Prospects for Refining *Kepler* TTV Masses Using *TESS* Observations

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Received 2018 October 10; revised 2019 February 4; accepted 2019 February 9; published 2019 March 14

Abstract

In this paper we investigate systems previously identified to exhibit transit timing variations in *Kepler* data, with the goal of predicting the expected improvements to the mass and eccentricity constraints that will arise from combining *Kepler* data with future data from the *Transiting Exoplanet Survey Satellite* (*TESS*) mission. We advocate for the use of the Kullback–Leibler (KL) divergence as a means to quantify improvements in the measured constraints. Compared to the original *Kepler* data, the *TESS* data will have a lower signal-to-noise ratio, rendering some of the planetary transits undetectable, and lowering the accuracy with which the transit mid-time can be estimated. Despite these difficulties, out of the 55 systems (containing 143 planets) investigated, we predict that the collection of short-cadence data by *TESS* will be of significant value (i.e., it will improve the mass uncertainty such that the KL divergence is \(\geq 0.1\)) for approximately 6–14 planets during the nominal mission, with the range primarily driven by the uncertain precision with which transit mid-times will be recovered from *TESS* data. In an extended mission this would increase to a total of approximately 12–25 planets.

Key words: planets and satellites: detection

1. Introduction

Transit timing variations (TTVs) are a powerful tool for measuring masses and eccentricities in multi-transiting systems (Agol et al. 2005; Holman & Murray 2005; Holman et al. 2010). The *Kepler* mission’s (Borucki et al. 2010) four years of nearly continuous photometric observations provided a rich data set containing hundreds of TTV measurements for transiting planets (Rowe et al. 2015; Holczer et al. 2016; Ofir et al. 2018). This data set has yielded a significant number of mass and eccentricity measurements in systems of small sub-Jovian planets that would otherwise be largely inaccessible to radial velocity characterization (e.g., Jontof-Hutter et al. 2016; Hadden & Lithwick 2017).

The *Transiting Exoplanet Survey Satellite* (*TESS*) mission (Ricker et al. 2015), successfully launched on 2018 April 18, represents the next generation in space-based transit missions. *TESS* will revisit the *Kepler* field in its second year of operation and the additional transit measurements of *Kepler* systems obtained by *TESS* during this time will provide an opportunity to improve planet mass and eccentricity constraints derived from *Kepler* transit timing data. If *TESS* operates beyond its nominal mission, it could revisit the *Kepler* field multiple times and further improve the precision TTV-derived constraints.

The goal of this paper is to quantify the expected improvements to the mass constraints\(^3\) that we predict will arise from combining *TESS* data with *Kepler* data, during both the nominal *TESS* mission as well as under various extended mission scenarios.

The systems we use in this work were previously observed to exhibit TTVs in *Kepler* data. Specifically, we work with the 55 systems fit by Hadden & Lithwick (2017, hereafter HL17).

This sample of multi-transiting *Kepler* TTV systems was initially selected from the Holczer et al. (2016) catalog on the significance of their TTV signals. HL17 fit the transit times of these systems derived by Rowe et al. (2015) using Markov chain Monte Carlo (MCMC) simulations to generate posterior samples of planets’ orbital elements and planet-to-star mass ratios. In this work, we take HL17’s computed posterior samples of orbital elements and planet-to-star mass ratios as the starting point of our analysis. Rowe et al. (2015) derived transit times from *Kepler* long-cadence data so the *Kepler* transit mid-time uncertainties discussed below are derived from 30-minute exposures.

In Section 2 we describe our expectations for the *TESS* data. In Section 3 we describe our statistical methods, in Section 4 we describe the systems likely to exhibit improved mass measurements under both nominal and extended *TESS* mission scenarios, and finally in Section 5 we discuss our results and conclusions.

2. Expected Observational Uncertainties

We computed the *TESS* magnitudes of the 55 multi-planet system hosts with the *tigenc* python module (Barclay 2017; Stassun et al. 2018), using \(J, H,\) and \(Ks\) magnitudes taken from the Exoplanet Archive. For each star, we estimated the photometric uncertainty in the *Kepler* data using *Kepler* magnitudes from the Exoplanet Archive and pre-launch noise characteristics from Kepler Science Center.\(^4\) We estimated the photometric uncertainty in a single *TESS* short-cadence measurement using the computed *TESS* magnitude and the expected noise characteristics from Ricker et al. (2015).

2.1. *TESS* Transit Time Uncertainties

In order to assess *TESS*’s contribution to TTV dynamical constraints, we need to estimate the precision of transit time measurements that can be derived from *TESS* light curves.

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\(^3\) TTV parameter inference generally exhibits strong correlations between planet masses and eccentricities (e.g., Lithwick et al. 2012). Therefore, we do not investigate improvements to eccentricity constraints separately since we do not expect situations in which eccentricity constraints are improved without corresponding improvements to planet mass constraints.

\(^4\) https://keplergo.arc.nasa.gov/CalibrationSN.shtml
Price & Rogers (2014), building on the work of Carter et al. (2008), derive analytic formulas for the variances and covariances of transit parameters when fitting light curve photometry. They give the following expression for transit mid-time uncertainty,

$$\sigma_\tau = \begin{cases} \frac{1}{\sqrt{Q}} \sqrt{\frac{T}{2}} \left(1 - \frac{T}{3\tau}\right)^{-1/2}, & \text{if } \tau \geq \mathcal{I}, \\ \frac{1}{\sqrt{Q}} \sqrt{\frac{T}{2}} \left(1 - \frac{\mathcal{I}}{3\tau}\right)^{-1/2}, & \text{if } \mathcal{I} > \tau, \end{cases}$$

where $T$ is the transit duration, $\tau$ is the transit ingress time, $Q = (\delta/\sigma)^2 T/\mathcal{I}$ is the signal-to-noise ratio (S/N) for a single transit with depth $\delta$ and photometric uncertainty $\sigma$, and $\mathcal{I}$ is the integration cadence. We can compute the ratio of uncertainty in transit mid-time between Kepler long-cadence and TESS short-cadence data,

$$\frac{\sigma_\tau, \text{TESS}}{\sigma_\tau, \text{Kepler}} = \begin{cases} \frac{\sigma_{\mathcal{I}_T}}{\sigma_{\mathcal{I}_T}} \sqrt{\frac{\mathcal{I}_k}{\mathcal{I}_T}} \sqrt{\frac{1 - \tau/3\mathcal{I}_k}{1 - \tau/3\mathcal{I}_T}}, & \text{if } \mathcal{I}_T > \tau, \\ \frac{\sigma_{\mathcal{I}_T}}{\sigma_{\mathcal{I}_K}} \sqrt{\frac{\mathcal{I}_k}{\mathcal{I}_T}} \sqrt{\frac{1 - \tau/3\mathcal{I}_K}{1 - \tau/3\mathcal{I}_T}}, & \text{if } \mathcal{I}_T \leq \tau \leq \mathcal{I}_K, \\ \frac{\sigma_{\mathcal{I}_K}}{\sigma_{\mathcal{I}_K}} \frac{1 - \mathcal{I}_K/3\tau}{1 - \mathcal{I}_K/3\tau}, & \text{if } \mathcal{I}_K < \tau, \end{cases}$$

where $\mathcal{I}_T = 2$ minutes and $\mathcal{I}_K = 30$ minutes are the TESS and Kepler cadences, respectively, and the $\sigma$ are photometric uncertainties in a 1 hr integration. Note that Equation (2) does not depend on transit depth or duration.

The upper left panel of Figure 1 shows the uncertainty ratio, Equation (2), as a function of ingress time for a representative Kepler target star as well as the distribution of ingress times for the HL17 sample of planets. Transits with ingress duration less than 30 minutes benefit significantly from a shift to TESS short cadence from Kepler long cadence, somewhat compensating for the reduced photometric performance of TESS relative to Kepler. The distribution of ratios of transit mid-time uncertainty, incorporating ingress time, Kepler and TESS magnitudes, is in the lower panel of Figure 1.

We estimate the expected TESS transit mid-time uncertainty for a particular planet as the product of the median mid-time uncertainty of the Kepler long-cadence transits of Rowe et al. (2015) and the uncertainty ratio predicted by Equation (2).

We note that the derivation of Equation (1) of Price & Rogers (2014) assumes uncorrelated photometric noise. As we use Equation (2) to scale the actual Kepler uncertainties, we implicitly include the effects of any correlated noise within the Kepler data, and we will only be underestimating the effects of correlated noise if the relative correlated error is much greater for TESS than for Kepler.

### 2.2. Detectability of Transits by TESS

Our approximation for expected TESS transit mid-time uncertainties, Equation (2), will fail when transits are undetectable or have very low S/N. We show in the Appendix that Equation (2) is approximately valid provided that the predicted transit S/N is $>3$. The larger photometric uncertainties of the TESS mission compared to the Kepler mission is likely to cause a number of the planets analyzed by HL17 to be undetectable by TESS. To illustrate this, in the bottom left panel of Figure 1 we plot the distribution of transit depths for the 143 TTV planets from HL17, and then in the bottom right panel of the same figure, plot the distribution of the predicted TESS transit depth fractional uncertainties (computed using the expressions in Appendix A of Price & Rogers 2014). The blue histogram shows the predicted depth uncertainties measured by fitting a single TESS transit. The orange and green histograms show depth uncertainties predicted by assuming all of the planets during the nominal and extended mission, respectively, are perfectly phase-folded.
required for obtaining useful transit timing information from the photodynamical modeling of TESS observations. The orange and green histogram in the bottom right panel of Figure 1 show predicted fractional transit depth uncertainties assuming that all planetary transits during the nominal and extended mission, respectively, are perfectly phase-folded.

There are 13, 24, and 56 planets to the left of the dashed line in the blue, orange, and green cases, respectively, i.e., these are the number of planets that would have depths detectable at the ~3σ level. We anticipate that the limiting transit S/Ns for which photodynamical modeling will be able to extract useful transit timing information lies somewhere between a pessimistic single-transit S/N = 3 and and optimistic phase-folded S/N = 3.

2.3. TESS Mission Scenarios

As described in Huang et al. (2018a), the TESS survey divides the sky into 26 partially overlapping sectors, each of which is observed for approximately one month during the two-year nominal mission. The first year of the mission targets the southern sky, hence the Kepler field, which is in the north, will be covered in year two. The Kepler field is centered at an ecliptic latitude of ~65°; lower portions of the field will be observed for contiguous intervals of ~27 days, most will receive contiguous intervals of ~54 days, and some of the higher portions will be covered for ~78 days during the nominal mission. We account for these differences in our simulations.

Bouma et al. (2017) and Huang et al. (2018b) discuss various extended TESS mission scenarios in terms of the overall number of planet discoveries that will be expected. In this study, we quantify the improvements to known multiplanet TTV systems. We consider six extended mission scenarios. We consider only three-year extensions of the TESS mission, which we summarize in Table 1. Our extended mission scenarios cover two possible camera configurations: the first, C4, has camera 4 is centered on the ecliptic pole as in the nominal mission and the second, C1, has camera 3 centered on the ecliptic pole and provides a larger area of sky with multiple pointings at the expense of coverage near the ecliptic equator. We also consider three possible extended mission pointing sequences: one in which TESS remains pointed in the northern ecliptic hemisphere for the entire extended mission (NNN), one in which TESS alternates hemispheres each year after starting in the north (NSN), and one in which TESS alternates hemispheres each year after starting in the south (SNS). As the Kepler field is in the north, these different scenarios have the effect of adding three, two, and one extra year(s), respectively, of observations on systems in the Kepler field. Hence the NNN scenarios (E3,NNN and E2,NNN) have the greatest likelihood of improving mass measurements for TTV systems in the Kepler field.

Table 1

| Scenario  | Nominal | Years | Pole Cam | Pattern |
|-----------|---------|-------|----------|---------|
| E1,N     | Nominal | 2     | C1       | NNN     |
| E2,N     | Nominal | 3     | C4       | NSN     |
| E3,N     | Nominal | 3     | C1       | NNN     |
| E3,SN    | Nominal | 3     | C1       | SNS     |
| E4,N     | Nominal | 3     | C1       | NNN     |

Note. All of the extended scenarios are in addition to the two-year nominal mission. Pole cam C4 and C1 indicate whether the 4th or 3rd cameras, respectively, are pointed toward the ecliptic pole. Patterns N and S indicate northern or southern hemisphere orientations, respectively.

3. Methods

We now describe the method we use to estimate the likely improvement in the measured mass from TESS observations, illustrating our method using the Kepler-36 system. In the top panel of Figure 2 we illustrate the original Kepler TTV data points for Kepler-36b, along with a sample of the posterior distribution of solutions (in gray) computed by HL17, extended out to the epoch of the TESS mission.

Simulated TESS data were repeatedly generated by randomly picking posterior samples to use as an underlying true model. Two such examples are plotted in the top panel of Figure 2 as a thick red and blue lines. Consider first the the red, low-mass model: this model was integrated with TTVFast (Deck et al. 2014) to the end of the E4,N extended TESS mission, and transits that fell within TESS’s simulated observational windows were paired with estimated uncertainties as described in Section 2.1 and plotted as red points and error bars in the middle panel of Figure 2. We integrated the rest of the posterior samples and computed χ2 for each set of transit times generated from the posterior sample relative to the simulated TESS data.

Next, we update the HL17 posterior samples of planet masses and orbits to reflect the new constraints of the simulated TESS observations. Assuming errors in transit time measurements are Gaussian and independent, the probability of measuring a χ2 value for a series of k transit times of χ2(θ) or greater for a particular set of planet parameters θ is given by

\[ 1 - F(\chi^2(\theta); k), \]

where \( F(\chi^2; k) \) is the cumulative distribution function of the chi-squared distribution with k degrees of freedom. We therefore resample the HL17 posteriors, accepting each set of planet parameters θ with probability \( 1 - F(\chi^2(\theta); k) \). We confirmed that this resampling method provided nearly identical results when compared to a more computationally demanding approach in which the entire system is refit to the combined constraints of TESS and Kepler data using MCMC. A sample of accepted solutions from the updated posterior is plotted in red in the middle panel of Figure 2.

The true solution selected in red in Figure 2 happened to be a low-mass solution, and this is reflected in the resultant mass distribution for Kepler-36c seen in the bottom panel of Figure 2. In contrast, an alternative true solution might be selected (blue lines and points in the top and middle panels of Figure 2) in which the solution is of higher mass, leading to the higher mass histogram in the bottom panel of Figure 2. For each system in our sample, we preform 50 iterations of the posterior updating scheme described above, randomly selecting a different true model from the HL17 posterior each time.
The KL divergence is defined as

$$D_{KL}(P||Q) = \int P(\mu) \log_2 \frac{P(\mu)}{Q(\mu)} d\mu$$  \hspace{1cm} (4)$$

and quantifies the amount of information provided by the new measurements, in units of bits (Kullback & Leibler 1951). The KL divergence is also known as the relative entropy of distribution $P$ with respect to distribution $Q$. In other words, $D_{KL}(P||Q)$ is a measure of the information gained when we update our beliefs from the probability distribution $Q$ (the planet mass posterior distribution when one has only Kepler data) to the new probability distribution $P$ (calculated with the additional information added by TESS data). A low KL divergence indicates nearly identical distributions. As an example, consider normal distributions with identical mean, i.e., $P(x) = N(0, \sigma_1)$ and $Q(x) = N(0, \sigma_2)$, then, the KL divergence from $Q$ to $P$ is

$$D_{KL}(P||Q) = \frac{1}{2\log 2} \left( \left( \frac{\sigma_1}{\sigma_2} \right)^2 - 1 \right) - \log_2 \frac{\sigma_1}{\sigma_2}.$$  \hspace{1cm} (5)$$

This quantity is 0 if, and only if, $\sigma_1 = \sigma_2$, otherwise it is positive. The KL divergence has previously been employed to inform the planning of exoplanet observations by, e.g., Loredo (2004) and Ford (2008).

For each planet, we compute the expected change, after obtaining TESS mission observations, to the relative entropy of its planet–star mass ratio posterior distribution (after marginalizing over all other model parameters). To estimate the KL divergence between two mass posteriors, we compute a kernel density estimation of each posterior using a bandwidth of $n^{-1/5}$ where $n$ is the number of samples, i.e., Scott’s Rule, and then approximate the integral as a Riemann sum over the range of masses, according to Equation (4).

To provide some intuition for the improvement in the measured mass that corresponds to a given KL divergence, we plot in Figure 3 some examples of systems with high, medium, and low KL divergences. Here we have plotted the Kepler mass posterior ($Q$) in gray and the updated posterior ($P$) in green. We emphasize that at this stage we are not considering the realistic detectability of the individual transits for these systems (this is done in subsequent sections), but rather we are illustrating idealized examples.

### 4. Results: Expected Improvements to Mass Constraints

We analyze each of the mission scenarios (nominal and extended) described in Section 2.3. For each system under each of the mission scenarios considered, we uniformly selected 50 different true models from the Kepler posterior and, following the procedure described in Section 3, generate updated posterior distributions. These updated posterior distributions provide a sample of $50$ $D_{KL}$ values for each planet.

In Figure 4 we plot the planetary KL divergence against the transit depth uncertainty of a companion planet.\(^{6}\) The results for

\(^{6}\) We plot the depth of the companion planet, not the target planet, because it is a detection of TTVs in the companion that allows constraints on the mass of the target planet. For planets having more than one transiting companion, multiple points are plotted with the same KL divergence at each of the different fractional depth uncertainties of the associated companions. As noted in Section 2.2, the practical analysis of individual systems is likely to be undertaken using a detailed photodynamical approach, which will extract the maximum information out of systems containing multiple planets with low-S/N transits.

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**Figure 2.** Top: original Kepler TTV data points along with the posterior distribution of solutions (in gray) computed by Hadden & Lithwick (2017) for Kepler-36, extended out to the epoch of the TESS mission. Two alternative true models (red and blue lines) have been selected from the posterior distribution. Middle: for each true solution in the top panel, TESS TTVs are simulated (red and blue points) assuming an $E_{\alpha,\beta}$ extended mission scenario and an updated posterior distribution (red and blue swaths) is calculated. Bottom: the posterior mass distributions of Kepler-36 c; the gray is for the Kepler-only data, and the red and blue correspond to the appropriate true solution illustrated in the middle panel. The histogram integrals are normalized to unity. The TESS TTV data is likely to substantially improve the precision of the mass constraints on Kepler-36 c, irrespective of whether the true model is of high or low mass.

**3.1. Kullback–Leibler (KL) Divergence**

To evaluate the degree of improvement represented by the updated planet mass posterior,\(^{5}\) $P$, over the HL17 planet mass posterior, $Q$, we computed the KL divergence from the Kepler-derived mass distribution to the simulated TESS distribution.

\(^{5}\) The TTV fits constrain the planet–star mass ratios, $\mu$, and not absolute planet masses, which depends on stellar mass constraints as well as the dynamical constraints. For simplicity, however, we refer the the planet–star mass ratio posteriors as planet mass posteriors.
The bottom system, with a small initial measured mass. We also indicate the ratio of the standard deviations of the subsequent sections of the KL divergences. In gray we plot the initial distributions provided in the right-hand panel of Figure 2. The top system, with high mass, shows negligible improvement to its measured mass. For the middle system we expect to benefit most from additional TESS data, in Figure 6 we plot the mass histograms from the Kepler mission (gray), the nominal TESS mission (cyan), and the \( E_{\text{4,NNN}} \) extended TESS mission (magenta) for all 25 planets. Given both the unfamiliarity of the KL divergence statistic, as well as the significant non-Gaussianities in the posterior distributions, in Figure 5 we plot the underlying quantity \( \sigma_{\text{mass, KL}} / \sigma_{\text{mass, T}} \) (i.e., the same quantity indicated in the labels of Figure 3) against the KL divergence from all 50 simulation iterations of the 25 planets in the right-hand panel of Figure 4, showing the results for each planet using a different color. We overplot (solid line) the expected results for a Gaussian distribution that has the same mean in the prior and posterior distributions. The scatter of points for each planet reflects the significant non-Gaussianity present in the distributions, as well as clarifying the overall scale of the variation seen in the mass measurement improvements (for a single planet) across the 50 different simulations performed for each.

To make explicitly clear which individual planets and systems we expect to benefit from additional TESS data, in Figure 6 we plot the mass histograms from the Kepler mission (gray), the nominal TESS mission (cyan), and the \( E_{\text{4,NNN}} \) extended TESS mission (magenta) for all 25 planets. We use solid lines for the histograms representing the median KL divergence case and dashed lines for the histograms corresponding to the upper limits of the horizontal error bars in extended mission scenarios, we present only this most optimistic scenario with three additional years of coverage).

The range indicated by the vertical bars illustrates the most pessimistic depth uncertainty (top of the bar) when assuming single-transit-only measurements to the most optimistic depth uncertainty (bottom of bar) for perfect stacking of all detectable transits. The horizontal bars indicate the central 68% of the spread of the KL divergences for each simulation.

As discussed in Section 2.2, a fractional transit depth uncertainty of \( \sim 3 \) is likely marginally detectable, given a known transiting planet. We overplot in Figure 4 a shaded box to indicate the region of parameter space that would have both significantly improved mass measurements (KL divergence \( \geq 0.1 \)) as well as being plausibly detectable (fractional transit depth uncertainty \( \sigma_p/\delta \lesssim 0.3 \)). We also include a more optimistic box, extending to a fractional transit depth uncertainty of \( \sigma_p/\delta \lesssim 1.0 \), since the ultimate photometric noise properties TESS will achieve are uncertain. We plot only the planets whose error bars touch the shaded box (omitted systems would likely be undetectable and/or have negligible improvements to their measured masses).

There are 14 planets across 7 systems that fall within the larger shaded box highlighted in Figure 4 in the nominal mission, and 25 planets across 10 systems in the extended, \( E_{\text{4,NNN}} \) mission; these systems are both easily detected by TESS and would have the greatest improvement to their mass measurements. For the more conservative restriction of \( \sigma_p/\delta < 0.3 \), these numbers fall to 6 planets across 4 systems in the nominal mission, and 12 planets across 6 systems in the extended, \( E_{\text{4,NNN}} \) mission.

Thus far, we have presented the results for the nominal mission and for the extended mission scenario \( E_{\text{4,NNN}} \). As expected, when we analyze in detail the alternative extended mission scenarios in Table 1 that contain fewer observation years in the northern hemisphere, we find that the number of planets with improved mass measurements is intermediate between the results of the nominal and \( E_{\text{4,NNN}} \) scenarios presented above. Altering the pole-centered camera from 4 to 3 had no significant impact on our results.

Given both the unfamiliarity of the KL divergence statistic, as well as the significant non-Gaussianities in the posterior distributions, in Figure 5 we plot the underlying quantity \( \sigma_{\text{mass, KL}} / \sigma_{\text{mass, T}} \) (i.e., the same quantity indicated in the labels of Figure 3) against the KL divergence from all 50 simulation iterations of the 25 planets in the right-hand panel of Figure 4, showing the results for each planet using a different color. We overplot (solid line) the expected results for a Gaussian distribution that has the same mean in the prior and posterior distributions. The scatter of points for each planet reflects the significant non-Gaussianity present in the distributions, as well as clarifying the overall scale of the variation seen in the mass measurement improvements (for a single planet) across the 50 different simulations performed for each.

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We see that planets such as Kepler 80-b and Kepler 80-c will benefit significantly from the acquisition of TESS short-cadence data.

As noted in Section 2.2, the practical analysis of individual systems is likely to be undertaken using a detailed photodynamical approach (e.g., Carter et al. 2011, 2012), particularly for the multi-planet systems identified in Figures 4–6 as being likely to be most improved by TESS observations.

5. Discussion

The main goal of this work has been to provide a quantitative estimate of the potential for the TESS to improve the mass (and eccentricity) constraints obtained from TTVs observed in Kepler systems.

We find the following.

1. Approximately 6–14 planets will have their measured masses (and eccentricities) significantly improved (i.e., their KL divergence is $\geq 0.1$) by measurements taken during the nominal TESS mission.

2. Approximately 12–25 planets (total) will have their measured masses and eccentricities significantly improved through measurements taken during a nominal+extended TESS mission.

We note that we have not evaluated other benefits that may flow from the observations of these (and other) systems at short cadence. In particular, the acquisition of such data may enable the detection (or constraint) of orbit evolution effects such as semimajor axis evolution driven by tidal effects. We refer the interested reader to the work of Christ et al. (2018) for a detailed examination of such issues.

M.J.H. and M.J.P. gratefully acknowledge NASA grants NNX12AE89G, NNX16AD69G, and NNX17AG87G, as well as support from the Smithsonian 2015–2017 Scholarly Studies program. M.G. gratefully acknowledges the Origins of Life Summer Undergraduate Research Prize Award Program. S.H. gratefully acknowledges the CfA Fellowship. The computations in this paper were run on the Odyssey cluster supported by the FAS Science Division Research Computing Group at Harvard University. This research has made use of the NASA Exoplanet Archive, which is operated by the California Institute of Technology, under contract with the National Aeronautics and Space Administration under the Exoplanet Exploration Program.

Facility: Exoplanet Archive.

Software: TTVFast (Deck et al. 2014), ticgen (Barclay 2017), batman (Kreidberg 2015).
Validity of Transit Mid-time Approximations

To understand whether the relation in Equation (2) for the uncertainty in the mid-time of the transits holds for the low S/N transits observed by TESS, we created synthetic light curves of the 25 planets plotted in Figure 4 using the batman python module (Kreidberg 2015). The light curves are sampled at a two-minute cadence and the transit properties are based on the planet periods, planet–star radius ratios, impact parameters, transit durations, and ingress times reported on the Exoplanet Archive. For simplicity, we ignore limb-darkening. We generate synthetic light curves with a mid-transit time of $T_c = 0$ by adding random Gaussian noise for 10 different levels of photometric noise logarithmically spaced between $\sigma_{\text{phot}}$, $Kep$, $\sigma_{\text{phot}}$, $TESS$, where $\sigma_{\text{phot}}$, $Kep$ and $\sigma_{\text{phot}}$, $TESS$ are the photometric precision of Kepler and TESS, respectively. We then attempt to recover the transit mid-time from each synthetic light curve by fitting the trapezoidal transit model presented in Price & Rogers (2014). We fit the transit mid-time by computing $\chi^2(T_c)$ over a grid of values spanning $\pm 7 \sigma_{T_c,\text{pred}}$, where $\sigma_{T_c,\text{pred}}$ is the predicted transit mid-time uncertainty based on the formula of Price & Rogers (2014). All transit parameters other than the mid-time are fixed, as we assume the planets’ transit depths and durations to be tightly constrained from Kepler photometry. We construct a posterior distribution, $p(T_c)$, for the transit mid-time from the grid of $\chi^2$ values such that $p(T_c) \propto \exp\left[-\frac{1}{2} \chi^2(T_c)\right]$ and then compute the standard deviation, $\sigma_{T_c}$, of this posterior distribution in order to compare it with the prediction, $\sigma_{T_c,\text{pred}}$, of the formula of Price & Rogers (2014). This procedure is repeated three times for each planet: first, for light curve data containing a single transit and then twice more for phase-folded data containing the number of transits expected during TESS’s nominal mission and extended mission $E_{4\text{,NNN}}$. The number of nominal and extended mission transits are taken to be $54d/P$ and $216d/P$, respectively, where $P$ is the planet’s orbital period.

The results are shown in Figure 7, where computed values of $\sigma_{T_c}/\sigma_{T_c,\text{pred}}$ are plotted against the expected transit depth fractional uncertainty, $\sigma_{\delta}/\delta$, predicted by the analysis of Price & Rogers (2014), which depends on the level of photometric noise as well as the particular properties of the transit. From the figure, we see that the formula of Price & Rogers (2014) underpredicts transit mid-time uncertainties for $\sigma_{\delta}/\delta \gtrsim 0.3$. 

Figure 6. Mass histograms of the 25 planets that are most improved through TESS observations (i.e., those planets from Figure 4). Gray bars are the original Kepler posterior, cyan is after a simulated TESS primary mission, and magenta is after the $E_{4\text{,NNN}}$ extended mission. Solid lines are the median results and the dashed lines correspond to the upper limits of the horizontal error bars in Figure 4. Values for the KL divergence are given in each panel, with the values for the dashed upper limit histograms being in parentheses.

Appendix

Validity of Transit Mid-time Approximations

To understand whether the relation in Equation (2) for the uncertainty in the mid-time of the transits holds for the low S/N transits observed by TESS, we created synthetic light curves of the 25 planets plotted in Figure 4 using the batman python module (Kreidberg 2015). The light curves are sampled at a two-minute cadence and the transit properties are based on the planet periods, planet–star radius ratios, impact parameters, transit durations, and ingress times reported on the Exoplanet Archive. For simplicity, we ignore limb-darkening. We generate synthetic light curves with a mid-transit time of $T_c = 0$ by adding random Gaussian noise for 10 different levels of photometric noise logarithmically spaced between $\sigma_{\text{phot}}$, $Kep$, $\sigma_{\text{phot}}$, $TESS$, where $\sigma_{\text{phot}}$, $Kep$ and $\sigma_{\text{phot}}$, $TESS$ are the photometric precision of Kepler and TESS, respectively. We then attempt to recover the transit mid-time from each synthetic light curve by fitting the trapezoidal transit model presented in Price & Rogers (2014). We fit the transit mid-time by computing $\chi^2(T_c)$ over a grid of values spanning $\pm 7 \sigma_{T_c,\text{pred}}$, where $\sigma_{T_c,\text{pred}}$ is the predicted transit mid-time uncertainty based on the formula of Price & Rogers (2014). All transit parameters other than the mid-time are fixed, as we assume the planets’ transit depths and durations to be tightly constrained from Kepler photometry. We construct a posterior distribution, $p(T_c)$, for the transit mid-time from the grid of $\chi^2$ values such that $p(T_c) \propto \exp\left[-\frac{1}{2} \chi^2(T_c)\right]$ and then compute the standard deviation, $\sigma_{T_c}$, of this posterior distribution in order to compare it with the prediction, $\sigma_{T_c,\text{pred}}$, of the formula of Price & Rogers (2014). This procedure is repeated three times for each planet: first, for light curve data containing a single transit and then twice more for phase-folded data containing the number of transits expected during TESS’s nominal mission and extended mission $E_{4\text{,NNN}}$. The number of nominal and extended mission transits are taken to be $54d/P$ and $216d/P$, respectively, where $P$ is the planet’s orbital period.

The results are shown in Figure 7, where computed values of $\sigma_{T_c}/\sigma_{T_c,\text{pred}}$ are plotted against the expected transit depth fractional uncertainty, $\sigma_{\delta}/\delta$, predicted by the analysis of Price & Rogers (2014), which depends on the level of photometric noise as well as the particular properties of the transit. From the figure, we see that the formula of Price & Rogers (2014) underpredicts transit mid-time uncertainties for $\sigma_{\delta}/\delta \gtrsim 0.3$. 

Figure 6. Mass histograms of the 25 planets that are most improved through TESS observations (i.e., those planets from Figure 4). Gray bars are the original Kepler posterior, cyan is after a simulated TESS primary mission, and magenta is after the $E_{4\text{,NNN}}$ extended mission. Solid lines are the median results and the dashed lines correspond to the upper limits of the horizontal error bars in Figure 4. Values for the KL divergence are given in each panel, with the values for the dashed upper limit histograms being in parentheses.

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Figure 7. Transit mid-time uncertainties, $\sigma_{T_c}$, recovered from fitting simulated light curves and normalized by $\sigma_{T_c,\text{pred.}}$, the uncertainty predicted by the formula of Price & Rogers (2014) derived from a Fisher information analysis vs. the predicted fractional depth uncertainty, $\sigma_{\delta_0}$, also computed from formulas of Price & Rogers (2014). Each point is computed by fitting a synthetic light curve generated by adopting the transit properties of one of the Kepler planets plotted in Figure 4 and a photometric noise level of $\sigma_{\text{phot}} \in [0.5\sigma_{\text{phot},\text{Kep}} \text{ to } 1.5\sigma_{\text{phot},\text{Tess}}]$. Symbol colors denote the Kepler system as in Figure 4. Different symbols are used for points computed using synthetic light curves corresponding to either a single transit (circles) or phase-folded data containing the number of transits expected during the nominal (square) or extended (diamond) missions.

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