Study of Hadronic Five-Body Decays of Charmed Mesons Involving $K_S^0$

The FOCUS Collaboration

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Preprint submitted to Elsevier Preprint 25 March 2022
Abstract

We study the decay of $D^0$ and $D_s^+$ mesons into five-body final states including a $K_S^0$ and report the discovery of the decay mode $D_s^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^+ \pi^-$. The branching ratio for the new mode is $\frac{\Gamma(D_s^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^+ \pi^-)}{\Gamma(D_s^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^-)} = 0.102 \pm 0.029 \pm 0.029$. We also determine the branching ratio of $\frac{\Gamma(D^0 \rightarrow K_S^0 \pi^+ \pi^+ \pi^-)}{\Gamma(D^0 \rightarrow K_S^0 \pi^+ \pi^-)} = 0.095 \pm 0.005 \pm 0.007$ as well as an upper limit for $\frac{\Gamma(D^0 \rightarrow K_S^0 K^- \pi^+ \pi^- \pi^+ \pi^-)}{\Gamma(D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^+ \pi^-)} < 0.054$ (90% CL). An analysis of the resonant substructure for $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^+ \pi^-$ is also performed.

PACS numbers: 13.25.Ft, 14.40Lb

More information on multibody final states in the charm sector is an essential ingredient for our ability to model decay rates and to further increase our understanding of the decay process in heavy quark systems. This is particularly important for the $D_s^+$ decays where a substantial part of its hadronic decay rate is still not identified. In this paper we extend our work [1] on four-body decays involving a $K_S^0$ to five-body decays involving a $K_S^0$. We have already published results on all charged five-body modes [2]. The FOCUS collaboration presents the first evidence of the decay mode $D_s^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^+ \pi^-$, measures an inclusive branching ratio for the mode $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^+ \pi^-$ relative to $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and places an upper limit on the mode $D^0 \rightarrow K_S^0 K^- \pi^+ \pi^-$. 

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Finally we present the first resonant substructure analysis of the decay mode $D^0 \rightarrow K^0_S \pi^+ \pi^+ \pi^- \pi^-$. The data were collected during the 1996-1997 fixed target run at Fermilab. Bremsstrahlung of electrons and photons with an endpoint energy of approximately 300 GeV produces photons which interact in a segmented beryllium-oxide target to produce charmed particles. The average photon energy for events which satisfy our trigger is $\approx 180$ GeV. Charged decay products are momentum analyzed by two oppositely polarized dipole magnets. Tracking is performed by a system of silicon vertex detectors [3] in the target region and by multi-wire proportional chambers downstream of the interaction. Particle identification is performed by three threshold Čerenkov counters, two electromagnetic calorimeters, a hadronic calorimeter, and two muon systems.

Five-body $D^0$ and $D^+_s$ decays are reconstructed using a candidate driven vertex algorithm [4]. A decay vertex is formed from the reconstructed charged tracks. The $K^0_S$ is also reconstructed using techniques described elsewhere [5]. The momentum information from the $K^0_S$ and the charged tracks is used to form a candidate $D$ momentum vector, which is intersected with other tracks to find the production vertex. Events are selected based on several criteria. The confidence level for the production vertex and for the charm decay vertex must be greater than 1%. The reconstructed mass of the $K^0_S$ must be within four standard deviations of the nominal $K^0_S$ mass. The likelihood for each charged particle to be a proton, kaon, pion, or electron based on Čerenkov particle identification is used to make additional requirements [6]. For pion candidates we require a loose cut that no alternative hypothesis is favored over the pion hypothesis by more than 6 units of log-likelihood. In addition, for each kaon candidate we require the negative log-likelihood kaon hypothesis, $W_K = -2 \ln(\text{kaon likelihood})$, to be favored over the corresponding pion hypothesis $W_\pi$ by $W_\pi - W_K > 2$. We also require the distance between the primary and secondary vertices divided by its error to be at least 10. Finally, in order to reduce background due to secondary interactions of particles from the production vertex, we require the secondary vertex to be located outside the target material.

For individual modes we apply additional analysis cuts. Due to the large combinatoric background for $D^0 \rightarrow K^0_S \pi^+ \pi^+ \pi^- \pi^-$, we increase the separation requirement of the secondary vertex from being just outside the target material to two standard deviations from the edge of the target material. Figure 1(a) shows the $K^0_S \pi^+ \pi^+ \pi^- \pi^-$ invariant mass plot for events that satisfy these cuts. The distribution is fitted with a Gaussian for the $D^0$ signal (1283±57 events) with the width and mass floated and a first degree polynomial for the background. Figure 1(b) shows the $K^0_S \pi^+ \pi^+ \pi^- \pi^-$ invariant mass plot for events originating from a $D^{*+} \rightarrow D^0 \pi^+$ decay.
Fig. 1. Invariant mass distributions for (a) $K^0_S \pi^+ \pi^- \pi^- \pi^-$, (b) $K^0_S \pi^+ \pi^- \pi^- \pi^-$ for $D^*$ tagged events, (c) $K^0_S K^0_S \pi^+ \pi^- \pi^\pm$, and (d) $K^0_S K^- \pi^- \pi^+ \pi^-$. The fits are described in the text.

The $D_s^+ \rightarrow K^0_S K^0_S \pi^+ \pi^- \pi^-$ mode is difficult to detect due to the relative inefficiency of $K^0_S$ reconstruction and that most of the time only the three pions define the secondary vertex. The confidence level that a pion track from the decay vertex intersects the production vertex must be less than 2%. We also require a reconstructed $D_s^+$ momentum of greater than 25 GeV/c. Figure 1(c) shows the $K^0_S K^0_S \pi^+ \pi^- \pi^-$ mass plot for events which satisfy these cuts. This is the first observation of this mode. We fit with a Gaussian (37±10 events) with mass and width allowed to float and a second degree polynomial for the background.

The decay $D^0 \rightarrow K^0_S K^- \pi^+ \pi^- \pi^-$ is Cabibbo suppressed, and we do not observe a signal in this mode. Thus we choose our analysis cuts by maximizing the quantity $S/\sqrt{B}$, where $S$ is the fitted yield from our Monte Carlo simulation of the mode, and $B$ is the number of background events in the signal region from data. Based on this optimization we require a reconstructed $D^0$ momentum of greater than 50 GeV/c. We also require the $D^0$ come from a $D^{*+}$ decay, that is $0.142 \text{ GeV/c}^2 < M_{D^{*+}} - M_D^0 < 0.149 \text{ GeV/c}^2$. Figure 1(d) shows the
resulting in $K^0_SK^-\pi^+\pi^+\pi^-$ invariant mass plot. As there is no apparent signal we report an upper limit branching ratio.

Table 1
Branching ratios for modes involving a $K^0_S$. All branching ratios are inclusive of subresonant modes.

| Decay Mode | Branching Ratio |
|------------|-----------------|
| $\Gamma(D^0\rightarrow K^0_SK^-\pi^+\pi^-\pi^-)$ | $0.095\pm0.005\pm0.007$ |
| $\Gamma(D^+\rightarrow K^0_SK^0\pi^+\pi^-\pi^-)$ | $0.102\pm0.029\pm0.029$ |
| $\Gamma(D^0\rightarrow K^0_SK^-\pi^+\pi^-\pi^-)$ | $<0.054$ (90% C.L.) |

We measure the branching fraction of the $D^0 \rightarrow K^0_S\pi^+\pi^+\pi^-\pi^-$ mode relative to $D^0 \rightarrow K^0_S\pi^+\pi^-$. The relative efficiency is determined by Monte Carlo simulation. The $K^0_S\pi^+\pi^-$ and $K^0_S\pi^+\pi^-\pi^-\pi^-$ channels are produced as an incoherent mixture of subresonant decays based on PDG information [7] and our analysis described below, respectively. We measure the $D^+_s \rightarrow K^0_SK^0\pi^+\pi^-\pi^-$ mode relative to $D^+_s \rightarrow K^0_SK^-\pi^+\pi^+$. We test for dependency on cut selection in both modes by individually varying each cut. The results are shown in Table 1, and we compare our measurement of the $D^0 \rightarrow K^0_S\pi^+\pi^+\pi^-\pi^-$ branching ratio with previous measurements in Table 2.

We studied systematic effects due to uncertainties in the reconstruction efficiency, in the unknown resonant substructure, and on the fitting procedure. To determine the systematic error due to the reconstruction efficiency we follow a procedure based on the S-factor method used by the Particle Data Group [7]. For each mode we split the data sample into four independent subsamples based on $D$ momentum and on the period of time in which the data was collected. These splits provide a check on the Monte Carlo simulation of charm production, of the vertex detector (it changed during the course of the run), and on the simulation of the detector stability. We then define the split sample variance as the difference between the scaled variance and the statistical variance if the former exceeds the latter. The method is described in detail in reference [11]. In addition, we split the data sample into three independent subsamples based on the location and geometry of the $K^0_S$ decay. We then calculate the $K^0_S$ reconstruction variance using the same procedure described for the split sample variance. We also vary the subresonant states in the Monte Carlo and use the variance in the branching ratios as a contribution to the systematic error. We also determine the systematic effects based on different fitting procedures. The branching ratios are evaluated under various fit conditions, and the variance of the results is used as an additional systematic error. Finally, we evaluate systematic effects from uncertainty in the absolute tracking efficiency of multi-body decays using studies of $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ and $D^0 \rightarrow K^-\pi^+$ decays. The systematic effects are then all added in quadrature to obtain the final systematic error.
Table 2
Comparison of this measurement of $D^0 \rightarrow K^0_S \pi^+ \pi^+ \pi^- \pi^-$ mode to previous measurements.

| Experiment            | Events | $\frac{\Gamma(D^0 \rightarrow K^0_S \pi^+ \pi^+ \pi^- \pi^-)}{\Gamma(D^0 \rightarrow K^0_S \pi^+ \pi^-)}$ |
|-----------------------|--------|---------------------------------------------------------------------|
| E831 (This Measurement)| 1283   | 0.095±0.005±0.007                                                   |
| PDG Average[7]        |        | 0.107±0.029                                                         |
| ARGUS[8]              | 11     | 0.07±0.02±0.01                                                     |
| CLEO[9]               | 56     | 0.149±0.026                                                         |
| E691[10]              | 6      | 0.18±0.07±0.04                                                     |

We do not observe a signal in the decay $D^0 \rightarrow K^0_S K^- \pi^+ \pi^- \pi^-$ and we calculate an upper limit for the branching ratio with respect to $D^0 \rightarrow K^0_S \pi^+ \pi^- \pi^-$. We evaluate the upper limit using the method of Rolke and Lopez [12]. We define the signal region as being within $\pm 2\sigma$ of the nominal $D^0$ mass, and the two sideband regions as $4-8\sigma$ above and below the $D^0$ mass. We observe 3 events in the signal region and 6 events in the sidebands, corresponding to an upper limit of 5.02 events (@90% CL).

We study systematic effects for this channel from cut variation and resonant substructure, and include these in our determination of the upper limit using the method of Cousins and Highland [13]. We determine the systematic error from cut variation by individually varying each cut, fitting the resulting distribution, and taking the variance between each branching ratio measurement as our systematic error. We also study systematic effects from our uncertainty in the resonant substructure of the mode by varying the subresonant states included in the Monte Carlo simulation, and used the variance in the resulting branching ratios as our systematic error. These two systematic effects are then added in quadrature to give a final relative systematic error of 26%.

We then determine the increase in our upper limit based on the equation:

$$\Delta U = \frac{1}{2} U^2 \sigma_{sys}^2 \frac{U + b - s}{U + b}$$

where $U$ is the original upper limit of events, $\sigma_{sys}$ is the percent systematic error determined above, $b$ is the number of events observed in the sideband region, and $s$ is the number of signal events observed. We calculate an upper limit of 5.64 events, corresponding to an upper limit for the branching ratio of:

$$\frac{\Gamma(D^0 \rightarrow K^0_S K^- \pi^+ \pi^- \pi^-)}{\Gamma(D^0 \rightarrow K^0_S \pi^+ \pi^- \pi^-)} < 0.054 \text{ (@90% CL).}$$

We have studied the resonance substructure in the decay $D^0 \rightarrow K^0_S \pi^+ \pi^+ \pi^- \pi^-$. We use an incoherent binned fit method [14] developed by the E687 Collabo-
ration which assumes the final state is an incoherent superposition of subresonant decay modes containing vector resonances. A coherent analysis would be difficult given our limited statistics. For subresonant decay modes we consider the lowest mass \( K^0_S \pi^- \) and \((\pi^+\pi^-)\) resonances, as well as a nonresonant channel: \( K^*\pi^+\pi^- \), \( K^0_S \rho^0\pi^- \), \( K^*\rho^+ \) and \( (K^0_S \pi^+\pi^-\pi^-)^{NR} \). All states not explicitly considered are assumed to be included in the nonresonant channel.

For the resonant substructure analysis of \( D^0 \to K^0_S \pi^+\pi^-\pi^-\pi^- \) we place additional cuts to enhance the signal to background ratio. We require the confidence level that a track from the decay vertex intersects the production vertex be less than 8%. We also require the \( D^0 \) to come from a \( D^{*+} \) decay, that is \( 0.144 \text{ GeV}/c^2 < M_{D^{*+}} - M_{D^0} < 0.148 \text{ GeV}/c^2 \), in order to reduce background and distinguish between \( D^0 \) and \( \overline{D}^0 \). Fig. 1(b) shows the \( K^0_S \pi^+\pi^-\pi^-\pi^- \) invariant mass plot for events which satisfy these cuts. We then determine the acceptance corrected yield into each subresonant mode using a weighting technique whereby each event is weighted by its kinematic values in three submasses: \( (K^0_S \pi^-) \), \( (\pi^+\pi^-) \), and \( (\pi^+\pi^+) \). No resonance in the \( (\pi^+\pi^+) \) submass exists, but we include it in order to compute a meaningful \( \chi^2 \) estimate of the fit.

Eight population bins are constructed depending on whether each of the three submasses falls within the expected resonance (in the case of \( \pi^+\pi^+ \), the bin is split into high and low mass regions). For each Monte Carlo simulation the bin population, \( n_i \), in the eight bins is determined and a matrix, \( T_{i\alpha} \), is calculated between the generated states, \( \alpha \), Monte Carlo yields, \( Y_\alpha \), and the eight bins \( i \):

\[
n_i = \sum_\alpha T_{i\alpha} Y_\alpha .
\]

The elements of the matrix, \( T \), can be summed to give the efficiency for each mode, \( \epsilon_\alpha \):

\[
\epsilon_\alpha = \sum_i T_{i\alpha} .
\]

The Monte Carlo determined matrix is inverted to create a new weighting matrix which multiplies the bin populations to produce efficiency corrected yields. The weight includes the contributions from the four combinations we have for each event. Each data event can then be weighted according to its values in the submass bins. Once the weighted distributions for each of the four modes are generated, we determine the acceptance corrected yield by fitting the distributions with a Gaussian signal and a linear background. Using incoherent Monte Carlo mixtures of the four subresonant modes we verify that our procedure is able to correctly recover the generated mixtures of the four modes.

The results for \( K^0_S \pi^+\pi^-\pi^-\pi^- \) are summarized in Table 3. The four weighted histograms with fits are shown in Fig. 2, where Fig. 2(e) is the weighted distribution for the sum of all subresonant modes. The goodness of fit is evaluated
Table 3
Fractions relative to the inclusive mode for the resonance substructure of the $D^0 \rightarrow K^0_S\pi^+\pi^+\pi^-\pi^-$ decay mode. These values are not corrected for unseen decay modes.

| Subresonant Mode | Fraction of $K^0_S\pi^+\pi^-\pi^+\pi^-$ |
|------------------|------------------------------------------|
| $(K^0_S\pi^+\pi^+\pi^-\pi^-)_{NR}$ | < 0.46 @90% CL |
| $K^*-\pi^+\pi^-\pi^-$ | 0.17±0.28±0.02 |
| $K^0_S\rho^0\pi^+\pi^-$ | 0.40±0.24±0.07 |
| $K^*-\rho^0\pi^+$ | 0.60±0.21±0.09 |

Fig. 2. $K^0_S\pi^+\pi^+\pi^-\pi^-$ weighted invariant mass for (a) $(K^0_S\pi^+\pi^+\pi^-\pi^-)_{NR}$, (b) $K^*-\pi^+\pi^-\pi^-$, (c) $K^0_S\rho^0\pi^+\pi^-$, (d) $K^*-\rho^0\pi^+$, (e) Inclusive sum of all four modes.

by calculating a $\chi^2$ for the hypothesis of consistency between the model predictions and observed data yields in each of the 8 submass bins. The calculated $\chi^2$ is 9.7 (4 degrees of freedom), with most of the $\chi^2$ contribution resulting from a poor Monte Carlo simulation of the $\pi^+\pi^+$ spectrum in the nonresonant channel.

We observe results similar to previous studies of five-body charm decays, with a small nonresonant component and the dominant mode of the form vector-
vector-pseudoscalar. Such a result has been predicted by theoretical discussion of a vector-dominance model for heavy flavor decays [15], which suggests that charm decays are dominated by quasi-two-body decays in which the $W^\pm$ immediately hadronizes into a charged pseudoscalar, vector or axial vector meson. Results consistent with the vector-dominance model have already been seen by FOCUS in five-body decays [2]. Such theoretical discussion raises the possibility that the resonant substructure for the decay $D^0 \to K^0_S \pi^+ \pi^+ \pi^- \pi^-$ is dominated by the quasi-two-body decay $K^*_0 - a_1^+$. To test this hypothesis we generate Monte Carlo simulations of this decay, assuming the $a_1^+$ has a width of 400 MeV/$c^2$ and decays entirely as an S-wave to $\rho^0 \pi^+$, and use our subresonant analysis procedure explained above. We observe yield fractions in each of the subresonant modes similar to the reported fractions from the data, suggesting our results are consistent with the decay being dominated by the $K^*_0 - a_1^+$ subresonant state.

In conclusion we have measured the relative branching ratios of many-body hadronic modes of $D^0$ and $D^+_s$ involving a $K^0_S$ decay and have presented the first evidence of the decay mode $D^+_s \to K^0_SK^0_S \pi^+ \pi^-$. We have also performed an analysis of the resonant substructure of the decay $D^0 \to K^0_S \pi^+ \pi^+ \pi^- \pi^-$. Finally we have placed an upper limit on the relative branching fraction of the Cabibbo suppressed decay $D^0 \to K^0_S K^- \pi^+ \pi^- \pi^-$. We acknowledge the assistance of the staffs of Fermi National Accelerator Laboratory, the INFN of Italy, and the physics departments of the collaborating institutions. This research was supported in part by the U. S. National Science Foundation, the U. S. Department of Energy, the Italian Istituto Nazionale di Fisica Nucleare and Ministero della Istruzione, Università e Ricerca, the Brazilian Conselho Nacional de Desenvolvimento Científico e Tecnológico, CONACyT-México, and the Korea Research Foundation of the Korean Ministry of Education.

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