On perturbative unitarity in the extended MSSM Higgs sector

M N Dubinin and E Yu Fedotova
Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 119991, Leninskie Gory, 1, Moscow, Russian Federation
E-mail: fedotova@theory.sinp.msu.ru

Abstract. Higgs sector of the minimal supersymmetric standard model (MSSM) extended by dimension-six operators, which are loop contributions in the expansion of the Coleman-Weinberg type potential, is considered. The presence of such additional contributions allows the reopening of phenomenological MSSM scenarios closed in the previous analysis. In order to restrict respective MSSM parameter regions, perturbative unitarity constraints must be satisfied. We find the analytical formula for quartic and trilinear couplings for the Higgs potential extended by dimension-six operators, compare results with the loop corrected constraints at high or finite \(\sqrt{s}\) with and without additional \(U^{(6)}\)-contributions, and show how the allowed regions in the parameter space are affected in these cases.

1. Introduction

These days, the Standard Model (SM) of particle physics works extremely well. A Higgs boson – the last critical SM prediction – was discovered at the LHC in 2012 [1]. However, due to known problems (SM RC fine-tuning, sources of CP-violation, dark matter candidate, etc.), the SM is considered as an effective theory at low-energy. An elegant solution to these perceived problems is provided by supersymmetry (SUSY). In this work, the most popular and well investigated SUSY model – a minimal supersymmetric standard model (MSSM) is considered. Its Higgs sector contains additional Higgs doublet \(\Phi_i^T = (-\omega_i^T, \frac{1}{\sqrt{2}} (v_i + \eta_i + i\chi_i), i = 1, 2\), where \(v_1 = v \cos \beta, v_2 = v \sin \beta, v^2 = v_1^2 + v_2^2 = 246^2\) GeV².

As soon as SUSY-particles are not observed, one can conclude that SUSY is strongly broken and the low-energy effective theory of the MSSM is a Two-Higgs-doublet model (2HDM). As any effective potential, 2HDM potential at the loop level can be written as [2]

\[
U(\Phi_1, \Phi_2) = U^{(2)} + U^{(4)} + U^{(6)} + U^{(8)} + \ldots,
\]

where

\[
U^{(2)} = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - [\mu_{12}^2(\Phi_1^\dagger \Phi_2) + h.c.],
\]

\[
U^{(4)} = \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_5/2(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + h.c.,
\]

\[
U^{(6)} = \kappa_1(\Phi_1^\dagger \Phi_1)^3 + \kappa_2(\Phi_2^\dagger \Phi_2)^3 + \kappa_3(\Phi_1^\dagger \Phi_1)^2(\Phi_2^\dagger \Phi_2) + \kappa_4(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)^2
\]
are the centre of mass three-momenta and the energies of the particles in the center of mass.

So the unitarity constraint is

$$\text{trilinear couplings of scalars, Mandelstam variable, } \delta$$

well-known unitarity constraint of the partial wave amplitude is

$$\text{arise (i) what do the perturbative unitarity conditions look like? (ii) what values of model parameters are allowed by perturbative unitarity?}$$

2. Perturbative unitarity constraints

We will study the unitarity constraints by computing the scalar scattering processes \(S_1S_2 \rightarrow S_3S_4\). The amplitude of this process is

$$M(s, t, u) = 16\pi \sum_{J=0}^{\infty} (2J+1)P_J(\cos \theta)a_J(s),$$

where \(s, t, u\) are Mandelstam variables, \(P_J\) are Legendre polynomials. The cross section in the massless limit

$$d\sigma/d\Omega = |M|^2/64\pi^2s, \quad \sigma = \frac{16\pi}{s} \sum_{J=0}^{\infty} (2J+1)|a_J(s)|^2$$

is proportional to the imaginary part of the amplitude in the forward direction (optical theorem)

$$\sigma = \frac{1}{s} \text{Im}[M(\theta = 0)].$$

So the unitarity constraint is \(|a_J|^2 = \text{Im}(a_J)\) for all \(J\). From \(\text{Re}(a_J)^2 + \text{Im}(a_J)^2 = |a_J|^2\) the well-known unitarity constraint of the partial wave amplitude is

$$|a_J|^2 \leq \frac{1}{2}.$$ 

A zeroth partial wave can be presented as (see e.g. [1])

$$a_0 = -\frac{2^{-1/2}(\delta_{12}+\delta_{64})}{16\pi} \left[ \frac{\lambda(s, m^2, m^2)\lambda(s, m^2, m^2)}{s} \right]^{1/2} \frac{\kappa^{1234} + \kappa^{125} \kappa^{345} \frac{1}{s-m^2}}{f_t(s, m^2_{1,5}) - \kappa^{145} \kappa^{235} f_u(s, m^2_{1,5})},$$

where the factor \(\delta_{ij}\) is 1 if particles \(\{i, j\}\) are identical, and zero otherwise, \(s = (p_1 + p_2)^2\) is the Mandelstam variable, \(m_5\) is the particle mass in a propagator, \(\lambda^{1234}\) and \(\kappa^{ijk}\) are quartic and trilinear couplings of scalars,

$$f_t(s, m^2_{1,5}) \equiv \frac{1}{s} \left[ \frac{\lambda(s, m^2_1, m^2_2)\lambda(s, m^2_3, m^2_4)}{s} \right]^{1/2} \log \left( \frac{m^2_2 + m^2_3 - m^2_5 - 2E_1E_3 + 2|p_1||p_3|}{m^2_1 + m^2_5 - m^2_2 - 2E_1E_3 - 2|p_1||p_3|} \right),$$

$$f_u(s, m^2_{1,5}) \equiv f_t(s, m^2_1, m^2_2, m^2_3, m^2_5),$$

$$\lambda(s, m^2_1, m^2_3) \equiv \frac{1}{s^2} \left( s^2 + m^4_1 + m^4_3 - 2m^2_1m^2_3 - 2sm^2_1 - 2sm^2_3 \right),$$

$$|p_{1(3)}| = \frac{1}{2} \sqrt{s\lambda(s, m^2_{1(3)}, m^2_{2(4)})}, \quad E_{1(3)} = \frac{s + m^2_{1(3)} - m^2_{2(4)}}{2\sqrt{s}}.$$ 

are the centre of mass three-momenta and the energies of the particles in the center of mass frame.
2.1. High-energy limit
A common approach of perturbative unitarity analysis is to consider a two-particle scattering matrix of scalars in the large center-of-mass energy limit. According to Lee-Quigg-Thacker theorem [3], in the high-energy limit ($\sqrt{s} \gg m_5$) the amplitudes for two-body scattering processes are equivalent to those with longitudinal gauge bosons up to terms of the order of $O(m_5^2/\sqrt{s})$. Thanks to Goldstone-boson equivalence theorem, external longitudinal gauge bosons can be replaced by the corresponding Goldstone modes. In other words, only point-interactions contribute to the Goldstone-Higgs-boson system, so partial-wave amplitudes depend only on self-couplings $\lambda_i$. The zeroth partial wave amplitude can be presented as [6]

$$a^2_{\text{HDM}} = \frac{1}{16\pi} \text{Diag}(X_{4\times4}, Y_{4\times4}, Z_{3\times3}, Z_{3\times3}),$$

where matrices $X_{4\times4}, Y_{4\times4}, Z_{3\times3}, Z_{3\times3}$ are defined by $\lambda_i$.

2.2. Finite energy
However, due to SUSY-particles interactions in the MSSM, trilinear couplings can be significant and large contributions can be present at smaller $\sqrt{s}$. So in the general case, one should calculate $a_0$ using the full expression specified by equation (9). The analytical formulae for trilinear and quartic couplings for the extended Higgs potential derived by the authors are rather cumbersome. The full list of expressions will be in an open access in future publication. An example of the coupling $\kappa_{hhh}$ is

$$\kappa_{hhh} = c_1v + c_2v^2,$$

where $(\lambda_{345} = \lambda_3 + \lambda_4 + \text{Re}\lambda_5)$

$$c_1 = -\lambda_1 s_\alpha c_\beta + \lambda_2 c_\alpha s_\beta - \frac{\lambda_{345}}{4} s_\alpha c_{\alpha+\beta} + \frac{\text{Re}\lambda_6}{2} s_\alpha (c_{\beta-\alpha} + 2c_{\beta+\alpha}) + \frac{\text{Re}\lambda_7}{2} c_\alpha (c_\alpha c_\beta - 3s_\alpha s_\beta),$$

$$c_2 = \frac{5}{2} \left[-\frac{\text{Re}\kappa_8 s_\alpha c_\beta^2 + \text{Re}\kappa_9 c_\alpha^2 s_\beta^2}{c_\alpha} + \left(\text{Re}\kappa_8 s_\alpha c_\beta^2 + \text{Re}\kappa_9 c_\alpha^2 s_\beta^2\right)c_{\alpha+\beta}\right] + \frac{1}{16}\left(|\kappa_3 + \kappa_5 + 2\text{Re}\kappa_9|s_\alpha c_\beta - (\kappa_4 + \kappa_6 + 2\text{Re}\kappa_9) c_\alpha s_\beta\right)\left(c_{\beta(\beta-\alpha) - 5c_2(a+\beta) - 4} + \frac{1}{32}\text{Re}(\kappa_7 + \kappa_11 + \kappa_13)\times [5c_3(a+\beta) - 3c_\beta - 3c_{3(\beta-\alpha) + c_3\beta - a - 3c_{\alpha+\beta}}].

3. Numerical analysis
Assume that only third generation of squarks ($\tilde{Q}, \tilde{U}, \tilde{D}$) are important, $M_{\tilde{Q}} \approx M_{\tilde{U}} \approx M_{\tilde{D}} \approx M_{\text{SUSY}}$. The SM-like Higgs boson is $h$ ($m_h=125$ GeV), so its couplings to SM particles are $g_{haa} \approx g_{had} \approx g_{hVV} \approx 1$ (alignment limit). In this approximation the free parameters are $m_A$ (a mass of CP-odd Higgs boson), $\tan\beta$ (the ratio of vacuum expectation values), $M_{\text{SUSY}}$ (the SUSY-breaking scale), $A_t, A_b$ (trilinear couplings to squarks), $\mu$ (a mass parameter of higgsino). They can be fixed using benchmark scenarios $m_h^{\text{max}}, m_h^{\text{mod}}, \text{light stop}, \text{light stau}, \tau-\text{phobic}$ [7] and low-$m_A$ [8] with varied parameters adjust in such a way that $m_h=125$ GeV in alignment limit. Note that additional contributions that come from dimension-six operators $U^{(6)}$ allow to open phenomenological MSSM scenarios with $m_A \sim 30 - 100$ GeV [8]. The calculation was performed within EFT-approach where 1- and 2-loop corrections to $\lambda_i$ [9,10] and 1-loop threshold corrections to $\kappa_i$ [8] are taken into account. The corresponding benchmark points (BPs) are presented in table 1 [11]. As one can see dimension-six operators $U^{(6)}$ change results insignificantly except the BP8 ones.
Table 1. Benchmark points for $m_h=125$ GeV in alignment limit. Here $m_t=173.2$ GeV, $m_b=4.2$ GeV, $m_Z=91.1876$ GeV, $m_W=80.385$ GeV, $\alpha_s(m_t)=0.118$, $G_F=1.16639 \times 10^{-5}$ GeV$^{-2}$; $\cos\theta_W = m_W/m_Z$, $g_2 = 8m_Z^2G_F/\sqrt{2}$, $g_2 = g \cos \theta_W$, $g_1 = g_2 \tan \theta_W$, $v^2 = 1/\sqrt{2}G_F$.

| Scenario | BP     | $M_{SUSY}$, GeV | $\mu$, GeV | $A_{t,b}$, GeV | $\tan \beta$ | $m_A$, GeV |
|----------|--------|-----------------|------------|---------------|--------------|------------|
| $m_h^{\text{max}}$ | BP1    | 1000            | 200        | 2482          | 6            | 3000       |
| $m_h^{\text{mod+}}$ | BP2    | 1000            | 200        | 1609          | 22           | 2500       |
| $m_h^{\text{mod-}}$ | BP3    | 1000            | 200        | -2167         | 6            | 3000       |
| light stop | BP4    | 500             | 350        | 1122          | 16           | 1500       |
| light stau | BP5    | 1000            | 500        | 1742          | 12           | 2000       |
| $\tau$-phobic | BP6    | 1500            | 2000       | 4635          | 7            | 3000       |
| low-$m_A$ | BP7$^{(4)}$ | 2000     | 6250       | 9000          | 2            | 28         |
|          | BP7$^{(6)}$ | 2000     | 5950       | 9000          | 2            | 28         |
|          | BP8    | 1000            | 9000       | 2350          | 5            | 90         |

Table 2. Max$|a_0|$ in high-energy limit and at finite energy $\sqrt{s}=13$ TeV.

| Approx. $\sqrt{s}$ & oper. | BP1 | BP2 | BP3 | BP4 | BP5 | BP6 | BP7$^{(4,6)}$ | BP8 |
|-------------------|-----|-----|-----|-----|-----|-----|--------------|-----|
| $\sqrt{s} \gg m_5$ | $U^{(4)}$ | 0.8526 | 0.7808 | 0.8496 | 0.7924 | 0.8003 | 0.8298 | 3.8364 | 48.6415 |
| $\sqrt{s} \sim m_5$ | $U^{(4)}$ | 0.0265 | 0.0314 | 0.0263 | 0.0316 | 0.0307 | 0.0267 | 0.0017 | 0.1060 |
| $\sqrt{s} \sim m_5$ | $U^{(6)}$ | 0.0265 | 0.0314 | 0.0263 | 0.0316 | 0.0307 | 0.0261 | 0.0017 | 0.3432 |

The numerical results of partial wave amplitude for BP1–BP8 are summarized in table 2. One can see that (i) the corresponding values differ significantly at high and finite energy, (ii) at $\sqrt{s} \sim m_5$ the results are more accurate and all BPs are appropriate.

4. Conclusions

In this paper, the perturbative unitarity constraints were considered for the MSSM Higgs sector extended by dimension-six operators. Corresponding analytical expressions for $s$-wave amplitudes of scattering processes in the Higgs sector were obtained. Numerical analysis performed for several benchmark MSSM scenarios demonstrates that the trilinear couplings can be significant and large contributions appear even at smaller $\sqrt{s}$. It was found that the perturbative unitarity results insignificantly change for the model regime $A_{t,b,\mu} \sim 1$ TeV and become stringent for $A_{t,b,\mu} \sim 5$ – 9 TeV. As a rule, the values of $A_{t,b,\mu}$ are acceptable up to 10 TeV, when a respective soft SUSY breaking terms are in the ”strong coupling” regime.

Acknowledgments

This work was supported by the Russian Science Foundation Grant No. 16-12-10280.

References

[1] Aad G et al. (ATLAS and CMS Collaborations) 2016 J. High Energy Phys. 1608 045
[2] Coleman S and Weinberg E 1973 Phys. Rev. D 7 1888
[3] Dubinin M N and Petrova E Yu 2017 Phys. Rev. D 95 055021 (Preprint 1612.03655)
[4] Goodsell M D and Staub F 2018 Eur. Phys. J. C 78 649
[5] Lee B W, Quigg C and Thacker H B 1977 Phys. Rev. D 16 1519
[6] Kamemura S and Yagyu K 2015 Phys. Lett. B 751 289-96
[7] Carena M, Heinemeyer S, Stal O, Wagner C E M and Weiglein G 2013 Eur. Phys. J. C 73 2552
[8] Dubinin M N and Petrova E Yu 2018 Int. J. Mod. Phys. A 33 1850150
[9] Akhmetzhanova E, Dolgopolov M and Dubinin M 2005 Phys. Rev. D 71 075008 (Preprint hep-ph/0405264)
[10] Carena M, Espinosa J R, Quiros M and Wagner C E M 1995 Phys. Lett. B 355 209 (Preprint hep-ph/9504316)