Procedure of experimental evaluation of nanoclass spacecraft
design parameters using the ground test equipment

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Abstract. As it is impossible to set high accuracy of the initial data for theoretical calculations of the design parameters, such as the mass-centering and inertial characteristics of nanosatellites, the problem of experimental determination of their actual values arises. For a number of reasons, the devices designed for large spacecraft are not appropriate for the small ones. This paper describes a device developed at Samara University for measuring the center of mass coordinates and moments of inertia specifically for CubeSat nanosatellites, as well as a technology for experimental evaluation of the design parameters of nanoclass spacecraft. The accuracy of determining the design parameters using the proposed device is confirmed by a series of experiments with standards.

1. Introduction
It is a very critical issue for design, dynamic analysis and effective control of mechanical systems, such as spacecraft, to know the design parameters, such as mass-centering and inertial properties. Accurate estimate of design parameters is even more important for nanoclass spacecraft, because of its small mass and dimensions.

The commonly used inertial properties include 9 parameters, namely center of mass coordinates (CM), the six independent components of the inertia tensor (the three moments of inertia (MOI) and the three products of inertia (POI)). In fact, inertial properties determine the dynamic behavior of the mechanical systems. Thus, accurate determination of inertial properties of nanoclass spacecraft, especially for controlled ones, has a crucial importance.

The approaches for determination the CM and inertia tensor of nanosatellite can be divided into several categories:

(1) numerical calculation method based on Computer Aided Design (CAD) models [1,2],
(2) experimental approach based on angular acceleration methods or oscillation methods [3,4],
(3) parameter identification methods based on vibration tests [5], or based on motion equations of an unconstrained object [6–8].

The first method based on CAD model is used at the design stage in order to estimate inertial properties. However, it is almost impossible to take into account the exact properties of all parts of nanosatellite. Sometimes small variations may result in significant changes in dynamics characteristics.

In the third method too many parameters are needed to be identified, they are usually complicate and need expensive hardware, furthermore, the error analysis is difficult. Moreover, some of these methods, such as [6,7] can be used only after nanosatellite has been launched into the orbit, but not before.

Therefore, there is a need to use an experimental approach. Acceleration methods include observation of dynamic response when the object is subjected by external force. The most common are three
approaches: rolling, falling weight, and running down [3,4]. In all these approaches there is significant influence of friction, drag, and nonlinearities, which affect inertia estimates a lot. Moreover, these methods estimate only moment of inertia about single axis, and not the product of inertia.

In oscillation methods the inertia is determined from the resulting period of oscillation, when the external periodic torque is applied to the system [3,4]. The oscillation methods are less affected by damping than acceleration methods; but, for some setups, the dynamics requires linearization, which introduces errors into the inertia estimates.

There are three most common oscillation methods: torsional, multifilar, and compound pendulums [9]. The torsional pendulum can be constructed by suspending the body by a single torsional spring, with known spring constant. The object is oscillated about a vertical axis aligned with the spring and the center of mass of the body [10]. The multifilar pendulum comprises a rigid body suspended by a set of flexible wires that oscillates around an axis passing through its center of mass. Nowadays, there is very popular trifilar pendulum which uses three equally spaced wires of length $L$ [11,12].

In some study, the products of inertia are neglected, because of the difficulty of direct measurement and their relatively small magnitude [13]. However, products of inertia, which are a measure of object mass unbalance, play a significant role in stability of mechanical system having spin motion. Products of inertia can be measured by spin balance method [14] and moment of inertia method, when MOI is measured about various axis and then POI is calculated.

Nowadays, there are several commercial products for inertia tensor estimation of the measured object. For instance, SMARTMechanical offers two products, InTenso and InTensino, which use non-linear pendulum method for testing large vehicles and their components, respectively. The payload is mounted on a platform suspended from rigid rods [15,16]. The InTenso series measure the weight, center of mass location in 3 axes and the full inertia tensor: moment of inertia in 3 axes and product of inertia, about single axis, and not the product of inertia.

Inertia Dynamics sells a torsion platform operating on the principle of an inverted torsional pendulum. It estimates the MOI of an object about the vertical rotation axis [17]. In order to obtain the tensor of inertia it is necessary to provide 6 measurements about different axis. This platform does not measure the center of mass.

Resonic offers the Resonic product lines. For example, the Resonic K system is a torsional platform together with a carrier adapter to support small test objects in one of twenty-four possible orientations. They use conventional air-bearing pendulum. The object is sequentially placed into a configuration specified by the unit’s software, followed by a measurement of the free oscillation response. Once all the data is collected, the system calculates the inertia tensor and CM location of the test object [18].

Space Electronics offers a variety of high-precision torsional platforms. Their KSR series offers the high accuracy and sensitivity. These systems use spherical air bearings to minimize rotational friction and force feedback for balancing and eliminating leveling errors. KSR series measures full mass properties [19]. Other systems include the XR and XKR [20], and the GB series [21] measures only moment of inertia.

Central Aerohydrodynamic Institute develops automated test rigs to compute the mass-inertial characteristics of moving objects. These test rigs are used in aviation and space engineering, the objects of which have masses from 10 to 5000 kg and measure all parameters within one test of the model [22].

All these commercially available systems are summarized in Table 1. Most of the above-mentioned products estimate only the MOI of the test object. In order to obtain the POI and CM location, Resonic requires employing twenty-four object orientations during testing [18], while Space Electronics suggests a method to calculate POI from MOI estimates requiring the rotation axis to pass through the CM of the object and the series of rotation axes to be exactly perpendicular to each other. Moreover, a lot of these commercial products require additional attachment devices in order to set the nanosatellite in necessary position.
Table 1. Commercial systems presently available, with Moment of Inertia Uncertainty (MOIu), Product of Inertia Accuracy (POIa), Centre of Mass Uncertainty (CoMu) and Approximate cost. Unreported values are noted as N/A.

| Brand Model                  | Mass (kg) | MOIu | POIa | CoMu          | Price (USD) |
|------------------------------|-----------|------|------|---------------|-------------|
| SMARTMechanical InTenso      | 500–3500  | 1%   | 0.5% | 1.5 - 3 mm    | $~400 k     |
| SMARTMechanical InTensino    | 7–400     | 1%   | 0.5% | 1.5 - 3 mm    | N/A         |
| Inertia Dynamics             | 11.4 max  | 0.5% | N/A  | –             | $~17 k      |
| Resonic K                    | light     | 1%   | N/A  | –             | N/A         |
| Space Electronics KSR330-6   | N/A       | 0.1% | N/A  | 0.025 mm      | $~345 k     |
| Space Electronics XR         | 114 max   | 0.25%| N/A  | N/A           | $~55 k      |
| Space Electronics XKR        | 2.3 max   | 0.1% | N/A  | N/A           | $~70 k      |
| Space Electronics GB         | 4000      | N/A  | N/A  | N/A           | $~115 k     |
| Space Electronics MP1100     | 500 max   | 0.25%| N/A  | 2.54 mm       | $~220 k     |
| IRANStMI test rig            | 800 - 1500| 3%   | –    | 2.5 mm        | N/A         |
| SCMiMI-1T test rig           | 50 – 1200 | 1%   | –    | 1 – 2 mm      | N/A         |
| SCMiMI-0.01T test rig        | 10 max    | 1%   | –    | 1 – 2 mm      | N/A         |

To solve the problem of determining CM and tensor of inertia of nanoclass spacecraft with high accuracy at low material costs, we developed the torsional platform on the inverted torsional pendulum principle at Samara University [23]. This platform provides the estimation of the center of mass coordinates of a CubeSat 1U-3U nanosatellite weighing from 1 to 10 kg with an error of no more than 0.5 mm in the building coordinate system and the estimation of moments of inertia with an error of no more than 1.5%.

2. Theoretical basics

In the study we used the following coordinate systems:

1. Torsional platform coordinate system (SXYZ)
   The torsional platform coordinate system is established with its origin S located on the plate of the torsional platform. SZ-axis is aligned with the axis of rotation, upward-directed. SX and SY axes are in plane of the rotating plate (or faceplate) of the torsional platform.

2. Reference coordinate system (Oxyz)
   The second coordinate system, referred to Reference coordinate system, is used to describe the CM of the nanosatellite. The coordinate system is fixed to the building coordinate system, which is aligned with the ribs of CubeSat. It’s origin O is chosen at the any convenient point of nanosatellite, for example, one of the vertices. Let be Ox-axis is along the longest rib of the nanosatellite.

3. Centroid coordinate system (Cxyz)
   Moving the origin of the Reference coordinate system to the CM of the nanosatellite, the Centroid coordinate system is obtained. The direction of each axis (Cxyz) in Centroid coordinate system is the same as that in Reference coordinate system Oxyz.

The inertia tensor of the measured body is defined in Centroid coordinate system and can be written in a form:

\[
I = \begin{bmatrix}
I_x & I_{xy} & I_{xz} \\
I_{xy} & I_y & I_{yz} \\
I_{xz} & I_{yz} & I_z \\
\end{bmatrix}
\]

(1)
We used the torsional pendulum principle for developing torsional platform at Samara University. Assuming that restoring torque of the spring is linearly proportional the angular displacement (\( \varphi \)) by a spring constant (\( k_s \)) and neglecting friction torque, for an object oscillating about a vertical axis aligned with the axis of rotation (SZ) and the center of mass of the body, the equation of motion can be written in the form [3,4]

\[
I_{sz} \ddot{\varphi} + k_s \varphi = 0, 
\]

where \( I_{sz} \) is the moment of inertia of the system about SZ-axis.

Eq. (2) is a second-order linear ODE and for initial displacement \( \varphi_0 \) and zero initial velocity (\( \dot{\varphi}_0 = 0 \)) has solution

\[
\varphi(t) = \varphi_0 \cos \left( \frac{k_s}{I_{sz}} t \right). 
\]

The system has a natural frequency \( \omega_n \) and a period of oscillation \( T \):

\[
\omega_n = \sqrt{\frac{k_s}{I_{sz}}}, 
\]

\[
T = 2\pi \sqrt{\frac{I_{sz}}{k_s}}. 
\]

Using (5) we can determine the moment of inertia about z-axis for known spring constant \( k_s \) and period of oscillation \( T \) as

\[
I_{sz} = \frac{T^2 \cdot k_s}{4\pi^2}. 
\]

Formula (6) allows us to calculate the moment of inertia of the whole system about the axis aligned with the axis of rotation.

If the center of mass of the nanosatellite does not align with the SZ-axis we can write

\[
I_{sz} = I_0 + I_z + m(X_C^2 + Y_C^2), 
\]

where \( I_0 \) is the own moment of inertia of the torsional platform, \( X_C, Y_C \) are coordinates of the nanosatellite CM in the torsional platform coordinate system.

The \( I_0 \) own moment of inertia of the torsional platform can be determined by

\[
I_0 = \frac{T_0^2 \cdot k_s}{4\pi^2}, 
\]

where \( T_0 \) is the period of the oscillation of the empty torsional platform.

Thus, from (7) and (8) we get the formula for estimation of the inertia moment of the nanosatellite:

\[
I_z = \frac{(T^2 - T_0^2) \cdot k_s}{4\pi^2} - m(X_C^2 + Y_C^2). 
\]

In order to estimate the two coordinates of the center of mass we need to provide 3 measurements, as shown at Figure 1:

1. the nanosatellite initial position,
2. the position, when nanosatellite is shifted to the distance \( \Delta X \) along SX-axis only,
3. the position, when nanosatellite is shifted to the distance \( \Delta Y \) along SY-axis only.

Thus, we get three periods of oscillation \( T_1, T_2, T_3 \) respectively. Using (7) and (8) we have

\[
\frac{T_1^2 \cdot k_s}{4\pi^2} = I_0 + I_z + m(X_C^2 + Y_C^2), 
\]

\[
\frac{T_2^2 \cdot k_s}{4\pi^2} = I_0 + I_z + m((X_C + \Delta X)^2 + Y_C^2), 
\]

\[
\frac{T_3^2 \cdot k_s}{4\pi^2} = I_0 + I_z + m(X_C^2 + (Y_C + \Delta Y)^2), 
\]
\( \frac{T_s^2 \cdot k_s}{4\pi^2} = I_0 + I_z + m \left( X_C^2 + (Y_c + \Delta Y)^2 \right). \) (12)

**Figure 1.** The scheme of the rotating plate of the torsional platform.

From (10), (11) and from (10), (12) we can write for \( X_C, Y_C \):

\[
X_C = \frac{(T_i^2 - T_s^2) k_s}{8\pi^2 m \Delta X} \cdot \frac{\Delta X}{2},
\]

\[
Y_C = \frac{(T_i^2 - T_s^2) k_s}{8\pi^2 m \Delta Y} \cdot \frac{\Delta Y}{2}.
\]

If the axes of the Reference coordinate system and the torsional platform coordinate system are in the same direction, we can estimate the values \( x_c, y_c \) by

\[
x_c = X_C - X_O,
\]

\[
y_c = Y_C - Y_O,
\]

where \( X_O, Y_O \) are origin \( O \) coordinates in the torsional platform coordinate system.

Thus, we obtained formulas for estimation of three parameters, namely, \( I_x, I_y, I_z \). In order to obtain \( I_x, I_y, I_z \) we have to put the nanosatellite in the corresponding position on the torsional platform. For estimation of products of inertia, we use moment of inertia method. We suggest to measure the moments of inertia along three axes which form the 45° angle with the positive direction of \( x, y, z \) axes, and then to calculate POI using formulas [4]:

\[
I_{xy} = \frac{(I_x + I_y)}{2} - I_4, \quad I_{xz} = \frac{(I_x + I_z)}{2} - I_5, \quad I_{yz} = \frac{(I_y + I_z)}{2} - I_6.
\]

Here \( I_4 \) is measured along axis between positive direction \( x \) and \( y \) axes; \( I_5 \) is measured along axis between positive direction \( x \) and \( z \) axes; \( I_6 \) is measured along axis between positive direction \( y \) and \( z \) axes.
3. Design of an instrumented torsional platform

The proposed torsional platform [23] consists of a frame and a balance beam (a rotation shaft with a rotation plate) (Figure 2). On the frame 1, the upper 2 and lower 3 radial rolling bearings are installed, located coaxially. Under the bearing 2 attachment unit 4 is installed to attach the torsion spring 5. The upper 7 and lower 8 spikes are installed on the shaft 6. The attachment point 9 of the torsion spring 5 is at the upper part of the spike 8. The upper part of the shaft 6 has a rotating plate 10. The rotating plate has a pattern of coordinate holes for installing the object 11. The frame 1 has adjustable screw supports 12. There is the arrow 13 on the torsional platform plate 10, and the optical encoder 14 is installed on the frame.

The shaft 6 hangs on the torsion spring 5, therefore, the bearings 2 and 3, that set the axis of rotation of the shaft 6, perceive only the radial load due to the mismatch of the common center of mass of the shaft 6, the torsional plate 10 and the object 11 with the axis of rotation of the shaft 6.

![Figure 2. The scheme of the torsional platform.](image)

Since the radial load on bearings 2 and 3 determines the moment of resistance to rotation of the shaft 6, in order to reduce it, the distance H between bearings 2 and 3 is set significantly greater than the deviation A of the total center of mass from the axis of rotation of the shaft 6.

The optical sensor is used to measure the oscillation period. It marks each passage of the balance beam through the equilibrium position. Data is written to the memory card, then it is processed. To perform the next measurement cycle, the information on the display is updated to zero by pressing the reset button or turning off the power of the microcontroller.

The working table of the balance beam is made in the form of a plate with a grid of coordinate holes for installing reference disks during adjustment (establishing the verticality of the axis of rotation of the balance beam) and calibration (determining the spring constant), as well as for installing attachment
blocks for basing objects. Nanosatellites (NS) are placed on the attachment blocks included in the platform kit.

The kit of the platform includes torsion springs with different spring constant and a set of standards (steel discs), the mass of which is determined with an accuracy of one hundredth of a gram.

This platform allows us to calculate a single moment of inertia passing through an axis parallel to the axis of rotation, so it is needed to conduct a series of at least 6 experiments in order to determine the whole tensor of inertia. The platform is equipped with a set of special attachment blocks that allows us to install a CubeSat 1U-3U nanosatellite in the required positions, namely horizontal, vertical and positions at 45° to the plane of the plate, as shown in figures 3 – 6.

The main difference from the similar platforms [17–19], which also use the torsional pendulum principle, is that the weight of the balance beam with the rotating plate and the object with the attachment
blocks is perceived by the torsion spring. The vertical position of the rotation axis is set by radial-load bearings. The load on the bearings is determined by the tipping moment and the distance between the bearings (base).

Since the base of this platform is almost an order of magnitude higher than the displacement of the center of mass of the loaded balancer (console) and precision bearings without lubrication are used, the errors caused by the damping of the balancer vibrations are close to the errors of platforms with air bearings. Moreover, for small and stable damping of oscillation, the damping can be taken into account when calculating and calibrating the spring constant of the platform. The spring constant depends on the tensile stress of the spring, which in turn depends on the load. Constant part of the load (load balancer and rotating plate) is taken into account automatically during calibration. The weights of equipment and objects can vary over a wide range from 1 to 10 kg. In addition, when determining the center of mass of the object, measurements are made at its different positions on the rotating plate and, accordingly, at different tipping moments acting on the balancer, which means different loads on the bearings. The influence of torsion tension and friction in bearings will be taken into account in the spring constant when determining the spring constant for all cases of loads and tipping moments acting during measurements.

The actual position of the axis of rotation of the balancer in the torsional platform coordinate system, as well as its verticality, is determined by standards with known parameters when making measurements in different quadrants of the rotating plate. The position of the platform does not change in the future, and the actual coordinates of the rotation axis are taken into account in the calculations.

With this procedure, the main source of errors in determining the center of mass coordinates and tensor of inertia are the geometric parameters of the attachment blocks and the nanosatellite, as well as the accuracy of installing the attachment blocks on the rotating plate. When determining the center of mass, such errors can be reduced by averaging. Measurements are made for nanosatellite orientations that differ by 180° in the plane of the rotating plate, and then averaged. When measuring moments of inertia, the accuracy of measurements is influenced a lot by the distance from the axis of rotation to the center of mass of the nanosatellite, as well as the accuracy of axis determination.

4. Experimental procedure

Procedure of experimental evaluation of nanosatellite spacecraft design parameters using proposed torsional platform consists of the following steps:

- Adjustment and calibration;
- Estimation of the center of mass coordinates;
- Estimation of the inertia tensor;
- Estimation of errors.

4.1. Adjustment and calibration

Preparation of the platform for operation involves setting the verticality of the axis of rotation and determining the spring constant for various loads on the plate and tipping moments.

The platform is adjusted using standards (steel discs). The standards are placed in turns diagonally from the axis of rotation for the same possible long distances and by adjusting the screw supports at the base of the platform the equal oscillation periods in symmetrical positions along each diagonal are achieved. Each diagonal is adjusted independently of the other diagonal. Ideally, the oscillation periods should coincide in all 4 directions.

Calibration is determination of the spring constant for various masses using the following procedure:

1. Set the standard of mass \( m \) (taking into account the mass of the centering pin) to the nearest diagonal coordinate hole to the axis of rotation (distance \( r_1 \)). Measure the period of oscillation \( T_1 \).

2. Shift the standard to the next hole from the axis of rotation along diagonal (the distance \( r_j \)). Measure the period of oscillation \( T_j \). Repeat step 2 for the next holes.

3. Calculate the value of spring constants by formulas:
\[ k_s = \frac{4\pi^2 m (r_i^2 - r_j^2)}{T_i^2 - T_j^2}, \quad i = 2, \ldots, n, \quad j = 1, \ldots, (n-1). \] (18)

4. Calculate the average value of spring constant.

4.2. Estimation of the center of mass coordinates

1. To measure the center of mass coordinates, install a rotating plate and check the adjustment of the platform.
2. Set on a rotating plate the horizontal attachment blocks (Ошибка! Источник ссылки не найден.).
3. Install the nanosatellite in the attachment blocks so that its \( x \) and \( y \) axes coincide with the corresponding \( X \) and \( Y \) axes of the torsional platform coordinate system. Measure the period \( T_1 \).
4. Shift the nanosatellite along the \( X \)-axis of the torsional platform by a distance \( \Delta X \). Measure the period \( T_2 \).
5. Return the nanosatellite to its original position and shift it along the \( Y \)-axis of the torsional platform by a distance \( \Delta Y \). Measure the period \( T_3 \).
6. Calculate the coordinates \( X_{c1}, Y_{c1} \) using the formulas (13), (14). Using formulas (15), (16) calculate \( x_{c1}, y_{c1} \).
7. Repeat steps 3–6 for the position of the nanosatellite rotated 180° in the plane of the torsional platform. Measure the periods \( T_4, T_5, T_6 \), respectively.
8. Calculate the coordinates \( X_{c2}, Y_{c2} \) by the formulas similar to (13), (14). Calculate \( x_{c2}, y_{c2} \) taking into account changes in the position of the origin \( O \) and direction of the axes of the nanosatellite by the formulas:

\[ x_{c2} = X_o - X_{c2}, \quad y_{c2} = Y_o - Y_{c2}. \] (19)

9. Average the results:

\[ x_c = (x_{c1} + x_{c2})/2, \quad y_c = (y_{c1} + y_{c2})/2. \] (20)

10. To determine the coordinate \( z_c \), rotate the nanosatellite relative to the \( Ox \)-axis by 90° and install it in attachment blocks, so that the \( z \)-axis is directed in the positive direction of the \( Y \)-axis of torsional platform. Measure the period \( T_7 \).
11. Shift the nanosatellite along the \( Y \)-axis of the torsional platform by a distance \( \Delta Y \). Measure the period \( T_8 \).
12. Calculate the coordinate \( Z_{c1} \) using the formula similar to (14). Calculate \( z_{c1} \) by formula similar to (16).
13. Repeat steps 10–12 for the position of the nanosatellite rotated 180° in the plane of the torsional platform. Measure the periods \( T_9, T_{10} \) respectively.
14. Calculate the coordinate \( Z_{c2} \) using the formula similar to (14). Calculate \( z_{c2} \) taking into account changes in the position of the origin and direction of the axes of the nanosatellite using the formula:

\[ z_{c2} = Y_o - Z_{c2}. \] (21)

15. Average the results:

\[ z_c = (z_{c1} + z_{c2})/2. \] (22)

Thus, the center of mass coordinates of the nanosatellite \( (x_c, y_c, z_c) \) in the Reference coordinate system \( Oxyz \) are determined.
4.3. Estimation of the inertia tensor

The moments of inertia are determined with the known center of mass coordinates of the nanosatellite \((x_c, y_c, z_c)\). When determining the moments of inertia, the attachment blocks must be installed on the faceplate of the balance beam so that the center of mass of the nanosatellite is as close as possible to the axis of rotation.

1. To measure moments of inertia, remove the rotating plate.
2. Install on the faceplate of the balance beam the diagonal-horizontal attachment blocks (Ошибка! Источник ссылки не найден.). Measure the oscillation period of the torsional platform with attachment blocks \(T_1\).
3. Install the nanosatellite in the attachment blocks so that the \(z\)-axis is directed vertically. Using the drawings of the attachment blocks and the rod depth gauge determine the projection of the center of mass of the nanosatellite \(X_{C1}, Y_{C1}\) on the plane of the faceplate. Measure the period \(T_2\).
4. Calculate the moment of inertia of the nanosatellite \(I_z\) by the formula:
\[
I_z = \frac{(T_2^2 - T_1^2)}{4\pi^2} k_s - m(X_{C1}^2 + Y_{C1}^2).
\] (23)
5. Install the nanosatellite in the attachment blocks so that the \(y\)-axis is directed vertically. Using the drawings of the attachment blocks and the rod depth gauge determine the projection of the center of mass of the nanosatellite \(X_{C2}, Y_{C2}\) on the plane of the faceplate. Measure the period \(T_3\).
6. Calculate the moment of inertia of the nanosatellite \(I_y\) by the formula:
\[
I_y = \frac{(T_3^2 - T_1^2)}{4\pi^2} k_s - m(X_{C2}^2 + Y_{C2}^2).
\] (24)
7. Install the nanosatellite in the attachment blocks so that the bisector of the angle between the positive direction of the \(y\) and \(z\) axes is directed vertically. Using the drawings of the attachment blocks and the rod depth gauge determine the projection of the center of mass of the nanosatellite \(X_{C3}, Y_{C3}\) on the plane of the faceplate. Measure the period \(T_4\).
8. Calculate the moment of inertia of the nanosatellite \(I_x\) by the formula:
\[
I_x = \frac{(T_4^2 - T_1^2)}{4\pi^2} k_s - m(X_{C3}^2 + Y_{C3}^2).
\] (25)
9. Install on the faceplate of the balance beam the vertical attachment blocks (Ошибка! Источник ссылки не найден.). Measure the oscillation period of the torsional platform with attachment blocks \(T_5\).
10. Install the nanosatellite in the attachment blocks so that the \(x\)-axis is directed vertically. Using the drawings of the attachment blocks determine the projection of the center of mass of the nanosatellite \(X_{C4}, Y_{C4}\) on the plane of the faceplate. Measure the period \(T_6\).
11. Calculate the moment of inertia of the nanosatellite \(I_x\) by the formula:
\[
I_x = \frac{(T_6^2 - T_5^2)}{4\pi^2} k_s - m(X_{C4}^2 + Y_{C4}^2).
\] (26)
12. Install on the faceplate of the balance beam the diagonal-vertical attachment blocks (Ошибка! Источник ссылки не найден.). Measure the oscillation period of a platform with attachment blocks \(T_7\).
13. Install the nanosatellite in the attachment blocks so that the bisector of the angle between the positive direction of the \(x\) and \(y\) axes is directed vertically. Using the drawings of the attachment
blocks determine the projection of the center of mass of the nanosatellite $X_{C5}, Y_{C5}$ on the plane of the faceplate. Measure the period $T_s$.

14. Calculate the moment of inertia of the nanosatellite $I_s$ by the formula:

$$I_s = \frac{(T_s^2 - T_r^2)^2}{4\pi^2} \cdot k_s - m(X_{C5}^2 + Y_{C5}^2).$$

15. Install the nanosatellite in the attachment blocks so that the bisector of the angle between the positive direction of the $x$ and $z$ axes is directed vertically. Using the drawings of the attachment blocks determine the projection of the center of mass of the nanosatellite $X_{C6}, Y_{C6}$ on the plane of the faceplate. Measure the period $T_g$.

16. Calculate the moment of inertia of the nanosatellite $I_o$ by the formula:

$$I_o = \frac{(T_o^2 - T_r^2)^2}{4\pi^2} \cdot k_o - m(X_{C6}^2 + Y_{C6}^2).$$

17. Calculate the products of inertia by formulas (17). Write the inertia tensor.

5. Estimation of errors

Several sources of measurement errors can be identified. One of these sources is the use of a model without taking into account the forces of resistance. Experiments have shown that the error introduced by the model is insignificant and is indirectly taken into account with the experimental determination of the spring constant.

The experimental determination of measured values is subject to errors from the uncertainty in determining the oscillation period, distance to the axis of rotation, and mass. The method of extracting the root from the sum of squares was used to estimate the absolute errors of indirect measurements:

$$
\Delta F = \sqrt{\sum_{i=1}^{n} \left( \frac{\partial F}{\partial u_i} \Delta u_i \right)^2},
$$

where $u_i$ are parameters, $\Delta u_i$ are the errors of parameters.

5.1. The error of spring constant

Spring constant is calculated using the formula (18). Therefore, the absolute error in determining the spring constant according to formula (29) can be calculated by the formula:

$$
\Delta k_s = 4\pi^2 \left( \frac{r_i^2 - r_j^2}{T_i^2 - T_j^2} \Delta m \right)^2 + \left( \frac{2mT_i (r_i^2 - r_j^2)}{(T_i^2 - T_j^2)^2} \Delta T_i \right)^2 +
\left( \frac{2mT_j (r_i^2 - r_j^2)}{(T_i^2 - T_j^2)^2} \Delta T_j \right)^2 +
\left( \frac{2mr_i}{T_i^2 - T_j^2} \Delta r_i \right)^2 +
\left( \frac{2mr_j}{T_i^2 - T_j^2} \Delta r_j \right)^2 \right)^{1/2},
$$

where $\Delta m$ is a mass error; $\Delta T_i, \Delta T_j$ are period errors; $\Delta r_i, \Delta r_j$ are distance errors.

Then the relative error of spring constant can be determined by the formula:

$$
\delta k_s = \sqrt{\left( \frac{\Delta m}{m} \right)^2 + \left( \frac{2T_i \Delta T_i}{T_i^2 - T_j^2} \right)^2 + \left( \frac{2T_j \Delta T_j}{T_i^2 - T_j^2} \right)^2 + \left( \frac{2r_i \Delta r_i}{r_i^2 - r_j^2} \right)^2 + \left( \frac{2r_j \Delta r_j}{r_i^2 - r_j^2} \right)^2}. 
$$

From (31) we can conclude that the greater the difference between the squares of periods and the squares of distances, the smaller the relative error in determining spring constant.

5.2. The error of center of mass coordinates
With the known error of spring constant, we can determine the error of the center of mass coordinates. The coordinate of the center of mass \( X_C \) is determined by the formula (13). Thus, the absolute error of the center of mass coordinates can be calculated by

\[
\Delta X_C = \left( \frac{T_2^2 - T_1^2}{8\pi^2 m\Delta X} \right)^2 + \frac{2T_kk_s\Delta T}{8\pi^2 m\Delta X} + \frac{2T_kk_s^2\Delta T_k}{8\pi^2 m\Delta X} + \frac{(T_2^2 - T_1^2)k_s\Delta m}{8\pi^2 m^2\Delta X^2} + \frac{(T_2^2 - T_1^2)k_s}{8\pi^2 m^2\Delta X^2} + \frac{1}{2} \Delta(\Delta X) \right)^{1/2}.
\]  

(32)

Absolute errors of the center of mass coordinates \( Y_C \) and \( Z_C \) are determined in the same way. When calculating the \( x_C, y_C, z_C \) coordinate errors, the error of the reference point \( O \) coordinates in the torsional platform coordinate system should be taken into account.

5.3. The error of the axial moments of inertia

With the known errors of spring constant and the center of mass coordinates \( x_C, y_C, z_C \) we can determine the error of the axial moments of inertia. The moment of inertia is determined by the formula

\[
|I_1| = \frac{1}{2} mX_y^2 + \frac{1}{2} mZ_x^2 + \frac{1}{2} mZ_z^2 + \frac{1}{2} k_s^2 \Delta T + \frac{1}{2} k_s^2 \Delta T + \frac{1}{2} k_s^2 \Delta T.
\]

(9).

Therefore, the absolute error of the axial moments of inertia has the form

\[
\Delta I_1 = 4\pi^2 \left( \frac{(T_2^2 - T_1^2)}{4\pi^2} \right)^2 + \frac{2Tk_kk_s\Delta T}{4\pi^2} + \frac{2Tk_kk_s^2\Delta T_k}{4\pi^2} + \frac{(X_C^2 + Y_C^2)^2}{4\pi^2} + \frac{2X_C^2 \Delta X_C^2}{4\pi^2} + \frac{2X_C^2 \Delta X_C^2}{4\pi^2} \right)^{1/2}.
\]  

(33)

Absolute errors of the moments of inertia \( I_x, I_y, I_4, I_5, \) and \( I_6 \), are determined similarly. The error of the products of inertia consists of the errors of the corresponding axial moments of inertia.

6. Validation experiments: determination of center of mass coordinates and the moment of inertia of a standard

This section shows the results of a series of experiments with a standard to illustrate the work of the proposed platform. The standard is a steel disk with the following characteristics: weight \( m=1.92428 \) kg; external radius 202.5 mm; internal radius 5 mm. This standard is installed on the plate using a centering pin weighing 25.633 g with the radius of 5 mm.

The spring constant of the torsional platform was determined using the procedure given in 4.1. A set of 4 experiments was made with different distances from the axis of rotation. The values of spring constant were calculated by the formula (18). The data is presented in table 2.

| № | \( r_j^2, \text{mm}^2 \) | \( T_j, \text{ms} \) | \( r_i^2, \text{mm}^2 \) | \( T_i, \text{ms} \) | \( k_s, \text{H} \cdot \text{m} \) |
|---|---|---|---|---|---|
| 1 | 800 | 2565.00 | 7200 | 2728.49 | 0.56927 |
| 2 | 800 | 2565.00 | 20000 | 3028.39 | 0.57024 |
| 3 | 800 | 2565.00 | 39200 | 3429.98 | 0.57005 |
| 4 | 7200 | 2728.49 | 20000 | 3028.39 | 0.57072 |
| 5 | 7200 | 2728.49 | 39200 | 3429.98 | 0.57020 |
| 6 | 20000 | 3028.39 | 39200 | 3429.98 | 0.56986 |
Thus, the experimental value of the spring constant is $k_s = 0.57006 \, H \cdot m$. It should be noted that the maximum deviation of the spring constant from the average value is 0.13%.

The center of mass of the standard is located in its geometric center. The geometric center was taken as the origin $O$ of the reference coordinate system. The theoretical moment of inertia of the standard and the centering pin is $I_T = 0.0098780 \, kg \cdot m^2$. Table 3 shows the values of the moment of inertia obtained as a result of experiments, the absolute error from the theoretical value, and the relative error. It can be noted that the relative error does not exceed 1.1% even for large distances of the center of mass from the axis of rotation. Table 4 shows the coordinates of the center of mass obtained experimentally. The table shows that the error in determining the coordinates does not exceed 0.33 mm.

### Table 3. Experimental determination of the moment of inertia of the standard.

| №  | $X_O$, mm | $Y_O$, mm | Average period, ms | The system moment of inertia, kg·m² | Experimental moment of inertia of the standard, kg·m² | Absolute error, kg·m² | Relative error, % |
|----|------------|------------|--------------------|-------------------------------------|---------------------------------------------------|------------------------|-------------------|
| 1  | 20         | 20         | 2565.00            | 0.095002                            | 0.009908                                          | 0.000030               | 0.31              |
| 2  | 60         | 60         | 2728.49            | 0.107498                            | 0.009926                                          | 0.000047               | 0.48              |
| 3  | 100        | 100        | 3028.39            | 0.132428                            | 0.009897                                          | 0.000018               | 0.19              |
| 4  | 140        | 140        | 3429.98            | 0.169880                            | 0.009910                                          | 0.000031               | 0.32              |
| 5  | 100        | 20         | 2805.89            | 0.113684                            | 0.009817                                          | -0.000007              | -0.07             |
| 6  | 180        | 20         | 3300.19            | 0.157266                            | 0.009776                                          | -0.000102              | -1.04             |
| 7  | 20         | 100        | 2805.08            | 0.113618                            | 0.009806                                          | -0.000072              | -0.73             |
| 8  | 20         | 180        | 3301.37            | 0.157379                            | 0.009888                                          | 0.000010               | 0.10              |
|    |            |            |                    | Average                             | 0.009868                                          | -0.000005              | -0.05             |

### Table 4. Experimental determination of the center of mass coordinates of the standard.

| №  | $X_O$, mm | $Y_O$, mm | $\Delta X$, mm | $\Delta Y$, mm | $X_c$, mm | $x_c$, mm | $Y_c$, mm | $y_c$, mm |
|----|------------|------------|-----------------|-----------------|------------|------------|------------|------------|
| 1  | 20         | 20         | 80              | 0               | 19.88      | 0.12       |            |            |
| 2  | 20         | 20         | 160             | 0               | 19.78      | 0.22       |            |            |
| 3  | 100        | 20         | 80              | 0               | 99.68      | 0.32       |            |            |
| 4  | 20         | 20         | 0               | 80              | 19.67      | 0.33       |            |            |
| 5  | 20         | 20         | 0               | 160             | 19.96      | 0.04       |            |            |
| 6  | 20         | 100        | 0               | 80              | 100.25     | -0.25      |            |            |
|    |            |            |                 |                 | Average    | 0.22       |            | 0.04       |

### 7. Conclusions

Thus, this paper describes the procedure of experimental evaluation of nanoclass spacecraft design parameters using the proposed torsional platform. It is given a detailed description of the design of the Samara University’s torsional platform for determining the center of mass coordinates and the moments of inertia. To confirm the stated accuracy, a set of experiments with the standards was performed and it
is shown that this platform can estimate the coordinates of the center of mass with an error of no more than 0.5 mm and the axial moments of inertia with an error not more than 1.5%.

Conducting experiments to determine the parameter of standard disks showed that in order to improve the accuracy of measurements, it is necessary to increase the accuracy of the object positioning, including minimizing backlash in the landing of the equipment on the plate and the nanosatellite in the attachment blocks.

Acknowledgment
The research was supported by the Russian Foundation for basic research (RFBR grant no. 20-08-00617).

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