Exploration of student’s flexibility in solving linear equation system and linear inequality problems

L D Permata, T A Kusmayadi and L Fitriana

Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Sebelas Maret, Indonesia.

E-mail: laelatul_dhian@yahoo.co.id

Abstract. Student’s flexibility was an important characteristic of problem solving. Flexibility could be explained as a skill to work flexibly in various situations. This study aimed to analyze, describe and provide information about student’s flexibility in solving linear equation system and linear inequality problems that could be used as a basis for improving the learning process. This study was a descriptive qualitative including twenty 11th high school students in Grobogan as its subjects. The study variables were indicators of IDEAL problem solver which students performed including identified problems, defined the problem, explored solution, acted on the strategies, and look back and evaluate. The data were taken of test and interview methods. The results showed that students with very good category in solving problem (25%) used two types of strategies and made a few mistakes, students with good category (15%) used four types of strategies and had difficulties to finish the third problem, students with enough category (35%) used three types of strategies and had problems in designing a plan to solve the third problem and students with poor category (25%) used two types of strategies. Most students were not able to design strategies to solve an unfamiliar problem.

1. Introduction

In the 21st century, communities and governments around the world will face severe complex problems in various fields and are required to make decisions that involve various considerations. In education field, the dimension of skills is one of dimensions that must be prepared to deal with these complex problems in addition to the dimensions of knowledge, character and metacognition. Skills are closely related to use and feedback from knowledge. Problem-solving is one of the skills or learning experiences of mathematics from the 21st century in addition to communication, collaboration, critical thinking, creativity and innovation [1].

The importance of problem-solving requires students to have that ability. Problem-solving is the ability obtained from a series of important activities in mathematics learning that can be used to solve other more complex problems and problems in the real world [2, 3, 4, 5]. Problem-solving is a process of individuals in finding solution to a problem by using the knowledge and skills that have been obtained previously and understanding to interpret the situation that is unfamiliar [6, 7]. Associated with that, then problem-solving can be interpreted as a process or an important capability that must always be developed by students in order to become a reliable problem solver.
An essential characteristic of problem-solving is where individuals can work flexibly and can change the pattern of work based on the situation. The ability to work flexibly is called flexibility. According to Krems, flexibility refers to the selection or modification of available problem-solving techniques, methods or strategies in accordance with the task or situation [8]. Flexibility can be defined as an individual’s ability to adjust problem-solving according to demands.

There are no certain methods or strategies to use. Bransford and Stein introduce IDEAL problem solver consisting of five indicators, including (1) identify problems: knowing and showing information and able to visualize situations that gave rise to opportunities for creative thinking, (2) define the problem: organize information and questions and can determine important information, (3) explore solutions: find or develop the necessary strategies that can be used, (4) act on the strategies: using numeracy, algebra or geometry for the chosen strategy and (5) look back and evaluate: check back the answers, determine other alternative solutions and discuss or develop answers to answer other situations [9]. IDEAL problem solver is a series of problem-solving activities that can be used to describe student’s thinking and problem-solving skills well [10].

In addition to the method, the strategy is an important thing that must be considered by students in solving a problem. Several strategies are known and can be offered to solve problems, including working backwards, finding pattern, adopting a different point of view, solving a simpler analogous problem, considering extreme cases, making a drawing, intelligent guessing and testing (also known as trial and error), accounting for all possibilities, organizing data, logical reasoning and using algebra [11,12]. All of these strategies are not intended to solve a problem. Sometimes just one strategy can solve the problem but sometimes it also requires more than one. Therefore, students must have the ability to choose and determine which strategies should be used to solve a problem.

Some researches discuss about flexibility, one of them is Dina, Amin and Masriah. They compare student’s flexibility based on adversity quotient [13]. Many researches discuss the flexibility of student’s relationships with certain categories but this study focuses on providing information about weaknesses, strengths and reasons for students regarding the selection of strategies that used for solving the problems given which can be used as material for consideration of selecting suitable or appropriate learning methods.

2. Method
This study used qualitative research methods with descriptive types. The subjects of this study were twenty 11th high school students from SMA Negeri 1 Kradenan and SMA Negeri 1 Wirosari. Subject’s school were selected based on the absorption capacity of linear program material in the UN SMA 2016/2017 so that subject’s ability was assumed to represent the average ability of students in Grobogan. The students used as subjects were students with heterogeneous learning abilities who were selected by random sampling technique. The variables of this study used the indicators that adopted from IDEAL problem solver. A description of IDEAL problem solver’s indicators that used in this study was shown in Table 1.

Table 1. Description of IDEAL Problem Solver.

| No | IDEAL’s indicators          | Description                                                        |
|----|-----------------------------|--------------------------------------------------------------------|
| 1  | Identify problem            | (a) Students write down information that is known and needed appropriately |
| 2  | Define goal                 | (a) Students organize the information and question                   |
|    |                             | (b) Students write down what is asked in the problems correctly      |
| 3  | Explore possible strategies | (a) Students choose the strategy used                               |
|    | and act                     | (b) Students write down problem-solving plan (table/model/graph) appropriately |
| 4  | Anticipate outcome          | (a) Students implement the plan                                     |
|    |                               | (b) Students write down the answers and computations appropriately  |
| 5  | Look back and learn         | (a) Students interpret the final answers obtained by making correct conclusion |
Data collection techniques in this study were tests and in-depth interviews. The test in this study used three problem-solving problems with two linear equations problems and one problem in linear inequality. The interview in this study aimed to explore subject’s information on the reasons for the selection and use of strategy in solving the three problems given. Triangulation was used to confirm the validity of the data by comparing test and interview results. The data analysis stage in this study included (1) provided tests of student’s problem-solving abilities regarding systems of linear equations and linear inequalities, (2) organized and analyzed test results, (3) conducted interviews with several students who use different strategies and (4) analyzed interview results.

3. Results and Discussion

3.1. Results

The results of problem-solving tests using the IDEAL problem solver show that out of twenty subjects can be classified into four categories, including 25% are categorized very well, 15% are categorized good, 35% are categorized enough and the rest are categorized poor. Most of the subjects make mistakes in solving the second problem and have difficulty in designing a plan to solve the third problem. Summary of problem-solving test’s the results from this study can be seen in Table 2.

Table 2. Percentage of problem-solving ability and type of strategies that arise

| Category  | Percentage | First Problem | Second Problem | Third Problem |
|-----------|------------|---------------|----------------|--------------|
| Very good | 25%        | (a) trial and error | (a) accounting for all possibilities | |
| Good      | 15%        | (a) trial and error | (a) accounting for all possibilities | |
|           |            | (b) working backward | (b) using algebra | |
|           |            | (c) using algebra | (c) using algebra | |
| Enough    | 35%        | (a) working backward | (a) using algebra | |
| Poor      | 25%        | (a) trial and error | (a) using algebra | |
|           |            | (b) using algebra | (b) using algebra | |

Table 2 shows the percentage of problem-solving ability that are classified using IDEAL problem solver and type of strategies that arise in solving the problems given. Subjects with very good categories use two types of strategies, subjects with good categories use four types of strategies. Other than that, subjects with enough categories use three types of strategies while subjects with poor categories use trial and error and using algebra to solve the problem given.

In the first problem, there are three strategies used by subjects. The success rate in solving the first problem reaches 100% that mean all subjects can answer the problem correctly without significant obstacles. The first problem is the problem of a two-variable linear equation system with the theme of purchasing school supplies as follows.

Santi want to shop for school uniforms and shoes at a school supply store to donate with the available money of 250,000.00 IDR. The price of each item in the store is available in the price list of goods so Santi can estimate what school equipment she can afford with the money she has and turns out Santi buy 5 items. Based on the price list one school uniform is 35,000.00 IDR and the price of a pair of shoes is 110,000.00 IDR. How many uniforms and shoes can Santi buy?

Of these problems, the following are presented three figure that show the results of student answers to the problem. In Figure 1, a trial and error strategy is shown from the subject of very good category (PAKA). Figure 2 shows the working backward strategy of enough category (SLS), while Figure 3 shows the strategy of using algebra from poor categories (NRAA).
From Figure 1, PAKA is able to solve the first problem well. PAKA chooses four school uniforms and a pair of shoes as the solution to the first problem. Figure 2 shows that SLS is able to solve problem 1 but experiences errors at the conclusion writing stage. It appears that the conclusions written by SLS have not answered the question completely. SLS only wrote four uniforms without writing down the many shoes purchased by Santi. Figure 3 shows that NRAA is able to choose strategy and write the solution to the first problem well. Then, the interview activities are conducted to explore and confirm the results of the test from PAKA, SLS and NRAA. The following are the results of interviews with PAKA on problem 1.

Question 1a: “Do you understand what the problem will be in the first problem? What do you understand?”

Answer (PAKA) 1a: “Yes ma’am, on problem one I had to find lots of school uniforms and shoes that Santi could buy.”

Question 1b: “What method do you choose to find many school uniforms and shoes that Santi can buy? How to?”

Answer (PAKA) 1b: “I tried several times, the most important thing was that Santi bought five items. For example, I took 2 shoes and 3 uniforms, apparently the answer was incorrect, then I took one shoe and 4 uniforms and it turned out the answer was correct.”

Question 1c: “How many times have you tried it? why did you choose that way?”

Answer (PAKA) 1c: “On the second trial, I thought that was the easiest method, because many items that Santi bought were only 5.”

|$\text{uniform price } = 35,000 \times 4 = 140,000$  
|$\text{shoe price } = 110,000 \times 1 = 110,000$ +  
|$250,000$  

**Conclusion:**
If Santi’s money is 250,000.00 IDR, then Santi can buy 4 uniforms and 1 shoe.

|$\text{Overall money - school shoes } = 250,000$  
|$\text{110,000 } = 140,000$  

$\Rightarrow 140. \text{ then, }$  
$\Rightarrow 140 \div 35$  
$\Rightarrow 4$  

**Conclusion:**
Santi can buy 4 uniforms.

![Figure 1](image1.png) Test results of PAKA on problem 1.  
![Figure 2](image2.png) Test results of SLS on problem 1.  
![Figure 3](image3.png) Test results of NRAA on problem 1.

For example, shoes: $x$ and uniform: $y$  
$x + y = 5 \ldots (D_1)$  
$35,000x + 110,000y = 250,000 \ldots (D_2)$  
Using elimination  
$\begin{align*} 
35,000x + 110,000y &= 250,000 \\
35,000x + 110,000y &= 250,000 \\
\hline 
75,000y &= 75,000 \\
75,000y &= 75,000 \\
\end{align*}$  

$y = \frac{75,000}{75,000} = 1$

![Example Table](example_table.png)
Interviews are also conducted on SLS about the use of working backward strategy to solve the first problem. The following are the results of interviews with SLS on problem 1.

**Question 2a**: “Do you understand what the problem will be in the first problem? What do you understand?”

**Answer (SLS) 2a**: “Yes, I understand, I wrote down the question How many uniforms and shoes did Santi buy”

**Question 2b**: “What method do you choose to find many school uniforms and shoes that Santi can buy? How to?”

**Answer (SLS) 2b**: “I chose the way to reduce the price of shoes first, because the price of the shoes is the most expensive, then after deducting the remaining IDR 140,000, if the money is deducted more, the money will not be left to buy a uniform, so only one shoe can be bought and the rest to buy uniforms.

**Question 2c**: “Why did you choose that way?”

**Answer (SLS) 2c**: “Because it’s easy to just subtract and divide without using the x and y symbols.”

**Question 2d**: “Try to look at the conclusion section! Is there something incomplete?”

**Answer (SLS) 2d**: “Yes, I forgot to write a lot of uniforms.”

Furthermore, interviews are also conducted on NRAA. The following are the results of interviews with NRAA on problem 1.

**Question 3a**: “Do you understand what the problem will be in the first problem? What do you understand?”

**Answer (NRAA) 3a**: “Yes, the question is about many items that Santi bought”

**Question 3b**: “What method do you choose to find many school uniforms and shoes that Santi can buy? How to?”

**Answer (NRAA) 3b**: “I use elimination and substitution”.

**Question 3c**: “why did you choose that way?”

**Answer (NRAA) 3c**: “Because this method is a way that teachers often use to answer such questions, so I use that method.”

**Question 3d**: “Do you understand that way? Are there difficulties?”

**Answer (NRAA) 3d**: “Yes I understand. I have no difficulty, the question is still an easy matter”

Based on the results of the interviews of the three subjects, two subjects choose that strategy because of the ease of implementation while the other one is familiar with the strategy used. The three subjects did not have difficulty in solving the first problem. PAKA solve the first problem with two trials without making mistakes and explain the work procedure well. SLS is able to solve the first problem but made a mistakes at the conclusion stage. Based on interview, SLS make mistake not because of the inability to understand but because of human error. Subject realize his mistake and gives clear explanation. On the other that, NRAA is able to make a plan to solve the problem and can execute the plan properly.

In the second problem, there are two strategies used. From 20 subjects, only 3 students are able to answer correctly. Most subjects make mistakes using inequalities on this problem. The second problem discusses the concept of linear inequality with the theme of mathematical test scores. The following is the second problem presented in student's mathematical problem-solving tests.

In four mathematical tests, Didi received scores of 90, 60, 45 and 35. If the average math score of Didi is greater than or equal to 60, what is Didi's fifth math test?

Based on these problems, the following is presented in Figure 4 which shows the results of student tests with the strategy of using algebra from subjects with poor category (NRAA). In addition, in Figure 5 shows a trial and error strategy which is used in a very good category (PAKA).
Based on the results of these tests, it appears in Figure 4 that NRAA wrote the solution of the second problem correctly but the concept used was different from the intended concept. NRAA makes a completion plan using the concept of equality. Figure 5 shows that PAKA has written the completion plan correctly. However, at the next stage PAKA changes the symbol and chooses a definite average score that meets. Furthermore, the interview activities are conducted to explore and confirm the results of NRAA and PAKA tests. The following are the results of interviews with NRAA on problem 2.

**Question 4a:** “Do you understand what the problem will be in the first problem? What do you understand?”

**Answer (NRAA) 4a:** “Yes, the question is Didi’s fifth score math test”

**Question 4b:** “Why did you choose that way?”

**Answer (NRAA) 4b:** “I use the average value formula, then I enter the test scores and many of the tests, I use that method because it’s the formula”

**Question 4c:** “The average math score of Didi written in question 2 is more than or equal to 60. Why do you use an equal sign? Do you know what sign should be used?”

**Answer (NRAA) 4c:** “Yes, I know that it uses more than or equal to, but if I use the sign, I’m confused, afraid of being wrong. So I give that sign in the conclusion”

**Question 4d:** “But Do you know that you should use equations or inequalities?”

**Answer (NRAA) 4d:** “Yes, I should use inequality”

**Question 4e:** “Has the teacher ever given such inequality questions? If not, what kind of problem is often given by the teacher?”

**Answer (NRAA) 4e:** “Never, the problem is that only the direct matter does not use stories.”

Interviews are also conducted on PAKA. The following is the result of an interview with PAKA on problem 2.

**Question 5a:** “Do you understand what the problem will be in the first problem? What do you understand?”

**Answer (PAKA) 5a:** “Yes, the fifth test score that must be obtained by Didi.”

**Question 5b:** “Why did you choose that way?”

**Answer (PAKA) 5b:** “I use the average value formula and choose an average value of 62, because 62 is more than 60”.

**Question 5c:** “What about 60, 61, 63, 64 and the average value that is more than or equal to the other 60?”

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**Figure 4.** Test results of NRAA on problem 2.

**Figure 5.** Test results of PAKA on problem 2.

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\[
\text{mean} = \frac{\text{Didi’s score}}{\text{many of tests}}
\]

\[
60 = \frac{90 + 60 + 45 + 75 + x}{5}
\]

\[
60 = \frac{230 + x}{5}
\]

\[
230 + x = 60.5
\]

\[
230 + x = 300
\]

\[
x = 300 - 230
\]

\[
x = 70
\]

**Conclusion:**

So, the fifth test result to reach an average of more than 60 is more than 70.

\[
\text{In order for the average math score of Didi is} 60: \\
\frac{90 + 60 + 45 + 75 + x}{5} = > 60
\]

\[
\frac{230 + x}{5} = 60
\]

So,

\[
\frac{230 + 80}{5} = 62
\]

\[
\frac{310}{5} = 62
\]

**Conclusion:**

In order for the average math score of Didi the average, Didi must get the 5th test value, namely 80 and above.
Answer (PAKA) 5c: “I only count one average score which is more than 60.”

Question 5d: “How many times have you read that question? Is the average score used more than 60?”

Answer (PAKA) 5d: “Yes, the average score used is more than 60”

Question 5e: “In the method on this first line (pointing to the answer), why do not you continue with the same symbol? Is it true that a symbol more than or equal to 60 is written like that”

Answer (PAKA) 5e: “I am confused and I do not know, so I only choose one value more than 60. Then look for the fifth score. I have never worked on a problem like that. I think writing symbol is more than or equal to 60 like that”

Question 5f: “What kind of questions do you often do about inequality?”

Answer (PAKA) 5f: “The questions given directly inequality do not use stories like this”

Based on the results of interviews with NRAA and PAKA about the second problem, it appears that the two subjects have never worked on a problem with a type like the second problem. Type of question like the second problem requires the ability to understand, organize, connect information in a structure and then solve it. In this case, it appears that the teacher only gives the direct type question which only requires numeracy, algebra or geometric capabilities of the student, whereas the problem with type like the second problem is rarely given. From the results of interviews, NRAA actually knows the written strategies but subject is unfamiliar with the inequality problem. NRAA prefers to perfect them at the conclusion stage. Based on interview, PAKA experiences errors in reading and understanding the problem given even though subject has written the structure/model correctly (there are errors in sign of inequality but the subject understood).

In the third problem, there are three strategies used. From 20 subjects, only 7 subjects solved the third problem correctly with accounting strategies for all possibilities by creating tables. Meanwhile, the strategies used by other subjects cannot solve the third problem with various obstacles. The following is the third problem presented in the student's mathematical problem-solving test regarding the length of the hair of the twins Cila and Cilo.

Cila and Cilo agreed to extend their hair for several years to come. In the third month the length of Cila’s hair is 16 cm while Cilo’s hair is 28 cm. Five months later, the length of Cila’s hair reaches 26 cm and Cilo’s hair is 36 cm. If the initial hair length of Cila and Cilo is measured in the first month and assume Cila and Cilo hair growth are the same every month. What is the length of their hair when it is the same length?

From the results of these tests, the following three images are presented which show the results of student answers to the problem. Figure 6 shows the answer of subjects with good categories (AMNA), while Figure 7 shows the strategy of using algebra by subjects with poor categories (NRAA).

| The-month | Hair length (cm) |  |
|-----------|-----------------|---|
|           | Cila | Cilo |
| 3         | 16  | 28  |
| 8         | 26  | 36  |
| 13        | 36  | 44  |
| 18        | 46  | 52  |
| 23        | 56  | 60  |
| 28        | 66  | 68  |
| 33        | 76  | 76  |

**Conclusion:**
It can be concluded from the data above that their hair will be 76 cm long, in the 33rd month.

For example, Cila: $x$; Cilo: $y$

Hair growth $x$

\[
x = \frac{26 - 16}{5} = 2 \text{ cm}
\]

Hair growth $y$

\[
y = \frac{36 - 28}{5} = 1.6 \text{ cm}
\]

Figure 6. Test results of AMNA on problem 3.

Figure 7. Test results of NRAA on problem 3.
Based on Figure 6, AMNA solve the third problem correctly without experiencing obstacles. AMNA find solution to the third problem by calculating all the possibilities of their hair growth with each interval of each element. On the other that in Figure 7, NRAA want to solve the third problem using algebra but has difficulty making mathematical models. Then, interview activities are conducted to explore and confirm the results of AMNA and NRAA tests. The following are the results of an interview with AMNA on problem 3.

**Question 6a**: “Do you understand what the problem will be in the first problem? Why did you choose that way?”

**Answer (AMNA) 6a**: “Yes, Because this is the easiest way, in the initial table the monthly increment is 5, the increase in Cila hair is 10 and the increase in hair is Cilo 8. Then the hair growth every month is the same, so I keep increasing until I find the same length.”

**Question 6c**: “Have you tried other ways to do it?”

**Answer (AMNA) 6c**: “I actually want to try using x and y, but I am confused when I want to make a model, so I use this method only.”

Interviews are also conducted on NRAA. The following are the results of interviews with NRAA on problem 3.

**Question 7a**: “Do you understand what the problem will be in the first problem? What do you understand?”

**Answer (NRAA) 7a**: “Yes, how long is the length of Cila’s hair and Cilo’s hair”

**Question 7b**: “Why did you choose that way?”

**Answer (NRAA) 7b**: “Because this method is often taught and used by teachers, so I tried to use this method, but I was confused when I wanted to make a mathematical model, I do not know which mathematical model was prepared using which information”

Based on the results of interviews with AMNA and NRAA, it seems that AMNA is more flexible in solving the third problem compared than NRAA. In this case, AMNA tried other strategies while NRAA did not. AMNA chose the accounting for all possibilities strategy after observing the increase in the moon and the growth of the Cila twins and Cilo hair. That’s means, AMNA has a good level of understanding and observation of the problems given. Meanwhile, NRAA focuses more on the ways taught and recommended by the teacher without thinking of other alternatives to solve the problem.

### 3.2. Discussion

The results of this study indicate that students have tried to solve the three problems given with strategies that are considered the easiest and most effective. In essence, problem-solving is not only related to finding solutions but rather in finding solutions effectively [14]. Furthermore, [15] states that students who solve problems with minimal instructions that will at least be able to easily become successful problem solvers. Thus, students with the ability to apply the most effective strategies are students who have greater flexibility than other students.

Strategies that are often used by students in completing the three problems given are trial and error, working backward and using algebra. Trial and error may not be included in mathematical procedures, but the concept of the strategy can be used to solve certain problems with less time than other strategies [16]. In trial and error strategy, students are faced with experiments to find solutions to the problems given. Meanwhile, working backward is a strategy that departs from predicting a solution that meets the given problem [17]. Maybe the two strategies are more effective to be used to solve the first problem than the strategy using algebra, but problem-solving using strategies using algebra can describe higher student thinking patterns. That is because, in solving problems using algebra students do a series of activities reading, understanding, investigating the relationship between elements in a given problem and making transformations into symbolic expressions, tables or others [18].

Based the results of the study, it appears that students make several mistakes. Most students make few error in solving the second problem, that is factual error. Factual error is an errors caused by lack
of factual information [19]. One factor that causes factual errors is a meaningless learning process and teachers quality in learning process [20, 21]. Besides factual errors, it is also found the fact that students did careless errors. Careless error is an error caused by lack of concentration or working too fast.

Flexibility can be developed through the learning process or instructional environments [22]. In addition, through the learning process can also minimize errors. The learning process is an important activity which is a pillar in constructing student's knowledge and skills. Therefore, teachers are expected to provide opportunities for students to carry out a series of meaningful learning activities that can develop their knowledge and skills. By developing students' knowledge and skills, students will solve various problems they face easily. This means that the flexibility of students will grow stronger and finally the students' ability to solve problems will be better.

4. Conclusion
Based on data analysis, it appears that out of twenty students can be classified into four categories, students with categories very good at solving problems (25%) using two types of strategies and making some mistakes, students with good categories (15%) use four types of strategies and having difficulty in solving the third problem, students with enough categories (35%) used three types of strategies and had difficulty in making plans to solve the third problem and students with poor categories (25%) used two types of strategies. Students make several error in solving the problem such as factual error and careless error. Most students are not able to solve an unfamiliar problem.

Data shows that there are four strategies that appear to solve the problem given. Students with very good categories are students who have stronger flexibility than other categories. Meanwhile, students with poor categories have the worst flexibility than the others.

The strategies used are the best strategies chosen by each student to solve the problem regardless of the effectiveness of using the strategy. In essence, problem-solving ability is not only related to student’s ability to solve problems flexibly, but also related to students’ ability to solve problems effectively. If students are able to solve problems flexibly and effectively, these students will easily become reliable problem solvers and can even be creative problem solvers. Student’s flexibility and effectiveness in solving problems can be trained and developed through meaningful learning processes.

Mathematics teachers are expected to be able to make a lesson that can be used to develop student’s skills that can be used as provisions in the future. Analyzing activities such as those mentioned above is one of tools that can be used by mathematics teachers to find out the strengths, weaknesses and flexibility of students in solving problems. So based on the results of that activities, teacher can find out what the learning system should be done to improve poor learning outcomes and minimizing errors. Furthermore, to follow up on this research, future researchers are expected to conduct research to find out the most appropriate learning methods for applying to students with diverse flexibility as the results shown in the study.

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