Tailoring Chirp in Spin-Lasers

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The usefulness of semiconductor lasers is often limited by the undesired frequency modulation, or chirp, a direct consequence of the intensity modulation and carrier dependence of the refractive index in the gain medium. In spin-lasers, realized by injecting, optically or electrically, spin-polarized carriers, we elucidate paths to tailoring chirp. We provide a generalized expression for chirp in spin-lasers and introduce modulation schemes that could simultaneously eliminate chirp and enhance the bandwidth, as compared to the conventional (spin-unpolarized) lasers.

Many advantages of lasers stem from their modulation response, in which refractive index and optical gain depend on carrier density $n$. Modulation $\delta n(t)$ thus generates both the intensity (photondensity) $\delta S(t)$ and frequency modulation $\delta \nu(t)$ of the emitted light. Such $\nu(t)$, known as chirp, is usually a parasitic effect associated with linewidth broadening, enhanced dispersion, and limiting the high bit-rate in telecommunication systems. Various approaches have therefore focused on low-chirp modulation: pulse shaping injection locking, temperature modulation, and employing quantum dots as the gain region. In conventional lasers for small signal analysis (SSA) in which the quantities of interest are decomposed into a steady state and modulated part $X = X_0 + \delta X(t)$, the chirp is given by

$$\delta \nu(t) = \left[ \Gamma g_0/(4\pi) \right] \alpha_0 \delta n(t),$$

where $\Gamma$ is the optical confinement factor, $g_0$ the gain coefficient, and $\alpha_0 = (\partial n_\tau/\partial n)/(\partial n_t/\partial n)$ is the linewidth enhancement factor[8] expressed in terms of complex refractive index $n = n_\tau + i n_t$ in the active region.

In the emerging class of semiconductor lasers, known as spin-lasers[2,3] with total injection $J = J_+ - J_-$ containing inequivalent spin up/down contributions ($J_+, J_-$), we expect additional possibilities for tailoring chirp. $J_+ \neq J_-$ is realized using circularly-polarized photoexcitation or electrical injection from a magnetic contact. The polarization of emitted light resolved in two helicities, $S = S^+ + S^-$, can be understood from the optical selection rules[22]. For example, in the quantum well-based spin-lasers with $J$ close to the lasing threshold, recombination of spin-up (spin-down) electrons and heavy holes yields $S^-$ ($S^+$) polarized light. Both amplitude modulation (AM) $\delta J(t)$ [see Fig. 1(a)] and polarization modulation (PM) $\delta P_J(t)$, of injection polarization[22] $P_J = (J_+ - J_-)/(J_+ + J_-)$, can be readily implemented. With PM the emitted light could be modulated even at a fixed $J$ and $n$. While Eq. 1 then suggests a chirp-free operation, we show that such a simple reasoning is not always true and suitable generalization for chirp in spin-lasers is required.

Our generalized picture reveals that AM and PM in spin-lasers enable both reduced chirp and enhanced modulation bandwidth, as compared to their spin-unpolarized ($P_J = 0$) counterparts. PM could also provide an efficient spin communication.

The chirp can be simply quantified by comparing the ratio of the central and first sidebands in the emitted light[23]. To visualize this effect, in Fig. 1(b) we show the spectrum of electric field which can be written as

$$E(t) \simeq E_0 [1 + \delta S(t)/(2S_0)] \Re \{ e^{i[2\pi m t + \phi(t)]} \}$$

where $E_0$ is a real amplitude of the field, the phase is $\phi(t) = 2\pi \int_0^t \frac{\delta v(t')}dt'$, and $v_0(\omega_0)$ is (angular) frequency of the output light. Using rate equations (REs) we calculate harmonic modulation with $\omega_m$ in SSA[27] and obtain $\phi(t) = \left[ \int_0^t \frac{\delta v(\omega_m)}{v_0} \sin(\omega_m t + \phi_\nu) \right]$, where $\phi_\nu = \arg(\delta v(\omega_m))$. The undesirable alteration to the original spectrum caused by chirp can be quantified by the ratio between the heights of the first sidebands with and without chirp. For spin-unpolarized lasers, an indentity

$$e^{i\phi} \sin x = \sum_{n=-\infty}^{\infty} J_n(\delta)e^{inx},$$

with asymptotic approxi-
mation \( \delta \ll 1 \) for Bessel functions \( J_n(\delta) \), leads to 

\[
\text{sideband height with chirp} \approx \sqrt{1 + 4 \left( \frac{\text{FM/IM}}{\text{IM}} \right)^2},
\]

where the ratio of frequency and intensity modulation index \( \text{FM/IM} \) can be expressed as 

\[
\text{FM/IM} = \left[ \frac{\delta\nu(\omega_m)/\nu_m}{\delta S(\omega_m)/S_0} \right].
\]

Equation (3) accurately gives the variation of the first sidebands in Fig. 1(b). The phase induced by the chirp also creates higher order sidebands further away. However, by the spin-polarized injection modulation, chirp and thus alteration of the spectrum can be suppressed.

To define chirp in spin-lasers, we recall that the generalization of the usual model of optical gain term \([21,26]\) is 

\[
g_0(n - n_{\text{trans}}) \rightarrow g_0(n_+ + p_+ - n_{\text{trans}}) = g_0[(3/2)n_+ + (1/2)n_- - n_{\text{trans}}],
\]

where \( g_0 \) is density-independent coefficient, \( n_{\text{trans}} \) the transparency density, and \( n_\pm (p_\pm) \) are electron (hole) spin-resolved density. Here 3:1 ratio of \( n_\pm \) contributions follows from the charge neutrality and the very fast spin relaxation of holes \( p_\pm = n/2 \) and this ratio reflects also the gain anisotropy for \( S^+ \) and \( S^- \).

For spin-lasers the generalization of Eq. (1) is then 

\[
\delta\nu(t) = \frac{3g_0}{4\pi} \left[ \frac{3}{2} \alpha_+ \delta n_+(t) + \frac{1}{2} \alpha_- \delta n_-(t) \right],
\]

where we focus on the spin-filtering regime \([\text{Fig. 1(a)}]\), \( J \in (J_{T1}, J_{T2}) \), and \( \alpha_+ = (\partial n_+/\partial n_\pm)/(\partial n_-/\partial n_\pm) \). For \( P_J > 0 \) the spin filtering implies \( S^- \) emitted light. When \( P_J = 0 \) (thus \( n_+ = n_- \)), Eq. (5) reduces to Eq. (1) since 

\[
\alpha_0 = (3\alpha_+ + \alpha_-)/4.
\]

While for \( P_J \neq 0 \), Eq. (6) is not always true (since \( \alpha_\pm \) depends on \( n_\pm \)), we still use it to relate \( \alpha_\pm \) and \( \alpha_0 \). This approximation is precise for \( J_T = J_{T2}/(1 - P_{J0}) \) where \( n_+ - n_- \rightarrow 0 \).

With typical spin-lasers, realized as vertical cavity surface emitting lasers \([27,28]\) in the spin-filtering regime it is accurate to use \( \eta_0 \) vanishing gain compression and spontaneous emission factors \( (\epsilon = \beta = 0) \). With a generalized chirp formulation \([\text{Eq. (5)}]\), we employ RE \([29]\) and SSA to obtain the results from Fig. 1(b). We confirm the chirp suppression in spin-lasers with the spectrum approaching the chirp-free case.

In conventional lasers, the chirp reduction is particularly important for high-frequency modulation where the transient chirp \( [\alpha \Delta \ln S(t)/dt] \) only weakly \( \epsilon \)-dependent is the dominant contribution. FM/IM is a constant 

\[
\frac{\delta\nu(\omega_m)/\nu_m}{\delta S(\omega_m)/S_0} = -\frac{i \alpha_0}{2},
\]

which provides both a simple way to experimentally extract \( \alpha_0 \) the linewidth enhancement factor \( \alpha_0 \), and a

![FIG. 2: (Color online) \( |\text{FM/IM}| \) normalized to the conventional value \( \alpha_0/2 \) for (a) AM and (b) PM, shown for \( \rho = \alpha_+/\alpha_- = 2 \) (solid) and \( \rho = 0.5 \) (dashed). Green (gray) curves represent only a small change for finite electron spin relaxation time \( \tau_s = \tau_r \). The regime of reduced chirp in spin-lasers (darker regions) is delimited with dotted lines for (c) AM and (d) PM. The circle in (a) and (c) for \( \rho = 0.5 \) represents the sampling point to generate Fig. 1(b). \( J_0 = 1.9J_T \) and \( P_{J0} = 0.5 \) are used in (a)-(d).](image)
FIG. 3: (Color online) SSA of CM. (a) Chirp-tailoring function $\kappa(\nu_m)$ and (b) modulation response $|R(\nu_m)/R(0)|^2$ are shown for $J_0 = 1.2 J_T$, $P_{J0} = 0.5$ and different $\rho$’s. The response of conventional laser (AM, $P_J = 0$) is given (gray/purple) for comparison.

completely removed using PM or AM. However, it is possible to achieve zero-chirp by introducing a scheme we term complex modulation (CM): one of the spin-resolved injections ($J_+$ for $P_{J0} > 0$) is the input signal, while the other is used only to cancel the chirp. From Eq. (8), the zero-chirp condition is $\delta \alpha_\pm(\omega_m)/\delta \nu_m = -3\alpha_+|/\alpha_\mp = -3\rho$, which can be satisfied by introducing a chirp-tailoring function $\kappa(\omega_m)$ obtained from SSA

$$\delta J_\pm(\omega_m) = \kappa(\omega_m) \delta J_\mp(\omega_m).$$ \hspace{1cm} (9)

Here $\delta J_\pm$ is the input modulation responsible for the modulation of emitted light $\delta S^\pm$, while the correction current $\delta J_\mp$ compensates the variation of the carrier density to reduce the chirp.

We next use SSA to consider the implications of CM on the modulation bandwidth, shown together with the chirp-tailoring function $\kappa$ in Fig. 3. The CM relaxation oscillation frequency, represented by the peak positions in Figs. 3(a) and (b) for $\rho \leq 1$, can be expressed as

$$\omega_{R}^{CM} \approx \{\Gamma_0 J_T (J/J_T - 1) (1-\rho)\}^{1/2},$$ \hspace{1cm} (10)

where $J_T = J_T/(1+P_{J0}/2)$ is the reduced threshold in a spin-laser\[^{23}\] The peak positions coincide for $|\kappa(\omega_m)|$ and for the modulation response function\[^{10}\] $R(\omega_m) = \{\delta^+/(\omega_m)/\delta J_\mp(\omega_m)\}$ because the character of $J_\pm(\omega_m)$ is $\kappa(\omega_m)$ propagates through $n_\pm$ and $S_\mp$ into $R(\omega_m)$. For $\rho > 1$, $|\kappa(\omega_m)|$ increases monotonically with $\omega_m$ showing no peak. Zero-chirp is not feasible for $\rho = 1$ since it is the same FM/IM as in conventional lasers [Eq. (3)].

By comparing $\omega_{R}^{CM}$ in Eq. (10) for $\rho \leq 1$ to $\omega_{R}^{AM}$ and $\omega_{R}^{PM}$ from Ref.\[^{16}\]$ $\omega_{R}^{AM,PM} \approx \{\Gamma_0 J_T (J/J_T - 1)\}^{1/2}$, we see that CM has narrower bandwidth than AM and PM (estimated by $\omega_{R}$), for the same $P_{J0}$. While CM provides a path for removing chirp, it may come at the cost of a reduced bandwidth. However, an optimized value of $\rho = 0.5$ in Fig. 3(b) yields simultaneously zero chirp and bandwidth enhancement, as compared to conventional lasers.

What about experimental feasibility to control chirp in spin-lasers? While CM has yet to be attempted, it can be viewed as a combination of AM and PM which individually already lead to an improved chirp (Figs. 1 and 2) and have been demonstrated in spin-lasers. Slow PM has been realized\[^{23}\] using a Soleil-Babinet polarization retarder at a fixed $J \in (J_{T1}, J_{T2})$. Fast PM ($\nu_m \sim 40 \text{ GHz}$) can be implemented with a coherent electron spin precession in a transverse magnetic field\[^{27}\] or a mode conversion in an electro-optic modulator\[^{23}\]. Recent advances in using birefringence for PM\[^{29}\] suggest that chirp reduction in spin-lasers could be feasible at higher injection, beyond the spin-filtering regime we have considered.

To further enhance the opportunities in spin-lasers, it would be helpful to utilize other gain media and achieve technologically important emission at 1.3 and 1.55 $\mu$m. We expect that our proposals will stimulate additional work toward understanding the spin-dependence of refractive index (already used for fast all-optical switching\[^{23}\]) and its implications for spin-lasers.

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We base SSA on the rate equations Eqs. (4)-(6) for conventional lasers and Eqs. (22) and (A1) for spin lasers in Ref. [23]. The parameters used for numerical calculations are given in Table I from Ref. [16].
We can infer that $\alpha_+ \neq \alpha_-$, since the Faraday angle, representing the asymmetry of the refractive indices for $S^\pm$, depends on $n$ [S. Crooker, D. Awschalom, J. Baumberg, F. Flack, and N. Samarth, Phys. Rev. B 56, 7574 (1997); R. Bratschitsch, Z. Chen, and S. T. Cundiff, Phys. Stat. Sol. (c) 0, 1506 (2003)].
If there is an emitted light with the other helicity, the chirp from $S^+$ signal, $\delta \nu^\prime(t)$, can be written analogously:
$$\delta \nu^\prime(t) = \Gamma g_0/(4\pi)[(1/2)\alpha_+^\prime \delta n_+(t) + (3/2)\alpha_-^\prime \delta n_-(t)].$$
Our approximation $\beta = 0$ throughout the paper, used also in Fig. [1] accurately captures the expected behavior for experiments on spin-lasers at 300 K, parametrized with $\beta \sim 10^{-5}$ in Ref. [9].
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