Bringing the apple to the Moon

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Abstract. Newtonian gravity can be regarded as a hypothetic - deductive system where the inverse square law is the starting point from which gravitational phenomena are deduced. This operational form of presenting gravity endorses problem solving and seems to be predominant in the teaching practice. In contrast, regarding phenomena as a source for the development of the theory is also possible, of course, and can be advantageous to scientific education since it deals with model conception and construction. This article intends to introduce undergraduates to Newtonian gravity using its empirical basis, i.e. the free fall and the planetary motion, to deduce the universal law of gravitation. It also steps into the modern interpretation of gravitational phenomena i.e. Einstein’s general relativity, including a discussion on the instantaneous action at a distance in this context. This didactic presentation of the Newtonian theory of gravity is designed to reach a threefold equality, similar to those applied in the method of separation of variables in partial differential equations, where $G$ is treated as a separation constant. By doing so, the universality of the gravitation constant emerges as a conclusion rather than a statement. A few historical remarks on the development of this topic are also highlighted.

1. Introduction
Newton’s universal law of gravitation is the current paradigm for teaching gravity in schools. It brings a way to compute an attraction force between two masses $m$ and $M$ and its simplest modern formulation reads

$$F = G \frac{mM}{r^2}$$

where $r$ is the distance between the centres of the two bodies, $F$ is the gravitational force and $G$ is the gravitational constant ($G = 6.67 \times 10^{-11}$ N m$^2$/kg$^2$). The universal law of gravitation is then taken as a fundamental postulate from which gravitational phenomena are deduced. This form of presenting gravity is predominant in today’s teaching practice since in most university physics courses [1-3] equation (1) is presented "ready to be used". This cut off is actually desirable when the point is to solve problems related to classical mechanics, the explanation of Kepler’s three laws being one important example. However, the reverse path is pedagogically helpful: regarding phenomena as a source for the development of theories can be instructive to scientific education since it deals with model conception and construction [4]. This reversal establishes firm grounds for science learning since the knowledge does not come arbitrarily to the students.

Part of this contribution is dedicated to the deduction of the universal law of gravitation using its empirical basis and letting the reasoning guide the mathematical procedures. It also explores some historical highlights on the origins of the gravitational paradigm presented to students today (section 2). As in the usual approach, a few basic characteristics of the Earth - Moon system are used in the analysis proposed here. The discussion fully integrates the universal law of gravitation with Newton’s three laws of movement in a way that makes the universality of the constant $G$ self-evident, being a conclusion rather than a statement.
In spite of predicting very precisely several characteristics of gravitationally bound systems (the dynamics of the solar system for instance) Newtonian gravity fails to describe the large structure of the universe and also is not able to predict the recent finding of gravitational waves that has just received the Nobel prize [5]. For these matters, the Einstenian view of gravity, i.e., the general relativity theory is required. Furthermore, modern cosmology counts on the concepts of dark matter, in respect to the galactic dynamics, and dark energy, which concerns to the accelerated expansion of the universe. Besides, the universal law of gravitation has some conceptual difficulties, acknowledged by Newton himself at the end of the Principia, concerning the assumption of the instantaneous action at a distance. Nonetheless, in the educational aspect, the study of Newtonian gravity offers a good opportunity to think scientifically.

The proposed didactic sequence (section 3) is formed by a line of thought designed to reach a threefold equality, similar to those applied in the method of separation of variables in partial differential equations [6], where \( G \) plays the role of the separation constant. The material in the didactic sequence forms one feasible route to the law of gravity. Only very basic Physics concepts are required: the kinematics of constant acceleration (either linear or centripetal), Newton’s three laws of movement, Kepler’s planetary laws [7], Galileo’s free fall and the concept of centre of mass, making the sequence suitable for advanced high school level (since differential calculus is not required). It is important to mention that the sequence presented here does not follow Newton’s own line of reasoning and that the modern terminology for mass, namely the conceptual distinction between inertial and gravitational mass, is used aiming a brief discussion of today’s interpretation of the theory of gravity i.e., Einstein’s general theory of relativity, done at section 4.

2. A few historical highlights

It is common, in the usual approach to Newtonian gravity, to teach equation (1) as being a sudden inspiration of Isaac Newton when he was at the Woolsthorpe Manor, in Lincolnshire, and noticed the free fall of an apple [8]. However, it is worth mentioning that the notorious apple was never cited in the Principia and that a mathematical expression like equation (1) was never written by Newton. The universal law of gravitation appears in book III – Propositions VII and VIII of the Principia where equation (1) is described in words without any mention to a gravitational constant [9].

In 1803, Siméon D. Poisson, may have been the first to write algebraically that the force of gravity is jointly proportional to the two interacting masses, \( mM \), and inversely proportional to the square distance, \( 1/r^2 \), in his Treatise on Mechanics [10]. In this text, the character \( f \) is used to refer to what is known today as the gravitational constant \( G \) in equation (1) and interpreted as an intensity factor defined as the attraction force between two unit masses apart a unit distance. The value of the gravitational constant was unknown at Poisson’s time albeit the 1798 Henry Cavendish research on the torsion balance was already published [11]. As a matter of fact, the Cavendish measurements aimed the determination of the mean mass density of the Earth and, although torsion balances can be used to measure the gravitational constant, the concept of a universal constant (such as \( G \)) was yet to come. By the way, in a textbook on mechanics for applicants to the École Polytechnique, the mathematician Henry Garret wrote, in 1853, that the force of gravity is given as \( F = 4\pi^2 \left[ \frac{a^3}{T^2} \right] \frac{m}{r^2} \). Explicit mention to Kepler 3th law is done but no mention to a universal constant is made [12]. H. Garret was related to Jules Verne and was a scientific consultant to Verne’s science fiction novels.

An estimate for the gravitational constant, still referred by \( f \) as in Poisson’s notation, based on the Earth’s density found by Cavendish, appeared in 1873 in a scientific communication by Cornu and Baille [13]. Several measurements of the gravitational constant surfaced during the nineteenth century turn [14] and the problem of the gravitational force as we know today, i.e. equation (1), appears in the works of John H. Poynting concerning Earth’s sciences [15]. In Poynting’s works, the gravitational constant, already referred by the symbol \( G \), is given as \( \frac{666}{506} \) in CGS units and this was the value known to Albert Einstein by the time general relativity was published in 1915 [16]. Figure 1 summarizes the remarks made in this section.
Another important characteristic of equation (1) is that the masses appearing on its numerator are referred as gravitational mass \((m_G)\) which, in modern science, stands for the ability to interact through a gravitational field. This is to be distinguished from the inertial mass \((m_I)\), appearing in Newton’s second law, which is a measure of inertia i.e. the resistance that an object presents to a change on its state of movement. It should be stressed that Newton did not distinguish these two mass concepts. For him, inertia and gravity are properties of mass which, in turn, is defined as quantity of matter [17]. The conceptual distinction between inertial and gravitational mass is important because they are logically independent and preserve causality in the application of Newton’s second law to problems involving gravity.

Figure 1 – A few extracts illustrating the works mentioned in section 2. (a) Poisson’s treatise on mechanics (b) the Cavendish paper (c) the scientific communication of Cornu and Baille (d) Poynting’s book on Earth’s sciences.

The roots of these different mass concepts are related to the equivalence principle, which can be mathematically expressed by \(m_I = m_G\), and traces back to times before Galileo Galilei [18]. One counterintuitive consequence of this principle is that all bodies fall with the same acceleration irrespectively to their masses. This was one of many important findings Galileo reported in the Dialogues [19]. In modern terms, the fact that free fall does not depend on the falling object’s mass consists of what is called the ‘weak equivalence principle’. The gravitational mass concept bridges
gravity towards a field theory and is crucial in Einstein’s general relativity. Actually, Einstein used the terms inertial mass and gravitational mass as suggested at the end of one of his 1907 paper [20]. At Einstein’s time the most successful quantitative testing of the equivalence principle was done by Roland Eötvös using a torsion balance [21].

3. The didactic sequence
A more detailed version of the didactic sequence can be found in reference [22]. The main steps concerning the deduction of equation (1) are presented in the next paragraphs. Figure 2 shows the schematics of the sequence. Any suspended object near the Earth surface, an apple for instance, will fall to the ground with constant acceleration when left free (figure 2a). According to Newton’s first law - inertia - a force must be acting on the object, otherwise it would keep motionless in its original position after being left. Remarkably, the fall movement is always uniformly accelerated and does not depend on the specific place where the experience is done. When left free, the movement of the apple is then driven by Newton’s second law – dynamics - which is written as:

$$F_{EA} = mg_E$$  \hspace{1cm} (2)

where $F_{EA}$ is the force of gravity that the Earth exerts on the apple and $m$ is the inertial mass of the apple. It should be emphasised that $g_E$ in equation (2) stands for acceleration, not gravitational field. It is also important to mention that the acceleration $g_E$ is always directed to the centre of the Earth (figure 2a).

![Figure 2](image)

**Figure 2** - The system treated in the didactic section. (a) A falling apple of mass $m$ near the surface of the Earth with acceleration $g_E$. (b) The Moon with velocity $v_M$ and acceleration $a_{EM}^M$. (c) The same apple falling on the surface of the Moon with acceleration $g_M$. (d) The Earth with velocity $v_E$ and acceleration $a_{EM}^E$. All the accelerations and velocities are to be computed in the centre of mass of the respective system.

If the orbital movement of the Moon is considered, one concludes from the first law - inertia - that there must be a force producing the centripetal acceleration on the Moon, otherwise, it would have a rectilinear uniform motion flying away from Earth and vanishing from the sky view. The orbiting Moon moves parallel to the Earth’s ground while the falling apple does not, hence their completely different types of movement. Yet, similarly to the case of the apple’s free fall, the Moon’s acceleration is directed to the centre of the Earth. Hence, one can deduce that the Earth produces a centripetal acceleration on all the objects around it. Having the forces bringing the apple to the Earth’s ground and the one keeping the Moon orbiting the Earth the same direction, one can apply Newton’s second law to the movement of the Moon in the same way it was done for the apple. Actually, this is part of an inductive argument made by Newton in the Principia (Book III -Proposition IV – Theorem IV) when he declares that "...the force by which the Moon
is retained in its orbit is that very same force we commonly call gravity …". Letting \( F_{EM} \) be the force the Earth exerts on the Moon, \( M_M \) the inertial mass of the Moon and \( a^E_M \) the acceleration of the Moon due to the Earth’s gravity (figure 2b) one has that:

\[ F_{EM} = M_M a^E_M \]

(3)

The experience of being on the surface of a massive giant sphere is just repeated if one could bring the apple the Moon and execute a free fall experiment with it up there (figures 2b and 2c). The similarity between the two situations compels us to consider that for an observer at the Moon’s surface, the apple movement will be qualitatively the same as the one near the Earth’s surface that is, uniformly accelerated (figure 2c). If \( g_M \) is the acceleration due to Moon’s gravity near its surface one has that:

\[ F_{MA} = m g_M \]

(4)

as was done for the terrestrial case (equation 2). In equation (4), \( F_{MA} \) is the force that the Moon exerts on the apple, \( m \) is the inertial mass of the apple and, as in equation (2), the symbol \( g_M \) stands for acceleration not gravitational field. This corresponds to Newton’s second law to the fall of the apple near the surface of the Moon and is a logical extrapolation of the validity of Newton’s laws outside Earth. Furthermore, an observer on the Moon’s surface would see the Earth in orbital motion. Accordingly, there should be a force acting on Earth given by:

\[ F_{ME} = M_E a^E_E \]

(5)

where \( M_E \) is the inertial mass of the Earth and \( a^E_E \) is the acceleration of Earth due to the Moon’s gravity (figure 1d). Being the accelerations \( a^E_M \) and \( a^E_E \) computed in the center of mass of the Earth-Moon system one has that

\[ \frac{|d^E_M|}{|d^E_E|} = \frac{M_E}{M_M} \]

(6)

which gives the same result for the accelerations ratio in (6) if Newton’s third law – action - reaction – is assumed to be valid for non-contact bodies, i.e., the instantaneous action at a distance hypothesis. In this case, the forces expressed in equations (3) and (5) should have the same magnitude and opposite directions (\( F_{MB} = -F_{ME} \)). In general, the accelerations \( \vec{a}^E_M \) and \( \vec{a}^E_E \) can be written in terms of the centre of mass and of the relative accelerations, \( \vec{A} \) and \( \vec{a} \) respectively, as:

\[
\begin{align*}
\vec{a}^E_M & = \vec{A} + \frac{1}{1+\left(M_M/M_E\right)} \vec{a} \\
\vec{a}^E_E & = \vec{A} - \frac{\left(M_M/M_E\right)}{1+\left(M_M/M_E\right)} \vec{a}
\end{align*}
\]

(7)

One has that \( \vec{A} = 0 \) in the centre of mass frame and, since the Moon is much lighter than the Earth \((M_M << M_E)\), equation (7) gives \( \vec{a}^E_M \sim \vec{a} \) and \( \vec{a}^E_E \sim -\left(M_M/M_E\right)\vec{a} \). That means the Earth-Moon relative acceleration is truly a very good approximation for the value of Moon’s acceleration, \( \vec{a}^E_M \), defined in equation (3). The study of the Moon’s orbital dynamics can then be done replacing the Moon’s acceleration with respect to the centre of mass by the Earth-Moon relative acceleration with the advantage that the latter is easily evaluated without the precise knowledge of the masses \( M_E \) and \( M_M \).

The value of the Earth - Moon relative acceleration, \( \vec{a} \), can be easily found from the radius, \( r \), and sidereal period, \( T \), of the Moon’s orbit:

\[ \vec{a} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} \]

(8)

where \( v = 2\pi r/T \) is Moon’s orbital velocity. A second relationship between \( r \) and \( T \) is available from Kepler’s third law:

\[ T^2 = k r^3 \]

(9)

where the pre-factor \( k \) is a proportionality constant. By merging equations (8) and (9) one finds that the centripetal acceleration in (8) should vary as the inverse square law:

\[ a = \frac{4\pi^2 r}{k T^2} \]

(10)

The application of equation (10) to the problem of the apple falling on Earth can be done by viewing the distance \( r \) as a variable and noting that the accelerations \( \vec{a} \) and \( g_E \) are both directed to the centre of the Earth (figures 2a and 2b). The centre of mass of the apple-Earth system is virtually the centre of the Earth and, therefore, the relevant distance to evaluate the apple’s acceleration near the Earth surface.
is the radius of the Earth, $R_E$. One has just to substitute $r$ by $R_E$ and $a$ by $g_E$ in equation (10), resulting in

$$g_E = \frac{4\pi^2 1}{k R_E^2}$$

(11)

Furthermore, one can simply replace the $a$ in equation (10) by $a_M^E$ and compare the ratio $[a_M^E / g_E]$ to the ratio $[r/R_E]$ dividing (10) by (11). One finds that:

$$a_M^E = \frac{R_E^2}{r^2} g_E$$

(12)

The numerical values involved in equation (12) were key to convince Newton of the inverse square law as he realized that the calculation of the ratio $[r/R_E]$ is $\sim 60$ while the ratio $[g_E / a_M^E]$ gives $\sim 3600$ which is $60^2$ as equation (12) suggests [9,23]. As said before, the approximation regarding the accelerations $(a_M^E \sim \ddot{a})$ permit us to take the relative acceleration in place of the acceleration of the Moon with respect to the centre of mass for the present purpose. If this was not the case, it would be much more difficult to reach the inverse square law since the passage from equation (10) to equation (12) would necessarily involve precise knowledge of the masses $M_E$ and $M_M$ values which were not available at Newton’s time. In a sense, it is fortunate that the Moon to Earth mass ratio is very small so the relative acceleration can be used by someone on Earth to establish the inverse square law without greater difficulties. By itself, equation (12) is a manifestation of the principle of universality since it links the acceleration of a terrestrial object, $g_E$, to the acceleration of a celestial body, $a_M^E$. It should be said that Newton argued that the universality of the Physical laws is a fundamental principle, as stated in his four rules of philosophizing [9,24]. The ratio expressed in equation (12) ought to be valid in the lunar situation given the similarity between the two cases (figure 2d). Thus, similarly to equation (12), one has that

$$a_M^E = \frac{R_E^2}{r^2} g_M$$

(13)

If equations (12) and (13) are used in equation (6) one can write, after a little algebra:

$$\frac{g_E R_E^2}{M_E} = \frac{g_M R_M^2}{M_M}$$

(14)

Equation (14) expresses that the value obtained by the combination of $g$, the acceleration of a falling object near the surface of a planet, $R$, the radius of the planet and $M$, the inertial mass of the planet, should not depend on where we are (Earth or Moon in this case). Therefore, it must have a universal value which can be called $G$. Consequently one has that,

$$G = \frac{g_E R_E^2}{M_E} = \frac{g_M R_M^2}{M_M}$$

(15)

or in general $G = g R^2 / M$. It is straightforward to write that

$$g = \frac{G M}{R^2}$$

(16)

Equation (16) can be applied to equations (2) or (4) to evaluate the force on an apple due to gravity on the surface of the Earth or on the surface of the Moon easily. Finally, the attraction force between the Earth and the Moon is found using equation (3) together with (12) and the definition of $g$ presented in equation (16) applied to Earth. One has that:

$$F_{EM} = G \frac{M_M M_E}{r^2}$$

(17)

which can be generalized so any two objects of inertial masses $m$ and $M$ apart a distance $r$, will sense a force whose magnitude is given by

$$F = G \frac{m M}{r^2}$$

(18)

What was just shown is that equation (18) can be deduced from the universality of the three laws of movement, including the instantaneous action at a distance hypothesis, plus Kepler’s third law. However, the above development do not mention what causes the bodies to attract one another and, as a matter of fact, Newton himself wrote in the general scholium at the end of the principia that “... we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power ...” [9]. Hence, equation (18) is not fully consistent with dynamics since it cannot be assigned to the cause of movement. Therefore, the universal law of gravitation would
be merely a corollary of Newtonian mechanics if it was not for the weak equivalence principle \((m^I = m^G)\) that makes equation (18) conceptually different from equation (1). The equivalence between gravitational and inertial masses bridges equation (18) into equation (1) substituting inertial mass in equation (18) by gravitational mass in equation (1). This final step imprints causality to the universal law of gravitation.

4. Discussion
Within the Newtonian view of nature, the force of gravity (equation 1) is an inference from the 1st law - inertia. In the above analysis, the inverse square law results from the combination of Kepler’s third law and the formulation of centripetal acceleration for circular motion. The proportionality of the force of gravity to the joint product of masses is related to the combination of the 2nd law - dynamics- and the concept of the centre of mass along with the 3rd law - action-reaction. Finally, the universality of the gravitational constant, \(G\), expressed mathematically in equation (15), plus the gravitational mass concept, brings closure to the universal law of gravity. A subtle suggestion in the assumption that the 3rd law can be applied to the force of gravity, is the existence of instantaneous action at a distance. However, Newton felt unease with this feature of the theory, as can be seen in the following quote from his letters to Bentley: "That one body may act upon another at a distance through a vacuum ... is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it..." [25]. It is the phenomenological range covered by Newtonian gravity that lead us to conclude for its validity (within the classical Physics limits).

Einstein’s general theory of relativity uncovers some aspects of gravity in ways unachievable to the Newtonian thinking. For instance, it solves the instantaneous action at a distance problem. This is done by reinterpreting gravity not in terms of a force, but considering that gravitational mass causes a local spacetime curvature that affects the movement of surrounding objects. This corresponds to an application of the 'strong equivalence principle', which says that the Physical laws in an accelerated frame should be no different from the ones experienced in the presence of a gravitational field. Therefore, relativistic effects, such as time dilation and length contraction, must be observed in a gravitational field even for slow objects, since gravity can be experienced in velocities much lower than the speed of light. For instance, the free fall of an apple on the Earth’s surface can actually be calculated from the bending of spacetime around Earth, without ever invoking a force law like equation (1) [26].

Strictly speaking, for Einstein, gravitational interaction exists through a spacetime – gravitational mass coupling so the fundamental concept for Einstein’s theory of gravity is the gravitational mass not the gravitational force; and if there is no force there is no action at a distance to be considered. Newton, however, could not agree with this picture because for him space and time are absolute and independent. Nevertheless, the universality of the gravitational constant, \(G\), along with gravitational and inertial masses concepts endure in general relativity. Finally, it should be mentioned that general relativity has an even larger phenomenological range and can be applied to problems Newtonian gravity fails to explain such as the large structure of the Universe and also the prediction of the recent finding of gravitational waves that has just received the Nobel prize; topics that are out of the scope of this paper.

5. Conclusion
The universal law of gravitation represents a great leap of scientific thought. By putting together the works of Johannes Kepler and Galileo Galilei, Isaac Newton promoted a profound impact not only in science but in the history of mankind. Although Newton was inspired by the free fall of an apple in formulating his theory of gravity, the Principia does not mention such fact. The apple parabola also appears in the works of Stukeley and also of Voltaire who dedicated part of his works to Newton during the age of Enlightenment [27]. The above presentation brings about one possible and simple route to the law of gravity. In this article, the universality of the constant \(G\) emerges as a conclusion of a reasoning involving all three Newton’s laws of movement plus the third Kepler law. It also brings \(G\), as defined in equation (17), to be essential to recognize the law of gravity as a fundamental and universal postulate. It can be said that it is the law of gravity that makes Newton’s laws of movement, which were developed
for Terrestrial objects, valid throughout. Furthermore, it is also important to mention that the universal law of gravitation is related to the instantaneous action at a distance hypothesis, i.e., that Newton’s third law is valid for non-contact interactions. This characteristic, together with the gravitational mass concept, can be seen as a rudimentary notion of gravitational field hence being important for recognizing equation (1) as fundamental as the other three Newton’s laws of movement and not merely as a corollary of them. On the other hand, the Einsteinian view of gravity substitute the picture of a classical force field by the space-time curvature originated by gravitational mass, thus, overcoming the difficulty of applying Newton’s third law to non-contact objects. In the educational aspect, the understanding of gravity, either Newtonian or Einsteinian, offers a good opportunity to think scientifically and this contribution intends to establish firm grounds on either paradigm. After all, the shift to a new paradigm is easier if old paradigms are completely and well understood. Otherwise, the conceptual frame of the old paradigm sticks to the student’s minds as prejudices making the acceptance, or comprehension, of new ideas more difficult. This should be prevented in Physics education. This teaching sequence has been used for first year university students. It is believed that advanced high school students may follow the proposed reasoning quite well and profit from the understanding of the conceptual frame and historical remarks presented in the paper.

6. References

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