Deformations of large $N = (4, 4)$ 2D SCFT from 3D gauged supergravity

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Abstract: Supersymmetric and non-supersymmetric deformations of large $N = (4, 4)$ SCFT with superconformal symmetry $D^1(2, 1; \alpha) \times D^1(2, 1; \alpha)$ are explored in the gravity dual described by a Chern-Simons $N = 8$, $(SO(4) \times SO(4)) \ltimes T^{12}$ gauged supergravity in three dimensions. For $\alpha > 0$, the gauged supergravity describes an effective theory of the maximal supergravity in nine dimensions on $AdS_3 \times S^3 \times S^3$ with the parameter $\alpha$ being the ratio of the two $S^3$ radii. We consider the scalar manifold of the supergravity theory of the form $SO(8, 8)/SO(8) \times SO(8)$ and find a number of stable non-supersymmetric $AdS_3$ critical points for some values of $\alpha$. These correspond to non-supersymmetric IR fixed points of the UV $N = (4, 4)$ SCFT dual to the maximally supersymmetric critical point. We study the associated RG flow solutions interpolating between these fixed points and the UV $N = (4, 4)$ SCFT. Possible supersymmetric flows to non-conformal field theories are also investigated. Additionally, a half-supersymmetric domain wall within this gauged supergravity is obtained.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence, Supergravity models.
AdS$_3$/CFT$_2$ correspondence is interesting in various aspects. Unlike in higher dimensional cases, much more insight to the AdS/CFT correspondence is expected since both gravity and field theory sides are well under control. It is also useful in the study of black hole entropy, see for example and . Until now, various gravity backgrounds implementing AdS$_3$/CFT$_2$ correspondence have been proposed. Some of them are obtained from Kaluza-Klein dimensional reductions of higher dimensional supergravities on spheres or other internal manifolds. The other are constructed directly within the three dimensional framework of Chern-Simons gauged supergravity, but, in some cases particularly for compact and non-compact gauge groups, higher dimensional origins are still mysterious.

One of the most interesting backgrounds for AdS$_3$/CFT$_2$ correspondence is string theory on AdS$_3 \times S^3 \times S^3 \times S^1$. The background is half-supersymmetric and dual to large $N = (4,4)$ SCFT in two dimensions, see for a classification of $N = 4$ SCFT in two dimensions. In string theory, this arises as a near horizon limit of the double D1-D5 brane system. The Kaluza-Klein spectrum for small $S^1$ radius has been computed in . Apart from the non-propagating supergravity multiplet in three dimensions, the spectrum contains massive multiplets of various spins. The full symmetry of AdS$_3 \times S^3 \times S^3$ is $D^1(2,1;\alpha) \times D^1(2,1;\alpha)$ whose bosonic subgroup is $SO(2,2) \times SO(4) \times SO(4)$ corresponding to the isometry of AdS$_3 \times S^3 \times S^3$, respectively. Additionally, the holography of large $N = 4$ SCFT has recently been studied in the context of higher spin AdS$_3$ dual.

Like in higher dimensions, it would be useful to have an effective theory in three dimensions that describes the above $S^3 \times S^3$ dimensional reduction. The AdS$_3 \times S^3 \times S^3$ background will become an AdS$_3$ vacuum preserving sixteen supercharges and $SO(4) \times SO(4)$ gauge symmetry, which is the isometry of $S^3 \times S^3$. This can be achieved by a gauged matter-coupled supergravity in three dimensions. The gauge group should contain the $SO(4) \times SO(4)$ factor. The natural construction should be the $N = 8$ gauged supergravity since the number of supersymmetry is exactly the same as that of the AdS$_3 \times S^3 \times S^3$ background. A theory describing supergravity coupled to massive spin-$\frac{1}{2}$ multiplets has been studied in which some critical points and a holographic RG flow have been discussed. The resulting theory is in the form of $N = 8$ gauged supergravity with compact $SO(4) \times SO(4)$ gauge group and $SO(8,n)/SO(8) \times SO(n)$ scalar manifold.

When coupled to massive spin-1 multiplets, the theory needs to accompany for massive vector fields. For a theory coupled to two spin-1 multiplets, the corresponding gauge group is a non-semisimple group $(SO(4) \times SO(4)) \rtimes T^{12}$. It has been argued that
the effective theory is the $N = 8$ gauged supergravity with $SO(8,8)/SO(8) \times SO(8)$ scalar manifold [14]. The gauging is a straightforward extension of the $SO(4) \ltimes T^6$ gauging of [13] in which the effective theory of six-dimensional supergravity reduced on $AdS_3 \times S^3$ has been given. Some supersymmetric vacua of the $(SO(4) \times SO(4)) \ltimes T^{12}$ gauged theory have already been identified in [10]. All of these vacua are related to the maximally supersymmetric vacuum by marginal deformations. The theory with only the $SO(4) \times SO(4)$ semisimple part of the gauge group being gauged has been study in [17], and the solution corresponding to a marginal deformation from $N = (4,4)$ to $N = (3,3)$ SCFT, describing a D5-brane reconnection, has been explicitly given.

In this paper, we will reexamine the full $(SO(4) \times SO(4)) \ltimes T^{12}$ gauging and look for other deformations apart from the marginal ones. This could be relevant for AdS$_3$/CFT$_2$ correspondence and black hole physics. The holographic study of the conformal symmetry $D^1(2,1;\alpha)$ is not only useful in the context of AdS$_3$/CFT$_2$ correspondence but also in AdS$_2$/CFT$_1$ correspondence. This is because the symmetry $D^1(2,1;\alpha)$ also arises in superconformal quantum mechanics [18, 19, 20]. The isometry of AdS$_2$ is $SO(2,1)$ which is a subgroup of the AdS$_3$ isometry $SO(2,2) \sim SO(2,1) \times SO(2,1)$. Accordingly, the superconformal symmetry in one dimension contains only a single $D^1(2,1;\alpha)$. The holographic study of AdS$_2$/CFT$_1$ correspondence directly from two dimensional gauged supergravity has not been performed extensively. This is in part due to the lack of gauged supergravities in two dimensions. Until now, only the maximal gauged supergravity and its truncation have appeared [21, 22]. Since AdS$_2$ can be obtained by dimensional reduction of AdS$_3$ on $S^1$ via a very-near-horizon limit [23, 24], the results obtained here might be useful in the study of deformations in $D^1(2,1;\alpha)$ superconformal mechanics.

The paper is organized as follow. In section 2, we will give a brief review of $N = 8$, $(SO(4) \times SO(4)) \ltimes T^{12}$ gauged supergravity along with some relations to the $N = (4,4)$ SCFT. Section 3 deals with a description of new critical points found in this work. In section 4, we will give some RG flow solutions describing deformations of the $N = (4,4)$ SCFT to other SCFTs in the IR. The possible supersymmetric flows to non-conformal theories are also explored in this section. We end the paper by giving some conclusions and discussions in section 5. The appendices summarize necessary ingredients needed in the construction of $N = 8$ theory and relevant formulae including the explicit form of some scalar potentials.
2. \( N = 8 \), \((SO(4) \times SO(4)) \ltimes T^{12}\) gauged supergravity in three dimensions

We now review the construction of \( N = 8 \) gauged supergravity with \((SO(4) \times SO(4)) \ltimes T^{12}\) gauge group. The theory has partially been studied before in [16]. We will explore the scalar potential of this theory in more details. Rather than follow the parametrization of \( SO(8,8)/SO(8) \times SO(8) \) coset manifold as in [16], we will use the parametrization similar to that of [23]. In this parametrization, it is more convenient to determine the residual gauge symmetry while the parametrization used in [16] gives a simple action of the translation generators \( T^{12} \) on scalar fields.

It has been argued in [14] that this theory is an effective theory of ten dimensional supergravity on \( AdS_3 \times S^3 \times S^3 \times S^1 \), or nine dimensional supergravity on \( AdS_3 \times S^3 \times S^3 \) for small \( S^1 \) radius, and describes the coupling of two massive spin-1 multiplets, containing twelve vectors, to the non-propagating supergravity multiplet of the reduction. All together, the resulting theory is \( N = 8 \) gauged supergravity with the scalar manifold \( SO(8,8)/SO(8) \times SO(8) \) and \((SO(4) \times SO(4)) \ltimes T^{12}\) gauge group.

The whole construction is similar to that given in [16] and [23]. We will work in the \( SO(8) \) R-symmetry covariant formulation of [12] with some relevant formulae and details explicitly given in appendix A. We first introduce the basis for a \( GL(16,\mathbb{R}) \) matrices

\[
(e_{mn})_{pq} = \delta_{mp} \delta_{nq}, \quad m, n, p, q = 1, \ldots, 16. \tag{2.1}
\]

The compact generators of \( SO(8,8) \) are then given by

\[
\begin{align*}
SO(8)^{(1)} : & \quad J_{IJ}^1 = e_{JI} - e_{IJ}, \quad I, J = 1, \ldots, 8, \\
SO(8)^{(2)} : & \quad J_{rs}^2 = e_{s+8,r+8} - e_{r+8,s+8}, \quad r, s = 1, \ldots, 8.
\end{align*} \tag{2.2}
\]

The non-compact generators corresponding to 64 scalars are identified as

\[
Y^{Kr} = e_{K,r+8} + e_{r+8,K}, \quad K, r = 1, \ldots, 8. \tag{2.3}
\]

In the formulation of [12], scalars transform as a spinor under \( SO(8)_R \) R-symmetry. It can be easily seen from the above equation that \( Y^{Kr} \) transform as a vector under \( SO(8)_R \) identified with \( SO(8)^{(1)} \) with generators \( J_{IJ}^1 \). We define the following \( SO(8)_R \) generators in a spinor representation by

\[
T^{IJ} = \begin{pmatrix}
\Gamma^{IJ} & 0 \\
0 & 0
\end{pmatrix} \tag{2.4}
\]

constructed from the \( 8 \times 8 \) \( SO(8) \) gamma matrices \( \Gamma^I \). We have defined

\[
\Gamma^{IJ} = -\frac{1}{4} \left( \Gamma^I (\Gamma^J)^T - \Gamma^J (\Gamma^I)^T \right) \tag{2.5}
\]
with the $8 \times 8$ gamma matrices $\Gamma^I$ are given in appendix A.

The gauge group $(SO(4) \times SO(4)) \rtimes T^{12}$ is embedded in $SO(8,8)$ as follow. We first form a diagonal subgroup of $SO(8) \times SO(8)$ with generators

$$SO(8)_{\text{diag}} : \quad J^{AB} = J_1^{AB} + J_2^{AB}, \quad A, B = 1, \ldots, 8. \quad (2.6)$$

The $SO(4) \times SO(4)$ part is generated by

$$SO(4)^+ : \quad j_1^{ab} = J^{ab},$$
$$SO(4)^- : \quad j_2^{\hat{a}\hat{b}} = J^{\hat{a}+4,\hat{b}+4}, \quad a, b, \hat{a}, \hat{b} = 1, \ldots, 4. \quad (2.7)$$

The “hat” indices refer to $SO(4)^-$. We now construct the translational generators $T^{28}$ as in (2.8)

$$t^{AB} = J_1^{AB} - J_2^{AB} + Y^{BA} - Y^{AB}$$

and identify $T^{12} \sim T^6 \times T^6$ generators as

$$t_1^{ab} = t^{ab}, \quad t_2^{\hat{a}\hat{b}} = t^{\hat{a}+4,\hat{b}+4}, \quad a, b, \hat{a}, \hat{b} = 1, \ldots, 4. \quad (2.9)$$

The gauge group is embedded in $SO(8,8)$ with a specific form of the embedding tensor. As shown in [26], there is no coupling among the $SO(4)^\pm$. The gauging is very similar to the $SO(4) \rtimes T^6$ gauged supergravity constructed in [15] with two factors of $SO(4) \rtimes T^6$. The embedding tensor is simply given by two copies of that given in [15]. We end up with two independent coupling constants

$$\Theta = g_1 \Theta_1 + g_2 \Theta_2. \quad (2.10)$$

where $\Theta_{1,2}$ describe the embedding of each $SO(4) \rtimes T^6$ factor of the full gauge group. We should note that supersymmetry allows for four independent couplings namely between the moment maps $g_1'(V(j_1^{ab}), V(t_1^{ab}))$, $g_2'(V(t_1^{ab}), V(t_1^{ab})))$, $g_3'(V(j_2^{\hat{a}\hat{b}}), V(t_2^{\hat{a}\hat{b}}))$ and $g_4'(V(t_2^{\hat{a}\hat{b}}), V(t_2^{\hat{a}\hat{b}}))$ in the T-tensor, see [15] and [16]. We have used a shorthand notation for $V^M_A$. However, the requirement that the theory admits a maximally supersymmetric vacuum at the origin of the scalar manifold imposes two conditions on the original four couplings. In more detail, the two conditions require $g_2' = -g_1'$ and $g_4' = -g_3'$. After rename the relevant couplings, we end up with the embedding tensor

$$\Theta_{abcd} = g_1 \epsilon^{\hat{a}\hat{b}\hat{c}\hat{d}} + g_2 \epsilon^{a\hat{b}c\hat{d}}. \quad (2.11)$$

This embedding tensor together with the formulae in appendix A and an explicit parametrization of the coset representative of $SO(8,8)/SO(8) \times SO(8)$ can be used to compute the scalar potential. We will analyze the resulting potential on submanifolds
of $SO(8,8)/SO(8) \times SO(8)$ invariant under some subgroups of $SO(4) \times SO(4)$ in the next section.

Before looking at the critical points, we give a review of the relation between $(SO(4) \times SO(4)) \rtimes T^{12}$, $N = 8$ gauged supergravity and $N = (4,4)$ SCFT. The semisimple part of the gauge group $SO(4)^+ \times SO(4)^-$ corresponds to the isometry of $S^3 \times S^3$. Together with the usual $SO(2,2)$ isometry of $AdS_3$, they constitute the bosonic subgroup $SO(2,1)_L \times SU(2)_L^+ \times SU(2)_L^- \times SO(2,1)_R \times SU(2)_R^+ \times SU(2)_R^-$ of the superconformal group $D^1(2,1;\alpha) \times D^1(2,1;\alpha)$ via the isomorphisms $SO(2,2) \sim SO(2,1)_L \times SO(2,1)_R$ and $SO(4)^\pm \sim SU(2)_L^\pm \times SU(2)_R^\pm$. The $\alpha$ parameter is identified with the ratio of the coupling constant $g_2 = \alpha g_1$. For positive $\alpha$, the theory describes the dimensional reduction of nine dimensional supergravity on $S^3 \times H^3$. For negative $\alpha$, it may possibly describe the reduction on $S^3 \times S^3$, $H^3$ where $H^3$ is a hyperbolic space in three dimensions.

The translational part $T^{12}$ of the gauge group describes twelve massive vector fields [26]. The massive vector fields will show up in the vacuum of the theory via twelve massless scalars in the adjoint representation of $SO(4) \times SO(4)$. These are Goldstone bosons for the $T^{12}$ symmetry since the vacuum is invariant only under $SO(4)^+ \times SO(4)^-$ not the full gauge group. We will see this when we compute the mass spectrum of scalar fields.

### 3. Some critical points of $N = 8$, $(SO(4) \rtimes SO(4)) \rtimes T^{12}$ gauged supergravity

We now look for critical points of the $N = 8$ gauged supergravity constructed in the previous section. Analyzing the scalar potential on the full 64-dimensional scalar manifold $SO(8,8)/SO(8) \times SO(8)$ is beyond our reach with the present-time computer. We then employ an effective method given in [27] to find some interesting critical points on a submanifold invariant under some subgroup of the gauge group. A group theoretical argument guarantees that the corresponding critical points are critical points of the scalar potential on the full scalar manifold. Even on these truncated manifolds, the explicit form of the potential is still very complicated. Therefore, in most cases, we refrain from giving the full expression for the potential.

At the trivial critical point with all scalars vanishing, the full gauge group $(SO(4) \times SO(4)) \rtimes T^{12}$ is broken down to its maximal compact subgroup $SO(4) \times SO(4)$ corresponding to the isometry of $S^3 \times S^3$. The 64 scalars transform under $SO(8) \times SO(8) \subset SO(8) \times SO(8)$.
SO(8) as $(8,8)$. Then, under the $SO(4)^+ \times SO(4)^- \subset SO(8)_{\text{diag}}$, they transform as

$$8 \times 8 = [(4^+, 1^+) + (1^-, 4^-)] \times [(4^+, 1^+) + (1^-, 4^-)]$$

$$= (1^+ + 6^+ + 9^+, 1^+) + (1^-, 1^- + 6^- + 9^-) + (4^+, 4^-) + (4^-, 4^+). \quad (3.1)$$

We can further decompose the above representations into $SU(2)_L^+ \times SU(2)_R^+ \times SU(2)_L^- \times SU(2)_R^-$ representations labeled by $(\ell^+_L, \ell^+_R, \ell^-_L, \ell^-_R)$ as follows:

$$8 \times 8 = (1, 1; 1, 1) + (3, 1; 1, 1) + (3, 3; 1, 1) + (3, 3; 1, 1)$$

$$+(1, 1; 1, 1) + (1, 1; 1, 3) + (1, 1; 3, 1) + (1, 1; 3, 3)$$

$$+(2, 2; 2, 2) + (2, 2; 2, 2). \quad (3.2)$$

The result precisely agrees with the representation content obtained from the $AdS_3 \times S^3 \times S^3$ reduction [3]. For conveniences, we also repeat the massive spin-1 multiplets $(0, 1; 0, 1)_S$ and $(1, 0; 1, 0)_S$ of the $AdS_3 \times S^3 \times S^3$ reduction in Table 1 and 2.

We can now compute the scalar potential by using the formulae in appendix [A]. After expanding the potential around $L = I$, we find the scalar mass spectrum at the maximally supersymmetric vacuum as shown in Table 3. The $AdS_3$ radius is given by $L = \frac{1}{\sqrt{-g_0}}$, and the value of the potential at this point is $V_0 = -64(g_1 + g_2)^2$. Using the relation $m^2 L^2 = \Delta(\Delta - 2)$ and $\Delta = h_R + h_L$, we can verify that the mass spectrum agrees with the values of $h_R$ and $h_L$ in Table 4 and 5. As mentioned before, there are twelve massless Goldstone bosons transforming in the adjoint representation $(1, 6) + (6, 1)$ of $SO(4) \times SO(4)$. Note also that there is a Minkowski vacuum at $g_2 = -g_1$ or $\alpha = -1$. 

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**Table 1:** The massive spin-1 multiplet $(0, 1; 0, 1)_S$.

| $h_R$ | $\frac{1}{1+\alpha}$ | $\frac{3+\alpha}{2(1+\alpha)}$ | $\frac{2+\alpha}{1+\alpha}$ |
|-------|----------------------|-----------------------------|----------------------|
| $\frac{\alpha}{1+\alpha}$ | $(0, 1; 0, 1)$ | $(0, 1; \frac{1}{2}, \frac{1}{2})$ | $(0, 1; 0, 0)$ |
| $\frac{3+\alpha}{2(1+\alpha)}$ | $(\frac{1}{2}, \frac{1}{2}; 0, 1)$ | $(\frac{1}{2}, \frac{1}{2}; \frac{1}{2}, \frac{1}{2})$ | $(\frac{1}{2}, \frac{1}{2}; 0, 0)$ |
| $\frac{2+\alpha}{1+\alpha}$ | $(0, 0; 0, 1)$ | $(0, 0; \frac{1}{2}, \frac{1}{2})$ | $(0, 0; 0, 0)$ |

**Table 2:** The massive spin-1 multiplet $(1, 0; 1, 0)_S$.

| $h_R$ | $\frac{1}{1+\alpha}$ | $\frac{3+\alpha}{2(1+\alpha)}$ | $\frac{2+\alpha}{1+\alpha}$ |
|-------|----------------------|-----------------------------|----------------------|
| $\frac{1}{1+\alpha}$ | $(1, 0; 1, 0)$ | $(1, 0; \frac{1}{2}, \frac{1}{2})$ | $(1, 0; 0, 0)$ |
| $\frac{3+\alpha}{2(1+\alpha)}$ | $(\frac{1}{2}, \frac{1}{2}; 1, 0)$ | $(\frac{1}{2}, \frac{1}{2}; \frac{1}{2}, \frac{1}{2})$ | $(\frac{1}{2}, \frac{1}{2}; 0, 0)$ |
| $\frac{2+\alpha}{1+\alpha}$ | $(0, 0; 0, 1)$ | $(0, 0; \frac{1}{2}, \frac{1}{2})$ | $(0, 0; 0, 0)$ |
Apart from the trivial critical point at \((3.1)\), we find that there are four singlets, two from the obvious ones \((1^+ \times 1^+, 1^- \times 1^-)\) and the other two from the product \((4^+ \times 4^-, 4^- \times 4^+)\). They correspond to the following non-compact generators

\[
\begin{align*}
\tilde{Y}_1 &= Y^{11} + Y^{22} + Y^{33} + Y^{44}, \\
\tilde{Y}_2 &= Y^{55} + Y^{66} + Y^{77} + Y^{88}, \\
\tilde{Y}_3 &= Y^{51} + Y^{62} + Y^{73} + Y^{84}, \\
\tilde{Y}_4 &= Y^{15} + Y^{26} + Y^{37} + Y^{48}.
\end{align*}
\] (3.3)

The coset representative is accordingly parametrized by

\[
L = e^{a_1 \tilde{Y}_1} e^{a_2 \tilde{Y}_2} e^{a_3 \tilde{Y}_3} e^{a_4 \tilde{Y}_4}. \tag{3.4}
\]

Apart from the trivial critical point at \(a_1 = a_2 = a_3 = a_4 = 0\), we find the following critical points.

- A \(dS_3\) critical point is at \(a_1 = \frac{1}{2} \ln \frac{g_1}{g_2 - g_1}, \) \(a_2 = \frac{1}{2} \ln \frac{g_2}{g_1 - g_2}\) and \(a_3 = a_4 = 0\). The cosmological constant is \(V_0 = \frac{64g_1g_2}{(g_1 - g_2)^2} \). This critical point has \(SO(4) \times SO(4)\) symmetry since \(a_3 = a_4 = 0\). Non-zero \(a_3\) or \(a_4\) would break \(SO(4) \times SO(4)\) to its diagonal subgroup.

- A non-supersymmetric \(AdS_3\) is given by \(a_1 = \frac{1}{2} \ln \frac{\sqrt{g_1 - 4g_2 - \sqrt{g_1^2 - 4g_2g_1}}}{2\sqrt{g_1}}\) and \(a_2 = a_3 = a_4 = 0\). The cosmological constant is

\[
V_0 = -32 \left[ g_1^2 + 4g_2^2 - 6g_1g_2 + (4g_2 - g_1)\sqrt{g_1(g_1 - 4g_2)} \right]. \tag{3.5}
\]
a_1 is real for \( g_1 > 0 \) and \( g_2 < 0 \), and the critical point is \( AdS_3 \), \( V_0 < 0 \), for \( g_1 > 0 \) and \( g_2 < -\frac{\sqrt{2} + 1}{2} g_1 \). An equivalent critical point is given by \( a_2 \neq 0 \) and \( a_1 = a_3 = a_4 = 0 \) but with \( g_1 \leftrightarrow g_2 \). For later reference, we will call this critical point \( P_1 \).

- Another non-supersymmetric critical point is at \( a_4 = \ln \frac{\sqrt{g_1 + \sqrt{3g_2}}}{\sqrt{g_1 - \sqrt{3g_2}}} \) with \( g_2 = \frac{1}{9} (\sqrt{13} - 2) g_1 \) and \( V_0 = -\frac{8}{3} (43 + 13\sqrt{13}) g_1^2 \). In this case, only a specific value of \( \alpha \) gives a critical point. The residual gauge symmetry in this case is \( SO(4)_{\text{diag}} \). We will label this critical point as \( P_2 \).

- There is another \( dS_3 \) critical point which is invariant under \( SO(4) \times SO(4) \) and characterized by

\[
\begin{align*}
a_1 &= \frac{1}{2} \ln \frac{g_2}{g_2 - g_1}, \\
a_2 &= \frac{1}{2} \ln \frac{g_1 - 2g_2}{g_1 - g_2}, \\
V_0 &= \frac{64g_2^2 (g_1^3 - g_1^2 g_2 - 8g_3^3)}{(g_1 - g_2)^3}. \quad (3.6)
\end{align*}
\]

\( a_1 \) and \( a_2 \) are real for \( g_1 < 0 \) and \( g_2 < g_1 \). In this range, we find \( V_0 > 0 \), so this critical point is \( dS_3 \).

The full scalar potential for the four scalars is given in appendix [3].

At this stage, we should note an interesting result discovered in [17] but with a compact gauge group \( SO(4) \times SO(4) \). This solution describes a marginal deformation of \( N = (4, 4) \) SCFT to \( N = (3, 3) \) SCFT and has an interpretation in term of a reconnection of D5-branes in the double D1-D5 system. The solution is also encoded in our present framework. In this case, we must set \( g_2 = g_1 \), or equivalently setting \( \alpha = 1 \) in order to get massless (marginal) scalars preserving the \( SO(4) \) diagonal subgroup of \( SO(4) \times SO(4) \).

Follow [17], we further truncate the four scalars to two via

\[
a_2 = a_1, \quad a_4 = -a_3. \quad (3.7)
\]

This is a consistent truncation for \( g_2 = g_1 \) since it corresponds to a fixed point of an inner automorphism that leaves the embedding tensor invariant [17]. We find a critical point at

\[
e^{a_1 + a_4} = 1 + \sqrt{1 - e^{2a_1}}, \quad V_0 = -256g_1^2 \quad (3.8)
\]

with the corresponding \( A_4 \) tensor given by

\[
A^{IJ}_1 = \text{diag} \left( -8g_1, -8g_1, -8g_1, 8g_1, 8g_1, 8g_1, -8g_1 \sqrt{4e^{-2a_1} - 3}, 8g_1 \sqrt{4e^{-2a_1} - 3} \right). \quad (3.9)
\]
\[
\begin{array}{|c|c|}
\hline
SO(4)^+ \times SO(4)^- & m^2 L^2 \\
\hline
(1, 1) & \frac{g_2 + \sqrt{g_1 (g_1 - 4g_2)}}{12g_2} \\
& - \frac{16g_2^2 + 20g_1g_2 - 6g_1^2 + 2(g_1 + 2g_2) \sqrt{g_1 (g_1 - 4g_2)}}{g_1^2 - 4g_1g_2 - 4g_2^2} \\
& + \frac{4g_2^2 + 14g_1g_2 - 3g_1^2 + (4g_2 - g_1) \sqrt{g_1 (g_1 - 4g_2)}}{2(g_1^2 - 4g_1g_2 - 4g_2^2)} \\
& - \frac{3g_1^2 - 30g_1g_2 + 12g_2^2 + 3(3g_1 - 4g_2) \sqrt{g_1 (g_1 - 4g_2)}}{2(g_1^2 - 4g_1g_2 - 4g_2^2)} \\
(6, 1) & 0 \\
(9, 1) & \frac{g_1^2 - 6g_1g_2 + (2g_2 - g_1) \sqrt{g_1 (g_1 - 4g_2)}}{4g_2 (2g_2 + g_1)} \\
(1, 1) & 0 \\
(1, 6) & - \frac{4g_2^2 - 24g_1g_2 - 8g_1^2 + (4g_1 - g_2) \sqrt{g_1 (g_1 - 4g_2)}}{g_1^2 - 4g_1g_2 - 4g_2^2} \\
(1, 9) & - \frac{4g_2^2 + 14g_1g_2 - 3g_1^2 + (4g_2 - g_1) \sqrt{g_1 (g_1 - 4g_2)}}{2(g_1^2 - 4g_1g_2 - 4g_2^2)} \\
(4, 4) & - \frac{12g_2^2 - 30g_1g_2 + 3g_1^2 + (9g_1 - 12g_2) \sqrt{g_1 (g_1 - 4g_2)}}{2(g_1^2 - 4g_1g_2 - 4g_2^2)} \\
\hline
\end{array}
\]

Table 4: The scalar mass spectrum of the \(SO(4) \times SO(4)\) critical point \(P_1\).

\[
\begin{array}{|c|c|}
\hline
SO(4) & m^2 L^2 \\
\hline
1 & 13.6358, 6.0931, 3.3703, 3.1180 \\
6 & 0_{(x18)} \\
9 & \frac{8}{29} (7\sqrt{13} - 12)_{(x9)}, \frac{4}{29} (5\sqrt{13} - 21)_{(x9)}, \\
& \frac{4}{29} (8 + 5\sqrt{13})_{(x9)}, \frac{4}{29} (19\sqrt{13} - 74)_{(x9)} \\
\hline
\end{array}
\]

Table 5: The scalar mass spectrum of the \(SO(4)\) critical point \(P_2\) for \(g_2 = \frac{\sqrt{15} - 2}{9} g_1\).

We can see that as long as \(a_1 \neq 0\), the \(N = (4, 4)\) supersymmetry is broken to \(N = (3, 3)\). We refer the reader to [] for the full discussion of this vacuum.

We now analyze the scalar masses at critical points \(P_1\) and \(P_2\) to check their stability. For critical point \(P_1\), it is useful to classify the 64 scalars according to their representations under the residual symmetry \(SO(4) \times SO(4)\). The result is shown in Table 4. Similar to the trivial critical point, there are 12 massless scalars corresponding to the broken \(T^{12}\) symmetry. The stability bound, or BF bound \(m^2 L^2 \geq -1\), is satisfied by \(-\frac{13 + 9\sqrt{2}}{2} g_1 < g_2 < -1 + \frac{\sqrt{1 + \sqrt{2}}}{2} g_1\).

For critical point \(P_2\), we can compute all scalar masses as shown in Table 5. It is easily seen that all masses satisfy the BF bound. There are 18 massless Goldstone bosons corresponding to the symmetry breaking \((SO(4) \times SO(4)) \times T^{12} \rightarrow SO(4)\).
3.2 Critical points on the $SO(2)_{\text{diag}} \times SO(2)_{\text{diag}}$ invariant manifold

We now proceed to consider a smaller residual symmetry $SO(2)_{\text{diag}} \times SO(2)_{\text{diag}} \subset SO(4)_{\text{diag}}$. Under $SO(2) \times SO(2)$, the $SO(4)$ fundamental representation $4$ decomposes according to $4 \rightarrow (2, 1) + (1, 2)$. Substituting this decomposition for $4^+$ and $4^-$ in (3.1) and taking the product to form a diagonal subgroup, we find that there are sixteen singlets given by the non-compact generators

$$
\begin{align*}
\bar{Y}_1 &= Y^{11} + Y^{22}, & \bar{Y}_2 &= Y^{33} + Y^{44}, & \bar{Y}_3 &= Y^{55} + Y^{66}, & \bar{Y}_4 &= Y^{77} + Y^{88}, \\
\bar{Y}_5 &= Y^{15} + Y^{26}, & \bar{Y}_6 &= Y^{37} + Y^{48}, & \bar{Y}_7 &= Y^{51} + Y^{62}, & \bar{Y}_8 &= Y^{73} + Y^{84}, \\
\bar{Y}_9 &= Y^{12} - Y^{21}, & \bar{Y}_{10} &= Y^{34} - Y^{43}, & \bar{Y}_{11} &= Y^{56} - Y^{65}, & \bar{Y}_{12} &= Y^{78} - Y^{87}, \\
\bar{Y}_{13} &= Y^{16} - Y^{25}, & \bar{Y}_{14} &= Y^{38} - Y^{47}, & \bar{Y}_{15} &= Y^{52} - Y^{61}, & \bar{Y}_{16} &= Y^{74} - Y^{83}.
\end{align*}
$$

(3.10)

The coset representative can be parametrized by

$$
L = \prod_{i=1}^{16} e^{a_i \bar{Y}_i}.
$$

(3.11)

Unlike the previous case, the scalar potential is so complicated that it is not possible to make the full analysis. However, with some ansatz, we find one non-trivial critical point at

$$
\begin{align*}
a_1 &= a_2 = \frac{1}{2} \ln 2, & a_3 &= -a_4 = \frac{1}{2} \ln \left( \frac{g_2 - 6g_1 + \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}}{2g_2} \right), \\
V_0 &= 64(8g_1^2 - g_2^2).
\end{align*}
$$

(3.12)

$a_3$ and $a_4$ are real for $g_1 > 0$ and $g_2 \geq -6g_1$. In this range, we find $V_0 < 0$ if $g_2 < -2\sqrt{2}g_1$. Therefore, it is possible to have an $AdS_3$ critical point. The residual symmetry is $SO(4) \times SO(2) \times SO(2)$. We will denote this critical point by $P_3$ for later reference.

The stability of this critical point can be verified from the scalar mass spectrum given in Table 3 in which $\alpha_i$ are eigenvalues of the submatrix

$$
\frac{1}{8g_1^2 - g_2^2} \begin{pmatrix}
-80g_1^2 & x_1 & x_2 \\
x_1 & -\frac{g_1^2}{4} & -\frac{2g_2^2}{3} \\
x_2 & -\frac{2g_1^2}{3} & -\frac{g_2^2}{3}
\end{pmatrix}
$$

(3.13)

with the following elements

$$
x_1 = 2\sqrt{2}g_1 \left( 6g_1 + g_2 - \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2} \right)
$$

and

$$
x_2 = 2\sqrt{2}g_1 \left( 6g_1 + g_2 + \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2} \right).
$$

(3.14)
Table 6: The scalar mass spectrum of the $SO(4) \times SO(2) \times SO(2)$ critical point $P_3$.

| $SO(4) \times SO(2) \times SO(2)$ | $m^2 L^2$ |
|-----------------------------------|----------|
| (4, 2, 1)                         | $-\frac{60g_1^2 - 14g_1g_2 + g_2^2 + (6g_1 - 3g_2)}{16g_1^2 - 2g_2^2} + \frac{36g_1^2 - 12g_1g_2 - 3g_2^2}{\sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}}$ |
| (4, 1, 2)                         | $-\frac{60g_1^2 - 24g_1g_2 + g_2^2 + (3g_2 - 6g_1)}{16g_1^2 - 2g_2^2} + \frac{36g_1^2 - 12g_1g_2 - 3g_2^2}{\sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}}$ |
| (4, 2, 1)                         | $-\frac{124g_1^2 - 3g_2^2 + (2g_2 - 6g_1)}{16g_1^2 - 2g_2^2} + \frac{36g_1^2 - 12g_1g_2 - 3g_2^2}{\sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}}$ |
| (4, 1, 2)                         | $-\frac{124g_1^2 - 3g_2^2 - (2g_2 - 6g_1)}{16g_1^2 - 2g_2^2} + \frac{36g_1^2 - 12g_1g_2 - 3g_2^2}{\sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}}$ |
| (1, 2, 1)                         | $\frac{6g_1^2 + 24g_1g_2 + 72g_2^2 + 2(g_2 - 6g_1)}{54g_1^2 - 2g_2^2} + \frac{36g_1^2 - 12g_1g_2 - 3g_2^2}{\sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}}$ |
| (1, 1, 2)                         | $\frac{8g_1^2 - g_2^2}{48g_1^2} + \frac{1}{\sqrt{g_2^2 - 8g_1^2}}$ |
| (9, 1, 1)                         | 0        |
| (6, 1, 1)                         | 0        |
| 2 × (1, 2, 2)                     | $\alpha_1$, $\alpha_2$, $\alpha_3$ |
| 2 × (1, 1, 1)                     | 0        |
| (1, 1, 1)                         | 0        |

Their numerical values can be obtained upon specifying the values of $g_1$ and $g_2$.

For all but (1, 1, 1) and (1, 1, 2) scalars, the masses are above the BF bound for $-6g_1 < g_2 < -2\sqrt{2}g_1$. The mass squares of (1, 1, 1) scalars are above the BF bound for $-6g_1 < g_2 < -4.47g_1$. For (1, 1, 2) scalars, the mass squares are above the BF bound for $-6g_1 < g_2 < \mathcal{X}$ with $\mathcal{X}$ being the first root of $p(\mathcal{X}) = 1088g_1^4 - 384g_1^3\mathcal{X} + 352g_1^2\mathcal{X}^2 - 144g_1\mathcal{X}^3 - 37\mathcal{X}^4 = 0$. This can be translated to the value of $\alpha$ by setting $\mathcal{X} = \alpha g_1$. The equation $p(\mathcal{X}) = 0$ gives the value of $\alpha = -5.93479$. The stability is obtained in the range $-6g_1 < g_2 < -5.93479g_1$ which is very narrow. Notice that for $g_2 = -6g_1$, we find $a_3 = a_4 = 0$, and the symmetry is enhanced to $SO(4) \times SO(4)$. It can be checked that this critical point indeed becomes critical point $P_1$ with $g_2 = -6g_1$.

### 3.3 Critical points on the $SU(2)^+_L \times SU(2)^-_L$ invariant manifold

One interesting deformation of $N = (4, 4)$ SCFT is the chiral supersymmetry breaking $(4, 4) \to (4, 0)$. The realization of this breaking in the D1-D5 system has been studied in [28]. Another gravity dual of $N = (4, 0)$ SCFT from string theory has been studied in [29], and the marginal perturbation driving $N = (4, 4)$ SCFT to the $N = (4, 0)$ SCFT has been identified in [30]. This supersymmetry breaking is not possible in the compact $SO(4) \times SO(4)$ gauging of [13] since there are no scalars which are singlets under a non-trivial subgroup of $SO(4) \times SO(4)$ in order to become the R-symmetry of $N = (4, 0)$. 


This is however possible in the present gauging. According to (3.2), we see that there are eight singlets under \(SU(2)_{1}^{+} \times SU(2)_{1}^{-}\) given by

\[
(1, 1; 1, 1) + (1, 1; 1, 1) + (1, 3; 1, 1) + (1, 1; 1, 3).
\]

They correspond to the following non-compact generators

\[
\begin{align*}
\hat{Y}_1 &= Y^{11} + Y^{22} + Y^{33} + Y^{44}, & \hat{Y}_2 &= Y^{12} - Y^{21} + Y^{34} - Y^{43}, \\
\hat{Y}_3 &= Y^{13} - Y^{31} - Y^{24} + Y^{42}, & \hat{Y}_4 &= Y^{14} - Y^{41} + Y^{23} - Y^{32}, \\
\hat{Y}_5 &= Y^{55} + Y^{66} + Y^{77} + Y^{88}, & \hat{Y}_6 &= Y^{56} - Y^{65} + Y^{78} - Y^{87}, \\
\hat{Y}_7 &= Y^{57} - Y^{75} - Y^{68} + Y^{86}, & \hat{Y}_8 &= Y^{58} - Y^{85} + Y^{67} - Y^{76}.
\end{align*}
\]  

(3.16)

We can parametrize the coset representative accordingly

\[
L = e^{b_1 \hat{Y}_1} e^{a_2 \hat{Y}_2} e^{a_3 \hat{Y}_3} e^{a_4 \hat{Y}_4} e^{b_5 \hat{Y}_5} e^{a_6 \hat{Y}_6} e^{a_7 \hat{Y}_7} e^{a_8 \hat{Y}_8}
\]  

(3.17)

in which \(b_1\) and \(b_5\) denote the \(SO(4) \times SO(4)\) singlets. Some critical points are given below.

- There is an \(AdS_3\) critical point characterized by

\[
a_2 = \cosh^{-1} \sqrt{\frac{g_1 + \sqrt{g_1(g_1 - 4g_2)}}{4g_1}},
\]

\[
V_0 = -32 \left[ g_1^2 + 4g_2^2 - 6g_1g_2 + (4g_2 - g_1) \sqrt{g_1(g_1 - 4g_2)} \right].
\]

(3.18)

The cosmological constant is the same as \(P_1\), but the residual gauge symmetry is just \(SO(4)^- \times SU(2)_{L}^{+} \times U(1)_{R}^{+}\) in which \(U(1)_{R}^{+} \subset SU(2)_{R}^{+}\).

- There is a \(dS_3\) critical point given by

\[
a_2 = \frac{1}{2} \ln \left[ \frac{\sqrt{g_1(2g_2 - g_1)} \left( g_1^2 - g_2^2 + 2g_1 \left( g_2 + \sqrt{g_2(2g_1 - g_2)} \right) \right)}{(g_1 - g_2) \left( g_1 + \sqrt{g_2(2g_1 - g_2)} \right)} \right],
\]

\[
a_6 = \frac{1}{2} \ln \frac{g_1 + \sqrt{g_2(2g_1 - g_2)}}{g_1 - g_2}, \quad V_0 = \frac{64g_1^2g_2^2}{(g_1 - g_2)^2}.
\]

(3.19)

This critical point is invariant under \(SU(2) \times SU(2) \times U(1)^2\) symmetry.
• There is another $dS_3$ vacuum given by $a_4 = a_3 = a_2$, $a_8 = a_7 = a_6$ and

$$b_1 = \ln \frac{g_2}{(g_2 - g_1) \cosh^3 a_2}, \quad b_5 = \ln \frac{g_1}{(g_1 - g_2) \cosh^3 a_6},$$

$$V_0 = \frac{64g_1^2g_2^2}{(g_1 - g_2)^2} \quad (3.20)$$

with $SU(2) \times SU(2)$ symmetry.

### 3.4 Critical points on the $SU(2)_{L_{\text{diag}}} \text{ invariant manifold}$

We further reduce the residual symmetry to $SU(2)_{L_{\text{diag}}} \subset SU(2)^+_L \times SU(2)^-_L$. Under $SO(4)_{\text{diag}}$, we already know that the 64 scalars transform as four copies of $1 + 6 + 9$. We can then further truncate to $SU(2)_{L_{\text{diag}}} \times SU(2)_{R_{\text{diag}}}$. They can be parametrized by the coset representative

$$L = \prod_{i=1}^{16} e^{a_i Y_i} \quad (3.21)$$

in which the non-compact generators are defined by

$$Y_1 = \frac{1}{2} (Y^{15} + Y^{26} + Y^{37} + Y^{48}), \quad Y_2 = \frac{1}{2} (Y^{16} - Y^{25} + Y^{38} - Y^{47}),$$

$$Y_3 = \frac{1}{2} (Y^{17} - Y^{35} - Y^{28} + Y^{46}), \quad Y_4 = \frac{1}{2} (Y^{18} - Y^{45} + Y^{27} - Y^{36}),$$

$$Y_5 = \frac{1}{2} (Y^{51} + Y^{62} + Y^{73} + Y^{84}), \quad Y_6 = \frac{1}{2} (Y^{52} - Y^{61} + Y^{74} - Y^{83}),$$

$$Y_7 = \frac{1}{2} (Y^{53} - Y^{71} - Y^{64} + Y^{82}), \quad Y_8 = \frac{1}{2} (Y^{54} - Y^{81} + Y^{63} - Y^{72}),$$

$$Y_9 = \frac{1}{2} (Y^{11} + Y^{22} + Y^{33} + Y^{44}), \quad Y_{10} = \frac{1}{2} (Y^{12} - Y^{21} + Y^{34} - Y^{48}),$$

$$Y_{11} = \frac{1}{2} (Y^{13} - Y^{31} - Y^{24} + Y^{42}), \quad Y_{12} = \frac{1}{2} (Y^{14} - Y^{41} + Y^{23} - Y^{32}),$$

$$Y_{13} = \frac{1}{2} (Y^{55} + Y^{66} + Y^{77} + Y^{88}), \quad Y_{14} = \frac{1}{2} (Y^{56} - Y^{65} + Y^{78} - Y^{87}),$$

$$Y_{15} = \frac{1}{2} (Y^{57} - Y^{75} - Y^{68} + Y^{86}), \quad Y_{16} = \frac{1}{2} (Y^{58} - Y^{85} + Y^{67} - Y^{76}). \quad (3.22)$$

From a very complicated potential, we find one non-supersymmetric critical point given by

$$a_6 = \ln \frac{\sqrt{g_2} - \sqrt{3}g_1}{\sqrt{g_2} + \sqrt{3}g_1}, \quad g_2 = (2 + \sqrt{13})g_1,$$

$$V_0 = -8(469 + 131\sqrt{13})g_1^2 \quad (3.23)$$

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which is invariant under $SU(2) \times U(1)$ symmetry.

Apart from $P_1$, $P_2$ and $P_3$, we have not given the complete mass spectra for other $AdS_3$ critical points. This is mainly because computing the full scalar masses for those critical points is much more involved. The stability of these critical points is uncertain without the full scalar masses. A partial check shows that at least the scalar masses for the singlets in each sector satisfy the BF bound. It could happen that some other scalars might have masses violating the bound. However, similar to the three stable critical points studied above, it is likely that the other critical points are stable for some values of $\alpha$.

4. Deformations of the $N = (4, 4)$ SCFT

In this section, we will study some RG flow solutions interpolating between the maximally supersymmetric $SO(4) \times SO(4)$ critical point in the UV and some of the non-supersymmetric critical points identified in the previous section. We will also consider supersymmetric flows to non-conformal field theories in the IR.

4.1 Supersymmetric deformations

We begin with supersymmetric solutions which can be obtained by finding solutions of the associated BPS equations. We have not found any supersymmetric critical point apart from the trivial one at $L = \mathbf{1}$, so we only expect to find flow solutions to non-conformal field theories. In these flows, the solutions interpolate between the UV point at which all scalars vanish and the IR with infinite values of scalar vev’s. Since supersymmetric solutions are of interest here, we need the supersymmetry transformations of fermions which in the present case are given by the non-propagating gravitini $\psi^I_{\mu}$ and the spin-$\frac{1}{2}$ fields $\chi^{iI}$. Their supersymmetry transformations are given by, see [12] for more details and conventions,

$$
\delta \psi^I_{\mu} = \mathcal{D}_{\mu} \epsilon^I + g A^I_{1\mu} \gamma^I \epsilon^J,
$$

$$
\delta \chi^{iI} = \frac{1}{2} (\delta^{IJ} 1 - f^{IJ})^i \mathcal{D}_J \epsilon^J - g A^J_{2i} \epsilon^J.
$$

These equations will be used to find supersymmetric solutions in the next subsections.

4.1.1 A supersymmetric flow to $SO(4) \times SO(4)$ non-conformal field theory

We first look for a simple solution preserving $SO(4) \times SO(4)$ symmetry. Accordingly, only $a_1$ and $a_2$ in equation (3.4) are turned on in order to preserve the full $SO(4) \times SO(4)$. Using the standard domain wall ansatz for the metric

$$
ds^2 = e^{2A} dx^2_{1,1} + dr^2
$$


with $A$ depending only on the radial coordinate $r$, we find the BPS equations

$$a_1' + 8g_1e^{2a_1}(e^{2a_1} - 1) = 0, \quad (4.4)$$

$$a_2' + 8g_2e^{2a_2}(e^{2a_2} - 1) = 0, \quad (4.5)$$

$$A' + 8\left[g_1e^{2a_1}(e^{2a_1} - 2) + g_2e^{2a_2}(e^{2a_2} - 2)\right] = 0 \quad (4.6)$$

where we have imposed the projector $\gamma_\nu\epsilon^I = -\epsilon^I$, $I = 2, 4, 5, 8$ and $\gamma_r\epsilon^I = \epsilon^I$, $I = 1, 3, 6, 7$. The $'$ denotes the $r$-derivative. The resulting solution is then half-supersymmetric with $N = (4, 4)$ Poincare supersymmetry in the dual two dimensional field theory. Equations (4.4) and (4.5) can be solved for $a_1$ and $a_2$ as an implicit function of $r$. The result is

$$r = c_1 - \frac{1}{16g_1}\left[e^{-2a_1} + \ln\left(1 - e^{-2a_1}\right)\right], \quad (4.7)$$

$$r = c_2 - \frac{1}{16g_2}\left[e^{-2a_2} + \ln\left(1 - e^{-2a_2}\right)\right] \quad (4.8)$$

with integration constants $c_1$ and $c_2$. Equation (4.6) can immediately be integrated to give $A$ as a function of $a_1$ and $a_2$. The result is

$$A = 2(a_1 + a_2) - \frac{1}{2}\ln(1 - e^{-2a_1}) - \frac{1}{2}\ln(1 - e^{-2a_2}). \quad (4.9)$$

In the UV, the dual field theory is conformal with $a_1 = a_2 = 0$. Near this point, the scalars behave as $a_1 \approx e^{-16g_1r} = e^{-2g_1rUV}\frac{r}{2+2\epsilon}$ and $a_2 \approx e^{-16g_2r} = e^{-2g_2rUV}\frac{r}{2+2\epsilon}$. We see that $a_{1,2} \to 0$ as $r \to \infty$. In this limit, we find $A' \approx 8(g_1 + g_2) = \frac{1}{L_{UV}}$ or $A \approx \frac{r}{L_{UV}}$ which gives the maximally supersymmetric $AdS_3$.

As $a_1, a_2 \to \infty$, we find $r \to$ constant as it should. Near $a_1, a_2 \to \infty$, equations (4.7) and (4.8) give $a_1 \approx -\frac{1}{4}\ln(32g_1r)$ and $a_2 \approx -\frac{1}{4}\ln(32g_2r)$. From equation (4.9), we find $A \approx a_1 + a_2 = -\frac{1}{4}\ln[(32r)^2g_1g_2]$. Accordingly, the metric becomes a domain wall in the IR

$$ds^2 = \frac{1}{32r\sqrt{g_1g_2}}dx_{1,1}^2 + dr^2. \quad (4.10)$$

The full bosonic symmetry is $ISO(1, 1) \times SO(4) \times SO(4)$ corresponding to non-conformal field theory with $N = (4, 4)$ supersymmetry.

However, flows of this type generally involve singularities. Various types of possible singularities have been classified in [32]. According to the result of [32], physical singularities are the ones at which the scalar potential is bounded from above. However, with the solution given above, the potential becomes infinite in this case. Therefore, the corresponding flow solution is not physically acceptable by the criterion of [32]. Since the framework we have used could be uplifted to ten dimensions via $S^3 \times S^3 \times S^1$ reduction, it is interesting to investigate whether this singularity is resolved in the full string theory.
4.1.2 A half-supersymmetric domain wall

We then look for a more general supersymmetric solution. The scalar sector of interest here is the $SU(2)_{L}^{+} \times SU(2)_{L}^{−}$ invariant one given in (3.17). We first relabel the scalars $(a_2, a_3, a_4, a_6, a_7, a_8)$ to $(b_2, b_3, b_4, b_6, b_7, b_8)$ in order to work with a uniform notation.

We begin with the BPS equations given by $\delta \chi^{\mu} = 0$

\begin{align*}
 b_1' & = -16g_1 e^{b_1} (e^{b_1} - \text{sech} b_2 \text{sech} b_3 \text{sech} b_4), \quad (4.11) \\
 b_2' & = -16g_1 e^{b_1} (e^{b_1} \cosh b_2 - \text{sech} b_3 \text{sech} b_4) \sinh b_2, \quad (4.12) \\
 b_3' & = -16g_1 \cosh b_2 \sinh b_3 (e^{b_1} \cosh b_2 \cosh b_3 - \text{sech} b_4), \quad (4.13) \\
 b_4' & = -16g_1 \cosh b_2 \cosh b_3 \sinh b_4 (e^{b_1} \cosh b_2 \cosh b_3 \cosh b_4 - 1), \quad (4.14) \\
 b_5' & = -16g_2 e^{b_5} (e^{b_5} - \text{sech} b_6 \text{sech} b_7 \text{sech} b_8), \quad (4.15) \\
 b_6' & = -16g_2 \sinh b_6 e^{b_3} (e^{b_5} \cosh b_6 - \text{sech} b_7 \text{sech} b_8), \quad (4.16) \\
 b_7' & = -16g_2 \cosh b_6 \sinh b_7 (e^{b_5} \cosh b_6 \cosh b_7 - \text{sech} b_8), \quad (4.17) \\
 b_8' & = -16g_2 \cosh b_6 \cosh b_7 \sinh b_8 (e^{b_5} \cosh b_6 \cosh b_7 \cosh b_8 - 1). \quad (4.18)
\end{align*}

where we have used the projection conditions $\gamma_{\epsilon} \epsilon^I = -\epsilon^I$, $I = 2, 4, 5, 8$ and $\gamma_{\epsilon} \epsilon^I = \epsilon^I$, $I = 1, 3, 6, 7$ as in the previous case. The gravitino variation $\delta \psi^{\mu}_{\epsilon}, \mu = 0, 1$, gives

\begin{align*}
 A' & = -8g_1 e^{b_1} \cosh b_2 \cosh b_3 \cosh b_4 (e^{b_1} \cosh b_2 \cosh b_3 \cosh b_4 - 2) \\
 & \quad -8g_2 e^{b_5} \cosh b_6 \cosh b_7 \cosh b_8 (e^{b_5} \cosh b_6 \cosh b_7 \cosh b_8 - 2). \quad (4.19)
\end{align*}

From these equations, we see that apart from the maximally supersymmetric point at $b_i = 0, i = 1, \ldots, 8$, there is a flat direction of the potential given by

\begin{align*}
 e^{-b_1} & = \cosh b_2 \cosh b_3 \cosh b_4, \quad e^{-b_5} = \cosh b_6 \cosh b_7 \cosh b_8 \quad (4.20)
\end{align*}

which leads to $V_0 = -64(g_1 + g_2)^2$. Equation (4.13) gives $A' = 8(g_1 + g_2)$ or $A = 8(g_1 + g_2)$ which is the $AdS_3$ solution with radius $L = \frac{1}{8(g_1 + g_2)}$. It can also be verified that the full (4, 4) supersymmetry is preserved. This should correspond to a marginal deformation of the $N = (4, 4)$ SCFT. There are no other supersymmetric critical points in this sector. Therefore, the flow breaking supersymmetry from (4, 4) to (4, 0) is not possible.

However, there is a half-supersymmetric domain wall solution similar to the dilatonic p-brane solutions of $N = 1, D = 7$ and $N = 2, D = 6$ gauged supergravities studied in [33]. It is remarkable that the full set of the above equations admits an analytic solution. The strategy to find the solution is as follow. We first determine $b_{2,3,4}$ as functions of $b_1$ and similarly determine $b_{6,7,8}$ as functions of $b_5$. $b_1$ and $b_5$ are
determined as functions of $r$ and can be solved explicitly. From (4.11) and (4.12), we find

$$\frac{db_2}{db_1} = \cosh b_2 \sinh b_2$$

(4.21)

which can be solved for $b_2$ as a function of $b_1$ giving rise to

$$b_2 = \coth^{-1} e^{-b_2-2c_1}.$$  

(4.22)

Using (4.11) and (4.13) together with $b_2$ solution from (4.22), we find

$$\frac{db_3}{db_1} = \frac{\sinh(2b_3)}{2 \left(1 - e^{2b_1+4c_1}\right)}$$

(4.23)

whose solution is given by

$$b_3 = \tanh^{-1} \frac{e^{b_1+2c_2}}{\sqrt{1 - e^{2b_1+4c_1}}}.$$ 

(4.24)

Combining (4.11) and (4.14) and substituting for $b_2$ and $b_3$ solutions give

$$\frac{db_4}{db_1} = -\frac{\cosh b_4 \sinh b_4}{(e^{4c_1} + e^{4c_2}) e^{b_4} - 1}.$$ 

(4.25)

We then find the solution for $b_4$

$$b_4 = \tanh^{-1} \frac{e^{b_1+2c_3}}{\sqrt{1 - e^{2b_1+4c_1}(e^{4c_1} + e^{4c_2})}}.$$ 

(4.26)

With solutions for $b_2$, $b_3$ and $b_4$, equation (4.11) becomes

$$b'_1 = 16 g_1 e^{b_1} \left( \sqrt{1 - e^{2b_1} (e^{4c_1} + e^{4c_2} + e^{4c_3})} - e^{b_1} \right).$$ 

(4.27)

This can be solved for $b_1$ as an implicit function of $r$. The solution is

$$r = -\frac{1}{32 g_1} \left[ 2e^{-b_1} \sqrt{1 - \beta_1 e^{2b_1}} + \ln \left[ e^{-2b_1} \left( (\beta_1 - 1)e^{2b_1} - 1 + 2e^{b_1} \sqrt{1 - \beta_1 e^{2b_1}} \right) \right] \right] + \text{constant}$$ 

(4.28)

where $\beta_1 = e^{4c_1} + e^{4c_2} + e^{4c_3}$.

We can solve (4.15) to (4.18) by the same procedure. The resulting solutions are given by

$$b_6 = \tanh^{-1} e^{b_5+2c_4}, \quad b_7 = \tanh^{-1} \frac{e^{b_5+2c_5}}{\sqrt{1 - e^{2b_5+4c_4}}},$$

$$b_8 = \tanh^{-1} \frac{e^{b_5+3c_6}}{\sqrt{1 - e^{b_5}(e^{4c_4} + e^{4c_5})}},$$

$$r = -\frac{1}{32 g_2} \left[ 2e^{-b_5} \sqrt{1 - \beta_2 e^{2b_5}} + \ln \left[ e^{-2b_5} \left( (\beta_2 - 1)e^{2b_5} - 1 + 2e^{b_5} \sqrt{1 - \beta_2 e^{2b_5}} \right) \right] \right] + \text{constant}$$ 

(4.29)
where $\beta_2 = e^{4c_4} + e^{4c_5} + e^{4c_6}$.

After substituting all of the $b_i$ solutions for $i = 2, 3, 4, 6, 7, 8$ in (4.19), we obtain

$$A' = \frac{16g_1e^{b_1}}{\sqrt{1 - \beta_1 e^{2b_1}}} + \frac{16g_2e^{b_5}}{\sqrt{1 - \beta_2 e^{2b_5}}} - \frac{8g_2e^{2b_5}}{1 - \beta_2 e^{2b_5}}$$

(4.30)

whose solution in terms of $b_1$ and $b_5$ is readily found by a direct integration using (4.11) and (4.15) including the solutions for the other $b_i$’s. The resulting solution is given by

$$A = b_1 + b_5 + \frac{1}{2} \tanh^{-1} \frac{e^{b_1}}{\sqrt{1 - \beta_1 e^{2b_1}}} + \frac{1}{2} \tanh^{-1} \frac{e^{b_5}}{\sqrt{1 - \beta_2 e^{2b_5}}} - \ln \left[1 - \beta_1 e^{2b_1}\right]$$

$$- \ln \left[1 - (1 + \beta_1) e^{2b_1}\right] - \ln \left[1 - \beta_2 e^{2b_5}\right] - \ln \left[1 - (1 + \beta_2) e^{2b_5}\right].$$

(4.31)

As $b_1, b_5 \to 0$, other scalars do not vanish for finite $c_i$. We then find that the solution will not have an interpretation in terms of the usual holographic RG flows. The solution is rather of the 1-brane soliton type, see [33] for a general discussion of $(D - 2)$-brane solitons in $D$ dimensions. It can also be verified that the $\delta \psi^r = 0$ condition precisely gives the Killing spinors for the unbroken supersymmetry $\epsilon^I = e^A e_0^I$ with the constant spinor $e_0^I$ satisfying $\gamma_r e_0^I = -e_0^I, I = 2, 4, 5, 8$ and $\gamma_r e_0^I = e_0^I, I = 1, 3, 6, 7$.

4.2 Non-supersymmetric deformations

We now study non-supersymmetric RG flow solutions interpolating between the $N = (4, 4)$ SCFT in the UV and some critical points found in the previous section. The solutions are essentially non-supersymmetric since they connect a supersymmetric to a non-supersymmetric critical point. Finding the corresponding solutions involve solving the full second order field equations for both the scalars and the metric in contrast to solving the first order BPS equations in the supersymmetric case. Although there are some examples of analytic supersymmetric flow solutions in three dimensions, in general, analytic solutions with many active scalars, even for the supersymmetric case, can be very difficult to find. Therefore, we will not expect to find any analytic solutions in the non-supersymmetric case but rather look for numerical flow solutions.

We begin with the scalar and metric Lagrangian which in our convention is given by a truncation of the bosonic part of the gauged Lagrangian given in [12]

$$\mathcal{L} = \frac{1}{2} R - \frac{1}{2} g_{ij} \partial_{\mu} \phi^i \partial^\mu \phi^j - V.$$  

(4.32)

In the rest of this section, we will work with the canonically normalized kinetic terms for scalars. With the standard domain wall ansatz for the metric in (4.3), the field
equations for $d$ scalars $\phi^i(r)$ and the metric function $A(r)$ can be found to be

$$\phi'' + 2A^i\phi'^i - \frac{\partial V}{\partial \phi^i} = 0,$$  \hspace{1cm} (4.33)

$$2A^2 - \sum_{i=1}^{d} (\phi'^i)^2 + 2V = 0,$$  \hspace{1cm} (4.34)

$$2A'' + 2A^2 + \sum_{i=1}^{d} (\phi'^i)^2 + 2V = 0.$$  \hspace{1cm} (4.35)

Note that equations (4.34) and (4.35) can be combined to

$$A'' = -\sum_{i=1}^{d} (\phi'^i)^2.$$  \hspace{1cm} (4.36)

There are indeed only $d + 1$ independent equations since (4.35) can be obtained by differentiating (4.34) and using (4.33).

Our flows mainly involve one active scalar. We will then concentrate on this case. The general solution to the second order field equation near the UV point is of the form

$$\phi = Ae^{-(d-\Delta)r/L} + Be^{-\Delta r}$$  \hspace{1cm} (4.37)

for which $d = 2$ in the present case. The first and second terms correspond to turning on an operator of dimension $\Delta$ and a vacuum expectation value (vev.) of the operator, respectively. In contrast to supersymmetric flows obtained by solving first order BPS equations which result in one of the two possibilities, there is some ambiguity in non-supersymmetric flows. Both of the two terms in the above equation arise in the behavior of $\phi$ near the UV point.

One way to solve this ambiguity is to recast the second order field equations into a first order form by introducing the generating function $W$ [34], see also a review [35],

$$\phi' = -\frac{\partial W}{\partial \phi},$$  \hspace{1cm} (4.37)

$$A' = W.$$  \hspace{1cm} (4.38)

$W$ is related to the scalar potential by the relation

$$V = \frac{1}{2} \left( \frac{\partial W}{\partial \phi} \right)^2 - W^2.$$  \hspace{1cm} (4.39)

With the generating function $W$ defined above, it is not difficult to show that (4.37) and (4.38) with an obvious generalization to $d$ scalars lead to (4.33) and (4.34).

In supersymmetric cases, $W$ becomes the true superpotential related to the $A^{IJ}$ tensor in our case, and flow solutions interpolate between critical points of $W$. For non-supersymmetric flows, the flows interpolate between some critical points of $V$ which
are not critical points of the superpotential. However, after the flow solutions (usually the numerical ones) are found, the corresponding generating function is obtained by solving (4.37). Given the generating function, the first order equation (4.37) can be used to determine the precise behavior near the UV point and eventually leads to the resolution of operator and vev deformations.

From [36], the generating function has an expansion near the UV point as

\[ W = -\frac{2(d-1)}{L} + \frac{(d-\Delta)}{2L} \phi^2 + \ldots \]  

for operator deformations and

\[ W = -\frac{2(d-1)}{L} + \frac{\Delta}{2L} \phi^2 + \ldots \]  

for vev deformations. Using (4.37), we find that near the UV point, normally at \( \phi = 0 \),

\[ \frac{\phi'}{\phi} \approx \begin{cases} \frac{d-\Delta}{L}, & \text{for operator deformations} \\ \frac{-\Delta}{L}, & \text{for vev deformations} \end{cases} \]  

(4.42)

Therefore, we can determine whether our flows are driven by turning on an operator or by a vev of an operator by using (4.42). A similar study of non-supersymmetric RG flows in \( N = 2 \) three dimensional gauged supergravity has also been done in [37].

4.2.1 Flow to \( SO(4) \times SO(4) \) CFT

The IR fixed point of this flow is critical point \( P_1 \) corresponding to a CFT with \( SO(4) \times SO(4) \) symmetry. The field equations can be consistently truncated to a single scalar \( \phi = a_1 \). We find the field equations

\[ \phi'' + 2\phi' A' - 64 \left( 4g_1^2 e^{4\phi} - 4g_1(g_1 - g_2) e^{2\phi} - 4g_1g_2 e^{\phi} \right) = 0, \]  

(4.43)

\[ 2A'^2 - \phi'^2 + 128 \left( g_1^2 e^{4\phi} - 2g_1(g_1 - g_2) e^{2\phi} - 4g_1g_2 e^{\phi} - g_2^2 \right) = 0. \]  

(4.44)

The flow solution will interpolate between \( \phi_{UV} = 0 \) in the UV and \( \phi_{IR} = \ln \sqrt{g_1(g_1 - 4g_2) - g_1} \) in the IR. A numerical solution to the above equations can be found, and an example solution for \( g_1 = 1 \) and \( g_2 = -4 \) is shown in Figure 1. It can be explicitly seen that the flow interpolates between \( \phi_{UV} = 0 \) and \( \phi_{IR} = 0.445681 \) at this value of \( g_1 \) and \( g_2 \). Figure 2 shows the behavior of \( \phi' \) along the flow. With \( g_1 = 1 \) and \( g_2 = -4 \), the scalar mass gives the dimension of the dual operator \( \Delta = \frac{4}{3} \). The \( AdS_3 \) radius in the UV is \( L_{UV} = \frac{1}{8g_1 + g_2} = \frac{1}{24} \). From Figure 2, we find that, near the UV point, \( \frac{\phi'}{\phi} = -15.984 \ldots = -\frac{0.666...}{L_{UV}} \) or \( \phi \sim e^{-\frac{2\phi}{L_{UV}}} \). Therefore, the flow is driven by a relevant operator of dimensions \( \frac{4}{3} \) and corresponds to a true deformation rather than a deformation by a vacuum expectation value. With these values of the coupling constants, the ratio of the central charges is \( \frac{c_{UV}}{c_{IR}} = 1.025 \).
4.2.2 Flow to $SO(4)$ CFT

We now consider an RG flow to critical point $P_2$ with residual gauge symmetry $SO(4)_{\text{diag}}$. In this case, the parameter $\alpha$ is positive, and the flow can be regarded as a flow to another $AdS_3 \times S^3 \times S^3$ background in the IR. The field equations are again consistently truncated to a single scalar which we will call $\phi_1$. Using $g_2 = \sqrt{\frac{13-2}{9}}g_1$ and $g_1 = 1$, we find

$$
\phi_{1 IR} = 1.8641, \quad L_{UV} = \frac{7 - \sqrt{13}}{32} \approx 0.1061, \quad \frac{c_{UV}}{c_{IR}} \approx 1.6422. \quad (4.45)
$$

That the flow is driven by this particular $SO(4)$ singlet among all of the four singlets can be seen by looking at the masses of the four singlets at the UV point. With the
above values of \( g_1 \) and \( g_2 \) or more precisely the value of \( \alpha \), only the singlet associated to our critical point is tachyonic corresponding to a relevant dual operator of dimension \( \Delta = 1.303 \).

The field equations are given by

\[
\begin{align*}
\phi''_1 + 2\phi'_1 A' + 8 \cosh^4 \frac{\phi_1}{2} & \left[ 4(g_1^2 - 6g_1g_2 + g_2^2) \sinh \phi_1 - 2(g_1 - 3g_2)^2 \sinh(2\phi_1) \right] \\
& + 16 \sinh \frac{\phi_1}{2} \cosh^3 \frac{\phi_1}{2} \left[ 5g_1^2 + 34g_1g_2 + 13g_2^2 + 4(g_1^2 - 6g_1g_2 + g_2^2) \cosh \phi_1 \\
& - (g_1 - 3g_2)^2 \cosh(2\phi_1) \right] = 0, (4.46)
\end{align*}
\]

\[
\begin{align*}
2A^2 - \phi'^2_1 - 16 \cosh^4 \frac{\phi_1}{2} & \left[ 5g_1^2 + 34g_1g_2 + 13g_2^2 + 4(g_1^2 - 6g_1g_2 + g_2^2) \cosh \phi_1 \\
& - (g_1 - 3g_2)^2 \cosh(2\phi_1) \right] = 0. (4.47)
\end{align*}
\]

A flow solution with \( g_1 = 1 \) is shown in Figure 3.

**Figure 3:** A solution for \( \phi_1 \) in a flow to \( SO(4) \) CFT with \( g_1 = 1 \).

Near the UV point, we find that \( \frac{\phi'_1}{\phi_1} = -6.5729 = \frac{-0.697}{\ell_{UV}} \) as shown in Figure 4. The flow is then driven by a relevant operator of dimension 1.303 since \( \hat{\phi}_1 \sim e^{-\frac{0.697}{\ell_{UV}}} \) near the UV point. As in the previous solution, the flow describes a true deformation of the UV SCFT.

### 4.2.3 Flow to \( SO(4) \times SO(2) \times SO(2) \) CFT

As in the previous subsections, we begin with the scalar field equations. It is easily verified that setting all but \( (a_1, a_2, a_3, a_4) \) to zero satisfies their field equations. The
It can be checked that a truncation $a_2 = a_1 = a$ is also consistent, but $a_4 = -a_3$ is not. So, together with the equation for $A$, we are effectively left with four equations to be solved. However, these four equations are still complicated. We will not attempt to give their solution here but simply end this section with some comments on the flow.

In terms of the $SO(4) \times SO(4)$ representations in the UV, $a_3$ and $a_4$ are combi-
nations of \((\mathbf{1}, \mathbf{1})\) and \((\mathbf{1}, \mathbf{9})\). From the value of \(g_1\) and \(g_2\) in the stability range, it can be checked that only the deformation dual to \(a\) is relevant. The flow should mainly be driven by the operator dual to \(a\). The deformations corresponding to \(a_3\) and \(a_4\) are given by vacuum expectation values of irrelevant operators since \(a_3\) and \(a_4\) have positive mass squares.

5. Conclusions and discussions

In this paper, we have studied \(N = 8\) gauged supergravity in three dimensions with a non-semisimple gauge group \((SO(4) \times SO(4)) \ltimes T^{12}\). The ratio of the coupling constants of the two \(SO(4)\)'s is given by a parameter \(\alpha\). For positive \(\alpha\), the theory describes an effective theory of ten dimensional supergravity reduced on \(S^3 \times S^3 \times S^1\). For negative \(\alpha\), on the other hand, the theory may describe a similar reduction on \(S^3 \times H^3 \times S^1\) in which \(H^3\) is a three-dimensional hyperbolic space. With \(\alpha = -1\), the cosmological constant is zero. This solution should describe a ten dimensional background \(M_3 \times S^3 \times H^3 \times S^1\) where \(M_3\) is the three dimensional Minkowski space.

We have studied the scalar potential and found a number of non-supersymmetric critical points. The trivial critical point with maximal supersymmetry is identified with the dual large \(N = (4, 4)\) SCFT in two dimensions. We have explicitly checked the stability of some non-supersymmetric critical points by computing the full scalar mass spectra at the critical points. They are perturbatively stable for some values of \(\alpha\) parameter in the sense that all scalar masses are above the BF bound. It is also interesting to see whether other critical points are stable or not. The RG flows interpolating between the large \(N = (4, 4)\) SCFT in the UV and non-supersymmetric IR fixed points have been given for the flows to \(SO(4) \times SO(4)\) and \(SO(4)\) CFT's. We have also resolved the ambiguity between the operator and vev deformations arising from solving the second order field equations and found that the flows are driven by turning on relevant operators.

Another result of this paper is half-supersymmetric domain wall solutions to \(N = 8\) gauged supergravity. For the domain wall preserving \(SO(4) \times SO(4)\) symmetry, the solution describes an RG flow from \(N = (4, 4)\) SCFT in the UV to a non-conformal \(N = (4, 4)\) field theory in the IR. The solution has however a bad singularity according to the criterion of \[32\]. For the solution preserving \(SU(2) \times SU(2)\) symmetry, the holographic interpretation is not clear. In the point of view of a \((D - 2)\)-brane soliton, the solution should describe a 1-brane soliton in three dimensions according to the general discussion in \[33\]. When uplifted to ten dimensions, the solution might describe some configuration of D1-branes. Hopefully, the solutions obtained in this paper might be useful in string/M theory context, black hole physics and the AdS/CFT correspondence.
The uplifted solution of the non-conformal flow preserving $SO(4) \times SO(4)$ symmetry is also necessary for the resolution of its singularity if the full ten-dimensional solution turns out to be non-singular.

Finally, the chiral supersymmetry breaking $(4,4) \rightarrow (4,0)$ found in [28] cannot be implemented in the framework of $N = 8$ gauged supergravity studied here. It would probably require a larger theory of $N = 16$ gauged supergravity with $(SO(4) \times SO(4)) \ltimes (T^{12}, \hat{T}^{34})$ gauge group studied in [14]. It would be very interesting to find the flow solution of [28] explicitly in the three dimensional framework. We hope to come back to these issues in future research.

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A. Useful formulae and details

For completeness, we include a short review of gauged supergravity in three dimensions in the formulation of [12]. The theory is a gauged version of a supersymmetric non-linear sigma model coupled to non-propagating supergravity fields. $N$-extended supersymmetry requires the presence of $N - 1$ almost complex structures $f^P$, $P = 2, \ldots, N$ on the scalar manifold. The tensors $f^{IJ} = f^{[IJ]}$, generating the $SO(N)$ R-symmetry in a spinor representation under which scalar fields transform, play an important role. In the case of symmetric scalar manifolds of the form $G/SO(N) \times H'$, they can be written in terms of $SO(N)$ gamma matrices. In our case, we use the $16 \times 16$ Dirac gamma matrices of $SO(8)$

$$\gamma^I = \begin{pmatrix} 0 & \Gamma^I \\ (\Gamma^I)^T & 0 \end{pmatrix}. \quad (A.1)$$

The $8 \times 8$ gamma matrices are explicitly given by

$$\begin{align*}
\Gamma_1 &= \sigma_4 \otimes \sigma_4 \otimes \sigma_4, \\
\Gamma_2 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_4, \\
\Gamma_3 &= \sigma_4 \otimes \sigma_1 \otimes \sigma_3, \\
\Gamma_4 &= \sigma_3 \otimes \sigma_4 \otimes \sigma_1, \\
\Gamma_5 &= \sigma_1 \otimes \sigma_2 \otimes \sigma_4, \\
\Gamma_6 &= \sigma_4 \otimes \sigma_1 \otimes \sigma_2, \\
\Gamma_7 &= \sigma_2 \otimes \sigma_4 \otimes \sigma_1, \\
\Gamma_8 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1. \quad (A.2)
\end{align*}$$
where
\[
\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\] (A.3)

According to our normalization, we find
\[
f^I_{K_r,L_s} = -\text{Tr}(Y_{L_s} [T^{IJ}, Y_{K_r}]).
\] (A.4)

Generally, the \(d = \dim(G/H)\) scalar fields \(\phi^i, i = 1, \ldots, d\) can be described by a coset representative \(L\). The useful formulae for a coset space are
\[
L^{-1}t^M L = \frac{1}{2} \mathcal{V}^M_{IJ} T^{IJ} + \mathcal{V}^M_{\alpha} X^\alpha + \mathcal{V}^M_A Y^A,
\] (A.5)

\[
L^{-1} \partial_i L = \frac{1}{2} Q^I_i t^{IJ} + Q^\alpha_i X^\alpha + e^A_i Y^A
\] (A.6)

where \(e^A_i, Q^I_i\) and \(Q^\alpha_i\) are the vielbein on the coset manifold and \(SO(N) \times H'\) composite connections, respectively. \(X^\alpha\)’s denote the \(H'\) generators.

Any gauging can be described by a symmetric and gauge invariant embedding tensor satisfying the so-called quadratic constraint
\[
\Theta_{PL} f^{KL} (M, \Theta_{N}) = 0,
\] (A.7)

and the projection constraint
\[
\mathbb{P}_{R_0} \Theta_{MN} = 0.
\] (A.8)

The first condition ensures that the gauge symmetry forms a proper symmetry algebra while the second condition guarantees the consistency with supersymmetry.

The T-tensor given by the moment map of the embedding tensor by scalar matrices \(\mathcal{V}^M_A\), obtained from (A.5), is defined by
\[
T_{AB} = \mathcal{V}^M_A \Theta_{MN} \mathcal{V}^N_B.
\] (A.9)

Only the components \(T^{IJ,KL}\) and \(T^{IJ,A}\) are relevant for computing the scalar potential. With our \(SO(8,8)\) generators, we obtain the following \(\mathcal{V}\) maps
\[
\mathcal{V}^{ab,IJ}_{A1} = -\frac{1}{2} \text{Tr}(L^{-1} J^a_i T^{IJ}), \quad \mathcal{V}^{ab,IJ}_{B1} = -\frac{1}{2} \text{Tr}(L^{-1} t^a_i T^{IJ}),
\]
\[
\mathcal{V}^{ab,Kr}_{A1} = \frac{1}{2} \text{Tr}(L^{-1} J^a_i Y^{Kr}), \quad \mathcal{V}^{ab,Kr}_{B1} = \frac{1}{2} \text{Tr}(L^{-1} t^a_i Y^{Kr}),
\]
\[
\mathcal{V}^{ab,IJ}_{A2} = -\frac{1}{2} \text{Tr}(L^{-1} J^a_2 T^{IJ}), \quad \mathcal{V}^{ab,IJ}_{B2} = -\frac{1}{2} \text{Tr}(L^{-1} t^a_2 T^{IJ}),
\]
\[
\mathcal{V}^{ab,Kr}_{A2} = \frac{1}{2} \text{Tr}(L^{-1} J^a_2 Y^{Kr}), \quad \mathcal{V}^{ab,Kr}_{B2} = \frac{1}{2} \text{Tr}(L^{-1} t^a_2 Y^{Kr})
\] (A.10)
where we have followed the convention of calling the semisimple part \( SO(4) \times SO(4) \) and the nilpotent part \( T^{12} \sim T^6 \times T^6 \) as \( \mathcal{A} \) and \( \mathcal{B} \) types, respectively. We then compute the T-tensor components

\[
T^{IJ, KL} = g_1 \left( \mathcal{V}_{A_1}^{ab, IJ} \mathcal{V}_{B_1}^{cd, KL} + \mathcal{V}_{B_1}^{ab, IJ} \mathcal{V}_{A_1}^{cd, KL} - \mathcal{V}_{B_1}^{ab, IJ} \mathcal{V}_{A_1}^{cd, KL} \right) \epsilon_{abcd} + g_2 \left( \mathcal{V}_{A_2}^{\hat{a}b, IJ} \mathcal{V}_{B_2}^{\hat{c}d, KL} + \mathcal{V}_{B_2}^{\hat{a}b, IJ} \mathcal{V}_{A_2}^{\hat{c}d, KL} - \mathcal{V}_{B_2}^{\hat{a}b, IJ} \mathcal{V}_{A_2}^{\hat{c}d, KL} \right) \epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}, \quad (A.11)
\]

\[
T^{IJ, Kr} = g_1 \left( \mathcal{V}_{A_1}^{ab, IJ} \mathcal{V}_{B_1}^{cd, Kr} + \mathcal{V}_{B_1}^{ab, IJ} \mathcal{V}_{A_1}^{cd, Kr} - \mathcal{V}_{B_1}^{ab, IJ} \mathcal{V}_{A_1}^{cd, Kr} \right) \epsilon_{abcd} + g_2 \left( \mathcal{V}_{A_2}^{\hat{a}b, IJ} \mathcal{V}_{B_2}^{\hat{c}d, Kr} + \mathcal{V}_{B_2}^{\hat{a}b, IJ} \mathcal{V}_{A_2}^{\hat{c}d, Kr} - \mathcal{V}_{B_2}^{\hat{a}b, IJ} \mathcal{V}_{A_2}^{\hat{c}d, Kr} \right) \epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}. \quad (A.12)
\]

The scalar potential can be computed by using the formula

\[
V = -\frac{4}{N} \left( A_{IJ}^1 A_{IJ}^1 - \frac{1}{2} N g_{ij} A_{IJ}^2 A_{IJ}^2 \right) \quad (A.13)
\]

in which the metric \( g_{ij} \) is related to the vielbein by \( g_{ij} = e^A_i e^A_j \). The \( A_1 \) and \( A_2 \) tensors appearing in the gauged Lagrangian as fermionic mass-like terms are given by

\[
A_{IJ}^1 = -\frac{4}{N - 2} T^{IJ, MN} + \frac{2}{(N - 1)(N - 2)} \delta^{IJ} T^{MN, MN}, \quad (A.14)
\]

\[
A_{IJ}^2 = \frac{2}{N} T^{IJ} + \frac{4}{N(N - 2)} f^M(l_m T^{IJ})_M + \frac{2}{N(N - 1)(N - 2)} \delta^{IJ} f^k l_k T^{KL, KL}. \quad (A.15)
\]

Finally, we repeat the condition for supersymmetric critical points. The residual supersymmetry is generated by the eigenvectors of the \( A_{IJ}^1 \) tensor with eigenvalues equal to \( \pm \sqrt{-\frac{V_0}{4}} \).
B. Explicit forms of the scalar potential

For $SO(4)_{\text{diag}}$ invariant scalars, the potential is given by

$$V = 4e^{6a_1}g_1^2\cosh^2(a_3 - a_4)\cosh^2(a_3 + a_4) [5 \cosh(2(a_1 - 2a_3))] + 8 \cosh(4a_3)$$
$$+ 5 \cosh[2(a_1 + 2a_3)] - 4 \cosh(2a_1)(7 + 2 \cosh(2a_3)\cosh(2a_4)) + 2 \cosh(4a_4) \times \cosh a_1 - 3 \sinh a_1 - 6 (\cosh(4a_3) - 4 \cosh(2a_3)\cosh(2a_4) - 6) \sinh(2a_1)]$$
$$+ 4e^{6a_2}g_2^2\cosh^2(a_3 - a_4)\cosh^2(a_3 + a_4) [5 \cosh(2(a_2 - 2a_3))] - 8 \cosh(4a_3)$$
$$+ 5 \cosh[2(a_2 + 2a_3)] - 4 \cosh(2a_2)(7 + 2 \cosh(2a_3)\cosh(2a_4)) + 2 \cosh(4a_4) \times \sinh a_2 - 3 \cosh a_2 - 6 (\cosh(4a_3) - 4 \cosh(2a_3)\cosh(2a_4) - 6) \sinh(2a_2)]$$
$$- 2e^{a_1+a_2+6(a_3+a_4)}g_1g_2 [86 \cosh(a_1 + a_2) - 64 \cosh(a_1 - a_2)\cosh(2a_3) + \cosh(2a_3)\cosh(6a_1)(\cosh a_1 - 3 \sinh a_1)(3 \cosh a_2 - \sinh a_2) + 16 \cosh a_1 \cosh(4a_3)\sinh a_2$$
$$+ \cosh(2a_4)[-64 \cosh(a_1 - a_2) + \cosh(6a_3)(3 \cosh a_1 - \sinh a_1) \times \cosh a_2 - 3 \sinh a_2 + 2 \cosh(2a_3)(37 \cosh(a_1 + a_2) - 19 \sinh(a_1 + a_2)))]$$
$$- 66 \sinh(a_1 + a_2) + 2 \cosh(4a_4)[8 \cosh a_2 \sinh a_1 + \cosh(4a_3) \sinh(a_1 + a_2)$$
$$- 3 \cosh(a_1 + a_2)] + [25 \cosh a_1 + a_2 - 27 \cosh a_2 \sinh a_1 + 2 \cosh(4a_3) \times$$
$$(3 \cosh a_1 - \sinh a_1)(\cosh a_2 - 3 \sinh a_2) - 35 \cosh a_1 \sinh a_2] \sinh(2a_3)\sinh(2a_4)$$
$$+ 2(\sinh a_1 + a_2 - 3 \cosh(4a_3)\sinh(4a_4) + \sinh(2a_3)\sinh(6a_4) \times$$
$$(3 \cosh a_2 - \sinh a_2)(\cosh a_1 - 3 \sinh a_1)]). \quad (B.1)$$

The potential for $SU(2)_L^+ \times SU(2)_L^-$ invariant scalars is given by, in notation of section $4$

$$V = 128 \left[ g_1^2e^{2b_1} \cosh^2b_2 \cosh^2b_3 \cosh^2b_4 \left( e^{b_1} \cosh b_2 \cosh b_3 \cosh b_4 - 1 \right)^2 \right.$$
$$+ g_2^2e^{2b_5} \cosh^2b_6 \cosh^2b_7 \cosh^2b_8 \left( e^{b_5} \cosh b_6 \cosh b_7 \cosh b_8 - 1 \right)^2 \left. \right]. \quad (B.2)$$

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