Tracing the gas at redshift 1.7-3.5 with the Lyman-α forest: the FLO approach

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ABSTRACT
We present FLO (From Lines to Over-densities), a new technique to reconstruct the hydrogen density field for the Lyα forest lines observed in high resolution QSO spectra. The method is based on the hypothesis that the Lyα lines arise in the low to intermediate density intergalactic gas and that the Jeans length is the typical size of the Lyα absorbers. The reliability of FLO is tested against mock spectra obtained from cosmological simulations. The recovering algorithm gives satisfactory results in the range from the mean density to over-densities of ∼30 and reproduces correctly the correlation function of the density field and the 1D power spectrum on scales between ∼20 and 60 comoving Mpc. A sample of Lyα forests from 22 high resolution QSO spectra is analysed, covering the redshift range 1.7 ≤ z ≤ 3.5. For each line of sight, we fit Voigt profiles to the lines of the Lyα forest, providing the largest, homogeneous sample of fitted Lyα lines ever studied. The line number density evolution with redshift follows a power-law relation: dn/dz = (166 ± 4) [(1 + z)/3.5]12.8±0.2 (1 σ errors). The two-point correlation function of lines shows a signal up to separations of ∼2 comoving Mpc; weak lines (log N(Hi) < 13.8) also show a significant clustering but on smaller scales (r ∼ 1.5 comoving Mpc). We estimate with FLO the hydrogen density field toward the 22 observed lines of sight. The redshift distribution of the average densities computed for each QSO is consistent with the cosmic mean hydrogen density in the analysed redshift range. The two-point correlation function and the 1D power spectrum of the δ field are estimated. They are both consistent with the analogous results computed from hydro-simulated spectra obtained in the framework of the concordance cosmological model. The correlation function shows clustering signal up to ∼4 comoving Mpc.

Key words: intergalactic medium, quasars:absorption lines, cosmology:observations, large-scale structure of Universe

1 INTRODUCTION
Semi-analytical and hydro-dynamical simulations (e.g., Cen et al. 1994; Zhang et al. 1997; Hernquist et al. 1996; Miralda-Escudé et al. 1996; Bi & Davidsen 1994; Davé et al. 1997; Zhang et al. 1997; Theuns, Leonard & Efstathiou 1998; Machacek et al. 2001) suggest that the Lyα forest arises from fluctuations of the low density intergalactic medium (IGM) that trace the underlying matter density field over cosmic time. The dynamical state of the low density IGM is governed mainly by the Hubble expansion and by gravitational instabilities. As a consequence, the physics involved is quite simple and mildly non-linear. The statistical analysis of the Lyα forest provides information on the dynamical growth and thermal state of the IGM, and on the correlation properties of the (dark) matter in the Universe. Correlations of the Lyα forest lines were detected with a 4 – 5σ confidence by various authors at typical scales ∆v ≲ 350 km s⁻¹ observing at high resolution individual lines of sight (Cristiani et al. 2001; Cristiani et al. 2001).
densities through the formula (Schaye 2001):

\[ n_{\text{HI}} \propto \frac{T_{0.4}^{-0.26} \delta^{1.5}}{1 + \delta}, \]

where \( \delta = n_{\text{HI}}/n_{\text{H}} \). This relation is used to compute the fraction of the mass in gas and \( \alpha \) depends on the ionisation history of the Universe. Equation 1 relies on three main hypotheses: (i) Ly\( \alpha \) absorbers are close to local hydrostatic equilibrium, i.e. their characteristic size will be typically of the order of the local Jeans length (\( L_J \)); (ii) the gas is in photo-ionisation equilibrium; (iii) the equation of state, \( T = T_0 (\delta + 1)^{\alpha} \), holds for the optically thin IGM gas (Hui & Gnedin 1997).

The procedure to recover the H density field from the list of Ly\( \alpha \) line column densities in a QSO line of sight, has been dubbed FLO (From Lines to Over-densities).

The paper is organised as follows: Section 2 describes the observed data sample used for our analysis and presents the statistical measures obtained for the fitted Ly\( \alpha \) lines; Section 3 introduces the hydrogen density field as a statistical estimator, and describes the construction algorithm; Section 4 presents the simulated spectra and the test of reliability of the method with this dataset; in Section 5 the new algorithm is applied to the observed data sample; finally, we draw our conclusions in Section 6.

The cosmological model adopted throughout this paper corresponds to a 'fiducial' \( \Lambda \)CDM Universe with parameters, at \( z = 0 \): \( \Omega_m = 0.26 \), \( \Omega_{\Lambda} = 0.74 \), \( \Omega_b = 0.0463 \), \( n_s = 0.95 \), \( s_8 = 0.85 \) and \( H_0 = 72 \, \text{km s}^{-1} \text{Mpc}^{-1} \) (the B2 set of parameters of Viel, Hachert, Springell 2004).

### Table 1. Summary of the main characteristics of our QSO sample. (see text.

| QSO    | \( z_{\text{em}} \) | J mag | \( \text{Ly} \alpha \) \( \Delta_j \) |
|--------|-----------------|-------|-------------------------------|
| HE1341-1020 | 2.139 | 18.68 | 1.658-2.097 |
| Q0122-380 | 2.203 | 17.34 | 1.711-2.150 |
| PKS1448-323 | 2.220 | 17.09 | 1.725-2.166 |
| PKS0237-23 | 2.233 | 16.61 | 1.737-2.179 |
| J2333-606 | 2.251 | 16.97 | 1.753-2.196 |
| HE0001-2340 | 2.267 | 16.74 | 1.765-2.213 |
| HE1122-1648 | 2.400 | 16.61 | 1.878-2.344 |
| Q0109-3518 | 2.407 | 16.72 | 1.884-2.350 |
| HE2217-2818 | 2.415 | 16.47 | 1.891-2.358 |
| Q0329-385 | 2.440 | 17.20 | 1.912-2.383 |
| HE1158-1843 | 2.451 | 17.09 | 1.921-2.394 |
| HE1347-2457 | 2.560 | 17.35 | 2.016-2.502 |
| Q0453-423 | 2.662 | 17.69 | 2.100-2.602 |
| PKS0329-255 | 2.698 | 17.88 | 2.125-2.636 |
| HE0151-4326 | 2.761 | 17.48 | 2.186-2.699 |
| Q0002-422 | 2.768 | 17.50 | 2.189-2.705 |
| HE2347-3432 | 2.878 | 17.12 | 2.281-2.814 |
| HS1946+7658 | 3.051 | 16.64 | 2.429-2.984 |
| HE0940-1050 | 3.088 | 17.08 | 2.463-3.020 |
| Q0420-388 | 3.122 | 17.44 | 2.489-3.053 |
| PKS2126-158 | 3.275 | 17.54 | 2.620-3.204 |
| B1422+231 | 3.620 | 16.22 | 2.911-3.543 |

2 OBSERVED DATA SAMPLE

Most of the observational data used in this work were obtained with the UVES spectrograph (Dekker et al. 2000) at the Kueyen unit of the ESO VLT (Cerro Paranal, Chile) in the framework of the ESO Large Programme (LP): “The Cosmic Evolution of the IGM” (Bergéron et al. 2004). Spectra of 18 QSOs were obtained in service mode with the aim of studying the physics of the IGM in the redshift range 1.7-3.5. The spectra have a resolution \( R \sim 45000 \) and a typical signal to noise ratio (SNR) of \( \sim 35 \) and 70 per pixel at 3500
and 6000 Å, respectively. Details of the data reduction can be found in Chand et al. (2004); Aracil et al. (2004).

We added to the main sample 4 more QSO spectra with comparable resolution and SNR:

- J2233-606 (Cristiani & D’Odorico 2000). Data for this QSO were acquired during the commissioning of UVES in October 1999.
- HE1122-1648 (Kim et al. 2002). Data for this QSO were acquired during the science verification of UVES in February 2000. The reduced and fitted spectrum was kindly provided to us by Tae-Sun Kim.
- HS1946+7658 (Kirkman & Tytler 1997). Data for this QSO were acquired with Keck/HIRES in July 1994.
- B1422+231 (Rauch et al. 1996). Data for this QSO were acquired with Keck/HIRES in 1996. The reduced and fitted spectrum was kindly provided to us by Tae-Sun Kim.

Table 1 summarises the main properties of our QSO sample. None of our QSOs is a Broad Absorption Line (BAL) object. Magnitudes are taken from the GSC-II catalogue (McLean et al. 2000). Figure 1 shows the distribution in redshift of the Ly\(\alpha\) forests for all the QSOs of the sample.

We considered for each QSO the redshift range between 1000 km s\(^{-1}\) red-ward of the Ly\(\beta\) emission, in order to avoid contamination by associated Ly\(\beta\) lines, and 5000 km s\(^{-1}\) blue-ward of the Ly\(\alpha\) emission to exclude the region affected by the proximity effect due to the ionising flux of the QSO. The coverage is good over the whole redshift range \(z \approx 1.7 - 3.5\), with most of the signal concentrated between \(z \sim 2\) and 2.5.

In Fig. 2 we show a portion of the Ly\(\alpha\) forest of the QSO HE0001-2340 compared with the same wavelength region in a mock spectrum extracted from the considered simulation box at \(z = 2\) (see section 2.1).

### 2.1 Creation of the line lists

All the lines in the Ly\(\alpha\) regions of the LP QSOs plus J2233-606 were fitted with the FITLYMAN tool of the ESO MIDAS data reduction package. In the case of complex saturated lines we used the minimum number of components to reach \(\chi^2 \leq 1.5\). Whenever possible, the other lines in the Lyman series were used to constrain the fit. The spectra of HE1122-1648, HS1946+7658, B1422+231 and all the simulated lines of sight were fitted with the VPFIT package. Both software tools model absorption features with a Voigt profile convolved with the instrument line spread function. The minimum H\(\text{I}\) column density detectable at 3\(\sigma\), at the lowest SNR of the spectra in our sample, is \(\log N(\text{H}\text{I}) \approx 12\) cm\(^{-2}\).

Metals in the forest were identified and the corresponding spectral regions were masked to avoid effects of line blanketing. We eliminated Ly\(\alpha\) lines with Doppler parameters \(b \leq 10\) km s\(^{-1}\), that are likely unidentified metal absorptions. In a total amount of 8435 fitted Ly\(\alpha\) lines, 368 (4.4\%) fall in the masked intervals, 1150 (13.6\%) are at less than 1000 km s\(^{-1}\) red-ward the Ly\(\beta\) emission or at less than 5000 km s\(^{-1}\) blue-ward the Ly\(\alpha\) emission, while 599 were eliminated because they have \(b \leq 10\) (7.1\%). The output of this analysis is a list of Ly\(\alpha\) lines for each QSO with central redshift, H\(\text{I}\) column density and Doppler parameter.

In the line fitting approach to the study of the Ly\(\alpha\) forest, each line is considered as the signature of an absorber. As a consequence statistical measures are computed with the population of absorption lines, representative of the population of absorbers. Our sample of fitted Ly\(\alpha\) lines is the largest, homogeneous sample ever gathered up to now. We will use it to compute the number density evolution with redshift and the two point correlation function of lines.
2.2 Line number density evolution

The line number density per unit of redshift is generally approximated as \( \frac{dn}{dz} = (\frac{dn}{dz})_0 (1 + z)^\beta \), where \( (\frac{dn}{dz})_0 \) is the local comoving line number density of the forest and the exponent \( \beta \) depends both on physical (redshift, column density interval) and instrumental (spectral resolution, decomposition of velocity profiles) factors.

In Fig. 3 we plot the result for the QSOs in our sample for the standard column density interval \( 13.64 < \log N(HI) < 17 \text{ cm}^{-2} \) in order to compare our statistics with the HST low redshift measurement\(^1\) (Weymann et al. 1998). The best fit to our data gives: \( \frac{dn}{dz} = (166 \pm 4)(1+z/3.5)^{2.8 \pm 0.2} \) (1\(\sigma\) errors). There is no substantial change in the trend with respect to previous results by Kim and collaborators (2001, 2002) who used smaller samples of UVES QSO spectra of the same quality. However, our points are systematically higher on the plot, with an increase in \( \log \frac{dn}{dz} \) amounting to \( \sim 0.03 \) at \( z \sim 2 \) up to \( \sim 0.1 \) at \( z \sim 3 \). The discrepancy arises from the fact that we have taken into account the decrease in the available redshift interval due to the presence of metal lines 'masking' the \( \text{Ly}\alpha \) features. High resolution spectra allow to identify a larger number of metal lines: in our sample these metal masks correspond to about 9 percent of the total redshift interval covered by the observed \( \text{Ly}\alpha \) forests.

Fig. 4 shows the number density evolution for two different \( \text{H}I \) column density ranges: \( 13 \leq \log N(HI) \leq 14 \) and \( 14.5 \leq \log N(HI) < 17 \text{ cm}^{-2} \). The linear fit in these intervals gives slopes of \( 1.9 \pm 0.2 \) and \( 3.8 \pm 0.4 \) for the weak and the strong lines selection, respectively. This trend was already noticed by Kim et al. (2002): stronger lines have a steeper number density evolution than the weaker ones.

2.3 Two-point correlation function of \( \text{Ly}\alpha \) lines

To study the clustering properties of our sample of \( \text{Ly}\alpha \) lines, we adopt the standard two point correlation function (TPCF) defined as the excess, due to clustering, of the probability \( dP \) of finding a \( \text{Ly}\alpha \) absorber in a volume \( dV \) at a distance \( r \) from another absorber: \( dP = \Phi_{\text{Ly}\alpha}(z)dV[1+\xi(r)] \), where \( \Phi(z) \) is the average space density of the absorbers as a function of \( z \).

Operatively this quantity is estimated with the formula (Peebles 1980):

\[
\xi(v) = \frac{N_{\text{obs}}(v)}{N_{\text{exp}}(v)} - 1,
\]

where \( N_{\text{obs}} \) is the observed number of line pairs with velocity separations between \( v \) and \( v+dv \), and \( N_{\text{exp}} \) is the number of pairs expected in the same range of separations from a random distribution in redshift. Since in this context peculiar velocities are negligible (see e.g. Rauch et al. 2003), we compute the correlation function in real space, measuring

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1. http://www.eso.org/midas
2. http://www.ast.cam.ac.uk/~rfc/vpfilt.html
3. The lower limit in column density is due to the fact that HST measurements have been transformed from equivalent width into \( \text{H}I \) column densities assuming a typical Doppler parameter of 30 km s\(^{-1}\)
Figure 5. Two point correlation function for the observed Lyα lines in the column density range \(12 < \log N(\text{H}i) < 17 \text{ cm}^{-2}\). In the bottom panel lines closer than one Jeans length have been merged into one line, see text. The dashed lines represent the 1σ confidence levels from a random distribution of lines.

Figure 6. Two point correlation function for the observed Lyα lines in two column density ranges as reported in the panels.

Figure 7. Two point correlation function for the observed Lyα lines in the column density range \(13.8 < \log N(\text{H}i) < 17\) and in two redshift ranges reported in the panels.

As already said in Section 1, the Jeans length \(L_J\) likely represents the typical size of IGM structures detected as Lyα absorptions. This length (varying from \(\sim 1.2\) to \(1.6\) comoving Mpc for the maximum and minimum redshift of our sample, \(z = 2.5\), a velocity separation \(\Delta v = 100 \text{ km s}^{-1}\) corresponds to \(\Delta r \approx 0.9\) comoving Mpc, in our fiducial cosmology. \(N_{\exp}\) is obtained by averaging the results of 1000 numerical simulations of the number of lines observed in each QSO spectrum. In particular, the set of line redshifts is randomly generated in the same redshift interval as the data according to the observed distribution \(\propto (1 + z)^\beta\), where we adopt the value \(\beta = 2.8\) found in the previous section. The same mock line lists are used to estimate the error on the observed correlation function by determining the 1σ standard deviation of the correlation functions of the randomly distributed lines. Lines closer than 0.3 comoving Mpc, are merged into a single line with redshift equal to the mean redshift, weighted with the column densities, and column density equal to the sum of the column densities. The minimal separation is set by the intrinsic blending due to the typical width of the lines (see Giallongo et al. 1996).

We compute the correlation function for the whole data set (Fig. 5) and for two column density cuts (Fig. 5) to investigate the clustering properties of strong and weak lines. Previous results (Cristiani et al. 1995; Lu et al. 1996; Cristiani et al. 1997) already showed a significant clustering signal for strong absorptions, which is confirmed and strengthened by our data. Furthermore, we also see a significant clustering for the weak lines, consistent with previous results by Misawa et al. (2004). The amplitude is about one order of magnitude lower than for the stronger lines but the clustering signal in the first bin is significant at the 7σ level.
consider the column density range for which the signal is preserved substantially at the same level inside themselves. The clustering signal is increased significance due to the smaller statistics. This is an indication that Lyα lines in the high-redshift range. Here the cut in column density is defined to correspond to a constant cut in density contrast, δ ≳ 3, corresponding to log N(H I) > 14.3 at z = 3.02, see text.

Two point correlation function for the observed Lyα lines in the high-redshift range. Here the cut in column density is defined to correspond to a constant cut in density contrast, δ ≳ 3, corresponding to log N(H I) > 14.3 at z = 3.02, see text.

sample, respectively) is also comparable with the clustering scale of the Lyα lines as shown in Figs. 3 (upper panel) and 4. In order to verify that the clustering signal we are detecting is not only due to structures internal to the absorbers, we perform the following test. Lines with separation less than the local L J are merged into a single line with column density equal to the sum of the column densities of the component features and redshift equal to the N(H I)-weighted mean of the component redshifts and the TPCF is re-computed. The result, reported in the lower panel of Fig. 5 shows that the clustering signal is preserved substantially at the same level of the one computed with all the lines, with a slightly decreased significance due to the smaller statistics. This is an indication that Lyα absorbers cluster among themselves and not only inside themselves.

The present data set is large enough to allow studying the evolution of the correlation function with redshift. We consider the column density range for which the signal is stronger, 13.8 < log N(H I) < 17, and we divide our sample in two parts. The first sub-sample is formed by objects with emission redshift z_em < 2.5, for which the average Lyα forest redshift is ⟨z_Lyα⟩ = 2.07, and the second sub-sample has objects with z_em > 2.5 and ⟨z_Lyα⟩ = 3.02. Results are shown in Fig. 7: the high redshift lines are less clustered than the low redshift lines. This apparent evolution with redshift is biased by the fact that the relation δ = log N(H I) is also z-dependent. Indeed, the same column density range selects objects with a lower density contrast at higher redshift (see eq. 1), explaining the lower clustering signal. To verify this effect, we selected lines on the ground of a constant density contrast, δ ≳ 3, which corresponds to log N(H I) > 13.8 at the average redshift of the low redshift sub-sample, and to log N(H I) > 14.3 at the higher average redshift. The correlation function for the latter sub-sample is shown in Fig. 8. Selecting the same kind of structures, there is no longer evidence of a significant evolution with redshift.

Tab 2 shows a detailed budget of the number of lines used to compute the TPCF in all the different selections described above.

3 INTRODUCING FLO

In Section 1 we have described: on the one hand, what are the main drawbacks of the two standard approaches (Voigt fitting and flux statistics) adopted to analyse the Lyα forest and derive statistical quantities describing the physical state of the IGM. On the other hand, the recovered H density field is introduced as a new robust estimator, whose statistical properties are in good agreement with those of the original density field, and which allows an easy comparison between observation and simulation results. The relation between the underlying H density field and the H I column densities measured for the observed absorption lines is summarised by eq. 1.

Before describing the FLO technique in details, it is important to recall the main hypotheses:

1. Lyα absorbers have typical sizes of the order of the local L J, which can be approximated as (Zaroubi et al. 2006): L J ≈ 1.33 (Ω_m h^2)^{-1/2} \left( \frac{T_0}{1.8 \times 10^2} \right)^{1/2} \left( \frac{1.6}{(\alpha + 1)} \right)^{1/2} \left( \frac{1 + z}{4} \right)^{-1/2} \text{Mpc}

in comoving units, where h ≡ H_0/100 km s^{-1} Mpc^{-1}, and the other parameters have been already defined;

2. the IGM gas is in the linear or slightly non-linear regime (log(δ + 1) ≈ 1) (Hui & Gnedin 1997).

In order to apply eq. 1 we have, first of all, to go through the Voigt fitting process of the Lyα forest absorptions in a QSO spectrum. Then, to transform the list of H I column densities of Lyα lines into the matter density field which generated them, we have to perform the following steps:

(i) the spatial separation between all the possible line pairs is computed and the minimum separation is compared with L J, computed at the N(H I)-weighted redshift mean of the pair;

(ii) if the two lines of the pair are more distant than the local L J, they are classified as two different absorbers, stored and deleted from the line list;

(iii) if the two lines are closer than the local L J, they are replaced in the line list by one line with a redshift equal to the N(H I)-weighted mean of the two redshifts and a column density equal to the sum of the two column densities;

(iv) the procedure is iterated until all the lines are converted into absorbers.
Voigt fitting complex absorption features, the total H\textsubscript{i} is low. On the other hand, we verify that, in the process of finding the minimum detectable column density in our data, one absorber with hydrogen density contrast correspondingly small lines is reduced, since their contribution to the weak lines is reduced, since their contribution to the correlation function.

With the introduction of this new statistical estimator, the drawbacks of the standard Voigt fitting approach are significantly reduced. On the one hand, the statistical weight of weak lines is reduced, since their contribution to the δ field is low. On the other hand, we verify that, in the process of Voigt fitting complex absorption features, the total H\textsubscript{i} column density is a much more robust quantity than the number of components. To this purpose, we compare the results of the line lists of a sub-sample of 12 QSOs adopted in the present work with the corresponding lists obtained with the VPFIT package, kindly provided to us by Tae-Sun Kim. The total number of lines in each line of sight is not conserved, in particular, significant differences are observed for the complex absorption systems where, in general, VPFIT fits more lines than FITLYMAN. Most of these discrepancies are due to the identification of low column density lines. However, the total column density in these complex absorbers appears to be much more stable between the two fitting methods.

In Fig. 9 we plot the comparison between FITLYMAN and VPFIT for one QSO of the sample, Q0109-3518. We divide the line of sight into redshift bins of width ∆r = 0.01; we sum both the number and the column density of lines falling closer than 1000 km s\textsuperscript{-1} blue-wards the Ly\textbeta emission or closer than 5000 km s\textsuperscript{-1} red-wards the Ly\textalpha emission; number of eliminated lines because one of the three previous conditions occurs; number of merged lines because their separation is less than the ∆r threshold indicated in the selection; number of merged lines not fulfilling the column density selection; number of lines used to compute the TPCF.

Table 2. Detailed budget of the number of lines used to compute TPCFs. The first column refers to the selection carried out, in terms of emission redshift of the considered objects.

| Selection | N\textsubscript{QSO} | N\textsubscript{tot} \textsuperscript{a} | N\textsubscript{mask} | N\textsubscript{comb} | N\textsubscript{prox} | N\textsubscript{col} | N\textsubscript{merg} | N\textsubscript{fin} |
|-----------|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| all QSOs  | 22                | 8435            | 368             | 644             | 1150            | 1955            | 380             | 4953            |
| 13.8 < log N(H\textsubscript{i}) < 17  \\
| ∆r > 0.3 com. Mpc                       |                  |                 |                 |                 |                 |                 |                 |
| all QSOs  | 22                | 8435            | 368             | 644             | 1150            | 1955            | 380             | 1319            |
| 12 < log N(H\textsubscript{i}) < 13.8  \\
| ∆r > 0.3 com. Mpc                       |                  |                 |                 |                 |                 |                 | 4781            |
| all QSOs  | 22                | 8435            | 368             | 644             | 1150            | 1955            | 380             | 170             |
| 12 < log N(H\textsubscript{i}) < 17  \\
| ∆r > 0.3 com. Mpc                       |                  |                 |                 |                 |                 |                 | 5930            |
| z\textsubscript{em} ≤ 2.5               \\
| 13.8 < log N(H\textsubscript{i}) < 17  \\
| ∆r > 0.3 com. Mpc                       | 11               | 3188            | 103             | 169             | 445             | 665             | 100             | 2042            | 381 |
| z\textsubscript{em} > 2.5               \\
| 13.8 < log N(H\textsubscript{i}) < 17  \\
| ∆r > 0.3 com. Mpc                       | 11               | 5247            | 265             | 475             | 705             | 1290            | 280             | 2911            | 766 |
| z\textsubscript{em} > 2.5               \\
| 14.3 < log N(H\textsubscript{i}) < 17  \\
| ∆r > 0.3 com. Mpc                       | 11               | 5247            | 265             | 475             | 705             | 1290            | 280             | 3345            | 332 |
| all QSOs  | 22                | 8435            | 368             | 644             | 1150            | 1955            | 380             | 2944            | 64             | 3472 |
| 12 < log N(H\textsubscript{i}) < 17  \\
| ∆r > 1 L\textsubscript{J}              |                  |                 |                 |                 |                 |                 |                 |                 |

\textsuperscript{a} total number of fitted Ly\textalpha lines; \textsuperscript{b} number of Ly\textalpha lines falling in the metal masks; \textsuperscript{c} number of Ly\textalpha lines with b < 10 or b > 100; \textsuperscript{d} number of lines falling closer than 1000 km s\textsuperscript{-1} red-wards the Ly\textbeta emission or closer than 5000 km s\textsuperscript{-1} blue-wards the Ly\textalpha emission; \textsuperscript{e} number of eliminated lines because one of the three previous conditions occurs; \textsuperscript{f} number of merged lines because their separation is less than the ∆r threshold indicated in the selection; \textsuperscript{g} number of merged lines not fulfilling the column density selection; \textsuperscript{h} number of lines used to compute the TPCF.

(2) transform the list of column densities of absorbers into a list of δ with eq. [4].

(3) bin the redshift range covered by the Ly\textalpha forest into steps of 1 L\textsubscript{J} and distribute the absorbers onto this grid, proportionally to the superposition between absorber size (which is again 1 L\textsubscript{J}) and bin. Empty bins are filled with one absorber with hydrogen density contrast corresponding to the minimum detectable column density in our data, log N(H\textsubscript{i}) = 12 cm\textsuperscript{-2} at the redshift of the bin.

(4) normalise the resulting δ field in order to have δ + 1 = 1.0 for the whole considered sample. This operation is necessary to recover the correct asymptotic behaviour of the correlation function.
stability of FLO against different fitting tools by applying it to the line lists obtained with VPFIT and with FITLYMAN for the 12 common QSOs. In Fig 10 we show the comparison between the two recovered fields by means of a contour scatter plot. The correlation is tight for all values of $\delta$, the scatter increases slightly for $\delta \lesssim 0$.

4 SIMULATED DATA SAMPLE

We use simulations run with the parallel hydro-dynamical (TreeSPH) code GADGET-2 based on the conservative ‘entropy-formulation’ of SPH (Springel 2005). They consist of a cosmological volume with periodic boundary conditions filled with an equal number of dark matter and gas particles. Radiative cooling and heating processes are followed for a primordial mix of hydrogen and helium. We assume a mean UV-Background (UVB) produced by QSOs and galaxies as given by Haardt & Madau (1996) with helium heating rates multiplied by a factor 3.3 in order to better fit observational constraints on the temperature evolution of the IGM. This background gives naturally a $\Gamma \sim 10^{-12}$ (H ionisation rate) at the redshifts of interest here (Bolton et al. 2003). The star formation criterion is a very simple one that converts in collision-less stars all the gas particles whose temperature falls below $10^4$ K and whose density contrast is larger than 1000 (it has been shown that the star formation criterion has a negligible impact on flux statistics). More details can be found in Viel, Haehnelt, Springel (2004).

We use $2 \times 400^3$ dark matter and gas particles in a 120 $h^{-1}$ comoving Mpc box (although for some cross-checks we analyse some smaller boxes of 60 $h^{-1}$ comoving Mpc). The gravitational softening is set to 5 $h^{-1}$ kpc in comoving units for all particles.

We stress that the parameters chosen here, including the thermal history of the IGM, are in reasonably good agreement with observational constraints including recent results on the CMB and other results obtained by the Ly$\alpha$ forest community (e.g. Viel, Haehnelt, Lewis 2006).

The 120 Mpc simulation box at $z = 2$ is pierced to create a set of 364 mock lines of sight covering a redshift range $\Delta z \approx 0.11$. For each of these lines of sight, we know the density contrast, the temperature, and the peculiar velocity pixel by pixel. Peculiar velocities are small, typically less than 100 km s$^{-1}$, and randomly oriented, so their contribution, e.g. to the correlation function, is in general negligible. However, since we want to compare the result of simulations and observations, we modify the redshifts of the density field ($z_{old}$) with the peculiar velocity field to obtain the density field in redshift space ($z_{new}$) using the formula $v_{pec}(z_{old}) = c(z_{new} - z_{old})/(1 + (z_{new} + z_{old})/2)$. We added to the simulated spectra a Gaussian noise $S/N=50$, in order to reproduce the observed average $S/N$ per pixel. The simulated lines of sight have been fitted with Voigt profiles using an automated version of VPFIT.

4.1 Reconstruction of the $\delta$ field

The Ly$\alpha$ lines in each simulated line of sight are selected to have, as in the case of observations, $b \geq 10$ km s$^{-1}$. We introduce a further constraint, $b \leq 100$ km s$^{-1}$, which is required by the fact that simulated spectra are not continuum fitted. Shallow and broad oscillations in simulated spectra are fitted as absorption lines with Doppler parameters of the order of thousands of km s$^{-1}$. In the real spectra these kind of oscillations are instead fitted with the continuum and $b$ parameters that large are not measured. The selected lines are grouped into absorbers and transformed into the corresponding density field following the procedure described in Section 3. In eq. 1 the values: $T_0 = 1.8 \times 10^4$ K and $\alpha = 0.6$
are adopted, which are consistent with an early re-ionisation epoch and are the ones inferred from the simulations.

The reconstructed field is compared with the original density field (i.e. the output of the simulation), which is also binned into $1 \times L_\perp$ steps. An upper threshold is adopted both for the true and the recovered $\delta$ field, $\delta_{\text{thr}} = 50$, since 99.95 percent of pixels in the simulated lines of sight have values $\delta \leq \delta_{\text{thr}}$ and the algorithm to recover the $\delta$ field (eq. 11) is valid for values of $\delta \leq \text{few} \times 10$. The upper cut is applied before the normalisation process.

The average values of the $\delta$ field considering all the 364 simulated spectra are: $\langle \delta \rangle \approx 0.9$ and 1.3 for the true and recovered field, respectively. The fields are normalised using these values.

Figure 11 shows the contour scatter plot of the original versus reconstructed density field. As can be seen from the figure, FLO reconstructs fairly well the original field above the mean density, while under-densities are underestimated, or not recovered, if they are below our lower threshold. Indeed, the lower horizontal tail observed in the scatter plot is due to the treatment of the empty bins during the absorber-field transformation. The upper horizontal tail is instead due to the cut applied to over-densities larger than the threshold, $\delta_{\text{thr}}$. Figure 12 shows the distribution of $\delta$ values in the true and recovered field. The peak at $\log(\delta + 1) \approx -0.83$ contains $\approx 53$ percent of all the points and it is due to the procedure that assigns to empty bins the value of $\delta$ corresponding to the redshift of the bin and to the minimum observed column density, $\log N(\text{H}^1) \approx 12 \text{ cm}^{-2}$. On the other hand, the small bump at $\log(\delta + 1) \approx 1.57$ is due to the upper cut applied to the recovered density field and it includes $\approx 0.4$ percent of the total number of points.

The transformation starts to recover more than half of the correct values of $\delta$ at $\log(\delta + 1) \approx -0.15$ and recovers all the $\delta$ within 30 percent in the range $-0.08 \lesssim \log(\delta + 1) \lesssim 1.45$. Clearly, we are not dealing correctly with the under-dense regions, even if they are above our observational detection limit. This is likely due to the fact that our primary hypothesis, the local hydrostatic equilibrium, is not valid for those regions. This was also discussed by Schaye (2001) and here we have the evidence that under the mean density the gas is still expanding.

In the next section, it will be shown that the inaccuracies in the under-density regime do not significantly affect statistical measures like the correlation function.

### 4.2 Two-point statistics of the $\delta$ field

We computed the correlation function for the original and recovered $\delta$ field, with the formula:

$$\xi_\delta(r) = \langle \delta(r + dr)\delta(r) \rangle, \quad (4)$$

where $r$ is the physical separation of two points in comoving Mpc. $\xi_\delta(r)$ quantifies the clustering properties of the considered field, showing a signal significantly different from zero at separations where the field presents structures (over or under-densities).

The bin size is the largest value of $L_\perp$ for our sample, $\approx 1.532$ comoving Mpc, corresponding to the minimum redshift.

Figure 13 plots the results of the correlation function for the true and recovered $\delta$ field. The value in each bin is the median value of 50 sample of 88 lines of sight obtained with a bootstrap technique from the 364 lines of sight of the total sample. This procedure is required in order to compare this result with the analogous one for the observed data (see Section 5.1). We have 22 observed spectra but each one covers a redshift range corresponding to about 4 simulated spectra. Error-bars are $1\sigma$, computed with the percentiles of the distribution of values in each bin. The recovered correlation...
We also estimate the one dimensional power spectrum of the hydrogen density contrast field, that is defined by the Fourier transform of the correlation function:

\[
\xi(r) = \frac{V}{2\pi} \int |\delta_k|^2 e^{-ikr} \, dk 
\]

\[
P_{\delta}^{1D}(k) = \langle |\delta_k|^2 \rangle 
\]

The power spectrum is computed adopting the Fast Fourier Transform (FFT) technique which requires that the field to be transformed is evenly sampled. To this purpose, we have re-binned the observed lines of sight to a constant step equal to the minimum Jeans length for the considered Lyα forest (corresponding to the maximum redshift). Then, the following steps have been applied:

1) a grid of wave-numbers is built in the Fourier space, starting from \( k_{\text{min}} = 2\pi/\Delta r \), where \( \Delta r \) is the length of a line of sight in comoving Mpc, and formed by \( n_{\text{pix}}/2 \) evenly spaced elements, where \( n_{\text{pix}} \) is the number of pixels of the original \( \delta \) field;

2) the Fourier transform of the \( \delta \) field is computed;

3) the products \( \delta_k \delta_{k'} \) are averaged in each bin;

4) the \( P_{\delta}^{1D}(k) \) is normalised by multiplying it for the line of sight length, \( \Delta r \);

5) the obtained \( P_{\delta}^{1D}(k) \) is smoothed on larger bins to reduce the noise;

6) the result is averaged over all the lines of sight.

Error bars are computed using a jackknife estimator on the whole sample of simulated lines of sight.

Fig. 14 shows the results of the computation of the corresponding \( \Delta^2(k) = k P_{\delta}^{1D}(k)/2\pi \) for the true and recovered simulated \( \delta \) fields. The two power spectra are consistent at the 3σ level on scales \( 20 \lesssim r \lesssim 60 \) comoving Mpc.
5 FLO APPLIED TO THE OBSERVED DATA SAMPLE

In Section 2.1, we have described how the line lists are compiled for the 22 high resolution QSO spectra forming our sample. The procedure explained in Section 3 is then applied to obtain the corresponding density contrast field for each line of sight. In the case of observations, we have to take into account the presence of the masked intervals covering regions occupied by metal absorption systems. We eliminate all the bins that are covered by more than 30% by masked intervals. Before the normalisation step, we apply an upper threshold as in the case of simulations ($\delta_{\text{thr}} = 50$) since we want to compare our result with the one obtained in Section 4.2. The pixels above the threshold correspond to $\sim 0.9$ percent of the total number of pixels. Figures 10 and 11 show the uncertainties associated with the use of a different fitting tool (in particular, VPFIT and FITLYMAN) and the ones intrinsic to FLO, respectively. Since the intrinsic errors turn out to be larger than those induced by the fitting technique, we can safely compare the results obtained from the simulations and those obtained from the data sample, presented in this section.

Figure 15 shows the average values of the recovered $\delta$ fields for 4 sub-samples built from the 22 observed lines of sight selected on the ground of the QSO emission redshifts. The spread of average $\delta$ values for the single QSOs forming the samples is also shown. There is no significant trend with redshift, as it is expected if the density field follows on average the evolution of the cosmic mean value.

5.1 Two point statistics of the $\delta$ field

The correlation function for the observed $\delta$ field is computed with the formula given in eq. [1] as for simulations.

The result is shown in Fig. 16. Here the value in each bin is obtained averaging all the sample, while the error bars are computed creating 50 samples of 22 lines of sight drawn out of our sample with a bootstrap technique and taking the percentiles of the distribution corresponding to $1\sigma$ errors. The bin size is $\simeq 1.6$ comoving Mpc which is the $L_1$ at the lowest redshift of the sample. The clustering signal is significant at more than the $3\sigma$ level in the first two bins ($r \lesssim 4.5$ comoving Mpc). We have superimposed to the data points the TPCF obtained from the recovered $\delta$ field of simulations. The two correlation functions are in very good agreement, confirming the validity of the cosmological parameters adopted in the simulation.

Since $P_{\delta}^{1D}(k)$ is very sensitive to cosmological parameters, it is very important to check if the prediction of such a function are in agreement with the observed values.

In the case of the observed spectra, the masked metal lines make the starting grid of pixels unevenly spaced, thus not fitted for the application of the FFT. To overcome this problem, as a 1st order approximation, the masked bins have been put to the average density. This procedure is based on the observation that the Ly$\alpha$ forest gas traces on average the average density and on the analogous method adopted in the computation of the power spectrum of the transmitted flux (Viel, Haehnelt, Springel 2004). Fig. 17 shows the result of this computation: the power spectrum obtained from our data is in excellent agreement with the one obtained from the density fields reconstructed with FLO from the simulated spectra based on a concordance cosmological model. Error bars are computed using a jackknife estimator on the whole sample of observed QSOs. The quantity which is generally compared with the model predictions is the 3D power spectrum, which is obtained from the 1D by differentiation, $P_{\delta}^{3D}(k) = -\frac{2\pi}{3} \frac{dP_{\delta}^{1D}(k)}{dk}$. A comparison of the rough estimates of $P_{\delta}^{3D}(k)$ from our observed and simulated data gives good agreement. However, we postpone a careful study of this quantity obtaining constraints on the cosmological parameters to a forthcoming paper.
6 CONCLUSIONS

We have presented results from the analysis of the largest sample of fitted Lyα lines obtained from 22 high resolution QSO spectra covering the redshift range between ~1.7 and 3.5.

In particular, we have computed:

1.a) the line number density evolution with redshift: for which we find \( \frac{dn}{dz} \sim (166 \pm 4)(1+z)^{2.8 \pm 0.2} \). While the redshift evolution is consistent with previous results, the normalisation is higher by a factor ranging from \( \sim 0.03 \) in \( \log(dn/dz) \) at \( z \sim 2 \) to 0.1 at \( z \sim 3 \). This difference is due to the improved treatment of the contamination by metal lines (amounting to \( \sim 9 \) percent of the redshift interval covered by the Ly\( \alpha \) forests), which is made possible by the high resolution and signal-to-noise ratio of our spectra. Consistently with [Kim et al. (2002)], we also find a steeper evolution for the stronger lines (\( 14.5 \leq \log N(H) < 17 \)) compared to the weak ones (\( 13 \leq \log N(H) < 14 \)).

1.b) the two-point correlation function (TPCF): which shows a significant clustering signal up to \( \sim 2 \) comoving Mpc for strong lines (\( 13.8 \leq \log N(H) < 17 \)), and also for weak lines (\( 12 \leq \log N(H) < 13.8 \)) although on smaller scales, \( \lesssim 1.5 \) Mpc. We then calculated the TPCF by grouping all the lines closer than the local Jeans length (the assumed typical size for the hydrogen absorbers in the IGM). The signal is still significant in the first bin (\( r \lesssim 2.5 \) Mpc).

1.c) the TPCF evolution with redshift for strong lines: we divided our sample in two sub-samples; the first one, formed by objects with \( z_{\text{em}} \lesssim 2.5 \), for which the average Ly\( \alpha \) forest redshift is \( \langle z_{\text{ly}\alpha} \rangle = 2.07 \), and the second one, formed by objects with \( z_{\text{em}} > 2.5 \), with \( \langle z_{\text{ly}\alpha} \rangle = 3.02 \). The TPCF computed for lines with \( 13.8 \leq \log N(H) < 17 \) in these two samples, show a trend of increasing clustering with decreasing redshift; this is an apparent evolution, due to the fact that the relation \( \delta - \log(N(H)) \) is z-dependent. Indeed a selection of lines tracing the same kind of structures (characterised by \( \delta > 3 \)) shows no evidence of a significant evolution with redshift of the TPCF.

In the second part of the paper, we have described FLO, a new algorithm to transform the measured \( H \) column densities of the Ly\( \alpha \) lines detected along a line of sight, into the underlying total \( H \) density field (and in particular, the density contrast, \( \delta \equiv n_H/(n_H^\text{eq}) - 1 \), field). The method is based on the assumption that Ly\( \alpha \) absorbers are in local hydrostatic equilibrium and, as a consequence, the Jeans length corresponds to their characteristic size. The aim of this study is to find a robust statistical estimator which allows a direct link to the physical properties of the gas and an easy comparison with the results of simulations. To test the effects of the transformation, we have used a set of 364 lines of sight obtained from a large N-body hydro-dynamical simulation run in a box of \( 120h^{-1} \) Mpc. For every line of sight we have both the density and velocity field pixel per pixel and the list of Voigt fitted Ly\( \alpha \) lines with central redshift, column density and Doppler parameter. Our results can be summarised as follows:

2.a) FLO recovers extremely well (within 30 percent) the over-densities up to \( \delta \sim 30 \) while it is not reproducing correctly the under-densities (more than 50 percent of \( \delta \) values are not recovered) even in the range above our resolution limit. This result suggests that the hypothesis of hydrostatic equilibrium is not valid for the under-dense regions that are likely still expanding. On the other hand, for the goal of our study, that is the computation of statistical properties of the IGM, the resulting \( \delta \) field gives satisfactory results when the two-point correlation function and the 1D power spectrum are considered. The comparison of the results obtained with the true \( \delta \) field of the simulation and with the one reconstructed from line column densities with our algorithm, are in very good agreement.

When applied to the observed data sample, the FLO technique gives the following results:

2.b) the redshift distribution of the average hydrogen density is consistent with the evolution of the cosmic mean hydrogen density in the redshift range covered by our QSO sample, supporting the fact that the Ly\( \alpha \) forest arises from fluctuations of the IGM close to the mean density;

2.c) the correlation function of the density field obtained from the observed spectra shows a significant clustering signal up to \( \sim 4 \) comoving Mpc and is consistent with the analogous result obtained for the recovered density field in a simulation based on the concordance cosmological model.

2.d) the one dimensional power spectrum of the \( \delta \) field obtained from the observed spectra is in very good agreement with the same result obtained from the recovered density field from the simulation based on the concordance cosmological model on scale lengths between \( \sim 2.5 \) and 63 comoving Mpc.

The algorithm presented in this work is particularly useful to extract information from observations in terms of overdensities, making it possible a more direct and handy comparison with simulations.

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