Holograms of conformal Chern–Simons gravity

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Work done with H. Afshar, B. Cvetković, S. Ertl and D. Grumiller
We study...

\[ S_{\text{CS}} = \frac{k}{4\pi} \int_{M^3} (\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma^3) \]

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- Holographic description: Partially massless gravitons, Brown–York responses, correlators... [arXiv:1107.xxxx]
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- This talk: What does the Weyl symmetry give rise to at the boundary?
Warm-up: (2+1)-dimensional EH gravity

\[ S_{EH} = \int_{M^3} \sqrt{-g} (R - 2\Lambda) \quad \text{Gauge symmetry:} \quad \delta g_{\mu\nu} = \nabla_{(\mu} \xi_{\nu)} \]
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Boundary conditions: \[ \text{[Brown, Henneaux '86]} \]

\[ g_{\mu\nu} = g^{\text{AdS}}_{\mu\nu} + h_{\mu\nu} = g^{\text{AdS}}_{\mu\nu} + \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}_{\mu\nu} \]
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Diffeos that preserve the BCs: \( \xi^\pm = \epsilon^\pm (x^\pm) + \mathcal{O}(y^2) \)
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\[ i\{L_n, L_m\} = (n - m)L_{n+m} + \frac{c}{12} (n^3 - n)\delta_{n+m} \]
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\[ i\{\bar{L}_n, \bar{L}_m\} = (n - m)\bar{L}_{n+m} + \frac{c}{12} (n^3 - n)\delta_{n+m} \]
Conformal Chern-Simons gravity

Gauge sym: \( \delta g_{\mu\nu} = \nabla_{(\mu} \xi_{\nu)} \), \( \delta g_{\mu\nu} = 2\Omega(x)g_{\mu\nu} \)
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Boundary conditions: [arXiv:1107.xxxx].

$$g_{\mu\nu} = e^{\phi(x,y)} \left( g_{\mu\nu}^{\text{AdS}} + h_{\mu\nu} \right)$$
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Gauge-trafo’s that preserve the BCs: depends on BC on \( \phi \).
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Weyl  \rightarrow  trivial
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\[ \phi \equiv f_{\text{fixed}}(x) + \ldots : \quad \text{diffeo} + \text{Weyl} \quad \rightarrow \quad \text{Vir}^2, \quad c = -\bar{c} \]

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Conservation of the Weyl charge \[\implies\] consistency conditions.
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Simplest case: \( \phi = \phi(x^+) \) and \( \Omega = \Omega(x^+) \)
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[\bar{L}_n, \bar{L}_m] &= (n - m) \bar{L}_{n+m} + \frac{\bar{c}}{12} (n^3 - n) \delta_{n+m} \\
[J_n, J_m] &= 2k n \delta_{n+m} \\
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Pure diffeos \( \implies \) Sugawara shift \( L_n \to L_n + \sum_k J_k J_{n-k} \).
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