Long-range interactions between dark-matter particles in a model with a cosmological, spontaneously-broken chiral symmetry

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Abstract

In a cosmological model with a chiral symmetry, there are two, dynamically-related spin-zero fields, a scalar $\phi$ and a pseudoscalar $b$. These fields have self-interactions. Spontaneous symmetry breaking results in a very massive scalar particle with $m_\phi \approx 5 \times 10^{11}$ GeV, and a nearly massless, (Goldstone-like) pseudoscalar particle with $0 < m_b < 2.7 \times 10^{-6}$ eV. One or both particles can be part of dark matter. There are coherent long-range interactions (at range $\sim 1/m_b > 10$ cm), from exchange of a $b$ particle between a pair of $b$ particles, a pair of $\phi$ particles, and between a $\phi$ and a $b$. We compare the strength of potentials for the different pairs to the corresponding gravitational potentials (within the same range $\sim 1/m_b$), and show that the new force dominates between a $b$ pair, that gravitation dominates between a $\phi$ pair, and that the potentials are comparable for a $\phi$-$b$ pair. The new interaction strength between a $b$ pair is comparable to the gravitational interaction between a $\phi$ pair; its possibly greater coherent effect originates in the possibility that the number density of a very light $b$ can be greater than that of a massive $\phi$. We consider these results in the context of recent speculations concerning possible effects of special forces between dark-matter particles on certain galactic, and inter-galactic, properties.

In this note, we point out the strength, and certain possible physical consequences, of coherent long-range interactions which can exist only between dark-matter particles. These interactions arise naturally in a model with a cosmological, spontaneously-broken chiral symmetry [1,2]. There have been a number of speculations about such forces. For example, one motivation [3] has been to
try to find a physical role for massless fields (dilatons) which exist in certain hypothetical, popular theories. Another recent speculation has been motivated \[4\] more directly, by certain empirical facts concerning galactic, and inter-galactic structure, which may conflict with the results of simulations carried out in the standard cosmological model (ΛCDM). As we point out below, results of our present investigation based upon a specific dynamical model, have marked analogies to these phenomenological considerations. \[4\]^F1.

In the chiral field theory \[5,6\], there is a self-interaction term \(\lambda(\phi^2+b^2)^2\). There is a spontaneous symmetry breaking at a very high energy scale \(\phi_c \lesssim M_P \approx 10^{19} \text{ GeV}\), the Planck mass. We have identified \(\phi\) with a scalar inflaton field. \[1,2\] Identification with the inflaton allows us to fix a dimensionless parameter \(\lambda \approx 3 \times 10^{-14}\), essentially empirically, \[7,8\] from the observed CMB fluctuations. The definite, unusual result is that one possible dark-matter particle is at a very high mass, \(m_{\phi} \sim 2\sqrt{2}\sqrt{\lambda}\phi_c \approx 5 \times 10^{11} \text{ GeV}\), for \(\phi_c \sim 10^{18} \text{ GeV}\). \[2\] This is also an implicit result of the phenomenology in ref. 4, where a classical scalar, (dark-energy) field with present magnitude of the order of \(M_P\) is coupled linearly to a fermionic, dark-matter field (unless the dimensionless coupling parameter, denoted by \(y\) there \[4\], is miniscule).

A new dynamical element which we introduced \[1\], is a further spontaneous symmetry breaking through a non-zero vacuum expectation value for the pseudoscalar \(b\) field, \(F_b \approx 5.5 \text{ eV}\). Thus, CP invariance is spontaneously broken. The low energy scale is independently fixed by a coupling of the \(b\) field to a neutrino (in particular, to the heaviest ordinary neutrino, presumably \(\nu_{\tau}\)). This neutrino then acquires a mass of \(g_{\nu}F_b \approx 0.05 \text{ eV}\), for a typical \(g_{\nu} \sim 10^{-2}\) \[2\]. Two further results follow from the model.

1. There is a natural possibility for generation of a significant \(\gtrsim 10^{-9}\) antineutrino-neutrino asymmetry in the early universe, as a consequence of CP non-invariance \[1\].

2. There is a residual vacuum energy density with a magnitude of \(\lambda F_b^4 \approx 2.7 \times 10^{-47} \text{ GeV}^4\), which can be identified with an effective cosmological constant \[1\]. (See Appendix.)

As a result of \(F_b \neq 0\), the self-interaction acquires trilinear coupling terms, \(4\lambda F_b b^3\) and \(4\lambda F_b \phi^2 b\), where we denote the particle quanta of the \(\phi\) and \(b\) fields by \(\tilde{\phi}\) and \(\tilde{b}\).^F2 These terms give rise to coherent long-range interactions, as illustrated in Fig. 1 for a pair of \(b\) particles. Below we give the effective, attractive (Yukawa)

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^F1 We thank Prof. P. J. E. Peebles for a communication about his concern for these questions.

^F2 These interaction terms in the Lagrangian density are formally odd under CP. We assume that a CP even coupling that would lead to rapid decay of \(\phi\) to a \(b\)-pair, is absent. (This is an explicit symmetry breaking by deletion, although explicit breaking is usually associated with addition of special terms.) Combinatorial factors arising from identical fields at the vertices are not included in the illustrative estimates below, nor are contact interactions.
potentials between a $b$ pair, a $\phi$-$b$ pair, and a $\phi$ pair, where the interacting particles are assumed to be nearly static i.e. with a very small, intrinsic relative velocity.

\[ b - b \quad - \frac{(4\lambda F_b)^2 e^{-m_b r}}{4m_b^2} \quad = \quad - \left( \frac{\lambda}{8\pi} \right) \frac{e^{-m_b r}}{r} \quad (1a) \]

\[ \phi - b \quad - \frac{(4\lambda F_b)^2 e^{-m_b r}}{4m_b m_\phi} \quad = \quad - \left( \frac{\lambda}{8\pi} \right) \left( \frac{F_b}{\phi_c} \right) \frac{e^{-m_b r}}{r} \quad (1b) \]

\[ \phi - \phi \quad - \frac{(4\lambda F_b)^2 e^{-m_b r}}{4m_\phi^2} \quad = \quad - \left( \frac{\lambda}{8\pi} \right) \left( \frac{F_b}{\phi_c} \right)^2 \frac{e^{-m_b r}}{r} \quad (1c) \]

At distances within $1/m_b$, with the exponential factor of order unity, we compare these attractive potentials (with $\phi_c \sim 0.1 M_P$ [2], $F_b \sim 5.5$ eV [1], $m_b \approx 2\sqrt{2}\sqrt{\lambda F_b}$) to the corresponding potentials from gravity:

\[ b - b \quad -G\frac{m_b^2}{r} = -8\lambda \left( \frac{F_b}{M_P} \right)^2 \frac{1}{r} \quad (2a) \]

\[ \phi - b \quad -G\frac{m_\phi m_b}{r} = -8\lambda \left( \frac{F_b \phi_c}{M_P^2} \right) \frac{1}{r} \quad (2b) \]

\[ \phi - \phi \quad -G\frac{m_\phi^2}{r} = -8\lambda \left( \frac{\phi_c}{M_P} \right)^2 \frac{1}{r} \quad (2c) \]

We have written the gravitational constant $G \approx 1/M_P^2$. Clearly, for $b-b$ the effective coupling strength in (1a), $\lambda/8\pi$, is much greater than the $8\lambda(F_b/M_P)^2$ in (2a). The strengths in (1b) and (2b) are comparable. For $\phi-\phi$, the gravitational strength in (2c) is much greater than that in (1c). An important comparison of strengths is between the new potential between a pair of very light, dark-matter $b$ particles in (1a), and the gravitational potential between a pair of very heavy, dark-matter particles in (2c). The ratio of (1a) to (2c) is independent of the coupling parameter $\lambda$; it is simply $1/64\pi(\phi_c/M_P)^2 \approx 1$. This is much like the result of the phenomenology in ref. 4; Eq. (18) there gives a ratio of a “fifth” force in the dark sector to the gravitational force, as $1/4\pi G\phi^2$, where $\phi$ is a (time-varying) dark-energy field with present magnitude $\lesssim M_P$.

For a mass $m_b \approx 2\sqrt{2}\sqrt{\lambda F_b} = 2.7 \times 10^{-6}$ eV, the range for the interaction of a pair of $b$ particles is $\sim 1/m_b \sim 10$ cm. The cross section for elastic scattering of an isolated pair, with very small, intrinsic relative velocity, is simply $\sigma = \lambda^2/16\pi m_b^2 \sim 1$ mb (up to approximately 1 b with enhancement from the square of a combinatorial factor). Note the compensation of the small factor $\lambda^2$ by the small mass in the dimensional factor $1/m_b^2$. To possibly have effects upon structure within a galactic dimension, the number density of such light particles must be very large.\(^3\) One likely effect is the formation of very massive dark-matter cores at the very center of many galaxies [9,10]. At larger distances from

\(^3\)For our hypothetical, very massive dark-matter particles $\phi$, a characteristic galactic number
the center, other possible structure effects have been the subject of conflicting claims, made under certain assumptions concerning collision cross sections and the motion of the dark-matter particles [11–14]. It is noteworthy that if there was relativistic motion of $b$ with momentum $p > m_b$ in the earliest epoch of structure formation, a relevant $\sigma$ reduced by $(m_b^2/p^2)^2 \simeq (10^{-4})^2$ still allows for a mean free path then less than 1 pc.

We consider the possibility that the $b$ mass is much smaller, but greater than the present Hubble parameter, and that the $b$ particles are essentially a zero-momentum condensate, with (galactic) number density similar to that for the massive $\phi$. If the potential from Fig. 1 extends to distances between galaxy groups, then the coherence factor for all $b$ particles is similar to (possibly even greater than) that for all $\phi$ particles. From the potentials in (1a) and (2c), the coherent effect of the new long-range force between $b$ dark matter is comparable to gravitation between $\phi$ dark matter (which dark matter gives the dominant contribution to the total, near-critical energy density).

In summary, the idea discussed here is that it is possible that dark matter has both a very massive and a very light particle content. With spontaneous breaking of CP invariance, the self-interaction of these dark-matter bosons can naturally generate attractive, coherent long-range potentials, which can augment the gravitational attraction between concentrations of large numbers of dark-matter particles.

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density is $\sim 10^{-12} \, \text{cm}^{-3}$. For a significant contribution to the total dark-matter energy density, a $b$-particle number density of the order of $10^{26}$ larger is necessary ($\sim 10^{14} \, \text{cm}^{-3}$). Note that this large factor is approximately the ratio of dynamical times $(1/F_b)/(1/\phi_c)$, and that both $F_b$ and $\phi_c$ are determined by dynamical considerations rather than the size of contributions to the dark-matter energy density. [1, 2] Such large number densities have been considered for a zero-momentum condensate of QCD axions, in which case they depend upon the square of a high energy scale. It is possible that a present large number density of $b$ particles could originate in production by a time-varying gravitational field [8] over an extended time interval during the early rapid expansion of the universe, say from $\sim 10^{-36}$ s [1] (where the expansion parameter $H(t)$ is of order of $10^{11}$ GeV, equivalent to the momentum of $\sim 10^{26}$ coherently-produced $b$ i. e. with coherent momenta $\sim 10^{-6}$ eV $\sim 1/R$, and $R$ the dimension at that time of the observable universe). Small primary, dark-matter fluctuations can also occur at a relatively late time, where $H(t)$ is $\gtrsim F_b$. Then, $\delta \rho / \rho \sim H |\delta b| / |\dot{b}| \gtrsim m_b / F_b \sim 3\sqrt{\lambda}$, for $|\delta b| \sim m_b$, and assuming that $|\dot{b}| \sim F_b^2$ can be relevant to an early epoch.

For example, $F_b$ may be at a (metastable) maximum of the effective potential for the $b$ field, where the second derivative (the effective, squared $b$ mass), is dynamically brought to near zero by some mechanism.

In the model of ref. 1, a long-range interaction between a pair of slowly-moving (dark-matter) neutrinos is also noted.

A remark of similar nature is made in section 5 of ref. 4. There also a second kind of dark-matter particle is considered, which is (initially) light; it is assumed to have relativistic motion.
Appendix

In the model with a cosmological, chiral symmetry [1], the magnitude of the residual vacuum energy density is estimated to be $|\rho_\Lambda| \sim | - \lambda F_b^4 | \sim 2.7 \times 10^{-47} \text{ GeV}^4$, with $\lambda$ and $F_b$ determined independently from the measured CMB fluctuations and from (a largest) ordinary neutrino mass, respectively. In order to have a positive effective cosmological constant, it is possible to consider an additional “gravitational” contribution of the form [15, 16] $(-1/48) F_{\mu \nu \sigma \rho} F^{\mu \nu \sigma \rho}$ where $\sqrt{-g} F_{\mu \nu \sigma \rho} = C \epsilon_{\mu \nu \sigma \rho}$, as introduced in the Lagrangian density by Duff and van Nieuwenhuizen [15]. The essential new hypothesis here, is that the scale of the constant $C = (m_b F_b / \sqrt{2})$; then the contribution to the vacuum energy density is $(1/4)(m_b F_b)^2 = 2 \lambda F_b^4$. The total vacuum energy density equivalent to an effective cosmological constant is $+\lambda F_b^4$. It is assumed that the additional contribution occurs in time only after the minimum of the effective potential is reached at $b = F_b$, and so a stable, non-zero mass $m_b = 2\sqrt{2}\sqrt{\lambda F_b}$. (The additional contribution is not negative as in [16], where $F_{\mu \nu \sigma \rho}$ is taken as purely imaginary because it is used to cancel an assumed positive vacuum energy density of unknown origin.) There is the possibility here of a unification of dark energy and dark matter, since $b$ particles may constitute the latter.

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Figure 1: Exchange of a very light $b$ particle generates a long-range potential between a pair of $b$ (with assumed very small, intrinsic relative velocity). The vertex strength in the chiral model with spontaneously-broken CP invariance ($F_b \neq 0$), is shown.