A complete survey of texture zeros in general and symmetric quark mass matrices

P.O. Ludl* and W. Grimus**

University of Vienna, Faculty of Physics
Boltzmanngasse 5, A–1090 Vienna, Austria

January 20, 2015

Abstract

We perform a systematic analysis of all possible texture zeros in general and symmetric quark mass matrices. Using the values of masses and mixing parameters at the electroweak scale, we identify for both cases the maximally restrictive viable textures. Furthermore, we investigate the predictive power of these textures by applying a numerical predictivity measure recently defined by us. With this measure we find no predictive textures among the viable general quark mass matrices, while in the case of symmetric quark mass matrices most of the 15 maximally restrictive textures are predictive with respect to one or more light quark masses.

*E-mail: patrick.ludl@univie.ac.at
**E-mail: walter.grimus@univie.ac.at
One of the most interesting questions of flavour physics is whether mixing angles are related to fermion mass ratios, like in the famous relation \( \sin \theta_c \simeq \sqrt{\frac{m_d}{m_s}} \) (1) between the Cabibbo angle \( \theta_c \) and the ratio of down-quark mass to strange-quark mass. Here and in the following, quark masses are denoted by \( m_q \) with \( q = d, s, b \) for the down-type quark masses and \( q = u, c, t \) for the up-type quark masses. It is an open question if equation (1) is only an empirical relation or if there is a deeper reason for it founded in a hitherto undiscovered theory of flavour. Obviously, since the CKM matrix \( U \) is defined as

\[
U = U_L^{(u)^\dagger} U_L^{(d)}
\]

with diagonalization matrices \( U_L^{(u)} \) and \( U_L^{(d)} \) given by

\[
U_L^{(d)^\dagger} M_d U_L^{(d)} = \text{diag} (m_d, m_s, m_b) \quad \text{and} \quad U_L^{(u)^\dagger} M_u U_L^{(u)} = \text{diag} (m_u, m_c, m_t)
\]

for the down-type and up-type quarks, respectively, the quark mass matrices \( M_d \) and \( M_u \) must have some structure in order to deduce relations like equation (1). The simplest attempt to achieve such relations is to place texture zeros in the mass matrices \[2\]. Apart from simplicity, texture zeros have the feature that they are practically synonymous with Abelian symmetries \[3\]. Unfortunately, even in this limited framework no clear-cut predictive model has emerged—see for instance \[4, 5\] for reviews and \[6\] for an attempt on a unified texture in both quark and lepton sector. Therefore, it is appropriate to perform a complete study of all possibilities, as was recently done in \[7\] for the lepton sector (see also \[8\]).

In the analysis of \[7\] the notion of “maximally restrictive” textures plays an important role. These have a maximal number of zeros in the pair \((M_d, M_u)\) in the sense that by placing one more zero into this pair it becomes incompatible with experimental data. It turned out that in the lepton sector the predictive power of general mass matrices with texture zeros is rather limited even for maximally restrictive textures. Actually, we find the same in the quark sector, as we will explain in more detail below. In view of this result, we perform an additional analysis with symmetric quark mass matrices \[1\]. In this paper, this is not more than a facile assumption in order to enhance predictivity \[2\] however, it could be motivated by left-right symmetric models \[10\] or by models based on \( SO(10) \)—see \[11\] for reviews—with renormalizable Yukawa couplings to scalar 10-plets and 126-plets, but not to scalar 120-plets which would introduce an antisymmetric component in the mass matrices \[12\].

Under weak-basis transformations the mass matrices transform as

\[
M_d \rightarrow V_L^{\dagger} M_d V_R^{(d)}, \quad M_u \rightarrow V_L^{\dagger} M_u V_R^{(u)}
\]

\[1\]In many papers the mass matrices are assumed to be Hermitian. It is true that one can always achieve this by separate weak-basis transformations on the right-handed quark fields, however, this is not a valid argument because in the first place it is the texture zeros which define a basis and performing a subsequent basis transformation will in general remove the texture zeros.

\[2\]For this purpose, other assumptions are possible as well, like for instance a scaling ansatz \[9\].
with unitary matrices $V_L$, $V_R^{(d)}$, $V_R^{(u)}$. Such transformations have no effect on the quark masses and the mixing matrix, i.e. they do not change the physical predictions. These weak-basis transformations are used in two ways. Firstly, it is well-known that with weak-basis transformations of equation (11) one can generate some zeros in $(M_d, M_u)$ which have, therefore, no predictive power at all [13]. Such cases we exclude a priori from our analysis—for more details on this issue see [7]. Secondly, if one wants to preserve the zeros in the mass matrices, the unitary matrices occurring in equation (11) have to be restricted to permutation matrices times diagonal matrices of phase factors. Weak-basis transformations where the unitary matrices in equation (4) are pure permutation matrices, i.e. “weak-basis permutations” [7], allow to divide the possible patterns of texture zeros in $(M_d, M_u)$ into equivalence classes with identical predictions and it is thus sufficient to treat one representative $(M_d^{(i)}, M_u^{(i)})$ of each equivalence class. Finally, with weak-basis transformations which are diagonal matrices of phase factors, i.e. by rephasing, one can then remove redundant phases from each representative $(M_d^{(i)}, M_u^{(i)})$ in order to obtain representatives with the minimal number of parameters.

Another important aspect of our general analysis is that we do not consider model realizations of the textures. Therefore, we cannot treat radiative corrections and we have to assume therefore that the quark masses and the CKM matrix can be reproduced with sufficient accuracy by tree-level mass matrices. Consequently, we take into account only non-singular mass matrices.

To test if a texture is compatible with the observations, we perform a $\chi^2$-analysis. We have ten physical observables: the six quark masses, the three mixing angles and the CP-violating phase. We have to check for each texture $(M_d^{(i)}, M_u^{(i)})$ whether it can reproduce the input data within experimental errors. Actually, for the mixing matrix $U$ we prefer to use the Wolfenstein parameters [14] $\lambda$, $A$, $\bar{\rho}$ and $\bar{\eta}$, as defined in [15]. In order to have a consistent set of input data, we have to fix a common energy scale $\mu$ at which the quark masses and mixing parameters are taken. We settle on the scale $\mu = M_Z$, the mass of the $Z$ gauge boson, which means that, if the texture zeros have a symmetry realization, then this symmetry is effective at the electroweak scale. This scale has the advantage that all observables used for the input, with the exception of the top quark mass, are measured at energies below $M_Z$ and, therefore, are evolved by the renormalization group equations of the Standard Model to the scale $\mu = M_Z$. Concretely, we take the input values at $M_Z$ from [17]; since in that paper the mixing angles and the CKM phase are given, we have to convert these into the Wolfenstein parameters. We display our input in table 1.

Now we formulate a criterion that a texture $(M_d^{(i)}, M_u^{(i)})$ is compatible with the data. We stipulate that the contribution of each observable to $\chi^2_{\text{min}}$, the minimum of $\chi^2$, is at most 25; this means that the deviation of the observable from its experimental value is at most $5\sigma$ [7]. Since we have ten input values, this implies $\chi^2_{\text{min}} \leq 250$.

Even if we know that a texture $(M_d^{(i)}, M_u^{(i)})$ is compatible with the data, we do not

---

3Within the Standard Model the relevant mass ratios $m_d/m_b$, $m_s/m_b$, $m_u/m_t$ and $m_c/m_t$ do not significantly change due to renormalization group running of the quark masses at scales from $\mu \sim 2 \text{ GeV}$ to $\mu \sim m_t$. This can be easily checked using the results of [16]. Moreover, within the Standard Model also the mixing angles do not run significantly. Therefore, if the effects of physics beyond the Standard Model are small at scales $\mu \lesssim m_t$, the analysis of texture zeros will not be affected by the exact choice of $\mu$.  

---

3
Table 1: The quark masses and the parameters of the CKM matrix in the \( \overline{\text{MS}} \) scheme at \( \mu = M_Z \) computed within the Standard Model in [17]. The quark masses are given in units of \( v = 174.104 \text{ GeV} \). We have transformed the mixing parameters \( \theta_{12}, \theta_{23}, \theta_{13} \) and \( \delta \) given in table 2 of [17] to the four Wolfenstein parameters \([15]\) \( \lambda, A, \bar{\rho} \) and \( \bar{\eta} \) assuming Gaussian error propagation.

| Quark Mass (\( 10^{-6} v \)) | Value (G) | Error (G) |
|-------------------------------|-----------|-----------|
| \( m_u \) | 7.4 | +1.5 \(-3.0\) |
| \( m_d \) | 1.58 | +0.23 \(-0.10\) |
| \( m_s \) | 3.12 | +0.17 \(-0.16\) |
| \( m_c \) | 3.60 | ±0.11 |
| \( m_b \) | 1.639 | ±0.015 |
| \( m_t \) | 9.861 | +0.086 \(-0.087\) |

\( \lambda = 0.22540 \pm 0.00070 \)

\( A = 0.828 \pm 0.014 \)

\( \bar{\rho} = 0.133 \pm 0.020 \)

\( \bar{\eta} = 0.350 \pm 0.015 \)

Let us assume that the model gives a good fit to the observables, i.e. \( \chi^2_{\text{min}} \) is sufficiently small. Now we want to pose the question whether the model is predictive with respect to the observable \( \mathcal{O}_i \). Loosely speaking, this means we want to investigate how much the prediction for \( \mathcal{O}_i \) can deviate from its mean value while varying \( x \) such that all \( P_j(x) \) with \( j \neq i \) remain close to \( \overline{\mathcal{O}}_j \). The numerical implementation is done in two steps [7]:

1. We define

\[
\tilde{\chi}^2_i(x) = \chi^2(x) - \chi^2_i(x),
\]

where we have removed the \( \chi^2 \)-contribution of the observable whose predictivity we want to investigate. With this \( \tilde{\chi}^2_i(x) \) we define a region in the parameter space via

\[
B_i = \{ x \mid \tilde{\chi}^2_i(x) \leq \chi^2_{\text{min}} + \delta \chi^2 \text{ and } \chi^2_j(x) \leq 25 \ \forall j \neq i \},
\]

\( \delta \) in case of asymmetric error intervals, \( \sigma_j \) is replaced by \( \sigma_{j\text{left}} \) and \( \sigma_{j\text{right}} \) for \( P_j(x) < \overline{\mathcal{O}}_j \) and \( P_j(x) \geq \overline{\mathcal{O}}_j \), respectively.

---

\[4\text{In case of asymmetric error intervals, } \sigma_j \text{ is replaced by } \sigma_{j\text{left}} \text{ and } \sigma_{j\text{right}} \text{ for } P_j(x) < \overline{\mathcal{O}}_j \text{ and } P_j(x) \geq \overline{\mathcal{O}}_j, \text{ respectively.} \]
where $\chi^2_{\text{min}}$ is the minimal value of the total $\chi^2(x)$ of equation (5) and $\delta \chi^2$ is a fixed parameter in the range $0 \leq \delta \chi^2 \lesssim 1$.

2. With $B_i$ we formulate the predictivity measure for $O_i$ as

$$\Delta(O_i) = \max_{x \in B_i} \chi^2_i(x).$$

(8)

Note that for $x \in B_i$ the $P_j(x)$ with $j \neq i$ are within the $5\sigma$ region of their experimental mean values $\overline{O}_j$, i.e. they are “close” to $\overline{O}_j$ in the sense of our compatibility criterion of a texture with the data. In equation (7), a non-zero $\delta \chi^2$ accelerates the convergence of the numerical maximization of $\Delta(O_i)$ [7]. In the present paper we have set $\delta \chi^2 = 0.1$.

Clearly, the smaller $\Delta(O_i)$ is, the better it is determined by the other observables. By choosing a bound $b^2$, we can define a predictivity criterion: for $\Delta(O_i) \leq b^2$ we say the model is capable to predict the observable $O_i$; in this case, its value deviates from its mean value by at most $b\sigma$. The choice of $b$ is rather arbitrary. We follow reference [7] and take $b = 10$. Thus our predictivity criterion is

$$\Delta(O_i) \leq 100.$$  

(9)

The results of our analysis are presented in two tables, table 2 for general mass matrices and table 3 for symmetric mass matrices. In the general case, removing those pairs of mass matrices $(M_d, M_u)$ whose texture zeros can be generated by weak-basis transformations and those with at least one singular matrix, we find 243 inequivalent classes with representatives $(M^{(i)}_d, M^{(i)}_u)$. Of these, 214 classes are compatible with the data and among them there are 27 maximally restrictive classes whose representatives $(M^{(i)}_d, M^{(i)}_u)$ are listed in table 2. None of these mass matrices are predictive in the sense discussed above.

For the symmetric mass matrices, we cannot apply general weak-basis transformations which generate texture zeros, without destroying the symmetry of the mass matrices. Thus we discard only those pairs $(M_d, M_u)$ which have at least one singular matrix and arrive in this way at 230 classes with representatives $(M^{(i)}_d, M^{(i)}_u)$. Out of these, 79 classes survive the $\chi^2$-test. Finally, these classes contain 15 maximally restrictive classes which are displayed in table 3. According to our predictivity criterion, 11 of these 15 textures are predictive with respect to one or more of the three light quark masses $m_u, m_d$ and $m_s$; the masses of the heavy quarks and the Wolfenstein parameters cannot be predicted.

At first sight, the last sentence seems to exclude approximate relations of the form of equation (1) or, more generally, relations of the form

$$f \left( \frac{m_1}{m_2}, W \right) = 0,$$

(10)

where $f$ denotes a function specified by the particular form of the texture, $m_1$ and $m_2$ are quark masses and $W$ is one of the Wolfenstein parameters. Clearly, if a relation of the type of equation (10) follows from a texture, then $W$ is predicted by $m_1/m_2$, but

\footnote{For the sake of simplicity, in this paragraph we confine ourselves to three observables.}
Table 2: The 27 maximally restrictive textures in general quark mass matrices. A matrix entry 0 denotes a texture zero, and entries 1 and 2 stand for real positive and complex parameters, respectively. None of these textures is predictive with respect to any observable.
mathematically we can turn this conclusion around and say that $m_1$ is predicted by $m_2$ and $W$ or $m_2$ is predicted by $m_1$ and $W$. However, numerically this will in general not be the case because of the different relative errors $\sigma_j/\bar{O}_j$ of the observables. From table 1 one finds that the relative errors of the light quark masses $m_u$, $m_d$ and $m_s$ are between 5% and 40%, the errors of the heavy quark masses are between 1% and 3% and the errors of the Wolfenstein parameters range from 0.3% for $\lambda$ to 15% for $\bar{\rho}$. For instance, for the famous relation (1), varying $m_d$ and $m_s$ around their experimental mean values within ranges of the order of magnitude of their respective experimental errors, one may well obtain values for $\lambda \equiv \sin\theta_c$ which, due to the small relative error $\sigma_\lambda/\lambda \sim 0.3\%$, may lie more than 10 sigmas off its mean value, i.e. $\Delta(\lambda) > 100$. Conversely, fixing $\lambda$ and one of the masses gives a prediction for the second mass which, since the relative errors of $m_d$ and $m_s$ are much larger, may very well be within ten sigmas of the experimental value, i.e. $\Delta(m_q) < 100$ for $q = d, s$. To summarize, even if there is a relation of the form (10), the predictivity analysis will in general not detect all involved observables. This has to be kept in mind in the assessment of the results displayed in table 3.

All of the 15 textures for symmetric quark mass matrices have four ($S_1, \ldots, S_7, S_{11}, S_{14}$) or five ($S_8, S_9, S_{10}, S_{12}, S_{13}, S_{15}$) independent texture zeros, the most predictive one being $S_{10}$:

$$S_{10}: \quad M_d \sim \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad M_u \sim \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(11)

which shows predictive power with respect to all of the three light quark masses. In the light of the discussion above, this does not mean that the texture $S_{10}$ has three independent predictions. Indeed, in this case there are only two independent ones, which can be formulated as $\sin\theta_{12} \simeq \sqrt{m_d/m_s}$, i.e. equation (11), and $|U_{ub}| \equiv \sin\theta_{13} \simeq \sqrt{m_u/m_t}$.

We emphasize once more that, after the consideration of general quark mass matrices, we have investigated symmetric (but not Hermitian) mass matrices, in which context we have discovered six viable textures with five texture zeros—see table 3. It is interesting to compare these six textures with the five viable Hermitian textures with five texture zeros discussed in the literature [18, 19]. For this comparison we use the table in [19] where the five Hermitian patterns I-V are listed and check if there are corresponding patterns in our table 3 with zeros in corresponding places after suitable weak-basis permutations. We find the correspondences II $\sim S_9$, III $\sim S_8$, IV $\sim S_{15}$ and V $\sim S_{10}$, while pattern I has no correspondence to viable texture zeros in symmetric quark mass matrices. This comparison reveals the fundamental difference between texture zeros in symmetric and Hermitian mass matrices.

**Summary:** In this paper we have performed a systematic and complete analysis of texture zeros in general and symmetric quark mass matrices. Among all the possible

---

6Note that for the mass matrices of equation (11) we find numerically $|(M_d)_{22}| \simeq m_b$, $|(M_u)_{22}| \simeq m_t$. For achieving the usual ordering with large elements having higher indices, one has to apply a permutation $2 \leftrightarrow 3$ to the indices.

7Note that in [19] the Hermitian mass matrices I-V are studied at $\mu = M_Z$ just as in this paper. Hence, the results can directly be compared.
Table 3: The 15 maximally restrictive textures in symmetric quark mass matrices. A matrix entry 0 denotes a texture zero, and entries 1 and 2 stand for real positive and complex parameters, respectively. Several of these textures are predictive with respect to some of the light quark masses.
texture zeros in general quark mass matrices, we identified the 27 maximally restrictive classes—see table 2—which, however, do not show predictive power with respect to any of the quark masses and mixing parameters. This is very similar to the situation of Dirac neutrinos, where texture zeros are predictive at most with respect to the smallest neutrino mass and, in one case, also to the Dirac phase $\delta$ of the lepton mixing matrix [7]. In other words, pure Abelian flavour symmetries effective at the electroweak scale, i.e. texture zeros but no further restrictions on the quark mass matrices, do not seem to contribute to the solution of the mass and mixing problem in the quark sector.

The case of texture zeros in symmetric quark mass matrices is more promising, since there the majority of the 15 maximally restrictive textures has some predictive power—see table 3. This may be compared to the case of Majorana neutrinos, where the neutrino mass matrix is symmetric. There the maximally restrictive textures are also more predictive than for the Dirac neutrino case [7].

Acknowledgments: This work is supported by the Austrian Science Fund (FWF), Project No. P 24161-N16. P.O.L. thanks Helmut Moser for his tireless servicing of our research group’s computer cluster on which the computations for this work were performed.

References

[1] R. Gatto, G. Sartori and M. Tonin, *Weak self-masses, Cabibbo angle, and broken $SU_2 \times SU_2$*, Phys. Lett. *28B* (1968) 128;
N. Cabibbo and L. Maiani, *Dynamical interrelations of weak, electromagnetic and strong interactions and the value of $\theta$*, Phys. Lett. *28B* (1968) 131.

[2] H. Fritzsch, *Weak interaction mixing in the six-quark theory*, Phys. Lett. B *73* (1978) 317;
H. Fritzsch, *Quark masses and flavor mixing*, Nucl. Phys. B *155* (1979) 189.

[3] W. Grimus, A. S. Joshipura, L. Lavoura and M. Tanimoto, *Symmetry realization of texture zeros*, Eur. Phys. J. C *36* (2004) 227 [hep-ph/0405016];
R. González Felipe and H. Serôdio, *Abelian realization of phenomenological two-zero neutrino textures*, Nucl. Phys. B *886* (2014) 75 [arXiv:1405.4263 [hep-ph]].

[4] H. Fritzsch and Z. z. Xing, *Mass and flavor mixing schemes of quarks and leptons*, Prog. Part. Nucl. Phys. *45* (2000) 1 [hep-ph/9912358];
M. Gupta and G. Ahuja, *Flavor mixings and textures of the fermion mass matrices*, Int. J. Mod. Phys. A *27* (2012) 1230033 [arXiv:1302.4823 [hep-ph]].

[5] H. Serôdio, *Yukawa sector of Multi Higgs Doublet Models in the presence of Abelian symmetries*, Phys. Rev. B *88* (2013) 056015 [arXiv:1307.4773 [hep-ph]].

[6] S. Sharma, P. Fakay, G. Ahuja and M. Gupta, *Clues towards unified textures*, Int. J. Mod. Phys. A *29* (2014) 1444005 [arXiv:1404.5726 [hep-ph]].
[7] P. O. Ludl and W. Grimus, *A complete survey of texture zeros in the lepton mass matrices*, JHEP 1407 (2014) 090 [Erratum-ibid. 1410 (2014) 126] arXiv:1406.3546 [hep-ph].

[8] P. M. Ferreira and L. Lavoura, *New textures for the lepton mass matrices*, Nucl. Phys. B 891 (2014) 378 arXiv:1411.0693 [hep-ph].

[9] R. N. Mohapatra and W. Rodejohann, *Scaling in the neutrino mass matrix*, Phys. Lett. B 644 (2007) 59 [hep-ph/0608111];
A. S. Joshipura and W. Rodejohann, *Scaling in the Neutrino Mass Matrix, mu-tau Symmetry and the See-Saw Mechanism*, Phys. Lett. B 678 (2009) 276 arXiv:0905.2126 [hep-ph];
M. Chakraborty, H. Z. Devi and A. Ghosal, *Scaling ansatz with texture zeros in linear seesaw*, Phys. Lett. B 741 (2014) 210 arXiv:1410.3276 [hep-ph];
A. Ghosal and R. Samanta, *Probing texture zeros with scaling ansatz in inverse seesaw*, arXiv:1501.00916 [hep-ph].

[10] J. C. Pati and A. Salam, *Lepton number as the fourth color*, Phys. Rev. D 10 (1974) 275 [Erratum-ibid. D 11 (1975) 703];
R. N. Mohapatra and J. C. Pati, *Left-right gauge symmetry and an isoconjugate model of CP violation*, Phys. Rev. D 11 (1975) 566;
R. N. Mohapatra and J. C. Pati, *A natural left-right symmetry*, Phys. Rev. D 11 (1975) 2558;
H. Fritzsch and P. Minkowski, *Parity conserving neutral currents and righthanded neutrinos*, Nucl. Phys. B 103 (1976) 61.

[11] M. C. Chen and K. T. Mahanthappa, *Fermion masses and mixing and CP violation in SO(10) models with family symmetries*, Int. J. Mod. Phys. A 18 (2003) 5819 [hep-ph/0305088];
A. S. Joshipura and K. M. Patel, *Fermion Masses in SO(10) Models*, Phys. Rev. D 83 (2011) 095002 arXiv:1102.5148 [hep-ph].

[12] N. Oshimo, *Antisymmetric Higgs representation in SO(10) for neutrinos*, Phys. Rev. D 66 (2002) 095010 [hep-ph/0206239];
W. Grimus and H. Kühböck, *Fermion masses and mixings in a renormalizable SO(10) × Z2 GUT*, Phys. Lett. B 643 (2006) 182 [hep-ph/0607197];
W. Grimus and H. Kühböck, *A renormalizable SO(10) GUT scenario with spontaneous CP violation*, Eur. Phys. J. C 51 (2007) 721 [hep-ph/0612132].

[13] G. C. Branco, D. Emmanuel-Costa and R. González Felipe, *Texture zeros and weak basis transformations*, Phys. Lett. B 477 (2000) 147 [hep-ph/9911418];
G. C. Branco, D. Emmanuel-Costa, R. González Felipe and H. Serôdio, *Weak basis transformations and texture zeros in the leptonic sector*, Phys. Lett. B 670 (2009) 340 arXiv:0711.1613.

[14] L. Wolfenstein, *Parametrization of the Kobayashi-Maskawa Matrix*, Phys. Rev. Lett. 51 (1983) 1945.
[15] K. A. Olive et al. (Particle Data Group), Chin. Phys. C 38 (2014) 090001.

[16] Z. z. Xing, H. Zhang and S. Zhou, Updated Values of Running Quark and Lepton Masses, Phys. Rev. D 77 (2008) 113016 [arXiv:0712.1419 [hep-ph]].

[17] S. Antusch and V. Maurer, Running quark and lepton parameters at various scales, JHEP 1311 (2013) 115 [arXiv:1306.6879 [hep-ph]].

[18] P. Ramond, R. G. Roberts and G. G. Ross, Stitching the Yukawa quilt, Nucl. Phys. B 406 (1993) 19 [hep-ph/9303320];
L. E. Ibanez and G. G. Ross, Fermion masses and mixing angles from gauge symmetries, Phys. Lett. B 332 (1994) 100 [hep-ph/9403338].

[19] W. A. Ponce, J. D. Gómez and R. H. Benavides, Five texture zeros and CP violation for the standard model quark mass matrices, Phys. Rev. D 87 (2013) 5, 053016 [arXiv:1303.1338 [hep-ph]].