**Classification of multi-site damage using support vector machines**

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**Abstract.** Pattern recognition is now well-known to be a powerful approach to addressing the higher levels of damage identification e.g. location and severity assessment of damage. However, a major problem in implementation for real structures is the need for training data associated with all possible damage states. Even if appropriate data were available for individual damage states, the combinatorial explosion in states which occurs when multiple simultaneous damages are present would usually prohibit a pattern recognition approach. One approach to the solution of this problem is to construct classifiers on the basis of single damage data which will generalise to multiple damage states; the current paper is a very preliminary step in this direction. In the first part, a comprehensive multiple damage feature database is established as the result of an experimental campaign on a full-sized aircraft wing structure; in the second part, a classifier based on the support vector machine paradigm is investigated. The paper also considers how data visualisation can shed light on which features are likely to generalise best from the single damage problem to the multiple damage case.

**1. Introduction**

It is more or less accepted that damage identification at Level 2 (damage location) of Rytter’s damage identification hierarchy [1] via statistical pattern recognition requires the adoption of a supervised learning approach, thus requiring the gathering of data from the structure in its damaged state. In the particular case that damage is believed to occur at only one of a finite discrete set of \( k \) locations, this would require the data to be gathered for each of the corresponding \( k \) damage states. In the more general case of damage occurring concurrently at more than one location, the naïve approach might be to gather damage data for all combinations of damage location. The number of states for which data would be required in order to cover all combinations thus grows exponentially with the number of locations \( k \). It is commonly accepted that damaged state data even for single site damage comes at a premium, and it would appear unwise to rely on the availability of damage data for all possible combinations of damage location in the general case. It is apparent that this lack-of-data problem, a major obstacle in the diagnosis of single-site damage, is a potentially critical issue in multiple-site damage location.

Despite the multi-site damage identification task representing an important and challenging problem in SHM it is one that has received relatively little dedicated attention in the literature. Where multiple damage identification has been addressed, it has typically been done with the aid of a law-based model. As an example, in [2] an essentially model-based approach is adopted that involves forming residuals between features drawn from the structure and the predictions of a representative FE
model. It is noted that the structures investigated in such studies are typically required to exhibit a low level of complexity.

One approach to the solution of this problem is to construct statistical classifiers on the basis of single damage data that will generalise to multiple damage states. The study presented in this paper is a preliminary step in this direction, with support vector classification applied to the multi-site damage identification task. The aim of the developed approach is to dramatically reduce the number of states for which training data is required, obviating the requirement for a law-based model or full damage state data. The study is experimental in nature, with the structure used being the wing of a Piper Tomahawk trainer aircraft. The features used are discordancy measures, extracted from transmissibility data.

The layout of the paper is as follows. The structure and the acquisition of experimental data are introduced in section §2. In section §3 an example of the data visualisation employed in selecting and evaluating individual features is provided. The development and training of a classifier built using support vector principles is described in §4. Finally, the results of the study are presented and discussed in §5-6.

2. Experimental data acquisition

2.1. Piper Tomahawk trainer aircraft wing

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The experimental structure considered in this study is an Aluminium aircraft wing, shown in situ in the laboratory in Figure 1. The wing is mounted in a cantilevered fashion on a substantial, sand-filled steel frame. The sensor network and data acquisition equipment may also be observed. The wing is in fact mounted upside-down in order to allow access to the inspection panels mounted on the underside of
the wing. The reasons for this are explained later. The wing includes various complicating features, including stiffening elements (both accessible and inaccessible), inspection panels, riveted and welded connections, as well as auxiliary structures such as aileron mounting points.

Fifteen PCB 353B16 piezoelectric accelerometers were mounted on the upper (as mounted) surface of the wing using ceramic cement. The location of the sensors, inspection panels and sub-surface stiffening elements (dotted lines) are shown schematically in Figure 2. The sensors were placed in an ad hoc fashion on the basis of previous experience, with no formalised sensor placement optimisation undertaken. The sensors were placed so as to form transmissibility ‘paths’ between sensor pairs. These are indicated in Figure 3, with the transmissibility paths denoted T1 to T13. Placement directly above stiffening elements was avoided as it was believed that such locations would offer poor observation of localised changes in structural flexibility.

![Figure 3. Sensor locations and resulting transmissibility paths](image)

Experimental data acquisition was performed using a DIFA SCADAS III unit controlled by LMS software running on a desktop PC. All measurements were recorded within a frequency range of 0-2048 Hz with a resolution of 0.5 Hz. The structure was excited with a band-limited white Gaussian signal using a Gearing and Watson amplifier and shaker mounted beneath the wing. Both the real and imaginary parts of the accelerance FRFs were recorded at 15 response locations using single-axis accelerometers. Five-average samples were recorded in all cases as this was found to offer a good compromise between noise reduction and acquisition time.

2.2. Damage introduction
In order to introduce damage in a repeatable, and in some sense realistic, way advantage was taken of the presence of inspection panels on the underside of the wing, with the wing being mounted upside-down to enable access. Five such panels were employed in the present study. The advantages of using the removal of a panel as a proxy for damage are that it is non-destructive; it is repeatable; and the primary effect of panel removal (a localised reduction in structural stiffness) is expected to be similar to the effect of introducing gross damage at the same location.

The disadvantage is that the repeatability is not perfect. During preliminary studies it was found that the removal and reattachment of the panel led to substantial variability in the FRF observations. Accordingly, care was taken in order to reduce this variability to as great an extent as possible by using a torque-controlled screwdriver and raking care over screw tightening. An attempt to accommodate the outstanding variability in the recorded data attributable to this and other sources of variability was made by introducing randomisation and repetition into the test sequence.

2.3. Test sequence
Two tests were conducted, resulting in two data sets being available with which to develop and test classifiers. A test sequence was developed for each test to reflect the testing objectives and to take
identified sources of measurement variability into account. Randomisation and blocking were applied in the specification of the test sequences in order to account for the effects of measurement noise and panel boundary condition variability.

Dataset A comprises 1000 normal state observations and 1000 single-site damage state observations. The primary objective for the dataset was to allow the training and validation of an SVM-based classifier. The crux of the problem here is to select features using observations of normal condition and single-site damage condition data only, but which are capable of generalising to the multi-site damage identification using data from a separate test. This is a demanding objective, and particular attention was paid to full randomisation of the panel boundary condition. ‘Randomisation’, as used here, refers to the removal and replacement of panels to ensure that the latent boundary condition variation (that which is present despite the use of a torque controlled screwdriver and care over the order of screw tightening) be represented in the dataset.

The testing set - Dataset B - comprised 1780 normal, 760 single-site damage and 1040 multi-site damage state observations. The primary objective of this set was to provide data with which to evaluate the classifier developed using Dataset A. Randomisation was limited to the removal and replacement of individual panels, rather than full randomisation of the panel boundary conditions.

For both tests, a large number of response measurements were taken with the aim of capturing random measurement variability. For Dataset A, repetition was applied to further enhance the dataset. In developing the test sequences, a balance is sought such that both random and systematic variabilities may be well represented in the datasets without exceeding a reasonable timescale for testing.

3. Visualisation of damage

Damage leads to the structural response of the structure deviating from that observed when it is in its initial, undamaged condition. The structural response features considered in this study are transmissibility spectra. By comparing examples of undamaged and damaged spectra, regions of the spectra that are sensitive to particular damage states may be identified. These regions form the basis of features that may subsequently be used to train a statistical damage classifier. In the interests of developing a statistical classifier, it is desirable that the feature set used is of low dimension.

Achieving a suitably concise feature set requires further condensation of the identified spectral region. In this study the additional data reduction is performed using a discordancy measure - the Mahalanobis squared-distance (MSD) - between the newly-presented (and therefore potentially-damaged) state and the previously-recorded undamaged state. The resulting features are the discordancy values associated with damage sensitive regions of the transmissibility spectra.

The MSD is a multivariate extension of the univariate discordancy measure. Discordancy measures allow deviations from normality to be quantitatively evaluated. A brief summary of the technique is given here: the technique is described in full in [3]. For a multivariate data set consisting of n observations in p variables, the MSD may be used to give a measure of the discordancy of any given observation. The scalar discordancy value $D_{\xi}$ of an observation $x_\xi$ is given by,

$$D_{\xi} = (x_\xi - \bar{x})\Sigma^{-1}(x_\xi - \bar{x})$$

where $\bar{x}$ is the mean of the sample observations and $\Sigma$ is the sample covariance matrix.

For this study a guided, manual approach was taken to feature selection, which proceeded as follows. First, discordancy plots were generated for all possible 20-spectral line windows using Dataset A. As each spectrum contains 4097 lines, 4078 such windows existed for each of the 13 spectra, resulting in a total of 53014 candidate features. Plotting the discordancy values across the feature set for each of the structural states allows the sensitivity of the features to be visualised, and a greatly reduced set of candidate features to be specified. Next, the transmissibility spectra corresponding to each remaining candidate feature were inspected. This allowed a degree of ‘engineering insight’ to be exercised. Decisions on the final feature set were made using
considerations relating, for example, to the perceived ‘robustness’ of each feature – a feature window displaying similar behaviour to other features in its immediate vicinity would be preferred to one that did not. By applying such considerations, a final set of 50 features was selected on the basis of Dataset A. Ten features were selected for each of the 5 panels and were labelled F1-F50.

The performance of the features selected using the single-site training damage observations of Dataset A when applied to the multi-site damage observations of Dataset B were first evaluated visually. An example of the visual evaluation of feature F32 is given in Figure 4.

![Figure 4. Visualisation of feature F32](image)

Figure 4 illustrates feature F32, which performed well in identifying both single-site and multiple-site removals involving panel P4. The first plot illustrates the clear distinction between the removal of panel P4 (the lower two spectra, dark grey) and the other states included in Dataset A (other single-site damage states in light grey and normal states in black) and this distinction lead to the selection of the feature. The second plot illustrates the same feature window for the transmissibility spectra of Dataset B. It is observed that the spectra behave in a very similar fashion to that found for Dataset A, both for the removal of panel P4 alone (shown as a solid line) and where panel P4 was one of several removed (shown as dashed lines). The clear distinction between the removal of panel P4 and other states is maintained. This is reflected in the discordancy values illustrated in the third plot. Observation numbers 1-2300 refer to normal state data; observations 2301-3300 are of single-site damage; and observations 3301-4600 refer to multiple-site damage. The feature ‘fires’ strongly for the observations.
of single panel removal, and similarly strongly for each of the states in which it was one of several panels removed (shaded background). This very encouraging level of performance was observed for the vast majority of the 50 identified features, and supports the hypothesis that features exist that are sensitive to both single-site and multi-site damage at particular locations.

4. Support vector classification
The second stage of the approach, addressed in this section, is the statistical modelling of the selected features. Multiple-class support vector machines (SVMs) are investigated for this purpose.

The application of SVM methods has received relatively little attention in the damage identification literature in comparison to that received by other pattern recognition approaches, notably neural network and nearest neighbour formulations. SVMs have, however, been demonstrated to possess several properties that suggest they may be well-suited to the damage identification task. They have been shown to be competitive with other methods when applied to real engineering datasets [4], and to generalise well from the small datasets usually encountered in damage identification problems. The SVM is also able to support different classes of discriminant function (for example linear, polynomial or radial basis functions) without requiring substantial modification of the basic learning algorithm. Finally, the SVM may be considered to be a universal approximator. This capability means that an SVM can represent any function to arbitrary accuracy, provided no limit is placed upon the number of free weights used. The advantage for the damage identification task is that the full ‘toolbox’ of discriminant functions is in principle available when training a damage classifier. The theoretical basis of the SVM is well known and is not included here. For the inquisitive reader, further information may be found in [5-7].

4.1. Multi-class support vector classification
In this paper SVMs are applied to a multiple class problem. Support vector machines are designed, without loss of generality, for the resolution of data into two classes. Extension to multi-class problems is a topic of on-going research, with approaches typically falling into two categories: those which seek to consider all classes at once using a ‘one-against-all’ classifier, and those that seek to construct a system of binary ‘one-against-one’ classifiers [8]. The strategy employed in this study is to train an ensemble of ‘one-against-one’ SVMs. This decision is informed in part by the computational cost of the approaches.

The computational cost involved in the application of support vector classification is largely attributable to the optimisation of solutions based upon a matrix \( H \). \( H \) is a \( (N \times N) \) square matrix, \( N \) being the number of observations employed for training. The iterative Matlab QP solver is used for performing the optimisation step in the current work. At each iteration step, multiplication and inversion of the matrix is required in order to compute the search direction. The computational complexity of this operation is of the order \( O(N^3) \). Computational cost considerations thus preclude the use of very large numbers of observations for training SVMs. As the number of observations required for training a ‘one-against-all’ classifier will typically be higher than that for a ‘one-against-one’ classifier in order to adequately represent the dataset, ‘one-against-one’ classifiers may be preferred on the grounds of computational cost.

4.2. Approach
The objective is to train an SVM-based classifier using normal and single-site damage state data from Dataset A, and to evaluate its performance when applied to multi-site damage data from a separate test, Dataset B. The classifier is judged on its ability to achieve a high rate of correct classification when presented with this previously unseen testing set containing observations from the structure in single-site, multiple-site and undamaged states. The task faced by the SVM classifier is challenging: not only is the classifier being asked to identify damage states on which it has not been trained, but it is expected to generalise between tests.
The classification architecture is based upon the dichotomous SVM extended to the multi-class problem. Each ‘classifier’ in fact comprises an ensemble of 6 binary SVM classifiers. The first SVM (labelled SVM1) seeks to separate damage-state data from normal-state data. Each observation is classified as either ‘undamaged’ or ‘damaged’. SVM1 thus acts as a damage detection step. Five further dichotomous SVMs (labelled SVM2-6) seek to indicate whether removal has occurred for each of the five panels in turn. Each SVM seeks to class an individual location as ‘undamaged’ or ‘damaged’ - SVM2 seeks to classify panel 1 as on or off, SVM3 relates to panel 2 etc. The classifiers were created using the MATLAB Support Vector Machine Toolbox [9]. The 50 features employed are log discordancy values derived from transmissibility data, as described in §2-3. Radial basis kernels were employed in the discriminant function. Normalisation of the data is recommended to aid the conditioning of the optimisation problem [10]. In this instance, each feature was normalised to the interval [0 1], with 1 being the maximum observed value of the feature.

4.3. Training, validation and testing sets

Conducting supervised learning in a principled fashion necessitates separating the data into three, non-overlapping sets: the testing set, validation set and training set. Each serves a purpose in the development and testing of the classifier:

- The training set is used to set the values of the classifier parameters
- The validation set is used to set the values of the classifier hyperparameters
- The testing set is used to verify that the developed classifier works for an independent set of observations.

For the radial basis kernel classifiers chosen in this instance two hyperparameter values must be set. These are the misclassification tolerance parameter $C$, and the kernel width $\alpha$. The values of these parameters were set in this instance by selecting a range of hyperparameter pairs ($C_j, \alpha_j$); training a classifier for each hyperparameter pair using testing set data drawn from Dataset A; and evaluating the performance of resulting classifiers when applied to validation set data, again drawn from Dataset A. The hyperparameter pair that returned the best performance when applied to the validation set was adopted for that classifier. A typical validation metric is the probability of correct classification of data in the validation set. In this instance, 100% correct classification was returned across the investigated hyperparameter space, offering little evidence for choosing one hyperparameter pair over another. As a result, the margin of separation and the number of support vectors used were also incorporated into the validation process. Validation was repeated for each of the six dichotomous classifiers, SVM1-6.

5. Results

The predictions of the developed classifier when presented with the 3580 observations contained in Dataset B are presented in confusion matrix form in Table 1. The results are summarised in terms of the probability of perfect, correct classification for each of the three sub-categories of structural states in Table 2. Perfect, correct classification is taken to mean is taken here to mean there were no false-positive of false-negative indications of damage for any of the five panels.

The classification outcomes presented in Tables 1 and 2 are highly encouraging. For the structure in its normal state, there were no false indications of damage, and 1769 of the 1800 observations tested were correctly classified as undamaged. The remaining 31 observations were returned as ‘unclassified’, with SVM1 indicating damage but SVM2-6 not indicating that damage had occurred on panels 1-5. This is taken as an indication that the classifier failed to generalise fully between tests, and may be attributable to the training set not fully representing the variability that may arise between tests. A second observation is that while the objective of SVM1 was damage identification, the 50 features used were selected based essentially upon damage location criteria. The criterion for selecting the features was that they should be indicative of damage on individual panels, rather than indicative of damage regardless of source.
### Table 1. Confusion matrix for Classifier 2 applied to testing

| Actual Class  | 1 | 2 | 3 | 4 | 5 |
|---------------|---|---|---|---|---|
|               |   |   |   |   |   |
|                |   |   |   |   |   |
|                |   |   |   |   |   |
| Panels Removed |   |   |   |   |   |
| 1              | 1769 | 0 | 0 | 0 | 0 |
| 2              | 0 | 152 | 0 | 0 | 0 |
| 3              | 0 | 0 | 150 | 0 | 0 |
| 4              | 0 | 0 | 0 | 152 | 0 |
| 5              | 0 | 0 | 0 | 0 | 149 |
| Unclassified   | 0 | 0 | 0 | 0 | 0 |

### Table 2. Summary of results for Classifier 2 applied to testing set

| Condition          | Correctly Classified (%) | Incorrectly Classified (%) | Unclassified (%) |
|--------------------|--------------------------|----------------------------|------------------|
| Undamaged          | 98.94%                   | 0.50%                      | 0.61%            |
| Single-site damage | 98.29%                   | 1.71%                      | 0.00%            |
| Multi-site damage  | 99.90%                   | 0.10%                      | 0.00%            |
The classifier performed exceptionally well for the single-site damage states, with only 5 non-
correct classifications out of 760 observations. Of these, three were returned as ‘unclassified’ with
SVM1 indicating damage but SVM5 failing to classify that panel 4 had been removed. A further two
observations were misclassified, with the classifier indicating the removals of panels 2 and 3 where in
fact only panel 2 had been removed. This was one of only two instances in which the classifier falsely
indicated damage at any location. The other was for the removal of panels 1 and 2, for which the
classifier indicated that panel 5 had also been removed.

In total, there were 128 observations (out of a total of 1040 observations) for which the classifier
trained solely on single-site data failed to perfectly identify multi-site damage states. Of these, 125
observations missed the removal of one panel but were otherwise correct. 118 of these observations
were due to removal of panel 5 having been missed; the remaining seven were due to panel 2 being
missed. The suggestion is that the feature set identified for indicating the removal of panel 5 is
comparatively ‘weaker’ than those for the remaining four panels. This will warrant further
investigation if the approach is to be applied for less severe damage scenarios for which less
discriminatory feature sets will be available. Alternative methods of feature selection and
normalisation may be sought in such cases.

Overall, the success of the classifier when applied to multi-site data is encouraging. It appears that
given a suitably discriminatory feature set, the SVM approach is capable of achieving an exceptionally
high level of correct classification for single-site damage, and a good level of classification for the
much more challenging task of identifying multi-site damage.

6. Discussion

In this paper SVMs are first shown to be applicable as the basis of multi-class classifiers for tackling
multiple damage location tasks. A family of binary support vector machines has been shown to be
capable of tackling a multi-class damage identification problem. The major contribution of the study is
to demonstrate that a classifier trained using normal condition and single-site damage data only may
be capable of identifying the presence of multi-site damage with a high degree of accuracy. This
hugely reduces the number of states for which damaged condition data is required.

The case investigated is challenging. However, the removal of inspection panels represents a
relatively gross level of damage, and the damage locations are coarsely dispersed. A question that
remains is how the method would cope in more challenging scenarios. This concern is promoted by
the difficulty experienced in achieving perfect diagnosis for panel P5. The features selected for
identifying this panel were somewhat weaker than those found for other panels. It is likely that the true
efficacy of the method will only become apparent when considering more challenging cases. It is also
foreseen that modelling choices will play a greater role as the discriminative capacity of the feature set
decreases, and that arriving at correct decisions with regard to the model structure and kernel choice,
for example, will subsequently be of greater significance in more marginal cases. Comparison of
options for multi-class identification and further investigation of validation methods are among the
areas that warrant further study.

A further point to consider is the existence or otherwise of an underlying physical explanation for
the success of the approach. The approach relies upon the presence of features that are sensitive to the
removal of one panel in isolation and which also ‘fire’ when that panel is one of several removed, but
little regard has been given here as to why this should be. A possible explanation arises through
considering the structure of the wing. The investigated section of the wing is made up of a thin
Aluminium top-sheet mounted on a framework of more substantial stiffening elements. This results in
the top-sheet being divided into many rectangular regions of flexible material, bounded on each side
by more rigid elements. A simple suggestion that may be made on the basis of engineering insight is
that the coupling between these regions may be rather weak, with the effect that stiffness changes on
individual regions would be expected to be most obvious in the response measurements taken from
that region and less so in responses taken from other locations. It is conceivable that this may lead to a
degree of independence between regions, which would, intuitively, aid the ability of features selected
using single-site damage data to generalise to multiple-site damage states. While this was not specifically considered during sensor placement it is notable from Figure 3 that 11 of the 13 ‘response’ sensors were placed within the same region as the panel they were employed to monitor.

The conclusion drawn is that while the approach pursued in this paper doesn’t require a law-based model for reference, its success still appears reliant on having some insight into the physical underpinnings of the observed phenomena. As such, a recommendation is made that research should be focused on investigating the physical basis for features extending from single-site to multi-site damage cases in the first instance, prior to further investigation of the application of statistical classification in such scenarios.

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References

[1] Rytter A. Vibration Based Inspection of Civil Engineering Structures [PhD Dissertation]: Aalborg University, Denmark; 1993.
[2] Ruotolo R, Surace C. Damage assessment of multiple cracked beams: Numerical results and experimental validation. Journal of Sound and Vibration. 1997;206(4):567-88
[3] Worden K, Manson G, Fieller NRJ. Damage detection using outlier analysis. Journal of Sound and Vibration. 2000;229(3):647-67.
[4] Worden K, Lane AJ. Damage identification using support vector machines. Smart Materials & Structures. 2001 Jun;10(3):540-7.
[5] Christianini N, Shawe-Taylor J. An Introduction to Support Vector Machines and other Kernel-based Learning Methods: Cambridge University Press 2000.
[6] Robinson J. Support Vector Machine Learning. Saarbrucken: VDM Verlag Dr. Muller Aktiengesellschaft & Co. KG 2008.
[7] Schölkopf B, Smola AJ. Learning with Kernels. Cambridge, Mass.: MIT Press Ltd 2002.
[8] Hsu CW, Lin CJ. A comparison of methods for multiclass support vector machines. IEEE Transactions on Neural Networks. 2002;13(2):415-25.
[9] Gunn SR. Matlab Support Vector Machine Toolbox. 2.1 ed: School of Electronics and Computer Science, University of Southampton 2001.
[10] Bishop CM. Neural Networks for Pattern Recognition 1995.