Hard collisions of photons: plea for a common language

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Abstract. An attempt is made to sort out ambiguities existing in the current usage of several basic concepts describing hard collisions of photons. It is argued that appropriate terminology is often a prerequisite for correct physics.

1 Motivation

Hard collisions of quasireal as well as virtual photons have recently received large experimental and theoretical attention, mainly due to the fact they offer new ways of testing perturbative QCD. The novel feature of such tests arises from the fact that the photon exhibits two apparently different faces: it acts as structureless particle and simultaneously as hadron-like object. At very short distances photon looks simpler than hadrons, but at experimentally accessible ones its hadron-like properties are essential.

This novel and intriguing aspect of photon physics has, however, also led to misunderstanding and confusion resulting mostly from unsettled or inappropriate terminology. It is perhaps not a coincidence that just those aspects which distinguish hard collisions of photons from those of hadrons involve ambiguous, unsuitable or obsolete notions and notation. Different names are used to denote the same content, but also conversely, a particular term is employed by different people to express different contents. The main purpose of this paper is to contribute to defining a set of notions and definitions which is unambiguous, sufficient but not redundant, and as exact as possible.

2 Basic facts

The two-facet appearance of the photon is due to the existence of the point-like coupling of photons to quark-antiquark pairs, described by pure QED. This coupling generates the inhomogeneous terms on the r.h.s. of the evolution equations

\[ \frac{d\Sigma(x, M)}{d\ln M^2} = \delta k_q + P_{qq} \otimes \Sigma + P_{qG} \otimes G, \]

\[ \frac{dG(x, M)}{d\ln M^2} = k_G + P_{qG} \otimes \Sigma + P_{GG} \otimes G, \]

\[ \frac{d\eta_{NS}(x, M)}{d\ln M^2} = \delta_{NS} k_q + P_{NS} \otimes \eta_{NS}, \]

where \( \delta_{NS} = 6n_f \langle e^4 \rangle - \langle e^2 \rangle^2 \), \( \delta = 6n_f \langle e^2 \rangle \)

\[ \Sigma(x, M) \equiv \sum_{i=1}^{n_f} \left[ q_i(x, M) + \eta_{\bar{q}}(x, M) \right], \]

\[ \eta_{NS}(x, M) \equiv \sum_{i=1}^{n_f} \left( e_i^2 - \langle e_i^2 \rangle \right) \left( q_i(x, M) + \eta_{\bar{q}}(x, M) \right), \]

describing the dependence of parton distribution functions (PDF) on the factorization scale \( M \). To order \( \alpha \) the splitting functions \( P_{ij} \) and \( k_i \) are given in powers of \( \alpha_s(M) \)

\[ k_q(x, M) = \frac{\alpha}{2\pi} \left[ k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi} k_q^{(1)}(x) + \cdots \right], \]

\[ k_G(x, M) = \frac{\alpha}{2\pi} \left[ \frac{\alpha_s(M)}{2\pi} k_G^{(1)}(x) + \frac{\alpha_s^2(M)}{2\pi} k_G^{(2)}(x) + \cdots \right], \]

\[ P_{ij}(x, M) = \frac{\alpha_s(M)}{2\pi} P_{ij}^{(0)}(x) + \frac{\alpha_s^2(M)}{2\pi} P_{ij}^{(1)}(x) + \cdots, \]

where \( k_q^{(0)}(x) = x^2 + (1-x)^2 \) and \( P_{ij}^{(0)}(x) \) are unique while all higher order splitting functions \( k_q^{(j)}, P_{ij}^{(j)}, j \geq 1 \) depend on the choice of the factorization scheme (FS). The equations (1-3) can alternatively be rewritten as evolution equations for \( q_i(x, M), \eta_{\bar{q}}(x, M) \) and \( G(x, M) \).

The structure function \( F_2^2(x, Q^2) \) is given as

\[ \frac{1}{x} F_2^2(x, Q^2) = q_{NS}(M) \otimes C_q(Q/M) + \frac{\alpha}{2\pi} \delta_{NS} C_{\gamma} + \langle e^2 \rangle \Sigma(M) \otimes C_q(Q/M) + \frac{\alpha}{2\pi} \langle e^2 \rangle \delta_{\Sigma} C_{\gamma} + \langle e^2 \rangle G(M) \otimes C_G(Q/M), \]

where the coefficient functions \( C_q, C_G, C_{\gamma} \) can be expanded in powers of \( \alpha_s \) taken at the renormalization scale \( \mu \):

\[ C_q(x, Q/M) = \delta(1 - x) + \frac{\alpha_s}{2\pi} C^{(1)}_q(x, Q/M) + \cdots, \]

\[ C_G(x, Q/M) = \frac{\alpha_s}{2\pi} C^{(1)}_G(x, Q/M) + \cdots, \]

\[ C_{\gamma}(x, Q/M) = C^{(0)}_{\gamma}(x, Q/M) + \frac{\alpha_s}{2\pi} C^{(1)}_{\gamma}(x, Q/M) + \cdots, \]
where \( \kappa(x) \equiv 8x(1-x) - 1 \) and \( \alpha_s \equiv \alpha_s(\mu) \). The lowest order contribution to \( C_\gamma \)

\[
C_\gamma^{(0)}(x, Q/M) = (x^2 + (1 - x)^2) \ln \frac{Q^2(1-x)}{M^2x} + \kappa(x) \tag{5}
\]
comes, similarly as \( k_q^{(0)}(x) \), from pure QED, which provides the lowest order contribution to \( F_2^q \) in the form

\[
\frac{1}{x} F_2^{QED}(x, Q^2) = \sum_{i=1}^{n_f} e_i^2 \left(g_i^{QED}(x, Q) + \kappa_i^{QED}(x, Q)\right) + \frac{\alpha}{2\pi} 6 n_f \langle e^4 \rangle C_\gamma^{(0)}(x, 1), \tag{6}
\]
where the quark distribution functions \( g_i^{QED}(x, M) \) satisfy the evolution equations (11) with the purely QED inhomogeneous splitting function \( k_q^{(0)}(x) \) only. The above formulae hold for \( n_f \) massless quark flavors, while for heavy quarks quark mass effects have to be taken into account.

3 Basic notions

In this Section I will review some of the notions and notation used for the description of hard scattering of photons, discussing their various connotations and overlaps.

3.1 Direct & resolved photon

Careful attention deserves already the interpretation of the basic concepts “direct” and “resolved” photon. For instance, in (11) one finds the following expression for the photon “wave function”

\[
|\gamma\rangle = c_{bare}|\gamma_{bare}\rangle + \sum_v c_v |V\rangle + \sum_q c_q |q\bar{q}\rangle + \sum_l c_l |l^+l^-\rangle, \tag{7}
\]
where the first sum runs over vector mesons, the second over \( q\bar{q} \) pairs and third over analogous pairs of leptons and antileptons. The coefficients \( c_v^2 \approx (\alpha/2\pi) \ln(\mu^2/m_v^2) \) and \( c_q^2 \approx (\alpha/2\pi) \ln(\mu^2/k_q^2) \) depend on the scale \( \mu \) “used to probe the photon” and satisfy the “unitarity” relation

\[
c_{bare}^2 = 1 - \sum_v c_v^2 - \sum_q c_q^2 - \sum_l c_l^2. \tag{8}
\]
In the language of (11) the first term on the r.h.s. of (11) defines the direct photon, whereas the remaining ones correspond to the resolved (either to partons or leptons) photon. The separation (11) of the photon state looks intuitively appealing, but must not be taken literally. In fact both relations (11) and (8) make sense only as shorthand for the statements concerning cross sections of the processes involving the initial photon. The terms “direct” and “resolved” are in fact not adjectives of the state of the photon, but of its interactions.

To illustrate this point in detail, let us consider the perturbatively calculable pure QED contribution to \( F_2^q(x, Q^2) \), i.e. we discard in (9) and (10) the contributions of vector mesons and take into account only electromagnetic interactions of quarks. The exact cross section for the lepton-antilepton production in DIS of electrons on the photon, described by the diagram in Fig. (1), is given to order \( \alpha \) and assuming \( Q^2 \gg m_t^2 \) as

\[
d\sigma(e^-\gamma \to e^-\bar{l}l) = \frac{2\pi\alpha^2}{xQ^4} F_{2,l}(x, Q^2) (1 + (1 - y)^2), \tag{9}
\]
where

\[
F_{2,l}(x, Q^2) = \frac{\alpha}{2\pi} 2e_i^4 x \left(k_q^{(0)}(x) \ln \frac{Q^2(1-x)}{m_l^2x} + \kappa(x)\right) \tag{10}
\]
and with the replacement \( e_i^2 \to 3e_i^2 \) similarly for the \( q\bar{q} \) pair production. In QED eq. (11) gives the exact result, but even there it makes sense to separate it into the parts

\[
F_{2,l}^{\gamma,\text{dir}}(x, Q^2) = \frac{\alpha}{2\pi} 2e_i^4 x \left[k_q^{(0)}(x) \ln \frac{Q^2(1-x)}{m_l^2x} + \kappa(x)\right] \tag{13}
\]
\[
F_{2,q}^{\gamma,\text{dir}}(x, Q^2) = \frac{\alpha}{2\pi} 6e_i^4 x \left[k_q^{(0)}(x) \ln \frac{Q^2(1-x)}{m_q^2x} + \kappa(x)\right]. \tag{14}
\]

The “large log” \( \ln(M^2/m_l^2) \) in (13) results from integrating the singular part, proportional to \( 1/\tau \), of the cross sections \( d\sigma(e^-\gamma \to e^-\bar{l}l)/dxdQ^2d\tau \) over the region of small lepton virtuality \( \tau \) (see Fig. (1)), the factorization scale \( M^2 \) defining the upper limit of this integration. The remaining part of this integral, depending on both \( M^2 \) and \( Q^2 \), together with the integral over the whole phase space of the regular part, yields (11). Analogously for the \( q\bar{q} \) production described by (12) and (13), the latter generating \( C_\gamma^{(0)}(x) \) in (11).

Defining generic lepton and quark distribution functions of the photon as

\[
l^{QED}(x, M) = \frac{\alpha}{2\pi} e_i^2 k_q^{(0)}(x) \ln \frac{M^2}{m_l^2}, \tag{15}
\]
\[
q^{QED}(x, M) = \frac{\alpha}{2\pi} 3e_i^2 k_q^{(0)}(x) \ln \frac{M^2}{m_q^2} \tag{16}
\]
allows us to express \( F_2^{\gamma,\text{dir}} \) in terms of quark and lepton distribution functions in the same way as \( F_2 \) for hadrons. The coefficients \( c_v^2, c_q^2 \) appearing in (11) are thus in fact just lepton and quark distributions functions \( l(x, M) \) and \( q(x, M) \) defined above, except that in the latter case \( m_q \) is replaced by the phenomenological parameter \( k_0 \) and the
Fig. 1. Feynman diagrams describing in pure QED the contributions of $\mu^+\mu^-$ and $q\bar{q}$ pairs to the cross section of DIS of electrons on the photon. Momentum fraction and virtuality of quarks and leptons entering the elastic scattering with an electron are denoted as $x$ and $\tau$.

QCD correction (discussed in the next subsection) are included. Expressing $C^2_{\text{bare}}$ as in \[3\] is thus nothing but a shorthand for defining the direct photon contribution, i.e. the sum of \[(13)\] and \[(14)\], by subtracting the resolved photon contributions \[(11) \rightarrow (12)\] from the full expression for $F_2^\gamma$:

$$F_2^{\gamma,\text{dir}} = F_2^\gamma - F_2^{\gamma,\text{res}} - F_2^{\gamma,\text{res}}$$

and similarly for other physical quantities.

Although the quark and lepton distribution functions do not characterize the state of the photon, they are universal, i.e. do not depend on the hard process in which the virtual quarks or leptons in Fig. 1 are involved. This crucial property implies that quark and lepton distribution functions are attributes of the photon and provides the basis for the predictive power of these concepts. In QED the decomposition of $F_2^\gamma$ into the resolved \[(13) \rightarrow (12)\] and direct \[(11) \rightarrow (12)\] photon contributions is actually not necessary since the full result \[(11)\] is known. Nevertheless, such decomposition useful even there, because it shows clearly the origin of the concept of quark and lepton distribution functions and illustrates the central fact that they describe cross sections rather than states!

Switching on QCD implies several modifications of the QED expression \[(1)\]:

- adding perturbative QCD corrections to quark distribution functions and introducing the gluon distribution function,
- adding perturbative QCD corrections to direct photon contribution $F_2^{\gamma,\text{dir}}$, i.e. generating $C^2_{\mathcal{G}(i)}$, $i \geq 1$,
- adding QCD corrections to quark and gluon coefficient functions $C_q$ and $C_{\mathcal{G}}$,
- including the so called hadron-like contribution, discussed in below,

but the physical meaning of PDF of the photon remains basically the same as in pure QED. Recall that for hadrons the very notion of PDF is based on the factorization theorem, which is a statement about cross sections. This theorem relies in turn on the validity of the KLN theorem, which guarantees absence of mass singularities in the sums of cross sections over the sets of degenerate initial and final states. The formal mathematical analogy between the UV renormalization of QCD and “IR renormalization group” technique used in \[3\] to define PDF of hadrons must not disguise the fact that the former deals with basic quantities of QCD lagrangian (fields, masses and charges), whereas the latter with cross sections of physical processes. Moreover, the standard UV renormalization of QCD actually precedes the factorization procedure. In other words, all fields, masses and charges entering the factorization procedure are renormalized quantities!

### 3.2 Factorization theorem and the “bare” PDF

In \[2\] the “bare” PDF of the photon are introduced but in this case the adjective “bare” has nothing to do with the standard UV renormalization of fields, masses and charges, and concerns IR behaviour of PDF, i.e. cross sections. Specifically, the unknown, nonperturbative “bare” PDF of hadrons are assumed to contain mass singularities which, according to KLN theorem, exactly cancel those due to homogeneous perturbative splitting of incoming and outgoing partons. Mass singularities of the “bare” PDF appear when we evaluate, as prescribed by the KLN theorem, cross sections of multiparton initial states, like for instance, an electron scattering on a pair of partons with parallel momenta $p_1, p_2$, degenerate with a single parton with momentum $p_1 + p_2$. The absorption of mass singularities of cross sections coming from perturbative splitting of single incoming partons in the “dressed” PDF is just an equivalent, and simpler, way of describing the result of such procedure. Nevertheless, for bound states the validity of such cancellation is nontrivial.

For the photon the same mechanism can be expected to operate for the “hadron-like” part of PDF, but not for the point-like one. For the latter the parallel logs resulting from the primary $\gamma \rightarrow q\bar{q}$ splitting are not cancelled by the singularity of the “bare” PDF, but cut-off by the confinement at $M_0$, analogously as in QED, where, however, this cut-off is provided by quark masses. The inherent ambiguity in the choice of $M_0$ then naturally relates the point-like and hadron-like parts of full PDF. Identifying the direct photon contributions, with the “bare” photon, as done, for instance in \[2\], is thus flawed.

The different nature of the UV renormalization of QCD quantities and IR “renormalization” of PDF is also the main argument for keeping the factorization and renormalization scales $M$ and $\mu$ as independent free parameters. The former sets the upper bound on the virtualities of quantum fluctuations taken into account, via the factorization theorem, in the definition of PDF, whereas the latter determines the lower bound on virtualities included in the renormalized charges, masses and fields. There is no reason, why these two scales should be identified.

### 3.3 Point-like & hadron-like

The terms point-like (PL) and hadron-like (HAD) have been used by the GRV group \[1\] to describe the separation of a general solution of the evolution equations

1. For discussion of this important point see \[2\].

2. See the next Subsection for definition of these notions.
into the particular solution of the full inhomogeneous equations and a general solution of the corresponding homogeneous one. A subset of the former resulting from the resummation of series of diagrams in Fig. 1 which start with the purely QED vertex \( \gamma \rightarrow q\bar{q} \) and vanish at \( M = M_0 \), are called point-like solutions. Due to the fact that \( M_0 \) is in principle arbitrary parameter, the separation of quark and gluon distribution functions into their point-like and hadron-like parts is, however, ambiguous. In general we can thus write \( (D = q, \bar{q}, G) \)

\[
D(x, M) = D^{PL}(x, M, M_0) + D^{HAD}(x, M, M_0).
\]

The main difference between these two components concerns their virtuality dependence. Whereas the hadron-like parts fall-off with \( P^2 \) rapidly and essentially independently of \( M^2 \), like \( (M_0^2/P^2)\gamma \), the point-like ones decrease much more slowly like \( \ln(M^2/P^2) \). Quantitative aspects of the separation are discussed in [3].

### 3.4 QED & QPM

Another notion often used in photon physics is the “Quark-Parton Model” (QPM) contribution. For \( \gamma \gamma \) processes it usually stands for the lowest order, purely QED contribution involving neither the PDF of the photon nor \( \alpha_s \).

For instance, for heavy quark production in \( \gamma \gamma \) collisions it comes from the diagram in the left part of Fig. 2 taken from [1], and similar diagram describes the lowest order, purely QED contributions to jet production in \( \gamma \gamma \) collisions as well. In both cases the denomination “QED” contribution is certainly accurate and unambiguous. The term QPM might seem appropriate for those processes, for which the lowest order contributions do involve PDF of the photon, but the lowest order parton level matrix element are independent of \( \alpha_s \), like, for instance, Drell-Yan dilepton production in double resolved photon contribution. But as nowadays all PDF used in such calculations do incorporate QCD effects in their scale dependence, the term “lowest order” QCD contribution is certainly more appropriate than “QPM”. Parton model had played indispensable role in the formulation of QCD, but is now so firmly embedded therein that there is little reason to use the denomination “QPM” for historical reasons only, when more appropriate terms are available.

3 Though not in practice if we want to describe the data. For instance, \( M_0 = 0.6 \) GeV in SaS1D parameterizations, whereas \( M_0 = 2 \) GeV in SaS2D ones.

### 3.5 Resolved photon à la DELPHI

Unfortunately, there is no universal agreement on the content of even the very basic notion of resolved photon contribution. Take, for instance, the photon structure function \( F_2^\gamma(x, Q^2) \) as measured at LEP. While OPAL, L3 and ALEPH use this term in the sense introduced in the Section 3.3, DELPHI associates it with the diagram in Fig. 3b, which corresponds to the convolution of PDF of the target photon with the cross section \( \sigma_{\gamma q} \) of the process \( \gamma(Q^2) + q \rightarrow q + G \), cut-off at \( p_T^\gamma \approx 2 \) GeV. In the standard terminology the regular part of \( \sigma_{\gamma q}/\mathrm{d}t \) integrated over the whole phase space of the emitted gluon plus the integral over the singular part from \( M^2 \) up to \( Q^2 \) gives the term proportional to \( q \otimes C_2^{(1)} \) in (1), whereas the integral over the singular part up to \( M^2 \) is included in \( q(x, M) \) and contributes to its scale dependence. On the other hand, and again contrary to the standard procedure, the “single resolved” photon contribution of Fig. 3a is summed with what DELPHI calls “QPM” contribution of Fig. 3a, regularized by means of \( (\text{constituent}) \) quark masses \( m_u = m_d = 0.3 \) GeV, \( m_s = 0.5 \) GeV, \( m_c = 1.5 \) GeV and \( m_b = 4.5 \) GeV. Note that in standard terminology the singular part of this contribution is included in the point-like part of quark distribution functions of the photon and becomes thus part of the resolved photon contribution, whereas its regular part goes to \( C_2^{(0)} \) and describes the lowest order direct photon contribution.

The fact that the DELPHI defines the “resolved photon” contribution to \( F_2^\gamma \) by this non-standard way is unfortunate, but the real problem with their treatment of \( \gamma \gamma \) collisions is the way they simulate genuine hadron-like (called “VMD” by DELPHI) contribution to \( F_2^\gamma(x, Q^2) \). Note that as their sum of QPM and single resolved photon contributions contains both the basic QED contribution and some QCD corrections to it, at least at related to the conventional single resolved photon contribution. For the genuine hadron-like part of PDF of the photon the diagram in Fig. 3b cut-off at \( p_T^\gamma \approx 2 \) GeV describes \( O(\alpha_s) \) correction to basic DIS process \( e+q \rightarrow e+q \), but not this lowest order contribution itself! Consequently, MC event generator TWOGAM used by DELPHI differs substantially from, for instance, HERWIG or PYTHIA event generators at \( x \lesssim 0.01 \), where the genuine hadron-like components of the photon PDF dominate.
3.6 Point-like & hadron-like: alternative use

The terms hadron-like and point-like are used by some theorists [14,15], as well as experimentalists [11,4] in a different sense than as introduced above, namely in the sense of resolved and direct photon contributions. This usage relies on formal mathematical similarity between the expressions for cross sections of hard collisions of hadrons and the resolved photon. The fact that PDF of the photon satisfy different evolution equations than those of hadrons is from this point of view of secondary importance.

The choice of terminology is a matter of convention, but we should avoid the present situation, where the terms “hadron-like” and “point-like” are used in two different senses. As there are good reasons for separating PDF of the photon into their genuine hadron-like and point-like parts, despite the inherent ambiguity of such separation, some notions should exist for this purpose. Since there is little justification to denote as “hadron-like” the contributions that have manifestly nothing to do with the existence and properties of hadrons, the terminology used in [11,4,13,4] is in my view preferable to that of [14,15]. Moreover, as we shall see in Section 8, the absence of unique interpretation of the term “hadron-like” may lead to unnecessary weakening of important experimental observations.

3.7 Anomalous, VMD, bare: who needs them?

The terminology introduced so far exhausts all concepts necessary for the description of hard collisions of photons, but three other terms, VMD, anomalous and bare photon are also widely used. However, for one reason or another, these notions are poor alternatives to the terms hadron-like, point-like and direct.

The term anomalous part of PDF of the photon is particularly unsuitable substitute for the term point-like part introduced above. The term anomalous itself has been coined in [14,15] to denote the result of a simple (from current point of view) calculation first done in [13] of purely QED contribution to cross section of the process \( \gamma^*(Q^2)\gamma(0) \to \text{hadrons} \) based on the box diagram. The fact that the resulting contribution to \( F_2^\gamma(x, Q^2) \), coinciding with (10) for \( \kappa = 0 \), turned out to be proportional to \( \ln Q^2 \) was rightly considered “essentially different” [13] from experimental results on deep inelastic scattering on hadrons, which showed approximate \( Q^2 \)-independence. This latter observation, combined with the idea of Vector Meson Dominance had naturally lead to the expectation of a similar scaling behaviour for \( F_2^\gamma(x, Q^2) \). Recall that [13] was written a few month before the birth of QCD, at the time when parton model was still in its infancy. The term “anomalous” was thus introduced to denote the behavior anomalous with respect to exact scaling of parton model predictions for proton structure function \( F_2^\gamma(x, Q^2) \). In QCD the logarithmic scaling violations are, on the other hand, commonplace and, moreover, stem from the same origin as those found in [13] for \( F_2^{\gamma, \text{QED}} \). From current point of view the term “anomalous” describes nothing anomalous, but on the contrary the behavior of \( F_2^\gamma \) which results from the standard QED coupling of photons to pairs of quarks and antiquarks. Despite its historical connotation, I see no compelling reason for retaining the term anomalous when the more appropriate term point-like is available and, indeed, used by part of physics community.

The hadron-like parts of PDF of the photon are often claimed (see, for instance, [13,4]) to be modeled by PDF of vector mesons and therefore called “VMD”. However, as there is no experimental information on PDF of vector mesons, the latter are actually approximated by those of pions, extracted from analyses of Drell-Yan processes in \( \pi p \) collisions. In view of huge differences between the role of vector mesons and pions in the Standard model, demonstrated among other things by large difference between the masses of pions and vector mesons, this further assumption is, however, difficult to justify. In any case what is assumed for this part of PDF is a rather general shape expected for meson states and the term hadron-like is thus clearly more appropriate.

As for the term “bare”, I have argued already in Section 3.3 why it has no role in the description of hard collisions of the photon and can only cause confusion when used as substitute for “direct”.

4 Graphical representation

Proper graphical representation of hard processes involving photon in the initial state is complicated by the interplay between the point-like part of the resolved photon contribution and the direct photon one. Before going into details, let me emphasize that the factorization theorem, on which the very concept of PDF is based, concerns cross sections, whereas conventional Feynman diagrams describe individual contributions to the corresponding amplitudes. For instance, Fig. 4 represents graphically the expression for the resolved photon contribution to inclusive cross section for the production of \( n \)-parton final state in \( \gamma p \) collisions, which has a generic form

\[
\sigma_{\gamma p}^{(n)} = \sum_{i,j} D_{ij/\gamma}(M) \otimes \sigma_{ij}^{(n)}(M) \otimes D_{j/p}(M),
\]  

(19)
where parton level cross sections are given as

$$
\sigma_{ij}^{(n)}(M) = \alpha_s \kappa \left( \sigma_{ij}^{(n)}(\text{LO}) + \alpha_s \sigma_{ij}^{(n)}(\text{NLO}) + \cdots \right)
$$

and $\kappa \geq 0$. The problem is that except for $\sigma_{ij}^{(n)}(\text{LO})$, all higher order cross sections in (20) involve integrals over the unobserved partons with subsequent subtraction of singular terms, as well as addition of loop corrections, which have different number of final state partons than the tree diagrams. As a result, only the lowest order cross section $\sigma_{ij}^{(n)}(\text{LO})$ can meaningfully be attached to the blobs representing PDF of the beam particles, as in Fig. 3a.

Whereas for hadrons the problem with the proper graphical representation appears first at NLO, for photon induced hard processes it appears once the resolved photon contribution is taken into account. The conventional way of graphical representation of quark distribution functions of the photon (see, for instance, Fig. 1a,c of [19]), reproduced in Fig. 3a,b, combines the solid blob, representing the hadron-like part and the standard Feynman diagram vertex $\gamma \to q\bar{q}$, standing for the point-like one. However, this representation of the point-like part of the resolved photon contributions is unsatisfactory because the diagram in Fig. 3a plays in fact double role. The evaluation of its contribution to the cross section for dijet production involves integration over the virtuality $\tau$, which is split into two parts in the manner described in Section 3. Consequently, part of the contribution of this diagram goes into the definition of the point-like part of quark distribution function and is thus included in the resolved photon contribution, whereas the other one defines the NLO direct photon one. The diagram in Fig. 3b cannot therefore be meaningfully associated to either the direct or resolved photon production. The representation of the point-like part of quark distribution functions depicted in Fig. 3b disregards also the fact that the point-like parts of quark distribution functions include resummation of the effects of multiple parton emissions off the primary $q\bar{q}$ pair. Finally, the point-like part exists also for the gluon distribution function of the photon, whereas Fig. 3b concerns quarks only.

Recently, however, the authors of [20] have come up with a good idea (see Fig. 4) how to represent graphically the point-like part of quark distribution functions of the photon which reflects its primary QED origin. Albeit basically correct, this suggestion goes only half-way in solving the problem of appropriate graphical representation of the point-like parts of PDF of the photon as it concerns point-like parts of quark distribution functions only. The obvious extension of this idea is to represent the point-like parts of PDF of the photon resulting from the resummation of cross sections corresponding to diagrams in Fig. 3 by the special blobs on the l.h.s. of the equation signs in this figure for both quarks and gluons. Moreover, I suggest discarding the lines representing beam particle remnants as the blobs themselves can take up their role. The full solid blobs would be reserved for genuine hadron-like parts of PDF of the photon. As, however, the point-like parts of quark distribution functions of the photon always appear accompanied by the corresponding direct photon contribution of the same order, we have to invent appropriate graphical representation of the latter as well. Because the direct photon contributions are process dependent, any such representation must connect the incoming photon not only to outgoing parton but also to the hard collision itself. For $F_2^{\gamma \gamma}$ possible such representation is suggested in Fig. 5b, along with the diagram a) describing the contribution of the point-like part of the resolved photon. The open blob connecting the initial photon and final quark to the exchanged probing photon represents graphically this process dependence and simultaneously suggests its relation to the solid blob in Fig. 5a. Let me reiterate that the above graphical representations are meaningful for the LO resolved photon contribution only.

There are, however, processes, like heavy quark production in $\gamma\gamma$ collisions, which do not involve at the lowest order of $\alpha_s$ the resolved photon contribution. In such cases the direct photon contribution is described by a simple Feynman diagram, like in the left part of Fig. 4.
5 “Leading” and “next-to-leading” orders

Existing QCD analyses of hard collisions of photons are burdened by the lack of clear separation of genuine QCD effects from those of pure QED origin. Take, for example, heavy quark production in $\gamma\gamma$ collisions discussed in [21]. Counting, as in [14,15], the lowest order, purely QED contribution of the left diagram in Fig. 3 as the “LO QCD” contribution is legitimate, but implies that the content of the term “NLO QCD approximation” is different for $F_{2,c\gamma}$ than, for instance, the analogous $F_2^p$. To perform QCD analysis of $F_{2,c\gamma}$ in a well-defined renormalization scheme requires working within the NLO (or higher) QCD approximation. Using the terminology of [14,15] the “NNLO QCD” approximation of $F_{2,c\gamma}$ would be required for the analysis of $F_{2,c\gamma}$ in a well-defined renormalization scheme!

To avoid misunderstanding it is in my view preferable to discard for the purpose of defining the terms “leading-order” and “next-to-leading order” QCD analysis the pure QED contributions.

6 Photon remnant

The photon remnant is another concept requiring careful use because it is employed in two distinct meanings. First, it simply denotes “something” flying roughly in the direction of the incoming photon. Used in this sense, photon remnant has similar content as proton remnant, except that the mean transverse momentum of partons making up the photon remnant is bigger for the point-like part than the hadron-like part of PDF of the photon. However, this difference is to large extent washed out by hadronization effects and there is thus little noticeable difference between the properties of photon remnant in these two classes of events.

Photon remnant plays a specific role within the framework of multiparton interactions (MI) models. Proper treatment of this additional source of soft particles requires distinguishing the hadron-like and point-like parts of PDF of the photon since for the additional partonic collisions beam remnants play the role of incoming particles. The assumption of uncorrelated multiple scatters, which lies at the heart of the MI model [22,23], amounts to assuming the factorization of the general multiparton distribution functions into products of single parton ones. This might be a good approximation for hadrons, or hadron-like component of the photon, at small to moderate $x$, but is less justified for the point-like one. Indeed, in both analyses [22,23] and their Monte-Carlo implementations in HERWIG and PYTHIA, multiple scatters are simulated only in events corresponding to hadron-like parts of PDF of the photon.

However, multiple parton scatters make in principle sense even for the point-like part of PDF of the photon. The difference between the MI model for hadrons and pointlike part of the photon is indicated in Fig. 8 which shows Feynman diagrams describing double parton interaction in pp and $\gamma\gamma$ collisions, in the latter case coming from the point-like parts of PDF of both photons. In pp collisions the standard way of simulating double parton scattering is to pick up from each of the protons any possible pair of partons $p_k^{(1)}, p_k^{(2)}, k = 1, 2$ with momentum fractions $x_k^{(1)}, x_k^{(2)}, k = 1, 2$. For instance, both partons from each of the protons in Fig. 3a can be quarks with the same small momentum fractions $x_1^{(j)} \simeq x_2^{(j)}, j = 1, 2$. In hadron-hadron collisions the two pairs of partons involved in double parton scattering are thus uncorrelated as far as their identity as well as momentum fractions $(x_1^{(1)}, x_1^{(2)}), (x_2^{(1)}, x_2^{(2)})$ are concerned (apart from the condition $x_1^{(k)} + x_2^{(k)} \leq 1, k = 1, 2$).

For contributions of the point-like parts of PDF of the photon the above approximation is manifestly invalid for the dominant part of the quark distribution functions, coming from the primary QED splitting $\gamma \to q\bar{q}$. For this contribution, depicted in Fig. 8b, both the parton species and momentum fractions are fully correlated as only $q\bar{q}$ pairs from both photons are allowed and, moreover, $x_2^{(1)} = 1 - x_1^{(1)}, k = 1, 2$. For the full point-like part of PDF of the photon this correlation is somewhat washed out, but it is clear that double parton distribution functions do not factorize into the product of single ones as assumed in standard formulation of the MI model.

7 Fluctuating photon

The concept of “fluctuating photon” is well-defined only within the framework of dispersion relations written first in [24] for moments of $F_2(x, P^2, Q^2)$. In [25] virtuality dependence of PDF of the photon was introduced using generalization of these dispersion relations to virtuality dependence of PDF $f_a(x, P^2, M^2)$ themselves

$$f_a^\gamma(x, M^2, P^2) = \sum_{\gamma} \left(\frac{m_\gamma^2}{P^2 + m_\gamma^2}\right)^2 \frac{4\pi \alpha}{F_\gamma^\gamma} \int_{y^\gamma} f_{a,\gamma}^{\gamma\gamma}(x, M^2, M_0^2) \tag{21}$$

$$+ \int_{M_0^2}^{M^2} \frac{dk^2}{k^2} \left(\frac{k^2}{k^2 + P^2}\right)^2 \alpha \sum_q 2\delta^2 f_a^{q\gamma}(x, M^2, k^2),$$

where the first sum runs over the vector mesons and the function $f_a^{q\gamma}(x, M^2, k^2)$ satisfies standard homogeneous
evolution equation with the boundary condition
\[ f_a^{\gamma}(x, k^2, \tau^2) = 3 \left( x^2 + (1 - x)^2 \right) (\delta_{aq} + \delta_{aq}). \]

Let me emphasize at this point that the difference between the VMD (hadron-like in my terminology) and anomalous (point-like) is not, as claimed for instance in [28], in the sign of the off-shellness of the $q\bar{q}$ pair to which the initial space-like photon couples. In both components the off-shellness of the $q\bar{q}$ pair is exactly the same as that of the original initial photon, i.e. negative.

The initial space-like photon involved in hard collisions at LEP and HERA interacts by first coupling to $q\bar{q}$ pairs of the same virtuality $P^2$ as the photon itself. The dispersion relations [27] allow us to express PDF (or, in general, cross sections) of these states as the sum of contributions associated with two types of singularities in the time-like region of the target photon virtuality $P^2$: discrete set of poles corresponding to vector mesons and continuum cut corresponding to free $q\bar{q}$ pairs. The phrase “photon fluctuates” make sense only as a shorthand for the preceding statement. There is also no principal difference between the contributions of vector mesons, in [29] associated with the “valence-driven structure”, and those of the free $q\bar{q}$ pairs, in [25] giving the “fluctuation-driven” contribution. The only difference is that latter is perturbatively calculable, whereas the former is not.

It is also worth emphasizing that the association [25] of the “fluctuation driven” (point-like in my terminology) contributions to $F_2^\gamma$ with the In $Q^2$ pattern of scaling violations holds for $\gamma_\gamma^+\gamma_\gamma^+$ only. The scale dependence of the point-like part of PDF of $\gamma_\gamma^+$ is typically hadron-like [30], i.e. without this ln $Q^2$ behaviour, yet it arises from exactly same “fluctuation driven” mechanism as the ln $Q^2$ behavior of point-like part of PDF of $\gamma_\gamma^+$.

8 OPAL results on the structure of photon

I will now discuss three recent OPAL papers bringing new information on the structure of the photon. Although the results of all three papers are phrased in similar words as evidence about the “hadronic” or “hadron-like” component of the photon, their true importance and impact is substantially different.

8.1 Jet production in $\gamma\gamma$ collisions

In [26] cross sections for dijet production in $\gamma\gamma$ collisions at $\sqrt{s_{\gamma\gamma}} = 161$ and 172 GeV were measured in the kinematical region $E_T \geq 3$ GeV and $x_\gamma \geq 0.1$ and confronted with the LO MC event generators PYTHIA and PHOJET using GRV and SaSID PDF of the photon. The data require the presence of double resolved photon contribution, but do not cover $x_\gamma \lesssim 0.01$ where the genuine hadron-like parts of PDF of the photon are expected to dominate. The conclusion of [26], namely that photons “appear resolved through its fluctuations into hadronic components” therefore implies that the term “hadronic” is used in the sense of [14, 15], i.e. as a substitute for “resolved”.

8.2 $F_2^\gamma(x, Q^2)$ at low $x$.

In [27] $F_2^\gamma(x, Q^2)$ has for the first time been measured for moderate $1.5 \leq Q^2 \leq 30$ GeV$^2$ and very small $x \gtrsim 0.002$. Comparison with existing parameterizations as well as pure QED contribution show convincingly, that the data are far above the contribution of the point-like parts of PDF of the photon and below $x \approx 0.05$ definitely require the presence of the genuine hadron-like part, roughly the size given by GRV or SaSID parameterizations! This is illustrated in Fig. 9, where the SaSID results are compared with pure QED ones for $Q^2 = 10$ GeV$^2$ and taking into account four quarks with masses $m_\mu = m_\tau = m_\pi = 0.32$ GeV, $m_\pi = 1.5$ GeV. Noting that the OPAL data (not shown) are roughly in agreement with the solid curve we conclude that although at very small $x$, even the point-like contribution starts to rise (the rise by itself is thus no evidence for the genuine hadron-like part of PDF of the photon), its magnitude insufficient to account for the data! The conclusion of this paper: “These results show that the photon must contain a significant hadron-like component at low $x$” thus concerns important evidence about the genuine hadron-like part of the photon, and not, as that of [26], merely about the resolved photon contribution.

8.3 Charm contribution to $F_2^\gamma(x, Q^2)$

In Fig. 10 the measurement [1] of $F_2^{\gamma,\pi}$ at $Q^2 = 20$ GeV$^2$ is compared to pure QED calculation using exact Bethe-Heitler formula for $m_\pi = 1.5$ GeV, as well as LO single resolved photon contribution evaluated with SaSID parameterization of PDF of the photon and plotted separately for their hadron-like and point-like parts. In the region $x \approx 0.05$ the data are significantly above all these calculations, as well as those (not shown) including NLO QCD effects in the single resolved photon contribution and $\mathcal{O}(\alpha_s)$ corrections to the Bethe-Heitler cross section in the direct photon channel. Although the error bar is large, it seems that in this case even including the genuine hadron-like contribution may not be sufficient to describe the OPAL data.

The term hadron-like component is used in [1] again as an equivalent of the resolved photon contribution. Thus, although it is true that [1] “the measurement suggests a non-zero hadron-like component of $F_2^{\gamma,\pi}$”, the data tell us
9 Summary and conclusions

Here is my proposal how to describe hard collisions of photons. Use the terms

- **resolved** and **direct** to distinguish contributions that do and do not involve PDF of the photon,
- **hadron-like** and **point-like** to distinguish two components of a general solution of the corresponding evolution equations, the latter coming from resummation of perturbative contributions of multiple parton emissions,
- **QED** to denote the contributions that involve neither $\alpha_s$ nor PDF of the photon,
- **LO and NLO** to denote approximations that contain first and first two nonzero powers of $\alpha_s$ in hard scattering cross sections or splitting functions.

Avoid, on the other hand, notions **bare, anomalous, VMD, QPM**, which are, for one reason or another, less suitable for description of hard collisions of the photon than those listed above.

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