Gauge Fixing in the Maxwell Like Gravitational Theory in Minkowski Spacetime and in the Equivalent Lorentzian Spacetime

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Abstract

In a previous paper we investigate a Lagrangian field theory for the gravitational field, which is there represented by a section \(\{g^\alpha\}\) of the coframe bundle over Minkowski spacetime \((M \simeq \mathbb{R}^4, \mathbf{g}, \overset{\circ}{D}, \tau_\mathbf{g}, \uparrow)\). Such theory, under appropriate conditions, has been proved to be equivalent to a Lorentzian spacetime structure \((M \simeq \mathbb{R}^4, \mathbf{g}, D, \tau_\mathbf{g}, \uparrow)\) where the metric tensor \(\mathbf{g}\) satisfies the Einstein field equation. Here, we first recall that according to quantum field theory ideas gravitation is described by a Lagrangian theory of a possible massive graviton field (generated by matter fields and coupling also to itself) living in Minkowski spacetime. The graviton field is moreover supposed to be represented by a symmetric tensor field \(\mathbf{h}\) carrying the representations of spin two and zero of the Lorentz group. Such a field, then (as it is well known) must necessarily satisfy the gauge condition given by Eq.(9) below. Next, we introduce an ansatz relating \(\mathbf{h}\) with the 1-form fields \(\{g^\alpha\}\). Then, using the Clifford bundle formalism we derive from our Lagrangian theory the exact wave equation for the graviton and investigate the role of the gauge condition given by Eq.(9) by asking the question: does Eq.(9) fix any gauge condition for the field \(\mathbf{g}\) of the effective Lorentzian spacetime structure \((M \simeq \mathbb{R}^4, \mathbf{g}, D, \tau_\mathbf{g}, \uparrow)\) that represents the field \(\mathbf{h}\) in our theory? We show that no gauge condition is fixed a priori, as it is the case in General Relativity. Moreover we investigate under which conditions we may fix Logunov gauge condition \(\overset{\circ}{D}_\gamma (\sqrt{-|\mathbf{g}|} g^\gamma) = 0\).
1 Introduction

In a previous paper\footnote{Please, consult the arXiv version of \cite{7} which corrects an error of the printed version.}, using the Clifford bundle formalism, a Lagrangian theory of the gravitational field as field in the Faraday sense, i.e., an object of the same ontology as the electromagnetic field living on a Minkowski spacetime structure $M = (\mathbb{R}^4, \hat{g}, \hat{D}, \tau, \uparrow)$ has been formulated\footnote{Minkowski spacetime will be called Lorentz vacuum, in what follows. Moreover in the 5-uple $(\mathbb{R}^4, \hat{g}, \hat{D}, \tau, \uparrow)$, $\hat{g}$ is a Minkowski metric, $\hat{D}$ is its Levi-Civita connection, $\tau$ is the volume element defining a global orientation and $\uparrow$ refers to a time orientation. The objects in the Lorentzian spacetime structure $L = (\mathbb{R}^4, g, D, \tau)$ have similar meanings. In what follows $\hat{g}$ denotes the metric of the cotangent bundle relative to the structure $M$. If $\hat{g} = \delta^\nu_\mu \partial_\nu \otimes \partial_\mu$ and $\hat{g} = \eta_{\alpha \beta} dx^\alpha \otimes dx^\beta$ then $\hat{g}_\nu^\mu \hat{g}_\xi^\nu = \delta^\xi_\mu$. Also $g$ denotes the metric of the cotangent bundle relative to the structure $L$ and if $\hat{g} = g_{\alpha \beta} \partial_\alpha \otimes \partial_\beta$ and $g = g_{\alpha \beta} dx^\alpha \otimes dx^\beta$, then $g^\alpha_\mu g_\xi^\alpha = \delta^\xi_\mu$. More details, if needed are given, e.g., in \cite{6}.}. The theory has been constructed on two assumptions. The first one is that the gravitational field is represented by a coframe $\{g^\alpha\}$, with $g^\alpha \in \sec \bigwedge^1 T^* M \hookrightarrow \sec \mathcal{C}(M, \hat{g})$ whose dynamics is encoded in Lagrangian density $\mathcal{L}^M$ (see Eq. 13 below) which is of the Yang-Mills type (containing moreover a gauge fixing term and an auto interaction term related to the “vorticity” of the fields). The theory is invariant under diffeomorphisms and under local Lorentz transformations of the coframe $\{g^\alpha\}$. The gravitational field couples universally with the matter fields and in such a way that the coupling resulting from the presence of energymomentum due to matter fields in some region of Minkowski spacetime distorts the Lorentz vacuum\footnote{A region of Minkowski spacetime void of matter fields will be called Lorentz vacuum.}. In much the same way that stresses in an elastic body produces plastic deformations in it \cite{15}. To present some additional details we need to introduce some notation. So, let $\{x^\nu\}$ be a set of global coordinates\footnote{If $\{x^\mu\}$ are global coordinate functions in the Einstein-Lorentz-Poincaré gauge, the coordinates of $\epsilon \in M$ in are $\{x^\mu\} := \{x^\mu(\epsilon)\}$, $\{x^\nu\} := \{x^\nu(\epsilon)\}$ and $x^\mu = \Lambda^\mu_\nu x^\nu$, with $\Lambda^\nu_\mu$ a proper and orthochronous Lorentz transformation.} for $M$ in the Einstein-Lorentz-Poincaré gauge associated to arbitrary inertial reference frame $\mathcal{I} = \partial / \partial x^0 \in \sec TM$. Let $\{\partial / \partial x^\nu\}$ be orthonormal basis for $TM$ and $\{\gamma^\mu = dx^\mu\}$ the corresponding dual basis for $T^* M$. We take $\gamma^\mu \in \sec \bigwedge^1 T^* M \hookrightarrow \sec \mathcal{C}(M, \hat{g})$. Of course, we have

$$\check{g} = \eta^{\alpha \beta} \gamma^\alpha \otimes \gamma^\beta,$$

and we recall that to each (non degenerated) metric tensor, say $\hat{g} \in \sec T^*_0 M$ there corresponds an unique invertible metric extensor field $\check{g} : \sec \bigwedge^1 T^* M \rightarrow \$
sec $\bigwedge^1 T^*M$, while the metric tensor $\hat{g} \in \text{sec} T_0^2 M$ is represented by the extensor field $\hat{g}^{-1}$. Our second assumption is that there are a plastic distortion field described by an extensor field $h : \text{sec} \bigwedge^1 T^*M \rightarrow \text{sec} \bigwedge^1 T^*M$ that distorts the cosmic lattice represented by the $\gamma^a$ producing the fields $g^a$ which may be used to introduce on on $T^*_0 M$ and $g \in \text{sec} T_0^2 M$ and such that the associated extensor fields are

$$g = h^1 h, \ldots g^{-1} = h^{-1} h^{-1}. \quad (2)$$

It is useful for application of the formalism we have in mind to suppose that the metric field $g$ is the pullback under a diffeomorphism of a metric field $\eta \in \text{sec} T_0^2 M$ according to the following scheme:

$$g = h^* \eta = \eta_{\alpha\beta} \theta^\alpha \otimes \theta^\beta \quad (3)$$

where

$$\theta^\alpha := h^* g^\alpha = h^{-1} (\gamma^\alpha) \quad (4)$$

$$\eta := \eta_{\alpha\beta} g^\alpha \otimes g^\beta \quad (5)$$

$$g(\theta^\alpha) \cdot \theta^\beta = h(\theta^\alpha) \cdot h(\theta^\beta) = \gamma^\alpha \cdot \gamma^\beta = \eta^{\alpha\beta} \quad (6)$$

Once the Levi-Civita connection $D$ of $g$ is introduced in the game, it is possible to show that the deformed gravitational fields $\theta^\alpha = h^* g^\alpha$ satisfy Maxwell like field equations (which follows from the variational principle [2]). Moreover, in our theory each nontrivial gravitational field configuration, i.e., one from which not all the $g^a$ are exact differentials can be interpreted as generating an effective Lorentzian spacetime $(M \simeq \mathbb{R}^4, g, D, \tau_g, \uparrow) \simeq (M \simeq \mathbb{R}^4, \eta, D = h^{-1} D, \tau_\eta, \uparrow)$ where $g$ satisfies Einstein equation 7.

Recall next that it is a physicist dream to construct a quantum theory for the gravitational field, where the quanta of the field are so called gravitons. In such (yet to be constructed) theory the gravitational field is supposed to be represented by a distribution valued symmetric field operator acting on the Hilbert space of the system. Classically that field is represented by a symmetric tensor (distribution)

$$h = h_{\alpha\beta} \theta^\alpha \otimes \theta^\beta \in \text{sec} T_0^2 M, \quad (7)$$

where $\theta^\alpha := dx^\mu \in \text{sec} \bigwedge^1 T^*M \rightarrow \text{sec} \mathcal{C}^\ell (M, \hat{g})$, with $\{x^\mu\}$ arbitrary coordinates covering $U \subset M$. Such a general field, as it is well known [11, 10] carries a direct sum of irreducible representations of the Lorentz group, one carrying spin two, one carrying spin one and two carrying spin zero. Now, consider the tensor field $h' \in \text{sec} T_1^1 M$

$$h' = h_{\beta\alpha} \frac{\partial}{\partial x^\alpha} \otimes dx^\beta \quad (8)$$

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7 Or by an effective teleparallel spacetime, see [2], the arXiv version.

8 Given arbitrary coordinate functions $\{x^\mu\}$ covering $U \subset M$ with coordinates $\{x^\mu\}$ such that $X^\mu (x) = x^\mu$, we write, as usual, $\{\frac{\partial}{\partial x^\mu}\}$ for the coordinate tangent vector fields and $\{dx^\mu\}$ for the coordinate cotangent covectors.  

3
If we impose that \( \text{div} h' = 0 \), i.e., the restriction
\[
\tilde{D}_\alpha h^\beta_\beta = 0, \tag{9}
\]
(where \( h^\alpha_\beta := \hat{g}^\gamma_\kappa h_{\kappa\beta}, \hat{g} = \hat{g}_{\alpha\beta}\theta^\alpha \otimes \theta^\beta \)) then the field \( h \) carries only the irreducible representations with spin 2 and one with spin zero of the Lorentz group. This restriction is the one appropriate for the description of gravitons with non null mass \( m \). Next we introduce the main purpose of this paper, which is to investigate (using the Clifford bundle formalism) the consequences of the ansatz

\[
h' = h = h^\beta_\beta \frac{\partial}{\partial x^\beta} \otimes dx^\beta = h^\alpha_\beta \frac{\partial}{\partial x^\alpha} \otimes dx^\beta = h^\alpha_\beta \gamma^\beta. \tag{10}
\]

We then show how to derive from our Lagrangian theory the exact wave equation for the graviton field and we obtain a reliable conservation law for the energy-momentum tensor of the gravitational plus the matter fields in Minkowski spacetime.

We also ask the question: does Eq. (9) fix any gauge condition for the field \( g \) of the effective Lorentzian spacetime structure \( (M \simeq \mathbb{R}^4, g, D, \tau_g, \uparrow) \) that is a well defined functional of the field \( h \) in our theory? We show that no gauge condition is fixed a priori, as it is the case in General Relativity (GR). Thus, writing \( g = g^\alpha_\beta \frac{\partial}{\partial x^\alpha} \otimes \frac{\partial}{\partial x^\beta} \) we do not need, e.g., to fix in our theory Logunov gauge condition

\[
\tilde{D}_\gamma \left( \sqrt{-\det(g_{\mu\nu})} g^{\gamma\kappa} \right) = 0, \tag{11}
\]

which, is indeed a result of a postulate in Logunov’s theory \[3, 4\]. Since Logunov thinks that Eq. (11) is very important, since according to him it fixes a unique solution of Einstein equations\[10\] once a matter distribution and a coordinate chart are given, thus eliminating (possible) ambiguities in predictions of experiments, we discuss briefly this issue.

2 The Wave Equation for the \( g^\alpha \)

We recall that the dynamics of the fields \( g^\alpha \) in a region of \( M \) is given by

\[
\mathcal{L} = \mathcal{L}_g^M + \mathcal{L}_m^M, \tag{12}
\]

where \( \mathcal{L}_m^M \) is the Lagrangian density of the matter fields and

\[
\mathcal{L}_g^M = -\frac{1}{2}dg^\alpha_\eta \wedge dg_{\eta\alpha} + \frac{1}{2} d\delta g^\alpha_\eta \wedge \delta g_{\eta\alpha} + \frac{1}{4} d(g^\alpha_\eta \wedge g_{\eta\alpha}) \wedge \star (d(g^\alpha_\eta \wedge g_{\eta\alpha})) + \frac{1}{4} m^2 g_{\alpha\eta} \wedge \star g^\alpha_\eta, \tag{13}
\]

\[\text{In}\ [7]\ we\ show\ explicitly\ how\ to\ determine\ the\ extensor\ field\ \( h \)\ once\ \( g \)\ is\ known\ in\ a\ given\ basis.\]

\[\text{Even\ for\ the\ case\ of\ a\ zero\ mass\ graviton.}\]
is invariant under local Lorentz transformations, which is a kind of gauge freedom, a crucial ingredient of our theory, as showed in [7].

The $g^\alpha$ couple universally to the matter fields in such a way that the energy momentum 1-form of the matter fields are given by

$$\star \eta^\alpha_M = \frac{\partial L^M_m}{\partial g^\alpha}. \quad (14)$$

Each one of the fields $g^\alpha$ in Eq. (13) resembles a potential of an electromagnetic field. Indeed, the first term is of the Yang-Mills type, the second term is a kind of gauge fixing term (analogous to the Lorenz condition for the gauge potential of the electromagnetic potential), and more important, the condition given by Eq. (9) is equivalent to

$$\delta \circ g^\alpha = 0. \quad (15)$$

Indeed, given the coordinates functions $\{x^\mu\}$ for $U \subset M$, $x^\mu (e^\beta) = x^\mu (g^\alpha = h^\alpha_\beta \gamma^\beta)$ and $\circ g^\alpha = \eta^\alpha_\beta \gamma^\alpha \otimes \gamma^\beta$ it is

$$\delta \circ g^\alpha = - \circ D \eta^\alpha = - \gamma^\kappa = \frac{\partial}{\partial x^\kappa} (h^\alpha_\beta \gamma^\beta)$$

$$= - (\circ D h^\alpha_\beta) \eta^\beta = \partial_\beta h^\alpha_\beta = 0 \quad (16)$$

Moreover, in general we also have

$$\delta \eta^\alpha = 0, \quad (17)$$

where for any $A_p \in \sec \Lambda^p T^* M \hookrightarrow \sec \mathcal{C}l(M, g)$, it is $\delta A_p := - \gamma^\kappa = \circ D \eta^\kappa = A_p$.

Also, the third term in the Lagrangian density is a self-interacting term, which is proportional to the square of the total ‘vorticity’ $\Omega = d g^\alpha \wedge g^\alpha$ associated to the 1-form fields $g^\alpha$. This shows that in the Lagrangian density the $g^\alpha$ does not couple with the energy-momentum tensor of the gravitational field, which according to the Lagrangian formalism is given by $\frac{\partial L^M}{\partial g^\alpha}$. We finally recall that as showed in details in [7], $\mathcal{L}_g := h^{*^{-1}} L^M_g$ has the same form of $\mathcal{L}^M_g$ with the substitutions $g^\alpha \mapsto \theta^\alpha$ and $\star \mapsto \star$ and differs (when the graviton mass is null) from the Einstein-Hilbert Lagrangian by an exact differential.

11We observe that the various coefficients in Eq. (13) have been selected in order for $L^M_g$ to be invariant under arbitrary local Lorentz transformations. This means, as the reader may verify that under the transformation $g^\alpha \mapsto u g^\alpha u^{-1}$, $u \in \sec \text{Spin}^e_3(M, g) \hookrightarrow \sec \mathcal{C}l(M, g)$, $L^M_g$ is invariant modulo an exact form.

12As usual we put $\circ D (h^\alpha_\beta dx^\beta \otimes \frac{\partial}{\partial x^\alpha}) := (\circ D h^\alpha_\beta) dx^\beta \otimes \frac{\partial}{\partial x^\alpha}$.

13On this respect see the discussion of [12].
Also, as showed in details, e.g., in \[2\] variation of \( \int L^M \) produces the following equations of motions

\[
d \eta^\alpha + \frac{1}{2} m^2 \star g^\alpha = - \star \nabla^\alpha \tag{18}
\]

with \(* t^\kappa \in \sec \wedge T^* M \hookrightarrow \mathcal{C}(T^* M, \mathfrak{g})\) and \(* S^\kappa \in \sec \wedge T^* M \hookrightarrow \mathcal{C}(T^* M, \mathfrak{g})\) given by

\[
* t^\kappa = \frac{\partial L^M}{\partial g^\kappa} = \frac{1}{2} \left[ (g^\kappa \eta \eta) \wedge \star d g^\alpha - d g^\alpha \wedge (g^\kappa \eta \eta \eta) \right] \\
+ \frac{1}{2} d \left( g^\kappa \eta \eta \eta \right) \wedge \star \eta \eta \eta + \frac{1}{2} \left( g^\kappa \eta \eta \eta \eta \eta \right) \wedge \star \eta \eta \eta + \frac{1}{2} d g^\alpha \wedge \eta \left( d g^\alpha \wedge g^\kappa \right) \\
- \frac{1}{2} d g^\alpha \wedge g^\alpha \wedge \left[ g^\kappa \eta \eta \eta \eta \eta \eta \right] - \frac{1}{2} \left[ g^\kappa \eta \eta \eta \eta \eta \eta \right] \wedge \eta \left( d g^\alpha \wedge g^\alpha \right), \tag{19}
\]

\[
* S^\kappa_M = \frac{\partial L^M}{\partial d g^\kappa} = - g^\kappa \wedge \star (d g^\alpha \wedge g^\kappa) + \frac{1}{2} g^\kappa \wedge \eta \left( d g^\alpha \wedge g^\alpha \right). \tag{20}
\]

For what follows we need also the following equivalent expression for the \(* S^\kappa \) obtained, e.g., in \[8\],

\[
* S^\kappa = \frac{1}{2} \left\{ - \left( (g^\kappa \eta) \eta \eta \eta \eta \eta \eta \right) \wedge \left( g^\kappa \eta \eta \eta \eta \eta \eta \right) \wedge \left( g^\kappa \eta \eta \eta \eta \eta \eta \right) \right\}. \tag{21}
\]

We write moreover

\[
* S^\kappa = - \frac{1}{2} \left[ d \star g^\kappa + \star R^\kappa \right] \tag{22}
\]

and insert this result in Eq.\((18)\) obtaining:

\[
- \frac{1}{2} d \star g^\kappa + \frac{1}{2} m^2 \star g^\kappa = - \left[ t^\kappa + \nabla^\kappa + \frac{1}{m^2} \star R^\kappa \right] \tag{23}
\]

Before proceeding we recall that we have the conservation law

\[
d \left[ t^\kappa + \nabla^\kappa + \frac{1}{m^2} \star R^\kappa - \frac{1}{2} m^2 \star g^\kappa \right] = 0, \tag{24}
\]

We now add the term \(- \frac{1}{2} \delta d \star g^\kappa \) to both members Eq.\((23)\) and next apply the operator \(*^{-1}\) to both sides of that equation, thus obtaining the equivalent equation:

\[
- \frac{1}{2} \delta d g^\kappa - \frac{1}{2} \delta \star g^\kappa + \frac{1}{2} m^2 \star g^\kappa = - \left[ t^\kappa + \nabla^\kappa + \frac{1}{2} \star R^\kappa \right]. \tag{25}
\]
We now recall the definition of the Hodge D’Alembertian, which in the Clifford bundle formalism is the square of the Dirac operator $\hat{\partial} := \hat{\partial}^\alpha \hat{D}_\alpha$ acting on sections of the Clifford bundle $[6]$, i.e.,

$$\hat{\partial}^\eta g^\kappa := (-\delta \eta - d \delta \eta)g^\kappa = \hat{\partial}^2 g^\kappa$$  \hspace{1cm} (26)

and recall moreover the following nontrivial decomposition $[6]$ of $\hat{\partial}^2$,

$$\hat{\partial}^2 g^\kappa = \hat{\partial} \cdot g^\kappa + \hat{\partial} \land \hat{\partial} g^\kappa,$$  \hspace{1cm} (27)

where $\Box := \hat{\partial} \cdot \hat{\partial}$ is the covariant D’Alembertian and $\hat{\partial} \land \hat{\partial}$ is the Ricci operator associated to the Levi-Civita connection $\hat{\nabla}$ of $\eta$. Moreover, we have

$$\hat{\partial} \land \hat{\partial} g^\kappa = R^\kappa = R^\kappa_\iota \theta^\iota,$$  \hspace{1cm} (28)

where $R^\kappa \in \sec \Lambda^1 T^* M \hookrightarrow \mathcal{C}^\ell (T^* M, \hat{g})$ are the Ricci 1-form fields and $R^\kappa_\iota$ are the components of the Ricci tensor.

This permits us to rewrite Eq.(25) as

$$\frac{1}{2} \Box g^\kappa + \frac{1}{2} m^2 g^\kappa = -\mathcal{T}^\kappa - \mathcal{T}^\kappa - \frac{1}{2} \delta \delta g^\kappa - R^\kappa = -T^\kappa.$$  \hspace{1cm} (29)

Thus, writing

$$\phi^\kappa_\iota := h^\kappa_\mu \frac{\partial \chi^\mu}{\partial x^\iota},$$  \hspace{1cm} (30)

$$g^\kappa = \phi^\kappa_\iota dx^\iota, T^\kappa = T^\kappa_\iota dx^\iota,$$  \hspace{1cm} (31)

and taking into account that $[6]$

$$\Box^n g^\kappa = (g^{\alpha \beta} D^\eta_\alpha D^\eta_\beta \phi^\kappa_\iota) dx^\iota$$  \hspace{1cm} (32)

we get from Eq.(29)

$$g^{\alpha \beta} D^\eta_\alpha D^\eta_\beta \phi^\kappa_\iota + m^2 \phi^\kappa_\iota = -2 T^\kappa_\iota,$$  \hspace{1cm} (33)

which is in our theory a possible form for the (covariant) equation for the graviton field in Minkowski spacetime. The last statement follows because $\hat{D}_\alpha$ can be easily be expressed in terms of the $D_\alpha$ using the formulas of the Appendix.

**Remark 1** If we recall that $[6]$ for an arbitrary $L \in \sec \Lambda^p T^* M$, $J \in \sec \Lambda^r T^* M$, $p \leq r$, we have:

$$dh^* L = h^* dL,$$

$$*d h^* L = h^* *d L,$$

$$h^* L \circ h^* J = h^* (L \circ J),$$  \hspace{1cm} (34)
we can immediately write from Eq.\textsuperscript{23} that

\[-\frac{1}{2} \frac{d \theta^\kappa}{\epsilon} + \frac{1}{2} m^2 \theta^\kappa = - \frac{1}{\epsilon} \left( t^\kappa + T^\kappa + m^2 \Theta^\kappa \right) \]  \hfill (35)

and also

\[\delta \left( T^\kappa + t^\kappa + \delta K^\kappa - \frac{m^2}{2} \Theta^\kappa \right) = 0, \]  \hfill (36)

where

\[\theta^\kappa = \epsilon \, g^\kappa, \quad t^\alpha = \epsilon \left( t^\kappa + \delta \eta^\kappa \right) = t^\kappa + \delta K^\kappa, \quad T^\alpha = \epsilon \, \Xi^\alpha. \]  \hfill (37)

Remark 2

Eq.\textsuperscript{36} express as we already anticipated a reliable conservation law for the total energy-momentum of the matter plus the gravitational field. However, take notice that in GR this result depends on the fixing of a cotetrad basis and changing it by a local Lorentz transformation changes accordingly the energy-momentum tensor of the gravitational field. In fact this last result has already been known since the work\textsuperscript{14} of Møller\textsuperscript{11}. Moreover, we see that imposing the Lorenz type gauge \(\delta \eta^\kappa = 0\) to the dynamic gravitational fields amounts to exclude the graviton energy density from the conservation law.

3 Which Gauge to Use for \(g\) in the Effective Lorentzian Spacetime?

We already recalled that our Lagrangian density differs for the Einstein-Hilbert Lagrangian by an exact form. But it can also be written as:

\[\mathcal{L}_g = -\frac{1}{2} (d\theta^\alpha \wedge \theta^\beta) \wedge \star (d\theta^\alpha \wedge \theta^\beta) + \frac{1}{4} d\theta^\alpha \wedge \theta^\alpha \wedge \star (d\theta^\alpha \wedge \theta^\beta) \]  \hfill (38)

which can be shown to be equivalent (modulus an exact differential) to

\[\mathcal{L}_g = -\frac{1}{2} (d\varphi^\alpha \wedge \varphi^\beta) \wedge \star (d\varphi^\alpha \wedge \varphi^\beta) + \frac{1}{4} d\varphi^\alpha \wedge \varphi^\alpha \wedge \star (d\varphi^\alpha \wedge \varphi^\beta) \]  \hfill (39)

where \(\{\varphi^\alpha\}\) is an an arbitrary coframe basis, not necessarily a \(g\) orthonormal one.

This permits us\textsuperscript{2} to obtain an equation analogous to Eq.\textsuperscript{18}, i.e.,

\[d \star S^\alpha + \frac{1}{\epsilon} t^\alpha + \frac{1}{2} m^2 \star \varphi^\alpha = - \star T^\alpha. \]  \hfill (40)

\textsuperscript{14}Which however did not use the present crystal clear formalism.
So, let us examine the structure of Eq. (40) in a coordinate basis \( \{ \vartheta^\mu = dx^\mu \} \). We immediately see that a conservation law (distinct from the previous one established above) in the effective Lorentzian spacetime structure (excluding the energy associated with the graviton mass) exists for \( \star (T^\mu + t^\mu) \) if

\[
\delta \vartheta^\mu = 0.
\] (41)

This of course, implies that

\[
\diamondsuit x^\mu = -d \delta x^\mu - \delta dx^\mu = 0,
\] (42)

i.e., the coordinates must be harmonic.

Now, if

\[
\begin{align*}
\delta \vartheta^\mu &= -\vartheta^\nu \varpi^\alpha \vartheta^\alpha \gamma^\nu \vartheta^\alpha \\
\diamondsuit x^\mu &= 0 \implies \Gamma^\mu_{\nu\alpha} g^{\nu\alpha} = 0
\end{align*}
\] (43)

we have

\[
\delta \vartheta^\mu = -\vartheta^\nu \partial_{x^\nu} \vartheta^\mu = -\vartheta^\nu \varpi^\alpha \vartheta^\alpha (\Gamma^\nu_{\nu\alpha} \vartheta^\alpha) = -\vartheta^\nu \varpi^\alpha \vartheta^\alpha = 0,
\] (44)

\[
\nabla x^\mu = 0 \implies \Gamma^\mu_{\nu\alpha} g^{\nu\alpha} = 0
\] (45)

Now, if \( \Gamma^\mu_{\nu\alpha} g^{\nu\alpha} = 0 \) we have

\[
\begin{align*}
D_\mu (\sqrt{-\det (g_{\mu\nu}) g^{\mu\nu}}) &= \frac{\partial}{\partial x^\mu} \left( \sqrt{-\det (g_{\mu\nu}) g^{\mu\nu}} \right) + \Gamma^\nu_{\mu\alpha} \sqrt{-\det (g_{\mu\nu}) g^{\mu\alpha}} \\
&= \frac{\partial}{\partial x^\mu} \left( \sqrt{-\det (g_{\mu\nu}) g^{\mu\nu}} \right) + \Gamma^\nu_{\mu\alpha} \sqrt{-\det (g_{\mu\nu}) g^{\mu\alpha}} \\
&= D_\mu \left( \sqrt{-\det (g_{\mu\nu}) g^{\mu\nu}} \right) + \Gamma^\nu_{\mu\alpha} \sqrt{-\det (g_{\mu\nu}) g^{\mu\alpha}}
\end{align*}
\]

and since \( D_\mu (\sqrt{-\det (g_{\mu\nu}) g^{\mu\nu}}) = 0 \) we get that

\[
\diamondsuit x^\mu = 0 \implies \nabla_\mu \left( \sqrt{-\det (g_{\mu\nu}) g^{\mu\nu}} \right) = \Gamma^\nu_{\mu\alpha} \sqrt{-\det (g_{\mu\nu}) g^{\mu\alpha}} = 0.
\] (46)

In particular in a coordinate basis \( \{ \gamma^\mu = dx^\mu \} \), where \( \{ x^\mu \} \) are global coordinates for \( M \) in Einstein-Lorentz-Poincaré gauge where the connection coefficients are \( \Gamma^\nu_{\mu\alpha} = 0 \) (see Eq. \( 10 \)) where \( \varpi^\alpha \vartheta^\alpha \gamma^\nu \vartheta^\alpha \) we have

\[

\diamondsuit x^\mu = 0 \implies \partial_\mu (\sqrt{-\det (g_{\mu\nu}) g^{\mu\nu}}) = \frac{\partial}{\partial x^\mu} \left( \sqrt{-\det (g_{\mu\nu}) g^{\mu\nu}} \right) = 0.
\]

15 Keep in mind that \( \varpi^\alpha = g_{\alpha\beta} dx^\beta \).
16 Recall that in GR Eq. (10) implies in a pseudo conservation law because in that theory (without a Minkowski spacetime interpretation, as here) \( S^\alpha \) are expressed in terms of connection-forms of the Levi-Civita connection of \( g \) and thus are not indexed forms. Details may be found in [8].
17 Recall that the energy-momentum conservation law of any Lorentz invariant field theory is unambiguously formulated global coordinates in Einstein-Lorentz-Poincaré gauge.
Also, for arbitrary non-harmonic coordinates functions \( \{ x^\mu \} \) we get (\( D_\mu \frac{\partial}{\partial x^\mu} = \Gamma^\kappa_\mu_\nu \frac{\partial}{\partial x^\nu} \), \( \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial x^\mu} \)) that
\[
\hat{\nabla} x^\mu = -\Gamma^\mu_\alpha_\nu g^\alpha_\nu = -\hat{\Gamma}^\mu_\alpha_\nu g^\alpha_\nu.
\]

(47)

Now, observe that the so-called Logunov gauge condition (Eq. (11))
\[
\hat{D}_\mu (\sqrt{-\det(g_{\mu\nu})} g^{\mu\nu}) = 0.
\]
does not imply that the coordinates are harmonic ones, for we have (using the formulas in the Appendix)
\[
\hat{D}_\mu (\sqrt{-\det(g_{\mu\nu})} g^{\mu\nu}) = D_\mu (\sqrt{-\det(g_{\mu\nu})} g^{\mu\nu}) + K^{\nu}_\mu_\kappa \sqrt{-\det(g_{\mu\nu})} g^{\mu\kappa}
\]
\[
= K^{\nu}_\mu_\kappa \sqrt{-\det(g_{\mu\nu})} g^{\mu\kappa}
\]
(48)

and thus
\[
\hat{D}_\mu (\sqrt{-\det(g_{\mu\nu})} g^{\mu\nu}) = 0 \Rightarrow K^{\nu}_\mu_\kappa \sqrt{-\det(g_{\mu\nu})} g^{\mu\kappa} = 0,
\]
(49)

and we see that under those conditions \(^{18}\) the allowed coordinate functions must always satisfy the constraint:
\[
\hat{\nabla} x^\mu = -\Gamma^\mu_\alpha_\nu g^\alpha_\nu = -\hat{\Gamma}^\mu_\alpha_\nu g^\alpha_\nu.
\]

(50)

Finally we comment that Logunov \(^{[3, 4]}\) imposed Eq. (11) as a gauge condition in his theory because he postulated (differently from what is the case here, see below)) that the relation between \( g^{\alpha_\beta}_{L} \) and the gravitational field \( h_{\alpha_\beta} \) is given by
\[
\begin{align*}
\zeta_{L}^{\alpha_\beta} := g^{\alpha_\beta} + h_{\alpha_\beta}.
\end{align*}
\]
(51)

Now, the second member of Eq. (51) implies immediately taking into account Eq. (52) that \( \hat{D}_\alpha (\zeta^{\alpha_\beta}_{L}) = 0 \) and thus from the second line in Eq. (76) (in Appendix) we get that \( \hat{D}_\alpha \left( \sqrt{-\det(g_{L\alpha_\beta})} g^{\alpha_\beta}_{L} \right) = 0. \)

In our theory, defining \( \{ \vartheta_\mu \} \) as the reciprocal basis of \( \{ \partial^\mu \} \) relative to \( g \), i.e.,
\[
g(\vartheta_\mu, \vartheta_\nu) = \delta^\nu_\mu,
\]
we can write directly from Eq. (52) that
\[
g = \eta_{\alpha_\beta} h^{-1}(\gamma_\alpha) \otimes h^{-1}(\gamma_\beta)
\]
\[
= g^{\alpha_\kappa} \vartheta_\kappa \otimes \vartheta_\nu,
\]
(52)

and clearly, differently from Logunov’s theory we have:
\[
\zeta^{\alpha_\kappa}_{L} \neq \zeta^{\alpha_\beta}_{L}
\]
(53)

\(^{18}\)The ones in Logunov theory.
from where it follows that there is no need to impose a priory any gauge condition (as it is the case in GR) for the "metric" field $g$ of the effective Lorentzian spacetime.

4 Conclusions

We showed that if gravitation is to be described by a massive graviton field living in Minkowski spacetime which is represented by a symmetric tensor field $h$ carrying the representations of spin two and zero of the Lorentz group and thus satisfying the gauge condition given by Eq. (9) then the effective Lorentzian spacetime structure that represents the gravitational field (under the ansatz given by Eq. (10)) of a given energy-momentum distribution is such that the field $g$ solving the effective Einstein-Hilbert equation (with cosmological constant)—as it is the case in GR—does not need to satisfy a priory any fixed gauge. We showed moreover that the Logunov gauge condition $\hat{D}_\gamma \left( \sqrt{-\det(g_{\mu\nu})} g^{\gamma\kappa} \right) = 0$ (which in his theory is indeed a postulate) does not hold in general in our theory without ad hoc hypothesis. If such a gauge is postulated it implies that the allowed coordinate functions to the ones satisfying Eq. (50). Moreover, we proved that the imposition of the Lorenz type gauge $\delta g^{\theta_\kappa}_\eta = \delta \eta^\kappa_\eta$ to the dynamic gravitational fields amounts to exclude the graviton energy density from the energy-momentum conservation law.

Logunov thought that the importance of the gauge condition $\hat{D}_\gamma \left( \sqrt{-\det(g_{\mu\nu})} g^{\gamma\kappa} \right) = 0$ in determining the effective Lorentzian spacetime generated by an energy-momentum distribution can be seen from the following example \cite{3, 4}. Let \{t, r, \theta, \varphi\} be the usual spherical coordinates in Minkowski spacetime. If we try to solve the (effective) Einstein-Hilbert equations (in the zero mass graviton case) for the field generated by a point mass at the origin of the coordinate system we get immediately that the following "metric" fields are solutions of those equations,

$$g_s = \left( 1 - \frac{2m}{r} \right) dt \otimes dt - \left( 1 - \frac{2m}{r} \right)^{-1} dr \otimes dr - r^2 (d\theta \otimes d\theta + \sin^2 \theta d\varphi \otimes d\varphi), \quad (54)$$

and

$$g_i = \left( \frac{r + \lambda - m}{r + \lambda + m} \right) dt \otimes dt - \left( \frac{r + \lambda + m}{r + \lambda - m} \right) dr \otimes dr - (r + \lambda + m)^2 (d\theta \otimes d\theta + \sin^2 \theta d\varphi \otimes d\varphi), \quad (55)$$

with $\lambda$ an arbitrary real parameter. Now, both solutions have the same asymptotic behavior when $r \to \infty$. Which one should we use for the descriptions of physical processes? It is important to emphasize that both metrics even if expressed in the same coordinates are diffeomorphically equivalent since it is possible to perform a coordinate trasformation in Eq. (54) which makes it in the new variables to have the apperance of Eq. (55). If we take into account that
the meaning of the coordinates in each one are different because we can know what the spacetime labels mean, only after we fix a metric on it. Specifically this statement means that those labels are associated with physical distances and time lapses measured by ideal rods and clocks in different ways.

But Logunov thinks that $g_s$ and $g_i$ given in the same coordinate basis even if diffeomorphically equivalent are physically distinguished through experiments and so fixing one of them as the correct one implies in the existence of an additional theoretical criterion and such a criterion does not exists in GR. He claims that the metric $g_i$ when $\lambda = 0$ that satisfies the condition $\hat{D}_\gamma \left( \sqrt{-\det(g_{\mu\nu})} g^{\gamma\kappa} \right) = 0$ is the only one that fits correctly all known data on solar system experiments. Does the method used by astronomers methods for determining the coordinates of their probes always fix those coordinates as being the spherical coordinates of Minkowski spacetime and fix the metric to be $g_s$? It is hard to believe in that possibility...

A last comment is in order. We start our considerations by postulating that the distortion field $h$ is symmetric since it has been constructed from the symmetric tensor field $h$. However, from the general theory of plastic deformations of the Lorentz vacuum presented in [2] it is quite clear that we can construct symmetric metric tensor fields associated to non symmetric $h$ fields. This observation shows that the quantum theory of the gravitational field must be more complex than one where the $g$ field is supposed to arise from the existence of a symmetric graviton field. We will return to this issue in another publication.

Appendix

Let $(M, \hat{g}, \hat{D})$ and $(M, g, D)$ be the two Lorentzian structures on the same manifold $M$ such that

$$\hat{D}_{\hat{g}} = 0, D_{\hat{g}} = 0,$$

with the non-metricity of $D$ relative to $\hat{g}$ being given by:

$$Q := -D_{\hat{g}}.$$

Let moreover the connection coefficients of $\hat{D}$ and $D$ in arbitrary coordinates $\{x^\mu\}$ covering $U \subset M$ be:

$$\hat{D}_{\partial_\alpha} dx^\rho = -\hat{\Gamma}^\rho_{\alpha\beta} dx^\beta, D_{\partial_\alpha} dx^\rho = -\Gamma^\rho_{\alpha\beta} dx^\beta,$$

and

$$Q_{\alpha\beta\sigma} = -D_{\alpha\beta\sigma},$$

Define the components of the strain tensor of the connection $D$ by:

$$S^\rho_{\alpha\beta} = 2\Gamma^\rho_{\alpha\beta} - 2\hat{\Gamma}^\rho_{\alpha\beta}.$$

19 In [2] it is directly derived from the variational principle and an appropriate Lagrangian the field equations for the plastic extensor field $h$.

20 More general formulas relating two arbitrary general connections may be found, e.g., in [6, 13].
Then
\[ Q_{\alpha\beta\sigma} = \frac{1}{2} (\hat{g}_{\mu\sigma} S_{\alpha\beta}^{\mu} + \hat{g}_{\beta\mu} S_{\alpha\sigma}^{\mu}), \] (60)
\[ S_{\rho}^{\alpha\beta} = \hat{g}^{\rho\sigma} (Q_{\alpha\beta\sigma} + Q_{\beta\sigma\alpha} - Q_{\sigma\alpha\beta}). \] (61)

Also,
\[ Q_{\alpha\beta\sigma} + Q_{\sigma\alpha\beta} + Q_{\beta\sigma\alpha} = S_{\alpha\beta\sigma} + S_{\sigma\alpha\beta} + S_{\beta\sigma\alpha}. \] (62)

where \( S_{\alpha\beta\sigma} = \hat{g}_{\rho\sigma} S_{\rho}^{\alpha\beta}. \)

Putting
\[ K_{\alpha\beta}^{\rho} = \frac{1}{2} S_{\rho}^{\alpha\beta}, \] (63)
we have
\[ K_{\alpha\beta}^{\rho} = \frac{1}{2} \hat{g}^{\rho\sigma} (D_{\alpha} \hat{g}_{\beta\sigma} + D_{\beta} \hat{g}_{\sigma\alpha} - D_{\sigma} \hat{g}_{\alpha\beta}) \] (64)

The relation between the curvature tensor \( R_{\mu}^{\rho} \alpha\beta \) associated with the connection \( D \) and the Riemann curvature tensor \( \hat{R}_{\mu}^{\rho} \alpha\beta \) of the Levi-Civita connection \( \hat{D} \) associated with the metric \( \hat{g} \) are given by:
\[ R_{\mu}^{\rho} \alpha\beta = \hat{R}_{\mu}^{\rho} \alpha\beta + J_{\mu}^{\rho} \alpha\beta, \] (65)
where:
\[ J_{\mu}^{\rho} \alpha\beta = \frac{1}{2} \hat{g}^{\rho\sigma} (D_{\alpha} \hat{g}_{\beta\sigma} + D_{\beta} \hat{g}_{\sigma\alpha} - D_{\sigma} \hat{g}_{\alpha\beta}). \] (66)

Multiplying both sides of Eq. (65) by \( \frac{1}{2} \theta^{\alpha} \wedge \theta^{\beta} \) we get for the curvature 2-forms of the two connections \( D \) and \( \hat{D} \):
\[ \mathcal{R}_{\mu}^{\rho} = \hat{\mathcal{R}}_{\mu}^{\rho} + \mathcal{J}_{\mu}^{\rho}, \] (67)
where we have written:
\[ \mathcal{J}_{\mu}^{\rho} = \frac{1}{2} J_{\mu}^{\rho} \alpha\beta \theta^{\alpha} \wedge \theta^{\beta}. \] (68)

The relation between the Ricci tensors \( \text{of the connections} \) \( D \) and \( \hat{D} \) is:
\[ R_{\mu\alpha} = \hat{R}_{\mu\alpha} + J_{\mu\alpha}, \] (69)
with
\[ J_{\mu\alpha} = \hat{D}_{\alpha} K_{\rho\mu}^{\rho} - \hat{D}_{\mu} K_{\rho\alpha}^{\rho} + K_{\rho\sigma}^{\rho} K_{\rho\mu}^{\sigma} - K_{\rho\mu}^{\rho} K_{\rho\sigma}^{\sigma} \] (70)
Recall that the connection \( \hat{D} \) plays with respect to the tensor field \( g \) a role analogous to that played by the connection \( D \) with respect to the metric tensor \( g \) and in consequence we shall have similar equations relating these two pairs of

\[ \text{For the Ricci tensor} \text{ } \text{Ricci } = R_{\mu\alpha} dx^\mu \otimes dx^\nu, \text{ } \text{we use the convention } R_{\mu\alpha} := R_{\mu}^{\rho} \alpha\rho. \]
objects. In particular, the strain of \( \hat{D} \) with respect to \( g \) equals the negative of the strain of \( D \) with respect to \( \hat{g} \), since we have:

\[
S^\rho_{\alpha \beta} = \Gamma^\rho_{\alpha \beta} + \Gamma^\rho_{\beta \alpha} - \hat{b}^\rho_{\alpha \beta} = -(\tilde{\Gamma}^\rho_{\alpha \beta} + \tilde{\Gamma}^\rho_{\beta \alpha} - b^\rho_{\alpha \beta}),
\]

where \( b^\rho_{\alpha \beta} = \tilde{\Gamma}^\rho_{\alpha \beta} + \Gamma^\rho_{\beta \alpha} \) and \( \hat{b}^\rho_{\alpha \beta} = \Gamma^\rho_{\alpha \beta} + \Gamma^\rho_{\beta \alpha} \). Furthermore, we have that:

\[
R^\rho_{\alpha \beta} = \frac{1}{2} \hat{g}^{\rho \sigma} (D_\alpha \hat{g}_{\beta \sigma} + D_\beta \hat{g}_{\alpha \sigma} - D_\sigma \hat{g}_{\alpha \beta})
= \frac{1}{2} \hat{g}^{\rho \sigma} (\hat{D}_\alpha \hat{g}_{\beta \sigma} + \hat{D}_\beta \hat{g}_{\alpha \sigma} - \hat{D}_\sigma \hat{g}_{\alpha \beta}).
\]

(71)

Now, recall that given arbitrary coordinates \( \{x^\alpha\} \) covering \( U \subset M \) and \( \{x'^\alpha\} \) covering \( V \subset M \) \((U \cap V \neq \emptyset)\) a set of functions \( A^\alpha_{\nu_1 \ldots \nu_s} (x^\alpha) \) is said to be the components of a relative tensor field of type \( \begin{pmatrix} r \\ s \end{pmatrix} \) and weight \( \omega \) if under the coordinate transformation \( x^\alpha \mapsto x'^{\beta} \) with Jacobian \( J = \det \left( \frac{\partial x'^\alpha}{\partial x^\alpha} \right) \) these functions transform as \( \ref{9} \)

\[
A^\alpha_{\nu_1 \ldots \nu_s} (x'^{\beta}) = J^\nu \frac{\partial x^\alpha}{\partial x'^\nu} \frac{\partial x'^\nu}{\partial x^\alpha} \frac{\partial x'^{\nu_1}}{\partial x^\nu} \ldots \frac{\partial x'^{\nu_s}}{\partial x^\nu} A^\nu_{\nu_1 \ldots \nu_s} (x^\alpha).
\]

(72)

The covariant derivative of a relative tensor field relative to a given arbitrary connection \( \nabla \) defined in \( M \) such that \( \nabla_{\alpha} dx^\mu = -L^\mu_{\nu \alpha} dx^\nu \) is given by

\[
\nabla_{\kappa} A^\mu_{\nu_1 \ldots \nu_s} = \frac{\partial}{\partial x^\kappa} A^\mu_{\nu_1 \ldots \nu_s} + L^\mu_{\nu \kappa} A^\nu_{\nu_1 \ldots \nu_s} - L^\kappa_{\nu \nu_1 \ldots \nu_s -1} \ldots \nu_s - \omega L^\kappa_{\nu \alpha} A^\nu_{\nu_1 \ldots \nu_s}.
\]

(73)

In particular we have

\[
\hat{D}_\alpha (\sqrt{-\det(\hat{g}_{\mu \nu})}) = \partial_\gamma (\sqrt{-\det(\hat{g}_{\mu \nu})}) - \tilde{\Gamma}^\rho_{\gamma \mu} \sqrt{-\det(\hat{g}_{\mu \nu})} = 0,
\]

\[
\hat{D}_\alpha \left( \frac{1}{\sqrt{-\det(\hat{g}_{\mu \nu})}} \right) = \partial_\gamma \left( \frac{1}{\sqrt{-\det(\hat{g}_{\mu \nu})}} \right) + \tilde{\Gamma}^\rho_{\gamma \mu} \frac{1}{\sqrt{-\det(\hat{g}_{\mu \nu})}} = 0,
\]

\[
D_\alpha (\sqrt{-\det(g_{\mu \nu})}) = \partial_\gamma (\sqrt{-\det(g_{\mu \nu})}) - \Gamma^\rho_{\gamma \mu} \sqrt{-\det(g_{\mu \nu})} = 0,
\]

\[
\hat{D}_\alpha \left( \frac{1}{\sqrt{-\det(g_{\mu \nu})}} \right) = \partial_\gamma \left( \frac{1}{\sqrt{-\det(g_{\mu \nu})}} \right) + \Gamma^\rho_{\gamma \mu} \frac{1}{\sqrt{-\det(g_{\mu \nu})}} = 0.
\]

(74)

Now, if we define

\[
\kappa := \frac{\sqrt{\det(g_{\mu \nu})}}{\det(g_{\mu \nu})},
\]

(75)
we can easily prove the following relations:

\[
K_{\rho\sigma}^\alpha = -\frac{1}{2} \delta^{\alpha\beta} D_\sigma g_{\alpha\beta} = \frac{1}{2} g^{\alpha\beta} \tilde{D}_\sigma g_{\alpha\beta} = \frac{1}{\kappa} \partial_\sigma (\kappa)
\]

\[
\delta^{\alpha\beta} K_{\alpha\beta}^\rho = \frac{1}{\kappa} \tilde{D}_\rho (\kappa g^{\rho\sigma}) = -\frac{1}{\sqrt{-\det(g_{\mu\nu})}} \tilde{D}_\rho (\sqrt{-\det(g_{\mu\nu})} g^{\rho\sigma})
\] (76)

\[
\delta^{\alpha\beta} K_{\alpha\beta}^\rho = \frac{1}{\kappa} \frac{1}{\kappa-1} \tilde{D}_\rho (\kappa^{-1} \tilde{g}^{\rho\sigma}).
\]

Another useful formulas valid for our particular connections \( \tilde{D} \) and \( D \) are:

\[
\tilde{D}_\alpha K_{\rho\sigma}^\beta = \tilde{D}_\beta K_{\rho\sigma}^\alpha
\]

\[
D_\alpha K_{\rho\sigma}^\beta = D_\beta K_{\rho\sigma}^\alpha.
\] (77)

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