Lorentz and CPT Tests in Atomic Systems

Robert Bluhm

Physics Department, Colby College, Waterville, ME 04901 USA

Abstract. A review of Lorentz and CPT tests performed in atomic systems is presented. A theoretical framework extending QED in the context of the standard model is used to analyze a variety of systems. Experimental signatures of possible Lorentz and CPT violation in these systems are investigated. Estimated bounds attainable in future experiments and actual bounds obtained in recent experiments are given.

INTRODUCTION

Many of the shaptest tests of Lorentz and CPT symmetry have been performed in atomic systems \cite{1, 2}. Although many of these experiments were performed in earlier decades, a new appreciation of them has emerged due to recent theoretical advances. The group of Kostelecký and collaborators have developed a consistent theoretical framework \cite{3, 4, 5} that permits a detailed investigation of Lorentz and CPT tests in particle and atomic systems. In the context of this framework, it is possible to look for new signatures of Lorentz and CPT violation in atomic systems that were previously overlooked.

In the next section, I begin with a brief review of Lorentz and CPT symmetry and some of the atomic experiments that test these symmetries. This is followed by a discussion of some theoretical ideas that have been put forward over the years for ways in which Lorentz symmetry and CPT might be violated in nature. Different theoretical approaches to searching for Lorentz violation are also described. However, the main focus of this work will be on the standard-model extension of Kostelecký and collaborators, as it is this model which permits the

---

\footnote{Presented at Symmetries in Subatomic Physics, Adelaide, Australia, March 2000.}
most detailed investigation of Lorentz and CPT tests and is applicable across different experiments. The QED sector of this theory will be presented and used to examine five different atomic or macroscopic systems: experiments in Penning traps [3], clock-comparison tests [4], experiments with hydrogen and antihydrogen [5], Lorentz and CPT tests with macroscopic spin-polarized materials [6] and muon experiments [7].

As a result of our investigations, several atomic experimental groups have recently reanalyzed their data or taken new data to obtain improved bounds on Lorentz and CPT violation. I will present summaries of the bounds that have been obtained as well as estimates of bounds that can be attained in future experiments.

**Lorentz and CPT Symmetry**

It appears that Lorentz symmetry and CPT are exact fundamental symmetries of nature [11]. All physical interactions appear to be invariant under continuous Lorentz transformations consisting of boosts and rotations and under the combined discrete symmetry CPT formed from the product of charge conjugation \( C \), parity \( P \), and time reversal \( T \). These symmetries are linked by the CPT theorem [12], which states that under mild technical assumptions all local relativistic field theories of point particles are symmetric under CPT. The CPT theorem predicts that particles and antiparticles should have exactly equal lifetimes, masses, and magnetic moments.

Numerous experiments confirm CPT and Lorentz symmetry to extremely high precision. The best CPT test listed by the Particle Data Group [1] compares the masses of neutral \( K^0 \) mesons with their antiparticles and obtains the figure of merit,

\[
r_K = \frac{|m_{K} - m_{\bar{K}}|}{m_{K}} \lesssim 10^{-18}.
\]  

(1)

The Hughes-Drever type experiments are generally considered the best tests of Lorentz symmetry. These experiments place very tight bounds on spatially anisotropic interactions [13].

**Experiments in Atomic Physics**

Many of the sharpest tests of CPT and Lorentz symmetry are made in atomic systems. For example, the Hughes-Drever type experiments typically compare two clocks or high-precision magnetometers involving different atomic species. The best CPT tests for leptons and baryons cited by the Particle Data Group are made by atomic physicists using Penning traps. These experiments have obtained a bound on the \( g \)-factor difference for electrons and positrons given by [14]

\[
r_e = \frac{|g_{e} - g_{e^+}|}{g_{avg}} \lesssim 2 \times 10^{-12}.
\]  

(2)
while experiments with protons and antiprotons have obtained bounds on the difference in their charge-to-mass ratios given by \[15\]

\[ r_{q/m}^P = \frac{|(q_p/m_p) - (q_{\bar{p}}/m_{\bar{p}})|}{(q/m)_{\text{avg}}} \lesssim 9 \times 10^{-11} . \quad (3) \]

Similarly, two proposed experiments at CERN intend to make high-precision comparisons of the 1S-2S transitions in trapped hydrogen and antihydrogen \[16\]. The 1S-2S transition is a forbidden transition that can only occur as a two-photon transition. It has a long lifetime and a small relative linewidth of approximately \(10^{-15}\). Atomic experimentalists believe that ultimately the line center might be measured to 1 part in \(10^3\) yielding a CPT bound

\[ r_{1S-2S}^H = \frac{\left| \Delta \nu_{1S-2S} \right|}{\nu_{1S-2S}} \lesssim 10^{-15} - 10^{-18} . \quad (4) \]

It is interesting to note that of all the experiments testing CPT it is the atomic experiments which have the highest experimental precisions. For example, in the meson experiments quantities are measured with precisions of approximately \(10^{-3}\), while in the atomic experiments frequencies are typically measured with precisions of \(10^{-9}\) or better. Nonetheless, the figure of merit in Eq. \(1\) is many orders of magnitude better than those in Eqs. \(2\) or \(3\). These differences are due in part to the fact that these experiments compare different physical quantities, such as masses, g factors, charge-to-mass ratios, and frequencies. What would obviously be desirable is a framework which puts these experiments on equal footing and allows better cross comparisons.

### Ideas for Violation

Different ideas for violation of Lorentz or CPT symmetry have been put forward over the years since the proof of the CPT theorem. To evade the CPT theorem one or more of its assumptions must be disobeyed. A sampling of some of the theoretical ideas that have been put forward include the following: nonlocal interactions \[17\], infinite component fields \[18\], a breakdown of quantum mechanics in gravity \[19\], Lorentz noninvariance at a fundamental level \[20\], spontaneous Lorentz violation \[3\], and CPT violation in string theory \[4, 21\].

To explore the experimental consequences of possible Lorentz or CPT violation, a common approach is to introduce phenomenological parameters. Some examples include the anisotropic inertial mass parameters in the model of Cocconi and Salpeter \[22\], the \(\delta\) parameter used in kaon physics \[23\], and the TH\(\epsilon\mu\) model which couples gravity and electromagnetism \[24\]. Another approach is to introduce specific lagrangian terms that violate Lorentz or CPT symmetry \[25, 26\]. These approaches have the advantages that they are straightforward and are largely model independent. However, they also have the disadvantages that their relation to experiments can be unclear and they can have limited predictive ability. To make
further progress, one would want a consistent fundamental theory with CPT and Lorentz violation. This would permit the calculation of phenomenological parameters and the prediction of signals indicating symmetry violation. No such realistic fundamental theory is known at this time. However, a candidate extension of the standard model incorporating CPT and Lorentz violation does exist.

**STANDARD-MODEL EXTENSION**

The standard-model extension of Kostelecký and collaborators is an effective theory based on the idea of spontaneous Lorentz-symmetry breaking [3]. It is also motivated in part from string theory [4]. The idea is to assume the existence of a fundamental theory in which Lorentz and CPT symmetry hold exactly but are spontaneously broken at low energy. As in any theory with spontaneous symmetry breaking, the symmetries become hidden at low energy. The effective low-energy theory contains the standard model as well as additional terms that could arise through the symmetry breaking process. A viable realistic fundamental theory is not known at this time, though higher dimensional theories such as string or M theory are promising candidates. A mechanism for spontaneous symmetry breaking can be realized in string theory because suitable Lorentz-tensor interactions can arise which destabilize the vacuum and generate nonzero tensor vacuum expectation values.

Colladay and Kostelecký have derived the most general extension of the standard model that could arise from spontaneous Lorentz symmetry breaking of a more fundamental theory, maintains SU(3)×SU(2)×U(1) gauge invariance, and is renormalizable [5]. They have shown that the theory maintains many of the other usual properties of the standard model besides Lorentz and CPT symmetry, such as electroweak breaking, energy-momentum conservation, the spin-statistics connection, microcausality, and observer Lorentz covariance. In addition to the atomic experiments described here, the standard-model extension has been used to analyze Lorentz and CPT tests with neutral mesons [27, 28], photon experiments [5, 25, 29], and baryogenesis [30].

**TESTS IN ATOMIC SYSTEMS**

To consider experiments in atomic physics it suffices to restrict the standard-model extension to its QED sector. The modified Dirac equation for a four-component spinor field \( \psi \) of mass \( m_e \) and charge \( q = -|e| \) in an electric potential \( A^\mu \) is

\[
(i\gamma^\mu D_\mu - m_e - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} + ic_{\mu\nu} \gamma^\mu D^\nu + id_{\mu\nu} \gamma_5 \gamma^{\mu\nu} D^\nu) \psi = 0
\]

(5)

Here, natural units with \( \hbar = c = 1 \) are used, and \( iD_\mu = i\partial_\mu - qA_\mu \). The two terms involving the effective coupling constants \( a_\mu \) and \( b_\mu \) violate CPT, while the three terms involving \( H_{\mu\nu}, c_{\mu\nu}, \) and \( d_{\mu\nu} \) preserve CPT. All five terms break Lorentz
invariance. Each particle sector in the standard model has its own set of parameters which we distinguish using superscripts. Since no Lorentz or CPT violation has been observed, these parameters are assumed to be small. A perturbative treatment in the context of relativistic quantum mechanics can then be used. In this approach, all of the perturbations in conventional quantum electrodynamics are identical for particles and antiparticles. However, the interaction Hamiltonians including the effects of possible Lorentz and CPT breaking are not the same.

**Penning-Trap Experiments**

Comparisons of the $g$ factors and charge-to-mass ratios of particles and antiparticles confined within a Penning trap have yielded the CPT bounds in Eqs. (2) and (3). These quantities are obtained through measurements of the anomaly frequency $\omega_a$ and the cyclotron frequency $\omega_c$. For example, $g - 2 = 2\omega_a/\omega_c$. These frequencies can be measured to $\sim 10^{-9}$ thereby determining $g$ to $\sim 10^{-12}$.

We have analyzed Penning-trap experiments with electrons and positrons and with protons and antiprotons [6]. We find that to leading order in the Lorentz-violating parameters there are corrections to the anomaly frequencies which are different for particles and antiparticles, while the cyclotron frequencies receive corrections that are the same for particles and antiparticles. Both frequencies have corrections which cause them to exhibit sidereal time variations. We also find that to leading order the $g$ factor has no corrections, and therefore the figure of merit $r_g \simeq 0$, even though there is explicit CPT breaking. Because of this, we have proposed using as an alternative figure of merit the relative relativistic energy shifts caused by Lorentz and CPT violation. This is a definition that can be used in any experiment and is consistent with neutral meson experiments, which use mass ratios.

Based on these observations, we proposed looking for two signals of Lorentz and CPT violation: one an instantaneous difference in anomaly frequencies for electrons and positrons, and the other sidereal-time variations in the anomaly frequency of electrons alone. Dehmelt’s group at the University of Washington has recently published the results of these observations [31]. In the first case, they reanalyzed existing data and obtained a figure of merit $r_{\omega_a}^e \lesssim 1.2 \times 10^{-21}$ from a bound on the difference in the electron and positron anomaly frequencies. In the second case, they analyzed more recent data for the electron alone and obtained a bound on sidereal time variations given by $r_{\omega_a,\text{diurnal}}^e \lesssim 1.6 \times 10^{-21}$. This corresponds to a bound on the combination of components $\tilde{b}_J^e \equiv b_J^e - md_{J0} - \frac{1}{2} \varepsilon_{JKL} H_{KL}^e$ defined with respect to a nonrotating coordinate system [4] given by $|\tilde{b}_J^e| \lesssim 5 \times 10^{-25}$ GeV.

Although no $g - 2$ experiments have been made for protons or antiprotons, there have been recent bounds obtained on Lorentz violation in comparisons of cyclotron frequencies of antiprotons and $H^-$ ions confined in a Penning trap [13]. In this case the sensitivity is to the parameters $c_{\mu\nu}$, and the figure of merit $r_{\omega_c}^H \lesssim 10^{-25}$ was obtained.
Clock-Comparison Experiments

The classic Hughes-Drever type experiments are atomic clock-comparison tests of Lorentz invariance [13]. These experiments look for relative changes between two “clock” frequencies as the Earth rotates. The “clock” frequencies are typically atomic hyperfine or Zeeman transitions. Using the standard-model extension, Kostelecký and Lane [7] have made an extensive analysis of these experiments. They have obtained approximate bounds on various combinations of the Lorentz-violating parameters from the published results of these experiments. For example, from the experiment of Berglund et al. the following bounds for the parameters $\tilde{b}_J$ have been found for the proton, neutron, and electron.

|          | $\tilde{b}_J$ |       |
|----------|---------------|-------|
| proton   | $b^p_J$       | $\sim 10^{-27}$ GeV |
| neutron  | $b^n_J$       | $\sim 10^{-30}$ GeV |
| electron | $b^e_J$       | $\sim 10^{-27}$ GeV |

Since certain assumptions about the nuclear configurations must be made to extract numerical bounds, these bounds should be viewed as good to within one or two orders of magnitude. To obtain cleaner bounds it is necessary to consider simpler atoms.

Hydrogen-Antihydrogen Experiments

We have analyzed the proposed experiments at CERN which will make high-precision spectroscopic measurements of the 1S-2S transitions in hydrogen and antihydrogen [8]. We find that the magnetic field plays an important role in the sensitivity of the 1S-2S transition to Lorentz and CPT breaking. For example, in free hydrogen in the absence of a magnetic field, the 1S and 2S levels shift by the same amount at leading order in hydrogen and antihydrogen. As a result of this, there are no leading-order corrections to the 1S-2S transition frequency in free H or $\bar{H}$.

In a magnetic trap, however, there are magnetic fields which mix the spin states in the four hyperfine levels. Since the Lorentz-violating couplings are spin-dependent, some of the 1S and 2S levels acquire energy corrections that are not equal. The transitions between these levels have leading-order sensitivity to Lorentz and CPT violation. However, these transitions are also field-dependent, making them prone to broadening in an inhomogeneous magnetic field. To be sensitive to leading-order Lorentz and CPT violation in 1S-2S transitions, experiments will have to overcome the difficulties associated with possible line broadening effects due to field inhomogeneities.

We have also considered measurements of the ground-state Zeeman hyperfine transitions in hydrogen and antihydrogen [8]. We find that certain transitions in a
hydrogen maser are sensitive to leading-order Lorentz-violating effects. These mea-
surements have now been made by Walsworth’s group at the Harvard-Smithsonian
Center using a double-resonance technique [32]. They have obtained bounds on
the Lorentz-violation parameters for the electron and proton. The bound for the
proton alone is $|\tilde{b}_p| \lesssim 10^{-27}$ GeV. This is an extremely clean bound and is now the
most stringent test of Lorentz and CPT symmetry for the proton.

Spin-Polarized Matter

Experiments at the University of Washington using a spin-polarized torsion
pendulum [33] are able to achieve very high sensitivity to Lorentz violation due to
the combined effect of a large number of aligned electron spins [9]. The experiment
uses stacked toroidal magnets with a net electron spin $S \simeq 8 \times 10^{22}$, but which
have a negligible magnetic field. The apparatus is suspended on a turntable and
a time-varying harmonic signal is sought. Our analysis shows that in addition to
a signal with the period of the rotating turntable, the effects of Lorentz and CPT
violation would induce additional time variations with a sidereal period caused by
Earth’s rotation. The University of Washington group has analyzed their data and
have obtained a bound on the electron parameters equal to $|\tilde{b}_e| \lesssim 1.4 \times 10^{-28}$ GeV
[34]. This is now the best Lorentz and CPT bound for the electron.

Muon Experiments

Despite the spectacular precision of recent Lorentz and CPT tests for protons
and electrons, it is important to keep in mind that particle sectors in the standard-
model extension might be independent of each other and should separately be
tested. The situation is analogous to CP tests where violation is observed only
in the neutral meson sector and not in the lepton or baryon sectors. A thorough
investigation of Lorentz and CPT symmetry must therefore probe as many possible
particle sectors as possible. For this reason, we also examine muon experiments,
which involve second-generation leptons. We find that there are several different
types of experiments that are sensitive to Lorentz and CPT. We have examined
both muonium experiments [35] and $g - 2$ experiments with muons that are being
conducted at Brookhaven [90].

Our results are that experiments measuring the frequencies of ground-state
Zeeman hyperfine transitions in muonium in a strong magnetic field are sensitive
to Lorentz and CPT violation. If bounds on sidereal time variations are obtained
at the 100 Hz level, then the Lorentz-violation parameter for the muon $\tilde{b}_\mu$ can be
bounded at the level of $|\tilde{b}_\mu| \leq 5 \times 10^{-22}$ GeV. We also find that in relativistic $g - 2$
experiments using positive muons with “magic” boost parameter $\delta = 29.3$, bounds
on Lorentz-violation parameters are possible at a level of $10^{-25}$ GeV. Experiments
looking for sidereal time variations in the muon anomaly frequency would yield
stringent new Lorentz and CPT bounds.
SUMMARY AND CONCLUSIONS

In summary, by using a general framework we are able to analyze Lorentz and CPT tests in a variety of atomic experiments. We find that experiments that have traditionally been considered Lorentz tests are also sensitive to CPT and vice versa. We find that it is also possible to make very precise tests of CPT in matter alone. Many of the bounds that have been obtained are well within the range of suppression factors associated with the Planck scale. The atomic experiments complement those in particle physics and together they are able to test the robustness of the standard model to increasing levels of precision.

ACKNOWLEDGMENTS

I would like to acknowledge my collaborators Alan Kostelecký, Charles Lane, and Neil Russell. This work was supported in part by the National Science Foundation under grant number PHY-9801869.

REFERENCES

1. See, for example, R.M. Barnett et al., Review of Particle Properties, Phys. Rev. D 54 (1996) 1.
2. V.A. Kostelecký, ed., CPT and Lorentz Symmetry (World Scientific, Singapore, 1999).
3. V.A. Kostelecký and S. Samuel, Phys. Rev. Lett. 63 (1989) 224; Phys. Rev. Lett. 66 (1991) 1811; Phys. Rev. D 39 (1989) 683; Phys. Rev. D 40 (1989) 1886.
4. V.A. Kostelecký and R. Potting, Nucl. Phys. B 359 (1991) 545; Phys. Lett. B 381 (1996) 389; V.A. Kostelecký, M. Perry, and R. Potting, Phys. Rev. Lett., in press, hep-th/991243.
5. D. Colladay and V.A. Kostelecký, Phys. Rev. D 55 (1997) 6760; Phys. Rev. D 57, 116002 (1998).
6. R. Bluhm, V.A. Kostelecký and N. Russell, Phys. Rev. Lett. 79 (1997) 1432; Phys. Rev. D 57 (1998) 3932.
7. V.A. Kostelecký and C.D. Lane, Phys. Rev. D 60, 116010 (1999).
8. R. Bluhm, V.A. Kostelecký and N. Russell, Phys. Rev. Lett. 82 (1999) 2254.
9. R. Bluhm and V.A. Kostelecký, Phys. Rev. Lett. 84 (2000) 1381.
10. R. Bluhm, V.A. Kostelecký and C.D. Lane, Phys. Rev. Lett. 84 (2000) 1098.
11. For reviews of CPT, see R.F. Streater and A.S. Wightman, PCT, Spin, and Statistics and All That, Benjamin Cummings, Reading, 1964; R. G. Sachs, The Physics of Time Reversal, University of Chicago, Chicago, 1987.
12. J. Schwinger, Phys. Rev. 82 (1951) 914; J.S. Bell, Birmingham University thesis (1954); Proc. Roy. Soc. (London) A 231 (1955) 479; G. Lüders, Det. Kong. Danske Videnskabernes Selskab Mat.fysiske Meddelelser 28, No. 5 (1954); Ann. Phys. (N.Y.) 2 (1957) 1; W. Pauli, in W. Pauli, ed., *Neils Bohr and the Development of Physics*, McGraw-Hill, New York, 1955, p. 30.

13. V.W. Hughes, H.G. Robinson, and V. Beltran-Lopez, Phys. Rev. Lett. 4 (1960) 342; R.W.P. Drever, Philos. Mag. 6 (1961) 683; J.D. Prestage *et al.*, Phys. Rev. Lett. 54 (1985) 2387; S.K. Lamoreaux *et al.*, Phys. Rev. A 39 (1989) 1082; T.E. Chupp *et al.*, Phys. Rev. Lett. 63 (1989) 1541; C.J. Berglund *et al.*, Phys. Rev. Lett. 75 (1995) 1879.

14. P.B. Schwinberg, R.S. Van Dyck, Jr., and H.G. Dehmelt, Phys. Lett. A 81 (1981) 119; R.S. Van Dyck, Jr., P.B. Schwinberg, and H.G. Dehmelt, Phys. Rev. D 34 (1986) 722; L.S. Brown and G. Gabrielse, Rev. Mod. Phys. 58 (1986) 233; R.S. Van Dyck, Jr., P.B. Schwinberg, and H.G. Dehmelt, Phys. Rev. Lett. 59 (1987) 26.

15. G. Gabrielse *et al.*, Phys. Rev. Lett. 82 (1999) 3198.

16. B. Brown *et al.*, Nucl. Phys. B (Proc. Suppl.) 56A (1997) 326; M.H. Holzscheiter *et al.*, Nucl. Phys. B (Proc. Suppl.) 56A (1997) 336.

17. P. Carruthers, Phys. Lett. B 26 (1968) 158.

18. A.I. Oksak and I.T. Todorov, Commun. Math. Phys. 11 (1968) 125.

19. S. Hawking, Commun. Math. Phys. 87 (1982) 395.

20. H.B. Nielsen and I. Picek, Nucl. Phys. B211 (1983) 269.

21. J. Ellis, N.E. Mavromatos, and D.V. Nanopoulos, Int. J. Mod. Phys. A 11 (1996) 146.

22. G. Cocconi and E. Salpeter, Nuovo Cimento 10 (1958) 646.

23. See for example, T.D. Lee and C.S. Wu, Annu. Rev. Nucl. Sci 16 (1966) 511.

24. A.P. Lightman and D.L. Lee, Phys. Rev. D 8 (1973) 364.

25. S.M. Carroll, G.B. Field, and R. Jackiw, Phys. Rev. D 41 (1990) 1231.

26. S. Coleman and S.L. Glashow, Phys. Rev. D 59, 116008 (1999).

27. B. Schwingenheuer *et al.*, Phys. Rev. Lett. 74 (1995) 4376; L.K. Gibbons *et al.*, Phys. Rev. D 55 (1997) 6625; R. Carosi *et al.*, Phys. Lett. B 237 (1990) 303; OPAL Collaboration, R. Ackerstaff *et al.*, Z. Phys. C 76 (1997) 401; DELPHI Collaboration, M. Feindt *et al.*, preprint DELPHI 97-98 CONF 80 (July 1997).

28. V.A. Kostelecký and R. Potting, in D.B. Cline, ed., *Gamma Ray–Neutrino Cosmology and Planck Scale Physics* (World Scientific, Singapore, 1993) (hep-th/9211116); Phys. Rev. D 51 (1995) 3923; D. Colladay and V. A. Kostelecký, Phys. Lett. B 344 (1995) 259; Phys. Rev. D 52 (1995) 6224;
V.A. Kostelecký and R. Van Kooten, Phys. Rev. D 54 (1996) 5585; V.A. Kostelecký, Phys. Rev. Lett. 80 (1998) 1818; Phys. Rev. D 61, 016002 (2000).

29. R. Jackiw and V.A. Kostelecký, Phys. Rev. Lett. 82 (1999) 3572; M. Pérez-Victoria, Phys. Rev. Lett. 83 (1999) 2518; J.M. Chung, Phys. Lett. B 461 (1999) 138.

30. O. Bertolami et al., Phys. Lett. B 395, 178 (1997).

31. R.K. Mittleman, I.I. Ioannou, H.G. Dehmelt, and N. Russell, Phys. Rev. Lett. 83 (1999) 2116; H.G. Dehmelt, R.K. Mittleman, R.S. Van Dyck, Jr., and P. Schwinberg, Phys. Rev. Lett. 83 (1999) 4694.

32. R. Walsworth et al, in preparation; see also this volume.

33. E.G. Adelberger et al., in P. Herczeg et al., eds., Physics Beyond the Standard Model, p. 717, World Scientific, Singapore, 1999; M.G. Harris, Ph.D. thesis, Univ. of Washington, 1998.

34. B. Heckel et al., private communication.

35. W. Liu et al., Phys. Rev. Lett. 82 (1999) 711.

36. R.M. Carey et al., Phys. Rev. Lett. 82 (1999) 1632.