Abstract

The nonperturbative modification of the evolution equation for QCD generating functional is applied to the analysis of the factorial moments of the intrajet particle distributions in QCD. Nonperturbative preasymptotic correction to the factorial moments scaling (F-scaling) is calculated. The results are compared with the available experimental data. The data show the possible presence of such a nonperturbative effect.
1 Introduction.

Physics of jets provides one of the most important testing grounds for practical applications of the theory of strong interactions, QCD. Perturbative evolution of a quark-gluon jet is now well investigated (see, for example, [1]). In particular, one of the striking predictions of perturbative QCD is an angular ordering in the intrajet evolution due to the color coherence in gluon emission. Nonperturbative aspects of jet evolution are considerably less clear. From experience of applying the sum rules to the analysis of characteristics of the heavy resonances [2] it is clear that when virtuality of radiated parton is of the order of 1 GeV it is necessary to take into account nonperturbative corrections due to background vacuum fields giving rise to QCD vacuum condensate. Technically this results in introducing additional soft interaction of hard parton currents with the soft medium parametrized by the condensate fields. A similar situation arises in describing an evolution of a jet in some nontrivial (e.g., nuclear) medium, so that inelastic collisions with it lead to dissipation of energy and momentum from the hard parton component. Although physically these situations are rather different, we expect the appropriate theoretical formalisms describing both cases to be rather similar.

In both cases a necessity of introducing new interaction vertices lead to the occurrence of new dimensional parameters. This in turn results in the specific scaling violations in the description of jet evolution, in which the perturbative interaction vertices are dimensionless (contain ratios of energies or virtualities, see [1]). Physically this could be interpreted as a beginning of nonperturbative process of the QCD string formation which takes energy from the perturbative component and pumps it into nonperturbative degrees of freedom. The first model describing the jet evolution taking into account the nonperturbative energy loss was proposed by I.M. Dremin [3]. It was based on an analogy with the physics of high-energy electromagnetic showers in a medium, where scaling-invariant evolution due to photon bremsstrahlung and electron-positron pair creation is accompanied by the scaling-violating scattering due to inelastic interactions of shower particles with the atoms of the medium leading to their ionization [4]. Later the corresponding modified QCD evolution equations were analytically solved in [3], where expressions for parton multiplicities in quark and gluon jets and the energy loss in the perturbative hard component were calculated. The discussion of this method, as well as of the relevant Monte-Carlo calculations [5], where nonperturbative component was described through the effective QCD lagrangian can be found in the review paper [6]. Serious defect of Dremin’s equations [3] is that there the color interference in intrajet evolution, known to be of crucial significance for the description of its characteristics, is not taken into account. The problem we are discussing in this paper is to build up a formalism providing an analytical description of intrajet evolution and taking into account both color coherence and nonperturbative energy loss.
This program was started in the previous work of the authors [8], where the modified jet evolution equations were solved in the leading logarithmic approximation (LLA) and the general evolution equations for QCD generating functional taking into account the nonperturbative dissipative corrections were written down. In [8] we considered only the simplest characteristics of the intrajet evolution such as the nonperturbative dissipative corrections to the intrajet particle multiplicities and distributions. In this paper we turn to a more detailed analysis of the jet particle content by calculating the nonperturbative dissipative corrections to the factorial moments characterizing it.

Let us remind that perturbative QCD predicts an asymptotic independence of the moments of the intrajet particle distributions on energy (KNO scaling) [1]. Closely related to the KNO scaling is a so-called factorial moments scaling (F-scaling) that holds in the high energy limit. A very clear review on this subject can be found, e.g., in [9], where one can also find the relevant references (see also a recent review [10]). The perturbative preasymptotic corrections to this scaling are due to the running of the coupling constant and thus proportional to $\sqrt{\alpha_s(Q^2)}$ (and therefore to $1/\sqrt{\ln Q^2}$). Below we shall restrict ourselves to the case of the frozen coupling constant, so the above-mentioned perturbative preasymptotic corrections are absent and the analysis is focused on the nonperturbative preasymptotic F-scaling violation due to dissipative effects in the jet’s evolution.

Let us note, that a modification of the evolution equations for QCD generating functional similar to those in [8] and introducing an additional inactivation of perturbative evolution were recently described in [12].

2 Generating functional for dissipative evolution

It is well known, that a formalism of generating functional [1] is indispensable in analyzing the properties of particle distributions in QCD jets. In particular, it provides a unique possibility of analytically accounting for such delicate properties of QCD jet evolution as color coherence and resulting angular ordering in the parton cascades. In the previous paper [8] we have proposed a nonperturbative generalization of the perturbative evolution equation for the QCD generating functional describing the intrajet particle distributions. A new element is to introduce a dimensional coupling between the perturbative partonic and nonperturbative soft modes resulting in additional “dissipative” damping of the perturbative intrajet evolution. The modified evolution equation in MLLA (modified leading logarithmic approximation) for the generating functional $G(p, \theta|u)$

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*We are grateful to I.M. Dremin for bringing our attention to this paper.*
$$\frac{\partial G(p, \theta | u)}{\partial \ln \theta} = \int dx \frac{\gamma_0^2}{K(x)} [G(xp, \theta | u) G((1-x)p, \theta | u) - G(p, \theta | u)] - \left(\frac{\beta}{p}\right)^{\alpha} p \frac{\partial G(p, \theta | u)}{\partial p}$$

(1)

where $p$ is an energy of the parton decaying into the two ones having energies $xp$ and $(1-x)p$ correspondingly, $\theta$ is an angle between the off-spring partons and $\beta$ is a dimensional coupling constant accounting for the coupling to the nonperturbative modes. In Eq. (1) $\gamma_0^2 = 2N_c \alpha_s/\pi$ is a running coupling, $(N_c = 3$ is a number of colors), $K(x) = 1/x - 1 + x(1-x)/2$ is an MLLA kernel for gluon splitting. The initial condition for this equation has the form $G(p_{\min}, \theta_{\min} | u) = u$, thus fixing the borderline for the possibility of perturbative evolution. From the requirement of probability conservation we get another constraint for the generating functional $G(p, \theta | 1) = 1$. The Eq. (1) is a further generalization of the one in [8], where only the case of $\alpha = 1$ was considered.

In the absence of dissipative effects (i.e. for $\beta = 0$) the evolution described by Eq. (1) is characterized by a specific scaling [9]. Namely, the generating functional $G(p, \theta | u)$ depends only on a certain combination of $p$ and $\theta$ having the form $y = \ln(p\theta/Q_0)$, where $Q_0 = p_{\min} \theta_{\min}$ is a minimal transverse momentum considered in terms of the perturbative evolution. This results, in particular, in the fact that both KNO scaling for intrajet multiparticle distributions and the (equivalent to it in the limit of high energy) F-scaling [9] hold in the limit of $y \to \infty$ (not only $p \to \infty$). However, this is no longer true when the additional "dissipative" term is taken into account. The presence of the new dimensional coupling $\beta$ leads, generally speaking, to the explicit violation of the perturbative scaling.

Below we shall consider only the effects of the first order in a parameter $\varepsilon = (\beta/p)^{\alpha}$, so we assume that the following decomposition takes place:

$$G(p, \theta | u) = G_0(y | u) - \varepsilon G_1(y | u)$$

(2)

so that the perturbative scaling for $G_0(p, \theta)$ and $G_1(p, \theta)$ still holds.

Substituting the decomposition (2) into the evolution equation Eq. (1), we get in the zeroth and first order in $\varepsilon$

$$\frac{\partial G_0}{\partial y} = \int dx \frac{\gamma_0^2}{K(x)} [G_0(y + \ln x) G_0(y + \ln(1-x)) - G_0(y)]$$

(3)

$$\frac{\partial G_1}{\partial y} = \int dx \frac{\gamma_0^2}{K(x)} \left[ G_0(y + \ln x) \frac{G_1(y + \ln(1-x))}{(1-x)^{\alpha}} + 
+ G_0(y + \ln(1-x)) \frac{G_1(y + \ln x)}{x^{\alpha}} - G_1(y) \right] + \frac{\partial G_0}{\partial y} ,$$

(4)

with the initial conditions $G_0(y = 0) = u$ and $G_1(y = 0) = 0$. Let us note, that physically the evolution equation in the parton branching angle $\theta$ Eq. (1) and the
evolution equations in $y$ Eqs. (3,4) have very different meaning. The former is really following the evolution of the intrajet distribution proceeding by emitting the gluons into decreasing opening angles. The latter ones are describing the evolution of our snapshot view of the cascade at a certain level. The initial condition here is formulated not at the origin of the parton tree, but at its very end. The condition $G_0(y = 0) = u$ actually means, that a parton having a minimal transverse momentum can not evolve at all. Larger $y$ mean that there is some phase space for degrading down to $Q_0$. In this sense the evolution in $y$ goes in the opposite direction to the more intuitive angular one. At the same time using $y$ as an evolution variable is crucial in formulating a scale-invariant picture of the perturbative cascade, so it will be this evolution that will be studied in the following. Let us also note, that we shall restrict our consideration to the simple case of pure gluodynamics and frozen coupling constant $\gamma_0 = const$.

Before proceeding to solving Eqs. (3,4) it is necessary to understand what are the integration limits in these equations. The most natural procedure is to demand the positivity of the arguments of the generating functions entering Eqs. (3,4). This is equivalent to the requirement of stopping the evolution completely at $p_t = Q_0$ (i.e. no particle could be emitted with $p_t < Q_0$). Then the integration over the energy fraction inherited by the daughter parton is restricted to the interval

$$\int dx \ldots = \int_{e^{-y}}^{1-e^{-y}} dx \ldots$$

In the high energy limit of large $y$, where we expect our basic perturbative treatment to hold, one can make a replacement

$$\int dx \ldots = \int_0^1 dx \ldots$$

provided that the corresponding integrals are convergent.

In general from Eq. (5) it is clear, that this constraint becomes increasingly important as we are moving towards the borderline of perturbative evolution, which corresponds to a starting point in the evolution equation in $y$. Thus the accuracy of the predictions following from the evolution equation Eq. (1) is to a certain extent limited by the necessity of fixing the initial conditions at the ”least perturbative” end of the intrajet evolution.

## 3 Perturbative Contribution to Moments

For reader’s convenience we briefly remind in this Section some important formulas in the solution [13] of the zeroth approximation Eq. (3) for the factorial moments of the intrajet particle distributions in the frozen coupling constant case $\gamma_0 = const$. Further details of this solution can be found in [13], [9].
Let us start with computing the mean multiplicity

\[ n_0(y) = \left. \frac{\partial G_0(y|u)}{\partial u} \right|_{u=1} \]  

(7)

From Eq.(3) and taking into account that \( G_0(y|1) = 1 \) we get a differential equation on mean multiplicity

\[ \frac{\partial n_0}{\partial y} = \gamma_0^2 \int dxK(x) \{ n_0(y + \ln(x)) + n_0(y + \ln(1 - x)) - n_0(y) \} \]  

(8)

with an obvious initial condition \( n_0(0) = 1 \). The solution is

\[ n_0(y) = e^{\gamma y} \]  

(9)

where \( \gamma \) is determined by a transcendent equation

\[ \gamma = \gamma_0^2 B(\gamma) \]  

(10)

\[ B(a) = \int_0^1 dxK(x) \{ x^a + (1 - x)^a - 1 \}, \quad a \neq 0, -1, -2, \ldots \]  

(11)

The function \( B(a) \) is plotted in Fig.1. The solution of Eq.(11) determines \( \gamma \) as a function of \( \gamma_0^2 \) (and therefore as a function of \( \alpha_s \)). The function \( \gamma(\alpha_s) \) is plotted in Fig. 2.

It is well known [9] that in the high energy limit the KNO-scaling, characterizing the intrajet evolution, is equivalent to the factorial moments F-scaling, so that in the expansion

\[ G_0(y|u) = 1 + \sum_{q=1}^{\infty} \frac{(u - 1)^q}{q!} F_q^{(0)} n_0(y)^q = 1 + \sum_{q=1}^{\infty} \frac{(u - 1)^q}{q!} F_q^{(0)} e^{\gamma y}, \quad y \to \infty \]  

(12)

where \( F_q^{(0)} \) are the factorial moments of the intrajet particle distribution, these factorial moments do not depend on \( y \). After substituting the above expansion into Eq.(3) one obtains a quadratic recurrent relation for the factorial moments

\[ F_q^{(0)} = \frac{\gamma_0^2}{\gamma q - \gamma_0^2 B(\gamma q)} \sum_{m=1}^{q-1} C_{m}^{q} F_m^{(0)} F_{q-m}^{(0)} R(\gamma m, \gamma(q-m)), \quad q > 1, \]  

(13)

where

\[ R(a, b) = \int dxK(x) x^a(1 - x)^b = B(a, b + 2) + \frac{1}{2} B(a + 2, b + 2), \]

\( B(a, b) \) being the Euler beta-function. The resulting values of the factorial moments are easily obtained by numerically solving Eq.(13). In Fig. 4 the values of \( F_q \) for some \( \alpha_s \) are shown.
4 Mean multiplicity

In this section we calculate the corrections to the mean multiplicity in the first order in $\varepsilon = \beta/p$, so let us take $n = n_0 - \varepsilon n_1$, where $n_1$ is a correction term that has to be calculated from the correction to the generating functional in Eq. (2)

$$n_1(y) = \frac{\partial G_1(y|u)}{\partial u} \bigg|_{u=1}$$

(14)

From Eq. (4) we get a differential equation on the correction to mean multiplicity

$$\frac{\partial n_1}{\partial y} = \gamma_0^2 \int_{e^{-\gamma}}^{1-e^{-\gamma}} dx K(x) \left\{ \frac{n_1(y + \ln x)}{x^\alpha} + \frac{n_1(y + \ln(1 - x))}{(1 - x)^\alpha} - n_1(y) \right\} + \gamma e^{\gamma y}$$

(15)

The above equation is a linear inhomogeneous one, so its solution is a sum of a general solution to a homogeneous equation and a particular solution to the inhomogeneous one. The solution of the homogeneous equation is proportional to $e^{\gamma_1 y}$, where $\gamma_1$ is a solution of

$$\gamma_1 = \gamma_0^2 B(\gamma_1 - \alpha)$$

(16)

In this case we have taken the integration limits to be $(0, 1)$, which here can be done without losing accuracy. From Eqs. (10,16), and the fact that $B(a)$ is a decreasing function of $a$, there follows an important inequality

$$\max(\gamma, \alpha) < \gamma_1 < \alpha + \gamma.$$  

(17)

for $\alpha > 0$ so that the solution to the homogeneous equation is dominating the high energy asymptotics $y \rightarrow \infty$. The function $\gamma$ is plotted in Fig.2 as a function of $\alpha_s$ and in Fig.3 as a function of $\alpha$. In the following we shall need only the high energy asymptotics of the solution, which is provided by the solution of the homogeneous equation $e^{\gamma_1 y}$:

$$n(y) = e^{\gamma y} - \left( \frac{\beta}{p} \right)^\alpha C_{\gamma_1} e^{\gamma_1 y} = e^{\gamma y} \left( 1 - C_{\gamma_1} \left( \frac{\beta}{Q_0} \right)^\alpha e^{(\gamma_1 - \gamma - \alpha)y} \right).$$

(18)

so the correction is known up to the prefactor which is determined from the initial conditions and can be found only from the full solution of Eq. (15). From the above equation and Eq. (17) we see that the relative magnitude of the dissipative correction is enhanced with respect to a naively expected product of the factors $\sim e^{-\alpha y}$ and $e^{-\gamma y}$, the latter obviously being a typical $1/n_0$ preasymptotic correction.

5 Factorial moments.

In this section we will calculate the nonperturbative corrections to the factorial moments of the intrajet particle distribution. Let us remind (see, e.g., [1] and
that the factorial moments $F_q$ are defined as coefficients in the expansion of the generating functional in a power series in the mean multiplicity

$$G(p, \theta|u) = 1 + \sum_{q=1}^{\infty} \frac{(u-1)^q}{q!} F_q n^q, \quad F_1 = 1. \quad (19)$$

To the first order in the small parameter $\varepsilon$ we get from Eq. (19)

$$G = 1 + \sum_{q=1}^{\infty} \frac{F_q^{(0)} - \varepsilon F_q^{(1)}}{q!} (n_0 - \varepsilon n_1)^q (u-1)^q = 1 + \sum_{q=1}^{\infty} \frac{F_q^{(0)} n_1 + F_q^{(1)} n_0}{q!} n_0^{q-1} (u-1)^q, \quad (20)$$

where $\varepsilon F_q^{(1)}$ is a dissipative correction to the $q$-th factorial moment and we have explicitly isolated the nonperturbative correction to the mean multiplicity calculated in the previous paragraph. From Eq. (20) we immediately read off the expression for the first order correction to the generating functional $G_1$:

$$G_1 = \sum_{q=1}^{\infty} \frac{q F_q^{(0)} n_1 + F_q^{(1)} n_0}{q!} n_0^{q-1} (u-1)^q = \sum_{q=1}^{\infty} \frac{\Phi_q(y)}{q!} (u-1)^q \quad (21)$$

where $\Phi_q(y) = (q F_q^{(0)} n_1 + F_q^{(1)} n_0) n_0^{q-1}$ and the "initial conditions" $F_1 = F_1^{(0)} = 1$ give $\Phi_1(y) = n_1(y)$. A complete calculation of $\Phi_q(y)$ is possible but very cumbersome. To the accuracy we are interested in it will be sufficient to work with the leading contribution to $\Phi_q(y)$, greatly simplifying the calculations and making the results transparent.

The full equation on $\Phi_q(y)$ reads

$$\Phi_q'(y) = \gamma_0^2 \int dx K(x) \left\{ \frac{1}{x^\alpha} \Phi_q(y + \ln x) + \frac{1}{(1-x)^\alpha} \Phi_q(y + \ln(1-x)) - \Phi_q(y) \right\} +$$

$$+ \gamma_0^2 \sum_{m=1}^{q-1} C_q^m F_q^{(0)} e^{\gamma(q-m)} \int dx K(x) \left\{ \frac{\Phi_m(y + \ln x)}{x^\alpha} (1-x)^\gamma(q-m) + (22) \right.$$ 

$$+ \frac{\Phi_m(y + \ln(1-x))}{(1-x)^\alpha} x^{\gamma(q-m)} \right\} + \gamma q F_q^{(0)} e^{\gamma y}. \quad (22)$$

The homogeneous part of Eq. (22) is the same as for in the equation for $n_1$ Eq. (15), so its solution has the same form $C_q e^{\gamma y}$. The solution to the inhomogeneous equation on $\Phi_q(y)$ is a sum of some exponents in $y$ (this immediately follows from the linearity of Eq. (22) and the structure of $\Phi_q$'s with $q' < q$). Proceeding this way we get $\Phi_q \sim \Phi_m e^{\gamma(q-m)}$ with $m < q$, where the sign $\sim$ means that $\Phi_q$ includes the terms from $\Phi_m e^{\gamma(q-m)}$. All such components together with the solution of
the homogeneous part of the equation give \( \Phi_q \sim e^{(q-m)\gamma y+\gamma_1 y} e^{\gamma y} (q \geq m \geq 1) \). From Eq. (17) it is clear that the leading term has the form \( e^{(q-1)\gamma y+\gamma_1 y} \). Let us define

\[
\Phi_q(y) = \Psi_q e^{((q-1)\gamma + \gamma_1) y} = \Psi_q e^{\gamma y}, \text{ where } \gamma_q = (q-1)\gamma + \gamma_1. \tag{23}
\]

Substituting this Ansatz to Eq. (22) we have

\[
\gamma_q \Psi_q = \gamma_0^2 \sum_{m=1}^{q-1} C_q^m F_q^{(0)} \int dx K(x) \left\{ x^{\gamma_m-\alpha} (1-x)^{\gamma(q-m)} + (1-x)^{\gamma_m-\alpha} x^{\gamma(q-m)} \right\} + \\
+ \gamma_0^2 \Psi_q \int dx K(x) \left\{ x^{\gamma_q-\alpha} + (1-x)^{\gamma_q-\alpha} - 1 \right\}, \quad q > 1 \tag{24}
\]

\( \Psi_1 = C_\gamma \). From Eq. (24) we immediately get a recurrent relation on \( \Psi_q \):

\[
\Psi_q = \frac{\gamma_0^2}{\gamma_q - \gamma_0^2 B(\gamma_q - \alpha)} \sum_{m=1}^{q-1} C_q^m \Psi_m F_q^{(0)} [R(\gamma_m - \alpha, \gamma(q-m)) + R(\gamma(q-m), \gamma_m - \alpha)]. \tag{25}
\]

Using Eqs. (21) and (23) one can extract \( F_q^{(1)} \) from \( \Psi_q \):

\[
q F_q^{(0)} \Psi_1 e^{\gamma y} + F_q^{(1)} e^{\gamma y} = \Psi_q e^{\gamma y} \Rightarrow \\
F_q^{(1)} = \left( \Psi_q - q F_q^{(0)} \Psi_1 \right) e^{(\gamma_q-\gamma) y}, \quad q \geq 1 \tag{26}
\]

(for \( q = 1 \) it is obvious that \( F_q^{(1)} = 0 \)). Let us further introduce a convenient notation \( \varepsilon_q \) by

\[
\varepsilon_q = \frac{\Psi_q}{F_q^{(0)}} - q, \tag{27}
\]

so that in the zeroth and first orders in \( \varepsilon = \left( \frac{\beta}{\rho} \right)^\alpha \) we get

\[
G = 1 + \sum_{q=1}^{\infty} F_q^{(0)} (1 - \varepsilon_q) \left( \frac{\beta \theta}{Q_0} \right)^\alpha \Psi_1 e^{(\gamma_q-\alpha-\gamma) y} n^q (u - 1)^q \tag{28}
\]

The function \( \varepsilon_q \) is plotted in Fig. 5.

In order to compare our final result with the existing experimental data on the factorial moments it is convenient to introduce another function \( \rho_q \)

\[
\rho_q = \frac{1 - F_q / F_q^{(0)}}{\varepsilon_q} = \left( \frac{\beta \theta}{Q_0} \right)^\alpha \Psi_1 e^{(\gamma_q-\alpha-\gamma) y} \tag{29}
\]

From this formula we see that the convenience of using \( \rho_q \) is that it directly measures the nonperturbative contribution to the factorial moments. From Eq. (24) we see, that our formalism predicts that \( \rho_q \) is positive and does not depend on \( q \). Some plots of \( \rho_q \) are shown in Figs. 6 and 7.
Let us now turn to the analysis of the existing experimental data on factorial moments of the intrajet particle distributions. From Eq. (29) it is clear that our main prediction is that if we substitute into Eq. (29) the experimentally measured factorial moments $F_{q}^{\text{exp}}$ and compute the quantity

$$\eta_{q}^{\text{exp}} = (1 - F_{q}^{\text{exp}} / F_{q}^{(0)}) = \rho_{q} \varepsilon_{q},$$

(30)

the difference between the experimental and purely perturbative factorial moments as measured by $\rho_{q}^{\text{exp}}$ should (a) be positive and (b) have no dependence on the moment’s rank $q$.

We have computed the function $\rho_{q}^{\text{exp}}$ for the experimental data on intrajet factorial moments from [14]. We see, that $\rho_{q}^{\text{exp}}$ is indeed positive thus signalling the presence of the contributions beyond the standard perturbative ones in the experimental data. In our treatment it is tempting to assume, that this difference is due to the influence of the soft nonperturbative modes on the process of intrajet’s evolution. From Fig. 6 we see, however, that the data show a clear dependence of $\rho_{q}^{\text{exp}}$ on $q$. This varies somewhat with varying $\alpha$, but always stays quite pronounced. This shows that our treatment of the nonperturbative effects is actually incomplete. However, this is something that had to be expected, because here the small parameter is actually $q \gamma$ [13], so for large enough $q$ the correction is already much larger than the "leading" perturbative term. Therefore with our accuracy we can meaningfully calculate only the corrections to the two first moments, and here the independence of $q$ as a starting approximation does not seem unreasonable. Let us also mention, that in order to get a complete understanding of the possible nonperturbative contributions, it is desirable to analyze the corresponding contributions brought in by conventional hadronization physics in the Monte-Carlo approach (actually the local parton-hadron duality we use seems to work quite well, see, e.g., the recent review [11]) and by taking into account the corrections originated by the running coupling. Finally, let us look at the sensitivity of our results to the choice of the value of the frozen perturbative coupling $\alpha_{s}$ and the parameter $\alpha$ parametrizing the coupling to the nonperturbative modes. In Fig. 6 we show ratio $\rho_{q}$ for 3 different values of $\alpha_{s}$ and in Fig. 7 we show the dependence of the ratio $\rho_{q}$ on the value of the parameter $\alpha$ fixing the functional form of the dissipative interaction. We see, that both dependencies are reasonably smooth.

6 Conclusions.

By using the previously developed formalism [3] we have computed a nonperturbative preasymptotic contribution to the factorial moments of the intrajet particle distribution violating the perturbative scaling. The presence of the additional nonperturbative contribution is supported by the existing experimental data on the factorial moments of the intrajet particle distributions. To arrive
at a final conclusion on the magnitude of the nonperturbative contribution one also has to take into account the contributions from hadronization stage (conventionally parametrized by some standard MC scheme), as well as those due to the running of the coupling constant.

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Figure captions

Fig. 1 The function $B(a)$ and a graphical calculation of $\gamma$ and $\gamma_1$ (see Eqs.(10, 16)) for $\alpha_s = 0.22$ ($\gamma_0^2 = 0.42$) and $\alpha = 1$.

Fig. 2 $\gamma_0$, $\gamma$ and $\gamma_1 - \alpha$ with respect to $\alpha_s$ obtained from Eqs.(10, 16).

Fig. 3 $\gamma_1$ dependence on $\alpha$. See Eqs.(16, 17).

Fig. 4 $F_q^{(0)}$ for three values of $\alpha_s$. See Eq.(13).

Fig. 5 $\varepsilon_q$ for different $\alpha_s$ and $\alpha$. See Eqs.(27, 28).

Fig. 6 $\rho_q$ dependence on $\alpha_s$, $\alpha = 1$. Based on OPAL data for gluon jets.

Fig. 7 $\rho_q$ dependence on $\alpha$ for $\alpha_s = 0.22$. Based on OPAL data for gluon jets.
Fig. 1
Fig. 2
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