Simulation of the dynamics of the PF-3 device’s plasma

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Abstract. We have simulated the motion of the current-drive plasma shell (CDPS) in the PF-3 facility – from discharge start to appearance of plasma focus. For this purpose, we modified one-component MHD model, taking Hall effect into account. The extended simulation matches CDPS “runaway” mode, obtained in the PF-3 facility.

1. Introduction

Plasma focus (PF) is a kind of pinch phenomena. Dense plasma focus appears as a result of plasma constriction, that takes place on the axis of facility. Through the cumulative heating and high plasma density, PF is the source of powerful X-ray, neutron and other emissions. Withal, the supersonic axial plasma jet was launched from PF in many experiments. The same jet may be useful for purpose of surface alloying [1], or material testing [2].

2. Model

The simulation is based on the model previously used in work [3], which assumes single-fluid ideal two-dimensional MHD approximation. It means using equations noted below:

$$\frac{\partial n}{\partial t} + \text{div}(n\vec{V}) = K(r, z, t)$$

$$\frac{\partial B_\theta}{\partial t} = -\frac{\partial}{\partial r}(V_r B_\theta) - \frac{\partial}{\partial z}(V_z B_\theta)$$

$$\frac{\partial (nV_r)}{\partial t} + \frac{nV_r^2}{r} + \frac{\partial (nV_z)}{\partial z} = -2 \frac{\partial}{\partial r}(\text{m}_i n T) + \frac{1}{4\pi \text{m}_i} \frac{\partial B_\theta}{\partial z} B_\theta$$

$$\frac{\partial (nV_z)}{\partial t} + \frac{nV_z^2}{r} + \frac{\partial (nV_r)}{\partial z} = -2 \frac{\partial}{\partial z}(\text{m}_i n T) + \frac{1}{4\pi \text{m}_i} \frac{\partial B_\theta}{\partial r} B_\theta$$

$$\frac{3}{2} \left( \frac{\partial nT}{\partial t} + \text{div}(nTV) \right) = -nT \text{div}(\vec{V})$$

where $n$ — concentration of ions; $V$ — velocity of plasma flow; $B_\theta$ — toroidal magnetic field; $r$ and $z$, — radial and axial coordinates correspondingly; $T$ — temperature of electrons, $m_i$ — ion mass, $K$ — function of three variables, that indicates ionization.

We presumed that plasma is singly ionized in terms of plasma dynamics. Ionization was considered in thin area near the insulator through $K(r, z, t)$ function of three variables. The set of equations was

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completed with equation of power supply circuit, and boundary conditions – zero speed on the surfaces, temperature equals to 3 eV, and magnetic field strength is continues:

\[ V_r = 0; V_z = 0; T = 3 \text{ eV}; \]

Magnetic field strength on the insulator surface is equal to:

\[ B_\theta = \frac{\mu_0 I}{2\pi r}; \]

\( I \) – discharge current.

The set of equations doesn’t contain dissipation. It means infinite conductivity to calculate the energy balance and plasma motion.

All simulations were built for Ne plasma with initial pressure \( P_0 = 1-1.5 \text{ Torr}, \) initial voltage \( U \) =8-9 kV, stored energy \( W \) =290-370 kJ

We varied pressure and parameters of power supply (initial voltage, capacitance and inductance); and we have reached the match of simulated plasma dynamics with one observed by optoelectronic transducer in the vicinity of pinch (within central circle in figure 1). On the contrary, we were unable to match the simulated signals of magnetic probes with the observations.

Magnetic probes show the difference of axial and radial plasma velocities. In the experiment radial propagation is faster than axial one. It means that an additional phenomenon should be taken into account.

There is another feature in simulations – simulated CDPS is more cylindrical with respect to experimental observed CDPS (figure 1). It means that its opening angle is noticeably less than one observed with optoelectronic transducer. Furthermore, there is the mode with extremely high opening angle (so called “runaway” mode) [4]. This feature isn’t explained by the ideal MHD only. Authors of the papers [5] and [6] explain “runaway” phenomenon through the Hall effect, caused by electrode evaporation. It results to lower gas sweeping; and then to faster radial propagation of CDPS. Axial propagation saves its normal speed, so we observe high opening angle.

We tried to build simulation based on extended MHD (xMHD) model.

First, we estimated how much is influence of the Hall effect.

Equation of magnetic field evolution (in terms of xMHD):
\[
\frac{\partial B_\theta}{\partial t} = -\frac{\partial}{\partial r} (V_r B_\theta) - \frac{\partial}{\partial z} (V_z B_\theta) + B_\theta \left( \frac{2\chi}{r} - \frac{\partial \chi}{\partial r} \right) \frac{\partial B_\theta}{\partial z} + B_\theta \frac{\partial \chi}{\partial z} \left( \frac{r}{B_\theta} + \frac{\partial B_\theta}{\partial r} \right)
\]

\[ - \chi = \frac{c^2 \chi_4}{4\pi}; \chi = \frac{1}{n_e e c}; \] - Hall parameter.

Hall effect preferably appears in areas of density drops. Low electron density and magnetic field drops are also important conditions. These conditions are most possible behind of CDPS. Let’s examine how important is the Hall effect. For this purpose we’ve compared current speed with flows one:

\[ V_{\text{car}} = \frac{f}{en_e} = \frac{c \text{rot} (\vec{B})}{4\pi en_e} \approx \frac{c B_\theta}{4\pi en_e \Delta_{\text{CDPS}}} \approx \frac{10^{22}}{n_e} \gtrsim V_{\text{flow}} \approx 10^7 \text{cm/s}; \]

This inequality is true when \( n_e \gtrsim 10^{15} \text{cm}^{-3} \).

In practice, most of current is concentrated in back surface of CDPS. It means \( \Delta_{\text{CDPS}} \) is less; so \( n_e \) might be more than \( 10^{15} \text{cm}^{-3} \). Under this condition we need to take the Hall effect into account.

It’s interesting to calculate evolution rate of the magnetic field strength caused by plasma flow and Hall effect:

\[ \left( \frac{\partial B_\theta}{\partial t} \right)_{\text{flow}} = -\frac{\partial}{\partial r} (V_r B_\theta) - \frac{\partial}{\partial z} (V_z B_\theta) \approx V_{\text{CDPS}} \frac{B_\theta}{\Delta_{\text{CDPS}}} \approx \frac{10^7 \cdot 10^4}{3} = 0.33 \cdot 10^{11} \text{G/s}; \]

\[ - V_{\text{CDPS}} \text{ is CDPS’s velocity and } \Delta_{\text{CDPS}} \text{ is its thickness. Both experiments and numerical simulation show } \Delta_{\text{CDPS}} = 3 \text{ cm} \text{[7]}. \]

\[ \left( \frac{\partial B_\theta}{\partial t} \right)_{\text{Hall}} = B_\theta \frac{\partial \chi}{\partial z} \left( \frac{B_\theta}{r} + \frac{\partial B_\theta}{\partial r} \right); \]

At 10 cm radii, Hall effect results to:

\[ \left( \frac{\partial B_\theta}{\partial t} \right)_{\text{Hall}} = -B_\theta \frac{c}{4\pi n_{e,\text{min}} e} \left( \frac{B_\theta}{\Delta_{\text{CDPS}}} \right) = 10^4 \cdot \frac{3 \cdot 10^{10}}{4\pi \cdot 10^{15} \cdot 1 \cdot 4.8 \cdot 10^{-10}} \cdot \frac{10^4}{3} = 0.17 \cdot 10^{12} \text{G/c}; \]

\( n_{e,\text{min}} \) is least concentration of electrons, which reaches to \( 10^{15} \text{cm}^{-3} \). As CDPS moves to axis, \( B_\theta \) increases. Thus we have to consider this member of equation at the late stage of discharge.

Unfortunately, it’s too hard to simulate Hall effect influence – it makes calculation unstable. But we have built simplified model, which allows us to calculate plasma dynamics through all the discharge. So we used the equation showed below for calculate field magnitude:

\[ \frac{\partial B_\theta}{\partial t} = -\left( k_x + k_x^* \right) \frac{\partial}{\partial r} (V_r B_\theta) - \frac{\partial}{\partial z} (V_z B_\theta); \]

- \( k_x \) and \( k_x^* \) - parameters of Hall effect rate. We had observed that \( k_x = 2.7 \); and \( k_x^* = 13.2 \) provides model matches experiments.
This model is extremely simplified, but it simulates plasma dynamics with high consistency to measurements.

3. Discussion
Figure 1 contains results of the simulation (both with Hall effect and without one). It shows strong difference between the cases. When Hall effect is applied, the “runaway” phenomena occurs. The axial spreading of CDPS is slower than radial one – because of Hall effect provides faster magnetic field radial advance (it transports magnetic field in addition to the plasma flow).

So Hall effect influences CDPS shape. But it also affects the temporal parameters of discharge. Figure 2 shows the difference of magnetic probes signals simulated. In the case of Hall effect is neglected, field was registered by four probes with slight time delay - about one microsecond. But with Hall effect, registration delays was more significant – about 3 µs from first to fourth signal. It’s noticeable, the same delays also observed experimentally (figure 3) [7].

The forming of plasma focus is shown in figure 4. For purpose of comparison with experiment, we have built image of integral distribution of density square (along a line of probing). The simulated image has the same shape as measured. It also has the same dimensions.

The simulation well corresponds with the real discharge plasma dynamics [7]. After the CDPS collision, the focus compression starts. At this moment, the current breaks rapidly (figure 3). The pinch formed near anode and then it lengthens due to zipper-effect. After that plasma axially leaves a plasma focus in the axial direction. And it forms the plasma jet (figure 4).

It’s noticeable, the jet contains strong magnetic field. So it’s possible, jet is magnetic-accelerated. In this case, jet’s radial propagation is slowly than axial – because of magnetic field is approximately proportional to 1/r (within the jet). Then magnetic pressure is highest on the edge of the jet.

Note, plasma jet contains dense “core” (on the axis), “magnetic bubble” (surrounds core), and dense sheath (which cover other parts). The core determines the upper limit of field magnitude (due to 1/r law). But the core is held by surrounded field. So it is subject of instabilities. It leads to some “clumps” (relatively dense plasma formation) appearance.

References
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