Gearbox Diagnostics Using Wavelet-Based Windowing Technique

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Abstract. In extracting gear box acoustic signals embedded in excessive noise, the need for an online and automated tool becomes a crucial necessity. One of the recent approaches that have gained some acceptance within the research arena is the Wavelet multi-resolution analysis (WMRA). However selecting an accurate mother wavelet, defining dynamic threshold values and identifying the resolution levels to be considered in gearboxes fault detection and diagnosis are still challenging tasks. This paper proposes a novel wavelet-based technique for detecting, locating and estimating the severity of defects in gear tooth fracture. The proposed technique enhances the WMRA by decomposing the noisy data into different resolution levels while data sliding it into Kaiser's window. Only the maximum expansion coefficients at each resolution level are used in de-noising, detecting and measuring the severity of the defects. A small set of coefficients is used in the monitoring process without assigning threshold values or performing signal reconstruction. The proposed monitoring technique has been applied to a laboratory data corrupted with high noise level.

1. Introduction
Early detection of a machine element failure is of interest and importance to industry. Gearboxes are considered the most widely used power transfer machine element. They are almost present in any machine. Depending on the place and the importance of the machine, an early detection of the onset of damage in a gear set and the evaluation of the remaining life could have crucial impact on productivity and safety. Vibration analysis is a broadly used condition monitoring technique for gears as well as for other mechanical system components such as bearings. A vibration transducer, usually an accelerometer, is fastened to the gearbox casing. The vibration signals obtained from the transducer can be integrated to produce velocity or even displacement if needed. Vibration signals are then sampled and processed in various ways to highlight specific aspects of the signal which can be used in the detection and diagnostic of the gearbox condition.

Many approaches and techniques were developed in the past for the gear diagnostics. Those techniques are either based on time domain analysis [1-5], frequency domain analysis [6-8], or time-frequency domain analysis that highlights the time and frequency components strength instantaneously. This advantage of the time-frequency technique has attracted many researchers working in gear diagnostics. Gear vibration varies throughout one gear cycle due to variations in gear meshing stiffness, transmission error, and gear damage. Wang and McFadden used the spectrogram for early failure detection in a gearbox [9]. The spectrogram represents the signal energy distribution over the frequency domain and at every instant of time. For a typical spectrum analysis the abnormal
disturbance in the signal energy at a single moment, e.g., damage at a single tooth, would be distributed over the time period of the gear rotation. However, the spectrogram would provide a higher sensitivity to changes of the short duration in the signal. Wang and Mcfadden found that the sensitivity of the spectrogram would be enhanced by removing the harmonics of the tooth meshing frequency before calculating the spectrogram. Later researchers directed their attention to the wavelet transform as it is powerful of detecting transients in non-stationary signals. The first attempt of using wavelet transform is reported by Wang et al. in 1995. They concluded that the wavelets are more suitable for gear damage detection than the spectrogram and the Gaussian enveloped oscillating wavelet is suitable for detecting various sizes of gear faults [10, 11]. Luo et al. used the wavelet transform as a band-pass filter around a number of natural frequencies in gear vibration power spectrum [12]. The technique presented enhanced the signal to noise ratio. The component peak values of the enhanced power spectrum components are used as indicators of condition and defect. However, the sensitivity of the approach to the degree of the tooth damage is not reported. Dalpiaz et al. compared the wavelet transform with the cepstrum and spectrum correlation density as being applied to vibration signals representing two damage severity levels of gear failure [13]. They found that the mean value of the wavelet cross-section is very sensitive to the presence and size of tooth crack. Lin and Qu introduced a de-noising method that is based on wavelet analysis to extract gear vibration features however, more work still needed to apply the technique on signals of faulty gears [14]. Baydar and Ball analyzed gear vibration and acoustics using a Gabor based wavelet [15]. They found that changes in the wavelet plots are clearly observed for cracks of 30% and more. More work is still needed to produce features from the wavelet coefficient images that are related to failure prognoses.

More recently, Saravanan et al. used Morlet wavelets to extract features from vibration signals of bevel gear. Then they used support vector machine, a classification technique based on statistical learning theory, to classify the extract features according to the gear tooth damage type, e.g., crack, wear and breakage [16]. However, the sensitivity of the technique for the damage type was not reported. A similar work was published by Rafiee et al. but using Daubechie 11 wavelet for feature extraction and the neural net technique for fault classification [17]. Jafarizadeh proposed a pre-processing time averaging technique before applying the technique [18]. The work presented in this paper considers using the Kasier’s window as a pre-processing step before applying the wavelet technique. The details of the technique are presented in section 5.

2. Experimental Setup and Laboratory Data

The experimental setup of the gear testing consists of one stage gearbox, two electric motors, a DC motor controller, and high power resistors, as illustrated in figure 1. The gearbox is constructed from a 127 mm steel square section welded to a 6.3 mm plate. It includes two identical mild steel gears of 10.2 mm width, 15 teeth, 14.5° pressure angle, and 15.87 mm pitch diameter. The two gear shafts are supported with bronze bushings. The gears and bushings are sump lubricated with Mobil SHC 634 synthetic lubrication oil.

The electric motors are 1 HP permanent magnet DC motors used for driving and loading the gearbox. The DC motor controller can drive the gearbox between 200 and 1400 rpm with a speed accuracy of 2%. The load motor applies the breaking torque, dissipating the energy through the resistors. The resistors can be arranged in various ways to provide a wider load torque range. A shunt resistor is connected to the high power resistors in series to measure the current through the load motor.

The vibration signal is first collected by a PCB-307A piezoelectric accelerometer that is mounted on the gearbox plat inline with the gear meshing force. The accelerometer signal is amplified by a PCB-482A power supply unit. The signal is then passed through a data acquisition card (Metabyte Dash-16F) which is controlled by software. A photo interrupter device is used in conjunction with a slotted disk to produce a pulse signal to trigger the A/D board and collect the data. This is required to facilitate time synchronous averaging. The data are then analyzed using Matlab software. The gear tooth crack is simulated using a thin saw cut at the tooth root. For the purpose of evaluating the
efficiency of the proposed monitoring tool, different experiment runs were performed for various sizes of gear tooth cracks at the same speed of 1221 RPM. The sampling frequency was set at 83.33 KHz that generates 4096 samples for each single shaft rotation. The data signal to noise ratio was enhanced using the time averaging technique and with the help of the optical sensors.

Figure 1. Experimental setup of gear vibration test.

3. Introduction to Windowing Wavelet Transform

The discrete wavelet transform (DWT) decomposes a signal \( f(t) \) at different resolution levels and represents the signal as a series of approximate \( c_j(k) \) and detail \( d_j(k) \) expansion [19], [20]:

\[
f(t) = \sum_{k} c_j(k) \phi(t-k) + \sum_{j=0}^{J-1} \sum_{k} d_j(k) 2^{j/2} \psi(2^j t-k)
\]  

(1)

The discrete wavelet coefficients measure the similarity between the signal and the selected wavelet \( \psi(t) \); hence provide a time-frequency localization of the signal. If the scaling function and wavelets form an orthonormal basis, then according to Parseval’s theorem, the energy of the signal can be partitioned in terms of the expansion coefficient.

For any proposed new signal monitoring tool, the following characteristics represent important features for evaluating the proposed tool:

- **Sparsity**: The expansion coefficients of any event should have most of the important information in the smallest number of coefficients so that the rest of coefficients are small enough to be neglected or set equal to zero. This is important for data management, compression and denoising.
- **Separation**: During abnormal conditions, defects can be modeled as a linear combination of signals with different characteristics; the expansion coefficients should clearly separate those signals. Features of interest that classify each defect should be separated and localized at different resolutions. This is important for detection and classification.
- **Super-resolution**: A defect free sample and its characteristic set of expansion coefficients should have much better resolution than that with a traditional basis system. This is likewise important for detection, classification and estimation.
- **Stability**: The expansion coefficients that are extracted from a reference signal should not be significantly changed by defects or noise. This is important in auto-monitoring application and data measurement.
- **Speed**: The numerical calculation of the expansion coefficients in the new system should be of order \( O(N) \) or \( O(N \log(N)) \). This is important for real-time application.

Significant improvement in the efficiency of any proposed monitoring tool can be achieved by increasing the similarity of the expansion system bases and different classes of gear tooth failures. The proposed technique uses a window function \( w(t) \) in order to enhance the resemblance of the selected
mother wavelet and processed signal. Hence, strengthen the important characteristics mentioned before. In this work, Kaiser’s window of length \( N \) is selected in the windowing process which is presented as:

\[
w[n] = \frac{I_0(\beta (1-(n-u)^2/u^2)^0.5)}{I_0(\beta (0,\alpha))}
\]

where \( I_0(x) \) is the modified Bessel function and \( u \) is the midpoint of the window function. The advantage of selecting Kaiser’s window comes from the ability to adjust the window shape by changing \( \alpha \) parameter [21], [22].

Using Mallat’s algorithm, the set of expansion coefficients resulted from a windowing version of the signal at certain resolution can be expressed as:

\[
wd_j(k) = \sum_{m} h_j(m - 2k) wc_{j+1}(m)
\]

Figure 2 highlights the advantage of using the Kaiser’s window with the wavelet analysis as applied to a test signal of 4.0 kHz sine wave. Figure 2a presents a direct application of the wavelet multi-resolution analysis where the data is processed through a rectangular window. Although large part of the test signal energy, as expected, is localized at the 4\( \text{th} \) resolution, a large number of coefficients are localized at adjacent resolutions and some of which have considerable large magnitudes. Simulation shows that maximum coefficient magnitude is close to 10 at the 3\( \text{rd} \) resolution and close to 20 at the 5\( \text{th} \) resolution. However, the proposed technique localizes all the coefficients in their true resolution level, as clearly shown in figure 2b, hence satisfies the sparsity, supper-resolution and separation requirements of the proposed coefficients-dependent monitoring tool.

![Wavelet expansion coefficients localized at 8 resolutions of a virtual reference signal](image)

**Figure 2.** Wavelet expansion coefficients localized at 8 resolutions of a virtual reference signal, (a) using rectangular window, (b) using Kaiser’s window

Figure 2b shows the wavelet expansion coefficients of local maxima extracted using Kaiser’s window \( wd_j(k) \). At the super-resolution, where the test signal component is reside, the number of coefficients is reduced to a very small set and the magnitude of the maximum coefficient is almost the same as compared with the one shown in figure 2a at the same resolution level. Furthermore, for denoising purposes a threshold value can be easily selected, if required, to eliminate the noise related
coefficients. Also, the difficulty of selecting the right mother wavelet is reduced due to utilizing Kaiser's window.

Figure 3 compares the results of monitoring the absolute value of the maximum coefficients extracted directly and with that use of the Kaiser's window. The comparison is applied on a simulated noise free non-stationary signal and a similar signal corrupted with high noise levels, as shown in figure 3. Direct application of the WMRA produces coefficients \( |d_5^{\text{Max}}| \) that can detect some variations in the signal magnitude; however they are not able to trace these variations accurately and localize them on time. On the other hand, the proposed technique shows a stable detailed coefficients \( |wd_5^{\text{Mag}}| \) that can detect accurately these variations and localize them on time. Hence reflect their magnitudes even with the presence of high noise levels.

![Figure 3](image-url)

**Figure 3.** The variation in the maximum coefficients at the 5th resolution of a signal using rectangular window and Kaiser's window, a- noise-free signal and b- signal corrupted with high noise level.

4. Gearbox Condition Monitoring Technique

Any defect in a gearbox can be considered as a linear combination of time variant signals with different characteristics. The coefficients-based monitoring technique should provide a small set of expansion coefficients that clearly detect and measure the severity of the defect.

As data slides into Kaiser's window, the magnitudes of the maximum coefficients, \( \text{Max}(wd_j)_{\text{Mag}} \), detected at different resolution levels represent important features to monitor and localize any deviation in the gear condition. The sampling rate, \( f_{\text{sam}} \), should be selected such that the healthy gear signal component is centered at the reference resolution level. The number of sampling points should be large enough to decompose the signal into resolution levels that cover the expected band where any defect spectra might be localized. The size of Kaiser’s window, \( W_N \), should not exceed the period of one gear tooth mesh and the data-sliding rate, \( N_\varepsilon \), should be kept very small compared to Kaiser’s window size, \( W_N \) to enhance the tool monitoring accuracy.

Figure 4 is a pictorial representation of the proposed technique. When Kaiser’s window is centered at \( w_{\varepsilon 1} \), the data processed represents a defect free signal. The information extracted will be similar to those extracted in the preprocessing stage from the healthy sample. As a new data set slides through Kaiser’s window at \( w_{\varepsilon 2} \), the proposed monitoring technique will generate expansion coefficients at certain resolutions that reflect the tooth failure event. The magnitude of the maximum coefficient, \( \text{Max}(wd_j)_{\text{Mag}} \), will change according to the severity of gear tooth fracture. The new magnitude of the maximum coefficient at each resolution \( \text{Max}(wd_j)_{\text{Mag}} \) can be easily detected and evaluated. These coefficients are localized in time at the center of Kaiser's window, \( w_{\varepsilon} \), since they were extracted from
a windowing version of the data. Then the dynamic behavior of the magnitude variation of the extracted signal and the location of the defected tooth can be estimated using the proposed technique.

![Kaiser's Window](image)

**Figure 4.** A pictorial representation of the proposed technique as data sliding through Kaiser's window.

### 5. Application and Results

A preprocessing stage is used to define the main features extracted from a healthy sample which will be used as a reference to monitor and diagnose any changes in the gear conditions. Figure 5 shows the results of the monitoring process under normal operation conditions, NOC, of a healthy gearbox. The vibration signal collected in laboratory of a healthy gear is shown in figure 5a. The normalized norm of the coefficients ($\| d_j \|_2$) at different resolutions is used to define the distribution of signal’s energy at 8th resolution levels during NOC. This feature vector will be used as a reference to detect any abnormal conditions in the gearbox. The maximum coefficients at the 4th resolution as data slides into Kaiser's window, as shown in figure 5c, are utilized to estimate the magnitude of defect in the gearbox and identify the tooth location. Monitoring abnormal operation conditions of a gearbox is shown in figure 6. The vibration signal collected of an unhealthy gear is shown in figure 6a. Any abnormal condition is detected as a change in the norm of the coefficients localized at different resolution in comparison with that representing NOC as shown in figure 6b. The variation in the magnitude of the maximum coefficients at the 4th resolution, as shown in figure 6c, will be used to estimate the magnitude of the tooth defect and to identify the tooth place.

![Normalized Norm](image)

**Figure 5.** Monitoring normal operation conditions of a healthy gearbox, a- vibration signal of a healthy gear, b- norm of the coefficients at different resolutions and c- maximum coefficients at the 4th resolution as data sliding through Kaiser's window.

![Maximum Coefficients](image)

**Figure 6.** Monitoring abnormal operation conditions of a 50% crack in gear tooth, a- vibration signal of unhealthy gear, b- norm of the coefficients at different resolutions and c- maximum coefficients at the 4th resolution as data sliding through Kaiser's window.
Comparing results extracted under NOC, as shown in figure 5, and that of defected gear, as shown in figure 6, one can notice a change in the magnitude of the normalized coefficients’ norm by 100% at the 4th and 5th and 200% at the 3rd resolutions. The magnitude of the coefficients related to gear tooth number 9 are increased from 2.0 under NOC to 6.0 at defected tooth location while other teeth show no change in their magnitudes.

The proposed technique is applied to monitor other sets of real gear vibration signals. Four data sets that represent healthy gear and three cracked tooth gears of 40%, 50%, and 70% crack sizes are used to evaluate the efficiency of the monitoring tool. Accurate results were reached that reflect the gear health condition, as shown in figure 7.

**Figure 7.** Monitoring and tracing fracture intensity increase in a gear tooth.

6. Conclusion
The proposed technique resolves the problem of selecting a suitable mother wavelet and defines a threshold for monitoring and evaluating gear tooth fracture. Processing vibration signals through Kaiser’s window increases the similarity between the signal under processing and the selected mother wavelet. The norm of the wavelet expansion coefficients can be used to detect any abnormal conditions in the gear teeth. The maximum coefficients localized as processing data through Kaiser’s window show clear features that can be used to measure and localize any damage in the gear tooth. The proposed tool is very sensitive to any variation in the signal and can be used in detecting early signs of any abnormal condition in the gearbox. Research is continuing to investigate the affect of the gearbox structure and gear types on the developed technique.

References
[1] Dalpiaz G Early 1990 Detection of Fatigue Cracks in Gears by Vibration Analysis Technique, Österreichische Ingenieur-and Architekten-Zeitschrift (OIAZ) Vol 135 (jg Heft).
[2] Ismail F, Martin H R and Omar F 1995 Statistical index for monitoring tooth cracks in a gearbox 15th Biennial Conference on Mechanical Vibration and Noise, ASME Design Engineering Division (Pub.) DE Vol 84 No 3 Pt A/2 (Boston, MA, USA) pp 1413-1418.
[3] Andrede F A, Isat I and Badi M N M 2001 A new approach to time-domain vibration condition monitoring: gear tooth fatigue crack detection and identification by the Kolmogorov–
[4] Andrede F A, Isat I and Badi M N M 2002 Gear condition monitoring by a new application of the Kolmogorov–Smirnov test Proceeding of the Institute of Mechanical Engineers–Part C Vol 215 pp 793-800.

[5] Kar C. and Mohanty A R 2006 Multistage gearbox condition monitoring using motor current signature analysis and Kolmogorov–Smirnov test Journal of Sound and Vibration Vol 290 pp 337-368.

[6] Mathew J 1989 Monitoring the Vibration of Rotating Machine Elements – an overview Diagnostics, Vehicle Dynamics and Special Topics ASME DeVol 18-5 pp 15-22.

[7] Randall R B 1981 Cepstrum Analysis Brue! Kjaer Technical Review No 3.

[8] Tang H, Cha J, Wang Y and Zhang C 1991 The Principle od Cepstrum and Its Application in Quantitative Fault Diagnostics of Gears Modal Analysis, Modeling, Diagnostics, and Control – Analytical and Experimental, ASME De-Vol 38 pp 141-144.

[9] Wang W J and McFadden P D 1993 Early Detection of Gear Failure by Vibration Analysis – I. Calculation of Time-Frequency Distribution Mechanical Systems and Signal Processing Vol 7 No 3 pp 193-203.

[10] Wang W J and McFadden P D 1995 Application of orthogonal wavelets to early gear damage detection Mechanical Systems and Signal Processing Vol 9 pp 497-507.

[11] Wang W J and McFadden P D 1996 Application of wavelets to gearbox vibration signals for fault detection Journal of Sound and vibration Vol 192 No 5 pp 927-939.

[12] Luo G Y, Osypiw D and Irle M 2000 Real-Time Condition Monitoring by Significant and Natural Frequencies Analysis of Vibration Signal with Wavelet Filter and Autocorrelation Enhancement Journal of Sound and vibration Vol 236 No 3 pp 413-430.

[13] Dalpiaz G, Rivola A and Rubini R 2000 Effectiveness and Sensitivity of Vibration Processing Techniques for Local Fault Detection in Gears Mechanical Systems and Signal Processing Vol 14 No 3 pp 387-412.

[14] Lin J and Qu L 2000 Feature Extraction Based on Morlet Wavelet and its application to Mechanical Fault Diagnosis Journal of Sound and Vibration Vol 234 No 1 pp 135-148.

[15] Baydar N and Ball A 2003 Detection of Gear Failures Via Vibration and Acoustic Signals using wavelet Transform Mechanical Engineering Signal Processing Vol 17 No 4 pp 787-804.

[16] Saravanan N, Siddabattuni V N S K and Ramachandran K I 2008 A comparative study on classification of features by SVM and PSVM extracted using Morlet wavelet for fault diagnosis of spur bevel gear box Expert Systems with Applications Vol 35 pp 1351–1366.

[17] Rafiee J, Tse P W, Harifi A and Sadeghi M H 2008 A novel technique for selecting mother wavelet function using an intelligent fault diagnosis system Expert Systems with Applications Preprint doi:10.1016/j.eswa.2008.05.052.

[18] Jafarizadeh M A, Hassannejad R, Ettefagh M M and Chitsaz S 2008 Asynchronous input gear damage diagnosis using time averaging and wavelet filtering Mechanical Systems and Signal Processing Vol 22 pp 172–201.

[19] Gaouda A M, Kanoun S H, Salama M M A and Chikhani A Y 2002 Wavelet-Based Signal Processing Techniques for Disturbance Classification and Measurement IEE Proceedings – Generation, Transmission and Distribution Vol 149 No 3 pp 310-318.

[20] Gaouda A M, El-Hag A, Abdel-Galil T K, Salama M M A and Bartnikas R 2008 On-Line Detection and Measurement of Partial Discharge Signals in a Noisy Environment IEEE Transaction on Dielectrics and Electrical Insulation Vol 15 No 4 pp 1162-1173.

[21] Sanjit K, Mitra and James F 1993 Kaiser Handbook for Digital Signal Processing (John Wiley & Sons).

[22] Burrus C S, McClellan J H, Oppenheim A V, Parks T W, Schafer R W and Schuessler H W 1994 Computer-Based Exercises for Signal Processing using Matlab Matlab Curriculum Series (Prentice hall).