Dynamics of BPS States in the Dirac-Born-Infeld Theory

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ABSTRACT

The Dirac-Born-Infeld action with transverse scalar fields is considered to study the dynamics of various BPS states. We first describe the characteristic properties of the so-called 1/2 and 1/4 BPS states on the D3 brane, which can be interpreted as F/D-strings ending on a D3-brane in Type IIB string theory picture. We then study the response of the BPS states to low energy excitations of massless fields on the brane, the scalar fields representing the shape fluctuation of the brane and U(1) gauge fields describing the open string excitations on the D-brane. This leads to an identification of interactions between BPS states including the static potentials and the kinetic interactions.

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I Introduction

The Dirac-Born-Infeld theory\cite{1,2} has recently drawn much attention in relation with the world-volume dynamics of Dp-branes\cite{3}-\cite{8}. The degrees of freedom involved with D-branes, consist of world-volume gauge fields and a number of scalar fields, representing the shape fluctuations of the Dp-brane embedded in higher dimensions. This may be understood from the fact that the $p+1$-dimensional Dirac-Born-Infeld action for Dp-brane is the dimensional reduction of the ten-dimensional supersymmetric Born-Infeld electromagnetism. The action allows static BPS configurations that saturate the so-called Bogomol'nyi bound. As shown in Ref. \cite{3}, they represent attachments of F(fundamental)-strings or D(Dirichlet)-strings to the Dp-brane. The dynamics of these states on the world-volume will be the main concern of this note.

To be specific, we shall focus on the case of D3-brane, whose 3 + 1-dimensional world-volume is immersed in ten spacetime so that the transverse space is six dimensions. As explained, for example, in Ref. \cite{3}, the electrically charged BPS states describe the intersection of the D3-brane with F-string, and the magnetic states describe the D-string that ends on the brane. There are also dyonic $(q, g)$-strings that may be viewed as the bound states of F-string and D-string. The magnitude of the scalar charge restricted by the BPS condition, can be identified with the tension of the attached string, while the direction of the string in the transverse space is specified by the $SO(6)$ direction of the scalar charge. Especially, the scalar charge determines how the D3-brane is pulled out by the string\cite{8}.

In this note, we shall first identify possible BPS states by investigating the energy functional of the Dirac-Born-Infeld theory. These will include 1/2-BPS states such as electric-pole, magnetic monopole, and dyonic states where the attached strings are directed all in one direction in the transverse six space. We shall also describe the properties of a 1/4 BPS state\cite{9} that represents a monopole and an electric-pole pulled out in mutually perpendicular directions in the transverse space. It will be shown that there are more general 1/4 BPS configurations. We present explicitly these solutions as well as their characteristic properties.

We then study the responses of various BPS states to asymptotic massless excitations by exploiting the classical field equations. As will be shown below, the motion of the center positions of F/D-strings will be governed by a generalized Lorentz force law. Furthermore, the acceleration will be accompanied by the electromagnetic and the scalar radiations. We, then, turn to the case where multiple strings of various charges are joined to the D3-brane with different $SO(6)$ angles. Of course, they may interact with each other via the electromagnetism or the deformation of the shape of the D3-branes. We shall first determine the static potentials between the F/D-strings as a function of their separation and charges. The nature of the scalar interaction will be explicitly exposed for various cases. In particular, when the static potential between the F/D-strings vanishes, the multistring states can be static; only kinetic interactions between the F/D-strings will appear in this case when they moving around on the parallel three space of the D3-brane. For generic non-BPS configurations, we shall also find the kinetic interactions as well as the static interactions.

Section II is devoted to the various BPS states including electric, magnetic and dyonic states that break half of the supersymmetries. We shall also discuss 1/4-BPS states that
leave four supersymmetries unbroken out of sixteen supersymmetries. We shall identify all
the charges and illustrate the shape of the solutions.

In Section III, the response of the BPS states to the world-volume massless fields will
be identified by studying the fields equations, which may be summarized by the force law
for the rigid motion of the F/D-strings. We will also discuss the radiations produced by
acceleration. We also study the response of the states to the incident scalar waves or
electromagnetic waves, and discuss the nature of the scattering of these waves by the BPS
configurations.

In Section IV, we will obtain the low-energy effective Lagrangian that describes all two
body interactions between various states. The static interactions together with the kinetic
interactions will be exploited based on the effective Lagrangian.

Section V comprises discussions and conclusions.

II Dirac-Born-Infeld Theory and its BPS States

We begin with the Dirac-Born-Infeld Lagrangian for a single D3-brane

\[ L = T \int d^3x (1 - K^{1/2}) \]  

(2.1)

with

\[ K = -\det(\eta_{\mu\nu} + \partial_\mu Y^I \partial_\nu Y^I + F_{\mu\nu}) \]  

(2.2)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) and the index \( I \) for the transverse space runs from 1 to 6.

As is well known, this Lagrangian is the dimensional reduction of the ten-dimensional pure
Born-Infeld action,

\[ L = T_{10} \int d^{10}x \left[ 1 - \left( -\det(\eta_{MN} + F_{MN}) \right)^{1/2} \right] \]  

(2.3)

to 3 + 1 dimensions where \( A_M \) is identified with the world-volume gauge potential for
\( M = 0, 1, \cdots, 3 \) and with the scalar fields \( Y_{M-3} \) for \( M = 4, 5, \cdots, 9 \).

Let us first consider the case where the attached string is directed in one direction, \( \hat{e}^I \) in the transverse space. As will be seen below, this includes the configurations of the
monopole, electric-pole, and dyon. One may here consistently set all the other perpendicular
components of \( Y^I \) to be zero except \( Y^I \hat{e}^I \), which we denote \( \phi \). The determinant \( K \) in
this case can be explicitly written as

\[ K = 1 - \dot{\phi}^2 - \mathbf{E}^2 + (\nabla \phi)^2 + \mathbf{B}^2 + (\mathbf{B} \cdot \nabla \phi)^2 - (\mathbf{B} \cdot \mathbf{E})^2 - (\mathbf{E} \times \nabla \phi + \dot{\phi} \mathbf{B})^2 \]  

(2.4)

where \( E_i = F_{i0} \) and \( B_i = \frac{1}{2} \epsilon_{ijk} F_{jk} \). To the quadratic order in its dynamical variables,
this Lagrangian corresponds to the conventional electromagnetism with a free scalar field.
This picture will be valid when the involved field strengths are weak enough. But the
nonlinearity in the region of strong fields will play an important role to understand the
brane dynamics.

\[ \text{We will use conventions } \eta_{\mu\nu} = \text{diag}(-, +, +, +) \text{ and } T = 1 \]
As discussed in Appendix in a more general setting, the above second-order Lagrangian may be turned into a first-order form,

\[ \mathcal{L} = \Pi \dot{\phi} - D \cdot \dot{A} - A_0 \nabla \cdot D - \mathcal{H} \tag{2.5} \]

where the Hamiltonian density \((\mathcal{H} \geq 0)\) is given by

\[ (\mathcal{H}+1)^2 = 1 + \Pi^2 + D^2 + (\nabla \phi)^2 + B^2 + (D \cdot \nabla \phi)^2 + (B \cdot \nabla \phi)^2 + (D \times B + \Pi \nabla \phi)^2. \tag{2.6} \]

In order to find the Bogomol’nyi bound\[10\] for the static configurations, we rewrite the Hamiltonian into the form,

\[ (\mathcal{H}+1)^2 = \Pi^2 + (D \times B + \Pi \nabla \phi)^2 + (\sin \xi B \cdot \nabla \phi - \cos \xi D \cdot \nabla \phi)^2 + (B - \cos \xi \nabla \phi)^2 + (D - \sin \xi \nabla \phi)^2 + (1 + \cos \xi B \cdot \nabla \phi + \sin \xi D \cdot \nabla \phi)^2. \tag{2.7} \]

From this expression, one may easily recognize that the Hamiltonian density is bounded below by\[11, 12\]

\[ \mathcal{H} \geq \cos \xi B \cdot \nabla \phi + \sin \xi D \cdot \nabla \phi. \tag{2.8} \]

The saturation of the bound occurs if the Bogomol’nyi equations,

\[ B = \cos \xi \nabla \phi, \quad D = \sin \xi \nabla \phi, \quad \Pi = 0, \tag{2.9} \]

together with the Gauss constraint, \(\nabla \cdot D = 0\), are satisfied. This set of equations may also be derived from the consideration of the supersymmetric variation of the gaugino\[3\], from which one finds that the BPS background satisfying the above equations preserves only half of the sixteen supersymmetries. A static solution of the Bogomol’nyi equation is found to be

\[ D = \frac{q^r}{4\pi r^2}, \quad B = \frac{g^r}{4\pi r^2}, \quad \phi = -\frac{q_s}{4\pi}, \tag{2.10} \]

with relations between charges, \(g = \cos \xi q_s\) and \(q = \sin \xi q_s\). Here we follow the conventional definitions

\[ Q_E = \oint_{r \to \infty} dS^i D_i, \quad Q_M = \oint_{r \to \infty} dS^i B_i \tag{2.11} \]

respectively for the electric and the magnetic charges. In addition we define the scalar charge by

\[ Q_S^l = \oint_{r \to \infty} dS^i \partial_i Y^l, \tag{2.12} \]

which is a six vector in the transverse space.

The shape of this solution seen in \(x_3 - \phi\) plane is depicted in Fig. 1 for \(q_s > 0\). The spike in the transverse direction represents the F/D string attached to the D3 brane. Namely, the F/D string pulls out the D3 brane in a smooth manner and the corresponding strength
given by the scalar charge is proportional to the tension of the string. The total energy defined by the spatial integration of the Hamiltonian density $\mathcal{H}$ is infinity since the string is stretched to the infinity in the transverse space. But we regulate it by restricting the domain of the integration by $r \geq |q_s| \varepsilon$. (The physics at $r = |q_s| \varepsilon$ will be determined by specifying consistent boundary data, whose dynamical aspects will be further discussed in the next Section.) In this case, the regulated mass of the configuration is

$$M(\varepsilon) = |q_s| l(\varepsilon),$$

with the transverse length $l(\varepsilon)$ of the string being $|\phi(\varepsilon) - \phi(\infty)| = 1/(4\pi \varepsilon)$. Since the scalar charge serves as the tension of the attached F/D-string, the mass agrees, as expected, with the tension multiplied by the length of the string.$^3$

The solution (2.10) describes a few distinctive cases. When the scalar charge is positive, it corresponds to the string stretched to the direction $-\hat{e}^I$, whereas the negative signature of the charge implies that the string is stretched to the opposite direction, i.e. $\hat{e}^I$. Since the unit vector $\hat{e}^I$ is in arbitrary direction in the transverse space, the F/D-string may then be in any transverse direction; the freedom in the signature of the scalar charge reflect the remaining possibility when an axis $\phi$ is chosen. There is also an angle parameter $\xi$. For $\xi = \pi/2, 3\pi/2$, the magnetic field vanishes and, consequently, the string is electrically charged. It is nothing but the fundamental string ending on the D3-brane. On the other hand, the magnetically charged string (D-string) corresponds to the case of $\xi = 0, \pi$. The remaining generic value of $\xi$ is for a bound state of F-string and D-string, i.e. a dyon. Furthermore, the signature of electric or magnetic charges carried by the attached string distinguishes whether one deals with string or anti-string; in other words, it correspond to the orientation of the F/D-string.
There are more generic BPS solutions than (2.10), which correspond to multi-strings located in various positions on the D3-brane. They are given by

\[
D = - \sum_n \nabla \frac{q_n}{4\pi|\mathbf{r} - \mathbf{x}_n|}, \quad B = - \sum_n \nabla \frac{g_n}{4\pi|\mathbf{r} - \mathbf{x}_n|}, \quad \phi = - \sum_n \frac{(q_s)_n}{4\pi|\mathbf{r} - \mathbf{x}_n|},
\]

(2.14)

with relations between charges, \(g_n = \cos \xi (q_s)_n\) and \(q_n = \sin \xi (q_s)_n\) for all \(n\). The sum of each dyon mass that is given by (2.13) will be the total mass of the configuration. Since they are BPS configurations, the scalar and the electromagnetic forces between any two F/D-strings are exactly canceled for all cases. There are two distinct situations in the two body interactions. One is the case where the two F/D-strings are stretched in the same direction in the transverse space. The signs of the scalar charges are the same and, hence, the scalar interaction is attractive. The cancellation of the forces in this case occurs since the electromagnetic force is repulsive with the same strength. The other corresponds to the situation where the spikes of the strings are in the exactly opposite directions. The scalar force is repulsive in this case, and the attractive electromagnetic force cancels it.

Having analyzed the aligned F/D-strings in the transverse space, let us now turn to the case where two transverse directions are allowed so that spikes of F/D-string may have an \(SO(6)\)-angle between them. Since a F/D string may have its spike into an arbitrary transverse direction specified by a unit transverse vector, the two unit vectors involved with two F/D strings form a plane in general in the transverse space. Let us denote the orthonormal unit vectors in this plane by \(\hat{e}_1^I\) and \(\hat{e}_2^I\). Then one may consistently set all the components of the scalar perpendicular to the plane to be zero. The nonvanishing component are restricted on the plane, which we denote by \(Y^I = \phi \hat{e}_1^I + \chi \hat{e}_2^I\). With these two component scalar fields, the determinant in (2.2) can be evaluated as

\[
K = 1 + B^2 + (\nabla \phi)^2 + (\nabla \chi)^2 + (B \cdot \nabla \phi)^2 + (B \cdot \nabla \chi)^2 + (\nabla \phi \times \nabla \chi)^2
- \dot{\phi}^2 - \dot{\chi}^2 - E^2 - (B \cdot E)^2 - (E \times \nabla \phi + \phi B)^2 - (E \times \nabla \chi + \chi B)^2
- (\dot{\phi} \nabla \chi - \dot{\chi} \nabla \phi)^2 - (E \cdot \nabla \phi \times \nabla \chi + \ddot{\phi} \nabla \chi \cdot B - \ddot{\chi} \nabla \phi \cdot B)^2.
\]

(2.15)

Although its complicated appearance, one may easily recognize that it does possess the \(SO(2)\)-rotational invariance in the transverse plane under the transformation, \(\phi' = \cos \alpha \phi - \sin \alpha \chi\) and \(\chi' = \sin \alpha \phi + \cos \alpha \chi\). In a Lorentz invariant form, it may be rewritten as

\[
K = 1 + \frac{1}{2} F_{mn}^2 - \frac{1}{2} \left( \frac{1}{8} \epsilon^{mnprqs} F_{pq} F_{rs} \right)^2 - \left( \frac{1}{48} \epsilon^{mnprqs} F_{mn} F_{pq} F_{rs} \right)^2,
\]

(2.16)

where the indices run from zero to five and \(F_{mn} = \partial_m A_n - \partial_n A_m\) with \((A_4, A_5) = (\phi, \chi)\). In this Lagrangian, one may consistently set, for instance, \(\chi\) to zero, and, then, the system is reduced to the case of one scalar discussed previously. The Bogomol’nyi equations in (2.9) are again serving as the condition for the BPS saturation. There are, of course, a family of Bogomol’nyi equations that are produced by \(SO(2)\) rotation; for example, the Bogomol’nyi equation (2.9) with \(\chi\) instead of \(\phi\) is obtained by the rotation with \(\alpha = \pi/2\). The solutions of these Bogomol’nyi equations are the 1/2-BPS states described above.
The system allows a new type of static Bogomol’nyi equations given by

\[
B = \nabla \phi, \quad D = \nabla \chi ,
\]

with the Gauss law constraint \( \nabla \cdot D = 0 \), where \( D \) is the momentum conjugate to \( A \) defined by \( D_i \equiv \partial \mathcal{L} / \partial \dot{A}_i \). The derivation of these equations is relegated to Appendix. From the supersymmetric viewpoint, the first equation in (2.17) breaks half of the sixteen supersymmetries of D3-branes and the second breaks further half of the remaining eight supersymmetries. The related BPS configurations with \( B,D \neq 0 \) leave only four supersymmetries unbroken and, hence, they are so-called 1/4-BPS states [9].

To show that the static solutions of the Bogomol’nyi equations do satisfy the original field equations, let us expand the Lagrangian density at the background that satisfies the Bogomol’nyi equations. With the Bogomol’nyi equations used for the background, one may easily find that the Lagrangian \( \mathcal{L} \) can be expanded as

\[
\mathcal{L} = -B^2 - (\bar{B} \cdot \delta B - \bar{E} \cdot \delta E + \nabla \bar{\phi} \cdot \nabla \delta \phi + \nabla \bar{\chi} \cdot \nabla \delta \chi) + O(\delta^2) ,
\]

(2.18)

where the quantities with a bar represent the solution of the Bogomol’nyi equations and we have used the fact that \( \bar{E} = \bar{D} \). When the Bogomol’nyi equations are further used, the terms in the parenthesis in (2.18) become purely surface terms and, consequently, the action is minimized for an arbitrary variation of the fields. This verifies that the background indeed satisfies the original field equations. A 1/4-BPS configuration can be found explicitly by solving the Bogomol’nyi equations:

\[
D = -\nabla \frac{g}{4\pi |r - x|} , \quad \chi = -\frac{q}{4\pi |r - x|} ,
\]

\[
B = -\nabla \frac{g}{4\pi |r - y|} , \quad \phi = -\frac{q}{4\pi |r - y|} ,
\]

(2.19)

where \( x \) and \( y \), respectively, denote the positions of the F-string and the D-string. To find the shape of the solution, we first note that \( Y^I \) is given by \( \hat{e}_I^1 \phi + \hat{e}_I^2 \chi \). Hence the spike of the monopole is directed to \( \hat{e}_I^1 \), while that of the fundamental string to the direction \( \hat{e}_I^2 \). Particularly, they are perpendicular to each other in the transverse space. The configuration of the above 1/4-BPS state is illustrated in Fig. 2.

Due to the BPS nature of the configuration, the static forces between the strings are again vanishing. Electromagnetic static force does not appear since one is charged electrically and the other carries only magnetic charge. Hence, the static scalar force should not exist. This can be understood as follows. The F-string possesses the scalar charge component that produces a nonvanishing gradient of \( \chi \)-field. But the D-string cannot feel the \( \chi \)-field since it carries \( \phi \)-charge only. Consequently, the scalar interaction does not appear in this case, and this fact can be stated more effectively by saying that the two scalar charges have different flavor indices.

Let us imagine the configuration where two 1/2-BPS dyons located at different positions are pulled out in two arbitrary transverse directions. In this case, the static force is not balanced in general, so the corresponding configuration is not a BPS state. However, when the transverse angle between the spikes takes on a special value determined by scalar
charges, the static force may disappear. Indeed, the most general 1/4 BPS solutions are given by

\[
D = -\nabla \sum_n \frac{q_n}{4\pi |r - x_n|}, \quad \chi = -\sum_n \frac{q_n}{4\pi |r - x_n|}, \\
B = -\nabla \sum_n \frac{g_n}{4\pi |r - x_n|}, \quad \phi = -\sum_n \frac{g_n}{4\pi |r - x_n|},
\]

where the number of dyons and their positions are arbitrary. In this configuration, each spike of the n-th \((q_n, g_n)\) dyon is in the transverse direction \(\frac{-q_n}{\sqrt{q_n^2 + g_n^2}} e_1 + \frac{-g_n}{\sqrt{q_n^2 + g_n^2}} e_2\). Namely, the directions of the spikes are not arbitrary because they are determined by their electromagnetic charges. The static forces of the configuration are again balanced, which will be analyzed in detail later on.

So far we have considered the static properties of 1/2-BPS states and 1/4-BPS state. More general configurations of D-strings, F-strings, or dyons are allowed in the theory, but they are, in general, not the BPS states. Since the static force exist in case of the generic non-BPS states, the corresponding configurations cannot be static. We shall exploit related dynamic aspects of F/D strings including the 1/4 BPS states in the next section.

III Response of a BPS State to Fields on a Brane

The F/D-strings dealt in the previous section, are solitonic configurations of fields, and they are characterized by their positions, charges and mass. Because they have charges, the configurations are expected to respond to asymptotic excitations on the D3-brane. In
general, these kinds of classical dynamics must be totally governed by the original field equations. However, when the asymptotic fields on the D3-brane are weak enough, the response of the string will be linear in the asymptotic fields, and governed by the linearized field equations in the background configuration of a single F/D string. Furthermore, in this weak field limit, the shape deformation of the well-separated F/D strings, will be negligible and each string will be moving collectively as an independent entity; this motion may be characterized by the time dependence of the center position of each F/D-string.

To analyze the weak-field response, let us expand the Lagrangian density (2.1) to the quadratic order of the small variations from a dyon solution. We choose the scalar field \( \phi = Y^I \hat{e}^I \) as before where \( \hat{e}^I \) is the direction of the scalar for the dyon. The remaining components of the scalar can also be fluctuating but it can be easily shown that they decouple from the dynamic modes of the dyon. Then, the resulting expression reads

\[
\mathcal{L} = -\bar{B}^2 + \frac{1}{2} \left( \delta \dot{\phi}^2 + (\delta E)^2 - (\nabla \delta \phi)^2 - (\delta B)^2 \right) + \frac{Z(r)}{2} (\cos \xi \delta B + \sin \xi \delta E - \nabla \delta \phi)^2 + O(\delta^3),
\]

(3.1)

where

\[
Z(r) = \frac{(\nabla \bar{\phi})^2}{1 + \bar{B}^2} = \frac{\bar{B}^2}{\cos^2 \xi(1 + \bar{B}^2)},
\]

(3.2)

and the quantities with a bar denote the background dyon solution satisfying the Bogomol'nyi equations (2.9). The corresponding linearized Euler-Lagrange equations can be found to be

\[
\frac{\partial^2}{\partial t^2} \delta \phi - \nabla^2 \delta \phi - \nabla \cdot Z(r) \delta \mathbf{G} = 0,
\]

(3.3)

\[
\frac{\partial}{\partial t} (\delta E - \nabla \times \delta \mathbf{B}) + \cos \xi \nabla \times Z(r) \delta \mathbf{G} + \sin \xi \frac{\partial}{\partial t} Z(r) \delta \mathbf{G} = 0,
\]

(3.4)

with the Gauss law constraint

\[
\nabla \cdot \delta \mathbf{E} + \sin \xi \nabla \cdot Z(r) \delta \mathbf{G} = 0,
\]

(3.5)

where \( \delta \mathbf{G} \) is the combination, \( \cos \xi \delta \mathbf{B} + \sin \xi \delta \mathbf{E} - \nabla \delta \phi \). For later convenience, one may rearrange the above set of equations as follows. When Eq. (3.5) multiplied by \( \sin \xi \) is added to Eq. (3.3), one obtains

\[
\frac{\partial^2}{\partial t^2} \delta \phi + \nabla \cdot W^{-1}(r) \delta \mathbf{G} = 0,
\]

(3.6)

where we have introduced a new function \( W(r) \equiv 1 + \bar{B}^2 \). Similarly, the remaining equations, (3.4) and (3.5), can be written as

\[
\frac{\partial}{\partial t} (\delta \mathbf{E} - \sin \xi \nabla \delta \phi) - \cos \xi \nabla \times W^{-1}(r) \delta \mathbf{G} - \sin \xi \frac{\partial}{\partial t} W^{-1}(r) \delta \mathbf{G} = 0,
\]

(3.7)

\[
\nabla \cdot (\delta \mathbf{E} - \sin \xi \nabla \delta \phi) - \sin \xi \nabla \cdot W^{-1}(r) \delta \mathbf{G} = 0,
\]

(3.8)
Let us first consider the case where the dyon accelerates constantly in weak asymptotically uniform fields. To find such a solution, we adopt the following ansatz:

\[ \delta A = -\frac{t^2}{2} a \cdot \nabla \hat{A} - ta \hat{A}_0 + \delta \hat{A}, \]

\[ \delta A_0 = -\frac{t^2}{2} a \cdot \nabla \hat{A}_0 - ta \cdot \hat{A} + \delta \hat{A}_0, \]

\[ \delta \phi = -\frac{t^2}{2} a \cdot \nabla \hat{\phi} + \delta \hat{\phi}, \]

where again the quantities with a bar refer to the dyonic background solution. The first terms in (3.9)-(3.11) are responsible for the bulk motion of the dyon while the second terms in (3.9) and (3.10) are the terms generated by the instantaneous Lorentz boost with the velocity \( ta \). Finally, the third terms are assumed to be time independent perturbations, which will be responsible for the remaining dynamical aspects of the dyon. When this ansatz is inserted into the equations, (3.6)-(3.8), one finds that they are reduced to

\[ \nabla \cdot [W^{-1} \delta \hat{G} - a \hat{\phi}] = 0, \]

\[ \nabla \times [W^{-1} \delta \hat{G} - a \hat{\phi}] = 0, \]

\[ \nabla \cdot (\delta \hat{E} - \sin \xi \nabla \hat{\phi}) = 0, \]

with \( \delta \hat{G} \) being \( \cos \xi \delta \hat{B} + \sin \xi (\delta \hat{E} + a \hat{A}_0) - \nabla \delta \hat{\phi} \). The most general solutions of these equations are provided if one solves the following equation

\[ (1 + \hat{B}^2)(\hat{\phi} + S_0)a = \cos \xi \delta \hat{B} + \sin \xi (\delta \hat{E} + a \hat{A}_0) - \nabla \delta \hat{\phi} = \delta \hat{G}, \]

together with the Gauss law (3.13), where \( S_0 \) is the integration constant and we have written the function \( W(r) \) explicitly. When viewed from \( r = \infty \), the equation (3.14) implies the force law,

\[ M(\epsilon)a = g B_0 + q E_0 - \frac{q s e^I Y_0^I}{\sqrt{1 - v^2}}, \]

where we have chosen the constant \( S_0 \) as \( l(\epsilon) \) and \( (B_0, E_0, Y_0^I) \) refer respectively to the asymptotic values of magnetic field, electric field, and the scalar fields. When the Lorentz symmetry of the system is used, the force law can be transformed into a covariant form:

\[ \frac{d}{dt} \left( \frac{[M(\epsilon) + q_s e^I Y_0^I] v}{\sqrt{1 - v^2}} \right) = g(B_0 - v \times E_0) + q(E_0 + v \times B_0) - q_s e^I \nabla Y_0^I \sqrt{1 - v^2} \]

where \( \mathbf{x} \) and \( \mathbf{v} \) are, respectively, the position and velocity of the dyon center \[19\]. The appearance of the factor, \( M(\epsilon) + q_s e^I Y_0^I \), on the left side can be understood from the fact that the change in the asymptotic value of the scalar effectively contributes to the mass by the corresponding length change of the F/D-string.

For general angle \( \xi \), the equations, (3.14) and (3.13) can be solved and the duality symmetry upon the interchange\[7, 13\],

\[ E \rightarrow B, \quad B \rightarrow -E, \quad q \leftrightarrow g, \]

(3.17)
may be explicitly verified. For simplicity, let us focus on the case of D-string (i.e. \( \xi = 0, \pi \)). The solution for the pure D-string reads

\[
\delta \tilde{\phi} = \hat{r} \cdot \mathbf{a} \varphi(r) + r \cdot \mathbf{C}, \quad \delta \tilde{A} = \hat{r} \times \mathbf{a} \varphi(r) - \frac{1}{2}[\mathbf{r} \times \mathbf{C} + l(\epsilon)\mathbf{r} \times \mathbf{a}], \quad \delta \tilde{A}_0 = r \cdot \mathbf{E}_0
\]

(3.18)

with the function \( \varphi \) being

\[
\varphi = \frac{g}{8\pi} \left(1 - \frac{[g/(4\pi r) - l(\epsilon)]^2}{r^2}\right).
\]

(3.19)

This solution involves seven integration constants. Six of them are given by arbitrary constant vectors, \( \mathbf{C} \) and \( \mathbf{E}_0 \). The remaining one involved with the function \( \varphi \) has been fixed by the requirement that \( \delta \tilde{\phi}(\epsilon) = o(\epsilon) \) as \( \epsilon \) goes to zero. The requirement implies that the D-string is moving without any significant deformation around \( r = |q_s|/\epsilon \) as \( \epsilon \) approaches zero. This boundary condition at \( r = |q_s|/\epsilon \) deserves further discussion. If we suppose that the string is extended to the region \( r < |q_s|/\epsilon \), the boundary condition implies that the segment of the string in the region \( r < |q_s|/\epsilon \) should always move with the same velocity as the string segment of \( r \geq |q_s|/\epsilon \). This implies physically that the asymptotic fields are responsible only for the motion of the latter string segment (\( r \geq |q_s|/\epsilon \)), since the movement of the former part is controlled by hand. Namely, the energy transfer from the asymptotic fields to the string should result in the energy gain of the string segment in \( r \geq |q_s|/\epsilon \), as will be illustrated explicitly, in the next section, by giving the effective theory of the F/D-string dynamics. This, in turn, means that the energy transfer across the boundary at \( r = |q_s|/\epsilon \) should be vanishing. When there exist many strings, the boundary conditions on each string at \( |r - x_n| = |(q_s)_n|/\epsilon \) may be specified in the same way, and then the resulting dynamics of the strings will be consistent with the conservation of energy and the unitarity requirement. (Note that our regularization is such that the transverse lengths of all strings are the same.) This will be further discussed in Sec. V in the context of many D3-branes where strings of finite length are naturally realized as connecting two D3-branes.

The accelerated charges should emit electromagnetic and as well as the scalar radiations. Using explicit solution, one finds that the radiated power densities for the electromagnetic and the scalar fields are, respectively, given by

\[
T_\text{em}^\text{I} = \frac{eI q_s^2 (a \cdot \hat{R})^2}{16\pi^2 R^2}, \quad T_\text{em} = \frac{(q^2 + g^2)(a \times \hat{R})^2}{16\pi^2 R^2}
\]

(3.20)

where vector, \( \mathbf{R} \), denotes the difference between the observational point and the position of F/D-string at retarded time. (A detailed discussions can be found in Ref. \[14, 15\] where an accelerated dyon in SU(2)-Yang-Mills-Higgs theory is dealt with.)

Let us now consider the motion of F/D-string due to more general asymptotic excitations. For our purpose, consideration of the harmonic time dependence \( e^{-i\omega t} \) is sufficient since the equations are linear. Eqs. (3.9) and (3.7), then, become

\[
\delta \phi = \frac{1}{\omega^2} \nabla \cdot \mathbf{b}
\]

(3.21)

\[
\delta \mathbf{A} = -\frac{1}{\omega^2} \left[ \cos \xi \nabla \times \mathbf{b} + i \sin \xi (\omega \mathbf{b} - \omega^{-1} \nabla \nabla \cdot \mathbf{b}) - i \omega \nabla \delta A_0 \right],
\]

(3.22)
with
\[ b = W^{-1}(r) \left[ \cos \xi \delta B + \sin \xi \delta E - \nabla \delta \phi \right]. \] (3.23)

There is no equation that constrains \( \delta A_0 \), which reflects the fact that \( \delta A_0 \) is a pure gauge degree of freedom. Inserting (3.21) and (3.22) into the right side of (3.23), we obtain the equation for \( b \) that reads
\[ (\nabla^2 + \omega^2) b + \omega^2 (\nabla \bar{\phi})^2 b = 0, \] (3.24)
where \( (\nabla \bar{\phi})^2 = q_s^2/(16\pi^2r^4) \) for a dyon. This equation is precisely the one dealt in Ref. [3] for the S-matrix analysis of D-string. The scattering analysis in Ref. [3, 16] shows that the low frequency wave from the asymptotic region does not pass through the throat of the F/D-string. This is the case where the Dirichlet boundary condition[17] of the string attachment to the D3-brane in the Type IIB string picture, is indeed realized. On the other hand, if the high frequency wave is incident upon the F/D-string, the wave freely travel through the throat of the F/D-string. Hence at high frequency or energy, the theory requires boundary data on the energy flux. For this reason, we shall restrict our discussion below to low enough energy dynamics, where the boundary flux is negligible.

IV Interactions between BPS States

As discussed previously, the Dirac-Born-Infeld theory allows multi-F/D-string configurations. These F/D-strings may interact via electromagnetism and the scalar field on the D3-brane. The static multi-string configuration may or may not be possible depending on whether their static potential vanishes or not. Since any BPS state that satisfies the Bogomol’nyi equations is static, the electromagnetic force should be exactly canceled by the scalar force. As will be shown below, this cancellation occurs if the charges carried by the strings satisfy certain conditions. If this condition does not hold for certain configurations, they cannot be static due to the existence of the static potential between strings. In this Section, we shall exploit the low-energy interactions between well separated F/D-strings, based on the dynamics of a single F/D strings discussed in the previous section.

Let us begin by the discussion on the free kinetic terms for the objects involved with many-string dynamics. The dynamical degrees of freedom for the effective theory will be asymptotic electromagnetic fields, six scalar fields and the center positions of F/D-strings. The kinetic terms of the asymptotic electromagnetic fields or the scalar fields can be easily obtained from the original Dirac-Born-Infeld Lagrangian. Since we are interested in low-energy dynamics, the linearized version of the Dirac-Born-Infeld Lagrangian will be enough to describe the excitations on the D3-branes. The kinetic terms for each F/D-string are also easily identified using the Lorentz symmetry of the system. The energy of a single F/D-string will become as \( M(\epsilon)/\sqrt{1 - \nu^2} \) when the static configuration is Lorentz-boosted with a velocity \( \nu \). This represents a moving F/D-string with a constant velocity. The kinetic term for this energy is evidently provided by \(-M(\epsilon)\sqrt{1 - \nu^2} \), which will serve as the free Lagrangian for F/D-string.
Let us turn to the interaction terms for the low-energy effective theory. The information on the interaction vertices is coded in the equations of motion obtained in the last Section. Namely, the interaction terms determine the forces in the F/D-string motion. They also serve as a source for the asymptotic fields.

The above discussion is summarized in the following action, 

$$I_{\text{eff}} = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} F_{\mu\nu} (\partial_{\mu} A_\nu - \partial_{\nu} A_\mu) - \frac{1}{2} \partial_{\mu} Y^I \partial^\mu Y^I \right\} - \int dt \sum_n M_n(\epsilon) \sqrt{1 - \dot{x}_n^2} + I_{\text{int}}(4.1)$$

with

$$I_{\text{int}} = \int dt \left[ - \sum_n q_n \epsilon_n^I Y^I \sqrt{1 - \dot{x}_n^2} - q_n (A^0 - \dot{x}_n \cdot A) - g_n (C^\mu - \dot{x}_n \cdot C) \right]$$

where $C_\mu(x)$, as a function of $F_{\mu\nu}$, is defined as

$$C^\mu(x) = - \int d^4x (n \cdot \partial)^{-1}(x, x') n_\nu \ast F^{\mu\nu}(x'), \quad (\ast F^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\delta} F_{\lambda\delta}).$$

Here, $n^\mu$ may be any fixed, spacelike, unit vector, and Green’s function $(n \cdot \partial)^{-1}$ is realized by

$$(n \cdot \partial)^{-1}(x, x') = \frac{1}{2} \int_0^\infty d\xi [\delta^4(x - x' - n\xi) - \delta^4(x - x' + n\xi)].$$

To be definite, we shall choose the location of the symmetric infinite Dirac string by $n^\mu = (0, \hat{n})$. The magnetic interaction terms given by the function $C_\mu(x)$ are borrowed from Schwinger’s Lagrangian formulation for both electric and magnetic charges. It is straightforward to verify that the equation of motion for an F/D-string (3.10) can be derived as the Euler-Lagrange equation from the above Lagrangian. Furthermore, the effective action describes all the low-energy processes involving electromagnetic and scalar excitations arising from the motion of the F/D-string. These include radiations from the accelerated F/D-strings, which may be shown to agree with the expression (3.20). Especially, the low-energy interactions between F/D-strings are understood as the mediation of the asymptotic fields of D3-branes. For well separated, slowly moving F/D-strings, these interaction can be clearly seen by integrating out the asymptotic fields. This is achieved by solving the field equations for the asymptotic fields and by inserting the solution into the action (4.1) [see Ref. [19] for detailed procedure.]. The resulting effective Lagrangian to the quadratic order in velocities, is given by

$$L_D = - \sum_n M_n(\epsilon) + \frac{1}{2} \sum_n M_n \dot{x}_n^2 - \frac{1}{16\pi} \sum_{n \neq m} (q_s)_n (q_s)_m \epsilon_n^I \epsilon_m^I \frac{|\dot{x}_{nm}|^2}{|x_{nm}|}$$

$$- \frac{1}{16\pi} \sum_{n \neq m} (q_n g_n - g_n q_m) \dot{x}_{nm} \cdot \omega(x_{nm})$$

$$- \frac{1}{16\pi} \sum_{n \neq m} [(q_s)_n (q_s)_m \epsilon_n^I \epsilon_m^I - q_n q_m - g_n g_m] \left\{ \frac{\dot{x}_n \cdot \dot{x}_m}{|x_{nm}|} + \frac{x_{nm} \cdot \dot{x}_n x_{nm} \cdot \dot{x}_m}{|x_{nm}|^2} \right\}$$

$$+ \frac{1}{8\pi} \sum_{n \neq m} \frac{(q_s)_n (q_s)_m \epsilon_n^I \epsilon_m^I - q_n q_m - g_n g_m}{|x_{nm}|},$$

(4.5)
with $x_{nm} \equiv x_n - x_m$, where $\omega^i(r)$ denotes the unit-monopole static vector potential (with a symmetrically-located infinite string),

$$\omega(r) = -\frac{\hat{n} \times \hat{r}}{r - \hat{n} \cdot r} + \frac{\hat{n} \times \hat{r}}{r + \hat{n} \cdot r}.$$ \hspace{1cm} (4.6)

The static potential term (the last term in $L_D$) between two F/D-strings vanishes if their charges satisfy the condition

$$J_{nm} \equiv \sqrt{q_n^2 + g_n^2} \sqrt{q_m^2 + g_m^2} \hat{e}_n^I \hat{e}_m^I - q_n q_m - g_n g_m = 0$$ \hspace{1cm} (4.7)

where we have used the relation $q_s = \sqrt{q^2 + g^2}$ between the scalar and electromagnetic charges of a single F/D-string. Namely, the strength of the static potential is solely determined by $J_{nm}$.

For generic configuration of two F/D-strings depicted in Fig. 3, the static potential does not vanish in general since the transverse angle $\theta$ can be arbitrary. But for BPS configurations discussed in Sec. 1, the static potential should, of course, be zero. Fig. 4 illustrates two cases with vanishing static potential. Fig. 4a describes two D-strings whose spikes are directed in the same direction of the transverse six-space. The magnetic force is canceled exactly by the attractive scalar force. The second configuration in Fig. 4, where a D-string and an anti-D-string are exactly in opposite direction in the transverse space, also satisfies the condition on the charges. The repulsive force between the scalar charges cancels the attractive magnetic force. We now turn to the case of 1/4-BPS states. For the 1/4-BPS state in Fig. 2, where the transverse direction of a F-string is perpendicular to
that of the D-string, the scalar force as well as the electromagnetic force totally disappear. For the more general 1/4-BPS states in (2.20), we note that
\[ \cos \theta_{nm} = \frac{q_n q_m + g_n g_m}{\sqrt{q_n^2 + g_n^2} \sqrt{q_m^2 + g_m^2}}, \] (4.8)
and, hence, the static potential again vanishes, i.e. \( J_{nm} = 0 \). Thus, the static potential vanishes for all 1/2 and 1/4 BPS states introduced in Sec. I.

When the BPS conditions on charges are satisfied, the Lagrangian (4.5) becomes purely kinetic. In other words, the classical trajectories are given in terms of the geodesic motions in the geometry defined by the kinetic term. For the 1/2 BPS-states of dyons (i.e. \( q_n g_m - q_m g_n = 0 \) with parallel or anti-parallel spikes for all \( n \) and \( m \) ), the Lagrangian is reduced to
\[ L_{1/2} = - \sum_n M_n(\epsilon) + \frac{1}{2} \sum_n M_n \dot{x}_n^2 - \frac{1}{16\pi} \sum_{n \neq m} (q_n q_m + g_n g_m) \frac{|\dot{x}_{nm}|^2}{|x_{nm}|}, \] (4.9)
where \( \sigma_{nm} \) denotes the signature of \( \hat{e}^I_n \hat{e}^I_m \) depending on the spikes involved are parallel or anti-parallel. Especially, this Lagrangian has been obtained in Ref. [20], when only parallel D-strings (\( \sigma_{nm} > 0 \)) are present on the D3-brane.

For the configuration of general 1/4-BPS states, again the static potential term again drops out from the effective Lagrangian:
\[ L_{1/4} = - \sum_n M_n(\epsilon) + \frac{1}{2} \sum_n M_n \dot{x}_n^2 - \frac{1}{16\pi} (g_n g_m + q_n q_m) \frac{|\dot{x}_{nm}|^2}{|x_{nm}|} \]
\[ - \frac{1}{16\pi} \sum_{n \neq m} (q_n g_n - g_n q_n) \dot{x}_{nm} \cdot \omega(x_{nm}). \] (4.10)

Especially, for the 1/4-BPS configuration of Fig. 2 (i.e. F-string and D-string whose spikes are perpendicular), even the kinetic interactions disappear and the related geometry becomes totally flat.
V Conclusions

In this note, the various BPS states have been identified by solving the Bogomol’nyi equations of the Dirac-Born-Infeld action with six transverse scalar (coordinates) fields. The basic elements of the static BPS configurations comprise electrically charged F-string, magnetically charged D-string and their dyonic bound stretched into the transverse six space. The more generic $1/2$ BPS states describe many (anti-)parallel strings with their charges restricted accordingly. We have also identified the generic $1/4$-BPS states including the case where the spikes of a F-string and a D-string are perpendicular in the transverse space.

The asymptotic excitations on D3-branes consist of electromagnetism and the six scalar fields. We have determined the response of a F/D-string by obtaining the equations of motion for the center position of the string. Based on this analysis, we have constructed the low-energy effective field theory involving many F/D-strings and the asymptotic fields on the D3-brane. When the massless degrees of freedom are integrated, the two-body interactions between strings for various configurations are fully identified.

We have found that the static potential between the two F/D-strings disappear for all cases of the $1/2$ and the $1/4$ BPS states discussed in Sec. I. The interactions of F/D-strings found in this note, are shown to be valid for low enough energies, whereas the description may be seriously affected for high enough energies. The existence of possible energy flow to the throat of a string at high energy scattering implies that the unitarity of the theory may even break down. But if the F/D-strings starting from a D3-brane end on another D3 brane at a finite transverse distance, the unitarity should be preserved because the energy from one brane that passes through the throat, reappears in the other brane. Because at least two D3-branes are involved here, the theory should be described by the non-Abelian Dirac-Born-Infeld action\cite{21, 22, 23}. In this non-Abelian version, it is expected that the two-body interactions of F/D-strings are determined by the dynamics of all the branes where the F/D-strings end. For example, in the case of two D strings ending commonly on the first and the second branes, the total interactions will be the sum of those on the first and the second branes. These indeed can be confirmed by using the $N = 4 \ SU(N_c)$ Super-Yang-Mills theory as a limiting version of the full-fledged non-Abelian Dirac-Born-Infeld actions\cite{24}. In this sense, we have obtained the interactions of F/D-strings from the view point of just one D3-brane where the F/D-strings end commonly.

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APPENDIX: Derivation of the Bogomol’nyi Equation for 1/4 BPS states

To obtain the Bogomol’nyi equations for the scalar system with $K$ in (2.10), let us first note that the determinant $K$ can be cast in the form,

$$K = 1 + W - u^a M_{ab} u^b,$$  

(1)

where $u^i = E^i$ for $i = 1, 2, 3$, $u^4 = \dot{\phi}$ and $u^5 = \dot{\chi}$. Since the determinant $K$ in (2.10) is quadratic in $u^a$, the matrix $M_{ab}$ do not depend on $u^a$ at all. For future use, let us give the explicit form of the function $W$ given by

$$W = B^2 + (\nabla \phi)^2 + (\nabla \chi)^2 + (B \cdot \nabla \phi)^2 + (B \cdot \nabla \chi)^2 + (\nabla \phi \times \nabla \chi).$$  

(2)

Using the definition $p_a \equiv \frac{\partial L}{\partial u^a}$ with $L = 1 - K^{1/2}$, one finds that the canonical momenta $p_a$ are related with $u^a$ by

$$p_a = \frac{M_{ab} u^b}{\sqrt{1 + W - u^c M_{cd} u^d}}.$$  

(3)

With help of the inverse $M^{ab}$ ($M_{ac} M^{cd} = \delta^d_d$), we find a relation

$$p_a M^{ab} p_b = \frac{u^a M_{ab} u^b}{1 + W - u^c M_{cd} u^d},$$  

(4)

Hence, $u^a$ is given in terms of $p_b$ by

$$u^a = \frac{M^{ab} p_b \sqrt{1 + W}}{\sqrt{1 + p_c M^{cd} p_d}}.$$  

(5)

The Hamiltonian, $H \equiv u^a p_a - L$ is then

$$H + 1 = \sqrt{1 + W} \sqrt{1 + p_c M^{cd} p_d} = \frac{1 + W}{\sqrt{1 + W - u^c M_{cd} u^d}}.$$  

(6)

The direct evaluation of the inverse $M^{ab}$ is too complicated, so we shall follow an alternative route. First, let us note that the static condition $u^4 = u^5 = 0$ is required for the minimization of the Hamiltonian (3). Using the static condition $u^4 = u^5 = 0$ that implies $M^{4b} p_b = M^{5b} p_b = 0$ (see (3)), the momenta $p_4$ and $p_5$ can be eliminated from the Hamiltonian $H$ in (3) with a little algebra. The resulting static Hamiltonian reads

$$H + 1 = \sqrt{1 + W} \sqrt{1 + D_i N^{ij} D_j},$$  

(7)

where $N^{ij}$ is the inverse of $3 \times 3$ matrix $M_{ij}$ defined by $M_{ij} N^{jk} = \delta^j_j$ and we have used the relation $D_i = p_i$ (for $i = 1, 2, 3$) following our earlier definition. It is relatively simple.
to find the inverse of the $3 \times 3$ matrix, $M_{ij}$. For example, the determinant, $\text{Det}M_{ij}$ can be evaluated as

$$\text{Det}M_{ij} = (1 + W)(1 + (\nabla \phi)^2 + (\nabla \chi)^2 + (\nabla \phi \times \nabla \chi)^2).$$

(8)

By the explicit evaluation of the inverse $N_{ij}$, one finds that our static Hamiltonian is explicitly given by

$$(\mathcal{H}+1)^2 = 1 + (\nabla \phi)^2 + (\nabla \chi)^2 + (\nabla \phi \times \nabla \chi)^2 + B^2 + D^2 + (B \cdot \nabla \phi)^2 + (B \cdot \nabla \chi)^2 + (D \cdot \nabla \phi)^2 + (D \cdot \nabla \chi)^2 + (D \times (D \times B))^2 + (\nabla \phi \times \nabla \chi \cdot D \times B)^2 \frac{1 + (\nabla \phi)^2 + (\nabla \chi)^2 + (\nabla \phi \times \nabla \chi)^2}{1 + (\nabla \phi)^2 + (\nabla \chi)^2 + (\nabla \phi \times \nabla \chi)^2}.$$ 

(9)

Note that this Hamiltonian with vanishing $\chi$ does agree with the one-scalar Hamiltonian in (2.6) when the static condition, $\dot{\phi} = 0$ [or equivalently $(1 + (\nabla \phi)^2)\Pi = \nabla \phi \cdot B \times D$], is used. Finally, the Hamiltonian (9) may be turned into the form,

$$(\mathcal{H}+1)^2 = (B - \nabla \phi)^2 + (D - \nabla \chi)^2 + (D \cdot \nabla \phi - B \cdot \nabla \chi)^2 + (1 + B \cdot \nabla \phi + D \cdot \nabla \chi)^2 + B^2 + D^2 + (B \cdot \nabla \phi)^2 + (B \cdot \nabla \chi)^2 + (D \cdot \nabla \phi)^2 + (D \cdot \nabla \chi)^2 + (D \times (D \times B))^2 + (\nabla \phi \times \nabla \chi \cdot D \times B)^2 \frac{1 + (\nabla \phi)^2 + (\nabla \chi)^2 + (\nabla \phi \times \nabla \chi)^2}{1 + (\nabla \phi)^2 + (\nabla \chi)^2 + (\nabla \phi \times \nabla \chi)^2},$$ 

(10)

where $S$ denotes the combination $B \times D - \nabla \phi \times \nabla \chi$. Thus the Hamiltonian is bounded by a surface terms by

$$\mathcal{H} \geq B \cdot \nabla \phi + D \cdot \nabla \chi,$$

(11)

where the saturation of the bound occurs if the claimed Bogomol’nyi equations (2.17) are satisfied with the Gauss law constraint $\nabla \cdot D = 0$.  

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