Chaotic brane inflation

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We illustrate a framework for constructing models of chaotic inflation where the inflaton is the position of a D3-brane along the universal cover of a string compactification. In our scenario, a brane rolls many times around a nontrivial one-cycle, thereby unwinding a Ramond-Ramond flux. These “flux monodromies” are similar in spirit to the monodromies of Silverstein, Westphal, and McAllister, and their four-dimensional description is that of Kaloper and Sorbo. Assuming moduli stabilization is rigid enough, the large-field inflationary potential is protected from radiative corrections by a discrete shift symmetry.

I. INTRODUCTION

Perhaps the simplest phenomenological model of inflation [1–3] is due to Linde’s monomial potential [4], which undergoes what is called “chaotic inflation” due to its expected behavior on large scales. Chaotic inflationary models are not obviously natural in the context of effective field theory precisely because of the requirement that the potential be sufficiently flat over super-Planckian field distances. In the quadratic model, the inflationary models are not obviously natural in the context of effective field theory precisely because of the requirement that the potential be sufficiently flat over super-Planckian field distances. In the quadratic model, the inflaton mass must be of order 10^{-26}\text{GeV}, which in turn increases the four dimensional Planck mass.

Here we will find a stringy realization of large-field inflation. As in brane inflation [8–10], the inflaton represents the position of a D3-brane in a six-dimensional compactification manifold, assumed to be sufficiently stable. The potential felt by the D3-brane due to the five-form compactification manifold, assumed to be sufficiently stable. In the probe approximation, the potential felt by a D3 must be exactly periodic, just as for an axion. Furthermore, the five-form flux takes quantized values over the five-cycle which is dual to the one-cycle. Assum ing rigid moduli stabilization, the flux potential is exactly quadratic in the discrete flux winding \( f_{\pi^5} F_5 \). By turning on the coupling of the D3 to the background five-form flux, the periodicity is lifted, and the discrete flux becomes a continuous parameter, contributing an exactly quadratic term to the potential. We now illustrate this with a simple example.

II. CHARGES IN COMPACT SPACES WITH NONTRIVIAL FIRST HOMOLOGY

As a warm-up example, let us consider a single electron and positron in the compact space \( S^2 \times S^1 \). The action and equations of motion are given by

\[
S = \int \frac{1}{4} F_{\alpha\beta} \wedge \ast F_{\alpha\beta} + e A_4 \wedge \delta_3(M^4),
\]

\[
d \ast dA_4 = e \delta_3(M^4),
\]

where \( M^4 \) is the oriented world lines of the charges. Integration and differentiation of the equations of motion require that the point-particle current \( \delta_3(M^4) \) satisfy

\[
\int_{S^2 \times S^1} \delta_3(M^4) = 0 \quad \text{and} \quad d \delta_3(M^4) = 0,
\]

or equivalently (see Appendix),

\[
S^2 \times S^1 \cap M^4 = 0 \quad \text{and} \quad \partial M^4 = \emptyset,
\]

respectively. We abuse the notation \( \cap \) to mean both the intersection and the winding number of the intersection, so the left-hand side of Eq. (4) should be read as stating that there are equally many positive points as negative points in the total intersection.

These equations simply state that no net charge can occupy a compact space, and electric current is conserved. We can either ensure that \( M^4 \) has no net time-like winding (as we have done), or add a diffuse background “jel-
charge to the action. A homogenous jellium contribution is just proportional to the spatial volume form,
\[
\delta_3(M^1) \rightarrow \delta_3(M^1) - n \frac{\text{vol}_3(S^2 \times S^1)}{\text{vol}_3(S^2 \times S^1)},
\]
with \( n = (S^2 \times S^1 \cap M^1) \). The uniform charge density cancels the tadpole.

Let us imagine that \( M^1 \) represents a single positive charge and a single negative charge. We can compute the potential between them by finding the Green’s function on this space. We expect the usual Coulomb interaction to be modified by two effects:

- Because the space is compact the potential will not be Coulombic at large distances.

- Because the space has nontrivial first homology \( H^1(S^2 \times S^1) = \mathbb{Z} \), the field strength will not be single-valued, but will depend on the winding of the particles’ paths.

It is the latter effect which we find useful here, as it enables one to change the electric flux on the \( S^1 \). It is straightforward to calculate the difference in flux caused by transporting \( \mathbf{one} \) of the two charges around the \( S^1 \). The transport of one of the particles around the one-cycle means that \( M^1 \) acquires winding number equal to one. The flux on the \( S^1 \) is measured by choosing a fixed time and \( z \) coordinate, and then integrating the dual field strength \( F_2 = \ast dA_1 \) over the \( S^2 \).

To calculate the change in flux caused by a single winding of a particle, let us define a 3-manifold (with boundary) \( M^3 \) which spans an interval in time \([t_i, t_f]\) times the full \( S^2 \) cycle. Then
\[
\int_{M^3} d \ast dA_1 = \int_{\partial M^3} \ast dA_1 = \int_{S^2} \ast dA_1 \bigg|_{t_i}^{t_f} = e \int_{M^3} \delta_3(M^1) = e \cdot M^1 \cap M^3 = e,
\]
and so
\[
\Delta F_2 \equiv e \ast \text{vol}_2(S^2) \bigg/ \text{vol}_2(S^2).
\]

Thus the electric field in the \( z \) direction changes by one unit each time a particle is transported around the circle in the \( z \) direction. A simple interpretation of this is that the charge drags the field lines around the cycle.

We can immediately write down the homological piece of the potential. If the metric is given by \( ds^2 = -dt^2 + R^2 (d\theta^2 + \sin^2 \theta d\phi^2) + dz^2 \), with \( z \equiv z + L \), then
\[
A_1(\Delta z) = \frac{\Delta z}{L} \frac{ez}{4\pi R^2} dt + \text{ single-valued part }
\]
where \( \Delta z \) is the \( z \) separation of the two charges as measured on the universal cover. The flux part of the electron potential is thus
\[
V(z) = \frac{e^2 z^2}{4\pi R^2 L} = \frac{e^2 z^2}{V_\perp},
\]
where \( V_\perp = \int \text{vol}_3(S^2 \times S^1) = 4\pi R^2 L \).

The flux potential cancels the jellium [22, 23] contribution. We can think of the jellium term in the potential as arising due to the finite compactification volume. A jellium term is required in the potential felt by a probe charge, since the field strength is then single-valued. But transport of a physical charge around a nontrivial cycle does not leave the field strength invariant, and so the probe charge is an inadequate description.

In a sense, one can say that the configuration space of charges and flux is not simply the product of the compact manifold and its first homology, but rather is a nontrivial fibration: one can change the flux by transporting charges around the one-cycles associated with them.

Although the potential is exactly quadratic classically (and even in perturbation theory), there are nonperturbative corrections. Pair production will eventually discharge any potential exceeding twice the electron mass. This is such a slow enough process that we can safely ignore it. Furthermore, adiabatic motion of a charge will never be able to wind more than one unit of flux because of avoided level crossing [6]. This is not a problem except on timescales long compared to \( m^{-1} \exp(mL) \), where \( m \) is the electron mass.

### III. INGREDIENTS FOR CHAOTIC BRANE INFLATION

The ingredients we will need is an F-theory compactification of Type IIB string theory which contains at least one mobile D3-brane. Furthermore, the six-dimensional transverse space must have a nontrivial first homology, i.e., \( H^1(M^6) = \mathbb{Z} \) or \( \mathbb{Z}_N \). Because the D3 moduli in the direction of the nontrivial one-cycles are lifted at tree level, these models may lack supersymmetry. All closed string moduli must be sufficiently stabilized, in order that inflation may take place in this background. It is further necessary that the periodic portion of the potential be flat enough that the full potential has only a single minimum. In the probe approximation, the D3 has a discreet “shift symmetry” associated with transport about the one-cycle, but this may not be sufficient to guarantee local flatness. As illustrated before, the inflationary potential exists due to nontrivial winding of the five-form flux about the homological one-cycle. The D3-brane moves classically through this cycle to unwind the flux. We will assume that the moduli stabilization is rigid enough to ignore the backreaction of the dynamical flux. This assumption is generically false in known warped flux compactifications [24–26], but such effects may actually flatten the potential [15].

The potential induced by brane monodromy is
\[
V(z) = \frac{\mu_{D3}^2 z^2}{M_{10,P}^4 L^6}
\]
where \( L^6 \) is the volume of the compact space, \( M_{10,P} \) is the ten-dimensional reduced Planck mass, and \( \mu_{D3} = M_{4,P}^4 \).
is the D3 charge. We assume the string coupling to be of order unity. In terms of the four-dimensional reduced Planck mass, \( M_P^2 = M_{10D}^2 L^6 \), and for a canonically normalized inflaton \( \phi = \frac{\sqrt{\mu_{D3}}}{3} \), we find the potential

\[
V(\phi) = \frac{\phi^2}{\mu_{D3} L^6} = \frac{\phi^2}{M_P L^3}.
\] (10)

To achieve reasonable density perturbations, the quadratic model needs the inflaton mass to obey

\[
m_\phi^2 \approx 10^{-11} M_P^2,
\] (11)

which requires the compactification scale to be

\[
L \approx \frac{10}{M_{10P}} \approx 10^4 M_P.
\] (12)

In terms of the inflaton, this scale corresponds to a field distance

\[
2\pi f_\phi = \sqrt{\mu_{D3}} L = \sqrt{\frac{M_P}{L}}.
\] (13)

Hence, successful large-field inflation will require the brane to undergo of order a few thousand revolutions, so any model must have first Homology large enough to permit this, i.e.

\[
H_1(M^6) = \mathbb{Z} \quad \text{or} \quad \mathbb{Z}_N
\] (14)

with \( N \gtrsim 2 \times 10^3 \). This rather large number can be relaxed by no more than two orders of magnitude by allowing the size of the one-cycle to be much larger than the natural scale \( \sqrt{\frac{\mu_{D3}}{L}} \).

In the \( \mathbb{Z} \) case, the quadratic approximation for \( V(\phi) \) must eventually break down, due to backreaction. If we assume the modulus \( L \) is very heavy, it can be written as \( L(\phi) \), which grows as more flux is wound on the transverse space. If this is the only effect of backreaction, the inflaton potential is flattened at large flux values, and reaches a maximum if \( dL(\phi)/d\phi \) exceeds \( \frac{4}{\phi^2} L(\phi)/\phi \).

However, a steepening [15] of the inflaton potential could instead occur due to kinetic coupling between the inflaton and \( L(\phi) \), say of the form

\[
\left( \frac{L(\phi)}{L(0)} \right)^n \phi^2.
\] (15)

for sufficiently negative \( n \). We will assume that the kinetic coupling is subdominant, and hence that backreaction leads to a flattening of the inflaton potential at sufficiently large field values \( \phi \sim N f_\phi \). Qualitatively speaking, a potential which is quadratic at small field values, and flat at large field values can be thought of as approximately sinusoidal over the range \( |\phi| \lesssim N f_\phi \).

The \( \mathbb{Z}_N \) case has extended periodicity \( z = z + NL \), and so the homological part of the potential in each of the above cases is approximately given by

\[
V(\phi) \approx \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{N f_\phi} \right) \right],
\] (16)

with

\[
\Lambda = \frac{\sqrt{N}}{2\pi^2 L}, \quad f_\phi = \frac{\sqrt{M_P}}{2\pi L}
\] (17)

where \( N \) represents either the backreaction scale or the size of the homology group. This scenario could be called natural brane inflation, following Refs. [27, 28], although it avoids the problems associated with large axion decay constants \( f_\phi \) by the appearance of the large factor \( N \) in the potential of Eq. (16), allowing \( f_\phi \ll M_P \ll N f_\phi \). Alternative approaches to this problem can be found in Refs. [29, 30].

We additionally require the single-valued part of the potential

\[
V_{s.v.}(\phi) \approx \lambda^4 \cos \left( \phi/f_\phi \right)
\] (18)

to be relatively flat, meaning \( \lambda \lesssim 1/L \).

To achieve 60 \( e \)-folds of slow-roll inflation, we must arrange the scalar field to initially have a super-Planckian vacuum expectation value, \( \phi \gtrsim 15 M_P \). The Hubble scale during inflation is then \( H \approx 10^{-5} M_P \), which is almost two orders of magnitude below the Kaluza-Klein scale \( 2\pi/L \). However, the four-dimensional potential will be of order \( V \approx 10^{-10} M_P^2 \), which exceeds the tension of a D3-brane by two orders of magnitude, opening the possibility for brane tunneling [31] or nucleation [32]. Because these are slow processes, our description remains valid. Indeed, brane nucleation\(^1\) could give rise to the mobile inflaton, although inflation will then end with brane-antibrane annihilation, but unlike Ref. [33], the bubble need not self-annihilate until after many laps are completed. The final D3-D3 annihilation will result in the formation of a cosmic-string network [34].

IV. DISCUSSION

We have provided a simple framework for large-field brane inflation. To construct realistic models, a number of hurdles must first be addressed, the most significant of which is moduli stabilization. However, because our framework relies on a nontrivial first homology group, much of the progress made on moduli stabilization of warped compactifications does not apply here. Another potential difficulty may arise in obtaining a flat enough periodic portion of the brane potential. If the modulation of the quadratic piece is too large, there may not be a long enough slow-roll trajectory. On the other hand, a periodic modulation of the inflaton potential can lead to detectable non-Gaussianity in the cosmic microwave background [35]. Finally, it is unlikely that supersymmetry can be unbroken in the models considered here, since the D3 moduli receive an explicit mass, rather than

\(^1\) This possibility is thoroughly considered in Refs. [20, 21].
from a spontaneous uplifting, say by the introduction of antibranes.

Nevertheless, a number of intriguing features arise here, foremost being a UV description of large-field inflation. The monodromies of this framework are extremely easy to visualize, being simply the motion of a (point-like) brane around a one-cycle. By traversing the cycle (perhaps several thousand times) a Ramond-Ramond flux is unwound, realizing either chaotic or natural inflation, both of which predict significant tensor modes in the cosmic microwave background.

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Appendix A: The de Rham delta function

Here we review a simple notation [36] appropriate for calculating the effects of localized sources coupled to gauge potentials. The new object is a singular differential form which we call the “de Rham delta function.”

1. Definition

On a $D$-dimensional oriented manifold $\mathcal{M}^D$ with $\mathcal{M}' \subseteq \mathcal{M}^D$ an oriented submanifold of dimension $r$, we define the de Rham delta function $\delta_{D-r}(\mathcal{M}')$ as follows:

$$\int_{\mathcal{M}^D} C_r \wedge \delta_{D-r}(\mathcal{M}') = \int_{\mathcal{M}^D \cap \mathcal{M}'} C_r,$$  \hspace{1cm} (A1)

where the pullback is implicit on the rhs. The subscripts denote the order for differential forms, and superscripts denote the dimension for manifolds. Stokes’ theorem then implies

$$\int_{\partial \mathcal{M}^D} C_{r-1} \wedge \delta_{D-r}(\mathcal{M}') = \int_{\partial \mathcal{M}^D} [dC_{r-1} \wedge \delta_{D-r}(\mathcal{M}')] + (-1)^{r-1} C_{r-1} \wedge d\delta_{D-r}(\mathcal{M}'),$$

and so

$$d\delta_{D-r}(\mathcal{M}') = (-1)^r \delta_{D-r+1}(\partial \mathcal{M}'),$$ \hspace{1cm} (A2)

where we have used the fact that $\partial(\mathcal{M}' \cap \mathcal{M}^s) = (\partial \mathcal{M}' \cap \mathcal{M}^s) \cup (-1)^{D-r}(\mathcal{M}' \cap \partial \mathcal{M}^s)$. Here $\cup$ is essentially the group sum of $r$-chains in $\mathcal{M}^D$. This definition of $\cup$ is equivalent to

$$\delta_{D-r}(\mathcal{M}^r \cup \mathcal{M}^r) = \delta_{D-r}(\mathcal{M}^r) + \delta_{D-r}(\mathcal{M}^r).$$  \hspace{1cm} (A3)

Following the definition we also find

$$\int_{\mathcal{M}^D \cap \mathcal{M}' \cap \mathcal{M}^r} C_{r+s-D} = \int_{\mathcal{M}^D \cap \mathcal{M}' \cap \mathcal{M}^r} C_{r+s-D} \delta_{D-r}(\mathcal{M}'),$$



which leads to the relation

$$\delta_{D-r}(\mathcal{M}') \wedge \delta_{D-s}(\mathcal{M}^s) = \delta_{2D-r-s}(\mathcal{M}' \cap \mathcal{M}^s).$$  \hspace{1cm} (A4)

This identity illuminates some generic features of submanifolds.

- The intersection of an $r$- and an $s$-dimensional submanifold in $D$ dimensions will generally be of dimension $r + s - D$.
- When the previous statement does not hold, integration on the intersection must vanish. This is because the intersection is not stable under infinitesimal perturbation (and not transversal).
- When two submanifolds each have odd codimension, the orientation of their intersection flips when the order of the manifolds is reversed. This is consistent with the Leibniz rule for the boundary operator given below Eq. (A2). As an example of this, consider two 2-planes in three dimensions, whose intersection is a line. The orientation of each plane is characterized by a normal vector, and the antisymmetric cross product of these is used to determine the orientation of the line of intersection.
- We should think of $\cap$ as being the oriented intersection operation from intersection homology which makes the above properties automatic. It is stable under infinitesimal perturbation of either submanifold.

2. Coordinate representation

The coordinate representation of $\delta_{D-r}(\mathcal{M}')$ is straightforward in coordinates where the submanifold is defined by the $D-r$ constraint equations,

$$q \in \mathcal{M}' \Rightarrow \lambda^i(x^1[q], ..., x^D[q]) = 0,$$  \hspace{1cm} (A5)

with $i = 1...D-r$, via

$$\delta_{D-r}(\mathcal{M}') = \delta^{(D-r)}(\lambda^i) \ d\lambda^1 \wedge ... \wedge d\lambda^{D-r},$$  \hspace{1cm} (A6)

where $\delta^{(D-r)}$ is the usual $(D-r)$-dimensional Dirac delta function. The well-known transformation properties of the Dirac delta function make this object automatically
a differential form. (Thus the only meaningful zeros of the $\lambda^i$ are transversal zeros, i.e., those where $\lambda^i$ changes sign in any neighborhood of the zero.) If a submanifold is $D$-dimensional, then the corresponding de Rham delta function is simply the characteristic function, $\delta_0(M^D) = \chi_{M^D}$, with

$$\chi_{M^D}(q) = \begin{cases} 1 & q \in M^D, \\ 0 & q \notin M^D. \end{cases} \quad (A7)$$

One may describe submanifolds with boundary by multiplication with this scalar de Rham delta function using Eq. (A4). As an example, if $M^1$ is the positive $x$ axis in $\mathbb{R}^3$, then

$$\delta_2(M^1) = \delta(y)\delta(z)\Theta(x)dy \wedge dz, \quad (A8)$$

where the characteristic function is the Heaviside function $\Theta(x)$. Notice that the orientation of this submanifold has been chosen to be along the $+z$ direction, consistent with Eq. (A2) and the fact that its boundary is $\text{minus}$ the point at the origin.

The $\lambda^i$ do not need to be well defined on the entire manifold, and in fact they only need to be defined at all in a neighborhood of $M^r$. Thus despite its appearance in Eq. (A6), $\delta_{D-r}(M^r)$ is not necessarily a total derivative. If all of the $\lambda^i$ are well defined everywhere, then $M^r$ is an algebraic variety. By Eq. (A2) and Eq. (A6) it can be seen that all algebraic varieties can be written globally as boundaries. It may be that the $\lambda^i$ are well defined only in a neighborhood of $M^r$, in which case $M^r$ is a submanifold. Near points not on $\partial M^r$ we may think of $M^r$ locally as a boundary, just as we may think of a closed differential form as locally exact. Nonorientable submanifolds will correspond to constraints that may be double valued, that is $\lambda^i$ may return to minus itself upon translation around the submanifold. We considered such cases in Ref. [36].

Another important case occurs when $M^r$ is only an immersion, i.e., it intersects itself. The $\lambda^i$ are path dependent here, as well. Consider the immersion $S^1 \subset \mathbb{R}^2$ defined by the constraint $\lambda = 2 \arcsin(y) - \arcsin(x) = 0$. This looks like a figure-eight centered on the origin of the plane. Clearly $\lambda$ is multivalued, and to get a complete figure-eight requires summing over two branches of $\lambda$. We suppress the sum in Eq. (A6). Notice that the figure-eight immersion satisfies

$$\delta_1(S^1) \wedge \delta_1(S^1) = \delta_2(0) - \delta_2(0) = 0.$$ 

The self-intersection of this immersion is twice the point at the origin, but since the orientation (sign) of the intersection is negative for exactly one of the two points of intersection, the total self-intersection vanishes. For even-codimension immersions, the sum over branches allows for nonzero self-intersection from cross terms. By the antisymmetry of the wedge product, one may show that the self-intersection of an immersion of odd codimension will always vanish, assuming $M^D$ is orientable. One may also compute the $n$th self-intersection of an immersion via

$$\delta_n(D-r)(\cap^n M^r) = \bigwedge_n \delta_{D-r}(M^r).$$

### 3. Geometry

If we introduce a metric on our manifold, we can measure the volume of our submanifolds with the volume element from the pull-back metric. We use $*_r$ to denote the Hodge star on $M^r$, so the induced volume element is $*_r \mathcal{V}$. Looking at the coordinate definition of $\delta_{D-r}(M^r)$ given in Eq. (A5), we see that each of the $\lambda^i$ is constant along the submanifold, and so $d\lambda^i$ is orthogonal to the submanifold. Thus, a good candidate for a volume element on $M^r$ is $*(d\lambda^1 \wedge ... \wedge d\lambda^{D-r})$. To remove the rescaling redundancy in the $\lambda^i$’s we may divide by $||\bigwedge_i d\lambda^i||$, where the norm of a differential form $C_r$ is defined by the scalar

$$||C_r|| = \sqrt{\epsilon_r (C_r \wedge *C_r)}.$$

Hence, we write

$$*_r \mathcal{V} = \frac{\epsilon_r \delta_{D-r}(M^r)}{||\delta_{D-r}(M^r)||}, \quad (A9)$$

where the pull-back is implicit on the rhs. Despite its appearance, $\frac{\epsilon_r \delta_{D-r}(M^r)}{||\delta_{D-r}(M^r)||}$ is an $r$-form living in all $D$ dimensions, although it is only well defined near $M^r$. This is because in Eq. (A9), the Dirac delta function coefficients cancel, leaving dependence only on the smooth $\lambda^i$’s. Because of this, we may define $d_{*_r \mathcal{V}} (\delta_{D-r}(M^r))$ using the full $D$-dimensional exterior derivative. This is well defined on $M^r$, and when restricting to points on the submanifold,

$$d_{*_r \mathcal{V}} (\delta_{D-r}(M^r)) = 0 \iff M^r \text{ is extremal.} \quad (A10)$$

From Eq. (A9) it is evident that

$$||\delta_{D-r}(M^r)|| = \epsilon_r \, *_r \delta_{D-r}(M^r).$$

More generally, the action by this Hodge star can be rewritten as

$$*_r F_s = \frac{F_s \wedge \delta_{D-r}(M^r)}{||\delta_{D-r}(M^r)||}, \quad (A11)$$

where $F_s$ is an $s$-form living on $M^r$ and the pull-back is implicit on the rhs.

### 4. Topology

One feature of the de Rham delta function is that Poincaré duality becomes manifest. Here we assume $M^D$ is orientable and compact with no boundary. Then Eq. (A2) tells us that $\delta_{D-r}(M^r)$ is closed if and only if
\( \mathcal{M}^r \) is a cycle, and \( \delta_{D-r}(\mathcal{M}^r) \) is exact if \( \mathcal{M}^r \) is a boundary. To complete this correspondence between the \( r \)th homology and the \((D - r)\)th de Rham cohomology of \( \mathcal{M}^D \), we need to show that

\[
\delta_{D-r}(\mathcal{M}^r) = df_{D-r-1} \quad \implies \quad \mathcal{M}^r = \partial \mathcal{M}^{r+1}. \tag{A12}
\]

But this is not true when torsion is present. Consider the manifold \( \mathbb{R}\mathbb{P}^3 = SO(3) \) which has a single nontrivial one-cycle \( \zeta^1 \), i.e. \( H_1(\mathbb{R}\mathbb{P}^3, \mathbb{Z}) = \mathbb{Z}_2 \). Since the group sum of two of these cycles is trivial, they must form a boundary, \( \zeta^1 + \zeta^1 = \partial \mathcal{M}^2 \) and so \( \delta_2(\zeta^1) = d\delta_1(\mathcal{M}^2) \) which means

\[
\delta_2(\zeta^1) = \frac{1}{2}d\delta_1(\mathcal{M}^2). \tag{A13}
\]

Since \( \zeta^1 \) is not a boundary, it is false to claim that all exact de Rham delta functions are Poincaré dual to boundaries. Only by using real coefficients can we make the statement that

\[
\mathcal{M}^r \cong \mathcal{M}^r \quad \iff \quad [\delta_{D-r}(\mathcal{M}^r)] \cong [\delta_{D-r}(\mathcal{M}^r)]. \tag{A14}
\]

De Rham’s theorem gives us the isomorphism between homology and cohomology

\[
H_r(\mathcal{M}^D; \mathbb{R}) \cong H^r(\mathcal{M}^D; \mathbb{R}),
\]

and Poincaré duality asserts that

\[
H_r(\mathcal{M}^D; \mathbb{R}) \cong H_{D-r}(\mathcal{M}^D; \mathbb{R}).
\]

The de Rham delta function provides the isomorphism

\[
H_r(\mathcal{M}^D; \mathbb{R}) \cong H^{D-r}(\mathcal{M}^D; \mathbb{R}). \tag{A15}
\]

In fact, if the cohomology basis is chosen such that

\[
\int_{\mathcal{M}^{D-r}} \omega^{(i)} = \delta^i,
\]

then

\[
[\delta_{D-r}(\mathcal{M}^r)] = [\omega^{(i)}], \tag{A16}
\]

where \( \mathcal{M}^r \) is the Poincaré dual of \( \mathcal{M}^{D-r} \), and together they satisfy

\[
\mathcal{M}^r \cap \mathcal{M}^{D-r} = \delta^i,
\]

i.e., their net intersection is a single positive point if \( i = j \), and is empty otherwise.

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