Larkin-Ovchinnikov state in resonant Fermi gas

Nobukatsu Yoshida and S.-K. Yip

Institute of Physics, Academia Sinica, Nankang, Taipei 115, Taiwan

(Dated: October 31, 2018)

Abstract

We construct the phase diagram of a homogeneous two component Fermi gas with population imbalance under a Feshbach resonance. In particular, we study the physics and stability of the Larkin-Ovchinnikov phase. We show that this phase is stable over a much larger parameter range than what has been previously reported by other authors.

PACS numbers: 03.75.Ss, 05.30.Fk, 34.90.+q
Trapped fermi gases offer us a wonderful opportunity to study strongly interacting fermion systems [1]. With Feshbach resonance, one is able to tune the effective interaction from weakly attractive for magnetic field above the resonance to strongly attractive below. This problem is also closely related to studies of quark and nuclear matter [2].

Recent attention has shifted to systems with unequal populations [3, 4, 5, 6]. The latter problem is analogous to the physics of a superconductor under the influence of an external Zeeman field, which provides a chemical potential difference $\mu_\uparrow - \mu_\downarrow \equiv 2h$ between the two species $\uparrow$ and $\downarrow$. In the weak-coupling limit, it was shown by Sarma [7] that the uniform state with population imbalance is unstable at zero temperature. By comparing the free energy of the normal state and the completely paired superconducting state, he concluded that, as the magnetic field is increased, there is a first order phase transition from the equal-population superconducting state (with gap $\Delta_0$) to the normal state at $h = \Delta_0/\sqrt{2}$. This implies that a system with given unequal numbers of the two spin species will either phase separate (with $h = \Delta_0/\sqrt{2}$), or be in the normal state (with $h > \Delta_0/\sqrt{2}$), depending on the imbalance. However, later, Fulde-Ferrell [8] and Larkin-Ovchinnikov [9] showed that there are better alternatives. Fulde and Ferrell (FF) considered a state with order parameter $\Delta(\vec{r}) = \Delta_0 e^{i\vec{q} \cdot \vec{r}}$, that is, pairs with finite momentum. The order parameter has a spatially varying phase with however the magnitude still constant in space. They showed that in certain parameter space this state has lower free energy than the states considered by Sarma. Larkin and Ovchinnikov (LO), however demonstrated that, at least in the small order parameter limit, states with certain choices of sinusoidal variations of order parameter (such as $\Delta(\vec{r}) = \Delta_0 \cos(qx)$ etc) are more energetically favorable than the FF state. Notice that in the LO state, the magnitude of the order parameter is no longer a constant in space. With decreasing magnetic field and hence decreasing population difference $n_d$, many authors showed that the LO state evolves into a state with a set of domain walls. This has been demonstrated for one [10], two [11], as well as in three [12] dimensions and also for a d-wave superconductor [13]. In the small population imbalance limit, the order parameter has a constant magnitude almost everywhere in space (with value identical to the state with no population difference), except near the domain walls. The phase of the order parameter changes by $\pi$ when these walls are crossed, and these are also the locations where the magnetization concentrates. This local magnetization arises from the occupation of bound states, available due to the presence of the domain walls. The physics of these domain walls is closely related to the $\pi$-junctions in
SFS (S=superconductor, F=ferromagnet) junctions [14], where for suitable parameters it is energetically favorable for the two superconductors to acquire a phase difference $\pi$.

Indeed, in three-dimensions, in the zero temperature and weak-coupling limit, Matsuo et al [12] demonstrated that the energy of the domain walls becomes negative for $h$ above $h_{dw} \equiv \frac{1}{2} \times 0.745\pi T_c \approx 0.665\Delta_0$ where $\Delta_0$ is the zero temperature gap for the completely-paired superfluid of equal populations. This $h_{dw}$ is less than the phase separation field $h_{ps} = \Delta_0/\sqrt{2} \approx 0.707\Delta_0$ (where the free energy of the normal and the completely-paired superfluid states are equal). Hence the LO state must be more stable than both the uniform superfluid and the normal state (at least) for $h_{dw} < h < h_{ps}$. For $h$ slightly above $h_{dw}$, the domain walls are far apart. The system resembles the uniform paired state except for the occasional domain walls. This state thus allows for the possibility of arbitrarily small population difference between the two species. This problem is quite analogous to the lower critical field $h_{c1}$ in type II superconductors, where the vortex energy becomes negative and vortices begin to penetrate the superconductor for $h$ slightly above $h_{c1}$. For increasing $h$, more and more domain walls are formed, the order parameter becomes more sinusoidal like and the state evolves smoothly to the picture given by LO [9].

For the resonant Fermi gas, it has been recognized that at intermediate coupling strengths, the uniform state with population imbalance must be unstable [3, 4]. Indeed this has also been demonstrated also by experiments [6]. However, the investigation into the actual phase diagram, that is, the question as to which phase appears where instead of the unstable uniform state, even for the case without a trap, cannot be regarded as complete, especially regarding the stability of the FFLO states. Many papers [15, 16] examined only phase separation, whereas some [4, 5, 17, 18, 19] considered in addition only the FF state with $\Delta \propto e^{i\vec{q}\cdot \vec{r}}$. However, none of these works actually investigated the LO states. They conclude that the FF state exists only in a very narrow region next to the normal state in the weak-coupling regime, and deduce then that phase-separation occurs throughout the rest of the entire region where the uniform phase is unstable. In particular, they conclude that, for small population differences, phase separation occurs unless the dimensionless coupling constant $1/k_f a \to -\infty$. However, as seen already in the above paragraph, this is simply an artifact of the FF state, which cannot smoothly go into the completely paired equal population state [20]. We expect that at least, in the weak-coupling limit, the state should be LO for any population imbalance below that of the normal state.
In this work, we shall investigate the stability of the LO state for arbitrary strength of the attractive interaction between the two species, thus generalizing previous works such as [11, 12] beyond the BCS limit. We shall concentrate in particular on the small $n_d$ limit. More specifically, we compare the critical chemical potential difference $h_{dw}$ at which the domain wall energy becomes negative, to the critical field $h_{ps}$ for phase separation where the free energy of the normal phase becomes equal to that of the completely paired equal population superfluid phase. We find that for $1/k_f a \lesssim (>) 0.845$, $h_{dw} < (>) h_{ps}$. Therefore we conclude that, for small $n_d$, the LO state is more stable than the phase separated state for $1/k_f a \lesssim -0.845$. By combining with previous results [3, 5, 23] (and with some reasonable extrapolations), we sketch the appropriate phase diagram for our system.

The mean-field Hamiltonian of our system can be written as

$$H = \int d^3\vec{r} \left\{ \sum_\sigma \left[ \frac{\hbar^2 \nabla^2 \psi_\sigma \psi_\sigma}{2m} - \mu_\sigma \psi_\sigma^\dagger \psi_\sigma \right] - \left[ \Delta^* (\vec{r}) \psi_\uparrow \psi_\uparrow + c.c. \right] - \frac{|\Delta (\vec{r})|^2}{g} \right\}$$

where $\sigma = \uparrow, \downarrow$ for the two species, $\psi_\sigma (\vec{r})$ their corresponding field operators, $\Delta (\vec{r})$ a position dependent order parameter and $g$ the coupling constant. For convenience below we shall also write $\mu_\uparrow = \mu + h$ and $\mu_\downarrow = \mu - h$.

This Hamiltonian can be diagonalized by the Bogoliubov transformation for a general inhomogeneous system [21]:

$$\begin{pmatrix} \psi_\uparrow (\vec{r}) \\ \psi_\downarrow (\vec{r}) \end{pmatrix} = \sum_J \begin{pmatrix} u_J (\vec{r}) \\ v_J (\vec{r}) \end{pmatrix} \begin{pmatrix} \alpha_J \\ \beta_J \end{pmatrix}$$

where $\alpha_J, \beta_J$ are annihilation operators for quasiparticles with spin $\uparrow$ and $\downarrow$ of the state labelled by a set of quantum numbers $J$. $u_J(\vec{r}), v_J(\vec{r})$ obeys $\int d^3\vec{r} \left[ |u_J(\vec{r})|^2 + |v_J(\vec{r})|^2 \right] = 1$ and the Bogoliubov-deGennes (B-dG) equation

$$\begin{pmatrix} -\frac{\hbar^2 \nabla^2}{2m} - \mu & \Delta (\vec{r}) \\ \Delta^* (\vec{r}) & \frac{\hbar^2 \nabla^2}{2m} + \mu \end{pmatrix} \begin{pmatrix} u_J (\vec{r}) \\ v_J (\vec{r}) \end{pmatrix} = E_J \begin{pmatrix} u_J (\vec{r}) \\ v_J (\vec{r}) \end{pmatrix}$$

Substituting eq (2) into eq (1) and using eq (3) to eliminate the derivatives $\nabla u_J$ and $\nabla v_J$’s, we obtain the ground state free energy

$$F = \int d^3\vec{r} \left\{ \sum_J \left[ -2E_J |v_J(\vec{r})|^2 + \frac{1}{V} \frac{|\Delta (\vec{r})|^2}{2\epsilon_J} \right] - \frac{m}{4\pi \hbar^2 a} |\Delta (\vec{r})|^2 \right\} + \sum_J (E_J - h) f (E_J - h)$$

where $\epsilon_J = \frac{|\Delta (\vec{r})|^2}{2\epsilon_J}$ and $f (E_J - h)$ is the Fermi-Dirac function.
The last term arises from the occupation of states $\alpha_j$ with $E_j < h$. Here $f$ is the Fermi function, $V$ the volume, and we have eliminated the interaction constant $g$ in favor of the scattering length $a$ via the relation $g = \frac{m}{(4\pi \hbar^2 a)} - \frac{1}{V} \sum_j \frac{1}{2a_j}$ where $\epsilon_j$ is the energy of the state $J$ in the normal phase. (Strictly speaking we should label the quantum states in the normal phase by another set of quantum numbers $J'$, but we shall not make such a distinction for simplicity in notations. See further below.) In eq (4), $\Delta(\vec{r})$ should be viewed as a variational parameter with respect to which $F$ has to be minimized.

We are interested in the domain wall energy for given $\mu$, $h$ and $a$. We evaluate this by calculating the energy difference between a state with a single planar domain wall and the uniform completely paired superfluid state. The latter is easy, since it is independent of $h$ and the B-dG equation (3) can be solved by Fourier transform. We obtained, for wavevector $\vec{k}$, the familiar quasiparticle energies $E_k = [(\epsilon_k - \mu)^2 + |\Delta|^2]^{1/2}$ (where $\epsilon_k \equiv \frac{\hbar^2 k^2}{2m}$) and hence the bulk free energy $F_S(\mu) = \sum_k [(\epsilon_k - \mu - E_k + \frac{m}{2\hbar^2 k^2} |\Delta|^2) - \frac{m}{2\hbar^2 k^2} |\Delta|^2]$. Minimizing with respect to $\Delta$ gives the usual BCS gap equation $-\frac{m}{2\hbar^2 k^2} \Delta = \Delta(1 - \sum_k \frac{1}{2E_k} - \frac{m}{2\hbar^2 k^2})$. We can also compute the corresponding density via $n = \int \frac{d^d k}{(2\pi)^d} [1 - \frac{\epsilon_k - \mu}{E_k}]$ and the corresponding Fermi wavevector $k_f \equiv (3\pi^2 n)^{1/3}$ and express $a$ in the dimensionless combination $k_f a$. These results are identical with those in [22]. We had also used $\Delta(z) = \text{const}$ instead of a domain wall in the procedure described below and verified numerically that we indeed obtained the same free energy density for the uniform state. [Equating this free energy to that of the normal state at the same $\mu$ and $h$, we obtained the phase separation field $h_{ps}$ discussed.]

Next we evaluate the planar domain wall energy. For this, we put our system in a box of dimensions $L_x, L_y = L_x, L_z \equiv L$. We assume that the order parameter varies only along the $z$ direction and thus Fourier transform the $x$ and $y$ coordinates. Calling the resulting wavevector $\vec{p}$, we rewrite $(u_j, v_j)$ as

\[
\begin{pmatrix}
u J(\vec{r}) \\
v J(\vec{r})
\end{pmatrix} = \frac{1}{L_x L^{1/2}} \begin{pmatrix}
u p_j(\vec{z}) \\
v p_j(\vec{z})
\end{pmatrix} e^{i\vec{p} \cdot \vec{r}_p}
\tag{5}
\]

where $\vec{r}_p$ is the component of $\vec{r}$ in the x-y plane, and $j$ is now a quantum number for the $z$ dependences. $u_{p,j}(\vec{z}), v_{p,j}(\vec{z})$ are dimensionless quantities obeying $\int dz (|u_{p,j}(\vec{z})|^2 + |v_{p,j}(\vec{z})|^2) / L = 1$. Eq (3) becomes

\[
\begin{pmatrix}
-\frac{\hbar^2 \partial^2}{2m} - \tilde{\mu} + \Delta(z) \\
\Delta^*(z) - \frac{\hbar^2 \partial^2}{2m} + \tilde{\mu}
\end{pmatrix}
\begin{pmatrix}
u p_j(z) \\
v p_j(z)
\end{pmatrix} = E_{p,j}
\begin{pmatrix}
u p_j(z) \\
v p_j(z)
\end{pmatrix}
\tag{6}
\]
where $\tilde{\mu} \equiv \mu - \frac{p^2}{2m}$. The energies $E_{p,j}$ and the wavefunctions $u_{p,j}(z), v_{p,j}(z)$ depend on $\vec{p}$ only through its magnitude $p$. The free energy can then be written as

$$F = \int dz \left \{ \frac{1}{L} \sum_{\vec{p}} \sum_{j}^{j_{max}} \left [ -2E_{p,j}|v_{p,j}(z)|^2 + \frac{|\Delta(z)|^2}{2\epsilon_{p,j}} \right ] - L^2 \frac{m}{4\pi \hbar^2 a} |\Delta(z)|^2 \right \} + \sum_{\vec{p},j} (E_{p,j}-h)f(E_{p,j}-h)$$

(7)

In this equation, we arrange the quasiparticle states for both the superfluid and the normal states as increasing function of the counting number $j$. We solve eq (6) by discretizing the $z$ coordinate. After the energies and the function $v_{p,j}(z)$ are calculated, we put it in eq (7) and calculate the free energy. For $\Delta(z)$, we limit ourselves to the one-parameter ansatz

$$\Delta(z) = \Delta_0 \tanh \left ( \frac{k_{\mu} z}{\beta} \right )$$

(8)

with $\beta$ as the variational parameter and $k_{\mu} \equiv (2m\mu)^{1/2}$. (Thus the width of the domain wall is given by $\beta k_{\mu}^{-1}$.) Here $\Delta_0$ is the bulk order parameter for the given $\mu$ and $a$ at $h = 0$, as the order parameter should approach this value far away from the domain wall. Our eq (8) is motivated by earlier investigations [11, 12], where their numerical results can be well fitted by a function of the form (8). (Since we are employing periodic boundary conditions in $z$, this ansatz actually introduces a sharp ($\beta = 0$) domain wall also at $z = \pm L/2$. However, this can be taken care of easily by removing the contributions due to this extra domain wall.)

We shall then input the ansatz eq (8) into eq (6) to solve for $E_{p,j}$. Our analysis of the free energy eq (7) is simplified by the following observations. At $h = 0$, since all quasiparticle energies are positive, $f(E_{p,j}) = 0$ and the free energy is given simply by the integral in eq (7). The free energy at finite $h$ of a given $\beta$ is related to that of $h = 0$ at the same $\beta$ by simply adding the negative term $\sum_{\vec{p},j}(E_{p,j}-h)f(E_{p,j}-h)$ due to the occupation of the quasiparticle states.

An example for our results for $1/k_{\mu}a = -1.0866, 1/k_f a = -1.0676$ and $\Delta_0/\mu = 0.2$ are discussed below. The bound state energies $E_b \equiv E_{p,j}$ are illustrated in Fig 1. We see that in general we have states below the continuum ($E_{p,j} < |\Delta_0|$ for $\tilde{\mu} > 0$, $E_{p,j} < \sqrt{|\tilde{\mu}|^2 + |\Delta_0|^2}$ for $\tilde{\mu} < 0$. It is the bound states that are essential: as we shall see, the relevant values of $h$ are below the gap edges). The $\beta = 0$ results were checked against the analytical values of $h$ in the Appendix. The bound state energy for a given $p$ decreases with the width of the domain walls, as one expects. The significance of this would be discussed again below.
The integrand of the first term in eq (7) is plotted in Fig 2. (The double hump structure for small β (≠ 0) is due to the fact that |Δ(z)| → 0 as z → 0: see eq (7)). As expected, the integrand decreases to a constant corresponding to the free energy density in the bulk for distances sufficiently far away from the domain wall. We can then evaluate the domain wall energy $F_{dw}$ at $h = 0$ by simply integrating this excess contribution over $z$. At $h = 0$, $F_{dw}$ is minimum at $β ≈ 3$. $F_{dw}$ is positive for all $β$’s, as expected since the uniform state should have a lower energy than a domain wall. For finite $h$, the free energies are evaluated by adding the negative term from bound state occupation as discussed above. The results are depicted in Fig 3. The free energy decreases due to the occupation of the bound states. Since the bound state energies are smaller for larger $β$, the wall energy decreases faster for larger $β$, shifting the $β$ for minimum wall energy to larger values with increasing $h$ (see Fig 4). The domain wall energy becomes negative at sufficiently large $h$ for all $β$’s. The important question is whether it will become negative for some $β$ at a value of $h$ which is less than that for phase separation. For the parameters under discussion, the domain wall energy first becomes negative for $β ≈ 5$ at $h ≡ h_{dw} ≈ 0.1376μ$, which is less than the phase separation value $h_{ps} ≈ 0.1391μ$. We thus conclude that for $1/k_f a = −1.0676$, when $n_d$ is infinitesimally small, the system is in the LO state but not the phase separated state.

At this point, it is of interest to compare our results more quantitatively with those in [12], who has assumed the quasiclassical limit in their calculations at the outset. They obtained the critical value for the sign change of the domain wall energy at $h = h_{dw} = \frac{1}{2} \times 0.745πT_c ≈ 0.665Δ_0$ (using $Δ_0 = 1.76T_c$). Our $h_{dw}$ is close to their value if we simply rewrite their result as $h_{dw}/μ = 0.665Δ_0/μ$ and substitute $Δ_0/μ = 0.2$. Also, their Fig 1 indicates a domain wall width near the critical field to be roughly given by $2Δ_0v_f/(πT_c)^2$ where $v_f = k_f/m$ is the Fermi velocity. Rewriting this expression as $≈ 1.25 \left(\frac{k_f}{k_{μ}}\right) \left(\frac{μ}{Δ_0}\right) k_{μ}^{-1}$ and simply substituting the values of $Δ_0/μ$ etc we again find very good agreement. Thus, even though we did not assume the quasiclassical limit at the outset, for $1/k_f a = −1.067$ our results can be understood from the quasiclassical limit calculations of [12] with extrapolations.

For increasing $1/k_f a$, both $h_{dw}$ and $h_{ps}$ increase (and the $β$ for the optimal domain wall decreases). However, $h_{dw}$ increases faster than $h_{ps}$, and eventually $h_{dw} < h_{ps}$ no longer holds. The situation is shown in Fig 5. If $h_{dw} > h_{ps}$, then the formation of domain wall is no longer favorable since phase separation already occurs at $h$ slightly above $h_{ps}$. We conclude then, for $n_d → 0_+$, the transition from the LO state to the phase separation state occurs at
\(1/k_f a \approx -0.845\). This point is indicated by the point A in Fig 6.

After locating this transition point for \(n_d \to 0_+\), we now attempt to construct a phase diagram for general \(n_d\) by combining the present results with previous ones in the literature. For \(n_d/n \neq 0\) and sufficiently negative \(1/k_f a\), the system is in the normal state \([3, 5]\).

We consider in turn the transition lines between this normal state and the LO and phase separated states. Assuming that the transition to the LO state is second order \([9]\) (c.f. below), the transition line between the normal and LO state can be found by solving the Cooper problem at finite wavevector \(\vec{q}\):

\[
- \frac{m}{4\pi \hbar^2 a} = \frac{1}{V} \sum_k \left[ \frac{1 - f(\epsilon_{\vec{k}+\vec{q}} - (\mu + h)) - f(\epsilon_{\vec{k}} - (\mu - h))}{\epsilon_{\vec{k}+\vec{q}} + \epsilon_{\vec{k}} - 2\mu} - \frac{m}{\hbar^2 k^2} \right]
\]

(9)

The transition line is determined by finding the optimal \(q\) corresponding to the weakest attractive interaction, i.e., the most negative \(1/k_f a\). We note here that, since eq (9) is obtained by linearizing in the order parameter, the same equation determines the second order transition line into the FF state \([9]\). Our result can be turned into a line in our phase diagram (an equation of \(1/k_f a\) versus \(n_d/n\)) by using \(k_f = (3\pi^2 n)^{1/3}\), with

\[
n = \frac{(2m)^{3/2}}{6\pi^2} \left[ (\mu + h)^{3/2} + (\mu - h)^{3/2} \right] \quad \text{and} \quad a_n = \frac{(\mu + h)^{3/2} - (\mu - h)^{3/2}}{(\mu + h)^{3/2} + (\mu - h)^{3/2}}.
\]

This gives the long-dashed line in Fig 6. Our numerical results (see also \([23]\)) agree well with \([5]\). It should however be remarked that the transition from the normal state to the LO state has also claimed to be first order \([24]\) at very low temperatures in three-dimensions. However, the difference between the actual transition line and the second order line is very small \([24]\), and we ignore this difference here.

For the transition line between the normal state and phase separation, we equate the free energy of the completely paired superfluid state \(F_S(\mu)\) to that of the normal state:

\[
F_N(\mu, h) = -V \frac{(2m)^{3/2}}{15\pi^2} \left[ (\mu + h)^{5/2} + (\mu - h)^{5/2} \right].
\]

This again yields a line \(1/k_f a\) as a function of \(h/\mu\) and hence \(n_d/n\), shown as the solid line in Fig 6. Our numerical result (see also \([23]\)) is also in good agreement with \([3]\). The two transition lines intersect at the point labelled by \(X\) at \(1/k_f a \approx -0.50\). Therefore the transition is to the LO (phase separation) state if \(1/k_f a\) is less (larger) than this value. Interpolating between points A and X, we thus conclude that the system is in the LO state for the shaded region to the right of the line XA, whereas phase separation occurs for the shaded region to the left of this line. The transition lines between this phase-separated state and the homogeneous superfluid (Bose-Fermi mixture)
phase for positive $1/k_fa$ have already been discussed elsewhere in the literature (\cite{3,15,16}, see also other references in \cite{23}), and we would not repeat the details here.

In conclusion, we studied the stability of the LO state in a homogeneous system, in particular in the limit of small population imbalance, by calculating the domain wall energies. Our investigation here is analogous to the determination of the lower critical field of the vortex phase of a type II superconductor. The determination of the detailed structures of the LO state, such as the question of the lattice structure of the domain walls (c.f. e.g. \cite{24}, analogous to the vortex lattice structure in type II superconductors), as well as the phase diagram in the presence of a trap (studied already partially in \cite{25}), are left for the future.

We thank C.-H. Cheng, C.-H. Pao and S.-T. Wu for their help in obtaining the transition lines from the normal state. This research was supported by the National Science Council of Taiwan under grant number NSC95-2112-M-054-MY3.

**Appendix – Bound states**

Due to the important role played by the bound states in the LO state, we give the analytic results for a sharp domain wall where $\Delta(z) = \Delta_0 \text{sgn}(z)$ (corresponding to $\beta \to 0$). It should be noted that, even though $|\Delta(z)|$ is a constant throughout in space, bound states still exist due to the sudden change in phase factor of the order parameter from $-\pi$ to $0$ at $z = 0$.

As already seen from eq (6), the bound state energy is a function of $\mu$ and $p$ only through the combination $\tilde{\mu}$. The B-dG equation (6) can be solved easily in this case since $\Delta(z)$ is piecewise constant. We require that the functions $u_{p,j}(z)$, $v_{p,j}(z)$ as well as their derivatives be continuous at $z = 0$. The calculation is similar to solving the Schrödinger equation in piecewise constant potentials. It is straightforward and we simply state the results. It is convenient to divide them into two regions:

1. $\tilde{\mu} > 0$. In this case states are bound when $E_b \equiv E_{p,j} < |\Delta_0|$. We find that there is only one bound state (thus correspondingly one bound state for each $\vec{p}$), with energy given by

$$\frac{E_b}{|\Delta_0|} = \frac{\sqrt{1 + 2\delta^2} - 1}{2\delta}$$

where $\delta \equiv |\Delta_0|/\tilde{\mu}$. We note here that the quasiclassical limit corresponds to $|\Delta_0| \ll \tilde{\mu}$ hence $\delta \ll 1$. In this limit, one can approximate the B-dG equation by the Andreev equation \cite{26} and the bound state energy vanishes (irrespective of the detailed positional dependence of
\( \Delta(z) \) so long as there is a \( \pi \) phase change: see e.g. [27] and also [28]). For small \( |\Delta_0|/\tilde{\mu} \), we have \( E_b \approx |\Delta_0|^2/2\tilde{\mu} \). (c.f. the mini-gap for bound states near the vortex core [21]). With increasing \( |\Delta_0| \) or equivalently decreasing \( \tilde{\mu} \), \( E_b/|\Delta_0| \) increases. For \( \delta \to \infty \), \( E_b \to |\Delta_0|/\sqrt{2} \).

(ii) \( \tilde{\mu} < 0 \). In this case the continuum starts at \( \sqrt{\tilde{\mu}^2 + |\Delta_0|^2} \). Again we find only one bound state, with

\[
\frac{E_b}{|\Delta_0|} = \sqrt{1 + 2\delta^2 + 1} \frac{1}{2\delta}
\]

where \( \delta \equiv |\Delta_0|/|\tilde{\mu}| \). For \( \tilde{\mu} = 0_- (\delta \to \infty) \), \( E_b \to |\Delta_0|/\sqrt{2} \) (c.f. case (i) above). For decreasing \( \tilde{\mu} \) (decreasing \( \delta \)), \( E_b \) increases. \( E_b \) reaches \( |\Delta_0| \) at \( \delta = 2 \). For \( \tilde{\mu} \to -\infty \), \( E_b \) approaches the continuum from below. It is remarkable that bound state exists even for \( \tilde{\mu} < 0 \). We note that in this regime we have always \( E_b > |\tilde{\mu}| \). A similar situation occurs also for the bound states at a vortex [29] for general coupling strength \( 1/k_f a \).

[1] C. A. Regal, M. Greiner, and D.S. Jin, Phys. Rev. Lett. 92, 040403 (2004); T. Bourdel et al, *ibid*. 93, 050401 (2004); C. Chin et al., *Science* 305, 1128 (2004); J. Kinast et al., *ibid*. 307, 1296 (2005); M. W. Zwierlein et al, *Nature (London)* 435, 1047 (2005); and references therein.

[2] R. Casalbuoni and G. Nardulli, Rev. Mod. Phys. 76, 263 (2004)

[3] C.-H. Pao, S.-T. Wu and S.-K. Yip, Phys. Rev. B 73, 132506 (2006); *ibid*. 74, 189901(E) (2006).

[4] D. E. Sheehy and L. Radzihovsky, Phys. Rev. Lett. 96, 060401 (2006)

[5] D. E. Sheehy and L. Radzihovsky, cond-mat/0607803

[6] M. W. Zwierlein, Andre Schirotzek, Christian H. Schunck, and Wolfgang Ketterle, *Science* 311, 492 (2006); G. B. Partridge, Wenhui Li, Ramsey I. Kamar, Yean-an Liao, and Randall G. Hulet, *ibid*. 311, 503 (2006); M.W. Zwierlein, C.H. Schunck, A. Schirotzek, and W. Ketterle, Nature 442, 54 (2006); Y. Shin, M. W. Zwierlein, C. H. Schunck, A. Schirotzek, and W. Ketterle, Phys. Rev. Lett. 97, 030401 (2006); G. B. Partridge, Wenhui Li, Yean-an Liao, Randall G. Hulet, M. Haque and H. T. C. Stoof, *ibid*. 97, 190407 (2006).

[7] G. Sarma, J. Phys. Chem. Solids, 24, 1029 (1963)

[8] P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

[9] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Éksp. Teor. Fiz. 47, 1136 (1964) [Sov. Phys. JETP 20, 762 (1965)].
K. Machida and H. Nakanishi, Phys. Rev. B 30, 122 (1984); A. Buzdin and S. Polonski, Sov. Phys. JETP 66, 422 (1987). These two papers assumed mean-field theory applies. See further K. Yang, Phys. Rev. B 63, 140511(R) (2001).

H. Burkhardt and D. Rainer, Ann. Phys. (Leipzig) 3, 181 (1994)

S. Matsuo, S. Higashitani, Y. Nagato and K. Nagai, J. Phys. Soc. Jpn. 67, 280 (1998)

A. B. Vorontsov, J. A. Sauls and M. J. Graf, Phys. Rev. B 72, 184501 (2005)

A. I. Buzdin, Rev. Mod. Phys. 77, 935 (2005)

M. M. Parish, F. M. Marchetti, A. Lamacraft and B. D. Simons, Nature Physics, 3, issue 2, 124 (2007)

Z.-C. Gu, G. Warner and F. Zhou, cond-mat/0603190

H. Hu and X.-J. Liu, Phys. Rev. A 73, 051603(R) (2006)

Lianye He, M. Jin and P. Zhuang, Phys. Rev. B 73, 214527 (2006)

Yan He, C.-C. Chien, Q. Chen and K. Levin, Phys. Rev. A 75, 021602(R) (2007)

S. Takada and T. Izuyama, Prog. Theor. Phys. 41, 635 (1969)

P. G. deGennes, Superconductivity of Metals and Alloys, 1989, Addision-Wesley.

J. R. Engelbrecht, M. Randeria and C. A. R. Sa de Melo, Phys. Rev. B, 55, 15153 (1997)

C.-H. Pao, S.-T. Wu and S.-K. Yip, cond-mat/0608501

C. Mora and R. Combescot, Phys. Rev. B 71, 214504 (2005); see also H. Shimihara, J. Phys. Soc. Jpn, 67, 736 (1998)

J. Kinnunen, L. M. Jensen and P. Törma, Phys. Rev. Lett. 96, 110403 (2006). K. Machida, T. Mizushima and M. Ichioka, ibid, 97, 120407 (2006)

A. F. Andreev, Zh. Eksp. Teor. Fiz. 46, 1823 (1964) [Sov. Phys. JETP 19, 1228 (1964)]

T. Löffwander, V. S. Shumeiko and G. Wendin, Supercond. Sci. Technol. 14, R53 (2001)

M. Atiyah, V. Patodi and I. Singer, Cambridge Philos. Soc. 77, 43 (1975)

R. Sensarma, M. Randeria and T.-L. Ho, Phys. Rev. Lett. 96, 090403 (2006)
FIG. 1: (color online) Bound state energies for $1/k_f a = -1.0676$, corresponding to $1/k_{\mu} a = -1.0866$, $\Delta_0/\mu = 0.2$, for widths of the domain walls given by $\beta k^{-1}$. Energies are normalized to $\mu$ and the momentum $p$ in the x-y plane normalized to $k_{\mu}$. Inset: bound state energies in log plot.
FIG. 2: (color online) Integrand of eq (7), in units of \( k_\mu^3 \mu A \) where \( A \) is the area, as a function of position \( z \) (in units of \( k_\mu^{-1} \)). Parameters are the same as in Fig 1.
FIG. 3: (color online) Domain wall free energy $F_{dw}$ per unit area $A$ (in units of $k_\mu^2 \mu$) versus $h$ (in units of $\mu$). Parameters are the same as in Fig [1]. Inset magnifies the region near $F_{dw} = 0$. 
FIG. 4: (color online) Domain wall free energy $F_{dw}$ per unit area $A$ (in units of $k^2\mu$) versus $\beta$. Parameters are the same as in Fig 1. Inset shows the detailed behavior for $h/\mu = 0.138$. 
FIG. 5: (color online) Comparison between $h_{dw}$ (dotted line) and $h_{ps}$ (full line) as a function of $1/k_f a$. Inset: $h_{dw} - h_{ps}$. 
FIG. 6: (color online) Phase diagram. N: normal state, BF: homogeneous superfluid (Bose-Fermi mixture) phase. Shaded regions to the right of XA: Larkin-Ovchinnikov state, shaded regions to the left of XA: phase separation.