Intra-Landau-level collective excitations in a bilayer disordered electronic system

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We investigate intra-Landau-level collective excitations in a bilayer disordered two-dimensional electron system exposed to a perpendicular magnetic field. The energy spectrum is calculated within the random phase approximation by taking into account electron-impurity scattering in the self-consistent Born approximation which includes consistent vertex corrections. Signatures of these bilayer excitations in drag and collective excitation measurements are identified.

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In bilayer two dimensional electron systems (2DES), an additional degree of freedom, associated with the layer index, produces collective excitations (CE), which have no counterpart in the individual 2DES [1]. New interlayer electron correlations arise which govern different ferromagnetic and antiferromagnetic phases of the quantum Hall states [2]. The bilayer CE also play an essential role in the dynamical screening of interlayer Coulomb interaction in the frictional drag effect both in zero [3, 4, 5] and finite magnetic fields [4, 6, 7, 8, 9].

The basic source of physical information on the fundamental properties of a system is the spectrum of CE. In zero magnetic field, the spectrum of CE in the bilayer 2DES without disorder effects has been studied comprehensively both in theory [10] and experiment [11]. The spectrum includes not only the optical wave with in-plane density fluctuations and the usual square-root dispersion, but also an additional acoustical branch with out-of-phase fluctuations and linear dispersion. In the finite magnetic fields, the dispersion of inter-Landau level (LL) CE with energies close to the multiples of the cyclotron energy was obtained analytically at the zero temperature limit [12]. Experimental measurements of the spectrum of inter-LL CE in bilayer 2DES were performed only recently: the principal Bernstein magnetoplasmon modes were observed by means of inelastic light scattering [13].

Of central importance, however, is the spectrum of CE in the extreme quantum limit when no kinetic energy change occurs in the 2DES. These CE with energies, $\hbar \omega$, below the cyclotron energy, $\hbar \omega_B$, are referred to as the intra-LL CE. In the fractional quantum Hall effect (FQHE) regime the intra-LL CE are the well-known magnetorotons in single [14] and double layer [15] 2DES with the dispersion minima at wave vectors $q \sim 1/\ell_B$ [16]. $\ell_B$ is the magnetic length. In disordered 2DES, a continuum of particle-hole excitations takes place within a broadened LL. In the individual disordered 2DES the spectrum of intra-LL CE has been studied by Antoniou et al. [17] for the lowest LL and it has been found that the plasmon-like CE emerge above the particle-hole continuum (PHC). It has been also predicted by Merkt [18] that in the disordered 2DES, electron-electron (e-e) interaction overcomes Kohn’s theorem [19] and results in a low-lying disorder-induced cyclotron resonance mode. Recently, the importance of intra-LL CE in the drag effect has been also discussed [8].

In the present paper we calculate the spectrum of intra-LL CE in the bilayer disordered 2DES in a perpendicular magnetic field at finite temperatures when a higher LL is partially filled. We treat e-e interaction within the random phase approximation (RPA) and electron-impurity (e-i) scattering in the self-consistent Born approximation (SCBA) [20], which includes the consistent vertex corrections to the response function of the individual 2DES [17]. We obtain that the whole spectrum of intra-LL CE is located in a finite wave vector interval and, depending on the system parameters, there can exist $l_0$ forbidden zones within this wave vector interval ($l_0$ is the Landau index of the outermost partially filled LL). Although our approach allows us to take into account coupling of the broadened LL at finite temperatures, the coupling is weak in the regime of our interest here. Therefore, the response function becomes small and the 2DES unable to respond in the forbidden wave vector zones around $q_r$ where $q_r$ are determined from the zeros of the Laguerre polynomial, $L_{l_0}(t_r) = 0$ and $t_r = (q_r \ell_B)^2/2$ ($r = 1, 2, ..., l_0$). In each allowed wave vector zone, we obtain the spectrum of bilayer intra-LL CE which consists of two excitations with in-phase and out-of-phase density fluctuations and with the energies depending on the interlayer spacing and the characteristics of individual 2DES.

The bilayer dielectric tensor $\varepsilon_{ij}(q, \omega)$ ($i, j = 1, 2$ are the layer indices) is given in terms of the irreducible electron polarization function $\Pi_{ij}(q, \omega)$, which is obtained from the solution of a matrix Dyson equation for the dynamically screened Coulomb interaction $V(q, \omega)$. It has the following usual form

$$
\hat{V}(q, \omega) = \tilde{v}(q) \left[1 - \tilde{v}(q) \hat{\Pi}(q, \omega)\right]^{-1} = \hat{v}(q) \tilde{v}^{-1}(q, \omega)
$$

where all quantities are $2 \times 2$ matrices, $\hat{v}$ denotes the bare Coulomb interaction tensor. For simplicity we assume that the layers are infinitely thin and use $v_{11} = v_{22} = v = 2 \pi e^2/\kappa_0 q$ and $v_{12} = v_{12} = \exp(-q\Lambda) v$, where $\kappa_0$ is the static dielectric constant and $\Lambda$ the in-
The components of the inverse dielectric tensor are given as $\varepsilon_{11,22}(q,\omega) = \varepsilon_{2,1}(q,\omega) / \varepsilon_{11}(q,\omega)$ and $\varepsilon_{12,21}(q,\omega) = \varepsilon_{21}(q,\omega) / \varepsilon_{11}(q,\omega)$. Here, $\varepsilon_{1,2}(q,\omega) = 1 - \varepsilon_{11,11,22}(q,\omega)$ and $\Pi_{11,22}(q,\omega)$ are the screening and polarization functions in each layer. The bilayer screening function $\varepsilon_{11}(q,\omega)$ is the determinant of the dielectric tensor, $\varepsilon_{11}(q,\omega) = \varepsilon_{11}(q,\omega) - \varepsilon_{12}(q,\omega) - \varepsilon_{22}(q,\omega)\Pi_{11}(q,\omega)\Pi_{22}(q,\omega)$.

For brevity we suppress the layer indices here, $G_{l}^{R,A}(E)$ are the electron retarded and advanced Green’s functions, dressed by e-i interaction. In the SCBA they are diagonal and given by $G_{l}^{R,A}(E) = 2 \left( E - E_{i} + \sqrt{(E - E_{i})^{2} - \Gamma_{0}^{2}} \right)^{-1}$ with the imaginary part of the square-root taken positive (negative) for the advanced (retarded) function. In the short-range impurity model, the half-width $\Gamma_{0} = \sqrt{\frac{2}{\hbar\omega_{B}m^{2}}}$ of $E_{i} = (l + 1/2)\hbar\omega_{B}$ LL is independent of the Landau index $l$ ($\tau$ is the transport relaxation time, determined from mobility $\mu$). The bare vertex functions $Q_{l}^{\ell}\ell(t)$ are given by the gauge invariant part of the in-plane form factor: $Q_{l}^{\ell}\ell(t) = (-1)^{l+1}e^{-tL_{l}^{\ell}(t)}L_{l}^{\ell}(t)$ where $t = (q\ell_{B})^{2}/2$ and $L_{l}^{\ell}(t)$ is the associated Laguerre polynomial. The denominators in the r.h.s. of Eq. (2) differ from one due to the vertex corrections $\delta\gamma_{l}^{ab}$ which are obtained consistently in the short-range impurity model with the assumption that the LL are clearly resolved [21]. The chemical potential $E_{F}(n, B, T, \mu)$ in the Fermi distribution function $f_{F}$ is determined implicitly by the electron density via $n = -2(2\pi^{2}\ell_{B}^{2})^{-1}\sum_{l=0}^\infty\int_{-\infty}^{\infty} dE f_{F}(E - E_{F})ImG_{l}(E)$. Here we use the electron Green’s functions which correspond to the Gaussian density of states, $ImG_{l}(E) = \sqrt{2\pi}\Gamma_{0}\exp\left(-2(E - E_{l})^{2}/\Gamma_{0}^{2}\right)$, without unphysical edges of the Landau band [22]. We assume spin degeneracy so the capacity of each LL is doubled.

It is seen in Fig. 1 that Re $\varepsilon(q,\omega)$ of the individual 2DES becomes negative for intermediate values of $\omega$ of the order of $\Gamma_{0}$. This requires two crossings of the Re $\varepsilon(q,\omega) = 0$ axis. The low-frequency zeros (LFZ) of Re $\varepsilon(q,\omega)$ always lie within the PHC where Im $\varepsilon(q,\omega)$ is large. The high-frequency zeros (HFZ) are close to or above the upper edge of the PHC where Im $\varepsilon(q,\omega)$ is small or exactly zero within the SCBA.

\[
\Pi(q,\omega) = \frac{1}{\pi\ell_{B}^{2}} \sum_{l,l'=0} Q_{l}^{\ell}\ell(t) \int_{-\infty}^{\infty} \frac{dE}{\pi} f_{F}(E - E_{F}) ImG_{l}^{F}(E) G_{l}^{R}(E + \hbar\omega) \left[ 1 - \delta\gamma_{l}^{ll}(E, E + \hbar\omega) \right]^{-1} \left[ 1 - \delta\gamma_{l}^{ll}(E, E + \hbar\omega) \right]^{-1} G_{l}^{A}(E - \hbar\omega) \left[ 1 - \delta\gamma_{l}^{ll}(E, E - \hbar\omega) \right]^{-1} \left[ 1 - \delta\gamma_{l}^{ll}(E, E - \hbar\omega) \right]^{-1},
\]

where

FIG. 1: The structure factor (solid line), the real (dashed line) and imaginary (dotted line) parts of dielectric function of the bilayer 2DES with the $\Lambda = 30$ nm spacing and for $l_{0} = 4$. The thin lines correspond to the individual 2DES in which Im $\varepsilon(q,\omega)$ differs from that of in the bilayer 2DES by a numerical factor and is not shown here.

The pronounced peaks of the dynamical structure factor $S(q,\omega) \propto -Im(1/\varepsilon(q,\omega))$ as a function of $\omega$ correspond to HFZ, while no peaks correspond to LFZ. Therefore, HFZ represent intra-LL CE of the system while LFZ cannot be interpreted as excitations. We believe, however, that these particle-hole states play an important role, particularly in realization of the frictional magnetodrag effect, and we calculate LFZ and present them below in the spectrum together with the intra-LL CE modes.

In the bilayer 2DES, there appears a pair of zeros instead of LFZ and HFZ of the single layer 2DES and Re $\varepsilon(q,\omega)$ becomes negative only within small regions, restricted by the two zeros in each pair. Therefore, the
bilayer intra-LL CE spectrum consists of two branches which correspond to the in-phase and out-of-phase intra-LL CE. Accordingly, one can see from Fig. I that the bilayer structure factor, instead of the single peak of $S(q, \omega)$ of the individual 2DES, shows a double-peak structure near the upper edge of the PHC, while again it demonstrates monotonic behavior at low energies.

In Fig. 2 we plot the spectra of intra-LL CE of individual 2DES at finite temperatures and for various values of electron mobility. Inclusion of the consistent vertex corrections allows us to reproduce the square-root like dispersion $17$ of intra-LL CE at small momenta. However, no exact $q \to 0$ limit exists in the spectrum. In the disordered 2DES the density of states remains finite in the center of the LL and the response function vanishes as $q^2$, even if $\omega$ is in the immediate vicinity of $\omega_B$. In the short wavelength limit, again the disordered 2DES does not respond since the response function vanishes exponentially at wavelengths much shorter than $\ell_B$. When the lowest LL is partially filled, there exists only one allowed wave vector zone, and our findings for $l_0 = 0$ are in agreement with Ref. $17$. For CE referred to the higher LL, one can see from Fig. 2 that the number of allowed wave vector zones can be multiple and increases with $\mu$. Near half-filling of the $l_0 = 4$ LL and at $T = 0.1$ K there exist only one, two, and three allowed zones for $\mu = 35, 200$, and 1000 T$^{-1}$, respectively. Notice that for $l_0 = 4$ the first two of the maximum four available wave vector gaps are located around $q, \ell_B = 0.8$ and 1.87. In the first zone close to $q = 0$, for all three values of $\mu$, the intra-LL CE emerge above the PHC in the middle of the zone where the energy shows a maximum. In this region the spectrum describes CE modes without damping and delta-function like peaks occur in $S(q, \omega)$. Towards the edges of the allowed zone the energy decreases. Within the PHC $\hbar \omega < 2 \Gamma_0$ near its upper edge, the peaks of $S(q, \omega)$ become broadened, and though the intra-LL CE modes are damped but still are well defined excitations. Exactly at the edges of the allowed zone, the group velocity shows anomalous behavior and becomes infinite. At these points CE merge with the particle-hole states, which are described by LFZ of $Re \, \varepsilon(q, \omega)$. Below this merging energy the damping is so strong that the zeros of $Re \, \varepsilon(q, \omega)$ cannot be interpreted as CE. The greatest maximum of energy and the largest width of the allowed wave vector zone are achieved, when the outermost LL is near half-filling, $\mu$ large, and $T$ low. In the higher zones the energy maximum is smaller and the intra-LL CE emerge above the PHC only in the second zone for the highest value of mobility.

In Fig. 3 we plot the dispersion relations of intra-LL CE in the first allowed wave vector zone for the bilayer 2DES. The thin lines represent the spectra in the single-layer limit.
and has a dipole oscillator strength. At small wave vectors the dispersion of in-phase modes is close to the square-root one while the out-of-phase modes demonstrate a smoother dispersion and achieve the maximum energy at larger values of $q$. The splitting energy, $\delta E$, decreases in $\Lambda$ which supports the tendency that within the limit of $\Lambda \to \infty$, the dispersion curves of in- and out-of-phase modes degenerate and coincide with that of single layer intra-LL CE mode. $\delta E$ has a minimum near the upper edge of the PHC and close to the right edge of the allowed wave vector zone. Towards the right edge of the zone, $\delta E$ increases slightly, while the increase is strong towards the left edge. The CE modes with $q$ close to the edges of the zone and with energies $\hbar \omega < 2 \Gamma_0$ are responsible for the enhancement of the frictional magneto-drag effect. The out-of-phase CE mode has relatively strong damping and contributes largely to the drag effect. However, in certain ranges of wave vectors close to the edges of the allowed zone, the in-phase modes develop and exist solely and will contribute to the drag effect in the absence of out-of-phase modes.

The corresponding bilayer structure factor for the $l_0 = 4$ LL is depicted in Fig. 1 which we calculate in the plasmon-pole approximation. For $q \ell_B = 0.33$ the splitting energy is approximately equal to $0.17 \Gamma_0$. For $\mu = 35$ T$^{-1}$ and $B \approx 3.22$ T we have $\Gamma_0 \approx 4.84$ K and this gives $\delta E$ more than 0.8 K. This is a quite measurable quantity, however, notice that at smaller wave vectors, $\delta E$ is appreciably larger (Fig. 3). As seen from Fig. 1 the out-of-phase peak of $S(q, \omega)$ is about six times lower than the in-phase mode peak. However, both the in- and out-of-phase CE modes have enough weight to be observed experimentally, and the signatures of these bilayer intra-LL CE in the infrared absorption and inelastic light scattering measurements should constitute asymmetric doublets.

We expect that the approximations we use here are adequate for the 2DES in which disorder dominates over Coulomb correlations. This is safer in the first allowed zone where $q \ell_B < 1$ and the RPA is an acceptable approximation even in the disorder free case [14]. In the disordered 2DES, the RPA is valid when no FQHE excitation gap is developed [17]. This is the case if the gap in the disorder free case, $\Delta_0$, is smaller than the typical disorder energy [24]. According to the Laughlin’s estimates [24], $\Delta_0 \approx 0.056E_c$ for the smallest fraction $\nu = 1/3$, for which $\Delta_0$ is maximal. Here, we consider moderate magnetic fields well below the FQHE threshold [26] and obtain the intra-LL CE referred to the higher LL. For $\mu = 35$ T$^{-1}$, the ratio $E_c / \Gamma_0$ is less than 20, and near half-filling of the $l_0 = 4$ LL, the system is well away from the regime where FQHE occurs. Thus, we expect that the regime considered here is favorable for the applicability of this theory and for the observation of these novel bilayer intra-LL CE with attractive many-body and disorder effects.

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