Evidence of Fueling of the 2000 New Economy Bubble by Foreign Capital Inflow:
Implications for the Future of the US Economy and its Stock Market

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Abstract

Previous analyses of a large ensemble of stock markets have demonstrated that a log-periodic power law (LPPL) behavior of the prices constitutes a qualifying signature of speculative bubbles that often land with a crash. We detect such a LPPL signature in the foreign capital inflow during the bubble on the US markets culminating in March 2000. We detect a weak synchronization and lag with the NASDAQ 100 LPPL pattern. We propose to rationalize these observations by the existence of positive feedback loops between market-appreciation / increased-spending / increased-deficit-of-balance-of-payment / larger-foreign-surplus / increased-foreign-capital-inflows and so on. Our analysis suggests that foreign capital inflow have been following rather than causing the bubble.

We then combine a macroeconomic analysis of feedback processes occurring between the economy and the stock market with a technical analysis of more than two hundred years of the DJIA to investigate possible scenarios for the future, three years after the end of the bubble and deep into a bearish regime. We conclude that the low interest rates and depreciating dollar are the indispensable ingredients for a lower sustainable burden of the global US debt structure and for allowing the slow rebuilding of an internationally competitive economy. This will probably be accompanied by a weak stock market on the medium term as the growing Federal deficit is consuming a large part of the foreign surplus dollars and the stock market is remaining a very risky and unattractive investment. Notwithstanding strong surge of liquidity in recent months orchestrated by the Federal Reserve, this macroeconomic analysis which incorporates an element of collective behavior is in line with our recent analyses of the bearish market that started in 2000 in terms of a LPPL “anti-bubble.” We project this LPPL anti-bubble to continue at least for another
year. On the short term, increased availability of liquidity (M1) and self-fulfilling bullish anticipations may hold the stock market for a while.

Key words: Foreign capital inflow, speculative bubble, new economy, dollar depreciation, depression

1 Introduction

According to more than a half-century of academic research, stock market prices move almost like a (slightly biased geometric) random walk [7,41]. Improvements accounting for the presence of heavy tails in the distribution of returns and of volatility persistence have not modified significantly the view in the academic literature that the stock market is generally (weakly) efficient and arbitrages away potential winning investment strategies [15]. A more elaborated understanding of potential gains was shown to be associated with risk: the return-risk trade-off theory holds that there are actually winning investment strategies but they come generally with increasing risks, so that a larger average return is a compensation for a larger risk [42]. This constitutes one of the central insights of modern financial economics. There are two main versions of this return-risk trade-off. In the first one, if a security’s expected price change is positive, it may just be the reward needed to attract investors to hold the asset and bear the associated risks. In the second version, profits earned by industrious investors, who are willing to work hard to gather information and to develop winning strategies, may be viewed as economic rents that accrue to those willing to engage in such activities. This entails that perfectly informationally efficient markets are an impossibility, for if markets are perfectly efficient, the return to gathering information is nil, in which case there would be little reason to trade and markets would eventually collapse [26]. Black proposed that “noise traders”, that is individuals who trade on what they think is information but is in fact merely noise, are the providers of the rents [5]. More generally, at any time there are always investors who trade for reasons other than accruing capital gains—for example, those with unexpected liquidity needs—and these investors are willing to “pay up” for the privilege of executing their trades immediately. In agent-based models, this competition with such “producers” and the speculative investors can be shown to provide
sustainable deviations from perfect market efficiency [63,49]. Recent work in behavioral finance has also made clear that, in order to understand market behaviors, one needs to recognize that humans are governed by a number of non-rational considerations, such as overconfidence, extrapolation from small samples of evidence, loss aversion (reluctance to realize losses), representativeness, mental accounting, relative reference levels and self-esteem issues (reluctance to admit an erroneous investment decision) [46–48]. Professional and private investors on the stock market have not waited for these academic developments to use so-called technical indicators (see for instance [18,4] and http://www.traders.com/). A significant component of the texture of stock market prices may thus be the result of patterns generated by such behavioral biases and by self-fulfilling technical indicators.

The most striking deviations from market efficiency are arguably the speculative bubbles followed by financial crashes. The post-mortem analysis of many financial bubbles (identified as such after their demise) have shown that their development follow essentially the same route. The following scenario underlying financial bubbles and crashes has repeated itself over the centuries and in many different locations since the famous tulip bubble of 1636 in Amsterdam, almost without any alteration in its main global characteristics [19,20,34,51].

(1) This first stage is characterized by positive economic indicators. The bubble starts smoothly with some increasing production and sales (or demand for some commodity), in a relatively optimistic market. Investors form positive expectations for the future and consequently buy the market, pushing it up.

(2) The attraction to investments with good potential gains then leads to increasing investments, possibly with leverage coming from novel sources of funds, often from international investors who are attracted by the potential for diversification and for boosting their revenue. This leads to further price appreciation.

(3) This in turn attracts less sophisticated investors. In addition, leveraging is further developed with small down payment (small margins), which lead to a demand for stock rising faster than the rate at which real money is put in the market. In this regime, foreign investors are even more attracted by the smaller margin requirements and the more flexible investment rules that are often accompanying a strong bullish market.

(4) At this stage, the behavior of the market becomes weakly coupled or practically uncoupled from real wealth (industrial and service) production. It is mostly the expectation of further capital gains which continues to attract investors, hoping to sell even higher what they buy at over-valued prices. The bubble is in regime of self-fulfilling expectations.

(5) As the price skyrockets, some investors start to cash in their gains and

\[\text{Not all these ingredients may be simultaneously present in a given bubble}\]
they and others question the sustainability of the process. As a consequence, the number of new investors entering the speculative market may finally lose its momentum and the market may enter a phase of larger nervousness, until a point when the instability is revealed and the market collapses by the positive feedbacks creating the selling rush.

This scenario applies essentially to all market bubbles ending in crashes, including old ones such as the many bubbles and bursts in the USA during the 19th century, the bubble in the USA in the 1920s ending in the Oct. 1929 crash (the US market was considered to be at that time an interesting “emerging” market with good investment potentialities for national as well as international investors). In the positive sentiment accompanying and being fueled by the developing bubble, explanations, appealing to rational reasoning, are offered to justify the increasing prices. The most famous one is the concept of a “new economy,” which are surfaced many times, including during the bubbles in the 1920s, during the 1960s “tronic” boom and during the recent bubble on the Internet and information technology ending in the crash of March 2000. See [3] for a recent survey of the macroeconomic issues that have developed as a result of the surprising economic performance of the 1990s expansion, including some sense of what is known and not known about accelerated productivity growth, the key driver of the new economy.

How can one go beyond this storytelling? A large literature has addressed the empirical detectability of bubbles (and in particular of “rational expectation bubbles”) in financial data (see [6,1] for surveys of this literature). Empirical research has largely concentrated on testing for exponential trends in the time series of asset prices and foreign exchange rates [13,62]. However, it has been shown that tests for exponential roots are quite unreliable in the presence of the more realistic variant of periodically collapsing rational expectation bubbles [14]. In fact, certain variants of bubble dynamics might look like the outcome of a stationary process [43]. They would, therefore, fool the testing strategy recommended by Hamilton and Whiteman [28] of comparing the stationarity properties of both asset prices and observable fundamentals. In a nutshell, the problem is that it is hard if not impossible to distinguish an exponential explosive bubble from an exponentially growing fundamental price. The following quote from Federal Reserve Chairman A. Greenspan illustrates a now famous view [25]: “We, at the Federal Reserve, recognized that, despite our suspicions, it was very difficult to definitively identify a bubble until after the fact, that is, when its bursting confirmed its existence. Moreover, it was far from obvious that bubbles, even if identified early, could be preempted short of the Central Bank inducing a substantial contraction in economic activity, the very outcome we would be seeking to avoid.”

A different line of attack has been suggested, based on an analogy with so-called critical phenomena in the statistical physics sense of critical phase tran-
sitions (see [53,32,37,31,52,51] and references therein). Two hallmarks of speculative bubbles have been documented: (i) super-exponential power law acceleration of the price towards a “critical” time $t_c$ corresponding to the end of the speculative bubble and (ii) log-periodic modulations accelerating according to a geometric series signaling a discrete hierarchy of time scales. Out-of-sample tests [36] have shown that, over many analyzed world markets in which 49 drawdown outliers were identified, the log-periodic power law (LPPL) precursors are present in 25 cases, qualifying these bubbles as endogenously generated. Restricting to the world market indices, 31 drawdown outliers have been identified of which 19 are preceded with clear LPPL precursors. Systematic non-parametric tests have also confirmed the relevance of LPPL structures for the detection of crashes or severe changes of regimes [66,55].

All these studies have been focused on financial time series taken one at a time, to test whether past market anomalies could be associated with incoming market instability. But, of course, the world is multi-dimensional and especially the financial world is multi-variate: interest rates, implied volatility, volumes, exchange rates, cross-sectional dependencies between equity and commodity time series should in principle all be integrated in a global analysis of speculative bubbles. One step in this direction was performed for the analysis the bearish market trends unraveling since the summer of 2000 [54,67], which showed that the majority of European and Western stock market indices as well as other stock indices exhibit practically the same LPPL “anti-bubble” structure as found for the USA S&P500 index.

Here, we present new evidence in favor of the LPPL theory of speculative bubbles by testing quantitatively one of the predictions of the scenario 1−5 presented above, which underlies financial bubbles and crashes. This prediction concerns the role of foreign investors in providing increasing amounts of funds, therefore driving up the stock market of the target country as the bubble develops. Many researchers have indeed stressed the impact of foreign investments in the fueling of bubbles, preparing the stage of financial instabilities and crashes [19]. Albuquerque et al. have analyzed the unparalleled increase in foreign direct investment to emerging market economies of the last 25 years [2]. Kim and Wei have shown that foreign investors outside Korea before and during the Korean currency crisis were more likely to engage in positive feedback trading strategies and were more likely to engage in herding than the branches/subsidiaries of foreign institutions in Korea or foreign individuals living in Korea [39]. Foreign investors’ strategies on future and option contracts have been found to have a destabilizing role [21]. Dahlquist and Robertsson found a strong link between foreigners’ trading and local mar-

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3 An “anti-bubble” is defined as a self-reinforcing generally severely declining price trajectory with self-similar expanding log-periodic oscillations, which are the symmetric to LPPL bubbles under a time reversal.
ket returns [8]. They also found that foreigners act like non-informed feedback traders as they increase their net holding in firms that have recently performed well. On the other hand, analyzing the effects of the 1987 stock market crash on the London markets, the bursting of the Japanese stock market and real estate market bubble in 1989-1990, the short term and long term consequences of the turbulence in the European exchange rate system in 1992 and 1993, the short term and long term effects of the Mexican devaluation in 1995 for Mexico and other emerging stock markets and the developments in South-East Asia in 1994-1997, Tesar and Werner purport that, while foreign investors at times set a financial market crisis in motion (especially currency crises), they have not been the underlying cause of the crises nor have they exacerbated these crises [56]. Dooley and Walsh survey the large literature on capital flows and discuss several possible origins of induced destabilization [9].

We propose to add on this literature by analyzing the relationship between the stock market in the late 1990s and the net foreign capital inflows in the USA, available from the Federal Reserve’s web site. The source is the U.S. Department of Commerce, Bureau of Economic Analysis. The data correspond to the net flux of US international transactions: investments of foreigners in the US are counted positive and US investments outside are deducted. The net flux corresponds to a given quarter and is seasonally adjusted. The time evolution of the US foreign assets net capital inflow \(I(t)\) per quarter since 1975 is shown in Fig. 1. The net inflow increased over the years, accelerating to very large values before crashing in the first quarter of 2001.

2 Testing for a LPPL signature in the foreign capital inflow

2.1 Log-periodic power law accelerating structure

We follow the methodology explained in our previous papers [52,51,36,66]. Specifically, we use the generalization of the LPPL formula in terms of the truncated second-order Weierstrass-type function [22]

\[
I(t) = A + B x^m + \Re\left(\sum_{n=1}^{2} C_n e^{i\psi_n x^{-s_n}}\right),
\]

where \(x = t_c - t\) with \(t_c\) the end of the bubble and \(\Re(\cdot)\) stands for the real part. The phases \(\psi_1\) and \(\psi_2\) are determined from the fit to the data. The exponent \(s_n\) is defined as \(s_n = m + in\omega\). Its real part is the exponent \(m\) which controls

\footnote{FRED II: http://research.stlouisfed.org/fred2/}
the overall power law dependence of the price trajectory. Its imaginary part \( \omega \) is the angular log-frequency of the log-periodicity. The component \( n = 2 \) allows for the existence of an harmonics.

This second-order Weierstrass-type function (1) proved to outperform our former specification in terms of a first-order LPPL function for modelling of the worldwide 2000-2002 anti-bubbles that started in mid-2000 [54,67]. The special case \( C_n = C/n^{m+0.5} \) recovers the zero-phase Weierstrass-type function, which captures well the five successive crashes within the 2000-2002 anti-bubble in the US S&P 500 index [68] and models well the ongoing UK real estate bubble [69].

We fit the seasonally adjusted quarterly net capital foreign inflow to the USA \( I(t) \) of foreign assets (transactions over a given quarter) from 1975 to the first quarter of 2001. We use the algorithm and the fitting procedure described in [68]. The optimization search was performed for a critical time \( t_c \) in a wide range of \( t_{\text{last}} \pm 1000 \) trading days by replacing \( t_c - t \) by \( |t_c - t| \) in fitting [54], where \( t_{\text{last}} \) is the date of the last point used in the fitting analysis.

Figure 2 shows the fit of the net inflow \( I(t) \) to Eq. (1). The predicted critical time is \( t_c = 2001/03/12 \), which is consistent with the real date of the inflow crash that occurred in the first quarter of 2001. Four well-developed log-periodic oscillations can be observed in Fig. 2. We can also identify their second-order harmonics, which we analyze further below. We have constrained the formula to have a non-negative exponent \( m \), to ensure that \( I(t) \) remains finite at \( t_c \). With this constraint, the fit chooses the lowest \( m = 0.01 \) that we allow. Slightly better but not significantly better fits are obtained by freeing \( m \), which then adjusts to values around \(-0.5\), while \( t_c \) is moved to 2002-2003. A negative exponent would lead to an unrealistic divergence of the flow and we thus reject this family of solution. This appears to be justified by the also unrealistic values of \( t_c \) obtained with negative \( m \) values. Such a small value of the exponent \( m \) means that the power law is equivalent to a logarithmic law \( \sim \ln(t_c - t) \), as proposed in [58–60]. The other parameters of the fit are \( \omega = 4.9, A = 7355, B = -6719, C_1 = 21.5 \) and \( C_2 = 16.2 \). The r.m.s. of the fit residuals is 22.810. Note that the comparable amplitudes of \( C_1 \) and \( C_2 \) imply the existence of a significant second-order harmonic in the log-periodic structures of the net inflow bubble.

We test for the robustness of this fit by studying the impact of inflation on the extracted log-periodic structures. We have deflated \( I(t) \) for inflation by the seasonally adjusted US consumer price index (CPI), also available at the database FRED II. The original monthly CPI were converted to quarterly CPI by averaging the three monthly CPIs in each relevant quarter. Figure 3 shows the time evolution of the deflated US foreign assets net capital inflow \( I(t) \) from 1975 to the first quarter of 2001 and its fit to the second-order Weierstrass-type

\[ C_n = C/n^{m+0.5} \]
function (1). Again, four significant log-periodic oscillations are observed in the figure, as well as their second-harmonic oscillations, which exhibit very similar characteristics as in Fig. 2. The predicted critical time is $t_c = 2001/03/14$, the power-law exponent is $m = 0.01$ constrained by the positivity condition, and the angular log-frequency is $\omega = 5.0$, which is essentially undistinguishable from the value obtained without deflating the price. The linear parameters are $A = 3963$, $B = -3609$, $C_1 = 15.3$ and $C_2 = 12.0$. The r.m.s. residual of the fit is 16.114. We conclude that inflation has a minor impact on the LPPL structure as both $I(t)$ and the deflated $I(t)$ exhibit the same accelerating oscillatory pattern, with the key parameters, $\omega$ and $t_c$, which are very close.

### 2.2 Robustness of the critical time $t_c$

The determination of $t_c$ is particularly important as it gives the estimated termination time of the speculative bubble as well as the most probable time for a crash or a change of regimes. Since the time evolution of a bubble is contaminated with noise and the sparseness of the data close to the critical point, it can be expected that the estimated $t_c$ may be quite sensitive to the length and end-point $t_{\text{last}}$ of the time series used for its determination, as well as to the properties of the noise [50,52,51]. In our previous studies, we often find that $t_c$ increases with $t_{\text{last}}$, which makes the prediction of $t_c$ unreliable. When the determination of $t_c$ is stable, this gives in general a reliable estimation of the end of the bubble. In the present case, we find a remarkable strong robustness of the prediction of $t_c$ when changing $t_{\text{last}}$ both on the un-deflated $I(t)$ and the deflated $I(t)$.

The results of the test on un-deflated $I(t)$ are presented in Table 1 with seven different values of $t_{\text{last}}$. For $t_{\text{last}}$ varying from 1999/10/01 to 2001/01/01, the six fits give almost the same solution with practically unchanging values for the seven parameters: $t_c$, $m$, $\omega$, $A$, $B$, $|C_1|$, and $|C_2|$. The predicted $t_c$ is found consistently in the first quarter of 2001. When data after 1999/07/01 are excluded ( $t_{\text{last}} \leq 1999/07/01$), the solution changes suddenly, with a new family of estimated $t_c = 1999/04/27$ and a much smaller value for the angular log-frequency $\omega = 1.8$. Such low value of $\omega$ can be interpreted as an artificial log-frequency stemming from the most probable noise decorating a power law [29].

Table 1 thus shows that one could have predicted precisely the crash date of the bubble on the net foreign inflow 1.5 year before the crash happened!

Similar results for the deflated $I(t)$ are presented in Table 2 for eight different values of $t_{\text{last}}$. For $t_{\text{last}}$ varying from 1999/07/01 to 2001/01/01, the seven fits give again almost the same solution with very close values of the seven parameters.
Table 1
Testing the robustness of fits with expression (1) of the seasonally adjusted quarterly net capital inflow \( I(t) \) not deflated for inflation. Notice the robustness of the prediction of the critical time \( t_c \) of the crash as well as that of the angular log-frequency \( \omega \) quantifying the log-periodicity.

| \( t_{last} \)       | \( t_c \)    | \( m \) | \( \omega \) | \( A \)  | \( B \)  | \( |C_1| \) | \( |C_2| \) | \( \chi \) |
|---------------------|--------------|--------|-------------|--------|--------|--------|--------|-------|
| 2001/01/01          | 2001/03/12   | 0.01   | 4.9         | 7355   | -6719  | 21.5   | 16.2   | 22.810|
| 2000/10/01          | 2001/02/14   | 0.01   | 4.9         | 7243   | -6616  | 21.5   | 16.4   | 22.824|
| 2000/07/01          | 2001/02/06   | 0.01   | 4.8         | 7267   | -6639  | 21.9   | 16.3   | 22.882|
| 2000/04/01          | 2001/03/04   | 0.01   | 4.9         | 7304   | -6672  | 22.1   | 16.5   | 22.960|
| 2000/01/01          | 2001/02/05   | 0.01   | 4.8         | 7165   | -6545  | 22.0   | 16.3   | 23.056|
| 1999/10/01          | 2001/01/31   | 0.01   | 4.8         | 7161   | -6542  | 22.0   | 16.3   | 23.169|
| 1999/07/01          | 1999/04/27   | 0.01   | 1.8         | 2541   | -2277  | 40.5   | 22.4   | 21.956|

parameters. One can observe that the values of \( t_c \) and \( \omega \) for the deflated \( I(t) \) in Table 2 are almost the same as for the un-deflated \( I(t) \) in Table 1. Notice, however, the predicted \( t_c \) using the un-deflated \( I(t) \) are closer to the actual time of the crash (in the foreign capital inflow), compared with those using the deflated \( I(t) \). This suggests that the un-deflated \( I(t) \) is a better measure for the bubble as herding investors are more sensitive to the actual price than their deflated value. The difference in values of the linear parameters \( A, B, |C_1| \) and \( |C_2| \) in Table 2 and in Table 1 simply reflect an affine transformation between the two fitted second-order Weierstrass-type functions. For \( t_{last} = 1999/04/01 \) and for smaller values, the solution branches to another family with incorrect critical times \( t_c \). These results show a rather amazing prediction power of (1) with respect to the determination of the critical time.

Additional tests on the goodness of the fits and on a non-parametric analysis of the log-periodicity are offered in the Appendix. In particular, we identify a rather strong periodicity in the residuals with a period of exactly one year. Since the formula (1) is describing a scale invariance pattern unfolding as a function of the time \( t_c - t \) towards a critical time, regular periodicities are just additional structures, not relevant here. Our non-parametric analysis of the log-periodicity gives a strong confirmation to our parametric fit (1) and shows that even higher harmonics of the fundamental angular log-periodic frequency \( \omega_1 \approx 5 \) are present in the data. The possible importance of harmonics in order to qualify log-periodicity is made also more credible by recent analyses of log-periodicity in hydrodynamic turbulence data [65,70] which have demonstrated the important role of higher harmonics in the detection of log-periodicity.
Table 2
Testing the robustness of fits and the prediction of critical time on the seasonally adjusted quarterly net capital inflow $I(t)$ deflated by seasonally adjusted consumer price index for inflation.

| $t_{last}$ | $t_c$   | $m$ | $\omega$ | $A$   | $B$   | $|C_1|$ | $|C_2|$ | $\chi$ |
|------------|---------|-----|---------|-------|-------|--------|--------|-------|
| 2001/01/01 | 2001/03/14 | 0.01 | 5.0     | 3963  | -3609 | 15.3   | 12.0   | 16.114|
| 2000/10/01 | 2001/02/25 | 0.01 | 5.0     | 3924  | -3573 | 15.3   | 12.0   | 16.166|
| 2000/07/01 | 2001/02/14 | 0.01 | 4.9     | 3926  | -3576 | 15.5   | 11.9   | 16.231|
| 2000/04/01 | 2001/03/29 | 0.01 | 5.0     | 3945  | -3592 | 15.6   | 12.1   | 16.254|
| 2000/01/01 | 2001/04/12 | 0.01 | 5.1     | 3976  | -3621 | 15.6   | 12.2   | 16.332|
| 1999/10/01 | 2001/03/25 | 0.01 | 5.0     | 3951  | -3598 | 15.7   | 12.2   | 16.412|
| 1999/07/01 | 2001/06/12 | 0.01 | 5.2     | 3960  | -3606 | 15.1   | 12.6   | 16.394|
| 1999/04/01 | 1997/04/23 | 0.01 | 4.1     | 2101  | -1905 | 15.0   | 11.2   | 16.303|

2.3 Comparison between the LPPL patterns in the NASDAQ 100 index and the foreign capital inflow

From the all-time high on the NASDAQ 100 Composite index of 5133 on March 10, 2000 to the low at 3321 on April 14, 2000, the NASDAQ 100 lost over 35%. Johansen and Sornette have shown [33] that this crash was preceded by a speculative bubble passing all LPPL tests, as defined in Refs. [52,51,36]. The analogy with the infamous crash of October 1929 was found to be striking: the belief in what was coined a “New Economy” both in 1929 and in the late 1990s made share-prices of companies with three digits price-earning ratios soar.

Table 3 presents a test of the robustness of the fits with the second-order approximation of the Weierstrass-type function (1) of the NASDAQ 100 index, obtained by varying the time interval of the analysis. We vary the start time of the window of analysis, keeping the ending time fixed at the date of March 10, 2000 (the date the all-time high was reached), as in [33]. The fits and predictions of $t_c$ are very robust with respect to $t_{first}$, confirming the conclusions obtained previously [33].

In view of the joint detection of LPPL patterns in the NASDAQ 100 Composite index and in the net foreign capital inflow to the USA, it is natural to ask whether the two are in any way related. If we compare the three key parameters $m, \omega$ and $t_c$, we find respectively: $m = 0.01 - 0.18, \omega = 7.0 \pm 1$ and $t_c = May 2000$ for the NASDAQ 100 index and $m = 0.01, \omega = 5.0$ and $t_c = March 2001$ for the foreign capital inflow. A smaller exponent $m$ implies a slower initial
Table 3
Tests of the robustness of the LPPL pattern on the bubble that developed on the NASDAQ 100 Composite index in the late 1990s. Each row corresponds to a different start time of the window of analysis, keeping the ending time fixed at the date of March 10, 2000. Each such time series is fitted with the first-order approximation of the Weierstrass-type function.

| $t_{\text{first}}$ | $t_c$    | $m$  | $\omega$ | $\phi$ | $A$   | $B$   | $C$   | $\chi$  |
|-------------------|---------|------|----------|--------|-------|-------|-------|---------|
| 1997/03/10        | 2000/05/25 | 0.18 | 7.41     | 0.08   | 10.50 | -0.93 | 0.02  | 0.0609  |
| 1996/11/29        | 2000/06/17 | 0.01 | 7.89     | 0.29   | 48.88 | -38.11| -0.06 | 0.0603  |
| 1996/08/22        | 2000/06/18 | 0.02 | 7.85     | 3.21   | 34.70 | -23.99| 0.05  | 0.0584  |
| 1996/05/14        | 2000/06/15 | 0.03 | 7.79     | 2.82   | 22.52 | -11.96| 0.05  | 0.0585  |
| 1996/02/05        | 2000/06/04 | 0.08 | 7.49     | 3.89   | 14.39 | -4.21 | -0.04 | 0.0574  |
| 1995/10/27        | 2000/05/17 | 0.15 | 6.95     | 3.36   | 10.99 | -1.34 | 0.02  | 0.0574  |
| 1995/07/19        | 2000/05/08 | 0.18 | 6.68     | 4.73   | 10.31 | -0.86 | -0.02 | 0.0565  |
| 1995/04/10        | 2000/03/24 | 0.33 | 5.55     | 3.35   | 8.98  | -0.18 | -0.01 | 0.0613  |
| 1994/12/30        | 2000/05/06 | 1.00 | 0.11     | 4.01   | 9.09  | -0.10 | -0.10 | 0.0637  |

acceleration finishing more abruptly close to $t_c$, hence suggests that the foreign capital inflow was lagging behind the NASDAQ 100 index with respect to the unfolding of the speculative bubble. The later date for $t_c$ found for the foreign capital inflow suggests a confirmation of this lag.

In order to attempt to capture further the existence of a possible synchronization between the two time series, we compare the pure log-periodic structures within the two time series. For this, we construct the log-periodic residues defined by $[I(t) - A - B(t_c - t)^m]/(t_c - t)^m$ for the two time series, in order to extract what should be pure log-periodic oscillations (if no other pattern and noise were present). We then standardized these residues to have unit variance, which provides us with the standardized residues $R_{\text{Nasdaq}}(t)$ and $R_I(t)$. Fig. 4 shows $R_{\text{Nasdaq}}(t)$ and $R_I(t)$, plotted as a function of their respective $\ln(t_c - t)$ (recall that the two time series have different critical times $t_c$). The thick (resp. thin) line and upper (resp. lower) horizontal scale correspond to the NASDAQ 100 index (resp. foreign capital inflow). We note that the last two log-periodic oscillations in the two time series before the crash seem to exhibit a synchronization, when taking into account a log-periodic lag. Indeed, the local peak in early 1999 in the NASDAQ 100 index residue $R_{\text{Nasdaq}}(t)$ corresponds approximately to the local peak in the foreign capital inflow residue $R_I(t)$ in early 2000. Similarly, the local peak in early 2000 in the NASDAQ 100 index residue $R_{\text{Nasdaq}}(t)$ corresponds to the local peak in the foreign capital inflow residue $R_I(t)$ in mid 2001. Notice the two logarithmic scales in terms of their respective $\ln(t_c - t)$ with different critical times $t_c$ for each time series. This
synchronization with a log-periodic lag suggests that the foreign capital inflow has been following a LPPL trajectory similar to that of the NASDAQ 100 index, indicating a speculative bubble. However, this signal on the foreign capital inflow has been lagging in a self-similar way behind the NASDAQ 100 index bubble. The reference to a log-periodic lag stresses that this lag was not constant in time but has been shrinking as the bubble approached its termination. The synchronization is however only approximate as the angular log-frequencies are different, indicating a transient effect present only at the end of the bubble. The fact that parallel log-periodicities can culminate at different critical times is made possible by the extended description in terms of a hierarchy of singularities offered by the quasi-Weierstrass functions [22].

3  “Egg-and-Chicken” source of the NASDAQ 100 bubble and of the net capital foreign inflow bubble to the USA

The evidence presented in the previous section suggests that foreign investors have been attracted by the capital gains offered by the bubble on the US stock market. They have followed the bubble rather than having been at its origin. The synchronization of the log-periodic patterns of the NASDAQ 100 index and of the foreign capital inflow however indicates that the foreign capital inflow may have amplified the bubble, in line with the standard scenario of a bubble described in the introduction.

We now attempt to elaborate a bigger picture embodying what could be the major elements linking foreign capital inflows and the stock market bubble. We also try to understand the positive feedbacks between the two processes and how they may be linked with other macroeconomic processes.

We start from the premise that the main mechanism behind the development of speculative bubbles is the strengthening of herding and over-confidence among investors, providing positive feedback which further enhances the expectation of future capital gains [19,20,34,51]. In the case of the internet and information bubble that ended in 2000, the flowchart shown in Fig. 5 summarizes the main processes involved and illustrate the cause and effects of the foreign capital inflow.

(i) Box 1 in Fig. 5: It is a well-documented fact that the private disposable income over expenditure has gone from its long-term average surplus of about 3% GDP per year in the early 1990s to a deficit approaching 6% GDP in 2000 [23]. By “private”, we refer to the aggregate of households and corporation.

(ii) By accounting balance, the surplus of private disposable income over expenditure is equal to the government balance (written as deficit) plus the
current balance of payment (income from exports minus expenditure to pay for imports): Private surplus = Government deficit + balance of payment.

(iii) Boxes 2 and 3 in Fig. 5: By (ii), the large private deficit has fostered a growing deficit of the balance of payment and a diminishing deficit of the Government turning to surplus in 2000. The larger spending of the private sector has profited to foreign manufacturers who have been increasingly successful in invading U.S. markets. The strengthening of the US dollar has also helped foreigners exports to the US. The growing deficit of the balance of payment has also resulted from the increased outsourcing of intermediate products.

(iv) Box 4 in Fig. 5: The annual deficit of the balance of payment has been growing and reached $500 billion (2% GDP). This deficit corresponds mechanically to a surplus for foreign exporting nations, which have accumulated trillions of dollars in reserves since the early 1970s. In 2001, the United States’ net debt to the rest of the world jumped to $2.3 trillion (this amounts to about 25% of GDP), a level double that recorded in 1999 [57]. Much of the increase reflects new borrowing undertaken by the country to finance its mounting current account deficit. A third of the change, however, could result from a simple accounting effect, the impact of a rising dollar on the decreasing value of U.S. assets held abroad.

(v) Box 5 in Fig. 5: To avoid pushing up their own currencies in order to keep their competitive hedge and because of the large interest rates enjoyed until 2000 on US treasury bonds, foreign central banks reinvested a significant part of their reserve denominated in US$ back in the US, leading to a large inflow of capital (see Figs.1 and 2). Thus, the spent US$ fueling the deficit of the balance of payment have been coming back as a boomerang in the form of investments by foreign countries [10].

(vi) Boxes 6 and 7 in Fig. 5: However, due to the decreasing deficit of the Federal government, a decreasing quantify of treasury bonds are been issued, letting the stock and real-estate markets (Fannie Mae, corporate bonds, NASDAQ 100) as the main siphoning tanks of the foreign capital inflows.

(vii) Boxes 1 and 8 in Fig. 5: This inflow of foreign capital together with an exuberant herding mood on the part of investors [47] has fueled the bubble in the later part of the 1990s.

(viii) Boxes 9 and 1 in Fig. 5: As a result of the impressive appreciation of the stock market, investors and households have felt richer, spending and consuming wantonly, further fueling the bubble by the feedback of this spending on economic activity, leading to a positive feedback amplification. This is the so-called wealth effect such that an increase in wealth directly causes households to increase their consumption and decrease their saving (see for instance [40] and references therein).

This implies that the 1990s expansion was powered in large part by a large increase in net spending by the private sector, which itself was fueled by the exhuberance and optimism resulting from the feedback of the growing stock
market via the wealth effect on spending. This also suggests a mechanism for the observed synchronization of the foreign capital inflow documented in this paper and the stock market LPPL pattern. First, the LPPL pattern of the foreign capital inflow documented here shows that foreign investors were following a herding behavior similar to that of national investors. Second, the feedback loops between market-appreciation / increased-spending / increased-deficit-of-balance-of-payment / larger-foreign-surplus / increased-foreign-capital-inflows rationalize our finding in Fig. 4 of a synchronization of the LPPL pattern taken as the qualifying signature of a speculative herding bubble.

Another interesting question is the following (we are indebted to J. Spanos for these remarks): why did it take so long (well into 2001) for the foreign capital to stop flowing into the US, while the US markets have crashed almost a year earlier? It seems that foreigners did not acknowledge the coming bear market in stocks until it was well under way. This implies that the foreigners did not cause the crash of the stock market. Their actions simply confirmed it after the fact, with a considerable lag. This confirmation came well before the 2001 attacks on the world trade center on 911. This is a strong evidence in proving that the attacks had nothing to do with the bear market which was already on its way and confirmed by foreign inflows. We also note that both the NASDAQ crash time and the foreign inflows crash time could have been forecasted as early as 1999.

4 Implications for the following years

Let us examine the implications of our findings and reasoning for the next years.

Two main variables have changed since 2000, when compared with the situation in the late 1990s described in the previous section 3.

First, the stock markets have declined substantially since their all-time highs in the first quarter of 2000. The NASDAQ 100 has lost close to 75% of its value at its dip in October 2002, and remains at a loss of close to 70% at the time of writing. The S&P500 has lost about 47% at its dip in October 2002 and remains negative by about 35%. The Dow Jones Industrial Average lost about 38% at its minimum in October 2002 and remains off by about 23%. The wealth effect has thus vanished.

Second, the Federal budget surplus of the last years of the 1990s has transformed again into a growing deficit. The private sector spending fury has abated but the private disposable income over expenditure remains in deficit and several percent of GDP below its long term average [24]. A radical change
of attitude to budget deficits has also occurred, which suddenly became respectable as a way to fight fears of recession [38]. In addition, after 911, private spending of households were encouraged at the highest level of the executive hierarchy as being patriotic.

Our own analysis for the period from 2000 to 2002 strongly suggest that the US stock markets has declined as a consequence of another herding process characteristic of bearish “anti-bubbles” [54,67,68].

The overheating of the speculative bubble led to a crash on the NASDAQ 100 in March-April 2000. As a consequence of vanishing of the wealth effect and of the self-reinforcing negative sentiments, the stock markets have gone down. As a result of the increasing Federal budget deficit, foreign capital have flowed again to buy the debts issued by the treasury, disrupting a part of the flow that was previously directed to the stock markets. As Fig. 1 and our analysis show, the crash on the foreign capital inflow coincide with the transition from surplus to deficit in the Federal budget.

There are several drastically different scenarios, proposed by various analysts and commentators, for the future evolution of the stock market and of the economy. Standard measures of valuations suggest that the market has not yet bottomed as it still appears significantly over-valued; currently, the price-to-earnings ratio is over 34, the dividend yield is 1.74%, and the price-to-book value is over three times. Compare this to the bottom in 1982, for which the price-to-earnings ratio of the S&P 500 was 7, the dividend yield was 6.3%, and the index was selling at book value.

However, there are strong forces willing to push up the confidence of investors, to foster the economy and by the same token the stock market, since the later is a confidence/sentiment thermometer. A first force is the increase in money supply. Since early 2001, the money supply M1, which is basically cash and checking accounts, has been rising at a 30% annual rate, with a deceleration in 2002 and then resuming an acceleration in the first part of 2003, as seen in Fig. 6. The size and growth of money supply is influenced (controlled?) by the Federal Reserve System through its direct control over the reserves of member banks, the discount rate and through open market operations. This increasing money supply, which is supposed to foster economic development, also finds its way in the stock market, because companies are not spending the money to boost capacity, as some industries like semiconductors are working at only 65% capacity utilization, and the overall capacity utilization rates are around 75% (compared to +90% in the late 1990s). Companies are also using this cheap money to re-leverage their balance sheets, similar to consumers switching their credit card debts to different cards with lower rates.

A second force is found in the behavior of foreign capital inflows. With the
growing availability of treasury debts, the enormous surplus of foreign central banks have found again a natural depository, which avoids the risk of inflating their own currency. However, the interests paid have now plummeted from above 6% to slightly above 1% per year. The bonds are attractive only on the basis of a speculative capital appreciation, no more by the paid interests. Therefore, naturally, foreign capital is attracted to the US stock markets. And here comes into play a confidence and herding game. Notwithstanding the stock market over-value, foreign capital (as well as national investors) would like to see the stock market re-appreciate since they have not many other choices to invest their surplus dollars. There is thus a growing availability of capital to hold prices from falling, at least for a while.

This may constitute a part of the explanation for the appreciation of the US stock market since the uncertainties with the war with Iraq in March 2003 fadded out. Will this continue? At this stage, since we view investors confidence and herding as an important and integral part of the self-organization of stock markets and of the economy, it is interesting to dwell more on quantitative measures of confidence. So-called market sentiment ratings are obtained through polls where responses are bullish, bearish or neutral on the market, which are regularly available [30]. During the entire time when the NASDAQ 100 dropped by more than 75% since March 2000, there was not one weekly reading showing more bears than bulls. It took the greatest terrorist act in U.S. history to finally register a week during which there were more bears than bulls, for the first time in 153 weeks. The plurality of bears over bulls was minuscule considering the magnitude of the events. The 911 catastrophe could only produce three consecutive weeks where bears outnumbered bulls, after which the bulls dominated the sentiment readings again. The study reported in Barron’s study on May 5th, 2003 gave the following categories and responses: Very Bullish (9%); Bullish (51.1%); Neutral (28.6%); Bearish (10.5%); and Very Bearish (0.8%). This is not the type of sentiment (60.1% bulls versus 11.3% bears) after three years of a nasty bear market! This overwhelming reading, already impressive in a bull market, is unheard of for a bear market.

Our reading of this surprising pervasive bullish sentiment is that it confirms that the private and foreign investors want the market to go up, but that there is so much uncertainty that “wishing” is different from “acting,” that is, investing. Investors are waiting for signs of confirmation of their bullish sentiment to drive the price up. They have already be burned severely by the crash in 2000 and the two years that followed. In particular, foreign investors have strong incentive to buy the US stock market as well as corporate bonds and the debts in US real estate market [69] in order to get a return on their surplus dollars above the ridiculous discount rate offered on treasury bonds. But such action would be warranted only if the market risk is not too high, hence the conflicted observations of a strong bullish sentiment in a depreciating stock market. The contradictory conclusions on the economic outlook and
these polls suggest that the natural herding behavior of investors will be even more predominant in the future and lead to highly volatile and unstable market behavior in the near future.

As the US stock market and economy were heating up in the late 1990s, the higher interest rates and stronger dollar were the natural instruments to attempt to avoid inflation and to try to stabilize growth but also resulted from the economic and stock market growth [61]. Actually, as we explained above, both led to the rather perverse effect of fueling further the bubble by (1) increasing the deficit of the balance of payment through the deterioration of competitiveness accompanying a strong dollar and (2) by the attractiveness of investing in the US for foreign capital in part obtained through the surplus of foreign countries on their balance of payment.

Now, the situation is different. The economy has been flirting with stagnation and depression several times in the last two years and the stock markets have been falling down; hence, the massive cuts in interest rates. Since May 2002, the strong dollar has been steadily losing ground against the major currencies, with an acceleration of this loss since November 2002, showing a cumulative loss of about 28%. As a consequence, investments in the US by foreigners is becoming less attractive due to the increasing exchange risk and the lowest interest payment.

Weighting these different ingredients, our prefered scenario for the future is the following.

- The private sector will continue spending more than its long-term average, as it is psychologically difficult to abandon habit acquired in good times (the glorious 1990s) and it is in addition almost considered as a patriotic act.
- The debt of the Federal Government as well as the private, municipal, corporate and local government debts will continue to rise, reinforcing further the US as the major deficit nation.
- As a consequence, interests will remain low to allow servicing of the payment of the interests of the debts, both of private sector and the government. This will continue to have the effect of further fueling the growth of liquidity by the mechanism of fostering loans refinancing on lower interest rates (mostly from residential real estate).

This consequence will also continue to be a source of the two first bullets, acting as a positive feedback loop. The central bank of the US is now compelled to peg short-term interest rates, promise to forewarn the marketplace of any intention to adjust the peg, and to guarantee continuous marketplace liquidity. Federal Reserve operations will continue to work by aggressively manipulating rates, yield spreads (by repurchasing long-term debts) and, increasingly, market perceptions to ensure these goals. This is
further reinforced by the perception that the proposed tax cuts and current low interest rate environment will further increase liquidity and turn the US economy around which will power the stock market even further ahead.

• The dollar will continue its descent as a mechanism to fight against the deficit of the current balance by boosting exports (which translate into cheaper imports for foreigners). A decrease of the dollar also provides a mechanical device to decrease the absolute value of the debt. This will accentuate the incentive for foreign central banks to sell progressively their dollars, but they cannot do it too fast to avoid losing the competitiveness of their currencies. There is thus a subtle balance between the economic competition giving rise to surplus in dollars, the corresponding importance of not having a strong currency and the present lack of attractiveness of the dollar. We thus envision a slow sell out of the US dollar, but only on a limited scale since the world is overflowed by dollars, which has replaced gold at the international reserve currency, since the breakdown of Bretton Woods (about 80% of the world’s free capital is invested in dollars, even though the US makes up only about 30% of the world’s economy).

• In view of these negative factors, foreign capital will be less attracted to the stock market.

From the point of view of detecting large scale cooperative behavior, Fig. 7 presents a long term view of the evolution of the US market proxied by the DJIA and its extrapolation in the past as explained in [35]. The continuous line shows the fit with our LPPL formula, a second-order Landau expansion described in [35,51], which is an extension of the formula (1) to describe such long time intervals. Fig. 8 shows the residual or difference between the realized DJIA and the fitted formula. The previous value of the residuals and the characteristic time scales before recovery suggest that one may wait for a year or two before the stock market recovers its long-term trend. This appears to be in line with the prediction that the US stock market will bottom during or at the end of the first semester of 2004, according to our previous analysis of the LPPL anti-bubble which started in 2000 [54].

The residuals plotted in Figure 8 show that the current bear market has been almost as severe as all other bear markets since 1950 but far less severe than the 1932 bear market. One scenario is that the market may have in fact bottomed out in October of 2002. However, if this is a deflationary bear market as the US was in 1932, then a huge decline is still ahead. The interest rate data so far point to a “deflationary” environment but this is far from conclusive as the Federal Reserve is currently waging war with deflation and is prepared to drive short term rates to zero to avoid this scenario. But doing so, it is fueling the credit bubble to unprecedented levels, developing another dimension of instability.

A very interesting additional information is provided by the behavior of the
main currencies against the US dollar. We have found unmistakable LPPL signatures of a speculative bubble which is presently developing on the EURO. Specifically, Figure 9 shows the EURO in US$ and a typical accelerating LPPL bubble pattern, which is suggestive of a speculative herding buying of EURO’s using US$. Figure 10 shows the EURO in Yen and again a significant accelerating LPPL pattern. In contrast, the Yen in US$ does not have any acceleration (nor has the US$ in Yen), even if a marginally significant log-periodicity may be observed, as shown in Figure 11. These three figures provide a remarkable message: the depreciation of the US$ is not just the undirected flight-for-safety of a herd fleeing from a looming catastrophe; it seems to be associated with a speculative bubble directed to what is felt (at least on the short- and medium-term) to be the new haven currency, the EURO.

5 Conclusion

Our main conclusions are the following.

(1) The “sacrifice” of the US$ and the stock market is the cost for a lower sustainable debt burden on the global US debt structure and for allowing the slow rebuilding of an internationally competitive economy. This is reinforced by the evidence for a speculative bubble developing on the EURO. We thus envision a continuation of the depreciation of the US$ that may reach unprecedented low levels.

(2) On the medium term, the stock market is not going to recover a strong bullish trend as the growing Federal deficit is consuming a large part of the foreign surplus dollars and the stock market is remaining a very risky and unattractive investment. In addition, the huge credit bubble, that the US has developed in the last decade and is increasingly fueled in the hope “to avoid deflation,” may be expected to burst and have severe consequences for the recovery of the economy.

(3) On the short term, the stock market may hold for a while as one of the main sink of a strong surge of liquidity and of the credit bubble, justified in the mind of investors by their sentiment and hopes.

The US has been growing as the major deficit nation in the world, attracting huge amounts of foreign capital. In parallel, it is also growing steadily by immigration, powered by a variety of factors. Studying the relationship between immigration and capital flows, Groznik has shown that, surprisingly, labor not only moves in the same direction as capital, but it also leads capital [27]. This finding is also found for various countries, periods and migration flow specifications. Thus, an important predictive variable for international capital flows is immigration flows. One strength of the USA has been its ability to attract people and capital. To what degree this inflow will continue to justify its un-
sustainable deficits (at all levels) is linked with its potential for development of new riches and remain to be seen.

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6 Appendix: goodness of fit and non-parametric analysis of log-periodicity

6.1 Testing the goodness of the fits

We present two tests assessing the goodness-of-fit of the seasonally adjusted quarterly US net capital inflow $I(t)$ of foreign assets with formula (1). They are based on the analysis of the fit residuals with the autocorrelation method [17] and the sign test [44]. Based on a rigorous mathematical proof [16], these two tests are complementary in their detection of possible remaining patterns in the residuals. The autocorrelation method is often considered to provide an estimate of the model error. The sign test is based on an intuition that the changes of sign of the residuals should be random for a good fit.

Assume that $\{y_i : i = 1, \cdots, n\}$ are $n$ observations of the true values $\phi_i$ at $n$ evenly spaced successive times $t_i$ and $f(t; \Theta)$ is a proposed model with $p$ parameters $\Theta$. If $\hat{\Theta}$ is an estimate of $\Theta$, then the residuals of the fit are

$$e_i = y_i - f(t_i; \hat{\Theta}) .$$

The one-lag covariance of the residuals is then defined

$$a = \frac{1}{n - 1} \sum_{i=1}^{n-1} e_i \times e_{i+1} ,$$

where the mean of $e_i$ is zero (this has been verified to be true in our residuals with extremely good accuracy $\sim 10^{-11}$). The unbiased estimate of the variance of the residuals is computed as

$$s^2 = \frac{1}{n - p} \sum_{i=1}^{n} e_i^2 .$$
By assuming that (1) the fit is made using least-squares, (2) the measurement errors \( \epsilon_i = y_i - \phi_i \) are centered random variables with the same variance, and (3) the data are uncorrelated, one can deduce the first order estimates for the model error \( \sigma_\delta \) or model bias, defined as the standard deviation of \( \phi_i - f(t_i; \hat{\Theta}) \), and the measurement error \( \sigma \), defined as the standard deviation of \( \epsilon_i \):

\[
\sigma^2 \equiv \text{Var}[y_i - \phi_i] \approx (1 - \frac{p}{n})s^2 - a , \\
\sigma^2_\delta \equiv \text{Var}[\phi_i - f(t_i; \hat{\Theta})] \approx \frac{4}{n}s^2 + (1 - \frac{p}{n})a .
\]

For \( p/n = 0 \), Eq. (5) recovers the standard zero order estimations. The key statistics of the sign test is the frequency \( f \) of changes of signs of the residuals \( e(t_i) \) given by [16]

\[
f \approx \frac{1}{2} - \frac{a}{\pi s^2} .
\]

The residuals of our fit are shown in Fig. 12. We have \( p = 5 \) free parameters since the four linear parameters are slaved [68]. We find \( a = -86 \) and \( s^2 = 546 \), which gives a one-lag correlation coefficient \( a/s^2 = -0.16 \). This yields \( \sigma^2 = 602, \sigma^2_\delta = -56 \). The negative value of \( \sigma^2_\delta \) reflects some anti-persistence in the residuals. The frequency of changes of signs given by (6) is \( f = 0.550 \), which is not too far from the value 0.5 for completely random residuals. We cannot however negate the existence of a significant residual structure, whose prominent characteristic is a strong yearly periodicity, as shown from the spectrum of the residuals presented Fig. 13.

### 6.2 Generalized \( q \)-analysis of the log-periodic structure

We check the the significance level of the extracted log-periodic pattern using a non-parametric analysis, called the \((H,q)\)-analysis which we have already explained and used in our previous works [64,66]. It consists in a generalization of the \( q \)-analysis [11,12]. The \((H,q)\)-derivative is defined by the following formula

\[
D_q^H f(x) \triangleq \frac{f(x) - f(qx)}{(x - qx)^H} ,
\]

such that \( D_q^{H=1} f(x) \) recovers the standard \( q \)-derivative \( D_q f(x) \). For a power law function \( f(x) = B x^m \), \( D_q^{H=m}[B x^m] = B(1 - q^m)/(1 - q)^m \) is constant. In a nutshell, the \((H,q)\)-analysis performs a kind of fractal derivative which is particularly sensitive to the presence of log-periodicity. The index \( q \) refers to the discrete scaling ratio used in the definition of the fractal \( q \)-derivative.
and $H$ is an exponent used to rescale the $q$-derivative. Scanning $q$ and $H$ provides an important test of the robustness of the log-periodic structure. A Lomb periodogram analysis [45] of the $(H,q)$-derivative allows one to detect the presence of log-periodicity and assess its significance level.

The analysis was performed on a $19 \times 19$ grid with $H$ ranging from $-0.9$ to $0.9$ evenly and $q$ varying from $0.05$ to $0.95$ evenly. Each node $(i,j)$ in the grid corresponds to a pair $(H(i), q(j))$, which is converted to a single number through the one-to-one map: $(i,j) \rightarrow 19 \times (i - 1) + j$. Figure 14 uses this number as the abscissa to show the corresponding angular log-frequency $\omega$ of the most significant Lomb peak in each Lomb periodogram for each of the scanned $(H,q)$ pairs of the $(H,q)$-derivative of the net foreign capital inflow $I(t)$ into the USA. Each cross corresponds to a pair of $(H,q)$.

The majority of the $\omega$’s cluster around $\omega_1 = 5$, which is consistent with the fundamental log-frequency extracted from the fit with equation (1). Furthermore, one can observe clusters of $\omega$’s at integer multiples $\omega_2 = 10$ and $\omega_3 = 15$ of this fundamental angular log-frequency. Some higher-order multiples $\omega_5 \approx 25$, $\omega_7 \approx 35$ and $\omega_{10} \approx 50$ (not shown) are also clearly visible. The deviations from the exact integer multiples are mostly due to small $q$’s which are most sensitive to finite-size effects.

The $(H,q)$-analysis confirms the presence of a very significant log-periodic structure in the bubble of the US net capital inflow of foreign assets.

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Fig. 1. Time evolution of the foreign net capital inflow $I(t)$ to the USA per quarter since 1975. The unit of $I(t)$ is in billion of US dollars while time $t$ is in calendar year.
Fig. 2. Fit of the time evolution of the foreign net capital inflow $I(t)$ in the USA from 1975 till the first quarter of 2001 when it reached its maximum, by a second-order Weierstrass-type function given by expression (1). The predicted critical time is \( t_c = 2001/03/12 \), the power-law exponent is \( m = 0.01 \), and the angular log-frequency is \( \omega = 4.9 \). The fitted linear parameters are \( A = 7355 \), \( B = -6719 \), \( C_1 = 21.5 \) and \( C_2 = 16.2 \). The r.m.s. of the residuals of the fit is 22.810.
Fig. 3. Same as Fig. 2 for the deflated foreign net capital inflow in the USA. The predicted critical time is $t_c = 2001/03/14$, the power-law exponent is $m = 0.01$, and the angular log-frequency is $\omega = 5.0$. The fitted linear parameters are $A = 3963$, $B = -3609$, $C_1 = 15.3$ and $C_2 = 12.0$. The r.m.s. of the residuals of the fit is 16.114.
Fig. 4. This figure shows the two log-periodic residues $R_{\text{NASDAQ}}(t)$ and $R_I(t)$ defined as $R(t) \equiv [I(t) - A - B(t_c - t)^m]/(t_c - t)^m$ for the the NASDAQ 100 index and for the foreign capital inflow. This construction extracts the log-periodic components of the two time series, while getting rid of their constant level, of their trend and power law acceleration. These two log-periodic residues are plotted as a function of their respective $\ln(t_c - t)$ (recall that the two time series have different critical times $t_c$). The thick (resp. thin) line and upper (resp. lower) horizontal scale correspond to the NASDAQ 100 index (resp. foreign capital inflow).
Fig. 5. Flowchart of the feedback loops between market-appreciation / increased-spending / increased-deficit-of-balance-of-payment / larger-foreign-surplus / increased-foreign-capital-inflows, which are proposed to explain the speculative bubble in the US stock markets and to rationalize the synchronization of the LPPL patterns shown in Fig. 4. According to this scenario, the 1990s expansion in the US was powered in large part by a large increase in net spending by the private sector, which itself was fueled by the exhuberance and optimism resulting from the feedback of the growing stock market via the wealth effect on spending.
Fig. 6. Growth of the money supply M1, which is basically cash and checking accounts. The outlying peak is the response to the 911 attack.
Fig. 7. Long term view of the evolution of the US market proxied by the DJIA and its extrapolation in the past as explained in [35]. The continuous line shows the fit with our LPPL formula extended to describe such long time intervals. See [35] and Chapter 10 of [51] for details.
Fig. 8. Residual or difference between the realized DJIA and the fitted formula shown in Fig. 7. The previous behaviors of the residuals and the characteristic time scales before recovery suggest that one may wait for a year or two before the stock market recovers its long-term trend.
Fig. 9. Time evolution of the EURO in US$ (thin fluctuated line) and its LPPL fit (thick smooth line). The fitted critical time is $t_c = 2003/10/30$ with a power-law exponent $m = 0.13$ and angular log-frequency $\omega = 12.0$. The r.m.s. of the fit residuals is $\chi = 0.0125$. The log-periodicity is highly significant with almost four oscillations.
Fig. 10. Time evolution of the EURO in Japanese Yen (thin fluctuated line) and its LPPL fit (thick smooth line). The fitted critical time is $t_c = 2003/09/04$ with a power-law exponent $m = 0.09$ and angular log-frequency $\omega = 7.3$. The r.m.s. of the fit residuals is $\chi = 1.635$. 
Fig. 11. Time evolution of the Yen in US$ (thin fluctuated line). The thick line is its fit with the LPPL formula (1) with $n = 1$ (a similar result is obtained with $n = 2$). The parameters of the fit are $t_c = 2003/10/30$, $m = 1.74$ and angular log-frequency $\omega = 11.8$. The r.m.s. of the fit residuals is $\chi = 9.78 \times 10^{-5}$. The value larger than 1 of the exponent $m$ implies the absence of any acceleration of the appreciation of the Yen in US$.
Fig. 12. Time evolution of the residuals $e(t)$ of the fit shown in Fig. 2. The one-lag correlation coefficient is $-0.16$ showing anti-persistence. The frequency of change of signs of the residuals is $f = 0.55$. 
Fig. 13. Spectrum of the residuals $e(t)$ shown in Fig. 12. The very significant peak corresponds to a periodicity of one year.
Fig. 14. Angular log-frequency $\omega$ of the most significant Lomb peak in each Lomb periodogram in the $(H, q)$-analysis of $I(t)$. The analysis was performed on a $19 \times 19$ grid with $H$ ranging from -0.9 to 0.9 evenly and $q$ varying from 0.05 to 0.95 evenly. Each node $(i, j)$ in the grid corresponds to a pair $(H(i), q(j))$, which is converted to a number in this plot through a one-to-one map: $(i, j) \rightarrow 19 \times (i-1) + j$. Each cross corresponds to a pair of $(H, q)$. Note that $W(H(7), q(19)) = 50.4$ is not plotted in the figure for the sake of better presentation.