A simplified method for the evaluation of the layer compression test using one 3D digital image correlation system and considering the material anisotropy by the equibiaxial Lankford parameter

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Abstract. The layer compression test is an established method for the characterization of sheet metal behavior under equibiaxial tension. Due to its ability to achieve high strains without necking and the simple experimental setup, the test is a welcome alternative to the more complex Bulge test for flow curve determination. In addition, the layer compression test provides further data for the calibration of yield loci models. However, the evaluation of the test is not trivial, since a tactile extensometer measurement incorporates the error of closing gaps between single sheet layers. Moreover, the sheet metal anisotropy is neglected. To overcome the above-mentioned drawbacks, a measurement and evaluation methodology based on the optical strain measurement of two orthogonally positioned 3D digital image correlation (DIC) systems was introduced recently. The method delivers reliable results but increases the complexity and costs of the test. Here, a simplification to this method is proposed. By the usage of only one 3D DIC system, the major in-plane strain is acquired from an online measurement. The minor in-plane strain is calculated posteriori by the consideration of the biaxial Lankford parameter. In this way, the anisotropy and thus the correct flow behavior can be studied by only one 3D DIC system. The proposed method is evaluated and discussed.

1. Introduction

The finite element analysis (FEA) is an indispensable tool in the field of virtual product development. It allows to assess the formability of sheet metal parts far before prototyping. In this way, technical and economic risks can be identified and eliminated already in the digital phase of the manufacturing process. That keeps the development costs low and reduces the necessary tool reworking steps to a minimum. However, the prediction quality of forming simulations is determined by the quality of the underlying constitutive models [1]. Furthermore, the experimental characterization and model parameter identification has a significant influence on the expected reliability of the simulation models [2].

The yield locus as well as the hardening curve are necessary models to describe the plastic behavior of metals. In metal forming and other applications where high strains are expected, the flow curve gained from a uniaxial tensile test is insufficient to fully cover the relevant strain range. To characterize the materials flow behavior beyond the point of uniform elongation in the uniaxial tensile test, many tests have been proposed in the past as the Bulge test [3], different types of shear tests [4],[5] and the layer compression test [6]. The latter test has gained an increased interest over the last years as it provides further data for the calibration of higher-order yield loci models, such as the yield stress and...
the Lankford parameter under equibiaxial tension. However, the evaluation of the test is not trivial, since a tactile strain measurement incorporates the error of closing gaps between single sheet layers. The same applies for the measurement of the machine crosshead displacement which is additionally influenced by the machine stiffness. Furthermore, in both cases the sheet metal anisotropy is neglected.

Merklein and Kuppert [7] introduced a measurement and evaluation methodology based on the optical strain measurement of two orthogonally positioned 3D DIC systems to overcome the typical drawbacks of the layer compression test. The method presented in [7] delivers reliable results but increases the complexity and costs of the test at the same time. Besides that, many laboratories are not equipped with several DIC systems.

Within this contribution a simplification to this method is proposed. By the usage of only one 3D DIC system, the major strain is acquired from an online measurement. The minor strain is calculated posteriori by the consideration of the Lankford parameter under equibiaxial tension. The Lankford parameter itself is obtained by a measurement of the final diagonals of each sheet disc as proposed by Barlat et al. [8]. In this way, the anisotropy and thus the correct flow behaviour can be studied by only one 3D DIC system. The proposed method is evaluated and discussed using an exemplary aluminium alloy sheet material with a pronounced anisotropy.

2. The layer compression test and its evaluation methods

2.1. Basic test principle
The layer compression test has not yet been standardized, but in many cases it is based on the DIN 50106 [9] standard for compression tests of solid cylinder specimens. Initially introduced by Pawelski [6] in 1967, the layer compression test reassembles a cylinder geometry by single sheet metal discs stacked on each other as shown in figure 1.

![Figure 1. Layer compression specimen before and after compression. Assuming isotropic material behaviour, the initial diameter \(d_0\) increases uniformly while the stack height \(h_0\) decreases [5].](image)

Under the assumption of isotropic material behavior, isochoric plastic flow and frictionless contact between the specimen surfaces and contact plates, the Cauchy stress tensor under uniaxial compression reads

\[
\sigma_{\text{uni.comp.}} \equiv \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -k_f \end{bmatrix},
\]

whereas the Cauchy stress tensor for equibiaxial tension is equal to

\[
\sigma_{\text{biax.tens.}} \equiv \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} = \begin{bmatrix} k_f & 0 & 0 \\ 0 & k_f & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\]

Since the hydrostatic part of the stress tensor has no influence on the plastic flow, the flow is controlled by the deviatoric part alone. An indifference between the stress deviator of both the uniaxial compression
given by equation (1) and the equibiaxial tension given by equation (2) may be evidenced through an additive decomposition of both stress tensors as shown in equation (5) and equation (7).

\[ \sigma = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{zy} & \sigma_z \end{bmatrix} = \sigma^{\text{dev}} + \sigma^{\text{vol}} \]  

(3)

\[ \sigma^{\text{vol}}_{\text{uni.com.}} = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) \mathbf{I} = k_f \cdot \begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} \]  

(4)

\[ \sigma^{\text{dev}}_{\text{uni.com.}} = \sigma_{\text{uni.com.}} - \sigma^{\text{vol}}_{\text{uni.com.}} = k_f \cdot \begin{bmatrix} 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix} \]  

(5)

\[ \sigma^{\text{vol}}_{\text{biax.tens.}} = \sigma_{\text{biax.tens.}} = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) \mathbf{I} = k_f \cdot \begin{bmatrix} 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 \end{bmatrix} \]  

(6)

\[ \sigma^{\text{dev}}_{\text{biax.tens.}} = \sigma^{\text{dev}}_{\text{biax.tens.}} - \sigma^{\text{vol}}_{\text{biax.tens.}} = k_f \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = k_f \cdot \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]  

(7)

Under the assumption of isotropic and isochoric plastic flow the diameter of the stacked cylinder increases uniformly with the plastic flow. The true stress \( k_f \) and the thickness strain \( \varepsilon_3 \) follows with

\[ k_f = \frac{F}{A} \implies k_f(h) = \frac{F}{A_0} \cdot \frac{h}{h_0} = \frac{F}{(d_0/2)^2 \pi} \cdot \frac{h}{h_0} \quad \text{and} \]  

(9)

\[ \varepsilon_3(h) = \ln \frac{h_0}{h}. \]  

(10)

By making use of the relationships given above, the crosshead displacement of a universal testing machine is commonly used for the evaluation of stress and strain in the layer compression tests [10], [11]. Alternatively, as suggested by the DIN 50106, the strain can be determined by measuring either the displacement of the compression plates or the compression of the sample directly using a tactile extensometer. A major disadvantage of height-based measuring approaches is, however, that the setting behavior between the individual sheet metal discs is included in the measurement. That can lead to a significant error in the derived experimental results.

2.2. Anisotropic evaluation

Another way to determine the strain is a direct inline measurement of the sample diameter during the compression test. Thereby the measuring error caused by closing gaps between the individual sheet layers can be eliminated. In addition, both diagonals of the discs can be evaluated by using a second orthogonally aligned extensometer and thus a possible anisotropy in the plastic flow can be considered, see figure 2.
Such setup is used for example by Geese et al. [13]. By a system of four orthogonal tactile extensometer sets the surface increase of four discs within a stack are evaluated independently. For the strain and stress calculation the measurement of all four systems is averaged.

Merklein and Kuppert [7] introduced a measurement and evaluation methodology based on the optical strain measurement of two orthogonally positioned 3D digital image correlation (DIC) systems. By making use of the proportionality between the tangential strain measured on the specimen surface perpendicular to the rolling and transverse direction, $\varepsilon_1$ and $\varepsilon_2$ respectively, the diameter increase is calculated from equation (11) as introduced by the authors

$$A = \left(\frac{\exp(\varepsilon_1) + \exp(\varepsilon_2)}{2} \cdot \frac{d_0}{2}\right)^2 \cdot \pi .$$

The proportionality assumption between the tangential strain and the diameter increase is based on the elementary relationship between the circumference $C$ and the diameter $d$ of a circle $C/d = \pi$. If the circumferential strain is evaluated on a reasonably small section, the following applies $C_0/d_0 = C_i/d_i$. The method presented by the authors delivers reliable results but increases the complexity and costs of the test at the same time. Besides that, many laboratories are not equipped with several DIC systems, which further limits the dissemination of this effective but complex measurement method.

![Figure 2](Figure 2. Layer compression specimen before and after compression. The non-uniform diameter increase is related to an anisotropy in the plastic flow.)

2.3. Proposed method

As already thoroughly discussed in [7], the use of only one 3D DIC system for the evaluation of the layer compression test implies the assumption of an isotropic deformation, since no information about the orthogonal direction is available. This assumption may be true for materials with no pronounced anisotropy, but for highly anisotropic sheet materials such as aluminum, significant deviations may occur. The main issue here is the incorrect determination of the current cross-sectional area of the deformed specimen. Therefore, the calculated strain as well as the stress may show significant discrepancies from the actual values.

In order to enable a correct evaluation of anisotropic materials with only one 3D DIC system during a layer compression test, a simplification of the method proposed by Merklein and Kuppert [7] is introduced in this paper. By combining the strain data acquired from an inline DIC measurement perpendicular to the rolling direction, $\varepsilon_1$, the strain in the transverse direction, $\varepsilon_2$, is calculated from a relationship between both values $\varepsilon_2 = f(\varepsilon_1)$. The functional relationship is obtained from post-deformation measurements of single - now elliptic - sheet metal discs in analogy to the evaluation method of the biaxial Lankford parameter described by Barlat et al. [8]. With this, the actual shape of the cross section can be incorporated into the calculation of strains and stresses. The methodology is presented in figure 3.
Figure 3. Proposed methodology for the evaluation of the layer compression test: a) Layer compression test with one 3D DIC system perpendicular to the rolling direction (RD), b) Posteriori evaluation of the functional relationship between $\varepsilon_1$ and $\varepsilon_2$, c) Calculation of the true stress from the elliptic cross-sectional area and the effective plastic stain under consideration of $\varepsilon_2 = f(\varepsilon_1)$.

By supplementing $\varepsilon_2$ in equation (11) with $\varepsilon_2 = f(\varepsilon_1) = r_b \cdot \varepsilon_1 + b$ equation (12) follows

$$A = \left(\frac{\exp(\varepsilon_1) + \exp(r_b \cdot \varepsilon_1 + b) \cdot d_0}{2}\right)^2 \pi,$$

whereby $r_b$ represents the Lankford parameter under equibiaxial tension and $b$ an offset factor. The stress is calculated from $\sigma = F/A$. Under the condition of isochoric flow, the effective von Mises plastic strain follows to

$$\bar{\varepsilon}_p,vM = \sqrt{\frac{2}{3} \left[ \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 \right]} = \sqrt{\frac{2}{3} \left[ \varepsilon_1^2 + (r_b \cdot \varepsilon_1 + b)^2 + (r_b \cdot \varepsilon_1 + b + \varepsilon_1)^2 \right]}.$$

3. Experimental procedures

3.1. Material and test setup

The studied material is a cold rolled AA6xxx aluminum alloy in W-temper with a nominal sheet thickness of 2.50 mm. The material is quenched between two cooled steel plates directly after solutionizing and tested within five minutes after quenching. In order to enable a quick conduction of the tests and to make the material easy to handle, the single discs of a diameter of 15 mm are pre-cut by laser and held in the sheet metal by a small rib. After quenching the single discs are removed from the sheet metal and layered to a stack of six discs with a height of 15 mm $(d/h = 1)$. The discs are positioned uniformly with respect to the rolling direction and held in position by a cyanoacrylate adhesive applied between the single layers. The experiments are performed at 25 °C on an electromechanically driven
MTS Criterion universal tensile testing machine with a maximal load capacity of 100 kN. In order to reduce the effect of friction a grease (DuPont Molykote) is applied as lubricant between the specimen faces and the compression plates. The strain fields at the specimen’s surface are accessed by a GOM Aramis 4M 3D DIC system positioned perpendicularly to the rolling direction. In addition, the displacements of both compression plates are tracked by three discrete points per plate. The force signal from the load cell is synchronized with the DIC system. The setup is presented in figure 4.

3.2. Evaluation
For comparison purposes the layer compression test will be exemplary evaluated by four different approaches:

- Isotropic evaluation by the averaged compression plate displacement by using equations (9) and (10).
- Isotropic evaluation by a digital extensometer applied between the second and the fifth disc and by using equations (9) and (10).
- Isotropic evaluation by using the averaged tangential surface strain of two discs acquired from the 3D DIC system by using equation (11) and assuming $\varepsilon_1 = \varepsilon_2$.
- Anisotropic evaluation by using the averaged tangential surface strain of two discs acquired from the 3D DIC system and the determined relationship $\varepsilon_2 = f(\varepsilon_1)$ by using equations (12) and (13).

The different evaluation approaches are depicted in figure 5.
4. Results

4.1. Lankford parameter at equibiaxial loading

Following the recommendations of Barlat et. al [9], the equibiaxial Lankford parameter $r_b$ is obtained by a measurement of the final diagonals of several distinctly compressed discs. The slope of the linear fit describing the relationship between the longitudinal $\varepsilon_1 (0^\circ$ with respect to the rolling direction) and transversal strains $\varepsilon_2 (90^\circ$ with respect to the rolling direction) gives the $r_b$-value, which is found to be 1.1734. The offset $b$ is found to be -0.0012. The dependency of $\varepsilon_2$ on $\varepsilon_1$ is depicted in figure 6 and given by equation (14)

$$\varepsilon_2 = 1.1734 \cdot \varepsilon_1 - 0.0012.$$  (14)

![Image of Figure 6](image_url)

**Figure 6.** Identified $r$-value at equibiaxial loading and the functional relationship between $\varepsilon_1$ and $\varepsilon_2$ obtained from layer compression experiments.

4.2. Stress-strain curves

The results of the four different evaluation approaches described in 3.2 are shown in figure 7. For both height-based methods ($\varepsilon_3$), a high level of correspondence between the stress-strain curves can be observed until a strain of $\approx 0.45$. Beyond this point both curves diverge and the extensometer-based result shifts progressively toward higher stresses. That indicates a stagnation of the locally measured strain and therefore leads, in accordance to equation (9), to higher stress values than those captured from the entire specimen height. In the case of results based on the tangential surface strain two main characteristics can be observed. First, the stiffness is higher compared to the height-based methods and the elastic regime corresponds to the Young’s modulus of aluminum ($\approx 70$ GPa). Secondly, the diameter-based methods give lower stresses whereby the isotropic evaluation converges at higher strains towards the solution based on the compression plate displacement. Both effects can be traced back to closing gaps between the individual layers with ongoing compression loading. The good correspondence of the isotropic diameter- ($\varepsilon_1$) and the isotropic height- ($\varepsilon_3$) evaluation at higher strains can be reasoned by a decreasing influence of the latter measurement by closing gaps at higher loads. The lower stress level of the anisotropic evaluation is caused by the correct incorporation of the higher cross-sectional area ($\varepsilon_1 < \varepsilon_2$) into the stress calculation. This in turn means, that in this case all isotropic evaluations ($\varepsilon_1 = \varepsilon_2$) overestimates the resulting stresses.
5. Conclusions
With the aim to increase the reliability of layer compression results evaluated by only one 3D DIC system, the standard evaluation method is enhanced by the materials anisotropic behavior. The consideration of the anisotropy is relying on the Lankford parameter which must be determined anyway for higher order yield loci models. Using the example of the investigated material, a non-negligible discrepancy between the isotropic and the anisotropic evaluation can be determined. This underlines the importance of the simple but nevertheless effective evaluation method proposed here. In addition, the yield stress at equibiaxial stress can be determined much more precisely than with a height-based evaluation method. In future work, the method should be compared with the results of an evaluation with two orthogonally positioned 3D DIC systems.

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