Super-Poissonian noise in a Coulomb blockade metallic quantum dot structure

V. Hung Nguyen and V. Lien Nguyen *

Theoretical Dept., Institute of Physics, VAST P.O.Box 429 Bo Ho, Hanoi 10000, Vietnam

The shot noise of the current through a single electron transistor (SET), coupled capacitively with an electronic box, is calculated, using the master equation approach. We show that the noise may be sub-Poissonian or strongly super-Poissonian, depending mainly on the box parameters and the gate. The study also supports the idea that not negative differential conductance, but charge accumulation in the quantum dot, responds for the super-Poissonian noise observed.

PACS numbers: 73.63.Kv, 72.70.+m, 73.23.Hk

Deviations of the shot noise (SN) from the full (Poissonian) value in nano-structures have been the subject of a great number of works, both experimental and theoretical. Mathematically, the measure of these deviations is the Fano factor $F_n$, defined as the ratio of the actual noise spectral density to the full SN-value, $2eI$, where $e$ is the elementary charge and $I$ is the average current. Physically, it is widely accepted that the Pauli exclusion and the charge interaction are the two correlations, which cause observed SN-deviations. While the Pauli exclusion always causes a suppression of SN, the charge correlation may suppress or enhance the noise, depending on the conduction regime. The typical non-Poissonian behaviors of SN can be found in resonant tunneling diodes (RTD), where the noise is partially suppressed (sub-Poissonian noise) at low bias voltages (pre-resonance) and becomes very large (super-Poissonian) in the negative differential conductance (NDC) region. For Coulomb blockade quantum dot (QD) structures, a suppression of SN, the charge correlation may be easily realized even in a positive differential conductance (PDC) region. Furthermore, in consistency with ref.6, our study supports the idea that the charge accumulation, not NDC, is ultimately responsible for the super-Poissonian noise observed.

FIG. 1: (a) Equivalent circuit diagram of the structure under study. (b) The normalized noise, $S(\omega)/2eI$, calculated from (8) is plotted as a function of frequency for some values of bias voltage $V$ (from top): 0.25, 0.26, and 0.27. Inset: the current $I(7)$ (dashed line, see the left axis) and the Fano factor (10) (solid line, see the right axis) as a function of bias voltage in the range $V_2 < V < V_3$ (see the text). The structure parameters: $C_m = C_3 = C_2 = C_1 \equiv C$, $R_3 = R_2 = R_1 \equiv R$, without gate.

*Corresponding author, E-mail: nvlien@iop.vast.ac.vn

Within the framework of the Orthodox theory, the state $|i>$ of the system under study is entirely determined by the numbers of excess electrons in two QDs, $n$ in $D$ and $m$ in $B$. At a given $(n,m)$-state, the free energy of the system can be written as:

$$F = Q_d^2/2C_d^* + Q_d^2/2C_b^* + Q_dQ_b/C_p^* - (C_1 + C_3)V^2/2 - C_qV_2^2/2 - n_qV$$

where $C_d^* = \Sigma/C_b; C_b^* = \Sigma/C_d; C_p^* = \Sigma/C_m$ with $\Sigma = C_dC_b - C_m^2, C_d = C_1 + C_2 + C_m + C_g, C_b = C_3 + C_m; Q_d = 2eI$. 

The equivalent circuit diagram of the structure studied is drawn in Fig.1(a), where the left QD ($D$) forms a SET, while the right QD ($B$) acts as an electronic box. Two QDs are coupled to each other by a capacitance $C_m$, but the electron tunneling between them is forbidden. The current through the SET depends not only on the bias voltage $V$ and the gate voltage $V_g$, but also on the charge state in the box. Such a SET-to-box coupling may produce an NDC as experimentally observed in ref.17.
In the system of interest there are six possible sequential electron transfers across three junctions (1, 2 and 3) upwards (+) or downwards (-). The change in free energy associated with these transfers can be deduced from eq.(1) as follows:

\[
\begin{align*}
\Delta F_1^± &= e^2 (1 ± 2n) / 2C_p ± me^2 / C_p^\ast, \\
\Delta F_2^± &= e^2 (1 ± 2n) / 2C_p ± me^2 / C_p^\ast, \\
\Delta F_3^± &= e^2 (1 ± 2n) / 2C_p ± me^2 / C_p^\ast.
\end{align*}
\]

(2)

At zero temperature the rate of a sequential electron transfer across any \( \nu \)-junction (\( \nu = 1, 2 \) or 3) is well-known:

\[
\Gamma_\nu = \theta (-\Delta F_\nu) |\Delta F_\nu| / (e^2 R_\nu),
\]

(3)

where \( \theta \) is the step function, \( R_\nu \) is the tunneling resistance of \( \nu \)-junction and \( \Delta F_\nu \) is the corresponding change in free energy defined in eq.(2).

Using expressions (2) and (3), one can solve the master equation (ME) or perform Monte-Carlo simulation to yield the current as a function of bias voltage \( V(I-V) \) characteristics) and further to calculate the noise. The Monte-Carlo method is very effective at finite temperature, but it does not allow us to calculate the noise in the important limit of low frequency. In this work we will discuss only the zero-temperature case, therefore, the ME method should be used. Denoting \( p(i) \) as the probability of the state \( |i\rangle = (n_i, m_i) \) of the system, the ME can be written in the matrix form:

\[
dp(t)/dt = M \hat{p}(t),
\]

(4)

where \( \hat{p}(t) \) is a column matrix of elements \( p(i, t) \) and \( M \) is an evolution matrix with elements defined as follows:

\[
M(i, j) = \Gamma_\nu^+ (j) + \Gamma_\nu^- (j) (\text{if } n_j = n_i - 1 \text{ and } m_j = m_i);
\]

\[
\Gamma_\nu^± (j) (\text{if } n_j = n_i + 1 \text{ and } m_j = m_i);
\]

\[
\Gamma_\nu^± (j) (\text{if } n_j = n_i \text{ and } m_j = m_i + 1);
\]

\[
\Gamma_\nu^± (j) (\text{if } n_j = n_i \text{ and } m_j = m_i - 1) + M(i, i) = -[\Gamma_\nu^+ (i) + \Gamma_\nu^- (i) + \Gamma_\nu^0 (i) + \Gamma_\nu^0 (i)].
\]

Solving the ME (4) under condition \( \sum_i p(i, t) = 1 \), we can further calculate the net current,

\[
I(t) = q_1 I_1(t) + q_2 I_2(t) + q_3 I_3(t),
\]

(5)

where \( I_\nu(t) = e \sum_i |\Gamma_\nu^+ (i) - \Gamma_\nu^- (i)| \) is the statistical average current through \( \nu \)-junction (\( \nu = 1, 2 \) or 3), the factors \( q_\nu \) are defined as:

\[
q_1 = (C_m C_2 + C_2 C_3) / \Sigma;
\]

\[
q_2 = (C_m C_1 + C_m C_3 + C_1 C_3) / \Sigma;
\]

and \( q_3 = C_m C_2 / \Sigma \) with \( \Sigma \) given in (1).

Next, the noise spectrum \( S(\omega) \) of the current \( I \) can be calculated in the way similar to that developed in refs.11,16:

\[
S(\omega) = 2 \sum_\nu q_\nu^2 A_\nu + 4e^2 \sum_\nu \sum_{ij} q_\nu q_\nu \left( |\Gamma_\nu^+ (i) - \Gamma_\nu^- (i)| \times B_{ij} \left[ \Gamma_\nu^+ (j) \mu^- \Gamma_\nu^- (j) \mu^- \right] \right)
\]

(6)

Here, \( A_\nu = e (I^+ \nu + I^- \nu) \) with \( I^± \nu = e \sum_i \mu_i \delta_{ij} \); the conditional probability \( p(i \leftarrow j) \) for having state \( |i\rangle \) at the time \( t = t + \tau \) under the condition that the state was \( |j\rangle \) at an earlier time \( t = 0 \) obeys the same ME as for the probability \( p(i, t) \); the stationary probability \( p_{\text{stat}}(i) \) of the state \( |j\rangle \) is state obtained from the state \( |j\rangle \) at \( t = 0 \) by transferring an electron across the \( \nu \)-junction upwards (+)/downwards (-); the tunneling rates \( \Gamma^\pm \nu \) and the factors \( q_\nu \) are defined in eqs.(3) and (5), respectively. Similarly, we can also obtain the noise expression for currents through junctions, \( I_1 \) or \( I_2 \).

Thus, using the tunneling rates (3), in principle, we can solve the ME (4) and further to calculate the current (5) and the noise (6). In practice, however, this ME can not be exactly solved with all possible states except some simple cases at low bias voltages. Let us consider such a simple case, when the SET is symmetrical, \( C_1 = C_2 \equiv C \) and \( R_1 = R_2 \equiv R \), and the box parameters are as follows: \( C_3 = C \), \( R_3 = R \), and \( C_m \) belongs to the range \((\sqrt{3} - 1)/5 \leq C_m \leq (\sqrt{3} + 1)/C \). The gate is neglected. With all these assumptions, in the way similar to that developed in refs.18,20 we can solve the ME (4) as well as calculate the current (5) and the noise (6) exactly in some ranges of low bias. Neglecting lengthy, but elementary, algebraic calculations the final results for the current can be reviewed as follows: (1) the Coulomb blockade region has the threshold voltage of \( V_0 = (e/2C)(C_m + C)/(5C_m + 3C) \); (2) In the next range of bias voltage, \( V_0 \leq V \leq V_1 \equiv (2/2C)(C_m + 2C)/(5C_m + 4C) \), the current has been found as \( I = e \Gamma_3^2 (1) \Gamma_3^0 (0) / (\Gamma_3^+ (1) + \Gamma_3^- (0)) \), where two states \( |1\rangle \equiv (-1, 0) \) and \( |0\rangle \equiv (0, 0) \) are written for short; (3) the current is equal to zero in the range of bias \( V_1 \leq V \leq V_2 \equiv (e/2C)(3C_m + C)/(5C_m + 3C) \) (second Coulomb blockade gap); and (4) in the last range of bias, \( V_2 \leq V \leq V_3 \equiv (e/2C)(3C_m + 2C)/(5C_m + 4C) \), where the ME (3) can be still solved exactly, the current is given by

\[
I = \frac{(a + b)cdh + bcgd}{cdh + bcd + (a + b)dh + (g + h)eb},
\]

(7)

where we introduce \( a = \Gamma_3^+ (0); b = \Gamma_3^- (0); c = \Gamma_3^+ (1); d = \Gamma_3^+ (2); g = \Gamma_3^- (3); \) and \( h = \Gamma_3^- (3) \). Two states \( |2\rangle \equiv (0, -1) \) and \( |3\rangle \equiv (1, -1) \) are also written for short.

Now, we can calculate the noise (6) in two ranges of bias, \( V_2 \leq V \leq V_1 \) and \( V_2 \leq V \leq V_3 \), where the current is finite and already known. As an example, we show the noise expression obtained in the last range of bias, \( V_2 \leq V \leq V_3 \):

\[
S(\omega) = 2(q_1^2 A_1 + q_2^2 A_2 + q_3^2 A_3) + 4e^2 D_r BD_c.
\]

(8)
Here, $A_1 = A_2 = eI$ ($I$ defined in (7)); $A_3 = 2e^2bcdh/e( cdh + bcd + (a + b)dh + (g + h)bc)$; $D_1$ is a row-matrix of four elements: \(q_1a + q_3b + q_2c + q_1d\) and \(q_2g - q_3h\); $D_2$ is a column-matrix of four elements: \(q_2(a + b)cdh/Q, (q_1acdh - q_3bcdh)/Q, (q_2bdcg + q_1bcgh)/Q\) and \(q_1(g + h)bc/Q\) with \(Q = cdh + bcd + (a + b)dh + (g + h)bc\); and \(\hat{B} = Re(i\omega \hat{I} - \hat{M})^{-1}\) with

\[
\begin{bmatrix}
   i\omega & a & c & 0 & 0 \\
   -a & i\omega & c & 0 & -b \\
   -b & 0 & i\omega & d & -g \\
   0 & 0 & -d & i\omega & g + h \\
\end{bmatrix}
\]

The expression (8) gives the SN of the net current I as a function of frequency $\omega$ and of bias V. In Fig.1(b), for example, we present the normalized noise, \(S(\omega)/2eI\), as a function of frequency, calculated at some bias voltages, for the structure with parameters given in the figure. Here and below, for symmetrical SETs of equal capacitances, $C$, and tunneling resistances, $R$, it is convenient to choose the elementary charge $e$, the capacitance $C$, and the resistance $R$ as basic units. The voltage, the current, and the frequency in figures are then measured in units of $e/C, e/C R$, and $(CR)^{-1}$, respectively. It seems from Fig.1b that the normalized noise obtained for the net current always decreases as the frequency increases and within the framework of the model considered there exists a large frequency limit: $S(\omega)/2eI \geq 0.5$.

Particularly, in the limit of zero frequency, when all the noises, for the net current and for currents through partial junctions, are coincident, we obtain an explicit expression for Fano factor:

\[
F_n = \frac{S(0)}{2eI} = 1 + \frac{2}{\pi} \frac{\sin \pi \left( \left( \frac{d + g + h}{a + b + c} \right) \right)}{d + g + h} \right) \frac{[d + g + h)]}{[a + b + c + d + g + h]}
\]

where the quantities $a, b, c, d, g$ and $h$ are defined in eq.(7). Clearly, this expression (10) shows that $F_n$ may be greater or smaller than 1, depending on relative values of two terms with opposite signs in the braces. In other words, we have exactly shown that at least for the simple case under study the SN may be super-Poissonian or sub-Poissonian, depending on the structure parameters and bias voltage. Such an interesting noise behavior can be seen in the inset of Fig.1(b), where, as an example, we present the current I (7) and the corresponding Fano factor $F_n$ (10) (valid in the range of bias $V_2 \leq V \leq V_3$) for the same structure as in the main figure. While the current monotonously increases (with an PDC), the noise is super-Poissonian ($F > 1$) at $V \leq 0.26$ and becomes sub-Poissonian at higher biases.

To extend calculations to higher biases and different varieties of structure parameters, we solve the ME (4) and calculate the current (5) and the noise (6) numerically. In Fig.2(a) we present obtained results of the current I (dashed line) and Fano factor $F_n$ (solid line) for the structure with the same parameters as in Fig.1(b) except the SET-to-box capacitance $C_m$. Apparently, the I-V characteristics obtained is very similar to that reported in [4] with a clear second Coulomb gap. Compared to this experiment, the calculation has been extended to higher bias voltages, where one more NDC region has been recognized. Along with such an I-V curve the Fano factor $F_n$ strongly varies with the bias $V$ and reaches super-Poissonian peaks, $F_n = 3.13$ and 3.01, at $V = 0.27$ and 0.43, respectively. Note that the lower value of $V$ belongs to a PDC region, while at the higher one we have an NDC. Statistics of numerical results for structures

![Figure 2: Numerical results: the current, calculated from (5) (dashed line, see the left axis) and the Fano factor, $F_n = S(0)/2eI$, calculated from (6) (solid line, see the right axis), are plotted against the bias voltage $V$. The structure parameters are the same as in Fig.1b except $C_m$, which is equal to 2$C$ in (a) and 10$C$ in (b).](image2.png)

![Figure 3: The gate effect: the current (dashed line) and Fano factor (solid line) are plotted against the gate parameter $C_m V_g$ for the same structure as that studied in Fig.2a at bias voltage $V = 0.44$.](image3.png)
the NDC regions are still clearly maintained in both figures, in Fig.2(b) the noise is sub-Poissonian in the whole range of bias voltages under study. The study demonstrates that by changing only \( C_m \) it is possible to get the noise as large as \( F \approx 100 \). Such a giant enhancement of noise has been suggested in a quantum shuttle at the shutting threshold\(^{21}\).

Results similar to those in Fig.2 have been also obtained when we change only the box parameter \( C_g \) or \( R_3 \). Noting again that the SET is still symmetrical, our study thus demonstrates an important role of the box in affecting both the I-V characteristics and the noise behavior of the SET.

All the results presented in Figs.1-2 are for the case without gate. The gate leads to an additional term in the free energy \( F(1) \) and simply makes numerical calculations little lengthier. As an example, the current (dashed line) and normalized zero-frequency noise (solid line), calculated at the bias \( V = 0.44 \), are plotted against the gate parameter \( C_g V_g \) in Fig.3 for the same structure as in Fig.2. The Fano factor decreases from the value of 1.59 (super-Poissonian) in the case without gate (\( C_g V_g = 0 \)) to the sub-Poissonian value of 0.5 at \( C_g V_g = 0.85 \) and then sharply rises to a large value of \( \approx 11.2 \). Note that as the gate parameter \( C_g V_g \) varies the changes of conductance and of noise are not always in accordance with each other: the noise may be either suppressed or enhanced in NDC regions. Experimentally, for the structure measured in,\(^{18}\) it was noted that the super-Poissonian peaks can be observed in only specific ranges of gate voltage.

The fact that a super-Poissonian noise is not necessarily accompanied by an NDC has been claimed by Song et al.\(^{14}\) and by Safonov et al.\(^{15}\). Comparing the I-V curves and the noises measured in a super-lattice diode and in a RTD, Song et al. concluded that not NDC, but charge accumulation in the well, responds for the super-Poissonian noise observed in RTD. Safonov et al., measuring the noise in resonant tunneling via interacting localized states, observed a super-Poissonian noise in the range of bias, where there is no NDC. They have also pointed out that the effect on noise of the Pauli exclusion principle and the Coulomb interaction are similar in most mesoscopic systems. For our structure of study, in solving the ME, we are able to exactly analyze the charge states of the dot and the box at bias voltages, where the super-Poissonian peaks are observed. Studies strongly support the idea\(^{16}\) that the charge accumulation in the dot causes the super-Poissonian noise observed.

In conclusion, we have calculated the current and the SN in a SET capacitively coupled to an electronic box, using the ME approach. In a particular case we were able to derive exact expressions for the I-V characteristics as well as the noise as a function of both frequency and bias voltage. For different varieties of structure parameters, including the gate, in a large range of bias voltage the calculation has been performed numerically. The obtained results show that the noise may be sub-Poissonian or strongly super-Poissonian, depending mainly on the box parameters and the gate. The super-Poissonian noise observed in the structure is not necessarily accompanied by an NDC. The study supports the idea that not NDC, but charge accumulation in the dot, responds for the super-Poissonian noise observed. Such an accumulation is accelerated by charge states in the box.

This work is in part supported by the Natural Science Council of Vietnam under grant 410901.

\(^{1}\) H. Bir, M.J.M. de Jong, and C. Schönenberger, Phys. Rev. Lett. 75, 1610 (1995)
\(^{2}\) G. Iannaccone, G. Lombardi, M. Macucci, and B. Pellegrini, Phys. Rev. Lett. 85, 3596 (1998)
\(^{3}\) V.V. Kuznetsov, E.E. Mendez, J.D. Bruno, and J.T. Pham, Phys. Rev. B 58, R10159 (1998)
\(^{4}\) A. Nauen, I. Hapke-Wurst, F. Hohls, U. Zeitler, R.J. Haug, and K. Pierz, Phys. rev. B 66, R161303 (2002)
\(^{5}\) M. Gattobigio, G. Iannaccone, and M. Macucci, Phys. Rev. B 65, 115337 (2002)
\(^{6}\) W. Song, E.E. Mendez, V. Kuznetsov, and B. Nielsen, Appl. Phys. Lett. 82, 1568 (2003)
\(^{7}\) Safonov, A.K. Savchenko, D.A. Bagrets, O.N. Jouravlev, Y.V. Nazarov E.H. Linfield, and D.A. Ritchie, Phys. Rev. Lett. 91, 36801 (2003)
\(^{8}\) V.Ya. Aleshkin, L. Reggiani, N.V. Alkeev, V.E. Lyubchenko, C.N. Ironside, J.M.L. Figueiredo, and C.R. Stanley, Semicond. Sci. Technol. 18, L1 (2003); 19, 665 (2004)
\(^{9}\) A. Nauen, F. Hohls, N. Maire, K. Pierz, and R.J. Haug, Phys. Rev. B 70, 033305 (2004)
\(^{10}\) S. Hershfield, J.H. Davies, P. Hyldgaard, C.J. Stanton, and J.W. Wikins, Phys. Rev. B 47, 1967 (1993)
\(^{11}\) A.N. Korotkov, Phys. Rev. B 49, 10381 (1994)
\(^{12}\) Ya.M. Blanter and M. Büttiker, Phys. Rev. B 59, 10217 (1999)
\(^{13}\) Ya.M. Blanter and M. Büttiker, Phys. Rep. 336, 1 (2000)
\(^{14}\) D.A. Bagrets and Yu.V. Nazarov, Phys. Rev. B 67, 085316 (2003)
\(^{15}\) X. Oriols, A. Trois, and G. Blouin, Appl. Phys. Lett. 85, 3596 (2004)
\(^{16}\) V. Hung Nguyen, V. Lien Nguyen, and P. Dollfus, Appl. Phys. Lett. 87, 123107 (2005)
\(^{17}\) C.P. Heij, D.C. Dixon, P. Hadley, and J.E. Mooij, Appl. Phys. Lett. 74, 1042 (1999)
\(^{18}\) A. Blanter and K.K. Likharev, *Mesoscopic Phenomena in Solids*, ed. B.I. Altshuler, P.A. Lee, and R.A. Webb (Amsterdam, Elsevier, 1991), p.173
\(^{19}\) The expression (4) with factors \( g \), indicated can be easily derived from the work done by voltage source considering the individual currents \( I(x) \).
\(^{20}\) V. Hung Nguyen, V. Lien Nguyen, and H. Nam Nguyen, J. Phys.: Condens. Matter 17, 1157 (2005)
\(^{21}\) T. Novotný, A. Donarini, C. Flindt, A.-P. Jauho, Phys. Rev. Lett. 92, 248302 (2004)