We present the first-ever cosmological constraints from a simulation-based inference (SBI) analysis of galaxy clustering from the new SimBIG forward modeling framework. SimBIG leverages the predictive power of high-fidelity simulations and provides an inference framework that can extract cosmological information on small non-linear scales, inaccessible with standard analyses. In this work, we apply SimBIG to the BOSS CMASS galaxy sample and analyze the power spectrum, $P(k)$, to $k_{\text{max}} = 0.5 h$/Mpc. We construct 20,000 simulated galaxy samples using our forward model, which is based on high-resolution $N$-body simulations and includes detailed survey realism for a more complete treatment of observational systematics. We then conduct SBI by training normalizing flows using the simulated samples and infer the posterior distribution of ΛCDM cosmological parameters: $\Omega_m, \Omega_b, h, n_s, \sigma_8$. We derive significant constraints on $\Omega_m$ and $\sigma_8$, which are consistent with previous works. Our constraints on $\sigma_8$ are 27% more precise than standard analyses. This improvement is equivalent to the statistical gain expected from analyzing a galaxy sample that is $\sim 60$% larger than CMASS with standard methods. It results from additional cosmological information on non-linear scales beyond the limit of current analytic models, $k > 0.25 h$/Mpc. While we focus on $P(k)$ in this work for validation and comparison to the literature, SimBIG provides a framework for analyzing galaxy clustering using any summary statistic. We expect further improvements on cosmological constraints from subsequent SimBIG analyses of summary statistics beyond $P(k)$.
than the amplitude of cosmic variance on scales smaller than $k > 0.1 \, h/Mpc$ (19–21). To correct for this effect, the weights of the “collided” galaxies missed by survey are assigned to their nearest angular neighbors (22, 23). Even for current analyses, these correction weights do not sufficiently correct the measured power spectrum (20). Furthermore, they are only designed and demonstrated for the power spectrum.

Meanwhile, additional cosmological information is available on non-linear scales and in higher-order statistics. Recent studies have accurately quantified the information content in these regimes using large suites of simulations. (24) and (25) used the Quijote suite of simulations to demonstrate that constraints on cosmological parameters, $\Omega_m, \Omega_b, h, n_s, \sigma_8$, improve by a factor of $\sim 2$ by including non-linear scales ($0.2 < k < 0.5 \, h/Mpc$) in power spectrum analyses. Even more improvement comes from including higher-order clustering information in the bispectrum. Similar forecasts for other summary statistics, e.g. marked power spectrum (26, 27), reconstructed power spectrum (28), skew spectra (29), wavelet statistics (30), find consistent improvements from including non-linear scales and higher-order clustering. Despite the growing evidence of the significant constraining power available in non-linear scales and higher-order statistics, it cannot be exploited by standard methods with PT.

Robustly exploiting non-linear and non-Gaussian cosmological information requires a framework that can both accurately model non-linear structure formation and account for detailed observational systematics. In this work, we present SimMulation-Based Inference of Galaxies (SimBIG), a framework for analyzing galaxy clustering that achieves these requirements by using a forward modeling approach. Instead of analyzing galaxy clustering using analytic models, a forward model approach uses simulations that model the full details of the observations.

In SimBIG, our forward model is based on cosmological $N$-body simulations that accurately models non-linear structure formation. We also use the halo occupation prescription for connecting the galaxy distribution to the dark matter distribution. Our forward model also takes advantage of the fact that many observational systematics can be more easily included in simulations (e.g. 31, 32) than corrected in the observations. With this forward model, we can rigorously analyze galaxy clustering on non-linear scales and with higher-order statistics.

To infer the cosmological parameters, our approach does not require sampling the posterior using an assumed analytic likelihood. We instead use simulation-based inference (SBI; see 33, for a review). SBI, also known as “likelihood-free inference”, enables accurate Bayesian inference using forward models (e.g. 34–37). Moreover, they leverage neural density estimation from machine learning (e.g. 34, 38) to more efficiently infer the posterior without sampling or making strong assumptions on the functional form of the likelihood.

In this work, we apply SimBIG to the CMASS galaxy sample observed by the Sloan Digital Sky Survey SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS; 39, 40). With the main goal of demonstrating the accuracy and potential of SimBIG, we use the power spectrum as our summary statistic. We present the cosmological constraints inferred from our analysis and compare them to previous constraints in the literature. In an accompanying paper (41, hereafter H22a), we present our forward model in further detail and the mock challenge that we conduct to rigorously validate the accuracy of SimBIG cosmological constraints.

Simulation-Based Inference of Galaxies SimBIG

Modern cosmological analyses use Bayesian inference to constrain the posterior distribution $p(\theta | x)$ of cosmological parameters, $\theta$, given observation $x$. In standard galaxy clustering analyses, the posterior is evaluated using Bayes’ rule. The likelihood is assumed to have a Gaussian functional form and evaluated using an analytic PT model.

SBI offers an alternative that requires no assumptions on the form of the likelihood. SBI only requires a forward model, i.e. a simulation to generate mock observations $x'$ given parameters $\theta'$. It uses a training dataset of simulated pairs $(\theta', x')$ to estimate the posterior. SBI has already been successfully applied to a wide range of inference problems in astronomy and cosmology (35, 36, 42–47).

In this work, we utilize SBI based on neural density estimation, where a neural network $q$ with parameters $\phi$ is trained to estimate $p(\theta | x) \approx q_\phi(\theta | x)$. In particular, we use “normalizing flow” models that are capable of accurately estimating complex distributions (48, 49). Below, we briefly describe our forward model and SBI framework.

A. Forward Model. SBI requires a forward model that is capable of generating mock observations which are statistically indistinguishable from the observations. We start with high-resolution $N$-body simulations from the Quijote suite (50). These simulations follow the evolution of $1024^3$ cold dark matter (CDM) particles in a volume of $(1 \, h^{-1} \text{Gpc})^3$ from $z = 127$ to $z = 0.5$ using the TreePM GADGET-III code. They accurately model the clustering of matter down to non-linear scales beyond $k = 0.5 \, h/Mpc$ (50).

To model the galaxy distribution, we identify gravitationally bound dark matter halos and populate them with galaxies using a flexible halo occupation framework. We identify halos using the ROCKSTAR phase-space-based halo finder (51), which accurately determines the location of halos and resolves their substructure (52). We then populate the halos using Halo Occupation Distribution (HOD) models that provide a statistical prescription for populating halos with galaxies based on halo properties such as their mass and concentration. In this work, we use a state-of-the-art HOD model that supplements the standard (53) model with assembly, concentration, and velocity biases. The extra features of our HOD model add additional flexibility that recent works suggest may be necessary to describe galaxy clustering (e.g. 54–56).

Once we have our galaxy distribution in the simulation box, we apply survey realism. We remap the box to a cuboid (57) and then cut out the detailed survey geometry of the BOSS CMASS SGC sample (see Materials and Methods). This includes masking for bright stars, centerpost, bad field, and collision priority (17, 18, 40). We apply fiber collisions by first identifying all pairs of galaxies within an angular scale of 62° then, for 60% of the pairs, removing one of the galaxies from the sample. Lastly, we trim the forward modeled galaxy catalog to match the 0.45 < z < 0.6 redshift range and angular range of the observations.

In total, our forward model has 14 parameters. 5 $\Lambda$CDM cosmological parameters, $\Omega_m, \Omega_b, h, n_s, \sigma_8$, that determine the matter distribution and 9 HOD parameters that determine the
Fig. 1. The SimBIG forward model produces simulated galaxy samples with the same survey geometry and observational systematics as the observed BOSS CMASS SGC galaxy sample. We present the 3D distribution of the galaxies from three different viewing angles. The colormap represents the redshift of the galaxies. In the top set of panels, we present the distribution of galaxies in the CMASS sample. In the bottom, we present the distribution of a simulated galaxy sample, generated from our forward model. The SimBIG galaxy samples are constructed from Quijote N-body dark matter simulations using an HOD model that populates dark matter halos identified using the Rockstar algorithm. The 3D distributions illustrate that our forward model is able to generate galaxy distributions that are difficult to statistically distinguish from observations. For more comparisons of the 3D distributions, we refer readers to [1].
Using simulations, however, this is not the case due to limitations on modeling observations at all scales. Even with galaxy catalogs if the forward model is capable of accurately encompassing the form priors on the cosmological parameters that conservatively impose uni-

SimBIG Forward Model

Fig. 2. Angular distribution of galaxies from the CMASS sample (top) and a galaxy sample generated using the SimBIG forward model (bottom). Comparison of the angular distributions highlights the detailed CMASS angular selection that we include in our forward model to account for observational systematics.

connection between galaxies and halos. For further details on our forward model, we refer readers to H22b. In the bottom panels of Fig. 1, we present the three-dimensional spatial distribution of galaxies in our forward model. We present the angular distribution of galaxies in our forward model in Fig. 2. The forward model accurately reproduces the survey geometry and angular footprint of the observed BOSS sample. For additional comparisons of the 3D distributions of galaxies in CMASS and our forward model, we refer readers to.

B. Training Dataset for Simulation-Based Inference. Using our forward model, we construct 20,000 simulated galaxy catalogs. They are constructed from 2,000 QUJOTE $N$-body simulations with 10 different sets of HOD parameters, sampled from a broad prior. The $N$-body simulations are arranged in a Latin Hypercube configuration, which therefore imposes uniform priors on the cosmological parameters that conservatively encompasses the Planck cosmological constraints (58).

In principle, SimBIG can be directly applied to the full galaxy catalog if the forward model is capable of accurately modeling observations at all scales. Even with $N$-body simulations, however, this is not the case due to limitations on mass and time resolution and inadequacies of halo occupation models. Instead, we use summary statistics of the galaxy sample, where we can impose cuts, e.g., based on physical scales, to which our forward model is accurate. Since the primary goal of this work is to present and demonstrate the SimBIG framework, we use the most commonly used summary statistic: the galaxy power spectrum multipole, $P_l(k)$. We also include the average galaxy number density of the sample, $\bar{n}_g$.

In this work, we use the redshift-space galaxy power spectrum monopole, quadrupole, and hexadecapole $(l = 0, 2, \text{ and } 4)$. We measure $P_0$, $P_2$, and $P_4$ for each of the simulated galaxy catalogs using the (59) algorithm. In this work, we impose a conservative $k < k_{\text{max}} = 0.5 h/$Mpc limit on the $P_l$, based on the convergence of matter clustering of the QUJOTE simulations (see H22b for further details). We also measure the power spectrum for the BOSS CMASS-SGC galaxy sample with the same algorithm. For the observed $P_l(k)$ we include systematics weights for redshift failures, stellar density, and seeing conditions, which are effects not included in our forward model but shown to be successfully accounted for using the weights (17, 23).

By using $P_l$, we can compare the constraints inferred using SimBIG with previous constraints (e.g. 10, 11) as further validation of SimBIG. To be further consistent with previous analyses, we include a nuisance parameter, $A_{\text{shot}}$, that is typically included to account for residual shot noise contribution (e.g. 9–11). In Fig. 3, we present $P_2(k)$ of our forward modeled galaxy catalogs. We randomly select 100 out of the total 20,000 power spectra for clarity. The left, center, and right panels present the monopole, quadrupole, and hexadecapole. These $P_l$ measurements and $\bar{n}_g$ serve as the training dataset $(\{\theta', x\})$ for our SBI posterior estimation using normalizing flows.

C. Simulation-Based Inference with Normalizing Flows. Normalizing flow models use an invertible bijective transformation, $f : z \rightarrow \theta$, to map a complex target distribution to a simple base distribution, $\pi(z)$, that is fast to evaluate. For SBI, the target distribution is the posterior, $p(\theta | x)$ while $\pi(z)$ is typically a multivariate Gaussian. The transformation $f$ must be invertible and have a tractable Jacobian so that we can evaluate the target distribution from $\pi(z)$ by change of variables. Since $\pi(z)$ is easy to sample and evaluate, we can also easily sample and evaluate the target distribution. A neural network with parameters $\phi$ is trained to obtain $f$.

Among the various normalizing flow-based neural density estimators now available in the literature, we use a Masked Autoregressive Flow (MAF; 34). MAF combines normalizing flows with an autoregressive design (60), which is well-suited for estimating conditional probability distributions such as a posterior. A MAF model is built by stacking multiple Masked Autoregressive Encoder for Distribution Estimation (MADE; 38) models so that it has the autoregressive structure of MADE models but with additional flexibility to describe complex probability distributions. We use the MAF implementation of the abif Python package (61, 62).

In training, our goal is to determine the parameters, $\phi$, of our normalizing flow, $q_\phi(\theta | x)$, so that it accurately estimates the posterior, $p(\theta | x)$. We can formulate this into an optimization problem of minimizing the Kullback-Leibler (KL) divergence between $p(\theta, x) = p(\theta | x)p(x)$ and $q_\phi(\theta | x)p(x)$,
We present the posterior distribution of the which measures the difference between the two distributions.

\[
\min_{\phi} D_{\text{KL}}(p(\theta, x) \mid\mid q_\phi(\theta \mid x)p(x)) = \min_{\phi} \int p(\theta, x) \log \frac{p(\theta, x)}{q_\phi(\theta \mid x)p(x)} \, d\theta \, dx
\]

\approx \min_{\phi} \sum_i \log p(\theta_i \mid x_i) - \log q_\phi(\theta_i \mid x_i)

\approx \max_{\phi} \sum_i \log q_\phi(\theta_i \mid x_i).

\]

Eq. 2 follows from the fact that the training dataset \(\{(\theta', x')\}\) is constructed by sampling from \(p(\theta, x)\) with our forward model.

We split the training data into a training and validation set with a 90/10 split, then maximize Eq. 4 over the training set. We use the ADAM optimizer (63) with a learning rate of 5 \times 10^{-4}. We prevent overfitting by stopping the training when the log-likelihood (Eq. 4) evaluated on the validation set fails to increase after 20 epochs. We determine the architecture of our normalizing flow through experimentation. Our final trained model has 6 MADE blocks, each with 9 hidden layers and 186 hidden units. For further details on the training procedure, we refer readers to H22b. Once trained, we estimate the posterior of our 5 cosmological, 9 HOD parameters, and 1 nuisance parameter, for the BOSS CMASS SGC \(P_t\) and \(\hat{n}_b\), by sampling our normalizing flow \(q_\phi(\theta \mid x)\).

Fig. 3. Power spectrum, \(P_t(k)\), measured from the simulated galaxy catalogs constructed using the SimBIG forward model. We present \(P_t(k)\) of 100 out of the total 20,000 catalogs for clarity. In each of the panels, we plot the monopole, quadrupole, and hexadecapole of the power spectrum (\(\ell = 0, 2, 4\)). For reference, we include \(P_t(k)\) measured from the BOSS CMASS SGC galaxy sample (black) with uncertainties estimated from H22b simulations. \(P_t\) is the most commonly used summary statistic of galaxy distribution that measures the two-point clustering. We use \(P_t\) in this work to showcase and validate the SimBIG framework and make detailed comparisons to previous works in the literature. The \(P_t\) of the SimBIG catalogs encompass the BOSS \(P_t\) and, thus, provide a sufficiently broad dataset to conduct SBI.

1. Results

We present the posterior distribution of the ΛCDM cosmological parameters, \(\Omega_m, \Omega_b, h, n_s, \sigma_8\), inferred from \(P_t(k)\) using SimBIG in Fig. 4. The posterior is inferred from the BOSS \(P_t(k)\) down to \(k_{\text{max}} = 0.5\,h/\text{Mpc}\). The diagonal panels present the marginalized one-dimensional posteriors for each parameter. The other panels present marginalized two-dimensional posteriors of different parameter pairs that highlight parameter degeneracies. We mark the 68 and 95 percentiles of the posteriors with the contours. We infer the posterior of HOD and nuisance parameters; however, we do not include them in the figure for clarity. Among the cosmological parameters, the SimBIG posterior significantly constrains \(\Omega_m\) and \(\sigma_8\). This is consistent with previous works that relied on priors from Big Bang nucleosynthesis or cosmic microwave background (CMB) experiments for the other parameters, \(\Omega_b\) and \(n_s\) (e.g. 10, 11). We infer \(\Omega_m = 0.292^{+0.055}_{-0.046}\) and \(\sigma_8 = 0.812^{+0.067}_{-0.063}\).

In the accompanying paper H22b, we present the validation of the SimBIG posterior using a suite of 1,500 test simulations. We construct the test suite using different forward models than the one used for our training data. They are constructed using different N-body simulations, halo finders, and HOD models. This is to ensure that the cosmological constraints we derive are independent of the choices and assumptions made in our forward model.

For validation, we conduct a mock challenge where we infer posteriors of the cosmological parameters for each of the test simulations. Since we know the true cosmological parameter values of the test simulations, we can access the accuracy and precision of the inferred posteriors. H22b reveals that SimBIG produces unbiased posteriors. On the other hand, the posteriors are conservative, i.e. they are broader than the true posterior. This is due to the limited number of N-body simulations used to construct our training dataset, which makes the estimate of the KL divergence (Eq. 2) noisy. Our constraint on \(\Omega_m\) is particularly conservative. Additional N-body simulations would significantly improve the precision of our posteriors.

Despite the fact that they are conservative, the \(\sigma_8\) posterior from SimBIG is significantly more precise than constraints from previous works. (10) analyzed the \(P_t\) of the BOSS CMASS galaxy sample using the PT approach with an analytic model based on effective field theory. For the CMASS SGC sample, with uniform priors on the cosmological parameters, and with \(k_{\text{max}} = 0.25\,h/\text{Mpc},\) (10) inferred \(\sigma_8 = 0.719^{+0.100}_{-0.065}\). With SimBIG, we improve \(\sigma_8\) constraints by 27% over the standard galaxy clustering analysis. We emphasize that this improvement is roughly equivalent to analyzing a galaxy survey ~60% larger than the original survey using PT.

Recently, (11) also analyzed the \(P_t\) of BOSS CMASS sample
Fig. 4. Left: Posterior of cosmological parameters inferred from $P_{\ell}$ using SimBIG. In the diagonal panels we present the marginalized 1D posterior of each parameter. The other panels present the 2D posteriors that illustrate the degeneracies between two parameters. The contours mark the 68 and 95 percentiles. We accurately analyze $P_{\ell}$ down to non-linear regimes, $k_{\text{max}} = 0.5 \, h/\text{Mpc}$, by using a simulation-based forward model that includes observational systematics. Right: We focus on the posteriors of $(\Omega_{m}, \sigma_{8})$, the parameters that can be most significantly constrained by galaxy clustering alone. We derive $\Omega_{m} = 0.292^{+0.055}_{-0.040}$ and $\sigma_{8} = 0.812^{+0.067}_{-0.068}$. Our $\sigma_{8}$ constraints are 27% tighter than the (10) $k_{\text{max}} = 0.25 \, h/\text{Mpc}$ PT constraint (orange).
but using a theoretical model based on a halo power spectrum emulator. Instead of using a galaxy bias scheme used by PT to connect the galaxy and matter distributions, (11) used halo power spectra predicted by an emulator and a halo occupation framework, similar to the HOD model in our forward model. We note that while the halo power spectrum emulator is trained using simulations, the approach in (11) does not forward model observational systematics. They also make the same assumptions on the form of the likelihood as PT analyses for their inference. For the CMASS SGC sample, with uniform priors on all cosmological parameters, and with $k_{\text{max}} = 0.25 h/$Mpc, (11) inferred $\sigma_8 = 0.790^{+0.083}_{-0.072}$. The (11) constraints are tighter than the (10) PT constraints because the halo occupation model provides a more compact framework for modeling galaxies. Nevertheless, with SimBIG, we improve on their $\sigma_8$ constraints by 13%.

SimBIG produces significantly tighter constraints on $\sigma_8$ because we are able to accurately extract cosmological information available on small, non-linear, scales. With our forward modeling approach, we can accurately model non-linear clustering and robustly account for observational systematics down to $k_{\text{max}} = 0.5 h/$Mpc. In both (10) and (11), they restrict their analysis to $k_{\text{max}} < 0.25 h/$Mpc due to the limitations of their analyses on smaller scales.

To further verify that our improvement comes from constraining power at $k > 0.25 h/$Mpc, we analyze $P_s$ to $k_{\text{max}} = 0.25 h/$Mpc using SimBIG. In Fig. 5, we present the SimBIG $k_{\text{max}} = 0.25 h/$Mpc posterior (blue) along with the posteriors from (10, orange) and (11, green). We focus our comparison on $\Omega_m$ and $\sigma_8$, the cosmological parameters that can be most competitively constrained with galaxy clustering alone. The contours again represent the 68 and 95 percentiles. We find overall good agreement among the posteriors. All of the posteriors are statistically consistent with each other. For $\sigma_8$, our $k_{\text{max}} = 0.25 h/$Mpc places a $\sigma_8 = 0.861_{-0.091}^{+0.079}$ constraint. This is significantly broader than our $k_{\text{max}} = 0.5 h/$Mpc constraint, which demonstrates that the constraining power is in fact from non-linear scales. Furthermore, the precision of the $k_{\text{max}} = 0.25 h/$Mpc SimBIG constraint is in excellent agreement with the (11) constraint. This serves as further validation of SimBIG, since (11) uses a similar halo occupation framework to model the power spectrum.

For $\Omega_m$, we infer broader posteriors than (10) and (11). As we discuss in H22b, this is due to the fact that the SimBIG normalizing flow is trained using a limited number of simulations. We use 20,000 forward modeled simulations; however, they are constructed from 2000 $N$-body simulations with different values of cosmological parameters. To estimate the expected improvement in the $\Omega_m$ constraints, we use the posterior “recalibration” procedure from (64). The re-calibration uses the posteriors inferred for the test simulations and their true parameter values. We calculate the local probability integral transform (65), a diagnostic of the inferred posteriors, and use this quantity to derive a weighting scheme that corrects the posteriors so that it matches the true posterior of the test simulations.

The re-calibration uses test simulations, so we do not use it for inference. However, it provides a bound for the SimBIG constraints if we were to have sufficient training simulations. The re-calibrated posterior constraints $\Omega_m = 0.284_{-0.021}^{+0.023}$. For reference, we mark the re-calibrated $\Omega_m$ constraint (black dotted) in Fig. 5. The re-calibrated $\Omega_m$ is in good agreement with both the (10) and (11) constraints. It is significantly tighter than the original SimBIG constraint and illustrates that additional training simulations would significantly improve the precision of the SimBIG $\Omega_m$ constraints.

Based on our $k_{\text{max}} = 0.5 h/$Mpc posterior, we infer $S_8 = \sigma_8 \sqrt{\Omega_m/0.3} = 0.802_{-0.092}^{+0.102}$ (and $0.797_{-0.076}^{+0.078}$ for our re-calibrated posterior). Multiple recent large-scale structure analyses have reported a “$S_8$ tension” with constraints from the (58) CMB analysis. They find significantly lower values of $S_8$ than Planck (66–73). PT analyses of BOSS also infer relatively low values of $S_8$. (10), for instance, infers $S_8 = 0.737_{-0.092}^{+0.110}$. This $S_8$ tension between constraints from large-scale structure and CMB analyses has motivated a number of works to explore modifications of the standard $\Lambda$CDM cosmological model (e.g., 74–77). We do not find a significant $S_8$ tension with the Planck constraints (58). However, given the statistical precision of our $S_8$ constraint, we refrain from more detailed comparison and discussion.

2. Conclusions

We present SimBIG, a forward modeling framework for analyzing galaxy clustering using SBI. As a demonstration of the framework, we apply it to the BOSS CMASS SGC, a galaxy sample at $z \sim 0.5$. We analyze the galaxy power spectrum multipoles ($P_l$), the most commonly used summary statistic of the galaxy spatial distribution, to showcase and validate the SimBIG framework.
SimBIG utilizes a full forward model of the CMASS sample, unlike standard approaches that use analytic models of the summary statistic. The forward model is based on high-resolution QUICHE N-body simulations that can accurately model the matter distribution on small scales. It uses halo modeling and a state-of-the-art HOD model with assembly, concentration, and velocity biases that provide a flexible mapping between the matter and galaxy distributions. The forward model also includes realistic observational systematics such as survey geometry and fiber collisions. With this forward modeling approach, we can leverage the predictive power of simulations to analyze small, non-linear, scales as well as higher-order clustering. It also provides a framework to account for systematics for any summary statistic.

Using the forward model, we construct 20,000 simulated CMASS-like samples that span a wide range of cosmological and HOD parameters. We measure $P_t$ and $\sigma_8$ for each of these samples and use the measurements as the training dataset for SBL. To estimate the posterior, we use neural density estimation based on normalizing flows. Using the training dataset, we train our normalizing flows by minimizing the KL divergence between its posterior estimate and the true posterior. Once trained, we apply our normalizing flow to the observed summary statistics to infer the posterior of 5 cosmological, 9 HOD, and 1 nuisance parameter.

Focusing on the cosmological parameters, we derive significant constraints on: $\Omega_m = 0.316^{+0.040}_{-0.036}$ and $\sigma_8 = 0.668^{+0.0324}_{-0.0254}$. Our $\sigma_8$ constraints are 27% tighter than the (10) constraints using a standard PT approach on the same galaxy sample. This improvement is roughly equivalent to increasing the volume of the galaxy survey by $\sim$60% for a standard PT analysis. The SimBIG constraints are inferred from $P_t$ out to $k_{\text{max}} = 0.5 h$/Mpc while the PT constraints are limited to $k_{\text{max}} = 0.25 h$/Mpc. The improvement is driven by the additional cosmological information on non-linear scales that SimBIG can robustly extract.

We also infer the posterior using SimBIG from $P_t$ with $k_{\text{max}} = 0.25 h$/Mpc and compare it to posteriors in the literature. In particular, the SimBIG $\sigma_8$ constraint for $k_{\text{max}} = 0.25 h$/Mpc are in excellent agreement with the constraint from the recent halo model based emulator analysis of (11). Since they use a similar halo occupation framework as SimBIG, this comparison firmly verifies the robustness of our constraints. The comparison also confirms that the improvement in SimBIG $k_{\text{max}} = 0.5 h$/Mpc constraints come from the non-linear regime. In the accompanying H22b, we present additional tests of SimBIG through a mock challenge. The tests use a suite of 1,500 test simulations constructed with different forward models to demonstrate that SimBIG produces unbiased cosmological constraints. H22b also presents further details on our forward model and discusses posterior constraints on HOD parameters.

SimBIG can also extract higher-order cosmological information. Standard galaxy clustering analyses primarily focus on two-point clustering statistics. Analyses of higher-order statistics have been limited to, e.g., the bispectrum (14, 15, 78), and even these analyses extract only limited cosmological information beyond linear scales. In subsequent work, we will use SimBIG to analyze the BOSS CMASS galaxies using higher-order statistics (the bispectrum) and non-standard observables that contain additional cosmological information: e.g. marked power spectrum, skew spectra, void probability functions, and wavelet-scattering-like statistics. We will also apply SimBIG to analyze field-level summary statistics that capture all the information in the galaxy field using convolutional and graph neural networks.

SimBIG can also be extended to upcoming spectroscopic galaxy surveys observed using DESI, PFS, Euclid, and Roman will probe unprecedented cosmic volumes over the next decade. They will produce the largest and most detailed three-dimensional maps of galaxies in the Universe. These surveys are already expected to provide the most precise constraints on cosmological parameters using standard analyses. SimBIG can further exploit the statistical power of these surveys to place even tighter constraints on cosmological parameters and produce the most stringent tests of the standard ΛCDM cosmological model and beyond.

Materials and Methods

**Observations:** SDSS-III BOSS. In this work, we analyze observations from the Sloan Digital Sky Survey SDSS-III (39, 40) Baryon Oscillation Spectroscopic Survey (BOSS) Data Release 12. In particular, we use the CMASS galaxy sample, which selects high stellar mass Luminous Red Galaxies (LRGs) over the redshift 0.43 < z < 0.7 (79). We restrict our analysis to CMASS galaxies in the Southern Galactic Cap (SGC) and impose a redshift cut of 0.45 < z < 0.6 and the following angular cuts: Dec > −6 and −25 < RA < 28. In upper panels in Fig. 1, we present the three-dimensional distributions of our CMASS SGC galaxy sample at three different viewing angles. We also present the angular distribution of the sample in Fig. 2. In total, our CMASS SGC galaxy sample contains 109,636 galaxies.

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