Relaxation time of the global order parameter on multiplex networks: The role of interlayer coupling in Kuramoto oscillators

Roberto F. S. Andrade, Thiago Albuquerque de Assis
Instituto de Física, Universidade Federal da Bahia, Salvador, Brazil
Alfonso Allen-Perkins, Juan Manuel Pastor
Complex System Group, Universidad Politécnica de Madrid, Madrid, Spain

This work presents a new formalism to study analytically the time scales of the global order parameter and the interlayer synchronization of general multilayer networks. We report results for a system of coupled Kuramoto oscillators on multiplexes. For two-layer multiplexes with an initially high degree of synchronization in each layer, the difference between the average phases in each layer is analyzed from two different perspectives: the spectral analysis and the nonlinear Kuramoto model. Both viewpoints confirm that the prior time scales are inversely proportional to the interlayer coupling strength. Thus, increasing the interlayer coupling always shortens the transient regimes of both the global order parameter and the interlayer synchronization. Surprisingly, the analytical results show that the convergence of the global order parameter is faster than the interlayer synchronization, and the latter is generally faster than the global synchronization of the multiplex. The formalism also outlines the effects of frequencies on the difference between the average phases of each layer, and it identifies the conditions for an oscillatory behavior. Computer simulations are in fairly good agreement with the analytical findings, and they reveal that the time scale of the global order parameter is half the size of the time scale of the multiplex, if not smaller.