A classical model of the Franson-type nonlocal correlation using coherent photon pairs

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Abstract
Nonlocal quantum correlation has been the main subject of quantum mechanics over the last century. Franson-type nonlocal correlation has been widely applied for quantum communications using an energy-time bin method of entangled photon pairs, where coincidence detection selectively controls measurement events for inseparable basis-product superposition. Here, a classical model of the Franson-type nonlocal correlation is proposed and analyzed for the same quantum features as in the entangled photon cases. Like conventional nonlocal correlation based on the particle nature of quantum mechanics, the wave nature of photons has also demonstrated the same quantum features using coherent photon pairs, where inherent phase information of each photon clarifies the coincidence detection-based quantum correlation. Owing to the rule of thumb in quantum mechanics that a photon never interferes with others, the classical model of Franson-type nonlocal correlation can be applied for a new realm of macroscopic quantum information.

Introduction
Quantum entanglement [1-15] is the key ingredient of quantum information technologies including quantum computing [17], quantum key distribution [18], and quantum sensing [19-21]. Based on the seminal paper of Bell inequality violation [2], quantum correlation between paired photons exceeds the classical limit violating the hidden variable theory [1]. In the Bell inequality violation [2,3], the definition of classicality is for independent and incoherent particles according to statistics. Quantum correlation violating the classical limit has been the main subject to study quantumness over the last several decades. Franson-type nonlocal correlation relates to the seemingly noninterfering interferometers, resulting in first-order interference-like fringes in the second-order intensity correlation between space-like separated individual photon pairs [22-28]. Based on the uniform intensity measured by all local detectors in both parties, the mechanism of the nonlocal fringe of two-photon intensity correlation has been veiled and left in a mysterious realm with respect to classical physics based on rationalism and determinism such as Newtonian mechanics. Here, a classical model of the typical Franson-type nonlocal correlation is presented to implement coherently the mysterious nonlocal correlation.

According to Copenhagen interpretation, quantum mechanics lies in the wave-particle duality, where measurements play an essential role [29,30]. Based on photon statistics, an anti-bunched photon stream or sub-Poisson photon distribution is a clear example of the quantum feature differentiating it from classical physics. Self-interference of a single photon in an interferometric system of a double-slit, beam splitter (BS), or Mach-Zehnder interferometer (MZI) is an ultimate compromise between two exclusive and definite realms of the particle and wave natures in quantum mechanics [31]. Due to the harmonic oscillation of a quantum particle such as a photon or an atom, introducing binary bases of a photon is inevitable especially for an interferometric system. Thus, two-photon correlation between individual photons or atoms results in four basis-product combinations in measurements. These combinations are of course separable and thus such correlation belong to a classical regime. A coincidence detection, however, modifies the measurement basis-product combinations for quantum superposition in an inseparable manner [22-28]. Thus, nonlocal two-photon correlation based on coincidence measurements belongs to a quantum regime. The particle nature of quantum mechanics well explains the quantum features that cannot be achieved by any classical means [32].

Recently, a new interpretation has been conducted to view deterministically the quantum features in the typical Franson scheme using the wave nature of quantum mechanics [33,34]. Although definite phase information cannot be allowed to deal with a single photon due to the uncertainty relation, however, assigning a
relative phase between the paired photons does not violate quantum mechanics. A typical example is the second-order nonlinear optics of the spontaneous parametric down conversion process (SPDC) governed by phase matching conditions in nonlinear optics [35,36]. Thus, the entangled photon pair, i.e., the signal and idler photons, must have a definite relative phase between them. This understanding of the phase information is the bedrock of the wave nature-based quantum interpretation of the Franson-type nonlocal correlation [33,34]. As a result, pure coherence optics can give the same nonlocal correlation features via coincidence detection. In this coherence interpretation, the nonlocal correlation cannot be mysterious anymore, but must be definite, except for the coincidence measurements [33].

Results

Figure 1 shows schematic of a classical model of the Franson-type nonlocal correlation using coherent photons. The linewidth of the laser L is narrower than the bandwidth $\Delta$ of the acousto-optic modulators (AOMs), where $\pm \Delta$ is achieved via a pair of synchronized and opposite frequency sweeping AOMs in a fast scanning mode. For random distributions of individual detuning $\delta f_j$ within $2\Delta$ bandwidth in a double-pass AOM scheme via a quarter-wave plate (QWP) for a polarization rotation, the AOM's scanning speed should be much faster than the generation rate of photon pairs for a $\Delta_j$-dependent random phase. Thus, the $\Delta_j (= \pm \delta f_j \tau)$-detuned coherent-photon pairs in the Inset of Fig. 1 have the signal and idler photon-pair relation in SPDC processes [35,36]. For each doubly-bunched photon pair according to Poisson statistics, a phase shift $\eta$ is added into one party.

However, this $\eta$ does not affect the MZI physics for the output photons as shown below. In the unbalanced MZIs in both parties, the matrix representations for the $j^{th}$ output photons are as follows:

$$E_1^j = \frac{E_0}{2} e^{-i\Delta_j}(e^{i\Delta_S} + e^{i\Delta_I}e^{i\psi})$$
$$= \frac{E_0}{2} e^{-i\Delta_j}e^{i\Delta_S}(1 + e^{i(\Delta_I S + \psi)}).$$  \hfill (1)
\[ E_2^j = \frac{iE_0}{2} e^{-i\Delta j} (e^{i\Delta S} + e^{i\Delta L e^{i\psi}}) = \frac{E_0}{2} e^{-i\Delta j} e^{i\Delta S} (1 - e^{i(\Delta L S + \psi)}), \]  
(2) 
\[ E_3^j = \frac{E_0}{2} e^{-i\Delta j} e^{i\Delta S} (e^{i\Delta S} + e^{i\Delta L e^{i\psi}}) = \frac{E_0}{2} e^{-i\Delta j} e^{i(\Delta S + \eta)} (1 + e^{i(\Delta L S + \psi)}), \]  
(3) 
\[ E_4^j = \frac{iE_0}{2} e^{-i\Delta j} (e^{i\Delta S} + e^{i\Delta L e^{i\psi}}) = \frac{E_0}{2} e^{-i\Delta j} e^{i(\Delta S + \eta)} (1 - e^{i(\Delta L S + \psi)}), \]  
(4) 
where \( \Delta L S = \Delta L - \Delta S \), \( \Delta j = \delta f_j \tau \), and \( \tau \) is the photon-propagation time delay between the long (L) and short (S) paths. Here, \( \Delta L S = \Delta L - \Delta S \) is the path-length difference-caused phase shift by this time delay \( \tau \). The absolute short-path lengths of the MZIs are slightly different but fixed. The long paths are controlled by piezoelectric transducers (PZTs). Thus, the corresponding local intensities are as follows:

\[ I_1^j = \frac{l_0}{2} (1 + \cos(\Delta L S + \psi)), \]  
(5) 
\[ I_2^j = \frac{l_0}{2} (1 - \cos(\Delta L S + \psi)), \]  
(6) 
\[ I_3^j = \frac{l_0}{2} (1 + \cos(\Delta L S' + \psi)), \]  
(7) 
\[ I_4^j = \frac{l_0}{2} (1 - \cos(\Delta L S' + \psi)), \]  
(8) 
where \( \Delta L S' = \Delta L - \Delta S + \delta_{SS} \), and \( \delta_{SS} \) is the initially given short path-length difference. Due to the symmetric cosine function, the sign of \( \delta_{SS} \) does not matter on the results. The results of Eqs. (5)-(8) are based on the coherence optics of each photon [33]. This coherence condition contains well satisfied experimental parameters used for nonlocal Franson correlations [23-28].

For the mean value of the local measurements in each output port, the bandwidth \( \Delta \) of the AOMs plays an essential role according to the MZI physics. For the coherence feature of each photon pair in both MZIs, the first condition of \( \delta f_L \ll \Delta \) and \( \frac{c}{\delta f_L} \gg \Delta L S \) must be met [33], where \( \delta f_L \) is the linewidth of the laser L. For the ensemble of photon pairs, the second condition of \( \frac{c}{\Delta} \ll \Delta L S \) must be met [33]. Here, the first (second) condition is for incoherence (coherence) feature of the MZIs. Under these conditions, mean measurements of Eqs. (5)-(8) become uniform, satisfying local randomness:

\[ \langle I_k \rangle = \left( \sum_{j=0}^{N} I_k^j \right) = \frac{(l_0)}{2}, \]  
(9) 
where \( k = 1,2,3,4 \).

The nonlocal correlation between local measurements in space-like separated parties is as follows:

\[ R_{14}^j(\tau) = I_1^j I_3^j = \frac{l_0^2}{4} (1 + \cos(\Delta L S + \psi))(1 + \cos(\Delta L S' + \psi)). \]  
(10) 
Equation (10) is the direct result of the intensity product between two individual measurements without coincidence detection. Thus, Eq. (1) belongs to classical physics, where the measurement basis product is separable. If coincidence detection is applied to Eq. (10), however, only S-S and L-L measurement-basis products are selectively chosen, resulting in the second-order nonlocal amplitude superposition, \( E_{AB} = E_A^j E_B^j \) [33]. Thus, \( R_{14}^j(\tau) = E_{AB}^j (E_{AB}^j)^* \) is obtained as a quantum feature. The role of the coincidence detection is for measurement modification in Franson-type nonlocal correlation. Thus, Eq. (10) is modified by the coincidence detection to:

\[ R_{14}^j(\tau) = \frac{l_0^2}{16} e^{-i2\Delta L e^{i(2\Delta S + \eta)}} (1 + e^{i(\Delta L S + \psi)})(1 - e^{i(\Delta L S' + \psi)})(c.c.) \]  
(10) 
\[ = \frac{l_0^2}{16} (1 - e^{i(\Delta L S + \Delta L S' + \psi + \psi)})(c.c.) \]  
(10) 
\[ = \frac{l_0^2}{8} (1 - \cos(\delta_{SS} + \psi + \psi)). \]  
(10)
In Eq. (11), the deletion of both terms \( e^{i(\Delta_{LS} + \psi)} \) and \( e^{i(\Delta_{LS}' + \psi)} \) is the direct results of the coincidence detection method, resulting in S-S and L-L basis-product superposition in an inseparable manner. Likewise, 
\[
R_{23}(\tau) = \frac{\nu^2}{8} (1 + \cos(\xi + \psi + \varphi)),
\]
(12)
where \( \Delta_{LS} + \Delta_{LS}' = \delta f_j \tau - \delta f_j \tau + \delta_{LL} + \delta_{SS} \) and \( \xi = \delta_{LL} + \delta_{SS} \). In Eqs. (11) and (12), the bandwidth \( \Delta \) does not affect the mean values due to the cross cancellation of \( \pm \delta f_j \). The correlation error in average is due to the laser linewidth. Satisfying a much narrower laser linewidth over the AOM bandwidth, thus, 
\[
\langle R_{14}(\tau) \rangle = \frac{\nu_0^2}{8} (1 - \cos(\xi + \psi + \varphi)),
\]
(13)
\[
\langle R_{23}(\tau) \rangle = \frac{\nu_0^2}{8} (1 + \cos(\xi + \psi + \varphi)).
\]
(14)
Because \( \xi \) is a fixed parameter, Eqs. (13) and (14) show the same fringe as observed in the typical Franson-type nonlocal correlations using SPDC-generated entangled photon pairs [26]. Thus, a classical model of the Franson-type nonlocal correlation is successfully presented for quantum features using the wave nature of photons without violating quantum mechanics.

Conclusion
A classical of the Franson-type nonlocal correlation was proposed and analyzed for the same quantum feature of inseparable basis-product superposition using coherent photon pairs generated from an attenuated laser. For this, two important conditions were set to satisfy both individual coherence and ensemble decoherence in each MZI. For this, a pair of synchronized and oppositely sweeping AOMs were used in a fast scanning mode, where the scanning speed should be much faster than the photon pair generation rate for detuning-dependent random phase generations. For the individual photon's coherence, the path-length difference of the MZI must be much shorter than the laser’s coherence length. For the incoherence MZI, the inverse of the AOM’s scanning bandwidth must be much shorter than the laser's coherence time as well. The present wave nature-based analysis resulted in the same nonlocal correlation fringe using an attenuated laser. Thus, the duality between the particle and the wave of a photon was demonstrated. Due to the rule of thumb in quantum mechanics that a photon never interferes with others [37], the present classical model of the Franson-type nonlocal correlation can be applied for macroscopic quantum information.

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