Singlet-doublet Majorana dark matter and neutrino mass in a minimal type-I seesaw scenario

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Abstract. In a bid to simultaneous explanation of dark matter (DM) and tiny but non-zero masses of left-handed neutrinos, we propose a minimal extension of the Standard Model (SM) by a vector-like fermion doublet and three right handed (RH) singlet neutrinos. The DM arises as a mixture of the neutral component of the fermion doublet and one of the RH neutrinos, both assumed to be odd under an additional $\mathbb{Z}_2$ symmetry. As a result, the DM emerges to be a dominantly Majorana particle and escapes from $Z$-mediated direct search constraints to mark a significant difference from singlet-doublet Dirac DM. The other two $\mathbb{Z}_2$ even heavy RH neutrinos give rise masses and mixing of light neutrinos via Type-I Seesaw mechanism. The particle content automatically allows us to extend the model by a gauged $U(1)_{B-L}$ symmetry, which is anomaly free and brings an additional portal between DM and SM particles. Relic density and direct search allowed parameter space for both the cases are investigated through detailed numerical scan, while collider search strategies are also indicated.

Keywords: dark matter theory, dark matter simulations, neutrino theory

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1 Introduction

Astrophysical observations like galaxy rotation curves, gravitational lensing, Cosmic Microwave Background (CMB) acoustic fluctuations etc. provide compelling evidences towards the existence of Dark Matter (DM) [1, 2], a form of matter that is electromagnetically inert and hence extremely difficult to detect, but can be inferred from its gravitational affects. In fact, satellite borne experiments like WMAP and PLANCK [3, 4], which measure anisotropies in CMB, established that DM constitute almost 85% of the total matter content and 26.8% of the total energy budget of the universe. Even after this tantalising hint, we have no answer to the question what DM actually is. DM as a fundamental particle answers many puzzles together like structure formation, self interaction, rotation curve etc., hence studied elaborately. Since no SM particle resembles the properties of a DM particle expected to have, it is believed that DM is essentially one or more particles beyond the Standard Model (BSM) content. Several BSM scenarios have been formulated to explain the particle nature.
of the DM, with additional field content and stabilising symmetry. Amongst different class of possibilities, Weekly Interacting Massive Particles (WIMP) has been the most popular due to its phenomenological richness, where DM can be explained as the thermal relics of the universe [5].

Another equally important puzzle in particle physics is the tiny neutrino mass which has been established by the solar and atmospheric neutrino oscillation experiments like T2K [6, 7], Double Chooz [8, 9]), Daya Bay [10–12], Reno [13] and MINOS [14, 15]. Besides, the nature of neutrinos, whether Dirac or Majorana, is also not known. Neutrinoless double beta decay experiments [16] perhaps will shed light onto it. Within the SM, neutrinos are assumed massless with no right handed (RH) neutrinos. Even if RH neutrinos are incorporated to the SM, the required Yukawa coupling to explain sub-eV neutrino mass through spontaneous symmetry breaking via Dirac mass term turns out to be as tiny as $10^{-12}$, almost six orders of magnitude smaller than the electron Yukawa coupling and seems pretty unnatural. Assuming that the neutrinos are Majorana, which violates lepton number by two units, the tiny neutrino masses can be realised via the dimension five gauge invariant effective Weinberg operator $LLHH/\Lambda$, where $\Lambda$ denotes the scale of new physics and $L, H$ are respectively the lepton and Higgs doublets of the SM [17]. After electroweak symmetry breaking (EWSB), the SM neutrinos acquire sub-eV masses given by $M_\nu = \langle H \rangle^2/\Lambda$. One possibility of generating this operator is to assume the presence of heavy RH neutrinos in the early universe, where the scale of new physics ($\Lambda$) is decided by the mass of RH neutrinos. Thus it is straightforward to see that for tiny neutrino mass of the order of $M_\nu \sim 0.1\text{eV}$, the new physics scale requires to be very heavy ($\Lambda \sim 10^{14}\text{GeV}$) when the involved couplings are of order one. This is usually referred as type-I seesaw mechanism [18–21].

While the origin of DM and neutrino mass is hitherto unknown, it is highly appealing and economical to find a model having simultaneous solution of both. In fact, such models are expected to have constrained parameter space in comparison to their individual counterpart and hence can be probed at ongoing and future terrestrial experiments. Motivated by this, here we consider a simple extension of the SM to explain simultaneously the sub-eV masses of neutrinos and DM content of the universe.

We consider a singlet-doublet WIMP like fermion DM [22–46]. The motivation of considering a singlet-doublet fermion DM has already been established; this is because a purely singlet case requires a higher dimensional effective operator for DM-SM interaction, which is mostly ruled out from direct search bound excepting for the Higgs resonance region, while the pure doublet case is also ruled out from relic density and direct search bound up to several TeVs of DM mass making the model inaccessible to probe. Our model consists of a vector-like fermion doublet $\Psi^T = (\psi^0, \psi^-)$ and three right handed neutrinos $N_{R_i}, i = 1, 2, 3)$. A $Z_2$ symmetry is imposed under which the doublet $\Psi$ and one of the right handed neutrinos, say $N_{R_1}$, are odd, while other particles are even. As a result there is mixing between the neutral component of the doublet and the singlet through the Yukawa interaction and DM emerges out to be a mixed state of the doublet $\psi^0$ and $N_{R_1}$ after EWSB. Due to Majorana mass of the RH singlet $N_{R_1}$, the DM is dominantly a Majorana particle. As a result it escapes the $Z$-mediated vector current direct search interaction and provides a distinction from the earlier vector like singlet-doublet DM [40–45]. The field content permits us to extend the model easily to a gauged $U(1)_{B-L}$ scenario, which allows an additional gauge mediated interaction for DM. We find the relic density and direct search allowed parameter space for both the cases and also indicate possible collider search strategies. The neutrino mass arises from the Yukawa interaction of $Z_2$ even RH neutrinos together with Majorana mass term in a minimal
Fields & $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_2$
\hline
VLfd & $\Psi = \begin{pmatrix} \psi^0 \\ \psi^- \end{pmatrix}$ & 1 & 2 & -1 & - \\
RHNs & $N_{R_1}$ & 1 & 1 & 0 & - \\
 & $N_{R_2}$ & 1 & 1 & 0 & + \\
 & $N_{R_3}$ & 1 & 1 & 0 & + \\
Higgs doublet & $H = \begin{pmatrix} w^+ \\ h+i z \sqrt{2} \end{pmatrix}$ & 1 & 2 & 1 & + \\
\hline

Table 1: Charge assignment of BSM fields with SM Higgs doublet under the gauge group $G \equiv G_{SM} \otimes Z_2$ where $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

Type-I Seesaw framework. Since two RH neutrinos take part in the seesaw, one of the light neutrino mass is exactly zero. The masses of RH neutrinos, including the one which constitutes DM, originate from the $U(1)_{B-L}$ symmetry breaking scale. We assume their masses to be of same order and derive constraints from lepton flavour violating processes like $\mu \rightarrow e\gamma$.

The paper has been arranged as follows. In section 2, we explain the details of the model, followed by a summary of different theoretical and experimental constraints. We discuss the relic abundance of dark matter in section 3 and direct detection in section 4. Then we discuss the gauged $U(1)_{B-L}$ extension of the model in section 5. We briefly summarise collider search strategy for both the cases in section 6. In section 7, we discuss the light neutrino mass and finally conclude in section 8.

2 The model for singlet-doublet Majorana DM

In this work the SM has been extended by one vector-like fermion doublet (VLFd) $\Psi^T = (\psi^0, \psi^-)$ (with hypercharge $Y = -1$, where we use $Q = T_3 + Y/2$) and three heavy right handed neutrinos (RHN) $N_{R_i} (i = 1, 2, 3)$, which are singlets under the SM gauge group. All the newly added particles are also singlets under $SU(3)_C$, i.e. colour neutral. An additional $Z_2$ symmetry is imposed under which $\Psi$ and $N_{R_1}$ are odd, while all other fields are even. It is well known that the stability of DM is ensured by some additional symmetry and $Z_2$ serves as the minimal one. The quantum numbers of the BSM fields under $SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_2$ are listed in table 1. The Lagrangian of the model (as guided by table 1) is given by:

$$L = L_{SM} + \bar{\Psi} (i \gamma^\mu D_\mu - M) \Psi + \bar{N}_{R_i} i \gamma^\mu \partial_\mu N_{R_i} - \frac{1}{2} M_{R_i} \bar{N}_{R_i} (N_{R_i})^c + h.c. + L_{yuk}. \tag{2.1}$$

Apart from kinetic pieces, it is straightforward to note that since $\Psi$ is a vector-like Dirac fermion, it possesses a bare Dirac mass term $M$, while all the three right handed neutrinos have Majorana mass $M_{R_i}$. Also worthy to note that $D_\mu$ denotes the covariant derivative involving the $SU(2)_L$ gauge boson triplet $W^a_\mu (a = 1, 2, 3)$ and $U(1)_Y$ gauge boson $B_\mu$ given by:

$$D_\mu = \partial_\mu - ig A^a_\mu \tau^a, W^a_\mu = ig Y B_\mu; \tag{2.2}$$

where $\tau^a$ are Pauli spin matrices (generators of $SU(2)$), $g$ and $g'$ denote $SU(2)$ and $U(1)$ coupling strength respectively. This ensures that $\Psi$ has $SU(2)$ gauge interaction with the SM.
We note that the Yukawa interaction plays the key role in this model and can be written as:

$$- \mathcal{L}_{\text{yuk}} = \left[ \frac{Y_1}{\sqrt{2}} \bar{\psi} \tilde{H} (N_R + (N_R)^c) + h.c. \right] + \left( Y_{j\alpha} \bar{N_R} \tilde{H}^\dagger L_\alpha + h.c. \right). \quad (2.3)$$

where $\tilde{H} = i\tau_2 H^*$ and $L$ denotes SM lepton doublet with indices $j = 2, 3$ and $\alpha = e, \mu, \tau$. $N_{R_i}$ being odd under $\mathbb{Z}_2$ has Yukawa coupling to fermion doublet $\Psi$ and determines the DM of the model after spontaneous symmetry breaking (SSB), as elaborated below. $N_{R_2}$ and $N_{R_3}$ being $\mathbb{Z}_2$ even, do not couple to $\Psi$, but couple to the SM lepton doublets and hence generate Dirac masses for SM neutrinos after SSB, which will be discussed in details later.

### 2.1 Masses and mixing of dark sector particles

Thanks to the Yukawa interaction given in (2.3), the electromagnetic charge neutral component of $\Psi$ viz. $\psi^0$ and $N_{R_i}$ mixes after the SM Higgs acquires vacuum expectation value (vev): $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$. The mass terms for these fields can then be written together as:

$$- \mathcal{L}_{\text{mass}} = M_{\psi^0} \psi^0_R + \frac{1}{2} M_{N_{R_i}} (N_{R_i}) + \frac{m_D}{\sqrt{2}} \left( \psi^0_L N_{R_i} + \overline{\psi^0_R} (N_{R_i})^c \right) + h.c. \quad (2.4)$$

where $m_D = \frac{Y_{i\mu}}{\sqrt{2}}$, where $v = 246$ GeV. Writing these mass terms in the basis $(\psi^0_R)^c$, $(\psi^0_L, (N_{R_i})^c)^T$, we get the following mass matrix:

$$\mathcal{M} = \begin{pmatrix} 0 & M & \frac{m_D}{\sqrt{2}} \\ M & 0 & \frac{m_D}{\sqrt{2}} \\ \frac{m_D}{\sqrt{2}} & \frac{m_D}{\sqrt{2}} & M_{R_1} \end{pmatrix}. \quad (2.5)$$

In the above equation, assuming $\mathcal{M}$ is symmetric, it can be diagonalised by a single unitary matrix $U(\theta) = U_{13}(\theta_{13} = \theta), U_{23}(\theta_{23} = 0), U_{12}(\theta_{12} = \frac{\pi}{4})$, which is essentially characterised by a single angle $\theta_{13} = \theta$. So we diagonalize the mass matrix $\mathcal{M}$ by $U \mathcal{M} U^\dagger = \mathcal{M}_{\text{Diag.}}$, where the unitary matrix $U$ is given by:

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & e^{i\pi/2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta \cos \theta \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta \cos \theta \\ \frac{1}{\sqrt{2}} \sin \theta & -\frac{1}{\sqrt{2}} \cos \theta \cos \theta \end{pmatrix}, \quad (2.6)$$

where the extra phase matrix is multiplied to make sure all the eigenvalues are positive.

The diagonalisation of the mass matrix (2.5) requires:

$$\tan 2\theta = \frac{2m_D}{M - M_{R_1}}. \quad (2.7)$$

The physical states that emerge are defined as $\chi_i = \frac{\chi_{iL} + (\chi_{iL})^c}{\sqrt{2}}$ ($i = 1, 2, 3$) and are related to the unphysical states as:

$$\chi_{1L} = \frac{\cos \theta}{\sqrt{2}} (\psi^0_L + (\psi^0_R)^c) + \sin \theta (N_{R_i})^c,$$

$$\chi_{2L} = \frac{i}{\sqrt{2}} (\psi^0_L - (\psi^0_R)^c),$$

$$\chi_{3L} = -\frac{\sin \theta}{\sqrt{2}} (\psi^0_L + (\psi^0_R)^c) + \cos \theta (N_{R_i})^c. \quad (2.8)$$
All the three physical states $\chi_1, \chi_2$ and $\chi_3$ are therefore of Majorana nature and their mass eigenvalues can be expressed respectively as,
\begin{align}
 m_{\chi_1} &= M \cos^2 \theta + M_{R_1} \sin^2 \theta + m_D \sin 2\theta, \\
 m_{\chi_2} &= M, \\
 m_{\chi_3} &= M_{R_1} \cos^2 \theta + M \sin^2 \theta - m_D \sin 2\theta.
\end{align}
(2.9)

In the small mixing limit ($\theta \to 0$), the eigenvalues can be further simplified as,
\begin{align}
 m_{\chi_1} &\approx M + \frac{m_D^2}{M - M_{R_1}}, \\
 m_{\chi_2} &= M, \\
 m_{\chi_3} &\approx M_{R_1} - \frac{m_D^2}{M - M_{R_1}}.
\end{align}
(2.10)

where we have assumed $m_D \ll M, M_{R_1}$. Hence it is clear that $m_{\chi_1} > m_{\chi_2} > m_{\chi_3}$ and $\chi_3$ becomes the stable DM candidate. We may note here that the analysis taken up before in [40–45], where the $Z_2$ odd doublet $\Psi$ mixes with a vector like singlet, providing a Dirac DM state with one heavy electromagnetically charged Dirac state as opposed to two heavy Majorana states here.

Using the relation $U M U^T = M_{\text{Diag.}}$, one can express $Y_1$, $M$ and $M_{R_1}$ in terms of the physical masses and the mixing angle as,
\begin{align}
 Y_1 &= \sqrt{2} \Delta M \sin 2\theta, \\
 M &= m_{\chi_1} \cos^2 \theta + m_{\chi_3} \sin^2 \theta, \\
 M_{R_1} &= m_{\chi_3} \cos^2 \theta + m_{\chi_1} \sin^2 \theta;
\end{align}
(2.11)

where $\Delta M = (m_{\chi_1} - m_{\chi_3})$. We can also see that in the limit of $m_D \ll M$, $m_{\chi_1} \simeq m_{\chi_2} = M$, where $M$ is the mass of electrically charged components $\psi^\pm$ of vector-like fermion doublet $\Psi$. The phenomenology of dark sector is therefore governed mainly by the three independent parameters, DM mass, splitting with the heavier neutral component, and doublet-singlet mixing:

\begin{equation}
\text{Dark Parameters : } \{ m_{\chi_3}, \Delta M = (m_{\chi_1} - m_{\chi_3}) \approx (m_{\chi_2} - m_{\chi_3}), \sin \theta \}. 
\end{equation}
(2.12)

2.2 Theoretical and experimental constraints

- **Perturbativity**: In order to maintain perturbativity of the model, Yukawa couplings should satisfy the following limits:
\begin{equation}
|Y_1| < \sqrt{4\pi}, \quad |Y_{\alpha j}| < \sqrt{4\pi}.
\end{equation}
(2.13)

- **LEP limits**: LEP exclusion bound on charged fermion mass $m_{\psi^\pm} = M > 102.7$ GeV [47]. The bound from LHC applies to a typical case of type III seesaw model, for which $m_{\psi^\pm} = M \gtrsim 800$ GeV [48, 49]. Note that we do not abide by the bound from LHC as the decay channels are widely different.
• **Relic Density and Direct Search of Dark Matter:** The observed number density of DM is constrained by the combined WMAP [3] and PLANCK [4] data as:

\[ 0.1166 \leq \Omega_{DM} h^2 \leq 0.1206. \tag{2.14} \]

For direct search, we have used the current stringent bounds from non-observation of DM at XENON-1T [50] (\(\sim 10^{-47}\) cm\(^2\)). We also note that the fluctuation recently observed at XENON 1T at \(\sim\) KeV scale [51] do not apply to our case.

3 Relic abundance of singlet-doublet Majorana Dark Matter

3.1 Annihilation/coannihilation processes and Boltzmann equations

The basic assumption for calculation of relic density of the DM here is to assume that DM is in equilibrium with thermal bath due to its non-negligible interaction with the SM particles in the early universe. It thereafter ‘freezes out’ from the hot soup of the SM particles via the number changing processes through which DM number density depletes as the universe expands to provide correct relic density. The dark sector consists of DM \(\chi_3\) as well as heavy neutral components \(\chi_1, \chi_2\) and charged components \(\psi^{\pm}\) (all odd under the dark symmetry \(Z_2\)). The number density of DM \((\chi_3)\) is therefore governed by its annihilation as well as coannihilations with other dark sector particles \((\chi_1, \chi_2\) and \(\psi^{\pm}\)) into SM final states. Feynman diagrams of relevant annihilation and coannihilation processes are shown in figure 1, figure 2 and figure 3. The DM-SM interaction terms which essentially contribute to the relic density has been detailed in appendix A.

The relic density of DM in this scenario can be estimated by solving the Boltzmann equation in the following form:

\[
\frac{dn}{dt} + 3H n = -\langle \sigma v \rangle_{\text{eff}} \left( n^2 - n_{eq}^2 \right), \tag{3.1}
\]

where \(n\) denotes number density of DM, i.e. \(n \sim n_{\chi_3}\) and \(n_{eq} = g(m_T^2/32\pi)^3/2 \exp(-m/T)\) denotes equilibrium distribution, which DM is initially subjected to. Then it freezes out depending on \(\langle \sigma v \rangle_{\text{eff}}\), which takes into account all number changing process listed in figure 1, figure 2

![Figure 1](image-url)
Figure 2: Coannihilation channels of DM ($\chi_3$) with $\chi_1$, $\chi_2$ and $\psi^\pm$.

and figure 3 as well as those annihilations involving $\chi_{1,2}$ (although the contribution is very small) and can be estimated as follows:

$$
\langle \sigma v \rangle_{\text{eff}} = \frac{g_3^2}{g_{\text{eff}}^2} \langle \sigma v \rangle_{\chi_3 \chi_3} + \frac{2 g_3 g_2}{g_{\text{eff}}^2} \langle \sigma v \rangle_{\chi_3 \chi_2} \left(1 + \frac{\Delta M}{m_{X_3}}\right)^2 \exp \left(-x \frac{\Delta M}{m_{X_3}}\right)
$$

$$
\approx \frac{g_3^2}{g_{\text{eff}}^2} \langle \sigma v \rangle_{\chi_3 \chi_3} + \frac{2 g_3 g_2}{g_{\text{eff}}^2} \langle \sigma v \rangle_{\chi_3 \chi_2} \left(1 + \frac{\Delta M}{m_{X_3}}\right)^2 \exp \left(-x \frac{\Delta M}{m_{X_3}}\right) + \frac{2 g_3 g_2}{g_{\text{eff}}^2} \langle \sigma v \rangle_{\chi_3 \chi_1} \left(1 + \frac{\Delta M}{m_{X_3}}\right)^3 \exp \left(-2x \frac{\Delta M}{m_{X_3}}\right)
$$

$$
+ \frac{g_3^2}{g_{\text{eff}}^2} \langle \sigma v \rangle_{\chi_2 \chi_2} \left(1 + \frac{\Delta M}{m_{X_3}}\right)^3 \exp \left(-2x \frac{\Delta M}{m_{X_3}}\right) + \frac{g_3^2}{g_{\text{eff}}^2} \langle \sigma v \rangle_{\chi_1 \chi_1} \left(1 + \frac{\Delta M}{m_{X_3}}\right)^3 \exp \left(-2x \frac{\Delta M}{m_{X_3}}\right) + \frac{g_3^2}{g_{\text{eff}}^2} \langle \sigma v \rangle_{\psi^+ \psi^-} \left(1 + \frac{\Delta M}{m_{X_3}}\right)^3 \exp \left(-2x \frac{\Delta M}{m_{X_3}}\right),
$$

(3.2)
where $\Delta M = m_i - m_{\chi_3}$ and $m_i$ denotes the mass of $\chi_1$, $\chi_2$ and $\psi^\pm$. Here we have defined $g_{\text{eff}}$ as the effective degrees of freedom given by,

$$g_{\text{eff}} = g_3 + g_2 \left( 1 + \frac{\Delta M}{m_{\chi_3}} \right)^2 \exp\left( -x \frac{\Delta M}{m_{\chi_3}} \right) + g_1 \left( 1 + \frac{\Delta M}{m_{\chi_3}} \right)^2 \exp\left( -x \frac{\Delta M}{m_{\chi_3}} \right) + g_4 \left( 1 + \frac{\Delta M}{m_{\chi_3}} \right)^2 \exp\left( -x \frac{\Delta M}{m_{\chi_3}} \right),$$

(3.3)

where $g_3$, $g_2$, $g_1$ and $g_4$ are the internal degrees of freedom of $\chi_3$, $\chi_2$, $\chi_1$ and $\psi^\pm$ respectively.

The dimensionless parameter $x$ is defined as $x = \frac{m_{\chi_3}}{T_f}$. We also note that the contributions from processes which do not directly involve DM, like $\psi^+\psi^-$ in effective annihilation $\langle \sigma v \rangle_{\text{eff}}$ is further Boltzmann suppressed by $\exp(-2x \frac{\Delta M}{m_{\chi_3}})$. The relic density of the DM ($\chi_3$) then can be given by [52–54]:

$$\Omega_{\chi_3} h^2 = \frac{1.09 \times 10^9 \text{GeV}^{-1}}{g^*_\text{str} M_{\text{Pl}}} \frac{1}{J(x_f)}$$

(3.4)

where $g_\ast = 106.7$ and $J(x_f)$ is given by

$$J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma v \rangle_{\text{eff}}}{x^2} dx.$$

(3.5)

Here $x_f = \frac{m_{\chi_3}}{T_f}$, where $T_f$ denotes the freeze-out temperature of the DM. We may note here that for correct relic $x_f \simeq 20$.

It is worthy to mention here that we have adopted a numerical way of computing annihilation cross-section and relic density by inserting the model into the package MicrOmegas [55], where the model files are generated using another package FeynRule [56, 57].

### 3.2 Parameter Space Scan

In order to understand the DM relic density, let us first study the dependence on important relevant parameters: the mass of DM ($m_{\chi_3}$), the mass splitting ($\Delta M$) between the DM $\chi_3$ and the next-to-lightest stable particle (NLSP) $\chi_2$ and the mixing angle $\sin \theta$. Note that the charged components of $\Psi$ namely $\psi^\pm$ which contribute dominantly to the coannihilation channels has the same mass as that of $\chi_2$, i.e., $m_{\chi_2} = m_{\psi^\pm}$. Variation of relic density of DM $\chi_3$ is shown in figure 4 as a function of its mass for different choices of $\Delta M = 1$–10 GeV, 10–30 GeV, 30–50 GeV, 50–100 GeV shown by different colour shades as in the inset of the figure and for different choices of $\sin \theta = 0.01, 0.1, 0.3, 0.5$ in the top left, top right, bottom left and bottom right panels respectively.

As seen from figure 4, when $\Delta M$ is small, relic density is smaller due to large coannihilation contribution from flavour changing $Z$-mediated processes as well as $W^\pm$ mediated
Figure 4: DM relic density as a function of DM mass ($m_\chi$) for different mass splitting $\Delta M$ between the DM and the NLSP (as mentioned in figure inset in GeV) for $\sin \theta = 0.01$ (top left panel), $\sin \theta = 0.1$ (top right panel), $\sin \theta = 0.3$ (bottom left panel) and $\sin \theta = 0.5$ (bottom right panel). Correct relic density region from PLANCK data ($0.1166 \leq \Omega_{DM}h^2 \leq 0.1206$) is indicated by the silver horizontal line.

processes (less Boltzmann suppression followed from eq. (3.2)). The resonance drop at $m_Z/2$ is seen due to $s$-channel off-diagonal $Z$ mediated coannihilation interactions. As none of these neutral current interactions are diagonal, we observe the resonance to be somewhat flattened rather than a sharp spike that would have been expected if the interactions were diagonal. These coannihilation channels dominantly contribute towards the relic density as long as the mass splitting between the DM and NLSP is small, e.g., for $\Delta M = 10$ GeV. As $\Delta M$ increases, these coannihilations become less and less effective, and Higgs mediated processes take over. For $\Delta M = 30$ GeV, both contributions are present comparable while for $\Delta M > 30$ GeV, the contributions from vector current (coannihilation) interactions are practically negligible and the Higgs mediated channels dominate. Consequently, we see a resonance drop at $m_h/2$, while the drop at $m_Z/2$ disappears. We have also observed that as long as $\Delta M$ is small and the coannihilation channels dominate, the effect of $\sin \theta$ on relic density is quite negligible. For smaller $\sin \theta$, the annihilation cross-section due to Higgs portal (see eq. (A.3)) is small leading to larger relic abundance, while for large $\sin \theta$, the effective annihilation cross-section is large leading to small relic abundance. However, this can only be observed when $\Delta M$ is sufficiently large enough and coannihilation processes are negligible.

In figure 4, we also show the correct relic density by the silver horizontal line. In figure 5, the correct relic density allowed parameter space has been shown in the plane of $\Delta M$ vs $m_\chi$ for wide range of mixing angle $\{\sin \theta = 0.001–0.01, 0.01–0.1, 0.1–0.2, 0.2–0.4, 0.4–0.6\}$, indicated
Figure 5: DM relic density $(0.1166 \leq \Omega_{DM} h^2 \leq 0.1206)$ allowed parameter space in the plane of $\Delta M$ vs $m_{\chi_3}$ for different ranges of $\sin \theta$ as mentioned in the figure inset. The shaded region in the bottom left corner is ruled out by LEP exclusion bound on charged fermion mass, $m_{\psi^\pm} = M > 102.7 \text{ GeV}$.

by different colours. We can see that in figure 5, there is a bifurcation around $\Delta M \sim 50 \text{ GeV}$, so the allowed plane of $m_{\chi_3} - \Delta M$ are separated in two regions: (i) the bottom portion with small $\Delta M$, where $\Delta M$ decreases with larger DM mass ($m_{\chi_3}$) and (ii) the top portion of the figure with large $\Delta M$, where $\Delta M$ increases slowly with larger $m_{\chi_3}$.

In region (i), given a specific range of $\sin \theta$, the annihilation cross-section decreases with larger DM mass $m_{\chi_3}$ (from annihilation diagrams) and hence more co-annihilation contribution is required to get correct relic density, resulting $\Delta M$ to decrease. This also implies that the region below the each coloured zone is under-abundant (small $\Delta M$ implying large co-annihilation for a given $m_{\chi_3}$), while the region above is over-abundant by the same logic. In this region the Yukawa coupling $Y_1$ which governs the annihilation cross-section is comparatively small since $Y_1 \propto \Delta M \sin \theta$ and $\Delta M$ is small. Also the annihilation cross-section decreases with increase in DM mass. Therefore, when DM mass is sufficiently heavy ($m_{\chi_3} > 1.2 \text{ TeV}$), annihilation becomes too weak to be compensated by the coannihilation even when $\Delta M \to 0$, producing over abundance. Hence, for small $\Delta M$, the allowed region has a maximum DM mass, as the region beyond $m_{\chi_3} \sim 1.2 \text{ TeV}$ is overabundant.

In region (ii), we note that, the co-annihilation contribution is much smaller due to large $\Delta M$, so the annihilation processes effectively contribute to the relic density. Annihilation processes are essentially gauge or Higgs mediated. We already noted that Higgs Yukawa coupling is proportional to both $\sin \theta$ and $\Delta M$ as $Y_1 \propto \Delta M \sin 2\theta$. Hence, for a given $\sin \theta$, larger $\Delta M$ leads to larger $Y_1$ and hence larger annihilation cross-section to yield under abundance, which can only be tamed down to correct relic density by having a larger DM mass. Also larger $\sin \theta$ requires smaller $\Delta M$ for the same reason. Therefore, the region above each coloured zone (allowed by relic density for a specific range of $\sin \theta$) is under abundant, while the region below each coloured zone is over abundant.

Let us come back to region (i) again and note that allowed parameter space indicates larger DM mass requires smaller and smaller $\Delta M$ and we reach a maximum DM mass ($\sim 1 \text{ TeV}$) for $\Delta M \to 0$. However, with $\Delta M \to 0$, the charged companions $\psi^\pm$ are degenerate.
Figure 6: Lower bound on $\Delta M$ as a function of $\sin \theta$ from Big-Bang Nucleosynthesis (BBN). The shaded region is allowed.

to DM and are stable. This is not acceptable as DM won’t be dark then. Hence, $\Delta M$ can not be arbitrarily small. We can put a lower bound on $\Delta M$ by requiring the charged partners $\psi^\pm$ of the DM to decay before the onset of Big Bang Nucleosynthesis ($\tau_{\text{BBN}} \sim 1$ sec.). The decay rate for the processes $\psi^\pm \to \chi_3 l^\pm \nu_l$ in the limit of small $\Delta M$ is given by:

$$\Gamma_{\psi^\pm} = \frac{1}{15(2\pi)^3} \frac{e^4 \sin^2 \theta (\Delta M)^5}{\sin^4 \theta_w M_W^4}.$$  \hspace{1cm} (3.6)

By requiring that the charged fermions should decay before the onset of BBN, we can get a lower bound on $\Delta M$ as,

$$\tau_{\psi^\pm} = \frac{1}{\Gamma_{\psi^\pm}} \leq \tau_{\text{BBN}} \sim 1 \text{ sec} \implies \left(\frac{\Delta M}{\text{GeV}}\right)^5 \geq \frac{6.4 \times 10^{-13}}{\sin^2 \theta}.$$  \hspace{1cm} (3.7)

In figure 6, we show the lower bound on $\Delta M$ for the range of $\sin \theta$ we used in our work. The region above the red line is allowed by the constraint. It is obvious that the bound is more stringent for smaller $\sin \theta$.

4 Direct detection of singlet-doublet Majorana Dark Matter

Among different possibilities of detecting DM, one major experimental procedure is direct DM search. Direct detection of the DM ($\chi_3$) at a terrestrial laboratory is possible through elastic scattering of the DM off nuclei via Higgs-mediated interaction represented by the Feynman diagram shown in figure 7. The presence of only Higgs mediated diagram for direct search makes this model crucially segregated from that of a vector like singlet-doublet DM as elaborated in [40–45]. Here, the DM being a Majorana fermion only has off diagonal

$^1$Semi-leptonic processes e.g. $\psi^\pm \to \chi_3 \pi^\pm$ are also possible, see for example [58]
Z-coupling and therefore do not contribute to direct search as it is very difficult to produce a heavier particle in the low energy scattering as in direct search experiment. The absence of Z mediation crucially alters the available parameter space of the model as we describe below. The corresponding vertex of $\chi_3\chi_3 h$ can be obtained from the Lagrangian $\mathcal{L}_{DM-Higgs}$ given by eq. (A.3). The cross section per nucleon for the spin-independent (SI) DM-nucleon interaction is then given by:

$$\sigma_{SI} = \frac{1}{\pi A^2 \mu_r^2} |\mathcal{M}|^2,$$

where $A$ is the mass number of the target nucleus, $\mu_r$ is the reduced mass of the DM-nucleon system and $\mathcal{M}$ is the amplitude for the DM-nucleon interaction, which can be written as:

$$\mathcal{M} = \left[ Z f_p + (A - Z) f_n \right],$$

where $f_p$ and $f_n$ denote effective interaction strengths of DM with proton and neutron of the nuclei used for the experiment with $A$ being mass number and $Z$ being atomic number. The effective interaction strength can then further be decomposed in terms of interaction with parton as:

$$f_{p,n} = \sum_{q=u,d,s} f^{p,n}_{Tq} \alpha_q \frac{m_{(p,n)}}{m_q} + \frac{2}{27} f^{p,n}_{TG} \sum_{q=c,b,t} \alpha_q \frac{m_{(p,n)}}{m_q};$$

with

$$\alpha_q = \frac{Y_1 \sin 2\theta m_q}{M_h^2} = \frac{\Delta M \sin^2 2\theta m_q}{v^2 M_h^2};$$

coming from DM interaction with SM via Higgs portal coupling. Further, in eq. (4.3), the different coupling strengths between DM and light quarks are given by Bertone et al. [1, 59] as $f^p_{Tu} = 0.020 \pm 0.004, f^p_{Td} = 0.026 \pm 0.005, f^p_{Ts} = 0.014 \pm 0.062, f^n_{Tu} = 0.020 \pm 0.004, f^n_{Td} = 0.036 \pm 0.005, f^n_{Ts} = 0.118 \pm 0.062$. The coupling of DM with the gluons in target nuclei is parameterised by:

$$f^{p,n}_{TG} = 1 - \sum_{q=u,d,s} f^{p,n}_{Tq}.$$

Using eqs. (4.1), (4.2), (4.3) and (4.4), the spin-independent DM-nucleon cross-section is given by:

$$\sigma_{SI} = \frac{4}{\pi A^2 \mu_r^2} Y^2 \sin^2 2\theta \left[ \frac{m_p}{v} \left( f^p_{Tu} + f^p_{Td} + f^p_{Ts} + \frac{2}{9} f^p_{TG} \right) + \frac{m_n}{v} \left( f^n_{Tu} + f^n_{Td} + f^n_{Ts} + \frac{2}{9} f^n_{TG} \right) \right]^2$$
Figure 8: [Left]: Direct detection cross section for the DM ($\chi_3$) confronted with bounds on spin-independent elastic scattering cross section by XENON-1T [50] over and above relic density constraint from PLANCK; [Right]: Correct DM relic density in $\Delta M - m_{\chi_3}$ plane constrained by XENON-1T bound. Different coloured points indicate different ranges of $\sin \theta$ as mentioned in figure inset. The shaded region in the bottom left corner of right panel plot is ruled out by LEP exclusion bound on charged fermion mass, $m_{\psi^{\pm}} = M > 102.7$ GeV.

In the above equation for DM-nucleon direct search cross-section, two parameters from model that enter are the Higgs-DM Yukawa coupling ($Y_1$) and singlet-doublet mixing parameter ($\sin 2\theta$), which can be constrained by requiring that $\sigma_{SI}$ is less than the current DM-nucleon cross-sections dictated by non-observation of DM in current direct search data. Recently, there has been a signal like event from electron recoil reported in XENON-1T data [51] observed at sub-GeV DM mass, which remains out of our scan.

In the left panel of figure 8, we confront the direct detection cross section obtained for the model as a function of DM mass, with bounds on spin-independent elastic scattering cross section from XENON-1T [50], shown by black dashed curve. It is worth mentioning that all points shown in left panel of figure 8 also satisfies relic density constraints from PLANCK. Different coloured patches indicate different ranges of mixing angle ($\sin \theta$) as indicated in figure panel. Obviously those regions that appear below the XENON-1T line can be allowed by the bound. It is obvious that $Y_1$ being proportional to $\sin \theta$ (see eq. (2.11)) and due to the explicit presence of $\sin 2\theta$ in the direct search cross section as in eq. (4.4), parameter space with smaller $\sin \theta$ survives the cut. This is what is shown in $\Delta M - m_{\chi_3}$ plane in the right hand side (r.h.s.) of figure 8, where we plot those points which simultaneously satisfy relic density [4] and direct search XENON-1T bound [50] together. It is seen that null observation from direct search crucially tames down the relic density allowed parameter space, which is evident when we compare the r.h.s. of figure 8 with that of figure 5, where only relic density allowed parameter space is depicted. It is seen in r.h.s. of figure 8, that $\sin \theta$ is correlated to DM mass and $\Delta M$. For example, $\sin \theta$ is very small for smaller DM mass with moderate $\Delta M$ ($\sin \theta \lesssim 0.2$ for $m_{DM} \sim 500$ GeV with $\Delta M \sim 20$ GeV shown by red and green points); while larger $\sin \theta \sim 0.6$ is allowed at higher DM mass $\sim 1000$ GeV, with very small $\Delta M \lesssim 2$ GeV (Cyan points). This is simply because, the direct search cross-section is proportional to $\sim Y_1 \sin 2\theta \sim \Delta M \sin^2 2\theta$, therefore larger $\sin \theta$ requires $\Delta M$ to be smaller to remain within correct direct search limit. However, due to larger coannihilation contribution with small $\Delta M$, the relic density drops below the PLANCK bound, unless we restore it to the correct ballpark by having larger DM mass (annihilation cross-section is
inversely proportional to DM mass). This feature crucially distinguishes the model at hand from vector like singlet-doublet scenario with Dirac dark matter, where the presence of $Z$ mediated direct search graph tames $\sin \theta$ to much smaller values like $\sim 0.05$ (for details see [40–45]). Higgs resonance $m_{\chi_3} \sim m_h/2$ is seen to satisfy both relic density and direct search bound, where $\Delta M$ can be very large having very small $\sin \theta \sim 0.2$.

5 Singlet-doublet Majorana DM in gauged $U(1)_{B-L}$ extension of the SM

5.1 The model

Due to the presence of three right handed neutrinos $N_{R_i}$ and the fermion doublet $\Psi$ being vector-like, the model is automatically $U(1)_{B-L}$ anomaly free if we assign one unit of B-L charge to each of these fields. This is because of the fact that in a gauged B-L theory with only SM fermion content, non-zero anomalies are associated with the following two triangular diagrams:

$$A_1[U(1)^3_{B-L}] = A_1^{SM}[U(1)^3_{B-L}] = -3,$$
$$A_2[(\text{Gravity})^2 \times U(1)_{B-L}] = A_2^{SM}[(\text{Gravity})^2 \times U(1)_{B-L}] = -3,$$

which are exactly cancelled by anomalies from three additional right handed neutrinos since,

$$A_1^{RHN}[U(1)^3_{B-L}] = 3,$$
$$A_2^{RHN}[(\text{Gravity})^2 \times U(1)_{B-L}] = 3.$$  

(5.1)

Motivated by this fact, we extend the gauge group of the model to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \otimes \mathbb{Z}_2$. Besides, one new complex scalar singlet $\Phi_{BL}$ is added with lepton number $-2$. The particle content and the corresponding quantum numbers under the symmetry of the model are listed in the table 2. Since two of the right handed neutrinos, say $N_{R_2}, N_{R_3}$ are chosen to be even under the imposed $\mathbb{Z}_2$ symmetry, they can couple to the SM lepton and Higgs doublets to explain non-zero masses and mixing of light neutrinos. On the other hand, the vectorlike fermion doublet $\Psi$ and $N_{R_1}$ are chosen to be odd under the imposed $\mathbb{Z}_2$ symmetry. As a result the DM emerges as a mixture of the neutral component of the doublet $\Psi$ viz. $\psi^0$ and $N_{R_1}$, similar to section 2. However, we notice certain differences in the mass matrix of dark sector neutral fermions in comparison to eq. (2.5) due to the conservation of $B-L$ charge. In the following we discuss in details the corresponding phenomenology.

Owing to the symmetry and charge assignments of the particles given in table 2, the Lagrangian of the Model can be given as:

$$\mathcal{L} = \bar{\Psi}(i\partial - M)\Psi + \bar{N}_{R_i}i\tilde{D}N_{R_i} + \mathcal{L}_{yuk} + \mathcal{L}_{Gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{SM};$$

(5.3)

where the covariant derivatives $D_\mu$ and $\tilde{D}_\mu$ are given by:

$$D_\mu = \partial_\mu - ig/2 \gamma.W_\mu - igY/2 B_\mu - ig_{BL}Y_{BL}Z_{BL},$$
$$\tilde{D}_\mu = \partial_\mu - ig_{BL}Y_{BL}(Z_{BL})_\mu.$$  

(5.4)

In the covariant derivative of $\Psi$, there is an additional term due to the lepton number assignment, i.e. its transformation under $U(1)_{B-L}$; $g_{BL}$ stands for $U(1)_{B-L}$ gauge coupling, which serves as an additional free parameter of the model. Note that $Y_{BL}$ can simply be replaced by the lepton number assignment as given in table 2.
Fields

\[ \Psi = \begin{pmatrix} \psi^0 \\ \psi^- \end{pmatrix} \]

\[ \begin{array}{cccc} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L} \otimes Z_2 \\ \end{array} \]

| Field | \[ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L} \otimes Z_2 \] |
|-------|---------------------------------------------------------------------|
| VLFd  | \[ \Psi = \begin{pmatrix} \psi^0 \\ \psi^- \end{pmatrix} \] |
| RHNs  | \[ N_{R_1} \] |
|       | \[ 1 \quad 1 \quad 0 \quad -1 \quad - \] |
|       | \[ N_{R_2} \] |
|       | \[ 1 \quad 1 \quad 0 \quad -1 \quad + \] |
|       | \[ N_{R_3} \] |
|       | \[ 1 \quad 1 \quad 0 \quad -1 \quad + \] |
| Higgs doublet | \[ H = \begin{pmatrix} w^+ \\ h + v + iz \sqrt{2} \end{pmatrix} \] |
|       | \[ 1 \quad 2 \quad 1 \quad 0 \quad + \] |
| Scalar Singlet | \[ \Phi_{BL} = \begin{pmatrix} \phi^{(')} \end{pmatrix} \] |
|       | \[ 1 \quad 1 \quad 0 \quad -2 \quad + \] |

*Table 2*: Charge assignment of BSM fields along with the SM Higgs doublet under the gauge group \( G \equiv G_{SM} \otimes U(1)_{B-L} \otimes Z_2 \), where \( G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \).

The Yukawa interaction of the model is given by:

\[ -L_{yuk} = \left[ Y_1 \overline{\Psi} H N_{R_1} + h.c. \right] + \left[ Y_2 \overline{N_{R_i}} H^\dagger L_{\alpha} + h.c. \right] + \left[ \frac{g_Y}{2} \Phi_{BL} \overline{N_{R_i}} (N_{R_i})^c + h.c. \right] ; \quad (5.5) \]

where \( \alpha = e, \mu, \tau \), \( j = 2, 3 \) and \( i = 1, 2, 3 \). Due to \( B-L \) conservation, the Yukawa interaction term \( \overline{\Psi} H N_{R_1} \) that was allowed in the earlier case (see eq. (2.3)) is no longer allowed. For the same reason, the bare Majorana mass terms of right-handed neutrinos are also not allowed. The masses of right-handed neutrinos as well as the neutral gauge boson \( Z_{BL} \) are generated from the vev of \( \Phi_{BL} \). Thus the gauge sector is augmented by a new gauge boson \( Z_{BL} \). The new gauge kinetic terms that appear in the Lagrangian constitute of,

\[ L_{Gauge} = -\frac{1}{4} (Z_{BL})_{\mu \nu} (Z_{BL})^{\mu \nu} - \frac{\epsilon}{2} (Z_{BL})_{\mu \nu} B^{\mu \nu}; \quad (5.6) \]

where \( Z_{\mu \nu}^{BL} \) represents the field strength of the \( U(1)_{B-L} \) gauge boson and is defined as:

\[ Z_{BL}^{\mu \nu} = \partial^\mu (Z_{BL})^\nu - \partial^\nu (Z_{BL})^\mu. \quad (5.7) \]

In the second term, \( \epsilon \) parametrises the kinetic mixing between the \( U(1)_{B-L} \) and \( U(1)_Y \) gauge sectors. Such a mixing term can be generated through quantum corrections and approximated at one loop as \( \epsilon \approx \frac{g_{YBL}^{2}}{16\pi^2} [60, 61] \). Since \( g_{YBL} \) has tight upper bound from ATLAS, such one loop mixing is very small compared to other relevant parameters of the model and the same has been neglected in rest of our analysis.

The Lagrangian of scalar sector is given by:

\[ L_{scalar} = |D_\mu H|^2 + |\Phi_{BL}|^2 - V(H, \Phi_{BL}) \quad (5.8) \]

where \( D_\mu \) and \( D_\mu \) are given as follows:

\[ D_\mu = \partial_\mu - i \frac{g}{2} \tau.W_\mu - ig' \frac{Y}{2} B_\mu \]
\[ D_\mu = \partial_\mu - ig_{BL} Y_{BL}(Z_{BL})_\mu. \quad (5.9) \]
The scalar potential is given by

\[ V(H, \Phi_{BL}) = -\mu_H^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2 \]

\[ - \mu_\Phi^2 (\Phi_{BL}^\dagger \Phi_{BL}) + \lambda_\Phi (\Phi_{BL}^\dagger \Phi_{BL})^2 + \lambda_{H\Phi} (H^\dagger H) (\Phi_{BL}^\dagger \Phi_{BL}). \]

We note here that \( H \) do not have any transformation under the extended symmetry, while \( \Phi_{BL} \) is a singlet under SM, the only gauge invariant terms that one can cook up are \( H^\dagger H \) and \( \Phi_{BL}^\dagger \Phi_{BL} \), resulting a simple scalar potential, where the only interaction term that can be written is \( (H^\dagger H) (\Phi_{BL}^\dagger \Phi_{BL}) \). \( \lambda_{H\Phi} \) turns out to be an important additional parameter that contributes to the phenomenology. We also note that for both \( H \) and \( \Phi_{BL} \) to acquire non-zero vevs, we need both \( \mu_H \) and \( \mu_\Phi \) to be positive.

We analyse the model as follows: scalar mixing in subsection 5.2, masses and mixing of dark sector particles in subsection 5.3, theoretical and experimental constraints in subsection 5.4, relic abundance of DM in subsection 5.5, direct detection in subsection 5.6 and finally show the allowed parameter space in the light of ATLAS bound on \( g_{BL} \) versus \( M_{Z_{BL}} \) in subsection 5.7.

### 5.2 Spontaneous symmetry breaking and physical scalars

At TeV scales \( \Phi_{BL} \) acquires a non-zero vev and breaks \( U(1)_{B-L} \) to identity. The non-zero vevs which spontaneously breaks \( G_{SM} \otimes U(1)_{B-L} \otimes Z_2 \) down to \( U(1)_Q \otimes Z_2 \) are given as:

\[
\langle \Phi_{BL} \rangle = \frac{v_{BL}}{\sqrt{2}}, \quad \langle H \rangle = \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right). \tag{5.10}
\]

The minimization conditions around the vev’s are given by:

\[
\frac{\partial V}{\partial H} \bigg|_{v} = 0 : \quad \mu_H^2 = \lambda_H v^2 + \frac{\lambda_{H\Phi} v_{BL}^2}{2},
\]

\[
\frac{\partial V}{\partial \Phi_{BL}} \bigg|_{v_{BL}} = 0 : \quad \mu_\Phi^2 = \lambda_\Phi v_{BL}^2 + \frac{\lambda_{H\Phi} v^2}{2}. \tag{5.11}
\]

Due to presence of \( (H^\dagger H) (\Phi_{BL}^\dagger \Phi_{BL}) \) interaction in the scalar sector, both weak states \( h \) and \( \phi \) mix with each other. Using above minimization conditions, the mass terms of the scalar sector can be expressed as:

\[
L_{\text{mass}} = \frac{1}{2} (h \phi) \begin{pmatrix} 2\lambda_H v^2 & \lambda_{H\Phi} v_{BL} \\ \lambda_{H\Phi} v_{BL} & 2\lambda_\Phi v_{BL}^2 \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix},
\]

\[
= \frac{1}{2} \begin{pmatrix} 0 & 0 \\ m_{h_1}^2 & m_{h_2}^2 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \tag{5.12}
\]

In order to obtain the mass eigenvalues, the flavor eigenstates are rotated by an orthogonal matrix as follows:

\[
\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix}; \tag{5.13}
\]

where \( h_1 \) and \( h_2 \) are the physical mass eigenstates. We identify \( h_1 \) to be the physical Higgs discovered in 2012 at LHC with mass \( m_{h_1} = 125 \text{ GeV} \) and \( m_{h_2} \) remains a scalar beyond...
the SM. How heavy \( h_2 \) requires to be is constrained from LHC data which we discuss in a moment. The CP odd states also mix with each other, but turns out to be massless states known as Goldstone Bosons. In unitary gauge they are accounted as the longitudinal modes of massive vector Bosons and do not enter into phenomenology explicitly. The scalar sector therefore accounts for three free parameters:

\[
\{ m_{h_2}, v_{BL}, \sin \beta \};
\]

which are constrained from Higgs data at Collider. We will discuss them in the next subsection. Other quartic couplings \( \lambda_H, \lambda_\Phi \) and \( \lambda_{H\Phi} \) can be expressed in terms of the physical parameters as:

\[
\lambda_H = \frac{m_{h_1}^2 \cos^2 \beta + m_{h_2}^2 \sin^2 \beta}{2v^2},
\]

\[
\lambda_\Phi = \frac{m_{h_1}^2 \sin^2 \beta + m_{h_2}^2 \cos^2 \beta}{2v^2},
\]

\[
\lambda_{H\Phi} = \frac{(m_{h_2}^2 - m_{h_1}^2) \sin 2\beta}{2v v_{BL}}.
\]

The broken \( U(1)_{B-L} \) gauge symmetry yields mass for \( Z_{BL} \) as:

\[
M_{Z_{BL}} = 2g_{BL} v_{BL}
\]

\( M_{Z_{BL}} \) and \( g_{BL} \) are constrained from both LEP and LHC which we shall address later. So it follows from eq. (5.16) that \( v_{BL} \) is no longer a free parameter. Instead, in the combined gauged and scalar sector, the free parameters involved are:

\[
\{ m_{h_2}, M_{Z_{BL}}, g_{BL}, \sin \beta \};
\]

As we will see in the later sections, these parameters play a crucial role in DM phenomenology in the \( U(1)_{B-L} \) extension of the SM model.

### 5.3 Masses and mixing of dark sector particles

After electroweak symmetry breaking the mass term of the neutral dark sector particles can be written as,

\[
-\mathcal{L}_{\text{mass}} = M \overline{\psi_R^0} \psi_R^0 + \frac{1}{2} M_{R_1} \overline{N_{R_1}} (N_{R_1})^c + m_D \overline{\psi_L^0} N_{R_1} + \text{h.c.},
\]

where \( m_D = \frac{Y_1(v)}{\sqrt{2}} \) with \( \langle v \rangle = 246 \text{ GeV} \) being the vacuum expectation value (vev) of the SM Higgs \( H \) and \( M_{R_1} = \frac{v_{BL}}{\sqrt{2}}, \) where \( v_{BL} \) is the vev of new scalar \( \Phi_{BL} \). Writing these mass terms in the basis \( ((\psi_R^0)^c, \psi_L^0, (N_{R_1})^c)^T \), we get the mass matrix:

\[
\mathcal{M} = \begin{pmatrix}
0 & M & 0 \\
M & 0 & m_D \\
0 & m_D & M_{R_1}
\end{pmatrix}.
\]

The above mass matrix of neutral dark sector particles can be diagonalized by using an orthogonal transformation: \( \mathcal{M}_{\text{diag}} = U \cdot \mathcal{M} \cdot U^T \), where \( U = U_{13}(\theta_{13}) \cdot U_{23}(\theta_{23}) \cdot U_{12}(\theta_{12}) \)
and $U_{13}(\theta_{13})$, $U_{23}(\theta_{23})$ and $U_{12}(\theta_{12})$ are taken as three Euler rotation matrices. Assuming $m_D \ll M, M_{R_1}$, the mass eigenvalues are given by:

$$m_{\chi_1} \approx M + \frac{m_D^2}{2(M - M_{R_1})},$$
$$m_{\chi_2} \approx -\left(M + \frac{m_D^2}{2(M + M_{R_1})}\right),$$
$$m_{\chi_3} \approx M_{R_1} \left(1 - \frac{m_D^2}{M^2 - M_{R_1}^2}\right).$$

(5.20)

From eqs. (5.19) and (5.20) we see that $\text{Tr} \mathcal{M} = M_{R_1} = \sum_{i=1}^{3} m_{\chi_i}$. Note that the above diagonalization is up to $O(\frac{m_D^2}{M^2 + M_{R_1}^2})$. The corresponding physical eigenstates can be given in terms of flavour eigenstates as:

$$x_{1L} = (c_{13}c_{12} + s_{13}s_{23}s_{12})(\psi_R^0)^c + (c_{13}s_{12} - s_{13}s_{23}c_{12})\psi_L^0 + (s_{13}c_{23})N_{R_1}^c,$$
$$x_{2L} = (-c_{23}s_{12})(\psi_R^0)^c + (c_{23}c_{12})\psi_L^0 + s_{23}N_{R_1}^c,$$
$$x_{3L} = (-s_{13}c_{12} + c_{13}s_{23}s_{12})(\psi_R^0)^c + (-s_{13}s_{12} - s_{23}c_{12}c_{13})\psi_L^0 + (c_{13}c_{23})N_{R_1}^c.$$

(5.21)

where we abbreviated $\cos \theta_{ij} = c_{ij}$ and $\sin \theta_{ij} = s_{ij}$, with \{ij : 12, 13, 23\}. The diagonalisation of the mass matrix requires:

$$\theta_{12} = \pi 4,$$
$$\tan 2\theta_{23} = -\frac{\sqrt{2}m_D}{M + M_{R_1}},$$
$$\tan 2\theta_{13} = -\frac{\sqrt{2}m_D}{M - M_{R_1} - \frac{m_D^2}{2(M + M_{R_1})}} \cos \theta_{23}.$$

(5.22)

Thus in the effective theory the dark sector comprises of three physical Majorana fermions $\chi_1, \chi_2, \chi_3$ defined as $\chi_i = \frac{\chi_{1,2,3} + e^{\imath \theta_i} \chi_{1,2,3}}{\sqrt{2}}$ (i = 1, 2, 3). We assume $m_{\chi_1} > m_{\chi_2} > m_{\chi_3}$, so that $\chi_3$ serves as a stable dark matter candidate. In the limit $m_D \ll M, M_{R_1}$, from eq. (5.22), we can further write,

$$Y_1 \approx \frac{\Delta M \sin 2\theta_{13}}{v},$$

(5.23)

where $\Delta M = |m_{\chi_1}| - |m_{\chi_3}| \approx |m_{\chi_2}| - |m_{\chi_3}|$. The mixing angle $\theta_{23}$ can be obtained using values of $m_D$ in the definition of $\theta_{13}$. Therefore the phenomenology of dark sector is governed mainly by the following three independent parameters: DM mass $m_{\chi_3}$, splitting with the heavier neutral components $\Delta M$ and mixing angle $\theta_{13}$. Thus the ultimate free parameters in the dark sector are:

$$\text{Dark Parameters} : \{ m_{\chi_3}, \Delta M, \sin \theta_{13}\}, \text{ or } \{M_{R_1}, M, \sin \theta_{13}\}. \quad (5.24)$$

\(^2\)Similar to eq. (2.6), the mass matrix (5.19) can be further rotated by a phase matrix $U_{ph}$ to make sure all the eigenvalues are positive.
5.4 Theoretical and experimental constraints

- **Stability of potential:** In order to maintain stable vacuum, the quartic terms of the scalar potential should obey following co-positivity conditions [62, 63]:

\[
\lambda_H \geq 0, \quad \lambda_H - 2\sqrt{\lambda_H}\lambda_P \geq 0.
\]

- **Perturbativity:** In order to maintain perturbativity of the model, Yukawa couplings should satisfy the following limits:

\[
|\lambda_H| < 4\pi, \quad |\lambda_H| < 4\pi, \quad |\lambda_{H}\phi| < 4\pi; \\
|Y_1| < \sqrt{4\pi}, \quad |Y_{aj}| < \sqrt{4\pi}, \quad |g_{BL}| < \sqrt{4\pi}.
\]

- **LEP limits:** LEP exclusion bound on charged fermion mass, \(m_{\psi^\pm} = M > 102.7\) GeV [47]. Again, we note that the bound from LHC has been evaluated for a typical case of type III seesaw model, \(m_{\psi^\pm} = M \gtrsim 800\) GeV [48, 49], which is not strictly applicable to our case.

- **Constraints on \(M_{Z\text{BL}}\):** LEP II data puts lower bound on \(M_{Z\text{BL}}/g_{BL} \geq 7\) TeV [64]. Corresponding bound from ATLAS and CMS at LHC Run 2 is more severe than LEP II, \(M_{Z\text{BL}} > 4.3\) TeV for \(g_{BL}\) of the same order as that of SM coupling [65–67]. However, this constraint can be relaxed for lower value of \(g_{BL}\). For \(M_{Z\text{BL}} = \mathcal{O}(1\text{TeV})\), the upper bound on \(g_{BL}\) can be as small as 0.009 [68].

- **Bounds on scalar singlet transforming under \(U(1)_{B-L}\):** In the extended scalar sector, the mixing angle \((\sin \beta)\) and the mass of the extra physical state \((m_{h_2})\) faces the following constraints: i) From \(W\) mass corrections at Next to Leading Order (NLO) [69]: For 250 GeV \(\leq m_{h_2} \leq 850\) GeV, one has 0.2 \(\leq \sin \beta \leq 0.3\). ii) For the requirement of perturbative unitarity [70]: \(\sin \beta \leq 0.2\) for \(m_{h_2} \geq 850\) GeV. iii) Direct search measurement of Higgs signal strength at LHC provides an upper limit on mixing angle \(|\sin \beta| < 0.36\) [70].

5.5 Relic abundance of dark matter

The DM-SM interaction terms which deplete the number density of dark sector particles in the gauged \(U(1)_{B-L}\) case has been discussed in appendix B. The additional relevant Feynman diagrams of annihilation and coannihilation processes over and above those already present in section 3 are shown in figure 9, figure 10 and figure 11.

Again we use MicroOmegas to calculate the relic density of DM. The plots for DM relic density \(\Omega h^2\) as a function of DM mass \(m_{DM} = m_{\chi_3}\) are shown in figure 12 for different mass splitting \(\Delta M\) between the DM and the NLSP and for a chosen mixing angle \(\sin \theta_{13}\). The main difference in this \(B - L\) extended case compared to section 3 is the presence of new resonances at \(m_{\chi_3} = m_{h_1}/2\) and \(m_{\chi_3} = m_{h_2}/2\). These resonances get prominent only when the mass difference \(\Delta M\) is sufficiently large such that the coannihilation processes are practically negligible. As we can see from figure 12, for small \(\Delta M\), the coannihilation through off-diagonal \(Z\) and \(W^\pm\) mediated interactions dominate. Apart from that in figure 12 we also see new resonances (in comparison to figure 4) occur at \(m_{\chi_3} = m_Z/2\) and \(m_{\chi_3} = m_{ZBL}/2\). Note that the resonance at \(m_{\chi_3} = m_Z/2\) is proportional to \(\sin \theta_{13}\). As a result in the limit \(\sin \theta_{13} \to 0\) and new particles, say \(h_2\) and \(Z_{BL}\) heavy enough we get back to the same situation as in figure 4.
Figure 9: Additional annihilation channels of the DM ($\chi_3$) to SM particles in $U(1)_{B-L}$ model.

Figure 10: Additional coannihilation channels of DM ($\chi_3$) with $\chi_1$, $\chi_2$ and $\psi^\pm$ in the $U(1)_{B-L}$ model.

Figure 11: Additional coannihilation channels of $\psi^+$ and $\psi^-$ that contribute to relic density of DM ($\chi_3$) in the $U(1)_{B-L}$ model.

In figure 12, we have chosen $M_{Z_{BL}} = 1.65$ TeV, $g_{BL} = 0.03$, $m_{h_2} = 300$ GeV and the mixing parameter of SM Higgs with the new B-L Higgs as $\sin \beta = 0.2$, consistent with the available constraints. Also the masses of the two $Z_2$ even right handed neutrinos are kept fixed as $M_{R_{2/3}} = 500$ GeV. As $\sin \theta_{13}$ increases, the Yukawa coupling between the doublet and the singlet increases, and hence the $h_1$ (SM-like Higgs) mediated interactions become more and more dominant. It is also clear from figure 12 that, irrespective of the mass difference $\Delta M$, with increasing $\sin \theta_{13}$ the annihilation rates increase making deeper resonance drops. Due to the presence of off-diagonal interactions in all cases, all resonance drops have been somewhat broadened up compared to the case of pure diagonal interactions.
Figure 12: DM relic density as a function of DM mass ($m_{\chi_3}$) for different mass splitting $\Delta M$ between the DM and the NLSP (shown by different coloured patches as indicated in figure inset) for fixed values of $\sin \theta_{13} = 0.01$ (top left panel), $\sin \theta_{13} = 0.1$ (top right panel), $\sin \theta_{13} = 0.3$ (bottom left panel) and $\sin \theta_{13} = 0.5$ (bottom right panel). Correct relic abundance from PLANCK data ($0.1166 \leq \Omega h^2 \leq 0.1206$) is shown by the thick horizontal silver line. The other parameters kept fixed are: $M_{Z_{BL}} = 1.65$ TeV, $g_{BL} = 0.03$, $m_{h_2} = 300$ GeV, $\sin \beta = 0.2$.

In figure 13, the correct relic abundance is plotted in the plane of $\Delta M$ vs $m_{\chi_3}$, where $\Delta M = (m_{\chi_1} - m_{\chi_3})$. Again, the main outcome remains almost similar as before, excepting the presence of additional peak at $m_{\chi_3} = M_{Z_{BL}}/2$ GeV due to $Z_{BL}$ resonance. Other resonances at $m_{Z}/2$, $m_{h_1}/2$, $m_{h_2}/2$ are also visible. Just before the $Z_{BL}$ resonance, we can see the effect of off-diagonal $Z_{BL}$ mediated interactions (see (B.3)).

We note here that in the limit $\sin \theta_{23} \rightarrow 0$ (alongwith $g_{BL} \rightarrow 0$, $\sin \beta \rightarrow 0$ and for very heavy $Z_{BL}$ and $h_2$), figure 13 reduces to figure 5, i.e. $U(1)_{B-L}$ extension boils down to the one without it.

5.6 Direct detection prospects

The DM candidate ($\chi_3$) in this model is a Majorana fermion, hence the $Z$ and $Z_{BL}$-mediated vector current interaction vanishes. Although there is a possibility of spin dependent scattering through axial vector interaction mediated by the vector bosons, the sensitivity and bounds are extremely weak. Therefore the prominent channel for direct detection of $\chi_3$ is through $H - \Phi_{BL}$ mixing, which results in spin-independent scattering of DM off nuclei. The Feynman diagram for such interaction is shown in figure 14. The spin-independent DM-nucleon elastic scattering cross-section is again given by eq. (4.1). However, in contrast to the previous case, here there are two propagators ($h_1$ and $h_2$) that can mediate the DM pair
Figure 13: DM relic density \((0.1166 \leq \Omega_{\text{DM}} h^2 \leq 0.1206)\) allowed parameter space shown in \(\Delta M - m_{\chi_3}\) plane for the \(U(1)_{B-L}\) model. Different coloured points indicate different ranges of \(\sin \theta_{13}\) as specified in the figure inset. The parameters kept fixed for the scan are \(M_{Z_{BL}} = 1.65\) TeV, \(g_{BL} = 0.03\), \(m_{h_2} = 300\) GeV, \(\sin \beta = 0.2\). The shaded region in the bottom left corner is ruled out by LEP exclusion bound on charged fermion mass, \(m_{\psi^\pm} = M > 102.7\) GeV.

production and hence direct detection is through the interference of two diagrams. So in this case the effective coupling strength \(\alpha_q\) is given by:

\[
\alpha_q = \frac{m_q}{v} \left( \frac{\lambda_a \cos \beta}{m_{h_1}^2} - \frac{\lambda_b \sin \beta}{m_{h_2}^2} \right),
\]

where

\[
\lambda_a = \frac{Y_1}{2} (s_{13} + s_{23} c_{13}) c_{13} c_{23} \cos \beta - \frac{y'_1}{2} c_{13}^2 c_{23}^2 \sin \beta,
\]

\[
\lambda_b = -\frac{Y_1}{2} (s_{13} + s_{23} c_{13}) c_{13} c_{23} \sin \beta - \frac{y'_1}{2} c_{13}^2 c_{23}^2 \cos \beta.
\]

In the numerical calculation we use the Yukawa coupling \(Y_1 \approx \Delta M \sin 2\theta_{13}/v\) as given by eq. \((5.23)\) and \(y'_1 = \sqrt{2} M_{R_1}/v_{BL} = 2 \sqrt{2} M_{R_1} g_{BL}/M_{Z_{BL}}\). So the direct search cross-section indirectly depends on \(\Delta M\), \(g_{BL}\) and \(M_{Z_{BL}}\) as well.

The relative minus sign between the two propagators comes from the orthogonal mixing matrix in eq. \((5.13)\). From eqs. \((4.1)\), \((4.2)\), \((4.3)\) and \((5.27)\), the spin-independent scattering cross-section is given by,

\[
\sigma^{SI} = \frac{\mu_r^2}{\pi A^2} \left( \frac{\lambda_a \cos \beta}{m_{h_1}^2} - \frac{\lambda_b \sin \beta}{m_{h_2}^2} \right)^2 Z m_p v \left( f_{T_u} p + f_{T_d} p + f_{T_s} + \frac{2}{9} f_{TG} \right)
\]

\[
+ (A - Z) m_n v \left( f_{T_u} n + f_{T_d} n + f_{T_s} + \frac{2}{9} f_{TG} \right)^2.
\]

Now we turn to the parameter space of the model consistent with direct search constraints. In left panel of figure 15, we have confronted the points satisfying relic density with
Figure 14: Feynman Diagram for elastic scattering of DM off nuclei at terrestrial laboratory in the $U(1)_{B-L}$ extended model.

Figure 15: [Left]: Spin-independent direct detection cross section of DM ($\chi_3$) with nucleon as function of DM mass (in GeV) for $U(1)_{B-L}$ model confronted with XENON-1T data over and above relic density constraint from PLANCK; [Right]: Correct DM relic density allowed parameter space of the model in $\Delta M - m_{\chi_3}$ plane constrained by XENON-1T bound. Different coloured points indicate different ranges of $\sin \theta_{13}$ as mentioned in the figure inset. The parameters kept fixed for the scan are $M_{Z_{BL}} = 1.65$ TeV, $g_{BL} = 0.03$, $m_{h_2} = 300$ GeV, $\sin \beta = 0.2$. The shaded region in the left hand corner of right hand plot is ruled out by LEP exclusion bound on charged fermion mass, $m_{\psi \pm} = M > 102.7$ GeV.

the spin independent elastic cross section obtained for the model as a function of DM mass. The XENON-1T bound is shown by dashed black line. Again, the region below this line satisfy both relic density as well as direct detection constraint. These points (satisfying relic density as well as direct detection constraint from XENON-1T) are shown in the right panel of figure 13 in the $\Delta M - m_{\chi_3}$ plane. Again we see that null observation from direct search crucially tames down the relic density allowed parameter space. The available parameter space of the $U(1)_{B-L}$ model is very similar to that without the gauge extension, excepting for the resonance regions at $m_{\chi_3} = m_{h_1/2}$ and $m_{\chi_3} = M_{Z_{BL}}/2$, where $\Delta M$ can be uncorrelated to DM mass.

5.7 ATLAS bound on $g_{BL} - M_{Z_{BL}}$

We now turn to find the allowed parameter space in the $\Delta M - m_{\chi_3}$ plane in light of ATLAS bound on $g_{BL}$ versus $M_{Z_{BL}}$. In the previous sections we kept $M_{Z_{BL}}$ fixed at 1650 GeV corresponding to $g_{BL} = 0.03$ compatible with ATLAS data [65]. As a result of choosing such a small value of $g_{BL}$, the effect of $Z_{BL}$ was only evident at resonance when $m_{\chi_3} \sim M_{Z_{BL}}/2$ (see fig-
Figure 16: [Left]: Parameter space satisfying relic density constraint from PLANCK ($0.1166 \leq \Omega_{DM} h^2 \leq 0.1206$) in the plane of $g_{BL} - M_{Z_{BL}}$ for $U(1)_{B-L}$ model; [Right]: Parameter space satisfying both relic density constraint from PLANCK and direct detection bound from XENON-1T in the plane of $g_{BL} - M_{Z_{BL}}$. The thick silver line shows the ATLAS bound on $g_{BL}$ vs $M_{Z_{BL}}$ [65] plane from non-observation of $Z_{BL}$ in collider data.

In the following we highlight the effect of $Z_{BL}$ mediated diagrams by varying the coupling and mass. We perform a scan by varying the model parameters in the following range:

$$
\begin{align*}
1 \text{ GeV} \leq m_{\chi_3} & \leq 2000 \text{ GeV} \\
1 \text{ GeV} \leq \Delta M & \leq 1000 \text{ GeV} \\
20 \text{ GeV} \leq M_{Z_{BL}} & \leq 4000 \text{ GeV} \\
0.001 \leq \sin \theta_{13} & \leq 0.6 \\
0.001 \leq g_{BL} & \leq 0.3 .
\end{align*}
$$

Other parameters kept fixed are: $\sin \beta = 0.2$ and $m_{\nu_2} = 300 \text{ GeV}$. Also the masses of the two $Z_2$ even right handed neutrinos are kept fixed as $M_{R_2/3} = 500 \text{ GeV}$.

We first show the constraint coming from non-observation of a new gauge boson ($Z_{BL}$) at LHC coming from ATLAS [65] analysis on $g_{BL}$ for corresponding values of $M_{Z_{BL}}$ shown by the silver thick line in figure 16. This indicates that points below the line with smaller $g_{BL}$ is allowed, while those above the line are discarded. The left plot shows points which satisfy relic density constraint from PLANCK ($0.1166 \leq \Omega_{DM} h^2 \leq 0.1206$) data and right plot shows the points which satisfy both relic density and direct search bounds from XENON 1T. Different colours indicate ranges of $\sin \theta_{13}$ as mentioned in figure inset. We then showcase the fate of the model when the bound from ATLAS is implemented on the parameter space in $\Delta M$ vs $m_{\chi_3}$ plane for different $g_{BL}$ values in figure 17. In the top panel we show the available parameter space in terms of different ranges of $\sin \theta_{13}$, while the same is shown in bottom panel for different ranges of $g_{BL}$ coupling for relic density and direct search allowed parameter space of the $U(1)_{B-L}$ model. For clarity in inferring how much parameter space gets discarded by the ATLAS bound, in the left panel we show relic density and direct search allowed points without ATLAS bound, while on the right panel, we show those after incorporating ATLAS bound [65].

We see from figure 17 when $\Delta M \lesssim 10 \text{ GeV}$, the contribution to relic density comes from annihilation, coannihilation and $Z_{BL}$ resonance with relatively smaller $g_{BL}$. As we go for further larger $\Delta M$, the coannihilation contribution to relic density decreases gradually.
Figure 17: [Top Left]: Parameter space satisfying relic density (PLANCK) and direct search (XENON-1T) bound in $\Delta M - m_{\chi^3}$ plane, different colours indicate different choices of $\sin \theta_{13}$; [Top Right]: Same as top left but additionally ATLAS bound on $g_{\text{BL}} - M_{Z_{\text{BL}}}$ [65] applied; [Bottom Panel]: Same as in the top panel, but different coloured points indicate different ranges of $g_{\text{BL}}$ coupling as mentioned in figure inset, with left (right) plot without respecting (with) ATLAS bound. The shaded region in the bottom left corner is ruled out by LEP exclusion bound on charged fermion mass, $m_{\psi^\pm} = M > 102.7$ GeV.

and gets compensated by $Z_{\text{BL}}$ exchange diagrams with increasing values of $g_{\text{BL}}$. Beyond $m_{\chi^3} = 1000$ GeV, the correlation between $\Delta M$ and $m_{\chi^3}$ is lost and relic is mostly dominated by Higgs and $Z_{\text{BL}}$ mediation. In the right panel of figure 17, we impose bound on $g_{\text{BL}} - M_{Z_{\text{BL}}}$ from ATLAS data. The upper bound on $g_{\text{BL}}$ by ATLAS data for lighter $Z_{\text{BL}}$ is extremely small (for eg., $M_{Z_{\text{BL}}} \sim O(1\text{TeV})$, upper bound on $g_{\text{BL}} \sim 0.009$ [68]). Consequently if we have to satisfy ATLAS bound, then all those resonance points with large $g_{\text{BL}}$ in the left panel of figure 17 upto $m_{\chi^3} \sim 500$ GeV are no longer there in the right panel of figure 17. It is only when $M_{Z_{\text{BL}}}$ becomes sufficiently large, so that $g_{\text{BL}}$ can take somewhat moderate values, we can see the $Z_{\text{BL}}$ resonance affects. That is why in the right panel of figure 17, such resonance points survive for $m_{\chi^3} \geq 500$ GeV. For $m_{\chi^3} > 1000$ GeV, the points which survive the ATLAS bound are mostly due to $Z_{\text{BL}}$ resonances with relatively large $g_{\text{BL}}$. Note that the direct detection cross section has mild dependency on these resonance points. Therefore, these resonance points for $m_{\chi^3} > 1000$ GeV also easily survive from XENON-1T bound.
Both the model frameworks studied here, have attractive signatures at the Large Hadron Collider (LHC) due to the presence of SM isodoublet. There exists different types of production processes and decay final states which can be categorized broadly into leptonic and hadronic final states. Leptonic final states are favoured over hadronic states for less SM contamination. All the heavier dark sector particles finally decay into the DM ($\chi^3$), which is missed in the detector and necessarily associate each final state with missing transverse energy ($E_T$) defined as:

$$E_T = -\sqrt{\left(\sum_{\ell,j,unc} p_x\right)^2 + \left(\sum_{\ell,j,unc} p_y\right)^2},$$

where the sum runs over all visible objects that include leptons ($\ell = e, \mu$) and jets, and unclustered components. Here we list some of the most important leptonic final states that the models offer. We will refer to the model without $U(1)_{B-L}$ as model I and the one with $U(1)_{B-L}$ extension as model II.

**Opposite sign dilepton ($\ell^+ \ell^- + E_T$).** The heavy charged component of $SU(2)_L$ doublet, $\psi^\pm$ (NLSP) can be produced via $(Z, \gamma)$ mediation in model I and $(Z, \gamma, Z_{BL})$ mediation in model II. Further they decay to leptonic final state via on-shell or off-shell $W^\pm$ mediator (depending on mass splitting $m_{\psi^\pm} - m_{\chi^3}$) and stable DM ($\chi^3$). As a result the process yields hadronically quiet opposite sign dilepton (OSD) plus missing energy ($\ell^+ \ell^- + E_T$) signature at collider as shown in the Feynman graph in the left panel of figure 18:

$$\text{OSD} + E_T: \quad p p \to \psi^+ \psi^-, (\psi^- \to \ell^- \nu_\ell \chi^3), \quad (\psi^+ \to \ell^+ \nu_\ell \chi^3); \quad \ell = \{e, \mu\}.$$

Also the production of $\chi_i \chi^3$ pair via $Z$ propagator in model I and $Z, Z_{BL}$ propagator in model II gives rise to OSD final state as shown in right panel of figure 18:

$$\text{OSD} + E_T: \quad p p \to \chi_i \chi^3 \quad (\chi_{1,2} \to \ell^- \ell^+ \chi^3); \quad \ell = \{e, \mu\}, \quad i = \{1, 2\}.$$

It is important to note that in model I, $i = 2$ is the only possibility [see appendix A and eq. (A.2) in particular]. Also note that the production of the heavy neutral components as
Three leptons ($\ell\ell\ell + E_T$). Hadronically quiet trilepton plus missing energy signature can be obtained from the production of heavy neutral, $\chi_{1,2}$ and charged fermions states, $\psi^\pm$ via $W^\pm$ mediator as shown in figure 19:

$$3\ell + E_T : \ p \ p \to \psi^\pm \chi_i, \ (\psi^\pm \to \ell^\pm \nu_\ell(p_T) \chi_3), \ (\chi_i \to \ell^- \ell^+ \chi_3); \ \ell = \{e, \mu\} \ i = \{1, 2\}.$$ 

Again, it is worth noting that although the production process is same in both model I and model II, subsequent decay of $\chi_i \to \chi_3 Z^*$ is only allowed for $\chi_2$ in model I and provides a way of distinguishing the two cases. The fact that no significant excess in hadronically quiet trilepton events are observed at LHC and the result agrees to SM contribution to a great extent puts a bound on the relevant parameters. From ATLAS data, following constraints can be obtained: $m_{\chi_{1,2}}, m_{\psi^\pm} < 270$ GeV, $m_{\chi_3} \lesssim 70$ GeV with BR $(\chi_{1,2} \to Z \chi_3) \gtrsim 60\%$ [83]. We may note that similar trilepton signature can also arise from Higgsino-Bino production in supersymmetric models, which have been studied in context of LHC data [72].

Four leptons ($\ell\ell\ell\ell + E_T$). The heavy neutral fermionic DM states, $\chi_{1,2}$ (NLSP) can be produced at LHC via $Z$ propagator in model I and $Z, Z_{BL}$ propagator in model II. The heavy states, $\chi_{1,2}$ further decay to leptonic final states via $Z$ and produce four leptons plus missing energy signature as shown in figure 20:

$$\ell\ell\ell\ell + E_T : \ p \ p \to \chi_i \chi_j, \ (\chi_{i,j} \to \ell^- \ell^+ \chi_3); \ \ell = \{e, \mu\}; \ i, j = \{1, 2\}.$$
Figure 20: $\ell\ell\ell\ell + E_T$ signal in model II at LHC ($i, j = \{1, 2\}, i \neq j$).

We should note here that there are two main issues of producing four lepton states: (i) We need to produce $\chi_1\chi_2$ pair, (ii) then $\chi_{1,2}$ both needs to decay via $Z$ to $\chi_3$. Now from interaction vertex in appendix A and eq. (A.2), we see that the decay of $\chi_1$ can’t occur to $\chi_3 Z^*$ unless the model is extended by $U(1)_{B-L}$ [see appendix B and eq. (B.2)], thus making the signal exclusive for the $U(1)_{B-L}$ extension. Apart, one may also have hadronically quiet six lepton states arising from the decay of $\chi_1 \rightarrow \chi_2 Z^*, \chi_2 \rightarrow \chi_3 Z^*$, followed by leptonic decays of the off-shell $Z$ from the same production process for both models with $U(1)_{B-L}$ extension and without that.

**Single lepton with jets ($\ell^\pm + jj + E_T$).** The leptons in the final state arise out of $W$ and $Z$ boson decays (see figures 18, 19, 20), which anyway could also decay to quark antiquark pair to yield jets. Therefore, apart from purely leptonic signatures, one may also have hadrons or jet-rich final states. For example, the charged fermion pair production can lead to single lepton with two jets plus missing energy signature when one off shell $W$ decays to hadronic final state (see figure 18). Obviously when both $W$ decay hadronically, one ends up with four (or more) jets. LHC being a QCD machine, hadronic final states are prone to huge SM QCD background and therefore disfavoured. In event analysis, segregating signal from SM background is an important task. Missing energy variable as introduced in eq. (6.1) play a crucial role, as in SM contributions to $E_T$ mainly arise from neutrinos and mistagging.

**Displaced vertex signature of $\psi^\pm$.** We already observed that a large region of available parameter space of the model relies on small $\Delta M$ (for example, see in the right panel of figure 8). The decay of $\psi^\pm$ is then phase space suppressed and can produce a displaced vertex, which can serve as a very crucial signature of the model. The decay length in its rest frame (following from eq. (3.6)) is given by,

$$L_0 = \frac{1.9 \times 10^{-2}}{(\Delta M^{\text{GeV}})^3 \sin \theta} \text{ cm}.$$

In figure 21, we show the decay length of $\psi^\pm$ as a function of $\Delta M$ for fixed $\sin \theta$ values depicted in different colours. We see that for $\Delta M < 10 \text{ GeV}$, the displaced vertex of $\psi^\pm$ can be significantly large to be detected at the collider. On the other hand, non-observation of a displaced vertex or a charge track will result in a bound on $\Delta M - \sin \theta$ plane.

**Effect of B-L gauge extension in $\psi^+ \psi^-$ pair production.** The effect of $U(1)_{B-L}$ gauge boson ($Z_{BL}$) mediation in $pp \rightarrow \psi^+ \psi^-$ production [84] is an important question and we
Figure 21: The decay length of $\psi^\pm$ as a function of mass difference $\Delta M$ for fixed $\sin \theta$ values.

Figure 22: [Left] The production cross-section of $\psi^+ \psi^-$ pairs at collider is shown as a function of $U(1)_{B-L}$ gauge boson mass, $M_{Z_{BL}}$ for fixed $m_{\psi^\pm} = 150$ GeV. Different coloured lines depict different cases: SM production cross-section is shown by black solid line; $U(1)_{B-L}$ case is shown for $g_{BL} = 0.03$ (green dashed line) and $g_{BL} = 0.3$ (red dashed line). [Right] The production cross-section of $\psi^+ \psi^-$ pairs at collider is shown as a function of $m_{\psi^\pm}$ with $M_{Z_{BL}} = 1.65$ TeV, $g_{BL} = 0.03$ (green dashed line) and $M_{Z_{BL}} = 4.4$ TeV, $g_{BL} = 0.3$ (red dashed line). Pure SM gauge boson mediated production cross-section (Model I) is also shown in black solid line.

discuss the main features here. We summarise our observations in figure 22. In the left panel of figure 22, we have shown the production cross-section of $\psi^+ \psi^-$ pair at LHC as a function of $M_{Z_{BL}}$ for fixed $m_{\psi^\pm} = 150$ GeV. On the right panel, we plot the production cross-section of $\psi^+ \psi^-$ as function of $m_{\psi^\pm}$, for two different combinations of $Z_{BL}$ parameters: \{$M_{Z_{BL}} = 1.65$ TeV, $g_{BL} = 0.03$\} (green dashed line) and \{$M_{Z_{BL}} = 4.4$ TeV, $g_{BL} = 0.3$\} (red dashed line), which agree to the current ATLAS bound. Only SM contribution with $Z_{SM} : (Z, \gamma)$ mediation is also shown by black solid line for comparison. It is evident from the figure 22, that for the smaller value $g_{BL} = 0.03$, the contribution from $Z_{BL}$ mediated production is negligible compared to SM and consequently green dashed and black lines fall on top of each other. However, with a moderate value of $g_{BL} = 0.3$, the production cross-section significantly improves with $Z_{BL}$ mediation, which is seen in red dashed line clearly separated from the other two. In the left plot we see that the effect of s-channel resonance in amplitude
∼ \frac{1}{s-M_{ZBL}^2} \) showing up at \( M_{ZBL} = 2m_{\psi^\pm} = 300\,\text{GeV} \) as the minimum subprocess center-of-mass energy required for this process to occur is \( \sqrt{s} = 300\,\text{GeV} \) with \( m_{\psi^\pm} = 150\,\text{GeV} \). The resonance is extended to account its finite decay width

\[ \sim \frac{1}{s-M_{ZBL}^2+iM_{ZBL} \Gamma_{ZBL}}. \]

The same effect is seen on the right panel plot where the resonance rise is visible at \( m_{\psi^\pm} = \frac{M_{ZBL}}{2} \sim 2\,\text{TeV} \) for the red dashed curve \( (g_{BL} = 0.3) \). To summarise, the effect of \( Z_{BL} \) mediation for the production of \( \psi^\pm \) pair, which contributes to opposite sign dilepton (OSD) plus missing energy signal, can only be realised at relatively larger values of gauge coupling \( g_{BL} \) and on-shell \( Z_{BL} \) production whenever possible, albeit that current experimental bound requires a higher \( Z_{BL} \) mass with larger \( g_{BL} \) coupling (see figure 16).

**Hadronically quiet OSD events at LHC.** We shall now briefly discuss the event level simulation for the OSD signal \( (\ell^+\ell^- + H_T) \) and estimate SM background contamination for the same final state. Our elaboration will be more indicative than exhaustive. For that, we refer to two different benchmark points with \( \Delta M = 15\,\text{GeV} \) and \( 300\,\text{GeV} \) keeping DM mass fixed at \( m_{\chi_3} = 150\,\text{GeV} \); important to note here that the first case applies to the model I without \( B-L \) extension where the second possibility with larger \( \Delta M \) is only allowed in model II with \( B-L \) extension (compare figure 8 to figure 17). For the analysis we generate the lhe file from the model implementation in FeynRule \[57\] and run it in Madgraph \[85\] to generate events and finally pass onto Pythia \[86\] for analysis. Following basic techniques are used in Pythia to mimic the actual collider environment:

- **Lepton isolation:** To identify a lepton \( (\ell = e, \mu) \) in the detector, one requires a minimum transverse momentum, which we keep as \( p_T > 20\,\text{GeV} \). We also require the pseudorapidity within \( |\eta| < 2.5 \), which ensures that leptons ejected centrally can only be observed in the detector. Separation of leptons from each other requires \( (\Delta R)^{\ell\ell} \geq 0.2 \) in \( \eta - \phi \) plane (where \( \Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \)). We further imposed \( (\Delta R)^{\ell j} \geq 0.4 \) to separate leptons from jets.

- **Jet identification:** Defining a jet \( (j) \) is an important issue at LHC environment. In the numerical simulation performed here, jets are formed in Pythia using cone algorithm inbuilt in PYCELL. A jet is then identified with all parton within a cone of \( \Delta R \leq 0.4 \) around a jet initiator with \( p_T > 20\,\text{GeV} \). We will finally require zero jet veto to ensure hadronically quiet final state.

- **Unclustered objects:** The unclustered objects consist of those objects, which neither qualify as jets nor identified as isolated leptons (following our previous definitions) and only contribute to missing energy. All final state objects with smaller transverse momentum \( 0.5 < p_T < 20\,\text{GeV} \) and larger pseudorapidity \( 2.5 < |\eta| < 5 \) are therefore identified as unclustered objects.

Three kinematic variables play a key role in the analysis: Missing Energy \( (E_T) \), Transverse Mass \( (H_T) \) and Invariant mass \( (m_{\ell\ell}) \); where the signal and background show different sensitivity. Missing energy has already been defined (eq. (6.1)), the other two are:

- **Transverse Mass \( (H_T) \).** Transverse mass of an event is identified to:

\[ H_T = \sum_{\ell,j} \sqrt{(p_x)^2 + (p_y)^2} = \sum_{\ell,j} p_T, \] (6.2)
Figure 23: Distribution of missing energy ($\hat{E}_T$) and transverse mass ($H_T$) for signal events and dominant SM background events at LHC with $\sqrt{s} = 14$ TeV.

where the scalar sum of transverse momentum runs over reconstructed objects like leptons ($\ell$) and jets ($j$).

• Invariant mass ($m_{\ell\ell}$). Invariant mass of opposite sign dilepton is defined by

$$m_{\ell^+\ell^-} = \sqrt{\left(\sum_{\ell^+\ell^-} p_x\right)^2 + \left(\sum_{\ell^+\ell^-} p_y\right)^2 + \left(\sum_{\ell^+\ell^-} p_z\right)^2}. \quad (6.3)$$

The normalised event distribution for OSD signal events $\ell^+\ell^- + (\hat{E}_T)$ at the two benchmark points with dominant SM background events are shown in figure 23 with missing energy ($\hat{E}_T$) in the left panel and transverse mass ($H_T$) on the right panel. In both graphs, we note that the peak for $\Delta M = 15$ GeV appear on the left side of SM background, while the one for $\Delta M = 300$ GeV is flatter and shifted towards high $\hat{E}_T/H_T$ value. It is then quite apparent, that segregating these two signals from SM background requires different selection cuts on $\hat{E}_T$, $H_T$ and $m_{\ell\ell}$, which are chosen as follows:

• Invariant mass ($m_{\ell\ell}$) cut: $m_{\ell\ell} < (m_Z - 15)$ GeV and $m_{\ell\ell} > (m_Z + 15)$ GeV is imposed to get rid of SM $Z$ boson contribution to OSD final state.

• $\hat{E}_T$ and $H_T$ cuts:
  
  - $\hat{E}_T < 30$ GeV, $H_T < 70$ for $\Delta M = 15$ GeV $< m_{W^\pm}$.
  - $\hat{E}_T > 100$ GeV, $H_T > 150$ for $\Delta M = 300$ GeV $> m_{W^\pm}$.

After imposing above cut-flow we list the signal and dominant SM background events in table 3 and table 4 respectively for luminosity $100 \text{fb}^{-1}$. We see that $W^+W^-$ production provides the most significant background for OSD at LHC, which couldn’t be tamed by the cuts used. This is surely the key reason for not being able to observe any signal excess over the huge SM background at LHC. The numbers of signal and SM background events thus obtained can provide the discovery reach of the signal for two benchmark points in terms of significance defined as $\sigma = \frac{S}{\sqrt{S+B}}$, where $S$ denotes signal events and $B$ denotes SM background events, shown as a function of luminosity in figure 24. It shows that 5$\sigma$ discovery reach is difficult to achieve for the model I without $U(1)_{B-L}$ characterised by low $\Delta M$ ($\mathcal{L} \sim 1500$ fb$^{-1}$), while the case with large $\Delta M$ in model II with $U(1)_{B-L}$ extension can be probed in near future with $\mathcal{L} \sim 150$ fb$^{-1}$. 

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### Table 3: Signal $(\ell^+\ell^- + (E_T))$ cross-section for the chosen benchmark points for $\sqrt{s} = 14$ TeV at LHC with luminosity $\mathcal{L} = 100 \text{ fb}^{-1}$ in Model I (without $B-L$) and Model II (with $B-L$) after the selection cuts employed.

| Model     | $m_{\chi^3}$ (GeV) | $\Delta M$ (GeV) | $\sigma_{\ell^+\ell^-X}$ (fb) | $E_T$ (GeV) | $H_T$ (GeV) | $\sigma_{\text{eff}}^{\ell^+\ell^-X}$ (fb) | $N_{\text{eff}}^{\ell^+\ell^-X}$ (@$\mathcal{L} = 10^2 \text{ fb}^{-1}$) |
|-----------|---------------------|------------------|-------------------------------|-------------|-------------|-------------------------------------------|--------------------------------------------------|
| Model I   | 150                 | 15               | 392.37                        | < 30        | < 70        | 6.23                                      | 623                                              |
| Model II  | 150                 | 300              | 9.48                          | > 30        | > 70        | 131.18                                    | 13118                                             |

### Table 4: Dominant SM background contribution to $\ell^+\ell^- + (E_T)$ signal events for $\sqrt{s} = 14$ TeV at LHC for luminosity $\mathcal{L} = 100 \text{ fb}^{-1}$. The SM background cross-section are quoted with next-to-leading order (NLO) level with appropriate K-factors [87].

| SM Bkg.     | $\sigma_{\ell^+\ell^-X}$ (fb) | $E_T$ (GeV) | $H_T$ (GeV) | $\sigma_{\text{eff}}^{\ell^+\ell^-X}$ (fb) | $N_{\text{eff}}^{\ell^+\ell^-X}$ (@$\mathcal{L} = 10^2 \text{ fb}^{-1}$) |
|-------------|---------------------------------|-------------|-------------|-------------------------------------------|--------------------------------------------------|
| $t\bar{t}$  | $36.69 \times 10^3$            | < 30        | < 70        | 6.23                                      | 623                                              |
| $W^+ W^-$   | $4.74 \times 10^3$             | > 100       | > 150       | 131.18                                    | 13118                                             |
| $Z Z$       | $0.25 \times 10^3$             | < 30        | < 70        | 0.53                                      | 53                                               |
| $W^+ W^- Z$ | 1.00                            | > 100       | > 150       | 0.01                                      | 1                                                |

### Figure 24: Signal significance $\sigma = \frac{S}{\sqrt{S+B}}$ of the benchmark points characteristic to model I (in blue) and model II (in red) at LHC with $\sqrt{s} = 14$ TeV, in terms of luminosity (fb$^{-1}$), subject to the selection criteria imposed in this analysis. 3σ and 5σ reach.
7 Non-zero masses and mixing of light neutrinos

The very construct of this model is motivated by the fact that we wish to have phenomenologically viable WIMP like DM and non-zero masses and mixing of light neutrinos in a minimal extension of the SM. This could be achieved by the presence of three RH neutrinos. While, one of them constitute the dark sector being odd under a stabilising $Z_2$ symmetry, the other two can contribute to neutrino sector. In this model, a tiny yet non-zero neutrino mass can be generated via Type I seesaw from the following terms in the Lagrangian (2.1),

$$- \mathcal{L}^\nu_{\text{mass}} \supset \left( Y_{j\alpha} \overline{N_{Rj}} H^\dagger L_{\alpha} + \text{h.c.} \right) + \left( \frac{1}{2} M_{Rj} \overline{N_{Rj}} (N_{Rj})^c + \text{h.c.} \right);$$

where $\alpha = e, \mu, \tau$ and $j = 2, 3$. After EW symmetry breaking, the SM Higgs acquires a vev to generate the Dirac mass terms for the neutrinos. In the gauged B-L scenario, the mass of all three right handed neutrinos are generated through the vev of the scalar $\Phi_{\text{BL}}$. So for simplicity we can consider the mass of two $Z_2$ even right handed neutrinos that take part in the seesaw to be quasi-degenerate and of the same mass scale as that of the $Z_2$ odd right handed neutrino taking part in the dark sector phenomenology. Without loss of generality, we assume the heavy Majorana mass matrix that take part in seesaw to be diagonal, i.e., $M_R = \text{Diag}(0, M_{R2}, M_{R3})$. In this basis, the light neutrino mass matrix obtained through Type-I seesaw is given as,

$$m_\nu = -m_D M_R^{-1} m_D^T$$

which is a complex $3 \times 3$ matrix and can be diagonalized by the PMNS matrix [88] as,

$$(m_\nu)^{\text{diag}} = U^T m_\nu U$$

where $(m_\nu)^{\text{diag}} = \text{Diag}(m_1, m_2, m_3)$ contains at least one zero eigenvalue.

The PMNS matrix $U$ is given by:

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13} \end{pmatrix} U_{ph}$$

where $c_{ij}$ and $s_{ij}$ stand for $\cos \theta_{ij}$ and $\sin \theta_{ij}$ respectively and $U_{ph}$ is given by:

$$U_{ph} = \text{Diag}(1, e^{-i \alpha/2}, 1)$$

where $\alpha$ is the CP-violating Majorana phase.

Using Casas-Ibarra parameterization [89], the Dirac mass matrix $m_D$ can be parametrized as,

$$(m_D)_{j\alpha} = \sqrt{M_{Rj} R_{ji}} \sqrt{m_i} U_{\alpha i}^T$$

where $m_i$ are the eigenvalues of the light neutrino mass matrix $m_\nu$ and $R$ is in general a $3 \times 3$ complex orthogonal matrix. Since in our case, $N_{R1}$ is decoupled from the spectrum, the corresponding Yukawa coupling $Y_{1\alpha}$ for a particulair flavour $\alpha$ in the Dirac mass matrix given by eq. (7.6) is zero, i.e.,

$$Y_{1\alpha} = \frac{1}{v} \left( \sqrt{M_{R1} R_{11}} \sqrt{m_1} U_{\alpha 1}^T \right) = \frac{1}{v} \left( \sqrt{M_{R1} R_{11}} \sqrt{m_1} U_{1\alpha}^T + \sqrt{M_{R1} R_{12}} \sqrt{m_2} U_{2\alpha}^T + \sqrt{M_{R1} R_{13}} \sqrt{m_3} U_{3\alpha}^T \right) = 0$$
At present, the oscillation experiments measure two mass square differences: namely solar ($\Delta m^2_{\odot}$) and atmospheric ($\Delta m^2_{\text{atm}}$) along with three mixing angles $\theta_{23}$, $\theta_{12}$ and $\theta_{13}$. Data indicates that $|\Delta m^2_{\text{atm}}| \gg \Delta m^2_{\odot}$, but depending on the sign of $\Delta m^2_{\text{atm}}$, two cases can arise.

- **Normal Hierarchy (NH):**

\[
\begin{align*}
\begin{cases}
m_1 &= 0 \\
m_2 &= \sqrt{\Delta m^2_{\odot}} \ll m_3 = \sqrt{\Delta m^2_{\text{atm}}}
\end{cases}
\end{align*}
\]  

(7.8)

In Normal Hierarchy (NH), the lightest mass eigenstate $m_1 = 0$. So, in order for l.h.s. of eq. (7.7) i.e., $Y_{1\alpha}$ to vanish, $R_{12}$ and $R_{13}$ must be zero, since $m_2$ and $m_3$ are non-zero. The orthogonality of $R$ then implies that $R_{11} = 1$ and $R_{21} = 0 = R_{31}$. The four remaining elements of $R$ viz., $R_{22}, R_{23}, R_{32}$ and $R_{33}$, form a $2 \times 2$ complex orthogonal matrix, defined by one complex angle $z$ [90]. Thus the structure of $R$ matrix in case of NH is reduced to the simple form:

\[
R = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos z & -\sin z \\
0 & \sin z & \cos z
\end{pmatrix}
\]  

(7.9)

The neutrino Dirac mass matrix obtained has the form:

\[
m_D = v \begin{pmatrix}
0 & 0 & 0 \\
Y_{2e} & Y_{2\mu} & Y_{2\tau} \\
Y_{3e} & Y_{3\mu} & Y_{3\tau}
\end{pmatrix}
\]  

(7.10)

where each element $Y_{\alpha j}$ of $m_D$ is given by eq. (7.6).

- **Inverted Hierarchy (IH):**

\[
\begin{align*}
\begin{cases}
m_3 &= 0 \\
m_1 &= \sqrt{\Delta m^2_{\text{atm}}} \\
m_2 &= \sqrt{\Delta m^2_{\text{atm}}} + \Delta m^2_{\odot}
\end{cases}
\end{align*}
\]  

(7.11)

In the case of Inverted Hierarchy (IH), we need to set $m_3 = 0$. So in order for l.h.s. of eq. (7.7) i.e., $Y_{1\alpha}$ to vanish, $R_{11}$ and $R_{12}$ must be zero. Again, orthogonality of $R$ demands $R_{13} = 1$ making the first row and the third column of $R$ trivial. The four remaining elements of $R$ viz., $R_{21}, R_{22}, R_{31}$ and $R_{32}$ then form a $2 \times 2$ complex orthogonal matrix, defined by one complex angle $z$. Thus the structure of $R$ matrix in case of IH is given by:

\[
R = \begin{pmatrix}
0 & 0 & 1 \\
\cos z & -\sin z & 0 \\
\sin z & \cos z & 0
\end{pmatrix}
\]  

(7.12)

Again we a get a Dirac mass matrix of the same structure as that of NH case, with each element given by eq. (7.6).
where, $\alpha_w$ is the weak coupling strength, $s_w$ is the sin of Weinberg’s angle, $m_\mu$ is the muon mass, $M_W$ is the mass of W boson and $\Gamma_\mu \approx 2.996 \times 10^{-19}$ GeV denotes the total decay width of muon. The factor $G^\mu\nu$ is given by,

$$G^\mu\nu = \sum_i U_{e\alpha i} U_{\mu i}^* G_\gamma(x_i) = \sum_j U_{eN_j} U_{\mu N_j}^* G_\gamma(x_N)$$

(7.14)

where, $x_i = \frac{m_i^2}{M_W^2}$ and $x_{N_j} = \frac{M_{N_j}^2}{M_W^2}$, where $i (j)$ runs over total number of light (heavy) physical neutrino states. $m_\nu(M_N)$ denotes the mass of light (heavy) physical neutrinos and $U_{e\alpha}(U_{\mu\alpha})$ represents the mixing matrix elements of light (heavy) neutrinos. The loop integration factor $G_\gamma(x)$ is given by,

$$G_\gamma(x) = \frac{x(2x^2 + 5x - 1)}{4(1 - x^3)} - \frac{2x^3}{2(1 - x^4)} \ln(x)$$

(7.15)

To study the dependence of this branching ratio on the right handed mass scale in light of Casas-Ibarra parameterization, we derive from (7.13) the following equation,

$$Br(\mu \rightarrow e\gamma) = \frac{\alpha_w^2 s_w^2}{256 \pi^2} \frac{m_\mu^4 m_\mu^4}{M_W^4 \Gamma_\mu M_R^2} G_\gamma^2(x_N)(|m_D^\dagger m_D|_{e\mu})^2$$

(7.16)

where $M_R$ denotes the mass of the right handed neutrino states. For simplicity we assume the two right handed neutrinos to be degenerate and $M_N = M_R$. The matrix element $(m_D^\dagger m_D)_{e\mu}$ for NH and IH respectively can be written using eq. (7.6) as,

$$\left.(m_D^\dagger m_D)_{e\mu}\right|_{NH} = M_R [(m_2 U_{e2} U_{\mu2}^* + m_3 U_{e3} U_{\mu3}^*) \cosh(2 \Im[z])$$

$$+ i \sqrt{m_2 m_3} (U_{e3} U_{\mu2}^* - U_{e2} U_{\mu3}^*) \sinh(2 \Im[z])]$$

(7.17)

$$\left.(m_D^\dagger m_D)_{e\mu}\right|_{IH} = M_R [(m_1 U_{e1} U_{\mu1}^* + m_2 U_{e2} U_{\mu2}^*) \cosh(2 \Im[z])$$

$$+ i \sqrt{m_1 m_2} (U_{e2} U_{\mu1}^* - U_{e1} U_{\mu2}^*) \sinh(2 \Im[z])]$$

(7.18)

where $U_{e\alpha}$ are the PMNS matrix elements parametrized as in eq. 7.4. In eq. (7.17) and 7.18, there are three free parameters namely $M_R$, $\Im[z]$ and $\alpha$; all other quantities being measured by oscillation experiments within a range. In left panel of figure 25, we have shown the $Br(\mu \rightarrow e\gamma)$ as a function of heavy neutrino mass $M_R$ and $\Im[z]$ taking all oscillation parameters within their $3\sigma$ range as given in [96, 97] in case of NH. The Majorana phase $\alpha$ is varied between 0 to $2\pi$. and the amplitude $|(m_D^\dagger m_D)_{e\mu}|$ is almost independent of phase $\alpha$. We confronted our result with current most stringent bound from MEG experiment $Br(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13}$ [98], represented by the contour in black colour in left panel of figure 25. The red contour shows the projected MEG-II sensitivity of $Br(\mu \rightarrow e\gamma) \sim 6 \times 10^{-14}$. The region above the black contour is ruled out by MEG experiment while the region below this contour provides us a wide allowed parameter space for $Br(\mu \rightarrow e\gamma)$ in the $M_R - \Im[z]$ plane simultaneously satisfying MEG limit and low scale neutrino phenomenology. Similar
result has been obtained for IH as well. In the right panel of figure 25, we have shown log[Br(µ → eγ)] for two particular values of Im[z], Im[z] = 0 and 10.

In the simplest scenario Im[z] = 0, the branching ratio is very less than the current sensitivity of worlds leading experiments like MEG for both NH and IH. For Im[z] = 10, the branching ratio is near to the current sensitivity. For intermediate values of Im[z], Br(µ → eγ) is below the current bound by MEG experiment, while for Im[z] > 10, Br(µ → eγ) is above the MEG limit for almost all mass range of MR upto 1000 GeV. As it can also be seen from the left panel of figure 25 that only for MR ≤ 10 GeV, Im[z] can take values upto 14. Naturalness and vacuum stability bounds can also be applied in principle as done in [99], but these bounds are extremely weaker for MR upto TeV scale.

8 Conclusion

In this paper, we have studied a minimal extension of the SM by adding a vector like fermion doublet Ψ and three singlet right handed neutrinos NRi to simultaneously address non-zero masses and mixing of light neutrinos as well as a phenomenologically viable dark matter component of the universe. An additional Z2 symmetry is required on top of the SM gauge symmetry to ensure the stability of the DM. Now, the Z2 symmetry crucially distinguishes the added fermions; for example, the vector-like fermion doublet and one of the three right handed neutrinos are assumed odd, while the rest are even. As a result the dark matter emerges as the lightest Majorana fermion from the mixture of the neutral component of the doublet Ψ and the singlet, which is odd under the same Z2. The other two right handed neutrinos being even under the Z2 symmetry couple to SM Higgs and generate non-zero masses for light neutrinos via type-I seesaw. The absence of either the doublet or the singlet (odd under Z2) make the DM absurdly constrained from relic density and direct search prospects. Therefore, one can simply see that the model under study is possibly the most economical one to simultaneously address neutrino mass and a phenomenologically viable DM candidate of the universe.
We studied the allowed parameter space of the model taking into account all annihilation and co-annihilation channels for DM mass ranging from 1 GeV to 1 TeV. The allowed parameter space is shown in the $\Delta M \sim m_{\chi_3}$ plane, where $m_{\chi_3}$ is the mass of the dark matter and $\Delta M$ is its mass difference with next to lightest dark sector particle. We confronted our results with recent data from both PLANCK and XENON-1T to obtain the correct parameter space satisfying both relic density and direct detection constraints. Since the DM is Majorana in nature, it escapes from the strong $Z$-mediated direct detection constraint. As a result we end up with relatively large singlet-doublet mixing. In particular, for DM mass of 1 TeV, the allowed singlet-doublet mixing can be as large as $\sin \theta \sim 0.6$. This crucially distinguishes the Majorana singlet-doublet DM from a vector like singlet-doublet DM. This feature also hasn’t been highlighted in earlier analysis of a similar model framework.

Since with three right handed neutrinos, the model is qualified for a anomaly-free gauged B-L extension, we studied how our results change in light of $U(1)_{B-L}$ gauge extension. Clearly, the model requires an additional complex scalar singlet to break the gauge group and the massive gauge boson $Z_{BL}$, further enhances the DM-SM coupling. The relic density allowed parameter space additionally enhances due to $Z_{BL}$ resonance in regions where $m_{DM} < m_{Z_{BL}}$. Also, the scalar sector mixes the SM doublet and additional singlet to produce two neutral scalar fields to mediate DM-SM interactions and enhance direct search possibility. The constraint on $g_{BL} - M_{Z_{BL}}$ from current LHC data is significant enough to ensure the coupling to be minuscule for relatively smaller $M_{Z_{BL}} \sim$ TeV, so that the DM signal at LHC doesn’t have any additional contribution from $Z_{BL}$ mediation to $\psi^\pm$ pair production. However with larger $M_{Z_{BL}} \sim$ 4 TeV, the coupling ($g_{BL}$) can be large enough to show up additional signal strength at LHC, which can be probed in its high luminosity run. It is worthy to mention that both the models offer variety of leptonic signatures like hadronically quiet opposite sign dilepton (OSD), trilepton or four lepton in association with missing energy. In a toy simulation for OSD events at LHC, we showed that it is easier to probe large $\Delta M$ regions of the model, characteristic to the $U(1)_{B-L}$ scenario than the small $\Delta M$ regions characteristic to the framework without $U(1)_{B-L}$. The model may also offer displaced vertex or stable charge track whenever the mass splitting $\Delta M$ between the charged companion and DM becomes very small.

Neutrino mass generation although fused naturally in this model, do not have direct influence on the dark sector. However, the RH neutrino mass turns out crucial for the neutrino sector, constrained from flavour changing decays like ($\mu \rightarrow e\gamma$). On the other hand, in the small mixing scenario, DM mass is dominantly controlled by the RH neutrino odd under $Z_2$ symmetry i.e. $m_{DM} \sim M_{R_i}$. Since in the context of $U(1)_{B-L}$ model, the Majorana masses of all the three RH neutrinos (including the one in the dark sector) are generated uniformly from the same symmetry breaking scale, we can treat them as a common parameter of the framework constrained by both dark sector and neutrino sector as a bridging ligand of the model.

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A DM-SM Interaction in model I

Expanding the covariant derivative of the Lagrangian given by eq. (2.1), we get the interaction term of $\psi^0$ and $\psi^\pm$ with the SM gauge bosons as follows:

$$
\mathcal{L}_{\text{int}} = \overline{\psi} i \gamma^\mu \left( -i g^2 \tau_\mu W^\mu - ig' Y^\mu_{--} B^\mu \right) \psi
= \left( \frac{e}{2 \sin \theta_W \cos \theta_W} \right) \overline{\psi} \gamma^\mu Z^\mu \psi^0
+ \frac{e}{\sqrt{2} \sin \theta_W} \left( \overline{\psi} \gamma^\mu W^\mu \psi^0 + \psi^+ \gamma^\mu W^- \psi^0 \right)
- e \psi^+ \gamma^\mu A^\mu \psi^-
- \left( \frac{e \cos 2 \theta_W}{2 \sin \theta_W \cos \theta_W} \right) \psi^+ \gamma^\mu Z^\mu \psi^-.
$$

(A.1)

where $g = \frac{e}{\sin \theta_W}$ and $g' = \frac{e}{\cos \theta_W}$ with $e$ being the electromagnetic coupling constant and $\theta_W$ being the Weinberg angle.

These interactions, when written in terms of the physical states becomes:

$$
\mathcal{L}_{\text{int}} = \left( \frac{e}{2 \sin \theta_W \cos \theta_W} \right) \left( - \cos \theta_{\chi_1} \gamma^\mu Z^\mu \chi_2 \chi_3 - \sin \theta_{\chi_1} \gamma^\mu Z^\mu \chi_3 \chi_3 + \text{h.c.} \right)
+ \frac{e}{\sqrt{2} \sin \theta_W} \left( \cos \theta_{\chi_1} \gamma^\mu W^\mu \psi^0 \chi_3 - \sin \theta_{\chi_1} \gamma^\mu W^\mu \psi^0 \chi_3 \right)
- e \psi^+ \gamma^\mu A^\mu \psi^- - \left( \frac{e \cos 2 \theta_W}{2 \sin \theta_W \cos \theta_W} \right) \psi^+ \gamma^\mu Z^\mu \psi^-.
$$

(A.2)

Another possibility of interaction between DM sector and the visible sector arises from the Yukawa interaction term $\sqrt{2} \overline{\psi} H^\dagger (N_{R_1} + (N_{R_2})^c)$ and its hermitian conjugate by expanding the SM Higgs $H$ around its vev. Writing in terms of physical bases,

$$
- \mathcal{L}_{\text{DM-Higgs}} = \frac{Y_1}{\sqrt{2}} \left[ \sin 2\theta (\overline{\chi}_1 h \chi_3 - \overline{\chi}_3 h \chi_1) + \cos 2\theta (\overline{\chi}_3 h \chi_3 + \overline{\chi}_1 h \chi_1) \right]
$$

(A.3)

Additionally, dark sector particles can annihilate into $Z_2$ even right handed neutrino $N_{R_2/3}$ and SM neutrinos via the Yukawa term $(\overline{Y_J} N_{R_1} \tilde{H}^\dagger L_{\alpha} + \text{h.c.})$ present in eq. (2.3). As it has been stated already, the lightest stable particle $\chi_3$ serves as the DM. The relic abundance of $\chi_3$ can be obtained through its annihilations to as well as through coannihilations with $\chi_1$, $\chi_2$ and $\psi^\pm$ to SM particles. The main processes which contribute to the relic abundance of DM are noted below:
The interaction terms of the dark and visible sector particles in the gauged $U(1)_{B-L}$ scenario can be obtained by expanding the kinetic terms of $\Psi$ and $N_{R_1}$ given in eq. (5.3) as the following,

$$
\mathcal{L}_{\text{int}} = \overline{\Psi} \gamma^\mu \left[ -i \frac{g}{2} \tau \cdot W_\mu - i g' \cdot B_\mu - i g_{BL} Y_{BL} (Z_{BL})_\mu \right] \Psi
+ \overline{N_{R_1}} \gamma^\mu (-i g_{BL} Y_{BL} (Z_{BL})_\mu) N_{R_1}
= \left( \frac{e}{2 \sin \theta_W \cos \theta_W} \right) \overline{\psi^0} \gamma^\mu Z_\mu \psi^0
+ \frac{e}{\sqrt{2} \sin \theta_W} \left( \overline{\psi^0} \gamma^\mu W_\mu^+ \psi^- + \psi^+ \gamma^\mu W_\mu^- \psi^0 \right)
- e \psi^+ \gamma^\mu A_\mu \psi^-
- \left( \frac{e \cos 2 \theta_W}{2 \sin \theta_W \cos \theta_W} \right) \overline{\psi^0} \gamma^\mu Z_\mu \psi^0
- g_{BL} \left[ \overline{\psi^0} \gamma^\mu (Z_{BL})_\mu \psi^0 + \psi^+ \gamma^\mu (Z_{BL})_\mu \psi^- + \overline{N_{R_1}} \gamma^\mu (Z_{BL})_\mu N_{R_1} \right].
$$

where $g = \frac{e}{\sin \theta_W}$ and $g' = \frac{e}{\cos \theta_W}$ with $e$ being the electromagnetic coupling constant, $\theta_W$ being the Weinberg angle and $g_{BL}$ is the $U(1)_{B-L}$ coupling constant. The other interaction is through the Yukawa interaction term $Y_1 \overline{\Psi} H N_{R_1}$, where we now have to also take into account...
the mixing between $H$ ans $\Phi_{BL}$. In terms of physical bases $\chi_1, \chi_2$ and $\chi_3$, the interaction terms of DM with the SM gauge bosons are given by:

\[
\mathcal{L}_{DM-SM} = \left( \frac{e}{2 \sin \theta_W \cos \theta_W} \right) \left[ (2 s_{23} s_{13} c_{13}) (\bar{\chi}_3 \gamma^\mu Z_\mu \chi_3 - \bar{\chi}_L \gamma^\mu Z_\mu \chi_L) 
+ (c_{23} c_{13} \bar{\chi}_L \gamma^\mu Z_\mu \chi_2 - c_{23} s_{23} \bar{\chi}_L \gamma^\mu Z_\mu \chi_L - s_{13} c_{23} \bar{\chi}_L \gamma^\mu Z_\mu \chi_3 + h.c.) \right]
+ \frac{e}{\sqrt{2} \sin \theta_W} \left[ \frac{1}{\sqrt{2}} \left( (c_{13} - s_{13} s_{23}) \bar{\chi}_L \gamma^\mu Z_\mu \chi_2 - (s_{13} + s_{23} c_{13}) \bar{\chi}_L \gamma^\mu Z_\mu \chi_L \right) \gamma^\mu W^+_\mu \psi^-_L
+ \frac{1}{\sqrt{2}} \left( (c_{13} + s_{13} s_{23}) \bar{\chi}_L \gamma^\mu Z_\mu \chi_2 - (s_{13} - s_{23} c_{13}) \bar{\chi}_L \gamma^\mu Z_\mu \chi_L \right) \gamma^\mu W^+_\mu \psi^-_R + h.c. \right]
+ \left( \frac{e}{\cos 2\theta_W} \right) \psi^+ \gamma^\mu Z_\mu \psi^-.
\]  
\text{Additionally we have the interactions of DM with } Z_{BL} \text{ as follows:}

\[
\mathcal{L}_{DM-Z_{BL}} = -g_{BL} \left[ (s_{23} s_{213} + c_{13} s_{23}) (\bar{\chi}_L \gamma^\mu (Z_{BL}) \mu \chi_3 
+ (s_{13} c_{23}^2 - s_{23} s_{213}) \bar{\chi}_L \gamma^\mu (Z_{BL}) \mu \chi_1 + s_{23} c_{23} \bar{\chi}_2 \mu \chi_2 + h.c.) 
+ \left( \frac{1}{2} s_{23} \bar{\chi}_L \gamma^\mu (Z_{BL}) \mu \chi_2 + h.c. \right)
+ \left( \frac{1}{2} s_{23} c_{23} \bar{\chi}_L \gamma^\mu (Z_{BL}) \mu \chi_3 + h.c. \right)
+ \frac{g_{BL}}{2} \psi^+ \gamma^\mu (Z_{BL}) \mu \psi^- \right].
\]

Here, we abbreviated $\sin 2\theta_{ij}$ and $\cos 2\theta_{ij}$ as $s_{2ij}$ and $c_{2ij}$ respectively. We note that in the limit $\sin \theta_{23} \rightarrow 0$ (along with $g_{BL} \rightarrow 0$), we get back to the interactions present in (A.2). DM-Scalar interaction also have additional channels from $H$ and $\Phi_{BL}$ mixing given by,

\[
-\mathcal{L}_{DM-Higgs} = \frac{y_1}{2} \left( h_1 \cos \beta - h_2 \sin \beta \right) \left[ (c_{13} - s_{13} s_{23}) \bar{\chi}_L \gamma^\mu (H) \mu \chi_3 + c_{23} \bar{\chi}_2 \mu \chi_2 + h.c. \right]
+ \frac{y_1'}{2} \left( h_1 \cos \beta + h_2 \sin \beta \right) \left[ (s_{13} c_{23} \bar{\chi}_L \gamma^\mu (H) \mu \chi_3 + c_{23} \bar{\chi}_2 \mu \chi_2 + h.c. \right]
\]

where $h_1, h_2$ are the two physical scalars of the model and $\beta$ represents $H - \Phi_{BL}$ mixing angle. The annihilation channels of dark matter in the $U(1)_{BL}$ extended case differs from the one without it, by having additional $Z_{BL}$ and an additional scalar present both in mediator as well as in final states. The following processes contributes to the relic abundance of the DM particle $\chi_3$ in this model with $U(1)_{BL}$ extension.
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