Plasma centrifuge for isotope separation with axial circulation caused by a traveling magnetic field

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Abstract. The isotope separation processes in the high-frequency plasma centrifuge operating in the withdrawal mode are studied. The rotation of weakly ionized plasma is provided by a rotating magnetic field, and the axial countercurrent flow (circulation) is caused by a traveling magnetic field. The dependences of the overall enrichment factor and the separation capacity of a plasma centrifuge on a product flow rate are calculated. The case when a separation column is operated in a symmetrical mode is considered. The optimum characteristics of the separation device are discussed. It is shown that there is the optimal value of a product flow rate at which the maximum value of the separative power of a plasma centrifuge is reached.

1. Introduction
Separation of stable isotopes is an important branch of the country's economy, serving nuclear industry power, basic and life sciences, medicine, agriculture, and many others. Without a number of stable isotopes, it is difficult to imagine the further development of nuclear power and its efficiency and safety, basic research in nuclear physics, diagnosis and treatment of many diseases. Interest in the separation of stable isotopes in the plasma phase is associated with inability to use gas centrifuges to enrich isotopes, if there are no suitable volatile compounds at room temperature which can be used as process gases [1-3]. In addition, the problem of reprocessing spent nuclear fuel from nuclear power plants can be solved, as suggested, with using rapidly rotating plasma [4, 5].

A new concept of a circulating plasma centrifuge (PC) is proposed in [6], in which rotation of a weakly ionized $^{20}\text{Ne}-^{22}\text{Ne}$ isotope mixture is provided by a rotating magnetic field, and the axial circulation flow multiplying the radial separation effect is created by a traveling magnetic wave. There were determined the hydrodynamic characteristics of a PC and the overall enrichment factor of the apparatus for the so-called total reflux operating mode. In practice, the separation unit operates in a mode when the initial isotope mixture enters the device and the enriched in a target component fraction is withdrawn.

In this paper, basing on the results of an analytical approach for calculation of the azimuthal $V_{\phi}(r)$ and axial $V_{r}(r)$ components of plasma velocities, the overall enrichment factor and the separation capacity of a PC operating in the working regime with a product flow were calculated. Notice that the analytical solution obtained in this research allows us not only to estimate the optimum parameters of a single separation device, but also to trace, using the found theoretical dependences, the ways of intensification the separation processes in a PC.
2. Design principles
A PC with an axial circulation flow excited by a traveling magnetic field is presented in Fig.1. The rotation of weakly ionized plasma is produced by a transverse HF magnetic field having the angular velocity $\omega_0$, generated by the axial current carrying rods, two of which are shown in the front view, and all six rods in the top view in Fig.1.

![Figure 1. Design of a plasma centrifuge with a rotating magnetic field and axial circulation stimulated by a traveling magnetic field. 1, 2, 3 - the feed, waste, and product flows in a PC, respectively; 4 – current carrying rods; 5 – a quartz camera; 6 – a flow of a light fraction; 7 – a flow of a heavy fraction; 8 - 3-phase HF power supply.](image)

The circulation flow is excited by a travelling magnetic field caused by a delay line which is a system of capacitors with a capacitance $C$. The radial magnetic field propagates along the axis of the discharge chamber with a phase velocity $V_{ph}$ and varies in space and time according to the law

$$B_t = B_i \cos[\omega_1(t - z/V_{ph})],$$

where $\omega_1$ is the circular frequency of a traveling magnetic wave, $B_i$ is the amplitude value of the magnetic induction of the traveling magnetic field, $z$ is the longitudinal coordinate.

A detailed description of the separation installation and the procedure to calculate the hydrodynamic characteristics of a PC with separate excitement of a centrifugal field of force forces...
and an axial countercurrent flow are given in [6]. The case was considered when the length of a PC is much larger than the radius of the discharge chamber. It was demonstrated that in the total reflux mode of operation of a PC, when the highest degree of separation is achieved, it is possible to obtain significant multiplication of the primary (radial) separation effect along the length of a PC due to the created an axial countercurrent flow.

A schematic drawing of a separating element is shown in Fig. 2. A separated mixture with a given flow rate $G$ and concentration of a target component $C_0$ enters a continuously working separation element. Withdrawals of heavy $W$, $C_W$ and light $P$, $C_P$ fractions occur near the upper and lower ends of the working chamber (Fig. 1), respectively. In this case, the longitudinal enrichment factors become lower in contrast to the case of the total reflux regime considered in [6].

![Figure 2. Schematic drawing of a separating element](image)

As is known, the separation device, in addition to the overall separation factor determining a degree of enrichment of a separated mixture, is characterized by the so-called “the separative power” or “separation capacity”. This value is related to a speed of a diffusion process and ultimately determines the cost of separation.

### 3. Estimation of separation performance

In the absence of circulation ($B_z = 0$, $V_z = 0$), the radial enrichment factor for the binary isotope mixture is determined by the expression [6]

$$
\varepsilon_0 = \frac{\left( \frac{C}{1-C} \right)_{r=R_0}}{\left( \frac{C}{1-C} \right)_{r=0}} - 1 = \int_{0}^{R_0} \varepsilon_r(r) dr, \quad (1)
$$

where $\varepsilon_r(r) = \frac{\Delta M V^2(r)}{9RT} , C$ is the averaged over a cross-section of a PC concentration of a heavy component, $\Delta M$ is a difference in the molecular weights of the separated isotopes, $T$ is a temperature of plasma, $\Re$ is the universal gas constant, $V^2(r)$ is the azimuthal velocity component.

The latter in the regime of moderate induced magnetic fields [5] and under condition when braking the medium in the Ekman layers on the lids of a working chamber is neglected [7, 8], is determined by the expression

$$
V^2 = \frac{B_0^2 R_0 \Re e_0}{16 \mu_0 \langle \eta \rangle 2 y(1 - y^2)}, \quad (2)
$$

where $B_0 = I_z 0 / 2$ is the amplitude value of induction for the vacuum rotating magnetic field, $I_z$ is the amplitude value of the surface density of an axial current induced a rotating magnetic field, $\mu_0$ is a
magnet constant, \( y = r/R_0 \), \( \langle \eta \rangle \) is an averaged value of the coefficient of dynamic viscosity, \( \chi \) is a dimensionless parameter depending on the magnet Reinolds number \( \text{Re}_m = \mu_0 \sigma R_0^2 \), \( \sigma \) is a plasma conductivity.

According to [1], the transport equation for a stripping part of a column can be written in the form

\[
(K_1 + K_2) \frac{dC}{dz} = K_3 C(1 - C) + P(C - C_p)
\]  

Eq. (3) differs from the transport equation used in [6] for the total reflux regime by the presence of an additional term characterizing the flow of a target component in a product flow. Here \( K_1, K_2, \) and \( K_3 \) are the coefficients defined by the relations as follows

\[
K_1 = 2\pi \int_0^{R_0} \psi^2(r)\rho Dr dr, \quad K_2 = 2\pi \int_0^{R_0} \rho Dr dr \quad \text{and} \quad K_3 = 2\pi \int_0^{R_0} \psi(r)\varepsilon(r) dr,
\]

where \( \psi(r) = \int_0^r \rho V_z(r) r dr \) is a flow function, \( \rho \) is a density of a separated mixture, \( D \) is a coefficient of radial diffusion.

Following [6], one can determine a plasma density distribution \( \rho \) along the radius of a working chamber as

\[
\rho = \rho_0 \exp(Ay),
\]

where \( A = \frac{\langle M \rangle B_0^2 R_0^2 \text{Re}_m^2}{2 \cdot 5129 T \nu_0^2 \langle \eta \rangle^{\chi^2}} \), \( \rho_0 \) is a density on a working chamber, \( T_0 \) is a temperature on a PC axis.

Now we introduce gas-filling density value \( \rho^* \) and the parameter \( V = \frac{\rho^* V_{z0} R_0}{2 \langle \rho D \rangle (1 + (A - 1) \exp(A))} \), where \( V_{z0} = \sigma B_0^2 R_0^2 k_e \omega_i \text{Re}_m / 12 \langle \eta \rangle (f_1^2 + f_2^2) \), \( k_e = \omega_i / V_{ph} \) [6]. Whereupon, one can demonstrate that the dependence of the dimensionless optimal circulation flow \( U = \frac{2 \langle \rho D \rangle (V_{z0} m)}{\rho^* R_0 A^2} \) on the compressibility parameter \( A \) in the total reflux mode of PC operation has the form shown in Fig.3. As expected, increase of the compressibility parameter \( A \) leads to increase of the optimal circulation flow rate.
Figure 3. Dependence of the dimensionless optimal velocity of circulation $U$ on the compressibility parameter $A$.

Integration of Eq. (3) under the assumption of small enrichment factors and the condition $C(1-C) \approx \text{const}$ leads to the following dependence of the enrichment factor over the length of the stripping part of a PC [2]

$$
\varepsilon_c = \frac{4}{\sqrt{2}} \varepsilon_0 \frac{L_p}{R_0} \frac{m}{1+m^2} N \frac{1-\exp(-a)}{a} \quad (5)
$$

where $L_p$ is a length of a stripping part of a column, $m = F/F_0$ is a dimensionless twofold circulation flow, $F = 2\pi \int_0^{R_0} |\rho V_0| r dr$, $F_0 = \sqrt{K_2/k_1}$, $k_1 = K_1/F$, $a = \frac{\theta(1-\theta)}{K_1+K_2} L_p$, $\theta = P/G$ is a cut, $N = 2 \int_0^1 y(1-y^2) \psi_0 dy$, $\psi_0 = \psi / V_0 \rho_0 R_0^2 = \exp(-Ay) \left[ \lambda (y^2-1)+1-y^4 \right]$, $\psi_0 = \Delta MB_0^4 R_0^2 \lambda^2 \psi_0 / (\pi 256 \eta^2 \mu_0^2 g \chi^2)$, $\lambda$ is a dimensionless coefficient defined in [6].

The obtained analytical dependence of the overall enrichment factor on the circulation value demonstrates the existence of the optimal countercurrent flow. It also confirms that the efficiency of multiplication of the primary (radial) separation effect depends to a significant extent on a circulation profile described by the dependence of the $N$ parameter on the shape of a stream function $\psi (y)$. The approach developed in the paper allows us to define the circulation profile at which the maximum overall enrichment factor is reached.

In the regime of the optimal by enrichment factor circulation flow ($m=1$) and a symmetrical scheme of working of a separation element ($\theta = 1/2$), the expression for the enrichment factor (5) takes the following form

$$
\varepsilon_c = \varepsilon_0 \frac{4L_p}{\sqrt{2}} N \left(1-\exp(-L g/8\pi)\right)/(L g /8\pi) \quad (6)
$$

where $L_p = L_p / R_0$, $g = G/\langle p D \rangle R_0$ . Note that in the case when $\theta = 1/2$ (the symmetric working regime of a PC, the value of a dimensionless product flow will be equal to $p = g /2$ .
Then the separation performance of a PC at moderate enrichments is defined as [9]

\[
\delta U = G \varepsilon^2 / 2 .
\]  

(7)

It is quite obvious that the separative power of a PC calculated by this way has the dimension of a flow.

The dependences of the normalized enrichment factor \( \varepsilon = \varepsilon / \varepsilon_0 \) and the separative power \( \delta U = \delta U / (\rho D) R_0 \varepsilon_0^2 \) on the dimensionless feed flow \( g \) for the case of neon isotope separation are presented in Fig. 4.

![Figure 4. Dependences of the enrichment factor \( \varepsilon \) and the dimensionless separative power of a PC \( \delta u \) on the dimensionless feed flow rate \( g=2p \).](image)

As would be expected, the enrichment factor of a PC operating in withdrawal regime decreases with the increase in a product flow. Unlike the enrichment factor, the dependence of the separative power of a PC on a dimensionless feed flow has an optimum, the value of which for the parameters specified in this research is equal to \( g_{opt} \approx 2 \).

Estimating the optimal feed flow as \( G_{opt} \approx 2(\rho D) R_0 \), one can find for natural abundance of neon isotopes at the characteristic temperature of a neutral gas \( T \approx 1000 K \) \( (\eta) \approx 10^{-4} \) kg/m·s the optimum value of a feed flow rate to a PC \( G_{opt} \approx 10^{-2} \) g/s. For \( B_0 \approx 1.3 \cdot 10^{-2} \) T, \( B_1 = 6.10^{-3} \) T, \( \omega_b = 2.10^3 \) 1/s, \( \omega_l = 2.10^6 \) 1/s, \( \sigma = 2.10^3 \) 1/Ω·m, \( \text{Re}_m = 0.6 \), \( \text{Re}_m = 12.6 \), \( \chi \approx 2 \), \( R_0 = 0.05 \) m, \( \Delta M = 2 \) kg/Kmol, \( L_p = 1 \) m, it will be obtained that \( \varepsilon_0 \approx 0.06 \) and \( \varepsilon(g = 0) \approx 11 \). In this case, the maximum separative power of a PC will be equal to \( \delta U_{max} \approx 5 \cdot 10^{-4} \) g/s.

4. Conclusions

1. Theoretical study of neon isotope separation in a PC with operating in a mode with a product flow is carried out. There were calculated the overall enrichment factor and the separative capacity of the apparatus in which the primary (radial) separation effect and its multiplication in axial direction are achieved by rotating and traveling magnetic fields, respectively.

2. The dependence of the characteristics under investigation on a feed flow rate is obtained for the symmetric mode of PC operation. The separative power of the installation having the
maximum separative power $\delta U_{\text{max}}$ at the value $G_{\text{opt}}$ of a feed flow rate for the given working parameters of a PC and the characteristics of the weakly ionized plasma are estimated.

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