Pixel-wise Amplitude Distribution Evaluation in Time Average Digital Holography

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Abstract. This article introduces a new holographic method for harmonic vibration measurement. The method called time average sweeping digital holography (TASDH) is based on the continuous shift of fringe pattern described by Bessel function over the object surface. The shift is done by phase modulation in one arm of holographic interferometer. Such temporal phase sweeping allows for evaluation of the vibration amplitude value independently in each pixel by application of intensity values cross-correlation of steady state and oscillating object. Main advantage of proposed technique with respect to other methods of digital holographic vibration is its ability to measure the amplitude of vibration without the risk of interference order mismatch. Thanks to above mentioned properties, the method allows measurement of amplitude modes with high slopes or discontinuous surfaces as well as partially shaded objects. The method was experimentally tested by measuring the bending beam cantilever. The standard deviation of the measured noise is below nanometer.

1. Introduction

Measurement of vibrations [1] is very important task in various technical and scientific disciplines. The most commonly used measuring instruments are laser Doppler vibrometers [2], instruments based on correlation analysis [3], ESPI (Electronic Speckle Pattern Interferometry) [4], or holographic interferometry [5]. The methods are either full-field or single-point, they provide dynamic measurement or measurement of modal shapes. Their measurement capabilities are limited in terms of maximum amplitude or frequency range. Therefore, particular attention should be paid to device parameters when selecting one for specific application.

This paper is focused on time average digital holography (TADH) [6]. Time averaging of holograms appears when the frequency of object oscillations is much higher than exposure time of camera. TADH is full-field method for harmonic vibrations measurement. One of the most significant advantages of TADH is unlimited frequency of measured vibration. The intensity map reconstructed from the time averaged digital hologram is modulated by the Bessel function where argument of the function is value of vibration amplitude. Maxima of the function creating Bessel fringe pattern play role of amplitude distribution contours. One of the most important tasks in TADH is to quantitatively determine the amplitude distribution from the Bessel function modulated fringe pattern. Commonly
used evaluation methods are based on fringe counting [7], inversion Bessel function [8] or phase shifting (PS). PS approach allows for accurate determination of interference phase where phase modulation by either a mirror mounted on a piezoelectric transducer (PZT) [9] or acoustic-optical modulator [10] is employed. Modulation depth introduced by the phase modulation leads to shift of the Bessel fringe pattern similarly like phase shifts in the case of cosine fringe pattern well known from PS interferometry. The result of PS evaluation method is very accurate phase value but bounded within $2\pi$ interval. In order to remove the $2\pi$ jumps, the spatial phase unwrapping must follow. Such spatial unwrapping can be very problematic or even impossible in some cases.

Easy use of PS approach is derived from the harmonic cosine nature of interference phenomenon. In TADH the intensity is modulated by Bessel functions and therefore the straightforward phase recovery in the very same way as for cosine fringes is not possible. Possible solution is to exploit nearly periodic nature of the Bessel functions and consider the modulation to be cosine. Such simplification must be compensated by application of correction function derived from the difference between cosine and Bessel approach.

Although PS methods are considered to be the most convenient, there are definitely limitations. On the one hand, the result needs to be corrected due to the difference between cosine and Bessel functions. Further, a spatial unwrapping algorithm is required to demodulate the wrapped phase field and "zero point" (oscillation node) must be determined - to which other pixels are relatively measured. PS method naturally fails in vibrational modes with high slopes that lead to very dense fringe patterns. In addition, the field of view mustn’t contain discontinuities and has to be free of shaded places, etc.

In order to address the above mentioned drawbacks, we propose time average sweeping digital holography (TASDH). The proposed method is also based on phase modulation of reference or object waves; however, the modulation depth is now continuously swept over certain range. Using the non-harmonic properties of a zero-order first kind Bessel function, vibration amplitude is determined absolutely in each pixel as a lag of the Bessel function at oscillating state behind the steady state. Therefore, no spatial phase unwrapping or Cosine-Bessel correction is necessary. Compared to PS methods, TASDH needs more acquired data, which is not a problem for today's computers. This paper presents the principle of TASDH, experimental evaluation done on bending beam cantilever and simple noise estimation is performed.

### 2. Recording and reconstruction of digital holograms

Digital holography (DH) includes the recording and reconstruction of optical waves. Capturing of hologram is done by recording of the superposition of the wave scattered from the surface of the object (object wave) $O$ with the known reference wave $R$. This captured interference pattern is called a digital hologram and can be described by an interference equation:

$$H = |O + R|^2 = |R|^2 + |O|^2 + O^*R + OR^*.$$  \hfill (1)

Hologram $H$ contains information about the intensity and phase of the waves and is stored in computer as a matrix of numbers.

The reconstruction process restores information about the amplitude and phase of the digital hologram. The digital hologram $H$ is at first multiplied by a numerical representation of the conjugated reference wave $R^*$. It results in a complex wave field in the holographic plane (with a coordinate notation $\xi$, $\eta$). This wave field is then numerically propagated in free space according to the laws of diffraction, and the resulting complex array is calculated at a reconstruction distance $d$ - called image plane (coordinates notations $x$, $y$). Propagation in the free space can be calculated by Fresnel transform:

$$U(x, y) = \mathcal{F}^{-1}\left\{H(\xi, \eta)R^*(\xi, \eta)\exp\left[-\frac{j\pi}{\lambda d}\left(\xi^2 + \eta^2\right)\right]\right\},$$  \hfill (2)
where $\mathcal{F}^{-1}$ denotes the inverse discrete Fourier transform and $\lambda$ is the wavelength of the laser. The both hologram and image plane are sampled by $N \times M$ pixels. Pixel extension in the hologram plane $\Delta x \times \Delta y$ is naturally defined by the digital sensor parameters, while the image plane dimensions $\Delta x$, $\Delta y$ are given by parameters of the reconstruction:

$$
\Delta x = \frac{\lambda d}{N \Delta \xi} \quad \text{and} \quad \Delta y = \frac{\lambda d}{M \Delta \eta}.
$$

(3)

The output of the reconstruction algorithm is a complex wave field with imaginary ($Im$) and real ($Re$) parts in the image plane from which the intensity and phase distributions can be calculated as:

$$
I(x, y) = \left| U(x, y) \right| \quad \text{and} \quad \phi(x, y) = \tan^{-1}\left( \frac{Im(U(x, y))}{Re(U(x, y))} \right).
$$

(4)

3. Principle of phase sweeping time average digital holography

Holographic recording of a harmonically oscillating object with an exposure time much longer then the period of measured object vibrations results in time average holography. Let us consider out-of-plane amplitude distribution $d(x, y, t) = D(x, y) \sin(\omega t)$ oscillating in time $t$, where $D$ is the amplitude of vibration and $\omega$ stands for the angular frequency. Such phase modulation of object wave results in $O(x, y, t) = O(x, y) \exp(j \phi(x, y) \sin(\omega t))$ and the intensity of the reconstructed image is proportional to magnitude of the first kind zero-order Bessel function $J_0$:

$$
I(x, y) = \left| J_0(\phi(x, y)) \right|.
$$

(5)

The argument of the Bessel function (further called interference phase) in equation (5) is proportional to the out-of-plane amplitude distribution:

$$
\phi(x, y) = D(x, y) 4\pi/\lambda.
$$

(6)

The TASDH requires secondary modulation of phase within the reference $R$ or the object $O$ beam. The frequency of the secondary modulation has the same frequency $\omega$ as the oscillating object and a modulation depth $\Omega$. Considering the object wave including the secondary phase modulation:

$$
O(x, y, t) = O(x, y) \exp(j \phi(x, y) \sin(\omega t)) \exp(j \Omega \sin(\omega t)).
$$

(7)

and substituting (7) into interference equation (1) modifies the equation (5) to be:

$$
I(x, y, \Omega) = \left| J_0(\phi(x, y) - \Omega) \right|.
$$

(8)

Naturally, no secondary modulation $\Omega = 0$ leads to equation (5). As follows from (8) the unambiguously loci of bright zero fringe appears where modulation of the secondary modulation and amplitude of vibration at point $(x, y)$ are in agreement:

$$
\phi(x, y) = \Omega.
$$

(9)

Sweeping of the modulation depth $\Omega$ gives an intensity signal for each pixel $(x, y)$ of the reconstructed intensity map as a function of the modulation depth (8). The modulation depth generated by the secondary phase modulator is not exactly known since it is defined by transfer function between user controllable electronic device and the real phase modulation of the light beam. Therefore the interference phase $\phi$ is computed as the shift between steady-state ($\phi = 0, \Omega = 0$) with intensity:

$$
I(x, y, 0) = \left| J_0(0) \right|
$$

(10)

and the oscillating state defined in equation (8). Although there are several strategies how to search for the shift, due to noise resistance cross-correlation formula (operator denoted by *):
\[ C(\hat{\Omega}) = I_0 * I = \int I_0(\Omega)I(\Omega + \hat{\Omega})d\Omega \]  

was used. In (11) the lag of the cross-correlation function \( \hat{\Omega} \) has the physical meaning of the modulation depth \( \Omega \). Lag of the cross-correlation function \( C(\hat{\Omega}) \) for its maximum indicates where the signals are best aligned and \( \hat{\Omega} \approx \varphi \). The interference phase therefore can be computed as:

\[ \varphi = \text{arg} \max (I_0 * I). \]

Hence, the out-of-plane amplitude in each pixel \( D(x, y) \) is computed as simple inversion of (6):

\[ D(x, y) = \varphi(x, y) \frac{\lambda}{4\pi}. \]

4. Measurement of bending beam cantilever

The schematics of experimental arrangement based on off-axis digital holography is provided in Figure 1. The object under investigation is a bending beam cantilever that is \( d=0.7 \text{ m} \) apart from the sensor of AVT Stingray digital camera. The sensor has \( N=M=2048 \) pixels with extensions \( \Delta \xi = \Delta \eta = 3.45 \mu \text{m} \). Light of wavelength \( \lambda=532\text{nm} \) was emitted by Nd:YAG laser. Superposition of the reference and the object wave (digital hologram) is captured by the digital camera.

\[ \text{Figure 1. Experimental arrangements for testing of TASDH: BS - beam splitter, NF - neutral density filter, SF - spatial filter, CO - collimating objective, OBJ - object, FG - arbitrary waveform generator, CAM - digital camera, M - mirror, PZT – piezoelectric transducer.} \]

The phase of the object wave is modulated by the object vibrations and also by oscillating mirror mounted on an electronically controlled piezoelectric transducer (PZT). The PZT mirror can also be placed in the reference arm. The object under investigation - bending beam cantilever – and the PZT are driven by a two channel waveform generator. One channel of the waveform generator sets the frequency and amplitude of the harmonic object vibrations while the second channel controls the PZT mirror.

In the first step, the reference state is measured. The object is in steady-state and the PZT is oscillating with frequency of 1000 Hz. The driving amplitude of PZT was swept in the range from -20 V_{pp} (peak-to-peak) to 20 V_{pp} with the step of 1 V_{pp}. The reconstructed intensity at any object point follows equation (8) with \( \varphi = 0 \). Results of the Bessel function fitting provides the transfer function between the electronic output signal (PZT driven amplitude) from the waveform generator and the
phase modulation depth of the light wave. The measured or fitted result can be used as reference for the measurement.

![Figure 2](image)

**Figure 2.** Reconstructed intensity maps with different phase modulation depth $\Omega$ creating a stack of images. Intensity profile in each pixel is described by Bessel function that is shifted according to amplitude of vibration with respect to steady state (reference). Reference and measured intensity profiles in pixel denoted by green mark are plotted on the right hand side of the figure.

In the second step we set the object frequency to 1000 Hz with 100 mV$_{pp}$ driving amplitude. Frequency of the PZT remains 1000 Hz and start phase of the PZT is aligned with the object vibration phase. The driving amplitude is again swept from -20 V$_{pp}$ to +20 V$_{pp}$ with the step of 1 V$_{pp}$ leading to sweeping of modulation depth. Maximum of cross-correlation function between the reference and oscillating states indicates the lag corresponding to the interference phase $\hat{\Phi} \approx \Phi$ as discussed in equation (12). This procedure is realized independently in every single pixel and using formula (13) the interference phase is converted to amplitude distribution, see Figure 3. Values along the white line in Figure 3a were plotted in Figure 3b and fitted with 8th order polynomial function. Difference between the measured and the fitted values is introduced in Figure 3c. Values represent noise of the measurement since such high spatial frequencies are not supposed to be real movement of the object. Standard deviation of the noise is 0.8 nm.

![Figure 3](image)

**Figure 3.** a) out-of-plane amplitude distribution measured by TASDH represented as false colour
image; b) measured values (blue) of vibration amplitude values along white line and fitted curve using 8th order polynomial (green); c) measured noise as a difference between measured and fitted values in b). Standard deviation of the noise is 0.8 nm.

**Conclusion**

Time average sweeping digital holography (TASDH) introduced in the paper is a powerful technique for evaluation of time averaged holograms. TASDH is used to quantitative measurement of harmonic vibration amplitudes. The reconstructed intensity image from the time average digital hologram is modulated by the Bessel function that is unambiguous function. For phase shifting techniques such disadvantage must be corrected while it is a key property for TASDH. The principle of TASDH is based on phase modulation of object or reference wave of a holographic interferometer. As the depth of phase modulation changes, the interference pattern continuously shifts over the surface of the object and due to the unambiguous behavior of the Bessel function, the lag of steady and oscillating state can be found in each pixel. The interference phase value is uniquely known and can therefore be easily converted into amplitude value. This is done independently in each pixel, so there is no need for spatial unwrapping. A beneficial feature of TASDH over other quantification methods based on TADH is the ability to measure vibration amplitudes without the risk of interference order errors. For this reason, the method can provide amplitude distribution of amplitudes with high slopes, discontinuities or in case of partially shaded objects. The correctness of the method was experimentally verified by measurement of bending beam cantilever where standard deviation of the measurement noise reached only 0.8 nm.

**Acknowledgement**

This work was supported by Czech Science Foundation Project No.: GACR GA 16-11965S.

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