Comparison of Modelling Tasks in Indonesian and Singaporean Mathematics Textbooks

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Abstract. The ability of students to apply mathematics has been regarded as an important goal of mathematics education in many countries, including Indonesia and Singapore. One of such abilities is the competence of mathematical modelling. Presenting students with mathematical modelling task is one of the efforts to improve their competence of mathematical modelling. This study aims to describe the characteristics of mathematical modelling task in Indonesian and Singaporean mathematics textbooks. This study used a qualitative approach with content analysis technique. The books analyzed in this study were an Indonesian mathematics textbook published by the government and a Singaporean mathematics textbook: Discovering Mathematics. Classification category for the modelling tasks is adapted from the classification made by Katja Maaß (2010). The results of analysis showed that there is no significant difference between Indonesian and Singaporean mathematics textbooks. Indonesian mathematics textbook contains contexts that are still small as well as Singaporean mathematics textbook. Therefore, more comprehensive research is needed with regard to curriculum content and learning strategies in Indonesia and Singapore.

1. Introduction

Education has an important role in developing individual competencies in the competitive era like today [1]. One of them is education in mathematics. The ability of students to apply mathematics has been regarded as an important goal of mathematics education in many countries, including Indonesia. One of the goals of mathematics education is that students have problem-solving skills which include the ability to understand problems, design mathematical models, complete models and interpret solutions obtained [2]. This means that the ability to solve problems involves the mathematical modelling process, which is understanding the problem, designing and completing mathematical models and interpreting the solutions obtained in order to solve problems in life (the real world). This indicates that the ability of students to apply mathematics to solve problems includes the modelling process. If students have passed the modelling process then it can be said that students have modelling competencies. Presenting students with mathematical modelling task is one of the efforts to improve their competence of mathematical modelling. Modelling task is a mathematical problem that involves the students in mathematical thinking, utilizing knowledge that has been learned and supporting their understanding of mathematical concepts that are currently being discussed (Dudley in [3]). Modelling tasks are intended to help students get a more positive attitude towards mathematics. Mathematical modelling will help students to understand and memorize mathematical content easily [4]. Therefore,
using modelling tasks in the textbooks can be used as a window to view students' mathematical experiences.

The Organization for Economic Cooperation and Development is one of the international organizations that organizes an international assessment, Programme for International Students Assessment (PISA), to measure mathematical literacy or students' ability to apply mathematics, which includes mathematical modelling. OECD released the 2015 PISA assessment results and reported that the performance of Indonesian students in mathematics was ranked 62 of the 69 countries involved, with an average score of 386 [5]. PISA questions had levels and each level had different levels of difficulty. Levels 5 and 6 expect students to work strategically using broad reasoning and formulate and communicate their actions and reflections regarding the interpretation, findings and suitability of the original situation. At level 5 and 6, the percentage of Indonesian students performance is still low at 0.7% (rank 60 of 69). This means that the small number of students reaching these two levels showed that the low ability of Indonesian students to do modelling for complex problems. This is different from the performance of Singaporean students who ranked far above Indonesia, with an average score of 564. When compared to the achievement of Singaporean students at level 5 and 6, the percentage of Singaporean students reaching both levels was very high, namely 34.8% (rank 1 from 69). Looking at the significant differences in modelling competencies between Indonesian and Singaporean students, it is based on the need for research on learning practices in both countries. Brewer and Stasz state that student learning opportunities are influenced by three dimensions, namely curriculum content, learning strategies used by teachers, and instructional resources used [6]. In this study will focus on instructional resources, in this case of textbooks namely mathematics textbooks.

Textbooks are the most commonly used learning resource in schools. Textbooks are one of the tools that can be used as students to obtain learning opportunities [7]. The importance of textbooks for learning is emphasized by the IES that places the textbooks as "potentially implemented curriculum" [8]. Textbooks are mediators of messages 'intended curriculum' that are set by the government with learning in class which is often seen as 'implemented curriculum'. Several studies [9] and [10] showed that the teachers tend to teach everything contained in textbooks and tend not to teach things that are not in the textbooks. Various studies show the relationship between textbooks and students' mathematical abilities. For example is the finding [7] which showed that the relationship between students' difficulties in solving contextual problems with the types of problems in textbooks. The same thing was found [11] which found that ability of students' problem solving were influenced by the types of questions in the textbooks they read.

The importance of textbooks as a learning resource in class has been mentioned in the regulations of the Indonesian government. Textbooks are the main learning resource for achieving basic competencies and core competencies and declared feasible by the Ministry of Education and Culture for use in education units. The textbook used as a mandatory reference by teachers and students in the learning process [12][13]. The existence of textbooks becomes the backbone of teaching and learning mathematics because how mathematics is compiled and presented in textbooks will influence how students build a mathematical idea.

Research on the analysis of textbooks has been widely carried out by previous researchers. Lisarani, Parta, and Chandra analyzed Indonesian and Singaporean mathematics textbooks by comparing the contextual feature of the worked examples in both books in each country [14]. The results of this study showed that 50% worked examples in Singaporean mathematics textbook are application worked example meanwhile only 16.7% those of Indonesian mathematics textbook. Meanwhile, Yang explained that Singaporean mathematics textbook emphasized all trigonometric concepts on righttriangle and further trigonometry (sine/cosine rules) while Indonesian mathematics textbook discussed more angles and concepts, trigonometry in righttriangles, and functions of trigonometric charts (which are more difficult than sine/cosine rules) [15]. In addition, the Singaporean mathematics textbook provides more mathematical questions that require a higher level of cognitive demand, while Indonesian mathematics textbook provides more questions that require a lower level of cognitive demand. This informs curriculum designers and/or textbook writers in
Indonesia, Singapore and other countries to consider and update the mathematics curriculum and/or math textbooks. Furthermore, Silalahi and Chang state that non-application problems are more used to convey knowledge to students before using applications related to student life [16]. Finally, a lot of suggestions are given to see some points of view, including the activities of teachers and students, namely how they will use mathematics textbooks in the classroom.

This study aims to describe the characteristics of mathematical modelling tasks in Indonesian and Singaporean mathematics textbooks. The task was analyzed using a modelling task framework adapted from Katja Maaβ [4]. Furthermore, this study extends previous research by analyzing Pythagoras presentations in Indonesian and Singaporean mathematics textbooks. In particular, it focused on the topic of modelling tasks on Pythagoras. Therefore, the research question is: how was Pythagoras discussed in the textbooks including worked example and exercise task from Pythagoras material in Indonesian and Singaporean mathematics textbooks?

2. Methods
This study used a qualitative approach with content analysis technique. The books analyzed in this study were an Indonesian mathematics textbook: Buku Siswa Matematika (BSM). This book was chosen as a representative of Indonesian mathematics textbooks because it is a compulsory book from the Indonesian government that is distributed nationally in the country. While Singapore, the chosen mathematics textbook was Discovering Mathematics (DM). This book was chosen as the representative textbook from Singapore because Discovering Mathematics (2nd Edition) is a series designed for students in secondary school and approved by the Singapore Ministry of Education for the latest secondary mathematics syllabus.

This study took a similar material focus from each textbook of the two countries, namely Pythagoras. Besides this material was chosen because Pythagoras was also closely related to the context of life, so the proportion of mathematical modelling tasks from Pythagorean material could be considered. Modelling tasks in this study focused on the worked example and exercise tasks that exist in Pythagoras chapter from the Indonesian and Singaporean mathematics textbooks.

The mathematics textbooks from the two countries are analyzed in two stages. In the first stage, modelling tasks in each textbook are grouped based on the presence or absence of the context in the modelling tasks. Furthermore, the second stage, modelling tasks that have the context would be analysed by using categories of the classification. This classification for modelling tasks is adapted from the classification made by Katja Maaβ [4]. This classification is focused on modelling activities needed to answer the worked example and exercise tasks. The following table 1 is the modelling task classification table.

| Type             | Name of the classification       | Categories of the classification |
|------------------|----------------------------------|---------------------------------|
| Worked example   | The focus on modelling activity  | Setting up the real model       |
| Exercise task    | Whole Understanding the situation| Mathematizing                   |
|                  |                                  | Working within mathematics      |
|                  |                                  | Interpreting                    |
|                  |                                  | Validating                      |

3. Results and Discussion
Based on the results of a comparative analysis of mathematics textbooks from Indonesian and Singaporean, the results in the following table are obtained. Based on Table 2, the results showed that almost all modelling tasks in the textbooks were identified as no-context modelling tasks, namely for worked example, 84.62% in BSM and 90% in DM while for exercise tasks, 88.14% in BSM and 81.67% in DM. Overall, the two textbooks each country has a small proportion of modelling tasks that contain context, i.e. 16 of 131 tasks (12.21%) in BSM and 23 of 130 tasks (17.7%) in DM.
Furthermore, from the results obtained in Table 2, context-modelling tasks were further analyzed to see the proportion of modelling tasks reviewed by modelling the required activities.

**Table 2. Frequency of no-context and context problem in textbooks.**

| Categories          | Textbooks                           | Indonesia (BSM) | Singapore (DM) |
|---------------------|-------------------------------------|-----------------|----------------|
|                     |                                     | n   | %      | n   | %      |
| **Worked Example**  |                                     |     |        |     |        |
| No-Context          |                                     | 11  | 84.62  | 9   | 90     |
| Context             |                                     | 2   | 15.38  | 1   | 10     |
| **Total**           |                                     | 13  | 100    | 10  | 100    |
| **Exercise Task**   |                                     |     |        |     |        |
| No-Context          |                                     | 104 | 88.14  | 98  | 81.67  |
| Context             |                                     | 14  | 11.86  | 22  | 18.33  |
| **Total**           |                                     | 118 | 100    | 120 | 100    |
| **All**             |                                     |     |        |     |        |
| No-Context          |                                     | 115 | 87.79  | 107 | 82.3   |
| Context             |                                     | 16  | 12.21  | 23  | 17.7   |
| **Total**           |                                     | 131 | 100    | 130 | 100    |

**Table 3. Frequency types of modelling activity of modelling task.**

| Categories          | Textbooks                           | Indonesia (BSM) | Singapore (DM) |
|---------------------|-------------------------------------|-----------------|----------------|
|                     |                                     | n   | %      | n   | %      |
| **Worked Example**  |                                     |     |        |     |        |
| Whole process       |                                     | 0   | 0      | 0   | 0      |
| Understanding the situation |               | 0   | 0      | 0   | 0      |
| Setting up the real model |                 | 0   | 0      | 0   | 0      |
| Mathematizing       |                                     | 0   | 0      | 0   | 0      |
| Working within mathematics |               | 0   | 0      | 0   | 0      |
| Interpreting        |                                     | 2   | 100    | 1   | 100    |
| Validating          |                                     | 0   | 0      | 0   | 0      |
| **Total**           |                                     | 2   | 100    | 1   | 100    |
| **Exercise Task**   |                                     |     |        |     |        |
| Whole process       |                                     | 0   | 0      | 1   | 4.55   |
| Understanding the situation |               | 2   | 14.29  | 0   | 0      |
| Setting up the real model |                 | 0   | 0      | 0   | 0      |
| Mathematizing       |                                     | 2   | 14.29  | 1   | 4.55   |
| Working within mathematics |               | 0   | 0      | 10  | 45.45  |
| Interpreting        |                                     | 8   | 57.13  | 8   | 36.36  |
| Validating          |                                     | 2   | 14.29  | 2   | 9.09   |
| **Total**           |                                     | 14  | 100    | 22  | 100    |
| **All**             |                                     |     |        |     |        |
| Whole process       |                                     | 0   | 0      | 1   | 4.35   |
| Understanding the situation |               | 2   | 12.5   | 0   | 0      |
| Setting up the real model |                 | 0   | 0      | 0   | 0      |
| Mathematizing       |                                     | 2   | 12.5   | 1   | 4.35   |
| Working within mathematics |               | 0   | 0      | 10  | 43.48  |
| Interpreting        |                                     | 10  | 62.5   | 9   | 39.13  |
| Validating          |                                     | 2   | 12.5   | 2   | 8.69   |
| **Total**           |                                     | 16  | 100    | 23  | 100    |

Based on Table 3, the results showed that almost all modelling tasks were identified as modelling tasks containing interpreting activities in Indonesian mathematics textbook, namely 100% worked
example and 57.13% exercise tasks. Overall, Indonesian mathematics textbook has a large proportion of modelling tasks that contain interpreting activity, namely 10 out of 16 tasks (62.5%) in BSM. While Singapore, 100% worked example contains interpreting and 45.45% exercise tasks contain working within mathematics. Overall, the Singaporean mathematics textbook has a large proportion of modelling tasks that contain working within mathematics activity, which is 10 out of 23 tasks (43.48%) in DM.

3.1. Modelling task in Buku Siswa Matematika
The Indonesian mathematics textbook begins with the presentation of material containing the real-world application of the Pythagorean Theorem. The number of modelling tasks in Indonesian students’ Mathematics textbook is 131 tasks, which consisted of 13 worked examples and 118 exercise tasks. Two of the 13 worked examples (15.38%) are context-modeling tasks and both are modelling tasks that focus on modelling activity, namely interpreting.

In the example of interpreting Figure 1, students asked to understand the problem situation first, which is to know the distance travelled and the average speed of the motorbike to find the travel time. From the problem, when students have obtained mathematical results in the form of travel time (in hours) i.e. 1.67 hours and 0.83 hours. Then students are expected to be able to interpret the mathematical results into minutes i.e. 70 minutes and 50 minutes, so that the mathematical results obtained are accordance with the context of the question being asked, which is how much time Wachid takes between picking up Dani and going directly to the beach, i.e. 70 - 50 = 20 minutes.

![Figure 1. Worked example that required students to make interpretation (translated version).](image-url)
exercise tasks that contain other modelling activities, namely understanding the situation, mathematizing, and validating each of the two tasks (14.29%).

In the example of exercise task number 2 in Figure 2, the problem expects students to arrive at the interpreting stage. However, to achieve this, students have been assisted with the first question 2(a) for understanding the real situation first, that is students asked to think of ways in determining the length of the lathe wire that supports the telephone pole. Students who understand the problem situation mean that students know the length of the lathe wire that asked is the hypotenuse of the right triangle formed. By using the Pythagorean theorem, students can calculate the length of the hypotenuse. However, from problem 2(a) there is still information has not been given, namely the distance between the wire and the pole on the ground. Next in problem 2(b), the information has been given, so students should be able to calculate the length of the hypotenuse of a right triangle. After students have the mathematical results obtained, students can reinterpret the results that the length of the hypotenuse is long wire.

![Figure 2. Exercise task that required students to make interpretation (translated version).](image1)

![Figure 3. Exercise task that required students to make interpretation (translated version).](image2)

In the exercise task number 5 in Figure 3, question 5(b) expects students to reach the interpreting stage. But before, question 5(a) asked students to draw a problem situation from the task by using a Cartesian field. This is a mathematizing process. Problems with this category can determine the real model and make a mathematical model. In addition, graphic solutions are also one of the solutions to this task [4]. Student answers in the form of graphs made in the Cartesian field are the solution to this task. After the student has successfully created an image in the Cartesian field, students can calculate the mathematical results of question 5(b) by referring to the picture. Until finally students can interpret the mathematical results that they have obtained to real solutions.

In the exercise task number 6 in Figure 4, the problem involves students to validate or check the answers they get. The question asks students to check whether the window frame is rectangular. This means that because the length of the two sides of the window is known, students can calculate the diagonal length of the window. Then the students check whether the diagonal length obtained is equal to the diagonal length given by the problem.

![Figure 4. Exercise task with validating (translated version).](image3)

3.2. Modelling task in Discovering Mathematics

The number of modelling tasks in Discovering Mathematics is 130 tasks, which consisted of 10 worked example and 120 exercise tasks. Nine of 10 worked examples (90%) are modelling tasks that have no context and only 1 worked example (10%) has context. One task of context-worked example is modelling tasks that have a focus on modelling activity, namely interpreting.
In the interpreting-worked example in Figure 5, it is almost the same as Indonesian mathematics textbook, but the question in Singaporean mathematics textbook is simpler. The length of the two sides righttriangle has been given so that students can directly calculate the hypotenuse (AB). After finding the mathematical solution, students expected to be able to interpret the hypotenuse is the length of the wire that asked for the problem.

**Figure 5.** Worked example that required students to make interpretation.

**Figure 6.** Exercise task with working within mathematics.

In addition to the worked example, exercise tasks are also presented in Singaporean mathematics textbook. The number of exercise tasks in DM is 120 tasks. Twenty two of 120 exercise tasks (18.33%) are contextual modelling tasks and the remaining, 81.67%, are no-context exercise tasks. Furthermore, from 22 exercise tasks that have a context, unlike Indonesia, the most proportion of modelling tasks in Singaporean mathematics textbook is modelling task with working within mathematics, i.e. 10 tasks (45.45%), and the number of exercise tasks that contain other modelling activities, i.e. 1 task (4.55%) mathematizing, 8 tasks (36.36%) interpreting, and 2 tasks (9.09%) validating. When compared to Indonesian mathematics textbook, Singaporean mathematics textbook has 1 task that requires whole process-modelling activity, while Indonesia does not exist.

In the exercise task Figure 6, the problem initially uses the context (wooden gate) in presenting the problem. But students have been given the form of a real picture model that has shown the righttriangle formed. The question asked students to calculate the length of KM as the length of the hypotenuse. This means that students who have obtained the results of calculations and find the length of KM, the students have completed until the stage of working within mathematics, without the need to continue the interpretation of the results into real problems.

**Figure 7.** Exercise task with mathematizing.

**Figure 8.** Exercise task with validating.

In the exercise task Figure 7, especially 7(a) is a problem that expects students to create mathematical models based on the given problem situation. Creating a mathematical model of a given problem is a modelling task that is categorized into mathematizing. An algebraical form that contains variable x is the right solution. In the exercise task Figure 8, the task asked students to achieve validating modelling activity. Students are expected to check that $a^2 + c^2 = b^2 + d^2$. For some students, to prove the problem is not an easy matter because it requires the critical thinking ability to see the relationship in the picture. Maαβ added that the category of validating modelling task asked students to think critically for mathematical form and provide reasons for their assessment [4].
4. Conclusions

Modelling tasks presented in Indonesian and Singaporean mathematics textbooks can be seen from the worked example and exercise tasks given in both textbooks. Based on the results of the textbooks analysis from two countries, apparently, there were no significant differences between Indonesian and Singaporean mathematics textbooks. Indonesian mathematics textbook contains contexts that are still the same as Singaporean mathematics textbooks. This study explained that the tasks in textbooks from both countries still tended to be insufficient for students to be actively involved in the mathematical modelling process. This can be seen from the proportion of no-context modelling tasks in each country's textbooks is very large. Based on research [11], the types of questions available in textbooks that students read will affect students' problem solving ability. But in reality, the problems in textbooks that are still largely no-context will also affect students' problem solving ability.

Therefore, more comprehensive research is needed with regard to curriculum content and learning strategies in Indonesia and Singapore. This is in line with Brewer and Stasz which say that student learning opportunities are not only influenced by instructional resources, but the curriculum content and learning strategies that used by the teacher also influence it [6]. In addition, Wijaya and Heuvelpanhuizen also mentioned that the learning strategies used by teachers are also important aspects that influence student achievement [17].

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