Modeling and optimization of dynamic absorber with viscous friction

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Abstract. The vibrations coming from outer sources can cause, in the most cases, important disturbances in mechanical systems functioning. For this reason, solutions for the mitigation of these negative effects are sought, one of them being the use of vibration dynamic absorbers. In the present work, a damped vibrating mechanical system is considered and a dynamic absorber with damping attached to it. The mechanical system is acted by a sinusoidal force. The mechanical system behavior is modeled by a system of four state-space equations, having as input data the outer force and as output data the elongations of the vibrating motions for the two masses, corresponding to the original system and to the dynamic absorber, respectively. In this paper, the analytical steady state solution for system functioning is determined. The absorber effect can be optimized by using this solution. For this purpose, a soft in MATLAB environment is conceived, soft which offers the possibility of computing the mass, the spring elastic constant and the absorber coefficient of viscous friction, so that the amplitude of the original system oscillations, caused by the input force, to be minimal.

1. Introduction
The mechanical systems are subjected to different exterior forces that could produce unpleasant effects like vibrations. One of the possible solutions for diminishing the amplitude of these vibrations is the attachment of a vibration dynamic absorber [1, 2]. There are several types of such dynamic absorbers, one of the most used being the absorber with viscous damping, which consists of a mass, a spring and a damper [3, 4]. The degree of mitigation for the amplitude of system mass oscillations depends on the values of absorber parameters (the mass, the elastic constant of the spring, the damper coefficient of viscous friction) [5-7]. In this work, the optimum values of these parameters are determined, based on the analytical solution for the amplitude of the system mass, which is minimized by using MATLAB optimization capacities.

2. The mechanical system and its mathematical model
The mechanical system has two subsystems. The first one is called primary system and it consists of a mass $M$, attached to a fixed element by using a spring, having the elastic constant $k$, and a damper, whose coefficient of viscous friction is $c$. Upon mass $M$, a sinusoidal force $F_0 \sin(\omega t)$ acts, which produces an oscillatory motion of mass $M$. In order to diminish the mass vibrations, a second subsystem, called attached system is used. It consists of a mass $m$, connected to mass $M$ by a spring,
whose elastic constant is $k_a$ and a damper characterized by a coefficient of viscous friction $c_a$, as shown in figure 1. This subsystem is a vibration dynamic absorber with viscous friction.

![Figure 1. The vibrating mechanical system provided with a dynamic vibrations absorber with viscous friction.](image)

When the displacements of masses $M$ and $m$ with respect to the initial position of static equilibrium are denoted by $x_1$ and $x_2$, respectively, the equations of system dynamics have the form:

$$M \ddot{x}_1 = -k{x}_1 - c_1 \dot{x}_1 - k_a (x_1 - x_2) - c_a (\dot{x}_1 - \dot{x}_2) + F_0 \sin(\omega_0 t)$$  

$$m \ddot{x}_2 = k_a (x_1 - x_2) + c_a (\dot{x}_1 - \dot{x}_2)$$  

By using the notations:

$$\dot{x}_1 = v_1 \quad \text{and} \quad \dot{x}_2 = v_2$$  

the system of second order differential equilibrium equations can be transformed into a system of four first order differential equations, expressed in explicit form:

$$\dot{v}_1 = \frac{1}{M} (c + c_a) v_1 + \frac{c_a}{M} v_2 - \frac{1}{M} (k + k_a) x_1 + \frac{k_a}{M} x_2 + \frac{F_0}{M} \sin(\omega_0 t)$$  

$$\dot{v}_2 = \frac{c_a}{m} v_1 - \frac{c_a}{m} v_2 + \frac{k_a}{m} x_1 - \frac{k_a}{m} x_2$$  

$$\dot{x}_1 = v_1$$  

$$\dot{x}_2 = v_2$$  

This is the system of state equations, which could be numerically integrated, by using the following initial conditions:

$$x_1(0) = 0, \quad x_2(0) = 0, \quad v_1(0) = 0 \quad \text{and} \quad v_2(0) = 0$$  

3. The analytical solution of the state equations system

The solution of a system of state equations, external stable, has two components. A transient component, which becomes zero in time, and a permanent component, which describes the system dynamics after the transient component disappearance. On the other side, the transient component is given by the general solution of the homogeneous system, while the permanent component represents the system particular solution. For finding the system particular solution, it is presumed its following form:

$$x_1 = a_1 \sin(\omega_0 t) + b_1 \cos(\omega_0 t)$$
\[ x_2 = c_1 \sin(\omega_0 t) + d_1 \cos(\omega_0 t) \]  
\[ v_1 = e_1 \sin(\omega_0 t) + f_1 \cos(\omega_0 t) \]  
\[ v_2 = g_1 \sin(\omega_0 t) + h_1 \cos(\omega_0 t) \]  

By substituting the previous solution and its derivatives in the state equations (4), (5), (6) and (7) and by identifying the coefficients of sinus and cosines functions, the following algebraic system is obtained:

\[ \frac{1}{M} (k + k_a) a_1 - \frac{k_a}{M} c_1 + \frac{1}{M} (c + c_a) e_1 - \omega_0 f_1 - \frac{c_a}{M} g_1 = \frac{F_u}{M} \]  
\[ \frac{1}{M} (k + k_a) b_1 - \frac{k_a}{M} d_1 + \omega_0 e_1 + \frac{1}{M} (c + c_a) f_1 - \omega_0 h_1 = 0 \]  
\[ - \frac{k_a}{m} a_1 + \frac{k_a}{m} c_1 - \frac{c_a}{m} e_1 + \frac{c_a}{m} g_1 - \omega_0 h_1 = 0 \]  
\[ - \frac{k_a}{m} b_1 + \frac{k_a}{m} d_1 - \frac{c_a}{m} f_1 - \omega_0 g_1 + \frac{c_a}{m} h_1 = 0 \]  
\[ - \omega_0 b_1 - e_1 = 0 \]  
\[ \omega_0 a_1 - f_1 = 0 \]  
\[ - \omega_0 d_1 - g_1 = 0 \]  
\[ \omega_0 c_1 - h_1 = 0 \]  

The displacements \( x_1 \) and \( x_2 \) depend on \( a_1, b_1, c_1 \) and \( d_1 \). The unknowns \( e_1, f_1, g_1 \) and \( h_1 \) given by equations (17), (18), (19) and (20) are substituted in equations (13), (14), (15) and (16). In this manner, it results an algebraic system with four equations, whose unknowns are \( a_1, b_1, c_1 \) and \( d_1 \).

\[ Aa_1 - Bb_1 - Cc_1 + Dd_1 = K \]  
\[ Ba_1 + Ab_1 - Dc_1 - Cd_1 = 0 \]  
\[ - Ea_1 + Fb_1 + Gc_1 - Hd_1 = 0 \]  
\[ - Fa_1 - Eb_1 + Hc_1 + Gd_1 = 0 \]  

where

\[ A = \frac{k + k_a}{M} - \omega_0^2; \quad B = \frac{c + c_a}{M} \omega_0; \quad C = \frac{k_a}{M}; \quad D = \frac{c_a}{M} \omega_0; \quad E = \frac{k_a}{m}; \quad F = \frac{c_a}{m} \omega_0; \quad G = \frac{k_a}{m} - \omega_0^2; \quad H = F = \frac{c_a}{m} \omega_0 \]  
\[ K = \frac{F_u}{M} \]  

After finding the solutions of the algebraic system, \( a_1, b_1, c_1 \) and \( d_1 \), the oscillations of the two masses, having expressions (9) and (10), could be also written in the following shape:

\[ x_1 = \sqrt{d_1^2 + b_1^2} \sin(\omega_0 t + \varphi_1), \quad \tan \varphi_1 = \frac{b_1}{a_1} \]
The analytical expressions of the amplitudes are determined by using the symbolical calculus in MATLAB.

4. Optimization of absorber parameters for a single value of the angular frequency

The dynamic absorber is attached to the primary system for minimizing its oscillations. The primary system and the force which acts upon mass \( M \) have known parameters. The objective is to find the values of the dynamic absorber parameters that lead to the minimum amplitude of mass \( M \) oscillations.

The following values of the parameters are considered:

\[
M = 1500 \text{ kg}; \quad k = 50000 \text{ N/m}; \quad c = 1000 \text{ Ns/m}; \quad F_0 = 0.05 \text{ N} \quad \text{and} \quad \omega_0 = 2\pi s^{-1}
\]  

(28)

For finding the values of mass \( m \), \( c_a \) and \( k_a \) that minimize the amplitude of mass \( M \), that is, minimize the amplitude of solution \( x_1 \), a computer programme based on \textit{fmincon} function is conceived in MATLAB, which gives the amplitudes of oscillations \( x_1 \) and \( x_2 \). The initial considered values of the parameters are:

\[
m = 100 \text{ kg}; \quad k_a = 500 \text{ N/m}; \quad \text{and} \quad c_a = 500 \text{ Ns/m}
\]  

(29)

The following limits are considered, so that the absorber mass to be much lower, compared to the system mass (not more than 10%):

\[
10 \text{ kg} \leq m \leq 100 \text{ kg}; \quad 100 \text{ N/m} \leq k_a \leq 100000 \text{ N/m}; \quad \text{and} \quad 100 \text{ Ns/m} \leq c_a \leq 10000 \text{ Ns/m}
\]  

(30)

The optimum resulted values are:

\[
m = 100 \text{ kg}; \quad k_a = 6432 \cdot 10^4 \text{ N/m}; \quad \text{and} \quad c_a = 100 \text{ Ns/m}
\]  

(31)

The amplitude obtained before the optimization is \( 62.751 \times 10^{-6} \) m and after optimization is \( 61.418 \times 10^{-6} \) m, that is half of the first assessment.

5. Optimization of the vibration dynamic absorber for different values of the angular frequency

The primary system with no absorber has a maximum amplitude of vibrations when

\[
\omega_b = \frac{1}{M} \sqrt{k^2 - \frac{c^2}{2}} \text{s}^{-1} = 5.754 \text{s}^{-1}
\]  

(32)

In this case, the amplitude of mass \( M \) has the maximum value

\[
A_{\text{max}} = 8.675 \cdot 10^{-6} \text{ m}
\]  

(33)

The optimization soft, for the angular frequency given by equation (32), leads to the following values of the absorber optimized parameters

\[
m = 100 \text{ kg}; \quad k_a = 3.3663 \cdot 10^3 \text{ N/m}; \quad \text{and} \quad c_a = 100 \text{ Ns/m}
\]  

(34)

for which the maximum amplitude of mass \( M \) decreases to the value

\[
A_{\text{min}} = 1.9927 \cdot 10^{-6} \text{ m}
\]  

(35)

The graph of mass \( M \) amplitude in terms of \( \omega_0 \) could be plotted by using the analytical expression of this function given by the mentioned soft. In figure 2 it is presented the variation of mass \( M \) amplitude with and without absorber.
Figure 2. The amplitude of the mass $M$ as a function of $\omega_b$.

The diagram presented in figure 2 shows the significant decrease of the maximum amplitude but, because of the absorber, there are two other high amplitude values.

\[ A_1 = 5.392 \times 10^{-6} m \text{ and } A_2 = 2.998 \times 10^{-6} m \]  \hspace{1cm} (36)

Figure 3. The optimized amplitude of the mass $M$.

The amplitudes $A_1$ and $A_2$ can be brought to closer values when the parameters values are optimized for $\omega_b$ close to $5.754 s^{-1}$, given by the equation (32). As an example, when it is considered

\[ \omega_b = 5.4 s^{-1} \]  \hspace{1cm} (37)
the following values result from the optimization soft:

\[ m = 100 \text{kg}; \quad k_a = 2.849 \times 10^3 \text{N/m}; \quad \text{and} \quad c_a = 100 \text{Ns/m} \]  

(38)

for which values \( A_1 \) and \( A_2 \) become:

\[ A_1 = 4.056 \times 10^{-6} \text{m} \quad \text{and} \quad A_2 = 3.956 \times 10^{-6} \text{m} \]  

(39)

but the minimum amplitude of mass \( M \) increases up to value

\[ A_{\text{min}} = 2.306 \times 10^{-6} \text{m} \]  

(40)

as shown in figure 3.

6. Conclusions

For diminishing the vibration amplitude of an oscillatory system, a vibration dynamic absorber with damping could be attached. In this work, the amplitude of vibrations is determined in terms of dynamic absorber parameters (the mass, the elastic constant of the spring, the damper coefficient of viscous friction). The optimum values of damper parameters, which minimize the vibration amplitude for any considered value of the disruptive force angular frequency, are determined by using the optimization capacities in MATLAB. A maximum amplitude of the system occurs when the dynamic absorber is not attached. For the angular frequency corresponding to that amplitude the optimum parameters of the dynamic absorber are determined. The results show that the maximum amplitude decreases 4.35 times. When the dynamic absorber is attached to the system, two other amplitudes with important values occur, but they are lower than the maximum amplitude. In order to get closer these two amplitudes (approximately equal), the values of absorber parameters are optimized for the angular frequency approximately equal to that which generates the maximum amplitude. In this manner, the absorber decreases the maximum amplitude 3.76 times, and the two previously mentioned amplitudes are half of the maximum amplitude. The absorber having these parameters could be used for a wide range of angular frequencies, but with a lower effectiveness than around the value which determines the maximum amplitude.

7. References

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