Enhancing the Reliability of Spectral Correlation Function with Distributed Computing

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Abstract. Various random time series used in signal processing systems are cyclostationary
due to the sinusoidal carriers, pulse trains, periodic motion, or physical phenomenon. The
cyclostationarity of the signal could be analysed by using the spectral correlation function
(SCF). However, SCF is considered high complex due to the 2-D functionality and the required
long observation time. The SCF could be computed in various methods however there are two
methods used in practice such as FFT accumulation method (FAM) and strip spectral
correlation algorithm (SSCA). This paper shows the benefit on the complexity and the
reliability due to the workload distribution of one processor over different cooperated
processors. The paper found that with increasing the reliability of the SCF, the number of the
cooperated processors to achieve the half of the maximum complexity will reduce.

1. Introduction

A random process \(X(t)\) is said to be wide-sense cyclostationary if its mean and autocorrelation
functions exhibit periodicity [1], [2] i.e.:
\[
m_X(t) = m_X(t + kT_0)
\]
(1)

\[
R_X(t_1,t_2) = R_X(t_1 + kT_0,t_2 + kT_0)
\]
(2)

Cyclostationarity is utilized essentially for the communication systems for recognition,
classification and detection [3]. Nevertheless, currently it is revolutionising the field of mechanical
signature analysis because of the hidden periodicity in the wide range of the mechanical systems such as
reciprocating mechanisms, gears, fans and electrical motors [4], [5], [6]. The cyclostationarity of
\(X(t)\) could be analysed by computing the spectral correlation function (SCF) that expressed as
following [7],

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\[
S_X^\alpha(f) = \lim_{\Delta f \to 0} \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} S_{X,t}^\alpha(t,f) \, dt
\]

(3)

where \( f \) is the spectral frequency, \( \alpha \) is the cyclic frequency, \( \Delta f \) and \( \Delta t \) are the spectral and the time resolution, respectively. \( S_{X,t}^\alpha(t,f) \) is the cyclic periodogram can be expressed as [3]:

\[
S_{X,t}^\alpha(t,f) = \frac{1}{T} X_T(t,f+\alpha/2) \cdot X_T^*(t,f-\alpha/2)
\]

(4)

where \( T = 1/\Delta f \) and

\[
X_{1/\Delta f}(t,\nu) = \int_{t-\Delta t/2}^{t+\Delta t/2} x(u) e^{-j2\pi\nu u} \, du
\]

(5)

representing the complex envelope of \( X(t) \) with center frequency \( \nu \) [3].

The time smoothing of (4) can be expressed in discrete-time as:

\[
S_{X,t}^\alpha(n,f) = \lim_{n \to \infty} \frac{1}{2n+1} \sum_{n=-n}^{n} X_T(n,f+\alpha/2) \cdot X_T^*(n,f-\alpha/2)
\]

(6)

where the complex envelope is represented as:

\[
X_T(n,f) = \sum_{k=-n/2}^{n/2} a(k) x(n-k) e^{-j2\pi f(n-k)T_s}
\]

(7)

where \( a(k) \) is the data tapering window with length \( T = N'T_s \), and \( T_s \) is the sampling period, \( T_s = \frac{1}{f_s} \) [8].

This paper aims to investigate the reliability of SCF over the complexity by using strip spectral correlation algorithm (SSCA). Then, it indicates the required complexity to evaluate more accurate SCF. Finally, the paper shows the advantage of distributing the workload of one processor over \( m \) cooperated processors.

The next section explains the representation of SSCA and the concept of the reliability. In section 3, the results and its analysis are presented. Finally, the conclusion is drawn in section 4.

2. Background

2.1 Strip Spectral Correlation Algorithm (SSCA)

In spite of the various approaches being utilized to compute SCF, FFT accumulation method (FAM) and SSCA are the two common approaches being used in practice [8]. The block diagram of SSCA in figure 1 starts with applying \( N' \) window on the received signal \( x(n) \). Next, the complex envelope \( X_T \) is computed by performing downconversion on the output of the \( N' \)-FFT. Then, SSCA computes (4) by multiplying \( X_T \) with the conjugate of \( x(n) \). In order to estimate SCF, as shown in [10], each output sample of the multiplication will be applied to \( N \)-FFT as shown in figure 2.

2.2 Complexity and reliability of SSCA

The implementation of \( S_X^\alpha(f) \) comprises of main and side cells. Figure 3 shows an ideal cell that represents the region of support of \( S_X^\alpha(f) \) main lobe with a bandwidth \( B = \frac{f_s}{2} \), width on the order of \( \Delta \alpha = \frac{1}{\Delta t} \) in cycle frequency and length in the order of \( \Delta f \) in frequency [10].

Gardner has demonstrated in [3] that in order to get more reliable representation of SCF i.e. reduce the random effects, the relation between the resolutions \( \Delta t \) and \( \Delta f \) should be as following:
\[ \Delta t \Delta f \gg 1 \]  \hspace{1cm} (8)

**Figure 1.** The strip spectral correlation algorithm block diagram [9].

**Figure 2.** The SSCA signal flow graph.

**Figure 3.** The region of support of SCF.

In (7), the complex envelope is a function of the frequency \( f \) that implies to the number of the first FFT points are
On the other hand, the 2-D function \( S_{nT}^\theta (n, \alpha) \) in (6) is a 2-D function in both of \( f \) and \( \alpha \) which implies to the number of the second FFT points are

\[
N' = \frac{f_s}{\Delta f}
\]  

(9)

where \( L = \frac{N'}{4} \) [8].

In order to satisfy the reliability condition in (8), \( \Delta \alpha \) and \( \Delta f \) should be close to zero. Thus, the number of FFT points will increase as stated in (9) and (10). The increasing in the number of the points will increase the number of complex multipliers.

This paper reduces the multiplication complexity for each cooperated processor by distributing the output of the first FFT on \( m \) cooperated processors. Then, each processor will compute the remaining steps of figure 1 to its assigned workload.

3. Results and Analysis

In this section, the reliability of the SCF will be evaluated at different levels of complexity. Then, the complexity of SSCA will be measured after distributing the workload on \( m \) cooperated processors. Firstly, the advantage of distributing the workload of SSCA is shown in Table 1. The table compares between the numbers of the complex multipliers required for both local and distributed computing. Secondly, in order to compute SCF at different levels of complexity, a sinusoidal signal is modulated by amplitude modulation (AM) with carrier frequency \( f_c = 2048 \text{ Hz} \), and sampled at \( f_s = 8192 \text{ Hz} \). Then, SCF is evaluated at different levels of complexity by selecting the spectral resolution to be \( \Delta f = 256 \), and \( \Delta \alpha \) is changed from 64, 16 to 2. Finally, distributed computing is applied in each case.

Figures 4, 5 and 6 represent the SCF at \( \Delta \alpha = 64 \), 16 and 2, respectively. As shown in the figures, the decreasing in \( \Delta \alpha \) reduces the random effects in the SCF representation. Figures 7, 8 and 9 show the effect of using distributed computing on the complexity of SSCA. According to the figures, the total number of complex multipliers is found around 3800, 17800 and 164000 complex multipliers when \( \Delta \alpha = 64, 16 \) and 2, respectively. The half reduction of the complex multipliers for \( \Delta \alpha = 64 \) is achieved with more than 15 cooperative processors while it is achieved for \( \Delta \alpha = 16 \) and \( \Delta \alpha = 2 \) with only 6 and 4 cooperative processors, respectively. Therefore, as the complexity increases the required number of the cooperated processors to reduce the maximum complexity to the half will reduce. According to the previous observation, the investigation concludes that with increasing the complexity a fewer number of cooperated processors are required to get more reliable SCF.

4. Conclusion

Reducing the random effects of SCF by increasing the reliability could reduce the probability of error. However, increasing the reliability will increase the complexity. This paper discussed the relation between the reliability and the complexity. Then, it is studied the advantage of distributing the workload of one processor on \( m \) cooperated processors. The paper found that with increasing the reliability, the required number of cooperated processors to reduce the half of the complexity maximum will decrease. That’s mean more reliable SCF evaluation could be achieved by distributing the workload on fewer number of cooperated processors.
Table 1. Comparison shows the computational complexity of SSCA between local and distributed computing.

| Comparison Section | Number of Complex Multipliers |
|--------------------|-------------------------------|
|                    | Local computing | Distributed computing |
| Data tapering      | \( N'N \)          | \( N'N \)          |
| \( N' \)-FFT       | \( \frac{N'N}{2} \log_2 N' \) | \( \frac{N'N}{2} \log_2 N' \) |
| Downconversion     | \( N'N \)          | \( N'N \)          |
| Sequences          | \( N'N \)          | \( N'N \)          |
| multiplication     | \( \frac{N'N}{2} \log_2 N \) | \( \frac{N'N}{2m} \log_2 N \) |
| \( N \)-FFT         | \( N'N \)          | \( N'N \)          |
| Total              | \( N'N(3 + \frac{1}{2} \log_2 NN') \) | \( N'N(\frac{m + 2}{m} + \frac{1}{2} \log_2 N'N^{1/m}) \) |

Figure 4. The SCF representation at \( \Delta f = 256 \) and \( \Delta \alpha = 64 \).
Figure 5. The SCF representation at $\Delta f = 256$ and $\Delta \alpha = 16$.

Figure 6. The SCF representation at $\Delta f = 256$ and $\Delta \alpha = 2$.

Figure 7. The complexity of SSCA with $\Delta \alpha = 64$. 
5. References
[1] Gardner W., Napolitano A., and Paura L., “Cyclostationarity: Half a century of research,” Signal Processing (Elsevier), vol. 86, no. 4, pp. 639-697, 2005.
[2] Gardner W. and Spooner C., “Signal interception: performance advantages of cyclic-feature detectors,” IEEE Transactions on Communications, vol.40, no.1, pp.149-159, Jan 1992.
[3] Gardner W., “Measurement of spectral correlation,” IEEE Trans. on Acoustics, Speech, and Signal Processing, vol. assp-34, no. 5, pp. 1111-1123, Oct 1986.
[4] Antoni J., “Cyclic spectral analysis in practice,” Mechanical Systems and Signal Processing, vol. 21, issue 2, pp. 597-630, February 2007.
[5] Antoni J., “Cyclostationarity by examples,” Mechanical Systems and Signal Processing, vol. 23, issue 4, pp. 987-1036, May 2009.
[6] Kilundu B., Chiementin X., Duez J. and Mba D., Cyclostationarity of Acoustic Emissions (AE) for monitoring bearing defects, Mechanical Systems and Signal Processing, vol. 25, issue 6, pp. 2061-2072, August 2011.
[7] Sutton P. D., Nolan K. E. and Doyle L. E., “Cyclostationary signatures in practical cognitive radio applications,” IEEE Journal on Selected Areas in Communications, vol. 26, no. 1, pp. 13-24, Jan 2008.
[8] Prithviraj V., Sarankumar B., Kalaiyarasan A., Chandru P. and Singh N., “Cyclostationary analysis method of spectrum sensing for cognitive radio,” 2011 2nd International Conference on Wireless Communication, Vehicular Technology, Information Theory and Aerospace & Electronic Systems Technology (Wireless VITAE), vol., no., pp.1-5, Feb. 28 2011-March 3, 2011.

[9] Simic D. C. and Simic J. R., “The strip spectral correlation algorithm for spectral correlation estimation of digitally modulated signals,” 4th Inter. Conf. Telecommunications in Modern Satellite, Cable and Broadcasting Services, vol.1, no., pp.277-280 vol.1, 1999.

[10] Roberts R.S., Brown W.A. and Loomis H.H., “Computationally efficient algorithms for cyclic spectral analysis,” IEEE Signal Processing Magazine, pp. 38-49, April 1991.