Theoretical search for collective effects in multiparticle production

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Abstract

The properties of QCD vacuum and of the confinement of quarks and gluons certainly influence the multiparticle production processes. Some phenomenological attempts of the consideration of related collective effects and the possibilities of their experimental detection are briefly discussed in this review. We consider in particular the correlation characteristics of pion systems, statistical and hydrodynamical analogies, the problem of a phase transition from a quark-gluon plasma to a multipion state and the possible modifications of the evolution equations of the quark-gluon jets. The presentation is somewhat simplified and could be interesting for those only entering the field.

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Foreword

This review is not a traditional one, which is seen already from its title. Usually a review contains a series of original papers providing a relatively exhaustive presentation. Here we are attempting to describe certain directions of theoretical search for the collective effects in the particle interactions at high energies. Using the modern terminology one could somewhat conventionally unify these directions and speak about some phenomenological attempts to find the nonperturbative effects which can not be described by the QCD perturbation theory. They are related in particular to the QCD vacuum structure and the confinement of quarks and gluons. Although the applied methods and the level of the results vary drastically, we have decided to give their brief description in the hope that their comparison could be useful for the development of new ideas and will lead to new suggestions. This is the reason for the presentation not being detailed. We only mention the final results and for the detailed explanations and calculations the reader should turn to the original papers. Our goal is to give a general perspective of several parallel approaches.

Additional motivation for this is a recollection of how lively Igor Evgenievich Tamm, to whose centenary this volume is dedicated, always faced new ideas and participated in their discussion. Moreover, the last paragraph of the review is directly related to the work of I.E. Tamm on the development of electron-photon showers in the medium and on the Cherenkov radiation, because there we discuss the hypothetical analogous effects in quark-gluon jets.

1. Introduction

A collision of elementary particles or nuclei at very high energies is usually accompanied by the production of many new particles. The multiparticle production processes were first discovered in cosmic rays and then on the accelerators. Their origin is a strong interaction of colliding particles which, according to the modern understanding (see [1-6]), is described as an interaction of quarks and gluons in the framework of Quantum Chromodynamics (QCD).

Due to the asymptotic freedom property of QCD, when the coupling constant becomes small at small distances, an approach based on the QCD
perturbation theory turned out to be extremely fruitful in the study of hard processes with large momentum transfer. Moreover, although it seems amazing, it turns out possible to describe a number of properties of the soft processes by taking into consideration higher order contributions in perturbation theory and conservation laws.

At the same time the problem of translating from a language of quarks and gluons into that describing the experimentally detectable hadrons is still solved either at the axiomatic or at the model level. In the first case one usually exploits a hypothesis of a local parton-hadron duality, when the momentum distributions of partons and hadrons are identified up to a multiplicative factor. This hypothesis is supported both by theoretical considerations (a formation of colorless parton clusters (preconfinement)) and by a number of experimental facts. The model approach is usually used in Monte Carlo modelling of a transition from partons to hadrons.

Despite of the impressive successes of the perturbative approach, one should not forget that the problem of confinement of color objects, quarks and gluons, still remains unsolved. In particular the influence of confinement on the properties of hadrons in multiparticle production processes could turn out to be nontrivial. One can not exclude, that the collective aspects of the system’s behavior as a whole and the specific properties of quark-gluon QCD vacuum could show up. For a theoretical description of such features the string models taking into account confinements of statistical physics (or, more generally, macroscopic approaches) are usually more relevant than the perturbative calculations. The study of the collective excitation modes of a hadron (quark-gluon) medium is a very complicated problem and is actually at the infancy stage.

To give an example of the possible effects let us turn to the analogies with the electrodynamics of a continuous media. An electric charge (e.g., electron), crossing a boundary of the two electrically neutral media, having different refractivity indices, radiates photons. The radiation properties are determined precisely by this difference, which describes the different collective properties of the two media. At sufficiently high electron velocities the Cherenkov radiation might also appear. Analogously, in the process of interaction of two hadrons or nuclei, the quarks that are hidden in one of them, can in principle "see" the target as a whole, i.e. can radiate as in the case when they pass through a color neutral medium. This is of course possible only at relatively small momentum transfer.
At the same time one should not forget that this analogy is not complete. The radiation of photon does not lead to a change of a charge of the radiating particle. On the contrary, the color current changes in the process of gluon emission, because the gluons are themselves color objects. The space-time analysis of the process is especially relevant in that situation [6].

Collective interactions can also be important in the case, when in the process of interaction a quark-gluon plasma is formed. Such a possibility is discussed already for a long time. The main criterion is an appearance of a sufficiently high hadronic density in the collisions of heavy nuclei.

The above-listed examples can only serve as guidelines in attempting to apply the methods of statistical physics to the problem of multiparticle production. Here it is a right place to stress that in the statistical physics as well as in the study of multiparticle production processes one often applies the same mathematical method. It is based on the calculation of correlation characteristics in the phase space. The similarity of the electro- and chromodynamics lagrangians allows to use certain analogies. At the same time their difference leads to important distinctions, one of them having been mentioned above. This shows up, in particular, in the similarity and distinctions of the corresponding equations.

We shall try to give a brief review of the theoretical attempts to analyze the problem of collective effects in particle physics. Unfortunately here it is difficult to choose some mainstream. The development of these ideas is rather following several directions, slowing down or completely stopping at certain obstacles. We shall begin with a brief review of a history of a subject and then give a more thorough discussion of the recent ideas.

2. Early history of a problem.
Statistical and hydrodynamical models

The processes in which a large number of particles is produced were discovered in the cosmic ray showers more than 60 years ago. The first attempt to describe them using the ideas of statistical physics and hydrodynamics dates back to Heisenberg [7]. Somewhat later the analogous attempts were made by Wataghin [8]. An active discussion of this approach began however after the appearance of the paper by Fermi [9] where he introduced a particular statistical model of the multiparticle production processes in nuclear collisions (a detailed description of the model and its further development is
given in the reviews [10,11]). According to the main assumption of the model the process of multiparticle production occurs via creation of a unique system, in which there establishes a thermodynamical equilibrium. The distributions of secondary particles are therefore described by the thermodynamical formulae for blackbody radiation. The legitimacy of such an assumption was discussed many times (see ref. [12] and the reviews [10,11]). It was agreed that if the models of this type are valid, this takes place only in the domain of relatively low energies. At higher energies (and higher multiplicities) the interaction of generated particles can lead to such an expansion of this unified system that can be described by the equations of hydrodynamics. This idea was put forward by Landau [13] and was later widely exploited in the papers by many authors (see the review [14]). Here it is worth mentioning, that the possibility of applying the thermodynamical formulae for the description of an ensemble of strongly interacting particles at realistic energies and multiplicities is of course far from being evident. The critical analysis of the basics of this approach can be found, for example, in the interesting paper [15].

According to the basic postulates of the statistical physics the Fermi model used an assumption of a dominant role of a phase space in the probability of a final state with \( n \) particles, when a quantum-mechanical matrix element is just a normalization factor. Let us write a general expression for this probability:

\[
P_n \sim \int |A_n|^2 \delta^4(\Sigma p_i - \Sigma p_f) \prod_f d^3 p_f, \tag{1}
\]

where \( A_n \) are the transition amplitudes, \( p_i \) and \( p_f \) are the four-momenta of initial and final particles. If we consider the transition amplitudes \( A_n \) as being independent of the final momenta \( p_f \), the integral over the phase space will factorially vanish with growing \( n \), because the mean particle momentum will be proportional to \( 1/n \), i.e.

\[
P_n \sim n^{-(3n-4)} \tag{2}
\]

at \( n \to \infty \) and fixed total energy \( E \) (the particle mass is neglected). The factorial behavior of analogous type is known in statistical physics as well. However, if an assumption on the weak amplitude dependence on the final momenta and particle number \( n \) can be justified, this can happen only at low
energies. With growing energy and number of created particles the strong interaction in such a quasiclassical system forces to describe its evolution rather as the hydrodynamical expansion of a blob of the nuclear matter. In the process of its expansion the temperature drops and the blob decays into final particles. This idea immediately explains an important experimentally known fact of the limited transverse momenta of the particles. Let us mention, that the quantum field theory is still unable to describe this phenomenon of the cut-off of large transverse momenta.

If one tries to take this into account phenomenologically in the general relation (1), it is necessary to note, that a "complete" theory should provide this transverse momenta cut-off in the integral (1) through the corresponding behavior of the amplitudes $A_n$. If (according to experiment) this cut-off happens at finite values of the transverse momenta, then instead of the estimate (2) one gets a much slower decrease with growing $n$:

$$P_n \sim n^{-(n-2)}.$$  

(3)

In this case the phase space does not already have a form of a $3n$-dimensional sphere and takes a form of an $n$-dimensional cylinder in this space (if one disregards the restrictions imposed by the conservation laws that slightly deform this cylinder).

Another substantial factor, which is not accounted for in the statistical approach to multiparticle production, is a rapid growth of a number of field theory diagrams contributing to the amplitude $A_n$ at large $n$. For example, for the process of transition from two gluons to $n$ gluons this growth even exceeds factorial one (see Table 1 in the review [16], where the following numbers can be found: at $n = 2$ one has 4 diagrams, and at $n = 8$ their number reaches already 10525900). Of course, the relative phases of different contributions are really essential and it is currently impossible to conclude how strongly the growth of a number of diagrams with increasing $n$ affects the estimate (3). However, this factor can not be neglected.

At high multiplicities this can lead, for example, to a change of an expansion parameter in quantum chromodynamics, describing the strong interactions, from the coupling constant $\alpha_s$ to its product at a factor of the order of $n$, and it will turn out that

$$|A_n|^2 \sim n!\alpha_s^n \sim (n\alpha_s)^n.$$  

(4)

In this connection one often speaks about a "cylindrical" phase space.
As a result, the law of decreasing of $P_n$ at large $n$ will change from the factorial (of Poisson type) distribution to the exponential one or to the one close to it. Precisely this type of behavior is currently being discussed in quantum chromodynamics [17].

Here the appearance of a new expansion parameter is clearly seen when one analyses the corresponding distributions over $n$ with the help of their moments [18,19]. It is easy to see, that the processes with high multiplicity $n$ determine the moments of multiplicity distribution of a high order $q$. For example, only the processes having the multiplicity $n > q$ contribute to the factorial moments of order $q$. One finds that precisely the quantity $q\alpha_s^{1/2}$, corresponding to the above-mentioned factor $na_s$, provides [18] an expansion parameter for the solutions of equations on the generating functions of $P_n$ distributions (see also the review [19]).

The appearance of this parameter actually supports the idea that at high multiplicities the collective interaction effects become essential although possibly not describable within the simplest statistical approach. Nevertheless the mathematical methods applied in the studies of many-body system (be it a statistical physics problem or a multiparticle production process) is the same and is based on the analysis of the distributions of particles and their correlations in the system under investigation. Thus we begin with its description.

3. Correlation characteristics and methods of description of multiparticle systems

As it was already stressed, the approaches used in the analysis of the properties of multiparticle systems are quite similar both in the statistical physics (to mention one particular example, in laser physics) and in the physics of multiparticle production at high energies. At fixed given particle number $n$ in some phase space volume $\Omega$ the system can be characterized by the probability density $W_n(1, 2, \ldots, n)$, where the arguments $1, 2, \ldots, n$ denote the corresponding (generally speaking, multidimensional) coordinates of these particles in the phase space. Such an approach is called exclusive.

However, it is often more convenient to use the so-called inclusive approach, where the total number of particles in the system is not fixed and

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4 It is interesting that the resulting distributions are not infinitely divisible (see [19]) which could also be related to the collective effects.
one considers just its $q$-particle characteristics. This is the way followed by the majority of the experiments studying the multiparticle production. In this case the inclusive densities $\rho_q$ are related to the experimentally measured inclusive differential cross-sections as follows:

$$\rho_q(\vec{p}_1,\ldots,\vec{p}_q) = \frac{1}{\sigma_{in}} \frac{d\sigma}{d^3p_1\ldots d^3p_q},$$

where $\sigma_{in}$ is a total cross-section of inelastic processes, and a distribution over the $3$-momenta of the final particles is considered. Of course, the less detailed distributions are also used, when one integrates over the certain momenta components. The integration of the inclusive densities over the total phase space volume $\Omega$ gives the non-normalized factorial moments:

$$\tilde{F}_q = \int_{\Omega} d^3p_1\ldots d^3p_q \rho_q(\vec{p}_1,\ldots,\vec{p}_q) = \langle n(n-1)\ldots(n-q+1) \rangle$$

$$= \sum_0^n n(n-1)\ldots(n-q+1)P_n = \langle n \rangle^q F_q,$$

where $P_n$ is a probability to find an $n$-particle state of a system (the so-called particle multiplicity distribution), and $F_q$ are the normalized factorial moments. The inclusive densities of order $q$ are given by a sum of the exclusive density of the same order and the integrals from the exclusive densities of higher order over all variables that are not taken into account, i.e. in formal notation

$$\rho_q(1,\ldots,q) = W_q(1,\ldots,q)$$

$$+ \sum_{m=1}^\infty \frac{1}{m!} \int_{\Omega} W_{q+m}(1,\ldots,q,q+1,\ldots,q+m) \prod_{j=1}^m d(q+j).$$

The inclusive densities $\rho_q$ are nonvanishing even if the particles are statistically independent. Therefore (analogously to the cluster decomposition in statistical mechanics) it is convenient to introduce the so-called cumulant correlation functions $C_q$, which vanish in the case when the particles are completely statistically independent [20-22]. The general formulae relating them to the inclusive densities are quite cumbersome (see, e.g., [23-24]). Therefore we shall reproduce only the formulae for the cases $q=2$ and $q=3$:

$$C_2(1,2) = \rho_2(1,2) - \rho_1(1)\rho_2(2),$$

$$C_3(1,2,3) = \rho_3(1,2,3) - \sum_{(3)} \rho(1)\rho_2(2,3) + 2\rho(1)\rho(2)\rho(3),$$
where it is clearly seen that the contributions from lower order correlations are subtracted from the higher order ones (the notation $\sum_{(a)}$ stands for the summation over three possible particle permutations).

All these results can be obtained in a unified form with the help of a generating functional

$$G(z) = 1 + \sum_{q=1}^{\infty} \frac{1}{q!} \rho_q(1, \ldots, n) z(1) \ldots z(q) \prod_{j=1}^{q} d(j),$$

where $z(j)$ is a subsidiary function depending on $\vec{p}_j$. Then

$$\rho_q(1, \ldots, q) = \frac{\delta^q G(z)}{\delta z(1) \ldots \delta z(q)} |_{z=0},$$

$$C_q(1, \ldots, q) = \frac{\delta^q \ln G(z)}{\delta z(1) \ldots \delta z(q)} |_{z=0}.$$  \hfill (11), (12)

At $z = \text{const}$ the generating functional becomes a generating function of the multiplicity distribution, and the variational derivatives in (11), (12) become the ordinary ones, which lead to the non-normalized factorial and cumulant moments of this distribution correspondingly. The normalized factorial moments $F_q$ and the cumulants $K_q$ are given by the formulae

$$F_q = \frac{1}{\langle n \rangle^q} \frac{d^q G(z)}{dz^q} |_{z=0},$$

$$K_q = \frac{1}{\langle n \rangle^q} \frac{d^q \ln G(z)}{dz^q} |_{z=0},$$

and the multiplicity distribution $P_n$ is

$$P_n = \frac{1}{n!} \frac{d^n G}{dz^n} |_{z=-1},$$

i.e. it is related to the generating function by the formula

$$G(z) = \sum_{n=0}^{\infty} (1 + z)^n P_n.$$ \hfill (13), (14), (15)

We see that differentiating the generating function one can compute both inclusive and exclusive characteristics of a system depending on a point $z$ in which the derivatives are taken.
As it is clear from all written above the direct computation of Feynman diagrams within the perturbative approach in quantum field theory is not well suited to the description of the multiparticle production processes. On the one hand the number of diagrams grows catastrophically with the increasing particle number, on another one in the perturbative approach one considers the matrix elements of the scattering operator for the transitions between the states having a fixed number of particles. Therefore even if a calculation at fixed multiplicity is made, one gets the exclusive quantities, whereas according to (7) for computing inclusive characteristics one has to perform the infinite summation of the exclusive probabilities. One usually tries to bypass these difficulties [1,6] either by resumming an infinite number of specially chosen (leading logarithms, etc.) terms in the perturbative series, or using the equation for the generating functions (their validity is again proved by comparing the results with the same series calculated up to the definite order in coupling constant). We should note that these approaches turned out to be quite successful and fruitful in predicting and describing many characteristics of the multiparticle production processes in quantum chromodynamics (see [1,6]). However this approach leaves unsolved the basic question whether the perturbation theory can reproduce all the effects corresponding to an interaction lagrangian in principle (in particular, the possible collective effects). For shedding light on the latter it seems more promising to exploit the analogies with statistical physics which we shall try to discuss in the subsequent sections.

The inadequacy of the perturbative calculations of given subsets of Feynman diagrams to the problem of finding the inclusive characteristics of the multiparticle production processes is due to an overwhelming complexity of a scattering operator in the basis of the eigenfunctions of the particle number. Therefore the attempts are made to find a more adequate representation. A well-known example of such kind can be the laser radiation, where it is preferable to use the basis of the coherent states. Below we shall consider the analogous attempts in the physics of the multiparticle production processes. Of course, in this case one often has to rely upon the specific model examples rather than upon the initial QCD interaction lagrangian.

In the study of correlations in the multipion systems it also proved fruitful to follow a direct analogy with hydrodynamics. The self-similarity of vortices in hydrodynamics corresponds to a growth of correlation functions at small scales. This led to the introduction of the notions of intermittency and
fractality, giving rise to the powerlike growth of the factorial moments (13) with decreasing phase space volume, in particle physics. We shall not discuss these questions (a detailed review can be found in ref. [24]).

4. Feynman-Wilson liquid. Statistical analogies

Each individual event in the particle interactions at high energies can be fully characterized by specifying (apart from the masses and quantum numbers) the three-dimensional momenta of the secondary particles. The endpoints of these vectors define a set of points, lying as a rule in the above-mentioned cylindrical phase space. The correlations in the position of these points are defined by the interaction lagrangian and conservation laws. A large enough set of such events can be considered as a statistical ensemble. In particular, Feynman [25] put forward the analogy with a usual liquid by assuming the presence of short-range correlations in the ensemble. This idea was further developed by Wilson [26], and after that the ensemble is called a Feynman-Wilson liquid. Let us stress, that in contrast to the Fermi model here one does not develop a new statistical interaction model, but attempts to consider the statistical properties of the ensemble of particles created in the interaction process at high energies using the analogy with statistical mechanics. This topic is discussed in large number of publications (from simple analogies to specific models, see refs. [11,27-37] and references therein). As the exclusive probabilities $P_n$ characterize the volume of the phase space filled with the ensemble of $n$-particle events, they play the role of a partition function for the canonical ensemble. The generation function defined by the relation (16) (in a general case it is a functional) is analogous to a grand canonical partition function. The role of the volume is played by the maximal rapidity

$$Y = \ln \frac{s}{m^2},$$

where $s$ is a total energy squared in the center of mass system (CMS), $m$ is a particle mass. It characterizes the size of the "cylinder" in the direction of the longitudinal momentum. The variable $z$ is related to activity (or a chemical potential $\mu = \ln(z + 1)$) in statistical mechanics. Thus one can calculate the "pressure" in such a liquid in a "thermodynamic limit" $Y \to \infty$ as

$$p(z) = \lim_{Y \to \infty} \frac{\partial \ln G(z, Y)}{\partial Y}$$

(18)
and its density as

$$\rho(z) = \lim_{Y \to \infty} \frac{\partial}{\partial z} \ln \frac{G(z,Y)}{Y}$$

(19)

and thus determine the equation of state (for details see [32-34]). The analysis of experimental data on $e^+e^-$ annihilation and non-diffractive hadron-hadron processes at high energies within this approach was carried out in ref. [34]. Using the formula (16) and the data on $P_n$ one can calculate the generating function $G(z,Y)$ at different energies $Y$. A linear extrapolation to $Y \to \infty$ (see Fig. 1) allows to compute the slopes for the different values of $z$ (i.e. for different chemical potentials). The resulting dependence of pressure on the chemical potential (see Figs. 2a and 2b) is especially interesting in relation to the problem of the phase transition in such a system. The existence of a phase transition should show up in the discontinuities of the pressure derivatives over $z$. From Figs. 2a and 2b it follows, that in $e^+e^-$ processes such discontinuities are absent, whereas in the hadron-hadron interaction the constancy of $p(z)$ at $z \leq 1$ turns into a rapid growth at large $z$. Such a difference in the behavior of the pressure is interesting by itself, although a conclusion on the existence of some phase transition would be clearly premature.

This problem could be approached from a different perspective [36,37] by studying the behavior of the zeroes of a grand canonical partition function in a complex $z$ plane as a function of a number of particles in the system under consideration. The point is that at finite energy one always has a certain maximally possible multiplicity $N$. Therefore the sum in the formula (16) terminates at this value of $N$, i.e. the generating functional becomes a polynomial from $z$ of degree $N$ and thus has $N$ zeroes in the complex plane. Lee and Yang have used these properties of a partition function [36,37,21] and formulated a method of locating the phase transition point by finding a point at the positive real $z$ axis to which the partition function zeroes converge at large $N$. The analysis of the model $e^+e^-$ events at the energy 1000 GeV performed in ref. [38] has shown that the zeroes of the partition function lie on the circle in the complex $z$ plane and really converge to a real $z$ axis at growing $N$. The real experimental data on $e^+e^-$ and $p\bar{p}$ interactions at high energies also lead to the zeroes of a generating function placed on the circle [40]. In the limiting point to which the zeroes locations converge one should have a singularity of a total generating function given

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5 More complicated configurations of zeroes were also discussed [39].
by the formula (16) (i.e. at $N \to \infty$). The methods of investigating the character of this singular point were proposed in ref. [41]. The question on in which cases the appearance of such singularity corresponds to a presence of a phase transition of some type in the particle production processes still remains open.

There were many theoretical attempts to describe this transition at the phenomenological level (see e.g. [29-33,35,42-47] and the paragraphs 4.2.4 and 4.2.5 in the review [24]). Like in all the phenomenological theories of critical phenomena the main problem is a choice of a corresponding order parameter and such an expression for the partition function, that at small values of this parameter the analytically solvable Ginzburg-Landau hamiltonian is recovered. For the order parameter one usually takes either a certain mean field (with the possible gaussian fluctuations) [29,33] or a fluctuating field of inclusive distributions [35]. In the latter case the order parameter is local in the momentum space, which actually corresponds to an intrinsically nonlocal approach in the usual space. In particular even the "free" Ginzburg-Landau hamiltonian leads here to strong correlations [35]. Within such a picture one can find the correlation characteristics described in the paragraph 3 and also the details of the behavior of pressure, density, etc. Although the indications of the presence of singularities in these quantities and also in the generating functionals were obtained, the specific values strongly depend on the particular assumptions.

Thus we shall not present a detailed account of this series of papers, and shall only briefly illustrate the idea at an example of the coherent states [30], which by definition realize the eigenstates of the annihilation operator

$$a(\vec{p})|\Pi\rangle = \Pi(\vec{p})|\Pi\rangle,$$

where $\Pi(\vec{p})$ is some (generally speaking, complex) function of $\vec{p}$. As has been already mentioned, the coherent states may provide a more convenient representation for the operators, characterizing the multiparticle production process, than the particle number representation (in analogy with laser physics). Of course, this does not mean, that the created system is always in the coherent state.

\[\text{\footnotesize 6 Although the same form of the hamiltonian can be assumed at large values of the order parameter too.}\]
In this case the inclusive density $\rho_q$ (5) takes the form

$$\rho_q(1, \ldots, q) = \int \delta\Pi|\Pi(1)|^2 \ldots |\Pi(q)|^2 e^{-F(\Pi)},$$  

(21)

where $F(\Pi)$ is an arbitrary functional analogous to the free energy in statistical physics. Here the generating function, which follows from (10) at $z=\text{const}$, is written as

$$G(z) = 1 + \sum_{q=1}^{\infty} \frac{z^q}{q!} \int_\Omega \rho_q \prod_{j=1}^{q} d(j)$$

$$= \frac{1}{N} \int \delta\Pi e^{-F(\Pi)} e^{z \int dp|\Pi(p)|^2},$$

(22)

where the normalization $N$ is fixed by the relation:

$$N = \int \delta\Pi e^{-F(\Pi)}.$$  

(23)

In the absence of the complete dynamical theory, which would allow to calculate $F(\Pi)$, one is tempted to use a phenomenological ansatz. An analogy with the phenomenological Ginzburg-Landau theory of superconductivity leads to the expression for $F(\Pi)$ of the form

$$F(\Pi) = \int dp[a|\Pi(p)|^2 + b|\Pi(p)|^4 + c|\frac{\partial\Pi}{\partial p}|^2].$$

(24)

The calculation of the thermodynamical quantities is then performed in accordance with the formulae (18), (19). Different models correspond to a different choice of the parameters $a, b, c$ and are considered in refs. [29,30,33,35]. As usual, the phase transition point is the one in which the parameter $a$ goes through zero.

In this case it is possible to describe a large variety of the states of a system, from coherent to chaotic ones. Therefore, the whole approach can be considered either as a convenient parameterization of the data or as an attempt to find out some dynamical features of processes. It is interesting to note, that the location of the singularity of the generating function is sensitive to the parameter choice. It moves from infinity in the case of a coherent state (Poisson distribution) approaching the point $z = 0$ (where the inclusive distributions are calculated, see eqs. (13), (14)) with increasing chaoticity. At the same time the model of coherent states can itself be modified taking into account the squeezed and correlated states (see, e.g., the review [48]).
5. Quark-gluon plasma and multipion states

The problem of the phase transition became especially acute, when there appeared an idea about the formation of the quark-gluon plasma in those collisions, where a high energy density is reached. The natural density scale in the hadronic matter is either the average nuclear or nucleon density or that based on purely dimensional arguments, namely, on the value of the QCD cut-off parameter. All these estimates give the values of the same order lying in the range from 0.15 to 0.5 GeV/fm$^3$. At much higher densities (exceeding 1-2 GeV/fm$^3$) one can expect the appearance of a plasma of quarks and gluons. The estimates show, that such energy densities can hopefully appear in nucleus-nucleus collisions at high energies, or in some fluctuations in hadron interactions. As the QCD coupling constant decreases at high temperatures and densities due to the asymptotic freedom property, one can hope that in this domain the weakly interacting system of quarks and gluons (plasma) can be theoretically described.

The evolution of quark-gluon matter with its subsequent transformation into hadrons is one of the most cardinal problems of QCD and demands the application of the methods of many-body theory. The high-temperature plasma behavior in a certain frequency domain is described by perturbative QCD, but at lower temperatures the description of the system is already given using the nonperturbative calculations on the lattice in QCD (in the vicinity of the phase transition) and the effective theories of meson and baryon fields in the hadronic phase. It is necessary to stress, that even in the high temperature domain the perturbative approach is applicable only in the lowest orders in perturbation theory at high frequencies and allows to calculate the energy density, pressure, plasmon damping, etc. [49,50]. However in the long-wavelength and lower temperature domain the perturbation theory is not applicable. In the phenomenological analysis, the influence of the long-wavelength modes is often described with the help of a notion of classical sources leading to the coherent states. At the same time the short-wavelength excitations are considered to be responsible for the quantum correlations, which in a thermalized system are averaged with the Planck distribution (see, e.g., [51]). All the higher correlations are often reduced to the products of the two-particle ones (analogously to the coupled pair approximation [52]). The relative contribution of these domains determines the relation between the chaotic and regular components of correlations, which
is also often used in the analysis of the experimental data (see, e.g., [52]).

The lattice QCD calculations have revealed the presence of a phase transition between the equilibrium quark-gluon and hadron phases at a temperature, which, according to the different estimates, is of the order of 100 - 160 MeV [53-56]. Let us note, that such a temperature was already mentioned as a maximally possible for the hadronic phase in the statistical bootstrap models that were developed in the sixties and seventies (see, e.g., [57]). The question concerning the order of the transition is not completely settled. The conclusions often depend even on the size of the lattice. However the latest investigations [58-60] give some ground to think, that in the theory with two massless dynamical quarks the transition is of the second order. At larger number of massless quarks it seems to be of the first order. However when the mass of the third (strange) quark grows, the transition is again becoming a second order one already for the masses that are even somewhat smaller than that of a strange quark. Thus the existence of such transition can be taken as a starting point in the construction of the phenomenological models.

The simplest situation from the point of view of its theoretical description would be the formation of an equilibrium quark-gluon plasma, then its expansion and accompanying cooling, the phase transition to the equilibrium hadronic matter and its subsequent expansion. This particular scenario lies in the foundation of the majority of papers analyzing the possible detection of the quark-gluon plasma in the experiments on heavy ion collisions. The central question here is doubtlessly whether the quark-gluon system, formed as a result of a heavy ion collision, has enough time to thermalize. This determines whether we have a hydrodynamical plasma expansion starting from some moment, or the process of conversion into hadrons is essentially non-equilibrium one.

Unfortunately the unified approach describing all the stages of the process still does not exist, and the description is quite fragmentary. Most often one deals with the particular models of this or that stage. Naturally this status of a problem will affect the subsequent presentation in this review. We shall begin with the description of the hadronic stage of a process. The dominant fraction of the created particles are the pions, which in the first approximation can be treated as practically massless and having relatively low energies in the center of mass system of colliding high energy particles.

One of the simplest effects could be the influence of the laws of conservation of quantum numbers on the properties of the pion system. Let us
consider for definitiveness the process of the collision of two high energy nucleons, where in the final state, apart from the two nucleons, a certain number of pions has been created. The total isotopic spin of the pion system is limited, according to the conservation laws, to the values $I = 0, 1, 2$, whereas in the general case it could take values up to $n_{tot}$, where $n_{tot}$ is a total number of pions. This fact significantly affects the pion charge distribution. Even if the distribution in the total pion number $n_{tot}$ is poissonian (as it happens when pions are produced by classical currents), it turns out, that the separate distributions of the charged $(n_+, n_-)$ or neutral $(n_0)$ pions are much wider than the Poisson one (see Fig.3, which is taken from ref. [61]). The experimental confirmation of this fact could be the "Centauro" events, where the charge particles are noticeably dominating, or "anti-Centauro" ones with a large number of neutral pions. Let us illustrate the idea by considering, following the quasiclassical approach of ref. [62], the production of many pions in a system with a zero isotopic spin. The characteristic initial assumption is a possibility of describing the pion system that radiates the final state pions as a classical field [61-66], i.e. that the number of pions per phase space cell is assumed to be big. According to the standard reduction formula, the amplitude of generation of $N$ pions by the source $J$ equals

$$A^{a_1,\ldots,a_n}(k_1,\ldots,k_n) = \lim_{k_n^2 \rightarrow m_n^2} \int Dp^a \int DJ^a W[J]$$

$$\exp(iS[\pi] + i \int d^4x \pi^a J^a) \prod_{n=1}^{N} \int d^4x_n e^{ik_n x_n} (-\partial_{x_n}^2 - m_n^2) \pi^{a_n}(x_n), \quad (25)$$

where the functional integration over $J$ corresponds to the averaging over the characteristics of the pion source. Let us notice, that the radiation of a classical current exactly reproduces the language of coherent states. The quasiclassical estimate of the amplitude, performed in the assumption on the axial symmetry of the initial interaction and on the isotopic symmetry of the pion system (i.e. of the zero total isospin) [62], leads to the two characteristic conclusions that also appear in other publications on this topic (in particular, such conclusions were reached in the above-mentioned paper [61]).

Firstly, only the distribution over the total number of pions is poissonian. The distributions over the number of neutral and charged pions are much wider (see Fig. 3). For the state with zero isospin a probability of finding $2n$
neutral pions in the system of $2N$ pions has a form $[67,68]$:  
\[ P(n, N) = \frac{(N!)^{2}2^{2n}(2n)!}{(n!)^{2}2^{2N}(2N + 1)!}. \quad (26) \]

At large $n, N$ we have a characteristic distribution $[69]$:  
\[ P(n, N) \sim (n/N)^{-1/2}. \quad (27) \]

Secondly, the conservation of the total isospin leads, for example, to the specific angular correlations between the particles with different charges. Let us give the characteristic formula for the correlation over the azimuthal angle $\varphi$, obtained in $[62]$ for the pions having zero rapidity:  
\[ \left( \frac{d\sigma^{\pi^{+}\pi^{-}}}{dk_{1}dk_{2}} \right) \left( \frac{d\sigma^{\pi^{+}\pi^{-}}}{dk_{1}dk_{2}} \right) = \frac{3}{10} \cos^{2}(\varphi_{1} - \varphi_{2}). \quad (28) \]

The experimental verification of such predictions is to our opinion very interesting.

We have already seen that only taking into account the isospin conservation within the framework of the quasiclassical approach can lead to the dramatic changes of the naive ideas concerning the character of particle multiplicity distributions. From the theoretical point of view the most interesting calculations are the ones attempting to use the effective pion lagrangians. In principle this gives a chance of getting the model-independent predictions.

Before describing the corresponding results, we feel necessary to make the following comment. The general feature of all the papers using the low-energy model lagrangians for the description of the distributions of particles and their correlations in the multiparticle production processes is an application of these lagrangians in the case, when the initial energy of the collision is quite high. The question concerning the possibility of neglecting the heavy modes and the interaction between the modes is quite nontrivial. Due to the practically insurmountable difficulties arising on the way of the complete solution of the problem, it seems reasonable to analyze the consequences of the most radical assumptions concerning the possible dynamical reasons causing the sharp charge asymmetry of the events where a large number of pions is generated.
In the recent literature a "disordered chiral condensate" was widely discussed as a possible asymmetry source in the production of charged and neutral pions in some fraction of the events (in particular, in the above-mentioned "Centauros") [70-77]. Let us remind, that the transformation properties of mesons with respect to the chiral transformations are determined according to the corresponding properties of the order parameter, which characterizes the spontaneous breaking of chiral symmetry and given, for example, by the average from the bilinear combination of quark fields

\[ \Phi \sim \langle \bar{q}LqR \rangle, \]  

where \( q_{R(L)} \) are the right (left) states of the massless quarks. For investigating the character of the singularity of thermodynamical functions in the vicinity of the phase transitions, it is desirable to find a solvable model having the same symmetry. Then, according to the universality principle based on the scale invariance near the critical point, the solutions of this model will have the same set of singularities. In such an approach the order parameter (29) can be rewritten in terms of a set of hadron fields having the same symmetry. Therefore, the multipion states become related to the massless quark fields and quark-gluon plasma, i.e. there appears a hope to describe a phase transition between these so differing phases within such a model. In the realistic case of two massless quark flavors the chiral field can be written in the form

\[ \Phi \sim \sigma \cdot \bar{1} + i \tau \cdot \bar{\pi}, \]  

where \( \sigma, \bar{\pi} \) are the real fields, \( \bar{\tau} \) are the standard Pauli matrices, the \( \pi \)-meson fields \( \bar{\pi} \) form an isotriplet and \( \sigma \) is an isosinglet.

In the case of \( SU(3) \) - algebra the number of such fields is already 18. They form the scalar and pseudoscalar nonets.

The dynamics of these degrees of freedom can be described by the lagrangian of the linear \( \sigma \) model

\[ L = \frac{1}{2} (\partial\sigma)^2 + \frac{1}{2} (\partial\bar{\pi})^2 - V(\sigma, \bar{\pi}), \]  

where \( V \) is a potential depending on the combination \( \sigma^2 + \bar{\pi}^2 \). In the standard version [78] the spontaneous symmetry breaking occurs via the formation of the nonzero vacuum average of the field \( \sigma \). The isotriplet fields remain massless, i.e. the pions are the goldstones of the chiral group.
Let us now assume [71], that in some region of space the vacuum orientation is different from the standard one, and, for example,

\[ \langle \sigma \rangle = f_\pi \cos \theta, \quad \langle \bar{\pi} \rangle = f_\pi \bar{n} \sin \theta, \]

where \( f_\pi = 93 \text{ MeV} \), and \( \bar{n} \) is a unit orientation vector of \( \bar{\pi} \). Such an assumption presupposes a specific scenario of the process, which is still not studied in details.

If the field \( \Phi \) is isotropic with respect to a direction on the 3-dimensional sphere in the 4-dimensional space with the angles defined as

\[ (\sigma, \pi_3, \pi_2, \pi_1) = (\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi \sin \eta, \sin \theta \sin \phi \cos \eta), \]

then for the probability distribution of a given state \( r \equiv \cos^2 \phi \) we have:

\[ \int_{r_1}^{r_2} dr P(r) = \frac{1}{\pi^2} \int_0^{2\pi} d\eta \int_0^\pi d\theta \sin^2 \theta \int_{\arccos \frac{r_1}{2}}^{\arccos \frac{r_2}{2}} d\phi \sin \phi, \]

as it was obtained for the first time in ref. [69] (see (3)):

\[ P(r) = \frac{1}{2\sqrt{r}}, \]

i.e. we have again returned to the formula (27). Thus the probability of finding an event with a fraction of the neutral pions being smaller than a certain given \( r_0 \) is

\[ W(r < r_0) = \sqrt{r_0}, \]

which constitutes 10 % even at \( r_0 = 0.01 \). The charge fluctuations in such a system are much larger than those that would follow from the Poisson distribution, where the distributions are concentrated around \( r = 1/3 \).

Let us notice that the configuration (32) is not a solution of the equations of motion and is therefore unstable. In order to give an accurate quantitative estimate it is necessary to take into account the presence of a domain wall separating the metastable configuration (32) from the surrounding normal vacuum. Unfortunately such calculations have not yet been performed.

To carry out more detailed investigations, it is necessary to specify the source of the fluctuations of the chiral order parameter. In ref. [70] this mechanism was considered in the context of a chiral phase transition, when
at increasing temperature the chiral symmetry restores at some temperature $T_c$. Earlier it was established [55], that the QCD chiral field corresponds to an $O(4)$-magnetic (which is natural for the choice of the order parameter in the form of Eq. (30)), for which one has a second order phase transition. Therefore a most natural mechanism for the generation of large scale fluctuations is a so-called "critical slowing". In the vicinity of a critical point the relaxation time of the long-wavelength degrees of freedom increases, and such fluctuations can thus generate an isotopic disbalance, as the system’s expansion can ”freeze” the appearing isotopic inhomogeneities. It turns out, however, that this mechanism is not effective. The reason is, that the tiny (on the hadronic mass scale) current quark masses generate nevertheless a sufficient pion mass $m_\pi$ (in fact, $m_\pi \sim T_c$), for the chiral fluctuations to be indistinguishable from the thermal ones.

This has led the authors of [70] to considering the alternative possibility, when the fluctuations that initially appear in the high-temperature (unbroken) phase, subsequently evolve according to the zero temperature equations of motion. Here the amplifying mechanism has quite remarkable character [70]. In the spontaneously broken phase of a linear $\sigma$ model with the condensate $v$ and the bare goldstone mass $\mu$, the zero mass of the goldstone is provided by the relation

$$m_\pi^2 = -\mu^2 + \lambda v^2 = 0.$$  \hspace{1cm} (37)

Above the transition point one always has long-wavelength fluctuations, for which $\langle \Phi^2 \rangle < v^2$, and during a certain time interval the mass of these fluctuations is negative. Correspondingly the long-range fluctuations will grow until the equations of motion restore the relation $\langle \Phi^2 \rangle = v^2$. Such a situation could take place for a rapidly expanding system. The resulting numerical solutions of the corresponding evolution equations [70,72-75] are somewhat ambiguous. We think that it would be premature to conclude, whether the fluctuations of the chiral order parameter can be amplified to an observable scale.

In several papers [63,64,66,69] the problem was considered within a nonlinear $\sigma$ model. Its lagrangian can be written in the form (the discussion of the nonlinear $\sigma$-model can be found, for example, in [79]):

$$L = \frac{1}{2f_\pi^2}(\partial_{\mu}\Phi_a)(\partial^{\mu}\Phi_a) - \lambda(\frac{1}{f_\pi^2}\Phi_a^2 - 1),$$ \hspace{1cm} (38)
where $\lambda$ is a Lagrange multiplier. In refs. [63,69] the classical solutions of the equations of motion of the nonlinear $\sigma$ model, describing the isotopic fluctuations of the chiral field, were found. Unfortunately this has not led to more detailed predictions, although the situation was clarified to some extent in [66,69].

An interesting attempt of obtaining the equations describing the quantum fluctuations of the order parameter was recently made in refs. [76,77]. The basic physical idea is that, as in the heavy nuclei collisions the longitudinal expansion of hadronic matter in the pionization domain at a scale of 8-10 fm dominates over the transverse one, the problem becomes essentially two-dimensional. In ref. [77] $SU(N_f)$ Wess-Zumino-Novikov-Witten lagrangian for the chiral field $U = \exp(2i\bar{\tau}\bar{\pi})$ was obtained as an effective 1+1-dimensional lagrangian. In this model the fluctuations of the order parameter can be computed exactly. In particular, for the two-point rapidity correlations at a given proper time $\tau$ one gets

$$
\langle U(\tau, Y) U^\dagger(\tau, Y') \rangle \sim \frac{1}{(\sqrt{2}\tau)^{4\Delta}} \cosh(Y - Y') - 1)^{2\Delta},
$$

where $\Delta = 3/20$. Such predictions could be tested experimentally. Let us however note once again, that a method of constructing the effective lagrangians which mixes the ideas originating both from the high- and low-energy regime, does not allow to get a consistent reduction. As a result the whole treatment is rather a guess subject to experimental verification.

Where should we look for the experimental signatures of the existence of a chiral condensate and what are they? The hints of how to answer the first part of the question follow from the fact, that "Centauros" were found in cosmic rays but were not found in accelerator experiments, although the Tevatron does already cover this energy domain. If we do not ascribe the whole effect to the specific conditions of the registration of cosmic rays and their composition, we have to assume, that the difference in the results originates from the fact, that in cosmic rays one studies the fragmentation region of large (pseudo) rapidities, whereas in the accelerator experiments we were up to now studying the pionization domain. According to the estimates of ref. [71] the "Centauro" type clusters should be looked for at Tevatron in the pseudorapidity range in CMS of the order of 4 and higher. This is one of the tasks of the T864 experiment. If the charge asymmetry is given by the formula (27), it should be quite visible.
As we have mentioned, apart from a distinct signal of charge asymmetry (27),(35) we also have got some conclusions on the correlation properties of such objects in azimuthal angle (28) and pseudorapidity (39). Moreover, the electromagnetic decay modes of the hadronic resonances can be sensitive [73,74] to the presence of such a condensate, because its orientation is misaligned with that of the electroweak vacuum as defined by the Higgs fields.

6. Collective effects in QCD jets

Let us turn now to QCD jets. From the point of view of field theory a typical situation, in which one uses the description in terms of the collective degrees of freedom, arises when the system is considered as composed from a classical subsystem (e.g., crystal, plasma, etc.) and corresponding quasiparticles (phonons, plasmons, etc.). In QCD it is natural to distinguish a domain of small distances (quarks and gluons as high energy modes) and large distances (hadronic modes). The high-energy phase is described by the QCD perturbation theory, but in order to extract quantitative predictions, we should be able to estimate the contribution of the low-energy modes to the characteristics that are traditionally computed within the perturbation theory. One of the most interesting objects to look at are here the hadron jets, generated by highly energetic quarks and gluons.

Let us begin with considering the possibility of the phenomenological account of the presence of low-energy modes (confinement) on the evolution of the quark-gluon jets that give rise to hadron jets. In the simplest approximation (leading logarithms) the perturbative evolution of quark-gluon jets is described by Gribov-Lipatov-Altarelli-Parisi equations (see, e.g., [1,6]). The structure of these equations is very similar to the system of equations describing the evolution of electromagnetic showers in matter [80]. The analogy becomes even more attractive, because the interaction with a low-energy subsystem is inherently present in electromagnetic showers: these are the inelastic losses on the ionization of the atoms of the medium by the particles from the shower. We are pleased to note, that precisely this problem was solved in the paper by S.Z. Belenky and I.E. Tamm [81], whose centenary this issue of the journal is devoted to. We are following the same line of investigations, but already in application to quark-gluon jets.

Let us assume, that the influence of confinement (long-range modes related to strings) on the evolution of hard quarks and gluons results in the
gradual loss of their energy going into the formation of pions. This process resembles the loss of the electrons from the shower into matter leading to the shower damping. As in the previous paragraph, we get the amplification of the low-energy modes and the attenuation of the hard component. In ref. [82] a modification of the GLAP equations was proposed, which treats the interaction with the low-energy modes in analogy with the ionization losses. If we consider for simplicity only a gluon subsystem, the modified equations take the form:

$$\frac{\partial D(E, \tilde{Y})}{\partial \tilde{Y}} = 2 \int_{E}^{\infty} \frac{dE'}{E'} P^{GG}(E, E')D^{G}(E, \tilde{Y})$$

$$- \int_{0}^{E} \frac{dE'}{E} P^{GG}(E', E)D^{G}(E, \tilde{Y}) + \beta_{G} \frac{\partial D(E, \tilde{Y})}{\partial E},$$

where $D^{G}(E, \tilde{Y})$ is a distribution function of gluons over the energy $E$ and "depth"

$$\tilde{Y} = \frac{1}{2\pi b} \ln \left(\frac{Q^2/\Lambda^2}{\ln(k^2/\Lambda^2)}\right); \quad b = \frac{33}{12\pi};$$

$Q^2, k^2$ are the initial and "current" invariant mass in a jet, $\Lambda$ is a cut-off parameter and $P^{GG}$ is a transition (gluon bremsstrahlung) probability

$$P^{GG} = C_{V} x(1 - x)[1 + \frac{1}{x^2} + \frac{1}{(1 - x)^2}]; \quad x = \frac{E'}{E};$$

$\beta_{G}$ is a dimensionful parameter determining the magnitude of the nonperturbative inelastic losses. The solution of the Eq. (40) reveals the maximum of the parton shower and rapid damping of the shower at low gluon energies due to conversion of gluons into hadrons which influence is phenomenologically described by the last term. No hadronization model was attempted in [82,83]. However, the solution of the Eq. (40) allows [83] to find the energy flow from a "hard" component to a soft one. For the gluon jet with the initial energy $E_{0}$ the average energy of a hard component at the depth $\tilde{Y}$ is equal to

$$\langle E(\tilde{Y}) \rangle = E_{0}[1 - \frac{\beta_{G}}{E_{0}} \sqrt{\frac{2c_{V}\ln(E_{0}/\beta_{G})}{2\sqrt{2c_{V}\tilde{Y}\ln(E_{0}/\beta_{G})}e^{-a\tilde{Y}}}} I_{1}(2\sqrt{2c_{V}\tilde{Y}\ln(E_{0}/\beta_{G})}e^{-a\tilde{Y}}),$$

where $\bar{a} = 101/18$, $I_{1}$ is a modified Bessel function.
We see that the picture is qualitatively attractive, but for providing more accurate predictions one should take into account the modification of the initial equations due to kinematical and interference restrictions on the perturbative evolution. It seems however quite probable, that the above refinements will significantly alter the distribution function and correlations, but the formula for the energy losses (43) will most probably remain essentially the same. The grounds for such a belief come from the derivation of (43) in [83], where we essentially used the energy conservation in the perturbative evolution. This phenomenological approach to parton’s hadronization is not complete since one should also describe how many hadrons and with which characteristics appear as a result of partons convolution. It is possible to do only with Monte Carlo models using even more phenomenological parameters.

An attempt of a field-theoretical realization of the above-described scheme was recently proposed in a series of papers [84,85]. The evolution of parton showers both in space-time and in energy-momentum phase space is analyzed. Some arguments in favor of rapid transition from partonic to hadronic stage are given. Therefore, the idea of a unified treatment of a high-energy subsystem and low-energy ”thermostat” is realized by a literal addition of the corresponding lagrangians of subsystem and interaction one:

\[ L = L_{QCD} + L_{eff} + L_{int}^{eff}, \]

where \( L_{eff} \) is a low-energy lagrangian accounting for the breaking of chiral and dilatational invariance [86,87] while the interaction lagrangian describing the processes at the intermediate scales interpolates between the two subsystems by phenomenologically introducing some effective cut-offs imitating the rapid parton-hadron conversion within a very narrow space-time region. In particular, if the gluon degrees of freedom are considered only, the lagrangian is chosen in the form:

\[
L = -\frac{1}{4}(G^a_{\mu\nu})^2 + \frac{1}{4}\xi(\chi)(G^a_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu \chi)^2 - b\left[\frac{1}{4}\chi_0^4 + \chi^4 ln\frac{\chi}{e^{1/4}\chi_0}\right],
\]

where

\[
\xi(\chi) = \theta(\chi)(\frac{\chi}{\chi_0})^3(4 - \frac{3\chi}{\chi_0}),
\]

and

\[
L = \frac{1}{4}(G^a_{\mu\nu})^2 + \frac{1}{4}\xi(\chi)(G^a_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu \chi)^2 - b\left[\frac{1}{4}\chi_0^4 + \chi^4 ln\frac{\chi}{e^{1/4}\chi_0}\right].
\]
\( \chi \) is a scalar glueball field, \( G_{\mu \nu}^a \) is a gluon field strength, and the interaction of the quantum gluon fields \( G_{\mu \nu}^a \) with the "classical" \( \chi \) has a form \( \xi(\chi)G^2 \). The non-vanishing gluon condensate at large scales is denoted by \( \chi_0 \) and the parameter \( b \) is related to the conventional bag constant \( B \) by \( B = b\chi_0^4/4 \). In the chiral limit, the potential has a minimum when \( \langle \chi \rangle = \chi_0 \) and equals the vacuum pressure \( B \) at \( \langle \chi \rangle = 0 \). From the point of view of the development of a gluon cascade the interaction lagrangian \( L^{\text{int}}_{\text{eff}} \) leads to the appearance of an energy flow (from the quantum component to the classical one) growing with increasing \( \tilde{Y} \) that reminds of the results of [83]. The numerical solutions of the corresponding kinetic evolution equations are indeed demonstrating this property. The elaborated Monte Carlo program provides characteristics of final hadrons in various reactions. The direction of investigations in [84,85] is rather appealing even though there appear several phenomenological parameters to be approved by later development. One can hope, that already in the nearest future the situation here will be clarified, and the problem of the nonperturbative effects in the evolution of the quark-gluon cascades will reach the quantitative level.

Let us mention an interesting possible analogy with a collective effect in the photon radiation. It is known, that the real part of an elastic scattering amplitude of hadrons becomes positive at large energies. In the terminology of classical physics this means, that the refractivity index of the hadronic medium exceeds one. In this case the phase velocity of a colored charge in the hadron medium can be higher than the speed of light. This can lead to the "color Cherenkov radiation" [88], which is analogous to the usual Cherenkov radiation, the theory for which was developed by Tamm and Frank [89]. A characteristic feature of such radiation will be its angular distribution with its typical "ring-like" structure (see the review [90] and the papers [91,92]). However, its intensity can be damped due to the small size of the target hadron and the absorption in the hadronic medium (an imaginary part of the scattering amplitude). Besides a change of the current in the course of emission of color objects could play its role (let us remind that the photon is electrically neutral and, unlike the gluon, does not change the emitting current at small recoils).

Nevertheless some events having a "ring structure" were observed in cosmic rays [93], and for larger statistics the processing of the experimental data
of the NA22 experiment gave indication [94] on a statistically significant contribution of such events. They show up as peaks in the pion distributions at the rapidities $|y| \sim 0.3$ in the center of mass frame. Several bands of energies are claimed [91,92] to satisfy the requirements of "color Cherenkov radiation". The continuation of the searches for the possible manifestations of this effect would be desirable.

Let us now consider another aspect of the physics of hadron jets that is also directly related to the considered collective effects. We are now discussing the passage of a fast particle, which subsequently forms the hadron jet, through the hadronic matter. A bright example of the effects that could be expected in this connection is a sharp decrease of the collisional energy losses in quark-gluon plasma near the phase transition point (jet quenching [95]). Really, from the Bjorken estimate for the energy losses of the parton in plasma at the unit length

$$\frac{dE}{dx} \sim 6\alpha_s^2 T^2 \ln \frac{4ET}{M^2} e^{-M/T} (1 + \frac{M}{T}),$$

(47)

it follows that for the quite realistic values of $\alpha_s = 0.2$, $T = 250$ MeV and $M = 500$ MeV the energy losses of a parton at unit length in plasma are 10 times smaller than the value of 1 GeV/fm expected for the usual hadronic matter, where the scale is determined by the string tension. The accuracy of the answer directly depends on how well-studied are the proper collective modes of a system. From classical plasma physics it is well known that the interaction of external probe with the system has a resonance character at the frequencies equal to the eigenfrequencies of the system. In spite of the work on the study of the collisional losses (see, e.g., [96]), the question concerning the quantitative estimate of the effect is to our opinion open. In particular one does not know sufficiently well the spectrum of collective excitations of the quark-gluon plasma (recently, new modes were found in [97]).

Let us also discuss one more interesting effect illustrating the deep differences between the effective nonabelian medium in different phases. Considering the propagation of a color charge in the external random chromoelectric field [98] a following formula for the change of the energy of a fast test particle was obtained:

$$\langle \frac{dE}{dt} \rangle = \frac{2\pi^{3/2} \alpha_s \langle \mathcal{E}^2 \rangle \tau_c}{m} \frac{C_A}{N_c^2 - 1},$$

(48)
where $\langle \mathcal{E}^2 \rangle$ is a mean square of an external field strength, $\tau_c$ is its characteristic correlation time, $m$ is a mass of the particle, $C_A$ is a corresponding Cazimir operator, $N_c$ is a number of colors (a gaussian chromoelectric field correlator is assumed). Most interesting is here even not a quantitative estimate of the possible losses, but a fact that in the QCD vacuum one gets $\langle \mathcal{E}^2 \rangle < 0$ (which is well known from the consideration of heavy quarkonia [99]), whereas the fluctuations of the color field in the QCD plasma naturally lead to $\langle \mathcal{E}^2 \rangle > 0$. Therefore we have a "stochastic cooling" (deacceleration) in the vacuum in the hadronic phase and the "stochastic heating" in e.g. QCD plasma, where in some way the stochastic color fields were generated.

It is necessary to mention the recent growth of interest to the different aspects of the physics of quark-gluon jets. We saw, how tightly the analysis of these problems is related to the collective properties of the nonabelian medium in which the jets propagate.

One of the most interesting is here a problem of coherent effects in the induced gluon bremsstrahlung during the passage of a fast particle through the nonabelian medium. We are speaking about the nonabelian analogue of Landau-Pomeranchuk effect which is well known in electrodynamics [100-102]. The physical essence of this effect is a damping of a soft photon radiation of a fast particle by a high energy particle due to a coherent influence of many scatterers. It is interesting that the theory of the electromagnetic effect was verified experimentally only in 1993 [103]! For the physics of quark-gluon jets the question of the intensity of the induced bremsstrahlung radiation is a central one. In particular, the above-mentioned estimates of the parton energy loss [96] are made under the assumption of the negligibly small energy losses at gluon radiation. It may seem that a coherent radiation in the QCD medium is simply impossible, because any radiated gluon changes the color of a test parton, that should exclude any possibility of "using" the multiple scattering for achieving the coherent action of a number of scatterers on the bremsstrahlung gluon radiation. This point of view was laid into a basis of calculations in [104], where for the energy losses at unit length the authors obtained

$$- \frac{dE}{dx} \sim \text{const} \cdot \alpha_s \mu^2,$$

where $\mu$ is a screening length in the medium, which is cardinally different from the electrodynamic result [100-102].

This statement was recently objected in the paper [105], where the au-
thors showed that a coherent action of a set of nonabelian scatterers can be achieved by taking into account the regeneration of the initial color by the gluon, emitted by the parent test parton. Ideologically this situation resembles the color coherence effect in the quark-gluon jets, where some gluons "see" only the total charge of a jet [6], which is analogous to the Chudakov effect [106], that is well known in the physics of electromagnetic showers. An account of the above mechanism the formula for the radiative energy losses in the nonabelian medium takes the form [105]

\[- \frac{dE}{dx} \sim \alpha_s \sqrt{\frac{E\mu^2}{\lambda_g}} \ln(\frac{E}{\lambda_g\mu^2}), \]

(50)

where \(\lambda_g\) is a mean free path of gluons. The formula (50) differs from the classical result of Migdal [101] by a logarithmic factors appearing due to the correct treatment of collisions with a large momentum transfer. An analogous factor also appears in the corresponding electrodynamic formula.

A problem of the parton energy losses in the nonabelian medium is of a decisive importance for the description of the spectrum of jets in the collisions with nuclei. The experimental study of hadron jets in such collisions has already a relatively long history. In particular, there exists a number of experimental results, which distinctly show the role of a multiple scattering in the formation of two-jet configurations [107]. The continuation of the studying of this problem will doubtlessly shed a light on the properties of the extremely complicated medium, in which the jets are born and where they propagate.

Here it is tempting to point out an analogy with the cosmic rays, due to which we obtain information on the structure of fields in the intergalactic space. It is possible, that a rich experience accumulated in the physics of cosmic rays, will also help in the study of the "cosmic rays" of the hadron physics, i.e. of the quark-gluon jets.

7. Discussion

We have tried to give a general description of the theoretical attempts to find the collective effects in the multiparticle systems formed in the collisions of high energy particles. In some way they are all having a phenomenological character and are inspired by the hope to reveal certain features of
confinement and QCD vacuum structure and their role in the multiparticle production process. The diversity of ideas, methods and approaches underlines the absence of a unifying picture, but doubtlessly is a necessary element on the way to its creation. We are sure that eventually it will appear. If we look only 60 years back, we shall see (see the article of E.L. Feinberg in this issue), that the very notion of a pion was still not accepted by everybody, and now we are trying to understand the properties of multipion states and their relation to the quark and gluon degrees of freedom of an excited hadronic medium. The absence of general conclusions means only that it is in the stage of formation and dynamical development.

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Figure Captions

Fig. 1. The best approximation of the functions $\ln G(z, Y)/Y$ depending on the energy $Y$ at various activities $z$ presented in ref. [34]. Let us mention, that the data from the experiments in the $Z$ - boson region are sufficiently well reproduced by the extrapolation of the straight lines drawn in the figure (see [41]).

Fig. 2. The extrapolated according to the procedure of Fig. 1 to the point $Y^{-1} = 0$ "experimental" values lead to the presented functional dependencies of a pressure (calculated from the formula (18)) on the activity $z$ for the cases a: electron-positron annihilation; b: non-diffractive hadron-hadron collisions. The figure is taken from ref. [34].

Fig. 3. The distributions in the total number of pions and separately in charged and neutral pions in the case of a pion system with zero isotopic spin. The figure is borrowed from ref. [61]
This figure "fig1-1.png" is available in "png" format from:

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