High-resolution spectroscopy of arbitrary light sources using frequency combs

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ABSTRACT

A powerful class of techniques utilizing frequency combs is that of multiheterodyne techniques. These techniques use each individual, evenly spaced, spectral line of a comb as a local oscillator to measure a source’s spectrum. By mixing an unknown source with that of a comb one can convert an optical signal into an electrical signal where standard radio-frequency (RF) electronics can be used. However, these techniques have been limited to measuring coherent sources, such as lasers, due to their inability to disambiguate signals that overlap in the down-converted intermediate frequencies (IF). This excludes most natural sources from being measured. In this manuscript, we present a new dual-comb technique that allows for the measurement of any arbitrary spectrum, even incoherent ones that span multiple comb lines.\textsuperscript{1,2} It is shown that using the same equipment required in a dual-comb experiment one can calculate a correlation function between the two channels that has all the information required for accurate reconstruction. We briefly present the theory followed by simulation and an RF comb experiment.

Keywords: spectroscopy, frequency comb, radiometry, infrared, terahertz, RF

1. INTRODUCTION

An optical frequency comb is a light source whose spectrum consists of perfectly evenly-spaced narrow lines. The success of frequency combs in spectroscopy has been due to multi-heterodyne techniques that utilize each individual spectral line as a local oscillator (LO). By beating a comb with an unknown optical source, one obtains a resultant spectrum that is an exact copy of the source at lower frequencies. Typically these down-converted spectra are designed to fall into the radio-frequency (RF) domain to allow for the use of traditional electronic sampling methods. Multi-heterodyne techniques have allowed for researchers to achieve high signal-to-noise (SNR) measurements of broadband spectra at high acquisition speeds, combining the benefits of multi-heterodyne techniques such as FTIR with that of laser spectroscopy. Examples of these techniques are dual-comb spectroscopy\textsuperscript{3–9} comb-referenced spectroscopy,\textsuperscript{10,11} and vernier spectroscopies.\textsuperscript{12–14} However, these techniques can only faithfully reconstruct the spectrum of a coherent source. This is because a broad incoherent source will result in overlapping intermediate frequencies (IFs) that result in spectral ambiguities. This drastically reduces the applicability of frequency comb systems for spectroscopy as most spectra are incoherent in nature. Of specific mention is radiometry - the measurement of the spectrum of a remote source. This class of problem is extremely prevalent in astronomy, and is completely untenable to these multi-heterodyne techniques.

In this manuscript we present a technique that is able to overcome this limitation and reconstruct the spectrum of any arbitrary light source, including those having spectra much broader than the comb line spacing. This technique draws inspiration from ptychography, an imaging technique that reuses a small bandwidth to piece-wise reconstruct a broad signal, and interferometric techniques that can uniquely resolve and separate individual comb lines.\textsuperscript{15–19} An outline of this technique is outlined in Figure 1. First, two frequency combs whose spectra are known are mixed with the spectrum that one wishes to measure onto two different detectors. The temporal signal measured by the detectors are divided into batches and the FFT calculated in order to produce their corresponding spectograms. Then using an appropriate inversion algorithm, one generates partial spectra corresponding to individual comb lines. One can then stitch together these partial spectra to obtain an accurate, high-resolution reconstruction of the source’s spectrum. An advantage of this technique is that...
Figure 1. Broad overview of the technique outlined in this paper. Utilizing two combs one mixes them with the arbitrary spectrum to measure and measure this beat signal via two power detectors. Using these signals spectrograms are computed and via an appropriate inversion algorithm the original spectrum can be reconstructed. This process works for any arbitrary spectra including incoherent spectrum broader than the comb spacing. Unlike other multi-heterodyne techniques relying on transmission experiments, this method can measure the spectrum of a remote source. Reprinted from Ref. 1.

Each step in the inversion process utilizes the same narrow bandwidth. We report here an algorithm capable of reconstructing the original spectrum along with proof of concept RF comb experiments. This technique is very general and can be conducted at any frequency provided suitable detectors and combs exist.

2. THEORY

The inversion algorithm can be broken down into the three following steps. First, split the time-domain signal into batches. We refer to the time in between each batches as slow-time and the time within each batch as fast time. Next, compute the Fourier transform of each batch individually in order to produce a complex spectrogram. Finally, the two spectrograms are multiplied together and the slow time Fourier transform of this signal is computed. The final result of these steps results in a signal that is proportional to the power spectral density of the source’s spectrum.

We choose to represent the combs and input signal as a superposition of exponentials,

\[ E_{ci}(t) = \sum_n E_{ci}^{(n)} e^{i\omega_{ci}^{(n)} t} \quad \text{and} \quad E_s(t) = \sum_m E_s^{(m)} e^{i\omega_s^{(m)} t} \]

where \( E \) is the field amplitude, \( \omega \) is the comb frequency, and the scripts \( c_i \) and \( c_s \) represent the comb and signal, respectively. One then defines the short time Fourier transform of the signal \( S \) as

\[ F(\omega, T) = \frac{1}{N_t} \sum_i e^{-i\omega t_i} S(t_i + T) \]

where \( t_i \) and \( T \) are short and fast time, respectively. The total time is thus \( t = t_i + T \). In an actual experiment, as outlined in Fig. 1, their exists an \( S \) for each detector and a corresponding spectrogram. Referring to these signals as \( S_1 \) and \( S_2 \) with corresponding spectrograms \( F_1 \) and \( F_2 \), one can define the correlation function \( C_n \) as,

\[ C_n(\omega_k) = \mathcal{F}_T[F_2(\omega_k - \Delta_n, T_j)F_1^*(\omega_k, T_j)](-\Delta_n) \]

where \( \Delta n \) is the separation between corresponding lines in the two combs and \( \mathcal{F}_T \) is the slow time Fourier transform. It can be shown that this function is statistically related to the power spectral density of the spectrum we wish to measure.\(^5\) Resulting in,

\[ \langle C_n(\omega_k) \rangle = \left| E_{ci}^{(n)} E_s^{(c)} \right|^2 \left| E_s(\omega_{ci}^{(n)} + \omega_k) \right|^2. \]

Thus, the spectrum around each comb line can be found by measuring this beat signal as outlined above and dividing out the comb line amplitudes. Like other dual-comb techniques this approach is susceptible to phase noise. However, this is not detrimental to its success as phase noise can be corrected during post-processing.\(^{20–25}\)
Here we present the reconstruction of a simulated terahertz source consisting of multiple lines that overlap in IF. The generated signal lies in the range of 4 - 5 THz and our combs span this region with repetition rates of 10 GHz and 10 GHz + 1 MHz. These values are typical of quantum cascade lasers. The peaks have full-width half maximums (FWHMs) of 100 MHz and are generated using a phase random walk process. Table 1 shows the power and the locations of the peaks. Of particular interest is the fourth column. This part of the table shows that peaks C and D are placed at the exact same frequency in the IF spectrum of comb 1. Therefore, traditional multi-heterodyne techniques are incapable of retrieving the original spectrum. The results of this simulation are shown in Fig. 2. In Fig. 2a the amplitude of the spectrogram for each channel are shown. The x-axis corresponds to slow time while the y-axis corresponds to the IF frequencies. In Fig. 2b the raw IF power spectral densities are shown. What can be seen is that for detector 1, the blue curve, peaks C and D lie on top of each other and are completely indiscernible from one another. In, addition for channel 2 peaks C and D merge together to form one broad feature. The results of applying our inversion algorithm are shown in Fig. 2c. The red curve shows the simulated input signal (i.e. the ground truth), while the blue signal shows the reconstruction. The reconstruction is in excellent agreement with the ground truth as verified by the computed residual.

| Line | Frequency (GHz) | Power (pW) | Offset from comb 1 (GHz) | Offset from comb 2 (GHz) |
|------|----------------|------------|-------------------------|-------------------------|
| A    | 4211           | 0.1        | 1                       | 0.669                   |
| B    | 4558           | 0.225      | -2                      | -2.366                  |
| C    | 4783           | 0.4        | 3                       | 2.612                   |
| D    | 4803           | 0.625      | 3                       | 2.610                   |

Table 1. Lines of the spectrum considered. Comb 1 spans 4-5 THz with a repetition rate of 10 GHz. Comb 2 has a repetition rate of 10 GHz + 1 MHz and has an additional offset of 0.3 GHz. Comb lines have a power of 1 mW per tooth.

Figure 2. a. Magnitudes of the recorded spectrograms as a function of slow time and IF frequency (10 MHz RBW, 0.45 ms measurement time). b. Raw signal power spectral densities, with contributions from beating with various lines labeled. c. Reconstructed signals calculated from equation (1), along with the actual spectrum. Reprinted from Ref. 2.
4. EXPERIMENT

Two microwave frequency comb experiments are demonstrated here as a proof-of-concept. The experimental setup for both are shown in Fig 3. The experiments differ in the specifics of the input signal. Comb 1 and 2 are offset free microwave frequency combs that span from DC up to 4 GHz. Comb 1 is a fixed comb with a repetition rate of 25 MHz while comb 2 has a tunable repetition rate. For the experiments demonstrated here the tunable comb is set to 24.95 MHz, resulting in a difference frequency of 50 kHz. The combs are then bandpass filtered to the region of interest. Also, the bandpass filters remove any comb teeth from the IF frequencies where the down-converted IF signal will be residing. After filtering, each comb is individually amplified and passed into two separate branches. Power combiners are used to join the signal paths together. The reference branch produces a signal that is the mixing of the two combs. This signal is the same signal that is measured in dual-comb spectroscopy. As alluded to in the theory section, this dual-comb signal will be used in order to normalize out any picket level variation from our combs. The signal branch mixes the comb with the desired input spectrum that is then incident onto two individual detectors. This output voltage of this signal branch is the signal used to generate the spectrograms. This voltage signals from each channel is digitized via a high-speed oscilloscope. If one closely inspects the experimental setup it seems that only one amplifier would be needed after the bandpass filter instead of utilizing one for each branch. The reasons for using two amplifiers is to prevent signal crosstalk. This is a problem that is exclusive to the microwave case where impedance mismatches cause unwanted signal reflection and contamination of the signals. Since amplifiers are high-directionality devices they serve to eliminate this interference.

In the first experiment we modulate the VCO in order to produce three peaks equally spaced at 5 MHz apart. This can be seen in Fig. 4a. With the signals having the same repetition rate as one of the combs all them appear at 9 MHz for the IF spectrum of comb 2. The inset of comb 1 shows that at the resolution of our measurement, the three peaks appear very close together in the IF between 3.6 and 3.7 MHz. Even with these lines being completely resolved in comb 1’s IF, one can only use this to determine the peak frequencies modulo the reprate. Shown in b is the actual spectrum measured via an RF spectrum analyzer with the reconstructed signal. As can be seen the technique presented here is capable of reconstructing this signal.

While the previous reconstruction was correct, because it consists of a few discrete lines it could have been inverted using vernier techniques. It does not display the full power of our technique. To do this, we consider a highly degenerate spectrum where the intermediate frequencies overlap multiple times. (See Figure 4) To generate this signal we oscillator near 2660 MHz and FM modulate its input with a linear ramp. This results in a broad spectrum spanning roughly 6 comb lines. It is important when doing so that the sweep rate is much higher than the measurement time of one’s system otherwise it will not appear as a broad signal, but rather a narrowband signal that changes from measurement to measurement. In Fig. 4, the raw spectra measured in each dual comb channel is shown. These signals do not resemble the true spectrum in any way, which is shown in Fig. 4. However, using this data and following the reconstruction procedure yields a spectrum that agrees
The spectrum has also been accurately reconstructed over six repetition rates, despite the fact that we are only using information up to half the combs’ repetition rates (12.5 A variable frequency element, in this case a voltage controlled oscillator (VCO) is used as the input signal.

![Raw power spectral densities](image1)

![Reconstructed overlapping IF spectrum](image2)

![Reconstructed broadband spectrum](image3)

Figure 4. a. IF spectrum of corresponding to the three peak case. b. Reconstructed and actual spectrum. c. IF spectrum corresponding to a signal that is 6 comb lines in width. d. Corresponding reconstruction of broad signal. In both cases if can be seen that it is impossible to infer the true spectrum only from the IF measurement due to overlapping features. However, these features are correctly reconstructed using the inversion algorithm utilizing a bandwidth of half the comb repetition rate.

5. CONCLUSION

We have demonstrated here theoretically, numerically, and in experiment a new frequency comb method that allows for the measurement of any arbitrary light source. This measured source can be incoherent and even broader than the comb spacing, opening the door to many applications that are not tenable to other multi-heterodyne spectroscopies. Of particular importance is that this technique requires no additional equipment as compared to what is normally used for dual-comb spectroscopy. It was shown that inability to resolve spectral features that overlap in IF is not a fundamental limit of heterodyne techniques and given the correct inversion algorithm two frequency combs can resolve any IF ambiguity.

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