Black Hole as a Bound State of Semi-classical Degrees of Freedom

Yuki Yokokura

iTHEMS Program, RIKEN, Wako, Saitama 351-0198, Japan

A black hole is considered as a bound state of semi-classical degrees of freedom with maximum gravity. For a configuration of those responsible for the area entropy, the information distribution determines the interior metric through the semi-classical Einstein equation. Then, the bound state has no horizon or singularity, and the interior is a continuous stacking of $AdS_2 \times S^2$ with an $AdS$ radius close to the Planck length and behaves like a thermal state at a near-Planckian local temperature. Integrating the entropy density over the interior volume reproduces the area law exactly. This indicates that the dynamics of gravity plays an essential role in the change of entropy from the volume law to the area law.

I. INTRODUCTION

Information has energy and curves spacetime. When a sufficient amount of information is gathered, a black hole is formed, which should be characterized more properly by the fact that the amount of information is measured by the surface area [1, 2], than by the existence of horizons. This is because the concept of information is quantum mechanical while in quantum theory the geometrical notion of horizons should emerge approximately only under a certain limit.

Such a view appears naturally among various approaches pursuing the idea that a black hole is a bound state of degrees of freedom responsible for the area entropy: string [3–5], discrete geometry [6–8], graviton condensation [9, 10], semi-classical dynamical modes [11–13], and so on (although their formation process is not well studied). Then, where do such degrees of freedom live? Around the “horizon” or somewhere inside? In this article, we explore a possibility of realizing the latter case in the semi-classical Einstein equation

$$G_{\mu\nu} = 8\pi G \langle \psi | T_{\mu\nu} | \psi \rangle. \quad (1)$$

We consider a spherical static black hole with mass $M = \frac{\alpha}{l_p}$ as a gravitational bound state of many semi-classical degrees of freedom, where $a \gg l_p \equiv \sqrt{\hbar G}$. As a simple trial, we examine a configuration in which the degrees of freedom are inside distributed uniformly in the radial direction, except for a small region around $r = 0$ where the semi-classical approximation may break down. That is, they are uniformly distributed in $l_p \ll r \leq R$, where the size of the bound state $R$ should be close to $a$. In fact, such a configuration can be obtained by growing a small black hole to a large one adiabatically in a heat bath and should be thermodynamically typical [14]. Here, however, we don’t specify the formation process or the details of the degrees of freedom except that their energy-momentum is represented by $\langle \psi | T_{\mu\nu} | \psi \rangle$ in (1).

In addition to its entropy being given by its area, it is natural to consider that its very strong gravity also characterizes a black hole. One way to express the strength of gravity is, from the equivalence principle, to use the acceleration $\alpha_n(r)$ required to stay at $r$. We here note two observations. First, in the Schwarzschild metric with $M = \frac{\alpha}{l_p}$, the acceleration increases as $r \to a$ and becomes $O\left(\frac{1}{l_p}\right)$ at $r \sim a + \frac{l_p^2}{a}$, where $\Delta r \sim \frac{l_p^2}{a}$ corresponds to the typical fluctuation of the mass, $\Delta M \sim T_H = \frac{\hbar}{4\pi a}$. Second, in quantum gravity, the limit of spacetime resolution is considered to be the Planck length $l_p$ [15–18], which is related to the existence of the maximum acceleration [19–23]. Therefore, the characteristic scale at $r$, which is given by $\alpha_n(r)^{-1}$, must be much longer than the Planck length for the semi-classical description to hold. Motivated by these, we thus characterize the uniformly distributed bound state with the semi-classically maximum entropy by $\alpha_n(r) \approx \text{const.} \approx O\left(\frac{1}{l_p}\right)$ for $l_p \ll r \leq R$, where $C$ is a large number of $O(1)$.

In the following, we use the two conditions to construct the interior metric of the bound state. The key step is to evaluate the energy density for the information distribution and apply it to (1). The metric means that the bound state is a dense object without singularity or horizon. We then show that the interior consists of many spherically excited quanta, and the structure is a continuous stacking of $AdS_2 \times S^2$ with $AdS$ radius $L \sim Cl_p$. Each of the quanta accelerates uniformly emitting radiation $L \sim Cl_p$. Each of the quanta accelerates uniformly emitting radiation and the state $|\psi\rangle$ behaves like a thermal state at a local temperature $T_{\text{loc}} = \frac{\hbar}{2l_p}$, leading to the equilibrium with a heat bath at $T_H = \frac{1}{2\pi a}$. Using thermodynamic relations, we evaluate the entropy density and sum it up over the interior volume to reproduce the area law. Finally, we discuss the self-consistency of the metric to (1) and find that the full $4D$ dynamics of the semi-classical degrees of freedom plays a key role in the bound state.

II. INTERIOR METRIC FROM THE INFORMATION DISTRIBUTION

A. Energy of information

Setting the interior metric for $l_p \ll r \leq R$ by

$$ds^2 = -e^{A(r)}dt^2 + B(r)dr^2 + r^2d\Omega^2, \quad (2)$$

we first determine $B(r)$. Suppose that there are $N$ quanta with local energy $\epsilon_{\text{loc}}$ and 1 bit of information
in the width $\Delta\tilde{r}$ around $r$. Here, $N$ is a number to be determined, and $(\tilde{r}, \tilde{t})$ is the local coordinate with $\Delta\tilde{t} = \sqrt{-g_{\tilde{t}\tilde{t}}(\tilde{r})} \Delta\tilde{t}$ and $\Delta\tilde{r} = \sqrt{g_{\tilde{r}\tilde{r}}(\tilde{r})} \Delta\tilde{r}$. Then, the total local energy, $N\epsilon_{loc} = 4\pi r^2 \Delta\tilde{r} (T_{\tilde{t}\tilde{t}})$, leads to

$$-\langle T_{\tilde{t}\tilde{t}}(r) \rangle = \frac{N\epsilon_{loc}}{4\pi r^2 \Delta\tilde{r}}, \quad (3)$$

where we used $g_{\tilde{t}\tilde{t}} = -1$ and $T_{\tilde{t}\tilde{t}} = T_{t\tilde{t}} = -T_{\tilde{t}t}$.

Here, the contribution to the ADM energy of the part within $r$ in a spherically symmetric system is given by

$$M(r) = 4\pi \int_0^r dr' r'^2 \langle -T_{\tilde{t}\tilde{t}}(r') \rangle, \quad (4)$$

where $\lim_{r \to \infty} M(r) = M$ and $(r^2 \langle T_{\tilde{t}\tilde{t}}(r) \rangle)_{r \to 0}$ is finite [24, 25]. From this and (3), the ADM energy $\Delta M$ for a 1-bit quantum located at $r$ is evaluated as

$$\Delta M = 4\pi r^2 \Delta\tilde{r} \left( \frac{\langle -T_{\tilde{t}\tilde{t}}(r) \rangle}{N} \right) = \frac{\epsilon_{loc}}{\sqrt{g_{\tilde{r}\tilde{r}}(r)}} \quad (5)$$

On the other hand, by using the uniform distribution condition and Bekenstein’s argument, we can obtain

$$\Delta M \sim \frac{\hbar}{r}. \quad (6)$$

The reason is as follows. First, the physical property at any radius $r$ is equivalent due to the uniform distribution, and the interior of a given $r$ is not affected by its exterior due to the spherical symmetry. Therefore, if the entire bound state of size $R \approx a$ behaves like a black hole of mass $\frac{\hbar}{2\sigma}$, then the region inside $r$ also behaves like a black hole of mass $\frac{\hbar}{2\sigma}$. Second, according to Bekenstein’s idea [1], a quantum with $\Delta M \sim \frac{\hbar}{r}$ has wavelength $\sim r$ and carries 1 bit of information about whether it would enter or not when forming a black hole of mass $\frac{\hbar}{2\sigma}$ since the wavelength and the black hole size are comparable. Thus, from the continuity of the uniform distribution, a 1-bit quantum at $r$ has energy (6).

Thus, by equating (5) and (6), we can evaluate the proper wavelength as $\lambda_{loc} \sim \frac{\hbar}{\sqrt{g_{rr}(r)}}$. Now, since there are $N$ quanta with 1 bit of information and this wavelength in the width $\Delta\tilde{r} \sim \lambda_{loc}$, the entropy per unit proper length can be estimated as

$$s(r) \sim \frac{N}{\Delta\tilde{r}} \sim \frac{N}{\sqrt{g_{rr}(r)}}. \quad (7)$$

Because the uniform distribution of information means that $s(r)$ is constant, we can set $g_{rr}(r) = \frac{L^2}{N_\sigma}$ with a constant $\sigma$. Then, the total entropy is obtained as

$$S = \int_{-l_p}^R dr \sqrt{g_{rr}(r)} s(r) \sim \int_0^a dr \frac{N\sqrt{g_{rr}(r)}}{r} \sim \frac{N a^2}{\sigma}. \quad (8)$$

For this to be consistent with $S \sim \frac{a^2}{r}$, we must have

$$g_{rr}(r) = B(r) = \frac{r^2}{2\sigma}, \quad \sigma \sim N l_p^2. \quad (9)$$

As a result, the entropy density (7) becomes $s(r) \sim \frac{\sqrt{g_{rr}}}{2\sigma} \sim \frac{\sqrt{N}}{l_p}$. Thus, $\sigma$ determines the entropy density, and $O(\sqrt{N})$ bits are packed per the proper Planck length. Note that the wavelength and local energy for 1 bit of information are given through (9) by $\lambda_{loc} \sim \sqrt{N l_p}$, $\epsilon_{loc} \sim \frac{m_p}{\sqrt{\sigma}}$, respectively, where $m_p \equiv \sqrt{\frac{\sigma}{G}}$.

### B. Interior metric

To fix $A(r)$, we consider the other condition, $\alpha_n(r) = O\left(\frac{1}{\sqrt{r}}\right)$, where $\alpha_n(r) = |g_{\mu\nu}\alpha_n^\mu\alpha_n^\nu|^\frac{1}{2}$, $\alpha_n^\mu = n^\mu \nabla_\nu n^\nu$ and $n^\mu \partial_\mu = (-g_{\mu\nu}(r))^{-\frac{1}{2}} \partial_\nu$. This can be expressed as

$$\alpha_n(r) = \frac{\partial_\nu \log \sqrt{-g_{\mu\nu}(r)}}{\sqrt{g_{\mu\nu}(r)}} = \frac{1}{\sqrt{2\sigma^2}}, \quad (10)$$

by introducing another constant $\eta$ satisfying $N\eta^2 = O(1) \gg 1$. Combining this and (9) provides

$$A(r) = \frac{r^2}{2\sigma^2} + A_0, \quad (11)$$

where $A_0$ is an integration constant.

Now, we study the properties of the interior metric (2) with (9) and (11) [26]. First, we have through (1)

$$-\langle T_{\tilde{t}\tilde{t}}(r) \rangle = \frac{1}{8\pi G l^2}, \quad (12)$$

$$\langle T^\nu_r(r) \rangle = \frac{2 - \eta}{\eta} \langle -\langle T_{\tilde{t}\tilde{t}}(r) \rangle \rangle \quad (13)$$

$$\langle T^\theta_\theta(r) \rangle = \frac{1}{16\pi G l^2 \eta^2} \quad (14)$$

as the leading ones for $r \gg l_p$. The energy density (12) reproduces through (4) the mass $M = \frac{\hbar}{2\sigma} \approx \frac{\hbar}{r}$ if the region outside $R \approx a$ is almost vacuum. Also, if we require that the radial pressure (13) be positive, $\eta$ must satisfy

$$0 < \eta < 2. \quad (15)$$

To hold $N\eta^2 = O(1) \gg 1$, therefore $N$ must be a constant of $O(1)$ much larger than 1, determined by dynamics (see Sec. V). Note that the tangential pressure (14) is almost Planckian, stabilizing the bound state against the gravity and violating the dominant energy condition.

The leading values of the curvatures for $r \gg l_p$ are semi-classically maximum [29]:

$$R = -\frac{2}{L^2}, \quad R_{\mu\nu} R^{\mu\nu} = \frac{2}{L^4}, \quad R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = \frac{4}{L^4} \quad (16)$$

$$L \equiv \sqrt{2\sigma^2} \sim \sqrt{N l_p} \gg l_p. \quad (17)$$

Note here that the bound state should have no singularity. Indeed, if we apply (4) and (12) to $r \sim \sqrt{N l_p}$, such a small semi-classical region has the energy $M(r \sim \sqrt{N l_p}) \sim \sqrt{N m_p}$. Therefore, we can expect that the
center region $0 \leq r \lesssim l_p$ has a small energy $\sim m_p$ and is described as a quantum-gravity regular state.

Now, let us fix the constant $A_0$ and the size $R$ by considering the smooth connection to the exterior. For simplicity, we neglect a small backreaction from vacuum fluctuations and set the exterior metric for $r \geq R$ as

$$ds^2 = -\left(1 - \frac{a}{r}\right)dt^2 + \left(1 - \frac{a}{r}\right)^{-1}dr^2 + r^2d\Omega^2.$$  

(18)

In order for (2) to connect to (18) at $r = R$ smoothly, each component of these metrics must agree at $r = R$. For $g_{rr}$, we have $g_{rr}(R) = \frac{4\sigma}{\eta} = (1 - \frac{\alpha}{R})^{-1}$, leading to

$$R \approx a + \frac{2\sigma}{\alpha}$$  

(19)

because of $\frac{\sigma}{\alpha} \ll 1$. Next, the condition $g_{tt}(R) = e^{\frac{2\sigma}{R}} + A_0 = 1 - \frac{R}{a} = \frac{2\sigma}{R}$ fixes $A_0$. ($g_{\theta\theta}$ and $g_{\phi\phi}$ connect trivially.) Thus, the interior metric is given by [26]

$$ds^2 = -\frac{2\sigma}{R^2}e^{-\frac{a^2}{2\sigma}}dt^2 + \frac{r^2}{2\sigma}dr^2 + r^2d\Omega^2.$$  

(20)

(19) indicates that the bound state has the surface at $r = R > a$ as the boundary between (20) and (18) and looks like a classical black hole from the outside [13, 30].

III. BEHAVIOR OF THE INTERIOR

A. Interior structure

The metric (20) originates from the quanta responsible for the entropy and other modes that may be induced self-consistently. To understand this more, let us focus on the former ones and study their behavior.

Because they are distributed uniformly in the radial direction, the interior region can be considered as a continuous concentric stacking of radially localized spherical excited states (like spherical thin shells as in the left of Fig.1). From the uniformness, we can assume that the shells, which may move radially, do not cross each other. Then, we can apply a model [31] to the metric (20) and show that a shell with position $r(t)$ moves according to (see Appendix A for the derivation)

$$\frac{dr(t)}{dt} = -\frac{2 - \eta\sqrt{2\sigma}}{\eta} r(t)e^{\frac{1}{2}A(r(t))}.$$  

(21)

Noting $ds^2 |_{r=r(t)} = -4\frac{\eta}{\sqrt{2\sigma}}e^{A(r(t))}dt^2$ in (20), we find that for the shells to be causal, $\eta$ cannot be smaller than 1. Therefore, $\eta$ is more restricted than (15):

$$1 \leq \eta < 2.$$  

(22)

The unit 4-vector for (21) with $\eta > 1$ is given by

$$u^\mu \partial_\mu = \frac{\eta}{2\sqrt{\eta - 1}}\left(e^{-\frac{1}{2}A} \partial_t - \frac{\eta}{\sqrt{B}} \partial_r\right),$$  

and the acceleration

$$\alpha_u \equiv |g_{\mu\nu}\alpha^\mu_u\alpha^\nu_u|^\frac{1}{2} \quad (\alpha^\mu_u \equiv u^\nu \partial_\nu u^\mu)$$  

is evaluated as

$$\alpha_u = \frac{\eta}{2\sqrt{\eta - 1}L}.$$  

(23)

which is constant and near Planckian. Thus, each shell accelerates uniformly in the radial direction.

We here examine the geometry around each shell. We first find from the Ricci scalar in (16) that the metric (20) is a warped product of $AdS_2$ with radius $L$ (17) and $S^2$ with radius $r$ [27]. Because the shells accelerate uniformly, it is natural to introduce the $AdS$-Rindler coordinate $(\tau, \xi)$ around each shell [32]. Consider an ingoing null line with label $\xi_0$ in the interior region and the shell with label $\xi$ approaching to it. Then, we can set

$$r(\tau, \xi)^2 = R^2 + \frac{2L}{\eta} \left[-\tau + L \log \left(\frac{\xi - \xi_0}{\eta R}\right)\right],$$  

(24)

$$t(\tau, \xi) = L \sqrt{1 + \frac{L^2}{(\xi - \xi_0)^2}}e^{\frac{1}{4}L^2},$$  

(25)

for $\xi - \xi_0 \sim L$ and write (20) as

$$ds^2 = -\frac{(\xi - \xi_0)^2}{L^2}d\tau^2 + \frac{\frac{\partial \xi}{\partial \tau}}{1 + \frac{1}{(\xi - \xi_0)^2}} + r(\tau, \xi)^2d\Omega^2.$$  

(26)

In this metric, a line with $\xi = \text{const.}$ has the acceleration

$$\alpha_\xi \equiv |g_{\mu\nu}\alpha^\mu_\xi\alpha^\nu_\xi|^\frac{1}{2} = \frac{1}{L^2} + \frac{1}{(\xi - \xi_0)^2},$$  

where $\alpha^\mu_\xi \equiv u^\nu \partial_\nu u^\mu$ and $u^\mu \partial_\mu = -(g_{\tau\tau})^{-\frac{1}{2}}\partial_\tau$. Comparing this to (23), we find the position of that shell as

$$\xi = \xi_0 + \frac{2\sqrt{\eta - 1}}{2 - \eta}L.$$  

(27)

This local setup can be applied to each of the continuous many shells. Thus, the interior has the structure like a continuous stacking of $AdS_2 \times S^2$, in which each shell approaches to its own “horizon” (see the right of Fig.1).

B. Thermal behavior

What happens to each shell? In $AdS$ space, a hyperbolic-trajectory observer with acceleration $\alpha = \text{const.} \geq \frac{1}{L}$ feels the Unruh temperature $T_U =$...
\( \frac{1}{2\pi} \sqrt{\frac{1}{2\pi} - \frac{a^2}{4}} \) [33, 34]. Therefore, the shell with \( \alpha = \alpha_u \), (23), has
\[
T_U = \frac{2 - \eta}{2\sqrt{\eta - 1} - 1} \frac{\hbar}{2\pi L},
\]
and the small region with (26) around the shell has the local temperature
\[
T_{\text{loc}} = \frac{\hbar}{2\pi L},
\]
which is the temperature with respect to the boost Hamiltonian generating the \( \tau \)-translation \( \partial_t \) [34]. (See the right of Fig.1 and Appendix B for the derivation.) This is a near-Planckian temperature and consistent with the energy scale for 1 bit, \( \epsilon_{\text{loc}} \sim \frac{m_c}{L} \).

To examine its origin, we check the energy flux \( J_{\text{loc}} \) through the shell. Considering the energy-momentum flow, \( j^\mu = -(T^\mu_{\nu})u^\nu \) of the shell along \( u^\nu \partial_{\nu} = (-g_{rr})^{-\frac{1}{2}} \partial_r \) with (27), and using the unit normal to the trajectory of the shell, \( m_u ds^\mu = \sqrt{g_{\xi\xi}} d\xi \), we have
\[
J_{\text{loc}} = 4\pi r^2 j^r m_u = \frac{1}{4G} \frac{2-\eta}{\eta - 1}.
\]
Here, we applied (1) and used \( G_{rr} = \frac{2}{\eta} \left( \frac{\partial_r \xi}{\xi_0} - \partial_r \xi r \right) \approx -\frac{\frac{2}{\eta} \partial_r \xi}{\xi_0} \) and (27). \( J_{\text{loc}} \neq 0 \) means that particles are created due to the self-gravitational energy of the shell, as a moving mirror consumes acceleration energy to create particles from the vacuum [35] (see the right of Fig.1). In the present case, however, the shells, particles and pressures are reflected in the metric (26).

Note that expressing (30) in terms of (28) and using
\[
L^2 \sim \frac{Nl^2}{2} \quad \text{(see (36))},
\]
we obtain \( J_{\text{loc}} \approx \frac{\pi^2}{N} T_0^2 \), which is 1D thermal radiation at \( T_U \) [36, 37]. On the other hand, in the \((t,r)\) coordinate (20), we have \( G_{tr} = 0 \) and no energy flow through a sphere of radius \( r \), and therefore the accelerating shells and the emitted radiations are in balance at each \( r \). Thus, a small region at \( r \) behaves like a 1D subsystem (in the radial direction) in local equilibrium at the local temperature \( T_{\text{loc}} \). (29).

IV. ENTROPY-AREA LAW AND HAWKING TEMPERATURE

A. Entropy again

In the local equilibrium, (13) should play a role of the equation of state, while \( (T^\theta_{\theta})(\gg (T^r_{r})) \) should have a qualitatively different origin (see Sec.V). Also, the 1D Gibbs relation
\[
T_{\text{loc}} s = p_{\text{id}} + p_{\text{ld}}
\]
should hold for \( p_{\text{id}} = 4\pi r^2 (-T^i_i) \) and \( p_{\text{ld}} = 4\pi r^2 (T^r_r) \) [38, 39]. From this, (12), (13), (17), and (29), we obtain
\[
s(r) = \frac{1}{2} \frac{2}{T_{\text{loc}} \eta} p_{\text{id}} = \frac{2\pi \sqrt{2\sigma}}{\hbar}.
\]
This reproduces the area law self-consistently:
\[
S = \int_0^R dr \sqrt{g_{rr}^2} s(r) = \frac{\pi a^2}{\hbar^2},
\]
where we used (9) and \( R^2 \approx a^2 \) (from (19)). This is a consequence of the semi-classical Einstein equation (1). Therefore, the entropy (33) corresponds to the number of states \( \{ |\psi \rangle \} \) that satisfy (1) for the metric (20) [27], and they are typically excited states at \( T_{\text{loc}} \) of the semi-classical degrees of freedom. Here, quantum-gravity effect does not contribute because of \( N \gg 1 \).

Thus, the information itself is stored inside, while the amount of the information is given by the surface area, due to the gravity satisfying (1) [13]. This result represents that the dynamics of gravity plays an essential role in the change of entropy from volume law to area law.

B. Equilibrium with a heat bath

Now, we derive Hawking temperature. We first note that the bound state evaporates in a vacuum region as
\[
\frac{dM(t)}{dt} = -\frac{2 - \eta}{\eta} \frac{\sigma}{Ga(t)^2},
\]
which is obtained by applying (20) and (21) to \( r(t) = R(t) \approx a(t) \equiv 2GM(t) \). This means the evaporation time scale \( \Delta t = \mathcal{O}(\frac{a^3}{N\hbar}) \), which is consistent with Hawking radiation [2]. The energy flux comes from the particle creation in the surface region \( R - \frac{\sigma}{a} \lesssim r \lesssim R [40] \). This is because, due to the exponentially large redshift in (20), the part below that region is frozen in time when viewed from the outside time \( t \).

Let’s suppose that the bound state is moved into a heat bath and becomes in equilibrium (see Fig.2). Then, we

FIG. 2: Tolman’s law between the surface region at \( T_{\text{loc}} \) and the heat bath at \( T \) (during \( \Delta t \ll \mathcal{O}(ae^{\sigma^2/\hbar^2}) \)).
bath at $r \gg a$, which are static each other, to obtain the equilibrium temperature $T$ as Hawking temperature [41]:

$$T_\infty = \sqrt{\frac{-g_{tt}(r = R)}{-g_{tt}(r \gg a)}} T_{\text{loc}} |_{\eta \to 2} \approx \frac{\hbar}{4\pi a}.$$  

(35)

where we used (17), (18), (19) and (29).

V. SELF-CONSISTENT VALUES OF $(\sigma, \eta)$

The results so far are based on the assumption that there exist the parameters $(\sigma, \eta)$ satisfying (1). We discuss briefly how to obtain them.

As a simple case, when the degrees of freedom are conformal, we can use the 4D Weyl anomaly [35] and solve the trace part of (1), which is independent of $|\psi\rangle$, to obtain [14] (see Appendix C for the derivation)

$$\sigma = \frac{8\pi l_2^2 c_W}{3\hbar^2}.$$  

(36)

Here, $c_W$ is the coefficient of $C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$ in the anomaly and plays a role of $N$ in (9).

A way to determine $\eta$ is to evaluate directly the renormalized energy-momentum tensor $\langle \psi | T_{\mu\nu} | \psi \rangle$ in the background metric (20) and find $\eta$ satisfying (1). This procedure can be done using the dimensional regularization and a perturbative technique [27]. Here, instead of explaining it, we present another possible path to $\eta$.

We first discuss the $AdS_2$ part of the interior geometry (26). This part is fixed only by $L \sim N l_2^2$ (from (17) and (36)) independently of $\eta$. Note here that the 4D Weyl anomaly, which plays the key role in determining (36) (see Appendix C and [28]), comes from vacuum fluctuations of all modes with arbitrary angular momentum, the full 4D dynamics of the fields [27, 35]. Therefore, the $AdS_2$ part with the large acceleration (10), tangential pressure (14) and curvatures (16) originates from such fluctuations without the information.

The $S^2$ part of (26) depends through $r(\tau, \xi)$ (24) on $\eta$, and $r(\tau, \xi)$ gives $G_{\tau\xi} \neq 0$ leading to $J_{\text{loc}} \neq 0$ (30). From (23), $\eta$ also determines the motion of the shell, the excited quanta responsible for the entropy. Thus, we can expect that $\eta$ contains the information $|\psi\rangle$, and $r(\tau, \xi)$ represents the excitation.

Thus, we can consider that the $AdS_2$ part as a “box” is state-independent, and the $S^2$ part as “particles in the box” is state-dependent. A way to determine $\eta$ is to construct a 2D effective model in which $r(\tau, \xi)$ is treated as a dynamical “dilaton field” in the $AdS_2$ space, and consider the boundary condition [33, 42] consistent with (24) and (30). Such a model should enable us to explain microscopically the entropy and the mechanism of $\eta \to 2$. We will study these in the future.

VI. CONCLUSION AND DISCUSSION

In this article, the black hole was described as the bound state with no horizon or singularity consisting of the semi-classical degrees of freedom, where their 4D dynamics was important and quantum-gravity effect was suppressed by $N \gg 1$. The distribution of information together with the semi-classically maximum gravity determined its interior structure as the continuous stacking of $AdS_2 \times S^2$. The interior degrees of freedom are excited at $T_{\text{loc}}$ to reproduce the entropy-area law, which should help to clarify the role of gravity in the holographic principle.

In addition to the construction of the 2D model mentioned above, there are still issues to be considered in the future. The first one is to examine the validity of the assumption that the spherical excitations don’t cross each other, which is related to how the energy of the bound state decreases in the vacuum region. Because the boundary at $r \gg a$ is open in such a case, the bound state will evaporate as fast as possible according to (34) [40]. Then, only $\eta$ in the surface region will approach to 1 (in a similar way to $\eta \to 2$ in the heat bath), since $\eta = 1$ corresponds to the fastest evaporation in the range (22). The shells composing the surface region behave lightlike (see the discussion just below (21)), and intersect ones just below. On the other hand, negative-energy ingoing null flow is induced around the surface [27, 35] and enters into the bound state, intersecting shells with $\eta > 1$. Therefore, the negative-energy flow and the positive-energy excitations (shells) should mix in the surface region. This should decrease the energy and be related to information recovery through interactions [27]. In order to describe these phenomena in the surface region, a generalized model is needed that can describe such crossing effects, while the other region can be described still by the metric (20) and responsible for the entropy (33).

The second one is about the corrections to the entropy-area law. In this paper, a method was proposed to identify the interior metric from the total entropy value and the internal information distribution. Therefore, by applying this to the various corrections to the area law that are thought to come from quantum gravity [43, 44], we should be able to study how the interior is modified.

The third one is to investigate the other possibility mentioned in Sec.I, that is, what interior structure would be obtained for a configuration where the degrees of freedom exist only around the “horizon”. It would then be interesting to study whether that configuration or the one considered in this paper is more plausible for e.g. dynamical stability or thermodynamic typicality.

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Appendix A: A model in [31] and derivation of (21)

For a self-contained presentation, we provide a review of the model in Sec.2.2. of [31], independently of its original motivation. Then, we derive the equation of motion for the shell, (21).

1. A review of the model

Consider a continuously distributed spherical matter. Generically, each part of it can move inward and outward in the radial direction. Let us consider $n$ inwardly moving thin spherical shells, each of which can emit something outwardly, and then construct the interior metric by taking their continuum limit. Here, for simplicity, we assume that the shells do not intersect each other. Then, we can describe the region between the $i$-th and $i+1$-th shells by the $i$-th Vaidya metric [25]:

$$ds_i^2 = - \left(1 - \frac{a_i(u_i)}{r_i}\right)du_i^2 - 2du_i dr + r^2d\Omega^2,$$  \hspace{1cm} (A1)

where $\frac{a_i}{r_i}$ is the contribution to the Bondi mass of the shells below/including the $i$-th one, $u_i$ is the local time in the region, $\frac{a_i}{r_i} = \frac{d\alpha_i}{dt}$ is the total Bondi mass, and $a_0 = 0$.

The two metrics on both sides of the $i$-th shell must be connected along its trajectory $r = r_i(u_i)$, leading to

$$\left(1 - \frac{a_i}{r_i}\right)du_i^2 + 2du_i dr_i \equiv \left(1 - \frac{a_{i-1}}{r_i}\right)du_{i-1}^2 + 2du_{i-1} dr_i \frac{dr_i}{du_i}. $$  \hspace{1cm} (A2)

Dividing this by $(du_i)^2$ and solving for $\frac{du_{i-1}}{du_i}$, we obtain

$$\frac{du_{i-1}}{du_i} = 1 - \frac{a_i}{r_i} + \frac{a_{i-1}}{r_i} \frac{\Delta a_i}{2} + O((\Delta a_i)^2).$$  \hspace{1cm} (A3)

Here, $\frac{dr_i}{du_i}$ represents the derivative along the $i$-th shell, and we expanded the solution for $\Delta a_i \equiv a_i - a_{i-1} \ll 1$ by noting that $1 - \frac{a_0}{r_i} > 0$ because $r_i > a_i$ and the shell cannot be spacelike. This relates $u_i$ and $u_{i-1}$ for given $a_i(u_i)$ and $r_i(u_i)$.

Using (A3) repeatedly, the redshift between the local time $u_i$ and the outside time $u_n = u$ are obtained as

$$\frac{du_i}{du} = \frac{du_{i-1}}{du} \cdots \frac{du_{n-2}}{du} \frac{du_{n-1}}{du} \frac{du_{n}}{du}.$$

$$\hspace{1cm} = \prod_{k=i+1}^{n} \left(1 - \frac{1}{r_k - a_k + r_k \hat{r}_k} \frac{\Delta a_k}{2}\right).$$  \hspace{1cm} (A4)

Taking the continuum limit $\Delta a_k \to 0$ and replacing $u_k \to u_{\alpha}$, $r_k(u_k) \to r_{\alpha}(u_{\alpha})$, and $a_k(u_k) \to a_{\alpha}(u_{\alpha})$ with a continuous label $\alpha$, (A4) becomes

$$\frac{du_{\alpha}}{du} \approx 1 - \frac{1}{2} \int_{\alpha}^{\alpha_{out}} \frac{du}{dr} \frac{a_{\alpha}'}{r_{\alpha}'} - a_{\alpha}' + r_{\alpha}'\hat{r}_{\alpha}' = exp \left[ -\frac{1}{2} \int_{\alpha}^{\alpha_{out}} \frac{du}{dr} \frac{a_{\alpha}'}{r_{\alpha}'} - a_{\alpha}' + r_{\alpha}'\hat{r}_{\alpha}' \right],$$  \hspace{1cm} (A5)

where $\alpha_{out}$ labels the outermost shell.

Because the shells are continuously distributed without crossing, the shell passing a position $r$ at a time $u$ is uniquely determined and so is the value $\alpha$. For a given $u$, therefore, $r$ can be used as $\alpha$, and (A5) becomes

$$\frac{du_{\alpha}}{du} = exp \left[ -\frac{1}{2} \int_{\alpha}^{R(u)} \frac{dr'}{r'} - a(u,r') + r'V(u,r') \right],$$  \hspace{1cm} (A6)

which relates $u_{\alpha}$, $r$, and $u$. Here, $R(u) = r_{\alpha_{out}}$ is the position of the outermost shell, $\frac{a(u,r)}{2G} \equiv \frac{r_{\alpha_{out}}(u(r))}{2G}$ represents the contribution to the Bondi mass of the region below $r$ at time $u$, and $V(u,r) \equiv \frac{dr_{\alpha}(u(r))}{du}.$  \hspace{1cm} means the velocity distribution of the continuous shells.

Let us construct the interior metric in the continuum limit. We consider the shell passing a spacetime position $(u,r)$ and evaluate the metric around the point as

$$ds_i^2 \approx - \left(1 - \frac{a_{\alpha}}{r_{\alpha}}\right)du_{\alpha}^2 - 2du_{\alpha} dr + r^2d\Omega^2,$$

$$\approx - \left(1 - \frac{a_{\alpha}(u,r)}{r_{\alpha}}\right)\left(\frac{du_{\alpha}}{du}\right)^2 du_{\alpha}^2 - 2\frac{du_{\alpha}}{du} dr + r^2d\Omega^2$$

$$\quad = - \frac{1}{C(u,r)} \left(\frac{du_{\alpha}}{du}\right)^2 du_{\alpha}^2 - 2\frac{d\alpha}{du} dr + r^2d\Omega^2,$$  \hspace{1cm} (A7)

where we used (A6) and defined

$$C(u,r) = \frac{1}{1 - \frac{a_{\alpha}(u,r)}{r_{\alpha}}},$$  \hspace{1cm} (A8)

$$D(u,r) = - \left(\frac{du_{\alpha}(u,r)}{du}\right) = - \int_{r}^{R(u)} dr' - a(u,r') + r'V(u,r').$$  \hspace{1cm} (A9)

2. Derivation of (21)

Now, we move on to the problem of applying this model to the metric (20). We first consider the stationary case of (A7) and express it in the $(t,r)$ coordinate:

$$ds^2 = - \frac{e^{D(r)}}{C(r)} dt^2 + C(r) dr^2 + r^2 d\Omega^2,$$  \hspace{1cm} (A10)

where $dt \equiv du + Ce^{-\frac{D(r)}{2}} dr$. Comparing (A10) to (2), we find

$$B = C, \hspace{0.5cm} A = D - \log C.$$  \hspace{1cm} (A11)
The first one gives from (9) and (A8)

\[ a(r) = r - \frac{2\sigma}{r}, \]  

(A12)

which, for \( r = R \), is consistent with (19). Considering the \( r \)-derivative of the second one in (A11) and applying (11), (A9) and (A12), the leading terms for \( r \gg l_p \) lead to

\[ V(r) = -(2 - \eta) \frac{\sigma}{r^2} = -(2 - \eta) \frac{1}{2C(r)} \]  

(A13)

Next, we study the motion of the shells. Using the definitions \( V(u, r) \equiv \frac{dr_u}{dr_a} \) and \( \frac{dr_a}{du} = \frac{e^{\frac{A(r_u)}{\sigma}}}{(r_u) \eta C} \) (from (A6)), we find that each shell moves according to

\[ \frac{dr_u}{du} \frac{dr_a}{dr_u} = -(2 - \eta) \frac{\sigma}{2C(r_a)} \]  

(A14)

We can express this as a vector field

\[ u^\alpha \partial_{\alpha} = K \left( \partial_u - \frac{2 - \eta}{2C} e^{\frac{A(r_u)}{\sigma}} \partial_r \right) \]  

(A15)

for a normalization \( K \), where the second one is expressed in the \((t, r)\) coordinate. This means that the equation of motion of the shell along \( r = r(t) \) is given by

\[ \frac{dr(t)}{dt} = \frac{2 - \eta}{\eta C(r(t))} e^{\frac{A(r(t))}{\sigma}} = -\frac{2 - \eta}{\eta \sqrt{C(r(t))}} \]  

(A16)

where we used (A11). This is (21).

**Appendix B: Derivation of \( T_{\text{loc}} \) (29)**

We here derive \( T_{\text{loc}} \) (29) [34]. Let us suppose that the small region with the metric (26) around a shell is in local equilibrium at a local temperature \( T_{\text{loc}} \) with respect to \( \partial_r \). The partition function can be expressed in terms of an Euclidean path integral over periodic fields for the Euclidean period \( \Delta T_E = h/T_{\text{loc}} \), where \( T_E \equiv i\tau \) [41]. This and the regularity of (20) require that no conical singularity occur when approaching the “horizon” \( (\xi \to \xi_0) \). On the other hand, the Euclidean 2D part \((\tau_E, \xi)\) of (26) becomes for \( \xi - \xi_0 = x \ll L \)

\[ ds_{E}^{(2D)} \approx dx^2 + a^2 d \left( \frac{\tau_E}{L} \right)^2 \]  

(B1)

Therefore, in order for the deficit angle \( \Theta_{\text{deficit}} = 2\pi - \frac{\pi}{T_{\text{loc}} L} \) to vanish, we must have (29).

**Appendix C: Derivation of \( \sigma \) (36)**

We provide a short review of the derivation of (36) [14]. The 4D Weyl anomaly is given by [35]

\[ \langle \psi | T^\mu_{\nu} | \psi \rangle = h(c_W F - a_W G + b_W \Box R) \]  

(C1)

Here, \( F \equiv C_{\alpha \beta \gamma \delta} C^{\alpha \beta \gamma \delta} = R_{\alpha \beta \gamma \delta} R^{\alpha \beta \gamma \delta} - 2 R_{\alpha \beta} R^{\alpha \beta} + \frac{1}{4} R^2 \) and \( G \equiv R_{\alpha \beta \gamma \delta} R^{\alpha \beta \gamma \delta} - 4 R_{\alpha \beta} R^{\alpha \beta} + R^2 \). \( c_W \) and \( a_W \) are positive constants determined by the matter content of the theory (for small coupling constants), while \( b_W \) also depends on the finite renormalization of \( R^2 \) and \( R_{\alpha \beta} R^{\alpha \beta} \) in the gravity action.

Then, using (16) and \( \Box R = O(r^{-4}) \) for (20), the trace of (1) provides

\[ \frac{2}{L^2} = \frac{8\pi l_p^3 c_W}{3L^4} \]  

(C2)

\[ \Rightarrow L^2 = \frac{16\pi l_p^3 c_W}{3} \]

at the leading order for \( r \gg l_p \). Combining this and (17) lead to (36). Note that the 4D Weyl anomaly is essential in that balancing both sides of (C2) yields (36) [28].

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The metric (20) is essentially the same as that obtained by different methods [14, 27, 28], in that the both have the same leading terms for $r \gg l_p$ of the accelerations, Einstein tensors and curvatures. Note that the present derivation of (9) and (11) represents the essential part of the metric in that it is independent of the exterior geometry, formation process, and matter content.

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