Search for invisible decays of $\pi^0, \eta, \eta'$, $K_S$ and $K_L$: a probe of new physics and test of the Bell-Steinberger relation

S.N. Gninenko

Institute for Nuclear Research, Moscow 117312

(Dated: September 9, 2014)

In the standard model the rate of the $\pi^0, \eta, \eta', K_S, K_L \to \nu\bar{\nu}$ decays is predicted to be extremely small. Therefore, observation of any of these mesons ($M$) decay into invisible final state would unambiguously signal the presence of new physics. The Bell-Steinberger relation connects CP and CPT violation in the mass matrix to CP and CPT violation in all decay channels of neutral kaons. It is a powerful tool for testing CPT invariance in the $K^0 - \bar{K}^0$ system, assuming that there are no significant undiscovered decay modes of either $K_S$ or $K_L$ which could contribute to the precision of the results. The $K_S, K_L \to \text{invisible}$ decays have never been tested and the question by how much these decays can influence the Bell-Steinberger analysis of the $K^0 - \bar{K}^0$ system still remains open. To clarify this question we propose an experiment to search for the $M \to \text{invisible}$ decays which is also capable to probe new physics. The experiment utilizes high energy hadronic beams from the CERN SPS and the charge exchange reactions of pion or kaon on nucleons of an active target, e.g. $\pi^-(K^-) + p \to \pi^0(\bar{K}^0) + \pi^-$. As a source of the well tagged $M$s emitted in the forward direction with the beam energy. If the decay $M \to \text{invisible}$ exists, it could be observed by looking for an excess of events with a quite specific signature: the complete disappearance of the beam energy in the detector. This unique signal of $M \to \text{invisible}$ decays allows searches for the decays $K_S, K_L \to \text{invisible}$ with the sensitivity in branching ratio $\text{Br}(K_S(K_L) \to \text{invisible}) \lesssim 10^{-8}$ ($10^{-6}$), and $\pi^0, \eta, \eta' \to \text{invisible}$ decays with sensitivity few orders of magnitude beyond the present experimental limits. This experiment is complementary to the one recently proposed for the search for invisible decays of dark photons and fits well with the present kaon physics program at CERN.

PACS numbers: 14.80.-j, 12.60.-i, 13.20.Cz, 13.35.Hb

I. INTRODUCTION

Experimental studies of invisible decays, i.e. particle transitions to an experimentally unobservable final state, played an important role both in development of the Standard Model (SM) and in testing its extensions [1]. It is worth it to remember the precision measurements of the $Z \to \text{invisible}$ decay rate for the determination of the number of lepton families in the SM. In recent years, experiments on invisible particle decays have received considerable attention. Motivated by various models of physics beyond the SM, they include searches for invisible decays of $\pi^0$ mesons at E949 [2], $\eta$ and $\eta'$ mesons at BES [3], heavy $B$ meson decays at Belle [4], BaBAR [5], BES [6] and invisible decays of the Upsilon(1S) resonance at CLEO [7], baryonic number violation with nucleon disappearance at SNO [8], BOREXINO [9], and KamLAND [10], see also Ref. [11], electric charge nonconserving electron decays $e^- \to \nu \bar{\nu}$ [12], neutron-mirror neutron oscillations at PSI [13] and the ILL reactor [14], and neutron disappearance into another brane world [15]. One could also mention experiments looking for extra dimensions with invisible decay of positronium [16, 17], and proposals for new experiments to search for muonium annihilation into two neutrino, $\mu^+e^- \to \nu\bar{\nu}$ [18], electric charge nonconservation in the muon decay $\mu^+ \to \text{invisible}$ [19], and mirror-type dark matter through the invisible decays of orthopositronium in vacuum [20].

The use of the (pseudo)scalar mesons ($M$), such as $\pi^0, \eta, \eta', K_S, K_L$, to search for new physics by looking for their decays into invisible final states has advantage, because in the standard model the rate of the $\pi^0, \eta, \eta', K_S, K_L \to \nu\bar{\nu}$ decays is predicted to be extremely small. For massless neutrino the decay $M \to \nu\bar{\nu}$ is forbidden kinematically by angular momentum conservation. Indeed, in the $M$-meson rest frame the neutrinos produced in the decay fly away in the opposite directions along the same line. Since the neutrino and antineutrino are massless, the projection of the sum of their spins on this line equals $\pm 1$. The projections of the orbital angular momentum of the neutrino on this line are equal to zero. Since in the initial state we had the scalar, the process is forbidden. For the case of massive neutrino their spins in the rest frame must be opposite and, hence, one of the them is forced to have “wrong” helicity. This results in the $M \to \nu\bar{\nu}$ decay rate to be proportional to the neutrino mass square:

$$\Gamma(M \to \nu\bar{\nu}) \sim \left(\frac{m_\nu}{m_M}\right)^2 \lesssim 10^{-16}$$  \hspace{1cm} (1)

Thus, we see that indeed, if the decay $M \to \text{invisible}$ is observed it would unambiguously signal the presence of new physics.

Another reason to search for, in particular, the $K_S, K_L \to \text{invisible}$ decays is motivated by the additional tests of the Bell-Steinberger relation [21]. This relation, obtained by using unitarity condition, connects CP and CPT violation in the mass matrix of the kaon
system, i.e. to parameters describing T and CPT non-invariance, to CP and CPT violation in all decay channels of neutral kaons, see e.g. [22, 26]. We know that only CPT appears to be an exact symmetry of nature, while charge conjugation (C), parity (P) and time reversal (T) are known to be violated. Hence, testing the validity of CPT invariance probes the basis of the standard model. The Bell-Steinberger relation remains to be the most sensitive test of the CPT symmetry. For example, the analyses of the KLOE Collaboration have reached the impressive sensitivity of \(-5.3 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\overline{K}^0} < 6.3 \times 10^{-19} \text{ GeV} at 95\% CL for the neutral kaons mass difference [27], see also [28].

Briefly, within the Wigner-Weisskopf approximation, the time evolution of the neutral kaon system is described by [27]:

\[ i \frac{d\Phi(t)}{dt} = H\Phi(t) = \left( M - i \frac{1}{c} \Gamma \right) \Phi(t) \quad (2) \]

where \( M \) and \( \Gamma \) are 2 \times 2 Hermitian matrices, which are time-independent, and \( \Phi(t) \) is a two-component state vector in the \( K^0 - \overline{K}^0 \) space. Denoting by \( m_{ij} \) and \( \Gamma_{ij} \) the elements of \( M \) and \( \Gamma \) in the \( K^0 - \overline{K}^0 \) basis, CPT invariance implies:

\[ m_{11} = m_{22} \quad (\text{or } m_{K^0} = m_{\overline{K}^0}) \quad \text{and} \quad \Gamma_{11} = \Gamma_{22} \quad (\text{or } \Gamma_{K^0} = \Gamma_{\overline{K}^0}) \quad (3) \]

The eigenstates of Eq. (2) can be written as

\[ K_{S,L} = \frac{1}{\sqrt{2(1 + |\epsilon_{S,L}|^2)}} \left( (1 + \epsilon_{S,L})K^0 \pm (1 - \epsilon_{S,L})\overline{K}^0 \right) \quad (4) \]

with

\[ \epsilon_{S,L} = \frac{1}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \left[ -i\text{Im}(m_{12}) - \frac{1}{2}(\Gamma_S - \Gamma_L) \right] \]

\[ \frac{1}{2}\text{Im}(\Gamma_{12}) \pm \frac{1}{2}(m_{K^0} - m_{\overline{K}^0} - \frac{i}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})) \equiv \epsilon \pm \delta \]

Unitarity condition allows to express the four elements of \( \Gamma \) in terms of appropriate combinations of the kaon decay amplitudes \( A_i \):

\[ \Gamma_{ij} = \sum_f A_i(f)A_j^*(f), \quad i,j = 1,2 = K^0, \overline{K}^0 \quad (6) \]

where the sum is over all the accessible final states.

\[ \frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i tan\phi_{SW} \left( \frac{\text{Re}(\epsilon)}{1 + |\epsilon|^2} - i\text{Im}(\delta) \right) \]

\[ = \frac{1}{\Gamma_S - \Gamma_L} \sum_f A_L(f)A_S^*(f), \quad (7) \]

where \( \phi_{SW} = \arctan[2(m_L - m_S)/(\Gamma_S - \Gamma_L)] \). One can see, that the Bell-Steinberger relation Eq. (7) relates a possible violation of CPT invariance \( m_{K^0} = m_{\overline{K}^0} \) and/or \( \Gamma_{K^0} = \Gamma_{\overline{K}^0} \) in the \( K^0 - \overline{K}^0 \) system to the observable CP-violating interference of \( K_S \) and \( K_L \) decays into the same final state \( f \). If CPT invariance is not violated, then \( \text{Im}(\delta) = 0 \). It is stressed that, any evidence for \( \text{Im}(\delta) \neq 0 \) resulting from this relation can only manifest the violation of CPT or unitarity [22]. It should be noted, that \( K_S, K_L \) decay parameters from the decay channels with the branching ratio \( \text{Br}(K_S \to f) = \Gamma(K_S \to f)/\Gamma_S \gtrsim 10^{-7} \) and \( \text{Br}(K_L \to f)/\Gamma_L/\Gamma_S \gtrsim 10^{-7} \) are within the present accuracy of Eq. (7) and contribute to the Bell-Steinberger analysis of the kaon system [23].

Generally, the advantage of the neutral kaon system is attributed to the fact, that only a few (hadronic) decay modes give significant contributions to the r.h.s. in Eq. (7). In particular, it is assumed that there are no significant contributions from invisible decay modes of either \( K_L \) or \( K_S \), which, however have never been experimentally tested. Therefore, the questions - what is the contribution from these decay modes? and how much would the errors on \( \text{Re}(\epsilon) \) and \( \text{Im}(\delta) \) increase if the invisible modes would have maximal CP violation? - are still open, see e.g. [29]. As long as this is not answered experimentally, further tests of CPT symmetry via Bell-Steinberger relations remain important.

One of the aims of this work is to show that the questions discussed before can be answered, at least partially, in the proposed experiment on searching for the still unexplored decay modes \( K_S, K_L \to \text{invisible} \) with a high energy \( K^\pm \) beams at the CERN SPS with the sensitivity in the branching fraction \( \text{Br}(K_S(K_L) \to \text{invisible}) \lesssim 10^{-8}(10^{-6}) \). The experiment has also a capability for a sensitive search for \( \pi^0, \eta, \eta' \to \text{invisible} \) decays and could improve the existing limits by more than an order of magnitude. The rest of the paper is organized in the following way. The method of the search and the experimental setup are described in Sec. II, background sources are discussed in Sec. III, and the expected sensitivity for the decay \( M \to \text{invisible} \) is presented in Sec. IV. Section V contains concluding remarks.

II. THE EXPERIMENT TO SEARCH FOR THE \( \pi^0, \eta, \eta', K^0 \to \text{invisible} \) DECAYS

Below for simplicity we will consider mainly the experiment on search for the \( K_S, K_L \to \text{invisible} \) decays. Application of these considerations to the search for the decays \( \pi^0, \eta, \eta' \to \text{invisible} \) with the same detector is straightforward.

The detector specifically designed to search for the \( K_S, K_L \to \text{invisible} \) decays is schematically shown in Fig. I. This experimental setup is complementary to the one recently proposed for the search for invisible decays of dark photons at the SPS at CERN [31, 32]. The experiment could employ, e.g. the CERN SPS H4 hadron beam, which is produced in the target T2 of the CERN SPS and transported to the detector in an evacuated beamline tuned to a freely adjustable beam momentum from
10 up to 300 GeV/c [30]. The typical maximal beam intensity at $\simeq 50$-100 GeV, is of the order of $\simeq 10^2 \pi^\pm$ and $\simeq 10^6 K^\pm$ for one SPS spill with $10^{12}$ protons on target. The typical SPS cycle for Fixed Target (FT) operation lasts 14.8 s, including 4.8 s spill duration. The maximal number of FT cycles is four per minute. The beam has high purity, the admixture of the other charged particles is below $10^{-2}$. The beam can be focused onto the spot of the order a few cm$^2$.

The method of the search is the following. The source of $K^0(\bar{K}^0)$ is the charge exchange reaction of high energy kaons on nucleons of an active target

$$K^- p \rightarrow K^0 n$$

$$K^+ n \rightarrow K^0 p \quad (8)$$

where the neutral kaon is emitted mainly in the forward direction with the beam momentum and the recoil nucleon carry away small fraction of the beam energy. Further, we will make no difference between these two reactions. The process of the $K^0$'s production and subsequent invisible decay $K^0 \rightarrow invisible$ is expected to be a very rare event, which occurs with the rate much smaller with respect to the total $K^0$ production rate. Hence, its observation presents a challenge for the detector design and performance.

The detector shown in Fig. 1 is equipped with the scintillating counters S1 and S2, defining the beam, an active target $T$ surrounded by a high efficiency electromagnetic calorimeter ECAL serving as a veto against photons and other secondaries emitted from the target at large angles, high efficiency forward veto counters V1 and V2, and a massive, hermetic hadronic calorimeter HCAL located at the downstream end of the setup to detect energy deposited by secondaries from the $K^- A \rightarrow anything$ primary interactions in the target. For searches at low energies Cherenkov counters to enhance the incoming hadron tagging efficiency can be used.

The reaction (8) occurs practically uniformly over the length of the target. The produced $K^0$, composed of equal portions of $K_S$ and $K_L$, either decay quickly in the $T$, or penetrates the Veto system without interactions and either decays in flight or interacts in the HCAL. If the $K_S$ and $K_L$ decay invisibly, then it is assumed that the final state particles also penetrate the rest of the detector without prompt decay into ordinary particles, which could deposit energy in the detector. The fraction of the primary $K^-$ energy deposited in the target is used to determine the position of the interaction vertex alone the beam direction. The HCAL is served as a dump to absorb completely the energy of secondary particles produced in the primary pion or kaon interaction in the target. In order to suppress background due to the detection inefficiency, the detector must be longitudinally completely hermetic. To enhance detector hermeticity, the hadronic calorimeter has the total thickness of $\simeq 28 \sqrt{\lambda_{int}}$ (nuclear interaction lengths) and placed behind the veto system, as shown in Fig. 1. The occurrence of $K_S, K_L \rightarrow invisible$ decays produced in $K^\pm$ interactions would appear as an excess of events with a signal in the $T$, see Fig. 1 and zero energy deposition in the rest of the detector, above those expected from the background sources. Thus, the signal candidate events have the signature:

$$S_{K^0 \rightarrow invisible} = T \cdot V1 \cdot V2 \cdot HCAL \quad (9)$$

and should satisfy the following selection criteria:

• The measured momentum of the incoming kaon should correspond to its selected value.
FIG. 2: Expected distributions of energy deposited by $\simeq 10^6 K^0$ with energy $\simeq 95$ GeV from the charge exchange reaction Eq. (8) in two (a) and four (b) consecutive HCAL modules. The peak at zero-energy in spectrum a) is due to the punch-through neutral kaons.

- The kaon should enter the target and the interaction vertex should be localized within the target volume.
- No energy deposition in the V1 and V2.
- The fraction of the beam energy deposited in the HCAL modules is consisted with zero.

III. BACKGROUND

The background reactions resulting in the signature of Eq. (9) can be classified as being due to physical- and beam-related sources. To perform full detector simulation in order to investigate these backgrounds down to the level $\lesssim 10^{-10}$ would require a prohibitively large amount of computer time. Consequently, only the following sources of background, identified as the most dangerous are considered and evaluated with reasonable statistics combined with numerical calculations:

- One of the main background sources is related to the low-energy tail in the energy distribution of beam hadrons. This tail is caused by the beam interactions with a passive material, such e.g. as entrance windows of the beam lines, residual gas, etc... Another source of low energy hadrons is due to their decays in flight in the beam line when the low decay pion or muon mimic the signature Eq. (9) in the detector. The uncertainties arising from the lack of knowledge of the dead material composition in the beam line are potentially the largest source of systematic uncertainty in accurate calculations of the fraction and energy distribution of these events. An estimation shows that the fraction of events with energy below $\lesssim 10$ GeV in the kaon beam tuned, e.g. to 50 GeV could be as large as $10^{-8} - 10^{-6}$. Hence, the sensitivity of the experiment could be determined by the presence of such particles in the beam, unless one takes special measures to suppress this background. To improve the high energy kaon selection and suppress background from the possible admixture of low energy particles, one can use a tagging system utilizing the magnetic spectrometer installed upstream of the detector, as schematically shown in Fig. 1.

- The fake signature of Eq. (9) could also arise when the $K^0$ from the reaction (8) occurred in the target is not detected due to the incomplete hermeticity of the HCAL. In this case, the produced $K^0$ punch through the HCAL without depositing energy above a certain threshold $E_{th}$. This effect is illustrated in Fig. 2a, which shows the distribution of energy deposited by $K^0$s produced at 95 GeV, in two consecutive HCAL modules ($\simeq 14 \lambda_{int}$). The distribution is obtained with GEANT4 simulations [33]. The peak of events at zero-energy in the spectrum is caused by the punch-through neutral kaons. These events with the sum of energy released in two HCAL modules below the threshold $E_{th} \simeq 0.5$ GeV are considered as zero-energy events. In Fig. 2b, one can also see that the similar distribution of energy deposited by $K^0$s in four consecutive HCAL modules ($\simeq 28 \lambda_{int}$) has no such zero-energy events.

The punch-through probability is defined roughly by $\simeq exp(-L_{HCAL}/\lambda_{int})$, where $L_{HCAL}$ is the HCAL thickness. It is $\lesssim 10^{-12}$ for the total thickness of the HCAL about $28 \lambda_{int}$. Since to perform
detector simulations down to this level of precision is not possible, the rough estimate of the HCAL non-hermeticity for high energy hadrons was cross-checked with GEANT4-based simulations in the following way. The low energy tail in the distribution of energy deposited in the full HCAL by $\sim 10^7$ simulated neutral kaons was fitted by a smooth polynomial function and extrapolated to the low energy region in order to evaluate the number of events below a certain threshold $E_{th}$, see Ref. [32] for more details. This procedure results in estimate of the HCAL non-hermeticity, defined as the ratio of the number of events below the threshold $E_{th}$ to the total number of incoming particles, $(E < E_{th})/n_{tot}$. For the energy threshold $E_{th} \approx 0.5$ GeV the non-hermeticity is found to be at the level $\lesssim 10^{-11}$ in a satisfactory agreement with the above estimate taking into account the accuracy of this procedure. This results in an overall conservative level of this background of $\lesssim 10^{-13}$ per incident beam kaon reaction in the target.

- Another type of background is caused by the low energy muon contamination in the beam due to, e.g. the $\pi, K$ decays in flight after passing the spectrometer. The background can be due the following event chain: the muon entering the detector could decay in flight into a low-energy electron and a neutrino pair, $\mu \to e\nu\bar{\nu}$ in the $T$. The decay electron then penetrates the V1 and V2 without being detected, and deposits all its energy in the HCAL, which is below the threshold $E_{th}$. For the $E_{th} \lesssim 0.5$ GeV, the muon energy should be well below 10 GeV, resulting in the probability for this event chain to be small, $P \lesssim 10^{-13}$. Similar background caused by the decays of the beam pions or kaons in the target was also found to be negligible.

- The fake signature of Eq. [34] could be due to the physical background: a muon scattering on proton, $\mu^- p \to \nu_p n$. Taking into account the corresponding cross-section and the probability for the recoil neutron to escape detection in the HCAL results in an overall level of this background of $\lesssim 10^{-14}$ per incoming kaon.

In Table I contributions from the all background processes are summarized for the kaon beam energy of 95 GeV. The total background is conservatively at the level $\lesssim 10^{-12}$ per incoming kaon. Taking into account that the cross section of the reaction [38] is in the range $10^{-4} - 10^{-3}$ of the total cross section, means that the search accumulated up to $\lesssim 10^{12}$ $\pi^-$ or $K^-$ events is expected to be background free and the branching fraction summarized below can be reached.

| Source of background                  | Expected level |
|---------------------------------------|----------------|
| punchthrough $K^0$'s                  | $\lesssim 10^{-14}$ |
| $\pi^-, K^-$ beam low energy tail     | $\lesssim 10^{-12}$ |
| HCAL non-hermeticity                 | $\lesssim 10^{-13}$ |
| $\mu^-, \pi^-, K^-$ in flight        | $\lesssim 10^{-13}$ |
| $\mu^-$ induced reactions            | $\lesssim 10^{-14}$ |
| Total (conservatively)               | $\lesssim 1.3 \times 10^{-13}$ |

IV. EXPECTED SENSITIVITY

To estimate the sensitivity of the proposed experiment a simplified feasibility study based on GEANT4 Monte Carlo simulations have been performed for 30-100 GeV pion and kaons. The ECAL is the hodoscope arrays of the lead-scintillator counters of the shashlyk type, see e.g. counters ($X_0 \approx 2$ cm), each of the size 36 $\times$ 36 $\times$ 400 mm$^3$, allowing accurate measurements of the lateral energy leak from the target. The target is a block of radiation hard plastic scintillator viewed by a photomultiplier. The veto counters are assumed to be 1-2 cm thick, high sensitivity LYSO crystal arrays with a high light yield of $\approx 3 \times 10^3$ photoelectrons per 1 MeV of deposited energy. It is also assumed that the veto’s inefficiency for the minimum ionizing particle (mip) detection is, conservatively, $\lesssim 10^{-4}$. The hadronic calorimeter is a set of four modules. Each module is a sandwich of alternating layers of iron and scintillator with the thickness of 25 mm and 4 mm, respectively, and with the lateral size 60 $\times$ 60 cm$^2$. Each module consists of 48 such layers and has the total thickness of $\approx 7X_0_{int}$. The number of photoelectrons produced by a mip crossing the module is in the range $\approx 150$-$200$ ph.e.. The energy resolution of the HCAL calorimeters as a function of the beam energy is taken to be $\frac{\sigma}{E} \approx \frac{E_{int}}{150}$ [39]. The energy threshold for the zero-energy in the HCAL is 0.5 GeV. The reported further analysis also takes into account passive materials from the DV vessel walls. To estimate the expected sensitivities we used simulations of the process shown in Fig. 1 first to calculate fluxes and energy distributions of mesons produced in the target by taken into account the relative normalization of the yield of different meson species $\pi^0 : \eta : \eta'$ and $K^0$ from the original publications [30, 57]. The cross section of $K^0$ production in reaction [38] can be expressed as [36]

$$
\frac{\sigma(K^- p \to K^0 n)}{dt} \simeq (1 - G t)(\exp[c_d t] + R^2 \exp[c_A t]) \\
-2R \cos \phi_+ - GT \cos \phi_-) \exp[(c_\rho + c_A)t/2]
$$

(10)

where $t$ is the four-momentum transfer squared, $G = (33.5 \pm 1.3)$ GeV$^{-2}$, $c_\rho = (15.5 \pm 0.3)$ GeV$^{-2}$, $c_A = (8.8 \pm 0.1)$ GeV$^{-2}$, $R = 0.83 \pm 0.05$, $\cos \phi_+ = -0.08 \pm 0.07$.
the efficiency of (8) can be determined in separate measurements with
signal reconstruction efficiency, and is in the range
of either length, tuned to obtain the total cross sections of meson pro-
duction, and depend on the beam energy [36, 37].

As mentioned before, the yield of particles in reaction (3) can be determined in separate measurements with
the same setup. In Fig. 3 an example of the expected distribution of energy deposited in the HCAL by neutral kaons from the reaction (8) is shown. The distribution is calculated for the beam energy of 95 GeV, the decay length $L \approx 0.1$ m and corresponds to the case when the $K_S, K_L$ decay length is $\gg L$. In this case, $K_S$ and $K_L$
mainly interact in the HCAL before they decay. One can see, that the $K^0$ energy distribution is peaked at max-
imal beam energy. The events are selected with the condi-
tions of Eq. (9), but without requirement of the absence
of the energy deposition in the HCAL. The distribution
is almost background free, it has Gaussian shape and al-

depend on the beam energy [31].

The calculated fluxes and energy distributions of mesons produced in the target were used to predict
the number of signal events in the detector. For a given number of primary kaons $N_{K^-}$, the expected number of $K_{S,L} \rightarrow invisible$ decays occurring within the decay length $L$ of the detector is given by

$$N_{K_{S,L}} = kN_{K^-} Br(K_{S,L} \rightarrow invisible) \int \frac{\sigma(K^- p \rightarrow K^0 n)}{dt} \left[ 1 - \exp \left(-\frac{LM_{K^0}}{P_{K^0} \tau_{K^0}}\right) \right] \zeta dt \quad (11)$$

where coefficient $k$ is a normalization factor that was
tuned to obtain the total cross sections of meson pro-
duction, $P_{K^0}$ and $\tau_{K^0}$ are the $K^0$ momentum and the
lifetime of either $K_S$ or $K_L$ at rest, respectively, $\zeta$ is
the signal reconstruction efficiency, and $L$ is the length of the
decay volume. In this estimate the average momentum
in the range $<p_{c.m.}> \approx 30 - 100$ GeV, $L \approx 10$ m, and
the efficiency $\zeta \approx 0.9$.

As mentioned before, the yield of particles in reaction (3) can be determined in separate measurements with

| Expected limit for branching ratio | Present limit |
|-----------------------------------|---------------|
| $Br(K_S \rightarrow invisible) \leq 10^{-8}$ | no |
| $Br(K_L \rightarrow invisible) \leq 10^{-8}$ | no |
| $Br(\pi^0 \rightarrow invisible) \leq 10^{-8}$ | $< 2.7 \times 10^{-1}$ [2] |
| $Br(\eta \rightarrow invisible) \leq 10^{-8}$ | $< 1.0 \times 10^{-4}$ [3] |
| $Br(\eta' \rightarrow invisible) \leq 10^{-6}$ | $< 5.2 \times 10^{-4}$ [4] |

These limits are given in Ref. [2] for the value $\frac{\Gamma(\eta' \rightarrow invisible)}{\Gamma(\eta' \rightarrow \gamma \gamma)}$ and are re-calculated for the ratios $\frac{\Gamma(\eta(\eta') \rightarrow invisible)}{\Gamma(\eta(\eta') \rightarrow hadrons)}$, respectively.

The obtained results can be used to impose constraints on the previously discussed decays of $\pi^0, \eta, \eta'$, $K^0$ into invisible final states. Using the relation $N_{90\%}^{\pi,\eta,\eta'} > N_{90\%}^{K_{S,L}}$, where $N_{90\%}^{K_{S,L}}$ (≈ 2.3 events) is the 90% C.L. upper limit for the number of signal events and Eq. (11), one can determine the expected 90% C.L. upper limits from the results of the proposed experiment. These bounds calculated for the total number of $10^{12}$ incident pions or kaons and the background free case, are summarized in Table II. Here we assume the exposure to the $\pi/K$ beam with the nominal rate is a few months, and that the invisible final states do not decay promptly into the ordinary particles, which would deposit energy in the Veto system or HCAL.

The statistical limit on the sensitivity of the proposed experiment is mostly set by the number of accumulated events. Thus, it is important to accumulate a large num-

FIG. 3: Expected distribution of the total energy deposited by $\approx 10^6 K^0$ with energy $\approx 95$ GeV from the reaction Eq. (8)
in four HCAL modules.

TABLE II: Upper limits on branching fractions of different decay modes into invisible final states calculated for the total number of $10^{12}$ incident pions or kaons (see text for details).
observation, several methods could be used to cross-check the result. For instance, to test whether the signal is due to the HCAL non-hermeticity or not, one could perform measurements with different HCAL thicknesses, i.e. with the one, two, three and four consecutive HCAL modules. In this case the expected background level can be obtained by extrapolating the results to an infinite HCAL thickness. The evaluation of the signal and background could be also obtained from the results of measurements at different beam energies.

An interesting hypothetical question related to the test of the Bell-Steinberger relation is: If the signal is observed, would it be possible in to test the CP violation in the invisible decays in this case? It it clear, that to check it directly, as in the case of the CP violating decay $K_L \to \pi \pi$, is difficult because the final state is assumed to be unobservable. However, one can perform measurements to see if there is variation of the signal for different length of the decay volume. For example, to cross-check whether the signal is mostly from the $K_L$ or $K_S$ decay, one could remove the decay volume DV and put the HCAL calorimeter behind the Veto system. This would not affect the main background sources and still allow the $K_S$ production, but with $K_L$ decays in front of the HCAL being suppressed. For the measurements with large $L$ to insure that there is no additional background due to the variation of the HCAL hermeticity, e.g. due to the large transverse fluctuation of hadronic final state, or due to unexpected (yet unknown) $t$-dependence of the charge exchange reactions at large $t$, the transverse HCAL size should be large enough. Finally note, that the presented analysis gives an illustrative order of magnitude for the sensitivity of the proposed experiment and may be strengthened by more detailed simulations of the experimental setup.

V. CONCLUSION

Due to their specific properties, neutral kaons are still one of the most interesting probes of physics beyond the standard model both from the theoretical and experimental viewpoints. In particular, the Bell-Steinberger relation remains to be the most probe of the CPT invariance in the $K^0 - \bar{K}^0$ system. It connects CP and CPT violation in the mass matrix of the kaon system to CP and CPT violation in all decay channels of neutral kaons, assuming that there are no significant undiscovered decay modes of either $K_S$ or $K_L$. However, the invisible decays $K_S, K_L \to \text{invisible}$ have never been investigated. And a still open question remains by how much these decays modes can influence the Bell-Steinberger analysis of the $K^0 - \bar{K}^0$ system? This makes the first experimental search for the decays $K_S, K_L \to \text{invisible}$ interesting and important.

We proposed to perform an experiment dedicated to the sensitive search for the still unexplored invisible decays of neutral kaons, $K_S, K_L \to \text{invisible}$ by using available 30-100 GeV kaons beams from the CERN SPS. The experiment is also capable for the sensitive search for decays $\pi^0, \eta, \eta' \to \text{invisible}$ with the SPS pion beams. If the $M \to \text{invisible}$ decays exist, they could be observed by looking for events with the unique signature - the total disappearance of the beam energy in the fully hermetic hadronic calorimeter. A feasibility study of the experimental setup shows that this unique signal of $M \to \text{invisible}$ decays allows searches for the decays $K_S, K_L \to \text{invisible}$ the with sensitivity in branching ratio $\text{Br}(K_S(K_L) \to \text{invisible}) \lesssim 10^{-8}(10^{-6})$, and $\pi^0, \eta, \eta' \to \text{invisible}$ decays with sensitivity few orders of magnitude beyond the present experimental limits. The sensitivity in branching ratios $\text{Br}(K_S(K_L) \to \text{invisible})$ is comparable with the branching ratios of $K_S, K_L$ decay modes which contribute to the present accuracy of the Bell-Steinberger analysis $^{23}$.

These results could be obtained with a detector optimized for several of its properties. Namely, i) the intensity and purity of the primary pion and kaon beams, ii) the high efficiency of the veto counters, and iii) the high level of hermeticity for the hadronic calorimeters are of importance. Large amount of high energy hadrons and high background suppression are crucial to improve the sensitivity of the search. To obtain the best limits, the choice of the energy and intensity suppression are crucial to improve the sensitivity of the search. To obtain the best limits, the choice of the energy and intensity of the beam, as well as background level should be compromised.

The proposed experiment is complementary to the one recently proposed for a sensitive search for dark photons decaying invisibly to dark-sector particles at the CERN SPS $^{31, 32}$. It also provides interesting motivations for the further kaon study and fits well with the present kaon physics program at CERN, see e.g. $^{39}$.

Acknowledgments

I would like to thank A. Ceccucci, P. Crivelli, N. Krasnikov, V. Matveev, V. Polyakov, and V. Samoylenko for useful discussions, and A. Dermenev and M. Kirsanov for their help in calculations.

[1] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).
[2] A. V. Artamonov et al. (E949 Collaboration), Phys. Rev. D 72, 091102 (2005).
[3] M. Ablikim et al. (BES Collaboration), Phys. Rev. 87, 012009 (2013).
[4] C.L. Hsu et al. (Belle Collaboration), Phys. Rev. D 86, 032002 (2012).
[5] B. Aubert et al., (BABAR Collaboration), Phys. Rev. Lett. 103, 251801 (2009).
[6] M. Ablikim et al. (BES Collaboration), Phys. Rev. Lett. 100, 192001 (2008).
[7] P. Rubin et al., (CLEO Collaboration), Phys. Rev. D 75, 031104 (2007).
[8] S.N. Ahmed et al., (SNO Collaboration), Phys. Rev. Lett. 92, 102004 (2004).
[9] H.O. Back et al., (Borexino Collaboration), Phys. Lett. B 563, 23 (2003).
[10] T. Araki et al., Phys. Rev. Lett. 96, 101802 (2006).
[11] V.I. Tretyak, V.Yu. Denisov, Yu.G. Zdesenko, JETP Lett. 79, 106 (2004), Pisma Zh. Eksp. Teor. Fiz. 79, 136 (2004); nucl-ex/0401022.
[12] H.V. Klapdor-Kleingrothaus, I.V. Krivosheina, and I.V. Titkova, Phys. Lett. B 563, 23 (2003).
[13] T. Araki et al., Phys. Rev. Lett. 96, 101802 (2006).
[14] V.I. Tretyak, V.Yu. Denisov, Yu.G. Zdesenko, JETP Lett. 79, 106 (2004), Pisma Zh. Eksp. Teor. Fiz. 79, 136 (2004); nucl-ex/0401022.
[15] H.O. Back et al., (Borexino Collaboration), Phys. Lett. B 563, 23 (2003).
[16] S.N. Gninenko, N.V. Krasnikov, and A. Rubbia, Phys. Rev. D 67, 075012 (2003).
[17] A. Badertscher, P. Crivelli, U. Gendotti, S.N. Gninenko, V. Postoev, A. Rubbia, V. Samoylenko and D. Sillou, Phys. Rev. D 75, 032004 (2007).
[18] S.N. Gninenko, N.V. Krasnikov, and A. Rubbia, Phys. Rev. D 67, 075012 (2003).
[19] A. Badertscher, P. Crivelli, U. Gendotti, S. Gninenko, A. Rubbia, JINST 5, P08001 (2010); arXiv:1005.4802 [hep-ex].
[20] J. S. Bell and J. Steinberger, In Wolfenstein, L. (ed.): CP violation, 42-57. (In Oxford International Symposium Conference on Elementary Particles, September 1965, 195-208, 211-222).
[21] J. Steinberger, "K\^\textquoteleft\textquoteright Decay and CP Violation", CERN 70-1 (1970).
[22] L. Maiani, "CP and CPT Violation in Neutral Kaon Decays", L. Maiani, G. Pancheri, and N. Paver, The Second Daphne Physics Handbook, Vol. 1, 2.
[23] G. D’Ambrosio, G. Isidori, and A. Pugliese, "CP and CPT measurements at DAΦNE", L. Maiani, G. Pancheri, and N. Paver, The Second Daphne Physics Handbook, Vol. 1, 2.
[24] P. Bloch and L. Tauscher, Annu. Rev. Nucl. Part. Sci. 53, 123 (2003).
[25] F. Ambrosino et al., [KLOE Collab.], JHEP 0612, 011 (2006) arXiv:hep-ex/0610034.
[26] M. Antonelli and G. D’Ambrosio, G. Isidori, and A. Pugliese, "CP and CPT measurements at DAΦNE", L. Maiani, G. Pancheri, and N. Paver, The Second Daphne Physics Handbook, Vol. 1, 2.
[27] P. Crivelli, A. Belov, U. Gendotti, S. Gninenko, A. Rubbia, JINST 5, P08001 (2010); arXiv:1005.4802 [hep-ex].
[28] A. Angelopoulou et al., J. Steinberger, "K\^\textquoteleft\textquoteright Decay and CP Violation", CERN 70-1 (1970).
[29] F. Binon et al., Il Nuovo Cimento 64 A, 89 (1981).
[30] V.N. Bolotov et al., Nucl. Phys. B 85, 158 (1975).
[31] V.N. Bolotov et al., Nucl. Phys. B 85, 158 (1975).
[32] S. Andreas et al., arXiv:1312.3309 [hep-ex].
[33] S. Agostinelli et al. (GEANT4 Collaboration), Nucl. Instrum. Meth. A 506, 250 (2003); J. Allison et al. (GEANT4 Collaboration) IEEE Trans. Nucl. Sc. 53, 270 (2006).
[34] G.S. Atoian, V.A. Gladyshev, S.N. Gninenko, V.V. Isakov, A.V. Kovzenev, E.A. Monich, A.A. Poblaguev, A.L. Proskuryakov, I.N. Semenyuk, V.G. Lapshin et al., Nucl. Instrum. Meth. A 320, 144 (1992).
[35] G.A. Alekseev et al., Nucl. Instrum. Meth. A 461, 381 (2001).
[36] F. Binon et al., Nucl. Instrum. Meth. A 64, 89 (1981).
[37] V.N. Bolotov et al., Nucl. Phys. B 85, 158 (1975).
[38] F. Binon et al., Z. Phys. C 9, 109 (1981).
[39] A. Ceccucci, "The Kaon Physics Programme at CERN" Proceedings, 5th International Seminar on High Energy Physics (Quarks 2008) 23-29 May 2008. Sergeiv Posad, Russia.