SAHA IONIZATION FORMULA AND THE VOIDS

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Abstract

The ultra-low density limit of Saha ionization formula suggests that, in this limit, matter would prefer to remain ionized. This has a very important implication for cosmic structures known as Voids. These are ultra-low density (much less than average density of matter in the Universe) regions in the galactic clusters and superclusters. The ionization formula implies that matter trapped in the Voids should be ionized. Therefore, we expect a very faint radiation glow from the Voids resulting from the motion of the charged particles.
I. INTRODUCTION

The Saha ionization formula [Saha 1920] has played a very important role in the development of astrophysics. The ultra-low density limit of this formula has been known for a long time [Feynman et al 1963]. In this limit, the formula suggests that the atoms in equilibrium prefer to remain in ionized state. This ionization, just from "expansion" as the density goes down, has been listed as one of the surprises in theoretical physics by Peierls [Peierls 1979]. In this paper we point that this ultra-low density limit of Saha ionization formula is very relevant for the cosmic structures known as Voids. These ultra low density (much less than average density of matter in the Universe) regions dominate the volume in the Universe. The ionization formula implies that matter trapped in the Voids should be ionized. Therefore, we expect a very faint radiation glow from the Voids resulting from the motion of the charged

II. ULTRA-LOW DENSITY LIMIT OF THE SAHA IONIZATION FORMULA

The ionization formula is given by,

\[ \frac{n_e n_i}{n_a} = \frac{1}{v_a} e^{-W/kT} \]  

(1)

In the equation above, \( n_e \), \( n_i \), \( n_a \) are the densities of electrons, ions and atoms(not ionized) respectively. \( W \) is the ionization potential, \( T \) is the temperature and \( k \) is the Boltzmann constant. The volume occupied by a bound electron at temperature \( T \) is represented by \( v_a \). It is, essentially, the volume contained within a thermal de Broglie wave length.

\[ v_a = \lambda_{th} \frac{3}{2} = \left( \frac{2\pi \hbar^2}{m_e kT} \right)^{3/2} \]  

(2)

Let us consider a box of volume \( V \) which, to start with, contains \( N \) number of hydrogen atoms. Let a fraction \( X \) of them be ionized. In this case, \( n_e = \frac{N}{V} X = n_i \) and \( n_a = (1-X) \frac{N}{V} \). Substituting these values in the ionization formula we obtain,

\[ \frac{X^2}{1-X} \frac{N}{V} \frac{1}{V} = \frac{1}{v_a} e^{-W/kT} \]  

(3)
From the equation above, we see that the fraction of charged particles in equilibrium increases when we increase the volume(i.e., decrease the density). In the ultra-low density ($\frac{N}{V} \to 0$ or $\frac{V}{N} \to \infty$), atoms would prefer to remain ionized [Feynman et al. 1963, Peierls 1979, Ghosh and Ghosh 1998]. Before we discuss the nature and consequences of the ultra-low density limit, let us introduce the cosmic structures known as Voids [Zeldovich et al. 1982, Sahni and Coles 1995] which are, essentially, underdense (less than the average density of matter in the universe) regions in galactic clusters and superclusters. This requires review of some aspects of standard cosmology and the theory of structure formation at the large scale. We review very briefly, in next section, the materials relevant for our discussion.

III. THE VOIDS

It is very well established through observation that at distance scales of the order of 200-300 megaparsec, the Universe is isotropic and homogenous. This means that if we pick up a region of the Universe of dimension 200-300 megaparsec at any distance and in any direction, it will contain the same amount of matter. Therefore, at this scale the density of matter can be considered to be constant. In Newtonian gravity, such a distribution of matter implies that at every point in space the potential and force are unbounded [Landau and Lifshitz 1975]. This dilemma is resolved in the General Theory of Relativity. For an isotropic and homogenous distribution of matter one assumes the Friedman-Robertson-Walker metric, given by the line element

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

in which the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{3} T_{\mu\nu}$$

(5)

takes the simple form [Landau and Lifshitz 1975],

$$\frac{\dot{a}^2 + ka^2}{a^4} = \frac{8\pi G}{3} \rho_0$$

(6)
where $a(t)$ is the scale factor, $\rho_0$ is the averaged constant density, and $k = 1, -1$ or 0, respectively for closed, open and flat universe. This equation along with the equation of state describes the isotropic and homogenous universe. The equations clearly show that the isotropic and homogenous distribution of matter can not be stable - the Universe expands. The constant density serves as a source term for the evolution of the scale factor. It does not give rise to attractive gravitational force. Therefore, the question that arises is: what is the source of gravity in the large scale? The answer is obtained as follows. When the mean free path of the particles is small, matter can be treated as an ideal fluid and the Newton’s equations governing the motion of gravitating collisionless particles in an expanding Universe can be written in terms of \( x = r/a \) (the comoving space coordinate), \( v = \dot{r} - Hr = a\dot{x} \) (the peculiar velocity field, \( H \) is the Hubble constant), \( \phi(x, t) \) (the Newton gravitational potential) and \( \rho(x, t) \) (the matter density). This gives us the following set of equations [Sahni and Coles 1995, Strauss and Willick 1995]. Firstly, the Euler equation,

\[
\frac{\partial(a\dot{v})}{\partial t} + (v \cdot \nabla_x) v = -\frac{1}{\rho} \nabla_x P - \nabla_x \phi \tag{7}
\]

Next the continuity equation

\[
\frac{\partial \rho}{\partial t} + 3H\rho + \frac{1}{a} \nabla_x (\rho v) = 0 \tag{8}
\]

And, finally the Poisson equation

\[
\nabla_x^2 \phi = 4\pi G a^2 (\rho - \rho_0) = 4\pi G a^2 \rho_0 \delta \tag{9}
\]

where \( \rho_0 \) is the mean background density and \( \delta = \rho/\rho_0 - 1 \) is the density contrast.

Therefore, at large scale, the source of gravity is not the average density \( \rho_0 \) but the density fluctuations, \( \delta \rho > 0 \). It is a subject of study in theory of structure formation as to what kind of density fluctuation would grow in time and lead to the formation of galaxies, and clusters and superclusters of galaxies [Padmanabhan 1993, Peebles 1980, Sahni and Coles 1995, Strauss and Willick 1995]. It is important to remember that at the scale of dimensions, 200 - 300 megapersecs, the Universe is homogenous and isotropic and acquires constant
density, and therefore, if in some subregion $\delta \rho > 0$, there must be some subregion where $\delta \rho < 0$, so as to reproduce the constant density profile. These domains with $\delta \rho < 0$ are known as Voids [Zeldovich et al 1982, Sahni and Coles 1995]. Note that for Voids $\delta \rho / \rho_0$ is always bounded bellow by $-1$. Such regions of Voids dominate the volume in the universe giving rise to cellular structures with the clusters and superclusters of galaxies forming string like walls around them. Existence of Voids are supported by direct observation as well as numerical simulation of hydrodynamic equations [Sahni and Coles 1995, Ryden and Mellot 1996, Hoyle and Vogeley 2001, Antonuccio-Delogu et al. 2000]. The observed Voids seem to have dimension of several (tens of) megapersecs.

**IV. CONCLUSION**

The Voids are the ultra-low density regions in the Universe, and these are the regions where one would expect to observe the consequences of ultra-low density limit of the Saha ionization formula. As discussed before, in this limit, atoms would prefer to remain ionized. At this stage, the important question is: what is the source of the ionization energy? There are the starlights but at high red shift, their intensity is very low. The most common source of ionization energy at high red shift is the lights from Quasars.

The second source which may sound somewhat speculative is the following. In the beginning the Voids expand faster than the Universe [Sahni and Coles 1995]. However, in the radiation domination era the whole Universe is kept in a single equilibrium state by the radiation field. During the decoupling of radiation, this equilibrium is destroyed, and in the process some ions may remain trapped in the Voids.

The Saha ionization formula implies that, at ultra-low density, once the ionization takes place there is hardly any chance for recombination [Feynman et al 1963, Peierls 1973, Ghosh and Ghosh 1998]. Therefore, the motion of the charged particles in the Voids should create a faint radiation glow.
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