ROTATING NON-KERR BLACK HOLE AND ENERGY EXTRACTION

CHANGQING LIU\textsuperscript{1,2}, SONGBAI CHEN\textsuperscript{3}, AND JILIANG JING\textsuperscript{1,3}

\textsuperscript{1} Department of Physics, Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University, Changsha, Hunan 410081, China; jljing@hunnu.edu.cn
\textsuperscript{2} Department of Physics and Information Engineering, Hunan Institute of Humanities Science and Technology, Loudi, Hunan 417000, China
\textsuperscript{3} Department of Physics, Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University, Changsha, Hunan 410081, China

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ABSTRACT

The properties of the ergosphere and energy extraction by the Penrose process in a rotating non-Kerr black hole are investigated. It is shown that the ergosphere is sensitive to the deformation parameter $\epsilon$ and the shape of the ergosphere becomes more oblate as parameter $\epsilon$ increases. It is of interest to note that, compared with the Kerr black hole, the deformation parameter $\epsilon$ can enhance the maximum efficiency of the energy extraction process greatly. Especially for the case of $a > M$, the non-Kerr metric describes a superspinning compact object and the maximum efficiency can exceed 60\%, while it is only 20\% for the extremal Kerr black hole.

Key words: black hole physics – gravitation

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1. INTRODUCTION

In four-dimensional general relativity, the no-hair theorem (Israel 1972, 1968; Carter 1971; Hawking 1972; Robinson 1975) guarantees that a neutral rotating astrophysical black hole is uniquely described by the Kerr metric, which only possesses two parameters, the mass $M$ and the rotational parameter $a$. For the Kerr black hole, the fundamental limit is the bound $a \leq M$, and the central singularity is always behind the event horizon due to the weak cosmic censorship conjecture (Penrose 1969). However, the hypothesis that the astrophysical black hole candidates are described by the Kerr metrics still lacks direct evidence, and the general relativity has been tested only for weak gravitational fields (Will 2006). In the regime of strong gravity, the general relativity could be broken down and astrophysical black holes might not be the Kerr black holes as predicted by the no-hair theorem (Caravelli & Modesto 2010; Johannsen & Psaltis 2011a; Bambi & Modesto 2011). Several parametric deviations from the Kerr metric have been suggested to study the observational signatures in both the electromagnetic (Hughes 2011; Psaltis & Johannsen 2011; Johannsen & Psaltis 2010a, 2010b, 2011b) and gravitational wave (Ryan 1995, 1997a, 1997b; Boffa & Cutler 2004; Brink 2008; Li & Lovelace 2008; Apostolatos et al. 2009) spectrum that differ from the expected Kerr signals.

Recently, Johannsen and Psaltis proposed a deformed Kerr-like metric (Johannsen & Psaltis 2011a) suitable for the strong field of the no-hair theorem, which describes a rotating black hole (we named it the non-Kerr black hole) in an alternative theory of gravity beyond Einstein’s general relativity. The non-Kerr black hole possesses the following novel features: there is no restriction on the value of the rotational parameter $a$ due to the existence of the deformation parameter $\epsilon$. Interestingly, for a positive parameter $\epsilon$, the non-Kerr black hole becomes more prolate than the Kerr black hole, and there are two disconnected horizons for high spin parameters, but there is no horizon when $a > M$. Therefore, for a negative parameter $\epsilon$, the non-Kerr black hole is more oblate than the Kerr black hole, and the horizon always exists for an arbitrary $a$ and the topology of the horizon becomes toroidal (Bambi & Modesto 2011, 2012).

\footnote{Corresponding author.}

The non-Kerr metric is an ideal spacetime to carry out strong-field tests of the no-hair theorem. Therefore, much effort has been dedicated recently to the study of the rotating non-Kerr black hole (Caravelli & Modesto 2010; Johannsen & Psaltis 2011a; Bambi & Modesto 2011, 2012; Bambi 2011; Pani et al. 2011; Johannsen 2012). In Chen & Jing (2012), we studied the properties of the thin accretion disk in the rotating non-Kerr spacetime and found that the presence of the deformation parameter $\epsilon$ changes the inner border of the disk, energy flux, conversion efficiency, radiation temperature, spectral luminosity, and spectral cutoff frequency of the thin accretion disk. Moreover, for the rapidly rotating black hole, the effect of the deformation parameter $\epsilon$ on the physical quantities of the thin disk becomes more distinct for the prograde particles and more tiny for the retrograde ones. These significant features in the mass accretion process may provide a possibility to test gravity in the regime of the strong field in the astronomical observations.

The power energy for active galactic nuclei, X-ray binaries, and quasars has always been a concern in high-energy astrophysics. Several mechanisms, e.g., the accretion disk model (Kozlowski et al. 1978; Shakura & Sunyaev 1973) and the Blandford–Znajek mechanism (Blandford & Znajek 1977) have been proposed to interpret how to extract energy from a black hole and the formation of the power jet. Furthermore, the Penrose process (Penrose 1969; Chandrasekhar 1983; Bhat et al. 1985; Parthasarathy et al. 1986) also provides an important method to extract energy from a black hole. The Penrose process was also extended to the five-dimensional supergravity rotating black hole (Prabhu & Dadhich 2010), higher dimensional black holes and black rings (Nozawa & Maeda 2005), and Hořava-Lifshitz gravity (Abdujabbarov & Ahmedov 2011). In this paper, we will investigate in detail the ergosphere of the non-Kerr black hole and how the deformation parameter $\epsilon$ of the non-Kerr black hole affects the negative energy state and the efficiency of the energy extraction.

The paper is organized as follows: in Section 2, we review briefly the metric of the rotating non-Kerr black hole proposed by Johannsen & Psaltis (2011a) to test the gravity in the regime of the strong field and then analyze the ergosphere structure. In Section 3, we investigate the efficiency of the energy extraction by using the Penrose process. Section 4 is devoted to a brief summary.
2. ROTATING NON-KERR BLACK HOLE SPACETIME

To test the gravity in the regime of the strong field, Johannsen & Psaltis (2011a), starting from a deformed Schwarzschild solution and applying the Newman–Janis transformation, constructed a deformed Kerr-like metric that describes a stationary, axisymmetric, and asymptotically flat vacuum spacetime. In the standard Boyer–Lindquist coordinates, the metric can be expressed as (Johannsen & Psaltis 2011a)

$$ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + g_{\theta\theta} d\theta^2 + g_{\phi\phi} d\phi^2 + 2g_{t\phi} dt d\phi,$$

(2.1)

with

$$g_{tt} = -\left(1 - \frac{2Mr}{\rho^2}\right)(1 + h), \quad g_{\phi\phi} = -\frac{2aMr}{\rho^2} (1 + h),$$

$$g_{rr} = \frac{\rho^2(1 + h)}{\Delta + a^2 h \sin^2 \theta}, \quad g_{\theta\theta} = \rho^2,$$

$$g_{\phi\phi} = \sin^2 \theta \left[\rho^2 + \frac{a^2(\rho^2 + 2Mr) \sin^2 \theta}{\rho^2} (1 + h)\right].$$

(2.2)

where

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2Mr + a^2, \quad h = \frac{\epsilon M^3 r}{\rho^2}.$$

(2.3)

The constant $\epsilon$ is the deformation parameter. The quantity $\epsilon > 0$ or $\epsilon < 0$ corresponds to the cases in which the compact object is more prolate or oblate than the Kerr black hole, respectively. As $\epsilon = 0$, the black hole is reduced to the usual Kerr black hole in general relativity. The horizons of the black hole are described by the roots of the following equation (Johannsen & Psaltis 2011a):

$$\Delta + a^2 h \sin^2 \theta = 0.$$  

(2.4)

Clearly, the radii of the horizons depend on $\theta$, which are different from that in the usual Kerr case. For the case of $\epsilon > 0$, two disconnected horizons exist for high spin parameters, but there is no horizon when $a > M$. However, for $\epsilon < 0$ the horizons never disappear for an arbitrary $a$ and the shape of the horizons becomes toroidal (Bambi & Modesto 2011, 2012).

The infinite redshift surface of a black hole is defined by the roots of $g_{tt} = 0$. For the non-Kerr black hole, we note that $g_{tt} = 0$ gives

$$1 + h = 0, \quad \text{or} \quad 1 - \frac{2Mr}{\rho^2} = 0.$$ 

(2.5)

Obviously, for the case of $\epsilon \geq 0$, the outer infinite redshift surface is determined by $M + (M^2 - a^2 \cos^2 \theta)^{1/2}$. For $\epsilon < 0$, it seems that both the positive root of $1 + h = 0$ and $1 - \frac{2Mr}{\rho^2} = 0$ are the infinite redshift surfaces of the non-Kerr black hole. However, the determinant of the metric $\sqrt{-g} = (1 + h)\rho^2 \sin^2 \theta$ vanishes, if $1 + h = 0$, which shows that the surface defined by $1 + h = 0$ is an intrinsic singularity. The singularity is indicated by infinite curvature and cannot be eliminated by coordinate transformation, and its Kretschmann scalar $\mathcal{K} = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$ and $\sqrt{-g}$ becomes zero or infinity. In Figure 1, we depict the positive root of the equations $1 + h = 0, 1 - (2Mr/\rho^2) = 0$, and $\Delta + a^2 h \sin^2 \theta = 0$ with different deformation parameter $\epsilon$. The figure also shows that the surface defined by $1 + h = 0$ is the intrinsic singularity, because if we choose the surface defined by $1 + h = 0$ as the outer infinite redshift surface, we find that the metric of the non-Kerr black hole does not satisfy $g_{tt} > 0, g_{\phi\phi} < 0$, and $g_{rr} > 0$ in the ergosphere and the causality of the spacetime is violated. Thus, the surface defined by $1 + h = 0$ cannot be the infinite redshift surface. In Figure 1, there is an intersection point of the three surfaces, and the value of the abscissa for the intersection point $\epsilon_p$ is $-4[M + (M^2 - a^2 \cos^2 \theta)^{1/2}]$. A turning point $\epsilon_p$ also exists for the event horizon surface. The value of the abscissa for the turning point is located at the position $(\partial \epsilon/\partial r) = 0$; thus, we get

$$10r^4 - 16Mr^3 + a^2 (7 + \cos(2\theta)) r^2 - a^4 (1 + \cos(2\theta)) = 0.$$ 

(2.6)

The maximum positive root of Equation (2.6) is

$$r_p = \frac{1}{2} \left[ \frac{-A}{2} + \sqrt{Z_1 + \left(\frac{3A^3}{4} - 2B - Z_1 + \frac{-A^3 + 4AB}{4\sqrt{Z_1}}\right)^{1/2}} \right].$$

$$Z_1 = \frac{A^2}{4} - \frac{2B}{3} + \frac{21/2(B^2 + 12C)}{3Z_{23}^{1/3}} + \frac{Z_{23}^{1/3}}{32^{1/3}}$$

$$Z_2 = 2B^3 + 27A^2 C - 72BC$$

$$+ \left[-(4B^2 + 12C) + (2B^3 + 27A^2 C - 72BC)^2\right]^{1/2}$$

$$A = -\frac{8M}{5}, \quad B = \frac{a^2(7 + \cos(2\theta))}{10}, \quad C = -\frac{a^4(1 + \cos(2\theta))}{10}.$$ 

(2.7)

Using Equations (2.6) and (2.7), we get the value of the abscissa for the turning point of the curve described by $\Delta + a^2 h \sin^2 \theta = 0$, which is

$$\epsilon_p = \frac{-\Delta \rho^4}{M^3 a^2 r^2 \sin^2 \theta} \bigg|_{r=r_p}.$$ 

(2.8)

The ergosphere is the region bounded by the event horizon $r_H$ and the outer stationary limit surface $r_\infty$. A novel feature of the ergosphere of a black hole is that the time-like killing vector becomes space-like crossing the infinite redshift surface. An observer moving along the time-like geodesics cannot remain stationary in the ergosphere due to the “frame dragging effect” (Will 2006). According to the properties of the ergosphere, the deformation parameter $\epsilon$ should satisfy the following relation:

$$-4[M + (M^2 - a^2 \cos^2 \theta)^{1/2}] \leq \epsilon \leq \frac{\Delta \rho^4}{M^3 a^2 r^2 \sin^2 \theta} \bigg|_{r=r_p}.$$ 

(2.9)

In this range of $\epsilon$, the non-Kerr black hole has an event horizon given by the largest root of $\Delta + a^2 h \sin^2 \theta = 0$ and an outer infinite redshift surface described by

$$r_H = M + (M^2 - a^2 \cos^2 \theta)^{1/2}.$$ 

(2.10)

When $\epsilon \geq \epsilon_p$ or $\epsilon < \epsilon_p$, there is no event horizon and the singularity becomes naked.

How the deformation parameter $\epsilon$ affects the shape of the ergosphere is described by Figure 2, which shows that the
Figure 1. Variation of the event horizon radius and the outer infinite redshift surface radius with the deformation parameter $\epsilon$ of the rotating non-Kerr black hole. The green, blue, and red lines correspond to the surfaces defined by $1 + h = 0$, $1 - (2Mr/\rho^2) = 0$, and $\Delta + a^2 h \sin^2 \theta = 0$, respectively. The dashed line is the location of the turning point of the surface, defined by $\Delta + a^2 h \sin^2 \theta = 0$. Here, we take $M = 1$.
(A color version of this figure is available in the online journal.)
ergosphere is sensitive to the deformation parameter $\epsilon$. For $a < M$ (such as $a = 0.87$), the non-Kerr black hole becomes more prolate than the Kerr black hole, and the ergosphere in the equatorial plane becomes thick as the parameter $\epsilon$ increases. It should be pointed out that, when $\epsilon$ exceeds $4.4676$, the horizons become disconnected. For the case of $a = M$, the inner and outer horizons that coincide at the north and south poles exist, and the thickness of the ergosphere decreases when the deformation parameter $\epsilon$ takes a bigger negative value. For $a > M$, $\epsilon$ can only take a negative value, and both the horizon and infinite redshift surface are not closed. A hole appears around the north and south poles in the range $\theta_{\text{hole}} \leq \arccos(a/M)$. Thus, a distant observer may see the central region of this compact object along the north or south pole. It is also interesting to note that the overspin compact object becomes increasingly thin and looks like a disk (Bambi & Modesto 2011) as the deformation parameter $\epsilon$ increases.

3. ENERGY EXTRACTION OF THE BLACK HOLE BY PENROSE PROCESS

In this section, we will discuss the Penrose process (Penrose 1969; Chandrasekhar 1983; Nozawa & Maeda 2005; Bhat et al. 1985; Parthasarathy et al. 1986), by which we can extract a rotational energy from the non-Kerr black hole. We focus our attention on how the deformation parameter $\epsilon$ of the non-Kerr black hole affects the negative energy state and the efficiency of the energy extraction.
### 3.1. The Negative Energy State of the Penrose Process

Let us now consider the trajectory of a test particle with the mass \( \mu \) on the equatorial plane. With the help of the time-like killing vector \( \xi^a = (\partial / \partial t)^a \) and space-like one \( \psi^a = (\partial / \partial \phi)^a \), we have the following conserved quantities along a time-like geodesics on the equatorial plane:

\[
E = -g_{ab} \xi^a u^b = \left( 1 + \frac{\epsilon M^3}{r^3} \right) \left( 1 - \frac{2M}{r} \right) u^t + \left( 1 + \frac{\epsilon M^3}{r^3} \right) \frac{2aM}{r} u^\phi, \tag{3.1}
\]

\[
L = g_{ab} \psi^a u^b = -\left( 1 + \frac{\epsilon M^3}{r^3} \right) \frac{2aM}{r} u^t + \left( r^2 + a^2 + \frac{a^2 M^3 (a^2 + r^2) \epsilon}{r^5} + \frac{2a^2 M}{r} \right) u^\phi, \tag{3.2}
\]

where \( u^b \) is the 4-velocity defined by \( u^b = dx^b / d\tau \) and \( \tau \) is the proper time for the spacetime. In Equation (3.1) or (3.2), the first equality is the basic definition of the energy or angular momentum (Bambi 2011), and the second equality describes the energy or angular momentum of the non-Kerr black hole. Moreover, we can introduce a new conserved parameter

\[
\kappa = g_{ab} u^a u^b, \tag{3.3}
\]

whose values are given by \( \kappa = -1, 0, 1 \) corresponding to the time-like, null, and space-like geodesics, respectively.

From Equations (3.1), (3.2), and (3.3), we can easily obtain the equation of motion

\[
\alpha E^2 - 2\beta E + \gamma = 0, \tag{3.4}
\]

with

\[
\alpha = \left( r^2 + a^2 + \frac{a^2 M^3 (a^2 + r^2) \epsilon}{r^5} + \frac{2a^2 M}{r} \right) \Gamma^{-1}, \tag{3.5}
\]

\[
\beta = L \left( 1 + \frac{\epsilon M^3}{r^3} \right) \frac{2aM}{r} \Gamma^{-1}, \tag{3.6}
\]

\[
\gamma = -L^2 \left( 1 + \frac{\epsilon M^3}{r^3} \right) \left( 1 - \frac{2M}{r} \right) \Gamma^{-1} - \frac{r^2 (r^3 + \epsilon M^3) (u^t)^2}{r^3 + a^2 \epsilon M^3} - \mu^2, \tag{3.7}
\]

where

\[
\Gamma = \left( 1 + \frac{\epsilon M^3}{r^3} \right) \left( \Delta + a^2 \epsilon M^3 \right). \tag{3.8}
\]

From Equation (3.4), we can obtain the energy \( E \)

\[
E = \frac{\beta + \sqrt{\beta^2 - \alpha \gamma}}{\alpha}, \tag{3.9}
\]

where we only choose \( +\sqrt{\beta^2 - \alpha \gamma} \) to ensure that the 4-momentum of the particle is future directed. In the Penrose process, the orbit of the particle with negative energy in the ergosphere is the key to extract energy from the non-Kerr black hole. When a particle enters the ergosphere, the time-like killing vector becomes a space-like one; thus, the energy of the particle \( E = -g_{ab} \xi^a u^b \) becomes negative. The orbit of the particle with the negative energy \( E \) must satisfy the conditions \( \alpha > 0 \), \( \beta < 0 \), and \( \gamma > 0 \), which can be achieved only if \( La < 0 \). In Figure 3, we describe the negative energy state \( E \) for the different deformation parameter \( \epsilon \) at a given location near the event horizon inside the ergosphere. We find that the negative energy \( E \) increases as the deformation parameter \( \epsilon \) increases for both cases \( a > M \) and \( a < M \).

According to the Penrose process, the mass of the black hole will change a quantity \( \delta M = E \) as a negative particle is injected into the central black hole. Clearly \( \delta M \) can be made as large as we wish by increasing the mass \( \mu \) of the injected particle.
However, there is a lower limit on $\delta M$ that could be added to the black hole corresponding to $\mu = 0$ and $u' = 0$ (Misner et al. 1973). Evaluating all of the required quantities at the horizon $r_H$, we can get the lower limit

$$E_{\text{min}} = \frac{L \left( 1 + \frac{\epsilon M^3}{r_H^4} \right) 2\mu M}{r_H^2 + a^2 + a^2 M^3 \left( \frac{a^2 + \epsilon^2}{r_H} \right) \epsilon + 2a^2 M}.$$  (3.10)

From Equation (3.10), we can conclude that, in order to extract energy from the black hole, the angular momentum of the injected particle must satisfy $L < 0$, and the deformation parameter $\epsilon$ influences the value of $E_{\text{min}}$.

### 3.2 Efficiency of the Energy Extraction Process

The efficiency of the energy extraction process is one of the most important questions in the energetics of the black hole. Thus, it is interesting to study how the deformation parameter $\epsilon$ affects the efficiency of the Penrose process (Penrose 1969; Chandrasekhar 1983; Bhat et al. 1985; Parthasarathy et al. 1986) for the non-Kerr black hole. To calculate the maximum efficiency of the energy extraction, we take the radial velocity to be zero. Let $U_i$ denote the 4-velocity of the $i$th particle of the locally non-rotating frame observer (Will 2006) at a given radius $r$, which can be expressed as

$$U_i = u'(1, 0, 0, \Omega_i),$$  (3.11)

with

$$u' = -\frac{E}{X_i}, X_i = -(g_{tt} + g_{t\phi} \Omega_i),$$  (3.12)

$$\Omega_i = -\frac{g_{t\phi}(1 + g_{tt}) + \left[ (1 + g_{tt}) \left( g_{t\phi}^2 - g_{tt} g_{\phi\phi} \right) \right]^{1/2}}{g_{\phi\phi} + g_{t\phi}},$$

where $\Omega_i$ is the angular velocity of the particle $i$ with respect to an asymptotic infinity observer. In the ergosphere, $\Omega_i$ takes the value in the range of $\Omega_- < \Omega < \Omega_+$, where

$$\Omega_{\pm} = \frac{-g_{t\phi} \pm \left( g_{t\phi}^2 - g_{tt} g_{\phi\phi} \right)^{1/2}}{g_{\phi\phi}}.$$  (3.13)

In the Penrose process, an incident particle $1$ with the rest mass $\mu_1 = 1$, i.e., $E_1 = 1$, splits into the particle $2$ absorbed by the black hole and the particle $3$ escaping to infinity. According to the conservative laws of the energy and angular momentum, we get

$$U_1 = \mu_2 U_2 + \mu_3 U_3.$$  (3.14)

The efficiency of the Penrose process is defined as

$$\eta = \frac{\mu_3 E_3 - E_1}{E_1} = \mu_3 E_3 - 1.$$  (3.15)

The maximum efficiency $\eta$ can be obtained by the choice of $\mu_2 U_2$ and $\mu_3 U_3$ (Bhat et al. 1985; Parthasarathy et al. 1986):

$$\mu_2 U_2 = k_2(1, 0, 0, \Omega_-)$$
$$\mu_3 U_3 = k_3(1, 0, 0, \Omega_+),$$  (3.16)

where $k_2$ and $k_3$ are constants to be determined. With the help of Equations (3.11), (3.12), (3.14), and (3.16), we find

$$\eta = \frac{(\Omega_1 - \Omega_-)(g_{tt} + g_{t\phi} \Omega_1)}{(\Omega_+ - \Omega_-)(g_{tt} + g_{t\phi} \Omega_+)} - 1.$$  (3.17)

When the incident particle $1$ splits at the horizon $r_H$, we thus obtain the maximum efficiency

$$\eta_{\text{max}} = \frac{\sqrt{1 + g_{tt}^2} - 1}{2}.$$  (3.18)

Now, we would like to analyze the effects of $\epsilon$ on the efficiency of the rapidly rotating non-Kerr black hole. We calculate the maximum efficiency with the numerical method and present the variation of the maximum efficiency in the energy extraction process with the deformation parameter $\epsilon$, which takes the value in the range (2.9) to guarantee that the non-Kerr black hole has the connected event horizon for a fixed $a$ in Figure 4. We also present the effects of the parameter $\epsilon$ on the maximum efficiency in Tables 1 and 2.

### Table 1

| $a$  | $\epsilon$ | $\eta_{\text{max}}$ | $\eta_{\text{max}}$ |
|------|------------|---------------------|---------------------|
| 0.2  | 0         | 0.25%               | 0.275%              |
| 0.4  | 0         | 0.5%                | 0.75%               |
| 0.6  | 0         | 0.75%               | 1.175%              |
| 0.8  | 0         | 1.0%                | 2.710%              |
| 0.9  | 0         | 1.5%                | 5.940%              |
| 0.99 | 0         | 20%                 | 9.01%               |

Note. Here, we set $M = 1$.

Now, we could like to analyze the effects of $\epsilon$ on the efficiency of the rapidly rotating non-Kerr black hole. We calculate the maximum efficiency with the numerical method and present the variation of the maximum efficiency in the energy extraction process with the deformation parameter $\epsilon$, which takes the value in the range (2.9) to guarantee that the non-Kerr black hole has the connected event horizon for a fixed $a$ in Figure 4. We also present the effects of the parameter $\epsilon$ on the maximum efficiency in Tables 1 and 2.
From Equation (3.18), we find that the maximum efficiency change of the energy extraction intuitively. Why is it not so?

\( \epsilon = - \epsilon \) gradually, by accretion or any other process, from \( a = M \) and from \( \epsilon > 0 \) to \( \epsilon < 0 \), we should expect a continuous change of the energy extraction intuitively. Why is it not so? From Equation (3.18), we find that the maximum efficiency \( \eta_{\text{max}} = (\sqrt{1 + g_{\text{tt}} - 1/2})_{\text{var}} \) is related to the event horizon \( r_H \) of the non-Kerr black hole. In Figure 5, by taking \( \epsilon = -0.001 \) and considering that the value of \( a \) changes gradually from \( a = 1 \) to 1.001, we find that the variation of the event horizon \( r_H \) is not continuous. Therefore, we cannot obtain a continuous change of the energy extraction for the process by taking \( a \) from \( a = M \) to \( a > M \) (or from \( \epsilon > 0 \) to \( \epsilon < 0 \)).

4. SUMMARY

In this paper, we present a detailed analysis of the properties of the ergosphere in the rotating non-Kerr black hole proposed recently by Johannsen & Psaltis (2011a) to test the gravity in the regime of the strong field in future astronomical observations. We now summarize our results as follows: (1) We present the restricted conditions on the deformation parameter \( \epsilon \) to guarantee that the non-Kerr black hole has the connected horizons (see Equation (2.9) and Figure 1). (2) We show that the ergosphere is sensitive to the deformation parameter \( \epsilon \) (see Figure 2), and the shape of the ergosphere becomes thick as the deformation parameter \( \epsilon \) increases. (3) We find that, compared with the Kerr black hole, the deformation parameter \( \epsilon \) not only enlarges the negative energy \( E \) (see Figure 3) but also enhances the maximum efficiency of the energy extraction process (see Tables 1 and 2 and Figure 4). Moreover, the maximum efficiency can exceed 60% for the non-Kerr compact objects with \( a > M \). The influence of the deformation parameter \( \epsilon \) on the maximum efficiency presents a good theoretical opportunity to distinguish the non-Kerr black hole from the Kerr one and to test whether or not the current black hole candidates are the black holes predicted by Einstein’s general relativity. However, we think that such a test is not possible at present.

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Figure 5. Variation of the event horizon with the rotating parameter \( a \) of a rotating non-Kerr black hole with \( M = 1 \) and \( \epsilon = -0.001 \).

(A color version of this figure is available in the online journal.)

The Maximum Efficiency \( \eta_{\text{max}} \) of the Energy Extraction Process in the Non-Kerr Black Hole Depends on the Parameter \( \epsilon \) for \( a > M \)

| \( \epsilon \) | \( a = 1.001 \) | \( a = 1.01 \) | \( a = 1.1 \) | \( a = 1.15 \) | \( a = 1.2 \) |
|---|---|---|---|---|---|
| -0.00001 | 60.739% | 59.954% | 52.885% | 49.487% | 46.406% |
| -0.0001 | 59.577% | 58.806% | 51.859% | 48.520% | 45.492% |
| -0.001 | 56.922% | 56.186% | 49.547% | 46.357% | 43.455% |
| -0.01 | 50.088% | 49.492% | 43.945% | 41.201% | 38.683% |
| -0.1 | 13.136% | 13.844% | 24.878% | 26.231% | 25.901% |
| -1 | 5.278% | 5.360% | 6.087% | 6.420% | 6.638% |

Note. Here, we set \( M = 1 \).

plane is much thicker than that of the extremal Kerr black hole \((a = 1, \epsilon = 0)\) in Figure 2. If the values of \( a \) and \( \epsilon \) change gradually, by accretion or any other process, from \( a < M \) to \( a > M \) and from \( \epsilon > 0 \) to \( \epsilon < 0 \), we should expect a continuous change of the event horizon intuitively. Why is it not so?