High Empirical Study of Edge Detection-Based Image Denoising Corrupted by the Additive White Gaussian Noise (WGN)

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Abstract. Denoising of images is one of the Sparky subjects in image manipulating. The goal behind new design approaches to the denoising of image chains is to alleviation the superinduced noise into minimal rate after adopting spatial and temporal areas. However, eliciting edges from denoising images consider the largest trouble that facing many of researchers. Many wavelet-based images denoising methods been proposed to elicit edges from the corrupted images. In this paper, denoising images can be actualized by thresholding the wavelets coefficients at the $\frac{1}{\sqrt{2}}$.

In addition, a new technique approach to the edge detection of images corrupted by the "White-Gaussian Noise" been proposed. This technique comprises two treads: First, all likely edge points elicited with the applying of the first and second partial derivatives. Second, edge detection based-gradients which, relying on the two-dimensional convolution-based on the theory of the finite impulse response (FIR) filter been attained. Here, the histograms of the $V/H$ image gradients can be exploited to create the essential threshold. This will facilitate the access to the convincing simulations in the process of image gradients detection. Experimental results show that the performance efficiency of our proposed technique was best comparing with the classical detection method in terms of blurriness and artifacts specifically, with areas that contain the edges.

Keywords: Image denoising, edge detection, wavelet transform, image gradients, finite impulse response.

1. Introduction

Used results of experiential monographs in image denoising, show many fruitful applications, for example: gradients and edges detection, textures, discontinuances, and the lackluster imaging. The paper was directly focused on eliciting the gradients and edges from the blurred images. The challenge is how to select the ideal manner to elicit those edges. For clarity, the edges are considered very important in perceptive imaging that have the ability in determining the landmarks of images. As such, wavelets transformations (WT) have the I for clustering of a singular value decomposition (K-SVD). This technique used to coach sparsifying lexicon for images corrupted by (WGN), dictating the corrupted patches to have sparse characterizations. Furthermore, the method in [1] is an ideal method that has the capability to deal with the implementations of the singular denoising images. The eventualities modes of the wavelets coefficients might be included to the following three types; Gaussian mixture, Gaussian mixer scale [5], and circular-symmetric Laplacian [6]. In general, the statistical modeling of the wavelets being empirically assessed. The authors in [7]...
proposed noise reduction technique to the digital images that corrupted with the following three types of noise; additive, multiplicative, and mixed. Accordingly, patches from an ideal images can be modeled as a linear ensemble of patches from images corrupted by the WGN's.

The technique in [8] is an adaptive procedure to the noisy image patches that corrupted by the WGN's. Meanwhile, the technique concentrates on points that wisely selected from a teeny and within an invariant sizes, to easily interpolate these patches with the neighboring pixels. Therefore, the interpolation process between the top, bottom, left, and right for the neighboring pixels after adopting the exact weights for these points will actualize an optimal accuracy and the stochastic error at each spatial position. The algorithm proposed in [9] is an appropriate threshold from images that corrupted by the WGN's based on wavelets coefficients, for the purpose of detection of uneasier.

It is important to understand how to separate edge curves or compensate for the significant image information. The following two algorithms (ADDA) and (PFA) are the newer techniques used for the above purpose. Typically, a various likely thresholds were numerated in accordance with the numbers of the existent acuminate gradients within the corrupted patches. Recently, many of algorithms have been suggested for detecting edges and gradients. Most of these algorithms were classed into the following two main categories; gradients and Laplacian. In the first manner, image gradients were detected after the looking to the minimal and maximal magnitudes for these gradients in the First-derivative of the corrupted image. Whereas in the second manner, edges were detected after inspection for zero-crossings in the Second – derivative of the corrupted image. In this paper, an efficient analysis to the image manipulation in gradient based-edge detection were executed. This follows with creating a suitable threshold from the histogram of image gradients. Further, an effective denoising method for removing an additive WGNs was proposed. Furthermore, an adaptive impulse filter for smoothing images that decomposed by the Haar wavelets also proposed. The organization of the paper is as follows. In Section 2, we give a brief discussion on related work of edge detection, sub-band wavelet decomposition, and the simulation of these bands. The proposed method, wavelet thresholding, edge detection using derivatives, and noisy images are explained in Section 3. Experimental results for image de-noising and edge detection with all simulation graphs are shown in section 4. Conclusion of the manuscript is summarized with section 5 by introducing the idea of the proposed method.

**Nomenclature**

- $K$, the vector dimension
- $f'(I)$, the first derivative
- $f''(I)$, the second derivative
- $N$, denote to the No. of input vectors to be test
- $H_{TH}$, universal threshold
- $\sigma$, the noise variance
- $\partial_y$, 1st vertical partial derivative
- $\partial_h$, 2nd horizontal partial derivative
- $I_D$, the de-noising input image
- $D$ denotes the decomposition structure of I/P image $I$
- $D_S$, the de-noising score
- $V_{WD, I_D}$ is the vector- norm of the wavelet decomposition structure of $I_D$.
- $V_{WD, I}$ is the vector-norm of the wavelet decomposition structure of $I$.
- $e(n)$, is a white Gaussian noise
2. Edge Detection and related work

Image edges are an essential domestic changes of density in a specific spatial locations of an image. They usually situate on the stropping between two different densities. The aim behind edges detection is to create a line of a vista from an image. An essential notable traits can be elicited from edges of corrupted images such as; lines, curves, and nooks. For clarity, image recognition is a high-level technique exploited for eliciting those notable traits. Actually, most of edge detectors are relied on detectors that have multiple-phases. These detectors exploit the medial idealizations to compute a filtered image using a specific kernel [10], attaining a match with defined edges [11], and using special threshold for the purpose of reducing the noise in the edge points. The authors in [12] explained that Canny edge detector is one of the most operator used for comparison for determining the performance of other detectors. The method in [13] used to optimize the detection of objects as well as region boundaries in natural scenes. The authors in [14], proposed an edge detection algorithm at sub-pixel level to make comparison with two other sub-pixel edge detectors. The goal behind this comparison is to assess the performance efficiency of the blurred edges. The method in [15], gives a better norm for detecting of the edge points. The above technique combines the following two methods; pixel-level method and sub-level method to elicit the aspired edges. The methods in [16-17] diagnosed, eliciting edges that only depend on the image gradients, will lead to blurred and broken edges. Furthermore, they developed a method to reduce both edge artifacts and contributions from the noninvolved image patches that sharpen the edges at the same time. The proposed method in [18] used to elicit the edge for images that are blurred by the WGN as well as to suppress the noise and accurately located the edges. The goal behind the method in [19] is to combine the interpolation of the essentially non-oscillatory (ENO) with the gradient-based edge detection to accomplish the sub-pixel edge detection. The method in [20], exploited for creating an appropriate threshold in an edge detection style. At first, the aspired findings in edges detection, based on the required implementation in [21]. In this context, the initial impact of fleecier-levels against both the total number of the edge points, and the precision of the counterparts points must be taken into consideration. The technique in [22], is a hybrid denoising method used curvelet and wavelet transformations to give streamlined representation, reconstruction of edges, and providing a perfect reconstruction in fleecier areas and mini patches respectively.

The authors in [23] proposed an active edge detection technique relied on the directional-WT, that detaining the remittent filtering and the simplicity of enumerations the filter design from the standard of the 2D-WT. An accurate analysis for processing of an image in a gradients based-edge detection system is carried out in [24], after the adopting of determining of a suitable threshold from the gradients histogram.

2.1. Sub-bands "Haar" Wavelet Decomposition.

Haar decomposition is devoted to the control of the huge number of pixels, which in turn ease of access to the target areas, this leads to a reduction of execution time of the implementation of programs. Figure 1 shows an example of high resolution CT images (432x368) pixels, 24-bit placed under the test after the adoption of discrete wavelet transform (DWT) with three subband-levels of the "Haar" wavelet decomposition.
2.2. Simulation of Sub-band "Haar" Wavelet decomposition.

Empirically, the coefficients of the \((LL, LH, HL, \text{ and } HH)\) subbands, which denotes to the change in image resolution can be evaluated by decomposing of a specific image using the wavelet transformations. Accordingly, introspecting of a multiple sub-bands frequencies to the decomposed image will facilitates in degrading of the additive noise. Therefore, the image coefficients in the "HH - subbands" can be exploited to assess an optimal threshold hopefully, to degrading the noise. Further, in case of compressing the corrupted image, the coefficients were closest to the weightings of the un-compressed image and without any losses in details. Here, the performance efficiency of our work waylays to how much of prominent edges can be elicited. Furthermore, an optimal threshold can be elicited from the histogram of both the "V&H" image gradients. So, any change in the orientation of these gradients will leads to change the histogram shape. This will greatly facilitate the process of assessing the mean square errors (MAE's) as illustrated in Table 1. The performance of our training procedure is measured after applying the following MSE via equation 1.

\[
\text{MSE} = \frac{1}{K \times N} \left( \sum_{n=1}^{N} \left( \| B_s - \tilde{B}_s \|^2 \right) \right)
\]

where, \(K\) is the vector dimension, and \(N\) denotes to the number of "1/P - Vectors" in the test. On the other hand, the corrupted images might be correctly recovered after concealing the areas that contains on the edges as well as tackling the artifacts. Here, the quality of images is assessed in terms of PSNRs and given via equation 2

\[
\text{PSNR} = 10 \log_{10} \left( \frac{255^2}{\text{MSE}} \right)
\]

Table 1. The MAEs of the vertical and horizontal image gradients

| Ratio | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Brain | MSE-VG | 0.068 | 0.025 | 0.007 | 0.004 | 0.001 | 0.001 | 0.001 | 0.001 | 0.000 |
|       | MSE-HG | 0.070 | 0.032 | 0.006 | 0.002 | 0.001 | 0.001 | 0.001 | 0.001 | 0.000 |
| Abdomen | MSE-VG | 0.025 | 0.006 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|       | MSE-HG | 0.025 | 0.008 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Figure 1. A CT- high resolution images: (a) Abdomen Pelvis image, (b) Brain image.
Figure 2 shows the simulation results of the detail of approximation image and the 3 — *levels* of the "Haar" DWT to the CT image.

![Figure 2. "Haar" wavelet-decomposition; (a) approximating detail, (b) L-3 Haar-decomposition, (c) L-2 Haar-decomposition, and (d) L-1 Haar-decomposition](image)

Referring to the results reported in Table 1 the performance of proposed exemplar is assessed by the simulation sketches shown in figure 3.

2.2.1 Simulation of Sub-band "Haar" Wavelet decomposition Details

DWT was found to performs the following four steps; spinning the computations of the WT, easy to implement, degrading the execution time, and curtailing the resources of the demand. The goal behind the DWT is to decompose an image into a series of lowest resolution images (*subbands*) and perfectly recovering the original image from these bands.
In other words, the I/P images with the "Haar – decomposition" were decomposed into "3 – levels". Each level, decomposes the entrance image into the following four subband frequencies LL, LH, HL, and HH. For example, the four subband-images at level-1 are; LL1, LH1, HL1, and HH1. These bands representing the coefficients of the approximation, vertical, horizontal, and the diagonal image detail, respectively. Fig. 4, shows the decomposition of the two-dimensional "Haar – WT" in which, each level contains all of the information that is necessary to reconstruct the image at the next higher level using the inverse WT. This is followed by the officiating of multi-levels wavelet decomposition.

3. The Proposed Method

Figure 5 shows the flow chart-steps of our proposed edge detection used for eliciting the meaningful edges within the complex areas in an image. Further, the technique used for detecting the points that lies on the
edges after the adopting of the first and the second partial derivatives. In practice, the edges in an image can be elicited by using the first and second partial derivatives.

![Flowchart of proposed edge detection method based-DWT](image)

**Figure 5.** The flow-chart of the proposed edge detection method based-DWT.

### 3.1 Wavelet thresholding

Briefly, hard thresholding employs the usual process of setting to zero those elements whose absolute values are lower than the threshold. The hard threshold is given via equation 3.

\[
H_{TH} \begin{cases} 
H_{TH} & \text{if} \quad |H_{TH}| > t_{TH} \\
0 & \text{if} \quad |H_{TH}| < t_{TH} 
\end{cases}
\]  

(3)

Universal threshold, \( H_{TH} \), which is proportional to the standard of the noise, is defined via equation 4.

\[
H_{TH} = \sigma \sqrt{2 \log M}
\]  

(4)
where $\sigma$ represents the noise variance, which is defined as: $\sigma = [(\text{median } |I_{i,j}|)/0.06745]^2$, where $l_{i,j}$, $eHH1$ sub-band thresholding.

The de-noising input image $I_D$ can be obtained by the thresholding of the wavelet coefficients. The denoising score in percentage is given via equation 5.

$$D_S = \left(\frac{V_{WD_D, l_D}}{V_{WD_D, I}}\right)^2 \times 100\%$$  \hspace{1cm} (5)

$D_S$ is the WT- decomposition construct of the entrance image $I$. Where, $V_{WD_D, l_D}$ is the "vector − norm" of the WT- decomposition construct of $I_D$. $V_{WD_D, I}$ is the "vector − norm" of the WT-decomposition construct of $I$. If, $I$ is "one − dimensional" signal with orthogonal WT, $D_S$ is downgraded to equation 6.

$$D_S = \frac{||l_D||^2}{||I||^2} \times 100\%$$  \hspace{1cm} (6)

The underlying model for noising signal is basically is given via equation 7.

$$S(n) = I(n) + \sigma \cdot e(n)$$  \hspace{1cm} (7)

Where, time $n$ is equally spaced. Assume that $e(n)$ is a WGN $N(0, 1)$, and assuming that the noise level $\sigma$ is to be equal to 1. The goal behind denosing is to quelling the portion of noise from signal $S$ as well as recovering $I$. Empirically, the predicted noise-level of the "$V, H, D - detail coefficients" at level-1 of the "Haar" decomposition is typically regards as a "noiser − coefficients" and can be easily neglected by using an analogical deviation equal to $\sigma$. The median absolute deviation (MAD) is a cubit of a robust statistical dissipating as well as robust estimator to the $\sigma$. MAM works better with distributions without a mean or variance, or is a good estimator of how spread out a set of data is. However, if the data isn’t normal, the MAD is one statistic be used instead of the standard deviations.

3.2 Edge-detection using derivatives.

In general, both the first partial derivative and the zero-crossing of the second partial derivative might be exploited to detect the more significant points that lies on the edges. The calculation of the first derivative can be assessed via equation 8.

$$f'(I) = \lim_{h \to 0} \frac{f(I+h) - f(I)}{h} = f(I + h) - f(I), \ldots \ldots \ldots \text{at: } h = 1$$  \hspace{1cm} (8)

Whereas the second derivative can be evaluated via equation 9.

$$f''(I) = \lim_{h \to 0} \frac{f''(I+h) - f''(I)}{h} = f''(I+1) - f''(I) = f(I + 2) - 2f(I + 1) + f(I) \ldots \ldots \text{at: } h = 1$$  \hspace{1cm} (9)

The gradient is a vector which has certain magnitude and direction:

$$\nabla I(i, j) = \begin{pmatrix} \partial_x I(i,j) \\ \partial_y I(i,j) \end{pmatrix}$$  \hspace{1cm} (10)

where, $\partial_x$ and $\partial_y$ are the vertical/horizontal partial derivatives.

In this work, the first derivative assumes a local maximum at an edge. For a continuous image, $I(i, j)$
Where \( i, j \) are the row and column abscissae, respectively. The magnitude of the gradients provides information about the strength of the edges. Both of amplitude and orientation of the image gradients can be guessed by using the following two partial derivatives; \( \partial_{x}I(i,j) \) and \( \partial_{y}I(i,j) \) and given via equation 11.

\[
|\nabla I(i,j)| = \sqrt{\left(\partial_{x}I(i,j)\right)^2 + \left(\partial_{y}I(i,j)\right)^2}
\]

(11)

For clarity, there is a 90° rotational direction between image gradients and edges. In other words, the rotational direction of edges be lagged by “90° – degrees” to the rotational direction of image gradients. Here, the gradient orientation is given via equation 12.

\[
\angle \nabla I(i,j) = \arctan \left( \frac{\partial_{y}I(i,j)}{\partial_{x}I(i,j)} \right)
\]

(12)

In case of the first partial derivative is maximum while the second derivative is zero, the magnitude of image gradients be highlighted, and easily distinguishing the edges in the entrance image \( I(i,j) \). Therefore, it is necessary to successfully implement a new strategy to the edge detection that be able to locate zeros in the second partial derivative of \( I(i,j) \). The “Laplacian – operator” (i.e., zero-crossing) is given via equation 13.

\[
\Delta I(i,j) = \partial_{(x^2)} I(i,j) + \partial_{(y^2)} I(i,j)
\]

(13)

Image gradients are traditionally evaluated and represented via equations 14, 15 while, the edge orientation is given via equation16.

\[
\nabla = \sqrt{g_{x}^2 + g_{y}^2}
\]

(14)

\[
\nabla = |g_{x}| + |g_{y}|
\]

(15)

\[
\alpha (i,j) = \sin^{-1} \left( \frac{G_{h}}{G_{v}} \right)
\]

(16)

The spatial gradient, \( I(i,j) = [G_{x}^2 + G_{y}^2]^{1/2} \), and edge direction, \( \alpha (i,j) = \sin^{-1} \left( \frac{G_{h}}{G_{v}} \right) \) are evaluated at each point.

3.3 Noisy Images.

Recently, intelligent process to the digital imaging acquisition, which aim at making imaging more sensitive to error, pleasant, and efficient by solving the problems of various types of noise and reflecting the soundest intensities of the veritable scene. Most of approaches that exploited in creating digital images (i.e., digital communication systems), fizzled not only generating of several types of noise but also compatibility with vision decoders. To simulate the noise impact on CT-images, various types of noise have been added to the images. Most of the suggested methods used in the noise-quelling, used a linear-filters (i.e., Gaussian-filter) for removing several types of noise. Median-filters (i.e., rank-filter) is closer to the linear-filters (i.e., average-filter) in which, local variations caused by cereals were reduced. Here, each \( O/I \)-pixel represents the average of the \( I/P \)-values of the neighboring pixels.

The authors in [25] described that median-filters are more efficient than that in averaging-filters. These filters successfully capable to eliminating all of the outliers (i.e., cereals, artifacts) without any affecting on the sharpness of images. The following simulation-sketches show the main comparison in using between the averaging-filter and the median-filter for eliminating the noise. For clarity, the additive-noise to the CT-images to be tested involves a random interspersion on pixels that is located in the patches that having a variant-intensities. Figure 6 shows the de-noising process to the “CT-image”. We added a certain type of noise to the original image hopefully to examine the performance efficiency of various type of filtering used
to remove these noises (see, Fig. 6a and Fig. 6b). Figure 6c shows the result of using averaging filter to the noisy image, where Fig. 6d shows the de-noised image after applying a median filter to the noisy image. In summary, it notices that in case of using a median-filter, the following four quandaries; noise, blurred-edges, blockness, and artifacts were tackled.

Figure 6. De-noising process; (a) original image, (b) noised image, (c) average image, and (d) de-noised image.

4. Image De-Noise Results

4.1 Removing Noise by Adaptive Filtering

Denoising based-wavelet transformation (WT) is one of the most important and general methods in analysis of image features. It considered as minute tools for dealing with the image coefficients at the decomposing-levels and within a high and low subband-frequencies. This will facilitate the compensation in losses for many of these coefficients. Further, WT use a local variance to localize the more significant image features as well as assessing these coefficients which, represent the change in the time series within a particular image resolution. However, in the case that the local variance is large, the Wiener-filter can execute fewer smoothing. whereas, in the case that the local variance is small, the filter can execute more smoothing. From the results we can see that the scores of the proposed method (adaptive-filter) are higher than the linear-filter. Here, the adaptive filter can easily maintain all of the detected edges and other significant fragments in an image. The most important part of this filter is to how take over all of the initial enumerations and how perfectly executing the designed filter to the entrance image. Finally, the proposed filter is more active in the case were the additive-noise (WGN) is constant. This useful feature enables us to find critical edges in a very short execution time. Figure 7 shows the introducing of Wiener filter to the CT-image that has had corrupted with the White Gaussian Noise.
4.2 Results of Edge Detection.

Our proposed method can faithfully preserve edges and also work well on the smooth areas. It is noted that the reconstructed CT-image has noticeable blurring, and artifacts specifically in the edge areas, due to its high complexity and thus low PSNR at this bit per pixel (bpp).

In most cases, eliciting of edges from the "V&H" image gradients in a non-smooth area will led to adducing some of the unsolicited defacements after enlarging the requested detection areas. Consequently, this leads to reduce somewhat in the quality of images. Fig 8 shows the detection process after resizing the interest area using a Canny detector.

The goal of our proposed method is how to be attaining an efficient detection to the more significant image gradients. This is achieved after the adopting of an appropriate threshold which elicited from the histogram of the CT- image gradients. In this context, our proposed technique being followed by regarded methods such as a "boundary match vector"", hopefully, to elicit the eldest number of image-gradients. Figure 9 presents an efficient style to the edges-detection after portioning the tested CT-image into a four quarters. The enumerations of these gradients were assessed after the adopting the formulas for both the mean-square errors (MSEs) and the peak-to-noise-ratios (PSNRs), and as summarized in Table 2. Referring to the results listed in Table 2, we noted that the results of our proposed method in the case, where the selected magnitude of the threshold is equal to 30%, and the boundary match vector angle is 0° are much better than the other cases. Furthermore, the results were very convincing and closer to the required values, as illustrated in the sketches of Fig. 10. In summary, images with a high visual quality can be actualized in the case, were the calculations of the MSERs are very low.
Figure 9. LLF Sub-band (Approximation detail) edge detection; (a) $1st - Q$ image, (b) $2^{nd} - Q$ image, (c) $3^{rd} - Q$ image, (d) $4^{th} - Q$ image, (e) $1st - Q$ edge-detection, (f) $2^{nd} - Q$ edge-detection, (g) $3^{rd} - Q$ edge-detection, (h) $4^{th} - Q$ edge-detection.

Table 2. The calculations of both "MSEs" and "PSNRs" for the V&H image-gradients

| Theta   | $\theta = 0^\circ$ | $\theta = +45^\circ$ | $\theta = -45^\circ$ |
|---------|--------------------|-----------------------|-----------------------|
| Threshold | 10% | 20% | 30% | 10% | 20% | 30% | 10% | 20% | 30% |
| MSE-VG | 0.0142 | 0.0064 | 0.0038 | 0.0158 | 0.0084 | 0.0056 | 0.0162 | 0.0054 | 0.0052 |
| MSE-HG | 0.010 | 0.0042 | 0.0026 | 0.022 | 0.0063 | 0.0044 | 0.0188 | 0.0066 | 0.0062 |
| PSNR-VG | 30.22 | 31.78 | 32.10 | 31.12 | 31.76 | 32.33 | 31.20 | 31.74 | 32.08 |
| PSNR-HG | 30.34 | 31.86 | 32.18 | 31.22 | 31.94 | 32.45 | 31.28 | 31.82 | 32.02 |

Figure 10. The MSE's against the threshold after changing the vector angle of boundary match.
5. Conclusions
In this paper, a high empirical technique for eliciting edges from an image is proposed. The tested image is firstly decomposed using a discrete wavelet transform (DWT) into a four subband-images (approximation and detail images). This technique was followed by the following five steps; image-smoothing, image-enhancement, thresholding, edge-localization, and edge-detection. The goal behind our proposed method is how to be attaining an efficient detection to the more significant image-gradients as well as detecting the points that lies on the edges by using the first and the second partial derivatives. This is achieved after the adopting of an appropriate threshold which elicited from the histogram of the CT-image gradients. In this context, our proposed technique being followed by regarded methods such as a “boundary match vector” hopefully, to elicit the eldest number of image-gradients. In this paper, the denoising input image is obtained by thresholding the wavelet-coefficients. This procedure will enable us to obtain an acceptable result in the process of eliciting most of the image-gradients. Experimental results show that the performance efficiency of our proposed technique comparing with the other classical operators is more efficient in terms of blurriness and artifacts especially, with areas that contain on edges.

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