On vortices in the tube flow of ideal media

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Abstract. Numerical simulation is presented, which models the evolution of the initial perturbation in axisymmetric subsonic flow of an ideal gas. The undisturbed flow is an axial flow perturbed by an azimuthal velocity, or a twist of the flow around the axis of symmetry. The twist leads to the establishment in the flow of an annular or ball vortex with increasing or decreasing radial axial velocity of the undisturbed flow, respectively.

1. Introduction

Many important hydrodynamic processes are based on vortex flows of liquid and gas, which can be ordered and chaotic. The study of this class of fluid flows is one of the important tasks of fluid mechanics, having a great application in the study of dynamic meteorology, hydrodynamic processes of ecology, ocean currents, atmospheric turbulence. An annular vortex in a liquid or gas is a vortex thread with axis curved to the form of a circle of a certain radius. Its axis is the axis of the annular vortex [1–4]. The motion of the medium occurs around this axis along closed current lines. The classical theory of ring vortices is presented in [1, 2]. The paper [5] provides a number of examples of natural formation of vortex motion of water and air masses caused by changes in the density of the medium at various points in space. The first researchers who paid attention to the effect of fluid and gas motion in the form of a ring vortex are Helmholtz [1], Kelvin, who laid the foundations for the study of rotational, or vortex, movements, and Lamb [2] who began to study the ring vortex directly. Since the 60-ies of the last century, the processes of formation and development of ring vortices have been studied by many well-known researchers – D.G. Akhmetov, O.P. Kisarov, A.A. Lugovtsova, A.T. Onufriev, V.F. Tarasov, A.A. Buzukov, V.T. Kouvola [6,7], and others.

We should mark that most of the works are devoted to experimental studies of ring vortices kinematics. Analytical solutions of the equations of motion are quite complex and do not always fully describe the behavior of the ring vortex [8, 9]. Almost all solutions based on the consideration of the equations of motion formulated in cylindrical coordinates, sometimes using a spherical coordinate system.

An interesting feature of this type of flows is that the moving ring vortex is resistant to destruction, possesses a lifting force and retains the energy reported to it for quite a long time. As a result, depending on the initial pulse, such a vortex passes a distance of tens of its diameters. This is important in chemical technology, for example, when certain reagents are delivered through the environment to a formerly specified location where other delivery methods are not possible to be used.
The issues of hydrodynamics of ordered vortex flows are currently given special attention due to their importance in most production processes. For example, in the field of combustion of gaseous fuels toroidal burners have become widespread, which provide a high intensity of heat at high temperature as a result of combustion of liquid and gaseous fuels with direct use of oxygen. At the same time, the products of combustion with a high degree of dissociation provides a significant convective flow of heat during the recombination on the colder surfaces [5]. The burners form a torch containing toroidal vortices, which contribute to the intensification and stabilization of the combustion process. Similar effects are used in the combustion chamber of a hybrid rocket engine. It should be noted that in the processes of formation and development of ordered structures which are considered in the internal problems of hydrodynamics, the phenomenon of vortex flow in the form of a ring attracts attention by the variety of physical consequences which arise due to its structure.

Due to the fact that the emergence of a toroidal vortex is possible in many processes occurring in nature and in production, its study using numerical simulation is a rather urgent problem. The goal of this work is to obtain ring and spherical ring structures based on the numerical solution of the euler equation system for different variants of the flow twist.

2. Problem formulation and computational method

Consider the problem of ideal fluid flow in a cylindrical channel with axial symmetry described by the equations:

\[
\begin{align*}
\rho \frac{\partial u_r}{\partial t} + \rho u_r \frac{\partial u_r}{\partial r} + \rho u_\varphi \frac{\partial u_\varphi}{\partial \varphi} + \rho \left( e + \frac{V^2}{2} \right) & = 0, \\
\rho \frac{\partial u_\varphi}{\partial t} + \rho u_r \frac{\partial u_\varphi}{\partial r} + \rho u_\varphi \frac{\partial u_\varphi}{\partial \varphi} \quad & = \rho(u_r')^2 + p, \\
\rho \frac{\partial u_z}{\partial t} + \rho u_r \frac{\partial u_z}{\partial r} + \rho u_\varphi \frac{\partial u_z}{\partial \varphi} \quad & = \rho(u_r')^2 + \rho u_z^2 + \rho u_\varphi^2 + p u_r', \\
\end{align*}
\]

(1)

To solve this problem, we make direct numerical simulations of equations (1). We cover the simulation box with a structured grid of 400 nodes along the length and 200 nodes along the radius. For calculations we use the finite volume method with computation of intercell fluxes by the Roe method [10]. This method utilises a linearized solution of the Riemann problem of the disintegration of discontinuity when calculating flows on the cell boundaries, which reduces the computational complexity approximately to the exact solution complexity level. To increase the order of the computational scheme, the method of reconstruction of the solution to the boundaries of cells with a minmod-type limiter of was used [11]. In general, the constructed method has a second order of accuracy of spatial variables. To speed up the calculations, the computational algorithm was partially parallelized using the OpenMP library.

As the initial data we take \( X \) axis-parallel flow, disturbed by angular component of velocity. Consider two cases:

1. The flow with negative velocity gradient

\[
u^x = \begin{cases}
1, & R_1 \leq r \leq R_2, \\
\frac{1}{R_3 - R_2} (R_3 - r), & R_2 < r \leq R_3, \\
0, & R_3 < r \leq R_4.
\end{cases}
\]

(2)
2. The flow with positive velocity gradient

\[ u^x = \begin{cases} 
0, & R_1 \leq r \leq R_2, \\
\frac{1}{R_3 - R_2} (r - R_2), & R_2 < r \leq R_3, \\
\frac{1}{R_3 - R_2}, & R_3 < r \leq R_4.
\end{cases} \]  

(3)

\( R_1 = 0.1 \) is the radius of the inner wall of the channel, the wall is needed because of \( 1/r \) multiplier in the calculation scheme. \( R_2 = R_1 + 0.3(R_4 - R_1), \) \( R_3 = R_4 - 0.3(R_4 - R_1), \) \( R_4 = 0.5 \) is the radius of the outer wall of the channel.

The disturbance is twisting of the initial flow:

\[ u^\varphi = \begin{cases} 
\Omega_{\text{max}}(r - R_4), & R_4 \geq r \geq \frac{R_1 + R_4}{2}, \\
-\Omega_{\text{max}}(r - R_1), & \frac{R_1 + R_4}{2} > r \geq R_1.
\end{cases} \]

where \( \Omega_{\text{max}} = 0.1. \)

Radial component of the velocity is \( u^r = 0. \) We should note that initial data correspond a shear flow with the shear layer width \( \Delta = R_3 - R_2. \) Boundary conditions are non-flow at the channel walls and periodic ones on the ends of the simulation box:

\[ u^r \mid_{r=R_1} = 0, \quad u^r \mid_{r=R_4} = 0, \quad u \mid_{x=0} = u \mid_{x=h}, \]

where \( h \) is the channel length,

\[ U = \begin{pmatrix}
\rho \\
p u^r \\
p u^x \\
p u^\varphi \\
\rho \left( e + \frac{V^2}{2} \right)
\end{pmatrix}.\]

3. Results. Vortex ball and vortex ring

Figure 1 shows the results of the problem (1)–(3). We see that the flow in consideration shows the instability with respect to perturbations of the angular speed, which leads to emergence of so called ball (right part of the figure) and annular (left part of the figure) vortices. The twisting of the flow with the positive velocity gradient led to a vortex emergence near the inner wall on the channel. If we neglect the small radius of the inner wall \( R_1 \ll 1 \) (numerical simulation also shows that the emergence of the vortex occurs with the decrease of \( R_1 \)), we can say that the vortex near the inner wall is ball one (figures 1–2, right side). The twisting of the flow with the negative velocity gradient leads to the emergence of the toroidal vortex, situated near the outer wall of the channel (figures 1–2, left side).

It is possible that the emergence of such secondary flows is connected to pressure redistribution due to laying upon the initial disturbance. As we can see from the equation for \( r \)-component of the velocity (1), \( \frac{\partial}{\partial r} p = \frac{\rho \left( u^r \right)^2}{r}. \) Thus, pressure redistribution takes place along the axis \( r: \) \( p = p(r). \) The pressure gradient emerged and disturbances layed upon the flow lead to the mixing of the medium layer moving with different velocities. As a result of this unstable mixing, a vortex arises in the \((r,x)\)-plane.

In general, the results presented (figure 1) has well-known numerical and experimental evidences [12] which are illustrated by figure 2.

Figure 3 shows the calculations made on different grids with \( n_r \times n_x = 200 \times 400; \) \( 300 \times 500; \) \( 400 \times 700. \) We can see that the solution is almost independent on the grid, i.e. the numerical solution shown approximate the exact solution because of grid convergence.
Figure 1. Establishment of a ball or annular vortex with azimuthal vorticity $\omega < 0$ or $\omega > 0$, consequently, and the twisting of $\hat{w}$.

Figure 2. A ball and annular vortices [12] observed.

Thus, the result is that the toroidal vortex is a secondary flow, arising at the twisting of the shear axisymmetric flow. The more axial velocity gradient is, the faster vortex arises (the flow becomes more unstable).

Probably, this property is the defining one at the emergence of the cigarette-smoke rings and the formation of water rings by dolphins. It is known, that in order to release a ring of cigarette smoke, the smoker needs to exhale a stream of smoke sharply and as quickly as possible. After the exhaled air gets into the external environment, where there are always disturbances of various types, the formation of a vortex ring occurs. Dolphins for the formation of water rings,
produce a "blow to the nose" in the liquid region, where the ring appears. Thus, they create a very thin jet.

We should note that it is possible that this type of disturbance (by means of the $u_{\phi}$ velocity component) is not a direct source of secondary flow development. A numerical experiment was made, the result of which is shown in figure 4, from which it follows that the secondary flow regime also occurs when the main flow is perturbed only by the $r$-velocity component. Moreover, this perturbation is deterministic and is described by harmonic functions

$$u^r \sim a(r) \sin(nx) + b(r) \cos(nx); \quad |a(r)|, |b(r)| \ll |u^x|.$$ 

Thus, it follows that the most likely is the transition to the secondary mode under the action of the $u^r$ velocity component. This $u^r$ component is induced by the $u^\phi$ component by which the flow is disturbed. That is, there is a so-called induced instability, when the flow character is influenced not directly by the introduced disturbance, but by the parameter that arose due to

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**Figure 3.** The flow diagram obtained on the grids of $n_r \times n_z = 200 \times 400; 300 \times 500; 400 \times 700$ cells.
Figure 4. Secondary flow resulting from the perturbation of the flow of the radial velocity component.

This perturbation. Thus, we obtain the instability of the standard shear flow, which is overturned by the $u^r$ velocity component, as would be expected.

Figure 5 shows the dependence of the flow regime change time on the dimensionless parameter $\frac{\omega H}{U}$ of the problem.

Figure 5. Dependence of the transition time to the secondary mode on the dimensionless parameter $\frac{\omega H}{U}$. 
Similar results are obtained when considering swirling flows. Here it is necessary to clarify what a swirling flow is. By swirling flow is understood to be the current in which the vorticity is constantly introduced from the outside. The flow in channels with twisting devices can be considered as examples of such flows [13]. Secondary regimes of such flows are characterized by the arising of pairs of vortices rotating in opposite directions.

Another application of the developed model is the simulation of flow in pipes. Simulation of flows in pipes is relevant in the problems of transportation of liquid and gaseous substances. for example, in oil and gas transportation problems.

It is known [14] that during transportation of any substances return flows occur in pipes, which increase the hydrodynamic resistance, pressure, etc. This has a negative impact on the condition of the pipeline-in the area of occurrence of the return flow, the pressure increases, which can even lead to damage/rupture of the pipeline.

According to the results of this study, we can say that the main destabilizing factor is the shear nature of the flow. The greater the value of \( \frac{d\omega}{dr} \) is, the faster the transition to the secondary mode (for more unstable) occurs. Moreover, the regime change is manifested with respect to the perturbations \( \delta u^r \), \( \delta u^\phi \) of the radial and axial velocity components.

4. Conclusions
The three-dimensional axisymmetric inviscid flow of a normal gas in the channel is numerically investigated. As a result of twisting and radial perturbations of the initial main flow, the occurrence of a secondary flow relative to the main shear flow is numerically detected. The secondary flow regime in the flow problem is either a toroidal or a ball vortex. The birth of a particular type of vortex depends on the sign of the flow velocity gradient.

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