}(t, \theta), \Omega_{\Psi_{0}}(z(t, \theta)) \right\}$$  \hspace{1cm} (4.32)

We assume that the remaining gauge fixing is of the linear type $\Psi = \Psi^{0}(\Phi(t, \theta)) + \theta \, \Psi^{1}(\Phi(t, \theta))$, with for simplicity $\Psi^{0} = 0$, cf. discussion in Section 2.1. We choose it so that it does not depend on the momenta $\Pi_{\alpha}(t, \theta)$. An exponential type of gauge fixing should be rewritten into a linear form, or should be included in the initial gauge fixing above.

We now introduce the gauge fermion functional

$$\psi = \int dt \, d\theta \, \Psi(z(t, \theta), \theta).$$  \hspace{1cm} (4.33)

Note that this indeed is a fermion. We have the following relation

$$(-1)^{\epsilon A} \frac{\partial \Psi}{\partial z^{A}(t, \theta)} \psi = \frac{\partial \Psi}{\partial z^{A}(t, \theta)} \psi = (-1)^{\epsilon A} \Psi(z(t, \theta), \theta) \frac{\partial \psi}{\partial z^{A}(t, \theta)} = \psi \frac{\partial \psi}{\partial z^{A}(t, \theta)}. \hspace{1cm} (4.34)$$

The path integral is given as

$$Z(J, z^{*}) = \int[dz] \exp \frac{i}{\hbar} \left[ W_{H}(z, z^{*} + \psi) + \int J_{A}(t, \theta) \, dt \, d\theta \, z^{A}(t, \theta) \right].$$  \hspace{1cm} (4.35)

As a first check, note that the Ward identity (2.85) may be derived by performing an ordinary BRST variation (with constant $\mu$):

$$\delta z^{A}(t, \theta) = \left\{ z^{A}(t, \theta), \Omega_{\Psi_{0}}(z(t, \theta)) \right\} \mu. \hspace{1cm} (4.36)$$

The phase space Master Equation may independently be derived from the fact that $W_{H}(z, z^{*})$ is BRST invariant:

$$0 = \int \left\{ \Omega_{\Psi_{0}}(z(t, \theta)), z^{A}(t, \theta) \right\} \, dt \, d\theta \frac{\delta}{\delta z^{A}(t, \theta)} \exp \frac{i}{\hbar} [W_{H}(z, z^{*})]$$

$$= \int dt' \, d\theta' \delta(t' - t) \delta(\theta' - \theta) \left\{ \Omega_{\Psi_{0}}(z(t', \theta')), z^{A}(t', \theta') \right\} \, dt \, d\theta \times \frac{\delta}{\delta z^{A}(t, \theta)} \exp \frac{i}{\hbar} [W_{H}(z, z^{*})]$$
\[\begin{align*}
&= -(-1)^{e_A} \int dt' \, d\theta' \, \delta(t' - t) \, \delta(\theta' - \theta) \, dt \, d\theta \\
&\quad \times \overrightarrow{\delta} \left( \delta z^A(t, \theta) \right) \left( \{ z^A(t', \theta'), \Omega_{\Phi_0}(z(t', \theta')) \} \exp \frac{i}{\hbar} \left[ W_H(z, z^*) \right] \right) \\
&= -(-1)^{e_A} \int dt' \, d\theta' \, \delta(t' - t) \, \delta(\theta' - \theta) \, dt \, d\theta \\
&\quad \times \overrightarrow{\delta} \left( \delta z^A(t, \theta) \right) \delta z^A_0(t, \theta) \exp \frac{i}{\hbar} \left[ W_H(z, z^*) \right] \\
&= -\Delta z \exp \left[ \frac{i}{\hbar} W_H(z, z^*) \right]. \quad (4.37)
\end{align*}\]

Note that the odd Laplacian separates into two pieces:
\[\Delta z = \Delta_{\Phi} + \Delta_{\Pi}, \quad (4.38)\]
each being on Darboux form, cf. Section 4.2.

The idea is now to formally integrate out all \(\Pi_0\) degrees of freedom. Although the functional integral may be undoable in closed form, we simply define the Lagrangian action \(W\) through
\[\exp \left[ \frac{i}{\hbar} W(\Phi, \Phi^*, \Pi_1, \Pi_1^*, \Pi_0^*) \right] = \int [d\Pi_0] \exp \left[ \frac{i}{\hbar} W_H(z, z^*) \right]. \quad (4.39)\]

This is manifestly supersymmetric under a supertranslation \(\Pi(\theta) \to \Pi(\theta - \theta_0)\). The reason why we do not integrate out the superpartners \(\Pi_1\) is – as we have seen earlier – that they are the Pfaffian ghosts. An integration over \(\Pi_1\) would in general produce delta functions in \(\Phi_1\) so that the action would turn singular. More precisely, one should require that the action is proper \([14]\). As it stands, eq. (4.39) is the most sensible way of defining the superfield Lagrangian action \(W\) at present. We also note that the introduction of superfield momentum sources \(\Pi_0^*\) and \(\Pi_1^*\) in the Hamiltonian path integral is a choice made by us. It is not required, but it makes the description more symmetric in phase space variables, and it simplifies the subsequent derivations.

Finally, this gives us the Lagrangian Master Equation for \(W\):
\[\Delta_{\Phi} \exp \frac{i}{\hbar} W(\Phi, \Phi^*, \Pi_1, \Pi_1^*, \Pi_0^*)] = \int [d\Pi_0] \Delta_{\Phi} \exp \left[ \frac{i}{\hbar} W_H(z, z^*) \right] = \int [d\Pi_0] (\Delta_{\Pi} - \Delta_{\Pi}) \exp \left[ \frac{i}{\hbar} W_H(z, z^*) \right] = 0. \quad (4.40)\]

where we have used the phase-space Master Equation (4.37) and the fact that the \(\Delta_{\Pi}\)-term is a total derivative in \(\Pi_0\).

While \(\Pi_1 = \int d\theta \, \Pi(\theta)\) is a manifest supersymmetric variable, the antisymplectic conjugate \(\Pi_0^*\) is not. In a Lagrangian formulation where \(\Pi_0\) is integrated out it is clear that one should consider \(\Pi_1\) as a auxiliary variable with no antifield attached. Apart from this, there is no difference in the formal structure between this superfield formulation, and the usual field-antifield formalism. All relations among antibrackets and between antibrackets and the \(\Delta\)-operator have analogous superfield extensions. Also BRST operators (be they classical or quantum) can therefore be constructed in the appropriate superfield form. It may be worthwhile to investigate that issue in further detail, but this would clearly take us beyond the scope the present paper.
5 Conclusion

As we have shown in this paper, it is possible to set up a manifestly BRST-symmetric operator formulation of the quantization of theories with first-class constraints by means of a straightforward superfield extension. The result has all the features one would have hoped for:

- The BRST operator $\hat{Q}$ and the Hamiltonian $\hat{H}$ enter in a unified manner through the nilpotent operator $\hat{Q} = \hat{Q} + \theta \hat{H}$.
- Each original field (operator) $\hat{z}_0^A(t)$, is unified in the corresponding superfield $\hat{z}^A(t, \theta) = \hat{z}_0^A(t) + \theta \hat{z}_1^A(t)$ with its BRST-transform $\hat{z}_1^A(t)$. In the path integral, this relationship holds on-shell. Both $z_0(t)$ and $z_1(t)$ are integrated over in the path integral, where now $z_1(t)$ act as Pfaffian ghosts.
- The superfield formalism naturally links BRST transformations with canonical transformations.
- The operator quantization can be carried through entirely in the superfield formulation, through exact operator relations at the superfield level.
- A phase-space superfield path integral can be set up, which reproduces the correct equations of motion, and which, upon integrating out the superfield partners $z_1^A$, reduce to the conventional phase space BFV path integral.
- A superfield generalization of the BFV Theorem can be proved through the use of a combination of a BRST transformation with a canonical transformation. On the subspace of original variables, it reduces to a proof of the usual BVF Theorem.
- By introducing sources for appropriate BRST transformations, and integrating out half of the symplectic phase space variables, we can formally derive a superfield analog of the field-antifield formalism.

The formalism we have set up seems to leave little room for an alternative formulation, but we have clearly by no means proved that the present construction gives a unique superfield formulation of the BRST quantization program. This holds in particular for the extension to the Lagrangian (field-antifield) formalism, where it is easy to imagine that more suitable schemes may be found. The singular nature of the solution to the superfield Master Equation is a consequence one would like to avoid. But it is not presently clear whether the obstacle is of fundamental or just technical nature.

Since we have constructed the appropriately gauge fixed phase-space path integral, one could also try to derive from it, following conventional lines, a relativistically covariant Lagrangian formulation for specific field theories. This would be of interest for comparison with known earlier attempts at writing a superfield version of the Lagrangian path integral for such gauge theories.

Our construction has turned out to identify the superspace behind the quantization of theories with first-class constraints precisely as one would have expected. Viewed as a two-dimensional space spanned by $t$ and the fermionic coordinate $\theta$, evolution in the $t$-direction is generated by the Hamiltonian as in eq. (2.34), while evolution in the $\theta$-direction is generated by the BRST operator, as in eq. (2.35). This provides us with a nice geometrical interpretation of BRST symmetries.
Anti-BRST symmetries can clearly be understood in a similar manner, while the imposition of both BRST and anti-BRST symmetries simultaneously will require a new and enlarged framework.

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A Alternative Derivation of the Superfield BFV Theorem

As we mentioned in Section 3.2, it is possible to prove the superfield analog of the BFV Theorem by means of variations induced by $Q_{\Psi}$ rather than by the original BRST charge $\Omega_{\Psi_0}$ itself. Since this perhaps is conceptually more satisfactory (in the superfield formulation we have seen that neither the BRST charge $\Omega$ nor the Hamiltonian $H$ play fundamental roles; only their “superfield” combinations in terms of $Q$ and $Q$ enter), we reproduce this alternative derivation here. It is clear from eq. (2.20), that we have to restrict ourself to “exponential” gauges $\Psi = \Psi'^{0} + \theta \Psi'^{1}$ with $\Psi'' = 0$ in order for $Q_{\Psi}$ to take over $\Omega_{\Psi_0}$’s dominant role. The basic ingredients in the proof are of course the same, and the proof itself leads to precisely the same conclusions as in Section 3.2. This is the first place where explicitly need both of the superspace derivatives (2.8).

We start again with the path integral expression

$$Z = \int [dz] \exp \left[ \frac{i}{\hbar} S(z) \right], \quad (A.1)$$

with the superfield action

$$S(z) = \int dt \, d\theta \left[ \frac{i}{2} z^A(t, \theta) \omega_{AB} Dz^B(t, \theta)(-1)^{e_B} - Q_{\Psi}(z(t, \theta), \theta) \right]$$

$$= - \int dt \, d\theta \left[ \frac{i}{2}(Dz^A(t, \theta)) \omega_{AB} z^B(t, \theta) + Q_{\Psi}(z(t, \theta), \theta) \right], \quad (A.2)$$

Consider now the combination of a generalized superfield BRST-like transformation, and a transformation with a superderivative:

$$\delta z^A(t, \theta) = \mu \{ Q_{\Psi}(t, \theta), z^A(t, \theta) \} - \mu \ Dz^A(t, \theta). \quad (A.3)$$

The first part of this transformation is what one might have guessed should have played the role of a superfield BRST transformation. In fact, this is not quite correct, since it does not leave the first (“kinetic energy”) part of the action (A.2) invariant. This is the origin of the compensating second piece of the transformation, which involves one of the superderivatives.

We choose the fermionic parameter $\mu$ to be

$$\mu = \frac{i}{2\hbar} \int dt \, d\theta \ E(\delta \Psi'). \quad (A.4)$$
This transformation is precisely tailored to yield no change in the action (A.2). It does, however, induce a non-trivial Jacobian:

\[ J - 1 = (-1)^A \int dt \, d\theta \, \delta(t - t') \, \delta(\theta - \theta') \, dt' \, d\theta' \, \frac{\delta^2}{\delta z^A(t', \theta')} \delta z^A(t, \theta) \]

\[ = \frac{i}{2\hbar} \int dt \, d\theta \, \left[ (E(\delta \Psi'))_1(t, \theta) - \{E(\delta \Psi')(t, \theta), Q\Psi(t, \theta)\} \right] . \quad (A.5) \]

While the last term \( \{E(\delta \Psi'), Q\Psi\} \), as we shall see shortly, could be absorbed into a modified superfield gauge boson, the first term \( E(\delta \Psi')_1 \) would spoil the interpretation as just a change in gauge fixing. This should not come as a surprise, as a change in \( \Psi \) in general involves a canonical transformation as well. Indeed, let us next perform a superfield canonical transformation of the kind

\[ \delta z^A(t, \theta) = \frac{1}{2} \left\{ E(\delta \Psi')(t, \theta), z^A(t, \theta) \right\} . \quad (A.6) \]

Due to the two compensating components of the superfield, the measure is left invariant. But from the action we get a change

\[ \delta S = - \frac{1}{2} \int dt \, d\theta \, \left[ \{E(\delta \Psi')(t, \theta), Q\Psi(t, \theta)\} + (E(\delta \Psi')_1(t, \theta) \right] . \quad (A.7) \]

Collecting the variations (A.5) and (A.7), we see that the two \( E(\delta \Psi')_1 \) terms cancel while the terms \( \{E(\delta \Psi'), Q\Psi\} \) add up to produce a term in the action of the form

\[ \delta S = - \int dt \, d\theta \, \{E(\delta \Psi')(t, \theta), Q\Psi(t, \theta)\} . \quad (A.8) \]

Finally one notices that this just corresponds to the variation of \( Q\Psi \) under the infinitesimal change \( \Psi' \to \Psi' + \delta \Psi' \):

\[ Q_{\Psi' + \delta \Psi} = Q_{\Psi} + \left\{ \int_0^1 \, d\alpha \, e^{\alpha \text{ad}_{\Psi}} \delta \Psi', Q_{\Psi} \right\} \]

\[ = Q_{\Psi} + \{E(\delta \Psi'), Q_{\Psi}\} . \quad (A.9) \]

Thus \( Z_{\Psi' + \delta \Psi} = Z_{\Psi} \). This establishes the proof of gauge independence based entirely on \( Q_{\Psi} \), rather than on the BRST operator \( Q_{\Psi} \).

**B Superconventions**

Our convention for the Berezin integration is

\[ \int d\theta \, 1 = 0 , \quad \int d\theta \, \theta = 1 . \quad (B.1) \]

The delta function can conveniently be represented as

\[ \delta(\theta) = \theta . \quad (B.2) \]

It satisfies

\[ \int F(\theta') \, \delta(\theta - \theta') \, d\theta' = F(\theta) = \int d\theta' \, \delta(\theta' - \theta) \, F(\theta') . \quad (B.3) \]
A superfield $z(\theta) = z_0 + \theta z_1$ has a functional derivative of opposite statistics $\epsilon(\frac{\delta}{\delta z(\theta)}) = \epsilon(z) + 1$:

$$\frac{\delta^l}{\delta z(\theta)} = \theta \frac{\delta^l}{\delta z_0} + (-1)^{\epsilon(z)} \frac{\delta^r}{\delta z_1}, \quad \frac{\delta^r}{\delta z(\theta)} = -\frac{\delta^r}{\delta z_0} \theta + \frac{\delta^l}{\delta z_1}, \quad (B.4)$$

where $z_0$ and $z_1$ are independent variables. This means that

$$\frac{\delta}{\delta z(\theta)} z(\theta') = \delta(\theta - \theta') = z(\theta) \frac{\delta}{\delta z(\theta')} \quad (B.5)$$

Right and left derivatives are connected via the formula

$$F \frac{\delta^r}{\delta z(\theta)} = (-1)^{\epsilon(F)(\epsilon(z)+1)+1} \frac{\delta^l}{\delta z(\theta)} F, \quad F \frac{\delta^r}{\delta z(\theta)} d\theta = (-1)^{\epsilon(z)(\epsilon(F)+1)} d\theta \frac{\delta^l}{\delta z(\theta)} F \quad (B.6)$$

Let us also finally mention the chain rule:

$$\frac{\delta}{\delta z_A(\theta)} F = \int \left( \frac{\delta^l}{\delta z_A(\theta)} z^B(\theta') \right) d\theta' \left( \frac{\delta^l}{\delta z^B(\theta')} F \right), \quad F \frac{\delta^r}{\delta z_A(\theta)} = \int \left( F \frac{\delta^r}{\delta z^B(\theta')} \right) d\theta' \left( z^B(\theta') \frac{\delta^r}{\delta z_A(\theta)} \right) \quad (B.7)$$

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