Stability Analysis of Pump Turbine Regulating System Based On Hopf Bifurcation Theory

Ming Zhuang\textsuperscript{1,a}, Tao Jiang\textsuperscript{2,b}, Yong Li\textsuperscript{3,c} and Linning Cao\textsuperscript{4,*}

\textsuperscript{1}Fujian Shuikou Power Generation Group Co.Ltd, Fuzhou, China
\textsuperscript{2}Anhui Xiangshuijian Pumped Storage Co.Ltd, Anhui, China
\textsuperscript{3}Hebei Zhanghewan Pumped Storage Co. Ltd, Hebei, China
\textsuperscript{4}College of Energy and Electrical Engineering, Hohai University, Nanjing, China

*Corresponding author e-mail: hhucln@163.com, \textsuperscript{a}Shuikuozm@163.com, \textsuperscript{b}jiangtao02521@163.com, \textsuperscript{c}leeyong_zhw@163.com.

Abstract. In order to study the stability of the pumped storage power station system, a model of the pump turbine regulating system under turbine operating conditions is established in this paper. Hopf bifurcation theory is applied to analyze the dynamic characteristics of pump turbine regulating system with different loads, and the results of theoretical analysis are verified by simulation calculation. The results show that the range of the regulator parameters increases significantly with the increase of the load disturbances of the regulating system, but the shape and direction of the bifurcation curves are basically the same. The change of the PID parameters will cause the change of the topology of the nonlinear system.

1. Introduction

The pump turbine regulating system for pumped storage power station is a complex system related to hydraulic, mechanical and electrical aspects, and basically shows significant nonlinear characteristics. If the linear model is adopted instead of the nonlinear dynamic action of the turbine and the water diversion system, unreasonable results may appear.

Hopf bifurcation theory is an important theory to study the stability of nonlinear systems. DJ Ling et al. used the bifurcation theory to study the Hopf bifurcation phenomenon caused by changing the regulator parameters in the case of saturated nonlinear links and disturbances, so as to explain the reasons for the system nonlinear oscillations [1, 2]. DY Chen et al. established the nonlinear mathematical model of the elastic water hammer of the hydraulic turbine regulating system, and applied the nonlinear dynamics theory to the stability analysis, which verified that Hopf bifurcation theory is of great significance for the stability analysis of the hydraulic turbine regulating system [3-6].

In this paper, the pump turbine regulating system of the pumped storage power station with different states is modeled, and the stable domain of the nonlinear system regulator parameters are obtained from the bifurcation curves. Through the simulation of the pump turbine regulating system, the dynamic characteristics of the pump turbine regulating system in different conditions are analyzed.
2. Pump turbine regulating system model

The elastic water hammer effect is considered in the model of water diversion system in this paper. The state-space equation form of transfer function can be described as follows:

\[
\begin{aligned}
    x_1 &= x_2 \\
    x_2 &= x_3 \\
    x_3 &= -a_1 x_2 + h
\end{aligned}
\] (1)

\[ q = (-a_1 b_2 + b_0) x_2 + b_2 h \] (2)

Where \( a_1 = 24/T_r^2 \), \( b_0 = -24/(h_w T_r^3) \), \( b_2 = -3/(h_w T_r) \), \( T_r \) is elastic time constant, \( h_w \) is characteristic coefficient of the water pipe of a drainage system, \( x_1, x_2 \) and \( x_3 \) are intermediate state variables, \( h \) is incremental turbine head deviation, \( q \) is incremental turbine flow deviation.

Considering that six transfer functions change with the operating conditions of the pump turbine regulating system, the whole system presents nonlinear characteristics, but the unit structure parameters remain unchanged. Six nonlinear expressions of transfer coefficients with turbine speed and water head can be obtained according to pump turbine synthetic characteristic curve [6, 7].

\[
\begin{aligned}
    e_x &= e_{xm} \sqrt{h + 1} \\
    e_q &= e_{qm} (h + 1) \\
    e_{hx} &= e_{hm} \\
    e_{qy} &= e_{qym} \sqrt{h + 1} \\
    e_{qhx} &= e_{qhm} / (x + 1)
\end{aligned}
\] (3)

Where \( e_{xm} \), \( e_{ym} \), \( e_{hm} \), \( e_{qym} \), \( e_{qhm} \) can be obtained according to the torque characteristics of the pump turbine.

First derivative differential equation of generator:

\[
\frac{dx}{dt} = (m_t - m_g) / T_a
\] (4)

Where \( T_a \) is time constant of unit inertial, \( m_t \) is relative dynamic torque, \( m_g \) is relative resistance torque.

In this paper, the parallel PID governor with practical differential link is considered, then its state-space equation form is:

\[
\begin{aligned}
    y &= [k_p (r - x) + y_i + k_d (r - x) - z - y] / T_y \\
    y_i &= k_i (r - x) \\
    z &= k_d (r - x) - z
\end{aligned}
\] (5)

Where \( T_y \) is engager relay time constant, \( x \) is incremental turbine speed deviation, \( k_p, k_i, k_d \) is proportional, derivative and integral gain.

According to Eq. (1-5), the model of pump turbine regulating system can be expressed as follows:
3. Stability analysis of the regulating system

3.1. Stable domain calculation of the regulating system with load disturbances

When considering the stability of the regulating system, the most unfavorable situation, namely the isolated state with full load, should be satisfied first. In full-load state, parameters of the unit are: $T_r=1.93$, $h_w=0.69$, $T_a=8.9$, $T_y=0.2$, $e_{x_m}=-1.38$, $e_{y_m}=0.84$, $e_{y_m}=1.61$, $e_{y_m}=0.34$, $e_{y_m}=0.91$, $e_{y_m}=0.66$. When the unit operates in isolation at full load, the stable domain is obtained by adding 5% and 10% rated load disturbances to the unit respectively, as shown in Fig. 1. With the increase of load disturbances, the variation domain of the regulator parameters increase, but the shape and direction of the bifurcation curves are basically the same. It can be seen that, with the increase of external load disturbances, in order to enable the system to restore the original stable state, the governor needs a larger adjustment range to adjust the system.

![Figure 1. Bifurcation curve formed by PI parameters of different load disturbances with $k_d=1$.](image)

3.2. Dynamic simulation of the regulating system with 10% load disturbance

3.2.1. System bifurcation caused by different load disturbances. When $k_i=1.0$ and $k_d=1.0$, under the condition that the system is disturbed by 5% and 10% rated load respectively, the nonlinear regulating system of the pump turbine is simulated, and the bifurcation diagram of the unit speed changing with the bifurcation parameter $k_p$ is shown in Fig. 2.
As shown in Fig. 2(a), when \( k_p \) is bifurcation parameter, there are two critical values \( k_{p1}^* = 0.77 \) and \( k_{p2}^* = 4.9 \) under the condition that the system is disturbed by 5% rated load. When \( k_{p1}^* < k_p < k_{p2}^* \), the system is in a stable state, and the initial deviation of the system state variable is stabilized at the equilibrium point after a period of time. When \( k_p \leq k_{p1}^* \) or \( k_p \geq k_{p2}^* \), the system has a stable limit cycle, that is, the system state variable is in the state of constant amplitude oscillation. Similarly, under the condition of 10% rated load disturbance, the dynamic characteristics of the system are basically consistent with that of 5% rated load disturbance, but the bifurcation point has an outward expansion trend.

![Bifurcation Diagram](image)

**Figure 2.** Bifurcation diagram of speed response for \( k_p \) with load disturbances.

### 3.2.2. Dynamic characteristics of the system when the bifurcation parameters are in a stable range.

When \( k_i = 1.0, k_d = 1.0, \) and bifurcation parameters are respectively set as \( k_p = 1.5 \) and \( k_p = 3.0 \), the time domain response of the unit speed when the system is disturbed by 10% rated load is shown in Fig. 3.

When the bifurcation parameter value in the stable domain, the system is asymptotically stable with the load disturbances, and when the bifurcation parameter is close to the bifurcation point, the number of unit speed oscillations increases and finally tends to be stable. However, the stable time becomes longer, so it can be seen that the closer the bifurcation parameter is to the critical value, the less stable the system is.

![Time Waveforms](image)

**Figure 3.** Time waveforms of speed response for \( k_{p1}^* < k_p < k_{p2}^* \).
3.2.3. Dynamic characteristics of the system when the bifurcation parameters are at critical values. When $k_i=1.0$, $k_d=1.0$, and $k_p$ is at the critical value, the time domain response of the unit speed and the phase trajectory are respectively made when the system is disturbed by 5% and 10% of rated loads, as shown in Fig. 4.

It can be seen from the comparison between Fig. 4(a) and (c) that when the bifurcation parameter $k_p$ is at the critical value, the unit speed is in the state of constant amplitude oscillation with different load disturbances, but the amplitude increases with the increase of load disturbances, but the oscillation frequency is basically the same. By comparing Fig. 4(b) and (d), it can be seen that when subjected to different load disturbances, the phase trajectory of the system will eventually converge to the limit cycle, but the position of the limit cycle is different.

![Graphs](image1.jpg)  
(a) Time waveform of speed with 10% rated load disturbance  
(b) Phase trajectory with 10% rated load disturbance  
(c) Time waveform of speed with 5% rated load disturbance  
(d) Phase trajectory with 5% rated load disturbance

**Figure 4.** Dynamic characteristics of nonlinear system with $k_p=k'_p$.

3.2.4. Dynamic characteristics of the system when the bifurcation parameters cross the critical values. When $k_i=1.0$, $k_d=1.0$, and the bifurcation parameter is set at $k_p=0.3$, the system is disturbed by 10% of rated load and the time domain response of the unit speed and phase trajectory are shown in Fig. 5. When the bifurcation parameters cross the critical values, the system is in an unstable state with the 10% load disturbance, and the unit speed shows obvious divergence over time.
4. Conclusion

In this paper, Hopf bifurcation theory is applied to theoretical analysis of the model of pump turbine regulating system of the pumped storage power station. According to the stable domain of PID parameters, the dynamic characteristics of the system in isolated state at full load are analyzed, and the correctness of the theoretical analysis is verified by simulation calculation.

The simulation results shows that with the increase of load disturbances, the variation domain of the regulator parameters increases, but the shape and direction of the bifurcation curves are basically the same. The change of PID parameters will lead to the change of the topology structure of the nonlinear system. When the PID parameters cross the bifurcation values, the system will have a stable limit cycle, that is, the regulating system is changed from gradual stability to stable constant amplitude oscillation, and the amplitude increases with the increase of load disturbances, but the oscillation frequency is basically the same.

References

[1] DJ Ling, ZY Shen, Bifurcation analysis of hydro-turbine governing system with saturation nonlinearity, Journal of Hydroelectric Engineering. 2007(06):126-131.
[2] DJ Ling, ZY Shen, The nonlinear model of hydraulic turbine governing systems and its PID control and Hopf bifurcation, Proceedings of the Csee. 2005(10):97-102.
[3] DY Chen, C Ding, DD Ba, XY Ma, Modeling and stability analysis of hydro-power system, Journal of Hydroelectric Engineering, 2014, 33(2): 235-241.
[4] DY Chen, Nonlinear dynamical analysis control and their applications, Northwest A&F University, 2013.
[5] DD Ba, P Yuan, DY Chen, C Ding, Modeling and analysis of nonlinear hydro-turbine governing system with complex penstocks, Journal of Drainage and Irrigation Machinery Engineering. 2012, 30(04):428-435.
[6] C Ding, DD Ba, DY Chen, P Yuan, Modeling and nonlinear dynamic analysis of Francis hydro-turbine governing system, Engineering Journal of Wuhan University. 2012, 45(2): 187-192.
[7] HQ Fang, Nonlinear control strategy for hydraulic turbine generating unit and its industrial application research, Hohai University. 2005.