Form-Finding Of Thomsen Surface Using Nonlinear Analysis Method

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Abstract. Tensioned fabric structure is the structure composed fabric surface as main material in tensioned fabric structure. Tensioned fabric structure with different surface form could be realized. Their variations as possible choice form of minimal surface for tensioned fabric structure have been studied. In this analysis, the form of Thomsen Surface has been used in this study. This study is to carry out form-finding computational analysis of Thomsen Surface for \( u=v=0.4 \) and \( u=v=1.0 \). The computational form-finding analysis used in this study is nonlinear analysis method. The surface of Thomsen Surface with variable \( u=v=0.4 \) and \( u=v=1.0 \) obtained in this study will provide an alternative shapes for designers to be considered for adoption in tensional fabric structures.

1. Introduction

Tensioned Fabric Structure (TFS) is a suitable structure to be applied in large space area, it also known as tensioned membrane structure. TFS composed fabric surface as main material. Fabric can prevent air, liquid and any particular thing from entering into it. Figure 1 shows example of TFS. [1] presented the fabrics used include PTFE coated plain weave glass-fibre fabrics, typically used for permanent TFS, and PVC coated plain weave nylon or polyester fabrics, typically used for temporary TFS. TFS with different surface form could be realized. Their variations as possible choice form of minimal surface for TFS have been studied. The form of Thomsen Surface has been used in this study. Thomsen Surface is a form of minimal surface. Thomsen surface have not been studied by other researcher. Besides, no other work on Thomsen Surface as idea in TFS has been found.
[2] has discussed minimal surface are not unique defined. The characteristic of minimal surface is belonging to minimal area with given boundary condition. It also described, the important role of minimal surface is mathematics and physics because the minimal area of minimal surface show the physical way. [3] have studied TFS becoming popular as an attractive way of providing temporary or semi-permanent cover for outdoor areas. Previous studies have shown that, [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16] and [17] have carried out form-finding using nonlinear analysis method in Catenoid, Helicoid, Scherk, Enneper, Oval, Costa, Moebius Strip, Monkey Saddle and Chen-Gackstatter TFS models.

The first step of design analysis in TFS is form-finding. Form-Finding is to determine initial equilibrium shape under prescribed pre-stress and boundary condition. Form-finding of tensioned fabric structures using nonlinear analysis method proposed by [17] and [18] has been used in this study.

Understanding of the possible form of Thomsen Surface will provide alternative shape for designers to be considered. TFS is highly suited to be used for realizing surface of new forms. However, none of the new example mentioned present any result on Thomsen Surface as load carrying members. In this paper, form-finding of Thomsen Surface with variable $u=v=0.4$ and $u=v=1.0$ have been carried out.

2. Generation of Thomsen Surface

![Thomsen Surface](image)

Figure 2 shows the form of Thomsen surface. The form of Thomsen surface can be obtained from [19] and [20]. Equation (1) shows the equation for Thomsen Surface.

$$X = \frac{\alpha fu + (1 + \beta^2) \sinh au \cos av}{a^2}, \quad Y = \frac{\alpha \sqrt{1 + \beta^2} v + \beta \cosh au \sin av}{a^2}, \quad Z = \sinh au \sin av$$

$$\alpha=1$$

$$\beta=0$$

$X$, $Y$ and $Z$ are the coordinates in $X$, $Y$ and $Z$ directions. For $u$ and $v$ = variables.
From this study, the program [21] has been used for the purpose of model generation. Aspect of modeling of surface of Thomsen Surface and form as well pre-stress pattern of the resulting TFS through form-finding using nonlinear analysis method are studied.

3. Computational Method using Nonlinear Analysis Method
The principle of nonlinear analysis method is based on [17]. The large displacement finite element formulation used for analysis of structural behaviour under external loads. Since the method can be used for both the converged shape problem and load analysis, the approach using nonlinear analysis is quite common. The basic equation used is expressed as follows:

\[
(\Delta K_L + K_G)u = r^{+\Delta}F - f^0
\]  

Where \(K_L\) is linear strain incremental stiffness matrix, \(K_G\) is nonlinear strain incremental stiffness matrix, \(f^0\) is vector internal forces, \(r^{+\Delta}F\) is load vector and \(u\) is vector of increment in displacement.

A nonlinear finite element analysis program by [17] for the analysis of tensioned fabric structures has been used in this study. The procedure adopted is based on the work as specified in [4]. 3-node plane stress element has been used as element to model the surface of TFS. All x, y and z translation of nodes lying along the boundary edge of the Thomsen surface have been restrained. The member pretension in warp and fill direction, is 2000N/m, respectively. The shear stress is zero.

Two stages of analysis were involved in the procedures of form-finding in one cycle proposed by [17]. First stage (denoted as SF1) is analysis which starts with an initial guess shape in order to obtain an updated shape for converged shape. The initial guess shape can be obtained from any pre-processing software and reference [17] is chosen for this study. This is then followed by the second stage of analysis (SS1) aiming at checking the convergence of updated shape obtained at the end of stage (SF1). During stage (SF1), artificial tensioned fabric properties, \(E\) with very small values are used. Both warp and fill tensioned fabric stresses are kept constant. In the second stage of (SS1), the actual values of tensioned fabric properties are used. Resulting warp and fill tensioned fabric stresses are checked at the end of the analysis against prescribed tensioned fabric stresses. Then, iterative calculation has to be carried out in order to achieve convergence where the criteria adopted is that the average of warp and fill stress deviation should be < 0.01. The resultant shape at the end of iterative step \(n\) (SSn) is considered to be in the state of converged shape under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed membrane stresses relative to the prescribed stress is negligibly small. Such checking of difference in the obtained and prescribed stresses has been presented in the form of total stress deviation in warp and fill direction versus analysis step. As a first shape for the start of form-finding procedure adopted in this study, initial guess shape is needed. For the generation of such initial guess shape, knowledge of the requirement of anti-clastic nature of TFS is used. The incorporation of anti-clastic feature into the model will help to produce a better initial guess shape.

4. Computational Result
Form-finding on TFS models in the form of Thomsen Surface model have been carried out. In this study, two variable of Thomsen Surface model has been used is \(u=v=0.4\) and \(u=v=1.0\). The numbers of nodes and triangular elements for Thomsen TFS model is 175 and 228, respectively.
4.1. Thomsen Surface, $u=v=0.4$

Figure 3. Initial Guess Shape of Thomsen Surface, $u=v=0.4$
Figure 3 shows initial guess shape of Thomsen Surface, \( u=v=0.4 \). Figure 4 shows the converged shape of Thomsen Surface, \( u=v=0.4 \). Figure 5 shows the graph for convergent curve of Thomsen Surface, \( u=v=0.4 \). The total warp and fill stress deviation is less than 0.01 for Thomsen Surface, \( u=v=0.4 \). From the result, the convergent curve of Thomsen Surface, \( u=v=0.4 \) is corresponding to equal tension surface.

4.2. Thomsen Surface, \( u=v=1.0 \)
Figure 6  Initial Guess Shape of Thomsen Surface, \( u=v=1.0 \)

Figure 7  Converged Shape of Thomsen Surface, \( u=v=1.0 \)
Figure 8 Convergent curve of Thomsen Surface, $u=v=1.0$

Figure 6 shows initial guess shape of Thomsen Surface, $u=v=1.0$. Figure 7 shows the converged shape of Thomsen Surface, $u=v=1.0$. Figure 8 shows the graph for convergent curve of Thomsen Surface, $u=v=1.0$. The total warp and fill stress deviation is less than 0.01 for Thomsen Surface, $u=v=1.0$. The convergent curve of Thomsen Surface, $u=v=1.0$ is corresponding to equal tension surface.

5. Conclusion

Form-Finding with surface in the form of Thomsen Surface with variable $u=v=0.4$ and $u=v=1.0$ have been carried out successfully using the procedure adopted which is based on nonlinear analysis method. Thomsen Surface with variable $u=v=0.4$ and $u=v=1.0$ obtained in this study will provide an alternative shapes for designers to be considered for adoption in tensioned fabric structures. The result from this analysis show that tensioned fabric structure in the form of Thomsen Surface is a structurally viable surface form to be considered by engineer.

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