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The boundary conditions of degressive proportionality

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Abstract

One of the basic socio-economic problems is the problem of distribution of benefits and dues. The historically shaped principles of justice direct the solution to the problem of the Aristotelian rule of proportionality. But applying this rule not always gives the expected, satisfactory solutions. This occurs, for instance, regarding the allocation of seats among the member states in the European Parliament. For this reason, the principle of degressive proportionality was legitimized. Although the definition appears in the context of the European Parliament, the application of the rule distribution may be much wider due to certain universal priorities it embodies. The principle of degressive proportionality, however, has a significant drawback, namely, it does not clearly provide a exact division. This lack of clarity results in the fact that within the rule there may appear many different proposals. The article is an attempt to evaluate some of the proposals by the criterion of drawing nearer the most, under certain assumptions, degressively proportional allocation to the proportional one and proposing rules for determining the boundary conditions for such allocation.

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1. Introduction

The division of property most commonly used in practice is derived from Aristotle principle of proportional allocation. It is unambiguous, and only in the case of indivisible goods there remains a

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problem of rounding the obtained values to integers. The solution to this problem can be found in many electoral regulations in force in the past and present. Numerous studies in this area, especially by American politicians, lived to see the formal rules and act as the legal standards which serve as the key to the distribution of seats in collegial bodies (Young, 2003).

Proportions of representation shall be set as nearly always in accordance with an aspect ratio of the population under the respective constituencies. It turns out that for large disparities in the population the principle of proportional allocation does not work. In such a situation it may happen that the smallest districts, with some methods of rounding the actual proportions to integer numbers, will be deprived of representation. Such a situation is the case, for example, in the current demographic structure of countries in the European Union. Some of the known methods of proportional allocation used in the distribution of seats in European Parliament would deprive Malta of any representation. This means that not always the principle of proportional division leads to the desired results. Therefore, in some cases one must also apply the solutions different from the classical rules of Aristotle (Lasier, 2012).

2. Degressive proportionality

The principle of degressive proportionality has been legally sanctioned in the art. Paragraph 9a. 2 of the Treaty of Lisbon. Not wanting to deprive the least populated country, which is Malta, of its representation at the collegiate body it was decided to move away from proportional in relation to population, manner of allocating seats (The Treaty of Lisbon, 2010): *The European Parliament shall be composed of representatives of the Union’s citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats.*

Guidelines for the understanding degressive proportionality can be found in the annex to the draft of resolution the European Parliament (Lamassoure, Severin, 2007). The rules include in it clarify the idea contained in the Treaty. The first of them, the principle of fair distribution is that a country with a larger number of people cannot get less seats than less populated country. The second, defined as the principle of relative proportionality, states that the bigger the country, the greater number of voters should its member of Parliament represent.

Taking this into account it can be concluded that the sequence $s_1, s_2, \ldots, s_n$ is degressively proportional with respect to $p_1, p_2, \ldots, p_n$ if and only if:

\begin{align}
\text{if } & p_1 \geq p_2 \geq \ldots \geq p_n, \text{ then } s_1 \geq s_2 \geq \ldots \geq s_n, \\
\text{and } & s_1 \geq s_2 \geq \ldots \geq s_n, \\
\text{if } & p_1 \geq p_2 \geq \ldots \geq p_n, \text{ then } \frac{p_1}{s_1} > \frac{p_2}{s_2} > \ldots > \frac{p_n}{s_n}.
\end{align}

This is clearly not the only formulation of the definition of degressive proportionality. The terms of the Treaty of Lisbon also allow for other possibilities for understanding this concept (Florek, 2012; Łyko, 2012), but the definition presented above is most common in the literature.

The said resolution also contains additional guidance concerning the distribution of seats between the Member States of the European Union. Inter alia, it stipulates that: “The minimum and maximum numbers set by the Treaty must be fully utilised to ensure that the allocation of seats in the European Parliament reflects as closely as possible the range of populations of the Member States.”
Given this recommendation, one should look for solutions among those that degressively proportionately divide the number of seven hundred and fifty or seven hundred fifty-one seats in such a way that so that the smallest country received six, and the largest ninety-six seats.

3. Proposed divisions

Of course, conditions that were formulated in this way do not guarantee the unambiguity of the division. For this reason one introduces, explicitly or not, in any concrete proposals for solutions additional criteria to eliminate or minimize ambiguities. In the literature there are a number of proposals of such criteria leading to different allocations of seats in European Parliament. Because of the wording used in the Treaty authors divide H = 750 or H = 751 seats.

Most natural way of the degressively proportional allocation of seats that takes into account the given boundary conditions is the use of integer linear programming methods (Serafini, 2012). Another approach is the use of spline methods (Ramirez et al, 2012). Finally, the next similar methods are the ones proposed by Ramirez (Ramirez et al, 2006) and Pukelsheim (Pukelsheim, 2007). They use a classic shift of proportionality and sticking in order to provide allocations that meet all of the boundary conditions. The results of these methods using the three classic rules of rounding to integers are given in the work of Misztal (Misztal, 2012a).

Part of the proposal does not include recommendations for the allocations to achieve the boundary conditions described in the Treaty. It is then said that in the place of equality there occur appropriate inequalities, recorded in the art. 9a. Among these allocations it is worth to mention an allocation created on the basis of the principle of reverse recursion (Misztal, 2012b), or an allocation that meets the demand for demographic stability (Lyko et al, 2010).

4. The boundary conditions

Determining the smallest and largest number of allocated seats itself generates a certain degression of the division. Indeed, denoting by m and M, respectively, the lowest and highest number of seats one determines at the same time smallest and largest number of voters represented by one Member of Parliament. These are the \( \frac{L}{m} \) and \( \frac{L}{M} \), where \( p = p_m \) and \( P = p_1 \). Condition (2) of the definition of degressive proportionality says that for every \( 1 < i < n \), \( \frac{P}{M} \) must belong to the interval \( \left( \frac{p}{m}, \frac{P}{P} \right) \), therefore assuming that the degression of the allocation is merely a consequence of boundary conditions one shall map the structure of the \( p_i \) proportionally on the \( s_i \). This solution seems to be very objective. Only the requirements of the smallest and largest number of seats are assumed and further construction is generally the consequence of adopting an undisputed rule of proportional division. Therefore, the boundary conditions become the only controllable parameters in performing the allocation.

The assumption of the proportional mapping of two intervals of different lengths implies the usage in the construction the theorem of Thales. The idea is to make the proportions in the section that represents the population the same as in the section presenting the allocated seats. Therefore, the problem boils down to defining the function of allocation.

\[ A_{prop}[p, P] \rightarrow [m, M] \] having the property that \( A_{prop}(p) = m \) \( A_{prop}(P) = M \) and satisfying condition

\[
\frac{A_{prop}(t) - m}{M - m} = \frac{t - p}{P - p} \quad \text{for each} \quad t \in [p, P].
\]

Transforming this equality, it is easy to see that the
What is more

\[ A_{\text{prop}}(t) = m + (t - p) \frac{M - m}{p - p}. \]

This function was considered in the work of Słomczyński and Życzkowski (Słomczyński, Życzkowski, 2012). Therefore, the sequence \( A(p_1), A(p_2), \ldots, A(p_n) \) is degressively proportional with respect to \( p_1, p_2, \ldots, p_n \), but it does not need to be a sequence whose elements are integers. According to the Cambridge Compromise it is sufficient that the proposed solution satisfies the condition of degressive proportionality before rounding to integers, so it may be a way to indicate the solution of the problem of allocation.

But there remains the problem of satisfying the third boundary condition – that is ensuring that the sum of allocated seats equals \( H \). In general,

\[ A(p_1) + A(p_2) + \ldots + A(p_n) \neq H \]

and

\[ [A(p_1)] + [A(p_2)] + \ldots + [A(p_n)] \neq H, \]

where \([A(p_i)]\) is a rounding to integers of the number \( A(p_i) \). In particular, the current demographic structure of European Union member states is that, taking the population of the countries as a sequence \( p_i \), even after rounding up of the numbers \( A(p_i) \) is the sum is less than 750.

Table 1. Values of allocation function \( A_{\text{prop}} \) in the period 2006-2011

| Year | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|------|------|------|------|------|------|------|
| 6    | 6,0703034 | 6,0751331 | 6,0808704 | 6,0881272 | 6,0969734 | 6,1042623 |
| 6,3965078 | 6,4075185 | 6,4169311 | 6,4227782 | 6,4299135 | 6,4280329 |
| 7,0309392 | 7,0269429 | 7,0238426 | 7,0223535 | 7,0237144 | 7,0208762 |
| 7,7535833 | 7,7609101 | 7,7602057 | 7,7856356 | 7,8053591 | 7,8065201 |
| 8,0730996 | 8,0586074 | 8,0469305 | 8,0381689 | 8,0280682 | 8,0059029 |
| 9,2894696 | 9,271221 | 9,2520964 | 9,238972 | 9,2230845 | 9,1281949 |
| 10,17251 | 10,290525 | 10,39072 | 10,452549 | 10,482404 | 10,496173 |
| 11,32166 | 11,350245 | 11,379913 | 11,419172 | 11,459473 | 11,48589 |
| 11,468235 | 11,478456 | 11,490492 | 11,513972 | 11,540748 | 11,55228 |
| 11,510232 | 11,537184 | 11,572775 | 11,623395 | 11,662181 | 11,690991 |
| 14,024052 | 13,989945 | 13,953977 | 13,934486 | 13,90585 | 13,842387 |
| 14,611612 | 14,653288 | 14,700263 | 14,760382 | 14,803308 | 14,837599 |
| 15,482126 | 15,565596 | 15,651155 | 15,754365 | 15,870854 | 15,956671 |
| 16,610874 | 16,61265 | 16,6 | 16,608852 | 16,615778 | 16,587572 |
| 16,80232 | 16,85552 | 16,96935 | 17,090426 | 17,16038 | 17,192907 |
| 17,087903 | 17,182244 | 17,2837 | 17,405401 | 17,528718 | 17,656435 |
| 17,151767 | 17,198244 | 17,229473 | 17,266599 | 17,305131 | 17,30822 |
| 17,761312 | 17,82747 | 17,885389 | 17,965025 | 18,043158 | 18,052823 |
Taking the above into consideration, the boundary conditions set in this way make finding the allocation that would satisfy them all and in the described way be most similar to proportional representation impossible. So to maintain this idea of allocation it is imperative to either change the boundary conditions, or under current assumptions seek for the allocation, corresponding to the highest extent, with a set specific criterion, to the sequence of $A(p_i)$. Such a criterion may be, for example, to minimize the sum of squared errors i.e. the sum of

$$\left( A_{\text{prop}}(p_1) - s_1 \right)^2 + \left( A_{\text{prop}}(p_2) - s_2 \right)^2 + \ldots + \left( A_{\text{prop}}(p_n) - s_n \right)^2,$$

where $s_1$, $s_2$, ..., $s_n$ is any, degressively proportional integer division with respect to the sequence $p_1$, $p_2$, ..., $p_n$ satisfying the conditions $s_1 = M$, $s_n = m$, $s_1 + s_2 + \ldots + s_n = H$.

Table 2. Square deviation of the allocation of selected proposals from the theoretical distribution generated by the function $A_{\text{prop}}$ determined on the basis of population in 2010

| NAQ   | DIQ   | LFS   | SFS   | PUK   | RAM   | MIS   |
|-------|-------|-------|-------|-------|-------|-------|
| 0.010871 | 0.010871 | 0.01087 | 0.010871 | 0.010871 | 0.010871 | 0.010871 |
| 0.183212 | 0.183212 | 0.183212 | 0.327146 | 0.183212 | 0.183212 | 0.183212 |
| 1.042188 | 1.042188 | 0.0004358 | 0.000436 | 0.000436 | 0.000436 | 1.042188 |
| 3.263515 | 3.263515 | 0.0374345 | 0.037434 | 0.037434 | 0.037434 | 0.650475 |
| 4.020396 | 4.020396 | 2.593E-05 | 0.989841 | 2.59E-05 | 2.59E-05 | 1.010211 |
| 9.785604 | 1.272824 | 0.760441 | 0.760044 | 0.016434 | 0.760044 | 0.016434 |
| 12.22323 | 0.246188 | 0.2538414 | 2.261495 | 0.246188 | 0.253841 | 0.253841 |
| 12.15143 | 0.236089 | 0.2643089 | 2.292529 | 0.264309 | 0.264309 | 0.264309 |
| 12.61869 | 0.305013 | 0.2004534 | 5.991334 | 0.200453 | 2.095894 | 0.200453 |
| 13.62342 | 0.477469 | 0.0954865 | 5.331522 | 0.095486 | 1.713504 | 0.095486 |
| 3.39439 | 0.024842 | 1.3400678 | 9.97052 | 1.340068 | 4.655294 | 4.655294 |
| 3.37767 | 0.026374 | 1.3511759 | 10.00078 | 1.351176 | 1.351176 | 4.675978 |
| 3.828563 | 0.001877 | 1.088534 | 9.261848 | 1.088534 | 4.175191 | 4.175191 |
| 2.520386 | 0.170097 | 1.9949519 | 11.64466 | 1.994952 | 5.819807 | 5.819807 |
| 1.423028 | 0.037213 | 0.6513983 | 7.879768 | 0.651398 | 3.265583 | 3.265583 |
2.743777 0.430907 0.1180368 5.492296 0.118037 5.492296 1.805167
1.711439 0.094999 2.8621202 13.62924 0.47856 7.245681 2.86212
1.108436 1.108436 0.897144 8.685853 0.897144 3.791499 0.897144
1.06433 0.001003 4.127658 9.190986 4.127658 9.190986 4.127658
7.654565 3.121188 7.6545645 7.654565 7.654565 22.72132 7.654565
84.49138 51.7237 84.49138 51.7237 84.49138 51.7237
153.5533 88.20331 153.5533 88.20331 153.5533 88.20331
337.6779 153.1652 337.6779 153.1652 337.6779 153.1652
337.6029 153.1152 337.6029 153.1152 337.6029 153.1152
341.6172 155.8225 341.6172 155.8225 341.6172 155.8225

Total 1352.691
Total 618.105
Total 223.775
Total 118.669
Total 243.152
Total 136.352
Total 212.010

NAQ = natural quotas, DIQ = divisor quotas (Serafini, 2012), LFS = large states favoring spline, SFS = small states favoring spline (Ramirez et al, 2012), RAM = (Ramirez et al, 2006), PUK = (Pukelsheim, 2007), MIS = (Misztal 2012b).

Source: own research.

It is evident that in this sense, the best proposition is the small states favoring spline division suggested by the Ramirez (Ramirez et al, 2012). Adopting the second idea i.e. the change in boundary conditions one can notice that the best solutions, with the current structure of population and keeping the total number of seats on the level \(H = 751\), is obtained for \(m = 8, M = 99\). Then rounding down the sequence \(A(p_i)\) for instance for the population in 2010 gives the sequence \(s_i\) to which \(s_1 = M\), \(s_n = m\) and \(s_1 + s_2 + ... + s_n = H\). In fact it is not degressively proportional, but it meets the conditions of Cambridge Compromise.

5. Conclusions

The principle of degressive distribution of seats in the European Parliament contained in the Lisbon Treaty does not specify clearly the rules for allocation. There are plenty of solutions that meet the specified conditions, which allows for different interpretations, and consequently, political negotiations. To avoid this, objective, clear-cut solutions should be pursued. One way to achieve this goal is to use the \(A_{\text{prop}}\) function in the allocation. Then you can unambiguously, with the accuracy of rounding to integers, determine the distribution that meets the conditions for allocation of a fixed number of seats to the smallest and the largest Member State. You cannot guarantee, however, the allocation of a predetermined total number of seats. Therefore, the application of the allocation function \(A_{\text{prop}}\) can be twofold. Firstly, its values can serve as a reference for the evaluation of specific proposals for the allocation of seats, which are presented in Table 2. Secondly, one can search for such specific boundary conditions which, with the current demographic structure, would be satisfied with no exceptions by the \(A_{\text{prop}}\) allocation function. One of such solutions is to distribute 751 seats among 27 Member States in a way that the smallest would get 8 and the largest 99 seats. It is just one of the proposals and that is why it seems that determining the appropriate boundary conditions is the key element in establishing the rules of degressive proportionality.

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