The concept of coded mask imaging in theory and in practice is reviewed, with particular emphasis on image reconstruction techniques. The techniques are simple in principle but become more complicated when one takes into account real, ‘as-built’, instruments, as opposed to idealised imaginary ones. Procedures are discussed with particular reference to the instruments of Integral.

1 Introduction

A coded mask telescope can be regarded as an imaging system in which the ‘image’ is so blurred that the point source response function (PSF) often extends over the whole of the detector plane. The ‘image’ referred to here, and the PSF which describes it \( (P_T) \), are simply a shadow of the mask, or of part of it. Image processing procedures can of course reduce or remove blurring and this is what is done by the various image reconstruction algorithms which are used with coded mask telescopes. After processing by such an algorithm one has a more normal response function – pyramidal or Gaussian for example. It is this post-processing PSF \( (P_{PP}) \) which is normally quoted for a coded mask telescope. \( P_{PP} \) is really the response of a pseudo-telescope which consists of a hardware component (often on a satellite) and a software component (usually on the ground).

Image processing techniques can de-blur an image, but even in conventional systems where the blurring is limited, they do so at the expense of a degradation in the noise level. In the Fourier domain, which provides a convenient way of removing the effects of a PSF which is spatially independent, the degradation results from the necessity of multiplying certain frequency components by large factors. For de-blurring algorithms based on matrix techniques, large terms in the inverse matrix play the same rôle.

We shall see below that, at least in principle, there are designs of coded mask systems which avoid any such selective amplification of Fourier terms and for which the inverse matrix has no exceptionally large elements. However, one still finds that each point in the image is subject to noise from most or all of the detector plane. This is an intrinsic problem of non-focussing instruments. In the absence of a mirror or lens to concentrate the flux, the signal must be collected from the whole of the detector and the background is harvested with the signal.

Despite having worked with coded masks telescopes for 25 years, it is the view of the author that, if at all possible, one should avoid their use! Systems which concentrate the flux will normally be preferable.

The fact remains though, that for certain X-ray and gamma-ray imaging applications coded
mask telescopes offer the best available solution. The most important alternatives are grazing incidence optics, tracking detectors (Egret, Glast ...) and Compton techniques. The first two are limited to low energies (<100 keV) and to high energies (>50 MeV) respectively. Compton techniques have a limited effective area (important when one is photon-limited - for burst studies, for example) and can never offer the very best energy resolution because the uncertainties in two or more detectors are involved. Even within the energy band for which narrow field instruments can use focussing optics, the wide field imaging capabilities of a coded mask system are often invaluable (e.g. the Beppo-Sax WFC).

2 Image reconstruction techniques

2.1 Correlation techniques

A method of image reconstruction widely used with coded mask telescopes is correlation of the recorded data pattern with some function $F$ which is a representation $P_T$ - i.e. of a shadow of the mask pattern. It can be shown that if $F$ has an appropriate scaling and offset, and if the observation is dominated by Gaussian noise on a uniform background, then this yields a minimum error estimate of the intensity of a single point source.

For this reason, the correlation image offers the highest sensitivity to a single point source; but it is not necessarily the best method for imaging a complex field. If $P_T$ is position independent, the post-processing PSF, $P_{PP}$, obtained in this way is the autocorrelation function of the mask pattern (with an offset and scaling). This is guaranteed to have a maximum value at zero shift, corresponding to the true source position. It may, however, have side peaks and spurious responses elsewhere ('side-lobes', or 'ghosts').

In practice for any reasonable mask pattern (an even for some less reasonable ones!) the correlation approach yields a robust, if imperfect, image.

Reconstruction by correlation is equivalent to multiplying a vector of observations (the intensities in detector pixels) by a reconstruction matrix $R$ in which the rows represent the different shifts of the mask pattern. But in cases where $P_T$ is position independent (and preferably cyclic), Fast Fourier Transform (FFT) techniques can be used to obtain the correlation image very efficiently. For certain mask designs, reconstruction by Hadamard or other transforms can be even faster than by FFT.

Working via the Fourier domain in principle allows compensation to be made for different coding efficiencies at different spatial frequencies. Wiener filtering does this with an optimum trade-off between errors due to residual side-lobes on the one hand and those due to noise-enhancement on the other.

2.2 Back Projection

A method of image reconstruction, useful when the number of events is small, is to project back from the detected position of each photon onto a map of the sky and to increment those pixels which are consistent with the origin of that photon. This is equivalent to forming an image by correlation with a 0/1 representation of the mask pattern.

2.3 Matrix Methods

If the autocorrelation of the coding pattern is not bi-valued with a single central peak and flat wings, then ghosts of bright sources will occur in correlation images. They can in principle be avoided by noting that at each of the $N_D$ points in the detector one measures a linear combination of the contributions from each point in the sky (plus background). Provided the
problem is not under determined, the inverse of the matrix representing this coding can be used to obtain the sky distribution which led to the observed data.

This process is equivalent to solving a set of linear equations for the intensities in $N_{\text{Sky}}$ sky pixels plus some number $N_{BG}$ of parameter describing the detector background ($N_{BG} = 1$ for a uniform but unknown background). Provided that among the $N_D$ measurements, there are at least $N'_D$ which are linearly independent of each other and if $N'_D = N_{Sky} + N_{BG}$, this is possible.

If $N'_D < N_{Sky} + N_{BG}$ more measurements are needed to obtain a unique solution. One approach is to make further observations with different mask patterns. Another, which is critically important for instruments like Integral-SPI with few detector elements, is to make, say, $n$ observations with different pointings in the same general direction - so-called ‘dithering’. One then has $nN_d$ observations. If the source fluxes and background parameters are assumed constant, the number of unknowns does not increase. Even if it is supposed that they vary according to some simple model, then one may still have gained.

If $N'_D$ (or $nN'_D$ with dithering) is greater than $N_{Sky} + N_{BG}$, then one must use the Moore-Penrose generalised inverse, which provides the best-fit solution to an over-determined set of equations.

As is well-known, inverse matrix methods tend to be unstable. Techniques are available which find a stabilised reconstruction matrix $R$ which is optimum according to the same Wiener criterion that is used in Fourier space. This stabilised matrix is a compromise between the $R$ matrix which corresponds to correlation imaging, which is best in the case of poor signal to noise ratio where side lobes are of secondary importance, and the inverse matrix with its perfect imaging but potentially disastrous noise amplification.

2.4 Non-linear techniques

All of the above techniques lead to an intensity estimate in each image pixel which is a linear combination of the data. When the number of detected events per pixel is small the statistics are not strictly Gaussian but Poissonian. Correlation involves combining many pixels, with both positive and negative multipliers and the differences tend to be small, but it can be argued that it is better to use non-linear methods which take the Poissonian nature of the noise into account.

There are other reasons for considering techniques which are not in the above sense linear. These reasons vary from the aesthetic to the pragmatic - from the appeal of the clearly defined assumptions of a Bayesian approach, via a preference for solutions without un-physical negative intensities, to the fact that certain techniques just ‘seem to work’. For lack of space, non-linear techniques will only be briefly mentioned here and not be reviewed in any detail.

The Poissonian nature of the statistics can be taken into account by using Maximum Likelihood (ML) techniques. ML naturally leads to a constraint that the intensity estimates shall be positive, but a non-negativity constraint can be imposed while adopting other optimisation parameters. Normally no assumption is made about the relative probability of different images, i.e. no prior probability function is used.

Maximum Entropy is a technique which seems to be well suited to coded mask image reconstruction, for which it has been widely discussed. It is an iterative Bayesian method, similar to maximum likelihood, but with a prior probability function (the entropy) which favours flatter images. Other iterative techniques, such ‘direct deconvolution’ have been proposed.

Certain techniques try to find a model, consistent with the data and comprising a limited number of components. Either the Pixon method or Wavelet techniques could be used in this way. Often the simplest description of the field (and hence in a sense the Maximum Entropy solution) is just a list of point sources. Supposing pure point source model leads to IROS.
3 Idealised (‘imaginary’) coded mask telescopes

Usually the mask pattern consists of pixels which are of equal size and which are either transparent or opaque, though the reconstruction methods discussed above are mostly not limited to such cases. In comparing alternative designs, one may use the concept of ‘coding power’[^1], which is a measure of the background-limited point source sensitivity obtainable with correlation techniques.

The coding power is essentially the root-mean-square deviation of the transmission of the mask about its mean value. As the transmission is limited to the range 0–1 one can quickly deduce that the highest value of the coding power, and hence the best point-source sensitivity in the detector background dominated case is obtained (a) if only the extreme values 0, 1 occur and (b) if the mean transparency is 50%. If sky-related events such as cosmic diffuse background and source counts make a significant contribution to the noise, then (a) remains true, though the optimum transparency may change[^13][^14][^15].

To avoid side lobes in correlation images (and to minimise the interdependence of image pixels irrespective of the reconstruction method) one should choose a mask pattern whose autocorrelation function is flat away from the central peak. If the coding can be contrived to be cyclic, this is possible by using patterns related to cyclic difference sets[^16][^17][^18]. As well as exhibiting no structure away from the central peak in the PSF in a correlation image, these patterns have 0/1 transmission and can have very close to 50% transparency. So they have optimum sensitivity.

For such mask designs all of the above mentioned linear reconstruction methods are equivalent[^a]. Thus in principle one has a way of designing a coded mask system which is as sensitive as possible and multiple ways of reconstructing the image, any one of which has imaging properties which are as good as possible.

4 Real systems

So much for the world of Dr. Pangloss[^b]. We can now proceed to strip off the various idealisations which lead to this fictional situation.

4.1 Detector sampling

Often in the literature it is implicitly assumed that the sources conveniently lie in directions such that the shadows of the mask elements are coincident with detector pixels which are equal either to the size of the mask elements or to a submultiple of that size. In general this will not be the case and a blurred form of the mask shadow will be recorded. Even if the detector readout is continuous, the recorded pattern will still generally be blurred, because the spatial resolution will not be infinitely good.

Provided that the detector response is independent of the position within the detector (except perhaps for binning associated with pixels which are some submultiple of the mask pixel size), then the same mask patterns continue to offer optimal properties. But in these circumstances it is the coding power of the blurred mask which matters and the sensitivity is reduced.

4.2 Cyclic coding

It is not always possible to ensure that the coding is cyclic; in fact it is not always desirable. In principle, cyclic coding can be achieved by having a mask which contains several repeats of a

[^a]: Here terms which are of the order $1/N$, where there are $N$ elements in the mask, and hence usually very small, are ignored.

[^b]: “Dans ce meilleur des mondes possibles ... tout est au mieux.” Voltaire, *Candide*, 1759.
Table 1: Imperfections which can arise in non-ideal systems

| Mask                                      | Detector                                      |
|-------------------------------------------|-----------------------------------------------|
| Non-cyclic                                |                                               |
| Closed element : imperfect absorption      | Detector finite position resolution           |
| Open element : imperfect transmission     | Detector efficiency non-uniformities          |
| Effects of mask finite thickness          | Detector response dependent on off-axis angle |
| Obstructions in the mask plane            | Detector background non-uniform               |
|                                          | Gaps in the detector plane                    |
| Other                                     | Dead/inactive pixels in the detector plane    |
| Shielding (collimation) imperfect         |                                               |
| Obstructions between the detector and the mask |                                               |
| Pointing errors; Pointing drift           |                                               |
| Leaks onto the detector from far outside the field of view | |

basic pattern and a detector which registers one cycle. But unless a fine-structured collimator covering the whole detector is used to reduce to zero the response outside a defined region, there will always be some source directions for which a partial shadow is recorded. Such collimators can be used but they attenuate the signal even within the ‘fully-coded’ region and the loss in sensitivity may be more important than the improvement in imaging properties.

If the coding is not truly cyclic, the optimum patterns mentioned in §3 are still sometimes selected (e.g. TTM, Integral ...) because they are well defined and are usually not worse than a random pattern. But when the recorded data does not correspond to a whole number of cycles of the the pattern shadow, they rapidly cease to be any better than a random pattern.

4.3 Other effects

Other respects in which real systems may differ from idealised ones are listed in Table 1. These effects lead to a raw PSF $P_T$ that differs greatly from the idealised response, which would just be a translation of a perfect mask pattern. What is worse, is that the differences tend to be a function of source position.

5 Real life data analysis

In principle a detailed knowledge of the instrument allows the coding matrix element for each detector pixel and each possible source direction to be determined. In theory, any of the more general of the methods outlined above (those which do not assume a position independent response function) can then deal with all of the effects in Table 1.

The matrices describing the response are of dimensions $(N_{Sky} + N_{BG}) \times nN'_D$ and so could potentially have up to $10^{12}$ elements for images of the order of $1000 \times 1000$ pixels. Techniques using detailed response matrices, including use of the stabilised pseudo-inverse matrix, are feasible for low resolution instruments with a limited number of detector pixels. The software for the SPI spectrometer of Integral, with 19 detector elements, depends heavily on this approach. But using such large matrices for the production of an image becomes impractical when the number of detector pixels and of resolvable sky pixels is large. For Integral, the ISGRI and PICSIT detectors of IBIS have 4096 and 16384 detector pixels respectively. With JEM-X the number rises to $>50000$.

Iterative Removal Of Sources (IROS) takes advantage of the fact that with systems with a large number of pixels the sensitivity to extended sources is poor. Consequently one is usually dealing with the problem of finding a limited number of point sources and determining their positions and intensities.

With IROS, the first stage is to obtain an image which is good enough to find the approximate position of the brightest source. The image does not need to be free of artifacts provided the most significant point corresponds to a real source. Typically approximations and short-cuts
are taken by filling in missing data with local averages and by making simplified corrections for non-uniformities and deficiencies. Generally Fourier techniques have to be used and these assume a position-independent response function, even though this may not strictly be the case. For smaller numbers of pixels the position-dependent response may be taken into account, but a comparatively coarse image sampling may be used to reduce the problem size.

Having identified the strongest source in the field, the position and intensity of that source (and the values of any unknown background parameters) are optimised by fitting in the data space. At this stage all the known imperfections in the coding and in the instrument are taken into account. After subtracting the data predicted for the fitted source, an image reconstruction based on the residual data is used to search for further sources. If more sources are found they are in turn subtracted, but importantly the parameters of all of the sources are re-optimised each iteration.

6 Conclusions

The fact that the source intensity will be a function of energy and perhaps of time has not been considered here. The above principles can be applied to successive subsets of the data in energy channel or time. In the former case, off-diagonal terms in the energy response matrix describing the entire hardware plus software ‘instrument’ must be taken into account in interpreting the spectra obtained.

It has here been possible discuss only a few of the vast range of different coded mask image reconstruction and data analysis techniques that have been suggested, and those only in outline. The objective has been review some of the key issues ahead of the Integral mission which will place into orbit no fewer than four telescopes using the coded mask technique. Experience with data from Integral will no doubt lead to the development of even more sophisticated and effective analysis techniques.

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