ABSTRACT
We propose a novel method for effective retrieval of multi-agent spatiotemporal tracking data. Retrieval of spatiotemporal tracking data offers several unique challenges compared to conventional text-based retrieval settings. Most notably, the data is fine-grained meaning that the specific location of agents is important in describing behavior. Additionally, the data often contains tracks of multiple agents (e.g., multiple players in a sports game), which generally leads to a permutational alignment problem when performing relevance estimation. Due to the frequent position swap of agents, it is difficult to maintain the correspondence of agents, and such issues make the pairwise comparison problematic for multi-agent spatiotemporal data. To address this issue, we propose a tree-based method to estimate the relevance between multi-agent spatiotemporal tracks. It uses a hierarchical structure to perform multi-agent data alignment and partitioning in a coarse-to-fine fashion. We validate our approach via user studies with domain experts. Our results show that our method boosts performance in retrieving similar sports plays—especially in interactive situations where the user selects a subset of trajectories compared to current state-of-the-art methods.

KEYWORDS
Retrieval and ranking, Multi-Agent Spatiotemporal Data, Data Alignment

ACM Reference Format:
Long Sha, Patrick Lucey, Stephan Zheng, Taehwan Kim, Yisong Yue, and Sridha Sridharan. 2018. Fine-Grained Retrieval of Sports Plays using Tree-Based Alignment of Trajectories. In Proceedings of ACM International Conference on Web Search and Data Mining, Los Angeles, California, USA, Feb 2018 (WSDM’18), 10 pages.
https://doi.org/10.1145/nnnnnn.nnnnnn

Figure 1: In this paper we focus on retrieving fine-grained multi-agent data in basketball using a tree-based alignment method. We focus on two types of retrieval tasks: (a) Given a four-second example play (blue is offensive team, green is defense and red is the ball – the small circle on each trajectory shows the end point) we use that as the input query and retrieve all plays that look similar to that query; and (b) Given the same play, the user selects only the trajectories of interest and the retrieval is based on that chosen subset.

1 INTRODUCTION
Research into “exemplar-based” and “sketched-based” approaches to image retrieval has recently surged [4, 43, 45]. The recent popularity of these fine-grained retrieval methods is due to the inadequacies of current “text-based” or “key-word” query-based methods. As a picture tells a thousand words, using examples or sketches which capture the fine-grained attributes that the user is interested in has shown to be superior to text-based searches.

We study the setting of fine-grained retrieval of multi-agent spatiotemporal data such as sports plays. A depiction of the “exemplar search” problem is shown in Figure 1(a). Given an input play of a specific length (say 4 seconds), the input query is then compared to the entire database of 4 second plays and a ranked list of most similar plays are then retrieved. Additionally, the user selects only the players that they are interested in (Figure 1(b)). Based on the subset of players, the system then retrieves a ranked list of similar plays based only on the selected trajectories. Previous work showed
that users much preferred the exemplar and sketched-based method over conventional keyword-based retrieval system [33]. Most crucially, the retrieval system allowed users to retrieve fine-grained plays in a matter of seconds, instead of days which can often be the case in practice in sports domains.

One major challenge in relevance estimation for multi-agent trajectories is that of alignment. Although low-dimensional compared to using an image-based representation, the inherent problem of using the raw multi-agent data is that of the misalignment due to the constant swapping of player positions over the course of a play (i.e., permutation problem). One approach to circumventing this issue is to utilize a preprocessing step which pre-aligned the multi-agent data to a template which allowed for quick local trajectory comparison.

In this paper, we propose an improved multi-agent data alignment method which gives improved fine-grained retrieval performance. Our proposed method uses a hierarchical structure to perform multi-agent data alignment and partitioning in a coarse-to-fine fashion. Our approach can be easily integrated into existing spatiotemporal retrieval pipelines. We validate our approach using over a wide range of retrieval tasks. Our user study results demonstrate significant benefits of our method over previous relevance estimation methods for sports play retrieval.

The rest of the paper is as follows. In Section 2 we describe the importance of aligning multi-agent data and why using the raw data is preferred. In Section 3, we explain our proposed tree-based multi-data alignment method, and Section 4 describes how this is implemented within a retrieval framework. Section 5 shows our results, and Section 6 gives the relevant related work. We conclude with a summary and discuss future work.\footnote{A demo video of our work can be viewed in https://www.dropbox.com/s/dhbtukok9z06tpk/WSDEM_video.mp4?dl=0}

## 2 MEASURING SIMILARITY VIA THE ALIGNMENT OF RAW MULTI-AGENT DATA

For effective retrieval to take place, we need an accurate and efficient similarity measure between multi-agent inputs. As shown in Figure 2(a), if we look at the raw positional data of a single team in basketball (i.e., 5 players) across a quarter of a match, given an initial ordering we can see that this ordering of the positional data contains little team structure as players tend to constantly switch positions. To counter this issue, we could exhaustively compare the pairwise distance between each player in the input query to a candidate play in the database which is manageable ($5! = 120$). But if we include the other team, the exhaustive approach starts to get prohibitive\footnote{In basketball, there are 5 players per team. To compare the offensive team trajectories (i.e., team with the ball), there are $5! = 120$ permutations or comparisons required. To include the defensive team, we square this - $(5!)^2 = 14,400$. We then add the ball trajectory comparison which yields $14,401$ comparisons. For other team sports such as soccer which have a higher number of players (i.e. 10 on-field players), the number of permutations are higher than the number of atoms in the universe - $(10!)^2$}. A solution to bypass the alignment issue is to use hand-crafted features [20, 35, 38, 40]. Alternatively, a more intuitive approach would be to compare plays as images as it is a visual data source and it has been employed previously for multi-agent data [28, 44, 47].

To employ an image-based approach we can do the following. Say we have a play across a window of time, like four-seconds, and the player and ball information is captured at 25 frames-per-second, we first quantize a court into a series of $1 \times 1$ foot cells (each one being a pixels). For a $94 \times 50$ feet basketball court, this would result in a $94 \times 50 \times 3$ RGB image ($D = 14,100$), with each channel being assigned to each team (offensive=blue, defense=green and ball=red). In addition to being high-dimensional, it is also lossy, meaning if we wanted to reconstruct the original signal this would be problematic as we have thrown away the temporal structure (i.e., we would not know which location each player was at in each frame). To maintain the original temporal structure, we could add another channel which would result is an extremely sparse and even higher-dimensional input signal – $D \approx 10^6$. For a retrieval system, this is highly undesirable as this would require us to store this high-dimensional data in addition to the raw data.

However, this approach is unnecessary when one considers that the original input data to create the image is already super compact and can be described via a dense matrix of spatial positions. For example, given the $x, y$ location of all 10 players and the $x, y$ of the ball, we can represent each frame as a $M = 23$ dimensional vector. Across $F$ frames, the multi-agent behavior can be represented as a $M \times F$ matrix – which in this case would be a dimensionality of $23 \times 100 = 2300$. To utilize the raw data, a solution is to align the raw positional data to a role template. This method was proposed by Lucey et al. [25], which dynamically assigns an unique role to each agent in each frame according to a single template.

Figure 2(b) shows the formation template of role-based alignment and (c) shows the aligned player positions once this method has been applied. The last plot clearly shows that some type of team structure is obtained. In terms of retrieval, this means that once the permutation matrix has been applied - only a single comparison between trajectories needs to be made. Additionally, only the permutation matrix needs to be stored and not a high-dimensional representation like an image.

Even though effective, as can been seen in Figure 2(c), the role-based method is suboptimal as it only picks up the coarse structure of a team. In this figure, we see the thin strips across the court which does not coincide with any meaningful interpretation of the game of basketball. A more meaningful representation would pick up the typical defensive and offensive structures. In the next section, we show how we can do this which yields better retrieval performance.

## 3 TREE-BASED ALIGNMENT

We now describe our main technical contribution. For effective retrieval using raw multi-agent data, accurate alignment is required. At a high-level, this means that we want to find the ordering of players in the input query to the candidate play which minimizes the difference between the two. Technically, this refers to finding the permutation matrix $P$ that minimizes the $L_2$ distance between all the agents in one team

$$\text{argmin}_P \| P(X_{\text{query}}) - X_{\text{play}} \|_2 \tag{1}$$

where $X_{\text{query}}$ is the matrix of the spatial positions of agents in the input query in one team with an initial agent order (i.e., that order
Fine-Grained Retrieval of Sports Plays using Tree-Based Alignment of Trajectories

WSDM’18, Feb 2018, Los Angeles, California, USA

Figure 2: (a) Given an initial ordering with a color corresponding to each player in a team, we show that a player’s position across a quarter of a game is quite random. (b) But if we align or permute the ordering at the frame-level to this template, we can (c) discover the hidden structure of the team. In this plot we show the alignment to a single template, but we will show later than using a tree of templates is more effective.

Figure 3: The reconstruction cost in Equation 4 at each layer. We set the minimum depth to 6 since it shows as the elbow point in this plot.

is fixed across that window of time), $X_{\text{play}}$ is the matrix of spatial positions of agents in order according to a pre-aligned template for a candidate play in the database, and permutation matrix $P()$ indicates the correspondence of agents between $X_{\text{query}}$ and $X_{\text{play}}$.

In terms of pre-aligning the data in the database, a similar approach is used but instead of just applying the permutation matrix to the input query, we find the permutation matrix between every input play in the database and the gold-standard template, $X_{\text{template}}$. To determine the gold-standard template, this can be either hand-crafted by a domain [25] or learnt in a data-driven method using the EM algorithm [2].

The above approach has yielded reasonable performance but it assumes the observed behavior is linear (i.e., single state). In complex scenarios like those that exist in a team sport like basketball, it is a more reasonable assumption that the behaviors are non-linear which consists of many states. As such, it would make intuitive sense that a superior approach is to learn a separate template for each of these states.

As the various game-states are not explicit, we can use hierarchical clustering to discover these latent states to provide better alignment. However, this presents a “chicken-or-the-egg” problem, as we cannot cluster the multi-agent data without it being aligned first. As such, we can apply a coarse-to-fine approach where we first align the data to a coarse template, and then using this initial alignment we can partition the data into finer states which provide templates which allow us to find a better alignment.

A feasible method to do this would be to use a tree approach, which iteratively executes two steps: alignment and data partitioning. The ultimate goal is to find a set of states/templates that can reasonably reconstruct the complex multi-agent behaviors, and align each data point with the corresponding states/template.

### 3.1 Step 1: Alignment

To start with, let us focus on the alignment step. The goal of our multi-agent alignment is to compute a permutation matrix $P()$ for each example which minimizes the distance between this example and the template of this state.

In each state, the template $X_{\text{template}}$ contains a canonical spatial ordering of agents. Our learning method utilizes a general EM approach, which learns the template in an unsupervised way. The template learning process in a given class is shown in Algorithm 1.

Once the template is obtained, the permutation matrix for a given example in this state can be computed:

$$\arg\min_P ||P(X) - X_{\text{template}}||^2$$  \hspace{1cm} (2)
where the clustering operation splits the data into more specific states responsive retrieval can occur. To aid with this balancing act, we so we have enough plays to retrieve but small enough so that similarity, and ii) the number of plays in each cluster high enough number of clusters is small, but still retaining high within-cluster retrieval, which means we need to balance two things: i) the total assignment cost.

As a clustering problem, we need to define the number of clusters in the given example, which is the cost of this agent-agent assignment.

Once the cost matrix \( D \) is computed, the Hungarian algorithm \[19\] is used to find the permutation matrix \( P() \) that minimizes the overall assignment cost.

\[
D(m, n) = ||X_{\text{template}}(m) - X(n)||_2
\]

where \( \mu_k \) represent the mean of the cluster that example \( X_i \) belongs to and \( \mu_{kn} \) indicates the mean of the closest neighbor cluster of example \( X_i \). Equation 5 measures the dissimilarity between neighboring clusters and how tightly the data is grouped within each cluster. When the number of clusters becomes too large, the similarity between neighboring clusters increases and the score \( E \) decreases as well. Thus, we want to maximize \( E \) to have the most discriminative clusters.

For the data partitioning in each node, we attempt \( K \)-means clustering with \( K \) equals to 2 to 10 and for each \( K \) we compute the score \( E \). The \( K \) that provides the maximal \( E \) will be selected to split the data in the current node.

### 3.3 Tree Growth

Since we can both align and cluster the multi-agent data, we can now learn the tree. To clarify the notation, we use the subscript and superscript to indicate the node index and layer index. \( X_{\text{template}}^l \) indicates the data group in the \( n \)th node of the \( l \)th layer, \( C^l_n \) indicates the classes found from that node \( X_{\text{template}}^l \) represents the template in that node and \( P^l_n \) is the permutation matrix computed by using that template. For every node, we first use the Algorithm 1 to align the data that is assigned to this node and then apply the clustering technique to split them into finer states.

It is worth noting that the templates in each node should also be aligned so that the consistency of agents permutation can be preserved. Thus, the new template of node \( n \) at layer \( l \) is aligned to its parent template in the previous layer \( X_{\text{template}}^l \) and \( P^l_n \) to its parent template \( X_{\text{template}}^{l-1} \).

Then the same process repeat for each node in our tree. Algorithm 2 summarizes the learning process of our tree-based method. During the learning process, the clusters \( C \) and templates \( T \) at each layer are stored for aligning process.

There are two stop criterion: 1) a pre-defined maximum number of examples in each leaf node, 2) a pre-defined depth. From Equation 4, we know that the reconstruction cost would reach minimum if we had infinite states \( |C| = \infty \). Thus, we aim to find a minimum
depth that can provide a considerably low cost. We plot the overall cost of Equation 4 at each layer in Figure 3 and set the minimum depth to 6 layers as it shows as the elbow point. A much deeper tree may be built for fast retrieval, but a 6-layer tree can achieve reasonable performance for alignment purpose. Figure 4 shows an example of one leaf node. The top left image shows the centroid of the leaf node and others shows the distribution heat map of ball and each agent.

In terms of applying the tree-based alignment, given an input play, the player permutation is aligned to the global template at the root node first, it then moves to a child node by finding the nearest neighbor and repeats the alignment again. The aligned data in our tree-based method can be expressed as:

$$X_{\text{aligned}} = P_l(...(P_1(P_0(X_{\text{raw}}))))$$  \hspace{1cm} (6)

where $P_l$ represents the permutation matrix at layer $l$. Essentially, such composition of permutation matrices yields the optimal ordering of the multiple agents. Figure 5 shows the location distribution of each aligned agent across a quarter. By contrast to Figure 2(c), our method reveals more meaningful structure at each side of the court.

### 3.4 Alignment Evaluation

Since better alignment should result in a more compressed input feature, we can evaluate our tree-based alignment via clustering and principle component analysis (PCA). To make clustering a fair comparison, instead of using the clusters generated by our approach inherently, K-means clustering is applied to both alignment methods. Given 100,000 frames, they are aligned with the role-based method and our method separately. Then, we apply K-means clustering to inspect the average within-cluster-error (WCE) with different K’s.

**Algorithm 2** Learning process of tree-based alignment

1. **procedure** LEARN TREE($X$)
2. $T = \emptyset, C = \emptyset$
3. **for** each layer $l$ **do**
4.   **for** each node $n$ **do**
5.     learn template $X_{\text{template},n}^l$ using Algorithm 1
6.     align to parent $X_{\text{template},n}^l = P_{ln}^{-1}(X_{\text{template},n}^l)$
7.     align data $X_i = P_{in}(X_i) \forall X_i \in X_n^l$
8.   **end for**
9.   store $[X_{\text{template},1}^l, ..., X_{\text{template},N_l}^l]$ in $T$
10. compute reconstruct loss with Eq. 4
11. **terminate** when stop criterion satisfies
12. **for** each node $n$ **do**
13.     Conduct K-means on $X_n^l$ with different $K$
14.     Select cluster set $C_n$ that maximizes $E$
15.     partition $X_n^l$ to child nodes according to $C_n$
16. **end for**
17. Store $[C_1^l, ..., C_{N_l}^l]$ in $C$
18. **end for**
19. **return** $T, C$
20. **end procedure**

**Figure 6:** The compressibility test results of using our tree-based alignment, role-based alignment and naive identity-based alignment. It shows our method provide the best compressibility.

$$WCE = \frac{1}{|X|} \sum_{C_k} \sum_{X_i \in C_k} ||X_i - \mu_k||_2$$  \hspace{1cm} (7)

where we abuse $C_k$ to indicate the $k$th cluster after K-means clustering, PCA is used to inspect the variance explained by different eigenvectors and we calculate the variance via

$$\text{Variance Explained} = \frac{\lambda_k}{\sum_{i=1}^{D} \lambda_i}$$  \hspace{1cm} (8)

where $\lambda_i$ is the $i$th eigenvalue that indicates the significance of the $i$th eigenvector.

Apart from the role-based alignment and our tree-based alignment, the naive identity-based alignment is also used as a baseline. Identity-based approach only compares the trajectories according to players’ identities which refer to an initial logical ordering (i.e., the player most like the point-guard ordered first, then the shooting-guard to the center - with this ordering fixed). Figure 6 (left) shows the result of our clustering test and on the right, shows the performance using PCA. Both results show that our alignment gives the better compressibility.

## 4 FINELY-GRANULAR RETRIEVAL SYSTEM

The prime motivation of obtaining better alignment of the raw multi-agent data is to achieve better fine-grained retrieval. As depicted back in Figure 1, instead of typing a textual description of the play, users can select an (a) example or modified example (b). The initial idea was first proposed in [33], where they utilized a simple hash-table by only clustering the ball trajectories. Although effective, that approach is not an optimal solution since such hashing ignores the information of players. In this section, we show that we obtain better fine-grain retrieval utilizing our tree-based approach. We call the approach in [33] as the Baseline Method.

The dataset that is used in this work is the SportVU basketball dataset. The dataset is captured by the STATS SportVU system [34], which generates location data for every player and ball at 25Hz, along with detailed logs for actions such as passes, shots, fouls, etc. The dataset is taken from 1300 games from the last two seasons of a professional basketball league. 1200 games are used as our database and queries are extract from the rest 100 games. The tracking data of each game is stored in a separate table and each row contains the information of one player at one frame, which are time, team ID, player ID, action ID and the $(x, y, z)$ location of the player or the
Within the tree, we traverse the path through the tree until a leaf.

We recruited ten people with strong basketball background to participate in our user studies. For each study, every person spent their first 5 minutes reading the instruction, which helped them to understand the images in Figure 10 and how to select relevant plays. After that, half an hour is allocated for each participant to finish all eight questions. Such procedure was repeated for each retrieval setting.

Each question contains one input query and interleaved retrieved results from two systems. If one result was returned by both systems, it would only be displayed once in our survey. Participants were asked to scan the results top-down and check the plays that they think are relevant to the input query.

Figure 9 displays the top-5 results for the baseline method and our tree-based method given the input query. This examples qualitatively highlights the benefit of our approach as it shows that the baseline method can not find the corresponding players correctly due to the imperfect alignment while our method maintains a high consistency between results and the query. In the next section, we show the results of an user study which quantitatively shows that the tree-based method yields better interactive fine-grain retrieval.

5 USER EXPERIMENTS

5.1 Experiment Design

To show the benefit of our tree-based alignment method to the previous method described in [33] - which we call the Baseline Method - we conducted a series of user studies which focused on the task of interactive fine-grain retrieval (i.e., where the user selects a subset of players within an example play). To enable a fair comparison, whilst also maintaining responsive retrieval times (< 1sec), we set the maximum size of plays within a leaf node to 2000, which generates a deeper tree with 314 individual leaf nodes/hashe entries. Both the baseline method and the tree-based method utilized 314 clusters.

Eight retrieval tasks were selected for our user study and we tested three different settings on each tasks:

1. Retrieval conditioned on all the players and the ball
2. Retrieval conditioned on the offense team and the ball
3. Retrieval conditioned on two selected players and the ball

Because the baseline method requires the ball trajectory for its hashing function, ball is included in each setting. An user study is conducted for each setting and Figure 10 shows the eight queries for the third setting.

We evaluated the retrieval quality via an interleaved evaluation, where the top 10 results returned by the baseline method and our method were combined via the Team-Draft Interleaving method [5] into a single ranking (see Figure 11). The combined ranking and its query were then displayed in an online survey form so that users could view the results top-down and select the relevant plays.

5.2 Procedure

We recruited ten people with strong basketball background to participate in our user studies. For each study, every person spent their first 5 minutes reading the instruction, which helped them to understand the images in Figure 10 and how to select relevant plays. After that, half an hour is allocated for each participant to finish all eight questions. Such procedure was repeated for each retrieval setting.

Each question contains one input query and interleaved retrieved results from two systems. If one result was returned by both systems, it would only be displayed once in our survey. Participants were asked to scan the results top-down and check the plays that they think are relevant to the input query.

5.3 Benchmark Results

Using the relevance feedbacks from the participants, we performed a benchmark comparison by using the average precision and the expected reciprocal rank of the first result, which are two standard retrieval evaluation metrics [6, 26, 30]. Let \( r_j \) denote the rank of the \( j \)-th relevant document, then the average precision of a ranking list can be computed by:

\[
\text{AvgPrec} = \frac{1}{\# \text{rel}} \sum_j \text{Prec} @ r_j
\]
The expected reciprocal rank is more sensitive to the efficiency of finding the first result while average precision is more recall-focused. For our user study, we computed both of them on the two ranking lists embedded in the interleaved ranking, and over the pooling of both top-10 results.

Table 1 shows the result of average precision in three different retrieval settings. In each setting, the top two rows show the mean average precision aggregated across all ten users for each method. It shows that our method has higher precision than the baseline method in all three settings, and the improvement of our method becomes larger when the queries become more specific (fewer agents are involved). The "Win/Loss" rows at bottom indicate how many individual participants have higher average precision with our method. It shows our tree-based approach wins in most users.

Table 2 compares expected reciprocal rank, which has the same structure to Table 1. Please note that since both methods may have the first relevant result at the same rank, a draw could occur sometimes. Thus, each cell in the "Win/Loss" rows may not sum to 10. Similarly, our method outperforms the baseline method especially in the third setting (S3). It shows that using one template cannot find the corresponding agents correctly when queries become very specific. The bar charts in Figure 12 highlight the overall "Win/Loss" in Table 1 and 2.
6 RELATED WORK

Information retrieval has a long research history in computer science domain [10, 31] and the majority of those previous studies focus on tokenized query format. Although tokenized query has been widely used to both text data and multimedia data [18, 42, 48], some research has shown that using free-form or “ad-hoc” queries can be significantly more user-friendly [26, 30]. One popular free-form query type in recent research is the exemplar-based/sketch-based query format [4, 43, 45] in image retrieval. Such query format enables users to issue the query at a more intuitive and precise level.

In terms of sports analytics domain, most works focus on evaluating and comparing players performance [8, 12], analyzing broadcasting videos [23] and discovering behavior patterns/styles [28, 39, 44]. Similar to other domains, the conventional approach of sports data retrieval still uses directory/taxonomy paradigm [37] to categorize sports plays [1, 7, 27]. Since multi-agent spatiotemporal data has been widely collected, using the sketch-based or exemplar-based query can be a more effective and user-friendly solution. The first formal spatiotemporal query paradigm is purposed by Sha et al. [33]. They developed a new interface that accepts exemplar-based or sketch-based queries for sports play retrieval. It was reported that their query format is much more effective and user-friendly than the conventional text-based retrieval.

From the technical perspective, the primary challenge of retrieving multi-agent spatiotemporal data is how to compare them effectively. Although similarity measure has been well investigated on trajectories and time series [9, 11, 36], most of them only focused on single trajectories rather than multi-agent ones. The seminal work of comparing multi-agent data called “role-based” representation [25, 41]. It uses a formation template to order the agents so that corresponding agents can be found between two samples. However, this method is suboptimal because using only one template is agnostic to those fine-grained behaviors.

Apart from the permutation alignment of multi-agent data, we still need to find the an effective similarity measure between individual pairs of trajectories. There are two main categories in trajectory comparison studies, one of them focuses on elastic measure that addresses shifting and warping issues in both time and space domains [9, 16, 17, 21], while the other group of research focuses on finding the most similar or dissimilar points between two trajectories, which ensures the robustness [15, 24]. Euclidean distance is used in our work because the experimental result in [33] shows that Euclidean distance is still the most effective metric for trajectory comparison in sports.

In all modern retrieval system, fast indexing is required for fast search through a large database. Hash table is one of the most popular approach to achieve this purpose [3, 22, 46]. Hash function is normally designed for specific domain and application, but in general, it aims to reduce the time cost. Similar to [33], our method uses the concept of locality sensitive hashing (LSH) [13, 14], which is designed to place similar samples into a same address. Such method has been applied in other settings where similarity measure or ranking is required [22, 32].
system tailored towards accurate and efficient sports play retrieval. A compressibility experiment showed that our tree-based method outperforms the state-of-the-art alignment method, and our full-stack retrieval system demonstrates its effectiveness in an user study where our approach achieves a higher precision than the baseline method. From a relevance estimation standpoint, even though our alignment has improved the similarity measure, the choice of distance metric can be improved. For instance, the concept of "inverse document frequency" cannot in information retrieval indicates that tokens that appear more frequent in documents are not as indicative of relevance as those more rare words. Thus, similar idea can be incorporated into trajectory distance measure. More generally, machine learning techniques can be applied to learn a better distance metric with appropriate training data. Apart from retrieval, our tree-based alignment could be applied to other important data mining tasks. Since a playbook is learned inherently by our tree, it can be used in game summarization and team/player characterization where those game states are required. Beyond sports, our method could be applied to a wide range of domains where multi-agent spatiotemporal data is involved. One example could be the crowd behavior analysis in surveillance domain.

REFERENCES

[1] Alina Bialkowski, Patrick Lucey, Peter Carr, Yisong Yue, and Iain Matthews. 2014. Win at home and draw away: Automatic formation analysis highlighting the differences in home and away team behaviors. In Proceedings of 8th Annual MIT Sloan Sports Analytics Conference.

[2] Alina Bialkowski, Patrick Lucey, Peter Carr, Yisong Yue, Sridha Sridharan, and Iain Matthews. 2014. Large-scale analysis of soccer matches using spatio-temporal tracking data. In Data Mining (ICDM), 2014 IEEE International Conference on. IEEE, 725–730.

[3] Roi Blanco, Giuseppe Ottaviano, and Edgar Meij. 2015. Fast and space-efficient entity linking for queries. In Proceedings of the Eighth ACIM International Conference on Web Search and Data Mining. ACM, 179–188.

[4] Tu Bui and John Collomosse. 2015. Scalable sketch-based image retrieval using color gradient features. In Proceedings of the IEEE International Conference on Computer Vision Workshops. 1–8.

[5] Oliver Chapelle, Thorsten Joachims, Filip Radlinski, and Yisong Yue. 2012. Large-scale validation and analysis of interleaved search evaluation. ACM Transactions on Information Systems (TOIS) 30, 1 (2012), 6.

[6] Oliver Chapelle, Donald Metlzer, Ya Zhang, and Pierre Grinspan. 2009. Expected reciprocal rank for graded relevance. In Proceedings of the thirtieth annual ACM symposium on Theory of computing. ACM, 621–630.

[7] Sheng Chen, Zongyuan Feng, Qunjia Lu, Behroz Mahasseni, Trevor Fire, Alan Fern, and Sinaia Tokevorder. 2014. Play type recognition in real-world football video. In Applications of Computer Vision (WACV), 2014 IEEE Winter Conference on. IEEE, 652–659.

[8] Shuo Chen and Thorsten Joachims. 2016. Modeling intransitivity in matchup and comparison data. In Proceedings of the ninth acm international conference on web search and data mining (WSDM). ACM, 227–236.

[9] Yueguo Chen, Mario A Nascimento, Beng Chin Ooi, and Anthony KH Tung. 2007. Spade: On shape-based pattern detection in streaming time series. In Data Engineering, 2007. ICDE 2007. IEEE 23rd International Conference on. IEEE, 786–795.

[10] Gobinda G Chowdhury. 2010.

[11] Philipp Eichmann and Emanuel Zgraggen. 2015. Evaluating subjective accuracy of relevance as those more rare words. Thus, similar idea can be incorporated into trajectory distance measure. More generally, machine learning techniques can be applied to learn a better distance metric with appropriate training data. Apart from retrieval, our tree-based alignment could be applied to other important data mining tasks. Since a playbook is learned inherently by our tree, it can be used in game summarization and team/player characterization where those game states are required. Beyond sports, our method could be applied to a wide range of domains where multi-agent spatiotemporal data is involved. One example could be the crowd behavior analysis in surveillance domain.

[12] Jennifer Listgarten, Radford M Neal, Sam T Rowres, and Andrew Emili. 2005. Multiple alignment of continuous time series. In Advances in neural information processing systems. 817–824.

[13] Huwenn Liu, Jiajie Xu, Kai Zheng, Chengfei Liu, Lan Du, and Xian Wu. 2017. Semantic-aware query processing for activity trajectories. In Proceedings of the Tenth ACM International Conference on Web Search and Data Mining. ACM, 283–292.

[14] Tie-Yan Liu, Wei-Ying Ma, and Hong-Jiang Zhang. 2005. Effective feature extraction for playback of american football video. Multimedia Modeling, 2005. MMM 2005. Proceedings of the 11th international. IEEE, 164–171.

[15] Yi Yang Lu, Qiufeng Liu, Tieniu Tan, and Weiming Hu. 2002. Semantic interpretation of object activities in a surveillance system. In Pattern Recognition, 2002. Proceedings. 16th International Conference on. Vol. 3. IEEE, 777–780.

[16] Tie-Yan Liu, Alina Bialkowski, Peter Carr, Stuart Morgan, Iain Matthews, and Yaser Sheikh. 2013. Representing and discovering adversarial team behaviors using player roles. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. 2796–2793.

[17] Cristopher D Manning and P Raghavan. [n.d.]. H. Schu. 2008. Introduction to Information Retrieval. (n.d.).

[18] Armand McMurren, Jenna Wiens, and John Gutttag. 2014. Automatically recognizing on-ball screens. In 2014 MIT Sloan Sports Analytics Conference.

[19] Andrew Miller, Luke Bornn, Ryan Adams, and Kirk Goldsberry. 2014. Factorized point process intensities: A spatial analysis of professional basketball. In International Conference on Machine Learning. 235–243.

[20] Peter J Rousseau. 1987. Silhouettes: a graphical aid to the interpretation and validation of cluster analysis. Journal of computational and applied mathematics. 20 (1987), 53–65.

[21] Gerard Salton and Michael J McGill. 1986. Introduction to modern information retrieval. (1986).

[22] Hinrich Schütze. 2008. Introduction to information retrieval. In Proceedings of the international communication of association for computing machinery conference.

[23] Tie-Yan Liu, Wei-Ying Ma, and Hong-Jiang Zhang. 2005. Effective feature extraction for playback of american football video. Multimedia Modeling, 2005. MMM 2005. Proceedings of the 11th international. IEEE, 164–171.

[24] Yi Yang Lu, Qiufeng Liu, Tieniu Tan, and Weiming Hu. 2002. Semantic interpretation of object activities in a surveillance system. In Pattern Recognition, 2002. Proceedings. 16th International Conference on. Vol. 3. IEEE, 777–780.

[25] Patrick Lucey, Alina Bialkowski, Peter Carr, Stuart Morgan, Iain Matthews, and Yaser Sheikh. 2013. Representing and discovering adversarial team behaviors using player roles. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. 2796–2793.
[43] Qian Yu, Feng Liu, Yi-Zhe Song, Tao Xiang, Timothy M Hospedales, and Chen-Change Loy. 2016. Sketch me that shoe. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 799–807.

[44] Yisong Yue, Patrick Lucey, Peter Carr, Alina Bialkowski, and Iain Matthews. 2014. Learning fine-grained spatial models for dynamic sports play prediction. In *Data Mining (ICDM), 2014 IEEE International Conference on*. IEEE, 670–679.

[45] Hua Zhang, Si Liu, Changqing Zhang, Wenqi Ren, Rui Wang, and Xiaochun Cao. 2016. Sketchnet: Sketch classification with web images. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 1105–1113.

[46] Shaoting Zhang, Ming Yang, Timothee Cour, Kai Yu, and Dimitris N Metaxas. 2015. Query specific rank fusion for image retrieval. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 37, 4 (2015), 803–815.

[47] Stephan Zheng, Yisong Yue, and Jennifer Hobbs. 2016. Generating Long-term Trajectories Using Deep Hierarchical Networks. In *Advances in Neural Information Processing Systems*. 1543–1551.

[48] Jinfeng Zhuang and Steven CH Hoi. 2011. A two-view learning approach for image tag ranking. In *Proceedings of the fourth ACM international conference on Web search and data mining*. ACM, 625–634.