Moduli-Induced Axion Problem

Tetsutaro Higaki
(KEK)

1208.3563 and 1304.7987
with K. Nakayama and F. Takahashi
Moduli-Induced Axion Probe for extra dimensions

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Cosmological test for **string models**

Axionic dark radiation (= relativistic DM):
\[ \Delta N_{\text{eff}} = O(0.1) \]
Key: Moduli problem in reheating

\[ \Phi \rightarrow a a \]

\(\Phi: \text{Moduli/Inflaton} \quad \text{a: Axion}\)

Axionic dark radiation exists even for \(m_\Phi >> 100\text{TeV}\).
Dark radiation
Dark radiation $\Delta N_{\text{eff}}$ in $\rho_{\text{rad}}$

$$\rho_{\text{rad}} = \left[ 1 + \frac{7}{8} N_{\text{eff}} \left( \frac{4}{11} \right)^{4/3} \right] \frac{\pi^2}{15} T^4$$

$N_{\text{eff}} = \Delta N_{\text{eff}} + 3.046$

$\Delta N_{\text{eff}}$: Dark radiation, $N_{\text{eff}}$: Effective neutrino number, 3.046: The SM value
Effective neutrino number $N_{\text{eff}}$

- Observations from Planck (95%): [Planck collaborations]

$$N_{\text{eff}} = 3.36^{+0.68}_{-0.64}, \quad 3.30^{+0.54}_{-0.51}, \quad 3.62^{+0.50}_{-0.48}$$

(CMB) (CMB+BAO) (CMB+$H_0$)
Effective neutrino number $N_{\text{eff}}$

- Observations from Planck (95%):  
  \[ N_{\text{eff}} = 3.36^{+0.68}_{-0.64}, \quad 3.30^{+0.54}_{-0.51}, \quad 3.62^{+0.50}_{-0.48} \]
  
  (CMB) \quad (CMB+BAO) \quad (CMB+H_0)

  Dark radiation is hinted, while tension between $H_0$ measurement and CMB/BAO.

See also [Hamann and Hasenkamp]: $N_{\text{eff}} = 3.66 \pm 0.30$ (1σ)  
(CMB+HST+C+BAO+WL)
Axions in string theory:
Dark radiation candidates
QCD axion: Strong CP and CDM

\[ \mathcal{L} \supset \frac{a}{32\pi^2 f_a} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu} G_{\nu\rho} \]

**Ultralight** \( \sim 10^{-6} \text{ eV} \) for \( f_a = 10^{12} \text{ GeV} \).

Original motivations:

- A solution of strong CP problem: \( \theta < 10^{-10} \)
- A candidate of CDM
Motivation for string theory

Unified theory including quantum gravity!

Gauge

Matter

Gravity

D-brane

$A_\mu$

$g_{\mu\nu}$

$\psi_{\text{matter}}$
Axions in string theory

- Axions via compactifications:

\[ a^i = \int_{\Sigma} C_n \]

\(C_n\): n-form gauge field for strings/branes.

They can be ultralight and very weak

- shift symmetry: \( a \rightarrow a + \delta \);
- solution of CP and/or CDM with \( f_a \sim M_{\text{string}} \) or \( M_P \).
Moduli

Reheating field:

• **String moduli:**

\[ \Phi^i = \text{Vol}(\Sigma^i_n) \]

**Long lifetime: Light + 1/M_{Pl}**

(if SUSY)

• Other possibilities:
  – Inflaton (also in non-SUSY if coupled to axions)
  – Open string state, e.g., SUSY-breaking fields
Moduli oscillation

The decay = Reheating + dark radiation.

\[ \Phi \rightarrow a \]

\[ \Phi \rightarrow A_\mu, \; H, \ldots \]
Moduli problems

• Non-SUSY moduli: $m_\Phi \leq m_{3/2} + \text{light axion}$

Moduli-Induced Axion problem:

$$\Delta N_{\text{eff}} \gg 0.1 : \Phi \rightarrow a.$$  

• SUSY moduli: $m_\Phi > m_{3/2}$

Moduli-induced gravitino problem:

$$\Omega_{DM} \gg 0.1 : \Phi \rightarrow \psi_{3/2} \rightarrow \chi_{DM}$$
Moduli-Induced Axion Problem;
moduli decay modes

We will use 4D N=1 SUGRA. 
\((M_{Pl} = 1)\)
Axion production via the kinetic term

\[ \mathcal{L}_{\text{kin.}} = K_{TT\bar{T}}(\partial_\mu a)^2; \quad K_T = \partial_T K \]

\[ \langle K_{TT\bar{T}} \rangle \Phi (\partial_\mu a)^2 + \cdots . \]

Moduli-axions coupling exists in general!

Φ: Moduli; K: invariant under δT = iα:

\[ T = \Phi + i\alpha; \quad K = K(T + T^\dagger) \]
The decay fractions of $\Phi$

$$\Gamma_a \equiv \Gamma(\Phi \rightarrow aa) = \frac{m_{\Phi}^3}{64\pi} \frac{K_{TT\bar{T}}^2}{K_{TT\bar{T}}^3}$$
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$$\Gamma(\Phi \rightarrow \text{radiation}) \sim \frac{m_\Phi^3}{4\pi}.$$
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\[ \Gamma(\Phi \rightarrow \text{radiation}) \sim \frac{m_{\Phi}^3}{4\pi} \]

$K \supset Z_{GM} H_u H_d$

$W \supset f_{\text{vis}} W^\alpha W_\alpha$

$\Gamma(\Phi \rightarrow HH) \sim \frac{m_{\Phi}^3}{8\pi} \frac{(\partial_T Z_{GM})}{Z_{H_u} Z_{H_d}}$,

\[ \Gamma(\Phi \rightarrow A_\mu A_\mu) \sim N_g \frac{m_{\Phi}^3}{128\pi} \frac{|\partial_T f_{\text{vis}}|^2}{\text{Re}(f_{\text{vis}})^2 K_{TT\bar{T}}} \]
The decay fractions of $\Phi$

$$\Gamma_a \equiv \Gamma(\Phi \rightarrow aa) = \frac{m_\Phi^3}{64\pi} \frac{K_{TT\bar{T}}^2}{K_{TT\bar{T}}^3}$$

$$\Gamma_a \sim \Gamma(\Phi \rightarrow \text{radiation}) \sim \frac{m_\Phi^3}{4\pi}.$$ 

$$\Delta N_{\text{eff}} = O(1)!$$

$$\therefore \rho a \sim \rho_{\text{radiation}} \sim \rho_\nu \text{ after moduli decay}$$
Constraint on decay widths of $\Phi$

$T_R = (1 - B_a)^{1/4} \left( \frac{\pi^2 g_*}{90} \right)^{-1/4} \sqrt{\Gamma_{\text{total}} M_P}$

$\Delta N_{\text{eff}} > 0.84$ ruled out:

$\text{Br}(\Phi \to 2a) > O(0.1)$

or

$T_R < O(1) \text{ MeV}$

[TH, Nakayama, Takahashi]
Two examples:
The problem and solution
1. No-scale a la Large Volume Scenario

The SM

Swiss-cheese Calabi-Yau

\[ K = -3 \log \left[ T + T^\dagger - \frac{1}{3} \left\{ |H_u|^2 + |H_d|^2 + (\bar{\nu} H_u H_d + \text{h.c.}) \right\} \right] + \cdots \]

\[ W = W_0; \quad f_{\text{vis}} = \text{const.} \]

\[ \Phi = \text{Re}(T) : \text{Volume modulus}; \quad a = \text{Im}(T) : \text{Axion}. \]
1. No-scale a la Large Volume Scenario

\[ \log(T^{3/2}) \sim 2\pi \xi \sim 10 \]

\[ m_{3/2} \sim \frac{1}{T^{3/2}}; \quad m_\Phi \sim \frac{1}{T^{9/4}}; \quad m_{\text{soft}} \sim \frac{1}{T^3} \]

\[ \sim 10^{11} \text{GeV}; \quad \sim 10^7 \text{GeV}; \quad \sim 1 - 10 \text{TeV}. \]

\[ K = -3 \log \left[ T + \bar{T} - \frac{1}{3} \left\{ |H_u|^2 + |H_d|^2 + (\bar{\phi}H_uH_d + h.c.) \right\} \right] + \cdots \]

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1. No-scale a la Large Volume Scenario

\[ B_a \equiv \text{Br}(\Phi \rightarrow 2a) = \frac{1}{2z^2 + 1}. \]

\[ \Gamma(\Phi \rightarrow 2a) = \frac{1}{48\pi M_P^2} m_\Phi^3, \quad \Gamma(\Phi \rightarrow HH) = \frac{2z^2 m_\Phi^3}{48\pi M_P^2}. \]

\[ \mathcal{L} = \frac{z}{\sqrt{6}} (\partial^2 \Phi) H_u H_d + \frac{2}{\sqrt{6}} \Phi (\partial a)^2. \]

\( \Phi = \text{Re}(T) \): Volume modulus; \( a = \text{Im}(T) \): Axion.
$\Delta N_{\text{eff}}$ in $(m_\Phi, z)$

$\Delta N_{\text{eff}} \sim 0.2$: $z \sim 3$

or

Many Higgses

(MSSM extension)
Mass and momentum

• $m_a \sim e^{-2\pi T} \sim 10^{-100000}$ eV,

• Decoupled from the SM: $f_a \geq M_{Pl}$

• $P_a \sim 0.1 - 1$ keV (today)
Implication in string theory?

\[ \mathcal{L} = z(\partial^2 \Phi) H_u H_d \]

\[ \Delta N_{\text{eff}} \sim 0.2 \]

\( z \sim 3: \) Strong bulk-brane coupling.

\( z = 1 \) for a Gauge-Higgs Unification

Many intersections (~ 6-9) among D-branes?
2. Ex-dim with two holes

Calabi-Yau

$T_0$ $T_1$ $T_2$

NP (D7/E3) on $T_1 + nT_2$

The SM (D7)

NP on the bulk

$K_{\text{moduli}} = -2 \log(\mathcal{V}); \quad \mathcal{V} = (T_0 + T_0^\dagger)^{3/2} - \kappa_1 (T_1 + T_1^\dagger)^{3/2} - \kappa_2 (T_2 + T_2^\dagger)^{3/2},$

$W_{\text{moduli}} = W_0 + Ae^{-\alpha T_0} + Be^{-\beta(T_1 + nT_2)}, \quad \alpha = \frac{2\pi}{N}, \quad \beta = \frac{2\pi}{M}.$
2. Ex-dim with two holes

\[ T = nT_1 - T_2 : \text{QCD Axion!} \]

Calabi-Yau

\[ \int d^2 \theta T_2 \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{c.c.} \]

The SM (D7)

NP (D7/E3) on \( T_1 + nT_2 \)

NP on the bulk

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K_{\text{moduli}} = -2 \log(\mathcal{V}); \quad \mathcal{V} = (T_0 + T_0^\dagger)^{3/2} - \kappa_1 (T_1 + T_1^\dagger)^{3/2} - \kappa_2 (T_2 + T_2^\dagger)^{3/2} ,
\]

\[
W_{\text{moduli}} = W_0 + A e^{-\alpha T_0} + B e^{-\beta (T_1 + nT_2)} , \quad \alpha = \frac{2\pi}{N}, \quad \beta = \frac{2\pi}{M}.
\]

[Choi, Jeong]
2. Ex-dim with two holes: Stabilization

\[ T = nT_1 - T_2 : \text{QCD Axion!} \]

- KKLT stabilization: \( D_{T_0} W \simeq D_{T_1 + nT_2} W \simeq \partial_T K \simeq 0 \).

\[
K_{\text{moduli}} = -2 \log(\mathcal{V}); \quad \mathcal{V} = (T_0 + T_0^\dagger)^{3/2} - \kappa_1 (T_1 + T_1^\dagger)^{3/2} - \kappa_2 (T_2 + T_2^\dagger)^{3/2},
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2. Ex-dim with two holes: Stabilization

\[ T = nT_1 - T_2 : \text{QCD Axion!} \] [Choi, Jeong]

- KKLT stabilization: \( D_{T_0} W \simeq D_{T_1 + nT_2} W \simeq \partial_T K \simeq 0. \)

- A sequestered uplifting: \( m_s \simeq \sqrt{2} m_{3/2}; \ m_\tilde{a} \simeq m_{3/2}. \)

\[ T = s + ia + \theta \tilde{a} \] with \( f_a \sim M_{\text{string}} \sim M_{\text{GUT}}. \)

\[
K_{\text{moduli}} = -2 \log(\mathcal{V}); \quad \mathcal{V} = (T_0 + T_0^\dagger)^{3/2} - \kappa_1 (T_1 + T_1^\dagger)^{3/2} - \kappa_2 (T_2 + T_2^\dagger)^{3/2},
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\]
2. Ex-dim with two holes: Stabilization

\[ T = nT_1 - T_2 : \text{QCD Axion!} \]

- KKLT stabilization: \( D_{T_0} W \approx D_{T_1 + nT_2} W \approx \partial_T K \approx 0. \)

- A sequestered uplifting: \( m_s \approx \sqrt{2} m_{3/2}; \ m_{\tilde{a}} \approx m_{3/2}. \)

\[ T = s + ia + \theta \tilde{a} \quad \text{with} \quad f_a \sim M_{\text{string}} \sim M_{\text{GUT}}. \]

- Mirage type soft masses: \( m_{\text{soft}} \approx \frac{m_{3/2}}{4\pi^2} \sim 1 - 10 \text{TeV}. \)

\[ K_{\text{moduli}} = -2 \log(V); \quad V = (T_0 + T_0^\dagger)^{3/2} - \kappa_1 (T_1 + T_1^\dagger)^{3/2} - \kappa_2 (T_2 + T_2^\dagger)^{3/2}, \]

\[ W_{\text{moduli}} = W_0 + Ae^{-\alpha T_0} + B e^{-\beta (T_1 + nT_2)}; \quad \alpha = \frac{2\pi}{N}, \quad \beta = \frac{2\pi}{M}. \]
2. Ex-dim with two holes: Decay

[TH, Nakayama, Takahashi]

- Saxion-decay into QCD axions; can be suppressed!

\[ \mathcal{L} = K_{TT\bar{T}} \xi_s (\partial a)^2; \]

\[ K_{TT\bar{T}} \propto (n^3 \kappa_1^2 - \kappa_2^2) \]

\[ \Gamma_a \approx \frac{(n^3 \kappa_1^2 - \kappa_2^2)^2}{768\pi \kappa_2^3} \frac{M_S^3}{M_P^2}; \]

\[ m_s \approx \sqrt{2} m_{3/2} \sim 100 \text{ TeV}; \]
2. Ex-dim with two holes: Decay

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\[ \Gamma_a \approx \frac{(n^3 \kappa_1^2 - \kappa_2^2)^2}{768\pi \kappa_2^3} \frac{M_S^3}{M_P^2} \]
\[ \Gamma(s \rightarrow gg) + \Gamma(s \rightarrow \tilde{g}\tilde{g}) \approx N g \frac{\kappa_2 M_S^3}{32\pi M_P^2} \]
\[ m_s \approx \sqrt{2}m_{3/2} \sim 100 \text{TeV}; \quad N_g = 12 \text{ for the MSSM.} \]
Mass and momentum if exists

- $m_a \sim 10^{-10} \text{ eV}$,

- $f_a \sim M_{\text{GUT}} \sim 10^{16} \text{ GeV}$.

- $P_a \sim 0.1 - 1 \text{ keV (today)}$
2. Ex-dim with two holes: Decay

- Saxion-decay into QCD axions; can be suppressed!

\[ \mathcal{L} = K_{TT\bar{T}} s \left( \partial a \right)^2; \]

\[ K_{TT\bar{T}} \propto \left( n^3 \kappa_1^2 - \kappa_2^2 \right) \]

\[ \Gamma_a \approx \left( \frac{n^3 \kappa_1^2 - \kappa_2^2}{768\pi \kappa_2^3} \right) \frac{M_S^3}{M_P^2} \]

\[ \Gamma(s \rightarrow gg) + \Gamma(s \rightarrow \tilde{g}\tilde{g}) \approx N_g \frac{\kappa_2}{32\pi} \frac{M_S^3}{M_P^2} \]

\[ m_s \approx \sqrt{2}m_{3/2} \sim 100\text{TeV}; \quad N_g = 12 \text{ for the MSSM.} \]
Symmetric Ex-dim: \( T_1 \Leftrightarrow T_2 \)

\[ \Delta N_{\text{eff}} < 0.1 \]
Conclusion
Cosmological test for string models

Axionic dark radiation (= relativistic DM): \( \Delta N_{\text{eff}} = O(0.1) \)
Key: Moduli problem in reheating

\[ \Phi \rightarrow aa \]

\[ \Phi: \text{Moduli/Inflaton} \quad a: \text{Axion} \]

Axionic dark radiation exists even for \( m_\Phi \gg 100\text{TeV} \).
Discussions on axion mass

Shift symmetry or $U(1)_{PQ}$ can be broken (to $Z_N$) by

• Flux compactifications/torsional geometry
• Stringy instantons (NOT QCD) – Light axion might appear, e.g., when

Adjoint states:
$$h^{1,0}(S), \ h^{2,0}(S) \neq 0$$

or

Many chiral states:
$$\text{Index}(\mathcal{D}) \gg 1.$$
Thank you!
Backup
Constraint on axion dark radiation

- Axion-photon conversion in the early universe

\[ \mathcal{L} = -\frac{1}{4} g_a a F_{\mu\nu} \tilde{F}^{\mu\nu} \]

Dark radiation = (Non-)QCD axion with large p

[1306.6518] [TH, Nakayama, Takahashi]
Axion-photon conversion

\[ \mathcal{L} = -\frac{1}{4} g_{\alpha\alpha} F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{\alpha} a \vec{E} \cdot \vec{B} \]

Axions mix with photons in the presence of magnetic field.

\[ M_{ij}^2 = \begin{pmatrix} \omega_p^2 & -g_{\alpha} B E \\ -g_{\alpha} B E & m_a^2 \end{pmatrix} \]

\[ \omega_p = \sqrt{\frac{4\pi\alpha n_e}{m_e}} \simeq 2 \times 10^{-14} \text{ eV} \ (1 + z)^{3/2} X_e^{1/2} \quad : \text{Plasma frequency} \]

\[ E \quad : \text{Axion energy} \]

Resonant and non-resonant conversion take place.

Yanagida and Yoshimura '88, Sikivie '83

The conversion rate depends on \( B_0 \).
Intergalactic magnetic field

Allowed region

\( B_0 = 10^{-16} \sim 10^{-9} \text{ G} \)

Durrer and Neronov 1303.7121
Typically, $g_a \lesssim 10^{-16} \text{ GeV}^{-1} \left( \frac{B_0}{1 \text{ nG}} \right)^{-1}$ for $\Delta N_{\text{eff}} = \mathcal{O}(0.1)$.
Radiation production (Main)

• $\Phi$ decays into Higgs via Giudice-Masiero-term:

$$K \supset Z_u|H_u|^2 + Z_d|H_d|^2 + g(T + T^\dagger) (H_uH_d + \text{h.c.})$$

$$\Gamma(\Phi \rightarrow HH) = \frac{m^3_\Phi}{8\pi} \frac{g^2_T}{Z_uZ_dK_{TT}}$$

• $\Phi$ decays into gauge fields via gauge coupling:

$$\int d^2\theta f_{\text{vis}} \mathcal{W}^\alpha \mathcal{W}_\alpha + c.c.$$  \hspace{1cm} N_g = 12 \text{ for the MSSM.}$$

$$\Gamma(\Phi \rightarrow A_\mu A_\mu) = N_g \frac{m^3_\Phi}{128\pi} \frac{\left|\partial_T f_{\text{vis}}\right|^2}{\left[\text{Re}(f_{\text{vis}})\right]^2 K_{TT}}$$
Solutions

• Suppress the Φ-a coupling:
  – Many visible modes/ geometry/ just open string axions

• Make (all closed string) axions massive:
  – $U(1)_A$ stückelberg coupling/ NP effects

• Change the final reheating field:
  – No moduli oscillation/ additional entropy production

Because axions are interesting:
  – A solution of strong CP and/or a CDM candidate
  – Ubiquitous in the 4D string vacua.
In Large Volume Scenario

Dark matter: Wino

for $m_{\text{soft}} = O(1-10) \text{ TeV}$

(With assumed R-parity)
Modulus decay

\[ \phi_1 \rightarrow H_u, H_d \]

\[ Br = O(1) \]
Modulus decay into Wino DM

$$\phi_1 \rightarrow H_u \rightarrow \tilde{W}^0 \tilde{h}_u$$

$$H_d \rightarrow O(10^{-3})$$
These process hardly depends on the branching fraction ($> 10^{-5}$).
\[ \Omega_{\text{wino}} h^2 \text{ (with } n_H = 1) \]

\[ (\Omega_{\text{CDM}} h^2)^{\text{obs}} \sim 0.1 \]

For \( z \sim 3 \), \( \Delta N_{\text{eff}} \sim 0.2 \)

\[ m_{\text{Wino}} \sim 700 \text{ GeV} \]

\[ m_{\tilde{W}} = 1/(\log(\mathcal{V}) \mathcal{V}^2) \]
Constraint on Wino-like DM mass

$700\text{GeV} \sim m_{\text{Wino}} < \mu \sim z \cdot m_{\text{Wino}}$. 

Hisano, Ishiwata, Nagata (2012)