Crack Initiation Evolution Under Triaxial Loading Conditions

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Abstract. In response to high demands for minerals and in the light of rapid advancement in instrumentation and testing facilities, mines, tunnels, and infrastructures are getting deeper at an annual increasing rate. For instance, Australia’s deepest Gwalia gold mine near Leonora is extending beyond 2 km depth in the hunt for more ore. In Europe, the Gotthard sub-alpine base tunnels exceeding 10 m wide are excavated at depths greater than 2 km. At depth, explosion-like fractures occur which are known as stress spalling, slabbing, or rock bursting due to high-pressure environment, high temperature, and low porosity. Although rockburst and spalling failures have both been observed in the past 50 years and investigated by numerous researchers, this phenomenon is yet mysterious and the physics behind is still not fully understood. Recent researches have suggested that the spalling strength can be related to the crack initiation point. Various methods have therefore been proposed to identify the crack initiation at laboratory scales based on stress-strain response. Limited models are also available based on applied stresses, but only under triaxial conditions. This study aims to compare the volumetric strain method (as a strain-based technique) with the improved Griffith and Hoek-Brown criterion (G-HB) (as a stress-based method) to determine the crack initiation stress threshold under triaxial loading conditions.

1. Introduction
The uniaxial compressive strength (UCS) test is a popular procedure used to measure the peak compressive strength and elastic constants of cylindrical rock samples. As shown in figure 1, the UCS stress-strain curve typically shows four distinct stages at various stress levels: crack closure, crack initiation (CI), unstable crack growth, and the peak strength stage [1, 2]. At the initial stage of loading, pre-existing voids and flaws dominate the mechanical behaviour of rocks [3]. After the closure of pre-existing cracks, stress-induced tensile/shear fractures start to initiate at about 30%-60% of the uniaxial compressive strength in a so-called crack initiation stage. This stage is of particular practical importance since recent studies suggest that the CI stress threshold can be safely used as a predictor of in-situ spalling and slabbing strength [1]. Moving up on the stress-strain curve after the CI level, crack tends to grow in an unstable fashion, and the crack coalesces eventually leads to strain localization and macroscale shear failure and axial splitting formations that cause permanent deformation at the crack damage (CD) stage and beyond. After the closure of pre-existing crack until the CI stress and well below the CD, which is also known as long-term strength as defined by [4], rock responds in an elastic fashion during which the elastic modulus and Poisson’s ratio can be determined (see figure 1).

Due to the practical implications of CI and CD stages, there is ample evidence and many published results on the development of microcracks and determination of strength characteristics at these stress levels [2, 5, 6, 7]. In particular, the literature is almost exclusively replete with various methods for the CI determination at laboratory scales including the development of the volumetric strain method, lateral
strain, extensional strain, crack volumetric strain, Poisson’s ratio, and the lateral strain response (LSR) methods [8, 9, 10, 11, 12, 1]. At low confinements (σ2 = σ3 ≤ 5 MPa), [13] proposed a linear CI criterion where the deviatoric stress (σ1 − σ3) becomes approximately 40% of UCS. This linear relationship, however, has shown to be of limited success when applied to higher confining pressures [14]. An improved CI criterion has therefore been more recently proposed by [14] based on the Griffith theory by introducing a frictional parameter (mci) from the Hoek-Brown failure model. Nevertheless, most of the available techniques in the literature are based on the stress-strain response of rock at zero confinements for which accurate determinations of elastic parameters (i.e. elastic modulus and Poisson’s ratio) is also deemed necessary, hence limiting their applications. Very limited stress-based models are proposed for the CI determination, especially under true triaxial conditions. In this research, the aim is to compare a stress-strain-based method (using the volumetric strain curve) against a stress-based technique (using the improved Griffith and Hoek-Brown method) for CI determination in a series of triaxial tests conducted with sandstone.

![Figure 1. Typical stress-strain behaviour under uniaxial test [1]](image)

### 2. Volumetric Strain Method

The volumetric strain method is one of the earliest attempts developed by [8] to determine the CI based on the volumetric strain dilatancy. It is observed that the onset of dilatancy (i.e. the CD stress level) can be readily determined when the volumetric strain curve deviates from its linearity. As in equation (1), they introduced volumetric strain (εv) as the summation of axial (ε1) and lateral strains (ε2 and ε3) in the direction of the major, intermediate and minor principal stresses, respectively. Under the uniaxial and triaxial conditions, both ε2 and ε3 are equal hence the volumetric strain can be obtained by the simplified equation (2). Further studies have revealed that by increasing the confining pressure, the volumetric strain increases slightly [15].

\[
\begin{align*}
ε_v &= ε_1 + ε_2 + ε_3 \\
ε_v &= ε_1 + 2ε_2
\end{align*}
\]
3. Improved CI Method Based on Griffith and Hoek-Brown Criterion

The Hoek-Brown failure criterion was developed based on the Griffith theory as a trial and error process to investigate the shear failure of isotropic rocks (i.e. either rock intact blocks or heavily jointed rock masses where there are a sufficient number of closely spaced discontinuities with similar surface characteristics) under triaxial stress conditions where \( \sigma_2 = \sigma_3 [16] \). Despite both methods being widely used in rock engineering design, neither Griffith nor Hoek-Brown criterion is capable of explaining the complex progressive coalition of tensile cracks at high confining stresses in brittle rocks [17]. At high confinements, two resistance components need to be further considered in Griffith’s theory; the resistance component of the crack initiation (\( \sigma_1 - \sigma_3 \)) and the frictional resistance of closed cracks, or \( \mu \sigma_1 \sigma_3 \) [14]. At the crack initiation level, the HB rock material constant (\( m_t \)) will be affected by the CI stress ratio (\( K \)) which is defined as the ratio of the CI stress (\( \sigma_{ci} \)) to the uniaxial compressive strength (\( \sigma_c \)). By considering the above modifications to Griffith and Hoek-Brown criteria, the following relationship was proposed by [14] to determine the CI at triaxial conditions at high confining stress domains.

\[
\sigma_1 - \sigma_3 = K \sigma_c \sqrt{m_{ci} \frac{\sigma_3}{\sigma_c} + \frac{1}{4} + \frac{1}{2} K \sigma_c}
\]

\[
m_{ci} = K m_t
\]

4. Test Results and Discussion

To determine the CI under uniaxial and triaxial loading conditions, sandstone bulk samples were extracted from a quarry in Queensland and were then cut, cored and ground according to ISRM standard recommendations. To measure the elastic constants, both the Brazilian and UCS tests were adopted.

The Brazilian test is commonly utilized to determine the indirect tensile strength of geomaterials in which two opposite radial loads, distributed over an arc angle of 10-15°, are applied on the outer boundary of a disc-shaped sample to split it into two halves [18, 19, 20, 21]. It can, however, be further adapted to determine rock elastic properties if radial and tangential strains at any arbitrary point inside the Brazilian disc is known [22]. The average indirect tensile strength of sandstone was obtained as \( \sigma_c = 3.7 \) MPa while the average UCS was measured as \( \sigma_u = 35 \) MPa.

4.1 CI under Uniaxial Compressive Strength

To determine the CI values in tested samples, the axial and lateral strain of the sandstone specimens were measured by an MTS axial and circumferential extensometers with a relative error of 2% in deformation measurements (figure 2). Before testing, a calibration process took place to convert the extensometer’s digital signals into displacement. To measure the change in specimen circumference (\( \Delta C \)) resulting from a change in the chain length (\( \Delta l \)) with an initial angle (\( \theta_i \)), the following formulation was used based on the schematic drawing of the circumferential extensometer (see figure 2):

\[
\Delta C = \frac{\pi \Delta l}{\sin \left( \frac{\theta_i}{2} \right) + \left[ \pi - \left( \frac{\theta_i}{2} \right) \right] \cos \left( \frac{\theta_i}{2} \right)}
\]

The loading rate was set to a strain rate of -0.025%/min and increasing after reaching the post-failure region. figure 3 represents the volumetric strain of the tested sandstone based on the axial and lateral strain measured by the extensometers according to equations 2 and 5. Following [8] model, the CI is the point where the volumetric strain curve deviates from its linearity which is corresponding to axial stress of 15 MPa, i.e. the corresponding CI stress ratio (\( K \)) is 0.42.
4.2 CI under Triaxial Loading

A novel hybrid true triaxial testing system (TTT) has been recently designed, fabricated, and set-up at the Geotechnical Engineering Center within the School of Civil Engineering of the University of Queensland, Australia (figure 4a). The system is capable of performing simultaneous true triaxial loading, permeability, hydraulic fracturing, and thermo-mechanical modelling under elevated temperatures of up to 100 degrees. The system is also configured to test three cubic sizes of rock-like materials from 50 to 200 mm. Sample deformation is monitored through 18 Linear Variable Differential Transformers (LVDT) mounted on the loading cell in all three directions [23].

For the triaxial testing stage of the current work, 50 mm cubic specimens were prepared and ground to insure smoothness and perpendicularity in all six faces. Each sample was then positioned inside the TTT loading cell as shown in figure 4b and tested under the stress states summarised in table 1. To avoid sample eccentricity while loading, a seating load of 8 kN was maintained in all three directions then a loading rate of 15 kN/min was applied simultaneously in all directions to satisfy the ISRM recommendations. The volumetric strain was measured through equation (1) concerning the sample strain in the directions of Z, Y, and X as plotted in figure 5. Several triaxial tests were carried out by increasing the intermediate and minor principal stresses from the seating load to the desired confinement and then the major principal stress was increased until failure.
From table 1 and figure 5, it can be deduced that the CI stress level increases with the increase of confinement from $\sigma_3 = 3$ MPa to $\sigma_3 = 40$ MPa while the ratio of CI/UCS remains almost unchanged in both techniques. More interestingly, however, the mode of CI can be seen to be very different at low confinement compared to higher confining pressures. At low confinements, the CI is developed in a splitting mode where the pre-existing cracks remain opened during the failure process, hence a higher local tensile stress will be developed around the perimeter leading to a sudden drop of volumetric strain at failure. In contrast, at higher confinements, the pre-existing cracks are closed which reduces the tensile stress regions leading to a sliding mode. Cracks will then initiate at the maximum tensile stress direction leading to the crack deflection out-of-plane and as a result, the volumetric strain is increased after failure.

To determine the CI through a modified Griffith and Hoek-Brown criterion, the Hoek-Brown criterion is plotted first where $m_t = 9$ at failure measured during the triaxial test. Referring to equation (4), the rock material constant at CI stress level is $m_{cl} = 3.78$. Accordingly, the new improved CI model is plotted as shown in figure 6. From table 1 and figures 5 and 6 it can be deduced that both models are similar where the CI stress level increased with the increase of confinements from 3 MPa to 20 MPa. At higher confinement of 40 MPa, the improved Griffith and Hoek-Brown models provide a slightly higher CI stress compared to the volumetric strain model.
Figure 5. CI stress at different confinements of $\sigma_3=3, 5, 7, 20$ and 40 MPa

Figure 6. Improved G-HB model for CI under triaxial loading condition
5. Conclusion
The increasing mining depth is associated with a high-pressure environment that induces explosion-like fractures known as stress spalling or slabbing. Spalling is the type of progressive failure that occurs in deep hard rocks and disturbs the short-term and long-term viability of mines and tunnels. Spalling failure is associated with the initiation of extensional cracks and therefore the crack initiation concept has been used as a predictor for spalling strength. The literature is replete with many cracks initiation stress-strain-based models while limited stress-based models are available in the literature. Unlike stress-based models, stress-strain-based models require an accurate measurement for the material elastic parameters. This study provides a quantitative comparison between a stress-strain-based model and a stress-based model proposed by [14] for the determination of crack initiation under triaxial loading conditions. The results show that both techniques provide similar values in the prediction of crack initiation stress level, especially at low confinements. However, further investigations show that under low confining stresses, crack initiates in a splitting mode while at high confinements, the predominant mode of crack becomes sliding.

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