Smallest QCD Droplet for Hydrodynamic Response and Multiparticle Correlations in pp Collisions

Seyed Farid Taghavi
Physik Department E62, Technische Universität München, James Franck Str. 1, 85748 Garching, Germany

We address the possible lower bound on the “system size” at which hydrodynamics is applicable. In the context of Gubser flow, we show that the existence of a consistent hydrodynamic solution translates the condition $\tau T \sim 1$ to a lower bound on system size for a fixed total transverse energy. We employ Gubser flow together with a simple model for initial state fluctuations to explain the experimentally observed multiparticle correlations for pp collisions and to inspect the total multiplicity bounds at which hydrodynamics works and has a clear signal for observation.

In 2010, CMS collaboration revealed a peculiar observation of long-range correlations in pp collisions [1] which is considered as a signature of collective evolution. Later on, this observation has been confirmed by different experimental collaborations for different small systems (pp, pAu, dAu, $^3\text{He}$Au, pPb) at LHC [2-5] and RHIC [6, 7]. Over the past years, there have been ongoing debates on the origin of the observed correlation. Efforts to explain the observed phenomena have been made from different perspectives e.g. to link the correlation to initial stages of the collision and/or to different descriptions of the collectivity in small systems (for review see [8]). The present letter belongs to the category of studies that intend to demonstrate the observed phenomena using the conventional hydrodynamics [9,15]. The strategy that we pursue in the present letter is the following: studying hydrodynamic evolution of the produced matter in pp experiment such that, first, it has essential features to explain the real data, and, second, it is still simple enough to monitor an event evolution anatomy clearly. To achieve this, the best choice in our opinion is the analytical solution for ideal hydrodynamic equations for causal hydrodynamics, Gubser flow, and perturbation on top of that [16,17]. Before proceeding, it is worth mentioning that the paper is based on the following assumption: the collective evolution is governed by hydrodynamics after a certain time, at least apparently, regardless of the hydrodynamic/non-hydrodynamic mode structure of the underlying physics [15,19,21].

The hydrodynamic equations are given by the energy-momentum conservation $\nabla_{\nu} T^\mu{}_{\nu} = 0$ where $T^\mu{}_{\nu} = (\epsilon + p)u^\mu u^\nu + p g^{\mu\nu} + \pi^{\mu\nu}$ is the energy-momentum tensor. Here, $\epsilon$, $p$, and $u^\mu$ ($u^\mu u_\mu = -1$) are the energy density, pressure and fluid velocity, respectively. The dissipative effects are encoded in the shear stress tensor $\pi^{\mu\nu}$. Imposing the symmetry $SO(3)_q \times SO(1,1) \times \mathbb{Z}_2$ on the system, the hydrodynamic equations are considerably simplified such that we are able to find an analytical solution for them. In this study, we focus on a background solution for an ideal conformal fluid $\hat{\epsilon}_0 = \hat{\epsilon}_0 / \cosh^{8/3} \rho$ ($\hat{\epsilon}_0$ is a free parameter) and an elliptic perturbation on top of that $\hat{\epsilon} \simeq \hat{\epsilon}_0 (1 + 4\lambda \delta_2 (\rho) y(\theta, \phi))$, $\hat{u}_\mu = (-1, \lambda \delta_\theta, \lambda \delta_\phi, 0)$ where $\lambda$ is a small parameter, $y(\theta, \phi) = -\sqrt{3}/8 Y_{2,0}/2 - \sqrt{3}/8 Y_{2,-2}$ and $\delta_\mu i \simeq v_2 (\rho) \delta_i y(\theta, \phi)$ for $i = \theta, \phi$ [10,17]. Most of the time the hyperbolic-cylindrical coordinates $(\tau, r, \phi, \eta)$ are used to study the boost-invariant fluids where $\tau$ is the proper time, $\eta$ is the space-time pseudorapidity and $(r, \phi)$ is the polar coordinate in the transverse plane.

The solution we have presented above, however, is written in $(\rho, \theta, \phi, \eta)$ where $q \tau = \text{sech} \rho / (\cos \theta - \text{tanh} \rho)$ and $q r = \sin \theta / (\cos \theta - \text{tanh} \rho)$. Here, $q$ is a free parameter. We refer to this coordinate as de Sitter coordinate. The hydrodynamic equations lead to a linear system of one dimensional equations for $\delta_2 (\rho)$ and $v_2 (\rho)$ which can be solved analytically for ideal hydrodynamics. The isotropic initial fluid velocity assumption at $\rho = \rho_{\text{hyd}}$ fixes the initial value of the equations to $\delta_2 (\rho_{\text{hyd}}) = 1$ and $v_2 (\rho_{\text{hyd}}) = 0$. The $\mathbb{R}^{1,3}$ metric in this coordinate system can be written as $ds^2 = \Omega^2 ds^2$ where $ds^2 = -d\tau^2 + dr^2 + r^2 d\sigma^2 + r^2 d\eta^2$ is the metric of $dS_3 \times \mathbb{R}$ space and $\Omega = \tau$ is the scale factor. Mostly, it is easier to work in $ds^2$. In such cases, we represent the quantities with a hat by taking the conformal scaling into account.

In this study, we explicitly ignore to present the Navier-Stokes (NS) equations solution. We found that the perturbation on top of Gubser flow is not stable for NS equations when the initial time of hydrodynamics is small (less than 1 fm/c). The problem could be cured in a causal hydrodynamics framework. However, to the best of our knowledge, the analytical solution for causal hydrodynamics with the Gubser symmetry has been found only in certain limits [22,23], and perturbation on top of causal Gubser flow has not been done yet. As a result, it would increase the complexity of the problem that would be against the prophecy of the present letter. Therefore, we estimate the effect of the shear viscosity by using the numerical studies of conformal second order hydrodynamics [24].

In order to compare the Gubser solutions with a real experiment, we translate the free parameters of the Gubser solution $(q, \hat{\epsilon}_0, \lambda, \rho_{\text{hyd}})$ to the physical quantities in an appropriate way. The physical quantities, which characterize our initial system, are the to-
tal transverse energy $\epsilon_{\text{tot}} = \int r dr d\phi \epsilon(\tau, r, \phi)$, the \textit{rms} radius $r_{\text{rms}}^2 = \frac{1}{3} \int r dr d\phi r^2 \epsilon(\tau, r, \phi)$, and ellipticity $\epsilon_2 = (1/2r_{\text{rms}}^2) \int r dr d\phi r^2 \cos(2\phi) \epsilon(\tau, r, \phi)$. The measure of the integrations, in the de Sitter coordinates reads as $\tau^2 \cosh^2 \rho_{\text{hyd}} \sin \theta d\theta d\phi$. Considering this measure together with the fact that the solution is initiated on the $\rho = \rho_{\text{hyd}}$ surface, we obtain $\hat{\epsilon}_0 = 3\epsilon_{\text{tot}} r_{\text{rms}}^2/(4\pi \cosh^{4/3}/\rho_{\text{hyd}})$, $1/q^2 = r_{\text{rms}}^2 (1 + 3 \tanh/k_{\text{hyd}})$, and $\lambda = \sqrt{(\sqrt{5}/3)} \epsilon_2$ up to leading order in the $\epsilon_2$ expansion.

Now, we would like to clarify the interpretation of $\rho_{\text{hyd}}$. Regarding fast hydrodynamization, numerous studies have been done so far. Specifically, the computations from gauge/gravity duality and kinetic theory indicate that the evolution of a boost-invariant system with ISO(2) in the transverse space is attracted to the hydrodynamic solutions after the time $\tau T \sim 1$. In our case, however, the temperature drops when we move from the center to the tail of the energy density. So it is plausible to assume that the hydrodynamization happens at different proper times depending on the temperature. This argument allows us to define the \textit{hydrodynamization surface}, the surface on which $\tau T$ is constant and in the order of the unity. Considering a typical heavy ion collision, some parts of the fireball are about to freeze out immediately after the initiation because the temperature of these parts is equal to the freeze-out temperature $T_{\text{fo}}$. For that reason, we choose that part of the medium as a reference point such that it is already hydrodynamized at time $T_{\text{hyd}}$. The condition $\tau T = \tau_{\text{hyd}} T_{\text{fo}}$ constant surface can be translated into the $\tau \epsilon^{1/4} = \tau_{\text{hyd}} \epsilon_{\text{fo}}^{1/4}$ via the equation-of-state (e.o.s.) $\epsilon = C_0 T^4$. Subsequently, by employing the background solution $\hat{\epsilon}_b(\rho) = \hat{\epsilon}_0 / \cosh^{3/2}$, one finds

$$r_{\text{rms}} \geq r_{\text{crit}},$$

for the hydrodynamization surface $\rho = \rho_{\text{hyd}}$. Here we have defined the following dimensionless quantities

$$\gamma = \tau_{\text{hyd}} \epsilon_{\text{fo}}^{1/4}, \quad \bar{\gamma} = r_{\text{rms}} \epsilon_{\text{tot}}^{1/2}.$$

Having found $\rho_{\text{hyd}}$, we are able to initiate the Gubser solution uniquely in terms of the physical quantities $(\epsilon_{\text{tot}}, r_{\text{rms}}, \epsilon_2, T_{\text{hyd}})$. In order to obtain the associated final particle distribution function, we employ Cooper-Frye prescription $dN/dp = -g/(2\pi)^3 \int p^\mu d\Sigma_\mu \exp[p_\mu u^\mu/T_{\text{fo}}]$ where $dp = d^3p/E$, and $\Sigma_\mu = (\rho, \theta_\rho(\rho, \phi), \phi, \eta)$ indicates the freeze-out surface. The surface is specified by equation $\hat{\epsilon}(\rho, \theta_\rho, \phi) = \hat{\epsilon}_0$ to find,

$$\cos \theta_\rho(\rho, \phi) = \tanh \rho + \frac{1}{q} \text{sech}^{1/3} \rho \left[ 1 + \lambda \delta_2(\rho) f(\rho, \phi, \bar{q}) \right].$$

where $f(\rho, \phi, \bar{q}) = \frac{1}{8} \sqrt{\frac{2}{3}} \left[ 1 + 3 \cos 2\phi - 6 \cos^2 \phi \cos^2 \theta_\rho(\rho) \right]$ and $\theta_\rho(\rho) = \cos \theta_\rho(\rho, \phi)$. Here, we have introduced another dimensionless quantity $\bar{q} = q (\hat{\epsilon}_0/\epsilon_{\text{tot}})^{1/2}$, which controls the overall shape of the freeze-out surface for unperturbed solution.

Initiating the evolution on $\gamma = \tau \epsilon^{1/4} = \tau_{\text{hyd}} \epsilon_{\text{fo}}^{1/4}$ surface leads us to an interesting conclusion. Let us define $\tau_{\text{fo}}$ as the time at which the last fluid cell of the system is frozen out. This implies $\tau < \tau_{\text{fo}}$ for any $\tau$ satisfying the equation $\epsilon_b(\tau, r) = \epsilon_{\text{fo}}$. On the other hand, $\tau_{\text{hyd}} = \gamma^{1/4}$ on the freezeout surface is a member of freeze-out surface too. Now we can simply see that there are possible cases such that $\tau_{\text{hyd}} > \tau_{\text{fo}}$ which is in contradiction to the definition of $\tau_{\text{fo}}$ and consequently the hydrodynamic solution $\epsilon_b(\tau, r)$ cannot exist. As an intuitive picture, one can consider a disk in the range $\epsilon < \epsilon_{\text{fo}}$ such that its internal parts are hydrodynamized while the outer parts are not. Referring to Eq. (1), we can simply find a criterion for the non-existence of $\rho_{\text{hyd}}$ (and corresponding $\epsilon_b(\tau, r)$) such as,

$$r_{\text{rms}} \geq r_{\text{crit}},$$

where $r_{\text{crit}} = (4\pi/3)^{1/2} \epsilon_{\text{tot}}^{1/2}$. In Fig. 1 the freeze-out surface of systems with three different sizes is shown, and the surface for a system in critical size is depicted in the right panel. The existence of the lower bound is based on a generic argument irrespective of the order used in hydrodynamic expansion. The bound (4) is compatible with the one obtained by numerical holographic computations for two colliding shock waves [27, 28]. To see that one can estimate the averaged initial energy density as $\hat{\epsilon}_{\text{init}} \sim \epsilon_{\text{tot}}/\pi r_{\text{rms}}^2$. Hence, by identifying $T_{\text{eff}} = (4\epsilon_{\text{init}}/3\pi^4)^{1/4}$, we obtain $r_{\text{rms}} T_{\text{eff}} \sim \gamma \sim 1$. For a given initial transverse energy, there is also an upper bound for the system size. This corresponds to the case that there is not enough energy density deposited into the given region for producing a deconfined matter. The criterion for such a case can be obtained by $\rho_{\text{hyd}} = \rho_{\text{max}}$ which indicates there is no “time” left for hydrodynamic evolution. Recalling the definition of $\rho_{\text{max}}$, 

\[ \text{FIG. 1. Freeze-out surface for systems with three different sizes where } r_{\text{crit}} = 0.1 \text{ fm, } T_{\text{hyd}} = 0.62 \text{ fm/c, } C_0 = 11. \]
we can find an upper bound for \( r_{\text{rms}} \)

\[
r_{\text{rms}} \leq R_{\text{crit}}, \tag{5}
\]

where the functionality of \( R_{\text{crit}} \) to \( \epsilon_{\text{tot}} \) can be found from

\[
\cos \theta_{\text{fo}}(\rho_{\text{hyd}}) = 1.
\]

In order to study the particle spectrum and its anisotropy, we divide events into two main categories. The events with size \( r_{\text{crit}} < r_{\text{rms}} < R_{\text{crit}} \) in which hydrodynamized matter, “QGP”, is formed and \( r_{\text{rms}} > R_{\text{crit}} \) which only contains hadrons. In this work, we skip those events with \( r_{\text{crit}} > r_{\text{rms}} \) which are not fully hydrodynamized. In the first case, there are at least a small region in the energy density (core) in which the system is in deconfined phase and hydrodynamized, but at the tail of the energy density (corona) the system is in hadronic phase. The spectrum of such a phase can be written as \( dN_{\text{QGP}}/dp = dN_{\text{core}}/dp + dN_{\text{corona}}/dp \). We assume the core part evolves with hydrodynamic equations while for the hadron parts we simply assume that a free streaming starts immediately after the initiation. The particle distribution of the corona also can be obtained by Cooper-Frye formula where the freeze-out surface is coincident with the hydrodynamization surface, \( \Sigma^\mu = (\rho_{\text{hyd}}, \theta, \phi, \eta) \). According to this picture, the particle distribution for a small perturbation \( \lambda \) is given by

\[
\frac{dN_{\text{QGP}}}{dp} \simeq \frac{dN_{\text{QGP}}}{dp} \bigg|_{\lambda=0} + \frac{d}{d\lambda} \frac{dN_{\text{core}}}{dp} \bigg|_{\lambda=0} \lambda. \tag{6}
\]

Ignoring the mass of the final particle, the particle distribution in the unit rapidity of an unperturbed flow can be obtained analytically, \( dN_{\text{QGP}}/dy_p |_{\lambda=0} = (g/C_0^{3/4}) \sqrt{3/2\pi} \rho_{\text{hyd}} \epsilon_{\text{fo}}^{1/4} r_{\text{rms}}^{1/2} \). The same computation for the events with \( r_{\text{rms}} > R_{\text{crit}} \) gives rise to the same result for \( dN_{\text{hadron}}/dp \). For a specific geometry and by fixing \( C_0 \) and \( \epsilon_{\text{fo}} \) related to the underlying physics property, we are able to relate the total final multiplicity to \( \epsilon_{\text{tot}} \). Furthermore, the interpretation of the inequality \( \{\} \) to particle distribution leads us to a lower bound for multiplicity when hydrodynamics is still applicable,

\[
n_{\text{tot}} \equiv \frac{dN}{dy_p} \geq n_{\text{crit}}. \tag{7}
\]

where \( n_{\text{crit}} = (4g/\pi C_0^{3/4}) \left[ r_{\text{hyd}}^{3/4} \right] \). Interestingly, the lower bound only depends on \( r_{\text{hyd}} \) for a given underlying physics parameters. The same bound for multiplicity is found for events \( r_{\text{rms}} < R_{\text{crit}} \) given as \( dN/dy_p > N_{\text{crit}}(r_{\text{rms}}) \). Unlike \( n_{\text{crit}} \), the lower bound \( N_{\text{crit}}(r_{\text{rms}}) \) depends on the system size. The bounds are shown by black curves in Fig. 2. In fact, for a fixed \( r_{\text{rms}} > 62 \text{ fm} \), \( r_{\text{rms}}^{1/2} \), the deconfined phase for the events in the range \( n_{\text{crit}} < dN/dy_p < N_{\text{crit}} \) is not formed.

We compute the second term of Eq. (5) to find out the hydrodynamic response, \( k_2(r_{\text{rms}}, n_{\text{tot}}) = (\partial v_2/\partial \epsilon_2)|_{\epsilon_2=0} \). It receives contributions form the integrand in Cooper-Frye formula and from the anisotropy in the freeze-out surface. Based on the model we have illustrated so far, we obtain a semi-analytical result for \( k_2 \) (a numerical integration over \( \rho_T \) and \( p \) should be performed). In Fig. 2 the contour plot of \( k_2(r_{\text{rms}}, n_{\text{tot}}) \) for \( \rho_{\text{hadron}} free \) streaming is depicted. Here, we have fixed \( \epsilon_{\text{fo}} = 0.18 \text{ GeV/fm}^3 \) and \( g = 2(N^3 - 1) = 16 \). As it can be seen, for a fixed multiplicity density, by approaching to the hadron phase region, \( k_2(r_{\text{rms}}, n_{\text{tot}}) \) vanishes.

Now, we introduce a simple and rather generic model for the fluctuations. Using this model together with \( k_2(r_{\text{rms}}, n_{\text{tot}}) \), we explore \( v_2 \{2\}, v_2 \{4\}, \) and \( v_2 \{6\}, \) observed by ATLAS and CMS for pp collisions \[\text{[3-5]}\]. Assume the rms radius fluctuation is not correlated with the ellipticity. This implies that we can consider the fluctuations as \( p_\epsilon(v_2; n_{\text{tot}}) \). Consequently

\[
p_\epsilon(v_2; n_{\text{tot}}) = \int \frac{dr_{\text{rms}}}{k_2} p_\epsilon(v_2/k_2) p_\epsilon(r_{\text{rms}}), \tag{8}
\]

where \( k_2 \equiv k_2(r_{\text{rms}}, n_{\text{tot}}) \). For a nearly Gaussian ellipticity distribution, one is able to write \( p_\epsilon(v_2) = \langle \epsilon^2 \rangle \exp\left[-(\epsilon^2/2\langle \epsilon^2 \rangle)\right] \). Where in the above, \( \langle \epsilon \rangle = \lambda_0 \langle \epsilon^2 \rangle \langle \epsilon^2 \rangle \) is the Laguerre polynomial, \( \Gamma_2 = -1/2 \langle \epsilon^4 \rangle \langle \epsilon^2 \rangle \) (kurtosis) and \( \Gamma_4 = \langle \epsilon^4 \rangle \langle \epsilon^2 \rangle \). Here \( \epsilon_2 \{2k\} \) is the cumulant of \( p_\epsilon(v_2) \) distribution \[\text{[31]}\]. Finally, one simply finds the following analytical expressions for first three cumulants of \( p_\epsilon(v_2; n_{\text{tot}}) \),

\[
c_2 \{2\} = 2\epsilon^2 \langle k_2^2 \rangle \rho,
\]

\[
c_2 \{4\} = 4\epsilon^4 \left[(2 + \Gamma_2)\langle k_2^2 \rangle - 2\langle k_2^2 \rangle \right],
\]

\[
c_2 \{6\} = 8\epsilon^6 \left[(6 + 9\Gamma_2 + \Gamma_4)\langle k_2^2 \rangle - 9(2 + \Gamma_2)\langle k_2^3 \rangle + 12\langle k_2^3 \rangle \right].
\]
where in the above \( \langle \cdots \rangle \) is averaging with respect to \( p_r(r_{\text{rms}}) \) distribution. In the present study, we simply consider a Gaussian distribution \( p_r(r_{\text{rms}}) = r_{\text{rms}}/\sigma_r \exp[-r_{\text{rms}}^2/(2\sigma_r^2)] \) for the rms radius. The width of the distribution depends on the center-of-mass energy via \( \langle r_{\text{rms}} \rangle^2 = \pi \sigma_r^2/2 = 2B \) where \( B = \sigma_{\text{inel}}^2(\sqrt{s})/14.3 \) \[33\]. The energy dependence of the pp inelastic cross section \( \sigma_{\text{inel}}^2(\sqrt{s}) \) can be found in \[34\].

In order to compare our hydrodynamic computations with real data, we justify two main simplifications assumed in this study: the first one is ignoring the effect of shear viscosity and the second is the conformal symmetry. We estimate the effect of shear viscosity by using the results of conformal second order hydrodynamic computations \[24\] in which the shear effect is evaluated as \( k_{\text{2isc}}^2 \approx k_2^2(1 - 1.2(\eta/s)) \). Due to the fact that this estimation is \( r_{\text{rms}} \) independent, it can be combined with the free parameter \( \sigma_r \) in Eqs. \[9\]. Regarding the conformal symmetry, although temperature dependence of \( C_0 = \epsilon/T^4 \) has dynamical contribution in the evolution of a non-conformal fluid, a suitably chosen “effective constant” \( C_0 \) should lead to sensible predictions for more realistic computations. For massless final particle distribution as a reasonable approximation for massive one, we find that the contribution of e.o.s. coefficient \( C_0 = \epsilon/T^4 \) always appears with \( \gamma \) as \( \gamma/C_0^{3/4} \). This indicates that it is always possible to “tune” the hydrodynamization time for model/data comparison. As we will see shortly, the extra contribution of \( \gamma \) on changing the duration of hydrodynamic evolution leads to a minor difference to final result.

For model/data comparison, we refer to Eqs. \[6\] to obtain

\[
\left( \frac{\epsilon_2[4]}{\epsilon_2[2]} \right)^4 - 2 = \left[ \left( \frac{\nu_2[4]}{\nu_2[2]} \right)^4 - 2 \right] \frac{\langle k_{\text{2}}^2 \rangle^2}{\langle k_2^2 \rangle^2}. \tag{10}
\]

One notes that for the case \( k_2 \) has no \( r_{\text{rms}} \) dependence we find \( \epsilon_2[4]/\epsilon_2[2] = \nu_2[4]/\nu_2[2] \) \[35\]. In Ref. \[3\], the kurtosis of the flow harmonic fluctuations \( \Gamma_2^3 = -\langle \nu_2[4]/\nu_2[2] \rangle^3 \) (translated into the “number of sources” \( N_s \)) for pp collision has been reported to be between -0.25 to -0.45 in the range of charged multiplicities, \( \langle N_{\text{ch}} \rangle \), around 80 to 160. The charged multiplicity in the above study has been obtained in the range \( |\eta| < 2.5 \). It means \( \langle N_{\text{ch}} \rangle \approx 5(2/3) dN/d\eta_p \) by assuming \( dN/d\eta_p \approx dN/d\eta \). With this considerations, one finds that the ratio \( \langle k_{\text{2isc}}^2 \rangle /\langle k_2^2 \rangle \) varies mildly around 0.86 in the range 80 < \( \langle N_{\text{ch}} \rangle < 150 \). Therefore, naïvely estimate the initial kurtosis \( \Gamma_2^3 \) to be around -0.5 to -0.66.

Now we compare our model with the ATLAS collaboration result for \( \nu_2[2] \) and \( \nu_2[4] \) obtained by peripheral subtraction and three-subevent event methods, respectively \[4\] \[5\] (Fig. 3), and \( \nu_2[6] \) obtained by CMS collaboration based on the standard multiparticle method \[2\] (Fig. 4). Here, the initial fluctuation parameters for all multiplicities are fixed to \( \sigma_r = 0.094 \) and \( \Gamma_2 = -0.63 \) and dissipative effect on hydrodynamic response is estimated for \( \eta/s = 0.16 \). As it is indicated in Fig. 3 except a rise around \( \langle N_{\text{ch}} \rangle \approx 80 \) in \( \nu_2[4] \), our hydrodynamic model fits to ATLAS data nicely. The smallness of \( \sigma_r \) guarantees that the perturbation on top of Gubser flow is valid for our computations. Our model shows very low dependence on center-of-mass energy compatible with data (\( \sqrt{s} = 5.02 \text{ TeV} \) comparison is not shown). Also the hydrodynamic response changes modestly for the systems with \( \gamma/C_0^{3/4} = \text{etc} \). For the parameters mentioned in the Fig. 3 the bound \( n_{\text{crit}} \) leads to the fact that \( \langle N_{\text{ch}} \rangle \gtrsim 3 \) for hydrodynamics applicability. On the other hand, referring to the distribution \( p_{\nu}(\nu_{\text{rms}}) \), one finds that around 3% of events have \( r_{\text{rms}} < 2 \text{ fm} \). As a consequence, according to \( N_{\text{crit}}(r_{\text{rms}}) \), “QGP” is not formed for more

**FIG. 3.** Comparing the Gubser based hydrodynamic model with the ATLAS data published in Refs. \[1\] \[5\].

**FIG. 4.** Comparing the Gubser based hydrodynamic model with the CMS data published at Ref. \[3\].
from Fig. 2, we see that $k_2$ is small for events with size $v_{rms} < 0.8$ fm. It indicates that, in the same range of multiplicity, the flow signal is very low for more than 50% of events. In Fig. 4, we compare $v_2(6)$, which is obtained from our model for different values of $\Gamma$, with the one from CMS experiment. Our model does not follow the data trend in an appropriate way. It might be due to the nonflow effect. In fact, there are evidences that nonflow effects are not fully removed from multiparticle correlations in small systems by using standard method [5]. The other possibility could be because of missing physics in our simple model. To this regards, further theoretical investigations and more accurate observations are needed.

In this letter, we introduced a Gubser flow based hydrodynamic model. We showed that there is a lower bound for a fluid droplet which indicates the smallest possible hydrodynamized system. For the initial state of pp collisions, we introduced a simple and rather generic model. This model together with the Gubser flow based hydrodynamic model was used to explain $v_2(2)$, $v_2(4)$, and $v_2(6)$ measured by ATLAS and CMS. We also discussed about the validity and feasibility range of hydrodynamic response in terms of charge multiplicity.

We thank A. Bilandzic for comments and discussions. We thank C. Mordasini and M. Lesch for comments. This project has received funding from the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation programme (grant agreement No 759257).

---

*s.f.taghavi@tum.de*

[1] V. Khachatryan et al. [CMS Collaboration], JHEP **1009**, 091 (2010) [arXiv:1009.4122 [hep-ex]].

[2] B. B. Abelev et al. [ALICE Collaboration], Phys. Rev. C **90**, no. 5, 054901 (2014) [arXiv:1404.2474 [nucl-ex]].

[3] V. Khachatryan et al. [CMS Collaboration], Phys. Lett. B **765**, 193 (2017) [arXiv:1606.06198 [nucl-ex]].

[4] M. Aaboud et al. [ATLAS Collaboration], Phys. Rev. C **96**, no. 2, 024908 (2017) [arXiv:1609.06213 [nucl-ex]].

[5] M. Aaboud et al. [ATLAS Collaboration], Phys. Rev. C **97**, no. 2, 024904 (2018) [arXiv:1708.03559 [hep-ex]].

[6] L. Adamczyk et al. [STAR Collaboration], Phys. Lett. B **747**, 265 (2015) [arXiv:1502.07652 [nucl-ex]].

[7] C. Aidala et al. [PHENIX Collaboration], Nature Phys. **15**, no. 3, 214 (2019) [arXiv:1805.02972 [nucl-ex]].

[8] M. Strickland, Nucl. Phys. A **982**, 92 (2019) [arXiv:1807.07191 [nucl-th]].

[9] P. Bozek, Phys. Rev. C **85**, 014911 (2012) [arXiv:1112.0915 [hep-ph]].

[10] P. Bozek and W. Broniowski, Phys. Rev. C **88**, no. 1, 014903 (2013) [arXiv:1304.3044 [nucl-th]].

[11] H. Niemi and G. S. Denicol, arXiv:1404.7327 [nucl-th].

[12] R. D. Weller and P. Romatschke, Phys. Lett. B **774**, 351 (2017) [arXiv:1701.07145 [nucl-th]].

[13] H. Mntysaari, B. Schenke, C. Shen and P. Tribedy, Phys. Lett. B **772**, 681 (2017) [arXiv:1705.03177 [nucl-th]].

[14] K. Gallmeister, H. Niemi, C. Greiner and D. H. Rischke, Phys. Rev. C **98**, no. 2, 024912 (2018) [arXiv:1804.09512 [nucl-th]].

[15] U. W. Heinz and J. S. Moreland, arXiv:1904.06592 [nucl-th].

[16] S. S. Gubser, Phys. Rev. D **82**, 085027 (2010) [arXiv:1006.0006 [hep-th]].

[17] S. S. Gubser and A. Yarom, Nucl. Phys. B **846**, 469 (2011) [arXiv:1012.1314 [hep-th]].

[18] P. Staig and E. Shuryak, Phys. Rev. C **84**, 044912 (2011) [arXiv:1105.0676 [nucl-th]].

[19] A. Kurkela, U. A. Wiedemann and B. Wu, arXiv:1905.05139 [hep-ph].

[20] P. Romatschke, Phys. Rev. Lett. **120**, no. 1, 012301 (2018) [arXiv:1704.08699 [hep-th]].

[21] P. Romatschke, JHEP **1712**, 079 (2017) [arXiv:1710.03234 [hep-th]].

[22] H. Marrochio, J. Noronha, G. S. Denicol, M. Luzum, S. Jeon and C. Gale, Phys. Rev. C **91**, no. 1, 014903 (2015) [arXiv:1307.6130 [nucl-th]].

[23] L. G. Fang, Y. Hatta, X. N. Wang and B. W. Xiao, Phys. Rev. D **91**, no. 7, 074027 (2015) [arXiv:1411.7767 [hep-ph]].

[24] D. Teaney and L. Yan, Phys. Rev. C **86**, 044908 (2012) [arXiv:1206.1905 [nucl-th]].

[25] P. M. Chesler and L. G. Yaffe, Phys. Rev. D **82**, 024006 (2010) [arXiv:0906.4426 [hep-th]].

[26] F. Cooper and G. Frye, Phys. Rev. D **10**, 186 (1974).

[27] P. M. Chesler, Phys. Rev. Lett. **115**, no. 24, 241602 (2015) [arXiv:1506.02299 [hep-th]].

[28] P. M. Chesler, JHEP **1603**, 146 (2016) [arXiv:1601.01583 [hep-th]].

[29] N. Abbasi, D. Allahbakhshi, A. Davody and S. F. Taghavi, Phys. Rev. C **98**, no. 2, 024906 (2018) [arXiv:1704.06295 [nucl-th]].

[30] H. Mehrabpour and S. F. Taghavi, Eur. Phys. J. C **79**, no. 1, 88 (2019) [arXiv:1805.04695 [nucl-th]].

[31] L. Ma, G. L. Ma and Y. G. Ma, Phys. Rev. C **94**, no. 4, 044915 (2016) [arXiv:1610.04733 [nucl-th]].

[32] N. Borghini, P. M. Dinh and J. Y. Ollitrault, Phys. Rev. C **64**, 054901 (2001) [nucl-th/0105040].

[33] U. Heinz and J. S. Moreland, Phys. Rev. C **84**, 054905 (2011) [arXiv:1108.5379 [nucl-th]].

[34] G. Antchev et al. [TOTEM Collaboration], Eur. Phys. J. C **79**, no. 2, 103 (2019) [arXiv:1712.06153 [hep-ex]].

[35] G. Giacalone, J. Noronha-Hostler and J. Y. Ollitrault, Phys. Rev. C **95**, no. 5, 054910 (2017) [arXiv:1702.01730 [nucl-th]].