Method to Evaluate Measuring Uncertainty of Probe Radius Compensation with Account for Variation in Measuring Surfaces

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Abstract. This article describes a methodology of evaluating the uncertainty of the contact measurement of freeform surfaces with coordinate measuring machines, taking into account the variation in the form of measured surfaces. This variation is due to machining uncertainties, which appear during production using a particular machining method and technology. Probabilistic estimates of errors of determination of contact points for freeform surfaces were obtained. Use of this model makes it possible to obtain interval estimates of errors with the required confidence level of probability and mathematical expectation. The relations of the probabilistic error estimates at any of the measured points on surface are obtained.

1. Introduction
In industry, coordinate measuring machines (CMMs) are widely used for the measurement of surfaces; they can be equipped with contact and non-contact measuring systems. CMMs are flexible measurement tools which allow one to measure various parts, including a free-form surface (The accuracy of which can be difficult to obtain, and for the production of which are used in the technologies of injection molding [1] and additive technologies [2]) with high accuracy. In addition to production necessity, precise measurements of parts allow different calculations for their work [3, 4]. If the accuracy of freeform surface measurement is the decisive factor for a product (e.g., blades of gas turbine engines (GTE)), the measurement is usually carried out by CMM implementing a contact measurement method. In the practice of controlling freeform profile parts, there are significant measurement uncertainties, which are difficult to confirm experimentally. In connection with this, it is actual to develop a method for estimating measurement uncertainties, which are necessary for the development of control programs for CMMs that provide the required accuracy.

The contact measuring method is performed by a contact of the probe tip with the surface of the measured part or product, which as a rule, has a spherical tip. After contact, the centre point of the measuring tip is fixed and then touch point is calculated. As a result of the set of measurements, a cloud of points is created. The cloud of points is used to define the surfaces, planes, lines and points of the parts, which are required to determine the geometric parameters of the product. When we measure surfaces with complex geometry, the accuracy of probing on CMMs is determined by many factors. One of the key factors affecting the accuracy of measurement is the radius of the probe tip (group of coordinate measuring system hardware) and curvature of part surface (a group of the workpiece features).
The greater the curvature of the measured surface and the larger the radius of the probe tip, the greater the measurement error of the determination of the actual part points is. The magnitude of the measurement error can be estimated by experiment (using an etalon surface) and by the method of mathematical modelling of the measuring process [5, 6].

Machined free form surfaces of work-piece are always different from their CAD-models by the value of form and location deviations [7]. Because of these variations, the coordinates of the probe tip, which are calculated to normal to CAD model from the centre point of the probe tip, are determined with an deviation. Figure 1 presents a measurement circuit for free-form surface with form and location deviations.

Figure 1. The error of surface 1 measurement by the probe tip 3 (4 - a touch point, 5 - a measured (calculated) point) caused by: a) discrepancy between the nominal direction of the normal 7 to CAD surface 2 and real direction at the point on surface; b) mismatch between the normal vector 8 to offset surface 6 and the measured surface 1.

In the practice of coordinate measurements of geometric parameters of parts, there are situations when the standard of comparison (the CAD-model) for measured surfaces is absent (Figure 1.b). In the case, the probe tip radius compensation is carried out on the normal to the offset surface as defined by the centre points of the probe tip [8].

This paper describes the method of evaluating the uncertainty of contact measurement of freeform surfaces (blade feather) with coordinate measuring machines (CMM). This method takes into account variations in the form of measured surfaces. This variation is due to machining uncertainties, which appear in the process of manufacture by the method of machining and technology. Our developed method allows simulating the form and location deviation of the surface and calculating the coordinates of touch points, probe tip centres and measured points.

2. The method of evaluating measurement error with consideration to variations in the form of measured surfaces

We developed a method to assess the accuracy of measurement in accordance with stochastic principles. Figure 2 shows a flowchart of the method that takes into account the degree of dispersion of the parameter of form and location deviations for free form surfaces. The method uses the Monte Carlo method with a sequential repetition of steps 3, 4 and 5. Further sections describe the steps of this procedure.

2.1. Mathematical description of surfaces

To build the coordinate measurements model it is necessary to describe mathematically the details of the measured surface and the respective CAD-model. Nevertheless, the condition of smoothness at all points of the described surfaces is required to be fulfilled. In this paper we use the tool, which is the standard for description of free-form surfaces with the CAD programs, the so-called NURBS-surface [9] and its equation can be written as:
\[
P(u, v) = \sum_{i=1}^{n} \sum_{j=1}^{m} h_{ij} \cdot P_{ij} \cdot N_{ik}(u) \cdot N_{jl}(v) \cdot \left/ \sum_{i=1}^{n} \sum_{j=1}^{m} h_{ij} \cdot N_{ik}(u) \cdot N_{jl}(v) \right.,
\]

where \( u \) is an option for calculation of spline points coordinates varying in the range of \([t_{i-1}, t_{i+1}]\);
\( h_{i} \) are homogeneous coordinates of setting points; \( P_{ij} \) are the coordinates \((x_{i}, y_{i}, z_{i})\) of the \( i \)-th setting point in three-dimensional space; \( N_{ik}(u), N_{jl}(v) \) are basic spline functions (or fitting) in the parametric direction; \( t \) are nodal values of parameter whereat the functions \( N_{ik}(u) \) are not zero; \( k \) is the degree of the piecewise spline in the direction of the parameter \( u \); \( v \) is an option for calculation of the coordinates of the spline points varying in the range of \([s_{l-1}, s_{l+1}]\); \( s \) are nodal parameter values where the functions \( N_{jl}(v) \) are different from zero; \( l \) is the degree of the piecewise spline in the direction of the parameter \( v \).

2.2. Simulated deviation of form and location

The points of the measured surfaces were simulated from the nominal by adding the form and location deviations to the coordinates of the nominal surfaces points. The error of the surface form at a high milling speed depends on the mechanical properties of the process material in the cutting area, which can be determined by the method described in [10]. The values of the form and location deviations were chosen on the basis of statistical observations for the series of blades.

Thus, the point coordinate of the measured surface can be expressed by the equation:

\[
\tilde{P}_{\text{meas}} = (\tilde{P}_{\text{CAD}} + N_{1\times3} \cdot \delta_F) \cdot R_{3\times3} + T_{1\times3},
\]

where \( \tilde{P}_{\text{meas}} \), \( \tilde{P}_{\text{CAD}} \) are coordinate vectors \((x, y, z)\) of measured (simulated) and nominal surface, respectively; \( N_{1\times3} \) is the normal vector at the \( \tilde{P}_{\text{CAD}} \) point; \( \delta_F \) is form deviation at \( \tilde{P}_{\text{CAD}} \) point; \( R_{3\times3} \), \( T_{1\times3} \) are rotate matrix and the translate vector coordinates of a \( \tilde{P}_{\text{CAD}} \) point.
The matrix $R_{3 \times 3}$ contains three angles of rotation around the coordinate axes. The $T_{1 \times 3}$ vector contains the values of displacement along the coordinate axes. In parts production practice there are two components of the form deviation, such as, systematic and random errors. Consequently, these two types of deviations were included in the model of the measured surface. Systematic deviations consist of the sinusoidal form deviation and the machining form deviation [11].

Thus, the total form deviation at each point can be written as the sum of three components:

$$\delta_F = \delta_s + \delta_m + \delta_r.$$

Consider each of these components.

2.2.1. Sinusoidal form deviation. Harmonic form deviation can be approximated using the composition of the sine and cosine functions by the next equation:

$$\delta_s = A \cdot \sin(w_x \cdot x + w_y \cdot y + \varphi_{\text{sin}}) + B \cdot \cos(w_x \cdot x + w_y \cdot y + \varphi_{\text{cos}}),$$

where $A$ and $B$ are the amplitudes of the sine and cosine components, respectively, $w_x$ is given by $\omega_x \cdot 2\pi / L_x$, $w_y$ is given by $\omega_y \cdot 2\pi / L_y$;

$\omega_x$, $\omega_y$ are frequencies of harmonic components along the x and y direction, respectively,

$L_x$, $L_y$ are bearing lengths along the x and y direction, respectively;

$\varphi_{\text{sin}}$, $\varphi_{\text{cos}}$ are angle phase of the sine and cosine.

2.2.2. Machining form deviation. The form deviation due to the machining process is the result of curvature measurement of the processed object when it is in contact with the surface of the cutting tool. The distribution of the error can be calculated basing on the values of the average curvature:

$$\delta_m = A_m \cdot (i_s - 0.5),$$

where $A_m$ is the maximum error of machining process.

The index $i_s$ is calculated by the equation:

$$i_s = (H - H_{\text{min}})/(H_{\text{max}} - H_{\text{min}}),$$

where $H$, $H_{\text{min}}$ and $H_{\text{max}}$ are values of the average curvature at the point on the surface, the minimum and maximum values of the average curvature [12] over the biparametric surface, respectively.

2.2.3. Random error. Random errors caused by numerous factors are small in their individual impact on the results and they cannot be taken into account during an experiment. Through multiple repeated measurements of the same non-random quantity the random measurement errors result in different instrumental data. Dispersion of the results of these instrumental data is usually subject to the Gaussian law. For the form deviation the value of the random component of the deviation $\delta_r$ is calculated irregularly with maximum amplitude of 0.005 mm.

2.3. Computation of touch point coordinates and probe tip center

We use a mathematical model permitting calculation of the point coordinates in measuring the gear surfaces in a contact coordinate-measurement machine [13]. Deflection of the coordinates of the point on the surface relative to the coordinates of the measured point characterizes the error of compensation of probe tip radius at the i-th measurement point:
\[ \delta_{\text{Recomp.}} = \| C_i - D_i \|, \]

where \( C_i \) are coordinates \((x, y, z)\) of the real point of contact of the probe tip with the surface;

\( D_i \) are coordinates \((x, y, z)\) of the measured point calculated by the equidistant surface or by CAD-model of reference surface.

3. Simulation Results
Simulation of the measurement process using the developed technique was carried out by the example of the suction and pressure sides of the blade of the GTE compressor.

A pen of GTE compressor blade is a blade airfoil portion placed in the stream of air (gas). The pen has two major surfaces: the suction and the pressure sides. The surfaces represent freeform surfaces.

Measurement surfaces and respective nominal surfaces of blade feather are simulated in MATLAB software package for a variety of coordinate points measured by CMMs DEA Global Performance 07.10.07 and the corresponding points taken from the CAD-model of the blade. The radius of the probe tip for measurement and experiments is 1 mm. One hundred surfaces of suction and pressure sides were modeled; the amplitudes of its form deviations were varying in the normal law. On the basis of statistical data of this type components measurement it was found that the maximum form deviation is not more than 0.2 mm. The range of \( A \) and \( B \) amplitudes (Equation 4) was 0.08 mm, the range of values of \( A_m \) (Equation 5) equals 0.1 mm. The phase and frequency (Equation 4) of harmonic component vary in the Equiprobable law. The phases range from 0º to 360º, the frequency is 1, 2, 3 and 4. Transposition matrix components changed under the normal law; displacement along the x-axis is ± 0.02 mm; the displacement along the y-axis is ± 0.2 mm; the displacement along the z-axis is ± 0.15 mm. The angles of rotation surface vary in the normal law; the angle around the x-axis is ± 9º; the angle around the y-axis is ± 3º; the angle around the z-axis is ± 6º.

Series of measurements and calculation of errors of the probe tip radius compensation at points were carried out with the use of the developed model for the 100 pressure sides and 100 suction sides. In measurement of surfaces we used the strategy of uniform distribution of control points on the measured surface, which is the simplest and widely used in practice. The distribution of errors at the points is subject to two basic laws: the normal and beta-distribution. Thereafter, expectation was calculated at each measured point, as well as, upper and lower estimate of the error of measurement of point coordinates at a confidence level of 99.73%. According to the obtained pointwise values of the middle, lower and upper boundaries of probe tip radius compensation error for the surfaces were defined using bilinear interpolation (Figure 3) in \( u-v \) parametric coordinates (Equation 1).

![Figure 3](image-url)

Figure 3. The probability evaluation of error values for the surface of the suction side

\( \text{a) with the use of CAD} \quad \text{b) with the use of equidistance; 1, 2, 3 are the middle, lower, upper surfaces of random function propagation.} \)

Thus, during the simulation we obtain dependencies, which allow determining the average, lower and upper propagation boundaries of the error of the probe tip radius compensation for any measured
point on the surface. Table 1 shows the marginal probability values of errors of determination of the measured points coordinates obtained by the processing of statistical data (the expectation, the lower and upper limits of error variation) for the considered surfaces.

**Table 1.** Probability values of errors of the measured points determination.

| The object of measurement  | The number of measured points, \( n_x \times n_y \) | Error with CAD, \( \mu \) m | Error without CAD, \( \mu \) m |
|---------------------------|---------------------------------|----------------|----------------|
|                           | Mean   | The lower bound | The upper bound | Mean   | The lower bound | The upper bound |
| The suction side of blade | 28×3   | 7,0             | 0,5             | 26,7   | 10,7             | 6,8             | 27,2             |
|                           | 56×3   | 8,3             | 0,6             | 33,3   | 9,9              | 4,6             | 25,8             |
| The pressure side of blade| 28×3   | 6,0             | 0,5             | 28,2   | 39,5             | 35,3             | 48,0             |
|                           | 56×3   | 7,8             | 0,7             | 30,3   | 28,5             | 21,2             | 35,8             |

As the table shows, the maximum value of errors of the determination of the compensation vector by the method of probe tip radius compensation using a CAD model increases. It is due to the fact that the probability to get to the area with a strong deviation from the standard increases. In case of computation of the coordinates of the measured points on the equidistant surface the limit error value is reduces with increasing points and the method becomes comparable in accuracy with the measurement by the method of CAD.

4. Conclusion

This article describes the developed method of obtaining probabilistic estimates of the errors of the probe tip radius compensation for the measurement of freeform surfaces with the coordinate measuring machine, which takes into account variations of the measured surfaces. The developed method can be used for propagation of measurement errors of geometrical parameters of engineering parts surfaces and also for the formation of the optimal sampling to control the geometry as a part of the optimizer.

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**References**

[1] Stepanenko I S and Khaimovich, A I 2017 GTE blade injection moulding modeling and verification of models during process approbation *IOP Conference Series: Materials Science and Engineering* vol 177 no 1 012093

[2] Smelov V G, Sotov A V and Agapovich A V 2016 Recovery technology features of aerospace parts by layering synthesis *Key Engineering Materials* vol 684 pp 316-322

[3] Zubanov V, Shabliy L and Krivcov A 2014 Centrifugal kerosene pump CFD-modeling *Research Journal of Applied Sciences* vol 9 no 10 pp 629-634

[4] Zubanov V, Shabliy L, Krivcov A 2014 Centrifugal kerosene pump CFD-modeling *Research Journal of Applied Sciences* vol 9 issue 10 pp 629-634

[5] Zakharov O V, Bobrovskij I N and Kochetkov A V 2016 Analysis of methods for estimation of machine workpiece roundness *Procedia Engineering* vol 150 pp 963 – 968

[6] Zakharov O V, Bobrovsky N M, Kochetkov A V, Grigoriev S N and Bobrovsky I N 2016 A sphericity measurement method based on the minimum measuring zone *AIP Conference Proceedings* (Electronic Materials vol 1785) 040094

[7] Savio E, De Chiffre L and Schmitt R 2007 Metrology of freeform shaped parts *CIRP Annals - Manufacturing Technology* vol 56 no 2 pp 810-835.
[8] Mayer J, Mir Y, Trochu F, Vafaeesefat A and Balazinski M 1997 Touch probe radius compensation for coordinate measurement using kriging interpolation Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture vol 211 no 1 pp 11-18.

[9] Piegl L and Tiller W 1997 The NURBS Book (Berlin: Springer-Verlag)

[10] Rajamohan G, Shunmugam M S and Samuel G L 2011 Practical measurement strategies for verification of freeform surfaces using coordinate measuring machines, Metrology and Measurement Systems vol XVIII no 2 pp 209-222.

[11] Rogers D F and Adams J A 1990 Mathematical Elements for Computer Graphics (2nd ed., McGraw-Hill Publishing Company)

[12] Shunmugam M S and Radhakrishnan M S 1976 Comparison of difference methods for computing the two-dimensional envelope for surface finish measurements Computed Aided Design vol 8 no 2 pp 89-83

[13] Bolotov M A, Pechenin V A and Ruzanov N V 2016 Uncertainties in measuring the compressor-blade profile in a gas-turbine engine Russian Engineering Research vol 36 no 12 pp 1058-1065