Influence of the photonic crystal medium on the self-interaction of electrons

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Abstract. The self-interaction of an electron in the photonic crystal medium is investigated. We show that modification of the self-interaction of electrons in photonic crystals is very significant and, as a consequence, at some photon frequencies the electron-light interaction becomes effectively strong.

1. Introduction
Photonic crystals (PC) are periodic systems that consist of dielectrics with different refractive indices. Its periodic dielectric function yields a photonic band structure with photonic band gaps. It has been shown that such a structure leads to quantum effects, including the coherent control of the spontaneous emission [1], the appearance of photon-atom bound states [2–5], the non-Markovian character of radiative decay [6], enhanced quantum interference effects [7], the localization of superradiance near the photonic band edge [8] and the modification of Lamb shift [2, 3, 9–13]. At the same time, in Ref. [14] it has been shown that there is a novel effect in PC medium which consists in the fact that the rest mass of an electron should change its value. Actually, this effect is another consequence of the periodic changing of the photon-matter interaction in the PC medium. The origin of the effect is the significant modification of the electromagnetic field in the PC medium. In this paper we investigate the influence of the modification of the photonic density of states (DOS) in PC on the self-interaction of electrons.

2. Methods
In our investigation we use the method [15] that allows one to solve the problem nonperturbatively. This method is based on the generalized dynamical equation (GDE) that has been derived in [15] as the most general dynamical equation consistent with the current concepts of quantum physics. Being equivalent to the Schrödinger equation in the case when the interaction in a quantum system is instantaneous, GDE allows one to extend dynamics to the case of nonlocal-in-time interactions. This equation provides a new insight into many problems in atomic physics [17–20], nuclear physics [21–24] and quantum optics [25, 26]. The method allows one to take into account from the very beginning that the contribution to the Green operator $G(z)$, which comes from the processes associated with the self-interaction of particles, has the same structure as the free Green operator $G_0(z)$. For this reason it is natural...
to replace $G_0(z)$ by the operator $G^{(v)}_0(z)$, which describes the evolution of the system when particles propagate freely or interact with vacuum, and, hence, has the structure

$$\langle n' | G^{(v)}_0(z) | n \rangle = \langle n' | n \rangle (z - E_n - C_n(z))^{-1},$$

with $|n\rangle$ being the eigenvectors of the free Hamiltonian $(H_0 | n \rangle = E_n | n \rangle)$. Other contributions are described by the operator $G^{(I)}(z) = G^{(v)}_0(z)M(z)G^{(v)}_0(z)$:

$$G(z) = G^{(v)}_0(z) + G^{(I)}(z) \equiv G^{(v)}_0(z) + G^{(v)}_0(z)M(z)G^{(v)}_0(z),$$

where the operator $M(z)$ describes the processes in which some particles interact each with other. The equations for $C(z)$ and $M(z)$ are derived from GDE. The equation for the function $C_n(z)$ referred to as the self-energy function reads

$$\frac{dC_n(z)}{dz} = - \langle n | M(z) \left(G^{(v)}_0(z)\right)^2 M(z) | n \rangle, \langle n | n \rangle = 1,$$

and the condition

$$z - E_n^{(0)} - C_n(z) = 0 \quad (1)$$

determines the physical masses of particles. In the case when we deal with an atom and $|n\rangle$ describes an atomic state, equation (1) determines the self-energy correction (the Lamb shift) to the energy $E_n$ of the state $|n\rangle$. An approximative solution of this equation is $E_n \equiv E_n^{(0)} + C_n^{(0)}(E_n^{(0)})$. For this approximation to be valid the variation of $C_n(z)$ in the energy interval between $E_n^{(0)}$ and $E_n$ must be negligible. This is the case for atoms in free space. In fact at leading order in $\alpha$ the equation for $C_n(z)$ is reduced to the equation [27,28]

$$\frac{dC_n^{(0)}(z)}{dz} = - \langle n | H_I \left(G^{(v)}_0(z)\right)^2 H_I | n \rangle, \langle n | n \rangle = 1, \quad (2)$$

with $H_I$ being the interaction Hamiltonian. By solving this equation with an appropriate boundary condition we arrive at the ordinary expressions for the self-energy shifts and widths of energy levels. However, in the case of atoms in the PC medium the variation of the self-energy function in the relevant vicinity of the point $z = E_n^{(0)}$ can be very significant and, as a result, the above approximation is invalid. In this case one has to derive the self-energy function from a nonperturbative solution of the equations for $C_n(z)$ and $M(z)$.

3. Photon density of states and the electron self-energy function in the PC medium

Let us consider the self-interaction of an electron in a PC. The Bloch structure of the photon states in the PC medium means that these states can be expanded in a set of Bloch states. The interaction Hamiltonian describing the light-matter coupling in the PC medium is $H_{pc}^I = -\frac{e}{m_e} \mathbf{p} \cdot \mathbf{A}_{pc}$, where $\mathbf{p}$ is the electron momentum, and $\mathbf{A}_{pc}$ is the PC-medium quantized vector potential (for details see [14]). Let us assume that the perturbation theory is applicable in this case, and the leading order solution for $M(z)$ is $M(z) = H_{pc}^I$. Equation (2) for the self-energy function is reduced in this case to the equation (here and below we use the unit system where $\hbar = c = \varepsilon_0 = 1$)

$$\frac{dC_n^{(0)}(z)}{dz} = - \sum_{\mathbf{p}' \mathbf{k} \varepsilon_\lambda} \left( \frac{\langle \mathbf{p}' \mathbf{H}_{pc}^I | \mathbf{p}' \mathbf{k} \varepsilon_\lambda \rangle | \mathbf{p}' n \mathbf{H}_{pc}^I | \mathbf{p} \rangle}{(z - m_e - \frac{p'^2}{2m_e} - \omega_{k_n})} \right) + \sum_{\mathbf{p}' n \mathbf{k} \varepsilon_\lambda} \left( \frac{\langle \mathbf{p}' \mathbf{H}_{pc}^I | \mathbf{p}' \mathbf{k} \varepsilon_\lambda \rangle | \mathbf{p} \mathbf{H}_{pc}^I | \mathbf{p} \rangle}{(z - m_e - \frac{p'^2}{2m_e} - |k|^2)} \right),$$

(3)

(2)
where \(|k,n\rangle\) is the photon in Bloch state with energy \(\omega_{kn}\). Actually, \(C_{\text{p}}^{\text{pc}}(z)\) is the difference between the self-energy function in a PC and that in vacuum. The second term of the right-hand part of equation (3) is \(dC_{\text{p}}^{\text{pc}}(z)/dz\) where \(C_{\text{p}}^{\text{pc}}(z)\) is the self-energy function of the electron in vacuum. The solution of equation (3) is

\[
C_{\text{p}}^{\text{pc}}(z) = \sum_{p'k,\varepsilon,\lambda} \left( \sum p |\mathcal{H}^{\text{pc}}_{p}| p'k,n,\varepsilon,\lambda \right) |p'k,n,\varepsilon,\lambda \rangle \langle p'k,n,\varepsilon,\lambda | \mathcal{H}^{\text{pc}}_{p} | p \rangle - \sum_{p'k,n} \left( \sum p |\mathcal{H}^{\text{pc}}_{p}| p'k,n,\varepsilon,\lambda \right) |p'k,n,\varepsilon,\lambda \rangle \langle p'k,n,\varepsilon,\lambda | \mathcal{H}^{\text{pc}}_{p} | p \rangle \]

The value of \(C_{\text{p}}^{\text{pc}}(z)\) at \(z = m_e + E_p\) is \(C_{\text{p}}^{\text{pc}}(z = m_e + E_p) = \left( 1 - \frac{p^2}{2m_e^2} \right) \delta m_{\text{pc}}\), with

\[
\delta m_{\text{pc}} = - \frac{2m_e^2}{p^2} \left( \sum_{p'k,\varepsilon,\lambda} \left( \sum p |\mathcal{H}^{\text{pc}}_{p}| p'k,n,\varepsilon,\lambda \right) |p'k,n,\varepsilon,\lambda \rangle \langle p'k,n,\varepsilon,\lambda | \mathcal{H}^{\text{pc}}_{p} | p \rangle - \sum_{p'k,n} \left( \sum p |\mathcal{H}^{\text{pc}}_{p}| p'k,n,\varepsilon,\lambda \right) |p'k,n,\varepsilon,\lambda \rangle \langle p'k,n,\varepsilon,\lambda | \mathcal{H}^{\text{pc}}_{p} | p \rangle \right)
\]

being the PC medium correction to the electron mass derived in [14]. Expression (4) for \(\delta m_{\text{pc}}\) can be simplified [14]:

\[
\delta m_{\text{pc}} = \frac{\alpha}{\pi^2} \int d\omega \frac{N_{\text{p}}(\omega) - \omega^2}{\omega^2},
\]

where \(N_{\text{p}}(\omega) = N_{\text{DOS}}(\omega)D_{\text{p}}(\omega)\) with \(N_{\text{DOS}}\) being the photon density of states and \(D_{\text{p}}(\omega) = \sum_{G,\lambda} |\langle \varepsilon_{G\lambda} | p_{k,n} \rangle|^2\) \((\varepsilon_{G\lambda})_p\) are the amplitudes of the components, which constitute the coefficients of the plane wave expansion of the Bloch wave function of the photon, \((\varepsilon_{G\lambda})_p\) are the polarization vector projections onto the electron momentum axis. In the similar way equation (3) can be rewritten in the form

\[
\frac{dC_{\text{p}}^{\text{pc}}(z)}{dz} = - \frac{\alpha}{\pi^2} \int d\omega \frac{N_{\text{p}}(\omega) - \omega^2}{\omega (z - m_e - E_p - \omega)^2}.
\]

The DOS describes the number of states per interval of energy at each energy level that are available to be occupied, and in vacuum is equal to \(\omega^2\). The modification of the electromagnetic field in the PC medium results in the fact that the DOS in the PC medium differs profoundly from that in vacuum. This in turn has a significant effect on the self-energy function of the electron. Further estimates can be made with the model of \(N_{\text{p}}(\omega)\) analogous to the one proposed in [14]. The results of our calculations of the self-energy function \(C_p(z)\) displayed in figure 1 show its strong dependence on \(z\). In the same way as it was made in Ref. [14] we can derive the formula for the self-energy function of the atomic hydrogen in the state \(|N\rangle = |n, j, l, m\rangle\)

\[
C_N(z) = \int d^3p \Psi_N^*(p) C_{\text{p}}^{\text{pc}}(z) \Psi_N(p),
\]

where \(\Psi_N(p)\) is the wave function in momentum space. Obviously, the above strong variation of the self-energy functions of electrons in the PC medium should result in a strong modification of the atomic self-energy function. In the case of atoms the self-energy function describes also the decay law of unstable states. The standard treatment of unstable states is based on the assumption that they correspond to poles of the analytic continuation of the Green operator. The positions of these poles are determined by equation (1). The approximative correction to the energy level determined by this equation is \(C_n^{(0)}(E_{n}^{(0)}) = \Delta E_n - \frac{1}{2} \Gamma_n\), with \(\Delta E_n\) and \(\Gamma_n\) being the self-energy shift and the natural width of the energy level of the state \(|n\rangle\) respectively, and the Green operator \(G_0^{(v)}(z)\) takes the form

\[
G_0^{(v)}(z) = \sum_{n} \frac{|n\rangle \langle n|}{z - E_n - \Delta E_n - \frac{1}{2} \Gamma_n},
\]

(5)
Figure 1. The self-energy function $C_{pc}^{\sigma}(z)$ of a free electron placed in PC medium. The calculations was performed with the model of $N_p(\omega)$ analogous to the one proposed in [14] with the parameters $n_{eff} = 4, \mu = 15$ eV, $\tau = 0.01$ eV, $\omega_0 = 1$ eV, $h_0 = 0.96, \sigma_0 = 0.07$ eV

which implies that the decay law is exponential and hence $\Gamma_n$ is equal to the spontaneous rate $\gamma_n$. For this quasi-stationary approximation to be valid the variation of the self-energy function should be negligible. However, as we have shown this is not the case for the self-energy functions of electrons and atoms in the PC medium. This means that the decay law of the unstable states of atoms placed in the PC medium can be nonexponential. Note in this connection that as it follows from the Fock-Krylov theorem, in general an unstable atomic state should be described by an energy distribution. Only in the case in which the variation in $z$ of the self-energy function near the position of the pole corresponding to the unstable state is negligible the energy distribution is of the Breit-Wigner shape and hence the state can be associated with some complex energy. However, as we have seen this is not the case when we deal with atoms in the PC medium. As no discrete energy level can be associated with the unstable state in this case it is not an eigenstate of any Hamiltonian. Therefore the methods which are based on the eigenstates of any Hamiltonian fail in this case. At the same time, the approach based on the generalized dynamical equation allows one to regard an unstable state as an energy distribution from the very beginning. The definition of the Green operator in generalized quantum dynamics is more general and remains valid even for the systems, where no Hamiltonian can be found as an operator generating dynamics of the system. From the results of calculations presented in figure 1 it follows that the differential of the self-energy function in the PC medium is $10^3$ times larger in magnitude than that in vacuum. On the other hand, as it follows from the equation for operator $M(z)$, there is a direct connection between the value of the differential of the leading-order vertex and the intensity of the light-matter interaction. This means that at some photon frequencies the electron-light interaction becomes effectively strong and equations for the self-energy function and the operator $M(z)$ should be solved nonperturbatively.

The above results show that the strong modification of the electromagnetic field in the PC medium gives rise to the fact that the electron self-energy function in a PC can significantly differ from that in vacuum. This modification of the electromagnetic field should also have effect on the light-matter interaction in the PC medium. In some cases this interaction can be efficiently strong. It is important because achieving the strong coupling regime is one of the ways of realizing a quantum interface between light and matter [29–32]. In the strong coupling regime the equation for $C(z)$ and $M(z)$ must be solved nonperturbatively. The above way of
achieving the strong coupling regime can be improved by using quantum dots as emitters in PC’s instead of atoms. In fact, transition dipole moments of energy levels of quantum dots can be much larger than that of atoms and these dipole moments themselves can be significantly increased by the modification of the electromagnetic field in the PC medium.

4. Conclusion
We have shown that the self-energy functions of electrons and atoms in the PC medium strongly variates with $z$. This has a significant effect on the character of their interaction with the electromagnetic field. Energy distributions of unstable states of atoms in PC’s can differ dramatically from the Breit-Wigner form. This in turn results in the fact that their decay law is in general nonexponential. The significant modification of the self-energy functions in the PC medium results also in the fact that the interaction of atoms with some modes of the electromagnetic field can be effectively strong.

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