MSSM phenomenology in the large $\tan \beta$ regime

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ABSTRACT

We discuss aspects of the low energy phenomenology of the MSSM, in the large $\tan \beta$ regime. We explore the regions of the parameter space where the $h_t$ and $h_b$ Yukawa couplings exhibit a fixed point structure, using previous analytic solutions for these couplings. Expressions for the parameters $A_t$ and $A_b$ and the renormalised soft mass terms are also derived, making it possible to estimate analytically the sparticle loop – corrections to the bottom mass, which are important in this limit.

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1 Introduction

Among the extensions of the Standard Model (SM), supersymmetry seems to provide the best grounds towards a unification of the fundamental interactions. This is not only because of the natural solution to the hierarchy problem [1], but also because of the correct prediction of \(\sin^2 \theta_W\), and the convergence of the three gauge couplings at a point, at a scale \(O(10^{16})\) GeV, unlike in the non–supersymmetric grand unified schemes [2]. The simplest supersymmetric extension of the theory is the Minimal Supersymmetric Standard Model (MSSM) which has the minimal number of fields and Yukawa couplings that is consistent with the content of the SM. However, even in the MSSM several arbitrary parameters also exist. These are the initial conditions for the Yukawa couplings and the scalar masses, which are expected to be fixed at an even more fundamental level, like string theory. A simplified approach to reduce the number of these variables, would be to assume universality of all scalar masses at the unification scale. Then, one is left with only five new arbitrary parameters in addition to those of the non–supersymmetric standard model. However, while this is consistent with the low \(\tan \beta\) regime, in the large \(\tan \beta\) limit of the theory (where \(h_t \approx h_b\)), in order to get the correct radiative electroweak symmetry breaking pattern [3] one is forced to depart from universality. This latter case, i.e. large \(\tan \beta\) and non-universal boundary conditions for the scalars, appears naturally in many string derived models. In fact it is generally expected that irrespectively of the fundamental theory, the low energy model will look much like the supersymmetric standard model with non–universal boundary conditions for the soft terms. Further theoretical expectations suggest that the Yukawa couplings of the third generation are of the order of the common gauge coupling at the unification scale. Moreover, specific grand unified groups suggest equalities of the form \(h_t = h_b = h_\tau\) at \(M_U\).

It is interesting to investigate whether radiative corrections can determine the Yukawa couplings and possibly the soft masses of the effective low energy theory. For example, the Yukawa couplings are running quantities from the unification scale \(M_U\) down to low energy. If they are relatively large at \(M_U\), their low energy values exhibit a fixed point structure [4] and they are rather insensitive to their initial conditions. The analysis of the above becomes more interesting by the fact that there is recent experimental evidence for a top quark with a mass of \(O(180\text{GeV})\) which is compatible with the prediction of the fixed point structure.

In the present letter we investigate the fixed point structure of previous analytic solutions for the \(h_t,h_b\) couplings [5, 6, 7]. We further extend the existing analytic solutions [8] for the scalar masses in the case of \(\tan \beta \gg 1\), by including the contribution from the \(A_t\) and \(A_b\) terms. This is of particular importance not only for the determination of the scalar mass spectrum itself, but for the correct theoretical computation of the fermion masses. In particular, when \(\tan \beta \gg 1\) the bottom mass receives large corrections from
sparticle loops [9, 10]. The tau lepton receives also corrections of the same type but they are less significant. A precise determination of these corrections would require the knowledge of the scalar masses involved in the computation. An analytic approach in both Yukawa and soft mass terms would make possible a systematic exploration of these corrections.

2 Yukawa coupling fixed point

In this section we are going to use the results of [8] for the top and bottom Yukawa couplings, in order to identify the regions where these couplings exhibit a fixed point behaviour and give simplified expressions that describe these fixed points. The renormalisation group equations for the top-bottom system (when ignoring the $h_{\tau}$ Yukawa coupling), read

$$\frac{d}{dt} h_t^2 = \frac{1}{8\pi^2} \left\{ 6 h_t^2 + h_b^2 - G_Q \right\} h_t^2$$

(1)

$$\frac{d}{dt} h_b^2 = \frac{1}{8\pi^2} \left\{ h_t^2 + 6 h_b^2 - G_D \right\} h_b^2$$

(2)

where

$$G_Q = \sum_{i=1}^{3} c_Q^i g_i^2, \quad G_D = \sum_{i=1}^{3} c_D g_i^2$$

(3)

Here $t = \ln Q$, where $Q$ is the energy scale, $c_Q^i = \left\{ \frac{13}{15}, 3, \frac{16}{3} \right\}$ and $c_B^i = \left\{ \frac{7}{15}, 3, \frac{16}{3} \right\}$ for $U(1)$, $SU(2)$ and $SU(3)$ respectively. Ignoring small $U(1)$ corrections, this system can be solved. Defining the parameters $x, y$ through

$$h_t^2 = \gamma_Q^2 x, \quad h_b^2 = \gamma_Q^2 y,$$

one can make the transformation

$$u = \frac{k_0}{(x - y)^{5/6}}, \quad dI = \frac{6}{8\pi^2} \gamma_Q^2 dt$$

(5)

where $\omega = x - y$ and $I(t) = \frac{3}{4\pi^2} \int_0^t \gamma_Q^2(t') dt'$. The parameter

$$k_0 = 4 \frac{x_0 y_0}{(x_0 - y_0)^{7/6}}$$

(6)

depends on the initial conditions $x_0 \equiv h_{t,0}^2$ and $y_0 \equiv h_{b,0}^2$. Then, one forms a differential equation for the new variable $u$, which can be solved in terms of hypergeometric functions. In particular, the $t - b$ Yukawa coupling solutions can be expressed in terms of the variable $u$ as follows [3]:

$$h_t^2 \equiv \frac{1}{2} \gamma_Q^2 \omega (\sqrt{1 + u} + 1)$$

(7)

$$h_b^2 \equiv \frac{1}{2} \gamma_Q^2 \omega (\sqrt{1 + u} - 1)$$

(8)
Note that we have “restored” the symmetry between the differential equations (1-2) and the solutions (7-8) by replacing the gauge factor $\gamma_Q^2$ with $\gamma_D^2$ in (8).

It is interesting to search for particular combinations of the Yukawa couplings which are rather independent of their initial values. To start our investigation we first obtain a simplified formula for the function $\omega(t) = x - y$, in the case $h_{b,t} \geq 1$

$$\omega(t) \approx \frac{x_0 - y_0}{\{2F_1^0 + \frac{7}{6}\sqrt{x_0y_0}I(t)\}^{12/7}}$$

(9)

where $2F_1^0$ is the value of the hypergeometric function at $u = u_0$. Then, we can use the relation between $x, y$ variables

$$\left(\frac{x - y}{x_0 - y_0}\right)^7 = \left(\frac{xy}{x_0y_0}\right)^6.$$  

(10)

to obtain the following expression,

$$\sqrt{xy} = \frac{\sqrt{x_0y_0}}{2F_1^0 + \frac{7}{6}\sqrt{x_0y_0}I(t)}$$

(11)

The above equation can also easily be obtained by a direct multiplication of the solutions (7,8) substituting $u$ from (8).

Note first that for a small difference between $h_t$ and $h_b$, $2F_1^0 \approx 1$, while the integral $I \geq 10$. Thus for $\sqrt{x_0y_0} \equiv h_{t,0}h_{b,0} \geq 1$ we can write

$$h_th_b \approx \frac{8\pi^2\gamma_Q\gamma_D}{\int \gamma_Q^2 dt}$$

(12)

This last expression tells us that we can get an approximate, model independent prediction for the product of $h_t, h_b$ couplings at the low energy scale provided we start with relatively large and comparable $h_{t,0}, h_{b,0}$ values at $M_U$. We note in passing, that it is possible to use Yukawa coupling constraints obtained at the Unification scale to eliminate one of the two parameters. Suitable constraints combined with (12), can determine the absolute values of both couplings. In [11] for example, it is shown that within spontaneously broken $N = 1$ supergravity models two generic types of such constraints are obtained. For a superpotential $W = h_{i,A^IB^C} \Phi_{iA} \Phi_{iB} \Phi_{iC}$ while assuming different scale structures for the various fields in the Kähler potential, there are multiplicative (duality invariant) constraints of the form $\prod_i h_{i,A^IB^C} = cst$ if of course $h_{i,A^IB^C} \neq 0$. Even if this constraint were applicable for the two Yukawa couplings which interest us here, it could not be useful however, due to the fixed point property of the product in relation (12). A more interesting situation arises in cases where the constraint applies to the ratio of the Yukawa couplings. A simple example is shown in [11] for two couplings with a Kähler potential having the two fields in the same no–scale structure. Thus if the higher theory can give a prediction about the ratio of the two
Yukawas at $M_U$, we can use this in conjunction with the analytic solution to extract information about the Yukawas at the weak scale.

Then the low energy values can be obtained using the relation (12) and the low energy ratio obtained from the analytic solution in the limit $h_{t,0}h_{b,0} > 1$,

$$\frac{h_t^2}{h_b^2} \approx \frac{\gamma_Q^2 \sqrt{1 + u} + 1}{\gamma_D^2 \sqrt{1 + u} - 1}$$

For the limit of interest, $u$ is given by the approximate formula

$$u \approx u_0\left\{1 + \frac{7}{6}p_0I(t)\right\}^{10/7}$$

with

$$u_0 = \left(\frac{2r_0}{(r_0^2 - 1)}\right)^2$$

and $r_0, p_0$ the ratio and product of $h_{t,0}, h_{b,0}$ couplings respectively. Thus in the case of $r_0 \to 1$, $u \to \infty$, and the ratio of the Yukawas runs also to a fixed point value determined approximately by the ratio $\gamma_Q/\gamma_D$.

In Figs. 1a, 1b and 1c, we show the fixed point structure of the Yukawa couplings and their product, when we use the analytic expressions without making any approximation on the hyper-geometric function. We have taken a common gauge coupling $a_G = 1/25.0$ at a unification scale $m_G = 1.35 \times 10^{16}$ GeV and a supersymmetry breaking scale $\approx 200$ GeV, leading to an $a_s(M_Z) \approx 0.112$. The couplings are presented at the top mass.

Fig. 1a shows $h_t$-low energy coupling versus the GUT $h_t^0$ values for the ratios $h_t^0/h_b^0 = 1.2$ and 1.001 denoted with stars and crosses respectively. For a wide $h_t^0$ range, the estimated $h_t(m_t)$ values differ at most by 1%.

Fig. 1b shows the bottom coupling versus $h_b^0$. Here we took the same region of initial values for $h_t^0$ for each of the two cases quoted, thus the two lines are interrupted before touching the contour. In Fig. 1c we plot the product $h_b h_t$ for the same input ratios as in fig 1a. In Fig. 1c, a rather interesting fixed point property is exhibited if $h_t^0 h_b^0 \geq 4$ were the estimated low energy values differ in less than 1%. On the other hand, it is remarkable that different initial $h_t^0 / h_b^0$ ratios accumulate exactly on the same curve at the weak scale. This is in a very good agreement with the equations that we derived for the description of the fixed points in terms of functions of the gauge couplings only.

\[\text{1 The fact that we use expressions up to only one-loop, as well as the approximations made in order to derive the analytic formulas for the couplings, cause small errors in the numerical values of } h_t \text{ and } h_b \text{ which however do not alter the validity of the results.}\]
$h_t$ - fixed point structure for $h_t^0/h_b^0 = 1.2, 1.001$

**Fig. 1a**

$h_b$ - fixed point structure for $h_t^0/h_b^0 = 1.2, 1.001$

**Fig. 1b**

$h_t h_b$ - fixed point structure for $h_t^0/h_b^0 = 1.2, 1.001$

**Fig. 1c**
3 Analytic Solutions for $A_t$ and $A_b$

The differential equations which govern the evolution of the soft scalar masses of the third generation and the two Higgs mass parameters $m_{H_1,2}$, are well known. In order to solve them, we need first a solution for the $A_t,b,\tau$ trilinear mass terms. Ignoring the $\tau$–Yukawa coupling, we can write the $A_t$ and $A_b$ evolution equations as follows

$$\frac{dA_t}{dt} = \frac{1}{8\pi^2} \left( 6h_t^2 A_t + h_b^2 A_b + \hat{G}_Q m_{1/2} \right)$$

$$\frac{dA_b}{dt} = \frac{1}{8\pi^2} \left( 6h_b^2 A_b + h_t^2 A_t + \hat{G}_B m_{1/2} \right)$$

(16)

where $\hat{G}_Q$ and $\hat{G}_B$ are given by

$$\hat{G}_Q = \frac{4\pi}{a_G} \sum_{i=1}^{3} \hat{c}_Q a_i^2, \quad \hat{G}_B = \frac{4\pi}{a_G} \sum_{i=1}^{3} \hat{c}_B a_i^2$$

(17)

where we have taken into account that the gluino masses are given by

$$M_i \approx \frac{a_i}{a_G} m_{1/2}$$

(18)

$a_G$ being the common coupling at unification scale. Since the small corrections that arise from the $U(1)$ factors may be ignored we have $\hat{G}_Q \approx \hat{G}_B$.

To solve this system we follow the lines of [8], where the system $M_U^2 - M_D^2$ (ignoring $A_t$, $A_b$) which has a similar structure has been solved [9]. We initially separate the running of the Yukawa couplings by rewriting $A_t$ and $A_b$ as

$$A_t = \tau X_t, \quad A_b = \sigma X_b,$$

(19)

where

$$\tau = \exp\left\{ \frac{3}{4\pi^2} \int_{t_0}^{t} h_t^2 dt' \right\}, \quad \sigma = \exp\left\{ \frac{3}{4\pi^2} \int_{t_0}^{t} h_b^2 dt' \right\}$$

(20)

Defining the $2 \times 2$ matrix

$$\mathcal{H}(t) = \gamma_Q (\sigma \tau)^{\frac{3}{16}} \left[ \begin{array}{cc} 0 & y_0(\frac{u}{y_0})^{\frac{17}{16}} \\ x_0(\frac{u}{y_0})^{\frac{17}{16}} & 0 \end{array} \right]$$

(21)

and the function

$$h(u) = \left( \frac{x_0}{y_0} \right)^{\frac{6}{17}} \left( \frac{\sqrt{1+u-1}}{\sqrt{1+u+1}} \right)^{\frac{17}{16}}$$

(22)

the $A_t - A_b$ system can be written as

$$\frac{d}{du} \left( \begin{array}{c} X_t \\ X_b \end{array} \right) = -\frac{1}{10} \frac{1}{\sqrt{u^2 + u}} \left[ \begin{array}{cc} 0 & h(u) \\ h(u)^{-1} & 0 \end{array} \right] \left( \begin{array}{c} X_t \\ X_b \end{array} \right) + \frac{m_{1/2}}{8\pi^2} \frac{dt}{du} \left( \begin{array}{c} \hat{c}_Q \tau \\ \hat{c}_B \sigma \end{array} \right)$$

(23)

\footnote{We come back to the system $M_U^2 - M_D^2$ in section 4.}
Note that $h(u)$ in the large $\tan \beta$ case, i.e. $h_{t,0} \sim h_{b,0}$ is approximately constant in most of the range of integration.

Then, in the case $u \gg 1$ we can write an approximate analytic solution as follows

$$A_t \approx \frac{\tau}{2} \left\{ (A_t^0 + h_0 A_b^0) \rho + (A_t^0 - h_0 A_b^0) \frac{1}{\rho} \right\} - m_{1/2} < J_\tau > \quad (24)$$

$$A_b \approx \frac{\sigma}{2h_0} \left\{ (A_t^0 + h_0 A_b^0) \rho - (A_t^0 - h_0 A_b^0) \frac{1}{\rho} \right\} - m_{1/2} < J_\sigma > \quad (25)$$

with

$$\rho = \left( \frac{\tan \phi}{\tan \phi_0} \right)^{\frac{1}{2}} \quad \sin 2\phi = (1 + u)^{-\frac{1}{2}}$$

while $h_0 = h(u \to \infty)$.

Furthermore, in the limit $u \gg 1$, the integrals $< J_{\tau,\sigma} >$ are given by

$$< J_\tau > = \tau \int_1^{h_0} \frac{\hat{G}_Q(s)}{\hat{\tau}(s)} \frac{1}{\rho(s)} ds \quad (26)$$

and similarly for $J_\sigma$. A simple inspection of the above formulae shows that for reasonable initial $A_t,b$ values the terms proportional to $m_{1/2}$ dominate. Thus, to a good approximation we may write $A_t \approx -m_{1/2} < J_\tau >$ and $A_b \approx -m_{1/2} < J_\sigma >$.

The semi-analytic expressions (24-25) are going to be used in the following sections, in order to compute the contributions to sparticle masses, as well as the corrections to the bottom mass from superparticle contributions.

4 Predictions for sparticle masses and comparison with the exact solutions of the RGE

Having obtained the solutions for $A_t$ and $A_b$ (thus for $M_U^2$ and $M_D^2$) we may calculate contributions to superparticle masses, by solving the system

$$\begin{align*}
M_U^2 &\equiv \tilde{m}_{Q_L}^2 + \tilde{m}_U^2 + m_{H_H}^2 + A_t^2 \\
M_D^2 &\equiv \tilde{m}_{Q_L}^2 + \tilde{m}_D^2 + m_{H_H}^2 + A_b^2
\end{align*} \quad (27)$$

with initial conditions $M_{(U,D)}^2 = \xi_{(U,D)} m_0^2$, where $m_0$ is a common scalar mass at the unification scale, $\xi_U \equiv \xi_{H_H} + \xi_Q + \xi_t$ and $\xi_D \equiv \xi_{H_H} + \xi_Q + \xi_b$. Taking into account the renormalisation group equations for the squark and Higgs fields$^3$, we obtain

$$\frac{dM_U^2}{dt} = \frac{1}{8\pi^2} \left\{ 6M_U^2 h_t^2 + M_U^2 h_b^2 - G_U^0 m_{1/2} \right\} + \frac{dA_t^2}{dt} \quad (28)$$

$^3$ In the renormalisation group equations of the squark and Higgs fields, for non-universal initial conditions, contributions proportional to $\frac{\alpha_t}{2\pi} S$ where $S(t) = \frac{\alpha_t(t)}{\alpha_t(0)} Tr [Y m^2]$ are obtained. However, these contributions cancel in the equations that describe the sums $M_U^2$ and $M_D^2$, due to the invariance of the $U(1)$ Yukawa Lagrangian. The non-universal initial conditions are still manifest, through the factors $\xi_i$. 

7
\[
\frac{dM_D^2}{dt} = \frac{1}{8\pi^2} \left\{ M_U^2 h_t^2 + 6M_b^2 h_b^2 - G_D^0 m_{1/2}^2 \right\} + \frac{dA_b^2}{dt} \tag{29}
\]

where \( G_U^0 = G_Q + G_{H_2} + G_{U^c} \) and \( G_D^0 = G_Q + G_{H_1} + G_{B^c} \). Taking into account the \( A_t \) and \( A_b \) contributions, the solution of the system to first order reads

\[
\frac{M_U^2}{m_0^2} \approx -\frac{\tau}{2}\left\{ (\xi_U + h_0 \xi_U) \rho + (\xi_U - h_0 \xi_U) \frac{1}{\rho} \right\} + \xi_1/2 < I_\tau > - < A_\tau > \tag{30}
\]

\[
\frac{M_D^2}{m_0^2} \approx -\frac{\sigma}{2h_0}\left\{ (\xi_U + h_0 \xi_U) \rho - (\xi_U - h_0 \xi_U) \frac{1}{\rho} \right\} + \xi_1/2 < I_\sigma > - < A_\sigma > \tag{31}
\]

where \( \xi_U \equiv \xi_{H_2} + \xi_Q + \xi_{U^c} \), \( \xi_D \equiv \xi_{H_1} + \xi_Q + \xi_{U^c} \). \( < I_\tau > \) is given by

\[
< I_\tau > = \tau \int_{t}^{t_0} \frac{G_U(t')}{\tau(t')} \frac{1}{\rho(t')} dt'. \tag{32}
\]

A similar expression is obtained for \( < I_\sigma > \) with the replacements \( \tau \rightarrow \sigma \) and \( G_U \rightarrow G_D \). Finally,

\[
< A_\tau > = 2\tau \int_{t}^{t_0} \left( \frac{A_t(s) A'_t(s)}{\tau(s) \rho(s)} \right) ds \tag{33}
\]

Here, \( A_t \) is approximately given by

\[
A_t(t) = -m_{1/2} \tau(t) \int_{t}^{t_0} \left( \frac{\hat{G}_Q(s)}{\tau(s)} \frac{1}{\rho(s)} \right) ds \tag{34}
\]

and

\[
A'_t(t) = -m_{1/2} \tau'(t) \int_{t}^{t_0} \frac{\hat{G}_Q(s)}{\tau(s)} \frac{1}{\rho(s)} + m_{1/2} \tau(t) \hat{G}_Q(t) \frac{1}{\tau(t)} \frac{1}{\rho(t)} \tag{35}
\]

where primes denote derivatives with respect to the logarithm of the scale \( t = \log(\mu) \). Similarly, \( < A_\sigma > \) is found by the proper substitutions.

Integrating the sums, we obtain

\[
M_U^2 - M_{U,0}^2 - C_U(t)m_{1/2}^2 = -6J_U - J_D + A_{t,0}^2 - A_{t,0}^2 \tag{36}
\]

\[
M_D^2 - M_{D,0}^2 - C_D(t)m_{1/2}^2 = -6J_U - J_D + A_{b,0}^2 - A_{b,0}^2 \tag{37}
\]

with \( J_I(t) = \int h_I^2 M_I^2 dt \), \( I = U, D \). Now, the unknown integrals \( J_I(t) \) can be expressed in terms of the already calculated functions, their initial conditions and known gauge functions, from the simple algebraic system (36,37). As an example, for the up–squark

\[
\hat{m}_Q^2 = (\xi_Q + C_Q(t)\xi_{1/2})m_0^2 - J_D(t) - J_U(t) + I_S' \tag{38}
\]

with \( I_S' \) representing the integral of the \( S \)–contribution in the case of non–universality and

\[
C_Q(t) = \sum_{i=1}^{3} \frac{C_i^Q}{2h_i \alpha^2_{iG}} \left( \alpha^2_{i}(t_1) - \alpha^2_{i}(t) \right) \tag{39}
\]
In table 1 we compare the superparticle masses obtained from the analytic formulae, with those when solving numerically the renormalisation group equations: In the first column, we give mass terms, for a numerical solution of the renormalisation group equations, and at a scale $\approx 250 \text{ GeV}$, when the heavier superparticle, which in our case is the gluino, decouples from the spectrum. The initial conditions we take are

$$
A_t^0 = A_b^0 = 1,
$$

$$
h_t^0 = 2.0, \quad h_t^0/h_b^0 \approx 1.1,
$$

$$
\xi_{H_1} = 4.0, \quad \xi_{H_2} = 1.0,
$$

$$
\xi_{d} = 1.0, \quad \xi_{u} = 1.0, \quad \xi_{q} = 1.0,
$$

and ignore for simplicity the $I_S$ contribution to scalar masses (of course we do the same in the analytical solutions). In the second column appear the analytic solutions, when we include the $A_t, A_b$ contributions, while in the third the solutions when these contributions are neglected. Finally, for comparison, in the fourth column we give the superparticle masses that are found numerically by the renormalisation group equations, when we ignore the $A_t, b$ contributions. Here we have taken $m_0 = m_{1/2} = 100 \text{ GeV}$ and the superparticle masses are given in GeV. The effect of the $\tau$ coupling has been ignored in the numerical solutions of the renormalisation group equations as well, for a better comparison. Comparing the analytic with the numerical solutions, we find that the total relative error is at most $2-3\%$. Part of this small error in the case where we include the $A_t, b$ contributions arises because we wanted to keep the expressions as simple as possible and therefore have ignored the contribution of the homogeneous part of the $A_{t,b}$ solutions in the superparticle masses (but not in the calculation of the $A$’s themselves). Had we included the effect of the $\tau$ Yukawa coupling in the numerical solution of the renormalisation group equations, the shift to the results is at most 10%. These observations are in agreement with the ones obtained in the Appendix A of [7].

Let us finally make the following observation: In the equation for the product of the Yukawa couplings, we see that the larger the Yukawas and the smaller the difference
between $h_t - h_b$, the stronger the fixed point behaviour becomes. Moreover, in the equations for the scalar masses, we observe that the smaller the difference between $h_t^0 - h_b^0$, the less important the contribution of the homogeneous part of the solution (which includes the main dependence from initial conditions for the Yukawa couplings, as the integrals $<I_{\tau,\sigma}>$ undergo a smaller change) becomes. This indicates that for low energy Yukawas which correspond to $h_t^0 = h_b^0$ and values close to the non-perturbative region for the couplings, one in principle expects to obtain the minimal variations of the effective potential under small displacements of the couplings. In [7] it has been shown that the effective potential exhibits a minimum related to the infrared fixed line, while for non–universal boundary conditions for the scalars, minima which correspond to $h_t = h_b$ will also be favored. This we think can be understood by looking at the regions where the expressions we have derived exhibit the strongest attraction to the fixed points.

5  Corrections to the bottom mass

It has been found that in the large $\tan\beta$ regime, there are large contributions to the running bottom quark mass $m_b$. These are given by [9, 10]

$$m_b = h_b v_1 (1 + \delta m_b).$$

where $\delta m_b = \epsilon_b \tan\beta$ and

$$\epsilon_b = \frac{\mu}{16\pi^2} \left( \frac{8}{3} g_3^2 M_g I(m_{\tilde{b},1}^2, m_{\tilde{b},2}^2, M_g^2) + h_t^2 A_t I(m_{\tilde{t},1}^2, m_{\tilde{t},2}^2, \mu^2) \right),$$

and the integral function $I(a, b, c)$ is given by

$$I(a, b, c) = \frac{ab \ln(a/b) + bc \ln(b/c) + ac \ln(c/a)}{(a - b)(b - c)(a - c)},$$

with $M_g$ and $m_{\tilde{b},i}$ ($m_{\tilde{t},i}$) being the gluino and sbottom (stop) eigenstate masses respectively.

To calculate these corrections we substitute the relevant expressions in the above formulae. In [8], expressions for the scalar masses have been derived (and these expressions are not sensitive to $A_t$, $A_b$). Here, we have derived an equation for $A_t$, while the gluino mass is approximately given by [8]. For the low energy value of $\mu$ we obtain the analytic expression

$$\mu^2 \approx -m_{\tilde{t}_2}^2 - \frac{1}{2} m_Z^2$$

while the renormalisation group equation can be integrated to give

$$\mu = \mu_G \frac{\mu_0}{u} \prod_{j=1}^{3} \left( \frac{\alpha_j}{\alpha_j} \right)^{c_{j/2b_j}}$$

(43)
\[ \tan \beta = \frac{m_1}{2} / \mu A_t I_1 (10^{-6}) I_2 (10^{-6}) \delta m_b \]

| \( \tan \beta \) | \( m_0 = m_{1/2} \) | \( \mu \) | \( A_t \) | \( I_1 (10^{-6}) \) | \( I_2 (10^{-6}) \) | \( \delta m_b \) |
|---|---|---|---|---|---|---|
| 58.1 | 100 | 124 | -147 | 12.8 | 18.1 | 0.40 |
| 59.3 | 150 | 192 | -208 | 6.2 | 8.5 | 0.41 |
| 60.0 | 200 | 253 | -264 | 3.8 | 5.1 | 0.42 |
| 60.3 | 250 | 310 | -316 | 2.6 | 3.4 | 0.43 |
| 60.6 | 300 | 365 | -364 | 1.9 | 2.5 | 0.43 |

**Table 2** : Bottom mass corrections for \( h_G \sim 2.0 \).

| \( \tan \beta \) | \( m_0 = m_{1/2} \) | \( \mu \) | \( A_t \) | \( I_1 (10^{-6}) \) | \( I_2 (10^{-6}) \) | \( \delta m_b \) |
|---|---|---|---|---|---|---|
| 55.5 | 100 | 119 | -153 | 12.1 | 18.0 | 0.34 |
| 56.6 | 150 | 184 | -216 | 5.9 | 8.4 | 0.35 |
| 57.1 | 200 | 243 | -274 | 3.5 | 5.0 | 0.36 |
| 57.4 | 250 | 297 | -328 | 2.4 | 3.4 | 0.37 |
| 57.5 | 300 | 348 | -377 | 1.8 | 2.5 | 0.37 |

**Table 3** : Bottom mass corrections for \( h_G \sim 1.0 \).

with \( \{ c_i^\mu \}_{i=1,2,3} = \{ \frac{3}{5}, 3, 0 \} \).

Then, we find the magnitude of the bottom corrections, for solutions at the fixed point \( (h_G = 2.0) \), as well as for small deviations from the fixed point \( (h_G = 1.0) \), while we keep \( h_t^0/h_b^0 \approx 1.1 \). The relevant quantities appear in tables 2 and 3. The masses in the tables are given in GeV.

As we increase the supersymmetry breaking scale, \( \tan \beta \) slightly increases, in order to get the same low energy parameters. (At the same time the unification scale drops slightly, while the inverse gauge coupling at the unification scale increases, by a small amount).

### 6 Conclusions

In this letter we have used simplified analytic solutions for the \( h_t, h_b \) Yukawa couplings in order to study the MSSM in the large \( \tan \beta \) regime. We have explored the regions of the parameter space which lead to a fixed point structure and derived the evolution of the Yukawas towards these fixed points. Using this information, one may identify the regions for the initial values of the Yukawa couplings which lead to the strongest attraction towards these infrared fixed points. Under these
considerations, top-bottom Yukawa coupling equality, and values of the couplings close to the non-perturbative regime seem to be favoured. Finally, we obtained corrections on the renormalised soft mass terms due to the evolution of the trilinear parameters $A_t$ and $A_b$. Using these results, we estimated analytically the sparticle loop – corrections to the bottom mass, which are important in the large $\tan\beta$ scenario. In agreement with previous calculations we find that the maximal corrections arise at the fixed point.

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References

[1] H. P. Nilles, Phys. Rep. 110(1984)1;
    G. G. Ross, *Grand Unified Theories*, Benjamin Cummings (1985);
    H. E. Haber and G. L. Kane, Phys. Rep. 117(1985)75;
    A. B. Lahanas and D. V. Nanopoulos, Phys. Rep. 145(1987)1;
    S. Ferrara, ed., “Supersymmetry” (North-Holland, Amsterdam, 1987);

[2] S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981)150;
    J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B249 (1990)441; Phys.
    Lett. B260 (1991) 131; U. Amaldi, W. de Boer and H. Fürstenau, Phys.
    Lett. B260 (1991) 447; P. Langacker and M. Luo, Phys.Rev. D44 (1991)
    817;

[3] K. Inoue et al., Prog. Theor. Phys. 68 (1982) 927; L.E. Ibáñez, Nucl.Phys.
    B218 (1983) 514; L.E. Ibáñez and C. López, Phys. Lett. B126 (1983) 54;
    Nucl.Phys. B233 (1984) 511; L. Alvarez-Gaume, J. Polchinsky and M. Wise,
    Nucl.Phys. B221 (1983) 495; L.E Ibáñez, C. López and C. Muñoz, Nucl.
    Phys. B256 (1985) 218.

[4] B. Pendleton and G. G. Ross, Phys. Lett. B98 (1981)291;
    C. T. Hill, Phys. Rev. D24(1981)291;
    M. Lanzagorta and G. G. Ross, Phys. Lett. B 349 (1995)319.

[5] E. G. Floratos and G.K. Leontaris, Phys. Lett. B336(1994)194;

[6] C. Kounnas, I. Pavel and F. Zwirner, Phys. Lett. B 335 (1994) 403.

[7] C. Kounnas, I. Pavel, G. Ridolfi and F. Zwirner, Phys. Lett. B 354 (1995)
    322.

[8] E. G. Floratos and G. K. Leontaris, hep-ph/9503455, IOA-320-95, to be
    published in Nucl. Phys. B.

[9] T. Banks, Nucl. Phys. B 303 (1988) 172;
    G. K. Leontaris, Phys. Lett. B 236 (1989) 179.

[10] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D 50(1994)7048; M. Carena,
     M. Olechowski, S. Pokorski, and C. E. M. Wagner, Nucl.Phys.B 426 (1994)
     269.

[11] P. Binetruy and E. Dudas, Nucl. Phys. B 442 (1995)21.