Students' validations on their solution in mathematical problem solving

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Students’ validations on their solution in mathematical problem solving

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Abstract. This study aims to explain how students constantly seek that their answers were correct mathematically and plausible during their problem solving processes and the implications of these activities. The results presented here correspond to a part of mathematical problem solving processes. The subjects of this study were 4 students of grade 8, a junior high school in Bandung. Grounded Theory was used to understand data and build the models it needs. This implies the need for analysis of field rubrics and notes step by step. During this analysis, it be used the method of constant comparative analysis and memo writing. During the study, it was observed that students’ mathematical problem solving behaviour, especially deal with validating a solution of a mathematical problem. Validating a solution is a stage of mathematical problem solving processes. In most cases, students try to validate their solution of mathematical problem that it was mathematically correct and by trying to find it plausible.

1. Introduction
Validation on a solution is an important aspect in mathematics education, especially in problem solving, and in problem solving activities, students are encouraged to look back [1]. The look back means validate of what was done to ensure that the truth of their work can convince themselves and others [2]. Encouraging students to validate a solution obtained is intended to help students to make sure that the solution they get are correct. Validation also encourages students to reflect on the activity that have been carried out and to provide a basis for subsequent learning. Usually validation is done at the end of problem solving activities [3]. Nevertheless, from an observation, it was shown that many students make a validation on a solution before submitting their work to the teacher is an unusual work done by students.

Validation on solutions can be viewed from two main perspectives, namely in relation to the verification and activity of students who do after obtaining the solutions. An observation states that students usually do not attempt to validate the results of their work when the results are obtained [4]. Usually, students ensure the truth of the solution using external sources, namely teacher approval. There are also students ensure the truth of the results of work using based on students' own understanding and experience. However, it is recommended that validation should be done in mathematically acceptable ways [5]. In other words, students need to be encouraged to question the results of their work and build their arguments to justify their work. For some students, it seems that a solution that looks right is satisfying for them, there is no need for validation anymore.
In the context of problem solving, students are expected to look back and provide certainty about the results they get. In some cases, validation is interpreted as verification. This verification by students is done at the end of the problem solving session. Nonetheless, it was found that students were not used to verifying the solution at the end of the problem solving process [4]. Validation on a solution during problem solving has also been considered implicitly when monitoring is directed. In working on problem solving, students indirectly bring ideas about the validation of solutions.

Several studies reveal that experts monitor their work by asking questions about the appropriateness and accuracy of the actions they take and the results they obtain [6,7]. Thus, it can be said that the validation of the solution is as a way of monitoring so that there is certainty and assurance that the ideas used and the solutions obtained are true and reasonable. The purpose of this study was to investigate the ways used by students to validate their solution in mathematical problem solving processes. The specific questions to be answered by this study was, “how do students validate the solutions produced in solving mathematical problems?”

2. Method
The purpose of this study was to build a model for how students validate their solution of non-routine mathematical problems. The subjects of this study were 4 students of grade 8, a junior high school in Bandung. Grounded Theory was used to understand data and build the models it needs. This implies the need for analysis of field rubrics and notes step by step. During this analysis, it be used the method of constant comparative analysis and memo writing.

During this research period, students were faced with mathematical problems and encouraged to explore in solving the problems proposed by the teacher. Rubric writing was a technique for recording mathematical experiences. During rubric writing, students need to write down the ideas and feelings they experienced during the process of solving this problem. "Rubric writing" was not a plausibility option of how a problem was handled but an article about mental processes that occur. Every time students finish learning, they are asked to complete their daily rubric for the day's problems. This rubric was an important aspect in this study because the student rubric was one of the main data sources for this study.

3. Result and discussion
In process of solving problem, there are three different stages. The first stage refers to building knowledge. Building knowledge includes issues related to building information and understanding, key ideas and gaining understanding. The second stage refers to building a solution and validating a solution. The last stage deals with improving a solution. Although not all students do all of these stages, it can be said that solutioning includes the main actions that students do in solving problems. Students do not always validate their work at the end of the problem solving process. They tend to close the process they do when a satisfactory solution has been obtained. Nonetheless, if we look closely, it appears that as long as the problem-solving process occurs, students constantly try to validate the results by looking for mathematical correctness and plausibility. Validation on a solution is usually done throughout a process of solving a problem.

When students solve a problem, they usually validate the results obtained by searching for correctness and sensibility. Mathematical correctness refers to situations where students verify that the results they get are mathematically correct. Sensibility refers to the situation in which the student tries to reassure that the idea he built is reasonable. Students look for mathematical correctness by searching for errors and inconsistencies in the process. To do this, they can review the process to ensure that the procedure chosen is feasible. Besides verification of procedures, students can also check whether general results can work in special cases. If the results obtained from special cases are as expected or matched with the previous data, then the results are acceptable.

This is a problem that is put forward to students on the first day.
Problem 1
Andi has 20 steel wire rods with @ 1 cm length. He wants to make a rectangle by connecting all the wires.

a. Which rectangular can be made by Andi?
b. Which rectangular has the largest area?

In problem 1, it is stated that Andi has 20 steel wire rods each measuring 1 cm long. He wants to make a rectangle by connecting all the wires. Students were asked to find the rectangles can be made by Andi, and then they ask to find the rectangles that has the largest area?

After students solve the above problem, they make a rubric about solving the problem, especially with regard to the way they convince themselves that the answer they are working on is correct. The following are some rubrics that represent the rubric that students make as soon as they complete the questions raised by the teacher.

Student 1 stated that she would check her results again, whether right or wrong before she submits it the teacher. I drew first all the rectangles that might have been made with 20 wire rods, then she searched broadly, and she said that it was right.

Student 2 stated that he drew all the rectangles that might be made with 20 wire rods, the result turns out that there were 5 rectangles in difference size. It turns out that the rectangle that had the largest area was the rectangle, with 25 cm². The student said that it is a square because the length of all sides is the same. He would check his work again by using the results that he had obtained. K = 2(5+ 5) = 20, correct. So the circumference was still 20 cm, and he said that his work was right.

Student 3 stated that she made a table consisting of 4 columns, namely the long column, the width column, the circumferential column, and the area column. The length and width must be 10, so she got 5 pairs of length and width and rectangle which had the largest area is 5 x 5, which was 25 cm². It was a square because the length of all sides is the same.

Student 4 stated that the half of the circumference was 10 cm. Then she stated that she found the rectangle which had the largest area was 5 x 5 rectangle, which the area was 25 cm². Then she checked again around the five rectangles and turns the perimeter constant, which is 20 m. From the long and wide pairs, it appears that the difference was getting smaller, the bigger was getting the multiplication. So, 6½ x 7½ < 49 and 99½ x 100½ <10000.

From the four rubrics made by the students above, it was obvious that students tried to make sure that what they are doing is mathematically correct. The student's effort showed that they were searching mathematical correctness. In searching for mathematical correctness the students were different. Student 1 tried to find the mathematical correctness by using pictures, while student 2 used the results obtained to be matched with what was known before, namely the size of the circumference of the rectangle. In contrast to student 1 and student 2, student 3 placed the search for mathematical correctness at every step in the settlement process by plausibility a table containing the length, width, circumference and extent of all rectangles that she might made. Student 4 showed that the problem solving was higher than the others. Besides searching the mathematical correctness, she tried to do "further solutions" rather than just the mathematical correctness, which is to utilize the information he obtained for other purposes, she has succeed in showing that a x a ≥ (a - b) x (a + b), where a ≥ b ≥ 0. The searching mathematical correctness includes efforts to ensure that the results obtained plausibility. Students obtain plausibility by explaining why the idea is true; by giving justification why the results must be accepted. Searching for this plausibility can be in the form of estimating the solution and comparing it to the solution that is obtained.

Problem 2
Amir has a rice fields. Amir and Agus go to the rice fields everyday. He wants this rice fields to be hoeed for planting rice. If the rice field were hoeed by Amir himself then it would take 6 days, but if the rice fields were hoeed by Agus himself then it would take 3 days. How many days are needed if the rice fields were hoeed by Mr. Amir and Agus together?

In problem 2, it is stated that Every day Amir and Agus go to the field. He wants this field to be hoeed to prepare for planting rice. If the rice field is hoeed by Amir himself then it will take 6 days, but
if the rice field is hoed by Agus himself then it will take 3 days. How many days is needed if the rice fields are hoed by Amir and Agus together?

After students solve the above questions, they make a rubric about solving the problem, especially with regard to the way they convince themselves that the answer they are working on makes sense. The following are some rubrics that represent the rubric that students make as soon as they complete the questions raised by the teacher.

Student 1 stated that the solution she got was 4½ because of an average of 6 days and 3 days. But the time is, how come it becomes longer than Agus did, it did not make sense. Surely there are these who are not right. Then she tried to count again and the results remain unreasonable, which was 4½.

Student 2 stated that the solution was definitely less than 3 days, because Agus himself was able to finish in 3 days, especially if assisted by Amir. But he did not know how many days exactly.

Student 3 stated that the result is definitely less than 3 days. If it was used the average, it must be wrong and did not make sense because the results are 4½ days. Then she used a rectangular image divided into 6 equal parts. On the first day Agus completed 1/3 of the rice field, which was 2 rectangles and Amir 1/6 of the rice field, which was 1 rectangle. So one day, 3 rectangles or ½ parts of the rice field are completed. Then all parts of the rice field will be finished within 2 days. Them she said that this makes sense because the result was less than 3 days.

Student 4 stated that the result is definitely less than 3 days. She tried to think that for one day, Agus himself could finish 1/3 of the rice field and Amir himself could finish 1/6 of the rice field. So, in one day Agus together with Pak Amir was able to complete 1/3 + 1/6 = 3/6 = ½ part of the rice field. Thus, if it was done together Agus and Amir the field (1 part) could be completed within 2 days and this was plausibility. Then she tried for another problem, if Agus himself was able to finish within 4 days and Amir himself 6 days then the time needed if done together was (1: (1/4 +1/6)) = 1: 5 / 12 = 12/5 = 2 2/5 days.

From the four rubrics made by the students above, it is obvious that students tried to make sure that the results they done were make sense. In the searching plausibility, the students looked uniform even though they were placed at different times. Uniformity looking for the plausibility is seen in the statement "the results must be less than 3 days". From the expression that the results must be less than 3 days it shows that they have used their reasoning correctly. Student 1 searched for plausibility at the time after the solution was obtained and failed to obtain the correct solution, while the other three friends used the first time to look for plausibility. Although the three of them put plausibility at the beginning before completing their work, student 2 was unable to get the idea of how to solve the problem, while student 3 and student 4 managed to get the solution correctly. Besides succeeding in finding plausibility and mathematical correctness, student 4 had also been able to show "further solutions" by trying to find a general pattern for the case.

Observation results show that students sometimes gain understanding and build results by reasoning about how the solution was built. For example, thinking about a process so as to provide a certain pattern might lead students to develop a formula that models the situation, as Fatimah has tried in the case of "hoeing the field" or in the case of "rectangular area". Reasoning like this method usually gives a clear understanding of why the results must be true and reasonable, therefore mathematical correctness and plausibility are obtained. In other words, when students get a solution with reasoning in this case understanding how this data is built, plausibility is usually automatically obtained. In such cases, students find that there is no need for further validation as they use a logical consequence of available information. Mathematical correctness and plausibility can be placed at almost the same time or one after another. After the formation of an idea or a result, students can check to see that this is mathematically correct. Then they can proceed by verifying whether the answer makes sense or not.

Finding mathematical correctness and plausibility is an activity where students are usually carried out in the whole process of solving a problem. The achievement of mathematical correctness and plausibility is a necessary condition in the sense that students prefer to work in contexts where the solutions they achieve and they build are true and reasonable. Building on the wrong idea or the wrong solution in a situation can endanger students in developing ideas and solutions in other situations.
Successful validation obtained through mathematical correctness and plausibility gives students the feeling that they are on the right track. Failure to reach mathematical correctness and plausibility requires students to review aspects of the process and try to correct their inconsistencies. Although the achievement of mathematical correctness and plausibility is a condition that allows students to move on to the next step by expanding advanced ideas or solutions, but in some cases, proceeding to the next process is not carried out either to relevant cases or to other domains, for various reasons.

Without validation on solutions, students have initiated a process even though it is defective or inaccurate. Alternatively, students can search for mathematical correctness and plausibility even though they are superficial. In the next case, students can accept reasons/arguments stating that validation is not necessary to be too strict but enough to provide certainty or assurance so that students continue on to the next job. Finally, the search for mathematical correctness and plausibility does not imply that the solution will always be validated satisfactorily, this can be because the validation carried out by students can be inaccurate. In some cases, students are aware of their inaccuracies and decide to repeat and correct their work. For example, students may not be able to fully explain why an idea is true or how to prove the truth. In another case, students do not care even though the ideas and solutions they obtain are inconsequential but they continue to progress to the next problem. When mathematical correctness and plausibility are not obtained by students in full, students can proceed to the next problem by delaying the mathematical correctness and plausibility search later, when new information and understanding is available.

4. Conclusion
In one case, students validate their solutions by searching a mathematical correctness rather than searching a plausibility, while in other case students prefer vice versa. The search for mathematical correctness and plausibility does not imply that the solution will always be validated satisfactorily, this can be because the validation carried out by students can be inaccurate. In some cases, students are aware of their inaccuracies and decide to repeat and correct their work.

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