Adaptive fuzzy fault-tolerant control of seat active suspension systems with actuator fault

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Abstract
This paper proposes an adaptive fuzzy fault-tolerant control method about seat active suspension systems. The significant indexes of seat active suspension system are ride comfort and safety. Considering the actuator failure of seat active suspension system, the problems of passengers’ ride comfort and safety are better solved. Two adaptive laws are designed to solve the two unknown coefficients in the actuator fault of the seat active suspension system. The fuzzy logic systems are applied to approximate the unknown terms in the system, and an adaptive fuzzy fault-tolerant controller is designed by using the backstepping method. In the closed-loop systems, all signals are bounded which is proved through Lyapunov stability theory. Finally, two different road surface disturbances are considered in the simulation, which proves the effectiveness of the proposed method.

1 | INTRODUCTION

With the rapid advances in science and technology, excellent control algorithm has appealed to the interest of numerous outstanding scholars. Many control methods are widely used. For example, PID control [1], robust control [2], optimal control [3], $H_{\infty}$ control [4], sliding model control [5], adaptive control [6], and backstepping control [7]. Since there are usually uncertain and unknown functions in the system under consideration, neural networks (NNs) control [8–11], FLSs control [12–16], and neuro-fuzzy control [17] are introduced to deal with these uncertainties. Fuzzy logic system is based on fuzzy logic and imitates human’s fuzzy comprehensive judgment and reasoning to deal with problems that are difficult to solve by conventional methods. The neural network is to imitate the biological neural system to deal with the problem. Because of the advantages of NNs and FLSs, they have been applied in many practical physical systems, such as hydraulic servo systems [18], robot operating systems [19], and vehicle active suspension systems. As a necessary tool for people’s daily travel, the ride comfort and driving safety of vehicles are closely related to us. Therefore, how to control the vehicle suspension system stability is a problem worthy of study.

Vehicle suspension system, which is composed of damping and spring, is a significant subsystem of vehicle system. It is usually divided into passive suspension systems [20], semi-active suspension systems [21] and active suspension systems [22]. Compared with the other two types of suspension system, the active suspension systems add a force generating device, which can actively suppress the impact of uneven road surface. Active suspension systems can also improve the riding comfort, driving stability and driving safety of the car. Therefore, the active suspension system is undoubtedly the most valuable suspension system, and more and more scholars pay attention to it. According to the model of vehicle suspension system, it is segmented into quarter active suspension system, semi-vehicle active suspension system and whole vehicle suspension system. The control method of vehicle active suspension system in finite frequency domain is proposed in [23]. [24] proposed an adaptive NNs control scheme for a quarter-car with time-varying constraints. Aiming at non-linear active suspension systems, a new model-free fuzzy logic controller is presented in [25]. An adaptive inverse optimal control is proposed in [26], and an observer is designed to estimate the unmeasurable state of the active suspension system. A fuzzy control technique [27] is proposed to resolve the input delay question of active suspension system. An event triggered control method [28] is applied to active suspension systems. A control-oriented hybrid model of quarter car semi-active suspension system is provided in [29]. A fault tolerant control method is proposed in [30] and applied to active...
suspension system. Although, so many excellent control algorithms have been applied to the vehicle active suspension system, the passenger comfort index is often indirectly obtained through the suspension vertical displacement response. The displacement of seat active suspension can directly reflect the comfort of passengers, which is more general.

As a system that links the seat and active suspension system together, the seat suspension system is more in line with the actual life. At the same time, considering the seat active suspension system can more intuitive response, the car displacement of the seat can better improve the comfort of passengers. Compared with the general active suspension, the seat suspension has more integrity and generality. Although the seat active suspension system has great advantages in improving ride comfort, there are few researches on it. An event triggering control based on time-varying full state constraints is proposed in [31]. A new optimal design method of output feedback controller controller is put forward in [32]. Compared with the comfort of passengers, the safety of suspension system cannot be ignored. For the active suspension systems, actuator and sensor fault have a great impact on the safety of the active suspension systems. In many practical physical systems, actuator and sensor failures have always been a hot issue, not only in vehicle active suspension system, but in many practical physical systems.

The method of adaptive fault-tolerant control is proposed by many scholars [33–35]. [33] studied the problem of finite time control for a class of uncertain non-linear systems with unknown actuator faults. A fault-tolerant control strategy for multiple automated guided vehicles is proposed in [34]. A fault-tolerant control method for electro-hydraulic system is proposed in [35]. In [36–38], some fault tolerant control strategies are proposed for quarter active suspension systems or half-car active suspension systems. Meanwhile, the corresponding control strategy is proposed for the vehicle active suspension systems in [39]. Therefore, considering the comfort and safety of passengers, it is of great significance to study the actuator fault of seat active suspension system.

An adaptive fuzzy fault-tolerant control method of seat active suspension systems with actuator faults is put forward in this article. The main contributions of the paper are as follows.

1) Considering the actuator fault problem of seat suspension system, two adaptive laws are used to approximate the corresponding fault parameters. At the same time, the fuzzy logic system is used to approximate the unknown terms. The controller is designed by the backstepping recursive method to reduce the calculation. It is more general.

2) Different from [24] [36], this paper considers the active seat suspension system, more directly considers the passenger’s riding comfort, rather than indirectly considers the vertical displacement of the active suspension in [24] and [36]. At the same time, on the premise of fully considering the ride comfort, this paper considers the actuator failure of the system, which further ensures the safety of the vehicle.

FIGURE 1 The model of seat active suspension system

2 | PROBLEM DESCRIPTION

2.1 | Model description

This paper considers the active seat suspension system, and model of the seat active suspension system is given in [40] as shown in Figure 1.

where \( m_1, m_2 \) and \( m_3 \) mean the mass of seat, suspension and wheel, respectively. \( f_1, f_2 \) and \( f_3 \) are the elastic force produced by the corresponding springs. \( d_1, d_2 \) and \( d_3 \) are the forces produced by corresponding damping. \( u \) is the control input. \( z_1, z_2 \) and \( z_3 \) represent the vertical displacement of seat, suspension and wheel, respectively. \( z_4 \) stands for road surface interference.

Remark 1. Because the passive suspension system is mainly composed of springs, shock absorbers and guide mechanisms, the requirements for the springs of different road surfaces are various under varying conditions, resulting in poor handling of the overall car. The control part of the active suspension system is equipped with a device that can produce twitch, which provides a way to suppress the impact of the road surface on the body and the tilt force of the body. In this way, in the face of different road driving process, the tire has a strong adhesion to the ground and the car has stronger stability. The disadvantage of active suspension system is that it has complex structure, high cost and inconvenient maintenance. At the same time, because of the complex structure, it will occupy some space in the car, but it will not affect the integrity of the vehicle.

The dynamic equation of active seat suspension system is given as:

\[
\begin{align*}
\dot{z}_1 &= -f_1 - d_1 \\
\dot{z}_2 &= f_1 + d_1 - f_2 - d_2 - u, \\
\dot{z}_3 &= f_2 + d_2 - f_3 - d_3 + u
\end{align*}
\]
where \( f_1 = g_1(z_1 - z_2), f_2 = p_1(z_2 - z_3) + p_2(z_2 - z_3)^3, f_3 = q_1(z_2 - z_3), d_1 = k_{d1}(z_1 - z_3), d_2 = k_{d2}(z_2 - z_3), g_1 \) and \( g_2 \) and \( d_2 = c_{d2}(z_2 - z_3) + c_{d2}(z_2 - z_3)^3, d_1 = k_{d2}(z_2 - z_3), g_1 \) and \( g_2 \) represent the elastic coefficients of seat spring and wheel spring, \( p_1 \) and \( p_2 \) stand for the car suspension stiffness coefficients of linear and non-linear term, \( k_{d1} \) and \( k_{d2} \) are the coefficients of seat damping and wheel damping, \( c_{d1} \) and \( c_{d2} \) are the vehicle suspension stiffness coefficients of linear and non-linear term.

The state variables are defined as \( x_1 = z_1, x_2 = z_2, x_3 = z_2, x_4 = z_2, x_5 = z_3, x_6 = z_3, \) where \( x_1, x_2, x_3 \) represent the vertical displacement of seat, suspension and wheel, respectively, \( x_2, x_4 \) and \( x_6 \) are the vertical speed of seat, suspension and wheel.

According to the above definitions and (1), expression for the state space about active seat suspension system can be expressed as

\[
\begin{cases}
    ̇x_1 = x_2 \\
    ̇x_2 = ax_3 + g_1(x) \\
    ̇x_3 = x_4 \\
    ̇x_4 = bu + g_2(x), \\
    ̇x_5 = x_6 \\
    ̇x_6 = g_3(x) + ku,
\end{cases}
\]

where \( g_1(x) = -q_1 x_1 - k_{d1} x_2 + k_{d1} x_3 / m_1, a = 1 / m_1, b = 1 / m_2, k = 1 / m_3, g_2(x) = (f_1 + d_1 - d_2 - d_3) / m_2, \) and \( g_3(x) = (f_2 + d_2 - d_3 - d_3) / m_3. \)

**Remark 2.** Different from [24], this paper considers the active seat suspension system, more directly considers the passenger’s riding comfort. At the same time, actuator failure is also considered. It not only considers the ride comfort, but ensures the safety of the vehicle. In [31], an event-triggered control method with full-state constraints is put forward. However, the problem of actuator failure is not considered. In article [38] and [41], Markov variable is introduced to solve the problem of actuator failure in suspension system. This paper designs two adaptive laws to solve the problem of unknown coefficient in failure system.

The actuator faults included deviation coefficient is given [42]

\[ u = \rho u_d + \psi_d, t \in [t_s, t_f], \]

where \( 0 \leq \rho \leq 1, \psi_d \) is an unknown constant, \( t_s \) and \( t_f \) are the start and end times when the actuator fails. The actuator fault model includes the following situations:

1) when \( \rho = 1 \) and \( \psi_d = 0 \), it means that there is not any actuator failure.

2) when \( 0 < \rho < 1 \) and \( \psi_d = 0 \), it shows partial actuator failure.

3) when \( \rho = 0 \) and \( \psi_d = 1 \), it implies the control inputs influence \( u_d \) no more.

### 2.2 Control problem and control objectives

Then, system (2) with actuator faults (4) becomes

\[
\begin{cases}
    \dot{x}_1 = x_2 \\
    \dot{x}_2 = ax_3 + g_1(x) \\
    \dot{x}_3 = x_4 \\
    \dot{x}_4 = bu_d + \psi_d + g_2(x),
\end{cases}
\]

There are unknown continuous functions in the seat suspension system, so we introduce fuzzy logic system to approximate it. For any continuous function \( b_j(x_j) \) in a compact set \( \Omega \), there is a fuzzy logic system \( W_{i}^{\infty} \Phi(x_j) \) such that

\[ b_j(x_j) = W_{i}^{\infty} \Phi(x_j) + \varepsilon_j, \]

where \( W_{i}^{\infty} \in \mathbb{R}^l \) is the weight vector, \( \varepsilon_j \) is the approximation error, with \( \varepsilon_j \geq 0, \| \varepsilon_j \| > 1. \Phi(x_j) = [\Phi_{i,j}, \Phi_{i,j}, \Phi_{i,j}, \cdots \Phi_{i,j}]^{T} \) is the basis function vector with

\[ \Phi_{i,j}(x_j) = \exp \left( -\left\| x_j - \hat{\theta}_j \right\|^2 / \kappa_j^2 \right), \]

where \( \kappa_j \) means the width of the Gaussian function, and \( \hat{\theta}_j \) represents the centre of the receptive field.

This paper puts forward an adaptive fuzzy fault-tolerant control method for seat active suspension systems; the ride comfort, safety problem, and stabilization time are essential parts of active seat suspension system.

1. Ride comfort is a significant index of the suspension system.
2. Considering the active seat suspension system, the vertical displacement of the seat should be as small as possible to provide better comfort for passengers.
3. This paper considers the actuator fault of the seat suspension system, so the system should be stable quickly after the fault occurs. The assumptions used are as follows:

**Assumption 1.** The expected trajectory \( y_d \) satisfies \( |y_d| \leq A \), where \( A > 0 \) is a constant, \( y_d \) and \( \dot{y}_d \) are bounded.

**Assumption 2.** Because there is a certain limit to the weight bearing of the car, the car body mass \( m_2 \) is limited to \( m_{2, min} \leq m_2 \leq m_{2, max} \), where \( m_{2, min} \) and \( m_{2, max} \) are constants.

**Assumption 3.** Active seat suspension system has only one failure at the same time.
3 | ADAPTIVE FUZZY CONTROL WITH ACTUATOR FAULT

3.1 | Adaptive controller design

In order to achieve the control goal, an adaptive fault-tolerant control method is proposed in the seat active suspension system which has the actuator failure. Coordinate transformation is introduced

\[ e_i = x_i - y_d \]
\[ e_i = x_i - \alpha_{i-1}, \quad i = 2, 3, 4, \]  

(8)

where \( y_d \) is the expected trajectory and satisfies \( |y_d| \leq A, A > 0 \) is a constant. \( \alpha_i \) are the virtual controller of seat active suspension system.

**Step 1:** According to (8), the derivative of \( e_i \) is given as

\[ \dot{e}_1 = \dot{x}_1 - j_d \]
\[ = x_2 - j_d \]
\[ = e_2 + \alpha_1 - j_d. \]

(9)

Select the following Lyapunov function

\[ V_1 = \frac{1}{2} e_1^2. \]

(10)

The time derivative of \( V_1 \) becomes

\[ V'_1 = e_1 e_1 \]
\[ = e_1 (e_2 + \alpha_1 - j_d). \]

(11)

The virtual controller \( \alpha_1 \) is given as

\[ \alpha_1 = -k_1 e_1 + j_d, \]

where \( k_1 > 0 \) is a designed constant. From (9), the time derivative of \( \alpha_1 \) is \( \dot{\alpha}_1 = -k_1 x_2 + k_1 j_d + j_d \).

Substituting (12) into (11), one gets

\[ V'_1 = -k_1 \dot{e}_1 + e_1 e_2. \]

(13)

**Step 2:** From (8), \( e_2 = x_2 - \alpha_1 \). The time derivative of \( e_2 \) is given as

\[ \dot{e}_2 = \dot{x}_2 - (-k_1 x_2 + k_1 j_d - j_d) \]
\[ = a x_3 + g_1 (x) - (-k_1 x_2 + k_1 j_d - j_d). \]

(14)

Choose the following Lyapunov function

\[ V_2 = V'_1 + \frac{1}{2} e_2^2 + \frac{1}{2} \dot{\hat{\theta}}_1^2, \]

where \( \eta_i > 0 \) \((i = 1, 2)\) are designed constants, \( \hat{\theta}_i (i = 1, 2) \) are the error weight vectors, which are defined as \( \hat{\theta}_i = \theta_i - \hat{\theta}_i \) with \( \hat{\theta}_i \) being the estimate of \( \theta_i \) as follows:

\[ \theta_i = \left\{ \left\| W_i \right\|^2, \quad i = 1, 2 \right\}. \]

(16)

The time derivative of \( V_2 \) yields

\[ V'_2 = V'_1 + \frac{1}{a} e_2 e_2 - \frac{1}{\eta_1} \dot{\hat{\theta}}_1 \dot{\hat{\theta}}_1 \]
\[ = \dot{V}' + \frac{1}{a} e_2 (a x_3 + g_1 (x) - (-k_1 x_2 + k_1 j_d - j_d)) - \frac{1}{\eta_1} \dot{\hat{\theta}}_1 \dot{\hat{\theta}}_1 \]
\[ = \dot{V}' + e_2 x_3 + e_2 b_1 (x_1) - \frac{1}{\eta_1} \dot{\hat{\theta}}_1 \dot{\hat{\theta}}_1, \]

(17)

where \( b_1 (x_1) = \frac{1}{a} (g_1 (x) - (-k_1 x_2 + k_1 j_d - j_d)). \)

Since \( b_1 (x_1) \) contains an unknown function, FLSSs can be used to approximate the unknown function. According to (6), one can obtained that

\[ b_1 (x_1) = W_1^a \Phi_1 (x_1) + e_1. \]

(18)

**Lemma 1.** For any \( x, y \in R^n \), there are following inequalities [43]

\[ x^T y \leq \frac{\|x\|^p}{p} + \frac{1}{q \alpha} \|y\|^q, \]

(19)

where \( a > 0, p > 1, q > 1 \) and \( (p - 1)(q - 1) = 1. \)

According to Lemma 1 and the definition of \( W_1^a \), the following inequality can be obtained

\[ e_2 b_1 = e_2 W_1^a \Phi_1 (x_1) + e_2 e_1 \]
\[ \leq \frac{1}{2} e_2^2 + \frac{1}{2} e_2^2 + \frac{1}{2} + \frac{1}{2} e_2^2 \dot{\hat{\theta}}_1 \Phi^T_1 (x_1) \Phi_1 (x_1), \]

(20)

The virtual controller \( \alpha_2 \) and parameter adaptive law \( \dot{\hat{\theta}}_1 \) are designed as

\[ \alpha_2 = -k_2 e_2 - \frac{1}{2} \dot{\hat{\theta}}_1 \Phi^T_1 (x_1) \Phi_1 (x_1) - e_1 \]
\[ \dot{\hat{\theta}}_1 = \frac{\eta_1}{2} \dot{\hat{\theta}}_1 \Phi^T_1 (x_1) \Phi_1 (x_1) - \sigma_1 \dot{\hat{\theta}}_1, \]

(21)

where \( k_2 > 0 \) and \( \sigma_1 > 0 \) are design parameters.

Substituting (20)-(21) into (17), one gets

\[ V'_2 = -k_1 e_1^2 + e_1 e_2 + \frac{1}{a} e_2 (a x_3 + a \alpha_2 + a b_1 (x_1)) - \frac{1}{\eta_1} \dot{\hat{\theta}}_1 \dot{\hat{\theta}}_1 \]
\[ \leq -k_1 e_1^2 - k_2 e_2^2 + e_2 e_3 + \sigma_1 \dot{\hat{\theta}}_1 \dot{\hat{\theta}}_1 + \frac{1}{2} + \frac{e_3^2}{2}. \]

(22)
Step 3: The error $e_3$ is defined as $e_3 = x_3 - \alpha_2$, and the derivative of $e_3$ is given as

$$\dot{e}_3 = \dot{x}_3 - \dot{\alpha}_2$$

$$= e_4 + \alpha_3 - \dot{\alpha}_2.$$  \hfill (23)

The Lyapunov function has been designed as follows:

$$V_3 = V_2 + \frac{1}{2}e_3^2.$$  \hfill (24)

Then, $\dot{V}_3$ is became

$$\dot{V}_3 = \dot{V}_2 + e_3 \dot{e}_3.$$  \hfill (25)

It is easy to know

$$\dot{V}_3 = \dot{V}_2 + e_3(e_4 + \alpha_3 - \dot{\alpha}_2).$$  \hfill (26)

where $\dot{\alpha}_2 = -(x_3 - \alpha_1)(k_2 + e_2 \hat{\theta}_1 \Phi^T_1(x_1) \Phi_1(x_1)) - x_2 + j_d$.

The virtual controller $\alpha_3$ is designed as

$$\alpha_3 = -k_3 e_3 + \hat{\alpha}_2 - e_2$$  \hfill (27)

where $k_3 > 0$ is a design parameter.

Substituting (27) into (26), one gets

$$\dot{V}_3 = \dot{V}_2 + e_3(e_4 + \alpha_3 - \dot{\alpha}_2)$$

$$\leq -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_3 e_4 + \frac{\sigma_1}{\eta_1} \hat{\theta}_1 \hat{\theta}_2$$

$$+ \frac{1}{2} + \frac{e_1}{2}.$$  \hfill (28)

Step 4: The error $e_4$ is defined as $e_4 = x_4 - \alpha_3$, and the derivative of $e_4$ is given as

$$\dot{e}_4 = \dot{x}_4 - \dot{\alpha}_3$$

$$= b \dot{u} + g_2(x) - \dot{\alpha}_3.$$  \hfill (29)

This paper considers the actuator fault problem of active seat suspension system, and the fault model is given as

$$u = \rho u_d + \psi_d.$$  \hfill (30)

Then, (29) becomes

$$\dot{e}_4 = \dot{x}_4 - \dot{\alpha}_3$$

$$= b \rho \dot{u}_d + \psi_d + g_2(x) - \dot{\alpha}_3.$$  \hfill (31)

There are the following definitions

$$s = \inf \frac{\rho}{r \geq 0}$$

$$w = \frac{1}{\bar{s}}$$

$$\zeta = \sup \psi,$$  \hfill (32)

where the designed adaptive laws are used to estimate the unknown parameters $w$ and $\zeta$.

Choose the Lyapunov function as follows:

$$V_4 = V_3 + \frac{1}{2}e_4^2 + \frac{1}{2\eta_2} \hat{\theta}_2^2 + \frac{s}{\eta_1} \hat{\theta}_3^2 + \frac{1}{\bar{r}_2} \hat{\zeta}^2.$$  \hfill (33)

The time derivative of $V_4$ becomes

$$\dot{V}_4 = \dot{V}_3 + e_4 \dot{e}_4 - \frac{1}{\eta_3} \hat{\theta}_2 \hat{\theta}_2 - \frac{s}{\eta_1} \hat{\theta}_3 \hat{\zeta} - \frac{1}{\bar{r}_2} \hat{\zeta}^2$$

$$= \dot{V}_3 + e_4(b \rho u_d + \psi_d + b_2(x)) + c - c$$

$$- \frac{1}{\eta_3} \hat{\theta}_2 \hat{\theta}_2 - \frac{s}{\eta_1} \hat{\theta}_3 \hat{\zeta} - \frac{1}{\bar{r}_2} \hat{\zeta}^2,$$  \hfill (34)

where $\hat{\omega}$ and $\hat{\zeta}$ are the estimate of $\omega$ and $\zeta$, respectively, $\eta_1 > 0$ and $\eta_2 > 0$ are designed constants, stands for the intermediate control law; $b_2(x) = g_2(x) - \dot{\alpha}_3$ is a unknown non-linear function.

The intermediate control law is designed as

$$c = k_4 e_4 + \frac{1}{2} e_4^2 \Phi_2^T(x_4) \Phi_2(x_4) + e_3 + \hat{\zeta} \tanh \left( \frac{e_4}{\bar{r}} \right),$$  \hfill (35)

where $k_4 > 0$ and $\bar{r} > 0$ are known constants. Similar to the above steps, define $b_2(x) = \frac{1}{2} e_4^2 \Phi_2^T(x_4) \Phi_2(x_4) + e_2$.

Similarly, it’s easy to get

$$e_4 \dot{b}_4 = e_4 \frac{1}{2} e_4^2 \Phi_2^T(x_4) \Phi_2(x_4) + e_4 e_2$$

$$\leq \frac{1}{2} e_4^2 + \frac{1}{2} e_4^2 + \frac{1}{2} + \frac{1}{2} e_4^2 \Phi_2^T(x_4) \Phi_2(x_4).$$  \hfill (36)

Then, one gets

$$\dot{V}_4 \leq \dot{V}_3 + e_4(b \rho u_d + \psi_d + b_2(x)) + \dot{c} - c$$

$$- \frac{1}{\eta_3} \hat{\theta}_2 \hat{\theta}_2 - \frac{s}{\eta_1} \hat{\theta}_3 \hat{\zeta} - \frac{1}{\bar{r}_2} \hat{\zeta}^2$$

$$\leq V_3 + e_4(b \rho u_d + |e_4| \zeta + e_4 e_4 + e_4 c$$

$$+ \frac{1}{2} e_4^2 + \frac{1}{2} e_4^2 + \frac{1}{2} + \frac{1}{2} e_4^2 \Phi_2^T(x_4) \Phi_2(x_4)$$

$$- \frac{1}{\eta_3} \hat{\theta}_2 \hat{\theta}_2 - \frac{s}{\eta_1} \hat{\theta}_3 \hat{\zeta} - \frac{1}{\bar{r}_2} \hat{\zeta}^2.$$  \hfill (37)
It can go further as

\[
V_4 \leq - \sum_{i=1}^{4} k_i \epsilon_i^2 - \sum_{i=1}^{2} \sigma_i \delta_i \dot{\delta}_i + 1 + \sum_{i=1}^{2} \frac{\epsilon_i^2}{2} + e_4 \epsilon + e_4 \mu \eta_d
- \frac{s}{r_1} \dot{\bar{w}} + \xi \left( |e_4| - e_4 \tanh \left( \frac{e_4}{\tau} \right) \right)
- \frac{1}{r_2} \dot{\xi} \left( -r_2 e_4 \tanh \left( \frac{e_4}{\tau} \right) + \dot{\xi} \right).
\]  

(39)

The actuator control input and parameter adaptive laws are designed as follows:

\[
u_d = - \frac{1}{\beta} \frac{e_4 \dot{\bar{w}}^2}{\sqrt{\epsilon_4^2 \dot{\bar{w}}^2 + \sigma^2}}
\]  

(40)

\[
\dot{\delta}_2 = \frac{\eta_2}{\beta} \epsilon_4 \Phi_2' (\chi_3) \Phi_2 (\chi_3) - \sigma_2 \dot{\delta}_2
\]  

(41)

\[
\dot{w} = r_1 e_4 - e_2 \dot{w}
\]  

(42)

\[
\dot{\xi} = r_2 e_4 \tanh \left( \frac{e_4}{\tau} \right) - e_1 \dot{\xi}
\]  

(43)

where \(e_1, e_2, \text{ and } \sigma_2\) are designed constants.

Substituting parameter adaptive laws into (39), it easy get

\[
V_4 \leq - \sum_{i=1}^{4} k_i \epsilon_i^2 - \sum_{i=1}^{2} \sigma_i \delta_i \dot{\delta}_i + 1 + \sum_{i=1}^{2} \frac{\epsilon_i^2}{2} + e_4 \epsilon + e_4 \mu \eta_d
- \frac{s}{r_1} \dot{\bar{w}} + \frac{sc}{r_1} \dot{\bar{w}} + \frac{c_1}{r_2} \dot{\xi} \dot{\xi}
+ \xi \left( |e_4| - e_4 \tanh \left( \frac{e_4}{\tau} \right) \right).
\]  

(44)

**Lemma 2.** The following inequality [44] holds for any \(\delta > 0\) and any \(\eta \in \mathbb{R}\)

\[
0 \leq |\eta| - \eta \tanh \left( \frac{\eta}{\delta} \right) \leq k_{\delta_1} \eta,
\]  

(45)

where \(k_{\delta_1} = 0.2785\) is a constant.

Then, one gets

\[
|e_4| - e_4 \tanh \left( \frac{e_4}{\tau} \right) \leq k \tau.
\]  

(46)

**Lemma 3.** The following inequality [45] holds:

\[
0 \leq |x| - \frac{x^2}{\sqrt{x^2 + \sigma^2(t)}} \leq \sigma(t),
\]  

(47)

where \(x\) is a variable, and \(\sigma(t)\) denotes a positive uniform continuous and bounded function satisfying \(\lim_{t \to \infty} \int_0^t \sigma(t)dt \leq \sigma, \text{ and } \mathcal{G} \text{ is a positive constant.}\)

From (40), it is easy to see

\[
e_4 \mu \eta_d \leq - \frac{\rho c^2 \dot{\bar{w}}^2 \sigma^2}{\sqrt{\epsilon_4^2 \dot{\bar{w}}^2 + \sigma^2}}
\]  

\[
\leq - \frac{sc^2 \dot{\bar{w}}^2 \sigma^2}{\sqrt{\epsilon_4^2 \dot{\bar{w}}^2 + \sigma^2}}
\]  

\[
\leq s \sigma - sc \dot{\bar{w}} e_2.
\]  

(48)

Substituting (46), (48) into (44), \(V_4\) becomes

\[
V_4 \leq - \sum_{i=1}^{4} k_i \epsilon_i^2 - \sum_{i=1}^{2} \sigma_i \delta_i \dot{\delta}_i + 1 + \sum_{i=1}^{2} \frac{\epsilon_i^2}{2} + s \sigma + k \xi \tau
+ \frac{sc}{r_1} \dot{\bar{w}} + \frac{c_1}{r_2} \dot{\xi} \dot{\xi}.
\]  

(49)

### 3.2 Stability analysis

**Theorem 1.** The seat active suspension with actuator failure is considered in (2). By design the virtual controller \(\alpha_i (i = 1, 2, 3)\), actual controller \(\nu_d\), intermediate control \(\epsilon\), and adaptive laws \(\dot{\delta}_i (i = 1, 2)\), \(\dot{\bar{w}}\), and \(\dot{\xi}\), all the signals of the seat active suspension system are bounded.

**Proof.** Based on Lemma 1, the adaptive laws can be obtained as follows:

\[
\frac{sc}{r_1} \dot{\bar{w}} \leq \frac{sc}{2r_1} \dot{\bar{w}}^2 - \frac{sc}{r_1} \dot{\bar{w}}^2
\]  

(50)

\[
\frac{\sigma_i}{\eta_i} \dot{\delta}_i \leq \frac{\sigma_i}{2\eta_i} \delta_i^2 - \frac{\sigma_i}{2\eta_i} \delta_i^2
\]  

(51)

\[
\frac{c_1}{r_2} \dot{\xi} \dot{\xi} \leq \frac{c_1}{2r_2} \xi^2 - \frac{c_1}{2r_2} \xi^2.
\]  

(52)

Then, \(V_4\) becomes

\[
V_4 \leq - \sum_{i=1}^{4} k_i \epsilon_i^2 - \sum_{i=1}^{2} \sigma_i \delta_i \dot{\delta}_i + \frac{sc}{2r_1} \dot{\bar{w}}^2 - \frac{sc}{2r_1} \dot{\bar{w}}^2 + \frac{c_1}{2r_2} \xi^2 + \frac{c_1}{2r_2} \xi^2 + k \xi \tau + \sigma
\]  

\[
\leq - CV_4 + D,
\]  

(53)
where \( C = \min\{2k_1, 2k_2/a, 2k_3, 2k_4, \sigma, \sigma, \sigma_j, j = 1, 2, 3, 4\} \) and
\[
D = \frac{1}{2} \sum_{i=1}^{3} \gamma_i^2 + \sigma + k_2 \tau + \frac{\sigma}{2n} \mathbf{y}^T \mathbf{y} + \frac{\sigma}{2} \mathbf{z}^T \mathbf{z} + \frac{1}{2} \sum_{j=1}^{3} \beta_j \mathbf{y}_j^2.
\]

By integrating \( V_4 \), it is easy to get
\[
V_4(t) \leq \left| V_4(0) - \frac{D}{C} \right| e^{-\frac{D}{C}} + \frac{D}{C}.
\]

Then, the errors \( e_i (i = 1, 2, 3, 4) \) are satisfied as follows
\[
|e_i| \leq \sqrt{2\left( \left| V_4(0) - \frac{D}{C} \right| e^{-\frac{D}{C}} + \frac{D}{C} \right)} = B.
\]

It is clearly that when \( t \to \infty \), the errors \( e_i (i = 1, 2, 3, 4) \) can be limited in a small neighbourhood. At the same time, \( |e_i| = |x_i| - |y_i|, |y_i| \leq A, \) and \( |e_i| \leq B, |x_i| = |e_i| + |y_i| \leq A + B. \) We can get \( x_2, x_3, x_4 \) are bounded similarly.

### 3.3 Zero dynamics analysis

In the previous controller design process, states \( x_i (i = 1, 2, 3, 4) \) have been considered and their boundedness has been proved. In this part, we will study \( x_5 \) and \( x_6 \), and their boundedness will be given. Substituting (40) into (3), one gets
\[
\dot{X} = MX + N,
\]
where
\[
M = \begin{bmatrix} 0 & 1 \\ -\frac{k_2}{m_3} & -\frac{k_3}{m_3} \end{bmatrix}, \quad N = \begin{bmatrix} 0 \\ \frac{k_2}{m_3} \dot{x}_4 + \frac{\sigma}{m_3} \dot{x}_6 + \gamma \end{bmatrix}, \quad x = \begin{bmatrix} x_5 \\ x_6 \end{bmatrix}^T,
\]
\[
\dot{y} = \begin{bmatrix} -\frac{k_2}{m_3} y_3 - \frac{k_3}{m_3} y_4 \end{bmatrix} \sqrt{\frac{\sigma^2}{2} + \frac{\gamma^2}{2}}.
\]
Then, \( \dot{y} \) is bounded with \( 0 < |y| \leq \tilde{y} \) and \( \tilde{y} \geq 0 \). Thus, \( N \) is also bounded. Denote \( M^T E + E^T M = -F \), where \( F \) and \( E \) are also positive definite symmetric matrices.

Define \( V = X^T E X \). One has \( V = X^T (M^T E + E^T M)X + 2X^T E N \). Based on Lemma 1, we have \( X^T E E N / t + tN^2 / N \), where \( t > 0 \). Then, it obtains
\[
\dot{V} \leq \omega_1 V + F_B,
\]
where \( \omega_1 = \lambda_{\min}(F) - \lambda_{\max}(E)/t > 0 \) and \( F_B = tN^2 \). The integral of (57) is obtained as
\[
V \leq \left( V(0) - \omega_1^{-1} F_B \right) e^{-\omega_1 t} + \omega_1^{-1} F_B \leq V(0) + \omega_1^{-1} F_B.
\]
Therefore, one gets \( x_i \leq \sqrt{V(0) + \omega_1^{-1} F_B} / \lambda_{\min}(E), \) \( i = 5, 6 \). It is obvious that \( x_5 \) and \( x_6 \) are bounded.

### 4 Simulation

To verify the effectiveness of the proposed method, an example with actuator fault of seat suspension systems is put forward and the parameters are designed as: \( k_{d1} = 170 N/m/s, m_1 = 80 kg, q_1 = 2000 N/m, m_2 = 40 kg, m_3 = 4500 kg, p_1 = 750 N/m, k_{d1} = 750 N/m/s, \) \( c_{z1} = 520 N/m/s, p_2 = 160000 N/m^2, \) \( a_2 = 190000 N/m, c_{z1} = 1385 N/m/s \). The designed parameters are choose as: \( c_1 = 0.8, c_2 = 0.8, k_2 = 0.8, k_3 = 0.8, \) \( k_4 = 0.8, \) \( \eta_1 = 2, \eta_2 = 2, \sigma = 10, \sigma_1 = 0.5, \sigma_2 = 0.5, r_1 = 20, r_2 = 20, \tau = 0.1 \).

The initial state values are selected as: \( \hat{\theta}_1 = 0.03, \hat{\theta}_2 = 0.03, \dot{\theta}_1 = 0.03, \dot{\theta}_2 = 0.03, (0.03, 0.03, 0.03, 0.03, 0.02) \), and \( \xi = 0.03 \).

This paper considers actuator failure of seat suspension system. Choose the following fault model:
\[
\begin{aligned}
&u = \rho u_\delta + \psi, \quad t \geq 3 \\
&u = u_\delta, \quad 0 \leq t < 3,
\end{aligned}
\]
where \( \rho \) and \( \psi \) are designed as \( \rho = 0.8 \) and \( \psi = 0.5 \).

This paper studies the seat suspension system. The vertical displacement of the seat is a very important standard, so two kinds of road surface interference are introduced. In order to further illustrate the effectiveness of the method, the passive suspension system is added for comparative verification.

**Case 1**: Choose lump road input:
\[
\begin{aligned}
\xi_0 &= \frac{1}{2} \beta (1 - \cos(\pi t)) \quad 0 < t \leq 1.25, \\
\xi_0 &= 0 \quad t > 1.25
\end{aligned}
\]
where \( \beta \) is the height of lump road and given as \( \beta = 0.002m \).

**Case 2**: Choose the road input as \( \xi_0 = 0.002 \sin(6 \pi t) \). In order to prove the effectiveness of the proposed method, compared with the control method using neural network to approximate the actuator fault parameters. Because this paper studies the seat suspension system, only the displacement and velocity corresponding to the suspension system are compared.

Then, the simulation results are given in Figures 2–15, where Figures 2–10 are renderings of the first case and the second case is shown in Figures 11–13. The parameter adaptive law and the system control input are shown in Figures 14 and 15.

Figure 2 represents the vertical displacement of the seat. It can be seen from the figure that after the interference, it converges quickly to a small neighbourhood. Compared with the passive suspension, the fluctuation is smaller and the convergence speed is faster, which ensures the ride comfort to a large extent. Similarly, the vertical velocity of the seat in Figure 3 and the speed and displacement of the suspension in Figures 5 and 4 converge to the small neighbourhood quickly compared with the passive suspension system.

Figures 6 and 7 represent the vertical displacement and vertical speed of the wheel of the seat suspension system, respectively. It is easy to get that they are bounded and converge to a...
FIGURE 2  Vertical displacement of seat suspension $x_1$

FIGURE 3  Vertical speed of seat suspension $x_2$

FIGURE 4  Vertical displacement of suspension system $x_3$

FIGURE 5  Vertical speed of suspension system $x_4$

FIGURE 6  Vertical displacement of wheel $x_5$

FIGURE 7  Vertical speed of wheel $x_6$
FIGURE 8  The seat suspension space $x_1 - x_3$

FIGURE 9  The suspension space $x_3 - x_5$

FIGURE 10  The errors $e_i$ ($i = 1, 2, 3, 4$)

FIGURE 11  Seat suspension system status $x_i$ ($i = 1, 2, 3$)

FIGURE 12  Seat suspension system status $x_i$ ($i = 4, 5, 6$)

FIGURE 13  The seat suspension space and suspension space
small neighbourhood. In Figures 8 and 9, the seat suspension space and suspension space of the seat suspension system are displayed. It can be easily seen, the errors in the seat active suspension system are shown in Figure 10.

Figures 11 and 12 show the state variables in the second case of seat suspension system, which can be clearly obtained. They can converge quickly and remain stable. It can be seen that the proposed method has faster convergence speed and smaller stable neighbourhood. Figure 13 shows that the seat suspension space and suspension space in the seat suspension system converge to the small neighbourhood quickly and keep stable under the continuous road interference. Two cases of parametric adaptive laws and control input for seat suspension systems are shown in Figures 14 and 15.

5 | CONCLUSIONS

This work focuses on the actuator failure of the seat suspension systems, two unknown parameters of the fault are solved by two adaptive laws. The FLSs are applied to approximate the unknown terms in the system, and the backstepping method is used to solve the problem of operation difficulty. The corresponding parameter adaptive law and the control input of the system are designed. In the closed-loop systems, all signals are bounded which is proved through Lyapunov stability theory. Finally, two different road surface disturbances are considered in the simulation. It can be clearly seen that the vertical displacement and speed of the seat and suspension system converge quickly to the small neighbourhood, which ensures the riding comfort and safety of passengers. In the future work, we will further study the excellent control algorithm, and consider the application of different cases of the system.

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