On the possibility of sourcing a mono-energetic $\bar{\nu}_e$ long baseline beta beam from bound beta decay

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Abstract

In this paper, the possibility of using fully stripped ions that can decay through bound beta decay to complement electron capture long baseline neutrino oscillation experiments is qualitatively analysed. The disadvantages of such a source are discussed through consideration of the technological challenges faced and the energy resolution required from the detector. It is concluded that ions that bound beta decay cannot be used as a source of mono-energetic anti-neutrinos in a realistic long baseline CP-even neutrino beam.
I. INTRODUCTION

Bound beta decay is the process in which, instead of ejecting an electron, as in usual beta decay, the electron becomes bound to the daughter ion. The anti-neutrino energy spectrum is therefore not continuous, instead it is a series of discrete mono-energetic spectra with the intensities dependent on the electron orbital occupied. Bound beta decay (BBD) was predicted in 1947 by Daudel et al. and was discussed theoretically by Bachall. However, it was not until 1992 that BBD was confirmed experimentally in $^{163}$Dy$^{66+}$ at the GSI Helmholtz Centre for Heavy Ion Research.

In the last couple of years, the BBD process has been considered for very short baseline neutrino oscillation experiments with very small detectors also; $L \sim 10$ m and detector mass $\sim 100$ g. Following works by Raghaven, it is proposed to use the processes

$$3^\text{H} \rightarrow 3^\text{He} + e^- + \bar{\nu}_e \quad \text{and} \quad 3^\text{He} + e^- + \bar{\nu}_e \rightarrow 3^\text{H} \quad (1.1)$$

as the emission and detector processes, respectively. The electron in each case is bound. By embedding the $^3$H and $^3$He atoms in a metal lattice, the detection process becomes resonantly enhanced with cross-sections up to 12 orders of magnitude larger than a non-resonant capture at the same energy. Oscillation of these ‘Mossbauer neutrinos’ has been demonstrated theoretically, and they have been studied in the context of determining the neutrino mass hierarchy without using matter effects and in the search of active-sterile neutrino oscillations, amongst other things. In this paper, however, only long baseline experiments will be considered and so this possible use of the BBD process will not be discussed further.

The use of the BBD process in long baseline neutrino oscillation experiments has recently been suggested, either for use in its own right or in view of complementing electron capture machine proposals. In this short article some practical problems with this proposal are identified and qualitatively analysed. The very low branching ratios of the BBD process; large proton number of the required ions, and the energy resolution of the far detector all indicate that such a machine is not practical.

II. BOUND BETA DECAYS

A continuum beta decay (CBD) is a transition between discrete stationary states of a parent and daughter nucleus - usually from ground state to ground state or low lying
excited state. The $\beta^-$ CBD process transforms a neutron bound within a nucleus into a proton accompanied by the creation of an electron and anti-neutrino:

$$^{A_Z}X \rightarrow ^{A_{Z+1}}Y + e^- + \bar{\nu}_e.$$

(2.1)

The decay electron is virtually always emitted to the continuum with capture in the outer orbitals strongly suppressed due to the weak bindings and small wave function overlaps. Capture into the inner orbitals is clearly forbidden by the Pauli principle. The electrons from $\beta^-$ for a fully ionised parent $^{A_Z}X^{Z^+}$, however, can be captured into a bound orbit:

$$^{A_Z}X^{Z^+} \rightarrow ^{A_{Z+1}}Y^{Z^+} + \bar{\nu}_e,$$

(2.2)

with the $\bar{\nu}_e$ being mono-energetic.

The kinematics for neutral and fully ionised atoms are not the same; corrections are necessary to compensate for electron binding energies. The CBD Q-value for ion with atomic mass $A$ and proton number $Z$ is defined as

$$Q_C = m_Y(A, Z+1) - m_X(A, Z),$$

(2.3)

where the mass of the ejected electron has been implicitly included. The $Q_C$ is therefore the kinetic energy available to the channel, for an atom with the full complement of orbital electrons. When the atom is stripped bare, the Q-value needs to be corrected by the difference in binding energies of the complete parent and daughter ions, $|\Delta B_{Y,X}^{BBD}|$. To get the BBD process Q-value, $Q_B^{Z^+}$, this needs to be further modified by the binding energy of the electron captured into orbital $n$ of the daughter nucleus, $|B_{n,Y}|$. In summary \cite{12}, we have:

$$Q_B^{Z^+} = Q_C + |B_{n,Y}| - |\Delta B_{Y,X}^{BBD}|$$

$$= Q_C^{Z^+} + |B_{n,Y}|.$$  

(2.4)

BBD is an important process in astrophysics owing to the heavily ionised environments. These corrections have therefore been tabulated for many ions \cite{13, 14}. $|\Delta B_{Y,X}^{BBD}|$ is the smaller correction to the $Q_C$, 17 keV for Thallium \cite{13}, for example. $|B_{n,Y}|$ is the larger but still small compared to $Q_C$: 99 keV for Thallium \cite{14}.

BBD can be thought of as the inverse process of electron capture. The relationship between the decay rates for BBD and CBD is therefore similar to that of CBD and electron capture. Taking BBD and CBD to have the same nuclear matrix elements and using phase
space arguments, for respective branching rates $\Gamma_C$ and $\Gamma_B$, the relative strength is given by

$$\frac{\Gamma_B}{\Gamma_C} \propto \frac{Q_B^2 |\psi_n(0)|^2}{f(Z, Q_C)}, \quad (2.5)$$

where

$$f(Z, Q_C) = \int_{Q_C-m_e}^{Q_C+m_e} E \sqrt{E^2 - m_e^2} (Q_C - E)^2 F(Z, E) dE \quad (2.6)$$

is the integral over phase space for CBD and $\psi_n(0)$ is the wave function for an electron in the $n$th orbital. It is seen that the strong $Q_C$ dependence for CBD means that this channel will not only dominate for high $Q_C$ but will have a substantial branching ratio at all but the very smallest $Q_C$. Since the rate for BBD is dependent on the modulus square of the orbital wave function, there is a proton number dependence which leads to substantial branching ratios for high $Z$ atoms, provided $Q_C$ is not too high. A more detailed calculation has been carried out in [12] and the branching ratios, in a full relativistic calculation, have been presented in [12]. The ratios presented in Tab. 1 were calculated using the expression for $\Gamma_{BBD}/\Gamma_{CBD}$ given in [15] without including radiative corrections.

### III. TECHNOLOGICAL CHALLENGES

In this section, the possibility of using BBD will be discussed and some of the technological challenges articulated. In particular, the demands on the acceleration chain and the impact on the anti-neutrino fluxes will be first focussed on. In the second part, the effect of the energy resolution of the detector will examined with the likely demands on the acceleration chain investigated. Although a concrete long baseline setup is not being proposed here, some ion ‘choices’ are presented in Tab. II to make the discussion more explicit. An optimal ion will have a half-life $\sim 1$ second [16]; however, the paucity of choice means the half-lives may be much higher. A scan of the database [17] for a selection of ions with single dominant decay channels and half-lives in the range $0.5 \text{ sec} < t_{1/2} < 8 \text{ min}$ was made. Very few ions matched the criteria.
| Ion  | Q-value (MeV) | Channel % | Half-life  | $\Gamma_{BBD}/\Gamma_{CBD}$ |
|------|--------------|-----------|-----------|---------------------------|
| $^{20}$O | 2.757 | 99.97 | 13.51 sec | $9.4 \cdot 10^{-5}$ |
| $^{34}$Si | 2.993 | 100 | 2.77 sec | $3.6 \cdot 10^{-4}$ |
| $^{52}$Ti | 1.831 | 100 | 1.7 min | $8.8 \cdot 10^{-3}$ |
| $^{56}$Cr | 1.506 | 100 | 5.94 min | $7.0 \cdot 10^{-3}$ |
| $^{55}$Cr | 2.603 | 99.96 | 3.497 min | $2.1 \cdot 10^{-3}$ |
| $^{62}$Fe | 2.023 | 100 | 68 sec | $4.5 \cdot 10^{-3}$ |
| $^{98}$Zr | 2.250 | 100 | 30.7 sec | 0.010 |
| $^{99}$Nb | 3.403 | 100 | 15.0 sec | $4.1 \cdot 10^{-3}$ |
| $^{120}$Cd | 1.760 | 100 | 50.8 sec | 0.026 |
| $^{121}$In | 2.434 | 100 | 23.1 sec | 0.014 |
| $^{206}$Tl | 1.533 | 99 | 4.199 min | 0.080 |
| $^{207}$Tl | 1.423 | 99.72 | 4.77 min | 0.138 |
| $^{209}$Tl | 1.832 | 98.8 | 2.20 min | 0.118 |

**TABLE I:** A selection of ions selected based on their half-lives and dominant decay channels. The quoted Q-values are for CBD and need to modified as discussed in Sec. II fully stripped ions.

**Acceleration and flux**

In a beta beam, the radioactive ions are accelerated then stored in a ring to decay. To source a useful flux from the storage rings requires an optimal half-life $O(1 \text{ sec})$. The half-life needs to be sufficiently long to minimise losses in the acceleration, but sufficiently short to source a useful flux once in a decay ring. This is one of the primary reasons why $^{18}$Ne, $^{8}$B, $^{6}$He and $^{8}$Li are excellent candidate ions. The ions put forward for electron capture machines and BBD machines are not optimal in that they have half-lives up to several minutes and so the number of useful neutrinos sourced is several orders to low \cite{18}. This problem could be dealt with R&D in the acceleration stage: increased production rates, reduction of losses during acceleration, and loosening of constraints on the duty factor could all lead to a boost in useful decay rate. An accumulation ring is also an option to compensate for the accelerator
complex dead time of approximately 8 seconds [19]. For electron capture machines, the aim is to choose ions with near 100% branching ratios. This is not a luxury available to BBD sources however.

The branching ratio for BBD is typically small unless the Q-value is very small or the proton number of the ion is large. However, if one wishes to source a long baseline experiment, very small Q-value ions are not an option (Tab. I). Low or modest branching ratios are therefore an intrinsic feature of BBD long baseline candidate ions. Achieving the necessary count rates is therefore very demanding; for example, consider $^{207}$Tl which has the highest branching ratio of the selected ions in Tab. I. $10^{18}$ useful decays is the target rate for any long baseline beta beam type experiment. If this could be achieved, one is still an order of magnitude short for the useful mono-energetic anti-neutrinos. In addition, to extract a useful BBD rate requires hydrogen-like atoms. A large proton number is likely which points to severe space charge issues, especially in the low energy part of the accelerator chain. These effects collectively force the need for an extra factor of 10 in production [21] requiring an extensive R&D program and large duty factors (up to 10%). For a fully stripped ion, vacuum losses are not a concern since the probability of the ion capturing an electron is effectively nil [19].

The ions considered in [7] could BBD, CBD and decay through electron capture. Four ions were identified with BBD Q-values ranging from 1.67 MeV to 2.46 MeV. The branching ratios were therefore low ($\sim 1\%$). The motivation behind this proposal was to use the BBD and electron capture spectra with the end part of the CBD spectrum to construct a ‘CP-even’ beam defined by

$$
\eta(E; \gamma) = \frac{\mathcal{F}(\nu_e)\sigma(\nu_\mu) - \mathcal{F}(\bar{\nu}_e)\sigma(\bar{\nu}_\mu)}{\mathcal{F}(\nu_e)\sigma(\nu_\mu) + \mathcal{F}(\bar{\nu}_e)\sigma(\bar{\nu}_\mu)} = 0 ,
$$

(3.1)

where $\mathcal{F}$ is an unoscillated neutrino flux and $\sigma$ is a cross-section. Such a strategy requires the separation of the neutrino and anti-neutrino events at the detector in addition to the separation of BBD and CBD events. One therefore needs to consider the characterisitics of the detector.
Detectors and energy resolution

In the previous section, a number of issues surrounding the production and acceleration were highlighted. BBD will now be examined in the context of the likely technology available to the beta beam class of machines and what energy resolutions are required.

For an ion boost $\gamma$, an energy $E_l$ in the laboratory frame is related to its rest frame counterpart by $E_l = 2\gamma E_r$. For a given accelerator, the maximum boost possible for an ion $^{A_Z X^N+}$ is given by

$$\gamma_{\text{ion}}^{\text{max}} = \frac{N}{A} \gamma_{p}^{\text{max}}, \quad (3.2)$$

where $\gamma_p^{\text{max}}$ is the maximum boost of the proton and $N$ is the number of electrons removed from the atom. For the 1 TeV machines available to beta beams, such as an upgraded Super Proton Synchrotron (SPS), $\gamma_p^{\text{max}} = 1066$. Ions that beta decay lie on the neutron-rich side of the line of stability on a Segre chart, and typically have $Z/A \sim 0.4 - 0.5$. Therefore, energies $\sim 1$ MeV in the rest frame correspond to energies $\sim 0.8$ GeV in the laboratory frame at maximum boost. In what follows, the lower limit, $\gamma_{\text{ion}}^{\text{max}} = 400$ is taken.

A beam source from ions that electron capture decay and bound beta decay will contain both neutrinos and anti-neutrinos. For such a strategy, it is therefore mandatory to discriminate the $\mu^-$ and $\mu^+$ events at the detector, as in the Neutrino Factory proposal. The Magnetised Iron Neutrino Detectors (MIND) studied for use with Neutrino Factories have thresholds $> 3$ GeV [22]. Neutrino energies set to first oscillation maximum will be below the MIND threshold for baselines $L < 1500$ km. Magnetised liquid argon detectors and totally active scintillator detectors [22] have been put forward as alternatives and could provide the techniques to deal with this issue. However, with only $\sim 1\%$ of the beam mono-energetic anti-neutrinos and the possibility of running electron capture and BBD ions separately, this is a mute point.

The shortest long baseline being considered for the future long baseline neutrino oscillation program is CERN-Frejus at 130 km. Using the current values of the oscillation parameters [22], the energy of first oscillation maximum for the $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ channel at 130 km is 0.25 GeV. With a boost $\gamma = 400$, $Q_B > 0.315$ MeV is necessary for the mono-energetic anti-neutrino flux to get placed on first oscillation maximum at Frejus. For $Z=90$, BBD will make up $\sim 75\%$ of the anti-neutrino flux. Therefore the CBD fraction will be at least $25\%$ for all cases in which the mono-energetic neutrinos are to be placed on first maximum. From
all the ions identified have much larger Q-values. The minimum CBD fraction, from these choices is a much higher 85%. All these ions could place a mono-energetic source on (or around) the first oscillation maximum for the CERN-Canfranc baseline ($L = 650$ km). The CERN-Boulby baseline ($L = 1050$ km) requires a minimum $Q_C \sim 2.5$ MeV. From the ions selected, no more than 1% BBD would be possible in this case. Therefore, in all conceivable cases, a substantial flux from the CBD is to be expected. If these are not separated then one is not exploiting the the mono-energetic nature of BBD neutrinos.

In the rest frame, the two channels are split by the difference between the CBD and the BBD Q-values, the electron binding energy $|B_{1,Y}|$. Therefore, for a detector with energy resolution $\Delta E$, to separate the channels one requires

$$\Delta E < 2\gamma |B_{1,Y}|.$$  \hspace{1cm} (3.3)

For example, $^{207}\text{Tl}^{81+}$ has $|B_{1,Y}| = 99$ keV. For a detector with $\Delta E = 150$ MeV, a boost $\gamma > 750$ is required. Since $|B_{1,Y}| \propto (Z + 1)^2$, where $Z$ is the proton number of the parent, the $\gamma$ factors required will be larger than this for other ions. With the accelerators expected to be available to the community, such as an upgraded SPS and the Tevatron, CBD and BBD cannot be separated for this example. A substantial portion of the anti-neutrino flux will always, therefore, be sourced from the CBD. If creating hydrogen like ions is problematic, the BBD neutrinos will be suppressed, or effectively reduced to nil. In that case, one would have a high Z anti-neutrino beta beam.

IV. CONCLUSIONS

In the present article, the possibility of using bound beta decays as a source of mono-energetic anti-neutrinos for a future long baseline neutrino oscillation experiment has been qualitatively analysed and a number of problem diagnosed. The bound beta decay process has been identified as a possible source of mono-energetic anti-neutrinos for a CP-even beam [7]. The required fluxes will be very hard to achieve, the low branching ratios and space charge restrictions mean that a target of $10^{18}$ useful decays per year (the standard minimum for beta beam related studies) will need major R&D work. Large duty factors will need to be accommodated which will increase the background component of the event rate. For anti-neutrinos and neutrinos in the same beam, either sourced from the same
ion or for two ion species circulating simultaneously, discrimination of $\mu^-$ and $\mu^+$ events is mandatory. For the energies considered, this will require innovative technologies such as magnetised totally active scintillator detector or magnetised liquid argon detectors. For ions that only beta decay and BBD, the two channels need to separated otherwise the mono-energetic nature of the BBD is not being exploited. For the largest binding energies and a energy resolution of $\Delta E = 150$ MeV, this requires a boost $\gamma > 750$. The LHC will be necessary and the event rate will be further diminished by the $1/\gamma$ dependence of the useful decay rate. In short, the principal reasons why a bound beta beam is not feasible is the inability to achieve a useful decay rate for the BBD channel, and the restrictive requirements on the energy resolution of the far detector.

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Note added:

During the final stages of the present work, a revised and more detailed version of [7] appeared.

[1] R. Daudel et al, C. R. Acad. Sci (Paris), 224,1427 (1947)
[2] J. N. Bachall, Phys. Rev. 124 (1961) 495.
[3] M. Jung et al., Phys. Rev. Lett. 69 (1992) 2164.
[4] H. Minakata and S. Uchinami New J. Phys. 8 (2006) 143 [arXiv:hep-ph/0602046]; H. Minakata, H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phy. Rev. D 76 (2007) 053004 [Erratum-ibid. D 76 (2007) 079901] [arXiv:hep-ph/0701151].

[5] R. S. Raghaven, arXiv:hep-ph/05111191; R. S. Raghaven, arXiv:hep-ph/0601079.

[6] E. K. Akhmedov, J. Kopp and M. Lindner, JHEP 0805 (2008) 005 [arXiv:0802.2513 [hep-ph]]; E. K. Akhmedov, J. Kopp and M. Lindner. [arXiv:0803.1424 [hep-ph]].

[7] A. Fukumi, I. Nakano, H. Nanjo, N. Saso, S. Sato and M. Yoshimura, J. Phys. Soc. Jpn. 78 (2009) 013201 [arXiv:hep-ex/0612047].

[8] For example, see M. Mezzetto Talk given at NOW 2008.

[9] J. Bernabeu, J. Burguet-Castell, C. Espinoza and M. Lindroos JHEP 0512 (2005) 014 [arXiv:hep-ph/0505504]; J. Bernabeu and C. Espinoza, Phys. Lett. B 664 (2008) 285 [arXiv:0712.1034 [hep-ph]].

[10] J. Sato [arXiv:hep-ph/0503144].

[11] M. Rolinec, J. Sato [arXiv:hep/0612148].

[12] D. Boutin, Dissertation Justus-Liebig Universität Giessen.

[13] J. P. Descalux, At. Data. Nucl. Data Tables 12 (1973) 311.

[14] W. R. Johnson and G. Soff, At. Data. Nucl. Data Tables 33 (1985) 405.

[15] M. Faber et al., Phys. Rev. C 80 (2009) 035503 [arXiv:0906.0959 [hep-ph]].

[16] B. Autin et al., J. Phys. G 29 (2003) 1785 [arXiv:physics/0306106].

[17] The Berkeley Laboratory Isotopes Project, http://ie.lbl.gov/education/isotopes.htm.

[18] M. A. Fraser, A survey of the introduction of rae-earth nuclei into the beta beam accelerator chain in order to attain a monochromatic neutrino beam, EURISOL DS task note 12-25-2008-0011, available from http://beta-beam.web.cern.ch/beta-beam/docs.htm.

[19] M. Lindroos, AIP Conf. Proc. 981 (2008) 93.

[20] M. Lindroos, J. Bernabeu, J. Burguet-Castell and C. Espinoza, PoS HEP2005 (2006) 365.

[21] M. Lindroos, private communication.

[22] The International Scoping Study for a Neutrino Factory - RAL-TR-2007024.

[23] T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 10 (2008) 113011 [arXiv:0808.2016 [hep-ph]].