Torsional vibration characteristic study of the grid-connected DFIG wind turbine

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Abstract. This paper studies the torsional vibration characteristics of the grid-connected doubly-fed induction generator (DFIG) wind turbine by small signal analysis method. Firstly a detailed small-signal stability union model of the grid-connected DFIG wind turbine is developed, including the mechanical system and electrical system. To study the dynamic characteristic of the blade, gearbox, low speed and high speed shafts, a three mass shaft model for the mechanical system is adopted. At the same time, small signal models of DFIG, the voltage source converter (VSC) and the transmission line of the electrical system are developed respectively. Then, through calculating the eigenvalues of the state matrix $A$ and the corresponding participation factors, the modal analysis is conducted in the shaft torsional vibration issues. And the impact of the system parameters including the series compensation capacitor, the flat-wave reactor, the PI parameters, especially the speed controller of generator rotor on shaft torsional vibration are discussed. The results show that the speed controller strengthens association between the mechanical system and the electrical system, and also produces a low-frequency oscillation mode.

1. Introduction
As an environmentally friendly resource, wind energy has been developed quickly in recent years [1-2], which has led to the explosive growth of wind turbines connected to the power grid. With obvious disadvantages of variable production, however, the large amount of wind turbines integrated to the grid will bring a great impact of low-frequency oscillation and wind turbine shaft torsional vibration on power system [3]. Worse still, the torsional vibration can cause a reduction in the lifespan of the wind turbine resulted from fatigue, even the shaft fault [4], which contributes critically to the stability and safe operation of power system. Therefore it is of greatly practical significance to study the wind turbine torsional vibration characteristics further.

The wind turbines are mainly categorized into two types: fixed speed and variable speed wind turbines. Doubly fed induction generator (DFIG)-based wind turbine is widely used nowadays due to its variable speed operation and power decoupling controllability [5-7]. DFIG is also one of the most economical types of wind turbines, which permits the regulation of reactive power with variable rotor speed. With the increasing use of DFIG, a multitude of researchers are keen on the study of DFIG. Reference [8] invented an effective rotor position phase lock loop (PLL) to track the maximum power point (MPPT) of DFIG. By developing a new control scheme for the rotor side converter of DFIG, inter-area oscillations caused by the long transmissions are damped in [9]. The impact of increased
penetration of DFIGs on the power system are studied in [10] by modelling DFIGs as synchronous machines. The researches mentioned above are mainly concentrated on studying the normal operation of DFIG, the impact of DFIGs on the power system and lack the ability of analysing the dynamic characteristics of the DFIG system.

Thus the modelling methods to study dynamic characteristics of DFIG-based wind turbines during the transient process are gaining increasing interests. References [11-13] developed a lumped-mass, a two-mass and a three-mass drive train model of wind turbine shaft system respectively to model the mechanical system of wind turbines exactly. An available model of DFIG is developed in [14] to study how the model parameters will influence the transient responses of DFIG-based wind plants. But the mechanical system and electrical system are both simplified during the investigation. A two-mass drive train model is adopted in [15] to investigate the machine dynamics of DFIG. Reference [16] use the modal analysis to characterize the small-signal behaviour of DFIG wind turbine, and research the DFIG intrinsic dynamics with the change of the series compensation system parameters, operating points, and some other factors. However, the aforementioned works are mainly studying the impact of the aerodynamic factors or the long transmission lines and neglect the influence from the power grid and the control system. Also, which parts of the wind power system and to what extent will the different system parameters influence the dynamic character of the mechanical system are unrevealed. In order to exactly analyze the torsional vibration characteristics of the mechanical system of DFIG, a three-mass shaft model, which consists of blades, a gearbox, a low speed shaft, a high speed shaft and an induction generator rotor in [17] is adopted. The small signal model of the voltage source converter (VSC) is developed, where the three subsystems, including a rotor side converter (RSC), a grid side converter (GSC) and a DC-link are modelled respectively. The transmission line and the flat-wave reactor together with the transformer are modelled as an equivalent RLC (resistor, inductor and capacitor) line and an equivalent RL (resistor and inductor) line respectively. Then, the union small signal model of DFIG consisting of seven modules is obtained. Through conducting modal analysis, the torsional vibration characteristics of the shaft are demonstrated in detail. With the multi-mass shaft model, the dynamic response of the shaft to the electrical and control system parameters is reflected clearly.

The main contributions of this paper are that it: i) develops a small signal model of DFIG-based wind power system for easy shaft torsional vibration analysis; ii) studies the influence of the electrical and control systems on the shaft torsional vibration and finds that only the speed controller will have an impact; iii) provides a new method for parameter design for DFIG.

2. Grid-connected DFIG Wind Turbine Modelling

The schematic diagram of the studied DFIG wind turbine is shown in figure 1. The wind turbine is connected to the induction generator via a gearbox. The stator side of the induction generator is connected to the infinite bus through a transmission line, including the impedance of transformer and cable. The rotor side of the induction generator is fed through a VSC to supply exciting voltage to the generator.

![Figure 1. Schematic diagram of the DFIG wind turbine connected to infinite bus system](image-url)
As shown in figure 1, the model of the grid-connected DFIG wind turbine consists of four parts: the drive train, the induction generator, the transmission line and the converter controller model.

2.1. Drive Train Model
To reflect the dynamic characteristics of the drive train, a three-mass model presented in [17] is used to model the mechanical system. The drive train model can be expressed as

\[
\begin{align*}
J_1 \frac{d}{dt} \omega_{ro} &= T_{nro} - K_1(\theta_1 - \theta_2) - D_1(\omega_{ro} - \omega_1) \\
J_2 \frac{d}{dt} \omega_2 &= -K_2(\theta_2 - \theta_1) - D_2(\omega_2 - \omega_1) \\
J_3 \frac{d}{dt} \omega_{gen} &= -T_{em} - K_3(\theta_3 - \theta_2) - D_3(\omega_{gen} - \omega_2) \\
\frac{d}{dt} \theta_1 &= \omega_{ro} \\
\frac{d}{dt} \theta_2 &= \omega_1 \\
\frac{d}{dt} \theta_3 &= \omega_{gen}
\end{align*}
\]

(1)

where \(J_1, J_2, J_3\) denote the moment of inertia of the blades, low speed shaft and the high speed shaft respectively; \(\omega_{ro}, \omega_1\) and \(\omega_{gen}\) denote the angular velocity of the three parts respectively; \(\theta_1, \theta_2\) and \(\theta_3\) are mechanical rotation angle of the three parts respectively; \(D_1, D_2\) and \(D_3\) are damping coefficient of the three parts respectively; \(K_1, K_2, K_3\) are elastic coefficient between the blades and the low speed shaft, the low speed and high speed shaft respectively; \(T_{nro}\) is input wind torque from the blade side; \(T_{em}\) is the electromagnetic torque of the generator.

Based on small signal analysis, the standard state space equation of (1) after it is linearized can be expressed as

\[
\begin{align*}
\dot{X}_{DT} &= A_{DT}X_{DT} + B_{DT}U_{DT} \\
Y_{DT} &= C_{DT}X_{DT} + D_{DT}U_{DT}
\end{align*}
\]

(2)

where subscript DT denotes the three-mass drive train model.

2.2. DFIG Model
In this paper, DFIG is modelled with the five-order model of an induction generator (IG), which is shown in [18]. Here only the linearized model of DFIG derived using small signal analysis is shown, whose standard state equation can be expressed as follows.

\[
\begin{align*}
\dot{X}_{DFIG} &= A_{DFIG}X_{DFIG} + B_{DFIG}U_{DFIG} \\
Y_{DFIG} &= C_{DFIG}X_{DFIG} + D_{DFIG}U_{DFIG}
\end{align*}
\]

(3)

where subscript DFIG denotes the double fed induction generator.

2.3. Transmission Line Model
The transmission is equivalent to a RLC line model, which is modelled the same way shown in Reference [17]. Then the standard state space equation can be written as

\[
\begin{align*}
\dot{X}_{TL} &= A_{TL}X_{TL} + B_{TL}U_{TL} \\
Y_{TL} &= C_{TL}X_{TL} + D_{TL}U_{TL}
\end{align*}
\]

(4)

where subscript TL denotes the transmission line.
The flat-wave reactor with the small transformer is equivalent to a RL line model, whose state space equation can be obtained through equation (4).

2.4. VSC Model

The VSC of DFIG consists of a RSC and a GSC, which are connected back-to-back via a DC-link. Because of the variable-frequency supply provided by the DFIG converter, the rotor angular frequency and synchronous angular frequency are decoupled, which realizes the operation of wind turbine with variable speed. The equivalent circuit of the VSC is shown in figure 2.

![Figure 2. Equivalent circuit of the DFIG converter](image)

2.4.1 DC-Link Model. The power balance equation about the DFIG converter is described by

\[ P_{DC} = P_r - P_g \]  

where \( P_r \) is the rotor-side active power, \( P_g \) is the grid-side active power, and \( P_{DC} \) is the active power of the capacitor in the dc link.

Then

\[ CV_{DC} v_{DC} = u_{dq} i_{dq} + u_{dq} i_{dq} - \left( u_{dq} i_{dq} + u_{dq} i_{dq} \right) \]  

After conversion by p.u. and linearization of equation (6), the standard state equation of DC-link can be obtained as follows:

\[ \dot{X}_{DC} = A_{DC} X_{DC} + B_{DC} u_{DC} \]  

where subscript DC denotes the DC-link.

2.4.2 RSC Controller Model. The RSC is responsible for regulating DFIG active power and terminal voltage based on stator flux vector orientation method. The block diagram of RSC controller is shown in figure 3, where \( u_{d}^s \) and \( u_{q}^s \) are used to control the active power and voltage respectively.

![Figure 3. Block diagram of RSC controller](image)

Assume that the converter operates fast enough so that the dynamics can be ignored and the controlled variable closely follows the given value.

During linearization at the steady state point, the following equations are obtained first.

\[ \Delta P_{r, ref} = 0 \]
\[ \Delta P_r = i_{d0}^r \Delta u_{d0}^r + u_{dq}^r \Delta i_{dq}^r + i_{dq}^r \Delta u_{dq}^r + u_{dq}^r \Delta i_{dq}^r \]
\[ \Delta Q_{r, ref} = 0 \]
\[ \Delta Q_r = i_{d0}^r \Delta u_{dq}^r + u_{dq}^r \Delta i_{dq}^r - i_{dq}^r \Delta u_{dq}^r - u_{dq}^r \Delta i_{dq}^r \]  

(8)
Then the standard state space equation of RSC can be expressed as follows

$$\dot{X}_r = A_r X_r + B_r u_r$$  \hspace{1cm} (9)

where subscript $r$ denotes RSC.

2.4.3 GSC Controller Model

The GSC is responsible for controlling the DC-link voltage as a constant value and the output reactive power of DFIG. The block diagram of GSC controller is shown in figure 4.

![Figure 4. Block diagram of GSC controller](image)

Similar to the RSC, the linearized equations of GSC can be expressed by

$$\Delta \dot{x}_i = \Delta V_{DC-ref} - \Delta V_{DC}$$
$$\Delta \dot{i}_{ig-ref} = K_{phg}(\Delta V_{DC-ref} - \Delta V_{DC}) + K_{shg}\Delta x_i$$
$$\Delta \dot{x}_g = \Delta i_{ig-ref} - \Delta i_{ig}$$
$$\Delta \dot{x}_g = K_{phg}(\Delta V_{DC-ref} - \Delta V_{DC}) + K_{shg}\Delta x_i - \Delta i_{ig}$$
$$\Delta \dot{x}_r = \Delta i_{ig-ref} - \Delta i_{ig}$$
$$\Delta \dot{x}_r = K_{phg}(\Delta V_{DC-ref} - \Delta V_{DC}) + K_{shg}\Delta x_i - \Delta i_{ig} + K_{shg}\Delta x_r$$

where $\Delta V_{DC-ref} = 0$ and $\Delta i_{ig-ref} = 0$.

The standard state space equation of GSC can be expressed as follows

$$\dot{X}_g = A_g X_g + B_g u_g$$  \hspace{1cm} (11)

where subscript $g$ denotes GSC.

3. Modal Analysis of Grid-connected DFIG Wind Turbine

3.1. Small-signal Stability Model

By solving the equations of the drive train, DFIG, transmission line and controller models simultaneously, the union model of the grid-connected DFIG wind turbine is obtained as follows:

$$\dot{X} = AX + Bu$$  \hspace{1cm} (12)

where $X = [X_{DT} \ X_{DFIG} \ X_{TL} \ X_{DC} \ X_r \ X_g]^T$.

![Figure 5. Schematic diagram of the small-signal model](image)
To study shaft torsional vibration issues of DFIG shown in figure 1, it is necessary to build the small-signal stability model of the system including three-mass shaft model, induction generator model, converter, transformer, transmission line model, and the power grid model. Figure 5 below is the schematic diagram of the small-signal model of the DFIG-based wind power system, and presents the interaction among different models. During the process of modelling, to facilitate interaction of the system components, the conversion between state variables, such as that of current and voltage, is indispensable. In this paper, a capacitor is introduced to transform the current deviation to the voltage deviation. In order to avoid accuracy losing resulted from the introduction of capacitor, the value of the capacitor is extremely small.

3.2. Modal Analysis
A 2MW, 690V doubly fed induction generator model is established in MATLAB/SIMULINK based on the models of the aforementioned wind turbine components. The parameters of the drive train, generator and transmission line are the same with those shown in [17], and the parameters of the VSC are listed in the Appendix.

| State variable | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|----------------|--------|--------|--------|--------|--------|
| $\Delta \theta_1$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.081  |
| $\Delta \theta_2$ | 0.000  | 0.000  | 0.000  | 0.445  | 0.043  |
| $\Delta \theta_3$ | 0.000  | 0.000  | 0.054  | 0.375  |        |
| $\Delta \theta_4$ | 0.000  | 0.000  | 0.000  | 0.081  |        |
| $\Delta \theta_5$ | 0.000  | 0.000  | 0.000  | 0.000  |        |

| Frequency(Hz) | Damping ratio |
|---------------|---------------|
| 80.914        | 0.031         |
| 19.918        | 0.118         |
| 11.807        | 0.437         |
| 12.427        | 0.0019        |
| 1.765         | 0.0026        |

| Table 1. Eigenvalues of matrix A |
|-------------------------------|
| Modal | Eigenvalue | Frequency(\text{Hz}) | Damping ratio |
| 1     | -15.55 ± 508.40i | 80.914 | 0.031 |
| 2     | -14.83 ± 125.15i | 19.918 | 0.118 |
| 3     | -36.05 ± 74.19i  | 11.807 | 0.437 |
| 4     | -0.15 ± 78.08i   | 12.427 | 0.0019 |
| 5     | -0.029 ± 11.09i  | 1.765  | 0.0026 |

| State variable | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|----------------|--------|--------|--------|--------|--------|
| $\Delta \psi_{gs}$ | 0.006  | 0.010  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{ds}$ | 0.007  | 0.037  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{qr}$ | 0.000  | 0.197  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{qg}$ | 0.001  | 0.363  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{dq}$ | 0.244  | 0.229  | 0.055  | 0.000  | 0.000  |
| $\Delta \psi_{v}$  | 0.245  | 0.192  | 0.044  | 0.000  | 0.000  |

| Modal | 0 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| Table 2. Participation factors of torsional modes |
|-----------------------------------------------|
| State variable | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|----------------|--------|--------|--------|--------|--------|
| $\Delta \theta_1$ | 0.251  | 0.220  | 0.020  | 0.000  | 0.000  |
| $\Delta \theta_2$ | 1.000  | 6.000  | 3.000  | 0.000  | 0.000  |
| $\Delta \theta_3$ | 4.000  | 5.000  | 5.000  | 0.000  | 0.000  |
| $\Delta \theta_4$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \theta_5$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{gs}$ | 0.273  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{ds}$ | 0.000  | 0.042  | 0.101  | 0.000  | 0.000  |
| $\Delta \psi_{qr}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{qg}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{dq}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{v}$  | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{x_1}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{x_2}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{x_3}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{x_4}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{x_5}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{x_6}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
| $\Delta \psi_{x_7}$ | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |

| Modal | 0 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
The eigenvalues of the system state matrix $A$ are calculated and shown in table 1 except the non-oscillation modes. As shown, there are five oscillation modes. Since the real parts of the eigenvalues are all negative, the system is stable. The participation factors of corresponding oscillation modes are shown in table 2, which reflect the relative participation of system components. The words marked in bold show that the eigenvalues are highly sensitive to the corresponding variables.

As shown in table 1 and 2, the frequencies of oscillation mode 1 and 2 are 80.914Hz and 19.918Hz. They are highly sensitive to $\Delta l_x$, $\Delta l_y$, $\Delta u_{x+y}$, $\Delta u_{x-y}$, in other words, they are greatly influenced by the inductance of the transmission line and series compensation capacitors. The oscillation mode 3, whose frequency is 11.807Hz, correlates strongly with many state variables. They are mainly affected by the stator flux $\Delta \psi_q$ and rotor flux $\Delta \psi_d, \Delta \psi_r$. The related variables of rotor side controller play a big role besides. The oscillation mode 4 is highly sensitive to the state variables $\Delta \theta_1, \Delta \omega_1$, so they are mainly affected by the torsion angle and speed of the low-speed shaft. In addition, it is also affected by the torsion angle and speed of the high-speed shaft to some degree. Accordingly, the oscillation mode 4 is the mechanical oscillation mode, and the oscillation frequency is 12.427 Hz. The oscillation mode 5 is also mechanical oscillation mode, and the oscillation frequency is 1.765 Hz. Since the number of natural frequencies of a multi-mass system are equal to the number of masses minus one [19], the drive train model in this paper has only two natural frequencies, which are 12.427 Hz and 1.765 Hz, and the oscillation mode 4 and 5 are the torsional modes.

4. Effect of Parameters

4.1. Effect of Series Compensation Capacitor
The effect of series compensation capacitor can be described by the series compensation level $k_{com}$ [16], which is expressed as

$$k_{com} = \frac{X_C}{X_L}$$

(13)

where, $X_C$ is the capacitive reactance of the transmission line; $X_L$ is the sum of the inductive reactance of the transmission line and the transformer.

Varying the value of $k_{com}$ (0.1–1) while keeping all other parameters at their original values, the frequency and damping ratio variations of the mechanical mode 4 and 5 are shown in table 3.

| $k_{com}$ | Mode 4 | Mode 5 |
|-----------|--------|--------|
|           | Frequency(Hz) | Damping ratio | Frequency(Hz) | Damping ratio |
| 0.1       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 0.2       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 0.3       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 0.4       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 0.5       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 0.6       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 0.7       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 0.8       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 0.9       | 12.43   | 0.0019  | 1.765       | 0.0026       |
| 1.0       | 12.43   | 0.0019  | 1.765       | 0.0026       |

As shown in table 3, the frequency and damping ratio of mode 4 and 5 almost remain unchanged with the value of $k_{com}$ increasing, which can be explained by table 2. The mechanical modes are mainly
affected by the angle and speed of the high and low-speed shafts, and the electrical parameters don’t cause significant effect on the mechanical modes.

At the same time, vary the value of the flat-wave reactor and the seven PI parameters of the converter controllers respectively. The frequencies and the damping ratios of mode 4 and 5 all remain unchanged, which are the same cases as that of the series compensation capacitor.

4.2. Effect of Speed Controller
As shown in figure 3, the RSC uses the stator active power as the control variable. If the rotor speed is used as the control variable, and apply a PI controller in the speed control scheme to track the reference speed, the association between rotor speed and PI parameters will be strengthened a lot. The block diagram of RSC with speed controller is shown in figure 6, and the speed controller is marked with dotted lines. The speed controller can be expressed as

$$\begin{align} 
\dot{\omega}_s &= \omega_{gen\_ref} - \omega_{gen} \\
P_{s\_ref} &= K_{p_{\omega}}\dot{\omega}_s + K_{i_{\omega}}\int \omega_s 
\end{align}$$

(14)

Figure 6. Block diagram of RSC with speed controller

The eigenvalues of the new system state matrix A are calculated and shown in table 4.

| Modes | Eigenvalues       | Frequency (Hz) | Damping ratio |
|-------|-------------------|----------------|--------------|
| 1     | -15.55 ± 508.40i  | 80.914         | 0.031        |
| 2     | -14.79 ± 125.16i  | 19.919         | 0.117        |
| 3     | -36.05 ± 74.19i   | 11.781         | 0.437        |
| 4     | -0.16 ± 78.13i    | 12.434         | 0.002        |
| 5     | -0.27 ± 13.25i    | 2.109          | 0.020        |
| 6     | -0.02 ± 2.55i     | 0.406          | 0.009        |

Compared with table 2, one more oscillation mode with frequency 0.406 Hz and damping ratio 0.009 can be found in table 4. Given that mode 1~3 have few changes and are not mechanical torsional mode, we just take mode 4~6 into consideration, whose corresponding participation factors are shown in table 5.

As shown in table 4, mode 4 has little changes in its frequency and damping ratio, while mode 5 changes with the frequency from 1.765Hz to 2.109Hz and the damping ratio from 0.0026 to 0.02 respectively, which can be explained from table 5. Mode 4 is mainly affected by low-speed shaft, while mode 5 is affected by high-speed shaft. Since the speed controller only enhances effect of PI parameters on rotor speed (of high-speed shaft), the state variable $\Delta x_8$ has more significant effect on
mode 5 and less on mode 4. In addition, it also can be obtained from Table 4 that the speed controller can increase system damping of mode 5.

**Table 5. Participation factors of torsional modes**

| State variables | Mode 4 | Mode 5 | Mode 6 | State variables | Mode 4 | Mode 5 | Mode 6 |
|-----------------|-------|-------|-------|-----------------|-------|-------|-------|
| $\Delta \theta_1$ | 0.0002 | 0.0398 | 0.4600 | $\Delta w_{av}$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta \theta_2$ | 0.4438 | 0.0495 | 0.0065 | $\Delta V_{gs}$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta \theta_3$ | 0.0551 | 0.2554 | 0.3102 | $\Delta i_{av}$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta \omega_1$ | 0.0002 | 0.0398 | 0.4600 | $\Delta V_{DC}$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta \omega_2$ | 0.4439 | 0.0495 | 0.0064 | $\Delta x_1$ | 0.0003 | 0.0048 | 0.0022 |
| $\Delta \omega_3$ | 0.0558 | 0.4106 | 0.0335 | $\Delta x_2$ | 0.0001 | 0.0006 | 0.0000 |
| $\Delta \psi_{qr}$ | 0.0001 | 0.0002 | 0.0000 | $\Delta x_3$ | 0.0001 | 0.0001 | 0.0000 |
| $\Delta \psi_{dr}$ | 0.0001 | 0.0018 | 0.0007 | $\Delta x_4$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta \psi_{qr}$ | 0.0001 | 0.0000 | 0.0000 | $\Delta x_5$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta \psi_{dr}$ | 0.0004 | 0.0007 | 0.0000 | $\Delta x_6$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta l_{fs}$ | 0.0000 | 0.0000 | 0.0000 | $\Delta x_7$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta l_{fs}$ | 0.0002 | 0.0002 | 0.0000 | $\Delta x_8$ | 0.0005 | 0.1550 | 0.3437 |
| $\Delta w_{av}$ | 0.0000 | 0.0000 | 0.0000 | — | — | — | — |

Mode 6 is mainly affected by the angle and speed of blades and high-speed shaft as well as PI controller, which are concerned with not only the mechanical system but also the electrical system. And the corresponding participation factors are 0.4600, 0.3102 and 0.3437. Therefore mode 6 is the electromagnetic oscillation mode.

![Figure 7](image)

(a) mode 4 changes with $k_{pw}$

(b) mode 5 changes with $k_{pw}$

**Figure 7.** Variation of frequencies and damping ratios of mode 4 and 5 with $k_{pw}$ increasing
As shown in figure 7, with the value of \( k_{pw} \) increasing, the frequencies of mode 4 and 5 remain unchanged, while the damping ratios increase significantly. Therefore the increase of \( k_{pw} \) can contribute to the torsional vibration suppression, especially the torsional mode affected by high-speed shaft.

Varying \( k_{in} \) of PI controller in the speed control scheme, the variation of frequencies and damping ratios of mode 4 and 5 is shown in figure 8.

![Graph showing variation of frequencies and damping ratios of mode 4 and 5 with \( k_{in} \) increasing](image)

(a) mode 4 changes with \( k_{in} \)

(b) mode 5 changes with \( k_{in} \)

**Figure 8.** Variation of frequencies and damping ratios of mode 4 and 5 with \( k_{in} \) increasing

As shown in figure 8, with the value of \( k_{in} \) increasing, the frequencies of mode 4 increase slightly, while the damping ratios first increase then decrease at the point \( k_{in} = 200 \). And mode 5 changes with the frequencies from 1.941Hz to 4.174Hz and the damping ratios from 0.0226 to -0.0028 respectively. Mode 5 becomes unstable since the damping ratios decrease to negative value, which means the damping is positive. It can be seen that the increase of \( k_{in} \) can cause the damping decrease, even excite the torsional vibration and cause the mechanical failure. In addition, the torsional mode affected by high-speed shaft is highly sensitive to \( k_{in} \).

In addition, compared to the original system without the speed control, variation of the other PI parameters of converter controller has certain effect on the torsional mode, but limited.

### 5. Conclusions

This paper describes the torsional vibration characteristics of DFIG-based wind turbine. Firstly a small-signal stability analysis model is established with drive train, DFIG, transmission line and converter controller models associated. The modal analysis is employed in the union system and two oscillation modes are found to be the mechanical mode. Then the effects of several parameters on the system modes are studied, especially that of the speed controller. The conclusions are drawn as follows.
Regarded to the DFIG-based wind turbine without speed controller, the torsional modes are mainly affected by the high-speed and low-speed shaft. Accordingly, system parameters, such as the series compensation capacitor, the flat-wave reactor and each PI parameter of converter controllers, do not have significant effect on the two mechanical modes.

The speed controller strengthens association between the mechanical system and the electrical system, and also produces a low-frequency oscillation mode, which proves to be the electromagnetic oscillation mode. In addition, the speed controller contributes to the torsional damping increase, especially for the mode affected by the high-speed shaft. As well, the torsional damping increases with the increase of the proportion coefficient, but decreases with the increase of the integration coefficient of the speed controller. And other parameters of converter controller have slight effect on the torsional modes.

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Appendix

| Parameters of DFIG converter | Value | Parameters of DFIG converter | Value |
|-----------------------------|-------|-----------------------------|-------|
| DC_link capacitor $C_{\text{DC}}/(10^{-4}\text{pu})$ | 1.337 | $K_{i3}$ | 256 |
| Grid-side equivalent reactance $X_{\text{con}}/\text{pu}$ | 0.55 | $K_{p\text{g}}$ | 15 |
| Grid-side equivalent reactance $R_{\text{con}}/\text{pu}$ | 0.006 | $K_{\text{idg}}$ | 65 |
| $K_{p1}$ | 0.6 | $K_{pg}$ | 15.8 |
| $K_{i1}$ | 80.6 | $K_{lg}$ | 145 |
| $K_{p2}$ | 0.26 | $K_{pw}$ | 1.36 |
| $K_{i2}$ | 6.0 | $K_{iw}$ | 256 |
| $K_{p3}$ | 1.36 | — | — |

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