A lattice study of $\Lambda_b$ semileptonic decay

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We present results from a lattice study of the semileptonic decay $\Lambda_b \to \Lambda_c \ell \nu$. We use $O(a^2, \alpha_s a^2)$ improved quenched lattices of the MILC collaboration \cite{2}. These are $O(\alpha_s a^2)$ improved $20^3 \times 64$ lattices, with $a^{-1} = 1.33$ GeV, as determined from $m_{p}$. We use three light quark masses near the strange quark mass, $\kappa_f = 0.1343, 0.1333, 0.1323$. Two heavy quark $\kappa$ values, 0.104 and 0.114 bracket the charm quark, and other two, 0.064 and 0.077 bracket the bottom. We use the clover action for the valence quarks, with a tadpole improved clover coefficient. The value for the tadpole improvement factor $u_0$ is taken from the Landau gauge fixed mean link. Fermilab formalism is used for the heavy quarks. Results are presented for 300 lattices for two-point functions, and 237 lattices for three-point functions.

1. INTRODUCTION

Current knowledge of the CKM matrix element $V_{cb}$ is derived from the mesonic decays $B \to D^{*} \ell \nu$ or $B \to D \ell \nu$. Experimental knowledge of the $\Lambda_b$ semileptonic decay can lead to an independent estimate of $V_{cb}$ if the effect of the strong interaction in the decay are understood, e.g., via lattice QCD. A first lattice study of the baryonic semileptonic decay was performed by the UKQCD collaboration \cite{1}. We report our initial results for the dominant form factors of this decay.

The semileptonic decay $\Lambda_Q \to \Lambda_Q' \ell \nu$ can be parametrized in terms of six form factors, $F_i$ and $G_i$, for $i = 1, 2, 3$.

\begin{equation}
\langle \Lambda_Q^{(r)}(v) | J_{\mu} | \Lambda_Q^{(r)}(v') \rangle = \bar{u}_Q^{(r)}(v) \gamma_{\mu} (F_1 - \gamma_5 G_1) + v_{\mu} (F_2 - \gamma_5 G_2) + v'_{\mu} (F_3 - \gamma_5 G_3) u_Q^{(r)}(v').
\end{equation}

Here $J_{\mu}$ is the weak current and $r,s$ are polarisation states of the baryons. Since both $\Lambda_b$ and $\Lambda_c$ are hadrons containing a single heavy quark, heavy quark effective theory (HQET) is applicable \cite{3}. Hence the matrix element is taken between baryons of a given velocity, and the form factors are functions of the scalar $\omega = v \cdot v'$. To leading order in HQET, the combinations $F_1 + v_0 F_2 + v'_0 F_3$ and $G_1$ involving the dominant form factors $F_1$ and $G_1$ can be written in terms of a single function, called the (baryonic) Isgur-Wise function, $\xi(\omega)$. This function is normalised at zero-recoil, $\xi(1) = 1$.

2. SIMULATION PARAMETERS

The simulations are performed on the Asqtad quenched lattices at $\beta = 8.00$ generated by the MILC collaboration \cite{2}. These are $O(\alpha_s a^2)$ improved $20^3 \times 64$ lattices, with $a^{-1} = 1.33$ GeV, as determined from $m_{p}$. We use three light quark masses near the strange quark mass, $\kappa_f = 0.1343, 0.1333, 0.1323$. Two heavy quark $\kappa$ values, 0.104 and 0.114 bracket the charm quark, and other two, 0.064 and 0.077 bracket the bottom. We use the clover action for the valence quarks, with a tadpole improved clover coefficient. The value for the tadpole improvement factor $u_0$ is taken from the Landau gauge fixed mean link. Fermilab formalism is used for the heavy quarks. Results are presented for 300 lattices for two-point functions, and 237 lattices for three-point functions.

3. TWO-POINT RESULTS

The dispersion relation is shown in Fig. 4. The fitted energy values agree very well with the expectation from the lattice dispersion relation. The chiral extrapolations for a fixed heavy quark mass are shown in Fig. 5. The baryon kinetic mass $M_2$ is estimated as $M_2 = M_1 + m_2 - m_1$. Where $M_2(1)$ and $m_2(1)$ are the baryon and heavy quark kinetic(rest) masses respectively. We use
3.2 \quad 3.4 \quad 3.6 \quad 3.8 \quad 4.0 \quad 4.2 \quad 4.4 \quad 4.6

0 \quad 0.5 \quad 1 \quad 1.5 \quad 2

Figure 1. Dispersion relation for $\kappa_h=0.114$, $\kappa_l=0.1323/0.1323$. The $E$ here is $E_1$, the parameter obtained from exponential fits. The line shows the lattice dispersion relation.

4. THREE-POINT RESULTS

Different form factors contribute to different matrix elements in the three-point function. For $\mu=0$, the dominant contribution to three-point functions comes from the vector form factors and for $\mu=i$, axial-vector form factor $G_1$ gives the dominant contribution. We present results for the Isgur-Wise function from vector as well as axial-vector data. $\Lambda_b$ is created at time 0 and $\Lambda_c$ is annihilated at time $t_x \equiv 16$ in lattice units. The time at which the current acts is varied, and we study three-point function as a function of this time $t_y \equiv t$. For the results presented here, the initial baryon is at rest and the final baryon is moving with different velocities giving different values for $\omega$. On the lattice, one is restricted to region near $\omega = 1$ as data starts getting noisy for high momenta. In this region, $\nu_0$ can be approximated by 1. Then for an initial baryon of mass $M'$ decaying to a final baryon of mass $M$ moving with a momentum $\vec{q}$, if we consider the sum of the co-efficients of $I$ and $\gamma_0$, for large $t_y$ and $t_x - t_y$, the three-point expression simplifies to

a linear fit for these extrapolations. In Fig. 2, we have shown the chirally extrapolated baryon mass as a function of the heavy quark mass, along with the corresponding meson masses taken from the MILC collaboration. Our values for $m_{\Lambda_b}$ and $m_{\Lambda_c}$ are 5.626(36)GeV and 2.300(27)GeV.

Figure 2. Chiral extrapolations of the measured heavy baryon masses to the $u$ quark. We have used the light quark kinetic mass $m_2$ for the fit.

Figure 3. The heavy baryon mass, plotted as a function of the heavy quark mass. Also shown are the heavy-light meson masses, taken from studies of the MILC collaboration. The bursts correspond to the $b$ and $c$ quark.

Figure 4. Isgur-Wise function from the vector current. $\kappa_{h1}$ is 0.114 for all these points, and the points corresponding to four different $\kappa_h$ are shown with four different symbols.
Figure 5. Isgur-Wise function from the axial-vector current. As before, \( \kappa_{h1} = 0.114 \) for all these points, and the points corresponding to four different \( \kappa_h \) are shown with four different symbols.

\[
C(t_\eta) = \frac{Z_l Z'_s(|\vec{q}|)}{16 M'E} e^{-t_\eta M'(E+M')} (F_1(\omega) + F_2(\omega) + F_3(\omega))/(E + M),
\]

where \( Z_l \) and \( Z'_s \) are known from the two-point functions. We fit this to a form \( A e^{-Bt} \) and consider the ratio

\[
\frac{A([M',0] \rightarrow (M,\vec{q})]}{A([M',0] \rightarrow (M,0))} = \left( \frac{F_1(\omega) + F_2(\omega) + F_3(\omega)}{F_1(1) + F_2(1) + F_3(1)} \right) \cdot \left( \frac{Z'_s(|\vec{q}|)}{Z'_s(0)} \right) \cdot \frac{(E + M)}{2E}.
\]

(3)

First factor on the RHS is \(^2\) the Isgur-Wise function \( \xi_{QQ'} \). The second and third factors are known from the two-point functions. The third factor may be approximated by 1 to 0.5 per cent accuracy. The second factor differs from 1 by up to 10\% over our range of \( \vec{q} \). The ratio is independent of the renormalization constant \( Z_V \) because we have the same heavy quark transition in both numerator and denominator.

Our results for the Isgur-Wise function from the vector current are shown in Fig. 4. The Isgur-Wise function obtained from the axial-vector current (\( \mu = i \) case) is shown in Fig. 5. The Isgur-Wise function seems to be quite insensitive to the heavy quark mass.

We have also studied the light quark mass dependence of the Isgur-Wise function. The Isgur-Wise function is expected to fall slower for smaller light quark masses, by a heuristic argument. We do see such a trend in this preliminary study, but it is very far from clear with the statistical errors we have. This is shown in Fig. 6.

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