Stability of nonlinear vibrations of plate protected from vibrations

M M Mirsaidov¹, O M Dusmatov² and M U Khodjabekov³

¹ Department of Theoretical and Construction Mechanics, Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 39 Kori Niyoziy str., 100000, Tashkent, Uzbekistan
² Department of Teaching Methods of Astronomy and Physics, Tashkent State Pedagogical University, 27 Bunyodkor str., 100185, Tashkent, Uzbekistan
³ Department of Construction Mechanics and Resistance of Materials, Samarkand State Architectural and Civil Engineering Institute, 70 Lolazor str., 140147, Samarkand, Uzbekistan

Email: misaidov1948@mail.ru, dusmatov62@bk.ru, uzedu@inbox.ru

Abstract. This work is devoted to solving the problem of the stability of nonlinear vibrations of the plate, which is protected from vibrations under the influence of kinematic excitations. A dynamic absorber is taken as an object to protect against vibrations. The dissipative properties of the plate material and the damping element of dynamic absorber are expressed by the Pisarenko-Boginich model of the hysteresis type. An analytical expression of the stability conditions was obtained using Lyapunov's first approximation method. A system of differential equations of normal form is obtained for nonlinear vibration of a plate with dynamic absorber, a characteristic equation is constructed, and it is shown on the basis of the Hurwitz criterion that the negativity of the real part of its roots ensures the stability of nonlinear vibrations of the protected plate.

Keywords: Plate, nonlinear vibration, dynamic absorber, stability condition, hysteresis, differential and characteristic equation, resonance.

1. Introduction

Today, it is important to work to reduce harmful vibrations in mechanical systems and to identify and investigate the factors that prevent their long-term perfect operation. In this regard, mathematical modeling, exploring the dynamics and stability, taking into account the nonlinear deformation of the motion of the elastic plate, protected from vibration, is one of the urgent problems.

There are a number of studies devoted to the problems of nonlinear vibrations of plates of different shapes, their stability, and reduce of vibrations. They are:

- the article [1] analyzes elastic dissipative characteristic of the hysteresis-type of a composite plate and conducts experiments for several types of materials. The resonant frequency is expressed analytically;
- in the work [2], the vibrations of a plate with elastic dissipative characteristics were studied under the influence of various random excitations. The roots of second moment and spectral densities were expressed analytically;
- in the work [3], the differential equation of motion of nonlinear vibrations of a plate of right-angled thin composite material under random forces is obtained on the basis of Hamilton's principle and stability of its stationary solution is studied using numerical methods. Equations of normal form were...
determined by the method of averaging, from which a stationary solution was found. The internal resonance event was expressed analytically in relation to the specific frequency in a 1: 2: 3 ratio, and the results of numerical calculations were analyzed:

- in the work [4], the energy losses in a right-angled elastic dissipative characteristic of hysteresis type plate with steel beams attached to both ends were determined and analyzed on the basis of analytical methods and experiments;

- the article [5] explores the stability of nonlinear vibrations and stationary motion of a thin composite plate under the influence of external forces. It analyzes the effectiveness of the influence of several factors on the stability of stationary motion. The numerical solutions obtained for the amplitude-frequency characteristic are compared with the analytical solutions;

- in the work [6], nonlinear motion of plate was modeled mathematically by energetic methods. When the initial conditions are small and there is no effect of external forces on the plate, a general solution of the differential equation of motion is found, analyzed, and this theorem is presented in the form;

- in the work [7], the vibrations of the elastic dissipative characteristic plate under the influence of wide-band random excitations were studied experimentally on a LDS V203 shaker machine. The analytical expression of the amplitudes was obtained depending on the system parameters and was analyzed for the aluminum plate;

- works [8-10, 14,15] describe the results obtained on the characteristics of different dynamic systems and the need to protect them;

- works [11–13, 16–18] considered the problems of vibration of various thin structures to be protected and their stability of motion;

- the monograph [19] is devoted to the study of mathematical modeling and dynamics of distributed parametric systems protected from vibrations.

Although each of these works has its advantages and disadvantages, they are widely used in solving theoretical and practical problems.

The results of the analysis show that the scope of work on creating a mathematical model, exploring the dynamics and stability, taking into account the nonlinear deformation of the motion of elastic plates protected from vibrations, are wide. Therefore, solving such problems is urgent.

2. Materials and Methods

The work considers the problem of exploring the stability of transverse vibrations of a plate with elastic dissipative characteristics of the hysteresis type under the influence of kinematic excitations in conjunction with a dynamic absorber mounted on it as an object to protect against vibrations. The differential equations of motion of the system under consideration required for this were obtained using classical methods of mechanics [19]. Since the dynamic absorber is mounted at one point on the plate, the effect of the dynamic absorber on the plate is expressed using Dirac’s delta function. In the derivation of the differential equation of nonlinear motion of the plate (in terms of bending and torsional moments), the hysteresis-type dissipative properties of the material are taken into account on the basis of Pisarenko-Boginich’s hypothesis. Since the natural vibrational forms of the plate satisfy the orthogonal condition, the differential equation of motion of elastic dissipative characteristic of the hysteresis type plate, protected from vibrations, is written as follows:

\[
\begin{align*}
\ddot{x}_{ik} + [1 + (c_0 + L_1)(-\eta_1 + j\eta_2) + (L_2 + L_3)(-\nu_1 + j\nu_2)]p_{ik}^2 x_{ik} - \\
- [1 + (D_0 + L_4)(-\theta_1 + j\theta_2)]d_{ik0} u_{ik0} x_2 = -d_{ik} w_0; \\
u_{ik0} \ddot{x}_{ik} + \ddot{x}_2 + [1 + (D_0 + L_4)(-\theta_1 + j\theta_2)]n^2 x_2 = -w_0,
\end{align*}
\]

(1)

where \(x_{ik}\) are displacements of \(ik\) points of plate; \(\rho, h\) are density and thickness of the plate; \(p_{ik}\) are natural frequencies of the plate; \(u_{ik} = u_{ik}(x, y)\) are natural modes of the plate; \(u_{ik0} = u_{ik0}(x_0, y_0)\) are quantities of natural modes of the plate at a point \((x_0, y_0)\) dynamic absorber is installed; \(\eta_1, \eta_2, \nu_1, \nu_2\) are harmonic linearization coefficients of dissipative characteristics of the material of the plate; \(x_2\) is
displacement of dynamic absorber relative to the point of plate on which it is installed; \( j^2 = -1; n = (\frac{c}{m})^{1/2} \) is natural frequency of the dynamic absorber; \( c, m \) are stiffness and mass of the elastic damping element of the dynamic absorber; \( \theta_1, \theta_2 \) are harmonic linearization coefficients of dissipative characteristics of the material of the element of dynamic absorber;

\[
\begin{align*}
    d_{ik} = & \frac{d_{1ik}}{d_{2ik}}; \\
    d_{2ik} = & \int_s \sum_{i=1}^{r} c_t x_{i k}^2 h^2 \left( \frac{\partial^2}{\partial x^2} (l_{11} l_{11}^{i}) + \frac{\partial^2}{\partial y^2} (l_{22} l_{22}^{i}) \right) dxdy; \\
    L_1 = & \frac{3D}{d_{2ik} \rho h p_{ik}^2} \sum_{i=1}^{r} c_t x_{i k}^2 h^2 \left( \frac{\partial^2}{\partial x^2} (l_{11} l_{11}^{i}) + \frac{\partial^2}{\partial y^2} (l_{22} l_{22}^{i}) \right) dxdy; \\
    L_2 = & \frac{6D(1 - \mu)}{d_{2ik} \rho h p_{ik}^2} \sum_{i=1}^{s_2} k_t x_{i k}^2 h^2 \left( \frac{\partial^2}{\partial x^2} (l_{11} l_{11}^{i}) + \frac{\partial^2}{\partial y^2} (l_{22} l_{22}^{i}) \right) dxdy; \\
    L_3 = & \frac{2D(1 - \mu)}{d_{2ik} \rho h p_{ik}^2} \sum_{i=1}^{s_3} \frac{\partial^2 u_{ik}}{\partial x^2} \frac{\partial^2 u_{ik}}{\partial y^2}; \\
    L_4 = & \sum_{n=1}^{s_1} D_n \cdot x_{n a}^{n}; \\
    D_0 \text{ and } D_n, (n^* = 1, ..., s_1) \text{ are parameters that depend on material of dynamic absorber, } c_t (i = 0, ..., r) \text{ and } k_t (i = 0, ..., s_2) \text{ are parameters that depend on material of the plate and they are determined by experiment [20]; } \mu \text{ is the Poisson ratio; } D = \frac{Er^2}{12(1-\mu^2)} \text{ is the cylindrical stiffness of the plate; } E \text{ is the Young modulus; } w_0 = w_0(t) \text{ is the acceleration of foundation.}
\]

It is required exploring the stability of the system of differential equations (1) representing the transverse vibrations of an elastic dissipative characteristic of the hysteretic type plate with the dynamic absorber under the influence of kinematic excitations \( w_0 \).

3. Results and discussion

If the foundation of an elastic dissipative characteristic of the hysteretic type plate, represented by equation (1), moves with the following acceleration in conjunction with a dynamic absorber:

\[
w_0 = \varepsilon \xi \cos \omega t,
\]

in that case the solution of the system of equations (1) can be sought in the following form, namely:

\[
x_{ik} = x_{ika}(t) \cos(\alpha t + \alpha_{ik}(t));
\]

\[
x_2 = x_{2a}(t) \cos(\omega t + \alpha_2(t)),
\]

where \( x_{ika}(t), x_{2a}(t), \alpha_{ik}(t), \alpha_2(t) \) are the amplitudes and phases of \( x_{ik}, x_2 \) and they are slowly changing functions; \( \varepsilon \xi \) is excitation amplitude; \( \omega \) is the system frequency.

The acceleration of foundation (2) and the solution (3) are putted into a system of differential equations of motion (1), note that \( x_{ika}(t), x_{2a}(t), \alpha_{ik}(t), \alpha_2(t) \) are slowly changing functions then we come to the following system of equations in the normal form corresponding to the system of differential equations of motion (1):

\[
\begin{align*}
    \frac{dx_{ika}}{dt} = & \frac{1}{2\omega} \left( d_{ik} e^2 \sin \alpha_{ik} - p_{ikx_{ika}R_2} + (a_1 \sin \beta + a_2 \cos \beta) d_{3ik} u_{ik0} x_{2a} \right); \\
    \frac{d\alpha_{ik}}{dt} = & \frac{1}{2 \omega \alpha_{ika}} \left( d_{ik} e^2 \cos \alpha_{ik} + (p_{ik} R_1 - \omega^2) x_{ika} + (a_2 \sin \beta - a_1 \cos \beta) d_{3ik} u_{ik0} x_{2a} \right); \\
    \frac{dx_{2a}}{dt} = & \frac{1}{2 \omega} \left( b_1 e^2 \sin \alpha_2 - a_2 b_2 x_{2a} - (R_1 \sin \beta - R_2 \cos \beta) p_{ik} u_{ik0} x_{ika} \right); \\
    \frac{d\alpha_2}{dt} = & \frac{1}{2 \omega x_{2a}} \left( b_1 e^2 \cos \alpha_2 + a_1 b_2 x_{2a} - x_{2a} \omega^2 - (R_1 \cos \beta + R_2 \sin \beta) p_{ik} u_{ik0} x_{ika} \right).
\end{align*}
\]
where \( a_1 = 1 - \theta_1(D_0 + L_4); a_2 = \theta_2(D_0 + L_4); b_1 = 1 - u_{i\kappa 0}d_{ik}; b_2 = n^2 + u_{i\kappa 0}^2d_{3ik}; R_1 = 1 - (c_0 + L_1)\eta_1 - (L_2 + L_3)\eta_2; R_2 = (c_0 + L_1)\eta_2 + (L_2 + L_3)\eta_2; \beta = a_2 - a_{ik} \).

Substituting \( \frac{d\mathbf{x}}{dt} = 0 \), \( \frac{d\mathbf{y}}{dt} = 0 \), \( \frac{d\eta_1}{dt} = 0 \), \( \frac{d\eta_2}{dt} = 0 \) into the system of equations of normal form (4), stationary vibrations are investigated.

If we vary the system of differential equations (4) in the normal form, then we will come to the characteristic equation of the following form:

\[ \lambda^4 + Y_1\lambda^3 + Y_2\lambda^2 + Y_3\lambda + Y_4 = 0, \]

where \( Y_1 = y_{11} - y_{22} - y_{33} - y_{44}; Y_2 = y_{11}y_{22} + y_{11}y_{33} + y_{11}y_{44} + y_{22}y_{44} + y_{33}y_{44} + y_{22}y_{33} - y_{34}y_{43} - y_{12}y_{21} - y_{12}y_{31} - y_{14}y_{41} + y_{24}y_{42} - y_{23}y_{32} - y_{3} = y_{22}(y_{34}y_{43} - y_{33}y_{44}) + y_{23}(y_{32}y_{44} - y_{34}y_{42}) + y_{24}(y_{42}y_{33} - y_{34}y_{42}) + y_{12}y_{21}(y_{33} + y_{44}) + y_{13}y_{31}(y_{22} + y_{44}) + y_{14}y_{41}(y_{22} + y_{33}) + y_{11}(y_{24}y_{42} + y_{23}y_{32} + y_{34}y_{43} - y_{22}y_{44} - y_{33}y_{44} - y_{22}y_{33}) - y_{12}(y_{24}y_{41} + y_{31}y_{23}) - y_{13}(y_{21}y_{32} + y_{34}y_{41}) - y_{14}(y_{31}y_{43} + y_{21}y_{42}); Y_4 = \frac{y_{11}(y_{22}(y_{34}y_{43} - y_{33}y_{44}) + y_{23}(y_{32}y_{44} - y_{34}y_{42}) + y_{24}(y_{42}y_{33} - y_{34}y_{42}))}{y_{11}y_{22} + y_{11}y_{33} + y_{11}y_{44} + y_{22}y_{44} + y_{33}y_{44} + y_{22}y_{33}} \).

If \( y_{11} = -p_{ik}^2 \frac{\partial(x_{i\kappa a})}{\partial x_{ka}}; y_{12} = x_{i\kappa a}(\omega^2 - p_{ik}^2 R_1); y_{13} = p_{ik}^2 \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} - \omega^2; y_{14} = -p_{ik}^2 R_2; y_{23} = p_{ik}^2 \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \cos\beta - \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \sin\beta; y_{24} = \frac{\omega^2}{x_{ka}^2} \cos\beta + \frac{\omega^2}{x_{ka}^2} \sin\beta; y_{33} = \frac{p_{ik}^2 u_{\kappa 0}^2 x_{i\kappa a} p_{ik}^2 (R_2 \cos\beta - R_2 \sin\beta)}{x_{ka}^2 (b_2 - b_2 a_2) x_{i\kappa a}^2} \cos\beta + \frac{\omega^2}{x_{ka}^2} \sin\beta; y_{34} = \frac{p_{ik}^2 u_{\kappa 0}^2 x_{i\kappa a}^2 (R_2 \cos\beta - R_2 \sin\beta)}{x_{ka}^2 (b_2 - b_2 a_2) x_{i\kappa a}^2} \cos\beta + \frac{\omega^2}{x_{ka}^2} \sin\beta; y_{44} = \frac{p_{ik}^2 u_{i\kappa 0}^2 x_{i\kappa a}^2 (R_2 \cos\beta - R_2 \sin\beta)}{x_{ka}^2 (b_2 - b_2 a_2) x_{i\kappa a}^2} \cos\beta + \frac{\omega^2}{x_{ka}^2} \sin\beta \right) \) under the condition that the real part of the roots of characteristic equation (5) is negative in order for the motion of an elastic dissipative characteristic of the hysteretic type plate with the dynamic absorber to be stable. Based on the Hurwitz criterion, it can be shown that the real part of the roots of the characteristic equation is negative, namely, this criterion is as following for the problem under consideration.

\[
\begin{align*}
Y_1 &> 0; \\
Y_2 &> 0; \\
Y_3 &> 0; \\
Y_4 &> 0; \\
Y_1 Y_2 Y_3 - Y_1^2 Y_4 - Y_2^2 &> 0.
\end{align*}
\]

In the system under consideration, if the material of an elastic damping element of the dynamic absorber has a linear elastic characteristic, the coefficients of the characteristic equation will be as follows:

\[
\begin{align*}
Y_{1+} &= p_{ik}^2 \left( R_2 + \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \right); \\
Y_{2+} &= \left( R_1 + \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \right)^2 + \left( R_1 + \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \right) \omega^2 p_{ik}^2 + \omega^4 + (b_2 - \omega^2)^2 + \left( R_1 + \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \right) p_{ik}^2 u_{\kappa 0}^2 d_{3ik}; \\
Y_{3+} &= \left( R_2 + \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \right)^2 \left( n^2 - \omega^2 \right)^2 + n^2 u_{i\kappa 0}^2 d_{3ik} p_{ik}^2; \\
Y_{4+} &= \left( R_1 + \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \right)^2 \left( n^2 - \omega^2 \right)^2 p_{ik}^4 + \left( R_1 + \frac{\partial(x_{i\kappa a})}{\partial x_{ka}} \right) p_{ik}^2 (n^2 - \omega^2) \omega^2 (\omega^2 - b_2) + \omega^4 (\omega^2 - b_2)^2.
\end{align*}
\]
Based on these coefficients, we obtain the following inequality for the fulfillment of the stability conditions (6):

$$\left( x_{ika} \frac{\partial (L_1 \eta_1 + L_2 \nu_1)}{\partial x_{ika}} \right)^2 - 4(L_1 \eta_2 + L_2 \nu_2) \frac{\partial (x_{ika}(L_1 \eta_2 + L_2 \nu_2))}{\partial x_{ika}} < 0. \quad (8)$$

In the system under consideration, if the plate material has a linear elastic characteristic, the coefficients of the characteristic equation will be as follows:

$$Y_{1*} = b_2 \left( a_2 + \frac{\partial (x_{2a})}{\partial x_{2a}} \right);$$

$$Y_{2*} = \left( a_1 \frac{\partial (x_{2a})}{\partial x_{2a}} + a_2 \frac{\partial (x_{2a})}{\partial x_{2a}} \right) b_2^2 - \left( a_1 + \frac{\partial (x_{2a})}{\partial x_{2a}} \right) b_2 \omega^2 + \omega^4 + \left( p_{ik}^2 - \omega^2 \right)^2 + \left( a_1 + \frac{\partial (x_{2a})}{\partial x_{2a}} \right) p_{ik}^2 \eta_{ik} d_{ik};$$

$$Y_{3*} = \left( a_2 + \frac{\partial (x_{2a})}{\partial x_{2a}} \right) \left( n^2 \left( p_{ik}^2 - \omega^2 \right)^2 + \omega^2 u_{ik} d_{ik} \right);$$

$$Y_{4*} = \left( a_1 \frac{\partial (x_{2a})}{\partial x_{2a}} + a_2 \frac{\partial (x_{2a})}{\partial x_{2a}} \right) \left( n^2 p_{ik}^2 - \omega^2 b_2 \right)^2 - \left( a_1 + \frac{\partial (x_{2a})}{\partial x_{2a}} \right) \left( n^2 p_{ik}^2 - \omega^2 b_2 \right) \left( p_{ik}^2 - \omega^2 \right)^2 + \omega^4 \left( p_{ik}^2 - \omega^2 \right)^2.$$ 

Based on these coefficients, we obtain the following inequality for the fulfillment of the stability conditions (6):

$$\left( \theta_1 (D_0 + L_4) + \frac{\partial (x_{2a} \theta_1)}{\partial x_{2a}} \right)^2 + 4 \theta_2 (D_0 + L_4) \frac{\partial (x_{2a} \theta_2)}{\partial x_{2a}} < 0. \quad (10)$$

Conditions (6) for an anelastic dissipative characteristic of the hysteresis type plate and dynamic absorber, condition (8) for an elastic dissipative characteristic of the hysteresis type plate and linear elastic characteristic dynamic absorber, condition (10) for linear elastic characteristic plate and an elastic dissipative characteristic of the hysteresis type dynamic absorber are counted stability condition.

4. Conclusion

1. Expressions of stability conditions have been developed that allow to determine the stability borders and fields of joint transverse vibrations of a plate with an elastic dissipative characteristic of the hysteresis type and dynamic absorber at different values of system parameters.

2. The analytical expression of stability conditions of the transverse vibration of a vibration-protected plate depends on the harmonic linearity coefficients, the natural frequency of the dynamic absorber and the stiffness of the elastic damping element, as well as the natural frequency, density and thickness of the plate.

3. When the analytical expression of the stability conditions is exactly equal to zero, it was shown that the formation of the boundaries of the stable and unstable fields of the joint transverse vibrations of the plate and the dynamic absorber with elastic dissipative characteristic of hysteresis type.

5. References

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