Next-to-leading-order QCD corrections to $Z \to \eta_c(\eta_b) + g + g$

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Abstract

In this paper, we carry out the next-to-leading-order (NLO) studies of $Z \to \eta_c(\eta_b) + g + g$ (labeled as “gg”) through the color-singlet (CS) state of $c\bar{c}(b\bar{b})[^1S^0_0]$, with the aim of assessing the impact of this process on $Z$ decaying into inclusive $\eta_c(\eta_b)$. We find the newly-calculated QCD corrections to the $gg$ process can notably enhance its leading-order (LO) results. To be specific, with the renormalization scale varying in $[m_{c,b}, m_Z]$, $\Gamma_{\eta_c gg}$ is increased by about 8-14 times, and about 1.5-2.0 times for $\eta_b$ production. Consequently, $\Gamma^{NLO}_{Z \to \eta_c + g + g}$ can reach up to about 40 – 70% of the LO results given by the CS dominant process $Z \to \eta_c + c + \bar{c}$, and about 30 – 40% for the $\eta_b$ case. Moreover, with the significant QCD corrections, the $gg$ process would exert crucial influence on the CS predictions of the $\eta_c(\eta_b)$ energy distributions. In conclusion, in the CS studies of $Z \to \eta_Q + X$ ($Q = c, b$), besides $Z \to \eta_Q[^1S^0_0] + Q + \bar{Q}$, the process $Z \to \eta_Q[^1S^0_0] + g + g$ can as well provide phenomenologically indispensable contributions.

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I. INTRODUCTION

Due to the experimental reconstruction difficulties, the observation of $\eta_c$ meson is scant comparing to that of $J/\psi$. For example, HERA, LEP II, and $B$ factories have accumulated copious $J/\psi$ yield data, while have not yet detected any evident event of inclusive $\eta_c$ production. In 2014, the LHC (LHCb group), which runs with a large center-of-mass proton-proton collision energy and a high luminosity, achieved the first measurement of inclusive $\eta_c$ yield [1]. Comparing to the theoretical results [2–13], the LHCb measured cross sections seem to almost be saturated by the color-singlet (CS) predictions alone, leaving very limited room for the color-octet contributions and thus posing a serious challenge to the nonrelativistic QCD (NRQCD) factorization [15]; however, Refs. [5, 6] pointed out NRQCD is still valid in describing the LHCb data. Note that, there are large uncertainties in the LHCb released data [1]. Therefore, more studies of inclusive $\eta_c$ yield in other processes and experiments with better precision are required to further assess the validity of NRQCD in $\eta_c$ production.

Heavy-quarkonium production in $Z$ boson decay, which has triggered extensive studies [16–41], provide a good chance for the study of $\eta_c$ production mechanism. At the LHC, large number of $Z$ events ($\sim 10^9$/year [34]) can be generated in one running year, with which the study of $Z$ decaying into heavy quarkonium has been an increasingly important area [44–46]. Furthermore, the upgrades of HE(L)-LHC will give birth to a higher collision energy (luminosity), largely improving the accumulated $Z$ yield events. In addition, the proposed future $e^+e^-$ collider, CEPC [47], equipped with “clean” background and enormous $Z$ production events ($\sim 10^{12}$/year), would also be beneficial to hunt $\eta_c$ yield through $Z$ decay. From these perspectives, precise measurements of $Z$ boson decay into inclusive $\eta_c$ look promising, and studying $Z \rightarrow \eta_c + X$ through the CS mechanism could help to explore whether there still holds the compatibility of the CS predictions with the future measurements.

In $Z \rightarrow \eta_c + X$, there exist two CS processes contributing at LO in $\alpha_s$, i.e., $Z \rightarrow \eta_c[^1S_0^1] + c + \bar{c}$ and $Z \rightarrow \eta_c[^1S_0^1] + g + g$. We can learn from Ref. [19], that $Z \rightarrow \eta_c + c + \bar{c}$ plays a leading role in the CS LO predictions because of the $c$-quark fragmentation; while, owing to the suppression of $m_c^2/m_Z^2$ [19], $Z \rightarrow \eta_c + g + g$ contributes just slightly at LO (less than 5% of the results of $Z \rightarrow \eta_c + c + \bar{c}$). However, considering the advent of the gluon-fragmentation

\footnote{$\eta_c$ is always established by its decaying into multiple hadrons, such as $p\bar{p}$, which is more difficult than the $J/\psi$ detection.}
structures at the next-to-leading-order (NLO) level, i.e., $Z \rightarrow q + \bar{q} + g^*; g^* \rightarrow \eta_c + g$ ($q = u, d, s$) and the loop-induced process $Z \rightarrow g + g^*; g^* \rightarrow \eta_c + g$, the uncalculated QCD corrections to $Z \rightarrow \eta_c + g + g$ are expected to provide considerable contributions, subsequently making the $gg$ process comparable with $Z \rightarrow \eta_c + c + \bar{c}$. Moreover, the $\eta_c$ energy distributions in $Z \rightarrow \eta_c + g + g$ and $Z \rightarrow \eta_c + c + \bar{c}$ may thoroughly be different. This can be understood by that the former process, together with the QCD corrections, are strongly suppressed by the factor $\frac{M_{\eta_c}^2}{E_{\eta_c}^2}$ for large $z$ [27, 42, 43], and thereby the $z$ value corresponding to the largest $\frac{d\Gamma}{dz}$ should be small; however, as a result of the $c$-quark fragmentation, the dominant contributions in $Z \rightarrow \eta_c + c + \bar{c}$ exist in the large $z$ region [19]. In view of these points, $Z \rightarrow \eta_c[1S_0^{[1]}] + g + g$ would be phenomenologically crucial for the decay of $Z$ boson into inclusive $\eta_c$, deserving a separate and precise investigation.

In contrast with $\eta_c$, the larger mass of $\eta_b$ would result in smaller typical coupling constant and relative velocity ($v$) between the constituent $b\bar{b}$ quarks, subsequently leading to better convergent results over the expansion in $\alpha_s$ and $v$. While, on the experimental side, $\eta_b$ has so far been observed only in $e^+e^-$ annihilation [48–51]. Taken together, in this article we will carry out the first NLO studies of $Z \rightarrow \eta_c(\eta_b)[1S_0^{[1]}] + g + g$, so as to provide a deeper insight into the $\eta_c(\eta_b)$ production mechanism.

The rest of the paper is organized as follows: In Sec. II, we give a description on the calculation formalism. In Sec. III, the phenomenological results and discussions are presented. Section IV is reserved as a summary.

II. CALCULATION FORMALISM

Within the NRQCD framework [15 52], the decay width of $Z \rightarrow \eta_c(\eta_b) + g + g$ can be factorized as

$$\Gamma = \hat{\Gamma}_{Z\rightarrow c\bar{c}(b\bar{b})[n] + g + g} \langle O_{\eta_c(\eta_b)}(n) \rangle,$$

where $\hat{\Gamma}_{Z\rightarrow c\bar{c}(b\bar{b})[n] + g + g}$ is the perturbative calculable short distance coefficients (SDCs), representing the production of a configuration of the $c\bar{c}(b\bar{b})[n]$ intermediate state. The universal nonperturbative long distance matrix element $\langle O_{\eta_c(\eta_b)}(n) \rangle$ stands for the probability of $c\bar{c}(b\bar{b})[n]$ into $\eta_c(\eta_b)$. In this paper, we focus only on the CS contributions, and accordingly $n$ takes on $^{1S_0^{[1]}}$. 

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FIG. 1: Representative Feynman diagrams for the NLO QCD corrections to $Z \rightarrow c\bar{c}(b\bar{b})[1^{1}S_{0}^{1}] + g + g$. “$q$” denotes the light quarks ($u,d,s$).

Up to NLO in $\alpha_{s}$, the SDC in Eq. (1) comprises three contributing components,

$$\hat{\Gamma}^{NLO}_{Z \rightarrow c\bar{c}(b\bar{b})[1^{1}S_{0}^{1}] + g + g} = \hat{\Gamma}_{\text{Born}} + \hat{\Gamma}_{\text{Virtual}} + \hat{\Gamma}_{\text{Real}},$$

where

$$\hat{\Gamma}_{\text{Virtual}} = \hat{\Gamma}_{\text{Loop}} + \hat{\Gamma}_{\text{CT}},$$

$$\hat{\Gamma}_{\text{Real}} = \hat{\Gamma}_{S} + \hat{\Gamma}_{HC} + \hat{\Gamma}_{H'C}.$$  \hspace{1cm} (3)

$\hat{\Gamma}_{\text{Virtual}}$ is the virtual corrections composed of the contributions from the one-loop diagrams ($\hat{\Gamma}_{\text{Loop}}$) and the counter terms ($\hat{\Gamma}_{\text{CT}}$). $\hat{\Gamma}_{\text{Real}}$ stands for the real corrections, containing the soft terms ($\hat{\Gamma}_{S}$), hard-collinear terms ($\hat{\Gamma}_{HC}$), and hard-noncollinear terms ($\hat{\Gamma}_{H'C}$). $\hat{\Gamma}_{\text{Real}}$ consists of two processes,

$$Z \rightarrow c\bar{c}(b\bar{b})[1^{1}S_{0}^{1}] + g + g + g,$$

$$Z \rightarrow c\bar{c}(b\bar{b})[1^{1}S_{0}^{1}] + g + q + \bar{q} \ (q = u, d, s).$$ \hspace{1cm} (4)

The diagrams for $\hat{\Gamma}_{\text{Born}}$, $\hat{\Gamma}_{\text{Virtual}}$, and $\hat{\Gamma}_{\text{Real}}$ are representatively shown in Fig. [1]. Note that, in calculating $Z \rightarrow c\bar{c}(b\bar{b})[1^{1}S_{0}^{1}] + g + g + g$, we apply the physical polarization tensor, $P_{\mu\nu}$,\footnote{$P_{\mu\nu} = -g_{\mu\nu} + \frac{k_{\mu}\eta_{\nu} + k_{\nu}\eta_{\mu}}{k \cdot \eta}$, where $k$ is the momentum of one of the three final gluons and $\eta$ is conveniently set as the momentum of one of the other two gluons in the final state.} for the polarization summation of the final gluons, thereby avoiding the considerations of the ghost diagrams.

To isolate the ultraviolet (UV) and infrared (IR) divergences, we adopt the dimensional regularization with $D = 4 - 2\epsilon$. The on-mass-shell (OS) scheme is employed to set the
FIG. 2: Cancellation of the divergences involved in the $\eta_c$ production. The superscripts “(2)” and “(1)” denote the $\epsilon^{-2}$- and $\epsilon^{-1}$-order terms, respectively.

renormalization constants for the heavy quark mass ($Z_m$), heavy quark filed ($Z_2$), and gluon filed ($Z_3$). The modified minimal-subtraction (\(\overline{\text{MS}}\)) scheme is used for the QCD gauge coupling ($Z_g$). The renormalization constants read ($Q = c, b$)

$$
\delta Z_m^{\text{OS}} = -3C_F \frac{\alpha_s N_c}{4\pi} \left[ \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln \frac{4\pi \mu^2}{m_Q^2} + \frac{4}{3} \right],
$$

$$
\delta Z_2^{\text{OS}} = -C_F \frac{\alpha_s N_c}{4\pi} \left[ \frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} - 3\gamma_E + 3\ln \frac{4\pi \mu^2}{m_Q^2} + 4 \right],
$$

$$
\delta Z_3^{\text{OS}} = \frac{\alpha_s N_c}{4\pi} \left[ (\beta'_0 - 2C_A) \left( \frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) - \frac{4}{3} T_F \left( \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln \frac{4\pi \mu^2}{m_Q^2} \right) \right],
$$

$$
\delta Z_g^{\overline{\text{MS}}} = -\frac{\beta_0}{2} \frac{\alpha_s N_c}{4\pi} \left[ \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln(4\pi) \right],
$$

(5)

where $\gamma_E$ is the Euler’s constant, $N_c = \Gamma[1 - \epsilon]/(4\pi \mu^2/(4m_{c,b}^2))$, $\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F m_f$, $\beta'_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$, $n_f = 5$, and $n_f = n_f - 2$ are the numbers of active quark flavors and light quark flavors, respectively. In SU(3), the color factors are given by $T_F = \frac{1}{2}$, $C_F = \frac{4}{3}$, and $C_A = 3$. In treating $\Gamma_{\text{Real}}$, we utilize the two-cutoff slicing strategy \cite{53} to subtract the IR divergences.

In our calculations, we adopt our Mathematica-Fortran package with the implementation of FeynArts \cite{54}, FeynCalc \cite{55}, FIRE \cite{56}, and Apart \cite{57} to deal with $\hat{\Gamma}_{\text{Virtual}}, \hat{\Gamma}_S$, and $\hat{\Gamma}_{\text{HC}}$; the FDC package \cite{58} is used to evaluate the hard-noncollinear part $\hat{\Gamma}_{\text{HC}}$. Taking $\eta_c$ for example, we visualize the cancellation of the $\epsilon^{-2(-1)}$-order divergences and the independence...
FIG. 3: The verification of the independence of the $\eta_c$’s SDCs on the cutoff parameters $\delta_s$ and $\delta_c$. The superscript “(0)” denotes the $\epsilon^0$-order terms. In the left diagram, $\delta_c = 2 \times 10^{-7}$, and $\delta_s = 1 \times 10^{-3}$ for the right one.

III. PHENOMENOLOGICAL RESULTS

The input parameters involved in the calculations are taken as

$$\alpha = 1/128, \quad m_c = 1.5 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$
$$m_Z = 91.1876 \text{ GeV}, \quad m_{q\bar{q}} = 0 \ (q = u, d, s),$$
$$\sin^2(\theta_W) = 0.23116. \quad (6)$$

To determine $\langle \mathcal{O}_{\eta_c(\eta_b)}(1S_0^{[1]}) \rangle$, we employ the relations to the radial wave functions at the origin,

$$\langle \mathcal{O}_{\eta_c(\eta_b)}(1S_0^{[1]}) \rangle = \frac{1}{2N_c} |R_{\eta_c(\eta_b)}(0)|^2, \quad (7)$$

where $|R_{\eta_c(\eta_b)}(0)|^2$ reads [14]

$$|R_{\eta_c}(0)|^2 = 1.16 \text{ GeV}^3,$$
$$|R_{\eta_b}(0)|^2 = 6.477 \text{ GeV}^3. \quad (8)$$

We summarize the predicted total decay widths of $Z \rightarrow \eta_c + g + g$ and $Z \rightarrow \eta_b + g + g$ in Tables. I and II respectively. For comparisons, the LO results of $Z \rightarrow \eta_c(\eta_b) + c\bar{c}(b\bar{b})$ are also included. Inspecting the two tables, one can observe
TABLE I: The total decay widths of $Z \rightarrow \eta_c + g + g$ (in units of KeV). “NLO” represents the sum of the contributions of LO terms and that of the QCD corrections. “$K$” and “$R$” refer to the ratios of $\Gamma_{gg}^{NLO}/\Gamma_{gg}^{LO}$ and $\Gamma_{c\bar{c}}^{NLO}/\Gamma_{c\bar{c}}^{LO}$, respectively, with “$gg(c\bar{c})$” denoting the process of $Z \rightarrow \eta_c + gg(c\bar{c})$.

| $\mu r$ (GeV) | $m_c$ (GeV) | $\Gamma_{gg}^{LO}$ | $\Gamma_{gg}^{NLO}$ | $K$ | $\Gamma_{c\bar{c}}^{LO}$ | $R$ |
|----------|----------|-----------|------------|-----|-------------|-----|
| 1.4      | 1.4      | 5.720     | 94.95      | 16.60 | 130.6 | 0.727 |
| 2$m_c$   | 1.5      | 4.828     | 67.32      | 13.94 | 99.90 | 0.674 |
|          | 1.6      | 4.122     | 48.89      | 11.86 | 77.80 | 0.628 |

$\mu r m_c$ (GeV) $\Gamma_{gg}^{LO}$ $\Gamma_{gg}^{NLO}$ $K$ $\Gamma_{c\bar{c}}^{LO}$ $R$

$\mu r m_c$ (GeV) $\Gamma_{gg}^{LO}$ $\Gamma_{gg}^{NLO}$ $K$ $\Gamma_{c\bar{c}}^{LO}$ $R$

TABLE II: The total decay widths of $Z \rightarrow \eta_b + g + g$ (in units of KeV). “NLO” represents the sum of the contributions of LO terms and that of the QCD corrections. “$K$” and “$R$” refer to the ratios of $\Gamma_{gg}^{NLO}/\Gamma_{gg}^{LO}$ and $\Gamma_{b\bar{b}}^{NLO}/\Gamma_{b\bar{b}}^{LO}$, respectively, with “$gg(b\bar{b})$” denoting the process of $Z \rightarrow \eta_b + gg(b\bar{b})$.

| $\mu r$ (GeV) | $m_b$ (GeV) | $\Gamma_{gg}^{LO}$ | $\Gamma_{gg}^{NLO}$ | $K$ | $\Gamma_{b\bar{b}}^{LO}$ | $R$ |
|----------|----------|-----------|------------|-----|-------------|-----|
| 4.6      | 4.6      | 2.515     | 3.717      | 1.478 | 13.35 | 0.278 |
| 4.7      | 2.383    | 3.441     | 1.444      | 12.23 | 0.281 |
| 4.8      | 2.260    | 3.193     | 1.413      | 11.23 | 0.284 |
| 4.6      | 1.052    | 2.080     | 1.976      | 5.584 | 0.372 |
| 4.7      | 1.007    | 1.964     | 1.950      | 5.172 | 0.380 |
| 4.8      | 0.965    | 1.857     | 1.925      | 4.796 | 0.387 |

i) $\Gamma_{Z \rightarrow \eta_c + g + g}^{LO}$ is less than 5% of $\Gamma_{Z \rightarrow \eta_c + c + \bar{c}}^{LO}$, implying that the $c\bar{c}$ process dominates over the $gg$ process at the LO accuracy in $\alpha_s$. However, after including the QCD corrections, the LO results of $gg$ would be enhanced to a large extent, as shown in the first figure of Fig. 4. This striking enhancement, which is attributed partially to the gluon-fragmentation structure ($g^* \rightarrow \eta_c + g$) occurring first at NLO, would lead to the comparableness of the $gg$ process with the $c\bar{c}$ one. (See the ratios in the “$R$” column of Tab. I.)

ii) As to $\eta_b$, $\Gamma_{Z \rightarrow \eta_b + g + g}^{LO}$ accounts for about 20% of $\Gamma_{Z \rightarrow \eta_b + b + \bar{b}}^{LO}$; the QCD corrections to the
FIG. 4: The total decay widths of $Z \to \eta_Q + g + g$ ($Q = c, b$) as a function of the renormalization scale $\mu_r$. $m_c = 1.5$ GeV and $m_b = 4.7$ GeV. “NLO” represents the sum of the contribution of the LO terms and that of the QCD corrections.

$gg$ process would enhance its LO results by about 1.5-2.0 times, then increasing the “20%” ratio up to about $30 - 40\%$. In addition, $\Gamma^{NLO}_{Z \to \eta_b + g + g}$ exhibits a more steady dependence than $\Gamma^{LO}_{Z \to \eta_b + g + g}$ on the renormalization scale $\mu_r$, as displayed in the second figure of Fig. 4.

In Fig. 3, the $\eta_c(\eta_b)$ energy distributions are drawn with $z$ defined as $\frac{2E_{\eta_c(\eta_b)}}{m_Z}$. It can be seen that,

i) The dominant contributions in $\Gamma^{LO}_{Z \to \eta_c + c + \bar{c}}$ arise from the region of $z \approx 0.7$, while the peak of $\frac{d\Gamma^{LO}_{Z \to \eta_c + g + g}}{dz}$ lies in the vicinity of $z \approx 0.2$. By incorporating the QCD corrections, $\frac{d\Gamma^{LO}_{Z \to \eta_c + g + g}}{dz}$ is notably enhanced, especially at the small- and mid-$z$ regions. As a result, adding the $gg$ contributions will greatly increase the $z$ distributions of $\eta_c$ in $Z \to \eta_c + c + \bar{c}$, which can be clearly seen by the huge discrepancy between the two lines referring to $c\bar{c}_{LO}$ with or without $gg_{NLO}$ in the two figures above of Fig. 5.

ii) Regarding $\eta_b$, there also exists an evident peak of $\frac{d\Gamma^{LO}_{Z \to \eta_b + b + \bar{b}}}{dz}$ around $z \approx 0.7$; in $Z \to \eta_b + g + g$ at LO, the mid-$z$ regions ($z \approx 0.5$) contribute dominantly. With the QCD corrections, which impose significant impacts on $\frac{d\Gamma^{LO}_{Z \to \eta_b + g + g}}{dz}$, the $gg$ process would evidently enhance the predicted $\eta_b$-energy distributions given by $Z \to \eta_b + b + \bar{b}$, as

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3 The impacts of the gluon-fragmentation structure, $g^* \to \eta_b + g$, are moderate due to the large mass of $\eta_b$; thus, the QCD corrections to $Z \to \eta_b + g + g$ appears milder than the $\eta_c$ case.
FIG. 5: The $\eta_Q$ ($Q = c, b$) energy distributions with $z$ defined as $\frac{2E_{\eta_Q}}{m_Z}$; “$gg(\bar{Q}Q)$” denotes the process of $Z \to \eta_Q + gg(\bar{Q}Q)$. $m_c = 1.5$ GeV and $m_b = 4.7$ GeV. “NLO” represents the sum of the contribution of the LO terms and that of the QCD corrections.

manifested by the large difference in height of the line of $\bar{b}b_{LO}$ and that of $\bar{b}b_{LO} + gg_{NLO}$ in the lower two figures in Fig. 5.

To summarize, our newly-calculated QCD corrections to $Z \to \eta_Q[^1S_0^{[1]}] + g + g$ ($Q = c, b$) could enormously enhance its LO results, and then greatly elevate the phenomenological significance of the $gg$ process by significantly increasing the CS predictions.

IV. SUMMARY

In this article, we achieve the first NLO studies of $Z \to \eta_Q + g + g$ ($Q = c, b$) through the CS state of $Q\bar{Q}[^1S_0^{[1]}]$. We find the newly-calculated QCD corrections can noticeably enhance its LO predictions of the total decay width, following which the $gg$ process would contribute comparably comparing to the CS dominant process $Z \to \eta_Q[^1S_0^{[1]}] + Q\bar{Q}$. Moreover, the NLO corrections would also to a large extent increase $\frac{d\Gamma_{Z \to \eta_Q + g + g}^{LO}}{dz}$, profoundly influencing the CS predictions of the $\eta_Q$ energy distribution. Therefore, to arrive at a strict CS prediction of
\[ Z \rightarrow \eta_Q + X, \text{ besides } Z \rightarrow \eta_Q[1S_0^{[1]}]+Q+\bar{Q}, \text{ it appears mandatory to take } Z \rightarrow \eta_Q[1S_0^{[1]}]+g+g \text{ into consideration as well.} \]

V. ACKNOWLEDGMENTS

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