Some effects of perturbed flight schedules to the performance of aircraft routings

Khusnul Novianingsih
Department of Mathematics education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudhi No. 229 Bandung 40154, Indonesia
E-mail: k_novianingsih@upi.edu

Abstract. In this paper, we observe some effects of schedule perturbations to the performance of aircraft routings. We show that moving departure time of flights to adding their block-time do not guarantee that the departure delay of the flights will be reduced. Contrarily, moving arrival time of flights always improve the performance of aircraft routings, especially for reducing arrival delay. The computational results also show that by adding more slacks, we obtain better performance of aircraft routings.

1. Introduction
Airline schedules are operated under uncertain conditions. Some factors such as bad weather and airport congestion can cause some flight schedule can be delayed or canceled. These affect to airline on-time performance which it is related to passenger satisfaction. To reduce the number of flight delays, airlines need to construct robust aircraft routings. Robust aircraft routings are aircraft route schedules that are arranged to minimize a specific robustness measure, as the expectation of departure delay, arrival delay, or propagated delay.

Recently, some researchers have shown that improving the robustness of aircraft routings can be done by perturbed flights. That is by moving the departure time or arrival time of flights earlier or later. Chiraphadanakul et al. [4] derive an optimization model to re-time departure time or arrival time of flight for minimizing the expectation of total arrival delay. AhmadBeygi et al. [2] construct an optimization model to re-time the departure time of flights for minimizing the expectation of total propagated delay. The similar model to minimize the expectation of total departure delay is proposed by Novianingsih et al [7]. An optimization model to built new aircraft routings that allow re-timing flights is built by Aloulou et al. [3].

The research above improve the robustness of aircraft routing through optimization models to determine the optimal perturbation. This research characterize the influence of perturbed flights analytically. As we know, only Schaefer et al. [8] who investigate the topic analytically. Unlike Schaefer et al. [8] who observe the perturbation of the original flight schedule to improve the operational performance of a given crew schedule, this paper inspect the effects of perturbed flight schedules to the aircraft routing operations. We obtain that moving departure time of flights does not always improve the performance of aircraft routings. But the arrival delay of flights will be reduced when the block-times are added. By simulation, we also show that more slack means better performance.
This paper is organized as follows. We discuss flight delay on an aircraft schedule in Section 2. Section 3 discuss the effect of perturbed flights to the performance of aircraft routings. We present computational results in Section 4. We give a conclusion in the last section.

2. Flight delay on an aircraft schedule

An aircraft schedule consists a set of aircraft routings that partitions the flights to be flow by a single aircraft. The aircraft scheduling problem is usually modeled as:

$$\min \{ cx : Ax = 1, x \in \{0,1\} \}, \quad (2.1)$$

where its columns matrix $A = (a_{fr})$ represent all possible aircraft routings in flight schedules, in which $a_{fr} = 1$ if flight $f$ in routing $r$, and $a_{fr} = 0$ otherwise. One routing consists a sequence of flights and it complies a number of maintenance requirements. The element $x_r$ of $x$ is a decision variable that has value 1 if routing $r$ is selected as a solution, and $x_r = 0$ otherwise. The element $c_r$ of $c$ is cost associated to routing $r$.

Let $F$ and $R$ be a set of flights and a set of aircraft routings, respectively. For $f \in F$, let $dep_f$ and $arr_f$ be the planned departure time of flight $f$ and the planned arrival time of flight $f$, respectively. Given an aircraft route $r$ which consists a sequence of flights $f_1, ..., f_n$.

Definition 2.1 Slack between two consecutive flights $f_i$ and $f_{i+1}$ in $r$ is defined as

$$s_{i,i+1} = c_{i,i+1} - m_{i,i+1},$$

where $ct_{i,i+1} = dep_{i+1} - arr_i$ is the planned connecting time between flight $f_i$ and $f_{i+1}$, and $m_{i,j}$ is the minimum required connecting time to serve flight $f_{i+1}$ after flight $f_i$.

Assume that in flight schedule operations we use a push-back recovery. Using this assumption, if any flight in $r$ is disrupted, the next flights in $r$ will be delayed until the planes that serve the flights are available [1]. Assume that the planes are always available to serve the flights in schedules. Suppose that we divide sources of a delay for flight $f_i$ into two categories: ground delay and block-time delay. A ground delay is a delay before an aircraft take-off, and it does not include propagated delay caused by the previous flight in the same routing. While a propagated delay is a flight delay caused by waiting for incoming aircraft, or a non-propagated delay is a delay caused by other reasons. A block-time delay is delay in the air, and we can view as the difference between the planned block-time of a flight with its actual block-time.

For aircraft routing $r$, let $\alpha_1, ..., \alpha_n$ and $\beta_1, ..., \beta_n$ be nonnegative random variables where $\alpha_i$ indicates departure delay of flight $f_i$ and $\beta_i$ indicates arrival delay of flight $f_i$, respectively. Then, the random departure time and the random arrival time of flight $f_i$ are given by $\gamma_i = dep_i + \alpha_i$ and $\delta_i = arr_i + \beta_i$, respectively. Let $\lambda_1, ..., \lambda_n$ be nonnegative random variables, where $\lambda_i$ represents ground delay of flight $f_i$ in $r$. Let $\mu_1, ..., \mu_n$ be nonnegative random variables, where $\mu_i$ indicates the block-time delay of flight $f_i$ relative to its planned block-time. Assume that the distributions of ground-time delay and block-time delay are independent of the time of day.

According Lan et al. [5], a delay can be decomposed into a propagated delay and a non-propagated delay. A propagated delay is a flight delay caused by waiting for incoming aircraft, and a non-propagated delay is a delay caused by other reasons. While a propagated delay is influenced by previous flights in an aircraft routing, a non-propagated delay more describes airport conditions where the flight depart from, such as weather condition and traffic intensities. Consider a departure delay of flight $f$ as decomposition of a propagated delay and a non-propagated. Let $pd_{i,i+1}$ be the propagated delay to flight $f_{i+1}$ caused by flight $f_i$. Since we do not have propagated delay for the first flight in $r$, then the departure delay of the first flight is calculated as

$$\alpha_1 = \lambda_1.$$
For \( i = 2, ..., n \), the departure delay of \( f_i \in r \) is determined as
\[
\alpha_i = \lambda_i + pd_{i-1,i}.
\]
A propagated delay to flight \( f_i \) occurs only if the arrival delay of flight \( f_{i-1} \) makes the new connection \( ct_{i-1,i} \) less than the minimum required connecting time. So, we calculate the propagated delay \( pd_{i-1,i} \) as
\[
pd_{i-1,i} = \max\{\beta_{i-1} - s_{i-1,i}, 0\}. \tag{2.2}
\]
On the other hand, the departure delay of flight \( f_{i-1} \) and its block-time delay will contribute to its arrival delay. So, we have
\[
\beta_{i-1} = \alpha_{i-1} + \mu_{i-1}. \tag{2.3}
\]
By substituting (2.3) to (2.2), we get
\[
pd_{i-1,i} = \max\{\alpha_{i-1} + \mu_{i-1} - s_{i-1,i}, 0\}. \tag{2.4}
\]
Finally, we have the following proposition as a formula to calculate the departure delay of flights along a path of an aircraft routing.

**Proposition 2.2** The departure delay of flight \( f_i \) in routing \( r \) is given by
\[
\alpha_1 = \lambda_1,
\]
and
\[
\alpha_i = \lambda_i + \max\{\alpha_{i-1} + \mu_{i-1} - s_{i-1,i}, 0\},
\]
for \( i = 2, ..., n \).

Given \( \lambda_1, ..., \lambda_n \) of arbitrary ground delays and \( \mu_1, ..., \mu_n \) of arbitrary block-time delays for flights \( f_1, ..., f_n \) in \( r \). The following proposition gives the bound of departure delay of flights in an aircraft routing.

**Proposition 2.3** For each \( f_i \) in \( r \),
\[
\lambda_i \leq \alpha_i \leq \sum_{k=1}^{i} \lambda_k + \sum_{k=2}^{i} |\mu_{k-1} - s_{k-1,k}|.
\]

**Proof.** Recall from Proposition 2.2,
\[
\alpha_i = \lambda_i + \max\{\alpha_{i-1} + \mu_{i-1} - s_{i-1,i}, 0\}.
\]
Since
\[
\max\{\alpha_{i-1} + \mu_{i-1} - s_{i-1,i}, 0\} \leq |\alpha_{i-1} + \mu_{i-1} - s_{i-1,i}|,
\]
and
\[
|\alpha_{i-1} + \mu_{i-1} - s_{i-1,i}| \leq \alpha_{i-1} + |\mu_{i-1} - s_{i-1,i}|,
\]
then
\[
\alpha_i \leq \lambda_i + \alpha_{i-1} + |\mu_{i-1} - s_{i-1,i}|. \tag{2.5}
\]
Substituting for \( \alpha_{i-1} \) in Equation(2.5),
\[
\begin{align*}
\alpha_i & \leq \lambda_i + (\lambda_{i-1} + \alpha_{i-2} + |\mu_{i-2} - s_{i-2,i-1}|) + |\mu_{i-1} - s_{i-1,i}| \\
& \leq \lambda_i + \lambda_{i-1} + \lambda_{i-2} + \alpha_{i-3} + |\mu_{i-3} - s_{i-3,i-2}| + |\mu_{i-2} - s_{i-2,i-1}| + |\mu_{i-1} - s_{i-1,i}| \\
& \quad \vdots \\
& \leq \sum_{k=1}^{i} \lambda_k + \sum_{k=2}^{i} |\mu_{k-1} - s_{k-1,k}| \tag{2.6}
\end{align*}
\]
If $pd_{i-1,i} = 0$, then $\alpha_i = \lambda_i$. □

Since $\beta_i = \alpha_i + \mu_i$, we established the following corollary.

**Corollary 2.4** For each $f_i$ in $r$,

$$
\lambda_i \leq \beta_i \leq \sum_{k=1}^{i} \lambda_k + \mu_i + \sum_{k=2}^{i} |\mu_{k-1} - s_{k-1,i}|.
$$

Especially, if $pd_{i-1,i} > 0$, for all $i = 2, \ldots, n$, then we have

$$
\alpha_i = \sum_{k=1}^{i} \lambda_k + \sum_{k=2}^{i} (\mu_{k-1} - s_{k-1,i}),
$$

and

$$
\beta_i = \sum_{k=1}^{i} \lambda_k + \mu_i + \sum_{k=2}^{i} (\mu_{k-1} - s_{k-1,i}).
$$

### 3. The Perturbed Schedule

Let $x$ and $y$ be a nonnegative vector in $\mathbb{R}^{|F|}$.

**Definition 3.1** The perturbed schedules $F + (x, y)$ is defined as the new flight schedules which obtain from $F$ where the departure time and the arrival time of each flight $f_i$ in $F$ are changed by $\text{dept}_i - x_i$ and $\text{arrt}_i + y_i$, respectively. Perturbed schedules $F + (x, y)$ is called as feasible perturbed schedules if the planned aircraft routings and the planned crew pairing in $F$ remain feasible under perturbed schedules $F + (x, y)$.

For flight $f_i$ in $F + (x, y)$, the new block time of flight $f_i$ is increased to $bt_i + x_i + y_i$. Hence, the perturbed random block-time delay of flight $f_{i-1}$ becomes $\mu'_i = \mu_i - x_i - y_i$. Similarly, the new slack for connecting flight $f_{i-1}$ and $f_i$ is reduced to $s'_{i-1,i} = s_{i-1,i} - x_i - y_i - 1$.

Let $r'$ be the feasible perturbed routing of $r$, that is the aircraft routing which all flight in $r$ are elements of $F + (x, y)$. Let $\alpha'_i$ and $\beta'_i$ be the random departure time and the random arrival delay of flight $f_i$ in $r$, respectively. Given a sequence of random ground delay $\lambda_1, \ldots, \lambda_n$ and a sequence of random block-time delay $\mu_1, \ldots, \mu_n$ for $f_1, \ldots, f_n$. According Proposition 2.2, the departure delay of flight $f_i \in r'$ is

$$
\alpha'_i = \lambda_i + \max\{\alpha'_{i-1} + \mu'_i - s'_{i-1,i}, 0\}.
$$

Since

$$
s'_{i-1,i} = s_{i-1,i} - x_i - y_i - 1,
$$

and

$$
\mu'_{i-1} = \mu_{i-1} - x_{i-1} - y_{i-1},
$$

then

$$
\alpha'_i = \lambda_i + \max\{\alpha'_{i-1} + \mu_{i-1} - s_{i-1,i} + x_i - x_{i-1}, 0\}.
$$

Equation (3.10) shows that re-timing arrival time of flights will not affect to their departure delays. Since the arrival delay is the sum of the departure delay and the block-time delay, then the shorter time of departure delay is the shorter time of arrival delay.
Since delays for flights \( f \) for Proposition 3.2 this condition is explain in the following relation.

\[
\alpha_i' = \sum_{k=1}^{i} \lambda_k + \sum_{k=1}^{i-1} (\mu_k - x_k - y_k) - \sum_{k=2}^{i} (s_{k-1,k} - y_{k-1} - x_k)
\]

\[
= \sum_{k=1}^{i} \lambda_k + \sum_{k=1}^{i-1} \mu_k - \sum_{k=2}^{i} s_{k-1,k} - \sum_{k=1}^{i-1} (x_k + y_k) + \sum_{k=2}^{i} (y_{k-1} + x_k)
\]

\[
= \sum_{k=1}^{i} \lambda_k + \sum_{k=1}^{i-1} \mu_k - \sum_{k=2}^{i} s_{k-1,k} + \sum_{k=1}^{i-1} (-x_k - y_k + y_k + x_{k+1})
\]

\[
= \sum_{k=1}^{i} \lambda_k + \sum_{k=1}^{i-1} \mu_k - \sum_{k=2}^{i} s_{k-1,k} + (x_i - x_1)
\]

Using Equation (2.8), the departure delay of flight \( f_i \) in the perturbed routing is given by the following relation.

\[
\alpha_i' = \sum_{k=1}^{i} (\lambda_i + \sum_{k=1}^{i-1} \mu_k - \sum_{k=2}^{i} s_{k-1,k} - \sum_{k=1}^{i-1} (x_k + y_k)) - \sum_{k=2}^{i} (s_{i-1,i} - x_k - y_{k-1})
\]

\[
= \sum_{k=1}^{i} (\lambda_i + \sum_{k=2}^{i} s_{i-1,i} - \sum_{k=1}^{i} (x_k + y_k)) + \sum_{k=2}^{i} (x_k + y_{k-1})
\]

\[
= \sum_{k=1}^{i} (\lambda_i + \sum_{k=2}^{i} s_{i-1,i} - \sum_{k=1}^{i} (x_k + y_k)) + \sum_{k=1}^{i-1} (x_{k+1} + y_k)
\]

\[
= \sum_{k=1}^{i} (\lambda_i + \sum_{k=2}^{i} s_{i-1,i} - (x_1 + y_1 + ... + x_i + y_i) + (x_2 + y_1 + ... + x_i + y_{i-1})
\]

\[
= \sum_{k=1}^{i} (\lambda_i + \sum_{k=2}^{i} s_{i-1,i} - (x_1 + y_k)
\]

Since \( x_1 \) and \( y_i \) are the non negative number, we can conclude that the moving flights in \( F \) will reduce the arrival delay of the flights in the routing. This condition is not true for moving departure time of flights. Equation (3.11) tell us that as long as the propagated delay is positive, the departure delay of flight \( f_i \) is influenced by the moving of both its flight and the first flight in the perturbed routing. It means that re-timing departure time of flights does not always improve the performance of aircraft routing. For a special case when the departure time and arrival time of each flight in a routing move in a constant number, we obtain that the departure delay before and after the re-timing are the equal. This condition is explain in the following proposition.

**Proposition 3.2** For \( \lambda_1, ..., \lambda_n \) of arbitrary ground delays and \( \mu_1, ..., \mu_n \) of arbitrary block-time delays for flights \( f_1, ..., f_n \), if \( x_i = c \) and \( y_i = c \) for \( i = 1, ..., n \) and \( c \) is any positif real number, then \( \alpha_i' = \alpha_i \) for \( i = 1, ..., n \).

**Proof.** Using Proposition 2.3, we have \( \alpha_1 = \lambda_1 \) and \( \alpha_1' = \lambda_1 \). Now, we get \( \alpha_i' = \alpha_i \). Since flight the departure time and the arrival time change of flight \( f_{i-1} \)
is $x_{i-1}$ and $y_{i-1}$, respectively and we have $x_{i-1} = y_{i-1} = c$, the arrival delay of flight $f_i$ is determined by

$$\beta_{i-1}' = \max\{\alpha_{i-1}' + \mu_{i-1} - x_{i-1} - y_{i-1}, 0\} = \max\{\alpha_{i-1}' + \mu_{i-1} - 2c, 0\}.$$ 

Since the propagated delay of flight $f_i$ caused by flight $f_{i-1}$ in $r'$ is

$$pd_{i-1,i}' = \max\{\beta_{i-1}' - s_{i-1,i}', 0\},$$ 

and $s_{i-1,i}' = s_{i-1,i} - y_{i-1} - x_i = s_{i-1,i} - 2c$, then

$$\alpha_{i}' = \lambda_i + \max\{\beta_{i-1}' - s_{i-1,i} + 2c, 0\}.$$ 

Next, we will consider two cases for $\beta'$. If we have $\beta_{i-1}' = 0$, then $\alpha_{i}' = \lambda_i$ for this case. Since $\alpha_{i-1}' + \mu_{i-1} \leq 2c$ and $\alpha_{i-1}' = \alpha_{i-1}$ according our assumption, then we get $\alpha_{i-1} + \mu_{i-1} \leq 2c$. Let consider two cases when $\alpha_{i-1} + \mu_{i-1} \leq 2c$. If $\alpha_{i-1} + \mu_{i-1} \leq 0$, then $pd_{i-1,i} = 0$. As a result, we have $\alpha_i = \lambda_i = \alpha_i'$. Otherwise, let we have $\alpha_{i-1} + \mu_{i-1} \leq 0$. Since $s_{i-1,i} = s_{i-1,i}' + 2c$ and $\beta_{i-1} = \alpha_{i-1} + \mu_{i-1} \leq 2c$, then we have $pd_{i-1,i} = 0$. Now, we obtain $\alpha_i = \lambda_i = \alpha_i'$. Let $\beta_{i-1}' > 0$. Then

$$pd_{i-1,i}' = \max\{\alpha_{i-1}' + \mu_{i-1} - s_{i-1,i}', 0\} = \max\{\alpha_{i-1}' + \mu_{i-1} - 2c - (s_{i-1,i} - 2c), 0\} = \max\{\alpha_{i-1}' + \mu_{i-1} - s_{i-1,i}, 0\}.$$ 

Since $\alpha_{i-1}' = \alpha_{i-1}$, we obtain $pd_{i-1,i}' = \max\{\alpha_{i-1} + \mu_{i-1} - s_{i-1,i}, 0\} = pd_{i-1,i}$. As the consequence, $\alpha_{i}' = \alpha_i$. Now, we complete the proof. $\square$

### 4. Computational results

We consider one-day flight schedules of an airline in Indonesia for computational study. The schedules consist of 287 flights which are covered by 92 aircraft routings. We construct a flight delay simulation by the following steps.

1. Generate a number of flight delays, each with its ground delay and block-time delay.
2. For each flight delay, apply push-back recovery strategy. If the new connecting time between the delayed flight and the connecting flight is less than the minimum required connecting time, the connecting flight will be delayed until the minimum connecting time is fulfilled.
3. Calculate total departure delays and total arrival delays.
4. Repeat step (1)-(3) for a number of iterations.
5. Calculate the average of total departure delays and total arrival delays.

The number of flight delays in Step (1) is generated randomly. We model the probability distribution of the ground delay and the block-time delay according one-year historical delay data of the airline. We refer the probability distribution of ground delays in Novianingsih et al. [7]. The distribution of ground delays is modeled as log-normal distribution:

$$f(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right), \quad x > 0,$$  

(4.13)

where $\mu$ and $\sigma$ are mean and standard deviation, respectively. In our case study, the logarithm of parameters in the log-normal distributions are $\mu \in [2.8 - 3.4]$ and $\sigma \in [0.4 - 0.6]$. The
distribution of block-time delays is modeled as uniformly distributed random numbers on the interval $[0 - 30]$. To perturb schedules, we allow departure times and arrival times of planned flight schedules to be moved earlier or later no more than 15 minutes in order to preserve passenger projection in flight schedule design. To observe the effect of additional slack to the performance of aircraft routing, we modify the departure times and the arrival times of the original flight schedules such that the new flight schedules will produce the aircraft routings with more slack than the original aircraft routings, but the original block-times do not change. The same technique is also applied to obtain the new flight schedules that will produce aircraft routings with more length of block-times than the original block-times and the same length of slacks with the original slacks. For both cases, we record total departure delay, total arrival delay, and 15-minute on time performance (15-OTP). According 15-OTP, a flight is delay if depart on a gate in more than 15 minutes after its schedule departure time.

The effect of additional slack and additional block-time are summarized on Table 1 and 2, respectively. From Table 1, we can see that the larger slack will result to the better aircraft routing performance. The same condition also occurs for the additional of block-time. The increasing block-time in aircraft routings will affect to the increasing of the aircraft routing performance. More block-times are the better performance.

| Table 1. The effect of additional slacks to the aircraft routings performance. |
|---------------------------------------------|
| Robustness measure | Additional slack (mins) |
|---------------------|-------------------------|
|                     | 0  | 5  | 10 | 15  |
| Total departure delay (mins) | 4320 | 3995 | 3422 | 2931 |
| Total arrival delay (mins)   | 4320 | 3995 | 3422 | 2931 |
| 15-OTP (percent)             | 71  | 78  | 85  | 91  |

| Table 2. The effect of additional block-time to the aircraft routings performance. |
|---------------------------------------------|
| Robustness measure | Additional block-time (mins) |
|---------------------|-----------------------------|
|                     | 0  | 5  | 10 | 15  |
| Total departure delay (mins) | 4320 | 3880 | 3600 | 3212 |
| Total arrival delay (mins)   | 4450 | 4450 | 4450 | 4450 |
| 15-OTP (percent)             | 71  | 79  | 87  | 92  |

We also consider four other cases of perturbations for each aircraft routing. In the first case, we set $x_1 = 10$ minutes and $x_i = x_{i-1}$ for $i = 2, 3, ...$. In Case 2, we set $x_1 = 5, x_i = \frac{3}{2}x_{i-1}$ for $i = 2, 3, ...$. In Case 3, we set $x_1 = 5, x_i = x_{i-1}$ for $i = 2, 3, ...$. In Case 4, we set $x_1 = 15, x_i = \frac{3}{2}x_{i-1}$ for $i = 2, 3, ...$. For all cases, we set $x_i = y_i$ for $i = 1, 2, ...$. We perform the same flight delay simulation for all cases.
Table 3. The effect of perturbed flights to the aircraft routings performance.

| Robustness measure       | Original Case |
|--------------------------|---------------|
|                          | 1 2 3 4       |
| Total departure delay (mins) | 4320 4320 4502 4320 3701 |
| Total arrival delay (mins)   | 4405 4380 4798 4391 4302 |
| 15-OTP (percent)            | 71 71 68 71 84 |

Using the results on the Table 3, we can see that the performance of aircraft routings are not improve when we move departure time and arrival time of flights in a constant number, and it ratifies the Proposition 3.2. The performance will continue to increase, if the additional of block-time of each flight in the aircraft routings is large than the reducing slack in its connection.

5. Conclusion

In this paper, we study some effects of flight schedule perturbations to the performance of aircraft routing. This research suggest that we can add block-times by moving departure time or arrival time of flights so that slacks in aircraft connections is reduced. We show that adding block-time of flights will not increase the arrival delay. However, the departure delay does not always decrease when the departure time of flights is changed. In reality, adding more block-time can affect to the increasing of crew costs. To obtain the perturbed schedules in aircraft routings that will not increase the crew costs, we should include the crew scheduling problem in the perturbed scheduled. This is our further research object.

Acknowledgment

This research was supported by Penelitian Produk Terapan from Ministry of Technology, and Higher Education Indonesia with contract number: 008/SP2H/LT/DRPM/IV/2017. The authors thank to Garuda Indonesia Airline for supporting the data.

References

[1] Abdelghany A, Ekollu G, Narasimhan R and Abdelghany K 2004 A proactive crew recovery decision support tool for commercial airline during irregular operations Ann. Oper. Res. 127 309
[2] AhmadBeygi S, Cohn A and Lapp M 2008 Decreasing airline delay propagation by re-allocating schedule slack (Michigan: University of Michigan)
[3] M.A. Aloulou M A, Haouari M and Mansour F Z 2013 A model for enhancing robustness of aircraft and passenger Transportation Research Part C 32 48
[4] Chiraphadanakul V and Bernhard C 2013 Robust flight schedules through slack re-allocation Euro J. Transport. Log. 2 277
[5] Lan S, Carke J P and Bernhart C 2006 Planning for robust airline operations: Optimizing aircraft routings and flight departure times to minimize passenger disruptions Transport. Sci. 40 15
[6] Lettovskiy L, Johnson E L and Nemhauser G L 2000 Planning for robust airline operations: Airline crew recovery Transport. Sci. 34 337
[7] Novianingsih K and Hadiani R 2016 Flight re-timing models to improve the robustness of airline schedules Thai J. Math. 14 49
[8] Schaefer A J and Nemhauser G L 2006 Improving airline operational performance through schedule perturbation Ann. Oper. Res. 114 3
[9] Sohoni M, Lee Y and Klabjan D 2011 Robust airline scheduling under block time uncertainty Transport. Sci. 45 451