Bose-Einstein Condensate and Spontaneous Breaking of Conformal Symmetry on Killing Horizons II

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Abstract. In a previous paper local scalar QFT (in Weyl algebraic approach) has been constructed on degenerate semi-Riemannian manifolds \( S^1 \times \Sigma \) corresponding to the extension of Killing horizons by adding points at infinity to the null geodesic forming the horizon. It has been proved that the theory admits a natural representation of \( PSL(2, \mathbb{R}) \) in terms of \(*\)-automorphisms and this representation is unitarily implementable if referring to a certain invariant state \( \lambda \). Among other results it has been proved that the theory admits a class of inequivalent algebraic (coherent) states \( \{ \lambda_\zeta \} \), with \( \zeta \in L^2(\Sigma) \), which break part of the symmetry, in the sense that each of them is not invariant under the full group \( PSL(2, \mathbb{R}) \) and so there is no unitary representation of whole group \( PSL(2, \mathbb{R}) \) which leaves fixed the cyclic GNS vector. These states, if restricted to suitable portions of \( \mathbb{M} \) are invariant and extremal KMS states with respect to a surviving one parameter group symmetry. In this paper we clarify the nature of symmetry breakdown. We show that, in fact, spontaneous symmetry breaking occurs in the natural sense of algebraic quantum field theory: if \( \zeta \neq 0 \), there is no unitary representation of whole group \( PSL(2, \mathbb{R}) \) which implements the \(*\)-automorphism representation of \( PSL(2, \mathbb{R}) \) itself in the GNS representation of \( \lambda_\zeta \) (leaving fixed or not the state).

1 Summary of some achieved results.

In \( \mathbb{M} \) local scalar QFT (in Weyl algebraic approach) is constructed on degenerate semi-Riemannian manifolds \( \mathbb{M} = S^1 \times \Sigma \) corresponding to the extension of future Killing horizons by adding points at infinity to the null geodesic forming the horizon. Above the transverse manifold \( \Sigma \) has a Riemannian metric inducing the volume form \( \omega_\Sigma \), whereas \( S^1 \) is equipped with the null metric. To go on, fix a standard frame (see section II A of \( \mathbb{M} \)) \( \theta \in (-\pi, \pi] \) on \( S^1 \) of \( \mathbb{M} = S^1 \times \Sigma \) – so that \( S^1 \) is realized as \( (-\pi, \pi] \) with the endpoints identified – and consider the \( C^* \)-algebra of Weyl \( W(\mathbb{M}) \) generated by non-vanishing elements \( V(\omega) \) in Eq. (12) in section II E in \( \mathbb{M} \), the smooth forms \( \omega \) of the space \( \mathcal{D}(\mathbb{M}) \) being the space of forms \( \epsilon_\psi \) defined in Eq.(1) in section II A in \( \mathbb{M} \). We shall exploit the group of \(*\)-automorphisms \( \alpha \) defined in Eq. (19) in \( \mathbb{M} \) representing the Möbius group \( PSL(2, \mathbb{R}) \) viewed as a subgroup of diffeomorphisms of \( S^1 \) and thus of \( \mathbb{M} \) (see Section III
\( \omega^g := g^* \omega \) being the natural pullback action of \( g \in \text{PSL}(2, \mathbb{R}) \) on forms. In the following \( \{ \alpha^X_t \}_{t \in \mathbb{R}} \) indicates the one-parameter subgroup associated with the vector field \( X \) on \( \mathcal{M} \) corresponding to an element of the Lie algebra of \( \text{PSL}(2, \mathbb{R}) \). In particular the vector field on \( \mathcal{M} \), \( \mathcal{D} \), generating a one-parameter subgroup of \( \text{PSL}(2, \mathbb{R}) \), is that defined in Eq. (16) in [1]. If \( \zeta \in L^2(\Sigma, \omega_\Sigma) \) and \( \lambda_\zeta \) is the coherent state on \( \mathcal{W}(\mathcal{M}) \) defined by means of (33) of Section IV B in [1], we indicate its GNS triple by \( (\delta_{\lambda_\zeta}, \Pi_{\lambda_\zeta}, \Psi_{\lambda_\zeta}) \). States \( \lambda_\zeta \) are constructed as follows with respect to \( \lambda := \lambda_0 \). The map \( V(\omega) \mapsto V(\omega) e^{i \int_0^1 \Gamma(\omega_\pm + \omega_\pm)}, \omega \in \mathcal{D}(\mathcal{M}) \), uniquely extends to a \(*\)-automorphism \( \gamma_\zeta \) on \( \mathcal{W}(\mathcal{M}) \) such that \( \gamma_\zeta \circ \alpha^X_t = \alpha^X_t \circ \gamma_\zeta \) for all \( t \in \mathbb{R} \). The function \( \Gamma := \ln |\tan(\theta/2)| \) and the \( \mathcal{D}\)-positive-frequency part of \( \omega, \omega_+ \), are respectively defined and discussed in section IV B (Eq. (32)) and in Lemma 3.1 of section III B of [1].

\[
\lambda_\zeta(w) := \lambda(\gamma_\zeta w), \quad \text{for all } w \in \mathcal{W}(\mathcal{M}).
\]

The state \( \lambda \) was defined in Eq. (13) in [1] and it is pure in accordance with Lemma A.2 in [2], because the real linear map \( K : \mathcal{S}(\mathcal{M}) \ni \psi \mapsto \psi_+ \) has dense range in the one-particle space \( \mathcal{H} \) by construction. It is clear that the GNS representation generated by \( \lambda_\zeta \) is irreducible if that of \( \lambda \) is irreducible. As a consequence every \( \lambda_\zeta \) is pure.

Among other results it has been proved that (theorems 4.1, 4.2, 5.1) the pure states \( \lambda_\zeta \) are inequivalent states and, the restrictions to the algebra localized at the “half circle times \( \Sigma \)”, give rise to different extremal KMS states at rationalized Hawking temperature \( T = 1/2\pi \) with respect to \( \{ \alpha^X_t \}_{t \in \mathbb{R}} \). If one is dealing with a bifurcate Killing Horizon of a black hole the “half circle times \( \Sigma \)” is noting but \( \mathcal{F}_+ \), the future right branch of the Killing horizon (see Section I and figure in [1]). In this case \( -\zeta^{-1} \mathcal{D} \), with \( \zeta^{-1} = \kappa \) being the surface gravity of the examined black hole, can be recognized as the restriction to the horizon to the Killing vector field defining Schwarzschild time, \( kT \) becomes proper Hawking’s temperature and the order parameter associated with the breakdown of symmetry can be related with properties of the black hole.

## 2 Spontaneous breaking of \( \text{PSL}(2, \mathbb{R}) \) symmetry.

As illustrated in further details below, a remarkable property of states \( \lambda_\zeta \) is that, for \( \zeta \neq 0 \), they break part of the symmetry, in the sense that each of them is not invariant under the full group \( \text{PSL}(2, \mathbb{R}) \) and so there is no unitary representation of whole group \( \text{PSL}(2, \mathbb{R}) \) which leaves fixed the cyclic GNS vector. We want here to clarify the nature of this breakdown of \( \text{PSL}(2, \mathbb{R}) \) symmetry, proving that, actually there is no unitary representation of \( \text{PSL}(2, \mathbb{R}) \) which implements the full group action of \( \text{PSL}(2, \mathbb{R}) \) no matter the issue of the invariance of \( \lambda_\zeta \) under \( \text{PSL}(2, \mathbb{R}) \). This leads to spontaneous breaking of \( \text{PSL}(2, \mathbb{R}) \) symmetry.

Let us focus on this issue from a general point of view. In the physical literature there are
several, also strongly inequivalent, definitions of spontaneous breaking of symmetry related to different approaches to quantum theories. We adopt the following elementary definition which is quite natural in algebraic QFT and is equivalent to the definition given on p.119 of the recent review on breaking symmetry theory (see the end of the paper for further comments).

Definition 1. Referring to a C*-algebra \( A \), one says that:

(a) A Lie group \( \mathcal{G} \) is a group of symmetries for \( A \) if there is a representation \( \beta \) of \( \mathcal{G} \) made of *-automorphisms of \( A \). If some notion of time evolution is provided, it is required that it corresponds to a one parameter subgroup of \( \mathcal{G} \).

(b) Assuming that (a) is valid, spontaneous breaking of \( \mathcal{G} \) symmetry occurs with respect to an algebraic state \( \mu \) on \( A \) and \( \beta \), if there are elements \( g \in \mathcal{G} \) not implementable unitarily in the GNS representation \( (\mathcal{H}_\mu, \Pi_\mu, \Psi_\mu) \) of \( \mu \), i.e. for those elements there is no unitary operator \( U_g \) on \( \mathcal{H}_\omega \) with \( U_g \Pi_\mu(a) U_g^\dagger = \Pi_\mu(\beta_g(a)) \) for all \( a \in A \).

Considering the inequivalent GNS triples \( (\mathcal{H}_\zeta, \Pi_\zeta, \Psi_\zeta) \), theorems 3.2 and 3.3 in [1] show that, if \( \zeta = 0 \) the group of automorphisms \( \alpha \) representing \( PSL(2, \mathbb{R}) \) can be unitarily implemented in the space \( \mathcal{H} = \mathcal{H}_0 \) and the cyclic vector \( \Psi := \Psi_0 \) of the GNS representation is invariant under that (strongly continuous) unitary representation of \( PSL(2, \mathbb{R}) \). Conversely, if \( \zeta \neq 0 \), \( PSL(2, \mathbb{R}) \) symmetry turns out to be broken. Indeed, theorem 4.1 states that each state \( \lambda_\zeta \) with \( \zeta \neq 0 \) is invariant under \( \{\alpha_\zeta^{(D)} \}_{t \in \mathbb{R}} \), but it is not under any other one-parameter subgroup of \( \alpha \) (barring those associated with \( cD \) for \( c \in \mathbb{R} \) constant). In the general case this is not enough to assure occurrence of spontaneous breaking of \( PSL(2, \mathbb{R}) \) symmetry as defined in Def.1.

In a different context, it is possible to show that – see section III.3.2 of [2] – if \( \mathcal{G} \) is Poincaré group or an internal symmetry group for a special relativistic system and the reference state \( \mu \) is a primary vacuum state over the net of algebras of observables, then non-\( \mathcal{G} \) invariance of the vacuum state implies – and in fact is equivalent to – spontaneous breaking of \( \mathcal{G} \) symmetry. Our considered case is far from that extent and thus there is no a priori guarantee for the occurrence of spontaneous breaking of \( PSL(2, \mathbb{R}) \) symmetry for states \( \lambda_\zeta \) with \( \zeta \neq 0 \) and the issue deserve further investigation. The following theorem give an answer to the issue.

Theorem 1. If \( L^2(\Sigma, \omega_\Sigma) \ni \zeta \neq 0 \), spontaneous breaking of \( PSL(2, \mathbb{R}) \)-symmetry occurs with respect to \( \lambda_\zeta \) and the representation \( \alpha \). (In particular, there is no unitary implementation of the nontrivial elements of the subgroup of \( PSL(2, \mathbb{R}) \) generated from the vector field \( \partial_\zeta \).

Proof. Referring to the GNS triple of \( \lambda_\zeta \) define \( \hat{V}_\zeta(\omega) := \Pi_\zeta(V(\omega)) \). The existence of a unitary

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1If this subgroup does not belong to the center of \( \mathcal{G} \), the self-adjoint generators of unitary representations of \( \mathcal{G} \) implementing \( \beta \), if any, give rise to constants of motion which may depend parametrically on time. It happens, for instance, for Poincaré-invariant systems, where the conserved self-adjoint operator \( K^i(t) \) associated with boost invariance along \( i \)-th axis depend parametrically on time. Indeed, on the appropriate domain, \( K^i(t) = K^i(0) - tP^i \) and it is conserved under Heisenberg evolution \( U_t K^i(t) U^\dagger_t = K^i(0) \).

2In the proof theorem 3.2 in all occurrences of the symbol \( PSL(2, \mathbb{R}) \) before the statement “...is in fact a representation of \( PSL(2, \mathbb{R}) \)” it has to be replaced by \( PSL(2, \mathbb{R}) \), it denoting the universal covering of \( PSL(2, \mathbb{R}) \).
implementation, \( L_g : \mathcal{H}_\zeta \to \mathcal{H}_\zeta \), of \( \alpha \) in the GNS triple \((\mathcal{H}_\zeta, \Pi_\zeta, \Psi_\zeta)\) implies, in particular, that

\[
L_g \hat{V}_\zeta(\omega)L_g^\dagger = \Pi_\zeta(\alpha_g(V_\zeta(\omega))) , \quad \text{for all } \omega \in \mathcal{D}(\mathbb{M}) \text{ and every } g \in \text{PSL}(2, \mathbb{R}) .
\]

By construction, \((\mathcal{H}_\zeta, \Pi, \Psi_\zeta)\) (notice that we wrote \( \Pi \) instead of \( \Pi_\zeta \)) is a GNS triple of \( \lambda \) (notice that we wrote \( \lambda \) instead of \( \lambda_\zeta \)) if

\[
\Pi : V(\omega) \mapsto \hat{V}_\zeta(\omega) := \hat{V}_\zeta(\omega) e^{-i\int M \Gamma(\zeta \omega_+ + \zeta \omega_-)} .
\]

In this realization \( \alpha \) can be unitarily implemented (theorem 3.2 in [3]): There is a (strongly continuous) unitary representation \( U \) of \( \text{PSL}(2, \mathbb{R}) \) such that

\[
U_g \hat{V}_\zeta(\omega)U_g^\dagger = \Pi(\alpha_g(V(\omega))) , \quad \text{for all } \omega \in \mathcal{D}(\mathbb{M}) \text{ and every } g \in \text{PSL}(2, \mathbb{R}) .
\]

Suppose that \( \alpha \) can be implemented also in \((\mathcal{H}_\zeta, \Pi_\zeta, \Psi_\zeta)\), where now \( \Pi_\zeta : V(\omega) \mapsto \hat{V}_\zeta(\omega) \), and let \( L \) be the corresponding unitary representation of \( \text{PSL}(2, \mathbb{R}) \) satisfying \((\ref{3})\). That equation together with \((\ref{4})\) entail that the unitary operator \( S_g := U_g^\dagger L_g \) satisfies

\[
S_g V(\omega)S_g^\dagger = e^{i c_{g,\omega}} V(\omega) ,
\]

\[
c_{g,\omega} := \int M \left( \zeta \left( \omega^{(g^{-1})} \right)_+ + \zeta \left( \omega^{(g^{-1})} \right)_+ - \zeta \omega_+ - \zeta \omega_- \right) \Gamma .
\]

Now, dealing with exactly as in the proof of (ii) of (b) of theorem 4.1 (where the role of our \( \text{PSL}(2, \mathbb{R}) \) of \( u \) was played by the operator \( U \) and the role of \( c_{g,\omega} \) was played by the simpler phase \( \int M (\zeta \omega_+ + \zeta \omega_-) \Gamma \)) one finds that \((\ref{5})\) entails that \( \langle \Psi_\zeta, S_g \Psi_\zeta \rangle \neq 0 \) and

\[
||S_g \Psi_\zeta||^2 = |\langle \Psi_\zeta, S_g \Psi_\zeta \rangle|^2 e^{\sum_{n,j} |\eta_{n,j}|^2} ,
\]

\[
\eta_{n,j} := -2i \int_M \Gamma(\theta) \zeta(s) u_j(s) \frac{\partial}{\partial \theta} \frac{e^{i\theta} - e^{i\theta}}{4\pi n} d\theta d\omega_\Sigma(s) .
\]

Above the real compactly-supported functions \( u_j \) defines a Hilbert base in \( L^2(\Sigma, \omega_\Sigma) \), and \((\theta_g, s_g) := g(\theta, s) \in M \) is obtained by the action of \( g \in \text{PSL}(2, \mathbb{R}) \) on \((\theta, s) \in M \) (obviously \( s_g = s \) since \( \text{PSL}(2, \mathbb{R}) \) acts on the factor \( S^1 \) of \( M = S^1 \times \Sigma \)). Now take \( g \in \{ \alpha^t \}_{t \in \mathbb{R}} \), the one-parameter subgroup of \( \text{PSL}(2, \mathbb{R}) \) generated by the vector field \( K := \frac{\partial}{\partial \theta} \), and realize the factor \( S^1 \) of \( M = S^1 \times \Sigma \) as \([-\pi, \pi] \) with the identification of its endpoints. In this case, obviously, \( \theta_g = \theta + t \). A direct computation shows that, for some \( j_0 \) with \( \int_\Sigma u_j(s) \zeta(s) \omega_\Sigma(s) = 0 \) (which does exist otherwise \( \zeta = 0 \) almost everywhere)

\[
|\eta_{2n+1,j_0}|^2 = C \left[ \frac{1}{2n+1} - \frac{\cos((2n+1)t)}{2n+1} \right]
\]

for some constant \( C > 0 \) independent form \( n \). The series of elements \(-\cos((2n+1)t)/(2n+1)\) converges for \( t \neq 0, \pm \pi \) (it diverges to \(+\infty\) for \( t = \pm \pi \)), whereas that of elements \( 1/(2n+1) \)
diverges to $+\infty$. Thus the exponent in (4) and $L_g$ cannot exist, if $g = \alpha_t^{(\infty)}$ with $t \neq 0$. □

Remark. The automorphism $\gamma_\zeta$ is a symmetry of the system because it commutes with time evolution $\alpha_t$. (It is worth noticing that, if restricting to real functions $\zeta$, $\zeta \mapsto \gamma_\zeta$ defines a group of automorphisms.) Since $\lambda_\zeta |_{W(\mathcal{F})} \neq \lambda_{\zeta'} |_{W(\mathcal{F})}$ for $\zeta \neq \zeta'$, and all these states are extremal $\alpha^{(\infty)}$-KMS states at the same temperature, following Haag (V.I.5 in [3]), we can say that spontaneous symmetry breaking with respect to $\gamma_\zeta$ occurs in the context of extremal KMS states theory.

3 Final comments on inequivalent approaches concerning symmetry breakdown

If $\mu$ is a state on the $C^*$-algebra $\mathcal{A}$, its Gelfand ideal, $\mathcal{I}_\mu$, consists of the elements $a \in \mathcal{A}$ such that $\mu(a^*a) = 0$. $\mathcal{I}_\mu$ plays a central role in GNS reconstruction procedure. Breakdown of symmetry could be investigated from another point of view – relying upon the invariance properties of Gelfand ideal – with some overlap with our approach. It is based on the following proposition.

Proposition 1. Let $\beta$ be a faithful $*$-automorphism representation of the group $\mathcal{G}$ on the $C^*$-algebra $\mathcal{A}$ with unit $\mathbb{I}$ and let $\mu : \mathcal{A} \to \mathbb{C}$ be a state with GNS triple $(\mathcal{H}_\mu, \Pi_\mu, \Psi_\mu)$.

The Gelfand ideal $\mathcal{I}_\mu$ is invariant under $\beta$ if and only if there is a $\mathcal{G}$-representation made of densely-defined operators $U_g : \Pi_\mu(\mathcal{A}) \to \Pi_\mu(\mathcal{A})$ which implements $\beta$ leaving fixed $\Psi_\mu$, i.e.

\[
(i) \quad U_g \Pi_\mu(a) U_g^{-1} = \Pi_\mu(\beta_g(a)) \quad \text{for all } a \in \mathcal{A}, g \in \mathcal{G} \quad \text{and} \quad (ii) \quad U_g \Psi_\mu = \Psi_\mu \quad \text{for all } g \in \mathcal{G}.
\]

If a representation $\mathcal{G} \ni g \mapsto U_g$ satisfying (8) exists the following holds.

(a) It is unique and the operators $U_g$ are completely determined by

\[
U_g \Pi_\mu(a) \Psi_\mu = \Pi_\mu(\beta_g(a)) \Psi_\mu, \quad \text{for all } a \in \mathcal{A} \text{ and } g \in \mathcal{G}.
\]

(b) Operators $U_g$ are (restrictions to the dense domain $\Pi_\mu(\mathcal{A})$ of uniquely determined) unitary operators on $\mathcal{H}_\mu$, if and only if $\mu$ is invariant under $\beta$.

Sketch of proof. From GNS theorem $\mathcal{I}_\mu = \{a \in \mathcal{A} \mid \Pi_\mu(a) \Psi_\mu = 0\}$. From it one easily proves that if $\mathcal{J}_\mu$ is $\beta$-invariant define a well-posed representation of $\mathcal{G}$ satisfying (8) (notice that, $\beta_g(\mathbb{I}) = \mathbb{I}$ and $\Pi_\mu(\mathbb{I}) = I$ so that (ii) in (8) holds true from (9)). Since (i) and (ii) in (8) entails (9), $\mathcal{J}_\mu = \{a \in \mathcal{A} \mid \Pi_\mu(a) \Psi_\mu = 0\}$ proves that the existence of a representation satisfying (8) implies $\beta$-invariance of $\mathcal{J}_\mu$ and (a) is valid as well. Proofs of (b) is based on the identity (from invariance of $\mu$ and GNS theorem) $||U_g \Pi_\mu(a) \Psi_\mu||^2 = \mu(\beta_g(a^*a)) = \mu(a^*a) = ||\Pi_\mu(a) \Psi_\mu||^2$. □

In view of that result, by a pure mathematical point of view, a strategy to distinguish several degrees of $\mathcal{G}$-symmetry breakdowns for a state $\mu : \mathcal{A} \to \mathbb{C}$ when $\mathcal{G}$ is represented, at algebraic level, by the $*$-automorphism representation $\beta$, could be the following.

1. (No symmetry breaking) Gelfand ideal is invariant under $\beta_g$ – i.e. (i) and (ii) in (8) are valid for an operator representation of $\mathcal{G}$.
(2) Gelfand ideal is invariant under $\beta$ – so that both (i) and (ii) in (S) hold true – but the induced action of $\beta$ in the GNS representation of $\mu$ is not unitarily implementable.

(3) Gelfand ideal is not invariant under $\beta_g$ – so that at least one of (i) and (ii) in (S) is not valid for any operator representation of $\mathcal{G}$ on the relevant domain.

In this paper we, instead, have adopted quite a different point of view (sharing however some overlap with the point of view illustrated above) entirely based on the definition of spontaneous breaking of symmetry defined on p. 119 of [4]. We considered only the problem of unitary (non)implementability of $(\mathit{PSL}(2,\mathbb{R}))$ symmetry. This is in accordance with the well-known general Wigner-Kadison notion of quantum symmetry described in terms of appropriate (projective) unitary or anti-unitary operators [5]. The three degree of symmetry breakdown one may consider in this context are the following, referring to $\mu, \mathcal{G}, \beta$ as before.

(U1) (No symmetry breaking) Algebraic symmetry $\beta$ is implementable for $\mu$ by means of unitary operators and the state $\mu$ is invariant under the action $\beta$ of the full group $\mathcal{G}$ – (i) and (ii) in (S) are valid for a unitary representation of $\mathcal{G}$.

(U2) (Symmetry breaking due to the cyclic vector) Algebraic symmetry $\beta$ is implementable for $\mu$ by means of unitary operators and the state $\mu$ is not invariant under the action $\beta$ of the full group $\mathcal{G}$ – (i), but not (ii) in (S), is valid for a unitary representation of $\mathcal{G}$.

(U3) (Spontaneous symmetry breaking) Algebraic symmetry $\beta$ is not implementable for $\mu$ by means of unitary operators – (i) in (S) does not hold for any unitary representation of $\mathcal{G}$.

In [1] we established, for states $\lambda_\zeta$ with $\zeta \neq 0$ the validity of either degree (U2) or (U3) concerning $\mathcal{G} = \mathit{PSL}(2,\mathbb{R})$ symmetry breaking. This paper shows that, actually, the strongest degree (U3) takes place.

\textit{A priori} this extent may be compatible with either (2) or (3) of the other scheme. A closed scrutiny would be necessary to examine this issue but it is far from the goal of this paper where only Wigner-Kadison unitary symmetries are considered since they are the only which preserve quantum probabilities and have direct physical meaning.

To conclude we notice that an example of the intermediate case (U2) is realized in the framework of [1] with respect to the Weyl algebra $\mathcal{W}(\mathbb{M})$, with $\mathbb{M} = \mathbb{S}^1$, and for the fully $\mathit{PSL}(2,\mathbb{R})$-symmetric state $\lambda_{\zeta=0}$. Breaking of symmetry at level (U2) occurs when extending the symmetry group from $\mathit{PSL}(2,\mathbb{R})$ to the infinite dimensional one $\mathcal{G} := \mathit{Diff}^+(\mathbb{S}^1)$.

References

[1] V. Moretti and N. Pinamonti: “Bose-Einstein condensate and Spontaneous Breaking of Conformal Symmetry on Killing Horizons”, J. Math. Phys. 46, 062303 (2005). [hep-th/0407256]

[2] B. S. Kay, R. M. Wald, “Theorems On The Uniqueness And Thermal Properties Of Stationary, Nonsingular, Quasifree States On Space-Times With A Bifurcate Killing Horizon”, Phys. Rept. 207, 49 (1991).
[3] R. Haag, “Local quantum physics: Fields, particles, algebras”, Second Revised and Enlarged Edition. Springer Berlin, Germany (1992).

[4] F. Strocchi, “Symmetry Breaking”, Lecture Notes in Physics, Springer Berlin, Germany (2005).

[5] R. Kadison, “Isometries of Operator Algebras”, Ann. Of Math. (2) 54, 325 (1951), C. Piron, “Foundations of Quantum Physics”, W. A. Benjamin, London (1976).