Quantum Memory Based on Λ-Atoms Ensemble with Two-Photon Resonance EIT

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We study the Λ-atoms ensemble based quantum memory for the storage of the quantum information carried by a probe light field. Two atomic Rabi transitions of the ensemble are coupled to the quantum probe field and classical control field respectively with a same detuning. Making use of the hidden symmetry analysis developed recently for the on-resonance EIT case (Sun, Li, and Liu, Phys. Rev. Lett. 91, 147903 (2003)), we show that the dark states and dark-state polaritons can still exist for the case of two-photon resonance EIT. Starting from these dark states we construct a complete class of eigen-states of the total system. A explicit form of the adiabatic condition is also given in order to achieve the memory and retrieve of quantum information.

PACS number: 03.67.-a, 42.50.Gy, 03.65.Fd

With great interest and quick development in quantum information science [1], the implementation of quantum memory becomes one of the particular challenges to quest a realistic system transporting or communicating quantum states between different nodes of quantum networks. Quantum optical systems with atoms appear to be very attractive since photons are ideal carriers of quantum information with very fast velocity; and the atoms represent long-lived storage and reliable processing units.

A well known quantum optical system is of the electromagnetically induced transparency (EIT) [2], which can be used to make a resonant, opaque medium transparent. The essential property of EIT is induced by atomic coherence and quantum interference. The discovery of EIT has led to the occurrence of new effects and new techniques including ultraslow light pulse propagation [3,4] and the light signal storage [5,6] in atomic vapor. A following idea is then how to use the EIT system to transport and communicate the quantum state between photons and atoms.

Conventionally the EIT system consists of a vapor of 3-level atoms with two classic optical fields (the probe and control fields) being one-photon on-resonance with the relevant atomic transitions [2,3,7]. Later, it is noticed that such an on-resonance EIT is not a prerequisite for achieving significant group velocity reduction. The EIT phenomenon can also occur when the frequency difference between the probe and control fields matches the two-photon transition between the two lower states of the Λ-type atoms [8–10] (for this case, it is called two-photon resonance EIT).

In order to implement the quantum memory and to transport the quantum states between photons and atomic ensemble, recently some people [5,6,11–14] have replaced the classical probe laser field by a weak quantum light field in the on-resonance EIT system with atomic collective excitations. Then, by adiabatically changing the coupling strength of the classic control field, they have demonstrated the possibility of coherently controlling the propagation of the quantum light pulses via the dark states and dark-state polaritons. Most recently, to avoid the spatial-motion induced decoherence, we have considered an on-resonance EIT system of "atomic crystal" with each atom fixed on a lattice site [15]. With discovery of the hidden dynamic symmetry, we have shown that such a system is a robust quantum memory to transport the quantum states between photons and atomic ensemble.

Since most works about the memory of quantum probe light field within an atomic ensemble are based on the on-resonance EIT with atomic collective excitations [11,12,15], we want to study a system under the case of two-photon resonance EIT with the method of atomic collective excitations. In a former paper [16] we have calculated the susceptibility and group velocity of the probe field under two-photon resonance. Our results show that the EIT phenomenon exists indeed and an ultraslow group velocity can be obtained. In this work, we study how the excitation system under two-photon resonance EIT serves as a robust quantum memory.

We consider an atomic ensemble consisted of $N$ 3-level atoms of Λ-type, which are coupled to two single-mode optical fields as shown in Fig. 1. The atomic levels are labelled as the ground state $|b\rangle$, the excited state $|a\rangle$ and the meta-stable state $|c\rangle$. The atomic transition $|a\rangle \leftrightarrow |b\rangle$ with energy level difference $\omega_{ab} = \omega_a - \omega_b$ is coupled to a quantum probe light field of frequency $\omega$ with the coupling coefficient $g$ and the detuning $\Delta_p = \omega_{ab} - \omega$, while the atomic transition $|a\rangle \leftrightarrow |c\rangle$ energy level difference $\omega_{ac}$ driven by a classical control field of frequency $\nu$ with the Rabi-frequency $\Omega(t)$ and the detuning $\Delta_c = \omega_{ac} - \nu$. For simplicity, the coupling coefficients $g$ and $\Omega$ are real and assumed to be identical for all the atoms in the ensemble. Under the two-photon resonance condition, that is, $\Delta_p \equiv \Delta_c$, the interaction Hamiltonian of total system can be written in the interaction picture as ($\hbar = 1$)

$$H_I = \Delta_c S + (g\sqrt{N}aA^\dagger + \Omega T_+ + h.c.)$$

in terms of the collective quasi-spin operators

$$S = \sum_{j=1}^N \sigma^{(j)}_{aa},$$
\[ A^\dagger = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{ab}^{(j)}, \]
\[ T_+ = \sum_{j=1}^{N} \sigma_{ac}^{(j)}. \]

Here \( \sigma_{\mu\nu}^{(j)} = |\mu\rangle_j \langle \nu| \) is the flip operator of the \( j \)-th atom from state \( |\mu\rangle_j \) to \( |\nu\rangle_j \) \( (\mu, \nu = a, b, c) \); and \( A^\dagger \) \( (a) \) is the creation (annihilation) operator of the probe field. In the large \( N \) and low atomic excitation limit with only a few atoms occupy states \( |a\rangle \) or \( |c\rangle \) [17], the quasi-spin wave excitations of the atoms behave as bosons since in this case they satisfy the bosonic commutation relation \( [A, A^\dagger] = 1 \).

![Diagram](image)

\[ |a\rangle \]
\[ \Delta_c \]
\[ g, \Omega \]
\[ \Omega, \nu \]
\[ |b\rangle \]
\[ |c\rangle \]

FIG. 1. The probe and control optical fields are respectively coupled to two atomic transitions with the same detuning \( \Delta_c \). That is, such a system consisted of 3-level \( \Lambda \)-atoms ensemble and two optical fields satisfies the two-photon resonance EIT condition.

We note that the above Hamiltonian is expressed in terms of the collective dynamic variables \( S, A, A^\dagger, T_+, \) and \( T_- = (T_+)^\dagger \). To properly describe the cooperative motion of the atomic ensemble stimulated by the probe and control fields, we consider the closed Lie algebra generated by \( A, A^\dagger, C, \) and \( C^\dagger \). To this end a new pair of collective excitation operators

\[ C = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{cc}^{(j)}, \quad C^\dagger = (C)^\dagger \]

are introduced here to form a closed algebra. In the large \( N \) and low excitation limit, the corresponding collective excitations also behave as bosons since they satisfy the bosonic commutation relation \( [C, C^\dagger] = 1 \). These quasi-spin collective excitations are independent of each other in the same limit because of the vanishing commutation relations

\[ [A, C] = 0, \quad [A, C^\dagger] = -T_-/N \rightarrow 0 \]

by a straightforward calculation. It’s noted that \( S \) is a Hermitian operator and has the commutation relations

\[ [S, A] = -A, \quad [S, A^\dagger] = A, \quad [S, C] = [S, C^\dagger] = 0, \]
\[ [S, T_{\pm}] = \pm T_{\pm}. \]

Moreover, it’s easy to prove the basic commutation relations

\[ [T_-, C] = -A, \quad [T_-, C^\dagger] = 0, \quad [T_-, A] = 0, \quad [T_-, A^\dagger] = C^\dagger. \]

The above relations define a dynamic symmetry hidden in our dressed atomic ensemble described by the semi-direct-product algebra \( SU(2) \otimes h_2 \) containing the algebra \( SU(2) \) (which is generated by \( T_- \), \( T_+ \), and \( T_3 = \sum_{j=1}^{N} (\sigma_{aa}^{(j)} - \sigma_{cc}^{(j)})/2 \)

as the third generator) and the algebra \( h_2 \) (generated by \( A, A^\dagger, C, \) and \( C^\dagger \)), since it follows that

\[ [SU(2), h_2] \subset h_2. \]

Actually, a similar semi-direct product algebra \( SU(2) \otimes h_2 \) has been found by us [15] in an "atomic crystal" under the case of the one-photon resonance EIT. In that paper, by means of this algebra and the spectral generating algebra method, we have obtained a complete class of eigen-states of the total system consisted of the atoms and optical fields. And a explicit form of adiabatic passage has been given to implement the quantum information memory and retrieve between the type of photons and the type of atomic collective excitations. In what follows, we will study how the system shown in this work can serve as a robust quantum memory.

We can define a polariton operator

\[ D = a \cos \theta - C \sin \theta, \]

where \( \theta(t) \) satisfies

\[ \tan \theta(t) = \frac{g N \sqrt{N}}{\Omega(t)}. \]

This polariton operator that mixes the optical field and the atomic collective excitations behaves a boson in the large \( N \) and low excitation limit since

\[ [D, D^\dagger] = 1. \]

Thus the Heisenberg-Weyl group \( h \) generated by \( D \) and \( D^\dagger \) is a symmetry group of the exciton-photon system. We introduce the state \( |0\rangle_p = |0\rangle_p \otimes |b^N\rangle \) where \( |0\rangle_p \)
is the vacuum of the electromagnetic field and \( |b^N\rangle = |b_1 b_2 \ldots b_N\rangle \) denotes all the \( N \) atoms staying in the ground states. The relations

\[
H_I |0\rangle = 0, \quad [D, H_I] = 0 \tag{12}
\]

hint to us that a degenerate class of eigen-states of \( H_I \) with zero eigen-value can be constructed naturally as follows:

\[
|d_n\rangle = [n!]^{-1/2} D^\dagger n |0\rangle, \quad n = 0, 1, 2, \ldots \tag{13}
\]

Using the Eq. (9), we can expand \( |d_n\rangle \) as

\[
|d_n\rangle = \sum_{m=0}^{n} \frac{(-1)^m}{m! (n-m)!} \cos^{n-m} \theta \sin^m \theta |c^m\rangle \otimes |n-m\rangle_p, \quad n = 0, 1, 2, \ldots \tag{14}
\]

where

\[
|c^m\rangle = [m!]^{-1/2} C^\dagger m |b^N\rangle \tag{15}
\]

represents there are \( m \) \( C \)-mode excitations in the atomic ensemble. Let us draw the energy level graphic as Fig. 2. It can be observed from Fig. 2 that the dark state \( |d_n\rangle \) is a linear combination consisted of the terms \( |b^N\rangle \otimes |n\rangle_p, |c\rangle \otimes |n-1\rangle_p, \ldots, |c^m\rangle \otimes |0\rangle_p \). In this work, we ignore all the atomic decays. Generally the decay related to the atomic excited state \( |a\rangle \) is much larger than the decay related to the meta-stable state \( |c\rangle \). However, the atomic parts of all the terms in the dark state only contain the single-atom lower states \( |b\rangle \) and \( |c\rangle \) so that the dark state is robust even if the physical decays are considered.

Physically, the above dressed state is cancelled by the interaction Hamiltonian, thus it is called a dark state and \( D \) is called a dark-state polariton (DSP). The DSP traps the electromagnetic radiation from the excited state due to the quantum interference cancelling. For the case with an ensemble of free moving atoms, the similar DSP was obtained in Refs. [6,11,12,15] to clarify the physics of the state-preserving slow light propagation in EIT associated with the existence of collective atomic excitations.

Now starting from these dark states \( |d_n\rangle \), we can use the spectrum generating algebra method to build other eigenstates for the total system. To this end we introduce the bright-state polariton operator

\[
B = a \sin \theta + C \cos \theta. \tag{16}
\]

It is obvious that

\[
[B, B^\dagger] = 1, \quad [D, B^\dagger] = [D, B] = 0. \tag{17}
\]

Evidently \([A, B] = [A, B^\dagger] = 0\) using the fact that \( A \) commutes with \( C \) and \( C^\dagger \) in the large \( N \) with low excitation limit.

It is straightforward to obtain the commutation relations

\[
[H_I, B^\dagger] = \varepsilon A^\dagger, \quad [H_I, A^\dagger] = \Delta_c A^\dagger + \varepsilon B^\dagger, \tag{18}
\]

where \( \varepsilon = \sqrt{g^2 N + \Omega^2} \). Notice that the system of dynamical equations (18) are not simply as the same as that of the case of on-resonance [15]. However, we can introduce the norm mode variables \( Q_{\pm} \):

\[
Q_{\pm} = \sqrt{\frac{\Theta \pm \Delta_c}{2\Theta}} A \pm \sqrt{\frac{\Theta \mp \Delta_c}{2\Theta}} B, \tag{19}
\]

where

\[
\Theta = \sqrt{\Delta^2 + 4\varepsilon^2}. \tag{20}
\]

What is crucial for our purpose is the commutation relations

\[
[H_I, Q_{\pm}^\dagger] = \varepsilon Q_{\pm}^\dagger, \tag{21}
\]

where

\[
\varepsilon_{\pm} = \pm \sqrt{\frac{\Theta \pm \Delta_c}{\Theta \mp \Delta_c}} \varepsilon. \tag{22}
\]

By introducing the norm mode transformation, it is almost the same as the case of on-resonance EIT. Based on these commutation relations we can construct the eigenstates

\[
|e(m, k; n)\rangle = [m! k!]^{-1/2} Q_{+}^m Q_{-}^k |d_n\rangle, \tag{23}
\]

as the dressed states of the total system. The corresponding eigen-values are
\[
E(m, k) = me_+ - ke_-, \quad m, k = 0, 1, 2, \ldots. \quad (24)
\]

In the following discussion, we consider whether the dark states of zero-eigen-value can work well as a quantum memory by the adiabatic manipulation. This means that we should consider how the adiabatic condition [18,19]

\[
\left| \frac{\langle e(m, k; n) | \partial_t | d_l \rangle}{E(m, k) - 0} \right| \ll 1, \quad (25)
\]

is satisfied for any \(m, k, n, l = 0, 1, 2, \ldots\). The eigenvalues of these instantaneous collective eigen-states are complicated and the corresponding energy levels can cross each other (including the dark states) when adiabatically varying the Rabi frequency \(\Omega(t)\). Fortunately, among all the terms \(\langle e(m, k; n) | \partial_t | d_l \rangle\), only the terms \(\langle e(0, 1; l) | \partial_t | d_l \rangle\) and \(\langle e(1, 0; l) | \partial_t | d_l \rangle\) do not vanish. In general, we calculate

\[
\langle e(m, k; n) | \partial_t | d_l \rangle = \frac{1}{\sqrt{m!k!n!l!}} (0) Q^+ Q^k D^n \partial_t D^l \langle 0 \rangle
\]

\[
= \frac{1}{\sqrt{m!k!n!l!}} (0) Q^+ Q^k D^n \partial_t D^l \langle 0 \rangle
\]

\[
= \frac{-i \theta}{\sqrt{m!k!n!l!}} (0) Q^+ Q^k D^n \partial_t D^l \langle 0 \rangle
\]

\[
= \frac{-i \theta}{\sqrt{m!k!n!l!}} (0) Q^+ Q^k D^n \partial_t D^l \langle 0 \rangle
\]

\[
= \sqrt{i} \delta \delta_{n,l} \left[ \delta_{m,0} \delta_{k,1} \sqrt{\frac{\Theta + \Delta}{2\Theta}} - \delta_{m,1} \delta_{k,0} \sqrt{\frac{\Theta - \Delta}{2\Theta}} \right]. \quad (26)
\]

From the above results, we can readily obtain the adiabatic condition:

\[
g \sqrt{N} \left| \frac{\langle \Theta + | \Delta_c | \rangle}{\sqrt{\Theta (\Theta - | \Delta_c |)}} \right| \varepsilon^3 \left| \Omega \right| \ll 1 \quad \text{by means of}
\]

\[
\dot{\theta} = -g \sqrt{N} \left| \Omega \right| / \varepsilon^2. \quad (28)
\]

When \(\Delta_c = 0\), the above equation (27) will reduce to

\[
g \sqrt{N} \left| \Omega \right| / \varepsilon^3 \ll 1, \quad (29)
\]

which is just as the same as the adiabatic condition under on-resonance EIT as shown in Ref. [15]. According to the Eq. (26), we know that: the dark state \(| d_l \rangle\) will not mix the states \(\langle e(m, k; n)\rangle\) under the adiabatic condition (27); and the dark state \(| d_l \rangle\) also can not mix the other degenerate state \(| d_l \rangle\) according to the adiabatic condition of degenerate states [15,19]. This means when the initial state is

\[
| \Phi(0) \rangle = \sum_n c_n(0) | d_n(0) \rangle, \quad (30)
\]

the total system will follow the superposition of dark-state

\[
| \Phi(t) \rangle = \sum_n c_n(0) | d_n(t) \rangle \quad (31)
\]

under the adiabatic evolution.

![FIG. 3. Dark state polariton](image)

Then we can implement the quantum information memory and retrieve between the type of photons and the type of atomic collective excitations. As shown in Fig. 2, we denote the dark state polariton \(D(t)\) as a vector in the \(a-C\) plane. Since the dark state is generated by \(D^\dagger\) as

\[
| d_n(t) \rangle = | n \rangle^{-1/2} D^\dagger(t) | 0 \rangle, \quad (32)
\]

then if \(\Omega(t)\) is changed adiabatically to make \(\theta(t) : 0 \rightarrow \frac{\pi}{2}\), one have \(D(t) : a \rightarrow -C\) and \(| d_n(t) \rangle\) will change from \(| b^n \rangle \otimes | n \rangle_p\) to \(| c^n \rangle \otimes | 0 \rangle_p\). Generally, the initial quantum state of the single-mode optical field is described by a density matrix

\[
\rho_p = \sum_{n,m} \rho_{nm} | n \rangle_p \langle m |, \quad (33)
\]

the transfer process generates a quantum state of collective excitations according to

\[
| b^n \rangle \langle b^n | \otimes \sum_{n,m} \rho_{nm} | n \rangle_p \langle m | \rightarrow \sum_{n,m} ( -1 )^{n+m} \rho_{nm} | c^n \rangle \langle c^n | \otimes | 0 \rangle_p \langle 0 |. \quad (34)
\]

After an inverse adiabatic control to make \(\theta : \frac{\pi}{2} \rightarrow 0\), the total quantum information will change from the type of the atomic ensemble to the type of photons:
\[ \sum_{n,m} (-1)^{n+m} \rho_{nm} |c^n\rangle \langle c^m| \otimes |0\rangle_{pp} |0\rangle \]
\[ \rightarrow |b^n\rangle \langle b^n| \otimes \sum_{n,m} \rho_{nm} |n\rangle_{pp} \langle m|. \] (35)

The quantum information can be adiabatically transferred from the optical field to the atomic ensemble, and vice versa. Such two adiabatic passages complement the "write" and "read" manipulation of quantum information. That is to say, the quantum information can be memorized in such an atomic ensemble.

In summary, we study the structure of the eigen-states and eigen-values of the collective exciton-photon system under the two-photon resonance EIT. Our results show that the dark states and dark-state polaritons can still exist for the case of two-photon resonance. We analyze in detail the possibility of a dark state being staying the initial form under the adiabatic evolution of systemic parameter \( \Omega(t) \). A precise adiabatic condition is presented in order to make sure that a dark state can not mix any other eigen-states. Then, with the help of the dark states under the two-photon resonance EIT system, we have described how one can transfer or communicate the quantum states between the type of photons and the type of atomic collective excitations by adiabatically changing the Rabi frequency \( \Omega(t) \) of the classical control laser field. Our results show that the two-photon resonance EIT system can be used as the same robust quantum memory as that under the case of one-photon resonance EIT.

This work is supported by the NSFC and the knowledge Innovation Program (KIP) of the Chinese Academy of Sciences. It is also founded by the National Fundamental Research Program of China with No. 001GB309310.

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[1] D. Bouwmeeste, A. Ekert and A. Zeilinger (Ed.) The Physics of Quantum Information (Springer, Berlin, 2000).
[2] S. E. Harris, Phys. Today 50(7), 36 (1997).
[3] L. V. Hau et al., Nature 397, 594 (1999).
[4] M. M. Kash et al., Phys. Rev. Lett. 82, 5229 (1999).
[5] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature 409, 490 (2001).
[6] D. F. Phillips et al., Phys. Rev. Lett. 86, 783 (2001).
[7] M. O. Scully, M. S. Zubairy, Quantum Optics (Cambridge Univ. Press, Cambridge, 1997).
[8] L. Deng, E. W. Hagley, M. Kozuma, and M. G. Payne, Phys. Rev. A 65, 051805(R) (2002).
[9] M. Kozuma et al., Phys. Rev. A 66, 031801(R) (2002).
[10] M. D. Lukin, Rev. Mod. Phys. 75, 457 (2003).
[11] M. D. Lukin, S. F. Yelin, and M. Fleischhauer, Phys. Rev. Lett. 84, 4232 (2000).
[12] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. 84, 5094 (2000).
[13] A. Andre, L.-M. Duan, and M. D. Lukin, Phys. Rev. Lett. 88, 243602 (2002).
[14] M. Fleischhauer and M. D. Lukin, Phys. Rev. A 65, 022314 (2002).
[15] C. P. Sun, Y. Li, and X. F. Liu, Phys. Rev. Lett. 91, 147903 (2003).
[16] Y. Li and C. P. Sun, quant-ph/0312093.
[17] Y. X. Liu, C. P. Sun, S. X. Yu, and D. L. Zhou, Phys. Rev. A 63, 023802 (2001).
[18] C. P. Sun, Phys. Rev. D 41, 1318 (1990).
[19] A. Zee, Phys. Rev. A 38, 1 (1988).