Minimal Flavour Violations and Tree Level FCNC

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Abstract

Consequences of a specific class of two Higgs doublet models in which the Higgs induced tree level flavour changing neutral currents (FCNC) display minimal flavour violation (MFV) are considered. These FCNC are fixed in terms of the CKM matrix elements and the down quark masses. The minimal model in this category with only two Higgs doublets has no extra CP violating phases but such a phase can be induced by adding a complex singlet. Many of the theoretical predictions are similar to other MFV scenario. The FCNC contribute significantly to $B$ meson mixing and CP violation. Detailed numerical analysis to determine the allowed Higgs contributions to neutral meson mixings and the CKM parameters $\bar{\rho}, \bar{\eta}$ in their presence is presented. The Higgs induced phase in the $B_{d,s}^0 \to \bar{B}_{d,s}^0$ transition amplitude $M_{12}^{d,s}$ is predicted to be equal for the $B_d$ and the $B_s$ systems. There is a strong correlation between phases in $M_{12}^{d,s}$ and $|V_{ub}|$. A measurable CP violating phase $\phi_s = -0.18 \pm 0.08$ is predicted on the basis of the observed phase $\phi_d$ in the $B_d$ system if $|V_{ub}|$ is large and close to its value determined from the inclusive b decays.

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I. INTRODUCTION

The Cabibbo Kobayashi Maskawa (CKM) matrix $V$ provides a unique source of flavour and CP violations in the standard model (SM). It leads to flavour changing neutral currents (FCNC) at the one loop level. $K$ and $B$ meson decays and mixing have provided stringent tests of these FCNC induced processes and the SM predictions have been verified with some hints for possible new physics contributions \cite{1, 2, 3}. Any new source of flavour violations resulting from the well-motivated extensions of the SM (e. g. supersymmetry) is now constrained to be small \cite{4, 5, 6}.

Several extensions of the SM share an important property termed as minimal flavour violation (MFV) \cite{7, 8}. According to this, all flavour and CP violations are determined by the CKM matrix even when the SM is extended to include other flavour violating interactions. In the extreme case (termed as the constrained MFV \cite{9}) the operators responsible for the flavour violations are also the same as in the SM. In more general situations, MFV models contain more operators with coefficients determined in terms of the elements of $V$. Some scenarios \cite{10} termed as the next to minimal flavour violation (NMFV) contain new phases not present in $V$.

A simple example of MFV is provided by a two Higgs doublet model with natural flavour conservation (NFC) \cite{11}. The discrete symmetry conventionally imposed to obtain NFC prevents any CP violation coming from the Higgs potential and the CKM matrix provides a unique source of CP and flavour violations in these models. The MFV in these models can be explicitly seen by considering the $B^q_0 \to \bar{B}^0_q$ ($q = d, s$) transition amplitude $M_{12}^q$ as an example. The charged Higgs boson in the model gives additional contributions to the SM amplitude and the dominant top quark dependent part can be written \cite{12} as

$$M_{12}^q = \frac{G_F^2 M_W^2 M_{B_q} B_q f_{B_q}^2 \eta_B(x_t)(V_{tb}^* V_{tq})^2}{12\pi^2} (1 + \kappa_H^+) \ ,$$

where

$$\kappa_H^+ \equiv \frac{1}{4S_0(x_t)} \frac{\eta_B(x_t, y_t)}{\eta_B(x_t)} (\cot^4 \theta S_{HH}(y_t) + \cot^2 \theta S_{HW}(x_t, y_t)) \ ,$$

$$\approx \frac{\eta_B(x_t, y_t)}{\eta_B(x_t)} (0.12 \cot^4 \theta + 0.53 \cot^2 \theta) \ ,$$

where $\eta_B$ are the QCD corrections \cite{13, 14}, $\tan \theta$ is the ratio of the Higgs vacuum expectation values and $x_t = \frac{m_t^2}{M_W^2}$, $y_t = \frac{m_t^2}{M_{H^+}^2}$. The functions appearing above can be found for example in \cite{14, 15} and the last line corresponds to the obtained numerical values in case of the charged Higgs mass $M_{H^+} = 200$ GeV. Flavour and CP violation are still governned by the same combinations of the CKM matrix elements that appear in the SM box diagram. The only effect of the charged Higgs boson is an additional contribution to the function $S_0(x_t)$. The same happens in case of other observables and one can parameterize all the FCNC induced processes in terms of seven independent functions in MFV models\cite{8}.
Two Higgs doublet models (2HDM) with NFC lead to MFV but they do not represent
the most generic possibilities. More general 2HDM will contain additional sources of CP
and flavour violation through the presence of FCNC. The principle of NFC now appears to
conflict with the idea of the spontaneous CP violation (SPCV) at low energy and both
cannot coexist together. Indeed, if NFC and the spontaneous CP violation are simultane-
ously present in multi-Higgs doublet models then the CKM matrix is implied to be real.
In contrast, the detailed model independent fits to experimental data require the Wolfen-
stein parameter \( \eta = 0.386 \pm 0.035 \) according to the latest fits by the UTfit collaboration.
Thus the CKM matrix is proven to be complex under very general assumptions.
Attractive idea of low energy SPCV can only be realized by admitting the tree level FCNC.
Independent of this, the 2HDM without NFC become phenomenologically interesting
if there is a natural mechanism to suppress FCNC. The phenomenology of such models has
been studied in variety of context.

This paper is devoted to discussion of models in which FCNC are naturally suppressed
and show strong hierarchy. Specifically, the FCNC couplings \( F^d_{ij} \) between the \( i \)
and the \( j \) generations obey
\[
|F^d_{12}| < |F^d_{13}|, |F^d_{23}|
\]
automatically suppressing the flavour violations in the \( K \) sector relative to \( B \) mesons. A
specific sub-class of these models has the remarkable property that the FCNC couplings
are determined completely in terms of the CKM matrix and the quark masses. These
models therefore provide yet another example of MFV in spite of the presence of FCNC.
The models to be discussed were presented long ago and the aim of the present
paper is to update constraints on them in view of the substantial experimental information
that has become available from the Tevatron and \( B \) factories.

The next section introduces the class of models we discuss and presents the structure
of the FCNC couplings. Section (III) is devoted to the analytic and numerical studies of the
consequences assuming that either the charged Higgs or a neutral Higgs dominates the \( P^0-\bar{P}^0 \)
\( (P = K, B_d, B_s) \) mixing. The last section summarizes the salient features of the
paper.

II. MODEL AND THE STRUCTURE OF FCNC

Consider the \( SU(2) \otimes U(1) \) model with two Higgs doublets \( \phi_a, (a = 1,2) \) and the
following Yukawa couplings:
\[
-L = \bar{Q}'_L \Gamma^d_a \phi_a d'_R + \bar{Q}'_L \Gamma^u_a \phi_a u'_R + H.c.
\]
\( Q'_{il}, (i = 1,2,3) \) represent three generations of weak doublets and \( u'_{ir}, d'_{ir} \) are the corre-
sponding singlets. Let us consider a class of models represented by a specific choice of
the matrices $\Gamma^d_a$ and their permutations:

$$
\Gamma^d_1 = \begin{pmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{pmatrix} ; \quad \Gamma^d_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix},
$$

(5)

where $x$ represents an entry which is allowed to be non-zero. We do not impose CP on eq.(4) allowing elements in $\Gamma^d_1, 2$ to be complex. The above forms of $\Gamma^d_a$ are technically natural as they follow from imposition of discrete symmetries on eq.(4), the simplest being a $Z_2$ symmetry under which only $Q'_3L$ and $\phi_2$ change sign.

The down quark mass matrix $M_d$ follows from eq.(5) when the Higgs fields obtain their vacuum expectation values (vev): $\langle \phi^0_1 \rangle = v_1$ and $\langle \phi^0_2 \rangle = v_2 e^{i\alpha}$. Let $V_{dL,R}$ be the unitary matrices connecting the mass (unprimed) and the weak basis $d'_L,R = V^d_{dL,R} d_{L,R}$. Then

$$
V^\dagger_{dL} M_d V_{dR} = D_d,
$$

(6)

$D_d$ being a diagonal matrix of the down quark masses $m_i$. $M_d$ obtains contributions from two different Higgs fields leading to the Higgs induced FCNC in the down quark sector. Eqs. (4-6) are used to obtain:

$$
-L_{FCNC} = \frac{(2\sqrt{2}G_F)^{1/2}}{\sin \theta \cos \theta} F^d_{ij} d_i^L d_j^R \phi^0 + H.C.,
$$

(7)

where $\tan \theta = \frac{v_2}{v_1}$ and

$$
\phi^0 \equiv \cos \theta \phi_2^0 e^{-i\alpha} - \sin \theta \phi_1^0
$$

(8)

is a specific combination of $\phi_{1,2}$ with zero vev. The orthogonal combination plays the role of the standard model Higgs. The strength of FCNC current is determined in the fermion mass basis by [21]:

$$
F^d_{ij} \equiv (V^L_{dL,2} \Gamma^d_2 v_2 e^{i\alpha} V^R_{dL})_{ij} = (V^L_{dL,3})_{i3} (V^R_{dL,3})_{j3} m_j,
$$

(9)

Note that the the specific texture of $\Gamma^d_1, 2$ allowed us to express $F^d_{ij}$ in terms of the left handed mixing and the down quark masses $m_j$ and the dependence on the unphysical $V_{dR}$ disappeared. The $F^d_{ij}$ depend on the left-handed mixing matrix $V_{dL}$ which is a priori unknown but would be correlated to the CKM matrix. One observes that

- independent of the values of elements of $V_{dL}$, the $F^d_{ij}$ display hierarchy given in eq.(3).

- all the FCNC couplings are suppressed if the off-diagonal elements of $V_{dL}$ are smaller than the diagonal ones. The model in this sense illustrates the principle of near flavour conservation [24]. This is a generic possibility in view of the strong mass hierarchy among quarks unless there are some special symmetries.

- $F^d_{ij}$ can be determined in terms of the CKM matrix elements for a specific structure of $M_u$ [23] given as follows:

$$
M_u = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix}.
$$

(10)
The above postulated structures of $M_{u,d}$ follow from discrete symmetries rather than being ad-hoc. Particular example can be:

$$(Q'_{1,2L}, \phi_1) \rightarrow \omega(Q'_{1,2L}, \phi_1) \ , \ u'_{1,2R} \rightarrow \omega^2 u'_{1,2R}.$$  

(11)

Here $\omega, \omega^2 \neq 1$ are complex numbers. The fields not shown above remain unchanged under the symmetry.

The particular form of $M_u$ as given above implies that $(V_{dL})_{3i} = V_{3i}$ as a result of which the $F_{ij}^d$ in eq.(9) are completely determined in terms of the CKM matrix $V$.

$$F_{ij}^d = V_{3i}^* V_{3j} m_j .$$  

(12)

As a consequence of eq.(11), $(M_u)_{33}$ gets contribution from $\phi_2$ while the first two generations from $\phi_1$ with no mixing with the third one. As a result, there are no FCNC in the up quark sector while they are determined as in eq.(12) in the down quark sector.

The tree level couplings of the charged Higgs $H^+ \equiv \cos \theta \phi_2^+ - \sin \theta e^{i\alpha} \phi_1^+$ can be read off from eq.(4) and are given by

$$(2\sqrt{2}G_F)^{1/2} H^+ \left\{ \bar{u}_R \hat{D}_u V d_L + \bar{u}_L (V D_d \tan \theta - \frac{1}{s_\theta c_\theta} V F^d) d_R \right\} + H.C. ,$$  

(13)

where $\hat{D}_u \equiv \text{diag.}(-m_u \tan \theta, -m_c \tan \theta, m_t \cot \theta)$.

It follows from eqs.(7,12,13) that all the Higgs fermion couplings are determined by the CKM matrix $V$ giving rise to MFV. There can however be an additional source of CP violation in the model. This can arise if the scalar-pseudoscalar mixing contains a phase. As noted in [23], the discrete symmetry of eq.(11) prevents this mixing in the Higgs potential even if one allows for explicit CP violation and a bilinear soft symmetry breaking term $\mu(\phi_1^\dagger \phi_2) + H.c.$.. Thus the minimal version of the model corresponds to the MFV scenario with no other CP violating phases present. CP violation in Higgs mixing can however be induced by adding a complex Higgs singlet field [23, 25]. In this case, there would be an additional phase which mixes the real and the imaginary parts of the Higgs $\phi_0$ defined in eq.(8). We will admit such a phase in our discussion.

There is an important quantitative difference between the present scenario and the general MFV analysis following from the effective field theory approach [7]. There the effective dominant FCNC couplings between down quarks are given by

$$(\lambda_{FC})_{ij} \approx \lambda_t^2 V_{3i}^* V_{3j} ,$$

where $\lambda_t$ denotes the top Yukawa coupling. The same factor controls the loop induced contributions here but the tree level flavour violations are given by eq. (12) which contains the same elements of $V$ but involves the down quark masses linearly. Its contribution is still important or dominates over the top quark dependent terms because of its presence at the tree level.
One could consider variants of the above textures and symmetry obtained by permutations of flavour indices. These variants lead to different amount of FCNC. Labeling these variants by $a$, one has three models with $F_{ij}^d(a) = V_{ai}^* V_{aj} m_j$, $(a = 1, 2, 3)$. Alternatively, one could also consider equivalent models in which FCNC in the $d$ quarks are absent while in the up quark sector they would be related to the CKM matrix elements and the up quark masses. The case $a = 3$ is special. It leads to the maximum suppression of FCNC in the 12 sector. We will mainly consider phenomenological implication of that case.

### III. EXPERIMENTAL CONSTRAINTS AND THEIR IMPLICATIONS

#### A. Basic Results

The strongest constraints on the model come from the $P^0 - \bar{P}^0$ ($P = K, B_d, B_s$) mixing. In addition to the SM contribution, two other sources namely, the charged Higgs induced box diagrams and the neutral Higgs $\phi^0$ induced tree diagram contribute to this mixing.

The charged Higgs leads to new box diagrams which follow from eq. (13). The last two terms of this equation are suppressed by the down quark masses (for modest $\tan \theta$) and the dominant contribution comes from the top quark. This term and hence the charged Higgs contributions remain the same as in 2HDM with NFC [12]. The contribution to the $B_q^0 - \bar{B}_q^0$ mixing is already given in eq. (1). The contribution to $\epsilon$ is given [15] by

$$\epsilon^{H^+} = \frac{G_F^2 M_W^2 f_K^2 m_K B_K A^2 \lambda^6 \tilde{\eta}}{6\sqrt{2} \pi^2 \Delta m_K} \left( f_1^H + f_2^H A^2 \lambda^4 (1 - \bar{\rho}) \right), \quad (14)$$

where functions $f_{1,2}^H$ can be read-off from expressions given in [15]. $\lambda, \tilde{\eta} \equiv \eta(1 - \frac{\lambda^2}{2}), \bar{\rho} \equiv \rho(1 - \frac{\lambda^2}{2})$ and $A$ are the Wolfenstein parameters. Contribution of $f_1^H$ to $\epsilon$ is practically negligible while the $f_2^H$ can compete with the corresponding term in the SM expression

$$\epsilon^{SM} = \frac{G_F^2 M_W^2 f_K^2 m_K B_K A^2 \lambda^6 \tilde{\eta}}{6\sqrt{2} \pi^2 \Delta m_K} (f_1(x_i) + f_2(x_i) A^2 \lambda^4 (1 - \bar{\rho})) \quad (15)$$

for moderate values of $\tan \theta$.

The neutral Higgs contributions to the above observables follow from eqs. (7) and (12). Define

$$\phi^0 \equiv \frac{R + iI}{\sqrt{2}} = \frac{(O_{Ra} + iO_{Ia})}{\sqrt{2}} H^0_\alpha \equiv |C_\alpha| e^{i\eta_\alpha} H^0_\alpha,$$

where $H^0_\alpha$ denote the mass eigenstates with masses $M_\alpha$. $\alpha = 1, 2, 3$ for the 2HDM while $\alpha = 1, \ldots, 5$ in the presence of a complex singlet introduced to induce the scalar-pseudo scalar mixing leading to phases $\eta_\alpha$ in the Higgs mixing $C_\alpha$. $O_{Ra,Ia}$ are elements of the mixing matrix. Using this definition and eq. (12) the neutral Higgs contribution to $M_{12}^0$ can be written as

$$(M_{12}^0)^H = \frac{5\sqrt{2} G_F m_b f_{B_d} f_{B_s}^2 B_{2q}}{12 \sin^2 2\theta M_\alpha^2} \left( \frac{m_{B_q}}{m_b + m_q} \right)^2 C_\alpha^2 (V_{3q}^* V_{33})^2 + O \left( \frac{m_q}{m_b} \right), \quad (16)$$
where we used the vacuum saturation approximation multiplied by the bag factor $B_{2q}$

$$< B^0_q | (q_L b_R)^2 | B^0 > = - \frac{5}{24} m_{B_q} f_{B_q}^2 B_{2q} \left( \frac{m_{B_q}}{m_b + m_q} \right)^2 .$$

The $O \left( \frac{m_s}{m_b} \right)$ refer to contributions coming from the $F^{d*}_{3q}$ terms in eq. (7). Using the vacuum saturation approximation and eq.(12), these terms are estimated to be only a few % of the first term in eq (16) for $q = s$ and much smaller for $q = d$. We do not display here the QCD corrections to $(M_{12}^d)^3$. Such corrections can be significant and play important role in the precise determination of the SM parameters. In contrast, the above expressions contain several unknowns of the Higgs sector because of which we prefer to simplify the analysis and retain only the leading terms as far as the Higgs contributions to various observables are concerned. The SM contribution is given by

$$M_{12}^{q,SM} = \frac{G_F m_W^2 m_{B_{q}} f_{B_{q}}^2 B_{q} \eta_{B}}{12 \pi^2} (V_{3q} V_{33})^2 S_0(x_t),$$

with $S_0(x_t) \approx 2.3$ for $m_t \approx 161$ GeV. Eqs. (16,17) together imply

$$k^d \equiv \left| \frac{(M_{12}^d)^H}{(M_{12}^d)^{SM}} \right| = \left( \frac{5 \sqrt{2} \pi |C_a|^2}{G_F M_W^2 \sin^2 2\theta} \right) \left( \frac{m_b}{M_\alpha} \right)^2 \frac{B_{2q}}{B_{q} \eta_{B}} \left( \frac{m_{B_{q}}}{m_b + m_q} \right)^2 + O \left( \frac{m_q}{m_b} \right) .$$

The neutral Higgs contribution to $\epsilon$ is given by

$$\epsilon_H^0 = \frac{5 G_F m_K f_K^2 B_{2K}}{12 \sin^2 2\theta \Delta m_K M_\alpha^2} \left( \frac{m_K}{m_s + m_d} \right)^2 \Im (F_{12}^d C_a)^2 ,$$

Using the expression of $F_{12}^d$ from eq.(12) and the Wolfenstein parameterization, one can rewrite the above equation as

$$\epsilon_H^0 \approx \frac{5 G_F m_K^2 f_K^2 B_{2K}}{12 \sin^2 2\theta \Delta m_K M_\alpha^2} \left( \frac{m_K}{m_s + m_d} \right)^2 |C_a|^2 A^4 \lambda^{10} [(1 - \hat{\rho})^2 + \hat{\eta}^2]^{1/2} \sin 2(\eta_{\alpha} - \beta) ,$$

where $\tan \beta = \frac{\eta}{\hat{\rho}}$ is one of the angles of the unitarity triangle. The Higgs contribution to $\epsilon$ is suppressed here by the strange quark mass and $\epsilon_H^0$ is practically negligible compared to $\epsilon^{SM}$:

$$| \frac{\epsilon_H^0}{\epsilon^{SM}} | \approx 3.810^{-4} \frac{B_{2K}}{B_K} \frac{|C_a|^2}{\sin^2 2\theta} \left( \frac{100 \text{GeV}}{M_\alpha} \right)^2 \frac{\sin 2(\eta_{\alpha} - \beta)}{\cos \beta + 0.1 \sin \beta} .$$

The neutral Higgs contribution to the $K^0 - \bar{K}^0$ mass difference is even more suppressed compared to its experimental value.

**B. Experimental Inputs**

Constraints on the present scheme come from several independent measurements. The complex amplitude $M_{12}^d$ is known quite well. The magnitude is given in terms of the $B_d^0 - \bar{B}_d^0$ mass difference \cite{20}:

$$\Delta M^d \equiv 2 |M_{12}^d| = (0.507 \pm 0.005) \text{ ps}^{-1} .$$
The phase $\phi_d$ is measured through the mixing induced CP asymmetry in the $B^0_d \to J/\psi K_S$ decay:

$$\sin \phi_d = 0.668 \pm 0.028 .$$  \hfill (23)

Likewise, the $B^0_s - \bar{B}^0_s$ mass difference is quite well determined:

$$\Delta M^s \equiv 2|M^s_{12}| = 17.77 \pm 0.12 \text{ ps}^{-1} .$$  \hfill (24)

The corresponding phase $\phi_s$ is determined $^{27}$ by the D0 collaboration $^{28}$

$$\phi_s = -0.70^{+0.47}_{-0.39} .$$  \hfill (25)

by combining their measurements of (1) the light and the heavy $B^0_s$ width difference (2) the time dependent angular distribution in the $B^0_s \to J/\psi \phi$ decay and (3) the semileptonic charge asymmetries in the $B^0$ decays.

The SM predictions for the above quantities depend on the hadronic and the CKM matrix elements. The determination of $\bar{\rho}, \bar{\eta}$ is somewhat non-trivial when new physics is present. The conventional SM fits use the loop induced variables $\epsilon, M^s_{12}, \phi_d$ for determining $\bar{\rho}, \bar{\eta}$. These variables are susceptible to new physics contributions. This makes extraction of $\bar{\rho}, \bar{\eta}$ model-dependent. It is still possible to determine these parameters and construct a universal unitarity triangle $^{29}$ for a unitary $V$ by assuming that the tree level contributions in the SM are not significantly affected by new physics. In that case, one can use only the tree level measurements for determining $\bar{\rho}, \bar{\eta}$.\footnote{Alternatively one can allow for NP contributions $^{1, 5, 6, 7, 8, 9, 10}$ in the loop induced processes while determining elements of $V$. The tree level observables are the moduli of $V$ and the unitarity angle $\gamma$.}

$$\lambda = |V_{us}| = 0.2258 \pm 0.0014 , \quad A = \frac{|V_{cb}|}{\lambda^2} = 0.82 \pm 0.014 ,$$  \hfill (26)

$$|V_{ub}|^{\text{excl.}} = 0.0034 \pm 0.0004 , \quad |V_{ub}|^{\text{incl.}} = 0.0045 \pm 0.0003 .$$

\(\gamma\) is determined from purely tree level decay $B \to D^*K^*$. We will use the UTfit average value\footnote{In terms of the Wolfenstein parameters,}

$$\gamma = (83 \pm 19)^\circ .$$  \hfill (27)

In terms of the Wolfenstein parameters,\footnote{Eqs.\((27)\) and \((28)\) provide a NP independent determination of $\bar{\rho}, \bar{\eta}$, e.g. with inclusive values in eq.\((26)\).}

$$\bar{\rho} = R_b \cos \gamma , \quad \bar{\eta} = R_b \sin \gamma ,$$

$$R_b \equiv (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} |V_{ub}| = 0.46 \pm 0.03 \quad \text{inclusive determination} ,$$

$$= 0.35 \pm 0.04 \quad \text{exclusive determination} .$$  \hfill (28)

$$\bar{\rho} = 0.06 \pm 0.15 , \quad \bar{\eta} = 0.46 \pm 0.03 .$$  \hfill (29)

$$\bar{\rho} = 0.06 \pm 0.15 , \quad \bar{\eta} = 0.46 \pm 0.03 .$$  \hfill (30)
One could use the above values of $\bar{\rho}, \bar{\eta}$ to obtain predictions of $\epsilon$ and $M_{12}^d$ in the SM. The errors involved are rather large but it has the advantage of being independent of any new physics contributing to these observables. This approach has been used for example in [2, 3, 4] to argue that a non-trivial NP phase is required if $|V_{ub}|$ is close to its inclusive determination. We will use an alternative analysis which also leads to the same conclusion.

The new physics contributions to the loop induced $\Delta F = 2$ observables is parameterized as follows

$$M_{12}^d = (M_{12}^d)^{SM}(1 + \kappa_q e^{i\sigma_q}) = \rho_q(M_{12}^d)^{SM} e^{i\phi_{NP}},$$

$$\epsilon = \rho_\epsilon \epsilon_{SM}.$$  \hspace{1cm} (31)

Model independent studies using the above or equivalent parameterization have been used to determine $\bar{\rho}, \bar{\eta}, \kappa_q, \sigma_q, C_\epsilon$ in number of different work [2, 4, 5, 6]. We will use the results from UTfit group whenever appropriate.

In view of the several unknown Higgs parameters, we make a simplifying assumption that only one Higgs contributes dominantly. We distinguish two qualitatively different situations corresponding to the dominance of the charged Higgs $H^+$ or of a neutral Higgs.

C. Charged Higgs dominance:

The effects of the charged Higgs on the $P^0 - \bar{P}^0$ mixing as well as on $\Delta F = 1$ processes such as $b \rightarrow s\gamma$ have been discussed at length in the literature [12, 14, 15, 30]. The present case remains unchanged compared to the standard two Higgs doublet model of type II as long as the down quark mass dependent terms are neglected in eq.(13). Just for illustrative purpose and completeness we discuss some of the restrictions on the charged Higgs couplings and masses in this subsection before turning to our new results on the neutral Higgs contributions to flavour violations.

The allowed values of $\bar{\rho}, \bar{\eta}$ in the presence of the charged Higgs follow from the detailed numerical fits in case of MFV scenario, e.g. fits in [4] give

$$\bar{\rho} = 0.154 \pm 0.032 \quad , \quad \bar{\eta} = 0.347 \pm 0.018 .$$  \hspace{1cm} (32)

We can substitute these values in the SM expressions for $\Delta M^d$ and $\epsilon$ to obtain [26]

$$\rho_d \equiv \frac{\Delta M^d}{(\Delta M^d)_{SM}} = 0.99 \pm 0.29 ,$$

$$\rho_\epsilon \equiv \frac{\epsilon}{\epsilon_{SM}} = 0.94 \pm 0.09 .$$  \hspace{1cm} (33)

This can be translated into bounds on $M_{H^+}$ and $\tan \theta$ using eqs.(11, 14) and eq.(15). The $2\sigma$ bounds following from eq.(33) are shown in Fig.(1). The constraints from $\epsilon$ are stronger and allow the middle (dotted) strip in the $M_{H^+} - \tan \theta$ plane. These are illustrative bounds and we refer to literature [12, 14, 15, 30] for more detailed results which include QCD corrections. Generally, there is sizable region in $\tan \theta, M_{H^+}$ plane (e.g. $\tan \theta \gtrsim 1 - 2$ in Fig.(1)) for which the top induced charged Higgs contribution to $\rho_d, \epsilon$ is not important. But the neutral Higgs can contribute to these observables in these regions as we now discuss.
D. Neutral Higgs dominance:

We label the dominating neutral Higgs field by $\alpha = H$ and retain only one term in eq.(16). Unlike in the previous case, the neutral Higgs contribution to $\epsilon$ (and the $K^0 - \bar{K}^0$ mass difference) is very small. It can contribute significantly to $M_{d,s}^{d,s}$ but these contributions are strongly correlated. Using eqs.(16,18) one finds that:

\[
\begin{align*}
    r &= \frac{\kappa_s}{\kappa_d} = \frac{B_{2s} B_{B_d}}{B_{2d} B_{B_s} (m_s + m_b)} \left( \frac{m_B}{m_B} \right)^2 \left( \frac{m_d + m_b}{m_B} \right)^2, \\
    \sigma_d &= \sigma_s = 2\eta_H.
\end{align*}
\]

(34)

This ratio does not involve most of the unknown parameters and is determined by masses and the bag parameters. The ratios of $B$ parameter in eq.(34) and hence $r$ is very close to 1. For example, the results in [31] for the bag parameters imply

\[
    r = 1.04 \pm 0.12.
\]

(35)

Assuming $r = 1$ leads to an important prediction:

\[
\frac{\Delta M^s}{\Delta M^d} = \left( \frac{\Delta M^s}{\Delta M^d} \right)^{SM}.
\]
This prediction holds good in various MFV scenario, e.g. SUSY MFV model at low $\tan \beta$. Here it remains true even in the presence of an extra phase $\eta_H$. The above prediction can be usefully exploited for the determination of one of the sides of the unitarity triangle:

$$R_t \equiv \sqrt{(1 - \rho)^2 + \bar{\eta}^2} = \frac{1}{\lambda} \frac{|V_{td}|}{|V_{cb}|},$$

$$= \frac{\xi}{\lambda} \sqrt{\frac{M_{B_u}}{M_{B_d}}} \sqrt{\frac{\Delta M_{B_d}}{\Delta M_{B_s}}} \approx 0.93 \pm 0.05,$$  \hspace{1cm} (36)

where $\xi = \frac{f_{B_s} f_{B_d}}{f_{B_u} f_{B_d}} = 1.23 \pm 0.06$. We used the SM expression, eq.(17) in the above equation and the approximation $|V_{ts}| = |V_{cb}|$.

The SM prediction for $\Delta M_s$ is independent of $\bar{\rho}, \bar{\eta}$. Using, $f_{B_s} \sqrt{B_s} = 0.262 \pm 0.035$ MeV, we obtain

$$\rho_s \equiv \left| \frac{\Delta M_s}{\Delta M_{SM}^s} \right| \approx 0.96 \pm 0.26. \hspace{1cm} (37)$$

The existing fits to the $\Delta F = 2$ processes in the presence of NP are carried out in the context of the MFV or NMFV scenario or in a model independent manner. Most of these assume that NP contributes significantly to $\Delta S = 2$ transition, particularly to $\epsilon$. This is not the case here. On the other hand the model independent fits neglect correlations between $\Delta M_d, \Delta M_s$ as present here. In view of this, we performed our own but simplistic fits in the present case. We use $\phi_d, \gamma, R_b, R_t, \rho_s$ and $\epsilon$ in the fits assuming all errors to be Gaussian. The expressions and the experimental values for these quantities are already given in respective equations. We use the standard model expression for $\epsilon$. We have used $r = 1$ in eq.(34) giving eq.(36) and $\rho_d = \rho_s \equiv \bar{\rho}$ and $\sigma_d = \sigma_s \equiv \sigma$. The above six observables are fitted in terms of the four unknowns $\bar{\rho}, \bar{\eta}, \bar{\rho}, \phi_d^{NP}$. The fitted values of the parameters are sensitive to $|V_{ub}|$. The accompanying table contains values of the fitted parameters and $1\sigma$ errors obtained in three cases which use (a) inclusive (b) exclusive and (c) average value of $|V_{ub}|$ as quoted in [32]. The predictions based on the average values agree within $1\sigma$ with the corresponding detailed model independent fits by the Utfit group: $\bar{\rho} = 0.167 \pm 0.051$, $\bar{\eta} = 0.386 \pm 0.035$. The values of $\bar{\rho}, \bar{\eta}$ in the fit directly determine the phase of $(M_{12}^d)_{SM}$:

$$\sin 2\beta_d = \frac{\bar{\eta}(1 - \bar{\rho})}{\sqrt{\bar{\eta}^2 + (1 - \bar{\rho})^2}}.$$

The phase $\phi_d$ as measured through $S(\psi K_S)$ is then given by

$$\phi_d = 2\beta_d + \phi_d^{NP},$$

where $\phi_d^{NP}$ is defined in eq.(31) and can also be written as

$$\tan \phi_d^{NP} = \frac{\kappa_q \sin \sigma_q}{1 + \kappa_q \cos \sigma_q}.$$

Results in the table imply that if $|V_{ub}|$ is close to the exclusive value then the present results are consistent with SM. If $V_{ub}$ is large and close to the inclusive value then $\phi_d^{NP}$ is non-zero
at $2\sigma$ level. This conclusion is similar to observations made \cite{2} on the basis of the use of $R_{b,\gamma}$ alone but with somewhat different input values then used here. A non-zero $\phi_{d}^{NP}$ (and hence $\sigma$) has important qualitative implication for the model under consideration. Non-zero $\sigma$ requires CP violating phase $\eta_{H}$ from the scalar-pseudoscalar mixing. As already remarked the minimal 2HDM with symmetry as in (11) cannot lead to such a phase and more general model with an additional singlet field will be required. Also the charged Higgs contribution by itself cannot account for such a phase.

At the quantitative level, $\tilde{\rho} \neq 1$ implies restrictions on the Higgs parameters, $M_H$, $|C_H|$, $\theta$. These parameters are simply related to $\kappa \equiv |\rho e^{i\phi_{d}^{NP}} - 1|$ which is related to the said parameters through eq.(18). Results in table imply $\kappa = 0.18 \pm 0.08$ if $|V_{ub}| = |V_{ub}^{incl}|$. The values of $M_H$ and $|C_H|^2$ which reproduce this $\kappa$ within $1\sigma$ range is shown in Fig.(1) for two illustrative values of $\tan \theta = 3, 10$. Both these values of $\tan \theta$ are chosen to make the charged Higgs contribution to $\kappa$ very small. Unlike general models with FCNC, relatively light Higgs is a possibility within the present scheme and there exist large ranges in $\theta$ and $C_H$ which allow this.

One major prediction of the model is equality of new physics contributions to CP violation in the $B_d$ and $B_s$ system. If the top induced charged Higgs contribution dominates then this CP violation is zero. In the case of the neutral Higgs dominance, the phases $\sigma_d$ and $\sigma_s$ induced by the Higgs mixing are equal see, eq.(34). Since the ratio $r$ in this equation is nearly one, let us write $r = 1 + \delta_r$ with $\delta_r \approx \pm O(0.1)$. Then $\phi_s^{NP}$ in eq.(38) can be approximated as

$$\tan \phi_s^{NP} \approx \tan \phi_d^{NP} \left[1 + \delta_r (1 - \cot \sigma \tan \phi_d^{NP})\right],$$

$$\approx (1 + \delta_r) \tan \phi_d^{NP} .$$

This prediction is independent of the details of the Higgs parameters. Its important follows from the fact the standard CP phase in the $B_s$ system is quite small, $\beta_s \sim -1.0^\circ$. Thus observation of a relatively large $\phi_s = 2\beta_s + \phi_s^{NP}$ will signal new physics. The predicted values of $\tan \phi_s$ based on eq.(39) and the numerical values given in table give

$$\tan \phi_s \approx -0.18 \pm 0.08 \quad \text{inclusive},$$
$$\approx 0.03 \pm 0.08 \quad \text{exclusive},$$
$$\approx -0.14 \pm 0.08 \quad \text{average} .$$

All these values are at present consistent with the experimental determination eq.(25), by the $D0$ collaboration \cite{28}. Significant improvements in the errors is foreseen in future at LHCb \cite{33} and relatively large $\phi_s$ following from the inclusive $V_{ub}$ can be seen. The above predictions show correlation with $V_{ub}$ and also with the CP violating phase $\phi_d$. So combined improved measurements of all three will significantly test the model. The predictions of $\phi_s$ in the present case are significantly different from several other new physics scenario allowing relatively large values for $\phi_s$ \cite{34}.
TABLE I: Determination of NP parameters and $\bar{\rho}, \bar{\eta}$ from detailed fits to predictions of the neutral Higgs induced FCNC. See, text for more details

|                | $|V_{incl.}|$  | $|V_{excl.}|$  | $|V_{average}|$ |
|----------------|--------------|--------------|---------------|
| $\bar{\rho}$  | 0.200 ± 0.039| 0.121 ± 0.042| 0.186 ± 0.039 |
| $\bar{\eta}$  | 0.391 ± 0.028| 0.320 ± 0.026| 0.378 ± 0.027 |
| $\rho_{d,s}$   | 0.96 ± 0.26  | 0.96 ± 0.26  | 0.96 ± 0.26   |
| $\sin\phi_{d}^{NP}$ | −0.18 ± 0.08 | 0.03 ± 0.08  | −0.14 ± 0.08  |

IV. SUMMARY

The general two Higgs doublet models are theoretically disfavored because of the appearance of uncontrolled FCNC induced through Higgs exchanges at tree level. We have discussed here the phenomenological implications of a particular class of models in which FCNC are determined in terms of the elements of the CKM matrix. This feature makes these models predictive and we have worked out major predictions of the scheme. Salient aspects of the scheme discussed here are

- Many of the predictions of the scheme are similar to various other models which display MFV. The tree level FCNC couplings are governed by the CKM elements and the down quark masses while the dominant part of the charged Higgs couplings involve the same CKM factors but the top quark mass. Both contributions can be important and there exists regions of parameters ($\tan \theta \gtrsim 2 - 3$) in which the former contribution dominates. Unlike general FCNC models, the neutral Higgs mass as light as the current experimental bound on the SM Higgs is consistent with the restrictions from the $P^0 - \bar{P}^0$ mixing, see Fig.(1).

- The neutral Higgs coupling to $\epsilon$ parameter is suppressed in the model by the strange quark mass. This prediction differs from the general MFV models where the top quark contributes equally to the $B^0 - \bar{B}^0$ mixing and $\epsilon$. Detailed fits to experimental data is carried out which determine the CKM parameters $\bar{\rho}, \bar{\eta}$ as displayed in the table.

- Noteworthy and verifiable prediction of the model is correlation (eq. (39)) between the CP violation in $B_s - \bar{B}_s, B_d - \bar{B}_d$ systems and $|V_{ub}|$ as displayed in the table.

- We have restricted ourselves to study of the the $\Delta F = 2$ flavour violations in this paper. The tree level FCNC would give rise to additional contributions to $\Delta F = 1$ processes and to new processes such as flavour changing neutral Higgs decays. Already existing information on the $\Delta F = 1$ and $\Delta F = 2$ processes can be very useful in identifying allowed parameter space and verifiable signatures of the model. Such a study will be taken up separately.
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