Does CPT violation affect the $B_d$ meson life times and decay asymmetries?

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Abstract

We study indirect CPT violating effects in $B_d$ meson decays and mixing, taking into account the recent constraints on the CPT violating parameters from the Belle collaboration. The life time difference of the $B_d$ meson mass eigenstates, expected to be negligible in the standard model and many of its CPT conserving extensions, could be sizeable ($\sim$ a few percent of the total width) due to breakdown of this fundamental symmetry. The time evolution of the direct CP violating asymmetries in one amplitude dominated processes (inclusive semileptonic $B_d$ decays, in particular) turn out to be particularly sensitive to this effect.

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The suggestion for two distinct lifetimes for the $B_d$ or $B_s$ meson mass eigenstates originated in parton model calculations \[1\], which, at that time, were limited by numerous uncertainties of hadronic ($f_B$, the bag parameter, top quark mass, ...) and weak parameters (CKM matrix elements). Many of these, however, cancel in the ratio

\[
\left( \frac{\Delta m}{\Delta \Gamma} \right)_d = \frac{8}{9\pi} \left( \frac{\eta_t}{\eta} \right) \left( \frac{m_t}{m_b} \right)^2 f(x_t) \tag{1}
\]

where $\Delta m_d(\Delta \Gamma_d)$ is the mass (width) difference of the $B_d$ meson mass eigenstates, $\eta_t, \eta$ are calculable perturbative QCD corrections, $x_t = \frac{m_t}{m_w}$ and

\[
f(x) = \frac{3}{2} \frac{x^2}{(1-x)^2} \ln x - \left( \frac{1}{4} + \frac{9}{4} \frac{1}{(1-x)} - \frac{3}{2} \frac{1}{(1-x)^2} \right) \tag{2}\]

Following the discovery of mixing in the $B_d$ system \[2\], $\Delta m_d$ was measured and $m_t$ was the only major source of uncertainty in the ratio. Using the then lower bound on $m_t$ it was shown \[3, 4\] that $\Delta \Gamma_d$ is indeed very small, while $\Delta \Gamma_s$, the width difference of the $B_s$ meson mass eigenstates could be rather large as is indicated by the scaling law \[4\]

\[
\left( \frac{\Delta \Gamma}{\Gamma} \right)_s = \left( \frac{X_{Bs}}{X_{Bd}} \right) \left( \left| V_{ts} \right|^2 \right) \left( \frac{\Delta \Gamma}{\Gamma} \right)_d \tag{3}\]

where $V_{ij}$ ’s are the elements of the CKM matrix and

\[
X_{B_q} = \langle B_q | [\bar{q} \gamma^\mu(1 - \gamma_5)b]^2 | B_q \rangle. \tag{4}\]

In the meanwhile, many advances have taken place with the discovery of the top quark and the determination of its mass \[5\] and more precise values for CKM matrix elements. Combining the new values with the above scaling laws, the width difference among the $B_d$ states is $(\Delta \Gamma/\Gamma)_d \approx 0.0012$, which is unobservable, but for $B_s$ eigenstates $(\Delta \Gamma/\Gamma)_s \approx 0.045$. More recent calculations using heavy quark effective theory and improved QCD corrections \[6, 7\] suggest that calculations based on the absorptive parts of the box diagram improved by QCD corrections give reasonable estimates for both $B_d$ and $B_s$ systems.
Nevertheless the possibility that there are loopholes in the above calculations cannot be totally excluded. For example, $\Delta \Gamma_q$ ($q = d$ or $s$) is determined by only those channels which are accessible to both $B_q$ and $\bar{B}_q$ decays. Its computation in the parton model may not be as reliable as the calculation of $\Gamma_q$, the total width which depends on fully inclusive decays and quark-hadron duality is valid.

In addition to the expected phenomena, one should, therefore, be prepared for unexpected effects and the final verdict on this subject should wait for experimental determination of $\Delta \Gamma$ from the B–factories, B–TeV or LHC–B. Many different suggestions for measuring $\Delta \Gamma_s$ have been put forward \cite{3, 4, 8}. It is believed that $(\Delta \Gamma/\Gamma)_d \sim 0.1$ can be measured at B–factories \cite{9} while $(\Delta \Gamma/\Gamma)_d \sim 0.001$ \cite{10} might be accessible at the LHC.

In this article we wish to emphasize that apart from dynamical surprises in the decay mechanism, a possible breakdown of the CPT symmetry contributes to $\Delta \Gamma/\Gamma$. The currently available constraints on CPT violating parameters \cite{11, 12} certainly allow this possibility. If this happens its effect will be more visible and detectable in the $B_d$ system which, in the electroweak theory, is expected to have negligible $(\Delta \Gamma/\Gamma)_d$. In other words the scenario with $(\Delta \Gamma/\Gamma)_d$ large not only due to hitherto unknown dynamics but also due to a breakdown of CPT is quite an open possibility. In the case of $(\Delta \Gamma/\Gamma)_s$ CPT violation may act in tandem with the already known electroweak dynamics to produce an even larger effect.

There are several motivations for drawing out a strategy to test CPT symmetry. From the experimental point of view all symmetries of nature must be scrutinized as accurately as possible, irrespective of the prevailing theoretical prejudices. It may be recalled that before the discovery of CP violation, there was very little theoretical argument in its favour.

There are purely theoretical motivations as well. First of all the CPT theorem is valid
for local, renormalizable field theories with well defined asymptotic states. It is quite possible that the theory we are dealing with is an effective theory and involving small nonlocal/ nonrenormalizable interactions. Further the concept of asymptotic states is not unambiguous in the presence of confined quarks and gluons. It has been suggested that physics at the string scale may indeed induce nonlocal interactions in the effective low energy theory leading to CPT violation \cite{13}. Moreover, modification of quantum mechanics due to gravity may also lead to a breakdown of CPT \cite{14}.

One of the major goals of the B–factories running at KEK or SLAC is to reveal CP violation in the B system. The discrete symmetry CPT has not yet been adequately tested for the B meson system, although there are many interesting suggestions to test it \cite{15, 16}. In all such works, however, the correlation between ∆Γ and CPT violation was either ignored or not adequately emphasized. It will be shown below that ∆Γ can in general be numerically significant even if CPT violation is not too large.

We consider the time development of neutral mesons $M^0$ (which can be $K^0$ or $D^0$ or $B^0_d$ or $B^0_s$) and their antiparticles $\bar{M}^0$. The time development is determined by the effective Hamiltonian $H_{ij} = M_{ij} - \frac{i}{2} \Gamma_{ij}$ with $M_{ij}$ and $\Gamma_{ij}$ being the dispersive and absorptive parts of the Hamiltonian, respectively \cite{17}. CPT invariance relates the diagonal elements

$$M_{11} = M_{22} \quad \text{and} \quad \Gamma_{11} = \Gamma_{22}. \quad (5)$$

A measure of CPT violation is, therefore, given by the parameter

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}} \quad (6)$$

which is phase convention independent. In order to keep the discussion simple we shall study the consequences of indirect CPT violation only. Since indirect CPT violation is a cumulative effect involving summations over many amplitudes, it is likely that its magnitude would be much larger than that of direct violation in a single decay amplitude. It is further assumed that CPT violation does not affect the off–diagonal elements of $H_{ij}$. 

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These assumptions can be justified in specific string models \[13\], where terms involving both flavour and CPT violations receive negligible corrections due to string scale physics. A further consequence of this assumption is that the usual SM inequality \(M_{12} \gg \Gamma_{12}\) holds even in the presence of CPT violation.

The eigenfunctions of the Hamiltonian are defined as

\[
|M_1\rangle = p_1|M^0\rangle + q_1|\bar{M}^0\rangle \quad \text{and} \quad |M_2\rangle = p_2|M^0\rangle - q_2|\bar{M}^0\rangle
\]

with the normalization \(|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1\). We summarize the consequences of the symmetries. We define

\[
\eta_1 = \frac{q_1}{p_1} = \left[\left(1 + \frac{\delta^2}{4}\right)^{1/2} + \frac{\delta}{2}\right] \frac{H_{21}}{H_{12}}^{1/2}
\]

\[\eta_2 = \frac{q_2}{p_2} = \left[\left(1 + \frac{\delta^2}{4}\right)^{1/2} - \frac{\delta}{2}\right] \frac{H_{21}}{H_{12}}^{1/2}\]

and note that CPT violation is contained in the first factor, while indirect CP violation is in the second factor with the square root. In many expressions we need the ratio \(\omega = \eta_1/\eta_2 = \frac{q_1p_2}{q_2p_1}\) which is only a CPT violating quantity. CPT conservation requires \(\text{Im}\ \omega = 0, \text{Re}\ \omega = 1\) and \(\eta_1 = \eta_2\).

The time development of the states is determined by the eigenvalues

\[
\lambda_1 = H_{11} + \sqrt{H_{12}H_{21}} \left[\left(1 + \frac{\delta^2}{4}\right)^{1/2} + \frac{\delta}{2}\right] \quad \text{and} \quad \lambda_2 = H_{22} - \sqrt{H_{12}H_{21}} \left[\left(1 + \frac{\delta^2}{4}\right)^{1/2} - \frac{\delta}{2}\right]
\]

which can be parametrized as \(\lambda_{1,2} = m_{1,2} - \frac{i}{2} \Gamma_{1,2}\). The quantities which occur in the asymmetries are \(\lambda_1 - \lambda_2 = \Delta m - \frac{i}{2} \Delta \Gamma\) and \(\Gamma = \frac{1}{2}(\Gamma_1 + \Gamma_2)\). To leading order in \((\Gamma_1/\Gamma_{12})/M_{12}\) they are expressed in terms of the CPT parameter

\[
y = \left(1 + \frac{\delta^2}{4}\right)^{1/2}
\]
as follows:

$$\Delta m = m_1 - m_2 = 2|M_{12}|(\text{Re } y + \frac{1}{2}\text{Re } \frac{\Gamma_{12}}{M_{12}}\text{Im } y)$$

$$\Delta \Gamma = \Gamma_1 - \Gamma_2 = 2|M_{12}|(\text{Re } \frac{\Gamma_{12}}{M_{12}}\text{Re } y - 2\text{Im } y).$$

(11)

In the CPT conserving limit $y = 1$ and the contribution to $\Delta m$ is large, overwhelming CPT violating corrections. The CPT conserving contribution to $\Delta \Gamma$, on the other hand, is suppressed by $\text{Re } \frac{\Gamma_{12}}{M_{12}}$. The purely CPT violating term dominating $\Delta \Gamma$ remains, therefore, an open possibility. In order to get a feeling for the magnitude of $\Delta \Gamma/\Gamma$, we use the small $\delta$ approximation and obtain $|\Delta \Gamma/\Gamma| = 0.5 \times (\Delta m/\Gamma) \times (\text{Re } \delta \times \text{Im } \delta)$.

Most of the measurements of $\Delta m/\Gamma$ have been carried out by assuming CPT conservation. If CPT is violated its magnitude could be somewhat different (see, e.g., Kobayashi and Sanda in [15]). Recently the Belle collaboration has determined $\Delta m$ with and without assuming CPT symmetry [12]. The two results, $\Delta m = 0.463 \pm 0.016$ and $0.461 \pm 0.008 \pm 0.016 \text{ ps}^{-1}$ respectively, do not differ appreciably from each other or from the average $\Delta m$ given by the particle data group (PDG). We shall, therefore, use throughout the paper $\Delta m/\Gamma = 0.73$, which is perfectly consistent with the PDG value. The relevant limits on CPT violating parameters from Belle are $|m_{B^0} - m_{\bar{B}^0}|/m_{B^0} < 1.6 \times 10^{-14}$ and $|\Gamma_{B^0} - \Gamma_{\bar{B}^0}|/\Gamma_{B^0} < 0.161$, which implies $|\text{Re } \delta| < 0.54$ and $|\text{Im } \delta| < 0.23$. A choice like $\text{Re } \delta \times \text{Im } \delta \sim 0.1$, consistent with the above bounds, would then yield $\Delta \Gamma/\Gamma$ of the order of a few $\%$, larger than the SM estimate by an order of magnitude. Moreover $\Delta \Gamma/\Gamma$ will be well within the measurable limits of LHC B, should $\delta$ happen to be much smaller.

The Belle limits are derived under the assumption that $\Delta \Gamma/\Gamma$ is negligible. We emphasize that for a refined analysis of CPT violation $\Delta m/\Gamma$ and $\Delta \Gamma/\Gamma$ along with $\delta$ should be fitted directly from the data. Such a combined fit may open up the possibility that $\delta$ could be somewhat larger than the above bounds. In our numerical analysis values consistent with the bounds as well as somewhat larger values will be considered.

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The time development of the states involves, now, the time factors

\[ f_-(t) = e^{-i\lambda_1 t} - e^{-i\lambda_2 t} \]  \hspace{1cm} (12)

\[ f_+(t) = e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t} \quad \text{and} \]

\[ \bar{f}_+(t) = \omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t} . \]  \hspace{1cm} (14)

The new feature is the presence of the factor \( \omega \) in the second and third of these equations.

The decays of an original \( |B^0\rangle \) or \( |\bar{B}^0\rangle \) state to a flavor eigenstate \( |f\rangle \) vary with time and are given by

\[ P_f(t) = \left| \langle f | B^0(t) \rangle \right|^2 = |f_+(t)|^2 N \left| \langle f | B^0 \rangle \right|^2 \]  \hspace{1cm} (15)

\[ \bar{P}_f(t) = \left| \langle f | \bar{B}^0(t) \rangle \right|^2 = |\bar{f}_+(t)|^2 N \left| \langle f | \bar{B}^0 \rangle \right|^2 \]  \hspace{1cm} (16)

\[ P_{\bar{f}}(t) = \left| \langle \bar{f} | B^0(t) \rangle \right|^2 = |\eta_1|^2 |f_-(t)|^2 N \left| \langle \bar{f} | B^0 \rangle \right|^2 \]  \hspace{1cm} (17)

\[ \bar{P}_{\bar{f}}(t) = \left| \langle \bar{f} | \bar{B}^0(t) \rangle \right|^2 = |f_- (t)|^2 N |\omega|^2 \left| \langle \bar{f} | \bar{B}^0 \rangle \right|^2 / |\eta_1|^2 \]  \hspace{1cm} (18)

where \( N^{-1} = |1 + \omega|^2 \) and the matrix elements on the right–hand side \( g = \langle f | B^0 \rangle, \bar{g} = \langle \bar{f} | \bar{B}^0 \rangle, \ldots \), are computed at \( t = 0 \) and have no time dependence. From these expressions it is evident that the five unknowns, \( \Gamma, \Delta m, \Delta \Gamma \) and \( \text{Re} \delta \) and \( \text{Im} \delta \) (or equivalently \( \text{Re} \omega \) and \( \text{Im} \omega \)), must be determined from the data. We emphasize that \( \Delta \Gamma \) must be treated as a free parameter, since in addition to the CPT violating contributions it may also receive contributions from new dynamics. In addition taking linear combinations of these decays, we can produce exponential decays accompanied by oscillatory terms which help in separating the various contributions. It may be recalled that the time dependent techniques for extracting these probabilities and the associated electroweak parameters from data are now being used extensively.

Different schemes for testing CPT violation suggested in the literature \[15, 16\] often involve observables specifically constructed for this purpose. Here we wish to point out...
that some of the observables involving the above probabilities, which are now being routinely measured at BABAR and BELLE are also sufficiently sensitive to CPT violation and have the potential of either revealing the breakdown of this fundamental symmetry or improving the limit on the CPT violating parameter. One such observable is the direct CP violating asymmetry in $B_d$ and $\bar{B}_d$ decays to flavor specific channels $f$ and $\bar{f}$, respectively [18], but with $f$ different from $\bar{f}$. The following ratio is at the center of current interest

$$a_{CP}^{\text{dir}}(t) = \frac{|\langle f|B^0(t)\rangle|^2 - |\langle \bar{f}|\bar{B}^0(t)\rangle|^2}{|\langle f|B^0(t)\rangle|^2 + |\langle \bar{f}|\bar{B}^0(t)\rangle|^2}$$

$$= \frac{|f_+(t)|^2 |g|^2 - |\bar{f}_+(t)|^2 |\bar{g}|^2}{|f_+(t)|^2 |g|^2 + |\bar{f}_+(t)|^2 |\bar{g}|^2}$$

(19)

In the SM or in any of its CPT conserving extensions, $\bar{f}_+(t) = f_+(t)$ and the asymmetry is time independent in general. The time independence holds even if $\Delta \Gamma$ happens to be large due to new dynamics or direct CPT violation and/or new physics influence the hadronic matrix elements. Time evolution of this asymmetry is, therefore, a sure signal of indirect CPT violation. Flavour specific B decays involving a single lepton or a kaon in the final state are possible candidates for this measurement.

This consequence is even more dramatic for decays dominated by a single amplitude in the SM, in which case $|g| = |\bar{g}|$ and $a_{CP}^{\text{dir}}(t)$ vanishes at all times. Purely tree level decays arising from the subprocess $b \to u_i\bar{u}_jd_k$ ($i \neq j$), penguin induced processes $b \to d_i\bar{d}_kd_k$, dominated by a single Penguin operator or inclusive semileptonic decays $b \to Xl^+\nu$ ($l = e$ or $\mu$ and X is any hadronic final state) are examples of such decays. The last process is particularly promising. A single amplitude strongly dominates the decay not only in the SM but also in many extensions of it. The large branching ratio ($\sim 20\%$ for $l = e$ and $\mu$) and reasonably large efficiency of detecting leptons is sufficient to ensure the measurement of this asymmetry at B-factories, provided it is of the order of a few percent.
For this class of decays the matrix elements along with their theoretical uncertainties cancel out in the ratio. Consequently in presence of indirect CPT violation the time dependent asymmetry is the same for all one–amplitude dominated processes and the statistics may be improved by including several channels. If a difference in the time dependence of various modes is observed, the assumption of one amplitude dominance will be questionable and new physics beyond the standard model leading to $|g| \neq |\bar{g}|$, in addition to indirect CPT violation, may be revealed.

In Figure 1 we present the asymmetry for a one amplitude dominated process as a function of time for $\text{Im} \delta = 0.1$ and $\text{Re} \delta = 0.1$ (solid curve, here $\Delta \Gamma / \Gamma = 0.004$), 0.5 (dotted curve, $\Delta \Gamma / \Gamma = 0.02$) and 1.0 (dashed curve, $\Delta \Gamma / \Gamma = 0.04$). Both the time evolution and the nonvanishing of the asymmetry are clearly demonstrated.

The correlation between $a_{CP}^{\text{dir}}$ and $\Delta \Gamma$ calls for a more detailed analysis. As has been noted $\Delta \Gamma$ is significantly different from the SM prediction only if $\text{Im} \delta \times \text{Re} \delta \neq 0$. The numerator and the denominator of the asymmetry are determined to be

$$D(t) = P_f(t) - P_{\bar{f}}(t)$$
$$= \left[ (|\omega|^2 - 1) \left( e^{-\Gamma_2 t} - e^{-\Gamma_1 t} \right) - 4 \text{Im} \omega e^{-\Gamma t} \sin \Delta mt \right] N$$

and

$$S(t) = P_f(t) + P_{\bar{f}}(t)$$
$$= \left[ (|\omega|^2 + 1) \left( e^{-\Gamma_2 t} + e^{-\Gamma_1 t} \right) + 4 \text{Re} \omega e^{-\Gamma t} \cos \Delta mt \right] N$$

A non-vanishing asymmetry can arise in various ways

i) $\text{Im} \omega \neq 0$, which requires $\text{Im} \delta \neq 0.0$,

ii) $\text{Re} \omega \neq 1.0$ and $\Delta \Gamma / \Gamma$ as small as in the SM,

or from a combination of the two possibilities. It is trivial to express $\omega$ in terms of $\delta$ and confirm that both $D(t)$ and $S(t)$ are modified from the SM prediction through $\delta$. When both numerator and denominator of the asymmetry are measured accurately, one
can determine separately real and imaginary parts of delta. This may indicate, albeit indirectly, that $\Delta \Gamma$ is unexpectedly large.

In order to have an idea of how large the effects can be, in Figure 2 we plot $D(t)$ as a function of time for the values $\text{Re} \delta = \text{Im} \delta = 0.1$ (the solid curve). $D(t)$ vanishes for $\text{Im} \delta = 0$ and has a relatively weak dependence on $\text{Re} \delta$, as illustrated by also plotting on the same figure the cases with $\text{Re} \delta = 0.5$ (the dotted curve) and 1.0 (the dashed curve). A similar study of $S(t)$ is presented in Figure 3. This quantity is fairly insensitive to $\text{Im} \delta$.

In order to estimate roughly the number of tagged B-mesons needed to establish a non-zero $D(t)$, we assume that at $t=0$ there is a sample of $N_0 \tag{21}$ tagged $B_0^d$ and $\bar{B}_0^d$. Let of number of semileptonic $B_0^d \tag{1}$ ($\bar{B}_0^d \tag{2}$ ) decays in the time interval $t=(1.0 \pm 0.1) \times \tau_B$ be $n(t) \tag{3}$ ($\bar{n}(t)$) (we assume the lepton detection efficiency to be $\sim 1$). By requiring

$$\frac{|n(t) - \bar{n}(t)|}{\sqrt{n(t)} + \sqrt{\bar{n}(t)}} \geq 3.0, \tag{22}$$

we obtain for $\text{Re} \delta = 0.1$ and $\text{Im} \delta = 0.1$, $N_0 \approx 2.0 \times 10^6$, a number which is realizable at B - factories after several years of run and certainly at the LHC. Including other flavour specific channels like $B_0^d \rightarrow K^+ + \chi$, which has a larger branching ratio ($\approx 70\%$), a measurable asymmetry may be obtained with a smaller $N_0$.

In the presence of indirect CPT violation, the time integrated asymmetry is obtained by integrating the numerator $D(t)$ and the denominator $S(t)$. This leads to

$$a_{\text{CP}}^{\text{dir}} = \left( (|\omega|^2 - 1) \frac{\Delta \Gamma}{\Gamma} - \frac{4 \text{ Im } \omega \times}{(1 + x^2)} \right) \left( 2 \left( |\omega|^2 + 1 \right) + \frac{4 \text{ Re } \omega}{(1 + x^2)} \right) \tag{23}$$

with $x = \Delta m / \Gamma$. In the standard model and for processes dominated by one amplitude the integrated asymmetry vanishes. In extensions of the SM in which the decays are no longer dominated by one amplitude the integrated asymmetry may be nonzero \cite{13}. Thus a nonzero integrated asymmetry points either to new physics (coming from additional
amplitudes) or to indirect CPT violation. In Figure 4 we present the variation of this observable with \( \text{Im} \delta \) for \( \text{Re} \delta = 0.1 \) (solid line) and 0.75 (dashed line).

Experimental studies of CPT violating phenomena can be combined with experiments that search for a \( \Delta \Gamma/\Gamma \). For example, one can consider untagged B mesons decaying to a specific flavour \([3, 4]\). The observable

\[
S_1(t) = P_f(t) + \bar{P}_f(t)
\]

which in the absence of CPT violation have a time dependence governed by two exponentials. If now CPT violation is also included, then an oscillation is superimposed on the exponentials. The original articles \([3, 4]\) considered \( B_s \) decays but the same properties hold for \( B_d \) meson decaying semileptonically or to specific flavour final states.

Looking at flavour non-specific channels there are results for \( a^\text{dir}_{CP} \) (also denoted by \( C_{\pi \pi} \)) from BABAR \([20]\) and BELLE \([21]\) for the channel \( B \to \pi^+ \pi^- \). In the SM using naive factorization this asymmetry turns out to be small \([22]\). It is interesting to note that although the Babar result is fairly consistent with the SM prediction, the BELLE result indicates a much larger asymmetry. It should, however, be noted that there are many theoretical uncertainties. Neither the magnitude of the penguin pollution nor the magnitude of the strong phase difference between the interfering amplitudes can be computed in a full proof way. Direct CP violation in flavour specific, charmless decays have also been measured \([23]\). Here the data is not yet very precise and the theoretical uncertainties are also large. In view of these uncertainties it is difficult to draw any conclusion regarding new physics effects. This underlines the importance of inclusive semileptonic decays which are theoretically clean and the branching ratios are much larger than any of the above exclusive modes.

\( B_d \) decays to CP eigenstates have been observed and established CP–violation via
time dependent measurements \( [24, 25] \). The golden example is \( B^0 \rightarrow \psi K_s \) where the time dependent asymmetry is proportional to \( \sin 2\beta \), where \( \beta \) (also denoted by \( \phi_1 \)) is an angle of the unitarity triangle. The current averaged value of this parameter is \( \sin 2\beta = 0.78 \pm 0.08 \). It is straight--forward to obtain the asymmetry in the presence of indirect CPT violation. An attempt to fit the data as in the SM would lead to an effective \( \sin 2\beta \) which is time dependent. We have checked that with \( \text{Re} \delta = 0.1 \) and \( \text{Im} \delta = 0.1 \), this effective \( \sin 2\beta \) varies between 0.74 and 0.84. We therefore conclude that if \( \sin 2\beta \) is determined with an accuracy of 5 \% or better, some hint of indirect CPT violation may be obtained. However, this observation cannot establish CPT violation unambiguously. Since CPT conserving new physics may change the phase of the \( B_d - \bar{B}_d \) mixing amplitude and/or the decay amplitudes and lead to similar effects.

Many other observables specifically constructed for the measurement of CPT violation \( [15, 16] \) have been suggested in the literature. It will be interesting to compare the sensitivities of these observables to the CPT parameter \( \delta \) with that of the observables considered in this paper, which are already being measured in the context of CP violation.

In summary, we wish to emphasize again that an unexpectedly large life time difference of \( B_d \) mesons, which is predicted to be negligible in the SM and many of its CPT conserving extensions, may reveal indirect CPT violation. Time dependence of the direct CP violating asymmetry for flavour specific decays, which is time independent and vanishes for decays dominated by only one amplitude may establish CPT violation as well as a large life time difference. The theoretically clean inclusive semileptonic decays having relatively large branching ratios might be particularly suitable in this context.

**Acknowledgements**

We wish to thank the Bundesministerium für Bildung and Forschung for financial support under contract No. 05HT1PEA9. One of us (EAP) thanks Mr. W. Horn for useful discussions. AD thanks the Department of Science and Technology, Government
of India for financial support under project no SP/S2/k01/97 and Abhijit Samanta for help in computation.

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Figure 1: The time evolution of the time dependent asymmetry ($a_{\text{CP}}^{\text{dir}}(t)$) for $\text{Im}\delta = 0.1$ for any one amplitude dominated process (in all figures $t$ is in units of $\tau_B/10$, where $\tau_B$ is the average $B^0$ life time). See text for the details.

Figure 2: The variation of $D(t) = \text{Prob}(B \to f) - \text{Prob}(\bar{B} \to \bar{f})$ as function of time for $\text{Im}\delta = 0.1$ for any one amplitude dominated process. See text for the details.
Figure 3: The variation of $S(t) = \text{Prob}(B \rightarrow f) + \text{Prob}(B \rightarrow \bar{f})$ as function of time. This quantity is sensitive to Re $\delta$ only. For comparison we have plotted for Re$\delta=0.0$ (SM) (the dotted curve) and Re$\delta = 0.5$ (the solid curve).

Figure 4: The variation of the time integrated asymmetry $a_{\text{dir}}^{\text{CP}}$ for a one amplitude dominated process vs Im $\delta$. See text for further details.