Nonperturbative light-front methods

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Abstract
Two methods for the nonperturbative solution of field-theoretic bound-state problems, based on light-front coordinates, are briefly reviewed. One uses Pauli–Villars regularization and the other supersymmetry. Applications to Yukawa theory and super Yang–Mills theory with fundamental matter are emphasized.

1. INTRODUCTION
There have been a number of calculations [1] using light-front coordinates [2] as a convenient means of attacking field-theoretic problems nonperturbatively, particularly in 1+1 dimensions. Efforts in more dimensions have generally been less successful, however, due to the need for regularization and renormalization. Here two approaches that include consistent regularization are briefly reviewed in the context of specific model calculations [3, 4]. One approach is Pauli–Villars (PV) regularization [5], where massive negative-metric particles are added to a theory to provide the necessary cancellations [6, 7]. The other is supersymmetry [8]. These are not the only approaches available on the light front; in particular, one can find the transverse lattice technique [9] and similarity transformations [10] discussed elsewhere in this volume.

The primary numerical method is discrete light-cone quantization (DLCQ) [11], in which one imposes a discrete momentum grid, with length scales $L$ and $L_\perp$, as $p^+ \rightarrow \pi n/L$ and $p^\perp \rightarrow \pi n_{\perp}/L_{\perp}$, and approximates integrals in the mass-squared eigenvalue problem with trapezoidal sums. The continuum limit $L \rightarrow \infty$ can be exchanged for a limit in terms of the integer resolution $K \equiv L P^+/\pi$, because light-cone momentum fractions $x_i \equiv p^+_i/P^+$ are measured in units of $1/K$.

For supersymmetric theories there is a supersymmetric version of DLCQ (SDLCQ) that preserves supersymmetry within the discrete approximation [12]. This is accomplished by discretizing the supercharge $Q^-$ and constructing the Hamiltonian $P^-$ from the superalgebra via the anticommutator: $P^- = \{Q^-, Q^\}$/$2\sqrt{2}$. This $P^-$ and the DLCQ $P^-$ are equivalent in the $K \rightarrow \infty$ limit.

The matrix eigenvalue problems that result from these discretizations are large but sparse. An efficient means for extracting a few lowest eigenvalues and their eigenvectors is the Lanczos algorithm [13]. In the case of PV-regulated theories, with their indefinite metrics, a special form [7] based on the biorthogonal algorithm [14] is required. In either case, the process is an iterative one that generates a tridiagonal matrix of much smaller size, which is easily diagonalized to yield approximate eigenvalues and eigenvectors. The number of iterations and the size of the tridiagonal matrix are determined by the rate of convergence. The primary difficulty that arises is that round-off error causes spurious copies to appear in the derived spectrum; however, there are techniques for removing them [15].

In the remaining sections we discuss an application of PV regularization to Yukawa theory and a study of supersymmetric QCD (SQCD) with a Chern–Simons (CS) term in the large-$N_c$ approximation.
2. YUKAWA THEORY

As the action of the PV-regulated Yukawa theory, we take

$$S = \int d^4x \left[ \frac{1}{2} (\partial_{\mu} \phi_0)^2 - \frac{1}{2} \mu_0^2 \phi_0^2 - \frac{1}{2} (\partial_{\mu} \phi_1)^2 + \frac{1}{2} \mu_1^2 \phi_1^2 + \frac{i}{2} \left( \psi_0 \gamma^\mu \partial_{\mu} - (\partial_{\mu} \psi_0) \gamma^\mu \right) \psi_0 - m_0 \psi_0 \psi_0 - \frac{i}{2} \left( \psi_1 \gamma^\mu \partial_{\mu} - (\partial_{\mu} \psi_1) \gamma^\mu \right) \psi_1 + m_1 \psi_1 \psi_1 - g(\phi_0 + \phi_1)(\psi_0 + \psi_1)(\psi_0 + \psi_1) \right],$$

(1)

with the subscript 0 indicating physical fields and 1 indicating PV fields. When fermion pairs are excluded, this action provides a light-cone Hamiltonian of the form

$$P^- = \sum_{i,s} \int dp \frac{m_i^2 + p_i^2}{p^+} (-1)^i b_{i,s}^\dagger(p) b_{i,s}(p) + \sum_j \int dq \frac{m_j^2 + q_j^2}{q^+} (-1)^j a_{j}^\dagger(q) a_{j}(q)$$

(2)

$$+ \sum_{i,j,k,s} \int dp dq \left\{ [V_{2s}(p,q) + V_{2s}(p + q,q)] b_{j,s}^\dagger(p) a_{k}^\dagger(q) b_{i,-s}(p + q) + [U_{j}(p,q) + U_{i}(p + q,q)] b_{j,s}(p) a_{k}^\dagger(q) b_{i,s}(p + q) + h.c. \right\},$$

where $a^\dagger$ creates a boson and $b^\dagger$ a fermion,

$$U_j(p,q) \equiv \frac{g}{\sqrt{16\pi^3}} \frac{m_j}{p^+ q^+}, \quad V_{2s}(p,q) \equiv \frac{g}{\sqrt{8\pi^3}} \frac{\epsilon_{2s}}{p^+ q^+}, \quad \epsilon_{2s} \equiv -\frac{1}{\sqrt{2}} (2s,i),$$

(3)

and

$$[a_i(q), a_j^\dagger(q')] = (-1)^i \delta_{ij} \delta(q - q'), \quad \{b_{i,s}(p), b_{j,s'}(p')\} = (-1)^i \delta_{ij} \delta_{s,s'} \delta(p - p').$$

(4)

The eigenfunction for the dressed-fermion state is expanded in a Fock basis as

$$\Phi_+(P) = \sum_{i} z_i b_{i+}^\dagger(P) |0\rangle + \sum_{ijs} \int dq dq_{12} f_{ijs}(q) b_{is}^\dagger(P - q)a_{j}^\dagger(q) |0\rangle$$

(5)

$$+ \sum_{ijks} \int dq_{1} dq_{2} f_{ijks}(q_{1},q_{2}) \frac{1}{\sqrt{1 + \delta_{jk}}} b_{js}^\dagger(P - q_{1} - q_{2}) a_{k}^\dagger(q_{1}) a_{k}^\dagger(q_{2}) |0\rangle + \ldots$$

It is normalized according to $\Phi_+^\dagger \cdot \Phi_+ = \delta(P' - P)$. The wave functions satisfy a coupled system of equations, derived from the fundamental mass-squared eigenvalue problem $P^+ P^- \Phi_+ = M^2 \Phi_+$ to be

$$m_i^2 z_i + \sum_{i'} (-1)^{i' + j} \int P^- dq \left\{ f_{i'i}(q) [V_{+}(P - q,q) + V_{+}^*(P,q)] + f_{i'i}(q) [U_{i'}(P - q,q) + U_{i'}(P,q)] \right\} = M^2 z_i,$$

(6)

$$\left[ \frac{m_i^2 + q_i^2}{1 - y} + \frac{\mu_i^2 + q_i^2}{y} \right] f_{ijs}(q) + \sum_{i'} (-1)^{i'} \left\{ z_{i'} \delta_{s,-} [V_{+}^*(P - q,q) + V_{-}(P,q)] + z_{i'} \delta_{s,+} [U_{i}(P - q,q) + U_{i}(P,q)] \right\}$$

(7)

$$+ 2 \sum_{i',k} \frac{(-1)^{i' + k}}{\sqrt{1 + \delta_{jk}}} \int P^- dq \left\{ f_{i'jk,s}(q,q') [V_{2s}(P - q - q',q') + V_{2s}(P - q,q')] + f_{i'jk,s}(q,q') [U_{i}(P - q - q',q') + U_{i}(P - q,q')] \right\} = M^2 f_{ijs}(q).$$
\[
\left[ \frac{m_i^2 + (q_{1\perp} + q_{2\perp})^2}{1 - y_1 - y_2} + \frac{\mu_k^2 + q_{1\perp}^2}{y_1} + \frac{\mu_k^2 + q_{2\perp}^2}{y_2} \right] f_{i'j'k}(q_1, q_2)
\]  
\[+ \sum_{i'} (-1)^{i'} \sqrt{1 + \delta_{i'k}} P^+ \left\{ f_{i'j'k}(q_1) [V_{-2k}(P - q_1, q_2) + V_{2k}(P - q_1, q_2)] + f_{i'j'k}(q_2) [U_i(P - q_1 - q_2, q_1) + U_i(P - q_1, q_2)] \right\} \]  
\[+ f_{i'j'k}(q_2) [U_i(P - q_1 - q_2, q_1) + U_i(P - q_1, q_2)] \]  
\[+ M^2 f_{i'j'k}(q_1, q_2) \]  
These equations can be approximated directly by DLCQ.

We now have a well-defined numerical problem. The PV particles are kept in the DLCQ basis and provide the necessary counterterms. The range of the now-finite transverse integrations is cut off by imposing \( p_{i\perp}^2 / x_i < \Lambda^2 \) for each particle in a Fock state, to reduce the matrix problem to a finite size. The transverse momentum indices \( n_x \) and \( n_y \) are limited by the transverse resolution \( N \). The bare parameters \( g \) and \( m_0 \) are fixed by fitting "physical" constraints, such as specifying the dressed-fermion mass \( M \) and its radius. The limits of infinite resolution, infinite (momentum) volume, and infinite PV masses can then be explored.

This process can be studied analytically in the case of a one-boson truncation \[3\]. The one-boson wave functions are

\[
f_{ij+}(q) = \frac{P^+}{M^2 - m_i^2 + q_{\perp}^2} \left\{ (\sum_k (-1)^{k+1} z_k) U_i (P - q, q) U_k (P, q) \right\},
\]

\[
f_{ij-}(q) = \frac{P^+}{M^2 - m_i^2 + q_{\perp}^2} \left\{ (\sum_k (-1)^{k+1} z_k) V_i (P - q, q) \right\}.
\]

The bare-fermion amplitudes and the coupling satisfy a pair of algebraic equations

\[
(M^2 - m_i^2)_{ij} = g^2 \mu_0^2 (z_0 - z_1) I_0 + g^2 m_i (z_0 m_0 - z_1 m_1) I_0,
\]

\[
+ g^2 \mu_0 [(z_0 - z_1) m_i + z_0 m_0 - z_1 m_1] I_0,
\]

with

\[
I_n = \int \frac{dy dq_{\perp}^2}{16\pi^2} \sum_{jk} \frac{(-1)^{j+k}}{M^2 - m_j^2 + q_{\perp}^2} \frac{1}{1-y} \frac{(m_j/\mu_0)^n}{y(1-y)^n},
\]

\[
J = \int \frac{dy dq_{\perp}^2}{16\pi^2} \sum_{jk} \frac{(-1)^{j+k}}{M^2 - m_j^2 + q_{\perp}^2} \frac{(m_j^2 + q_{\perp}^2)/\mu_0^2}{y(1-y)^2} = M^2 \mu_0 I_0.
\]

The solution is

\[
g^2 = -\frac{(M + m_0)(M + m_1)}{(m_1 - m_0)(\mu_0 I_1 \pm MI_0)}, \quad \frac{z_1}{z_0} = \frac{M + m_0}{M + m_1}.
\]

Ref. \[3\] contains many subsequent results.

For a two-boson truncation, the solution is no longer analytic, but the coupled equations can be reduced to eight equations for the two-particle amplitudes only, which are of the form

\[
\left[ M^2 - m_i^2 + q_{\perp}^2 \right] f_{ij}(y, q_{\perp}) = \frac{g^2}{16\pi^2} \sum_{a} \frac{I_{0a}(y, q_{\perp})}{1-y} f_{a'j}(y', q_{\perp}')
\]

\[
+ \frac{g^2}{16\pi^2} \sum_{a,b'} \int_0^1 dy' dq_{\perp}' f^{(0)}_{j, a'b'}(y, q_{\perp}; y', q_{\perp}') f_{a'b'}(y', q_{\perp}')
\]

\[
+ \frac{g^2}{16\pi^2} \sum_{a,b'} \int_0^{1-y} dy' dq_{\perp}' f^{(2)}_{j, a'b'}(y, q_{\perp}; y', q_{\perp}') f_{a'b'}(y', q_{\perp}'),
\]
with the angular dependence removed via $\sqrt{P^+} f_{ij+}(q) = f_{ij+}(y, q_{\perp})$ and $\sqrt{P^+} f_{ij-}(q) = f_{ij-}(y, q_{\perp}) e^{i\phi}$. Here $I$ is a computable self-energy. $J^{(0)}$ is the kernel due to bare-fermion intermediate states, and $J^{(2)}$ is the kernel due to two-boson intermediate states.

Although one could impose a transverse cutoff and discretize these equations as per DLCQ, alternative quadratures are more efficient. In particular, the transverse momentum $q_{\perp}$ can be mapped to a finite range that compresses the wave function’s tail to a relatively small region, so that a Gauss–Legendre quadrature can yield a good approximation. A comparison of results is given in Fig. 1.

Fig. 1: The Yukawa coupling as a function of the bare fermion mass for the two-boson and (exactly soluble) one-boson truncations. The dressed-fermion mass is $M = \mu_0$, and the PV masses are $m_1 = \mu_1 = 15 \mu_0$. The longitudinal resolution is $K = 20$; the transverse resolutions $N$ are specified in the legend.

3. SUPERSYMMETRIC QCD

As the action for (2+1)-dimensional $\mathcal{N} = 1$ SQCD-CS theory, we consider

$$S = \int d^3 x \text{Tr} \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_{\mu} \xi^\dagger D^\mu \xi + i \bar{\Psi} D_{\mu} \Gamma^\mu \Psi 
- g \left[ \bar{\Psi} \Lambda \xi + \xi^\dagger \Lambda \Psi \right] + \frac{i}{2} \Lambda \Gamma^\mu D_{\mu} \Lambda + \frac{\kappa}{2} \epsilon^{\mu\nu\lambda} \left[ A_\mu \partial_\nu A_\lambda + \frac{2i}{3} g A_\mu A_\nu A_\lambda \right] + \kappa \bar{\Lambda} \Lambda \right\}. \quad (15)$$

The adjoint fields are the gauge boson $A_\mu$ (gluons) and a Majorana fermion $\Lambda$ (gluinos); the fundamental fields are the Dirac fermion $\Psi$ (quarks) and a complex scalar $\xi$ (squarks). The CS coupling $\kappa$ induces a mass for the adjoint fields without breaking the supersymmetry; this reduces formation of the long strings characteristic of super Yang–Mills theory. The covariant derivatives are

$$D_{\mu} \Lambda = \partial_\mu \Lambda + ig [A_\mu, \Lambda], \quad D_{\mu} \xi = \partial_\mu \xi + ig A_\mu \xi, \quad D_{\mu} \Psi = \partial_\mu \Psi + ig A_\mu \Psi. \quad (16)$$

The supersymmetry transformations are

$$\delta A_\mu = \frac{i}{2} \bar{\epsilon} \Gamma_\mu \Lambda, \quad \delta \Lambda = \frac{1}{4} F_{\mu\nu} \Gamma^{\mu\nu} \epsilon, \quad \delta \xi = \frac{i}{2} \bar{\epsilon} \Psi, \quad \delta \Psi = -\frac{1}{2} \Gamma^{\mu} \bar{\epsilon} D_{\mu} \xi. \quad (17)$$
We reduce this theory to 1+1 dimensions by taking the fields to be independent of the transverse coordinate \( x^2 \).

As usual, there are constraints and not all of the fields are dynamical. To separate the dynamical fields, we first introduce components for the Fermi fields and the supercharge as

\[
\Lambda = \left( \lambda, \dot{\lambda} \right)^T, \quad \Psi = \left( \psi, \dot{\psi} \right)^T, \quad Q = (Q^+, Q^-)^T.
\]

Then, in light-cone gauge \((A^+ = 0)\), the constraints are

\[
\begin{align*}
\partial_+ \dot{\lambda} &= -\frac{i g}{\sqrt{2}} \left[ A^2, \lambda \right] + i \xi \psi^\dagger - i \lambda \psi^\dagger, \\
\partial_+ \dot{\psi} &= -\frac{i g}{\sqrt{2}} A^2 \psi + \frac{g}{\sqrt{2}} \lambda \xi - \kappa \lambda / \sqrt{2}, \\
\partial_-^2 A^- &= g \left\{ i [A^2, \partial_- A^2] + \frac{1}{\sqrt{2}} \left\{ \lambda, \lambda \right\} + \kappa \partial_- A^2 - i \hbar \partial_- \xi \psi^\dagger + i \xi \partial_- \xi^\dagger + \sqrt{2} \psi \dot{\psi} \right\}.
\end{align*}
\]

When these constraints are used to eliminate the nondynamical fields, the supercharge becomes

\[
Q^- = g \int dx^- \left\{ 2^{3/4} \left[ i [A^2, \partial_- A^2] - \kappa \partial_- A^2 + \frac{1}{\sqrt{2}} \left\{ \lambda, \lambda \right\} \right] \frac{1}{\partial_-} \lambda \right. \\
\left. - \frac{1}{\sqrt{2}} \left( i \sqrt{2} \xi \partial_- \xi^\dagger - i \sqrt{2} \partial_- \xi \xi^\dagger + 2 \psi \dot{\psi} \right) \frac{1}{\partial_-} \lambda - 2 \left( \xi^\dagger A^2 \psi + \psi^\dagger A^2 \xi \right) \right\}.
\]

The mode expansions of the dynamical fields are

\[
\begin{align*}
A_{ij}^2(0, x^-) &= \frac{1}{\sqrt{4 \pi}} \sum_{k=1}^\infty \frac{1}{\sqrt{k}} \left( a_{ij}(k) e^{-ik\pi x^- / L} + a_{ij}^\dagger(k) e^{ik\pi x^- / L} \right), \\
\lambda_{ij}(0, x^-) &= \frac{1}{2 \pi 2L} \sum_{k=1}^\infty \left( b_{ij}(k) e^{-ik\pi x^- / L} + b_{ij}^\dagger(k) e^{ik\pi x^- / L} \right), \\
\xi_i(0, x^-) &= \frac{1}{\sqrt{4 \pi}} \sum_{k=1}^\infty \frac{1}{\sqrt{k}} \left( c_i(k) e^{-ik\pi x^- / L} + c_i^\dagger(k) e^{ik\pi x^- / L} \right), \\
\psi_i(0, x^-) &= \frac{1}{2 \pi 2L} \sum_{k=1}^\infty \left( d_i(k) e^{-ik\pi x^- / L} + d_i^\dagger(k) e^{ik\pi x^- / L} \right).
\end{align*}
\]

The creation and annihilation operators obey the commutation relations (for finite \( N_c \))

\[
\begin{align*}
\left[ a_{ij}, a_{kl}^\dagger \right] &= \left( \delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right), \\
\left\{ b_{ij}, b_{kl}^\dagger \right\} &= \left( \delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right), \\
\left[ c_i, c_j^\dagger \right] &= \delta_{ij}, \\
\left\{ c_i, c_j^\dagger \right\} &= \delta_{ij}, \\
\left\{ d_i, d_j^\dagger \right\} &= \delta_{ij}, \\
\left\{ d_i, d_j^\dagger \right\} &= \delta_{ij}.
\end{align*}
\]

The solutions that we obtain are meson-like states \( f_{ij}^\dagger(k_1) a_{i1i2}^\dagger(k_2) \ldots b_{i_{n-1}i_{n+1}}^\dagger(k_{n-1}) \ldots f_{ij}^\dagger(k_n) \right\}_0 \rangle \), where \( f^\dagger = c^\dagger \) or \( d^\dagger \), and glueball states \( \text{Tr}[a_{i1i2}^\dagger(k_1) \ldots b_{i_{n-1}i_{n+1}}^\dagger(k_{n-1}) \ldots f_{ij}^\dagger(k_n)] \right\}_0 \rangle \). Because of the supersymmetry, either could be a boson or a fermion. Because we work in the large-\( N_c \) limit, there is no mixing between these states, and they are composed of single traces. This simplifies the calculation, particularly with respect to the size of the matrices that are diagonalized; however, study of baryons will require finite \( N_c \) or additional approximations. To help reduce the size of the calculation further, there is an additional \( \mathbb{Z}_2 \) symmetry \([^10] \) \( a_{ij}(k, n^+) \rightarrow -a_{ij}(k, n^+) \), \( b_{ij}(k, n^+) \rightarrow -b_{ij}(k, n^+) \). This divides the states between those with even and odd numbers of gluons, and we diagonalize in each sector separately. A collection of results can be found in Ref. \[^{[13]} \]. The spectrum for mesons shows the existence of light states at strong coupling, a feature found previously in other supersymmetric theories \([^17] \).
4. CONCLUSION
Methods for the nonperturbative solution of multidimensional field theories are now available; they do, however, require more development. A number of calculations will soon be undertaken to further demonstrate and improve this capability. In Yukawa theory, the two-fermion sector is of interest, particularly for a pseudo-scalar coupling with which the deuteron might be modelled. Quantum electrodynamics is immediately treatable with the same techniques; the anomalous magnetic moment of the electron can be computed. The full (2+1)-dimensional SQCD-CS theory can be solved, including finite-$N_c$ corrections to allow baryons and meson-glueball mixing. This work should bring us closer to the goal of being able to solve for hadron properties in quantum chromodynamics.

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