NUMERICAL SOLUTION OF THE PROCES OF HYDROPLANING AT WHEELS WITH ENVELOPE

M Stan¹, P Stan²

¹University Of Pitesti, Arges, Romania, email: marinica.stan@upit.ro
²University Of Pitesti, Arges, Romania, email: petre.stan@upit.ro

Abstract. In this paper is presented a study for the envelope’s properties settlement regarding the process of hydroplaning. The phenomenon of hydroplaning is calculable with the aid of the equations of motions for the viscid fluid in streamline regime for thin layers of fluid supposed to crush. Conclusively this paper offers the base of determinations prequisites to settle the envelope’s properties concerning to the phenomenon of hydroplaning.

1. Characteristic quantities of the contact area between the envelope and the rolling track

If the envelope has inside air with pressure equal with atmospheric pressure, the experiment shows that in usual limits of quaquaversal distortion practically the envelope doesn't take over quaquaversal forces, that is to say that in this case it's acting like a perfectly elastic membrane at encasing. The experiment shows also that at loading of the envelope with charge \( G \) and at the appearance of the quaquaversal distortion between envelope \( R \)'s ray and the radius of running \( r \), the width \( B \) of the contact zone remains the same (figure. 1). For a tyre filling pressure given as \( p \) read at the pressure gauge it is seen that once with the charge \( G \)'s growing it also grows the length of the contact zone having an almost rectangular shape of \( A = BL \) area. The contact between the envelope [1] and the path is made through the bars of profile from area \( A \) and the path is made through the bars of profile from area \( A = BL \), but the effective contact area \( A_e \) between the bars of profiles and the path is smaller than area \( A \) and we denote

\[
\lambda = \frac{A_e}{A} = \frac{A_e}{BL} < 1 \quad (1)
\]

the using of the contact zone's surface coefficient.

If the effective pressure between the bars of profiles and the path is denoted with \( p_e \) then the envelope's loading force \( G \) may be exprimed with relation

\[
G = pA = p_e A_e \quad (2)
\]

From relations (1) and (2) may be exprimed the effective pressure

\[
p_e = \frac{G}{A_e} = \frac{G}{\lambda A} = \frac{G}{\lambda BL} \quad (3)
\]

The tyre studied contains profiles \( \delta_1 \) and rectangulars \( \delta_2 \) with dimensions \( mk \) and \( nk \), where \( k \) is the pace of the meshwork for the numerical integration of the pressure [2] in the water course expelled from underthe profiles and \( m \) and \( n \) are natural numbers.
For each profile [3] recomes from surface $A$ an enclosure of width $a$, so that the first output factor in the contact area is:

$$\lambda_1 = \frac{n^2k^2}{(nk + 2a)^3}. \quad (4)$$

The second output factor in the contact area has the following expression:

$$\lambda_2 = \frac{mnk^2}{(mk + 2a)(nk + 2a)} \quad (5)$$

2. The liquid's expelling time from under the bar of circular profile

If on the bed bearer is found out a liquid $\delta$ with the dynamic viscosity $\eta$ forming a layer of thickness $H$, then the circular basal plane $f - f$ of the bar of profile [4], before entering in contact with surface $S - S$ of the path of run, is due to expel through crushing this layer of liquid (figure. 2). In figure 3 are represented two particular situations of the profile block $\delta$’s surface from figure 2 which are a square $\delta_1$ and a rectangle $\delta_2$, in which was layed a meshwork with pace $k$ having $n = 4$ for the square and $n = 4; m = 6$ for the rectangle.

The forces [5] that make the expell posible are:

$$F_1 = p_\alpha n^2k^2, \quad F_2 = p_\epsilon mnk^2 \quad (6)$$

The surface $S - S$ of the path of run is considered to be the flat surface that passes through the bed bearer’s rugosity peaks. It is noticed that at the moment when the bar of profiles’ circular basal plane $f - f$ is getting closer of surface $S - S$ in a parallel motion until the distance between them is deleted and the bed bearer’s rugosity peaks are starting to get into the bar of profile's material, the liquid from under the surface $f-f$ can still be expelled trough the bed bearer's random assigned micro channels.
Figure 2. The rolling path with liquid.

Figure 3. Particular situations of the surface of the profile block.

To express the expelling flow liquid from under the bar of profiles in this limit situation is entered a mathematical plan \( m - m \), parallel with \( S - S \) plan and situated at a distance \( H_{\text{min}} \) below this, so that the flow of expulsion expressed by computation among the parallel surfaces \( m - m \) and respective \( f - f \) to be equal with the real flow expulsion through the micro channels from the bed bearer’s rugosity [6]. If in the axis system \( Oxyz \) is defined a solid surface \( \delta = \delta(x, z) \) which displaces after axis \( Ox \) with constant speed \( \bar{u} = u \vec{i} \) and after axis \( Oy \) with constant speed \( \bar{v} = v \vec{j} \), the space among \( \delta \) and a plan also solid \( xOz \) being full with viscid fluid of \( \eta \) dynamic viscosity, with the very little value \( \delta \) in report with the common zone of \( \delta = \delta(x, z) \) and \( xOz \), therefore is known the pressure equation [6] between \( \delta \) and \( xOz \) is:

\[
\frac{\partial}{\partial x} \left( \delta^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \delta^3 \frac{\partial p}{\partial z} \right) = 12\eta v - 6\eta u \frac{\partial \delta}{\partial x} .
\]

In the case from figure 2 we have \( u = 0 \), \( \delta = h = \text{constant} \) and \( \bar{v} = -v \vec{j} \) and equation (6) becomes:

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = -\frac{12\eta v}{h^3} .
\]
The expression in limited differences of this equation on the meshwork from figure 3 for the arbitrary point \(i,j\) is:

\[
p_{i+1,j} + p_{i-1,j} + p_{i,j+1} + p_{i,j-1} - 4p_{i,j} = \frac{-12\eta v}{k^2} \tag{9}
\]

or with

\[
v = -\frac{dh}{dt}, \tag{10}
\]

\[
p_{i+1,j} + p_{i-1,j} + p_{i,j+1} + p_{i,j-1} - 4p_{i,j} = \frac{12\eta k^2 dh}{h^3 dt} = T. \tag{11}
\]

If is applied equation (11) in the \(\delta_1\) surface’s meshwork points is obtained the equations system:

\[
\begin{align*}
p_2 + p_3 - 4p_1 &= T \\
2p_1 + p_4 - 4p_2 &= T \\
p_1 + p_4 + p_5 - 4p_3 &= T \\
p_2 + 2p_3 + p_6 - 4p_4 &= T \\
p_3 + p_6 - 4p_5 &= T \\
p_4 + 2p_5 - 4p_6 &= T
\end{align*} \tag{12}
\]

It is working with relative pressures so on the \(\delta_1\) surface’s contour the relative pressure is zero. It is kept the notation \(p\) also for the relative pressure for making it easier to write.

The previous system [7] solutions are:

\[
p_1 = -0.6875T, \quad p_2 = -0.875T, \tag{13}
\]

\[
p_3 = -0.875T, \quad p_4 = -1.125T, \tag{14}
\]

\[
p_5 = -0.6875T, \quad p_6 = -0.875T. \tag{15}
\]

The medium pressure [8] under surface \(\delta_1\) is

\[
p_m = \frac{2p_1 + p_2 + 2p_3 + p_4 + 2p_5 + p_6}{25} = 0.29T \tag{16}
\]

If equation (10) is applied in the \(\delta_2\)’s surface meshwork points is obtained the equation system:

\[
\begin{align*}
p_2 + p_4 - 4p_1 &= T \\
p_1 + p_3 + p_5 - 4p_2 &= T \\
2p_2 + p_6 - 4p_3 &= T \\
p_1 + p_5 + p_7 - 4p_4 &= T \\
p_2 + p_4 + p_6 + p_8 - 4p_5 &= T \\
p_3 + 2p_3 + p_9 - 4p_6 &= T \\
p_4 + p_8 - 4p_7 &= T \\
p_5 + p_7 + p_9 - 4p_8 &= T \\
p_6 + 2p_8 - 4p_9 &= T
\end{align*} \tag{17}
\]

We have:

\[
p_1 = -0.775T; \quad p_2 = -1.11T \tag{18}
\]
\[
p_3 = -1.19T,\ p_4 = -0.99T \\
p_5 = -1.475T;\ p_6 = -1.54T \\
p_7 = -0.775T;\ p_8 = -1.11T;\ p_9 = -1.19T.
\]

The medium pressure under surface \( \delta_2 \) is
\[
p_{m_2} = \frac{2p_1 + 2p_2 + p_3 + 2p_4 + 2p_5 + p_6 + 2p_7 + 2p_8 + p_9}{35} = -0.4682T
\]

3. The equation of the evolution of the liquid feather from under the envelope and the hydroplaning phenomenon’s definition

In figure 3 is represented the contact zone between the envelope and the path at rolling on wet path with ones for possible situation of the bar of profile temporally in which this is found out in the contact zone of length \( L \). These, in position 1 the bar of profile entered into the contact zone of length \( L \) and below this is found out a liquid layer of thickness \( H \) which must expel it through crush on the periphery of profile \( \delta \) and in this it can enter into contact with the bed bearer.

![Figure 4. The contact zone between the tyre and the wet path.](image)

From relations (6), (16) and (22) is obtained
\[
p_{m_1} = p_{e_1} = -0.295T;
\]
\[
p_{m_2} = p_{e_2} = -0.4682T,
\]
and the with relation (11) we obtain the equations
\[
p_{e_1} = -0.295 \frac{12h^2 k dh}{h^3 dt}, \tag{24}
\]
\[
p_{e_1} = -0.4682 \frac{12h^2 k dh}{h^3 dt}. \tag{25}
\]
which are explained this way:
\[
dt_1 = -\frac{0.295 \cdot 12h^2 k dh}{p_{e_1} h^3}, \tag{26}
\]
respectively
\[
dt_2 = -\frac{0.4682 \cdot 12 \eta k^2}{p_{c_1}} \frac{dh}{h^3}
\]

The time in which the layer of liquid’s consistency decreases from value \( H \) to value \( H_{\text{min}} \) is in those two cases [8]

\[
t_1 = -\frac{0.295 \cdot 12 \eta k^2}{p_{c_1}} \frac{H_{\text{max}}}{h} \frac{dh}{h^3} = \frac{1.77 \eta k^2}{p_{c_1}} \left( \frac{1}{H_{\text{min}}^2} - \frac{1}{H^2} \right)
\]

\[
t_2 = -\frac{0.4682 \cdot 12 \eta k^2}{p_{c_1}} \frac{H_{\text{max}}}{h} \frac{dh}{h^3} = \frac{2.8 \eta k^2}{p_{c_1}} \left( \frac{1}{H_{\text{min}}^2} - \frac{1}{H^2} \right)
\]

The wheel’s center \( O \), represented in figure 1 is displacing with speed \( v_a \), and therefore the bar of profile’s position is changing in the contact zone of length \( L \), the bar abiding in the same time non-moving after direction \( S - S \) belonging to the bed bearer surface’s plan, so in position 2 from figure 3 is represented a certain situation in which the bar of profiles has position \( x \) in the contact zone and still has to expel a liquid layer of thickness \( h \). After a time \( t \) from the entrance in the contact zone given by formula (28) and (29) position 3 of the bar of profile [9] in the contact zone is \( l \) given by

\[
l_1 = v_a t_1; l_2 = v_a t_2,
\]

situation in which the bar of profile’s plan \( f - f \) touched the plan \( S - S \) of the bed bearer rugosity peaks, and from the viewpoint of the liquid from under plane \( f - f \) ‘s expelling phenomenon, there was an equivalent thickness of liquid \( H_{\text{min}} \) which can be expelled through the bed bearer rugosity’s micro channels. For the reason that opening with position 3 and continuing then with position 4 the bar of profiles has expelled completely the liquid among planes \( f - f \) and \( S - S \), it is considered that in these positions can be developed dry friction forces between surfaces \( f - f \) and \( S - S \).

On the contrary, on contact zone of length \( l \), cannot be developed dry friction forces because the bars of profiles are suspended on a layer of liquid called feather of water, the phenomenon of this feather of water’s appearance carrying the name of hydroplaning or hydroplaning.

If we replace the time from relation (30) into relation (28) and (29) it is obtained the feather of water evolution’s equation depending on the wheel’s run \( v_a \):

\[
l_1 = \frac{1.77 \eta k^2}{p_{c_1}} \left( \frac{1}{H_{\text{min}}^2} - \frac{1}{H^2} \right) v_a^2; l_2 = \frac{2.8 \eta k^2}{p_{c_1}} \left( \frac{1}{H_{\text{min}}^2} - \frac{1}{H^2} \right) v_a^2.
\]

The envelope is behaving as a perfect elastic to flexure membrane also in the contact zone of length \( L \), looker at the different positions of the bars of profiles on vertical direction concerning the variation of position \( h \) of these between \( H \) and \( H_{\text{min}} \), fortiori how much values \( h \) are very little in report with length \( L \).

4. The critical velocity of hydroplaning

From relation (28) and (29) it is noticed that the length \( l \) of the watery feather formed below the envelope grows in line with wheel’s speed \( v_a \). The critical velocity \( v_{\text{crit}} \) is defined for the situation when \( l = L \), what means that the support of the envelope on bed bearer is made exclusively by dint of a liquid layer which separates completely the bars of profiles of the surface \( S - S \) of the bed bearer and in this case, watching relation (26) the frictional forces \( F_f \) that can develop between the tyre and the bed bearer are null. The critical velocity of hydroplaning [10] is calculated from relation
\[ L = \frac{1.77 \eta k^2}{\rho c} \left( \frac{1}{H_{\text{min}}^2} - \frac{1}{H^2} \right) v_{\text{crit}}, \quad (32) \]
\[ L = \frac{2.8 \eta k^2}{\rho c} \left( \frac{1}{H_{\text{min}}^2} - \frac{1}{H^2} \right) v_{\text{crit}}. \quad (33) \]

The term \( \frac{1}{H^2} \) can be neglected in reports with term \( \frac{1}{H_{\text{min}}^2} \) for all situations that appear at hydroplaning and so the critical velocity has the expression:

\[ v_{\text{crit}} = \frac{L \rho c H_{\text{min}}^2}{1.77 \eta k^2}. \quad (34) \]

The other critical velocity has the expression:

\[ v_{\text{crit}} = \frac{L \rho c H_{\text{min}}^2}{2.8 \eta k^2}. \quad (35) \]

If we hold the expense of relation (3) we obtain for critical velocity the formula:

\[ v_{\text{crit}} = \frac{G H_{\text{min}}^2}{1.77 \eta k^2 \lambda B}, \quad (36) \]

respectively

\[ v_{\text{crit}} = \frac{B H_{\text{min}}^2}{2.8 \eta k^2 \lambda B}. \quad (37) \]

5. Numerical results

We efectuate a calculus [4] example for a tyre with width \( B = 150mm \), filled with \( G = 2500N \). We take water viscosity \( \eta = 1.31 \cdot 10^{-3} Ns/m^2 \) at \( 0^\circ C \).

If we take pace \( k = 6mm \) then from figures and 3 results a square profile with leg \( nk = 4 \cdot 6 = 24mm \) and a rectangular profile with bigger leg \( mk = 6 \cdot 6 = 36mm \) and smaller leg \( nk = 24mm \). With a value \( a = 3mm \) from formules (4) results \( \lambda_1 = 0.64 \) and \( \lambda_2 = 0.68 \).

We calculate the critical velocities of hydroplaning for the two profiles with formula (36), (37) and obtain:

\[ v_{\text{crit}} = \frac{G H_{\text{min}}^2}{1.77 \eta k^2 \lambda B} = 44.92 \frac{m}{s} = 161.7 \frac{km}{h}. \quad (38) \]
\[ v_{\text{crit}} = \frac{B H_{\text{min}}^2}{2.8 \eta k^2 \lambda B} = 26.72 \frac{m}{s} = 96.22 \frac{km}{h}. \quad (39) \]

6. Conclusions

The phenomenon of hydroplaning is calculable with the aid of the motional equations of the viscid fluid in streamline regime for thin layers of fluid supposed to crush. As physical characteristic size of this phenomenon is entered the minimal conventional thickness of the fluid \( H_{\text{min}} \), who is expelled among the bar of profile and the bed bearer in the moment when the bar of profile touches the bed bearer rugosity's peaks. In this moment the expulsion is produced through the micro channels from the bed bearer and possible through the micro channels from the emplacement surface of the bar of profile. These expulsion possibilities are exprimed through thickness \( H_{\text{min}} \), as through the planes that are approaching relatively, the emplacement plane of the bar of profile and the bed bearer's plane would not have rugosities and between them would be liquid of thickness \( H_{\text{min}} \). Through the introduction of the evolution function of the watery feather from under the envelope at running on wet bed bearer and
through the critical velocity’s calculus this offered the possibility to express the diminution of the adherence to bed bearer of the envelope depending on the wheel's running speed as the complete detachment of the wheel from the path.

Conclusively this paper offers the scientific base of the experimentations prequisited to settle the envelope's properties concerning to the phenomenon of hydroplaning. The phenomenon of hydroplaning is calculable with the aid of the equations of motions for the viscid fluid in streamline regime for thin layers of fluid supposed to crush.

References
[1] Hara V and Stan M 2007 The equations of the process of hydroplaning at wheels with envelope having the roller track from bars of detached profiles (Pitesti: Pitesti University Press) p 180
[2] Stan M 2004 Mechanics of Fluids (Bucuresti: EDP Bucuresti) p134
[3] Stan M and Popa D 2005 Fluid mechanics problems (Bucuresti: EDP Bucuresti) p 240
[4] Persson B 2001 Theory of rubber friction and contact mechanics (Journal of Chemical Physics, vol. 115, no. 8) p 3840–3861
[5] Wies B and Roeger B 2009 Influence of pattern void on hydroplaning and related target conflicts (Tire Science & Technology, vol. 37, no. 3) p 187–206
[6] Kersys A 2013 Research of the influence of tire hydroplaning on directional stability of vehicle (American Review of Respiratory Disease, vol. 28) p. 374–380
[7] Wu Z and Zong Z 2014 Mie-Gruneisen mixture Eulerian model for underwater explosion (Engineering Computations, vol. 31, no. 3) p. 425–452
[8] Martin C S 1966 Hydrodynamics of Tire Hydroplaning (Georgia Institute of Technology, Final Report) p 166-187
[9] Bramono D P and Racz L M 1998 Numerical Flow-Visulization of Slurry in a Chemical Mechanical Planarization Process (CMP-MIC Conf) p 185-192
[10] Clark S K 1971 Mechanics of Pneumatic Tires (Monograph Number 122. National Bureau of Standards, Washington, D.C.) p 231-240