Roughness measurement with nanoscale resolution by symmetric array of optical vortices

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Abstract. In present report we review the principles and applications of interference vortex method for the real time determination of polished optical surface roughness for transparent and reflecting materials with using of symmetric Laguerre-Gaussian array as a probe beams and reference beams with low topological charge. High spatial resolution caused by interference of vortices and their phase sensitivity, which is automatically analyzable to retrieve the 2D and 3D shape of micro- and nanostructured surfaces is applicable for non-destructive roughness testing of thin films and solid microstructures. The longitudinal and transverse resolution down to 1.75 nm and 7 nm respectively for visible light sources is achieved by the proposed method. The dependence of the rotational angle of the resulting interference pattern in form of two-petal laser beams on the optical path difference and sample thickness for two singular beams superposition is considered in detail.

1. Introduction

Precise determination of surface roughness and relief is essential task for the manufacturing of optical and mechanical components with high degree of quality and smoothness in engineering, metrology research and material science applications [1]. For these purposes a great number of instruments and measuring principles were developed last two decades. In general, they are divided into contact and non-contact [2] methods of interaction with the test sample. The first principle uses stylus probe detector and is based only on mechanical interaction with an object. This method is inapplicable to the wide range of practical tasks because the surface may be damaged or received parameters are not adequate. In particular it concerns such cases as biological systems imaging, polymer structures analysis or metrology of thin film layers.

In contrast, optical methods of profilometry and surface analysis has a lot of advantages, among which: non-contact and non-destructive interaction with test sample, relatively high resolution, accuracy and flexible control. The principle of optical profilometry is based on interferometric measurements of optical field, consisting of amplitude and phase information received from the surface. Phase images
may be considered as two-dimensional phase distribution or as optical path difference of interfering beams [3].

Currently the most common methods include the measurement of light modulation envelope and phase estimation or use its combination. The phase of the electromagnetic wave is determined by using interferometer with considering of refractive index of the material in case of transparent object under investigation. Phase estimation method is the most suitable data processing method in 3D optical microscopy and profilometry with using of both smooth plane waves and beams with phase singularities carrying helical wavefront.

A practical application of singular beams in the vortex scanning optical imaging allows to study, for example, surface geometry and optical density of the sample by analysis of singular phase transformation [4] and depends on the features of incident beam and aperture of optical systems [5]. In manuscripts [5-7] authors demonstrated a new solution for visualization and characterization of nanometer structures called Optical Vortex Scanning Microscope where the sample is scanned by moving vortex. This study demonstrates the response of the optical vortex imbedded in focused Gaussian beam to the shift from the critical plane inside the object arm of interferometer. Scanning procedure enables to cover all area of surface and plot a vortex trajectory, which has a characteristic way of movement and depends on thickness of the probe. Further research of Optical Vortex Scanning Microscope conducted with developing of analytical models and phase retrieval algorithms [7]. In the recent research [8] authors describe both theoretical and experimental results of imaging system using movable optical vortex, where the image of the probe was combined with the structured singular beam. Hence, the phase distribution after the object may be recovered with quite good accuracy. Thus, using of optical vortices in imaging and microscopy opens new perspectives for development of relief retrieval algorithms and new optical instruments design [9-11].

By analyzing of captured interference patterns in form of vortex spiral or any other retrievable light structures [12], we can extract information about phase shift between superposed beams and, as result, about surface roughness and relief or sample thickness and the task for optimization of such procedures on practice is the purpose of given research.

2. Phase-difference analysis of rough specimens by vortex beam array

There are two ways to analyze phase of probe singular beam after spreading through the measured sample: firstly, a direct phase-shift measurement by interference with Gaussian beam (plane-wave and spherical) and secondly, by the mutual interference of optical vortex beams carrying opposite-signed, but equally charged vortices [12]. The superposed singular beams create a new optical structure, known as two-petal laser beams, whose spatial rotation indicate change of phase [13]. This principle has become a basis for employing two-petal beam as a probe beam and further is discussed.

In current research we consider analytically an interference of singular beams with the wavelength \( \lambda \) carrying optical vortices of topological charge \( l \). One of the important features of the singular beams is the screw dislocation of wave front expressed as beam phase spatial dependence in the following form:

\[
\Phi(\varphi, z) = kz + l\varphi ,
\]

where \( z \) is the propagation direction, \( \varphi \) is the azimuthal angle at the beam cross section and \( k \) denotes wavenumber in a free space. In simplified form optical vortex with topological charge \( l \) may be expressed in terms of Laguerre-Gaussian mode \( LG_l^0 \) with zero radial index.

Let us consider the propagation of the paraxial \( LG_l^0 \) beam along the z-axis. The transverse profile \( E_x \) of the beam has a wavenumber \( k = nk_o \), where \( k_o = \frac{2\pi}{\lambda} \) is a wavenumber in a free space and \( n = \sqrt{\varepsilon} \) – refractive index of medium (for convenience it can be assumed \( n=1 \)). In the paraxial approximation,
The vortex position, by definition, is the point of the obstruction, which was described in [1]. A particular solution to the paraxial wave equation (2) for the radially-symmetric Laguerre-Gaussian vortex beam can be written as follows [14]:

\[
E_x = \frac{\alpha_0}{w} \left( \frac{r \sqrt{2}}{w} \right)^{|l|} \mathcal{L}^l \left( \frac{2r^2}{w^2} \right) \exp \left( -\frac{r^2}{w^2} \right) \exp \left( -\frac{ikr^2}{2z(1+z_R/z^2)} \right) \times \exp \left( i(2m+l-1)\arctan \left( \frac{z}{z_R} \right) \right),
\]

where \( \alpha_0 \) is the beam waist at the plane in \( z = 0 \), \( \mathcal{L} \) is the generalized Laguerre polynomial, \( z_R = \frac{k \omega_0^2}{2} \).

denotes Rayleigh length of the beam and current beam radius is expressed as \( w = \omega_0 \sqrt{1+\left( \frac{z}{z_R} \right)^2} \).

Topological charge of vortex beam is introduced as: \( l = \pm 1, \pm 2, \ldots \). The vortex position, by definition, is described by the equation \( \Re \tilde{E}_x(x, y, z) = \Im \tilde{E}_x(x, y, z) = 0 \). In equation (1) we suppose azimuthal angle is: \( \phi = \arg(x + iy) \). To introduce diffraction orders caused by diffraction grating, we will follow the next explanation, which was described in [15]: when an incident paraxial beam with topological charge \( l \) intersects the central part of square grating along the normal to its plane, an angular spectrum of diffracted waves with directions of spreading determined by angle:

\[
\theta_x = \frac{k}{d_x} \lambda, \quad \theta_y = \frac{m}{d_y} \lambda,
\]

which depend on the diffraction orders \( k \) and \( m \), wavelength \( \lambda \) and the grating periods \( d_x \) and \( d_y \) for \( x \) and \( y \) directions respectively. Here we have also a transformation of radius \( r \) which depending now on the \( \theta_x \) and \( \theta_y \) by the shift coefficient \( a_x \) and \( a_y \) for two-dimensional grating. We will restrict ourselves on the same grating periods \( d_x = d_y = d \) for each direction \( (x, y) \):

\[
\begin{align*}
    r &\rightarrow \tilde{r}_{k,m} = \sqrt{(x-a_x)^2 + (y-a_y)^2}, \\
    \phi &\rightarrow \tilde{\phi}_{k,m} = \arg \left[ (x-a_x) + i(y-a_y) \right],
\end{align*}
\]

where, in case of regular square 2D diffraction grating, \( a_x = k z \theta_x \), \( a_y = m z \theta_y \) – is the position of the \( k^{th} \) and \( m^{th} \) diffracted beam axis on the observation plane in corresponding directions (the condition \( \theta_{k,m} \ll 1 \), is implied, thus we may use only low diffraction orders). To perform diffraction over 2D grating we have replaced \( r \) and \( \phi \) in (3) by equation (5) and summarized on the diffraction orders \( k \) and \( m \). For the small \( \theta_{x,y} \) we assume that the additional phase, which arose due to the beam inclination is negligible in equation (3). Then the resulting observed field pattern may be expressed as:
\[ I(r, z) = \left| \sum_{k} \sum_{m} E_{\ell} (r_{k, m}, z) \right|^2, \]  

(6)

where \( k \) and \( m \) is the pair of minimal and maximal numbers of the diffraction orders along \( x \) and \( y \) directions respectively. The diffracted beams are depicted in figure 1. In practice, only first orders with maximal energy may be used.

Numerical calculation of intensity distribution for the array of \( 3 \times 3 \) vortex beams \((k = 0, \pm 1, \ m = 0, \pm 1)\) with \( l = 1 \) and typical axial minima produced by two-dimensional diffraction grating is shown on figure 1 (a) (other diffraction orders were cut by a diaphragm). Phase pattern is shown in figure 1 (b) depicts vortex phase as a helix.

**Figure 1.** Multiple array of Laguerre-Gaussian beams produced by 2D diffraction grating. Vortex topological charge does not depend on diffraction order and described by azimuthal index \( l = \pm 1 \) translated to the observation plane. The Cartesian coordinate system \((x, y, z)\) is associated with the grating so that the \( z \) axis is normal to the observation plane [15] and coincides with grating periods along corresponding axes.

Singular beam phase is extremely sensitive to changes of optical path between superposed waves. When the optical path of two beams spreading, for example, in arms of interferometer, are equal, it means that waves have the same waist radius and curvature of wavefront. But small variations of optical or geometrical path, which are the same in case, when \( n = 1 \) provoke well-known relative phase shift whether is observable via interference spiral behaviour. Let us consider a case when singular beam propagates through the free space between mirrors of non-equable Mach-Zehnder interferometer and interfere with Gaussian beam. In the object arm we may slightly change the optical path \( \Delta = 0 \div \lambda \), where \( \lambda \) is the wavelength.

**Figure 2.** Numerically calculated intensity distribution \((\text{Re} E(r, \varphi))^2\) \((a)\) of array of Laguerre-Gaussian beams with dimensionless node numbers: radial index \( m = 0 \) and vortex topological charge, defined by azimuthal index \( l = \pm 1 \) and its phase \( \text{Im} E(r, \varphi) \) \((b)\) with random optical path in each partial beam, which does not exceed \( \lambda \). Other beam parameters are the following: \( \omega_o = 180 \ mcm, \ \lambda = 632.8 \ nm, \ z = 2 \ cm \).

The simplest way for rapid and computationally non intensive analysis is using of direct data from image sensors and cameras with minimal adjustment and transformations. For this purpose, we propose employing of highly sensitive probe vortex beams superposition with the reference one, which contains singularity with opposite sign [9-12].
Making one full turn intensity pattern coincides with itself at geometrical path difference equal to wavelength \( \lambda \). Note, that whole picture coincides only ones for topological charges \( l = \pm 1 \). Thus, the symmetry of the pattern caused by topological charges of interfering singular beams. Further increasing of topological charge induces multiplication of high intensity spots around of image centre and is not essential for our application. So we restrict ourselves only on superposition of single charged optical vortices for two reasons: at first, interference pattern of single charged vortices has well defined intensity minimum and only one symmetrical axis thus it may be automatically defined with high accuracy. Secondly, the beam spot in this case has much less radius and performs beam focusing easier due to structural stability of optical vortices with topological charge \( l = \pm 1 \). Therefore this method can be used as addition to direct vortex phase analysis as quick draft regime of profile measurement. On estimated evaluation phase rotation measurement allows to achieve vertical resolution down to 1.75 \( nm \) for He-Ne laser source with \( \lambda = 632.8 \text{ nm} \) and less than 1 \( nm \) for blue light laser. Rough measurements with resolution \( \approx 4 \text{ nm} \) can be performed by processing of interference pattern formed by two coherent optical vortices with opposite signs. The resulting intensity and phase distribution for different \( \Delta \) is depicted in figure 3.

\[ 7 \text{ mm} \]

Figure 3. Theoretically calculated intensity distribution of array of interfered Laguerre-Gaussian beams (a) with topological charge \( l = \pm 1 \) and its phase (b) for different optical path after the reflecting from the specimen with rough surface relief. Other beam parameters are following: \( \omega_0 = 180 \text{ mcm} \), \( \lambda = 632.8 \text{ nm} \), \( z = 2 \text{ cm} \).

3. Conclusion
Development of new methods for phase shift analysis in application to thickness measurement is the quite important issue in fundamental and applied research in optical non-invasive metrology. A great attention had been paid recently to the development of optical methods and caused, first of all, by the fact that in field of applicable measurement technics in nano-research and the nano-systems industry was not limited from the point of view of image quality improving, but also had a great importance for such tasks as recording information in the optical environment: lithography and nanostructuring as well as optical manipulation. In particular, great attention was attracted by the possibility of using interferometric methods for investigating stepped structures in industry and, as result, their application to such interdisciplinary research as biophotonics.

In practice, the array of vortex beams may be produced with Spatial Light Modulator (SLM) and Diffractive Optical Element (DOE). All of them are quite customizing, however SLM needs some extra optical scheme and hardware and software equipment, this complicates their integration into real devices. Square diffracting grating is a high efficiency instrument for introducing symmetric singular beams array in simultaneous measurements. Similarly to the DOE, 2D grating allows to reproduce an array of Laguerre-Gaussian beams with arbitrary azimuthal and radial indexes on its input. As result, each partial beam carrying the same optical vortex expresses the same response to the thickness variation via changing of optical path \( \Delta = 0 \div \lambda \), where \( \lambda \) is the wavelength. The interference of two identical arrays with opposite-signed vortices broke the circular beam symmetry and two-petal beams arise.

We have analytically considered evaluation of singular beam optical phase features and its sensitivity to geometrical path changes. We have shown that the distinguishable spiral phase rotation occurs at \( \lambda/300 \). Proposed technique may be applied to optically transparent and reflecting surfaces exceed the limitations of optical wide field microscopy. Furthermore, the probe beam phase retrieval becomes
easier and automatically analyzable without using classical interference patterns and their Fourier transforms [5]. The vortex beams superposition approach enables to scale it for successfully deploying in the measurements of stepped structures as thin films, coatings and roughness of substrates in noncontact operation regime, as well as in area of biophotonics and crystal optics. Automatic processing of both vortex spiral interferograms and two-petal beam intensity distribution with typical edge dislocation allow to achieve a vertical resolution down to 1.75 nm. The experimental evaluation of measurement accuracy and finding of optimal phase retrieval procedure is a task of future investigations.

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