Enhancement of dark matter capture by neutron stars in binary systems

Lionel Brayeur† and Peter Tinyakov‡

Service de Physique Théorique, Université Libre de Bruxelles, 1050 Brussels, Belgium

We study the capture of dark matter particles by neutron stars in close binary systems. By performing a direct numerical simulation, we find that there is a sizeable amplification of the rate of dark matter capture by each of the companions. In case of the binary pulsar PSR J1906+0746 with the orbital period of 4 hours the amplification factor is $\sim 3.5$. This amplification can be attributed to the energy loss by dark matter particles resulting from their gravitational scattering off moving companions.

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I. INTRODUCTION

If dark matter (DM) particles interact with nucleons they must accumulate in stars [1,2]. Their subsequent annihilation or collapse into a black hole inside the star may produce observable effects whose non-detection may be used to impose constraints on dark matter properties.

In ordinary stars these effects are difficult to detect. Given the direct constraints on the DM-to-nucleon cross section [3,4], the DM capture rate in ordinary stars is too low to eventually form a black hole. Likewise, the heat produced in annihilations of DM particles is by many orders of magnitude smaller than that resulting from the nuclear reactions. Only if the annihilation of DM particles goes into neutrinos, the ones produced in the Sun can be in principle observed by the neutrino detectors [5–7].

The accumulation of DM is more efficient in compact objects like neutron stars and white dwarfs. Unlike ordinary stars, these have no internal heat sources and the heat produced by DM annihilations can, in principle, be detected [8–14]. Another possible observable consequence of the dark matter accumulation in compact stars is its eventual collapse into a black hole [8,14–17]. Both the DM annihilation and collapse depend crucially on the amount/rate of the DM captured by the star, so an increase in the capture rate is a potentially important factor which may result in stronger constraints on the DM properties.

In order to be captured, a DM particle has to lose a part of its energy and become gravitationally bound to the star. This energy loss may result from the DM scattering off the star nucleons. For this mechanism to work, a DM particle has to cross the star surface. Another mechanism is operative in binary systems where the gravitational field is time-dependent. In this case DM particles may lose their energy by means of the gravitational interaction without crossing the star surface. In the context of the Solar system this phenomenon, known as the gravitational slingshot, is routinely used to accelerate/decelerate spacecraft of interplanetary missions. This mechanism affects larger number of particles than the scattering off the nucleons. Deceleration of DM particles by moving companions of binary systems may, therefore, increase the number of gravitationally bound DM particles and amplify their capture rate.

In the Solar system the effect of the planets (notably, Jupiter) on the dark matter capture by the Sun has been quantitatively studied in Refs. [15,20] and was shown to be small. The smallness is due to the combination of two factors: (i) the inverse process (acceleration of DM particles by the gravitational slingshot) prevents a substantial accumulation of DM and (ii) the Jupiter velocity is much smaller than that of DM particles, so only a tiny part of the DM phase space is affected.

Both arguments are not directly applicable to close binary systems involving neutron stars. While the re-acceleration of DM particles still prevents an infinite DM accumulation, this effect may be less important in view of a higher capture rate by neutron stars as compared to the Sun. On the other hand, the velocities of companions are comparable to those of DM particles, so that the whole phase space of DM is affected. Thus, an unsuppressed effect of the companion motion on the DM capture rate in such systems may be expected.

In this paper we calculated numerically the amplification of DM capture rate in close binary systems involving neutron stars. As a prototype we considered the binary pulsar PSR J1906+0746 composed of two neutron stars of approximately $1.3 M_\odot$ with the period of orbital motion of 4 hours. In this system the velocity of the companions is 270 km/s, which is comparable to a typical velocity of DM particles in the Galaxy.

We have found that in this binary system each of the companions captures DM at a rate $\sim 3.5$ times higher than if it were an isolated star. The enhancement can be attributed to the combination of two factors: the velocity of the star relative to the DM distribution, and the scattering off the moving companions, i.e., the gravitational slingshot. One can see that the first factor, calculable analytically, is smaller than one and for the above binary system equals 0.57. Thus, the amplification due to the gravitational slingshot alone is about $\sim 6$. We also found
that the amplification factor decreases when the orbital velocity of the companions becomes much smaller than the typical velocity of the DM particles, in agreement with earlier studies.

The paper is organized as follows. In Sect. II we describe the numerical procedure used to calculate the dark matter capture rate in a binary system. In Sect. III we present our results and conclusions.

II. NUMERICAL PROCEDURE

Since the velocities of DM particles and neutron stars are comparable, for most of the particle trajectories the interaction with both companions has to be taken into account. No analytic approximation seems possible in this case, and particle trajectories has to be calculated numerically.

In principle, the numerical calculation of the capture rate is straightforward: one injects particles according to the Maxwellian distribution, traces them until they either cross the surface of the neutron star and get captured or leave to infinity, and determines the capture rate from the fraction of captured particles. In practice, a substantial gain in calculational time is achieved by treating some parts of this process analytically.

Consider first the motion of the binary system itself. For simplicity, we assume that it is composed of two neutron stars of equal mass $M$ moving on a circular orbit. From the Newton equations one finds

$$\frac{G_N M}{4 R^2 \omega^2} = 1,$$

where $R$ is the radius of the orbit and $\omega$ is the frequency of the orbital motion of the neutron stars. The motion of a DM particle in the vicinity of this binary system is most naturally described in units where lengths are measured in units of $R$ and times are measured in unit of $1/\omega$. In these units the velocity of the neutron stars is $v = \omega R = 1$. Assuming the parameters of the binary pulsar PSR J1906+0746, in the above units the velocity of light is $c = 1127 \omega R$, while a typical velocity of DM particles in the Galactic halo is $v_{\text{DM}} \sim 270 \text{ km/s} \approx \omega R$. Note that the radius of the neutron stars is much smaller than $R$,

$$R_{\text{NS}} \simeq 12 \text{ km} \simeq 2 \times 10^{-5} R. \quad (2)$$

Let us now turn to particle trajectories. Defining two distance scales $R_{\text{max}} \gg R$ and $R_0 \ll R$, we separate each trajectory in three different regions: (i) the exterior of the sphere $r = R_{\text{max}}$ which we will refer to as the injection sphere (ii) the small spherical regions of the radius $R_0$ around the neutron stars, and (iii) the interior of the injection sphere excluding the vicinities of the neutron stars.

In the region (i) the binary system may be approximated by a single central mass $2M$, so the particle equations of motion can be solved analytically. Given the asymptotic velocity $v_{\infty}$ and the impact parameter $\rho$, the parameters of the particle trajectory at $r = R_{\text{max}}$ are

$$v = \sqrt{v_{\infty}^2 + \frac{4G N M}{R_{\text{max}}}}. \quad (3)$$

$$\sin \psi = \frac{\rho v_{\infty}}{R_{\text{max}} v}, \quad (4)$$

where $v$ is the particle velocity and $\psi$ is the angle between the velocity and the internal normal to the injection sphere. We generate the asymptotic parameters $v_{\infty}$ and $\rho$ according to the distribution

$$dF \propto \rho v_{\infty}^3 \exp \left( \frac{-3v_{\infty}^2}{2v_{\text{DM}}^2} \right) d\rho dv_{\infty}. \quad (5)$$

The extra power of $v_{\infty}$ appears here because we are interested in flux rather than the density of particles.

For each generated $v_{\infty}$ and $\rho$ we then calculate the parameters of the trajectory $v$ and $\psi$ at the boundary of the injection sphere $r = R_{\text{max}}$. These parameters should be supplemented with the position of the entry point (two parameters) and the orientation of the projection of the velocity in the plane tangent to the sphere. We choose these additional parameters randomly. The resulting set of parameters fixes the initial conditions for propagation of the particle trajectory inside region (iii).

Inside the injection sphere the spherical symmetry is no longer a good approximation. In this region we propagate the particle trajectories numerically until they either escape outside (beyond $200R$) with positive energy or get within the distance $R_0$ from one of the neutron stars.

In the region (ii), i.e., close to one of the neutron stars, the gravitational force of the remote companion can be neglected, and the problem once again becomes tractable analytically. Taking the initial conditions from numerical simulations, the trajectory can be continued inside $r = R_0$ and the point of the closest approach to the neutron star can be determined. It is expressed in terms of the particle energy per mass $E$ and the angular momentum per mass $J$ in the reference frame where the star is at rest,

$$R_{\text{min}} = \frac{J^2}{4} \left\{ 1 + \sqrt{1 + 8J^2/E^2} \right\}^{-1},$$

where the units $R = \omega = 1$ were used. In this way the fraction of particles that get closer than a certain distance to one of the neutron stars can be computed.

Since the probability for a particle to actually cross the neutron star surface in numerical simulations is very small in view of the small value of $R_{\text{NS}}$, eq. (2), it is not realistic to measure the fraction of such particles directly with a reasonable accuracy. The problem can be avoided by noting that in the gravitational potential of a single star the cross section for crossing a sphere of radius $R_s$ is equal to

$$\sigma(R_s) = \pi R_s^2 \left( 1 + \frac{2G N M}{R_s v_{\text{NS}}^2} \right). \quad (6)$$
where $v_{\text{as}}$ is the asymptotic velocity of a particle. For the actual parameters of the neutron star system the second term dominates when $R_s \ll R$. One thus expects that for a sufficiently small $R_s$, the cross section of crossing the sphere $r = R_s$ scales linearly with $R_s$. Since the dependence on the velocity factorizes in this regime, the linear behavior with $R_s$ is maintained after averaging over particle velocities. Measuring the fraction of particles crossing $r = R_s$ at different values of $R_s$ small enough for the linear scaling, one may then extrapolate to the actual neutron star radius.

We note in passing that eq. (6) also explains why the capture rate by a single neutron star decreases with the increasing relative velocity of the star and dark matter: with the first term inside the brackets on the r.h.s. neglected, the cross section is proportional to $1/v_{\text{as}}^2$. Depending on the direction of the particle velocity the effect of the neutron star motion can have both signs. Upon averaging over the Maxwellian distribution the net result is the decrease of the capture rate with the increasing velocity of the neutron star.

### III. RESULTS AND CONCLUSIONS

In our simulations we did not fix the absolute normalization of the distribution of injected particles. Instead, for each value of the parameters we performed two identical simulations: one with moving neutron stars and one with the neutron stars fixed to their positions at a given moment of time. In the second case all the effects due to the star motion disappear, and the capture rate equals, to a good approximation, twice the capture rate of a single isolated neutron star. The amplification factor thus equals the ratio of the number of particles captured in these two simulations. Clearly, this ratio does not depend on the absolute normalization of particle distributions.

In each of the two simulations we measured the number of particles that got closer than $R_s$ to one of the neutron stars, as a function of $R_s$. The results are presented in Fig. 1. The parameters of the simulation were as follows: $R_{\text{max}} = 10 R_s$, $R_0 = 10^{-3} R_s$, and the number of thrown particles was $10^8$. One can see that the expected linear dependence with $R_s$ is well reproduced. Fitting both dependencies with a linear function and taking the ratio of the two coefficients we find that the amplification factor equals $3.5 \pm 0.1$.

The amplification factor depends on the parameters of the binary system, as it certainly has to go to one for slowly moving stars. The neutron star motion (assuming stars of equal mass) is determined by their mass and rotation period. When the trajectories of DM particles are expressed in the units of $R$ and $1/\omega$, the change of the binary system parameters translates into the change of asymptotic velocity of the DM particles expressed in terms of $R\omega$, and the value of the capture radius that corresponds to the actual radius of the neutron star. As we have seen, the latter parameter is irrelevant. Thus, the only important parameter is the ratio of the asymptotic velocity of DM to that of the neutron stars. Equal values of this parameter imply equal amplification factors.

![Graph showing the dependence of the number of "captured" particles on the ratio of the neutron star radius to the capture radius](image)

**FIG. 1:** The dependence of the number of “captured” particles (i.e., those crossing the sphere $r = R_s$) on the capture radius $R_s$ for moving (upper line) and static (lower line) neutron stars. The points represent numerical simulation with 1-$\sigma$ statistical errors. The lines show fits with the linear dependence. The actual radius of the neutron star is shown by the vertical dashed line. The number of injected particles was $10^8$.

We have calculated the amplification factors for several values of the binary system periods, assuming the masses of the companions equal $M = 1.3 M_\odot$. The results are presented in Table. I. The maximum amplification corresponds to the case when the velocities of the companions and the asymptotic velocity of dark matter particles are comparable, while when the neutron stars move much slower than the DM particles, the amplification factor approaches 1. The amplification factor also decreases for short periods when the neutron star velocity becomes larger than that of DM particles, because the suppression due to the motion of the neutron stars relative to dark matter (cf. eq. (6)) is no more compensated by the gain in the phase space of DM affected by the gravitational slingshot effect.

| period | amplification |
|--------|-------------|
| 4h     | 3.5         |
| 8h     | 4.3         |
| 16h    | 2.8         |
| 32h    | 1.5         |

**TABLE I:** The dependence of the amplification factor on the period of the binary system.

To summarize, we have shown that the motion of the neutron stars in close binaries leads to the amplification of the DM capture rate by a factor of up to 3-4 depending on the period of the binary system. Thus, binary systems are favorable place to put constraints on dark matter properties that follow from its capture in compact stars.
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[1] W. H. Press and D. N. Spergel, Astrophys. J. 296, 679 (1985).
[2] A. Gould, Astrophys. J. 321, 560 (1987).
[3] Z. Ahmed et al. [CDMS-II Collaboration], Phys. Rev. Lett. 106, 131302 (2011).
[4] E. Aprile et al. [XENON100 Collaboration], arXiv:1104.2549
[5] G. Jungman and M. Kamionkowski, Phys. Rev. D 51, 328 (1995).
[6] S. Nussinov, L. T. Wang and I. Yavin, JCAP 0908, 037 (2009).
[7] A. Menon, R. Morris, A. Pierce, N. Weiner, Phys. Rev. D82, 015011 (2010).
[8] I. Goldman and S. Nussinov, Phys. Rev. D 40, 3221 (1989).
[9] C. Kouvaris, Phys. Rev. D 77, 023006 (2008).
[10] F. Sandin and P. Ciarcelluti, arXiv:0809.2942
[11] G. Bertone and M. Fairbairn, Phys. Rev. D 77, 043515 (2008).
[12] M. McCullough and M. Fairbairn, arXiv:1001.2737
[13] C. Kouvaris, P. Tinyakov, Phys. Rev. D82, 063531 (2010).
[14] C. Kouvaris, P. Tinyakov, Phys. Rev. D83, 083512 (2011).
[15] A. de Lavallaz and M. Fairbairn, Phys. Rev. D 81, 123521 (2010).
[16] C. Kouvaris, P. Tinyakov, Phys. Rev. Lett. 107, 091301 (2011).
[17] S. D. McDermott, H. -B. Yu, K. M. Zurek, arXiv:1103.5472
[18] J. Lundberg, J. Edsjo, Phys. Rev. D69, 123505 (2004).
[19] A. H. G. Peter, Phys. Rev. D79, 103531 (2009).
[20] A. H. G. Peter, Phys. Rev. D 79, 103533 (2009).