Creation of particles in a cyclic universe driven by loop quantum cosmology

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We consider an isotropic and homogeneous universe in loop quantum cosmology. We assume that the matter content of the universe is dominated by dust matter in early time and a phantom matter at late time which constitutes the dark energy component. The quantum gravity modifications to the Friedmann equation in this model indicate that the classical big bang singularity and the future big rip singularity are resolved and are replaced by quantum bounce. It turns out that the big bounce and recollapse in the herein model contribute a cyclic scenario for the universe. We then investigate the effects of quantum fields propagating on this cosmological background. By solving the Klein-Gordon equation for a massive and non-minimally coupled scalar field in the primordial region, we study the quantum theory of fields undergoing cosmological evolution towards the late time bounce. By using the exact solutions to describe the quantum fields at early and late time phases we obtain the density of created particles at late time. We find that the density of created particles is negligible comparing with the quantum background density at Planck era, hence, the effects of the quantum particle production do not lead to modification of the future bounce.

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I. INTRODUCTION

Quantum field theory (QFT) in curved space-time is the theory of quantum fields propagating on a classical background \cite{113}. This theory has provided a good approximate description of quantum phenomena in a regime where the quantum effects of gravity do not play a dominant role, but the effects of curved space-time may be significant. In particular, this theory was applied to the description of quantum phenomena occurring in the early universe or close to the black holes. Nevertheless, in the regimes arbitrarily close to the classical singularities where the space-time curvatures reach Planckian scales, quantum effects of gravity cannot be neglected, hence, the theory of quantum fields in classical curved space-time is no longer valid. Therefore, it is expected that the quantum nature of the space-time would have to be taken into account while studying the QFT in Planck era.

The theory of quantum gravity is one of the major open problems in physics, even though by now there are some interests in loop quantum gravity (LQG) \cite{410}. Loop quantum cosmology (LQC), is one possible approach to investigating quantum gravity effects in the early universe by using the quantization techniques from LQG, which provides a number of concrete results \cite{711}. In particular, LQC predicts that the quantum modification of space-time geometry at early times replaces the big bang singularity by a big bounce when the energy density of the universe approaches the Planck energy \cite{1214}. While the model shows the key role played by quantum effects in resolution of big bang singularity at early time, the question whether quantum gravity effects can also be manifested in large scale cosmology has been investigated in Refs. \cite{1516}.

According to several astrophysical observations, our universe is undergoing a state of accelerating expansion. Such experiments indicate that the matter content of the universe, leading to the accelerating expansion, must contain dark energy component (with equation of state $p_{DE} = w_{d} \rho_{DE}$ where $w_{d} < -1/3$) which constitutes 68\% of the total matter content of the universe \cite{17}. The recent astrophysical data as observed by the Planck (using baryon acoustic oscillation (BAO) and Wilkinson Microwave Anisotropy Probe polarization low-multipole likelihood (WP) data), for a constant $w_{d}$ and a universe with the flat spatial section, indicates that $w_{d} = -1.13^{+0.24}_{-0.25}$ at 2\sigma \cite{17}. We will thus suppose from now on, suggested by the Planck’s result, that dark energy is phantom, that is $w_{d} < -1$. When the universe is expanding, the density of dark matter decreases more quickly than the density of dark energy, and eventually the matter content of the universe becomes dark energy dominant at late times. Therefore, dark energy might play an important role on the implications for the fate of the universe \cite{15}. In the context of dark energy cosmology, there have been many classical investigations into the possibility that expanding universe can come to a violent end at a finite cosmic time, experiencing a singular fate. However, the effects of LQC correction might impose an upper bound for the density of dark energy at late time, that lead to resolving the future singularities and replacing them by a quantum bounce. Interestingly, this indicates that the scale factor of the universe undergoes contracting and expanding phases periodically, so that the universe can possess an exactly cyclic evolution \cite{1516}.

One restriction in experiencing the quantum gravity
is that the Planck energy scale is very far beyond the reach of standard experiments such as particle accelerators. Ultra-high energy phenomenas, possibly capable of probing the Planck scale, have occurred at early universe. Although we cannot re-do the early universe, we can witness its consequences. While the theory of quantum fields propagating on classical FLRW background is well-known, there has been some attempts to develop the theory of test quantum fields propagating on a quantum cosmological space-time where the background geometry is governed by the LQC model of the homogeneous and isotropic universe [19–22]. These studies have also provided an extension of the quantum theory of cosmological perturbations to the Planck era [21]. Furthermore, by a similar study of the QFT on a Bianchi I quantum space-time, it has been provided some phenomenological insight into the effects of the quantum nature of geometry on the propagation of test fields and the possible violation of the local Lorentz symmetry [20]. These effects might be possibly observed in some cosmological experiments. A general prediction of the QFT in a curved background at early universe is that particles can be created by time-dependent gravitational fields [23]. Interestingly, a cyclic universe in LQC which initiated from a big bounce and then expanding towards a future bounce, may provide a background with no unique vacuum state. This implies a mechanism for quantum particle production, and further provides a circumstance in which one can measure and observe interesting QFT phenomena at Planck era in an expanding universe. This constitutes our main goal to investigate within this paper.

In this paper we study the quantum fields propagating on a cosmological background governed by the improved dynamics framework LQC. The paper is organized as follows. In section II we consider a Friedmann-Lemaître-Robertson-Walker (FLRW) universe dominated by a dust matter in the past and a phantom dark energy component in the future; then, by employing LQC modifications to the Friedmann equation, we show that, the classical big bang and big rip singularities are resolved and are replaced by quantum bounces at Planck era, one in early time and the other at late time. It will be further shown that such expanding and recollapsing phases lead to a cyclic behaviour for the universe. In section III we study the QFT in quantum cosmological background we presented in section II. We will then pay a particular attention on the mechanism of particle production in the universe at late time. Finally, we will present the conclusions and the results of our work in section IV.

II. QUANTUM COSMOLOGICAL SCENARIO

We consider a FLRW universe whose matter content constitutes a dust matter with the density $\rho_{\text{matt}}$, and a dark energy with the density $\rho_{\text{DE}}$; its total energy density $\rho$ reads

$$\rho = \rho_{\text{matt}} + \rho_{\text{DE}}$$  \hspace{1cm} (2.1)

The total energy density $\rho$ must satisfy the conservation equation:

$$\dot{\rho} + 3H(\rho + p) = 0$$  \hspace{1cm} (2.2)

Furthermore, we assume that there is no interaction between dark matter and dark energy components, so that, each component must also satisfy the local conservation equation:

$$\dot{\rho}_{\text{matt}} + 3H\rho_{\text{matt}} = 0$$  \hspace{1cm} (2.3)

$$\dot{\rho}_{\text{DE}} + 3H(\rho_{\text{DE}} + p_{\text{DE}}) = 0$$  \hspace{1cm} (2.4)

In effective loop quantum cosmological scenario, evolution equations of the universe are given by the modified Friedmann equation [13]

$$H^2 = \frac{\kappa}{3\rho}(1 - \frac{\rho}{\rho_{\text{crit}}})$$  \hspace{1cm} (2.5)

and the Raychaudhuri equation

$$\dot{H} = -\frac{\kappa}{2}(\dot{p} + p)(1 - 2\frac{\rho}{\rho_{\text{crit}}})$$  \hspace{1cm} (2.6)

where $\kappa := 8\pi G$ and $\rho_{\text{crit}} = 0.41\rho_{\text{Pl}}$ is the upper bound for the energy density of the universe provided by quantum gravity ($\rho_{\text{Pl}}$ is the Planck energy density) [14].

Our goal in this section is to construct a cyclic cosmological scenario in LQC (see for example [15]). Therefore, we consider an expanding phase for the universe initiated at a preferred instant of time, $t_i = 0$, being the initial quantum bounce resulted from loop-quantum-gravity effects at early time [12]. Then, in the far future when the density of the dark energy components becomes large and comparable with the Planck energy, modifications to the Friedmann equation given in LQC, again give rise to a quantum bounce for the fate of the universe. Within this scenario, we can select two cosmological phases which, as in QFT, can play the role of ‘in’ and ‘out’ regions for the quantum (scalar) fields propagating on the quantum background; they are, respectively, the ‘initial’ and ‘late time’ bounces.

In this model we assume that the matter content of the universe is dominated by dust matter ($\rho \approx \rho_{\text{matt}}$) at early time, and is dominated by dark energy with phantom component (with the density $\rho \approx \rho_{\text{DE}}$) at late time. We consider that transition from matter dominant phase to dark energy dominant phase occurs at the present time $t = t_0$. Let us describe these two cosmological regimes in more details:

(i) In the region $0 < t < t_0$, we assume that the matter content of the universe is dominated by a dust fluid whose density is $\rho_{\text{matt}}$ with the pressure $p_{\text{matt}} = w_M\rho_{\text{matt}} = 0$. Then, from Eq. (2.3) we obtain

$$\rho(a) \approx \rho_{\text{matt}} = \rho_{\text{crit}} \left(\frac{a}{a_i}\right)^{-3}$$  \hspace{1cm} (2.7)
In this region, we assume that at some very early time \( t = t_i = 0 \), the universe begins to expand; in other words, at \( t_i = 0 \), we have that \( a(t = 0) = a_i \) and \( \dot{a}(t = 0) > 0 \). Therefore, from the modified Friedmann equation \( (2.5) \), we have that the energy density of the universe has its maximum value \( \rho(0) = \rho_{\text{crit}} \) at \( t = 0 \), where it is at a minimum scale \( a(0) = a_i \). Then, for \( t > 0 \) the energy density of the universe (i.e., dust component) decreases until present time \( t = 0 \), when the energy density and the scale factor of the universe are \( \rho(t_0) = \rho_0 \) and \( a(t_0) = a_0 \), respectively. By integrating the Friedmann equation \( (2.5) \), we obtain time evolution of the energy density in this region as

\[
\rho(t) = \rho_{\text{crit}} \left(1 + 3 \beta \rho_{\text{crit}} t^2 \right)^{-1}. \tag{2.8}
\]

Then, by substituting this in Eq. \( (2.7) \), time evolution of the scale factor \( a(t) \) of the universe in this regime is obtained as:

\[
a(t) = a_i \left(1 + 3 \frac{\kappa \rho_{\text{crit}} t^2}{\rho_0} \right)^{1/3}. \tag{2.9}
\]

(ii) In the region \( t_0 < t < t_b \), the universe becomes dark energy dominant: \( \bar{\rho} := \rho(t > t_0) \approx \rho_{\text{DE}} \). The dark energy component is assumed to be a fluid satisfying the equation of state parameter \( w_d = p_{\text{DE}}/\rho_{\text{DE}} \). Then, from equation of conservation \( (2.4) \) we find the density of dark energy component as

\[
\dot{\bar{\rho}} \approx \rho_{\text{DE}} = \rho_0 \left(\frac{\dot{a}}{a_0} \right)^{-\beta}, \tag{2.10}
\]

where \( \dot{a}(t) := a(t > t_0) \) and \( \beta = 3(1 + w_d) < 0 \) (for a phantom fluid with \( w_d \lesssim -1 \)). This implies that the energy density of the universe, in this region, increases until it reaches an upper bound \( \rho_{\text{crit}} \) in the future. By integrating the Friedmann equation \( (2.5) \) for this case, we find the time dependent energy density \( (2.10) \) as

\[
\bar{\rho}(t) = \rho_{\text{crit}} \left[1 + \frac{\kappa}{12} \rho_{\text{crit}} \beta^2 (t_b - t)^2 \right]^{-1}. \tag{2.11}
\]

This equation indicates that, in the far future, at some times \( t = t_b \) the energy density of the universe reaches its maximum \( \bar{\rho}(t_b) = \rho_{\text{crit}} \) and the universe bounces; \( \dot{a}(t_b) = 0 \). By replacing \( \dot{a}(t_b) = a_b \), at which \( \bar{\rho} = \rho_{\text{crit}} \), in Eq. \( (2.10) \), we obtain the scale factor \( a_b \) of the universe at the future bounce:

\[
a_b = a_0 \left(\frac{\rho_{\text{crit}}}{\rho_0} \right)^{\frac{1}{3\beta}}. \tag{2.12}
\]

Consequently, we can obtain the evolution of the scale factor in the late time era by substituting \( \bar{\rho}(t) \) in \( (2.10) \) from Eq. \( (2.11) \):

\[
\dot{a}(t) = a_b \left[1 + \frac{\kappa}{12} \rho_{\text{crit}} \beta^2 (t_b - t)^2 \right]^{-1}. \tag{2.13}
\]

By setting \( \rho(t_0) = \rho_0 \) in Eq. \( (2.11) \) at present time, the time \( t_b \) at which the universe hits a bounce in the future is determined:

\[
t_b = t_0 + \frac{\sqrt{12}}{\kappa \rho_0 \beta^2} \left(1 - \frac{\rho_0}{\rho_{\text{crit}}} \right) \approx t_0 + \frac{6H_0}{\kappa \rho_0 |\beta|}. \tag{2.14}
\]

Considering that at present time \( t = t_0 \) we have \( \rho_0/\rho_{\text{crit}} \ll 1 \), the Friedmann equation \( (2.5) \) can be approximated as \( H^2_0 \approx \kappa \rho_0/3 \). Putting this in Eq. \( (2.14) \) we obtain

\[
t_b \approx t_0 + \frac{2}{|\beta| H_0}. \tag{2.15}
\]

In this equation, by setting \( t_0 \) as \( t_0 \sim 1/H_0 \), we obtain \( t_b \sim (1 + 2/|\beta|) t_0 \), indicating that the universe will hit a bounce in some time \( (2/|\beta|) t_0 \) in the future.

From the matching condition at \( t = t_0 \) for the two (past and future) cosmological regions we have that the time derivative of the scale factors satisfy \( \dot{a}(t_0) = \dot{a}(t_0) \) which corresponds to \( H(t_0) = \dot{H}(t_0) = H_0 \). Suppose that \( \rho_0/\rho_{\text{crit}} \ll 1 \) at present time, the Hubble rate takes its classical limit, \( H_0^2 \approx \kappa \rho_0/3 \). This leads to the matching of energy components of the universe in two cosmological regions: \( \rho(t_0) = \bar{\rho}(t_0) = \rho_0 \).

Figure 1 presents a numerical solution for the scale factor of the universe, governed by Eqs. \( (2.5) \) and \( (2.6) \), for the rescaled choices of parameters \( \beta = 3(1 + w_d) = -6.6 \), \( \rho_0/\rho_{\text{crit}} = 0.005 \), \( a_i = 0.56 \) and \( a(0) = 1 \). This shows an oscillatory behaviour for the scale factor in the whole evolution of the universe.
III. QUANTUM FIELD THEORY

We consider the cyclic cosmological background, resulted from LQC, which was presented in the previous section. In this section, we study the propagation of quantum fields on this background space-time. Then, we investigate circumstances for creation of quantum particles near the future bounce.

A. Scalar fields on quantum FLRW background

Let us consider a real (inhomogeneous) scalar field $\phi(t, \vec{x})$ on a FLRW background, whose Lagrangian is

$$\mathcal{L}_\phi = \frac{1}{2} \left( g^{ab} \partial_a \phi \partial_b \phi - m^2 \phi^2 + \xi \mathcal{R} \phi^2 \right), \quad (3.1)$$

where $\mathcal{R}$ is the curvature of the gravitational system; the coupling constants $\xi = 0$ and $\xi = 1/6$ denote respectively, to the cases of minimally and non-minimally coupled scalar fields. By variation of the Lagrangian above with respect to $\phi$, we obtain the Klein-Gordon equation

$$\left( \Box + m^2 - \xi \mathcal{R} \right) \phi(t, \vec{x}) = 0. \quad (3.2)$$

Performing the Legendre transformation, one gets the canonically conjugate momentum for the test field $\phi$, denoted by $\pi_\phi$, on a $t = \text{const}$ slice. Then, for the pair $(\phi, \pi_\phi)$, the classical solutions of the equation of motion (coming from (3.1)) can be expanded in Fourier modes:

$$\phi(t, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3 k \phi_k(t) e^{i \vec{k} \cdot \vec{x}}, \quad (3.3)$$

$$\pi_\phi(t, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3 k \pi_k(t) e^{i \vec{k} \cdot \vec{x}}, \quad (3.4)$$

where the Fourier coefficients $\phi_k(t)$ and $\pi_k(t)$ satisfy the commutation relation $[\phi_k, \pi_{k'}] = \delta(\vec{k} + \vec{k'})$, and the reality conditions $\phi_k = \bar{\phi}_{-\vec{k}}$ and $\pi_k = \bar{\pi}_{-\vec{k}}$ [2].

In the conformal form, the FLRW metric reads

$$g_{ab} dx^a dx^b = C(\eta) \left( -d\eta^2 + d\vec{x}^2 \right), \quad (3.5)$$

with $\eta$ being the conformal time where $C^{1/2}(\eta)d\eta = dt$. For convenience, we introduce the auxiliary field given by $\chi := a\phi$, so that, each mode of the form $\phi_k := \chi_k/a$ satisfies the equation

$$\chi_k'' + \Omega_k^2(\eta) \chi_k = 0. \quad (3.6)$$

The functions $\chi_k$ satisfy the initial conditions at a particular moment of time $\eta_i$, being the preferred mode functions which determine the (initial) vacuum state, or the lowest energy state.

In this paper, we will consider a non-minimal coupling scalar field, i.e., $\xi = 1/6$. In this case, the frequency reduces to

$$\Omega_k^2(\eta) = k^2 + m^2 a^2(\eta). \quad (3.7)$$

In order to find the evolution of quantum fields in our cosmological background in all times, we need to consider both cosmological regions (i.e., $0 < t < t_0$ and $t_0 < t < t_b$) as described in section II.

It is difficult to solve the Klein-Gordon equation for the general forms [2,9] and [2,13] of the scale factor, so that we will consider some simplifications. To do that, we first divide the entire evolution of the universe into two phases: one which characterizes the “primordial bounce” phase ($t \to 0$), for which it is possible to solve the Klein-Gordon equation, so that, the solution naturally contains the structure of the vacuum state of quantum fields (say, the ‘in’ region). The other which characterizes the “late time bounce” phase ($t \to t_b$). Then, we will investigate the possibility of the particle creation at the late time bounce (i.e., the ‘out’ region).

(i) In the ‘primordial phase’, close to the initial bounce where $t \to 0$, one may expand the general expression of the scale factor (2.9) as

$$a(t) = a_i (1 + s^2 t^2) + \mathcal{O}(t^4), \quad (3.10)$$

where $s^2 := \kappa \rho_{\text{crit}}/4$. In terms of the conformal time $\eta$, by using the relation $a^{-1}dt = d\eta$, we can rewrite the scale factor (3.10) as

$$a(\eta) \approx a_i \text{sec}^2(2as\eta). \quad (3.11)$$

This indicates that as $t \to 0$, then $\eta \to \eta_i = 0$ at the initial bounce; $a(\eta_i) = a_i$. Then, for small $\eta$, close to the initial bounce, we can expand Eq. (3.11) and take the first order terms:

$$a(\eta) \approx a_i + s^2 a_i^3 \eta^2, \quad (3.12)$$

from which we have (to the first order terms)

$$\Omega_k^2(\eta) \approx \omega_k^2 + 2m^2 a_i^4 s^2 \eta^2. \quad (3.13)$$

where $\omega_k^2 := k^2 + m^2 a_i^2$. For later convenience, let us define new variables $x := \sqrt{2\sqrt{2}msa_i} \eta$ and $\tilde{\omega}_k^2 := \omega_k^2/2\sqrt{2}msa_i^2$; in terms of these variables, we rewrite the Klein-Gordon equation (3.6) for the frequency (3.13) as

$$\frac{\partial^2 \chi_k}{\partial x^2} + \left( \tilde{\omega}_k^2 + \frac{x^2}{4} \right) \chi_k(x) = 0. \quad (3.14)$$
Then, by expanding the hypergeometric functions in the form

\[ \chi_k(x) = e^{-\frac{i\bar{k}^2}{2}} \left[ \frac{1}{2} F_1(p, q, ix^2/2) + c_2 x \frac{1}{2} F_1(p, q, ix^2/2) \right] , \tag{3.15} \]

where \( c_{1,2} \) are constants, \( p_1 = (i\bar{k}^2/2 + 1/4), \) \( q_1 = 1/2; \) and \( p_2 = (i\bar{k}^2/2 + 3/4), \) \( q_2 = 3/2. \) Let us rewrite the solution \( \chi_k(x) \) in the form \[ \chi_k(x) = c_1 \chi^{(1)}_k(x) + c_2 \chi^{(2)}_k(x) . \tag{3.16} \]

Then, by expanding the hypergeometric functions \( F_1(p, q, ix^2/2) \) as series, we obtain

\[ \chi^{(1)}_k = 1 - \bar{k}^4 \frac{x^2}{2} - \frac{1}{12} \frac{\bar{k}^8}{6} \frac{x^4}{4} + \ldots , \tag{3.17} \]

\[ \chi^{(2)}_k = x - \bar{k}^4 \frac{x^3}{6} - \frac{1}{20} \frac{\bar{k}^8}{30} \frac{x^5}{4} + \ldots . \tag{3.18} \]

Considering only to the order \( x^3 \) for odd series (and \( x^2 \) for even series), Eq. \( \chi_k(x) \) reduces to

\[ \chi^{(1)}_k(x) \approx c_1 \cos(\bar{k}x) + c_2 \sin(\bar{k}x) . \tag{3.19} \]

By choosing \( c_1 = 1/\sqrt{2\omega_i} \) and \( c_2 = i\bar{k}/\sqrt{2\omega_i}, \) we can rewrite \( \chi_k(x) \) in terms of the conformal time \( \eta, \) as a typical normalized quantum vacuum state:

\[ \chi_k(\eta) \approx \frac{1}{\sqrt{2\omega_i}} e^{i\omega_i \eta} , \tag{3.20} \]

which is in fact the initial vacuum state at \( \eta \to \eta_i = 0. \) Moreover, with the same choices of \( c_{1,2} \) in Eq. \( \chi_k(x) \) we have the solution for the quantum modes at any time. Notice that, the modes \( \chi_k(\eta) \) are the negative-frequency solution with respect to \( \eta, \) therefore, all modes with equal \( \omega \) are complex solutions of Eq. \( \chi_k(x) \) such that, for a chosen mode function \( v_k(\eta) \) of that equation, the general solution \( \chi_k(\eta) \) can be expressed as

\[ \chi_k(\eta) = \frac{1}{\sqrt{2}} \left[ A_k^- v_k(\eta) + A_k^+ v_k^*(\eta) \right] , \tag{3.21} \]

where \( A_k^\pm \) are constants of integration (being the so-called “annihilation” and “creation” operators with respect to the mode \( \chi_k \) in quantum theory), and the mode function \( v_k(\eta) \) is given by

\[ v_k(\eta) = \frac{1}{\omega_i} e^{-i\omega_i \eta} , \tag{3.22} \]

which is normalized due to \( W[v_k, v_k^*] = 2i \) given in Eq. \( \chi_k(\eta) \). Imposing the reality condition \( \chi^*_k(\eta) = \chi^\dagger_{-k}(\eta) \) into Eq. \( \chi_k(x) \) we get \( A_k^\pm = (A_{-k}^\pm)^*; \) then by using Fourier series we can express the expansion of \( \chi(\eta, \bar{x}) \) as

\[ \chi(\eta, \bar{x}) = \frac{1}{(2\pi)^2} \int d^3k \chi_k(\eta, \bar{x}) , \tag{3.23} \]

where \( \chi_k(\eta, \bar{x}) \) is given by

\[ \chi_k(\eta, \bar{x}) = A_k^- v_k(\eta) e^{i\bar{k} \cdot \bar{x}} + A_k^+ v_k^*(\eta) e^{-i\bar{k} \cdot \bar{x}}. \tag{3.24} \]

In the summation above we have changed the variables as \( \bar{k} \to -\bar{k}. \)

(ii) At the ‘late time phase’ as \( t \to t_0 (or \eta \to \eta_b), \) the scale factor \( a(\eta), \) in terms of proper time, tends to its maximum at the bounce:

\[ a(\eta) = a_b , \tag{3.25} \]

where \( a_b \) is given by Eq. \( \Omega^2(\eta) = k^2 + m^2 a_b^2 = \omega_b^2 . \tag{3.26} \]

The solution \( \chi_k(\eta) \) to the Klein-Gordon equation \[ \chi_k(\eta) \] for each mode with the frequency \( \omega_k \) can be expanded as

\[ \chi_k(\eta) = \frac{1}{\sqrt{2}} \left[ B_k^- u_k(\eta) + B_k^+ u_k^*(\eta) \right] , \tag{3.27} \]

in which a new complete orthonormal set of modes \( u_k(\eta) \) was considered as

\[ u_k(\eta) = \frac{1}{\sqrt{a_b}} e^{-i\omega_b \eta} . \tag{3.28} \]

Notice that, similar to the ‘in’ region in the primordial phase, \( u_k(\eta) \) herein this phase, defines the final vacuum state in the ‘out’ region as \( \eta \to \eta_b \). The \( B_k^\pm \) are constants of integration; similarly they are the annihilation and creation operators in quantum theory, but, with respect to the new vacuum state \( u_k(\eta). \) Therefore, similar to \( \chi_k(\eta, \bar{x}) \) we can express the solution \( \chi_k(\eta, \bar{x}) \) as

\[ \chi_k(\eta, \bar{x}) = B_k^- u_k(\eta) e^{i\bar{k} \cdot \bar{x}} + B_k^+ u_k^*(\eta) e^{-i\bar{k} \cdot \bar{x}} , \tag{3.29} \]

where we again applied \( \bar{k} \to -\bar{k} \) in the second term.

Figure 2 shows typical numerical solution to the Klein-Gordon equation \[ \chi_k(\eta) \] for the field in the herein cyclic cosmological model, for sub-planckian values of the mass \( m \) and wavenumbers \( k. \) It presents an oscillatory behaviour for scalar field evolving in the universe between the initial and final bounces. When those parameters become trans-planckians, the oscillations give places to asymptotically divergent behaviour, as could be expected.
the dark energy dominant region.) Furthermore, continuity condition at \( \eta = \eta_c \), from one era to another implies that the (conformal) time derivative of mode functions must satisfy
\[
\frac{\partial \hat{\chi}^{(\text{in})}_k(\eta_c, \vec{x})}{\partial \eta} = \frac{\partial \hat{\chi}^{(\text{out})}_k(\eta_c, \vec{x})}{\partial \eta}.
\]
(3.33)

From the matching conditions (3.32) and (3.33) we obtain
\[
\hat{B}^-_k = \alpha_k \hat{A}^-_k + \beta_k \hat{A}^+_k,
\]
(3.34)
and
\[
\hat{B}^+_k = \alpha_k \hat{A}^+_k + \beta_k \hat{A}^-_k.
\]
(3.35)

These relations present the Bogolyubov transformations in which the old annihilation and creation operators \( \hat{A}_k^\pm \) are expressed through the new operators \( \hat{B}_k^\pm \), with the \( \eta \)-independent (complex) Bogolyubov coefficients:

\[
\alpha_k = \frac{\sqrt{\omega_b}}{2 \sqrt{\omega_i}} \left( 1 + \frac{\omega_i}{\omega_b} \right) e^{-i(\omega_i - \omega_b)\eta_c},
\]
(3.36)
\[
\beta_k = \frac{\sqrt{\omega_b}}{2 \sqrt{\omega_i}} \left( 1 - \frac{\omega_i}{\omega_b} \right) e^{-i(\omega_i + \omega_b)\eta_c}.
\]
(3.37)

Using these coefficients we can express the primordial basis \( v_k(\eta) \) in terms of the late time one, \( u_k(\eta) \), as
\[
v_k(\eta) = \alpha_k u_k(\eta) + \beta_k u^*_k(\eta).
\]
(3.38)

Since \( v_k(\eta) \) and \( u_k(\eta) \) are normalized, it follows that
\[
\alpha_k \alpha_k^* - \beta_k \beta_k^* = 1.
\]
(3.39)

The coefficient \( \beta_k \) is associated with the created particles. More precisely, in the Heisenberg picture, the initial vacuum state \( |0_{\text{in}}\rangle \) in the ‘in’ region is the state of the system for all time. The physical number operator which counts particles in the ‘out’ region reads \( \hat{N}_k = \hat{B}^-_k \hat{B}^+_k |1\rangle \). Then, the mean number of \( u_k \)-mode particles in the state \( |0_{\text{in}}\rangle \) is given by
\[
N_k = \langle 0_{\text{in}} | \hat{N}_k | 0_{\text{in}} \rangle = \beta_k \beta_k^* = \frac{\omega_b}{4 \omega_i} \left( 1 - \frac{\omega_i}{\omega_b} \right)^2,
\]
(3.40)

which is to say that the vacuum state of the \( v_k \) modes contains \( N_k \) particles in the \( u_k \) mode. For a massless scalar field, \( \omega_i = \omega_b = |\vec{k}|c \), thus, \( \alpha_k = 1 \), \( \beta_k = 0 \) and the mean density \( N_k \) is zero; therefore, no particle is produced in the case of a massless scalar field in the ‘out’ region.

C. The energy density of created particles

The relation (3.40) is the average number of late time particles per spatial volume and per wave number \( k \). The
energy density of the \( \ell \) particles in the vacuum state \( |0_{\text{m}}\rangle \) for each mode \( \nu k \) reads \( \rho_0 = \left( \frac{1}{2} + \ell \right) \omega_i \), where \( w_i/2 \) is the zero-point energy. Since the chosen quantum state corresponds to the ‘in’ vacuum \( |0_{\text{m}}\rangle \), then in the ‘out’ region \( (\eta \to \eta_b) \) the combined mean density of total particles for each mode \( k \) (after subtracting the zero-point energy) is \( \rho_k = N_k \omega_i \). Then, for all modes (per unit volume), we obtain the density of late time particles as
\[
\rho_p = \int_0^{\infty} \frac{d^3k}{(2\pi)^3} |\beta_k|^2 \omega_i
\]
\[
= \int_0^{\infty} \frac{dk}{8\pi^2} k^2 \omega_b \left( 1 - \frac{\omega_i}{\omega_b} \right)^2 . \tag{3.41}
\]
At small scales close to the initial bounce (‘in’ region) we can approximate \( m \omega_i \ll 1 \), thus, \( \omega_i \approx k \). Then, Eq. (3.41) reduces to
\[
\rho_p \approx \frac{1}{8\pi^2} \int dk k^2 \omega_b \left( 1 - \frac{k}{\omega_b} \right)^2 \]
\[
= \frac{1}{8\pi^2} \left[ \omega_b k \left( \frac{k^2}{2} - \frac{m^2 \omega_b^2}{4} \right) - \frac{k^4}{2} + \frac{m^2 \omega_b^2}{4} \ln(2k + 2\omega_b) \right] . \tag{3.42}
\]
This integral has obviously, a logarithmic divergence when it tends to the ultraviolet limit, \( k \to \infty \), so that it is necessary to regularize it. Using the \( n \)-wave method developed in Ref. [27], which consists in subtracting terms obtained by expanding \( \rho_k \) in powers of \( k^{-2} \),
\[
\rho_k^{\text{ren}} = \rho_k - E_k^{(0)} - E_k^{(1)} - \frac{1}{2} E_k^{(2)} \tag{3.43}
\]
in which we have defined
\[
E_k^{(p)} = \lim_{n \to \infty} \frac{\partial^p \rho_k^{(n)}}{\partial(n^{-2})^p} , \tag{3.44}
\]
where \( \rho_k^{(n)} = \rho_k(nk, nm)/n \), and \( n \) is the parameter that characterizes the order of the divergence. Therefore, \( E_k^{(0)}, E_k^{(1)} \) and \( E_k^{(2)} \) eliminate, respectively, the logarithmic, quadratic and quartic divergences. Following the regularisation procedure corresponding to a full renormalisation of the coupling constants [28, 29], we have that
\[
\rho_k = \omega_b - 2k + \frac{k^2}{\omega_b} \tag{3.45}
\]
which follows that \( \rho_k^{(n)} = \rho_k \); then, we find the renormalised energy as [25]
\[
\rho_k^{\text{ren}} = \rho_k - E_k^{(0)} = 0 , \tag{3.46}
\]
which is zero. Vanishing energy density of created particles indicates that quantum field effects associated with the cosmological dynamics at late time do not change the nature of quantum gravity bounce in the far future.

IV. CONCLUSIONS AND OUTLOOK

In this work, we considered a flat FLRW universe whose matter content is dominated by a dust fluid at early time and a phantom dark energy component at late time. We employed loop quantum cosmology to govern the geometry of space-time at Planck era. In particular, quantum field effects modified the dynamics of the universe in the primordial and late-time phases, thus, resolved the big bang singularity in the past and the big rip singularity in the future and replaced them by quantum bounce. This indicated that the resulting quantum bounce and recollapse in our herein model contribute a cyclic scenario for the universe.

We investigated the quantum theory of scalar fields on the resulting cosmological background. By solving the Klein-Gordon equation, for a massive and non-minimally coupled scalar field, undergoing cosmological evolution, we obtained exact solutions to the quantum fields evolving between the past and the future bounces. The transition from matter dominant phase to the dark energy dominant phase led to different frequencies for the modes of the quantum fields in the initial and final phases. Consequently, this gave rise to the differences in vacuum states at late time bounce (the ‘out’ region) from the one at the primordial bounce (the ‘in’ region); this non-uniqueness of the vacuum state is accompanied by quantum particle production in the ‘out’ region. We then computed the density of the quantum particles created at late time, on approach to the future bounce where the energy density and the scale factor of the universe take their maximum value and remain finite. We showed that the density of created particles is negligible comparing with the quantum background density at Planck era. We concluded that the effects of the quantum particle production do not lead to modification of the future bounce.

One important issue for the model analysed here is the possibility of observational traces of the quantum phase and of the oscillatory behaviour of the universe. This would require, for example, to compute the evolution of perturbations in the model. However, we must face an important limitation: We have no classical covariant formulation of the equations of motion governing the evolution of the universe. Strictly speaking, this analysis would require to return to the full quantum model in performing the perturbative analysis, what implies in considerable technical difficulties (c.f. Refs. [21, 22]).

This drawback is less severe in the case of gravitational waves (see Ref. [30]), since there is no coupling of the spin 2 modes with the scalar and vectorial modes (which are directly connected to the matter content), at least at linear level. If the gravitational waves, represented by the propagation of the traceless, transverse field \( \delta g_{ij} = h_{ij} \), obeys the same equation as in the usual general relativity
case, that is,
\[ \mu'' + \left( k^2 - \frac{a''}{a} \right) \mu = 0, \tag{4.1} \]
where \( h_{ij} = a \epsilon_{ij} \mu \), with \( \epsilon_{ij} \) being the polarisation tensor and \( a(\eta) \) the scale factor, we would have the same equation as for a massless scalar field propagating in the geometry determined by the background equations [31]. We would have in this case, qualitatively, the same behaviour as displayed in figure 2 for the massive field, but with an accumulative divergence for transplackian values of \( k \). However, it is not sure that equation (4.1) remains the same in the present case. Moreover, the transplanckian regime would require a full quantum analysis, including for the background. Due to these important difficulties to surmount, we postpone a perturbative analysis for future researches.

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