Research on Evaluation Method of Storage Reliability of a Control Cabin

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Abstract. A problem that is difficult to assess for storage cabin storage reliability. According to the historical data of the control cabin, the life distribution function was assumed, and the model parameters were estimated by the maximum likelihood estimation method and the minimum chi-square estimation method respectively, and different storage life distribution functions were obtained. Then compare and analyze different storage life distribution functions to determine the storage life distribution function of the control cabin. Finally, the Person's chi-square goodness-of-fit test was used to verify the results. The life distribution function of the control cabin is the I-type maximum value distribution, and the storage life is 18.12 years when the confidence is 0.90 and the reliability is 0.95. It provides a basis for subsequent life extension maintenance work and compares different storage reliability assessment methods.

Keywords: Control cabin; Storage reliability; Maximum likelihood estimation; Minimal chi-square estimate

1. Introduction
As the core component of precision guided munitions, the control cabin is the "brain and central nervous system" of precision guided munitions. Due to its typical characteristics of “long-term storage, one-time use”, accurate assessment of its storage life is of great significance for the reliable use of precision guided munitions. In recent years, the evaluation of the storage life of the control cabin had been a hot topic of research [1-2], but most of them were concentrated on the evaluation of the components [3-4], and the evaluation of the control cabin system was less. Based on the historical monitoring data accumulated during the long-term storage of the control cabin, this paper used the maximum likelihood estimation method and the minimum chi-square estimation method to evaluate its storage reliability and determine its life distribution function to provide a basis for subsequent life extension maintenance.

2. Historical data
As a special weapon, ammunition has the typical characteristics of "long-term storage, one-time use". After the ammunition was shipped from the factory, it usually needed to be stored in the warehouse for a period of time. Due to the influence of the storage environment, the performance indicators of its components would decrease.
In order to identify the quality changes of the control cabin at different stages of the storage period, it was necessary to periodically check the control cabin during storage. The data obtained by the test is the success or failure type data, and since the specific failure time was not known, the detection data of the control cabin is incomplete data. The actual test data is shown in Table 1:

**Tab. 1 Control cabin fault data**

| Storage month | Number of samples tested | Number of failures | Failure ratio |
|---------------|-------------------------|-------------------|--------------|
| 30            | X1                      | Y1                | 0.004242     |
| 33            | X2                      | Y2                | 0.001038     |
| 36            | X3                      | Y3                | 0            |
| 48            | X4                      | Y4                | 0            |
| 54            | X5                      | Y5                | 0.007568     |
| 59            | X6                      | Y6                | 0.003878     |
| 63            | X7                      | Y7                | 0.005051     |
| 74            | X8                      | Y8                | 0.004337     |
| 78            | X9                      | Y9                | 0.005119     |
| 81            | X10                     | Y10               | 0.001001     |
| 85            | X11                     | Y11               | 0.003846     |
| 88            | X12                     | Y12               | 0.011294     |
| 102           | X13                     | Y13               | 0.004114     |
| 113           | X14                     | Y14               | 0.022657     |

It can be seen from Table 1 that the actual fault detection data has zero failure rate at some detection time points. In order to save the calculation cost, the data shown in Table 2 can be obtained.

**Tab. 2 Finished control cabin fault data**

| Storage month | Number of samples tested | Number of failures | Failure ratio |
|---------------|-------------------------|-------------------|--------------|
| 31.4754       | X1                      | Y1                | 0.002961     |
| 53.42787      | X2                      | Y2                | 0.006846     |
| 60.98718      | X3                      | Y3                | 0.004461     |
| 76.1702       | X4                      | Y4                | 0.004761     |
| 84.56919      | X5                      | Y5                | 0.005616     |
| 102           | X6                      | Y6                | 0.004114     |
| 113           | X7                      | Y7                | 0.022657     |

The data type can be equivalent to:

Note $t = (t_1, \cdots, t_k)$ is the detection time point. After the storage time $t_i$, the $n_i$ control cabin was taken for testing. Among them, the $X_i$ failed, and the data is obtained:

$$ (t_i, n_i, X_i), \quad i = 1, 2, \cdots, k $$

Among them, $0 < t_1 < t_2 < \cdots < t_k$, each test is independent of each other.

3. Evaluation method

For the incomplete detection of fault data in the control cabin storage process, the reliability of the storage was evaluated by the maximum likelihood estimation method and the minimum chi-square estimation method. The maximum likelihood estimation method gives a conservative estimation of the distribution parameters and the reliable lifetime. The complete detection data is used in the evaluation process, without any deletion, and the distribution life of the control cabin is expected to be obtained under real conditions. The minimum chi-square estimation method combines the goodness-of-fit theory
to process the detection data, and gives the estimation of the distribution parameters and the reliable lifetime. At the same time, the distribution function is tested.

3.1. Distribution function model
Considering the physical failure mechanism of the control cabin, it is assumed that the possible forms of life distribution have exponential distribution, Weibull distribution, type I maximum value distribution and type II maximum value distribution. The distribution function expression is as follows:

Index distribution: \( F(t) = 1 - \exp(-\frac{t}{\theta}) \)

Weibull distribution: \( F(t) = 1 - \exp(-\frac{t}{\eta})^m \)

Type I maxima distribution: \( F(t) = \exp(-\exp(-\frac{t-\mu}{\sigma})) \)

Type II maximum value distribution: \( F(t) = \exp(-\frac{t}{\eta})^{-m} \)

3.2. Maximum likelihood estimation
The storage time is \( t_i \), the number of samples detected at this time is \( n_i \), and the number of failures is \( x_i \), then the likelihood function is:

\[
L = \prod_{i=1}^{k} F(t_i, \lambda_i^{\theta})^{n_i} \left(1 - F(t_i, \lambda_i^{\theta})\right)^{n_i - x_i}
\]

Among them, the exponential distribution \( \lambda = \theta \), the Weibull distribution \( \lambda = (\eta, m) \), the I-type maximum value distribution \( \lambda = (\mu, \sigma) \), and the type II maximum value distribution \( \lambda = (\eta, m) \).

By solving \( \frac{\partial \ln L}{\partial \lambda_i} = 0 \), the maximum likelihood estimate of the distribution parameters can be obtained.

3.3. Pearson goodness of fit test
Pearson proposed a chi-square statistic to test whether a set of independent samples belong to the same distribution family with specific properties. The variable sample composite Pearson chi-square statistic is of the form [5]:

\[
\chi^2(\lambda) = \sum_{i=1}^{k} \frac{(X_i - n_i p_i(\lambda))^2}{n_i p_i(\lambda)}
\]

When \( n_i \to \infty \), the limit distribution of \( \chi^2(\lambda) \) is the \( \chi^2 \) distribution with a degree of freedom \( k-1 \), that is, \( \chi^2(\lambda) \sim \chi^2_{k-1} \). When the test is performed, since the true value of \( \lambda \) is unknown, the estimated amount of \( \lambda \) is used instead of \( \lambda \) to calculate \( \chi^2(\lambda) \).

Given a significance level \( \alpha \), there are:

\[
P(\chi^2(\lambda) \geq \chi^2_{k-1}(1-\alpha) \mid H_0) \leq \alpha
\]

When \( \chi^2(\lambda) \geq \chi^2_{k-1}(1-\alpha) \) is established, a small probability event occurs, rejecting the null hypothesis.
3.4. Minimal chi-square estimation

The minimal chi-square estimation refers to the parameter $\hat{\lambda}$ obtained by minimizing the statistic as the best estimate of the true value $\lambda$, i.e.:

\[ \chi^2(\hat{\lambda}) = \min\{\chi^2(\lambda) : \lambda \in \Lambda\} \]

A very small chi-square estimate of $\hat{\lambda}$ can be obtained by solving the following equations.

\[
\frac{\partial \chi^2(\lambda)}{\partial \lambda_j} = \sum_{i=1}^{k} \left\{ -\frac{2n_i (X_i - n_i p_j(\lambda))}{n_i p_j(\lambda)} - \frac{(X_i - n_i p_j(\lambda))^2}{n_i p_j^2(\lambda)} \right\} \frac{\partial p_j(\lambda)}{\partial \lambda_j} \\
= \sum_{i=1}^{k} \left\{ \frac{-X_i^2 + n_i^2 p_j^2(\lambda)}{n_i p_j^2(\lambda)} \right\} \frac{\partial p_j(\lambda)}{\partial \lambda_j} \\
= \sum_{i=1}^{k} \left\{ 1 - \left( \frac{f_i}{p_j(\lambda)} \right)^2 \right\} \frac{n_i \cdot \partial p_j(\lambda)}{\partial \lambda_j}
\]

Therefore, the minimum chi-square estimate of $\hat{\lambda}$ is the solution of the following equation:

\[ \sum_{i=1}^{k} \left\{ 1 - \left( \frac{f_i}{p_j(\lambda)} \right)^2 \right\} \frac{n_i \cdot \partial p_j(\lambda)}{\partial \lambda_j} = 0, \quad j = 1, 2, \cdots, s \]

4. Storage life assessment

The maximum reliability estimation method and the minimum chi-square estimation method were used to evaluate the storage reliability of the control cabin based on the compiled control cabin fault detection data.

The results of the reliability assessment of the control cabin using the maximum likelihood estimation method are shown in Table 3:

| Distribution function                  | Estimated parameter | Reliable life |
|----------------------------------------|---------------------|---------------|
| Index distribution                     | $\theta=999.503$    | $t_{0.95}=51.2678$ |
| Weibull distribution                   | $\eta=3001.0626$    | $t_{0.95}=81.5877$ |
| $m=0.8239$                            |                     |               |
| Type I maxima distribution             | $\mu=59.0419$       | $t_{0.95}=23.5961$ |
| $\sigma=32.3060$                      |                     |               |
| Type II maximum value distribution     | $\eta=276497.2337$  | $t_{0.95}=202.6761$ |
| $m=0.1520$                            |                     |               |

Comparing the evaluation results in Table 3, it can be concluded that the storage reliability distribution function of the control cabin is a type I maximum value distribution, and the storage life of the control cabin at a confidence level of 0.90 and a reliability of 0.95 is determined to be 23.591 years.

The results of the evaluation of the reliability of the control cabin storage using the minimum chi-square estimation method are shown in Table 4:
Comparing the evaluation results in Table 4, it can be concluded that the storage life distribution function of the control cabin is a type I maximum value distribution, and the storage life of the control cabin at a confidence level of 0.90 and a reliability of 0.95 is 18.1170 years.

By comparing the evaluation results in Tables 3 and 4, it can be determined that the storage reliability of the control cabin obeys the type I maximum value distribution and explains the difference of different evaluation methods.

Finally, the Pearson goodness-of-fit test is carried out to verify the correctness of the minimum chi-square estimation method. A is taken as 0.05, and the goodness-of-fit p values of the four distributions are calculated respectively. The calculation results are shown in Table 5:

### Tab. 5 Goodness of fit p value

| Distribution function                  | Goodness of fit p value |
|----------------------------------------|-------------------------|
| Index distribution                     | 0.000479                |
| Weibull distribution                   | 0.000258                |
| Type I maxima distribution             | 0.001078                |
| Type II maximum value distribution     | 0.0001550               |

The higher the p value, the better the fitting effect. It can be seen from Table 5 that the fitting goodness p value of the type I maximum value distribution is the highest, which verifies that the storage reliability of the control cabin obeys the type I maximum value distribution. The storage life of the control cabin at a confidence level of 0.90 and a reliability of 0.95 was determined to be 18.1170 years.

### 5. Conclusion

According to the fault detection data of the control cabin, the storage reliability of the control cabin was evaluated by the maximum still estimation method and the minimum chi-square estimation method, and the life distribution function was determined as the type I maximum value distribution, and the results were verified by Pearson's goodness of fit theory. The method accurately estimates the storage life under the standard storage environment of the control cabin, determines the storage life distribution function, verifies the rationality of the method, and provides reference for other engineering applications.

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