A Multifractal Detrended Fluctuation Description of Iranian Rial-US Dollar Exchange Rate

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Abstract

The multifractal properties and scaling behaviour of the exchange rate variations of the Iranian rial against the US dollar from a daily perspective is numerically investigated. For this purpose the multifractal detrended fluctuation analysis (MF-DFA) is used. Through multifractal analysis, the scaling exponents, generalized Hurst exponents, generalized fractal dimensions and singularity spectrum are derived. Moreover, contribution of two major sources of multifractality, that is, fat-tailed probability distributions and nonlinear temporal correlations are studied.

Keywords: Multifractality, Scaling, Rial-dollar exchange rate, Financial markets.

1 Introduction

For more than two decades, there has been considerable interest in the investigation of the scaling behaviour on fractal models. The pioneering work [1] on fractals introduced the concept of fractals and showed some relation between self-similar fractals and self-affine fractals. Self-affine fractals [2, 3, 4, 5] constitute random and complicated structure and have been applied to a broader range of problems, such as the Eden and ballistic deposition model [6, 7, 8, 9], mountain heights, clouds, coast lines, and cracks. Among other examples of many fractal models, the self-avoiding random walk, random resistor, polymer bonds, turbulences, chaotic motions can be mentioned [2, 4, 5, 10, 11, 12, 13], etc. Specially, the real data from different financial markets show apparent multifractal properties [14, 15, 16, 17, 18, 19, 20, 21, 22]. Moreover, recently, financial analysis of
foreign exchanges has become one of the outstanding topics in econophysics \cite{23}. Many of these researches apply multifractal analysis framework as a basic framework. In universal multifractal framework, the statistics of the data are fully described with some parameters, taking into account two complementary aspects of financial time series: the multiple scaling and the Pareto probability distributions, which is a generic feature of multifractal processes \cite{24}.

It has been shown that there are two main factors leading to multifractal behaviour of financial time series, nonlinear time correlations between present and past events and the heavy-tailed probability distributions of functions. Based on ref. \cite{21}, for the stocks, the main contribution to multifractality comes from a broad distribution of returns while a long memory present in this kind of data contributes only marginally. It should be noted that the nature of correlations leading to the multifractal dynamics of the variations is strongly nonlinear and, curiously, cannot be simply related to some well-known correlation type like a slowly decreasing volatility autocorrelation with an imposed daily pattern. For example, one even has to consider the nonlinear dependencies in the volatility itself in order to reveal how the temporal correlations contribute to multifractality in the stock market and foreign exchange data.

In this paper, the rial-dollar exchange rate data is studied with the focus on their fractal properties. The multifractal detrended fluctuation analysis is applied which is a well-established method of detecting scaling behaviour of time series. In Section 1, theoretical backgrounds including MFDFA method, sources of multifractality, multifractality finger prints and strength of multifractality are reviewed. Data are described in Section 2. Numerical results are presented in Section 3 and finally, conclusions are given in Section 4.

\section{Theoretical backgrounds}

\subsection{Method}

Detrended fluctuation analysis (DFA) is a scaling analysis technique providing a simple quantitative parameter—the scaling exponent $\alpha$-to represent the correlation properties of a time series \cite{25}. The advantage of DFA over many techniques are that it permits the detection of long-range correlations embedded in seemingly non-stationary time series, and also avoids the spurious detection of apparent long-range correlations that are an artifact of non-stationarity. Additionally, the advantages of DFA in computation of $H$ over other techniques (for example, the Fourier transform) are:

- inherent trends are avoided at all time scales;
local correlations can be easily probed.

To implement the DFA, let us suppose there is a time series, \( N(i) (i = 1, ..., N_{\text{max}}) \). The time series \( N(i) \) is integrated:

\[
y(j) = \sum_{i=1}^{j} [N(i) - \langle N \rangle]
\]

where:

\[
\langle N \rangle = \frac{1}{N_{\text{max}}} \sum_{j=1}^{N_{\text{max}}} N(i).
\]

Next \( N(i) \) is broken up into \( K \) non-overlapping time intervals, \( I_n, \) of equal size \( \tau \) where \( n = 0, 1, ..., K - 1 \) and \( K \) corresponds to the integer part of \( N_{\text{max}}/\tau \). In each box, the integrated time series is fitted by using a polynomial function, \( y_{\text{pol}}(i) \), which is called the local trend. For order-\( l \) DFA (DFA-1 if \( l = 1 \), DFA-2 if \( l = 2 \), etc.), the \( l \)-order polynomial function should be applied for the fitting. The integrated time series \( y(i) \) is detrended in each box, and calculated the detrended fluctuation function:

\[
Y(i) = y(i) - y_{\text{pol}}(i).
\]

For a given box size \( s \), the root mean square fluctuation is calculated:

\[
F(s) = \sqrt{\frac{1}{N_{\text{max}}} \sum_{i=1}^{N_{\text{max}}} [Y(i)]^2}
\]

The above computation is repeated for box sizes \( s \) (different scales) to provide a relationship between \( F(s) \) and \( s \). A power law relation between \( F(s) \) and \( s \) indicates the presence of scaling: \( F(s) \sim s^\alpha \). The parameter \( \alpha \), called the scaling exponent or correlation exponent, represents the correlation properties of the signal: if \( \alpha = 0.5 \), there is no correlation and the signal is an uncorrelated signal \([25]\); if \( \alpha < 0.5 \), the signal is anticorrelated; if \( \alpha > 0.5 \), there are positive correlations in the signal. In the two latest cases, the signal can be well approximated by the fractional Brownian motion law \([26]\).

For a further characterization of data it is meaningful to extend Eq. (15) by considering the more general fluctuation functions \([27]\). Simply, it is achieved by averaging over all boxes to obtain the \( q \)th order fluctuation function

\[
F_q(s) = \left[ \frac{1}{2N_{\text{max}}} \sum_{i=1}^{N_{\text{max}}} (F^2(s))^{q/2} \right]^{1/q},
\]

where, in general, the index variable \( q \) can take any real values except zero. If the analyzed signal develops fractal properties, the fluctuation function reveals power-law scaling

\[
F_q(s) \sim s^{h(q)}
\]
for large \( s \). The scaling exponents \( h(q) \) can be then obtained by observing the slope of log-log plots of \( F_q \) vs. \( s \). The family of the exponents \( h(q) \) describe the scaling of the \( q \)-th order fluctuation function. For positive values of \( q \), \( h(q) \) exponents describe the scaling behaviour of boxes with large fluctuations while those of negative values of \( q \), describe scaling behaviour of boxes with small fluctuations [28]. For stationary time series, the exponent \( h(2) \) is identical to the Hurst exponent. Thus the exponents \( h(q) \) are called as the generalized Hurst exponents [28]. For monofractal time series which are characterized by a single exponent over all scales, \( h(q) \) is independent of \( q \), whereas for a multifractal time series, \( h(q) \) varies with \( q \). This dependence is considered to be a characteristic property of multifractal processes [28].

The \( h(q) \) obtained from MF-DFA is related to the Renyi exponent \( \tau(q) \) by

\[
qh(q) = \tau(q) + 1. \tag{7}
\]

Therefore, another way to characterize a multifractal series is the singularity spectrum \( f(\alpha) \) defined by [26]

\[
\alpha = h(q) + qh'(q), \quad f(\alpha) = q[\alpha - h(q)] + 1, \tag{8}
\]

where \( h'(q) \) stands for the derivative of \( h(q) \) with respect to \( q \). \( \alpha \) is the Hölder exponent or singularity strength which characterizes the singularities in a time series. The singularity spectrum \( f(\alpha) \) describes the singularity content of the time series. Finally, it must be noted that \( h(q) \) is different from the generalized multifractal dimensions

\[
D(q) \equiv \frac{\tau(q)}{q - 1} = \frac{qh(q) - 1}{q - 1}, \tag{9}
\]

that are used instead of \( \tau(q) \) in some papers. While \( h(q) \) is independent of \( q \) for a monofractal time series with compact support, \( D(q) \) depends on \( q \) in that case.

### 2.2 Sources of multifractality

Generally, there are two different types of sources for multifractality in time series: (i) due to different long-range temporal correlations for small and large fluctuations, and (ii) due to fat-tailed probability distributions of variations. Both of them need a multitude of scaling exponents for small and large fluctuations. Two procedure is followed to find the contributions of two sources of multifractality and to indicate the multifractality strength: (i) shuffling, and (ii) phase randomization. Shuffling procedure preserves the distribution of the variations but destroys any temporal correlations. In fact, one can destroy the temporal correlations by randomly shuffling the corresponding time series of variations. What then remains are data with exactly the same fluctuation distributions but without memory. The shuffling procedure consists of the following steps.
(i) Generate pairs \((p, q)\) of random integer numbers (with \(p, q \leq N\)) where \(N\) is the total length of the time series to be shuffled.

(ii) Swap entries \(p\) and \(q\).

(iii) Repeat two above steps for \(20N\) times. (This step ensures that ordering of entries in the time series is fully shuffled.)

In order to study the contribution of the fat-tailed variations on the multifractality, the surrogate data are used. In fact, the non-Gaussianity of the distributions can be weakened by creating the phase-randomized surrogates [29]. The Phase randomization steps are:

(i) Take discrete Fourier transform of time series.

(ii) Multiply the discrete Fourier transform of the data by random phases.

(iii) Perform an inverse Fourier transform to create a phase randomized surrogates.

Phase randomization preserves the amplitudes of the Fourier transform but randomizing the Fourier phases. This procedure eliminates nonlinearities, preserving only the linear properties of the original time series [30].

2.3 Multifractality finger prints

One can see that in the whole \(q\)-range the generalized Hurst exponents \(h(q)\) can be fitted well by the formula

\[
h(q) = \frac{1}{q} - \frac{\ln|a^q + b^q|}{q \ln 2}
\]

which corresponds to \(\tau(q) = -\ln(a^q + b^q)/\ln 2\). This formula can be obtained from a generalized binomial multifractal model [31]. Instead of choosing \(a\) and \(b\), the Hurst exponent \(h(1)\) and the persistence exponent \(h(2)\) could be chosen. From knowledge of two moments, all the other moments follow. Here the formula is used only to show that the infinite number of exponents \(h(q)\) can be described by only two independent parameters, \(a\) and \(b\). These two parameters can then be regarded as multifractal finger prints for a considered time series.

2.4 Strength of multifractality

In the generalized binomial multifractal model, the strength of the multifractality of a time series can be characterized by the difference between the
maximum and minimum values of $\alpha$, $\alpha_{\text{max}} - \alpha_{\text{min}}$. When $q \frac{dh(q)}{dq}$ approaches zero for $q$ approaching $\pm \infty$, then $\Delta \alpha = \alpha_{\text{max}} - \alpha_{\text{min}}$ is simply given by

$$\Delta \alpha = h(-\infty) - h(\infty) = \frac{\ln(b) - \ln(a)}{\ln 2}.$$  

(11)

It must be noted that this parameter is identical to the width of the singularity spectrum $f(\alpha)$ at $f = 0$. The wider singularity spectrum the richer multifractality.

3 Data Analysis

The data which is analyzed are the time series of the daily closing exchange rate logarithmic variations (that is, $\ln(P(t))/\ln(P(t+1))$) for the time period 24th September 1989, to 15th November 2003. So that our database consists of 4369 exchange rates and 4368 daily variations. The sources of this data is the central bank of the islamic republic of Iran. In Fig. 1 a time series corresponding to daily values of the Iranian rial-US dollar exchange rates in mentioned period is presented. A great increment in dollar price is seen about 16th May 1995 because of Iranian government decision on lifting the ban on foreign exchanges price variations. The Iranian government has managed to keep the exchange rate stable at around 8000 rials per US dollar ever since March 2000. Also, Table 1 provides summary statistics of logarithmic variations of exchange rates. According to data in Table 1, a

Table 1: Mean, standard deviation, skewness, and kurtosis of rial-dollar exchange rate variations.

| Mean | Std.Dev. | Skewness | Kurtosis |
|------|----------|----------|----------|
| 0.00055 | 0.0117  | -1.2504 | 49.925 |

negatively large skew is seen. The probability distribution function of variations also show a high degree of peakedness and fat tails relative to a normal distribution. Thus there is a clear departure from Gaussian normality. The departure from a Gaussian Cumulative Distribution Function (CDF) can be clearly seen in Fig. 2, where the CDF of variations against a Gaussian CDF is depicted.

4 Results

The fluctuation functions $F_q(s)$ for timescales ranging from 3 days to $N/5$ are calculated, where $N$ is the total length of the time series, and for $q$ varying between -10 and 10, with a step of 0.5. Fig. 3 shows the MF-DFA2
fluctuations $F_q(s)$ for various $q$’s.

A crossover with great magnitude (like as a phase transition) in fluctuation function is seen for negative $q$ values in the range $30 < s < 65$. The position of crossover doesn’t have sensitivity to decreasing or increasing $q$ values. The only interest behaviour is the asymptotic behaviour of $F_q(s)$ at large times $s$. One can clearly observe that above the crossover region, the $F_q(s)$ functions are straight lines in the double logarithmic plot, and the slopes increase slightly when going from high positive moments towards high negative moments (from the top to the bottom in Fig. 3).

For the sake of better studying the large fluctuations, randomized data (both of reshuffled and surrogate data) have been used. Fig. 4 indicates that, the magnitude of change in crossover for reshuffled data is very large relative to the surrogate data. In fact, one can say that such an effect originates mainly from temporal correlations. Moreover, the position of crossover is intended to left (about $s \simeq 4$) because of randomizing.

Monofractal time series are associated with a linear plot $\tau(q)$, while multifractal ones possess the spectra nonlinear in $q$. The highest nonlinearity of the spectrums, the strongest multifractality in time series. Calculations indicate that the time series of exchange rate variations can be of multifractal nature. In order to visualize the scaling character of the data, in Fig. 5, the corresponding multifractal spectra is shown. Fig. 5 shows three examples of $\tau(q)$ for the original (solid), surrogate (dotted) and reshuffled (dashed) data. The nonlinearity of $\tau(q):s$ is much weaker for the modified time series than for the original ones. Additionally, surrogate data show less nonlinearity based on Fig. 5 and therefore, their contribution to multiscaling relative to reshuffled data is less.

The $h(q)$ spectra has been fitted in the range $-10 \leq q \leq 10$ for original, reshuffled and surrogate series by Eq. (10). Representative example for original series is shown in Fig. 6. The dotted line in Fig. 6 is obtained by best fits of $h(q)$ by Eq. (10). The respective parameters $a$ and $b$ for original, reshuffled and surrogate series are listed in Table 2. It is notable that in each single case, the $q$ dependence of $h(q)$ for positive and negative values of $q$ can be characterized very well by the two parameters, and all fits remain within the error bars of the $h(q)$ values.

Table 2: Multifractality fingerprint (parameters $a$ and $b$) and strength for original, reshuffled and surrogate data.

| Time series     | a   | b   | $\Delta \alpha$ |
|-----------------|-----|-----|-----------------|
| Original data   | 0.03| 1.07| 3.54            |
| Reshuffled data | 0.51| 0.93| 0.60            |
| Surrogate data  | 0.69| 0.8 | 0.15            |
It is seen that the strength of multifractality in rial-dollar exchange rate variations is very powerful. Moreover, multifractality strength in randomized data decreases specially in surrogate data based on values in Table 2. In order to visualizing and better understanding the strength of multifractality for original, reshuffled and surrogate data, the singularity spectrum of series are shown in Fig. 7. Both the widths of the \( f(\alpha) \) spectra in each randomized data are much smaller than for the original one. This behaviour of the reshuffled time series confirms that the persistent autocorrelations play an important role in multiscaling of the price variations. But, The spectra for the surrogates are typically much narrower than for the reshuffled data which can be interpreted as an evidence of the influence of extremely large non-Gaussian events on the fractal properties of the time series.

5 Conclusions

The multifractal properties of the Iranian rial-US dollar exchange rate logarithmic variations has been studied in this paper through multifractal detrended fluctuation analysis. It is shown that the time series for exchange rate variations exhibit the characteristics that can be interpreted in terms of multifractality. Its degree expressed by e.g. the widths of the singularity spectra \( f(\alpha) \) indicate a strong multifractality. Moreover, although the most multifractality of the exchange rate variations data is due to different long-range correlations for small and large fluctuations, the shape of the probability distribution function also contributes to the multifractal behaviour of the time series.
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Figure 1: Daily closure rial-dollar exchange rates history (1989-2003).
Figure 2: Cumulative distribution function of rial-dollar exchange rate variations against a Gaussian cumulative distribution.

Figure 3: The multifractal fluctuation function $F_q(s)$ obtained from multifractal DFA2 for variations of rial-dollar exchange rates in the period 1989 to 2003.
Figure 4: The multifractal fluctuation function $F_q(s)$ obtained from multifractal DFA2 for randomized (reshuffled and surrogate) variations of rial-dollar exchange rates in the period 1989 to 2003.

Figure 5: Comparison of the multifractal spectra $\tau(q)$ of the original and randomized exchange rate variations: original (solid), surrogate (dotted) and reshuffled (dashed) time series.
Figure 6: The generalized Hurst exponents $h(q)$ for the rial-dollar exchange rate variations in period 1989 to 2003. The fitted curve has been shown by dotted line.

Figure 7: Comparison of the singularity spectra for original and randomized data: original (solid), reshuffled (dotted) and surrogate (dashed) time series.