LETTER TO THE EDITOR

Properties of the short period CoRoT-planet population I:

Theoretical planetary mass spectra for a population of stars of 0.8 to 2 solar masses and orbital periods of less than 20 days.

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ABSTRACT

Context. We study the planet populations in the discovery window of the CoRoT-space-telescope scheduled for launch on December 27th. We base the prediction on ‘first principles’ calculations of planet formation in the framework of the planetesimal hypothesis.

Aims. To provide a-priori planetary initial mass functions for confrontation with the CoRoT-planet discoveries in the entire range of sensitivity of the CoRoT instrument, i.e. for all giant planets and down to terrestrial planet masses.

Methods. We construct a comprehensive set of static complete-equilibrium core-envelope protoplanets with detailed equations of state and opacity and radiative transfer by convection and radiation. Protoplanets are calculated for host-star masses of 0.8 to 2 solar masses and orbital periods of 1 to 16 days. We subsequently check the stability of the planetary population by a series of methods.

Results. We find the static planetary populations to be stable and thus a plausible ensemble to predict the planetary IMF for orbital periods in the specified range.

Conclusions. We predict bimodal planetary initial mass functions with shapes depending on orbital period. The two main maxima are around a Jupiter mass and about 50 earth masses. We predict an abundant population of Hot Neptunes and a large population of planets that fill the solar-system gap of planetary masses between Neptune and Saturn.

Key words. planets:formation – exoplanets – planets: mass – solar system:formation – initial mass function

1. Introduction

Planets form in circumstellar disks which are a necessary step in star-formation. Presently two mechanisms are in discussion: formation via a disk-instability and via planetesimals, see Wuchterl et al. (2000). Uncertainties about the actual nebula conditions at the era of planet formation and an incomplete understanding of relevant physical processes (e.g. of planetesimal formation and of the various types of orbital migration) limit the predictive power of conventional formation theories. In preparation for the CoRoT-transit-search for exoplanets, e.g. Baglin et al. (2002) we have developed a new synoptic approach to planet formation, Pečnik (2003), Pečnik & Wuchterl (2005), Broeg (2006).

2. Synopsis of protoplanets

Broeg (2006) calculated all planets and protoplanets in hydrostatic and thermal equilibrium for stellar masses of 0.4, 0.8, 1 and 2 M⊙ and orbital periods of 1, 4, 16 and 64 d (0.02, 0.049, 0.124 and 0.313 AU for the solar case). The ranges are chosen to cover CoRoT’s discovery space. Planetary models are spherically symmetric, consist of a rigid, constant density core and a gaseous envelope in hydrostatic and thermal equilibrium. Planetesimal accretion provides an energy input at the core surface that is transferred through the envelope by radiation and convection. Detailed equations of state and opacities for roughly solar composition are used with ‘interstellar’ dust opacities. Orbital distance enters via the thermal and tidal effects of the star. Assuming that no core size is favoured and that gravitationally stable nebulae provide the pressures for embedding the planets, statistical properties of the planet population are provided, in particular planetary mass spectra. Here we use the CoRoT Mark I v1.1 ensemble of planetary models, Broeg (2006), Mark I for short. Mass spectra and solution manifolds are available online, Broeg, C. (2006), see Broeg & Wuchterl (2006) for an application to the large-core Pegasi-planet HD 149 026 b. Here we apply the method to theoretically predict what planets are in CoRoT’s search space. We restrict the discussion to close-in planets with orbital periods of < 16 d for two reasons: (1) they are in CoRoT’s discovery window for the first run with a foreseen continuous observing period of 30–60 d and hence with a completeness limit for planets detected by at least three transits at about 15 days, (2) there is a qualitative change in the planet-population for periods of ≃ 16 d, Broeg (2006) and planet formation being dominated by hydrostatic processes for smaller periods, Broeg & Wuchterl (2006).
3. Planetary Mass Spectra for a typical CoRoT field

Our calculations of the planet population show a strong dependence of the planetary properties on stellar mass, \( M_\odot \) (Broeg 2006). We therefore account for the types of stars CoRoT will observe. Given the Mark 1 stellar mass-grid, we model a typical CoRoT-field with a mixture of 56\%, 37\%, 7\% of 2, 1, and 0.8 \( M_\odot \) stars, resp.. This roughly represents a grouping of stellar spectral types A + F, F + G and K, resp.. As noted by Broeg (2006) Mark 1 has a relatively sparse sampling of the stellar mass-dependence but is already based on \( \approx 10^6 \) planetary models and thus by far the largest survey of planet-formation. We do not include the Mark 1 results for 0.4 \( M_\odot \) (M-stars) here. To account for the uncertainties in planetesimal-formation and -accretion theory we use the results for all core-accretion rates given in Mark 1, i.e. \( 10^{-6}, 10^{-4} \), and \( 10^{-2} \) \( M_\text{Earth} \text{yr}^{-1} \). The resulting theoretical planetary mass spectra for a CoRoT-star-stellar population mix, the CoRoT-Mix for short, are shown in Fig. 1. We note that this a-priori prediction of the planetary initial mass function naturally predicts known empirical ‘planetary’ masses. Given the fact that the only particular macroscopic input masses are the stellar masses, that is non-trivial. For a general discussion of the properties we refer to Broeg (2006) and focus on the particularities of the CoRoT-Mix in the following.

The multi-peak structure in the mass functions follows the general double peaked structure described by Broeg (2006) but is complicated by mixing the mass spectra of different host stars. This results in the structures displayed in Fig. 1. Averaging the mass spectra using a finer sampling in host star mass would most likely revert the mass spectra to a double peaked structure. At all periods the most frequent planets appear with the first peak near the typical planetary masses (Neptune and Jupiter) become more well defined farther from the host stars. A key feature at all periods is a rich population of ‘Hot Neptunes’ that always significantly outnumbers the Jupiters.

Table 1. Characteristic quantities of the CoRoT-Mark 1 v1.1 theoretical planetary population for a solar mass host star, from Broeg (2006) for various orbital periods, \( T_{\text{orb}} \). Semi-major axis, \( a \) and the radiative equilibrium nebula-temperature, \( T_{\text{n}} \) result from the host star mass and luminosity, resp.. \( M_1 \) and \( M_2 \) give characteristic values for peaks in the planetary mass spectra. \( P_1 \) and \( P_2 \) are the corresponding typical nebula pressures.

For reference and an accompanying study of the impact of particle mass-loss processes characteristic numbers are given for the pure solar mass results of CoRoT-Mark 1 in Tab. 1.

4. Stability of the planetary population

The CoRoT Mark 1 planet population is based on an equilibrium concept, much like the stellar main sequence where stars are in complete i.e. hydrostatic and thermal equilibrium. The advantage is that whatever the formation history of the planet and the nebula was, it does not influence the planetary properties. Thus whether planets migrated to their final position or had undergone giant impacts or mass-loss events during the formation era does not change our prediction as long as the respective conditions of the planetary equilibria are realised — i.e. nebulae are sufficiently diverse — as the planets approach their final masses. The disadvantages are (1) that because the method is based on time-independent models, we do not automatically have information about the stability of the equilibria used in the construction and (2) because it is based on planet formation theory we do not account for modifications of the planetary properties during planetary evolution in to the present, in particular and most important...
mass loss, that might distort the mass spectra. The second issue is discussed in an accompanying letter, the question of stability in the following sections. For HD 149 026 b, Broeg & Wuchterl (2006) showed that planet formation is a completely hydrostatic process in that case. They gave a complete formation-history including a radiation fluid-dynamical calculation with convection and detailed, particle in box planetesimal accretion to ages of about 100 Ma. Because that is presently beyond computational feasibility for the entire Mark 1 population of $\sim 10^6$ planets we have addressed the stability question by a series of simplified model systems: (1) linear analysis of isothermal and polytropic protoplanets, (2) non-linear fluid dynamics for isothermal protoplanets, (3) quasi-hydrostatic stellar evolution type calculations to investigate non-linear secular or thermal stability and (4) radiation-fluid dynamical calculations with convection.

5. Connections between mass spectra and linear dynamical stability analysis

A comprehensive linear analysis of isothermal protoplanets was given by Schönke, J. (2005) who found two types of linear instabilities that operate in protoplanets for sufficiently high-nebula-densities and are damped by large cores. One characteristic peak in the mass spectra obviously results from structures in the outer envelope, which are in a state describable as the onset of self-gravity. These models were found for a certain region in the parameter set of core mass and gas pressure at the core surface (cf. ‘region IV in Pečnik & Wuchterl (2005) or Broeg (2006) and have all the same characteristic mass, leading to the peak in the spectra. The linear stability analysis (cf. Schönke, J. (2005)) shows a clear connection between this class of models and a dynamical instability. Almost all models are linearly unstable, except for a small strip in the low pressure regime. What kind of consequences for the spectra follow from this result can not be answered completely from the standpoint of the linear analysis because no information about the dynamical evolution can be gathered from it. It is likely that a part of the unstable models will end up in the stable low pressure strip, a behaviour which the non-linear stability analysis shows (cf. Pečnik (2005), Pečnik & Wuchterl (2006)). At least we can say that a ‘stability-corrected’ mass spectrum would reduce the ‘strength’ of the peak significantly. The appearance of the described unstable models mainly depends on the available envelope volume (i.e. the Hill radius) and therefore on the orbital distance or period. As the Hill radius becomes small (i.e. orbital distance or period are small), the region where unstable envelope structures are built up shifts to smaller core masses and the spectral peak broadens. Because of the quantitative differences between isothermal and detailed non-isothermal protoplanets it is nor possible to presently decide which peak in the Mark 1 spectra will suffer the flattening, likely the more massive one of the two main peaks but possibly both.

6. Non-linear stability of short-period isothermal proto-planets

Isothermal solution manifolds Pečnik & Wuchterl (2005) for short-period (P=1...4d) proto-planets (PPs) have different non-linear stability properties than those of long-period PPs (see Pečnik, B. and Wuchterl, G. (2007) for dynamical properties summary, and Pečnik (2005) for $a = 5$ AU orbital region details). Unlike long-period PPs, the critical core mass ($M_{crit}$) for short-period isothermal PPs gets larger for smaller orbital periods. Isothermal Hot-Jupiter’s $M_{crit}$ also strongly depends on the temperature and the density of the surrounding nebula. Additionally, massive subcritical short-period PPs are more linearly stable than long-period counterparts Schönke, J. (2005).

Fluid-dynamical calculations at orbital periods of 1 and 4d for a solar mass primary, show that the non-linear stability properties of isothermal subcritical PPs with temperatures of $T_1=1260$ K and $T_2=1600$ K are almost identical, which hints that most of these equilibria might be thermally stable. Short-period subcritical PPs with gas density at the core surface ($\rho_{cs}$) of up to around 10 kg m$^{-3}$ are stable and just oscillate around the equilibrium if perturbed. Subcritical PPs with higher $\rho_{cs}$ will make a transition towards a corresponding solution with lower $\rho_{cs}$, and could either loose or gain envelope mass in the process. Solution with still higher $\rho_{cs}$ will eject more than 95% of its envelope. Finally, a compact PP (a PP with highest $\rho_{cs}$) will remain stable, but will include an outgoing wind, supersonic at the outer boundary. Depending on the level of envelope compactness, mass loss due to this wind could be anything from completely negligible to a significant percent of the envelope mass within a single sound-crossing time. Additional effect of this wind is “puffing-up” of the outer PP’s stratifications. All of these properties suggest that massive stable subcritical PPs are formed more easily at small orbital distances (than at larger orbital distances), and that quasi-hydrostatic evolutionary paths are available up to the $M_{crit}$.

7. Quasi-hydrostatic calculations and secular/thermal stability

The previous sections were dedicated to the isothermal case that is computationally very efficient but cannot provide quantitative agreement with present day giant planet masses. To analyze the stability of the Mark 1 population directly, i.e. with the time-dependent versions corresponding to the the static Mark 1 equilibria, a full stellar evolution type calculation is necessary. Such calculations, with constitutive relations identical to the Mark 1, have been performed for selected parts of the planetary manifolds for orbital periods of 1 and 4 days for the solar mass case. The result is that in all cases calculated, i.e. for core masses to $10^{23}$ kg and total masses up to about 50 earth masses the protoplanets contract and loose less than 30% of their total mass within the 100 Ma calculation period.

8. Non-linear convective radiation-fluid-dynamic calculations

Finally we performed full non-linear-radiation fluid calculations for selected protoplanetary models in continuation of Broeg & Wuchterl (2006). These calculations are the largest computational effort and have been done in parts of the CoRoT-Mark 1 planet population that are thought to be particularly unstable from both the population analysis, Broeg (2006) which finds the planets at largest orbital distance to more likely become dynamical and the isothermal linear and non-linear studies, Schönke, J. (2005), Pečnik (2005), that find the high envelope mass, high nebula pressure, low core mass protoplanets to be the most likely ones to suffer an instability. Because we are primarily interested in the stability here, unlike Broeg & Wuchterl (2006) we use a constant nebula state for the outer boundary condition — in practice that means an unlimited mass reservoir instead of a feeding zone with given mass.
We used the equations of Wuchterl & Tscharnuter (2003) with planetary boundary conditions, Wuchterl (1991). The dynamic evolution of 6 planets of the Mark 1 population for a 16d period at 1 M⊙ and core masses of 0.47 and 1.11 Mearth was calculated with static models as initial condition. These core masses are relatively low compared to the typical critical masses for close-in planets, see Broeg (2006) to select potentially unstable planets. It turned out that the envelope mass average of the first close-in planets, see Broeg (2006) to select potentially unstable planets. It turned out that the envelope mass average of the first close-in planets, see Broeg (2006) to select potentially unstable planets. It turned out that the envelope mass average of the first close-in planets, see Broeg (2006) to select potentially unstable planets. It turned out that the envelope mass average of the first close-in planets, see Broeg (2006) to select potentially unstable planets. It turned out that the envelope mass average of the first close-in planets, see Broeg (2006) to select potentially unstable planets. It turned out that the envelope mass average of the first close-in planets, see Broeg (2006) to select potentially unstable planets. We therefore followed a two-fold strategy. We studied simplified problems (linear isothermal stability) in a comprehensive way and sampled a hierarchy of more detailed models - nonlinear dynamical stability of isothermal PP, nonlinear secular stability of realistic PP and did a limited set of non-linear radiation fluid-dynamical calculations at positions that were indicated as particularly unstable in the more comprehensive but simpler models. While the simpler models provide a detailed and synoptic view, they indicate instabilities in some parts of the planet manifolds. Approximately 1/3 to 1/6, say, of the more massive envelopes with small core are unstable and likely corresponding to realistic Mark 1 planets. The few detailed models show that improved physics, in particular time-dependent convection acts stabilising at the investigated places an indicates stability for all cases calculated. In particular a non-linear development of a case with linear adiabatic dynamic instability indicated by globally low first adiabatic exponents (down to 1.25) is damped out by convection and the respective planets remain static and thus stable, valid elements of the Mark 1 planet population.

9. Close-in Convective Protoplanets

The fact that ‘realistic’ close-in exoplanets turn out to be more stable than estimated by isothermal models can be explained by the fact that the envelopes are largely convective. Largely convective protoplanets, Wuchterl (1993), Ikoma et al. (2001) damp dynamical instabilities in their large convection zones, Wuchterl (1995) and are favouring accretion, Wuchterl et al. (2000).

Thus as far as the sample calculations are representative of the ensemble, the planets of the CoRoT Mark 1 population are stable. The onset of apparent adiabatic dynamical instability in 2/6 models is damped by convection.

10. Discussion

The CoRoT-Mark 1 theoretical planet populations rely on the implicit assumption that there is no preferred scale in the masses of planetary cores and no preferred range of nebula pressures. Together this requires to assume a diversity of gravitationally stable nebulae. That does not imply arbitrary ranges for core-masses and nebula pressures but sufficiently wide ranges so that the planetary core-envelope equilibria can ‘draw’ any required value from a nebula at a given orbital radius. If some cores that are required in the population do not form, the respective planets would not be able to form and thus not appear in the observed mass spectra. Presently we do neither have a sufficiently comprehensive overview about the diversity of nebula properties from observations nor from theory. Thus the CoRoT-Mark 1 predictions rely on an assumption of nebula diversity. Presently the best argument that there is indeed a huge diversity of nebular conditions comes from the observation of extreme planets as GQ Lupi b and HD 149 026 b.

The second key assumption to derive the mass spectra is that the planets are static. While planet formation has been found to be a hydrostatic process under many conditions and for most of the time, there are important exceptions and there is no guarantee that the results from studies of planet formation for a rather restricted set of conditions can be generalised to the diverse nebula properties required by the Mark 1 population.

A general study of stability for the entire ensemble of ~ 10⁶ ‘realistic’ protoplanets is presently beyond our computational reach. We therefore followed a two-fold strategy. We studied simplified problems (linear isothermal stability) in a comprehensive way and sampled a hierarchy of more detailed models - nonlinear dynamical stability of isothermal PP, nonlinear secular stability of realistic PP and did a limited set of non-linear radiation fluid-dynamical calculations at positions that were indicated as particularly unstable in the more comprehensive but simpler models. While the simpler models provide a detailed and synoptic view, they indicate instabilities in some parts of the planet manifolds. Approximately 1/3 to 1/6, say, of the more massive envelopes with small core are unstable and likely corresponding to realistic Mark 1 planets. The few detailed models show that improved physics, in particular time-dependent convection acts stabilising at the investigated places an indicates stability for all cases calculated. In particular a non-linear development of a case with linear adiabatic dynamic instability indicated by globally low first adiabatic exponents (down to 1.25) is damped out by convection and the respective planets remain static and thus stable, valid elements of the Mark 1 planet population.

11. Conclusions: CoRoT’s planets

Based on the assumption that diverse gravitationally stable protoplanetary nebulae are produced by the star formation process we predict the short period, \( T_{\text{orb}} < 16 \text{d} \) planet population of a typical CoRoT field: (1) they closely follow the CoRoT-Mark 1 planet population applied to the stellar population mix of the field, i.e. show planetary initial mass functions (IMFs) as given in Fig. 1. (2) these IMFs have to be corrected for particle loss process to arrive at the epoch of observation mass function; these corrections are discussed in a parallel letter. (3) typical field planetary IMFs depend on orbital period and have the largest mass peak around a Jupiter mass, we refer to Broeg (2006) for more predictions of properties of single-host-star-mass planetary IMFs. (4) the broad peaks in the stellar population mix IMFs point to a large population with masses of \( 1 \text{ to } 3 \times 10^{26} \text{kg} \), (50 – 160 earth-masses). (5) there is an abundant population of Hot Neptunes and an important population filling the solar-system mass gap between Neptune and Saturn.

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References

Baglin, A., Auvergne, M., Barge, P., et al., 2002, in ESA SP-485: Stellar Structure and Habitable Planet Finding, ed. B. Battrick, F. Favata, I. W. Roxburgh, & D. Galadi, 17–24
Broeg, C., 2006, PhD thesis, Universität Jena
Broeg, C. & Wuchterl, G., 2006, in Tenth Anniversary of 51 Peg-b: Status of and prospects for hot Jupiter studies, ed. L. Arnold, F. Bouchy, & C. Moutou, 70–77
Broeg, C., 2006, Mass spectra CoRoT Mark 1 v1.1, online at http://www.astro.uni-jena.de/corot/
Ilkova, M., Emori, H., & Nakazawa, K., 2001, ApJ, 553, 999
Pečnik, B., 2005, PhD thesis, Ludwig-Maximilians Universität, München
Pečnik, B. & Wuchterl, G., 2005, A&A, 440, 1183
Pečnik, B. & Wuchterl, G., 2006, MNRAS, submitted
Pečnik, B. and Wuchterl, G. 2007, MNRAS, submitted, submitted
Pečnik, B., 2003, Master’s thesis, Univ. Zagreb
Schönke, J. 2005, Master’s thesis, Universität Jena
Wuchterl, G. 1991, Icarus, 91, 39
Wuchterl, G. 1993, Icarus, 106, 323
Wuchterl, G. 1995, Earth Moon and Planets, 67, 51
Wuchterl, G., Guillot, T., & Lissauer, J. J. 2000, Protostars and Planets IV, 1081
Wuchterl, G. & Tscharnuter, W. M. 2003, A&A, 398, 1081