Abstract

We generalize Cohen, Gómez-Rodríguez, and Satta’s (2011) parser to a family of non-projective transition-based dependency parsers allowing polynomial-time exact inference. This includes novel parsers with better coverage than Cohen et al. (2011), and even a variant that reduces time complexity to $O(n^6)$, improving on prior bounds. We hope that this piece of theoretical work inspires design of novel transition systems with better coverage and better run-time guarantees.

1 Introduction

Non-projective dependency trees are those containing crossing edges. They account for 12.59% of all training sentences in the annotated Universal Dependencies (UD) 2.1 data (Nivre et al., 2017), and more than 20% in each of 10 languages among the 54 in UD 2.1 with training treebanks. But modeling non-projectivity is computationally costly (McDonald and Satta, 2007).

Some transition-based dependency parsers have deduction systems that use dynamic programming to enable exact inference in polynomial time and space (Huang and Sagae, 2010; Kuhlmann et al., 2011). For non-projective parsing, though, the only tabularization of a transition-based parser is, to our knowledge, that of Cohen et al. (2011). They define a deduction system for (an isomorphic variant of) Attardi’s (2006) transition system, which covers a subset of non-projective trees. The exact inference algorithm runs in $O(n^7)$ time, where $n$ denotes sentence length.

In this paper, we show how Cohen et al.’s (2011) system can be modified to generate a new family of deduction systems with corresponding transition systems. In particular, we present three novel variants of the degree-2 Attardi parser, summarized in Fig. 1 (our technique can also be applied to generalized Attardi (2006) systems; see §3.2). The first two bring non-projective coverage for UD 2.1 to as high as 95.99% by adding extra transitions, and yet retain the same time complexity. The third reduces time complexity for exact inference to $O(n^6)$ and space complexity from $O(n^5)$ to $O(n^4)$, while still improving empirical coverage from 87.24% to 93.16%.¹

Code and full statistics for all treebanks can be found at https://github.com/tzshi/nonproj-dp-variants-naacl2018.

These theoretical improvements are a step towards making recent state-of-the-art results in transition-based parsing with exact inference (Shi et al., 2017) extensible to practical non-projective parsing, by exemplifying the design of transition systems with better coverage on highly non-projective datasets and, for one variant, bringing the runtime complexity one level closer to feasibility.

2 Transition-based Parsing

We first introduce necessary definitions and notation.

2.1 A General Class of Transition Systems

A transition system is given by a 4-tuple $(C, T, c^s, C_T)$, where $C$ is a set of configurations, $T$ is a set of transition functions between configurations, $c^s$ is an initialization function mapping an input sentence to an initial configuration, and $C_T \subseteq C$ defines a set of terminal configurations.

¹Faster exact inference algorithms have been defined for some sets of mildly non-projective trees (e.g. Pitli et al. (2013); see Gómez-Rodríguez (2016) for more), but lack an underlying transition system. Having one has the practical advantage of allowing generative models, as in Cohen et al. (2011), and transition-based scoring functions, which have yielded good projective-parsing results (Shi et al., 2017); plus the theoretical advantage of providing a single framework supporting greedy, beam-search, and exact inference.
We employ a tripartite representation for configurations: \((\sigma, \beta, A)\), where the three elements are as follows. \(\sigma\) and \(\beta\) are disjoint lists called the stack and buffer, respectively. Each dependency arc \((h, m)\) in the resolved arcs set \(A\) has head \(h\) and modifier \(m\). For a length-\(n\) input sentence \(w\), the initial configuration is \(c^i(w) = ([], [0, 1, ..., n], \varnothing)\) where the 0 in the initial buffer denotes a special node representing the root of the parse tree. All terminal configurations have an empty buffer and a stack containing only 0.

Indexing from 0, we write \(s_i\) and \(b_j\) to denote item \(i\) on the stack (starting from the right) and item \(j\) on the buffer (from the left), respectively. We use vertical bars to separate different parts of the buffer or stack. For example, when concerned with the top three stack items and the first item on the buffer, we may write \(\sigma|s_2|s_1|s_0\) and \(b_0|\beta\).

### 2.2 Attardi’s (2006) System

We now introduce the widely-used Attardi (2006) system, which includes transitions that create arcs between non-consecutive subtrees, thus allowing it to produce some non-projective trees. To simplify exposition, here we present Cohen et al.’s (2011) isomorphic version.

The set of transitions consists of a shift transition \(\text{sh}\) and four reduce transitions \(\text{re}\). A shift moves the first buffer item onto the stack:

\[
\text{sh}[(\sigma, b_0|\beta, A)] = (\sigma|b_0, \beta, A).
\]

A reduce transition \(\text{re}_{h,m}\) creates a dependency arc between \(h\) (head) and \(m\) (modifier) and reduces \(m\). For example,

\[
\text{re}_{s_0,s_1}[(\sigma|s_1|s_0, \beta, A)] = (\sigma|s_0, \beta, A \cup \{(s_0, s_1)\}) .
\]

Row 1 of Fig. 1 depicts the four Attardi reduces. The distance between \(h\) and \(m\) in a \(\text{re}_{h,m}\) transition is called its degree. A system limited to degree-1 transitions can only parse projective sentences. As shown in Fig. 1, Attardi’s (2006) system has two degree-2 transitions \(\text{re}_{s_0,s_2}\) and \(\text{re}_{s_2,s_0}\) that allow it to cover 87.24% of the non-projective trees in UD 2.1. More generally, an Attardi system of degree \(D\) adds \(\text{re}_{s_0,s_D}\) and \(\text{re}_{s_D,s_0}\) to the system of degree \(D - 1\).

### 3 Improving Coverage

A key observation is that a degree-\(D\) Attardi system does not contain all possible transitions of degree within \(D\). Since prior empirical work has ascertained that transition systems using more transitions with degree greater than 1 can handle more non-projective treebank trees (Attardi, 2006; Gómez-Rodríguez, 2016), we hypothesize that adding some of these “missing” reduce transitions into the system’s inventory should increase coverage. The challenge is to simultaneously...
maintain run-time guarantees, as there exists a known trade-off between coverage and complexity (Gómez-Rodríguez, 2016). We want to use Cohen et al.’s (2011)’s exact-inference algorithm for Attardi-based degree-D non-projective dependency parsing systems, which was previously analyzed as having $O(n^{3D+1})$ time complexity. Our contribution is systems that improve the degree-2 Attardi (2006) system’s non-projective coverage, and yet (i) one has degree 3 but still the same $O(n^7)$ runtime as Cohen et al. (2011), rather than $O(n^{3.3+1})$; and (ii) another has degree 2 but better runtime than Cohen et al.’s (2011) system.

Here, we first sketch the existing exact inference algorithm, and then present our variants.

### 3.1 Cohen et al.’s (2011) Exact Inference

The main idea of the algorithm is to group transition sequences into equivalence classes and construct longer sequences from shorter ones. Formally, for $m \geq 1$, Cohen et al. (2011) define a length-$m$ computation as a sequence of $m$ applications of transitions to configurations: $c_0 \xrightarrow{T_1} c_1 \cdots \xrightarrow{T_m} c_m$, where $t_i \in T$ and $t_i(c_{i-1}) = c_i$ for $i \in 1..m$. As depicted in Fig. 2, a length-$m$ I-computation $[h_1, i, h_2, h_3, j]$ is any length-$m$ computation where (1) $c_0 = (\sigma)[h_1, i]A$ and $c_m = (\sigma)[h_2][h_3, j]\beta', A')$ for some $\sigma$, $\beta$, $\beta'$, $A$, and $A'$; and (2) for all $k \in 1..m$, $c_k$’s stack has $\sigma$ as base and length at least $|\sigma| + 2$. Only condition (1) is relevant to this paper: it states that the net effect of an I-computation is to replace the rightmost item $h_1$ on the stack with items $h_2$ and $h_3$, while advancing the buffer-start from $i$ to $j$.

The dynamic programming algorithm is specified as a deduction system, where each transition corresponds to a deduction rule. The shift rule is:

$$sh : [h_1, i, h_2, h_3, j] \rightarrow [h_3, j, h_3, j, j + 1].$$

Each reduce rule combines two I-computations into a larger I-computation, e.g. (see Fig. 3):

$$re_{s_0, s_1} : [h_1, i, h_2, h_3, k] \rightarrow [h_1, i, h_2, h_3, j] [h_3, k, h_4, h_5, j].$$

With the side condition that $h_4$ modifies $h_5$. Other reduce transitions have similar deduction rules, with the same two premises, but a different conclusion depending on the reduced stack item. As an illustration:

$$re_{s_2, s_0} : [h_1, i, h_2, h_3, k] \rightarrow [h_1, i, h_2, h_4, j].$$

The goal of deduction is to produce the I-computation $[\epsilon, 0, \epsilon, 0, \epsilon]$, using the shift and reduce deduction rules starting from the axiom $[\epsilon, 0, \epsilon, 0, 1]$, corresponding to the first and mandatory shift transition moving the root node from buffer to stack. $\epsilon$ stands for an empty stack or buffer. As analyzed by Cohen et al. (2011), direct tabularization for this deduction system takes $O(n^5)$ space and $O(n^8)$ time. With adaptation of the “hook trick” described in Eisner and Satta (1999), we can reduce the running time to $O(n^7)$.

### 3.2 Our New Variants

In this section, we modify Cohen et al.’s (2011) set of reduce deduction rules to improve coverage or...
time complexity. Since each such deduction rule corresponds to a reduce transition, each revision to the deduction system yields a variant of Attardi’s (2006) parser. In other words, generalization of the deduction system gives rise to a family of non-projective transition-based dependency parsers.

We first explain why there are exactly nine reduce transitions \( \mathcal{R} = \{ \text{re}_{0,0,1}, \text{re}_{1,0,0}, \text{re}_{0,0,2}, \text{re}_{2,0,0}, \text{re}_{0,1,2}, \text{re}_{2,0,1}, \text{re}_{0,0,1}, \text{re}_{0,0,2} \} \) that can be used in Cohen et al.’s (2011) exact inference algorithm, without allowing a reduction with head \( b_i \) for \( i \geq 1 \). (Note that Cohen et al.’s (2011) reduce rules are precisely the first four elements of \( \mathcal{R} \).) From Fig. 3 we infer that the concatenation of I-computations \([h_i, i, h_2, h_3, k]\) and \([h_3, k, h_4, h_5, j]\) yields a configuration of the form \((\sigma[h_2][h_4][h_5], j, \beta, A)\). For the application of a reduce rule to yield a valid I-computation, by condition (1) of the I-computation definition, first, the head and modifier must be selected from the “exposed” elements \( h_2, h_3, h_5, \) and \( j \), corresponding to \( s_2, s_1, s_0, b_0 \), respectively; and second, the modifier can only come from the stack. \( \mathcal{R} \) is precisely the set of rules satisfying these criteria. Further, every reduce transition from \( \mathcal{R} \) is compatible with Eisner and Satta’s (1999) “hook trick”. This gives us the satisfactory result that the \( O(n^7) \) running time upper bound still holds for transitions in \( \mathcal{R} \), even though one of them has degree 3.

Next, we consider three notable variants within the family of \( \mathcal{R} \)-based non-projective transition-based dependency parsers. They are given in Fig. 1, along with their time complexities and empirical coverage statistics. The latter is computed using static oracles (Cohen et al., 2012) on the UD 2.1 dataset (Nivre et al., 2017).\(^7\) We report the global coverage over the 76,084 non-projective sentences from all the training treebanks.

One might assume that adding more degree-1 transitions wouldn’t improve coverage of trees with non-crossing edges. On the other hand, since their addition doesn’t affect the asymptotic run-time, we define \text{ALLDEG1} to include all five degree-1 transitions from \( \mathcal{R} \) into the Attardi (2006) system. Surprisingly, using \text{ALLDEG1} improves non-projective coverage from 87.24% to 93.32%.

Furthermore, recall that we argued above that, by construction, using any of the transitions in \( \mathcal{R} \) still preserves the original \( O(n^7) \) run-time upper bound for Cohen et al.’s (2011) exact inference algorithm. We therefore define \text{ALL} to include all 9 reduce transitions in \( \mathcal{R} \); it runs in time \( O(n^7) \) despite the fact that \( \text{re}_{0,0,2} \in \mathcal{R} \) has degree 3, a significant improvement over the best previously-known bound for degree-3 systems of \( O(n^{10}) \). Moreover, as shown in Fig. 1, this variant improves non-projective coverage to 95.54%.

Now, if our goal is to reduce run-time, we can start with an Attardi (2006) system of degree 1 instead of 2, which, as previously mentioned, can only handle projective sentences, but which has runtime \( O(n^7) \), which we show as follows. Since \( s_2 \) cannot be reduced, when concatenating \([h_1, i, h_2, h_3, k]\) and \([h_3, k, h_4, h_5, j]\), the larger I-computation we deduce will be either \([h_1, i, h_2, h_4, j]\) or \([h_1, i, h_2, h_5, j]\), so that the first three indices of the conclusion item remain the same as those of the first premise. In addition, the only remaining deduction rule, a shift, produces deduction items of the form \([h_1, j, h_1, j, j+1]\). Hence, all derivable items will be of the form \([h_1, i, h_1, h_3, j]\), with only four unique indices, instead of five. It follows that the exact inference algorithm for this variant runs in \( O(n^6) \) time, improving from \( O(n^7) \). The tabularization takes \( O(n^4) \) space, a reduction from the original \( O(n^5) \) as well. In terms of empirical coverage, this new system can handle 93.16% of the non-projective sentences in UD 2.1, more than Attardi’s (2006) system, but fewer than our other two variants.

Generally, for a degree-\( D \) Attardi (2006)-based system, one may apply our first two variants to improve its non-projective coverage while maintaining the previously-analyzed \( O(n^{3D+1}) \) time complexity, or the third variant to reduce its time complexity down to \( O(n^{3D}) \), and space complexity from \( O(n^{2D+1}) \) to \( O(n^{2D}) \).
4 Conclusion

We have introduced a family of variants of Cohen et al.’s (2011) Attardi-based transition system and its associated dynamic programming algorithm. Among these, we have highlighted novel algorithms that (1) increase non-projective coverage without affecting computational complexity for exact inference, and (2) improve the time and space complexity for exact inference, even while providing better coverage than the original parser. Specifically, our ALLs_{0}s_{1} runs in \( O(n^6) \) time and \( O(n^4) \) space (improving from \( O(n^7) \) and \( O(n^5) \), respectively) while providing coverage of 93.16% of the non-projective sentences in UD 2.1.

Exact inference for transition-based parsers has recently achieved state-of-the-art results in projective parsing (Shi et al., 2017). The complexity improvements achieved in this paper are a step towards making their exact-inference, projective approach extensible to practical, wide-coverage non-projective parsing.

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