Three-phase Power Flow Calculation of Low Voltage Distribution Network Considering Characteristics of Residents Load

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Abstract. In the traditional three-phase power flow calculation of the low voltage distribution network, the load model is described as constant power. Since this model cannot reflect the characteristics of actual loads, the result of the traditional calculation is always different from the actual situation. In this paper, the load model in which dynamic load represented by air conditioners parallel with static load represented by lighting loads is used to describe characteristics of residents load, and the three-phase power flow calculation model is proposed. The power flow calculation model includes the power balance equations of three-phase (A, B, C), the current balance equations of phase 0, and the torque balancing equations of induction motors in air conditioners. And then an alternating iterative algorithm of induction motor torque balance equations with each node balance equations is proposed to solve the three-phase power flow model. This method is applied to an actual low voltage distribution network of residents load, and by the calculation of three different operating states of air conditioners, the result demonstrates the effectiveness of the proposed model and the algorithm.

1. Introduction
Low-voltage distribution network is a power supply district which connects to the electricity customers directly. Its operation has great influences on the reliability and quality of power users[1]. At present, because of the coexistence about three-phase load and single-phase load, unbalance problem is serious. It’s mentioned that different states of dynamic loads may cause different unbalance problems. They strongly affect the quality of user’s power supply. In the low-voltage distribution network, there is a large number of single-phase air conditioning loads. When the air conditioning starts, three-phase imbalance degree will increase greatly. Therefore, considering operation states of dynamic loads such as air conditioner is of great significance for the optimization in power system.

In power flow calculation, constant power load model is widely used. It is divided into single-phase, two-phase and three-phase constant power load model in [1]; In [2], three-phase constant power load model is adopted; [3-4] use single-phase constant power load standing for the constant power load. Furthermore, the load mode of constant power, constant current and constant impedance put into use in [5]. In [6], three-phase constant power load is converted into three three-sequence current injected into the network. Three-phase power load is expressed as three-phase current in [7]. [8] adopts linear circuit elements to represent the motor. [9] develops the three-phase power flow based on electric vehicle (EV) demands. In the above three-phase power flow calculation, the load which does not change with load’s operation states cannot reflect actual load characteristics. For instance, the air...

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conditioner is in star-up operation state or steady operation state, the power of it will have great changes comparing with each other. Therefore, using constant power load model to evaluate the three-phase unbalance degree is not reasonable.

This paper adopts the load model in which dynamic load represented by air conditioners parallel with static load represented by lighting loads in section 2. Dynamic load is changing with operation states. And then the paper builds three-phase power flow calculation model based on power balance equations of three-phase(that is phase A, phase B and phase C), the current balance equations of phase 0, and torque balancing equation in section 2. Alternating iterative algorithm of three-phase power flow is used to combine node balancing equations and torque balancing equations of induction motor in section 3. Finally, the simulation result and discussions are reported through a case of actual low-voltage distribution area in section 4.

2. Three-phase Power flow calculation model considering residents load characteristics

2.1. Residents load model

In the low-voltage distribution network, residential load often includes the dynamic load. When the air conditioner (ACR) starts, the value of its current and power is several times than it is in the steady operation condition. Using constant power load model, results of three-phase power flow calculation are great different from the actual power grid state. Therefore, we build load mode in which dynamic load represented by air conditioners parallel with static load represented by lighting load. In the air conditioning load, we should separate the steady operation state and start-up state respectively.

2.1.1. Model of the air conditioner load

A. stable operation state

When the ACR is in normal working condition, we use the induction motor equivalent circuit model of double-cage rotor [10] to stand for it:

![Figure 1. Equivalent circuit model of ACR](image)

Where \( \dot{V} \) is the voltage of system; \( r_3 \), \( x_A \), and \( x_m \) are the stator resistance, stator reactance and magnetizing reactance; \( r_1 \), \( x_1 \), \( r_2 \) and \( x_2 \) are the lower-cage rotor resistance, mutual leakage reactance, upper-cage rotor resistance and upper-cage rotor reactance(in steady operation state, lower-cage reactance neglected is too small); \( s \) is the slip ratio of the induction motor; \( K_H \) is conversion capacity ratio, that is the ratio of system capacity reference to the induction motor reference as follows (1):

\[
K_H = S_B \cdot S_{sys}^{-1}
\]

The ACR absorbing power from the distribution network can be expressed as follows [11]:

\[
R_m(s) + jX_m(s) = K_H(r_3 + jx_A) + jK_H x_m / [jK_H x_1 + (K_H r_1 \cdot s + jK_H x_2)]
\]

(2)

\[
P_m + jQ_m = V^2 \cdot (R_m - jX_m)^{-1} = V^2 (R_m + jX_m) \cdot (R_m^2 + X_m^2)^{-1}
\]

(3)

Where \( R_m + jX_m \) is the ACR’s equivalent circuit impedance of induction motor connecting to the distribution network, which is the function of slip ratio \( s \); \( P_m + jQ_m \) is the power absorbing from the distribution network.

From (1) ~ (3), we need to calculate the value of slip ratio to get the absorbing power. From the torque balance equation expressed in (4), we can get the slip ratio[12]. Combined with (1)~(3), then the absorbing power can be get. In (4), \( P_m \), \( P_c \) and \( K_L \) are mechanical power, electromagnetic
power and load rate. For household ACRs, mechanical power $P_m$ is often expressed as the cubic function of slip ratio as (5).

$$P_m - P_e = 0$$  
$$P_m = K_L[(1 - s)^3] \cdot K_r^{-1}$$  

We need the electromagnetic power $P_e$ to get the slip value since we can get the $P_m$ by (5). When the induction motor is in steady working condition, electromagnetic power of two cages can be converted into absorbing electromagnetic power of rotor resistances. Next, we introduce how to get the electromagnetic power. Firstly, we get Thevenin equivalent circuit values on the left of the dash lines as (6)–(7) expressed. $\hat{E}_{eq}$ expresses electromotive force and $R_{eq} + jX_{eq}$ expresses equivalent impedance. Based on electric circuit theory, voltage of $\hat{V}'$ in Figure 1 can be expressed as (8).

$$R_{eq} + jX_{eq} = K_H(r_A + jx_A)/jK_Hx_m$$  
$$\hat{E}_{eq} = (R_{eq} + jX_{eq})(K_H(r_A + jx_A))^{-1} \cdot \hat{V}$$  

$$\hat{V}' = \frac{\hat{E}_{eq}[K_H \frac{r_H}{s} / K_H(\frac{r_2}{s} + jx_2) + jK_Hx_1 + (R_{eq} + jX_{eq})]}{K_H \frac{r_H}{s} / K_H(\frac{r_2}{s} + jx_2)}$$  

Then electromagnetic power absorbed by the lower-cage rotor and the upper-cage rotor can be expressed as (9). So the total electromagnetic power absorbed by rotor resistance is follows as (10). In total, as long as the node voltage connected to the ACR is already known, we can get slip ratio by (4)–(10). Since slip ratio can be known, we can get the ACR power absorbing from distribution network using (1)–(3).

$$\begin{aligned}
\begin{cases}
P_{el} = V^{12} \cdot (K_Hr_1)^{-1} \\
P_{e2} = V^{12} \cdot K_Hr_2 \cdot ([K_Hr_2 \cdot s^{-1}]^2 + [K_Hx_2]^2)^{-1} \cdot (s)^{-1}
\end{cases}
\end{aligned}$$  

$$P_e = P_{el} + P_{e2}$$  

B. start-up operation state

When ACR starts, the frequency of rotor is so great that lower-cage rotor has more leakage reactance than upper-cage rotor. Therefore, rotor current is mainly concentrated on upper-cage rotor and we can still ignore the lower-cage rotor reactance. Motor equivalent circuit still can use Figure 1. When ACR starts, slip ratio can use approximate value '1' to express. In (11), (12), equivalent resistant and absorbing power of the load can be calculated.

$$R_{im} + jX_{im} = K_H(r_A + jx_A) + jK_Hx_m / (jK_Hx_1 + K_Hx_2 + jK_Hx_2)$$  

$$P_{im} + jQ_{im} = V^n(R_{im} + jX_{im}) \cdot (R_{im}^2 + X_{im}^2)^{-1}$$  

2.1.2. Model of the statistic load. The cubic model is used for the static load represented by lighting as follows:

$$\begin{aligned}
\begin{cases}
P_s = P_s' \cdot V \cdot V^{-1} \cdot s^{33} \\
Q_s = Q_s' \cdot V \cdot V^{-1} \cdot s^{23}
\end{cases}
\end{aligned}$$  

where $P_s$ and $Q_s$ are static load’s active power and reactive power connected to the network; $V$ is node voltage amplitude; $V_s$ is rated voltage; $P_s$ and $Q_s$ are rated active power and reactive power.
2.1.3 Model of total resident load. In the low-voltage distribution network, the total load is the sum of
dynamic load and statistic as (14) expressed. In (14), P_L and Q_L are total active power and reactive power.
\[
\begin{align*}
    P_L &= P_{im} + P_s \\
    Q_L &= Q_{im} + Q_s
\end{align*}
\]  

(14)

2.2. Three phase power flow calculation model
Assuming that the low-voltage distribution network has n nodes and distribution transformer’s node
number is n. Distribution transformer is chosen as balance node. The three phrase voltage amplitude
and phase are determined. We build three phase power flow calculation model of load node as Figure 2
shows, in which phase A,B,C load nodes use power balance equations, the phase 0 uses KCL equation.
Three-phase model can be expressed as (15):
\[
\begin{align*}
    &P_{iA/B/C} + JQ_{iA/B/C} = (V_{iA/B/C} - \hat{V}_{i,0}) \sum_{j=1}^{n} [(V_{jA/B/C} - \hat{V}_{j,0})(Z_{ij})]^{-1} \\
    &\sum_{j=1}^{n} (\hat{V}_{i,0} - \hat{V}_{i,p})(Z_{ij})^{-1} + \sum_{p=0,B,C} \sum_{j=1}^{n} (\hat{V}_{i,p} - \hat{V}_{j,p})(Z_{ij})^{-1} = 0
\end{align*}
\]  

(15)

where \(P_{i,p}\) and \(Q_{i,p}\) are node i, phase p active power and reactive power absorbing from power grid;
\(\hat{V}_{i,p}\) is the phase p voltage of node i; \(\hat{V}_{j,p}\) is node j, the phase p voltage; \(Z_{ij}\) is phase A,B,C impedance
between node i and node j (we set three phases have the same impedance in here); \(Z_{ii,0}\) is phase 0
impedance between node i and j; n is the sum of the node.

Figure 2. Three-phase line model

In the three phase power flow calculation model, the node of transformer’s low side has to consider
the transformer ratio. So model is different from the mode of (15), as Figure 3 shows. In the transformer’s low side, the three-phase and the phase 0 all use KCL equation as follows:
\[
\begin{align*}
    \hat{k}(V_{iA/B/C} - k\hat{V}_{iA/B/C})(Z_{ii})^{-1} &= \sum_{j=1}^{n} [(V_{jA/B/C} - \hat{V}_{j,0})(Z_{ij})]^{-1} \\
    \sum_{j=1}^{n} (\hat{V}_{i,0} - \hat{V}_{i,0})(Z_{ij})^{-1} &= \hat{V}_{i,0} \cdot Z_{ii}^{-1} + \sum_{p=0,B,C} \sum_{j=1}^{n} [\hat{k}(V_{i,p} - k\hat{V}_{j,0})(Z_{ij})]^{-1} = 0
\end{align*}
\]  

(16)

where \(\hat{k} = ke^{j\theta}\) is the transformer ratio of high voltage side to low voltage side, \(\theta\) is voltage phase shift
angle of transformer high voltage to low voltage; \(\hat{V}_{i,p}\) is the phase p voltage of transformer high side;
\(\hat{V}_{i,0}\) is the phase p voltage of transformer low side; \(Z_{ii}\) is equivalent impedance of transformer; \(Z_{ij}\) is phase A,B,C impedance between transformer low side node i and line node j; \(Z_{ii}\) is ground impedance
of transformer low side.

Through (15) to (16), we can get 8(n-1) equations by separating real and imaginary parts. We set
the node n as slack bus. Except slack bus, each node has 8 variables including phase A,B,C,0 voltage
amplitude and phase angle in node 1~node (n-1). In total, there are 8(n-1) variables. Therefore, we
can calculate by Newton-Raphson algorithm to solve (15)–(16) and get value of variables.
3. Alternating iterative three phase power flow algorithm
In this algorithm, firstly, we can get absorbing power of load nodes by (1) ~ (14). Then, each node voltage can be calculated by (15) and (16). Lastly, through repeated alternated iteration, we get the result until the load power equation and voltage equation convergence. Steps are shown as follows:
Step1: Getting operational data and circuit wiring diagram of resident low-voltage distribution area;
Step2: Set the initial voltage. Using Newton-Raphson algorithm, calculate the slip ratio and power of rotor in steady operation state by (1) ~ (10) or only power in start-up operation state by (11) ~ (12). Then calculate the power of static load by (13). Finally get the total power of every node by (14);
Step3: Using Newton-Raphson algorithm, calculate voltage amplitude and phase of a,b,c,0 phase;
Step4: Back to the step3 and step4, calculate by repeated alternated iteration until power flow convergence. Get the power and voltage of each node;
Step5: Calculate line loss, load rate, and unbalance degree to reflect the area operation condition.

4. Case study
In order to verify the accuracy of three-phase unbalanced power flow algorithm considering resident distribution network’s load characteristics, we select 7 nodes actual low-voltage distribution network to test. Connection layout of the distribution network is shown as Figure 4.
As Figure 4 shows, distribution transformer capacity is \( S_{\text{v}} = 200 \text{kVA} \); its ratio is \( 10/0.4 \); it uses Dyn11 connection. According to the actual distribution network data, phase 0 distribution line parameters are 1.5 times than other phase. In node 1~5, the dynamic load and cubic static load in different phase are connected to each node. In the same node, capacity ratio of dynamic load to static load is 60% to 40%. Power in the rated condition are shown in Table 1. We adopt the ACR load model of in section 2, and its relevant parameters of ACR are given as Table 2.

![Figure 4. Connection layout of the distribution network](image)

**Table 1.** The dynamic load and statistic load phase of each node

| Node | Phase | ACR load | Statistic load |
|------|-------|----------|----------------|
|      |       | Rated active power (kW) | Rated power capacity (kVA) | Rated active power (kW) | Rated reactive power (kVar) |
| 1    | B     | 8.50     | 10.00          | 6.00             | 2.91             |
| 2    | C     | 27.20    | 32.00          | 19.20            | 9.30             |
| 3    | B     | 12.75    | 15.00          | 9.00             | 4.36             |
| 4    | A     | 6.80     | 8.00           | 4.80             | 2.32             |
| 5    | A     | 4.25     | 5              | 3.00             | 1.45             |

**Table 2.** Parameters of ACR equivalent model

| \( r_A \) | \( x_A \) | \( r_1 \) | \( x_1 \) | \( r_2 \) | \( x_2 \) | \( x_m \) | \( K_1 \) |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.025     | 0.08      | 0.028     | 0.04      | 0.07      | 0.03      | 5.00      | 0.80      |

To analyse three-phase unbalanced power flow calculation in different operation states, we choose following three kinds states respectively:
1) ACRs in all nodes are in stable operation state;
2) ACR start in node 5, the others does not change;
3) ACRs start in node 4 and node 5 at the same time, the others does not change.

By alternating iterative calculation, we can calculate the node voltage amplitude as Figure 5 ~ Figure 7 shows. Phase A voltage amplitude is smaller in state 2) and 3). In addition, phase A voltage amplitude of nodes in state 3) is smaller than state 2). It illustrates that voltage amplitudes reduce.
when more ACRs start. Phase B and phase C voltage amplitudes don’t have great influences. In order to illustrate different working states have influences on the slip ratio of other nodes, node 1 ~ node 3 are chosen. Effects are shown in Figure 8. Air conditioning have a small influence on slip ratio of node1~node3 in figure, since that they are in the other feeder compared to node 4 and node 5. Slip ratio of node 1 is greater than node 2, 3 in any operation state. We can get electricity law that if the node is farther away from the head of feeder line, the slip ratio is greater. Compared with the operation state 1), slip ratio of node 1 and node 2 is greater in operation state 2) and 3). It shows the speed of electric motor reduces following with states. What’s more, slip ratio of nodes in condition 3) is greater than condition 2). In a word, when the ACR starts, it leads to other ACRs speed reducing. If more ACRs start, the speed will reduce greater.

At the same time, the result is shown about low-voltage area in different conditions as Table 3 follows. When working operation is changing from state 1) to state 3), line loss, loading rate and phase A loading rate are becoming greater. when ACRs start, power of a certain phase is much greater and leads to that loading rate of this phase increase and then line loss increase significantly.

By Symmetrical Component, asymmetric three-phase voltage is decomposed into symmetric voltage including positive sequence, negative sequence and zero sequence[13]. Then we calculate the zero sequence (ZS) and negative sequence (NS) unbalance degree percentage through method in national standard (GB/T 15543-2008) [14] as Table 4 shows. The Chinese standards of power quality set 2% as upper limit. When the air conditioning load of each node is in a stable state, imbalance is not exceeded. However, node 4 and node 5 voltage unbalance degree exceeded the maximum (2%) if there are star-up conditioners. This situation is bad for the operation of power grid and has harmful effects to voltage quality in power supply.

**Table 4. Voltage unbalance degree in different operation states**

| Conditions       | Line loss/kW | Three-phase load rate of distribution transformer/% | A-phase load rate of distribution transformer/% |
|------------------|--------------|---------------------------------------------------|-------------------------------------------------|
| Operation state 1 | 5.0142       | 32.9888                                           | 15.8875                                         |
| Operation state 2 | 13.0406      | 47.0588                                           | 59.1826                                         |
| Operation state 3 | 34.0467      | 69.6250                                           | 126.9508                                        |
5. Summary
This paper focuses on load characteristics and builds three-phase power flow calculation model based on power balance equations, the current balance equations, and the torque balancing equations of induction motors. Then the paper proposes alternating iterative algorithm including each node balance equations and induction motor torque balance equations. By case study which changes air conditioner operating states, we get conclusions as follows:

1) The load model in which dynamic load represented by air conditioners parallel with static load represented by lighting loads can describe the load characteristics accurately and reflect actual working conditions of distribution network.

2) The alternating iteration algorithm of three-phase power flow has a good convergence and it is easy to program;

3) Compared with the air conditioning load stable working state, the line loss of the distribution, three-phase load rate and the set phase load rate increase significantly when there is air conditioner starts;

4) Nodes voltage have greater unbalance degrees if air conditioners start up, which may cause some nodes voltage imbalance degrees exceed upper limits of national standards. The situation is harmful to residential users.

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