$B_{s,d}^* \rightarrow \mu^+\mu^-$ and its impact on $B_{s,d} \rightarrow \mu^+\mu^-$

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ABSTRACT: The decay of $B_{s,d}^* \rightarrow \mu^+\mu^-$ and its impact on $B_{s,d} \rightarrow \mu^+\mu^-$ is studied here. The $\mu^+\mu^-$ decay widths of vector mesons $B_{s,d}^*$ are about a factor of 700 larger than the corresponding scalar mesons $B_{s,d}$. The obtained ratio of the branching fractions $Br(B_{s,d}^* \rightarrow \mu^+\mu^-)/Br(B_{s,d} \rightarrow \mu^+\mu^-)$ is about $0.3 \times 10^{-10}$. At the same time, the hadronic contribution $B_{s,d} \rightarrow B_{s,d}^* \gamma \rightarrow \mu^+\mu^-$ is estimated too. The relative increase of the amplitude of $B_{s,d} \rightarrow \mu^+\mu^-$ is about $(0.01 \pm 0.006) \sqrt{\Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma)/100 \text{ eV}}$. If we choose $\Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma) = 2 \text{ eV}$, the branching fractions of the vector mesons to lepton pair are $(6.2 \pm 0.6) \times 10^{-10}$ and $(1.7 \pm 0.2) \times 10^{-11}$ for $B_s^*$ and $B_d^*$ respectively. If we choose $\Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma) = 200 \text{ eV}$, the updated branching fractions of the scalar mesons to muon pair are $(3.78 \pm 0.25) \times 10^{-9}$ and $(1.09 \pm 0.09) \times 10^{-10}$ for $B_s$ and $B_d$ respectively. Further studies on $B_{s,d}^*$ are usefully here, including dielectron decay, two-body decay with $J/\psi$, and so on.
1 Introduction

The leptonic decays of the $B_{s,d}$ mesons play an important role in the standard model (SM) and the new physics (NP) [1, 2]. They are highly suppressed in the SM since the flavor changing neutral current decays are generated through W-box and Z-penguin diagrams. Furthermore, the branching fractions of leptonic decays of scalar meson undergo an additional helicity suppression factor by $m_\mu^2/M_S^2$, where $m_\mu$ and $M_S$ denote masses of the muon lepton and the scalar meson, respectively. The suppression factor can be removed in some NP models, such as the two-Higgs doublet models [3], the minimal supersymmetric standard model (MSSM) [4], the next minimal supersymmetric standard model (NMSSM) [5], the dark matter [6], the universal extra dimensional model [7], lepton universality violation model [8], the four generations fermion [9], and so on [10]. The branching fractions of $B_{s,d} \rightarrow \mu^+\mu^-$ measured by the CMS and LHCb Collaborations [2] and predicted within the SM [1] with NNLO QCD [11] and NLO EW [12] corrections included are collected in Table 1.

Since the experimental branching fractions of $B_{s,d} \rightarrow \mu^+\mu^-$ are measured from the dimuon distributions by the CMS and LHCb Collaborations [2], the process $B_{s,d}^* \rightarrow \mu^+\mu^-$ will enhance the dimuon distributions for the mass splitting are about 45 MeV between $B_{s,d}$ and $B_{s,d}^*$. In another hand, the hadronic contribution $B_{s,d} \rightarrow B_{s,d}^*\gamma \rightarrow \mu^+\mu^-$ are missed in the theoretical prediction [1]. So we study $B_{s,d}^* \rightarrow \mu^+\mu^-$ and its impact on $B_{s,d} \rightarrow \mu^+\mu^-$ within SM here. The process $B_s \rightarrow B_s^*\gamma \rightarrow \mu^+\mu^-\gamma$ was considered in Ref. [13]. Recently, $B_{s,d}^* \rightarrow \mu^+\mu^-$ are considered in Ref. [14, 15]. And hadronic contribution from charmonium in $B \rightarrow K^{(*)}\ell^+\ell^-$ and $B \rightarrow X_s\gamma$ had been studied in Refs. [16, 17].
Within the SM, effective Lagrangian related with \( b \to s \) is given in Ref. \cite{18, 19}

\[
\mathcal{L} = \mathcal{N} \left[ C_{7}^{eff} (\mu f) \mathcal{O}_{7}^{\mu} + C_{9} (\mu f) \mathcal{O}_{9}^{\mu} + C_{10} (\mu f) \mathcal{O}_{10}^{A} \right],
\]

(2.1)

where \( \mathcal{N} = \frac{G_{F}}{\sqrt{2}} V_{tb} V_{ts}^{*} \frac{e^{2}}{4 \pi^{2}} \), and the operators \( \mathcal{O}_{7,9,10} \) read as

\[
\mathcal{O}_{7}^{\mu} = \frac{2 i m_{b} (p_{\mu}^{\nu} + p_{\nu}^{\mu})}{(p_{\mu} + p_{\nu})^{2}} (\bar{s} \gamma_{\rho} P_{R} b) (\bar{\mu} \gamma^{\rho} \mu),
\]

(2.2)

\[
\mathcal{O}_{9}^{\mu} = (\bar{s} \gamma_{\rho} P_{L} b) (\bar{\mu} \gamma^{\rho} \mu),
\]

(2.3)

\[
\mathcal{O}_{10}^{A} = (\bar{s} \gamma_{\rho} P_{L} b) (\bar{\mu} \gamma^{\rho} \gamma_{5} \mu),
\]

(2.4)

where \( P_{L} = (1 - \gamma_{5}) / 2 \), \( P_{R} = (1 + \gamma_{5}) / 2 \). And the Wilson coefficients are \( C_{7,9,10}(\mu f) = (-0.316, 4.403 - 0.47i, -4.493) \) at \( \mu f = m_{b} = 4.5 \text{ GeV} \) \cite{15}. The superscript \( \gamma, V, \) and \( A \) denote the contributions from photon, vector current, and axial vector current, respectively.

The relations between the quark level operators and the meson are described as

\[
\langle 0 | \bar{s} \gamma^{\mu} b | B_{s}^{*}(q, e) \rangle = m_{B_{s}} f_{B_{s}} e^{\mu},
\]

(2.5)

\[
\langle 0 | \bar{s} \sigma^{\mu\nu} b | B_{s}^{*}(q, e) \rangle = -i f_{B_{s}}^{T}(q^{\mu} e^{\nu} - e^{\mu} q^{\nu}),
\]

(2.6)

\[
\langle 0 | \bar{s} \gamma^{\mu} \gamma_{5} b | B_{s}(q) \rangle = -i f_{B_{s}} q^{\mu},
\]

(2.7)

where the three decay constants \( f_{B_{s}} \), \( f_{B_{s}}^{T} \), and \( f_{B_{s}}^{T} \) depend on the renormalization scale. Their relations have been studied in Ref. \cite{20} in the heavy-quark limit. Ignoring the mass difference between \( B_{s} \) and \( B_{s}^{*} \) and the high order QCD corrections, we have

\[
f_{B_{s}}^{T} = f_{B_{s}}^{T} = f_{B_{s}}.
\]

(2.8)

Then the amplitudes of \( B_{s}^{*}(B_{s}) \to \mu^{+} \mu^{-} \) are \cite{13}

\[
\mathcal{M}(B_{s}^{*} \to \mu^{+} \mu^{-}) = f_{B_{s}}^{T} \frac{N}{2} m_{B_{s}} \bar{\mu} / e \left[ C_{9}^{eff} + C_{10} \right] \mu,
\]

\[
\mathcal{M}(B_{s} \to \mu^{+} \mu^{-}) = -i f_{B_{s}} N C_{10} m_{\mu} \bar{\mu} \gamma_{5} \mu,
\]

(2.9)
\[ C_{eff}^V = C_9 + 2 \frac{m_b}{m_{B_s}^*} C_{7}^{eff}. \]  \hfill (2.10)

The helicity suppression factor \( m_\mu^2 / m_{B_s}^2 \) in the decay width is removed in the vector meson decay. Then we can get the decay widths of \( B_s^* (B_s) \rightarrow \mu^+ \mu^- \)

\[
\Gamma(B_s^* \rightarrow \mu^+ \mu^-) = \frac{G^2 \alpha_{em}^2}{96\pi^3} |V_{tb} V_{ts}^*|^2 \left( |C_{10}|^2 + |C_{eff}^V|^2 \right) m_{B_s}^3 f_{B_s}^2 \left( 1 + \mathcal{O}(m_\mu^2 / m_{B_s}^2) \right)
\]

\[
\Gamma(B_s \rightarrow \mu^+ \mu^-) = \frac{G^2 \alpha_{em}^2}{16\pi^3} |V_{tb} V_{ts}^*|^2 |C_{10}|^2 m_{B_s}^2 m_{B_s} f_{B_s}^2 \left( 1 + \mathcal{O}(m_\mu^2 / m_{B_s}^2) \right)
\]

\[
\frac{\Gamma(B_s^* \rightarrow \mu^+ \mu^-)}{\Gamma(B_s \rightarrow \mu^+ \mu^-)} = \frac{|C_{10}|^2 + |C_{eff}^V|^2}{6|C_{10}|^2 m_\mu^2 m_{B_s} f_{B_s}^2} \left( 1 + \mathcal{O}(m_\mu^2 / m_{B_s}^2) \right)
\]  \hfill (2.11)

The ratio of decay width is about 700 for \( B_s^* \) and \( B_d^* \) both.

3 The impact of \( B_s^* (B_d^*) \rightarrow \mu^+ \mu^- \) on \( B_s (B_d) \rightarrow B_s^* (B_d^*) \gamma \rightarrow \mu^+ \mu^- \)

Meanwhile, \( B_s^* (B_d^*) \) will impact on the leptonic decay of \( B_{s,d} \) through the loop contribution \( B_{s,d} \rightarrow B_{s,d} \gamma^* \rightarrow \mu^+ \mu^- \). The Feynman diagrams are given in Fig. 1. The vertex of \( B_{s,d} \rightarrow B_{s,d} \gamma \) is given as the operator [21, 22]

\[
\mathcal{M}_{B_{s,d} \gamma} = \sum_{q=s,b} B_s^* |ie e q(q(p_q)\gamma\mu q(p_q)|B_s >
\]

\[
= \sum_{q=s,b} |e^\mu_p q^\gamma |B_s^* |ie e q(q(p_q)\frac{i\sigma_{\mu\nu}}{2m_q} q(p_q)|B_s >.
\]
We can simplify the matrix element \( < B_s^* | \bar{q}(p) \sigma_{\mu \nu} q(p) | B_s > \) with the procedure in Refs. [23, 24],

\[
\mathcal{M}_{B_s^* B_s \gamma} = \sum_{q=s, b} \frac{-e e_q}{2 m_q} e^\mu_\gamma e^\nu_\gamma p^\alpha_{\gamma B_s^*} p^\beta_{B_s} \sum_{q=s, b} \left( \frac{e e_q}{m_q} \right) \mathcal{F}.
\]  

(3.1)

\( \mathcal{F} \) is related with the wave functions of \( B_s^* \) and \( B_s \) [23], and it is \( \mathcal{F} = \langle B_s | j_0(p) \gamma r | B_s^* \rangle \sim 1 \) in the non-relativistic limit [25]. We can rewrite Eq. (3.1) with

\[
\mathcal{M}_{B_s^* B_s \gamma} = i \frac{g_{B_s^* B_s \gamma}}{m_{B_s^*}} e^{\mu \nu \alpha \beta} e^\mu_\gamma e^\nu_\gamma e^\alpha_\gamma e^\beta_\gamma \sum_{q=s, b} \left( \frac{e e_q}{m_q} \right) \mathcal{F}.
\]  

(3.2)

Where the dimensionless vector-scalar-photon coupling constant \( g_{B_s^* B_s \gamma} \) is related with the magnetic moments of \( b \) and \( s \) quarks. And the phase factor \( i \) is consistent with the amplitude of \( \gamma^* \rightarrow VP \) in Ref. [26].

There are ultraviolet (UV) logarithmical divergences in the evaluation of loop integrals. Then we introduce a cut off regularization scheme for the UV divergence integral

\[
\int \frac{d^4 q}{(2\pi)^4} \left( \frac{1}{(q_i^2 - m_i^2)(q_j^2 - m_j^2)} \right) \cdot \int \frac{d^4 q}{(2\pi)^4} \left[ \frac{1}{(q_i^2 - m_i^2)(q_j^2 - m_j^2)} \right] \cdot \left( \frac{1}{(q_i^2 - (m_i + \Lambda)^2)(q_j^2 - (m_j + \Lambda)^2)} \right),
\]  

(3.3)

where \( i, j = B_s^* \), \( \gamma \) or \( \mu \), and \( q_i \) corresponds the momentum of \( i \) in loop. \( \Lambda \ll M_W \) for the amplitude is UV finite when \( W \) boson are involved. Since the hadronic contribution will be suppressed when \( \sqrt{q_i^2 - m_i} \gg \Lambda_{QCD} \), \( \Lambda \) is about several \( \Lambda_{QCD} \). The cut off regularization scheme is similar Pauli-Villars regularization scheme but acts on two propagators. The Pauli-Villars regularization scheme of the UV divergence integral is the same with the form factor \( \mathcal{F} \) which is introduced in the \( B_s B_s^* \gamma \) vertex in Ref. [27]

\[
\mathcal{F} = \left( \frac{\Lambda^2 - m_i^2}{\Lambda^2 - q_i^2} \right) \cdot \left( \frac{\Lambda^2 - m_j^2}{\Lambda^2 - q_j^2} \right),
\]  

(3.4)

for

\[
\frac{1}{q_i^2 - m_i^2} \cdot \mathcal{F} = \frac{1}{q_i^2 - m_i^2} - \frac{1}{q_i^2 - \Lambda^2},
\]  

(3.5)

But here it acts on the UV divergence term only and the two propagators. Then the soft contribution will be maintained in our calculation.

\footnote{The decay constants defined of vector meson in Eq. (2.5) is different with Eq.(44) in Ref. [23] with an additional \( i \).}
Figure 2. $R(\Lambda)$ of $B_s \to \mu^+\mu^-$ defined in Eq. (3.6) as a function of the cut off energy.

The amplitude from $B_s \to B_s^* \gamma \to \mu^+\mu^-$ can be written as

$$
\mathcal{M}(B_s \to B_s^* \gamma \to \mu^+\mu^-) = \frac{i e N_{g_{B_sB_s^*\gamma}} R(\Lambda)}{f_{B_s}} m_\mu \bar{\mu} \gamma_5 \mu.
$$

(3.6)

$m_\mu$ reappears in the amplitude of leptonic decay of scalar meson. The factor $R(\Lambda)$ serves as a function of the high energy cut is shown in Fig. 2. More information about the factor $R(\Lambda)$ are given in the Appendix.

Only the $C_{10}$ term is taken into account in the calculation of the NNLO QCD [11] and NLO EW [12] corrections of $B_{s,d} \to \mu^+\mu^-$ within the SM [1]. But the contribution from $B_s^*$ is missed in the previous calculation, which is also related with $C_7$ and $C_9$ terms.

Compared with Eq. (2.9), the previous amplitude is added a factor $F$,

$$
F(B_s^*) = \frac{\mathcal{M}(B_s \to B_s^* \gamma \to \mu^+\mu^-)}{\mathcal{M}(B_s \to \mu^+\mu^-)} = \frac{c_{g_{B_sB_s^*\gamma}} f_{B_s}}{c_{10} f_{B_s}} e\ g_{B_sB_s^*\gamma} R(\Lambda)
$$

(3.7)

We can estimate $g_{B_sB_s^*\gamma}$ in several ways, including the heavy-quark and chiral effective theories [28, 29] with the radiative and pion transition widths of $D^{*+}$, light cone QCD sum rules [30, 31], and the radiative M1 decay widths of $B_s^* \to B_s \gamma$ from potential model [25, 32]. The radiative M1 decay width of $B_s^* \to B_s \gamma$ is

$$
g_{B_sB_s^*\gamma} = -m_{B_s^*} \left( \frac{12\pi}{F_\gamma^3} \Gamma(B_s^* \to B_s \gamma) \right)^{1/2}.
$$

(3.8)

The predicted M1 widths are 0.15 – 400 eV and 10 – 300 eV for $B_s^* \to B_s \gamma$ and $B_d^* \to B_d \gamma$, respectively [22, 24, 25, 32, 33].
4 Numerical Result

The parameters in the numerical calculation are chosen as [34]

\[
\Lambda = 1.2 \text{ GeV}, \\
m_b = 4.2 \text{ GeV}, \\
\alpha_{em} = 1/137. \tag{4.1}
\]

The branch fraction of \( B_{s,d}^* \) weak decay is much less than the M1 decay, \( \Gamma_{tot}(B_{s,d}^*) \approx \Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma) \). We can get the ratio

\[
\frac{\text{Br}(B_{s,d}^* \rightarrow \mu^+\mu^-)}{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)} = \frac{(0.34 \pm 0.03) \times \text{eV}}{\Gamma(B_{s,d} \rightarrow B_{s,d}\gamma)} \\
\frac{\text{Br}(B_d^* \rightarrow \mu^+\mu^-)}{\text{Br}(B_d \rightarrow \mu^+\mu^-)} = \frac{(0.33 \pm 0.03) \times \text{eV}}{\Gamma(B_d^* \rightarrow B_d\gamma)}. \tag{4.2}
\]

The main uncertainty resulting from the value of \( f_{B_{s,d}} \). The distribution of the dimuon invariant mass measured by CMS and LHCb Collaborations in Ref. [2] should include the contributions of \( B_{s,d}^* \rightarrow \mu^+\mu^- \). If \( \Gamma(B_{s,d}^* \rightarrow B_s\gamma) = 1 \text{ eV} \) [25], we can get \( \frac{\text{Br}(B_{s,d}^* \rightarrow \ell^+\ell^-)}{\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-)} = 0.34 \) for \( \ell = e, \mu \). Then \( B_{s,d}^* \rightarrow e^+e^- \) may be searched by CMS and LHCb experiments with larger data samples.

In another hand, if \( \Gamma(B_{s,d}^* \rightarrow B_s\gamma) \sim 100 \text{ eV} \), we can find that the amplitude of \( B_{s,d} \rightarrow \mu^+\mu^- \) will be modified by the contributions of \( B_{s,d}^* \) with a factor

\[
F(B_{s,d}^*) = (0.011 \pm 0.006) \sqrt{\frac{\Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma)}{100 \text{eV}}}. \tag{4.3}
\]

The main uncertainty resulting from the value of \( \Lambda \). Then the new predictions of \( \Gamma(B_{s,d} \rightarrow \mu^+\mu^-) \) are

\[
\text{Br}(B_{s,d} \rightarrow \mu^+\mu^-) = (36.6 \pm 2.3) \times \left( 1 + (0.023 \pm 0.012) \sqrt{\frac{\Gamma_{tot}(B_{s,d}^*)}{100 \text{eV}}} \right) \times 10^{-10}, \\
\text{Br}(B_d \rightarrow \mu^+\mu^-) = (10.6 \pm 0.9) \times \left( 1 + (0.023 \pm 0.012) \sqrt{\frac{\Gamma_{tot}(B_{d}^*)}{100 \text{eV}}} \right) \times 10^{-11}.
\]

If \( \Gamma(B_{s,d} \rightarrow B_s\gamma) = 200 \text{ eV} \), this factor will increase the decay width \( \Gamma(B_{s,d} \rightarrow \mu^+\mu^-) \) by a factor \( (3.3 \pm 1.7)\% \), which is about a factor of 10 larger than the neglect NLO EW correction factor 0.3% at the decay width in Ref. [1]. And the corresponded \( g_{B_{s,d}B_s\gamma} = -1.5 \), about a factor of 15 larger than the \( e_qe = -1/3\sqrt{4\pi\alpha_{em}} = -0.10 \). The \( \Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma) \) may be measured through two-body decay \( B_{s,d}^* \rightarrow J/\psi + M \) by CMS and LHCb.

5 Summary

In summary, \( B_{s,d}^* \rightarrow \mu^+\mu^- \) in the dimuon distributions and the hadronic contribution \( B_{s,d} \rightarrow B_{s,d}\gamma \rightarrow \mu^+\mu^- \) are studied here. The \( \mu^+\mu^- \) decay widths of vector mesons \( B_{s,d}^* \) are about a factor of 700
larger than the scalar mesons $B_{s,d}$. The obtained ratio of the branching fractions $Br(B_{s,d}^+ \to \mu^+ \mu^-)/Br(B_{s,d} \to \mu^+ \mu^-)$ is about $0.3 \times 10^{-11}$ or $0.1 \times 10^{-11}$. At the same time, the hadronic contribution $B_{s,d} \to B_{s,d}^* \gamma \to \mu^+ \mu^-$ is calculated too. The amplitude of $B_{s,d} \to \mu^+ \mu^-$ is enhanced by a factor of $0.01 \sqrt{\Gamma(B_{s,d}^* \to B_{s,d} \gamma)/100}$ eV. If we choose $\Gamma(B_{s,d}^* \to B_{s,d} \gamma) = 2$ eV, the branching fractions of the vector mesons to lepton pair are $5.3 \times 10^{-10}$ and $1.6 \times 10^{-11}$ for $B_s^*$ and $B_d^*$ respectively. If we choose $\Gamma(B_{s,d}^* \to B_{s,d} \gamma) = 200$ eV, the updated branching fractions of the scalar mesons to muon pair are $(3.78 \pm 0.25) \times 10^{-9}$ and $(1.09 \pm 0.09) \times 10^{-10}$ for $B_s$ and $B_d$ respectively. Further studies on $B_{s,d}^*$ are usefully here, including dielectron decay, two-body decay with $J/\psi$, and so on.

Acknowledgments

We would like to thank K.T. Chao, Y.Q. Ma, C. Meng, Q. Zhao, and S.L. Zhu for useful discussions. This work is supported by the National Natural Science Foundation of China (Grants No. 11375021, 11575017), the New Century Excellent Talents in University (NCET) under grant NCET-13-0030, the Major State Basic Research Development Program of China (No. 2015CB856701), and the Fundamental Research Funds for the Central Universities.

6 Appendix: $R(\Lambda)$

$R(\Lambda)$ of $B_s \to B_s^* \gamma \to \mu^+ \mu^-$ defined in Eq. (6.6) is given as:

$$R(\Lambda) = \frac{1}{32\pi^2 m_{B_s}^2 m_{\mu}^2} \left\{ 3 m_{B_s}^2 m_{\mu}^2 - 2 m_{\mu}^2 (m_{B_s}^2 - m_{B_s^*}^2)^2 \right\} C_0 \left\{ m_{B_s}^2, m_{\mu}^2, m_{\mu}^2, m_{B_s^*}^2, 0, m_{\mu}^2 \right\}$$

$$+ m_{B_s}^2 (m_{\mu}^2 - m_{B_s^*}^2) \left\{ B_0 \left( 0, m_{\mu}^2, m_{B_s^*}^2 \right) - B_0 \left( 0, (\Lambda + m_{\mu}^2), (\Lambda + m_{B_s^*}^2) \right) \right\}$$

$$+ \left\{ m_{B_s}^2 \left( 2 m_{\mu}^2 + m_{B_s^*}^2 \right) + 2 m_{\mu}^2 m_{B_s^*}^2 \right\} \left\{ B_0 \left( m_{\mu}^2, m_{\mu}^2, m_{B_s^*}^2 \right) - B_0 \left( m_{\mu}^2, (\Lambda + m_{\mu}^2), (\Lambda + m_{B_s^*}^2) \right) \right\}$$

$$+ m_{\mu}^2 \left( 3 m_{B_s^*}^2 - 2 m_{B_s^*}^2 \right) \left\{ B_0 \left( m_{\mu}^2, 0, m_{\mu}^2 \right) - B_0 \left( m_{\mu}^2, \Lambda^2, \Lambda + m_{\mu}^2 \right) \right\} \right\} \right\}$$

(6.1)

The scalar functions $B_0$ and $C_0$ are given in Ref. [35–37]. As a numerical fit between $0.5 - 2$ GeV, we can get

$$R(\Lambda) = 0.022 + 0.062 \times \ln \left( \frac{\Lambda + m_{B_s}}{m_{B_s}} \right).$$

(6.2)

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