Ants: Mobile Finite State Machines

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Abstract

Consider the Ants Nearby Treasure Search (ANTS) problem introduced by Feinerman, Korman, Lotker, and Sereni (PODC 2012), where \( n \) mobile agents, initially placed at the origin of an infinite grid, collaboratively search for an adversarially hidden treasure. In this paper, the model of Feinerman et al. is adapted such that the agents are controlled by a (randomized) finite state machine: they possess a constant-size memory and are able to communicate with each other through constant-size messages. Despite the restriction to constant-size memory, we show that their collaborative performance remains the same by presenting a distributed algorithm that matches a lower bound established by Feinerman et al. on the run-time of any ANTS algorithm.

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1 Introduction

"They operate without any central control. Their collective behavior arises from local interactions." The last quote is arguably the mantra of distributed computing, however, in this case, “they” are not nodes in a distributed system; rather, this quote is taken from a biology paper that studies social insect colonies [15]. Understanding the behavior of insect colonies from a distributed computing perspective will hopefully prove to be a big step for both disciplines.

In this paper, we study the process of food finding and gathering by ant colonies from a distributed computing point of view. Inspired by the model of Feinerman et al. [11], we consider a colony of \( n \) ants whose nest is located at the origin of an infinite grid that collaboratively search for an adversarially hidden food source. An ant can move between neighboring grid cells in a single time unit and can communicate with the ants that share the same grid cell. However, the ant’s navigation and communication capabilities are very limited since its actions are controlled by a randomized finite state machine (FSM) — refer to Section 1.2 for a formal definition of our model. Nevertheless, we design a distributed algorithm ensuring that the ants locate the food source within \( O(D + D^2/n) \) time units w.h.p., where \( D \) denotes the distance between the food source and the nest. It is not difficult to show that a matching lower bound holds even under the assumptions that the ants have unbounded memory (i.e., are controlled by a Turing machine) and that the ants know the parameter \( n \).

1.1 Related Work

Our work is strongly inspired by Feinerman et al. [11, 10] who introduce the aforementioned problem called ants nearby treasure search (ANTS) and study it assuming that the ants (a.k.a. agents) are controlled by a Turing machine (with or without space bounds) and that communication is allowed only in the nest. They show that if the \( n \) agents know a constant approximation of \( n \), then they can find the food source (a.k.a. treasure) in time \( O(D + D^2/n) \). Moreover, Feinerman et al. observe a matching lower bound and prove that this lower bound cannot be matched without some knowledge of \( n \). In contrast to the model studied in [11, 10], the agents in our model can communicate anywhere on the grid as long as they share the same grid cell. However, due to their weak control unit (a FSM), their communication capabilities are very limited even when they do share the same grid cell (see Section 1.2). Notice that the stronger computational model assumed by Feinerman et al. enables an individual agent in their setting to perform tasks way beyond the capabilities of a (single) agent in our setting, e.g., list the grid cells it has already visited or perform spiral searches (that play a major role in their upper bound).

Distributed computing by finite state machines has been studied in several different contexts including population protocols [4, 5] and the recent work of Emek and Wattenhofer [9] from which we borrowed the agents communication model (see Section 1.2). In that regard, the line of work closest to our paper is probably the one studying graph exploration by FSM controlled agents, see, e.g., [12].

Graph exploration in general is a fundamental problem in computer science. In the typical case, the goal is for a single agent to visit all nodes in a given graph. As an example, the exploration of trees was studied in [8], the exploration of finite undirected graphs was studied in [14, 16], and the exploration of strongly connected digraphs was studied in [7, 1]. When a

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1 We say that an event occurs with high probability, abbreviated by w.h.p., if the event occurs with probability at least \( 1 - n^{-c} \), where \( c \) is an arbitrarily large constant.
A deterministic agent is exploring a graph, memory usage becomes an issue. With randomized agents, it is well-known that random walks allow a single agent to visit all nodes of a finite undirected graph in polynomial time [2]. The speed up gained from using multiple random walks was studied by Alon et al. [3]. Notice that in an infinite grid, the expected time it takes for a random walk to reach any designated cell is infinite.

Finding treasures in unknown locations has been previously studied, for example, in the context of the classic cow-path problem. In the typical setup, the goal is to locate a treasure on a line as quickly as possible and the performance is measured as a function of the distance to the treasure. It has been shown that there is a deterministic algorithm with a competitive ratio 9 and that a spiral search algorithm is close to optimal in the 2-dimensional case [6]. The study of the cow-path problem was extended to the case of multiple agents by López-Ortiz and Sweet [13]. In their study, the agents are assumed to have unique identifiers, whereas our agents cannot be distinguished from each other (at least not at the beginning of the execution).

1.2 Model

We consider a variant of [11]’s ANTS problem, where \( n \) mobile agents are searching for an adversarially hidden treasure in the infinite grid \( \mathbb{Z}^2 \). The execution progresses in synchronous rounds, where each round lasts 1 time unit. At time 0 (the beginning of the execution), all agents are positioned in a designated grid cell referred to as the origin (say, the cell with coordinates \((0,0) \in \mathbb{Z}^2\)). We assume that the agents can distinguish between the origin and the other cells.

The main difference between our variation of the ANTS model and the original one lies in the agents’ computation and communication capabilities. In both variants, all agents run the same (randomized) protocol, however, under the model considered in the current paper, the agents are controlled by a randomized finite state machine (FSM). This means that the individual agent has a constant memory and thus, in general, cannot store its coordinates in \( \mathbb{Z}^2 \). On the other hand, in contrast to the model considered in [11], the communication of our agents is not restricted to the origin. Specifically, under our model, an agent \( a \) positioned in cell \( c \in \mathbb{Z}^2 \) can communicate with all other agents positioned in cell \( c \) at the same time. This communication is quite limited though: agent \( a \) merely senses for each state \( q \) of the finite state machine, whether there exists at least one agent \( a' \neq a \) in cell \( c \) whose current state is \( q \). Notice that this communication scheme is a special case of the one-two-many communication scheme introduced in [9] with bounding parameter \( b = 1 \).

The distance between two grid cells \( (x,y), (x',y') \in \mathbb{Z}^2 \) is defined with respect to the \( \ell_1 \) norm (a.k.a. Manhattan distance), that is, \(|x-x'| + |y-y'|\). The two cells are called neighbors if the distance between them is 1. In each round of the execution, agent \( a \) positioned in cell \( (x,y) \in \mathbb{Z}^2 \) can either move to one of the 4 neighboring cells \( (x,y+1), (x,y-1), (x+1,y), (x-1,y) \), or stay put in cell \( (x,y) \). The former 4 position transitions are denoted by the corresponding cardinal directions \( N,S,E,W \), whereas the latter (stationary) position transition is denoted by \( P \) (standing for “stay put”).

Formally, the agents’ protocol is captured by the 3-tuple

\[
\Pi = \langle Q, s_0, \delta \rangle ,
\]

where \( Q \) is the finite set of states; \( s_0 \in Q \) is the initial state; and \( \delta : Q \times 2^Q \rightarrow 2^{Q \times \{N,S,E,W,P\}} \) is the transition function. At time 0, all agents are in state \( s_0 \) and positioned in the origin. Assuming that at time \( t \geq 0 \), agent \( a \) is in state \( q \in Q \) and positioned in cell \( c \in \mathbb{Z}^2 \), the state \( q' \in Q \) of \( a \) at time \( t + 1 \) and its position transition \( \tau \in \{N,S,E,W,P\} \) are chosen uniformly
at random from pairs \((q', \tau) \in \delta(q, Q_a)\), where \(Q_a \subseteq Q\) contains state \(p \in Q\) if and only if there exists some (at least one) agent \(a' \neq a\) such that at time \(t\), agent \(a'\) is in state \(p\) and positioned in cell \(c\).

The goal of the agents is to locate an adversarially hidden treasure, i.e., to bring at least one agent to the cell in which the treasure is positioned. The performance measure of our algorithms is their run-time expressed in terms of the number \(n\) of agents and the distance \(D\) between the origin and the treasure. It is important to point out that neither of the two parameters \(n\) and \(D\) is known to the agents (who cannot even store them in their very limited memory).

## 2 Parallel Rectangle Search

In this section, we introduce the collaborative search strategy \textbf{RectSearch} that depends on an emission scheme, which divides all agents in the origin into search teams of size five and emits these teams continuously from the origin until all search teams have been emitted. We delay the description of our emission scheme until Section 3 and describe for now the general search strategy (without a concrete emission scheme). Whenever a team is emitted, four agents become \textit{guides} — one for each cardinal direction — and the fifth one becomes an \textit{explorer}. Now each guide walks into its respective direction until it hits the first cell that has not been occupied by a guide before (this might later in the execution require to first traverse empty cells until the block of active guides is found and then walk to the end of this block). The explorer follows the north guide and when they hit the first not yet covered cell \((0, d) \in \mathbb{Z}^2\) for some \(d > 0\), the explorer starts a rectangle search by first walking south-west towards the west guide. When it hits a guide, the explorer changes its direction to south-east, then to north-east, and finally to north-west. This way, the explorer traverses all cells in distance \(d\) from the origin (and in passing also almost all cells in distance \(d + 1\)).

Whenever the explorer meets a guide on its way, the respective guide moves further outwards to the next empty cell — hopping over all the other guides on its way if any — and waits there for the explorer’s next appearance. When the explorer has finished its rectangle by reaching the north guide again, it moves north — together with the north guide — to the first empty cell and starts another rectangle search there. All other guides have as well reached their target positions in the same distance from the origin and a new search can begin. Figure 1 gives an illustration of the process for a single search team. We will now describe the individual aspects of the \textbf{RectSearch} algorithm in a more precise and formal way.

**Emission Scheme.** Initially, all \(n\) agents are located at the origin. Until all agents have been emitted, an emission scheme emits new search teams consisting of five agents from the origin satisfying the following two properties: (i) no two search teams are emitted at the same time \(t\) and (ii) until all search teams have been emitted, the number of search teams emitted until time \(t\) is \(f_n(t)\) for some emission function \(f_n\).

Whenever a search team is ready, four of the five agents become \textbf{NewGuides} — one for each cardinal direction — and walk outwards in their corresponding directions, while the fifth one becomes a \textbf{NewExplorer} and follows the north \textbf{NewGuide} (see below for a detailed description of the agent types).

**Agent Types.** In the rest of the paper, we will refer to several different types of agents. Since there is only a constant number of different types, these can be modelled by having
Figure 1: An explorer (E) starts a rectangle search in distance $d$ (here $d = 3$) from the origin $O$ at the north guide, visits all guides (G) in distance $d$ in a counter-clockwise fashion and ends at the north guide. Whenever the explorer meets a guide, the guide moves outwards in its cardinal direction. When the explorer completes its search by arriving again at the north guide, both agents walk outwards together to the next distance to be searched. (The red numbers indicate the order in which the explorer moves.)

individual finite automata for the various types. We use six different types and explain their specific behavior in the following: Guide, NewGuide, MovingGuide, Explorer, NewExplorer, and MovingExplorer. Since agents of the three Guide-types are associated with a cardinal direction, we will use the term “outwards” to indicate their respective direction. We subsume the types Explorer, NewExplorer, and MovingExplorer under the name exploring agents.

NewGuide. A NewGuide moves outwards until it hits the first cell containing a Guide. From then on, it continues outwards until it hits a cell that contains neither a Guide nor a MovingGuide, and stops in this cell, becomes Guide and waits for an Explorer to visit. The NewGuides of the first search team stop on the first cell outwards from the origin.

NewExplorer. A NewExplorer acts like a NewGuide unless that when it hits a cell that contains neither a Guide nor a MovingGuide, it moves one cell west and becomes an Explorer.

Guide. A Guide remains dormant unless its cell is visited by an Explorer whereafter it moves a cell outwards and becomes a MovingGuide.

MovingGuide. A MovingGuide moves outwards until it hits a cell that does not contain a Guide. Then it stops there, becomes a Guide again, and waits for an Explorer to visit.
**MovingExplorer.** A MovingExplorer initially moves north along with “its” MovingGuide. It does so until it hits the first cell that does not contain a Guide. Then it moves one cell west and becomes an Explorer.

**Explorer.** An Explorer does the bulk of the actual search process by moving along the sides of a rectangle using Guides on its way to change direction. Initially, it is positioned one cell west of a north Guide. It moves diagonally south-west by alternatingly moving one field south and one field west until it hits a cell containing the west Guide whereafter it changes its movement direction to south-east, north-east, and finally north-west when hitting the respective Guides. When the Explorer arrives at the north Guide after it has completed the movement along the rectangle, it moves one cell north (in parallel with the north MovingGuide) and becomes a MovingExplorer.

2.1 Analysis

We denote by level $d$ the set of all cells in distance $d$ from the origin. We say that a cell in distance $d$ is explored when it is visited by an Explorer exploring level $d$. An Explorer is said to start a rectangle search in level $d$ at time $t$ if it moves west from the cell $(0,d)$ (containing the north Guide) at time $t$. It finishes a rectangle search in level $d$ at time $t$ if it moves onto cell $(0,d)$ (containing the north Guide) from the east at time $t$. The start time $t^s_d$ and finish time $t^f_d$ of a level $d$ are given by the time when an Explorer starts or finishes a rectangle search in level $d$, respectively. An Explorer explores distance $d$ or performs a rectangle search in level $d$ at time $t$, if it has started a rectangle search in level $d$ at time $t' < t$ and has not yet finished it at time $t$.

The design of RectSearch ensures that regardless of which emission scheme is used, the subset of Guides in every cardinal direction occupy a contiguous segment of cells. It also ensures the following two observations.

**Observation 1.** For any level $d > 0$, $t^f_d - t^s_d = 8d$.

**Observation 2.** $t^s_d - t^s_{d'} \geq d - d'$ for every $d \geq d'$.

*Proof.* Let $e$ and $e'$ be the agents exploring levels $d$ and $d'$, respectively. If $e = e'$, then the assertion holds trivially, so assume that $e \neq e'$. When agent $e$ arrives to cell $(0,d')$ as a Moving-Explorer at some time $t'$, agent $e'$ must have already started exploring level $d'$ as otherwise, agent $e$ would have explored this level; that is, $t' > t^s_{d'}$. It will take agent $e$ at least $d - d'$ more time units to get to cell $(0,d)$ and start exploring level $d$.

We are now ready to prove the following lemma essential for the correctness of our algorithm.

**Lemma 3.** Outside the origin, no two agents of the same type occupy the same cell at the same time.

*Proof.* First observe that no two NewGuides or NewExplorers can ever be in the same cell since they are emitted in different rounds and no agent can become NewGuide or NewExplorer again. Combining Observations 1 and 2 with the fact that no two NewExplorers are emitted from the origin at the same time, we conclude that two MovingExplorers cannot occupy the same cell at the same time and therefore, neither can two Explorers. A similar argument establishes the assertion for MovingGuides. □
Notice that a NewExplorer and a MovingExplorer (or a NewGuide and a MovingGuide) may share a cell, but this does not affect our algorithm since they are distinguishable.

For the sake of a clearer run-time analysis, we analyze RectSearch employing an ideal emission scheme with emission function \( f_n(t) = t \), i.e., a new search team is emitted from the origin every round. We do not know how to implement such a scheme, but in Section 3 we will describe an emission scheme with an almost ideal emission function of \( f_n(t) = \Omega(t - \log n) \).

Consider some cell \( c \) in level \( d \leq n \). Observe that under the assumption that \( f_n(t) = t \), the \( d \)th emitted explorer will start exploring level \( d \) at time \( 2^d \) unless some previously emitted explorer already did so. Therefore, cell \( c \) is explored in time \( O(d) \) as promised. So, it remains to prove that \( c \) is explored in time \( O(d^2/n) \) for every cell \( c \) in level \( d > n \). The following lemma plays a key role in this regard.

**Lemma 4.** Consider an Explorer when it finishes exploring level \( d \) and becomes a MovingExplorer at time \( t_f^d \). Then at that time, every other MovingExplorer is at distance at least \( d + 8 \) from the origin.

**Proof.** Let \( e \) be an Explorer that finishes exploring level \( d \) and becomes a MovingExplorer at time \( t_f^d \) and consider some other MovingExplorer \( e' \) at that time. Let \( d' \) be the last level explored by agent \( e' \) prior to time \( t_f^d \). Since at time \( t_f^d \), agent \( e' \) must have been at distance less than \( d \) from the origin and since every level larger than \( d \) takes longer time to explore than level \( d \), it follows that \( d' < d \). Combining Observations 1 and 2 we conclude that during the time interval \([t_s^d, t_f^d]\), agent \( e' \) managed to complete the exploration of level \( d' \) and move \( d - d' + 8(d - d') \geq d - d' + 8 \) steps outwards as a MovingExplorer.

**Corollary 5.** The distance between any two MovingExplorers is at least 8.

Let \( t_0 \) be the first time at which there are no NewExplorers anymore and note that \( t_0 = O(n) \). Corollary 5 implies that once all NewExplorers are gone, most of the explorers are busy exploring new cells.

**Observation 6.** At any time \( t > t_0 \), at least \( 7/8 \) of the exploring agents are Explorers.

**Corollary 7.** At any time \( t > t_0 \), \( \Omega(n) \) new cells are being explored.

The main theorem of this section can now be established.

**Theorem 8.** Employing an emission scheme with \( f_n(t) = t \), RectSearch locates the treasure in time \( O(D + D^2/n) \).

**Proof.** The case of \( D \leq n \) was already covered, so assume that \( D > n \). Since the Guides occupy a contiguous segment of cells, it follows that no cell in level larger than \( D + n/5 < 2D \) is explored before the exploration of level \( D \) is completed. The number of cells in the first \( 2D \) levels is \( O(D^2) \). The assertion is completed by Corollary 7 ensuring that all these cells will be explored by time \( t_0 + O(D^2/n) = O(D^2/n) \).

## 3 An Almost Optimal Emission Scheme

In this section, we introduce the emission scheme ParallelTeamAssignment that w.h.p. guarantees an emission function of \( f_n(t) = \Omega(t - \log n) \). Plugging ParallelTeamAssignment into the RectSearch strategy described in the previous section can be viewed as if after time \( k \log n \), a new search
team is emitted every constant number of rounds. By Theorem 8 this results in an almost optimal run-time of $O(D + D^2/n + \log n)$ w.h.p.

In Section 4 we describe the search strategy GeomSearch, that when combined with RectSearch yields an optimal run-time of $O(D + D^2/n)$.

The main goal of this section is to establish the following theorem.

**Theorem 9.** Employing the ParallelTeamAssignment emission scheme, RectSearch locates the treasure in time $O(D + D^2/n + \log n)$ w.h.p.

Our first goal is to describe the process FastSpread, where $n$ agents spread out along the east ray $R$ consisting of the cells $(x, 0)$ for $x \in \mathbb{N}_{>0}$ such that eventually for a prefix of the ray $R$ every cell contains a single agent.

The idea is that in every round each agent tosses a fair coin and moves east with probability 1/2 or stays put in its current cell otherwise. When an agent senses that it is the only agent in its cell, it marks itself as ready and stops moving. To avoid cells being left without any agents, the agents check before moving that not all agents in the corresponding cell decided to move and stays put if that is the case. Furthermore, when an agent walks onto a cell with an agent already marked as ready, it moves one cell further east regardless. We refer to a cell that has once been visited by a ready agent as a ready cell.

**Lemma 10.** For every positive integer $s \leq 6n$ and for every constant $k$, the first $s/6$ cells of the ray $R$ are ready after $s + k \log n$ rounds w.h.p.

**Proof.** Let $X_a$ be the random variable that counts the number of moves a not ready agent $a$ made towards east. Since the probabilities of $a$ moving forward are not independent, we study a weaker process where the number of movements for $a$ is dominated by $X_a$. Let $c$ be the cell occupied by $a$. Note that if $a$ is the only not ready agent occupying $c$, then it moves forward with probability 1. Let $a_0, \ldots, a_z$ be the set of agents occupying $c$, assume that $z \geq 2$ and let $a = a_i$. In the weaker process, $a_i$ only moves forward if agent $a_{i+1} \pmod{z}$ decides to stay put. In other words, the probability of $a$ successfully moving forward is $1/4$.

Let $X'_a$ be the random variable that counts the number of moves $a$ made east in the weaker process. Now assume that $a$ is not ready in round $s + k \log n$. Then $E[X'_a] \geq 1/4 \cdot (s + k \log n)$. By applying a Chernoff bound we get that

$$P(X'_a < 2/3 \cdot E[X'_a]) < P(X'_a < 2/3 \cdot (s + k \log n)) < O(n^{-k}) .$$

Since $X_a \geq X'_a$, for any agent $a$ that is not ready on round $s + k \log n$, the distance to the origin is at least $s/6$ w.h.p. \qed

Intuitively, the aforementioned process can be seen as parallel leader election. Since we want to describe an efficient emission scheme, it remains to show how the process can be used to quickly emit search teams consisting of five agents from the origin. To enable the FastSpread procedure to elect five different kind of agents per search team, we dedicate every fifth cell to a specific kind of agent. As an example, every cell in distance $d \equiv 1 \pmod{5}$ is dedicated to an Explorer. After an Explorer is alone in a cell using the FastSpread procedure described above, it collects its search team in the following manner: it first takes one step east where a leader election for a Guide takes place. If the corresponding Guide cell is occupied by a Guide that is marked ready, they both move outwards to collect the next Guide. Otherwise, the Explorer waits until the leader election is over. After the Explorer (accompanied by the collected Guides)
Figure 2: First, the $n$ agents executing the ParallelTeamAssignment protocol form a ray of single agents in cells $(0, 1), \ldots (0, n)$. After the innermost Explorer $e$ in cell $(0, 1)$ (denoted with red circle) is ready, it starts collecting its search team. Assuming that all the agents to join its search team are ready, after five rounds, $e$ enters the cell dedicated to the Explorer $e'$ of the next search team. Then the innermost flag of $e'$ is turned on and in the sixth round $e$ returns to cell $(0, 5)$, where its search team is waiting for it.

collected all four Guides needed for the search team, the team walks to the origin from where it will then be emitted into the four cardinal directions. We refer to the FastSpread protocol combined with the collection of the agents as ParallelTeamAssignment.

To prevent two search teams from entering the origin at the same time, we keep track of the innermost search team. In particular, we assign a flag to the agents in the leftmost cell that has not been collected. Every time an agent successfully moves to east by a coin toss in the leader election procedure, the flag is turned off on the respective agent. During the collection phase after all the agents have been collected, the Explorer makes an additional move east to the cell dedicated to elect the next Explorer and turns on the flag for all agents occupying this cell. Then the Explorer turns its flag off and moves back to its Guides. While the Explorer passes flag on, the Guides in its search team wait for its return. An Explorer executes the collection process only if it has the innermost flag activated. The use of the grid by the ParallelTeamAssignment protocol is illustrated in Figure 2.

**Corollary 11.** Assume that $n$ agents start executing ParallelTeamAssignment protocol in round 0. Then in round $8s + k \log n$, at least $\lfloor \min\{s, n\}/5 \rfloor$ search teams have entered the origin w.h.p.

**Proof.** By Lemma 10 the first $s$ cells are ready w.h.p. at time $t_0 = 6s + k \log n$, which indicates that the agents in these cells are ready to perform their collection process latest at time $t_0$. In addition, in each round after $t_0$, the Explorer flagged as innermost and occupying one of the first $s$ cells moves east. Therefore, latest in round $s + 6s + k \log n = r'$, all Explorers with distance of at most $s$ from the origin have been flagged as innermost. Since the Explorer of the successive search team is flagged after the previously innermost search team is ready to walk back to the origin, at least $\lfloor s/5 \rfloor$ search teams have started walking at round $r'$. Furthermore, every search team has to walk at most $s$ steps towards the origin and therefore, in round $8s + k \log n$ all of the $\lfloor \min\{s, n\}/5 \rfloor$ search teams, consisting of agents distance at most $s$, have reached the origin w.h.p.

By Corollary 11 the emission function $f_n(t)$ provided by the ParallelTeamAssignment protocol satisfies $f_n(t) = \Omega(t - \log n)$ and therefore, Theorem 9 follows.
4 Optimal Rectangle Search

In this section, we will present the search strategy \textit{HybridSearch} that locates the treasure with optimal run-time of \(O(D + D^2/n)\). This is achieved by, combining \textit{RectSearch} employing the \textit{ParallelTeamAssignment} with the randomized search strategy \textit{GeomSearch} that is fast only if the treasure is close to the origin.

The search strategy \textit{GeomSearch} is suited to locate the treasure very quickly if it is located close to the origin, more precisely if \(D \leq \log(n)/2\). Initially, each of the \(n\) agents chooses uniformly at random one of the four quarter-planes that it will be searching. We will explain the strategy exemplary for an agent “responsible” for the north-east quarter-plane. The other three types operate analogously in their respective quarter-plane.

Initially, the agent moves one cell to the east. From then on, it moves a geometrically distributed number of steps east following which it moves a geometrically distributed number of steps to the north. More precisely, with probability \(1/2\) the agent moves further and otherwise stops walking in the current direction. Both these processes can be realized in our model by having two state transitions where one of them moves the agent further while the other one ends the current walk.

\begin{lemma}
If \(D \leq \log(n)/2\), then \textit{GeomSearch} locates the treasure in time \(O(D)\) w.h.p.
\end{lemma}

\begin{proof}
Consider some cell \(c\) at distance \(d \leq \log(n)/2\) from the origin and fix some agent \(a\). Let \(X_a\) be a random variable that captures the length of the walk of agent \(a\) and observe that \(X_a\) obeys a negative binomial distribution so that

\[ P(X_a = k) = (k + 1) \cdot 2^{-(k+2)}. \]

Recalling that \(a\) has already moved one step, we conclude that the probability that \(a\) moves up to distance \(d\) is

\[ P(X_a = d - 1) = d \cdot 2^{-d-1} \geq 2^{-d-1} = \Omega(1/\sqrt{n}). \]

Since all cells at distance \(d\) from the root have the same probability of being explored by \(a\) and since there are \(O(\log n)\) such cells, it follows that \(a\) explores cell \(c\) with probability at least \(\Omega\left(\frac{1}{\sqrt{n \log n}}\right)\). Therefore, the probability that none of the agents explores cell \(c\) is at most

\[ \left(1 - \Omega\left(\frac{1}{\sqrt{n \log n}}\right)\right)^n < e^{-\Omega\left(\frac{\sqrt{n}}{\log n}\right)}. \]

The assertion follows.
\end{proof}

We can now combine the two search strategies \textit{GeomSearch}, which is optimal for \(D \leq \log(n)/2\), and \textit{RectSearch} employing \textit{ParallelTeamAssignment}, which is optimal for \(D = \Omega(\log n)\), into the \textit{HybridSearch} strategy as follows.

At the beginning of the execution, each agent tosses a fair coin to decide whether it participates in \textit{RectSearch} or \textit{GeomSearch}. Let \(n_r\) and \(n_g\) be the number of agents participating in \textit{RectSearch} and \textit{GeomSearch}, respectively and observe that \(n_r, n_g \geq n/3\) w.h.p. Then the agents enter according states so that they do not interfere with each other anymore. One group executes \textit{GeomSearch} and locates the treasure w.h.p. in time \(O(D)\) if \(D \leq \log(n)/2\) and the other group executes \textit{RectSearch} locates the treasure w.h.p. in time \(O(D + D^2/n)\) if \(D = \Omega(\log n)\), thereby establishing the main theorem.

\begin{theorem}
\textit{HybridSearch} locates the treasure in time \(O(D + D^2/n)\) w.h.p.
\end{theorem}
5 Conclusions.

In this paper, we establish a tight $O(D + D^2/n)$ upper bound on the time it takes for $n$ FSM-controlled agents to locate a treasure hidden at distance $D$ from the origin. Combined with the lower bound of Feinerman et al., our result demonstrates that by allowing the agents to use a very primitive mean of communication, one can get rid of the requirement for a super-constant memory and an approximation of $n$. This last observation may come as good news to anyone interested in studying the communication between collaborating entities.

The aforementioned upper bound is based on a randomized algorithm that locates the treasure in finite time with probability 1 (i.e., a Las Vegas algorithm) and in time $O(D + D^2/n)$ w.h.p. It is not difficult to extend our analysis showing that the run-time of our algorithm holds also in expectation.

As mentioned in Section 1.2, we assume that the agents operate in a synchronous environment. Given the natural motivation for our model (studying ant colonies), it may be desirable to lift this assumption. Although this issue is beyond the scope of the current extended abstract, note that our algorithm can be adapted to work in a fully asynchronous environment as long as the model is changed so that an agent’s communication range is extended to include its neighboring cells on top of its own cell. Whether the ANTS problem can be solved within the same asymptotic time bounds in an asynchronous environment without extending the communication range is an interesting open question.

One may wonder if locating the treasure is indeed the “right” goal for agents whose navigation capabilities are so weak (cannot even store their own coordinates): Can an agent locating the treasure find its way back to the origin? Can we guarantee that all agents eventually return to the origin? To that end, notice that our algorithm can be modified to ensure a positive answer for these two questions. In fact, we can guarantee that the treasure finder returns to the origin in time $O(D + D^2/n)$ w.h.p. and that all agents return to the origin in time $O(D + D^2/n + \log n)$ w.h.p. Getting rid of the extra logarithmic term in the latter bound seems to be an interesting challenge. In any case, the treatment of these secondary goals is deferred to the full version.

Finally, it is important to point out that our algorithm is not inspired by any observations regarding the real behavior of ants. (In fact, we will be surprised if the RectSearch strategy that lies at the heart of our algorithm fits an exploration pattern used by real ants.) As such, we do not claim that our results explain any natural phenomenon, but rather attempt to advance the understanding of the power and limitations of a basic nature-inspired model.
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