Spin Polarization at Semiconductor Point Contacts in Absence of Magnetic Field

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Semiconductor point contacts can be a useful tool for producing spin-polarized currents in the presence of spin-orbit (SO) interaction. Neither magnetic fields nor magnetic materials are required. By numerical studies, we show that (i) the conductance is quantized in units of $2e^2/h$ unless the SO interaction is too strong, (ii) the current is spin-polarized in the transverse direction, and (iii) a spin polarization of more than 50% can be realized with experimentally accessible values of the SO interaction strength. The spin-polarization ratio is determined by the adiabaticity of the transition between subbands of different spins during the transport through the point contacts.

KEYWORDS: point contact, Rashba, spin-orbit interaction, conductance quantization, spin filter, Landau-Zener

Generating spin-polarized currents in semiconductors is an important issue for the development of spin-based electronics, “spintronics.” To manipulate electron spins, the Rashba spin-orbit (SO) interaction is useful since its strength is locally controllable by applying an electric field. Several spin-filtering devices for producing the spin currents have been proposed utilizing the SO interaction, e.g., three-terminal devices related to the spin Hall effect, a triple-barrier tunnel diode, a one-dimensional system with a magnetic field, and a three-terminal device for the Stern-Gerlach experiment using a nonuniform SO interaction.

In the present paper, we theoretically study the ballistic transport through a semiconductor point contact (quantum wire with a narrow constriction) in the presence of Rashba SO interaction. In its absence, it is well known that the conductance is quantized in units of $2e^2/h$ when the constriction changes gradually in space. By numerical studies, we show that the conductance is quantized even with the SO interaction and that the current is spin-polarized in the transverse direction, in the absence of magnetic field. We demonstrate that the polarization ratio can be more than 50% in InGaAs heterostructures. The spin current is obtained generally, e.g., in GaAs heterostructures, if the condition later described by eq. (9) is fulfilled with not only Rashba but also Dresselhaus SO interaction. As spin filters, two-terminal devices with point contacts are easy to fabricate on semiconductors, compared with other devices that have been proposed.

We consider a two-dimensional electron gas confined in the z direction. The electric field in the z direction results in the Rashba SO interaction,

$$H_{RSO} = \frac{\alpha}{\hbar} (p_y \sigma_x - p_x \sigma_y),$$

(1)

where $\sigma_x$ and $\sigma_y$ are Pauli matrices. (The Dresselhaus SO interaction is discussed later.) We use a dimensionless parameter, $k_{\alpha}/k_F$, where

$$k_{\alpha} = m \alpha / \hbar^2$$

(2)

with $m$ being the effective mass and $k_F$ is the Fermi wavenumber. In InGaAs heterostructures, $\alpha = (3 \sim 4) \times 10^{-11}$ eV m and $\Delta_R \equiv 2k_\alpha \alpha = 15 \sim 20$ meV [$k_{\alpha}/k_F = \Delta_R/(4\hbar^2) \approx 0.1$], Electron-electron interaction and impurity scattering are neglected. Electrons propagate in a quantum wire along the x direction, with width $W_0$ in the y direction. A hard-wall confinement potential $U(y)$ is assumed, $U(y) = 0$ for $-W_0/2 < y < W_0/2$ and $\infty$ otherwise. For a narrow constriction around $x = y = 0$, we consider an extra potential at $-L_1 < x < L_2$, which is analogous to that adopted in ref. 15:

$$V(x, y) = \frac{V_0}{2} \left(1 + \cos \frac{\pi x}{L_x}\right)$$

$$+ E_F \sum_{\pm} \left[\frac{y - y_\pm(x)}{\Delta}\right]^2 \theta(\pm[y - y_\pm(x)]),$$

(3)

with

$$y_\pm(x) = \frac{W_0}{4} \left(1 - \cos \frac{\pi x}{L_x}\right),$$

(4)

where $L_x = L_1 ( -L_1 < x < 0)$ and $L_x = L_2 (0 < x < L_2)$. $\theta(t)$ is a step function [$\theta(t) = 1$ for $t > 0$, 0 for $t < 0$]. For fixed $x$, $V(x, y)$ is flat at $y_-(x) < y < y_+(x)$ and grows with increasing $|y - y_\pm(x)|$ at $y > y_+(x)$ or $y < y_-(x)$. On a line of $y = 0$, the potential height is given by $(V_0/2)[1 + \cos(\pi x/L_x)]$, being maximal at $x = 0$. In the present paper, we fix $W_0 = 4\Delta$ and $\Delta = \lambda_F$, where $\lambda_F$ is the Fermi wavelength ($\lambda_F = 2\pi/k_F$).

Numerical calculations are performed using a tight-binding model on a square lattice ($-L_1 < x < L_2$, $-W_0/2 < y < W_0/2$), following refs. 15 and 16. The transmission coefficients are evaluated for incident electrons from the left side of the constriction ($x < -L_1$) to the right ($L_2 < x$), using the Green function’s recursion method. They yield the conductance $G$ through the Landauer formula.

Figure 1 shows the calculated results when $L_1 = L_2 = 4\Delta$. In Fig. 1(a), we plot $G$ as a function of $V_0$, potential height at $x = y = 0$. The conductance quantization is clearly observed when $k_{\alpha}/k_F = 0.25$ (solid line) as well as in the absence of SO interaction (broken line). The conductance quantization is broken for $k_{\alpha}/k_F > 0.5$ (not shown here), which is addressed later. We divide the output current into two components, one carried by spin-up electrons in the y direction ($S_y = 1/2$) and the other by spin-down electrons ($S_y = -1/2$). Figure 1(b) presents each conductance, $G_{\pm}$, as a function of $k_{\alpha}/k_F$. $V_0 = 0.7E_F$, where the total conductance is at the first plateau ($G = G_+ + G_- = 2e^2/h$). The spin polarization

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in the $y$ direction, $(G_+ - G_-)/(G_+ + G_-)$, increases with an increase in $k_0/k_F$. Note that incident electrons are unpolarized in eight conduction channels per spin direction. These results indicate that (i) the point contact works as a spin filter, (ii) the polarization ratio is about 30% with experimental values of $k_0/k_F ≈ 0.1$ in this case, and (iii) the conductance is still quantized.

These calculated results can be understood as follows. We divide the Hamiltonian into two parts: $H = H_0 + H'$,

$$H_0 = \frac{1}{2m}(p_x^2 + p_y^2) - \frac{\alpha}{\hbar} p_x \sigma_y + V(x, y) + U(y)$$

$$H' = \frac{\alpha}{\hbar} p_y \sigma_x$$

We treat $H'$ as a perturbation. When $V(x, y)$ is independent of $x$, the eigenstates and eigenvalues of $H_0$ are given by

$$\psi_{n,k,\pm} = e^{ikx} \varphi_n(y) \chi_\pm$$

$$E_{n,\pm}(k) = \frac{\hbar^2}{2m}(k \mp k_0)^2 - \frac{\hbar^2}{2m}k_{\alpha}^2 + \varepsilon_n$$

respectively, where $\varphi_n(y)$ are eigenstates of $\frac{p_y^2}{(2m)} + V(y) + U(y)$ with eigenvalues $\varepsilon_n (n = 1, 2, 3, \cdots)$. $\chi_{\pm}$ are eigenstates of $\sigma_y$, representing spin-up or -down states. The dispersion relations of $E_{n,\pm}(k)$ (subbands) are schematically shown in Figs. 2(a) and 2(b). The Fermi level is denoted by horizontal line $A$ in the quantum wire outside of the constriction, where $V(x, y) = 0$. (The number of channels is three per spin direction in the figures, which is smaller than that in our numerical studies.) There are two situations regarding the crossings between $E_{n,\pm}(k)$ and $E_{n',\pm}(k)$ $(n < n')$. When $k_0/k_F < (3/16)(\lambda_F/W_0)^2$, all the crossings take place above $E_F$ [Fig. 2(a)]. When $k_0/k_F > (3/16)(\lambda_F/W_0)^2$, some of them appear below $E_F$ [Fig. 2(b)].

Now we consider the transport of electrons through a constriction when the conductance shows the first plateau. Except in the vicinities of the above-mentioned crossings, we assume an adiabatic transport in which the wavefunction $\psi_{n,k,\pm}$ changes gradually remaining the quantum numbers of transverse motion $n$ and of spin $\pm$. The wavenumber $k$ changes with $x$, which is determined by an intersection between the subband and $E_F$; positive $k$’s for incident electrons. Let us begin with the case of Fig. 2(a). As electrons propagate from the wire to a narrow constriction ($x < 0$), the subbands shift upwards. Alternatively, we move $E_F$ downwards in Fig. 2(a). Similarly, at $x > 0$, we move $E_F$ upwards. [Separations between the subbands increase (decrease) with $x$ at $x < 0$ ($x > 0$), which is not shown in the figure.] Before the injection into the constriction (position $A$), there are six conduction modes, $(1, \pm), (2, \pm)$ and $(3, \pm)$. At position $B$, modes $(1, \pm)$ and $(2, \pm)$ are conducting, whereas modes $(3, \pm)$ have been completely reflected. At the narrowest region (position $C$), only modes $(1, \pm)$ exist. At $x > 0$, the modes propagate with the transmission probability of unity, which results in the conductance quantization, $G = 2e^2/h$. The small perturbation of $H'$ does not play an important role. No spin polarization is observed in this case.

In the case of Fig. 2(b), the situation is different. Electrons pass by the crossings twice, once at $x < 0$ and once at $x > 0$. Let us look at the crossing between modes
the crossing. In more gradual point contacts (smaller x should be evaluated at $\alpha / \hbar$).

A term of $(\alpha / \hbar) p_y \sigma_y$ in $H_{\text{SO}}$ is taken into account (a) only at $x > 0$ or (b) only at $x < 0$, whereas the other term, $-(\alpha / \hbar) p_x \sigma_x$, is considered in the whole region. The thin lines indicate $G_\pm$ when both the terms are present in the whole region.

(1, −) and (2, +). These subbands are mixed by the perturbation $H'$, as shown in Fig. 2(c). Hence these modes change to each other with a transition probability $P$ when electrons pass through the crossing. Around the first pass ($x < 0$), both modes are occupied by electrons just before the modes cross. Then spin-up electrons are flipped to spin-down with probability $P$, while spin-down electrons are flipped to spin-up with the same probability. Accordingly, no spin polarization takes place. Around the second pass ($x > 0$), on the other hand, mode (2, +) is empty while mode (1, −) is full of electrons just before the modes cross. Then spin-down electrons in the latter mode are spin-flipped to spin-up in the former mode with probability $P$. The spin-up electrons in mode (1, +) are transmitted through the constriction without passing by any mode crossing. Consequently we obtain the spin-polarization ratio of $| (1 + P) - (1 - P) | / 2 = P$.

The transition probability $P$ is evaluated using the Landau-Zener theory:  
\[ P = 1 - \exp(-2\pi \lambda), \tag{10} \]
where $\lambda = J^2/(\hbar |v|)$ represents the degree of adiabaticity, $J = |(2, +)|H'|[1, −] \rangle$ and $v = \hbar^2 [E(2, +) - E(1, −)]$, the velocity of the change of level spacing. $P = 1$ for $\lambda = \infty$ in the adiabatic limit, whereas $P = 0$ for $\lambda = 0$ in the sudden-change limit. If $V(x, y)$ is a hard-wall potential with width $W(x)$ in the $y$ direction, instead of by eq. (3), $\lambda$ is estimated as
\[ \lambda = k_u \left| \frac{1}{W} \frac{dW}{dx} \right|^{-1}, \tag{11} \]
apart from a numerical factor. $W$ and its derivative should be evaluated at $x$ where electrons pass by the crossing. In more gradual point contacts (smaller $\left| \frac{1}{W} \frac{dW}{dx} \right|$), electrons pass through the crossing more adiabatically (larger $\lambda$). Then the larger spin-flip probability $P$, and thus the larger spin-polarization ratio, is expected.

In spite of the spin polarization, the total conductance is not affected by the mode crossings, unless the SO interaction is too strong. In Fig. 2(b), both the derivatives of $E_{1, −}(k)$ and $E_{1, +}(k)$ are positive at their crossing. Since the group velocities have the same sign, the transition from one mode to the other is not accompanied by a reflection (forward scattering). If the SO interaction were so strong that the crossing occurred at $k < k_u$, backward scattering could take place, which would destroy the conductance quantization. The condition that the backward scattering does not take place is given by $2h^2 k_u^2 / m < \varepsilon_2 - \varepsilon_1$, or
\[ k_u / k_F < (\sqrt{3}/4)(\lambda_F / W) \tag{12} \]
for the hard-wall confinement of width $W(x)$. In eq. (12), $W$ is the width of confinement where modes (1, −) and (2, +) cross ($\lambda_F < W < W_0$). [In our numerical studies, this condition seems to be satisfied with $k_u/k_F < 0.25$. We could still observe a spin-polarized current when eq. (12) does not hold, as discussed later.]

We have so far presented a simple theory to explain the numerical results in Fig. 1, on the basis of the adiabatic approximation for the transport through the point contact and perturbative treatment of $H'$ in eq. (6). To verify this theory, we further perform numerical studies and present the results in Figs. 3 and 4.

In our theory, the spin-flip by $H'$ is important for the spin polarization during the transport from a narrow region to a wide region ($x > 0$). To confirm this idea, we examine two situations in Fig. 3: (a) $H'$ is present only at $x > 0$ or (b) only at $x < 0$. Indeed we observe the spin polarization in situation (a), whereas the polarization is not visible in situation (b).

The adiabaticity of the spin-flip transition can be controlled by changing the shape of point contacts, e.g. $(L_1, L_2)$ in our model. With increasing $L_2$, $\left| \frac{1}{W} \frac{dW}{dx} \right|$ decreases in eq. (11) and hence $P$ in eq. (10) increases.
As a result, a larger spin-polarization is expected. Figure 4(a) shows the numerical results with (A) \((L_1, L_2) = (4\lambda_F, 4\lambda_F), (B) (4\lambda_F, 8\lambda_F),\) and (C) \((4\lambda_F, 12\lambda_F).\) The polarization ratio increases with an increase in \(L_2,\) in accordance with our theory. In case (C), we observe a polarization of 60% with experimental values of \(\alpha\) in InGaAs heterostructures. Around crossings appear below \((n, +)\) with \(n > 1,\) in the spin-flip processes. If the obtained current is injected into another point contact in the absence of SO interaction, the height of the first plateau is suppressed to \(\sim (2 - P)c^2/\hbar\) when the polarization ratio is \(P.\)

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