Computer simulation of the torque distribution system of a hybrid heavy truck in conditions of slippage of one of the wheels

E O Gurjanova¹, D N Demyanov²

¹ Moscow Technological University, Vernadsky Avenue 78, Moscow, the Russian Federation
² Kazan Federal University, Naberezhnye Chelny Institute, 423812, Russia, Naberezhnye Chelny, Prospekt Syuyumbike 10A
demyanovdn@mail.ru

Abstract. Mathematical and computer models for the motion of a hybrid heavy truck in conditions of wheel slip have been developed. Algorithms for the redistribution of the torque between the wheels to provide controllability and stability of motion along the guided path have been proposed. The efficiency of the developed algorithms is confirmed by the results of computer simulation in the MATLAB / Simulink system.

1. Introduction
The use of hybrid transmissions is currently one of the most effective ways to improve the consumer properties of heavy vehicles. The appearance of cars equipped with AWD-system and the system "start-stop" can significantly improve the controllability of the car and increase its fuel efficiency [1]. Moreover, AWD system is more comfortable for the driver than ESC-system, as it does not involve the use of a brake. However, a significant disadvantage of the AWD system is its high complexity. Thus, the problem of developing control algorithms for AWD-system is very actual.

2. Mathematical modeling
Consider a four-wheeled car equipped with a hybrid transmission. The torque to the rear axle comes from the hybrid powerplant, the front axle is fitted with motor-wheels. The distribution of torque between the wheels of one axle is carried out by means of the limited slip differential [2].

To describe the motion of a heavy vehicle, we use the four-wheel model described in [3]:

\[
\begin{align*}
\dot{m} \cdot x &= (F_{xfr} + F_{xfr}) \cdot \cos(\delta) + F_{xrl} + F_{xrr} - (F_{xfr} + F_{xfr}) \cdot \sin(\delta) + m \cdot \dot{y} \cdot y; \\
\dot{m} \cdot y &= F_{xrl} + F_{xrr} + (F_{xfr} + F_{xfr}) \cdot \sin(\delta) + (F_{xfr} + F_{xfr}) \cdot \cos(\delta) - m \cdot \dot{x} \cdot x; \\
\dot{J}_z \cdot \psi &= l_f \cdot (F_{xfr} + F_{xfr}) \cdot \sin(\delta) + l_f \cdot (F_{xfr} + F_{xfr}) \cdot \cos(\delta) - l_r \cdot (F_{xrl} + F_{xrl}) + \\
&+ \frac{l_r}{2} \cdot (F_{xfr} - F_{xfr}) \cdot \cos(\delta) + \frac{l_r}{2} \cdot (F_{xfr} - F_{xfr}) + \frac{l_r}{2} \cdot (F_{xfr} - F_{xfr}) \cdot \sin(\delta).
\end{align*}
\]

(1)

Here is denoted: \( x, y \) – displacement in the longitudinal and transverse directions, respectively; \( \psi \) –
yaw angle; \( F_{\text{ax}} \) – the reaction force at the contact point of the wheel with the road, where the first index denotes longitudinal or transverse direction of force, the second index denotes the rear or the front axle, and the third index – the left or right side of the vehicle; \( m \) – vehicle weight; \( J_c \) – mass moment of inertia with regard to normal axle; \( \delta \) – the steering angle of front wheels; \( l_f, l_r \) – the distance from the center of mass of the vehicle to the front and rear axle respectively; \( l_w \) – the wheel space.

The magnitude of the longitudinal and transverse reaction force can be determined by the formulas, described in the work [4]:

\[
F_x = C_{\sigma} \cdot \frac{\sigma}{1+\sigma} \cdot f(\lambda) ; \quad F_y = C_{\alpha} \cdot \tan(\alpha) \cdot f(\lambda) .
\]

Here: \( F_x, F_y \) – longitudinal and transverse component of the response reaction respectively; \( C_{\sigma} \) – normal tyre stiffness; \( C_{\alpha} \) – lateral tyre stiffness; \( \sigma \) – the index of the wheel skidding; \( \alpha \) – the slippage angle; \( f(\lambda) \) – a function, describing the wheel parameters, numerical values of which is given in the work [4].

The slippage value for each of the wheels is determined by the formula:

\[
\sigma = \frac{r \cdot \omega - x}{r \cdot \omega} .
\]

Here: \( r \) – rolling radius; \( \omega \) – the angular velocity of the wheel.

The slippage coefficient is used to determine the condition of the wheel. Herewith, they distinguish two modes of a vehicle movement: the lead and the braking condition. In the lead condition the full slippage will take place at the slipping wheel and when the vehicle is stationary \((r = 0)\), and at the braking condition – during the wheel slip movement \((r = \infty)\) [5].

The slippage angles for the wheels of the front or rear axle are determined by the formulas [5]:

\[
\alpha_f = \delta - \frac{y + l_f \cdot \psi}{x} ; \quad \alpha_r = \frac{y - l_r \cdot \psi}{x} .
\]

Here: \( \alpha_f, \alpha_r \) – the slippage angles for the wheels of the front and rear axle respectively.

The rotational rates of the wheels can be determined with the help of the equation of rotational motion dynamics:

\[
\begin{align*}
J_w \cdot \omega_{\phi} &= T_{\phi} - T_{\psi} - r \cdot F_{g}\phi ; \\
J_w \cdot \omega_{\psi} &= T_{\phi} - T_{\psi} - r \cdot F_{g}\psi ; \\
J_w \cdot \omega_{d\phi} &= T_{d\phi} - T_{d\psi} - r \cdot F_{sr}\phi ; \\
J_w \cdot \omega_{d\psi} &= T_{d\phi} - T_{d\psi} - r \cdot F_{sr}\psi .
\end{align*}
\]

It is indicated here: \( J_w \) – mass moment of inertia of the wheel; \( \omega_{d\phi}, \omega_{d\psi} \) – the rotational speed of the wheel, where the first index denotes the front or the rear axle, the second index – the left or the right wheel; \( T_{d\phi}, T_{d\psi} \) – torque and braking torque respectively, affecting the wheel, where the second index denotes the front or the rear axle, and the third index – the left or the right wheel.

The torque for the motor-wheels of the front axle comes from independent drives. The torques on the rear axle wheels are determined by the equations:

\[
T_{d\phi} = \frac{T_m}{2} - q \cdot T_{\text{clutch}} ; \quad T_{d\psi} = \frac{T_m}{2} + q \cdot T_{\text{clutch}} .
\]

It is indicated here: \( T_{d\phi}, T_{d\psi} \) – the torque of the left and the right wheel respectively; \( T_d \) – general torque affecting the axle; \( q \) – the index of differential blocking; \( T_{\text{clutch}} \) – clutch torque.

The system of equations (1) – (6) enables to describe the motion of a heavy-duty vehicle taking into account the possible slippage of the wheels. By completing this system with the ratios to determine the coefficient of differential blocking and the torque of motor-wheels of the front axle, one can get a closed system of equations, which enables to simulate the motion of a heavy-duty vehicle.
3. Computer modeling

On the basis of the equations (1) – (6) in the MATLAB/Simulink system a computer model of the motion of the hybrid heavy truck in conditions of one of the wheels slippage was designed.

The following parameter values, close to the real characteristics of ‘KAMAZ’ family have been chosen: \( m = 46000 \text{ kg} \); \( l_w = 2.04 \text{ m} \); \( l_f = 3 \text{ m} \); \( l_r = 4.27 \text{ m} \); \( r = 0.55 \text{ m} \); \( C_{\sigma} = 490000 \text{ N/m} \); \( C_\alpha = 431200 \text{ N/m} \); \( J_w = 13.5 \text{ kg}\cdot\text{m}^2 \).

The system of torque distribution is modeled as a logical unit. Its input variables are the general torque, the speed of the vehicle, the state of charge and the voltage of the electric battery. The output variables are the value of the torque, affecting each wheel \( T_{df}, T_{dfr}, T_{drl} \).

The algorithm used to control the drives of the front axle motor-wheels is described in detail in [6]. The control of differential blocking is performed in the following way: the electronic control unit detects the moment when the wheel slips and redistributes the total torque to the other wheel.

Figure 1 shows the results of one of the computer experiments in which the movement of a hybrid truck with AWD system and without it was reproduced.

![Figure 1 – The dependence of wheel rotating speed on time](image)

The results of the simulation allow us to conclude that the use of the described control algorithms makes it possible to increase controllability and stability of a truck on different types of surfaces in the of wheel slippage.

4. Results

A mathematical and computer model for the motion of a hybrid heavy truck was developed in conditions of wheel slippage. We proposed algorithms for controlling motor-wheels and the pattern of change for the differential blocking coefficient, that provide improved controllability and stability. The results of the study can be used in practice in the development of active safety systems for hybrid trucks.

5. Acknowledgements

The work is executed at financial support of RFBR (grant № 17-08-00516).

References

[1] Qui L, Qian L, Zomorodi H, Pisu P 2017 *Global optimal energy management control strategies for connected four-wheel-drive hybrid electric vehicles* IET Intelligent Transport Systems
Vol 11 Iss 5 pp 264-272

[2] Andreev A F, Vantsevich V V, Lefarov A H 1987 *Differentials of wheeled vehicles* (M.: Machinostroenie) p 176

[3] Rajamani R 2011 *Vehicle dynamics and control* (Springer) p 500

[4] Dugoff H, Fancher P S, Segal L 1969 *Tire performance characteristics affecting vehicle response to steering and braking control inputs. Final Report* (US National Bureau of Standards) p 105

[5] Kiencke U, Nielsen L 2005 *Automotive Control Systems* (Springer) p 521

[6] Van Е V, Gurjanova Е O, Demyanov D N 2017 *Development of control strategies by transmission elements of a hybrid truck in the case of boundary conditions* XXVIII International innovation-oriented conference of young researchers and students (IIOCYRS - 2016) pp 219-222