Helioseismology has provided very precise information about the solar internal sound speed and density throughout most of the solar interior. The results are generally quite close to the properties of standard solar models. Since the solar oscillation frequencies do not provide direct information about temperature and composition, the helioseismic results to not completely rule out an astrophysical solution to the discrepancy between the predicted and measured neutrino fluxes from the Sun. However, such a solution does appear rather implausible.

1 Introduction

The persistent discrepancy between the measured fluxes of solar neutrinos and the predictions of solar models (e.g. Bahcall for a recent review, see Bahcall, these proceedings) has led to doubts about the reliability of solar-model calculations. Indeed, a large number of suggestions have been made for changes to the properties of the model which might reduce the predicted neutrino fluxes.

The neutrino flux depends principally on the temperature and composition profiles of the solar core but provide little detailed information about the solar interior. In the last decade far more information on the internal structure of the Sun has been obtained from observations of solar oscillations, providing tight constraints on solar models. Recent “standard” solar models are generally in reasonable agreement with the helioseismic inferences. Here I consider the extent to which this agreement argues against an astrophysical solution to the solar neutrino problem. To be definite, I base the analysis on Model S of Christensen-Dalsgaard et al. This was computed with OPAL equation of state and opacity, using the Bahcall & Pinsonneault nuclear parameters, and including settling of helium and heavy elements computed with the Michaud & Proffitt diffusion coefficients. The predicted neutrino capture rates in the $^{37}$Cl and $^{71}$Ga experiments are 8.2 and 132 SNU, respectively, while the flux of high-energy $^8$B neutrinos is $5.9 \times 10^6$ cm$^{-2}$ s$^{-1}$, quite similar to, for example, the corresponding model of Bahcall & Pinsonneault.

In common with other current models of the Sun the model is based on significant simplification, neglecting possible hydrodynamical phenomena in the convectively stable part of the Sun, associated e.g. with instabilities or motion induced by convective penetration. Indeed, it is well established that the Sun
becomes unstable to low-order, low-degree g modes during its evolution\[3\] the non-linear development of the instability might conceivable involve mixing of the solar core.\[4\] Other instabilities, leading to mixing, could be associated with the spin-down from the commonly assumed early rapid rotation\[5\].

2 Inversion for the Solar Internal Sound Speed

The observed solar oscillations are adiabatic almost everywhere, to a very good approximation. Thus their frequencies depend on the distribution of mass, pressure $p$ and density $\rho$, and on the adiabatic compressibility $\Gamma_1 = (\partial \ln p / \partial \ln \rho)_s$, the derivative being at fixed specific entropy $s$. Assuming that the model is spherically symmetric and in hydrostatic equilibrium, the frequencies are completely determined by specifying $\rho(r)$ and $\Gamma_1(r)$, $r$ being the distance to the solar centre, whereas they do not depend directly on temperature. The observed modes are essentially standing acoustic waves, with frequencies depending predominantly on the adiabatic sound speed $c$ given by $c^2 = \Gamma_1 p / \rho$. Departures from this simple description of the oscillations, occurring very near the solar surface, can be eliminated in the analysis of the frequencies.

The differences $\delta \omega_i = \omega_i^{(\text{obs})} - \omega_i^{(\text{mod})}$ between the observed and computed frequencies of the $i$-th mode can be linearized around the model, resulting in

$$\delta \omega_i = \omega_i \frac{\delta c^2}{c^2} + \frac{1}{\omega_i} \int_0^R K_{c^2,\rho}^i(r) \frac{\delta c^2}{c^2}(r) \, dr + \frac{1}{\omega_i} \int_0^R K_{\rho,c^2}^i(r) \frac{\delta \rho}{\rho}(r) \, dr + \frac{F_{\text{surf}}(\omega_i)}{Q_i} + \epsilon_i. \quad (1)$$

Here the integrals extend to the surface radius $R$ of the Sun, $\delta c^2$ and $\delta \rho$ are differences between the Sun and the model in $c^2$ and $\rho$, the kernels $K_{c^2,\rho}^i$ and $K_{\rho,c^2}^i$ are known functions, $F_{\text{surf}}(\omega_i)$ results from the near-surface errors in the model, and $\epsilon_i$ is the error in the observed frequencies. This equation forms the basis for inferring the corrections to solar models\[3\]. The principle is to make linear combinations of eqs (1) with coefficients $c_i(r_0)$ chosen such that the sums corresponding to the last three terms on the right-hand side of eqs (1) are suppressed, while in the first term the averaging kernel $K_{c^2,\rho}(r_0, r) = \sum_i c_i(r_0) K_{c^2,\rho}^i(r)$ is localized in the vicinity of $r = r_0$. The corresponding combination of the left-hand sides, $\sum_i c_i(r_0) \delta \omega_i / \omega_i$, then clearly provides a localized average of $\delta c^2 / c^2$ near $r = r_0$.

Several extensive sets of helioseismic data are now available, including initial results from the GONG network\[18\] and the SOI/MDI project\[19\] on the SOHO satellite. The results presented here are based on a combination of frequencies from the BiSON network\[20\] and the LOWL instrument\[21\] however,
analyses of other, independent sets give results that are generally consistent with those presented here. The inferred difference in squared sound speed between the Sun and the model is shown in Fig. 1. Each point corresponds to an average of $\frac{\delta c^2}{c^2}$, weighted by $K_{c^2}(r_0, r)$; however, as indicated these averages are relatively well localized in $r$. Also, the propagated data errors are small. Thus the procedure has succeeded in providing precise and well resolved measures of the sound-speed errors in the model, even quite close to the centre of the Sun. The differences are evidently highly significant; nonetheless, it is striking that the model reproduces the solar sound speed to within a small fraction of a per cent. This has been achieved without any explicit adjustment of parameters to fit the model to the observations.

From the results shown in Fig. 1 we can evidently reconstruct an estimate of the actual solar sound speed. Tests have shown that this is largely independent of the choice of reference model, even for models differing rather more from the structure of the Sun than the model used here. Thus it is possible from the observed frequencies to determine quite precisely the dependence of sound speed on position in the Sun.

3 Relevance to the Solar Neutrino Problem

The support for the standard solar model provided by the helioseismic results, at least to the level of precision relevant to the current state of neutrino measurements, might argue against solutions to the solar neutrino problem in terms of non-standard solar models. Indeed, such models have generally been
ruled out by helioseismology. These include models with partial mixing of the core, or with energy transport in the core from motion of hypothetical weakly interacting massive particles (WIMPs). A measure of the structure of the solar core is provided by the separation $d_{nl} = \nu_{nl} - \nu_{n-1l+2}$ in cyclic frequencies $\nu$ between modes differing by 1 in radial order $n$ and by 2 in degree $l$, for low-degree modes. For standard solar models the computed $d_{nl}$ is very close to the observed value. Core mixing increases $d_{nl}$ whereas the inclusion of WIMPs reduces it, such that in both cases models where the neutrino fluxes are reduced to near the measured values are inconsistent with the observed $d_{nl}$. Thus these models are effectively ruled out.

This exemplifies the fact that while helioseismology constrains the mechanical properties such as sound speed and density, other aspects of the model cannot be uniquely determined. Indeed, $c^2 \propto T/\mu$, where $T$ is temperature and $\mu$ is the mean molecular weight; hence $T$ and $\mu$ can be varied individually, as long as their ratio is unchanged. For example, this might be achieved by changing $\mu$ and $T$ through mixing and the inclusion of WIMPs, respectively. Only by invoking the physics of stellar interiors, such as information about energy transport and production, is it possible to determine, e.g., the variation of temperature and composition, and hence to compute the neutrino flux expected from the helioseismically determined model. Since the inferred sound speed is so close to the standard solar model, a substantial reduction in the neutrino flux while keeping the model consistent with helioseismology can be achieved only by modifying several aspects of the model computations.

Which modifications might be contemplated? The most questionable assumption in computations of standard models is probably the absence of motion in the solar interior; thus it is natural to consider the possibility of mixing, modifying the distribution of composition. However, as argued above, any change in $\mu$ must be compensated by changes in $T$. This in turn requires modification of the description of energy generation and transport. It is conceivable that there are significant errors in the opacities; the basic rate of energy generation is probably rather well known, although one cannot entirely exclude problems with the treatment of nuclear screening. Evidently, inclusion of non-standard processes allows substantially more freedom in the modelling; examples are energy transport by WIMPs or waves, or departures from thermal equilibrium in the present Sun, perhaps caused by a recent mixing episode.

Models with such modifications to the physics permit fairly substantial reductions in the computed neutrino flux, while keeping the sound speed consistent with the helioseismic evidence, although typically drastic and perhaps
unrealistic reductions in opacity are required. A particularly careful analysis of this nature was carried out by Antia & Chitre, who were able to reduce the computed capture rates to near the observed values. However, in common with other astrophysical attempts at a solution it was not possible to match all neutrino measurements simultaneously.

To account for the details of the measured neutrino fluxes Cumming & Haxton proposed slow mixing of the core at such a rate that $^3$He is not in nuclear equilibrium, hence shifting the balance between the different components of the pp chains. This would unavoidably lead to homogenization of the hydrogen abundance over a substantial region which, unless other modifications were invoked, would result in a sound-speed profile inconsistent with the helioseismic results. Even allowing opacity reductions by a factor of up to eight it was not possible to obtain an acceptable sound speed. However, further tests taking into account the modified energy generation rate caused by the redistribution of $^3$He are still required.

4 Conclusions

Given the success of the standard solar model in predicting the solar sound speed, it is tempting to assume that the models are equally successful in determining the temperature and composition. Indeed, it might appear unreasonable if the Sun were to have substantially different temperature and composition profiles, arranged in such a way as to give the same sound speed as in the standard model. Models of this nature can be constructed, although only with very substantial changes to the assumed physics of the solar interior, carefully adjusted to avoid changes in the sound speed. Thus, it is perhaps natural to conclude that the predictions of the neutrino production rates are robust, and that therefore the neutrino discrepancies reflect a need for revision of our understanding of the physics of the neutrino.

However, given the fundamental implications of such a conclusion, it should not be accepted prematurely. Further investigations, taking into account plausible errors in the helioseismic results as well as in the physics of the solar interior, are required to place firmer limits on predicted neutrino fluxes consistent with the helioseismic results.

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