Numerical simulations of relativistic magnetic reconnection with Galerkin methods

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Abstract. We present the results of two-dimensional magnetohydrodynamical numerical simulations of relativistic magnetic reconnection, with particular emphasis on the dynamics of Petschek-type configurations with high Lundquist numbers, $S \sim 10^5 - 10^8$. The numerical scheme adopted, allowing for unprecedented accuracy for this type of calculations, is based on high order finite volume and discontinuous Galerkin methods as recently proposed by Dumbser & Zanotti (2009). The possibility of producing high Lorentz factors is discussed, by studying the effects produced on the dynamics by different magnetization and resistivity regimes. We show that Lorentz factors close to $\sim 4$ can be produced for a plasma magnetization parameter $\sigma_m = 20$. Moreover, we find that the Sweet-Parker layers are unstable, generating secondary magnetic islands, but only for $S > S_c \sim 10^8$, much larger than what is reported in the Newtonian regime.

1. Introduction

Relativistic magnetic reconnection is a high-energy process converting magnetic energy into heat and plasma kinetic energy over short timescales. It is supposed to play a fundamental role in the magnetospheres of pulsars (Uzdensky 2003; Gruzinov 2005); at the termination shock of a relativistic striped pulsar wind (Pétri & Lyubarsky 2007); in soft gamma-ray repeaters (Lyutikov 2003, 2006); in gamma-ray burst jets (Drenkhahn & Spruit 2002; Barkov & Komissarov 2010; McKinney & Uzdensky 2010; Rezzolla et al. 2011); in accretion disc coronae (di Matteo 1998; Schopper et al. 1998; Jaroschek et al. 2004).

In spite of the initial optimistic expectations that relativistic magnetic reconnection could provide very fast reconnection rates, evidence has emerged over the years that the relativistic Petschek reconnection should not be considered as a mechanism for the direct conversion of the magnetic energy into the plasma energy and the reconnection rate would be at most 0.1 the speed of light, contrary to what originally suggested by Blackman & Field (1994). Fundamental progresses in the numerical modeling of relativistic magnetic reconnection have been recently obtained by Watanabe & Yokoyama (2006); Zenitani et al. (2009b,a). However, two major numerical limitations still prevent realistic astrophysical applications of numerical schemes specifically devoted to relativistic magnetic reconnection. The first limitation is due to the difficulty in reaching sufficiently high magnetization parameters $\sigma_m$, while the second limitation is due to the difficulty in treating physical systems with very high Lundquist numbers $S$. 


In this work, by adopting the innovative numerical method presented in Dumbser & Zanotti (2009), we show the results of numerical simulations in the very high Lundquist numbers regime, $S \sim 10^5 - 10^8$, showing that that Sweet-Parker current sheets are unstable to super-Alfvenically fast formation of plasmoid chains, but only for $S > S_c \sim 10^8$.

2. Physical set up and numerical approach

The initial model that we have considered is built on Harris model, as reported by Kirk & Skjæraasen (2003), and it reproduces a current sheet configuration in the $x$–$y$ plane. Gas pressure and density are given by $p = p_0 + \sigma_m p_0 [p_0 \cosh^2(2x)]^{-1}$, $\rho = \rho_0 + \sigma_m \rho_0 [p_0 \cosh^2(2x)]^{-1}$, where $p_0$ and $\rho_0$ are the constant values outside the current sheet, whose thickness is $\delta = 1$. The magnetic field changes orientation across the current sheet according to $B_y = B_0 \tanh(2x)$, where the value of $B_0$ is given in terms of the magnetization parameter $\sigma_m = B_0^2 / (2\rho_0 \Gamma_0^2)$. All over the grid there is a small background uniform resistivity $\eta_b$, except for a circle of radius $r_\eta = 0.8$, defining a region of anomalous resistivity of amplitude $\eta_{i0} = 1$. The resistivity can be written as

$$\eta = \begin{cases} \eta_b + \eta_{i0} \left[2(r/r_\eta)^3 - 3(r/r_\eta)^2 + 1\right] & \text{for } r \leq r_\eta, \\ \eta_b & \text{for } r > r_\eta, \end{cases}$$

(1)

where $r = \sqrt{x^2 + y^2}$. The velocity field is initially zero, while the electric field is given by $E_z = \eta (\partial B_y/\partial x)$. We have considered the case with $p_0 = 1$, $\rho_0 = 1$. The Lundquist number for every model is $S = v_A L / \eta_b$, where $L$ is the length of the initial current sheet, while $v_A^2 = B_0^2 / (\gamma \rho + B_0^2)$ is the relativistic Alfven velocity.

A well known and challenging feature of the relativistic resistive magnetohydrodynamics equations is that the source terms in the three equations for the evolution of the electric field become stiff in the limit of high conductivity. To cope with this difficulty, we have applied the strategy described by Dumbser & Zanotti (2009), who used the so called high order $P_N P_M$ methods, which combine high order finite volume methods and discontinuous Galerkin finite element schemes in a more general framework (Dumbser et al. 2008b,a). The numerical grid consists of an unstructured mesh composed of triangles which are clustered along the current sheet. The grid extension is given by $[-50,50] \times [-150,150]$. We have used periodic boundary conditions at $y_{\text{min}}$ and $y_{\text{max}}$, while zeroth order extrapolation is applied at $x_{\text{min}}$ and $x_{\text{max}}$.

3. Results

The dissipated magnetic energy, triggered by the anomalous resistivity, produces an increase of both the thermal and kinetic energy. The latter results in the acceleration of the plasmoid along the direction of the magnetic field, which is more efficient for higher magnetizations. However, as the region around the anomalous resistivity becomes more and more rarefied, the conversion of magnetic energy into thermal energy becomes more efficient than the conversion into kinetic energy and the Lorentz factor reaches a saturation. This effect is shown in Fig. 1.

Fig. 2, on the other hand, shows the development of an tearing-like instability for very high Lundquist numbers configurations. No sign of instability is visible in
Figure 1. Time evolution of the Lorentz factor for models with increasing magnetizations $\sigma_m$ and $S \sim 10^5$.

Figure 2. Generation of plasmoid chains in very high Lundquist numbers configurations. Left and right panels report, respectively, the color map of the rest mass density at time $t = 60$ for two models having $S = 5.36 \times 10^7$ and $S = 2.68 \times 10^8$. 
simulations with Lundquist numbers as large as $S \sim 10^7$ (left panel), while a chain of magnetic islands is produced for $S > S_c \sim 10^8$ (right panel). Our results, combined with those by [Samtaney et al., 2009], indicate that, in the transition to the relativistic regime, the critical Lundquist number increases from $S_c \gtrsim 10^4$ to $S_c \gtrsim 10^8$. Such a conclusion may have deep implications for high Lundquist number reconnection in realistic astrophysical conditions, and it requires further investigations.

4. Conclusions

By adopting high order discontinuous Galerkin methods as proposed by [Dumbser & Zanotti, 2009], we have found that Lorentz factors up to $\sim 4$ can be obtained for plasma parameters $\sigma_m$ up to 20 in systems undergoing relativistic magnetic reconnection with Lundquist number $S \sim 10^5$. When $S$ is larger than a critical value $S_c \sim 10^8$, the Sweet-Parker layer becomes unstable, generating a chain of secondary magnetic islands (Zanotti & Dumbser, 2011).

Acknowledgments

I am grateful to Luciano Rezzolla for useful discussions. Numerical simulations where performed on the National Supercomputer HLRB-II installed at Leibniz-Rechenzentrum.

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