Improving students' mathematical inductive reasoning ability through reflective learning Model

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Abstract. Mathematical inductive reasoning ability is one of the higher-order thinking skills that students must have. Mathematical inductive reasoning includes the process of drawing to conclude, estimating answers or solutions, and providing explanations of existing models, facts, properties, relationships, or patterns. This research aims to analyze mathematical inductive reasoning ability between students who received a reflective learning model and those who received conventional learning. The research was conducted at one of the senior high school in Tangerang Selatan. The method used in this research is a quasi-experimental method with Randomized Subjects Post-test Only Control Group Design. The instrument used was a mathematical inductive reasoning ability test. The research results based on the analysis using t-test showed that the students' mathematical inductive reasoning ability who are taught by reflective learning model higher than students taught by the conventional model. This matter visible from the mean score of the results tests students' mathematical inductive reasoning who taught with reflective learning model is 75.50 and who taught with a conventional model is 67.68. In conclusion, this research results that mathematics learning with a reflective learning model can improve the students' mathematical inductive reasoning ability better than using conventional learning models.

1. Introduction
Reasoning ability is one of the mathematics learning objectives in the national curriculum that students must own. Reasoning ability also plays an important role in the world of education. It can be seen in the students' learning process in a class—reasoning ability used by students consciously or unconsciously in the learning process, especially in learning mathematics. Students need reasoning ability in understanding mathematics, so it can be said that learning mathematics and reasoning ability are two things that cannot be separated. Namely, learning mathematics can be understood through reasoning, and reasoning can be trained and improved through learning mathematics. Students'reasoning ability is used when students are required to solve problems and draw conclusions that involve students' logical thinking processes. Therefore, students' reasoning ability needs to be improved. Ways to train and improve students’ reasoning ability can be done in learning mathematics's process, so learning mathematics is also very important. Also, Sukardjono stated that mathematics is a way or method of thinking and reasoning [1]. Thus, it can also be said that if a student wants to have good reasoning ability, the student needs to understand mathematics well as well because reasoning with mathematics learning cannot be separated.

In fact, students' reasoning abilities are still relatively low. This can be seen from the results of international studies such as Trends in International Mathematics and Science Research (TIMSS).
2015, the average percentage of Indonesian students in the cognitive domain of reasoning is 15% below the international average of 25% [2]. From the results of the pre-research conducted, it was found that the percentage of students’ mathematical inductive reasoning ability on three indicators, namely generalizing, estimating answers, and explaining the existing models, facts, properties, relationships, or patterns that were tested were 33.33%, 43.75%. And 25.00%. Of the three indicators tested, it was clear that a very small percentage was in the third indicator, followed by the first indicator. In indicators 1 and 3, most of the students did not answer the questions given, and some students gave explanations and drew conclusions but were not related to the problems given. This explains that the students’ mathematical inductive reasoning ability in these schools is still relatively low.

Based on the results of the interview, it was found that one of the factors that caused the students’ low mathematical inductive reasoning ability was the lack of interest in learning from students. The learning model applied in the classroom is still teacher-centered. Also, in giving math problems, teachers often provide easy and routine questions that only require students to solve them according to the examples given without knowing what they have finished. This results in passive students, especially in building their knowledge through scientific activities during the learning process. Therefore, researchers use a reflective learning model as a solution to low reasoning ability by designing teaching materials and instruments to be implemented in learning in the research to be carried out.

Reflective learning has many advantages, especially when used in matters relating to inductive reasoning ability. Reflective learning is a learning model that involves reflective thinking in the process. Students learn what they are dealing with during the reflective thinking process, and students will assume, assess, and apply their understanding. This will be very useful for students if it occurs regularly because students will arrive at a more in-depth understanding, change students’ thinking patterns, and in the end, students can solve problems with their thoughts. In other words, a reflective learning model that is carried out routinely in the learning process will help students be better prepared to process their experiences so that they become materials for students to grow and develop in solving problems, especially in mathematics learning. That is when students’ mathematical inductive reasoning ability will increase. Reflective thinking processes can empower students to identify the ability to identify mathematical concepts or formulas, arrange solutions with various strategies, predict completion, and the ability to conclude and evaluate [3]. In order for this research to be more focused, to provide a clearer picture of the problem under research, and based on the background of the problems that have been raised, the problem formulation in this research is: Is the students’ mathematical inductive reasoning ability of students who use a reflective learning model higher than the students' mathematical inductive reasoning ability who in their learning use conventional learning models.

2. Literature Review
2.1 Mathematical Inductive reasoning ability
Reasoning ability is closely related to everyday life. This is following R.G. Soekadijo, who stated that reasoning is close and means very close to inferences, arguments, and evidence [3]. Therefore, everyone must have the ability to reason. According to Sumarmo, reasoning comes from English, namely reasoning, which means achieving logical conclusions based on facts and relevant sources [4]. Sadiq argues that reasoning is a process or a thinking activity to draw a conclusion or a thought process to make a new true statement based on several statements whose truth has been proven or previously assumed [5]. Thus, the term reasoning can be defined as a thinking activity to conclude. Reasoning ability takes place when a person thinks about a problem or solves a problem.

The material in learning mathematics is closely related to students’ mathematical reasoning ability. Therefore students who have good reasoning ability will be able to understand mathematics material well as well. The NCTM also states that learning programs at the kindergarten through 12 levels should allow students to [6]:

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1) Recognizing reasoning and proof as very basic aspects of mathematics.
2) Conducting and investigate mathematical conjectures.
3) Developing and evaluate mathematical arguments and proofs.
4) Selecting and using different types of reasoning and various methods of proof.

From the description above, it can be concluded that mathematical reasoning ability is a person's ability to use his thinking with facts, models, or existing solutions to conclude, solve, and solve a mathematical problem. Mathematical reasoning can be classified into two types, namely inductive reasoning and deductive reasoning. Inductive reasoning is a thought process that tries to connect known facts or special events to a general conclusion [4]. Fadjar Sadiq defines inductive reasoning as an activity of a thought process that seeks to link known facts or special evidence to a general conclusion [7]. From this description, the essence of inductive reasoning is logical thinking's process in drawing general conclusions from specific problems.

Increasing the ability of inductive reasoning is an important thing for students because by using this ability, students can predict the phenomena that occur and problems that occur in learning, especially mathematics learning. This is expected to help students adapt more effectively to the various problems they face. From the description above, it can be concluded that the mathematical inductive reasoning ability is a general logical conclusion based on a specific problem, where the problems are in the form of mathematical problems.

In terms of the characteristics of the conclusion drawing process, inductive reasoning includes the following activities [8]:

1) Transductive reasoning, namely the process of concluding limited observations and applied to certain cases;
2) Analogical reasoning, namely the process of drawing conclusions based on the similarity of processes or data;
3) Generalized reasoning, namely the process of drawing general conclusions based on limited data;
4) Estimating answers, solutions, or trends: interpolation and extrapolation;
5) Explaining existing models, facts, characteristics, relationships, or patterns;
6) Using relationship patterns to analyze situations and construct conjectures.

In this research, the indicators of mathematical inductive reasoning used are as follows:

1) Transductive reasoning, namely the process of concluding limited observations applied to certain cases.
2) Estimating answers, solutions, or trends: interpolation and extrapolation.
3) Explaining existing models, facts, characteristics, relationships, or patterns.

2.2. Reflektif Learning Model
Reflective thinking makes students more aware of what they are learning and provides the possibility for a deeper understanding of each learning process. Reflective thinking is a thinking activity that allows a person to activate his thinking ability to do or understand a learning material with the experiences he has gone through. Reflective learning rests on thinking reflective and is closely related to the human brain's workings. Boud et al. define reflective as an intellectual and affective activity where individuals explore their experiences to achieve new understandings and appreciations [9]. Also, reflective thinking can provide a stimulus for a person's thinking so that he is accustomed to solving problems with solutions he gets himself with a thought process that is following the subject matter.

Pollard suggests seven characteristics of reflective teaching, namely [10]:

1) Implementing activeness by paying attention to the objectives and consequences, as well as technical means and efficiency.
2) Applying in a cycle (spiral) process, the teacher monitors, assess, and continuously revises learning activities.
3) Demanding ability in evidence-based classroom research methods to support the progressive development of higher teaching standards.
4) Demanding an attitude of open-mindedness, responsibility, and sincerity.
5) Carrying out based on teacher consideration or assessment through evidence-based assessment and knowledge from various studies.
6) Professional learning and personal achievement can be enhanced through collaboration and dialogue with peers.
7) Allowing teachers to creatively bridge or construct an external framework that is developed into learning.

Coughlan emphasized that students who engage in reflective learning have an active role in learning and recognize their responsibility to be lifelong students [9]. In reflective learning, students act as the main character who carries out all the instructions needed to understand the subject matter. In the application of reflective learning in the classroom, there are several important things to say. From the explanation above, it can be concluded that reflective learning is a learning process that prioritizes student activeness so that students are accustomed to testing and investigating problems, especially math problems and analyzing individual experiences they experience, and facilitating learning from these experiences.

One of the reflective learning models is reflective learning developed by The International Commission on the Apostolate of Jesuit Education (ICAJE) [11]. Reflective learning consists of three main elements, namely, experience, reflection, and action. For these three elements to be applied properly, a pre-learning element is needed, namely context, and a post-learning element, namely evaluation [12]. In a complete application, there are five steps of reflective learning, namely: context, experience, reflection, action, and evaluation.

3. Research Method
This research was conducted at SMK Islamiyah Ciputat, located at Jalan Pendidikan 1 Ciputat, South Tangerang. The research method used is Quasi-Experimental. Researchers cannot fully control other factors that will arise, such as interest factors, intelligence factors, motivational factors, and learning mode factors that affect control and experimental variables. The research design used was randomized subjects post-test only control group design [14]. The population in this research were all students of class XI SMK Islamiyah Ciputat. The sampling technique in this research was using random cluster sampling. After conducting Cluster Random Sampling, the experimental group was selected (which in learning uses a reflective learning model), namely class XI TKJ 3 with 30 students, and another class selected to be the control group (in learning using conventional learning models), namely class XI TKJ 2 with 32 students.

The research instrument used was a mathematical inductive reasoning ability test in a written form description test. Before being used, the test was tested first to determine its validity, reliability, difficulty expectation, and distinction. The data in this research were obtained from the post-test results of students' mathematical inductive reasoning ability. Before testing the hypothesis on the data obtained, the prerequisite analysis is first tested, namely:

3.1. Normality Test
The normality test is carried out to find out whether the sample under research comes from a normally distributed population or not. The normality test of the data from the results of this research used the Shapiro-Wilk test (W-test) with SPSS software. The formulas used in the Shapiro-Wilk test are [15]:

\[ d = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 - \frac{1}{n} \sum_{i=1}^{n} x_i^2 \]

\[ n \quad : \text{Jumlah data yang akan diujikan} \]
b. Barrier (k) W-test:

\[ k = \begin{cases} n \frac{n}{2} & \text{if } n \text{ = Even} \\ \frac{n-1}{2} & \text{if } n \text{ = Odd} \end{cases} \]

c. Formula \( W_{\text{count}}(W) \):

\[ W = \frac{1}{d} \left( a_i \left( x_{[n-i+1]} - x_{[i]} \right) \right)^2 \]

3.2. Homogeneity Test

The homogeneity test is carried out to determine the similarity of variances between two situations or populations. If the homogeneity test results show the similarity of variance (homogeneous), it meant that the experimental class and the control class have the same ability. The test carried out is using the Levene test using software assistance. The formula used in the Levene test is as follows [15]:

\[ W_o = \frac{\sum_i (\bar{z}_i - \bar{z}.)^2 / (g - 1)}{\sum_j (\bar{z}_{ij} - \bar{z}_i .)^2 / \sum_i (n_i - 1)} \]

with

\[ \bar{z}_i = \frac{\sum x_{ij}}{n_i} \text{ dan } \bar{z} = \frac{\sum \bar{z}_{ij}}{\sum n_i} \]

Data from the post-test results were analyzed using the two-mean difference test (t-test). This test is carried out on the condition that the two data analyzed must be normally distributed. The formula for testing the hypothesis using the t-test statistic at the significant level \( \alpha = 0.05 \) is as follows [16]:

If homogeneous, the data will be analyzed using the t-test as follows:

\[ t_{hit} = \frac{x_1 - x_2}{s_g \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \]

where \( s_g = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} \)

However, if it is not homogeneous, then the data will be analyzed by t'-test as follows

\[ t'_{hit} = \frac{x_1 - x_2}{s'_g} \text{ with } s'_g = \sqrt{\frac{s_1^2 + s_2^2}{n_1 + n_2}} \]

where \( s'_g = \sqrt{\frac{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}{n_1 + n_2}} \)

with \( n_1 + n_2 - 2 \), \( n \) is the number of students

4. Result

The data used in this research results from the mathematical inductive reasoning ability test tested in two sample classes after the two classes have been given different treatments in the learning process. The data on the final results of students' mathematical inductive reasoning ability in the experimental class using the reflective learning model and control class students whose learning uses conventional learning models are presented in Table 1 below.

**Tabel 1.** Students’ mathematical inductive reasoning ability in experiment and control class.

|                | Experimental | Control |
|----------------|--------------|---------|
| N              | 30           | 32      |
| Minimum        | 42           | 38      |
| Maximum        | 96           | 92      |
| Mean           | 75.50        | 67.28   |
| Median         | 77.00        | 69.00   |
| Std. Deviation | 14.799       | 15.956  |

Table 1 shows that the highest individual mathematical inductive reasoning ability is in the experimental class, while the lowest individual mathematical inductive reasoning ability is in the
control class. The average score obtained by the experimental class was higher than the mean score of students in the control class. Besides, it can be seen that the standard deviation and variance values of the experimental class are smaller than the standard deviation and variance values of the control class. In the experimental class, the difference between the frequency at each interval is much different, while in the control class, the frequencies at each interval are close to each other, so the standard deviation of the control class is greater than the standard deviation in the experimental class. This shows that the control class's values distribution is more varied, while the experimental class groups at values above average.

Indicators of students' mathematical inductive reasoning ability in this research consisted of three indicators, namely transductive reasoning (a), estimating answers, solutions or tendencies: interpolation and extrapolation (b), and providing explanations of models, facts, properties, relationships, or patterns there (c). The scores of students' mathematical inductive reasoning ability in the experimental class and control class in terms of each indicator can be seen visually in Figure 1 below.

Figure 1 shows that the average score per indicator of students' mathematical inductive reasoning ability in the experimental class is higher than the average score per indicator of students' mathematical inductive reasoning ability in the control class. A very significant difference is found in transductive reasoning indicators, where the experimental class is 73.33%, while the control class is 59.72%. Another case with indicators of mathematical inductive reasoning ability is estimating answers, solutions, or trends: interpolation and extrapolation. The experimental class's average score on the indicators predicts answers, solutions, or trends: interpolation and extrapolation of 81.67%, while the average in the control class is 77.60%. The average percentage score on the indicator explains the model, facts, nature, relationship, or pattern in the experimental class is higher than the control class, which is 73.33%, while the control class is only 67.71%.

The normality and homogeneity tests that have been done show that the test scores of mathematical inductive reasoning ability in the experimental and control classes are normally distributed and homogeneous, so hypothesis testing is carried out using the t-test. Data from the results of statistical hypothesis testing using the t-test are presented in Table 2 below.
Table 2. Hypothesis test results of mathematical inductive reasoning ability students of experiment class and control class.

| Nilai             | Equal variances assumed | T   | Df  | Sig.  |
|-------------------|-------------------------|-----|-----|-------|
|                   |                         | 2.099 | 60 | .020  |
|                   | Equal variances not assumed | 2.104 | 59.994 | .020  |

From the different test results, the mean difference between the experimental class and the control class for mathematical inductive reasoning ability shows to reject H0 and accept H1. Based on the resulting significance value is smaller than the specified α value (0.020 < 0.050), it can be concluded that the average mathematical inductive reasoning ability of experimental class students using the reflective learning model is higher than the average mathematical inductive reasoning ability of control class students, which uses conventional learning.

5. Discussion

The results showed that students' mathematical inductive reasoning ability taught using a reflective learning model were higher than those taught using conventional learning models. The average score of students' mathematical inductive reasoning ability with the reflective learning model was also higher than the conventional learning model. Based on the results of the post-test data hypothesis analysis using the t-test in the experimental class and the control class showed that the average mathematical inductive reasoning ability of the experimental class students who were taught using a reflective learning model was higher than the mathematical inductive reasoning ability of students in the control class who were taught using conventional learning model. This is also supported by the results obtained on each indicator developed in the research.

Inductive reasoning mathematical ability of students in transductive indicator shows that the experimental class students have been able to conclude from the information received on the matter. Students can also prove the truth of the conclusions they make by using complete steps and processes. The conclusions made by students are also complete and arranged in detail so that the conclusions made are following the problems presented. Students in the control class indicate that the conclusions on students' answers are also correct but tend to give direct conclusions after knowing the truth. On indicators of estimating answers, solutions, or trends: interpolation and extrapolation show that the experimental class students can answer correctly. The estimated answer is also accompanied by the reasons and evidence students receive from the question's information. It is for this reason that students can accurately estimate the answer. In control class students, the reasons made were still not following the information provided by the questions.

The interesting and unique findings that occur in this research can be seen in the indicators that explain the existing models, facts, properties, relationships, or patterns showing that students in the experimental class can solve problems related to a model in group data using the concept of the intersection of two straight lines. So far, almost no students search for a group data model using the concept of the intersection of two lines. They usually use the mode formula from known group data. Thus, students who use the reflective learning model can use previous learning experiences and connect the relationships between mathematics concepts, namely the concept of line equations in geometry and the mode of statistics. Examples of questions and unique answers given by students in solving mathematical inductive reasoning ability test questions are as follows:

Problems example:
"Ahead of the K-Pop Fanbase researched active users of social media in the area. From this research, it was found that the relationship between K-Pop fans and their age. This data can be seen in the following Table 3."
Table 3. The relationship between K-Pop fans and their age.

| Age (Years) | Frequency |
|-------------|-----------|
| 9 – 12      | 16        |
| 13 – 16     | 23        |
| 17 – 20     | 40        |
| 21 – 24     | 34        |
| 25 – 28     | 20        |
| 29 – 32     | 12        |
| 33 – 36     | 3         |
| 37 – 40     | 2         |

Examples of unique answers given by students are as follows:

Figure 2. Examples of unique student answers.

Figure 2 shows the answers to the post-test questions from students in the experimental class using a reflective learning model. When viewed from the answer, students understand what concepts are used to solve the problems given. Students analyze the questions well and know the meaning of the questions by first looking for the given group data mode. They look for the mode by using an irregular solution, namely using a formula, but by drawing a histogram from the known data, then determining the model using the concept of the intersection of two lines drawn from the lower and upper limits of the model class. From the value of the model, students provide facts following what the questions want. From the answers given by students, it can be seen that the reflective learning that is applied makes students better understand the concept of the material they are learning and can use previous learning experiences, namely the concept of line equations in solving problems related to the concept of mode for group data. In the control class, students who use conventional learning, the answers are given only look at the table provided and determine the mode directly.
This happens because of the reflective learning model. In the learning process, there is an experience step. In the experience step, the experimental class students solve problems directly so that the learning process is more meaningful. Students also look for their information to complement the material that students have obtained, after which students also make conclusions from the learning they are doing. Reflective learning also includes action steps that allow students to make conclusions. In reflective learning, there is also a reflection step. In that step, students review the subject matter, experiences, responses, and thoughts that students have made in the previous step and are trained to predict answers. Besides, students’ reflective learning plays an active role in the learning process. Students' ability to explain existing models, facts, properties, relationships, or patterns increases because, in reflective learning, there are steps of experience, reflection, and action [13]. This is following the opinion of Kurniawati [3], who said that the reflective thinking process could empower students to identify the ability to identify mathematical concepts or formulas, the ability to arrange solutions with various strategies, the ability to predict completion, the ability to conclude and evaluate.

Based on the description above, it can be seen that the reflective learning model applied by the researcher in the experimental class can improve students' mathematical inductive reasoning ability better than the control class. In the reflective learning process, students experience how to solve a problem with the thoughts they get and conclude. Students are also trained to apply these experiences to solve new problems with the experiences students have gone through so that the learning that students do is more meaningful and more controlled by students.

This is following the opinion of Pollard [11] regarding the characteristics of reflective learning, and Coughlan emphasized that students who are involved in reflective learning have an active role in learning and realize their responsibility to be students for life [10]. In reflective learning, students act as the main character who carries out all the instructions needed to understand the subject matter. In implementing reflective learning in the classroom, several important things need to be conveyed. Reflective learning prioritizes student activities so that students are used to testing and investigating problems, especially math problems. Then students analyze individual experiences they have experienced and facilitate learning from these experiences.

6. Conclusion
The reflective learning model applied in the experimental class can improve mathematical inductive reasoning ability better than conventional classes. Students' mathematical inductive reasoning ability in the experimental class taught using a reflective learning model had a higher average score than students in the control class whose learning used conventional learning models. This follows the percentage of each indicator achieved by the experimental class and control class students on the given test. The percentage of achievement obtained by experimental class students on the three indicators tested was higher than the percentage of achievement of the three control class students’ three indicators. Learning using a reflective learning model was more effective than conventional learning in improving students’ mathematical inductive reasoning ability. This is because this learning trains students to find their strategies to solve a mathematical problem. Reflective learning also directs students to learn from experience and reflects on these experiences into new knowledge that students use for further learning so that students’ thinking will develop.

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