The method for approximate determination of the heat pipes finned radiators optimal geometric characteristics

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Abstract. The method is proposed for approximate optimization of the geometric characteristics of heat pipes finned radiators, the heat from which is removed by natural convection. The mass of the radiator is used as the objective function for a given transmitted heat flux from its surface. The mathematical model developed to calculate the optimal geometric characteristics of radiators made of flat round fins, as well as fins made of square plates.

1. Introduction
One common way to remove heat from heat pipe condensers is through natural convection. In order to increase the transmitted heat flux, finned radiators are installed in the condensation zone. When designing them, the problem arises of optimizing their mass and dimensions. The solution to the optimization problem will reduce the mass of metal or transfer more heat from the surface of the radiator with its constant mass.

Unlike other types of finned cooling or heating devices (such as, for example, heating convectors), the surface temperature of the wall of the heat pipe condenser remains constant in length, which facilitates a mathematical description of the heat transfer process and optimization of the radiator geometric parameters.

2. Physical picture of the process
When heat is removed using a radiator located on the heat pipe condenser, heat is transferred from the condenser wall by the heat conduction through the metal of the fin and is then removed from its surface by convection. The efficiency of heat transfer by the heat conduction decreases with decreasing thickness and increasing fin height. In the space between the fins there is a naturally convective movement of air. The speed of air will be determined primarily by the difference in densities in the ambient air and the air near the heated fin. With a sufficiently large distance between the fins, the heat transfer intensity is almost constant and weakly depends on the fins height. It can be described by the relations used to describe natural convection on a vertical plate.

Hydrodynamic and thermal boundary layers arise on the vertical surface of the fin. As the distance between the fins decreases, the boundary layers of air close in the space between them. As a result, the speed and temperature profiles are reconstructed, the aerodynamic drag increases, air flow rate and speed decrease. Thus, when the fins approach, the heat transfer decreases sharply and heat flux removed from the surface of the radiator fins.
In addition to convective heat transfer, there is radiation heat exchange between the surface of the radiator and the objects surrounding it. As calculations [1] show, at a radiator surface temperature of 90 – 120 °C, the proportion of the radiant heat flux in the total heat flux from the radiator surface does not exceed 20%.

3. Statement of the optimization problem
The choice of the objective function is determined by the tasks facing consumers of the cooling system and is described in previous works of the authors [2]. The geometric characteristics of the finned radiator are shown in figure 1.

![Figure 1](image)

Figure 1. The geometric characteristics of the heat pipe condenser finned radiator: 
D – fin diameter, d – heat pipe radiator tube diameter, 
s – the distance between fins, 
L – the radiator length, 
δ₁ – fin thickness, δ₂ – thickness of the heat pipe casing.

In the case under consideration, the objective function is the minimum value of the mass of the radiator $M$ for a given transmitted heat flux from its surface $Q$. The length $L$ of the heat pipe condenser finned radiator is not a constant value.

The optimization problem statement is as follows. The radiator transmits heat flux $Q$. It is necessary to find the optimal values of the length of the radiator $L$, the equivalent diameter of the fins $D$ (the diameter of the circle, equal in area to the fin having a rectangular or square shape), the distance between the fins $s$, and the thickness of the fins $\delta₁$ at which the minimum value of the mass $M$ of the finned radiator is achieved. The diameter and wall thickness of the casing of the heat pipe, the density of the material of the fins and the heat pipe casing, the coefficient of thermal conductivity of the fins, the temperature of the wall of the condenser and ambient air are known.

The mass of the finned radiator can be calculated as the sum of the mass of the part of the heat pipe on which the radiator is applied and the mass of the fins ($M = M_p + M_f$):

$$M = \rho_p \cdot F_p \cdot \delta_p + \rho_f \cdot F_f \cdot \delta_f \cdot \frac{L}{s + \delta_f}$$

(1)

Here $F_p = \pi \cdot d \cdot L$; $F_f = 0.25 \cdot \pi \cdot (D^2 - d^2)$ are the tube area of the heat pipe radiator, on which the fins are applied, and the area of the fins, m². The equation $\frac{L}{s + \delta_f}$ is the number of radiator fins, pc.

The greatest difficulty is the calculation of the transmitted heat flux of the radiator $Q$, which is included in the above optimization problem as a limitation. Previously, a mathematical model was developed for calculating the transmitted heat flux for radiators with round fins, as well as fins from rectangular plates with different geometric characteristics [3]. The model has the following features.
The fin temperature is considered constant in its height. The temperature difference between the fins and ambient air is defined as:

$$T_f - T_\infty = (T_w - T_\infty) \cdot \eta \quad (2)$$

where $T_f$, $T_\infty$, $T_w$ – average temperature of the fin surface, air temperature in the room, the temperature of the base of the fin (taken equal to the surface temperature of the heat pipe tube), °C; $\eta$ - fin efficiency.

The heat transfer coefficient in the space between the fins is taken as a constant value and is calculated according to the known criterial dependences of the Nusselt number on the Grashof and Prandtl numbers for natural convection on a vertical plate. It is considered that the presence of the tube does not change the heat transfer coefficient between the fins.

Fin efficiency is calculated using the formula proposed in the A. Piir book [4]:

$$\eta = \text{th}\left[m \cdot l \cdot (D / d)^{1/3}\right] - \left[m \cdot l \cdot (D / d)^{1/3}\right]^{-1} \quad (3)$$

where $m = \left[\frac{2 \cdot \Pi}{\Lambda_f \cdot \delta_t}\right]^{1/2}$ is the dimensionless complex, and $l = 0.5 \cdot (D - d)$ is the effective fin height, m.

In the case, if the fin is square or rectangular, then in determining its effectiveness it comes down to a round fin. On the contrary, if the edge is round, it reduces to a square to determine its height.

In this work, in calculating the heat flux from the surface of the radiator, only natural convection is taken into account. To simplify the problem, radiation is not taken into account, although its accounting is also possible using the proposed method. In this case, the transmitted heat flux can be determined as:

$$Q_{tl} = \bar{\alpha} \cdot (T_0 - T_\infty) \cdot \left[F_p + \eta \cdot 2 \cdot F_t \cdot \frac{L}{s + \delta_t}\right] \quad (4)$$

Let's single out the factors that enter into the expressions for $M$ and $Q$, and therefore affect the result of optimization.

- Parameters of the heat pipe casing on which the fins are fixed: the heat pipe condenser finned radiator length $L$, heat pipe casing diameter $d$; wall thickness of the heat pipe casing $\delta_t$; heat pipe casing material density $\rho_p$.
- Fins parameters: fin diameter $D$; fin thickness $\delta_f$; the distance between the fins $s$; fin material density $\rho_f$; thermal conductivity of the fin $\Lambda_f$.
- Parameters of environments: coolant vapor temperature in the condensation zone of the heat pipe $T_0$; air temperature in the room where the radiator is installed $T_\infty$.

Most of them are constant parameters set for each case under consideration. These include: air and coolant temperatures, tube and fin materials. Independent variables are the height of the fins $D$, their thickness $\delta_f$, the distance between the fins $s$, as well as the length of the radiator $L$. In general, restrictions can also be imposed on the length of the radiator and the height of the fins, which cannot be excessively large, as well as on the thickness of the fins, since they cannot be too thin due to the need to ensure their mechanical strength.

We will transform the optimization problem with reducing the number of variables.

Consider the expressions for the objective function (1) and the limitation on the transmitted heat flux (4). Both mass and thermal power are directly proportional to the length.
We introduce the notation:

\[ Z = \frac{D^2 - d^2}{4} \cdot \frac{1}{s + \delta_i} \]  

(5)

Then the mass and thermal power can be represented as:

\[ Q = \pi \cdot \alpha \cdot \Delta T \cdot L \cdot (d + 2 \cdot Z \cdot \eta) \]  

\[ M = \pi \cdot \rho_l \cdot \delta_i \cdot L \cdot (A \cdot d + Z) \]  

(6)

(7)

Here the equation \( A = \frac{\rho_p \cdot \delta_2}{\rho_l \cdot \delta_i} \) has a constant value calculated as the ratio of the products of the thicknesses and densities of the fins and the heat pipe casing.

Expressing \( L \) from equation (6) and substituting it in (7), we obtain the expression for the mass of the radiator, in which the value \( Q \) is a known constant:

\[ M = \frac{Q \cdot \delta_i \cdot \rho_l}{\alpha \cdot \Delta T} \cdot \frac{A \cdot d + Z}{d + 2 \cdot Z \cdot \eta} \]  

(8)

Thus, the objective function no longer depends on the length.

Based on the type of objective function, we can analyze the influence of the fins thickness on its minimum value. It can be shown that with a decrease in its thickness, a monotonous decrease in the objective function is proportional to the square root of its value, even though the efficiency of the fin decreases.

If we assume that the fins thickness is significantly less than the distance between the fins, which is actually performed in practice, then the dependence of the mass of the radiator on the fins thickness can be represented as follows:

\[ M = C_1 \cdot \frac{\delta_i}{C_2 + C_3 \cdot \eta} = C_1 \cdot \frac{\delta_i}{C_2 + C_4 \cdot \sqrt{\delta_i}} \]  

(9)

Here \( C_1 \cdot C_4 \) are values independent of the fins thickness. The form of the equation shows that for any values of these functions, the mass increases monotonically with increasing thickness of the fin, and at \( C_3 = 0 \) than \( M \sim \delta_i \), and at \( C_2 = 0 \) than \( M \sim \sqrt{\delta_i} \). Thus, the smaller the fin thickness, the smaller the mass value. Then, the minimum fin thickness is determined only by technological and strength considerations. The fin thickness is a constant and a parameter of our problem. It is not an independent variable, but it is the source data for solving the optimization problem.

In the resulting expression, two independent variables remain – the diameter and the distance between the fins. We can find the distance between the fins at which the previously described sharp decrease in the heat transfer coefficient between the fins for a given diameter of the fins begins, that is, connect these two values. The Polhausen solution is used for the problem of natural convection on a vertical plate [5]:

\[ s = 2 \cdot C_1 \cdot D^{0.25} \cdot \left( \frac{\beta \cdot g \cdot \Delta T}{4 \cdot \nu^2} \right)^{-1/4} \]  

(10)

The constant \( C_1 \) determines the dimensionless thickness of the boundary layer on the surface of the fin. When \( C_1 = 3 \) the boundary layers in the space between the fins on the upper edge of the fins begin to close [5]. We consider that a significant decrease in heat transfer occurs with a significantly smaller distance between the fins, when the maximum values of the velocities in the hydrodynamic boundary layer on the plate are closed, i.e. at \( C_1 = 1 \). The issue of meaning \( C_1 \) requires further
clarification in the future. It can be carried out joint analysis based on the results of [6] and [5], or on the basis of a numerical solution of the problem of natural convection in the space between the fins. As a result, an expression for the distance between the fins $s$ are obtained as a function of the fin height associated with its equivalent diameter $D$. The expression $Z(D)$, which is part of the objective function, for a small fins thickness will now take the view:

$$Z(D) = 0.25 \cdot (D^2 - d^2) \left( \frac{\beta \cdot g \cdot \Delta T}{4 \cdot \nu^3} \right)^{1/2} \cdot \frac{1}{2 \cdot C \cdot D^{0.2}}$$  \hspace{1cm} (11)$$

As a result of our reasoning, a formula for the objective function (9) is obtained, which depends on only one independent variable $D$. Having determined the optimal value, we can calculate the optimal values of the other two arguments. Optimal radiator length is:

$$L = \frac{Q}{\pi \cdot \alpha \cdot \Delta T} \cdot \frac{1}{d + 2 \cdot \eta \cdot Z(D)}$$  \hspace{1cm} (12)$$

The optimal distance between the fins can be found by the equation (10).

Thus, equations (6), (8), (10) – (12) describes the optimization problem posed earlier.

The proposed optimization method is approximate. This is due to the fact that the distance at which the boundary layers are closed is taken as the optimal distance between the fins and after that there is a rapid decrease in the heat transfer coefficient and the transmitted heat flux. In fact, the optimal distance between the fins $s$ will be slightly lower than the values calculated by the above method. This is due to the fact that with a decrease of the distance between the fins $s$ not only the heat transfer coefficient decreases, but also the number of fins on the heat pipe tube increases, and, therefore, the heat transfer surface area increases.

The method is approximate also because it is based on an approximate model that does not take into account radiation. Nevertheless, it is accurate enough for estimates, takes into account many factors and may well be refined.

4. Results and discussion

The analysis of the objective function shows, as well as the performed calculations show that it always has a minimum.

Optimization calculations were carried out under the following conditions. The transmitted heat flux from the radiator was 200 W. Heat pipe was made of steel. Its outer diameter was 20 mm, wall thickness of the heat pipe casing was 1 mm. The air temperature was 20 °C. Heat pipe tube wall temperature was 90 °C.

It was found that the change in the fins thickness in a fairly wide range (from 0.1 mm up to 0.4 mm) does not affect the optimal values of other geometric parameters, and the minimum mass of the radiator increases as as shown in figure 2.

Calculations of radiators with fins made of steel, copper and aluminum were carried out. The fin thickness in all cases was 0.2 mm. The optimum mass of the radiator with steel fins is about one and a half times higher than that of fins made of copper or aluminum and is achieved with a significantly lower equivalent diameter as shown in figure 3.

Despite the difference in density, the fins of aluminum, copper and steel have approximately the same characteristics. This is due to the fact that the mass of the radiator is mainly determined by the mass of the heat pipe radiator tube, and the mass of the fins in it is a small fraction.

The calculations showed that the optimum value of the finning characteristics is strongly influenced by the diameter and wall thickness and the density of the material of the heat pipe casing, on which the finned radiator is mounted.
The proposed method can be refined by stricter accounting for the reduction of the heat transfer coefficient in the space between the fins while decreasing the distance between them in the mathematical model, the Elenbaas [6] and Chen’s [7] formulas are used. In addition, it is possible to take into account the radiant heat flux from the surface of the radiator. However, taking into account all these factors, the calculations will be significantly more complicated. The advantage of this method is its clarity and simplicity.

1. The above method allows you to approximately determine the optimal dimensions of the heat pipe radiator: the diameter of the fins, the distance between them, their thickness, as well as the length of the radiator for a given diameter of the thickness of the heat pipe casing, materials of the fins and heat pipe and the temperature of the wall of the condenser and the ambient air.

2. The mass of the radiator has an unconditional minimum at a certain ratio of the geometric parameters of the radiator.

3. Changing the thickness of the fins in a fairly wide range (from 0.1 mm to 0.4 mm) does not affect the optimal values of other geometric parameters, but affects the radiator mass.

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