Near Infrared Spectroscopy of M Dwarfs. II.
H$_2$O Molecule as an Abundance Indicator of Oxygen

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Abstract

Based on the near infrared spectra (R \approx 20000) of M dwarfs, oxygen abundances are determined from the ro-vibrational lines of H$_2$O. Although H$_2$O lines in M dwarfs are badly blended each other and the continuum levels are depressed appreciably by the collective effect of numerous H$_2$O lines themselves, quantitative analysis of H$_2$O lines has been carried out by referring to the pseudo-continua consistently defined by the same way on the observed and theoretical spectra. For this purpose, the pseudo-continuum on the theoretical spectrum has been evaluated accurately by the use of the recent high-precision H$_2$O line-list. Then, we propose a simple and flexible method of analyzing equivalent widths (EWs) of blended features (i.e., not necessarily limited to single lines) by the use of a mini curve-of-growth (CG), which is a small portion of the usual CG around the observed EW. The mini CG is generated by using the theoretical EWs evaluated from the synthetic spectrum by exactly the same way as the EWs are measured from the observed spectrum. The observed EW is converted to the abundance by the use of the mini CG, and the process is repeated for all the observed EWs line-by-line or blend-by-blend.

In cool M dwarfs, almost all the oxygen atoms left after CO formation are in stable H$_2$O molecules, which suffer little change for the uncertainties due to imperfect modelling of the photospheres. Moreover, the thermal velocity of H$_2$O is most probably larger than the micro-turbulent velocity because of its lower molecular weight, and the uncertainty of the micro-turbulent velocity will have relatively minor effect on the abundance determination. Then the numerous H$_2$O lines are excellent abundance indicators of oxygen. The oxygen abundances are determined to be log $A_O$ ($A_O = N_O/N_H$) between -3.5 and -3.0 in 38 M dwarfs, but cannot in four early M dwarfs in which H$_2$O lines are detected only marginally. The resulting log $A_O/A_C$ plotted against log $A_C$ appears to be systematically smaller in the carbon-rich M dwarfs, showing the different formation histories of oxygen and carbon in the chemical evolution of the Galactic disk. Also, $A_O/A_{Fe}$ ratios in most M dwarfs are closer to the solar $A_O/A_{Fe}$ ratio based on the classical high oxygen abundance rather than on the recently downward revised low value.

Key words: Molecular data – Stars : abundances – Stars : atmospheres – Stars : fundamental parameters – Stars : low mass

1. Introduction

Presence of water in stellar photospheres has been predicted based on the theory of thermochemistry (e.g. Russell 1934), and the observational confirmation has been done on a Mira variable star at the infancy of the infrared spectroscopy (Kuiper 1962). However, detailed observations of water vapor in celestial objects have been hampered by the obscuration of the water vapor in the Earth’s atmosphere. For this reason, the most clear demonstration of the water vapor in stellar atmospheres has been done by the balloon-born telescope known as Stratoscope II on several red giant and supergiant stars (Woof et al. 1964). This pioneering undertaking had been taken over after 30 years by the Infrared Space Observatory ISO (Kessler et al. 1996), which revealed that water exists everywhere in the Universe. Now, water (H$_2$O) may be assumed to be the third abundant molecule next to H and CO in the Universe.

Meanwhile, important progress on the high resolution infrared spectroscopy has been achieved already in the 1970’s, despite the rather noisy infrared detectors at that time. This has been made possible with
the Fourier Transform Spectroscopy (FTS) pioneered by Connes (1970) and further developed at Kitt Peak National Observatory (e.g., Ridgway & Brault 1984). Based on observation of cool M giant Mira variable stars with the FTS of KPNO 4m Telescope, a detailed study on the high resolution H$_2$O spectra on celestial object outside the solar system has been done by Hinkle & Barnes (1979) for the first time, with the use of the molecular data on H$_2$O known at that time. Their analysis based on the curve-of-growth method revealed complicated behaviors of H$_2$O spectra, and showed the presence of a very cool layer different from the pulsating photospheric layer. This first attempt at analyzing the spectra of H$_2$O in M-type Miras showed that the H$_2$O spectra contain a wealth of information on the coolest outer atmosphere where stellar photosphere and cool circumstellar layers interact.

Next detailed study on high resolution celestial H$_2$O spectra has been initiated with the detection of many H$_2$O lines on sunspot umbral spectra with the FTS of the National Solar Observatory at Kitt Peak (Wallace & Livingston 1992; Wallace et al. 1995), followed by detailed laboratory analyses of hot H$_2$O spectra (Polyansky et al. 1997; Zobov et al. 2000; Tereszchuk et al. 2002). Then detailed laboratory and theoretical works on hot water have been continued by many groups as reviewed by Bernath (2002). As the fruits, an extended line-list of hot water has been continued by many groups as reviewed by Bernath (2002).

On the other hand, H$_2$O molecule has been recognized as an important source of opacity in cool gaseous mixture such as the photospheres of M-type stars (e.g., Tsuji 1966; Auman 1967), and the H$_2$O opacity has been applied to initial attempts of constructing model photospheres of M dwarfs (e.g., Auman 1969; Tsuji 1969). The observational manifestations of water in M dwarfs have been done from low resolution spectra and progressive importance of H$_2$O as an infrared opacity source in cooler M dwarfs has been well established (e.g., Bertrman & Reid 1987; Tinney et al. 1993; Jones et al. 1994).

The high resolution spectroscopy has been more difficult for such faint objects as M dwarfs until recently. Even with the FTS, M dwarfs were too faint and we know only one pioneering attempt at analyzing the infrared high resolution spectra of M dwarfs by Mould (1978). However, recent progress in infrared spectroscopy with the new infrared detectors finally made it possible to observe infrared spectra of faint objects including M dwarfs at high resolution (e.g., Onenhag et al. 2012). We started to explore such a new possibility opened by the progress of infrared spectroscopy with the use of the echelle mode of InfraRed Camera and Spectrograph, IRCS (Kobayashi et al. 2000) at Subaru.

Although the fundamental parameters such as the effective temperatures needed for spectroscopic analysis were poorly known for M dwarfs until recently, recent progress of stellar interferometry has finally resolved this problem and accurate effective temperatures are now known based on the measured angular diameters. We found that the effective temperatures, $T_{\text{eff}}$, based on the interferometry (Boyajian et al. 2012 and references cited therein) and supplemented by those based on the infrared flux method for cooler M dwarfs (Tsuji et al. 1996), show a fairly tight correlation with the absolute magnitudes at 3.4 µm, $M_{3.4}$, based on the WISE data (Wright et al. 2010). Then we proposed a simple method to infer $T_{\text{eff}}$ from $M_{3.4}$ available to almost all the M dwarfs we have observed (Tsuji & Nakajima 2014; hereafter be referred to as Paper I).

Another problem in the quantitative analysis of the spectra of M dwarfs is that the continuum levels cannot be seen because of the veil opacity due to numerous molecular lines, especially of H$_2$O. For this difficulty, we showed that the spectroscopic analysis of M dwarfs can be done by referring to the pseudo-continuum levels both on the observed and theoretical spectra. This is possible since the pseudo-continuum level on the theoretical spectrum can be evaluated accurately thanks to the recently developed high precision H$_2$O line database (Paper I). It can be shown that the quantitative analysis of the spectrum referring to the pseudo-continuum is essentially the same as that referring to the true-continuum.

In this paper, we use the observed data introduced in Paper I (as for detail, see its Table 1) in which we analyzed the CO spectra. In this paper, we focus our attention to the H$_2$O spectra. We first summarize the basic physical parameters and model photospheres of M dwarfs we study in this paper (section 2). Then, we examine the H$_2$O spectra in the $K$ band region (section 3). We analyze a portion of the spectral region with the strong H$_2$O features in the midst of the 1.9 µm band (section 4) and another portion with relatively sparse H$_2$O blends in the tail of the H$_2$O 2.7 µm band (section 5). We will show that H$_2$O is an excellent abundance indicator of oxygen in M dwarfs (subsection 6.1) and oxygen abundances in 38 M dwarfs have been determined (subsection 6.2). We summarize the method of abundance analysis which is essentially very simple (subsection 6.3). We close our discussion with a prospect on the spectroscopy of H$_2$O in stellar spectra other than M dwarfs (subsection 6.4).

2. Fundamental Parameters and Model Photospheres of M Dwarfs

We summarize the basic parameters of our 42 program stars in Table 1. Following the object names in the first column, the spectral types given in the second column are from Joy & Abt (1974) for 29 M dwarfs (those beginning with dM in Table 1), from SIMBAD for the 12 cases (those beginning with M), and unknown for the remaining one object (GJ 768.1C). The values of $T_{\text{eff}}$ and log $g$ given in the third and fourth columns, respectively, are those used in our analysis of CO in Paper I. In dis-
cussing the values of $T_{\text{eff}}$, however, we used the WISE All-Sky Release in Paper I to obtain $M_{3.4}$, the absolute magnitude based on the WISE W1 flux, throughout. Later we notice that there is a new version referred to as the AllWISE Catalog. According to the AllWISE Explanatory Supplement\(^1\), the W1 and W2 photometric sensitivity is better in the AllWISE Catalog than in the WISE All-Sky data we have applied in Paper I. However, it is also noted that the sources brighter than $W1 < 8$ mag may have better photometry in the WISE All-Sky Catalog than in the AllWISE Catalog.

We have examined the AllWISE data and find that the results for some K and M dwarfs we have used as calibrators of $M_{3.4} - \log T_{\text{eff}}$ relation in Paper I were changed to unreasonable values that may suffer the saturation effect or even some objects disappeared from the database. As a result, the number of calibration sources using $T_{\text{eff}}$ values by interferometry (Boyajian et al. 2012) decreases from 27 to 15 and those using $T_{\text{eff}}$ values by the infrared flux method (Tsuji et al. 1996) from 9 to 7. But we add three new M dwarfs for which angular diameters are measured anew (von Braun et al. 2014) and thus the number of calibration stars is $15 + 7 + 3 = 25$ as shown in Tables 10 & 11 (Appendix I). These data of Tables 10 & 11 are plotted in Fig. 18 (Appendix I), on which the mean $M_{3.4} - \log T_{\text{eff}}$ relation from Fig. 1 in Paper I is reproduced. Despite some changes in the WISE database, the AllWISE data are consistent with the mean $M_{3.4} - \log T_{\text{eff}}$ relation based on the WISE All-Sky Catalog. We reanalyze a few objects with $F_{3.4} > 8$ mag in Table 5 of Paper I with the data of the AllWISE, but the resulting $T_{\text{eff}}$ values agree within 10 K with the results based on the WISE All-Sky data. Since most of M dwarfs in Table 5 of Paper I are brighter than $F_{3.4} = 8$ mag, we decide to use the results of Table 5 in Paper I to all the objects\(^2\).

We also apply the same model photospheres used in the analysis of CO in Paper I, and these models are given in the fifth column of Table 1. The model photospheres are designated by cloud type/abundance case/$T_{\text{eff}}$/log $g$ as in our model database referred to as the unified cloudy model (UCM)\(^3\). The cloud type is defined to characterize the thickness of the dust cloud formed in the photospheres of cool dwarfs. The dust cloud forms at the condensation temperature $T_{\text{cond}}$ and dissolves at a critical temperature $T_{\text{cr}}$ because dust particles become too large and precipitate. Thus dust cloud exists in the region of $T_{\text{cr}} \lesssim T \lesssim T_{\text{cond}}$, where $T_{\text{cond}}$ is fixed thermochromically. Then, thickness of the cloud depends on $T_{\text{cr}}$ which is a free parameter in our UCMs and used to define the cloud type (as for details see Tsuji 2002; 2005). The cloud type includes a limiting case of no dust cloud (referred to as a clear case, C). Generally, dust clouds are formed in the photospheres of cool dwarfs with $T_{\text{eff}}$ below about 2600 K, and hence all our present sample are dust-free. Also, case a abundance is based on the classical solar abundance with log $A_{\text{C}} = -3.40$\(^4\) and log $A_{\text{O}} = -3.08$ (see Table 1 in Tsuji 2002), and case c on the downward revised solar C & O abundances of log $A_{\text{C}} = -3.61$ and log $A_{\text{O}} = -3.31$ (Allende Prieto et al. 2002). We used the UCM grid in our preliminary analysis, but generated specified model for $T_{\text{eff}}$ and log $g$ of each object in the subsequent analysis. These models are referred to, for example, as Ca3570c489 implying a clear model without dust cloud, with the case a abundance, $T_{\text{eff}} = 3570$ K, and log $g = 4.89$.

Table 1: Fundamental parameters and model photospheres (p.27).

3. H$_2$O in the Spectra of M Dwarfs

3.1. Molecular Data of H$_2$O

We first try to identify H$_2$O transitions by referring to the laboratory data by Zobov et al. (2008), but we soon notice that the BT2-HITEMP2010 database (Barber et al. 2006; Rothman et al. 2010), which we have already examined in Paper I, provides fairly accurate line positions of H$_2$O lines. It has generally been thought that the ab initio approach to the molecular structure and spectroscopic data provides large line-lists but their accuracy especially on the line positions cannot be very high. However, the recently computed line-list by Barber et al. (2006) attained an accuracy to be used for identifications of H$_2$O lines at last. Since intensity data are also required for abundance analysis, we decide to use the BT2-HITEMP2010 data in the following analysis. The accurate laboratory data should certainly be useful if we are to study radial velocities, for example.

Under the high density of the photospheres of M dwarfs, pressure broadening plays an important role. Usually, collision half-width $\gamma$ is represented by

$$\gamma = \gamma_0 \frac{p}{p_0} \left( \frac{T_0}{T} \right)^n,$$

where $\gamma_0$ is the collision half-width measured at a reference temperature $T_0$ (e.g. 296 K) and gas pressure $p_0$. As to pressure broadening coefficients for H$_2$O perturbed by H$_2$ and He relevant to stellar photospheres, there are some experimental (e.g., Steyert et al. 2004, Faure et al. 2013) and theoretical (e.g., Gamache et al. 1996) works for pure rotational transitions. We have compared the resulting collision half-widths by the H$_2$ and He broadening with those by the air broadening for the case of CO and found that they are not drastically different in Table 6 of Paper I. The case of H$_2$O is found to be more or less the same, and we assume a median value of $\gamma_0 = 0.08$ cm$^{-1}$ atm$^{-1}$

\(^1\) http://wise2.ipac.caltech.edu/docs/release/allwise/expsup.

\(^2\) This also applies to the values of $T_{\text{eff}}$ of three objects for which angular diameters have been measured (von Braun et al. 2014) after Paper I was completed, for consistency of our analysis with that of Paper I. Also, we wonder why $T_{\text{eff}}$ of GJ876 by the new interferometric measurement deviates so large from the mean $M_{3.4} - \log T_{\text{eff}}$ relation, as shown in Fig. 18 (Appendix I; also see Table 14 in Paper I), as the case of GJ 725B (see Fig. 1 in Paper I).

\(^3\) http://www.mtk.iaa.s.u-tokyo.ac.jp/~tsuji/export/ucm2.

\(^4\) We use the notation: $A_{\text{El}} = N_{\text{El}}/N_{\text{H}}$, where $N_{\text{El}}$ and $N_{\text{H}}$ are the number densities of the element El and hydrogen, respectively.
by the air broadening (e.g., Rothman et al. 2010) for all the H$_2$O lines in M dwarfs.

### 3.2. H$_2$O Spectra in the K Band Region

Water molecule is a tri-atomic asymmetric top molecule having three normal vibration modes with $\nu_1 = 3651.7$ cm$^{-1}$, $\nu_2 = 1595.0$ cm$^{-1}$, and $\nu_3 = 3755.8$ cm$^{-1}$ (Herzberg 1945). In the K band region, the H$_2$O 2.7 $\mu$m band is mainly composed of the $\nu_1$ and $\nu_3$ fundamentals and 2$\nu_3$ overtone, while the H$_2$O 1.9 $\mu$m band mainly of the combination bands $\nu_2 + \nu_3$ and $\nu_1 + \nu_2$. The model spectra of H$_2$O (with resolution of $R \approx 2000$) based on the BT2-HITEMP2010 line-list for four M dwarf model photospheres of $T_{\text{eff}} = 2800, 3200, 3600, \text{and} 4000$ K, assuming the solar metallicity ($\text{case } a$), are shown in Fig. 1. The 1.9 $\mu$m band is still very weak in the model of $T_{\text{eff}} = 4000$ K and strengthens towards cooler models. The 2.7 $\mu$m band is already visible weakly in the model of $T_{\text{eff}} = 4000$ K and appears to be very strong in cooler models.

We choose two portions noted as region A (20296–20391 Å) and region B (22515–22935 Å) in Fig. 1 for our analysis of H$_2$O spectra. At the top of Fig. 1, the atmospheric window (the K band window) in which atmospheric transmission is larger than 50% under conditions appropriate for Mauna Kea, Hawaii, is indicated based on the data given by Cox (1999). Both the regions A and B are in the atmospheric window. The region A is in the midst of the 1.9 $\mu$m band where H$_2$O lines are strong. We select this particular region because the atmospheric absorption appears to be relatively weak by visual inspection of the observed spectra. The region B is disturbed by both the 1.9 and 2.7 $\mu$m bands least, but nevertheless some H$_2$O features are found in this region in later M dwarfs. Since H$_2$O lines in the region A are very strong in later M dwarfs and the pseudo-continua are depressed by as much as 15% from the true continuum, we think it’s useful to analyze the H$_2$O lines without such heavy blendings for comparison. The H$_2$O blends are sparsely distributed in the region B and we survey a larger spectral range to measure a modest number of H$_2$O blends.

By the way, we have confined our analysis of CO lines to the bandhead region of the CO 2-0 band, and it is confirmed in Fig. 1 that this region adjacent to the region B is disturbed by both the H$_2$O 1.9 and 2.7 $\mu$m bands least and that it may be more difficult to analyze CO spectra in other regions, because of the severe blending of H$_2$O lines.

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**Fig. 1:** Model spectra of M dwarfs in the K band region (p.14).

5 It is of course not possible to see the true-continuum level in the observed spectrum of M dwarf, but it can be estimated with the help of the theoretical spectrum, as will be shown in subsection 4.3.

### 3.3. H$_2$O Spectra in the Region A

As examples of the observed spectra in the region A near 2.0 $\mu$m, the cases of four M dwarfs are shown in Fig. 2. We select candidates of H$_2$O blends to be measured for abundance analysis and indicate them by arrows with reference numbers A01–A17. The total number of H$_2$O lines in the region A (20296–20391 Å) from BT2-HITEMP2010 line-list with the cut off at the integrated intensity $S(T = 3000$ K) $\approx 3.10^{-27}$ cm$^{-1}$ molecule$^{-1}$ is 69909 lines or about 736 lines per 1 Å interval, and we only show the strong line(s) near the position indicated by each arrow in Table 2. Following the reference number and the observed wavelength (in vacuum) in the columns 1 and 2, respectively, the wavelength, assignments of the upper and lower vibrational levels and of the upper and lower rotational levels, log $gf$ value, and the lower excitation potential (in cm$^{-1}$) from the BT2-HITEMP2010 are given successively after the column 3. The vibrational level is defined by the three quantum numbers $v_1 v_2 v_3$ representing the vibrational states of the three fundamental vibrations. The rotational level is defined by the three parameters $J, K_a, \text{and } K_c$: $J$ is the rotational quantum number of the total angular momentum, $K_a$ and $K_c$ are the quasi-quantum numbers that in the limiting cases of the prolate and oblate symmetric rotors would become the quantum numbers of the angular momenta associated with the respective figure axes (King et al. 1943).

In the early M dwarf GJ 338A shown in Fig. 2a, H$_2$O lines are still weak as can be inferred from Fig. 1, but almost all the features shown by the arrows are definitely confirmed as due to H$_2$O lines in this dM0.5 dwarf. However, in other early type M dwarfs, GJ 380 (dM0.5), GJ 820B (dM0), GJ 884 (dM0.5), and HIP 12961 (M0), these H$_2$O features are not necessarily very clear and cannot definitely be identified, even though some of the H$_2$O features can be recognized. Except for these four M dwarfs, the features indicated by the arrows are definitely confirmed as due to H$_2$O lines in 38 M dwarfs out of our 42 M dwarfs. For examples, H$_2$O features are well developed in the dM2.5 dwarf GJ 15A shown in Fig. 2b, and increasingly stronger in later M dwarfs as in the dM4 dwarf GJ 611B shown in Fig. 2c. In the coolest M dwarf GJ 406 in our sample, H$_2$O features are very strong as shown in Fig. 2d. The spectrum of the B type dwarf Regulus (α Leo, B7V) shown in Fig. 2e reveals the telluric lines mostly due to water vapor. Even though the region A is in the midst of the H$_2$O 1.9 $\mu$m band, the telluric H$_2$O lines are not so strong in this region A within the atmospheric K window, and have been removed in the observed spectra of M dwarfs rather well during the data reduction with Regulus as calibrator.

Finally, we measure the equivalent widths (EWS) of the H$_2$O blends by referring to the pseudo-continuum of each $\mu$m...
object. We select those features whose profiles are well
defined both on their left and right wings and, as a result,
not all the blends marked by arrows in Fig. 2 are mea-
sured. The resulting values of $\log (W/\lambda)_{\text{obs}}$ (where $W$ is
equivalent width) for the H$_2$O blends with the reference
numbers indicated are given in Table 3.

Fig. 2: H$_2$O in the region A of the observed spectra
(p.15).

Table 2: H$_2$O blends in the region A (p.28).

Table 3: $\log (W/\lambda)_{\text{obs}}$ of H$_2$O blends in the region A
(for online version only) (p.29).

3.4. H$_2$O Spectra in the Region B

As an example of the observed spectrum in the region B
near 2.3 $\mu$m, the case of the coolest M dwarf in our sam-
ples, GJ 406 (dM6.5), is shown in Fig. 3. In this region,
we may not expect to find many stellar H$_2$O lines from
Fig. 1. However, we already know that there are many
weak H$_2$O features in the region of CO 2-0 band just ad-
justed to the region B (Paper I) and we find that there
are considerable number of weak features possibly due to
H$_2$O blends in our higher resolution spectra ($R \approx 20000$
compared to $R \approx 2000$ of Fig. 1) of later M dwars. We
mark these features again by arrows in Fig. 3 with refer-
ence numbers B01–B27. The features marked by ar-
rows are in fact mostly identified as due to stellar H$_2$O
lines by referring to the BT2-HITEMP2010 line-list. The
transition(s) that give the large contribution(s) to each
feature are given in Table 4, in the same format as in
Table 2. The total number of H$_2$O lines in the region B
(22515 – 22935 Å) from BT2-HITEMP2010 line-list with
the cut-off at $S(T=3000K) \approx 3.1 \times 10^{-27}$ cm$^{-1}$ molecule$^{-1}$ is
317621 lines or about 756 lines per 1 Å interval. Thus it
is not possible to measure the isolated single line of H$_2$O
even in this region where blendings are expected to be
not so heavy compared to the region A$^7$, and we had to
analyze the blended H$_2$O features throughout this paper.

In the spectrum of the region B shown in Fig. 3, some
strong features are known as due to atomic lines of Ca I
in the sunspot umbral spectrum (Wallace & Livingston
1992), and these features are also noted on Fig. 3. The
spectrum of Regulus is also shown by the solid (green or
grey) line in Fig. 3. There are some strong telluric lines
in the region B, and they are due to telluric CH$_4$ and N$_2$O
rather than H$_2$O (Mohler 1955). The effect of these tel-
 luric absorption has again been removed by Regulus as

\footnote{But the line density in the region B is nearly the same or even
larger compared to that in the region A. This is because there are
many weak lines even in the region where strong H$_2$O lines are
scarcely as in the region B.

als remain occasionally, while inspection of the corrected
spectra in the regions A and B does not show such resid-
uals.

We also measure the equivalent widths (EWs) of the H$_2$O blends by referring to the pseudo-continuum of each
object. We can measure the weak H$_2$O blends in the re-

gion B in 20 out of 42 M dwars in our sample. These M
dwarfs are mostly later than dM3.5 except for the dM2.5
dwarfs pair GJ 797B or are M dwars with $T_{\text{eff}}$ lower than
3500 K. The resulting values of $\log (W/\lambda)_{\text{obs}}$ for the H$_2$O
blends with the reference numbers indicated are given in
Table 5.

Fig. 3: H$_2$O in the region B of the observed spectrum
of GJ 406 (p.16).

Table 4: H$_2$O blends in the region B (p.30).

Table 5: $\log (W/\lambda)_{\text{obs}}$ of H$_2$O blends in the region B (for
online version only) (p.31).

4. Analysis of H$_2$O Lines in the Region A

4.1. Mini Curves-of-Growth Analysis of H$_2$O Blends

Of the four parameters that characterize stellar spec-
tra, the effective temperature and gravity are known (see
Table 1 based on Tables 3, 4, & 5 of Paper I), and
we assume the micro-turbulent velocity to be $\xi_{\text{micro}}$
= 1 km sec$^{-1}$ as in Paper I. Then, the only unknown pa-
parameter to be determined from the spectra is abundance.

Given that the carbon abundances are already known in
Paper I (reproduced in Table 6), rough estimation of the
oxygen abundances can be possible. For example, we use
a starting oxygen abundance (logarithmic) to be

$$ \log A_O^0 = \log A_C + 0.30, $$

(2)

where $\log A_C$ is the logarithmic carbon abundance from
Paper I. Note that this relationship is taken from the solar
case (Allende Prieto et al. 2002).

Then, we apply a simple method using mini curve-of-
growth blend-by-blend: We generate synthetic spectra for
$\log A_O^0$ and $\log A_O^0 + \delta$ with the carbon abundance and the
specific model photosphere for each object from Paper I
(also noted in Table 1). We generally assume $\delta = \pm 0.1$ at
the beginning. For the H$_2$O blends for which EWs are
measured (Table 3), we evaluate EWs, $W(\delta)$ ($\delta = 0, \pm 0.1$),
from the three synthetic spectra we have just prepared,
by exactly the same way as we measure the EWs from the
observed spectrum. Especially, we refer to the pseudo-
continua generally defined consistently both on the ob-
served (see Fig. 2) and predicted spectra.

Now, we have a theoretical mini curve-of-growth defined
by $\log (W(\delta)/\lambda)$ vs. $\delta = -0.1, 0.0, \text{ and } +0.1$, where
the values of $W(\delta)$ are evaluated from the synthetic spec-
tra as noted above. The value of the abundance correction
$\Delta \log A_O$ to be consistent with the observed value of
$\log (W(\delta)/\lambda)_{\text{obs}}$ from Table 3 can be determined by the
use of this mini curve-of-growth. This process is repeated
blend-by-blend for all the H$_2$O blends measured. If the
resulting values of $\Delta \log A_O$ for many blends appear to be less than -0.1, for example, we generate a synthetic spectrum for $\delta = -0.2$ and reanalyze the same data by applying mini curves-of-growth defined by $\log (W/\lambda)_{\text{obs}}$ vs. $\delta = -0.2, -0.1, \text{and } 0.0^\circ$. Finally, we derive the mean abundance correction from the values of $\Delta \log A_O$ by all the measured blends.

As an example of the above noted procedure, theoretical mini curves-of-growth for seven selected H$_2$O blends on the spectrum of GJ 229 are shown by the solid lines in Fig. 4: The reference number of each blend (see Table 3) is shown for the corresponding mini CG, on which the filled circle indicates the values of the observed log ($W/\lambda$)_{obs} (read on the ordinate of Fig. 4) and the derived abundance correction $\Delta \log A_O$ (read on the abscissa). The resulting seven values of $\Delta \log A_O$ and their mean value are shown by the filled circles and dashed line, respectively, in Fig. 5d.

Some other examples of the resulting $\Delta \log A_O$ values plotted against the observed values of $\log (W/\lambda)_{\text{obs}}$ are shown in Fig. 5 for relatively early M dwarfs, in the order of the decreasing effective temperatures. The number of H$_2$O blends measured is rather small in the early M dwarfs such as GJ 338B (Fig. 5a) and GJ 205 (Fig. 5b), and this is because H$_2$O lines are still weak and difficult to measure (see Fig. 2). For this reason, the scatter of the data is rather large and accuracy of the results cannot be very high. The number of H$_2$O blends measured increases for M dwarfs with lower $T_{\text{eff}}$ values, and H$_2$O blends of modest strength in these M dwarfs can be measured fairly accurately. Especially, most of the 17 selected blends in Fig. 2 can be used in M dwarfs such as GJ 411 (Fig. 5i) and GJ 436 (Fig. 5j), and reasonably accurate results can be obtained for these cases.

More or less similar results are obtained in later M dwarfs shown in Fig. 6. However, in the coolest end including GJ 777B (Fig. 6h) and GJ 406 (Fig. 6j), the numbers of blends measured decrease. This should be because the blending is so severe that the line profiles of the H$_2$O blends cannot be defined well and the number of H$_2$O blends measured has decreased. Nevertheless, it appears that the mini curves-of-growth analysis on the selected blends works reasonably well even for later M dwarfs in which the continua are depressed by as much as 15 % (subsection 4.3). In Figs. 5 & 6, a remarkable feature is that the abundance corrections $\Delta \log A_O$ plotted against the observed values of $\log (W/\lambda)_{\text{obs}}$ show little systematic effect and distributed nearly horizontally. As a result, the mean abundance correction can be well determined. We will discuss on this result further in subsection 6.1.

The logarithmic abundance correction $\Delta \log A_O^{A}$, the resulting logarithmic oxygen abundance

$$\log A_O^{A} = \log A_O^{i} + \Delta \log A_O^{A},$$

and the number of H$_2$O blends used for each M dwarf, $N_A$, are given in the third, fourth, and fifth columns, respectively in Table 6, following the object’s name and the logarithmic carbon abundance log $A_C$ from Paper I in the first and second columns, respectively.

Fig. 4: Examples of mini curves-of-growth for H$_2$O blends in GJ 229 (p.17).

Fig. 5: Derived values of the abundance correction $\Delta \log A_O^{A}$ from H$_2$O blends in the region A (early M) (p.18).

Fig. 6: Derived values of the abundance correction $\Delta \log A_O^{A}$ from H$_2$O blends in the region A (late M) (p.18).

Table 6: Mean values of the abundance correction $\Delta \log A_O^{A}$ and the resulting log $A_O^{A}$ from H$_2$O blends in the region A (p.32).

4.2. Synthetic Spectra

We compare the observed spectra with the predicted ones based on the oxygen abundances log $A_O^{A}$ obtained from the H$_2$O blends in the region A for all the M dwarfs in Table 6, and some results are shown in Fig. 7. In the early M dwarf GJ 338B, the observed H$_2$O blends (filled circles) are rather weak but most of them can be reproduced well by the predicted spectra (solid line). For this reason, we are sure that we have certainly identified H$_2$O features in this early M dwarf. In other early M dwarfs including GJ 380, GJ 820B, GJ 884, and HIP 12961, we can recognize a few strong blends as due to H$_2$O, but other weak features cannot be explained well as due to H$_2$O lines, and we decide to exclude such cases from our analysis. In other 38 M dwarfs, the observed H$_2$O blends are reproduced by the predicted spectra fairly well in general, even if the matchings of the observed and predicted spectra are not perfect. Since there may be some unknown blends other than H$_2$O and some noise due to imperfect cancellations of the atmospheric lines, for example, the perfect matching may anyhow be difficult.

The candidates of H$_2$O blends used for analysis are reproduced from Fig. 2 and shown by the arrows in Fig. 7c, but not all of these candidates are used in the actual analysis. For example, in the worst case GJ 377B, only five blends ($\log (W/\lambda)_{\text{obs}}$ given in Table 3) are used in the analysis as in Fig. 6h. In this case, the pseudo-continuum can be defined (Fig. 7f) but many H$_2$O features are so strong and blendings are quite severe. For this reason, the profiles cannot reach the pseudo-continuum level and thus are not well defined to meet our criterion to accept the blends for analysis. This result implies that it is difficult to analyze the spectrum composed of the blends of many strong lines even if the pseudo-continuum can be defined. For this reason, we will analyze H$_2$O blends in the region B where the blendings are not so severe (section 5).

We also evaluate $\chi^2$ values by

$$\chi^2 = \frac{1}{N - 1} \sum_{i=1}^{N} \left( \frac{f_{\text{obs}}^i - f_{\text{cal}}^i}{\sigma_i} \right)^2,$$

where $f_{\text{obs}}^i$ and $f_{\text{cal}}^i$ are the observed and predicted spectra, both normalized by their pseudo-continua. $N$ is the
number of data points and $\sigma_i$ is the noise level obtained from the $S/N$ ratio (Table 1 of Paper I) assumed to be independent of $i$. We evaluate $\chi^2$ values for the predicted spectra based on $\log A_O^A$ and $\log A_O^A \pm 0.05$ where $\log A_O^A$ is the oxygen abundances obtained above (fourth column of Table 6) and 0.05 is about the probable error of the resulting $\log A_O^A$. The resulting values of $\chi^2$ for $\log A_O = \log A_O^A - 0.05, +0.0$, and $+0.05$ are shown as $\chi^2_1$, $\chi^2_0$, and $\chi^2_2$ in the sixth, seventh, and eighth columns, respectively, in Table 6. We confirmed that the $\chi^2$ value for $\log A_O = \log A_O^A$ is smaller than those for $\log A_O$ values deviating by about the probable error from $\log A_O^A$ for each object, except for a few cases. This result implies that our oxygen abundance $\log A_O^A$ is consistent with the spectral synthesis analysis in most cases.

We now examine the case that the $\chi^2$ value suggests a possible poor fitting. In the worst case GJ 406, $\chi^2$ value is quite large and larger for our resulting oxygen abundance $\log A_O^A$ than for reduced oxygen abundance (Table 6). We show the observed and predicted spectra for this M dwarf for a) $\log A_O = \log A_O^A - 0.05$ $(\chi^2 = 13.204)$ and b) $\log A_O = \log A_O^A + 0.05$ $(\chi^2 = 10.510)$ in Fig. 8a and 8b, respectively. Also shown are the H$_2$O blends actually used for the analysis of this M dwarf by the arrows (see Table 3). Inspection of Fig. 8 reveals that the fittings of the H$_2$O blends used for the analysis (those indicated by the arrows) are in fact better in Fig. 8a with larger $\chi^2$ value than in Fig. 8b with smaller $\chi^2$ value. The reason why $\chi^2$ value is smaller in Fig. 8b should be due to better fittings in the features not used in our analysis. To confirm this, we exclude the spectral region not used for our analysis by masking the region between 20354.5 and 20385.2˚A in the features not used in our analysis. To confirm this, we now convinced that the result of the mini CG analysis is consistent with the $\chi^2$ analysis.

![Fig. 7: Observed and predicted spectra of H$_2$O in the region A of six M dwarfs (p.19).](image)

![Fig. 8: Observed and predicted spectra of H$_2$O in the region A of GJ 406 (p.20).](image)

### 4.3. The True- and Pseudo-Continua

The analyses so far are based on the spectra normalized by the pseudo-continua. This is because the true-continuum cannot be seen on the observed spectrum. In the predicted spectrum, the true-continuum is also not shown explicitly, but it can easily be evaluated by considering the continuous opacity alone in computation. The true-continuum levels obtained in this way and the predicted spectra normalized by the true-continua for GJ 338A, GJ 15A, GJ 611B, and GJ 406 are shown in Fig. 9 by the dotted and solid lines, respectively. The pseudo-continuum levels are also shown by the dashed lines on the predicted spectra. The ratios of pseudo- to true-continua, $F_{pc}/F_{tc}$, are estimated to be 0.986, 0.950, 0.841, and 0.856 for the model spectra of GJ 338A, GJ 15A, GJ 611B, and GJ 406, respectively.

Although the true-continuum level cannot be seen in the observed spectrum, it can be estimated from the pseudo-continuum level in the observed spectrum by assuming the same $F_{pc}/F_{tc}$ ratio for the observed spectrum as for the predicted spectrum. This is possible since the pseudo-continua for the observed spectra are defined well as in the predicted spectra in all the M dwarfs of Fig. 9. The observed spectra of GJ 338A, GJ 15A, GJ 611B, and GJ 406 normalized by the true-continua estimated in this way are shown by the filled circles connected by the dotted lines in Fig. 9. From this result, we now know that the pseudo-continua of the observed spectra in GJ 338, GJ 15A, GJ 611B, and GJ 406 are depressed by 1.4, 5.0, 15.9, and 14.4%, respectively, from the true-continua.

![Fig. 9: True- and pseudo-continua in four representative M dwarfs (p.21).](image)

### 5. Analysis of H$_2$O Lines in the Region B

#### 5.1. Mini Curves-of-Growth Analysis of H$_2$O Blends

In the region B, H$_2$O lines can be measured in 20 M dwarfs as mentioned in subsection 3.4, and we carry out the mini curve-of-growth analysis blend-by-blend as in subsection 4.1. Some examples of the resulting $\Delta \log A_O$ values plotted against the observed values of $\log (W/\lambda)_{obs}$ are shown in Fig. 10 for the same dwarfs in Fig. 6 for comparison. In general, considerable number of blends can be measured in dM3.5 - dM4 dwarfs which are the major constituents of our sample. The number of blends measured is rather small in the dM2.5 dwarf GJ 797B-NE and more difficult in M dwarfs with $T_{eff} \gtrsim 3500$ K. Actually, we measure several possible H$_2$O blends in the M2 dwarf GJ 250B ($T_{eff} = 3567$ K), but other predicted H$_2$O blends cannot be definitively identified and we decide not to include such a case in our analysis. On the other hand, even in M dwarfs with $T_{eff} \lesssim 3500$ K, it is not always possible to find sufficient number of H$_2$O lines to be analyzed. For example, we cannot obtain oxygen abundances in GJ 411 ($T_{eff} = 3465$ K) and GJ 436 ($T_{eff} = 3416$ K) from H$_2$O lines in the region B. We investigate the reason for this and notice that the oxygen abundances in these stars are about $\log A_O \approx -3.4$ (Table 6), which are near the lower limit in our M dwarfs (Fig. 17). For this reason, H$_2$O lines are rather weak in these objects. Thus, the detection of weak H$_2$O lines depends not only on temperature but also on oxygen abundance.

Even in the cooler dwarfs, the blendings are not so severe and the pseudo-continua are well defined. In fact, the depressions of the continuum levels estimated by the way outlined in subsection 4.3 are 3.7% in the dM2.5 dwarf GJ 797B-NE and 4.8% even for our coolest sample GJ 406. For this reason, measurements of the EWs are easier in the region B than in the region A, and a large number of lines can be measured for such cooler dwarfs including GJ 611B, GJ 406, respectively.
Fig. 10: Derived values of the abundance correction \( \Delta \log A^B_O \) from \( H_2O \) blends in the region B (for the same objects as in Fig. 6) (p.22).

Table 7: Mean values of the abundance correction \( \Delta \log A^B_O \) and the resulting \( \log A^B_O \) from \( H_2O \) blends in the region B (p.33).

### 5.2 Synthetic Spectra

Since the \( H_2O \) blends are sparsely distributed in the region B, we examine a selected region (about one fourth of the region B) shown in Fig. 11 where relatively large number of \( H_2O \) blends are found together. The candidates of the \( H_2O \) blends to be analyzed are reproduced from Fig. 3 and shown by the arrows in Fig. 11c, but again not all these candidates are actually analyzed as in the region A. In this region B, the central depths of \( H_2O \) features are only about 5% or so in the earlier sample such as GJ 797B-NE and GJ 687, and never reach 15% even in the latest M dwarf in our sample GJ 406. The predicted spectra based on the values of \( \log A^B_O \) shown by the solid lines generally reproduce the observed ones shown by the filled circles but some weak features are rather noisy. For such sparsely distributed \( H_2O \) blends, \( \chi^2 \) test may not be useful.

Fig. 11: Observed and predicted spectra of \( H_2O \) in the region B of six M dwarfs (p.23).

### 5.3 Comparison of the Results from the Regions A and B

The differences of the values of the logarithmic oxygen abundances \( \log A^A_O \) based on the regions A and B are given in the seventh column of Table 7. The differences appear to be less than the probable errors of the analysis in the region A as well as in the region B in most cases. We also plot the values of the logarithmic oxygen abundances \( \log A^B_O \) based on the regions B against those based on the region A in Fig. 12 for 20 objects for which analyses are done both in the regions A and B. The results from the regions A and B show no systematic difference and agree well in general.

The result outlined above implies that the analysis of spectra whose continua are depressed by as much as 15% can provide the results not much different from the spectra whose continua are depressed by less than 5%. Thus, it is confirmed that the spectral analysis referring to the pseudo-continua works well, and spectra whose continua are depressed by molecular veil opacity can be analyzed only if the pseudo-continua can be predicted accurately by the use of proper molecular data and if the EWs can be measured well by referring to the pseudo-continua.

Fig. 12: Comparison of the logarithmic oxygen abundances from the regions A and B (p.24).

### 6. Discussion

#### 6.1. \( H_2O \) as an Abundance Indicator of Oxygen

The major difficulty in stellar abundance analysis is that the derived abundance depends on model photosphere used. For example, the solar carbon and oxygen abundances are revised downward by about 50% (Allende Prieto et al. 2002) against the classical values (e.g., Anders & Grevesse 1989). This revision is based on a new solar model photosphere referred to as the 3D model in which a time-dependent dynamical model for the convective zone has been incorporated. It is true that the classical 1D models are too simple for the real solar photosphere and further sophistication would certainly be welcome. However, verification of the sophisticated models may be by no means easy.

For comparison, we remember another major revision of solar abundance in the past: The downward revision of iron abundance by about 50%. This revision has been achieved by the use of Fe II lines whose \( g_f \)-values were measured accurately for the first time instead of the Fe I lines used for a long time (Holweger et al. 1990; Biémont et al. 1991). In solar photosphere, most iron atoms are singly ionized rather than neutral. For this reason, the Fe\( ^+ \) abundance is relatively insensitive to the change of the physical condition in the line-forming region and hence will not be affected by imperfect modelling of the solar photosphere. Thus, iron abundance could have been determined accurately by the use of the Fe\( ^+ \) lines for the first time, and the discrepancy between the solar iron abundance and the meteoritic iron abundance (Anders & Grevesse 1989) could have been resolved. This result was further confirmed by Grevesse & Sauval (1999) who succeeded to reconcile abundance results from the FeI and FeII lines with the use of a new solar model photosphere. The abundance determination depends on many factors, of which model photosphere is a crucial one. For this reason, stellar abundance should better be determined from the species that consumes the larger portion of an element and we refer such species as the major species for the element.

In Fig. 13, the abundances of some molecules are plotted
against the depth in four model photospheres of $T_{\text{eff}} = 2800, 3200, 3600, \text{and } 4000 \text{K}$ (the same models as used in Fig. 1). Also, we plot

$$y(\text{CO}) = P_{\text{CO}}/(P_{\text{H}} + 2P_{\text{H}_2}),$$

(5)
on logarithmic scale in Fig. 13 by dotted lines. If almost all the carbon atoms are in CO, $\log y(\text{CO}) \approx \log A_{\text{C}} \approx -3.4$. This expectation is satisfied in almost all the region of our four models in Fig. 13 and this fact confirms that CO is the major species of carbon in all our cases. Thus the carbon abundance can be determined accurately from CO, the major species of carbon, as noted in Paper I. We also plot $y(\text{H}_2\text{O}) = P_{\text{H}_2\text{O}}/(P_{\text{H}} + 2P_{\text{H}_2})$, (6) by dashed lines. If almost all the oxygen atoms left after CO formation are in H$_2$O, $\log y(\text{H}_2\text{O}) \approx \log (A_{\text{O}} - A_{\text{C}})$. Inspection of Fig. 13 shows that this case is realized for cooler models of $T_{\text{eff}} = 3200$ and 2800 K, and H$_2$O is the major species of oxygen in these cases. We confirm that the same situation applies to the models of $T_{\text{eff}} \lesssim 3400 \text{K}$ (as for details of the molecular abundances in these models, see the UCM database$^3$). In the model of $T_{\text{eff}} = 3600$, $P_{\text{H}_2\text{O}} \approx P_{\text{OH}}$ and H$_2$O is no longer the major species of oxygen in the models of $T_{\text{eff}} \gtrsim 3600 \text{K}$. Thus, the oxygen abundances determined from H$_2$O may depend somewhat on models in early M dwarfs of $T_{\text{eff}} \gtrsim 3600 \text{K}$. Since H$_2$O lines are anyhow weak in the early M dwarfs, OH may also be used as another abundance indicator of oxygen. Except for the early M dwarfs, H$_2$O is the major species of oxygen and can be an excellent abundance indicator of oxygen in M dwarfs.

Another advantage of H$_2$O as abundance indicator may be that its thermal velocity is rather large because of its light weight. In fact, thermal velocity of H$_2$O at $T = 3000 \text{K}$ is $\xi_{\text{th}} = 1.66 \text{km sec}^{-1}$. This is considerably larger than the micro-turbulent velocity we have assumed ($\xi_{\text{micro}} = 1 \text{km sec}^{-1}$). Since actual turbulent velocity in the photospheres of M dwarfs is not likely to be larger than the value we assumed (see e.g., Bean et al. 2006), Doppler broadening of H$_2$O lines should be dominated by the thermal velocity. This possibility may be consistent with the result that the abundance corrections $\Delta \log A_{\text{O}}$ plotted against $\log (W/\lambda)_{\text{obs}}$ in Figs. 5, 6, and 10 show little tilt or nearly horizontal as a mean, as noted in subsection 4.1. If Doppler broadening is dominated by the turbulence as in (super)giant stars, the abundance corrections based on the saturated lines depend largely on the assumed turbulent velocity while those based on the weak lines are nearly independent of the assumed velocity, at least for the case of the unblended single lines. In our case of the blended lines in which several hundreds of lines per 1 Å interval are overlapping, it is not known how EWs are influenced by the micro-turbulent velocity, and this problem will be investigated further hopefully with higher resolution spectra.

Since the discovery that the turbulence plays dominant role in the line broadening in stellar spectra (Struve & Elvey 1934), how to determine the turbulent velocity has been one of the major problems in abundance analysis. In fact, one reason why the results of the abundance analyses by different authors do not agree well in general is due to differences of the turbulent velocities applied by different authors. We might have emphasized somewhat too much on the effect of micro-turbulence in the analysis of CO in Paper I even though CO is heavier than H$_2$O and hence the relative role of the thermal velocity is not so large as in H$_2$O. Now, so far as the spectral analysis of H$_2$O in M dwarfs is concerned, the uncertainty due to the micro-turbulent velocity is rather minor because of the larger thermal velocity, and this fact can be regarded as an additional advantage of H$_2$O as an abundance indicator of oxygen in M dwarfs.

Fig. 13: The major species of carbon and oxygen in the photospheres of M dwarfs (p.24).

### 6.2. Oxygen Abundances in M Dwarfs

We take the weighted mean (with the numbers of lines used as weights) of the oxygen abundances resulting from the analyses of the regions A (section 4) and B (section 5) for 20 M dwarfs for which the two regions have been analyzed. The results and those from the region A alone (18 objects) are summarized in Table 8 together with the carbon abundances from Paper I. Also, the classical and more recent solar carbon and oxygen abundances by Anders & Grevesse (1989) and Asplund et al. (2009), respectively, are included in Table 8 for comparison.

The resulting values of $\log A_{\text{O}}$ are plotted against the values of $\log A_{\text{C}}$ in Fig. 14. The larger filled circles are based on the mean oxygen abundances from the regions A and B while the smaller filled circles from the region A alone. The two representative solar values are shown by $\circ$ marks. The initial assumed values of $\log A_{\text{O}} = \log A_{\text{C}} + 0.30$ are represented by the solid line in Fig. 14. The oxygen abundances are systematically lower than the assumed initial values in the carbon-rich M dwarfs. This tendency can be shown more clearly by plotting the values of $\log A_{\text{O}}/A_{\text{C}}$ (given in the fourth column of Table 8) against the values of $\log A_{\text{C}}$ as illustrated in Fig. 15. Clearly abundances of oxygen relative to carbon are systematically smaller in M dwarfs with the larger carbon abundances. This result agrees well with the result by Gustafsson et al. (1999) who have determined carbon abundances in 80 F and G dwarfs (and oxygen abundances from Edvardsson et al. 1993) in the larger metallicity range with the use of the forbidden [CI] line, which is expected to be less sensitive to the photospheric structure and non-LTE effect compared to the other carbon abundance indicators for F and G dwarfs.

The carbon abundance may approximately represent the metallicity usually measured by the iron abundance. Then, Fig. 15 indicates that the $A_{\text{O}}/A_{\text{C}}$ ratios are larger at lower metallicities and gradually decrease in higher metallicities. Thus, Fig. 15 suggests different formation histories of carbon and oxygen in the evolution of the Galaxy. After the large production of oxygen by Type
II supernovae/hypernovae in the early Galaxy, the decrease of oxygen and carbon with increasing metallicity in the metallicity range of Fig.15 is due to Type I supernovae which produce more iron, but the decrease of carbon may be tempered by the contribution of carbon from AGB stars (Nomoto et al. 2013). However, the actual site of carbon production is still unclear as discussed in detail by Gustafsson et al. (1999). In view of the advantage of carbon production is still unclear as discussed in de-

bon may be tempered by the contribution of carbon from novae which produce more iron, but the decrease of car-

in the metallicity range of Fig.15 is due to Type I super-

crease of oxygen and carbon with increasing metallicity

II supernovae/hypernovae in the early Galaxy, the de-

metallicities \[\text{[Fe/H]} = \log \frac{A_{\odot}}{A_{\text{Fe}}}\] from the liter-

ature (Mould 1978; Önehag et al. 2012; Neves et al. 2013) in Table 9. The iron abundances \[\log A_{\text{Fe}}\] are obtained assuming solar value of \[\log A_{\odot}^{\text{Fe}} = -4.5\] (Asplund et al. 2009) and, together with our oxygen abundances, O/Fe ratios are obtained. The resulting values of \[\log A_{\text{O}}/A_{\text{Fe}}\] are plotted against [Fe/H] in Fig.16. Except for a few case, O/Fe ratios show a systematic increase towards lower [Fe/H], confirming the well known excess production of oxygen in metal-poor era. The two solar results are shown by the \(\odot\) marks: The higher O/Fe case (marked with 1) is based on the classical solar abundance (Anders & Grevesse 1989) and the O/Fe ratio follows the general trend shown by the M dwarfs. To the contrary, the lower O/Fe ratio (marked with 2) based on the recent downward revised oxygen abundance (Asplund et al. 2009) does not follow the general trend shown by the M dwarfs, or the solar oxygen abundance is atypical for its metallicity compared with the nearby M dwarfs which is typical unevolved stars in the Galactic disk. On the other hand, it was shown that the solar abundances including oxygen are typical for its metallicity, compared to the elemental abundances in unevolved stars in the solar neighborhood (Edvardsson et al. 1993). The classical solar oxygen abundance (Anders & Grevesse 1989) is consistent with this result while the downward revised result (Allende Prieto et al. 2002; Asplund et al. 2009) is contradicting with this result.

So far, evolution of chemical elements in the Universe has been studied mostly from the abundance analyses of G type dwarfs including the halo sample, and there should be some reasons for this choice. For example, these stars are relatively well understood with the Sun as the prototype compared to very hot and very cool stars and, especially, many metallic lines used for abundance analysis can be observed in their spectra. However, one drawback to use G type dwarfs as tracers of the chemical history of the Galaxy, is that the photospheres of these stars may possibly be polluted by accretion of the metal-rich matter through encounter with the interstellar gas clouds during the evolution of the Galaxy, as has been pointed out first by Yoshii (1981). In this case, the accreted matter will not be diluted well because of the shallow convective zone in the G dwarfs, and the photospheric abundances will suffer appreciable changes from the values at their birth. Recently, observational support for this possibility has been presented by showing that the metallicity in halo G dwarfs is indeed larger by about 0.2 dex compared with that in K dwarfs of the same kinematic characteristics but having deeper convective zone (Hattori et al. 2014). Such a dilution effect may be most efficient in M dwarfs whose envelopes are wholly convective and the photospheric abundances may not be changed very much from the values at their birth. For this reason, M dwarfs could be the best tracers of the Galactic chemical evolution if chemical analysis can be extended to halo M dwarfs in the near future, and M dwarfs will provide unique contribution to our understanding on the element formation in the early Universe.

Finally, frequency distribution of oxygen abundances in 38 M dwarfs is shown in Fig.17. The oxygen abundances \[\log A_{\text{O}}\] in M dwarfs are mostly between -3.5 and -3.0. On the other hand, a larger fraction of M dwarfs hosting planet(s) (those noted by \(\dagger\) in Table 8) are oxygen-rich, which is consistent with the results for carbon in Paper I and for metallicity in F, G, K, and M stars (e.g., Fischer & Valenti 2005; Johnson & Apps 2009).

Table 8: Carbon and oxygen abundances in M dwarfs (p.34).
Table 9: Oxygen-to-Iron ratios in M dwarfs (p.34).

6.3. Method of Abundance Analysis

The spectra of cool stars including M dwarfs are apparently very complicated, and abundance analysis has been deemed difficult in general. We have done some trials in Paper I and in this paper to simplify the abundance analysis in M dwarfs, and we summarize our attempts.

In stellar abundance analysis, the so-called curve-of-growth (CG) method played an important role as detailed in the textbooks of classical astrophysics (e.g. Unsöld 1955). In its refined form, the so-called universal curve-of-growth, the growth of the equivalent width (EW) broadened by the Doppler and damping effects has been expressed in unified curves in dependence on the effective number of species producing the line. With this curve-of-growth, the observed EW could be transformed to the effective number of species and hence to the abundance of
element producing the line. This method has widely been applied to the stellar abundance determinations until the middle of the previous century. Despite the simplicity and wide applicability of the CG method, however, it has some drawbacks. For example, the effect of the photospheric structure could not be taken into account because of the simplified assumptions needed to compute EWs universally (i.e., for all the types of stars). Also, applications are limited to a single line with well defined profile which can be described by the Voigt profile.

To relax the problems in the CG method noted above, the so-called spectral synthesis (SS) method has been widely used in recent years. In this method, any model photosphere can be used in computing the synthetic spectrum including the blended features. Then the observed spectrum is compared with the predicted one characterized by several parameters including abundance, which are to be determined from the best fit between the observed and predicted spectra. Despite the wide use of the SS method, however, the stellar abundance determinations by different authors are still not well converged to a definite result (e.g., Lebzelter et al. 2012). One problem is that the different line-broadenings such as the rotation, micro- and macro-turbulence could not be separated on the synthetic spectrum. In this respect, the CG method has a definite advantage in that the EWs depend on the micro-turbulent velocity but not on the other line broadening, and thus micro-turbulence could be well separated from the other broadenings by the analysis of EWs with the CG method. Also, the SS as well as CG method is based on the prerequisite that the synthetic spectra as well as EWs are measured by referring to the true-continuum.

We believe that our analysis applied in subsection 4.1 has relaxed all the restrictions involved in the analyses noted above. As to the problem of continuum, we have noticed that the spectral analysis can be carried out by referring to the pseudo-continuum if it can be evaluated accurately on the theoretical spectrum, and recent progress in molecular database indeed makes such an approach possible (Paper I). Also, analysis is not limited to a single line but any blended features can be used. What is important is that the observed and theoretical spectral features are defined by exactly the same way by referring to the same reference (i.e., either the true- or pseudo-continuum). The abundance cannot directly be obtained from the observed EW of the spectral feature selected, but can be obtained easily with the use of a simple curve-of-growth. This CG is not necessary to cover the large range of EW as in the classical CG, but it should cover the restricted portion of the CG around the observed EW. Such a mini CG can be prepared theoretically by the use of EWs evaluated from the synthetic spectra including all the lines contributing to the spectral features to be analyzed. For this purpose, it is sufficient to generate three synthetic spectra for slightly different abundances, from which theoretical EWs are evaluated by exactly the same way as EW is measured on the observed spectrum. Then abundance can be obtained for the observed EW from the three-point mini CG. This analysis is repeated line-by-line or blend-by-blend for all the spectral features measured.

The mini CG method can best be applied to H$_2$O for which the pseudo-continuum level is also determined by H$_2$O itself. Then, the mini CG analysis directly provides the oxygen abundance. For comparison, in the case of CO discussed in Paper I, the pseudo-continuum level is determined not by CO itself but by H$_2$O and also EWs of CO measured include the contamination due to H$_2$O. For this reason, derived carbon abundance includes the effect of H$_2$O. Even though this effect is already included in the synthetic spectra from which EWs used for mini CG are evaluated, the result depends on the oxygen abundance assumed. For this reason, it is required to repeat the above processes by using the revised oxygen abundance if accurate result is required. In our Paper I, however, this process was skipped since the blending H$_2$O lines are rather weak and will have little effect.

The micro-turbulent velocity can be determined if necessary: For this purpose, we repeat the mini CG analysis with several assumed values of the micro-turbulent velocity. The resulting abundance corrections from different lines (or blends) depend little on the observed EWs only if the assumed micro-turbulent velocity is correct. In our analysis of M dwarfs, this requirement is almost satisfied for almost all the cases (see Figs. 5, 6, & 10) for the assumed micro-turbulent velocity, but this is because the thermal broadening dominates over the turbulent broadening as noted in subsection 6.1. If the turbulent broadening is dominating, the micro-turbulent velocity is determined so that the abundance corrections depend little on the observed EWs, and some examples are shown for the case of red giant stars using unblended lines (e.g., Tsuji 2008). It is still to be shown if the micro-turbulent velocity can be determined from the blended spectral features by the method outlined above.

We believe that our abundance analysis is essentially very simple and flexible enough to be easily applied to complicated blended spectra with depressed continuum, and we hope to have shown that the abundance analysis of cool stars is not especially difficult compared to that of other types of stars.

6.4. H$_2$O in Stellar Spectra other than M Dwarfs

We have shown that H$_2$O molecule can be an excellent abundance indicator of oxygen in M dwarfs. Then, we wonder if this is true for other types of stars including M type giants and supergiants which show H$_2$O lines in their spectra. For example, H$_2$O lines are prominent in the spectra of Mira type variables for which detailed spectroscopic analysis has been done by Hinkle & Barnes (1979), who showed that H$_2$O lines are composed of multiple components originating from different layers reflecting the complicated dynamical structures of their atmospheres. Then H$_2$O lines are primarily useful as the probes of the atmospheric structures of Mira type variables and abundance analysis would be more difficult.

Spectra of water vapor were observed in red giants and supergiants other than Mira type variables, but in the ways somewhat unexpected. For example, the announce-
ment to have detected H$_2$O lines in the early M supergiant Betelgeuse (α Ori, M2Iab) by Woolf et al. (1964) was so unexpected at that time that it could not be appreciated correctly for a long time (Tsuji 2000). Also, H$_2$O lines were observed in the K giant Arcturus (α Boo, K2IIIp) with high resolution in the atmospheric window near 12 μm (Ryde et al. 2002) and in the K giant Aldebaran (α Tau, K5III) with low resolution by ISO (Tsuji 2001). Such results are difficult to understand within the framework of the classical theory of stellar photospheres. Further, in non-Mira M giant stars, H$_2$O spectra were observed by ISO but showed excess absorption that could not be accounted for by the LTE model photospheres (e.g., Decin et al. 2003). Such problems must be solved before H$_2$O molecule can be used as an abundance indicator of oxygen in red giant and supergiant stars.

Probably the rarefied and extended atmospheres of high luminosity stars are more complicated to be described by the classical theory of stellar photospheres. Recent observations revealed that H$_2$O and CO are in fact found not in the photospheres but in the warm molecular layers extended out to $\approx 1.5 R_\odot$ in M giant and supergiant stars (e.g., Ohnaka 2004; Montargès et al. 2014). Moreover, it was found recently from the aperture synthesis imaging of Betelgeuse that such molecular layers or clouds are asymmetrically extended and show large scale time-dependent dynamical structure (Ohnaka et al. 2011). Thus, interpretation of H$_2$O spectra in red giant and supergiant stars would require hybrid model atmosphere consisting of components with different dynamical characteristics, and H$_2$O spectra will serve primarily as probes of the outer atmospheric structure of high luminous cool stars.

7. Concluding Remarks

We have shown in the present paper and in Paper I that CO and H$_2$O spectra of M dwarfs can be excellent abundance indicators of carbon and oxygen, respectively, and carbon and oxygen abundances in dozens of M dwarf stars have been determined rather well. This conclusion may appear to be somewhat unexpected in view of the difficulties so far experienced in the attempts to analyze the spectra of M dwarfs. However, a new possibility of determining chemical abundances in cool dwarf stars has been opened by the recent progress as follows:

First, the progress in observations is remarkable. For examples, infrared spectra of high quality can be obtained by the use of new infrared detectors, fundamental parameters such as the effective temperatures are determined from the directly measured angular diameters, and high precision photometric and astrometric data on M dwarfs are available especially from the space observations.

Second, the traditional abundance analysis appears to be difficult in M dwarfs whose continuum levels are depressed by the numerous molecular lines. However, given that the pseudo-continuum levels defined by the numerous molecular lines can be evaluated accurately with the use of the recent molecular database, spectral analysis referring to the pseudo-continuum results in an identical result with that referring to the true-continuum, as we have shown in the present paper.

Third, abundance analysis depends on model photospheres, which are unfortunately by no means perfect yet because of the extreme complexity of the real photospheres. For this reason, it is useful to do abundance analysis to be free from imperfect modelling of the photospheric structure as far as possible. In the photospheres of M dwarfs, CO abundance is identical with the abundance of carbon itself in almost all the M dwarfs and H$_2$O abundance is almost the same with $A_{\odot} - A_C$ except for early M dwarfs (subsection 6.1). Then, CO and H$_2$O abundances depend little on the model applied. For this reason, carbon and oxygen abundances can best be determined in M dwarfs rather than in other types of stars.

Thanks to the favorable conditions noted above, the abundance analysis of M dwarfs based on CO and H$_2$O has been done rather well within the framework of the classical LTE analysis. This shows a marked contrast to the case of molecular spectra in high luminous cool stars including K - M giant and supergiant stars, in which classical LTE analysis encountered serious difficulties (subsection 6.4). As expected, LTE analysis is applicable to the high density and compact photospheres of low luminous stars rather than to the low density and extended atmospheres of high luminous stars. Although we certainly hope that the analysis of molecular spectra can be done consistently both in low and high luminous stars in the near future, we now take the advantage that the spectra of M dwarfs can be analyzed on the basis of the well established classical theory of spectral line formation, and we hope that M dwarfs will be used more widely as the probes of the chemical abundances in the Universe.

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This research has made use of the VizieR catalog access tool and the SIMBAD database, both operated at CDS, Strasbourg, France, and of the RECONS database in www.recons.org.

Computations are carried out on common use data analysis computer system at the Astronomy Data Center, ADC, of the National Astronomical Observatory of Japan.

Appendix 1. The AllWISE vs. the WISE All-Sky Catalogs

The absolute magnitudes at 3.4 μm, $M_{3.4}$, based on the WISE W1 fluxes given in the AllWISE Catalog are shown in Tables 10 and 11, while those in the WISE All-Sky Release were shown in Tables 3 and 4 of Paper I. The $M_{3.4} - \log T_{\text{eff}}$ plots based on the data in Tables 10 and 11
are illustrated in Fig. 18 and the dashed line is the exact copy from that given in Fig. 1 of Paper I. As noted in section 2, the dashed line fits the plots in Fig. 18 quite well, and can be applied to the data of both the AllWISE and the WISE All Sky Catalogs.

Fig. 18: The absolute magnitude $M_{3.4}$ based on the AllWISE Catalog plotted against log $T_{\text{eff}}$ (p.26).

Table 10: $M_{3.4}$ and $T_{\text{eff}}$ based on the interferometry (p.35).

Table 11: $M_{3.4}$ and $T_{\text{eff}}$ based on the infrared flux method (p.35).

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Fig. 1. Spectra of M dwarfs ($R = 2000$) in the $K$ band region based on the model photospheres of $T_{\text{eff}}/\log g = a) 4000 \, K/4.5$, b) $3600 \, K/4.75$, c) $3200 \, K/5.0$, and d) $2800 \, K/5.25$. The H$_2$O 1.9 and 2.7 $\mu$m bands are increasingly stronger in cooler models, but the CO second overtone bands are rather masked by H$_2$O bands in the cooler models. The regions A and B, which are in the atmospheric window shown at the top of the figure, are selected for detailed analysis of H$_2$O lines. The region A is in the midst of H$_2$O 1.9 $\mu$m bands while the region B suffers the effects of both the H$_2$O 1.9 and 2.7 $\mu$m bands least.
Fig. 2. The spectra of M dwarfs in the region A roughly in the order of the decreasing effective temperatures: a) GJ338A, b) GJ15A, c) GJ611B, and d) GJ406. Also shown is e) Regulus used for calibration. The candidates of H$_2$O blends for detailed analysis are indicated by arrows with the reference numbers A01–A17 to Tables 2 & 3.
Fig. 3. The spectrum of GJ 406 (red or black) as an example of the spectrum in the region B. The spectrum of Regulus (green or grey) is used as the calibrator of the wavelength and the atmospheric absorption. The candidates of H$_2$O blends for detailed analysis are indicated by arrows with the reference numbers B01–B27 to Tables 4 & 5. The strong lines in Regulus are CH$_4$ and N$_2$O of telluric origins.
Fig. 4. Mini curves-of-growth for seven selected H$_2$O blends in GJ 229, and the reference numbers to Table 3 are shown for the corresponding mini CG. The filled circle on each mini curve-of-growth indicates the observed value of log \((W/\lambda)_{obs}\) (from Table 3 and read on the ordinate) and the resulting value of the abundance correction \(\Delta \log A_O\) (read on the abscissa and plotted on Fig 5d by filled circle).
Fig. 5. The resulting logarithmic abundance corrections $\Delta \log A_O$ by the mini curves-of-growth analysis for the $\text{H}_2\text{O}$ blends in the region A are plotted against the observed values of $\log (W/\lambda)_{\text{obs}}$. The dashed line shows the mean value of $\Delta \log A_O$. In this figure, relatively early M dwarfs are shown in the order of decreasing effective temperatures.

Fig. 6. The same as in Fig. 5 but for later M dwarfs in the order of decreasing effective temperatures.
Fig. 7. Comparison of the observed (filled circles) and predicted (solid lines) spectra based on the oxygen abundances $\log A^\Delta_O$ in Table 6 for six selected M dwarfs: a) GJ 338B (dM0.5), b) GJ 229 (dM2.5), c) GJ 526 (dM0), d) GJ 797B-NE (M2.5), e) GJ 324B (M4), f) GJ 777B (M4.5).
Fig. 8. Comparison of the observed (filled circles) and predicted (solid lines) spectra for GJ 406: a) For the oxygen abundance $\log A_O = -3.40$ in Table 6 resulting $\chi^2 = 13.204$, but fittings are fine for the blends used for the analysis (indicated by the arrows). b) For the oxygen abundance $\log A_O = -3.40 - 0.05$, resulting $\chi^2 = 10.510$, but fittings are worse for the blends used for the analysis (indicated by the arrows). Then the part of the spectrum not used for the analysis (between 20354.2 and 20385.2 Å; filled by yellow or grey in the figure) is excluded from the $\chi^2$ test. The results are $\chi^2 = 11.119$ and $\chi^2 = 11.227$ for a) $\log A_O = -3.40$ and b) $\log A_O = -3.40 - 0.05$, respectively, confirming that the mini CG analysis and the $\chi^2$ analysis are consistent.
Fig. 9. Model spectra normalized by their true-continua shown by the dotted lines are shown by the solid lines for four model spectra. The observed spectra of the four stars normalized by their pseudo-continua (dashed lines) in Fig. 2 are shown here re-normalized by their true-continua estimated by applying the pseudo-to-true continua ratios of the model spectra. In this way, the depressions of the continuum levels of the four stars are estimated even though the true-continua can never be seen on the observed spectra.
The resulting logarithmic abundance corrections $\Delta \log A_O$ by the mini curves-of-growth analysis for the H$_2$O blends in the region B plotted against the observed values of $\log (W/\lambda)_{\text{obs}}$. The dashed line shows the mean value of $\Delta \log A_O$. Compared with Fig. 6 for the same objects, larger numbers of blends can be used especially for later types.
Fig. 11. Comparison of observed (filled circles) and predicted (solid lines) spectra based on the oxygen abundances determined from H$_2$O blends in the region B for six M dwarfs: a) GJ 797B-NE (M2.5). b) GJ 687 (dM4). c) GJ 324B (M4). d) GJ 777B (M4.5). e) GJ 611B (M4). f) GJ 406 (dM6.5). The strong feature at 22614 Å in each observed spectrum is due to Ca I line. But this Ca I line is not included in our line-list and the feature at the position of the Ca I line on the predicted spectra is not due to Ca but to H$_2$O.
Fig. 12. Oxygen abundances determined from the region B are plotted against those from the region A. The results from the different regions agree within the probable errors in most objects.

Fig. 13. Molecular abundances plotted against the optical depths in four model photospheres of $T_{\text{eff}}/\log g = \text{a)} 4000 \text{ K}/4.5$, b) 3600 K/4.75, c) 3200 K/5.0, and d) 2800 K/5.25. Also, $y_{\text{CO}}$ and $y_{\text{H}_2\text{O}}$ are defined by equations (5) and (6), respectively, in the text. If almost all the carbon atoms are in CO (i.e., if CO is the major species of carbon), $y_{\text{CO}} \approx A_C$. If almost all oxygen atoms left after CO formation are in H$_2$O (i.e., if H$_2$O is the major species of oxygen), $y_{\text{H}_2\text{O}} \approx A_O - A_C$. It can be confirmed that CO is the major species of carbon in all the models and H$_2$O in the models of $T_{\text{eff}} \lesssim 3400 \text{ K}$. 
Fig. 14. The oxygen abundances $\log A_O$ are plotted against the carbon abundances $\log A_C$ in 38 M dwarfs. The classical high solar case is indicated by the upper $\odot$ and the recent downward revised solar case by the lower $\odot$. The assumed initial relation of $\log A_O = \log A_C + 0.30$ is shown by the solid line.

Fig. 15. The oxygen-to-carbon ratios are plotted against the carbon abundances $\log A_C$ in 38 M dwarfs, and in the Sun. The oxygen-to-carbon ratios are smaller at the higher carbon abundances, showing that carbon productions are more effective at higher metallicities.

Fig. 16. The oxygen-to-iron ratios are plotted against $[\text{Fe/H}] = \log A_{\text{Fe}}^\star - \log A_{\odot}^\star$ in M dwarfs and the Sun. The oxygen-to-iron ratio of the classical oxygen abundance (Anders & Grevesse 1989) follows the general trend shown by the M dwarfs (marked with 1), but that of the recent downward revised result (Asplund et al. 2009) does not follow the general trend (marked with 2). Thus, if the recently recommended low oxygen abundance is correct, the solar oxygen abundance is atypical for its metallicity.

Fig. 17. Frequency distribution of M dwarfs against $\log A_O$. Note that nine M dwarfs are hosting planet(s).
Fig. 18. The absolute magnitudes at 3.4 µm, \( M_{3.4} \), plotted against \( \log T_{\text{eff}} \), where \( M_{3.4} \) values are based on the AllWISE Catalog (see Tables 10 and 11) instead of the WISE All-Sky Catalog applied in Paper I. The probable errors of the AllWISE Catalog are larger than those of the WISE All-Sky Catalog in general. The values of \( T_{\text{eff}} \) based on the interferometry and infrared flux method are shown by the filled and open circles, respectively. The mean \( M_{3.4} - \log T_{\text{eff}} \) relation shown by the dashed line is exactly the same as that in Fig. 1 of Paper I.
Table 1. Fundamental Parameters and Model Photospheres for the Program Stars.

| obj.  | Sp. Type | $T_{\text{eff}}$ | $\log g$ | model  |
|-------|----------|------------------|----------|--------|
| GJ 15A | dM2.5   | 3567             | 4.890    | Ca3570c489 |
| GJ 105B | dM4.5   | 3360             | 4.954    | Ca3360c495 |
| GJ 166C | dM4e    | 3337             | 4.972    | Ca3340c497 |
| GJ 176 | dM2.5e  | 3616             | 4.804    | Ca3620c480 |
| GJ 179 | dM3.5e  | 3476             | 4.877    | Ca3480c488 |
| GJ 205 | dM3    | 3801             | 4.710    | Ca3800c471 |
| GJ 212 | dM1    | 3757             | 4.748    | Ca3760c475 |
| GJ 229 | dM2.5  | 3707             | 4.766    | Ca3710c487 |
| GJ231.1B | M3.5  | 3442             | 4.897    | Ca3440c490 |
| GJ 250B | M2     | 3567             | 4.827    | Ca3570c483 |
| GJ 273 | dM4    | 3415             | 4.915    | Ca3420c492 |
| GJ 324B | M4     | 3382             | 4.938    | Ca3380c494 |
| GJ 338A | dM0.5  | 3907             | 4.709    | Ca3910c471 |
| GJ 338B | dM0.5  | 3867             | 4.709    | Ca3870c471 |
| GJ 380 | dM0.5  | 4081             | 4.643    | Ca4080c464 |
| GJ 406 | dM6.5e | 2800             | 5.170    | Cc2800c517 |
| GJ 411 | dM2    | 3465             | 4.857    | Cc3470c486 |
| GJ 412A | dM2    | 3497             | 4.843    | Cc3500c484 |
| GJ 436 | dM3.5  | 3416             | 4.797    | Cc3420c480 |
| GJ 526 | dM3    | 3618             | 4.784    | Cc3620c478 |
| GJ 581 | dM4    | 3442             | 4.959    | Ca3440c496 |
| GJ 611B | M4     | 3202             | 5.063    | Cc3200c506 |
| GJ 649 | dM2    | 3660             | 4.784    | Ca3660c478 |
| GJ 686 | dM1    | 3538             | 4.842    | Ca3540c484 |
| GJ 687 | dM4    | 3413             | 4.811    | Ca3410c481 |
| GJ 725A | dM4    | 3407             | 4.837    | Ca3410c484 |
| GJ 725B | dM4    | 3337             | 4.972    | Cc3340c497 |
| GJ 768.1C | —      | 3470             | 4.881    | Ca3470c488 |
| GJ 777B | M4.5   | 3310             | 4.991    | Ca3310c499 |
| GJ 783.2B | M4    | 3370             | 4.949    | Ca3370c495 |
| GJ 797B-NE | M2.5  | 3473             | 4.878    | Ca3470c488 |
| GJ 797B-SW | M2.5  | 3473             | 4.878    | Ca3470c488 |
| GJ 809 | dM2    | 3692             | 4.720    | Ca3690c472 |
| GJ 820B | dM0    | 3932             | 4.679    | Ca3930c470 |
| GJ 849 | dM3.5  | 3580             | 4.821    | Ca3580c482 |
| GJ 876 | dM4.5  | 3458             | 4.888    | Ca3460c489 |
| GJ 880 | dM2.5  | 3713             | 4.716    | Ca3710c472 |
| GJ 884 | dM0.5  | 3850             | 4.720    | Ca3850c472 |
| GJ 3348B | M4    | 3476             | 4.876    | Ca3480c488 |
| HIP 12961 | M0    | 3890             | 4.709    | Ca3890c471 |
| HIP 57050 | M4    | 3464             | 4.884    | Ca3460c488 |
| HIP 79431 | M3    | 3592             | 4.815    | Ca3590c482 |

* from Paper I.
Table 2. H$_2$O Lines in the Region A.

| Ref. | $\lambda_{obs}$ | $\lambda$ | $v_1^*v_2^*v_3^*$ | $v_1^*v_2^*v_3^*$ | $J$ | $K_a^*$ | $K_c^*$ | $J$ | $K_a^*$ | $K_c^*$ | log $g_f$ | L.E.P. $\dagger$ |
|------|-----------------|------------|-------------------|-------------------|-----|--------|--------|-----|--------|--------|-----------|------------|
| A01  | 20298.6         | 0.1       | 0.10              | 0.1              | 15  | 8      | 8      | 16  | 8      | 9      | -5.195    | 5949.217 |
|      | 20298.428       | 0.32      | 0.21              | 0.9              | 3   | 7      | 10     | 3   | 8      | 5.049  | 8358.617 |
|      | 20298.838       | 0.11      | 0.00              | 0.19             | 11  | 9      | 20     | 11  | 10     | -5.074 | 6664.142 |
| A02  | 20306.4         | 0.11      | 0.00              | 0.17             | 0   | 17     | 18     | 0   | 18     | -5.284 | 3319.448 |
|      | 20306.355       | 0.11      | 0.00              | 0.17             | 1   | 17     | 18     | 1   | 18     | 4.676  | 3319.448 |
| A03  | 20308.6         | 0.11      | 0.00              | 0.16             | 1   | 10     | 17     | 6   | 11     | -4.909 | 4291.907 |
|      | 20308.479       | 0.21      | 0.01              | 0.15             | 5   | 11     | 16     | 5   | 12     | 4.637  | 5310.240 |
|      | 20308.775       | 0.21      | 0.01              | 0.14             | 4   | 10     | 15     | 4   | 14     | 4.638  | 4894.586 |
| A04  | 20312.4         | 0.51      | 0.04              | 0.8              | 3   | 5      | 9      | 3   | 6      | 4.389  | 7577.863 |
|      | 20312.344       | 0.51      | 0.04              | 0.7              | 2   | 5      | 8      | 2   | 6      | -5.487 | 7212.158 |
|      | 20312.398       | 0.61      | 0.05              | 0.6              | 1   | 6      | 7      | 1   | 7      | -4.778 | 8162.491 |
|      | 20312.555       | 0.41      | 0.03              | 0.11             | 2   | 10     | 12     | 2   | 11     | -4.344 | 6517.811 |
| A05  | 20316.7         | 0.21      | 0.01              | 0.15             | 3   | 12     | 16     | 3   | 13     | -5.068 | 5089.336 |
| A06  | 20324.3         | 0.31      | 0.01              | 0.12             | 3   | 9      | 13     | 11  | 2      | 4.634  | 5654.780 |
| A07  | 20331.2         | 0.31      | 0.02              | 0.13             | 2   | 11     | 14     | 2   | 12     | 4.937  | 5786.852 |
|      | 20331.287       | 0.13      | 0.12              | 0.5              | 1   | 5      | 6      | 1   | 6      | 4.838  | 7216.297 |
| A08  | 20334.5         | 0.41      | 0.03              | 0.11             | 3   | 9      | 12     | 3   | 10     | 4.410  | 6767.303 |
| A09  | 20337.1         | 0.12      | 0.00              | 0.14             | 1   | 14     | 15     | 1   | 15     | 4.682  | 6077.105 |
| A10  | 20339.8         | 0.22      | 0.01              | 0.12             | 1   | 12     | 13     | 1   | 13     | 4.426  | 7098.995 |
| A11  | 20342.6         | 0.11      | 0.00              | 0.18             | 2   | 16     | 19     | 2   | 17     | -4.828 | 4331.067 |
| A12  | 20347.5         | 0.22      | 0.01              | 0.12             | 11  | 9      | 13     | 2   | 12     | -5.560 | 7355.381 |
| A13  | 20352.932       | 0.51      | 0.04              | 0.9              | 2   | 8      | 10     | 2   | 9      | -4.301 | 7542.146 |
| A14  | 20363.6         | 0.31      | 0.02              | 0.12             | 4   | 8      | 13     | 4   | 9      | 4.567  | 5781.958 |
| A15  | 20369.5         | 0.11      | 0.00              | 0.17             | 5   | 12     | 18     | 5   | 13     | 5.330  | 4606.168 |
| A16  | 20383.3         | 0.31      | 0.02              | 0.14             | 1   | 13     | 15     | 1   | 14     | -4.458 | 5827.338 |
| A17  | 20387.3         | 0.21      | 0.01              | 0.18             | 9   | 19     | 9      | 9   | 10     | -4.746 | 7470.305 |
|      | 20387.338       | 0.41      | 0.03              | 0.12             | 0   | 12     | 13     | 0   | 13     | -4.318 | 6456.296 |

* reference number to Fig. 2.
† in Å and in vacuum.
‡ in cm$^{-1}$. 
| obj    | A01  | A02  | A03  | A04  | A05  | A06  | A07  | A08  | A10  | A11  | A12  | A13  | A14  | A15  | A16  | A17  |
|--------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| GJ15A  | -4.957 | -4.803 | -4.671 |       | -5.018 | -5.177 | -5.046 | -4.994 |       | -5.018 |       |       |       |       |       |       |
| GJ105B | -4.757 | -4.685 | -4.522 | -4.405 | -4.815 | -4.711 | -4.854 | -4.779 | -4.739 |       | -4.772 |       |       |       |       |       |
| GJ16C  | -4.760 | -4.709 | -4.521 | -4.375 | -4.740 | -4.595 | -4.772 |       |       |       |       |       |       |       |       |       |
| GJ176  |       |       |       | -4.981 | -4.837 | -4.707 | -5.070 | -5.233 | -5.163 | -5.034 |       | -4.987 |       |       |       |       |
| GJ179  |       |       | -4.837 | -4.689 | -4.545 | -4.958 | -4.856 | -5.061 |       | -4.906 |       | -4.667 | -4.928 | -4.673 |       |       |
| GJ205  |       | -4.986 |       |       | -5.026 |       |       |       |       |       |       |       |       |       |       |       |
| GJ212  |       | -5.052 | -5.099 |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ229  |       | -5.032 | -5.106 | -4.847 |       | -5.186 | -5.207 |       |       |       |       |       |       |       |       |       |
| GJ231.1B | -4.871 | -4.796 | -4.617 | -4.479 |       | -4.846 | -4.778 | -4.924 | -4.851 | -4.803 | -4.670 |       | -4.840 | -4.778 | -4.977 | -4.798 |       |
| GJ250B |       | -4.864 | -4.802 |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ273  | -4.779 |       | -4.562 | -4.417 |       | -4.787 | -4.731 | -4.870 | -4.767 |       |       | -4.537 | -4.805 |       |       |       |       |
| GJ324B | -4.834 | -4.762 | -4.599 | -4.475 |       | -4.827 | -4.861 | -4.943 | -4.878 | -4.836 | -4.695 | -4.636 | -4.879 |       |       |       |       |
| GJ338A |       |       |       |       | -5.213 |       |       |       |       |       |       |       |       |       |       |       |
| GJ338B |       |       |       |       |       | -5.590 | -5.626 |       |       |       |       |       |       |       |       |       |
| GJ406  | -4.723 | -4.438 | -4.304 |       |       | -4.693 | -4.556 | -4.724 | -4.636 |       | -4.590 | -4.639 |       |       |       |       |
| GJ411  |       | -4.832 | -4.681 | -4.564 |       | -4.996 | -4.933 | -5.056 | -5.020 | -4.935 | -4.809 | -4.691 | -4.934 | -4.630 |       |       |
| GJ412A |       | -4.919 | -4.808 | -4.666 |       | -5.137 |       |       |       |       |       |       |       |       |       |       |
| GJ436  | -4.963 | -4.919 | -4.754 | -4.618 |       | -5.091 | -5.008 | -5.134 | -5.055 | -5.065 | -4.885 | -4.731 | -4.978 | -4.647 |       |       |
| GJ526  | -4.905 | -4.829 | -4.696 |       |       | -5.122 | -5.131 | -5.169 | -5.064 | -4.932 | -5.030 |       |       |       |       |       |
| GJ581  | -4.918 | -4.865 | -4.667 | -4.506 |       | -4.929 | -5.022 | -4.941 |       | -4.764 | -4.873 |       |       |       |       |       |
| GJ611B | -4.714 |       | -4.490 | -4.388 | -4.936 | -4.732 | -4.690 | -4.852 | -4.719 | -4.650 | -4.481 | -4.702 |       |       |       |       |
| GJ649  |       | -5.041 | -5.050 | -4.867 |       | -5.289 | -5.279 | -5.402 | -5.336 | -5.219 | -5.013 | -5.252 |       |       |       |       |
| GJ686  | -4.851 | -4.777 | -4.538 |       |       | -4.841 | -4.869 | -4.909 | -4.942 | -4.900 | -4.747 | -4.629 | -4.868 | -4.586 |       |       |
| GJ687  | -4.866 |       | -4.658 | -4.516 |       | -4.837 | -4.734 | -4.892 | -4.860 | -4.794 | -4.703 |       |       |       |       |       |
| GJ725A | -4.834 | -4.794 | -4.599 | -4.447 |       | -4.813 | -4.768 | -4.862 | -4.813 | -4.743 | -4.664 | -4.560 | -4.801 |       |       |       |
| GJ725B | -4.810 | -4.725 | -4.580 | -4.411 |       | -4.903 | -4.870 | -4.956 | -4.934 | -4.909 | -4.788 | -4.645 | -4.874 |       |       |       |
| GJ786.1B |       |       |       | -4.670 | -5.520 |       |       |       |       |       |       |       |       |       |       |       |
| GJ77B  | -4.649 |       |       | -4.362 |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ783.2B |       |       |       |       | -4.522 | -4.417 |       |       |       |       |       |       |       |       |       |       |
| GJ797B-NE |       |       |       |       |       | -4.805 | -4.716 | -4.884 | -4.797 | -4.715 | -4.625 | -4.540 | -4.827 |       |       |       |
| GJ797B-SW |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| HIP57050 |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| HIP79431 |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

Table 3. Log (W/λ)\textsubscript{obs} of H\textsubscript{2}O Blends in the Region A of 38 M Dwarfs\textsuperscript{1}.

\textsuperscript{1} The spectroscopic data for the representative line(s) in the H\textsubscript{2}O blends through Ref. nos. A01 to A17 are given in Table 2 for the corresponding Ref. nos. Complete listing of this table is available in the online version of the journal.

\* The equivalent widths are measured by referring to the pseudo-continuum.
Table 4. H$_2$O Lines in the Region B.

| Ref. | $\lambda_{\text{obs}}$ | $\lambda$ | $v_1^*v_2^*v_3^*$ | $v_1$ | $v_2$ | $v_3$ | $J$ | $K_a$ | $K_c$ | $J'$ | $K_a'$ | $K_c'$ | log $gf$ | L.E.P.$^\dagger$ |
|------|------------------|---------|------------------|-------|-------|-------|-----|-------|-------|-----|-------|-------|--------|----------|
| B01  | 22517.5          | 22517.619 | 0.20 | 0.00 | 0.00 | 17 | 8 | 9 | 16 | 5 | 12 | -5.910 | 3639.538 |
| B02  | 22528.7          | 22528.588 | 0.11 | 0.10 | 0.10 | 18 | 8 | 10 | 17 | 6 | 11 | -5.283 | 5965.714 |
| B03  | 22537.6          | 22537.447 | 1.00 | 0.00 | 0.00 | 20 | 7 | 14 | 19 | 4 | 15 | -5.410 | 4851.821 |
| B04  | 22545.3          | 22545.037 | 0.11 | 0.10 | 0.10 | 16 | 6 | 11 | 15 | 4 | 12 | -6.015 | 4736.242 |
| B05  | 22560.1          | 22560.082 | 1.00 | 0.00 | 0.20 | 14 | 9 | 5 | 13 | 7 | 6 | -6.016 | 4643.871 |
| B06  | 22564.2          | 22564.066 | 0.01 | 0.00 | 0.00 | 15 | 9 | 7 | 14 | 7 | 8 | -5.924 | 3646.337 |
| B07  | 22579.5          | 22579.191 | 0.01 | 0.00 | 0.10 | 21 | 11 | 10 | 20 | 10 | 11 | -5.462 | 8202.864 |
| B08  | 22585.6          | 22585.514 | 0.11 | 0.01 | 0.10 | 15 | 5 | 11 | 14 | 3 | 12 | -5.741 | 4814.832 |
| B09  | 22592.7          | 22592.391 | 0.30 | 0.01 | 0.10 | 21 | 11 | 10 | 20 | 10 | 11 | -5.291 | 4296.031 |
| B10  | 22604.4          | 22604.469 | 0.11 | 0.00 | 0.00 | 16 | 8 | 9 | 15 | 6 | 10 | -5.188 | 5807.121 |
| B11  | 22621.4          | 22621.393 | 0.11 | 0.10 | 0.00 | 20 | 7 | 13 | 19 | 5 | 14 | -5.011 | 6725.862 |
| B12  | 22628.6          | 22628.605 | 0.11 | 0.10 | 0.14 | 13 | 11 | 13 | 1 | 12 | -6.054 | 3654.049 |
| B13  | 22639.4          | 22639.680 | 1.00 | 0.00 | 0.19 | 15 | 4 | 14 | 8 | 15 | -5.718 | 4201.859 |
| B14  | 22647.4          | 22647.348 | 0.01 | 0.00 | 0.20 | 20 | 8 | 12 | 19 | 6 | 13 | -5.040 | 5199.596 |
| B15  | 22649.3          | 22649.543 | 0.21 | 0.20 | 0.14 | 3 | 11 | 13 | 1 | 12 | -5.868 | 3235.958 |
| B16  | 22681.9          | 22682.061 | 0.01 | 0.00 | 0.14 | 13 | 3 | 11 | 13 | 1 | 12 | -6.348 | 2042.311 |
| B17  | 22709.1          | 22709.215 | 0.11 | 0.00 | 0.18 | 10 | 17 | 6 | 11 | -5.290 | 4291.907 |
| B18  | 22724.7          | 22724.508 | 0.11 | 0.10 | 0.15 | 6 | 10 | 14 | 4 | 11 | -5.585 | 4396.051 |
| B19  | 22740.6          | 22740.486 | 0.21 | 0.20 | 0.13 | 4 | 10 | 12 | 2 | 11 | -5.895 | 4967.491 |
| B20  | 22763.1          | 22762.836 | 0.01 | 0.00 | 0.13 | 9 | 4 | 12 | 7 | 5 | -6.617 | 2613.104 |
| 200 | 22763.025         | 22763.592 | 0.30 | 0.10 | 0.19 | 11 | 8 | 18 | 10 | 9 | -5.555 | 7293.308 |
| B21  | 22768.1          | 22768.193 | 0.01 | 0.00 | 0.00 | 12 | 10 | 2 | 11 | 8 | 3 | -6.765 | 2522.265 |
| B22  | 22791.5          | 22791.547 | 0.11 | 0.10 | 0.18 | 6 | 12 | 17 | 4 | 13 | -5.052 | 5680.547 |
| B23  | 22801.6          | 22801.865 | 0.01 | 0.00 | 0.15 | 6 | 10 | 14 | 4 | 11 | -5.740 | 2746.023 |
| B24  | 22839.2          | 22838.883 | 0.11 | 0.10 | 0.13 | 4 | 10 | 12 | 2 | 11 | -6.056 | 3386.379 |
| B25  | 22907.9          | 22907.998 | 0.11 | 0.10 | 0.13 | 3 | 10 | 12 | 1 | 11 | -6.539 | 3386.052 |
| B26  | 22917.9          | 22918.090 | 0.01 | 0.00 | 0.00 | 18 | 7 | 11 | 17 | 5 | 12 | -5.093 | 4174.288 |
| B27  | 22931.0          | 22930.801 | 0.11 | 0.10 | 0.16 | 7 | 9 | 15 | 5 | 10 | -5.317 | 5015.704 |

$^*$ reference number to Fig. 3.
$^\dagger$ in cm$^{-1}$.
$^\dagger$ in A and in vacuum.
Table 5. Log \((W/\lambda)_{\text{obs}}^*\) of H\(_2\)O Blends in the Region B of 20 M Dwarfs\(^\dagger\).

| obj.      | B01   | B02   | B03   | B04   | B05   | B06   | B07   | B08   | B09   | B10   | B11   | B12   | B13   | B14   | B15   | B16   | B17   | B18   | B19   | B20   | B21   | B22   | B23   | B24   | B25   | B26   | B27   |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| GJ105B    | -5.229| -5.078| -5.268|       |       |       | -5.262|       |       |       | -5.152| -5.128| -5.454| -5.245| -5.322|       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ166C    | -5.191| -5.060| -5.351| -4.884| -5.172| -5.216| -5.274| -5.140| -5.133| -5.244| -5.081| -5.138| -4.981| -5.105| -4.910| -5.322| -5.017| -5.038|       |       |       |       |       |       |       |       |       |       |       |       |
| GJ179     | -5.439| -5.321| -5.470| -5.430| -5.392| -5.194|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ241.1B  | -5.421| -5.307| -5.432| -5.322|       | -5.189| -5.433| -5.290|       |       | -5.102| -5.432| -5.302| -5.495|       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ273     | -5.526| -5.249| -5.320| -5.259|       |       | -5.023| -5.256| -5.320|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ324B    | -5.297| -5.269| -5.399| -5.555| -5.207|       | -5.505|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ406     | -5.091| -4.973| -5.210| -5.071|       | -5.212|       |       |       |       | -5.106| -5.158| -5.286| -5.317| -5.465| -5.103| -5.050|       | -5.273| -5.022| -5.186|       |       |       |       |       |
| GJ581     | -5.304| -5.404|       | -5.157|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ611B    | -4.918| -5.064| -5.255| -5.267| -5.217| -5.398| -4.893|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ687     | -5.249|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ725A    | -5.323| -5.227| -5.386| -5.306| -5.380| -5.600| -5.041| -5.400| -5.269|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ725B    | -5.117| -5.096|       | -5.246| -5.374| -5.588| -5.011| -5.371| -5.295| -5.349| -5.261|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ768.1B  | -5.262|       | -5.183|       |       | -4.999|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ777B    | -5.106| -5.128| -5.183| -5.130| -5.350|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| GJ876     | -5.243|       | -5.350|       |       | -5.507|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
| HIP57050  | -5.290| -5.210|       | -5.311| -5.277| -5.605| -5.115| -5.426| -5.389|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |

\(^*\) The equivalent widths are measured by referring to the pseudo-continuum.

\(^\dagger\) The spectroscopic data for the representative line(s) in the H\(_2\)O blends through Ref. nos. B01 to B27 are given in Table 4 for the corresponding Ref. nos. Complete listing of this table is available in the online version of the journal.
| obj. | log \( A_C \) | \( \Delta \log A_{O} \) | \( \log A_{O} \) ± p.e. | \( N_A \) | \( \chi^2 \) | \( \chi^2 \) | \( \chi^2 \) |
|------|-------------|----------------|----------------|-------|-------------|-------------|-------------|
| GJ15A | -3.601 | -0.012 | -3.221 ± 0.013 | 10 | 3.947 | 2.644 | 3.987 |
| GJ105B | -3.468 | +0.026 | -3.142 ± 0.033 | 12 | 7.660 | 7.554 | 9.413 |
| GJ166C | -3.366 | +0.039 | -3.027 ± 0.043 | 8 | 14.971 | 12.498 | 11.775 |
| GJ176 | -3.352 | -0.101 | -3.153 ± 0.021 | 10 | 8.747 | 4.841 | 7.720 |
| GJ179 | -3.427 | -0.074 | -3.201 ± 0.021 | 12 | 7.146 | 4.549 | 6.724 |
| GJ205 | -3.116 | -0.152 | -2.968 ± 0.046 | 5 | 2.519 | 4.037 | 9.772 |
| GJ212 | -3.296 | -0.101 | -3.097 ± 0.027 | 7 | 2.553 | 2.134 | 4.696 |
| GJ229 | -3.271 | -0.133 | -3.104 ± 0.022 | 7 | 1.969 | 2.122 | 6.164 |
| GJ231.1B | -3.572 | +0.037 | -2.935 ± 0.035 | 14 | 4.839 | 5.106 | 7.874 |
| GJ250B | -3.413 | -0.078 | -3.191 ± 0.031 | 12 | 3.962 | 2.618 | 4.624 |
| GJ273 | -3.403 | -0.015 | -3.118 ± 0.023 | 11 | 15.290 | 12.533 | 14.772 |
| GJ324B | -3.360 | -0.103 | -3.163 ± 0.041 | 14 | 9.319 | 6.069 | 9.544 |
| GJ338A | -3.587 | -0.030 | -3.317 ± 0.026 | 4 | 1.757 | 1.700 | 2.668 |
| GJ338B | -3.578 | -0.034 | -3.312 ± 0.058 | 6 | 1.542 | 2.167 | 4.166 |
| GJ406 | -3.612 | -0.088 | -3.400 ± 0.023 | 10 | 10.510 | 13.204 | 22.045 |
| GJ411 | -3.671 | -0.001 | -3.372 ± 0.030 | 14 | 7.784 | 5.624 | 8.727 |
| GJ412A | -3.849 | +0.019 | -3.530 ± 0.029 | 10 | 6.765 | 4.423 | 7.272 |
| GJ436 | -3.634 | -0.082 | -3.416 ± 0.018 | 15 | 7.506 | 3.676 | 7.974 |
| GJ526 | -3.553 | -0.025 | -3.278 ± 0.026 | 11 | 6.650 | 4.851 | 9.105 |
| GJ581 | -3.560 | -0.015 | -3.275 ± 0.021 | 11 | 9.629 | 7.148 | 10.703 |
| GJ611B | -3.762 | +0.137 | -3.325 ± 0.053 | 13 | 11.089 | 10.892 | 12.478 |
| GJ649 | -3.544 | -0.094 | -3.338 ± 0.026 | 12 | 2.819 | 1.304 | 3.019 |
| GJ686 | -3.497 | -0.033 | -3.230 ± 0.030 | 8 | 3.416 | 3.157 | 5.112 |
| GJ687 | -3.428 | -0.090 | -3.218 ± 0.030 | 14 | 13.681 | 6.855 | 14.452 |
| GJ725A | -3.584 | +0.035 | -3.249 ± 0.037 | 13 | 5.835 | 6.069 | 8.940 |
| GJ725B | -3.605 | +0.046 | -3.259 ± 0.053 | 14 | 9.752 | 9.755 | 12.871 |
| GJ768.1C | -3.504 | -0.028 | -3.232 ± 0.022 | 12 | 1.789 | 1.278 | 1.913 |
| GJ777B | -3.239 | -0.045 | -2.984 ± 0.078 | 5 | 2.560 | 2.104 | 2.164 |
| GJ783.2B | -3.413 | -0.003 | -3.116 ± 0.051 | 13 | 12.113 | 10.850 | 13.617 |
| GJ797B-NE | -3.535 | -0.053 | -3.288 ± 0.021 | 13 | 2.152 | 1.265 | 1.954 |
| GJ797B-SW | -3.506 | -0.057 | -3.263 ± 0.024 | 12 | 2.127 | 1.255 | 1.999 |
| GJ809 | -3.550 | -0.118 | -3.368 ± 0.031 | 8 | 3.943 | 1.809 | 2.146 |
| GJ849 | -3.270 | -0.116 | -3.086 ± 0.016 | 11 | 8.274 | 5.173 | 11.741 |
| GJ876 | -3.355 | -0.089 | -3.144 ± 0.030 | 12 | 5.098 | 3.155 | 5.331 |
| GJ880 | -3.348 | -0.133 | -3.181 ± 0.034 | 6 | 2.890 | 1.897 | 5.065 |
| GJ3348B | -3.400 | -0.004 | -3.104 ± 0.030 | 10 | 1.309 | 1.097 | 1.460 |
| HIP7050 | -3.259 | -0.093 | -3.052 ± 0.038 | 13 | 13.003 | 8.453 | 13.002 |
| HIP79431 | -3.296 | -0.175 | -3.111 ± 0.014 | 11 | 8.007 | 3.314 | 13.338 |

* from Paper I.
† The correction to be applied to the initial assumed value of \( \log A_{O} = \log A_{C} + 0.30 \).
‡ Number of H₂O blends used in the analysis (see Table 3 as for the Ref. Nos. and EWs of the blends used).
§ \( \chi^2 \) value for the fitting of the predicted spectrum with \( \log A_{O} \) - 0.05 and the observed one.
‖ \( \chi^2 \) value for the fitting of the predicted spectrum with \( \log A_{O} \) + 0.05 and the observed one.
⊥ \( \chi^2 \) value for the fitting of the predicted spectrum with \( \log A_{O} \) in the fourth column and the observed one (see Fig. 7).
Table 7. Oxygen Abundances ($\log A^B_O$) from the H$_2$O Lines in the Region B.

| obj.          | $\Delta \log A^A_O$ | $\Delta \log A^A_O$ ± p.e. * | $\Delta \log A^B_O$ | $\log A^B_O$ ± p.e. | $N_B$ † | diff § |
|--------------|----------------------|--------------------------------|----------------------|----------------------|---------|--------|
| GJ 105B      | +0.026               | -3.142 ± 0.033                 | +0.040               | -3.128 ± 0.050       | 13      | +0.014 |
| GJ 166C      | +0.039               | -3.027 ± 0.043                 | +0.033               | -3.033 ± 0.074       | 18      | -0.006 |
| GJ 179       | -0.074               | -3.201 ± 0.021                 | -0.044               | -3.171 ± 0.062       | 16      | +0.030 |
| GJ 231.1B    | +0.037               | -3.235 ± 0.035                 | +0.028               | -3.244 ± 0.064       | 15      | -0.009 |
| GJ 273       | -0.015               | -3.118 ± 0.023                 | -0.034               | -3.137 ± 0.047       | 17      | -0.019 |
| GJ 324B      | -0.103               | -3.163 ± 0.041                 | -0.112               | -3.172 ± 0.040       | 17      | -0.009 |
| GJ 406       | -0.088               | -3.400 ± 0.023                 | -0.086               | -3.398 ± 0.032       | 20      | +0.002 |
| GJ 581       | -0.015               | -3.275 ± 0.021                 | +0.024               | -3.236 ± 0.047       | 10      | +0.039 |
| GJ 611B      | +0.137               | -3.325 ± 0.053                 | +0.087               | -3.375 ± 0.058       | 21      | -0.050 |
| GJ 687       | -0.090               | -3.218 ± 0.030                 | -0.093               | -3.221 ± 0.034       | 12      | -0.003 |
| GJ 725A      | +0.035               | -3.249 ± 0.037                 | +0.004               | -3.280 ± 0.050       | 19      | -0.031 |
| GJ 725B      | +0.046               | -3.259 ± 0.053                 | -0.001               | -3.306 ± 0.062       | 22      | -0.047 |
| GJ 768.1C    | -0.028               | -3.232 ± 0.022                 | +0.037               | -3.167 ± 0.056       | 11      | +0.065 |
| GJ 777B      | -0.045               | -2.984 ± 0.078                 | -0.092               | -3.031 ± 0.037       | 18      | -0.047 |
| GJ 783.2B    | -0.003               | -3.116 ± 0.051                 | -0.035               | -3.148 ± 0.051       | 22      | -0.032 |
| GJ 797B-NE   | -0.053               | -3.288 ± 0.021                 | -0.034               | -3.269 ± 0.049       | 8       | +0.019 |
| GJ 797B-SW   | -0.057               | -3.263 ± 0.024                 | -0.045               | -3.251 ± 0.036       | 7       | +0.012 |
| GJ 876       | -0.089               | -3.144 ± 0.030                 | -0.065               | -3.120 ± 0.056       | 7       | +0.024 |
| GJ 3348B     | -0.004               | -3.104 ± 0.030                 | +0.041               | -3.059 ± 0.063       | 12      | +0.045 |
| HIP 57050    | -0.093               | -3.052 ± 0.038                 | -0.084               | -3.043 ± 0.047       | 19      | +0.009 |

* results from H$_2$O Lines in the region A (see Table 6).
† The correction to be applied to the initial assumed value of $\log A^0_O = \log A^C_O + 0.30$ ($\log A^C_O$ is from Paper I and reproduced in the second column of Table 6).
‡ Number of H$_2$O blends used in the analysis (see Table 5 as for the Ref. Nos. and EWs of the blends used).
§ diff = $\log A^B_O - \log A^A_O$. 
| obj.          | log $A_C$ ± p.e.* | log $A_O$ ± p.e.† | log $A_O/A_C$ |
|--------------|-------------------|-------------------|---------------|
| GJ 15A       | -3.60 ± 0.11      | -3.11 ± 0.01A     | 0.29          |
| GJ 105B      | -3.47 ± 0.06      | -3.14 ± 0.04      | 0.33          |
| GJ 166C      | -3.37 ± 0.14      | -3.03 ± 0.06      | 0.34          |
| GJ 176‡      | -3.35 ± 0.07      | -3.15 ± 0.02A     | 0.20          |
| GJ 179‡      | -3.43 ± 0.11      | -3.18 ± 0.04      | 0.24          |
| GJ 205       | -3.12 ± 0.08      | -2.97 ± 0.05A     | 0.15          |
| GJ 212       | -3.30 ± 0.11      | -3.10 ± 0.03A     | 0.20          |
| GJ 229       | -3.27 ± 0.07      | -3.10 ± 0.02A     | 0.17          |
| GJ 231.1B    | -3.57 ± 0.05      | -3.24 ± 0.05      | 0.33          |
| GJ 250B      | -3.41 ± 0.10      | -3.19 ± 0.03A     | 0.22          |
| GJ 273       | -3.40 ± 0.11      | -3.13 ± 0.04      | 0.27          |
| GJ 324B      | -3.36 ± 0.13      | -3.17 ± 0.04      | 0.19          |
| GJ 338A      | -3.59 ± 0.04      | -3.32 ± 0.03A     | 0.27          |
| GJ 338B      | -3.58 ± 0.04      | -3.31 ± 0.06A     | 0.27          |
| GJ 406       | -3.61 ± 0.10      | -3.40 ± 0.03      | 0.21          |
| GJ 411       | -3.67 ± 0.06      | -3.37 ± 0.03A     | 0.30          |
| GJ 412A      | -3.85 ± 0.04      | -3.53 ± 0.03A     | 0.32          |
| GJ 436‡      | -3.63 ± 0.06      | -3.42 ± 0.02A     | 0.22          |
| GJ 526       | -3.55 ± 0.04      | -3.28 ± 0.03A     | 0.27          |
| GJ 581‡      | -3.56 ± 0.05      | -3.26 ± 0.03      | 0.30          |
| GJ 611B      | -3.76 ± 0.03      | -3.36 ± 0.06      | 0.41          |
| GJ 649‡      | -3.54 ± 0.04      | -3.34 ± 0.03A     | 0.21          |
| GJ 686       | -3.50 ± 0.04      | -3.23 ± 0.03A     | 0.27          |
| GJ 687       | -3.43 ± 0.09      | -3.22 ± 0.03      | 0.21          |
| GJ 725A      | -3.58 ± 0.09      | -3.27 ± 0.05      | 0.32          |
| GJ 725B      | -3.61 ± 0.08      | -3.29 ± 0.06      | 0.32          |
| GJ 768.1C    | -3.50 ± 0.08      | -3.20 ± 0.04      | 0.30          |
| GJ 777B      | -3.24 ± 0.16      | -3.02 ± 0.05      | 0.22          |
| GJ 783.2B    | -3.41 ± 0.10      | -3.14 ± 0.05      | 0.28          |
| GJ 797B-NE   | -3.54 ± 0.04      | -3.28 ± 0.03      | 0.25          |
| GJ 797B-SW   | -3.51 ± 0.09      | -3.26 ± 0.03      | 0.25          |
| GJ 809       | -3.55 ± 0.04      | -3.37 ± 0.03A     | 0.18          |
| GJ 849‡      | -3.27 ± 0.09      | -3.09 ± 0.02A     | 0.18          |
| GJ 876‡      | -3.36 ± 0.13      | -3.14 ± 0.04      | 0.22          |
| GJ 880       | -3.50 ± 0.07      | -3.18 ± 0.03A     | 0.17          |
| GJ 3348B     | -3.40 ± 0.11      | -3.08 ± 0.05      | 0.32          |
| HIP 57050‡   | -3.26 ± 0.12      | -3.05 ± 0.04      | 0.21          |
| HIP 79431‡   | -3.24 ± 0.12      | -3.11 ± 0.01A     | 0.12          |
| the Sun1§    | -3.44 ± 0.04      | -3.07 ± 0.05      | 0.37          |
| the Sun2§    | -3.57 ± 0.05      | -3.31 ± 0.05      | 0.26          |

* From Paper I.
† For the case with A attached, the same as log $A_O^A$ in Tables 6. Otherwise, the weighted mean of log $A_O^A$ and log $A_O^B$ from Tables 6 and 7, respectively.
‡ Planet hosting M dwarf.
§ Anders & Grevesse (1989).
¶ Asplund et al. (2009).
We apply different Errors estimated from those of the photometric data.

The spectral types for M dwarfs we analyze in this paper are by Joy & Abt (1974) and those for other objects from Tsuji et al. (1996).

Based on new angular diameter measurements by von Braun et al. (2014), which has been published after Paper I.

Table 10. Fundamental Parameters Based on T eff by the Interferometry.

| obj.   | sp.ty.* | p(msec)† | F 3.4 (mag)‡ | M 3.4 (mag)§ | F 3.4 (mag)¶ | M 3.4 (mag)∥ | T eff  |
|--------|---------|----------|-------------|-------------|-------------|-------------|--------|
| GJ 15A | dM2.5   | 278.76 ± 0.77 | 3.853 ± 0.099 | 6.08 ± 0.10 | 3.872 ± 0.381 | 6.10 ± 0.39 | 3563 ± 11 |
| GJ 53A | G5Vp    | 132.38 ± 0.82 | 3.326 ± 0.141 | 3.94 ± 0.15 | 3.311 ± 0.487 | 3.92 ± 0.50 | 5348 ± 26 |
| GJ 75  | K0V     | 99.33 ± 0.53  | 3.753 ± 0.091 | 3.74 ± 0.10 | 3.725 ± 0.278 | 3.71 ± 0.29 | 5398 ± 75 |
| GJ 176#| dM2.5e  | 107.83 ± 2.85 | 5.434 ± 0.066 | 5.60 ± 0.12 | 5.486 ± 0.177 | 5.65 ± 0.23 | 3679 ± 77 |
| GJ 205 | dM3     | 176.77 ± 1.18 | 3.743 ± 0.120 | 4.98 ± 0.13 | 3.790 ± 0.519 | 5.03 ± 0.53 | 3801 ± 9  |
| GJ 412A| dM2     | 206.27 ± 1.00 | 4.638 ± 0.085 | 6.21 ± 0.10 | 4.641 ± 0.312 | 6.21 ± 0.32 | 3497 ± 39 |
| GJ 436 | dM3.5   | 98.61 ± 2.33  | 5.987 ± 0.052 | 5.96 ± 0.10 | 6.017 ± 0.110 | 5.99 ± 0.16 | 3416 ± 53 |
| GJ 526 | dM3     | 185.49 ± 1.10 | 4.372 ± 0.095 | 5.71 ± 0.11 | 4.191 ± 0.426 | 5.53 ± 0.44 | 3618 ± 31 |
| GJ 551 | M5.5V   | 771.64 ± 2.60 | 4.195 ± 0.086 | 8.63 ± 0.09 | 4.207 ± 0.331 | 8.64 ± 0.34 | 3054 ± 79 |
| GJ 570A| K4V     | 171.22 ± 0.94 | 3.159 ± 0.012 | 4.33 ± 0.02 | 3.474 ± 0.387 | 4.64 ± 0.40 | 4507 ± 58 |
| GJ 581 | dM4     | 160.91 ± 2.62 | 5.694 ± 0.055 | 6.73 ± 0.09 | 5.743 ± 0.094 | 6.78 ± 0.13 | 3442 ± 54 |
| GJ 631 | K0V     | 102.55 ± 0.40 | 3.797 ± 0.110 | 3.85 ± 0.12 | 3.882 ± 0.325 | 3.94 ± 0.33 | 5337 ± 41 |
| GJ 649#| dM2     | 96.67 ± 1.39  | 5.502 ± 0.065 | 5.43 ± 0.10 | 5.559 ± 0.139 | 5.49 ± 0.17 | 3690 ± 45 |
| GJ 687 | dM4     | 220.84 ± 0.94 | 4.397 ± 0.094 | 6.12 ± 0.10 | 4.398 ± 0.268 | 6.12 ± 0.28 | 3413 ± 28 |
| GJ 725A| dM4     | 280.18 ± 2.18 | 4.498 ± 0.226 | 6.74 ± 0.24 | 4.065 ± 0.494 | 6.30 ± 0.51 | 3407 ± 15 |
| GJ 809 | dM2     | 141.87 ± 0.64 | 4.501 ± 0.088 | 5.26 ± 0.10 | 4.518 ± 0.272 | 5.28 ± 0.28 | 3692 ± 22 |
| GJ 876#| dM4.5   | 213.28 ± 2.12 | 4.844 ± 0.077 | 6.49 ± 0.10 | 4.863 ± 0.243 | 6.51 ± 0.26 | 3129 ± 19 |
| GJ 880 | dM2.5   | 146.09 ± 1.00 | 4.432 ± 0.080 | 5.26 ± 0.09 | 4.396 ± 0.353 | 5.22 ± 0.37 | 3713 ± 11 |

Table 11. Fundamental Parameters Based on T eff by the Infrared Flux Method.

| obj.   | sp.ty.* | p(msec)† | F 3.4 (mag)‡ | M 3.4 (mag)§ | F 3.4 (mag)¶ | M 3.4 (mag)∥ | T eff  |
|--------|---------|----------|-------------|-------------|-------------|-------------|--------|
| GJ 273-L**| dM4  | 262.98 ± 1.39 | 4.723 ± 0.074 | 6.82 ± 0.09 | 4.787 ± 0.223 | 6.89 ± 0.23 | 3150 ± 95 |
| GJ 406 | dM6.5e  | 419.10 ± 2.10 | 5.807 ± 0.055 | 8.92 ± 0.07 | 5.836 ± 0.138 | 8.95 ± 0.15 | 2800 ± 85 |
| GJ 644C| M7V     | 148.92 ± 4.00 | 8.588 ± 0.023 | 9.45 ± 0.08 | 8.619 ± 0.023 | 9.48 ± 0.08 | 2640 ± 80 |
| GJ 752A| M3V     | 170.36 ± 1.00 | 4.466 ± 0.078 | 5.62 ± 0.09 | 4.379 ± 0.262 | 5.54 ± 0.27 | 3475 ± 105 |
| GJ 752B| M8V     | 170.36 ± 1.00 | 8.465 ± 0.023 | 9.62 ± 0.04 | 8.317 ± 0.023 | 9.47 ± 0.04 | 2250 ± 70 |
| GJ 884 | dM0.5   | 121.69 ± 0.69 | 4.424 ± 0.087 | 4.85 ± 0.10 | 4.324 ± 0.273 | 4.75 ± 0.29 | 3850 ± 115 |
| GJ 3849| M9V     | 95.00 ± 5.70  | 10.431 ± 0.203 | 10.32 ± 0.15 | 10.428 ± 0.022 | 10.32 ± 0.15 | 2130 ± 65 |

* The spectral types for M dwarfs we analyze in this paper are by Joy & Abt (1974) and those for other objects from Tsuji et al. (1996).
† Parallax by Hipparcos (van Leeuwen 2007), except for GJ 406 and GJ 644C by RECONS and GJ 3849 by Gliese & Jahreiss (1991).
‡ W1 band flux centered at 3.4 μm from the WISE All-Sky Release (Wright et al. 2010).
§ Absolute magnitude at 3.4 μm based on the F 3.4 in the preceding column.
¶ W1 band flux centered at 3.4 μm from the AllWISE Catalog (Wright et al. 2010).
∥ Errors estimated from those of the photometric data.
** We apply different T eff values - low and high - for this object, and L implies that this is the case of low T eff (see subsection 3.1).