Estimation of spatial correlation for seismic design of buildings

V A Pshenichkina, A A Churakov, T V Yereschenko, I A Ivanov
Volgograd State Technical University, 1 Akademicheskaya street, Volgograd, 400074, Russia

E-mail vap_hm@list.ru

Abstract. The paper discusses a solution for the probabilistic problem of high-rise buildings oscillation exposed to seismic loads in form of a six-component random vector. Two groups of matrices for the dynamic response factors to help estimate the effects of spatial correlation between generalized coordinates and correlation in the various seismic vector components on design stresses in load-bearing building structures have been obtained. The spatial model of a building configured as a thin-wall compound bar provides a solution for the probabilistic problem in analytical form.

Introduction
The assurance of structural reliability of buildings located in seismically hazardous areas is one of the consistently relevant construction problems. Seismic loads affecting buildings and constructions are random in nature and, in the general case, should be viewed as space-time random fields, and for high-rise facilities – as six-component random vectors. The use of spatial probabilistic models for buildings requires the estimation of two types of correlation: correlation between the components of vectors of dynamic environmental effects and correlation of generalized coordinates.

The issue of factoring in the mutual correlation between various degrees of freedom of constructions when exposed to seismic loads was first raised in the paper by Bolotin V.V. [1]. It is resolved on a case-by-case basis and depends on the attenuation parameters of ground motions in the earth foundation and oscillations of the construction itself, as well as the mutual layout of frequencies and nature of spectral densities of environmental effects.

Spatial systems in most cases are characterized by a spectrum of relatively close natural frequencies. Besides, when the maximum spectral density of loads is located at a sufficient distance from the natural frequencies of a system, the main and accessory coefficients of the correlation matrix of generalized coordinates show the same pattern, which means that correlation links of generalized coordinates tend to increase [2].

The processing of vector components for a seismic ground-motion field shows that the correlation links between them account for less than 30% [3]; which is why they are considered statistically independent. The paper by Bierbreier A.N., Petrenko A.V. [4] establishes that for constructions, such NPPs, statistically in dependent components of original accelerograms, once they pass through an elastic system (building), may cause a high correlation of floor-based accelerograms. At the same time, note should be taken that floor-based accelerograms (FA) result from the passing of a vector random function of seismic foundation acceleration through a dynamic system, and FA correlation is
in specifically determined by the presence of correlation between the components of initial seismic acceleration.

For residential and public buildings, the effects of correlation on seismic response remains unexplored for the time being. The use of these factors may lead to a quantitative revaluation of dynamic loads.

**Goals and objectives of study**
The goal of this study is to assess the influence of mutual correlation between the components of a seismic ground-motion vector, as well as between different degrees of freedom of the construction itself on the design magnitude of seismic load affecting high-rise buildings.

The paper consists of three parts:
- modeling a random vector of seismic load;
- solving the problem of oscillation of a building as a thin-wall compound bar, obtaining the correlation function of generalized coordinates, matrices of dynamic amplification factors and inertia loads in consideration of the two types of correlation;
- analysis of the results obtained.

**Probabilistic seismic design of a building subject to multicomponent seismic load**
To explore the problem of influence of the two types of correlation on the design magnitude of seismic load, the analytical spatial model of a building has been developed featuring a thin-wall compound bar (figure 1) [5].

Seismic impact is regarded as the acceleration vector of progressive motion and rotation of earth foundation.

\[
\ddot{U}(t) = \left[ \ddot{X}_{01}(t), \ddot{X}_{02}(t), \ddot{X}_{03}(t), \ddot{a}_{01}(t), \ddot{a}_{02}(t), \ddot{a}_{03}(t) \right],
\]  

(1)
Here, \( \tilde{X}_{0h}(t), \tilde{a}_{0h}(t) \) are the permanent and permanently bound random functions.

Considering a relatively short length of the building in plan, we will only factor in the horizontal effects of seismic forces. As such, the components \( \tilde{X}_{03}, \tilde{a}_{01}, \tilde{a}_{02} \) are assumed equal to zero, and the seismic load applied to the thin-wall compound bar is expressed as a three-component vector random function

\[
\tilde{X}_h(t) = \begin{bmatrix} \tilde{X}_{01}(t), \tilde{X}_{02}(t), \tilde{a}_{03}(t) \end{bmatrix} = [\tilde{X}_1(t), \tilde{X}_2(t), \tilde{X}_3(t)]
\]

(2)

with zero mathematical expectations, correlation function and spectral density in form of the matrices \( K_{hl}^X(\tau) \) and \( S_{hl}^X(w) \) \( (h, l = 1, 2, 3) \).

The equations of spatial oscillation of the thin-wall compound bar in the generalized coordinates, subject to attenuation, are expressed as follows:

\[
\ddot{\Phi}_{pk} + 2c_{pk} \dot{\Phi}_{pk} + \lambda_{pk} \Phi_{pk} = -\tilde{H}_{pk}(t), \quad k = 1, 2, \ldots, \infty; \quad p = \xi, \eta, \theta
\]

(3)

where \( \xi, \eta, \theta \) – the directions of the main linear and angular displacements of the flexural center of the thin-wall compound bar; \( \Phi_{pk}(t) \) – the vector of stochastic generalized coordinates; \( \tilde{H}_{pk}(t) \) – the vector of generalized accelerations;

\( \lambda_{pk} \) – the modal frequencies of flexure-torsion oscillations;

\( c_{pk} = c_p \lambda_{pk}/2 \) – the adjusted dissipation;

\( c_p \) – the loss factor.

The statistical characteristics of generalized coordinates are obtained with the canonical decomposition method invented by V.S. Pugachev [6]. For the canonical decomposition of the input vector random function \( \tilde{X}_h(t) \), the vector \( \tilde{U}_h(w) \) must be transformed vis-à-vis the vector \( \tilde{V}_h(w) \) with non-correlated components.

The canonical decomposition of the vector random function of seismic load is written as:

\[
\tilde{X}_h(t) = \sum_{\nu=1}^{M} \sum_{r=1}^{3} a_{rh}^\nu (V_{rs}^\nu \sin w_{\nu,t} + V_{rc}^\nu \cos w_{\nu,t}),
\]

(4)

where \( a_{hl}^\nu \) – the adjustment factors;

\( V_{rs}^\nu, V_{rc}^\nu \) – the random non-correlated components with mathematical expectations equal to zero, and pairwise identical dispersions \( M[V_{rs}^\nu] = M[V_{rc}^\nu] = 0; \quad D[V_{rs}^\nu] = D[V_{rc}^\nu] = D_r^\nu; \quad a_{rh}^\nu \sin w_{\nu,t}, a_{rh}^\nu \cos w_{\nu,t} \) – the input coordinate functions.

Then the output vector random function of the generalized coordinates can also be presented as canonical decomposition

\[
\ddot{\Phi}_{hk} = \sum_{\nu=1}^{M} \sum_{r=1}^{3} a_{rh}^\nu \left[ \ddot{V}_{rs}^\nu \psi_{pks}^\nu(t) + \ddot{V}_{rc}^\nu \psi_{pke}^\nu(t) \right]
\]

(5)

The input coordinate functions \( \psi_{pks}^\nu(t), \psi_{pke}^\nu(t) \) are obtained by solving the equations (4) containing the input coordinate functions in their right-hand part.
We obtain the correlation functions of the generalized coordinates. With \( t = t' \), this gives us the expressions for dispersions. The dispersions (correlation factors) for various components of the output vector random function \( \tilde{\phi}_{hk}(t) \) are expressed as follows

\[
K_{\phi \xi \eta j}(t, t') = \sum_{v=1}^{M} D_{v}^{T} \alpha_{12}^{T} a_{12}^{T} \cos(\phi_{12}^{T} - \phi_{12}^{T}) \left[ \psi_{\xi j}(t) \psi_{\xi j}(t') + \psi_{\eta j}(t) \psi_{\eta j}(t') \right],
\]

\[
K_{\phi \xi \theta j}(t, t') = \sum_{v=1}^{M} D_{v}^{T} \alpha_{12}^{T} \cos(\phi_{12}^{T} - \phi_{12}^{T}) \left[ \psi_{\xi j}(t) \psi_{\theta j}(t') + \psi_{\eta j}(t) \psi_{\theta j}(t') \right],
\]

\[
K_{\phi \eta \theta j}(t, t') = \sum_{v=1}^{M} \left[ \psi_{\eta j}(t) \psi_{\eta j}(t') + \psi_{\eta j}(t) \psi_{\eta j}(t') \right].
\]

The expression (6) allows obtaining two groups of matrices of dynamic response factors for individual forms of oscillations \( \beta_{p q i j}(\lambda_{p}, \lambda_{j}, t) \).

The first group includes the dynamic response factors obtained by the factoring in the correlation between the generalized coordinates of each seismic vector component

\[
\beta_{p q i j}^{2}(\lambda_{p}, \lambda_{q}, t) = \lambda_{p}^{2} \lambda_{q}^{2} K_{\phi p q i j}(t, t).
\]

The second group includes the dynamic response factors based on the correlation between various components of the vectors of environmental effects

\[
\beta_{p q i j}^{2}(\lambda_{p}, \lambda_{q}, t) = \lambda_{p}^{2} \lambda_{q}^{2} K_{\phi p q i j}(t, t).
\]

The influence of correlation can be assessed based on the example of a 16-floorshear wall frame building (fig.2). The building is rectangular in plan and has the overall dimensions of 61.4×16.4m. The centerlines of columns form 6×6m and 3×3m cells. The building consists of a 4.2m high basement, sixteen 3.3m high useful floors and a 4.8m high top utility floor. The walls are made of B25 concrete, and the columns are made of B30 concrete. Reinforcement bars are A400 class. The column dimensions are 0.4×0.4m. The wall thickness is 0.2m.

The spectral densities of the components of the vector random function for seismic ground motion is approximated by using the functions

\[
S_{h h}(x) = D_{h}^{2} \frac{2}{\pi} a_{h}^{2} \frac{m_{h}^{2} + w_{h}^{2}}{m_{h}^{4} + 2 a_{h} w_{h}^{2} + w_{h}^{4}},
\]

where \( m_{h}^{2} = \alpha_{h}^{2} + \beta_{h}^{2} \); \( a_{h} = \alpha_{h}^{2} - \beta_{h}^{2} \); \( D_{h} \)-the dispersion of the \( h \)-component.

The in-phase \( C_{h l}^{x}(w) \) and out-of-phase \( Q_{h l}^{x}(w) \) components of mutual spectral densities \( S_{h l}(w) \) are written as

\[
C_{h l}^{x}(w) = A_{h l} \cos(\phi_{h l}(w))
\]

\[
Q_{h l}^{x}(w) = A_{h l} \sin(\phi_{h l}(w))
\]

The seismic load parameters \( \alpha_{h}, \beta_{h}, A_{h l}, \phi_{h l}(w) \) are assumed according to [3]. The standards are
\[ \sigma_1 = \sigma_2 = \sqrt{D_1} = 0.25 \text{ m/c}^2 \text{ (for a 7-degree earthquake)}, \quad \sigma_3 = \sqrt{D_3} = 0.0077 \text{ rad/c}^2. \]

The spectral density factors are
\[ \alpha_1 = 6.1 \text{ c}^{-1}; \quad \alpha_2 = 7 \text{ c}^{-1}; \quad \alpha_3 = 6.5 \text{ c}^{-1}; \beta_1 = 14 \text{ c}^{-1}; \quad \beta_2 = 16 \text{ c}^{-1}; \quad \beta_3 = 16.5 \text{ c}^{-1}. \]

The loss factor is \( c_p = 0.1 \). The amplitudes and phase angles of mutual spectral densities are
\[ A_{12} = A_{13} = A_{23} = 0.02; \quad \phi_{12} = \phi_{13} = \phi_{23} = 0.4 \pi; \quad \gamma_1 = \gamma_2 = \gamma_3 = 0.35 \text{ c}^{-1}. \]

### Figure 2. Layout plan of a 16-floor building

The frequencies of free flexure-torsion building oscillations for the first 4 tones are provided in table 1.

#### Table 1. Frequencies of free oscillations

| Tone number | Frequencies, s\(^{-1}\) |
|-------------|--------------------------|
|             | \( \lambda_\xi \) | \( \lambda_\eta \) | \( \lambda_\theta \) |
| 1           | 1.112                   | 2.137                   | 8.628                   |
| 2           | 10.78                   | 13.88                   | 55.37                   |
| 3           | 29.48                   | 38.68                   | 154.71                  |
| 4           | 55.84                   | 75.38                   | 302.36                  |

The dispersion of inertia seismic load is calculated according to the formula:
\[
\langle S_{pqij}^2(t) \rangle = \lambda_{pi}^2 \lambda_{qj}^2 \pi \quad m_p^2 p_i(z) q_j(z) K_{\phi_{piqj}}(t, t),
\]

\[
(11)
\]

where \( m_\xi = m_\eta = m \) – the mass per length unit of the bar; \( m_\theta \) – the linear moment of mass inertia.

With \( p=q \), the matrix elements (11) determine the inertia load according to the formulae of oscillations from each of the seismic vector components and the additional inertia load obtained by factoring in the correlation between various forms.

With \( p \neq q \), the matrix elements (11) determine the inertia load based on the forms of oscillations obtained by factoring in the correlation between the components of the vector of seismic load.

The earthquake response of a construction is calculated separately for each form of oscillations and for their combinations.
The design values of transverse and lengthwise forces, bending moments, normal and tangential stresses in load-bearing structures produced by inertia seismic load are obtained as mean root square values of the component force in the cross-section in question:

$$R_s = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} R_{si,j}^2},$$

(12)

where $R_{si,j}$ – the forces or stresses in the target cross-section for each $i$-th and $j$-th form of motion caused by seismic load $S_{p_iq_j} (i, j = 1, ..., N; p, q = \xi, \eta, 0)$; $N$ – the number of oscillation forms taken into consideration in the design.

The development of the building design shows that additional stresses in load-bearing elements of the thin-wall compound bar obtained by factoring in the correlation of generalized coordinates account for 15-20% of the design stresses obtained without such correlation. These additional stresses based on the correlation of the components of the seismic vector account for 30-35%.

**Summary**

The stochastic spatial model of a building explored herein allows us not only to take into account the statistical variability of forces and stresses in building structures, but also to estimate the extent of influence of two correlation types for the operation multicomponent seismic load. Thus, it has been established based on the analytical calculation that the hypothesis of statistical independence of the accelerogram components adopted in the theory of seismic stability leads to an understatement of the design forces by 30-35%. At the same time, the correlation of the degrees of freedom for a spatial system depends to a large extent on the amount of eccentricity between the flexural center and the geometric center of a building in plan. For horizontally symmetrical buildings, the correlation of generalized coordinates is minimal accounting for 5-10%.

The areas of further studies: assessing the influence of correlation on the stress-strain condition of structures of residential and civil seismically stable buildings of various structural types, subject to the effects of statistical variability of the earth foundation characteristics.

**References**

[1] Bolotin V V 1961 *Statistical Methods of Construction Mechanics* (Stroiizdat, Moscow).
[2] Pshenichkina V A 1996 *Probabilistic Calculation of Dynamic Impacts on High-Rise Buildings* (VGASU, Volgograd).
[3] Nikolayenko N A, Nazarov Yu P 1999 *Dynamics and Seismic Stability of Constructions* (Stroiizdat, Moscow).
[4] Bierbreier A N, Petrenko A V 2004 *Statistical Independence of Seismic Accelerogram Components on the Ground and on Building Floors and its Importance for Seismic Stability of NPP Equipment* (Theses of CJTI, SPb) 29 40-64.
[5] Pshenichkina V A, Belousov A S, Kuleshova A N, Churakov A A 2010 *Reliability of Buildings as Spatial Systems Exposed to Seismic Impacts* (VGASU, Volgograd).
[6] Pugachev V S 1960 *Theory of Random Functions* (Fitmatgiz, Moscow).