Cooling prediction of motorized spindle based on multivariate linear regression

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Abstract. In order to reduce the influence of heat produced by motorized spindle on the machining performance, cooling of motorized spindle need to be studied. Based on the multiple linear regression theory, the motorized spindle cooling model was established according to the temperature conditions of different cooling water temperatures and flow rates for the cooling process of the motorized spindle temperature. The validity of the model was verified by three verification methods. Based on the experimental data, the numerical simulation model of the motorized spindle cooling is established. It is effective and reliable to verify the mathematical model of the motorized spindle cooling with three test methods. The results showed the cooling prediction model of motorized spindle was relatively stable.

1. Introduction
The motorized spindle system is the most important component of machining equipment, which is one of the core components. Rotate speed of motorized spindle can reach tens of thousands of revolutions per minute, which make the spindle produce a lot of heat, including rotor and bearing[1]. The heat generated is not easy to disperse, which directly affects the machining accuracy of the motorized spindle. Therefore, it is necessary to cool the motorized spindle system.

The research on the heating and cooling of motorized spindle is a hot spot now. Many scholars at home and abroad are doing research on this aspect. Mansinghs found that the circulating cooling water channel can effectively reduce the temperature of the motorized spindle, and used the three-dimensional fluid model to analyze the influence of the water cooling system under different cooling pipes on the temperature field distribution of the motorized spindle[2]. Bern Bossmanns proposed the finite element difference model to describe the energy distribution of high-speed motorized spindle, analyzed the heat transfer mechanism of the spindle, analyzed the heat generation and heat transfer of the spindle in theory and simulation, and carried out experimental verification on the grinding spindle[3]. A dynamic model of a single-stage metal hydride cooling system working have been established by Jean Gabriel Sezgin and so on[4]. E.Uhlmanna established the cooling model of the spindle system by introducing convective heat transfer boundary conditions between the spindle components and verified the experiment[5]. Huang designed a spindle thermal deformation compensation control device for improving machining tolerance, and indirectly cooled the motorized spindle, thereby improving the machining accuracy of the machine tool. According to the principle of flow-solid-heat coupling of motorized spindle[6].

In summary, studying the cooling strategy scheme of the motorized spindle is of great significance for improving the working performance of the motorized spindle. Based on the multiple linear regression theory, the cooling law of the motorized spindle is numerically based on the data. Combined to establish a model for predicting the motorized spindle cooling system. The research
content of this paper provides the necessary theoretical support and method basis for further optimizing the heat transfer characteristics of the motorized spindle and improving its working performance.

2. Multiple linear regression theory

Multiple linear regression analysis means that there are two or more independent variables in the regression analysis \((X_1 - X_n)\), looking for multiple independent variables \((X_1 - X_n)\) and dependent variable \(Y\). This method of analysis is very simple and can be used to build relevant models based on the experimental data. The multiple linear regression analysis model is as shown in formula (1):

\[
Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n + \epsilon
\]  

(1)

Where \(\beta_0\) is a constant term, indicating when all explanatory variables \(X_i\) is 0, the explained variables \(Y\) is a predicted value of the average of all values. \(\beta_1, \beta_2, \ldots, \beta_n\) are the independent variable coefficients of the regression model. \(\beta_i\) represents the average change of the explained variable \(Y\) for each unit change in \(X_i\) without changing other explanatory variables. \(\epsilon\) is the random variable value of the error term, which is the degree of variation in \(Y\) that cannot be explained by the linear relation of \(n\) explanatory variables. The heating matrix of \(n\) random equations is expressed as follows:

\[
Y = X\beta + \epsilon
\]  

(2)

In the formula, if \(X\) is full rank, the general least squares estimate can be adopted as follows:

\[
\beta = (X'X)^{-1}X'Y
\]  

(3)

In order to establish a multiple linear regression equation, it is necessary to select the independent variables that have obvious influence on the dependent variable among various influencing factors, so that the dependent variable can be well expressed. In this process, the factors that have little effect on the dependent variable are automatically screened out. For the establishment of a good regression model, it is necessary to test the performance of each performance to ensure that the model is accurate and usable. There are three main methods of inspection, named the R test, T test and F test, which are testing the coefficient and significant regression coefficient. Sex test, regression equation significance test residual analysis.

The judgment coefficient test is also called the goodness of fit test (R test), and the statistical method is used to measure the fitting degree of the sample regression to the sample observation value. \(TSS = \sum(Y_i - \bar{Y})^2\) is the sum of the total deviation squared, \(ESS = \sum(\hat{Y}_i - \bar{Y})^2\) is regression sum of squares, and \(RSS = \sum(\hat{Y}_i - \hat{Y}_i)^2\) is residual sum of squares, the formula is as follows:

\[
R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}
\]  

(4)

Where \(R^2\) is the coefficient of determination. The closer this statistic is to 1, the higher the fitting degree of the model is.

Regression equation significance test (F test), under the condition that the original hypothesis \(H_0\) is established, the statistic formula is as follows:

\[
F = \frac{ESS / k}{RSS / (n - k - 1)}
\]  

(5)

The formula is the \(F\) distribution that obeys the degree of freedom \((k, n - k - 1)\). By given the level of significance \(\alpha\). The general value is 0.05 or 0.1, that is, the confidence is 90% or 95% or 0.95), the critical value \(F_\alpha(k, n - k - 1)\) is obtained. The value of the statistic \(F\) can be calculated
from samples. To determine whether the overall linear relationship of the original equation is significant, the original hypothesis $H_0$ can be rejected (or accepted) by $F \geq F_{\alpha}(k, n-k-1)$.

Regression coefficient significance test (T test) constructed $t$ statistics as follows:

$$
t = \frac{\hat{\beta}_j - \beta_j}{\sqrt{c_{jj}} e} \sim t(n-k-1)
$$

(6)

In the regression coefficient significance test, for a certain variable $X_j (j = 1, 2, ..., k)$. The null hypothesis and alternative hypothesis of the design are $H_0 : \beta_j = 0$ , $H_1 : \beta_j \neq 0$. By given the level of significance $\alpha$, the critical value $t_{2 \alpha}(n-k-1)$ is obtained. To determine whether the corresponding explanatory variables should be included in the model, the original hypothesis $H_0$ can be determined by $|t| \geq t_{2 \alpha}(n-k-1)$.

3. Cooling prediction model of motorized spindle

The motorized spindle cooling system mainly uses the circulating cooling water in the cooling water jacket to take away the heat generated by the stator and the rotor of the motorized spindle to achieve the cooling effect, and the cooling method is mainly in the form of convection. The basic calculation formula for convective heat transfer is obtained by the Newtonian cooling formula, and the convective heat transfer amount is:

$$\varphi = hA(T_{\text{spindle}} - T_{\text{water}})
$$

(7)

Where $\varphi$ is the heat flow, $h$ is convective heat transfer coefficient, $A$ is the wall area; $T_{\text{spindle}}$ is the motorized spindle temperature, $T_{\text{water}}$ is cooling water temperature.

Forced convection heat transfer:

Laminar flow: $h = (Nu \cdot \lambda) / l = (0.664 \text{Re}^{1/2} \text{Pr}^{1/3} \lambda) / l$

(8)

Turbulence: $h = (Nu \cdot \lambda) / l = (0.037 \text{Re}^{4/5} \text{Pr}^{1/3} \lambda) / l$

(9)

The Reynolds number needs to be calculated when the cooling water flows, and the flow state of the cooling water is analyzed by the Reynolds number. The Reynolds number of the flowing liquid in the pipe can be based on the following criteria:

$$Re = ul / \nu
$$

(10)

Where $u$ is the average flow rate of cooling water, $l$ is the equivalent hydraulic diameter for cooling the water jacket, $\nu$ is the kinematic viscosity of the fluid. We get $Re > 4000$. The entire flow rate is in a turbulent state. According to formula (7)-(10), the $T_{\text{spindle}}$:

$$T_{\text{spindle}} = (\varphi l)^{4/5} (0.037 u^{4/5} \text{Pr}^{1/3} \lambda A - T_{\text{water}})
$$

(11)

Based on the above relationship of $u$, $T_{\text{water}}$ and $T_{\text{spindle}}$, $T_{\text{spindle}} = a_1 u^{4/5} + a_2 T_{\text{water}}$. To speed $n$ and the axial position of the motored spindle $x$ for adjustments, fit from experimental data.

From the heat conduction theory, the relationship between the spindle temperature, the cooling water flow rate, and the cooling water temperature is known. $T_{\text{spindle}} = a_1 x_{1}^{4/5} + a_2 T_{\text{water}}$. To speed $n$ and the axial position of the motored spindle $x$ as an adjustment, you can think of the equation as $T_{\text{spindle}} = a_1 x_{1} + a_2 x_{2}$. Therefore, the equation is transformed into a multiple linear regression problem to solve.
According to the data, the spindle of the motorized spindle temperature $T$ is the dependent variable, the cooling water flow rate $u$, motorized spindle speed $n$, cooling water temperature $T_{water}$ and the axial position of the motorized spindle $x$. Data entry are independent variables. After getting the function relationship, you need to estimate the parameters, assuming $n$ independent observed data $(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{im}, y_i), i=1,2,\ldots, n$. The regression coefficient to be determined is $\beta_0, \beta_1, \beta_2, \ldots, \beta_m$, as shown by the least squares formula 12:

$$\min Q(\beta_0, \beta_1, \ldots, \beta_m) = \sum_{i=1}^{n} \left( y_i - (\beta_0 + \beta_1 x_{i1} + \cdots + \beta_m x_{im}) \right)^2$$ \hspace{1cm} (12)

Find the estimated value $\hat{\beta} = (X^T \cdot X)^{-1} \cdot X^T \cdot Y$ and find it as follows:

$$\hat{\beta} = \left[ \frac{1}{x_{i1}} \ldots \frac{1}{x_{im}} \right] \cdot \left[ \begin{array}{c} y_1 \\ \vdots \\ y_n \end{array} \right] \hspace{1cm} (13)$$

Find the estimated value, as in equation 14:

$$\hat{y} = \beta_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_m x_m$$ \hspace{1cm} (14)

Fitting error $e = y - \hat{y}$. It is called the sum of squared residuals. The regression equation can be obtained as follows:

$$T_{spindle} = 1.7898 + 0.9796u^{-4/5} + 0.8837T_{water} + 0.0008u + 0.0551x - 0.0002x^2$$ \hspace{1cm} (15)

Among them, the applicable range of each independent variable in the model is as follows: cooling water velocity and cooling water temperature, the spindle speed is 8000r/min, 15000r/min and 22000r/min. The values of axial position x are 62mm, 96mm, 128mm, 164mm, 200mm and 238mm.

Next you need to test the model:

(1) Judgment coefficient test: the model summary is shown in Table 1. $R = 0.919$ and $R^2 = 0.845$ both are close to 1, indicating that the goodness of fit of the equation is higher, and the independent variable can explain the 84.5% change of the dependent variable.

| model | $R$ | $R^2$ | Adjusted $R^2$ | Error of standard estimation |
|-------|-----|-------|----------------|-----------------------------|
| 1     | 0.919 | 0.845 | 0.845          | 0.55902                    |

(2) The saliency test of the regression coefficient: the probability value of the T statistic is 0.00 < 0.01, because of the introduction of the independent variable, the significance value of the significance is much less than 0.01. The coefficient table of the model is shown in Table 2:

| model                        | Non standardized coefficient | Standardization coefficient | T  |
|------------------------------|------------------------------|-----------------------------|----|
| Constant                     | 1.059                        | 0.286                       | 3.697 |
| Cooling water velocity       | 1.560                        | 0.092                       | 16.993 |
| Cooling water temperature    | 1.124                        | 0.018                       | 63.586 |
| Spindle speed                | 0.000781                     | 0                           | 26.238 |
| Axial arrangement point      | 0.0021                       | 0                           | 6.717  |
The T item in the table shows the significant difference of each coefficient to the model. The four values are not zero, indicating that the four coefficients are significantly different from zero, and the linear relationship between the interpreted variable and the explanatory variable is very significantly, all should be retained in the regression equation.

(3) Regression equation significance test and residual analysis, the F statistic obtained according to SPSS is 25.37, the linear relationship between the interpreted variable and the explanatory variable is very significant; the maximum value of the residual is -1.34197, the minimum value is 1.19296, the next figure shows the normalized residual normal distribution map. It can be seen from the figure that the residual distribution is consistent with the residual test, and the average level of the residuals fluctuates around zero.

4. Conclusion
Based on the multiple linear regression theory, the cooling model of the motorized spindle was established by regression analysis. The validity of the model was verified by three verification methods. The research content of this paper mainly focused on the thermal characteristics of the motorized spindle and the model establishment of the motorized spindle cooling. The research results can lay a good data support and method basis for realizing more effective motorized spindle cooling control and motorized spindle temperature transformation prediction. It provides an effective theoretical basis for ultimately improving the machining accuracy and working performance of the motorized spindle.

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