Dephasing of a qubit coupled with a point-contact detector

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The dephasing of a qubit coupled with a point-contact detector is theoretically studied. We calculate the time evolution of the reduced density matrix of qubit by using the perturbation expansion. We show that the dephasing rate is proportional to the temperature at zero bias-voltage, while it is proportional to the bias-voltage when the bias-voltage is large. We also evaluate the dephasing rate by using the real time renormalization group method and show that the higher order processes of the particle-hole excitation enhances dephasing of qubit.

Much attention has been devoted in recent years to quantum computation and quantum information, which are the new technology of information processing based on quantum mechanical principles [1]. The dephasing of a qubit is one of the fundamental problems in quantum information sciences. Gruvitz has studied the dephasing of a qubit in a double-dot(DD) caused by the continuous measurement by a point-contact(PC) detector [3,4]. He showed that the collapse and the role of the observer in quantum mechanics can be resolved experimentally via a non-destructive continuous monitoring of a single quantum system. However, extensive theoretical studies on the dephasing of qubit, such as the temperature dependence and bias-voltage dependence, are needed to construct the reliable quantum information processing system.

In this paper, we study the dephasing of a qubit coupled with a PC detector. Using the lowest order approximation, we show that the dephasing rate is proportional to the temperature at zero bias-voltage while it is proportional to the bias-voltage in the limit of large bias-voltage. We also evaluate the dephasing rate by using the real time renormalization group method developed by Shoeller [3,4] and show that the dephasing rate is enhanced by the higher order processes of the particle-hole excitation.

The system we consider is a DD coupled with a PC detector. The PC is placed near the upper dot as shown in Fig. 1 [3,4]. The barrier height of the PC, therefore the tunneling current through PC, is modified by the electron state of DD. We assume that current can flow if and only if the electron in the DD occupies the lower dot. In our system, the dephasing of qubit is caused by the interaction between the qubit and the PC.

We can map the electron state of the DD into that of a single quantum system. However, extensive theoretical studies on the dephasing of qubit, such as the temperature dependence of the particle-hole excitation, respectively. \( \rho_{\text{tot}} \) is the density matrix of the total system consisting of the qubit and PC, \( T_{\text{PC}} \) denotes tracing out the degrees of reservoirs.

The time evolution of the total density matrix \( \rho_{\text{tot}}(t) \) is described by the von Neumann equation, \( i\hbar \dot{\rho} = [H, \rho] \).

\[
\begin{align*}
H &\equiv H_{\text{qb}} + H_{L} + H_{R} + H_{\text{int}}, \\
&= \frac{\epsilon_{\mu}}{2} \sigma_{z} + \sum_{l} \epsilon_{l} c_{l}^{\dagger} c_{l} + \epsilon_{r} c_{r}^{\dagger} c_{r} + \frac{1}{2}(\sigma_{z} + 1) \sum_{l,r}(\Omega_{l} c_{l}^{\dagger} c_{r} + \text{h.c.}).
\end{align*}
\]

Here \( H_{\text{qb}} \), \( H_{L(R)} \) and \( H_{\text{int}} \) are the Hamiltonians of the qubit, the left(right) reservoir of PC and qubit-PC interaction, respectively. \( \epsilon_{l(r)} \) are the energy levels in the left(right) reservoir, and \( \Omega \) is the tunneling matrix element of the PC. When the upper dot is occupied \( (\sigma_{z} = -1) \), the tunneling current does not flow through the PC. Since our interest is in the dephasing of the qubit, we neglect the tunneling between upper and lower dots.

![FIG. 1. The double-dot (qubit) coupled with a point-contact detector. \( \mu_{L} \) and \( \mu_{R} \) are the chemical potential of the left and right reservoirs of point-contact and \( eV \) is the difference between them. The filled circle represents the electron in a double-dot. The height of the potential barrier of the point-contact is modified by the electron state of the double-dot. The current through the point contact flows only when the electron occupies the lower dot.](image-url)

We consider the following reduced density matrix of the qubit:

\[
\rho = T_{\text{PC}} \rho_{\text{tot}} = \begin{pmatrix}
\rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\
\rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow}
\end{pmatrix},
\]

where \( \rho_{\text{tot}} \) is the density matrix of the total system consisting of the qubit and PC, \( T_{\text{PC}} \) denotes tracing out the degrees of reservoirs.
Due to interaction with the detector, the time evolution of the reduced density matrix $\rho(t)$ is not unitary and it approaches the statistical mixture represented by the diagonal matrix in the limit of $t \to \infty$. Therefore, we can evaluate the dephasing of the qubit by calculating the time evolution of the off-diagonal element of reduced density matrix $\rho_{\uparrow\downarrow}(t)$.

We assume that the density matrix of the total system at the initial time $t = 0$ is in product form and the qubit and reservoirs of PC are decoupled, and the reservoirs are in thermal equilibrium. We then have

$$\rho_{\text{tot}}(0) = \rho(0) \otimes \rho_{L0} \otimes \rho_{R0},$$

where

$$\rho_{L0} = \frac{e^{-\sum_i (\epsilon_i - \mu_L) c_i^\dagger c_i/k_BT}}{\text{Tr}e^{-\sum_i (\epsilon_i - \mu_L) c_i^\dagger c_i/k_BT}},$$

$$\rho_{R0} = \frac{e^{-\sum_i (\epsilon_i - \mu_R) c_i^\dagger c_i/k_BT}}{\text{Tr}e^{-\sum_i (\epsilon_i - \mu_R) c_i^\dagger c_i/k_BT}}.$$  

The time evolution of the off-diagonal element $\rho_{\uparrow\downarrow}(t)$ obeys the generalized master equation defined by

$$\left[\frac{d}{dt} + i\frac{\epsilon_G}{\hbar}\right]\rho_{\uparrow\downarrow}(t) = \int_0^t dt'\rho_{\uparrow\downarrow}(t')\Sigma(t-t'),$$

where $\Sigma$ is a ‘self-energy’ which describes the qubit-detector interactions.

We first calculate the self-energy $\Sigma(t-t')$ by using the perturbation expansion with respect to the interaction term $H_{\text{int}}$. In the lowest order approximation, the self-energy is given by

$$\Sigma(t-t') = \exp[-i\epsilon_G(t-t')]\gamma(t-t'),$$

where $\gamma(t-t')$ is the propagator of the particle-hole excitation defined as

$$\gamma(t-t') = \alpha\left(\frac{\pi k_BT}{\hbar}\right)^2 \times \frac{\cos[eV(t-t')/\hbar]}{\sinh^2[\pi k_BT(t-t') - iD/\hbar]}.$$  

Here $D$ is the high frequency cutoff and $\alpha$ is the dimensionless conductance of the PC defined as $\alpha = 2\Gamma^2 N_L N_R$, where $N_L(R)$ is the density of states in the left(right) reservoir. Suppose that the reduced density matrix varies very slowly compared with the time scale of the life time of the particle-hole excitation $\hbar/k_BT$, $\rho_{\downarrow\uparrow}(t)$ is written as

$$\rho_{\downarrow\uparrow}(t) = \rho_{\downarrow\uparrow}(0)e^{-\alpha\nu c^\dagger c/\hbar - \Gamma t},$$

where

$$\Gamma = \int_0^\infty dt\Sigma(t).$$

The dephasing rate, or the decay rate of $\rho_{\uparrow\downarrow}(t)$, is given by $\text{Re}\Gamma$. Since the dephasing is caused by the particle-hole excitation in reservoirs, it depends on the temperature $T$ and bias-voltage $V$ as shown in Fig. 2.

![Fig. 2. Dephasing rate of the qubit as a function of bias-voltage $V$ applied to the point contact. Solid, dotted and dashed lines represent the dephasing rate for $T = 50, 100$ and 150mK, respectively. The high frequency cutoff is assumed to be $hD = 20K$.](image)

It is easy to show that, in the lowest order approximation for the self-energy, the dephasing rate at $V = 0$ is given by

$$\text{Re}\Gamma = \alpha\frac{\pi k_BT}{\hbar}$$

and is proportional to the temperature $T$ as shown in Fig. 2. This can be explained as follows. The dephasing rate is determined by the number of particle-hole excitations in the reservoirs which interact with the qubit. The number of particle-hole excitations at $V = 0$ is proportional to the temperature $T$. On the other hand, the number of particle-hole excitations for $eV \gg k_BT$ is proportional to the bias-voltage $V$. Indeed, the dephasing rate for $eV \gg k_BT$ is given by

$$\text{Re}\Gamma = \alpha\frac{1}{2}\frac{eV}{\hbar}$$

and is proportional to $V$ as shown in Fig. 2.

Next we go beyond the lowest order approximation. We employ the real time renormalization group (RTRG) method developed by Schoeller [5,6]. Following Schoeller, we introduce the short-time cutoff $t_c$ in the propagator of the particle-hole excitation by

$$\gamma_{\downarrow\uparrow}(\tau) = \gamma(\tau)\delta(\tau - t_c),$$

where $\delta(t)$ is a step function. In Laplace space, the self-energy $\Sigma(z)$ is expressed by

$$\Sigma(z) = \Sigma_{t_c}(z) + F_{t_c},$$

where $\Sigma_{t_c}(z)$ includes only the time scales which are precisely smaller than $t_c$ and $F_{t_c}$ consisting of the other time
scales is the functional of renormalized correlation function $\gamma_{tc}$, vertex $g_{tc}$ and frequency $\omega_{tc}$.

Let us increase the cutoff by an infinitesimal amount $t_c \rightarrow t_c + dt_c$. The change of $\gamma_{tc}(\tau)$ is given by

$$d\gamma(\tau) = \gamma_{tc}(\tau) - \gamma_{tc+dt_c}(\tau) = \gamma(t_c)\delta(\tau - t_c)dt_c.$$  \hfill (18)

The change of the self-energy $\Sigma_{tc+dt_c}(z)$ caused by $d\gamma(\tau)$ can be expressed by $d\gamma$, $g_{tc}$ and $\omega_{tc}$. The change of $g_{tc}$ and $\omega_{tc}$ due to $d\gamma(\tau)$ can also be represented by $d\gamma$. As we increase $t_c$, the long time scale of $\gamma$ is renormalized. Since $\gamma(\tau)$ is a decreasing function of $\tau$ with the life time $h/k_B T$, $\Sigma_{tc}$ is zero for $t_c \rightarrow \infty$. Therefore, the self-energy $\Sigma(z)$ in Laplace space is expressed as

$$\Sigma(z) = \lim_{t_c \rightarrow \infty} \Sigma_{tc}(z).$$  \hfill (19)

The renormalization group (RG) equations is obtained by calculating the renormalization of $\omega_{tc}$, $g_{tc}$ and $\Sigma_{tc}$ due to the infinitesimal deviation $d\gamma_{tc}$. The straightforward calculation leads to the following RG equations:

$$\frac{d\Sigma_{tc}(z)}{dt_c} = \int_0^\infty dt \left( \frac{d\gamma_{tc}}{dt_c} \right) (t) a_{tc}(t)b_{tc}(0)$$  \hfill (20)

$$\frac{d\omega_{tc}}{dt_c} = i \int_0^\infty dt \left( \frac{d\gamma_{tc}}{dt_c} \right) (t) g_{tc}(t)g_{tc}(0)$$  \hfill (21)

$$\frac{dg_{tc}}{dt_c} = 0$$  \hfill (22)

$$\frac{da_{tc}}{dt_c} = \int_0^\infty dt \int_{-\infty}^0 dt' \left( \frac{d\gamma_{tc}}{dt_c} \right) (t - t') \times [a_{tc}(t)g_{tc}(0)g_{tc}(t') - g_{tc}(t)a_{tc}(0)g_{tc}(t')]$$  \hfill (23)

$$\frac{db_{tc}}{dt_c} = - \int_0^\infty dt \int_{-\infty}^0 dt' \left( \frac{d\gamma_{tc}}{dt_c} \right) (t - t') \times g_{tc}(t)g_{tc}(0)b_{tc}(t'),$$  \hfill (24)

where

$$\left( \frac{d\gamma_{tc}}{dt_c} \right) (\tau) = \gamma(t_c)\delta(\tau - t_c).$$  \hfill (25)

We numerically solve the above RG equations with the initial condition $\omega = \epsilon_c/h$ and $g = a = b = 1$ at $t_c = 0$. Once we obtain the self-energy in Laplace space, the off-diagonal element of reduced density matrix in Laplace space is given by

$$\rho_{\uparrow\downarrow}(z) = \frac{\rho_{\uparrow\downarrow}(0)}{z - i\epsilon_c - \Sigma(z)}.$$  \hfill (26)

Finally, we obtain $\rho_{\uparrow\downarrow}(t)$ by performing an inverse Laplace transformation.

We define the dephasing time $\tau_{dep}$ as the time when the absolute value $|\rho_{\uparrow\downarrow}(t)|$ becomes one-half of its initial value $|\rho_{\uparrow\downarrow}(0)|$. In Fig. 3, we plot the dephasing rate defined as $1/\tau_{dep}$ at $V = 0$ as a function of $\alpha$. For small $\alpha$, the result agrees well with that obtained by the lowest order approximation. For large $\alpha > 0.2$, however, the dephasing rate becomes larger than that calculated in the lowest order approximation. As shown in Fig. 3, the higher order processes of the particle-hole excitation enhances the dephasing rate, since the number of particle-hole excitations increases due to the quantum fluctuation described by those higher order processes.

![Fig. 3](image-url)  

In conclusion, we have studied the dephasing of a double-dot qubit coupled with a point-contact detector. The time evolution of the reduced density matrix of the qubit is calculated by using the perturbation expansion. In the lowest order approximation of the self-energy, we show that the dephasing rate is proportional to the temperature at $V = 0$, while it is proportional to the bias-voltage at large bias-voltage, $eV \gg k_B T$. The real time renormalization group method is also applied to evaluate the dephasing rate of the qubit. We show that the dephasing rate is enhanced by the higher order processes of the particle-hole excitation.

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