Synchronization of Two Coupled Crystal Oscillators

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Abstract

Synchronization is the phenomenon of two or more autonomous oscillators oscillating with the same frequency due to mutual interactions or common external forces. In this study, we focused on crystal oscillators, which are utilized in a wide variety of electrical circuits. When independently oscillating crystals in each circuit are synchronized in-phase, it is conceivable that simultaneous information processing is possible. To observe synchronization, we designed a circuit containing two Pierce oscillators with a branching path for mutual interactions. Then, we show that in-phase oscillation of the Pierce circuits can be induced by controlling the coupling strength.

1. Introduction

Synchronization of autonomous oscillators is observed in diverse fields of nature, science, engineering, and so on. Theoretical studies of synchronization of the limit cycle and chaotic oscillators have been carried out for many years [1]. From the experimental side, synchronization phenomena are observed in various oscillators, e.g., Stirling engines [2], metronomes [3] and candles [4]. Synchronization phenomena rely on the assumption that the oscillators are coupled with each other. There is an adjustment of the rhythm of the oscillators due to their weak interaction.

We conducted experiments on synchronizing the frequencies of two coupled crystal oscillators. Crystal oscillators are widely used as the clocks of CPUs. Multiple oscillators may be used in the near future to control processing units, which are divided into different parts. Synchronized operation of these clocks will be essential. If their clock outputs are desynchronized, protocol will be required to process information between processing units. Towards such applications, we present an experimental study on the synchronization of two crystal oscillators.

2. Methods

2.1 LTspice simulation

Two Pierce oscillators, each of which was composed of a crystal (XTAL1, XTAL2), an operational amplifier (op amp) (XOP1, XOP2), the input resistance of the op amp (R2, R5), the feedback resistor (R3, R6) of the op amp, a feedback resistor (R1, R4) from an output node, and two capacitors (C1 and C2, C3 and C4), and a bidirectional path composed of two capacitors (C5 and C6) and a resistor (R7) were implemented in the LTspice simulator (Ver. IV, Linear Technology Corp.) as shown in Fig. 1. The parameter settings are summarized in Table 1.

For the operational amplifier, the power-supply voltage source and the threshold voltage source were set to VDD = 10 [V] and Vth = 5 [V], respectively. The initial node voltages were set to n1 = 6.1 [V] and n2 = 6.2 [V]. To observe the op-
Table 1: Parameters of the circuit schematic

| Circuit elements on LTspice | Value          |
|-----------------------------|----------------|
| $R_1$ and $R_4$             | 20 [$\text{M\Omega}$] |
| $R_2$ and $R_5$             | 10 [$\text{K\Omega}$] |
| $R_3$ and $R_6$             | 1 [$\text{M\Omega}$] |
| $C_1$, $C_2$, $C_3$, and $C_4$ | 30 [$\text{pF}$]       |
| XTAL$_1$                    | 1.05 [$\text{MHz}$]   |
| XTAL$_2$                    | 1.06 [$\text{MHz}$]   |
| XOP$_1$ and XOP$_2$         | LT1354            |

| Coupling path                | Value          |
|-------------------------------|----------------|
| $R_7$                         | 1 - 100 [$\text{K\Omega}$] |
| $C_5$ and $C_6$               | 10 [$\text{pF}$] |

Figure 2: Time series of $v_1$ (pink line) and $v_2$ (green line) measured from two Pierce oscillators in the case of low coupling strength ($R_7 = 1$ [$\text{K\Omega}$])

For a crystal oscillator device, an analog inverter, a feedback resistor, and two capacitors. These crystals oscillate around 3.579 [$\text{MHz}$] (XTAL$_3$ and XTAL$_4$). Inverters (74HCU04AP TOSHIBA) (INV$_1$ and INV$_2$) amplify oscillations from the crystals and operate as buffers (INV$_3$ and INV$_4$) to generate output signals ($v_3$ and $v_4$). Capacitors ($C_7$, $C_8$, $C_9$, and $C_{10}$), adjust the oscillation frequency for each crystal. Feedback resistors ($R_8$ and $R_9$) return currents from the outputs. A bidirectional path consists of a resistor, two capacitors, and diodes. The resistor adjusts the coupling strength between the Pierce oscillators. Coupling capacitors ($C_{11}$ and $C_{12}$) transfer the mutual interaction. Schottky diodes (D$_1$ and D$_2$) slightly rectify the interactions. The coupling strength is adjusted by a potentiometer ($R_{11}$). The parameters of the electric components are summarized in Table 2.

2.2 Hardware implementation

To observe synchronization due to mutual interactions between crystal oscillators, we implemented two Pierce oscillators connected by a bidirectional path with a resistor to adjust the coupling strength between these oscillators in a discrete circuit as shown in Fig. 4. Each Pierce oscillator is composed of a crystal oscillator device, an analog inverter, a feedback resistor, and two capacitors. These crystals oscillate around 3.579 [$\text{MHz}$] (XTAL$_3$ and XTAL$_4$). Inverters (74HCU04AP TOSHIBA) (INV$_1$ and INV$_2$) amplify oscillations from the crystals and operate as buffers (INV$_3$ and INV$_4$) to generate output signals ($v_3$ and $v_4$). Capacitors ($C_7$, $C_8$, $C_9$, and $C_{10}$), adjust the oscillation frequency for each crystal. Feedback resistors ($R_8$ and $R_9$) return currents from the outputs. A bidirectional path consists of a resistor, two capacitors, and diodes. The resistor adjusts the coupling strength between the Pierce oscillators. Coupling capacitors ($C_{11}$ and $C_{12}$) transfer the mutual interaction. Schottky diodes (D$_1$ and D$_2$) slightly rectify the interactions. The coupling strength is adjusted by a potentiometer ($R_{11}$). The parameters of the electric components are summarized in Table 2.
2.3 Experimental procedure

First, the Pierce circuits are made to asynchronously oscillate. Then, we adjust the coupling strength of the bidirectional path to observe the operating characteristics of the proposed circuit. To quantify the degree of synchronization between these Pierce circuits, we calculated correlation the value $r$ between the outputs from these oscillators as

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

where $x$ and $y$ represent output signals. We recorded the output waveforms from these oscillators by an oscilloscope (Keysight InfiniiVision DSOX2014A). Then, we measured the output voltages 10 times while changing the value of $R_{10}$ from 0.0 [KΩ] to 6.0 [KΩ] in steps of 1.5 [KΩ] and plotted their average with the standard deviation indicated by error bars.

3. Results

Figure 5 shows output waveforms measured from the two coupled crystal oscillator circuits, which are not synchronized when the coupling strength is low ($R_{10} = 1$ [KΩ]). Figure 6 shows a Lissajous plot of the two outputs in case of no synchronization.

Figure 7 shows output waveforms measured from the two coupled crystal oscillator circuits which are synchronized when the coupling strength is high ($R_{10} = 2$ [KΩ]). Figure 8 shows a Lissajous plot of the two outputs in the case that the two coupled crystal oscillators are synchronized with small phase difference.

Figure 9 shows dependence of the correlation coefficient of the two outputs on the the resistance $R_{10}$. The two coupled crystal oscillators are synchronized when $R_{10}$ becomes about 2 [KΩ]. As the coupling strength increases, the cross-correlation coefficient approaches 1.0, which indicates that...
the two crystal oscillators synchronize in-phase. Figure 9 also shows that when the two coupled crystal oscillators are synchronized, the standard deviation decreases.

4. Discussion

In this section, we compare the results of the LTspice simulation and circuit experiments to investigate synchronization. This is because it is difficult to estimate crystal parameters and to build equations for Pierce nonlinear inverter circuit. In both cases, when the coupling strength was low, the Pierce circuits oscillated asynchronously and when the coupling strength was increased, synchronization of the circuits was observed.

In the SPICE simulation, when the natural frequencies are slightly different (< 1%) between the two crystals, other circuit components have the same parameters, and the coupling strength is strong, they are synchronized with a 0.007π phase shift as shown in Fig. 3. Furthermore, phase differences are increased by parameter detuning in the circuit experiments because discrete components have error ranges, for example, resistances have ±5% range, capacitors have ±5% range, and crystals have ±0.65 [Hz] range. Under these conditions, the phase shift is around 0.09π when $R_{10}$ is 2 [kΩ] in Fig. 7. From these results, the difference in the circuit parameters between the simulation and circuit experiments generates the phase difference between coupled crystal oscillators.

5. Conclusion

In this study, we observed the synchronization of crystal oscillators with mutual interactions. We first designed two Pierce crystal oscillator circuits with a branched signal path that can adjust the coupling strength between them. After confirming that the two crystals independently oscillate, we adjusted the coupling strength in the mutual interaction path. Under sufficient interaction strength, these crystal oscillators synchronize in phase. These results suggest that coupled crystal oscillators may enable distributed electrical circuits on a board to simultaneously process information mutual synchronization. Because weak common noise generally promotes the synchronization of weakly coupled oscillators, we will examine other ways approaches as the noise-induced synchronization [5] of coupled or uncoupled crystal oscillators in future works.

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