Exploration of Student Learning Obstacles in Solving Fraction Problems in Elementary School

Mimi Hariyani* Tatang Herman Didi Suryadi Sufyani Prabawanto
Universitas Pendidikan Indonesia, INDONESIA

Abstract: This research aims to explore students’ learning obstacles in solving fraction problems in elementary school. This qualitative research used a case study method. The research subjects involved 30 third-grade elementary school students from two different schools in Bandung City, Indonesia. The instruments utilized were test and non-test. The test technique was done by giving fractional material questions, while the non-test technique was in the form of interviews. The data collected were then analyzed employing the three stages of the Miles and Huberman model, including data reduction, data presentation, and drawing conclusions. The study findings revealed that students experienced epistemological obstacles, where students experienced limited knowledge, ranging from the basic concepts of fractions, fractional arithmetic operations, and fraction problem-solving. Based on these findings, teachers can construct learning designs with appropriate didactic situations as a follow-up to minimize the occurrence of similar learning obstacles in future fractional learning materials.

Keywords: Elementary school, epistemological obstacles, fraction problem-solving, learning obstacles.

Introduction

Fractions are one of the most important materials that form the basis for learning mathematics at the elementary and middle school levels. According to National Council of Teachers of Mathematics (NCTM) that Fractions material is also a core and a challenge in the basic mathematics curriculum (NCTM, 2000). It is because this material is the basis of various subsequent materials, such as decimal numbers, rational numbers, ratios, social arithmetic, and various other materials, which are studied starting from elementary school, high school, and even college levels (Bailey et al., 2012; Booth & Newton, 2012; Fritz et al., 2019; Pedersen & Bjerre, 2021; Rosli et al., 2020; Siegler et al., 2012; Van Hoof et al., 2017). Therefore, it is necessary to instill the proper concept of fractions when students are in elementary school. A good understanding of fractions also plays a major role in everyday life, such as reading recipes, calculating prices during sales, and others (Reyna & Brainerd, 2007).

Every learning cannot be separated from the existence of problems, including learning mathematics. Various problems are experienced by teachers and students in learning mathematics. One of the most urgent problems is the existence of learning obstacles (Fauzi & Suyadi, 2020; Fritz et al., 2019). Learning obstacle is one of the realities impacting didactic design studies (Suryadi, 2019). Learning obstacles can be seen from the behavior that describes learning obstacles. For example, low learning outcomes are not worth the effort. The indications of learning obstacles are manifested directly in the form of behavior.

Learning obstacles experienced by students often occur when students are faced with new concepts that are completely different or have never been studied before (Fuadiah & Suryadi, 2017). Another obstacle also stems from the unpreparedness of students to explore new materials and concepts. Furthermore, some students experience poor learning memory towards learning mathematics. Some of them also avoid learning mathematics. In addition, students need extra understanding to understand mathematical material.

© 2022 The Author(s). Open Access - This article is under the CC BY license (https://creativecommons.org/licenses/by/4.0/).
Previous studies did not reveal in detail the learning obstacles of students in solving fractions problems in elementary school, especially from aspects of the basic concepts of fractions, fractional arithmetic operations, and fraction problem-solving. Thus, the teacher must try to overcome or minimize these learning obstacles so that learning objectives can be achieved as expected.

One form of effort that can be carried out is to analyze in-depth student learning obstacles in solving mathematical problems, especially fractions. Insights about learning obstacles will be a breath of fresh air for teachers to minimize learning problems in the classroom. On the other hand, learning obstacles will naturally be experienced by students. In practice, learning obstacles are influenced by three factors: ontogeny (mental readiness to learn), didactic (due to teacher teaching), and epistemology (student knowledge that has a limited application context). These three factors need to be investigated more deeply to obtain valid information. Therefore, further research is needed. For this reason, this study aims to explore further students’ learning obstacles in solving fraction problems.

**Literature Review**

Fractions material in elementary school begins in the third grade of elementary school. Since the second grade of elementary school, students have started to learn about fraction symbols, but the material has not been discussed in detail and in-depth. Learning about fractions in elementary schools starts with developing the meaning of fractions, developing the concept of fractions, comparing fractions and equivalent fractions, and operating fractions (Jones, 2011).

Moreover, there is a hierarchical level of fraction understanding based on the seven abilities that make up the understanding of fractions in elementary school students (Nicolau & Pitta-pantazi, 2016). The seven skills encompass the introduction of fractions, mathematical definitions and explanations for fractions, arguments and justifications about fractions, relative magnitudes of fractions, fraction representation, the connection of fractions with decimals, percentages and divisions, and reflection during solving fractions problems. Then, third graders also understand the meaning and use of fractions to represent parts of a whole, parts of sets, and placing fractions on a number line. This understanding is then expanded and refined throughout the elementary and secondary (NCTM, 2000).

According to Van de Walle, there are several key ideas regarding the introduction of the concept of fractions: (1) Fractional parts are equal-sized parts of the whole (unit); (2) Fractional parts have special names that indicate how many parts of the size are needed to make one. For example, it takes three fractions of a third to make one; (3) The more fractional parts needed to make one, the smaller the fractional part. For example, one-eighth is less than one-fifth; (4) The denominator of the fraction indicates how much the whole (one) has been divided to get the type of part discussed. Thus, the denominator is a divisor; (5) Two equivalent fractions are two ways to describe the same quantity by using fractional parts of different sizes (Van de Walle et al., 2010).

In the learning process, students often experience obstacles in learning called learning obstacles. Learning obstacles affect the student’s learning process greatly. In practice, the analysis of learning obstacles is included in a series of analyses of the intellectual framework of didactic research and the learning trajectory and the gap in the relationship between teachers and students (learning gap) (Sidik et al., 2021).

Gaston Bachelard initiated the concept of epistemological obstacles in the philosophy of science (Rheinberger, 2005). This idea was later developed by Brousseau into didactic mathematics as a learning obstacle. According to Brousseau, based on the causes, learning obstacles are categorized into three types: (1) ontogenic obstacles, namely barriers related to the stage of mental development of children according to biological age development. In this case, some abilities are required for age-related cognitive purposes. If the inhibition stems from slow mental development (and not from a pathological situation), it will disappear with the growth; (2) didactical obstacle, namely constraints that arise from the teacher’s choice to carry out his teaching practice in a certain context. These barriers can be avoided by developing alternative learning approaches (Brousseau calls didactic engineering); (3) epistemological obstacles, namely the barrier that cannot be avoided because of their important role in constructing the concept of knowledge itself.

Based on the background and literature review, this study aims to explore students’ learning obstacles in solving fraction problems in elementary school. Research questions are: (1) How are learning obstacles affected to the understanding the concept of fraction? (2) How are learning obstacles concerned to operation procedure for adding and subtracting fractions? (3) How are learning obstacles related to the fraction problem-solving?
Methodology

Research Design

This study used a qualitative approach. Qualitative research has a relationship with the ideas or views of the subject under study (Fauzi & Suyadi, 2020; Sugiyono, 2017). The method employed was a case study. Case studies are research methods to explain certain phenomena, such as individuals, programs, processes, and others (Gall et al., 2010; Salimi et al., 2021). Therefore, this case study was deemed an appropriate method to explore in detail students’ learning obstacles in solving fractional problems in elementary school.

Sample and Data Collection

A total of 30 fifth-grade students (aged 10 - 11 years) in two different schools were involved as participants. The detail is as follows: 16 public elementary school students in large schools and 14 public elementary school students in small schools, both in urban areas. The researchers chose the group of students because the fraction material had been given in fifth grade. Meanwhile, the instruments used included tests and non-tests. The test instrument was in the form of questions related to understanding fractions, and the non-test instrument was in-depth interviews to strengthen the data obtained.

The researchers traced students' obstacles from the aspect of understanding the concept by giving tests to 30 fifth-grade students who had studied fractions. A total of 12 questions were given to students related to the concept of fractions and fractional operations. The questions were designed to identify students’ obstacles in understanding fractions material, covering aspects of understanding the concept of fractions, operating procedures for addition and subtraction of fractions, and problem-solving, which were adapted from the diagnostic instrument for learning obstacles in mathematics according to (Depdiknas, 2007) and (Widdiharto, 2008).

The researchers provided problems related to procedures, principles, and problem-solving strategies in addition and subtraction operations involving fractions to obtain accurate data regarding students’ understanding of fractions. The instruments passed qualitative validation in terms of material, construction, and language, involving two mathematics education experts and two elementary school teachers. After revisions, the instrument was declared feasible for use according to the research objectives. Table 1 presents some possible student obstacles.

Table 1. Aspects of Possible Sources of Student Obstacles

| The possible source of the problem | Indicator | Test items |
|-----------------------------------|-----------|-----------|
| Understanding the concept of fractions | Unable to compare fractions with the same denominator | 1a |
| | Unable to compare fractions with unequal denominators | 1b |
| | Unable to identify an equivalent fraction | 1c |
| | Unable to sort fractions with unequal denominators from smallest to largest | 2 |
| | Unable to convert fractions to decimals and percent | 3 |
| Operation procedure for adding and subtracting fractions | Unable to perform the operation of adding fractions with the same denominator | 4a |
| | Unable to perform the operation of subtracting fractions with the same denominator | 4b |
| | Unable to perform the operation of adding fractions with unequal denominators | 4c |
| | Unable to perform the operation of subtracting fractions with unequal denominators | 4d |
| | Unable to perform the operation of adding mixed fractions | 4e |
| Problem-solving | Unable to represent a problem in everyday life in the form of a mixed number addition operation and the right strategy to solve it | 5 |
| | Unable to represent a problem in everyday life in the form of mixed fraction subtraction operations and the right strategy to solve it | 6 |

The test was given to all students who agreed to run the test. Each student got a question sheet and answer sheet and was not allowed to use counting aids. Students were free to choose the number of questions they wanted to solve first. The implementation of this test was also supervised by the teacher in the class. The time provided was 60 minutes, considering that the number of questions was quite large.

The researchers gave an initial score on a scale of 0 to 4 for each item. Thus, the overall score was 48. This score was given based on the level of the obstacle of each item. These scores were then converted to 0 and 1. The criteria: it was 0 if the previous score was 0, 1, or 2, and it was 1 if the previous score was 3 or 4. This conversion considered that students scoring 0 – 2 did not fully understand the meaning of the test questions or were wrong in concepts or procedures. Meanwhile, students who got scores of 3 were generally considered to have answered correctly. Although there were still calculation errors, they were considered to have understood the purpose of the test questions, used concepts with the correct procedures and could use the right concepts and procedures.
Analyzing of Data

Data analysis in this study used the Miles and Huberman model, carried out in three stages: (1) data reduction, where the researchers recorded all student responses in answering questions related to fractional material; (2) data presentation, namely, the researchers began to classify the types of student responses and identify them based on their obstacles; (3) drawing conclusions, where the researchers analyzed in detail the types of students’ learning obstacles in fractions material in third-grade elementary schools based on the theory of learning obstacles and then draws conclusions (Miles & Huberman, 1994). To conduct the reliability in this study, it has been reevaluated by the experts (Emzir, 2012).

Findings / Results

The analysis was carried out based on the components of the obstacle aspect to describe how many students had obstacles based on the mean score for each number of questions. This analysis was then continued by determining the percentage of students who experienced obstacles in each aspect. Overall, the researchers saw almost evenly distributed obstacles in the aspects of concepts, procedures, and problem-solving. Table 2 provides information on students’ obstacles based on the source of the problem related to fractional material.

Table 2. Student Performance in Each Aspect of the Obstacle

| N = 30 | Fraction concept | Calculation operation procedure | Solution to problem |
|--------|------------------|---------------------------------|---------------------|
|        | 5                | 5                              | 2                   |
| Number of questions | Mean | Median | Standard deviation |
| 5 | 59% | 59.17 | 10.74 |
| 5 | 57.33% | 60.83 | 8.62 |
| 2 | 72.92% | 72.92 | 3.75 |

Table 2 shows that the most obstacles occurred in the problem-solving aspect, which was applying the concept of fractions and operating procedures to count fractions understood by students (72.92%) and the concept of fractions, which was the initial introduction to fractions (59%).

Understanding the Concept of Fractions

On average, 59% of students experienced this first problem with a fairly large standard deviation of 10.74. It was due to the significant difference between each indicator (see Table 3).

Table 3. Percentage of Students Who Met the Indicators in the First Obstacle Aspect

| Indicator | Question number | The number of students who met the indicators | Percentage |
|-----------|-----------------|---------------------------------------------|------------|
| Unable to compare fractions with the same denominator | 1a | 10 | 33.33 |
| Unable to compare fractions with unequal denominators | 1b | 27 | 90 |
| Unable to identify equivalent fraction | 1c | 26 | 86.67 |
| Unable to sort fractions with unequal denominators from smallest to largest | 2 | 17 | 56.67 |
| Unable to convert fractions to decimals and percent | 3 | 10 | 33.33 |

Based on Table 3, it can be said that there were still many students who could compare fractions with the same denominator and convert fractions into decimals and percentages. The highest error occurred in questions that asked students to compare fractions with unequal denominators (see Figure 1).

Compare the following fractions by giving a sign (<, >, or =)!

Explain your answer with the solution steps!

\[
\frac{1}{2} \quad \cdots \quad \frac{1}{3}
\]

Figure 1. Subject 1’s Answer
Subject 1 compared the two fractions by paying attention to the denominator. Subject 1 concluded that the fraction $\frac{1}{2}$ is less than $\frac{1}{3}$ because the number 2 is smaller than 3. Subject 1 explained the reason as follows:

Researcher : "I want to ask about your answer to this number 1. What is question number 1(b)?
Subject 1 : "Compare the fractions $\frac{1}{2}$ and $\frac{1}{3}$. Which one is bigger?"
Researcher : "Then, what is your answer?"
Subject 1 : "The bigger one is $\frac{1}{3}$ ma'am."
Researcher : "Why is that? Please explain your reasoning!"
Subject 1 : "Judging by the numbers below, ma'am."
Researcher : "What is the number below called?"
Subject 1 : "Denominator, ma'am."
Researcher : "Yes, then? What are the denominators of the two fractions?
Subject 1 : "Because 3 is greater than 2, $\frac{1}{3}$ is greater than $\frac{1}{2}$."

Subject 1 assumed that the way to compare fractional numbers is the same as whole numbers. It indicates that the student still had limited knowledge about comparing fractions. There were 90% of students who experienced problems in this matter.

The second highest error was when students were asked to identify equivalent fractions (see Figure 2).

*Compare the following fractions by giving a sign (<, >, or =)!*

*Explain your answer with the solution steps!*

\[
\frac{2}{4} \text{  and  } \frac{4}{8}
\]

As many as 86.67% of students answered incorrectly on the question. It means that only four of the 30 students answered correctly. In this case, students were still affected by the previous problem, where students were asked to compare two fractions by giving a sign (<, >, or =). Students assumed that comparing is only greater or less and ignored the equal sign.

*Figure 2. Answers by Subjects 2 and 3*

Subjects 2 and 3's answers showed that they only focused on comparing the denominators of the two fractions without considering the numerators. Therefore, they did not realize that the two fractions were equivalent. After observing all the answers, students did not understand the equivalence of fractions or equivalent fractions.

Furthermore, obstacles in sorting fractions with unequal denominators from the smallest to the largest were still experienced by more than half of the students (see Figure 3).
Arrange the following fractions from smallest to largest!

Explain your answer with the solution steps!

\[ \frac{3}{4}, \frac{3}{8}, \frac{1}{4}, \frac{1}{2}, \frac{5}{8} \]

Subject 4 sorted the fractions from the smallest to the largest by paying attention to the order of the numerator and denominator. Subject 4 started sorting from the fraction with the smallest numerator and denominator. If there were the same numerator, subject 4 chose the smallest denominator first until the largest order with the largest numerator and denominator was obtained. It denotes that subject 4 did not yet understand that fractions can be sorted by equating the denominators first. The fractions will be easier to sort if the units are the same. Constraints like this were experienced by 56.67% of students.

Operation procedure for adding and subtracting fractions

In this component, there were five questions to diagnose whether students understood the operation procedure for adding and subtracting fractions with five indicators to determine whether students had obstacles in this aspect. As a result, on average, students who experienced obstacles in this aspect were 52.67%, with 80% of students unable to perform arithmetic subtraction operations with unequal denominators, followed by indicators performing addition operations with unequal denominators, 73.33% (see Table 4).

| Indicator                                               | Question number | The number of students who met the indicators | Percentage |
|---------------------------------------------------------|-----------------|---------------------------------------------|------------|
| Unable to perform the operation of adding fractions with the same denominator | 4a              | 6                                           | 20         |
| Unable to perform subtraction operation with the same denominator | 4b              | 7                                           | 23.33      |
| Unable to perform addition operations with unequal denominators | 4c              | 22                                          | 73.33      |
| Unable to perform subtraction operations with unequal denominators | 4d              | 24                                          | 80         |
| Unable to perform the operation of adding mixed fractions | 4e              | 20                                          | 66.67      |

The Table 4 results display that most students could not perform addition and subtraction operations on fractions with unequal denominators and could not perform mixed addition operations (question numbers 4c, 4d, 4e). In addition, a small number of students did not master the operation of addition and subtraction of fractions with the same denominator (question numbers 4a and 4b). In many answers, students applied the properties of addition and subtraction of whole numbers to the addition and subtraction of fractions. Besides, the arithmetic operation procedure had not been fully mastered by students well (see Figure 4).

Calculate the addition and subtraction of the following fractions!

Explain your answer with the solution steps!

\[ \frac{2}{3} - \frac{1}{4} = \ldots \]

\[ \frac{2}{3} - \frac{1}{4} = \ldots \]

Figure 3. Subject 4's Answer

Figure 4. Answers by Subjects 5 and 6
Subject 5’s answer shows that subject 5 had not understood the subtraction of fractions with unequal denominators. Subject 5 immediately subtracted the numerator from the numerator and the denominator to the denominator. When the student met questions in the form of 3-4, subject 5 made the result equal 1. It indicates that subject 5 also did not understand integer operations. Furthermore, in the answer by subject 6, it can be seen that subject 6 had tried to equate the denominator by using the least common multiple, but subject 6 did not continue to elaborate on each of the numerators.

Students also experienced similar answers when answering questions about adding fractions with unequal denominators. It can be seen in the following image.

![Figure 5. Answers by Subjects 7 and 8](image)

Similar to subject 7, subject 8 performed the operation of adding fractions with unequal denominators by adding up each of the numerators with the numerator and the denominator with the denominator. Furthermore, subject 8 first equated the denominator using the least common multiple, but the student also had not been able to decipher the numerator. They immediately added up the two numerators so that the results obtained were incorrect. Misperceptions about the concept of arithmetic operations were experienced by many students. On average, 52.67% did not understand this concept, even though some had understood arithmetic operations procedures.

**Solution to problem**

The researchers gave two forms of everyday problems related to fractions to see students' obstacles in representing a problem context in fractional arithmetic operations and the appropriate procedures and strategies to solve them. Overall, 83.33% of students had obstacles in this aspect (see Table 5).

| Indicator                                                                 | Question number | The number of students who met the indicators | Percentage |
|---------------------------------------------------------------------------|-----------------|---------------------------------------------|------------|
| Unable to represent a problem in everyday life in the form of a mixed number addition operation and the right strategy to solve it | 5               | 25                                          | 83.33      |
| Unable to represent a problem in everyday life in the form of mixed fraction subtraction operations and the right strategy to solve it | 6               | 25                                          | 83.33      |

The problem posed in this aspect is that students have encountered or often encounter daily. Contextual problems like this are believed to develop the stage of formal operational thinking. The problems posed require students to understand the meaning of the questions first, identify problems, construct a mathematical object, and then determine strategies to obtain solutions. The results found were that almost all students experienced obstacles in solving mathematical problems (see Figure 6).

*A farmer has just harvested his garden. After weighing, the weight of peanuts is $15\frac{3}{4}$ kg and the weight of soybeans is $7\frac{2}{5}$ kg. What is the total weight of the farmer's crop? Explain your answer with the solution steps!*

![Figure 6. Subject 9's Answer](image)

Subject 9 had tried to convert mixed fractions into improper fractions correctly in this problem. However, when performing addition operations on mixed fraction numbers, subject 9 again made the mistake of adding directly between the numerator and the numerator and the denominator with the denominator. Subject 9 and almost all students
experienced confusion when faced with problem-solving problems like this. The findings revealed that 25 students (83.33%) had obstacles in understanding and solving this problem. Some students tried to add integers with integers correctly but still failed to add fractions with unequal denominators.

In another problem, the same problem was also given with a different operation, namely the subtraction operation. However, similar to the previous problem, students also had obstacles in understanding the problem, determining schemes and strategies for solving it and choosing the right algorithm.

Discussion

Introducing fractions after students understand integers is a challenge for teachers. Understanding integers, both basic concepts, properties, and operating procedures on integers, often still overshadows students when dealing with fractions. This kind of problem is also known as integer bias (Avgerinou & Tolmie, 2020; Puntambekar et al., 2005; Van Hoof et al., 2013). It is where students tend to let the knowledge of integers interfere with the concepts of rational numbers, including fractions. When introduced to rational numbers, there are some aspects where the rules of natural numbers no longer apply, leading to systematic errors. It was even stated that this problem would still exist even though the students had understood the new concept well (Van Hoof et al., 2020).

Based on the study results on students’ obstacles in understanding fractions, students had some learning barriers in every aspect. In this case, 60% of students had not understood the basic concepts of fractions. The main problem experienced by students in understanding the concept of fractions is especially seen when comparing two fractions with unequal denominators. Most students compared fractions with different denominators by paying attention to the denominators. Students assumed that comparing fractions is the same as comparing whole numbers. This problem is the main epistemological obstacle to students because students still experience limited knowledge in understanding the basic concepts of fractions (Arrigo et al., 2011; Siswanto, 2020; Suryadi, 2019; Wahyuningrum & Suryadi, 2017).

The arithmetic operation procedure is also one of the sources of problems in fractional material. In this case, more than 50% of students could not complete fractional arithmetic operations, especially in fractions with unequal denominators and mixed fractions. The possibility of this erroneous concept was due to students’ lack of understanding in interpreting that every unit of fractions added or subtracted must be the same size. So far, the knowledge they have received is that to perform operations on fractions with unequal denominators, they must first equalize the denominators without understanding the reason behind them. The limited understanding of this basic concept becomes a major problem for students’ arithmetic operations (Imaroh, 2021; Lestiari & Suryadi, 2020).

More than 83% of students had an obstacle in solving problems. The general obstacle found is that students could not translate the information in the problem, so students did not understand the proper procedure to describe the condition of the problem. When they had been able to translate the information on the problem, the next difficulty was determining the right procedures and strategies to find the expected results. In problem-solving involving fractions, students tended to have difficulty because, basically, they did not fully understand the basic concepts and operating procedures for fractions. Therefore, it is necessary first to master the basic concepts of fractions, followed by fractional operation procedures, so that students can easily solve the problem related to fractions (Hairunnisah, 2016; Rivai, 2015).

Conclusion

Based on the discussion above, it can be concluded that students experienced epistemological obstacles in fractional material, where students experienced limited knowledge, ranging from basic concepts of fractions, fractional arithmetic operations, and solving fractional problems. In the aspect of the basic concept of fractions, students experienced obstacles when comparing two fractions with unequal denominators, where students compared the denominators of fractions as in whole numbers. In fractional counting operations, students experienced obstacles because they did not understand the concept of equating the denominator when performing addition and subtraction operations on fractions. Meanwhile, in solving fractional problems, the obstacles are that students could not translate the information in the questions and had obstacles in determining the right procedures and strategies to find the expected results.

Recommendations

Insights about student learning obstacles in the matter of fractions are crucial to be understood by mathematics teachers, especially in elementary schools. Based on these findings, teachers can design learning designs with appropriate didactic situations as a follow-up to minimize the occurrence of similar learning obstacles in future fractional learning materials. However, this research is limited to fractional material ranging from basic concepts to the addition and subtraction of fractions. Therefore, it is recommended that further researchers conduct similar research on various other materials, not only in elementary schools but also in universities. Furthermore, the results of this study become the basis for teachers and other researchers to determine effective learning designs to minimize the various learning obstacles found. This research can also be used as a reference for further research, for example, regarding the appropriate didactic design on fractional material and various qualitative and quantitative studies related to fractional material in elementary schools.
Limitations

Research only focuses on fractional operations, there are many other studies that have not been revealed. This research is limited in scope, the results and conclusions cannot be generalized.

Acknowledgements

The authors express their deepest gratitude to the Graduate School, Universitas Pendidikan Indonesia and the 5000 Doctoral Program of the Ministry of Religion, Republic of Indonesia.

Authorship Contribution Statement

Hariyani: Conceptualization, design, analysis, writing, drafting the manuscript. Herman: Conceptualization, reviewing, supervision, final approval. Suryadi: Conceptualization, reviewing, supervision. Prabawanto: Conceptualization, reviewing, supervision

References

Aliustağlo, F., Tuna, A., & Biber, A. Ç. (2018). Misconceptions of sixth grade secondary school students on fractions. *International Electronic Journal of Elementary Education, 10*(5), 591–599. https://doi.org/10.26022/iejee.2018541308

Arrigo, G., Sharagli, S., Frapolli, A., Frigerio, D., & Villa, O. (2011). Epistemological and didactic obstacles: The influence of teachers’ beliefs on the conceptual education of students. *Mediterranean Journal for Research in Mathematics Education, 10*(1–2), 61–102. https://bit.ly/3Q640lk

Avgerinou, V. A., & Tolmie, A. (2020). Inhibition and cognitive load in fractions and decimals. *British Journal of Educational Psychology, 90*(S1), 240–256. https://doi.org/10.1111/bjep.12321

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology, 113*(3), 447–455. https://doi.org/10.1016/j.jecp.2012.06.004

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper’s doorman? *Contemporary Educational Psychology, 37*(4), 247–253. https://doi.org/10.1016/j.cedpsych.2012.07.001

Braithwaite, D. W., Pyke, A. A., & Siegler, R. S. (2017). A computational model of fraction arithmetic. *Psychological Review, 124*(5), 603–625. https://doi.org/10.1037/rev0000072

Christou, K. P. (2015). Natural number bias in operations with missing numbers. *ZDM, 47*(5), 747–758. https://doi.org/10.1007/s11858-015-0675-6

Depdiknas. (2007). *Tes Diagnostik* [Diagnostic Test]. Departemen Pendidikan Nasional [Ministry of Education]. https://bit.ly/3OyWEAS

Emzir. (2012). *Metodologi penelitian kualitatif: Analisis data* [Qualitative research methodology: Data analysis]. Raja Grafindo. https://bit.ly/3H0ggq92

Fauzi, I., & Suyadi, D. (2020). The analysis of students’ learning obstacles on the fraction addition material for five graders of elementary schools. *Al Ibtida: Jurnal Pendidikan Guru MI, 7*(1), 33–45. https://doi.org/10.24235/al.ibtida.snj.v7i1.6020

Fritz, A., Haase, V. G., & Räsänen, P. (2019). *International handbook of mathematical learning difficulties: From the laboratory to the classroom*. Springer. https://doi.org/10.1007/978-3-319-97148-3

Fuadiah, N. F., & Suryadi, D. (2017). Some difficulties in understanding negative numbers faced by students: a qualitative study applied at secondary schools in Indonesia. *International Education Studies, 10*(1), 24–38. https://doi.org/10.5539/ies.v10n1p24

Gall, M. D., Gall, J. P., & Borg, W. R. (2010). *Applying educational research* (6th ed.). Pearson.

Hairunnisah, H. (2016). *Komampuan pemecahan masalah siswa pada pokok bahasan pecahan di kelas VA SD Negeri 100890 Gunungtua Kab. Padanglawas Utara* [Students’ problem solving abilities on the subject of fractions in class VA SD Negeri 100890 Gunungtua Kab. Padanglawas Utara] [Undergraduate thesis, Institut Agama Islam Negeri Padangsidimpuan]. Theses IAIN Padangsidimpuan. http://etd.iain-padangsidimpuan.ac.id/1940/

Imaroh, N. A. (2021). Analisis kesulitan siswa sd kelas iv dalam menyelesaikan soal operasi hitung pecahan [Analysis of the difficulties of fourth grade elementary school students in solving fraction count operation problems]. *JP3M: Jurnal Penelitian Pendidikan dan Pengajaran Matematika, 7*(2), 87–96. https://bit.ly/3NlK2fv

İskenderoğlu, T. A. (2017). The problems posed and models employed by primary school teachers in subtraction with fractions. *Educational Research and Reviews, 12*(5), 239–250. https://doi.org/10.5897/ERR2016.3089
Jones, J. C. (2011). *Visualizing elementary and middle school mathematics methods*. Wiley Global Education.

Lestari, D. E., & Suryadi, D. (2020). Analisis kesulitan operasi hitung bentuk aljabar [Analysis of the difficulty of arithmetic operations of algebraic forms]. *JURING: Journal for Research in Mathematics Learning*, 3(3), 247–258. https://bit.ly/3OFkCB

Mcmullen, J., Laakkonen, E., Hannula-sormunen, M., & Lehtinen, E. (2015). Modeling the developmental trajectories of rational number concept(s). *Learning and Instruction*, 37, 14–20. https://doi.org/10.1016/j.learninstruc.2013.12.004

Miles, M. B., & Huberman, A. M. (1994). *An expanded sourcebook: Qualitative data analysis* (2nd ed.). SAGE Publications.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. https://bit.ly/2HI3GE

Nicolaou, A. A., & Pitta-pantazi, D. (2016). Hierarchical levels of abilities that constitute fraction understanding at elementary school. *International Journal of Science and Mathematics Education*, 14, 757–776. https://doi.org/10.1007/s10763-014-9603-4

Pedersen, P. L., & Bjerre, M. (2021). Two conceptions of fraction equivalence. *Educational Studies in Mathematics*, 107(1), 135–157. https://doi.org/10.1007/s10649-021-10030-7

Puntambekar, S., Hubsher, R., & Hübscher, R. (2005). Tools for scaffolding students in a complex learning environment: What have we gained and what have we missed? *Educational Psychologist*, 40(1), 1–12. https://doi.org/10.1207/s15326985ep4001_1

Reyna, V. F., & Brainerd, C. J. (2007). The importance of mathematics in health and human judgment: Numeracy, risk communication, and medical decision making. *Learning and Individual Differences*, 17(2), 147–159. https://doi.org/10.1016/j.lindif.2007.03.010

Rheinberger, H.-J. (2005). Gaston Bachelard and the notion of “phenomenotechnique”. *Perspectives on Science*, 13(3), 313–328. https://doi.org/10.1162/106361405774288026

Rivai, S. (2015). Analisis kesalahan menyelesaikan pengurangan pecahan di SDN 6 Bulango Selatan Kabupaten Bone Bolango [Error analysis completing fraction subtraction at SDN 6 Bulango Selatan, Bone Bolango Regency]. *Journal of Education/ Jurnal Kependidikan*, 12(1), 1–11. https://bit.ly/3QHo0X7

Rosli, R., Goldsby, D., Owuegbuzie, A. J., Capraro, M. M., Capraro, R. M., & Gonzalez, E. G. Y. (2020). Elementary preservice teachers’ knowledge, perceptions and attitudes towards fractions: A mixed-analysis. *Journal on Mathematics Education*, 11(1), 59–76. https://eric.ed.gov/?id=EJ1237785

Salimi, M., Dardiri, A., & Sujarwo. (2021). The Profile of students’ social skills of bengawan solo elementary nature school. *European Journal of Educational Research*, 10(1), 211–226. https://doi.org/10.12973/eu-jer.10.1211

Sidik, G. S., Suryadi, D., & Turmudi, T. (2021). Learning obstacle on addition and subtraction of primary school students: analysis of algebraic thinking. *Education Research International*, 2021, 1–10. https://doi.org/10.1155/2021/5935179

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Suls, J., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, 23(7), 691–697. https://doi.org/10.1177/0956797612441011

Siswanto, R. D. (2020). Analysis epistemological obstacle students in completing mathematical stories based on cognitive styles. *Journal of Innovation in Elementary Education*, 6(1), 1–12. https://doi.org/10.22236/jipd.v6i1.133

Sugiyono. (2017). *Metode penelitian pendidikan: Pendekatan kuantitatif, kualitatif dan R&D* [Educational research methods: Quantitative, qualitative and R&D approaches]. Alfabeta.

Suryadi, D. (2019). *Landasan filosofis penelitian desain didaktis (DDR)* [Philosophical foundations of didactic design research (DDR)]. Gapura Press.

Vanvalkoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *Journal of Mathematical Behavior*, 31(3), 344–355. https://doi.org/10.1016/j.jmathb.2012.02.001

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Allyn & Bacon.

Van Hoof, J., Degrande, T., Ceulemans, E., Verscha, L., & Van Dooren, W. (2018). Learning and Individual Differences Towards a mathematically more correct understanding of rational numbers: A longitudinal study with upper elementary school learners. *Learning and Individual Differences*, 61, 99–108. https://doi.org/10.1016/j.lindif.2017.11.010

Van Hoof, J., Linjen, T., Verschaffel, L., & Van Dooren, W. (2013). Are secondary school students still hampered by the
natural number bias? A reaction time study on fraction comparison tasks. *Research in Mathematics Education, 15*(2), 154–164. https://doi.org/10.1080/14794802.2013.797747

Van Hoof, J., Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2017). The transition from natural to rational number knowledge. In D. C. Geary, D. B. Berch, R. J. Ochsendorf & K. M. Koepke (Eds.), *Acquisition of complex arithmetic skills and higher-order mathematics concepts* (pp. 101–123). Academic Press. https://doi.org/10.1016/B978-0-12-805086-6.00005-9

Van Hoof, J., Verschaffel, L., De Neys, W., & Van Dooren, W. (2020). Intuitive errors in learners’ fraction understanding: A dual-process perspective on the natural number bias. *Memory and Cognition, 48*(7), 1171–1180. https://doi.org/10.3758/s13421-020-01045-1

Wahyuningrum, A. S., & Suryadi, D. (2017). Epistemological obstacles on the topic of ratio and proportion among junior high school students. *Journal of Physics: Conference Series, 895*(1), 12066. https://doi.org/10.1088/1742-6596/895/1/012066

Widdiharto, R. (2008). *Diagnosis kesulitan belajar matematika siswa smp dan alternatif proses remidinya* [Diagnosis of mathematics learning difficulties for middle school students and alternative remedial processes]. Departemen Pendidikan Nasional [Ministry of Education]. https://bit.ly/3xOhWUb