PREPARING FOR AN EXPLOSION: HYDRODYNAMIC INSTABILITIES AND TURBULENCE IN PRESUPERNOVAE

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ABSTRACT

Both observations and numerical simulations are discordant with predictions of conventional stellar evolution codes for the latest stages of a massive star’s life before core collapse. The most dramatic example of this disconnect is in the eruptive mass loss occurring in the decade preceding Type IIn supernovae. We outline the key empirical evidence that indicates severe pre-supernova instability in massive stars, and we suggest that the chief reason that these outbursts are absent in stellar evolution models may lie in the treatment of turbulent convection in these codes. The mixing length theory that is used ignores (1) finite amplitude fluctuations in velocity and temperature and (2) their nonlinear interaction with nuclear burning. Including these fluctuations is likely to give rise to hydrodynamic instabilities in the latest burning sequences, which prompts us to discuss a number of far-reaching implications for the fates of massive stars. In particular, we explore connections to enhanced pre-supernova mass loss, unsteady nuclear burning and consequent eruptions, swelling of the stellar radius that may trigger violent interactions with a companion star, and potential modifications to the core structure that could dramatically alter calculations of the core-collapse explosion mechanism itself. These modifications may also impact detailed nucleosynthesis and measured isotopic anomalies in meteorites, as well as the interpretation of young core-collapse supernova remnants. Understanding these critical instabilities in the final stages of evolution may make possible the development of an early warning system for impending core collapse, if we can identify their asteroseismological or eruptive signatures.

Key words: instabilities – meteorites, meteors, meteoroids – nuclear reactions, nucleosynthesis, abundances – stars: mass-loss – supernovae: general – turbulence

1. INTRODUCTION

A significant subset of supernovae (SNe) appear to have suffered heavy and episodic pre-SN mass loss (Smith et al. 2011a). This has not been explained by standard stellar evolutionary models, nor have the effects of episodic mass loss been included in them (see Langer 2012 and Maeder & Meynet 2000). This may partly be because stellar evolution calculations are one dimensional and have large time steps that damp dynamic effects; in addition, many do not even progress past carbon burning. Such an approach may be valid for lower masses; however, at higher masses, several physical processes conspire to cause problems: (1) luminosities near the Eddington limit, (2) adiabatic exponent γ approaching 4/3 (or less) as a result of formation of electron–positron pairs, (3) complex and unstable nuclear burning (quasi-equilibrium and nuclear statistical equilibrium during oxygen and silicon burning), and (4) accelerated evolution caused by copious energy loss by neutrinos. All of these effects can become important and problematic if the stars suffer hydrodynamic instability, and these effects could lead to significant errors in models of pre-SN stars.

There is empirical evidence of pre-SN hydrodynamic instability in both evolved massive stars in the Milky Way and the Local Group, and in the dense circumstellar material (CSM) that must exist around Type IIn SNe and related events (see Section 2 for details). In the Milky Way and in other local galaxies, the massive stars that are most notoriously observed to experience violent episodic mass loss are the luminous blue variables (LBVs; Conti 1984; Humphreys & Davidson 1994; Smith et al. 2004, 2011c; Clark et al. 2005). This class of objects unifies a number of different specific types of blue supergiant variable stars, not all of which are necessarily in identical evolutionary phases, and most of which have persisted without a SN explosion for centuries. Most relevant are the so-called giant eruptions of LBVs, exemplified by the nineteenth century Great Eruption of η Carinae (see Section 2.1), although η Car is perhaps the most extreme example of the phenomenon. Extragalactic Type IIn SNe, named for their narrow H emission lines, are unique among SNe because they require CSM that is massive enough to decelerate the fast SN ejecta, allowing radiation from slow, heated material to dominate the observed spectrum. For the shock interaction to occur simultaneously and veil the broad lines from the underlying SN photosphere, this CSM must reside very close to the star (within a few 1015 cm), and its ejection by the star must therefore have been synchronized to occur within only a few years of core collapse. This synchronicity, fully appreciated only in the past decade or so, hints at some as-yet-unidentified phenomenon associated with the final nuclear burning phases in the star’s life. We explore this topic in more detail in this paper.

It has been proposed that eruptive LBV-like events are connected to SNe IIn (Smith et al. 2007, 2008, 2010a; Gal-Yam et al. 2007), but this has been controversial. It is in direct conflict with stellar evolution models for massive stars, all of which currently assume that massive stars with $M_{\text{ZAMS}} \gtrsim 40 M_\odot$ at near-solar metallicity will shed their H envelopes through steady winds to make Wolf–Rayet stars before core collapse as SNe of Type Ibc.4 With currently adopted mass-loss rates (see Smith

4 This conflict with stellar evolution models persists even if we admit that the “LBV-like” pre-SN eruptions inferred for SNe IIn may not necessarily be from the same unstable stars that make up nearby LBVs (many of which have not exploded within centuries of a past giant LBV eruption). The conflict is mainly in the substantial mass of H left in the envelopes of these very massive stars at the time of death.
Despite the giant eruptions themselves are rarely observed because they are infrequent and considerably fainter than SNe, a large number of LBVs and spectroscopically similar stars in the Milky Way and Magellanic Clouds are surrounded by massive shell nebulae, indicating previous eruptions with a range of ejecta masses from 1 to 20⊙ (Clark et al. 2005; Smith & Owocki 2006; Wachter et al. 2010; Gvaramadze et al. 2010). Thus, LBV mass loss is inferred to be important in late evolution of massive stars.

In the past, LBVs have been cast as super-Eddington winds driven by an increase in the star’s bolometric luminosity (Humphreys & Davidson 1994; Shaviv 2000; Owocki et al. 2004; Smith & Owocki 2006), but there is growing evidence that some of them are explosive hydrodynamic ejections (see Smith 2008, 2013). Of course, these two cases (long-lived super-Eddington winds or sudden hydrodynamic explosions) may both operate.

2.2. Type IIn Supernovae and Pre-SN Mass Loss

As noted earlier, the narrow H lines in SNe IIn require very dense CSM ejected shortly before core collapse. As the SN blast wave expands into the CSM, the SN ejecta are decelerated and the slow CSM is heated (e.g., Chugai et al. 2004; Chugai & Danziger 1994). Through conservation of momentum, one can infer the mass of CSM required to decelerate the fast SN ejecta (Smith et al. 2010a). Special cases of this are the super-luminous SNe IIn (SN 2006gy, SN 2006ft, SN 2003ma, etc.) where the bolometric luminosity of the CSM interaction demands very large masses up to 20M⊙ ejected a few years before core collapse (Smith & McCray 2007; Smith et al. 2007, 2008, 2010a; Woosley et al. 2007; Rest et al. 2011). For the characteristic Type IIn spectrum and high luminosity from CSM interaction to occur immediately after explosion, the CSM must be very close (within few to several 1013 cm) of the star; from the widths of narrow lines in spectra (typically 100–600 km s−1; Smith et al. 2008, 2010a; Kiewe et al. 2012), the mass ejection must have occurred just a few years earlier. This strongly hints that something violent (i.e., hydrodynamic) may be happening to these stars during O and Ne burning, such that the star’s outer layers are already affected by the impending core collapse.

2.3. Earlier Mass Loss

Some SNe IIn (and even some other SN types) show indications of more distant CSM at radii of 103 AU (1.5 × 1016 cm) or more, suggesting heavy mass loss that occurred over a longer timescale than just a few years or a decade (at 100 km s−1 it takes 150 years to reach a thousand AU). The evidence for this is that some SNe IIn remain luminous and show signatures of strong CSM/shock interaction for years as the shock continues to overtake more distant CSM ejected in centuries to millennia before the SN. Many SNe IIn show bright infrared echoes from

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2.3. Earlier Mass Loss

Some SNe IIn (and even some other SN types) show indications of more distant CSM at radii of 10^3 AU (1.5 × 10^16 cm) or more, suggesting heavy mass loss that occurred over a longer timescale than just a few years or a decade (at 100 km s\(^{-1}\) it takes 150 years to reach a thousand AU). The evidence for this is that some SNe IIn remain luminous and show signatures of strong CSM/shock interaction for years as the shock continues to overtake more distant CSM ejected in centuries to millennia before the SN. Many SNe IIn show bright infrared echoes from

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distant, dusty CSM heated by the SN luminosity, with massive shells again residing at radii of up to a few light years (Fox et al. 2011). Some non-IIn explosions show infrared echoes and may have dense CSM shells; an illustrative example is SN 1987A, where the SN blast wave began crashing into a CSM ring located 0.2 pc from the star after a delay of 10 years. Thus, both persistent CSM interaction and infrared echoes suggest that heavy mass loss is not limited to only the few years preceding core collapse. On timescales of 100–1000 years or more, this may indicate that enhanced episodic mass loss may be linked to C burning and possibly even He burning. Binary interaction may play a role. These more distant (older) CSM shells are harder to detect, so we do not have a reliable census of what fraction of all SN progenitors experience this phenomenon.

2.4. Progenitor Eruptions

There have now been two clear direct detections of a precursor eruption, seen as a transient source just a few years before the SN. SN 2006jc was the first object clearly seen to have a brief outburst 2 years before a SN. The precursor event in 2004 had a peak luminosity similar to that of η Car (absolute magnitude of −14) and was fairly brief (only a few weeks; Pastorello et al. 2007). No spectra were obtained for the transient source, but the SN explosion 2 years later had strong narrow emission lines of He, indicating dense CSM (Pastorello et al. 2007; Foley et al. 2007).

A much more vivid case was SN 2009ip, mentioned earlier. It was initially discovered and studied in detail as an LBV-like outburst in 2009, again with a peak absolute magnitude near −14 and a spectrum similar to LBVs. A quiescent progenitor star was detected in archival Hubble Space Telescope (HST) data taken 10 years earlier, which indicated a very massive star of 50–80 $M_\odot$ (Smith et al. 2010b; Foley et al. 2011). The object then experienced several similar eruptions over three years that looked like additional LBV eruptions (unlike SN 2006jc, detailed spectra of these precursor outbursts were obtained), culminating in a final SN explosion in 2012 (Mauerhan et al. 2013; Smith et al. 2014). The SN light curve was double-peaked, with initially faint bump (−15 mag) that had very broad emission lines (probably the SN ejecta photosphere), and it rose quickly 40 days later to a peak of −18 mag, when it looked like a normal SN IIn (caused by CSM interaction, as the SN crashed into the slow material ejected 1–3 years earlier; see Mauerhan et al. 2013; Smith et al. 2014). A number of detailed studies of the SN have now been published (Mauerhan et al. 2013; Prieto et al. 2013; Pastorello et al. 2013; Fraser et al. 2013; Smith et al. 2013, 2014; Margutti et al. 2014; Ofek et al. 2013a). A related case is the Type IIn SN 2010mc (Ofek et al. 2013b), which had a double-peaked light curve that was nearly identical to SN 2009ip (Smith et al. 2013, 2014), but did not have the same extensive pre-SN observations or a detection of precursor outbursts. While there was initially some debate about the nature of these objects, their late-time data revealed them to be true core-collapse events, and Smith et al. (2014) demonstrated that their double-peaked light curves were well explained as core-collapse SNe from blue supergiants, but with strong CSM interaction.

Another possible case of a detected pre-SN outburst is the historical object SN 1961V (Smith et al. 2011c; Kochanek et al. 2011); its pre-1961 photometry may have indicated a precursor outburst, but it was far less clearly delineated than in SN 2006jc and SN 2009ip. In any case, if we take the full class of SNe IIn, which represent 8%–9% of all core-collapse SNe in large galaxies (Smith et al. 2011a), the phenomenon is far too common to be caused by the pulsational pair instability that operates in very massive stars (as subsequently discussed).

2.5. Progenitor Star Detections

We now have four directly detected quiescent progenitors of SNe IIn with rough mass estimates. (1) SN 2005gj with an implied initial mass of roughly 60 $M_\odot$ (Gal-Yam et al. 2007; Gal-Yam & Leonard 2009), (2) SN 1961V with a very luminous progenitor indicating an initial mass of order 100–150 $M_\odot$ or more (Smith et al. 2011c; Kochanek et al. 2011), (3) SN 2009ip (50–80 $M_\odot$; see above), and (4) SN 2010jl with a likely progenitor mass of > 30 $M_\odot$ (Smith et al. 2011b). In this last case, however, the SN has not yet faded, so the candidate progenitor might be a massive young cluster; if so it still suggests a massive star above 30 $M_\odot$ (Smith et al. 2011b). All four cases suggest progenitor stars that are much more massive than the typical red supergiant progenitors of SNe II-P (Smartt 2009). (This does not, however, preclude the possibility that some lower-mass stars produce SNe IIn as well, since the most luminous LBV-like progenitors are the easiest to detect.)

2.6. Statistics

From a volume-limited sample in a survey with controlled systematics, Type IIn SNe appear to make up roughly 8%–9% of all core-collapse events (Smith et al. 2011a). To get a sense of what this might mean, we note that it corresponds roughly to the ratio of the number of stars in the range 30–100 $M_\odot$ to the total number in 8–100 $M_\odot$ (Arnett 1996, Table 14.4). SNe IIn are the most diverse and poorly understood of all SN types—any SN type can appear as a Type IIn if it has dense CSM. Some SNe IIn appear to be Type Ia with CSM, and some appear to be low-energy electron capture SNe from 8–10 $M_\odot$ stars, but the majority appear to be from more massive LBV-like stars (see, e.g., Smith 2014 and references therein).

These statistics indicate that the precursor eruptions leading to SNe IIn are far too common for all of them to be attributable to the pulsational pair instability (Arnett 1996, Section 11.7; Woosley et al. 2007; Heger & Woosley 2002; Chatzopoulos & Wheeler 2012), which is expected to occur only in very rare, very massive stars with initial masses around 100 $M_\odot$ or more. Some other instability must operate in most SNe IIn.

The relative rarity of SNe IIn is also important. As we consider implications for the dominant physical mechanism that may lead to pre-SN eruptions, we must be mindful that the majority of SNe (≈90%) do not suffer violent precursor outbursts that are powerful enough to yield dense, observable CSM—and it is important to know why. Is the relevant instability significant only at higher initial mass and luminosity (Section 3, 4.1, 4.2)? Does the small fraction of SNe IIn reflect rare interactions in a binary system at the right separation (e.g., Section 4.3) or some other special circumstance? The process is rare but not

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6 While our manuscript was in the review process, Fraser et al. (2013) reported the direct detection of another pre-SN outburst in archival data one year before the Type IIn-P event SN 2011ht.

5 With no bolometric correction, this is a luminosity of $\sim 10^{41}$ erg s$^{-1}$.

7 The Type IIn spectrum of SN 2006jc is quite significant. Strong narrow He lines (and very weak H lines) indicate a He-rich/H-poor CSM. This, in turn, implies that the progenitor probably had a relatively compact stellar radius. The fact that SN 2006jc exhibited the same type of pre-SN eruptions as seen in SNe IIn indicates (1) that an extended stellar radius and encounters with a companion may not be the only explanation for precursor outbursts and (2) that something deeper is at work. Woosley et al. (2007) suggested that the precursor outburst of SN 2006jc might have been a pulsational pair instability eruption.
too rare. Do stars of lower initial mass (e.g., $10^{-30} M_\odot$) that make the majority of SNe also encounter similar instabilities before core collapse, but they are less powerful and do not cause hydrodynamic ejection in these stars? Is there a continuum in energy and mass ejection in pre-SN eruptions that extends below what can readily be detected in bright SNe IIn, such that some fraction of "normal" SNe may also suffer pre-SN instability? If the pre-SN activity is too weak to cause severe mass ejection, does it nevertheless have potential implications for the initial conditions for core collapse (Section 4.4)? Although these questions are inspired by observations and address some issues, observations alone do not yet yield definitive answers. In the next section, we discuss these issues from a theoretical perspective.

3. INSIGHTS FROM THREE-DIMENSIONAL SIMULATIONS OF TURBULENCE IN LATE STAGES

In parallel to the observational progress outlined in Section 2, theoretical progress—in the form of direct numerical simulations of two- and three-dimensional turbulence in late-burning stages of stars—has provided valuable insights into the physical nature of this evolution (Baz`an & Arnett 1994, 1998; Asida & Arnett 2000; Meakin & Arnett 2006, 2007; Arnett & Meakin 2011a; Viallet et al. 2013). These simulations indicate that one-dimensional models, as currently implemented, will not capture essential aspects of turbulent convection (especially its time-dependent and fluctuating nature) or its influence on stellar structure and evolution. This, in turn, may lead to inaccuracies in pre-SN stellar structure that are fundamentally important.

In what follows, we use the following common terms in a precise way.

1. Perturbation: a change in a variable that is small in comparison with its average value, so that linear perturbation theory is valid.
2. Fluctuation: a change in a variable that is comparable to its average value such that nonlinear terms are important and linear analysis is invalid.
3. Eruption: an event in which the rate of change of the average value of a variable (the “background”) is comparable to that of fluctuations. Such events are often associated with mass loss because, by this definition, the energy of the eruption is comparable to the internal energy and, hence, the binding energy of the star.

In the stellar context, eruptions often lead to mass loss. They may be especially violent and significant in stars near the Eddington limit with loosely bound envelopes.

3.1. An Example of O-burning in Three-dimensions

Significant progress has been made in understanding such turbulent, dynamic behavior theoretically, by careful analysis of well-resolved three-dimensional numerical simulations. Meakin & Arnett (2007) examined the burning of $^{16}$O through eight turnover times in a $23 M_\odot$ star, a few days before core collapse (as estimated by one-dimensional models). There was no other significant burning on the grid, and the initial state was early in oxygen shell burning. The initial evolution in this three-dimensional simulation was a rapid readjustment in which a dynamically self-consistent set of pressure and velocity fluctuations grew from low-level noise (turbulent kinetic energy increased from less than $10^{42}$ to more than $10^{47}$ erg) to form a convective region. The lack of consistent turbulent fluctuations is a common characteristic of one-dimensional models. They are not self-consistent regarding the dynamics of convection. Even well-chosen guesses at initial velocities do not help because they must be properly phased relative to entropy fluctuations, information we do not have from one-dimensional models. The natural state of the convection, which develops in another turnover time, is dynamic; and it is structured in a complex way in both space and time, as demanded by the fluid flow equations in the turbulent regime.

As the simulation in Meakin & Arnett (2007) advanced, the convection proceeded with pulses of turbulent kinetic energy. The burning was mild, almost constant, and the fluctuations in kinetic energy and luminosity instead came from the turbulent flow. Arnett & Meakin (2011b) show that this dynamic behavior is similar to that generated in the Lorenz (1963) model of a convective roll, well known in meteorology and dynamic systems theory. During this early mild stage of oxygen burning, the fluctuations did not yet induce significant changes in the energy generation rate, although fluctuations in turbulent kinetic energy and in enthalpy flux give variations of more than 50%.

3.2. Simulating a More Extreme Case

Of particular relevance are the subsequent results of Arnett & Meakin (2011a) in which it was shown that two-dimensional simulations, of combined O and Si burning shells in a $23 M_\odot$ star, evolved from a quasi-steady state into eruptive behavior. The eruptions are violent and probably cause extensive mass loss but do not blast the core completely apart. In this more extreme case, the fluctuations in temperature induced by turbulent flow do affect the energy generation rate and allow coupling between the flow, the oxygen burning, and the silicon burning. This is a new and complex problem, and the time dependence of burning may be of fundamental importance to observed transient sources. The prediction of this eruptive behavior is surprising, being entirely different from that inferred from one-dimensional stellar evolution codes, for which O and Si burning stages are assumed to be quasi-static. However, indications of this violent behavior have been seen in all hydrodynamic simulations of Si burning that could show it (Baz`an & Arnett 1998; Meakin 2006; Arnett & Meakin 2011b). Another example of violent, nondisruptive behavior is the pulsational electron–positron pair instability SN models mentioned in Section 2.6, but they are extremely rare because they are limited to the massive cores arising from stars with extremely high initial mass. Unsteady nuclear burning may be spread across a wider range of initial mass.

3.3. The Importance of Fluctuations

The earlier calculations we reviewed illustrate that there is an important limitation in the standard one-dimensional approach to the late stages of stellar evolution. It ignores finite amplitude fluctuations that have nonlinear interactions with the nuclear burning. Here, fluctuations differ from perturbations in that the nonlinear terms are not small enough to discard; thus, the assumption of quasi-static burning is invalid.

Linear stability theory (e.g., Murphy et al. 2004) will not capture this behavior because the driving and damping terms are nonlinear (they involve second- and third-order correlations between fluctuations; Arnett & Meakin 2011b). The problem gets worse (the nonlinear effects get bigger) as neutrino emission accelerates the evolution of the star in the final moments approaching core collapse (Fowler & Hoyle 1964). Unno et al. (1989) have stressed that the standard version of stability
theory now used is missing a potentially important term: the dynamic effects of convection, which is on the same level as the terms giving rise to the excitation mechanisms for the standard instabilities (e.g., the $\epsilon$, $\kappa$, and $\delta$ mechanisms; Unno et al. 1989, p. 241).

Arnett & Meakin (2011b) have derived a turbulence model as a possible replacement for mixing length theory (MLT), with the constraint that it reproduce the dynamic, fluctuating behavior seen in the three-dimensional simulations (Meakin & Arnett 2007). It includes two advances that were published in the Western literature only after the original work of Erika Böhm-Vitense (Böhm-Vitense 1958): the Lorenz model (Lorenz 1963) and the Kolmogorov damping at small scales (Kolmogorov 1962); see Arnett & Meakin (2011b) for details. We will rederive the Böhm-Vitense equation of MLT, upon which most stellar evolutionary codes are based, and we will show that it artificially suppresses fluctuations. This illustrates, explicitly and analytically, that conventional stellar evolution ignores a major dynamical aspect of late stages of stellar evolution—the same stages during which the existence of eruptions and explosive mass loss now have firm observational evidence (see Section 2), as well as confirmation in the multidimensional fluid-dynamic simulations (Arnett & Meakin 2011a).

### 3.4. MLT and the Lorenz Model

It has long been known that nothing like an MLT blob appears in multidimensional well-resolved simulations (e.g., Freytag et al. 1996; Stein & Nordlund 1998; Bazán & Arnett 1998); yet, MLT has proven so useful that it is still the algorithm that is used in most stellar evolution codes (for a review, see Langer 2012).

Perhaps there is some truth in the mathematics of the MLT equations, if not the physics of the blob picture. If so, then it should be possible to derive the same (or very similar) equations from a conceptual picture that is more consistent with the behavior seen in the simulations. We do this in the next section.

The mathematical basis for MLT as used in stellar evolution is the Böhm-Vitense cubic equation (Vitense 1953; Böhm-Vitense 1958), which may be written as follows:

$$x^3 + \frac{8}{9} U (2Ux + x^2 - W),$$

where $x = \sqrt{(\nabla - \nabla_e)}$ and $W = \nabla T - \nabla_e$ (see Kippenhahn & Weigert 1990, Section 7.2 for this notation). This equation is constructed from three equations (their Equations (7.6), (7.14), and (7.15)):

1. $v^3 = u_0^3 (\nabla - \nabla_e) \frac{\ell_m^2}{8 H_P}$
2. $\nabla e - \nabla u = 2U \sqrt{\nabla - \nabla_e}$
3. $(\nabla - \nabla_e)^2 = \frac{8}{9} U (\nabla e - \nabla e)$

Here $u_0^3 = g\beta_T H_P$, and

$$U = \frac{3acT^3}{C_P \rho^2 k \ell_m^2} \sqrt{\frac{8 H_P}{g\beta_T}},$$

where $H_P$ is the pressure scale height, $\ell_m$ the mixing length, $\beta_T$ the coefficient of thermal expansion, $C_P$ the specific heat at constant pressure, and the other symbols have their usual meaning. The expressions $\nabla$, $\nabla_e$, $\nabla_u$, and $\nabla e$ represent the dimensionless temperature gradients $((\partial \ln T/\partial \ln P)_i)$, for the background, the convective element, the adiabat, and that temperature gradient required to carry all the luminosity, respectively.

### 3.5. The Vortex Model

MLT is not unique; we illustrate this with another theoretical model that can produce a cubic equation like that of Böhm-Vitense (Equation (1)). To construct an alternative, we begin with the Lorenz (1963) model of a convective roll (see Figure 1), which has been shown to exhibit key properties of the simulation behavior as noted earlier (Arnett et al. 2009; Arnett & Meakin 2011b). This model includes chaotic behavior and finite fluctuations in velocity and luminosity. The dissipation is assumed to be that implied by the Kolmogorov cascade, which affects the effective Peclet number for the flow (Arnett & Meakin 2011b). Transformation of variables for nondimensionality leads to the following:

$$\tau = \frac{t K}{\ell K},$$
$$X = \frac{u}{(2/\ell K)},$$
$$Y = \frac{T_3 (g/2\Gamma KT_0)}{},$$
$$Z = \frac{T_4 (g/2\Gamma KT_0)}{}.$$

where $\sigma = \Gamma/K$ is the Prandtl number, $\Gamma$ is the inverse of the effective viscous damping time, $K = \nu_T (2/\ell)^2$ is the thermal relaxation time, $\nu_T$ is the thermal diffusivity, $t$ is the time variable, $\ell$ is the diameter of the convective roll, and $u$ is the convective speed around the roll. The aspect ratio of the roll is $a$ so that $b = 4/(1 + a^2)$ deals with the excess in vertical over horizontal heat conduction. Lorenz took $b = 8/3$ and $\sigma = 10$. These values give behavior that is similar to that found in three-dimensional simulations; see Arnett & Meakin (2011b) for further discussion.

$^{10}$ The Peclet number is the ratio of the advective transport rate to the diffusive transport rate. In a turbulent medium, its definition requires some care because of the turbulent cascade.

$^{11}$ The Prandtl number is the ratio of the momentum diffusivity to the thermal diffusivity.
Figure 1 illustrates the temperatures around the roll. The mean temperature at the midplane of the roll is \( T_0 \) and the amplitude of the variation in background temperature in the vertical direction is \( T_1 \). The fluctuating temperature amplitude in the horizontal direction is \( T_3 \); in the vertical direction, it is \( T_2 \). It is convenient to define the temperature amplitude difference \( T_4 = T_1 - T_2 \). These are “potential temperatures,” variations that are relative to the adiabatic run of temperature.

We have the following:

\[
\frac{dX}{d\tau} = -\frac{X}{2} + \sigma Y \tag{7}
\]

\[
\frac{dY}{d\tau} = -XYZ + rX - Y \tag{8}
\]

\[
\frac{dZ}{d\tau} = XY - bZ, \tag{9}
\]

where \( r = (g\ell/2CP T_1) \sigma \) is the Rayleigh number.\(^{12}\) The only functional change with respect to the original set of equations (Lorenz 1963) is the dissipation term in the \( dX/d\tau \) equation, with \( \sigma X \rightarrow X/2 \). This is because, for stellar dimensions, viscosity is effective at the end of the turbulent cascade, not at the integral scale of the roll.

Equations (7), (8) and (9) define a vortex model for convection if we use the identifications made in Section 3.6 for stellar variables. Numerical solution of these equations in the context of hydrodynamic stellar structure gives rise to chaotic behavior similar to that found by Lorenz (1963); see Arnett & Meakin (2011b).

3.6. The Steady Vortex Model

To make contact with MLT, we take the steady-state solutions of the vortex model, which we call the “steady vortex” model. We then have three algebraic equations:

\[
X^2 = 2\sigma Y, \tag{10}
\]

\[
rX = XZ + Y, \tag{11}
\]

\[
XY = bZ, \tag{12}
\]

as in Lorenz (1963) for his steady-state case. In MLT, \( X|X| \rightarrow X^2 \) by definition. Converting back to dimensional variables (see Figure 1 and Arnett & Meakin 2011b), the first equation becomes the following:

\[
\frac{uT_2}{T_0} = \frac{2}{\ell} \frac{T_3}{T_0}, \tag{14}
\]

where

\[
\frac{v_T}{\ell} = \frac{4aT^3}{pc_P 3 \beta k^4}, \tag{15}
\]

To keep the algebra concise, we define

\[
\bar{U} = \left( \frac{2H_P}{\ell_m} \right)^2 \frac{v_T}{H_P u_0}, \tag{16}
\]

which is similar to the mixing length quantity \( U \) previously defined,

\[
U = \frac{9\sqrt{2}}{8} \bar{U}. \tag{17}
\]

The third equation becomes the following:

\[
\frac{uT_3}{T_0} = \frac{2}{\ell} \frac{T_1 - T_2}{T_0}. \tag{18}
\]

To once again make contact with MLT variables, we equate finite differences of amplitudes to derivatives. The temperature field in the vortex model is two-dimensional (horizontal and vertical), not one-dimensional (radial) as in MLT. The horizontal derivative in density (temperature) is needed to drive a buoyant torque, so we identify the following:

\[
\frac{T_1}{T_0} \frac{\ell}{2} = \left( \nabla - \nabla_e \right)/H_P \tag{19}
\]

and Equation (13) becomes the following:

\[
u^2 = u_0^2 (\nabla - \nabla_e) \frac{\alpha_d \ell^3}{4H_P^2}, \tag{20}
\]

which is to be compared to Equation (2). With the appropriate choice of mixing length \( \ell_m \) they are identical. This implies

\[
\ell_m = \sqrt{2a_d \ell} \approx \sqrt{1.6 \ell} \approx 1.26 \ell. \tag{21}
\]

In the Lorenz model, \( \ell \) has a precise meaning: the diameter of the roll; in MLT, \( \ell_m \) is an adjustable parameter of order \( H_P \). Here, \( u_0 \) appears as a natural velocity scale, where \( u_0^2 = g\beta T H_P \).

We further identify the vertical temperature gradient relative to the adiabatic gradient by the following:

\[
\frac{T_2}{T_0} \frac{\ell}{2} = \left( \nabla_e - \nabla_a \right)/H_P, \tag{22}
\]

and the imposed vertical gradient relative to adiabatic as follows:

\[
\frac{T_1}{T_0} \frac{\ell}{2} = \left( \nabla_r - \nabla_a \right)/H_P. \tag{23}
\]

Using Equations (19) and (22) with (14) gives the following:

\[
\nabla_e - \nabla_a = 2\bar{U} \sqrt{\nabla - \nabla_e}, \tag{24}
\]

which is to be compared to Equation (3). The two factors have many components in common; equating them implies the following:

\[
\ell_m = \frac{3}{2} (2\alpha_d)^{\frac{1}{2}} \ell \approx 1.7 \ell, \tag{25}
\]

\(^{12}\) The Rayleigh number indicates the onset of buoyant convection.
which relates the MLT mixing length to the diameter of the convective roll. This differs from Equation (21), but not drastically. This contradiction stems from the fact that in MLT, one adopts the length of a blob mean free path as being the same as its size for radiative cooling (Arnett et al. 2010); this is related to the geometric factor problem.

Using Equations (19), (22), and (23) with (18) gives

\[(\nabla - \nabla_e)^2 = b^2 2\bar{U}(\nabla_e - \nabla),\]

which is to be compared with Equation (4). These two equations have two differences. First, Equation (4) has a factor \(\nabla_e - \nabla\), while Equation (26) has an apparently different factor \(\nabla_e - \nabla_e\). This equation arises from the condition that the total flux is the sum of convective and radiative fluxes (flux of turbulent kinetic energy is ignored in both cases). Both theoretical models require this to be true, but they define the radiative flux differently in the convective zone. Outside the convective region, \(\nabla_e = \nabla = \nabla_L\); thus, there is no difference. Inside the convective region, the Lorenz model takes the whole region to be the convective “element” (this resembles the simulations), so that the radiative flux is that of the element. MLT assumes that the element is distinct from the background and that the radiative flux is that of the background. This difference is large only in the superadiabatic region, where neither theory resembles simulations nor is self-consistent (see above). In most stars, the thickness of the superadiabatic region is determined by the hydrogen recombination region and is much less than a pressure scale height. Such details are ignored in both MLT and the vortex model, but they are prominent in simulations (Stein & Nordlund 1998; Nordlund et al. 2009).

The second difference is in the coefficients multiplying these factors; equating them gives the following:

\[\frac{8}{9} U = b^2 2\bar{U} \quad (27)\]

Removing the common factors gives the following:

\[\ell_M = \frac{1}{b}(2\alpha_d)^{\frac{1}{2}} \ell \approx \frac{3}{8}(1.6)^{\frac{1}{2}} \ell \approx 0.422\ell. \quad (28)\]

While the choices (Equation (21), (25), and (28)) for mixing length needed for equality are surprisingly similar, they are not identical given the different conceptual foundations used. Note that Equations (21) and (25) are necessarily inconsistent if we make the reasonable choice of using the \(a_d\) determined by simulations. At this level of precision, MLT is inconsistent with hydrodynamics.

The Lorenz model is mathematically precise for the single mode of flow that was chosen; simulations show that several low-order modes (~5) dominate, such that a single mode picture is oversimplified; e.g., see Arnett & Meakin (2011b). The MLT derivation has a plethora of factors of two and of geometry, which are not unique, and MLT has adjustable parameters. It is unlikely that this difference in factors (of the order of unity) is significant.

Both MLT and Lorenz convection assume symmetry between upflows and downflows; this is inconsistent with simulations, which due to up-down asymmetry, have non-zero acoustic \((F_P)\) and kinetic energy \((F_K)\) fluxes (Meakin & Arnett 2007; Arnett et al. 2009; Viallet et al. 2013). The asymmetry increases for increasing stratification.

3.7. Böhm-Vitense Equation Recovered

As we have seen, the steady vortex model produces three basic algebraic equations almost identical to those making up the Böhm-Vitense version of MLT. Combining Equations (20), (24), and (26) gives an equation strikingly akin to Equation (1) (the Böhm-Vitense cubic equation),

\[x^3 + 2\bar{U}(2\bar{U}x - W) = 0. \quad (29)\]

These differ in (1) \(U\) and \(\bar{U}\) as discussed earlier, and (2) the lack of an \(x^2\) term given the different definition of radiative flux in a convective zone. The \(x^2 = \nabla - \nabla_e\) term may be neglected in both the large \(x\) and small \(x\) limits, and it has only a modest effect in the transition region. Equations (1) and (29) have the same asymptotic behavior and similar constant factors. Physically, this transition between large and small \(x\) corresponds to that between the adiabatic gradient and the radiative gradient. The central regions of massive stars are never in this transition region; it occurs in the Sun (and other stars) near the surface, associated with the hydrogen ionization zone. On purely mathematical grounds, the steady vortex model must give comparable agreement with observations as MLT. Inside a stellar evolutionary code, the two are almost identical, and any small differences may be removed by slight adjustments of assumptions involving flow patterns, or mixing length and geometric parameters, for example.

The mathematical equivalence of MLT and the steady vortex model via the Böhm-Vitense equation should serve as a warning. The physical pictures used in the two models are different; therefore, the interpretation of MLT in terms of physical processes is not unique. The steady vortex picture may prove more useful for physical interpretation—to the extent that it has connections to actual solutions of the Navier–Stokes equations in the form of three-dimensional numerical simulations.

Both MLT and the steady vortex picture lack fluctuations. For the steady vortex picture, this is because use of the steady-state solutions removes the terms that cause chaotic behavior (the Lorenz strange attractor). These terms do not exist in MLT.

Thus, Equations (10), (11), and (12) (the steady vortex model) have no chaotic behavior while Equations (7), (8), and (9) (the dynamic vortex model) do. This explicitly demonstrates a fundamental difference between MLT and the more general dynamic vortex model. It is equivalent to using a damping (to get the steady-state solution), and ignoring the dynamic aspects of turbulence. This neglect has potentially important consequences for models of the late stages of stellar evolution, which we discuss in the next section.

The dynamic vortex model, which is more directly based on the three-dimensional fluid dynamics equations than on MLT, allows a more direct way to generalize stellar evolution codes by including additional physics, and a better conceptual base to deal with the physics involved. It can be shown that use of an acceleration equation in a vortex picture provides a way to include dynamics, boundary layers, and the turbulent cascade into a simple convection model (W. D. Arnett et al., in preparation). In addition, it provides a natural link to the Richardson–Kolmogorov cascade and gives a balance between driving and damping of turbulent flow.

Stellar evolutionary models are based on an implicit assumption of stability. The most common tool for exploring instability is linear perturbation analysis (Cox 1980; Unno et al. 1989), which deals with convection poorly (although perhaps adequately for the relatively stable main-sequence phase). The
driving term for turbulence is quadratic, and the damping term is cubic; neither is linear, so linear analysis is blind to them. However, numerical hydrodynamics is not so blind, and such simulations clearly deviate from stellar evolution models in the latest stages of a massive star’s life, for which turbulent convection plays a dominant role. In this paper, we have precisely identified the source of the problem—the chaotic behavior that is implicit in a convective roll. Steady-state solutions become irrelevant when they are unstable; the Lorenz (1963) model of a convective roll, with its strange attractor (chaotic behavior), illustrates this. The three-dimensional simulations have similar chaotic behavior that give rise to convective pulses (Meakin & Arnett 2007) and perhaps eruptions (Arnett & Meakin 2011a).

Why not just simulate the whole star? The combination of the need for high resolution to capture the turbulence (high numerical Reynolds number is required) and sufficient volume to include both the core and active burning shells, results in a computational problem that strains present-day computer facilities. This is currently being attempted (C. Meakin & W. D. Arnett, in preparation), but it is appropriate now to examine the implications of this new perspective.

4. IMPLICATIONS

For models to provide a coherent picture of the observations, we need some process to be able to inject $10^{48}$ to $10^{50}$ erg\textsuperscript{13} in the final couple of years before core collapse (i.e., during Ne, O, and Si burning). We may need something similar with lower energy to be more prolonged (several hundred years) during C burning and possibly even He burning. While Ne burning is weak and Si burning complex, both C and O burning release about $5 \times 10^{47}$ erg g\textsuperscript{−1} of fuel burned (or equivalently, $10^{51}$ erg per $M_\odot$ of fuel burned), so that a modest amount of nuclear burning could directly provide the energy needed. This is relevant to the issue of shell instabilities (Arnett & Meakin 2011a).

Must the energy be supplied directly, or could it be the result of a process that is less than completely efficient? For core collapse to occur, at least $1.5 \times 10^{51}$ erg (1.5 bethes) must be released by nuclear burning to convert helium burning products into iron-peak nuclei.\textsuperscript{14} This is much larger than the gravitational binding energy of the star; to avoid explosion and mass loss this energy must be radiated away.

Neutrinos can remove the energy, but vigorous convection is demanded. It has long been known (Fowler & Hoyle 1964; Arnett 1996) that, because of the more sensitive temperature dependence of thermonuclear heating in comparison with neutrino cooling, a convection zone in thermal balance will have heating at the bottom and cooling at the top. This generates an entropy gradient that drives convective transport of heat upward. Preparing the core for collapse requires turbulent convection to transport 1.5 bethes of energy, a factor of 15 more than the largest value needed to explain the observed pre-collapse ejection of mass. Any inefficiency in the transport process can therefore contribute to the injection of energy into a star’s outer envelope. Depending on how much energy is diverted, the results can range from modest to catastrophic. It would take less than 7% of 1.5 bethes to supply the largest eruption energy ($10^{50}$ erg) required.

Next, we highlight six ways in which the dynamical instability associated with turbulent convection in late-burning stages might profoundly influence the star’s pre-SN evolution, the stellar structure at the moment of collapse, the observational interpretation of SN remnants, and isotopic abundances in meteorite pre-solar grains.

4.1. Enhanced Mass Loss

A characteristic of turbulence is the presence of chaotic fluctuations in luminosity and velocity. Convective velocities rise from $\sim 0.5 \text{ km s}^{-1}$ in He burning to 100 km s\textsuperscript{−1} in O burning, and 300 km s\textsuperscript{−1} in Si burning (Meakin & Arnett 2007; Arnett & Meakin 2011a). This is subsonic in the core (mach number $\sim 0.01$ to 0.1) but not in the envelope. Simulations show vigorous wave production at convective boundaries (Meakin & Arnett 2007). Waves generated by vigorous turbulence travel outward and dissipate (e.g., steepen into shocks); consequently, the envelope becomes an absorber for this energy. If the envelope is already near Eddington luminosity, it can rid itself of this extra energy only by dynamical expansion and mass loss. For a red giant structure, this would occur on a pulsation (sound travel) timescale, and it appear as an eruption. For a more condensed structure (a star stripped of its H envelope, for example), it might be seen as a vigorous wind.\textsuperscript{15}

Quataert & Shiode (2012) have discussed this mechanism of enhanced pre-SN mass loss driven by waves propagating through the star’s envelope as a result of furious convection during O and Ne burning. This type of mechanism has the potential to explain strong winds that may produce dense CSM in the few years leading up to core collapse, although it is unclear whether it can account for the most violent explosive events that occur.

4.2. Unsteady or Explosive Burning

The episodic nature of the observed pre-SN mass loss suggests that, in addition to strong winds that are quasi-steady state by definition, we should consider hydrodynamic instabilities in shell burning with energy injection by explosive/unsteady nuclear burning. Both would be in operation once neutrino cooling dominates over photon cooling (i.e., in the months and centuries before core collapse) from C burning to Si burning. There should be a wide variety of behavior because the instabilities and the wave transport will be different for different core-envelope structures and rotational rates, both of which also depend upon the history of any binary interaction.

To make a bomb requires an energy source and an ignition mechanism to release that energy. The 1.5 bethes that are available from burning the ashes of He burning to iron-peak nuclei is a more than ample source; only a small fraction is required. For an ignition mechanism, we already have the instability found by Arnett & Meakin (2011a). That simulation \textsuperscript{13} This range of energy comes from the most extreme events ($\eta$ Car. SN 2006gy’s precursor eruption) to the least extreme LBV-like giant eruptions (e.g., V12 in NGC 2403, SN 2000ch, precursors of SN 2006jc and SN 2009ip), plus a number of objects in between. These exhibit a range of $\sim 10^{48}$ in ejected mass at similar outflow speeds (see Smith et al. 2011c). Individual eruptions with energy well below $10^{49}$ erg may exist, but they could be difficult to detect in external galaxies. In contrast, energy injection of $10^{50}$ erg or less may be absorbed by the stellar envelope and might not lead to the hydrodynamic ejection that gives rise to a brief optical transient (e.g., Dessart et al. 2010).

\textsuperscript{14} A mass of $1.5 M_\odot$ converted from $\text{C}^{12}$ and $\text{O}^{16}$ releases about $5 \times 10^{17}$ ergs g$^{-1}$ in burning to $\text{Si}^{28}$ (Fowler & Hoyle 1964), or $5 \times 10^{17} \times 3 \times 10^{33} = 1.5 \times 10^{51}$ ergs. Further burning could release slightly more energy, but it is countered by neutrino loss from nuclear weak interactions (Arnett 1996).

\textsuperscript{15} Kleier & Kasen (2014) have recently argued that the features of some strange SNe could be explained by energy input into the oxygen shell during Si burning.
was terminated because the matter was flowing off the limited grid but already had violently excited the lowest order mode available to fluctuations, with every indication of continued growth. Mild shocks were already forming. Despite the severe challenge to computer resources, it is important to repeat these simulations in three-dimensions, on a $4\pi$ steradian grid, extended to later times (C. Meakin et al., in preparation). It is also important that the hydrodynamic algorithms used be nondamping; although anelastic, low Mach-number, and implicit methods may allow larger time steps (Viallet et al. 2011; Almgren et al. 2010), they must be validated to ensure that they give negligible artificial damping\footnote{This is a difficult and subtle problem; see Brown et al. (2012); Vasil et al. (2013).} for this problem, and that they seamlessly transition from turbulent to explosive flow.

There may be other instabilities to be found; the published library of well-resolved simulations of pre-collapse evolution is still small. While oxygen shell burning seems quasi-stable over eight turnovers (Meakin & Arnett 2007), it is vigorous; the possibility of eruption after a longer time remains open. Both C and Ne burning are more difficult to simulate directly because of their slower burning (which implies that more time steps are needed to calculate them over significant evolutionary times). We do not yet know whether they harbor nonlinear instabilities that would become evident over the timescales needed to consume C and Ne.

4.3. Triggering Violent Binary Interactions

There is another avenue for extremely sudden and violent events to occur. Recent observations of main-sequence O-type stars have shown definitively that the majority of massive stars (roughly two out of three) are born in binary systems with a separation small enough that the two stars will interact before they die (Sana et al. 2012; Chini et al. 2012; Kiminki et al. 2012). The fate of this interaction depends on the initial separation: The closest systems will exchange mass or merge on the main sequence, while binary systems with wider separations will interact only when the more massive star evolves off the main sequence and expands to fill its Roche lobe as a supergiant.

If the aforementioned mechanisms (unsteady or explosive burning, waves generated by vigorous convection) are able to inject an amount of energy into the star’s envelope that is not quite sufficient to completely unbind the envelope, the result may be a swelling of the star’s hydrostatic radius (i.e., a large pulsation). In that case, dramatic events may ensue if the star is in a binary system with an orbital period ranging from tens of days to a few years (depending on eccentricity). If the SN progenitor increases its radius significantly, a companion star that previously had been too distant to interact may suddenly find itself to be the victim of mass transfer, a merger, or a violent collision if it is in an eccentric orbit.

Consider the case of $\eta$ Car, where something similar is known to have occurred: $\eta$ Car is a binary with a 5.5 year orbital period and an eccentricity $e = 0.9-0.95$. Smith & Frew (2011) showed that brief ~100 day brightening events occurred at times of periastron, and Smith et al. (2011b) argued that a stellar collision must have occurred because the periastron separation was much smaller than the required photospheric radius at that time. Mauerhan et al. (2013) (1) pointed out that the multiple brief peaks in the few years leading up to the 2012 explosion of SN 2009ip were reminiscent of $\eta$ Car’s events and (2) suggested that violent periastron encounters may play a role. The SN impostor SN 2000ch may be yet another example (Pastorello et al. 2010; Smith et al. 2011c). A related idea was suggested by Chevalier (2012), who also included the intriguing possibility of mergers with a neutron star companion leading to SNe IIn, although he found that such events are probably too rare to account for the observed frequency of SNe IIn.

The possibility of energy injection that leads to envelope inflation might act to enhance the frequency of any such merger/collision events, because the progenitor suddenly finds itself to have a larger radius and is more able to interact with more widely separated companion stars (here, “suddenly” means a time comparable to the orbital period). More important, if the increase in stellar radius is a result of energy injected during C, Ne, O, or Si burning phases, it provides a physical reason to expect such merger or collision events to be synchronized to within only a few years\footnote{The actual duration of these phases of C, Ne, O, and Si burning might be modified by their dynamic behavior.} before core collapse.

4.4. Preparing the Core Structure for Collapse

Numerical simulations have yet to reliably produce successful SN explosions in a self-consistent way (Arnett 1996; Kitaura et al. 2006; Burrows et al. 2007; Liebendorfer et al. 2001a, 2001b; Rampp et al. 2002; Bursa et al. 2003; Thompson et al. 2003; Liebendorfer et al. 2005; Sumiyoshi et al. 2005; Fryer & Young 2007; Lentz et al. 2012). While debate continues over details involved in these calculations (Janka 2012), a unifying characteristic is that all results are highly sensitive to the core structure of the model progenitor. For example, this progenitor structure determines the rate at which mass rains down onto the proto-neutron star after core collapse (see O’Conner & Ott 2013, and their discussion of “compactness”; Ugliano et al. 2012). Real SNe are observed to explode nonetheless. Thus, it seems prudent to ask: what if the fault lies not so much in the details of the method of computing neutrino transport, the geometry (two-dimensional versus three-dimensional), or other aspects of the explosion, but in the structure of the progenitor itself?

The progenitor models most commonly used by the groups doing core collapse simulations are those of Woosley and collaborators (e.g., Woosley et al. 2002; Heger et al. 2000; Woosley & Weaver 1995), all of which use MLT. There is not yet good detailed agreement in the final stellar structure among the various evolution groups (see Georgy et al. 2012; Chieffi & Limongi 2013; B. Paxton 2013, private communication), even though they all use some version of MLT (this may be related to differing mixing algorithms). What effect might a proper treatment of convection and its associated hydrodynamic instabilities have on the stellar structure of a SN progenitor? Observations dictate that at least in some stars, pre-SN instabilities inject energy into the star’s envelope that leads to significantly enhanced mass loss or even explosive ejection. This is not predicted by evolution models (but is indicated by the hydrodynamic simulations), yet something must be providing substantial extra energy on short timescales before collapse. One may surmise that successful mass ejection before core collapse is most likely to occur in more massive stars, given that they are closer to the Eddington luminosity and have more loosely bound envelopes. What would happen, then, in lower-mass progenitor stars if the same process occurs but where the injected energy is insufficient to cause
observables eruptive/explosive mass loss? As in Section 4.3, one might infer that the outer layers may swell and that binary interaction might be more likely, or alternatively, that the mass loss might be less vigorous and the consequences less easily observed.

We will not know what the detailed changes will be until realistic three-dimensional simulations illustrate the strongly nonlinear behavior. However, some general trends are clear. For example, both the enhanced mass loss and unsteady burning move matter out of the potential well, changing the central condensation of the core and making the progenitors easier to explode by the neutrino-transport mechanism (O’Conner & Ott 2013; Ugliano et al. 2012). The “compactness” parameter of O’Conner & Ott (2013) is essentially the gravitational potential at the edge of the core: in hydrostatic equilibrium, it acts as a pivot point for the density profile, so that higher central density, and lower compactness, is accompanied by lower mantle density (see Figure 10.4 and 10.7 in Arnett 1996). The mapping of initial zero-age main sequence mass to mass of the core at collapse may also be altered. This is a complex problem because of unaddressed issues in one-dimensional evolution relating to convective boundaries (Meakin & Arnett 2007; Viallet et al. 2013) as well as large uncertainties in the treatment of mass loss (Smith 2014).

Turbulent fluctuations in the collapsing cores will also break spherical symmetry. They will be a result of both instantaneous driving by current burning and historical flows during the approach to collapse. Estimates for the amplitude and the form of these fluctuations are available. In the most advanced published simulation (Arnett & Meakin 2011a), there seems to be a complex interaction between O and Si burning shells, both wildly turbulent, driving pulsational modes of the core and mantle. The fluctuations (1) are a factor of 10 or more larger than perturbations used to test core collapse simulations and (2) are of global rather than local scale (see Fryer & Young 2007; Wongwathanarat et al. 2010, 2013 for simulations with lower amplitude fluctuations). Such fluctuations may lead to enhanced energy flow, and to increasing neutrino luminosity, so they may make explosion more likely. Since the posting of a preprint of our paper, simulations have confirmed our basic suggestion; Couch & Ott (2013) have explored just one aspect of this problem and have found encouraging results. Using the Arnett & Meakin (2011a) data, they imposed a modest nonradial velocity (momentum conserving), corresponding to an initial convective flow in the mantle of their collapsing core. At 100 ms after core collapse, this material passed through the stalled shock; by 200 ms, the core is exploding. This small modification of the velocity field was sufficient to change a dud into a successful explosion by allowing the revival of the shock, showing the critical importance of realistic initial models for core collapse.

4.5. Mixing in Presupernovae: Meteoritic Grains

The unsteady burning and its resulting eruptions may significantly change the mixing history of the progenitor. Convective boundaries are not sacrosanct; there is mixing across the boundaries even in quasiastatic phases (Meakin & Arnett 2007; Močák et al. 2008, 2009). Eruptions add to this, and turbulent mixing is faster and different from the diffusive convection algorithm currently in use.

Isotopes (nucl ei with the same proton number Z but different neutron number N) are good records of thermonuclear history because they are resistant to changes caused by chemistry (different Z). The development of precise microprobes and the discovery of grains in meteorites whose crystallization predated the solar system has given a new window into nucleosynthesis. The changed picture of presupernova evolution sketched earlier should have profound implications for the interpretation of isotopic anomalies in pre-solar grains in meteorites (e.g., see Pignatari et al. 2013; Travaglio et al. 1999; Lugaro et al. 2001). We have not only possible changes in the bulk character of the initial models and the explosion mechanism to consider, but also new processes. First, turbulent fluctuations involve not only thermodynamic fields (e.g., temperature or density) and velocity, but also composition. Each turbulent burning zone will introduce a “cosmic scatter” in the detailed abundances; see Figure 4 of Meakin & Arnett (2007). Second, advective mixing often proceeds by “plumes,” which allow matter from one level (and characteristic temperature and composition) to punch through other layers to mix in yet another level (with different composition), without homogenizing intermediate levels. This layer jumping may be more likely as the advection becomes more and more violent. There seem to be unresolved issues in isotopic anomaly interpretation that this might help. Third, these two new processes would occur before the explosion shock from core collapse, which ejects the mantle from the core. Thermonuclear processing during this explosion would therefore be altered.

4.6. Mixing Before, During, and after the Explosion: Young SN Remnants

Most of the interpretation of the observed asymmetries and mixing in young SN remnants has proceeded with the assumption that the explosion itself drove the asymmetries, rather than them being already developed prior to explosion. For core collapse, this presents a problem. To form a collapsed core implies a radial compression factor of 100 or so. Asymmetries grow on compression, but decrease on expansion (this is the fundamental problem of inertial confinement fusion; Lindl 1998). Core asymmetries will tend to be made smaller (more spherical) on expansion by a large factor as compared with preexisting asymmetries in the mantle. In addition, the mantle asymmetries are likely to be increased by local explosive burning.

Young core-collapse SN remnants may contain evidence for asymmetries directly related to turbulent fluctuations in pre-collapse phases. Tantalizing new maps of abundances in Cas A are now available (Milisavljevic & Fesen 2013; Isensee et al. 2012), which call for detailed comparison with multidimensional simulations (Ellinger et al. 2013). Preexisting density fluctuations in the mantle seem indicated (Isensee et al. 2012). Such fluctuations will be caused by the type of turbulent convection in the progenitor that we previously described. Advection turnover occurs on a timescale slightly longer than the explosive timescale, so that mixing will be partial, and will tend to preserve (in a distorted way) any layering in abundance. The more thoroughly burned matter will have higher entropy, and it will punch through overlying layers in some regions. Any $^{56}$Ni produced will decay and heat the plasma, giving a characteristic modification (Ni bubbles) of the explosive outflow (Milisavljevic & Fesen 2013).
5. CONCLUSIONS

Both observations of pre-SN mass loss and direct numerical simulations of convection are discordant with predictions of conventional stellar evolution codes for the late stages of massive star evolution. In this paper, we have discussed the observed nature of the most dramatic discrepancies (pre-SN eruptions), and we have proposed that the problem in one-dimensional stellar evolution codes lies in the treatment of turbulent convection. In particular, one-dimensional codes ignore (1) finite amplitude fluctuations in velocity and temperature and (2) their nonlinear interaction with nuclear burning. For most of a star’s life, this is probably a reasonable approximation (except perhaps near convective boundaries); however, in the latest phases of evolution, such fluctuations can become catastrophic in massive stars. The fluctuations are not allowed to occur in conventional one-dimensional codes that impose steady-state behavior, so their associated pre-SN eruptions are not predicted.

To derive MLT from a more general hydrodynamic formulation, the following assumptions are needed: (1) the damping is consistent with the Kolmogorov cascade, (2) the large scale dynamics could be approximated by a Lorenz convective roll, and (3) a steady-state solution was appropriate. Although the first two are consistent with the three-dimensional simulations and with experimental work on turbulent flows, the last is not. Use of MLT, therefore, underestimates dynamic behavior by artificially damping fluctuations in velocity and temperature.

This may be adequate (or not) through H and He core burning, but the traditional treatment of convection becomes increasingly unrealistic in the late stages of stellar evolution, which are accelerated by neutrino losses. This stage of cooling coincides with an evolutionary phase for which we have strong observational evidence of vigorous eruptive mass loss. In contrast, convection for photon-cooled stages (H and He burning) is less strongly modified, although treatments of boundary layers, composition gradients, and discontinuities remain questionable.

Carbon burning is an interesting transitional stage; it is the first stage to be cooled primarily by neutrino emission, and the vigor of carbon burning depends upon the abundance of $^{12}$C resulting from the previous helium burning (Arnett 1972b). The $^{12}$C abundance also depends on the algorithm used for convective mixing (Arnett 1972a). The precise reaction rates remain elusive; see, e.g., Gai (2013); Brune & Sayre (2013). For small values of $^{12}$C (massive cores), carbon will burn nonconvectively (and feebly), followed by weak neon burning (carbon burning produces $^{20}$Ne, the fuel for neon burning). For large values of $^{12}$C, carbon burning will be vigorous, lasting $\sim 3 \times 10^{11}$ s ($10^7$ years), to be compared with oxygen burning, which lasts for $\sim 9 \times 10^8$ s (30 years) (Arnett 1974). Carbon burning occurs at a lower temperature; it has a lower rate of neutrino cooling to balance than oxygen burning and can thus last longer. These values are quoted for helium cores, which are more relevant to SNbc events. While oxygen burning may be directly simulated in three-dimension, turbulent carbon burning, over timescales long enough for carbon exhaustion, remains beyond present computational capability. At present, the behavior of massive stellar cores during carbon burning rests on one-dimensional models and speculation, but observations suggest that there may be enhanced episodic mass loss during this stage (see Section 2.3).

We conclude that the use of MLT and diffusive convection are likely to be a dominant cause of the substantial discrepancies between one-dimensional stellar models and actual observed pre-supernova stars. Realistic inclusion of turbulence in stellar codes is a frontier topic, as is the inclusion of rotation, binary interaction, and eruptive mass ejection, with which it interacts. Strong empirical evidence for eruptions and violent binary interaction occurring in the few years before core collapse suggests a link to instability from turbulent convection in the latest burning phases. Both interpretation of isotopic anomalies in pre-solar grains in meteorites and initial models for gravitational collapse should be strongly influenced by the likely alteration of progenitor structure.

We note that if the surface layers of a SN progenitor star are modified before the impending explosion in a predictable way, it now becomes conceivable to design an early warning system for core collapse. The key would be to recognize the observational signature resulting from energy deposition caused by the type of mechanisms discussed herein. They could manifest as a sudden swelling of the star, sudden onset of violent binary interaction, or eruptive mass loss. Less extreme cases may in principle be recognized by their asteroseismological signatures. Some of these eruptive effects have been observed in stars that have not yet experienced core collapse (e.g., nearby LBV stars that experienced eruptions in past centuries or millennia), so the observational patterns of pre-SN outbursts need to be studied in more detail. Identification of pre-SN stars would be useful for detection of their neutrinos, gravitational waves, and early stages of nearby SNe and core collapse. Present and future transient surveys should be examined with this in mind.

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