Systematic bias in the estimate of cluster mass and the fluctuation amplitude from cluster abundance statistics

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(Received 2005 November 22; accepted 2006 February 1)

Abstract

We revisit the estimate of the mass fluctuation amplitude, \( \sigma_8 \), from the observational X-ray cluster abundance. In particular, we examine the effect of the systematic difference between the cluster virial mass estimated from the X-ray spectroscopy, \( M_{\text{vir, spec}} \), and the true virial mass of the corresponding halo, \( M_{\text{vir}} \). Mazzotta et al. (2004) recently pointed out the possibility that \( \alpha_M = M_{\text{vir, spec}}/M_{\text{vir}} \) is systematically lower than unity. We perform the statistical analysis combining the latest X-ray cluster sample and the improved theoretical models and find that \( \sigma_8 \sim 0.76 \pm 0.01 + 0.50(1 - \alpha_M) \) for \( 0.5 \leq \alpha_M \leq 1 \), where the quoted errors are statistical only. Thus if \( \alpha_M \sim 0.7 \), the value of \( \sigma_8 \) from cluster abundance alone is now in better agreement with other cosmological data including the cosmic microwave background, the galaxy power spectrum and the weak lensing data. The current study also illustrates the importance of possible systematic effects in mapping real clusters to underlying dark halos which changes the interpretation of cluster abundance statistics.

Key words: cosmology: theory — dark matter — galaxies: clusters: general — X-rays: galaxies

1. Introduction

Recent progress in observational cosmology has made it possible to determine precise values of cosmological parameters including the matter density parameter \( \Omega_M \), the cosmological constant \( \Omega_\Lambda \), the dimensionless Hubble constant \( h \equiv H_0/(100 \, \text{km s}^{-1} \, \text{Mpc}^{-1}) \), and the mass fluctuation amplitude at \( 8 \, h^{-1} \, \text{Mpc} \), \( \sigma_8 \). For instance, Spergel et al. (2003) obtained \((\Omega_M, \Omega_\Lambda, h, \sigma_8) = (0.29 \pm 0.07, 0.71 \pm 0.07, 0.72 \pm 0.05, 0.9 \pm 0.1)\) from the first-year data of
the Wilkinson Microwave Anisotropy Probe (WMAP) under the assumption of a spatially flat universe, $\Omega_M + \Omega_\Lambda = 1$. These estimates slightly vary when combined with other observational probes such as the galaxy power spectrum, weak lensing data and the Hubble diagram of Type Ia supernovae, but the value of $\sigma_8$, which is the main focus of the present paper, is always larger than 0.8. For instance, Tegmark et al. (2004) concluded that $\sigma_8 = 0.89 \pm 0.02$ and $\Omega_M h = 0.213 \pm 0.023$.

Cluster abundance has been known as yet another useful probe of the value of $\sigma_8$ (White, Efstathiou & Frenk 1993). In a decade ago, the methodology seemed to have almost established a value of $\sigma_8 = 0.9 \sim 1.0$ for standard $\Lambda$CDM (Lambda-dominated Cold Dark Matter) cosmology (e.g., Viana & Liddle 1996; Eke et al. 1996; Kitayama & Suto 1996; Kitayama & Suto 1997; Kitayama, Sasaki & Suto 1998) when combined with a simple self-similar model mass–temperature (M-T) relation of X-ray clusters ($M \propto T^{3/2}$; see Kaiser 1986). Seljak (2002), however, showed that the use of the observed M-T relation (Finoguenov et al. 2001), rather than the simple self-similar M-T relation, leads to a much lower value:

$$\sigma_8 = (0.77 \pm 0.07)(\Omega_M/0.3)^{-0.44}(\Gamma/0.2)^{0.08}, \quad (1)$$

where $\Gamma$ is the shape parameter of the CDM power spectrum.

His result was also confirmed later by Shimizu et al. (2003). They constructed the M-T relation from a combined analysis of X-ray luminosity-temperature relation and temperature function of clusters, and found $\sigma_8 = 0.7 \sim 0.8$. The derived M-T relation is marginally consistent with the observed ones (Finoguenov et al. 2001; Allen et al. 2001), but is in clear conflict with the simple relation as $M \propto T^{3/2}$. Therefore it seems now clear that cluster abundance combined with the reliable M-T relation (Finoguenov et al. 2001), rather than the simple self-similar M-T relation, leads to a much lower value of $\sigma_8$ than the other cosmological indications.

Given the above situation, it is important to recall that the conventional modeling of galaxy clusters in terms of their mass or temperature is oversimplified; non-sphericity, inhomogeneity and substructure in the intracluster medium would give rise to both random and systematic variations to cluster properties with respect to the simple model predictions. Figure 1 is the improved version of the plot presented before by one of us (Suto 2002; Suto 2003), which summarizes a wide range of practical, and quite different, definitions of dark halos that are directly related to cosmology, but cannot be directly observed and of galaxy clusters. Of course they are closely related, but any simple one-to-one correspondence is unrealistic, and should be understood as a working hypothesis. One has to improve the working hypothesis continuously in order to increase the reliability of cluster abundance statistics.

In this context, it is important to note that Mazzotta et al. (2004) and Rasia et al. (2005) pointed out a potentially important source for the systematic bias in estimating temperature and mass of X-ray clusters, which motivated our current study. Indeed galaxy clusters may consist of multi-phase temperature structure, and it is not a straightforward task to define the
Abell clusters
selected by eyes
from the photographic plates
galaxies satisfying
$m_1 < m < m_2 + 2

SZ clusters
inverse Compton of CMB
$\Delta I_{\text{sz}} = n_s T_a R c_1$

Halos in N-body simulations
friend-of-friend method FOF(0.2)
spherical overdensity SO(324)
on-spherical identification

Press-Schechter halos
nonlinear spherical collapse
$\Delta_{\text{vir}} = 18 \pi^2$

X-ray clusters
thermal bremsstrahlung
$S = n_e^2 T^{1/2} R_{c1}$

overall single temperature which characterizes the cluster as a whole (see also Vikhlinin 2005).

Usually X-ray observers estimate the temperature of a cluster by a single temperature
fit to the observed X-ray spectrum of the cluster. Let us call it the spectroscopic temperature, $T_{\text{spec}}$. It is natural, and indeed has been (implicitly) assumed that $T_{\text{spec}}$ is equivalent to the emission-weighted temperature, $T_{\text{ew}}$, in which the locally defined temperature $T$ in a small region of the cluster is averaged over with a weight of its emission measure. More specifically, it is given by

$$T_{\text{ew}} \equiv \frac{\int T n^2 \Lambda(T) dV}{\int n^2 \Lambda(T) dV},$$

where $n$ and $T$ are the gas density and temperature, $\Lambda(T)$ is the cooling function ($\propto \sqrt{T}$ for thermal bremsstrahlung), and the integration is over the entire cluster volume. Mazzotta et al. (2004), however, noticed that $T_{\text{spec}}$ significantly underestimates the value of $T_{\text{ew}}$ if clusters have multi-phase temperature structure. Since relatively cool clumps in a cluster exhibit many prominent emission lines, any single temperature spectroscopic fitting to the cluster naturally tends to be biased toward such low temperature clumps. In addition, Mazzotta et al. (2004) showed that the systematic underestimate occurs also in the case of thermal bremsstrahlung alone even without considering contributions of emission lines. The authors introduced a spectroscopic-like temperature $T_{\text{sl}}$:

$$T_{\text{sl}} \equiv \frac{\int T n^2 T^{-0.75} dV}{\int n^2 T^{-0.75} dV},$$

which reproduces $T_{\text{spec}}$ within a few percent for simulated clusters hotter than a few keV (assuming Chandra or XMM-Newton detector response functions). Rasia et al. (2005) per-
formed a more systematic study of the relation between $T_{ew}$ and $T_{sl}$ using a sample of clusters from SPH simulations with radiative cooling and heating, and found that $T_{sl} = (0.70 \pm 0.01)T_{ew} + (0.29 \pm 0.05)$ keV for $2 \text{ keV} \lesssim T_{ew} \lesssim 13$ keV. We note here that Mathiesen & Evrard (2001), based on their adiabatic simulations, also noticed earlier that $T_{spec}$ tends to be lower than $T_{ew}$, while the systematic difference is somewhat smaller than that found by Rasia et al. (2005).

As already discussed in Rasia et al. (2005), the above result should have a significant impact on the estimate of $\sigma_8$ from cluster abundance. If one simply uses $T_{sl}$, instead of $T_{ew}(> T_{sl})$ in converting the temperature to the underlying halo mass, one would underestimate the true mass and the amplitude of the halo mass function, leading to an underestimation of $\sigma_8$ as well. Furthermore, several numerical simulations indicate that the assumption of hydrostatic equilibrium itself, applied with the use of $T_{ew}$, tends to underestimate the cluster mass by $\sim 20\%$ (e.g., Muanwong et al. 2002; Borgani et al. 2004; Rasia et al. 2004). Taken together, they may therefore account for the systematically smaller value of $\sigma_8$ derived from cluster abundance as described in the above. In reality, however, a reliable prediction requires a more careful treatment of the selection function and the statistical analysis of the observational sample. This is exactly what we will conduct below.

We do not attempt to find the best-fit set of cosmological parameters, but rather focus on the precise determination of $\sigma_8$. Thus in this paper, we adopt a conventional $\Lambda$CDM model with the following parameters; the matter density parameter $\Omega_M = 0.27$, the cosmological constant $\Omega_{\Lambda} = 0.73$, and the dimensionless Hubble constant $h_{70} = h/0.7 = 1$. We denote natural and decimal logarithms by $\ln$ and $\log$, respectively.

2. Method

The estimate of $\sigma_8$ from cluster abundance requires a variety of theoretically and/or observationally calibrated relations among mass, luminosity and temperature of clusters. Since our main interest here is the effect of the difference between $T_{ew}$ and $T_{spec}$, we carefully re-examine those relations that directly involve the cluster temperature. Otherwise we adopt the conventional modeling, following, but improving wherever possible, the procedure of Ikebe et al. (2002).

2.1. Mass function of dark matter halos

Recent numerical simulations significantly advanced the understanding of mass function of dark matter halos, and provide several fitting formulae that are more accurate than their analytic counterpart (Press & Schechter 1974). In the present paper, we adopt the result of Jenkins et al. (2001). The formula is based on the SO(324) halos which are identified when their spherical over-density within the virial mass, $M_{vir}$, exceeds 324 times the mean matter background density, $\bar{\rho}$:
\[
\frac{dn}{d\ln M_{\text{vir}}} = 0.316 \exp\left(-|\ln \sigma^{-1} + 0.67|^{3.82}\right) \frac{\bar{\rho}}{M_{\text{vir}}} \frac{d\ln \sigma^{-1}}{d\ln M_{\text{vir}}}, \tag{4}
\]

\[
\sigma^2(M_{\text{vir}}) = 4\pi \int P(k) \hat{W}^2(k; R_{\text{vir}}) k^2 dk, \tag{5}
\]

\[
\hat{W}(k; R_{\text{vir}}) = \frac{3}{(kR_{\text{vir}})^3} \left[ \sin(kR_{\text{vir}}) - kR_{\text{vir}} \cos(kR_{\text{vir}}) \right], \tag{6}
\]

where \( P(k) \) is the linear power spectrum of matter fluctuations, and \( R_{\text{vir}} \equiv (3M_{\text{vir}}/4\pi\bar{\rho})^{1/3} \).

### 2.2. Mass-temperature relation of X-ray clusters

The most uncertain procedure in estimating \( \sigma_8 \) from cluster abundance is how to relate the dark matter halos, which are not directly observable, to the actually observed X-ray clusters. Strictly speaking, there is no reason why one can rely on any one-to-one mapping between (simulated) halos and X-ray clusters; non-sphericity, substructure, and merging history, among others, should be taken into account to specify their individual properties (e.g., Taruya & Suto 2000; Komatsu et al. 2001; Suto 2002; Jing & Suto 2002; Suto 2003; Kitayama et al. 2004). Nevertheless it is common to characterize halos and clusters merely as a function of their mass and temperatures, respectively, and to relate them on the basis of an empirically determined M-T relation (e.g., Shimizu et al. 2003). As described in Introduction, this procedure is fairly successful. Nevertheless the resulting conclusion should be interpreted with caution if one wants to take its precision and accuracy seriously.

Let us make clear a few different definitions of mass and temperature of clusters in order to specify our assumptions on their mutual relation. For simulated clusters, one can compute the emission-weighted temperature, \( T_{\text{ew}} \) (eq. [2]), and the spectroscopic temperature, \( T_{\text{spec}} \). The mass of observed clusters within a radius \( R \) is usually derived on the assumption of hydrostatic equilibrium:

\[
M(R) = -\frac{k_B T_{\text{gas}}(R) R}{G \mu m_H} \left[ \frac{d \ln n_{\text{gas}}(R)}{d \ln R} + \frac{d \ln T_{\text{gas}}(R)}{d \ln R} \right], \tag{7}
\]

where \( k_B \) is the Boltzmann constant, \( G \) is the gravitational constant, \( \mu \) is the mean molecular weight, \( m_H \) is the proton mass, and \( n_{\text{gas}} \) and \( T_{\text{gas}} \) are the gas density and temperature, respectively. We define \( M_{\text{vir, ew}} \) and \( M_{\text{vir, spec}} \) as those evaluated at a radius within which the mean over-density is 324 when we use \( T_{\text{ew}} \) and \( T_{\text{spec}} \), respectively, for \( T_{\text{gas}} \) in equation (7). To be more strict, our \( T_{\text{ew}} \) and \( T_{\text{spec}} \) correspond to \( T_{\text{ew}}(R_{\text{vir}}) \) and \( T_{\text{spec}}(R_{\text{vir}}) \) if the temperature profile is taken into account.

For definiteness and simplicity, we assume that

\[
T_{\text{spec}} = \alpha_T T_{\text{ew}}, \tag{8}
\]

where \( \alpha_T \) is a constant. According to Rasia et al. (2005), \( \alpha_T \sim 0.7 \) is favored from numerically simulated clusters. Equations (7) and (8) alone imply \( M_{\text{vir, spec}} = \alpha_T M_{\text{vir, ew}} \). Taking account of the additional possibility that \( M_{\text{vir, ew}} \) is systematically lower than the actual virial mass \( M_{\text{vir}} \).
of simulated clusters (e.g., Muanwong et al. 2002; Borgani et al. 2004; Rasia et al. 2004), we simply relate $M_{\text{vir, spec}}$ to $M_{\text{vir}}$ as

$$M_{\text{vir, spec}} = \alpha_M M_{\text{vir}},$$

where the proportional constant $\alpha_M$ accounts for both the difference of $T_{\text{spec}}$ and $T_{\text{ew}}$ and that of $M_{\text{vir, ew}}$ and $M_{\text{vir}}$. If $M_{\text{vir, ew}} = M_{\text{vir}}$, then $\alpha_M = \alpha_T$.

Incidentally it is interesting to note that equation (9) with $\alpha_M \sim 0.6$ accounts for the well-known systematic difference between $M_{\text{vir, spec}}$ and the lensing mass estimate in galaxy clusters (e.g., Wu 2000; Schmidt et al. 2001) if the latter is equal to the actual virial mass. Another possible consequence of such a systematic difference in temperature may be found in the interpretation of the Sunyaev-Zel’dovich effect observations, where one needs to distinguish the mass-weighted temperature and the X-ray emission-weighted or spectroscopic temperature (e.g., Yoshikawa et al. 2000; Yoshikawa et al. 2001; Komatsu & Seljak 2001).

Next we fit the M-T relation to the cluster sample of Pointecouteau et al. (2005) using a power-law model of the form:

$$\frac{M_{\text{vir, spec}}}{10^{14} h(z)^{-1} M_\odot} = A_{\text{vir}} \left(\frac{T_{\text{spec}}}{5 \text{ keV}}\right)^p,$$

where

$$h(z) = h_{70} \sqrt{\Omega_M (1+z)^3 + (1 - \Omega_M - \Omega_\Lambda)(1+z)^2 + \Omega_\Lambda}.$$ 

In practice, we follow Shimizu et al. (2003), and convert the values of mass $M_{200} \pm \Delta M_{200}$ in their paper to $M_{\text{vir}} \pm \Delta M_{\text{vir}}$ using the redshift-dependent overdensity threshold estimated from the spherical collapse model (e.g. Kitayama & Suto 1996), and assuming the universal density profile of the hosting halo. We adopt that the inner power-law index is equal to 1, and use the concentration parameter, $c_{200}$, listed in Pointecouteau et al. (2005). In addition, we scale the masses by $h(z)$, which corrects the evolutionary effect due to the observed redshifts of the clusters following Allen et al. (2001) and Arnaud et al. (2005). We perform the fit on the log $T$-log $M$ plane, and find that $A_{\text{vir}} = 10^{0.829 \pm 0.018}$ and $p = 1.74 \pm 0.10$. The best fit M-T relation and the observational data are plotted in figure 2.

2.3. Fitting Procedure

We search for the best-fit $\sigma_8$ from the maximum likelihood analysis of X-ray cluster number density on the luminosity and temperature plane, following Ikebe et al. (2002). We assume that at a given spectroscopic temperature the cluster luminosity follows the log-normal distribution:

$$p_L(\log L|T_{\text{spec}}) d\log L = \frac{1}{\sqrt{2\pi}\sigma_{\log L}^2} \exp \left\{ -\frac{[\log L - \log L(T_{\text{spec}})]^2}{2\sigma_{\log L}^2} \right\} d\log L,$$

around the mean of the logarithm of the luminosity, $\log L(T_{\text{spec}})$, for a given temperature $T_{\text{spec}}$, where $\sigma_{\log L}$ is its standard deviation.
Fig. 2. Mass-temperature relation of the cluster sample by Pointecouteau et al. (2005) (symbols with error bars). Also plotted is the best-fit power-law that we adopt in the present analysis (eq. [10]).

The predicted number of clusters per unit logarithmic luminosity and unit logarithmic temperature, $N(L, T_{\text{spec}})$, is then given by

$$N(L, T_{\text{spec}}) d\log L d\log T_{\text{spec}} = \frac{dn}{dM_{\text{vir}}} \bigg|_{z=0.046} \frac{dM_{\text{vir}}}{dT_{\text{spec}}} V_{\text{max}}(L) p_L \langle \log L | T_{\text{spec}} \rangle \frac{dT_{\text{spec}}}{d\log T_{\text{spec}}} d\log L d\log T_{\text{spec}},$$

(13)

where we estimate the mass function at the median redshift value of the cluster sample ($\langle z \rangle = 0.046$), and $dM_{\text{vir}}/dT_{\text{spec}}$ is computed from equation (10). The maximum comoving volume, $V_{\text{max}}(L)$, is given by

$$V_{\text{max}}(L) = \frac{\Delta \Omega}{4\pi} \int_0^\infty dz \frac{dV}{dz} \int_{f_{\text{lim}}}^\infty \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp \left[ -\frac{(f-f_0)^2}{2\sigma_i^2} \right] df,$$

(14)

where $\Delta \Omega = 8.14 \, \text{sr}$ is the total sky coverage, $dV/dz$ is a volume element per unit redshift per unit solid angle of the sky, $f_{\text{lim}} = 2.0 \times 10^{-11} \, \text{erg \, s}^{-1} \, \text{cm}^{-2}$ is the observational flux limit in the $0.1$–$2.4$ keV band, $\sigma_i = 10^{-12} \, \text{erg \, s}^{-1} \, \text{cm}^{-2}$ is a typical flux measurement error. We set the average flux as $f_0 = L/4\pi d_L^2(z)$, where $d_L(z)$ is the luminosity distance at $z$.

If the number of clusters with a given luminosity and temperature obeys the Poisson distribution, the corresponding likelihood function reduces to (e.g., Cash 1979)

$$\ln \mathcal{L} = \sum_i \ln N(L_i, T_i) - \int N(L, T_{\text{spec}}) d\log L d\log T_{\text{spec}} + \text{const.},$$

(15)

where $L_i$ and $T_i$ are the observed luminosity and spectroscopic temperature of the $i$-th clus-
ter, respectively. The summation is taken over the observed clusters with temperature larger than our adopted threshold $T_{\text{min}}$. In practice, we consider three values, $T_{\text{min}} = 1.4$, $3.0$, and $5.0 \text{ keV}$ in order to see the extent to which our correction for the flux-limit in the observational sample changes the conclusion (see the next section). The integration in the second term of equation (15) is performed for $41.5 < \log (L/h^{-2}\text{erg s}^{-1}) < 45.5$ and $T_{\text{min}} < T_{\text{spec}} < 11.2 \text{ keV}$.

By maximizing the likelihood function given above, we are practically fitting the observed X-ray temperature function (XTF) and the luminosity-temperature relation simultaneously. This method has the following advantages over the conventional chi-square fitting to the XTF data alone; 1) the Malmquist bias of the observed luminosity-temperature relation is corrected using the observed XTF, 2) it is free from any bias arising from binning a small number of data.

The above procedure is essentially the same as that of Ikebe et al. (2002) except that we adopt the observed M-T relation (eq. [10]) and the Jenkins mass function (eq. [4]) in equation (13). The integration over the temperature in equation (15) of Ikebe et al. (2002) is omitted because the temperature measurement errors are already incorporated in the observed M-T relation. We have further checked that the fitted values of $\sigma_8$ will be unchanged within $\pm 0.01$ even if we artificially introduce the intrinsic scatter of $\sqrt{\langle(\Delta T/T)^2\rangle} \sim 0.05$ into the adopted M-T relation.

Since the main focus of the present paper is the systematic bias on the value of $\sigma_8$ arising from errors in cluster mass measurements, we do not intend to repeat time-consuming multi-dimensional fit already performed by Ikebe et al. (2002). Instead, we fix the luminosity-temperature relation of clusters. Specifically we adopt the fitting result of Ikebe et al. (2002), PS(Flat, $T > 3 \text{ keV}$) in their Table 3 where they assume the analytic Press-Schechter mass function, the flatness of the universe and $T_{\text{spec}} > 3 \text{ keV}$:

$$\log L_{0.1-2.4\text{keV}}(T_{\text{spec}}) = 42.19 + 2.44\log(T_{\text{spec}}/1\text{keV}) - 2\log h. \quad (16)$$

In the above expression, $\log L_{0.1-2.4\text{keV}}(T_{\text{spec}})$ is the mean logarithmic X-ray luminosity of clusters in the 0.1–2.4 keV band for $T_{\text{spec}}$. We also adopt that $\sigma_{\log L} = 0.23$ based on their results. We have made sure that the amplitude, the slope and the scatter in the above fit are not affected by our use of the Jenkins mass function and the observed M-T relation.

Figure 3 shows the observed luminosity-temperature relation of the sample (Ikebe et al. 2002). The dashed line indicates the straightforward power-law fit to those data. Our adopted luminosity-temperature relation (solid line) takes into account the flux-limit of the observations (i.e., the Malmquist bias), and thus has a systematically lower amplitude than a direct fit to the observed data.
Fig. 3. The luminosity–temperature relations of clusters. Symbols with error bars represent the cluster sample of Ikebe et al. Solid line is our adopted power-law model (eq. [16]) with the corresponding log-normal errors (shaded region). Dashed line indicates the direct fit to the data without taking account of the Malmquist bias.

3. Results

We perform the analysis for $\alpha_M = M_{\text{vir, spec}}/M_{\text{vir}} = 1, 0.9, 0.8, 0.7, 0.6$ and 0.5. The values of $\sigma_8$ maximizing the likelihood function are summarized in table 1. The clear general trend is that the best-fit $\sigma_8$ systematically increases as $\alpha_M$ decreases and $T_{\text{min}}$ increases; if $\alpha_M$ becomes smaller, the mass of the corresponding halo, $M_{\text{vir}}$ is indeed larger than the naive estimate $M_{\text{vir, spec}}$. Therefore the resulting amplitude of the observed mass function becomes larger, which requires larger $\sigma_8$. This is exactly expected. On the other hand, the weak trend with respect to $T_{\text{min}}$, if real, might indicate that the sample is not yet completely corrected for the Malmquist bias, and/or that the sample is still statistically limited. Table 1 indicates that the best-fit value for $T_{\text{min}} = 3$ keV is roughly given as

$$\sigma_8 = 0.76 \pm 0.01 + 0.50(1 - \alpha_M).$$

(17)

The quoted errors represent the statistical error only, and the systematic error due to the difference of $T_{\text{min}}$ amounts to ±0.02. Thus the systematic difference of the spectroscopic and the true virial mass $\alpha_M \equiv M_{\text{vir, spec}}/M_{\text{vir}} \sim 0.7$ indeed reconciles the discrepancy of $\sigma_8$ between the cluster abundance and Tegmark et al.’s result, for instance.

To exhibit the goodness-of-fit of our derived parameters, we plot the cumulative number

9
Table 1. Best-fit values of $\sigma_8$ for different $\alpha_M$ and $T_{\text{min}}$.

| $\alpha_M$ | $> 1.4 \text{ keV}$ | $> 3 \text{ keV}$ | $> 5 \text{ keV}$ |
|------------|----------------------|-------------------|-------------------|
| # of clusters | 61                   | 51                | 26                |
| 1          | $0.76^{+0.02}_{-0.01}$ | $0.78 \pm 0.02$  | $0.79 \pm 0.02$  |
| 0.9        | $0.79^{+0.02}_{-0.01}$ | $0.81^{+0.02}_{-0.01}$ | $0.82^{+0.02}_{-0.01}$ |
| 0.8        | $0.83^{+0.02}_{-0.01}$ | $0.85 \pm 0.02$  | $0.86 \pm 0.02$  |
| 0.7        | $0.88 \pm 0.02$      | $0.90 \pm 0.02$  | $0.91 \pm 0.02$  |
| 0.6        | $0.94 \pm 0.02$      | $0.96 \pm 0.02$  | $0.97^{+0.02}_{-0.03}$ |
| 0.5        | $1.02^{+0.03}_{-0.02}$ | $1.03^{+0.03}_{-0.02}$ | $1.04 \pm 0.03$ |

Counts of clusters as a function of $T_{\text{spec}}$ in figure 4:

$$N(> T_{\text{spec}}) = \int_{T=\infty}^{T=T_{\text{spec}}} d\log T \int_{L=0}^{L=\infty} d\log L \, N(L,T).$$

(18)

Given the simplified assumptions of single power-law fits both to the observational M-T and to the underlying luminosity-temperature relations, the fits are in reasonable agreement. Note that because the horizontal axis of figure 4 is $T_{\text{spec}}$, the effect of $\alpha_M$ does not look appreciable in the resulting curves. In reality, however, the relation to the underlying halo mass is very different, and this is why one needs larger values of $\sigma_8$ to compensate the effect.

We would also like to call attention to the fact that the quality of the fit degrades at $T_{\text{spec}} < 3 \text{ keV}$. This may possibly be due to some unknown systematic effects in the observational sample of Ikebe et al. (2002). While not obvious in the cumulative distribution like figure 4, there are few clusters observed around $T_{\text{spec}} \sim 5 \text{ keV}$ in their sample (see also Figure 6 of Shimizu et al. 2003). Furthermore, the theoretical curves at $T_{\text{spec}} < 3 \text{ keV}$ are highly sensitive to how the flux limit of the sample is corrected when modeling the XTF and the luminosity-temperature relation. For these reasons, we prefer to rely on the results of $T_{\text{min}} > 3 \text{ keV}$ even though the difference due to the choice of $T_{\text{min}}$ is not big (table 1). Definitely, more reliable conclusions on $\sigma_8$ should still need future well-controlled statistical samples of clusters.

4. Summary and discussion

We have shown that the systematic underestimate bias of the spectroscopic and emission-weighted temperatures $\alpha_M \equiv M_{\text{vir, spec}}/M_{\text{vir}}$ may reconcile the discrepancy of the values of $\sigma_8$ between the cluster abundance and the other cosmological analyses if $\alpha_M \sim 0.7$.

Another equally important lesson that we have learned from this analysis, however, is that the apparent agreement with independent observational estimates does not justify the use of any crude but conventional assumptions. If we compare the value of $\sigma_8$ alone, the latest estimate $\sigma_8 \approx 0.9$ by WMAP is indeed in good agreement with that obtained from cluster abundance argument a decade ago (Viana & Liddle 1996; Eke et al. 1996; Kitayama & Suto 1996; Kitayama 2000...
Fig. 4. Cumulative number counts of the X-ray clusters as a function of $T_{\text{spec}}$: Left $\alpha_M = 1$, Middle $\alpha_M = 0.8$ Right $\alpha_M = 0.6$. The shaded histogram indicates the range of the observational data of Ikebe et al. (2002) with the Poisson errors. Solid, dashed and dotted curves show the results from our best-fit models for $T_{\text{min}} = 5$ keV, 3 keV and 1.4 keV.

It is often inevitable to introduce simple, reasonable, but nevertheless inaccurate, assumptions in modeling and analyses of cosmological observations since cosmological objects cannot be predicted from the first principle of physics. Naturally we would like to conclude that those assumptions are justified when they lead to the same values of the cosmological parameters derived from independent dataset and analysis. In most cases, the above procedure may not be wrong, but still one has to keep in mind that the procedure as a whole cannot be justified strictly because of the mere agreement since the goal is not to determine the precise
value of parameters, but to improve the understanding of the underlying physical processes involved.

In this sense, the estimate of $\sigma_8$ that we have presented in this paper is certainly indicative, but may not be the final answer. To reach more reliable conclusions, one needs to understand the quantitative degree and the physical origin of the systematic difference between spectroscopic and emission-weighted temperatures, in addition to the improved statistical sample obviously. Furthermore, the possible difference between the epochs of cluster formation and observation may still be a source of systematic errors in the measured $\sigma_8$ (Kitayama & Suto 1997; Ikebe et al. 2002). Hopefully the recent progress of numerical hydrodynamic simulations in cosmology and a variety of proposals of galaxy surveys at intermediate and high redshifts will significantly improve the situation in near future. For that direction, the independent careful analysis of the systematics in the Sunyaev-Zel’dovich and lensing cluster samples plays a very important and complementary role.

We thank Eiichiro Komatsu and an anonymous referee for valuable suggestions which improved the earlier manuscript of the present paper. We are also grateful to Elena Rasia and Klaus Dolag for useful comments in the early phase of this work, and to Gus Evrard and Joe Mohr for discussions. This research was partly supported by Grant-in-Aid for Scientific Research of Japan Society for Promotion of Science (Nos. 14102004, 14740133, 15740157, 16340053).

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