Research of pneumatic cylinder actuator's dynamics

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Abstract. Pneumatic actuators are widely used in industry as the most reliable automation tools. The performance of automated systems depends on the speed of pneumatic actuators. Therefore, the development of methods for studying the dynamic processes of pneumatic actuators is an urgent task. This paper proposes a mathematical model of a pneumatic actuator of the hoist, which is convenient for computer calculations using universal mathematical packages. Friction forces on transient processes in a pneumatic actuators are investigated as an example of the application of a mathematical model.

1. Introduction
Increasing the performance of pneumatic actuators means increasing their responsiveness. Since pneumatic actuators are part of technological equipment, the time of their actuation is included in the total time of the operating cycle of technological equipment, affecting its performance [1, 2].

The problem of increasing the productivity of pneumatic actuators is directly related to the development of methods of dynamic research of technological equipment, with the creation of methods of dynamic calculation of pneumatic mechanisms [1].

The most important directions in the development of pneumatic actuators include research into the dynamics of pneumatic actuators.

Papers [1-4] are devoted to the dynamics of pneumatic systems. A more complete list of papers devoted to the dynamics of pneumatic actuators can be found in [1].

In [1], the dynamics of a one-way pneumatic actuator is described by a system of two equations for a forward stroke

\[ m \frac{d^2 y_1}{dt^2} = S(p_1 - p_a) - F_p, \]  
\[ \frac{dp_1}{dt} = \frac{k \mu_s c_1 K p_a / R T_m}{S y_1} \varphi(\sigma_1) - \frac{k p_1}{y_1} \frac{dy_1}{dt}, \]

and two equations for the return

\[ m \frac{d^2 y_2}{dt^2} = -S(p_2 - p_a) + F_p, \]  
\[ \frac{dp_2}{dt} = \frac{k \mu_s c_2 K p_2}{S(y_1 + L + y_2) p_m} \varphi(\sigma_a / \sigma_2) - \frac{k p_2}{(y_1 + L + y_2) y_2} \frac{dy_2}{dt}. \]
where \( m_p \) – reduced to the piston mass of the mechanism units, set in motion by the rod; \( y_1 \) and \( y_2 \) – displacement of the piston, respectively, during forward and reverse strokes; \( S \) – area of the piston from the side of the working cavity of the pneumatic cylinder; \( p_1 \) and \( p_2 \) – pressure in the working cavity of the pneumatic cylinder, respectively, during forward and reverse travel; \( p_a \) – atmospheric pressure; \( F_p \) – the value of the drag force reduced to the piston; \( k \) – adiabatic index; \( \mu_s \) – flow rate coefficient of the hole through which air enters the working cavity of the pneumatic cylinder; \( S_s \) – area of the hole through which air enters the working cavity of the pneumatic cylinder; \( p_m \) is the pressure of the gas in the pipeline; \( R \) – gas constant of the gas; \( T_m \) – absolute temperature of the gas coming from the main; \( K \) – parameter determined by the formula
\[
K = \sqrt{\frac{2k}{k-1}};
\]
\( \varphi(\sigma_1), \varphi(\sigma_2) \) and \( \varphi(\sigma_a) \) – flow functions,
\[
\varphi(\sigma_1) = \sqrt{\frac{\sigma_1^{2/k} - \sigma_1^{(k+1)/k}}{\sigma_1^{2/k} - \sigma_1^{(k+1)/k}}}, \quad \varphi(\sigma_2) = \sqrt{\frac{\sigma_2^{2/k} - \sigma_2^{(k+1)/k}}{\sigma_2^{2/k} - \sigma_2^{(k+1)/k}}}, \quad \varphi(\sigma_a/\sigma_2) = \sqrt{\frac{(\sigma_a/\sigma_2)^{2/k} - (\sigma_a/\sigma_2)^{(k+1)/k}}{(\sigma_a/\sigma_2)^{2/k} - (\sigma_a/\sigma_2)^{(k+1)/k}}};
\]
\( \sigma_1, \sigma_2 \) and \( \sigma_a \) – pressure ratios, \( \sigma_1 = p_1/p_m; \sigma_2 = p_2/p_m; \sigma_a = p_a/p_m \).

In equation (2), the values of the flow rate function \( \varphi(\sigma_1) \) are
\[
\varphi(\sigma_1) = \sqrt{\frac{\sigma_1^{2/k} - \sigma_1^{(k+1)/k}}{\sigma_1^{2/k} - \sigma_1^{(k+1)/k}}} \text{ for } \sigma_* < \sigma_1 < 1 \\
\varphi(\sigma_1) = \varphi(\sigma_*) \text{ for } 0 < \sigma_1 \leq \sigma_*,
\]
where \( \sigma_* \) is the critical pressure ratio, \( \sigma_* = \left[ \frac{2k}{(k+1)} \right]^{1/(k-1)} \).

In equation (2), the values of the flow rate function \( \varphi(\sigma_a/\sigma_2) \) is equal to
\[
\varphi(\sigma_a/\sigma_2) = \sqrt{\frac{(\sigma_a/\sigma_2)^{2/k} - (\sigma_a/\sigma_2)^{(k+1)/k}}{(\sigma_a/\sigma_2)^{2/k} - (\sigma_a/\sigma_2)^{(k+1)/k}}} \text{ for } \sigma_* < (\sigma_a/\sigma_2) < 1 \\
\varphi(\sigma_a/\sigma_2) = \varphi(\sigma_*) \text{ for } 0 < (\sigma_a/\sigma_2) \leq \sigma_*.
\]

When solving the system of equations (1) and (2) for the forward stroke and the system of equations (3) and (4) for the reverse stroke, it is necessary to use two formulas for the flow function, respectively (5) and (6) for the forward stroke and (7) and (8) for forward travel. It complicates solving the task.

2. Pneumatic lift drive

Single-acting pneumatic hoists are widely used. In single-acting pneumatic actuators, the working stroke is performed by compressed air entering the piston cavity (figure 1 (a)), and the reverse stroke (figure 1 (b)) occurs under the action of gravity, while the compressed air is escapes into the atmosphere.
3. Mathematical model of the lift pneumatic drive. Rise

Let us carry out a mathematical description of a single-acting pneumatic actuators, the diagram of which is shown in figure 1. In a mathematical description, the processes occurring in a pneumatic actuator are be considered as quasi-stationary, air is considered as a perfect gas and heat exchange with the environment is neglected. The compressed air pressure in the line is taken as a constant value.

The equation of motion of the piston with a forward stroke and constant reduced mass mp can be written in the form

$$m_{p1} \frac{dv_{p1}}{dt} = S(p_1 - p_a) - F_p,$$

(9)

where $m_{p1}$ – mass of the platform, product, pin and piston when moving up (with a forward stroke); $v_{p1}$ - speed of the piston in the forward stroke; $p_1$ – pressure of the gas in the piston cavity of the pneumatic cylinder during the forward stroke.

The relationship between the displacement $y_1$ and the speed $v_{p1}$ of the piston is determined by the formula

$$v_{p1} = \frac{dy_1}{dt}.$$

(10)

To determine the dependences $y_1(t)$, $v_{p1}(t)$ in $p_1(t)$, the system of equations (9) and (10) must be supplemented with one more equation. We obtain this equation from the energy equation for gas of variable mass in a pneumatic cylinder without heat exchange [3]

$$kR T_M G_{M1}(p_1) = y_1 S \frac{dp_1}{dt} + k S p_1 v_{p1},$$

(11)

where $k$ – adiabatic index; $R$ – gas constant of the gas; $T_M$ – absolute temperature of the gas coming from the main; $G_{M1}(p_1)$ - mass flow rate of the gas entering the pneumatic cylinder from the line during the forward stroke.

Considering the process of filling the working cavity of pneumatic cylinder with gas as quasi-stationary, the mass flow rate of gas $G_{M1}(p_1)$, coming from the main to the working cavity of the pneumatic cylinder can be determined for each moment of time $t$ by the formula obtained for the steady motion of perfect gas [3]

$$G_{M1}(p_1) = \mu_c S c_1 \sqrt{\frac{p_m}{R M}} \left( \frac{p_{c1}(p_1)}{p_m} \right)^{1/k} \frac{2 k}{k - 1} \left[ 1 - \left( \frac{p_{c1}(p_1)}{p_m} \right)^{(k-1)/k} \right],$$

(12)

where $\mu_c$ – flow rate coefficient of the hole through which air enters the working cavity of the pneumatic cylinder; $p_m$ – pressure of the gas in the pipeline; $p_{c1}(p_1)$ - pressure in the gas stream flowing into the working cavity of the pneumatic cylinder with pressure $p_1$.

The formula for determining gas pressure in a jet flowing into the piston cavity of a pneumatic cylinder can be written using logical operators (for example, in the computer mathematical system Mathcad) as

$$p_{c1}(p_1) = p_1 (p_1 > p_{k1}) + p_{k1}(p_1 \leq p_{k1}),$$

(13)

where $p_{k1}$ – critical pressure in the jet flowing into the piston cavity of the pneumatic cylinder, at which the gas velocity is equal to the local speed of sound, is determined by the formula

$$p_{k1} = p_m \left( \frac{2}{k+1} \right)^{k/(k-1)}.$$

(14)

If the condition in the bracket ($p_1 > p_{k1}$) is satisfied, then bracket and the first term on the right-hand side of formula (13) take the value 1, while the condition in the bracket($p_1 \leq p_{k1}$) is not satisfied, therefore this bracket and the second term on the right parts of formula (13) take the value 0. And vice versa, if the condition in the parenthesis ($p_1 > p_{k1}$) is not met, then this parenthesis and the first term on the right side of formula (13) take the value 0, while the condition in the parenthesis ($p_1 \leq p_{k1}$) is
satisfied and, therefore, this bracket and the second term on the right-hand side of formula (13) take the value 1.

The system of three equations (9) - (11), taking into account formulas (12) - (14), is a mathematical model of a one-way pneumatic actuator for a forward stroke, will be brought to the Cauchy form [5-8]

\[
\frac{dy_1}{dt} = \frac{u_{p1}}{y_1}; \quad (15)
\]

\[
\frac{du_{p1}}{dt} = \frac{1}{m_p} [S(p_1 - p_a) - F_p]; \quad (16)
\]

\[
\frac{dp_1}{dt} = \frac{k}{y_1} \left\{ \frac{RT_M G_M(p_2)}{S} - p_1 u_{p1} \right\}. \quad (17)
\]

To solve the mathematical model, the system of equations (15) - (17) should be supplemented with the initial conditions (the values of the sought variables at the initial moment of time):

\[
y_1(t_0) = y_{10}; \quad (18)
\]

\[
u_{p1}(t_0) = u_{p1,0}; \quad (19)
\]

\[
p_1(t_0) = p_{d1}. \quad (20)
\]

The air pressure in the working cavity of the pneumatic cylinder at the beginning of movement can be found from equation (24) under the condition of a stationary piston \(u_{p1} = 0 \) and \( \frac{du_{p1}}{dt} = 0 \)

\[
p_{d1} = a + \frac{F_{p1}}{S}. \quad (21)
\]

4. Mathematical model of the lift pneumatic drive. Sinking

Let us carry out a mathematical description of a one-way pneumatic actuator, the diagram of which is shown in figure 2 during the reverse stroke.

The equation of motion of the piston return stroke at a constant reduced mass \(m_p\) can be written in the form

\[
m_{p2} \frac{du_{p2}}{dt} = F_{p2} - S(p_2 - p_p), \quad (22)
\]

where \(m_{p2}\) – mass of the platform, product, rod and piston when moving down (during the reverse stroke); \(u_{p2}\) - speed of the piston during the return stroke; \(p_2\) - pressure in the piston cavity of the pneumatic cylinder during the return stroke; \(p_p\) - atmospheric pressure; \(F_{p2}\) – force of gravity acting on the platform, product, rod and piston.

Dependence between displacement and piston speed is determined by the following formula

\[
u_{p2} = \frac{dy_2}{dt}, \quad (23)
\]

where \(y_2\) – piston coordinate during the return stroke, shown in figure 2.

It is necessary to supplement one more equation to solve the system of equations (22) and (23). We obtain this equation from the energy equation for gas of variable mass in a pneumatic cylinder without heat exchange [3]

\[-kRT_G M_2 (p_2) = (L_x + y_{10} + y_{20} - y_2) S \frac{dp_2}{dt} - kS p_2 u_{p2}. \quad (24)
\]

where \(G_{M2}(p_2)\) - mass flow rate of gas flowing out of the working cavity of the pneumatic cylinder into the atmosphere during the return stroke; \(L_x\) - piston stroke; \(y_{10}\) – value of the \(y_1\) coordinate in the initial position of the piston during the forward stroke; \(y_{20}\) – value of the \(y_2\) coordinate at the initial position of the piston during the return stroke.

The minus sign appeared on the left side of the energy equation in comparison with the energy equation for the forward stroke, since the left side represents the energy that gas carries out from the
cavity of the pneumatic cylinder, and during the forward stroke, energy entered the cavity of the pneumatic cylinder together with the gas.

Mass flow rate of gas $G_{M2}(p_2)$ flowing from the working cavity of the pneumatic cylinder into the atmosphere is a function of the pressure $p_2$ in the piston cavity and is determined by the formula

$$G_{M2}(p_2) = \mu_{c2} S_{c2} \frac{p_2}{\sqrt{R T(p_2)}} \left( \frac{p_{c2}(p_2)}{p_2} \right)^{1/k} \sqrt{\frac{2 k}{k-1}} \left[ 1 - \left( \frac{p_{c2}(p_2)}{p_2} \right)^{(k-1)/k} \right],$$

(25)

where $\mu_{c2}$ – flow rate coefficient of the hole through which air flows out of the working cavity of the pneumatic cylinder into the atmosphere; $S_{c2}$ – area of the hole through which air flows out of the working cavity of the pneumatic cylinder into the atmosphere; $T(p_2)$ – gas temperature in the piston cavity of the pneumatic cylinder during the return stroke; $p_{c2}(p_2)$ – pressure in the gas stream flowing out of the working cavity of the pneumatic cylinder with pressure $p_2$ into the atmosphere.

The formula for determining gas pressure in the jet flowing from the piston cavity of pneumatic cylinder into the atmosphere can be written using logical operators (for example, in the computer mathematical system Mathcad) in the form

$$p_{c2}(p_2) = p_a(p_a > p_{k2}(p_2)) + p_{k2}(p_a \leq p_{k2}(p_2)).$$

(26)

where $p_{k2}(p_2)$ – critical pressure in the jet flowing from the piston cavity of the pneumatic cylinder into the atmosphere.

The critical pressure $p_{k2}(p_2)$ in the jet flowing out of the piston cavity of the pneumatic cylinder into the atmosphere is a function of the pressure in the piston cavity and is determined by the formula

$$p_{k2}(p_2) = p_2 \left( \frac{2}{k+1} \right)^{(k-1)/k}.$$  

(27)

Temperature and pressure of gas in the working cavity before the return stroke of the piston depend on the duration of the stay of the piston after the end of the forward stroke of the piston. With a short duration of the piston stay, the gas temperature and pressure before the return stroke are equal, respectively, to the temperature and gas pressure at the end of the forward stroke. With an average duration of piston standing, the temperature and pressure of the gas in the working cavity before the return stroke are equal, respectively, to the temperature and pressure of the gas in the line.

Further, in the calculations, we take the temperature and pressure of the gas in the working cavity before the return stroke equal to the temperature and pressure of the gas in the pipeline. Assuming the process of gas outflow from the working cavity during the reverse stroke as adiabatic, as well as during the forward stroke of the piston, we can write

$$T(p_2) = T_m \left( \frac{p_2}{p_m} \right)^{(k-1)/k};$$

(28)

$$\frac{dy_2}{dt} = u_{p2};$$

(29)

$$\frac{du_{p2}}{dt} = \frac{1}{m_{p2}} \left[ F_{p2} - S(p_2 - p_a) \right];$$

(30)

$$\frac{dp_2}{dt} = \frac{k}{(\mu_s + \mu_{20} + \mu_{2a} - \mu_2)} \left[ p_2 u_{p2} - \frac{RT(p_2) G_{c2}(p_2)}{S} \right].$$

(31)

To solve the mathematical model, the system of equations (29) - (31) should be supplemented with the initial conditions (values of the sought variables at the initial moment of time):

$$y_2(t_0) = y_{20};$$

(32)

$$u_{p2}(t_0) = u_{p2,0};$$

(33)

$$p_2(t_0) = p_{d2}.$$  

(34)
The air pressure in the working cavity of the pneumatic cylinder at the beginning of the movement (reverse stroke) can be found from the equilibrium condition of the stationary piston

\[ p_{d2} = p_a + \frac{F_{p2}}{S} \]  

(35)

5. Calculation results

The program has been developed for the calculation in the computer mathematical system Mathcad of the mathematical model of pneumatic actuator of a single-acting hoist.

Figures 2-4 show results of calculations of a pneumatic actuator of a single-acting hoist for two values of friction forces in parts of the payload (9800 N) and pressure in the line (8 bar).

Figure 2 shows the graphs of the piston displacement versus time. From the graphs (figure 2) it follows that the dependences of the piston displacement on time when moving up and down with different values have a monotonic character close to linear. It can be seen from the graphs (figure 2) that the time of the working stroke of the piston also increases with increasing friction force.

Figure 2. Dependence of the piston piston movement on time for two values of friction forces, in fractions of the payload: 0%; 20%. (a) - to move the piston up; (b) - for downward movement of the piston.

Figure 3 shows the graphs of the piston speed versus time. From the graphs (figure 3) it follows that the dependences of the piston speed on time have an oscillatory character, at the beginning of the movement the amplitude of the oscillations has a maximum value, then the oscillations die out. The maximum values of the amplitude of oscillations of the piston speed correspond to the lower values of the friction force.

Figure 3. Dependence of the piston speed on time for two values of friction forces, in fractions of the payload: 0%; 20%; (a) - to move the piston up; (b) - for downward movement of the piston.

Figure 4 shows graphs of the dependence of the pressure in the piston cavity of the pneumatic cylinder on time. From the graphs (figure 4) it follows that the dependences of the pressure in the piston cavity of the pneumatic cylinders are oscillatory. When moving up at the beginning of the movement, the amplitude of the oscillations has a maximum value, then the oscillations quickly decrease the
amplitude of the oscillations. When the piston moves downward with a lower friction force, the vibration amplitude practically does not change, and when the piston moves with a higher friction force, the vibration amplitude slowly decreases.

![Figure 4. The dependence of the pressure in the piston cavity on time for two values of friction forces, in fractions of the payload: 0%; 20%; (a) - to move the piston up; (b) - for downward movement of the piston.](image)

6. Conclusions

The obtained mathematical model of single-acting pneumatic cylinder actuator makes it possible to simplify the study of the dynamics of pneumatic actuators using universal computer mathematical systems. The mathematical model of a pneumatic drive with single-acting pneumatic cylinder allows at the design stage to study the effect on the dynamics of changing the parameters of the pneumatic actuator, as well as to select the optimal values of the drive parameters.

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