The Olympic Medals Ranks, Lexicographic Ordering, and Numerical Infinites

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The International Olympic Committee (IOC) does not produce any official ranking of the countries participating in the Olympic Games. However, it does publish tables showing the medals won by athletes representing each country. The convention used by the IOC to order the countries in this unofficial rank is the following. First, countries are sorted by the number of gold medals won. If the number of gold medals won by two or more countries is the same, the number of silver medals is taken into consideration, and then the number of bronze. If the countries have an equal number of gold, silver, and bronze medals, then equal ranking is given and the countries are listed alphabetically by their IOC country code (for instance, in the 2010 Winter Olympics held in Vancouver, China and Sweden each won 5 gold, 2 silver, and 4 bronze medals; both countries have the 7th place in the rank, but China is higher in the table). Table 1 shows countries sorted by this rank at the Sochi 2014 Olympic Games (the first ten countries). We will call this rank R1.

However, there are several methods for ranking countries (some of them are illustrated in Tables 2 and 3 showing the best 10 countries for each rank; for more countries see, e.g., [4]). First, in many countries ranking by the total number of Olympic medals is very popular. This rank (R2) gives equal ratings to gold, silver, and bronze medals. So, if country A has won $g_A$ gold, $s_A$ silver, and $b_A$ bronze medals, then its rank is the sum

$$R2(A) = g_A + s_A + b_A.$$  

Because R2 assigns the same weight to gold, silver, and bronze medals, there have been several proposals to improve this way of counting by introducing weights for medals. For instance, the Fibonacci weighted point system (this method is shown in Table 2 as R3) uses the following weights: gold gets 3 points, silver 2 points, and bronze 1 point; these weights are called the 3:2:1 system. Thus

$$R3(A) = 3g_A + 2s_A + b_A.$$  

Table 2 shows that Norway and United States have the same rank R3, but in the rank Norway has a higher position because it has won more golds (the same situation holds for Switzerland and Sweden). To make gold medals more precious, the exponential weighted point system assigns 4 points to gold, 2 points to silver, and 1 point to bronze—the 4:2:1 system. The variation used by the British press during the Olympic Games in London in 1908 used the weights 5:3:1. There exist also systems 5:3:2, 6:2:1, 10:5:1, etc.

Other rankings use completely different ideas. For instance, one method counts all the medals won (weighted or not), counting separately the medals for each individual athlete in team sports. Another uses an improvement rank based on the percentage improvement attained by countries with respect to the previous Games results. There exist ranks built in comparison to expectations. Among them there are predictions based on previous results (in the Games or other competitions) and predictions using economics, population, and a range of other criteria.

Another interesting proposal is to calculate the rank by dividing the number of medals by the population of the country. The column R4 in Table 3 shows the total number of medals won by a country per 10 million people. Whereas criteria R1–R3 yield similar results, criterion R4 puts different countries, mainly those with relatively small populations, at the top. In fact, Norway, with 26 medals and a population of approximately 5 million people, is the best in this ranking. In general, the countries that top the list have small populations in comparison, for instance, with the United States and the Russian Federation. The number of medals per $100$ billion of the gross domestic product (GDP) of the country (this rank is called R5 in Table 2) also favors smaller countries.

In this note, I do not discuss the advantages and disadvantages of various ranks. Instead, we consider a purely mathematical problem regarding a difference between the unofficial International Olympic Committee rank R1 and the other ranks R2–R5. In fact, although ranks R2–R5 produce numerical coefficients for each country that allow one to rank-order the countries, rank R1 does not produce any number that can be used for this purpose. This rank uses the lexicographic ordering, used in dictionaries to order words: first words are ordered with respect to the first symbol in the word, then with respect to the second one, and so on. In working with the rank R1 we have words that consist of three symbols $g_A$, $s_A$, $b_A$ and, therefore, their length $w = 3$.

I show, however, that there is a procedure for computing rank R1 numerically for each country and for any number of medals. Moreover, the computation can be generalized from words consisting of three symbols to words having a general finite length $w$ and used in situations that require lexicographic ordering.
How Can We Compute the Rank R1 for Any Number of Medals?

Evidently, in the rank R1, gold medals are more precious than silver ones, which in turn are better than the bronze ones. An interesting issue arises. Let us consider Belarus and Austria, which occupy the 8th and 9th positions, respectively. Belarus has 5 gold medals and Austria only 4. The fact that Austria has 8 silver medals and Belarus has none is not taken into consideration. Austria could have respectively. Belarus has 5 gold medals and Austria only 4.

Can we quantify what these words, more important, mean? Can we introduce a counter that would allow us to compute a numerical rank of a country using the number of gold, silver, and bronze medals in such a way that the higher gold, silver, and bronze medals in such a way that the higher
counter that would work for any number of medals.

More formally, I wish introduce a number \(n(g_A, s_A, b_A)\), where \(g_A\) is the number of gold medals, \(s_A\) is the number of silver medals, and \(b_A\) is the number of bronze won by a country \(A\). This number should be calculated so that, for countries \(A\) and \(B\), we have

\[
n(g_A, s_A, b_A) > n(g_B, s_B, b_B), \quad \text{if} \quad \begin{cases} g_A > g_B, \\
    g_A = g_B, s_A > s_B, \\
    g_A = g_B, s_A = s_B, b_A > b_B.
\end{cases}
\]

As mentioned earlier, \(n(g_A, s_A, b_A)\) should not depend on the upper bound \(K > \max\{g_A, s_A, b_A\}\) for the number of medals of each type that can be won by each country.

As a first try in calculating \(n(g_A, s_A, b_A)\), let us assign weights to \(g_A, s_A\), and \(b_A\) as is done in the positional numeral system with a base \(\beta\):

\[
n(g_A, s_A, b_A) = g_A \beta^2 + s_A \beta^1 + b_A \beta^0 = g_A s_A b_A
\]

For instance, in the decimal positional numeral system with \(\beta = 10\), the record \(n(g_A, s_A, b_A) = g_A 10^2 + s_A 10^1 + b_A 10^0 = g_A s_A b_A\) provides the rank of the country \(A\). However, we see immediately that this does not solve our problem, because it does not satisfy condition (1). In fact, if a country has more than 11 silver medals, then formula (3) implies that these medals are more important than one gold. For instance, the data

\[
g_A = 2, s_A = 0, b_A = 0, \quad g_B = 1, s_B = 11, b_B = 0.
\]

As it is difficult to make numerical computations with infinity (symbolic computations can be done with nonstandard analysis, see [11]) because in the traditional calculus \(\infty\) absorbs any finite quantity, and we have, for instance,

\[
\infty + 1 = \infty, \quad \infty + 2 = \infty.
\]

A Numerical Calculator of the Rank R1 Involving Infinites

To construct a numerical calculator of a medal ranking involving infinite numbers, let us recall the difference between numbers and numerals: a numeral is a symbol or a group of symbols that represents a number. The difference between them is the same as the difference between words and the things to which they refer. A number is a concept that a numeral expresses. The same number can be represented by different numerals. For example, the symbols “7,” “seven,” and “VII” are different numerals, but they all represent the same number.