Modeling of a vibrating machine with spatial oscillations of the working body

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Abstract. In modern industry, vibration technological machines with unbalanced vibration exciters (vibrating conveyors, vibrating screens, vibrating crushers, etc.) are widely used. The variety of vibration technological machines typical schemes leads to a need to take into account the dynamic system features in the mathematical models that are being created. The aim of this work is to create a spatial mathematical model and determine the dynamic system unknown parameters of a vibrating screen experimental sample with two self-synchronizing unbalanced vibration exciters that create the working body spatial motion. The mathematical model motion equations are derived using the Lagrange equations of the second kind. Using the obtained experimental data (natural frequencies and logarithmic damping decrement), the mathematical model mass-geometric parameters and the damping parameters values were calculated. The investigation result is a verified mathematical model of a vibrating screen sample with two self-synchronizing unbalanced vibration exciters.

1. Introduction

In modern industry, vibrating technological machines (vibrating conveyors, vibrating screens, vibrating crushers, etc.) are widely used [1-3]. Nowadays, self-synchronizing unbalanced vibration exciters based on asynchronous electric motors with a squirrel-cage rotor [1, 3, 4] are widely used as drives of vibration technological machines. In the induction motors with the elastic system of a vibrating machine interaction, various nonlinear effects are often observed, manifested in the form of jumps in the working body oscillations frequencies and amplitudes [2, 3, 5, 6]. When using several unbalanced vibration exciters, the possibility of self-synchronization phenomenon usage is crucial [1, 3, 7].

In most cases, modern vibrating technological machines with unbalanced vibration exciters operate in a resonant mode, when the frequency of forced vibrations exceeds the natural frequency of the working body [1, 4]. This mode allows to ensure the machine vibration stability in a wide range of load parameters. However, due to the need to overcome resonant frequencies, it is necessary to use electric motors with excess power [8, 9]. This leads to the fact that in the operating mode the drive motor is significantly underloaded, as a result of which energy consumption increases and its service life decreases. In addition, the excess electric drive power narrows the frequencies range of unbalances synchronous rotation when using several self-synchronizing vibration exciters [5, 7].

One of the creating energy-efficient vibrating machines principles is based on the use of working body resonant modes of vibration. At the same time, the required masses of unbalances and electric
motor power are significantly reduced, which leads to an increase in the efficiency and service life of vibration exciters [3, 10, 11]. However, maintaining such an operation mode requires the use of automated systems for collecting, processing and analyzing information about the current state of the machine dynamic characteristics with the control actions simultaneous formation [3, 10-12]. The creation of rational systems for automatic control for vibration machines is based on a mathematical model of a machine, which should take into account the essential features of its dynamic properties (dynamic system with an engine interaction, self-synchronization various types stability areas, dynamic system to regulation response, etc.).

The purpose of this study is to develop a design scheme and a mathematical model of a vibrating screen with two self-synchronizing unbalanced vibration exciters that can create a working body spatial motion. To achieve this goal, the following main tasks are solved in the work: the machine design scheme formation and its modeling based on the motion differential equations, the parameters identification of the machine design scheme based on the experimental studies results.

2. Calculation model

The existing designs of screens, basically, use the layout of two unbalanced vibration exciters, the axes of rotation of which are perpendicular to the vertical plane passing through the technological axis of the machine (the main movement of the processed material direction) [1, 4, 13]. A feature of the machine design considered in this work is that the axes of both unbalanced vibration exciters rotation are located in vertical planes parallel to the technological axis of the machine. Figure 1 shows a model of ULS2010.12W vibrating screen manufactured by Kroosh Technologies Ltd, where it is indicated: 1 - vibration exciters; 2 - working body; 3 - springs of elastic suspension, 4 - movable support of the bed; X - the direction of the technological axis of the machine [14].

Due to the screen symmetry relative to the vertical plane passing through the technological axis of the machine, in figure 2 only one (right) half of the calculation scheme is presented. The working body movement is described relative to the global coordinate system OXYZ, the beginning of which at rest coincides with the position of the center of mass of the system C. The working body is modeled by an absolutely rigid body of mass M of length 2Lx and width 2Ly. The position of the unbalances and the spring fixing points are set in the local system O’xyz, rigidly connected with the working body and the reference point coinciding with the origin of the system OXYZ, and the axes OY and O’y coincide, and the axes OX and O’x form an angle ψ - the angle of the working body inclination. The inertia moments
of the working body around the axes \(Ox, Oy, Oz\) are designated \(J_x, J_y, J_z\), respectively. Angular vibrations are described using the Krylov-Bulgakov angles \([15, 16]\), which in figure 2 to simplify the figure are presented as the angles of working body rotation \(\alpha, \beta, \gamma\) around the axes \(OX, OY, OZ\). As a result, the working body position relative to the global coordinate system can be specified using six coordinates: three displacements \(x, y, z\) and three angles \(\alpha, \beta, \gamma\).

The fastening points of the springs are located in the working body corners, and are displaced by a distance \(z_p\) along the \(Oz\) axis, which determines the level of oscillations along the \(OX\) and \(OY\) axes with rotations \(\beta\) and \(\alpha\), respectively connected. It is believed that each of the elastic suspension springs has linear characteristics of stiffness in three mutually perpendicular directions with the coefficients \(k_x, k_y, k_z\), which in the design scheme is represented by three elastic elements whose axes are parallel to the global coordinate system, and \(k_i = k_i\) (in figure 2 and hereinafter these coefficients are denoted as \(k_{xy}\)). Damping in the system is only due to the energy dissipation in the springs and is described by a linear viscous friction model with coefficients \(b_x, b_y, b_z\), and \(b_1 = b_2 = b_3\).

In the design scheme, each vibration exciter has one unbalance of mass \(m_i\), eccentricity \(r_i\) and inertia moment \(J_{ii}\) \((i = 1, 2\) is the vibration exciter number) fixed in the \(zO'y\) plane. Each unbalance position in the local coordinate system \(O'xyz\) is described using an additional local coordinate system \(O_1_\xi_\eta_\zeta\). The position of the local coordinate system \(O_1_\xi_\eta_\zeta\) is specified by the radius vector \(\rho_i = (0, \rho_i \cos \delta_i, \rho_i \sin \delta_i)\), where \(\rho_i\) and \(\delta_i\) are the modulus and the inclination angle of the radius vector to the positive direction of the \(O'y\) axis, and the inclination angle \(\theta_i\) of the \(O_1_\xi\) axis to the \(O'x\) axis, measured from the \(O'x\) axis positive direction counterclockwise. The unbalance position angles \(\varphi_i\) are counted from the \(O_1_\zeta\) axis negative direction counterclockwise.

The vibration exciters asynchronous electric motors have torque characteristics \(L_i\) To determine the direction of the motors rotation, the parameter \(\sigma_i = \pm 1\) is used, where the positive value corresponds to the rotation direction of the \(i\)-th unbalance counterclockwise, and the negative value corresponds to the clockwise direction.

To connect the local coordinate systems \(O_1_\xi_\eta_\zeta\) and \(OXYZ\), the rotation matrix \(T\) is used:

\[
T = \begin{pmatrix}
\cos(\theta + \psi) - \beta \sin(\theta + \psi) & -\cos(\theta + \psi)(\alpha \sin \theta + \gamma \cos \theta) & \sin(\theta + \psi) + \beta \cos(\theta + \psi) \\
\alpha \sin \theta + \gamma \cos \theta & 1 & 0 \\
-\sin(\theta + \psi) - \beta \cos(\theta + \psi) & \sin(\theta + \psi)(\alpha \sin \theta + \gamma \cos \theta) & \cos(\theta + \psi) - \beta \sin(\theta + \psi)
\end{pmatrix}
\]

The equations of the system motion, which are obtained using the Lagrange equation of the second kind and geometric rotation matrices, have the form:

\[
M \ddot{q} + B \dot{q} + Kq = F(q, \dot{q})
\]  

(2)

where \(q^T = (x, y, z, \alpha, \beta, \gamma, \varphi_1, \varphi_2)\) is the column vector of displacements, \(M, K\) and \(B\) are symmetric matrices \((8x8)\) of masses, stiffness rates and damping, respectively, \(F(q, \dot{q})\) – is the column vector of nonlinear functions describing the disturbing effect arising from the unbalances rotation.

The components of the mass matrix \(M\) are as follows:

\[
M_{21} = M_{31} = M_{32} = M_{62} = M_{87} = 0; \quad M_{11} = M_{22} = M_{33} = m_i + m_2 + M;
\]

\[
M_{41} = \sum_{i=1,2} m_i (\rho_i \cos \delta_i \sin \psi - r_i \cos(\theta + \psi) \sin \theta \sin \varphi_i); \quad M_{51} = \sum_{i=1,2} m_i (\rho_i \cos \delta_i \sin \psi - r_i \cos(\theta + \psi) \sin \theta \sin \varphi_i);
\]

\[
M_{61} = \sum_{i=1,2} m_i (-\rho_i \cos \delta_i \cos \psi - r_i \cos(\theta + \psi) \cos \theta \sin \varphi_i);
\]

\[
M_{(i+6)1} = m_i r_i \sin \varphi_i [\sin(\theta + \psi) + \beta \cos(\theta + \psi)] - \cos(\theta + \psi) \cos \varphi_i (\alpha \sin \theta + \gamma \cos \theta); \quad M_{42} = -\sum_{i=1,2} m_i \rho_i \sin \delta_i; \quad M_{(i+6)2} = m_i r_i \cos \varphi_i; \quad M_{43} = \sum_{i=1,2} m_i (\rho_i \cos \delta_i \cos \psi + r_i \sin(\theta + \psi) \sin \theta \sin \varphi_i);
\]
The components of the mass matrix \(K\) are as follows:

\[
K_{ij} = K_{ji} = K_{4i} = K_{6i} = K_{3j} = K_{5j} = K_{6j} = K_{5j} = K_{7j} = K_{8j} = 0, \quad j = 1...8;
K_{11} = K_{22} = 4k_{x}z_{p}^2; K_{33} = 4k_{y}L_{y}^2 \cos^2 \psi + 4k_{y}z_{p}^2 (z_{p}^2 + L_{y}^2 \sin^2 \psi);
K_{55} = 4k_{z}z_{p}^2 \sin^2 \psi + L_{z}^2 \cos^2 \psi + 4k_{z}z_{p}^2 (z_{p}^2 + L_{z}^2 \sin^2 \psi);
K_{66} = 4k_{z}L_{z}^2 \sin^2 \psi + 4k_{z}(L_{z}^2 + L_{z}^2 \cos^2 \psi); K_{51} = -4k_{y}z_{p} \cos \psi;
K_{42} = 4k_{y}z_{p}; K_{53} = 4k_{y}z_{p} \sin \psi; K_{64} = 2L_{y}(k_{y} - k_{x}) \sin 2\psi.
\]

The components of the mass matrix \(B\) are as follows:

\[
B_{21} = B_{31} = B_{41} = B_{51} = B_{61} = B_{72} = B_{82} = B_{23} = B_{34} = B_{43} = B_{54} = B_{65} = B_{76} = B_{87} = 0, \quad j = 1...8;
B_{11} = B_{22} = 4b_{x}z_{p}; B_{33} = 4b_{y}z_{p}; B_{44} = 4b_{z}L_{z}^2 \cos^2 \psi + 4b_{z}z_{p}^2 (z_{p}^2 + L_{z}^2 \sin^2 \psi);
B_{55} = 4b_{z}z_{p}^2 \sin^2 \psi + L_{z}^2 \cos^2 \psi + 4b_{z}z_{p}^2 (z_{p}^2 + L_{z}^2 \sin^2 \psi);
B_{66} = 4b_{z}L_{z}^2 \sin^2 \psi + 4b_{z}(L_{z}^2 + L_{z}^2 \cos^2 \psi); B_{11} = -4b_{y}z_{p} \cos \psi;
B_{42} = 4b_{y}z_{p}; B_{53} = 4b_{y}z_{p} \sin \psi; B_{64} = 2L_{y}(b_{y} - b_{x}) \sin 2\psi.
\]

The \(F(q, \dot{q})\) vector components of nonlinear functions are as follows:
\[ F_1 = \sum_{i=1}^{2} m_i r_i \phi_i (2\cos(\theta + \psi)\hat{\beta}\sin \phi_i - \cos \phi_i [\hat{\alpha}\sin \theta + \hat{\gamma}\cos \theta]) + \phi_i [\cos \phi_i (\sin(\theta + \psi) + \beta \cos(\theta + \psi)) + \cos(\theta + \psi) \sin \phi_i (\alpha \sin \theta + \gamma \cos \theta)]; \]
\[ F_2 = -\sum_{i=1}^{2} m_i r_i \phi_i^2 \sin \phi_i; \]
\[ F_3 = g (m_i + m_\gamma + M) + \sum_{i=1}^{2} m_i r_i \phi_i (2\sin(\theta + \psi)[\cos \phi_i (\hat{\alpha}\sin \theta + \hat{\gamma}\cos \theta) - \hat{\beta}\sin \phi_i]) + \phi_i [\cos(\theta + \psi) \cos \phi_i - \sin(\theta + \psi) (\alpha \sin \phi_i \sin \theta + \gamma \sin \phi_i \cos \theta + \beta \cos \phi_i)]; \]
\[ F_4 = \sum_{i=1}^{2} m_i r_i \phi_i (\rho_i \cos \delta_i + r_i \sin \phi_i [2\hat{\alpha}\cos \phi_i \sin^2 \theta - 2\hat{\beta}\sin \theta \sin \phi_i + \hat{\gamma}\cos \phi_i \sin 2\theta]) + \phi_i [\rho_i \{\cos \delta_i \cos \phi_i \cos \theta + \sin \phi_i \sin \phi_i \} - \sin \theta (\rho_i \cos \delta_i + r_i \sin \phi_i) \times \times [\alpha \sin \phi_i \sin \theta + \gamma \sin \phi_i \cos \theta + \beta \cos \phi_i]); \]
\[ F_5 = \sum_{i=1}^{2} m_i r_i \phi_i (2r_i \cos \phi_i - \rho_i \cos \phi_i \cos \delta_i) [\cos \phi_i (\hat{\alpha}\sin \theta + \hat{\gamma}\cos \theta) - \hat{\beta}\sin \phi_i] + \phi_i [\rho_i \cos \phi_i \cos \delta_i \sin \theta - \{r_i \cos \phi_i - \rho_i \sin \delta_i \cos \theta\} (\alpha \sin \phi_i \sin \theta + \gamma \sin \phi_i \cos \theta + \beta \cos \phi_i)]; \]
\[ F_6 = \sum_{i=1}^{2} m_i r_i \phi_i^2 (2\cos \theta [\rho_i \cos \delta_i + r_i \sin \phi_i]) [\cos \phi_i (\hat{\alpha}\sin \theta + \hat{\gamma}\cos \theta) - \hat{\beta}\sin \phi_i] - \phi_i [\rho_i \cos \phi_i \cos \delta_i \sin \theta + \cos \theta (\rho_i \cos \delta_i + r_i \sin \phi_i) (\alpha \sin \phi_i \sin \theta + \gamma \sin \phi_i \cos \theta + \beta \cos \phi_i)]; \]
\[ F_{i6} = \sigma_{\rho_i} M_{s_i} - \sigma_{L_i} L_i + 2m_i r_i^2 (\cos \phi_i [\alpha \sin \theta + \gamma \cos \theta] - \beta \sin \phi_i) \times \times [\phi_i [\cos \phi_i (\alpha \sin \theta + \gamma \cos \theta) - \hat{\beta}\sin \phi_i], \]

where \( L_i \) is the \( i \)-th electric drive driving moment, \( M_{s_i} \) is the rotation resistance moment of the \( i \)-th unbalance, the index \((i + 6) \) is equal to 7 or 8 depending on \( i = 1.2 \).

3. System parameters identification
The system parameters \( J_x, J_y, J_z, k_{xy}, k_z, z_p \) were determined from the equation
\[ \det(K_0 - M_0 p^2) = 0, \]

where \( p^2 = (p_1^2 \ p_2^2 \ p_3^2 \ p_4^2 \ p_5^2 \ p_6^2)^T \) is the column vector of squares of natural frequencies, the number of which corresponds to the working body degrees of freedom number, \( M_0 \) and \( K_0 \) are the matrices of masses and stiffness, which are obtained from the matrices \( M \) and \( K \) included in the equations of motion of the working body (2), at \( \phi_i = \rho_i = 0 \), \( B = 0 \) and \( F(q, \dot{q}) = 0 \).

The (7) is an equation of the sixth degree relative to the square of the natural frequency. To determine the parameters \( J_x, J_y, J_z, k_{xy}, k_z, z_p \), the inverse problem of dynamics is solved, when the components of the matrices \( M_0 \) and \( K_0 \) are calculated from the values of natural frequencies known from the experiment. From the solutions obtained, those in which the values of \( k_z \) differ significantly from the known one are discarded, as well as those that do not satisfy physical and geometric representations. The damping parameters were estimated using the experimentally obtained envelope of damped oscillations by varying the logarithmic damping coefficient \( \varsigma \) in the equation of the form \( A(t) = A_0 e^{-\varsigma t/\tau} \). The viscous friction coefficient was calculated by the formula \( b = 2\varsigma M p \), where \( p \) is the corresponding natural vibration frequency.

The required system parameters were determined from specially set experiments performed on an experimental sample of a vibrating screen manufactured by REC «Mekhanobr-Tekhnika» at an angle of the working body inclination \( \psi = 0 \) (figure 3).
Oscillations of the working body were excited in the direction of one of the global coordinate system axes of the vibrating machine by hitting the working body with a dynamometric hammer. The vibrations of the working body were measured using piezoaccelerometers of the AP2038P-50 type. The data obtained were processed in the Wolfram Mathematica 10 software package, the envelopes of damped oscillations were constructed, and the center of mass vibration acceleration spectra along the global coordinate system axes were calculated using the fast Fourier transform. As a result of processing the experimental data, 6 vibrations natural frequencies of the vibrating machine working body were calculated
\[ p_1 = 6.5 \text{ Hz}, \quad p_2 = 6.35 \text{ Hz}, \quad p_3 = 7.5 \text{ Hz}, \quad p_4 = 10.2 \text{ Hz}, \quad p_5 = 12.6 \text{ Hz}, \quad p_6 = 9.3 \text{ Hz}, \]
according to the values of which the following values were obtained system parameters:
\[ J_x = 0.456 \text{ kg} \cdot \text{m}^2, \quad J_y = 0.478 \text{ kg} \cdot \text{m}^2, \quad J_z = 1.036 \text{ kg} \cdot \text{m}^2, \quad k_{xy} = 10475 \text{ N} \cdot \text{m}^{-1}, \quad k_z = 13141 \text{ N} \cdot \text{m}^{-1}, \quad z_p = 0.054 \text{ m}, \]
as well as the coefficients damping system \[ b_x = 35.5 \text{ N} \cdot \text{s} \cdot \text{m}^{-1}, \quad b_{xy} = 38.8 \text{ N} \cdot \text{s} \cdot \text{m}^{-1}. \]
The model parameters \( \rho_1 = 0.23 \text{ m} \) and \( \delta_1 = -0.07 \text{ rad}, \quad \delta_2 = \pi + 0.07 \text{ rad} \) were calculated from the known screen size and distance \( z_p \).

4. Mathematical model verification
The verification obtained mathematical model was carried out by comparing the calculated and experimental dependences of the steady oscillations acceleration amplitudes along the axes \( OX, OY, OZ \) on the unbalances rotation frequency. To conduct the experiment, a three-axis accelerometer (AP2038P-10) and two encoders (E40H8-2500-6-L5) on the axes of vibration exciters were fixed on a laboratory sample of a vibrating screen. The motors were recorded from a frequency converter with a current with a frequency \( f \) ranging from 10 to 60 Hz, varied discretely with a variable step after the establishment of system oscillations. As a result of processing the experimental data, the maximum amplitudes of accelerations and the averaged frequency of rotation of the unbalances \( \Phi \) of steady-state oscillations were obtained at each frequency of the supply voltage, which is shown by dots in figure 4.

![Figure 3. Photographs of experimental sample of vibrating screen.](image)

![Figure 4. The amplitudes of the acceleration of steady-state oscillations along OX axis.](image)
When calculating the dependence of the acceleration amplitude on the unbalance rotation frequency, the supply voltage frequency change by the converter was taken into account using the modified Kloss formula [10]:

\[
L(\Phi_c) = \frac{2M_C}{(\Delta\omega - \Phi_c)^2 + (\omega_s - \omega_{cr})^2}.
\]

where \( L_{cr} = 1.232 \) N·m is the critical moment of the electric motor, \( \omega_{cr} = 115 \text{ rad·s}^{-1} \) is the frequency at which the critical moment is reached, \( \omega_s = 2\pi f_0 K^{-1} = 157 \text{ rad·s}^{-1} \) is the no-load frequency, where \( f_0 = 50 \text{ Hz} \) is the supply voltage frequency, \( K = 2 \) is the electric motor number of poles, \( \Delta\omega = 2\pi f_1 K^{-1} \) is the torque characteristic displacement of the electric motor caused by a change in the frequency of the current \( f \). In the range \( \Delta\omega = 0 \ldots 188 \text{ rad·s}^{-1} \) (which corresponds to a change in the supply frequency in the range \( f = 0 \ldots 60 \text{ Hz} \)). The calculation results are shown by lines in figure 4-6.

5. Results and conclusions
As a result of the study, a vibrating screen design scheme and a mathematical model with the spatial motion of the working body were developed, taking into account the torque characteristics of vibration exciters asynchronous electric motors and their rotation speed frequency regulation. The model describes such features of the dynamic screening system as the variable inclination angle of vibration exciters axes of rotation, the possibility of changing the inclination angle of the working body, the connectivity of angular and longitudinal vibrations. It should be noted that the calculated and experimental data match is quite accurate. The existing discrepancies between the model and the experiment can be explained by the natural difference in the parameters of the drive electric motors.
and unbalances, as a result of which one unbalance lags behind the other during synchronous rotation, which leads to the appearance of a small disturbing force in the direction of the $OY$ axis. In the mathematical model, the motors and unbalances are exactly the same and oscillations along the $OY$ axis occur only in case of a change in the synchronous rotation type of the unbalances.

The obtained mathematical model verification was carried out using comparison of the steady oscillations acceleration amplitudes dependences along the main axes of the installation.

Solutions comparison of the obtained motion differential equations system showed satisfactory coincidence of the calculated and experimental characteristics of the system, which, among other things, indicates the correct parameters determination of the vibrating screen laboratory sample.

The results obtained in carrying out numerical experiments correspond to the physical concept of the processes occurring in the dynamic system of a vibrating technological machine.

The proposed mathematical model and universal algorithm for determining the parameters of a vibrating machine can be used to study the behavior of vibrating technological machines with a similar design scheme.

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