Examination of a repetitive process control system

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Abstract: In the present work, the application of repetitive filters in the robust process control systems is examined. The functionality of the proposed system and the improved performance, robust performance and filtering properties has been proven.

1. Introduction

By increasing the performance requirements of the modern technological process control systems, there is considerable interest in the classes of non-stationary control systems in the conditions of a priori uncertainty and unmodulated dynamics, some of which are the following known classes of systems: robust systems; systems with parametric compensation; systems with interference absorption; systems with reduced sensitivity; systems with variable structure; extreme systems; repetitive systems; intelligent control systems (using neural networks, fuzzy multi-theory, genetic algorithms, chaos theory, etc.).

There are differences between them in structural and parametric terms. They are predominantly governed by the class of control objectives, the type of control and interfering objective, and the controlled objective. In the case where the parametric disturbances in the objects are dominant and there are periodic external disturbances whose nominal parameters are known in advance but which fluctuate operationally around the nominal ones and one of the main requirements for the performance of the system is the resilience and the robust resilience of the system, application of robust repetitive systems is recommended as an effective.

In this context, the aim of the current work is to study the properties and applicability of the repetitive filters in robust systems and systems with robust properties. [1].

2. Robust systems and systems with robust properties.

In literature are known systems with internal model RMM [1-3], systems with interference absorption DAS [1], and gain scheduling control system GSC [1], as well as the methods and the algorithms for their synthesis.

The free parameter method solves the task of analytical synthesis of robust systems RMM (fig.1) with an internal model for controlled objects in the conditions of visibility uncertainty, under the criteria: (1): robust stability and robust performance of the system for a predetermined in the synthesis parameter set $II$, satisfying the requirements (1.a); local criteria for quality $LCQ$ (1.b) – process with minimal integral squared error.

$$Q = Q(Q^*, F, p)$$

$$R_{,u,1}(p) = \frac{Q(p)}{1-G^*(p)Q(p)}$$

and initial conditions for synthesis – a priori know $G^*$, $\Pi$, $\sigma$, where: $Q^*(p)$ classic regulator (2) with filte $F, (p)$ (4) [2], corresponding to a nominal model of the controlled object $G$ by the set LCQ: $R_{,u,1}(p)$ (3) – robust regulator with an internal model of the object $G^*$. Every $l$ element or $f, (p)$ (4) should meet the requirements (5), where $\pi$ are the poles $G^*$ in the right half-space.

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frequency of the closed, a representative trend of the system error in the structure 
(5.a) 
\[
\frac{d}{dp} f_j(p) |_{p_0} = 0, \quad (j = 1, \ldots, m) 
\]  
(5.b) 
The method of the equilibrium partial absorption equation [2] solves the task for analytical synthesis of systems (fig.2), absorbing the influence of wave structured disturbances on the adjustable magnetite, for controlled objects under a priori uncertainty, under the criteria (6): minimum Euclidean norm (6.a) of the systems (fig.3) has two additive constituents: 
- frequency of the closed process control, modeled using the nominal model 
- transmitting fluent of the closed, top-class classical system.

\[ A(p) = k^+ Q^+ (p) \] 
(9)

\[ k^+ = \frac{\int R \cdot (j \omega_{class}^+) \cdot G^+ (j \omega_{class}^+) \cdot Q_+ (j \omega_{class}^+) - 1}{\int R \cdot (j \omega_{class}^+) \cdot G^+ (j \omega_{class}^+) \cdot Q_+ (j \omega_{class}^+) \cdot \omega_{class}^+} \] 
(10)

The method of the compensation equation of the parametric balance [1] solves the task of analytical synthesis of gain scheduling control system with combined G(C) (fig.3) parametric compensation for controlled objects under a priori uncertainty, under the criteria (12): constant value (corresponding to optimal system setup under \( \sigma \)) of the transmission coefficient \( k_{\omega^+} \) of the regulatory body (12.a); constant value (corresponding to optimal system setup under \( \sigma \)) of the transmission coefficient \( k_{CLASS} \) of the system (12.b); LCQ (12.c) (process with reordering set \( \sigma = const \) etc.)

\[ y = C x + E \begin{pmatrix} u_+ \ u_+ \end{pmatrix} + F \xi \] 
(7)

Corresponding to the disturbance absorbing system (fig.2) “the classical” parametric disturbed system is with regulator \( R^+ \) in process control, modeled using the \( G \{ p, \xi \} \) in real operational conditions (or with adequate computer simulation). \( y^+, u^+ \) are signal external interference aggregated on the assignment channel \( y^+ \) and/or the loading \( u_+ \) and \( \xi^+ \) are aggregated internal additive or multiplicative parametric and/or structural disturbances with a wave structure in process control. Analytical design of the absorber \( A \) in the structure (fig.2) of the system is according (9), where the fluent \( Q^+ (p) \) is determined by a known methodology [1], and the optimum gain value \( k^+ \) is determined based on (10) or (11). \( \omega_{class}^+ \) - frequency of the closed classical system, \( \omega_{class}^+ \) - frequency of the closed, disrupted, interference-absorbing system; \( G^+ \) - a model of disruption at the highest level process control; \( \Phi_{class}^+ \) - transmitting fluent of the closed, top-class classical system.

Fig.2. Disturbance absorbing system

Analytical design of the absorber \( A \) in the structure (fig.2) of the system is according (9), where the fluent \( Q^+ (p) \) is determined by a known methodology [1], and the optimum gain value \( k^+ \) is determined based on (10) or (11). \( \omega_{class}^+ \) - frequency of the closed classical system, \( \omega_{class}^+ \) - frequency of the closed, disrupted, interference-absorbing system; \( G^+ \) - a model of disruption at the highest level process control; \( \Phi_{class}^+ \) - transmitting fluent of the closed, top-class classical system.

\[ y^+(p) \] 
\[ A(p) = k^+ Q^+ (p) \] 
(9)

\[ k^+ = \frac{\int R \cdot (j \omega_{class}^+) \cdot G^+ (j \omega_{class}^+) \cdot Q_+ (j \omega_{class}^+) - 1}{\int R \cdot (j \omega_{class}^+) \cdot G^+ (j \omega_{class}^+) \cdot Q_+ (j \omega_{class}^+) \cdot \omega_{class}^+} \] 
(10)

The method of the compensation equation of the parametric balance [1] solves the task of analytical synthesis of gain scheduling control system with combined G(C) (fig.3) parametric compensation for controlled objects under a priori uncertainty, under the criteria (12): constant value (corresponding to optimal system setup under \( \sigma \)) of the transmission coefficient \( k_{\omega^+} \) of the regulatory body (12.a); constant value (corresponding to optimal system setup under \( \sigma \)) of the transmission coefficient \( k_{CLASS} \) of the system (12.b); LCQ (12.c) (process with reordering set \( \sigma = const \) etc.)

Initial conditions for synthesis: a priori known \( \sigma \), type of regulator, where: \( R^* (p) \leftrightarrow G^* (p) \), \( l = l^+, s = s^+ \); \( y^+ \), \( s \) - are signal external interference aggregated on the assignment channel \( y^+ \) and/or
loading $s$, and $\xi$ are aggregated internal additive or multiplicative parametric and/or structural interferences in process control; $l, y, q, r$ - position of the valve control system $PO$; adjustable magnitude and cost of output at $PO$; $\ell_{\ell^{(\xi)}}, l_{\ell^{(\xi)}}, \ell_{\ell^{(\xi)}}$ - compensation variables; $\ell_{\ell^{(\omega)}}, \ell_{\ell^{(\omega)}}$ - compensation variables for $PO$ with equal and linear flaw characteristics; $\nu_s$, (13) - $V$ - loading changes.

$$
\begin{aligned}
\n\n\begin{cases}
\n\mathcal{L}_{x^{(\xi)}}, k_{\xi}, \mathcal{L}_s, \\
\mathcal{L}_{x^{(\xi)}}, a_{x^{(\xi)}}, \mathcal{L}_s,
\end{cases}
\end{aligned}
$$

(13)

FIG.4. Gain scheduling control system

3. Repetitive control and ML filters.

The repetitive control is effectively applied in systems for controlled technological objects characterized by periodic signal interference [4,5]. In the structure of the repetitive regulator, the internal model in the ML (Memory Loop) [6-11] $M_{\lambda}(p)$ (fig.4 and fig.6) is used, which consists of a periodic signal generator allowing compensation of periodic disturbances with a predetermined and fixed value of the period $T_r$. The base ML has the ability to "memorize" the frequency of "cut-off" and effectively to counteract it, through its specific structure as a dynamic system. There are dependency bridges for it (14)-(16):

$$
\begin{aligned}
\n\n\begin{cases}
\n\xi^\ast(p) = \xi(p) + \xi^\ast(p) \\
M_{\lambda}(p) = \xi^\ast(p) / \xi(p) = 1 / 1 - e^{-r_t}, \\
\xi^\ast(p) = M_{\lambda}(p) \xi(p) = \left(1 - e^{-r_t}\right) \xi(p)
\end{cases}
\end{aligned}
$$

FIG.4. Base ML filter

FIG.5. Improved ML filter

In the literature is well-known and improved [9] ML (fig.5). It differs from the basic one [6-11] because it uses a model of delay, but in a different structure. Unlike basic one (17), the improved ML-filter (19) is a sustainable dynamic system.

$$
\begin{aligned}
\n\n\begin{cases}
\n\xi^\ast(p) = \xi(p) & (17) \\
\xi^\ast(p) = \frac{1}{2 - e^{-r_t}} & (18) \\
M_{\lambda}(p) = \frac{\xi^\ast(p)}{\xi(p)} = \frac{1}{2 - e^{-r_t}} & (19)
\end{cases}
\end{aligned}
$$

The values of the regulator setting parameters in the controlled technological objects systems are subject to the aim and the criteria presented to the system in its synthesis. They are not a function of the respective ML-filter, which is added to the regulator. In this sense, the design of the ML-filter in the control systems is autonomous and is not related to the synthesis of the regulator. For designing ML-filters with memory is used "rejecting module bandpass filter criterion". The essence of this method is a sequential frequency correction. The design filter criterion is defined by (20).

$$
\begin{aligned}
\n\mathcal{M}_{\lambda_{\omega}}(j \omega) = \left\{ \begin{array}{ll}
0 & \forall \omega \in \Delta \omega, \\
\omega_{\lambda_{\omega}} < \omega < \omega_{\lambda_{\omega}} & \forall \omega \in [0, \omega_{\lambda_{\omega}}], \\
1 & \forall \omega \in [\omega_{\lambda_{\omega}}, \infty),
\end{array}ight.
\end{aligned}
$$

4. Robust repetitive control systems with internal model. Synthesis.

1. They are set below (chart 1): nominal model $G^\ast(p)$ of a minimal-phase object $G(p)$; an embarrassed object (model) at the top $G^\ast(p)$; synthesis criteria (1.a); period $T_r$ of constant intermittent disturbance $d = \sin (\omega_r t) = \text{const}$. 

2. It is necessary to synthesize: robust $R_p$ regulator (free parameter method); repetitive ML-filter (19), (20); compensation variables $\nu_{\ell^{(\xi)}}, \nu_{\ell^{(\omega)}}$ (13) (method of the parametric balance equation); absorber $A^{-\omega}(p)$ of the wave structure disturbances (equilibrium partial absorption of the disturbances method).

Based on the synthesis methods and the advanced ML-filter described above, the configuration of robust repetitive systems is based on development in the structure (fig.1) using the principle of consistent correction of
\[ R_u \text{ with } MC \text{-filter, compensation variables } V_{\xi}^{**}, \]
\[ V_{a}^{**} \text{ and/or absorber } A^{-}(p) \text{. In fact, four completely new systems are illustrated respectively:} \]
- in fig.7 – robust repetitive internal model control \( RMREP \), where the controlled algorithm \( R_u^{RM} (p) \) is presented analytically with (21), and the results of the synthesis are plotted in table 1;
- in fig.8 – robust stabilization repetitive \( RMSCREP \), where the controlled algorithm \( R_u^{RMSCREP} (p) \) is presented analytically with (22), and the results of the synthesis are plotted in table 1;
- in fig.9 – robust repetitive disturbance absorbing \( RMDASREP \), where the controlled algorithm \( R_u^{RMDASREP} (p) \) is presented analytically with (23), and the results of the synthesis are plotted in table 1;
- in fig.10 – robust stabilization repetitive disturbance absorbing \( RMSCDASREP \), where the controlled algorithm \( R_u^{RMSCDASREP} (p) \) is presented analytically with (24), and the results of the synthesis are plotted in table 1.

| Table 1. Results of the synthesis |
|----------------------------------|
| **initial conditions of synthesis** | **G*(p)** | **G*(p)** | **control valves CV** |
| ---------------------------------|---------------|---------------|---------------------|
| permanent intermittent disturbance | \( T_s = 100 \text{ s} \) \( d = \sin (\omega_r \ t) = \sin (0.0628 \ t) = \text{const} \) | \( \frac{1,25 e^{-10}}{10 p + 1} \) | equal percent control valves ECV \( \omega_r = 0.0628 \text{ rad/s ; } T_s = 100 \text{ s} \) |
| system synthesis results | \( R_u (p) \) | \( 1.25 \left( \frac{1 + 0.909 p + \frac{1}{11 p}}{10 p + 1} \right) \) | \( 0.08666 \text{ p}^2 + 5p + 1 \) |
| compensation variable | \( V_{\xi}^{**} \) | \( 0.5 \left( e^{a^{(i+1)}} - 1 \right) \) | \( c^{(i)} \) | |
| \( A^{-}(p) \) | | | |

\[ R_u^{RM} (p) = MC \left( R_u + \frac{R_u'}{p} = G^* \right) \] (21)

\[ R_u^{RMSCREP} (p) = MC \left( R_u + \frac{R_u'}{p} = G^* \right) \] (22)

\[ R_u^{RMSCDASREP} (p) = MC \left( R_u + \frac{R_u'}{p} = G^* \right) \] (23)

\[ R_u^{RMSCDASREP} (p) = MC \left( R_u + \frac{R_u'}{p} = G^* \right) \] (24)

5. Analysis and quality assessment. Comparative analysis.

The four synthesized systems are modeled and simulated under the same conditions. The simulation model results- time and frequency characteristics for closed \( \Phi, \) and for open \( W \) control systems are shown in fig.11 and fig.12. The system features are visualized in nominal parametric mode \( (G = G^*, \ d = 0, \ fig.11) \) and in perturbation mode on the upper limit \( (G = G^*, \ d = 0, \ fig.12) \) without the presence of a permanent intermittent external signal disturbance \( d \). The following symbols
are used: \( h_\phi \) - transient functions, \( \phi (j\omega) \), \( W(j\omega) \) -
frequency characteristics of closed and open systems.

Quality analysis [1,12] in nominal parametric mode of
the systems (fig.11) confirms that all four systems meet
LCQ, reflecting the adjustment times (fig.11.a) and their
stocks by module and phase (fig.11.b).

Quality analysis in perturbation mode on the upper limit
systems (fig.12) confirms that systems remain stable, re-
reflecting the times regulation (fig.12.a) and their gain
margin and phase margin (fig.12.b).

![Time characteristics - nominal systems](image1)

**Fig.11.a.** Time characteristics - nominal systems

![Frequency characteristics - nominal systems](image2)

**Fig.11.b.** Frequency characteristics - nominal systems

![Robust analysis of the characteristics of the open
loop systems](image3)

**Fig.13.a.** Robust analysis of the characteristics of the open
loop systems

![Time analysis of the filtering properties systems](image4)

**Fig.14.** Time analysis of the filtering properties systems

6. Robust analysis.

By definition, the system analyzed is: robust stability
and robust performance [1] for the set \( \Pi \) (1.a), if it
meets the requirements (29) u (31) for the entire fre-
quency range. If the nominal \( W^{\phi} \) and the perturbed
\( W^{\phi,0} \) are open systems (25), so the circle \( \pi \) (26) with
radius \( r^\pi \) (27) and with centers in points \( \omega_s \), depicting
the uncertainty in the process control, so the
requirements for the robust stability and robust performance can also be expressed in the characteristics of an open system with (28) and respectively with (30).

\[ W' = R, G^*; \] \[ W'' = R, G'' \] \hspace{1cm} (25)

\[ \pi(j\omega) \in W(j\omega), (\forall \omega, \omega \in [0, \infty)) \] \hspace{1cm} (26)

\[ r'(\omega), \] \[ R(\omega), G^*(\omega) \] \hspace{1cm} (27)

\[ RS(\omega) \Rightarrow |1 + G^*(\omega)R(\omega)| > r'(\omega), \] \[ (\forall \omega, \omega \in [0, \infty)) \] \hspace{1cm} (28)

\[ RS(\omega) \Rightarrow \left| \eta^*(\omega) \tau^*_s(\omega) \right| < 1, \] \[ (\forall \omega, \omega \in [0, \infty)) \] \hspace{1cm} (29)

\[ \text{RP}(\omega) \Rightarrow \left| 1 + G(\omega)R(\omega) \right| > 1 + G^*(\omega)R(\omega) - r'(\omega), \] \[ (\forall \omega, \omega \in [0, \infty)) \] \hspace{1cm} (30)

\[ \text{RP}(\omega) \Rightarrow \left| \eta^*(\omega) \tau^*_s(\omega) + |\varepsilon^*(\omega)\nu(\omega)| \right| < 1, \] \[ (\forall \omega, \omega \in [0, \infty)) \] \hspace{1cm} (31)

The results (fig.13) from the robust analysis of the characteristics of the open loop \( W(j\omega) \) (fig.13.a) and close loop \( \Phi_j(j\omega) \) (fig.13.b-c) systems, analytically prove that under uncertainty \( \xi, \xi^0 \) (table 1): the repetitive systems (fig.7-fig.10) meet the requirements (28)-(31) both for robust stability and robust performance.

7. Time analysis

A time analysis of the filtering properties of the four (fig.7-fig.10) synthesized systems with "test" periodic effects was also carried out [11], by modeling the systems and evaluating the simulation results by time criteria. The four systems were modeled with nominal parametric mode. Their transitional functions \( h_1(t) \) of periodic impact \( d(t) \) (table 1) are simulated. The time characteristics of the systems as a result of their parallel simulation for a "test" frequency \( \omega_\text{s} = 0.0628 \text{ rad/s} \) are illustrated in fig.14 – under disturbance \( d(t) \) on the assignment channel \( y^* \) (fig.14.a), as well as in parallel along the three channels \( y^*, y \in f \) (fig.14.b).

The results of the study of the four systems with a time analysis of the robust filtering properties with respect to the quality evaluation for performance of the process confirm that the repetitive systems (fig.7, fig.8, fig.9, fig.10) have a reduced sensitivity to external harmonic impact \( d(t) \) with frequency \( \omega_\text{s} = 0.0628 \text{ rad/s} \).

Conclusion

The present study examines and analyzes the results of the research on the peculiarities and properties of the repetitive robust systems. Results of research are presented, which are confirmation and proof of their quality. The stability, performance, accuracy, robust performance and stability, filtering properties of external periodic effects of this class of systems are all proven. In comparative terms, the properties of robust repetitive (fig.7), robust stabilization repetitive (fig.8), robust repetitive disturbance absorbing (fig.9) and robust stabilization repetitive disturbance absorbing (fig.10) control systems of the process \( G \), described in table 1, using the methods for: ● Performance analysis in nominal and parametric mode of the systems (fig.11, fig.12); ● time analysis of the filtering properties of systems with "test" periodic effects (fig.14); ● robust analysis of the behavior of uncertainty systems \( \Pi, G, G^*, G''^{\xi}, G''^{\xi^0} \) of the object (fig.13).

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