Accurate evaluation of hadronic uncertainties in spin-independent WIMP–nucleon scattering: Disentangling two- and three-flavor effects

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We show how to avoid unnecessary and uncontrolled assumptions usually made in the literature about soft $SU(3)$ flavor symmetry breaking in determining the two-flavor nucleon matrix elements relevant for direct detection of WIMPs. Based on $SU(2)$ Chiral Perturbation Theory, we provide expressions for the proton and neutron scalar couplings $f_u^S, f_n^S$ and $f_d^S, f_n^S$ with the pion–nucleon $\sigma$-term as the only free parameter, which should be used in the analysis of direct detection experiments. This approach for the first time allows for an accurate assessment of hadronic uncertainties in spin-independent WIMP–nucleon scattering and for a reliable calculation of isospin-violating effects. We find that the traditional determinations of $f_u^S - f_d^S$ and $f_n^S - f_d^S$ are off by a factor of 2.

I. INTRODUCTION

Establishing the nature of dark matter (DM) is one of the fundamental open problems in particle physics and cosmology. A weakly interacting massive particle (WIMP) is an excellent candidate since, for masses in the GeV to TeV range, it naturally provides a relic abundance consistent with that required of DM. Direct detection experiments aim at measuring recoil energy deposition consistent with that required of DM. The dimensionless Wilson coefficients $C_i$, encode unresolved dynamics at energy scales higher than the cutoff $\Lambda$, which is of the order of the mass of the lightest high-energy particles that get integrated out.

In this paper we stress a point that has been overlooked in the literature and investigate its important implications. Information on nucleon matrix elements involving just $u$- and $d$-quarks have so far been extracted from an empirical formula based on soft flavor $SU(3)$ symmetry breaking. This prevents the possibility to assign any reliable theory uncertainty to these predictions. Here we show how to properly relate two-flavor dependent quantities to phenomenology in a rigorous, model-independent way based on Chiral Perturbation Theory (ChPT), the effective field theory of QCD at low energies. In particular, we disentangle two-flavor observables from matrix elements involving the strange quark, which can be more reliably determined from lattice QCD computations. We clarify the role of the input parameters in the SI WIMP–nucleon cross section in such a way that hadronic uncertainties can now be accurately assessed. While the impact of the pion–nucleon $\sigma$-term $\sigma_{\pi N}$ has been emphasized before, here we work out its effects devoid of unnecessary $SU(3)$ assumptions. Better convergence is a distinctive feature of the two-flavor chiral expansion in $M_\pi/\Lambda_\chi$ as compared to its three-flavor analog, which involves $M_\pi/\Lambda_\chi$ corrections, with $\Lambda_\chi \approx 1$ GeV the typical scale of chiral symmetry breaking. Moreover, starting from ChPT in its $SU(2)$ formulation allows for the well-controlled calculation of isospin-breaking effects, whose incorporation is crucial in the context of isospin-violating DM.

In the next sections we provide all the formulae that should be used in phenomenological analyses, provide updated expressions for the scalar couplings to $u$- and $d$-...
quarks, and illustrate the role of hadronic uncertainties in the SI WIMP–nucleon cross section as a function of the Wilson coefficients for quark scalar and gluon effective operators.

II. SPIN-INDEPENDENT CROSS SECTION AND CHIRAL PERTURBATION THEORY

In terms of the contributions from the dynamical degrees of freedom at the hadronic scale relevant for direct detection, the SI cross section for elastic Dirac WIMP scattering on a nucleon \((N \in \{p, n\})\) has the form (cf. \cite{13,18,20})

\[
\sigma_{NI} = \frac{\mu_X^2}{4 \Lambda^4} \left( \sum_{q=u,d,s} C_{qq}^{SS} f_q^N - 12 \pi C_{gg}^S f_Q^N \right) + \sum_{q=u,d} C_{qq}^{VV} f_{Q}^N, \tag{3}
\]

with \(\mu_X = m_X m_N / (m_X + m_N)\) and scalar (vector) couplings \(f_q^N\) \((f_Q^N)\). For heavy quarks, the parameter \(f_Q^N\) is induced by the gluon operator as discussed in \cite{21}. Accordingly, the Wilson coefficient \(C_{qq}^{SS}\) encodes matching corrections from integrating out \(c-, b-,\) and \(t\)-quarks as well as possible new heavier strongly interacting particles. The vector coefficients simply count the valence quarks in a proton or a neutron, i.e. \(f_u^N = f_d^N = 2, f_s^N = 2,\) while the scalar couplings measure the contribution of the quark condensates to the mass of the nucleon

\[
\langle N|qq|qq\rangle = f_q^N m_N. \tag{4}
\]

In the literature (see, e.g. \cite{12,22,24}) \(f_u^N\) and \(f_d^N\) are usually determined from the so-called strangeness content of the nucleon

\[
y = \frac{2 \langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}, \tag{5}
\]

and another quantity

\[
z = \frac{\langle N|\bar{u}u - \bar{s}s|N\rangle}{\langle N|\bar{d}d - \bar{s}s|N\rangle}. \tag{6}
\]

The combination of \(y\) and \(z\) then permits the reconstruction of \(f_u^N\) and \(f_d^N\). \(y,\) in turn, is usually determined from \(\sigma_{\pi N}\) based on \(SU(3)\) ChPT \cite{25}, an approach by itself afflicted with large uncertainties from the \(SU(3)\) expansion. More crucially, it is not possible to attach a reliable error to the estimate \(z \approx 1.49\) in \cite{9,22} commonly employed in the literature since it originates from leading-order fits to the baryon spectrum, whose inadequacy had already been demonstrated in \cite{26,27}. Nevertheless, this value for \(z\) has been widely used (see e.g. \cite{12,23,24}) without any attempt to quantify its inherent systematic uncertainty.

All these shortcomings can be avoided by using directly \(SU(2)\) ChPT. The starting point is the chiral expansion of the nucleon mass in the presence of strong isospin violation \cite{28,29}

\[
m_N = m_0 - 4c_1 M^2_\pi - \frac{e^2 F^2}{2} (f_1 \pm f_2 + f_3) \tag{7}
\]

\[
\pm 2 B c_5 (m_d - m_u) - \frac{g_A^2 (2M^3_\pi + M^3_\sigma)}{32 \pi F^2} + O(M^4),
\]

where the upper (lower) sign refers to proton (neutron), \(B\) is related to the pion masses according to

\[
M^2_\pi = B(m_u + m_d) + 2e^2 F^2 Z + O(m_q^2),
\]

\[
M^2_\sigma = B(m_u + m_d) + O(m_q^2), \tag{8}
\]

\(F_\pi\) denotes the pion decay constant, \(e = \sqrt{4\pi\alpha}\) the electric charge, \(g_A\) the axial coupling of the nucleon, and \(c_1, c_5, f_{1-3}, Z\) are low-energy constants, which encode short-distance effects.

The scalar couplings follow from (7) by means of the Feynman–Hellmann theorem \cite{30,31}

\[
\langle N|m_q \bar{q}q|N\rangle = m_q \frac{\partial m_N}{\partial m_q} \quad \text{with} \quad q \in \{u,d\}, \tag{9}
\]

resulting in

\[
f_u^N = -\frac{2B}{m_N} m_u \left[ 2c_1 \pm c_5 + \frac{9g_A^2 M_\pi}{128 \pi F^2} \right],
\]

\[
f_d^N = -\frac{2B}{m_N} m_d \left[ 2c_1 \pm c_5 + \frac{9g_A^2 M_\pi}{128 \pi F^2} \right], \tag{10}
\]

where \(M_\pi = (2M_\pi^+ + M_\pi^-)/3\) denotes an average pion mass. Next, we define \(\sigma_{\pi N}\) as the average value of \(1/2 \langle N|(m_u + m_d) (\bar{u}u + \bar{d}d)|N\rangle\) between proton and neutron,\(^2\) which leads to the identification

\[
\sigma_{\pi N} = -4c_1 M^2_\sigma - \frac{9g_A^2 M^2_u M_\pi}{64 \pi F^2} + O(M^4_\pi). \tag{11}
\]

This expression can be derived from (7), rewritten in terms of \(\hat{m} = (m_u + m_d)/2\) and the quark-mass difference, via another Feynman–Hellmann relation

\[
\sigma_{\pi N} = \frac{1}{2} \left( \frac{\partial m_p}{\partial \hat{m}} + \frac{\partial m_n}{\partial \hat{m}} \right). \tag{12}
\]

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\(^1\) If the WIMP is a Majorana fermion, the right-hand side of (4) has to be multiplied by a factor of 4. In (3), contributions from tensor twist-2 operators are not present since we restrict ourselves to operators up to dimension 7.

\(^2\) At this order in the chiral expansion the expressions for proton and neutron even coincide.
In this way, we obtain the following result for the scalar couplings
\[
m_N f_u^N = \frac{\sigma_{\pi N}}{2} (1 - \xi) \pm B c_5 (m_d - m_u) \left(1 - \frac{1}{\xi}\right),
\]
\[
m_N f_d^N = \frac{\sigma_{\pi N}}{2} (1 + \xi) \pm B c_5 (m_d - m_u) \left(1 + \frac{1}{\xi}\right),
\]
\[
\xi = \frac{m_d - m_u}{m_d + m_u} = 0.36 \pm 0.04, \quad (13)
\]
where again the upper (lower) sign refers to proton (neutron) and we used \(m_u/m_d = 0.47 \pm 0.04\) from \([25, 26]\).

Taking particle masses from \([32, 33]\) and \(B c_5 (m_d - m_u) = (-0.51 \pm 0.08)\) MeV according to the electromagnetic proton-neutron mass difference \((m_p - m_n)^{em} = (0.76 \pm 0.3)\) MeV from \([27]\),\(^4\) we find
\[
f_u^N = \frac{\sigma_{\pi N}(1 - \xi)}{2 m_N} + \Delta f_u^N, \quad f_d^N = \frac{\sigma_{\pi N}(1 + \xi)}{2 m_N} + \Delta f_d^N,
\]
\[
\Delta f_u^p = (1.0 \pm 0.2) \cdot 10^{-3}, \quad \Delta f_u^n = (-1.0 \pm 0.2) \cdot 10^{-3},
\]
\[
\Delta f_d^p = (-2.1 \pm 0.4) \cdot 10^{-3}, \quad \Delta f_d^n = (2.0 \pm 0.4) \cdot 10^{-3}.
\]
\[(14)\]

Expressing \(c_1\) by means of \([11]\) can be understood as resuming higher chiral orders. We have verified this procedure explicitly at fourth order in the chiral expansion \([26, 35, 50]\), with low-energy constants from \([40]\) for a numerical analysis. Our result shows that once \(\sigma_{\pi N}\) is fixed, \(f_u^N\) and \(f_d^N\) can be inferred immediately, with both chiral expansion and isospin violation fully under control. This is crucial in order to accurately evaluate hadronic uncertainties in SI direct detection.

The importance of these findings for isospin-violating DM can be nicely illustrated by considering the difference between proton and neutron couplings
\[
\begin{align*}
f_u^p - f_u^n &= (1.9 \pm 0.4) \cdot 10^{-3},
\end{align*}
\]
\[
\begin{align*}
f_d^p - f_d^n &= (-4.1 \pm 0.7) \cdot 10^{-3},
\end{align*}
\]
\[(15)\]
where we used \([13]\) directly, so that \(\sigma_{\pi N}\) and \(c_1\) drop out and the remaining uncertainty is generated by \(c_5\) and \(m_u/m_d\). Comparing this result to the most recent estimate \([24]\)
\[
\begin{align*}
f_u^p - f_u^n &= 4.3 \cdot 10^{-3}, \quad f_d^p - f_d^n = -8.2 \cdot 10^{-3},
\end{align*}
\]
\[(16)\]
we see that the traditional approach overestimates isospin violation by a factor of 2. As the difference between proton and neutron couplings is proportional to \(c_5\), which measures the quark-mass contribution to the proton-neutron mass difference, this implies that the indirect reconstruction of this quantity by means of \(y\) and \(z\) fails by 100%.

A precise determination of the crucial \(\sigma_{\pi N}\) is still an open issue. Ongoing efforts involve lattice QCD calculations at (nearly) physical values of the pion mass and refined phenomenological analyses. For a compilation of recent lattice results we refer to \([24, 41–43]\) and references therein. The extraction of \(\sigma_{\pi N}\) from \(\pi N\) scattering requires an analytic continuation into the unphysical region \([44]\), which is extremely sensitive to small shifts in the isoscalar amplitude, so that even isospin-breaking effects may become important. On the experimental side, new information about threshold \(\pi N\) scattering has become available over the last years thanks to accurate measurements in pionic atoms \([45, 46]\). These results led to a precision extraction of the \(\pi N\) scattering lengths \([17, 18]\), which are extremely valuable in stabilizing the analytic continuation.\(^5\) For these reasons, a systematic analysis of \(\pi N\) scattering fully consistent with unitarity, analyticity, and crossing symmetric along the lines of \([52, 54]\), respecting the new pionic-atom input, will help clarify the situation concerning the phenomenological determination of \(\sigma_{\pi N}\) \([55, 56]\).

Traditionally, the strangeness coupling \(f_s^N\), or, equivalently, the strangeness content \(y\), has been determined from \(\sigma_{\pi N}\) based on \(SU(3)\) ChPT \([25]\), incurring large uncertainties both from \(\sigma_{\pi N}\) and the \(SU(3)\) expansion. In view of recent lattice results, where contrary to the lightest quarks \(m_s\) is close to its physical value, a large strangeness content as sometimes inferred from \(\sigma_{\pi N}\) becomes increasingly unlikely. In the following, we adopt the average from \([43]\)
\[
f_s^N = 0.043 \pm 0.011, \quad (17)
\]
which takes into account the details of each lattice calculation in the averaging procedure.

Finally, the coupling for the heavy quarks is \([21]\)\(^6\)
\[
f_Q^N = \frac{2}{27} (1 - f_u^N - f_d^N - f_s^N). \quad (18)
\]

### III. NUMERICAL ANALYSIS

We first compare our results for the light-quark couplings to the traditional approach (see \([12, 22]\)), as a function of \(\sigma_{\pi N}\). Since in the latter case the \(u-\) and

\(^3\) In the isospin limit, this reduces to \(m_N f_u^N = m_N f_d^N = \sigma_{\pi N}/2\), as expected \([13]\).

\(^4\) Within uncertainties, this estimate for \(c_5\), originating from an analysis of the Cottingham sum rule \([24]\), is consistent with a recent determination from a subtracted version of this sum rule with the subtraction constant estimated from nucleon polarizabilities \([33]\), an extraction from \(p n \rightarrow d n^0\) \([34]\), and lattice calculations, see \([37]\) and references therein.

\(^5\) In addition, these results for the scattering lengths nicely illustrate the sensitivity of the \(\sigma\)-term extraction to small changes in the isoscalar amplitude, as the isospin-breaking corrections \([49]\) translated to \(\sigma_{\pi N}\) according to \([51]\) would lead to a shift of more than 5 MeV.

\(^6\) For a discussion of \(f_Q^N\) at higher orders in \(\alpha_s\) we refer to \([58, 59]\).
cise determination of This shows that one can take proper advantage of a pre-
the band for a given value of \( \sigma \) compared to our approach. More importantly,
observe a moderate shift of the central value or a change
the inherent uncertainty may be even larger. \( \sigma \) extracted from hadron masses in analogy to [9], to
\( \pm 36 \) shift between the leading-order value
expansion, we simply take a 30% error. In fact, the large
without resorting to higher-order calculations for
z, our approach is the only way to achieve reliable error

\[
y = \frac{m_N f_s^N 2 m}{\sigma_{\pi N} m_N}.
\]

Without resorting to higher-order calculations for \( z \), as
usually done in the literature, it is impossible to provide
a reliable uncertainty estimate for this quantity. Based
on general expectations of the convergence of the \( SU(3) \)
expansion, we simply take a 30% error. In fact, the large
shift between the leading-order value \( \sigma_0 \approx 26 \text{ MeV} \), as
extracted from hadron masses in analogy to [1], to \( \sigma_0 \approx (36 \pm 7) \text{ MeV due to higher chiral orders indicates that}
the inherent uncertainty may be even larger.

As shown in Fig. 1, for both determinations of \( y \) we observe a moderate shift of the central value or a change
in slope, compared to our approach. More importantly,
the band for a given value of \( \sigma_{\pi N} \) shrinks drastically.
This shows that one can take proper advantage of a pre-
cise determination of \( \sigma_{\pi N} \), with accurate error estimates,
only within our framework, since otherwise the need for
strangeness input thwarts the transition to the two-flavor
scalar couplings. Due to the arbitrariness in estimating
the uncertainties of this strangeness input, especially of
\( z \), our approach is the only way to achieve reliable error
enha brilliance in estimating

\[
\Sigma = C_{dd}/C_{uu} = 0 \quad \text{and} \quad \sigma_{\pi N} = 50 \text{ MeV}.
\]

This illustrates the maximally possible isospin violation induced by scalar operators. Color coding as in Fig. 1

\[\text{FIG. 2: Ratio of proton and neutron cross section for } C_{dd}^{sS} = C_{ss}^{sS} = C_{VV}^{s} = 0 \quad \text{and} \quad \sigma_{\pi N} = 50 \text{ MeV}. \]

Our central results are the expressions given in [19] and [20] based on \( SU(2) \) ChPT. We have provided
values for these coefficients, as a function of the pion-
nucleon \( \sigma \)-term, without any reference to an \( SU(3) \)
exansion and consistently incorporating isospin-violating
d- couplings are reconstructed from two strangeness

\[
f_{u,d}^{p,n} \quad \text{and} \quad f_{d}^{p,n} \quad \text{as a function of } \sigma_{\pi N} \quad \text{according to (14) (red) compared to the traditional approach, with } y \text{ either derived from } \sigma_{\pi N} \quad \text{or the lattice value (17) for } f_{s}^{N} \quad \text{(blue). In both plots, the upper (lower) bands refer to } d- \text{ (u-)quark couplings.}
\]

As shown in Fig. 1, for both determinations of \( y \) we observe a moderate shift of the central value or a change
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scalar couplings. Due to the arbitrariness in estimating
the uncertainties of this strangeness input, especially of
\( z \), our approach is the only way to achieve reliable error
estimates.

Constraining the Wilson coefficients \( C_{dd}^{sS} \) and \( C_{ss}^{sS} \), see [2] and [3], allows one to gain information about
DM-Higgs operators from direct detection [60], by
proper renormalization group evolution, matching corrections [21], and mixing [61], from the low-energy hadronic
scale up to the scale \( \Lambda \) of New Physics [62]. The impact of
our results compared to the traditional approach becomes
most pronounced in the context of isospin violation. In
the absence of vector operators, the ratio \( C_{dd}^{sS}/C_{uu}^{sS} \) is
the quantity responsible for isospin-violating effects. Models
with isospin violation in the scalar sector have been con-
sidered e.g. in [2, 13, 63–65]. It has been argued that even in the Constrained MSSM isospin violation could
be large enough to be detected in experiment [12]. In
Fig. 2, we show the ratio of SI WIMP–proton and WIMP–
neutron cross sections as a function of \( C_{dd}^{sS}/C_{uu}^{sS} \), assuming
that all other Wilson coefficients are zero. Again, we see that using our approach the uncertainties reduce
dramatically, while hinting at smaller isospin-violating ef-
fects than expected before, see [15] and [14].

IV. CONCLUSIONS

In this article we have presented a novel approach to
determine the proton and neutron scalar couplings \( f_{p,n}^{p,n} \) and \( f_{d}^{p,n} \), which are key input quantities for direct DM
searches. Our central results are the expressions given in [19] and [20] based on \( SU(2) \) ChPT. We have provided
values for these coefficients, as a function of the pion-
nucleon \( \sigma \)-term, without any reference to an \( SU(3) \)
exansion and consistently incorporating isospin-violating

\[
d_{i} = \langle \phi \rangle \quad \text{for } \sigma \text{ (17) for } f_{s}^{N} \quad \text{(blue). In both plots, the upper (lower) bands refer to } d- \text{ (u-)quark couplings.}
\]

As shown in Fig. 1, for both determinations of \( y \) we observe a moderate shift of the central value or a change
in slope, compared to our approach. More importantly,
the band for a given value of \( \sigma_{\pi N} \) shrinks drastically.
This shows that one can take proper advantage of a pre-
cise determination of \( \sigma_{\pi N} \), with accurate error estimates,
effects. Thus removing an additional source of theoretical uncertainty that had so far been overlooked in the literature, our results permit an honest assessment of hadronic uncertainties in DM detection without uncontrolled approximations.

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