1 Experiment, Observation and Astronomy

Astronomers observe.

Most other sciences permit experiment of some kind, where cause is manipulated and effect monitored; as astronomers, however, we are denied this luxury and must learn what we can of the Universe by eavesdropping on Nature as she murmurs to herself. Denied the luxury of experiment, we rely on experience guided by intuition to interpret, and so bring order, to our observations. Our experience is so far removed from the environment we are studying, however, that it is a poor guide to our imagination.

An important lesson of astronomy is that the ways we see determine the things we know. We thought the Universe a different place when, before Galileo, we saw it only through the naked eye: the real failure of the Ptolemaic model was only manifest when we saw that the planets had disks and that Venus passed through a full set of phases. Each increase in sensitivity, spatial or temporal resolution, or spectral range has brought with it new puzzles and surprises, and ultimately new insights and understanding. Progress in astronomy has historically been associated with new or improved observational perspectives.

Thus, to compensate for the vast difference between our own and Natures laboratories, we have learned to rely on a multiplicity of observational perspectives: we view the Universe in as many different ways as we can in the hope that the multiplicity of distinct perspectives will lead us to a more reliable view of Nature’s workings.

When completed, the gravitational wave detectors now proposed or under construction will provide us with a perspective on the Universe fundamentally different from any we have come to know. With this new perspective comes the hope that new insights and understandings of Nature will emerge. The proposed acoustic detectors, with spherical geometries and operating at millikelvin temperatures, are particularly well-suited to study gravitational radiation in the 1–3 KHz band. In this brief report I review some sources of particular
interest for these detectors.

2 The detector

For the purpose of describing the sensitivity to astrophysical gravitational radiation sources of the next generation acoustic detectors, I assume an array of TIGA\textsuperscript{1},\textsuperscript{2},\textsuperscript{3} “spheres” operated together as a broadband detector. Each element in the array has a different resonant frequency $f_0$ and a bandwidth $\Delta f_0 \simeq f_0/10$; over this bandwidth the one-sided noise power spectral density in either $h_+$ or $h_\times$ is constant. The resonant frequencies and bandwidths of the different array elements are arranged so that they adjoin with negligible overlap so that broadband detector composed of these separate elements covers the band from 850 Hz to 2650 Hz with a one-sided noise power spectral density of approximately $10^{-46}\text{Hz}^{-1}$ in either $h_+$ or $h_\times$. Finally, outside of this band I assume that the noise power is effectively infinite.

3 Lumpy neutron stars

Among the more interesting gravitational radiation sources for a high frequency detector like that described in section 2 is a rapidly rotating, non-axisymmetric neutron star. Setting aside pulsar spin-down over the course of an observation as well as the time-varying Doppler shift arising from Earth’s rotation on its axis and revolution about the Sun, the radiation from such a lumpy neutron star, rotating about a principle moment of inertia with rotational period $1/f_{\text{rot}}$, is

$$h_+ = \frac{4}{r} \frac{1 + \cos^2 i}{2} (2\pi f_{\text{rot}})^2 \epsilon I_3 \cos 4\pi f_{\text{rot}} t \quad (1)$$

$$h_\times = \frac{4}{r} \cos i (2\pi f_{\text{rot}})^2 \epsilon I_3 \sin 4\pi f_{\text{rot}} t \quad (2)$$

where $I_j$ are the neutron star’s principal moments of inertia, rotation is about the $I_3$ axis, $I_2 - I_1 = \epsilon I_3$ and $i$ is the inclination angle between the neutron star angular momentum and the detector line-of-sight. The mean square signal-to-noise (averaged over all $i$) in our hypothetical detector for an observation of duration $T$ is

$$\langle \rho^2 \rangle = 4.5 \times 10^4 \frac{T}{\text{yr}} \frac{10^{-46}}{S_h(f)} \left( \frac{10\text{Kpc}}{r} \right)^2 \left( \frac{f_{\text{rot}}}{\text{KHz}} \right)^4 \left( \frac{\epsilon}{10^{-6}} \right)^2 \left( \frac{I_3}{10^{45}\text{gcm}^2} \right)^2 . \quad (3)$$

In order to interpret the signal-to-noise ratio we can evaluate the false alarm rate: the probability that, in a fixed observation, noise alone will result
in a signal-to-noise ratio above a fixed threshold. A search for a continuous-wave source in a time series of duration \( T \) can achieve a frequency resolution no better than \( 1/T \). The filtered detector output in each frequency “bin” of width \( \Delta f = 1/T \) is independent. The filtered output can be further decomposed into an in-phase and quadrature-phase component (relative to an arbitrary, monochromatic reference signal), which are also independent. The total signal-to-noise, which is the Pythagorean sum of the signal-to-noise of the in-phase and quadrature filtered components, is Rayleigh distributed:

\[
P(\rho) = \rho \exp \left(-\rho^2/2\right);
\]

consequently, the probability that \( \rho < \rho_0 \) is

\[
C(\rho_0) = 1 - \exp \left(-\rho_0^2/2\right)
\]

and the probability that, across all bins in a bandwidth \( f \), all the signal-to-noise ratios are less than \( \rho_0 \) is

\[
P(\rho < \rho_0) = C(\rho_0)^fT.
\]

The false alarm rate (i.e., the probability that at least one bin has \( \rho > \rho_0 \)) is thus \( 1 - C(\rho_0)^fT \). Assuming (as we have in section 2) a detector bandwidth of 1150 Hz and an observation period of 1 yr, \( \rho > 7.5 \) corresponds to a false alarm rate (odds that one or more of the \( 3.6 \times 10^{10} \) “bins” are above threshold owing to noise) of 2%, or 1 false alarm every 50 y\(^{-1}\).

The likely maximum strength of the neutron star crust provides an upper bound \( \epsilon \equiv 10^{-6} \). Beyond this upper bound, theory provides no guidance on \( \epsilon \). Observation, on the other hand, does. Gravitational radiation emission contributes to the spin-down of a rapidly rotating neutron star; thus, the observed spin-down rate of any millisecond pulsar limits \( \epsilon \) for that pulsar. Observed millisecond pulsars have very low spin-down rates and correspondingly low upper-bounds on \( \epsilon \): no observed millisecond pulsar admits an \( \epsilon \) greater than a few times \( 10^{-8} \) (Blair, private communication).

Were it known that for some part of the millisecond neutron star population \( \epsilon \) did take on a reasonably large value then one could use the expected ms neutron star space density \( n \sim 10^{-6} \text{pc}^{-3} \) to estimate the expected number with signal-to-noise greater than some threshold \( \rho_0 \). Let’s do that. Suppose that 1% of ms neutron stars have \( \epsilon \simeq 10^{-7} \) and rotational frequencies in the range \( 100 \lessdot f_{\text{rot}}/\text{Hz} \lessdot 1000 \). Assume that these are distributed in frequency so that \( dn/df \propto f^{-2} \), where \( n \) is the number space density. Take the galactic disk radius to be 15 Kpc, with Sol 8 Kpc from the galactic center, and the ms pulsar disk scaleheight \( H \) to be 300 pc (the same as for LMXBs).
Restricting attention to those sources with $\rho > 7.5$ in a 1 y observation we can expect

- 27 sources out to the near edge of the galactic disk in the band $850 \text{ Hz} < f_{gw} < 1 \text{ KHz}$;
- 240 sources in the band $1 \text{ KHz} < f_{gw} < 1.8 \text{ KHz}$;
- 350 sources throughout the galaxy in the band $1.8 \text{ KHz} < f_{gw} < 2 \text{ KHz}$.

A precessing neutron star will also radiate gravitationally in a manner very similar to that of a non-axisymmetric neutron star rotating about a principal axis. For the small wobble angles that might be expected in a realistic source, the radiation amplitudes are typically much smaller than for the case studied here. The interested reader may find details in [9, 10].

4 Coalescing compact binaries

Coalescing binary neutron star systems are a significant gravitational wave source for the ground-based interferometric gravitational wave detectors like LIGO and VIRGO. These systems radiate most of their energy at low frequencies, making them less suited as sources for the proposed acoustic detectors. Nevertheless, it is instructive to consider how observable compact binary systems are to the proposed acoustic detectors.

The amplitude signal-to-noise ratio for binary coalescence in an omnidirectional detector is given by

$$\rho = 8 \left( \frac{r_0}{d_L} \right) \left( \frac{M}{1.2 \, M_\odot} \right)^{5/6} \Theta \zeta(f_{\text{coal}}),$$

where $r_0$ is a characteristic distance that depends only on the source waveform and the detector noise power spectrum, $d_L$ is the binary’s luminosity distance, $\Theta$ depends on the angular orientation of the binary with respect to the line of sight to the detector and is of order unity, $\zeta$ describes the fraction of the detector bandwidth that is filled by radiation from the inspiraling binary system, and $f_{\text{coal}}$ is the orbital frequency (not the gravitational wave frequency) where the inspiral ends and the coalescence begins. For binaries with equal mass components,

$$f_{\text{coal}} \simeq 710 \text{ Hz} \frac{2.8 \, M_\odot}{M},$$

(8)
where \( M \) is the binary system’s total mass, and for the detector described in section 3,

\[
\zeta(f) = \begin{cases} 
0 & \text{if } f < f_{\text{min}}/2, \\
f_{\text{min}}^{4/3} - (2f)^{-4/3} & \text{if } f_{\text{min}} < f/2 < f_{\text{max}}, \\
1 & \text{if } f > f_{\text{max}}/2
\end{cases}
\]  

(9)

\( f_{\text{min}} = 850 \text{ Hz} \),

(10)

\( f_{\text{max}} = 2650 \text{ Hz} \), and

(11)

\[
r_0 = 28 \text{ Mpc} \left( \frac{10^{-46} \text{ Hz}}{S_n} \right).
\]  

(12)

For neutron star binaries (\( m_1 = m_2 = 1.4 \, M_\odot \)) \( \zeta \) is approximately 1/2.

The overlap function \( \zeta \) taken together with the relationship between the system mass and the coalescence frequency \( f_{\text{coal}} \) shows immediately that the \textit{inspiral radiation} from binaries with total mass greater than about 4.7 \( M_\odot \) will not leave any imprint on the proposed detectors (however, see the discussion of black hole formation in section 3). Similarly, the value of \( r_0 \) for the proposed detector suggests that, in any event, binaries not much further than the Virgo cluster will be visible. Assuming that the rate density of neutron star binary inspiral \( \equiv 3 \, \text{y}^{-1} \) at 200 Mpc, neutron star binary coalescence within 20 Mpc is not expected more frequently than once every 330 y. The inspiral of “conventional” compact binaries is thus not a source of interest for the advanced proposed acoustic detectors.

What of unconventional binary systems, however? Recent results from the MACHO collaboration suggest that perhaps 50% of the galactic halo mass is in dark objects with \( m \sim 0.3 \, M_\odot \). Assuming that

- a fraction \( w \simeq 50\% \) of the halo mass (\( M_{\text{halo}} \) greater than or approximately equal to \( 6 \times 10^{11} \, M_\odot \)) is in MACHOs,
- all of the MACHOs are mass \( m \simeq 0.3 \, M_\odot \) neutron stars or black holes,
- a fraction \( x \simeq 30\% \) of the MACHOs are bound in symmetric binaries,
- this binary population was formed at the same time that the galaxy was formed (age \( T \simeq 10^{10} \, \text{y} \)), and
- the MACHO binary population is coalescing today at a steady rate,

then the rate of MACHO binary coalescence in the halo is given by

\[
\tilde{N} = 15 \, \text{y}^{-1} \left( \frac{x}{30\%} \frac{w}{50\%} \frac{M_{\text{halo}}}{6 \times 10^{11} \, M_\odot} \frac{10^{10} \, \text{y}}{T} \right)
\]  

(13)
If we take the distance to the typical halo binary to be $50\text{Kpc}$, then the amplitude signal-to-noise in our antenna array of a typical halo inspiral is

$$\rho \simeq 510.$$  

(14)

Thus, under these (optimistic!) assumptions, a TIGA antenna array would see virtually all halo binary inspirals: 15 strong ($\rho \simeq 500!$) events per year.

5 Black hole formation

When a binary whose total mass $M$ is greater than the maximum neutron star mass coalesces the likely outcome is a mass $M$ black hole. The time dependence of the spacetime strain owing to the excitation of fundamental quadrupole mode of the resulting blackhole is, at late times, given by

$$m(t) = e^{-\pi ft/Q} \sin (2\pi ft)$$  

(15)

where $Q$ and $f$ are related to the black hole mass and angular momentum.

In general there are five fundamental quadrupole modes as well as a succession of “overtones.” The fundamental modes are degenerate if the black hole is non-rotating (Schwarzschild) and are split otherwise. The overtones are at higher frequency and are also more strongly damped than the fundamental modes. To give a rough estimate of the detectability of the gravitational radiation from the late stages of black hole formation, let’s ignore both rotation and the overtones and assume, for convenience, that black hole excitation is concentrated in a single mode; then

$$f \simeq 12\text{ KHz} \left(\frac{M_{\odot}}{M}\right)$$  

(16)

$$Q \simeq 2.$$  

(17)

For Schwarzschild black holes, the proposed detector is sensitive to formation of black holes with mass between 4.5 and 14.1 $M_{\odot}$. For comparison, the mass of Cygnus X-1 is estimated to be in the range 5–10 $M_{\odot}$, LMC X-3 is about 10 $M_{\odot}$, and A0620-00 is about 3.8 $M_{\odot}$; so, the formation of astrophysical black holes in our antenna array’s “mass range” is reasonable.

Assuming a quadrupole radiation pattern for the black hole we can relate the “efficiency” of the black hole formation (ratio of the total energy radiated to the black hole mass) to the rms (averaged over all orientations of the black hole with respect to the detector line-of-sight) strain induced in each element of our antenna array:

$$h_{\text{rms}}(t) = 2\sqrt{\frac{\epsilon}{\rho}} m(t)$$  

(18)
Numerical modeling\textsuperscript{4} of Schwarzschild black hole head-on collisions give efficiencies of $10^{-4}$. For collisions of rotating black holes with significant orbital angular momentum the efficiencies might be significantly higher — perhaps 1\% or greater.

The mean square amplitude signal-to-noise ratio of Schwarzschild black hole formation in our hypothetical detector is thus given by\textsuperscript{3}

$$p^2 \simeq 34 \left( \frac{\epsilon}{10^{-4}} \right) \left( \frac{20 \text{ Mpc}}{r} \right)^2 \left( \frac{M}{13 M_\odot} \right)^3 \left( \frac{10^{-46} \text{ Hz}^{-1}}{S_h(f)} \right).$$ (19)

Even under the pessimistic assumption that there is no increase in efficiency for the coalescence of orbiting black holes our antenna array is sensitive to, we can expect to observe a typical $10 M_\odot$ black hole formation event with a signal-to-noise of several out to the Virgo cluster.

The rate of black hole formation is entirely uncertain; however, most astrophysicists see no reason why the same mechanisms that make neutron stars do not also make black holes at approximately the same rate. By our present understanding of formation mechanisms, this is not a high rate even at the distance of the Virgo cluster; so, evidence of black hole formation at any measurable rate would require a significant change in our understanding of stellar evolution.

\section*{6 Supernovae}

Theoretical models of stellar core collapse, and the corresponding gravitational wave luminosity, have a long and checkered history: estimates of the gravitational wave luminosity have at different times ranged over more than four orders of magnitude. It is not simply the luminosity that is unknown: the waveforms themselves are also uncertain, leading to a further difficulty in estimating the detectability of this source. Nevertheless, it is still possible to evaluate what is required of stellar core collapse in order that it be observable in our hypothetical detector.

Suppose that the waveform from supernovae is given by

$$h_+ = \frac{2M_\odot}{r} \alpha f_+ m(t)$$ (20)

$$h_\times = \frac{2M_\odot}{r} \beta f_\times m(t)$$ (21)

$\alpha$ and $\beta$ are constants, $f_+$ and $f_\times$ are functions of the relative orientation of the source with respect to the detector, and $m(t)$ is some function of time which
we leave undetermined for now. The power radiated into each polarization mode is given by

\[
\dot{E}_+ = \alpha^2 \langle f_+^2 \rangle M_\odot^2 |\tilde{m}|^2
\]
\[
\dot{E}_\times = \beta^2 \langle f_\times^2 \rangle M_\odot^2 |\tilde{m}|^2,
\]

where <> signifies an average over the sphere.

Now assume that equal power is radiated into the two polarization modes. Then we can write \(\alpha\) and \(\beta\) in terms of a single parameter \(\epsilon\) as

\[
\alpha^2 = \frac{\epsilon}{2M_\odot} \langle f_+^2 \rangle \int dt |\dot{m}|^2
\]
\[
\beta^2 = \frac{\epsilon}{2M_\odot} \langle f_\times^2 \rangle \int dt |\dot{m}|^2.
\]

In terms of \(\epsilon\) the power radiated into the + and \(\times\) polarization states is thus

\[
\dot{E}_+ = \dot{E}_\times = \frac{\epsilon M_\odot |\tilde{m}|^2}{2} \int dt |\dot{m}|^2.
\]

Now let us return to consider the time dependence of the waveform \(m(t)\). Note that

\[
\int dt |\dot{m}(t)|^2 = \int df (2\pi f)^2 |\tilde{m}(f)|^2,
\]

where \(\tilde{m}\) is the Fourier transform of \(m\). Assume that there is equal radiated power in equal bandwidths out to a frequency \(f_0\); then \(|\tilde{m}|^2 f^2\) is constant for \(f < f_0\) and zero for \(f > f_0\). The mean-square signal-to-noise ratio (averaged over the source orientation relative to the detector) in the detector described in section 3 is then

\[
\langle \rho^2 \rangle = \frac{\epsilon M_\odot}{r^2} \frac{1}{2\pi^2 S_n f_0} \left( \frac{1}{f_\min'} - \frac{1}{f_\max'} \right)
\]

where

\[
\begin{align*}
    f_\min' &= \max \left[ f_\min, \min(f_0, f_\min) \right] \\
    f_\max' &= \max \left[ f_\min, \min(f_0, f_\max) \right].
\end{align*}
\]

For the detector described in section 3, \(f_\min\) is 850 Hz and \(f_\max\) is 2650 Hz. If we assume that \(f_\min < f_0 \simeq 1\) KHz < \(f_\max\), then

\[
\langle \rho^2 \rangle \simeq 420, \frac{\epsilon}{10^{-4}} \left( \frac{100 \text{ Kpc}}{r} \right)^2 \frac{10^{-46} \text{ Hz}^{-1}}{S_n} \frac{10^3 \text{ Hz}}{f_0} \frac{850 \text{ Hz}}{f_\min} \left( 1 - \frac{f_\min}{f_0} \right)
\]

\[8\]
Current calculations (which, bear in mind, are still unable to successfully describe the supernova explosion) suggest, in the range $10^{-9} - 10^{-8} M_\odot$, with peak power in the 200–300 Hz band. Thus, without a very optimistic efficiency and a substantially wider signal bandwidth, we can't expect to be able to observe supernovae much beyond our own galaxy, and certainly not out to the Virgo cluster.

7 Conclusions

There are several interesting opportunities for an array of millikelvin acoustic gravitational antennae covering the 1–3 KHz frequency band. In particular, we can expect that the radiation from population of rapidly rotating neutron stars with oblateness on order 1/10 that allowed by the crustal breaking strength would be observable throughout the galaxy in a one year observation. Local supernovae, neutron star binary inspiral, or black hole formation, while infrequent, would be a serendipitous radiation source from our own galaxy or, perhaps, as far as the Virgo cluster. Finally, a very speculative source — coalescence of a population of compact MACHOs in the galactic halo — would be observable with large signal-to-noise ratio.

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