Dark aspects of massive spinor electrodynamics

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Received March 28, 2014
Revised May 30, 2014
Accepted June 5, 2014
Published July 1, 2014

Abstract. We investigate the cosmology of massive spinor electrodynamics when torsion is non-vanishing. A non-minimal interaction is introduced between the torsion and the vector field and the coupling constant between them plays an important role in subsequential cosmology. It is shown that the mass of the vector field and torsion conspire to generate dark energy and pressureless dark matter, and for generic values of the coupling constant, the theory effectively provides an interacting model between them with an additional energy density of the form \( \sim 1/a^6 \). The evolution equations mimic \( \Lambda \)CDM behavior up to \( 1/a^3 \) term and the additional term represents a deviation from \( \Lambda \)CDM. We show that the deviation is compatible with the observational data, if it is very small. We find that the non-minimal interaction is responsible for generating an effective cosmological constant which is directly proportional to the mass squared of the vector field and the mass of the photon within its current observational limit could be the source of the dark energy.

Keywords: dark energy theory, gravity

ArXiv ePrint: 1305.4438v3
1 Introduction

One of the most intriguing discovery of modern cosmology is the acceleration of the Universe [1, 2]. A standard approach is to assume that dark energy of repulsive nature is causing the current acceleration. Many candidates of the dark energy have been proposed [3–5], among which the cosmological constant is the most accepted one. Along with yet another unidentified constituent of the Universe called dark matter, they compose standard cosmological model, ΛCDM [6]. Even though the extreme fine-tuning of the cosmological constant [7] has been an unsatisfactory theoretical feature of the model, and many alternatives to explain the smallness of the cosmological constant as the source of dark energy have been proposed, it is remarkable that so far the observable Universe can be well addressed with the ΛCDM model.

In this paper, we investigate cosmology of massive spinor electrodynamics with torsion. The theory consists of massive vector field interacting with the Dirac spinor in the Einstein-Cartan space-time, and we find that both of these fields and torsion contribute to the energy densities of dark energy and pressureless dark matter at late times.

Spinor electrodynamics in curved space-time is an old subject and there are extensive amounts of literature on the subject, describing both classical and quantum aspect [13]. But as far as cosmology is concerned, vector sector and spinor sector were considered separately so far. On the spinor side, it has been known for some time that spinor fields could play some important roles in the evolution of the Universe [14–18]. For example, in ref. [15] it is shown that a spinor field can accommodate any desired behavior of its energy density if an appropriate self-interaction of the spinor field is introduced. For the Dirac spinor mass term, the energy density shows a cosmological behavior exactly like a pressureless dark matter term. In ref. [16] it is investigated whether fermionic sources could be responsible for accelerated periods during the evolution of a universe after a matter field would guide for the decelerated period. In refs. [17, 18] the authors have shown that it is possible to simulate perfect fluid and dark energy by means of a nonlinear spinor field.

On the other hand, the cosmology with vector fields received more attention recently and various models have been proposed to account for dark energy [19–23]. Nonlinear electromagnetism in cosmology was considered in ref. [24], where it was shown that the addition of a non-linear term to the Lagrangian of the electromagnetic field yields a fluid with an asymptotically super-negative equation of state, causing an accelerated expansion of the universe. Universe filled with a massive vector field non-minimally coupled to gravitation was
proposed, and the cosmology yields a dark energy component which is proportional to the mass of the vector particle [25]. In ref. [26], several (non)-minimally coupled vector field models with a potential for the vector field were considered and some model mimics ΛCDM expansion at late times. In particular, the possibility of understanding dark energy from the standard electromagnetic field, without the need of introducing new physics was explored recently [27–30] and it was shown that the presence of a temporal electromagnetic field on cosmological scales generates an effective cosmological constant which can account for the accelerated expansion of the universe.

In this paper we attempt investigations on the cosmological consequences of the massive spinor electrodynamics where both spinor and vector sectors come into play, but they are non-minimally coupled to gravitation. One could naively expect that the pressureless dark matter term [15] coming from the spinor sector and the effective cosmological constant [27–30] coming from the massless vector sector could combine to yield a dark energy model whose cosmological evolution mimics that of the standard ΛCDM model. However, one finds out immediately that this anticipation is not met by an explicit check of the equations of motion, because the theory has an interaction between the spinor and the vector field. It turns out that such a difficulty can be overcome by introducing a massive vector field and torsion [31, 32] component along with a specific non-minimal interaction between them into the theory. They can intervene between the two sectors and alleviate the difficulties coming from the interaction. Consequently, we find that cosmology of massive spinor electrodynamics provides a dark energy model where the dark energy and dark matter are interacting with each other through the electromagnetic interaction. It is found that the model allows an asymptotic de Sitter acceleration, which is an attractor. When dark energy dominance has taken place at late times, the spinor provides dark matter density, whereas the massive vector field and torsion contribute to both dark energy and dark matter densities. One of the interesting aspects as a consequence is that the dark energy density is directly proportional to the mass squared of the vector fields. Another distinctive feature coming from the cosmology of massive spinor electrodynamics is that the dynamics allows a very small fraction of the energy density evolving as $\sim 1/a^6$ at late times. We check whether the presence of this energy density is consistent with the observational data by giving a detailed analysis of how much of such a deviation from ΛCDM can be allowed.

The paper is organized as follows: in section 2, we give a classical formalism of massive spinor electrodynamics in curved space-time with torsion. In section 3, we investigate the cosmology of massive spinor electrodynamics and show that it can provide a field theoretical model of dark energy and dark matter. We find de Sitter solution and perform an asymptotic expansion to show that an additional energy density component of the form $\sim 1/a^6$ is present in this approach. In section 4, we present data analysis in order to test whether ΛCDM + (1/a^6) can be consistent with observations. Section 5 includes conclusion and discussions.

2 Massive spinor electrodynamics with torsion

We start with a brief summary of torsion. Let us consider the connection

$$\Gamma_\mu^\rho = \{\rho_{\mu}\nu\} - K_{\mu\nu}^{\rho}, \quad (2.1)$$
where $\{^{\rho}_{\mu\nu}\}$ is the Christoffel connection and the contortion $K_{\mu\nu}^{\rho}$ is given by the torsion tensor $S_{\rho\mu\nu} = \Gamma_{[\mu\nu]}^{\rho}$ via

$$K_{\mu\nu}^{\rho} = - \left( S_{\rho\mu\nu} + S_{\rho\nu\mu} + S_{\nu\rho\mu} \right).$$

The above connection (2.1) satisfies the metricity condition, $\nabla_{\mu} g_{\nu\rho} = 0$. It is important to note that $K_{\alpha\beta\gamma}$ is anti-symmetric for last two indices and $S_{\alpha\beta\gamma}$ anti-symmetric for first two indices. Now, we can generally decompose the contortion tensor (2.2) into a traceless part and trace $\{33–35\}$:

$$K_{\mu\nu\rho} = \tilde{K}_{\rho\mu\nu} - \frac{2}{3} \left( \delta_{\rho\mu} S_{\nu} - g_{\mu\nu} S_{\rho} \right),$$

where $\tilde{K}_{\rho\mu\nu} = 0$ and $S_{\nu}$ is the trace of the torsion tensor, $S_{\nu} = S_{\mu\nu}$. Making use of the connection $\Gamma_{\rho\mu\nu}$ with (2.3), we can write curvature scalar as follows

$$R = R(\{\}) - 4 \left\{ \nabla_{\mu} S_{\mu} - \frac{8}{3} S_{\mu} S_{\mu} - \tilde{K}_{\nu\rho\alpha} \tilde{K}_{\alpha\nu\rho} \right\},$$

with the obvious notation that quantities with $\{\}$ are those which are constructed with Christoffel connection only.

Next, let us consider an action given by (in units of $\sqrt{1/8\pi G} \equiv M_p = 1$)

$$S' = \int d^4 x \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_{\mu} A^{\mu} \right] + \int d^4 x \sqrt{-g} L_D,$n

$$L_D = - \frac{1}{2} \left( \nabla_{\mu} \tilde{\psi} \gamma^{\mu} \tilde{\psi} - \tilde{\psi} \gamma^{\mu} \nabla_{\mu} \tilde{\psi} \right) - m_f \tilde{\psi} \tilde{\psi} + A_{\mu} \tilde{\psi} \gamma^{\mu} \tilde{\psi}.$$

The above action describes a massive vector field which is known as Proca field interacting with Dirac fermion in curved spacetime. For the field strength, we can choose the gauge invariant non-minimal coupling prescription $[31, 32]$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu},$$

or the minimal coupling, $\tilde{F}_{\mu\nu} = \nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu} = F_{\mu\nu} - 2 S_{\rho\mu\nu} A_{\rho}$, which is not gauge invariant. For the case of Proca field, the term $\sim S_{\mu\nu}^{\rho} A_{\rho}$ is harmless, since there is no gauge invariance. However, the minimal coupling encounters problems in the massless limit, because with this term present the field strength is no longer gauge invariant in that limit. Therefore, we adopt non-minimal expression (2.7) which allows a smooth massless limit.\(^1\) The limit involves another important modification of the action (2.5) itself. In the flat spacetime, the Proca theory is quantum mechanically consistent as long as it couples to a conserved current, because the condition $\partial_{\mu} A_{\mu} = 1/m^2 \partial_{\mu} J^{\mu} = 0$ eliminates the unwanted ghost state of the vector field. However, the model has problem when massless limit is considered, that is, a singularity is encountered in the limit $[37]$. This is reflected in the fact that the limiting Lagrangian $-\frac{1}{4} F^2$ with gauge symmetry is unsuitable without a gauge prescription

\(^1\)The non-minimal prescription adopted here might confront conceptual difficulty because vector fields are decoupled from torsion, although they are spinning particles. The situation could be improved by adding an explicit interaction term between the vector field and torsion $[36]$. However, such interactions do not play any role, because $F_{\mu\nu} = 0$, for the background cosmological configuration in our case. It is also pointed out that as far as cosmology is concerned, the two approaches do not make any difference.
and massless spinor electrodynamics is not recovered in that limit. To recover the internal symmetry structure smoothly in the limit, an auxiliary condition which behaves like a gauge fixing term in the massless limit \((\sim (\partial \cdot A)^2)\) is imposed. In curved background, we introduce

\[
S_{gf} = -\frac{1}{2\alpha} \int d^4x \sqrt{-g} \left[ \left( \nabla_\mu A^\mu \right)^2 + 4\xi S_\mu A^\mu \nabla_\nu A^\nu + 4\xi^2 (S_\mu A^\mu)^2 \right], \quad S = S' + S_{gf}. \tag{2.8}
\]

A couple of comments are in order regarding the above auxiliary condition (or gauge fixing term in the massless limit) for the massive vector field. In flat space with vanishing torsion, this term reduces to \(-\frac{1}{2\alpha} (\partial \cdot A)^2\) which guarantees a smooth massless limit \(m \to 0\) for the propagator, and massless spinor electrodynamics is recovered in the limit. \(\alpha \to \infty\) gives the ordinary Proca theory. When \(\xi = 1\), three terms of (2.8) combine into a single covariant derivative with torsion connection \((\sim (\nabla_\mu A^\nu)^2)\) which corresponds to a minimal extension to spacetime with torsion. \(\xi \neq 1\) can be regarded as a non-minimal coupling between the vector field and torsion. Even though \(\xi = 1\) corresponds to extending the gauge fixing term straightforwardly to curved space with torsion, there is no a priori reason to favor this choice and we consider general values of \(\xi\) which turns out to be important to account for dark energy. Since \(\xi \to -\xi\) can always be compensated by \(S_\mu \to -S_\mu\), we will assume \(\xi \geq 0\) without loss of generality. If we rescale \(S_\mu \to \xi^{-1} S_\mu\), \(\xi\)-dependence disappears in (2.8), but reappears in (2.4). We stick to the expression of (2.8).

For the Dirac part, we use the gamma matrices given by

\[
\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{2.9}
\]

with \(\{\gamma^a, \gamma^b\} = -2\eta^{ab}\), \(\eta^{ab} = \text{diag.}(−1, 1, 1, 1)\). The covariant derivative of the spinor and its dual are given by

\[
\nabla_\mu \psi \equiv \partial_\mu \psi - \Gamma_\mu \psi \tag{2.10}
\]

and

\[
\nabla_\mu \bar{\psi} \equiv \partial_\mu \bar{\psi} + \bar{\psi} \Gamma_\mu, \tag{2.11}
\]

where \(\Gamma_\mu\) is the connection on the spinor given by

\[
\Gamma_\mu = \frac{1}{4} \omega^{ab}_\mu \gamma_a \gamma_b. \tag{2.12}
\]

The spin connection is given by

\[
\omega^{ab}_\mu = e^a_\nu \left( \partial_\mu e^{b\nu} + \Gamma^{b\nu}_\mu e^{c\rho} \right), \tag{2.13}
\]

where the tetrad is defined by

\[
g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}. \tag{2.14}
\]

One can check that the spin connection satisfies \(\omega^{a\mu}_\mu b = -\omega^{b\mu}_\mu a\). It is useful to decompose the spin connection (2.13) into two parts,

\[
\omega^{ab}_\mu = \omega^{ab}_\mu (\{\}) - e^a_\nu e^{b\rho} K^{\nu}_{\mu\rho}. \tag{2.15}
\]

Then, one can show that eq. (2.6) becomes [38]

\[
\mathcal{L}_D = \mathcal{L}_D(\{\}) + \frac{i}{4} \bar{\psi} \gamma^{[\rho} \gamma^{\nu} \gamma^\mu \psi K_{\mu\nu\rho}. \tag{2.16}
\]
Hence, the Dirac spinor interacts only with the totally anti-symmetric components of the torsion.

The equations of motions for $g_{\mu\nu}$ are given by

$$G_{\mu\nu}(\{} \rangle = T_{\mu\nu}(T, A) + T_{\mu\nu}(D),$$  \hspace{1cm} (2.17)

where

$$T_{\mu\nu}(T, A) = \frac{8}{3} \left[ S_\mu S_\nu - \frac{1}{2} g_{\mu\nu} S_\alpha S^\alpha \right] + \left[ \tilde{K}_{\alpha}(\mu) \tilde{K}_\beta(\nu) \right] - \frac{1}{2} g_{\mu\nu} \tilde{K}_\alpha \tilde{K}_\beta \tilde{K}^{\alpha\beta} \right]$$  \hspace{1cm} (2.18)

$$+ \left[ F_{\mu\alpha\beta} F_{\nu}^{\alpha\beta} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right] + m^2 \left[ A_\mu A_\nu - \frac{1}{2} g_{\mu\nu} A^\alpha A_\alpha \right]$$  \hspace{1cm} (2.19)

$$+ g_{\mu\nu} \left[ \frac{1}{2} \left( \nabla_\mu A^\alpha \right)^2 + A^\nu \nabla_\nu \left( \nabla_\alpha A^\alpha + 2 \xi S_\alpha A^\alpha \right) - 2 \xi^2 \left( S_\alpha A^\alpha \right)^2 \right]$$  \hspace{1cm} (2.20)

and

$$T_{\mu\nu}(D) = \frac{i}{4} \left[ (D_\mu \bar{\psi}) \gamma_\nu \psi + (D_\nu \bar{\psi}) \gamma_\mu \psi - \bar{\psi} \gamma_\mu \gamma_\mu D_\nu \psi - \bar{\psi} \gamma_\nu \gamma_\mu D_\mu \psi \right] + g_{\mu\nu} \mathcal{L}_D(\{} \rangle),$$  \hspace{1cm} (2.21)

where $\mathcal{L}_D$ is given in (2.6) and we introduced $D_\mu = \nabla_\mu - i A_\mu$. Equation of motions for $A_\mu$ is given by

$$0 = \frac{1}{\sqrt{-g}} \partial_\nu \left( \sqrt{-g} F^{\mu\nu} \right) + m^2 A^\mu$$

$$- \frac{1}{\alpha} \left[ \nabla_\mu \left( \nabla_\nu A^\nu \right) - 2 \xi S_\mu \nabla_\nu A^\nu + 2 \xi \nabla_\mu \left( S_\nu A^\nu \right) - 4 \xi^2 S_\mu S_\nu A^\nu \right] - \bar{\psi} \gamma_\mu \psi,$$  \hspace{1cm} (2.22)

and the equation coming from variation of the torsion $S_\mu$ is

$$S^\mu = - \frac{3 \xi}{4 \alpha} \left[ A^\mu \nabla_\nu A^\nu + 2 \xi A^\mu S_\nu A^\nu \right],$$  \hspace{1cm} (2.23)

and for the totally antisymmetric component $K_{[\mu\nu\rho]}$, we have

$$K_{[\mu\nu\rho]} = \frac{i}{4} \bar{\psi} \gamma_\rho \gamma_\nu \gamma_\mu \psi.$$  \hspace{1cm} (2.24)

The Dirac equation is given by

$$\left( i \gamma^\mu \nabla_\mu - m_f + \gamma^\mu A_\mu \right) \psi = - \frac{i}{4} \gamma_\rho \gamma_\nu \gamma_\mu \gamma^\rho \gamma^\nu \gamma^\mu \psi K_{[\mu\nu\rho]},$$  \hspace{1cm} (2.25)

3 FRW cosmology

To discuss Friedman cosmology in the flat Robertson-Walker space-time, consider a metric of the form

$$ds^2 = -dt^2 + a^2(t) dx^i dx^i,$$  \hspace{1cm} (3.1)
where \( a(t) \) is the scale factor of our universe. Isotropy and homogeneity permit only the following non-vanishing components of torsion \([39]\)

\[
S_{10} = S_{20} = S_{30} = h(t)/2, \quad \tilde{K}_{[ijk]} = \epsilon_{ijk}k(t)/6.
\]

(3.2)

Here \( h(t) \) and \( k(t) \) are unknown functions of time whose dynamics are governed by the equations of motion. Before proceeding, we notice that the second ansatz of eq. (3.2) and (2.23) yields

\[
k(t) = -\frac{1}{4}\bar{\psi}\gamma_0\gamma^5\psi,
\]

(3.3)

which vanishes for spinor configurations (see below) which respect spatial isotropy. Therefore, the totally antisymmetric part of torsion is dropped from now on. Then, RW metric gives the following non-vanishing connection components

\[
\left\{0 \atop ij\right\} = a\dot{a}\delta_{ij}, \quad \left\{i \atop j0\right\} = \frac{\dot{a}}{a}\delta_{ij},
\]

(3.4)

and the tetrad with

\[
e_a^0 = (1, a, a, a), \quad e_a^i = \left(1, \frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right),
\]

(3.5)

which in turn produces the following non-vanishing components of the spin connection (2.13):

\[
\omega_{x^i 0}(\{\}) = \omega_{x^i j}(\{\}) = \dot{a}\delta_{ij}.
\]

(3.6)

Then, we have from eq. (2.12)

\[
\Gamma_0(\{\}) = 0, \quad \Gamma_{x^i}(\{\}) = \frac{\dot{a}}{2}\gamma_0\gamma_i.
\]

(3.7)

For the gauge field, we choose a non-vanishing temporal gauge field for isotropic configuration \([27–30]\)

\[
A_\mu = (f(t), 0, 0, 0).
\]

(3.8)

Using these ingredients, eqs. (2.17), (2.21), (2.22), and (2.24) become \((H = \dot{a}/a)\)

\[
3H^2 = -\frac{1}{2\alpha}\left[\dot{f} + 3(H + \xi h)f\right]^2 + 3h^2 - \frac{m^2}{2}f^2 - f\bar{\psi}\gamma^0\psi + mf\bar{\psi}\psi
\]

(3.9)

\[
-3H^2 - 2\dot{H} = \frac{1}{2\alpha}\left[\dot{f} + 3(H + \xi h)f\right]^2 + 3h^2 - \frac{m^2}{2}f^2 - f\bar{\psi}\gamma^0\psi
\]

(3.10)

\[
0 = \partial_t \left[\dot{f} + 3(H + \xi h)f\right] - 3\xi h \left[\dot{f} + 3(H + \xi h)f\right] + \alpha m^2f + \alpha\bar{\psi}\gamma^0\psi
\]

(3.11)

\[
\frac{2\alpha}{\xi} \frac{h}{f} = \dot{f} + 3(H + \xi h)f
\]

(3.12)

\[
0 = \dot{\psi} + \frac{3}{2}H\psi + i\gamma^0mf\psi - if\psi.
\]

(3.13)

In deriving eqs. (3.9) and (3.10), eq. (3.12) was used.

Let us discuss solutions of the evolution equations. Eq. (3.13) gives

\[
\bar{\psi}\psi = \frac{A}{a^3}, \quad \psi^\dagger\psi = \frac{B}{a^3},
\]

(3.14)
where $A$ and $B$ are constants. They are related by $A = B(>0)$ for spinor configuration $\psi = (\psi_1, 0, 0, 0)$ and by $A = -B(<0)$ for $\psi = (0, 0, 0, \psi_4)$ with constants $A$ and $B$. Each respects isotropy with $\bar{\psi}\gamma^i\psi = 0$. This solution is independent of the rest of the equations. We choose $A = B$ from here on, for which the fermionic mass term in eq. (3.9) contributes positive energy density. For the remaining four evolution equations, one can explicitly check that only three of them are independent for the three variables $a(t)$, $f(t)$ and $h(t)$. We also notice that the first term in the right hand side of (3.9) corresponds to equation of state $\omega = -1$, middle three terms to $\omega = 1$, and the last term to pressureless matter. However, one is not to be led to the conclusion that the first term is a constant and the combined middle three terms behaves $\sim 1/a^6$, because these are true only when each term is considered separately.

We first discuss an existence of an asymptotic de Sitter phase which is obtained by neglecting the decaying energy densities of spinor contributions. Assume $\alpha$ to be negative ($\alpha = -|\alpha|$) and define

$$\sqrt{\Lambda(t)} \equiv \sqrt{\frac{1}{2|\alpha|}} \left[ \dot{f} + 3(H + \xi h) f \right].$$

(3.15)

Then, eqs. (3.9) and (3.10) suggest that $\Lambda$ behaves as a cosmological constant ($\rho = -p$) given by

$$\Lambda = \frac{|\alpha| m^2}{3\xi^2},$$

(3.16)

and both $h$ and $f$ are constants being related by

$$3h_1^2 = \frac{m^2}{2} f_*^2.$$  

(3.17)

Eqs. (3.11) and (3.12) produce a relation which is satisfied by $\Lambda$ given above when the spinor contributions are neglected. The remaining job is to check whether the Hubble parameter $H$ obtained from (3.12) is consistent with (3.9). Equating both equations and using (3.17), we have

$$h_*^4 - \left(2 - \frac{1}{\xi^2}\right) \frac{\alpha m^2}{9\xi^2} h_*^2 + \frac{\alpha^2 m^4}{81\xi^4} = 0,$$

(3.18)

from which we have

$$h_*^2(\xi) = \frac{|\alpha| m^2}{18\xi^4} \left[ 1 - 2\xi^2 \pm \sqrt{1 - 4\xi^2} \right], \quad f_*(\xi) = \frac{\sqrt{|\alpha|}}{\sqrt{6} \xi^2} \left( 1 \pm \sqrt{1 - 4\xi^2} \right).$$

(3.19)

Therefore, $|\xi|^2 \leq 1/4$ and we have $f_*(\xi) \geq 0$. Note that the cosmological constant in (3.16) is directly proportional to mass squared of the vector field and diverges when $\xi \to 0$. This is to be compared with the massless case [27–30] where $\Lambda$ is a priori undetermined constant except its dependence on the gauge fixing parameter $\alpha$.

To proceed further, we first note that in the constant solution with $h = h_*$ and $f = f_*$ the spinor contribution and time dependence of each energy densities are neglected. To account for these, we consider an expansion in terms of negative powers of the scale factor around the solution (3.19) in the dark energy dominance epoch,

$$f = f_*(\xi) \left( 1 + \frac{f^{(3)}}{a^3} + \frac{f^{(6)}}{a^6} + \cdots \right), \quad h = h_*(\xi) \left( 1 + \frac{h^{(3)}}{a^3} + \frac{h^{(6)}}{a^6} + \cdots \right).$$

(3.20)
Then, we adopt the following systematic method: (i) Substitute the expressions (3.20) and (3.12) into (3.9) and (3.10) to find expansions of the energy density \( \rho = \rho^{(0)} + \rho^{(3)} + \rho^{(6)} + \cdots \) and similarly for pressure \( p \). (ii) Identify each term in the energy density and pressure with perfect fluid with barotropic equation of state and impose \( B/m \) expansion coefficient not included. Therefore, one can say that the expansions make sense as long as \( B/m \) terms are given by (\( \xi f > 1 \/ 2 \))

\[
f^{(3)} = \left( -\frac{\sqrt{3|\alpha|}}{2|\alpha| + 3\xi^2 f_s^2} + \frac{\sqrt{3|\alpha|}}{\sqrt{|\alpha|} f_s} \right) \frac{B}{m^2} , \tag{3.21}
\]

\[
f^{(6)} = C_f^{(1)} \frac{B}{m^2} f^{(3)} + C_f^{(2)} f^{(3)}^2 + \cdots , \tag{3.22}
\]

and

\[
h^{(3)} = f^{(3)} + \left( -\frac{3\xi^2 f_s}{2|\alpha| + 3\xi^2 f_s^2} \right) \frac{B}{m^2} , \tag{3.23}
\]

\[
h^{(6)} = f^{(6)} - \frac{1}{2} \left( h^{(3)} - f^{(3)} \right) \left( h^{(3)} - 3f^{(3)} \right) + \cdots , \tag{3.24}
\]

with

\[
C_f^{(1)} = \frac{3\xi^2 f_s}{2|\alpha|} \left( -\frac{2|\alpha| + \xi^2 f_s^2}{2|\alpha| + 3\xi^2 f_s^2} \right) \left( \frac{2\sqrt{6}|\alpha| - 3\xi^2 f_s}{2\sqrt{6}|\alpha| + \xi^2 f_s} \left( 8\sqrt{|\alpha|} - \sqrt{6}f_s \right) \right) , \tag{3.25}
\]

\[
C_f^{(2)} = \frac{3\xi^2 f_s}{|\alpha|} \left( \frac{2|\alpha| + \xi^2 f_s^2}{2|\alpha| + 3\xi^2 f_s^2} \right) \left( \frac{2\sqrt{6}|\alpha| - 3\xi^2 f_s}{2\sqrt{6}|\alpha| + \xi^2 f_s} \right) \left( 8\sqrt{|\alpha|} + 3\sqrt{6}f_s \right) , \tag{3.26}
\]

Putting these expressions into eqs. (3.9) and (3.10), we obtain the following evolution equations (keeping terms up to \( 1/a^6 \))

\[
3H^2 \simeq \frac{|\alpha|m^2}{3\xi^2} + \frac{m_f + \frac{4|\alpha| f_s}{2|\alpha| + 3\xi^2 f_s^2}}{a^2} B f_s f^{(3)} + \frac{2|\alpha| + 3\xi^2 f_s^2}{a^2} B f_s f^{(3)} , \tag{3.27}
\]

\[
-3H^2 - 2\dot{H} \simeq \frac{-|\alpha|m^2}{3\xi^2} + \frac{2|\alpha| + 3\xi^2 f_s^2}{2|\alpha| + 3\xi^2 f_s^2} B f_s f^{(3)} a^6 . \tag{3.28}
\]

We make a couple of comments regarding the above expansions. The first one is that all of the above expansions including energy density and pressure are given in powers of \( B/m^2 \), \( x^{(3n+3)} \sim \frac{3H}{m} \), \( x^{(3n)} = (f^{(3)} , h^{(3)} , \rho^{(3n)} , p^{(3n)}) \). This feature persists for the higher expansion coefficient not included. Therefore, one can say that the expansions make sense as long as \( B/m^2 << 1 \), and in this case, contributions from the higher order terms rapidly decay with the expansion of the Universe and can be neglected. Second one is that the evolution
equations mimic ΛCDM behavior up to $1/a^3$ term. Therefore, higher order terms starting from $1/a^6$ in eqs. (3.27) and (3.28) can be interpreted as a deviation from ΛCDM, and if they are very small, it may be possible that such deviation is compatible with observational data. In the next section, we perform data analysis in order to check this explicitly. For that purpose, we only keep expansions up to $1/a^6$.

Before passing, we make the following remark. The numerators in the previous expansions vanishes for $\xi = 1/2$, and this case must be treated separately. One can check that $A$ also has to be zero and the expansion coefficients $f^{(3n)}_i, h^{(3n)}_i (n = 1, 2 \cdots)$ cannot be determined uniquely, that is, they are arbitrary. This does not define a consistent expansion and this case is discarded. For general values of $\xi < 1/2$, $1/a^6$ term contributes a positive energy density for $m_f < \lesssim M_p$.

4 Data analysis

In this section we explore whether or not having additional energy density of the form $\rho_s = \rho_s \sim 1/a^6$ is consistent with current observations and if it is, what is the permissible density parameter $\Omega_s$. We use the recent observational data such as type Ia supernovae (SN), baryon acoustic oscillation (BAO) based on large-scale structure of galaxies, cosmic microwave background radiation (CMB), and Hubble constant. For spatially flat ΛCDM model with the additional energy density, the Friedmann equation is given by

$$\frac{H^2(z)}{H_0^2} = \Omega_r (1 + z)^4 + \Omega_m (1 + z)^3 + \Omega_\Lambda + \Omega_s (1 + z)^6,$$  

(4.1)

where $z \equiv a_0/a - 1$ is the redshift with $a(t)$ the cosmic expansion scale factor, $H \equiv \dot{a}/a$ is the Hubble parameter with $H_0$ its present value, and $\Omega_i$ with $i = r, m, \Lambda, s$ indicates the current density parameter for radiation, matter, cosmological constant, and the deviation, respectively. We assume that $\Lambda$ has relaxed to its constant value and include radiation component. In our model we have four free parameters, which are denoted as $\theta = (h, \Omega_b, \Omega_\Lambda, \Omega_s)$; $h$ is a normalized present-day Hubble parameter, $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$; $\Omega_b$ is the baryon density parameter; the density parameter of cold dark matter is given as $\Omega_c = 1 - \Omega_b - \Omega_\Lambda - \Omega_s$ and $\Omega_m = \Omega_b + \Omega_c$. During exploring the parameter space, we use $\log_{100} \Omega_s$ as the free parameter for the deviation with a prior $\log_{100} \Omega_s > -25$. The lower bound is chosen as the limit where models with and without the deviation cannot be distinguished within the current observational precision. To obtain the likelihood distributions for model parameters, we use the Markov chain Monte Carlo (MCMC) method based on Metropolis-Hastings algorithm to randomly explore the parameter space that is favored by observational data [40, 41]. The method needs to make decisions for accepting or rejecting a randomly chosen chain element via the probability function $P(\theta|D) \propto \exp(-\chi^2/2)$, where $D$ denotes the data, and $\chi^2 = \chi^2_{\text{SN}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}} + \chi^2_{\text{H}_0}$ is the sum of individual chi-squares for SN, BAO, CMB, and $H_0$ data (defined below). During the MCMC analysis, we use a simple diagnostic to test the convergence of MCMC chain: the means estimated from the first (after burning process) and

\[ \rho(3) = \left( \frac{m_f}{M_p} + \frac{4\xi_0^2}{3M^2} \right) M_p B. \]

For $B M^2_p < \lesssim 1, m_f < \lesssim M_p$, and $\mathcal{O}(\xi, \mathcal{O}(\xi)) \sim 1$, that we are interested in, $\rho(3) \sim M_p B = \frac{n}{M_p} m^2 M_p^2 \ll \Delta M_p^2$ cannot account for the whole of the dark matter density but only a very small fraction of it. Therefore, unknown dark matter sector has to be added separately in this approach and we assume that this is the case from here on.
the last 10% of the chain are approximately equal to each other if the chain has converged (see appendix B of ref. [42]).

For SN data set, the Union2.1 sample with 580 members is used [43]. We use the chi-square that is marginalized over the zero-point uncertainty due to absolute magnitude and Hubble constant [44, 45]:

\[ \chi_{\text{SN}}^2 = c_1 - c_2^2 / c_3, \] (4.2)

where

\[ c_1 = \sum_{i=1}^{580} \left[ \frac{\mu(z_i; \theta) - \mu_{\text{obs}}(z_i)}{\sigma_i} \right]^2, \quad c_2 = \sum_{i=1}^{580} \frac{\mu(z_i; \theta) - \mu_{\text{obs}}(z_i)}{\sigma_i^2}, \quad c_3 = \sum_{i=1}^{580} \frac{1}{\sigma_i^2}, \] (4.3)

\[ \mu_{\text{obs}}(z_i) \] and \( \sigma_i \) the distance modulus and its error of SN at redshift \( z_i \), respectively, \( \mu(z; \theta) = 5 \log[(1 + z)r(z)] \) the model distance modulus, and \( r(z) \) the comoving distance at redshift \( z \),

\[ r(z) = \frac{c}{H_0 \sqrt{\Omega_k}} \sin \left[ \sqrt{\Omega_k} \int_0^z \frac{H_0}{H(z')} \, dz' \right]. \] (4.4)

Here \( c \) is the speed of light, and \( \Omega_k \) is the current density parameter for spatial curvature (\( \Omega_k = 0 \) in this paper).

We use an effective distance measure which is related to the BAO scale [46],

\[ D_V(z) \equiv \left[ r^2(z) \frac{cz}{H(z)} \right]^{\frac{1}{3}} \] (4.5)

and a fitting formula for the redshift of drag epoch \((z_d)\) [47]:

\[ z_d = \frac{1291 (\Omega_m h^2)^{0.251}}{1 + 0.659 (\Omega_m h^2)^{0.828}} \left[ 1 + b_1 (\Omega_b h^2)^{b_2} \right], \] (4.6)

where

\[ b_1 = 0.313 (\Omega_m h^2)^{-0.419} \left[ 1 + 0.607 (\Omega_m h^2)^{0.674} \right], \quad b_2 = 0.238 (\Omega_m h^2)^{0.223}. \] (4.7)

As the BAO parameter, we use the six numbers of \( r_s(z_d)/D_V(z) \) extracted from the Six-Degree-Field Galaxy Survey [48], the Sloan Digital Sky Survey Data Release 7 and 9 [49, 50], and the WiggleZ Dark Energy Survey [51], where \( r_s(z) \) is the comoving sound horizon size. These BAO data points were used in the WMAP 9-year analysis [52]. Since the sound speed of baryon fluid coupled with photons \((\gamma)\) is given as

\[ c_s^2 = \frac{\dot{\rho}_s}{\dot{\rho}_s} = \frac{1}{3} \frac{\dot{\rho}_s + \dot{\rho}_b}{\dot{\rho}_s + \dot{\rho}_b} = \frac{1}{3} \left[ 1 + (3\Omega_b/4\Omega_\gamma) a \right], \] (4.8)

the comoving sound horizon size before the last scattering becomes

\[ r_s(z) = \int_0^t c_s dt' / a = \frac{1}{a} \sqrt{3} \int_0^{1/(1+z)} \frac{da}{{a^2 H(a)[1 + (3\Omega_b/4\Omega_\gamma) a]^{\frac{1}{2}}}}, \] (4.9)
where \( \Omega_\gamma = 2.469 \times 10^{-5} h^{-2} \) for the present CMB temperature \( T_{\text{cmb}} = 2.725 \) K. The BAO data points together with an inverse covariance matrix between measurement errors can be written in vector and matrix forms as \([52]\)

\[
d_{\text{BAO}} = \begin{pmatrix} r_s(z_d)/D_V(z = 0.1) \\ D_V(z = 0.35)/r_s(z_d) \\ D_V(z = 0.57)/r_s(z_d) \\ r_s(z_d)/D_V(z = 0.44) \\ r_s(z_d)/D_V(z = 0.60) \\ r_s(z_d)/D_V(z = 0.73) \end{pmatrix}
= \begin{pmatrix} 0.336 \\ 8.88 \\ 13.67 \\ 0.0916 \\ 0.0726 \\ 0.0592 \end{pmatrix},
\quad (4.10)
\]

\[
C_{\text{BAO}}^{-1} = \begin{pmatrix} 4444.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 34.602 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20.661157 & 0 & 0 & 0 \\ 0 & 0 & 0 & 24532.1 & -25137.7 & 12099.1 \\ 0 & 0 & 0 & -25137.7 & 134598.4 & -64783.9 \\ 0 & 0 & 0 & 12099.1 & -64783.9 & 128837.6 \end{pmatrix}.
\quad (4.11)
\]

The chi-square is given as \( \chi^2_{\text{BAO}} = d_{\text{BAO}}^T C_{\text{BAO}}^{-1} d_{\text{BAO}} \).

We also use the CMB distance priors based on WMAP 9-year data for testing our model (see ref. \([52]\) for the detailed description). The first distance measure is the acoustic scale \( l_A \) defined as

\[
l_A = \pi \frac{r(z_*)}{r_s(z_*)}.
\quad (4.12)
\]

The decoupling epoch \( z_\ast \) can be calculated from the fitting function \([53]\):

\[
z_\ast = 1048 \left[ 1 + 0.00124 (\Omega_b h^2)^{-0.738} \right] \left[ 1 + g_1 (\Omega_m h^2)^{g_2} \right]
\quad (4.13)
\]

where

\[
g_1 = \frac{0.0783 (\Omega_b h^2)^{-0.238}}{1 + 39.5 (\Omega_b h^2)^{0.763}},
\quad g_2 = \frac{0.560}{1 + 21.1 (\Omega_b h^2)^{1.81}}.
\quad (4.14)
\]

The second distance measure is the shift parameter \( R \) which is given by

\[
R(z_\ast) = \frac{\sqrt{\Omega_m H_0^2}}{c} r(z_\ast).
\quad (4.15)
\]

According to WMAP 9-year observations \([52]\), the estimated values and inverse covariance for the three parameters \((l_A, R, \text{ and } z_\ast)\) are given as

\[
d_{\text{CMB}} = \begin{pmatrix} l_A(z_\ast) \\ R(z_\ast) \\ z_\ast \end{pmatrix}
= \begin{pmatrix} 302.40 \\ 1.7246 \\ 1090.88 \end{pmatrix},
\quad C_{\text{CMB}}^{-1} = \begin{pmatrix} 3.182 & 18.253 & -1.429 \\ 18.253 & 11887.879 & -193.808 \\ -1.429 & -193.808 & 4.556 \end{pmatrix}.
\quad (4.16)
\]

The chi-square is given as \( \chi^2_{\text{CMB}} = d_{\text{CMB}}^T C_{\text{CMB}}^{-1} d_{\text{CMB}} \).

In order to impose a tight constraint on the Hubble constant parameter, we apply a Gaussian prior on the Hubble constant, \( H_0 = 73.8 \pm 2.4 \) km s\(^{-1}\) Mpc\(^{-1}\) (68% CL) \([54]\). The chi-square is given as \( \chi^2_{H_0} = [(H_0 - 73.8)/2.4]^2 \).

Figure 1 shows one- and two-dimensional likelihood distributions of model parameters in the flat \( \Lambda \)CDM model with the deviation. Using the MCMC method, we constrain
our model parameters $\theta = (h, \Omega_b, \Omega_A, \Omega_s)$ with the combined data sets. The results are shown as green contours for SN+CMB+$H_0$ data sets and red for SN+CMB+BAO+$H_0$. As expected, the joint analysis with all combination of data sets gives tighter constraints on parameters. To be consistent with current observations, the amount of the additional energy density should be small with $\Omega_s \lesssim 10^{-13}$. In the $h$- and $\Omega_b$-likelihood distributions, the deviation model (red) appears to favor the parameter regions similar to those of $\Lambda$CDM model (gray curves). However, we note that the model favors regions of larger Hubble constant and smaller baryon density as the amount of the deviation increases. Interestingly, the current observations prefer the deviation with positive energy density with $\Omega_s \sim 10^{-14}$, with a minimum chi-square smaller than the $\Lambda$CDM one, and the preference for positive deviation is amplified by adding BAO data points (see bottom-right panel of figure 1). For SN+CMB+BAO+$H_0$ data sets, the best-fit locations in the parameter space are $\theta = (h, \Omega_b, \Omega_A, \Omega_s) = (0.738, 0.0433, 0.708, 2.36 \times 10^{-14})$ with $\chi^2_{\text{min}} = 567.96$ (minimum chi-square) for the deviation model, and $\theta = (0.692, 0.0472, 0.711, 0)$ with $\chi^2_{\text{min}} = 572.31$ for $\Lambda$CDM model, respectively. We also present the likelihood results for the deviation model with a fixed value of $\Omega_s = 10^{-14}$ (yellow), which shows different $h$-$\Omega_b$ likelihood, compared with the case of freely varying $\Omega_s$ (red contours). As stated above, fixing the deviation component with $\log_{10} \Omega_s = -14$ favors Hubble constant (baryonic matter density) that is slightly larger (smaller) by the amount of about 5% than $\Lambda$CDM value (yellow and gray curves).

Very recently, the CMB data of Planck satellite has been publicly available together with a new result for cosmological parameter estimation [55, 56]. Since the cosmological parameters used in this paper are consistent with the Planck results within the observational precision, we expect that our constraint on the positive deviation will not change much even with the new observational data. We defer a more detailed analysis using Planck data for future research.

5 Conclusion and discussions

In this paper, we investigated cosmology of massive spinor electrodynamics with torsion which is non-minimally coupled with the vector field. The massive vector field and torsion cooperate together to generate dark energy and dark matter. Depending on the values of the vector-torsion coupling constant, the cosmology reveals several novel aspects which are not present in the massless spinor electrodynamics alone.

The analysis shows that the theory effectively provides dark energy and pressureless dark matter with additional energy density of the form $\rho_s \sim 1/a^6$. Comparisons with observations show that its existence is consistent, even though only a very small portion of energy density $\Omega_s \sim 10^{-14}$ is allowed. If we apply this data analysis to (3.27), we see that $B/m^2 \sim 10^{-7}$ for $O(\xi), O(|\alpha|) \sim 1$. Perhaps, the importance of the deviation lies not in the magnitude of its portion but in the possibility that there could be other type of component in the current Universe.

The restriction $|\xi| < 1/2$ and dependence of cosmology on $\xi$ are hard to be understood intuitively. But $\xi$ can be introduced in a different context. If we consider a different model where the trace of the torsion field $S_\mu$ is replaced with another massive vector field $B_\mu$ along with its field strength, then, in eq. (2.8) $\xi$ can be interpreted as a coupling constant between the two vectors $A_\mu$ and $B_\mu$. The ensuing cosmology will show the same qualitative behavior, because the field strength of $B_\mu$ is zero for isotropic cosmology, and it can duplicate the cosmological behavior of $S_\mu$. But a new restriction on $\xi$ now depending on the mass of $B_\mu$.
will emerge and it would be worthwhile to explore this aspect in detail in connection with a possible meaning of $\xi$ in cosmology.

In eq. (3.27), the dark energy contribution comes from the massive vector field and torsion whereas dark matter also contains a contribution from the vector-spinor interaction term. Therefore, cosmology of massive spinor electrodynamics effectively provides an interacting dark energy and dark matter model whose interaction is via the well-known standard vector-spinor interaction. It would be interesting to compare with other models in the literature [58] and to explore whether massive spinor electrodynamics can be another but a more realistic field theoretical model of interacting dark energy.
We only considered non-minimal torsion coupling with the vector field. There exist a couple of other sources of non-minimal couplings and extensions one could associate with massive spinor electrodynamics. The first one is to include direct interactions between the vector field and curvatures \[59\]. The other is to assume non-minimal couplings of torsion with fermionic sector \[60\] and non-vanishing of the totally anti-symmetric torsion components. Taking these into account may open up new possibilities for cosmology.

It remains to be seen whether the cosmology of massive spinor electrodynamics discussed in this paper can provide a viable description of dark energy of current Universe. But if this is possible, then, our analysis predicts some deviation from ΛCDM with additional contributions of the energy density and pressure, \(\rho_s = p_s \sim 1/a^6\) given in eqs. (3.27) and (3.28), even though they occupy only a very small fraction of our Universe. But at least, its existence is favored with a slightly better statistical probability than ΛCDM as is discussed in section 4.

The last discussion concerns the expression of dark energy in terms of mass of the vector field given by eq. (3.16). With a choice of negative \(\alpha\), the dark energy density which is responsible for a repulsive force is proportional to mass squared of the vector field. This is attributed to an existence of the cosmological solution associated with classical background configuration \(A = (f_*,0,0,0), S_\mu = (h_*,0,0,0),\) and \(\bar{\psi} = \psi = 0\), in which the specific non-minimal coupling effectively transforms cosmological term of (3.15) into (3.16) with the relation (3.17) being satisfied. For values like \(\mathcal{O}(|\alpha|) \sim \mathcal{O}(\xi) \sim 1, m \sim 10^{-61} M_p\) can yield the current dark energy density, and it is within the allowed range of the photon mass limit \(\mu \lesssim 10^{-62} \text{kg} \quad [61, 62]\). It can be made more flexible with adjustments of the parameters \(\alpha\) and \(\xi\), in which case it can encompass the massive dark photon \[63\]. Therefore, massive (dark) QED can be a realistic model for the dark energy and if this is the case, our result is suggesting that nonvanishing mass of the photon or dark photon could be responsible for the dominant component of our Universe.

Acknowledgments

We would like to thank anonymous referee for valuable suggestions, and Young-Hwan Hyun, Joohan Lee, Tae Hoon Lee, Seokcheon Lee, Taeyoon Moon, and Wanil Park for useful discussions. P. O. was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by Ministry of Education, Science and Technology (2010-0021996) and by a NRF grant funded by the Korean government (MEST) through the Center for Quantum Spacetime (CQUeST) of Sogang University with Grant No. 2005-0049409. C.G.P. was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT and Future Planning (No. 2013R1A1A1011107) and was partly supported by research funds of Chonbuk National University in 2012.

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