Determination of Freeze-out Conditions from Lattice QCD Calculations*

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Abstract: Freeze-out conditions in Heavy Ion Collisions are generally determined by comparing experimental results for ratios of particle yields with theoretical predictions based on applications of the Hadron Resonance Gas model. We discuss here how this model dependent determination of freeze-out parameters may eventually be replaced by theoretical predictions based on equilibrium QCD thermodynamics.

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1. Introduction

One of the main motivations for the beam energy scan (BES) at RHIC is to explore the QCD phase diagram at non-vanishing baryon chemical potential and to collect evidence for or against the existence of a critical point at a certain pair ($T, \mu_B$) of temperature ($T$) and baryon chemical potential ($\mu_B$) values. Whether or not a phase transition at a parameter set ($T_{cp}, \mu_{cp}$) exists is one of the major uncertainties in our understanding of the QCD phase diagram.

In the vicinity of a critical point various thermodynamic quantities will show large fluctuations. However, even if equilibrated, the hot and dense matter created in a heavy ion collision will expand and cool down. Fluctuations of thermodynamic quantities thus, in general will not be characteristic for a specific ($T, \mu_B$) point in the QCD phase diagram. The situation may, however, be different for fluctuations of conserved charges that freeze-out at ($T_f, \mu_f$) and will not change afterwards. For this reason the analysis of event-by-event fluctuations of baryon number,

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electric charge, and strangeness as well as their higher order cumulants play a central role in the interpretation of thermal conditions created in the BES at RHIC. They provide unique information about the thermal conditions at the time of chemical freeze-out. In fact, this is quite generally the case and is not only restricted to fluctuations in the vicinity of \((T_{cp}, \mu_{cp})\). It also is the case at any point on the freeze-out line mapped in the BES. In particular, the cumulants of fluctuations of conserved charges will also provide information on critical behavior at \(\mu_B = 0\), if the freeze-out points are close to the "true" chiral phase transition that exists in QCD for vanishing quark mass values and describes a line \(T_c(\mu_B)\) in the phase diagram. Whether or not fluctuation observables will be more sensitive to a possibly existing critical endpoint at \((T_{cp}, \mu_{cp})\) or, for instance, the chiral transition at \(T_c(\mu_B \approx 0)\) crucially depends on the proximity of the freeze-out parameters \((T_f, \mu_f)\) to the critical region of the corresponding critical points. In fact, the current determination of freeze-out parameters based on Hadron Resonance Gas (HRG) model calculations [1] and the determination of the QCD crossover and chiral transition lines, which are known from lattice calculations in leading order \((\mu_B/T)^2\) [2, 3], suggest that freeze-out and transition lines differ more as \(\mu_B\) increases. This situation is illustrated in Fig. 1.

2. Freeze-out parameter

It is common practice in heavy ion phenomenology to determine the chemical freeze-out parameters and their dependence on beam energy by comparing experimentally measured particle yields with HRG model calculations [1]. In fact, this approach seems to be quite successful and reliable. It is, however, evident that this approach is conceptually unsatisfactory and must fail, when freeze-out happens close to a critical point in the QCD phase diagram where the dependence of thermodynamics on \(T\) and \(\mu_B\) is more complex than in a HRG. Clearly one eventually wants to compare experimental observables with theoretical predictions based on (equilibrium) QCD.

Figure 1. Phase diagram of QCD in the space of temperature, baryon chemical potential and light quark mass (left) and the freeze-out line determined from a comparison of ratios of particle yields and hadron resonance gas model calculations (right). Also shown in the right hand figure are results for the chiral phase transition line calculated in lattice QCD to leading order in the square of the baryon chemical potential [2].
Extracting information on particle yields at finite $T$ directly from QCD is difficult, if not impossible. However, the experimental measurements of fluctuation observables and their higher order cumulants [4], which all probe thermal conditions at freeze-out, and the improved theoretical calculations of fluctuations of conserved charges in lattice regularized equilibrium QCD thermodynamics [5, 6] make it now possible to determine freeze-out conditions directly from QCD. We will outline in the following a determination of $T_f$ and $\mu_f$ at different values of the beam energy. For simplicity we ignore possible, small non-zero values of the electric charge and strangeness chemical potentials. We also will ignore complications that may arise from the limited phase space in which fluctuation observables are being analyzed experimentally. Our point here is a conceptual one! We present this discussion for the case of baryon number fluctuations but will later on generalize it to the case of electric charge fluctuations.

2.1. The baryon chemical potential at freeze-out

The $n$-th order cumulants of net baryon number fluctuations, $\chi^B_n$, can be calculated in lattice QCD at vanishing baryon chemical potential as suitable derivatives of the pressure $p/T^4$. For small, non-zero values of $\mu_B$ this allows then to calculate cumulants from a Taylor series expansion in $\mu_B/T$,

$$\chi^B_{n,\mu} = \sum_{k=0}^{\infty} \frac{1}{k!} \chi^B_{k+n}(T) \left( \frac{\mu_B}{T} \right)^k$$

with

$$\chi^B_n = \frac{1}{VT^3} \left. \frac{\partial^n \ln Z}{\partial (\mu_B/T)^n} \right|_{\mu_B=0}.$$  

(1)

Appropriate ratios of these cumulants are related to shape parameters of the probability distribution of net baryon number, i.e., the mean value $M_B$, variance $\sigma_B$, skewness $S_B$ and kurtosis $\kappa_B$. In particular, one has

$$\frac{\sigma_B^2}{M_B} = \frac{\chi^B_{2,\mu}}{\chi^B_{1,\mu}}, \quad S_B = \frac{\chi^B_{3,\mu}}{\chi^B_{2,\mu}}, \quad \kappa_B \sigma_B^2 = \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}}.$$  

(2)

Let us consider the Taylor expansion for the simplest even-odd ratio of cumulants, $\chi^B_{2,\mu}/\chi^B_{1,\mu}$. In next to leading order one finds,

$$\frac{\sigma_B^2}{M_B} \equiv \frac{\chi^B_{2,\mu}}{\chi^B_{1,\mu}} = \frac{T}{\mu_B} \left[ 1 + \frac{1}{2} \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}} (\mu_B/T)^2 + \ldots \right].$$  

(3)

A similar relation holds for $\chi^B_{3,\mu}/\chi^B_{2,\mu}$. To leading order the ratios of even and odd cumulants thus determine directly the ratio of $\mu_B$ and $T$ at the time of freeze-out, $\sigma_B^2/M_B = (T_f/\mu_f)(1 + O((\mu_f/T)^2))$. The coefficient of the next-to-leading order correction is small for all temperatures; current lattice QCD calculations suggest $\chi^B_4/\chi^B_2 < 1.5$ for all temperatures. Therefore, the systematic errors that arise from ignoring this correction also remains small for a broad range of beam energies covered in the BES at RHIC. In fact, the systematic error is at most 2% at $\sqrt{s_{NN}} = 200$ GeV and rises to about 20% at $\sqrt{s_{NN}} = 39$ GeV.

Even-odd ratios of cumulants are good observables to determine the value of the baryon chemical potential at freeze-out.

We give results for $\mu_f/T_f$ based on measurements of $\chi^B_{2,\mu}/\chi^B_{1,\mu}$ by the STAR collaboration [4] in Table 1. These compare quite well with HRG model calculations.
Table 1. The ratio of baryon chemical potential and temperature at freeze-out determined from measurements of the ratio of squared variance and mean value of net proton number fluctuations by comparing to lattice QCD calculations of corresponding cumulants of net baryon number fluctuations (third column). Results are given for the two largest values of the beam energy scan. The second error in the third column gives an estimate for the systematic error that arises from neglecting next-to-leading order corrections in the Taylor expansion (Eq. (3)). The last column gives the result for $\mu_f/T_f$ obtained by comparing measured particle yields with HRG model calculations.

| $\sqrt{s_{NN}}$ | STAR $\chi_B^2/\chi_B^{(1)}$ | QCD $\mu_f/T_f$ | HRG $\mu_f/T_f$ |
|-----------------|-----------------------------|-----------------|----------------|
| 200             | 5.3(9)                      | 0.190(30)(4)    | 0.183          |
| 63.4            | 2.35(42)                    | 0.43(8)(3)      | 0.43           |

2.2. The freeze-out temperature

While the ratio of even-odd cumulants is most sensitive to the baryon chemical potential, the ratio of even-even cumulants is, at leading order, determined only by $T_f$. For small values of the baryon chemical potential a low order Taylor series thus again is sufficient. E.g., one finds for the ratio of fourth and second order cumulants,

$$\kappa_B \sigma_B^2 = \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B} \left[ 1 + \frac{1}{2} \chi_{4,\mu}^{B\mu} (T) (\mu_B/T)^2 + \ldots \right],$$

where we explicitly point out the $T$-dependence of cumulants at $\mu_B = 0$. A potential difficulty in the determination of the freeze-out temperature from measurements of $\kappa_B \sigma_B^2 = \chi_{4,\mu}^B/\chi_{2,\mu}^B$ is that lattice QCD calculations [5] suggest that this quantity varies rapidly only in the crossover region but shows little variation at low temperature where it stays close to unity. The discretization errors inherent in these calculations are, however, still too large to allow a direct comparison with experimental data. This will change when improved calculations with a better fermion discretization scheme and closer to the continuum limit will be completed [6].

3. Freeze-out conditions from electric charge fluctuations

The discussion presented in the previous section carries over to fluctuations of other conserved charges, e.g., electric charge or strangeness. The former is of particular interest, as it may soon be accessible experimentally. It will also avoid the problems that arise from the fact that the conserved net baryon number is not accessible directly in a heavy ion experiment. What is measured instead is the fluctuation of proton number, which may change even after freeze-out [9]. Also theoretically, electric charge fluctuations allow a more concise determination of freeze-out parameters as $\kappa_Q \sigma_Q^2$ shows a characteristic variation with temperature also in the hadronic phase [5].

A measurement of $\chi_{4,\mu}^Q/\chi_{2,\mu}^Q$ will allow to determine the freeze-out temperature.

More precisely, it determines a line $T_f(\mu_B)$ in the QCD phase diagram; the additional measurement of an even-odd cumulant ratio will then fix $\mu_B \equiv \mu_f$ and calculations of further ratios will provide consistency checks.

In Fig. 2(left) we show results for the quadratic fluctuations of net electric charge and compare this to HRG model calculations [7]. Preliminary results for $\chi_{4,\mu}^Q/\chi_{2,\mu}^Q$ [8] are shown in Fig. 2(right). We stress that the latter require a careful cut-off analysis, which already for quadratic electric charge fluctuations is difficult [7].
Fig. 2 shows that QCD results for quadratic electric charge fluctuations are consistent with HRG model calculations only for temperatures $T \leq 160$ MeV. Preliminary results for the quartic fluctuations [8] suggest that the ratio $\chi_4^Q/\chi_2^Q$ differs strongly from HRG results at $T \simeq 160$ MeV, i.e. $\chi_4^Q/\chi_2^Q)_{HRG} \simeq 1.7$, and is less than unity.

4. Conclusion

Even-odd ratios of cumulants of conserved charge fluctuations allow to determine the baryon chemical potential at freeze-out. A more precise experimental determination of $\kappa_B \sigma_B^2$ as well as $\kappa_Q \sigma_Q^2$ and improved lattice QCD results for these observables will not only allow to determine the freeze-out temperature $T_f$, it will also provide information on the deviation of the freeze-out line $T_f(\mu_B)$ from the crossover line $T_{pc}(\mu_B)$ and the chiral phase transition line $T_c(\mu_B)$.

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