Hierarchies from D-brane instantons in globally defined Calabi-Yau Orientifolds

Mirjam Cvetič and Timo Weigand

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, USA

We construct the first globally consistent semi-realistic Type I string vacua on an elliptically fibered manifold where the zero modes of the Euclidean D1-instanton sector allow for the generation of non-perturbative Majorana masses of an intermediate scale. In another class of global models, a D1-brane instanton can generate a Polonyi-type superpotential breaking supersymmetry at an exponentially suppressed scale.

I. INTRODUCTION

The quest for a natural generation of hierarchies is a classic theme in particle physics. Prominent examples of hierarchical mass scales are, amongst others, intermediate scale Majorana masses for right-handed neutrinos or the $\mu$-term and supersymmetry breaking scale in the minimal supersymmetric Standard Model. In the past year important conceptual progress has been made towards realizing these hierarchies due to new D-brane instantons [1] of genuinely string theoretic origin with apparently no field theory analogs. In type II string compactifications with D-branes the mentioned couplings are typically forbidden perturbatively, but under specific conditions they can be generated by D-brane instantons whose strength is exponentially suppressed by the classical instanton action with respect to the string scale. This transforms the mass hierarchies into logarithmic ones. Further work in this direction includes [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

While local realisations of this mechanism have been found, see, e.g., [8], an important outstanding challenge lies in the construction of globally consistent chiral string vacua which exemplify explicitly the non-perturbative origin of the respective hierarchies. The main difficulty is due to tadpole cancellation conditions corresponding to Gauss law: the total charge of the D-branes on the compact internal manifold has to vanish. Typically this requires additional D-branes in a hidden sector. These can introduce extra fermionic instanton zero modes in form of open strings between the D-branes and the instanton that “annihilate” the instanton effect. Furthermore, the classical instanton action given by the volume of the instanton depends on the shape or size of the manifold, i.e. the moduli fields; these are constrained by the D-brane sector supersymmetry conditions. Thus, even before tackling full moduli stabilisation, the desired range of exponentially suppressed couplings has to be compatible with the supersymmetry constraints.

In this letter we provide the first classes of semi-realistic globally consistent string vacua with D-branes where some hierarchical couplings can be realised by D-brane instantons. We choose the framework of Type I string theory with magnetized D9-branes compactified on a specific Calabi-Yau threefold $X$. The Euclidean D1-branes (E1-instantons) wrap isolated holomorphic curves on $X$. For a suitable choice of compactification manifold the charged zero mode structure of the instanton can be engineered to allow for the existence of specific non-perturbative couplings. Our focus is to demonstrate that it is indeed possible to satisfy all global consistency conditions and maintain the required instanton zero mode structure while the desired suppression scale is compatible with the D-brane sector supersymmetry constraints. To this aim we have not attempted to build a fully-realistic Standard Model sector but content ourselves with SU(5) Grand Unified (GUT) toy models with four chiral families. While the models have phenomenological drawbacks, such as a large number of Higgs pairs and chiral exotics, they demonstrate for the first time an explicit global realisation of the described instanton physics along with a sufficiently complex particle physics sector. The present framework can be further refined to build ever more realistic string vacua with D-instanton effects.

Our first class of examples realizes intermediate scale Majorana masses. In the second class of examples instantons generate a Polonyi-type superpotential in a hidden sector which breaks supersymmetry at an exponentially suppressed scale. These techniques can readily be applied to realize other instanton effects such as the generation of $\mu$-terms [1] or certain GUT Yukawa couplings [12].

II. E1-INSTANTONS IN TYPE I VACUA

We consider Type I compactifications on an internal Calabi-Yau threefold $X$. The gauge sector is defined in terms of stacks of $M_a = n_a \times N_a$ spacetime-filling D9-branes wrapping the whole of $X$ (and their orientifold images), where $\sum a n_a N_a = 16$. These D9-branes can carry rank $n_a$ holomorphic vector bundles $V_a$ whose structure group $U(n_a)$ breaks the original gauge group $U(M_a)$ associated with the coincident D9-branes to the commutant $U(N_a)$ [15, 16]. Stacks of $N_i$ D5-branes wrapping the holomorphic curve $\Gamma_i$ on $X$ are described by the sheaf $i_* O|_{\Gamma_i}$ supported on $\Gamma_i$ and carry gauge group $Sp(2N_i)$.

The massless open string spectrum is encoded in various cohomology (or rather extension [17]) groups associated with the respective bundles on the branes as summarized in table I with the goups of degree 2 (1) counting (anti-)chiral superfields. For details see [16]. Cancellation of D5-tadpoles requires $\sum a N_a c_2(V_a) - \sum_i N_i \gamma_i = -c_2(TX)$, and absence of global anomalies is ensured if $\sum a N_a c_1(V_a) \in H^2(X, 2\mathbb{Z})$. 

TABLE I: Massless spectrum of Type I compactifications.

| reps. | \( \prod_i SU(N_a) \times U(1)_a \times \prod_i SP(2N_c) \) |
|-------|--------------------------------------------------|
| \((\text{Sym}_{\mathbb{C}_a})_2(\xi)\) | \(H^s(X, P^2 ker(V_a))\) |
| \((\text{AntiSym}_{\mathbb{C}_a})_2(\xi)\) | \(H^s(X, \bigoplus^2 ker(V_a))\) |
| \((N_a, N_b)_{1(a), 1(b)}\) | \(H^s(X, V_a \otimes V_b)\) |
| \((\mathbb{N}_a, \mathbb{N}_b)_{-1(a), 1(b)}\) | \(H^s(X, V_a^\vee \otimes V_b)\) |
| \((N_a, 2N_b)_{1(a)}\) | \(H^s(\Gamma, V_a^\vee | r_1 \otimes K_{1/2}^1)\) |

TABLE II: Fermionic zero modes in D9-E1 sector.

| state | rep | cohomology |
|-------|-----|------------|
| \(\lambda_a\) | \((N_a, 1_E)\) | \(H^0(P^1, V_a^\vee (-1|r_1)\) |
| \(\overline{\lambda}_a\) | \((\mathbb{N}_a, \mathbb{N}_b)\) | \(H^1(P^1, V_a^\vee (-1)|r_1)\) |

The vector bundles must be supersymmetric with respect to the O9-plane. For stable holomorphic bundles this amounts to satisfying the D-flatness condition inside the Kähler form, \(f_a = \frac{n}{3!} \int_X J \wedge J \wedge - \ell_a \int_X J \wedge \left( ch_2(V_a) + \frac{n_a}{24} c_2(T) \right) \), to be positive. Here
\[
\widetilde{f}_a = \frac{n}{3!} \int_X J \wedge J \wedge - \ell_a \int_X J \wedge \left( ch_2(V_a) + \frac{n_a}{24} c_2(T) \right).
\]

The superpotential of the four-dimensional \(N \equiv 1\) supersymmetric effective action receives non-perturbative corrections due to E1-instantons on a holomorphic curve \(C_{\mathbb{P}^4}\). For the first related work in this context see [1]. The universal part of the E1-instanton measure is given by \(d^4x d^2\theta\) as is necessary for contributions to the superpotential. To avoid complications due to additional instanton bosonic and fermionic deformation zero modes, we focus on E1-branes wrapping rigid \(\mathbb{P}^1_i\), where these modes are absent.

In the presence of D9/D5-branes additional fermionic zero modes arise which are charged under the gauge group on the branes [13, 3, 4]. Consider first the zero modes in the D9-E1 sector. In analogy to the D9-D5 spectrum they should be described by the cohomology groups \(H^s(C, V_a^\vee | C \otimes K_C^{1/2})\). The crucial difference as compared to the spectrum between pairs of D-branes is that only states corresponding to chiral operators (from a worldsheet perspective) are present [3, 4] and that the four-dimensional polarisation of the fermionic zero modes is given by a single Grassmann number. As a result, only the even degree cohomology group \(H^0(C, V_a^\vee | C \otimes K_C^{1/2})\) corresponds to actual fermionic zero modes in the representation \((N_a, 1_E)\), while the would-be modes classified by \(H^1(C, V_a^\vee | C \otimes K_C^{1/2})\) are not realised physically. Zero modes in the conjugate representation \((\mathbb{N}_a, 1_E)\) are associated with \(H^0(C, V_a^\vee | C \otimes K_C^{1/2}) = H^1(C, V_a^\vee | C \otimes K_C^{1/2})\). The last equality follows from Serre duality on \(C\). All this is summarized in Table [1].

III. E1-INSTANTONS ON ELLIPTIC CY3

We now present a class of globally defined supersymmetric models exhibiting such instanton effects. We choose a Calabi-Yau threefold \(X\) which is elliptically fibered (with fiberation \(p\)) over the del Pezzo surface \(dP_r\) for \(r = 4\). For detailed information on this type of geometries see, e.g., [20]. For concrete algebraic geometry formulae relevant in our context see [13]. The Kähler form \(J\) enjoys the expansion \(J/\ell_a^2 = r_s \sigma + r_1 \pi^s l + \sum_{i=1}^{4} r_i \pi^s E_i\) (c.f., Appendix A of [13]) in terms of the fibre class \(\sigma\), the hyperplane class \(l\) as well the class \(E_i\) of the 4 \(\mathbb{P}^1\) inside \(dP_4\) obtained as the blow-up of certain singularities in \(\mathbb{C}P^2\).

This type of geometry is particularly suitable for constructing models exhibiting the desired non-perturbative corrections: each bundle \(V_a\) can locally be engineered in such a way as to lead to the correct number of charged zero modes with the instanton. This setup can then be promoted to a globally defined model if the tadpole...
cancellation condition can be satisfied without introducing additional zero modes between the instanton and the filler D5-branes. To demonstrate the existence of globally consistent models in this context it suffices to consider branes carrying line bundles $L_a$ with first Chern class $c_1(L_a) = g_a \sigma + \pi^* \zeta_a$ with $\zeta_a \in H^2(dP_4, \mathbb{Z})$. For the concrete expressions of all higher Chern characters and the Euler characteristic $\chi(L_a)$ in terms of $c_1(L_a)$ see sections 3.3 and 3.4 of [12], while the full cohomology groups $H^*(X, \mathbb{C})$ can be computed from appendix B in [19].

We consider instantons wrapping certain rigid non-horizontal $\mathbb{P}^1$s in the $dP_3$ surface $\pi^* E_4$, which is obtained as the pullback of the $\mathbb{P}^1 E_4$ in the base. Specifically, the $\mathbb{P}^1$s are taken not to intersect the base of $dP_3$. It turns out that one can always take the filler D5-branes to wrap curves $\Gamma_i$ that do not hit at least some of the $\mathbb{P}^1$s in that class, thus introducing no extra zero modes for this subset of instantons.

### A. Majorana masses

In models with instanton-generated Majorana masses a particular set of magnetized D9-branes engineers the Standard Model (or a GUT version of it), while the right-handed neutrinos $N_R$ arise as the bi-fermionic matter between a pair of $U(1)$ stacks with gauge groups $U(1)_b$ and $U(1)_c$. For $N_R$ transforming as, say, $(-1_b, 1_c)$, Majorana masses can be generated in the presence of precisely two charged fermionic instanton zero modes of type $\lambda_b$ and $\overline{\lambda}_c$. This guarantees that the coupling $\lambda_b N_R \overline{\lambda}_c$ in the instanton class. This will be detailed elsewhere.

For realisations of the $U(1)_b$ and $U(1)_c$ stack in terms of single D9-branes carrying line bundles $L_b$ and $L_c$, the zero mode constraints translate into $h^i(\mathbb{P}^1, L_b(1)_{E_4}) = (2, 0) = h^i(\mathbb{P}^1, L_c(1)_{E_4})$. For the described subclass of $\mathbb{P}^1$s in $\pi^* E_4$ not intersecting the horizontal $E_4$ in $dP_3$, this can be shown to correspond to $\zeta_b \cdot E_4 = -2 = -\zeta_c \cdot E_4$. Since our primary aim is to find global embeddings of the instanton sector, we content ourselves with the realisation of a GUT toy version of the Standard Model sector, namely to engineer it as a $U(5)$ theory from $N_a = 5$ D9-branes with line bundle $L_a$. In table III we give a representative example of an SU(5) model of the described type. All D5-brane tadpoles are cancelled by including also stacks of $N_i$ un magnetized D5-branes on curves $\Gamma_i$ with total D5-brane charge $\sum N_i \gamma_i = 41 F + \pi^*(16 l - 12 E_1)$, with $F$ the fibre class. Furthermore, 12 un magnetized D9-branes are required to cancel the D9-brane tadpoles. One can check that the D-flatness conditions allow for solutions inside the Kähler cone, e.g., for $r_\sigma = 0.97, r_l = 10.49, r_1 = -7.27$ and $r_2 = r_3 = r_4 = -1.00$. The spectrum can be computed from table IV and the formulae of [15, 16]. It contains four chiral families of $\overline{10}$ counted by $H^*(X, (L_1^\vee)^2)$ together with additional 5 and $\overline{3}$ from the $a - b$ and $a - c$ sector as well as from the filler D5- and D9-branes. Only part of them can be interpreted as matter 5 and Higgs pairs once Yukawa couplings are taken into account. Four chiral generations transforming as $(-1_b, 1_c)$ in the spectrum correspond to right-handed neutrinos $N_R$. The zero mode structure of the $E1$ instanton ensures that a superpotential of the form $W = x M_4 \exp(-\frac{2 \pi \nu}{\alpha g_s} \overline{\mathcal{V}}_{\overline{N_R}} N_R^\vee N_R)$ can be generated, where $x$ is an $O(1)$ factor from the exact computation. The above value of the Kähler moduli corresponds to $\overline{\mathcal{V}}_a = 8.15 \ell_6^3$ and an instanton volume $\mathcal{V}_{\overline{N_R}} = -r_4 = 1$. Then for $\alpha g_s \sim 0.04$, corresponding to $M_s \sim 10^{18}$ GeV, the Majorana mass is $O(10^{10})$ GeV. Thus, we demonstrated that in a global setup the exponential suppression can indeed be engineered at the required intermediate scale. The full answer involves summing up all instanton contributions associated with all suitable curves, possibly along the lines of [21]. Note that potentially dangerous vanishing theorems can be bypassed by suitably distributing the D5-branes such that they introduce additional zero-modes on some of the rigid $\mathbb{P}^1$ of the instanton class. This will be detailed elsewhere.

### B. Polonyi model

A fascinating application of stringy instanton effects lies in the context of realising supersymmetry breaking at a naturally suppressed scale. In the probably simplest such scenario supersymmetry is broken by a Polonyi-type superpotential of the form $W = c \Phi$ for a superfield $\Phi$. The possible generation of such tadpoles for open string fields by D-brane instantons was first pointed out in [1] and discussed recently in local setups in [22, 23]. As a variant of the configuration leading to Majorana masses, the Polonyi field $\Phi$ arises in the bi-fundamental sector of two massive U(1) gauge groups $U(1)_b$ and $U(1)_c$ of charge, say, $(-1_b, 1_c)$, while the linear superpotential is generated by an E1-instanton with the two zero modes $\lambda_b$ and $\overline{\lambda}_c$. In table IV we present a global embedding of this scenario into an $SU(5)$ GUT-type vacuum with 4 chiral generations of $\overline{10}$ and, as before, further 5 and $\overline{3}$.
The tadpole constraints require the introduction of five-brane stacks of total class \( \sum_n N_n \gamma_n = 28 F + \sigma \cdot \pi^*(2 E_2) \). The bundle configuration is D-flat inside the Kähler cone e.g., for \( r_0 = 1.59, r_1 = 12.67, r_1 = -2.11, r_2 = -5.00, r_3 = -4.00, r_4 = -6.00, \) for which \( f_4 = 32.87 \delta_i^a \). The zero mode structure of an instanton wrapping a non-horizontal \( \mathbb{P}^1 \) curve of the type described in \( \pi^* E_4 \) allows for the generation of a Polonyi-type term for \( \Phi \) of the form \( W = x M_s^2 \exp(-S_{E_1}) \Phi \). For Kähler moduli of the above D-flat value, the instanton volume is \( \text{Vol} = 6 \), and the Polonyi-term is suppressed by \( \exp(-S_{E_1}) = \exp(- \frac{2 \pi^a_i \delta_i^a \text{Vol}}{\alpha_{GUT} f_a}) \approx 2.8 \times 10^{-11} \). Its F-term breaks supersymmetry dynamically at the scale \( F_\Phi \sim 10^{-11} M_s^2 \). The particular virtue of the D-brane instanton lies in engineering this hierarchical scale naturally from the string theoretical point of view [22, 23]. For gravity mediation this can lead to soft supersymmetry breaking masses in the TeV range for \( M_s \sim (10^{17} - 10^{18}) \) GeV and a suitable matter Kähler potential, e.g. as in [23]. Gauge mediated supersymmetry breaking can also occur at the loop-level due to perturbative superpotential couplings of the type \( \Phi \cdot 5_M \cdot \bar{5}_M \) where \( 5_M, \bar{5}_M \) represent messenger fields. These effects have an extra suppression \( \frac{4 \pi}{\alpha_{GUT}} \) factor.

We conclude with a few remarks related to the interplay of closed sector moduli stabilization and the D-instanton induced Polonyi terms. The closed string sector Kähler moduli can in principle be stabilized due to the strong gauge dynamics associated with the additional \( Sp(2 N_i) \) gauge group factors. Their large negative beta functions induce gaugino condensation and typically result in “race-track”-type Kähler moduli stabilisation, e.g. along the lines of [24]. The one-loop threshold corrections to the gauge kinetic function can lead also to complex structure moduli stabilisation, possibly in connection with fluxes. These global constructions therefore provide an intriguing framework where strong gauge dynamics can yield a vacuum solution with stabilized closed sector moduli. For unbroken supersymmetry in the closed string sector the D-instanton induced Polonyi term provides an exponentially suppressed SUSY breaking and possibly even “uplifting mechanism” [25, 26, 27].

Acknowledgements: We thank R. Blumenhagen, K. Bobkov, S. Kachru, J. Kumar, R. Richter, E. Sharpe and E. Silverstein for discussions, and especially R. Donagi and T. Pantev for discussions on elliptic fibrations. This research was supported by DOE Grant EY-76-02-3071.

| Bundle | N | \( c_2(L) = q \sigma + \pi^*(\zeta) \) |
|-------|---|---------------------------------|
| \( L_a \) | 5 | \( \pi^*(-2 E_1 + 2 E_2 - E_3) \) |
| \( L_b \) | 1 | \( -2 \sigma + \pi^*(l - E_1 - E_3 - E_4) \) |
| \( L_c \) | 1 | \( -4 \sigma + \pi^*(l - E_1 - E_3 - E_4) \) |

TABLE IV: A U(5) \( \times U(1) \) \( \times U(1) \) Polonyi-type model.