Negative temperature is cool for cooling

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In this work, we study an autonomous refrigerator composed of three qubits [Phys. Rev. Lett. 105, 130401 (2010)] operating with one of the reservoirs at negative temperatures, which has the purpose of cooling one of the qubits. We find the values of the lowest possible temperature that the qubit of interest reaches when fixing the relevant parameters, and we also study the limit for cooling the qubit arbitrarily close to absolute zero. We thus proceed to a comparative study showing that reservoirs at effective negative temperatures are more powerful than those at positive temperatures for cooling the qubit of interest.

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I. INTRODUCTION

In a 2010 work, Linden et al. (Ref. [1]) studied three models for cooling qubits or arbitrary quantum systems. The first model uses only qubits, the second one a qubit and a qutrit with nearest-neighbor interactions, and the third one a single qutrit. The model using only qubits is a self-contained or autonomous refrigerator (AR) with three interacting qubits, one of them being the object to be cooled. As shown in Ref. [1], under the condition of perfect insulation of the qubit to be refrigerated, this model presents no fundamental cooling limit, and thus, depending on the parameters used, the qubit of interest can be arbitrarily cooled towards absolute zero.

In this work, we study the AR consisting only of qubits as proposed by Linden et al. in Ref. [1], putting it to work with one of the reservoirs at negative temperature. Negative absolute temperatures were initially considered in 1951 when Purcell first produced spin states with population inversion [2]. In 1956, Ramsey theoretically studied these states, treating them as thermodynamic equilibrium states [3]. More than 60 years after the experiment performed by Purcell, other experiments involving negative temperatures followed [4, 5], which drew attention to this topic. While some authors defend the existence of negative thermodynamic temperatures, some works associate them with non-equilibrium states and thus refer to them as effective (or apparent) negative temperatures, as opposed to equilibrium states at positive thermodynamic temperatures [6–17]. As a general property, experimental realizations of reservoirs with effective negative temperatures, such as the ones we will consider in this work, require population inversion [4, 5, 18, 19]. As we will show, the qubit-based model AR has advantages when operating with one of the reservoirs at negative temperature compared to operating only with conventional reservoirs at positive temperatures.

This work is organized as follows: In Section II, we present the model for the study of AR considering bosonic reservoirs, for which the temperature is always positive, and fermionic reservoirs, which can reach negative temperatures. In Section III, we present our results, showing that the use of a fermionic reservoir with negative temperatures has an advantage over the use of only bosonic reservoirs. Finally, in Section IV, we present our conclusions about the comparative study of the AR involving positive and negative temperatures.

II. MODEL

The AR considered here consists of three qubits, each in contact with a different reservoir [1]. The first qubit, which is the qubit to be cooled, is put in contact with a cold reservoir at temperature $T_c$. The second qubit, analogously to the classical refrigerator, plays the role of the spiral and is put in contact with a reservoir at "room" temperature $T_r$, while the third qubit works as the engine staying in contact with a hot reservoir at temperature $T_h$. For the AR to work properly, considering only reservoirs at positive temperatures, the relations $T_c<T_r<T_h$ and $E_3 = E_2 - E_1$ - see Fig. 1(a), must be satisfied [1].

In the weak coupling limit and the Markovian regime, the dynamics governing the AR is dictated by the master equation

$$\frac{\partial \rho}{\partial t} = -i[H_0 + H_{\text{int}}, \rho] + \frac{1}{2}\sum_{k=1}^{3} \Gamma_k^+ (2\sigma_{-,k} \rho \sigma_{+,k} - \{\sigma_{+,k} \sigma_{-,k}, \rho\}) + \frac{1}{2}\sum_{k=1}^{3} \Gamma_k^0 (2\sigma_{+,k} \rho \sigma_{-,k} - \{\sigma_{-,k} \sigma_{+,k}, \rho\}), \hspace{1cm} (1)$$

where $H_0$ and $H_{\text{int}}$ are respectively the free qubits Hamiltonian and the three-body interaction Hamiltonian, given by

$$H_0 = \frac{1}{2}E_1 \sigma_{z,1} + \frac{1}{2}E_2 \sigma_{z,2} + \frac{1}{2}E_3 \sigma_{z,3} \hspace{1cm} (2)$$

and

$$H_{\text{int}} = g(\sigma_{-,1} \sigma_{+,3} + \sigma_{+,1} \sigma_{-,3}). \hspace{1cm} (3)$$
In the above equations, $\sigma_{-k}$, $\sigma_{+k}$, and $\sigma_{z,k}$ ($k = 1,2,3$) are respectively the lowering, raising, and $z$ Pauli operators for the $k$th qubit; $E_1$, $E_2$, and $E_3$ are the energy gaps for each qubit, and $g$ is the coupling constant. Also, we let $\Gamma_k^\pm = \gamma_k(1 \pm n_{\alpha_k})$, where the signal "+" ("−") is for a bosonic (fermionic) reservoir, and $\Gamma_k^n = \gamma_k n_{\alpha_k}$ ($\alpha_1 = c$, $\alpha_2 = r$, and $\alpha_3 = h$), with $\gamma_k$ being the dissipation rate for the $k$th qubit and $n_{\alpha_k}$ being the average excitation number of the reservoir at temperature $T_{\alpha_k}$. Note that Eq. (1) describes the dynamics dictated by either bosonic [20, 21] or fermionic [22, 23] thermal reservoirs. For bosonic thermal reservoirs $n_{\alpha_k} = 1/(e^{E_k/T_{\alpha_k}} - 1)$, while for fermionic reservoirs $n_{\alpha_k} = 1/(e^{E_k/T_{\alpha_k}} + 1)$. As said before, negative temperatures are characterized by population inversion, which occurs when $n_{\alpha_k} > 1/2$, and it is only possible for fermionic reservoirs. To find the final temperature of qubit 1, we numerically calculate the asymptotic equilibrium state of Eq. (1). We perform these numerical calculations using the quantum optics toolbox [24, 25].

III. RESULTS

To compare the AR operation at positive temperature with its operation at negative temperature, we must consider the same parameters that characterize the AR, such as coupling constant, energy gaps and dissipation rates. For clarity, let us start detailing the positive temperature case by fixing the parameters used in Ref. [1]. Thus, we fixed the energies gaps $E_1 = 1$, $E_2 = 5$, and $E_3 = 4$, which is a special case of the more general relation $E_3 = E_2 - E_1$ to AR work properly, and the temperatures $T_c = 1, 1.5, 2$, and $T_c = 2$. Finally, we fixed the dissipation rates $\gamma_1 = \gamma_2 = \gamma_3 = 1$ and let $T_h$ vary from 1 to 10. Cooling is achieved when $T_h < T_c$, which means that the temperature $T_1$ of the qubit 1 is lower than the temperature $T_c$ of the cold reservoir - See Fig. 2(a). In Fig. 2(b) we show how the temperature $T_1$ drops by increasing $T_h$. Note from Fig. 2(b) that the temperature of qubit 1 stabilizes at a constant value no matter how high $T_h$ is. We numerically found that the corresponding lowest temperatures reached by the qubit 1 are $T_1 = 0.9486$ (when $T_c = 1$), $T_1 = 1.4054$ (when $T_c = 1.5$), and $T_1 = 1.867$ (when $T_c = 2$). These lowest possible temperatures, when fixing the set of parameters, are dictated by the population of the ground state $n_{\alpha_1}$, which is given by $n_{\alpha_1} = 1/(1 + e^{-E_1/T_1})$. Solving for temperature, we get

$$T_1 = \frac{E_1}{\ln\left(\frac{1}{1 - n_{\alpha_1}}\right)}.$$  

(4)

From this equation, we see that $T_1$ can reach any value since $0.5 < n_{\alpha_1} < 1.0$. However, due to imperfect insulation, the heat flow to qubit 1 prevents it from reaching temperatures arbitrarily close to absolute zero. In fact, in our numerical calculations, we found that if we start at low temperatures as $T_c < 0.48$, then $T_1 - T_c > 0$, meaning that the refrigerator no longer works. As we shall see, using the same parameters but considering one of the reservoirs at negative temperatures, we can cool the qubit 1 below $T_c = 0.48$.

Next, we use the perfect insulation condition for the temperature of qubit 1 to get as close to absolute zero as possible [1]. This condition is obtained by isolating qubit 1 from its reservoir, thus letting $\gamma_1 \to 0$. This condition allows us to reach lower temperatures for qubit 1 than that shown in Fig. 2(b) and obtain the following

Figure 1. Schematic representation of possible level configuration for the qubit-based AR cooler. (a) Level configurations used in Ref. [1] for positive temperatures, where the condition $E_3 = E_2 - E_1$ is required. (b) A possible level configuration with $E_3 = E_2 = E_1$ for negative temperatures, where the condition $E_3 = E_2 - E_1$ is no longer needed. In (a) and (b), $|g\rangle_k$ and $|e\rangle_k$ are the ground and excited states of the $k$th qubit, respectively.

Figure 2. (a) $T_1 - T_c$ versus $T_h > 0$ for three different values of $T_c$: $T_c = 1$ (dot green line), $T_c = 1.5$ (dash red line) and $T_c = 2$ (solid blue line). Refrigeration occurs for $T_1 - T_c < 0$. (b) $T_1$ behavior for those three initial conditions given in (a). Note that the lowest temperature of the qubit $T_1$ does not decrease any further, no matter how high $T_h$ becomes. The lowest values reached by $T_1$ are $T_1 = 0.9486$ (when $T_c = 1$), $T_1 = 1.4054$ (when $T_c = 1.5$) and $T_1 = 1.867$ (when $T_c = 2$).
analytical expression for the cooling temperature [1]

\[ T_1 = \frac{T_c}{1 + \frac{E_3}{E_1}(1 - \frac{T_c}{T_h})}. \tag{5} \]

The above formula tells us, under the condition of perfect insulation, it is possible to cool down to absolute zero, provided that \( E_3/E_1 \to \infty \).

Now we turn our attention to negative temperatures. In this case, we must indicate which reservoir will be at negative temperatures. Knowing that the engine is modeled by qubit 3, which is in contact with a reservoir at hot temperature \( T_h \), it is natural to replace the reservoir of qubit 3 by a reservoir at \( T_h < 0 \) since bodies at negative temperatures are known to be hotter than any body at positive temperature as it scales, from cold to hot, according to \( +0 \text{ K},...,+300 \text{ K},...,+\infty \text{ K}, -\infty \text{ K},...,−300 \text{ K},...,0 \text{ K} \). In view of that, one might think that instead of using negative temperatures it would suffice to use arbitrarily hot reservoirs at positive temperatures. However, we saw from Fig. 2(b) that this is not the case: no matter how high \( T_h \) is, there is a limit achieved by \( T_1 \). Remarkably, we will show that under the same conditions, i.e., the same set of parameters characterizing the AR, the lowest possible temperature of qubit 1 cooled using positive temperatures can be pushed even lower using a hot reservoir at negative temperatures.

As said before, to compare the cooling process at either negative or positive temperatures, we put the AR to work with the same parameters. The steady state is found using the Eq. (1), but now considering a fermionic reservoir with \( n_3 > 0.5 \) (\( T_h < 0 \)). In Fig. 3(a), we show the difference \( T_1 - T_c \) as a function of \( T_h < 0 \). Interestingly, if we compare with Fig. 2(a) for \( T_h > 0 \), we see that at negative temperatures the steady-states are always cooled, no matter the initial temperature we choose for \( T_h < 0 \). Also, remembering that for \( T_h < 0 \) the higher temperatures ranges according to \((−\infty,0)\) in Fig. 3(b) we see that the higher the temperature \( T_h \) the lower is the temperature of qubit 1.

Note that the temperature reached by qubit 1 is yet given by equation Eq. (4). Again, imperfect insulation of qubit 1 prevents it from reaching temperatures arbitrarily close to absolute zero, as also shown in Figs. 3(a) and 3(b), similarly to what happens in the case of \( T_h > 0 \). To the set of parameters we are using, these values are \( T_1 = 0.7805 \) (when \( T_c = 1 \)), \( T_1 = 1.1615 \) (when \( T_c = 1.5 \)), and \( T_1 = 1.5568 \) (when \( T_c = 2 \)). Interestingly, note that the lowest temperatures \( T_1 \) in the case \( T_h < 0 \) are lower than the temperatures \( T_1 \) corresponding to the case \( T_h > 0 \). That is true for all reference temperatures \( T_c \) according to our numerical calculation and as can be seen in few examples shown in the table of Fig. 4(a).

Taking \( T_c \) as the reference temperature, we can calculate the percentage decrease of \( T_1 \) in relation to \( T_c \) for both negative and positive temperatures. In Fig. 4(b) we show the corresponding percentages for both \( T_h > 0 \) (blue) and \( T_h < 0 \) (red).

Fig. 4 makes it clear that negative temperatures are more effective for cooling a qubit, as it drives qubit 1 to lower temperatures by a large percentage. According to our numerical calculations, for this set of parameters, which is the same as used for positive temperatures, when the reference temperature \( T_h \) is as low as \( T_c = 0.48 \) the AR stops cooling if \( T_h > 0 \). However, at \( T_h < 0 \) the AR continues to cool down until qubit 1 reaches the limit of \( T_1 = 0.0275 \), that is, an order of magnitude lower.

We can also ask what happens when we impose the condition for perfect insulation, \( \gamma_1 \to 0 \). In this case, we can demonstrate that Eq. (5) remains valid. Alternatively, we can write Eq. (5) as

\[ T_1 = \frac{T_c}{1 + \frac{E_3}{E_1}(1 - \frac{T_c}{T_h})}. \tag{6} \]

Eq. (6) tells us that, as in the case of positive temperatures, it is possible to cool a qubit to temperatures arbitrarily close to absolute zero. Note, however, that for cooling a qubit toward zero, since \( T_c/T_h < 1 \), while for positive temperatures, Eq. (5), we need to resort only to the ratio \( E_3/E_1 \to \infty \), for negative temperatures we can have \( T_c > |T_h| \). Thus, it is possible to take advantage of negative temperatures by choosing \( T_c/T_h \to \infty \).

Regarding the ratio \( E_3/E_1 \), there is an important remark: positive temperatures require the condition \( E_3 =
calculations, AR stops working if \( T \) both positive and negative temperatures for the pumping heat or \( T \) effective temperatures allow arbitrary configurations, such as needed for AR to work. On the other hand, negative temperatures and in the condition of perfect insulation where \( T \) is very small, such that the final temperature of \( T \) is obtained when \( E/Q \). Nevertheless, for negative temperatures and in the condition of perfect insulation where Eq. (5) applies, even if we let \( E/Q \) we can still make the ratio \( T/T \) very large, thus cooling qubit 1 as close to absolute zero as we want.

### IV. CONCLUSION

In this work we consider the autonomous quantum refrigerator making use of three qubits, one qubit to be cooled and the other two playing the role analogous to the spiral and the engine in a classical refrigerator, as proposed in Ref. [1]. Making a comparative study of the functioning of this refrigerator in environments with either positive temperature or negative temperatures, we show that under the same conditions, that is, with the same parameters that characterize the autonomous quantum refrigerator, the use of negative temperature brings advantages in relation to the use restricted to positive temperatures. According to our numerical simulations, negative temperatures allow a greater cooling range, surpassing the positive temperature operation by about 20% for some parameters, see Fig. 4(a). Furthermore, while restricting the environments at positive temperatures, the steady-state of qubit 1, which ends cooled at temperature \( T \), depends on the temperature \( T \) of the hot reservoir, see Fig. 2(a), by using the hot environment at negative temperature the cooling of qubit 1 occurs for all \( T \), see Fig. 3(a). Also, by imposing the condition of perfect insulation, that is, isolating the qubit 1 to be cooled from its environment and letting it interact directly with the other qubits, we show that, as in the case of positive temperatures, the qubit 1 can be cooled to a temperature \( T \) arbitrarily close to absolute zero. However, even for perfect insulation, it is still possible to demonstrate an advantage of using negative temperatures. In fact, for \( T > 0 \), it is necessary to greatly increase the ratio between the energy \( E_3 \) of the third qubit and the energy \( E_1 \) of the qubit 1 to be cooled to bring its temperature \( T \) to values close to absolute zero. On the other hand, using \( T < 0 \) is possible to set the ratio \( E_3/E_1 \) to reasonably small values but let the ratio \( T/T \) very large, such that the temperature of qubit 1 to be cooled approaches absolute zero. To summarize, using \( T < 0 \) to cool a qubit is more powerful than \( T > 0 \).

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