Sflavor mixing map viewed from a high scale in
supersymmetric SU(5)

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ABSTRACT: We study flavor violation in a supersymmetric SU(5) grand unification scenario in a model-independent way employing mass insertions. We examine how the quark and the lepton sector observables restrict sfermion mixings. With a low soft scalar mass, a lepton flavor violating process provides a stringent constraint on the flavor structure of right-handed down-type squarks. In particular, $\mu \rightarrow e\gamma$ turns out to be highly susceptible to the 1–3 and 2–3 mixings thereof, due to the radiative correction from the top Yukawa coupling to the scalar mass terms of $\mathbf{10}$. With a higher scalar mass around the optimal value, in contrast, the quark sector inputs such as $B$-meson mixings and hadron electric dipole moment, essentially determine the room for sfermion mixing. We also discuss the recent deviation observed in $B_s$ mixing phase, projected sensitivity of forthcoming experiments, and ways to maintain the power of leptonic restrictions even after incorporating a solution to fix the incorrect quark–lepton mass relations.
1. Introduction

The Large Hadron Collider (LHC) has started finally, which we hope will be the first machine to produce supersymmetric particles directly. At this stage, experimental input that is still playing a major role in probing the soft supersymmetry breaking sector and that will keep doing so even in the LHC era, is the flavor changing neutral current (FCNC) and $CP$ violating processes. From this data, one can extract information on the potential new sources of flavor and $CP$ violations in the soft supersymmetry breaking terms (see e.g. [1] and papers that cite it). A model of supersymmetry breaking/mediation, possibly in conjunction with a model of flavor, should be compatible with this information. In particular, the past two years have seen new measurements of $B_s - \overline{B}_s$ mixing, both its size [2, 3] and its phase [4, 5, 6] (the latter still with low precision), which provide new important restrictions on the mixing between the second and the third families of down-type squarks [7, 8, 9, 10]. On the other hand, a new experiment is going to explore the lepton flavor violation (LFV) decay mode $\mu \to e\gamma$, squeezing its branching ratio down to the level of $10^{-13}$ [11], two orders of magnitude lower than the current upper bound. Therefore, it can be regarded as timely to update an analysis on supersymmetric flavor violation.
An interesting option in this style of model-independent analysis is to work with a grand unified theory (GUT). We take the SU(5) group for example. Since a single irreducible representation contains both quarks and leptons, their flavor structures are related. This enables us to use both quark sector and lepton sector processes to look into a single source of flavor violation. It is entertaining to see which observable is supplying a tighter constraint. The outcome can serve as a hint concerning which sector has a higher prospect for discovery of FCNC mediated by sparticles. For the scalar masses and trilinear couplings to obey the GUT symmetry, the scale of supersymmetry breaking mediation should be higher than the GUT scale. We suppose that this scale \( M^* \) is given by the reduced Planck scale \( M_{\text{Pl}}/\sqrt{8\pi} \sim 2 \times 10^{18} \text{ GeV} \), or very close to it, as is the case in a gravity mediation scenario.

This work is by no means the first attempt in this direction \cite{12, 13, 14, 15, 16, 17, 18}. Most notably, there is a recent article that has performed an analysis in a similar framework \cite{16}. Three differences are worth mentioning. First, we use the aforementioned \( \mu \rightarrow e\gamma \) decay mode to constrain the 1–3 and the 2–3 mixings, in addition to \( \tau \rightarrow e\gamma \) and \( \tau \rightarrow \mu\gamma \) which were considered in Ref. \cite{16}. This seemingly unrelated process becomes relevant, and highly restrictive in some cases, thanks to the radiative correction to the \( 10 \) representation scalar mass matrix from the top Yukawa coupling and the Cabibbo–Kobayashi–Maskawa (CKM) mixing \cite{19}. As a matter of fact, this mechanism has long been known and included in many of the preceding model studies \cite{12, 20, 21}. Yet, this is the first instance of taking it into account in a model independent analysis allowing for general flavor mixing of sfermions, as far as we know. Second, the authors of Ref. \cite{16} assume that the quark and the lepton mass eigenstates at the GUT scale are aligned to a high degree. This may or may not be the case if a solution is incorporated for fixing the wrong quark–lepton mass relations. Especially, the first and the second families are subject to unlimited misalignment in general \cite{12}. We propose a method to overcome this obstacle to some extent. Third, we elucidate the importance of the gaugino to scalar mass ratio as a key parameter governing relative strengths of the hadronic and the leptonic flavor violations. We show expansions and shrinks of the territory ruled by each of the two sectors. In addition to these refinements, we include remarks concerning the latest hint of anomaly in the mixing phase of the \( B_s \)-meson \cite{22, 23}.

This paper is organized as follows. In Section 2, we spell out basics of flavor physics in a supersymmetric SU(5) GUT model. Section 3 presents the procedure of numerical analysis and the experimental inputs. In Section 4, we exhibit the exclusion plot of each mass insertion, and discuss how one can interpret the plot conservatively when the lagrangian has non-renormalizable terms for accommodating the first and the second family fermion masses. This section also has a collection of upper bounds on the sfermion mixings, as well as deviations in selected \( CP \) asymmetries allowed by the other constraints. With a summary, we conclude in Section 5. One can find notations of the soft supersymmetry breaking terms and the mass insertion parameters in the appendix.
2. SU(5) GUT and FCNC

2.1 GUT relation between squark and slepton mixings

Let us begin by reviewing basic elements of a supersymmetric SU(5) grand unification model, that are relevant to flavor physics. The superpotential has the Yukawa couplings and the right-handed neutrino mass terms,

\[
W_{\text{GUT}} \supset -\frac{1}{4} \varepsilon_{abcde} \lambda^i_{ij} T^{ab}_i H^e + \sqrt{2} \lambda^i_{ij} H_a T^{ab}_i F^e_j - \lambda^i_{ij} N_i F^e_j H^a + \frac{1}{2} M_N^{ij} N_i N_j. \tag{2.1}
\]

Matter fields in \( \mathbf{10} \) and \( \mathbf{5} \) representations are denoted by \( T \) and \( F \), respectively, \( 5 \) and \( 5 \) Higgses by \( H \) and \( \overline{H} \), respectively, and a right-handed neutrino by \( N \). The indices \( a, \ldots, e \) run over components of the fundamental representation of SU(5), and \( i, j = 1, 2, 3 \) indicate the family. Obviously, \( \lambda_U \) and \( M_N \) are symmetric matrices while \( \lambda_D \) and \( \lambda_N \) are not. The above Yukawa couplings, by themselves, predict mass unification of down-type quarks and charged leptons at the GUT scale:

\[
m_e = m_d, \quad m_\mu = m_s, \quad m_\tau = m_b. \tag{2.2}
\]

Among these, the third relation is consistent with measurements at low energies, while the first two are not. One way to explain this discrepancy is to make corrections to relatively smaller masses by including the following non-renormalizable terms \( W_{\text{NR}} \):

\[
W_{\text{NR}} = \frac{1}{4} \varepsilon_{abcde} \left( f_{ij}^{ab} T^{cd}_i H^e \right)^2 + \frac{1}{2} M_N^{ij} N_i N_j.
\]

These terms will contribute to the Yukawa couplings of the effective theory below the GUT scale, expressed in terms of the SM fields as

\[
W_{\text{SSM}} = Q^T Y_U \overline{U} H_u + Q^T Y_D \overline{D} H_d + L^T Y_E \overline{E} H_d + L^T Y_N N H_u + \frac{1}{2} N^T M_N N, \tag{2.4}
\]

where the fields denoted by uppercase letters are components of the GUT multiplets,

\[
T_i \simeq \{ Q, \overline{U}, \overline{E} \}_i, \quad \overline{F}_i \simeq \{ \overline{D}, L \}_i. \tag{2.5}
\]

The Yukawa couplings appearing in the superpotential of (2.4) are related to those in (2.1) and (2.3) by

\[
Y_U = \lambda_U + \xi \left( \frac{3}{5} f_1 + \frac{3}{20} f_2^S + \frac{1}{4} f_2^A \right), \tag{2.6a}
\]

\[
Y_D = \lambda_D - \xi \left( \frac{3}{5} h_1 - \frac{2}{5} h_2 \right), \tag{2.6b}
\]

\[
Y_E^T = \lambda_D - \xi \left( \frac{3}{5} (h_1 + h_2) \right), \tag{2.6c}
\]

\[
Y_N^T = \lambda_N + \xi \frac{3}{5} h_N, \tag{2.6d}
\]

\[
-3-
\]
where the superscripts $S$ and $A$ denote the symmetric and the antisymmetric part of the given matrix, respectively. The small number $\xi$ is defined by

$$\xi \equiv 5 \sigma M_s \approx 10^{-2},$$  \hspace{1cm} (2.7)

where $\sigma$ is the vacuum expectation value (VEV) of $\Sigma$, expressed as in

$$\langle \Sigma \rangle = \sigma \text{ diag}(2, 2, 2, -3, -3).$$  \hspace{1cm} (2.8)

The contribution from the non-renormalizable terms makes the difference,

$$Y_D - Y_E^T = \xi h_2,$$ \hspace{1cm} (2.9)

and this can account for the first and the second family quark and lepton masses.

For this purpose, Ref. [16] does not make use of the $O(\xi)$ corrections, but they rely on Georgi-Jarlskog mechanism [25]. Their scenario corresponds to a case in our work where the quark and the lepton mass eigenbases coincide, i.e. $U_L = U_R = 1$ in the formalism spelled out below.

Note that the proton lifetime depends on the structure of non-renormalizable operators [26], thereby imposing a restriction on the parameters appearing in (2.3). There are corners of the parameter space in conflict with proton decay experiments. The present work is not specific to a particular pattern of those terms and is valid provided that they are Planck-suppressed.

In order to discuss flavor violation coming from the sfermion sector, one should fix the basis of matter supermultiplets. One can choose a basis of $T_i$ and $\bar{T}_i$ fields such that

$$Y_U = V_Q^T \bar{Y}_U U_Q^*, \quad Y_D = \bar{Y}_D, \quad Y_E = U_L^T \bar{Y}_E U_R^*, \quad Y_N = U_L^T V_L^T \bar{Y}_N,$$  \hspace{1cm} (2.10)

where the hat on a matrix signifies that the given matrix is diagonal with positive elements [27], $V_Q$ and $V_L$ are unitary matrices in the standard parametrization [28, 29] each with three mixing angles and one phase, and $U_Q$, $U_L$, and $U_R$ are general unitary matrices. Note that $Y_U$ may not be a symmetric matrix, unlike $\lambda_U$. In this basis where $Y_D$ is diagonal, $Y_E$ may not be diagonalized in general due to the difference (2.9), and it should be decomposed into the above form using $U_L$ and $U_R$. These two unitary matrices describe the mismatch between the down-type quark and the charged lepton mass eigenstates, arising from breakdown of the Yukawa unification $Y_E^T = Y_D$ which is a consequence of $SU(5)$ at the renormalizable level. Since $U_L$ and $U_R$ are crucial in correlating hadronic and leptonic processes, we need to examine their structures. We can estimate the size of an off-diagonal element of $Y_E$ in the unit of the tau Yukawa coupling,

$$\frac{Y_E - \bar{Y}_D}{|Y_E|_{33}} \approx \frac{-\xi h_2^T}{m_\tau/(v \cos \beta)} \approx -\cos \beta h_2^T,$$  \hspace{1cm} (2.11)

where $v \approx 170$ GeV is the Higgs VEV. Notice the suppression by the factor $\cos \beta$ for high $\tan \beta$. Assuming that each element of $h_2$ is not larger than $O(1)$, one can obtain approximate magnitudes of 1–3 and 2–3 mixings [12],

$$[U_L]_{3a} \approx -\cos \beta [h_2]_{3a}, \quad [U_L]_{a3} \approx \cos \beta [U_L h_2^2 U_R^\dagger]_{a3},$$

$$[U_R]_{3a} \approx -\cos \beta [h_2^1]_{3a}, \quad [U_R]_{a3} \approx \cos \beta [U_L^\dagger h_2^2 U_R^\dagger]_{a3},$$ \hspace{1cm} (2.12)
for $a = 1, 2$. Note that they are suppressed by $\cos \beta$. The other entries of $U_L$ and $U_R$ can be of $\mathcal{O}(1)$. Finally, we relate the fields to the down-quark and charged lepton mass eigenstates as

$$Q = q, \quad U = U_Q^T \bar{u}, \quad E = U_R^T \bar{e}, \quad D = \bar{d}, \quad L = U_L^T \bar{l}. \quad (2.13)$$

This leads us to the superpotential,

$$W_{SSM} = q^T [V_Q^T \hat{Y}_V] \pi H_u + q^T [\hat{Y}_D] \bar{d} H_d + l^T [\hat{Y}_E] \bar{e} H_u + \frac{1}{2} N^T M_N N. \quad (2.14)$$

One can notice that $V_Q$ is the CKM matrix at the GUT scale. If $M_N$ is diagonal in this basis, one also has $V_L = U_{PMNS}^T$. Otherwise, the lepton mixing matrix receives additional rotations for diagonalizing $M_N$.

Let us turn to the soft supersymmetry breaking sector. The SU(5) symmetry relates the soft supersymmetry breaking terms of squarks and sleptons in a single GUT multiplet. The scalar mass terms are given by

$$-\mathcal{L}_{\text{soft}} \supset \bar{F} m_F^2 F + T m_T^2 T + \bar{F} \Sigma m_{\Sigma}^2 F + T \Sigma m_{\Sigma}^2 T + \cdots, \quad (2.15)$$

in which the higher dimensional terms involving $\Sigma$ are suppressed by $\mathcal{O}(\xi)$. In terms of these soft mass parameters of the GUT multiplets, one can express the soft scalar mass matrices of the SM fields as

$$m_Q^2 = m_T^2 + \frac{1}{10} \xi m_T^2, \quad m_U^2 = m_T^2 - \frac{2}{5} \xi m_T^2, \quad m_E^2 = m_T^2 + \frac{3}{5} \xi m_T^2, \quad (2.16a)$$

$$m_D^2 = m_T^2 + \frac{2}{5} \xi m_T^2, \quad m_L^2 = m_T^2 - \frac{3}{5} \xi m_T^2, \quad (2.16b)$$

using (2.5). From these expressions and (2.13), one can see that the mass insertion parameters of down-type squarks and sleptons at the GUT scale are linked by

$$\delta^l_{LL} = U_L \delta^d_{RR} U_L^T + \mathcal{O}(\xi), \quad (2.17a)$$

$$\delta^l_{RR} = U_R \delta^d_{LL} U_R^T + \mathcal{O}(\xi). \quad (2.17b)$$

We can notice two possible sources of deviation from the naive equalities [14],

$$\delta^l_{LL} = \delta^d_{RR}, \quad \delta^l_{RR} = \delta^d_{LL}. \quad (2.18)$$

One is the higher dimensional terms in (2.15), which makes the $\mathcal{O}(\xi)$ corrections, and the other is $U_L$ and $U_R$, the unitary transformations parametrizing the misalignment between the down-type quark and the charged lepton mass eigenstates. The former type of corrections is negligible compared to the typical size of a scanning mass insertion parameter appearing later on. On the other hand, these corrections might be comparable to the renormalization group (RG) contribution to $\delta^l_{RR}$. Unless they are tuned in such a way that they cancel out the RG-generated $\delta^l_{RR}$, they nevertheless do not undermine the importance of $\mu \rightarrow e\gamma$ constraint. The latter needs more consideration. Obviously, $U_L$ and $U_R$ depend
on $h_2$ through $Y_E$. If $h_2$ is diagonal in the basis where $Y_D$ is diagonal, $U_L$ and $U_R$ are unit matrices, and (2.18) becomes a fairly good approximation correlating squark and slepton flavor mixings. If $h_2$ is not diagonal, the correlation gets loose, but in many cases, LFV processes can still give meaningful restrictions on the down-type squark mixings, thanks to the suppression of 1–3 and 2–3 mixings shown in (2.12). Examples of this situation will be presented in Section 4.2.

In a similar way, the GUT symmetry links the scalar trilinear coupling terms of squarks and sleptons so that their chirality-flipping mass insertions have the relations,

$$
\delta_{LR}^t = U_L \delta_{LR}^t U_R^\dagger + O(\xi) \times A_0 \langle H_d \rangle/\tilde{m}_l^2,
$$

(2.19)

where $A_0$ is the overall scale of the $A$-terms and $\tilde{m}_l$ is the average slepton mass. In what follows, we do not use this expression since we will ignore the $A$-term contributions to flavor violating processes.

2.2 RG running of scalar masses

RG running from one scale down to a lower scale generates off-diagonal elements of a scalar mass matrix. For our purpose, we need to consider two intervals of scale: from $M_\star$ to $M_{GUT}$, and from $M_{GUT}$ (via $M_R$) to $M_{SUSY}$. The former is needed to determine the boundary condition to give on the soft supersymmetry breaking terms at the GUT scale, and the latter is to connect the given boundary condition with low energy observables.

First, we think of running between $M_\star$ and $M_{GUT}$. Using one-loop approximation, the RG-induced off-diagonal elements can be written as

$$
\Delta g_{m_T^2} \simeq -\frac{2}{(4\pi)^2} [3\lambda^*_U \lambda_{U}^T + 2\lambda_{D} \lambda_{D}^T] (3m_0^2 + |A_0|^2) \ln \frac{M_\star}{M_{GUT}},
$$

(2.20a)

$$
\Delta g_{m_F^2} \simeq -\frac{2}{(4\pi)^2} [4\lambda_{D}^\dagger \lambda_{D} + \lambda_{N}^\dagger \lambda_{N}] (3m_0^2 + |A_0|^2) \ln \frac{M_\star}{M_{GUT}},
$$

(2.20b)

where $m_0$ is the scalar mass and $A_0$ is the trilinear scalar coupling. Let us focus on the mass matrix of $T$ fields, which feeds into the mixings of left-handed squarks and right-handed sleptons. From (2.6a), (2.10), (2.16a), and (2.20a), one can obtain the following form of RG contribution to the $LL$ squark mixing at the GUT scale,

$$
(\delta_{ij}^d)_{LL} \simeq -\frac{6}{(4\pi)^2} [V_Q^\dagger \hat{Y}_U V_Q]_{ij} \frac{3m_0^2 + |A_0|^2}{\tilde{m}_l^2 (M_{GUT})} \ln \frac{M_\star}{M_{GUT}} + O(\xi).
$$

(2.21)

The $O(\xi)$ correction in the second term is not necessarily smaller than the first term coming from the CKM mixing and the large top quark Yukawa coupling. Neither is it very likely, however, that they cancel out leading to a value much smaller than the first term. That is, the left-handed squark mixing in the above expression, without the $O(\xi)$ correction, can be regarded as the minimal value of $(\delta_{ij}^d)_{LL}$ that is expected in a supersymmetric SU(5) model with the cutoff at $M_\star$. Let us record the CKM matrix dependence of the above minimal mass insertions,

$$
(\delta_{12}^d)_{LL} \sim V_{td}^* V_{ts} \sim \lambda^5, \quad (\delta_{13}^d)_{LL} \sim V_{td}^* V_{tb} \sim \lambda^3, \quad (\delta_{23}^d)_{LL} \sim V_{ts}^* V_{tb} \sim \lambda^2.
$$

(2.22)
where we also express them as powers of $\lambda$, sine of the Cabibbo angle.

Using (2.17b), one can get the right-handed slepton mixing from (2.21). Again, we drop the $O(\xi)$ term in (2.17b), assuming that it does not conspire with the first term to result in a drastic cancellation. If $U_R$ is an identity matrix, $(\delta_{ij}^l)^{RR}$ has the same pattern as (2.22). Otherwise, one should take the misalignment into account. As (2.12) shows that the 1–3 and 2–3 mixings are suppressed, one can rephrase (2.17b) into

\[ (\delta_{a3}^l)^{RR} = [U_R]_{ab} (\delta_{33}^d)^{LL} [U_R]_{3a}^* + O(\cos^2 \beta \delta_{LL}^d), \quad a, b = 1, 2, \]

(2.23)

where $[U_R]_{ab}$, the upper-left $2 \times 2$ submatrix of $U_R$, is approximately unitary. [Supposing universal scalar masses at $M_*$, one actually has another term of the form $[U_R]_{a3} (\delta_{33}^d)^{LL} [U_R]_{33}^*$ where (with an abuse of notation what we here call) $(\delta_{33}^d)^{LL}$ is given by setting $i = j = 3$ in (2.21). In what follows we discard this term although it can be larger than what is kept in the above equation. Even if it happens to be non-negligible, it generically enlarges the rate of $\mu \to e\gamma$, only to reinforce the sensitivity of this LFV channel.] Keeping only the powers of $\lambda$, one can schematically rewrite this as

\[ (\delta_{13}^l)^{RR} \sim [U_R]_{11} \lambda^3 + [U_R]_{12} \lambda^2, \quad (\delta_{23}^l)^{RR} \sim [U_R]_{21} \lambda^3 + [U_R]_{22} \lambda^2. \]

(2.24)

The mixing between the first and the second families, described by $[U_R]_{ab}$, is not particularly restricted to be small. There can be small, large, or no mixing. One finds that $(\delta_{a3}^l)^{RR}$ is generically not much smaller than $\lambda^3$, unless the mixing is fine-tuned in such a way that the two terms cancel out in either of (2.24). For example, the mixing angle should be tuned between $-\lambda \pm \lambda^2$ in order to have $|(\delta_{13}^l)^{RR}| \lesssim \lambda^4$.

Next, we should turn to the running below $M_{GUT}$. Before examining an off-diagonal entry of a scalar mass matrix, let us recall the running of a diagonal element since a mass insertion parameter is normalized by it. Squark and slepton masses at $M_{SUSY}$ are approximately related to the GUT scale variables by

\[ \tilde{m}_d^2(M_{SUSY}) \approx (1 + 6x) m_0^2, \quad \tilde{m}_e^2(M_{SUSY}) \approx m_0^2, \]

(2.25a, 2.25b)

with the definition of gaugino to scalar (squared) mass ratio,

\[ x \equiv M_{1/2}^2/m_0^2. \]

(2.26)

The squark mass increases considerably by the gaugino mass contribution. The slepton mass actually receives a small correction from the gaugino mass, but it can be ignored for later discussions. These facts will be crucial to understanding parameter dependence of a constraint.

Unless $\tan \beta$ is extremely high, an off-diagonal element of $m_D^2$ does not run significantly, while running of the left-handed squark mass matrix makes the difference [32],

\[ \Delta_{\mu}[m_D^2]_{ij} \simeq -\frac{2}{(4\pi)^2} [V_Q^\dagger \tilde{Y}_U V_Q]_{ij} (3m_0^2 + |A_0|^2) \ln \frac{M_{GUT}}{M_{SUSY}}. \]

(2.27)
Using these facts and (2.25a), we can associate squark mass insertions at $M_{\text{SUSY}}$ to those at $M_{\text{GUT}}$ as

$$(\delta_{ij}^d)_{RR}(M_{\text{SUSY}}) \approx \frac{(\delta_{ij}^d)_{RR}(M_{\text{GUT}})}{1 + 6x},$$

$$\tag{2.28a}$$

$$(\delta_{ij}^d)_{LL}(M_{\text{SUSY}}) \approx \frac{(\delta_{ij}^d)_{LL}(M_{\text{GUT}}) + q_{ij}}{1 + 6x},$$

$$\tag{2.28b}$$

with the definition

$$q_{ij} \equiv \Delta_s[m^2_{\tilde{Q}}]_{ij}/m^2_0.$$  \hspace{1cm} (2.29)

In a parallel way, one can relate slepton mass insertions at a low scale to those at a high scale by

$$(\delta_{ij}^l)_{RR}(M_{\text{SUSY}}) \approx \frac{(\delta_{ij}^l)_{RR}(M_{\text{GUT}})}{1 + 6x},$$

$$\tag{2.30a}$$

$$(\delta_{ij}^l)_{LL}(M_{\text{SUSY}}) \approx \frac{(\delta_{ij}^l)_{LL}(M_{\text{GUT}}) + l_{ij}}{1 + 6x},$$

$$\tag{2.30b}$$

using (2.25b) and the definition $l_{ij} \equiv \Delta_s[m^2_{\tilde{l}}]_{ij}/m^2_0$ with the radiative correction to the off-diagonal slepton mass matrix entries $[33],$

$$\Delta_s[m^2_{\tilde{l}ij}] \approx -\frac{2}{(4\pi)^2} \left[V_{\tilde{L}}^\dagger \tilde{Y}_N \tilde{V}_L\right]_{ij} (3m^2_0 + |A_0|^2) \ln \frac{M_{\text{GUT}}}{M_R}.$$ 

$$\tag{2.31}$$

This estimate is based on the assumption that the right-handed neutrinos are degenerate so that they are integrated out at a single scale $M_R$. If they are not degenerate, it is modified to involve mixings, phases, and eigenvalues of $M_N$ (see e.g. [13]). Even in this case, it has been shown that one can use the above form of expression by replacing $M_R$ with the largest eigenvalue of $M_N$, if there is a large hierarchy among the right-handed neutrino masses [13]. Unlike the quark sector, we do not yet have much information on the neutrino Yukawa couplings. They can be of $O(1)$ in the case of heavy right-handed neutrinos, or extremely small if the neutrino masses are of Dirac type. Even if we suppose that seesaw mechanism is working, a vast range of right-handed neutrino mass scale is possible, from around the GUT scale down to the weak scale. Although the lepton mixing angles have been measured to an extent, they cannot be directly related to the mixing matrix $V_L$ due to the additional degrees of freedom in $M_N$, the Majorana right-handed neutrino mass matrix. Moreover, the hierarchy of neutrino masses is unknown yet. As the magnitude of $l_{ij}$ in one model can greatly differ from another, we choose to drop it in the following analysis. Therefore, the results shown later are legitimate only for a scenario where right-handed neutrinos are light enough for $l_{ij}$ to be negligible in (2.30b). (For a study on a case with a large neutrino Yukawa coupling and a specific boundary condition on the soft terms, see e.g. [18, 34, 35].)

Nevertheless, there are circumstances where one can tell consequences of non-negligible $l_{ij}$. Here, we assume that $U_L$ is a unit matrix. This assumption will be relaxed in Section 4.2. If neutrino Yukawa couplings are large, they affect not only the running below, but also above $M_{\text{GUT}}$, of $m^2_{\tilde{l}}$. Thus, $(\delta_{ij}^l)_{LL}(M_{\text{GUT}})$ is decomposed into two pieces,

$$(\delta_{ij}^l)_{LL}(M_{\text{GUT}}) \approx (\delta_{ij}^l)_{LL}(M_s) + \alpha l_{ij},$$

$$\tag{2.32}$$
where the first term represents possible flavor non-universality at the reduced Planck scale, and the second is the RG contribution with

\[
\alpha \equiv \frac{\ln(M_s/M_{GUT})}{\ln(M_{GUT}/M_R)}.
\] (2.33)

What we will do in the following sections is to search for a set of viable values of \((\delta^l_{ij})_{LL}(M_{GUT})\), imposing experimental constraints. In terms of the variables in (2.32), we can interpret this procedure in two different ways: we fix \(l_{ij}\) and scan over \((\delta^l_{ij})_{LL}(M_s)\), or the other way around. As an example of the first option, suppose that one studies a neutrino mass model in which the neutrino Yukawa matrix is given, but there is a room for flavor mixing in the soft supersymmetry breaking terms. In this case, one can easily guess the allowed region of \((\delta^l_{ij})_{LL}(M_{GUT}) = (\delta^l_{ij})_{RR}(M_{GUT})\) from the one shown in Section 4.1 using (2.30b): shift the region by \(-l_{ij}\). This method is applicable to a model with non-degenerate right-handed neutrinos as well. Regarding the second option, one can imagine a situation where the only source of \(\mathcal{F}\) mixing is the neutrino Yukawa matrix, i.e. \((\delta^l_{ij})_{LL}(M_s) = 0\). Under this condition, (2.30b) can be rewritten as

\[
(\delta^l_{ij})_{LL}(M_{GUT}) \approx \frac{\alpha}{1 + \alpha} (\delta^l_{ij})_{LL}(M_{SUSY}),
\] (2.34)

which relies on the degeneracy of right-handed neutrinos. Obviously, the allowed region of \((\delta^l_{ij})_{LL}(M_{GUT})\) is given by shrinking the one in Section 4.1 by the factor \(\alpha/(1 + \alpha)\).

In this subsection, we used one-loop estimates to understand the qualitative behaviors of squark and slepton mixings, but we numerically solve RG equations for quantitative analysis in the subsequent sections.

3. How to impose constraints on scalar mixings

3.1 Scheme

One popular way to constrain sfermion mixings in a model-independent fashion is to scan over one mass insertion parameter at a time, while setting the other parameters to zero. The practical reason to assume all but one of the parameters to be zero is that it is difficult or impossible to take more than one complex mass insertions as free variables and plot the allowed volume. Despite its makeshift motive, this strategy works as long as the parameter being swept by itself makes the dominant contribution to the process in consideration. However, there are cases where presence of another mass insertion amplifies the contribution from the scanned parameter, thereby rendering the constraint from a process much tighter.

A well known example is \(B \to X_s\gamma\). For instance, a single \((\delta^d_{23})_{RR}\) insertion contributes to this decay via the gluino loop shown in Fig. 1 (a). If one takes into account nonzero \((\delta^d_{33})_{RL}\) insertion as well, the diagram in Fig. 1 (b) with double insertions can make an additional contribution [10, 36], whose amplitude is enhanced by \(\tan\beta\) relative to the single insertion graph due to the chirality flip on the gluino propagator. The reason for including the double insertion diagram, namely considering nonzero \((\delta^d_{33})_{RL}\) in addition to the \((\delta^d_{23})_{RR}\)
under inspection, is not only that it can significantly increase the $B \to X_s \gamma$ branching ratio, but also that $(\delta^d_{23})_{RL} \equiv m_b(A - \mu \tan \beta)/\tilde{m}_d^2$ is generically present and therefore it should not be ignored. An $s \to d$ equivalent has been used in the study of $\epsilon'/\epsilon_K$ [37].

Another example is $B_s - \overline{B_s}$ mixing. This process is affected by $(\delta^d_{23})_{RR}$ as well. However, the $B_s - \overline{B_s}$ mixing constraint on $(\delta^d_{23})_{RR}$ greatly depends on the size of $(\delta^d_{23})_{LL}$ [36], and therefore it matters what value of the $LL$ insertion we choose when we are focusing on the $RR$ mixing. Apart from the simple-minded choice of vanishing $LL$ insertion, one option is to set $(\delta^d_{23})_{LL} = (\delta^d_{23})_{RR}$ [7], which may be expected from a left-right symmetry. Another well-motivated value of $(\delta^d_{23})_{LL}$ is the one generated by RG running from the scale where the boundary condition is given down to the sparticle mass scale [8]. This value is shown in a rather obscure form in (2.28b) and (2.21). It comes from the CKM mixing of quark Yukawa couplings and is expected even with universal soft supersymmetry breaking terms at $M_*$. It should be reasonable to expect at least this amount of $LL$ insertion, even if one allows for general non-universal boundary condition, which is the case in this work.

In the framework of supersymmetric GUT, the story can be extended in a more interesting way. The aforementioned parameter $(\delta^d_{23})_{RR}$ is related to $(\delta^l_{23})_{LL}$ at the GUT scale, and it can lead to LFV. An obvious decay mode is $\tau \to \mu\gamma$ [13, 14, 15, 16]. It can serve as another constraint on $(\delta^d_{23})_{RR}$, under the assumption of SU(5) grand unification. A less obvious mode is $\mu \to e\gamma$. Due to the GUT symmetry, the CKM mixing leads to an off-diagonal element of the scalar mass matrix of the entire 10 members, while they run from the reduced Planck scale down to the GUT scale. With the help of $(\delta^l_{13})_{RR}$ produced in this way, one can complete a diagram for $\mu \to e\gamma$ with triple mass insertions shown in Fig. 2. This diagram receives $m_\tau/m_\mu$ enhancement relative to the usual chargino loop since it is proportional to $(\delta^d_{33})_{RL}$ [12, 20, 38, 39]. Therefore it can give a strong restriction on $(\delta^d_{33})_{LL}$ and thus on $(\delta^d_{23})_{RR}$.

The above examples illustrate how much a constraint on a given mass insertion parameter can be strengthened due to the presence of another insertion. Then, the question
would be what the reasonable default value of a mass matrix element is, while a particular mass insertion is being scanned. In this work, we take the following scheme for choosing the default value of a soft supersymmetry breaking parameter: by default, the $10$ soft scalar mass matrix elements are set to the RG-induced values from the top Yukawa coupling and the CKM mixing, and the off-diagonal components of $\mathbf{M}$ mass matrix are set to zero; we ignore scalar trilinear couplings supposing that a loop graph arising from a nontrivial $A$-term does not accidently cancel the contributions considered later.

Using this scheme, we carry out a numerical analysis taking the following steps. From the Yukawa couplings and gauge couplings at the weak scale, those at the GUT scale are computed by solving the one-loop RG equations. In this process, neutrino Yukawa couplings are ignored. After reaching the GUT scale, we move to the basis of $q$, $\bar{u}$, $\bar{d}$, $l$, and $\bar{e}$ such that $Y_d$ and $Y_e$ are diagonal. We assume that $U_L$ and $U_R$ are identity matrices, and thus $q$, $\bar{d}$, $l$, $\bar{e}$ are identical to their uppercase counterparts in (2.13). In terms of the superpotential parameters, this corresponds to the case where $h_2$ in (2.3) is such that it reproduces the observed down-type quark and charged lepton masses, and is diagonal in the basis where $Y_D$ is diagonal. Consequences of relaxing this assumption will be discussed in Section 4.2. In this super-CKM basis of down-type quarks and charged leptons at the GUT scale, we set the soft mass matrix of squarks to the form,

$$m^2_q = m^2_0 \begin{pmatrix} 1 & (\delta^d_{12})_{LL} & (\delta^d_{13})_{LL} \\ (\delta^d_{12})_{LL} & 1 & (\delta^d_{23})_{LL} \\ (\delta^d_{13})_{LL} & (\delta^d_{23})_{LL} & 1 \end{pmatrix}, \quad m^2_d = m^2_0 \begin{pmatrix} 1 & 0 & (\delta^d_{13})_{RR} \\ 0 & 1 & (\delta^d_{23})_{RR} \\ (\delta^d_{13})_{RR} & (\delta^d_{23})_{RR} & 1 \end{pmatrix}, \quad (3.1)$$

and we determine the slepton soft masses using (2.16) neglecting the $O(\xi)$ corrections. The other scalar masses including those of Higgses are universally put to $m_0$. The trilinear scalar couplings are set to zero. With these boundary conditions given at the GUT scale, the one-loop RG evolution of the lagrangian parameters is performed down to the weak scale.

In order to fill out the mass matrices of scalars, charginos, and neutralinos, we determine $\mu$ from the electroweak symmetry breaking condition, choosing the positive sign. We have numerically checked that changing the sign of $\mu$ does not make a substantial difference. Then, we have all the sparticle mass matrices needed to calculate flavor and $CP$ violation quantities. We do not use mass insertion approximation, but employ mass eigenvalues and mixing matrices, thereby taking account of multiple insertion graphs automatically. For a quark sector amplitude, we keep only gluino loops, and disregard parametrically suppressed corrections from neutralino, chargino, and charged Higgs exchanges.

Regarding patterns of the mass insertion parameters in (3.1), we consider the four cases displayed in Table 1. A parameter indicated as ‘free’ is a variable to be scanned over, and the other three are fixed at the respective specified numbers, according to the policy outlined above. Those numbers have been obtained by solving the RG equations for the soft scalar mass matrices with universal boundary conditions at the reduced Planck scale down to the GUT scale in a supersymmetric SU(5) model with minimal field content [12]. In this procedure, we have ignored effects of non-renormalizable operators on the running of scalar mass matrices. The size of $(\delta^d_{ij})_{LL}$ depends on $m_0$, $M_{1/2}$, and $\tan\beta$, where $M_{1/2}$ is the unified gaugino mass at $M_{\text{GUT}}$. This dependence is taken into account in a plot for
a different set of input parameters, although the change from the value shown in the table is insignificant.

| Fig. | $|\delta_{12}^d|_{LL}$ | $|\delta_{13}^d|_{LL}$ | $|\delta_{23}^d|_{LL}$ | $|\delta_{13}^d|_{RR}$ | $|\delta_{23}^d|_{RR}$ |
|------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 4    | $4.8 \times 10^{-5}$ | $1.5 \times 10^{-3}$ | $7.4 \times 10^{-3}$ | 0                    | free                 |
| 5    | $4.8 \times 10^{-5}$ | $1.5 \times 10^{-3}$ | $7.4 \times 10^{-3}$ | free                 | 0                    |
| 6    | $4.8 \times 10^{-5}$ | $1.5 \times 10^{-3}$ | free                 | 0                    | 0                    |
| 7    | $4.8 \times 10^{-5}$ | free                 | $7.4 \times 10^{-3}$ | 0                    | 0                    |

Table 1: Values of mass insertion parameters to be given as boundary conditions at the GUT scale, for the case with $m_0 = 220$ GeV, $M_{1/2} = 180$ GeV, and $\tan\beta = 5$. The phase of a fixed $|\delta_{13}^d|_{LL}$ is equal to $\arg(-V^*_{ti}V_{sj})$, as can be expected from (2.21). The first column points to the plot of each free variable.

### 3.2 Observables

We summarize observables from the quark sector and how we use them as constraints, in Table 2.

The mass splittings of $B^0$ and $B_s$ mesons have been measured with high precision. The error of $\Delta M_{B_s}$ is 0.8% and that of $\Delta M_{B_s}$ is 0.7%. However, their theoretical prediction from short-distance physics is not so precise. The main obstacle stems from $f_{B_s}/f_{B_d}^0$ which enters the hadronic matrix element of $B^0\bar{B}^0$ ($B_s\bar{B}_s$) mixing, parametrizing long-distance QCD effects. The present uncertainty in lattice QCD calculation is around 30% (see e.g. [43] and references therein). A popular way to avoid this large uncertainty is to take the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ since the error in $(f_{B_s}^2/B_d)$ is much smaller. Still, the SM prediction of the mass difference ratio has an uncertainty of about 40% due to the

| Observable          | Measured value | Imposed constraint |
|---------------------|----------------|--------------------|
| $\Delta M_{B_d}$    | $0.507 \pm 0.004$ ps$^{-1}$ [23] | $0.507$ ps$^{-1}$ ± 30% |
| $\sin 2\beta$      | $0.681 \pm 0.025$ [23] | $2 \sigma$ |
| $\cos 2\beta$      | $> -0.4$ [40] | |
| $B(B \to X_d\gamma)$ | $(3.1 \pm 0.9^{+0.5}_{-0.5}) \times 10^{-6}$ [41] | $[5 \times 10^{-7}, 10^{-5}]$ |
| $\Delta M_{B_s}$    | $17.77 \pm 0.10 \pm 0.07$ ps$^{-1}$ [3] | $17.77$ ps$^{-1}$ ± 30% |
| $\phi_{B_s}$        | $-0.76^{+0.37}_{-0.33}, -2.37^{+0.33}_{-0.37}$ [23] | $[-1.26, -0.13] \cup [-3.00, -1.88]$ |
| $B(B \to X_s\gamma)$ | $(352 \pm 23 \pm 9) \times 10^{-6}$ [23] | $2 \sigma$ |
| $s_{CP}^{\phi_K}$   | $0.39 \pm 0.17$ [23] | $2 \sigma$ |
| $|\epsilon_K|$      | $(2.232 \pm 0.007) \times 10^{-3}$ [29] | $|\epsilon_K^{\text{SUSY}}| < |\epsilon_K^{\text{exp}}|$ |
| $\phi_i/\epsilon_K$ | $(1.66 \pm 0.26) \times 10^{-3}$ [29] | $|\epsilon_i/\epsilon_K^{\text{SUSY}}| < |(\epsilon_i/\epsilon_K)^{\text{exp}}|$ |
| $|d_{ij}|$           | $< 6.3 \times 10^{-26} \text{ e cm}$ [42] | |

Table 2: Constraints from the quark sector on sfermion mixing. An empty third column means that the second column is used as is.
errors in the CKM matrix elements [43]. As a comprehensive way to embrace the above uncertainties, we require that each of computed $\Delta M_{B_d}$ and $\Delta M_{B_s}$ falls within 30% of its central value, fixing $f_{B_d}^2 B_{B_d}$ and $f_{B_s}^2 B_{B_s}$. In spite of the seemingly loose conditions, we will find that these requirements play impressive roles, given higher soft scalar mass. The uncertainty decreases with the progress of lattice QCD, and is estimated to be reducible down to 8–10% with 6–60 tera flops year of computing power [44]. The improved constraint from this smaller error is considered as well.

Although $\sin 2\beta$ does not suffer from uncertainty in the $\Delta B = 2$ matrix element, its SM prediction depends on $V_{ub}$, which has a sizable error. When we require $\sin 2\beta$ to be within the 2 $\sigma$ range of its experimental value, we allow for a 2 $\sigma$ variation in $|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}$ [29] as well. As with the magnitude of mixing, we estimate effects of an improved measurement of $\sin 2\beta$ at a super $B$ factory, under the assumption that it will converge to its SM value. We use 2% as a projected error of $|V_{ub}|$ and 0.005 as a standard deviation of $\sin 2\beta$ [45]. Given these smaller errors, the central values of $|V_{ub}|$ and $\sin 2\beta$, if they remain as they are now, become inconsistent with each other, reflecting the present tension between them. Under this condition, the future $\sin 2\beta$ measurement would appear to exclude the SM, and therefore it would be hard to evaluate the influence of its improved precision. However, there is a claim that the tension can be reconciled within the SM [46]. We do not regard this as a signal of new physics, and assume that $|V_{ub}|$ will decrease so that it becomes compatible with the present $\sin 2\beta$.

As for $\phi_{B_s}$, the phase of $B_s$-$\overline{B}_s$ mixing, we try two distinct ways of imposing the constraint: (a) using the latest data from DØ at 90% confidence level (CL); (b) employing the 90% CL range recently reported by the Heavy Flavor Averaging Group (HFAG). Regarding option (b), we choose the one obtained with constraints from flavor-specific $B_s$ lifetime and $B_s$ semileptonic asymmetry. (What is denoted by $\phi_{B_s}$ in this work is $\phi_{J/\psi}$ in the notation of HFAG.) We present both of these cases as they lead to very different impressions of the results—the DØ range includes the SM prediction of $\phi_{B_s}$ and hence it still works as a bound on the room for new physics, while the HFAG range lies outside the SM value, thereby indicating the size of extra contribution required to account for the discrepancy [47, 48]. In order to compare the power of $\phi_{B_s}$ measurements at LHCb with that of LFV, we suppose that the future central value of $\phi_{B_s}$ is given by the SM, despite the current hint of new physics at the level around 2 $\sigma$. We assume that the error of $\phi_{B_s}$ will be 0.009 at 10 fb$^{-1}$ [49].

Measurement of the inclusive branching fraction $B(B \to X_d \gamma)$ had not been available until its preliminary result was recently reported from BaBar [41]. The precision is still low. Considering the experimental and theoretical uncertainties, we take modest upper and lower bounds guesstimated from the exclusive branching fraction $B(B \to \rho/\omega \gamma) = (1.18 \pm 0.17) \times 10^{-6}$ [23]. Unlike $B \to X_d \gamma$, the branching ratio of $B \to X_s \gamma$ has been measured with a high precision. We impose a 2 $\sigma$ constraint on it.

We use QCD factorization [50] to evaluate $S_{CP}^{\phi K}$, the sine term coefficient in the time-dependent $CP$ asymmetry of $B \to \phi K$ [51, 52]. This approach has a source of hadronic uncertainty stemming from regularizing a divergent integral in the annihilation contribution. We follow the original prescription in Ref. [50], i.e. we replace $\int_0^1 dy/y$ by
we are using a rather conservative bound.

Finally, we examine $d_n$, the neutron electric dipole moment (EDM). Recently it has been pointed out that this observable can be greatly influenced if there are both $LL$ and $RR$ down-type squark mixings at the same time [53]. In order to evaluate $d_n$, we add contributions through the down quark EDM, down quark chromoelectric dipole moment (CEDM), and strange quark CEDM.

We use the constraints from the lepton sector listed in Table 3. The second column shows the present 90% CL upper bound on each mode. The third column is the prospective upper bound from future experiments. The new limit on $\mu \to e\gamma$ is the goal of MEG at 90% CL. Also, higher sensitivity to $\tau \to e\gamma$ and $\tau \to \mu\gamma$ is anticipated from a super $B$ factory. In Section 4.1, we choose to use $10^{-8}$ as the future limit on $B(\tau \to e\gamma)$ and $B(\tau \to \mu\gamma)$, between the two numbers in each row of the table. If one wants to use $2 \times 10^{-9}$ instead, the result can be obtained easily: multiply the upper bound on a given mass insertion from $\tau \to e\gamma$ or $\tau \to \mu\gamma$, by $1/\sqrt{3}$.

Imposing the conditions enumerated above, we estimate possible deviations in additional observables of interest, shown in Table 4. The first three measure time-dependent $CP$ asymmetries in radiative $B$ decays. The definition of $S_{CP}^{K^*\gamma}$ is given by [59],

$$A_{K^*\gamma}(t) = \frac{\Gamma(B_{d}(t) \to K^*\gamma) - \Gamma(B_{d}(t) \to K\gamma)}{\Gamma(B_{d}(t) \to K^*\gamma) + \Gamma(B_{d}(t) \to K\gamma)}$$

$$= A_{CP}^{K^*\gamma} \cos(\Delta M_{B_d} t) + S_{CP}^{K^*\gamma} \sin(\Delta M_{B_d} t). \tag{3.2}$$

| Mode               | Present bound    | Future bound    |
|--------------------|------------------|-----------------|
| $B(\mu \to e\gamma)$ | $1.2 \times 10^{-11}$ [54] | $10^{-13}$ [11] |
| $B(\tau \to e\gamma)$ | $1.1 \times 10^{-7}$ [55] | $10^{-8}$ [56], $2 \times 10^{-9}$ [45] |
| $B(\tau \to \mu\gamma)$ | $4.5 \times 10^{-8}$ [57] | $10^{-8}$ [56], $2 \times 10^{-9}$ [45] |

**Table 3:** Constraints from radiative LFV decay modes.

| Observable         | Measured value       |
|--------------------|----------------------|
| $S_{CP}^{K^*\gamma}$ | $-0.19 \pm 0.23$ [23] |
| $S_{CP}^{\rho\gamma}$ | $-0.83 \pm 0.65 \pm 0.18$ [58] |
| $S_{CP}^{B_d\to K^*\gamma}$ |                             |
| $A_{CP}^{b\to s\gamma}$ | $0.004 \pm 0.037$ [23] |
| $A_{CP}^{b\to d\gamma}$ |                             |
| $A_{CP}^{b\to (s+d)\gamma}$ |                             |

**Table 4:** Monitored observables. Precisions attainable at a super $B$ factory are summarized in Table 6.
Note that the time-dependent $CP$ asymmetry in $B_d \to K^*\gamma$ in our convention has the sign opposite to that in the above reference. This observable is sensitive to a new $CP$ violating phase in the right-handed $b \to s$ transition, such as coming from $(\delta_{23}^d)^{RR}$. We define $S_{CP}^{d*\gamma}$ in a parallel way by replacing $K^*$ with $\rho$ in the expression. This can serve as a $b \to d$ analog of $S_{CP}^{K^*\gamma}$, affected by $(\delta_{13}^d)^{RR}$. One might as well use $S_{CP}^{B_d\to K^*\gamma}$ to investigate $(\delta_{13}^d)^{RR}$, and we record its variation. The rest three are direct $CP$ asymmetries in radiative $B$ decays, $B \to X_s\gamma$, $B \to X_d\gamma$, and $B \to X_{s+d}\gamma$, whose definitions can be figured out by setting $t=0$ in the above equation. They are complementary to the preceding observables in the sense that they can probe left-handed $CP$ violating new physics such as $(\delta_{23}^d)_{LL}$ and $(\delta_{13}^d)_{RR}$. We quote the measured value of each observable if available.

4. Results

4.1 Viable region of each mass insertion

As a preparation for reading plots of the GUT scale mass insertions, we sketch the process amplitudes in terms of these variables. This will help us understand how a figure changes as a parameter is modified. Keeping only factors of interest, the LFV decay amplitudes can be roughly put in the form,

$$A(\tau \to \mu \gamma) \propto \mu \tan \beta \cdot (\delta_{23}^d)^{LL}(M_{\text{GUT}}),$$

$$A(\mu \to e \gamma) \propto \frac{m_\tau \mu \tan \beta}{m_\mu^2} \cdot \frac{(\delta_{13}^d)^{RR}(M_{\text{GUT}}) \cdot (\delta_{32}^d)^{LL}(M_{\text{GUT}})}{m_S^2},$$

where $m_S$ is the typical mass of a slepton, chargino, or neutralino in the loop. The first factor in the second line is $(\delta_{33}^d)^{RL} \equiv m_\tau(A - \mu \tan \beta)/m_\mu^2$ rewritten with (2.25b). We used (2.30) to replace the other mass insertions by the GUT scale quantities. As to hadronic observables, let us pick up $\Delta M_{B_s}$ as an example; other constraints can be understood in a similar fashion. The $B_s$–$\bar{B}_s$ transition amplitude depends on $(\delta_{23}^d)^{AA}(\delta_{23}^d)^{BB}/m_S^2$ with $A, B = L, R$. For instance, we can use (2.28) to recast one of these combinations at $M_{\text{SUSY}}$ as

$$\left.\frac{(\delta_{23}^d)^{LL}(\delta_{23}^d)^{RR}}{m_S^2}\right|_{M_{\text{SUSY}}} \approx \frac{[(\delta_{33}^d)^{LL}(M_{\text{GUT}}) + q_{ij}] \cdot (\delta_{23}^d)^{RR}(M_{\text{GUT}})}{(1 + 6x)^2 m_S^2},$$

where $m_S$ is the typical mass of a squark or gluino in the loop. Note that $q_{ij}$ from (2.29) is nearly independent of $m_0^2$. Therefore, as we vary $m_0$ and $M_{1/2}$, the scaling property of (4.2) is determined by its denominator. The other two combinations with $A = B = L, R$ scale in the same way.

With these ingredients at hand, we begin to interpret the results. First of all, let us look at the result of a two-parameter scan to get a taste of two mass insertions. In each of the four plots shown in Figs. 3, there are 1–3 and 2–3 mixings with different chiralities. The gaugino mass $M_{1/2}$ at the GUT scale is chosen in such a way that the gluino mass becomes 500 GeV at the weak scale. The scalar mass $m_0$ at $M_{\text{GUT}}$ is set to two different values that can elucidate complementarity of the quark and the lepton sector processes. In
The left column, $m_0$ is taken to be 220 GeV, so that the first and the second family right-handed down-type squarks have the same mass as the gluino at the weak scale. (The third family is slightly lighter.) This is a benchmark case often encountered in the literature on supersymmetric flavor violation. In the right column, we change $m_0$ to 600 GeV. If one fixes the $\delta$ parameters at $M_{GUT}$, this $m_0$ maximizes gluino loop contribution to $B$-meson...
mixing for the gaugino mass chosen here. We elaborate on this point later. On the plots, each mass insertion parameter is treated as a real number.

In each of Figs. 3 (a) and (b), the two axes are \( (\delta_{13}^d)_{LL} = (\delta_{13}^d)^*_{RR} \) and \( (\delta_{23}^d)_{RR} = (\delta_{23}^d)_{LL} \) at the GUT scale. Here we restrict the horizontal axis to a range much narrower than the vertical axis since we are especially interested in the effect of RG contribution, but otherwise the \( LL \) mixing can be arbitrary. One can find that \( (\delta_{23}^d)_{RR} \) is constrained by \( \tau \to \mu \gamma \) and \( \Delta M_{B_s} \). Interestingly, the two plots show different relative significance of these two constraints. In Fig. (a), \( \tau \to \mu \gamma \) is stronger than \( \Delta M_{B_s} \), i.e. a large portion of the region allowed by the latter is excluded by the former. In Fig. (b), the order of importance appears to be reversed. Although it is early to draw a conclusion since these plots restrict the mass insertions to be real, it is obvious that \( \Delta M_{B_s} \) gets tighter while \( \tau \to \mu \gamma \) becomes looser if \( m_0 \) is changed from 220 GeV to 600 GeV. As \( m_0 \) increases, the LFV constraints get relaxed because \( m_S \) gets bigger in (4.1). In fact, \( \mu \) in a numerator grows as well, but the growth of \( m_S^2 \) in the denominator wins. In spite of heavier sparticles, hadronic constraints get relatively more stringent. To understand why, pay attention to (4.2). Since we have fixed the gluino mass while raising \( m_0 \), a squark is heavier than a gluino, and we should substitute (2.25a) for \( m_S^2 \). Then, the gluino loop contribution to \( B_s - \overline{B}_s \) mixing scales like \( x/(1 + 6x)^3 \). This factor increases as we decrease \( x \) inversely proportional to \( m_0^2 \), unless \( x \) is smaller than 1/12, the maximum point. For \( x \lesssim 1/12 \), the squarks are so heavy that they begin to decouple from low-energy processes as in split supersymmetry. Note that \( x = 0.67 \) on the left plot and \( x = 0.09 \) on the right. This explains the narrower \( \Delta M_{B_s} \) band on the right plot.\(^1\) Other quark sector processes are enhanced in a similar way, as will be shown later. As can be expected from Fig. 2, \( \mu \to e \gamma \) restricts the product \( (\delta_{13}^d)_{LL} (\delta_{23}^d)_{RR} \), resulting in the hyperbolas on the plane. Therefore, the \( \mu \to e \gamma \) limit on \( (\delta_{23}^d)_{RR} \) varies depending on the size of \( (\delta_{13}^d)_{LL} \). If \( (\delta_{13}^d)_{LL} = 0 \), \( (\delta_{23}^d)_{RR} \) is free, but the restraint grows severer as \( |(\delta_{13}^d)_{LL}| \) increases. A special case with RG-induced \( (\delta_{13}^d)_{LL} \), marked by the vertical hatched strip, will be detailed shortly. With increasing \( m_0 \), \( \mu \to e \gamma \) is doubly suppressed by \( m_0^2 m_S^2 \) in (4.1b). This expands the area within the hyperbola in the plots. The width of the \( \Delta M_{B_s} \) band is mainly due to the current uncertainty in the \( B_s - \overline{B}_s \) mixing matrix element around 30%. The projected bound with 8% uncertainty is depicted by the two white lines around \( (\delta_{23}^d)_{RR} = 0 \). In Fig. (a), it is not yet as tight as the present \( \tau \to \mu \gamma \) constraint which will be even tighter in the future. However, in Fig. (b), the reduced hadronic uncertainty makes the \( \Delta M_{B_s} \) bound more restrictive than \( \tau \to \mu \gamma \) at a super \( B \) factory. Nevertheless, it should be kept in mind that \( \tau \to \mu \gamma \) becomes more sensitive as \( \tan \beta \) grows while \( B_s - \overline{B}_s \) mixing does not.

We do the same exercise with a different mixture of mass insertions, \( (\delta_{13}^d)_{RR} = (\delta_{13}^d)_{LL} \) and \( (\delta_{23}^d)_{LL} = (\delta_{23}^d)_{RR}^* \), to get Figs. 3 (c) and (d). The vertical range is set around the magnitude generated by RG running. Here \( (\delta_{23}^d)_{RR} \) is bounded by \( \Delta M_{B_d} \), \( \sin 2\beta, \tau \to e \gamma \), and \( B \to X_d \gamma \). In Fig. (c), the current limits from \( \Delta M_{B_d} \) and \( \tau \to e \gamma \) are comparable to

\(^1\) A similar discussion is given in Ref. [34] in a different context, in a more qualitative way. In their scenario, the 2-3 squark mixing arises from large neutrino Yukawa couplings. They state that squark loop effects on \( B_s \) mixing can be more significant for higher \( m_0 \). However, they do not mention at what point of \( m_0 \) this trend stops and squark loops begin to decouple.
each other, which are stronger than \( B \rightarrow X_d \gamma \). In Fig. (d), \( \Delta M_{B_s} \) with the aid of \( \sin 2\beta \) leaves a band which is much narrower than that allowed by \( \tau \rightarrow e\gamma \). Also, the \( B \rightarrow X_d \gamma \) bound moves inside the \( \tau \rightarrow e\gamma \) bound. These changes, as well as enhancement of the other quark sector processes, can be understood in the same way as the difference between Figs. (a) and (b). The inner two white lines around \( \delta_{13}^{d\gamma} \leftrightarrow 0 \) indicate the limit from \( \Delta M_{B_s} \) with 8% uncertainty in the \( B^0 - \bar{B}^0 \) mixing matrix element, and the outer two white lines with short thin lines attached to them arise from \( \sin 2\beta \) at a super \( B \) factory. The projected limits from \( \Delta M_{B_d} \) and \( \tau \rightarrow e\gamma \) are expected to maintain the present tendency of relative strengths. That is, they are comparable to each other in Fig. (c) and the former is stronger than the latter in Fig. (d). Again, the hyperbolas come from \( \mu \rightarrow e\gamma \). Boundaries by the neutron EDM appear in Fig. (d) even though all the mixings are real. Extra contribution to \( d_n \) arises from the combination \( \delta_{d\gamma}^{13} \leftrightarrow 0 \) of which the \( LL \) insertion picks up a non-vanishing phase from the CKM matrix, running from the GUT scale down to the weak scale. A specific case with RG-induced \( \delta_{23}^{d\gamma} \), indicated by the horizontal hatched strip, will be discussed later.

Having grasped a picture of how different constraints act on two mass insertions, let us examine cases where one of the insertions originates from RG running from the reduced Planck scale to the GUT scale.

We display in Figs. 4, complex versions of Figs. 3 (a) and (b) with the \( LL \) insertions fixed at the numbers shown in Table 1. They indicate regions of \( \delta_{23}^{d\gamma} = \delta_{23}^{d\gamma} \leftrightarrow 0 \) inconsistent with observations. This time, \( \tan \beta \) is varied as well as \( m_0 \). Let us walk through them starting from Fig. 4 (a). A light gray (yellow) area is allowed by \( \Delta M_{B_s} \) but not by \( \phi_{B_s} \) from DØ. A gray (cyan) area is allowed by both. However, most of it is ruled out by the LFV processes, as we have already noticed in Fig. 3 (a). One can guess that this should be the case even in the near future, comparing the zone surrounded by the white curves and the thin circles with their centers at the origin. In particular, the \( \mu \rightarrow e\gamma \) data from the MEG experiment should be able to kill all the parameter space except for the tiny disk around the origin. It deserves a remark that \( \mu \rightarrow e\gamma \) plays an important role here. Being a 2–3 mixing, \( \delta_{23}^{d\gamma} \leftrightarrow 0 \) is normally associated with the \( \tau \rightarrow \mu\gamma \) process. For example, Ref. [16] discusses interplay between leptonic and hadronic constraints in a similar context, but they use only \( \tau \rightarrow \mu \) transitions to restrict \( \delta_{23}^{d\gamma} \leftrightarrow 0 \). This difference arises from the strategy of setting the mass insertion parameters. Their default value of a mass insertion is zero, while our default is the one which is minimally expected from RG running. Therefore, they do not find \( \mu \rightarrow e\gamma \) limiting a 2–3 mixing as is obvious from Fig. 3 (a). We believe that our choice of mass insertions is more reasonable in a scenario where the soft terms are generated around the Planck scale such as gravity mediation. It may be argued that the RG-induced \( 10 \) scalar mixing is not always guaranteed to be sizeable since the cutoff scale can happen to be low close to the GUT scale. This is true. However, a low cutoff would threaten the validity of making a connection between the quark and the lepton flavors in the first place. Non-renormalizable operators shown in (2.3) and even higher order terms, generically, give \( \mathcal{O}(1) \) contributions to the quark and the lepton Yukawa couplings, thereby erasing any trace of their connection in the flavor space as a single GUT multiplet. It should be remembered that an RG-induced \( LL \) insertion is not only critical to \( \mu \rightarrow e\gamma \), but
also to $B_s$-$\bar{B}_s$ mixing. Indeed, the presence of $(\delta_{23}^d)_{LL}$ is rendering the $\Delta M_{B_s}$ constraint on $(\delta_{23}^d)_{RR}$ tighter [7, 8, 36]. If it were not for $(\delta_{23}^d)_{LL}$, the gray region would look like the one in Fig. 6 (a), where the contribution to $\Delta M_{B_s}$ from $(\delta_{23}^d)_{LL}$ is not enhanced by $(\delta_{23}^d)_{RR}$. Another noticeable point is that the information on $\phi_{B_s}$ from LHCb can play an important role in shaping the allowed region. A particular pleasure with this constraint that it does not suffer from the hadronic uncertainty that plagues $\Delta M_{B_s}$. Other observables of interest

Figure 4: Constraints on the complex plane of $(\delta_{23}^d)_{RR}$, with $(\delta_{23}^d)_{LL}$ generated from RG running between the reduced Planck scale and the GUT scale. For each LFV process, the thick circle is the present upper bound and the thin circle is the prospective future bound. A light gray (yellow) region is further consistent with the present upper bound and the thin circle is the prospective future bound. A light gray (yellow) region is further consistent with the present upper bound and the thin circle is the prospective future bound. A light gray (yellow) region is further consistent with the present upper bound and the thin circle is the prospective future bound. A light gray (yellow) region is further consistent with the present upper bound and the thin circle is the prospective future bound. A light gray (yellow) region is further consistent with the present upper bound and the thin circle is the prospective future bound. A light gray (yellow) region is further consistent with the present upper bound and the thin circle is the prospective future bound.
related to 2–3 mixing are \( S_{CP}^{φK} \) and \( d_n \). The area excluded by each of them is depicted. Note that \( d_n \) depends on the phase of \( (δ_{23}^{d})_{LL} \) as well as on its size. Although the phase of the first term in (2.21) is fixed by the CKM matrix elements, the \( O(ξ) \) correction is unknown and may influence the phase of the entire insertion. Varying the phase of \( (δ_{23}^{d})_{LL} \) amounts to rotating the \( d_n \) band on the plot around the origin. Finally, the dotted lines are contours of \( S_{CP}^{φKγ} \). From them, one can read off its largest possible deviation that is consistent with the other experimental inputs. Further information on this \( CP \) asymmetry is collected in Section 4.4.

Now that we have recognized the general structure of a plot, we try different values of parameters. First, \( \tan β \) is doubled from 5 to 10 in Fig. 4(b). The \( ∆M_{B_s} \) belt does not change very much since its dependence on \( \tan β \) is negligible. Each LFV circle halves and becomes tighter. This is evident from (4.1), where each decay amplitude is proportional to \( \tan β \). The gluino loop diagrams contributing to each of \( d_n \) and the \( B \rightarrow φK \) decay are also proportional to \( \tan β \), and the allowed region shrinks as \( \tan β \) increases. Next, we change the scalar mass parameter. In Fig. 4(c), we raise \( m_0 \) to 600 GeV, a value optimized for \( B \)-meson mixings. For the reason already explained, the LFV constraints turn weaker while the \( B_s \rightarrow \bar{B}_s \) mixing belt shrinks on the plot. Other quark sector processes are boosted as well. Because of this, the impressions of hadronic and leptonic bounds undergo a sea change from Fig. (a) to (c). The current \( μ \rightarrow eγ \) limit gets so much relaxed that it does not exclude any region compatible with \( ∆M_{B_s} \) and \( φ_{B_s} \). One can also notice that \( d_n \) has become much more powerful. Its limit on the imaginary part of \( (δ_{23}^{d})_{RR} \) is stronger than any other bound on the plot. Indeed, the combination of \( d_n \) and \( B_s \rightarrow \bar{B}_s \) mixing leaves nothing to do for \( τ \rightarrow μγ \). In the future case, \( B_s \rightarrow \bar{B}_s \) mixing looks more restrictive than \( τ \rightarrow μγ \) and \( μ \rightarrow eγ \), particularly thanks to improved precision of \( φ_{B_s} \) at LHCb. This should be contrasted with the situation in Fig. (a) where the LFV constraints, both at present and in the future, are stronger than the hadronic ones. Lastly, we consider higher \( m_0 \) and \( \tan β \) in Fig. (d). Each observable changes according to its \( \tan β \) dependence already mentioned. For the first time, the \( B \rightarrow X_sγ \) bound becomes visible. Until now, its branching fraction has not been sufficiently disturbed by the new physics contribution given by Fig. 1(b). One reason is that this diagram does not interfere with the SM one since they lead to different photon helicities. In Fig. 4(d), however, higher \( m_0 \) and \( \tan β \) cooperate to enhance the supersymmetric amplitude. Still, \( B \rightarrow X_sγ \) is not very restrictive. Its role should be more significant for \( \tan β \) much higher than 10. The range of \( S_{CP}^{Kγ} \) predicted in each case is summarized in Table 6.

A remark is in order regarding the recent reports on \( φ_{B_s} \) which reveal a small but interesting disparity between the combined fit and the SM prediction [22, 23]. Once we take the 90% CL range of \( φ_{B_s} \) from HFAG, instead of that from DØ, the gray regions change to those surrounded by the thick black curves. Since the HFAG result demands new physics contribution to \( φ_{B_s} \), the origin on the plane is positioned outside the thick black boundary. On the other hand, the mass insertion can be compatible with the LFV data only around the origin. This conflict leads to a restriction in an attempt to understand the new fit result of \( φ_{B_s} \) with an \( RR \) mixing. The trouble is more serious with lower \( m_0 \) and/or higher \( \tan β \). As was explained above, lower \( m_0 \) enhances the LFV branching ratios
while suppressing supersymmetric contributions to $B_s - \overline{B}_s$ mixing. Also, higher $\tan\beta$ gives rise to higher LFV rates. Figs. (a) and (b) tell us that these cases are disfavored by LFV in combination with $\phi_{B_s}$. The tension between LFV and $\phi_{B_s}$ is relaxed for higher $m_0$ used in the lower plots. Indeed, one can find a small intersection of the $\phi_{B_s}$ area and the $\tau \rightarrow \mu \gamma$ disk in each of Figs. (c) and (d). This region could be accessed by measurements of $\tau \rightarrow \mu \gamma$ and $\mu \rightarrow e\gamma$ in the near future. However, the neutron EDM becomes a new obstacle to the zone favored by $\phi_{B_s}$ as higher $m_0$ reinforces hadronic constraints. The limit from $d_n$ grows more serious for higher $\tan\beta$. The problem can be eased by modifying the size and phase of $(\delta^d_{23})_{LL}$ at the GUT scale to rotate the $d_n$ band as already mentioned.

At this point we stop considering 2–3 mixing of $\tau$, and apply the same procedure to the 1–3 sector. We present in Figs. 5, complex versions of Figs. 3 (c) and (d) with the $LL$ insertions fixed at the values listed in Table 1. They exhibit constraints on $(\delta^d_{13})_{RR} = (\delta^d_{13})_{LL}$. We start out with Fig. 5 (a). The light gray (yellow) belt is compatible with $\Delta M_{B_d}$, which is further reduced by $\sin 2\beta$ into the gray (cyan) region. The resulting area is completely consistent with $B \to X_d\gamma$. The width of this area is comparable to the diameter of the circle from $\tau \to e\gamma$. The restriction from $\mu \to e\gamma$ is so strong that it rules out most of the gray zone. The $\mu \to e\gamma$ disk in this plot is smaller than that in Fig. 4 (a). The reason is that the decay amplitude is proportional to $(\delta^d_{23})_{RR} \sim \lambda^2$ there, but to $(\delta^d_{13})_{RR} \sim \lambda^3$ there. In a few years, improved lattice QCD should be able to narrow the $\Delta M_{B_d}$ belt down to the one between the two white curves, whose width is again comparable to the diameter of the future $\tau \to e\gamma$ disk. If this narrowed belt is complemented by measurement of $\sin 2\beta$ at a super $B$ factory, the combined constraint could be comparable to or stronger than the future $\tau \to e\gamma$ bound. The MEG constraint is so tight that the circle appears to be a single dot at the origin. The radius of this circle can be looked up in Table 5. The dotted curves are contours of $S^\tau_{CP}$. We present its shift that can be expected obeying other constraints in Section 4.4. The rest three plots are for the cases with (b) higher $\tan\beta$, (c) higher $m_0$, and (d) higher $m_0$ and $\tan\beta$, respectively. They can be understood in the same way as each corresponding figure in Figs. 4 was. Let us stress again that with higher $m_0$, the sensitivity of hadronic observables to the GUT scale mass insertions is reinforced while that of LFV is weakened. In Figs. (c) and (d), the combination of $\Delta M_{B_d}$ and $\sin 2\beta$ essentially determines the viable areas. This trend is expected to be maintained by a super $B$ factory. One can notice that $d_n$ in Figs. (c) and (d), and $\epsilon'/\epsilon_K$ in Fig. (d), begin to be visible due to increased $m_0$. These quantities are susceptible to the imaginary parts of $(\delta^d_{13})_{RR}(\delta^d_{32})_{RL}(\delta^d_{31})_{LL}$ and $(\delta^d_{13})_{RR}(\delta^d_{33})_{RL}(\delta^d_{32})_{LL}$, respectively, although they are not playing important roles here. Figs. 5 (b) and (d) show that the amplitude of $B \to X_d\gamma$ is enhanced by higher $\tan\beta$. One can see the reason replacing $s$ by $d$ in Fig. 1 (b).

For the sake of completeness, we report restrictions on the 10 sector mixings as well, which can be represented by $(\delta^d_{ij})_{LL} = (\delta^d_{ij})_{RR}$. In Figs. 6, we examine the 2–3 mixing. Comparing them with Figs. 4, one can notice that $B_s - \overline{B}_s$ mixing is not as restrictive on $(\delta^d_{23})_{LL}$ as $(\delta^d_{23})_{RR}$. There, the gluino loop contribution from $(\delta^d_{23})_{RR}$ to the $\Delta B = 2$ transition was enhanced by $(\delta^d_{23})_{LL}$ from radiative correction. By contrast, $(\delta^d_{23})_{LL}$ here is not reinforced by $(\delta^d_{23})_{RR}$ which is set to zero. Nonetheless, $\Delta M_{B_s}$ and $\phi_{B_s}$ exclude part of the plane in Figs. 6 (c) and (d) where their sensitivities to the GUT scale squark mixing
Figure 5: Constraints on the complex plane of $(\delta_{13}^d)^{RR}$, with $(\delta_{13}^d)^{LL}$ generated from RG running between the reduced Planck scale and the GUT scale. For each LFV process, the thick circle is the present upper bound and the thin circle is the prospective future bound. A light gray (yellow) region is allowed by $\Delta M_{B_S}$, given 30% uncertainty in the $\Delta B = 2$ matrix element, and a gray (cyan) region is further consistent with $\sin 2\beta$. The white curves running along the belt mark a possible improved constraint from $\Delta M_{B_d}$ with 8% hadronic uncertainty. The other white lines running across the belt display a measurement of $\sin 2\beta$ at a super $B$ factory. Of the two sides of a $\cos 2\beta$ curve or a white $\sin 2\beta$ curve, the excluded one is indicated by thin short lines.

are maximized. These constraints should be strengthened in the future. The white curves with short thin lines attached to them mark an improved $\phi_B$ measurement at LHCb. They appear in all the four cases. The other white curves, appearing in Figs. (c) and (d), represent the projected $\Delta M_{B_d}$ limit. Another outstanding point is that $B \to X_s \gamma$ is excluding larger area of the $(\delta_{23}^d)_{LL}$ plane than $(\delta_{23}^d)_{RR}$. Recall that the supersymmetric diagram arising from an $LL$ mixing is added to the SM piece since they have the same
Figure 6: Constraints on the complex plane of $(\delta\phi_{12}^1)_{LL}$, with $(\delta\phi_{12}^1)_{LL}$ and $(\delta\phi_{12}^2)_{LL}$ generated from RG running between the reduced Planck scale and the GUT scale. For each LFV process, the thick circle is the present upper bound and the thin circle is the prospective future bound. A light gray (yellow) region is allowed by $\Delta M_{Bs}$, given 30% uncertainty in the $\Delta B = 2$ matrix element, and a gray (cyan) region is further consistent with $\phi_{B_s}$ from DØ. A thick black curve shows $\phi_{B_s}$ from HFAG. The white curves without short thin lines mark a possible improved constraint from $\Delta M_{B_d}$ with 8% hadronic uncertainty. The white curves with short thin lines attached to them display a measurement of $\phi_{B_s}$ at LHCb. Thin short lines attached to a curve indicate the excluded side.

Concerning $\tau \rightarrow \mu \gamma$, this is because the decay is dominated by neutralino loop here, but by chargino loop there. The chargino loop, if present, generically has higher effectiveness per mass insertion size, than the neutralino loop. One can find that $\mu \rightarrow e\gamma$ also occurs. It is caused by a
neutralino loop graph proportional to \((\delta_{23})_{RR}(\delta_{31})_{RR}\) with RG-induced \((\delta_{31})_{RR}\). However, it is not strengthened by the factor \(m_\tau/m_\mu\), which accounts for the lower branching ratio than in Figs. 4. Despite being moderate, the present and future LFV bounds are still disallowing portions of the parameter space. The dotted contours show \(A_{CP}^{b\to s\gamma}\), the direct \(CP\) asymmetry in \(B \to X_s\gamma\). The numerical value of its variation is shown in Table 4, together with that of another related \(CP\) asymmetry, \(A_{CP}^{b\to(s+d)\gamma}\).

Now, we switch to the HFAG fit of the phase of \(B_s-\bar{B}_s\) mixing. In Figs. 4 (a) and (b), we cannot find a point which falls within the 90% CL range of \(\phi_{B_s}\), even if we allow for an \(O(1)\) squark mixing. Favored regions appear in Figs. (c) and (d), where hadronic processes are enhanced. As those regions involve a large 2–3 mixing of left handed down-type squarks, they are likely to give a large modification to \(B \to X_s\gamma\), in particular for high \(\tan\beta\). In an attempt to account for the negative value of \(\phi_{B_s}\) with an \(LL\) mixing, one could have a bigger hope, given a large mixing, higher \(m_0\), and low \(\tan\beta\). Even if this scenario is realized, \(\tau \to \mu\gamma\) and \(\mu \to e\gamma\) will be hard to observe even at a super \(B\) factory or MEG.

Finally, we proceed to the exclusion plots on the complex plane of \((\delta_{13})_{LL}=(\delta_{13})_{RR}\) in Figs. 7. Let us compare these with those in Figs. 5. A gray (cyan) region here is larger. A significant portion of a gray zone is cut out by \(B \to X_d\gamma\) [60]. The LFV circles are significantly bigger. Each of the above facts can be explained in a parallel fashion as we did in the previous paragraph. The dotted contours are values of \(A_{CP}^{b\to d\gamma}\). Its discussion will follow in a later part. Note that there are cases where the future LFV data may kill part of the area that is compatible with \(B\) physics measurements. An example is shown in Fig. 7 (b) with lower \(m_0\) and higher \(\tan\beta\). Yet, constraints mostly come from the hadronic sector.

We finish this subsection with a remark on the sizes of the allowed regions shown in the preceding figures. We use mass insertion parameters at the GUT scale as the horizontal and vertical axes. Therefore, one should be careful in comparing a plot in this paper with one from another work, when the latter is using mass insertions at the weak scale. If the weak scale variable is a squark mass insertion, one should convert our plots using (2.28) beforehand.

4.2 Non-renormalizable operators and leptonic constraints

In the numerical analysis of the previous subsection, we have been employing the naive relations (2.18). Now, we should discuss how the results will change if we relax this simplification and generalize the correlation of mass insertions to (2.17). One could easily guess that the one-to-one correspondence between a hadronic and a leptonic channel should be disturbed. Yet, it is not completely broken as will be shown below. Tau decay modes still limit 1–3 and 2–3 mixings of squarks of either chirality, albeit to a reduced extent. Similarly, \(\mu \to e\gamma\) remains a constraint on the \(RR\) insertions.

Let us think about how a tau decay bound should be modified. We first focus on \(RR\) insertions, and then on \(LL\). If there is an \(RR\) mixing, a tau decay amplitude is dominated by the chargino loop which is proportional to \((\delta_{13})_{LL}\) for \(\tau \to e\gamma\) or \((\delta_{23})_{LL}\) for \(\tau \to \mu\gamma\).
Neglecting the $\mathcal{O}(\xi)$ term in (2.17a), one has
\begin{equation}
(\delta_{\alpha\beta}^I)_{LL} = |U_L|_{ab} (\delta_{\alpha\beta}^{dI})_{RR} [U_L]_{33}^* + \mathcal{O}(\cos^2 \beta \delta_{RR}^{dI}), \quad a, b = 1, 2, 3.
\end{equation}

Using (2.12), the smallness of mixing of the third family with the other two. By the same token as for (2.23), there can be another term $|U_L|_{a3} (\delta_{\alpha3}^d)_{RR} [U_L]_{33}^* \sim 1.5 \times 10^{-5}/\cos \beta + 0.17 \lambda_N^2 \cos \beta$ where we use the (3,3) component of (2.20b) for its estimation. For small
neutrino Yukawa couplings, which we assume in the numerical analysis, this term is negligible even compared to the smallest upper bound that can be found in Table 5. For large $\lambda_N$, this should be an uncertainty in relating squark and slepton mixings, apart from that stemming from running below $M_{\text{GUT}}$. The mixing between the first and the second families, parametrized by $[U_L]_{ab}$, is not limited to be small. For instance, consider the case where $(\delta^d_{23})_{RR}$ is nonzero while $(\delta^d_{13})_{RR}$ is zero, as in Figs. 4. Here, $\tau \to \mu \gamma$ provides a significant constraint on $(\delta^d_{23})_{RR}$ if $U_L$ is a unit matrix. Otherwise, it might happen that the association of $(\delta^d_{23})_{LL}$ with $(\delta^d_{23})_{RR}$ is weakened by the factor $[U_L]_{22}$, or in the worst case, is completely broken for $[U_L]_{22} = 0$. Although $\tau \to \mu \gamma$ does not occur in this extreme situation, $\tau \to e \gamma$ does since $(\delta^d_{23})_{RR}$ gives rise to it through $(\delta^d_{13})_{LL}$ due to the approximate unitarity of $[U_L]_{ab}$. This argument, for a general $[U_L]_{ab}$, can be summarized in the form,

$$
|\langle \delta^l_{13}\rangle_{LL}|^2 + |\langle \delta^l_{23}\rangle_{LL}|^2 \approx |\langle \delta^d_{13}\rangle_{RR}|^2 + |\langle \delta^d_{23}\rangle_{RR}|^2 + \mathcal{O}(\cos^2 \beta \langle \delta^d_{RR}\rangle^2),
$$

(4.4)

which determines $B(\tau \to (e + \mu) \gamma)$. The mass insertions appearing above are all at the GUT scale. Note that the current experimental bounds on $B(\tau \to \mu \gamma)$ and $B(\tau \to e \gamma)$ differ only by a factor of 2.4. Therefore, once one combines these two, one can always give an upper bound on each of $(\delta^d_{23})_{RR}$ and $(\delta^d_{13})_{RR}$, almost independent of $U_L$. The error caused by non-vanishing $1\text{–}3$ or $2\text{–}3$ mixing in $U_L$, is diminished below 10% even for $\tan \beta$ as low as 3. If one wants to apply this conservative constraint to the case of Figs. 4, the radius of each thick $\tau \to \mu \gamma$ circle should be enlarged by a factor of 1.9. The thick $\tau \to e \gamma$ circles in Figs. 5 should be expanded by a factor of 1.2. Similarly, the future bounds can be modified: multiply each by $\sqrt{2}$. Even in this case, tau decays remain severe constraints on sfermion mixings.

The same prescription can be applied to the tau decay bound on an $LL$ mixing. Except that the amplitude is dominated by a neutralino loop, we can repeat the above line of reasoning with $L$ and $R$ exchanged. In this case, a possible additional term in (2.23) arising from $(\delta^d_{23})_{LL}$ discussed in Section 2.2, is negligible relative to an upper limit from $\tau \to \mu \gamma$ or $\tau \to e \gamma$ shown in Figs. 6 and 7. One can obtain a region permitted by $\tau \to (e + \mu) \gamma$ in Figs. 6, multiplying the radius of a thick $\tau \to \mu \gamma$ circle by 1.9. The expansion factor for Figs. 7 is 1.2. Again, each future bound should be multiplied by $\sqrt{2}$.

Unlike the tau decay modes, $\mu \to e \gamma$ is more involved, and the following method is applicable only to an $RR$ insertion. The dominant contribution comes from the triple insertion graph in Fig. 2. Including the diagram with opposite chirality structure, we find that the decay rate is proportional to

$$
d \equiv |\langle \delta^l_{13}\rangle_{RR}\langle \delta^d_{32}\rangle_{LL}|^2 + |\langle \delta^l_{13}\rangle_{LL}\langle \delta^d_{32}\rangle_{RR}|^2.
$$

(4.5)

The $\mu \to e \gamma$ data supplies an upper limit on this quantity. One can use (4.4) to show that

$$
d \gtrsim \min \{ |\langle \delta^l_{13}\rangle_{RR}|^2, |\langle \delta^d_{23}\rangle_{RR}|^2\} \cdot |\langle \delta^d_{13}\rangle_{RR}|^2 + |\langle \delta^d_{23}\rangle_{RR}|^2
$$

(4.6)

ignoring the term suppressed by $\cos^2 \beta$. In contrast to $\tau \to (e + \mu) \gamma$, $\mu \to e \gamma$ depends on the new pivotal factors, $(\delta^d_{13})_{RR}$ and $(\delta^d_{32})_{RR}$. Ignoring the non-renormalizable operators, we had their values equal to those of $(\delta^d_{13})_{LL}$ and $(\delta^d_{32})_{LL}$ in Table 1, respectively. As to how
\((\delta_{13}^{L})_{RR}\) and \((\delta_{23}^{R})_{RR}\) change after the non-renormalizable operators are turned on, there are three logical possibilities: (a) each value remains at the same order of magnitude; (b) either is very small and the other is not; (c) both are vanishingly small. In Case (a), one can use (4.6) in order to translate the upper limit on \(d\) to those on \((\delta_{13}^{L})_{RR}\) and \((\delta_{23}^{R})_{RR}\), nearly independent of \(U_{L}\). We have seen that both \((\delta_{13}^{L})_{RR}\) and \((\delta_{23}^{R})_{RR}\) are at least of the same order as \((\delta_{13}^{L})_{LL}\) from (2.24)—otherwise, they should belong to Case (b) or (c). Thus, the \(U_{L}\)-independent upper bound on each of \((\delta_{13}^{L})_{RR}\) and \((\delta_{23}^{R})_{RR}\), should be given by a \(\mu \rightarrow e\gamma\) ring in Figs. 4. That is, Figs. 4 are not modified even with this conservative interpretation, while the \(\mu \rightarrow e\gamma\) circles in Figs. 5 should be replaced by those in Figs. 4. In Case (b), the bound inevitably depends on \(U_{L}\). As above, consider the scenario where \((\delta_{23}^{R})_{RR}\) is non-vanishing while \((\delta_{13}^{L})_{RR}\) vanishes. In addition, suppose that \((\delta_{23}^{R})_{RR}\), for example, happens to be highly suppressed. Then, (4.3) and (4.5) lead to

\[
d \approx |(\delta_{13}^{L})_{RR}|^2 |(\delta_{23}^{R})_{RR}|^2 |U_{L}|_{22}^2.
\]

The branching ratio scales like \(|U_{L}|_{22}^2\). Therefore, a \(\mu \rightarrow e\gamma\) circle in Fig. 4 should be enlarged by the factor \(1/|U_{L}|_{22}|\). However, we have learned in Section 2.2 that Case (b) is not realized unless the mixing angle in \([U_{R}]_{ab}\) is fine-tuned. In Case (c), which requires a conspiracy of \(\lambda_{U}, \lambda_{D}, h_{1}, h_{2}, f_{1}, \) and \(f_{2}\) in (2.3), as well as the soft terms, \(\mu \rightarrow e\gamma\) does not serve as a constraint.

In the last part of Section 2.2, we discussed consequences of large neutrino Yukawa couplings assuming \(U_{L}\) to be an identity matrix. We considered two cases: one where neutrino Yukawa couplings are fixed, and the other where boundary condition at \(M_{s}\) is fixed at a universal set of values. Here, let us examine how those results change if we relax the condition on \(U_{L}\). For the first case, we include \(l_{ij}\) into (4.4) to obtain

\[
|\langle \delta_{13}^{L}\rangle_{RR}|^2 + |\langle \delta_{23}^{R}\rangle_{RR}|^2 \approx |\langle \delta_{13}^{L}\rangle_{LL} - l_{13}|^2 + |\langle \delta_{23}^{R}\rangle_{LL} - l_{23}|^2,
\]

where \((\delta_{13}^{L})_{RR}\) and \((\delta_{13}^{L})_{LL}\) are at \(M_{GUT}\) and \(M_{\text{SUSY}}\), respectively. Unless \(l_{ij}\) is small enough compared to the bound on \(|\langle \delta_{ij}^{L}\rangle_{LL} = \langle \delta_{ij}^{L} \rangle_{RR}^{*}|\) presented in the previous subsection, the limit on the left hand side is appreciably weakened. Note that a model with \(l_{ij}\) that large is likely to be ruled out by LFV data. The second case is more promising. One can extend (2.34) in the style of (4.4), to have

\[
|\langle \delta_{13}^{L}\rangle_{RR}|^2 + |\langle \delta_{23}^{R}\rangle_{RR}|^2 \approx \left(\frac{\alpha}{1 + \alpha}\right)^2 \times |\langle \delta_{13}^{L}\rangle_{LL}|^2 + |\langle \delta_{23}^{R}\rangle_{LL}|^2.
\]

Therefore, the upper bounds on \((\delta_{23}^{R})_{RR}\) and \((\delta_{13}^{L})_{RR}\) attained from (4.4), are further scaled down by \(\alpha/(1 + \alpha)\).

Recently, an alternative approach to settling down the uncertainties posed by the non-renormalizable operators has been reported [17]. Their work in progress makes use of dependence of the proton lifetime on the coefficients of the operators [26] in order to find a pattern among them. We would say that our strategy is more generic in the sense that it relies only on the condition that the non-renormalizable operators are Planck-suppressed, although it may not be as predictive as their anticipated outcome.
suppose Case (a) therein, i.e. we do not envisage a fine tuning among contributions to RR insertion using the method described in the previous subsection. As for $U$ constraint depends on $\tau$ of $U$ right one is for treatment. A variation may also be caused by choosing $2\sin^2\beta$ central value will coincide with the SM prediction. We make the same supposition about used the HFAG fit (which would be a very interesting outcome on its own $m_{\tau}$ second and third columns indicate the values of $m_{\tau}$, $M_{1/2}$, and $\tan\beta$, used in Figs. 4–7. Regarding an RR mixing, if there are two numbers separated by a dash, the left one is for $U_L = 1$ and the right one is for $U_L \neq 1$ obeying (2.12). If the two numbers are the same, it is written only once. We do the same for an LL mixing on which the alignment condition is given through $U_R$ instead of $U_L$. For a general $U_R$, we drop the $\mu \rightarrow e\gamma$ constraint as we do not have a systematic way to impose it.

4.3 Summary of bounds

The restrictions on down-type squark mixings at the GUT scale, graphically shown in Section 4.1, are condensed in a numerical form in Table 5. Each number is the maximum distance of a point from the origin on the corresponding figure that satisfies all the constraints considered in the present work. As for $\phi_{B_s}$, we use the DØ result, which is marked in gray (cyan) in Figs. 4 and 6. We would be left with no solution in many cases if we used the HFAG fit (which would be a very interesting outcome on its own [34, 35, 47, 48]). In order to estimate the power of $\phi_{B_s}$ measurement at LHCb, we suppose that its future central value will coincide with the SM prediction. We make the same supposition about $\sin 2\beta$.

Those upper bounds are subject to change of parameters or scheme of uncertainty treatment. A variation may also be caused by choosing $2 \times 10^{-9}$ instead of $10^{-8}$ as the reach of $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ searches at a super $B$ factory. In particular, the strength of a LFV constraint depends on $U_L$ and $U_R$. We take into account the $U_L$ dependence of a maximal RR insertion using the method described in the previous subsection. As for $\mu \rightarrow e\gamma$, we suppose Case (a) therein, i.e. we do not envisage a fine tuning among contributions to $\delta$. 

\begin{table}
\begin{tabular}{|c|c|c|c|}
\hline
Mixing & Fig. & Present & Future \\
\hline
$|\langle \delta_{23}^{d} \rangle_{RR}(M_{\text{GUT}})|$ & 4 & \\
(a) & $7.6 \times 10^{-2} - 1.4 \times 10^{-1}$ & $1.3 \times 10^{-2}$ \\
(b) & $3.8 \times 10^{-2} - 7.1 \times 10^{-2}$ & $8.1 \times 10^{-3}$ \\
(c) & $1.7 \times 10^{-1}$ & $4.0 \times 10^{-2}$ \\
(d) & $8.3 \times 10^{-2} - 1.5 \times 10^{-1}$ & $3.9 \times 10^{-2} - 4.3 \times 10^{-2}$ \\
\hline
$|\langle \delta_{13}^{d} \rangle_{RR}(M_{\text{GUT}})|$ & 5 & \\
(a) & $2.7 \times 10^{-2} - 1.4 \times 10^{-1}$ & $2.5 \times 10^{-3} - 1.2 \times 10^{-2}$ \\
(b) & $1.6 \times 10^{-2} - 7.0 \times 10^{-2}$ & $1.5 \times 10^{-3} - 7.3 \times 10^{-3}$ \\
(c) & $4.7 \times 10^{-2}$ & $1.1 \times 10^{-2}$ \\
(d) & $5.0 \times 10^{-2}$ & $1.1 \times 10^{-2}$ \\
\hline
$|\langle \delta_{23}^{d} \rangle_{LL}(M_{\text{GUT}})|$ & 6 & \\
(a) & $\mathcal{O}(1)$ & $\mathcal{O}(1)$ \\
(b) & $0.6 - \mathcal{O}(1)$ & $0.3 - 0.4$ \\
(c) & $0.7$ & $0.3$ \\
(d) & $0.5$ & $0.3$ \\
\hline
$|\langle \delta_{13}^{d} \rangle_{LL}(M_{\text{GUT}})|$ & 7 & \\
(a) & $0.6$ & $0.3$ \\
(b) & $0.6$ & $0.1 - 0.3$ \\
(c) & $0.1$ & $0.06$ \\
(d) & $0.1$ & $0.06$ \\
\hline
\end{tabular}
\caption{Upper limit on the size of each mass insertion of down-type squarks at the GUT scale. The second and third columns indicate the values of $m_{\tau}$, $M_{1/2}$, and $\tan\beta$, used in Figs. 4–7. Regarding an RR mixing, if there are two numbers separated by a dash, the left one is for $U_L = 1$ and the right one is for $U_L \neq 1$ obeying (2.12). If the two numbers are the same, it is written only once. We do the same for an LL mixing on which the alignment condition is given through $U_R$ instead of $U_L$. For a general $U_R$, we drop the $\mu \rightarrow e\gamma$ constraint as we do not have a systematic way to impose it.}
\end{table}
If a LFV restriction is important, relaxing the assumption of $U_L = 1$ increases the upper limit of the given insertion. Concerning the limit on an $LL$ insertion, we follow the same procedure to evaluate the dependence of a tau channel on $U_R$, while we keep $\mu \to e\gamma$ only for $U_R = 1$. Even if $U_R$ is unity, however, it turns out that the leptonic data does not cause a big additional reduction in the bounds set by the hadronic inputs, under the conditions considered in this work. A lepton sector constraint should be looser if we allow for a different $U_R$. Therefore, the quoted numbers are not greatly influenced by a change of $U_R$.

### 4.4 Possible alterations in observables

With the region of each mass insertion obtained in Section 4.1, we estimate a possible difference of an affected observable from its SM value. The result is summarized in Table 6. Four of them have already been displayed as contours on each figure indicated in the table. Note that what has been shown as contours is the value of the observable, not the deviation from the SM prediction. We use the same set of constraints as in Section 4.3.

Under the present conditions, there are still $CP$ asymmetries that might potentially have a discrepancy bigger than the precision attainable at a super $B$ factory. They are $S_{CP}^{K^*\gamma}$, $S_{CP}^{\rho\gamma}$, $A_{CP}^{b\to s\gamma}$, and $A_{CP}^{b\to (s+d)\gamma}$. They show larger possible alterations for higher $m_0$, while $A_{CP}^{b\to d\gamma}$ doesn’t follow this tendency. Being hadronic observables, their sensitivity to the GUT scale flavor violation is amplified for higher $m_0$, as was explained in Section 4.1, although they are more severely restricted by other quark sector processes for the same reason. As we did for Table 5, we take account of uncertainties due to a misalignment between quarks and leptons of the lighter two families. In this case, we obtain the values after the dash signs, which can be larger than the estimates for perfect alignment.

We repeat the same task with the prospective future inputs. One may expect the present deviations, provided that no constraint is seriously violated in a future experiment. With lower $m_0$, $S_{CP}^{K^*\gamma}$ and $S_{CP}^{\rho\gamma}$ will not show a signature detectable at a super $B$ factory, even if quark–lepton misalignment is allowed, while $A_{CP}^{b\to s\gamma}$ and $A_{CP}^{b\to (s+d)\gamma}$ might reveal a hint. With higher $m_0$, search for a supersymmetric effect in $S_{CP}^{K^*\gamma}$ becomes feasible as well.

In the case with the $(\delta_{13}^d)_{LL}$ mixing, its effect on $A_{CP}^{b\to (s+d)\gamma}$ is negligible so that the variation is at most about 0.5%, because the channel $B \to X_{s+d}\gamma$ is dominated by $B \to X_{s}\gamma$. We include $S_{CP}^{B_s \to K^*\gamma}$ in the table as well for reference.

Among the $CP$ asymmetries mentioned above, $S_{CP}^{K^*\gamma}$ and $S_{CP}^{\rho\gamma}$ are sensitive to $RR$ mixings of squarks, and thus are closely related to LFV. Recall that $RR$ mixings give rise to much higher LFV rates than $LL$, as we have seen in Section 4.1. This motives us to look into allowed ranges of those two $CP$ asymmetries as functions of LFV branching ratios.

First, we show the correlation between $S_{CP}^{K^*\gamma}$ and $B(\tau \to \mu\gamma)$ in Figs. 8, each of which results from the same set of mass insertions as the corresponding plot in Figs. 4. Every point on the figures satisfies the current $\Delta M_{B_s}$ and $B(B \to X_{s}\gamma)$ constraints. The upper limit on $\tau \to \mu\gamma$ from $\mu \to e\gamma$ has been deduced from the contours in Figs. 4. In Figs. 8 (a) and (b), what restricts $S_{CP}^{K^*\gamma}$ at present is $\tau \to \mu\gamma$, and in the future $\mu \to e\gamma$ at MEG should take over. In Figs. (c) and (d), $d_n$, in addition to $\tau \to \mu\gamma$, is playing an important
Table 6: Maximal departure of each observable from its SM value given the present and the future constraints. The second and third columns indicate the plot on which we calculate the observable. Of the two deviations separated by a dash in a cell, the left one is for $U_L = 1$ and the right one is for $U_L \neq 1$ obeying (2.12), for the first three $CP$ asymmetries. Those two types of deviations should be regarded as the same if only one is written. For the rest, the alignment condition is given through $U_R$ instead of $U_L$. For a general $U_R$, we drop the $\mu \to e\gamma$ constraint as we do not have a systematic way to impose it.

| Deviation Mixing | Fig. | Present | Future | Future precision |
|------------------|------|---------|--------|------------------|
| $|\Delta S_{CP}^{K^\ast\gamma}|$ | 4    | (a) 0.04–0.07 | 0.007 | 0.02 |
| $(\delta_{23})_{RR}$ |      | (b) 0.04–0.07 | 0.007 |      |
|                    |      | (c) 0.18 | 0.04 |      |
|                    |      | (d) 0.16–0.26 | 0.07–0.08 |      |
| $|\Delta S_{CP}^{\eta\gamma}|$ | 5    | (a) 0.06–0.30 | 0.006–0.03 | 0.10 |
| $(\delta_{13})_{RR}$ |      | (b) 0.06–0.28 | 0.006–0.03 |      |
|                    |      | (c) 0.21 | 0.05 |      |
|                    |      | (d) 0.39 | 0.09 |      |
| $|\Delta S_{CP}^{B_{\tau\to K^\ast\gamma}}|$ | 5    | (a) 0.06–0.28 | 0.006–0.03 |      |
| $(\delta_{13})_{RR}$ |      | (b) 0.06–0.28 | 0.006–0.03 |      |
|                    |      | (c) 0.17 | 0.03 |      |
|                    |      | (d) 0.32 | 0.05 |      |
| $|\Delta A_{CP}^{b\to s\gamma}|$ (%) | 6    | (a) 1.3 | 1.3 |      |
| $(\delta_{23})_{LL}$ |      | (b) 1.9–2.3 | 1.0–1.4 | 0.4 |
|                    |      | (c) 3.3 | 1.7 |      |
|                    |      | (d) 5.2 | 2.8 |      |
| $|\Delta A_{CP}^{b\to(s+d)\gamma}|$ (%) | 6    | (a) 1.3 | 1.3 |      |
| $(\delta_{23})_{LL}$ |      | (b) 1.8–2.2 | 0.9–1.3 | 0.6 |
|                    |      | (c) 3.2 | 1.6 |      |
|                    |      | (d) 5.1 | 2.7 |      |
| $|\Delta A_{CP}^{b\to d\gamma}|$ (%) | 7    | (a) 16 | 7 |      |
| $(\delta_{13})_{LL}$ |      | (b) 57 | 5–15 |      |
|                    |      | (c) 7 | 3 |      |
|                    |      | (d) 15 | 6 |      |
Figure 8: Correlation between $S_{CP}^{K^*\gamma}$ and $B(\tau \to \mu\gamma)$ obtained by varying $(\delta_{21}^d)^{RR}$, with $(\delta_{ij}^d)^{LL}$ generated from RG running between the reduced Planck scale and the GUT scale. A light gray (yellow) point is disfavored by neutron EDM, while a gray (orange) point is not, and a black (blue) point satisfies the future $\Delta M_{Bs}$ and $\phi_{Bs}$ constraints. The dashed horizontal line marks the 2 $\sigma$ range of $S_{CP}^{K^*\gamma}$, and its SM value is the solid horizontal line. The present and the future limits on $\tau \to \mu\gamma$ and $\mu \to e\gamma$ are indicated by the vertical lines.

in accordance to this change, while keeping the positions of the vertical lines for $\mu \to e\gamma$.

Second, let us move to the correlation between $S_{CP}^{\rho\gamma}$ and $B(\tau \to e\gamma)$, displayed in Figs. 9, which correspond to the parameter space considered in Figs. 5. We discard any point that is incompatible with the present data of $\Delta M_{B_d}$, $\sin 2\beta$, or $\cos 2\beta$. The upper limit on $\tau \to e\gamma$ from $\mu \to e\gamma$ has been inferred as we did in the preceding paragraph. For lower $m_0$ shown in Figs. 9 (a) and (b), $\mu \to e\gamma$ provides the limits on $S_{CP}^{\rho\gamma}$ both currently and in the future. The MEG bound is not visible on the plane since it restricts
$B(\tau \to e\gamma) \lesssim 7 \times 10^{-11}$. For higher $m_0$ in Figs. (c) and (d), possible range of $S_{CP}^{\rho\gamma}$ is determined by the other hadronic observables, with little help from the lepton sector. The way to convert these plots to those for $U_L \neq 1$ is almost the same as above: relabel the horizontal axis as $B(\tau \to (e + \mu)\gamma)$ instead of $B(\tau \to e\gamma)$, and change the upper bounds on $\tau \to e\gamma$ to those on $\tau \to (e + \mu)\gamma$. A difference from the above case is that one should also multiply the $\mu \to e\gamma$ limit on $B(\tau \to e\gamma)$ by $24 \sim \lambda^{-2}$.

The latest interest in the phase of $B_s - \bar{B}_s$ mixing leads us to examine its modification
Table 7: Maximal departure of $\phi_{B_s}$ from its SM value under the present and the future constraints except for those on itself. The second and third columns indicate the relevant plot. Of the two deviations separated by a dash in a cell, the left one is for $U_L = 1$ and the right one is for $U_L \neq 1$ obeying (2.12), for the RR mixing. Those two types of deviations should be regarded as the same if only one is written. In the case with $(\delta_{23}^d)_{LL}$, the alignment condition is given through $U_R$ instead of $U_L$. For a general $U_R$, we drop the $\mu \rightarrow e\gamma$ constraint as we do not have a systematic way to impose it.

| Deviation Mixing | Fig. | Present | Future |
|------------------|------|---------|--------|
| $|\Delta \phi_{B_s}|$ | (a) 0.05–0.08 | 0.01 |
| $(\delta_{23}^d)_{RR}$ | (b) 0.02–0.04 | 0.004 |
| | (c) 0.08 | 0.08 |
| | (d) 0.05 | 0.05 |
| $|\Delta \phi_{B_s}|$ | (a) 0.05 | 0.05 |
| $(\delta_{23}^d)_{LL}$ | (b) 0.02–0.03 | 0.004–0.008 |
| | (c) 0.57 | 0.33 |
| | (d) 0.32 | 0.12 |

that can be caused by new physics. We lift the constraint on $\phi_{B_s}$ while keeping the others used in Section 4.3, and record its variation allowed by the other bounds in Table 7. The difference between the announced central value and the SM prediction is about 0.7. From the table it appears that cases with lower $m_0$ and/or large RR mixing (but small LL mixing) are disfavored by $\phi_{B_s}$. In the case of RR insertion with higher $m_0$, the primary barrier is the neutron EDM as is evident from Figs. 4 (c) and (d). Let us remind the reader that this situation can be ameliorated by multiplying $(\delta_{23}^d)_{LL}$ by an $O(1)$ complex factor at $M_{GUT}$. With the LL insertion and higher $m_0$, on the other hand, Figs. 6 (c) and (d) show that $B \rightarrow X_s \gamma$ and $S_{CP}^{\phi K}$ exclude a major part of the region preferred by the HFAG fit, although there are still remaining parts that are responsible for the large difference in $\phi_{B_s}$ recorded in the table.

In Figs. 10, we investigate how LFV constrains $\phi_{B_s}$ by means of correlation plots, focusing on the RR insertion case considered in Figs. 4. At first, let us consider only the leptonic constraints. In this case, LFV and the latest $\phi_{B_s}$ fit are better reconciled for higher $m_0$ depicted in Figs. (c) and (d). Obviously, lower $\tan \beta$ is preferable since a LFV limit gets tighter for higher $\tan \beta$. However, if one takes the neutron EDM bound seriously, the light gray (yellow) points are discarded while the gray (orange) points remain, and therefore it becomes harder to account for $\phi_{B_s}$ with an RR mixing. Remember that one can apply this result to a popular benchmark scenario in which the soft terms at $M_*$ are flavor-blind and all the right-handed squark mixings are supposed to originate from large neutrino Yukawa couplings, as we discussed in the last part of Section 2.2. The recipe is to multiply each LFV branching fraction by $(1 + \alpha^2)/\alpha^2$ with $\alpha$ in (2.33). This factor arises from the additional running of slepton masses from $M_{GUT}$ down to $M_R$, and strengthens LFV as the result.
Figure 10: Correlation between $\phi_{B_s}$ and $B(\tau \to \mu \gamma)$ obtained by varying $(\delta_{23}^f)_{RR}$, with $(\delta_{ij}^f)_{LL}$ generated from RG running between the reduced Planck scale and the GUT scale. A light gray (yellow) point is disfavored by neutron EDM, while a gray (orange) point is not. The dashed horizontal line marks the 90% CL range of $\phi_{B_s}$, and its SM value is the solid horizontal line. The present and the future limits on $\tau \to \mu \gamma$ and $\mu \to e \gamma$ are indicated by the vertical lines.

One can be more optimistic in viewing the same correlation plots. For example, the neutron EDM constraint may be weakened if there is also a non-vanishing complex $LL$ mass insertion at $M_*$, or one might simply choose to ignore the constraint due to its hadronic uncertainties. Then, it might be that the present status of $\phi_{B_s}$ is hinting at a LFV process occurring at a rate that can be explored in the near future. Notice that this scenario works best when the value of $x$ defined in (2.26), is around 1/12, as we discussed in Section 4.1.
5. Conclusions

We imposed hadronic and leptonic constraints on sfermion mixing in a class of supersymmetric models with SU(5) grand unification. We did not particularly assume that the sfermion mass matrices have a universal form at any scale, but rather that any off-diagonal entry may be nonzero, which is generically the case in gravity mediated supersymmetry breaking. Those off-diagonal elements are encoded in the dimensionless mass insertion parameters in terms of which we express experimental bounds on flavor non-universality at the GUT scale. While fixing the gluino mass to 500 GeV at the weak scale, we tried two different boundary conditions on the diagonal components of the soft scalar mass matrix at $M_{\text{GUT}}$: lower $m_0 = 220$ GeV and higher $m_0 = 600$ GeV. We varied $\tan\beta$ from 5 to 10 as well. For lower $m_0$, we have found that the upper limit on an $RR$ mixing is essentially determined by a LFV decay mode both at present and in the near future. This is true even when one introduces non-renormalizable terms to accommodate the lighter down-type quark and charged lepton masses. In particular, the apparently unrelated mode $\mu \rightarrow e\gamma$ turns out to be remarkably sensitive to a mixing involving the third family. This sensitivity will be much higher with the progress of the MEG experiment. For higher $m_0$, the situation turns the other way around so that the hadronic constraints, such as $B$-meson mixing and neutron EDM, dominate. Also in the near future, measurements at the LHCb and a super $B$ factory, with the aid of improved lattice QCD, should be able to probe an $RR$ mixing, with a sensitivity higher than that of a LFV experiment. Concerning the $LL$ mixings, they are mostly restricted by hadronic data from $B$ physics, although LFV supplies additional information if $m_0$ is low and $\tan\beta$ is high. These findings unveil a nice complementarity of the quark and the lepton sector processes showing their strengths and weaknesses, depending on the gaugino to scalar mass ratio. We included discussions on the consequences of the discrepancy recently observed in the $B_s$-meson mixing phase.

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A. Notations

The scalar mass terms in the soft supersymmetry breaking sector of the minimal super-
symmetric standard model are given by
\[-\mathcal{L}_{\text{soft}} \supset Q^\dagger m_Q^2 Q + U^T m_U^2 U^* + \bar{E}^T m_E^2 E^* + \bar{D}^T m_D^2 D^* + L^\dagger m_L^2 L, \quad (A.1)\]
where the uppercase letters denote the scalar components of the SM superfields embedded in \(T\) and \(\bar{F}\) as in (2.5). Consider a basis where the down-type quark and the charged lepton Yukawa matrices are diagonalized by superfield rotations. The scalars in this basis, denoted by lowercase letters, are related the above fields by (2.13). Therefore, their mass matrices are connected to those above by the basis change,
\[m_q^2 = m_Q^2, \quad m_u^2 = U m_U^2 U^\dagger_Q, \quad m_e^2 = U_R m_E^2 U_R^\dagger, \quad m_d^2 = m_D^2, \quad m_l^2 = U_L m_L^2 U_L^\dagger. \quad (A.2)\]

Suppose that the squark and slepton mass terms are given by,
\[-\mathcal{L} \supset \tilde{d}^\dagger_{\!Ai} [m_{\tilde{d}AB}^2]_{ij} \tilde{d}_{BJ} + \tilde{e}^\dagger_{\!Ai} [m_{\tilde{e}AB}^2]_{ij} \tilde{e}_{BJ}, \quad (A.3)\]
in the basis where the down-type quark and the charged lepton mass matrices are diagonal. The sfermion mass matrices include contributions from the Yukawa couplings, the \(\mu\) term, the \(D\) terms, the soft scalar mass terms, and the \(A\) terms. In terms of the mass matrices, mass insertion parameters are defined by [30]
\[\begin{align*}
(\delta_{ij}^d)_{AB} &\equiv \frac{[m_{\tilde{d}AB}^2]_{ij}}{\bar{m}_d^2}, \quad (\delta_{ij}^l)_{AB} \equiv \frac{[m_{\tilde{e}AB}^2]_{ij}}{\bar{m}_e^2}, \quad (A, i) \neq (B, j), \\
(\delta_{ii}^d)_{AA} &\equiv (\delta_{ii}^l)_{AA} = 0,
\end{align*} \quad (A.4)\]
where \(A, B = L, R\) denote the chiralities, \(i, j = 1, 2, 3\) are the family indices, and \(\bar{m}_d^2\) and \(\bar{m}_e^2\) are the average sfermion masses [1]. In this work, we heavily rely on the mass insertion notation defined above to discuss the flavor structure of squarks and sleptons. Yet, we do not use mass insertion \textit{approximation} to compute physical amplitudes, but work with mass eigenstates and mixing matrices.

Normally, as its name implies, a mass insertion is a quantity that should be defined at the scale of the particle mass. Therefore, a squark or a slepton mass insertion is considered at the sparticle mass scale or at the weak scale. This is the case in the previous paragraph. In this work, we borrow this notation to deal with the scalar mass matrices at the GUT scale: a GUT scale mass insertion is an off-diagonal entry of a soft scalar mass matrix divided by the averaged diagonal element, in the basis where the Yukawa matrix is diagonal. Following this definition, we have
\[\begin{align*}
(\delta_{ij}^d)_{LL} &= \frac{[m_{\tilde{d}LL}^2]_{ij}}{\bar{m}_d^2}, \quad (\delta_{ij}^d)_{RR} = \frac{[m_{\tilde{d}RR}^2]_{ij}}{\bar{m}_d^2}, \\
(\delta_{ij}^l)_{LL} &= \frac{[m_{\tilde{e}LL}^2]_{ij}}{\bar{m}_e^2}, \quad (\delta_{ij}^l)_{RR} = \frac{[m_{\tilde{e}RR}^2]_{ij}}{\bar{m}_e^2},
\end{align*} \quad (A.5)\]
for \(i \neq j\).

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