Spectra, triple and quartic gauge couplings in a Higgsless model

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Spectra, triple and quartic gauge couplings of the Higgsless model with gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ defined in warped space are explored with numerical method. We extend the equation of motions, boundary conditions and formalism of multi-gauge-boson vertices to the Hirn-Sanz scenario. By assuming the ideally delocalized fermion profile, we study the spectra of vector bosons as well as the triple and quartic gauge couplings among vector bosons. It is found that mass spectra can be greatly modified by the parameters of QCD power corrections. Meanwhile, the triple and quartic gauge couplings can deviate from the values of the standard model to at least $\pm 10\%$ and can saturate the LEP2 bounds. We find the triple gauge couplings of $Z\bar{W}W$ can be 50% smaller than the unitarity bounds. While the triple gauge couplings of $\bar{Z}WW$ is 20% smaller than the unitarity bounds, which might challenge the detection of $\bar{Z}$ via s channel at LHC if $m_{\bar{Z}} > 500$ GeV.

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INTRODUCTION

Electroweak symmetry breaking mechanism is one of the most important questions of the standard model (SM) which will be investigated by the LHC soon. Following its low energy QCD counterpart, and when only massless photon and massive weak bosons are considered, the electroweak symmetry breaking sector can be effectively described by the electroweak chiral Lagrangian (EWCL) $^{1, 2, 3, 4, 5}$ in a model-independent way. Recently, it was proposed by $^6$ to make a global analysis on the electroweak symmetry breaking sector by incorporating triple gauge coupling measurements. It was found that the two-point chiral coefficients $\alpha_0$, $\alpha_1$, and $\alpha_8$ are tightly constrained from Z-pole data as well as W boson mass and top quark mass measurements. However, the triple and quartic gauge couplings are loosely bounded, which can make the QCD-like models to be consistent with electroweak precision data. At the same time, driven by the measurement of three-point and four point chiral coefficients at LHC and future colliders $^{7, 8, 9, 10, 11, 12, 13}$, it is necessary to extend the theoretical study from two-point vertices to three-point and four-point vertices of the EWCL. Therefore it will be interesting to look at those strongly coupled models where might have large deviations in the triple and quartic gauge couplings.

It is of great importance to measure the triple gauge couplings at LEP2 and Tevatron $^{14, 15}$, because such a measurement is the direct proof of the non-Abelian structure of the standard model. It can help to measure the size of $W$ and $Z$ weak bosons, and to understand to what extent these weak bosons can be interpreted as the composite or elementary objects. Meanwhile, it can remove the theoretical background for the discovery of new physics, for example to remove the SM background (say, $Z$ decaying to a WW pair) for the Higgs decaying to a WW pair.

Up to now, both LEP2 and Tevatron have measured the triple gauge couplings by observing diboson events. D0 and CDF collaborations from Tevatron have released their results on the single W production events $^{16}$, WW pair production events $^{17}$ and WZ pair production events $^{18}$. But the most stringent bounds on the triple gauge couplings are from LEP2 W pair production events. Till now, LEP2 already accumulate more than 40,000 W pair events $^{19, 20, 21, 22, 23, 24, 25, 26}$. Total cross section, angular distributions and spin density matrix have been analyzed. The component of $\rho_{00}$ of the spin density matrix clearly show the existence of the longitudinal component of the W bosons. Currently, the most stringent bounds on triple gauge couplings are extracted from LEP2 data $^{23}$. The bounds from LEP2 $^{8, 9, 10, 19, 21, 22, 23, 24, 25, 26}$ on three-point chiral coefficients are a few $10^{-2} \pm 10^{-2}$ in one-parameter fit and $10^{-1} \pm 10^{-1}$ in two-parameter fit.

It is quite natural to ask whether there exists a concrete example in strongly interacting model where
three-point functions can have large deviations from the predictions of the SM. The answer provided in this paper is affirmative, we explicitly show that the deviation can be as large as 10%.

It is well-known that it is extremely difficult to tackle strongly interacting dynamics. Currently the only reliable tool is the lattice calculation. Computing two-point functions with lattice method is relatively easy. However, computing three-point and four-point functions is expensive and difficult.

Recently, the development in AdS/CFT correspondence provides a powerful tool to tackle strongly interacting dynamics. By assuming the AdS/CFT correspondence, it is possible to explore the strongly interacting gauge theories by studying the weakly interacting gravity theories. Spontaneous symmetry breaking can be realized in the gravity theories by imposing the boundary conditions. For example, the Higgsless model, the electroweak symmetry breaking is realized with boundary conditions imposed on the ultraviolet (UV) and infrared (IR) branes. These boundary conditions can be interpreted as composite Higgs developing vacuum expectation values on the boundary branes. Such a model can be viewed as the AdS dual of a walking technicolor-like model.

Higgsless model, as its name indicated, has no Higgs boson as the particle content. In the SM, the presence of the Higgs boson (lighter than 1 TeV or so) is necessary to guarantee the unitarity of the scattering amplitudes of longitudinal gauge bosons to any energy scale above TeV. The Higgsless models suggest that the absence of Higgs is still possible and workable. The unitarity of longitudinal gauge boson scattering amplitudes can be preserved by including the contributions from the Kaluza-Klein (KK) states of vector bosons. It was found that by assuming the delocalized fermion profile or ideally delocalized fermion profile, Higgsless models can be consistent with precision data. With this assumption, KK resonances in the model are fermiophobic and can escape all experimental bounds on W and Z'. To discover the signature of the Higgsless models with ideally delocalized fermions, vector boson scattering processes will be the key detection means. As pointed out by Ref. [40, 41], the smoking gun for the Higgsless model at LHC is a narrow resonance of W (W is the next KK resonance of the charged vector boson, which can understood as the W_R in the left-right model) in the WZ channel. Therefore, it is useful to examine spectra and the couplings of KK resonances in the Higgsless model to the vector bosons of the SM. In this paper, we will examine both charged and neutral vector excitations as well as the triple gauge couplings of ZWW (Z is the next KK resonance of the neutral vector boson, which can be understood as the Z_R in the left-right hand model) and ZWW vertices, which might be useful for phenomenological study on the searching for new spin-1 resonances.

The multi-gauge boson vertices of Higgsless models with bulk gauge symmetry SU(2) × SU(2) have been studied in both the flat and warped space-time metrics, as shown in Ref. [45]. Triple gauge boson vertex ZWW is also studied in the Gauge-Higgs unification models, as shown in [46]. Although the analytic expressions are provided in these works, the study on the strength of KK vector bosons couplings to the SM vector bosons still lacks. This work is to provide a comprehensive study on the spectra of new particles and the couplings of the neutral and charged KK vector bosons to the vector bosons of the SM.

This work is also motivated by the work of Hirn and Sanz, where the S parameter can be negative in some region of parameter space and the vector and axial gauge bosons may feel different effective metrics by including a higher dimensional left-right kinetic mixing term. Therefore it is interesting to study the effects of these effective metrics to mass spectra as well as triple and quartic gauge couplings. Recently there are also some controversies about the sign of the S parameter in Hirn and Sanz model. It is claimed that S is positive in this model if the contribution of higher-dimensional operators is subleading. This does not contradict with Hirn and Sanz model, because the left-right kinetic mixing term is important to induce different effective metrics for vector and axial sector.

Therefore, in this paper by assuming ideally delocalized fermion sector and by including a gauge kinetic mixing term to the Higgsless model (which we call Hirn-Sanz scenario), we derive the equation of motion (EOM), boundary conditions (BC) and formalism of multi-gauge-boson vertices. By using the shooting method, we study mass spectra as well as the triple and quartic gauge couplings of vector bosons. We find that the deviation from the prediction of the SM in the triple gauge couplings can reach ±10% by parameter r, and ±5% by parameter O_X, both of which is within the reach of LHC and ILC. We also find the triple gauge couplings of ZWW can be 50% smaller than the unitarity bounds. While
the triple gauge couplings of $\bar{Z}WW$ is 20% smaller than the unitarity bounds, which might challenge the detection of $\bar{Z}$ via s channel at LHC.

The paper is organized as follows. In section II, the Higgsless model in Hirn-Sanz scenario is briefly reviewed. In section III, the formulas for triple and quartic gauge couplings in Higgsless model are provided. In section IV, numerical results are presented. We end this paper with conclusions and discussion.

**HIGGSLESS MODEL IN HIRN-SANZ SCENARIO**

Higgsless models are defined on the AdS$_5$ spacetime background. The metric of the spacetime reads

$$ds^2 = \frac{R^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right),$$  \hspace{1cm} (1)

where the $z$ is defined on the interval $[R, R']$. The UV brane is located at $z = R$, and the IR boundary is located at $z = R'$. For fermion sector, we assume the ideally delocalized fermion \[^{[39]}\], then the fifth profile of the SM fermions can extend in the bulk and guarantee the S parameter small enough and the couplings to the KK resonances to vanish. Since we consider the electroweak symmetry breaking, below we drop the $SU(3)_c$ gauge group of the SM. Weak gauge bosons are defined in the bulk, and can be interpreted as the composite objects.

The Lagrangian with $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group in 5D reads as

$$\mathcal{L} = \int d^4xdz \sqrt{g} \left( -\frac{1}{2g^2_L(z)} Tr[W_{MN,L}W^{MN}_L] - \frac{1}{2g^2_R(z)} Tr[W_{MN,R}W^{MN}_R] - \frac{1}{4g^2_B(z)} B_{MN}B^{MN} \right),$$  \hspace{1cm} (2)

where the indices $M, N = (\mu, 5)$. The parameters $g_L$, $g_R$, and $g_B$ are gauge couplings of $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$, respectively. These couplings have mass dimension $-1/2$. $W_{MN,L}$, $W_{MN,R}$, and $B_{MN}$ are strength tensors of the gauge fields. $\sqrt{g}$ is the Jacobian factor of the 5D space-time. $\Omega$ is an auxiliary field, which transforms as a bifundamental field under the gauge groups $SU(2)_L \times SU(2)_R$ and plays a role as the nonlinearly realized 5D Goldstone boson \[^{[47]}\]. The gauge kinetic term can also be realized in the linearly realized Higgs fields by a dimension-6 operator. However, in this article, we neglect the kinetic term of $\Omega$ field and the bulk mass terms of vector bosons.

In the Hirn-Sanz scenario, the gauge couplings can be parametrized as

$$\frac{1}{g^2_L} = \frac{f_V + f_A}{2g_0},$$  \hspace{1cm} (3)

$$\frac{1}{g^2_R} = r_r \frac{f_V + f_A}{2g_0},$$  \hspace{1cm} (4)

$$\frac{1}{g^2_{LR}} = \frac{f_V - f_A}{2g_0},$$  \hspace{1cm} (5)

$$\frac{1}{g^2_B} = r_b \frac{f_V + f_A}{2g_0},$$  \hspace{1cm} (6)

where $f_X$ are defined as

$$f_X = \exp \left( \frac{O_X}{2d(d-1)} z^{2d} \right),$$  \hspace{1cm} (7)
where $X = V$ and $A$. For the sake of simplicity, we use universal parameters $O_X$ to describe gauge couplings $1/g_{1L}^2$, $1/g_{1R}^2$, $1/g_B^2$, and $1/g_{BR}^2$. Parameters $O_V$ and $O_A$ are introduced to describe the QCD power corrections for the two-point functions, and they can be related to the gluonic and fermionic condensates \cite{47}. In the Higgsless model, parameters $O_V$ and $O_A$ can be interpreted as the techni-gluonic and techni-fermionic condensates. The parameters $O_V$ and $O_A$ make it possible to study the cases where gauge couplings can have the fifth dimensional dependence, and they can be either positive or negative. According to the study of Ref. \cite{49}, these terms modify the QCD OPE behavior. The parameter $g_0$ is a dimensional constant to measure the strength of gauge couplings in 5D, which can be fixed by the normalization conditions of photon and W boson as well as by fixing the coupling $g_{W} W = e$.

We introduce $r_r$ and $r_b$ to describe the relative strength of gauge couplings, respectively. Here we simply assume that $r_r$ and $r_b$ are independent of the fifth dimension. The parameters $r_r$ and $r_b$ are assumed to be positive to avoid ghost-like behavior. Then, the gauge couplings $1/g_{1L}^2$, $1/g_{1R}^2$, and $1/g_B^2$ are always positive for any value of $O_X$. While the coupling of kinetic mixing term $1/g_{BR}^2$ can be either positive or negative, depending on the values of $O_V$ and $O_A$. The $r_r$ corresponds to the $1/k$ as introduced in \cite{45}.

To derive EOM and BC, we adopt the unitary gauge by setting the fifth component of the gauge fields to zero. In the construction of Higgsless models, the symmetry of the model is further broken by BC. The boundary conditions at UV brane ($z = R = R' \exp (-b/2)$) are given as

$$
\frac{1}{g_{1L}^2} \partial_z W^a_{\mu,L} + \frac{1}{g_{1R}^2} \partial_z W^a_{\mu,R} = 0, \quad W^{1,2}_{\mu,R} = 0
$$

$$
\frac{1}{g_{BR}^2} \partial_z B_{\mu,L} + \frac{1}{g_{BR}^2} \partial W^3_{\mu,L} + \frac{1}{g_{BR}^2} \partial W^3_{\mu,R} = 0, \quad B_{\mu,L} - W^3_{\mu,R} = 0,
$$

(8)

which break the $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$. The boundary conditions at IR brane ($z = R'$) are given as

$$
\left(\frac{1}{g_{1L}^2} + \frac{1}{g_{1R}^2}\right) \partial_z W^a_{\mu,L} + \left(\frac{1}{g_{BR}^2} + \frac{1}{g_{BR}^2}\right) \partial_z W^a_{\mu,R} = 0, \quad W^a_{\mu,L} - W^a_{\mu,R} = 0, \quad \partial_z B_{\mu} = 0,
$$

(9)

which break the $SU(2)_L \times SU(2)_R$ to the custodial symmetry $SU(2)_D$. We notice that the kinetic mixing term of vector boson affect the boundary conditions, as indicating by the terms proportional to $1/g_{BR}^2$. After imposing all these symmetry breaking boundary conditions, the remnant symmetry both in the bulk and on the boundaries is the $U(1)_{EM}$ gauge symmetry.

Following the compactification and dimensional reduction procedure, the charged vector bosons in 5D can be decomposed with the corresponding charged massive vector fields in 4D,

$$
W^\pm_{\mu,L}(x, z) = \sum_{k=1}^{\infty} \psi^L_k(z) W^k_{\mu,L}(x),
$$

(10)

$$
W^\pm_{\mu,R}(x, z) = \sum_{k=1}^{\infty} \psi^R_k(z) W^k_{\mu,R}(x),
$$

(11)

where $W^k_{\mu,L}(x)$ are W and $\tilde{W}$ bosons and their KK excitations. The neutral vector fields in 5D can be decomposed with the corresponding neutral massless and massive vector fields in 4D,

$$
B_{\mu}(x, z) = \psi^B_\gamma(z) \gamma_{\mu}(x) + \sum_{k=1}^{\infty} \psi^B_k(z) Z^k_{\mu}(x),
$$

(12)

$$
W^{3}_{\mu,L}(x, z) = \psi_L^3(z) \gamma_{\mu}(x) + \sum_{k=1}^{\infty} \psi_L^3(z) Z^k_{\mu}(x),
$$

(13)

$$
W^{3}_{\mu,R}(x, z) = \psi_R^3(z) \gamma_{\mu}(x) + \sum_{k=1}^{\infty} \psi_R^3(z) Z^k_{\mu}(x),
$$

(14)
where $\gamma_\mu(x)$, $Z_5^F(x)$ are 4D fields of photon, Z boson, $\tilde{Z}$ and their KK excitations. To describe the profile of charged vector bosons in the fifth dimension, wave functions have two branches: $\psi^{L_\pm}_k(z)$ and $\psi^{R_\pm}_k(z)$. For each neutral particle, wave functions have three branches: $\psi^B_k(z)$, $\psi^{L_3}_k(z)$ and $\psi^{R_3}_k(z)$.

The wave function $\psi_k(z)$ are determined by the on-shell equation of motions. For the mode functions of charged W boson and their KK excitations, the equation of motion read

$$\partial^2 \psi^{L_\pm}_k + \left( \partial \ln J_5 + c \partial \ln \frac{1}{g^2_L} + (1-c) \partial \ln \frac{1}{g^2_{LR}} \right) \partial \psi^{L_\pm}_k + \frac{g^2_L}{g^2_{LR}} \partial \ln \frac{g^2_L}{g^2_{LR}} \partial \psi^{R_\pm}_k = \frac{-J_4}{J_5} M^2_{W_k} \psi^{L_\pm}_k,$$

(15)

$$\partial^2 \psi^{R_\pm}_k + \left( \partial \ln J_5 + c \partial \ln \frac{1}{g^2_R} + (1-c) \partial \ln \frac{1}{g^2_{LR}} \right) \partial \psi^{R_\pm}_k + \frac{g^2_R}{g^2_{LR}} \partial \ln \frac{g^2_R}{g^2_{LR}} \partial \psi^{L_\pm}_k = \frac{-J_4}{J_5} M^2_{W_k} \psi^{R_\pm}_k.$$

(16)

For the mode functions of neutral vector bosons, the equation of motion read as

$$\partial^2 \psi^{L_3}_k + \left( \partial \ln J_5 + c \partial \ln \frac{1}{g^2_L} + (1-c) \partial \ln \frac{1}{g^2_{LR}} \right) \partial \psi^{L_3}_k + \frac{g^2_L}{g^2_{LR}} \partial \ln \frac{g^2_L}{g^2_{LR}} \partial \psi^{R_3}_k = \frac{-J_4}{J_5} M^2_{Z_k} \psi^{L_3}_k,$$

(17)

$$\partial^2 \psi^{R_3}_k + \left( \partial \ln J_5 + c \partial \ln \frac{1}{g^2_R} + (1-c) \partial \ln \frac{1}{g^2_{LR}} \right) \partial \psi^{R_3}_k + \frac{g^2_R}{g^2_{LR}} \partial \ln \frac{g^2_R}{g^2_{LR}} \partial \psi^{L_3}_k = \frac{-J_4}{J_5} M^2_{Z_k} \psi^{R_3}_k,$$

(18)

$$\partial^2 \psi^B_k + \left( \partial \ln J_5 + \partial \ln \frac{1}{g^2_B} \right) \partial \psi^B_k = \frac{-J_4}{J_5} M^2_{Z_k} \psi^B_k,$$

(19)

with

$$\frac{1}{c} = 1 - \frac{g^2_{R}g^2_{T}}{g^2_{LR}}.$$  

(20)

To simplify the expression, we introduce a shorthand convention

$$\partial \ln \frac{1}{g^2_i} = \frac{-\partial g^2_i}{g^2_i}.$$  

(21)

which is independent of the sign of $g^2_i$, with $(i = L, R, B, LR)$, which is also true for $J_5$.

The $J_4$ and $J_5$ are defined as

$$J_4 = \sqrt{g} g^{\mu\nu} g^{\mu'\nu'},$$

(22)

$$J_5 = \sqrt{g} g^{\mu\nu} g^{55}.$$  

(23)

For AdS$_5$ case, we have

$$J_4 = R \exp\{-b/2\}/z,$$

(24)

$$J_5 = -R \exp\{-b/2\}/z.$$  

(25)

We observe that the operator $Tr[W^{MN}_{L,R}W^{MN}_{R}]$ induces the entangling terms in the Eqs. (15)-(19), which are the last terms in the left-hand side of these equations. When $1/g^2_{LR}$ (or $f_V = f_A$) equal zero, the contributions of kinetic mixing terms to boundary conditions as well as EOMs vanish.

Another fact is that the constant $g_0$ does not affect either EOM or boundary conditions, which means that it does not affect the mass spectra of vector bosons.

By using the boundary conditions of symmetry breaking given in Eqs. (8) and Eqs. (11)-(14), the UV boundary conditions for wave function $\psi$ at $z = R$ are determined by Eq. (8) as

$$\left( \frac{1}{g^2_L} \partial_z \psi^{L_\pm}_n + \frac{1}{g^2_{LR}} \partial_z \psi^{R_\pm}_n \right) |_{z=R} = 0, \psi^{R_\pm}_n |_{z=R} = 0,$$

$$\left( \frac{1}{g^2_R} \partial_z \psi^{L_3}_n + \frac{1}{g^2_{LR}} \partial_z \psi^{R_3}_n + \frac{1}{g^2_B} \partial_z \psi^B_n \right) |_{z=R} = 0, (\psi^{R_3}_n - \psi^B_n) |_{z=R} = 0.$$

(26)
And the IR boundary conditions for wave function $\psi$ at $z = R'$ are determined by Eq. (25) as

\[
\left[ \left( \frac{1}{g_L} + \frac{1}{g^R_L} \right) \partial_z \psi_n^{L_z, L_3} + \left( \frac{1}{g_R} + \frac{1}{g^R_R} \right) \partial_z \psi_n^{R_z, R_3} \right] \Big|_{z=R'} = 0,
\]

\[
(\psi_n^{L_z, L_3} - \psi_n^{R_z, R_3}) \Big|_{z=R'} = 0, \quad \partial_z \psi_n^{B_z} \Big|_{z=R'} = 0. \tag{27}
\]

We observe that if the gauge couplings have non-trivial dependence on $z$, they can affect both boundary conditions and equation of motions of KK modes in a non-trivial way.

Since there is an extra $\mathbb{U}(1)_{B-L}$ sector in the Higgsless model and there is no extra advantage for formulating two-, three-, and four-point functions by using the vector and axial vector fields, we use the left and right vector fields in both EOM and boundary conditions. However, we find that Eqs. (26, 27) can reduce to those in vector and axial-vector fields when the gauge symmetry is taken as $SU(2)_L \times SU(2)_R$ and $g_L = g_R$ is imposed. In the original Hirn-Sanz scenario, the QCD corrections can be interpreted as that vector and axial gauge bosons can feel different metrics. Here we adopt the interpretation of the QCD corrections as the modification of the gauge couplings $1/g_L^2, 1/g_R^2, \text{ and } 1/g_{LR}^2$, as shown in the EOM and boundary conditions given above.

The normalization conditions for wave functions of neutral and charged bosons are given as

\[
\int_z J_4 \left( \frac{\psi_{n, \psi_n^{L_3}}^{L_z}}{g^2_L(z)} + 2 \frac{\psi_{n, \psi_n^{R_3}}^{R_z}}{g^2_R(z)} \right) = 1, \tag{28}
\]

\[
\int_z J_4 \left( \frac{\psi_{n, \psi_n^{L_3}}^{L_z}}{g^2_L(z)} + 2 \frac{\psi_{n, \psi_n^{R_3}}^{R_z}}{g^2_R(z)} \right) = 1, \tag{29}
\]

which guarantee the canonically normalized kinetic terms of vector bosons. In our numerical method, we choose to find eigen-wavefunctions with the shooting method from IR brane to UV brane. We need to choose an initial value of $\psi_n^{L_z}$ as input in order to determine the whole wavefunctions. The normalization condition is essential to remove the arbitrariness in the magnitude of $\psi_n^{L_z}$. However, there still exists an ambiguity on the sign of $\psi_n^{L_z}$. Therefore we always choose a positive $\psi_n^{L_z}$ in our numerical calculation.

**FORMALISM FOR MULTI-GAUGE BOSON VERTICES**

The triple gauge couplings are induced from the non-Abelian structure of the model in 5D. Below we consider the couplings of $W$ pair to neutral vector bosons, which can be expressed straightforwardly into two terms:

\[-i k_{NW} N_{\mu \nu} W^\mu_W W_\nu^\nu - ig_{NW} (N_{\mu W^\mu W^\nu} - N_{\mu W^\nu W^\mu}). \tag{30}\]

These two terms are part of the more general parametrization of triple gauge couplings [14], since we have not included higher dimensional operators nor the operators violating $C$ and $CP$ symmetries in the original Lagrangian given in Eq. (4).

The effective couplings $k_{NW}$ and $g_{NW}$ are determined by the integral over the fifth dimension as

\[
\int_z J_4 \left( \frac{\psi_{n, \psi_n^{L_z}}^{L_3}}{g^2_L(z)} + \frac{\psi_{n, \psi_n^{R_z}}^{R_3}}{g^2_R(z)} \right) = k_{NW}, \tag{31}
\]

\[
\int_z J_4 \left( \frac{\psi_{n, \psi_n^{L_z}}^{L_3}}{g^2_L(z)} + \frac{\psi_{n, \psi_n^{R_z}}^{R_3}}{g^2_R(z)} \right) = g_{NW}. \tag{32}
\]

In order to fix the free parameter $g_0$, we introduce the charge universality condition

\[g_{\gamma WW} \equiv c. \tag{33}\]
We find vertices of photon with charged vector bosons are the same, which is expected from the symmetry breaking pattern, as demonstrated by the first column in Table IV and Table V. We can extend the above formalism straightforward to study the triple gauge couplings of $W$ to all neutral vector bosons.

The quartic gauge couplings are also induced from the non-Abelian structure of the Higgsless model in 5D. The quartic gauge couplings of $WWWW$ can be cast into two terms

$$- g_{WWWW}^2 \left( W^+ \cdot W^+ W^- \cdot W^- - W^+ \cdot W^- W^+ \cdot W^- \right),$$

where the coupling of $WWWW$ is defined as

$$\int z J_4 \left( \frac{\psi^L \psi^L \psi^R \psi^R}{g_L^2(z)} + 2 \frac{\psi^L \psi^R \psi^R}{g_{LR}^2(z)} \right) = g_{WWWW}^2.$$

It is easy to extend the coupling $g_{WWWWW}$ to $g_{WWWW}^2$.

The quartic gauge couplings of $ZZWW$ can be cast into two terms

$$- g_{ZZWW}^2 \left( Z \cdot ZW^+ \cdot W^- - Z \cdot W^+ Z \cdot W^- \right),$$

where the coupling of $ZZWW$ is defined as

$$\int z J_4 \left( \frac{\psi^L \psi^R}{g_L^2(z)} + 2 \frac{\psi^L \psi^R}{g_{LR}^2(z)} + \frac{\psi^R \psi^R}{g_R^2(z)} \right) = g_{ZZWW}^2.$$

We neglect other quartic gauge couplings, like $ZZWW$ in this paper.

We observe that the $U(1)_{E-4}$ gauge kinetic term does not contribute three-point nor four-point vertices, due to its Abelian nature and there is no self-interaction terms in its gauge kinetic term.

In warped spacetime, since all vertices are determined by the parameters of the model, we can examine the unitarity violation for the $WW$ scattering process by introducing the following two parameters:

$$\delta U^{WW}_E \equiv g_{WWWW}^2 - \sum_N g_{NNWWW} g_{NNWWW},$$

$$\delta U^{WW}_E \equiv 4g_{WWWWW}^2 - 3 \sum_N g_{NNWWW} g_{NNWWW} \frac{M_N^2}{M_W^2}.$$

$U^{WW}_E$ determines the magnitude of the term of $E^4/m_W^4$, and $U^{WW}_E$ determines the magnitude of the term of $E^2/m_W^3$.

**NUMERICAL ANALYSIS**

Before resorting to numerical solution, it is instructive to count the number of free parameters in the model. We have 6 free parameters in total: $b, r_r, r_{r_l}, O_V, O_A$, and $d$. In the following study, we fix $b = 24$ and $d = 2$ for simplicity of the numerical analysis. $b = 24$ corresponds to the UV cutoff set at $10^4$ TeV or so.

We use the following experimental data to determine eigenvalues and eigen-wavefunctions as well as the TGC of $g_{WW}$:

$$m_Z = 91.19 \text{GeV}, \ e = 0.3124, \ \sin^2 \theta_W = 0.2315.$$
dependence of gauge couplings on fifth dimension as trivial case. This point will serve as the reference point to compare and contrast other points.

For point 2, the gauge couplings have 5D profile while the gauge kinetic mixing term vanishes at tree-level. For point 3, the gauge couplings have 5D profile and the kinetic mixing term contributes to both the EOM and the boundary conditions. Points 4 and 5 are similar to points 2 and 3. The difference is that the gauge couplings increase with the increase of $z$ for points 2 and 3, while the gauge couplings decrease with the increase of $z$ for points 4 and 5.

To select these points, we impose $O_A > O_V$ to satisfy Witten’s positivity condition [50]. We deliberately choose $r_b$ to guarantee the mass ratio of $M_2^U$ close to its experimental value. The large $r_b$ corresponds the small $g_{2B}$. For these five point case study, we fix $r_r = 1$.

The parameter $\Lambda_{EW}$ is about 300 GeV. To determine the mass spectra, we fix the IR brane on the point $z = 1$ by choosing $\Lambda_{EW} R' = 1$ and the UV brane on the point $z = \epsilon_+ = \exp\{-b/2\}$. By fixing the mass of $z$ boson we can find the parameter $\Lambda_{EW}$. The value of $\sqrt{\Lambda_{EW} g_0}$ demonstrates these points indeed locates at weakly coupled region.

|        | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ |
|--------|-------|-------|-------|-------|-------|
| $r_b$  | 2.85  | 2.850 | 2.50  | 2.45  | 2.45  |
| $O_V$  | 0.00  | 10.00 | 0.00  | -10.00| -10.00|
| $O_A$  | 0.00  | 10.00 | 10.00 | -10.00| 0.00  |
| $\Lambda_{EM}(\text{GeV})$ | 275.24 | 208.03 | 201.72 | 481.14 | 271.81 |
| $\sqrt{\Lambda_{EM} g_0} \times 10^3$ | 5.85  | 6.24  | 5.78  | 5.57  | 5.62  |

**TABLE I**: Selected points in parameter space to show spectra and couplings

The mass spectra of vector bosons are listed in Table II and Table III, for the neutral and charged vector bosons, respectively. Compared with the point 1, the point 2-like gauge coupling dependence can decrease the mass difference of vector bosons, while the point 4-like gauge coupling dependence can increase the mass difference of vector bosons. Obviously, QCD-like corrections can modify the spectra dramatically. The spectra of point 3 is similar to point 2, especially for higher KK excitations. This means that the sign of $1/g_{2LR}^2$ can make differences for lower KK excitations. The spectra of point 5 is quite different from those of point 4. This means that the sign of $1/g_{2LR}^2$ can make differences for all KK excitations.

Table IV lists the TGC of $WWN$, while Table IV lists the TGC of $\bar{W}WN$. In point 1, we can have the exact identity $g_{NWW} = k_{NWW}$. In other points, such an identity is broken negligibly, which can be understood simply from Eq.(31-32), where only $g_{LR}$ terms have some minor difference and left and right gauge boson profiles difference is not big enough to break the identity heavily. Therefore we only list $g_{NWW}$ type TGC.

The coupling $g_2$ in the SM is given as 0.5786, the deviation can reach to $+4.4\%$, $4.3\%$, $+2.7\%$, $+0.3\%$, and $+1.3\%$ for each points, respectively. Such deviations can saturate the LEP2 TGC measurement bounds. We observe that $W$ pair has larger couplings with photon and $Z$ boson than other neutral vector bosons. Couplings of $NWW$ to higher KK excitation decrease quickly with the increase of KK number.

The pattern of the couplings of $\bar{W}$ pair to neutral vector bosons becomes more complicated. There
are several comments in order. 1) The charge universality is guaranteed, which is realized when only we include kinetic term consistently in both equations of motion and boundary conditions. This serves as one crucial check for our numerical approach. 2) Generally speaking, the $\tilde{W}$ pair strongly couple to $\tilde{Z}$ and $\gamma'$. 3) The tendency of that the couplings of the $\tilde{W}$ pair to neutral bosons decrease when KK number of neutral vector bosons goes to higher is also demonstrated by Table V, though not as quickly as the cases for $W$ pair.

Fig. 1 is to show the dependence of $ZWW$ on the parameter $O_X$ and $r_r$. We select three cases with 1) $O_V = O_A = O_X$, 2) $O_V = O_X, O_A = 0$, and 3) $O_V = 0, O_A = O_X$ to show the dependence of $g_{ZWW}$ on $O_X$. In Fig. 1, $r_r$ is fixed to 1. We follow [47] to vary $O_X$ from $-15$ to 15. There are several comments in order for Fig. (1a): (1) When $O_X$ is smaller than 0, the difference in $g_{ZWW}$ between these three cases is small. While when $O_X$ is larger than 0, with the increase of $O_X$, $g_{ZWW}$ decreases in case 1) and case 2) while is increase in case 3). The $O_X$ can modify the $g_{ZWW}$ up to $\pm 5\%$. (2) We fix $O_A = O_V$ and select three cases to show the effects of $O_X$ while $r_r$ can vary from 0.07 to 8.89: 1) with $O_X = 0$, 2) with

| $W$ | $\tilde{W}$ | $W'$ | $\tilde{W}'$ | $W''$ | $\tilde{W}''$ |
|-----|-------------|------|-------------|------|-----------|
| $P_1$ | 80.68 | 680.48 | 1075.27 | 1539.78 | 1952.37 |
| $P_2$ | 80.22 | 325.05 | 931.00 | 1563.76 | 1776.53 |
| $P_3$ | 80.36 | 499.21 | 903.47 | 1516.71 | 1761.84 |
| $P_4$ | 80.20 | 1569.89 | 1795.07 | 3038.59 | 4459.40 |
| $P_5$ | 80.55 | 888.50 | 1063.81 | 1716.73 | 2519.27 |

TABLE II: The spectrum of charged vector bosons. The unit of mass is GeV.

| $\gamma$ | $Z$ | $\tilde{Z}$ | $\gamma'$ | $Z'$ | $\tilde{Z}'$ | $\gamma''$ | $Z''$ | $\tilde{Z}''$ |
|---------|-----|-------------|-----------|------|-------------|-----------|------|-------------|
| $P_1$ | 0 | 91.19 | 675.48 | 699.67 | 1080.69 | 1534.32 | 1560.53 | 1958.02 | 2397.57 |
| $P_2$ | 0 | 91.19 | 320.60 | 340.57 | 934.90 | 1091.76 | 1111.45 | 1567.81 | 1772.18 |
| $P_3$ | 0 | 91.19 | 370.33 | 503.56 | 907.34 | 1064.98 | 1134.48 | 1520.87 | 1723.24 |
| $P_4$ | 0 | 91.19 | 1560.07 | 1614.09 | 1805.69 | 3029.12 | 3075.59 | 3411.93 | 4449.22 |
| $P_5$ | 0 | 91.19 | 755.63 | 896.75 | 1068.71 | 1557.74 | 1723.85 | 1934.74 | 2397.04 |

TABLE III: The spectrum of neutral vector bosons. The unit of mass is GeV.
Meanwhile, the $O_X = 10$, and 3) with $O_X = -10$. It is obviously that the $g_{ZWW}$ can be either larger or smaller than the prediction of the SM. The deviation from the prediction of the SM can even reach to 100% when $r_r$ is larger than 8. To obtain this large deviation we fix $r_b = 1$, which cannot guarantee that the $m_W$ equals to its experimental value. But after adjusting $r_b$ to make $m_W$ equal to its experimental value, we find the deviation can be larger than a few 10%.

The unitary bounds of the coupling $\bar{Z}WW$ can be put as [43]

$$u_{\bar{Z}WW} = g_{\bar{Z}WW}\frac{m_Z}{\sqrt{3}M_Z},$$

The unitary bounds of the coupling $g_{ZWW}$ can be put as [40]

$$u_{ZWW} = g_{ZWW}\frac{m_Z^2}{\sqrt{3}M_W M_W}.$$  

To show the deviation from this unitarity estimate, we define

$$r_{\bar{Z}WW} = \frac{g_{\bar{Z}WW}}{u_{\bar{Z}WW}},$$

$$r_{ZWW} = \frac{g_{ZWW}}{u_{ZWW}}.$$  

Fig. 2 shows the dependence of $r_{\bar{Z}WW}$ and $r_{ZWW}$ on $O_X$ and $r_r$. In Fig. 2 we select three cases with 1) $O_V = O_A = 0$, 2) $O_V = O_A = 10$, and 3) $O_V = O_A = -10$ to show the dependence of $g_{\bar{Z}WW}$ and $g_{ZWW}$ on $r_r$. Fig. 2a) shows that $r_{\bar{Z}WW}$ is smaller than 0.25 for case 2), 12% for case 1), and 0.035% for case 3). The underlying reason for smallness of $r_{\bar{Z}WW}$ is related with the almost degeneracy of $\bar{Z}$ and $\gamma'$, as demonstrated in Table III and Table IV. Fig. 2b) shows that $r_{ZWW}$ is closer to its unitarity bound value than $r_{\bar{Z}WW}$ does. When $r_r$ is close 1, the value of $r_{\bar{Z}WW}$ and $r_{ZWW}$ approach their maximum values.

The deviation of QGC for four $W$ vertex is about $+6.7\%$, $+8.0\%$, $+4.2\%$, $-0.3\%$, and $+1.5\%$, from the value of SM, 0.4356. The deviation of QGC for $ZZW$ vertex is about $+9.9\%$, $+12.5\%$, $+6.7\%$, $+0.6\%$, and $+3.0\%$, from the value of SM, 0.3348. It is remarkable that the QGC of four $W$ are larger than 1.2, as shown in Table V.

Fig. 3 shows the quartic gauge couplings $g_{WWW}^4$ and $g_{W}^4W$ versus $O_X$. It is obvious that the quartic gauge coupling $g_{WWWWW}^4$ is almost unchanged when $O_X$ is smaller than 0, while it changes when $O_X$ is larger than 0. Such a behavior is similar to the case of $g_{ZWZ}$, which can be interpreted as the consequence of unitarity. The $g_{WWW}^4$ (decreases steeply with the increase of $O_X$ for case 1) and case 2); while increase mildly with the increase of $O_X$ for case 3). The coupling $g_{W}^4W$ can be larger than one due to the fact that the wavefunctions of $W$ can be larger than one. In order to guarantee the interactions of four $W$ is weak, Fig. 3b implies that $O_X$ should be larger than 10 or so.

It is impossible to sum infinite number of KK resonances to examine the unitarity violation. In the realistic calculation, we sum neutral vector bosons up to the first 12 KK excitations for $WW$ and $\bar{W}W$ scattering processes to examine the unitarity violation. The parameters $U_E^W$ and $U_E^W$ as well as $U_{E'}^W$ and $U_{E''}^W$ are listed in Table VI.

We notice that unitarity is almost exact for both $WW$ and $\bar{W}W$ scattering processes for the points 1, 2, and 4. For points 3 and 5, we need to sum more contributions of KK excitations. Generally speaking, the $U_E^W$ is smaller than $U_{E'}^W$ for both $WW$ and $\bar{W}W$ when we sum the same number of KK resonances. Meanwhile, the $U_{E'}^W$ for $WW$ cases are smaller than that for $\bar{W}W$ cases. The $U_{E''}^W$ for $WW$ cases are smaller than that for $\bar{W}W$ cases. The unitarity violation parameters show that unitarity is guaranteed by KK resonances quite well.

CONCLUSION AND DISCUSSIONS

In this paper, we parametrize the Higgsless models in the Hirn-Sanz scenario. We study the effects of QCD improved background metric to the spectra of KK resonances and find the spectra can be
FIG. 1: Triple gauge couplings, $g_{ZW W}$, vary with $O_X$ and $r_r$. In fig. 1a), we select three cases to show the effects of $O_X$: 1) $O_V = O_A = O_X$, 2) $O_V = O_X, O_A = 0$, and 3) $O_V = 0, O_A = O_X$. In fig. 1b), we select three cases to show the effects of $r_r$: 1) $O_V = O_A = 0$, 2) $O_V = O_A = 10$, and 3) $O_V = O_A = -10$. For both fig. 1a) and 1b), $r_b$ is fixed as 1. In figure 1a, the LEP2 two parameter fit upper bound is plotted, while in figure 1b, both the upper and lower bounds are plotted. Future colliders' bounds on $g_{ZW W}$ can reach $10^{-3}$ and are not depicted.

|   | $g_\gamma$ | $g_2$ | $g_2'$ | $g_2''$ | $g_2'''$ | $g_3$ | $g_3'$ | $g_3''$ | $g_3'''$ | $g_4$ | $g_4'$ | $g_4''$ | $g_4'''$ |
|---|-----------|-------|--------|---------|---------|------|--------|---------|---------|------|-------|--------|---------|
| $P_1$ | 0.3124 | 0.6042 | -0.0364 | -0.0281 | 0.0000 | -0.0014 | -0.0010 | -0.0000 | -0.0002 | 0.0000 | 0.0000 |
| $P_2$ | 0.3124 | 0.6036 | -0.0775 | -0.0479 | 0.0040 | 0.0072 | 0.0060 | 0.0000 | 0.0008 | 0.0006 | 0.0000 |
| $P_3$ | 0.3124 | 0.5940 | -0.0111 | -0.0526 | -0.0057 | 0.0015 | 0.0066 | -0.0038 | 0.0002 | 0.0002 | -0.0004 | 0.0000 |
| $P_4$ | 0.3124 | 0.5802 | -0.0109 | -0.0094 | -0.0014 | -0.0037 | -0.0032 | -0.0003 | -0.0017 | -0.0015 | -0.0004 | -0.0008 |
| $P_5$ | 0.3124 | 0.5862 | -0.0032 | -0.0222 | -0.0109 | 0.0003 | -0.0004 | -0.0052 | 0.0000 | -0.0014 | -0.0009 | -0.0001 |

TABLE IV: Triple gauge couplings of $NWW$, up to the first 12 neutral vector bosons.

significantly modified. We also study the three-point and four-point couplings of vector bosons in the parameter space. Roughly speaking, 10% deviations in TGC and QGC from the prediction of the SM can occur, which are within the detection capability of the LHC and ILC. Though it is hard to detect the KK excitations, the measurement of these TGC and QGC can provide complementary information on vector boson structure of the underlying theories.
FIG. 2: The ratio of triple gauge couplings, $r_{ZW \bar{W}}$ and $r_{ZWW}$, vary with $r_\tau$ from 0.07 to 8.89. We fix $O_V = O_A$ and select three cases to show the effects of $O_X$: 1) $O_X = 0$, 2) $O_X = 10$, and 3) $O_X = -10$. Here the $r_b$ is fixed to 1.

| $\bar{g}_\gamma$ | $\bar{g}_Z$ | $\bar{g}_2$ | $\bar{g}_{\gamma'}$ | $\bar{g}_{Z'}$ | $\bar{g}_{2'}$ | $\bar{g}_{\gamma''}$ | $\bar{g}_{Z''}$ | $\bar{g}_{2''}$ | $\bar{g}_{\gamma'''}$ | $\bar{g}_{Z'''}$ | $\bar{g}_{2'''}$ |
|------------------|-------------|-------------|----------------------|-----------------|-----------------|---------------------|-----------------|-----------------|---------------------|-----------------|-----------------|
| $P_1$ 0.3124     | 0.2229      | 1.3360      | 1.1144               | -0.0925         | 0.4284          | 0.3416              | -0.0170         | 0.0072          | 0.0033              | 0.0007          | 0.0001          |
| $P_2$ 0.3124     | 0.2969      | 0.7910      | 0.6349               | -0.0416         | 0.1836          | 0.1431              | 0.0002          | 0.0055          | 0.0036              | 0.0014          | -0.0018         |
| $P_3$ 0.3124     | 0.2363      | 0.1350      | 1.7072               | -0.0346         | 0.0954          | 0.5356              | 0.0060          | 0.0035          | 0.0069              | 0.0054          | -0.0004         |
| $P_4$ 0.3124     | 0.1028      | 1.5638      | 1.3228               | -0.4006         | 0.5832          | 0.4711              | -0.1663         | -0.0288         | -0.0309              | -0.0004         | -0.0006        |
| $P_5$ 0.3124     | 0.1625      | 0.2171      | 2.0184               | -0.2039         | 0.0963          | 0.7591              | -0.0257         | -0.0012         | -0.0376              | 0.0277          | -0.0016        |

TABLE V: Triple gauge couplings of $N\bar{W}W$, up to the first 12 neutral vector bosons.

On the new resonances sector, we notice that the couplings of $Z\bar{W}W$ and $NWW$ vertices are usually smaller than the unitarity bounds. According to the simulation in Ref. [40], although the coupling of $Z\bar{W}W$ is suppressed a little bit compared with the unitary bounds, the total cross section for the production of $\bar{W}$ is large enough for its detection at the LHC. According to the simulation of Ref. [43], with the total integrated luminosity of 300 fb$^{-1}$, it is possible to discover $\bar{Z}$ or $\gamma'$. However, in the model we have studied here, the couplings of $NWW$ are suppressed at least 25% for $\bar{Z}$ (correspondingly the cross section will be suppressed by one order or so) due to the almost degeneracy of $\bar{Z}$ or $\gamma'$. Therefore
TABLE VI: Part of TGC, QGC, and unitarity violation parameters.

|   | $g_{WW}$ | $g_{WW}^2$ | $g_{WW}$ | $g_{WW}$ | $U_{E4}$ | $U_{E4}$ | $U_{E4}$ |
|---|---------|---------|---------|---------|----------|----------|----------|
| $P_1$ | -0.0532 | 0.4648 | 3.4834 | 0.3679 | 0.0000 | 0.0000 | 0.0000 |
| $P_2$ | -0.1094 | 0.4704 | 1.2705 | 0.3765 | 0.0000 | 0.0000 | 0.0004 |
| $P_3$ | -0.0626 | 0.4537 | 3.3847 | 0.3572 | 0.0003 | 0.0623 | 0.0011 |
| $P_4$ | -0.0165 | 0.4345 | 5.0552 | 0.3370 | 0.0000 | 0.0265 | 0.0015 |
| $P_5$ | -0.0257 | 0.4422 | 4.9050 | 0.3449 | 0.0003 | 0.1335 | 0.0300 |

FIG. 3: Quartic gauge couplings, $g_{WW}^2$ and $g_{WW}^2$, vary with $O_X$. We select three cases to show the effects of $O_X$: 1) $O_V = O_A = O_X$, 2) $O_V = O_X, O_A = 0$, and 3) $O_V = 0, O_A = O_X$. Here the $r_b$ and $r_r$ are fixed to 1.

it might challenge the discovery of $Z$ or $\gamma'$ via s-channel at the LHC if $\tilde{Z}$ or $\gamma'$ are larger than 500 GeV. Recently, H.J. He et al. [41] simulated the reconstruction of $W'$ in a minimal Higgsless model at LHC with $PP \rightarrow W_0Z_{0jj} \rightarrow \nu 3\ell jj$ scattering process and found that there exists a very large cancellation between fusion and non-fusion contributions, both of which are gauge dependent. Hence, the gauge-invariant results are substantially smaller than those given in the naive sum rule approach [40, 43] in the unitary gauge. Nevertheless, the detection of new resonances, like $W'$, according to the results of [41], is still rather promising at LHC.

In the phenomenological models of Refs. [47], adding new operators in 5D Lagrangian seems arbitrary
and higher order effects seems uncontrollable due to the lack of well-established power counting rules. Following the standard construction of the chiral effective Lagrangian, we should also introduce terms like $L_{MN} D^M D^N$ and $R_{MN} D^M D^N$ ($D^M$ is the standard covariant differential operator defined in 5D) in our consideration, which are counted as of the same order as the kinetic mixing term and can affect our results on TGCs and QGCs. Since the signs and magnitudes of these effective couplings are unknown, the contributions of these operators can shift our results in either directions. However, in the present work, even without these operators, we observe large deviations from the predictions of the SM.

In comparison, in the holographic models rooted in D-brane configurations of string theory, like the Sakai-Sugimoto model \cite{51, 52}, the large-N and curvature corrections are well-defined and calculable. In the Sakai-Sugimoto model, the spontaneous symmetry breaking can be naturally realized with brane picture. Therefore it is interesting to extend our study to Sakai-Sugimoto model.

Notes: When this work is almost finished, we noticed that triple gauge couplings are also analyzed by \cite{53} in a technicolor model with Sakai-Sugimoto background metric.

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