**Quantitative phase spectroscopy**

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**Abstract:** Quantitative phase spectroscopy is presented as a novel method of measuring the wavelength-dependent refractive index of microscopic volumes. Light from a broadband source is filtered to a ~5 nm bandwidth and rapidly tuned across the visible spectrum in 1 nm increments by an acousto-optic tunable filter (AOTF). Quantitative phase images of semitransparent samples are recovered at each wavelength using off-axis interferometry and are processed to recover relative and absolute dispersion measurements. We demonstrate the utility of this approach by (i) spectrally averaging phase images to reduce coherent noise, (ii) measuring absorptive and dispersive features in microspheres, and (iii) quantifying bulk hemoglobin concentrations by absolute refractive index measurements. Considerations of using low coherence illumination and the extension of spectral techniques in quantitative phase measurements are discussed.

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1. Introduction

Quantitative phase microscopy (QPM) has developed into an effective tool for measuring spatial and temporal properties of semitransparent samples, especially in vitro cells and microfluidic systems [1]. Recent interest has turned toward the wavelength dependence of refractive index in the context of QPM. Park et al. demonstrated the ability to measure pure solutions of bovine serum albumin and hemoglobin using a white light source and seven color filters with various center wavelengths [2]. They further showed that it was possible using three specific wavelengths to measure spatial distributions of hemoglobin within individual blood cells [2]. Fu et al. used a two-wavelength QPM system to measure and map dispersion in live HeLa cells [3]. Multi-wavelength illumination has also been employed to aid phase unwrapping [4] and to decouple refractive index from cell thickness [5]. All of these methods exploit dispersive effects in QPM at small numbers of discrete spectral points.

Here, we introduce quantitative phase spectroscopy (QPS) as a method of investigating spatially-resolved dispersion in microscopic samples. QPS combines microscopic quantitative phase measurements with a rapidly-tunable broadband source. In addition to producing spatially-resolved quantitative dispersion information, QPS phase images can be spectrally averaged to reduce coherent noise. We demonstrate the utility of this new technique by characterizing dispersion differences between fluorescent and non-fluorescent microspheres. We also use this instrument to measure the absolute refractive index as a function of wavelength in bulk hemoglobin solutions.

2. Theory

2.1. Quantitative phase spectroscopy (QPS)

QPM is readily extended into QPS by either sweeping the center wavelength of a narrowband source or using a tunable filter with a broadband source and acquiring interferometric data at multiple discrete wavelength steps. While swept source lasers and some tunable filters can have extremely narrow output lines, this paper focuses on the specific application of an acousto-optic tunable filter with a supercontinuum source. The resulting illumination spectra are broad enough (~5 nm) that temporal coherence effects are evident in the off-axis interferograms.

2.2. Low-coherence off-axis interferometry

Consider the intensity recorded by a digital camera from interference between monochromatic sample and reference fields: 

\[ I(k, \vec{r}) = I_S(k, \vec{r}) + I_R(k, \vec{r}) + I_{INT}(k, \vec{r}), \]

where \( k = \frac{2\pi}{\lambda} \) and \( \vec{r} \) is a two-dimensional spatial vector. The first two terms are the intensities of each individual field and \( I_{INT} \) is an interferometric term containing cross correlated information about the two fields. The interference term may be written as: 

\[ I_{INT} \propto \text{Re} \left\{ e^{i(k_0 + k_0 + \Delta \varphi(k, \vec{r}))} \right\}, \]

where \( \vartheta \) is a vector of the Cartesian-coordinate angular difference between the sample and reference plane waves; \( \delta z \) is the average optical path length (OPL) difference between the two interferometer arms; and \( \Delta \varphi(k, \vec{r}) \) encompasses the spatially- and wavelength-dependent phase delays, relative to the average OPL, imparted by the semitransparent sample. Extending the interference term to describe a source with a finite spectral extent, modeled here as a Gaussian distribution, yields a detected interference term that is integrated across the wavenumber spectrum:

\[ I_{det}(k_0, \vec{r}) \propto e^{-\frac{(k-k_0)^2}{2\delta k^2}} \text{Re} \left\{ e^{i(k_0 + k_0 + \Delta \varphi(k, \vec{r}))} \right\} dk. \]
Here, \( k_0 = 2\pi / \lambda_0 \) is the center wavenumber and \( \delta k \) is a measure of the spectral bandwidth of the illumination. If the dispersion of \( \Delta \varphi \) is small relative to this spectral bandwidth, \( \Delta \varphi(k, \vec{r}) \) simplifies to \( \Delta \varphi(k_0, \vec{r}) \), in which the wavenumber is treated as constant across this integral. Integrating the remaining terms yields

\[
I(k_0, \vec{r}) \propto \text{Re} \left\{ e^{i\Delta \varphi(k_0, \vec{r})} e^{-i\theta \delta z} e^{-i\theta \delta z / 4} \right\}.
\] (2)

The first exponential term in Eq. (2) describes the relative phase delays of the semitransparent sample object at location \( \vec{r} \); the second exponential term is the high-frequency spatial carrier; and the third exponential term describes the complex amplitude coherence envelope that modulates the interferometric information. It is important to note that both the angular carrier frequency and the spatial fringe visibility dictated by the coherence envelope are set by \( \theta \), and thus a fixed number of fringes are visible for a given spectral bandwidth \( \delta k \). As can be seen in Figs. 1(A)–1(B) and 1(E)–1(F), an increase of \( \delta k \) from 1 nm to 5 nm reduces the number of fringes visible, and in Figs. 1(B)–1(C) and 1(F)–1(G), a 20 \( \mu m \) change in bulk path delay \( \delta z \) between the two interferometer arms causes the fringes to shift to a different lateral section of the field of view. The primary consequence of such a finite spectral bandwidth in the context of off-axis interferometry is that the phase sensitivity at the edges of the field of view decreases as the width of the complex envelope decreases [6]. Several groups have investigated the use of diffractive optical elements to alleviate this effect when using a large spectral bandwidth [7,8].

2.3. Spectral averaging of phase measurements

As illustrated in Figs. 1(D) and 1(H), traditional off-axis QPM with a large spectral bandwidth results in a narrowed field of view due to the low temporal coherence; however, this also permits high-SNR phase measurements with reduced coherent noise. On the other hand, quantitative phase images with a narrow bandwidth source benefit from a wide field of view but often suffer from coherent artifacts arising from speckle and spurious reflections. In a QPS setup using narrow-band illumination, coherent noise artifacts can be significantly improved by spectral averaging of phase measurements. In cases where the phase delay of light...
transmitted through a material is linear in $k$ (i.e., no chromatic dispersion), averaging the phase images across a range of spectral points, $k'_0$, yields

$$\overline{\Delta \phi(k'_0, \vec{r})} = \sum_{i=1}^{N} \frac{\Delta \phi(k'_i, \vec{r})}{N}, \quad (3)$$

where $k'_0 = \text{mean}(k'_1, k'_2, k'_3, \ldots, k'_N)$. The resulting phase image $\overline{\Delta \phi(k'_0, \vec{r})}$ maintains both the wide field of view and extended axial range afforded by narrowband illumination at each $k_0$ while reducing the magnitude of coherent artifacts to a similar extent as would be achieved by illuminating with the full spectral bandwidth. It is important to note that when higher order $k$-dependencies of phase are present (e.g., material dispersion), $\overline{\Delta \phi(k'_0, \vec{r})}$ is a first-order approximation of $\Delta \phi(k'_i, \vec{r})$ and a more detailed calculation is needed to determine the average $k'_0$ over the spectral illumination range.

2.4. Calculation of relative and absolute refractive index

The relative phase delays, written as $\Delta \phi(k_0, \vec{r})$ in the first exponential term in Eq. (2), are related to the optical path length and absolute refractive index as $\Delta \phi(k_0, \vec{r}) / k_0 = \Delta n(k_0, \vec{r}) \cdot d(\vec{r}) = \Delta \text{OPL}(k_0, \vec{r})$. By rearranging this equation to solve for $\Delta n$, it becomes clear that calculating relative dispersion requires knowledge of the object thickness $d(\vec{r})$ and that calculation of the absolute refractive index at a specific point requires a calibrated refractive index standard at another point within the field of view against which to reference the phase measurement [1]. With both of these reference quantities known, the refractive index as a function of wavelength at any point within the field of view can be calculated as

$$n_2(k_0, \vec{r}) = \frac{\Delta \phi(k_0, \vec{r})}{k_0 d(\vec{r})} + n_1(k_0, \vec{r}), \quad (4)$$

where $\Delta \phi$ is the difference in phase between points 1 and 2, $n_1$ is the reference point with a previously-characterized refractive index spectrum, and $n_2$ is the refractive index spectrum at the point of interest. It is important to note that in order to calculate the absolute refractive index, $\Delta \phi(k_0, \vec{r})$ must be accurately unwrapped and is limited to a range of phase delays defined by the coherence length of the source.

3. Experimental setup and methods

QPS is implemented using a modified Mach-Zehnder off-axis interferometer similar to that described previously [4]. Light from a supercontinuum laser source is spectrally filtered to a narrow band by a rapidly-tunable acousto-optic filter, coupled into a single-mode fiber, and linearly polarized before entering the interferometer. The interferometer contains adjustable path length-matching optics and compensating glass coverslips in the reference arm to offset differences in bulk dispersion arising from the sample and other interferometer optical elements. The tuning of the spectral filter is synchronized to the camera acquisition by software triggering, thus enabling an arbitrary spectral sweep and dwell pattern that is configurable based on the specific application. Off-axis interferograms acquired at each center wavenumber, $k_0$, are digitally processed individually to yield two-dimensional quantitative phase images; all images are referenced to a suitable background point within the field of view.
For the experiments demonstrated in this paper, the filtered output of a supercontinuum laser (SC450-2-AOTF, Fianium) was tuned from 500 to 720 nm in 1 nm steps with a FWHM bandwidth ranging from 3.8 nm at $\lambda_0 = 460$ nm to 8.2 nm at $\lambda_0 = 720$ nm. Tuning was performed by an acousto-optic tunable filter with input RF waveforms synthesized by a programmable frequency synthesizer (iDDS-2-SE, Isomet). Interferograms were acquired with a high speed CMOS camera with 20 $\mu$m square pixels (Fastcam SA4, Photron) using a 2.5 ms integration time and a 20 ms delay between tuning steps, resulting in a 5.5 s acquisition time for a sweep that comprises the majority of the visible spectrum. Imaging and wavefront matching optics consisted of 100$\times$, 0.9 NA dry objectives with a 1mm working distance (Nikon) in 4F image relay configuration with an F = 200 mm achromatic tube lens. The angle between the reference and sample beams at the detector was set to $\theta_x, \theta_y \approx 6.25$ mrad, yielding a fringe frequency of 1 cycle/4px at $\lambda_0 = 500$ nm and 1 cycle/5.76 px at $\lambda_0 = 720$ nm. This range of carrier fringe frequencies was chosen to optimally use the camera’s spatial frequency bandwidth for off-axis interferometry [9].

The acquired interferograms were each processed to yield quantitative phase images according to methods described in our previous work [1]. It is important to note that the fringe modulation frequency in term 2 of Eq. (2) is a function of $k_0$ and thus changes as the center illumination wave number is tuned. Therefore the spatial filtering step in the digital phase retrieval algorithm must be tailored for each acquired interferogram based on previous system characterization.

4. Experimental results & discussion

4.1. Coherent noise reduction

A custom-molded phase target was used to demonstrate coherent noise reduction across a wide field of view. The phase target was molded by casting PDMS (polymethylsiloxane, Sylgard 184, Dow Corning) on top of a custom fabricated photomask (Photo Sciences Inc.) with 90 nm nominal thickness chrome features, consisting of the words ‘BIOS LAB’, on a
glass substrate. After molding, the PDMS was peeled off of the photo mask and placed feature-side-up on a microscope coverslide. A quantitative phase image of this sample, acquired using $\lambda_0 = 603$ nm, $\delta \lambda = 5.4$ nm is shown in Figs. 2(A) and 2(D). This image suffers from coherent noise and exhibits degraded phase information at the edges of the field of view, as predicted by Eq. (2). Figures 2(B) and 2(E) show a composite phase image of the target computed by spectrally-averaging 220 quantitative phase images acquired in 1 nm intervals across the 500–720 nm bandwidth. The effective center wavelength, computed as $\lambda' = 2\pi / k'_{\lambda_0}$ (see Subsection 2.3), is $\lambda'_0 = 603$ nm. While the field of view is comparable for both images, the coherent noise is significantly reduced in the spectrally-averaged phase image. The remaining variations in measured thickness that are still visible are primarily due to smooth changes in the molded PDMS thickness. Note that the improved SNR reveals the bottom half of the first letter “B”, which is overwhelmed by phase noise in Fig. 2(A). The standard deviation of a flat background area denoted by the star symbols in Fig. 2(D) and 2(E) are $\sigma = 27.7$ mrad and $\sigma = 8.6$ mrad, respectively. This demonstrates a factor of $\sim 3.2 \times$ reduction in the amplitude of spatially-dependent coherent noise artifacts achieved by spectral averaging.

For comparison, we consider an alternative method of reducing coherent artifacts by illuminating the sample with the entire spectral range in a single shot, corresponding to an even lower degree of coherence. However, according to Eq. (2) and as illustrated in Fig. 1, only a few fringes would be visible when using traditional off-axis QPM. The area of high-SNR phase measurements would be limited to a fraction of the total field of view. Figures 2(C) and 2(F) simulate the phase image field that would be visible if illuminated with a spectral range of 500–720 nm simultaneously.

Fig. 3. (A) Phase images of a fluorescent 10μm polystyrene microsphere, $\lambda'_0 = 594$ nm; scale bar is 5 μm. (B) Phase profile taken from dashed line (A). (C) Published absorption peak of fluorescent microsphere [10]. (D) Theoretical and measured absolute refractive index spectra of fluorescent and non-fluorescent microspheres, calculated from the points circled in (B).

4.2. Phase measurements of absorptive features

According to the Kramers-Kronig relation, absorption peaks in a sample give rise to nonlinear features in the real refractive index [11,12]. In this section, we examine the ability of QPS to recover spectroscopic data from phase images using fluorescent (F-8833, Life Technologies) and non-fluorescent (Duke Scientific) 10μm polystyrene microspheres as samples. The
microspheres were immersed in index-matching oil (Series A, Cargille Labs), $n_o = 1.5900$, and imaged using QPS as described above. Figure 3(A) shows the quantitative phase map of the fluorescent microsphere, $\Delta \varphi$, calculated by averaging phase images from 500 to 700 nm, $\lambda_0' = 594$ nm. Using a background point as a reference for the relative delays through the center of the microsphere (red encircled points in Fig. 3(B)), the absolute refractive index was calculated according to Section 2.4 by adding the computed $\Delta n$ to the refractive index spectrum of the immersion oil. Figure 3(C) shows the absorption peak published for the fluorescent microsphere [10]. Figure 3(D) shows the measured and theoretical refractive indices of the fluorescent (blue line) and non-fluorescent (red line) microspheres. The theoretical curve for the non-fluorescent microsphere is simply the known RI of polystyrene as a function of wavelength, while the RI of the fluorescent microsphere was calculated by adding the theoretical RI of the fluorophore, as calculated by a truncated Kramers-Kronig relation [11,12], to the RI of polystyrene. The strong modulation introduced by the presence of the absorbing fluorophore is easily visible here, and shows good agreement to the theoretical predictions.

![Fig. 4. (A) Phase image of PDMS chamber for bulk fluid RI spectroscopy, scale bar 25 μm. (B) Measured RI dispersion spectra for serial dilutions of oxy-hemoglobin and PBS. (C) Refractive index of measured fluids as a function of hemoglobin concentration. Blue diamonds correspond to values measured by QPS; Red triangles to values measured by a commercial refractometer. $\lambda_0 = 589$ nm; fit trend line: $n = 1.3362 + 0.1496C$, $R^2 = 0.9962$.](image)

4.3. Hemoglobin dispersion measurements

The final proof of principle experiment presented here is the application of QPS to acquire quantitative refractive index spectra of bulk fluids. A custom-molded PDMS chamber (Fig. #164551 - $15.00 USD #164551 - $15.00 USD
4(A)) with a $d = 10.5\mu m$ step height is first filled with deionized H$_2$O with a known refractive index [13]. Since the difference in phase, $\Delta \phi$, between points on the top and bottom surfaces of this chamber can potentially exceed $2\pi$, the mold was engineered to have a smooth slope of $\sim 54.74^\circ$ over the step between the PDMS area and the fluid-filled area (arrow in Fig. 4(A)) in order to enable accurate phase unwrapping [1]. The refractive index of the PDMS is treated as the sample to be measured, and is calculated using Eq. (4). The resulting spectrum is fit to a $1/k$ trend line, which is subsequently used as the reference refractive index of PDMS.

After characterizing the chamber, the water is replaced with the fluid of interest. In these experiments, we measure serial dilutions of ferrous-stabilized hemoglobin powder (H0267, Sigma-Aldrich) dissolved in phosphate buffer solution (PBS), forming oxy-hemoglobin solutions [14] with biologically-relevant concentrations of 331 g/L, 165 g/L, 110 g/L, 55 g/L, and 0 g/L (PBS). Figure 4(B) shows the refractive index spectra calculated for each of these solutions from the differential phase measurements averaged over $4 \times 4\mu m$ areas, as indicated by the white squares shown in Fig. 4(A). The noise associated with this measurement, expressed by the spatial standard deviation across the points used in the measurement, is shown for each solution by the dashed gray lines. This standard deviation value monotonically decreases with increasing concentration from an average of $1.59 \times 10^{-3}$ for PBS to $6.34 \times 10^{-4}$ for 331 g/L oxy-hemoglobin.

To validate these measurements, the refractive index of these solutions at $\lambda_0 = 589$ nm as measured with QPS (blue diamonds in Fig. 4(C)) were compared to RI values of the same samples measured by a commercial refractometer (RFM300, Bellingham & Stanley) at $\lambda_D = 589.3$ nm (red triangles in Fig. 4(C)). These measurements show good agreement and also compare well with a recent report of direct multi-wavelength refractometry measurements of oxy-hemoglobin solutions [15]. Upon fitting the concentration-dependent measurements of refractive index data to the form originally described by Barer et al. [16]: $n = n_0 + \alpha C$, where $C$ is in g/mL, a trend line of $n = 1.3362 + 0.1496C$ was found with an $R^2$ coefficient of 0.9962 and a root-mean-square-error of $1.450 \times 10^{-3}$. The concentration-dependent coefficient $\alpha$ closely matches the fit ($\alpha = 0.148$), reported by Zhernovaya et al. [15].

5. Summary

In this paper, we have presented QPS as a novel method of investigating refractive index features with high spatial and spectral resolution. We developed a theoretical basis to model the tradeoff between coherence length and lateral field of view in off-axis holography, and described a method of spectral averaging to reduce coherent artifacts in quantitative phase images that circumvents this tradeoff. This approach was experimentally demonstrated using a transparent PDMS phantom in which we calculate an $\sim 3.2 \times$ reduction of spatial noise to $\sigma = 8.6$ mrad when using a 220 nm spectral range, corresponding to a path length of 807 picometers. We then applied QPS to microspheres with different absorptive properties and observed distinct features in the measured RI spectra that agree with the predictions of the Kramers-Kronig relation. Finally, we measured the RI spectra of multiple oxy-hemoglobin serial dilutions using a custom-molded PDMS chamber with a fixed step height and a sloped step edge. These measurements yielded a linear dependence over a biologically-relevant concentration range that agrees with previously published direct refractometry measurements of bulk hemoglobin samples. These results both validate the method of QPS for investigating spatially-resolved RI spectra and also suggest the utility of this technique for quantifying molecular compositions based on unique absorptive and dispersive features.

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