Superfluid current disruption in a chain of weakly coupled Bose–Einstein condensates

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Abstract. We report the experimental observation of the disruption of the superfluid atomic current flowing through an array of weakly linked Bose–Einstein condensates. The condensates are trapped in an optical lattice superimposed on a harmonic magnetic potential. The dynamical response of the system to a change of the magnetic potential minimum along the optical lattice axis goes from a coherent oscillation (superfluid regime) to a localization of the condensates in the harmonic trap (‘classical’ insulator regime). The localization occurs when the initial displacement is larger than a critical value or, equivalently, when the velocity of the wavepacket’s centre of mass is larger than a critical velocity dependent on the rate of tunnelling between adjacent sites.

Atomic Bose–Einstein condensates have been either loaded or produced in periodic potentials opening up the possibility of investigating new phenomena tuning the degree of coherence in the system. Experiments have explored regimes ranging from the coherent matter wave emission from a condensate loaded on a vertical standing wave [1], to the observation of number squeezed states [2], the demonstration of a one-dimensional Josephson junction array with a linear chain of condensates produced in an optical lattice [3], and the recent observation of a quantum phase transition in a condensate loaded in a 3D optical lattice [4]. The dynamical behaviour of coherent matter waves in periodic potentials has also been the subject of extensive

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theoretical work. Different phenomena have been predicted including the formation of bright solitons and the appearance of various forms of instability where the usual long-range phase coherence properties are lost [5]–[11].

In this paper we report on the experimental observation of the disruption of superfluidity in a linear array of $^{87}$Rb condensates trapped in the combined potential of an optical standing wave superimposed on a harmonic magnetic trap.

We observe a transition from a regime in which the wavepacket coherently oscillates in the array to another one in which the condensates stop in the optical potential sites and lose their relative phase coherence. We also observe the appearance of an intermediate regime where no oscillations are visible while the relative phase coherence is not completely lost. In a previous experiment we have investigated the regime of small periodic potential height, showing that in this regime and for large atom numbers, the mechanism for superfluidity breakdown was in agreement with predictions based on the onset of the Landau instability [12]. In the present experiment we explore the dynamics of the system in the case of a large periodic potential height, where a modulational instability is expected to play a fundamental role [8, 9]. Furthermore, one can study the overall coherence of the system, observing the phase evolution in the interferogram of the expanded array of condensates.

The experimental set-up has already been described in [3]: a dilute vapour of $^{87}$Rb atoms confined in a Ioffe-type magnetic trap in the $(F = 1, m_F = -1)$ state is Bose condensed by rf evaporative cooling. Slightly above the condensation threshold, we superimpose on the harmonic magnetic trap a 1D optical standing wave. We then continue the evaporation process through the phase transition temperature. The magnetic trap has a cylindrical symmetry, with frequencies $\omega_z = 2\pi \times 9$ Hz and $\omega_r = 2\pi \times 90$ Hz along the axial and radial directions, respectively.

The optical standing wave is superimposed on the magnetic potential, along the $z$-axis, and is created with a retroreflected collimated laser beam, detuned $\sim 3$ nm to the blue of the rubidium $D_1$ transition at $\lambda = 795$ nm. The resulting periodic potential is $V_L = V_0 \cos^2(2\pi z/\lambda)$: the valleys of the potential are separated by $\lambda/2$ and the interwell energy barrier $V_0 = s E_R$ can be controlled by varying the intensity of the laser beam. $E_R = h^2/(2m \lambda^2)$ is the kinetic energy of an atom of mass $m$ recoiling after absorbing one lattice photon. Typically, in our experiments, $s$ ranges from 3 to 14 $E_R$; for such values, the condensate chemical potential $\mu$, that ranges from 1.8 to 3.7 $E_R$, is smaller than $V_0$ and the system realizes an array of weakly coupled condensates driven by an external harmonic field.

We drive the system out of equilibrium by a sudden displacement ($t_{dis} \ll 2\pi/\omega_z$) of the magnetic potential along the lattice axis. The ensuing dynamics is revealed by turning off, after different evolution times, both the magnetic and optical traps and imaging the atomic density distribution after an expansion time $t_{exp} = 27.8$ ms. During the expansion the different condensates, originally located at the nodes of the laser standing wave, overlap and interfere allowing us to monitor their relative phases [13].

For small displacements we observe a superfluid regime that has been extensively studied both at zero temperature [3] and in the presence of a significant thermal fraction [14]. The condensates released from the traps show an interference pattern, consisting of three peaks, separated by a distance $d = 2ht_{exp}/m \lambda$, oscillating in phase, thus revealing the long-range coherence of the condensates across the entire optical lattice.

When we increase the displacement of the magnetic trap centre for a fixed height of the optical barriers we observe a different behaviour: the centre of mass of the atomic sample no longer oscillates but slowly moves towards the centre of the magnetic potential where it
subsequently stops. This is shown in figure 1 where we report the evolution of the centre-of-mass position of the atomic cloud inside the combined trap for a barrier height of $5 \, E_R$ and displacements of 30 and 120 $\mu$m. We reconstruct the centre-of-mass motion in the trap from the images of the expanded cloud assuming ballistic expansion. We take data points at intervals of time small enough to allow us to assume that the condensate is moving in the combined trap with constant velocity during each time interval.

For the larger displacement we also observe a vanishing of the visibility of the interference peaks demonstrating the loss of long-range coherence across the array.

In figure 2 we report the evolution of the axial width $R_z$ of the interferogram central peak normalized to the peak separation $d$ for the same conditions as in figure 1. While in the small-amplitude coherent oscillation the axial width remains constant, in the larger-displacement case
it spreads out reaching the peak separation in $\sim 120$ ms when the interferogram is completely washed out. We remark that the vanishing of the interferogram is a dynamical effect and is not due to scattering of lattice photons. Indeed in the absence of displacement or even for small displacements we do not observe a deterioration of the interference contrast on the timescale of the experiment, as shown in the right part of figure 2.

We have studied the crossover between these two dynamical regimes, repeating the experiment varying the displacement for different optical lattice heights $V_0$. In figure 3 we summarize the experimental results: the closed circles correspond to values of displacement and lattice depth for which we observe a coherent oscillation; the open circles identify conditions where we have the destruction of the oscillation and a slow, overdamped, motion toward the centre of the magnetic trap. In the latter case, only for very large displacements do we observe a complete vanishing of the interferogram.

We compare our results with the prediction for the onset of the dynamical instability recently proposed in [8] using the one-dimensional discrete non-linear Schrödinger equation (DNLS) model [5]. This treatment gives an analytical expression for the ‘critical’ velocity $v_{cr}$ for entering the dynamical instability regime. The 1D system is predicted to undergo a sudden transition from a regime with coherent oscillations (superfluid regime) to one with pinning (insulator regime) when the phase difference between adjacent condensates $\Delta \phi(t)$ reaches $\pi/2$. In this case the relative phases start to run independently with different velocities, the phase coherence through the array is lost and the system behaves as an insulator. The system reaches this regime for a critical velocity

$$v_c = \frac{K \lambda}{\hbar}$$

(1)

where $K$ is proportional to the rate of tunnelling between adjacent sites of the optical potential. In a harmonic potential this corresponds to a critical displacement:

$$\Delta z_{cr} = \frac{\lambda}{2\sqrt{2K/\Omega}}$$

(2)

where $\Omega = \frac{1}{2}m\omega_z^2(\lambda)^2$ describes the magnetic potential energy of the condensates. Since the tunnelling rate $K$ depends on $V_0$, the critical displacement should depend on the height of the interwell potential: the greater $V_0$ is, the lower the tunnelling rate and the critical displacement are.

In figure 3 we report the DNLS prediction for the critical displacement (continuous curve5). The theoretical curve divides well the two dynamical regions observed in the experiment.

From the experimental data (open circles in figure 3) we can also extract the maximum velocity reached by the system before entering the instability regime. In figure 4, we show the experimental maximum velocities measured as a function of the optical potential height, in qualitative agreement with the theoretical prediction of equation (1) (continuous curve).

We believe that the 1D model gives a reliable estimate for the onset of the loss of superfluidity; however, it does not quantitatively describe the degradation of the interferogram in the present experimental set-up. In fact, in the experiment, for values of displacement just above the predicted $\Delta z_{cr}$ we could not observe a complete disruption of the interferogram, but we saw the appearance of more complex structures within the three visible peaks. We believe

5 The critical displacement has been obtained from equation (2) using for the parameter $K$ the results of a numerical simulation provided by A Trombettoni.
that this could be due to the excitation of radial modes that are obviously not included in the simple 1D model. A possible way to include such modes is to resort to a full numerical solution of the 3D Gross–Pitaevskii equation [15].

In conclusion, we have experimentally observed the transition between different regimes in an array of Josephson junctions realized with BECs trapped in a one-dimensional optical lattice. The transition occurs between a superfluid and an insulator regime and is accompanied by a loss of coherence through the array even though each condensate in the array is still described by a coherent state. The experimental findings have been compared with the prediction of a one-
dimensional theoretical model [8] based on a DNLS where the transition is due to a dynamical instability taking place when the eigenfrequencies of the excitation spectrum become imaginary. This model is found to qualitatively describe the instability onset which occurs at a critical value of the displacement or, equivalently, when the velocity of the wavepacket’s centre of mass is larger than a critical velocity dependent on the rate of tunnelling through the optical barriers. We have also observed that close to the transition the 1D model fails to describe the experimental results where the system behaves like an insulator but coherence is still present through the array. We attribute this behaviour to the excitation of radial modes that would require a full 3D simulation of the GPE.

A quantitative comparison between the critical velocity for the dynamical instability and the speed of sound in the presence of the optical lattice in the regime of high optical lattice is still lacking as far as we are aware. We think that it will prove very helpful for shedding light on the role of these two instability regimes already discussed for the case of small lattice height in [9].

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