Supersonic, subsonic and stationary filaments in the plasma focus

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Abstract. Filaments in the plasma focus were investigated using a model of plasma with the London current. These structures involve a forward current that flows along the surface of a tangential discontinuity and reverse induction currents in the surrounding plasma, including those that flow over the surface of discontinuity, where the magnetic field reverses its direction. Supersonic filaments demonstrated the capture of plasma by the London current, and in subsonic and stationary filaments, the London current expelled the plasma.

1. Introduction
The plasma focus, which is used in important technological applications to produce rigid radiations and hot plasma flows, is observed to have current filaments with diameters that vary between submillimeter and millimeter ranges [1–3]. The ratio of the diameter of a filament to its length is a small parameter that enables us to consider the filaments as quasicylindrical structures (or as cylindrical structures in the zero approximation with respect to the small parameter).

It has been assumed that filaments can emerge as a result of radiative magnetohydrodynamic instability [4] or their formation can be due to inhomogeneous ionization at the stage of breakdown, and then a multistage ionization enhances plasma density inhomogeneities [5].

In [1], it has been proposed to consider paired plasma vortices to describe the filaments. In [6], stationary filaments were related to helical magnetic field lines. In [7], stationary current filaments were also considered as the states of equilibrium plasma. Then, [8, 9] proposed a model for stationary and moving filaments that involved tangential discontinuity over the surface of which the current flew to induce reverse currents. This paper aims to study the filaments in the plasma model with the London current within the region of parameter values that have not been previously investigated, as well as to discuss plasma capture by the London current and other features of the filaments.

2. Basic equations
The theory is based on dissipationless equations of two-fluid plasma hydrodynamics. These equations have been first used in superconductivity theory [10], which established a relation for the London current \( \vec{j} = -e^2 n_e \vec{A} / m_e c \), where \( \vec{j} \) is the density of the London current, \( n_e \) is the concentration of electrons, and \( \vec{A} \) is the vector potential of the electromagnetic field.
We will use the plasma hydrodynamics equations. The characteristic scale of plasma and field inhomogeneity is \( L \approx c/\omega_p \), where \( \omega_p \) is the electron plasma frequency. In this case, the theory of general relativity [11] or variational principles [12] can be used to substantiate the plasma equations. If the hydrodynamic velocity of plasma is much smaller than the current velocity, a transition occurs to the one-fluid model of plasma with the London current.

A simple model of plasma with the London current is a one-fluid model of quasineutral fully ionized plasma [12]:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{\nabla P}{\rho} - \frac{Ze^2}{2m_emc^2} \nabla \vec{A}^2, \quad \nabla \times \nabla \times \vec{A} = -\frac{4\pi Ze^2}{m_emc^2} \rho \vec{A},
\]

(1)

where \( Ze \) and \( m_i \) are the charge and mass of the ion, \( m_e \) is the mass of the electron, and \( c \) is the speed of light. In system (1), pressure \( P(\rho) \) is the specified function of density \( \rho \), and \( \vec{v} \) is the plasma velocity. Consider the plasma to be adiabatic, i.e., \( P(\rho) \rho^{5/3} = \rho_0^{5/3} \), where \( P_0 \) and \( \rho_0 \) are the unperturbed pressure and density of the plasma. Besides, from the last equation of the system it follows that \( \nabla \cdot (\rho \vec{A}) = 0 \). Hence, at \( \vec{A} \perp \nabla \rho \), there is the Coulomb calibration for the vector potential \( \nabla \vec{A} = 0 \), which is used below.

In this paper, the filaments are considered as cylindrical structures that can move with a constant velocity \( D \) perpendicular to the filament axis. Then, system (1) is reduced to two equations for two independent variables \( \rho \) and \( A_z \), where \( A_z \) is the component of vector potential in cylindrical coordinates.

Let us use dimensionless variables: \( \rho/\rho_0 \rightarrow \rho, \quad \omega_p r/c \rightarrow \tau, \quad Ze^2 A_z^2 / 2m_emc^2 v_{s0}^2 \rightarrow a^2, \quad D/v_{s0} \rightarrow D \). Here, \( r \) is the cylindrical coordinate, \( \omega_p \) and \( v_{s0} \) are the electron plasma frequency and speed of sound in unperturbed plasma.

Then, equations (1) take the form of

\[
-\frac{1}{\tau} \frac{d}{d\tau} \left( \tau \frac{da}{d\tau} \right) = \rho a, \quad a^2 = \frac{D^2}{2} \left( 1 - \frac{1}{\rho^2} \right) + \frac{3}{2} \left( 1 - \rho^{2/3} \right),
\]

(2)

We will seek the solution of system (2) that contains tangential discontinuity. Recall that the combined pressure of plasma and magnetic field is preserved via tangential discontinuity. Note that there is a plasma flow for a cylindrical structure that moves over the tangential discontinuity. Therefore, an approximate relation \( P + \left( \nabla \times \vec{A} \right)^2 / 8 \pi \approx \text{cons} \tan t \) can be used in this case, provided that the plasma flow over the discontinuity is small.

Let us introduce a coordinate \( \tau_{\ast} \) to designate the position of the tangential discontinuity. At \( \tau < \tau_{\ast} \), the solution has only the plasma pressure because the magnetic field vanishes. We will also introduce the coordinate \( \tau_\ast \rightarrow \tau \rightarrow \tau_{\ast} \) to designate a discontinuity where the magnetic field reverses its direction. Herewith, all other variables remain continuous such as plasma density \( \rho_\ast = \rho(\tau_{\ast}) \). For the dimensionless parameters, the condition of smallness of the plasma flow over a tangential discontinuity will take the form of \( D \rho_{\ast}^{-1} \ll 1 \), where \( \rho_{\ast} \) is the plasma density on the inner side of the discontinuity.

3. Results and discussion

Equations (2) were solved numerically together with the equation for the tangential discontinuity under the condition of smallness of the plasma flow over the tangential discontinuity. The results of
the calculations are given in figures 1–4. The solutions are determined by the filament movement velocity \( D \), position of the discontinuity \( \tau_* \), where the magnetic field changes its direction stepwise, and density \( \rho_* \) at this discontinuity.

First, let us discuss subsonic filaments, which are a particular case of stationary filaments. Here, we present the results for a stationary cylindrical filament (see figure 1 and figure 2). Note that the current flows over the surface of tangential discontinuity in the direction of the axis of the cylindrical system, and induction currents flow around the tangential discontinuity in the reverse direction.

Let us assess the parameters of the presented solution. For the concentration of electrons around the cylindrical structure, \( n_e \approx 2 \cdot 10^{17} \text{ cm}^{-3} \), the characteristic scale of plasma and field inhomogeneity is \( L \approx 15 \mu\text{m} \), i.e., the discussed solutions are attributed to submillimeter structures. The maximal value of magnetic field in the solution in figure 2 is approximately 0.1 MG, if the plasma temperature around the filament is taken to be equal to 100 eV. The current flowing over the surface of the tangential discontinuity is 100 A according to the order of magnitude.

Note also that as \( \rho_* \) or \( \tau_* \) decrease, the maximal values of plasma density and magnetic field increase (the latter increases up to the megagauss magnitude) inside a subsonic or stationary filament [8].

Let us now discuss supersonic filaments. Figures 3 and 4 present the results for a supersonic cylindrical filament, the structure of which also has a tangential discontinuity, and along its surface, the forward current flows to induce reverse currents.

Figure 3 shows a local maximum of plasma density at the coordinate of \( \tau_* \). The maximum is related to the maximum of current velocity, which in turn is proportional to the parameter \( a \). The maximum of current velocity is on the coordinate of \( \tau_* \) for supersonic as well as for subsonic and stationary solutions. From system (2), it follows that the maximum of plasma density in the supersonic solution corresponds to that of current velocity. For the subsonic solution, the minimum of plasma density corresponds to the maximum of current velocity. In other words, the London current captures the plasma in the supersonic solution (figure 3) and expels it in the subsonic or stationary solution (figure 1).

With an increase of \( \rho_* \) or a decrease of \( \tau_* \), the maximal values of plasma density and magnetic field increase inside the supersonic filament [9].

![Figure 1. Distribution of plasma density over the radius in a stationary filament.](image1)

![Figure 2. Distribution of magnetic field over the radius in a stationary filament.](image2)
4. Conclusions
A simple model of plasma using the London current provided solutions for the cylindrically moving and stationary filaments. The filaments contain a tangential discontinuity, over which the current flows and induces reverse currents around it. The magnetic fields inside these filaments can reach megagauss values. The obtained solutions for cylindrical filaments proved to be in the submillimeter range.

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