Top Quark Loop Corrections to the Neutral Higgs Boson Production at the Fermilab Tevatron

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ABSTRACT

We calculate the $O(\alpha m_t^2/m_H^2)$ corrections arising from diagrams involving the top-quark loops to the light neutral Higgs boson production via $q\bar{q} \rightarrow WH$ at the Fermilab Tevatron in both the standard model and the minimal supersymmetric model. In contrast to the QCD correction which increases the tree-level cross section, the corrections imply a few percent reduction in the production cross section relative to the tree-level results.

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I. INTRODUCTION

The Higgs boson is the only particle of the Standard Model (SM) which has not been discovered so far. The direct search in the LEP experiments via the $e^+e^- \rightarrow Z^*H$ yields a lower bound of $\sim 77.1$ GeV on the Higgs mass [1]. This search is being extended at present LEP2 experiments, which will explore up to a Higgs boson mass of about 95 GeV via $e^+e^- \rightarrow ZH$ by the year 2000 [2]. Much higher masses will be explored by the CERN Large Hadron Collider (LHC). Since it will be some years before the LHC comes into operation it is worth considering whether the Higgs boson can be discovered from the existing hadron collider, the Tevatron. Much study has been made in the detection of a Higgs boson at the Tevatron [3]. It was recently pointed out [4] that a light Higgs boson of mass $60 \text{GeV} \leq m_H \leq 130 \text{GeV}$ can be observable at the Tevatron with CM energy $\sqrt{s} = 2 \text{TeV}$ and sufficient integrated luminosity, $30-100 fb^{-1}$, through the production subprocess $q\bar{q}' \rightarrow WH$, followed by $W \rightarrow \ell \nu$ and $H \rightarrow b\bar{b}$. Since the expected number of events is small, it is important to calculate the cross section as accurately as possible. In Ref. [5] the $O(\alpha_s)$ QCD correction to the total cross section to this process have been calculated, and the QCD correction were found to be about $12\%$ in the $\bar{MS}$ scheme at the Fermilab Tevatron and the LHC in the SM. In general, the SM electroweak corrections are small comparing with the QCD correction. Beyond the SM, the electroweak corrections might be enhanced, since more Higgs bosons with stronger couplings to top or bottom quarks are involved in some new physics models; for example, the minimal supersymmetric model (MSSM) [6] [7], which predict that the lightest Higgs boson $h_0$ be less than 140GeV. Therefore, it is worthwhile to calculate the electroweak corrections to the light Higgs boson production via $q\bar{q}' \rightarrow Wh_0$. In this paper we present the calculation of the top quark loop correction of order $\alpha m_t^2/m_W^2$ to the Higgs boson production at the Fermilab Tevatron in both the SM and the MSSM. These corrections arise from the virtual effects of the third family (top and bottom) of quark, neutral and charged Higgs bosons. And we shall present the complete calculations of the electroweak radiative corrections to this process in a future publication [8].
The Feynman diagrams for the lightest Higgs boson production via $q(p_1)ar{q}'(p_2) \to W(k_1)h_0(k_2)$, which include the top quark loop corrections of order $\alpha m_t^2/m_W^2$ to the process $qar{q}' \to Wh_0$, are shown in Fig 1. We perform the calculation in the 't Hooft-Feynman gauge and use dimensional regularization to all the ultraviolet divergences in the virtual loop corrections utilizing the on-mass-shell renormalization, in which the fine-structure constant $\alpha$ and the physical masses are chosen to be the renormalized parameters, and the finite parts of the counterterms are fixed by the renormalization conditions. As far as the parameters $\beta$ and $\alpha$, for the MSSM we are considering, they have to be renormalized, too. In the MSSM they are not independent. Nevertheless, we follow the approach of Mendez and Pomarol in which they consider them as independent renormalized parameters and fixed the correponding renormalization constant by a renormalization condition that the on-mass-shell $H^+\ell\nu\ell$ and $h_0\ell\ell$ couplings keep the forms of Eq.(3) of Ref. to all order of perturbation theory.

We define the Mandelstam variables as

\begin{align*}
\hat{s} &= (p_1 + p_2)^2 = (k_1 + k_2)^2 \\
\hat{t} &= (p_1 - k_1)^2 = (p_2 - k_2)^2 \\
\hat{u} &= (p_1 - k_2)^2 = (p_2 - k_1)^2.
\end{align*}

(1)

The relevant renormalization constants are defined as

\begin{align*}
m^2_{W_0} &= m^2_W + \delta m^2_W, \quad m^2_{Z_0} = m^2_Z + \delta m^2_Z, \quad (2) \\
\tan \beta_0 &= (1 + \delta Z_\beta) \tan \beta, \quad \sin \alpha_0 = (1 + \delta Z_\alpha) \sin \alpha, \quad (3) \\
W^\pm_0 &= Z^{1/2}_W W^\pm + i Z^{1/2}_{H^\pm} \partial^\mu H^\pm, \quad H^\pm_0 = (1 + \delta Z_{H^\pm})^{1/2} H^\pm, \quad (4) \\
h_0 &= (1 + \delta h_0)^{1/2} h + Z^{1/2}_{h_0} H, \quad H_0 = (1 + \delta Z_H)^{1/2} H + Z^{1/2}_{H h_0} \quad (5)
\end{align*}

Taking into account the $O(\alpha m_t^2/m_W^2)$ corrections, the renormalized amplitude for $q\bar{q}' \to Wh_0$ can be written as
\[ M_{\text{ren}} = M_0 + \delta M^{\text{self}} + \delta M^{\text{vertex}}, \]  

where \( M_0 \) is the amplitude at the tree level, \( \delta M^{\text{self}} \) and \( \delta M^{\text{vertex}} \) represent the corrections arising from the self-energy and vertex diagrams, respectively. \( M_0 \) is given by

\[ M_0 = \frac{e^2 m_W \sin(\beta - \alpha)}{\sqrt{2}(m_W^2 - \hat{s}) \sin \theta_w^2} \bar{d}(p_2) \gamma \not{P} L u(p_1), \]  

where \( P_{L,R} \equiv (1 \mp \gamma_5)/2 \). \( \delta M^{\text{self}} \) is given by

\[ \delta M^{\text{self}} = \left[ \frac{\delta m_W^2 + (m_W^2 - \hat{s}) \delta Z_W}{\hat{s} - m_W^2} \right] M_0 \]

\[ + \frac{N_e e^4 m_W \sin(\beta - \alpha)}{288 \sqrt{2} \pi^2 \hat{s} \sin \theta_w^2} \left[ 6\hat{s} - 3m_W^2 - 2(\hat{s} - m_W^2) \right] \times B_0(0, m_b^2, m_t^2) + 3(m_t^4 - \hat{s} m_t^2 + 2\hat{s}^2) B_0(\hat{s}, m_b^2, m_t^2) \]  

\[ \bar{d}(p_2) \gamma \not{P} L u(p_1) \]  

with

\[ \delta m_W^2 = \frac{N_e e^2 m_t^2}{96 \pi^2 \sin \theta_w^2} \left[ -2 + 2B_0(0, m_b^2, m_t^2) - B_0(m_W^2, m_b^2, m_t^2) - 4B_0(0, m_t^2, m_t^2) \right] \]

\[ + \frac{m_t^2}{m_W^2} \left[ B_0(0, m_b^2, m_t^2) - B_0(m_W^2, m_b^2, m_t^2) \right], \]  

\[ \delta Z_W = \frac{N_e e^2}{288 m_W^4 \pi^2 \sin \theta_w^2} \left[ 2m_W^4 + 3m_t^4 B_0(0, m_b^2, m_t^2) - 3(m_t^4 + 2m_W^4) \right] \times B_0(m_W^2, m_b^2, m_t^2) + 3m_W^2 (m_t^4 + m_t^2 m_b^2 - 2m_W^4) G(m_W^2, m_b^2, m_t^2), \]

Here and below, \( B_0, C_0, C_i \) and \( C_{ij} \) is the two-point and three-point scalar integrals, definitions for which can be found in Ref. \[11\] and \( G \) is the derivative of \( B_0 \) which is expressed as

\[ G(M^2, M_1^2, M_2^2) = \frac{\partial B_0(k^2, M_1^2, M_2^2)}{\partial k^2} \bigg|_{k^2=M^2}. \]

\( \delta M^{\text{vertex}} \) is given by

\[ \delta M^{\text{vertex}} = M_0 \left[ \frac{1}{2} \delta Z_{h_0} + \frac{\delta m_W^2 - \delta m_Z^2}{2(m_Z^2 - m_W^2)} + \frac{\delta m_Z^2}{m_Z^2} + \frac{\delta m_W^2}{m_W^2} \right] \]

\[ + \cot(\beta - \alpha)(Z_{h_0}^{1/2} + \sin \beta \cos \beta \delta Z_{\beta} - \tan \alpha \delta Z_{\alpha}) \]

\[ + f_1^{\text{vertex}} \bar{d}(p_2) \gamma \not{P} L u(p_1) \]

\[ + f_2^{\text{vertex}} \bar{d}(p_2) \gamma \not{P} L u(p_1) \gamma \not{P} L u(p_1) \epsilon \cdot p_1 \]

\[ + f_3^{\text{vertex}} \bar{d}(p_2) \gamma \not{P} L u(p_1) \epsilon \cdot p_2, \]  

\[ (12) \]
with

$$\delta m_Z^2 = \frac{N_c e^2 m_t^2}{3 \cos \theta_w^2 \pi^2} \left[ \frac{B_0(0, m_t^2, m_t^2)}{6} - \frac{B_0(0, m_t^2, m_t^2)}{16 \sin \theta_w^2} + \frac{2 \sin \theta_w^2 B_0(0, m_t^2, m_t^2)}{9} \right] 
- \frac{B_0(m_Z^2, m_t^2, m_t^2)}{6} - \frac{B_0(m_Z^2, m_t^2, m_t^2)}{32 \sin \theta_w^2} + \frac{2 \sin \theta_w^2 B_0(0, m_t^2, m_t^2)}{9}, \tag{13}$$

$$\delta Z_{h_0} = -\frac{N_c e^2 \cos^2(\alpha) \csc^2(\beta) m_t^2}{32 m_W^2 \pi^2 \sin \theta_w^2} \left[ B_0(0, m_{h_0}^2, m_t^2) 
+ (m_{h_0}^2 - 4m_t^2) G(m_{h_0}^2, m_t^2, m_t^2) \right], \tag{14}$$

$$Z_{H h_0}^{1/2} = \frac{N_c e^2 m_t^2 \cos(\alpha) \csc^2(\beta) \sin(\alpha)}{32 (m_H^2 - m_{h_0}^2) m_W^2 \pi^2 \sin \theta_w^2} \left[ -2m_t^2 - 2m_t^2 B_0(0, m_t^2, m_t^2) 
+ (m_{h_0}^2 - 4m_t^2) B_0(0, m_t^2, m_t^2) \right]. \tag{15}$$

$$\delta Z_{\beta} = \frac{\delta m_Z^2 - \delta m_W^2}{2(m_Z^2 - m_W^2)} - \frac{\delta m_Z^2}{2m_Z^2} + \frac{\delta m_W^2}{2m_W^2} - \frac{1}{2} \delta Z_{H^\pm} - \frac{m_W}{\tan \beta} Z_{W H^\pm}^{1/2}, \tag{16}$$

$$\delta Z_{\alpha} = -\sin^2 \beta \delta Z_{\beta} + \frac{\delta m_Z^2 - \delta m_W^2}{2(m_Z^2 - m_W^2)} - \frac{\delta m_Z^2}{2m_Z^2} + \frac{\delta m_W^2}{2m_W^2} - \frac{1}{2} \delta Z_{h_0} + \frac{\cos \alpha}{\sin \alpha} Z_{H h_0}^{1/2}, \tag{17}$$

$$\delta Z_{H^\pm} = \frac{N_c g^2 m_t^2}{32 m_W^2 \pi^2} \left[ -B_0(m_{H^\pm}^2, m_b^2, m_t^2) \cot^2(\beta) 
+ (m_t^2 - m_{H^\pm}^2) \cot^2(\beta) G(m_{H^\pm}^2, m_b^2, m_t^2) \right], \tag{18}$$

$$Z_{H^\pm W}^{1/2} = \frac{N_c g^2 m_t^2 \cot(\beta)}{32 m_{H^\pm}^2 \pi^2 m_W^2 \pi^2} \left[ m_W^2 B_0(0, m_t^2, m_t^2) 
+ (m_{H^\pm}^2 - m_t^2) B_0(0, m_t^2, m_t^2) \right], \tag{19}$$

$$f_{\text{vertex}}^{1} = \frac{-N_c e^4 m_t^2}{32 \sqrt{2} m_W \pi^2 (m_W^2 - \hat{s}) \sin \theta_w^4} \left[ -2 B_0(\hat{s}, m_b^2, m_t^2) 
+ (-2m_t^2 - \hat{t}) C_0(0, m_h^2, m_W^2, \hat{s}, m_t^2, m_t^2, m_b^2) 
+ (-2m_W^2 - \hat{t}) C_1(0, m_W^2, \hat{s}, m_h^2, m_t^2, m_b^2, m_t^2) 
+ (-m_{h_0}^2 - 2\hat{t}) C_2(0, m_{h_0}^2, \hat{s}, m_h^2, m_t^2, m_b^2, m_t^2) 
+ 4 C_{00}(m_W^2, \hat{s}, m_{h_0}^2, m_t^2, m_b^2, m_t^2) \right]. \tag{20}$$
The parton luminosity, which is defined as
\[
\sqrt{\text{luminosity}} = \frac{N_c e^2 m_t^2}{16 \sqrt{2} m_W \pi^2 (m_W^2 - \hat{s}) \sin \theta_w^4} [-C_0(m_{h_0}^2, m_W^2, s, m_t^2, m_b^2, m_{t_0}^2)
\]
\[\quad - C_1(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)
\]
\[\quad - 3C_2(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)
\]
\[\quad - 2C_{12}(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)
\]
\[\quad - 2C_{22}(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)]
\]
\[f_{\text{vertex}} = \frac{N_c e^2 m_t^2}{16 \sqrt{2} m_W \pi^2 (m_W^2 - \hat{s}) \sin \theta_w^4} [-C_2(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)
\]
\[\quad - 2C_{12}(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)
\]
\[\quad - 2C_{22}(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)]
\]
\[f_{\text{vertex}} = \frac{N_c e^2 m_t^2}{16 \sqrt{2} m_W \pi^2 (m_W^2 - \hat{s}) \sin \theta_w^4} [-C_2(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)
\]
\[\quad - 2C_{12}(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)
\]
\[\quad - 2C_{22}(m_W^2, s, m_{h_0}^2, m_t^2, m_b^2, m_{t_0}^2)]
\]
\[21\]

The corresponding amplitude squared for the process \(qq' \rightarrow Wh_0\) can be written as
\[
\sum |M_{\text{ren}}|^2 = \sum |M_0|^2 + 2Re \sum (\delta M^{\text{self}} + \delta M^{\text{vertex}}) M_0^\dagger
\]
where the bar over the summation recalls average over initial partons spins. The cross section of process \(qq' \rightarrow Wh_0\) is
\[
\dot{\sigma} = \int_{t_{\text{min}}}^{t_{\text{max}}} \frac{1}{16 \pi \hat{s}^2} \sum_{\text{spins}} |M|^2 d\hat{t}
\]
\[24\]
with
\[
\hat{t}_{\text{min}} = \frac{m_{h_0}^2 + m_W^2 - \hat{s}}{2} - \sqrt{(\hat{s} - (m_{h_0} + m_W)^2)(\hat{s} - (m_{h_0} - m_W)^2)/2}
\]
\[25\]
\[
\hat{t}_{\text{max}} = \frac{m_{h_0}^2 + m_W^2 - \hat{s}}{2} + \sqrt{(\hat{s} - (m_{h_0} + m_W)^2)(\hat{s} - (m_{h_0} - m_W)^2)/2}
\]

The total cross section of \(PP \rightarrow qq' \rightarrow Wh_0\) can be obtained by folding the \(\dot{\sigma}\) with the parton luminosity
\[
\sigma(s) = \int_{(m_{h_0} + m_W)/\sqrt{s}}^{1} dz \frac{dL}{dz} \dot{\sigma}(qq' \rightarrow Wh_0 \text{ at } \hat{s} = z^2 s)
\]
\[26\]
where \(\sqrt{s}\) and \(\sqrt{\hat{s}}\) is the CM energy of \(PP\) and \(qq'\), respectively, and \(dL/dz\) is the parton luminosity, which is defined as
\[
\frac{dL}{dz} = 2z \int_{z}^{1} \frac{dx}{x} f_{q/P}(x, q^2) f_{q'/P}(z^2/x, q^2)
\]
\[27\]
where \(f_{q/P}(x, q^2)\) and \(f_{q'/P}(z^2/x, q^2)\) are the parton distribution function.
III. NUMERICAL RESULTS

In the following we present some numerical results. In our numerical calculations, the SM parameters were taken to be \( m_W = 80.33\,\text{GeV} \), \( m_Z = 91.187\,\text{GeV} \), \( m_t = 176\,\text{GeV} \), \( m_b = 4.5\,\text{GeV} \) and \( \alpha(m_W) = \frac{1}{128} \). Moreover, we use the relation [7] between the Higgs boson masses \( m_{h_0,H,A,H^\pm} \) and parameters \( \alpha, \beta \) at one-loop, and choose \( m_{h_0} \) and \( \tan \beta \) as two independent input parameters. As explained in Ref. [10], there is a small inconsistency in doing so since the parameters \( \alpha \) and \( \beta \) of Ref. [7] are not the ones defined by the conditions given by Eq.(3) of Ref. [10]. Nevertheless, this difference would only induce a higher order change [11]. We will limit the value of \( \tan \beta \) to be in the ranges \( 2 \leq \tan \beta \leq 30 \), which are consistent with ones required by the most popular MSSM model with scenarios motivated by current low energy data (including \( \alpha_s, R_b \) and the branching ratio of \( b \to s\gamma \)).

In Fig.2 we present the tree-level total cross sections versus the Higgs boson mass in both the SM and the MSSM for the different values of \( \tan \beta \), using the CTEQ3L parton distributions [12]. Figure 2 shows the total cross sections in the SM always are larger than ones in the MSSM, and they are almost same only for \( \tan \beta = 2 \), or the mass of the Higgs boson approach to 130 GeV.

In Fig.3 we shows the top quark loop corrections of order \( \alpha m_t^2/m_W^2 \) to the total cross sections. From Fig.3 one sees that in the SM the corrections are not sensitive to the mass of the Higgs boson and amounts to \( 1\% \sim 2\% \) reduction in the cross section. And in the MSSM the corrections depend strongly on the values of \( m_{h_0} \) for all \( \tan \beta \). Especially for \( \tan \beta > 2 \), such correction can reach about \( -4\% \) when \( m_{h_0} = 60\,\text{GeV} \), but the correction is only about \( -1\% \) if \( m_{h_0} = 130\,\text{GeV} \). Since QCD correction increases the tree-level total cross sections by about 12\%, it is necessary for an accurate calculation of the cross sections to include the top quark loop corrections, which typically imply a few percent reduction in the cross sections.

In conclusion, we have calculated that the top quark loop corrections of order \( \alpha m_t^2/m_W^2 \) to the neutral Higgs boson production via \( q\bar{q}' \to WH \) at the Fermilab Tevatron in the SM and the MSSM. In contrast to the QCD corrections, such corrections reduce the tree-level total cross sections by about \( 1\% \sim 2\% \) in the SM, and \( 1\% \sim 4\% \) in the MSSM.
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FIG. 1. Feynmann diagrams for the process $qq' \rightarrow Wh_0$. 
FIG. 2. Tree-level cross sections as a function of the Higgs boson mass of the process $q\bar{q}' \rightarrow Wh_0$ with $\sqrt{s} = 2TeV$ at Tevatron.
FIG. 3. Relative corrections $\delta \sigma / \sigma_0$ as a function of the Higgs boson mass of the process $q\bar{q}' \rightarrow Wh_0$ with $\sqrt{s} = 2TeV$ at Tevatron.