Two-loop Barr-Zee type Contributions to $(g-2)_\mu$ in the MSSM

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Abstract

We consider the contribution of a two-loop Barr-Zee type diagram to $(g-2)_\mu$ in the minimal supersymmetric standard model (MSSM). At relatively large $\tan \beta$, we show that the contribution of light third generation scalar fermions and neutral CP-even Higgs, $h^0(H^0)$, can easily explain the very recent BNL experimental data. In our analysis $(g-2)_\mu$ prefers negative $A_f$ and positive $\mu$. It is more sensitive to the chirality flipping $h^0(H^0)f_R^* f_L$ rather than chirality conserving couplings.

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I. INTRODUCTION

The recent measurement of the anomalous magnetic dipole moment (MDM) of a muon, \( a_\mu \equiv (g_\mu - 2)/2 \), by Brookhaven E821 collaboration, showed a 2.6\( \sigma \) deviation from the standard model (SM) calculation.

\[
\delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (43 \pm 16) \times 10^{-10}.
\]

Although the experimental error and the theoretical uncertainties in the hadronic contribution is still large [2], this deviation can be a harbinger of new physics beyond the SM. There have already been extensive discussions on the implication of \( \delta a_\mu \) in a variety of models. These include SUSY theories [3], extended Higgs sectors [4], extra dimensions [5], extra gauge bosons and leptoquarks [6], model independent analysis [7], etc...

The minimal supersymmetric standard model (MSSM) is the leading candidate for new physics beyond the SM. In the MSSM, the approximate one loop contribution to \( \delta a_\mu \) is given by [3]:

\[
\delta a_\mu^{\text{SUSY}} = 13 \times 10^{-10} \left( \frac{100\text{GeV}}{m_{\text{SUSY}}} \right)^2 \tan \beta \text{ sign}(\mu)
\]

As can be seen from the above equation, the BNL experimental data can easily be accommodated with positive \( \mu \), large \( \tan \beta \) and light charginos, neutralinos and smuons.

On the other hand, as one of the popular SUSY models, the effective SUSY models [8] have been widely studied. In effective SUSY models, the 1st and 2nd generations of sfermions are very heavy and only the 3rd generation sfermions are light enough to be relevant for low energy and collider phenomenology, thereby easily evading the SUSY FCNC and SUSY CP problems. However, the light 2nd generation sleptons favored by \( (g - 2)_\mu \) [3] can put a severe challenge to effective SUSY models, since the 1st and 2nd generation of sleptons are expected to be at the same scale in such models.

In this paper we will study the implications of a large \( \delta a_\mu \) in the framework of effective SUSY models. We will show that the neutral CP-even Higgs, \( h^0(H^0) \), together with relatively light 3rd generation sfermions can easily explain the \( \delta a_\mu \) for large \( \tan \beta \) by contributing to muon MDM through two-loop Barr-Zee type diagrams shown in Fig. 1. In our analysis, it is found that \((g - 2)_\mu\) prefers negative soft trilinear coupling \( A_f \) and positive \( \mu \). It is more sensitive to the chirality flipping \( h^0(H^0)\tilde{f}_R\tilde{f}_L \) originating both from F-term and soft trilinear terms rather than chirality conserving F and D terms.

II. BARR-ZEE TYPE DIAGRAM CONTRIBUTION TO \((g - 2)_\mu\)

The Barr-Zee type diagrams were first studied in the literature [9] a long time ago. In particular they were shown to give large contributions to the electric dipole moment of the electron and neutron [10]. The generic Barr-Zee type diagram we are considering is shown in Fig. 1 where \( S \) denotes a generic scalar and \( V \) a gauge boson. In the upper loop \( \gamma-S-V \) both fermions and sfermions can be exchanged. The fermion contribution has been studied recently in the Two Higgs Doublet Model [11]. In this study, it is shown that a sizable contribution to \((g - 2)_\mu\) can be obtained for relatively light pseudo-scalar \( A^0 \) and/or light
CP-even scalar $h^0$. This fermion contribution turns out to be small in the MSSM since both $A^0$ and $h^0$ are heavier than $\approx 90$ GeV.

We consider the sfermion contribution to the effective vertex $\gamma - S - V$ in the upper loop of Fig. 1. We limit ourselves to the internal photon exchange ($V = \gamma$) in the Barr-Zee diagram, since the $V = Z$ and $V = W^\pm$ diagrams are expected to be suppressed by their masses and also a small coupling in case of $V = Z$. We assume that CP is conserved, and then only $(A^0 \bar{f}_i f_j)_{i \neq j}$ is allowed and in this case only the CP-even Higgs $S = h^0, H^0$ can contribute to $\gamma - S - V$ while $\gamma - A^0 - V$ vanishes.

The Yukawa interaction of Higgs fields with fermions or scalar fermions can be written generically by

$$
\mathcal{L}_{\text{int}} = - \frac{g \lambda_f}{2m_W} S \bar{f} f - \frac{g \lambda_{\tilde{f}}}{2m_W} S \bar{\tilde{f}} \tilde{f}.
$$

With this generic coupling, and owing to electromagnetic gauge invariance, the effective vertex $h^0 (H^0) - \gamma - \gamma$ can be written as:

$$
\Gamma^{\mu\nu}(q) = -i (g^{\mu\nu} q \cdot k - q^\mu k^\nu) \frac{g e^2}{(4\pi)^2 m_W} \sum_f N_c \bar{\tilde{f}} Q_{\tilde{f}} \frac{\lambda_{\tilde{f}}}{m_S^2} \int_0^1 dx \frac{x(1 - x)}{x(1 - x)q^2 - m_{\tilde{f}}^2}.
$$

Here $N_c$ is the color factor and $Q_{\tilde{f}}$ and $m_{\tilde{f}}$ are respectively the electric charge and the mass of the internal sfermion.

When deriving the above equation, we kept only the linear term in $k$ because the MDM is obtained in the soft photon limit. Note that the Lorentz structure is manifestly gauge invariant and it is the only possible form factor which contributes to MDM in the CP conserving theory. The above vertex is connected to the external muon line by integrating over the $h^0 (H^0), \gamma$ and $\mu$ propagators. In that integration we neglected the terms proportional to $m_\mu^2$ and kept only terms proportional to $q^2$. After performing the second integration, $(g - 2)_\mu$ takes the following form:

$$
\delta a_\mu = -\frac{\alpha}{2\pi} \left( \frac{G_F m_\mu^2}{4\sqrt{2} \pi^2} \right) \lambda_\mu^S \sum_f N_c \bar{\tilde{f}} Q_{\tilde{f}} \frac{\lambda_{\tilde{f}}}{m_S^2} \mathcal{F}(z_{\tilde{f}S}),
$$

where $z_{fS} = m_f^2 / m_S^2$, $\lambda_{\mu(h^0,H^0)}^{(\mu,h^0)} = \{-\sin \alpha, \cos \alpha\} / \cos \beta$. The loop function $\mathcal{F}(z)$ is given by:

$$
\mathcal{F}(z) = \int_0^1 dx \frac{x(1 - x) \log \frac{z}{x(1 - x)}}{z - x(1 - x)}.
$$

The asymptotic form of $\mathcal{F}(z)$, which may be useful for later discussion, is given by:

$$
\mathcal{F}(z) = \begin{cases} 
-\log z + 2 & z \ll 1 \\
0.344 & z = 1 \\
\frac{1}{6z} \log z + \frac{5}{3} & z \gg 1.
\end{cases}
$$

In the flavor eigenstates, the interactions between $h^0 (H^0) - \tilde{f}^* - \tilde{f}$ are given by
\[ \mathcal{L} = -\frac{g}{2m_W \sin \beta} m_t (\mu^* \sin \alpha + A_t \cos \alpha) h^0 \bar{t}_R^* t_L + h.c \]
\[ -\frac{g}{2m_W \sin \beta} m_t (-\mu^* \cos \alpha + A_t \sin \alpha) H^0 \bar{t}_R^* t_L + h.c \]
\[ + \frac{g}{2m_W \cos \beta} m_b (\mu^* \cos \alpha + A_b \sin \alpha) h^0 \bar{b}_R^* b_L + h.c \]
\[ + \frac{g}{2m_W \cos \beta} m_b (\mu^* \sin \alpha - A_b \cos \alpha) H^0 \bar{b}_R^* b_L + h.c, \] (8)

where we did not include the chirality conserving F and D-term contributions, which do not give any enhancement to \( \delta a_\mu \). The corresponding formula for \( \bar{\tau} \) interactions with \( h^0 (H^0) \) can be obtained by replacing \( b \) by \( \tau \) in (8). After diagonalization of the sfermion mass matrix the flavor eigenstates \( \bar{f}_{L,R} \) in (8) can be expressed in terms of the mass eigenstates \( \bar{f}_{1,2} \). The \( \lambda_{\bar{f}} \) in (5) can be easily read from (8).

We stress that the quantity inside the parenthesis in (5)

\[ \frac{G_F m^2_\mu}{4\sqrt{2}\pi^2} = 23.32 \times 10^{-10} \] (9)

is the magnitude of the SM electroweak contribution to \((g - 2)_\mu\). It is also interesting to note that this is also the order of magnitude of \( \delta a_\mu \).

The two-loop expression (5) is suppressed by the electroweak coupling \( \alpha/4\pi \sim 1/1722 \). To compensate this suppression and achieve the required magnitude to explain the experimental deviation, some enhancement factors are required. We note that this enhancement factors are provided by: i) the large \( \tan \beta \) enhancement in the down type Yukawa coupling \( \lambda_{\mu} \) and also \( \lambda_{\bar{f}} \), ii) large positive \( \mu \) and/or large negative \( A_f \), iii) if the splitting between the internal scalar mass \( m_{h^0 (H^0)} \) and internal sfermions mass \( m_{\bar{f}} \) is large one can have additional enhancement from the loop function \( F \), which follows from (7).

III. NUMERICAL RESULTS

In this section we will discuss our numerical results in the framework of the effective SUSY model. As stated in the introduction, in such models, the 1st and 2nd generation sfermions are very heavy while the 3rd generation sfermions can be light without any conflict with low energy constraints. It follows that the one-loop contribution to \((g - 2)_\mu\) in effective SUSY model is suppressed by the heaviness of smuons. Therefore we do not include the one-loop contributions in our numerical analysis. When diagonalising the mass matrix of the 3rd generation sfermions, we neglect the small D-terms and assume that left-handed and right-handed soft mass parameters are degenerate, that is:

\[ m^2_{Q, s} = m^2_t = m^2_b = m^2_{L, A} = m^2_{\tau} \equiv \tilde{m}^2. \] (10)

Those assumptions allow maximal mixings between left and right chirality states. In the absence of chirality conserving terms \( h^0 (H^0) \bar{f}_{L,R}^* \bar{f}_{L,R} \), the Lagrangian given in (8) can be written easily in terms of the sfermion mass eigenstates by replacing \( h^0 (H^0) \bar{f}_{L,R}^* \bar{f}_{L,R} \) by \( \pm \sin 2\theta_f h^0 (H^0) \bar{f}_{1,2} \bar{f}_{1,2} \). One can see that maximal sfermion mixings lead to maximal
The MSSM Higgs sector is parametrized by the mass of the CP-odd $M_A$ and $\tan \beta$ while the top quark mass and the associated squark masses enter through radiative corrections\[^{[12]}\]. In our study we will include the leading corrections only, where the light Higgs mass is given by:

$$m_{h^0,H^0}^2 = \frac{1}{2} \left[ m_{AZ}^2 \mp \sqrt{m_{AZ}^4 - 4m_A^2m_Z^2 \cos^2(2\beta) - 4\epsilon \left( m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 \right)} \right]$$

$$\tan(2\alpha) = \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2 + \epsilon / \cos(2\beta)} \tan(2\beta) - \frac{\pi}{2} \leq \alpha \leq 0 \ . \quad (11)$$

with

$$m_{AZ}^2 = m_A^2 + m_Z^2 + \epsilon \ , \quad \epsilon = \frac{3G_F m_t^4}{\sqrt{2}\pi^2} s_\beta^4 \log \left( \frac{m_t m_{\tilde{Q}_3}}{m_t^2} \right) \ . \quad (12)$$

Our inputs are fixed in the following ranges: $2 \leq \tan \beta \leq 60$, $100 \leq M_A \leq 250$ GeV, $500 \leq \mu \leq 6000$ GeV, $-6000 \leq A_0 \leq -500$ GeV, $100 \leq m_{\tilde{f}_1} \leq 200$ GeV. In figures 2, 3, 4 and 5, the number beside each line represents the number of standard deviation from the central value of $\delta a_\mu$. Since we need large LR mixing, we need some degree of cancellation between $LL$ and $LR$ components to achieve the above range for light sfermions. Therefore, all the four entries of the sfermion mass matrices have the same or der of magnitude. In Fig.2 we show the allowed regions in the $(\tan \beta, \mu)$ and $(\tan \beta, A_0)$ plane. We observe that the central value of $\delta a_\mu$ can be easily accommodated with relatively large $\tan \beta$ and large $\mu$ or $A_0$. Even $\mu$ and $A_0$ around 500 GeV, are compatible with $\delta a_\mu$.

In Fig. 3 and 4 it can be seen that light sfermion $m_{\tilde{f}_1}$ and light CP-odd $M_A$ are favored both for moderate and large $\tan \beta$. We have checked that the dominant contribution comes from the heavy CP-even scalar rather than light one. Although the former is suppressed by its mass the enhancement due to the loop function is more important.

In Fig. 5 we show the $\delta a_\mu$ dependence on $\mu$ and $A_0$ for light CP-odd scalar and light sfermion, $M_A = m_{\tilde{f}_1} = 100$ GeV for $\tan \beta = 50$ (left panel) and $\tan \beta = 35$ (right panel). For large $\tan \beta$, even $\mu \sim -A_0 \sim 0.5$ TeV can accommodate $\delta a_\mu$. For moderate $\tan \beta$, we need at least $\mu \sim -A_0 \sim 1$ TeV to explain $\delta a_\mu$.

**IV. DISCUSSIONS AND CONCLUSIONS**

In this paper we considered the contribution of two-loop Barr-Zee type diagram with scalar fermion and neutral Higgs exchange to $(g - 2)_\mu$ in the effective SUSY model. It turns out that only the neutral higgs ($S = h^0, H^0$) and photon exchange ($V = \gamma$) give sizable contributions. It is possible to produce a large enough value to account for the discrepancy of $\delta a_\mu$ between the SM prediction and the BNL measurement. We showed that moderate to large $\tan \beta$ (30 to 50) and large mixing between the left-right sfermions can easily explain $\delta a_\mu$ without invoking the lightness of the 2nd generation smuon.
We found that the sfermions should be light enough (< 200 GeV) to be produced in the future experiments, such as the Tevatron Run II or LHC.

In our scenario, since the light stop is relatively light, it can contribute to the FCNC process $b \rightarrow s\gamma$ through the one-loop chargino and stop contribution. In addition, the charged Higgs mass is of the same order as the pseudoscalar higgs mass $M_A$, its contribution is also relevant. However, to study the MSSM contribution to $b \rightarrow s\gamma$, we need to specify the $SU(2)$ gaugino mass, $M_2$, which enters the chargino mass matrix. Since the parameter space we considered is independent of $M_2$, we do not further investigate $b \rightarrow s\gamma$ in our scenario. We just note that the positive $\mu$ which is favoured by $\delta a_\mu$ in our analysis is also favoured by $B(b \rightarrow s\gamma)$. This is because for $\mu > 0$ the chargino contribution interferes destructively with the SM and charged Higgs contribution, which can help evade strong $B(b \rightarrow s\gamma)$ constraint.

**Note Added:** While we were finishing this work, we received a paper [13] dealing with similar two-loop Barr-Zee type diagrams. However, in our paper we consider the neutral higgs and photon exchange, whereas [13] analyses the charged Higgs and $W^\pm$ gauge boson contributions. Their result seems to prefer the same sign of $\mu$ and $A_0$ as ours, which means that their result and our result can be added constructively. This can further increase the allowed parameter regions where BNL data can be accommodated.

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FIG. 1. Generic Barr-Zee type two-loop diagram.

FIG. 2. Contours of the two-loop sfermion contribution to $\delta a_\mu$ in $(\tan \beta, \mu)$ plane with $M_A = 100, m_{\tilde{f}_1} = 100, A_0 = -3000$ GeV (left panel), and in $(\tan \beta, A_0)$ plane with $M_A = 100, m_{\tilde{f}_1} = 100, \mu = 3000$ GeV (right panel).
FIG. 3. Contours of the two-loop sfermion contribution to $\delta a_\mu$ in $(\tan \beta, m_{\tilde{f}_1})$ plane with $M_A = 100$, $\mu = 3000$, $A_0 = -3000$ GeV (left panel), and in $(\tan \beta, M_A)$ plane with $A_0 = -3000$, $m_{\tilde{f}_1} = 100$, $\mu = 3000$ GeV (right panel).

FIG. 4. Contours of the two-loop sfermion contribution to $\delta a_\mu$ in $(m_{\tilde{F}_1}, \mu)$ plane with $M_A = 100$, $A_0 = -3000$ GeV and $\tan \beta = 50$ (left panel), and in $(m_{\tilde{F}_1}, A_0)$ plane with $\mu = 3000$, $M_A = 100$ GeV and $\tan \beta = 50$ (right panel).
FIG. 5. Contours of the two-loop sfermion contribution to $\delta a_\mu$ in $(A_0, \mu)$ plane with $M_A = 100$, $m_{\tilde{f}} = 100$ GeV and $\tan \beta = 50$ (left panel), and in $(A_0, \mu)$ plane with $m_{\tilde{f}} = 100$, $M_A = 100$ GeV and $\tan \beta = 35$ (right panel).