Heat transfer and axisymmetric stagnation point flow due to a shrinking vertical plate in a nanofluid with slip effects

M A Kardri¹,², N Bachok², N M Arifin² and F M Ali²

¹Faculty of Computer & Mathematical Sciences, Universiti Teknologi MARA, Perak Branch, Tapah Campus, 35400 Tapah Road, Perak, Malaysia
²Department of Mathematics and Institute for Mathematical Research, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia

E-mail: mahan702@perak.uitm.edu.my

Abstract. The problem on steady axisymmetric stagnation point flow with velocity slip due to a shrinking vertical plate in a nanofluid was studied. This problem was focussing on the first-order and second-order velocity slip effects on the governing parameters, such as mixed convection parameter \( \sigma \) and nanoparticle volume fraction \( \phi \). Three types of nanofluids were considered in this study which known as Copper (Cu), Alumina (Al₂O₃) and Titania (TiO₂) with the Prandtl number, \( Pr = 6.2 \). In order to solve the problem, solver bvp4c in Matlab has been applied to solve the numerical part. Before the numerical phase, the governing system of partial differential equations was transformed first into ordinary differential equations by similarity transformation. Then, the observation was done to study the effects of first and second order velocity slip parameter, \( \Lambda \) and \( \Lambda \), mixed convection parameter \( \sigma \) and nanoparticle volume fraction \( \phi \) on heat transfer and fluid flow. Dual solutions exist for a certain range of mixed convection parameter \( \sigma \). It is observed that when the nanoparticle volume fraction increase, the shear stress on the shrinking sheet also increase, same goes for heat transfer rate regarding the first-order and the second-order velocity slip parameters, \( \Lambda \) and \( \Lambda \). The rate of heat transfer can be raised when the magnitude of the first-order and the second-order velocity slip parameter, \( \Lambda \) and \( \Lambda \), decrease.

1. Introduction

Industries nowadays always searching for findings that can give an improvement to their research and development. One of the beneficial study that may contribute to raise the efficiency in production industries is on axisymmetric stagnation point flow problem involving the process on extrusion of plastic sheets, continuous stretching of plastic films and artificial fibres, polymer extrusion and the cooling of metallic plate [1].

Hiemenz [2] was the first researcher who pointed out the problem of steady two-dimensional stagnation point flow and obtained its exact solution. Homann [3] then analysed the axisymmetric problem as an extension to the problem by Hiemenz [2]. Both considered the situation where no-slip condition were applied on the solid surface but in this study, we are interested to examine the slip effects on mixed convection parameter and nanoparticle volume fraction.

In recent years, the problem of slip effect of different flows is widely getting attention by researchers. One of them is Wang [4] who solved the first-order velocity slip case on stagnation point
flows problem. From his observations, the high slip parameter value will influence the flow characteristics. He was also interested to study the moving plate case [5] where the slip is highly affected the changes on velocity profiles and surface resistances.

A new second-order velocity slip model from kinetic theory was found by Wu [6] with the results almost similar to the numerical solution of linearized Boltzmann equation in Knudsen number. Wang [7] then extended the problem to shrinking sheet case with axisymmetric stagnation flow. He found that in two-dimensional case, the solutions are non-unique and no solution for higher shrinking rates. After his pioneering work, considerable attention has been given by researchers to the flow field over a stagnation point flow towards stretching/shrinking sheet. Wang and Ng [8] continued the study by considering the first-order velocity slip on a heated vertical plate. Similar to the problem studied by Wang and Ng [8], Roșca and Pop [9] investigated the mixed convection stagnation point flow past a vertical flat plate with enhancement to the second-order velocity slip.

In the last five years, we can see that the study on axisymmetric stagnation point flow is rising among researchers. Among them that we can highlight here is on cylinder case [10-12], moving plate [13-14], Magnetohydrodynamic [11], rotational case [15-16] and slip effects [17-19]. Other than that, the problem on unaxisymmetric stagnation point flow also started to gain attention from researchers for the problem on cylinder [20-21] and Magnetohydrodynamic [20].

The objective of the present study is to obtain the solution to the problem of second-order velocity slip for axisymmetric stagnation point flow on vertical plate over a shrinking sheet, with slip effect. The results are then compared with Wang and Ng [8] and Soid et al. [17] for validity. The problem in this present study considers the nanofluid model proposed by Tiwari and Das [22]. The numerical results obtained from nanoparticle volume fraction \( \phi \), mixed convection parameter \( \sigma \) and slip parameters, on the fluid velocity component, temperature distribution, skin friction coefficient and local Nusselt number were discussed in detail from their respecting graphs. Recently, Kardri et al. [19] studied similar problem related to stretching sheet but to date, there is still no attempt has been made to study the problem on shrinking sheet in nanofluid.

2. Mathematical analysis

Three-dimensional axisymmetric stagnation point flow was considered in this study on a vertical plate over a shrinking surface with slip effects. The inclusion of nanofluid was highlighted in this study to observe their effects on the heat transfer and fluid flow. Figure 1 portrayed the symmetrical stagnation point flow about the \( z \)-axis and the \( x \)-axis located in the opposite direction to the gravity.

![Figure 1. Shrinking case flow.](image)

The shrinking parameter \( \lambda \) used in this study is \( \lambda < 0 \) for shrinking case. The velocities on the surface in the \( x \) and \( y \) directions represent by \( \lambda u_0(x) \) and \( \lambda v_0(y) \). Far from the plate, the velocity distributions are \( u_0(x) = ax \), \( v_0(y) = ay \) and \( w_0(z) = -2az \), where \( a > 0 \) is the flow strength as \( z \to \infty \). The surface temperature is \( T_\infty(x) = T_0 + T_\infty x \), where wall temperature is \( T_0 \) and \( T_\infty \) is ambient
temperature. The governing equations written are as follows (refer to Wang and Ng [8] and Roşca and Pop [9])

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}
\]

\[
\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \beta g (T - T_\infty), \tag{2}
\]

\[
\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \tag{3}
\]

\[
\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right), \tag{4}
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right), \tag{5}
\]

subject to the boundary conditions

\[
u = \lambda \nu_u(x) + \nu_{slip}(y), \quad T = T_w(x) + T_{slip}(z) \text{ on } z = 0, \tag{6}
\]

\[
\frac{u}{\alpha}, \quad \frac{v}{\alpha}, \quad \frac{w}{\alpha} \rightarrow -2\alpha, \quad T \rightarrow T_\infty \text{ as } z \rightarrow \infty,
\]

where \(T_{slip} = C \partial T/\partial z\) and \(C\) is the temperature jump coefficient. The velocity components \(u = a\eta f(\eta), \quad v = a\eta f'(\eta)\) and \(w = -2\alpha \eta f'(\eta)\) has been identical proved to satisfy the equation (1) from derivation. Modification of equation (4) is needed to find the pressure \(p\), from \(p - p_\infty = \mu \partial w/\partial z - \rho w^2/2\). The slip velocities at the shrinking surface denoted by \(u_{slip}\) and \(v_{slip}\) (refer to Wu [6]) are defined as

\[
u_{slip}(y) = A \frac{\partial v}{\partial z} + B \frac{\partial^2 v}{\partial z^2}, \quad v_{slip}(y) = A \frac{\partial v}{\partial z} + B \frac{\partial^2 v}{\partial z^2}, \tag{7}
\]

where \(A\) and \(B\) are constants where \((B < 0)\) (Soid et al. [17]).

By employing the transformation technique, the following ordinary differential equations were obtained from equations (2), (3) and (5) such that

\[
\frac{1}{(1 - \phi)^{2.5}} \left[ (1 - \phi) + \phi \rho_{nf} / \rho_f \right] f''' + 2 f'' + 1 - f'^2 + \sigma \theta = 0, \tag{8}
\]

\[
\frac{k_{nf} / k_f}{(1 - \phi) + \phi (\rho \rho_C p)} \left[ \rho C_p \right] f' + Pr (2 f' f' - f'') \theta = 0, \tag{9}
\]

subject to boundary conditions (6), we obtain

\[
f(0) = 0, \quad f'(0) = \lambda + \Delta f''(0) + \Lambda f'''(0), \quad \theta(0) = 1 + K \phi(0), \quad \theta(\eta) \rightarrow 0, \quad \nu(\eta) \rightarrow 0, \quad \text{as } \eta \rightarrow \infty \tag{10}
\]

where \(\alpha_{sf}\) is the thermal diffusivity of the nanofluid, \(\mu_{sf}\) is the dynamic viscosity of the nanofluid, \(\rho\) and \(\rho_{sf}\) are the density of the fluid and nanofluid, respectively, \(T\) is the temperature of the nanofluid, \(g\) is the gravitational acceleration, \(\beta\) is the thermal expansion coefficient, \(Pr = \nu/\alpha\) is the Prandtl number, \(\Delta = A \sqrt{a/n} \) where \(\Delta > 0\) is the first-order velocity slip parameter, \(\Lambda = B \alpha / \nu < 0\).
where $\Lambda < 0$ is the second-order velocity slip parameter, $K = C\sqrt{|a|/v}$ where $K > 0$ is the temperature parameter and $\sigma$ is the mixed convection parameter, $\sigma = g\beta T_0/\alpha^2$.

The similarity transformation approach has been used

$$\eta = z\sqrt{|a|/v}, \quad T - T_\infty = \Delta T \theta(\eta).$$

where $\eta$ is the similarity variable, $\Delta T = T_0 - T_\infty$ is the characteristic temperature and $\theta(\eta)$ is the dimensionless parameter.

The skin friction coefficients $C_f$ and the local Nusselt number $Nu_x$ are the two physical components that are really important in this study which defined as

$$C_f = \frac{\tau_w}{\rho_f u_c^2} f''(0), \quad Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)},$$

where $\tau_w$ is the skin friction coefficient, and $q_w$ is the heat flux from the surface of the plate given by

$$\tau_w = \mu(\frac{\partial u}{\partial z})_{z=0}, \quad q_w = -k_{nf}(\frac{\partial T}{\partial z})_{z=0}.$$

The similarity transformation are by substituting equation (11) into equations (12) and (13), we obtain

$$C_f \ Re_x^{1/2} = \frac{1}{(1-\varphi)^{1/2}} f''(0), \quad Nu_x \ Re_x^{-1/2} = \frac{k_{nf}}{k_f} \theta'(0),$$

where $Re_x = u_c x/\nu_f$ is the local Reynolds number.

### 3. Results and Discussion

This study aims to observe the characteristics of heat transfer and fluid flow, as well as the effect of slip parameters to the governing parameters. The partial differential equations were reduced to non-linear ordinary differential equations using similarity transformation. Then, the non-linear ordinary differential equations are solved numerically through numerical computations using Matlab bvp4c function. The parameters involved are nanoparticle volume fraction $\varphi$, mixed convection parameter $\sigma$, first-order velocity slip parameter $\Delta$ and second-order velocity slip parameter $\Lambda$. The nanoparticle volume fraction $\varphi$ used is ranging from 0 to 0.2 where $\varphi = 0$ is for a regular (Newtonian) fluid. The thermophysical properties of water and three nanoparticles namely, Copper (Cu), Titania (TiO$_2$) and Alumina (Al$_2$O$_3$) were taken from Oztop and Abu-Nada [23] with Prandtl number of $Pr = 6.2$ (for water). The results obtained from this study were compared to Wang and Ng [8] and Soid et al. [17] for regular Newtonian fluid case. Favorable agreement was achieved from results validation as shown in Table 1 and the results was presented graphically.

| $\Delta$ | Wang and Ng [8] | Soid et al. [17] | Present results |
|---------|-----------------|-----------------|-----------------|
|        | $f''(0)$        | $-\theta'(0)$   | $f''(0)$        | $-\theta'(0)$   |
| 0       | 1.31194         | 0.45110655      | 1.3119377       | 0.45110655      |
| 0.1     | 1.21009         | 0.46867805      | 1.2100866       | 0.46867805      |
| 1       | 0.61730         | 0.52304734      | 0.6172996       | 0.52304734      |
| 5       | 0.17928         | 0.54568595      | 0.1792836       | 0.54568595      |
Figures 2-3 demonstrate the effect of mixed convection parameter $\sigma$ for a shrinking case $\lambda = -1$. In Figure 2 shows the variations of $f''(0)$ and $-\theta'(0)$ for the first-order velocity slip parameter $\Delta$ with the mixed convection parameter $\sigma$ in a Copper-water nanofluid. We found that, the range of solution for mixed convection parameter $\sigma$ increases with the increase of the first-order velocity slip parameter $\Delta$. The existence of the critical value for mixed convection parameter $\sigma_c < 0$ and the values of $\sigma_c$ decrease in the shrinking case as the first-order velocity slip $\Delta$ increases. Dual solutions exist from the saddle-node bifurcation at $\sigma_c$ where the first solution branch seems leading to larger values of $\sigma$ and the second solution branch terminating as the mixed convection parameter $\sigma$ approaches zero.

![Figure 2](image1.png)

**Figure 2.** Variation of $f''(0)$ and $-\theta'(0)$ for the first-order velocity slip parameter $\Delta=0.1,1,5$ with $\sigma$ when $\Lambda = -0.3$, $K = 0.2$, $\varphi = 0.1$ and $\lambda = -1$ for Copper.

![Figure 3](image2.png)

**Figure 3.** Variation of $f''(0)$ and $-\theta'(0)$ for the second-order velocity slip parameter $\Lambda = -0.1, -1, -5$ with $\sigma$ when $\Lambda = 5$, $K = 0.2$, $\varphi = 0.1$ and $\lambda = -1$ for Alumina.

Figure 3 illustrates the variations of $f''(0)$ and $-\theta'(0)$ with $\sigma$ for second-order velocity slip $\Lambda$ examined on Alumina. This behaviour indicating that when the magnitude of the second-order parameter $\Lambda$ increases, the solution range for both figures decrease. As reported by Soid et al. [17], for a similar problem, an interesting behaviour has been found in Figure 3(a) such that the pattern of each curve is significantly differ when $\Lambda = -0.1$, the curve is forming a parabolic shape and for $\Lambda = -5$, the curve tends form an ellipse. However, when $\Lambda = -1$, the curve seems to form a straight line. The first solution of $\Lambda = -0.1$ and $\Lambda = -1$ give the larger value of $f''(0)$ compared to the second solution as predicted, but it totally differs for $\Lambda = -5$. 


Figure 4. Different nanoparticles with $\sigma = -0.5, \Delta = 5, \Lambda = -0.3, K = 0.2$ and $\lambda = -1$ for variation of skin friction coefficient and local Nusselt number.

Figure 5. Alumina with different values of first-order velocity slip parameter $\Delta$ when $\sigma = -0.5$ and $\Lambda = -0.3$ for variation of skin friction coefficient and local Nusselt number.

Figure 6. Alumina with different values of second-order velocity slip parameter $\Lambda$ when $\sigma = -0.5$ and $\Delta = 0.3$ for variation of skin friction coefficient and local Nusselt number.

Figure 4 exhibits the skin friction coefficient and local Nusselt number for different nanoparticles and figures 5-6 illustrate the different values of first and second order velocity slip, $\Delta$ and $\Lambda$, respectively for Alumina. Based on figures 4-5, we can see that when the nanoparticle volume fraction $\phi$ increases, the skin friction coefficient and local Nusselt number will increase as well. In figure 4(a), Copper gives the highest value of skin friction coefficient, compared to Titania and Alumina that are slightly differ to each other. Copper also gives the highest heat transfer rate compared to the other two nanoparticles as seen in figure 4(b) and Titania shows the lowest result for heat transfer rate due to the
conduction mode domination. This situation proved that Titania has the lowest thermal conductivity compared to Copper and Alumina.

From Figure 5, we found that the decrease of the first-order velocity slip $\Lambda$ will increase the skin friction coefficient and heat transfer rate. Figure 6 pictures that increasing the second-order velocity slip $\Lambda$, will increase the heat transfer rate and decrease the skin friction coefficient.

![Figure 7](image1)

**Figure 7.** Alumina with effect of the nanoparticle volume fraction $\varphi$ when $\Delta = 5, \Lambda = -0.3, \sigma = -2, K = 0.2$ and $\lambda = -1$ on the velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$.

![Figure 8](image2)

**Figure 8.** Titania with effect of the mixed convection parameter $\sigma$ when $\Delta = 5, \Lambda = -0.3, K = 0.2, \varphi = 0.1$ and $\lambda = -1$ on the velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$.

An analysis has been made to observe the velocity and temperature profiles, $f'(\eta)$ and $\theta(\eta)$, respectively for three different nanofluids towards the parameters $\varphi, \sigma$ and $\Lambda$. Figures 7-9 portray that the boundary layer thickness of the second solution is larger than the first solution using different values of nanoparticle volume fraction $\varphi$, mixed convection parameter $\sigma$ (in opposing flow) and second-order velocity slip parameter $\Lambda$.

Figure 8(a) describes that decreasing the mixed convection parameter $\sigma$, will decrease the first solution and increasing the second solution for the velocity profile. However, opposite results were obtained for temperature profile as shown in figure 8(b). The dual velocity profile of figure 8(a) and 9(a) depict the velocity increases with increases of $\sigma$ and $\Lambda$ corresponding to a faster mixed convection parameter $\sigma$ and second-order slip parameter $\Lambda$ velocity for the first solution and vice-versa for the second solution. The velocity and temperature profiles satisfy the far field boundary conditions in equation (10) asymptotically; thus supporting the validity of the numerical results obtained and the existence of the dual solutions displayed in figures 2-3.
Figure 9. Copper-water with effect of the second-order velocity slip $\Lambda$ when $\sigma = -2$, $\Delta = 5$, $K = 0.2$, $\varphi = 0.1$ and $\lambda = -1$ on the velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$.

4. Conclusion

A study has been conducted to solve the problem of axisymmetric stagnation point flow with slip effects for shrinking case due to a vertical plate with the existence of three types of nanofluid particles Copper (Cu), Titania (TiO$_2$) and Alumina (Al$_2$O$_3$). The heat transfer and fluid flow behavior were observed and the following results are derived:

- mixed convection parameter $\sigma$ for a certain range of solution gives dual solutions,
- increases of $f''(0)$ and $-\theta'(0)$ for the first-order velocity slip parameter $\Delta$ will increase their range of solutions but decreases their critical values $\sigma_c$,
- increases of second-order velocity slip parameter $\Lambda$ magnitude will decrease the range of solution for $f''(0)$ and $-\theta'(0)$ but increases their critical values $\sigma_c$,
- increases of the nanoparticle volume fraction $\varphi$, will enhance the shear stress on the shrinking sheet for both slip parameters and increase the heat transfer rate,
- inclusion of Copper into the base water fluid gives the best result to the skin friction and heat transfer coefficients compared to Alumina and Titania,
- the rate of heat transfer can be raised when the magnitude of the first-order and second-order velocity slip, $\Delta$ and $\Lambda$, decreases.

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