Analysis of pultrusion process for thick glass/polyester composites: transverse shear stress formations

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Abstract In the present work process-induced residual stress development is described for a 100 × 100 mm pultruded square profile made of glass/polyester. A thermo-chemical model developed in MATLAB is coupled with a mechanical model developed in ABAQUS. The temperature and degree of cure distributions are calculated for three different preheating temperatures. In the mechanical model, a parameter study is examined to investigate the effect of total volumetric shrinkage ($V_{sh}$) and the coefficient of thermal expansion (CTE) in rubbery state on the residual stresses. The non-uniform internal constraints in the part yield in an internal shear deformation during the process. The transverse shear stress and compressive normal stress levels decrease significantly as compared with the tensile normal stresses with an increase in preheating temperature. The change in $V_{sh}$ and CTE in rubbery state affects the shear stresses and compressive stresses significantly, whereas the tensile stresses are less affected.

Keywords Pultrusion, Transverse shear stress, Cure behavior, Thermo-mechanical, Finite difference method, Finite-element method

Introduction Pultruded products have increasingly been used in electrical, construction, transportation, and wind energy industries as load carrying pre-fabricated components as well as thermal and electrical insulation elements. One example is the wind turbine blade root reinforcement which is a relatively thick fiber-reinforced polymer composite manufactured by pultrusion. Another one is the thick pultruded rods used in transformers and electric motors as an electrical insulator. In order to have a better product reliability of the pultruded parts during the service life, the product property development during the process needs to be understood and described well. Therefore, it is necessary to characterize the pultrusion process to manufacture high quality pultruded profiles with minimum defects or improved product performance (mechanical, thermal, or electrical). Although pultrusion process is a continuous and automated process, the involved multi-physics make the process difficult to control. A schematic view of the thermosetting pultrusion process is depicted in Fig. 1. The thermosetting resin is gradually cured in the heated die in which the status of the resin changes from liquid (viscous) to rubbery (gel) and finally to solid (glassy). The cured profile is pulled by a pulling mechanism and cut into desired length by a saw.

One of the main challenges in pultrusion process is the process-induced residual stresses since they may cause manufacturing-induced defects in the product such as premature cracking, delamination, and shape distortions which has a negative effect on the product performance. The degree of cure gradients through the thickness due to temperature lags

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and chemical exotherms can be important in thick composite parts, and may cause additional residual stresses. The residual stresses in the transverse to the fiber direction even might be more critical for unidirectional (UD) composites since their transverse strength is dominated by the matrix. The transverse shear strength is very important since it has relatively a low value, e.g. the transverse shear strength of a UD glass/epoxy composite was found to be around 7 MPa using short-beam shear tests.

Keeping the multi-physics and large amount of variables involved in the pultrusion process in mind, a satisfactory experimental analysis for the production requires considerable time which is obviously not a cost-efficient approach. Therefore, a numerical process simulation tool is essential to address the main challenges in pultrusion such that the pultrusion industry can use the developed models to avoid expensive trial-and-error approaches for designing new products and process conditions.

The residual stress formation in fiber-reinforced composites has been investigated extensively in literature by considering the cool-down stage of the curing process in which the stresses evolving during the curing were neglected. Transient heat transfer causes thermal gradients which results in differential polymerization, shrinkage, and modulus development of the matrix material in the through-thickness direction and generate residual stresses. The evolution of macroscopic in-plane residual stresses was investigated for thick thermoset laminates resulting from temperature and degree of cure gradients. These studies mainly concentrated on the normal stress development, whereas shear stress evolution has not been considered.

Apart from the studies mentioned above, the residual stresses for pultrusion processes have been studied in literature. A square UD glass epoxy profile (25.4 × 25.4 mm) was analyzed in. A UD graphite/epoxy rod was studied in which the diameter of the rod was 9.5 mm. A process simulation was performed for an industrially pultruded rectangular hollow profile (64 × 27 mm) with a thickness of 3 mm containing both UD roving and continuous filament mat layers. The predicted residual shape distortions were found to be around 7 MPa using short-beam shear tests.

As compared with the aforementioned literature on pultrusion process, a much thicker UD-reinforced profile (100 × 100 mm) is studied in this work. The focus is given not only on the transverse normal stresses but also on the transverse shear stress formation which has not been investigated quantitatively up to now. The state-of-the-art process model developed specifically for pultrusion processes is being implemented in the present work. This model predicts the temperature and degree of cure distributions at steady state in a three-dimensional (3D) domain and the stresses are calculated in a 2D domain. The proposed modeling approach is therefore computationally fast and accurate for the estimation of transverse stresses and deformations together with thermal and cure history. The effect of preheating temperature as well as the coefficient of thermal expansion (CTE) of the resin in rubbery state on the stresses is studied. Moreover, in order to understand the effect of low profile additives on the residual stresses, a parameter analysis is carried out by changing the total volumetric shrinkage value of the polyester matrix.

**Thermal model**

The 3D steady state heat transfer equation for the composite and die is solved using the control volume-based finite difference (CV/FD) method in MATLAB. The advective term due to the pulling speed of the composite part and the source term due to exothermic reaction (internal heat generation) are included in the heat transfer model (Eq. 1)). These terms are discarded for the heat transfer equation solved for the die.

\[
\frac{\rho C_v}{\rho_C} \left( \frac{\partial T}{\partial x_1} \right) = k_{x_1} \frac{\partial^2 T}{\partial x_1^2} + k_{x_2} \frac{\partial^2 T}{\partial x_2^2} + k_{x_3} \frac{\partial^2 T}{\partial x_3^2} + q \quad (1)
\]

where \( T \) is the temperature, \( u \) is the pulling speed, \( \rho \) is the density, \( C_v \) is the specific heat and \( k_{x_1} \), \( k_{x_2} \), and \( k_{x_3} \) are the thermal conductivities along \( x_1 \), \( x_2 \), and \( x_3 \) directions, respectively. Here, \( x_1 \) is the pulling or longitudinal direction; \( x_2 \) and \( x_3 \) are the transverse directions for the UD pultruded part. The internal heat generation is expressed as:

\[
q = (1 - V_f) \rho_p H_f R_\sigma(\alpha, T) \quad (2)
\]
where \( H_r \) is the total heat of reaction for the resin, \( \rho \) is the resin density, \( V_f \) is the fiber volume fraction, \( \alpha \) is the degree of cure, and \( R_r(\alpha, T) \) is the reaction of cure which can also be defined as the rate of \( \alpha \), i.e. \( da/dt \). The expression for the cure kinetics is given as:

\[
R_r(\alpha, T) = \frac{da}{dt} = A_o \exp(-E_a/RT)\alpha^n(1 - \alpha)^m \tag{3}
\]

where \( A_o \) is the pre-exponential constant, \( E_a \) is the activation energy, \( R \) is the universal gas constant, \( m \) and \( n \) are the order of reaction (kinetic exponents).

The computational domain of the thermal model is depicted in Fig. 2 for the pultrusion process of a 100 × 100 mm glass/polyester profile. Only a quarter of the pultrusion domain is considered due to symmetry. The cross-sectional mesh of the composite part consists of 400 control volumes (20 × 20) and 0.01 m is used for the control volume size in the pulling direction (\( x_1 \)-direction). This results in total of 640000 control volumes (20 × 20 × 1600). A steel die is used for the die block. The pulling speed is set to 100 mm/min. Material models for the composite are taken from15. Two heating zones having prescribed set temperatures 110 °C and 140 °C are defined. At the inlet of the pultrusion die, the temperature of all the nodes attached to the composite is set to a preheating temperature \( T_{\text{preheat}} \) (Fig. 2) as described in Fig. 3. A general purpose finite element package ABAQUS is employed for the mechanical model.

### Table 1 Thermal properties used in the thermo-chemical model29,30.

| Material      | \( \rho \) [kg/m³] | \( C_p \) [J/kg K] | \( k_x \) [W/m K] | \( k_y \) [W/m K] |
|---------------|--------------------|--------------------|-------------------|-------------------|
| Polyester     | 1100               | 1830               | 0.17              | 0.17              |
| Glass fiber   | 2560               | 670                | 11.4              | 1.04              |
| Steel die     | 7833               | 460                | 40                | 40                |

### Table 2 Cure kinetics parameters used in Eq. (3) and the total heat of reaction used in Eq. (2)29.

| \( A_o \) [1/s] | \( E_a \) [kJ/mol] | \( m \) | \( n \) | \( H_r \) [kJ/kg] |
|----------------|--------------------|--------|--------|-----------------|
| 7.5581 × 10⁸   | 82.727             | 0.63   | 1.847  | 175             |

### Mechanical model

In the 2D mechanical analysis of the pultrusion process, the cross section of the composite is moved along the pulling direction during the process (Lagrangian frame) meanwhile tracking the corresponding temperature and degree of cure profiles already calculated in the 3D thermo-chemical analysis (Eulerian frame) as described in Fig. 3. A general purpose finite element package ABAQUS is employed for the mechanical model. Since the cross sectional dimensions are much smaller than the total length of the pultrusion line in the pulling direction (\( x_1 \)-direction), a plane strain assumption is made for the residual stress analysis in which no strain component is taken into account in the pulling direction (the out of plane strain is assumed to be zero in Eq. (11)). An eight-node biquadratic generalized plane strain quadrilateral elements (CPE8R) with reduced integration are used. The temperature and degree of cure distributions calculated using the CV/FD in MATLAB are mapped to the nodal points of the 2D cross section defined in the mechanical model in ABAQUS (20 × 20 elements at the cross section). Symmetry boundary conditions are used at the symmetry surfaces and 2D rigid body surfaces are employed at the die–part interface. A mechanical contact condition is defined which allows separation at the interface while restricting any expansion of the composite beyond the tool interface. The friction force at the contact condition is assumed to be zero (sliding condition).
User-defined subroutines are developed in ABAQUS for the constitutive material models. The elastic modulus of the resin is modeled using the modified cure hardening instantaneous linear elastic (CHILE) approach proposed by Johnston\(^\text{12}\) and the corresponding expression is seen in Eq. (4).

\[
E_r = \begin{cases} 
E_0 & ; \quad T^* \leq T_{C1} \\
A_e \exp(K_e T^*) & ; \quad T_{C1} < T^* < T_{C2} \\
E_t + \frac{T^* - T_{C2}}{T_{C3} - T_{C2}}(E_t - E_r) & ; \quad T_{C2} < T^* < T_{C3} \\
E_r & ; \quad T_{C3} \leq T^*
\end{cases}
\tag{4}
\]

where \(T^*\) represents the difference between the instantaneous glass transition temperature \(T_g\) and the resin temperature \(T\), i.e. \(T^* = T - T_g\). \(A_e\) and \(K_e\) are the constants for the exponential term, \(T_{Cp}\) are defined as the critical temperatures and \(E_t\), \(E_r\) and \(E_\alpha\) are the corresponding elastic modulus values, respectively.\(^\text{22}\) More specifically, \(E_r\) and \(E_\alpha\) can be considered as the elastic modulus in the viscous and glassy state, respectively. The glass transition temperature \(T_g\) can be defined as a function of cure degree\(^\text{13}\) and expressed as:

\[
T_g = T^0 + a_1 T \cdot \alpha
\tag{5}
\]

where \(T^0_g\) is the glass transition temperature at \(\alpha = 0\) and \(a_1\) is a fitting constant. The parameters used in Eq. (4) are taken\(^\text{22}\) from\(^\text{13}\) and listed in Table 3. For Eq. (5), \(T^0_g = 0\) and \(a_1 = 135^\circ\text{C}\) are used\(^\text{13}\).

The effective mechanical properties as well as thermal and chemical shrinkage strains of the UD pultruded profile are calculated using the self consistent field micromechanics (SCFM) approach which is a well known and documented technique in the literature\(^\text{14}\). The details of the SCFM are presented in Appendix 1. The mechanical properties of the glass fiber and polyester resin are given in Table 4. An incremental linear elastic approach is implemented utilizing the user-defined subroutines in ABAQUS to calculate the displacements and stresses. The total incremental strain \(\varepsilon^\text{tot}\), which is composed of the incremental mechanical strain \(\varepsilon^\text{mech}\), thermal strain \(\varepsilon^\text{th}\), and chemical strain \(\varepsilon^\text{ch}\), is given in Eq. (6). Here, the incremental

\[
\varepsilon^\text{mech} = \varepsilon^\text{mech} + \varepsilon^\text{th} + \varepsilon^\text{ch}
\tag{6}
\]

The thermal strains are calculated as:

\[
\varepsilon^\text{th} = \alpha_{CTE} \Delta T
\tag{7}
\]

where \(\beta\) is the rubbery state multiplier and \(\alpha_{CTE}\) is the CTE in glassy state and \(\Delta T\) is the temperature increment. It should be noted that the resin CTE in rubbery state is known to be much larger than the CTE in glassy state\(^\text{14,26}\), i.e. \(\beta \geq 1\).

The chemical shrinkage strain increments for the resin \(\varepsilon_{\text{ch}}\) are calculated as a function of the total volumetric shrinkage \(\varepsilon_{\text{vd}}\) and degree of cure increment \(\Delta \alpha\) using a uniform contraction for a unit cell in the resin and as expressed as:\(^\text{25}\):

\[
\varepsilon_{\text{ch}} = \sqrt{1 + \Delta \alpha \cdot V_{\text{vd}} - 1}
\tag{8}
\]
The incremental stress tensor ($\tilde{\sigma}_{ij}$) is then calculated using the material Jacobian matrix ($J$) based on the incremental mechanical strain tensor ($\tilde{\varepsilon}_{mech}$) (Eq. (10)) in ABAQUS. The linear elastic model is actually a convenient approach for the pultrusion process since the curing takes place in a relatively short time (in the order of minutes) where the viscoelastic effects are less pronounced.

$$\tilde{\sigma}_{ij} = J \tilde{\varepsilon}_{mech}$$  \hspace{1cm} (10)

The stress and strain tensors are updated at the end of the each time increment ($n$) as in Eqs. (11) and (12), respectively.

$$\sigma_{ij}^{n+1} = \sigma_{ij}^n + \tilde{\sigma}_{ij}^n$$  \hspace{1cm} (11)

$$\varepsilon_{ij}^{n+1} = \varepsilon_{ij}^n + \tilde{\varepsilon}_{ij}^n$$  \hspace{1cm} (12)

Results and discussions

Effect of $T_{preheat}$

The temperature and degree of cure distributions are calculated from the point at which part enters the pultrusion die ($x_1 = 0$ m) till $x_1 = 16$ m. A parametric study is carried out using different preheating temperature ($T_{preheat}$). The temperature and degree of cure distributions are shown in Figs. 4–7 for different $T_{preheat}$ values namely 20 °C, 40 °C and 60 °C. It is seen from Fig. 6 that larger portion of the cross section is cured with a degree of cure value above 0.85 with higher $T_{preheat}$. The inner region is not cured at the die exit ($x_1 = 1$ m). However, the curing of the inner portion takes place at post die since the temperature of the outer region is still high to initiate the exothermic reaction of the inner region. At $x_1 = 16$m, it is seen

![Figure 4: Temperature distribution at pultrusion die exit ($x_1 = 1$ m) for different $T_{preheat}$ values.](image)

![Figure 5: Temperature distribution at $x_1 = 16$ m for different $T_{preheat}$ values.](image)

![Figure 6: Degree of cure distribution at pultrusion die exit ($x_1 = 1$ m) for different $T_{preheat}$ values.](image)
from Fig. 7 that the degree of cure is almost 1 (fully cured) for \( T_{\text{preheat}} \) values 40 °C and 60 °C, whereas some portion of the cross section is not fully cured for the case with \( T_{\text{preheat}} = 20 \) °C having a degree of cure value around 0.85.

The corresponding temperature, degree of cure, and transverse modulus evolution at the center (Point A) and top (Point B) of the part are depicted in Fig. 8. It is seen that relatively faster curing takes place at Point A (center) for \( T_{\text{preheat}} = 40 \) °C (at \( x_1 = 2 \) m) and \( T_{\text{preheat}} = 60 \) °C (at \( x_1 = 3 \) m) than \( T_{\text{preheat}} = 20 \) °C (from \( x_1 = 4 \) m to \( x_1 = 7 \) m). The temperature at the post die is not sufficient for the exothermic reaction with \( T_{\text{preheat}} = 20 \) °C. This yields in lower peak temperature and hence a slower curing rate of Point A as seen in Fig. 8 (middle). The effect of temperature and degree of cure on the elastic modulus are seen in Fig. 8 (bottom). It is seen that the elastic modulus of a fully cured region (Point B) can have a decrease during the process due to the temperature increase in the inner regions (Point A).

The predicted transverse normal stress \( (\sigma_{x_2}) \) and transverse shear stress \( (\tau_{x_3}) \) distributions at \( x_1 = 16 \) m are shown in Figs. 9 and 10, respectively. Note that \( \beta = 2.5 \) for Eq. (8) and \( V_{\text{sh}} = 0.06 \) for Eq. (9) are used in the mechanical model. It is seen that higher stress values are obtained for the case with \( T_{\text{preheat}} = 20 \) °C where \( \tau_{x_3} \) is found to be more critical than \( \sigma_{x_2} \). The reason behind that is the larger cure and temperature gradients yielding in larger elastic modulus gradients through the thickness for lower \( T_{\text{preheat}} \) values (see Fig. 8). A value of approximately 8.76 MPa is found for \( \tau_{x_3} \) which is larger than the highest value for \( \sigma_{x_2} = 6.92 \) MPa with \( T_{\text{preheat}} = 20 \) °C.

The curing lag between Point A and Point B seen in Fig. 8 causes internal mechanical constraints to the inner region which cures later as compared with the outer region of the pultruded part. During curing, the outer region shrinks toward symmetry surfaces and the elastic modulus increases while no deformation takes place at the inner region. The degree of mechanical constraint exerted by the cured and solidified outer region is hence influenced by the temperature of the outer surface since higher temperatures yield in lower elastic modulus at the glass state. In other words, harder mechanical constraint takes place for lower temperature of the cured region and softer constraint is the case for higher temperature at glass state. As a result, higher stresses prevail for \( T_{\text{preheat}} = 20 \) °C since the temperature of the outer region is approximately 60 °C when the inner region (Point A) starts curing (see Fig. 8). The predicted stress levels decrease with an increase in \( T_{\text{preheat}} \) since more uniform curing is accommodated in the part.

**Figure 7** Degree of cure distribution at \( x_1 = 16 \) m for different \( T_{\text{preheat}} \) values.

**Figure 8** Evolution of temperature (top), degree of cure (middle), and elastic modulus (bottom) for Point A and Point B.

**Effect of CTE in rubbery state**

The characterization of CTE in rubbery state is a difficult task which needs a careful investigation of the dimensional change during curing. Although \( \beta = 2.5 \) was proposed in 24 and used also in the present work in Section 4.1 by utilizing the 2D mechanical model in ABAQUS, a parameter study is carried out by employing different values for \( \beta \) in Eq. (8) to simulate the effect of CTE in rubbery state on residual stress evolution during the process. The evolution of normal and shear stresses at locations, where they are the maximum, are depicted in Figs. 11 and 12, respectively together with corresponding \( T \) and \( T_c \) for \( T_{\text{preheat}} = 20 \) °C. Note that \( V_{\text{sh}} = 0.06 \) is used in the simulations. It is seen that an increase in \( \beta \) results in an increase in the residual stress magnitude. This can be explained by the competition between the thermal expansion and chemical shrinkage during curing (see Eq. (6)) such that lower CTE provides smaller thermal strains and hence the chemical shrinkage strain increments become larger. The
In order to simulate the effect of low profile additive content, the $V_{sh}$ used in Eq. (9) are varied from 0 to 0.1 in the mechanical model. Note that the value for $\beta$ is taken as 2.5.

The through thickness distribution of $\varepsilon_{22}$ and $\varepsilon_{23}$ at $x_i = 16$ m are shown in Fig. 13. As expected, the stress levels gradually decrease with a decrease in $V_{sh}$. As compared to the base case, i.e. $V_{sh} = 0.06$, $\varepsilon_{23}$ as well as $\varepsilon_{22}$ (compressive) are almost doubled for $V_{sh} = 0.1$. The change in tensile stresses ($\varepsilon_{22}$) at the through thickness distribution of $\varepsilon_{22}$ and $\varepsilon_{23}$ at $x_i = 16$ m for different $T_{preheat}$ values.

Effect of $V_{sh}$

For polyester resins, the $V_{sh}$ value can vary between 0.06 to 0.1\(^2\). This value can be further decreased to 0 by mixing the unsaturated polyester with low profile or “shrink-reducing” additives\(^3\). In order to simulate the effect of low profile additive content, the $V_{sh}$ used in Eq. (9) are varied from 0 to 0.1 in the mechanical model. Note that the value for $\beta$ is taken as 2.5. The through thickness distribution of $\varepsilon_{22}$ and $\varepsilon_{23}$ at $x_i = 16$ m are shown in Fig. 13. As expected, the stress levels gradually decrease with a decrease in $V_{sh}$. As compared to the base case, i.e. $V_{sh} = 0.06$, $\varepsilon_{23}$ as well as $\varepsilon_{22}$ (compressive) are almost doubled for $V_{sh} = 0.1$. The change in tensile stresses ($\varepsilon_{22}$) at the through thickness distribution of $\varepsilon_{22}$ and $\varepsilon_{23}$ at $x_i = 16$ m for different $T_{preheat}$ values.
The effect of CTE in rubbery state was investigated and larger CTE resulted in lower residual stress levels. It is postulated that this is caused by the increased contribution of thermal expansion to the process-induced strain increment (Eq. (6)) for larger CTE resulted in lower residual stress levels. It is postulated that this is caused by the increased contribution of thermal expansion to the process-induced strain increment (Eq. (6)) for larger CTE in rubbery state. As a consequence, the chemical shrinkage strain increment was compensated. In addition, the effect of $V_{sh}$ was also examined and $\tau_{12}$ and compressive $\sigma_{22}$ changed significantly as a function of $V_{sh}$ whereas there was a relatively negligible change in tensile $\sigma_{12}$ at the center of the part.

It is therefore concluded that taking relatively low transverse shear strength of UD composites (3–7 MPa) into account, the residual shear stresses in the transverse directions need to be carefully analyzed for thick UD pultruded profiles since they may initiate matrix cracking in the manufactured part.

Disclosure statement
No potential conflict of interest was reported by the authors.

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Figure 13 Transverse normal stress $\sigma_{12}$ distribution along the path AB on the symmetry line (left) and transverse shear stress $\tau_{12}$ distribution along the path AC on the diagonal line (right) for $T_{preheat}$ = 60 °C with different $V_{sh}$ values.

References
1. I. Baran: ‘Pultrusion: State-of-the-art Process Models’, 2015, Shropshire, UK, Smithers Rapra.
2. I. Baran: ‘Modelling the pultrusion process of off shore wind turbine blades;’ PhD thesis, Technical University of Denmark, 2014.
3. M. R. Wisnom, M. Gigliotti, N. Ersoy, M. Campbell and K. D. Potter: ‘Mechanisms generating residual stresses and distortion during manufacture of polymer-matrix composite structures;’ Composites Part A: Applied Science and Manufacturing, 2006, 37, 522–529.
4. I. Baran, K. Cinar, N. Ersoy, R. Akkerman and J. H. Hattel: ‘A review on the mechanical modelling of composite manufacturing processes;’ Archives of Computational Methods in Engineering, 2016, 1–31.
5. T. A. Bogetti and J. W. Gillespie: ‘Process-induced stress and deformation in thick-section thermostet composite laminates;’ Journal of Composite Materials, 1992, 26, 626–660.
6. E. Sideidis and G. A. Papadopoulos: ‘Short-beam and three-point-bending tests for the study of shear and flexural properties in unidirectional-fiber-reinforced epoxy composites;’ Journal of Applied Polymer Science, 2004, 93, (1), 63–74.
7. H. T. Hahn and N. J. Pagano: ‘Curing stresses in composite laminates;’ Journal of Composite Materials, 1975, 9, 91–106.
8. Y. Weitsman: ‘Residual thermal stresses due to cool-down of epoxy-resin composites;’ Journal of Applied Mechanics, 1979, 46, 563–567.
9. D. Adolf and J. E. Martin: ‘Calculation of stresses in crosslinking polymers;’ Journal of Composite Materials, 1996, 30, 13–34.
10. P. Sunderland, W. Yu and J. A. Manson: ‘A thermoviscoelastic analysis of process-induced internal stresses in thermoplastic matrix composites;’ Polymer Composites, 2001, 22, (5), 579–592.
11. D. Kevin and P. W. R. Beaumont: ‘The measurement and prediction of residual stresses in carbon-fibre/polymer composites;’ Composites Science and Technology, 1997, 57, (11), 1445–1455.
12. A. Johnston: ‘An integrated model of the development of process-induced deformation in autoclave processing of composite structures;’ PhD thesis, University of British Columbia, Vancouver, Canada, 1997.
13. E. Ruiz and F. Trochu: ‘Numerical analysis of cure temperature and internal stresses in thin and thick rtm parts;’ Composites Part A: Applied Science and Manufacturing, 2005, 36, 806–826.
14. I. Baran, C. C. Tutum, M. W. Nielsen and J. H. Hattel: ‘Process induced residual stresses and distortions in pultrusion;’ Composites Part B: Engineering, 2013, 51, 148–161.
15. P. Carlone, I. Baran, J. H. Hattel and G. S. Palazzo: ‘Computational approaches for modelling the multi-physics in pultrusion process;’ Advances in Mechanical Engineering, Article ID, 2013, 301875, 1–14.
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16. I. Baran, J. H. Hattel and R. Akkerman: ‘Investigation of process induced warpage for pultrusion of a rectangular hollow profile’, Composites Part B: Engineering, 2015, 68, 365–374.

17. I. Baran, R. Akkerman and J. H. Hattel: ‘Modelling the pultrusion process of an industrial I-shaped composite profile’, Composite Structures, 2014, 118, 37–48.

18. A.A. Safonov and A.Yu. Konstantinov: ‘Numerical evaluation of residual manufacturing deformations in complex pultruded composite profiles’, In 20th International Conference on Composite Materials, Copenhagen, Denmark, 19–24th July 2015.

19. S. V. Patankar: ‘Numerical heat transfer and fluid flow’, 1980, New YorkMcGraw-Hill.

20. I. Baran, J. H. Hattel and C. C. Tutum: ‘Thermo-chemical modelling strategies for the pultrusion process’, Applied Composite Materials, 2013, 20, 1247–1263.

21. I. Baran: ‘Pultrusion processes for composite manufacture’, in Advances in Composites Manufacturing and Process Design, (ed. P. Boisse), 2015, Cambridge, UKWoodhead Publishing.

22. I. Baran, R. Akkerman and J. H. Hattel: ‘Material characterization of a polyester resin system for the pultrusion process’, Composites Part B: Engineering, 2014, 64, 194–201.

23. M. W. Nielsen, J. H. Hattel, T. L. Andersen, K. Branner and P. H. Nielsen: ‘In situ measurement using fbgs of process-induced strains during curing of thick glass/epoxy laminate plate: experimental results and numerical modeling’, Wind Energy, 2013, 16, 1241–1257.

24. J. M. Svamborg and J. A. Holmberg: ‘Prediction of shape distortions part I. fe-implementation of a path dependent constitutive model’, Composites Part A: Applied Science and Manufacturing, 2004, 35, 711–721.

25. N. Ersoy, T. Garstka, K. Potter, M. R. Wisnom, D. Porter, M. Clegg and G. Stringer: ‘Development of the properties of a carbon fibre reinforced thermostetting composite through cure’, Composites Part A: Applied Science and Manufacturing, 2010, 41, 401–409.

26. L. Khoun, T. Centea and P. Hubert: ‘Characterization methodology of thermoset resins for the processing of composite materials case study: Cycom 890™ epoxy resin’, Journal of Composite Materials, 2010, 44, 1397–1415.

27. T. F. Starr: ‘Pultrusion for engineers’, 2000, Cambridge, Woodhead PublishingCRC Press.

28. M. Haider, P. Hubert and L. Lessard: ‘Cure shrinkage characterization and modeling of a polyester resin containing low profile additives’, Composites Part A: Applied Science and Manufacturing, 2007, 38, 994–1009.

29. Y. R. Chachad, J. A. Roux, J. G. Vaughan and E. Arafat: ‘Three-dimensional characterization of pultruded fiberglass-epoxy composite materials’, Journal of Reinforced Plastics and Composites, 1995, 14, 495–512.

30. V. Kosar and Z. Gomzi: ‘Crosslinking of an unsaturated polyester resin in the mould: modelling and heat transfer studies’, Applied Mathematical Modelling, 2010, 34, 1586–1596.

Appendix 1

The mechanical properties of the UD fibers are assumed to be transversely isotropic and described by five independent elastic constants: Young’s modulus and Poisson’s ratio in the transverse direction (E_v and ν_v), respectively) and in the longitudinal direction (E_L and ν_L), and the shear modulus in the longitudinal direction (G_L). The resin has an isotropic Young’s modulus (E_r), Poisson’s ratio (ν_r), and a shear modulus (G_r). Based on the fiber volume fraction (V_f), the effective mechanical properties of the UD laminate are calculated in the following using the SCFM approach proposed in1.

**Longitudinal Young’s Modulus:**

\[
E_L = E_r V_f + E_L (1 - V_f) + \frac{4 (1 - V_f) (k_f V_f (1 - V_f) + G_r)}{(k_f + G_r) (1 - 2k_f V_f)}
\]  
(A1)

where \(k_f\) and \(k_r\) are the isotropic plane strain bulk modulus for fiber and resin, respectively and expressed as:

\[
k_f = \frac{E_f}{2(1 - \nu_f - 2\nu_f')}
\]  
(A2)

\[
k_r = \frac{E_r}{2(1 - \nu_r - 2\nu_r')}
\]  
(A3)

**Shear Modulus:**

\[
G_{13} = G_{13} = \frac{G_r (G_{12} + G_r) + (G_{12} - G_r) V_f}{G_{12} + G_r - 2G_r V_f}
\]  
(A4)

\[
G_{13} = \frac{G_r (k_r + (G_{12} - G_r) V_f)}{G_{12} + G_r - 2G_r V_f}
\]  
(A5)

where

**Transverse Young’s Modulus:**

\[
E_r = E_r \left(\frac{1}{4(k_f V_f)^2} + \frac{(4G_r)^2}{(1-k_f V_f)^2}\right)
\]  
(A6)

**Poisson’s Ratios:**

\[
\nu_{13} = \nu_{13} = \nu_F (1 - V_f) + \nu_r (1 - V_f) \left(\frac{V_f - \nu_r (k_r - k_f) G_r (1 - V_f) V_f}{(k_f + G_r) (1 - k_f V_f)}\right)
\]  
(A9)

The effective CTEs of a UD laminate in the longitudinal direction (\(\alpha_L\)) and in the transverse directions (\(\alpha_T = \alpha_a\)) are obtained as:

\[
\alpha_L = \frac{\alpha_f E_f V_f + \alpha_r E_r (1 - V_f)}{E_f V_f + E_r (1 - V_f)}
\]  
(A10)

\[
\alpha_T = \alpha_L + \frac{(\alpha_f V_f + (\nu_f,\alpha_r) (1 - V_f) - (\nu_v,\nu_f,\nu_r) V_f)}{E_f V_f + E_r (1 - V_f)}
\]  
(A11)

where \(\alpha_f\) and \(\alpha_r\) are the CTE of the fiber in the longitudinal and transverse directions, respectively, and \(\alpha_L\) is the CTE of the resin.

The effective incremental chemical shrinkage strains of a UD laminate in the longitudinal direction (\(\varepsilon_L^a\)) and in the transverse directions (\(\varepsilon_T^a = \varepsilon_r^a\)) are written as:

\[
\varepsilon_L^a = \varepsilon_r E_r (1 - V_f)
\]  
(A13)

\[
\varepsilon_T^a = \varepsilon_L (1 - V_f) - (\nu_v,\nu_f,\nu_r) V_f + \nu_r (1 - V_f) \varepsilon_r^a
\]  
(A14)