Constraints on ΛCDM extensions from the SPT-3G 2018 EE and TE power spectra

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We present constraints on extensions to the $\Lambda$CDM cosmological model from measurements of the $E$-mode polarization autopower spectrum and the temperature-$E$-mode cross-power spectrum of the cosmic microwave background (CMB) made using 2018 SPT-3G data. The extensions considered vary the primordial helium abundance, the effective number of relativistic degrees of freedom, the sum of neutrino masses, the relativistic energy density and mass of a sterile neutrino, and the mean spatial curvature. We do not find clear evidence for any of these extensions, from either the SPT-3G 2018 dataset alone or in combination with baryon acoustic oscillation and Planck data. None of these model extensions significantly relax the tension between Hubble-constant, $H_0$, constraints from the CMB and from distance-ladder measurements using Cepheids and supernovae. The addition of the SPT-3G 2018 data to Planck reduces the square-root of the determinants of the parameter covariance matrices by factors of 1.3–2.0 across these models, signaling a substantial reduction in the allowed parameter volume. We also explore CMB-based constraints on $H_0$ from combined SPT, Planck, and ACT DR4 datasets. While individual experiments see some indications of different $H_0$ values between the $TT$, $TE$, and $EE$ spectra, the combined $H_0$ constraints are consistent between the three spectra. For the full combined datasets, we report $H_0 = 67.49 \pm 0.53$ km s$^{-1}$ Mpc$^{-1}$, which is the tightest constraint on $H_0$ from CMB power spectra to date and in 4.1$\sigma$ tension with the most precise distance-ladder-based measurement of $H_0$. The SPT-3G survey is planned to continue through at least 2023, with existing maps of combined 2019 and 2020 data already having $\sim 3.5 \times$ lower noise than the maps used in this analysis.

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I. INTRODUCTION

Measurements of the cosmic microwave background (CMB) provide a unique opportunity to learn about the early universe and its evolution over cosmic time. A combination of satellite and ground-based observations have provided a sample-variance-limited view of CMB temperature anisotropy down to few-arcminute scales, beyond which foreground signals dominate [1–4]. The snapshot of conditions in the early universe provided by the CMB has been crucial in establishing the six-parameter $\Lambda$CDM model as the standard model of cosmology.

Despite its achievements, some questions regarding the $\Lambda$CDM model remain open, such as: is the preference for different cosmologies between large and small angular-scale CMB data physical [5–9]? What is the origin of the tension between high- and low-redshift measurements of the expansion rate, and can simple model extensions reconcile it [10,11]? The persistence of these and other tensions, as well as unsolved fundamental physics problems, such as the nature of dark matter and dark energy, is a key motivation for further theoretical study of cosmology.
Measurements of the CMB polarization on intermediate and small angular scales present an excellent opportunity to investigate these questions. The E-mode polarization auto-power spectra ($EE$) and the temperature-E-mode cross-power spectra ($TE$) contain as much information as the temperature power spectrum ($TT$) [15], with extragalactic foregrounds relatively dimmer at small angular scales [16–18]. Thus CMB polarization observations can act both as an important consistency check on the stringent constraints derived from temperature data, and as a source of additional and complementary information on the $\Lambda$CDM model and its extensions. Improving these measurements is one focus of contemporary ground-based CMB experiments. Precision measurements out to few-arcminute scales have been carried out recently by the Atacama Cosmology Telescope (ACT) [3], POLARBEAR [19], and the South Pole Telescope (SPT) [[7,20], hereafter D21].

D21 presented $TE$ and $EE$ power spectrum measurements from the 2018 observing season of the SPT-3G 1500 deg$^2$ survey. From the SPT-3G 2018 bandpowers, D21 inferred an expansion rate of $H_0 = 68.8 \pm 1.5$ km s$^{-1}$ Mpc$^{-1}$, under the $\Lambda$CDM model, in line with other contemporary CMB experiments [2,10] and lower than the distance-ladder measurement of Riess et al. [[11], hereafter R20] using Cepheids and supernovae. In this paper we consider the implications of the D21 $TE$ and $EE$ bandpowers for extensions to the $\Lambda$CDM model. We assess whether these extensions help reconcile the tension between high- and low-redshift probes of the Hubble constant.

Specifically, we utilize the SPT-3G 2018 bandpower measurements to constrain models with a strong impact on the damping tail, by allowing the effective number of neutrino species, $N_{\text{eff}}$, to vary from the standard model prediction and by breaking big-bang nucleosynthesis (BBN) consistency to change the primordial helium abundance, $Y_p$. We also constrain the sum of neutrino masses, $\Sigma m_\nu$, the effective mass of one additional sterile neutrino, $m^\text{eff}_\nu$, and spatial curvature, $\Omega_K$. While the SPT-3G 2018 bandpowers alone can constrain each of these cosmological extensions, we also look at joint constraints when combined with data from the Planck satellite and baryon acoustic oscillation (BAO) data. After presenting the constraints these datasets place on each model, we investigate the results for $H_0$ more closely and discuss any relevant degeneracies in the full parameter space. Motivated by the higher values of $H_0$ inferred from the $EE$ spectra of contemporary CMB experiments [D21], we look at constraints on the expansion rate from combined measurements of the temperature versus polarization spectra across multiple experiments. Furthermore, we report the tightest constraint on $H_0$ from CMB power spectra to date by combining the temperature and polarization spectra from these datasets, and reevaluate the Hubble tension.

When analyzing the expansion rate constraints, we choose to compare the CMB results to the distance-ladder measurement of R20 using Cepheids and supernovae, because of the high precision on $H_0$. We note that the distance-ladder data calibrated using the tip of the red giant branch (TRGB) by Freedman et al. [21] agrees with contemporary CMB experiments as well as R20, although the TRGB and Cepheid approaches lead to significantly different distances to some supernova-host nearby galaxies [21]. There are also independent, if more uncertain, constraints on $H_0$ using time-delay cosmography [22,23]. However, for simplicity, we restrict the comparisons in this work to the most precise local measurement of $H_0$ from R20.

This paper is structured as follows. In Sec. II we review the datasets used in this work and the likelihood used to obtain cosmological parameter constraints. We report constraints on $\Lambda$CDM extensions and evaluate their inferred expansion rates in Sec. III. We scrutinize Hubble constant constraints from temperature and polarization spectra in Sec. IV before concluding in Sec. V.

II. DATASETS AND FITTING METHODOLOGY

A. The SPT-3G 2018 $EE/TE$ dataset

This work explores the cosmological implications of the first power spectrum measurements from the SPT-3G instrument, which were presented by D21. The E-mode autospectrum and temperature-E-mode cross-spectrum bandpowers are based on observations of a 1500 deg$^2$ region taken over four months in 2018 at three frequency bands centered on 95, 150, and 220 GHz, which result in polarized map depths of 29.6, 21.2, and 75 $\mu$K-arcmin (averaged across 1000 $< \ell < 2000$), respectively. The $EE$ and $TE$ bandpowers span the angular multipole range 300 $\leq \ell < 3000$. Despite the truncated 2018 observing season, the SPT-3G 2018 bandpowers improve on previous SPT results across 300 $\leq \ell \leq 1400$ for $EE$ and 300 $\leq \ell \leq 1700$ for $TE$ [7] and are sample-variance dominated at $\ell < 1275$ and $\ell < 1425$ for $EE$ and $TE$, respectively. The bandpowers provide precise measurements on the angular scales where hints of physics beyond the standard model may hide.

We adopt the likelihood used in D21, which accounts for the effects of the aberration due to relative motion with respect to the CMB rest frame [24], super-sample lensing [25], polarized foregrounds, uncertainty in the calibration of the bandpowers, and uncertainty in the beam measurements. As in D21, we place priors on many of these terms, which are listed in Table I. We refer the reader to D21 for a detailed discussion of the likelihood. As reported in Sec. VI of D21, the cosmological constraints from the SPT-3G 2018 dataset are robust with respect to the choice of priors on the nuisance parameters. We confirm that this remains true for the combination of the SPT-3G and Planck datasets.
TABLE I. The Gaussian priors listed here are used for the SPT-3G parameter constraints. The list of parameters with priors includes the optical depth to reionization $\tau$, mean-field lensing convergence $\kappa$, the amplitude $A_{50}^E$ (in $\mu K^2$) at 150 GHz and spectral index $\alpha_{EE}^E$ of polarized Galactic dust, the $EE$ power of Poisson-distributed point sources $D_{5000}^{0.95 \times 0.95}$, (in $\mu K^2$), absolute temperature calibration factor $T_{\text{cal}}^i$, and absolute polarization calibration factor $E_{\text{cal}}^i$.

| Parameter | Prior |
|-----------|-------|
| $\tau$    | 0.0543 ± 0.0073 |
| $10^3 \kappa$ | 0 ± 0.45 |
| $A_{50}^E$ | 0.095 ± 0.012 |
| $\alpha_{EE}^E$ | $-2.42 \pm 0.02$ |
| $A_{90}^E$ | 0.184 ± 0.072 |
| $\alpha_{EE}^E$ | $-2.42 \pm 0.02$ |
| $D_{5000}^{0.95 \times 0.95}$ | 0.041 ± 0.012 |
| $D_{5000}^{150 \times 150}$ | 0.0115 ± 0.0034 |
| $D_{5000}^{220 \times 220}$ | 0.048 ± 0.014 |
| $D_{5000}^{0.95 \times 150}$ | 0.0180 ± 0.0054 |
| $D_{5000}^{0.95 \times 220}$ | 0.0157 ± 0.0047 |
| $D_{5000}^{150 \times 220}$ | 0.0190 ± 0.0057 |
| $\tau^{150 \text{GHz}}_{\text{cal}}$ | 1 ± 0.0049 |
| $\tau^{220 \text{GHz}}_{\text{cal}}$ | 1 ± 0.0050 |
| $\tau^{220 \text{GHz}}_{\text{cal}}$ | 1 ± 0.0067 |
| $E_{150 \text{GHz}}^{\text{cal}}$ | 1 ± 0.0087 |
| $E_{120 \text{GHz}}^{\text{cal}}$ | 1 ± 0.0081 |
| $E_{220 \text{GHz}}^{\text{cal}}$ | 1 ± 0.016 |

Baryon acoustic oscillation (BAO) measurements provide information about the expansion history of the universe at late times, which is particularly useful to break degeneracies in the CMB data for model extensions that affect the late-time dynamics [27,28]. This class of models is of particular interest in the context of the Hubble tension. We use BAO measurements from the BOSS MGS and 6dFGS surveys, which have mapped the low-redshift universe in great detail [29–31]. We also include the BOSS measurements of the Lyman–$\alpha$ forest and quasars at higher redshifts [32]. Together these datasets provide a detailed view of the expansion history of the universe across $0.2 < z < 3.5$.

C. BAO datasets

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D. Fitting methodology

We produce cosmological constraints using the Markov Chain Monte Carlo (MCMC) package COSMOMC [33]. COSMOMC uses the Boltzmann code CAMB [34] to calculate CMB power spectra at each point in parameter space. We use the following parameters to describe the $\Lambda$CDM model: the density of cold dark matter, $\Omega_{ch} h^2$; the baryon

1https://pole.uchicago.edu/public/data/dutcher21.
2https://lambda.gsfc.nasa.gov/product/spt/index.cfm.
3https://cosmologist.info/cosmomc/.
4https://camb.info/.
density, $\Omega_b h^2$; the optical depth to reionization, $\tau$; the (approximated) angular scale of the sound horizon at decoupling, $\theta_{MC}$; the amplitude of primordial density perturbations, $A_s$, defined at a pivot scale of 0.05 Mpc$^{-1}$; and the scalar spectral index, $n_s$.

D21 presented constraints on the \Lambda CDM model from the SPT-3G 2018 dataset individually and jointly with Planck and BAO data. We expand that analysis by considering one- and two-parameter extensions to the \Lambda CDM model, drawn from these five parameters: the effective number of neutrino species, $N_{\text{eff}}$; the primordial fraction of baryonic mass in helium, $Y_P$; the sum of neutrino masses, $\Sigma m_\nu$; the effective mass of sterile neutrinos, $m_{\nu,\text{sterile}}^{\text{eff}}$; and the spatial curvature, parametrized by $\Omega_K$. The uncertainties reported in this work on these core \Lambda CDM parameters are 68\% confidence levels.

The optical depth to reionization is constrained primarily by the reionization bump at $\ell < 10$ in polarization. Since these angular scales are not probed by the ground-based CMB experiments in this work, we adopt a Planck-based prior of $\tau = 0.0543 \pm 0.007$ \cite{0928-4367-2003-12-015} for all chains that do not include Planck data. Without this prior, the ground-based CMB constraints show the expected degeneracy between $\tau$ and the amplitude of primordial density perturbations. We point out that Aiola et al. \cite{Aiola:2020yiy} use the prior $\tau = 0.065 \pm 0.015$, which is why we report slightly different results for ACT DR4.

When reporting joint constraints from SPT-3G 2018, Planck, and ACT DR4, we ignore correlations between different datasets, unless we combine Planck and ACT DR4 temperature data, in which case we restrict the angular multipole range of the latter to $\ell > 1800$ as recommended by Aiola et al. \cite{Aiola:2020yiy}. The SPT-3G footprint is approximately 1/17th of the Planck observation region, and the Planck polarization spectra are not sample-variance dominated on any of the angular scales probed by SPT-3G, which further reduces the correlation between the two band power measurements. A simple simulation of the modes measured by SPT-3G and Planck seeking to approximate these two features of the data shows that the correlation is at most at the 10\% level and drops off with increasing $\ell$. We therefore judge correlations between SPT-3G and Planck data to be negligible and ignore them. The two ground-based surveys, SPT-3G and ACT DR4, observe different parts of the sky.

III. COSMOLOGICAL CONSTRAINTS

We now present constraints on extensions to \Lambda CDM. We begin by looking at three extensions that test for new light relics or inconsistencies with BBN: varying the effective number of neutrino species, $N_{\text{eff}}$ (Sec. III A); varying the primordial helium abundance, $Y_P$ (Sec. III B); or varying both parameters (Sec. III C). We then turn our attention to questions about neutrino mass, and examine constraints on the sum of neutrino masses, $\Sigma m_\nu$ (Sec. III D), and an effective sterile neutrino mass, $m_{\nu,\text{sterile}}^{\text{eff}}$ (Sec. III E). Finally, we discuss the implications of the SPT-3G 2018 data for the spatial curvature parameter, $\Omega_K$, in Sec. III F. We highlight key results in this section and refer the reader to Appendix B for tables containing the full cosmological parameter constraints. We only report constraints for the full SPT-3G 2018 dataset, finding that the consistency between low and high angular multipole moments seen in D21 for \Lambda CDM also extends to the cosmological models considered here.

A. Effective number of neutrino species, $N_{\text{eff}}$

The relativistic energy density in the early universe can be parametrized by $N_{\text{eff}}$, which is normalized to equal three for a thermal distribution of the three neutrino species in the standard model of particle physics. The expected value is $N_{\text{eff}} = 3.044$, as there is a small non-thermal contribution to the neutrinos from electron-positron annihilation \cite{35,36,35,36}.

There are a plethora of hypothesized particles that might change the observed $N_{\text{eff}}$, such as axionlike particles, hidden photons, gravitinos, or massless Goldstone bosons; the exact change in $N_{\text{eff}}$ depends on the nature of the particle and its coupling to the standard model \cite{13,38}.

We present constraints from SPT-3G 2018 data on $\Lambda$CDM + $N_{\text{eff}}$ in Table V. We find

\begin{equation}
N_{\text{eff}} = 3.70 \pm 0.70,
\end{equation}

which is within 0.9\sigma of the standard model prediction of 3.044. As you can see in Fig. 1, in CMB data constraints, higher values of $N_{\text{eff}}$ tend to lead to higher values of $H_0$; the slightly raised $N_{\text{eff}}$ value translates into a higher expansion rate, $H_0 = 73.5 \pm 5.2$ km s$^{-1}$ Mpc$^{-1}$. While this is consistent with the distance-ladder measurement of $H_0$ by R20 (0.05\sigma), the large uncertainty on the result means it is also consistent with CMB-based $H_0$ values in \Lambda CDM. As noted in Table II, this model barely changes the quality of fit compared to \Lambda CDM ($\Delta\chi^2 = -0.2$).

The reported central value for $N_{\text{eff}}$ is consistent with, although higher than, the corresponding Planck and ACT DR4 values by 1.1\sigma and 1.7\sigma, respectively. For the latter shift, we point out that our MCMC analysis of ACT DR4 yields $N_{\text{eff}} = 2.34 \pm 0.43$, which is less than the standard model prediction. The shift to lower $N_{\text{eff}}$ compared to \Lambda CDM in ACT DR4 is accompanied by shifts along the degeneracy directions in $\Omega_b h^2$ by $-0.0097$ and $n_s$ by $-0.048$. The constraints based on SPT-3G 2018 move in the opposite way along these same degeneracy axes, which places the central values of $\Omega_b h^2$ and $n_s$ 0.082 and 0.039

\footnote{In our MCMC analysis we have assumed the standard model value of $N_{\text{eff}} = 3.046$ based on Abazajian et al. \cite{37}. However, this small change has a negligible impact on the results of this paper.}
higher than in ΛCDM, respectively, and $N_{\text{eff}}$ slightly above the standard model prediction.

Two tensions have been noted between Planck data and low-redshift measurements: in ΛCDM one infers lower values of $H_0$ and higher values of $S_8$ compared to the standard model prediction. Contours indicate the 68% and 95% probability regions.

Table II. Improvement to the quality of fit for the cosmological models considered with respect to ΛCDM, $\Delta \chi^2 = \chi^2_{\Lambda \text{CDM}+} - \chi^2_{\Lambda \text{CDM}}$. We have run 10 minimizers without the annealer for each model and find that the $\chi^2$ of the best three runs typically span a range of the order of 0.1. We also list the extra degrees of freedom (d.o.f.) added by each model extension compared to ΛCDM.

| Model                                      | $\Delta \chi^2$ | Additional d.o.f. |
|-------------------------------------------|------------------|-------------------|
| ΛCDM + $N_{\text{eff}}$                  | -0.2             | 1                 |
| ΛCDM + $Y_P$                              | 0.1              | 1                 |
| ΛCDM + $N_{\text{eff}} + Y_P$             | -1.8             | 2                 |
| ΛCDM + $\Sigma m_{\nu}$                  | 0.0              | 1                 |
| ΛCDM + $m_{\nu,\text{sterile}} + N_{\text{eff}}$ | 0.1              | 2                 |
| ΛCDM + $\Omega_K$                         | -0.3             | 1                 |

$N_{\text{eff}}$ and correspondingly high values of $H_0$ that overlap with the distance-ladder measurement in R20 (the horizontal grey bands). However, such high values of $N_{\text{eff}}$ and $H_0$ are ruled out by the Planck data (black contours), so the tension persists, although at lower significance due to the larger uncertainty on $H_0$ when varying $N_{\text{eff}}$ for CMB data. The $S_8$ value for each sample in the SPT-3G chains is represented by the color, with the color range chosen to represent the 3σ range of the cosmic shear analysis by Heymans et al. [39]. Notably, $S_8$ varies perpendicular to the main degeneracy direction in the data, thus allowing $N_{\text{eff}}$ to vary does little to reduce the tension in constraints of $S_8$.

The right panel of Fig. 1 shows the constraints on $N_{\text{eff}}$ and $H_0$ from the SPT-3G 2018 and Planck data. The full results are listed in Table V. In particular, the joint constraint on the effective number of neutrino species is

$$N_{\text{eff}} = 2.95 \pm 0.17,$$

which is within 0.6σ of the standard model prediction. Adding the SPT-3G 2018 bandpowers to the Planck data tightens the $N_{\text{eff}}$ and $H_0$ constraints by 11% and reduces the square-root of the determinants of the parameter covariance matrices in this 7-parameter model by a factor of 1.5 (see Table III).

B. Primordial helium abundance, $Y_P$

The primordial helium abundance is a direct measure of the equilibrium abundance of neutrons relative to protons
during BBN, when the reactions that interconvert them become slow compared to the expansion rate. Virtually all neutrons end up in helium atoms during this period. The equilibrium abundance when these reactions freeze out depends on all known forces and as such measurements of the primordial helium abundance are a powerful probe of our understanding of particle physics.

The CMB anisotropies are sensitive to the helium abundance because helium’s first electron has a higher binding energy than hydrogen’s, which means that the helium recombination happens earlier than hydrogen. As a consequence, increasing the helium abundance lowers the free electron density during hydrogen recombination. The presence of fewer free electrons reduces the likelihood for Thomson scattering. The photon mean-free path is increased, leading the CMB power spectra at high ℓ to be suppressed as structure on small scales is washed out. Therefore, CMB power spectrum measurements can leverage the change in the Silk damping scale to constrain Yp.

The constraints from the SPT-3G 2018 bandpowers on ΛCDM + Yp are given in Table V. We find

\[ Y_p = 0.225 \pm 0.052, \]  

which is consistent with the BBN prediction of 0.2454 at 0.4σ [221]. The SPT-3G 2018 helium constraint is also consistent with the latest CMB results from Planck (0.3σ, [10]) and ACT DR4 (0.5σ, [2]), as well as recent measurements of H II regions of metal-poor galaxies (0.4σ, [40]). Current measurements of the primordial helium abundance are consistent with BBN expectations. The change to the quality of fit for this model compared to ΛCDM is insignificant (Δχ^2 = 0.1, see Table II).

We look at joint constraints from SPT-3G 2018 and Planck (see Table V). As noted in Table III, the addition of SPT-3G 2018 data to the Planck data reduces the square-root of the determinants of the parameter covariance matrices in this 7-parameter model by a factor of 1.4, yielding

\[ Y_p = 0.234 \pm 0.012. \]  

This measurement is consistent with the BBN prediction of 0.2454 (note the BBN prediction varies with the ΛCDM parameters) at 0.9σ, as well as the H II region-based measurement of Aver et al. [40] (0.9σ).

C. Effective number of neutrino species and primordial helium abundance, N_{eff} + Y_p

We now look at the constraints when simultaneously varying N_{eff} and Y_p. Since BBN makes precise predictions for the primordial helium abundance as a function of the effective number of neutrino species and other parameters, the constraint on N_{eff} in Sec. III A implicitly assumes that any extra relativistic species are present during both BBN and recombination. Simultaneously varying N_{eff} and Y_p removes this assumption and allows for independent constraints on the relativistic energy density during each epoch.

We present the constraints SPT-3G 2018 places on ΛCDM + N_{eff} + Y_p in Table V and show the marginalized 1D and 2D posterior probabilities for N_{eff} and Y_p in the left panel of Fig. 2. We find

\[ N_{eff} = 5.1 \pm 1.2, \]  

\[ Y_p = 0.151 \pm 0.060. \]  

The central value of N_{eff} is 1.7σ higher than the standard model prediction of 3.044, while the Y_p value is 1.6σ lower than the ΛCDM prediction of 0.2454; the parameters shift along the degeneracy direction in the N_{eff} vs Y_p plane as shown in the left panel of Fig. 2. The plot also shows that consistency with BBN, as well as departures to Y_p values far below the BBN expectation, are compatible with the SPT-3G data. The fit quality improves by only Δχ^2 = −1.8 compared to ΛCDM for two additional parameters (see Table II). The mild preference is driven by the data at ℓ < 800; removing the lower multipoles shifts the best-fit values toward the ΛCDM expectations. These angular-scales have been well-measured by Planck, which does not share this trend. Similar to Sec. III A, we find that the shifts in the values of N_{eff} and Y_p lead to increases in Ω_{ch} and n_s by 0.026 and 0.020 compared to ΛCDM, respectively.

The left panel of Fig. 2 compares the posteriors in the N_{eff} vs Y_p plane from SPT-3G 2018, Planck, and ACT DR4. As should be expected, all three show a similar degeneracy axis, where increasing N_{eff} decreases Y_p. The central value of the SPT-3G 2018 constraint is higher along the N_{eff} axis (and lower along the Y_p axis) than Planck, which in turn is higher than ACT DR4. Our central value of
$N_{\text{eff}}$ is 1.8σ higher than the Planck value, and larger than the ACT DR4 value by the same amount (although it is lower than Planck, its associated uncertainty is larger). The $Y_P$ value from SPT-3G is lower than the Planck and ACT DR4 ones by 1.5σ and 1.0σ, respectively.

To quantify the agreement between SPT-3G 2018 and Planck in the full parameter space, we calculate the $\chi^2$ of the differences in the mean values of the parameters using the inverse of the sum of parameter covariance matrices. We use a combined parameter, $10^9A_s e^{-2\tau}$, to account for the Planck-based $\tau$ prior used in the SPT-3G constraints. Thus the comparison covers seven parameters ($\Omega_b h^2, \Omega_c h^2, \theta_{MC}, 10^9A_s e^{-2\tau}, N_{\text{eff}}, Y_P$). We find $\chi^2 = 12.3$ between the SPT-3G 2018 and Planck datasets, which corresponds to a probability to exceed (PTE) of 9%. This is within the central 95% confidence interval [2.5%,97.5%] and we conclude that the two datasets are consistent with one another.

The same comparison for SPT-3G 2018 and ACT DR4 yields $\chi^2 = 17.8$, which translates to a PTE of 1%. This low PTE is driven by differences in the preferred baryon density. The $\Omega_b h^2$ value for ACT DR4 is 2.6σ below the SPT-3G 2018 result. The low baryon density inferred from ACT DR4 has been previously noted by Aiola et al. [2], who explain that the shift is related to degeneracies over the limited angular multipole range probed. Removing $\Omega_b h^2$ from the comparison reduces the $\chi^2$ to 12.7 and raises the PTE to 5%. Outside of the noted variation in the preferred baryon density with ACT DR4, we conclude that the parameter constraints in the $\Lambda$CDM + $N_{\text{eff}} + Y_P$ model are consistent across the three experiments.

The SPT-3G 2018 primordial helium abundance constraint is 1.6σ lower than the most precise measurement based on the H II regions of metal-poor galaxies [40]. While the SPT-3G 2018 data alone allow for very high expansion rates in the $\Lambda$CDM + $N_{\text{eff}} + Y_P$ model extension, $H_0 = 80.4 \pm 7.2$ km s$^{-1}$ Mpc$^{-1}$, the addition of Planck data significantly tightens the $H_0$ constraint and pulls the value down to $H_0 = 67.7 \pm 1.8$ km s$^{-1}$ Mpc$^{-1}$. We discuss the results with Planck in more detail below.

Comparison in the $\Lambda$CDM + $N_{\text{eff}} + Y_P$ model shows the improvement across successive SPT power spectrum
measurements. We compile the 1D marginalized posterior for $N_{\text{eff}}$ as constrained by SPT-SZ, SPTpol, and SPT-3G 2018 for this two-parameter extension in the right panel of Fig. 2. Across three generations of experiments from SPT-SZ to SPTpol to SPT-3G 2018, the uncertainty on the effective number of neutrino species has shrunk from $\sigma(N_{\text{eff}}) = 1.9$ to 1.4 to 1.2. Furthermore, we note that the SPT-SZ and SPTpol datasets were based on nearly complete multiyear surveys, whereas the SPT-3G 2018 data was recorded over a four-month period (half of a typical observing season) and data is still being collected.

Joint constraints from SPT-3G 2018 and Planck are given in Table V. Adding the SPT-3G to Planck data reduces the square-root of the determinants of the parameter covariance matrices in this 8-parameter model by a factor of 1.7 (see Table III), signalling a substantial reduction in the allowed parameter volume. For SPT-3G 2018 and Planck, we report

$$N_{\text{eff}} = 3.13 \pm 0.30,$$

$$Y_p = 0.230 \pm 0.017. \quad (6)$$

These values are offset from their standard model predictions by 0.3$\sigma$ and 0.1$\sigma$, respectively. The mean of the helium fraction posterior is 0.7$\sigma$ less than the H II region-based measurement of Aver et al. [40].

**D. Neutrino masses, $\Sigma m_\nu$**

The neutrino sector is one of the least understood areas of the standard model of particle physics. Determining neutrino hierarchy and the mechanism by which neutrinos attain their mass are key questions. CMB observations allow us to constrain the sum of neutrino masses, $\Sigma m_\nu$, and are complementary to terrestrial experiments, which have so far measured the squared mass splittings and the sign of one splitting [41–43].

We present the constraints on $\Lambda$CDM + $\Sigma m_\nu$ placed by SPT-3G 2018 alone and in combination with BAO and Planck data in Table VI. SPT-3G 2018 alone constrains $\Sigma m_\nu$ to 0.69 ± 0.67 eV, with an upper limit of $\Sigma m_\nu < 2.0$ eV at 95% confidence. We report no change to the quality of fit for this model compared to $\Lambda$CDM (see Table II).

We add BAO measurements to improve the $\Sigma m_\nu$ constraint. The low-redshift BAO points significantly reduce the large degeneracy between the expansion rate today and sum of the neutrino masses that exists in the SPT-3G data alone; the uncertainty on $H_0$ drops from 5.3 to 0.70 km s$^{-1}$ Mpc$^{-1}$ as can be seen in columns 1 and 3 of Table VI. The upper limit from on $\Sigma m_\nu$ SPT-3G plus BAO is

$$\Sigma m_\nu < 0.30 \text{ eV (95\% CL)}. \quad (7)$$

This limit is weaker than the 95% CL upper limits of 0.13 eV and 0.24 eV set by Planck and ACT DR4 in combination with BAO measurements, respectively. We show the associated marginalized 1D posteriors for all three datasets in the left panel of Fig. 3. As can be seen there, some of the difference in the upper limits is due to where

![FIG. 3.](image-url)
the posteriors peak, with the SPT-3G posterior reaching its maximum at \( \sim 0.11 \text{ eV} \).

We highlight the interplay between the joint constraints from SPT-3G 2018 and BAO data on the sum of the neutrino masses \( \Sigma m_\nu \), Hubble constant \( H_0 \), and a parameter describing the amplitude of density perturbations today, \( S_8 \), in the right panel of Fig. 3. Massive neutrinos offer no resolution to the Hubble tension: increasing the neutrino mass lowers the expansion rate inferred from the CMB and increases the gap between early- and late-time probes. The combination of the SPT-3G 2018 and BAO datasets rules out \( H_0 > 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) at 2.9\( \sigma \), leaving a 3.5\( \sigma \) rift to the most recent distance-ladder measurement by R20 (indicated in grey in the figure). It is interesting to note that in the \( \Sigma m_\nu, H_0, S_8 \) space shown, the measurements of R20 and Heymans et al. \[39\] lie in the same direction relative to the Planck constraints; increasing the value of \( H_0 \) at fixed \( \Sigma m_\nu \) also decreases the inferred \( S_8 \) value, thus improving the consistency with the local measurements of \( H_0 \) from R20 and of \( S_8 \) from Heymans et al. \[39\].

The parameter constraints from combining SPT-3G 2018, Planck, and BAO data on \( \Lambda \text{CDM} + \Sigma m_\nu \) are shown in Table VI. The addition of Planck power spectrum data reduces the upper limit on \( \Sigma m_\nu \) by more than a factor of two to:

\[
\Sigma m_\nu < 0.13 \text{ eV (95\% CL)}.
\]

The Planck large-scale temperature data adds information from both the late time integrated Sachs-Wolfe effect and the observed peak smoothing, which depends on the amount of gravitational lensing. Previous works have noted that one reason the Planck data favor low neutrino masses is the excess peak-smoothing observed in the Planck TT bandpowers \[10,44\]. Removing the Planck TT bandpowers (keeping Planck TE and EE) from the data combination relaxes the upper limit by 50\% to \( \Sigma m_\nu < 0.20 \text{ eV} \). As an approximate estimate of how much information is added by the SPT-3G data, we calculate the ratio for the square-root of the determinants of the parameter covariance matrices when adding the SPT-3G 2018 dataset to Planck (including the TT bandpowers) and BAO data to be 1.3 (see Table III). Adding the SPT-3G data to the Planck and BAO data thus substantially reduces the allowed parameter volume.

E. Sterile Neutrinos, \( m_{\nu, \text{sterile}}^{\text{eff}} \)

Sterile neutrinos are a hypothesized species of neutrinos that do not interact through the weak force, only gravitationally. We investigate the model formulated by the Planck collaboration, which we describe briefly here (for more details see Planck Collaboration et al. \[10,45,46\]). Motivated by the results of Acero et al. \[43\], we assume minimal neutrino masses in the normal mass hierarchy, which we approximate as two massless and one massive active neutrino with a mass of 0.06 eV. To these we add one massive sterile neutrino with an abundance and distribution across momentum arising from its mixing with active neutrinos.

We consider both a thermal distribution and, as in the Dodelson-Widrow (DW) mechanism \[47\], a distribution proportional to that of the active neutrinos with a scaling factor dependent on the mixing angle between the active and sterile neutrinos. Since the two scenarios are cosmologically equivalent, we sample over the effective mass \( m_{\nu, \text{sterile}}^{\text{eff}} = 94.1 \Omega_{\nu, \text{sterile}} h^2 \text{ eV} \), which maps to the physical mass according to \( m_{\nu, \text{sterile}}^{\text{physical}} = m_{\nu, \text{sterile}}^{\text{eff}} (\Delta N_{\nu, \text{eff}})^{\alpha} \), where \( \Delta N_{\nu, \text{eff}} \) is the deviation of the effective number of neutrino species from the standard model prediction, and \( \alpha = -3/4 \) for a thermal distribution of sterile neutrino momenta or \( \alpha = -1 \) for the DW mechanism.

Sterile neutrinos with physical masses \( \gtrsim 10 \text{ eV} \) become non-relativistic well before recombination and, depending on their mass, mimic warm or cold dark matter. To avoid this regime, we focus our analysis on the region in \( (N_{\nu, \text{eff}}^{\text{st}}, m_{\nu, \text{sterile}}^{\text{eff}}) \) space that corresponds to a physical mass of \( m_{\nu, \text{sterile}}^{\text{physical}} < 2 \text{ eV} \), assuming a thermal distribution of sterile neutrino momenta. Since sterile neutrinos in this region of parameter space would be relativistic at last-scattering, we would expect them to increase \( N_{\nu, \text{eff}} \).

We present the constraints the SPT-3G 2018 dataset places by itself and in combination with BAO on \( \Lambda \text{CDM} + \Sigma m_\nu \) in Table VII. The SPT-3G 2018 dataset is consistent with the null hypothesis of no sterile neutrinos, constraining \( \Delta N_{\nu, \text{eff}} < 1.8 \) and \( m_{\nu, \text{sterile}}^{\text{eff}} < 1.5 \text{ eV} \) at 95\% confidence. Including BAO data tightens these 95\% CL limits to:

\[
\Delta N_{\nu, \text{eff}} < 1.6, \\
m_{\nu, \text{sterile}}^{\text{eff}} < 0.50 \text{ eV}.
\]

As noted in Table II, we find that the quality of fit for this model does not change significantly from \( \Lambda \text{CDM} \) (\( \Delta \chi^2 = 0.1 \)). The Planck and ACT DR4 datasets also yield no evidence for sterile neutrinos: in combination with BAO data we infer \( \Delta N_{\nu, \text{eff}} < 0.29 m_{\nu, \text{sterile}}^{\text{eff}} < 0.24 \text{ eV} \) from Planck and \( \Delta N_{\nu, \text{eff}} < 0.58, m_{\nu, \text{sterile}}^{\text{eff}} < 0.32 \text{ eV} \) from ACT DR4.

We plot the constraints placed by SPT-3G 2018 + BAO in the \( N_{\nu, \text{eff}}^{\text{st}} vs m_{\nu, \text{sterile}}^{\text{eff}} \) plane in Fig. 4, where the degeneracy of these parameters with \( H_0 \) can be observed. We report \( H_0 = 71.6 \pm 2.2 \text{ km s}^{-1} \text{ Mpc}^{-1} \), which is higher than the \( \Lambda \text{CDM} \) value due to the increase in the effective number of neutrino species, similar to Sec. III A. While an increase to \( N_{\nu, \text{eff}} \) of the size needed to reconcile late- and early-time probes of \( H_0 \) is allowed by the SPT-3G 2018 dataset, it is disfavored by Planck \[10\].

---

\[6\]The results only change slightly if we assume the DW scenario for this prior instead of a thermal distribution of sterile neutrino momenta.
Joint constraints from SPT-3G 2018, Planck, and BAO data on sterile neutrinos are given in Table VII. We find 95% CL upper limits of

\[
\Delta N_{\text{eff}} < 0.30,
\]

\[
m_{\nu, \text{sterile}}^{\text{eff}} < 0.20 \text{ eV}.
\]  

The addition of Planck data reduces the upper limit on \(N_{\text{eff}}\) five-fold, and as a result tightens the posterior on \(H_0\) to \(68.30 \pm 0.70 \text{ km s}^{-1} \text{ Mpc}^{-1}\). The CMB-preferred value of \(H_0\) remains in tension with the distance-ladder measurement of R20 at 3.5\(\sigma\). Finally, as an indicator of the extent to which SPT-3G data reduces the allowed parameter volume in the 8-dimensional space, we once again calculate the square-root of the determinants of the parameter covariance matrices, finding a reduction by a factor of 1.6 when adding the SPT-3G 2018 dataset to Planck and BAO data.

**F. Spatial curvature, \(\Omega_K\)**

Inflation in the early universe should suppress any primordial spatial curvature, leading to a flat universe today to well below the precision of current measurements. While primary CMB observations can test this assumption, they suffer from geometric degeneracies which limit their precision. The Planck dataset prominently gives support for a closed universe at well over 2\(\sigma\) when considering primary CMB data alone. However, adding CMB lensing or BAO data drives the posterior back to \(\Omega_K = 0\) [10].

We report constraints on \(\Lambda CDM + \Omega_K\) from SPT-3G 2018 alone and jointly with BAO data in Table VIII. From SPT-3G 2018 alone we determine \(\Omega_K = 0.001^{+0.018}_{-0.019}\). This is perfectly consistent with a flat universe. We highlight that the marginalized confidence interval for \(\Omega_K\) is close to the precision of the Planck data (\(\Omega_K = -0.044^{+0.018}_{-0.015}\)). The precision of this result is not simply a reflection of the quality of the SPT-3G 2018 dataset, but also due to increasing slope of the degeneracy between \(H_0\) and \(\Omega_K\) observable in Fig. 5. This model barely changes the quality of fit compared to \(\Lambda CDM\) (\(\Delta \chi^2 = 0.3\), see Table II).

With the primary CMB information alone, spatial curvature is degenerate with the Hubble constant; the geometric impact of an open universe on the distance to the last-scattering surface can be compensated for by a higher expansion rate. Adding BAO information breaks this degeneracy, and for SPT-3G 2018 plus BAO data we report

\[
\Omega_K = -0.0014 \pm 0.0037. 
\]  

The central value is consistent with flatness at 0.4\(\sigma\). The BAO data also reduces the error on the \(H_0\) determination from \(\sigma(H_0) = 8.5 \text{ km s}^{-1} \text{ Mpc}^{-1}\) by a factor of 11 to \(\sigma(H_0) = 0.76 \text{ km s}^{-1} \text{ Mpc}^{-1}\) for the SPT-3G 2018 dataset. The combination of SPT-3G 2018 and BAO data constrains
seen in Table III, this extension shows the largest improvement from the SPT-3G data. The joint constraint on $H_0$ is $60.6 \pm 3.4$ km s$^{-1}$ Mpc$^{-1}$, which is $3.5\sigma$ lower than the distance-ladder measurement by R20.

Combining the two CMB datasets with BAO information yields

$$\Omega_K = 0.0009 \pm 0.0018,$$

which is consistent with flatness ($0.5\sigma$). The addition of BAO data also tightens the $H_0$ constraint to $68.05 \pm 0.67$ km s$^{-1}$ Mpc$^{-1}$. This value is in tension with the latest distance-ladder measurement at $3.5\sigma$.

### IV. $H_0$ FROM TEMPERATURE AND POLARIZATION DATA

We now turn our attention to the observation made by D21 that current $EE$ power spectrum measurements are consistent with comparatively high values of $H_0$. Fits to the $EE$ power spectra from SPT-3G 2018, SPTpol, Planck, and ACT DR4 yield $H_0 = 76.4 \pm 4.1$, $73.4 \pm 3.3$, $69.9 \pm 2.7$, and $71.8 \pm 4.4$ km s$^{-1}$ Mpc$^{-1}$, respectively [(1,2,7) D21]. These values are all within $\leq 1.1\sigma$ of the distance-ladder measurement of $H_0$ by R20. As stated by D21, this inconsistency between cosmological constraints derived from temperature and polarization data might hint at new physics to resolve the Hubble tension.

Although an interesting lead, the current evidence for such an inconsistency in individual experiments is low (see D21 Sec. 7, Planck Collaboration et al. [10] Sec. III, Choi et al. [3] §12). To increase the statistical weight, we combine the measured bandpowers from recent experiments at the likelihood level and present constraints based only on the $TT$, $TE$, or $EE$ spectra. For the $TT$ results we use SPT-SZ, Planck, and ACT DR4 $TT$, $TE$, and $EE$ spectra.

![Comparison of the 2D marginalized posteriors from joint constraints from collections of $TT$ (dark grey; SPT-SZ, Planck, ACT DR4 $\ell > 1800$), $TE$ (red; SPT-3G 2018, Planck, ACT DR4), and $EE$ (blue; SPT-3G 2018, Planck, ACT DR4) power spectra for each $\Lambda$CDM parameter vs $H_0$. The solid black contours show constraints from the combination of $TT$, $TE$, and $EE$ spectra from SPT-3G 2018, Planck, and ACT DR4. The light grey band indicates the $2\sigma$ interval of the distance-ladder measurement of $H_0$ by R20. Despite the raised expansion rate inferred from each individual $EE$ spectrum, the joint result is consistent with the $TT$ and $TE$ data and remains in $2.2\sigma$ tension with the low-redshift measurement of $H_0$. The low acoustic scale value inferred from the $EE$ spectra is driven by the Planck data (see Fig. 5 of Planck Collaboration et al. [10]). Contours indicate the 68% and 95% probability regions.](image)
Planck, and ACT DR4 data, with the ACT DR4 spectrum limited to the multipole range \( \ell > 1800 \) as recommended by Aiola et al. [2] in order to avoid correlations with the Planck data. For the \( TE \) and \( EE \) spectra, we combine the SPT-3G 2018, Planck, and ACT DR4 data. The parameter posteriors for the three sets of spectra are plotted in Fig. 6 and tabulated in Table IV. The joint constraints on the expansion rate for the three cases are \( H_0 = 68.85 \pm 0.97 \text{ km s}^{-1} \text{ Mpc}^{-1} \) for \( TT \)-only, \( H_0 = 67.95 \pm 0.94 \text{ km s}^{-1} \text{ Mpc}^{-1} \) for \( TE \)-only, and \( H_0 = 69.2 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1} \) for \( EE \)-only. There is no significant shift toward higher expansion rates in the polarization data. We note that the result from the combined \( EE \) data is lower than the value inferred from each individual dataset. As discussed by Addison [48] and shown by Fig. 1 of that work, this is because the ground-based experiments are most consistent with the lower end of the Planck \( H_0 \) parameter ellipses. We conclude that the temperature and polarization constraints paint a consistent picture of a low expansion rate, and do not suggest possible explanations for the gap between the Cepheid and supernova distance-ladder measurements of R20 and CMB data.

In the late stages of completing this work, Addison [48] published a similar, though more extensive, analysis investigating the \( H_0 \) constraints produced by combining \( EE \) power spectra of different experiments. While Addison [48] use the SPTpol 500d bandpowers, their results are fairly similar to ours. Addison [48] report a combined constraint on \( H_0 \) of \( 68.7 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1} \) which is consistent with our result of \( 69.2 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Note that the results are not independent, as they use the same data from Planck and ACT DR4. Moreover, the SPTpol and SPT-3G 2018 datasets produce similar cosmological constraints by themselves as pointed out by D21, which is partly due to the shared sky area between the two surveys.

We combine the SPT-3G 2018, Planck, and ACT DR4 temperature and polarization spectra to obtain the most precise constraint of \( H_0 \) from CMB power spectra to date.\(^7\) We report \( H_0 = 67.49 \pm 0.53 \text{ km s}^{-1} \text{ Mpc}^{-1} \). This result is 4.1\( \sigma \) lower than the low-redshift measurement of \( H_0 = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1} \) by R20; the Hubble tension remains.

\(^7\)We exclude SPT-SZ and SPTpol from this comparison due to the shared survey area with SPT-3G.

### V. CONCLUSION

In this work, we have presented constraints on cosmological models beyond ΛCDM using the SPT-3G 2018 power spectra, paying attention to the results in the context of the Hubble tension. The multifrequency \( EE \) and \( TE \) bandpowers from SPT-3G provide a high-precision measurement of the CMB at intermediate and small angular scales. As such, the bandpowers allow us to place tight constraints on physics beyond the standard model. We look for evidence of models with additional (or fewer) light and free-streaming degrees of freedom, or with nonstandard BBN helium production. Introducing \( N_{\text{eff}} \) as a free parameter, we determine \( N_{\text{eff}} = 3.70 \pm 0.70 \) from SPT-3G 2018 data, which is consistent with the standard model prediction of 3.044 at 0.9\( \sigma \). Instead varying \( Y_P \), we find \( Y_P = 0.225 \pm 0.052 \), which agrees well with the BBN prediction of 0.2454. Varying the two parameters simultaneously yields \( N_{\text{eff}} = 5.1 \pm 1.2 \) and \( Y_P = 0.151 \pm 0.060 \). Both values are within 2\( \sigma \) of their ΛCDM values. When adding the SPT-3G data to Planck, the constraints tighten to \( N_{\text{eff}} = 1.3 \pm 0.3 \) and \( Y_P = 0.230 \pm 0.017 \). For the \( \Lambda \text{CDM} + N_{\text{eff}} \) model, the SPT-3G data tighten the Planck-only constraints on \( N_{\text{eff}} \) and \( H_0 \) by 11\%. We see no significant evidence for new light relics or inconsistencies with BBN.

We also look at the implications of the SPT-3G 2018 data for the sum of the neutrino masses. Joint constraints from SPT-3G 2018 and BAO data limit the sum of neutrino masses to \( \sum m_\nu < 0.30 \text{ eV} \) at 95% confidence. Adding the Planck power spectrum data reduces the 95% CL limit to \( \sum m_\nu < 0.13 \text{ eV} \).

We explore the possibility of an additional sterile neutrino, while assuming minimal masses in the normal hierarchy for the three known neutrino species. From the SPT-3G 2018 data alone we derive a 95% CL upper limit on the effective mass of \( m_{\nu,\text{sterile}} \) < 1.5 eV and on the increase to the effective number of neutrino species of \( \Delta N_{\text{eff}} < 1.8 \). Adding BAO data significantly tightens these constraints to \( \Delta N_{\text{eff}} < 1.6 \) and \( m_{\nu,\text{sterile}} < 0.50 \text{ eV} \).

The SPT-3G 2018 dataset is consistent with a flat universe. We find \( \Omega_K = 0.001^{+0.018}_{-0.019} \), which is comparable to the precision of Planck data. Adding Planck and BAO data refines the constraint by an order of magnitude to \( \Omega_K = 0.0009 \pm 0.0018 \).

Varying \( N_{\text{eff}} \) or \( \Omega_\Lambda \) allows for higher values of \( H_0 \) with the SPT-3G 2018 data. In the first case, the higher values of
$H_0$ are connected to the slight preference for higher values of $N_{\text{eff}}$ as well as increased uncertainties compared to $\Lambda$CDM constraints. The increase in uncertainty is the main effect in the curvature case, where the uncertainty on $H_0$ is increased by a factor of 5.3. In both cases, the higher values of $H_0$ are disfavored by the addition of Planck or BAO data.

We find that adding SPT-3G 2018 to Planck data reduces the square root of the determinants of the parameter covariance matrices by factors of 1.3–2.0 across the cosmological models considered, signaling a substantial reduction in the allowed parameter volume.

We update the recent work of Addison [48], and combine SPT-3G 2018, Planck, and ACT DR4 at the likelihood level and report joint constraints on $H_0$ using only the $EE$ spectra. We find $H_0 = 69.19 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$, which is 2.2σ lower than the distance-ladder measurement of R20. We evaluate the significance of the Hubble tension by combining all spectra of the aforementioned datasets to produce the constraint on $H_0$ from CMB power spectra to date: $H_0 = 67.49 \pm 0.53$ km s$^{-1}$ Mpc$^{-1}$. This value is in 4.1σ tension with the most precise distance-ladder measurement [R20].

While the SPT-3G 2018 dataset provides a detailed view of the small-scale CMB polarization anisotropy, the data were obtained during a four-month period of the SPT-3G survey, during which approximately half of the detectors were inoperable. The SPT-3G survey is planned to continue through at least 2023, with existing maps from the combined 2019 and 2020 observing seasons already having $\sim 3.5 \times$ lower noise than the maps used in this analysis. The bandpowers from the full SPT-3G survey will significantly improve measurements of the damping tail of the $TE$ and $EE$ spectra, enabling tight constraints on physics beyond the $\Lambda$CDM model.

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APPENDIX A: LENSING CONVERGENCE ON THE SPT-3G SURVEY FIELD

The matter density field between us and recombination lenses the CMB and changes the observed power spectrum. One non-trivial consequence of this for surveys that do not cover a large fraction of the sky is super-sample lensing, i.e., the distortion of the CMB caused by matter-fluctuation modes with wavelengths larger than the survey field. This effect can be accounted for by adding a term to the covariance matrix or by marginalizing over the mean convergence across the survey field, $\bar{\kappa}$ [25]. While both yield the same results, we have chosen the latter approach in this work because it has the advantage of returning information on the local matter density across the survey field. As such, introducing $\bar{\kappa}$ as a variable in the MCMC chains can help us better understand the data and provide context when comparing the SPT-3G results to those of other experiments.

We account for super-sample lensing in our likelihood analysis by modifying the model spectrum, $C_\ell(p)$, based on a number of parameters $p$ to $\tilde{C}_\ell(p, \bar{\kappa})$ via

$$\tilde{C}_\ell(p, \bar{\kappa}) = C_\ell(p) + \frac{\partial C_\ell(p)}{\partial \ln \ell} \frac{\bar{\kappa}}{\ell^2}. \quad (A1)$$

Note that the definition of $\bar{\kappa}$ is of opposite sign to D21, matching Motloch and Hu [54]. All cosmological constraints presented in this work have been derived using a
Gaussian prior centered on zero with width $4.5 \times 10^{-4}$ as shown in Table I. The prior width is based on the geometry of the survey field [25].

Due to the limited sky fraction observed by SPT-3G, $\kappa$ is degenerate with $\theta_{MC}$ as can be seen in Fig. 7. This degeneracy was already noted by Motloch and Hu [54] using the example of the SPTpol 500d dataset, which for this purpose is similar to the SPT-3G dataset. The $\kappa - \theta$ degeneracy can be broken by imposing a prior on $\kappa$ centered on zero with a width that depends on the field area, as we have done throughout this work.

Motloch and Hu [54] demonstrate that the $\kappa - \theta$ degeneracy can be broken without resorting to a prior on $\kappa$ through the inclusion of Planck data, which due to its large sky coverage is insensitive to super-sample lensing. Combining SPT-3G 2018 and Planck data yields an estimate of the mean convergence on the SPT-3G survey field,

$$10^3 \kappa_{\text{SPT-3G}} = -1.60 \pm 0.56.$$  \hspace{1cm} (A2)

This inferred $\kappa$ is $2.9\sigma$ away from zero and would imply that the SPT-3G footprint coincides with a local under-density. If we include the expected $\Lambda$CDM cosmic variance across this field size (0.45 as mentioned above), this becomes a $2.2\sigma$ event.

We run SPT-3G-only chains imposing this result as a prior on $\kappa$ instead of the zero-centred prior used throughout this work (see Table I). As expected, the $\theta_{MC}$ prior constraint shifts high to $1.04126 \pm 0.00078$, which is close to the Planck result ($1.04090 \pm 0.00031$). The central values of other $\Lambda$CDM parameters only shift slightly ($\lesssim 0.1\sigma$). The inferred $H_0$ changes from $68.8 \pm 1.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ to $69.2 \pm 1.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

### APPENDIX B: PARAMETER TABLES

We present the full parameter constraints from SPT-3G 2018 alone and in combination with BAO and Planck data on $\Lambda$CDM extensions in the following tables. We show results for $\Lambda$CDM + $N_{\text{eff}}$, $\Lambda$CDM + $Y_p$, and $\Lambda$CDM + $N_{\text{eff}} + Y_p$ in Table V. We show constraints on $\Lambda$CDM + $\Sigma m_\nu$ and $\Lambda$CDM + $m_{\nu, \text{sterile}}$ in Tables VI and VII, respectively. Constraints on $\Lambda$CDM + $\Omega_k$ are given in Table VIII.

### TABLE V. Constraints on $\Lambda$CDM model extensions $N_{\text{eff}}, Y_p,$ and $N_{\text{eff}} + Y_p$ from SPT-3G 2018 alone and jointly with Planck.

| Parameter | SPT-3G 2018 | SPT-3G 2018 + Planck | SPT-3G 2018 | SPT-3G 2018 + Planck | SPT-3G 2018 | SPT-3G 2018 + Planck |
|-----------|-------------|-----------------|-------------|-----------------|-------------|-----------------|
| $N_{\text{eff}}$ | 3.70 ± 0.70 | 2.95 ± 0.17 | — | — | — | — |
| $Y_p$ | 0.225 ± 0.052 | 0.234 ± 0.012 | — | — | 0.151 ± 0.060 | 0.230 ± 0.017 |

Free

$\Omega_m h^2$ | 0.02275 ± 0.00048 | 0.02232 ± 0.00020 | 0.02231 ± 0.00050 | 0.02229 ± 0.00019 | 0.02256 ± 0.00049 | 0.02230 ± 0.00020 |

$\Omega_b h^2$ | 0.1232 ± 0.0097 | 0.1183 ± 0.0027 | 0.1152 ± 0.0037 | 0.1197 ± 0.0013 | 0.141 ± 0.016 | 0.1210 ± 0.0045 |

$10^3 \theta_{MC}$ | 1.03913 ± 0.00089 | 1.04086 ± 0.00039 | 1.0390 ± 0.0018 | 1.04034 ± 0.00051 | 1.0345 ± 0.0027 | 1.0400 ± 0.0011 |

$10^3 A_s e^{-2\pi}$ | 1.828 ± 0.041 | 1.873 ± 0.016 | 1.824 ± 0.038 | 1.876 ± 0.012 | 1.866 ± 0.046 | 1.879 ± 0.018 |

$\eta_s$ | 1.038 ± 0.046 | 0.9629 ± 0.0079 | 0.984 ± 0.044 | 0.9615 ± 0.0068 | 1.019 ± 0.046 | 0.9627 ± 0.0079 |

Derived

$H_0$ | 73.5 ± 5.2 | 66.8 ± 1.3 | 68.4 ± 1.7 | 67.20 ± 0.63 | 80.4 ± 7.2 | 67.7 ± 1.8 |

$\Omega_\Lambda$ | 0.726 ± 0.028 | 0.6833 ± 0.0095 | 0.704 ± 0.022 | 0.6839 ± 0.0083 | 0.743 ± 0.027 | 0.6854 ± 0.0099 |

$\sigma_8$ | 0.812 ± 0.030 | 0.804 ± 0.010 | 0.786 ± 0.020 | 0.8058 ± 0.0077 | 0.829 ± ± | 0.808 ± ± |

$S_8$ | 0.774 ± 0.042 | 0.826 ± 0.015 | 0.780 ± 0.041 | 0.827 ± 0.015 | 0.765 ± 0.042 | 0.827 ± 0.014 |

Age/Gyr | 13.22 ± 0.63 | 13.90 ± 0.18 | 13.84 ± 0.10 | 13.822 ± 0.034 | 12.32 ± 0.80 | 13.75 ± 0.27 |
### TABLE VI. Combined constraints on ΛCDM model extension $\Sigma m_e$ from the SPT-3G 2018, Planck, and BAO datasets.

|            | SPT-3G 2018 | SPT-3G 2018 + Planck | SPT-3G 2018 + BAO | SPT-3G 2018 + Planck + BAO |
|------------|-------------|----------------------|-------------------|-----------------------------|
| **Free**   |             |                      |                   |                             |
| $\Omega_0 h^2$ | 0.02239 ± 0.00033 | 0.02239 ± 0.00014   | 0.02244 ± 0.00032 | 0.02246 ± 0.00012          |
| $\Omega_A h^2$ | 0.1179 ± 0.0042   | 0.1197 ± 0.0013     | 0.1152 ± 0.0019   | 0.11885 ± 0.00099          |
| $10^9 A_s e^{-2r}$ | 1.03907 ± 0.000822  | 1.04070 ± 0.000290  | 1.03956 ± 0.000666 | 1.04082 ± 0.000270         |
| $n_s$       | 0.980 ± 0.026    | 0.9662 ± 0.0043     | 0.997 ± 0.018     | 0.9682 ± 0.0037            |
| $\Sigma m_e$ | <2.0           | <0.29                | <0.30             | <0.13                      |
| **Derived**|             |                      |                   |                             |
| $H_0$       | 62.7 ± 5.3      | 67.1 ± 1.1           | 68.02 ± 0.70      | 67.92 ± 0.52                |
| $\Omega_\Lambda$ | 0.61 ± 0.11     | 0.681 ± 0.015       | 0.6991 ± 0.0087   | 0.6924 ± 0.0067             |
| $\sigma_8$  | 0.686 ± 0.089   | 0.801 ± 0.021       | 0.774 ± 0.025     | 0.810 ± 0.011               |
| $S_8$       | 0.764 ± 0.045   | 0.825 ± 0.016       | 0.775 ± 0.027     | 0.820 ± 0.013               |
| Age/Gyr     | 14.11 ± 0.27    | 13.820 ± 0.059      | 13.847 ± 0.052    | 13.779 ± 0.027              |

### TABLE VII. Combined constraints on ΛCDM model extension $m_{e, sterile}^{eff}$ from the SPT-3G 2018, Planck, and BAO datasets.

|            | SPT-3G 2018 | SPT-3G 2018 + Planck | SPT-3G 2018 + BAO | SPT-3G 2018 + Planck + BAO |
|------------|-------------|----------------------|-------------------|-----------------------------|
| **Free**   |             |                      |                   |                             |
| $\Omega_0 h^2$ | 0.02284 ± 0.00042 | 0.02248 ± 0.00014   | 0.02281 ± 0.00039 | 0.02256 ± 0.00013          |
| $\Omega_A h^2$ | 0.1278 ± 0.0079   | 0.1210 ± 0.0019     | 0.1269 ± 0.0077   | 0.1201 ± 0.0018             |
| $10^9 A_s e^{-2r}$ | 1.03858 ± 0.000822  | 1.04052 ± 0.00032   | 1.03877 ± 0.00078 | 1.04066 ± 0.00031           |
| $n_s$       | 1.042 ± 0.036    | 0.9690 ± 0.0053     | 1.038 ± 0.031     | 0.9725 ± 0.0050             |
| $\Delta N_{eff}$ | <1.8           | <0.30                | <1.6              | <0.30                       |
| $m_{e, sterile}^{eff}$ | <1.5           | <0.44                | <0.50             | <0.20                       |
| **Derived**|             |                      |                   |                             |
| $H_0$       | 71.0 ± 4.4     | 67.47 ± 0.81        | 71.6 ± 2.2        | 68.30 ± 0.70                |
| $\Omega_\Lambda$ | 0.686 ± 0.044   | 0.680 ± 0.011       | 0.7020 ± 0.0086   | 0.6911 ± 0.0065             |
| $\sigma_8$  | 0.741 ± 0.063   | 0.787 ± 0.021       | 0.777 ± 0.030     | 0.798 ± 0.013               |
| $S_8$       | 0.753 ± 0.047   | 0.813 ± 0.018       | 0.774 ± 0.031     | 0.810 ± 0.014               |
| Age/Gyr     | 13.16 ± 0.41    | 13.713 ± 0.073      | 13.20 ± 0.37      | 13.687 ± 0.085              |

### TABLE VIII. Combined constraints on ΛCDM model extension $\Omega_K$ from the SPT-3G 2018, Planck, and BAO datasets.

|          | SPT-3G 2018 | SPT-3G 2018 + Planck | SPT-3G 2018 + BAO | SPT-3G 2018 + Planck + BAO |
|----------|-------------|----------------------|-------------------|-----------------------------|
| **Free** |             |                      |                   |                             |
| $\Omega_0 h^2$ | 0.02241 ± 0.00033 | 0.02251 ± 0.00015   | 0.02243 ± 0.00033 | 0.02242 ± 0.00014          |
| $\Omega_A h^2$ | 0.1162 ± 0.0055   | 0.1184 ± 0.0014     | 0.1149 ± 0.0038   | 0.1192 ± 0.0013             |
| $10^9 A_s e^{-2r}$ | 1.03956 ± 0.000812  | 1.04086 ± 0.000300  | 1.03960 ± 0.00073 | 1.04075 ± 0.00028           |
| $\Omega_K$ | $0.001_{-0.019}^{+0.018}$ | $-0.020 ± 0.011$     | $-0.0014 ± 0.0037$ | $0.0009 ± 0.0018$           |
| **Derived**|             |                      |                   |                             |
| $H_0$   | 70.8 ± 8.5    | 60.6 ± 3.4           | 68.11 ± 0.76      | 68.05 ± 0.67                |
| $\Omega_\Lambda$ | 0.710 ± 0.046   | 0.630 ± 0.032       | 0.704 ± 0.011     | 0.6918 ± 0.0059             |
| $\sigma_8$ | 0.794 ± 0.030   | 0.789 ± 0.012       | 0.788 ± 0.017     | 0.8082 ± 0.0077             |
| $S_8$   | 0.772 ± 0.068  | 0.897 ± 0.039       | 0.785 ± 0.027     | 0.818 ± 0.012               |
| Age/Gyr | 13.65 ± 0.92   | 14.57 ± 0.39        | 13.88 ± 0.16      | 13.751 ± 0.077              |
CONSTRAINTS ON $\Lambda$CDM EXTENSIONS FROM THE PLANCK 2015 RESULTS

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