Approximation method of the speed-torque mechanical characteristics using the spline interpolation of various electrical motors

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Abstract. The design of the electric drives of the on-board installation requires motors of an adequate power. The dimensioning of the electric drive takes into account the mechanical loads and it provides the value of the torque which loads the electrical motor shaft. The selection of the appropriate electrical motors is based on the speed-torque mechanical characteristics, \( n = u(M) \), where \( n \) is the angular speed in \( \text{rpm} \) and \( M \) is the load torque in \( \text{N/m} \). Drawing the \( n = u(M) \) characteristic, one can notice that the function is not injective, therefore we employ its ‘rotated’ version, i.e. the \( M = f(n) \) function. In this way we are able to mathematically define the \( f(...) \) function, and further on to deduce its approximation using the spline functions. As a case study we use an asynchronous motor for which the calculus is based on the Freidzon method that uses relative values. The spline approximation uses an original data processor which outputs various types of information in a wide range of text, image and computer code formats. Using the experimentally acquired data as input information in the spline based processor we have a more accurate approximation of the speed-torque mechanical characteristics. The previously mentioned methods are useful for the rapidly evaluation of the mechanical characteristics of the motors, that is an important criterion in the marine engineering decision making process.

1. Introduction
The importance of the automatic use of the on-board installations and mechanisms has strongly increased due to the new concept regarding the autonomous ships. Accordingly, the importance of the automatic electric drives has also increased.

The electric drive represents the operation through which commands are done depending on the machines’ work regime (mechanisms, mechanical devices, pneumatics, hydraulics) by the use of the electric energy, [1]. In comparison with the pneumatic, hydraulic and other types of actions, the electric drive has several strengths, such as:
- easy supply with electrical energy;
- large range of speeds without special reductors;
- fine adjustment of the speed, in a wide range of values, promptly achieved;
- starting, stopping and the reversal of a turn, may be done in a simple, fast and easy way;
- relative high efficiency;
- possibility to include them in automatic systems;
- easy and inexpensive fixing and maintenance.

![Figure 1. A typical drive system.](image)

All these features make the electric drive to be preferred in most of the industrial processes, being adapted to the wide types of conditions demanded by the various technological processes. An electric drive is manufactured using systems of electric motors, made of an assembly of devices which transforms the electric energy. The principal components of an electric drive system are presented in figure 1, [2].

2. Theoretical background

The properties of the electrical machines and work mechanisms determine the behaviour of the drive systems which they are a part of. These properties are emphasized by the mechanical characteristics functional dependence between the electrical drive and the work mechanism, according to the technological process to be made. This is why in electrical drive technique is useful to classify electrical machines and work mechanisms according to the form of their mechanical characteristics.

The mechanical characteristics of the electrical motors represent the dependence of the angular speed with respect to the resistance torque, $M_s$. Because the mechanical characteristics can be the same for work mechanisms belonging to various industrial processes, the classification of work mechanisms is done independently of their destination and is rather done according to the dependence of the $M_s$ torque to the aforementioned parameters.

Every electrical machine has an infinity of mechanical static characteristics from which only one is a natural mechanical static characteristic. It is defined as the locus of the steady functioning points at various loads and angular speeds, at the nominal voltage and nominal frequency without other electrical and electronic components, such as rheostats, coils, condensers, in the electrical supply circuitry.

The asynchronous motor with short circuited rotor, of normal construction, has a low starting torque, being able to start only at low-value loads. Most of the electrical drives require high starting loads, therefore it is necessary to develop a motor with a squirrel cage, high electrical resistance rotor (tall bars or double squirrel cage motor solutions). These motors have parameters, including the output drive torque, which are depending on the slip. This is why there cannot be expressed the natural mechanical characteristics in an analytical form, similar to the ones of the coiled rotor motors.

The squirrel cage motor natural characteristic is unique for each unit. Normally it is provided by the manufacturer or it can be experimentally deduced.

If we don’t have these opportunities to assess the natural characteristic, various scientists conceived methods to approximate the variation of the natural characteristic, being conceived mathematical laws of variation which accurately approximate the real characteristic.

The natural mechanical characteristics of an asynchronous motor with short circuited rotor, with amendable start, used in naval electrical drives can be calculated within a reasonable margin with the so called "general equation" of electrical naval engines mechanical characteristics, [3]:
\[ \nu = (1 - b \cdot \mu)^x = (1 - b \cdot m)^x \]  

(1)

where

\[ \nu = \frac{n}{n_0} = \frac{n}{n_1} \] is the angular speed in relative size;

\[ \mu = \frac{m}{M} \] is the torque in relative size;

\[ b = 1 - \left(1 - \frac{n}{n_0}\right)^\frac{1}{2} \] is variable, depending on the slip, where

\[ s \equiv \frac{n_0 - n}{n_0} = \frac{n_1 - n}{n_1} \] is the nominal slip;

\[ n_0 \] is the no-load speed (synchronisation speed);

\[ x \] is an exponent which depends on the motor type; for an asynchronous motor with large slip

\[ x \in \left[ \frac{1}{3}, \frac{1}{2} \right] \]

The approximation of the natural mechanical characteristic, depending on the voltage and frequency variation may be computed using the following expression which uses the relative sizes:

\[ m = \frac{\delta \cdot r_i' \cdot u^2}{f \cdot z^2(s)} \]  

(2)

where:

\[ \delta = \frac{1}{\cos(\phi_n - r_i)} ; \quad z^2(s) = \alpha \cdot s + \beta + \frac{\gamma}{s} \]

\[ \alpha = (c_1 \cdot r_i)^2 + f^2 \cdot (x_1 + c_1 \cdot x_e)^2 ; \quad \beta = r_i \cdot r_i' ; \quad \gamma = (c_1 \cdot r_i')^2 + \left(\frac{b \cdot r_i \cdot r_1'}{f}\right)^2 \]

\[ c_1 = 1 + b \cdot x_1 ; \quad c_2 = 1 + b \cdot x_2 ; \quad b = \frac{i}{u - i_0 \cdot x_1} \]

where \( f \) the input current frequency in relative size.

The reactive and active components of the fixed coil current are:

\[ i_r = u \cdot \frac{c \cdot (x_1 + c_1 \cdot x_e) \cdot s \cdot f + b \cdot c_1 \cdot r_1'^2}{z^2(s)} ; \quad i_a = u \cdot \frac{c_2 \cdot r_i \cdot s + r_i'^2 + b^2 \cdot r_i \cdot r_i'^2}{z^2(s)} \]

The current of the fixed coil is

\[ i = \sqrt{i_r^2 + i_a^2} = \frac{u}{z^2(s)} \cdot \left[ c \cdot (x_1 + c_1 \cdot x_e) \cdot s \cdot f + \frac{b \cdot c_1 \cdot r_1'^2}{f} \right] + \left(\frac{c_2 \cdot r_i \cdot s + r_i'^2 + b^2 \cdot r_i \cdot r_i'^2}{s \cdot f^2}\right)^2 \]

If we denote by

\[ N(s) = \sqrt{c \cdot (x_1 + c_1 \cdot x_e) \cdot s \cdot f + \frac{b \cdot c_1 \cdot r_1'^2}{f}} + \left(\frac{c_2 \cdot r_i \cdot s + r_i'^2 + b^2 \cdot r_i \cdot r_i'^2}{s \cdot f^2}\right)^2 \]

it results
\[ i = \frac{u}{z'(s)} \cdot N(s). \]

The slip is
\[ s = \frac{\delta \cdot r_1' \cdot u^2 - m \cdot f \cdot \beta \pm \sqrt{(m \cdot f \cdot \beta - \delta \cdot r_1' \cdot u^2)^2 - 4 \cdot \gamma \cdot m^2 \cdot f^2}}{2 \cdot m \cdot f \cdot \alpha}. \]

The critical slip, the starting couple, the maximum as well as \( \cos(\phi) \) are computed using the following expressions:
\[ s_m = \pm \sqrt{\frac{\gamma}{\alpha}}, \quad m_{\text{max}} = \frac{\delta \cdot r_1' \cdot u^2}{f \cdot (\beta \pm 2 \cdot \sqrt{\gamma \cdot \alpha})}, \quad m_p = \frac{\delta \cdot r_1' \cdot u^2}{f \cdot (\alpha + \beta + \gamma)}, \]
\[ \cos(\phi) = \frac{1}{\sqrt{1 + \tan^2(\phi)}}, \quad \tan(\phi) = \frac{I_s}{I_u}. \]

To see the influence of the resistance and the motor’s reactance onto its functioning, in figures 2 and 3 are presented the variations of the following parameters with respect to the motor power, [4].

**Figure 2.** Rotors resistance and fixed coils functions P.

**Figure 3.** Rotors reactance and fixed coils functions P.

The aforementioned parameters are:
\[ \frac{R_1}{Z_{1N}} - \text{resistance of the fixed coil in relative size;} \]
\[ \frac{R_2}{Z_{1N}} - \text{resistance of the rotor in relative size;} \]
\[ \frac{X_{11}}{Z_{1N}} - \text{reactance of the fixed coil in relative size;} \]
\[ \frac{X_{12}}{Z_{1N}} - \text{reactance of the rotor in relative size;} \]
\[ \frac{X_{m}}{Z_{1N}} - \text{reactance of magnetizing in relative size;} \]
\[ Z_{1N} = \frac{U_{1N}}{I_{1N}} \text{ is the impedance of the motor;} \]
\[ U_{1N} \text{ - nominal voltage in the fixed coil phase;} \]
\[ I_{1N} \text{ - nominal current in the fixed coil phase.} \]

For asynchronous motors with double squirrel cage or with deep slotted armature, the resistance and the reactance of the rotor which depend on the nominal frequency aren’t remaining constant because of the pellicular effect. Their values are adjusted using the relations:
\[ r_k = k_r \cdot r_{20}, \quad x_k = k_i \cdot x_{20}, \]
where:
\[ r_k, x_k \text{ are the actual values of the rotor resistance and of the rotor reactance;} \]
\[ r_{20}, x_{20} \text{ are the rotor resistance and the rotor reactance for the null-load value;} \]
\[ k_r, k_i \text{ are coefficients used for corrections, their values being extracted from some characteristics.} \]

3. Original computer based method
The mechanical characteristics may be regarded as a general type diagram whose variation is given by a series of points. The coordinates of the points may be either computed, or acquired using experimental studies.

![Figure 4. Mechanical load characteristic approximated using spline functions.](image)

A general method used to approximate a diagram and to express it in an analytical form is presented in [5, 6]. The method is based on spline functions, expressed along the \( i, i+1 \) interval as a third order polynomial, i.e.
\[ s_{i,i+1}(x) = A_i \cdot (x-x_i)^3 + B_i \cdot (x-x_i)^2 + C_i \cdot (x-x_i) + D_i. \]  

(3)

In this case the general function may be
\[ f(x) = \sum_{i=1}^{N} \left[ H(x-x_i) - H(x-x_{i+1}) \right] \cdot s_{i,i+1}(x), \quad \text{or} \quad f(x) = \sum_{i=1}^{N} \left[ H(x-x_i) \cdot H(x_{i+1} - x) \right] \cdot s_{i,i+1}(x), \]

(4)

where \( H(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases} \) is Heaviside’s step function.
Let us consider an asynchronous motor with star connection, whose mechanical characteristic is presented as a set of points experimentally acquired, the results of the interpolation process being presented in figure 4.

4. Discussion
The calculus relations presented in section 2 were used to generate a series of theoretical mechanical characteristics, figure 5.

![Figure 5. Mechanical load characteristics for S=1.0; S=0.8; S=0.6; S=0.4; S=0.2; S=0.0 slip values.](image)

Freidzon’s calculus relations allow general approaches with a certain fair accuracy, relations which may be used for a wide range of types of motors and a wide range of motors’ powers.

5. Conclusions
An important issue in marine electrical engineering is the accurate selection of the motor drives to be used for most of the naval installations. One of the major criteria is based on the mechanical characteristic of the electrical motors. This is why, the concepts employed to rapidly assess the
mechanical characteristics must use the strengths of the previously presented methods. In this way, both methods have strengths and weak points.

The spline approximation method is accurate, its precision depending on the accuracy of the experimentally acquired data. However, it offers a particular characteristic specific to the motor under testing.

The main strength of the calculus based on Freidzon method resides in its generality, being applicable for AC and DC motors of various ranges of powers. However, its accuracy is under the precision of a dedicated solution based on spline approximation.

The spline approximation method as well as the Freidzon method, [3], are useful for the rapidly evaluation of the speed-torque mechanical characteristics of the motors, which is an important criterion in the marine engineering decision making process, [7].

6. References
[1] Ion C. Ionita and Jimbu Apostolache 1986 *On-board naval equipment, Construction and utilization* (Bucharest: “Technics” Publishing House).
[2] Dordescu M and Gheorghiu S 2015 *Naval electric drive systems* (Constanta: “Nautica” Publishing House) ISBN 978-606-681-054-8.
[3] Freidzon I-R 1979 *Electric drives for naval mechanisms* (Bucharest: “Technics” Publishing House).
[4] Seracin E and Popovici D 1985 *Electric drives technique* (Bucharest: “Technics” Publishing House).
[5] Dordescu M and Oanta E 2018 *IOP Conference Series: Materials Science and Engineering* 400 042015 doi:10.1088/1757-899X/400/4/042015.
[6] Oanta E, Pescaru A and Micu A 2018 Proc SPIE 10977, Advanced Topics in Optoelectronics, Microelectronics, and Nanotechnologies IX 109771R doi: 10.1117/12.2323169.
[7] Raicu A, Oanta E and Sabau A 2017 *IOP Conference Series: Materials Science and Engineering* 227 012108 doi:10.1088/1757-899X/227/1/012108.
[8] Panait C 2017 *HORESEC project*, Romanian Ministry of Research and Innovation, CCCDI-UEFISCDI, project number PN-III-P1-1.2-PCCDI-2017-04-04/31PCCDI/2018, within PNCDI III.

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