Abstract

Perturbative relations between pole and running heavy quark masses, defined in the Minkowski regions, are considered. Special attention is paid to the appearance of the kinematic $\pi^2$-effects, which exist in the coefficients of these series. The estimates of order $O(\alpha_s^4)$ QCD corrections are presented.

1 Introduction

Among important parameters of QCD are the masses of $c$, $b$ and $t$-quarks, which are more heavy than $N_L=3,4,5$ number of lighter ones. They can be defined either as the poles of the corresponding renormalised heavy-quark propagators at $q^2 = M^2_{(N_L+1)}$ in the Minkowski space-like region or as the running masses $\overline{m}_{(N_L+1)}(\mu^2)$ in the $\overline{MS}$-scheme. Their scale-dependence is described by the solution of the following equation

$$\frac{\overline{m}_{(N_L+1)}(s)}{\overline{m}_{(N_L+1)}(\mu^2)} = \exp\left[ \int_{\alpha_s(\mu^2)}^{\alpha_s(s)} \frac{\gamma_{m_{(N_L+1)}}(x)}{\beta(x)} \, dx \right]$$

where $a_s(s) = \alpha_s(s)/\pi$ and $\alpha_s(s)$ is the QCD coupling constant of the $\overline{MS}$-scheme, fixed in the Minkowski reference point $s > \overline{m}^2_{(N_L+1)}$, and the renormalization group functions $\gamma_{m_{(N_L+1)}}(x)$ and $\beta(x)$ are defined as

$$\gamma_{m_{(N_L+1)}}(a_s) = \frac{d \ln \overline{m}_{(N_L+1)}(\mu^2)}{d \ln \mu^2} = - \sum_{i \geq 0} \gamma_i(N_L) a_s^{i+1}$$

$$\beta(a_s) = \frac{da_s(\mu^2)}{d \ln \mu^2} = - \sum_{i \geq 0} \beta_i(N_L) a_s^{i+2}.$$  

The coefficients $\beta_i(N_L)$ and $\gamma_i(N_L)$ (apart of the coefficient $\gamma_0$) depend from $N_L + 1$ number of active flavours. Note, that for the $\overline{MS}$-scheme heavy quarks
running masses $\overline{m}_{(N_L+1)}(\mu^2)$ the Minkowskian normalization point $\mu^2 = \overline{m}_{(N_L+1)}^2$ is frequently used (see, e.g., Ref. [1]). In this case the definition of $\overline{m}_{(N_L+1)}(\overline{m}_{(N_L+1)}^2)$ may be geometrically illustrated by finding the intersection of the curve, which represent the inverse logarithmic scale-dependence of the squared running mass, with the bisectrix of the angle, formed by positive axises $0 \leq \overline{m}_{(N_L+1)}^2 \leq \infty$ and $0 \leq \mu^2 \leq \infty$. The relations between pole and running heavy quark masses we will be interested read

$$M_{(N_L+1)} = \overline{m}_{(N_L+1)}(\overline{m}_{(N_L+1)}^2) \sum_{n=0}^{4} t_n^M(N_L) a_s^n(\overline{m}_{(N_L+1)}^2) .$$

(4)

Note, that in the process of comparison of theoretical predictions for the $e^+e^-$-annihilation Euclidean time-like characteristic, namely Adler D-function, with its experimental-motivated behaviour [2] heavy quark pole masses were defined in the MOM-scheme, while heavy quark running masses were defined at the Euclidean scale $\mu^2 = Q^2$. The similar mixed MOM-\ MS-scheme prescriptions are also widely used to analyse heavy-quark mass dependent effects in characteristics of deep inelastic scattering (see e.g. [3], [4]). However, the processes, which may be observed at LHC, are described by theoretical predictions in the time-like region of energies. In view of this it is important to study relations between different most commonly used definitions of heavy quark masses and to derive the relations between pole and running heavy quark masses, tied to the Euclidean and Minkowski regions of momentum transferred. This problem was analysed in Ref.[5] with the help of the special Källen-Lehman type representation. Here we will consider this approach in more detail, presenting additional arguments in favour of theoretical background of the investigations, performed in the work mentioned above. We will also update estimates of the order $O(\alpha_s^4)$ terms in the relation of Eq.(4), which were obtained in Ref.[5] using the extended to the mass-dependent case effective-charges inspired massless approach, elaborated in Ref.[6].

2 Comments on application of the dispersion relations

Let us discuss the subject of applicability of the Källen-Lehman type spectral representations within the context of perturbative QCD. The well-defined

\begin{footnotesize}
\footnote{We are grateful to G.B. Pivovarov for the discussion of this topic.}
\end{footnotesize}
dispersion relation for the $e^+e^-$-annihilation Adler function is well known

$$D_V(Q^2) = -Q^2 \frac{d \Pi_V(Q^2)}{dQ^2} = Q^2 \int_0^\infty \frac{R(s)}{(s + Q^2)^2} ds$$  \hspace{1cm} (5)$$

where $\Pi_V(Q^2)$ is the photon vacuum polarization function and $R(s) \sim Im \Pi_V$. The two-point function of the scalar quark currents $m_{(N_L+1)} \bar{\psi}_q \psi_q$ has the imaginary part, which defines the scalar Higgs boson decay width into quark-antiquark pairs. In this case it is possible to write-down the following representation [7]:

$$D_S(Q^2) = -Q^2 \frac{d}{dQ^2} \left[ \frac{\Pi(Q^2)}{Q^2} \right] = Q^2 \int_0^\infty \frac{R_S(s)}{(s + Q^2)^2} ds$$  \hspace{1cm} (6)$$

which faces no problems in the region where the asymptotic freedom property of QCD holds. The same equation was used in Ref. [5] to extend the massless procedure of the estimates of higher-order perturbative corrections to the Euclidean quantities [6] to the case of Eq. (6), which contains the dependence from the square of running mass $m_{(N_L+1)}(Q^2)$ defined in the Euclidean region. However, as was shown in Ref. [8], the dispersive relation of Eq. (6) is valid within perturbative sector only and can not be proved on the level of rigour, considered in Ref. [9]. Indeed, it was shown in Ref. [8] that in the low-energy region Eq. (6) is ill-defined and contains fictitious $\Lambda^2_{QCD}/Q^2$-term. It reflects the failure to remove the infinities from $\Pi_S(0)$. The well-defined dispersive relation, which do not contain this term, can be written down through the second derivative of the scalar correlator [10]. It leads to the following Euclidean function

$$\mathcal{D}_S(Q^2) = 2Q^2 \int_0^\infty \frac{sR_S(s)}{(s + Q^2)^3} ds$$  \hspace{1cm} (7)$$

Note, however, that its perturbative expansion differs from the one, which corresponds to the Euclidean part of perturbative series for $\Gamma(H^0 \rightarrow \bar{q}q)$, generated by the ill-defined in non-perturbative sector expression of Eq. (6). Moreover, the application of the “approximate” dispersion relation from Eq. (6) fixes the kinematic $\pi^2$-contributions to the coefficients of the perturbative series for $\Gamma(H^0 \rightarrow \bar{q}q)$ both in the expanded [5] and summed up [8, 11] forms. Note, that the idea of the summation of $\pi^2$-terms at lowest order of QCD was proposed and used over thirty five years ago in the works of Refs. [12, 13, 7].
3 Dispersion relations for the pole and running heavy quark masses

Consider now the following “approximate” dispersion model of Ref.\[5\] for the heavy quark pole masses

\[
M_{(NL+1)} = \frac{1}{2\pi i} \int_{-e^{i\pi}(m_{(NL+1)}^2+ie)}^{e^{i\pi}(m_{(NL+1)}^2+ie)} ds' \int_{0}^{\infty} \frac{T(s)}{(s+s')^2} ds
\]

with the spectral density defined as \(T(s) = m_{(NL+1)}(s)\sum_{n=0}^{4} t_n^M a_n^e(s)\). It can be obtained from the dispersion-type expression for the Euclidean series

\[
F(Q^2) = m_{(NL+1)}(Q^2) \sum_{n=0}^{4} f_n^E(N_L) a_n^e(Q^2) = Q^2 \int_{0}^{\infty} \frac{T(s)}{(s+Q^2)^2} ds
\]

where \(m_{(NL+1)}(Q^2)\) and \(a_e(Q^2)\) are the heavy quark masses and the QCD coupling constant which are “running” in the Euclidean region. The application of Eq.\([5]\) allows one to fix the relations between coefficients \(f_n^E(N_L)\) and \(t_n^M(N_L)\) of the perturbative series in the time-like and space-like regions as \(f_0^E = t_0^M, f_1^E = t_1^M, f_2^E(N_L) = t_2^M(N_L) + e_2(N_L), f_3^E(N_L) = t_3^M(N_L) + e_3(N_L), f_4^E(N_L) = t_4^M(N_L) + e_4(N_L)\). The kinematic \(\pi^2\)-terms enter the derived in Ref.\([5]\) explicit expressions for the \(e_i(N_L)\)-contributions, namely

\[
e_2(N_L) = \frac{\pi^2}{6} t_0^M \gamma_0 (\beta_0 + \gamma_0) = 5.89435 - 0.274156 N_L
\]

\[
e_3(N_L) = \frac{\pi^2}{3} \left\{ t_1^M (\beta_0 + \gamma_0) \left(\beta_0 + \frac{\gamma_0}{2}\right) + t_0^M \left[ \frac{\beta_1 \gamma_0}{2} + \gamma_1 (\beta_0 + \gamma_0) \right] \right\}
\]

\[
e_4(N_L) = \pi^2 \left\{ t_2^M (\beta_0 + \frac{\gamma_0}{2}) + t_1^M \left[ \frac{\beta_1}{2} (3 \beta_0 + \gamma_0) + \frac{\gamma_1}{3} (2 \beta_0 + \gamma_0) \right] \right\}
\]

\[
+ t_0^M \left[ \frac{\beta_2 \gamma_0}{6} + \frac{\gamma_1}{3} \left( \beta_0 + \frac{\gamma_0}{2} \right) + \gamma_2 \left( \frac{\beta_0}{2} + \frac{\gamma_0}{3} \right) \right] \}
\]

\[
+ 7 \pi^4 \left( \frac{\gamma_0}{60} t_0^M \gamma_0 (\beta_0 + \gamma_0) (\beta_0 + \gamma_0) \right) \]

\[
= 2272.02 - 403.951 N_L + 20.6768 N_L^2 - 0.315898 N_L^3
\]

Their \(N_L\)-dependence result from \(N_L\)-dependence of the coefficients \(\beta_i(N_L)\) with \(i \geq 0\) in Eq.\([5]\), \(\gamma_i(N_L)\) with \(i \geq 1\) in Eq.\([2]\) and \(t_2^M\) in Eq.\([3]\), which has the following numerical form \([14]\)

\[
t_2^M = 13.44396 - 1.041367 N_L
\]
and comes from the analytical expression of Ref.\[15\], confirmed by the independent calculations of Ref.\[16\]. Notice, that the results of Refs.\[15\],\[16\] contain the explicit dependence from \(\zeta_2 = \pi^2/6\)-terms. The discussions presented above clarify that the part of these \(\pi^2\)-terms, explicitly visible in the formulae of Refs.\[15\],\[16\], appear from the analytical continuation effect of Eq.\(\text{(10)}\). This our claim can be generalised to the level of \(\xi^M_3\)-corrections, evaluated analytically in Ref.\[14\] and semi-analytically in Ref.\[17\]. In this case kinematic \(\pi^2\)-contributions are determined by Eq.\(\text{(11)}\). The coefficients of the relation between heavy quark Euclidean masses, defined in the MOM on-shell, and \(\overline{\text{MS}}\)-scheme masses, contain only remaining transcendental terms, typical to the on-shell scheme calculations.

4 Estimates of \(\alpha^4_s\) corrections

We consider now two perturbative series, namely the one of Eq.\(\text{(4)}\) and the related to it relation

\[
M_{(N_L+1)} = \overline{m}_{(N_L+1)}\left(M^2_{(N_L+1)}\right) \sum_{n=0}^{4} v^M_n(N_L) a^s_n(M^2_{(N_L+1)}) \ . \quad (15)
\]

Keeping in mind that for \(0 \leq n \leq 3\) the values of the explicit dependence from \(N_L\) of the coefficients \(v^M_n(N_L)\) is already known \[14\], \[1\], we will study the problem of estimates of the \(\alpha^3_s\) and \(\alpha^4_s\) coefficients, using the effective-charges (ECH) inspired approach, developed and used in Refs.\[6\], \[5\]\(^2\). It is known that the applications of this approach in the Euclidean region at the level of \(\alpha^3_s\) and \(\alpha^4_s\) corrections give correct signs and in order of magnitude estimates of the perturbative contributions to the number of physical quantities (see e.g. \[6\], \[5\], \[22\], \[23\]). As to the application of this procedure to the Minkowskian quantities, two ways are possible. The first, prescribes to apply the procedure of estimates in the Euclidean region and add explicitly calculable kinematic \(\pi^2\)-terms afterwards. Within the second way one may use the procedure of estimates in the Minkowski region directly. It should be noted, that both ways are leading to reasonable predictions of signs and numerical values of perturbative series for physical quantities. Moreover, in the case of direct application of this approach in the Minkowski region, the order \(\alpha^4_s\) estimates are sometimes even closer to the results of the explicit calculations (see e.g. Ref. \[22\]). However, in the

\(^2\) The method of ECH was proposed and developed in Refs.\[18\],\[19\] and independently in Ref.\[20\] (see also Ref. \[21\]).
latter case the estimates do not reproduce the known values of the analytical continuation effects, similar to the ones of Eq. (11) and Eq. (13). Note, that their precise knowledge is important for applying different approaches of re-summations of these contributions (see e.g. [24]-[27], [8], [11]). Following two ways mentioned above we first estimate the values of $t_M^3(N_L)$ coefficients and compare them with the results for $t_M^{\text{exact}}(N_L)$ obtained in Refs. [14], [17]. Satisfied by this comparison we are going one step further and estimate $t_M^4(N_L)$-coefficients, taking into account the numerical expressions for $t_M^{\text{exact}}(N_L)$. The concrete numbers are presented in Table 1. One can see, that the estimates obtained give correct signs and order of magnitude estimates for the values of $t_M^3(N_L)$-terms. Thus, one may hope that the estimates for $t_M^4(N_L)$ are not far from reality. We present now concrete numbers for the coefficients of the series of Eq. (4), where for the $\alpha_s^4$-coefficients we use the estimates $t_{ECH}^4(N_L)$ from Table 1:

$$M_c \approx \overline{m}_c(\overline{m}_c^2) \left[ 1 + \frac{4}{3} a_s(\overline{m}_c^2) + 10.3 a_s^2(\overline{m}_c^2) + 116.5 a_s^3(\overline{m}_c^2) + 1281 a_s^4(\overline{m}_c^2) \right]$$  \hspace{1cm} (16)$$

$$M_b \approx \overline{m}_b(\overline{m}_b^2) \left[ 1 + \frac{4}{3} a_s(\overline{m}_b^2) + 9.28 a_s^2(\overline{m}_b^2) + 94.4 a_s^3(\overline{m}_b^2) + 986 a_s^4(\overline{m}_b^2) \right]$$ \hspace{1cm} (17)$$

$$M_t \approx \overline{m}_t(\overline{m}_t^2) \left[ 1 + \frac{4}{3} a_s(\overline{m}_t^2) + 8.24 a_s^2(\overline{m}_t^2) + 73.6 a_s^3(\overline{m}_t^2) + 719 a_s^4(\overline{m}_t^2) \right]$$ \hspace{1cm} (18)$$

The similar relations for Eq. (15) with on-shell normalizations of running parameters read

$$M_c \approx \overline{m}_c(M_c^2) \left[ 1 + \frac{4}{3} a_s(M_c^2) + 13 a_s^2(M_c^2) + 156 a_s^3(M_c^2) + 1853 a_s^4(M_c^2) \right]$$ \hspace{1cm} (19)$$

$$M_b \approx \overline{m}_b(M_b^2) \left[ 1 + \frac{4}{3} a_s(M_b^2) + 12 a_s^2(M_b^2) + 131 a_s^3(M_b^2) + 1460 a_s^4(M_b^2) \right]$$ \hspace{1cm} (20)$$

$$M_t \approx \overline{m}_t(M_t^2) \left[ 1 + \frac{4}{3} a_s(M_t^2) + 11 a_s^2(M_t^2) + 107 a_s^3(M_t^2) + 1101 a_s^4(M_t^2) \right]$$ \hspace{1cm} (21)$$

| $N_L$ | $t_3^{\text{exact}}$ | $t_3^{ECH}$ | $t_3^{ECH \: \text{direct}}$ | $t_4^{ECH}$ | $t_4^{ECH \: \text{direct}}$ |
|------|----------------------|-------------|-----------------------------|-------------|-----------------------------|
| 5    | 73.6366              | 58.0645     | 48.4906                     | 719.339     | 710.016                     |
| 4    | 94.4175              | 100.74      | 78.243                      | 986.097     | 1045.5                      |
| 3    | 116.504              | 147.303     | 111.315                     | 1281.05     | 1438.75                     |

Table 1: The estimates for $t_M^3(N_L)$, $t_M^4(N_L)$. 

The similar relations for Eq. (15) with on-shell normalizations of running parameters read

$$M_c \approx \overline{m}_c(M_c^2) \left[ 1 + \frac{4}{3} a_s(M_c^2) + 13 a_s^2(M_c^2) + 156 a_s^3(M_c^2) + 1853 a_s^4(M_c^2) \right]$$ \hspace{1cm} (19)$$

$$M_b \approx \overline{m}_b(M_b^2) \left[ 1 + \frac{4}{3} a_s(M_b^2) + 12 a_s^2(M_b^2) + 131 a_s^3(M_b^2) + 1460 a_s^4(M_b^2) \right]$$ \hspace{1cm} (20)$$

$$M_t \approx \overline{m}_t(M_t^2) \left[ 1 + \frac{4}{3} a_s(M_t^2) + 11 a_s^2(M_t^2) + 107 a_s^3(M_t^2) + 1101 a_s^4(M_t^2) \right]$$ \hspace{1cm} (21)$$

6
The results presented in Eq. (20) give the following ratios of the squares of running and pole $b$-quark masses

$$\frac{m_b^2(M_b^2)}{M_b^2} = 1 - \frac{8}{3} a_s(M_b^2) - 18.5559 a_s^2(M_b^2) - 175.797 a_s^3(M_b^2) - 1684 a_s^4(M_b^2)$$

(22)

where the last term is fixed by the result of application of the ECH-motivated approach with adding kinematic $\pi^2$-contributions at the final step. In the case when the Euclidean and kinematic $\pi^2$ corrections are summed up at the intermediate steps, the last coefficient in Eq. (22) should be changed from $-1684$ to $-1835$. Note, that in the process of analysing the uncertainties of QCD predictions for $\Gamma(H^0 \rightarrow b\bar{b})$, preformed in the work of Ref. [28], we used slightly lower estimate, namely $-1892$. The difference is explained in part by smaller number of significant digits taken into account in the values of coefficients, which enter in the procedure of corresponding estimates. However, this difference between the values of estimated order $O(\alpha_s^4)$ contributions are not so numerically important. Other possible physical applications, like the comparison with the renormalon-based analysis of asymptotic behaviour of perturbative series in Eq. (19)-Eq. (21) (for the related theoretical discussions one can see Refs. [29]-[31]) are beyond the scope of this study.

5 Acknowledgements

The results described above were presented by one of us (ALK) at the International Conference “Problems of Theoretical and Mathematical Physics”, dedicated to the 100th anniversary of the birth of N.N. Bogolyubov, Dubna, August 23-27, 2009, and at the 9th International Symposium on Radiative Corrections: Applications of Quantum Field Theory to Phenomenology (RADCOR-2009), October 25-30, 2009, Ascona, Switzerland. It is the pleasure to thank the Organizers of these scientific events for hospitality and partial financial support. We are grateful to A.P. Bakulev, M. Y. Kalmykov and S.V. Mikhailov for their interest to our work, and to J. Bluemlein for the questions, which stimulated us to describe in more detail the origin of the appearance of the $\pi^2$-analytical continuation effects in the relations of heavy quark masses, defined in the Minkowski region. We also wish to thank N.V. Krasnikov for discussions of the topics concerning different applications of the Källen-Lehman representations. The work is done as the research, planned within the program of the Grants RFBR 08-01-00686, 08-02-01184 and President of RF NS-1616.2008.2 and NS-378.2008.2.
References

[1] Chetyrkin K.G., Kuhn J.H. and Steinhauser M. // Comput. Phys. Commun. 2000. V.133. P.43.

[2] Eidelman S.I., Jegerlehner F., Kataev A.L. and Veretin O.L. // Phys.Lett. B. 1999. V.454. P.369.

[3] Bierenbaum I., Blumlein J. and Klein S. // Nucl. Phys. B. 2009. V.820 P. 417.

[4] Forte S., Laenen E., Nason P. and Rojo J. // [arXiv:1001.2312 [hep-ph]].

[5] Chetyrkin G., Kniehl B.A. and Sirlin A. // Phys. Lett. B. 1997. V.402. P.35.

[6] Kataev A.L. and Starshenko V.V. // Mod.Phys.Lett.A.1995.V.10. P.235.

[7] Gorishnii S.G., Kataev A.L. and Larin S.A. // Sov. J. Nucl. Phys. 1984. V.40. P.329. [Yad. Fiz. 1984. V.40. P.517.]

[8] Broadhurst D.J., Kataev A.L. and Maxwell C.J. // Nucl. Phys. B. 2001. V.592. P.247.

[9] Bogolyubov N. N., Medvedev B. V. and Polivanov M.C. // Problems of the theory of dispersive relations, First printed by the permission of authors at The Institute of Advanced Study, Princeton, USA, 1956; Reprinted edition, Dubna, JINR, 2009.

[10] Becchi C., Narison S., de Rafael E. and Yndurain F.J. // Z. Phys. C. 1981. V.8. P.335.

[11] Bakulev A.P., Mikhailov S.V. and Stefanis N.G. // Phys. Rev. D. 2007. V.75. 056005; [Erratum-ibid. D. 2008. V.77. 079901.]

[12] Radyushkin A.V. // Preprint JINR E2-82-159, 1982; JINR Rapid Commun. 1996. V.78. P.96.

[13] Krasnikov N.V. and Pivovarov A.A. // Phys.Lett.B. 1982. V.116. P.168.

[14] Melnikov K. and v. Ritbergen T. // Phys. Lett. B. 2000. V.482. P.99.

[15] Gray N., Broadhurst D.J., Grafe W. and Schilcher K. // Z. Phys. C. 1990. V.48. P.673.
[16] Fleischer J., Jegerlehner F., Tarasov O.V. and Veretin O.L. // Nucl. Phys. B. 1999. V.539. P.671; [Erratum-ibid. B. 2000. V.571. P.511.]

[17] Chetyrkin K.G. and Steinhauser M. // Nucl.Phys.B. 2000. V.573. P.617.

[18] Grunberg G. // Phys. Lett. B. 1980. V.95. 70.; [Erratum-ibid. B. 1982. V.110. 501.]

[19] Grunberg G. // Preprint Ecole Polytechnique-82-0720. 1982.; Phys. Rev. D. 1984. V.29. 2315.

[20] Krasnikov N.V. // Nucl. Phys. B. 1981. V.192. 497.; Yad. Fiz. 1982. V.35. 1594.

[21] Kataev A.L., Krasnikov N.V. and Pivovarov A.A. // Nucl. Phys. B. 1982. V.198. 508.; [Erratum-ibid. B. 1997. V.490. 505.]

[22] Baikov P.A., Chetyrkin K.G., and Kuhn J.H. // Phys.Rev.Lett. 2006. V.96. 012003.

[23] Baikov P.A., Chetyrkin K.G. and Kuhn J.H. // Phys. Rev.Lett. 2008. V.101. 012002.

[24] Pivovarov A.A. // Sov. J. Nucl. Phys. 1991. V.54. P.676. [Yad. Fiz. 1991. V.54. P.1114.]; Z. Phys. C. 1992. V.53. P.461.

[25] Le Diberder F. and Pich A. // Phys. Lett. B. 1992. V.286. P.147.

[26] Ahrens V., Becher T., Neubert M. and Yang L.L. // Eur. Phys. J. C. 2009. V.62. P.333.

[27] Bakulev, A. P. // Phys. Part. Nucl. 2009. V.40. P.715.

[28] Kataev A.L. and Kim V.T. // Proceedings of Science. PoS. ACAT08. 2009. 004.

[29] Beneke M. and Braun V.M. // Nucl. Phys. B. 1994. V.426. P.301.

[30] Bigi I.I.Y., Shifman M. A., Uraltsev N.G. and Vainshtein A.I. // Phys. Rev. D. 1994. V.50. P.2234.

[31] Hoang A.H., Jain A., Scimemi I., and Stewart I.W. // Phys. Rev. Lett. 2008. V.101. 151602.