A Robot Compliant Joint Control Development and Simulation using a Linear Control Approach

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Abstract. Biological systems have inspired that robot have to demonstrate a capability to perform safe physical human robot interactions. This property is vital for ensuring that a robot can conduct their tasks successfully without risking or damaging both the human and the robot. A solution to this issue is to introduce a compliance control strategy. This strategy requires compliant actuator and sensor system to transfer the external forces or torques to the robot system. The external torque is sensed using a torque sensor where the observed torque is then used for compliance control policies. This is not a trivial problem in control system. This paper proposes a linear control technique applied on a robot compliant joint system. The mechanical system of a 1 DOF link is modelled using the Lagrangian function. The 1 DOF link mathematical model is used to simulate and verify the effectiveness of the proposed control strategy. A natural and controlled behaviors of the compliant joint are presented. The simulation results suggest that the proposed control strategy can effectively be used for a compliance robot joint. The parameter values in a closed loop control system obtained from the PD controller are \( k_p = 120 \) and \( k_d = 40 \).

1. Introduction

A robot manipulator is an important precision machine in modern industry, medical operations [1], underwater plant applications [2-3], anthropomorphic application [4], satellite outer space [5], rehabilitation robot [6], and others. However, the manipulator robot that exists in most industries requires a particular place that safe from human reach and the environment. Because the robot structure is rigid and heavy is the main reason in that case. So, robots in a limited environment [7] must be controlled very well, in order to have a safe interaction with the humans and environment [8-10].

At the present stage, a safe robot control strategy is required. A solution that can be used to solve this problem is introducing the compliance control strategy. This control strategy requires a robot torque sensor within the robot actuator to measure an external force or torque. The external force that read by the torque sensor will be used as a reference for compliance control policies. To implement compliance control to the control system is not a trivial problem. This paper, we propose a linear control technique...
[11-14] for controlling a robot compliant joint system. So that, the compliance control can be properly applied to the robot manipulator.

In this study, analysis and simulation of manipulator robot will be present. And a linear control technique is designed and analyzed to reach zero steady state error and minimum overshoot. The validity of the proposed method is demonstrated through simulation studies with a typical 1-degrees of freedom (1-DOF) manipulator robot. The 1 DOF manipulator robot is selected to implement the strategy control in order to the proposed control strategy can effectively be used for a compliance robot joint.

The purpose of this study is to use the PD control to stabilize 1 DOF manipulator robot to reach the desired position and give security to the environment when the robot interacts directly with humans. Modeling systems that near the actual system can be explained how to find the final dynamic equation using the Lagrange function [15-16] and the control design of the 1 DOF manipulator robot with linear control in Section II. Results and discussion are given in Section III and finally, Section IV presents the Conclusions.

2. Method

2.1. The lagrangian equation and mathematical modelling of manipulator

Figure 1 is research plan that will be proven by simulation to know the result of system response, to get the simulation result of the robot manipulator system response then, the first searched dynamic equation of 1 DOF manipulator robot. To get the dynamic equation of manipulator robot, in this research used the Lagrangian method to solve the mathematical equation of the mechanical system 1 DOF manipulator robot. So from the dynamic equation of motion has been found then, it can be simulated to get the result of system response from 1 DOF manipulator robot. The simulation result will then be analyzed more deeply for know system response from manipulator robot.

2.1.1. Lagrange equation. A mathematical modelling of 1 DOF robot manipulator should be derived to map the torque into the joint angle. In this study, the Lagrange equation is used to describe the system modelling. The equation of dynamic system motion Euler-Lagrange can be formulated as follows:

\[
Q = \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}} \right) - \frac{\partial \mathcal{L}}{\partial q} \quad (1)
\]

Where notation \( Q \) is a generalized force or torque. And \( q \) is the coordinate system of generalized system movements.

The basic principle of the Lagrange equation is to use the formula of total kinetic energy (K) and potential energy (P) to get the dynamic motion equation of manipulator robot. So the Lagrange equation for the case of 1 DOF manipulator robot is notated to be \( \mathcal{L} \) and is calculated by
\[ L = K(q) - P(q) \] (2)

Where the kinetic energy function in term of the generalized coordinate \( q \). And the potential energy is generalized in term of the generalized coordinate \( q \).

2.1.2. Mathematical modelling of manipulator. For the total kinetic energy of the 1 DOF manipulator robot consist of translation kinetic and rotational kinetic energy. Where the kinetic energy of translation is notated as \( K_T \) and the kinetic energy of rotation is notated as \( K_R \). However, for the kinetic energy of translational motion, the translational velocity is projected on the x-axis and the y-axis, so it is calculated by

\[
K = K_T - K_R \\
= \frac{1}{2}mv^2 + \frac{1}{2}I\dot{\theta}^2 \\
= \frac{1}{2}m(v_x^2 + v_y^2) + \frac{1}{2}I\dot{\theta}^2
\] (3) (4) (5)

From the geometric transformation, the projection coordinates center point of arm weight on the x-axes and y-axes is

\[
x = l_c \cos \theta \\
y = l_c \sin \theta
\] (6) (7)

From equation (6) and equation (7), so it can be derived to

\[
\frac{dx}{dy} = v_x = -l_c \sin \theta \\
\frac{dx}{dy} = v_y = -l_c \cos \theta
\] (8) (9)

By substituting the equation (8) and the equation (9) into the equation (5) then a new equation will be defined as

\[
K = \frac{1}{2}ml_c^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2
\] (10)

While to know the potential energy of 1 DOF manipulator robot, in general, can be formulated as follows

\[
P = mgh
\] (11)

Where \( h \) is the height of the projection of the center point of the weight on the y-axis, therefore the potential energy can be calculated by

\[
P = mgl_c \sin \theta
\] (12)

After known the kinetic energy equation and the potential energy equation of the derivation that has been done, the kinetic energy and potential energy of the DOF 1 robot manipulator system can be rewritten in the Lagrange equation by substituting the equation (10) and the equation (12) into the equation (2). So, it can be formulated as follows.

\[
L = \left( \frac{1}{2}ml_c^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2 \right) - (mgl \cos \theta)
\] (13)
From the equation (13) it can be seen that the system manipulator robot 1 DOF consists only of one coordinate system is rotation coordinate system, because from the equation (1) in the case of 1 DOF manipulator dynamics system, force or torque is generalized $Q = \tau$ and coordinate is generalized $q = \theta$, so from the equation can be reformulated into Euler Lagrange equation which can be written as follows

$$\tau = \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta}$$  \hspace{1cm} (14)

From equation (14) shows that torque changes only in manipulator robot systems. Where in this system the torque function $\tau$ depends on the dynamic state $\dot{\theta}$ and $\theta$ only. So this equation is called inverse dynamic equation. Because in addition to depending on dynamic state conditions but also serves to calculate the torque requirements of a robot manipulator. And here are the derivatives in the form of differential equations so that the components $\frac{\partial \mathcal{L}}{\partial \dot{\theta}}$ is derived as follows

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = m l^2 \dot{\theta} + l \dot{\theta}$$  \hspace{1cm} (15)

Afterward, $\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right)$ is derived as follows

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) = m l^2 \ddot{\theta} + l \ddot{\theta}$$  \hspace{1cm} (16)

And the last component $\frac{\partial \mathcal{L}}{\partial \theta}$ as follows

$$\frac{\partial \mathcal{L}}{\partial \theta} = -mgl \cos \theta$$  \hspace{1cm} (17)

So the final equation of the manipulator robot is formulated as follows

$$\tau = (m l^2 + l) \ddot{\theta} + mgl \cos \theta$$  \hspace{1cm} (18)

2.2. Design of the control techniques

By using kinematic inverse, the desired path of the manipulator robot can be calculated. To instruct the manipulator robot substitute the kinematic joint into the provided motion equation. Then the manipulator robot will move such as commands and will reach the desired path. However, because of disturbances and non-modelled phenomena, making the movement of manipulator robot to achieve the desired path cannot reach properly. Techniques to minimize and also reduce the error is called a control technique.

![Figure 2. Feedback control algorithm.](image)

2.2.1. Open and closed loop control. A robot is a system consisting of an actuator at each joint $i$ to practice a torque to move the link of manipulator robot $(i)$. To make each joint of the manipulator robot to follow the desired motion, the required torque instruction must be provided. The required torques that make the manipulator robot to move the desired motion can be calculated by the equation of motion and is formulated as
\[ Q_c = D(q_d) \dot{q}_d + H(q_d, \dot{q}_d) + G(q_d) \]  \hspace{1cm} (19)

Where \( c \) is control and \( d \) is desired. In other words, the equation (19) can cause the actuator \( Q_c \) move the link of manipulator robot at the desired path \( q_d \). We expect the robot to move like the desired motion, however, there is no system to minimize any possible error. Because it's called an open loop control. Now assume that at the time a manipulator robot isn’t always moving without any interference from external. Therefore there will be a difference between the actual joint variables and also the desired values. The difference is commonly called error and is calculated by

\[ e = q - q_d \]  \hspace{1cm} (20)

\[ \dot{e} = \dot{q} - \dot{q}_d \]  \hspace{1cm} (21)

While a closed loop control or a feedback control algorithm is read the actual variables and comparing with the desired values. The formula of the closed loop control can be defined as

\[ Q_c + k_D \dot{e} + k_P e = D(q_d) \dot{q}_d + H(q_d, \dot{q}_d) + G(q_d) \]  \hspace{1cm} (22)

In the equation (22) \( k_P \) and \( k_D \) is a gain control with always a constant value. Then the actual joint variable \((q, \dot{q})\) with the desired value \((q_d, \dot{q}_d)\) will be compared to get the desired value. Therefore Fig (2) shows the control method that will be applied to the manipulator robot.

**2.2.2. Linear control technique.** The Proportional Integral Derivative control algorithm uses position errors, derivative errors and integral errors to develop control laws. So, the PID control has a general form can be formulated as follows

\[ Q = k_P e + k_I \int_0^t e dt + k_D \dot{e} \]  \hspace{1cm} (23)

Where \( e = q - q_d \) is an error signal. \( k_P, k_I, \) and \( k_D \) is a gain with always a positive value. In proportional form, the PID control law is simplified as follows

\[ Q = k_P e + Q_d \]  \hspace{1cm} (24)

While the integral form. The PI control law is

\[ Q = k_P e + k_I \int_0^t e dt \]  \hspace{1cm} (25)

And the final form of PD control law is

\[ Q = k_P e + k_D \dot{e} \]  \hspace{1cm} (26)

From this linear control technique, it will find the final formula for controlling the 1 DOF manipulator robot to reach the desired value.

**2.3. Manipulator robot with linear control technique**

In the controller design, the linear control PD will be used to stabilize 1 DOF robotic manipulator to achieve the desired value. However, to prove the validity, we will apply a new control command to change the dynamic equation (19) of the robot manipulator to produce the actual joint variable \((\theta)\). The equation is

\[ Q = Q_c - k_D \dot{q}_d - k_P q_d + k_P q \]  \hspace{1cm} (27)
Then, the required torques to move the manipulator robot to reach desired value is formulated as
\[
\tau_c = (ml^2 + I) \ddot{\theta}_d + mgl \cos \theta_d
\]  
(28)

In equation (28) it can be clarified that \( Q \) and \( Q_c \) are generalized notations to \( \tau \) and \( \tau_c \) so that equation (19) and equation (29) can be substituted into the equation (27) to generate the new formula as follows
\[
(ml^2 + I) \ddot{\theta} + mgl \cos \theta = (ml^2 + I) \ddot{\theta}_d + mgl \cos \theta_d - k_D \dot{q}_d - k_D \dot{q} - k_P q_d + k_P q
\]
(29)

From the equation (29) then it can be determined the case position control, where the case position is
\[
\dot{\theta}_d = 0
\]
\[
\theta_d = constant
\]

So with the case position control, the final equation of 1 DOF manipulator robot with linear control approach become
\[
(ml^2 + I) \ddot{\theta} + mgl \cos \theta = mgl \cos \theta_d - k_D \dot{q}_d - k_D \dot{q} - k_P q_d + k_P q
\]
(30)

From equation (30) can be simulated to find out how the control response of robot manipulator by applying linear control approach with PD control.

3. Results and discussion
The simulation results can be confirmed that linear control can be applied to the robot manipulator. However, before providing a control on the manipulator robot system, how the system response of manipulator robot if a natural force or no control is provided at 1 DOF robot manipulator.

In Figure 3 shows system response when a mechanical system of 1 DOF manipulator robot is not given any control then the natural phenomena of the dynamic system state, the system response of 1 DOF robot manipulator continues to oscillate because there is no internal force and external forces that found on the robot manipulator.

![Figure 3. Joint angel response without control.](image)

In this section some experiments have been done to know the performance of linear control approach on the 1 DOF manipulator robot. And with value gain of proportional dan derivative which has been arranged gradually until it gets a stable system, so it can be observed how the performance of the linear
control in reducing the amount of error that occurred when the robot link moves to reach the desired value.

The validity of the proposed PD controller for reducing the maximum errors which has been done with the best value $k_p = 120$, $k_D = 40$. And can be shown in Figure 4 and Figure 5.

![Figure 4. Joint angle response, $k_p = 120$, $k_D = 40$.](image)

![Figure 5. Experimental results for the theta errors with $k_p = 120$, $k_D = 40$.](image)

From Figure 4 and Figure 5 it can be seen that by value $k_p = 120$ and $k_D = 40$, the system response of 1 DOF manipulator robot has successfully reach desired value. Where that happens in dynamic systems error $= 1.5^\circ$ from the desired value. So with a gain of proportional and derivative, 1 DOF manipulator robot can be controlled effectively. As for consideration what will happen to dynamic system 1 DOF manipulator robot arm when given different gain value. The results can be seen in the next experiment.

![Figure 6. Joint angle response, $k_p = 500$, $k_D = 40$.](image)
Figure 7. Experimental results for the theta errors with change $k_P = 500$, $k_D = 40$.

Figure 6 and Figure 7 are show that the change gain value of $k_P = 500$. The system response of 1 DOF manipulator robot shows error $= 0.5^\circ$, but this system response has an overshoot until $5.5^\circ$ and then system response stable at the desired value.

Figure 8. Joint angel response, $k_P = 10$, $k_D = 40$.

Figure 9. Experimental results for the theta errors with change $k_P = 10$, $k_D = 40$. 
The next experiment shows the change in a gain value of $k_p = 10$. From Figure 8 and Figure 9 show that system response cannot reach desired value successfully. And error always increasing and getting bigger.

![Graph](image1)

**Figure 10.** Joint angle response, $k_p = 120, k_D = 200$.

![Graph](image2)

**Figure 11.** Experimental results for the theta errors with change $k_p = 120, k_D = 200$.

In Figure 10 and Figure 11 are show the change in a gain value of $k_D = 200$. With larger gain value the system response of 1 DOF manipulator robot is unstable.

![Graph](image3)

**Figure 12.** Joint angle response, $k_p = 120, k_D = 2$. 
The final experiment shows the change in gain value $k_p = 2$. By giving smaller value the system response of 1 DOF manipulator robot come to pass to reach desired value and then drop from the maximum overshoot value then stable on the desired value. The system response can be seen in Figure 12 and Figure 13.

4. Conclusions

From several experiments that realized. Can be concluded by using linear control approach on 1 DOF manipulator robot. The system response can reach desired value and for the next study can be implemented directly on the compliance robot joint by adding more joints.

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