MSTW PDFs and impact of PDFs on cross sections at Tevatron and LHC

Graeme Watt

CERN PH-TH

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**Introduction**

Talk mostly based on two recent papers (with some updates):

- G. Watt,
  "*Parton distribution function dependence of benchmark Standard Model total cross sections at the 7 TeV LHC*
  
  [JHEP 09 (2011) 069, arXiv:1106.5788]

- R. S. Thorne and G. Watt,
  "*PDF dependence of Higgs cross sections at the Tevatron and LHC: response to recent criticism*
  
  [JHEP 08 (2011) 100, arXiv:1106.5789]

"Impact of PDFs on cross sections at Tevatron and LHC"

PDFs $\Rightarrow$ cross sections at the Tevatron and LHC.
Cross sections at the Tevatron and LHC $\Rightarrow$ PDFs.
MSTW 2008 PDFs [http://projects.hepforge.org/mstwpdf/]

MSTW 2008 NLO PDFs (68% C.L.)

\[ Q^2 = 10 \text{ GeV}^2 \]

\[ Q^2 = 10^4 \text{ GeV}^2 \]

G. Watt
MSTW 2008 PDFs [http://projects.hepforge.org/mstwpdf/]

A. D. Martin, W. J. Stirling, R. S. Thorne, G. Watt

- “Parton distributions for the LHC”
  [Eur. Phys. J. C 63 (2009) 189, arXiv:0901.0002]

- “Uncertainties on $\alpha_S$ in global PDF analyses and implications for predicted hadronic cross sections”
  [Eur. Phys. J. C 64 (2009) 653, arXiv:0905.3531]

- “Heavy-quark mass dependence in global PDF analyses and 3- and 4-flavour parton distributions”
  [Eur. Phys. J. C 70 (2010) 51, arXiv:1007.2624]

- “The effects of combined HERA and recent Tevatron $W \rightarrow \ell\nu$ charge asymmetry data on the MSTW PDFs”
  [DIS 2010 proceedings, arXiv:1006.2753]
Data sets fitted in MSTW 2008 NLO analysis [arXiv:0901.0002]

| Data set | $\chi^2 / N_{pts}$ |
|----------|------------------|
| H1 MB 99 $e^+p$ NC | 9 / 8 |
| H1 MB 97 $e^+p$ NC | 42 / 64 |
| H1 low $Q^2$ 96–97 $e^+p$ NC | 44 / 80 |
| H1 high $Q^2$ 98–99 $e^-p$ NC | 122 / 126 |
| H1 high $Q^2$ 99–00 $e^+p$ NC | 131 / 147 |
| ZEUS SVX 95 $e^+p$ NC | 35 / 30 |
| ZEUS 96–97 $e^+p$ NC | 86 / 144 |
| ZEUS 98–99 $e^-p$ NC | 54 / 92 |
| ZEUS 99–00 $e^+p$ NC | 63 / 90 |
| H1 99–00 $e^+p$ CC | 29 / 28 |
| ZEUS 99–00 $e^+p$ CC | 38 / 30 |
| H1/ZEUS $e^\pm p$ $F_2^{charm}$ | 107 / 83 |
| H1 99–00 $e^+p$ incl. jets | 19 / 24 |
| ZEUS 96–97 $e^+p$ incl. jets | 30 / 30 |
| ZEUS 98–00 $e^\pm p$ incl. jets | 17 / 30 |
| DØ II $p\bar{p}$ incl. jets | 114 / 110 |
| CDF II $p\bar{p}$ incl. jets | 56 / 76 |
| CDF II $W \to l\nu$ asym. | 29 / 22 |
| DØ II $W \to l\nu$ asym. | 25 / 10 |
| DØ II $Z$ rap. | 19 / 28 |
| CDF II $Z$ rap. | 49 / 29 |

| Data set | $\chi^2 / N_{pts}$ |
|----------|------------------|
| BCDMS $\mu p$ $F_2$ | 182 / 163 |
| BCDMS $\mu d$ $F_2$ | 190 / 151 |
| NMC $\mu p$ $F_2$ | 121 / 123 |
| NMC $\mu d$ $F_2$ | 102 / 123 |
| NMC $\mu n/\mu p$ | 130 / 148 |
| E665 $\mu p$ $F_2$ | 57 / 53 |
| E665 $\mu d$ $F_2$ | 53 / 53 |
| SLAC ep $F_2$ | 30 / 37 |
| SLAC ed $F_2$ | 30 / 38 |
| NMC/BCDMS/SLAC $F_L$ | 38 / 31 |
| E866/NuSea pp DY | 228 / 184 |
| E866/NuSea pd/pp DY | 14 / 15 |
| NuTeV $\nu N$ $F_2$ | 49 / 53 |
| CHORUS $\nu N$ $F_2$ | 26 / 42 |
| NuTeV $\nu N \times F_3$ | 40 / 45 |
| CHORUS $\nu N \times F_3$ | 31 / 33 |
| CCFR $\nu N \to \mu\mu X$ | 66 / 86 |
| NuTeV $\nu N \to \mu\mu X$ | 39 / 40 |
| All data sets | 2543 / 2699 |

- Red = New w.r.t. MRST 2006 fit.
Input parameterisation in MSTW 2008 NLO fit

At input scale $Q_0^2 = 1$ GeV$^2$:

\[
\begin{align*}
 xu_v &= A_u x^{\eta_1} (1 - x)^{\eta_2} (1 + \epsilon_u \sqrt{x} + \gamma_u x) \\
 xd_v &= A_d x^{\eta_3} (1 - x)^{\eta_4} (1 + \epsilon_d \sqrt{x} + \gamma_d x) \\
 xS &= A_S x^{\delta_S} (1 - x)^{\eta_S} (1 + \epsilon_S \sqrt{x} + \gamma_S x) \\
 x(d - \bar{u}) &= A_\Delta x^{\eta_\Delta} (1 - x)^{\eta_S + 2} (1 + \gamma_\Delta x + \delta_\Delta x^2) \\
 xg &= A_g x^{\delta_g} (1 - x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g x) + A_{g'} x^{\delta_{g'}} (1 - x)^{\eta_{g'}} \\
 x(s + \bar{s}) &= A_+ x^{\delta_S} (1 - x)^{\eta_+} (1 + \epsilon_S \sqrt{x} + \gamma_S x) \\
 x(s - \bar{s}) &= A_- x^{0.2} (1 - x)^{\eta_-} (1 - x/x_0)
\end{align*}
\]

- $A_u$, $A_d$, $A_g$ and $x_0$ are determined from sum rules.
- 28 parameters allowed to go free to find best fit,
  20 parameters allowed to go free for error propagation.
Compare to input parameterisation in HERAPDF fits

Input parameterisation \((Q_0^2 = 1.9 \text{ GeV}^2)\) in HERAPDF1.0/1.5

\[xu_\nu = A_{u_\nu} x^{B_{q_\nu}} (1 - x)^{C_{u_\nu}} (1 + E_{u_\nu} x^2)\]
\[xd_\nu = A_{d_\nu} x^{B_{q_\nu}} (1 - x)^{C_{d_\nu}}\]
\[x\bar{u} = A_{\bar{q}} x^{B_{\bar{q}}} (1 - x)^{C_{\bar{u}}}\]
\[x\bar{d} = A_{\bar{q}} x^{B_{\bar{q}}} (1 - x)^{C_{\bar{d}}}\]
\[x\bar{s} = 0.45 x\bar{d}\]
\[xs = x\bar{s}\]
\[xg = A_g x^{B_g} (1 - x)^{C_g}\]

- **10 parameters** for central fit and “experimental” uncertainties, additional “model” and “parameterisation” uncertainties.
- **4 more params.** for HERAPDF1.5 NNLO (2 for \(g\), 1 each for \(u_\nu, d_\nu\)).
Dynamic tolerance: different for each eigenvector

- Outer (inner) error bars give tolerance for 90% (68%) C.L.
Impact of Tevatron Run II jet data on high-$x$ gluon

- Run II jet data prefer **softer** gluon at high $x$ than Run I.
NuTeV/CCFR dimuon cross sections and strangeness

\[
\frac{d\sigma}{dx dy}(\nu_\mu N \rightarrow \mu^+ \mu^- X) = B_c \, A \, \frac{d\sigma}{dx dy}(\nu_\mu N \rightarrow \mu^- c X)
\]
\[\propto |V_{cs}|^2 \xi S(\xi, Q^2) + |V_{cd}|^2 \ldots\]

- \(\nu_\mu\) and \(\bar{\nu}_\mu\) cross sections constrain \(s\) and \(\bar{s}\).
**W+charm as a probe of strangeness** [CMS PAS EWK-11-013]

- Dominant $\bar{s}g \rightarrow W^+ \bar{c}$ and $sg \rightarrow W^- c$.
- 5% from $\bar{d}g \rightarrow W^+ \bar{c}$, 15% from $dg \rightarrow W^- c$.

$$R_{c}^{\pm} \equiv \frac{\sigma(W^+ + \bar{c})}{\sigma(W^- + c)} = 0.92 \pm 0.19{\text{(stat.)}} \pm 0.04{\text{(syst.)}}$$

$$R_{c} \equiv \frac{\sigma(W + c)}{\sigma(W + \text{jets})} = 0.143 \pm 0.015{\text{(stat.)}} \pm 0.024{\text{(syst.)}}$$

$x(s + \bar{s})(x, Q^2 = 2 \text{ GeV}^2)$:

| Ratio   | MCFM (MSTW08)        | MCFM (CT10)            | MCFM (NNPDF2.1)    |
|---------|----------------------|------------------------|--------------------|
| $R_{c}^{\pm}$ | $0.881^{+0.022}_{-0.032}$ | $0.915^{+0.006}_{-0.006}$ | $0.902 \pm 0.008$ |
| $R_{c}$   | $0.118^{+0.002}_{-0.002}$ | $0.125^{+0.013}_{-0.007}$ | $0.103 \pm 0.005$ |
Dependence on strong coupling $\alpha_S(M_Z^2)$ [MSTW, arXiv:0905.3531]

MSTW 2008 NNLO ($\alpha_S$) PDF fit

- **Experimental error** on best-fit $\alpha_S(M_Z^2)$ using same method applied to determine the tolerance for each eigenvector.
\[ \Delta \chi^2_{\text{global}} \] as a function of \( \alpha_S(M^2_Z) \) for the NNLO global fit

\[ \alpha_S(M^2_Z) = 0.1171^{+0.0014}_{-0.0014} \quad (68\% \text{ C.L.})^{+0.0034}_{-0.0034} \quad (90\% \text{ C.L.}) \]

- Additional theory uncertainty (\( \lesssim |\text{NNLO} - \text{NLO}| = 0.003 \)).
- cf. \( \alpha_S(M^2_Z) = 0.1184 \pm 0.0007 \) [S. Bethke, arXiv:0908.1135].
Impact of $\alpha_S$ on SM Higgs uncertainty versus $M_H$

- Enhanced “PDF+$\alpha_S$” uncertainty compared to “PDF only”.

G. Watt
Heavy-quark mass dependence [MSTW, arXiv:1007.2624]

Impact of (pole-mass) $m_{c,b}$ variation on LHC cross sections

- Vary $m_c = 1.40 \pm 0.15$ GeV $\Rightarrow$ just over 1% change in $\sigma_{W,Z}$.
- Vary $m_b = 4.75 \pm 0.25$ GeV $\Rightarrow$ negligible change (0.1%).

| LHC, $\sqrt{s} = 7$ TeV | $\sigma_W$ | $\sigma_Z$ | $\sigma_H$ |
|-------------------------|---------|---------|---------|
| PDF only uncertainty    | $+1.7\%$ | $+1.7\%$ | $+1.1\%$ |
|                         | $-1.6\%$ | $-1.5\%$ | $-1.6\%$ |
| PDF+$\alpha_S$ uncertainty | $+2.5\%$ | $+2.5\%$ | $+3.7\%$ |
|                         | $-1.9\%$ | $-1.9\%$ | $-2.9\%$ |
| PDF+$\alpha_S+m_{c,b}$ uncertainty | $+2.7\%$ | $+2.9\%$ | $+3.7\%$ |
|                         | $-2.2\%$ | $-2.4\%$ | $-2.9\%$ |

- Only slight increase in uncertainty on $\sigma_{W,Z}$, no impact on $\sigma_H$. 
Impact of combined HERA I data [arXiv:0911.0884] (R. Thorne)

- Changes not large enough to warrant an immediate update.
Heavy quark contribution to DIS structure function $F_2$

- **General-mass variable flavour number scheme (GM-VFNS)**
  - Interpolates between two well-defined regions ($H \equiv c, b$):
    - **FFNS** for $Q^2 \leq m_H^2$, **ZM-VFNS** for $Q^2 \gg m_H^2$.
  - Ambiguous up to $O(m_H^2/Q^2)$ terms $\Rightarrow$ theory uncertainty.
Impact of GM-VFNS variations [R. Thorne, arXiv:1006.5925]

**Status of MSTW PDF analysis**

- Benchmark
- $W$ and $Z$ production
- Higgs, top and jet production
- $\alpha_S$ from DIS

### Summary

Impact of GM-VFNS variations

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#### $F_2^{charm}$ at NLO (fixed PDFs)

- **Figure**
- **Graphs**
- **Curves**
- **Legend**

#### $F_2^{charm}$ at NNLO (fixed PDFs)

- **Figure**
- **Graphs**
- **Curves**
- **Legend**

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G. Watt
Impact of GM-VFNS variations [R. Thorne, arXiv:1006.5925]

Effect on $g$ and $u$ at NLO

Effect on $g$ and $u$ at NNLO
Background and motivation for benchmark exercise

- Various fitting groups currently produce PDF sets: **MSTW, CT, NNPDF, HERAPDF, AB(K)M, (G)JR.**
- Quantifying and understanding differences *between* groups is as (or more) important as continued improvements *within* groups.
- Recent work initiated by activities of **LHC Higgs Cross Section Working Group** and **PDF4LHC Working Group**.
- Use most recent public NLO PDFs from all fitting groups to calculate LHC benchmark processes: $W^\pm$, $Z^0$, $t\bar{t}$, $gg \rightarrow H$.

**Aims:**
- Establish degree of compatibility and identify outliers.
- Compare cross sections at *same* $\alpha_s$ values.
- To what extent are differences in predictions due to *different* $\alpha_s$ values used by each group, rather than differences in PDFs?
- Results initially presented in talk by **G.W.** at PDF4LHC meeting at CERN on **26th March 2010** and formed basis for subsequent **PDF4LHC Interim Report** [arXiv:1101.0536].
- Subsequent update and extension to NNLO [G.W., arXiv:1106.5788].
Status of PDFs from different groups in March 2010

- Consider only *public* sets, where “public” ≡ available in LHAPDF.
- Then LHAPDF V5.8.2 (released 18th March 2010).
- Highlight major differences in data and theory between groups:

|                  | MSTW08 | CTEQ6.6 | NNPDF2.0 | HERAPDF1.0 | ABKM09 | GJR08/JR09 |
|------------------|--------|---------|----------|------------|--------|------------|
| HERA DIS         | ✔      | ✔       | ✔        | ✔          | ✔      | ✔          |
| Fixed-target DIS | ✔      | ✔       | ✔        | ✔          | x      | ✔          |
| Fixed-target DY  | ✔      | ✔       | ✔        | ✔          | x      | ✔          |
| Tevatron W,Z     | ✔      | ✔       | ✔        | ✔          | x      | ✔          |
| Tevatron jets    | ✔      | ✔       | ✔        | ✔          | x      | ✔          |
| GM-VFNS          | ✔      | ✔       | x        | ✔          | x      | ✔          |
| NNLO             | ✔      | x       | x        | x          | x      | ✔          |

- “Global” ≡ includes all five main categories of data.
- GJR08 *almost* global but restrictive “dynamical” parameterisation.
- Three groups with NLO global fits, but only one at NNLO. Approx. NNLO for jets, massive $O(\alpha_S^3)$ NC and $O(\alpha_S^2)$ CC DIS.
- CTEQ6.6 only uses Tevatron Run I data, not Run II.
- NNPDF2.0 inadequate through use of ZM-VFNS for DIS.
Status of PDFs from different groups in September 2011

- Now **LHAPDF V5.8.6** (released 2nd August 2011).
- Highlight major differences in data and theory between groups:

|                              | MSTW08 | CT10  | NNPDF2.1 | HERAPDF1.5 | ABKM09 | GJR08/JR09 |
|------------------------------|--------|-------|----------|------------|--------|------------|
| HERA DIS                     | ✔      | ✔     | ✔        | ✔          | ✔      | ✔          |
| Fixed-target DIS             | ✔      | ✔     | ✔        |            | ✔      | ✔          |
| Fixed-target DY              | ✔      | ✔     | ✔        | ✔          | ✔      | ✔          |
| Tevatron $W, Z$              | ✔      | ✔     | ✔        | ✔          | ✔      | ✔          |
| Tevatron jets                | ✔      | ✔     | ✔        | ✔          | ✔      | ✔          |
| GM-VFNS                      | ✔      | ✔     | ✔        | ✔          | ✔      | ✔          |
| NNLO                         | ✔      | ✗     | ✔        | ✔          | ✔      | ✔          |

- **CT10** uses both Tevatron Run I and Run II data.
- Only **CT10, NNPDF2.1** and **HERAPDF** use combined HERA I.
- Only **HERAPDF1.5** uses preliminary combined HERA II data.
- **NNPDF2.0 (ZM-VFNS)** → **NNPDF2.1 (GM-VFNS)**, now allowing meaningful comparison to other NLO global fits.
- **NNPDF2.1** and **HERAPDF1.5** now provided at NNLO.
Default values of $\alpha_S(M^2_Z)$ used by different fitting groups

- $\alpha_S(M^2_Z)$ for MSTW08, ABKM09 and GJR08/JR09 fitted.
- $\alpha_S(M^2_Z)$ for other groups applied as an external constraint.
- Smaller symbols indicate alternative $\alpha_S(M^2_Z)$ values provided.
- Fitted NLO $\alpha_S(M^2_Z)$ always larger than NNLO $\alpha_S(M^2_Z)$: attempt by fit to mimic missing higher-order corrections.
Ratio of NLO quark–antiquark luminosity functions

\[
\frac{\partial L \Sigma_q(q\bar{q})}{\partial \hat{s}} = \frac{1}{s} \int_{t}^{1} \frac{dx}{x} \sum_{q=d,u,s,c,b} [q(x, \hat{s})\bar{q}(\tau/x, \hat{s}) + (q \leftrightarrow \bar{q})], \quad \tau \equiv \frac{\hat{s}}{s}
\]

- Relevant values of \(\sqrt{\hat{s}} = M_{W,Z}\) are indicated: good agreement for global fits (left), but more variation for other sets (right).
\[ \Sigma_q (q\bar{q}) \text{ luminosity at LHC (} \sqrt{s} = 7 \text{ TeV)} \]

- NNLO trend between groups similar to NLO (apart from HERAPDF).

G. Watt (September 2011)
NLO $W^\pm$ and $Z^0$ total cross sections versus $\alpha_S(M_Z^2)$

- Global fits in good agreement for $\sigma_{W^\pm}$ and $\sigma_{Z^0}$ (left plots).
- Small PDF uncertainties in predictions for $W/Z$ ratio:
  \[
  \frac{\sigma_{W^+} + \sigma_{W^-}}{\sigma_{Z^0}} \sim \frac{u(x_1) + d(x_1)}{0.29 u(\tilde{x}_1) + 0.37 d(\tilde{x}_1)}
  \]
NNLO $W^{\pm}$ and $Z^0$ total cross sections versus $\alpha_S(M_Z^2)$

- HERAPDF1.5 closer to global fits at NNLO for $\sigma_{W^\pm}$ and $\sigma_{Z^0}$ (left plots).
- $W/Z$ ratio insensitive to NNLO corrections (and $\alpha_S$):
NLO $W^+$ and $W^-$ total cross sections versus $\alpha_S(M_Z^2)$

- Slightly more spread in separate $\sigma_{W^+}$ and $\sigma_{W^-}$.
- Reflected in $W^+/W^-$ ratio:

$$\frac{\sigma_{W^+}}{\sigma_{W^-}} \sim \frac{u(x_1)\bar{d}(x_2)}{d(x_1)\bar{u}(x_2)} \sim \frac{u(x_1)}{d(x_1)}$$
NNLO $W^+$ and $W^-$ total cross sections versus $\alpha_S(M_Z^2)$

- HERAPDF1.5 closer to global fits at NNLO for $\sigma_{W^+}$ and $\sigma_{W^-}$ (left plots).
- $W^+/W^-$ ratio insensitive to NNLO corrections (and $\alpha_S$):
Consolidate two cross section measurements (and their ratio).

- Luminosity uncertainty of 3.4% (ATLAS) or 4% (CMS).
- Know correlation of both data and theory (from PDFs).
NNLO $W^\pm$ vs. $Z^0$ and $W^+$ vs. $W^-$ total cross sections

- Correlation of ellipse $\iff$ uncertainty in ratio of cross sections.

- Largest uncertainty in ATLAS/CMS total cross-section ratios from acceptance calculation $\Rightarrow$ compare to theory within acceptance.
NNLO $W^\pm$ vs. $Z^0$ and $W^+$ vs. $W^-$ fiducial cross sections

[http://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/STDM-2011-06/]

- NNLO comparisons now possible using FEWZ or DYNNLO codes.
• MSTW08 has input $x_u \propto x^{0.29 \pm 0.02}$ and $x_d \propto x^{0.97 \pm 0.11}$.

• First PDF constraint from LHC data ($\rightarrow$ NNPDF2.2).

Many other groups assume equal powers \(\Rightarrow\) potential bias.
Wide spread in predictions using different NNLO PDF sets.
Differential cross sections: $d\sigma(\ell^+)/d\eta_\ell$ and $d\sigma(\ell^-)/d\eta_\ell$

[http://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/STDM-2011-06/]

- ATLAS provide differential cross sections for $W^+$ and $W^-$ with information on correlated systematic uncertainties.
  - Potentially more useful for PDF fits than simply $A_\ell(\eta_\ell)$. 

G. Watt
\( W^\pm \rightarrow \ell^\pm \nu \) charge asymmetry at the Tevatron

- **Outstanding issues to be resolved concerning Tevatron data**, particularly when split up into \( p_T^\ell \) bins \([\text{MSTW, arXiv:1006.2753}]\).
- **Current plan**: consider only inclusive \( p_T^\ell \) bin, try to fit nuclear effects in deuteron structure functions simultaneously with PDFs.
Exclusion limits at 95% C.L. for SM Higgs boson

| Mass (GeV) | SM $\sigma$/$\sigma_{SM}$ |
|-----------|----------------------------|
| Tevatron   | 156 - 177, 0.08 - 0.09     |
| ATLAS      | 146 - 232, 0.02 - 0.03     |
|            | 256 - 282, 0.04 - 0.04     |
|            | 296 - 466, 0.04 - 0.07     |
| CMS        | 145 - 216, 0.02 - 0.03     |
|            | 226 - 288, 0.03 - 0.04     |
|            | 310 - 340, 0.04 - 0.05     |

- $\sigma_{SM}$ uses MSTW 2008 PDFs.

[TEVNPHWG, arXiv:1107.5518]
Tevatron Higgs exclusion limits: a critical appraisal

[Baglio, Djouadi, Ferrag, Godbole, arXiv:1101.1832]

Phys. Lett. B 699 (2011) 368
Tevatron Higgs exclusion limits: a critical appraisal

[Baglio, Djouadi, Ferrag, Godbole, arXiv:1101.1832]
\( gg \rightarrow H \) total cross sections versus SM Higgs mass \( M_H \)

**HERAPDF1.5** and **NNPDF2.1** results agree with **MSTW08**.
$gg \to H$ total cross sections versus $\alpha_s(M_Z^2)$

- $\alpha_s(M_Z^2)$ values can only partly explain low $\sigma_H$ for ABKM09.
• Relevant values of $\sqrt{s} = M_H$ are indicated.
Ratio of gluon–gluon luminosity functions

- Relevant values of $\sqrt{s} = M_H, 2m_t$ are indicated.
Top-pair production at the Tevatron and LHC

- ∼80% of $\sigma_{t\bar{t}}^{\text{NLO}}$ from $gg$ at LHC (7 TeV), cf. ∼15% at Tevatron.
- Compare NLO and various NNLO approximations for total $\sigma_{t\bar{t}}$ (pb) for $m_t = 173$ GeV [Kidonakis, Pecjak, arXiv:1108.6063]:

| Calculation             | Tevatron          | LHC (7 TeV)        |
|-------------------------|-------------------|--------------------|
| NLO                     | 6.74 $^{+0.36+0.37}_{-0.76-0.24}$ | 160 $^{+20+8}_{-21-9}$ |
| Aliev et al. [arXiv:1007.1327] | 7.13 $^{+0.31+0.36}_{-0.39-0.26}$ | 164 $^{+3+9}_{-9-9}$ |
| Kidonakis [arXiv:1009.4935] | 7.08 $^{+0.00+0.36}_{-0.24-0.24}$ | 163 $^{+7+9}_{-5-9}$ |
| Ahrens et al. [arXiv:1105.5824] | 6.65 $^{+0.08+0.33}_{-0.41-0.24}$ | 156 $^{+8+8}_{-9-9}$ |

- First uncertainty is perturbative ($\mu_R,F$ variation etc.).
- Second uncertainty is the MSTW08 PDF error at 90% C.L.
$t\bar{t}$ total cross sections versus $\alpha_S(M_Z^2)$ at the LHC

- **NNLO (approx.)** using HATHOR code [Aliev et al., arXiv:1007.1327].
- Compare to single most precise current LHC measurements.
  - **CMS**: $\sigma_{t\bar{t}} = 164 \pm 3\,(\text{stat.}) \pm 12\,(\text{syst.}) \pm 7\,(\text{lumi.}) \text{ pb (e/}\mu+\text{jets+b-tag)}$ [CMS PAS TOP-11-003]
  - **ATLAS**: $\sigma_{t\bar{t}} = 179.0 \pm 9.8\,(\text{stat.+syst.}) \pm 6.6\,(\text{lumi.}) \text{ pb (using kinematic information of lepton+jets events)}$ [ATLAS-CONF-2011-121]
  - Tevatron: $m_t = 173.2 \pm 0.9 \text{ GeV}$ [TEVEWWG, arXiv:1107.5255].

Increasing $m_t$ by 2 GeV decreases predicted $\sigma_{t\bar{t}}$ at LHC by 6%.

G. Watt
JETS

PV: Any PDF should reproduce jet data if being used for Higgs

Closest observable to Higgs in terms of Luminosity, kinematics and power of coupling!

[D. de Florian, talk at “Higgs Hunting 2011”, Orsay, France, 28th July 2011]
Jets as a discriminator of the high-$x$ gluon distribution

**JETS**

**PV:** Any PDF should reproduce jet data if being used for Higgs

Closest observable to Higgs in terms of Luminosity, kinematics and power of coupling!

\[ g \xrightarrow[\sigma]{} \text{jet} \]

\[ g \xrightarrow[2\text{-loop}]{} \text{jet} \]

\[ g \xrightarrow[t,b]{} H \]

[D. de Florian, talk at “Higgs Hunting 2011”, Orsay, France, 28th July 2011]

- **Problem:** NNLO $\hat{\sigma}$ unknown, approximate with NLO $\hat{\sigma}$ and 2-loop threshold corrections [Kidonakis, Owens, hep-ph/0007268].
- Jet cross sections calculated with **FASTNLO** [Kluge, Rabbertz, Wobisch, hep-ph/0609285]: includes 2-loop threshold corrections.
- Take different scale choices $\mu_R = \mu_F = \mu = \{p_T/2, p_T, 2p_T\}$ as some indication of the theoretical uncertainty.
Status of MSTW PDF analysis

Benchmark

W and Z production

Higgs, top and jet production

αS from DIS

Summary

Scale dependence and size of 2-loop threshold corrections

**Ratio w.r.t. LO for different μ**

CDF Run II inclusive jet data (kT, D = 0.7)

(K-factor = Ratio w.r.t. LO using MSTW08 NNLO PDFs)

| K-factor range | pT (GeV) | Ratio w.r.t. LO |
|----------------|----------|-----------------|
| 0.0 < |y|JET| ≤ 0.1 | 10^5 | 1.0 |
| 0.1 < |y|JET| < 0.7 | 10^6 | 1.2 |
| 0.7 < |y|JET| ≤ 1.1 | 10^7 | 1.2 |
| 1.1 < |y|JET| < 1.8 | 10^8 | 1.2 |
| 1.6 < |y|JET| ≤ 2.1 | 10^9 | 1.4 |

- μR = μF = 0.5•pT
- μR = μF = 1.0•pT
- μR = μF = 2.0•pT

Solid lines: NLO D = 2-loop threshold

**Ratio w.r.t. NLO with μ = pT**

CDF Run II inclusive jet data (kT, D = 0.7)

(Ratio w.r.t. NLO using MSTW08 NNLO PDFs)

| K-factor range | pT (GeV) | Ratio w.r.t. NLO |
|----------------|----------|-----------------|
| 0.0 < |y|JET| ≤ 0.1 | 10^5 | 1.0 |
| 0.1 < |y|JET| < 0.7 | 10^6 | 1.2 |
| 0.7 < |y|JET| ≤ 1.1 | 10^7 | 1.2 |
| 1.1 < |y|JET| < 1.8 | 10^8 | 1.2 |
| 1.6 < |y|JET| ≤ 2.1 | 10^9 | 1.4 |

- μR = μF = 0.5•pT
- μR = μF = 1.0•pT
- μR = μF = 2.0•pT

Solid lines: NLO D = 2-loop threshold

- K-factor with μ = pT more uniform across |y|JET| bins than μ = pT/2.
- Scale dependence stabilised by inclusion of 2-loop threshold corrections.
- Lack of exact NNLO should **not** prevent use of jet data in PDF fits.
Inclusive jet production at the Tevatron and LHC

- **LHC jets**: generally lower $x_T$, no correlated systematics.
- **Current best constraint on high-$x$ gluon from Tevatron jets.**

[M. Wobisch et al., arXiv:1109.1310]

[MSTW, arXiv:0905.3531]
Treatment of correlated systematic uncertainties

- Important to account for *correlated* systematic uncertainties of experimental data points [CTEQ6, hep-ph/0201195]:

\[
\chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \left( \frac{\hat{D}_i - T_i}{\sigma_i^{\text{uncorr.}}} \right)^2 + \sum_{k=1}^{N_{\text{corr.}}} r_k^2,
\]

where \( \hat{D}_i \equiv D_i - \sum_{k=1}^{N_{\text{corr.}}} r_k \sigma_{k,i}^{\text{corr.}} \) are *shifted* data points.
- Trade-off between systematic shifts \( r_k \) and fitted parameters.
- More traditional form (with hidden systematic shifts):

\[
\chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \sum_{j=1}^{N_{\text{pts.}}} (D_i - T_i) (V^{-1})_{ij} (D_j - T_j),
\]

where \( V_{ij} = \delta_{ij} (\sigma_i^{\text{uncorr.}})^2 + \sum_{k=1}^{N_{\text{corr.}}} \sigma_{k,i}^{\text{corr.}} \sigma_{k,j}^{\text{corr.}} \).
- \( \chi^2 \) definition similar to Eq. (1) used by MSTW and CTEQ.
- \( \chi^2 \) definition similar to Eq. (2) used by ABKM and NNPDF.
Values of $\chi^2/N_{\text{pts.}}$ for different NNLO PDFs and scale choices:

| NNLO PDF   | $\alpha_s(M_Z^2)$ | $\mu = p_T/2$       | $\mu = p_T$         | $\mu = 2p_T$        |
|------------|---------------------|----------------------|----------------------|----------------------|
| MSTW08     | 0.1171              | 1.39 (+0.35)         | 0.69 (−0.45)         | 0.97 (−1.30)         |
| NNPDF2.1   | 0.1190              | 0.68 (−0.77)         | 0.71 (−2.02)         | 0.71 (−3.46)         |
| HERAPDF1.0 | 0.1145              | 2.37 (−2.65)         | 1.48 (−3.64)         | 1.29 (−4.12)         |
| HERAPDF1.0 | 0.1176              | 2.24 (−0.48)         | 1.13 (−1.60)         | 1.09 (−2.23)         |
| HERAPDF1.5 | 0.1176              | 1.61 (+1.22)         | 0.77 (+0.30)         | 1.06 (−0.39)         |
| ABKM09     | 0.1135              | 1.53 (−4.27)         | 1.23 (−5.05)         | 1.44 (−5.65)         |
| JR09       | 0.1124              | 0.75 (+0.13)         | 1.26 (−0.61)         | 2.20 (−1.22)         |

Numbers in brackets are the systematic shift ("$-r_{\text{lumi.}}$") for the 5.8% luminosity uncertainty.

Highlight in *italics* if $|r_{\text{lumi.}}| \in [1, 3]$ and in **bold** if $|r_{\text{lumi.}}| > 3$.

Optimal $\chi^2$ for ABKM09 requires data to be normalised downwards by $\sim 30\%$, i.e. 5-σ luminosity shift.
Description of Tevatron $W/Z$ total cross sections

- CDF/DØ measurements of $W/Z$ cross sections are dominated by $\sim 6\%$ luminosity uncertainty (common to jet cross sections).
- All NNLO PDFs in good agreement with $W/Z$ cross sections.
- Can use Tevatron $W/Z$ cross sections as a luminosity monitor: demand agreement with theory prediction to effectively remove normalisation uncertainty from jet cross sections.
- Done automatically in MSTW08 fit by fitting CDF $d\sigma_Z/dy_Z$.  

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\end{itemize}
Data/theory ratio for MSTW08 and ABKM09

CDF Run II inclusive jet data ($k_T$, D = 0.7)
(data points before systematic shifts, show total errors)

| $|y^{\text{JET}}|$ | Data/Theory | $p_T^{\text{JET}}$ (GeV) |
|-----------------|-------------|--------------------------|
| 0.0 < $|y^{\text{JET}}|$ < 0.1 | Data/Theory | $p_T^{\text{JET}}$ (GeV) |
| 0.1 < $|y^{\text{JET}}|$ < 0.7 | Data/Theory | $p_T^{\text{JET}}$ (GeV) |
| 0.7 < $|y^{\text{JET}}|$ < 1.1 | Data/Theory | $p_T^{\text{JET}}$ (GeV) |
| 1.1 < $|y^{\text{JET}}|$ < 1.6 | Data/Theory | $p_T^{\text{JET}}$ (GeV) |
| 1.6 < $|y^{\text{JET}}|$ < 2.1 | Data/Theory | $p_T^{\text{JET}}$ (GeV) |

$|H_R| = |H_F| = 1.0 \times p_T^{\text{JET}}$

NNLO PDFs, 76 data points
- MSTW08, $\chi^2 = 34$
- ABKM09, $\chi^2 = 68$

Inner error bars: only uncorrelated
Outer error bars: total (add in quadrature)

[ABM, arXiv:1105.5349]

- ABM studies: data lie above theory even after refitting.
  But still 15% increase in $\sigma_H$ ($M_H = 165$ GeV) at Tevatron.
- Would be interesting to see impact of Tevatron jet data on ABKM09 fit with constrained CDF/DØ normalisation.
Description of CDF II inclusive jet \((k_T)\) data [hep-ex/0701051]

- More realistic \(\chi^2\) computation without complication of including \(W/Z\) data: simply constrain \(|r_{\text{lumi.}}| < 1\).
- Values of \(\chi^2/N_{\text{pts.}}\) for different NNLO PDFs and scale choices:

| NNLO PDF    | \(\alpha_S(M_Z^2)\) | \(\mu = p_T/2\) | \(\mu = p_T\) | \(\mu = 2p_T\) |
|-------------|----------------------|-----------------|---------------|----------------|
| MSTW08      | 0.1171               | 1.39            | 0.69          | 0.97           |
| NNPDF2.1    | 0.1190               | **0.68**        | 0.81          | 1.29           |
| HERAPDF1.0  | 0.1145               | 2.64            | 2.15          | 2.20           |
| HERAPDF1.0  | 0.1176               | 2.24            | 1.17          | 1.23           |
| HERAPDF1.5  | 0.1176               | 1.61            | **0.77**      | 1.06           |
| ABKM09      | 0.1135               | 2.55            | 2.76          | 3.41           |
| JR09        | 0.1124               | **0.75**        | 1.26          | 2.21           |

- Highlight in **bold** if \(\chi^2/N_{\text{pts.}} < 0.83\), i.e. 90\% C.L. region.
Distribution of pulls and systematic shifts

\[ \chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \left( \frac{\hat{D}_i - T_i}{\sigma_i^{\text{uncorr.}}} \right)^2 + \sum_{k=1}^{N_{\text{corr.}}} r_k^2 \]

- Plot distribution of \( \chi^2 \) contributions from each of two terms:

**Pulls, \((\hat{D}_i - T_i)/\sigma_i^{\text{uncorr.}}\)**

**Systematic shifts, \(r_k\)**

- **MSTW08**: both distributions follow Gaussian behaviour.
- **ABKM09**: broader tail for pulls, non-Gaussian systematic shifts.
Treatment of $F_L$ correction for NMC data

- Recent claim that bulk of MSTW/ABKM difference explained by $F_L$ for NMC data [Alekhin, Blümlein, Moch, arXiv:1101.5261].

$$\frac{d^2\sigma}{dx \, dQ^2} \simeq \frac{4\pi \alpha^2}{x \, Q^4} \left[ 1 - y + \frac{y^2/2}{1 + R(x, Q^2)} \right] F_2(x, Q^2)$$

- ABKM fit NMC cross sections, MSTW fit NMC $F_2$ corrected for $R = \sigma_L/\sigma_T \simeq F_L/(F_2 - F_L)$, where [NMC, hep-ph/9610231]:

$$R(x, Q^2) = \begin{cases} R_{\text{NMC}}(x) & \text{if } x < 0.12 \\ R_{\text{1990}}(x, Q^2) & \text{if } x > 0.12 \end{cases}$$

| ABKM09               | MSTW08               |
|----------------------|----------------------|
| Fit NMC cross section| Fit NMC $F_2$        |
| $Q^2 \geq 2.5$ GeV$^2$, $W^2 \geq 3.24$ GeV$^2$ | $Q^2 \geq 2$ GeV$^2$, $W^2 \geq 15$ GeV$^2$ |
| Fit empirical higher-twist | Neglect higher-twist |
| Separated beam energies | Averaged beam energies |
| Correlated systematics | Neglect correlations |
| 3 input gluon parameters | 7 input gluon parameters |
| No jet data           | Tevatron jet data    |
Effect of NMC $F_L$ treatment on $\alpha_S(M_Z^2)$ and $\sigma_H$

| NNLO PDF      | $\alpha_S(M_Z^2)$ | $\sigma_H$ at Tevatron | $\sigma_H$ at 7 TeV LHC |
|---------------|-------------------|------------------------|------------------------|
| MSTW08        | 0.1171            | 0.342 pb               | 7.91 pb                |
| Use $R_{1990}$ for NMC $F_2$ | 0.1167            | $-0.7\%$               | $-0.9\%$               |
| Cut NMC $F_2$ ($x < 0.1$) | 0.1162            | $-1.2\%$               | $-2.1\%$               |
| Cut all NMC $F_2$ data | 0.1158            | $-0.7\%$               | $-2.1\%$               |
| Cut $Q^2 < 5$ GeV$^2$, $W^2 < 20$ GeV$^2$ | 0.1171            | $-1.2\%$               | $+0.4\%$               |
| Cut $Q^2 < 10$ GeV$^2$, $W^2 < 20$ GeV$^2$ | 0.1164            | $-3.0\%$               | $-1.7\%$               |
| Fix $\alpha_S(M_Z^2)$ | 0.1130            | $-11\%$                | $-7.6\%$               |
| Input $xg > 0$, no jets | 0.1139            | $-17\%$                | $-4.9\%$               |
| ABKM09        | 0.1135            | $-26\%$                | $-11\%$                |

- $\alpha_S$ and $\sigma_H$ insensitive to treatment of NMC $F_L$.
- Similar stability found by NNPDF [arXiv:1102.3182], but using a fixed $\alpha_S(M_Z^2)$.
- Conclusion: jets stabilise fit (lessen sensitivity to details).
Common lore that DIS-only fits prefer low $\alpha_S$. Is it true?

ABKM09: $\alpha_S(M_Z^2) = 0.1135 \pm 0.0014$, cf. MSTW08: $0.1171 \pm 0.0014$.

- **Answer**: Not all DIS data sets prefer low $\alpha_S(M_Z^2)$ values.
- True only for BCDMS, and for E665 and SLAC ep data.
- NMC, SLAC ed and HERA data prefer high $\alpha_S(M_Z^2)$ values.
Correlation between $\alpha_S$ and gluon distribution

- Known that $\alpha_S$ is **anticorrelated** with low-$x$ gluon through scaling violations of HERA data: $\frac{\partial F_2}{\partial \ln(Q^2)} \sim \alpha_S g$. Then $\alpha_S$ is **correlated** with high-$x$ gluon through momentum sum rule.

- **MSTW08**: $\alpha_S(M_Z^2) = 0.1171 \pm 0.0014$ [arXiv:0905.3531].
- Positive input gluon: $\alpha_S(M_Z^2) = 0.1157$, but $\Delta \chi^2_{\text{global}} = 63$. 
What is $\alpha_S$ from only DIS in the MSTW08 NNLO fit?

[Studies prompted by question from G. Altarelli, December 2010]

- **Global fit**: $\alpha_S(M_Z^2) = 0.1171 \pm 0.0014$ [arXiv:0905.3531].
- **DIS-only fit** gives $\alpha_S(M_Z^2) = 0.1104$ (BCDMS-dominated), but input $xg < 0$ for $x > 0.4$ due to lack of data constraint. $\Rightarrow F_2^{\text{charm}} < 0$ and $\chi^2/N_{\text{pts.}} \sim 10$ for Tevatron jets.
- DIS-only fit fixing high-x gluon parameters gives $\alpha_S(M_Z^2) = 0.1172$.
- DIS-only fit without BCDMS gives $\alpha_S(M_Z^2) = 0.1193$.
- Global fit without BCDMS gives $\alpha_S(M_Z^2) = 0.1181$.
- **Conclusion**: Tevatron jet data vital to pin down high-x gluon, giving smaller low-x gluon and therefore larger $\alpha_S$ in the global fit compared to a DIS-only fit, at the expense of some deterioration in the fit quality of the BCDMS data.
Input values to world average $\alpha_S(M_Z^2)$ [S. Bethke, arXiv:0908.1135]

- $\tau$-decays (N3LO)
- Quarkonia (lattice)
- $Y$ decays (NLO)
- DIS $F_2$ (N3LO)
- DIS jets (NLO)
- $e^+e^-$ jets & shps (NNLO)
- Electroweak fits (N3LO)
- $e^+e^-$ jets & shapes (NNLO)

- “DIS $F_2$” from BBG06 [Blümlein, Böttcher, Guffanti, hep-ph/0607200].
- Non-singlet analysis: free of assumptions on gluon (in principle).
Non-singlet QCD analysis of DIS data \[\text{[BBG06, hep-ph/0607200]}\]

| Order   | $\alpha_S(M_Z^2)$ (expt.) |
|---------|----------------------------|
| NLO     | $0.1148^{+0.0019}_{-0.0019}$ |
| NNLO    | $0.1134^{+0.0019}_{-0.0021}$ |
| NNNLO   | $0.1141^{+0.0020}_{-0.0022}$ |

- Fit $F_2^p$ and $F_2^d$ for $x > 0.3$ (neglect singlet contribution), and $F_2^{NS}$.
- But singlet makes up about:
  - $10\%$ of $F_2^p$ at $x = 0.3$,
  - $2\%$ of $F_2^p$ at $x = 0.5$.

**Exercise:** perform MSTW08 NNLO DIS-only fit to $F_2^p$ and $F_2^d$ for $x > 0.3$ (282 points, 160 from BCDMS).

$\Rightarrow \alpha_S(M_Z^2) = 0.1103 \begin{pmatrix} 0.1130 \end{pmatrix}$ without (with) singlet included.
(Lower than BBG06 due to lack of $y > 0.3$ cut on BCDMS.)

**Conclusion:** low value of $\alpha_S(M_Z^2)$ found by BBG06 due to
(i) dominance of BCDMS data and (ii) neglect of singlet.

**Closest possible to reliable extraction of** $\alpha_S(M_Z^2)$ **from DIS is**
MSTW08 NNLO combined analysis of DIS, DY and jet data:

$\alpha_S(M_Z^2) = 0.1171 \pm 0.0014 \text{ (68\% C.L.)} \pm 0.0034 \text{ (90\% C.L.)}$
Summary

- **MSTW08** still fairly current: no immediate update planned.
- The LHC is starting to provide useful input data for PDF fits.
- Now reasonably good agreement between *global* fits from **MSTW08**, **CT10**, and **NNPDF2.1**, all using GM-VFNS.
- More variation with other PDF sets using more limited data sets and/or restrictive input PDF parameterisations.
- (But **HERAPDF1.5 NNLO** is surprisingly close to **MSTW08**.)
- Tevatron jet data are important to pin down the high-$x$ gluon, with indirect effect on the value of $\alpha_S(M_Z^2)$ extracted.