Analyzing the Effect of Slowly Variable Parameters on the Adaptive Active Control Strategy

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Abstract. This paper reveals that the adaptive active control strategy used to completely synchronize two chaotic systems with unknown parameters is not suitable to those systems with slowly variable parameters, such as electronic neuron systems. Simulation results show that two electronic neuron systems can be phase synchronized only by using adaptive active control strategy.

1. Introduction
Synchronization in chaotic systems and its applications have been extensively investigated [1-6]. Most of the discussions on drive-driven type synchronization and its applications are under a limitation: all the parameters of drive and response systems are precisely known, and the controller can be constructed by these known parameters. However, some parameters of systems cannot be exactly known in advance, these uncertainties will destroy the synchronization. Adaptive synchronization may play an essential role in unknown parameters systems. In Ref. [7-9], active control techniques have been proposed and used to synchronize two identical or different systems with known parameters. In Ref. [10,11], the parameter update law is introduced to improve known active control techniques based on Lyapunov stability theorem, and to synchronize two system with unknown constant parameters, the results show that it is effective.

In this paper, the adaptive active control method reported in Ref. [10,11] is applied to two electronic neurons[12], in which there are two slow variables. The results show that it fails in complete synchronizing two systems.

2. Electronic neuron model and control strategy
The electronic neurons are constructed based on biological experiments and on numerical analysis of isolated neurons models, whose properties are designed to emulate the membrane voltage characteristics of the individual neurons by using analog circuit. The nonlinear differential equations of ENs have the form as follows:
\[
\dot{x} = ay + bx^2 - cx^3 - dz + I \\
\dot{y} = e - fx^2 - y - gw \\
\dot{z} = \mu(-z + S(x + h)) \\
\dot{w} = \nu(-kw + r(y + l))
\]

(1)

Where, \(a, b, c, d, I, e, f, g, \mu, S, \nu, h, l\) and \(k, r\) are the parameters, \(x(t)\) is the membrane voltage in the model, \(y(t)\) represents a “fast” current in the ion dynamics, we choose \(\mu << 1\), so \(z(t)\) is a “slow” current, \(w(t)\) represents an even slower dynamical process for \(\nu < \mu << 1\), and it is included because a slow process, such as the calcium exchange between intracellular stores and the cytoplasm was found to be required in H-H modeling to fully reproduce the observed chaotic oscillations of STG neurons.

Let \(a = 1, b = 3, c = 1, d = 0.99, I = 3.15, e = 1.01, f = 5.0128, g = 0.0278, \mu = 0.0021, S = 3.966, h = 1.605, \nu = 0.0009, k = 0.9573, r = 3.0, l = 1.619\), the behavior of ENs is chaotic [12]. The main parameters used in controlling the modes of spiking and bursting activity of the model are the dc external current \(I\) and the constant parameters \(\mu\) and \(\nu\) of the slow variables. Fig.1 shows chaotic time series of the four variables. In numerical simulation, the double precision fourth-order Runge-Kutta method with integration time step 0.01 was used, the initial condition is (1.0,-1.0, 3.0,-3.0). In each realization, the data for \(n < 10^4\) are ignored to avoid transients.

**Figure 1.** The time series of the dynamical variables \(x(t), y(t), z(t), w(t)\) of ENs model, and various 3D projections \((x(t), y(t), z(t)), (x(t), y(t), w(t)), (x(t), z(t), w(t))\) of the 4D phase space orbits.

From Fig.1, we can see how \(w\) modulates the length of the bursts in \(x\), each two local minimum in the global oscillations of \(w\) coincides with a short burst period of \(x\). From various 3D projections in Fig.1, it is showed that systems are more complex by adding variable \(w\).
In ENs systems, the constants $\mu$ and $\nu$ are main parameters to control slowly variables, in order to discuss the effect of slow variable parameters on the adaptive active control method, we synchronize two ENs systems with unknown parameters $\mu$ and $\nu$. The driving and driven systems are defined as follows:

\begin{align*}
\dot{x}_1 &= y_1 + 3x_1^2 - x_1^3 - 0.99z_1 + I \\
\dot{y}_1 &= 1.01 - 5.0128x_1^2 - y_1 - 0.0278w_1 \\
\dot{z}_1 &= \mu(-z_1 + 3.966(x_1 + 1.605)) \\
\dot{w}_1 &= \nu(-0.9573w_1 + 3(y_1 + 1.619))
\end{align*}

and

\begin{align*}
\dot{x}_2 &= y_2 + 3x_2^2 - x_2^3 - 0.99z_2 + I + u_1 \\
\dot{y}_2 &= 1.01 - 5.0128x_2^2 - y_2 - 0.0278w_2 + u_2 \\
\dot{z}_2 &= \mu'(-z_2 + 3.966(x_2 + 1.605)) + u_3 \\
\dot{w}_2 &= \nu'(-0.9573w_2 + 3(y_2 + 1.619)) + u_4
\end{align*}

where $\mu, \nu, \mu', \nu'$ are unknown parameters of Eq.(2) and Eq.(3), respectively, and $u_1, u_2, u_3, u_4$ are control functions. Let $\mu' = \mu + \delta\mu, \nu' = \nu + \delta\nu$, in which $\delta\mu, \delta\nu$ are unknown constants. The error terms are defined as follows:

\begin{align*}
e_1 &= x_2 - x_1 \\
e_2 &= y_2 - y_1 \\
e_3 &= z_2 - z_1 \\
e_4 &= w_2 - w_1
\end{align*}

The dynamical equations for errors are given as:

\begin{align*}
\dot{e}_1 &= e_2 - 0.99e_3 + 3(x_2^2 - x_1^2) - (x_2^3 - x_1^3) + u_1 \\
\dot{e}_2 &= -5.0128(x_2^2 - x_1^2) - e_2 - 0.0278e_4 + u_2 \\
\dot{e}_3 &= \mu(e_3 + 3.966(e_1 + h)) + \delta\mu(-z_2 + 3.966(x_2 + h)) + u_3 \\
\dot{e}_4 &= \nu(-0.9573e_4 + 3(e_2 + I)) + \delta\nu(-0.9573w_2 + 3(y_2 + I)) + u_4
\end{align*}

We can take a Lyapunov function for Eq. (5),

\[ V = \frac{1}{2}e^T e + \frac{1}{2}(\hat{\mu}^2 + \hat{\nu}^2) + \frac{1}{2}(\tilde{\delta}\mu^2 + \tilde{\delta}\nu^2) \]

where $e(t) = [e_1, e_2, e_3, e_4]^T$, $\hat{\delta}\nu = \delta\nu - \hat{\delta}\nu$, $\hat{\delta}\mu = \delta\mu - \hat{\delta}\mu$, $\hat{\mu} = \mu - \hat{\mu}, \hat{\nu} = \nu - \hat{\nu}$, and $\hat{\mu}, \hat{\nu}, \hat{\delta}\mu, \hat{\delta}\nu$ are estimated values of the unknown constants, respectively.

We choose controllers as:
\[ u_1 = -e_2 + 0.99e_1 - 3(x_2^2 - x_1^2) + (x_2^3 - x_1^3) \]
\[ u_2 = ge_4 + 5.0128(x_2^2 - x_1^2) \]
\[ u_3 = -3.966\hat{\mu}(e_1 + h) - \hat{\delta}\mu(-z_2 + 3.966(x_2 + h)) \]
\[ u_4 = -3\hat{\nu}(e_2 + l) - \hat{\delta}\nu(-0.9573w_2 + 3(y_2 + l)) \]
\[ \hat{\mu} = 3.966(e_1 + h)e_3 \]
\[ \hat{\nu} = 3(e_2 + l)e_4 \]
\[ \hat{\delta}\mu = (-z_2 + 3.966(x_2 + h))e_3 \]
\[ \hat{\delta}\nu = (-0.9573w_2 + 3(y_2 + l))e_4 \]

and parameter estimation update laws are as follows:

\[ \frac{dV}{dt} = -e_2^2 - \mu e_3^2 - ve_4^2 < 0 \]

with this choice, we have the time derivative of \( V \) along the solutions of Eq. (5)

3. Discussion and Conclusion
In order to verify theory analysis, the time courses of corresponding membrane voltages of two EN systems are given in Fig. 2. In numerical simulation, set \( I = 3.15 \), the true values of “unknown” parameters are \( \mu = 0.0021, \nu = 0.0009; \mu' = 0.0024, \nu' = 0.00091 \); the initial estimate values of the unknown constants are \( \hat{\mu} = 0.002, \hat{\nu} = 0.00088, \hat{\delta}\mu = 0.0001, \hat{\delta}\nu = 0.00001 \), respectively. The initial condition for integrating Eq.(2) and Eq.(3) are \((1.0,-1.0,3.0,-3.0)\) and \((0.5,1.0,-2.0,2.0)\).

![Figure 2. The time courses of corresponding membrane voltages of two EN systems](image)
The results show that phase synchronization is achieved between the corresponding membrane voltages of two systems only. In the Ref. [10, 11], complete synchronization is achieved between two Chen systems and PR circuits respectively by adaptive active control strategy. Why complete synchronization can’t be achieved between two ENs systems?

Drawing the time courses of corresponding slow variables of two ENs systems in Fig.3, We find that the properties of slow variable $z_2$ and $w_2$ in driven system have been changed, and there are burst phenomena in time courses of variable $z_2$ and $w_2$ between two local minimum in the global oscillations of variables $z_1, w_1$ of drive system.

![Figure 3](image)

**Figure 3.** The time courses of corresponding slow variables of two ENs

The numerical analyses show that the slow variables of driven system have been remarkably changed as the adaptive active control strategy is used, which results in that complete synchronization can’t be achieved between two ENs. So the adaptive active control strategy is not suitable to the systems with slow variables parameters.

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