Filter for strangeness in $J^{PC}$ exotic four–quark states

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Abstract

Symmetrization selection rules for the decay of four–quark states to two $J = 0$ mesons are analysed in a non–field theoretic context with isospin symmetry. The OZI allowed decay of an isoscalar $J^{PC} = \{1, 3, \ldots \}^{-+}$ exotic state to $\eta' \eta$ or $f'_0 f_0$ is only allowed for four–quark components of the state containing one $s \bar{s}$ pair, providing a filter for strangeness content in these states. Decays of four–quark $a_0$ states are narrower than otherwise expected. If the experimentally observed $1^{-+}$ enhancement in $\eta \pi$ is resonant, it is qualitatively in agreement with being a four–quark state.

Keywords: symmetrization, selection rule, four–quark state, decay, $J^{PC}$ exotic

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Ever since the original work in the MIT bag model, it has been recognized that multi–quark states containing strange quarks can often have lower energies than those with only the equivalent light (up or down) quarks, leading to the prediction of the stability of strangelets. For four–quark ($q\bar{q}q\bar{q}$) states, the same conclusion was reached in potential models.

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In this Letter symmetrization selection rule II [3], i.e. the case of isospin symmetry, is exhaustively analysed for the decay of four–quark states to two $J = 0$ hybrid or conventional mesons in QCD, expanding the earlier analysis [3]. Decay topologies of (hybrid) mesons and glueballs to two (hybrid) mesons were considered before [3]. The possibility of six–quark or higher multi–quark states is not considered. It is shown that certain decays signal the presence of strangeness in decaying $J^{PC}$ exotic four–quark states, providing an experimental tool to verify the claimed presence of strangeness in these states. Decays also allow us to distinguish between the hybrid, glueball or four–quark character of a decaying $J^{PC}$ exotic state. There are also implications for non–exotic four–quark states.

We first consider states built only from isospin $\frac{1}{2}$ quarks, i.e. $u$ and $d$ quarks. For four–quark states $A$ we are free to choose any basis to construct the flavour state. Labelling the quarks as $q_1\bar{q}_2q_3\bar{q}_4$, and grouping $q_1\bar{q}_2$ and $q_3\bar{q}_4$ (denoted by $X$ and $Y$) together, the four–quark flavour state is

$$|I_AI_XI_Y\rangle \equiv \sum_{I_XI_Y} \langle I_AI_X|I_XI_YI_Y\rangle|X\rangle|Y\rangle \tag{1}$$

where we summed over all isospin projections$^{[3]}$. States can be verified to satisfy the orthonormality condition $\langle I_AI_XI_Y|I_Y'\rangle = \delta_{I_AI'_A}\delta_{I_XI'_X}\delta_{I_YI'_Y}$. In this Letter we consider four–quark states with integral isospin. When $I_A = 0$, the physical state is a linear combination of $|0000\rangle$ and $|0011\rangle$. For $I_A = 2$, the physical state is $|2011\rangle$. Thus in both cases $I_X = I_Y$. When $I_A = 1$, the physical state is a linear combination of $|1011\rangle$, $|1100\rangle$ and $|1110\rangle$. For $I_A = 1$, we define new states $|1I_A\pm\rangle \equiv \frac{1}{\sqrt{2}}(|1I_A01\rangle \pm |1I_A10\rangle)$. The presence of $s\bar{s}$ pairs is now explored. By convention, we choose a single strange pair to correspond to labels $q_3 = s$ and $\bar{q}_4 = \bar{s}$, so that 1 and 2 still labels $u, d$ quarks. The four–quark state is $|I_AI_XI_Xs\bar{s}\rangle \equiv |X\rangle s\bar{s}$, either isovector or isoscalar. Another possibility is $|00c\bar{c}s\bar{s}\rangle \equiv c\bar{c}s\bar{s}$. For two strange pairs, the state is $|00s\bar{s}ss\rangle \equiv s\bar{s}s\bar{s}$. Other states are obtained by freely interchanging strange, charm and

$^{1}$ Because $X$ and $Y$ are merely labels, the states will be constructed to be representations of the label group, i.e. either symmetric or antisymmetric under $X \leftrightarrow Y$ exchange. Models where the dynamics are truncated in such a way that $q_1\bar{q}_2$ occur in one meson, and $q_3\bar{q}_4$ in another, i.e. where four–quark states are viewed as molecules of mesons, are not included in our discussion. This is because, e.g. for an $\eta\pi$ molecule, one can define $q_1$ and $\bar{q}_2$ to be in $\eta$. Label symmetry requires that $q_1$ and $\bar{q}_2$ can also be in $\pi$. But this is impossible by assumption. It should be noted that in QCD there is nothing special about $q_1\bar{q}_2$ as opposed to $q_3\bar{q}_4$, so that $X \leftrightarrow Y$ exchange is allowed.
Isospin 2 four–quark:

\[ |000\rangle \frac{1}{\sqrt{2}} (u \bar{u} + d \bar{d}) s \bar{s} \]
\[ |1011\rangle \frac{1}{\sqrt{6}} (-u \bar{d}d \bar{u} + d \bar{u}u \bar{d} + d \bar{d}u \bar{d} + u \bar{u}d \bar{d}) \]

Isospin 1 four–quark:

\[ |101\rangle \frac{1}{\sqrt{2}} (d \bar{d}u \bar{u} - u \bar{d}d \bar{u}) \]
\[ |10+\rangle \frac{1}{\sqrt{2}} (u \bar{u}u \bar{d} + d \bar{d}u \bar{d}) \]
\[ |10–\rangle \frac{1}{\sqrt{2}} (u \bar{u}d \bar{d} - d \bar{d}u \bar{u}) \]
\[ |101s\rangle \frac{1}{\sqrt{2}} (u \bar{d} - d \bar{u}) s \bar{s} \]

Table 1: Explicit neutral four–quark flavour states.

bottom quarks. Explicit forms for some of the neutral states are given in Table 1.

We shall be interested in decay and production \( A \leftrightarrow BC \) processes in the rest frame of \( A \). For simplicity we shall usually refer to the decay process \( A \rightarrow BC \), but the statements shall be equally valid for the production process \( A \leftarrow BC \). The decay of an isospin \( I_A \) four–quark state to two states with integral isospins \( I_B \) and \( I_C \) is considered \(^4\). The strong interactions include all interactions described by QCD. The quarks and antiquarks in \( A \) are assumed to travel in all possible complicated paths going forward and backward in time and emitting and absorbing gluons until they emerge in \( B \) and \( C \). We shall restrict \( B \) and \( C \) to angular momentum \( J = 0 \) states with valence quark–antiquark content and arbitrary gluonic excitation, i.e. to hybrid or conventional mesons. \( B \) and \( C \) can be radial excitations or ground states, with \( J^P = 0^– \) or \( 0^+ \). If \( C–\)parity is a good quantum number, \( J^{PC} = 0^{–+}, 0^{+-}, 0^{++} \) or \( 0^{––} \) are allowed. Since \( 0^{–+} \) and \( 0^{++} \) ground state meson states \( B \) and \( C \) are most likely to be allowed by phase space, they are used in the examples.

Assume that states \( B \) and \( C \) are identical in all respects except, in principle, their flavour and their equal but opposite momenta \( p \) and \( -p \). Hence \( B \) and \( C \) have the same parity, \( C–\)parity, radial and gluonic excitation, as well as the same internal structure. However, they are not required to have the same energies or masses \(^3\). One possible example is \( \eta \) and \( \pi \).

The decay amplitude is a product of the flavour overlap \( \mathcal{F} \) and the “remaining” overlap. We shall be interested in the exchange properties of \( \mathcal{F} \) when the labels that specify the flavour of the states \( B \) and \( C \) are formally exchanged, denoted by \( B \leftrightarrow C \). In cases where \( \mathcal{F} \) is non–zero and transforms into itself, which will be of particular interest, define \( \mathcal{F}_{B\leftrightarrow C} = f \mathcal{F} \).
If a quark (or antiquark) in A ends up in the particle with momentum \( p \), there is also the possibility that it would end up in the particle with momentum \( -p \). Hence for a given topology in Figure 1, e.g. 6a, there are in principle two topologically distinct amplitudes. Furthermore, each of topologies 4–6 is separately distinct. They are labelled analogous to earlier conventions \[3\].

It is possible to omit the following proof of the results of this Letter and continue directly to the statement of the results, which can be found where Table 2 is discussed in the text.

The flavour state of a \( q \bar{q} \) pair is

\[
|H⟩ = \sum_{\bar{h}h} H_{\bar{h}h} |\bar{h}⟩|h⟩ \quad \text{where} \quad H_{\bar{h}h} = ⟨I_H I_H^z | \frac{1}{2} h \frac{1}{2} - \bar{h}⟩(-1)^{\frac{1}{2} - h} \tag{2}
\]

and \(|\frac{1}{2}⟩ = u, \ |−\frac{1}{2}⟩ = d, \ |\frac{1}{2}⟩ = \bar{u} \) and \(|−\frac{1}{2}⟩ = \bar{d}⟩. This just yields the usual \( I = 1 \) flavour \( u \bar{u} = 1 \sqrt{2} (u \bar{u} − d \bar{d}) \) for \( I = 0 \). The advantage of this way of identifying flavour is that any pair creation or annihilation that takes place will do so with \( I = 0 \) pairs \( \frac{1}{\sqrt{2}} (u \bar{u} + d \bar{d}) = \frac{1}{\sqrt{2}} \sum_{\bar{h}h} δ_{\bar{h}h} |\bar{h}⟩|h⟩ \) being formed out of the vacuum, making the operator trivial.

In order to illuminate the method, we discuss the case where only \( u, d \) quarks participate in the decay. The presence of strange quarks only simplifies the overlap. From Eqs. \[3\] and \[4\], the flavour overlap \( F \) is

\[
\sum_{a_1 a_2 a_3 a_4 b c} \langle I_A I_A^z | I_X I_X^z I_Y I_Y^z | I_B I_B^z | I_C I_C^z \rangle \langle I_X I_X^z | \frac{1}{2} a_1 \frac{1}{2} - a_2 \rangle (-1)^{\frac{1}{2} - a_1} \langle I_Y I_Y^z | \frac{1}{2} a_3 \frac{1}{2} - a_4 \rangle (-1)^{\frac{1}{2} - a_3} \\
\times \langle I_B I_B^z | \frac{1}{2} b \frac{1}{2} - b \rangle (-1)^{\frac{1}{2} - b} \langle I_C I_C^z | \frac{1}{2} c \frac{1}{2} - c \rangle (-1)^{\frac{1}{2} - c} \text{ KD} \tag{3}
\]

where “KD” is a set of Kronecker delta functions that specifies how the quark lines connect in the decay topology. Specialize to topology 6a as an example. From Figure 1 “KD” is \( \delta_{a_1 b} \delta_{a_2 d} \delta_{a_3 a_4} \delta_{b c} \). If one formally interchanges all labels \( B \) and \( C \) in Eq. \[4\], it can be verified that \( F \to (-1)^{I_X+I_B+I_C} F \). Since the overlap is non-zero only when \( I_Y = 0 \) (due to the \( q_3 \bar{q}_4 \) pair annihilating), it follows by conservation of isospin that \( I_A = I_X \), so that \( F \to iF \), where \( i \equiv (-1)^{I_A+I_B+I_C} \). Thus \( f = i \). This, as well as the fact that the overlap vanishes when \( I_Y = 1 \), are indicated in Table 2.

Let \( C_A^0 \) be the C–parity of a neutral state \( A \). For charged states (with no C–parity), we
Figure 1: Connected topologies.

Figure 2: Disconnected topologies.
assume that at least one of the states in the isomultiplet it belongs to has a well-defined C-parity, denoted by $C_0^A$. G-parity conservation $G_A = G_BG_C$ and the relation $G = (-1)^IC_0^A$ imply that $C_0^A = i$, as was noted in section 2.2 of ref. [3].

It was shown in Eq. 3 of ref. [3] that the decay vanishes, called a symmetrization selection rule, if the parity $P_A = -f$. If $f = i$, then $P_A = -f = -i = -C_0^A$, i.e. state A is CP odd. Since states B and C both have $J = 0$, it follows by conservation of angular momentum that an $L$-wave decay would necessitate $J_A = L$. Hence states A have $J^PC = 0^+, 1^-, 2^+, 3^+, \ldots$, which are all exotic $J^PC$ not found in the quark model, so that these states are not conventional mesons. A charged state A (with no C-parity) should have a neutral isopartner with the foregoing $J^PC$. If $f = -i$, the same reasoning shows that states A have non-exotic $J^PC = 0^{++}, 1^{--}, 2^{++}, 3^{--}, \ldots$.

The results of our analysis for topologies 4–6 are summarized in Table 2. For topology 7 in Figure 2 the flavour overlap has in general no simple transformation properties under $B \leftrightarrow C$ exchange, corresponding to lack of symmetrization selection rules. Topology 8 is discussed further below. Topologies 4–6 are called “connected” and are allowed by the Okubo–Zweig–Iizuka (OZI) rule [3], while topologies 7–8 are “disconnected” and suppressed by the OZI rule. In the topology in Figure 1 under consideration an entry $i$ indicates that the decay of the corresponding four-quark component vanishes for $J^PC = 0^+, 1^-, 2^+, 3^+, \ldots$ four-quark states. Ditto for an entry $-i$, except that the four-quark state has $J^PC = 0^{++}, 1^{--}, 2^{++}, 3^{--}, \ldots$. It immediately becomes clear that the decay of the four-quark states with the $J^PC$ just mentioned is less than what one would naively expect, making them more stable.

To make the use of Table 2 clear, we consider the example to the decay of an isovector $1^{-+}$ state to $\eta \pi$ in topologies 4–6. The $1^{-+}$ state is a linear combination of flavour wave functions $|1I^z_A11\rangle, |1I^z_A+\rangle, |1I^z_A-\rangle$ and $|1I^z_A1s\bar{s}\rangle$. Referring to Table 2, the $|1I^z_A11\rangle$ component decays in topology 4 only, $|1I^z_A-\rangle$ in topology 5 only and $|1I^z_A1s\bar{s}\rangle$ in topology 5 only. The $|1I^z_A+\rangle$ component does not decay.

The implications of Table 2 for the two $J^PC$ sequences are now analysed.

**Decay of $J^PC = 0^+, 1^-, 2^+, 3^+, \ldots$ four-quark states to two $J = 0$ mesons:**

We arrive at the following conclusions:

1. If $I_A = 2$ or $I_A = I_B = I_C = 1$, contributions from all four-quark topologies vanish.
They also vanish for all hybrid meson and glueball topologies [3]. If \( I_A = 0 \) and \( I_B = I_C = 1 \), contributions from all connected four-quark topologies vanish. They also vanish for the connected hybrid meson topology [3].

2. Contributions from all “non – fall apart” connected topologies 6 vanish.

3. If \( I_A = 0 \) and \( I_B = I_C = 0 \), and the decay is non–vanishing, this comes from either a single \( s\bar{s} \) four–quark component which decays via “fall apart” connected topology 5 or from disconnected topologies. Also note that the decay cannot come from connected hybrid meson decay [3]. Assuming the OZI rule that disconnected topologies are suppressed, one discovers that a non–vanishing decay only comes from a single \( s\bar{s} \) four–quark component. This isolates the presence of an \( s\bar{s} \) component in the state, i.e. acts like a strangeness filter. It has been noted [3] that \( u\bar{u}, d\bar{d} \) components of a four–quark state can in perturbation theory be expected to mix substantially via single gluon exchange with \( s\bar{s} \), although flavour mixing of this kind has been found to be \( \lesssim 10\% \) in a model calculation [4].

4. If \( I_A = 1 \) and \( I_B \neq I_C \), decay does not come from the \( |1I_A^z+\rangle \) component.

Examples: There are no examples involving \( \pi\pi \) final states that are not forbidden by well–known selection rules of QCD, e.g. G–parity or \( CP \) conservation, or generalized Bose symmetry. Hence there is no new selection rules arising from item 1. From the last two items we obtain the following examples:

Item 5: Isoscalar \( 1^{-+}, 3^{-+}, \ldots \rightarrow \eta' \eta, f_0'f_0 \) indicates a four–quark component with a single \( s\bar{s} \) in the initial state.

Item 6: Isovector \( 1^{-+}, 3^{-+}, \ldots \rightarrow \eta\pi, \eta'\pi, f_0a_0, f_0'a_0 \) does not come from a \( |1I_A^z+\rangle \) component in the initial state.

Decay of \( J^{PC} = 0^{++}, 1^{--}, 2^{++}, 3^{--}, \ldots \) four–quark states to two \( J = 0 \) mesons:

In the cases that \( I_A = 1 \) and \( I_B \neq I_C \) some contributions vanish, making the states narrower than otherwise expected.

Examples: Isovector \( 0^{++}, 2^{++}, \ldots \rightarrow \eta\pi, \eta'\pi, f_0a_0, f_0'a_0 \) is narrower than otherwise expected.

The decays can only be found to vanish by symmetrization selection rules if the quark
structure of the decay is analysed. Models which only analyse decay at the hadronic level, do not incorporate the selection rule: The decay of four–quark \( a_0(980) \rightarrow \eta \pi \) was recently modelled at the hadronic level [7].

The validity of the preceding discussion should be viewed within the context of the restrictions on the final states \( B \) and \( C \) discussed earlier.

This concludes the main results of this Letter. A few final remarks are in order.

If one does not assume isospin symmetry [8], i.e. considers both QCD and QED, the initial four–quark states with different isospin will in general mix, yielding a complicated behaviour for the flavour overlap under \( B \leftrightarrow C \) exchange in Table 2. There are two exceptions. Firstly, for doubly charged states \( A \) (those with \( I^z_A = \pm 2 \) in the third column of Table 1) \( f = i \) in topologies 4 and 5. Secondly, for all decays in topologies 6, \( f = i \). Hence the symmetrization selection rule remains valid in these cases even without isospin symmetry. One can verify that each of these cases is an application of symmetrization selection rule I of ref. [3]: the case without isospin symmetry.

Consider topology 8 where two “raindrops” or a “half–doughnut” is created from the vacuum after the four–quark state has annihilated. There are similar topologies for an initial meson or glueball [3]. These topologies can be analysed without the need for isospin symmetry. The “half–doughnut” can be shown to apply only for decays already known to vanish by \( CP \) conservation or Bose symmetry [3]. From the symmetrization selection rule III of ref. [3], decay in “raindrop” topologies vanish in those cases where the \( B \leftrightarrow C \) exchanged diagram is topologically distinct from the original diagram.

It needs to be emphasized that this Letter analyses the flavour structure of various decay topologies in a generic way, which should subsume the treatments of numerous models of QCD. However, it is not a field theoretic treatment, and can hence not be regarded as predictions of QCD as a field theory. This becomes evident when one studies the following condition for the validity of our conclusions. We assume that states \( B \) and \( C \) are identical in all respects except, in principle, their flavour. Although this requirement is needed here, it is not sufficient, as a recent field theoretical analysis demonstrates [3]: The requirement is not needed for at least on–shell \( \eta \) and \( \pi \) states \( B \) and \( C \) in a certain energy range and for certain quark masses.

A candidate state \( \hat{\rho}(1405) \) with width \( 333 \pm 50 \) MeV, decaying to \( \eta \pi \), and possibly to \( \eta' \pi \),
has been reported \footnote{10}. It is interesting to note that a quark model calculation finds the lightest $1^{-+}$ four–quark state at 1418 MeV, although it is an isoscalar with flavour wave function \ket{000\bar{s}s} \footnote{2}. The isovector state is heavier \footnote{11}. If the $\hat{\rho}(1405)$ is resonant and has a substantial branching ratio of $\eta\pi$, this decay mode may discriminate against the hybrid interpretation of the state. This is because only the (presumably suppressed) OZI forbidden hybrid meson topology contributes \footnote{3, 9}. We predict that the OZI allowed decays of an isovector $1^{-+}$ only arise from certain four–quark components, so that the detection of substantial branching ratios in $\eta\pi$ or $\eta'\pi$ signals such a component.

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Table 2: Behaviour of the (non–vanishing) flavour overlap \( \mathcal{F} \) for the decay of the indicated four–quark state to two mesons under \( B \leftrightarrow C \) exchange, i.e. \( \mathcal{F}_{B \leftrightarrow C} = f \mathcal{F} \), in the topology under consideration. The symbol \( \nexists \) denotes that \( \mathcal{F} \) has no simple transformation properties under \( B \leftrightarrow C \) exchange, so that there is no symmetrization selection rule. If a state is not indicated for a given topology it means that \( \mathcal{F} \) vanishes. When decay is not allowed by isospin conservation, \( \mathcal{F} = 0 \) as expected. This happens when \( I_A \neq I_B + I_C \) or \( I_A^z \neq I_B^z + I_C^z \), or when \( I_A = I_B = I_C = 1 \) and \( I_A^z = I_B^z = I_C^z = 0 \). \( \dagger \mathcal{F} \neq 0 \) only if \( I_B = I_C \). \( \ddagger \mathcal{F} \neq 0 \) only if \( I_B^z = I_C^z = 0 \). \( \nexists \mathcal{F} \neq 0 \) only if \( I_B = I_C = 0 \). \( \text{II} \mathcal{F} \neq 0 \) only in topology 6a.

| Isospin 0 four–quark | Isospin 1 four–quark | Isospin 2 four–quark |
|----------------------|----------------------|----------------------|
| Top. | State | \( f \) | Top. | State | \( f \) | Top. | State | \( f \) |
| 4 | \( |0000\rangle \) | \( i \) | 4 | \( |1I_A^z11\rangle \, \dagger \) | \( -i \) | 4 | \( |2I_A^z11\rangle \) | \( i \) |
| | \( |0011\rangle \) | \( i \) | | \( |1I_A^z+\rangle \, \dagger \) | \( i \) | | \( |2I_A^z11\rangle \) | \( i \) |
| | \( |00ss\rangle \, \nexists \) | \( i \) | | \( |1I_A^z-\rangle \, \nexists \) | \( i \) | | | | |
| 5 | \( |0000\rangle \, \nexists \) | \( i \) | 5 | \( |1I_A^z11\rangle \, \nexists \) | \( i \) | 5 | \( |2I_A^z11\rangle \) | \( i \) |
| | \( |0011\rangle \, \nexists \) | \( i \) | | \( |1I_A^z+\rangle \, \dagger \) | \( i \) | | \( |2I_A^z11\rangle \) | \( i \) |
| | \( |00ss\rangle \) | \( \nexists \) | | \( |1I_A^z-\rangle \, \dagger \) | \( -i \) | | | | |
| | \( |c\bar{c}s\rangle \, \nexists \) | \( \nexists \) | | \( |1I_A^z1s\bar{s}\rangle \, \dagger \) | \( \nexists \) | | | | |
| 6a,b | \( |00ss\rangle \) | \( i \) | 6a,b | \( |1I_A^z+\rangle \) | \( i \) | | | | |
| | \( |0000\rangle \) | \( i \) | | \( |1I_A^z-\rangle \) | \( i \) | | | | |
| | \( |00ss\rangle \, \nexists \) | \( i \) | | \( |1I_A^z1s\bar{s}\rangle \, \text{II} \) | \( i \) | | | | |
| | \( |c\bar{c}s\rangle \) | \( \nexists \) | | \( |1I_A^z11\rangle \) | \( i \) | | | | |
| | \( |00ss\rangle \) | \( \nexists \) | | \( |1I_A^z+\rangle \) | \( i \) | | | | |
| | \( |0000\rangle \) | \( i \) | | \( |1I_A^z-\rangle \) | \( i \) | | | | |
| 6c,d | \( |0011\rangle \) | \( i \) | | \( |1I_A^z11\rangle \) | \( i \) | | | | |
| | \( |00ss\rangle \, \nexists \) | \( i \) | | \( |1I_A^z+\rangle \) | \( i \) | | | | |