Ellipsograph of Archimedes
as a simple LEGO construction

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Abstract
We report on a simple LEGO construction that can draw an ellipse by using the concept of trammel of Archimedes.

School curriculum related to elementary geometry usually focuses on “compass and ruler” constructions where the traditional Euclidean construction steps are allowed to generate a figure. While it is possible to construct any point of an ellipse that is given by its parameters, school curriculum seldom covers any methods how to efficiently draw an arc or the full ellipse by home-made mechanical constructions.

To draw an ellipse on your own, several options exist. One of them, the gardener method is mentioned in many textbooks (see [5] for a reference), and it is based on the locus definition, namely:

Given two fixed points $F_1$, $F_2$ called the foci and a distance $2a$ which is greater than the distance between the foci, the ellipse is the set of points $P$ such that the sum of the distances $|PF_1|$, $|PF_2|$ is equal to $2a$:

$$E = \{ P \in \mathbb{R}^2 : |PF_1| + |PF_2| = 2a \}.$$ 

By using this definition, the gardener puts down two stakes and loops a piece of rope around them. Using a stick, he pulls the loop taut and marks the points around a curve.

In this article we consider a different method, namely the ellipsograph (or trammel) of Archimedes [1] that consists of two shuttles which are confined to perpendicular channels or rails and a rod which is attached to the shuttles by pivots at fixed positions along the rod.

We remark that such constructions are well-known and very popular as commercial toys. Also as LEGO constructions there are several options. A recent YouTube video at https://www.youtube.com/watch?v=E5V9C-9dxUc contributed by Brixe63 shows a complicated mechanism that is definitely based on the same idea, but consists of several
LEGO bricks. Another approach (among several others) can be found on Rebrickable at https://rebrickable.com/mocs/MOC-4096/JKBrickworks/trammel-of-archimedes/#details, contributed by JKBrickWorks, it uses 47 parts. In our contribution—which is based on [2]—we focus on minimizing the number of parts, and, at the same time, we focus on drawing the ellipse, not just the building up the motion.

Figure 1: An ellipsograph

Fig. 1 shows how the ellipsograph is built. Easy sliding is assured by using two flat tiles for the shuttles. Their collision-free arrival in the channels is solved by using 1 × 4 flat tiles. However, their somewhat long size has some drawback, namely, that the shuttles require a bigger distance to each other, as seen in Fig. 2.

Figure 2: A closer look on the ellipsograph

We use 24 LEGO parts listed in Fig. 3 to build the construction and a G2 type pen refill (see https://en.wikipedia.org/wiki/Ballpoint_pen#/media/File:Ball_point_pen_refills_en.png for an overview on pen refills). We highlight that this construction well harmonizes with a set of other LEGO bricks, described in [4] at the GitHub page [3]; it allows the students to draw a high variety of other algebraic curves. On the other hand, static
Table 1: Shopping list of the LEGO bricks. Buying components in different colors may be more expensive. Prices are listed in €, as of 30 November 2020, at brickowl.com.

| Brick | Brick | Brick | Brick | Brick | Flat tile | Pin | Base plate | Total |
|-------|-------|-------|-------|-------|-----------|-----|------------|-------|
| 1 x 1 | 1 x 2 | 1 x 6 | 2 x 4 | 1 x 8 | 1 x 4 | 15M | 16 x 16 | 3005 3004 3009 3001 3008 2431 32278 2780 6098 |
| 0.01€ | 0.01€ | 0.01€ | 0.01€ | 0.01€ | 0.01€ | 0.01€ | 0.01€ | 0.06€ |
| 2 x | 2 x | 5 x | 4 x | 3 x | 2 x | 1 x | 4 x | 1 x | 24 |
| 0.02€ | 0.02€ | 0.05€ | 0.04€ | 0.03€ | 0.02€ | 0.04€ | 0.06€ | 0.97€ |

balancing of our contribution is an important part: a counterweight is required to ensure stable motion (see Fig. 3).

Figure 3: LEGO bricks used for the ellipsograph

We emphasize that the LEGO ellipsograph can be cheaply built by ordering parts from Internet stores. Table 1 shows some recent prices at brickowl.com by selecting the cheapest color variant of the required parts at the time of writing this paper. Teachers and students should be aware that a suitable pen refill is also required, but it should be available at an affordable price below 1€. Thus the total minimal price of our ellipsograph is about 2€.

Our contribution can draw almost 100% of a quite big ellipse that fits on an A4 sheet of paper as it can be observed in Fig. 4 and 5. A very small part of the arc cannot be drawn because the shuttles get too close in that part of the
movement. As future work we address solving this problem, too.

![Figure 4: An ellipse drawn with the ellipsograph](image)

Finally, we point the reader to LEGO’s official computer aided design tool, LEGO Digital Designer, available at [https://www.lego.com/en-us/ldd](https://www.lego.com/en-us/ldd). We used this program to visualize our concept in a digital way. The LXF file that contains our work can be downloaded at [https://matek.hu/zoltan/eg.lxf](https://matek.hu/zoltan/eg.lxf). Also the building instructions can be checked out at [https://matek.hu/zoltan/Building%20Instructions%20%5Beg%5D.html](https://matek.hu/zoltan/Building%20Instructions%20%5Beg%5D.html).

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![Figure 5: The ellipse and the ellipsograph](image)
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Figure 6: The ellipsograph sketched up with LDD