Interface effects of strange quark matter

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QCD Phase Diagram [McLerran2009_NPB195-275]
Strange quark matter (SQM)

Bodmer first suggested a low energy nuclear state called “collapsed nuclei” [Bodmer1971_PRD4-1601]; Witten reported on the stability of SQM consisting of approximately equal numbers of u, d and s quarks, suggesting that SQM could indeed be stable even at zero external pressure [Witten1984_PRD30-272].

If Witten-Bodmer hypothesis is true, there exists stable lumps of SQM with the baryon number $A \approx 2 \sim 10^{57}$:

**Strangelets (A<10^7)**
Comparing with nuclei, strangelets have: lower charge-to-mass ratio; larger mass; smaller radius; spherical shape; ...

**Nuclearites**
[Rujula_Glashow1984_Nature312-734]; Meteorlike Compact Ultradense Objects (CUDO) [Rafelski_Labun_Birrell2013_PRL110-111102]; ...

**Strange stars (A ≈ 10^{57})**
Comparing with traditional neutron stars, strange stars have: no crust; different mass-radius relations; smaller radii; higher rotational frequencies; ...
Energy per baryon

The shaded region correspond to the results obtained with $B^{1/4} = 152 \pm 7$ MeV. [Xia_Peng_Zhao_Zhou2016_PRD93-085025]

Berger_Jaffe1987_PRC35-213:

$$A_{\text{min}}^\text{meta} = \left( \frac{2c_{\text{surf}}}{3(m_n - \epsilon_0)} \right)^3$$

$$c_{\text{surf}} \equiv 4\pi\sigma R^2 / A^{2/3} \equiv 4\pi\sigma \rho^2$$

$R \equiv \rho A^{1/3}$

The shaded region correspond to the results obtained with $B^{1/4} = 152 \pm 7$ MeV. [Xia_Peng_Zhao_Zhou2016_PRD93-085025]
Energy excess per baryon

The shaded region corresponds to the results obtained with $B_1^{1/4} = 152 \pm \text{7 MeV}$. 

[Xia_Peng_Zhao_Zhou2016_PRD93-085025] 
[Alford_Han_Reddy2012_JPG39-065201]
Unstable SQM

\[ E/N \text{ [MeV]} \]

Tidal deformability

\[ \sigma = 1 \text{ MeV/fm}^2 \]  
\[ \sigma = 5 \text{ MeV/fm}^2 \]  
\[ \sigma = 10 \text{ MeV/fm}^2 \]  
\[ \sigma = 20 \text{ MeV/fm}^2 \]  
\[ \sigma = 30 \text{ MeV/fm}^2 \]  
\[ \sigma = 50 \text{ MeV/fm}^2 \]

Mass \( (M_\odot) \)

Gibbs

Maxwell

GW170817
Estimations of surface tension

**Lattice QCD**: Huang, Potvin, Rebbi, Sanielevici, Alves, Brower, de Forcrand, Lucini, Vettorazzo, et al.

For **vanishing** chemical potentials!

**Effective models:**
- **Linear sigma model** [Palhares_Fraga2010_PRD82-125018, Pinto_Koch_Randrup2012_PRC86-025203, Kroff_Fraga2015_PRD91-025017], **Nambu-Jona-Lasinio (NJL) model** [Garcia_Pinto2013_PRC88-025207, Ke_Liu2014_PRD89-074041], **three-flavor Polyakov-quark-meson model** [Mintz_Stiele_Ramos_Schaffner-Bielich2013_PRD87-036004], and **Dyson-Schwinger equation approach** [Gao_Liu2016_PRD94-094030]

\[ \sigma = 5\sim30 \text{ MeV/fm}^2 \]

**Quasiparticle mode** [Wen_Li_Liang_Peng2010_PRC82-025809]

\[ \sigma = 30\sim70 \text{ MeV/fm}^2 \]

**NJL model** adopting the MRE method [Lugones_Grunfeld_Ajmi2013_PRC88-045803]

\[ \sigma = 145\sim165 \text{ MeV/fm}^2 \]

For **color-flavor locked SQM**, dimensional analysis suggests:

\[ \sigma \approx 300 \text{ MeV/fm}^2 \]

**For magnetized SQM**, \( \sigma \) has a different value in the parallel and transverse directions with respect to the magnetic field [Lugones_Grunfeld2017_PRC95-015804]
Equivparticle model

The Lagrangian density with quark mass scaling [Xia_Peng_Chen_Lu_Xu2014_PRD89-105027]

The confinement parameter is connected to the string tension $\sigma_0$, the chiral restoration density $\rho^*$, and the sum of the vacuum chiral condensates [Peng_Chiang_Yang_Li_Liu1999_PRC61-015201]. The perturbative strength parameter $C$ is linked to the strong coupling constant $\alpha_s$.

5. The one-gluon-exchange interaction was further included by Chen et al.: $m_T = \frac{D}{m^{1/3}} - Cn^{1/3}$; [Chen_Gao_Peng2012_CPC36-947]

6. The quark matter symmetry energy was considered by Chu and Chen: $m_T = \frac{D}{m^{1/3}} - \tau\delta D_1n^{\alpha}e^{-\beta n}$; [Chu-Chen2014_ApJ780-135]
Strangelets in MFA

For spherically symmetric strangelets, the Dirac spinor of quarks is

\[ \psi_{n\kappa m}(\mathbf{r}) = \frac{1}{r} \left( \begin{array}{c} iG_{n\kappa}(r) \\ F_{n\kappa}(r) \sigma \cdot \hat{r} \end{array} \right) Y_{jm}^{l}(\theta, \phi) \]

radial wave functions spinor spherical harmonics

\[ \kappa = (-1)^{j+l+1/2}(j + 1/2) \]

Dirac equation

\[
\begin{pmatrix}
V_i + V_S & -\frac{d}{dr} + \frac{\kappa}{r} \\
\frac{d}{dr} + \frac{\kappa}{r} & V_i - V_S - 2m_{i0}
\end{pmatrix}
\begin{pmatrix}
G_{n\kappa} \\
F_{n\kappa}
\end{pmatrix}
= \varepsilon_{n\kappa}
\begin{pmatrix}
G_{n\kappa} \\
F_{n\kappa}
\end{pmatrix}
\]

Mean field scalar and vector potentials

\[ V_S = m_I(n_b), \]
\[ V_i = \frac{1}{3} \frac{dm_I}{dn_b} \sum_{i=u,d,s} n_i^s + e q_i A_0. \]
Density profiles

\[ n_0 = 0.099 \text{ fm}^{-3} \]
\[ n_d = 0.19 \text{ fm}^{-3} \]
\[ n_u = 0.099 \text{ fm}^{-3} \]
\[ n_s = 0.0055 \text{ fm}^{-3} \]

Oertel_Urban2008_PRD77-074015
Surface structures

\[ n_0 = 0.11, 0.099, 0.13 \text{ fm}^{-3} \]
Energy per baryon

The multiple reflection expansion (MRE) method: The average effects due to quark depletion are treated with a modification to the density of states, i.e.,

\[ N'_i(p) = 6 \left[ \frac{p^2 v}{2\pi^2} + f_s \left( \frac{p}{m_i} \right) ps + f_c \left( \frac{p}{m_i} \right) c \right], \]

\[ f_s(x) = -\frac{\eta_s}{4\pi^2} \arctan \left( \frac{1}{x} \right), \]

\[ f_c(x) = \frac{\eta_c}{12\pi^2} \left[ 1 - \frac{3}{2} x \arctan \left( \frac{1}{x} \right) \right]. \]

[Madsen1994_PRD50-3328, ...]

\[ N_i = \int_0^{\nu_i} N'_i(p) dp \]

\[ M = \sum_{i=u,d,s} \int_0^{\nu_i} \sqrt{p^2 + m_i(n_b)^2} N'_i(p) dp + M_{ch} \]

Red solid: \( \eta_s = 1, \eta_c = 1. \)
Energy per baryon

Liquid-drop type formula:
\[
\frac{M}{A} = \frac{E_0}{n_0} + \frac{\alpha_S}{A^{1/3}} + \frac{\alpha_C}{A^{2/3}}
\]
\[
\sigma = \alpha_S \left( \frac{n_0^2}{36\pi} \right)^{1/3},
\]
\[
\lambda = \alpha_C \left( \frac{n_0}{384\pi^2} \right)^{1/3}.
\]

[Oertel_Urban2008_PRD77-074015]

\[
f_c(x) = \frac{\eta_c}{12\pi^2} \left[ 1 - \frac{3}{2} x \arctan \left( \frac{1}{x} \right) \right].
\]

[Madsen1994_PRD50-3328, ...]

| Parameters | Bulk properties | MFA | MRE method | \(\sigma_{\text{MFA}} / \sigma_{\text{MRE}}\) |
|------------|----------------|-----|-------------|---------------------|
| C          | \(\sqrt{D}\)   | \(n_0\) | \(E_0 / n_0\) | \(f_S\) | \(\alpha_S\) | \(\alpha_C\) | \(\sigma\) | \(\lambda\) | \(\alpha_S\) | \(\alpha_C\) | \(\sigma\) | \(\lambda\) | \(\sigma_{\text{MFA}} / \sigma_{\text{MRE}}\) |
| 0.4        | 129            | 0.11 | 850.91     | 0.20 | 56 | 177 | 2.7 | 5.49 | 190.5 | 86.1 | 9.247 | 2.67 | 0.29 |
| 0.7        | 129            | 0.099 | 918.94     | 0.056 | 54 | 172 | 2.4 | 5.12 | 173.5 | 85.7 | 7.681 | 2.54 | 0.31 |
| 0.7        | 140            | 0.13 | 995.77     | 0.14 | 61 | 185 | 3.3 | 6.03 | 191.1 | 90.9 | 10.18 | 2.96 | 0.32 |
Energy per baryon

The multiple reflection expansion (MRE) method: The average effects due to quark depletion are treated with a modification to the density of states, i.e.,

\[
N_i'(p) = 6 \left[ \frac{p^2 v}{2\pi^2} + f_s \left( \frac{p}{m_i} \right) ps + f_c \left( \frac{p}{m_i} \right) c \right],
\]

\[
f_s(x) = -\frac{\eta_s}{4\pi^2} \arctan \left( \frac{1}{x} \right),
\]

\[
f_c(x) = \frac{\eta_c}{12\pi^2} \left[ 1 - \frac{3}{2} x \arctan \left( \frac{1}{x} \right) \right].
\]

[\text{Madsen1994_PRD50-3328, ...}]

\[
N_i = \int_0^{\nu_i} N_i'(p) dp
\]

\[
M = \sum_{i=u,d,s} \int_0^{\nu_i} \sqrt{p^2 + m_i(n_b)^2} N_i'(p) dp + M_{ch}
\]

Red solid: \(\eta_s = 1, \eta_c = 1\).

Blue dashed: \(\eta_s = 0.3, \eta_c = 0.1\).
Charge-to-mass ratio and Strangeness per baryon
Ratio of root-mean-square radius to baryon number

\[ r_0 = \frac{\langle r^2 \rangle^{1/2}}{A^{1/3}} \text{ (fm)} \]

- \( C = 0.4, \sqrt{\bar{D}} = 129 \text{ MeV} \)
- \( C = 0.7, \sqrt{\bar{D}} = 129 \text{ MeV} \)
- \( C = 0.7, \sqrt{\bar{D}} = 140 \text{ MeV} \)
Summary

Based on an equivparticle model, we study the interface effects in strangelets adopting mean-field approximation (MFA). It is found that

1. the surface tension and curvature term of strange quark matter (SQM) become larger for larger confinement strength and smaller perturbative strength;
2. if SQM is absolutely stable and a strange star can reach 2 solar mass, the surface tension is \( \sim 2.4 \text{ MeV/fm}^2 \);
3. the MRE method overestimates the surface tension and underestimates the curvature term, which can be fixed be introducing proper damping factors; . . .

Thank you!!!
The single-particle levels for u-quarks