On Modeling of Lifetime Data Using Akash, Shanker, Lindley and Exponential Distributions

Abstract
The statistical analysis and modeling of lifetime data are crucial for statisticians and research workers in almost all applied sciences including engineering, biomedical science, insurance, and finance, amongst others. The two important and popular one parameter distributions for modeling lifetime data are exponential and Lindley distributions. Shanker et al. [1] observed that there are many lifetime data where these distributions are not suitable from theoretical and applied point of view. Recently Shanker [2,3] has introduced two one parameter lifetime distribution namely “Akash distribution” and “Shanker distribution” for modeling lifetime data. In the present paper the relationships and comparative studies of Akash, Shanker, Lindley and exponential distributions, their distributional properties and estimation of parameter have been discussed. The applications, goodness of fit and theoretical justifications of these distributions for modeling life time data through various examples from engineering, medical science and other fields have been discussed and explained.

Keywords: Akash distribution; Shanker distribution; Lindley distribution; Exponential distribution; Statistical properties; Estimation of parameter; Goodness of fit

Introduction
In reliability analysis the time to the occurrence of event of interest is known as lifetime or survival time or failure time. The event may be failure of a piece of equipment, death of a person, development (or remission) of symptoms of disease, health code violation (or compliance). The modeling and statistical analysis of lifetime data are crucial for statisticians, research workers and policy makers in almost all applied sciences including engineering, medical science/biological science, insurance and finance, amongst others.

In statistics literature a number of lifetime distributions for modeling lifetime data-sets have been proposed. In this paper, the main objective is to have a critical and comparative study on one parameter lifetime distributions namely, Akash, Shanker, Lindley and exponential and their applications for modeling lifetime data-sets from engineering, medical sciences, and other fields of knowledge.

Akash, Shanker, Lindley and Exponential Distributions
Akash distribution introduced by Shanker [2] for modeling lifetime data from engineering and medical science is a two-component mixture of an exponential (θ) distribution and a gamma (2,θ) distribution with their mixing proportions $\frac{\theta}{\theta_1}$ and $\frac{1}{\theta_1}$ respectively. Shanker [2] has discussed its various mathematical and statistical properties including its shape, moment generating function, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability, amongst others. Shanker et al. [3] has detailed study about modeling of various lifetime data from different fields using Akash, Lindley and exponential distributions and concluded that Akash distribution gives better fit in most of the lifetime data. Shanker [5] has also obtained a Poisson mixture of Akash distribution named, “Poisson-Akash (PAD)” for modeling count data.

Shanker distribution introduced by Shanker [2] for modeling lifetime data from engineering and medical science is a two-component mixture of an exponential (θ) distribution and a gamma (2,θ) distribution with their mixing proportions $\frac{\theta}{\theta_2}$ and $\frac{1}{\theta_2}$ respectively. Shanker [3] has discussed its various mathematical and statistical properties including its shape, moment generating function, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability, amongst others. Shanker [6] has also obtained a Poisson mixture of Shanker distribution named, “Poisson-Shanker (PSD)” for modeling count data.

Lindley [7] distribution is a two-component mixture of an exponential (θ) distribution and a gamma (2,θ) distribution with their mixing proportions $\frac{\theta}{\theta_1}$ and $\frac{1}{\theta_1}$ respectively. A detailed study about its various mathematical properties, estimation of parameter and application showing the superiority of Lindley distribution over exponential distribution for the waiting times before service of the bank customers has been done by Ghitany et al. [8]. A number of researchers have studied in detail the
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In statistical literature, exponential distribution was the first widely used lifetime model in areas ranging from studies on the lifetimes of manufactured to research involving survival or remission times in chronic diseases. The main reason for its wide usefulness and applicability as lifetime model is partly because of the availability of simple statistical methods for it and partly because it appeared to be suitable for representing the lifetimes of many things such as various types of manufactured items.

Let $T$ be a continuous random variable representing the lifetimes of individuals in some population. The expressions for probability density function, $f(t)$, cumulative distribution function, $F(t)$, hazard rate function, $h(t)$, mean residual life, $m(t)$, mean $\mu_t$, variance $\sigma_t^2$, coefficient of variation (C.V), coefficient of Kurtosis $\gamma$, and index of dispersion $\gamma$ of Akash and Shanker distributions are summarized in Table 1 and that of Lindley and exponential distributions are in Table 2.

A table of values for coefficient of variation (C.V), coefficient of Skewness $\gamma$, coefficient of Kurtosis ($\beta_2$), and index of dispersion ($\gamma$) for Akash, Shanker and Lindley distributions for varying values of their parameter are summarized in the Table 3.

The conditions under which Akash, Shanker and Lindley distributions are over-dispersed ($\mu^2 < \sigma^2$), equi-dispersed ($\mu^2 = \sigma^2$), and under-dispersed ($\mu^2 > \sigma^2$) are summarized in Table 5.

The graphs of C.V, $\sqrt{\beta_1}$, $\beta_2$ and $\gamma$ of Akash, Shanker and Lindley distributions for varying values of the parameter $\theta$ are shown in Figure 1.

Parameter Estimation

Estimation of the parameter of Akash distribution

Assuming $(t_1, t_2, t_3, ..., t_n)$ be a random sample of size $n$ from Akash distribution, the maximum likelihood estimate (MLE) $\hat{\theta}$ and the method moment estimate (MOME) $\hat{\theta}$ of $\theta$ is the solution of the following cubic equation.

$T\theta^3 - 6T\theta^2 - 6 = 0$, where $T$ is the sample mean

Estimation of the parameter of Shanker distribution

Let $(t_1, t_2, t_3, ..., t_n)$ be a random sample of size $n$ from Shanker distribution. The maximum likelihood estimate (MLE) $\hat{\theta}$ of $\theta$ is the solution of the following non-linear equation.

$2T \theta^2 - 2T \theta + \frac{\sum_{i=1}^{n} (\theta + t_i)}{n} = 0$

The method of moment estimate (MOME) $\hat{\theta}$ of $\theta$ is the solution of the following cubic equation.

$T \theta^3 - \theta^2 + T \theta - 2 = 0$, where $T$ is the sample mean.

Estimation of the parameter of Lindley distribution

Assuming $(t_1, t_2, t_3, ..., t_n)$ be a random sample of size $n$ from Lindley distribution, the maximum likelihood estimate (MLE) $\hat{\theta}$ and the method moment estimate (MOME) $\hat{\theta}$ of $\theta$ is given by

$\hat{\theta} = -\frac{(T-1)^2 + 8T}{2T} = \frac{1}{T}$, where $T$ is the sample mean.

Applications and Goodness of Fit

In this section the goodness of fit test of Akash, Shanker, Lindley and exponential distributions for following sixteen real lifetime data-sets have been discussed.

In order to compare the goodness of fit of Akash, Shanker, Lindley and exponential distributions, $-2 \ln L$, AIC (Akaike Information Criterion), AICC (Akaike Information Criterion Corrected), BIC (Bayesian Information Criterion), and K-S Statistics (Kolmogorov-Smirnov Statistics) for all sixteen real lifetime data-sets have been computed and presented in Table 5. The formulae for computing AIC, AICC, BIC, and K-S Statistics are as follows:

$AIC = -2 \ln L + 2k$, $AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$, $BIC = -2 \ln L + k \ln n$ and $D=\sup_{x} |F_n(x) - F_0(x)|$, where $k$ is the number of parameters, $n$ is the sample size and $F_n(x)$ is the empirical distribution function.

The best distribution is the distribution which corresponds to lower values of $-2 \ln L$, AIC, AICC, BIC, and K-S statistics.

The best fitting has been shown by making $-2 \ln L$, AIC, AICC, BIC, and K-S Statistics in bold.

Conclusions

In this paper an attempt has been made to find the suitability of Akash, Shanker, Lindley and exponential distributions for modeling real lifetime data from engineering, medical science and other fields of knowledge. A table for values of the various characteristics of Akash, Shanker, and Lindley distributions has been presented for varying values of their parameter which reflects their nature and behavior. The conditions under which Akash, Shanker, Lindley and exponential distributions are over-dispersed, equi-dispersed, and under-dispersed have been given. The goodness of fit test of Akash, Shanker, Lindley and exponential distributions for sixteen real lifetime data-sets have been presented using Kolmogorov-Smirnov test to test their suitability for modeling lifetime data.

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Table 1: Characteristics of Akash and Shanker Distributions.

| Akash Distribution | Shanker Distribution |
|--------------------|----------------------|
| \( f(t) = \frac{\theta^2}{\theta^2 + 2} \left(1 + \frac{1}{\theta^2 + 2}\right) e^{-\theta t} \) | \( f(t) = \frac{\theta^2}{\theta^2 + 1} e^{-\theta t} \) |
| \( F(t) = 1 - \left[1 + \frac{\theta t(\theta + 1)}{\theta^2 + 2}\right] e^{-\theta t} \) | \( F(t) = 1 - \left[1 + \frac{\theta t}{\theta^2 + 1}\right] e^{-\theta t} \) |
| \( h(t) = \frac{\theta^2}{\theta^2 + 1 + \theta t^2} \) | \( h(t) = \frac{\theta^2}{\theta^2 + 1 + \theta t^2} \) |
| \( m(t) = \frac{\theta^2 + 4 \theta t + \theta^2 + 6}{\theta^2 + 1 + \theta t^2} \) | \( m(t) = \frac{\theta^2 + 4 \theta t + \theta^2 + 6}{\theta^2 + 1 + \theta t^2} \) |
| \( \mu'_1 = \frac{\theta^2 + 6}{\theta^2 + 1} \) | \( \mu'_2 = \frac{\theta^2 + 4 \theta^2 + 2}{\theta^2 + 1} \) |
| \( \mu'_2 = \frac{\theta^2 + 16 \theta^2 + 12}{\theta^2 + 1 + \theta t^2} \) | \( \mu'_2 = \frac{\theta^2 + 16 \theta^2 + 12}{\theta^2 + 1 + \theta t^2} \) |
| \( C.V = \frac{\sigma}{\mu'_1} \) | \( C.V = \frac{\sigma}{\mu'_2} \) |
| \( \sqrt{\beta_1} = \frac{2(\theta^6 + 30 \theta^4 + 36 \theta^2 + 24)}{(\theta^4 + 16 \theta^2 + 12)^{1/2}} \) | \( \sqrt{\beta_1} = \frac{2(\theta^6 + 30 \theta^4 + 36 \theta^2 + 24)}{(\theta^4 + 16 \theta^2 + 12)^{1/2}} \) |
| \( \beta_2 = \frac{3(3 \theta^8 + 128 \theta^6 + 408 \theta^4)}{(576 \theta^4 + 240)} \) | \( \beta_2 = \frac{3(3 \theta^8 + 128 \theta^6 + 408 \theta^4)}{(576 \theta^4 + 240)} \) |
| \( \gamma = \frac{\sigma^2}{\mu'_1} \) | \( \gamma = \frac{\sigma^2}{\mu'_1} \) |
Table 2: Characteristics of Lindley and Exponential Distributions.

| Lindley Distribution | Exponential Distribution |
|----------------------|--------------------------|
| \( f(t) = \frac{\theta^2}{\theta+1} \frac{1}{t+1} e^{-\theta t} \) | \( f(t) = \theta e^{-\theta t} \) |
| \( F(t) = 1 - \frac{\theta+1}{\theta+1} e^{-\theta t} \) | \( F(t) = 1 - e^{-\theta t} \) |
| \( h(t) = \frac{\theta^2 (1+t)}{\theta+1+\theta t} \) | \( h(t) = \theta \) |
| \( m(t) = \frac{\theta+2+\theta t}{\theta (\theta+1+\theta t)} \) | \( m(t) = \frac{1}{\theta} \) |
| \( \mu_1' = \frac{\theta+2}{\theta (\theta+1)} \) | \( \mu_1' = \frac{1}{\theta} \) |
| \( \mu_2 = \frac{\theta^2 + 4\theta + 2}{\theta^2 (\theta+1)^2} \) | \( \mu_2 = \frac{1}{\theta^2} \) |
| \( CV = \frac{\sqrt{\theta^2 + 4\theta + 2}}{\mu_1} \) | \( CV = \frac{\sigma}{\mu_1} = 1 \) |
| \( \sqrt{\beta_1} = \frac{2 \left( \theta^2 + 6\theta^2 + 6\theta + 2 \right)}{\left( \theta^2 + 4\theta + 2 \right)^{3/2}} \) | \( \sqrt{\beta_1} = 2 \) |
| \( \beta_2 = \frac{3 \theta^4 + 24\theta^3 + 84\theta^2 + 32\theta + 8}{\left( \theta^2 + 4\theta + 2 \right)^2} \) | \( \beta_2 = 9 \) |
| \( \gamma = \frac{\sigma^2}{\mu_1} = \frac{\theta^2 + 4\theta + 2}{\theta (\theta+1)(\theta+2)} \) | \( \gamma = \frac{\sigma^2}{\mu_1} = \frac{1}{\theta} \) |
### Table 3: Values of $\mu_1$, $\sqrt{\beta_1}$, $\sqrt{\beta_2}$, $\beta_2$, and $\gamma$ of Akash, Shanker and Lindley distributions for varying values of the parameter $\theta$.

| $\theta$ | Values of $\theta$ for Akash Distribution | Values of $\theta$ for Shanker Distribution | Values of $\theta$ for Lindley Distribution |
|----------|-------------------------------------------|-------------------------------------------|-------------------------------------------|
| 0.01     | $\mu_1$ 299.990, $\sqrt{\beta_1}$ 1.155, $\beta_2$ 5.000, $\gamma$ 100.007 | $\mu_1$ 199.990, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 100.005 | $\mu_1$ 199.010, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 100.493 |
| 0.05     | $\mu_1$ 59.950, $\sqrt{\beta_1}$ 1.153, $\beta_2$ 4.997, $\gamma$ 20.033 | $\mu_1$ 39.950, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 20.005 | $\mu_1$ 39.048, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 20.493 |
| 0.1      | $\mu_1$ 29.900, $\sqrt{\beta_1}$ 1.149, $\beta_2$ 4.987, $\gamma$ 10.066 | $\mu_1$ 19.901, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 10.033 | $\mu_1$ 19.091, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 10.049 |
| 0.3      | $\mu_1$ 9.713, $\sqrt{\beta_1}$ 1.115, $\beta_2$ 4.897, $\gamma$ 3.522 | $\mu_1$ 34.208, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 3.465 | $\mu_1$ 5.897, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 3.668 |
| 0.5      | $\mu_1$ 5.556, $\sqrt{\beta_1}$ 1.084, $\beta_2$ 4.785, $\gamma$ 2.284 | $\mu_1$ 12.691, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 2.178 | $\mu_1$ 3.333, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 2.267 |
| 1        | $\mu_1$ 2.333, $\sqrt{\beta_1}$ 1.165, $\beta_2$ 4.834, $\gamma$ 1.381 | $\mu_1$ 3.222, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 1.167 | $\mu_1$ 1.500, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 1.167 |
| 1.5      | $\mu_1$ 1.294, $\sqrt{\beta_1}$ 1.388, $\beta_2$ 5.473, $\gamma$ 1.009 | $\mu_1$ 1.306, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 0.775 | $\mu_1$ 0.833, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 0.781 |
| 2        | $\mu_1$ 0.833, $\sqrt{\beta_1}$ 1.614, $\beta_2$ 6.391, $\gamma$ 0.776 | $\mu_1$ 0.639, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 0.567 | $\mu_1$ 0.600, $\sqrt{\beta_1}$ 1.414, $\beta_2$ 6.000, $\gamma$ 0.583 |
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Table 4: Over-dispersion, equi-dispersion and under-dispersion of Akash, Shanker, Lindley and exponential distributions for varying values of their parameter $\theta$.

| Distribution | Over-Dispersion $\left(\mu<\sigma^2\right)$ | Equi-Dispersion $\left(\mu=\sigma^2\right)$ | Under-Dispersion $\left(\mu>\sigma^2\right)$ |
|--------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Akash        | $\theta<1.515400063$                         | $\theta=1.515400063$                         | $\theta>1.515400063$                         |
| Shanker      | $\theta<1.171535555$                         | $\theta>1.171535555$                         | $\theta>1.171535555$                         |
| Lindley      | $\theta>1.170086487$                         | $\theta>1.170086487$                         | $\theta>1.170086487$                         |
| Exponential  | $\theta<1$                                   | $\theta=1$                                   | $\theta>1$                                   |

Figure 1: Graphs of C.V, $\sqrt{\beta_1}$, $\gamma$ and $\gamma$ of Akash, Shanker and Lindley distributions for varying values of the parameter $\theta$. 

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Table 5: MLE’s, -2ln L, AIC, AICC, BIC, K-S Statistics of the fitted distributions of data-sets 1-16.

| Data 1 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 1.355445  | 163.73             | 165.73  | 165.79 | 169.93 | 0.355  |
| Shanker| 0.956264  | 162.28             | 164.28  | 164.34 | 166.42 | 0.346  |
| Lindley| 0.996116  | 162.56             | 164.56  | 164.62 | 166.70 | 0.371  |
| Exponential | 0.663647   | 177.66             | 179.66  | 179.73 | 181.80 | 0.402  |

| Data 2 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 0.043876  | 950.97             | 952.97  | 953.01 | 955.58 | 0.184  |
| Shanker| 0.029252  | 980.97             | 982.97  | 983.01 | 985.57 | 0.238  |
| Lindley| 0.028859  | 983.11             | 985.11  | 985.15 | 987.71 | 0.242  |
| Exponential | 0.014635   | 1044.87            | 1046.87 | 1046.91| 1049.48| 0.357  |

| Data 3 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 0.041510  | 227.06             | 229.06  | 229.25 | 230.20 | 0.107  |
| Shanker| 0.027675  | 231.06             | 233.06  | 233.25 | 234.19 | 0.145  |
| Lindley| 0.027321  | 231.47             | 233.47  | 233.66 | 234.61 | 0.149  |
| Exponential | 0.013845   | 242.87             | 244.87  | 245.06 | 246.01 | 0.263  |

| Data 4 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 0.013514  | 1255.83            | 1257.83 | 1257.87| 1260.43| 0.110  |
| Shanker| 0.009009  | 1251.19            | 1253.34 | 1253.38| 1255.60| 0.097  |
| Lindley| 0.008970  | 1251.34            | 1253.34 | 1253.38| 1255.95| 0.098  |
| Exponential | 0.004505   | 1280.52            | 1282.52 | 1282.56| 1285.12| 0.190  |

| Data 5 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 0.030045  | 794.70             | 796.70  | 796.76 | 798.98 | 0.184  |
| Shanker| 0.020031  | 788.57             | 790.57  | 790.63 | 792.28 | 0.133  |
| Lindley| 0.019841  | 789.04             | 791.04  | 791.10 | 793.32 | 0.134  |
| Exponential | 0.010018   | 806.88             | 808.88  | 808.94 | 811.16 | 0.198  |

| Data 6 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 0.119610  | 981.28             | 983.28  | 983.31 | 986.18 | 0.393  |
| Shanker| 0.079746  | 1033.10            | 1035.10 | 1035.13| 1037.99| 0.442  |
| Lindley| 0.077247  | 1041.64            | 1043.64 | 1043.68| 1046.54| 0.448  |
| Exponential | 0.040060   | 1130.26            | 1132.26 | 1132.29| 1135.16| 0.525  |

| Data 7 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 0.013263  | 803.96             | 805.96  | 806.02 | 810.01 | 0.298  |
| Shanker| 0.008843  | 764.62             | 766.62  | 766.69 | 768.06 | 0.246  |
| Lindley| 0.008804  | 763.75             | 765.75  | 765.82 | 767.81 | 0.245  |
| Exponential | 0.004421   | 744.87             | 746.87  | 746.94 | 748.93 | 0.166  |

| Data 8 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 0.013423  | 609.93             | 611.93  | 612.02 | 613.71 | 0.280  |
| Shanker| 0.008949  | 579.51             | 581.51  | 581.60 | 583.29 | 0.220  |
| Lindley| 0.008910  | 579.16             | 581.16  | 581.26 | 582.95 | 0.219  |
| Exponential | 0.004475   | 564.02             | 566.02  | 566.11 | 567.80 | 0.145  |

| Data 9 | Model     | Parameter Estimate | -2ln L  | AIC    | AICC   | BIC    | K-S Statistic |
|--------|-----------|--------------------|---------|--------|--------|--------|---------------|
| Akash  | 0.310500  | 887.89             | 889.89  | 889.92 | 892.74 | 0.198  |
| Shanker| 0.210732  | 847.37             | 849.37  | 849.40 | 852.22 | 0.132  |
| Lindley| 0.196045  | 839.06             | 841.06  | 841.09 | 843.91 | 0.116  |
| Exponential | 0.106773   | 828.68             | 830.68  | 830.72 | 833.54 | 0.077  |

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| Data Set 1: The data set represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith & Naylor [19]. |

| Data 10 | Akash | 0.050293 | 354.88 | 356.88 | 357.02 | 358.28 | 0.421 |
| Shanker | 0.033569 | 325.74 | 327.74 | 327.88 | 329.14 | 0.351 |
| Shanker | 0.033021 | 323.27 | 325.27 | 325.42 | 326.67 | 0.345 |
| Shanker | 0.016779 | 305.26 | 307.26 | 307.40 | 308.66 | 0.213 |

| Data 11 | Akash | 1.165719 | 115.15 | 117.15 | 117.28 | 118.68 | 0.156 |
| Shanker | 0.033569 | 325.74 | 327.74 | 327.88 | 329.14 | 0.351 |
| Shanker | 0.033021 | 323.27 | 325.27 | 325.42 | 326.67 | 0.345 |
| Shanker | 0.016779 | 305.26 | 307.26 | 307.40 | 308.66 | 0.213 |

| Data 12 | Akash | 0.024734 | 658.04 | 660.04 | 660.08 | 662.65 | 0.163 |
| Shanker | 0.016492 | 635.26 | 637.26 | 637.30 | 639.86 | 0.042 |
| Shanker | 0.016360 | 638.07 | 640.07 | 640.12 | 642.68 | 0.058 |
| Shanker | 0.016245 | 658.04 | 660.04 | 660.08 | 662.65 | 0.163 |

| Data 13 | Akash | 0.050293 | 354.88 | 356.88 | 357.02 | 358.28 | 0.421 |
| Shanker | 0.033569 | 325.74 | 327.74 | 327.88 | 329.14 | 0.351 |
| Shanker | 0.033021 | 323.27 | 325.27 | 325.42 | 326.67 | 0.345 |
| Shanker | 0.016779 | 305.26 | 307.26 | 307.40 | 308.66 | 0.213 |

| Data 14 | Akash | 1.165719 | 115.15 | 117.15 | 117.28 | 118.68 | 0.156 |
| Shanker | 0.033569 | 325.74 | 327.74 | 327.88 | 329.14 | 0.351 |
| Shanker | 0.033021 | 323.27 | 325.27 | 325.42 | 326.67 | 0.345 |
| Shanker | 0.016779 | 305.26 | 307.26 | 307.40 | 308.66 | 0.213 |

| Data 15 | Akash | 0.024734 | 658.04 | 660.04 | 660.08 | 662.65 | 0.163 |
| Shanker | 0.016492 | 635.26 | 637.26 | 637.30 | 639.86 | 0.042 |
| Shanker | 0.016360 | 638.07 | 640.07 | 640.12 | 642.68 | 0.058 |
| Shanker | 0.016245 | 658.04 | 660.04 | 660.08 | 662.65 | 0.163 |

| Data 16 | Akash | 0.024734 | 658.04 | 660.04 | 660.08 | 662.65 | 0.163 |
| Shanker | 0.016492 | 635.26 | 637.26 | 637.30 | 639.86 | 0.042 |
| Shanker | 0.016360 | 638.07 | 640.07 | 640.12 | 642.68 | 0.058 |
| Shanker | 0.016245 | 658.04 | 660.04 | 660.08 | 662.65 | 0.163 |

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Data Set 2: The data is given by Birnbaum & Saunders [20] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data (X × 10^{-3}) are presented below (after subtracting 65).

|     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 5   | 25  | 31  | 32  | 34  | 35  | 38  | 39  | 40  | 42  | 43  | 43  |     |
| 43  | 44  | 44  | 47  | 48  | 48  | 49  | 49  | 49  | 51  | 54  | 55  | 55  |
| 55  | 56  | 56  | 56  | 58  | 59  | 59  | 59  | 59  | 63  | 63  | 64  |     |
| 64  | 65  | 65  | 65  | 66  | 66  | 66  | 66  | 67  | 67  | 67  | 68  |     |
| 69  | 69  | 69  | 71  | 71  | 72  | 73  | 73  | 73  | 74  | 74  | 76  |     |
| 76  | 77  | 77  | 77  | 77  | 77  | 79  | 79  | 80  | 81  | 83  | 83  |     |
| 84  | 86  | 86  | 87  | 90  | 91  | 92  | 92  | 92  | 92  | 93  | 94  | 97  |
| 98  | 98  | 99  | 101 | 103 | 105 | 109 |     |     |     |     |     |     |

Data Set 3: The data set is from Lawless (1982, p-228). The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests.

|     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 17.88 | 28.92 | 33.00 | 41.52 | 42.12 | 45.60 | 48.80 | 51.84 | 55.56 | 67.80 |     |     |
| 68.44 | 68.64 | 68.88 | 84.12 | 93.12 | 98.64 | 105.12 | 127.92 | 128.04 | 173.40 |     |     |

Data Set 4: The data is from Picciotto [21] and arose in test on the cycle at which the Yarn failed. The data are the number of cycles until failure of the yarn.

|     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 86  | 146 | 251 | 653 | 98  | 249 | 400 | 292 | 131 | 169 | 175 | 176 | 76  |
| 264 | 15  | 364 | 195 | 262 | 88  | 264 | 157 | 220 | 42  | 321 | 180 | 198 |
| 38  | 20  | 61  | 121 | 282 | 224 | 149 | 180 | 325 | 250 | 196 | 90  | 229 |
| 166 | 38  | 337 | 65  | 151 | 341 | 40  | 40  | 135 | 597 | 246 | 211 | 180 |
| 93  | 315 | 353 | 571 | 124 | 279 | 81  | 186 | 497 | 182 | 423 | 185 | 229 |
| 400 | 338 | 290 | 398 | 71  | 246 | 185 | 188 | 568 | 55  | 55  | 61  | 244 |
| 20  | 284 | 393 | 396 | 203 | 829 | 239 | 236 | 286 | 194 | 277 | 143 | 198 |
| 264 | 105 | 203 | 124 | 137 | 135 | 350 | 193 | 188 |     |     |     |     |

Data Set 5: This data represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal [22].

|     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 12  | 15  | 22  | 24  | 24  | 32  | 32  | 33  | 34  | 38  | 38  | 43  | 44  |
| 48  | 52  | 53  | 54  | 54  | 55  | 56  | 57  | 58  | 59  | 60  | 60  |     |
| 60  | 60  | 61  | 62  | 63  | 65  | 65  | 67  | 68  | 70  | 70  | 72  | 73  |
| 75  | 76  | 76  | 81  | 83  | 84  | 85  | 87  | 91  | 95  | 96  | 98  | 99  |
| 109 | 110 | 121 | 127 | 129 | 131 | 143 | 146 | 146 | 175 | 175 | 211 | 233 |
| 258 | 258 | 263 | 297 | 341 | 341 | 376 |     |     |     |     |     |     |

Data Set 6: This data is related with behavioral sciences, collected by Balakrishnan N et al. [23]: The scale "General Rating of Affective Symptoms for Preschoolers (GRASP)" measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to this scale. A study conducted by the authors in a city located at the south part of Chile has allowed collecting real data corresponding to the scores of the GRASP scale of children with frequency in parenthesis.

|     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 19(16)| 20(15)| 21(14)| 22(9)| 23(12)| 24(10)| 25(6)| 26(9)|     |     |     |     |     |
| 27(8)| 28(5)| 29(6)| 30(4)| 31(3)| 32(4)| 33  | 34  | 35(4)| 36(2)| 37(2)|     |     |
| 39  | 42  | 44  |     |     |     |     |     |     |     |     |     |     |
Data Set 7: The data set reported by Efron [24] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT).

| 6.53 | 7 | 10.42 | 14.48 | 16.10 | 22.70 | 34 | 41.55 | 42 | 45.28 | 49.40 | 53.62 | 63 |
| 64 | 83 | 84 | 91 | 108 | 112 | 129 | 133 | 133 | 139 | 140 | 140 | 146 |
| 149 | 154 | 157 | 160 | 160 | 165 | 146 | 149 | 154 | 157 | 160 | 160 | 165 |
| 173 | 176 | 218 | 225 | 241 | 248 | 273 | 277 | 297 | 405 | 417 | 420 | 440 |
| 523 | 583 | 594 | 1101 | 1146 | 1417 |   |   |   |   |   |   |   |

Data Set 8: The data set reported by Efron [24] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

| 12.20 | 23.56 | 23.7 | 25.9 | 31.98 | 37 | 41.35 | 47.38 | 55.46 | 58.36 | 63.47 | 68.46 | 78.3 |
| 74.5 | 81.43 | 84 | 92 | 94 | 110 | 112 | 119 | 127 | 130 | 133 | 140 | 146 |
| 155 | 159 | 173 | 179 | 194 | 195 | 209 | 249 | 281 | 319 | 339 | 432 | 469 |
| 519 | 633 | 725 | 817 | 1176 |   |   |   |   |   |   |   |   |

Data set 9: This data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee & Wang [25].

| 0.08 | 2.09 | 3.48 | 4.87 | 6.94 | 8.66 | 13.11 | 23.63 | 0.20 | 2.23 | 3.52 | 4.98 | 6.97 |
| 9.02 | 13.29 | 0.40 | 2.26 | 3.57 | 5.06 | 7.09 | 9.22 | 13.80 | 25.74 | 0.50 | 2.46 | 3.64 |
| 5.09 | 7.26 | 9.47 | 14.24 | 25.82 | 0.51 | 2.54 | 3.70 | 5.17 | 7.28 | 9.74 | 14.76 | 6.31 |
| 0.81 | 2.62 | 3.82 | 5.32 | 7.32 | 10.06 | 14.77 | 32.15 | 2.64 | 3.88 | 5.32 | 7.39 | 10.34 |
| 14.83 | 34.26 | 0.90 | 2.69 | 4.18 | 5.34 | 7.59 | 10.66 | 15.96 | 36.66 | 1.05 | 2.69 | 4.23 |
| 5.41 | 7.62 | 10.75 | 16.62 | 43.01 | 1.19 | 2.75 | 4.26 | 5.41 | 7.63 | 17.12 | 46.12 | 1.26 |
| 2.83 | 4.33 | 5.49 | 7.66 | 11.25 | 17.14 | 79.05 | 1.35 | 2.87 | 5.62 | 7.87 | 11.64 | 17.36 |
| 1.40 | 3.02 | 4.34 | 5.71 | 11.79 | 18.10 | 18.10 | 1.46 | 4.40 | 5.85 | 8.26 | 11.98 | 19.13 |
| 1.76 | 3.25 | 4.50 | 6.25 | 8.37 | 12.02 | 2.02 | 3.31 | 4.51 | 6.54 | 8.53 | 12.03 |   |
| 20.28 | 2.02 | 3.36 | 6.76 | 12.07 | 21.73 | 2.07 | 3.36 | 6.93 | 8.65 | 12.63 | 22.69 |   |

Data Set 10: This data set is given by Linhart & Zucchini [26], which represents the failure times of the air conditioning system of an airplane.

| 23 | 261 | 87 | 7 | 120 | 14 | 62 | 47 | 225 | 71 | 246 | 21 | 42 | 20 |
| 5 | 12 | 120 | 11 | 3 | 14 | 71 | 11 | 14 | 11 | 16 | 90 | 1 |
| 16 | 52 | 95 |   |   |   |   |   |   |   |   |   |   |

Data Set 11: This data set used by Bhaumik et al. [27], is vinyl chloride data obtained from clean up gradient monitoring wells in mg/l.

| 5.1 | 1.2 | 1.3 | 0.6 | 0.5 | 2.4 | 0.5 | 1.1 | 8 | 0.8 | 0.4 | 0.6 | 0.9 | 0.4 |
| 2 | 0.5 | 5.3 | 3.2 | 2.7 | 2.9 | 2.5 | 2.3 | 1 | 0.2 | 0.1 | 0.1 | 1.8 |
| 0.9 | 2 | 4 | 6.8 | 1.2 | 0.4 | 0.2 |

Data set 12: This data set represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by Ghitany et al. [8] for fitting the Lindley [7] distribution.

| 0.8 | 0.8 | 1.3 | 1.5 | 1.8 | 1.9 | 1.9 | 2.1 | 2.6 | 2.7 | 2.9 | 3.1 | 3.2 |
| 3.3 | 3.5 | 3.6 | 4.0 | 4.1 | 4.2 | 4.2 | 4.3 | 4.3 | 4.4 | 4.4 | 4.6 | 4.7 |
| 4.7 | 4.8 | 4.9 | 4.9 | 5.0 | 5.3 | 5.5 | 5.7 | 5.7 | 6.1 | 6.2 | 6.2 | 6.2 |
| 6.3 | 6.7 | 6.9 | 7.1 | 7.1 | 7.1 | 7.1 | 7.4 | 7.6 | 7.7 | 8.0 | 8.2 | 8.6 |
| 8.6 | 8.6 | 8.8 | 8.8 | 8.9 | 8.9 | 9.5 | 9.6 | 9.7 | 9.8 | 10.7 | 10.9 | 11.0 |
| 11.0 | 11.1 | 11.2 | 11.2 | 11.5 | 11.9 | 12.4 | 12.5 | 12.9 | 13.0 | 13.1 | 13.3 | 13.6 |
| 13.7 | 13.9 | 14.1 | 15.4 | 15.4 | 17.3 | 17.3 | 18.1 | 18.2 | 18.4 | 18.9 | 19.0 | 19.9 |
| 20.6 | 21.3 | 21.4 | 21.9 | 23.0 | 27.0 | 31.6 | 33.1 | 38.5 |

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Data Set 13: This data is for the times between successive failures of air conditioning equipment in a Boeing 720 airplane, Proschan [28].

| Data     | Value |
|----------|-------|
| 74       | 57    |
| 48       | 29    |
| 502      | 12    |
| 21       | 29    |
| 386      | 59    |
| 27       | 153   |
| 26       |       |
| 326      |       |

Data set 14: This dataset represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross & Clark [29].

| Data | Value |
|------|-------|
| 1.1  | 1.4   |
| 1.3  | 1.7   |
| 1.9  | 1.8   |
| 1.6  | 2.2   |
| 1.7  | 2.7   |
| 4.1  | 1.8   |
| 1.5  | 1.2   |
| 1.4  | 3     |
| 1.7  | 2.3   |
| 1.6  | 2     |

Data Set 15: This data set is the strength data of glass of the aircraft window reported by Fuller et al. [30].

| Data  | Value    |
|-------|----------|
| 18.83 | 20.8     |
| 21.657| 23.03    |
| 23.23 | 24.05    |
| 24.321| 25.5     |
| 25.52 | 25.8     |
| 26.69 | 26.77    |
| 26.78 |          |
| 27.05 | 26.77    |
| 29.9  | 31.11    |
| 33.2  | 33.73    |
| 33.76 | 33.89    |
| 34.76 | 35.75    |
| 35.91 | 36.98    |
| 37.08 |          |
| 39.58 | 44.045   |
| 45.29 | 45.381   |

Data Set 16: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm Bader & Priest [31,32].

| Data | Value |
|------|-------|
| 1.312| 1.314 |
| 1.479| 1.552 |
| 1.700| 1.803 |
| 1.861| 1.865 |
| 1.944| 2.006 |
| 2.021| 2.027 |
| 2.055| 2.063 |
| 2.098| 2.140 |
| 2.179| 2.224 |
| 2.240| 2.270 |
| 2.253| 2.272 |
| 2.274|       |
| 2.301| 2.301 |
| 2.359| 2.382 |
| 2.426| 2.434 |
| 2.435| 2.478 |
| 2.490| 2.511 |
| 2.514| 2.535 |
| 2.535|       |
| 2.554| 2.566 |
| 2.570| 2.586 |
| 2.629| 2.633 |
| 2.642| 2.648 |
| 2.684| 2.697 |
| 2.726| 2.770 |
| 2.773|       |
| 2.800| 2.809 |
| 2.818| 2.821 |
| 2.848| 2.880 |
| 2.954| 3.012 |
| 3.067| 3.084 |
| 3.090| 3.096 |
| 3.128|       |
| 3.233| 3.433 |
| 3.585| 3.858 |

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Conflict of Interest

None.

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