Research on optimal meshing speed difference based on shift success probability for dog Clutch in automated manual transmission

Shaohua Sun, Maosen Gong1, Baogui Wu and Yupeng Zou
College of Mechanical and Electrical Engineering, China University of Petroleum (East), Qingdao 266000, China

1 Email: gmsen@qq.com

Abstract. The engagement theory for dog Clutch was analyzed and the dynamic model of its engagement process was established. The uncertain result of shift success or failure was described from meshing probability and the shift probability model of dog clutch used in AMT for electric vehicle was established. Afterwards, the factors affecting shift success probability were analyzed by Matlab/Simulink simulation and the optimal meshing speed differences of different gears are defined. Finally, the validity and accuracy of theoretical and simulation analysis were verified by experiments. The results can be used for designing shift strategy as well as improving shift quality for dog clutch meshing mechanism which used in AMT.

1. Introduction
Due to the advantages of short shifting time, simple structure, accurate torque transfer, large capacity and low cost, the dog clutch can replace synchronizer to become the best meshing mechanism in AMT for electric vehicle[1-3].

Lei Yulong researched the contact mechanics of dog Clutch based on its engagement process [4-5]. Zhang Mingsong established a dog Clutch model by SolidWorks and showed the conditions of its engagement and release[6]. Tong Xiaohui explored the relative position of the active and passive components in the engagement process of dog Clutch for a hybrid bus[7].

This paper focuses on the dog Clutch of EV AMT, analyzes its engagement theory, and establishes its mathematical model. Then, the simulation model for shift success probability is established and the influence factors are analyzed. Finally, the optimal speed difference between the meshing gear and sleeve is determined during gear shift process, which can provide theoretical guidance for designing motor speed control algorithm in the dynamic shifting process.

2. Dog clutch meshing mechanism
In order to establish an effective system model, the following assumptions are required: 1) The system consists of inelastic inertia elements. 2) There is only one degree of freedom for each rotating part. Converting the parameters of the meshing gear and its front end to this gear, and converting the parameters of engaging sleeve and its rear end to the sleeve. The model is shown in Figure 1.
Figure 1. Simplify model of dog Clutch.

Figure 2. dog Clutch shifting engagement process.

3. Analysis and modeling for dog Clutch engagement theory

It is divided into four phases: initial free-sliding, tooth surface impact, tooth surface friction and spline engaging. Figure 2 shows the engagement process.

3.1. Dynamic model of dog Clutch engagement process

3.1.1. Initial free-sliding phase. Because of no locking device, gear can be shift at any angular velocity difference \( \Delta \omega_0 \) between engaging sleeve and meshing gear. The friction force of axial sliding is:

\[
F_\mu = \mu F_{\text{sling}}
\]

The duration of the initial free-sliding phase is:

\[
t_1 = \frac{2x_m}{F_{\text{act}} - F_\mu}
\]

The angular velocity of engaging sleeve remains constant during shift process. Because of the power loss caused by gear churning and bearing friction, the speed of meshing gear will decrease [8]. It is considered as an uniform deceleration process. At the end of this phase, angular velocity difference between the two spline teeth \( \Delta \omega_1 \) is:

\[
\Delta \omega_1 = \Delta \omega_0 - \left( \frac{T_{\text{gloss}}}{J_1} - \frac{T_{\text{sling}}}{J_2} \right)t_1
\]

The value of \( J_1 \) and \( J_2 \) can be obtained from the following formula:

\[
J_1 = i_s \left( J_{\text{c}} + J_0 \right)
\]

\[
J_2 = J_{aw} + J_{ps} + \left( 2J_{di} + 2J_{wh} + m_r r_3 \right) / i_s^2
\]

3.1.2. Tooth surface impact phase. When the normal distance reaches \( 10^{-5} \) m, the two spline teeth directly contact, and the axial speed of the sleeve is rapidly reduced to zero. Then the angular velocity difference between the two teeth is reduced from \( \Delta \omega_1 \) to \( \Delta \omega_2 \). The axial displacement of the engaging sleeve is ignored during impact process [9-10].The angular acceleration of meshing gear and engaging sleeve meets Eq.(6):

\[
\alpha_{\text{m}}(t) = (-1)^i \text{sgn} [\Delta \omega(t)] \frac{\mu r_n F_{\text{imp}}(t)}{J_i \sin \beta} \quad t \in [t_1, t_2], \ i = 1, 2
\]

The duration of impact is \( t_2-t_1 \approx 0 \). According to the momentum theorem:
The variation of angular velocity difference between two teeth after impact can be expressed as:

$$
\Delta \omega_{\text{imp}} = \int_{t_1}^{t_2} \left[ \alpha_i(t) - \alpha_j(t) \right] dt = - \text{sgn}(\Delta \omega_2) \frac{\mu r_i m_j v_i t_{f} \mu r_i F_{\text{act}} - \mu r_i F_{\text{slip}}}{J_i J_j}
$$

At time $t_2$, the angular velocity difference between the two spline teeth $\Delta \omega_2$ is:

$$
\Delta \omega_2 = \Delta \omega_1 + \int_{t_1}^{t_2} \left[ \alpha_i(t) - \alpha_j(t) \right] dt = \text{sgn}(\Delta \omega_1) \max \left(0, |\Delta \omega_1| - |\Delta \omega_{\text{imp}}| \right)
$$

The absolute value of the relative variation for contact position can be expressed as:

$$
|\Delta \varphi_{\text{imp}}| = \left| \int_{t_1}^{t_2} \left[ \varphi_i(t) - \varphi_j(t) \right] dt \right| \leq \left( t_2 - t_1 \right) |\Delta \omega_1| - \frac{\mu r_i m_j v_i t_{f} \mu r_i F_{\text{act}} - \mu r_i F_{\text{slip}}}{J_i J_j}
$$

If $|\Delta \omega_1| \leq |\Delta \omega_{\text{imp}}|$, the angular velocity of the two spline teeth can be synchronized during this phase.

3.1.3. Tooth surface friction phase. When the axial velocity of the engaging sleeve is stable at zero, it is considered that the two spline end faces contact, and tooth surface friction phase begins. At this time, the amount of overlap between the end faces is $\varphi_{\text{start}}$. When the angular velocity is synchronized or the two end faces disengage, the phase ends. At this time, the amount of overlap between the end faces is $\varphi_{\text{end}}$.

Assuming that the angular velocities achieve synchronization at time $t_3$, the angular acceleration and angular velocity of meshing gear and engaging sleeve are expressed as:

$$
\alpha_i = (-1)^i \text{sgn}(\Delta \omega_1) \frac{\mu r_i F_{\text{act}}}{J_i \sin \beta} \quad i = 1, 2, t \in [t_2, t_2 + t_f)
$$

$$
\omega_i(t) = \omega_i(t_2) + \int_{t_2}^{t_2 + t_f} \alpha_i(t) dt \quad i = 1, 2, t \in [t_2, t_2 + t_f)
$$

If $\omega_i(t_2) = \omega_2(t_2)$ in Eq.(12), the duration of this phase can be known from (1), (2), (3), (9) and (11):

$$
t_f = \frac{\max \left(0, |\Delta \omega_1| - |\Delta \omega_{\text{imp}}| \right)}{\sin \beta \ J_i J_j}
$$

The relative variation of spline contact position around circumference during the time $t_f$ is:

$$
\Delta \varphi_f = \left| \int_{t_2}^{t_2 + t_f} \left[ \varphi_i(t) - \varphi_j(t) \right] dt \right| = \frac{\max \left(0, |\Delta \omega_1| - |\Delta \omega_{\text{imp}}| \right)}{2 \sin \beta \ J_i J_j}
$$

At the end of this phase, there are three possibilities: 1) $\Delta \omega_2 = 0$, $\varphi_{\text{end}} \neq 0$; 2) $\Delta \omega_2 = 0$, $\varphi_{\text{end}} = 0$; 3) $\Delta \omega_2 \neq 0$, $\varphi_{\text{end}} = 0$. The schematic diagram of successful and failed shift is shown in Figure 3.
In the first case, the rotating speed achieves synchronization in this phase, and the two tooth end faces are in permanent contact. They can not engage and gear shift fails. The second and third cases indicate that the overlap between two spline tooth end faces has been eliminated at the end of this phase, and gear shift is bound to succeed whether the angular velocity difference $\Delta \omega_3$ is zero or not.

### 3.1.4. Spline engaging phase.

At this stage, the axial force of engaging sleeve is the same as the initial sliding phase. During engagement process, $\Delta \omega_3$ will rapidly decrease to zero due to the impact of the two teeth, causing a brief torsional vibration, and the peak value can be obtained from the following formula:

$$ T_{\text{tor peak}} = |\Delta \omega_3| \sqrt{\frac{k_{\text{tor}} J_1 J_2}{J_1 + J_2}} $$

(15)

### 3.2. Shift probability model of dog Clutch

In this paper, the random factor $\xi$ is introduced to indicate the relative position of the two spline tooth end faces along circumference at the beginning of the surface friction phase. $\xi$ is evenly distributed over $[0, 2\pi/n]$. The overlap $\phi_{\text{start}}$ determines the angle which the meshing gear needs to rotate relative to the engaging sleeve when shift success. At the beginning of the tooth surface friction phase, the relationship between $\xi$ and $\phi_{\text{start}}$ is shown in Figure 4.

At the beginning of tooth surface friction, $\phi_{\text{start}}$ can be expressed by $\xi$ as:

$$ \phi_{\text{start}} = \frac{\pi}{n} \Phi - \xi \quad 0 \leq \xi < \frac{2\pi}{n} - \Phi $$

$$ 0 \quad \frac{2\pi}{n} - \Phi \leq \xi < \frac{2\pi}{n} $$

(16)

The conditions for successful engagement can be expressed as:

$$ \phi_{\text{start}} \leq \Delta \phi_f $$

(17)

In this paper, shift success probability under a certain speed difference is defined as the gear shift capacity on this speed difference. Its expression is as follows:

$$ p_{\text{succ}} = P(\phi_{\text{start}} \leq \Delta \phi_f ) = \begin{cases} 
\frac{n_\Delta \phi_f}{2\pi} + p_0 & \Delta \phi_f \leq \frac{2\pi}{n} \Phi \\
1 & \Delta \phi_f > \frac{2\pi}{n} \Phi 
\end{cases} $$

(18)

Where $p_0$ represents the probability that the two spline teeth will directly engage without contact, and the value can be expressed as:

$$ p_0 = \frac{n \Phi}{2\pi} $$

(19)

According to Eq. (14), (16) and (17), it can be known that:

$$ p_{\text{succ}} = \begin{cases} 
p_0 & |\Delta \omega_1| - |\Delta \omega_{\text{imp}}| \leq 0 \\
p_0 + p_1 & 0 < |\Delta \omega_1| - |\Delta \omega_{\text{imp}}| \\
\frac{2\pi - \eta_\beta J_1 J_2}{n J_1 J_2} \sqrt{\frac{2 \mu r F_{\text{act}}}{\sin \beta}} & |\Delta \omega| - |\Delta \omega_{\text{imp}}| \leq 0 \\
1 & |\Delta \omega| - |\Delta \omega_{\text{imp}}| > 0 \\
\frac{2\pi - \eta_\beta J_1 J_2}{n J_1 J_2} \sqrt{\frac{2 \mu r F_{\text{act}}}{\sin \beta}} & |\Delta \omega| - |\Delta \omega_{\text{imp}}| \leq 0 \\
\frac{n J_1 J_2 \left(|\Delta \omega_1| - |\Delta \omega_{\text{imp}}| \right)^2 \sin \beta}{4 \pi r_{\text{act}} F_{\text{act}} (J_1 + J_2)} & |\Delta \omega| - |\Delta \omega_{\text{imp}}| > 0 
\end{cases} $$

(20)

(21)
Since $\varphi_{\text{fstart}}$ is a random variable, $\Delta \omega_3$ is also a variable determined by $\varphi_{\text{fstart}}$ and $\Delta \omega_0$.

$$\Delta \omega_0 = \left\{ \begin{array}{ll}
\frac{2\pi}{n} - \Phi \leq \xi < \frac{2\pi}{n} \\
\sqrt{\Delta \omega_2^2 - 2\left( \frac{2\pi}{n} - \Phi - \xi \right) \frac{J_1 + J_2 \mu J_1 F}{J_1 J_2} \sin \beta} \\
\frac{2\pi}{n} - \Phi - \Delta \varphi_f \leq \xi < \frac{2\pi}{n} - \Phi
\end{array} \right. \quad (22)$$

When $0 \leq \frac{2\pi}{n} - \Phi - \Delta \varphi_f$, gear shift fails.

4. Determination of the optimal meshing speed difference

In the paper, the nonlinear Sigmoid function is used to replace the signal-judgment function $\text{sgn} (x)$ in the model:

$$\text{sgn} (x) \approx 2\sigma \left( \frac{\xi}{\sigma} \right) - 1 = \frac{2}{1 + e^{-\xi}} - 1 \quad (23)$$

In Eq. (23), $\sigma$ is a smooth coefficient of the Sigmoid function. The shift capacity simulation results of the 1st, 2nd, and 3rd gears at various speed differences are shown in Figure 5 and 6.

As can be seen from Figure 5, the $p_{\text{succ}}$ and $\Delta \omega_1$ is a nonlinear relationship, which gradually increases with $|\Delta \omega_1|$ increasing. When $|\Delta \omega_1|$ increases to a critical value, $p_{\text{succ}}$ is maintained at the maximum of 1 and is symmetric about $\Delta \omega_1=0$. The larger $|\Delta \omega_1|$ is, the larger the relative displacement along the circumference $\xi$ is. At the end of this phase, the possibility of complete elimination of $\xi$ is greatly increased, and the shift success probability rises. However, when $|\Delta \omega_1|$ increased to the critical value, $\xi$ is completely eliminated at the end of the phase, no matter how $|\Delta \omega_1|$ increases, it will no longer affect the shift success probability. With the same $\Delta \omega_1$, $p_{\text{succ}}$ is not equal for each gear. This probability is not only related to the moment of inertia of the system in each gear, but also affected by gear ratio.

![Figure 5. Relation curve of $p_{\text{succ}}$ and $\Delta \omega_1$.](image5)

![Figure 6. Relation curve of $p_{\text{succ}}$ and $\Delta \omega_0$.](image6)

In the initial free sliding phase, the two gears are both uniformly decelerated and rotated. The movement of the curve in Figure 6 relative to the curve in Figure 5 depends on the relationship between the absolute value of angular acceleration change for $|\Delta \omega_1|$ increasing and $|\Delta \omega_2|$; 1) If $|\Delta \omega_1| > |\Delta \omega_2|$, the curve in Figure 6 moves to the right; 2) If $|\Delta \omega_1| < |\Delta \omega_2|$, the curve moves to the left. The situation in the paper belongs to the latter.

$\Delta \omega_0$ when $p_{\text{succ}}$ just reaches 1 is the optimal meshing speed difference $\Delta \omega_{\text{opt}}$. At this speed difference, not only the dog Clutch can be successfully engaged, but also the torsional vibration caused by $\Delta \omega_3$ during spline engaging phase can be minimized. The $\Delta \omega_{\text{opt}}$ is expressed as:
\[
\Delta \omega_{\text{opt}} = \frac{J_1 T_{\text{start}} - J_2 T_{\text{start}}}{J_1 J_2} \left[ \frac{2 r_s m_s}{r_1 F_{\text{act}} - \mu F_{\text{slip}}} + \frac{\mu F_{\text{slip}}}{\sin \beta} \right] + \frac{\mu F_{\text{slip}}}{\sin \beta} \left[ \frac{2 r_s m_s}{r_1 F_{\text{act}} - \mu F_{\text{slip}}} + \frac{2 \pi - n \Phi}{n} \right] \\
\]

Figure 7 is a relation curve between \( \Delta \omega_3 \), \( \Delta \phi_{\text{start}} \) and \( \xi \), when the spline engagement phase begins in shifting to 3rd gear process.

**Figure 7.** Relation between \( \Delta \omega_3 \), \( \Delta \phi_{\text{start}} \) and \( \xi \).

**Figure 8.** The data of 1st gear.

### 5. Experimental research

The shift results of each gear in different speed differences were statistically analyzed by bench test. The speed difference \( \Delta \omega_1 \) at which the two spline end faces start to contact is calculated by a speed sensor installed on the input shaft and output shaft of the transmission. Table 1 shows the experimental results of 1st gear, where the data includes \( \Delta \omega_1 \), the total number of shifts \( N \), the number of successful shifts \( K \), and the frequency of successful shifts \( K/N \).

| Serial number | \( \Delta \omega_1 \) (rad/s) | \( N \) | \( K \) | \( K/N \) |
|---------------|----------------------------|-------|-------|---------|
| 1             | 0–0.4                      | 348   | 70    | 0.2011  |
| 2             | 0.4–1.3                    | 343   | 81    | 0.2362  |
| 3             | 1.3–2.2                    | 340   | 110   | 0.3235  |
| 4             | 2.2–3.1                    | 239   | 115   | 0.4812  |
| 5             | 3.1–4.0                    | 225   | 135   | 0.6000  |
| 6             | 4.0–4.9                    | 254   | 210   | 0.8268  |
| 7             | 4.9–                       | 524   | 524   | 1       |

The shift success probability interval in each \( \Delta \omega_1 \) interval can be figured out according to the law of large numbers, and as follows:

\[
\frac{K}{N_i} - \frac{1}{\sqrt{4 \alpha N_i}} \leq \bar{p} \leq \frac{K}{N_i} + \frac{1}{\sqrt{4 \alpha N_i}}
\]

Where \( \alpha \) is 0.1, the confidence of the above estimation is \( 1-\alpha=0.9 \). Simulation data, experimental data and confidence interval of 1st gear are drawn on the same graph, as shown in Figure 8.

It can be seen that the experimental and simulation data are identical, and the \( K/N \) of the 1st gear calculated by the data in each \( \Delta \omega_1 \) interval is within the confidence interval.

### 6. Conclusion

(1) The engaging result of is uncertain, which is affected by the gear ratio, the moment of inertia of the transmission system and the initial angular velocity difference at the beginning of gear shifting.
(2) There is a definite minimum value for the shift success probability. This value is determined by the backlash after the dog Clutch splines fully engage, indicating the possibility of direct engagement success without the teeth surfaces impact and friction during gear shifting process.

(3) The relationship between the shift success probability and the initial angular velocity difference is nonlinear. The probability increases with the increase of the angular velocity difference, but when the angular velocity difference reaches a certain critical value, the probability reaches the maximum value 1 and does not change.

(4) The optimal meshing speed difference can not only ensure the successful engagement of the dog Clutch, but also minimize the torsional vibration caused by the residual speed difference in the process of the spline engaging and improve the shift quality.

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The attachments
\( J_1 \) - The equivalent moment of inertia of the meshing gear;
\( J_2 \) - The equivalent moment of inertia of the engaging sleeve;
\( \omega_1 \) - The angular velocity of the meshing gear;
\( \omega_2 \) - The angular velocity of the engaging sleeve;
\( F_{act} \) - The shifting force acting on the engaging sleeve;
\( F_f \) - The interaction force between the spline tooth end face of the meshing gears;
\( \Delta \omega \) - The angular velocity difference between the meshing gears;
\( \Delta \omega_0, \Delta \omega_1, \Delta \omega_2, \Delta \omega_3 \) - The angular velocity difference between the meshing gears when each phase of the engagement process begins;
\( t_1, t_2, t_3, t_4 \) - The end of each phase of the meshing process;
\( T_{loss} \) - The torque loss of the engaging sleeve and the rotating parts connected to the sleeve.
$F_{\mu_1}$ - The axial friction between the engaging sleeve inner spline and the splined hub outer spline;
$T_{\text{gloss}}$ - The torque loss of the meshing gear and the rotating parts connected to the gear.
$T_{\text{torpeak}}$ - The peak torque of torsional vibration at the last sliding phase;
$v_{1_{-}}$ - The axial speed of the engaging sleeve at the end of the initial sliding phase;
$k_{\text{tor}}$ - The contact stiffness of the spline tooth end face;
$\phi_{\text{fstart}}$ - The end face overlap at the beginning of the tooth surface friction phase;
$\phi_{\text{fend}}$ - The end face overlap at the end of the tooth surface friction phase;
$F_{\text{imp}}$ - The impact force between the spline teeth when tooth surfaces collide;
$p_{\text{suc}}$ - The probability of successful shifting engagement.