Mass-matrix ansatz and constraints on $B^0_s - \bar{B}^0_s$ mixing in 331 models,

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February 5, 2008

Abstract

Comparing the theoretically predicted and measured values of the mass difference of the $B^0_s$ system, we estimate the lower bound on the mass of the $Z'$ boson of models based on the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ gauge group. By assuming zero-texture approaches of the quark mass matrices, we find the ratio of the measured value to the theoretical prediction from the Standard Model and the $Z'$ contribution from the 331 models of the mass difference of the $B^0_s$ system. We find lower bounds on the $Z'$ mass ranging between 1 TeV and 30 TeV for the two most popular 331 models, and four different zero-texture ansätze. The above results are expressed as a function of the weak angle associated to the $b - s - Z'$ couplings.

1 Introduction

Although the Standard Model (SM) [1] is considered as an effective low energy theory that should be embedded into a more fundamental theory, many of the SM predictions have been successfully tested by precision measurements. The latter impose strong restrictions to new physics contributions associated to any extension of the SM [2]. Thus, small deviations between the experimental data and the SM predictions allow to set stringent limits on new physics from a more fundamental theory that contains new types of matter and interactions at the TeV scale. It will be explored with the new generation of accelerators and detectors like the forthcoming Large Hadron Collider (LHC) [3]. Among the possible extensions of the SM, the models with gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, also called 3-3-1 models [4, 5], arise as an interesting alternative with new physics content and some motivating features. First of all, from the cancellation of chiral anomalies [6] and asymptotic freedom in QCD, the 3-3-1 models can explain why there are three fermion families. Secondly, since the third family is treated under a different representation, the large mass difference between the

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heaviest quark family and the two lighter ones may be understood [7]. Thirdly, the models have a scalar content similar to the two Higgs doublet model (2HDM), which allow to predict the quantization of electric charge and the vectorial character of the electromagnetic interactions [8, 9]. Also, these models contain a natural Peccei-Quinn symmetry, necessary to solve the strong-CP problem [10, 11]. Finally, the model introduces new types of matter relevant to the next generations of colliders at the TeV energy scales, which do not spoil the low energy limits at the electroweak scale.

In the SM, the Flavor Changing Neutral Currents (FCNC) are strongly suppressed with respect to the charged-current weak interactions, which follows from the experimental data on neutral meson decays and the mass difference in meson systems exhibiting particle-antiparticle mixing [12]. In particular, some extensions of the SM produce new FCNC contributions at tree level, as for example some models with an extra neutral $Z'$ boson, which represents an stringent limit for new physics. Although not all models with new neutral $Z'$ bosons exhibit additional FCNC contributions [13], many interesting ones contain FCNC effects at tree level [14, 15, 16]. In 3-3-1 models, the contributions in meson systems have been considered before [17] in $K^0 - \overline{K^0}$, and $B_d^0 - \overline{B_d^0}$ systems, which induce the flavor changing transitions $s \leftrightarrow d$ and $b \leftrightarrow d$, respectively, while no information other than a lower bound associated to the $b \leftrightarrow s$ transition was available for the $B_s^0 - \overline{B_s^0}$ system. However, the $b - s$ sector was recently confirmed in the $B_s$ mixing by both CDF and DØ [18]:

CDF: \[ \Delta M_s = 17.33^{+0.42}_{-0.21} \text{ ps}^{-1}, \]

DØ: \[ \Delta M_s = 19.0 \pm 1.215 \text{ ps}^{-1}. \]

Ref. [16] use the following averages

\[ \Delta M_s^{\text{exp}} = 17.46^{+0.47}_{-0.3} \text{ ps}^{-1}, \]
\[ \Delta M_s^{\text{SM}} = 19.52 \pm 5.28 \text{ ps}^{-1}, \] (1)

for the experimental and SM prediction, respectively. Since the study of $B$ physics has been an important tool to extract information on CP violation and new physics [19], we will use the above data for the mass difference of the $B_s$ system to explore the FCNC contribution induced by the $Z'$ boson in the two most popular 3-3-1 models. However, since FCNC contribution in these models are very sensitive to the rotations of the fermionic spectrum to mass eigenstates, it is necessary to implement some criterion to fix the values of the components of the rotation matrices, and to get numerical predictions on the meson mass difference. In contrast to other studies in $D^0$, $K^0$ and $B_d^0$ systems [17], we will consider various cases for the rotation matrix, including the texture-zero approaches, where an ansatz on the texture of the fermion mass matrices is adopted in agreement with the measured masses and mixing angles of the Cabibbo-Kobayashi-Maskawa matrix. An additional motivation to study the $B_s$ system comes from the fact that the $b - s$ sector induces the maximum flavor-changing contribution, as will be confirmed in this work. This offers a good opportunity to extract information on new physics at low energy.
Eq. (1) shows good agreement between the experimental data and the SM one-loop prediction of \( \Delta M_s \), however, due to the hadronic parameters, the SM prediction contains a large uncertainty which we use to find allowed regions for the mass of the \( Z' \) boson and the weak angle associated to the \( b-s-Z' \) coupling by assuming four specific forms in the rotation matrix of the quark mass.

2 The 331 spectrum

The fermionic structure is shown in Tab. 1 where all leptons transform as \((3, X^L_\ell)\) and \((1, X^R_\ell)\) under the \((SU(3)_L, U(1)_X)\) sector, with \(X^L_\ell\) and \(X^R_\ell\) the \(U(1)_X\) generators associated with the left- and right-handed leptons, respectively; while the quarks transform as \((3^*, X^L_{q_m^*})\), \((1, X^R_{q_m})\) for the first two families, and \((3, X^L_{q_3})\), \((1, X^R_{q_3})\) for the third family, each one with its \(U(1)_X\) values for the left- and right-handed quarks. The quantum numbers \(X_\psi\) for each representation are given in the third column from Tab. 1, where the electric charge is defined by

\[
Q = T_3 + \beta T_8 + X I, \tag{2}
\]

with \(T_3 = 1/2\text{diag}(1, -1, 0)\), \(T_8 = (1/2\sqrt{3})\text{diag}(1, 1, -2)\) and \(\beta = -1/\sqrt{3}\) and \(-\sqrt{3}\), where the first case contains the Foot-Long-Truan model (FLT) [20] and the second contains the Pisano-Pleitez-Frampton model (PPF) [4, 5].

For the scalar sector, we introduce the triplet field \(\chi\) with vacuum expectation value (VEV) \(\langle \chi \rangle^T = (0, 0, \nu_\chi)\), which provides the masses of the third fermionic components. In the second transition, it is necessary to introduce two triplets \(\rho\) and \(\eta\) with VEV \(\langle \rho \rangle^T = (0, \nu_\rho, 0)\) and \(\langle \eta \rangle^T = (\nu_\eta, 0, 0)\), in order to give masses to the quarks of up- and down-type, respectively [21].

In the gauge boson spectrum associated with the group \(SU(3)_L \otimes U(1)_X\), we are just interested in the physical neutral sector that corresponds to the photon, \(Z\), and \(Z'\), which are written in terms of the electroweak basis for \(\beta = -1/\sqrt{3}\) and \(-\sqrt{3}\) as [22]

\[
A_\mu = S_W W^3_\mu + C_W \left( \beta T_W W^8_\mu + \sqrt{1 - \beta^2 T^2_W} B_\mu \right), \\
Z_\mu = C_W W^3_\mu - S_W \left( \beta T_W W^8_\mu + \sqrt{1 - \beta^2 T^2_W} B_\mu \right), \\
Z'_\mu = -\sqrt{1 - \beta^2 T^2_W} W^8_\mu + \beta T_W B_\mu, \tag{3}
\]

where the Weinberg angle is defined as [22]

\[
S_W = \sin \theta_W = \frac{g_X}{\sqrt{g_L^2 + (1 + \beta^2) g_X^2}} \tag{4}
\]

and \(g_L, g_X\) correspond to the coupling constants of the groups \(SU(3)_L\) and \(U(1)_X\), respectively.
| representation | $Q_\psi$ | $X_\psi$ |
|----------------|---------|---------|
| $q_m^* L = \begin{pmatrix} d_m^* \\ -u_m^* \\ J_m^* \end{pmatrix}_L$ | $\begin{pmatrix} -\frac{1}{3} \\ -\frac{1}{6} + \frac{\sqrt{3} \beta}{2} \end{pmatrix}$ | $X_{q_m^*}^L = \frac{1}{6} + \frac{\beta}{2\sqrt{3}}$ |
| $d_m^* R; u_m^* R; J_m^* R : 1$ | $-\frac{1}{3}; \frac{2}{3}; \frac{1}{6} + \frac{\sqrt{3}}{2} \beta$ | $X_{d_m^* u_m^* J_m^*}^R = -\frac{1}{3}; \frac{2}{3}; \frac{1}{6} + \frac{\sqrt{3}}{2} \beta$ |
| $q_3 L = \begin{pmatrix} u_3 \\ d_3 \\ J_3 \end{pmatrix}^L : 3$ | $\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ \frac{1}{6} - \frac{\sqrt{3} \beta}{2} \end{pmatrix}$ | $X_{q_{(3)}}^L = \frac{1}{6} - \frac{\beta}{2\sqrt{3}}$ |
| $u_3 R; d_3 R; J_3 R : 1$ | $\frac{2}{3}; -\frac{1}{3}; \frac{1}{6} - \frac{\sqrt{3} \beta}{2}$ | $X_{u_3 d_3 J_3}^R = \frac{2}{3}; -\frac{1}{3}; \frac{1}{6} - \frac{\sqrt{3} \beta}{2}$ |
| $\ell_j L = \begin{pmatrix} \nu_j \\ e_j \\ E_j^{Q_1} \end{pmatrix}^L : 3$ | $\begin{pmatrix} 0 \\ -\frac{1}{2} - \frac{\sqrt{3} \beta}{2} \end{pmatrix}$ | $X_{\ell_j}^L = -\frac{1}{2} - \frac{\beta}{2\sqrt{3}}$ |
| $e_j R; E_j^{Q_1}^{R}$ | $-1; -\frac{1}{2} - \frac{\sqrt{3} \beta}{2}$ | $X_{e_j E_j}^R = -1; -\frac{1}{2} - \frac{\sqrt{3} \beta}{2}$ |

Table 1: Fermionic content for three generations with $\beta = -1/\sqrt{3}, -\sqrt{3}$. We take $m^* = 1, 2$ and $j = 1, 2, 3$
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3 Neutral Couplings

Using the fermionic content in weak eigenstates from Tab. 1, we obtain the neutral coupling for the SM quarks [22]

$$L^{NC}_D = \frac{g_L}{2C_W} \left[ \overline{Q}^0 \gamma_\mu \left( g_v^{Q^0} - g_a^{Q^0} \gamma_5 \right) Q^0 Z^\mu + \overline{Q}^0 \gamma_\mu \left( \tilde{g}_v^{Q^0} - \tilde{g}_a^{Q^0} \gamma_5 \right) Q^0 Z^{\mu'} \right],$$

(5)

where $Q^0 : U^0 = (u, c, t)^0, D^0 = (d, s, b)^0$ for up- and down-type quarks, respectively. The vector and axial-vector couplings of the $Z$ boson are

$$g_v^{U^0} = \frac{1}{2} - 2Q_{U^0} S^2_W, \quad g_a^{U^0} = \frac{1}{2},$$

$$g_v^{D^0} = -\frac{1}{2} - 2Q_{D^0} S^2_W, \quad g_a^{D^0} = -\frac{1}{2},$$

(6)

with $Q_{U^0, D^0}$ the electric charge of each quark given by Tab. 1; while the corresponding couplings to $Z'$ are given by

$$\tilde{g}_{v,a}^{U^0} = g_x C_W \frac{1}{2g_L T_W} \left[ \frac{1}{\sqrt{3}} \left( diag(1, 1, -1) + \frac{\beta T^2_W}{\sqrt{3}} \right) \pm 2Q_{U^0} \beta T^2_W \right],$$

$$\tilde{g}_{v,a}^{D^0} = g_x C_W \frac{1}{2g_L T_W} \left[ \frac{1}{\sqrt{3}} \left( diag(1, 1, -1) + \frac{\beta T^2_W}{\sqrt{3}} \right) \pm 2Q_{D^0} \beta T^2_W \right],$$

(7)

which are written for $\beta = -1/\sqrt{3}$ and $-\sqrt{3}$. In particular, for the $Z'$ coupling in the neutral Lagrangian in Eq. (5), we can write

$$L^{Z'} = \frac{g_L}{2C_W} \left[ \overline{Q}^0 \gamma_\mu \left( \tilde{\epsilon}_{L,R}^{Q^0} P_L + \tilde{\epsilon}_{L,R}^{Q^0} P_R \right) Q^0 Z^\mu \right],$$

(8)

where $\tilde{\epsilon}_{L,R}^{Q^0} = (1/2)(\tilde{g}_v^{Q^0} \pm \tilde{g}_a^{Q^0})$, and $P_{L,R} = (1/2)(1 \mp \gamma_5)$ the chiral projectors. Using the neutral $Z'$-couplings from Eq. (7), the new chiral couplings $\tilde{\epsilon}_{L,R}^{U^0, D^0}$ are written as follows

$$\tilde{\epsilon}_L^{U^0, D^0} = \frac{g_x C_W}{2g_L T_W} \left[ \frac{1}{\sqrt{3}} diag(1, 1, -1) + \frac{1}{3} \beta T^2_W \right],$$

$$\tilde{\epsilon}_R^{U^0, D^0} = \frac{g_x C_W}{g_L T_W} \left[ Q_{U^0, D^0} \beta T^2_W \right].$$

(9)

On the other hand, we will consider linear combinations among the three families of quarks to obtain couplings in mass eigenstates

$$Q^0 = R_Q Q,$$

(10)

where $Q : U = (u, c, t), D = (d, s, b)$ denotes the quarks in mass eigenstates, $Q^0$ in weak eigenstates and $R_Q$ the rotation matrix that diagonalize the Yukawa mass terms. Thus, we can write the Eq. (8) as
\[ \mathcal{L}^{Z'} = \frac{g_L}{2C_W} \left[ \overline{Q} \gamma^\mu \left( \tilde{B}^Q_L P_L + \tilde{B}^Q_R P_R \right) Q Z'_\mu \right], \] (11)

where the chiral couplings in mass eigenstates are defined as

\[ \tilde{B}^Q_{L,R} = R^Q_{Q' L,R} \tilde{e}_R. \] (12)

Because of the fact that \( \tilde{e}_R \) in Eq. (9) is family independent, the right-handed couplings remain flavor-diagonal in the mass eigenbasis, such that \( \tilde{B}^Q_{L,R} = \tilde{e}_R \). However, due to the \( \text{diag}(1,1,-1) \) term from Eq. (9) (family dependent couplings), we obtain non-diagonal components in the left-handed couplings \( \tilde{B}^Q_L \) in Eq. (12), which is sensitive to the form of the rotation matrix \( R_Q \). In order to have a predictive model, we adopt a different ansatz on the texture of the quark mass matrices in agreement with the six quark physical masses and the four physical parameters of the CKM matrix. The \( SU(3)_L \otimes U(1)_X \) Lagrangian for the Yukawa interaction between quarks is

\[- \mathcal{L}_{Yuk} = \sum_{m=1}^{2} q_{m^*L} \left[ \Gamma_{m^*D} \eta D^0_R + \Gamma_{m^*U} \rho U^0_R + \Gamma_{m^*J} \chi J^0_R \right] + h.c, \] (13)

with \( \eta \) and \( \rho \) being the two scalar triplets necessary to give masses to the SM fermion spectrum from Table I and \( \chi \) the scalar triplet that gives masses to the new extra fermions \( J_{1,2,3}, E_{1,2,3} \), as explained in Sec. 2. Thus, we are not interested in the couplings of \( \chi \). \( \Gamma_{\phi}^Q \) are the Yukawa interaction matrices. Taking into account only the \( SU(2)_L \) sector (which lies in the two upper components of each scalar triplet), and omitting the couplings of \( \chi \), the mass eigenstates of the scalar sector can be written as \[ H = \begin{pmatrix} \phi_1^0 \\ h_3^0 + \nu \mp i\phi_3^0 \end{pmatrix} = \rho S_\beta - \eta^* C_\beta, \]

\[ \phi = \begin{pmatrix} h_2^0 \\ -h_4^0 \mp ih_1^0 \end{pmatrix} = \rho C_\beta + \eta^* S_\beta, \] (14)

where \( \eta^* \) denotes the conjugate representation of \( \eta \), \( \tan \beta = \nu_\rho / \nu_{\eta} \) and \( \nu = \sqrt{\nu_{\rho}^2 + \nu_{\eta}^2} \). Thus, after some algebraic manipulation, the neutral couplings of the Yukawa Lagrangian can be written as

\[- \mathcal{L}_{Yuk}^{(0)} = \left[ \tilde{D}_L^0 (M_{\phi^0}) D_R^0 + \tilde{U}_L^0 (M_{\phi^0}) U_R^0 \right] \left( 1 + \frac{h_3^0 \mp i\phi_3^0}{\nu} \right) \]

\[ + \left[ \tilde{D}_L^0 (\Gamma_{\phi^0}) D_R^0 + \tilde{U}_L^0 (\Gamma_{\phi^0}) U_R^0 \right] (h_4^0 \pm ih_1^0) + h.c, \] (15)

where the fermion masses and Yukawa coupling matrices are given by

\[ M_{\phi} = \nu (\Gamma_1 C_\beta + \Gamma_2 S_\beta) \quad \text{and} \quad \Gamma_{\phi} = \Gamma_1 S_\beta - \Gamma_2 C_\beta, \] (16)
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where $\Gamma_1 = \Gamma_\eta$ and $\Gamma_2 = \Gamma_\rho$. The Lagrangian from Eq. (15) is equivalent to the two-Higgs-doublet model (2HDM) Lagrangian [23], which exhibits FCNC due to the non-diagonal components of $\Gamma$. In the literature, there are various approaches on the zero-textures of the quark mass matrices $M_{q\phi}$ from Eq. (16), where the most popular are listed as follows

1. **Fritzsch ansatz**: In the basis $U^0(D^0) = (u^0(q^0), c^0(s^0), t^0(b^0))$ the quark mass matrices in the Fritzsch ansatz are defined as [24]

$$M_{q\phi} = \begin{pmatrix} 0 & |D_q| & 0 \\ |D_q| & 0 & |F_q| \\ 0 & |F_q| & |C_q| \end{pmatrix},$$

with $|C_q| \approx m_{t,b}$, $|F_q| \approx \sqrt{m_{t,b}m_{c,s}}$ and $|D_q| \approx \sqrt{m_{u,d}m_{c,s}}$, where $m_q$ corresponds to the physical mass of the quarks. The above ansatz is diagonalized by the following rotation matrices for both the up- and down-type quarks

$$R_q = \begin{pmatrix} 1 & \sqrt{m_{u,d}/m_{c,s}} & -\sqrt{m_{u,d}/m_{t,b}} \\ -\sqrt{m_{u,d}/m_{c,s}} & 1 & \sqrt{m_{c,s}/m_{t,b}} \\ 0 & -\sqrt{m_{c,s}/m_{t,b}} & 1 \end{pmatrix}.$$  

2. **Matsuda-Nishihura ansatz**: This texture takes the same form as Eq. (17), but with $|B_q| = m_{c,s}$ in the (2,2) component of the mass matrix [25]. This form have the following rotation matrices

$$R_q = \begin{pmatrix} 1 & \sqrt{m_{u,d}/m_{c,s}} & \sqrt{m_{c,s}m_{u,d}/m_{t,b}} \\ -\sqrt{m_{u,d}/m_{c,s}} & 1 & \sqrt{m_{c,s}m_{u,d}/m_{t,b}} \\ -\sqrt{m_{c,s}m_{u,d}/m_{t,b}} & -\sqrt{m_{u,d}/m_{t,b}} & 1 \end{pmatrix}.$$  

The above ansatz was reconsidered by the authors in ref. [26], where the parameter $C_q$ in the (3,3) component is taken as a free parameter. In particular, they define the ratio $x_q = C_q/m_{t,b}$, such that the experimental values of the CKM matrix are derived by fine tuning of the parameter $x_q$. Thus, the non-zero components of the mass matrix takes the form $|D_q| = \sqrt{m_{c,s}m_{u,d}/x_q}$ in the (1,2) components, $|B_q| = m_{t,b}(1-x_q) + m_{c,s} - m_{u,d}$ in the (2,2) component, and $|F_q| = (m_{t,b}x_q + m_{u,d})(m_{t,b}x_q - m_{c,s})(1-x_q)/x_q$ in the (2,3) components, where the hierarchy $m_{c,s} \ll C_q < m_{t,b}$ is required. The rotation matrix is

$$R_q = \begin{pmatrix} 1 & \sqrt{m_{u,d}/m_{c,s}} & \sqrt{m_{c,s}m_{u,d}/m_{t,b}(1-x_q)} \\ -\sqrt{m_{u,d}/m_{c,s}} & 1 & \sqrt{m_{c,s}m_{u,d}/m_{t,b}(1-x_q)} \\ -\sqrt{m_{c,s}m_{u,d}/m_{t,b}(1-x_q)} & -\sqrt{m_{u,d}/m_{t,b}(1-x_q)} & 1 \end{pmatrix}.$$  

The authors in ref. [26] obtain the values $x_u = 0.9560$ and $x_d = 0.9477$. 


3 Matsuda ansatz: Another consistent possibility is to consider different texture assignment for the up- and down-type quarks, as follows \[ [27, 28] \]

\[
\hat{M}_q^0 = \begin{pmatrix}
0 & |D_q| & |D_q|
|D_q| & |B_q| & |F_q|
|D_q| & |F_q| & |B_q|
\end{pmatrix},
\]

with \(|B_U| = (m_t + m_c - m_u)/2\), \(|F_U| = (m_t - m_c - m_u)/2\) and \(|D_U| \approx \sqrt{m_dm_u}/2\) for the up sector, while for the down sector the structure is \(|B_D| = (m_b + m_s - m_d)/2\), \(|F_D| = (m_s - m_b - m_d)/2\) and \(|D_D| \approx \sqrt{m_sm_d}/2\). The above textures are diagonalized by \[ [28] \]

\[
R_U = \begin{pmatrix}
\frac{c'}{\sqrt{2}} & 0 & \frac{s'}{\sqrt{2}} \\
-\frac{s'}{\sqrt{2}} & 1 & \frac{c'}{\sqrt{2}} \\
-\frac{c'}{\sqrt{2}} & \frac{s'}{\sqrt{2}} & \frac{c'}{\sqrt{2}}
\end{pmatrix}; \quad R_D = \begin{pmatrix}
c & s & 0 \\
-s & c & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

where

\[
c = \sqrt{\frac{m_s}{m_d + m_s}}; \quad s = \sqrt{\frac{m_d}{m_d + m_s}}; \\
c' = \sqrt{\frac{m_t}{m_t + m_u}}; \quad s' = \sqrt{\frac{m_u}{m_t + m_u}}.
\]

4 \(B^0_s - \bar{B}^0_s\) mixing constraints

The left-handed coupling in Eq. (12) contains non-diagonal components, which induce mixing between the neutral \(Z\) boson and quarks from different families. This will produce new physics contributions to the mass difference in neutral meson systems as for example in Kaons \(K^0 - \bar{K}^0\), Bottom \(B^0_s - \bar{B}^0_s\) and Bottom-strange \(B^0_s - \bar{B}^0_s\) mesons, each one induced by the \(s - \bar{d}, d - \bar{b}\) and \(s - \bar{b}\) transition, respectively. In particular, we take the most recent data of the \(B^0_s\) difference mass given by Eq. (1) in order to constraint new physics induced by the \(Z\) interaction. The ratio between the experimental value and the SM prediction in Eq. (1) is \[ [15] \]

\[
\frac{\Delta m^\text{exp}_s}{\Delta m^\text{ME}_s} = \left| 1 + 3.57 \times 10^5 e^{2i\phi^{sb}_L} \left( \frac{M_Z}{M^{\text{obs}}_L} \right)^2 \right| = 0.894 \pm 0.243,
\]

with \(\tilde{B}^{sb}_L\) the \(s\bar{b}\) component of \(\tilde{B}^D_L\) defined by Eq. (12), and \(\phi^{sb}_L\) the weak phase. The above data constrain the values of the \(Z\) mass and the weak phase assuming different ansatz in the texture of the mass matrices of the quarks, as discussed in Sec. 3. For the rotation matrix \(R_D\) in the down sector, we consider the Fritzsch ansatz \((R_F)\) in Eq. (18), the Matsuda-Nishihura ansatz \((R_{MN})\) in Eq. (20), and the Matsuda ansatz \((R_M)\) in Eq. (22). In order to achieve a complete comparison, we also consider the flavor-changing contribution assuming that \(\left| \tilde{B}^{sb}_L \right| = |V_{tb}V_{ts}^*|\), with \(V_{tb(ts)}\) the \(t - b(t - s)\) component of the CKM matrix, where we use the values \(|V_{tb(ts)}| = 0.77(4.06 \times 10^{-4})\) \[ [18] \]. We use the notation \(R_{CKM}\) for this last case.
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Figure 1: Allowed regions at 1σ C.L. of the $Z'$-mass and the weak phase in models with (a) $\beta = -1/\sqrt{3}$ and (b) $-\sqrt{3}$ for different ansatz, where $R_M$ is the rotation matrix in the Matsuda ansatz, $R_{MN}$ in the Matsuda-Nishiura ansatz, $R_F$ in the Fritzsch ansatz, and $R_{CKM}$ assuming $|\tilde{B}_L^b| = |V_{tb}V_{t\alpha}|$.

Figs. 1 show plots of the contours at 1σ C.L for both (a) $\beta = -1/\sqrt{3}$ and (b) $-\sqrt{3}$ models, and for each ansatz of the rotation matrix, where we use the following data at the $Z$ scale

$$M_Z = 91.1876 \pm 0.0021 \text{ GeV}; \quad S_W^2 = 0.23113 \pm 0.00033;$$
$$m_u(M_Z) = 1.38 \text{ MeV}; \quad m_d(M_Z) = 3.05 \text{ MeV}; \quad m_c(M_Z) = 0.626 \text{ GeV};$$
$$m_s(M_Z) = 58.04 \text{ MeV}; \quad m_b(M_Z) = 2.89 \text{ GeV}; \quad m_t(M_Z) = 171.8 \text{ GeV}.$$  (25)

The regions below each curve correspond to excluded points in the $M_{Z'} - \phi_{L}^{sb}$ plane. We can also see excluded regions in the center of each curve, as shown in the plots. The minimum values for $M_{Z'}$ are found when $\phi_{L}^{sb} = \pi/2$. In particular, the ansatz $R_{CKM}$ induces the lowest bounds with values $M_{Z'} \approx 1 \text{ TeV}$, while the Matsuda ansatz $R_M$ leads high values with bounds from $M_{Z'} \approx 10 \text{ TeV}$ for $\beta = -1/\sqrt{3}$ models, and $M_{Z'} \approx 30 \text{ TeV}$ for $\beta = -\sqrt{3}$. Although the flavor-changing contribution is very sensitive to the rotation matrix, we find points that overlap regions from different ansatz in the curves of the plots. For example, in fig. 1(a), we find the same value $M_{Z'} = 10 \text{ TeV}$ for $R_M, R_{MN}$ and $R_F$ if $\phi_{L}^{sb} = 5\pi/12, 5\pi/24$ and $\pi/12$, respectively.

The differences exhibited by each curve in the above figures arise from the size of the mixing components of the couplings $\tilde{B}_L^D$ for each ansatz. In tab. 2 we compare the non-diagonal components of the left-handed coupling in the down sector. We also compare the FCNC contribution for the $u \bar{c}$ component in the up sector, as shown in the last line from Tab. 2. First of all, we observe that the maximum mixing resides in the $b-s$ sector, which is
Table 2: Magnitudes of the Left-Handed couplings for different ansatz of the rotation matrix. We consider 331 models with $\beta = -1/\sqrt{3}$, and $\beta = -\sqrt{3}$.

about one order of magnitude bigger than other flavor-changing transitions, like for example in $K^0 - \bar{K}^0$, $B^0_d - \bar{B}^0_d$ and $D^0 - \bar{D}^0$ systems, which induce the flavor changing transitions $s \leftrightarrow d$, $b \leftrightarrow d$, and $u \leftrightarrow c$, respectively. Secondly, the Matsuda ansatz yields the biggest couplings, so that in order to control the low energy limits exhibit by Eq. (24), it is necessary to impose stronger restrictions to the new physics contribution induced by this ansatz, such as seen in figs. [11].

In addition to the low energy differences shown by the plots in the above figures, the different sizes of the coupling $\tilde{B}_L^D$ leads to different predictions of the decay width of the $Z'$ boson into quarks. In particular, the flavor-changing width can be written as

$$\Gamma_{Z'\rightarrow q'q} = \frac{g_\text{L}^2 M_{Z'}}{16\pi C_W^2} \left[ \left( g_{qq'}^a \right)^2 + \left( g_{qq'}^b \right)^2 \right] = \frac{g_\text{L}^2 M_{Z'}}{8\pi C_W^2} \left[ \left( \tilde{B}_L^{qq'} \right)^2 \right],$$

(26)

From Tab. 2 it is evident that the main source of flavor-changing decay is $Z' \rightarrow b\pi$, with a decay probability of about 80% bigger than others flavor-changing decays. A detailed study of FCNC decays is carried out in ref. [29] in the Matsuda ansatz. Other ansätze, as the ones considered here, will yield lower values in the width.

5 Conclusions

In the framework of the 3-3-1 models, we have described the contribution to the mass difference $\Delta M_s$ in $B_s$ meson systems. These models behave as a purely left-handed neutral flavor-changing model. Using the recent experimental data and the SM one-loop prediction of $\Delta M_s$, we found bounds for the mass of the $Z'$ boson in the Foot-Long-Truan model (FLT) and the Pisano-Pleitez-Frampton model (PPF). The lowest values of $M_{Z'}$ are found when the weak angle associated to the $b - s - Z'$ coupling is $\phi_{L}^{sb} = \pi/2$. By assuming four different ansätze in the texture of the mass matrices of the quarks, we obtained plots of the allowed regions in the $M_{Z'} - \phi_{L}^{sb}$ plane. We considered the Fritzsch ansatz ($R_F$), the Matsuda-Nishihura ansatz ($R_{MN}$), and the Matsuda ansatz ($R_M$) for the rotation matrix $R_D$ in the down sector. We also assumed another alternative, where $|\tilde{B}_L^{sb}| = |V_{ts}^* V_{ts}|$. Lower bounds from $M_{Z'} \approx 1$ TeV to $\approx 10$ TeV in the FLT model, and from $M_{Z'} \approx 2$ TeV to $\approx 30$ TeV in
the PPF model are found for each ansatz of the rotation mass matrix. Since the Matsuda ansatz leads to the biggest size of the left-handed couplings as shown by tab. 2, this ansatz exhibit stronger low energy limits than the other ansätze. Also, the Matsuda texture yields a bigger probability of flavor-changing decay for the $Z'$ boson than the other ansätze.

This work was partially supported by Fundación Banco de la República and by ALFA-EC funds through the HELEN programme. F. Ochoa would like to thank F. Schrempp for helpful discussions and hospitality at DESY, where part of this work was developed in the framework of the HELEN programme.

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