While the Kobayashi–Maskawa model of CP violation passed its first crucial precision test in $B \rightarrow J/\psi K_S$, other CP asymmetries in $B$ and $B_s$ decays will have to be measured in order to critically test and overconstrain the model in an unambiguous way. On another front, the chirality of weak $b$-quark couplings has not yet been carefully tested. We discuss recent proposals for studying both the chiral and CP-violating phase structures of these couplings in $B \rightarrow D^* \rho$ and $B \rightarrow D^* a_1$.

1 Introduction

The Kobayashi–Maskawa model for CP violation was suggested thirty years ago to explain the tiny CP non-conservation observed in $K$ decays. In the past two years the model passed in a remarkable way its first crucial test in $B$ decays when a large CP asymmetry was measured in $B^0 \rightarrow J/\psi K_S$, in excellent agreement with expectations. The great virtue of this decay mode is the absence of hadronic uncertainties in predicting the mixing induced asymmetry in terms of a fundamental phase parameter $\beta \equiv \phi_1$ of the Standard Model. This opens a new era, in which other CP asymmetries in $B$ and $B_s$ decays will have to be measured in order to test the weak phase structure of $b$ quark couplings in an unambiguous way. Hopefully, this will lead to a point where deviations from the simple CKM framework will be observed. An important step in this direction would be a measurement of the phase $\gamma \equiv \phi_3$, usually associated with CP violation in direct decays. Several methods along this line, in which experimental progress has been made recently, were discussed at this meeting.

The most accessible experimental tests for $\gamma$, in processes such as $B \ (B_s) \rightarrow \pi \pi \ (K K)$ and $B \ (B_s) \rightarrow K \pi$, involve theoretical hadronic uncertainties due to penguin amplitudes, which may be resolved by applying approximate symmetries such as isospin or flavor SU(3) including SU(3)
breaking effects. Other tests, including \( B^\pm \to DK^\pm \) which are free of such uncertainties, usually require a larger number of \( B \) mesons than produced so far. Our present discussion will focus on two theoretically clean studies of \( B^0 \to D^- \rho^+ \) and \( B^0 \to D^- a_1^+ \), where time-dependent CP asymmetries are sensitive to the weak phase \( 2\beta + \gamma \), combining the mixing phase \( 2\beta \) and the decay phase \( \gamma \). While these studies are very challenging from an experimental point of view, and require much more data than accumulated so far, we will explain first what can be learned from existing data of these processes, without performing time-dependent measurements. In particular, we will show how to test the left-handed chirality of the weak \( b \) quark coupling, for which very little evidence exists.

Our motivation for calling for chirality tests of \( b \) quark coupling is both phenomenological (1) as well as theoretical (2):

1. The very small charged current \( b \) quark couplings \( |V_{cb}| = 0.04 \), \( |V_{ub}| = 0.003 - 0.004 \), are more sensitive to new types of interactions, such as right-handed currents, than the lighter quarks’ couplings.

2. An artificial left-right asymmetry is introduced by hand in the Standard Model in order to account for the low energy weak interaction phenomenology. Ultimately, left-right symmetry may be restored at high energies if parity violation and the observed quark mass hierarchy (also introduced by hand in the Standard Model in terms of arbitrary Yukawa couplings) have a common origin, then it may be expected that right-handed couplings grow with quark masses and are larger for the \( b \) quark than for \( s \) and \( d \) quarks.

Our discussion will start with chirality tests for the \( b \) coupling and will end with studies of CP violation. In Section 2 we study helicity amplitudes in \( B \to D^* \rho \), pointing out the success of predicting these amplitudes using factorization and heavy quark symmetry. Application of the same assumptions to \( B \to D^* a_1 \) is shown in Section 3 to permit a test of \( V-A \) for the \( b \) quark coupling. In Section 4 time-dependent CP asymmetries in \( B \to D^* \rho \) and \( B \to D^* a_1 \) are studied in order to learn \( 2\beta + \gamma \), while Section 5 concludes.

2 The decay \( \bar B^0 \to D^{*+} \rho^- \)

2.1 Helicity amplitudes

The decays \( \bar B^0 \to D^{*+} \to D^0 \pi^+ \) \( \rho^- \to \pi^- \pi^0 \), in which each of the two vector mesons decays to two spinless particles whose momenta are measured, can be used to study the vector meson polarization. Using an angular momentum decomposition, the decay amplitude can be written in terms of three helicity amplitudes, \( H_0 \), \( H_+ \), \( H_- \), corresponding to the three polarization states of the vector mesons,

\[
A = \frac{3}{2\sqrt{2}\pi} \left[ H_0 \cos \theta_1 \cos \theta_2 + \frac{1}{2} (H_+ e^{i\phi} + H_- e^{-i\phi}) \sin \theta_1 \sin \theta_2 \right].
\]

Here \( \theta_1 \) and \( \theta_2 \) are the angles between each of the two vector mesons’ momenta in the \( B \) rest frame and the momenta of the corresponding daughter particles in the decaying vector mesons’ rest frame; \( \phi \) is the angle between the \( D^* \) and \( \rho \) decay planes. We use a convention in which the normalized decay angular distribution is given by \( |A|^2 \),

\[
\frac{1}{\Gamma} \frac{d^3\Gamma}{d\cos \theta_1 \cos \theta_2 d\phi} = |A|^2 \Rightarrow |H_0|^2 + |H_+|^2 + |H_-|^2 = 1.
\]

The decay distribution is symmetric under \( (H_0, H_+, H_-) \to (H_0^*, H_+^*, H_-^*) \), implying that rates into left and right polarizations, \( |H_-|^2 \) and \( |H_+|^2 \) respectively, are indistinguishable.
Namely, one cannot distinguish in this process between left- and right-polarized vector mesons. As will be explained in the next section, this follows from the lack of a parity-odd observable when each of the vector mesons decays into two spinless particles. Thus, while the rate into the longitudinally polarized state \(|H_0|^2\) can be measured, only the magnitude of \(|H_+|^2 - |H_-|^2\) is measurable, but not its sign.

2.2 Factorization and heavy quark symmetry

The three helicity amplitudes \(H_{0,\pm}\) can be calculated using factorization and heavy quark symmetry.\(^{15}\) Factorization implies

\[
\langle D^* \rho | H_{\text{eff}} | \bar{B} \rangle \propto \langle D^* | V_{\mu} - A_\mu | \bar{B} \rangle \langle \rho | V^\mu | 0 \rangle ,
\]

where \(\langle \rho | V^\mu | 0 \rangle \propto \epsilon^\mu\). In the heavy quark symmetry limit the \(V - A\) current matrix element can be written in terms of a single form factor multiplying a purely kinematic factor,

\[
\langle D^* (v', \epsilon') | V_{\mu} - A_\mu | \bar{B} (v) \rangle \propto i \epsilon_{\mu \nu \alpha \beta} v'^\nu v^\alpha \epsilon'^\beta + \epsilon'_\mu (1 + v' \cdot v') - v'_\mu \epsilon' \cdot v ,
\]

where \(v \equiv p/m\). In this approximation, the three normalized helicity amplitudes can be written in terms of meson masses. For a \(\bar{c} \gamma_\mu (1 - \gamma_5)b\) current one finds\(^1^{16}\)

\[
H_0 = \left( 1 + \frac{4y}{y + 1} \epsilon^2 \right)^{-\frac{1}{2}} , \quad H_\pm = \left( 1 \mp \sqrt{\frac{y - 1}{y + 1}} \right) \epsilon \left( 1 + \frac{4y}{y + 1} \epsilon^2 \right)^{-\frac{1}{2}} ,
\]

where \(y \equiv (m_B^2 + m_{D^*}^2 - m_\rho^2)/2m_Bm_{D^*} = 1.476\), \(\epsilon \equiv m_\rho/(m_B - m_{D^*}) = 0.236\). Thus, one obtains the values

\[
H_0 = 0.940 , \quad H_+ = 0.125 , \quad H_- = 0.318 .
\]

These predictions of the Standard Model apply to \(\bar{B}^0\) decays, while in \(B^0\) decays the values of \(H_+\) and \(H_-\) are interchanged. In the case of a \(\bar{c} \gamma_\mu (1 + \gamma_5)b\) current, the roles of \(H_+\) and \(H_-\) are interchanged.

The following values were reported very recently by the CLEO collaboration for \(\bar{B}^0 \rightarrow D^{**} \rho\):\(^{17}\)

\[
|H_0| = 0.941 \pm 0.009 \pm 0.006 ,
\]

\[
|H_+| \text{ or } |H_-| = 0.107 \pm 0.031 \pm 0.011 ,
\]

\[
|H_-| \text{ or } |H_+| = 0.322 \pm 0.025 \pm 0.016 .
\]

The collaboration quotes a value for \(|H_+|\) which is smaller than \(|H_-|\), assuming that the \(D^*\) predominantly carries the chirality of the \(c\) quark, as would follow from a \(\bar{c} \gamma_\mu (1 - \gamma_5)b\) coupling. While it is impossible to check this assumption in this experiment, it is important to verify it elsewhere.

The experimental results\(^1^{17}\) are in very good agreement with the amplitudes calculated in Eq.\(^6\). In particular, the measured value of \(|H_0|\) agrees with the Standard Model prediction at a very high precision. The agreement between theory and experiment for \(|H_\pm|\) is somewhat surprising, since one expects sizable deviations from factorization in these amplitudes which are by themselves subleading in \(1/m_b\). In principle, order \(1/m_b\) corrections to factorization in these amplitudes could be as large as the amplitudes themselves. There are some hints in the data\(^1^{17}\) for possibly nonzero relative final state interaction phases between \(H_\pm\) and \(H_0\), which would indicate deviations from factorization. We note that, in spite of this outstanding agreement between factorization predictions and experiment, measurements of \(|H_\pm|\) cannot distinguish between \(V - A\) and \(V + A\) currents. The present experimental precision in the measured values of \(|H_\pm|\) allows also for an admixture of the two opposite chiralities in the \(b \rightarrow c\) coupling.
3 The decay $B^0 \rightarrow D^{*+}a_1^-$

3.1 What's unique about $B \rightarrow D^*a_1$?

In the previous Section we noted that in $B$ meson decays to two vector mesons, each of which decays to two spinless particles whose momenta are measured, one cannot distinguish between left- and right-polarized vector mesons. Therefore, these processes are unsuitable for chirality tests for the $b$ quark coupling. A chirality measurement for one of the two decay particles requires that the particle decays subsequently to a three body final state. The sequence of arguments proving this statement is straightforward: (a) Chirality is a parity-odd quantity, (b) Hadronic quantities multiplying the chirality in the decay distribution must be parity-odd, (c) A pseudoscalar quantity containing the smallest number of hadron momenta is a triple product, $\vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$. Thus, chirality measurements can be performed in $B$ meson decays into a vector meson and an axial-vector meson, which decays subsequently to three pseudoscalars. We note in passing that decays into two vector mesons, one of which decays to three pseudoscalars, do not contain sufficient invariants to permit such a measurement.\[18]

This leads us to the decay $B^0 \rightarrow D^{*+}a_1^-$, in which the $a_1$ is observed through $a_1^- \rightarrow \pi^- \pi^- \pi^+$. This decay mode is unique in the following sense.\[19] A triple product $\vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$ is not only parity-odd but also time-reversal-odd, which requires a nonzero phase due to final state interactions. Usually, such a phase is incalculable and would render measurements which cannot be interpreted theoretically in a simple manner. In the case of $a_1 \rightarrow \pi^- \pi^- \pi^+$, the decay occurs through an interference of two intermediate $\rho^0$ states, the amplitudes of which are equal by isospin. Thus, the final state phase is calculable in terms of the $\rho$ width.

3.2 Distinguishing between left-handed and right-handed helicities

The decay amplitude for $B^0 \rightarrow D^{*+}a_1^-$, $a_1^- \rightarrow \pi^- \pi^- \pi^+$ is written in terms of weak helicity amplitudes $H_i'$, in analogy with \[11\],

$$A(B^0 \rightarrow D^{*+}\pi^- (p_1)\pi^-(p_2)\pi^+(p_3)) = \sum_{i=0,+,−} H_i' A_i .$$

(8)

The strong amplitude $A_i$ involves two terms, corresponding to two possible ways of forming a $\rho$ meson from $\pi^+\pi^-$ pairs, each of which can be written in terms of two invariant amplitudes:

$$A(a_1(p,\varepsilon) \rightarrow \rho(p',\varepsilon')\pi) = A(\varepsilon \cdot \varepsilon') + B(\varepsilon \cdot p')(\varepsilon' \cdot p) ,$$

(9)

convoluted with the amplitude for $\rho^0(\varepsilon') \rightarrow \pi^+(p_i)\pi^-(p_j)$, which is proportional to $\varepsilon' \cdot (p_i - p_j)$. One finds\[19\]

$$A(a_1^- (p,\varepsilon) \rightarrow \pi^- (p_1)\pi^- (p_2)\pi^+(p_3)) \propto C(s_{13}, s_{23})(\varepsilon \cdot p_1) + (p_1 \leftrightarrow p_2) ,$$

(10)

$$C(s_{13}, s_{23}) = [A + Bm_{a_1}(E_3 - E_2)]B_p(s_{23}) + 2AB_p s_{13} ,$$

(11)

where $s_{ij} = (p_i + p_j)^2$, $B_p(s_{ij}) = (s_{ij} - m_\rho^2 - i\rho \Gamma_\rho)^{-1}$, and pion energies are given in the $a_1$ rest frame. The amplitudes $A$ and $B$ are related to $S$- and $D$-wave $\rho\pi$ amplitudes. When neglecting the small $D$-wave amplitude\[12\] they obey\[20\]

$$B = -A \left(1 - \frac{m_\rho}{E_\rho}\right) \frac{E_\rho}{m_\rho p^2_{\rho'}} .$$

(12)

Defining an angle $\theta$ between the normal to the $a_1$ decay plane, $\hat{n}$, and the direction opposite to the $D^*$ in the $a_1$ rest frame, one calculates the $B \rightarrow D^*3\pi$ decay distribution,

$$\frac{d\Gamma}{ds_{13}ds_{23}d\cos\theta} \propto |H_0'|^2 \sin^2\theta|\vec{J}|^2 + (|H_+|^2 + |H_-|^2)^2 \frac{1}{2}(1 + \cos^2\theta)|\vec{J}|^2$$

$$+ (|H_+|^2 - |H_-|^2) \cos\theta \text{ Im}[\vec{J} \times \vec{J}^* \cdot \hat{n}] ,$$

(13)
where
\[ \mathcal{J} = C(s_{13}, s_{23})\vec{p}_1 + C(s_{23}, s_{13})\vec{p}_2 \quad (14) \]

A fit to the angular decay distribution enables separate measurements of the three terms \(|H_0'|^2, |H_+|^2 + |H_-|^2\) and \(|H_+|^2 - |H_-|^2\). We note that when calculating the quantity \(\mathcal{J}\) free of any parameter, using Eq. (12), we have only assumed for the \(a_1\) an \(S\)-wave \(p^0\pi^-\) structure, without using the \(a_1\) resonance shape and width, which would have involved a large uncertainty. A small \(D\)-wave correction can also be incorporated in the calculation.

### 3.3 Factorization and a chirality test

In the heavy quark symmetry and factorization approximation, using (5) where \(y \equiv (m_{b^*}^2 + m_{D'}^2 - m_{a_1}^2)/2m_Bm_{D'} = 1.432, \quad \epsilon \equiv m_{a_1}/(m_B - m_{D'}) = 0.376\), the results for a \(\bar{c}\gamma_\mu(1 - \gamma_5)b\) current are
\[ H_0' = 0.866, \quad H_+ = 0.188, \quad H_- = 0.463. \quad (15) \]

These values, which depend somewhat on \(m_{a_1}\), can be verified by measuring the decay distribution (13).

In order to define a measure for the sensitivity of determining the chirality of the \(b\) quark coupling, let us consider the following \(P\)-odd up-down asymmetry of the \(D^*\) momentum direction with respect to the \(\bar{b}\) current:
\[ A = \frac{\int_0^{\pi/2} \frac{d\Gamma}{d\theta} - \int_{\pi/2}^{\pi} \frac{d\Gamma}{d\theta}}{\Gamma}. \quad (16) \]

One has
\[ A = \frac{3}{4} \left( \frac{\langle |\mathcal{J}|^2 \rangle}{\langle |\mathcal{J}|^2 \rangle} \right) \frac{|H_+|^2 - |H_-|^2}{|H_0'|^2 + |H_+|^2 + |H_-|^2}, \quad (17) \]

and integration over the entire Dalitz plot gives
\[ A = -0.237 \frac{|H_+|^2 - |H_-|^2}{|H_0'|^2 + |H_+|^2 + |H_-|^2}. \quad (18) \]

Measuring this asymmetry determines \(|H_+|^2 - |H_-|^2\). Using Eq. (15), one obtains \(A = 0.042\).

The sign of the asymmetry provides an unambiguous signature for a \(V - A\) coupling, in contrast to \(V + A\) which would yield an opposite sign. In the center-of-mass frame of \(\pi^-\pi^-\pi^+\) the \(\bar{B}^0\) and \(D^{**}\) prefer to move in the hemisphere defined by the direction \(\vec{p}(\pi^-)_{\text{fast}} \times \vec{p}(\pi^-)_{\text{slow}}\). In order to measure an asymmetry at this level one needs about 5000 identified \(B \rightarrow D^*a_1\) events. A very large sample of 18400 ± 1200 partially reconstructed events was reported recently by the BaBar collaboration, in which the \(a_1\) was reconstructed via the decay chain \(a_1^- \rightarrow \rho_0\pi^-, \quad \rho_0 \rightarrow \pi^+\pi^-\) while the \(D^*\) was identified by a slow pion. A correspondingly smaller sample of fully reconstructed events seems sufficient for an up-down asymmetry measurement. A more precise measurement of \(|H_+|^2 - |H_-|^2\) than from the asymmetry alone may be obtained by fitting data to the energy- and angle-dependent decay distribution given in Eq. (13).

### 4 Determining \(2\beta + \gamma\) in time-dependent decays

Both \(\bar{B}^0 \rightarrow D^{**}\rho^-\) and \(\bar{B}^0 \rightarrow D^{**}a_1^-\) belong to a class of processes, which also contains \(\bar{B}^0 \rightarrow D^+\pi^-, D^{**}\pi^-, \quad D^+\rho\), from which the weak phase \(2\beta + \gamma\) can be determined with no hadronic uncertainty. Using the well-measured value of \(\beta\) this would fix \(\gamma\). The difficulty in these methods lies in having to measure a very small time-dependent interference between \(b \rightarrow c\bar{u}d\) and doubly-CKM-suppressed \(\bar{b} \rightarrow \bar{u}\bar{c}\bar{d}\) transitions, where \(|V_{ub}V_{cd}/V_{cb}V_{ud}| \approx 0.02\).
In decays to $D^{+}\pi^{-}$, $D^{*+}\pi^{-}$, $D^{+}\rho^{-}$ the resulting analyses are sensitive to the square of a doubly-CKM-suppressed amplitude; a precise knowledge of which is very challenging. In decays to two vector mesons, $B^0 \to D^{*+}\rho^{-}$, one avoids the need to determine this small quantity by using an interference between helicity amplitudes of CKM-allowed and doubly-CKM-suppressed decays. This was claimed\cite{24} to improve the sensitivity, but requires a detailed angular analysis in addition to time-dependent measurements. The feasibility of using an angular analysis for measuring the helicity amplitudes in the dominant CKM-allowed channel was demonstrated by the CLEO collaboration\cite{17} as discussed above. It will require considerably more statistics to measure the time and angular dependent interference of helicity amplitudes with such disparate magnitudes. Here we will assume that sufficient statistics is gained,\cite{25} and will describe this method for determining $2\beta + \gamma$, first in $\bar{B}^0 \to D^{*+}\rho^-\pi^-$ and then in $\bar{B}^0 \to D^{*+}a_1^-\pi^0$ where a discrete ambiguity in the weak phase will be shown to be resolved.

4.1 $\bar{B}^0(t) \to D^{*+}\rho^-$

It is convenient to write the amplitude $A \equiv A(\bar{B}^0 \to D^{*+}(\to D^0\pi^+)\rho^-((\to \pi^-\pi^0))$ in a linear polarization basis (a so-called transversity basis\cite{16,26}), in which the $D^*$ and $\rho$ transverse polarizations are either parallel or perpendicular to one another, $H_{||,\perp} = (H_+ \pm H_-)/\sqrt{2}$, and to similarly expand $a \equiv A(B^0 \to D^{*+}\rho^-)$ in terms of $h_{0,||,\perp}$:

$$A = \frac{3}{2\sqrt{2\pi}} (H_0 g_0 + H_{||} g_{||} + iH_{\perp} g_{\perp}) , \quad a = \frac{3}{2\sqrt{2\pi}} (h_0 g_0 + h_{||} g_{||} + ih_{\perp} g_{\perp}) , \quad (19)$$

$$g_0 = \cos \theta_1 \cos \theta_2 , \quad g_{||} = \frac{1}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \cos \phi , \quad g_{\perp} = \frac{1}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \sin \phi . \quad (20)$$

The transversity amplitudes can be written as

$$H_t = |H_t| \exp(i\Delta_t) , \quad h_t = |h_t| \exp(i\delta_t) \exp(i\gamma) . \quad (21)$$

The time-dependent rate for $\bar{B}^0(t) \to D^{*+}\rho^-$ has the general form

$$\Gamma(t) \propto e^{-\Gamma t} \left[ |a|^2 + |a|^2 \right] + (|a|^2 - |a|^2) \cos \Delta t + 2\text{Im} \left( e^{-i2\beta} a^* \right) \sin \Delta t \right]$$

$$= e^{-\Gamma t} \sum_{t \leq t'} (\Lambda_{tt'} + \Sigma_{tt'} \cos \Delta t + \rho_{tt'} \sin \Delta t) g_t g_{t'} . \quad (22)$$

Each of the coefficients in the sum can be measured by performing a time-dependent angular analysis. Denoting $\Phi \equiv 2\beta + \gamma$, this determines the following quantities:

$$|H_0|^2 , \quad |H_0||H_{\perp}| \sin(\Delta_0 - \Delta_\perp) , \quad |H_{||}||H_{\perp}| \sin(\Delta_- - \Delta_\perp) , \quad |H_t||h_t| \sin(\Phi + \Delta_t - \delta_t) , \quad (t = 0, ||, \perp) ,$$

$$|H_{\perp}||h_0| \cos(\Phi + \Delta_{\perp} - \delta_0) - |H_0|h_{\perp} \cos(\Phi + \Delta_0 - \delta_{\perp}) , \quad |H_{\perp}||h_0| \cos(\Phi + \Delta_{\perp} - \delta_{\perp}) - |H_0|h_{\perp} \cos(\Phi + \Delta_0 - \delta_0) . \quad (23)$$

One does not rely on knowledge of the small $|h_0|^2$ terms\cite{24} in which uncertainties would be large. Decays into the charge-conjugate state $D^{*-}\rho^+$ determine similar quantities, where $\Phi$ is replaced by $-\Phi$. It is then straightforward to show that this overall information is sufficient for determining $|\sin \Phi|$. However, the sign of $\sin(2\beta + \gamma)$ remains ambiguous.

4.2 What is new in the time-dependence of $\bar{B}^0(t) \to D^{*+}a_1^-$?

The amplitudes $A' \equiv A(\bar{B}^0 \to D^{*+}(3\pi)_{a_1})$ and $a' \equiv A(B^0 \to D^{*+}(3\pi)_{a_1}^-)$ are written in analogy with\cite{19}:

$$A' = \sum_{t=0,||,\perp} H_t A_t , \quad a' = \sum_{t=0,||,\perp} h_t A_t . \quad (24)$$
of the angles $\theta$ and $\phi$, one has calculable complex amplitudes $A_t$ defined in Eq. (10). These are functions of $\theta$ defined above, an angle $\chi$ describing a common angle of rotation for the three pions in the $a_1$ decay plane, and an angle $\psi$ determined by the $D^*$ decay plane. The latter defines the angle between the two intersection lines of the $D^*$ decay plane and of the $a_1$ decay plane with a plane perpendicular to the $D^*$ direction.

One measures $\Gamma(\bar{B}^0(t) \to D^{*+}(3\pi)^-_a)$ and $\Gamma(\bar{B}^0(t) \to D^{*-}(3\pi)^+_a)$, with time-dependence as in Eq. (22), as a function of $\theta$ and $\psi$ while integrating over $\chi$. Instead of the product of geometrical functions $g_t g_t$ in $B \to D^*\rho$, the sum in Eq. (22) now involves calculable functions of the angles $\theta$ and $\psi$, defined as $R_{ij} \equiv (1/2\pi) \int d\chi \text{Re}(A_i A_j^*)$ and $I_{ij} \equiv (1/2\pi) \int d\chi \text{Im}(A_i A_j^*)$, $(i,j = 0, \parallel, \perp)$. The nine independent functions are given by

$$
\begin{align*}
R_{00} &= \frac{1}{2} \sin^2 \theta |\vec{J}|^2, \\
R_{\parallel \perp} &= \frac{1}{2} (1 - \sin^2 \theta \sin^2 \psi) |\vec{J}|^2, \\
R_{0 \perp} &= \frac{1}{4} \sin \psi \sin 2\theta |\vec{J}|^2, \\
R_{\parallel \parallel} &= \frac{1}{2} (1 - \cos^2 \psi \sin^2 \theta |\vec{J}|^2, \\
I_{00} &= \frac{1}{4} \cos \psi \sin 2\theta |\vec{J}|^2, \\
I_{0 \perp} &= \frac{1}{4} \sin 2\psi \sin^2 \theta |\vec{J}|^2, \\
I_{\parallel \parallel} &= \frac{1}{4} \sin 2\psi \sin^2 \theta |\vec{J}|^2,
\end{align*}
$$

(25)

where $J_n^2 \equiv (1/2)\text{Im}[(\vec{J} \times \vec{J}^*) \cdot \hat{n}]$.

The complex amplitudes $A_t$, in contrast to the real functions $g_t$, imply that one can measure both real and imaginary interference terms between transversity amplitudes $H_t$ and $h_t$. This includes terms similar to those in Eq. (23) in which the cosines and sines are interchanged. These additional terms provide information which enables resolving the ambiguity in the sign of $\sin(2\beta + \gamma)$.

The advantage of $B \to D^*a_1$ in determining unambiguously the CP-violating phase $2\beta + \gamma$ can be traced back to the parity-odd measurable that occur in this process but not in $B \to D^*\rho$. As noted, $|H_{t+}^0|^2 - H_{t-}^0|^2 = 2\text{Re}(H_{t+} H_{t+}^*)$ is P-odd, and so is $\text{Im}[e^{2i\beta}(H_{t+} H_{t+}^* + H_{t-} H_{t-}^*)]$. These terms, which do not occur in the time-dependent rate of $\bar{B}^0 \to D^{*+}\rho^-$, do occur in $\bar{B}^0(t) \to D^{*+}a_1^-$ multiplying a P-odd function of $\theta, \cos \theta \text{Im}[(\vec{J} \times \vec{J}^*) \cdot \hat{n}]$. A practical advantage of $\bar{B}^\to \to D^{*+}a_1^-$ over $\bar{B}^0 \to D^{*+}\rho^-$ is the occurrence of only charged pions in the first process. A slight disadvantage of the first process may be an intrinsic uncertainty in the amplitudes $A_t$ calculated in Eq. (10), due to a possible small D-wave $\rho\pi$ amplitude.

5 Conclusion

Predictions of factorization and heavy quark symmetry for helicity amplitudes in $\bar{B}^0 \to D^{*+}\rho^+$ agree very well with experiment, but do not distinguish between positive and negative helicities. Parity-odd measurable in hadronic $B$ decays are quite rare. We identify such a measurable in $\bar{B}^0 \to D^{*+}a_1^+$ in terms of the up-down asymmetry of the $D^*$ momentum direction with respect to the $a_1$ decay plane. Measurement of this asymmetry using current data can test the chirality of the weak $b$ quark coupling. Time-dependent CP asymmetry measurements in $B \to D^*\rho$ and $B \to D^*a_1$, which entail the potential for a clean determination of $2\beta + \gamma$, require considerably more data than acquired so far. Study of $\bar{B}^0(t) \to D^{*+}a_1^+$ complements that of $\bar{B}^0(t) \to D^{*+}\rho^+$, and resolves a discrete ambiguity in the CP-violating phase.
Acknowledgments

I thank Dan Pirjol and Daniel Wyler for an enjoyable collaboration on a study of $B \to D^*a_1$, and David London, Nita and Rahul Sinha for pointing out the virtue of $B \to D^*\rho$. I am grateful to the CERN Theory Division and the Enrico Fermi Institute at the University of Chicago for their kind hospitality. This work was supported in part by the United States Department of Energy through Grant No. DE FG02 90ER40560.

References

1. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
2. A. B. Carter and A. I. Sanda, Phys. Rev. D 23, 1567 (1981); I. I. Bigi and A. I. Sanda, Nucl. Phys. B 193, 85 (1981).
3. BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 89, 201802 (2002); Belle Collaboration, K. Abe et al., Phys. Rev. D 66, 071102 (2002).
4. M. Gronau, Phys. Rev. Lett. 63, 1451 (1989); D. London and R. L. Peccei, Phys. Lett. B 223, 257 (1989).
5. J. L. Rosner, these Proceedings.
6. T. Nakadaira, these Proceedings.
7. G. Hamel de Monchenault, these Proceedings.
8. T. Tomura, these Proceedings.
9. S. Swain, these Proceedings.
10. M. Gronau and D. Wyler, Phys. Lett. B 265, 172 (1991); M. Gronau, Phys. Lett. B 557, 198 (2003) and references therein.
11. M. Gronau, B Decays, ed. S. Stone (World Scientific, Singapore, 1994), p. 644; T. G. Rizzo, Phys. Rev. D 58, 055009 (1998).
12. Particle Data Group, K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002).
13. J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 566 (1975).
14. J. G. Körner and G. R. Goldstein, Phys. Lett. B 89, 105 (1979).
15. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000); C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. 87, 201806 (2001).
16. J. L. Rosner, Phys. Rev. D 42, 3732 (1990).
17. CLEO Collaboration, S. E. Csorna et al., hep-ex/0301028, submitted to Phys. Rev. D.
18. M. Gronau, Y. Grossman, D. Pirjol and A. Ryd, Phys. Rev. Lett. 88, 051802 (2002); M. Gronau and D. Pirjol, Phys. Rev. D 66, 054008 (2002).
19. M. Gronau, D. Pirjol and D. Wyler, Phys. Rev. Lett. 90, 051801 (2003).
20. N. Isgur, C. Morningstar and C. Reader, Phys. Rev. D 39, 1357 (1989); M. Feindt, Z. Phys. C 48, 681 (1990).
21. A fit to the complete energy and angular dependence[13] would be a more sensitive probe for the chirality than the asymmetry $A$.
22. BaBar Collaboration, B. Aubert et al., Conference Report BABAR-CONF-02/10, hep-ex/0207085, see also ARGUS Collaboration, H. Albrecht et al., Z. Phys. C 48, 543 (1990); CLEO Collaboration, M. S. Alam et al., Phys. Rev. D 45, 21 (1992), 50, 43 (1994).
23. I. Dunietz, Phys. Lett. B 427, 179 (1998); D. A. Suprun, C. W. Chiang and J. L. Rosner, Phys. Rev. D 65, 054025 (2002).
24. D. London, N. Sinha and R. Sinha, Phys. Rev. Lett. 85, 1807 (2000).
25. F. Wilson, these Proceedings.
26. I. Dunietz, H. R. Quinn, A. Snyder, W. Toki and H. J. Lipkin, Phys. Rev. D 43, 2193 (1991).