Optimization of the Design of Water Distribution Systems for Variable Pumping Flow Rates

Araceli Martín-Candileo 1*, David Santillán 1, Ana Iglesias 2 and Luis Garrote 1

1 ETSI Caminos, Canales y Puertos. Departamento de Ingeniería Civil, Hidráulica, Energía y Medio Ambiente, Universidad Politécnica de Madrid. Calle Profesor Aranguren 3, 28040 Madrid, Spain; david.santillan@upm.es (D.S.); l.garrote@upm.es (L.G.)
2 Departamento de Economía Agraria y CEIGRAM, Universidad Politécnica de Madrid, 28040 Madrid, Spain; ana.iglesias@upm.es
* Correspondence: araceli.martin@upm.es

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Abstract: Water supply systems need to be designed in an efficient way, accounting for both construction costs and operational energy expenditures when pumping is required. Since water demand varies depending on the moment’s necessities, especially when it comes to agricultural purposes, water supply systems should also be designed to adequately handle this. This paper presents a straightforward design methodology that using a constant flow rate, the total cost is equivalent to that of the variable demand flow. The methodology is based on the Granados System, which is a very intuitive and practical gradient based procedure. To adapt it to seasonal demand, the concepts of Equivalent Flow Rate and Equivalent Volume are presented and applied in a simple case study. These concepts are computationally straightforward and facilitate the design process of hydraulic drives under demand variability and can be used in multiple methodologies, aside from the Granados System. The Equivalent Flow Rate and Equivalent Volume offer a solution to design procedures that require a constant flow regime, adapting them to more realistic design situations and therefore widening their practical scope.

Keywords: water distribution systems; optimization; design; pump operation; demand variability

1. Introduction

In September 2015, the United Nations published the 2030 Agenda for Sustainable Development [1], intending to eradicate poverty in all its forms by setting up 17 goals and 169 targets to foster sustainable development through economic, social, and environmental aspects. Goal 6—Water Availability —aims to achieve universal access to drinkable water and sanitation. This Goal is very much related to Goal 7—Sustainable Energy—that aims to ensure sustainable energy for all. In order to achieve the challenges of water and energy, improving the design of hydraulic system is crucial, to ensure efficiency in the use of the resources. The areas in most need of action have less developed water resources systems and scarce economic resources. In these areas, the design strategies need to account for construction costs and the operation of the system, especially when pumping is required, linking energy to water. Design methodologies of water supply systems need to be practical and intuitive for practitioners, and at the same time, reliable and affordable for the users.

Mala-Jetmarova et al. [2] provide an excellent summary of the state of the art in water distribution designs, providing a clear classification of the optimization methods according to the objectives of the optimization (e.g., single objective, multi objective, and others) or the calculation method (e.g., stochastic, heuristic, and others). In more than 70% of the documented choices, the
single objective “least-cost” is selected, and many design proposals consider only construction costs, which leads to an incomplete vision of the water drive. This is a crucial error, since in 1994 Kim and Mays already reported that “for some utilities 90% of the total budget is for energy required for pumping” [3]. From 1980s onwards, some designers already integrate both the construction and the operating cost into the design procedure. Gessler and Walski [4] proposed a construction costs formulation including excellent details of digging costs and others. Alperovits and Shamir [5] detailed pumping costs calculated from an empirical approach that depends on the required power, providing practical results. Examples of recent designers that include both operational and construction costs are Ostfeld [6], who uses the software EPANET for the design; Pérez-Sánchez et al., Kang and Lansey, Jin et al. [7–9], and Samani and Mottaghi [10] carry out a “least cost assessment”, using hybrid programming. The remaining 30% of the known methodologies are multi objective proposals. The multi objective approaches are more widely followed in the past three decades and they consider the cost analysis and other aspects such as pressure deficit [11] or excess [12], greenhouse emissions [13], or water quality [14], among others. An excellent summary of the different approaches in multi objective methods is included in the work by Reed et al. [15].

In terms of the calculation process, evolutionary programming techniques like genetic algorithms have been widely used for water distribution optimization, for example by Marchi et al. [16], Kadu et al. [17], or Van Dijk et al. [18]. Nevertheless, most efforts in modern models deal with the computational processes. According to Goulter [19] most of the optimization models that are developed in research are not being used in the real practical design. The main reason of the limited use is not because the models do not work, as Walski et al. [20] proves, but mainly because of the challenges to interpret and use them in practical terms. In contrast, gradient search techniques are less common but they have convincingly shown that they can yield near optimal solutions for water networks [21]. They are also more intuitive than other design and optimization procedures.

On the other hand, due to climate change, population migrations, etc. [22], demand variations are becoming more extreme, increasing variability throughout the year [23,24]. This seasonal variation makes it even more complex for designers to properly conceive efficient water supply systems. For these reasons, designs that can manage changes in demand are desirable [25,26]. Babayan et al. [27] introduce this issue in their optimization using the standard deviation in the demand calculation. In studies like Granados et al. [28], safety coefficients are determined after a sensitivity analysis of the variables influencing the demand variation, especially for agricultural purposes. A different approach is used in Kapelan et al. [29], where the robustness is maximized generating random flow situations and calculating the probability to meet the required conditions at all nodes. Babayan et al. [30] compare the safety coefficient perspective to the approach of a random demand adjusting to a certain probability, and conclude that while both visions have similar results, the latter is more computationally expensive.

As suggestions for future works that will contribute to the crucial issue of the optimization of the design of water distribution systems for variable pumping flow rates, researches could also consider the assessment of the water sources and reclaimed water that can be used to balance water supply and demand [31].

The objective of this research is to develop a methodology that can overcome the limitations that a variable water demand regime brings to the design of a water supply system and that can easily be understood facilitating a practical approach. We propose two new design procedures that account for the variable demand, the Equivalent Flow Rate and the Equivalent Volume. The methodology proposed is based on a gradient based procedure, since it is very intuitive and not computationally demanding, therefore easily translated into practical terms. To illustrate the application, a case study is presented.
2. Materials and Methods

The total cost of a hydraulic impulsion depends mainly on the cost of construction of the pipe, the cost of construction of the pumping station, and the cost of the energy required for the pumps to work during the entire lifespan of the system. Other costs, such as maintenance and operation, may be neglected since they do not depend directly on pump or pipe sizing.

$$\text{Total Cost} = \text{Pipe Construction} + \text{Pumping Station Construction} + \text{Energy}$$

The Pipe Construction Cost increases as the pipe diameter widens, since the price of the tubes increases, and so too does the excavation and installation cost due to the difficulties that arise through transportation and assembly.

The construction cost of the pumping station can be considered relatively independent of that of the chosen pipe, since the different pump models usually have a similar cost and the group and stage configurations depend on the variability of the demand and not on the pipe features.

The cost of energy depends mainly on the volume of water to be elevated and also on the head and the pump efficiency. The head depends on the diameter of the pipe, since the larger the pipe diameter, the less head loss. In this way, using a larger diameter reduces the pumping cost. The efficiency of the pump is a variable whose relationship with the other factors had not been studied in depth until recently. As a general rule, it was assumed that the larger the pump is, the better performance it had. Nevertheless, Martin-Candilejo et al. [32] have addressed this issue in depth concluding that there is a direct relationship between the flow rate and the pump efficiency: The pump efficiency is better for greater discharge flow rates, reaching an asymptote for 90%.

Figure 1 is a scheme of the cost distribution depending on the pipe diameter. Since the pipes diameters do not form a continuous series, but rather are discrete values in the market, the traditional way of solving the problem has consisted various alternatives of diameters and pumps that meet the technical requirements and then each alternative is evaluated to select the one with the lowest cost.

Figure 1. Simplified scheme of the costs of the water drive depending on the diameter $\varnothing$ of the pipeline: As the diameter increases the construction costs rise but the head losses decrease, implying a reduction in the energy cost.

The procedure proposed in this work is based on the following principle: The head loss can be reduced by increasing the pipe diameter. This means savings in the energy cost but also an increase in the cost of construction. This can be studied from a unitary perspective: Comparing the cost of reducing one meter of head loss by increasing the pipe diameter with the savings in energy expenditure by not having to elevate water that one extra meter. This is the change Gradient concept, that was developed by Granados [33,34] as part of his pipe network optimization method, the Granados’ System. The Change Gradient only makes sense when the
flow rate is constant. Therefore, for more realistic situations, this paper has developed the new concepts of the Equivalent Flow Rate and the Equivalent Volume for two different procedures for the design of a hydraulic drive.

2.1. Change Gradient Concept

The Change Gradient [33] is defined as the cost of reducing one meter of head loss by increasing the pipe diameter from \( \phi_i \) to the next bigger one \( \phi_{i+1} \):

\[
GC_q^{\phi_i \rightarrow \phi_{i+1}} = \frac{p_i - p_{i+1}}{\Delta h_{i+1}^q - \Delta h_i^q},
\]

(1)

\( p_i \), \( p_{i+1} \) are prices of pipes of length \( L \) and diameters \( \phi_i \) and \( \phi_{i+1} \) respectively and \( \Delta h_i^q \), \( \Delta h_{i+1}^q \) are the head losses of a pipe of length \( L \) and diameters \( \phi_i \) and \( \phi_{i+1} \) for a given \( q \) flow rate. This is represented in Figure 2.

![Figure 2. Change Gradient concept. Increasing the pipe’s diameter means a reduction in the head loss along the pipeline, but it has the extra cost of the wider and more expensive tube. The Change Gradient is the cost of reducing the head loss by 1 meter.](image)

If, by instance, the head loss is calculated using Manning’s formulation [34], the length of the pipe is canceled out from the Change Gradient’s expression, and it stands as:

\[
GC_q^{\phi_i \rightarrow \phi_{i+1}} = \frac{\pi}{\frac{L}{n_i^{16/3}}} \left( \frac{\phi_i^{20/3} q_i^2}{\phi_{i+1}^{20/3} q_{i+1}^2} \right) = \frac{\pi^2}{n_i^{16/3}} \left( \frac{1}{\phi_i^{16/3}} - \frac{1}{\phi_{i+1}^{16/3}} \right) \left( \frac{p_i - p_{i+1}}{q_i^2} \right),
\]

(2)

being \( K_{GC} \):

\[
K_{GC} = \frac{\pi^2}{n_i^{16/3}} \left( \frac{1}{\phi_i^{16/3}} - \frac{1}{\phi_{i+1}^{16/3}} \right).
\]

where \( L \) is the pipe length, \( p_i \) and \( p_{i+1} \) are the prices of the pipes of a diameter \( \phi_i \) and \( \phi_{i+1} \), respectively; \( n_i \) and \( n_{i+1} \) are the Manning coefficients for the roughness of pipes \( i \) and \( j \), respectively; and lastly \( q_i \) is the flow rate through the pipe.

Regarding the available diameters, each pipe manufacturer only offers a finite number of commercial diameters for each type of pipe. This means that the designer must adjust to the series of diameters offered for that pipe type.

Sometimes the pipes are conformed by two or more twin pipes, usually connected in parallel, that start from the same point and reach the same destination, as Figure 3 represents. In these cases, the flow rate through the drive \( q \) is distributed between these pipes, passing through each of them \( q/nt \), where \( nt \) is the number of equal pipes that make up the drive. The price of
these pipes is nt times the price of an independent pipe. The Change Gradient corresponding to this type of situations follows the following expression:

$$GC_{\varnothing_1 \to \varnothing_j} = \frac{\pi^2}{n^2 \cdot 2^{2/3}} \left( \frac{1}{s_{\varnothing_1}^{5/3} \cdot \frac{1}{s_{\varnothing_j}^{5/3}}} \right) \frac{1}{(q/nt)^2} = nt^3 \cdot GC_{\varnothing_1 \to \varnothing_j}.$$  

(3)

Figure 3. Parallel pipe configuration of the hydraulic drive.

2.2. Energy Cost

During the operation of a pumping station it is common that different discharges are pumped to meet the needs of the demand. The energy consumed in each of these situations is different, depending not only on the flow pumped, but also on the height pumped, the electromechanical efficiency, the duration of this pumping situation and the unit price of the energy.

If a simple case is analyzed, it can be assumed that there are n pumping situations that differ by the flow rate pumped ($q_1$, $q_2$, $q_i$, ..., $q_n$). In this case, the annual cost for the energy consumed $c_E$ can be obtained from the following expression [34] (the first and second equality are in International System units, whilst the third one depends on the following chosen units):

$$c_E = \Sigma E_i \cdot p_i = \Sigma_{i=1}^n q_i \cdot h_i \cdot \frac{1}{\mu_{B_i}} \cdot \frac{1}{\mu_{M_i}} \cdot t_i \cdot p_i = \Sigma_{i=1}^n 9.81 \cdot \frac{V_i}{3600} \cdot h_i \cdot \frac{1}{\mu_{B_i}} \cdot \frac{1}{\mu_{M_i}} \cdot p_i,$$  

(4)

where $c_E$ is the annual cost of energy consumed by pumping (€); E is the annual energy consumed in the situation i (kWh); $p_i$ is the unit price of energy in the situation i (€/kWh); $q_i$ is the specific weight of the pumped liquid (for water 9.810 N/m³); $h_i$ is the height pumped in the situation i (m); $\mu_B$ and $\mu_M$ are the pump and engine efficiency in the situation i, respectively; $t_i$ is the annual duration of situation i (hours); and lastly $V_i$ is the annual volume (m³) pumped during situation i.

This $c_E$ cost is repeated every year during the lifespan of the pipe. To obtain the capitalized cost, the annual costs must be integrated at present value into a single global energy cost. The total cost of the energy spent during the useful life of the pipes capitalized at the beginning would therefore be $C_E$:

$$C_E = f_A \cdot c_E.$$
\( f_a \) is called discount factor and it depends mainly on the discount rate \( i \), the lifespan of the pipe \( n_a \) and the construction period duration \( n_c \). A generalist definition of \( f_a \) [34] would be:

\[
f_A = \frac{(1 + i)^{n_a} - 1}{(1 + i)^{n_a} i} \times \frac{1}{(1 + i)^{n_c}}.
\]

### 2.3. Optimization for Constant Flow Rate

This situation is given when the pumping station works with a constant flow rate. This means the operating point of the pump does not change. In this way, all the other variables that affect the energy cost (\( q_v, h_v, n_v, M \) and \( p \)) are constant. In this case, the flow rate \( q_v \) can be calculated, as well as the Change Gradients \( GC_{q_i \rightarrow q_{i+1}} \) associated to the increase of diameter \( \phi_i \) to the next bigger one \( \phi_i = \phi_{i+1} \). Likewise, as much as the cost of pumping energy is concerned, since the pump always works at the same operating point, the pump efficiency \( n_v \), the engine efficiency \( M \) and the pumping height \( h_v \) remain constant (they can be simplified to just \( h_v, n_v, M \)), and the total cost of energy \( C_E \) can be obtained with the following expression:

\[
C_E = f_A C_E = f_A \sum_{1}^{n} \frac{V_i}{3600} h_i \frac{1}{\mu B_i} \frac{1}{\mu M_i} p_i = f_A 9.81 \frac{\sum V_i}{3600} h \frac{1}{\mu B} \frac{1}{\mu M} p .
\]

From the previous expression, making \( h = 1m \) and \( \sum V_i = V \) (which is the total annual volume pumped), the cost of the energy required for each meter of elevation \( C_{E_1} \) can be calculated as:

\[
C_{E_1} = \frac{C_E}{h} = f_A 9.81 \frac{V}{3600} \frac{1}{\mu B} \frac{1}{\mu M} p .
\]

(5)

With the exception of the pump performance \( n_v \), the other variables in the previous equation are actually data: \( V \) is the total volume to be pumped, \( p \) is the unit price of the energy that has been hired, and \( f_A \) is calculated from the discount rate \( i \), the service life \( n_w \) of the water pipeline and the construction period \( n_c \). Regarding the engine efficiency \( M \), although it varies theoretically depending on the engine model chosen and the operating point of the pump, the variations in engine performance are so small that it can be considered constant across different models and manufacturers [32].

Therefore, the above equation can be simplified by grouping all the data and parameters that have fixed values in the coefficient \( KC_{E_1} \), which can be considered constant for each case analyzed:

\[
C_{E_1} = KC_{E_1} \frac{1}{\mu B} ,
\]

(6)

\[
KC_{E_1} = f_A 9.81 \frac{V}{3600} \frac{1}{\mu M} p .
\]

(7)

In the previous expression, it is shown that the only variable that affects is the pump efficiency, \( \mu B \), which will depend on the pump model chosen and the operating point (and it is yet unknown).

For this simple case of constant pumping flow, a design procedure for selecting the pipe diameter is established through the following argument:

- If \( GC_{q_i \rightarrow q_{i+1}} < C_{E_1} \Rightarrow \) Diameter \( \phi_{i+1} \) is preferable to \( \phi_i \) since the cost of reducing 1 m the head loss by passing from \( \phi_{i+1} \) to \( \phi_i \) is cheaper than the cost of pumping that additional meter.

- If \( GC_{q_i \rightarrow q_{i+1}} > C_{E_1} \Rightarrow \) Diameter \( \phi_i \) is preferable to \( \phi_{i+1} \) since the cost of reducing 1 m the head loss by passing from \( \phi_{i+1} \) to \( \phi_i \) is more expensive than the cost of pumping that additional meter.
If this comparison is started on the diameter $\varnothing_1$, which is the smallest in the series of diameters available for that pipe model and manufacturer (that meets the maximum velocity condition along the network), the process leads to the optimum diameter for the water drive.

It has already been indicated that the value of $C_E$ depends on the pump model chosen, and more specifically, on its efficiency $\eta$ at the point of operation. Therefore, theoretically, the pump should first be chosen so that the pipe diameter can be selected. The $\eta$ efficiency at the operating point can only be obtained when the operating point is known, which depends on the diameter of the pipe. Therefore, the pipe diameter should first be known to obtain the operating point of the pump and thus its efficiency.

Therefore, starting by either selecting the pump, or the pipe diameter, the process involves iterating to find the optimal pipe diameter for each specific pump model. If the pump model changes, even slightly, the operating point would change and so would the pipe diameter.

In order to break this vicious circle, here we propose a more direct calculation process. It consists of solving the pump efficiency that each diameter requires to be competitive. As it is shown in the following equation, diameter $\varnothing_i$ will be the optimum (this means it should not be substituted by to the next diameter $\varnothing_{i+1}$ of the series) when:

$$GC^q_{\varnothing_i \rightarrow \varnothing_{i+1}} > C_{E1} \Rightarrow GC^q_{\varnothing_i \rightarrow \varnothing_{i+1}} > KC_{E1} \frac{1}{\mu_B} \Rightarrow \mu_B > \frac{KC_{E1}}{GC^q_{\varnothing_i \rightarrow \varnothing_{i+1}}} \Rightarrow \text{Keep } \varnothing_i$$

otherwise:

$$GC^q_{\varnothing_i \rightarrow \varnothing_{i+1}} < C_{E1} \Rightarrow GC^q_{\varnothing_i \rightarrow \varnothing_{i+1}} < KC_{E1} \frac{1}{\mu_B} \Rightarrow \mu_B < \frac{KC_{E1}}{GC^q_{\varnothing_i \rightarrow \varnothing_{i+1}}} \Rightarrow \text{Move to } \varnothing_{i+1}.$$  

(8)

This means that, whenever there is a pump on the market whose efficiency can be greater than the calculated $\mu_B$, the optimum diameter will be $\varnothing_i$. In the event that no commercial pump can reach that performance because it is very high, it will be necessary to move to the next diameter $\varnothing_{i+1}$.

Therefore, to apply this method it is necessary to know the maximum performance that pumps can reach. Martin-Candilejo et al. [32] studied the optimum pump efficiency of 226 commercial pumps. After their assessment, they obtained an empiric relationship between the optimum $\mu_B$ and the flow rate, $q$. They presented Equation (10) to calculate the estimated optimum pump efficiency depending on the discharge flow. In their work, they also expose Equation (11) to determine the maximum expected value of the optimum $\mu_B$.

$$\mu_B^{\text{Average}} = 0.1286 \ln (2.047 \ln q - 1.7951) + 0.5471; \text{ } r^2 > 98\%,$$  

(10)

$$\mu_B^{\text{Maximum}} = 0.0576 \ln (2.047 \ln q - 1.7951) + 0.741; \text{ } r^2 > 90\%,$$  

(11)

being $q$ the circulating flow rate in liters per second (L/s).

Of course, only diameters that meet the minimum limitations required will be selected. These limitations might be maximum velocity, pressure [35,36], among others.

2.4. Optimization for Variable Flow Rate

Pumping stations that are designed to work with variable flow rates are more frequent in civil engineering applications, since they allow a better adjustment of the flow rates to those demanded at any time, avoiding in this way the situation of pumping a flow greater than the one required, and therefore, reducing the head losses with the consequent saving of energy. For this more realistic situation, the method proposed previously raises two problems:

- The Change Gradient can only be calculated at a constant flow rate.
- The energy cost $C_E$ is calculated for a constant pump performance and head.
To solve these two issues, this paper proposes two new methods called the Equivalent Flow Rate and the Equivalent Volume methods explained below.

2.4.1. Concept and Calculation of the Equivalent Flow Rate

The equivalent flow rate \( q_{eq} \) can be defined as a theoretical flow rate for which, if all the volume \( V \) required in a year was pumped at this discharge, the cost of the pumping energy would be the same as the cost of pumping at a variable flow rate regime.

For the following reasoning, it is convenient to use a theoretical example. Assuming that the required annual volume \( V \) is pumped through a specific pipe and following a variable flow distribution, the annual energy cost \( c_r \), applying the Equivalent Flow Rate definition, would result in:

\[
c_E = \sum_{k}^{n} 9.81 \frac{V_k}{3600} h_k \frac{1}{\mu_{B_k}} \frac{1}{\mu_{M_k}} p_k = 9.81 \frac{V}{3600} \frac{h_{Eq}}{\mu_{B_{Eq}}} \frac{1}{\mu_{M_{Eq}}} p_{Eq},
\]

being \( V \) the annual volume of water pumped; \( h_{eq} \) the pumping head for the theoretical operating point corresponding to \( q_{eq} \) \( \mu_{B_{Eq}} \) and \( \mu_{M_{Eq}} \) the pump and engine efficiency at the theoretical operating point corresponding to \( q_{eq} \) \( n \) the number of periods of different flow rate, and \( p_{eq} \) the theoretical unit price of the energy with which \( q_{eq} \) would be pumped. Figure 4 represents this idea the concept of the equivalent flow rate.

![Figure 4. Equivalent Flow Rate definition.](image)

For this analysis, the cost of pumping energy should be separated into two parts. On the one hand, the cost corresponding strictly to the geometric height, and on the other hand, the cost corresponding to the head losses. In this way, knowing that \( h_k = h_G + h_{eq} \), the previous equation would be:

\[
c_E = \sum_{k}^{n} 9.81 \frac{V_k}{3600} (h_G + \Delta h_k) \frac{1}{\mu_{B_k}} \frac{1}{\mu_{M_k}} p_k = 9.81 \frac{V}{3600} (h_G + \Delta h_{Eq}) \frac{1}{\mu_{B_{Eq}}} \frac{1}{\mu_{M_{Eq}}} p_{Eq},
\]

where \( h_G \) is the geometric height and \( h_{eq} \) is the head losses when \( q_{eq} \) is circulating.

Some simplifications can be made in the previous equation. To begin with, the motor efficiency and the unit price of energy can be assumed as constant. These assumptions are quite close to reality, since, as it has been already indicated, motor efficiencies vary very little in different operating regimes, and it is common to hire a flat rate of energy. With the above simplifications, the expression would be:

\[
V h_G + V \Delta h_{Eq} = \sum_{k}^{n} \frac{\mu_{B_{Eq}}}{\mu_{B_k}} V_k h_G + \sum_{k}^{n} \frac{\mu_{B_{Eq}}}{\mu_{B_k}} V_k \Delta h_k.
\]
Now, a new simplification must be made, and this one is more debatable. It consists of assuming a constant pump efficiency. This implies that all $\eta_k$ are equal, including $\mu_{BEq}$. This is a purely operational assumption and does not correspond to reality, since the pump’s performance varies significantly for the different operating points. This simplification is made in order to prevent from future iterations to obtain the Equivalent Flow Rate, and it is later shown to be unnecessary.

The general expression would be:

$$V h_G + V \Delta h_{Eq} = \sum_{i=1}^{n} V_k h_G + \sum_{i=1}^{n} V_k \Delta h_k \quad \Rightarrow \quad V \Delta h_{Eq} = \sum_{i=1}^{n} V_k \Delta h_k.$$

On the other hand, the various head losses associated to different flow rates can be expressed as below using Manning’s expression:

$$\Delta h_k = \frac{L_n}{\pi^2} \frac{2^{20/3} q_k^2}{h_k^{16/3}} = \alpha q_k^2.$$

Applying this to the previous equation, the new general expression is:

$$V \alpha q_{Eq}^2 = \sum_{i=1}^{n} V_k \alpha q_k^2,$$

and it can finally be simplified to obtain the Equivalent Flow Rate:

$$q_{Eq} = \left[ \frac{\sum_{i=1}^{n} V_k q_k^2}{V} \right]^{1/2}, \quad (14)$$

where $q_{Eq}$ is the Equivalent Flow Rate. It is a constant flow that implies the same energy cost as that of a variable flow regime; $q_k$ is the flow pumped in the operating situation $k$; $V_k = q_k \times t_k$, is the annual volume in operating situation $k$; $t_k$ is the annual time spent in pumping $q_k$; and at last, $V$ is the annual volume pumped, calculated as $V = \sum_{i=1}^{n} V_k$.

The previous expression shows that the Equivalent Flow Rate is an average flow rate of the variable regime. From this point of view, it is an intermediate discharge that should correspond to an operating point close to the optimum of the pump, in a way that the real variable flow values are located around the sides of this optimum.

It should be remembered that the previous equivalent flow rate has been obtained by making three assumptions: that the engine efficiency, that the unit price of energy, and that the pump efficiency are the same for any operating point. The first two are logical and respond to the reality of most cases. The third one is, however, conceptually incorrect and it would theoretically require a second approximation of the Equivalent Flow Rate once the pipe diameter has been obtained and the pump selected. However, our simulations let us affirm that this first approximation of the equivalent flow rate is already sufficient, since the pump performances, although different, compensate between the different operating points if the pump is properly selected.

2.4.2. Optimization Procedure Using the Equivalent Flow Rate Method

The Equivalent Flow Rate has been developed to calculate the Change Gradients when the discharge regime is variable. As previously explained, each Change Gradient is associated with a specific flow rate. Therefore, when the flow regime is variable, it is necessary to use the Equivalent Flow Rate to calculate them. In this way, the expression of the Change Gradient will change to:

$$GC_{q_{i} - q_{j}}^{q_{Eq}} = \frac{P_i - P_j}{\Delta h_i^{q_{Eq}} - \Delta h_i^{q_{Eq}}}.$$

Using the concept of the Equivalent Flow Rate in the calculation of the cost of energy per meter lifted, the previous equation turns into:

$$C_E = \frac{C_E}{\Delta h_{Eq}} = f_A \ 9.81 \ \frac{V}{3600} \ \frac{1}{\mu_{BEq}} - \frac{1}{\mu_{BEq}^{eq}} - \mu_{BEq}^{eq} = K_C^{q_{Eq}} \ \frac{1}{\mu_{BEq}}.$$
It has already been discussed that the engine efficiency and the unit price of energy can be assumed equal for any operating point. For this reason, coefficient $KC_{E1}$ does not really depend on the value of the Equivalent Flow Rate, so it will remain as $KC_{E1}$.

Therefore, the Equivalent Flow Rate allows the calculation of the Change Gradients and the Energy Cost per meter pumped for a variable discharge regime. The comparison between the two of these two terms allows to obtain the optimum diameter of the pipeline, following the same reasoning that was previously described: Diameter $\phi_i$ will be optimum (e.g., it should not be passed onto the next diameter of the series $\phi_{i+1}$) when:

$$GC_{\phi_i \rightarrow \phi_{i+1}} > C_{E1} \Rightarrow GC_{\phi_i \rightarrow \phi_{i+1}} > KC_{E1} \frac{1}{\mu_{BEq}} \Rightarrow \mu_{BEq} > \frac{KC_{E1}}{GC_{\phi_i \rightarrow \phi_{i+1}}} \Rightarrow \text{Select } \phi_i.$$

As a summary, the method would be:

1. **Equivalent Flow Rate is calculated:**
   $$q_{Eq} = \sqrt[n]{\frac{\sum_k^n V_k q_k^2}{V}}.$$

2. With $q_{Eq}$, the Change Gradients are calculated for each increase in diameter (starting with the smaller in a commercial list and passing onto the very next one):
   $$GC_{\phi_i \rightarrow \phi_{i+1}} = \pi^2 \frac{n^2}{2^{2/3}} \left(\frac{1}{\phi_i^{16/3}} - \frac{1}{\phi_{i+1}^{16/3}}\right) \frac{1}{q_{Eq}^2}.$$

3. Parameter $KC_{E1}$ is calculated:
   $$KC_{E1} = f_A 9.81 \frac{V}{3600} \frac{1}{\mu_{M}} \frac{1}{\mu_{P}}.$$

4. The required pump efficiency corresponding to each diameter change is calculated:
   $$\mu_B = \frac{KC_{E1}}{GC_{\phi_i \rightarrow \phi_{i+1}}}.$$

5. When the pump efficiency reaches a value that can be easily found in the pump market, diameter $\phi_i$ is selected.

6. In case the pump efficiency seems too high to reach, it is preferable to select a bigger diameter $\phi_{i+1}$. The initial investment will be greater, but the cost of pumping energy will be lower. This reduces the risk in the event of rises in energy price, different from the predictions made during design phase. As a reference for the expected value of the pump efficiency use Equation (10).

To sum up, the optimization procedure proposed in this section is based on the two following premises, which have been demonstrated during the development of the work itself:

- The optimum diameter of the water drive depends only on the pump performance.
- The cost of the full water drive (pipes and pumps) depends only on the pipe diameter.

Therefore, the optimum design of a water drive depends on one only variable, and that is the flow rate. If, as usual, the flow rate is variable, the Equivalent Flow Rate can be used, whose formulation is an original innovation of this work.

2.4.3. Concept and Calculation of the Equivalent Volume

If a constant flow rate $q_m$ is chosen to design the pipe line (preferable $q_m = 1$ m$^3$/s to make calculations easier), $V_{Eq}^{Sm}$ is the total Equivalent Volume of water that needs to be pumped at that flow rate $q_m$ to make the final cost of pumping equal to what it would be pumping at a variable regime of flow rates. For the calculation of the Equivalent Volume $V_{Eq}^{Sm}$ , a similar reasoning to the one followed for the Change Gradient will be applied: The final cost will always be referred to the cost of elevating the water at 1m height $C_{E1}$. Applying this approach in Equation (5), $C_{E1}$ can be expressed in terms of $V_{Eq}^{Sm}$ as below:
\[ C_{E1} = f_A \times 9.81 \times \frac{\nu_{\text{im}}}{3600} \times \frac{1}{\mu_B} \times \frac{1}{\mu_M} \times p = K_{CV} \times \frac{\nu_{\text{eq}}}{\mu_B}, \]  
(15)

\[ K_{CV} = f_A \times 9.81 \times \frac{1}{3600} \times \frac{1}{\mu_M} \times p. \]

Therefore, the concept of the Equivalent Volume is expressed as follows.

\[ C_{Ei} = \sum C_{E_1} \times q_k = C_{E_1} \times q_m = K_{CV} \times \frac{\nu_{eq}}{\mu_B}. \]  
(16)

Apart from that, in the situation that a flow rate \( q_i \) is circulating, an increase of the pipe diameter from \( \phi_i \) to \( \phi_{i+1} \) would mean a reduction of the head loss \( \Delta h_{\phi_i \to \phi_{i+1}} \) that, using Manning’s expression, only depends on the flow discharge \( q_1 \). To simplify the terminology \( \Delta h_{\phi_i \to \phi_{i+1}} \) will be referred as \( \Delta h_{\phi_k} \).

\[ \Delta h_{\phi_k} = \Delta h_{\phi_i \to \phi_{i+1}} = \Delta h_{\phi_i} - \Delta h_{\phi_{i+1}} = \beta \times q_k^2, \]

being \( \beta \) a constant deduced from the invariable terms in Manning’s formula (see Equation (13)).

This reduction of the head loss translates in a savings in the energy cost \( C_{eq} \). To express \( C_{eq} \), the same simplifications that were used for the Equivalent Flow Rate will be made: the engine efficiency, that the unit price of energy, and that the pump efficiency are the same for any operating point. Once again, the first two assumptions are close to reality, but the third one is not; however further simulations have shown that the pump performances, although different, compensate between the different operating points. This being said, the savings in energy costs can be expressed as it follows.

\[ C_{eq} = f_A \times 9.81 \times \frac{\nu_k}{3600} \times \Delta h_{\phi_k} \times \frac{1}{\mu_B} \times \frac{1}{\mu_m} \times p_k = K_{CV} \times V_k \times \Delta h_{\phi_k} \times \frac{1}{\mu_B} \]

(17)

Additionally, any variance of the head loss associated to a flow rate can be expressed in terms of a different discharge as:

\[ \Delta h_{\phi_k} = \beta \times q_k^2 \to \Delta h_{\phi_k} = \beta \frac{q_m^2}{q_{m_k}} \times q_k^2 = \Delta h_{\phi_m} \times \frac{q_k^2}{q_{m_k}^2}. \]

Introducing the previous expression in Equation (16), \( C_{eq} \) is then:

\[ C_{eq} = K_{CV} \times V_k \times \Delta h_{\phi_m} \times \frac{q_k^2}{q_{m_k}^2} \times \frac{1}{\mu_B}. \]

However, as it was expressed in the Equivalent Volume definition, this needs to be analyzed from the perspective of pumping water at 1 m height so that, later on, the Change Gradient methodology can be followed. For this unitary point of view, \( C_{E_i} \) is obtained from \( C_{eq} \) by:

\[ C_{E_i} = \sum C_{eq} \times \Delta h_{\phi_m} = K_{CV} \times V_k \times \frac{q_k^2}{q_{m_k}^2} \times \frac{1}{\mu_B}. \]

Introducing this conclusion in Equation (13), the Equivalent Volume formulation can be deduced:

\[ V_{eq} \times \frac{\nu_{eq}}{\mu_B} = K_{CV} \times V_k \times \frac{q_k^2}{q_{m_k}^2} \times \frac{1}{\mu_B}. \]

Therefore, the Equivalent Volume is:

\[ V_{eq} = \frac{\nu_{eq} \times \sum V_k \times q_k^2}{q_{m_k}^2}. \]

This expression is simplified if the virtual constant discharge of design \( q_m \) takes the value of \( q_m = 1 \text{m}^3/\text{s} \). For this value, the Equivalent Volume is simplified as:

\[ V_{eq}^{q_m=1 \text{m}^3/\text{s}} = \sum V_k \times q_k^2. \]  
(18)
2.4.4. Optimization Procedure using the Equivalent Volume Method

The Equivalent Volume has been deduced to calculate the Change Gradients when the discharge regime is variable. As a summary, the process is:
1. Choose any value for a virtual constant flow rate \( q_m \). It is recommended that \( q_m = 1 \) m\(^3\)/s. All of the following steps will be shown for this value.
2. For the chosen \( q_m \) calculate the Change Gradients series, correspondent to the change of one pipe diameter from a commercial catalogue to the immediate bigger one:

\[
G_{C_{\phi_j \rightarrow \phi_{j+1}}}^{q_m=1} = \frac{2\pi^2}{n^2 2^{20/3}} \left( \frac{1}{\phi_j^{16/3}} - \frac{1}{\phi_{j+1}^{16/3}} \right).
\]

3. Calculate the Equivalent Volume for \( q_m \):

\[
V_{E_{q_m}}^{q_m} = \sum V_k q_k^2 = \sum V_k q_k^2.
\]

4. Calculate \( K_{CE1} \) for \( V_{E_{q_m}}^{q_m} \):

\[
K_{CE1} = \frac{9.81 f_A P}{3600 \mu_m V_{E_{eq}}} V_{eq}^{q_m}.
\]

5. Calculate the required pump efficiency needed to each diameter change:

\[
\mu_B = \frac{K_{CE1}}{G_{C_{\phi_j \rightarrow \phi_{j+1}}}^{q_m=1}}.
\]

6. At the point, the same reasoning as for the constant discharge situation is followed: When the pump efficiency reaches a value that could easily be found in the market (e.g., between 80–85%), select diameter \( \phi_j \). For those cases of uncertainty, it is preferable to select a bigger diameter \( \phi_{j+1} \). The initial investment will be bigger but the risk of additional cost in case of a higher energy price will be diminished.

3. Results and Discussion

Case study: Parallel pipes using the Equivalent Flow Rate

As a simple example of the applications of the Equivalent Flow Rate concept, a theoretical case of a hydraulic drive has been studied. The drive consists of two parallel pipes of 500 m long. The water drive serves agricultural purposes, and the total area to supply is 3000 ha. The system has no regulation at the end of the drive, as Figure 5 illustrates, and therefore, the flow rate will vary depending on the demand of the month, therefore, the scheme is similar to Figure 3 (without regulation tank at the end of the circuit). As it is expected, this demand is higher for the warmer periods (reaching its peak in July), and will stop during winter, conforming in this way, the hydrological year. The demand distribution is shown in Table 1 and the average monthly flow rate \( q_k \) is calculated below.

This illustrates a classical case very common in the professional practice, which is the elevation of water to a higher deposit. In practice, the designing would firstly require the pipeline price collection among the local manufacturers as well as a proper demand study, so that the design flow rate can be obtained. With this data, the procedure will follow the steps enumerated in Section 2.4.2.
The new approach derived in this study and presented in the Methodology section is applied in the case study, where the circulating flow rate is variable, and this prevents from direct calculation (see data in Table 1). Therefore, to begin with, applying Equation (14), a constant flow rate distribution that is equivalent in energy costs to the real one is obtained: The Equivalent Flow Rate is calculated, as shown in Table 2.

Table 2. Equivalent flow rate calculation from Equation (13).

|        | April     | May       | June      | July      | August    | September | \( q_{eq} \) |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|--------------|
| \( V_k \cdot q_k^2 \) | 502,347   | 3,763,682 | 4,018,776 | 12,702,426| 3,763,682 | 502,347   | 1.237        |

Once the homogeneous demand distribution has been obtained, thanks to the Equivalent Flow Rate, Granados System can be applied. As it was previously explained, the methodology consists of comparing the cost of investing in buying a bigger pipe or pumping greater head losses, and choosing whatever is cheaper. This analysis is made through the Change Gradient concept, and it requires a commercial series of diameters and its prices. In Table 3, it was included a list of diameters and its correspondent prices as a representation of a commercial catalog. These prices have been estimated based on average values for the selected material, which in this case stainless steel has been preferred. For Manning Coefficient, the selection of the friction factor a value of 0.0085 has been assumed for stainless steel pipes in good state. In this way, all Change
Gradients are calculated following Equation (3). Table 3 shows the resulting $GC_{\phi_i \rightarrow \phi_{i+1}}^\text{eq}$ for each diameter onto the next one.

Table 3. Commercial diameter series accompanied by their correspondent price. Change Gradient calculation and optimization process by calculating the required efficiency of the pump.

| Pipe Catalog | Optimization |
|--------------|--------------|
| Stainless Steel | Change Gradient | Efficiency |
| Diameter | Price | One Pipe | Parallel Pipes | $\mu_B$ |
| mm | €/m | €/m | €/m | % |
| 300 | 57.5 | 11.6 | 46.6 | 179937% |
| 400 | 63.9 | 220.8 | 883.3 | 9487% |
| 500 | 87.1 | 998.0 | 3992.1 | 2099% |
| 600 | 115.6 | 3185.7 | 12,742.9 | 658% |
| 700 | 146.6 | 8117.1 | 32,468.6 | 258% |
| 800 | 178.1 | 25,318.6 | 101,274.6 | 83% |
| 900 | 222.3 | 50,705.5 | 202,822.1 | 41% |
| 1000 | 265.8 | 100,827.2 | 403,308.8 | 21% |
| 1100 | 311.6 | 193,612.7 | 774,450.8 | 11% |
| 1200 | 360.8 | 313,524.1 | 1,254,096.4 | 7% |
| 1300 | 407.7 | 517,682.7 | 2,070,730.7 | 4% |
| 1400 | 455.1 | |

These $GC_{\phi_i \rightarrow \phi_{i+1}}^\text{eq}$ need to be compared with the cost of pumping the full annual volume at 1 meter height. As it was previously explained, because this annual cost depends on the pump efficiency $\mu_B$, the analysis turns into seeing what $\mu_B$ needs to be for the pumping option to be cheaper than building a greater pipe. For this analysis KCE1 is calculated, following Equation (7), as it is shown in Table 4. In this case study, it has been assumed that the construction period would last for two years, the useful life of the installation will be 25 years, and that a flat rate is hired, being 0.12 €/kWh the accorded price for the energy. The engine efficiency is taken as 0.93 and, in order to update the cost along the useful life, 4% is assumed as the discount rate. All of these values are estimated as usual values in the field.

Table 4. Energy cost constant factors and calculation of the invariable part of C$_{EI}$.

| Energy Cost |
|-------------|
| Energy Price | Construction Years | Useful Life | Engine Efficiency | Discount Rate | Discount Factor | KCE1 Constant |
| $p_e$ | $n_c$ | $n_u$ | $\mu_M$ | i | fa | KCE1 |
| (€/kWh) | (years) | (years) | (%) | (%) | (€/m) |
| 0.12 | 2 | 25 | 0.93 | 0.04 | 14.44 | 83,796 |

Once all $GC_{\phi_i \rightarrow \phi_{i+1}}^\text{eq}$ and KCE1 have been calculated, the following step is to obtain the efficiency required for the pumps so that the energy cost is smaller than the construction cost. As it can be observed in Table 3 and in Figure 6, a diameter of 800 mm requires a pump efficiency of 83% which is a value that can easily be found in the market. Per contrary, a diameter of 700 mm is too small because it would require a pump efficiency of 258%, which is completely irrational. On the other hand, a pipe of 900 mm only requires 41% efficiency, so it is already too wide and it
is more convenient to pump the head losses produced by an 800 mm pipe, than to invest in a wider pipe (as a 900 mm, per instance). Therefore, the optimum pump diameter of the water drive would be 800 mm wide.

![Pump efficiency for each diameter](image)

**Figure 6.** Pump efficiency required for each diameter. For the construction cost to be competitive with the energy cost, each pipeline designed with diameter Ø requires a pump with a minimum efficiency of $\mu_B$. The smaller the pipe, the better (since the construction costs will be smaller), but the required $\mu_B$ should be available in the market. Therefore, diameter Ø 800 mm is selected since it is the smallest of the commercial catalog meeting realistic performance conditions.

As it can be seen in Figure 7, the Change Gradient shows that, starting the design from small pipes, it is very cheap and convenient to move to bigger diameters, but the construction cost will rise up as the pipe gets wider. At some point the cost of increasing the pipe diameter to reduce the head loss will be too high, and it will be better to pump the water, as long as the pump efficiency has a reasonable value.

![Change Gradient and pipe size](image)

**Figure 7.** Change Gradient and pipe diameter in the application case.

However, it is important to remind that the simplicity of the Granados’ System requires a constant demand distribution, and it would not be possible to apply for a variable flow rate as the one of this study case, were it not for the Equivalent Flow Rate concept, which is a novelty of this research study. Combined with the Granados’ System, the application of the Equivalent Flow
Rate (or the Equivalent Flow Rate) give practice engineers a simple design alternative to obtain the main initial figures to work with.

Figure 8 (a) and (b) show the variable real demand distribution compared with the constant equivalent one, and in it can be observed that both distributions mean the same annual cost for the energy required in the water drive. As Figure 9 shows, the minimum total cost of the installation is achieved with diameter 800 mm, and for that reason it is the one to be selected for the design of the water drive. The graph agrees with the result previously given by the method. In Figure 8 (c) and (d), the separated construction and energy costs are shown.

![Flow Rate Distribution](image1)

**Flow Rate Distribution**

(a)

![Annual energy cost of the head losses](image2)

**Annual energy cost of the head losses**

(b)

![Construction cost](image3)

(c)

![Energy cost throughout the full life of the water drive](image4)

(d)

**Figure 8.** (a) Flow rate distribution in the application case of two parallel tubes and (b) annual accumulated cost of the energy required for the head losses. (c) Construction cost of the installation depending on the pipe diameter. (d) Accumulated energy cost throughout the entire lifespan of the installation depending on the pipe diameter.
Figure 9. Total accumulated cost of the water drive throughout the entire lifespan of the installation depending on the pipe diameter. Dimeter Ø800 provides the smallest total cost for the construction and operation altogether, and therefore it is the one selected, as the method previously anticipated.

4. Conclusions

The study presents a practical approach that allows to calculate variable flow rates in a practical way since: (a) actual computational resources allow to work with static variables although the mathematical approach of the Granados’ System is dynamic; and (b) it is efficient since it only needs a few operations up to the optimum, making the proposed procedure extremely computationally straightforward. The approach avoids the major computational inconvenience of dynamic programming that may limit the use in practical designs.

Our novel concepts applied in the proposed approach of the Equivalent Flow Rate and the Equivalent Volume offer engineers a solution to the main constrain of the cost gradient techniques, which rely on constant design flow rates. The uncertainties regarding the future water demand and the evolution of the energy price are a major concern in nowadays water supply designs and there is still effort to make to accurately determine what the demand curve will be in the forthcoming years; however, these two concepts allow the designer to include various different flow rates along the time, contributing to the mitigation of this issue regarding the design. Future research should focus on probabilistic methods to determine the demand pattern for the forthcoming years.

The present procedure is similar in concept to the ratio used by [19] in their proposal for optimization of the design of water supply systems. However, their system also depends on a constant flow rate, and the novel concepts of this paper of the Equivalent Flow Rate and Equivalent Volume is also applicable for their algorithm.

The applications of the present methodology are adequate for branched networks, including those with a twin pipe configuration, penstock and hydroelectric power plants, and the approach is also extensible to channel design. Regardless of the fact this report does not present a solution for all networks, it covers a wide range of configurations. As Walski [37] states “chances that a single optimization approach will work for all types of problem is unlikely, and it is somewhat understandable that practicing engineers look skeptically at models that claim to optimize pipe selection”.

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