Interpretation of the atmospheric muon charge ratio in MINOS

PHILIP SCHREINER1,2 AND MAURY GOODMAN1 FOR THE MINOS COLLABORATION
1 Argonne National Lab, 2 Benedictine University
maury.goodman@anl.gov

Abstract: MINOS is the first large magnetic detector deep underground and is the first to measure the muon charge ratio with high statistics in the region near 1 TeV. An approximate formula for the muon charge ratio can be expressed in terms of \( \epsilon_\pi = 115 \text{ GeV}, \epsilon_K = 850 \text{ GeV} \) and \( E_\mu \cos \theta \). The implications for K production in the atmosphere will be discussed.

Introduction

Defining \( A \)

\[
A = \frac{0.14E_{\mu}^{-2.7}}{\text{cm}^2 \text{s sr GeV}}
\]

the muon energy spectrum underground can be expressed as [2]:

\[
\frac{dN}{dE_\mu} = A \left\{ \frac{1}{1 + \frac{1.1E_\mu\cos \theta}{\epsilon_\pi}} + \frac{0.054}{1 + \frac{1.1E_\mu\cos \theta}{\epsilon_K}} \right\}
\]

(2)

where the two terms give the contributions of charged pions and kaons; the values of \( \epsilon = m\chi_0/\tau \) are close to the energies where the probability of meson interaction and decay are equal: \( \epsilon_\pi = 115 \text{ GeV} \) and \( \epsilon_K = 850 \text{ GeV} \). The zenith angle is \( \theta \) and in experiments with a flat overburden, the largest muon intensity comes at \( \cos \theta = 1 \). It is possible to generalize this equation to separately consider \( \mu^+ \) and \( \mu^- \) by introducing constants \( f_\pi \) and \( f_K \):

\[
\frac{dN^+}{dE_\mu} = A \left\{ \frac{f_\pi}{1 + \frac{1.1E_\mu\cos \theta}{115 \text{ GeV}}} + \frac{0.054 \times f_K}{1 + \frac{1.1E_\mu\cos \theta}{850 \text{ GeV}}} \right\}
\]

(3)

\[
\frac{dN^-}{dE_\mu} = A \left\{ \frac{1 - f_\pi}{1 + \frac{1.1E_\mu\cos \theta}{115 \text{ GeV}}} + \frac{0.054 \times (1 - f_K)}{1 + \frac{1.1E_\mu\cos \theta}{850 \text{ GeV}}} \right\}
\]

The measured charge ratio in a bin of surface muon energy and zenith angle can then be expressed as

\[
\frac{N^{+\mu}}{N^{-\mu}} = \left\{ \frac{f_\pi}{1 + \frac{1.1E_\mu\cos \theta}{115 \text{ GeV}}} + \frac{0.054 \times f_K}{1 + \frac{1.1E_\mu\cos \theta}{850 \text{ GeV}}} \right\}
\]

\[
\left\{ \frac{1 - f_\pi}{1 + \frac{1.1E_\mu\cos \theta}{115 \text{ GeV}}} + \frac{0.054 \times (1 - f_K)}{1 + \frac{1.1E_\mu\cos \theta}{850 \text{ GeV}}} \right\}
\]

An interesting feature of Equation 3 is that it depends only on the product \( E_\mu \cos \theta \) and not otherwise on \( E \) or \( \theta \). This combination of terms controls the relative portions of interaction and decay for both \( \pi^\pm \) and \( K^\pm \). Thus at a fixed value of \( E_\mu \cos \theta \) the ratio of \( \mu^+ \) from \( \pi^\pm \) and \( K^\pm \) is constant. Equation 3 postulates an energy independent \( \pi^+/\pi^- \) ratio related to \( f_\pi \), an energy independent \( K^+/K^- \) ratio related to \( f_K \) and an energy independent \( \pi/K \) ratio embodied in the 0.054. These are all reasonable as a consequence of Feynman scaling. (At energies above 10 TeV, charm and the changing cosmic ray chemical composition could also affect the muon charge ratio.) In this simple but powerful parameterization, the charge ratio depends only on the relative number of \( \pi^\pm \) and \( K^\pm \) that contribute to muons measured underground. The relative contribution to the muon intensity from \( \pi^\pm \) and \( K^\pm \) is affected by the relative probability of hadron interaction and decay, which vary with both energy and angle. It varies as a function of energy because of the hadron lifetimes, and it varies as a function of angle because of the interaction probability in the atmosphere.
But as can be seen from inspection of Equation 3, at any fixed value of \( E_{\mu}^{\text{surface}} \cos \theta \), the ratio of muons from \( \pi \) and K decay is also fixed, even if E or \( \theta \) is varied.

**Measurement of \( E_{\mu}^{\text{surface}} \cos \theta \) underground**

The energy of a muon underground can be related to the energy at the surface by \( E_{\mu}^{\text{surface}} = E_{\text{underground}} + E_{\text{loss}} \). The energy loss for muons in MINOS has been calculated using the energy loss equation, \( dE/dx = a(E) + b(E) \times E \). Many high energy muons which do reach the detector do not bend enough in MINOS’ magnetic field to measure the curvature and hence charge. This can be quantified in terms of the maximum detectable momentum, which depends on the geometry of the track through the detector [3]. A magnetic detector underground can measure the charge of muons that have the lowest energies \( E_{\text{underground}} \) when they reach the detector.

MINOS has made measurements of the charge ratio in the far detector [1] which is 710 meters underground or 2070 m.w.e., and also has yielded a preliminary result using its near detector [4] which is 100 m underground or 225 m.w.e. (Note that the far detector is under a small ridge.)

As a crude approximation, we can consider the case when: 1) the parameter \( a \) is constant, 2) the parameter \( b \) is zero, 3) the rock density is constant in all directions, 4) the surface is flat and 5) the maximum detectable momentum is negligible compared to the energy loss from the surface. The first four assumptions are equivalent to saying that the energy loss is proportional to the overburden which only depends on zenith angle. In that case \( E_{\text{loss}} = E_{\min} \cos \theta \) where \( E_{\min} \) is the minimum energy loss for a vertical cosmic ray muon.

The fifth assumption implies that the underground energy of muons used for the charge ratio measurement in MINOS was much smaller than their surface momentum. The MINOS maximum detectable momentum, defined at one sigma, is discussed in these proceedings [3] and a cut was chosen at 2.2 sigma.

In the limit corresponding to these assumptions, the frequency distribution in \( E_{\mu}^{\text{surface}} \cos \theta \) that we would measure is a delta function at \( E_{\min} \). At large zenith angles, the surface energy increases due to the \( \cos \theta \) dependence of the slant depth, but \( E_{\mu}^{\text{surface}} \cos \theta \) is constant.

This can be compared with the measured \( E \) and \( E_{\mu}^{\text{surface}} \cos \theta \) distributions from MINOS for the far detector in Figure 1 and for a preliminary result from the near detector in Figure 2. Both \( E_{\mu}^{\text{surface}} \cos \theta \) distributions are much narrower than the corresponding \( E_{\mu}^{\text{surface}} \cos \theta \) distributions. The largest contribution to the width of the measured \( E_{\mu}^{\text{surface}} \cos \theta \) distribution for the far detector is the \( b(E) \) term in the energy loss, while the largest contribution to the much narrower near detector distribution is the larger ratio of maximum detectable momentum to energy loss in the overburden.

**Implication for Kaons**

The charge ratio of muons that we measure from \( \pi \) decay is \( r_{\pi} = \frac{f_{\pi}}{1 - f_{\pi}} \) and from kaon decay \( f_{K} = \frac{f_{K}}{1 - f_{K}} \). The value 0.054 is related to the relative kaon/pion intensity in Gaisser’s model.[2]

A fit was made to MINOS data and the published L3+C ratio data [5] to this parameterization for \( r \). A chi-square fit was done in 178 bins of \( E_{\mu}^{\text{surface}} \cos \theta \) vs. \( \cos \theta \). The fit gave \( f_{\pi} = 0.555 \pm 0.002 \) and \( f_{K} = 0.667 \pm 0.007 \). The errors in \( f_{\pi} \) and \( f_{K} \) are correlated. \[1\] Using these numbers, the charge ratio versus \( E_{\mu}^{\text{surface}} \cos \theta \) is plotted in Figure 3. These values for \( f_{\pi} \) and \( f_{K} \) imply that the muon charge ratio from pion decay is \( r = 1.25 \) and that the muon charge ratio from kaon decay is about \( r = 2.0 \).

As noted above, there is some \( E_{\mu}^{\text{surface}} \cos \theta \) variation in the MINOS far detector data at a value of \( E_{\mu}^{\text{surface}} \cos \theta \) where the curve in Figure 3 is varying. We have performed a fit solely to the MINOS far detector data, and obtain \( f_{\pi} = 0.549 \pm 0.012 \) and \( f_{K} = 0.702 \pm 0.049 \), which imply \( r_{\pi} = 1.22 \) and \( r_{K} = 2.36 \). The errors on \( f_{\pi} \) and \( f_{K} \) in this fit are almost completely correlated and are shown in Figure 4. A fit which also incorporates the preliminary MINOS near detector charge ratio will be presented at the 30th ICRC.

Equation 3 has asymptotic values at both low energy and high energy. Note that both val-
values have contributions from both $\pi$ and K decays. For the above MINOS-only fit, the low $E_\mu^{surface}\cos\theta$ charge ratio is $r = 1.27$ and the high $E_\mu^{surface}\cos\theta$ charge ratio is $r = 1.45$. The parameterization does contain enough physics to (so far) satisfactorily represent the world’s underground muon charge ratio data. Furthermore, it is a reasonable way to parameterize the energy dependency of the charge ratio, and seems preferable to a linear fit in $E$ or $\log E$. We suggest that the new MINOS data, equation 3, and the fitted values for the model parameters near 1 TeV will be useful in the next round of atmospheric neutrino modeling in this energy range. It is also now apparent that future precise muon charge ratio data in the 0.2-0.6 TeV energy range will be useful for verifying the $E_\mu^{surface}\cos\theta$ dependency of the charge ratio. Our fits to $r_\pi$ give values near expectation [6]. Our fit to the $r_K = K^+/K^-$ charge ratio in atmospheric showers yield values just above 2. It is clearly difficult to directly measure this ratio. Eq. 3 provides a direction for future study of this subject.

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References

[1] P. Adamson et al., “Measurement of the Atmospheric Muon Charge Ratio at TeV Energies with MINOS,” to be published; S. Mufson for MINOS, “Measurement of the Atmospheric Muon Charge Ratio with the MINOS Far Detector”, these proceedings.

[2] T. Gaisser, “Cosmic Rays and Particle Physics”, Cambridge University Press, 1990.

[3] M. Goodman for MINOS, “Maximum Detectable Momentum in MINOS”, these proceedings.

[4] J.K. de Jong for MINOS, “Measurement of the Atmospheric Muon Charge Ratio with the MINOS Near Detector”, these proceedings.

[5] P. Achard et al., Phys. Lett. B 598, 15 (2004).

[6] V. Naumov, private communication.
Figure 3: Behavior of Equation 3 for $E_{\text{surf}} \cos \theta$ from 1 GeV to 10 TeV. The critical energies of $\pi$’s and $K$’s are shown. The regions where the MINOS near and far detectors are sensitive are also indicated. Values of $f_\pi$ and $f_K$ from the MINOS/L3+C fit were used.

Figure 4: Distribution of $\chi^2$ surface in the $f_\pi - f_K$ plane, showing the error of the MINOS only fit and the correlation between $f_\pi$ and $f_K$. 
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