Chiral Symmetry: Pion-Nucleon Interactions in Constituent Quark Models

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We study the interactions of an elementary pion with a nucleon made of constituent quarks and show that the enforcement of chiral symmetry requires the use of a two-body operator, whose form does not depend on the choice of the pion-quark coupling. The coordinate space NN effective potential in the pion exchange channel is given as a sum of terms involving two gradients, that operate on both the usual Yukawa function and the confining potential. We also consider an application to the case of quarks bound by a harmonic potential and show that corrections due to the symmetry are important.

1. INTRODUCTION

The pion nucleon (πN) form factor is present in a wide variety of situations and plays an important role in many hadronic processes. For instance, in elastic πN scattering, it yields corrections to tree diagrams because the intermediate baryon states are off-shell. In the case of nucleon-nucleon interactions, on the other hand, it corresponds to an effective size that modifies the one pion exchange potential (OPEP) at short distances.

The πN form factor at intermediate energies is determined by the cooperation of two complementary mechanisms, associated with both the meson cloud that surrounds the nucleon and with its quark structure. In the former case, the relevant interactions may be represented by diagrams with point-like nucleons, such as those of fig. 1, which were recently investigated in the framework of chiral perturbation theory [1].

The description of the intrinsic extension of the nucleons, on the other hand, requires models where quarks are bound by means of bags [2], effective gluon interactions [3], non-
relativistic potentials or other mechanisms. Here, again, chiral symmetry is expected to play an important role, but its implementation may prove to be more subtle. Even if one starts from a chiral Lagrangian for free particles, the kinematical or dynamical approximations performed in the course of a calculation involving bound systems may effectively break the symmetry. This situation is similar to the case of electromagnetic interactions with deuterons or other nuclei, where gauge invariance is achieved with the help of exchange currents. These currents are associated with binding effects, since they arise from the coupling of the external photon with the fields that keep the nucleons together. Therefore, at least for non-relativistic constituent quark models, one may expect the full implementation of chiral symmetry also to require the inclusion of processes involving simultaneously the pion probe and the binding fields.

The main purpose of this work is to study the role of chiral symmetry in the pion-nucleon vertex, using a model in which constituent quarks are confined by a generic scalar non-relativistic potential and coupled to elementary pions. We begin by deriving an effective NN potential, that is afterwards used to extract the \( \pi N \) form factor.

Our presentation is divided as follows. In sect.2 we introduce the basic formalism and in sect.3 we present the effective chiral Lagrangians which describe the interactions. The pion vertices are constructed in sect.4, and the effective NN potential is obtained in sect.5. In sect.6 we apply our results to the case of a harmonic confining potential and present conclusions in sect.7.

II. BASIC FORMALISM

In this section we display our basic equations, with the purpose of establishing the notation. The variables \( \vec{r}_i \) refer to individual quarks, whereas \( \vec{R}, \vec{\rho} \) and \( \vec{\lambda} \) are collective and internal coordinates of the nucleon, given by

\[
\vec{R} = \frac{1}{3} (\vec{r}_1 + \vec{r}_2 + \vec{r}_3),
\]  

(1)
\[ \vec{\rho} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2), \]  
\[ \vec{\lambda} = \frac{1}{\sqrt{6}} (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3). \]  

The centre of mass and relative coordinates of the two nucleon system are denoted respectively by \( \vec{S} \) and \( \vec{X} \) and related to the individual coordinates \( \vec{R}_a \) and \( \vec{R}_b \) by

\[ \vec{S} = \frac{1}{2} (\vec{R}_a + \vec{R}_b), \]  
\[ \vec{X} = \vec{R}_a - \vec{R}_b. \]

The effective Schrödinger equation for the two nucleon system is written as

\[
\left\{ \nabla^2 S + \nabla^2 X + \frac{P_S^2}{4M} + E_X \right\} |N_a, N_b\rangle = V_{NN} (\vec{X}) |N_a, N_b\rangle,
\]

where \( P_S \) is the total momentum , \( |N_a\rangle \) describes the collective motion of nucleon \( a \) and \( V_{NN} \) is the effective potential. For the six quark system, on the other hand, we have

\[
\left[ \sum_{i=1}^{6} \left( \frac{\nabla^2_i}{2m} \right) + \frac{P_S^2}{4M} + E \right] |q_1q_2q_3; q_4q_5q_6\rangle = V_{qq} (q_1q_2q_3; q_4q_5q_6) |q_1q_2q_3; q_4q_5q_6\rangle.
\]

In this work we are interested in the pion-nucleon form factor, which is related to the potential at long and intermediate distances. Therefore we assume that the six quark state may be decomposed into two clusters

\[ |q_1q_2q_3; q_4q_5q_6\rangle = |q_1q_2q_3\rangle \otimes |q_4q_5q_6\rangle. \]

This assumption corresponds to the idea that there are no quark exchanges between the two nucleons. For the quark-quark interaction, we write

\[ V_{qq} = W + \Pi, \]
where $W$ is a short ranged confining potential and $\Pi$ is a long ranged function due to the exchange of pions.

In agreement with the short-range nature of the confining potential and the cluster decomposition of the six quark system, we assume that $W$ operates only inside each nucleon, and have

$$W \cong W_a + W_b ,$$

(10)

where

$$W_i = W_i \left( \vec{\rho}_i, \vec{\lambda}_i \right) .$$

(11)

Our last approximation consists in assuming that pion exchanges are more relevant to interactions between quarks in different nucleons than between quarks within a single cluster. Formally, this corresponds to

$$\Pi \cong \Pi_{ab} = \sum_v \sum_y \Pi^{(v,y)} ,$$

(12)

where the indices $v$ and $y$ refer to quarks in nucleons $a$ and $b$ respectively.

In order to display the quark structure of the nucleon, we write

$$|q_1q_2q_3\rangle = |N_a \left( \vec{R}_a \right) \rangle |I_a \left( \vec{\rho}_a, \vec{\lambda}_a \right) \rangle ,$$

(13)

where $|N_a\rangle$ and $|I_a\rangle$ correspond to the collective and internal wave functions. The latter is determined by the equation

$$\left[ - \frac{\nabla^2_{\rho_a}}{2m} - \frac{\nabla^2_{\lambda_a}}{2m} + W_a \right] |I_a\rangle = \epsilon_a |I_a\rangle .$$

(14)
Using this expression in eq.(7), we have
\[
\left[ \frac{\nabla_S^2}{4M} + \frac{\nabla_X^2}{M} + \frac{\vec{P}_S^2}{4M} + E_X \right] |N_a, N_b \rangle |I_a \rangle |I_b \rangle = \Pi_{ab} |N_a, N_b \rangle |I_a \rangle |I_b \rangle ,
\] (15)

with
\[
E_X = E - \epsilon_a - \epsilon_b .
\] (16)

Multiplying eq.(15) by \( \langle I_b | \langle I_a | \), integrating over the internal coordinates and comparing with eq.(6), we obtain the following effective potential:
\[
V_{NN} (\vec{X}) = \int d\Omega \langle I_b | \langle I_a | \Pi_{ab} |I_a \rangle |I_b \rangle ,
\] (17)

where
\[
d\Omega = d\vec{\rho}_a d\vec{\lambda}_a d\vec{\rho}_b d\vec{\lambda}_b .
\] (18)

The \( \pi N \) form factor modifies the OPEP at short distances and may be extracted from the effective potential. Denoting it by \( G(k^2) \), the modified OPEP may be written as
\[
V_\pi (\vec{X}, [G]) = \left( \frac{g_{\pi NN}}{2M} \right)^2 \vec{T}^{(a)} \cdot \vec{T}^{(b)} \vec{\Sigma}^{(a)} \cdot \vec{\Sigma}^{(b)} . \vec{\nabla} \vec{\Sigma}^{(a)} \cdot \vec{\nabla} \vec{\Sigma}^{(b)} . \vec{\nabla} \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{-i\vec{k} \cdot \vec{X}}}{k^2 + \mu^2} G^2(k^2) ,
\] (19)

where \( g_{\pi NN} \) is the \( \pi N \) coupling constant, \( M \) and \( \mu \) are the nucleon and pion masses and \( \vec{\Sigma} \) and \( \vec{T} \) are the nucleon spin and isospin operators. Evaluating the gradients, we get
\[
V_\pi (\vec{X}, [G]) = \frac{1}{3} \left( \frac{g_{\pi NN} \mu}{2M} \right)^2 \frac{\mu}{4\pi} \vec{T}^{(a)} . \vec{T}^{(b)} \left\{ \vec{\Sigma}^{(a)} \cdot \vec{\Sigma}^{(b)} [U_0 (X, [G]) - D (X, [G])] \\
+ [3 \vec{\Sigma}^{(a)} \cdot \vec{X} \vec{\Sigma}^{(b)} . \vec{X} - \vec{\Sigma}^{(a)} \cdot \vec{\Sigma}^{(b)} ] U_2 (X, [G]) \right\} ,
\] (20)
where $D, U_0$ and $U_2$ are functionals of $G$, given by

\begin{align}
D(X, [G]) &= \frac{2}{\pi\mu^3} \int_0^{\infty} dk k^2 G^2 \left( k^2 \right) j_0 (kX) \tag{21} \\
U_0(X, [G]) &= \frac{2}{\pi\mu} \int_0^{\infty} dk \frac{k^2}{k^2 + \mu^2} G^2 \left( k^2 \right) j_0 (kX) \tag{22} \\
U_2(X, [G]) &= \frac{2}{\pi\mu^3} \int_0^{\infty} dk k^4 \frac{k^2}{k^2 + \mu^2} G^2 \left( k^2 \right) j_2 (kX) \tag{23}
\end{align}

The inversion of these results yields

\begin{align}
G^2 \left( k^2 \right) &= \mu^3 \int_0^{\infty} dXX^2 D(X, [G]) j_0 (kX) , \tag{24} \\
G^2 \left( k^2 \right) &= \frac{\mu^{l+1}}{k^l} \left( k^2 + \mu^2 \right)^l \int_0^{\infty} dXX^2 U_l (X, [G]) j_l (kX) . \tag{25}
\end{align}

In a more synthetic notation, we may also write

\begin{align}
V_{\pi} \left( \vec{X}, [G] \right) &= \hat{T}^a \cdot \hat{T}^b \left[ -\hat{\Sigma}^a \cdot \hat{\Sigma}^b V_0^- \left( \vec{X}, [G] \right) + S_{12} V_2^- \left( \vec{X}, [G] \right) \right] , \tag{26}
\end{align}

where

\begin{align}
V_i^- \left( \vec{X}, [G] \right) &= \frac{1}{6\pi^2\mu^2} \left( \frac{g_{\pi NN\mu}}{2M} \right)^2 \int_0^{\infty} dk \frac{k^4}{k^2 + \mu^2} G^2 \left( k^2 \right) j_i (kX) . \tag{27}
\end{align}

In this case, the form factor is given by

\begin{align}
G^2 \left( k^2 \right) &= 12\pi\mu^2 \left( \frac{2M}{g_{\pi NN\mu}} \right)^2 \left( k^2 + \mu^2 \right)^2 \int_0^{\infty} dXX^2 V_i^- (X, [G]) j_i (kX) . \tag{28}
\end{align}
III. DYNAMICS

We assume that the scalar confining potential $W$ is associated with a field $S$, that may represent effectively the self interactions of gluons as, for instance, in the case of non-topological solitons [6–8]. Alternatively, it may be associated with fluctuations of the chiral scalar field $\sigma'$, considered long ago by Weinberg [9]. The important point to stress, however, is that our calculation is completely independent of the meaning attached to this scalar field. For the pion sector we adopt the non-linear sigma model and the basic Lagrangian is written as

$$\mathcal{L} = \left[ \frac{1}{2} \left( \partial_{\mu} S \partial^{\mu} S - m^2_s S^2 \right) - V(S) \right] + \left[ \frac{1}{2} \left( \partial_{\mu} \phi \partial^{\mu} \phi + \partial_{\mu} \sigma \partial^{\mu} \sigma \right) + f_\pi^2 \sigma^2 \right] + \mathcal{L}_q, \quad (29)$$

where $\phi$ and $f_\pi$ are the pion field and decay constant, whereas $\sigma$ corresponds to the function $\sigma = \sqrt{f_\pi^2 - \phi^2}$. Formally, $V(S)$ represents a potential associated with self interactions of the scalar field, but it has no direct role here. The Lagrangian $\mathcal{L}_q$ represents both the quark sector and its interactions with the bosonic fields.

There are many possible forms for the Lagrangian $\mathcal{L}_q$, two of which are widely employed in the literature. In one of them the pion-fermion coupling is pseudo-vector (PV), whereas in the other it is pseudo scalar (PS). In the case of PV coupling, one has

$$\mathcal{L}_q^{PV} = \bar{\psi} i \gamma_{\mu} D^{\mu} \psi - m \bar{\psi} \psi + \frac{g}{2 m} \bar{\psi} \gamma_{\mu} \gamma_5 \tau \psi \cdot D^{\mu} \phi - g_s S \bar{\psi} \psi, \quad (30)$$

where $\psi$ and $m$ are the constituent quark field and mass, $g$ is the pion-quark coupling constant and $g_s$ represents the coupling of the quark to the scalar. In this expression the pion and nucleon covariant derivatives are given by [10]

$$D^{\mu} \phi = \partial_{\mu} \phi - \frac{1}{\sigma + f_\pi} \partial^{\mu} \sigma \phi, \quad (31)$$

$$D^{\mu} \psi = \left[ \partial^{\mu} + i \frac{\vec{\tau}}{f_\pi \left( \sigma + f_\pi \right)} \left( \vec{\phi} \times \partial^{\mu} \vec{\phi} \right) \right] \psi. \quad (32)$$
For PS coupling, on the other hand, the chiral Lagrangian for the fermion sector is

\[
\mathcal{L}_q^{PS} = \bar{q}i\partial q - \bar{q} \left( \sigma + i\vec{\gamma} \cdot \vec{\phi} \right) q - \left( \frac{g_s}{f_\pi} \right) \bar{q}\left( \sigma + i\vec{\gamma} \cdot \vec{\phi} \gamma_5 \right) q, \tag{33}
\]

where \(q\) is a quark field that transforms linearly.

On general grounds one knows that, in the framework of chiral symmetry, results should not depend on the choice of \(\mathcal{L}_q\) \[11,12\]. However, this point is not always appreciated in particular calculations and we would like to stress it in this problem. Therefore, we adopt both forms of \(\mathcal{L}_q\) and demonstrate explicitly, in the next section, that our results do not depend on how the symmetry is implemented.

**IV. PION VERTICES**

In this section we evaluate the operators needed to construct the effective NN interaction. In order to motivate our approach we recall that, in general, chiral symmetry is realized by means of families of diagrams organized according to loop and momentum counting rules. For instance, when two free point-like nucleons interact through pion fields, the simplest chiral family involves just a single diagram, associated with the one pion exchange potential (OPEP) \[13\]. In the case of PV coupling, the \(\pi N\) vertex used to construct the OPEP is proportional to the matrix element

\[
\Gamma_{\pi N} = \frac{g}{2m} \bar{u}(\vec{p}') k \gamma_5 \tau u(\vec{p}), \tag{34}
\]

where \(k = p' - p\). Using the equations of motion for the nucleons, we may rewrite \(\Gamma_{\pi N}\) as

\[
\Gamma_{\pi N} = g \bar{u}(\vec{p}') \gamma_5 \tau u(\vec{p}), \tag{35}
\]
which is the expression one would obtain from the PS Lagrangian. Hence both couplings yield the same result. This kind of equivalence, which must hold for all chiral families of processes, is true for the OPEP only if the nucleon wave functions are exact solutions of the equation of motion. When this does not happen, the PV and PS couplings do not yield the same results, indicating that the single pion exchange no longer constitutes an autonomous chiral family.

In the case of composite nucleons, this result means that single pion exchanges between constituent quarks within different bags will not, in general, be chiral symmetric. Thus the implementation of the symmetry for models based on non-relativistic quarks requires families of diagrams which are more complex and involve necessarily the binding potential.

In this work the quarks are bound by a scalar field and the simplest chiral family of diagrams that encompasses binding effects is related to the process $\pi q \rightarrow Sq$, which we study in the sequence. Its amplitude is denoted by $T_\chi$ and given by the diagrams displayed in fig.2. For PV coupling there are just the direct (d) and crossed (x) diagrams, whereas for PS coupling one has three possibilities, including a contact term.

In the PV coupling scheme, the amplitude $T_\chi$ is written as

$$T_\chi = -i g_s \frac{g}{2m} \tau_\alpha \bar{u} (\vec{p}') \left[ \frac{\hat{p}_d + m}{p_d^2 - m^2} \gamma_5 + \frac{k \gamma_5}{p_x^2 - m^2} \right] u (\vec{p}),$$

with

$$p_d = p + k,$$  \hspace{1cm} (37)

$$p_x = p' - k.$$

In order to show that this result is equivalent to that produced in the PS approach, we use the Dirac equation and rewrite $T_\chi$ as

$$T_\chi = -i g_s g \tau_\alpha \bar{u} (\vec{p}') \left[ \frac{\hat{p}_d + m}{p_d^2 - m^2} \gamma_5 + \frac{k \gamma_5}{p_x^2 - m^2} + \frac{1}{m} \gamma_5 \right] u (\vec{p}).$$  \hspace{1cm} (38)
This expression is the same one would obtain from the PS Lagrangian, with the last term within the square brackets being due to the contact term in fig 2. This result shows that the PV and PS schemes yield identical results when the equations of motion for the external quarks can be used. On the other hand, it also indicates that, as in the case of the OPEP, these two approaches are not fully equivalent when one deals with off-shell constituent quarks. Thus, in this case, the inclusion of first order effects in the scalar field improves the OPEP description, but does not correspond to a complete solution of the problem. In fact, such a full solution would require interactions in all orders of the potential.

Using the Dirac equation, one obtains a somewhat simpler form for the amplitude

$$T_\chi = -i g_s g_\alpha \bar{u}(\vec{p}) \left[ \frac{k}{p_d^2 - m^2} + \frac{k}{p_s^2 - m^2} + \frac{1}{m} \right] \gamma_5 u(\vec{p}) .$$ (39)

The $\pi N$ form factor is associated with the diagram shown in fig. 3A, which involves an amplitude $T_\chi$ for each nucleon. However we cannot use directly eq.(39) in the evaluation of the NN potential, for this would produce an amplitude containing disconnected parts. In order to avoid this, we must consider only the positive frequency irreducible part of $T_\chi$, which is shown in fig. 3B, and the proper NN interaction is given by the diagrams (1-4) of fig. 3C.

For the one-body ($\pi q$) vertex in quark $v$, we adopt the form given by eq.(35), and have

$$\Gamma^{(v)}_{\pi q} = [g_\alpha \gamma_5]^{(v)} .$$ (40)

The two body operator, on the other hand, is associated with the proper pion diquark ($\pi d$) amplitude of fig.3B, which does not contain positive energy intermediate states. In order to isolate these contributions, we write the quark propagator as

$$\frac{\vec{p} + m}{p^2 - m^2} = \frac{1}{2E} \left[ \frac{1}{p^0 - E} \sum_s u^s(\vec{p}) \bar{u}^s(\vec{p}) + \frac{1}{p^0 + E} \sum_s v^s(-\vec{p}) \bar{v}^s(-\vec{p}) \right] ,$$ (41)
where

\[ E = \sqrt{\vec{p}^2 + m^2}. \] (42)

Thus the contribution from the positive energy states is

\[ T_{(+)} = -ig_s g_{\tau \alpha} \bar{u} (\vec{p}') \left[ \frac{(\hat{p}_d + m)}{2E_d (p^0_d - E_d)} + \frac{(-\hat{p}_x + m)}{2E_x (p^0_x - E_x)} \right] \gamma_5 u (\vec{p}) , \] (43)

with

\[ \hat{p}_i = (E_i, \vec{p}_i) , \]
\[ E_i = \sqrt{\vec{p}_i^2 + m^2} , \] (44)

for \( i = d, x \).

The diagrams of fig.3C involve scalar interactions between two quarks. If one were dealing with just a perturbative exchange of a scalar particle of mass \( m_s \), as in the case of sigmas in nuclear physics, the evaluation of this part of the diagram would yield a potential of the form

\[ \tilde{W}_P(q_w) = \frac{g_s^2}{q_w^2 - m_s^2} , \] (45)

where the subscript \( P \) stands for perturbation and \( q_w \) is the exchanged momentum,

\[ q_w = p'_w - p_w. \] (46)

In the case of quarks, the scalar interaction, represented by a formal momentum-space function \( \tilde{W}(q_w) \), is highly non-linear and its Fourier transform corresponds to the confining potential in configuration space.
\[ W(r) = \int \frac{d\mathbf{q}_w}{(2\pi)^3} e^{-i\mathbf{q}_w \cdot \mathbf{r}} \tilde{W}(q_w). \] (47)

With this definition, the proper \( \pi d \) amplitude for the quarks \( v \) and \( w \) of fig.3B is

\[
T_{\pi d}^{(vw)} = -i\tilde{W}(q_w) \left[ g\tau_\alpha \bar{u}(\vec{p}'') \left( \frac{k}{p_x^2 - m^2} - \frac{(-\slash{p}_d + m)}{2E_d (\slashed{p}_d^0 - E_d)} + \frac{\slash{k}}{p_x^2 - m^2} \right) \right]^{(v)} \left[ \bar{u}(\vec{p}') u(\vec{p}) \right]^{(w)}. \] (48)

Using the Dirac equation, we have

\[
T_{\pi d}^{(vw)} = -i\tilde{W}(q_w) \left[ g\tau_\alpha \bar{u}(\vec{p}'') \left( \frac{\gamma^0 (E_x - E_d)}{2E_d E_x} + \frac{1}{m} \right) \right]^{(v)} \left[ \bar{u}(\vec{p}') u(\vec{p}) \right]^{(w)}. \] (49)

Excluding the free spinors from this result, we obtain the proper \( \pi d \) vertex as

\[
\Gamma_{\pi d}^{(vw)} = \tilde{W}(q_w) \left[ g\tau_\alpha \left( \frac{\gamma^0 (E_x - E_d)}{2E_d E_x} + \frac{1}{m} \right) \right]^{(v)} \left[ \bar{u}(\vec{p}') u(\vec{p}) \right]^{(w)}, \] (50)

where \( I^{(w)} \) is the identity matrix for the pure scalar vertex in quark \( w \).

**V. EFFECTIVE POTENTIAL**

In order to calculate the effective potential, we insert the \( \pi q \) and \( \pi d \) vertex functions between free spinors and use the following correspondences between relativistic and non-relativistic amplitudes:

\[
\bar{u}(\vec{p}_v') \Gamma_{\pi q}^{(v)} u(\vec{p}_v) \xrightarrow{n.r.} -\frac{g}{2m} \tau_\alpha^{(v)} \sigma^{(w)} : (\vec{p}_v' - \vec{p}_v). \] (51)
\begin{equation}
\bar{u}(\vec{p}_w') \bar{u}(\vec{p}_v') \Gamma_{\pi d}^{(vw)} u(\vec{p}_w) u(\vec{p}_w) \frac{n_\gamma}{2} \bar{W}(q_w) \left\{ g_{\alpha\beta} \left[ \frac{\tilde{\sigma}^{(v)} \cdot \tilde{q}_w}{2m^2} \right] \right\} f^{(w)}.
\end{equation}

Using eqs.(51) and (52), we write the non-relativistic amplitudes corresponding to the diagrams of fig.3C, in the centre of mass (CM) frame of the NN system, as

\begin{equation}
t^{(v,y)}_{qq} = \left( \frac{g}{2m} \right)^2 \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \left[ \frac{1}{k^2 + m_\pi^2} \right] \left[ \frac{1}{\tilde{\tau}^{(y)} \cdot \vec{k}} \right],
\end{equation}

\begin{equation}
t^{(vw,y)}_{dq} = -\left( \frac{g}{2m} \right)^2 \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \left[ \frac{1}{\tilde{\tau}^{(y)} \cdot \vec{k}} \right] \left[ \frac{1}{\tilde{q}_w \left( \bar{W}(q_w) \right)} I^{(w)} \right] \left[ \frac{1}{\tilde{\tau}^{(y)} \cdot \vec{k}} \right],
\end{equation}

\begin{equation}
t^{(v,yz)}_{qd} = \left( \frac{g}{2m} \right)^2 \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \left[ \frac{1}{\tilde{\tau}^{(y)} \cdot \vec{k}} \right] \left[ \frac{1}{\tilde{q}_z \left( \bar{W}(q_z) \right)} I^{(z)} \right],
\end{equation}

\begin{equation}
t^{(vw,yz)}_{dd} = -\left( \frac{g}{2m} \right)^2 \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \left[ \frac{1}{\tilde{\tau}^{(y)} \cdot \vec{k}} \right] \left[ \frac{1}{\tilde{q}_z \left( \bar{W}(q_z) \right)} I^{(z)} \right],
\end{equation}

These results are related to the potentials between quarks and diquarks in momentum space by

\begin{equation}
<\vec{p}_1'...\vec{p}_6'|V|\vec{p}_1...\vec{p}_6> = -(2\pi)^3 \delta \left( \vec{p}_1' + ... + \vec{p}_6' \right) - \left( \vec{p}_1 + ... + \vec{p}_6 \right) t.
\end{equation}

The Fourier transform of this expression yields a potential in coordinate space, that is local and given by

\begin{equation}
<\vec{r}_1'...\vec{r}_6'|V|\vec{r}_1...\vec{r}_6> = \delta(\vec{r}_1' - \vec{r}_1)...\delta(\vec{r}_6' - \vec{r}_6) \Pi (\vec{r}_1...\vec{r}_6),
\end{equation}

where \Pi is the potential due to pion exchanges, as defined in section 2. Using

\begin{equation}
\vec{r}_{ij} = \vec{r}_i - \vec{r}_j
\end{equation}

\text{13}
and results (53-56), we obtain the potentials in configuration space

$$
\Pi_{qq}^{(v,y)} = - \left( \frac{g}{2m} \right)^2 \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \sigma_i^{(v)} \sigma_j^{(y)} \left[ \int \frac{dk}{(2\pi)^3} \frac{k^i k^j}{k^2 + m^2} e^{-i\vec{k} \cdot \vec{r}_{vy}} \right], \quad (60)
$$

$$
\Pi_{dq}^{(vw:y)} = \left( \frac{g}{2m} \right)^2 \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \sigma_i^{(v)} \sigma_j^{(y)} \left[ \int \frac{dq}{(2\pi)^3} q_i^{(v)} \left( \bar{W} \left( q_w \right) \right) e^{-i\vec{q} \cdot \vec{r}_{wv}} \right]
\times \left[ \int \frac{dk}{(2\pi)^3} \frac{k^i}{k^2 + m^2} e^{-i\vec{k} \cdot \vec{r}_{vy}} \right], \quad (61)
$$

$$
\Pi_{qd}^{(v,yz)} = - \left( \frac{g}{2m} \right)^2 \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \sigma_i^{(v)} \sigma_j^{(y)} \left[ \int \frac{dq}{(2\pi)^3} q_i^{(y)} \left( \bar{W} \left( q_w \right) \right) e^{-i\vec{q} \cdot \vec{r}_{wv}} \right]
\times \left[ \int \frac{dk}{(2\pi)^3} \frac{1}{k^2 + m^2} e^{-i\vec{k} \cdot \vec{r}_{vy}} \right],
$$

$$
\Pi_{dd}^{(vw,yz)} = \left( \frac{g}{2m} \right)^2 \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \sigma_i^{(v)} \sigma_j^{(y)} \left[ \int \frac{dq}{(2\pi)^3} q_i^{(y)} \left( \bar{W} \left( q_w \right) \right) e^{-i\vec{q} \cdot \vec{r}_{wv}} \right]
\times \left[ \int \frac{dk}{(2\pi)^3} \frac{1}{k^2 + m^2} e^{-i\vec{k} \cdot \vec{r}_{vy}} \right], \quad (62)
$$

The full potential due to pion exchanges is thus given by

$$
\Pi^X_{ab} = \sum_v \sum_v \Pi_{qq}^{(v,y)} + \sum_v \sum_v \Pi_{dq}^{(vw,y)} + \sum_v \sum_v \Pi_{qd}^{(v,yz)}
+ \sum_v \sum_v \Pi_{dd}^{(vw,yz)} \quad (63)
$$

where the symbol \( \Sigma_{i,j} \) indicates a sum over \( i \) and \( j \), with \( i \neq j \).

This is the pion exchange potential between quarks in different nucleons. In order to obtain the effective potential, we use this result in eq.(17) and have

$$
V_{NN} \left( \vec{R}_a, \vec{R}_b \right) = \left( \frac{g}{2m} \right)^2 \int d\Omega \left\langle I_b \right| \left\langle I_a \right| \left[ \sum_v \sum_v \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \left[ \sigma_i^{(v)} \cdot \nabla \bar{U} \left( \vec{r}_{vy} \right) \right] 
- \sum_v \sum_v \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \left[ \sigma_i^{(v)} \cdot \nabla \left( \frac{W \left( \vec{r}_{wv} \right)}{m} \right) \right] \left[ \sigma_j^{(y)} \cdot \nabla \bar{U} \left( \vec{r}_{vy} \right) \right] 
+ \sum_v \sum_v \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \left[ \sigma_i^{(v)} \cdot \nabla \bar{U} \left( \vec{r}_{vy} \right) \right] \left[ \sigma_j^{(y)} \cdot \nabla \left( \frac{W \left( \vec{r}_{zy} \right)}{m} \right) \right] 
- \sum_v \sum_v \bar{\tau}^{(v)} \cdot \bar{\tau}^{(y)} \left[ \sigma_i^{(v)} \cdot \nabla \left( \frac{W \left( \vec{r}_{wv} \right)}{m} \right) \right] \bar{U} \left( \vec{r}_{vy} \right) \left[ \sigma_j^{(y)} \cdot \nabla \left( \frac{W \left( \vec{r}_{zy} \right)}{m} \right) \right] \right] \left\rangle_{\tilde{I}_a} \right| \left\langle \tilde{I}_b \right| \right\rangle_{\tilde{I}_a} \right|_{\tilde{I}_b},
$$

14
where \( W(\vec{r}) \) is the configuration space confining potential and \( U(\vec{r}) \) is the Yukawa function

\[
U(\vec{r}) = \frac{4\pi}{\mu} \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{-i\vec{k}\cdot\vec{r}}}{k^2 + \mu^2} = \frac{e^{-\mu r}}{\mu r}.
\]

This is the main result of this work. The first term within the curly brackets is the usual OPEP between quarks whereas the other ones correspond to corrections due to chiral symmetry, in the form of gradients of the confining potential. An interesting feature of this result is that all the terms of the effective potential contain two gradients, reflecting the fact that they come from a uniform expansion in momentum space, as expected from a calculation based on chiral symmetry.

\section*{VI. APPLICATION}

In this section we apply the results from the previous section to the case of a nucleon composed by three quarks bound by a harmonic potential of the form

\[
W(\vec{r}) = \frac{1}{2} Kr^2.
\]

In order to make the structure of our calculation more transparent, we allow different confining constants for the two nucleons.

The internal nucleon wave function is given by

\[
|I\rangle = |\vec{\rho}, \vec{\lambda}, S^z, T^z, C\rangle = \frac{\alpha^3}{\pi^{3/2}} e^{-\frac{\lambda^2}{4}} (\vec{\rho^2} + \vec{\lambda^2}) |S^z\rangle |T^z\rangle |C\rangle
\]

where \( \vec{\rho} \) and \( \vec{\lambda} \) are Jacobi coordinates and \( S^z \), \( T^z \) and \( C \) are spin, isospin and color states. The color component \( |C\rangle \) is totally antisymmetric with respect to quark permutations and
the same happens with the full wave-function. The constant \( \alpha \) represents the size of the nucleon, is given by \( \alpha^2 = \sqrt{3Km} \) and related to the binding energy per nucleon by \( \omega = \frac{\alpha^2}{m} \).

The action of the quark spin and isospin operators over the nucleon wave function is related to the corresponding collective operators by

\[
\tau_i^{(v)} |S^z, I^z\rangle = \frac{1}{3} T_i |S^z, I^z\rangle + \ldots ,
\]

(69)

\[
\sigma_i^{(v)} |S^z, I^z\rangle = \frac{1}{3} \Sigma_i |S^z, I^z\rangle + \ldots ,
\]

(70)

\[
\sigma_i^{(v)} \tau_j^{(v)} |S^z, I^z\rangle = \frac{5}{9} \Sigma_i \Sigma_j |S^z, I^z\rangle + \ldots ,
\]

(71)

\[
\sigma_i^{(v)} \tau_j^{(w)} |S^z, I^z\rangle = -\frac{1}{9} \Sigma_i T_j |S^z, I^z\rangle + \ldots v \neq w ,
\]

(72)

where we have omitted non-nucleon states on the right hand side. Using these results in eqs.(60-64), we obtain the effective potential operator in spin and isospin spaces

\[
V_{NN} (\vec{R}_a, \vec{R}_b) = - \left( \frac{g}{2m} \right) \frac{5}{9} \alpha^2 \alpha^2 \frac{\bar{\tau}^{(a)} \cdot \bar{\tau}^{(b)} \Sigma_i \Sigma_j}{\pi^2 \pi^3} \int d\vec{k} e^{-\alpha^2 (\vec{p}_a^2 + \vec{\lambda}_a^2) - \alpha^2 (\vec{p}_b^2 + \vec{\lambda}_b^2)} - i\vec{k} \vec{r}_{vy}.
\]

(73)

The vector \( \vec{r}_{vy} \) that enters these expressions may be written as linear combinations of \( \vec{R}, \vec{\rho} \) and \( \vec{\lambda} \)

\[
\vec{r}_{vy} = \vec{X} + (c_{vp} \vec{p}_a + c_{vl} \vec{\lambda}_a - c_{vp} \vec{p}_b - c_{vl} \vec{\lambda}_b),
\]

(74)
where the coefficients $c_{ij}$ have the values $c_{1\rho} = \sqrt{\frac{2}{3}}$, $c_{2\rho} = -\sqrt{\frac{2}{3}}$, $c_{3\rho} = 0$, $c_{1\lambda} = \sqrt{\frac{2}{6}}$, $c_{2\lambda} = \sqrt{\frac{1}{6}}$, $c_{3\lambda} = -\sqrt{\frac{2}{3}}$ and obey the relationships

$$c_{i\rho}^2 + c_{i\lambda}^2 = \frac{2}{3}, \quad (c_{i\rho} - c_{j\rho}) c_{i\rho} + (c_{i\lambda} - c_{j\lambda}) c_{i\lambda} = 1.$$  

These results allow the various gaussian integrations to be performed and we obtain

$$I_{(v,y)} = \frac{\alpha_b^6}{\pi^3 \pi^3} \int d\Omega e^{-\alpha_b^2 (\vec{p}_v^2 + \vec{\lambda}_v^2) - \alpha_b^2 (\vec{p}_y^2 + \vec{\lambda}_y^2) - i\vec{k} \cdot (c_{uv} \vec{p}_a + c_{v\lambda} \vec{\lambda}_a - c_{uv} \vec{p}_b - c_{v\lambda} \vec{\lambda}_b)}$$
$$= e^{-Ak^2},$$

$$I_{(v,v)} = \frac{\alpha_b^6}{\pi^3 \pi^3} \int d\Omega e^{-\alpha_b^2 (\vec{p}_v^2 + \vec{\lambda}_v^2) - \alpha_b^2 (\vec{p}_v^2 + \vec{\lambda}_v^2) - i\vec{k} \cdot (c_{uv} \vec{p}_a + c_{v\lambda} \vec{\lambda}_a - c_{uv} \vec{p}_b - c_{v\lambda} \vec{\lambda}_b)}$$
$$\times \left[ (c_{wp} - c_{vp}) \vec{p}_a + (c_{w\lambda} - c_{v\lambda}) \vec{\lambda}_a \right]^i$$
$$= i\frac{k_i}{2\alpha_b^2} e^{-Ak^2},$$

$$I_{(v,zy)} = \frac{\alpha_b^6}{\pi^3 \pi^3} \int d\Omega e^{-\alpha_b^2 (\vec{p}_v^2 + \vec{\lambda}_v^2) - \alpha_b^2 (\vec{p}_v^2 + \vec{\lambda}_v^2) - i\vec{k} \cdot (c_{uv} \vec{p}_a + c_{v\lambda} \vec{\lambda}_a - c_{uv} \vec{p}_b - c_{v\lambda} \vec{\lambda}_b)}$$
$$\times \left[ (c_{zp} - c_{yp}) \vec{p}_b + (c_{z\lambda} - c_{y\lambda}) \vec{\lambda}_b \right]^j$$
$$= -i\frac{k_j}{2\alpha_b^2} e^{-Ak^2},$$

$$I_{(w,zy)} = \frac{\alpha_b^6}{\pi^3 \pi^3} \int d\Omega e^{-\alpha_b^2 (\vec{p}_w^2 + \vec{\lambda}_w^2) - \alpha_b^2 (\vec{p}_v^2 + \vec{\lambda}_v^2) - i\vec{k} \cdot (c_{uv} \vec{p}_a + c_{v\lambda} \vec{\lambda}_a - c_{uv} \vec{p}_b - c_{v\lambda} \vec{\lambda}_b)}$$
$$\times \left[ (c_{wp} - c_{vp}) \vec{p}_a + (c_{w\lambda} - c_{v\lambda}) \vec{\lambda}_a \right]^i \left[ (c_{zp} - c_{yp}) \vec{p}_b + (c_{z\lambda} - c_{y\lambda}) \vec{\lambda}_b \right]^j$$
$$= \frac{k_i k_j}{2\alpha_b^2 2\alpha_a^2} e^{-Ak^2},$$

where

$$A = \left( \frac{1}{6\alpha_a^2} + \frac{1}{6\alpha_b^2} \right).$$

Thus all configuration space integrals entering eq.(73) become proportional to

$$U^{ij} (\vec{X}, \alpha_a, \alpha_b) = \frac{4\pi}{\mu} \int \frac{d\vec{k}}{(2\pi)^3} \frac{k_i k_j}{k^2 + \mu^2} e^{-i\vec{k} \cdot \vec{X} - Ak^2}. \quad (82)$$
Using these results in eq.(73), we obtain

\[ V_{NN}(\vec{X}, \alpha_a, \alpha_b) = \left( \frac{g}{2m^3} \right)^2 \left( 1 + \frac{\alpha_a^2}{3m^2} \right) \left( 1 + \frac{\alpha_b^2}{3m^2} \right) \frac{\mu}{4\pi} \times \vec{T}^{(a)} \cdot \vec{T}^{(b)} \vec{\Sigma}^{(a)} \cdot \vec{\nabla} \vec{\Sigma}^{(b)} \cdot \vec{\nabla} U(\vec{X}, \alpha_a, \alpha_b). \]  

where

\[ U(\vec{X}, \alpha_a, \alpha_b) = \frac{4\pi}{\mu} \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{(-i\vec{k} \cdot \vec{X} - Ak^2)}}{k^2 + \mu^2} = \frac{2}{\pi\mu} \int \frac{dkk^2e^{-Ak^2}}{k^2 + \mu^2}j_0(kX). \]  

This integral may be performed analytically and we have

\[ U(\vec{X}, \alpha_a, \alpha_b) = \frac{ae^{A\mu^2}}{\mu X} \left[ \sinh \mu X - \frac{1}{2} e^{-\mu X} \text{erf} \left( \frac{X}{2\sqrt{A}} - \mu \sqrt{A} \right) \right. \]
\[ \left. - \frac{1}{2} e^{\mu X} \text{erf} \left( \frac{X}{2\sqrt{A}} + \mu \sqrt{A} \right) \right]. \]  

For large values of \( X \) this integral becomes

\[ U(\vec{X}, \alpha_a, \alpha_b) \xrightarrow{x \to \infty} e^{\frac{\mu^2}{\alpha_a^2}} e^{\frac{\mu^2}{\alpha_b^2}} \frac{e^{-\mu X}}{\mu X}, \]  

after using \( \text{erf}(\infty) = 1 \).

The potential therefore reduces to

\[ V_{NN}(\vec{X}) = \left( \frac{g}{2m^3} \right)^2 \frac{\mu}{4\pi} \left( 1 + \frac{\alpha_a^2}{3m^2} \right) e^{\frac{\mu^2}{\alpha_a^2}} \left( 1 + \frac{\alpha_b^2}{3m^2} \right) e^{\frac{\mu^2}{\alpha_b^2}} \times \vec{T}^{(a)} \cdot \vec{T}^{(b)} \vec{\Sigma}^{(a)} \cdot \vec{\nabla} \vec{\Sigma}^{(b)} \cdot \vec{\nabla} \left( \frac{e^{-\mu X}}{\mu X} \right). \]  

Comparing this result with the usual expression for the OPEP, we have
\[
\frac{5}{3} \frac{g}{2m} \exp\left(\frac{\mu^2}{6\alpha^2}\right) \left(1 + \frac{\alpha^2}{3m^2}\right) = \frac{g_{\pi NN}}{2M_N}.
\] (88)

Going back to eq. (83), we write

\[
V_{NN}(\vec{X}) = \left(\frac{g_{\pi NN}}{2M}\right)^2 \vec{T}^{(a)} \cdot \vec{T}^{(b)} \vec{\Sigma}^{(a)} \cdot \vec{\nabla} \vec{\Sigma}^{(b)} \cdot \vec{\nabla} \int \frac{dk}{(2\pi)^3} \frac{e^{-i\vec{k} \cdot \vec{X} - (k^2 + \mu^2)A}}{\vec{k}^2 + \mu^2}.
\] (89)

This expression allows the \(\pi N\) form factor to be identified as

\[
\bar{G}(k) = \exp\left(-\frac{\vec{k}^2 + \mu^2}{6\alpha^2}\right).
\] (90)

Thus one learns that, in the case of a harmonic confining potential, the \(\pi N\) form factor is not modified by the binding corrections, since it is the same as one would obtain by considering only the diagram 1 of fig. 3C.

On the other hand, eq. (88) allows one to write the effective pion-quark coupling constant as

\[
g_{\text{eff}} = g \left(1 + \frac{\omega}{3m}\right),
\] (91)

with \(\omega = \sqrt{\frac{3K}{m}}\), indicating that it is influenced by binding corrections. In order to interpret this result, we go back to the PS Lagrangian given by eq. (33) and note that, within the approximations considered here, it is equivalent to having the field \(S\) replaced by a mean value such that

\[
\frac{g_{\pi}}{f_\pi} \langle S \rangle = g \frac{\omega}{3m}
\] (92)

and this corresponds to the effective mass

\[
m_{\text{eff}} = g f_\pi \left(1 + \frac{\omega}{3m}\right).
\] (93)
In the case of the PV Lagrangian, on the other hand, eq.(92) produces a shift in the quark mass which translates into a change of the coupling constant when the effective equation of motion is used in the $\pi N$ vertex.

The shift in the effective mass is due to the potential energy associated with each particle, given by the expectation value of the function $\frac{K^2 r^2_i}{2}$ (and not $\frac{K^2 r^2_{ij}}{2}$!), which yields $\frac{\omega}{3}$. Alternatively, one may note that the total potential energy in the nucleon is $\frac{3}{2}\omega$; of this amount, $\frac{1}{2}\omega$ corresponds to the energy of the CM system and hence $\frac{\omega}{3}$ is available for shifting each quark mass.

VII. CONCLUSIONS

In this work we have studied the role of chiral symmetry in the $\pi N$ form factor, assuming the nucleon to be a constituent quark cluster, bound by a scalar field. The symmetry was implemented by means of two different Lagrangians, for PS and PV pion quark couplings, which led to identical results, as expected in a consistent symmetrical calculation.

The implementation of chiral symmetry in a system of bound quarks requires the use of both one and two quark operators, the latter corresponding to binding corrections. This gives rise to a nucleon-nucleon effective potential that is a uniform second order polynomial in the pionic and internal momenta. In configuration space, it contains two gradients, acting on either the Yukawa function or on the confining potential.

In order to assess the properties of this chiral effective potential in a particular model, we considered the case of a harmonic confining force characterized by a frequency $\omega$. In this instance, the shape of the $\pi N$ form factor is not modified by the symmetry, since the harmonic wave function is an exponential and hence does not change its form upon derivation. On the other hand, the $\pi N$ coupling constant receives chiral contributions of the order $\frac{\omega}{m}$, $m$ being the constituent quark mass. The fact that in many models we find $\omega \sim m$ means that these corrections are significant and tend to favour smaller values of
the coupling constant. In some baryon models, pion exchanges are used to generate spin
dependent forces and hence changes in the coupling constant may influence the observables.

It is important to stress that our calculation deals only with the internal part of the form
factor and, in particular, effects associated with the pion cloud were not considered. Therefore
the phenomenological implications of our study cannot be fully explored at present, but
the consistent inclusion of cloud effects is part of our programme.

However, even with its present limitations, our work provides an insight on how the
symmetry works in bound systems. It is a general feature of chiral models that fermion
masses and coupling constants to pions be constrained by the Goldberger-Treiman relation.
Therefore one expects that any chiral interaction which modifies the coupling constant should
also shift the constituent quark mass. In the case of the harmonic potential, we have shown
that this coherent picture emerges from the inclusion of binding effects.
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Figure Captions

Fig.1. Pion cloud contribution to the $\pi N$ form factor; pions and nucleons are represented by dashed and continuous lines respectively.

Fig.2. Chiral amplitude for the process $\pi q \rightarrow Sq$, where the scalar boson is represented by a wavy line and (PV) and (PS) stand for pseudovector and pseudoscalar couplings.

Fig.3. A: interaction between two clusters; B: one quark irreducible pion-diquark vertex, where the propagator with the insertion (+) corresponds to positive frequency states; C: the proper part of the NN interaction.
