Privacy of Distributed Optimality Schemes in Power Networks

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Abstract—The increasing participation of local generation and controllable demand units within the power network motivates the use of distributed schemes for their control. Simultaneously, it raises two issues; achieving an optimal power allocation among these units, and securing the privacy of the generation/demand profiles. This study considers the problem of designing distributed optimality schemes that preserve the privacy of the generation and controllable demand units within the secondary frequency control timeframe. We propose a consensus scheme that includes the generation/demand profiles within its dynamics, keeping this information private when knowledge of its internal dynamics is not available. However, the consumption profiles may be inferred using knowledge of its internal model. We resolve this by proposing a privacy-preserving scheme which ensures that the generation/demand cannot be inferred from the communicated signals. For both proposed schemes, we provide analytic stability, optimality and privacy guarantees and show that the secondary frequency control objectives are satisfied. The presented schemes are distributed, locally verifiable and applicable to arbitrary network topologies. Our analytic results are verified with simulations on a 140-bus system, where we demonstrate that the proposed schemes offer enhanced privacy properties, enable an optimal power allocation and preserve the stability of the power network.

Index Terms—Privacy, smart grids, automatic frequency control, power system control.

NOMENCLATURE

Main symbols used within the paper.

Functions & Indices
\[ \dot{x} \quad \text{time derivative of function of time } x \]
\[ x^* \quad \text{equilibrium point of variable } x \]

Sets
\[ \tilde{E} \quad \text{set of communication lines in the Primal-Dual scheme} \]
\[ E \quad \text{set of transmission lines} \]
\[ N \quad \text{set of buses} \]
\[ N^G \quad \text{set of generation units at bus } j \]
\[ N^L \quad \text{set of controllable demand units at bus } j \]
\[ N^p \quad \text{set of buses preceding bus } j \text{ in the power network} \]
\[ N_j^s \quad \text{set of buses succeeding bus } j \text{ in the power network} \]
\[ E_j^s \quad \text{set of communication lines in the Extended Primal-Dual and Privacy-Preserving schemes} \]
\[ \tilde{N} \quad \text{set of prosumption units} \]

Variables & Parameters
\[ \eta_{ij} \quad \text{power angle difference between bus } i \text{ and bus } j \]
\[ \tilde{H}_j, H \quad \text{incidence matrices associated with the communication graphs of the Primal-Dual, and the Extended Primal-Dual and Privacy-Preserving schemes respectively} \]
\[ \omega_j \quad \text{frequency deviation at bus } j \]
\[ \psi_{i,j} \quad \text{integral of power command difference between units } i \text{ and } j \]
\[ \tilde{s}_j \quad \text{vector with all generation and controllable and uncontrollable demand units at bus } j \]
\[ \zeta_j \quad \text{aggregate demand minus generation at bus } j \]
\[ B_{ij} \quad \text{line susceptance between bus } i \text{ and bus } j \]
\[ d_{k,j} \quad \text{demand of the } k\text{th uncontrollable demand unit at bus } j \]
\[ D_j \quad \text{frequency damping at bus } j \]
\[ M_j \quad \text{generator inertia at bus } j \]
\[ n \quad \text{privacy-enhancing signal} \]
\[ p^c_j \quad \text{power command at bus } j \]
\[ p^L_{k,j} \quad \text{power command variable at } k\text{th prosumption unit at bus } j \]
\[ p_k^L \quad \text{kth uncontrollable demand unit at bus } j \]
\[ p_{k,j}^M \quad \text{kth mechanical power injection at bus } j \]
\[ p_{ij} \quad \text{power transfer from bus } i \text{ to bus } j \]
\[ x_{k,j} \quad \text{internal states of } k\text{th mechanical power injection unit at bus } j \]

I. INTRODUCTION

A. Motivation and Literature Survey

The increasing penetration of renewable sources of generation is expected to cause more frequent...
generation-demand imbalances within the power network, which may harm power quality and even cause blackouts [2]. Controllable demand is considered to be a means to address this issue, since loads may provide a fast response to counterbalance intermittent generation [3]. However, the increasing number of such active units makes traditionally implemented centralized control schemes expensive and inefficient, motivating the adoption of distributed schemes. Such schemes offer many advantages, such as scalability, reduced expenses associated with the necessary communication infrastructure and enhanced reliability due to the absence of a single point of failure.

The introduction of controllable loads and local renewable generation raises an issue of economic optimality in the power allocation. In addition, the introduction of smart meters for the monitoring of generation and demand units poses a privacy threat for the citizens, since readings may be used to expose customers daily life and habits, by inferring the users energy consumption patterns and types of appliances [4]. For example, this issue led the Dutch Parliament to prohibit the deployment of smart meters until the privacy concerns are resolved [5], as well as several counties and cities in California to vote for making smart meters illegal in their jurisdictions [6]. These concerns motivate the design of distributed schemes that will simultaneously achieve an optimal power allocation and preserve the privacy of local prosumption profiles.

In recent years, various studies considered the use of decentralized/distributed control schemes for generation and controllable demand with applications to both primary [7], [8], [9], [10] and secondary [11], [12], [13], [14] frequency regulation, where the objectives are to ensure generation-demand balance and that the frequency attains its nominal value at steady state respectively. In addition, the problem of obtaining an optimal power allocation within the secondary frequency control timeframe has received broad attention in the literature [15], [16], [17]. These studies considered suitably constructed optimization problems and designed the system equilibria to coincide with the solutions to these problems. In many studies, the control dynamics were inspired from the dual of the considered optimization problems [13], [18], [19]. Such schemes, usually referred to in the literature as Primal-Dual schemes, yield an optimal power allocation and at the same time enable the satisfaction of operational constraints. Alternative distributed schemes, which ensure that frequency attains its nominal value at steady state by using the generation outputs, have also been proposed [20], [21]. However, the use of real-time knowledge of the generation and controllable demand in the proposed schemes may compromise the privacy of prosumers.

The topic of preserving the privacy of generation and demand units has recently attracted wide attention in the literature. Different types of privacy concerns, resulting from the integration of information and communication technologies in the smart grid, are mentioned in [22]. In addition, [23] analyzes various smart grid privacy issues and discusses recently proposed solutions for enhanced privacy, while [24] proposes a privacy-preserving power request scheme. In addition, [25] uses the differential privacy framework to provide privacy guarantees and [26] studies the effect of differential privacy on smart metering data. Moreover, homomorphic encryption has been used in [27] to enable the direct connection and exchange of data between electricity suppliers and final users, while preserving the privacy in the smart grid. A privacy-preserving aggregation scheme is proposed in [28] which considers various security threats. A network equivalent approach is developed in [29] which preserves the information privacy of integrated electricity and heat systems. The use of energy storage units to preserve the privacy of user consumption has been considered in [30] and [31]. Furthermore, [32] and [33] aim to simultaneously preserve the privacy of individual agents and enable an optimal power allocation using homomorphic encryption and differential privacy respectively. Both approaches result in suboptimal allocations, which suggest a trade-off between optimality and privacy. Several existing techniques that aim at preventing disclosure of private data are also discussed in [34].

Although the problems of preserving the privacy of power prosumption and obtaining an optimal power allocation in power networks have been independently studied, to the authors best knowledge, no study has managed to simultaneously achieve these objectives. In addition, the impact of such schemes on the stability and dynamic performance of the power grid has not been investigated. This study aims to jointly consider these objectives within the secondary frequency control timeframe.

Contribution: This paper studies the problem of providing optimal frequency regulation within the secondary frequency control timeframe while preserving the privacy of generation and controllable demand profiles. We first propose an optimization problem that ensures that secondary frequency regulation objectives, i.e., achieving generation-demand balance and frequency attaining its nominal value at steady state, are satisfied. In addition, to facilitate the interpretation of our privacy results, we define two types of eavesdroppers; (i) naive eavesdroppers, that do not possess/make use of knowledge of the system dynamics to analyze the intercepted information and (ii) informed or intelligent eavesdroppers that use knowledge of the underlying system dynamics to infer the prosumption profiles.

We consider a distributed scheme that has been extensively studied in the literature, usually referred to as the Primal-Dual scheme, that enables an optimal power allocation and the satisfaction of system constraints, and explain why it causes privacy issues. Inspired by the Primal-Dual scheme, we propose the Extended Primal-Dual scheme that incorporates a distributed controller at each privacy-seeking unit of the power grid. The latter replaces the communication of prosumption profiles with a consensus signal providing privacy against naive eavesdroppers. However, we explain how intelligent eavesdroppers may infer the prosumption profiles using the communicated signal trajectories and knowledge of the underlying system dynamics. To resolve this, we propose the Privacy-Preserving scheme, which incorporates two important features into the Extended Primal-Dual scheme, such that privacy against intelligent eavesdroppers is achieved. In particular, the proposed scheme continuously alters the speed of response of each controller, making model
based inference inaccurate. Moreover, it adds bounded noise to the prosunption information within each controller, with a maximum magnitude proportional to the local frequency deviation. The latter yields changes in all controllers when a disturbance occurs, making it hard to detect the origin of the disturbance. These properties ensure that the Privacy-Preserving scheme guarantees the privacy of the prosunption units against intelligent eavesdroppers. On the other hand, due to its additional features, the Privacy-Preserving scheme could potentially result in slower convergence, since the controllers response speed is reduced. For both proposed schemes, we provide analytic stability guarantees and show that an optimal power allocation is achieved at steady state. In addition, the proposed schemes are distributed and applicable to arbitrary network topologies, while the proposed conditions are locally verifiable.

Our analytic results are illustrated with numerical simulations on the NPCC 140-bus system which validate that the proposed schemes enable an optimal power allocation and satisfy the secondary frequency regulation objectives. In addition, we demonstrate how the Extended Primal-Dual and the Privacy-Preserving schemes offer privacy of the prosunption profiles against naive and intelligent eavesdroppers respectively.

To the authors best knowledge, this is the first study that:
(i) Jointly studies the privacy, optimality and stability properties of distributed schemes within the secondary frequency control timeframe.
(ii) Proposes distributed schemes that yield an optimal power allocation and simultaneously preserve the privacy of the prosunption profiles. In particular, the proposed schemes offer privacy guarantees against naive (Extended Primal-Dual scheme) and informed (Privacy-Preserving scheme) eavesdroppers respectively. For the proposed schemes, we show that stability is guaranteed and that the secondary frequency control objectives are satisfied.

Paper structure: In Section II we present the dynamics of the power network, the considered optimization problem and the problem statement. In Section III we examine the Primal-Dual scheme and discuss its privacy issues. In Sections IV and V we present the proposed Extended Primal-Dual and Privacy-Preserving schemes respectively and provide our main analytic results. In Section VI we validate our main results through numerical simulations on the NPCC 140-bus system. Finally, conclusions are drawn in Section VII. The proofs of the main analytic results (Theorems 1, 2, Propositions 1–6 and Lemma 1) are omitted due to space restrictions and are provided in [35].

Notation: Real numbers and the set of n-dimensional vectors with real entries are denoted by \( \mathbb{R} \) and \( \mathbb{R}^n \) respectively. The \( p \)-norm of a vector \( x \in \mathbb{R}^n \) is given by \( \|x\|_p = (|x_1|^p + \cdots + |x_n|^p)^{1/p} \), \( 1 \leq p < \infty \). A function \( f : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is said to be locally Lipschitz continuous at \( x \) if there exists some neighbourhood \( X \) of \( x \) and some constant \( L \) such that \( \|f(x) - f(y)\| \leq L\|x - y\| \) for all \( y \in X \), where \( \| \cdot \| \) denotes any \( p \)-norm. A matrix \( A \in \mathbb{R}^{n \times n} \) is called diagonal if \( A_{ij} = 0 \) for all \( i \neq j \). In addition, \( A \preceq 0 \) indicates that the matrix \( A \) is negative semi-definite. The image of a vector \( x \) is denoted by \( \text{Im}(x) \). The cardinality of a discrete set \( S \) is denoted by \( |S| \). A set \( B \) is a proper subset of a set \( A \) if \( B \subset A \) and \( B \neq A \). For a graph with sets of nodes and edges denoted by \( N \) and \( E \) respectively, we define the incidence matrix \( H \in \mathbb{R}^{|N| \times |E|} \) as follows:

\[
H_{ij} = \begin{cases} 
+1, & \text{if } i \text{ is the positive end of edge } j \in B, \\
-1, & \text{if } i \text{ is the negative end of edge } j \in B, \\
0, & \text{otherwise.}
\end{cases}
\]

An illustrative example of the incidence matrix of a simple graph is presented in Fig. 1. We use \( 0_n \) and \( 1_n \) to denote \( n \)-dimensional vectors with all elements equal to 0 and 1 respectively. Finally, for a state \( x \in \mathbb{R}^n \), we let \( x^* \) denote its equilibrium value.

II. Problem Formulation
A. Power Network Model
We describe the power network by a connected graph \( (N, E) \) where \( N = \{1, 2, \ldots, |N|\} \) is the set of buses and \( E \subseteq N \times N \) the set of transmission lines connecting the buses. The term \((i, j)\) denotes the link connecting buses \( i \) and \( j \). The graph \( (N, E) \) is assumed to be directed with an arbitrary direction, so that if \((i, j) \in E\) then \((j, i) \notin E\). For each \( j \in N\), we define the sets of predecessor and successor buses by \( N_j^- = \{k : (k, j) \in E\} \) and \( N_j^+ = \{k : (j, k) \in E\} \) respectively. It should be noted that the form of the considered dynamics is unaffected by changes in the graph ordering and the results presented in this paper are independent of the choice of direction. The following assumptions are made for the network:
1) Bus voltage magnitudes are \( |V_j| = 1 \) per unit for all \( j \in N \).
2) Lines \((i, j) \in E\) are lossless and characterized by the magnitudes of their susceptances \( B_{ij} = B_{ji} > 0 \).
3) Reactive power flows do not affect bus voltage phase angles and frequencies.
4) The relative phase angles are sufficiently small such that the approximation \( \sin \theta_{ij} = \theta_{ij} \) is valid.

The first three assumptions are standard in secondary frequency regulation studies [8], [9], [11], [13], [14], [36], [37], [38]. These assumptions usually hold in medium to high voltages, associated with transmission systems, where lines are dominantly inductive and voltage variations are small and tightly controlled. The fourth assumption is valid when the network operates in nominal conditions, where relative phase
angles are small. It should be noted that the theoretical results presented in this paper are validated with numerical simulations in Section VI, on a comprehensive power network model.

We use the swing equations to describe the rate of change of frequency at buses [39]. In particular, at each bus we consider a set of generation and controllable and uncontrollable demand units. This motivates the following system dynamics:

\[
\dot{\omega}_j = \omega_j - \omega_j, \quad (i, j) \in \mathcal{E}, \quad (1a)
\]

\[
M_j \dot{\omega}_j = \sum_{k \in \mathcal{N}_j^G} p_{kj}^M - \sum_{k \in \mathcal{N}_j} d_{kj}^e - \sum_{k \in \mathcal{N}_j} G_{kj} \delta_{kj} - D_j \dot{\omega}_j - \sum_{i \in \mathcal{N}_j} p_{ij} + \sum_{j \in \mathcal{N}_j^p} p_{ij}, \quad j \in \mathcal{N}, \quad (1b)
\]

\[
p_{ij} = B_{ij} \eta_{ij}, \quad (i, j) \in \mathcal{E}. \quad (1c)
\]

In system (1), variable \(\omega_j\) represents the deviation of the frequency at bus \(j\) from its nominal value, namely 50 Hz (or 60 Hz). Variable \(p_{kj}^M\) represents the mechanical power injection associated with the \(k\)th generation unit at bus \(j\). Moreover, \(d_{kj}^e\) denotes the demand associated with the \(k\)th controllable load at bus \(j\). \(\mathcal{N}_j^G\) and \(\mathcal{N}_j^p\) represent the sets of generation units and controllable loads, which are jointly referred to as active elements or active units, at bus \(j\) respectively. Each of these units is associated with a privacy-seeking user or entity. The set of active units at bus \(j\) is given by \(\mathcal{N}_j = \mathcal{N}_j^G \cup \mathcal{N}_j^p\). The variable \(p_{kj}^M\) represents the uncontrollable demand associated with the \(k\)th active unit at bus \(j\). Furthermore, the time-dependent variables \(\eta_{ij}\) and \(p_{ij}\) represent, respectively, the power angle difference and the power transmitted from bus \(i\) to bus \(j\). The quantities \(B_{ij}\) represent the line susceptances between buses \(i\) and \(j\). Finally, the positive constants \(D_j\) and \(M_j\) represent the generation damping and inertia at bus \(j\) respectively. The generation and consumption will be jointly referred to as prosumption.

Remark 1: An alternative, but equivalent, representation of (1) could include a single variable at each bus representing the aggregation of uncontrollable demand. We opted to associate uncontrollable loads with active units to facilitate the study of their privacy properties. The benefits of this representation are evident in Sections IV and V. Note that when no uncontrollable load is associated with some generation or controllable demand unit, then \(p_{kj}^M = 0\).

B. Generation and Controllable Demand Dynamics

We will study the behavior of the power system under the following dynamics for generation and controllable loads,

\[
\tau_{kj} \dot{x}_{kj} = -x_{kj} + m_{kj} (u_{kj} - \omega_j), \quad k \in \mathcal{N}_j^G, \quad j \in \mathcal{N}, \quad (2a)
\]

\[
p_{kj}^M = x_{kj} + h_{kj} (u_{kj} - \omega_j), \quad k \in \mathcal{N}_j^G, \quad j \in \mathcal{N}, \quad (2b)
\]

\[
d_{kj}^e = -h_{kj} (u_{kj} - \omega_j), \quad k \in \mathcal{N}_j^p, \quad j \in \mathcal{N}, \quad (2c)
\]

where \(x_{kj} \in \mathbb{R}\) represents the internal state, and \(\tau_{kj} > 0\) and \(m_{kj} > 0\) the time and droop constants associated with generation unit \(k\) at bus \(j\) respectively. The positive constant \(h_{kj}\) represents the damping associated with active unit \(k\) (generation or controllable load) at bus \(j\). In addition, \(u_{kj}\) represents the control input to the \(k\)th active unit at bus \(j\), the dynamics of which are discussed in the following sections. It should be noted that generation and controllable demand units, as well as their inputs, evolve in continuous time.

We consider first-order generation dynamics and static controllable demand for simplicity and to keep the focus of the paper on developing a privacy-preserving scheme. More involved generation and demand dynamics could be considered by applying existing results (e.g., [8], [11], [19]).

For convenience, we define the vectors \(p_j^M = [p_{kj}^M]_{k \in \mathcal{N}_j^G}\), \(d_j^e = [d_{kj}^e]_{k \in \mathcal{N}_j^p}\), \(p_j^L = [p_{kj}]_{k \in \mathcal{N}_j}, p^M = [p_j^M]_{j \in \mathcal{N}}, d^e = [d_j^e]_{j \in \mathcal{N}}\) and \(p^L = [p_j^L]_{j \in \mathcal{N}}\).

C. Prosumption Cost Minimization Problem

In this section we form an optimization problem that aims to minimize the costs associated with generation and controllable demand and simultaneously achieve generation-demand balance. The considered optimization problem is described below.

A cost \(\frac{1}{2} q_{kj} (p_{kj}^M)^2\) is incurred when the generation unit \(k\) at bus \(j\) produces a power output of \(p_{kj}^M\). In addition, a cost \(\frac{1}{2} q_{kj} (d_{kj}^e)^2\) is incurred when controllable load \(k\) at bus \(j\) adjusts its demand to \(d_{kj}^e\). It should be noted that quadratic cost functions are considered since these can locally approximate general convex cost functions. The optimization problem is to obtain the vectors \(p^M\) and \(d^e\) that minimize the cost associated with the aggregate generation and controllable demand and simultaneously achieve power balance. The considered optimization problem is presented below.

\[
\min_{p^M, d^e} \sum_{j \in \mathcal{N}} \left( \sum_{k \in \mathcal{N}_j^G} \frac{1}{2} q_{kj} (p_{kj}^M)^2 + \sum_{k \in \mathcal{N}_j^p} \frac{1}{2} q_{kj} (d_{kj}^e)^2 \right)
\]

subject to

\[
\sum_{j \in \mathcal{N}} \left( \sum_{k \in \mathcal{N}_j^G} p_{kj}^M - \sum_{k \in \mathcal{N}_j^p} d_{kj}^e - \sum_{k \in \mathcal{N}_j} p_{kj}^L \right) = 0. \quad (3)
\]

The equality constraint in (3) requires all the uncontrollable loads to be matched by the generation and controllable demand, such that generation-demand balance is achieved. The equality constraint also guarantees that the frequency attains its nominal value at equilibrium, which is a main objective of secondary frequency control. The latter follows by summing (1b) at steady state over all buses, which yields \(\sum_{j \in \mathcal{N}_j} D_j \dot{\omega}_j = 0\), and noting that frequency synchronizes at equilibrium from (1a).

Remark 2: The optimization problem (3) does not consider any power line constraints. The incorporation of such constraints has been studied in the literature [14], [16], where suitable approaches have been developed. We opted not to include power line constraints in this study to keep its focus on the privacy aspect of the control design.
D. Eavesdropper and Privacy Definitions

In this section, we define the two considered eavesdropper types, inspired from [40], and present two notions of privacy to facilitate the interpretation and intuition of our results.

Definition 1: An eavesdropper is a person or entity that aims to extract private information by intercepting the signals communicated to and from generation and controllable demand units. Eavesdroppers are classified as follows:

(i) Naive eavesdroppers, who possess knowledge of:
   (K1) All signals communicated to and from a given unit, for which it aims to obtain private information.
(ii) Informed or intelligent eavesdroppers, who possess knowledge of K1 and:
   (K2) The underlying control dynamics of the system.

Definition 1 presents two types of eavesdroppers, based on whether they make use of knowledge of the underlying system dynamics to infer private information. In particular, naive eavesdroppers have no knowledge of the system model on whether they make use of knowledge of the underlying system dynamics to infer private information. They only try to overhear sensitive information. Informed eavesdroppers analyze the intercepted signals using knowledge of the underlying dynamics. It is intuitive to note that privacy against intelligent eavesdroppers implies privacy against naive eavesdroppers but not vice versa.

Remark 3: It should be noted that naive and intelligent eavesdroppers are assumed to possess knowledge of K1 and K1 and K2 respectively throughout the considered time duration. In addition, it is assumed that naive and intelligent eavesdroppers stay as such throughout the considered time duration, i.e., knowledge of K2 is not gained or lost by eavesdroppers during the considered timeframe.

Below we provide a definition of a private prosumption trajectory and profile, used throughout the rest of the manuscript. We remind that $s^*$ denotes the equilibrium value of $s$, i.e., $s^* = \lim_{t \to \infty} s(t)$.

Definition 2: The following two notions of prosumption privacy are considered:

(i) A prosumption trajectory is called private against an eavesdropper type if the knowledge available to the eavesdropper does not allow the estimation of $s(t)$, $t \geq 0$, $s \neq s^*$.

(ii) A prosumption profile is called private against an eavesdropper type if the knowledge available to the eavesdropper does not allow the estimation of its trajectory and steady state values, i.e., of $s(t)$, $t \geq 0$.

The considered privacy definition implies that a prosumption trajectory is private when an eavesdropper cannot accurately estimate its initial condition and values when not at steady state. The privacy of a prosumption profile requires in addition the privacy of its steady state value. The distinction between the two notions enables privacy guarantees based on different conditions (see Section V-B).

E. Problem Statement

This paper aims to design control schemes that enable stability and optimality guarantees and at the same time preserve the privacy of all active units. The problem is stated below.

Problem 1: Design a control scheme that:

(i) Preserves the privacy of the prosumption profiles against intelligent eavesdroppers.
(ii) Enables asymptotic stability guarantees.
(iii) Uses local information and locally verifiable conditions.
(iv) Yields an optimal steady-state power allocation.
(v) Applies to arbitrary connected network configurations.

Problem 1 aims to design a control scheme that enables stability guarantees, ensures an optimal power allocation at steady state, and guarantees the privacy of the generation/demand profiles against informed eavesdroppers, following Definitions 1 and 2. In addition, we aim to design a scheme that relies on locally available information and locally verifiable conditions, to enable scalable designs. Finally, it is desired that the proposed scheme is applicable to general network topologies.

III. PRIMAL-DUAL SCHEME

In this section we examine a distributed scheme that has been widely studied in the literature [8], [13], [16], [18], usually referred to as the Primal-Dual scheme, that enables an optimal power allocation, and discuss its resulting privacy issues.

To describe the Primal-Dual scheme, we consider a connected communication graph $(\mathcal{N}, \mathcal{E})$, where $\mathcal{E}$ represents the set of communication links among the buses, i.e., $(i,j) \in \mathcal{E}$ if buses $i$ and $j$ communicate. In addition, we let $\tilde{H}$ be the incidence matrix of $(\mathcal{N}, \mathcal{E})$ and define the variable $\zeta_j = \mathbf{1}_{|\mathcal{N}|}^T \R_j^D + \mathbf{1}_{|\mathcal{N}|}^T \R_j^E - \mathbf{1}_{|\mathcal{N}|}^T \R_j^M$ for all $j \in \mathcal{N}$. The prosumption input dynamics are given by

\begin{align}
\dot{\hat{\psi}}_j &= \tilde{H}^\top \hat{\rho}^j, \\
\hat{\rho}^j &= \zeta - \tilde{H} \psi, \\
u_{k,j} &= p_j^f, k \in \mathcal{N}_j, j \in \mathcal{N},
\end{align}

where the diagonal matrices $\hat{\Gamma} \in \mathbb{R}^{(|\mathcal{E}| \times |\mathcal{E}|)}$ and $\Gamma \in \mathbb{R}^{(|\mathcal{N}| \times |\mathcal{N}|)}$ contain the positive time constants associated with (4a) and (4b) respectively and $p_j^f$ is a power command variable associated with bus $j$ and shared with communicating buses. In addition, variable $\psi$ is a state of the Primal-Dual scheme that integrates the difference in power command variables between communicating buses. The input for all active elements at bus $j$ is given by the local power command value $p_j^f$, via (4c).

The Primal-Dual scheme aims to achieve two objectives through its dynamics. Firstly, it aims to ensure that the secondary frequency control objectives, i.e., ensuring generation/demand balance and the frequency attaining its nominal value, are satisfied at steady state. The latter follows by summing (1b) and (4b) at steady state over all $j \in \mathcal{N}$, which yield $\sum_{j \in \mathcal{N}} \zeta_j = 0$ and $\sum_{j \in \mathcal{N}} (\theta^f_j + \bar{\delta}^E_j + D \omega^f_j - p_j^M) = 0$, respectively, which implies that $\omega^f = 0$ from the synchronization of frequency at equilibrium, as follows from (1a). The dynamics in (4a) enable the synchronization of the power command variables at steady state. This property is useful to provide an optimality interpretation of the system’s equilibria. It should be noted that the stability and optimality of the Primal-Dual scheme (4)
a wide class of generation/demand dynamics, including those in (2), have been analytically shown in the literature (e.g., [8]).

Remark 4: A shortcoming of the Primal-Dual scheme (4) is the requirement for real-time knowledge of the generation and demand from all active units in the network. In practice, this would require the transmission of this information to a central controller at each bus, in order to calculate \( \zeta_j \), exposing the local generation/demand profiles to a naive eavesdropper who intercepts these signals. The latter compromises the privacy of the prosumption profiles.

IV. EXTENDED PRIMAL-DUAL SCHEME

In this section, we aim to resolve the privacy issues associated with the Primal-Dual scheme. To achieve this we propose the Extended Primal-Dual scheme, which enables the privacy of the prosumption profiles against naive eavesdroppers and simultaneously retains the beneficial properties of the Primal-Dual scheme in terms of being distributed and achieving an optimal power allocation at steady state.

A. Extended Primal-Dual Scheme

In this section we present a scheme that aims to improve the privacy properties of the generation/demand profiles. In contrast to (4), which includes a controller at each bus, the proposed scheme employs a controller at each privacy-seeking unit (generator or controllable load). We demonstrate that the presented scheme offers privacy against naive eavesdroppers and simultaneously enables an optimal power allocation.

To describe the new scheme, we consider a communication network characterized by a connected graph \((\mathcal{N}, \mathcal{E})\), where \(\mathcal{N} = \bigcup_{j \in \mathcal{N}_j} N_j\) represents the set of active units within the power network and \(\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}\) the set of connections. Moreover, we let \(H \in \mathbb{R}^{(\mathcal{N} \times |\mathcal{N}|, |\mathcal{E}|)}\) be the incidence matrix of \((\mathcal{N}, \mathcal{E})\). In addition, the following variables are defined for compactness in presentation,

\[
\begin{align*}
\tilde{s}_j^T &= \left( -p^M \right)^T, \quad \left( \tilde{d}_j^c \right)^T, \quad j \in \mathcal{N}, \quad (5a) \\
\tilde{y}_j &= s_j + \tilde{p}_j^c, \quad j \in \mathcal{N}, \quad (5b)
\end{align*}
\]

where \(\tilde{y} \in \mathbb{R}^{(\mathcal{N})}\) is a vector with all generation and controllable and uncontrollable demand units.

The proposed Extended Primal-Dual scheme, is presented below

\[
\begin{align*}
\bar{\Gamma} \bar{\psi} &= H^T \bar{p}^c, \quad (6a) \\
\Gamma \tilde{p}^c &= \tilde{y} - H \psi, \quad (6b) \\
u &= p^c, \quad (6c)
\end{align*}
\]

where \(\bar{\Gamma} \in \mathbb{R}^{(\tilde{E} \times |\tilde{E}|)}\) and \(\Gamma \in \mathbb{R}^{(\mathcal{N} \times |\mathcal{N}|)}\) are diagonal matrices containing the positive time constants associated with (6a) and (6b) respectively, and \(p^c_{k,j}\) corresponds to the power command variable associated with active unit \(k\) at bus \(j\), that is also used as the input to (2) following (6c). A schematic representation of the system (1), (2), (5), (6) is provided in Fig. 2.

Remark 5: The proposed Extended Primal-dual scheme assumes communication among prosumption units by considering the connected graph \((\mathcal{N}, \mathcal{E})\). Note that when the communication of prosumption is considered for the Primal-Dual scheme, then its communication topology (i.e., a meshed network at bus level with a star structure within each bus to enable communication from prosumption units towards the bus controller) is a special case to that of the Extended Primal-dual scheme. It should also be noted that, apart from connectivity, no assumption is made on the topology of the communication network. The latter allows practical aspects to be considered in the design of the communication network (e.g., communication among buses could be at bus level only).

Following the Extended Primal-Dual scheme, privacy-seeking users share power command signals instead of their...
generation and demand values. Hence, the prosumption profiles are not communicated towards local controllers. The latter suffices to ensure privacy against naive eavesdroppers, since inferring the prosumption profiles from the power command variables would require knowledge of the underlying dynamics. The privacy of prosumption profiles against naive eavesdroppers under the **Extended Primal-Dual scheme** is demonstrated in the following proposition.

**Proposition 1:** Consider any supply unit \( k,j \) implementing the Extended Primal-Dual scheme (6). Then, its prosumption profile \( \tilde{s}_{k,j} \) is private against eavesdroppers with knowledge of \( K_1 \).

**Remark 6:** Proposition 1 demonstrates that the Extended Primal-Dual scheme ensures the privacy of prosumption profiles against naive eavesdroppers. In addition, the Extended Primal-Dual scheme achieves the same objectives as the Primal-Dual scheme, both being inspired by suitable dual decomposition approaches. In particular, it ensures generation/demand balance and that the frequency attains its nominal value at steady state. The latter follows by summing (1b) and (6b) at steady state over all units in \( \mathcal{N} \) and \( \tilde{N} \), which yield \( \sum_{j \in \mathcal{N}} \tilde{s}_{k,j} - D_j \omega_j^* = 0 \) and \( \sum_{j \in \tilde{N}} \tilde{s}_{k,j} = 0 \) respectively, allowing to deduce that \( \omega^* = 0 \) due to (1a). The second objective is to ensure that power command variables are synchronized at steady state, which is achieved through (6a). Hence, the Extended Primal-Dual scheme achieves the objectives of the Primal-Dual scheme and additionally ensures privacy against naive eavesdroppers.

**B. Equilibrium Analysis**

We now provide a definition of an equilibrium point to the interconnected dynamical system (1), (2), (5), (6).

**Definition 3:** The point \( \alpha^* = (\eta^*, \psi^*, \omega^*, x^*, p^c, \tilde{s}) \) defines an equilibrium of the system (1), (2), (5), (6) if all time derivatives of (1), (2), (5), (6) are equal to zero at this point.

We will make use of the following equilibrium equations for (1), (2), (5), (6).

\[
\begin{align*}
0 &= \omega_j^* - \omega_{j^*}, (i,j) \in \mathcal{E}, \\
0 &= \sum_{i \in \mathcal{N}^j} p_{ij}^* + \sum_{j \in \mathcal{N}^j} p_{j^*}^*, j \in \mathcal{N}, \\
0 &= -x_{k,j} + m_{k,j}(u_{k,j} - \omega_j^*), k \in \mathcal{N}^G_j, j \in \mathcal{N}, \\
0 &= H^T \tilde{s}, \\
0 &= \tilde{s}^* - \tilde{H} \psi^*, \\
\end{align*}
\]

where the variables \( p^*, p^M, p^c, u^*, \tilde{s}^* \) satisfy

\[
\begin{align*}
p_{ij}^* &= B_{ij} \eta_{ij}^*, (i,j) \in \mathcal{E}, \\
p_{k,j}^* &= x_{k,j} + h_{k,j}(u_{k,j}^* - \omega_j^*), k \in \mathcal{N}^G_j, j \in \mathcal{N}, \\
d_{j}^* &= -h_{k,j}(u_{k,j}^* - \omega_j^*), k \in \mathcal{N}^L_j, j \in \mathcal{N}, \\
\tilde{u}_{k,j} &= p_{k,j}^c + x_{k,j} + h_{k,j}(u_{k,j}^* - \omega_j^*), k \in \mathcal{N}^L_j, j \in \mathcal{N}.
\end{align*}
\]

The following lemma characterizes the equilibria of (1), (2), (5), (6).

**Lemma 1:** The equilibria of (1), (2), (5), (6) satisfy \( \omega^* = 0 \) and \( p_{c}^* \in \text{Im}(1_{\tilde{N}^G}) \).

Lemma 1 demonstrates that the presented scheme ensures that the frequency attains its nominal value at equilibrium, which is a main objective of secondary frequency control. In addition, it shows that power command variables share the same value at steady state. The latter can be used to enable an optimal power allocation, as demonstrated in the following section.

**C. Optimality and Stability Analysis**

The following proposition provides necessary and sufficient conditions that ensure that the equilibrium values of \( p^M \) and \( d^c \) are global solutions to the optimization problem (3).

**Proposition 2:** Let \( q_k_j(m_{k,j} + h_{k,j}) = 1, k \in \mathcal{N}^G_j, j \in \mathcal{N} \) and \( q_k_j h_{k,j} = 1, k \in \mathcal{N}^L_j, j \in \mathcal{N} \). Then, the equilibrium values \( p^M, p^c \) and \( d^c \) of system (1), (2), (5), (6) globally minimize the optimization problem (3).

Proposition 2 follows directly from the KKT conditions [41]. It demonstrates how the controller gains in generation and controllable load units should be designed such that an optimal power allocation is ensured. Hence, we deduce that the Extended Primal-Dual scheme (6) enables an optimal power allocation.

The following theorem provides global asymptotic stability guarantees for (1), (2), (5), (6).

**Theorem 1:** Solutions to (1), (2), (5), (6) globally asymptotically converge to the set of its equilibria, where \( \omega^* = 0 \).

Theorem 1 guarantees the convergence of solutions to (1), (2), (5), (6) to the set of its equilibria. In addition, the Extended Primal-Dual scheme is locally verifiable and applicable to arbitrary network configurations. Furthermore, the presented scheme guarantees the privacy of the prosumption profiles against naive eavesdroppers. Noting also that Proposition 2 demonstrates how optimality may be achieved at steady state, it follows that the Extended Primal-Dual scheme satisfies all objectives of Problem 1, except from ensuring privacy against intelligent eavesdroppers.

**D. Discussion**

The scheme presented in this section extends the Primal-Dual scheme (4) by including a controller at each unit contributing to secondary frequency control. The Extended Primal-Dual scheme results in the transmission of power command signals instead of prosumption signals, which enables privacy against naive eavesdroppers, as demonstrated by Proposition 1. On the other hand, the interaction between an increased number of controllers may result in slower convergence. The proposed scheme yields an optimal power allocation, ensures that frequency attains its nominal value at steady state and guarantees the global stability of the
power network as follows from Proposition 2, Lemma 1 and Theorem 1 respectively. However, the Extended Primal-Dual scheme (6) does not ensure the privacy of generation and demand profiles against intelligent eavesdroppers. In particular, an intelligent eavesdropper may use the communicated power command trajectories and knowledge of the underlying power command dynamics to infer the prosumption profiles by reversing (6b), i.e., using \( \tilde{s} = \Gamma \hat{p}^c + H \psi \). In the next section, we present a scheme that aims to resolve this issue.

V. PRIVACY-PRESERVING SCHEME

In this section we present a scheme that aims to preserve the beneficial properties of the Extended Primal-Dual scheme described in the previous section and simultaneously guarantee the privacy of the generation/demand profiles against intelligent eavesdroppers.

A. Privacy-Preserving Scheme

The proposed scheme, which shall be referred to as the Privacy-Preserving scheme, incorporates a privacy-enhancing signal \( n \) in the power command dynamics, as follows

\[
\begin{align*}
\tilde{\Gamma} \psi &= H^T \hat{p}^c, \quad (8a) \\
\Gamma \hat{p}^c &= \tilde{s} - H \psi + n, \quad (8b) \\
u &= \hat{p}^c. \quad (8c)
\end{align*}
\]

In (8) above, the locally Lipschitz, privacy-enhancing signal \( n = [n_i]_{i \in \mathcal{N}} \), where \( n_i = [n_{k,j}]_{k \in \mathcal{N}_k} \), adapts the derivative of the power command variables to enable enhanced privacy properties.

The design of the signal \( n \) is crucial in providing enhanced privacy properties and simultaneously enabling stability and optimality guarantees for the Privacy-Preserving scheme (8). Some desired properties of the privacy-enhancing signal \( n \) are: (i) to permit the existence of equilibria, by taking a constant value when the states of the system are at equilibrium, and (ii) to enable an optimality interpretation of the resulting equilibria. Both objectives can be achieved if \( n \) is zero at steady state since in this case the equilibria of (1), (2), (5), (8), and (1), (2), (5), (6) are identical.

The following design condition is imposed on the privacy-enhancing signal \( n \). As demonstrated below, this condition ensures the privacy of the prosumption profiles against intelligent eavesdroppers and allows stability and optimality to be deduced. It should be noted that the trajectories of \( n \) are in general non-unique.

**Design Condition 1**: The privacy-enhancing signals satisfy

(i) \( n_{k,j}(t) = -\xi_k(t)\hat{p}_{k,j}(t) \)

(ii) \( |n_{k,j}(t)| < \beta_{k,j} |o_j(t)| \)

Moreover, the positive design constants \( \beta_{k,j} \) satisfy

\[
\left| \begin{array}{c}
h_{k,j} - D_j/|N_j| \\
h_{k,j} + \beta_{k,j}/2
\end{array} \right| \leq 0, \quad k \in \mathcal{N}_j, j \in \mathcal{N}.
\]

Design Condition 1 splits the privacy-enhancing signal \( n \) to two other signals, \( n^d \) and \( n^l \), that serve different purposes. The signal \( n^d_{k,j} \) is proportional to the power command derivative \( \hat{p}_{k,j} \) with a non-negative, time-varying gain \( \xi_k \) designed such that \( \xi_k < \beta_{k,j} \) is satisfied at all times. The latter adjusts the rate at which the power command variables respond to external signals and makes any prior estimates of the power command model inaccurate. Hence, a potential eavesdropper utilizing model-based observations will produce inaccurate results. The component \( n^l \) introduces a noise signal \( \xi \) that is mixed with the generation/demand values. The latter offers improved privacy properties since: (i) the generation/demand profile information in the controller is distorted, and (ii) it perturbs the communicated signals of all controllers when a disturbance occurs, making it harder to detect the origin of the disturbance from a change in the transmitted signal. Design Condition 1(ii) restricts the magnitude of \( n^l \) in relation with the magnitude of the local frequency. The values of \( \beta_{k,j} \) are selected to satisfy the linear matrix inequality (LMI) in Design Condition 1 such that convergence is guaranteed, as demonstrated in Theorem 2 later on. These properties enable the privacy of prosumption against intelligent eavesdroppers since the same power command trajectories result from a (wide) class of prosumption profiles due to different potential trajectories of the privacy-enhancing signal \( n \). The latter is analytically demonstrated in Section V-B below. In addition, note that since all communicated power command signals synchronize at steady state, their equilibrium values do not convey any information about local generation/demand.

**Remark 7**: The bounds \( \beta_{k,j} \) and \( \hat{\beta}_{k,j} \) associated with \( n^d_{k,j} \) and \( n^l_{k,j} \) respectively are interdependent through the LMI in Design Condition 1. Hence, there is a trade-off between the maximum allowed derivative of the gain \( \xi_k \) and the maximum magnitude ratio between the signal \( n^d_{k,j} \) and the local frequency \( o_j \).

The latter can be used for design purposes by placing different weights on the associated bounds, and hence the effect of signals \( n^d_{k,j} \) and \( n^l_{k,j} \).

**Remark 8**: The implementation of the proposed Extended Primal-Dual and Privacy-Preserving schemes requires suitable monitoring and communication capabilities and an increased number of controllers. These requirements are facilitated by the significant improvement in monitoring, control and communication technologies and their implementation on smart power grids [42], [43]. It should also be noted that privacy issues are mostly motivated as side effects of these improvements which in most cases coincide with enhanced control capabilities.

B. Privacy Analysis

In this section, we present our main privacy results regarding the proposed Privacy-Preserving scheme. First, we clarify that for an intelligent eavesdropper, K1 implies knowledge of all power command signals communicated to and from a considered unit. In addition, K2 implies knowledge of the Privacy-Preserving scheme dynamics (8).

It should be noted that \( n^l \) (and similarly \( t \)) is treated as time-dependent variables rather than random variables, following the assumption that \( n \) is locally Lipschitz. The latter is made for simplicity and to avoid a diversion of the paper focus from the privacy properties of the proposed schemes.
The following proposition demonstrates that the proposed scheme preserves the privacy of the prosumption profiles against intelligent eavesdroppers.

**Proposition 3:** Consider any supply unit \( k,j \) implementing the Privacy-Preserving scheme (8). Then, its prosumption profile \( \tilde{s}_{k,j} \) is private against intelligent eavesdroppers with knowledge of \( K1 \) and \( K2 \).

Proposition 3 provides privacy guarantees for the prosumption profiles when the Privacy-Preserving scheme is implemented. The latter demonstrates that the proposed scheme satisfies objective (i) within Problem 1.

A reasonable case to be considered is when intelligent eavesdroppers gain knowledge of the steady-state value of the privacy-preserving signal \( n \), i.e., have the following knowledge:

(K3) The steady state value of the privacy-preserving signal \( n \), i.e., \( \lim_{\infty} n(t) = 0_{n(|N|)} \).

The following proposition shows that the prosumption trajectories are private against eavesdroppers with knowledge of \( K1, K2 \) and \( K3 \). We remind that the definition of a private trajectory is provided in Definition 2(i).

**Proposition 4:** Consider any supply unit \( k,j \) implementing the Privacy-Preserving scheme (8). Then, its prosumption trajectory is private against intelligent eavesdroppers with knowledge of \( K1, K2 \) and \( K3 \).

Note that Proposition 4 does not guarantee the privacy of the prosumption at steady state, since knowledge of the variable \( \psi \) may yield the equilibrium values of \( \tilde{s} \) from (7e). However, since \( \psi \) results from integrating the differences between communicated power command variables, any inaccuracy on determining these variables will lead to growing deviations between the estimated and true values of \( \psi \), compromising the reliability of such estimate.

Stronger privacy guarantees may be obtained, such that the prosumption profiles are kept private when eavesdroppers have knowledge of \( K3 \), by relaxing \( K1 \). In particular, we consider the case where an eavesdropper does not have full knowledge of the information communicated to a considered unit, i.e., has knowledge of:

(K4) A proper subset of the power command signals communicated to and from a given unit, for which it aims to obtain private information.

The following proposition guarantees the privacy of prosumption profiles when intelligent eavesdroppers have knowledge of \( K2, K3 \) and \( K4 \).

**Proposition 5:** Consider any supply unit \( k,j \) implementing the Privacy-Preserving scheme (8). Then, its prosumption profile \( \tilde{s}_{k,j} \) is private against intelligent eavesdroppers with knowledge of \( K2, K3 \) and \( K4 \).

Proposition 5 enables privacy guarantees of the prosumption profile, when knowledge of the steady state value of \( n \) is available. However, it assumes that the intelligent eavesdropper does not possess full knowledge of the information communicated to and from the considered privacy-seeking unit. Note that the latter case might describe eavesdroppers associated with some prosumption unit that communicates with the considered privacy-seeking unit, under specific conditions on the communication network topology such that \( K4 \) is satisfied.

Remark 9: The presented privacy results hold when either of \( n' \) or \( n'' \) is neglected in \( n \). However, their combined impact keeps additional information associated with the prosumption profiles private. In particular, the presence of \( n' \) results in a change on all controllers after a power disturbance, making it difficult to infer its origin from the power command signals. In addition, \( n'' \) makes model based inference inaccurate, and hence difficult to have a reasonable range estimate of the prosumption magnitude, i.e., obtain an estimate with a margin of error analogous to the magnitude of \( n'_k,j \).

C. Optimality and Stability Analysis

In this section we provide analytic optimality and stability guarantees for system (1), (2), (5), (8).

The following proposition extends Proposition 2 by demonstrating that Design Condition 1 enables an optimal power allocation.

**Proposition 6:** Let Design Condition 1, \( q_k(j|m_{k,j} + h_{k,j}) = 1, k \in N_{f}^k, j \in N \) and \( q_k h_{k,j} = 1, k \in N_{f}^k, j \in N \) hold. Then, the equilibrium values \( p^{M,*} \) and \( d^{*,*} \) of system (1), (2), (5), (8), globally minimize the optimization problem (3).

Proposition 6 demonstrates that when Design Condition 1, and the gain conditions provided in Proposition 2 hold, then the Privacy-Preserving scheme yields an optimal power allocation. The latter follows trivially from Proposition 2, since the privacy-enhancing signal \( n_{k,j} \) is zero at steady state from Design Condition 1, which results in identical equilibrium points for (1), (2), (5), (8) and (1), (2), (5), (6).

The following theorem demonstrates that when Design Condition 1 holds, then the set of equilibria of (1), (2), (5), (8), is attracting. The latter shows that the proposed Privacy-Preserving scheme does not compromise the stability of the power network.

**Theorem 2:** Let Design Condition 1 hold. Then, the solutions of (1), (2), (5), (8), globally asymptotically converge to the set of its equilibria, where \( o^* = \theta_{0|N} \).

Theorem 2 guarantees the convergence of solutions to (1), (2), (5), (8), to the set of its equilibria. In addition, the dynamics of (1), (2), (5), (8), are distributed, applicable to arbitrary network configurations and locally verifiable. Moreover, as demonstrated in Section V-B, the Privacy-Preserving scheme enables the privacy of prosumption profiles against informed eavesdroppers. Finally, as demonstrated in Proposition 6, the presented scheme enables an optimal power allocation among generation and controllable demand. Hence, all objectives of Problem 1 are satisfied.

VI. SIMULATION ON THE NPCC 140-BUS SYSTEM

In this section, we validate our analytic results with simulations using the Power system toolbox [44] on MATLAB.

A. Test System Description

We use the Northeast Power Coordinating Council (NPCC) 140-bus interconnection system. This model is more detailed and realistic than the considered analytical model, including voltage dynamics, line resistances, and a transient reactance.
generator model. The test system consists of 93 load buses and 47 generation buses and has a total real power of 28.55 GW.

B. Simulation Results

For our simulations, controllable demand was considered in 20 load buses, where at each bus the number of controllable loads was randomly selected from an integer uniform distribution with range [90, 180]. A single generation unit was added at each of 20 generation buses. In addition, quadratic cost functions were considered for generation and controllable demand following the description in (3). The values for \( q_{k,j}, k \in N_f, j \in N \) were selected from a uniform distribution with range [50, 250]. For the simulation, a step change in demand of magnitude 0.2 per unit (100 MW) at 10 randomly selected loads at each of buses 2 and 3 was considered at \( t = 1 \) second. The time step for the simulations, denoted by \( \Delta T \), was set at 10 ms.

The system was tested under the four control schemes described below:

(i) An Integral action scheme, where generation units and controllable loads integrate the local frequency with the controller gains selected to be inversely proportional to their respective cost coefficients.

(ii) The Primal-Dual scheme, described by (4).

(iii) The Extended Primal-Dual scheme that we proposed, described by (6).

(iv) The Privacy-Preserving scheme that we proposed, described by (8) and Design Condition 1. First suitable values for \( \hat{q}_{k,j}, \hat{\beta}_{k,j}, k \in N_f, j \in N \) were selected in accordance with the LMI in Design Condition 1. The values of \( \hat{\xi}_{k,j}(t) \) were then randomly selected at each time step such that \( (\hat{\xi}_{k,j}(t) - \hat{\xi}_{k,j}(t - \Delta T))/\Delta T \) lied in \([ - \hat{\beta}_{k,j}, \hat{\beta}_{k,j}]\) following Design Condition 1(i). In addition, the values of \( \hat{n}_{k,j}(t) \) were randomly selected at each time step from the uniform distribution \( [ - \beta_{k,j}[\omega_j], \beta_{k,j}[\omega_j]] \) such that Design Condition 1(ii) was satisfied.

In schemes (ii)-(iv), the dynamics of the implemented generation and controllable demand units followed from (2) and the controller gains were selected such that the optimality conditions presented in Propositions 2 and 6 were satisfied. The communication network associated with scheme (ii) had the same structure as the power network. A random connected communication network was generated when schemes (iii) and (iv) were implemented. For consistency, the same sets of randomly selected parameters were considered in all simulations.

The frequency response at a randomly selected bus (bus 18) is depicted in Fig. 3. From Fig. 3, it follows that the frequency converges to its nominal value at all simulated cases. The latter suggests that the proposed Extended Primal-Dual and Privacy-Preserving schemes yield a stable response. Note also that the frequency returns to within 0.01 Hz from its nominal value in less than two minutes, which is well within the secondary frequency control timeframe. Nevertheless, the

3The details of the simulation model can be found in the Power System Toolbox data file datanp48.
eavesdroppers, we considered an observer scheme that aims to infer the controllable demand using a model of the power command dynamics and knowledge of the power command signals. In particular, by evaluating the power command derivative and the value of $\psi$, an eavesdropper may attempt to observe the generation and controllable demand profiles by reversing (6b), i.e., using $\tilde{s} = \Gamma \dot{p}_c + H\psi$. Figure 6 demonstrates the result from such observer scheme for the same three loads considered in Fig. 5, when the Extended Primal-Dual and Privacy-Preserving schemes are implemented. From Fig. 6, it follows that an intelligent eavesdropper may obtain the controllable demand profiles when the Extended Primal-Dual scheme is applied. By contrast, the application of the Privacy-Preserving scheme ensures that the demand is private against intelligent eavesdroppers, since the retrieved information is distorted by the signal $n_k,j$.

The outcomes of Section VI are synopsized in Table I, which provides a comparison between the properties of each of the considered schemes in terms of enabling convergence, an optimal power allocation and prosumption privacy against naive and intelligent eavesdroppers.

### Table I
| Property / Scheme               | (i) | (ii) | (iii) | (iv) |
|--------------------------------|-----|------|-------|------|
| Convergence                    | ✓   | ✓    | ✓     | ✓    |
| Optimality                     | X   | ✓    | ✓     | ✓    |
| Privacy - Naive Eavesdroppers  | ✓   | X    | ✓     | ✓    |
| Privacy - Intelligent Eavesdroppers | ✓   | X    | X     | ✓    |

C. Statistical Validation

This section aims to offer statistical validation to our simulation results. In our simulations we randomly selected (a) the
number of controllable loads at each of 20 load buses, (b) the loads where the step change in demand was applied, (c) the cost coefficient parameters $q_{k,j}$, $k \in N^u_j$, $j \in N$, and the values for $\hat{x}_{k,j}(t)$ and $\hat{y}_{k,j}(t)$ associated with scheme (iv) and (d) the structure of the (connected) communication graph for schemes (iii) and (iv).

To validate our results for a broad range of randomly selected parameters, we repeated the presented simulations 100 times for each scheme and compared their performance on the following three properties, (a) the frequency at the end of the simulation duration, (b) the synchronization of the marginal cost values at the end of the simulation duration, (b) the ability of eavesdroppers to infer prosumption profile values. The results are shown in Table II. Rows 1 and 2 of Table II demonstrate that in all cases the frequency converges to its nominal value, although schemes (iii) and (iv) result in slower frequency convergence. Nevertheless, in all cases the frequencies converge to within 0.015 Hz from the nominal value within three minutes, which is well within the secondary frequency control timeframe. To produce the third and fourth rows of Table II, we removed the top and bottom 5% of the marginal cost values, to avoid outliers. We then obtained the average and minimum frequency at the end of the simulation duration, the marginal cost variance, the maximum difference between the average and any non-outlier marginal cost values, and the mean absolute prosumption inference error.

| Property / Scheme | (i) | (ii) | (iii) | (iv) |
|-------------------|-----|------|-------|------|
| Mean freq. (Hz)   | -0.0001 | -0.0001 | -0.012 | -0.013 |
| Min freq. (Hz)    | -0.001 | -0.0003 | -0.014 | -0.015 |
| Marginal Cost - Variance | $2 \times 10^{-5}$ | $2 \times 10^{-8}$ | $10^{-6}$ | $7 \times 10^{-7}$ |
| Marginal Cost - Max Difference from Mean | 0.012 | $3 \times 10^{-4}$ | $2 \times 10^{-3}$ | $2 \times 10^{-3}$ |
| Mean Absolute Prosumption Inference Error | N/A | 0 | 0 | 0.14 |

The results demonstrate that for scheme (iii) this approach always allows to infer prosumption while for scheme (iv) there exists a significant mean inference error. Hence, the presented statistical results validate the theoretical results in this paper.

VII. CONCLUSION

We have considered the problem of enabling an optimal power allocation and simultaneously preserving the privacy of generation and controllable demand profiles within the secondary frequency control timeframe. To enhance the intuition on our results, two types of eavesdroppers were defined; naive eavesdroppers that do not possess/make use of knowledge of the internal system dynamics to analyze the intercepted signals and intelligent eavesdroppers that use knowledge of the underlying dynamics to infer the privacy-sensitive prosumption profiles. We proposed the Extended Primal-Dual scheme, which implements a controller at each privacy-seeking unit in the power grid to provide improved privacy properties. The proposed scheme enables privacy guarantees against naive eavesdroppers. However, the generation/demand profiles may be inferred by intelligent eavesdroppers using the communicated signal trajectories and information on the underlying system dynamics. To resolve this issue, we proposed the Privacy-Preserving scheme, which shares the structure of the Extended Primal-Dual scheme but also incorporates a privacy-enhancing signal at each controller. The latter continuously adjusts the response speed of the controllers, making model based observations inaccurate, and disturbs the generation/demand profile information within the controllers.
enabling privacy against intelligent eavesdroppers. For both proposed schemes, we provide analytic stability, optimality and privacy guarantees. Our presented results are distributed, locally verifiable and applicable to general network configurations. The applicability of the proposed schemes is demonstrated with simulations on the NPCC 140-bus system where we show that stability is preserved, and improved privacy properties and an optimal power allocation are attained.

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