Rare top decay $t \rightarrow c\bar{l}l$ as a probe of new physics

J.L. Díaz-Cruz$^1$, A. Diaz-Furlong$^1$, R. Gaitán-Lozano$^{2,a}$, J.H. Montes de Oca Y$^{2,b}$

$^1$Cuerpo Académico de Partículas, Campos y Relatividad Facultad de Ciencias Físico-Matemáticas, BUAP, Apdo. Postal 1364, C.P. 72000 Puebla, Pue., México
$^2$Departamento de Física, FES-Cuautitlán, UNAM, C.P. 54770, Estado de México, México

Received: 8 May 2012 / Revised: 23 July 2012 / Published online: 28 August 2012
© Springer-Verlag / Società Italiana di Fisica 2012

Abstract The rare top decay $t \rightarrow c\bar{l}l$, which involves flavor violation, is studied as a possible probe of new physics. This decay is analyzed with one of the simplest Standard Model extensions with additional gauge symmetry formalism. The considered extension is the Left–Right Symmetric Model, including a new neutral gauge boson $Z'$ that allows one to obtain the decay at tree level through Flavor- Changing Neutral Currents (FCNC) couplings. The neutral gauge boson couplings are considered diagonal but family non-universal in order to induce these FCNC. We find $BR(t \rightarrow c\bar{l}l) \sim 10^{-13}$ for the range $1 \text{ TeV} \leq M_{Z'} \leq 3 \text{ TeV}$.

1 Introduction

The last year we have witnessed the impressive work of LHC, which has reached a luminosity that has allowed us to test the Standard Model (SM) at extraordinary levels [1–3]. In particular, LHC has provided notable bounds on the SM Higgs boson mass [4, 5]. After the discovery of the top quark at Fermilab Tevatron Collider, experimental attention has been turned on the examination of its production mechanisms and decay properties. Within the SM, the top quark production cross section is evaluated with an uncertainty of the order of $\sim 15\%$, while it is assumed to decay to a $W$ boson and a $b$ quark almost $100\%$ of the time. With higher energy, as planned, the LHC will also become an amazing top factory, allowing to test the top properties, its couplings to SM channels and rare decays. Because about $10^7$–$10^8$ top pairs will be produced per year, rare decays with B.R. of order $10^{-5}$–$10^{-6}$ may be detectable, depending on the signal. For the $W$ boson coupling to fermion pairs ($t\bar{d}W^\pm$), the structure is proportional to the CKM element $V_{td}$ in the framework of the SM. Therefore, the decay $t \rightarrow b+W$ dominates its branching ratio. Radiative corrections to this mode are of order $10\%$ and are difficult to detect at hadron collider, but may be at the reach of the International Linear Collider (ILC). Top quark decays from FCNC, such as $t \rightarrow c\gamma$, $t \rightarrow cg$, $t \rightarrow cz$ and $t \rightarrow c\phi$, have been studied, both in the context of the SM and new physics. In the SM, the branching ratio of FCNC top decays is extremely suppressed. The rare top quark decay $t \rightarrow c+\gamma$ was calculated [6–13], the result implied a suppressed branching ratio, less than about $10^{-10}$, which was confirmed when subsequent analysis, included the correct top mass value, and gave $BR(t \rightarrow c+\gamma) = 5 \times 10^{-13}$ [14]. The decays $t \rightarrow c+Z$ and $t \rightarrow c+g$ were also calculated and the resulting branching ratios obtained were turned out to be $BR(t \rightarrow c+Z) = 1.3 \times 10^{-13}$ and $BR(t \rightarrow c+g) = 5 \times 10^{-11}$ [15, 16]. The top-charm coupling with the SM Higgs $\phi^0$ could be induced at one-loop level with a resulting branching ratio $BR(t \rightarrow c+\phi^0) \approx 10^{-15}$ [17, 18]. The FCNC top decays involving a pair of vector bosons in the final state, $t \rightarrow cVV$, can also be of interest [19]. Although one could expect such modes to be even more suppressed than the ones with a single vector boson, the appearance of an intermediate scalar resonance, as in the previous case, could enhance the branching ratio. Furthermore, it also seems possible to allow the tree-level decay $t \rightarrow b+W$, at least close to threshold, because of the large top quark mass [20]. The top decay into the light quarks $t \rightarrow W+d(s)$ is suppressed, as they are proportional to $V_{td(s)}$ [21]. Probably for this reason, the SM corrections to this mode have not been studied, though the QCD corrections should be the similar for both modes. However, it may be possible to get a large enhancement that could even make it detectable at the ILC in extensions of the SM. Some typical results for the top decays in the SM are summarized in Table 1. This table also includes, for comparison, the results for top branching ratios from models beyond the SM, in particular from the THDM-III [22–26] and SUSY [27], which will be discussed in this work. Another interesting mode is
the decay \( t \to c l^+ l^- \), which could be mediated by a vector resonance. Within the SM one could expect a \( BR(t \to c l^+ l^-) \approx BR(t \to c Z) \cdot BR(Z \to l^+ l^-) < 10^{-10} \). Thus, this mode offers the possibility to test extensions of the SM that include an additional vector boson \( Z' \), which could have SM-like couplings to \( t l^+ l^- \), but enhanced coupling \( tcZ' \). In this paper, we evaluate this decay mode within a particular extension that are well motivated and produce interesting signals. The so-called Left–Right Symmetric Model with Non-Universal extra gauge bosons, where there are strong constraints for transitions involving the 1st and 2nd generations, but admit larger effects for transitions involving the 3rd generation. We obtain \( BR(t \to c l^+ l^-) < 10^{-10} \). This paper is organized as follows. In Sect. 2, the parametrization of the couplings of \( Z' \) neutral gauge boson and the relevant details of the Non-universal \( Z \) model. The evaluation of the decay width for \( t \to c l^+ l^- \) and the corresponding \( BR(t \to c l^+ l^-) \) are done in Sect. 3. Finally, our conclusions appear in Sect. 4.

2 Flavor-changing neutral currents from family non-universal couplings

The Left–Right symmetric model (LRSM) is considered in order to include extra gauge bosons [28–32]. The gauge symmetry group of the LRSM is \( SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \). In literature, several models contain extra neutral gauge bosons through increasing the gauge symmetry group [33] but LRSM is one of the simplest model based on physical motivation. In this work, exotic fermions are not included in fermion field contain, only the SM fermions.

The notation for parameters and formalism introduced by Langacker and Plumacher have been used in this work [34]. Then, the couplings for neutral gauge bosons with fermions are given by

\[
-\mathcal{L}_{NC} = e J^\mu_{EM} A_\mu + \sum_{a=1}^{2} g_a J^\mu_a Z_{a\mu},
\]

where \( Z_{1\mu} \) is the usual electroweak neutral gauge boson, \( Z_{2\mu} \) is the neutral gauge boson associated with the additional gauge symmetry and \( g_{1,2} \) are their respective gauge coupling constants. The invariance under parity operation requires that \( g_1 = g_2 \). The \( Z_{a\mu} \) and \( J_{a\mu} \) are written in gauge eigenstate basis. The general form of the \( J_{2\mu} \) current is

\[
J_{2\mu} = \sum_h \sum_{i,j} \sum_{h,i,j} \epsilon_f \epsilon_{hij} P_h f_{i,j},
\]

where \( i, j = 1, 2, 3 \), \( h = L, R \) and \( P_{L,R} = (1 \mp \gamma_5)/2 \). \( \epsilon_{hij} \) are model depending parameters described below. FCNC can be introduced through the \( \epsilon_{hij} \) parameters, when family non-universal assumption is done. The flavor diagonal and family universal couplings mean that \( \epsilon_{hij} = Q^f_{h} \delta_{ij} \), where \( Q^f_{h} \) denotes the chiral charges and depends on the model. For LRSM, the chiral charges are given by

\[
Q^f_L = -\sqrt{\frac{2}{3}} \left( \frac{1}{2\alpha} \right) (B - L)_i
\]

and

\[
Q^f_R = \sqrt{\frac{2}{3}} \left( \alpha T^3_R - \frac{1}{2\alpha} (B - L)_i \right),
\]

where \( B \) and \( L \) denote the baryon and lepton numbers of fermion \( i \), respectively. \( T^3_R \) is the third component of its right-handed isospin in the \( SU(2)_R \) group and \( \alpha^2 = (1 - 2\sin^2\theta_W)/\sin^2\theta_W \). If the \( Z_2 \) couplings are diagonal but they are family non-universal, then \( \epsilon_{hij} = \chi_h Q^f_h \delta_{ij} \), here the repeated indices do not denote a sum. After changing to mass eigenstate basis, the up-quark sector current is

\[
J^u_{2\mu} = \sum_h (\bar{u}_a Z_{ah} V^T_{ah} V^\dagger_{ur} P_h \left( \begin{array}{c} u \\ c \\ t \end{array} \right)).
\]

Analogously in the case of the down-quark sector. However, for simplicity, we shall assume that down-quark sector has non-mixing, then left-handed and right-handed CKM matrix can be \( V_{CKM} \approx V_{dL} \) and \( V_R \approx V_{dR} \), respectively. Therefore, the following 3 \( \times \) 3 matrices are defined:

\[
B^L_{u} \equiv V^T_{CKM} \epsilon^L_{u} V_{CKM}
\]

and

\[
B^R_{u} \equiv V^T_{R} \epsilon^R_{u} V_{R}.
\]
The usual and well known CKM matrix parametrization in terms of the Wolfenstein parameters, $A$, $\lambda$, $\eta$, $\delta$ are used in this work [35]. For $V_R$ matrix, the parametrization given by Zhang et al. is taken into account [36],

$$V_R = P_UP_D,$$

in which

$$P_U = \text{diag}(s_u e^{2i\theta_2}, \rho s_u e^{2i\theta_3}),$$

and

$$P_D = \text{diag}(s_d e^{i\theta_1}, s_t e^{-i\theta_2}, s_b e^{-i\theta_3}).$$

The $s_q$ guarantee that up-type and down-type quark mass matrix elements are positive. The $\theta_i$ phases come up as well as it happens in CKM matrix.

Finally, electroweak and extra neutral gauge bosons are mixed by $\xi$ parameter. Since $\xi \ll 1$, $Z_1$ and $Z_2$ can be identified with physical gauge bosons $Z$ and $Z'$, respectively.

### 3 Rare top decay $t \to c\bar{l}l$

3.1 Decay width for $t \to c\bar{l}l$

The branching ratio for the rare top decay $t \to c\bar{l}l$ is calculated by using the formalism introduced in the previous section. The diagram for this decay is shown in Fig. 1. If $q^2 \ll M_{Z'}^2$, then the average of amplitude is

$$|\mathcal{M}_{t\to c\bar{l}l}|^2 = \frac{g_1^4}{8M_{Z'}^4} \left[ B_1(q_1 \cdot k_1)(q_2 \cdot k_2) + B_2(q_1 \cdot k_2)(q_2 \cdot k_1) + B_3 m^2_l (q_1 \cdot q_2) - B_4 m_c (k_1 \cdot k_2) - B_5 m_c m^2_l \right],$$

and the decay width becomes

$$\Gamma_{t\to c\bar{l}l} = \frac{g_1^4 m_t^4}{(16\pi)^3 M_{Z'}^4} \sum_{i=1}^5 a_i B_i,$$

where the $a_i$ and $B_i$ are explicitly written in Appendix A.

![Fig 1 Tree-level Feynman diagram for rare top decay mediated by $Z'$ extra gauge boson](image)

3.2 Parameter space

Three scenarios are analyzed in order to explore the behavior of the family non-universal parameters. All scenarios assume that the $Z'$ couplings with the leptons and down-type quarks are flavor diagonal and family universal, that is, $(x^L_{u,R})_{ij} = (x^L_{l,R})_{ij} = \delta_{ij}$. The scenarios are:

1. **Left-handed family non-universality for top quark.** The right-handed up-type quarks are family universal and only the last family for left up-type quarks has family non-universal coupling with $Z'$ [37, 38]. $(x^L_{u,R})_{ij} = \delta_{ij}$, $(x^L_{d})_{11} = (x^L_{d})_{22} = 1$ and $(x^L_{d})_{33} = x$. The parameter $x$ must be close to 1, but it is not exactly 1 in order to obtain FCNC from $Z'$ boson.

2. **Left–Right handed family non-universality for top quark.** The third generation of the right-handed and left-handed up-type quarks are family non-universal, $(x^L_{u,R})_{11} = 1$, $(x^L_{u,R})_{22} = 1$ and $(x^L_{u,R})_{33} = x$.

3. **General left-handed family non-universality for up-type quarks.** Three families of the left-handed up-type quarks have family non-universal couplings $(x^L_{u,R})_{11} \equiv x_1$, $(x^L_{u,R})_{22} \equiv x_2$ and $(x^L_{u,R})_{33} \equiv x_3$. As in first scenario, right-handed up-type quarks are family universal.

3.3 Numerical result and discussion

We have taken from Particle Data Group the central values of the SM parameters [39]. Then, numerical values of the chiral charges are shown in Table 2. For all considered scenarios in previous section, Appendix A contains the analytical expressions and numerical values of the $B_{L,R}^{u,d}$ parameters.

In the case of the scenario 1 and 2, the branching ratio of the rare top decay $t \to c\bar{l}l$ has been obtained as function of the family non-universal parameter $x$ and the $Z'$ mass, see Figs. 2, 3, 4 and 5. In the case of the scenario 3, the branching ratio depends on the parameters $x_{31} = x_3 - x_1$ and $x_{32} = x_3 - x_2$. Figure 6 is obtained by using the representative lower bound of the $Z'$ mass, which is around 1 TeV [39].

As mentioned before, the family non-universal couplings are important to including FCNC from extra gauge bosons.
In both scenarios 1 and 2, branching ratio and family non-universal parameter shows a decreasing behavior for values of the $Z'$ mass greater than 1 TeV. However, the results for scenario 2 are 10 times bigger than the results for scenario 1.

A prominent feature of right-handed CKM matrix obtained in the branching ratio is its the increment.

Table 2 Numerical values of the chiral charges for leptons and quarks obtained by using (3), (4) and $\sin^2 \theta_W = 0.2316$

| Chiral charge | LRSM |
|--------------|------|
| $Q^L_e$      | -0.0847 |
| $Q^R_e$      | 0.5048 |
| $Q^L_d$      | -0.0847 |
| $Q^R_d$      | -0.6744 |
| $Q^L_u$      | 0.2543 |
| $Q^R_u$      | -0.3352 |

We finally discuss the branching ratio in the scenario 3. The branching ratio not only depends on two parameters as above, but two more parameters are added for the general family non-universality. However, the unitary of the CKM matrix allows us to write the branching ratio as function of $x_{21} = x_3 - x_1$ and $x_{21} = x_2 - x_1$. The domain of the $(x_{21}, x_{31})$ shall be $[-1, 1] \times [-1, 1]$ in order to have closed values between the family non-universal parameters, $x_{1,2,3}$, and close to 1. The region in the domain not allowed is bounded by line $x_{31} = 0.9746x_{21}$ and $x_{31} = 0.9746x_{21} + \ldots$
1.4404. Out of this region, there still exist some values of the $x_{31}$ and $x_{21}$ for which the $x_{1,2,3}$ could be negative. It can be controlled when we constrain the parameters as $x_3 > x_{31}$ and $x_{21} > x_{31} - x_3$.

We have obtained the result that the branching ratio for rare top decay is of the order of $10^{-13}$. Our results for this branching ratio are consistent with the experimental bound reported by [40] which is

$$Br(t \rightarrow c l^+ l^-) \approx Br(t \rightarrow c Z) Br(Z \rightarrow l^+ l^-) \leq 10.77 \times 10^{-4}.$$

4 Conclusion

We find the branching ratio of the rare top decay at tree level for three possible scenarios. The branching ratio is between $10^{-13} \sim 10^{-12}$. The right-handed CKM matrix contribution in scenario 2 helps to give an arise in the branching ratio. However, for any scenario is still very suppressed.

We also obtain a allowed region for the family non-universal parameters in scenario 3. The allowed parameters keep the branching ratio positive. Finally, we remark that the branching ratio of the rare top decay is very suppressed in all scenarios.

Acknowledgements This work is supported in part by PAPIIT project IN117611-3 and Sistema Nacional de Investigadores (SNI) in México. J.H. Montes de Oca Y. is thankful for support from the post-doctoral DGAPA-UNAM grant.

Appendix A: Analytical expressions and numerical values

Below we give the complete analytic formulas for the family non-universal parameters in three considered scenarios. We also present the analytic expression of the integrals $I_{1,...,5}$.

A.1 Family non-universal couplings

The general expressions for the $B_i$ coefficients are given by

$$B_1 = 2([B_L^u]^2 |B_L^d|^2 + |B_L^u|^2 |B_R^d|^2),$$  \hspace{1cm} (A.1)

$$B_2 = 2([B_L^u]^2 |B_L^d|^2 + |B_L^d|^2 |B_R^u|^2),$$  \hspace{1cm} (A.2)

$$B_3 = \text{Re}(B_L^u B_R^u)(|B_L^d|^2 + |B_R^u|^2),$$  \hspace{1cm} (A.3)

$$B_4 = \text{Re}(B_L^u B_R^d)(|B_L^d|^2 + |B_R^d|^2),$$  \hspace{1cm} (A.4)

and

$$B_5 = 2[\text{Re}(B_L^u B_L^d B_R^u B_R^d) + \text{Re}(B_L^u B_L^d B_R^u B_R^d)].$$  \hspace{1cm} (A.5)

The matrix elements for the scenario 1 are

$$[B_L^u]_{32} = Q_L^u (x - 1) V_{tb} V_{cb}^*, \hspace{1cm} (A.6)$$

$$[B_R^u]_{32} = 0, \hspace{1cm} (A.7)$$

$$[B_L^d]_{33} = Q_L^d x,$$  \hspace{1cm} (A.8)

Then, we write the expressions of the $B_{1,...,5}$ in terms of the Wolfenstein parameter, taking $V_{tb} = 1$ and $V_{cb} = A \lambda^2$,

$$B_1 = 2(Q_L^u Q_L^d) (x - 1)^2 A^2 \lambda^4,$$  \hspace{1cm} (A.9)

$$B_2 = 2(Q_L^u Q_L^d) (x - 1)^2 A^2 \lambda^4,$$  \hspace{1cm} (A.10)

$$B_3 = 0,$$  \hspace{1cm} (A.11)

$$B_4 = Q_R^u Q_L^u (x - 1) A \lambda^2 (|Q_L^d|^2 + |Q_R^d|^2),$$  \hspace{1cm} (A.12)

and

$$B_5 = 0.$$  \hspace{1cm} (A.13)

For the scenario 2

$$[B_L^u]_{32} = Q_L^u (x - 1) A \lambda^2,$$  \hspace{1cm} (A.14)

$$[B_R^u]_{32} = Q_R^u (x - 1) s_1 s_c A \lambda^2,$$  \hspace{1cm} (A.15)

$$[B_L^d]_{33} = Q_L^d x,$$  \hspace{1cm} (A.16)

Then

$$B_1 = 2(x - 1)^2 A^2 \lambda^4 [(|Q_L^u Q_L^d)|^2 + (Q_R^u Q_L^d)|^2],$$  \hspace{1cm} (A.17)

$$B_2 = 2(x - 1)^2 A^2 \lambda^4 [(|Q_L^u Q_R^d|^2 + (Q_R^u Q_R^d)|^2),$$  \hspace{1cm} (A.18)

$$B_3 = Q_R^u Q_L^u (x - 1) s_{12} s_c A^3 \lambda^6 [(Q_L^u|^2 + (Q_R^u|^2)],$$  \hspace{1cm} (A.19)

and

$$B_4 = Q_R^u Q_R^d (x - 1) A \lambda^2 (|Q_L^d|^2 + |Q_R^d|^2),$$  \hspace{1cm} (A.20)

Finally, for scenario 3

$$[B_L^u]_{32} = Q_L^u (x_1 V_{td} V_{cd}^* + x_2 V_{ts} V_{cs}^* + x_3 V_{tb} V_{cb}^*),$$  \hspace{1cm} (A.22)

$$[B_R^u]_{32} = 0,$$  \hspace{1cm} (A.23)

and

$$[B_L^d]_{33} = Q_L^d x,$$  \hspace{1cm} (A.24)
The $a_i$ coefficients in (12) contain the charged lepton and charm quark contribution. They are given by

$$a_1 = (1 - \mu_2) I_2 - I_3,$$

$$a_2 = (1 - \mu_2) I_4 - I_5,$$

$$a_3 = 2 \mu_1 (2 I_1 - I_2 - I_4),$$

$$a_4 = 2 \sqrt{\mu_2} [(1 + 2 \mu_1 - \mu_2) I_1 - I_2 - I_4],$$

$$a_5 = -4 \sqrt{\mu_2} \mu_1 I_1,$$

where $\mu_1 = \frac{m_{\text{top}}}{m_{\text{lep}}} \text{ and } \mu_2 = \frac{m_{\text{charm}}}{m_{\text{lep}}}$, see Table 4. The $I_i$ are integrals in the three phase space with analytical solution

$$I_1 = \frac{1}{2} (1 - \mu_1^2) + \mu_1 \ln \mu_1,$$

$$I_2 = \frac{1}{3} (1 - \mu_1)^3,$$

$$I_3 = \frac{1}{4} (1 - \mu_1) \left(1 + \mu_1 + \mu_1^2 + \mu_1^3\right) + \mu_1^2 \ln \mu_1,$$

$$I_4 = \frac{1}{6} (1 - \mu_1) \left(2 + 5 \mu_1 - \mu_1^2\right) + \mu_1 \ln \mu_1,$$

$$I_5 = \frac{1}{12} (1 - \mu_1) \left(\mu_1^3 - 5 \mu_1^2 + 13 \mu_1 + 3\right) + \mu_1 \ln \mu_1.$$
21. J.L. Diaz-Cruz, R. Gaitan-Lozano, G. Lopez-Castro, C.E. Pagliarone, Phys. Rev. D 77, 094010 (2008)
22. J.L. Diaz-Cruz, R. Noriega-Papaqui, A. Rosado, Phys. Rev. D 69, 095002 (2004)
23. J.L. Diaz-Cruz, A. Rosado, Rev. Mex. Fis. 53, 396 (2007)
24. R. Martínez, J.A. Rodriguez, M. Rozo, Phys. Rev. D 68, 035001 (2003)
25. R. Díaz, R. Martínez, J.A. Rodriguez, Phys. Rev. D 63, 095007 (2001)
26. A.E. Cárcamo, R. Martínez, J.A. Rodriguez, Eur. Phys. J. C 50, 935 (2007)
27. J.L. Díaz-Cruz, in Proceedings of the 5th Mexican Workshop of Particles and Fields. AIP Conference Proceedings, vol. 359, Puebla, México (AIP, New York, 1995), pp. 178–201
28. R.N. Mohapatra, J.C. Pati, Phys. Rev. D 11, 566 (1975)
29. R.N. Mohapatra, J.C. Pati, Phys. Rev. D 11, 2558 (1975)
30. G. Senjanovic, R.N. Mohapatra, Phys. Rev. D 12, 1502 (1975)
31. G. Senjanovic, R.N. Mohapatra, Phys. Rev. D 23, 165 (1981)
32. R.N. Mohapatra, in CP Violation, ed. by C. Jarlskog (World Scientific, Singapore, 1989) (for a review)
33. P. Langacker, Rev. Mod. Phys. 81, 1199 (2009)
34. P. Langacker, M. Plumacher, Phys. Rev. D 62, 013006 (2000)
35. L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983)
36. Y. Zhang, H. An, X. Ji, R.N. Mohapatra, Nucl. Phys. B 802, 247 (2008)
37. A. Arhrib, K. Cheung, C.W. Chiang, T.C. Yuan, Phys. Rev. D 73, 075015 (2006)
38. J. Drobnak, S. Fajfer, J.F. Kamenik, J. High Energy Phys. 0903, 077 (2009). 0812.0294 [hep-ph]
39. K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010), and 2011 partial update for the 2012 edition
40. V.M. Abazov et al. (D0 Collaboration), Phys. Lett. B 701, 313 (2011). arXiv:1103.4574 [hep-ex]