An improved harmonics detection method based on sliding discrete Fourier transform for three-phase grid-tie inverter system

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Abstract: Harmonic currents in a grid-tie inverter system not only increase the inverter loss but also reduce the inverter efficiency. Harmonic detection is the precondition of harmonic control. Among the various methods, sliding discrete Fourier transform (SDFT) based algorithms has found wide and popular applications due to advantages like simplicity and excellent selectively filtering properties. However, SDFT suffers from disadvantages like slow dynamics and large memory occupation. In order to alleviate these drawbacks, an improved SDFT is proposed to detect the system harmonics. The proposed SDFT not only maintains the advantages of simplicity and selectively filtering properties but also features fast transients, around 1/6 fundamental cycle for typical harmonics in three-phase application, which is much shorter than the one-cycle settling time of the conventional SDFT. Simulation and experimental results show the validate effectiveness of the proposed method.

Keywords: sliding discrete Fourier transform (SDFT), harmonics detection, grid-tie inverter, three-phase system

Classification: Energy harvesting devices, circuits and modules

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1 Introduction

With the rapid development of power electronic technology, microelectronics technology and material technology, inverter control research become more and more important, and the high performance inverter systems keep improving [1, 2, 3, 4]. Harmonic currents in the inverter controller not only increase the loss, but also reduce the efficiency. Therefore, detecting key harmonic quickly and accurately has great significance for harmonic analysis and treatment.

Various harmonic detection methods have been researched in the literatures [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12], which can be generally categorized into time-domain and frequency-domain methods. Typical time-domain methods based on the instantaneous reactive power theory (pq-theory), which only estimate the fundamental signal and detect the rest harmonics as a whole. Therefore, it is not flexible to detection a single harmonic component. [1, 2] A CDSC based methods is proposed in [3], which construct by a series of DSC operators and consists of high-order delay buffers. For extracting each harmonic, different sets of DSC operators are required, which can increase the system complexity, computational effort and storage memory overhead especially when many harmonic components are to be extracted in applications like the selective active power filter.
Frequency-domain methods typically refer to the Fourier transform based techniques. The fast Fourier transform (FFT) implements the DFT in a different form to reduce the computational burden and is widely used for harmonic monitoring and metering [6], but the sample point must be $2^N$, hence, the algorithm is inflexible. The SDFT essentially calculates a DFT on a sample-by-sample basis with the window shifting every sampling instant for a fixed number of samples, usually just one for simplicity. And many research focus on SDFT stability [11, 12], in [11], the author proposed an algorithm which contains two SDFT sequence to improve the stability and dynamic performance of the SDFT, however, it will increase the computational and storage memory burden greatly. And in [12], and Goertzel algorithm is adopted to increase the stability of the SDFT, but it can not lower the dynamic response and storage memory burden.

The major drawback of the above Fourier-based harmonic detection methods, as evaluated in [7, 8, 9, 10, 11, 12], is the slow dynamic responses, requiring at least one-cycle settling time and it must occupy all sample point of a fundamental cycle.

To alleviate the aforementioned drawbacks of slow dynamics and, an improved SDFT strategy is proposed. The key idea is to reconfigure and reconstruct the conventional SDFT according to the specific harmonic scenario of the three-phase system, and improving system dynamics and reduce the memory occupation of the harmonic detection algorithm.

Finally, a simulation model and experiment prototype is built in laboratory, and the results verify the proposed method is valid and has fast dynamic response.

2 Sliding discrete Fourier transformation

In the complex frequency domain, the periodic signal can be expressed as a complex exponential series of equation (1).

$$x(t) = \sum_{k=-\infty}^{+\infty} X_k \cdot e^{j2\pi f_k t}$$  \hspace{1cm} (1)

For a given signal $x(t)$. The discrete fourier transformation (DFT) is defined as equation (2), where $k$ represents the harmonic order, and the spectrum $X_k$ after DFT transformation is as shown in equation (2). After the $X_k$ is obtained, the $k$-th harmonic $x_k(t)$ can be obtained by the equation (3).

$$X_k = \frac{1}{T} \int_{0}^{T} x(t) \cdot e^{-j2\pi f_k t} \, dt$$  \hspace{1cm} (2)

$$x_k(t) = X_k \cdot e^{j2\pi f_k t}$$  \hspace{1cm} (3)

Discretization of equation (2):

$$X_k = \frac{1}{N} \sum_{i=0}^{N-1} x(i) \cdot e^{-j2\pi f_k i}$$  \hspace{1cm} (4)

Equations (2) and (4) are harmonic detection equations based on DFT. However, the calculation of DFT according to them requires a large amount of calculation. Therefore, the sliding discrete fourier transformation (SDFT) has been proposed to reduce the amount of computation of the DFT algorithm.
According to the idea of sliding window iteration, $X_k$ is obtained by the summation of $N$ consecutive sampling points, which represents the length of the sliding window. When there is a new sampling point, the window is shifted to the right by one bit. At this time, the expression of the last sampling time $X_k(n - 1)$ and the current sampling time $X_k(n)$ are as follows:

$$X_k(n - 1) = \frac{1}{N} \sum_{i=n-N}^{n-1} x(i) \cdot e^{-jk\frac{2\pi}{N}}$$  \hspace{1cm} (5)$$

$$X_k(n) = \frac{1}{N} \sum_{i=n-N+1}^{n} x(i) \cdot e^{-jk\frac{2\pi}{N}}$$  \hspace{1cm} (6)$$

Most of the terms in the two expressions are the same, hence, $X_k(n)$ can be iterated by $X_k(n - 1)$, i.e.:

$$X_k(n) = e^{-jk\frac{2\pi}{N}}X_k(n - 1) + \frac{1}{N} x(n) - \frac{1}{N} x(n - N)$$  \hspace{1cm} (7)$$

With this method, only one subtraction and two multiplications are required for each calculation, and a new calculated value is obtained. This greatly accelerates the updating speed of sampling data and improves the system tracking load current conversion capability and to calculate the accuracy of the reference current. However, when the load current steps, it requires a whole grid fundamental period to track the new load current correctly. Moreover, it requires storing $N$ sampling points for each phase.

Discretizing the equation (7), the transfer function of the input periodic function $x(n)$ to the extracted $k$ - $th$ harmonic signal $x_k(n)$ is:

$$H_S(z) = \frac{1}{N} \frac{1 - z^{-N}}{1 - z^{-1}e^{-jk\frac{2\pi}{N}}}$$  \hspace{1cm} (8)$$

The $H_S(z)$ can be divided into three parts as follows:

$$H_S(z) = \frac{1}{\lambda} \frac{(1 - z^{-N})}{h_c(z)} \left( \frac{1}{1 - z^{-1}e^{-jk\frac{2\pi}{N}}} \right)$$  \hspace{1cm} (9)$$

It can be seen that the SDFT transfer function consists of a comb filter $H_c(z)$, a characteristic frequency resonator $H_k^c(z)$, and an amplitude calibration coefficient $\lambda$.

At the same time, it can be seen that the comb filter $H_c(z)$ has a fundamental period delay unit $z^{-N}$, which will lead the SDFT algorithm to have an inherent delay of the fundamental period.

Moreover, comb filter introduces $N$ zeros, which are centered at integer multiples of the fundamental frequency and are evenly distributed on the unit circle, as shows in Fig. 1(a). The introduced zeros guarantee that complete rejections of the harmonics at the corresponding frequencies. This property can be better visualized by the notches, i.e., zero amplitude, in the frequency response of the comb filter in Fig. 1(b).

The specific frequency resonator $H_k^c(z)$ is used to amplify the signal at the $k$ - $th$ harmonic frequency. It can be seen from the transfer function that $H_k^c(z)$ has a pole at the $k$ - $th$ harmonic, which can be canceled with the corresponding zero of the comb filter, so that the spectrum of the $k$ - $th$ harmonic, which filtered by
the comb filter, can be preserved. And ensure that the SDFT algorithm can extract the required $k$-th harmonics separately. And the conventional SDFT can be expressed as Fig. 2.

It can be seen that the comb filter determines the introduced zeros and the set of harmonics to be rejected. In the conventional DFT, the comb filter $H_c(z)$ introduces $N$ zeros equidistantly located at the $z$-domain unit circle to reject all the harmonic components of integer multiples of the fundamental frequency. However, some of the zeros may be unnecessary and redundant.

### 3 Proposed harmonic detection method

Due to the shortcomings of SDFT, i.e. poor dynamic response and large memory occupation, caused by the comb filter. Therefore, to improve the performance of SDFT, the key is to improve the performance of the comb filter. And, in a three-phase power systems, the typical harmonic scenario is the non-triple odd harmonics, hence, the comb filter can reconfigure according to the typical harmonic scenario of three-phase system.

#### 3.1 Typical harmonic scenario of three-phase system

Generally speaking, a balanced three-phase system only contains $(6k \pm 1)$-th ($k = 0, 1, 2 \ldots$) harmonics, because many power conversion processes (i.e., diode rectifier loads) involved in industrial applications produce harmonic components at these frequencies. Among them, the $6k + 1$-th harmonic is the positive sequence component, and the $6k - 1$-th harmonic is the negative sequence component. Hence, it can be expressed with:
After these typical harmonics are Clark transformed:

\[
\begin{bmatrix}
i_a(t) \\
i_b(t) \\
i_c(t)
\end{bmatrix}
=
\begin{bmatrix}
\sum_{n=0}^{+\infty} i_h(t) \cos(\omega t + \phi_h) \\
\sum_{n=0}^{+\infty} i_h(t) \cos(\omega t + \phi_h + \frac{2\pi}{3}) \\
\sum_{n=0}^{+\infty} i_h(t) \cos(\omega t + \phi_h - \frac{2\pi}{3}) \\
\sum_{n=0}^{+\infty} i_h(t) \cos(\omega t - \frac{2\pi}{3}) \\
\sum_{n=0}^{+\infty} i_h(t) \cos(\omega t + \frac{2\pi}{3})
\end{bmatrix}
\]

\[
(10)
\]

It can be seen that in the \(a\beta\) coordinate system, these harmonics are all transformed into \(6k + 1\) harmonics when considering the negative frequency.

### 3.2 Improving sliding discrete Fourier transformation

According to the analysis in the previous section, the three-phase typical harmonics current is transformed to \(6k + 1\) harmonics in the \(a\beta\) coordinate system.

Then, when the improving SDFT algorithm is used for the transformed current signal, a comb filter of \(6k + 1\) \((k = 0, \pm 1, \pm 2, \pm 3)\) harmonics is needed. Hence, the comb filter can be designed as:

\[
H_c(z) = (1 - e^{j2\pi 6k}) = 1 - \frac{1}{2} z^{-k} - j \frac{\sqrt{3}}{2} z^{-\frac{k}{2}}
\]

\[
(12)
\]

The frequency response of the comb filter is shown as Fig. 3 in the entire frequency range.

It can be seen that the comb filter can filter the \(...-11, -5, 1, 7, 13\ldots\) harmonics out, and the result is exactly the same as the harmonics of the Clark transformed three-phase typical harmonic.

### 3.3 Implement of the proposed improving SDFT

It can be seen that the comb filter is a complex filter, which is not easy to implement in practical digital system. In the power system, the imaginary unit \(j\) can be
understood as the real signal is rotated 90° ahead, which is consistent with the two coordinate axes of the \(\alpha\beta\) coordinate system, that is, \(i_\beta = ji_\alpha\). Therefore, in the practical digital system, the implementation of the complex comb filter is shown as Fig. 4:

It can be seen from the above analysis that since the delay unit of the improving comb filter is \(z^{-\frac{1}{6}}\), the dynamic delay should be 1/6 fundamental period.

When applying the improved SDFT algorithm, the entire harmonic extraction process is shown in Fig. 5:

Compared the calculated phase, the storage capacity and the inherent delay with the conventional SDFT harmonic detection method and the proposed improving SDFT harmonic detection method, the results are presented in the following Table I:

It can be seen that the proposed harmonic detection method has a great advantage, the memory space occupied by the processor will be greatly reduced,
which is \( \frac{1}{6} \) of the traditional SDFT method. At the same time, the calculated phase will be reduced to \( \frac{2}{3} \), which makes the number of calculations to achieve greatly reduced. And the rapidity of harmonic detection will be 6 times that of the conventional SDFT method, which is helpful to enhance the dynamic response ability of the system.

4 Simulation and experiment result

4.1 Simulation design and results

To verify the proposed reference current detection method, simulation is made in Matlab/simulink.

Construct a three-phase signal composed of fundamental wave, negative sequence 5th, 11th, 17th harmonic, positive sequence 7th, 13th, 19th harmonic, and reduce its amplitude to 30% at 0.04 s, the input waveform is as shown in Fig. 6(a):

First verify the ability of the improved SDFT algorithm to detect single harmonics. The proposed improved SDFT method is used for 5th harmonic detection, and the comparison of the detection result and the input 5th harmonic is shown in Fig. 6(b).

Similar to single-harmonic detection, when all harmonics are detected, the improved SDFT can fully extract the harmonic components of the three-phase current at steady state, and at the same time, the improved SDFT can respond quickly to the input changes. The dynamic adjustment time is still about 3.3 ms as shown in shown in Fig. 6(c). which is down to \( \frac{1}{6} \) of the traditional SDFT. The dynamic response of improving SDFT is greatly improved.

4.2 Experiment design and results

In order to verify the engineering practical value of the proposed improving SDFT algorithm, a experimental prototype is built in labtory. The controller uses
DSP2812 to sample the load current signal by using the built-in 12-bit AD. The DA is expanded as the output of the calculation result. The input signal is sent to the DSP, and is output to the oscilloscope through the DAC module after being calculated by proposed improving SDFT. The load selection three-phase rectifier bridge with resistance.

And proposed improving SDFT are used to carry out the fundamental current, the 5th harmonic, the 5–25th harmonics, the fundamental and the 5–25 harmonic of the load current. The results is shown in Fig. 7.

It can be seen from the figure that when the proposed SDFT is used to detect harmonics, the required fundamental wave as Fig. 7(a) and each harmonic as Fig. 7(b)(c) can be completely extracted and the complete load current can be reconstructed as Fig. 7(d). At the same time, it can be seen that when the load changes dynamically The 6k + 1 SDFT can complete the dynamic response process after 1/6 fundamental period, which is consistent with the theoretical analysis and the the previous simulation results.

Fig. 6. Simulation results (a) input signal (b) comparison of the 5-th harmonics in input signal and detection result (c) comparison of the all harmonics in input signal and detection result
5 Conclusion

In this paper, an effective harmonics detection method based on improving SDFT is presented. First, according to the transfer function of SDFT, the essence of SDFT is composed of comb filter, resonator and adjustment factor. After in-depth analysis of the characteristics of comb filter, the corresponding improvement method is obtained. The proposed method can configure the corresponding comb filter according to different load types. Then, according to the typical harmonic of the three-phase system, the corresponding improved SDFT algorithm is designed, and the implementation in the practical digital system is designed in detail. Finally, the simulation model and experimental prototype are built. The simulation and experimental results verify the effectiveness of the proposed improving SDFT. The method proposed in this paper can be used in three-phase grid-tie inverter applications in which highly nonlinear currents are involved, and improve system control effect of harmonics and reduce THD of inverter output current.

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Fig. 7. Experiment results (a) fundamental current (b) 5th harmonics (c) 5–25 harmonics (d) fundamental and 5–25th harmonics