Spatial point pattern analysis of the local current distribution on the surface of multi-tip field emitters

S V Filippov¹, S Carapezzi²,³, E O Popov¹, A G Kolosko¹
¹Ioffe Institute, ul. Polytechnitscheskaya 26, St.-Petersburg, 194021, Russia
²Department of Physics and Astronomy, University of Bologna, Viale Berti Pichat 6/2, 40127 Bologna, Italy
³ARCES-DEI, University of Bologna, Viale Risorgimento 2, 40136 Bologna, Italy.

Abstract. A further development of the method for obtaining local characteristics of multi-tip field emitter using spatial point pattern analysis was presented. The field emission characteristics of nanocomposite emitter single-walled carbon nanotubes / polystyrene were calculated. Two sets of emission sites with different current load distribution were discovered. The application of the of spatial point pattern analysis allow to determine the spatial distribution of emission sites over the cathode surface. The results of the analysis confirm the cluster character of the emitting sites distribution on the cathode surface.

1. Introduction
The development of effective field emitters is one of the trends in the modern vacuum nanoelectronics. Now they have found application as electron sources in computer tomography, X-ray tubes, compact mass spectrometers and other devices [1-3]. The most promising emitters are microscopic structures with a distributed current load - arrays of conducting tips created using various technological methods. The main disadvantage of such structures is the thermal instability of individual emission sites on their surface, which undergo an excessive current load during the field emission and explode in an unpredictable manner, creating a vacuum discharge in the interelectrode gap. Therefore, an optimized distribution of the current load over the emission sites is one of the main technological issues for creating efficient cathodes.

The solution of this problem is to control the geometric parameters of the tips (radius, height and relative spatial arrangement) using the following techniques: scanning electron microscopy, scanning anode field emission microscopy, field emission scanning microscopy. However, all these techniques are capable of providing information on a limited portion of the cathode surface, while modern field cathodes have an area in square centimeters. In this latter case, because the analysis is required to be carried out already for the entire cathode with thousands of tips as a whole, it is preferable to use the IMLS (Integrated Measurement System with Luminescent Screen) method with Field Emission Projector (FEP).

In previous works [4-6], we reported about a method for estimating microscopic emission characteristics based on online processing of macroscopic electrical signals and corresponding field emission images obtained via IMLS. In [4] the correlation of the total brightness level of the luminescence pattern with the emission current level was shown. The approach presented in [5] made it possible to construct histograms of the brightness distribution of the emission sites and of the current load density of the surface regions. These experimental distributions were numerically compared with
the ones of the simulated "ideal emitter". A histogram of the current load distribution for emission sites for a multi-tip emitter based on multiwall carbon nanotubes / polystyrene was presented in [6].

In the present paper, we investigated the current load distribution of a single wall carbon nanotubes / polystyrene (SWCNT / PS) based emitter. Additionally, we studied the spatial maps of emission sites obtained by the IMLS method by means of the spatial point pattern (SPP) analysis.

Spatial analysis is a statistical technique generally used to examine spatial entities. In case of point patterns (PPs), it is used to explore their degree of spatial clustering or ordering or randomness. It is well known that the geometry of the ensemble of individual emitters has the influence on the electron emission efficiency in large area field emitters. Then, the use of SPP analysis to study the spatial relationships between emission sites can be helpful to fully grasp their overall emission properties. SPP analysis was already used in [7] to study the spatial properties of ensembles of nanoscaled field emitters.

2. Experimental

The cathode was a metal tablet 10 mm in diameter with a SWCNT / PS nanocomposite film deposited on it. The diameter of the nanotubes was 2-4 nm, and the length was up to 10 µm. To measure field emission properties, the computerized FEP described in [4] was used. A phosphor glass screen covered with tin oxide was used as an anode. The distance between the cathode and the anode was 350 µm. The residual pressure in the projector's chamber was no more than 3·10⁻⁵ Pa.

To obtain current-voltage characteristics (IVCs), a high-voltage power supply with a pulse frequency of 50 Hz was used. The pulses of a half-sine wave voltage of 10 ms duration were applied to the cathode. The amplitude of the pulses was kept constant for each of the selected levels of emission current.

3. Discussion.

The sample was examined at two levels of the emission currents: 320 µA and 530 µA (see figure 1a).

The obtained IVCs (see figure 1b) were processed using the Elinson equation (1) [8].

$$I = A_e 1,4 \cdot 10^{-6} \varphi^{-1} \exp(10,17 / \sqrt{\varphi}) F^2 \exp(-6,49 \cdot 10^9 \varphi^{3/2} / F)$$

where $A_e$ - the effective emission area, $F=\gamma U/d_{sep}$ - the electric field strength, and $\varphi$ - the work function of the emitter, which here was assumed to be of 4,6 eV, $d_{sep}=350$ µm, $\gamma$ – the effective field enhancement factor (FEF). The corresponding plots in the Fowler-Nordheim coordinates are shown in the inset figure 1 b.

![Figure 1 (a, b). Emission current levels during the experiment (a). Corresponding current-voltage curves (b) and Fowler-Nordheim plots present on the inset.](image-url)
1710 and 1490 for current levels of 320 µA and 530 µA, respectively. The decreasing of γ with the increasing of the current level is due to the inclusion of new lower emission sites.

We assumed that the brightness level of each registered emission site $Y_{iloc}^m$ on the luminescence image was proportional to the corresponding local emission current $I_i$ at each instant of time. Thus, the current load of the sites could be calculated using the proportionality factor $C$, which is calculated from the total current $I$ and the total brightness $\sum Y_{iloc}^m$. To find $C$, the values $I$ and $\sum Y_{iloc}^m$ recorded at the beginning of the experiment was used. Using the maximum brightness values of the sites $Y_i^m$, the distributions of the emitting tips were constructed for their optimum brightness and the optimum current load (see figure 2 a, b):

$$Y_i^m = \max(Y_{iloc}^m)$$

$$I_i = Y_i^m C = Y_i^m \left( \frac{I}{\sum Y_{iloc}^m} \right)_{t=0}$$

The plotted histograms represent for two current levels the emission activity of all the tips without the influence of adsorbates. Two clearly distinguished maxima on each histogram indicate the presence of at least two sets of emission sites with different field enhancement factor distribution.

On the insets of figure 2, the registered glow patterns on the luminescent phosphor screen of FEP are shown, which exhibit instantaneous brightness distribution of active emission sites over the emitter surface. The point diagrams of the optimum site distribution over the surface, corresponding to the state of the emitter "without adsorbates", are also shown.

![Figure 2 (a, b). Histograms of current load of emission sites for 320 µA (a) and 530 µA (b) current levels. The corresponding spatial distribution and registered brightness maxima of emission sites are shown on the insets.](image)

Point diagrams of the optimal site distribution were subjected to SPP analysis. An analysis of the emission site arrangement was carried out using the pair-correlation function $g(r)$ [9, 10]. The behavior of the function $g(r)$ allows to identify the spatial structure of the emission sites: randomly located, clustered or hyper-dispersed. By definition of the pair correlation function, $g(r)=1$ in the case of complete spatial randomness, for which the site distribution is uniform and independent from each other. The value $g(r)>1$ could be indicative of the tendency of the sites to cluster. On the contrary, $g(r)<1$ could imply a relatively regular placement of sites.

Figure 3a, b shows the $g(r)$ function of the PPs of the emission sites for two different current levels, 320 µA and 530 µA. In both cases, the experimental $g(r)$ is greater than 1 in the smaller $r$ range ($r<0.028$, for figure 3a and $r<0.045$ for figure 3b), and such departure is statistically significant (the probability that this happens in case of a random PP is less than 2%). This is indicative of clustering of the PPs in
these ranges, where the order of magnitude of the cluster size is given by the upper value of $r$ such that $g(r)>1$ and $g(r)$ lies outside the envelopes ($r=0.028$, correspondent to 280 µm, for PP of current level 320 µA, and $r=0.045$, correspondent to 450 µm, for PP of current level 530 µA). The first maximum of $g(r)$ is located at $r\approx 0.01$ for the PP of current level 320 µA, and at $r\approx 0.016$ for the PP of current level 530 µA. Such $r$ values represent the most frequent distance between emission centers in each PP.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Plots of the computed $g$-function for all emission sites at 320 µA (a) and 530 µA (b) current levels (short dotted black line). The short dashed red line represents the complete spatial randomness value of $g(r)$. The solid blue line are the upper and lower bounds of the generated simulation envelopes.}
\end{figure}

Since the experimental histograms of the current load had two clearly distinguished peaks, we divided the emission sites into two sets. For a current of 320 µA, set 1 – corresponded to emission sites with $I_i>1.83$ µA and set 2 – to emission sites with $I_i<1.83$ µA. For a current of 530 µA, the separating boundary was set at around 2.35 µA. The corresponding $g(r)$ functions are shown in figure 4.

It is apparent that the PPs associated to these two sets possess different features, as reflected from the diversity of the behavior of their $g$ functions. For both current levels, the PP of set 2 is clustered just in the smaller $r$ range, this range being $r<0.02$ (200 µm) for current level of 320 µA, and $r<0.03$ (300 µm) for current level of 530 µA. For the PP of set 1 $g(r)>1$ in a statistical significant way for all $r$ values, that is the clustering is present at all the distances between the emission sites. However, in the range of clustering of set 2 the associated $g$ function has mostly larger values than the $g$ function of set 1. That is, the clustering is more evident for set 2 on this smaller $r$ range. Finally, the $g$ function has a first maximum only for the set 1, located approximately at $r=0.014$, for which the same interpretation applies that was given for the $g$-function of the whole PP.
Figure 4 (a, b, c, d). Plots of the computed g-function for two sets of emission sites for 320 µA (a, c) and 530 µA (b, d) current levels (short dotted black line). The short dashed red line represents the CSR value of $g(r)$. The solid blue line are the upper and lower bounds of the generated simulation envelopes.

4. Conclusion

Emission characteristics of the SWCNT / PS nanocomposite emitter were studied. The values of the threshold field and the effective field enhancement factor for two levels of emission current were determined. SPP analysis of glow patterns obtained using FEP was carried out. The results showed a tendency of the emission sites to cluster over the cathode surface, with order of magnitude of the clusters being 280 µm for the current level of 320 µA, and slightly larger - 450 µm for the current level of 530 µA. The obtained histograms of the distribution of emission sites over the current load showed the presence of two sets of emission sites with different field enhancement factor distribution. The SPP analysis applied to these two sets showed that they possessed distinctive features. The sites with high current load (set 1) showed indication of clustering at all the distances between the emission sites. For the emission sites with low current load (set 2) the clustering was present only on smaller $r$ range, the
range being dependent on the current level, but in this range the degree of clustering was higher than for the sites with high current load.

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