Wavefront sensing is a set of techniques providing efficient means to ascertain the shape of an optical wavefront or its deviation from an ideal reference. Owing to its wide dynamical range and high optical efficiency, the Shack–Hartmann wavefront sensor is nowadays the most widely used of these sensors. Here we show that it actually performs a simultaneous measurement of position and angular spectrum of the incident radiation and, therefore, when combined with tomographic techniques previously developed for quantum information processing, the Shack–Hartmann wavefront sensor can be instrumental in reconstructing the complete coherence properties of the signal. We confirm these predictions with an experimental characterization of partially coherent vortex beams, a case that cannot be treated with the standard tools. This seems to indicate that classical methods employed hitherto do not fully exploit the potential of the registered data.
light is a major carrier of information about the universe around us, from the smallest to the largest scale. Three-dimensional objects emit radiation that can be viewed as complex wavefronts shaped by diverse features, such as refractive index, density or temperature of the emitter. These wavefronts are specified by both their amplitude and phase; yet, as conventional optical detectors measure only (time-averaged) intensity, information on the phase is discarded. This information turns out to be valuable for a variety of applications, such as optical testing, interferometric methods based on the superposition of two beams with a well-defined relative phase; (b) methods based on the measurement of the wavefront slope or wavefront curvature and (c) methods based on the acquisition of images followed by the application of an iterative phase-retrieval algorithm. Notwithstanding the enormous progress that has already been made, practical and robust wavefront sensing still stands as an unresolved and demanding problem.

Actually, there exists a diversity of methods for wavefront reconstruction, each one with its own pros and cons. Such methods can be roughly classified into three categories: (a) interferometric methods based on the superposition of two beams with a well-defined relative phase; (b) methods based on the measurement of the wavefront slope or wavefront curvature and (c) methods based on the acquisition of images followed by the application of an iterative phase-retrieval algorithm. Notwithstanding the enormous progress that has already been made, practical and robust wavefront sensing still stands as an unresolved and demanding problem.

The time-honoured example of the Shack–Hartmann (S–H) wavefront sensor surely deserves a special mention; its wide dynamical range, high optical efficiency, white light capability and ability to use continuous or pulsed sources make of this setup an excellent solution in numerous applications.

The operation of the S–H sensor appeals to the intuition, giving the overall impression that the underlying theory is obvious. Indeed, it is often understood in an oversimplified geometrical-optics framework, which is much the same as assuming full coherence of the detected signal. By any means, this is not a complete picture: even in the simplest instance of beam propagation, the coherence features turn out to be indispensable.

It has been recently suggested that S–H sensing can be reformulated in a concise quantum notation. This is more than an academic curiosity, because it immediately calls for the application of the methods of quantum state reconstruction. Accordingly, one can verify right away that wavefront sensors may open the door to an assessment of the mutual coherence function, which conveys full information on the signal.

In this paper, we report the first experimental measurement of the coherence properties of an optical beam with a S–H sensor. To that end, we have prepared several coherent and incoherent superpositions of vortex beams. Our strategy can efficiently disclose that information, whereas the common S–H operation fails in the task.

Results

S–H wavefront sensing. The working principle of the S–H wavefront sensor can be elaborated with reference to Fig. 1. An incoming light field is divided into a number of sub-apertures by a microlens array that creates focal spots, registered in a camera, typically a charge-coupled device (CCD). The deviation of the spot pattern from a reference measurement allows the local direction angles to be derived, which in turn enables the reconstruction of the wavefront. In addition, the intensity distribution within the detector plane can be obtained by integration and interpolation between the foci.

Unfortunately, this naïve picture breaks down when the light is partially coherent, because the very notion of a single wavefront becomes somewhat ambiguous: the signal has to be conceived as a statistical mixture of many wavefronts. To circumvent this difficulty, we observe that these sensors provide a simultaneous detection of position and angular spectrum (that is, directions) of the incident radiation. In other words, the S–H is a pertinent example of a simultaneous unsharp position and momentum measurement, a question of fundamental importance in quantum theory and about which much has been discussed.

Rephrasing the S–H operation in a quantum parlance will prove pivotal for the remaining discussion. Let $\rho$ be the coherence matrix of the field to be analysed. Using an obvious Dirac notation, we can write $G(x,x') = \langle x' | \rho | x \rangle = \text{Tr}(\rho | x' \rangle \langle x |)$, where $| x \rangle$ is a vector describing a point-like source located at $x$ and $\text{Tr}$ is the matrix trace. Thereby, the mutual coherence function $G(x,x')$ appears as the position representation of the coherence matrix. As a special case, the intensity distribution across a transversal plane becomes $I(x) = \text{Tr}(\rho | x \rangle \langle x |)$. Moreover, a coherent beam of complex amplitude $U(x)$, can be assigned to a ket $| U \rangle$, such that $U(x) = \langle x | U \rangle$.

To simplify, we restrict the discussion to one dimension, denoted by $x$. If the setup is illuminated with a coherent signal $U(x)$, and the $ith$ microlens is $\Delta x_i$ apart from the S–H axis, this microlens feels the field $U(x - \Delta x_i) = \langle x | \exp(-i\Delta x_i P)| U \rangle$, where $P$ is the momentum operator. This field is truncated and filtered by the aperture (or pupil) function $A(x) = \langle x | A \rangle$ and Fourier transformed by the microlenses before being detected by the CCD camera. All this can be accounted for in the form

$$U'(\Delta p_i) = \langle A | \exp(-i\Delta p x) \exp(-i\Delta x_P) | U \rangle,$$

where $x$ is the position operator and we have assumed that the $ith$ pixel is angularly displaced from the axis by $\Delta p_i$. The intensity measured at the $ith$ pixel behind the $ith$ lens is then governed by a Born-like rule

$$I(\Delta x_i, \Delta p_i) = \text{Tr}(\rho | \pi_{ij} \langle \pi_{ij} |),$$

with $| \pi_{ij} \rangle = \exp(i\Delta x_i P)\exp(i\Delta p x)| A \rangle$. As a result, each pixel performs a projection on the position- and momentum-displaced aperture state, as anticipated before.

Some special cases of these aperture states are particularly appealing. For point-like microlenses, $A(x) \rightarrow \delta(x)$ and $| \pi_{ij} \rangle \rightarrow | x = \Delta x_i \rangle$ (that is, a position eigenstate): they produce broad diffraction patterns and information about the transversal...
complex amplitudes are lost. Conversely, for very large microlenses, \( A(x) \to 1 \) and \( |\pi_{ij}\rangle \to |p = \Delta p\rangle \) (that is, a momentum eigenstate): they provide a sharp momentum measurement with the corresponding loss of position sensitivity. A most interesting situation is when one uses a Gaussian approximation: now \( A(x) = \exp(-x^2/2) \), which implies \( |\pi_{ij}\rangle \to |x_j\rangle \), that is, a coherent state of amplitude \( x_j = \Delta x_j + i\Delta p_j \). This means that the measurement in this case projects the signal on a set of coherent states and hence yields a direct sampling of the Husimi distribution.

This quantum analogy provides quite a convenient description of the signal: different choices of CCD pixels and/or microlenses can be interpreted as particular phase-space operations.

**S–H tomography.** Unlike the Gaussian profiles discussed before, in a realistic setup the microlens apertures do not overlap. If we introduce the operators \( \Pi_{ij} = |\pi_{ij}\rangle \langle \pi_{ij}| \), the measurements describing two pixels belonging to distinct apertures are compatible whenever \( [\Pi_{ij}, \Pi_{i'j'}] = 0 \), \( i \neq i' \), which renders the scheme informationally incomplete. Signal components passing through distinct apertures are never recombined and the mutual coherence of those components cannot be determined.

Put differently, the method cannot discriminate signals comprised of sharply localized non-overlapping components. Nevertheless, these problematic modes do not set any practical restriction. As a matter of fact, spatially bounded modes (that is, with vanishing amplitude outside a finite area) have an unbounded Fourier spectrum and so, an unlimited range of transversal momenta. Such modes cannot thus be prepared with finite resources and they must be excluded from our considerations: for all practical purposes, the S–H performs an informationally incomplete measurement and any practically realizable transformation of the setup and sensor noise make the actual state to differ from the true.

To proceed further in this matter, we expand the signal as a finite superposition of a suitable spatially unbounded computational basis (depending on the actual experiment, one should use plane waves, Laguerre–Gaussian beams and so on). If that basis is labelled by \( |k\rangle \) (\( k = 1, \ldots, d \), with \( d \) being the dimension), the complex amplitudes are \( \langle x|k\rangle = \psi_k(x) \). Therefore, the coherence matrix \( \rho \) and the measurement operators \( \Pi_{ij} \) are given by \( d \times d \) non-negative matrices. A convenient representation of \( \Pi_{ij} \) can be obtained directly from equation (2), viz,

\[
(\Pi_{ij})_{mn} = \psi_{n,i}(\Delta p_j)\psi_{m,j}^*(\Delta p_i),
\]

where \( \psi_{n,i}(x) \) is the complex amplitude at the CCD plane of the \( i \)th lens generated by the incident \( m \)th basis mode \( \psi_m \).

This idea can be illustrated with the simple yet relevant example of square microlenses: \( A(x) = \text{rect}(x) \). We decompose the signal in a discrete set of plane waves \( \psi_m(x) = \exp(-ip_kx) \), parametrized by the transverse momenta \( p_k \). This is just the Fraunhofer diffraction on a slit, and the measurement matrix is

\[
(\Pi_{ij})_{mn} = \text{sinc}(\Delta p_i + p_m)\text{sinc}(\Delta p_j + p_n)e^{ip_m - p_n}\Delta x_i.
\]

The smallest possible search space consists of two plane waves (which is equivalent to a single-qubit tomography). By considering different pixels \( j \) belonging to the same aperture \( i \), linear combinations of only three out of the four Pauli matrices can be generated from equation (4). For example, a lens placed on the \( S-H \) axis (\( \Delta x_i = 0 \)) fails to generate \( \sigma_i \), and at least one more lens with a different \( \Delta x_i \) needs to be added to the setup to make the tomography complete.

This argument can be easily extended: the larger the search space, the more microlenses must be used. In this example, the maximum number of independent measurements generated by the \( S-H \) detection is \( (2M + 1)d - 3M \), for \( M \) lenses. A \( d \)-dimensional signal—a spatial qudit—can be characterized with about \( M \sim d/2 \) microlenses. This should be compared with the \( d \) qudits required for the homodyne reconstruction of a photonic qudit.

**Experiment.** We have validated our method with vortex beams. Consider the one-parameter family of modes specified by the orbital angular momentum \( l \), \( V_l = \langle r, \varphi|V_l\rangle \propto e^{il\varphi} \), where \( (r, \varphi) \) are cylindrical coordinates. In our experiment, the partially coherent signal

\[
\rho_{\text{true}} = |V_{-3} - \frac{i}{2} V_{-5}\rangle\langle V_{-3} - \frac{i}{2} V_{-5}| + \frac{1}{2} |V_3\rangle\langle V_3|
\]

was created: that is, modes \( V_{-5} \) and \( V_{-3} \) are coherently superposed, while \( V_3 \) is incoherently mixed. Figure 2 sketches the experimental layout used to generate equation (5). Imperfections of the setup and sensor noise make the actual state to differ.
from the true state. Calibration and signal intensity scans are presented in Fig. 3.

The coherence matrix of the true state is expanded in the seven-dimensional (7D) space spanned by the modes \( V_\ell \), with \( \ell \in \{-9, -6, -3, 0, +3, +6, +9\} \). The resulting matrix elements are plotted in Fig. 4.

To reconstruct the state we use a maximum likelihood algorithm\(^{27,28}\), whose results are summarized in Fig. 4. The main features of \( \rho_{\text{true}} \) are nicely displayed, which is also confirmed by the high fidelity of the reconstructed state \( F(\rho_{\text{true}}, \rho) = \text{Tr}[\sqrt{\rho} \rho_{\text{true}} \sqrt{\rho}] = 0.98 \). The off-diagonal elements detect the coherence between modes, whereas the diagonal ones give the amplitude ratios between them. The reconstruction errors are mainly due to the difference between the true and the actually generated state.

To our best knowledge, this is the first experimental measurement of the coherence properties with a wavefront sensor. The procedure outperforms the standard S–H operation, both in terms of dynamical range and resolution, even for fully coherent beams. For example, the high-order vortex beams with strongly helical wavefronts are very difficult to analyse with the standard wavefront sensors, while they pose no difficulty for our proposed approach.

The dynamical range and the resolution of the S–H tomography are delimited by the choice of the search space \( \{ |k_\ell| \} \) and can be quantified by the singular spectrum\(^{29}\) of the measurement matrix \( \Pi_k \). For the data in Fig. 4, the singular spectrum (which is the analogue of the modulation transfer function in wave optics) is shown in Fig. 5. Depending on the threshold, around 20 out of the total of 49 modes spanning the space of \( 7 \times 7 \) coherence matrices can be discriminated. The modes outside this field of view are mainly those with significant intensity contributions out of the rectangular regions of the CCD sensor. Further improvements can be expected by exploiting the full CCD area and/or using a CCD camera with more resolution, at the expense of more computational resources for data post-processing.

**3D imaging.** Once the feasibility of the S–H tomography has been proven, we illustrate its utility with an experimental demonstration of 3D imaging (or digital propagation) of partially coherent fields.

As it is well known\(^{16}\), the knowledge of the transverse intensity distribution at an input plane is, in general, not sufficient for calculating the transverse profile at other output planes. Propagation requires the explicit form of the mutual coherence function \( G_{\text{in}} \) at the input to determine \( I_{\text{out}} \):

\[
I_{\text{out}}(\mathbf{x}) = \int_{-\infty}^{\infty} h(\mathbf{x}, \mathbf{x}') h^*(\mathbf{x}, \mathbf{x}'') G_{\text{in}}(\mathbf{x}', \mathbf{x}'') d\mathbf{x}' d\mathbf{x}''.
\]

Here \( \mathbf{x}' (\mathbf{x}'') \) and \( \mathbf{x} \) are the coordinates parameterizing the input and output planes, respectively, and \( h(\mathbf{x}, \mathbf{x}') \) the response function accounting for propagation.

The dependence of the far-field intensity on the beam coherence properties is evidenced in Fig. 6 for coherent, partially coherent and incoherent superpositions of vortex beams. Once the coherence matrix is reconstructed, the forward/backward spatial propagation can be obtained using tools of diffraction theory and, consequently, the full 3D spatial intensity distribution at an output plane is obtained.

**Figure 4 | Vortex-beam coherence-matrix reconstruction.** Real \( \Re \) and imaginary \( \Im \) parts of the coherence matrix for the true state \( \rho_{\text{true}} \) (upper panel) and for the reconstructed \( \rho \) (lower panel). The reconstruction space is spanned by vortex modes with \( \ell \in \{-9, -6, -3, 0, +3, +6, +9\} \). The nonzero values of \( \Im \rho_{-6,-3} \) and \( \Im \rho_{-3,-6} \) describe coherences between the modes \( |V_{-6}\rangle \) and \( |V_{-3}\rangle \) and the phase shift \( \pi \) between them. The very small values of \( \rho_{-6,0} \), \( \rho_{-3,0} \) and \( \rho_{-6,3} \) and \( \rho_{-3,3} \) come from the incoherent mixing of \( |V_{-6}\rangle \) and \( |V_{-3}\rangle - (i/2)|V_{-6}\rangle \). The fidelity of the reconstructed coherence matrix is \( F = 0.98 \).
distribution can be computed. In particular, the intensity profile at the focal plane of an imaging system can be predicted from the S–H measurements. This has been experimentally confirmed, as sketched in Fig. 7. We prepared the partially coherent superposition \(|V_4 + V_{-4} + k|V_0\rangle \langle V_0|\) and characterized it by the S–H tomography method. The reconstructed

**Figure 6 | Influence of the spatial coherence on the far-field intensity distribution.** We have considered different mixtures of the modes |V₄⟩, |V₋₄⟩ and |V₀⟩ and calculated the associated intensity distribution as a Fraunhofer diffraction pattern. (a) Fully coherent superposition |V₄ + V₋₄ + 0.4V₀⟩ ⟨V₄ + V₋₄ + 0.4V₀|; (b) incoherent mixture |V₄⟩ ⟨V₄⟩ + |V₋₄⟩ ⟨V₋₄⟩ + 0.4|V₀⟩ ⟨V₀|; and (c) partially coherent mixture |V₄ + V₋₄⟩ ⟨V₄ + V₋₄⟩ + 0.4|V₀⟩ ⟨V₀|.

**Figure 7 | Digital 3D imaging.** The prediction of the far-field intensity distribution is compared with a direct intensity measurement. The partially coherent vortex beam |V₄ + V₋₄⟩ ⟨V₄ + V₋₄⟩ + k|V₀⟩ ⟨V₀| was generated (with a beam diameter of 4.9 mm) with a fixed parameter k (unknown before the reconstruction). Upper, middle and lower panels correspond to the S–H tomography, standard S–H measurement and direct intensity measurement, respectively. Upper left: real and imaginary parts of the reconstructed \(\rho\) in the 7D space spanned by the vortices \(V_c\) with \(c \in \{-6, -4, -2, 0, 2, 4, 6\}\). Upper right: calculated far-field intensity distribution \(I_{\rho}\) based on the reconstructed \(\rho\) propagated to the focal plane of the lens (\(f = 500\) mm). Middle left: intensity distribution (in arbitrary units) and wavefront as measured by the standard S–H sensor. Middle right: calculated far-field intensity distribution \(I_{\text{std}}\) using the standard S–H wavefront reconstruction and the transport of intensity equation included in the sensor (HASO). Bottom left: schematic picture of the direct intensity measurement at the lens focal plane. Bottom right: the result of the direct intensity measurement \(I_{\text{CCD}}\) at the focal plane with a CCD camera.
coherence function (upper left) was digitally propagated to the focal plane of a lens and the intensity distribution at this plane was calculated (upper right) and compared with the actual CCD scan in the same plane (lower right). Excellent agreement between the predicted and measured distributions was found.

We emphasize that the standard S–H operation fails in this kind of application. Indeed, we measured the intensity and wavefront of the target vortex superposition with a standard S–H sensor (middle left) and propagated the measured intensity to the focal plane using the transport of intensity equation (middle right). To quantify the result, we compute the normalized correlation coefficient \( C(U, H) = \sum_{i,j} I_{U,i} I_{H,j} / \sqrt{\sum_{i} I_{U,i}^2 \sum_{j} I_{H,j}^2} \) of the measured intensity with the prediction: the result, \( C(U_{\text{data}}, I_{\text{CCD}}) = 0.47 \), confirms the inability of the standard S–H approach to cope with the coherence properties of the signal. This has to be compared with the result for the S–H tomography: \( C(I_{\text{trans}}, I_{\text{CCD}}) = 0.89 \), which supports its advantages.

Discussion

We have demonstrated a non-trivial coherence measurement with a S–H sensor. This goes further than the standard analysis and constitutes a substantial leap ahead that might trigger potential applications in many areas. Such a breakthrough would not have been possible without reinterpreting the S–H operation as a simultaneous unsharp measurement of position and momentum. This immediately allows one to set a fundamental limit in the experimental accuracy.

Moreover, although the S–H has been the thread for our discussion, it is not difficult to extend the treatment to other wavefront sensors. For example, let us consider the recent results for temperature deviations of the cosmic microwave background. The anisotropy is mapped as spots on the sphere, representing the distribution of directions of the incoming radiation. To get access to the position distribution, the detector has to be moved and, in principle, such a scanning brings information about the position and direction simultaneously: the position of the measured signal before detection is delimited by the scanning aperture, whereas the direction of the signal comes from is revealed by the detector placed at the focal plane. When the aperture moves, it scans the field repeatedly at different positions. This could be an excellent chance to investigate the coherence properties of the relic radiation. To our best knowledge, this question has not been posed yet. Quantum tomography is especially germane for this kind of application. Indeed, we measured the intensity and correlation coefficient \( \rho = \sum \rho_{ij} \rho_{ij} / \rho_{0} / \rho_{0} \), with an amplitude spatial light modulator (OPTO SLM), with a resolution of 640 × 480 pixels, each pixel being 9.9 µm × 9.9 µm in size. Because of microlens array imperfections, CCD-induced aberrations (of the f/4 optics (aberrations of the collimating optics are negligible), calibration of the detector must be carried out. The holographic part of the setup provided this calibration wave. S–H data from the calibration wave and the partially coherent beam are shown in Fig. 3. The beam axis position in the microlens array coordinates was adjusted with a Gaussian mode. The detection noise is mainly due to the background light, which is filtered out before reconstruction.

Reconstruction. The reconstruction was done in the 2D space spanned by the \( V \) modes with \( i \in \{-9, -6, -3, 0, 3, 6, 9\} \). In all, 49 real parameters had to be reconstructed. The data come from CCD areas belonging to seven microlens around the beam axis; each one of them comprise 11 × 11 pixels, which means 847 data samples altogether. An iterative maximum likelihood algorithm was applied to estimate the true coherence matrix of the signal.

Dynamical range and resolution. The errors of the S–H tomography can be quantified by evaluating the covariances of the parameters of the reconstructed coherence matrix \( \rho \). In the absence of systematic errors, the Cramer–Rao lower bound works pretty well. Let us decompose the \( d \times d \) coherence matrix \( \rho \) (\( d \) is just the dimension of the search space) and the measurement operators \( \Pi_{k} \) in an orthonormal matrix basis \( \Gamma_{k} \) (\( k = 1, \ldots, d \)) \( \text{Tr}(\Gamma_{k}^{T} \Gamma_{j}) = \delta_{kj} \), namely

\[
\rho = \sum_{k} \rho_{k}\Gamma_{k}
\]

so that the Born-like rule (2) can be recast as a system of linear equations

\[
I_{k} = \sum_{j} \rho_{k}^{*}\rho_{j}
\]

On using a single index \( x \) to label all possible microlenses/CCD–pixel combinations \( x \equiv (i, j) \), equation (8) can be concisely expressed in the matrix form

\[
I = \mathbf{P}\mathbf{r}
\]

where \( I \) is the vector of measured data, \( r \) is the vector of coherence-matrix parameters and \( P_{k} = p_{k}^{T} \) is the tomography matrix.

Obviously, for ill-conditioned measurements, the reconstruction errors will be larger and vice versa. By applying a singular-value decomposition to the measurement matrix \( \mathbf{P} = \mathbf{USV}^{T} \), equation (9) takes the diagonal form

\[
I' = \mathbf{S}'\mathbf{r}'
\]

where \( r' = \mathbf{V}'r \) and \( I = \mathbf{U}'I \) are the normal modes of the problem and the corresponding transformed data, respectively. The singular values \( S_{k} \) are the eigenvalues associated with the normal modes, so the relative sensitivity of the tomography to different normal modes is given by the relative sizes of the corresponding singular values. With the help of equations (9) and (10), the errors are readily propagated from the detection \( I \) to the reconstruction \( r \).

Drawing an analogy between equation (10) and the filtering by a linear spatially invariant system, the singular spectrum \( S_{k} \) and the sum of the singular values \( S_{k} \) are the discrete analogues of the modulation transfer function and the maximum of the point spread function, respectively. Hence we define the dynamical range (or field of view) of the S–H tomography as the set of normal modes with singular values exceeding a given threshold. The sum of the singular values then describes the overall performance of the S–H tomography setup. When some of the singular values are zero, the tomography is not informationally complete and the search space must be readjusted.

Far-field intensity. In the experiment on 3D imaging, the partially coherent vortex beam \( |V_{\theta} > \) was realized using a plane-wave phase profile modulation by a special vortex phase mask (RPC Photonics). Finally, the field in equation (5) was prepared by mixing the two vortex modes in a beam splitter. During the state preparation, special care was taken to reduce any deviation between the true and target states. This involved minimizing aberrations as well as imperfections of the spatial light modulator, resulting in distortions of the transmitted wavefront.

The second beam \( |V_{\phi} > \) was obtained through a plane-wave phase profile modulation by a special vortex phase mask (RPC Photonics). Finally, the field in equation (5) was prepared by mixing the two vortex modes in a beam splitter.
6.45 μm × 6.45 μm each). Second, the same vortex superposition was subject to the S–H tomography using the S–H sensor (Flexible Optical) and the reconstruction of the coherence matrix in the 7D subspace spanned by the vortices V, with ρ = 1, 2, 3, 4, 5, 6. Once ρ is reconstructed, the far-field intensity was computed using equation (6), where the focusing is described by the Fraunhofer diffraction response function. The predicted intensity was found to be in an excellent agreement with the direct sampling by the Olympus CCD camera. Finally, the Flexible Optical S–H sensor was replaced by a HASO3 S–H detector. The intensity and wavefront of the prepared vortex beam was measured and the far-field intensity was computed by resorting to the transport of intensity performed by the HASO software. Resampling was done to match the resolution of the HASO output to the resolution of the Olympus CCD camera.

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Acknowledgements
This work was supported by the Technology Agency of the Czech Republic (Grant TE01020229), the Czech Ministry of Industry and Trade (Grant FR-TI1/364), the IGA Projects of the Palacky University (Grants PRF_2012_005 and PRF_2013_019) and the Spanish MINECO (Grant FIS2011-26786).

Additional information
Competing financial interests: The authors declare no competing financial interests.

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How to cite this article: Stoklasa, B. et al. Wavefront sensing reveals optical coherence. Nat. Commun. 5:3275 doi: 10.1038/ncomms4275 (2014).