Complete bandgaps in one-dimensional left-handed periodic structures

Ilya V. Shadrivov, Andrei A. Sukhorukov, and Yuri S. Kivshar
Nonlinear Physics Centre, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia

Artificially fabricated structures with periodically modulated parameters such as photonic crystals offer novel ways of controlling the flow of light due to the existence of a range of forbidden frequencies associated with a photonic bandgap. It is believed that modulation of the refractive index in all three spatial dimensions is required to open a complete bandgap and prevent the propagation of electromagnetic waves in all directions. Here we reveal that, in a sharp contrast to what was known before and contrary to the accepted physical intuition, a one-dimensional periodic structure containing the layers of transparent left-handed (or negative-index) metamaterial can trap light in three-dimensional space due to the existence of a complete bandgap.

PACS numbers: 42.70.Qs, 41.20.Jb, 78.67.Pt

Photonic crystals are artificial materials with a periodic modulation in the dielectric constant which can create a range of forbidden frequencies called a photonic bandgap. Photons with frequencies within the bandgap cannot propagate through the medium. This unique feature can alter dramatically the properties of light, enabling control of spontaneous emission in quantum devices and light manipulation for photonic information technology. Photonic bandgap structures can also be found in nature, and they explain the color diversity of some of the living creatures.

Complete two-dimensional (2D) and three-dimensional (3D) bandgaps can be realized in photonic crystals, where the refractive index is periodically modulated in two or three dimensions, respectively. Such modulation is necessary to satisfy the Bragg condition simultaneously for all propagation directions, requiring that phase accumulation per period is close to a multiple of $2\pi$, so that the waves reflected at different interfaces between the regions with low and high refractive indexes interfere constructively and wave propagation is prohibited for any incidence angle. Manufacturing of 3D photonic crystals still remains a technological challenge due to the requirements of large index contrast and high fabrication precision.

The simplest periodic structure, both in geometry and manufacturing, is a one-dimensional stack of two types of layers which differ in the dielectric constant. However, such structures may only possess partial bandgaps for certain ranges of propagation directions. Here we study the scattering properties of one-dimensional periodic structures containing layers made of the so-called left-handed metamaterials (LHMs)—artificially created composites which are characterized by simultaneously negative dielectric permittivity and magnetic permeability. Such materials are transparent and can bend light in the opposite direction to normal reversing the way in which refraction usually works. We demonstrate that specially designed one-dimensional structures with negative refraction may exhibit a complete three-dimensional bandgap.

First, we show that a layered structure made of two alternating layers of left-handed materials and conventional dielectrics can exhibit a complete two-dimensional bandgap, i.e., it does not support any propagating TE or TM waves and is therefore opaque for any angle of propagation in the plane. As a direct consequence of such bandgap, the radiation of waves with given polarization by any source placed anywhere inside this structure is prohibited, and the two-dimensional density of states (DOS) is zero. This result is in sharp contrast to directional reflection in conventional layered dielectric structures, which reflect only electromagnetic waves launched from air or a low-index medium. In the periodic structures with usual dielectrics a source dipole can emit radiation of both polarizations, indicating that the complete gap is absent and propagating TE and TM modes are always present. The out-coupling of radiation from a periodic structure can vanish only at certain interfaces (see Ref. and references therein), whereas the radiation along the layers, as well as DOS, is never zero. We reveal the physical effects which lead to such fundamental differences between periodic structures with conventional and left-handed layers. In the final part of the paper, we suggest a design of a one-dimensional periodic structure consisting of three alternating layers made of conventional dielectric and left-handed materials which possesses a complete three-dimensional bandgap.

We consider a one-dimensional periodic structure created by layers (with the thickness $d_1$ and $d_2$) of two different materials with dielectric permittivities $\epsilon_{1,2}$ and magnetic permeabilities $\mu_{1,2}$, respectively, as shown in Fig. First, we study the propagation of the TE-polarized electromagnetic waves which have the component of the electric field parallel to the layers ($E = E_y$); all results can be easily generalized for the case of the TM polarized waves. We consider the wave propagation in the $(x, z)$ plane characterized by the wavevector $k = (k_x, 0, k_z)$. The TE-polarized waves are described by the linear Helmholtz-type equation for the electric field component,

$$\Delta E + \frac{\omega^2}{c^2} n^2(x) E - \frac{1}{\mu(x)} \frac{\partial}{\partial x} \frac{\partial E}{\partial x} = 0,$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2$ is the two-dimensional Laplacian, $n^2(x) = \epsilon(x)\mu(x)$ is the square of refractive...
In a one-dimensional periodic structure the propagating waves have the form of Bloch modes, for which the electric field amplitudes satisfy the periodicity condition, \( E(x + \Lambda, z) = E(x, 0) \exp(iK_b + ik_z z) \), where \( \Lambda = d_1 + d_2 \) is the period of the structure. Here \( K_b \) is the dimensionless Bloch wave number which defines the wave transmission across the layers, and its dependence on the wavevector component along the layers \((k_z)\) can be found explicitly for two-layered periodic structures (see, e.g., Refs. [4, 12]),

\[
2 \cos(K_b) = \text{Tr}(M) = 2 \cos(k_{1z}d_1) \cos(k_{2z}d_2) - \left( \frac{k_{2x}\mu_1}{k_{1x}\mu_2} + \frac{k_{1x}\mu_2}{k_{2x}\mu_1} \right) \sin(k_{1z}d_1) \sin(k_{2z}d_2).
\]

Here \( \text{Tr}(M) \) is the trace of the transfer matrix \( M \) characterizing the wave scattering in a periodic structure [4], \( k_{jx} = k_j(1 - k_j^2/k_i^j)^{1/2} \) are the \( x \)-components of the wavevector in the first \((j = 1)\) and second \((j = 2)\) media, and \( k_i = \omega n_i/c \) are wavenumbers in each media with refractive indexes \( n_j \). For completeness, we mention that the dispersion relation of the TM polarized waves is obtained by replacing \( \varepsilon \leftrightarrow \mu \) in Eq. (2).

Solutions of the dispersion relation [2] with both real \( k_z \) and \( K_b \) correspond to Bloch waves which can propagate through the periodic structure, whereas complex \( k_z \) or \( K_b \) indicate the presence of bandgaps in the spectrum where the wave propagation is prohibited. A complete bandgap occurs if for all real \( k_z \), the \( K_b \) remains complex. It was recently shown [11, 12] that novel partial bandgaps can appear in structures made of alternating layers of LHM and normal dielectrics when the condition of zero average refractive index is satisfied for particular propagation angles \((k_z)\), \( k_{1x}d_1 + k_{2x}d_2 = 0 \), which is possible because \( k_z \) is positive in conventional dielectrics and it is negative in left-handed materials. However, we find that this requirement is neither sufficient nor necessary to obtain complete bandgaps.

In order to emphasize the importance of our findings presented below, first we recall the basic physics which explains why one-dimensional periodic structures containing materials of the same type (i.e., normal dielectrics) do not possess a complete three-dimensional bandgap. Analyzing the effects associated with the wave scattering in Bragg gratings [4, 13], we come to the conclusion that the only phenomenon which always allows for the wave propagation in the 1D dielectric periodic structures, and which cannot be suppressed by a choice of the structural parameters, is the wave guiding by optically dense layers. Indeed, it is well known that a dielectric waveguide with the core made of an optically dense medium always supports a fundamental mode. However, as was shown recently [14], the fundamental mode can be absent if the core is made of LH metamaterial. And it is this property of a LH waveguide that allows us to introduce a novel type of one-dimensional periodic structures with a complete bandgap.

\textbf{Why does the usual dielectric waveguide always support the fundamental mode, and why a metamaterial waveguide does not?} The condition for the guided waves to exist, defined by the dispersion relation for the modes in a slab waveguide, has a simple physical meaning: the round-trip accumulation of phase due to wave propagation across the layer, \( 2\phi_{\text{prop}} \), including the phase retardation upon the total internal reflection, \( 2\phi_{\text{refl}} \), should be equal to a multiple of \( 2\pi \). The phase change due to the total internal reflection is negative for both types of waveguides, and depending on the angle of incidence it varies from \( 0 \) to \(-\pi \). A difference between the conventional and LHM waveguides appears due to the phase accumulated by the wave propagating across the layer. In usual dielectrics, the wave is forward, i.e. the phase accumulated along the direction of energy flow is positive. As a result, there always exists an angle of wave propagation, such that the total phase change vanishes, and at least one mode always exists in a dielectric waveguide. In a LHM waveguide, the wave is backward, and the phase change \( \phi_{\text{prop}} \) is negative. Then, one can choose the parameters in such a way that for all angles of wave propagation (greater than the angle of the total internal reflection), the total phase change is \(-4\pi < \phi = 2(\phi_{\text{refl}} + \phi_{\text{prop}}) < -2\pi \), and no guided modes exist.

Based on the results presented above we can construct...
the one-dimensional periodic structure which possesses a complete bandgap for one polarization. Indeed, we need to choose the layer thickness such that the guided modes are absent in the waveguides formed by the layers in the structure, and by varying the material parameters we should avoid the transmission resonances \[ \text{[13]}. \] Our analysis shows that it is indeed possible to find such structures and, for example, the complete gap appears for the following set of parameters: \( \epsilon_1 = \mu_1 = 1, \) \( \epsilon_2 = -6, \) \( \mu_2 = -1.38, \) \( d_1^{(0)} = 1.5\lambda/2\pi, \) \( d_2^{(0)} = 1.4\lambda/2\pi, \) where \( \lambda = 2\pi c/\omega \) is the vacuum wavelength. Then we study the dependence of the bandgap spectrum on the structure period for a fixed ratio \( d_1/d_2 = d_1^{(0)}/d_2^{(0)} \) and Fig. 2(top) shows the bands (colored) where the wave propagation is possible. For the TE modes [see Fig. 2(left, top)], there exists a range of periods for which the propagation is completely prohibited for all possible \( k_s, \) since the corresponding \( K_0 \) are complex. This means, in particular, that if we consider an end-fire generation problem and launch the waves along the layers towards the structure, they will be completely reflected, as schematically shown in Fig. 2. Such a regime is impossible for any type of the conventional dielectric gratings.

However, there is no complete bandgap for the TM-polarized waves propagating in the same structure. In the optimal case with \( d_{1,2} \approx d_{1,2}^{(0)}, \) we have only one angle of propagation possible (i.e. the only value of \( k_s \) with real \( K_0 \)). This is the Brewster angle for which there is no reflection of TM waves at the interfaces. From the electromagnetic duality principle we find that taking the structure with LH Metamaterial characterized by \( \epsilon_2^{\text{new}} = \mu_2 \) and \( \mu_2^{\text{new}} = \epsilon_2, \) we can obtain a complete bandgap for the TM polarized waves. Most remarkably, the complete bandgap for each polarization exists for rather broad ranges of the structure parameters, see Fig. 3. The regions for the complete TE and TM bandgaps are symmetric with respect to the line \( \epsilon_2 = \mu_2, \) and this is a consequence of the electromagnetic duality.

One-dimensional structures with a complete bandgap for one of the polarizations can be used to form an electromagnetic cavity. To study the main features of the wave localization due to the presence of a complete band-gap, we analyze the field of a line current \( J \) running along the \( y \)-axis inside the structure at the position \( r_0 = (x_0, z_0) \) in the \( r = (x, z) \) plane. It follows from the Maxwell’s equations that the electric field can be expressed as \( E(x, z) = i\omega J \mu(x_0, z_0) G(x, z)/c^2, \) where \( G \) is the Green’s function found as a solution of the following equation,

\[
\Delta G + \frac{\omega^2}{c^2} n^2(x) G - \frac{1}{\mu(x)} \frac{\partial \mu \partial G}{\partial x \partial x} = 4\pi \delta(r - r_0). \tag{3}
\]

The total emitted power per unit length of the line current \( J \) is

\[
W = -\frac{\omega J^2 \mu(x_0, z_0)}{2c^2} \text{Im} \left[ G(x_0, z_0) \right], \tag{4}
\]

and this quantity is proportional to the local density of states (LDOS) \[ \text{[15]}. \] The density of states (DOS) of the structure, which is an integral of LDOS over the Brillouin zone, characterizes the radiation efficiency of multiple sources located at different positions. The DOS becomes zero only if radiation in any direction in the plane is prohibited, indicated the presence of a complete 2D band-gap. We plot the dependence of the DOS on the magnetic permeability of negative index material \( \mu_2 \) for fixed \( \epsilon_2 = -6 \) and \( d_{1,2} = d_{1,2}^{(0)} \) in Fig. 4 which clearly demonstrates that a two-dimensional band-gap exists for the TE-polarized waves within a certain range of media parameters. Within the bandgap region the radiation is suppressed and the Green’s function is exponentially localized for any source position [see Fig. 4(top, middle)]. Outside the bandgap, different propagating Bloch modes are excited, and the Green’s function is not localized [see Fig. 4(top, left and right)].
After the comprehensive study of the two-layer periodic systems and the properties of the complete bandgaps supported by one-dimensional hybrid structures, we are able to suggest the case when the complete bandgap may appear for both polarizations thus allowing the existence of the absolute bandgap. Indeed, to do this we should consider more sophisticated case of a three-layer periodic structure in order to suppress the conditions for the existence of the Brewster angle which prevents us from creating a complete bandgap in two-layer structures. The Brewster-angle transmission resonance can be easily eliminated by introducing a third layer in the structure, thus allowing the existence of a complete three-dimensional bandgap for all waves propagating inside a specially designed one-dimensional structure. To demonstrate this quite unique property, we choose the structure with the parameters $\epsilon_2 = \mu_3 = -6$, $\epsilon_3 = \mu_2 = -1.38$, $d_1 = 1.5\lambda/(2\pi)$, $d_2 = d_3 = 0.7\lambda/(2\pi)$ possesses an absolute three-dimensional bandgap, and the Green’s function corresponding to this three-layer structure is presented in Fig. 5.

In conclusion, we have revealed a novel and highly non-trivial property of left-handed metamaterials with negative refraction: A one-dimensional periodic structure containing layers made of a left-handed metamaterial can trap light in three dimensions due to the existence of a complete photonic bandgap. This finding is in a sharp contrast with the fundamental concepts of the conventional physics of photonic crystals where complicated structures with two- and three-dimensional periodicity are required. We believe that our results suggest new directions for the future applications of metamaterials for microwave, Terahertz frequencies, and visible light as fabrication technologies become available.

We thank C. Soukoulis, P. Belov, I. Gabitov, S. Mingaleev, V. Shalaev, and H. Winful for useful discussions.

---

[1] J. D. Joannopoulos, P. R. Villeneuve, and S. Fan, Nature 386, 143 (1997).
[2] P. Lodahl, A. Floris van Driel, I. S. Nikolaev, A. Irman, K. Overgaag, D. Vanmaekelbergh, and W. L. Vos, Nature 430, 654 (2004).
[3] P. Vukusic and J. R. Sambles, Nature 424, 852 (2003).
[4] P. Yeh, Optical Waves in Layered Media (John Wiley & Sons, New York, 1988).
[5] J. N. Winn, Y. Fink, S. H. Fan, and J. D. Joannopoulos, Opt. Lett. 23, 1573 (1998).
[6] D. N. Chigrin, A. V. Lavrinenko, D. A. Yarotsky, and S. V. Gaponenko, Appl. Phys. A 68, 25 (1999).
[7] S. D. Hart, G. R. Maskaly, B. Temelkuran, P. H. Prideaux, J. D. Joannopoulos, and Y. Fink, Science 296, 510 (2002).
[8] K. Busch and S. John, Phys. Rev. E. 58, 3896 (1998).
[9] I. Alvarado-Rodriguez, P. Halevi and A. S. Sánchez, Phys. Rev. E, 63, 056613 (2001); 65, 039901(E) (2002).
[10] J. R. Zurita-Sánchez and A. S. Sánchez, and P. Halevi, Phys. Rev. E 66, 046613 (2002); M. Wuhs, L. G. Suttorp, and A. Lagendijk, Phys. Rev. E 69, 016616 (2004).
[11] D. R. Smith, J. B. Pendry and M. C. K. Wiltshire, Science 305, 788 (2004).
[12] I. S. Nefedov and S. A. Tretyakov, Phys. Rev. E 66, 036611 (2002).
[13] J. Li, L. Zhou, C. T. Chan, and P. Sheng, Phys. Rev. Lett. 90, 083901 (2003).
[14] P. St. J. Russel, T. A. Birks, and F. D. Lloyd Lucas, In: Confined Electrons and Photons, Eds. E. Burstein and C. Weisbuch, (Plenum, New York, 1995), p. 585.
[15] I. V. Shadrivov, A. A. Sukhorukov, and Yu. S. Kivshar, Phys. Rev. E 67, 057602 (2003).
[16] P. Sheng, Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena, (Academic Press, San Diego, 1995).