Quantum quenches and work distributions in ultra-low-density systems

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We present results on quantum quenches in systems with a fixed number of particles in a large region. We show that the typical differences between local and global quenches present in systems with regular thermodynamic limit are lacking in this low-density limit. In particular, we show that in this limit local quenches may not lead to equilibration to the new ground state, and that global quenches can have power-law work distributions ("edge singularities") typically associated with local quenches for finite-density systems. We also show that this regime allows for large edge singularity exponents beyond that allowed by the constraints of the usual thermodynamic limit. This large-exponent singularity has observable consequences in the time evolution, leading to a distinct intermediate power-law regime in time. We demonstrate these results first using local quantum quenches in a low-density Kondo-like system, and additionally through global and local quenches in Bose-Hubbard, Aubry-André, and hard-core boson systems in the low-density regime.

Introduction. Motivated by remarkable experimental progress in realizing and exploring non-equilibrium physics in cold-atom systems [1], there has been increasing interest in the dynamics of thermally isolated systems [2]. Despite the rapidly growing body of research in this class of non-equilibrium dynamics, many aspects are still poorly understood. For example, what type of equilibration can be expected for various types of local and global quenches? Another question involves the overlap distribution, closely related to the work distribution [3–5] for a quantum quench. What is the typical form of the distribution of overlaps of the initial state with the final eigenstates? What are the effects of various overlap distributions and work distributions on dynamical (time-evolving) quantities?

In the experimental settings suitable for exploring non-equilibrium physics, such as cold atoms and semiconductor nanostructures like quantum wells, a common situation is to have a fixed number of particles in a large spatial region. This contrasts sharply with the solid-state notion of the thermodynamic limit, where large regions are filled with a constant density. The study of non-equilibrium issues (e.g., quenches and work distributions) in such situations, where the usual thermodynamic limit is not applicable, is clearly of topical importance but has been near-absent in the non-equilibrium theory literature.

In this work, we focus on this ultra-low-density limit — fixed number of particles, arbitrary large sizes. We present a study of quenches in a system which is the counterpart of the Kondo model in this low-density regime. We present several dynamical aspects which, through calculations in a few other low-density systems, we show to be generic features of quantum quenches in this limit.

One peculiarity of this limit is a blurring of differences between the consequences of local versus global quenches. Another striking result involves the overlap distribution,

$$|\langle 0 | \phi_{in}^{(f)} \rangle|,$$

where $|\Psi(0)\rangle = |\phi_{0}^{(i)}\rangle$ is the initial state (ground state of initial Hamiltonian), and $m$ indexes the eigenstates of the final Hamiltonian. We show that this quantity is dominated by a power-law decay, $\sim m^{-\alpha}$, generically for quenches involving low-density systems. The associated "edge singularity" in the work distribution has large power-law exponents which would not be compatible with the usual thermodynamic limit. This in turn has remarkable consequences on the real-time evolution: in the evolution of observables away from their initial value, there appears an intermediate power-law regime between the initial perturbative time period and the large-time steady-state behavior.

Since the fixed-number large-size limit is applicable to many experimental non-equilibrium setups, these results are expected to be relevant to experimental situations realized or realizable in the near future.

Kondo-like model. The main system we use for demonstrating these general results involves a few ($N_{c}$) mobile fermions ("conduction electrons") in a tight-binding closed chain (Fig.1). One site of the lattice is Kondo-coupled to a single spin-$\frac{1}{2}$ "impurity". The Hamiltonian is

$$H = - \sum_{i,s} (c_{i,s}^\dagger c_{i+1,s} + h.c.) + J \hat{S}_{imp} \cdot \vec{S}_0,$$

where $\hat{S}_{0} = \sum_{s,s'} c_{i,s}^\dagger c_{i,s'}^{s'c_{0,s}}$ is the spin on site $i = 0$ ($s$, $s'$ are spin indices), and $i \in [0, L-1]$ is the site index. We study quenches of $J$, i.e., local quenches, starting from the ground state at $J = J_{f}$ and studying the dynamics after changing $J$ instantaneously to its new value $J_{f}$. The ground state is a
spin singlet, and quenches of $J$ preserve the spin, so that all dynamics is confined to the spin singlet sector.

In Fig. 1 we summarize the equilibrium physics of the $N_c = 1$ system. In an infinite chain, in the ground state, the fermion is localized around the impurity-coupled site ($i = 0$) with localization length $\xi$. $\xi$ decreases with increasing $J_f$. At large $J$ (regime C), the itinerant fermion is almost completely localized at site 0 ($\xi \ll 1$). However, for any finite size $L$, there is a boundary-sensitive small-J regime (regime A) where the fermion cloud extends over the whole system ($\xi \gtrsim L$). For $1 < N_c < L$, we naturally get additional features, but the same general behavior persists in the three regimes.

Observables. We will present time dependences of the occupancy $n_0(t)$ of site $i = 0$ for the Kondo-like system, and of the Loschmidt echo $\mathcal{L}(t) = \langle \Psi(0) | \Psi(t) \rangle^2$. The observable $n_0(t)$ is of obvious importance for the model [7], while $\mathcal{L}(t)$ is well-defined for any model and is closely related to the work distribution [8,9]. Despite the nonlocal nature of the Loschmidt echo, there exist proposals for experimentally measuring this quantity, and related quantities have been measured [6].

Lack of equilibration to new ground state in local quenches. The final value at which an observable $\mathcal{O}$ saturates is given by $\langle \mathcal{O} \rangle_{DE} = \sum_m |\langle \Psi(0) | \phi_m(t) \rangle|^2 |\langle \phi_m(0) | \phi_m \rangle|$, the so-called “diagonal ensemble” (DE) value [7]. In Fig. 2(a) we show the time dependence of $n_0(t)$ after a quench within the C region. We note that $n_0(t)$ reaches the DE value $\langle n_0 \rangle_{DE}$ relatively rapidly, and then shows ‘revivals’ at roughly periodic intervals of $t \sim L/2$. The DE value where $n_0(t)$ saturates is markedly different from the ground state value of $n_0$ for $J = J_f$. This seemingly contradicts the intuition that a local quench in a large system should lead to relaxation to the final ground state value, because the energy pumped into the system by a local quench is a $O(L^{-1})$ effect. The reason this does not happen in the C→C quenches is that the itinerant electron only occupies a small number of sites near the impurity position. Thus, most of the lattice sites do not play any role in the dynamics, and cannot serve as a bath to absorb the disturbance at site 0. This effect is not restricted to $N_c = 1$, but is true for finite number $N_c > 1$ of fermions for $L \rightarrow \infty$ [8].

In Fig. 2(b) we show $\langle n_0 \rangle_{DE}$ as a function of $J_f$ for fixed $J_f$. The $\langle n_0 \rangle_{DE}$ values deviate significantly from the $J = J_f$ equilibrium values for most $J_f$, $J_f$ combinations. Fig. 2(c) shows, through $L$-dependences of $\langle n_0 \rangle_{DE}$, that the lack of equilibration in quenches to C or B regions is not a finite-$L$ effect.

This effect represents a loss of the distinction between local and global quenches, which is a generic feature of the $L \rightarrow \infty$ limit with finite particle number.

Overlap Distributions. Fig. 3(a-d) summarize overlap distribution behaviors in quantum quenches between different regimes of the system [11] for $N_c = 1$. These behaviors can be derived from detailed consideration of the eigenfunctions [8].

In C→C quenches, the ground state overlap $\langle \phi_0^{(i)} | \phi_m^{(f)} \rangle$ is much larger than the others, while the small $m \neq 0$ overlaps have the form $\propto \sin(\pi m/L)$ [8]. The most remarkable feature is the power-law behavior, $\langle \phi_0^{(i)} | \phi_m^{(f)} \rangle \sim m^{-\alpha}$, in quenches starting from or ending in the A region. The exponent $\alpha$ is 2 for A→A quenches and 1 for A→C quenches.
These power law behaviors are a generic phenomenon; we have found such power-law overlap distributions in several other systems in the low-density limit, both for local and global quenches. (The behavior is particularly clean for the \( N_c = 1 \) system because of its simplicity.) Fig. 3(e) shows the overlap distribution for the same model with \( N_c = 3 \) fermions. There are now additional structures, but the dominant overlaps follow a clear power law. Fig. 3(e) shows the overlap distribution for a Bose-Hubbard chain at low density \(^8\). Again, there are interesting additional structures, but the dominant overlaps follow a clear power law (\( \sim m^{-\alpha} \)).

**Work distribution.** The overlap distribution is related to

\[
p(\omega) = \sum_m \delta(\omega - \epsilon_m) \left| \langle \phi_0^{(i)} | \phi(f) \rangle \right|^2
\]

where \( \epsilon_m = E_m^{(f)} - E_0^{(f)} \) are the final eigenenergies measured from the final ground state energy. This is the so-called work distribution \(^3\), except for a shift between \( \omega \) and the usual work variable. (The energy prior to the quench plays no role in the temporal dynamics and so is not relevant for this work.) The work distribution is related to the Loschmidt echo: \( \mathcal{L}(t) = |\int d\omega p(\omega) e^{i\omega t}|^2 \). Since \( \mathcal{L}(0) = 1 \) by definition, \( p(\omega) \) must be normalized.

At large sizes (but constant particle number), \( p(\omega) \) can be treated as a continuous function starting from \( \omega = \Delta \), the finite-size gap, which vanishes at large \( L \). We have found that, for quantum quenches in low density systems, the work distribution generically has behavior \( p(\omega) \sim p_0 \omega^{-b} \) for \( \omega > \Delta \), with large exponents \( b > 1 \). These power-law divergences are analogs of what would be called “X-ray edge singularities” in systems with a regular thermodynamic limit. In finite-number systems, \( p(\omega) \) remains normalized despite the singularity as \( \Delta \to 0 \) because the magnitude of \( p(\omega) \) also vanishes \( (p_0 \to 0) \) in the large-size limit, due to the vanishing density.

This contrasts sharply to systems with the usual thermodynamic limit where density remains constant as \( L \to \infty \), and \( p(\omega) \) itself is a well-defined non-vanishing quantity in the limit. This constrains the singularity \( p(\omega) \sim \omega^{-b} \) to have smaller exponent, \( b < 1 \) (e.g., \( \omega^2 \)). The low-density systems of interest here have no such constraint; a central result of the present work is that super-linear singularities \( (b > 1) \) are signatures of low-density systems.

For the model \(^1\) with \( N_c = 1 \), \( p(\omega) \sim \omega^{-5/2} \) (\( A \to A \)) and \( p(\omega) \sim \omega^{-3/2} \) (\( A \to C \)). \((b > 1 \) in both cases.) Fig. 3(f) shows the work distribution for a global interaction quench in the Bose-Hubbard chain, with the delta function regularized as gaussian. There is a power law with super-linear \( (b > 1) \) singularity. This is another example of the loss of distinction between global and local quenches in the low-density limit, as “edge singularities” are normally associated only with local quenches for finite-density systems \(^4\).

We have also found super-linear singularity exponents in other low-density systems \(^8\), e.g., quenches of the strength/position of a weak trapping potential for a Bose-Hubbard system, quenches of on-site potentials and hopping strengths for hard-core bosons in a ladder geometry, and quenches of quasi-disorder potential strengths in an Aubry-André \(^9\) system.

**Role of the density of states.** For the model \(^1\) with \( N_c = 1 \), the behavior \( \sim m^{-\alpha} \) implies energy-dependence \( \sim \omega^{-\alpha/2} \) for the overlap distribution. Together with a factor of \( \omega^{-1/2} \) from the 1D single-particle density of states, this leads to \( p(\omega) \sim \omega^{-\alpha-1/2} \), i.e., \( b = 5/2(3/2) \) for \( A \to A(C) \) quenches.

This argument can be generalized: if the overlap distribution follows \( m^{-\alpha} \) and the density of states in the relevant lower-energy part of the spectrum behaves as \( \rho(\omega) \sim \omega^\gamma \), the work distribution \( p(\omega) \sim \omega^{-b} \) will have exponent \( b = 2\gamma + 2\alpha \). For single-particle systems, we have \( \gamma = -1/2 \) in 1D, as in the above example. For a generic system, however, the many-body density of states does not necessarily behave as a power law. We have found cases (Bose-Hubbard chain with trap) where an approximate power-law region with exponent \( \gamma \) in \( p(\omega) \) leads to an approximate power law in \( p(\omega) \) with exponent \( b = 2\gamma + 2\alpha \). Also, if \( \alpha = 1/2 \), any power-law form of \( p(\omega) \) implies a linear edge singularity \( p(\omega) \sim \omega^{-1} \). In this case, a super-linear edge singularity can only happen with some non-power-law form of \( p(\omega) \). This occurs in the Bose-Hubbard chain case of Fig. 3(e.f) \(^8\).

**The intermediate-time \( \sim t^3 \) region.** The appearance of larger powers in the edge singularity has novel consequences for real-time dynamics. We have identified an intermediate-time power-law region in the dynamics of the Loschmidt echo (and other observables), that appears as a direct consequence of the large-power edge singularity.

At initial times after a quench, observables and \( \mathcal{L}(t) \) evolve away from their initial value quadratically with time, \( \sim t^2 \), as can be explained from generic perturbative arguments. We have found that, when \( p(\omega) \) has a large-exponent singularity, there is a region of time (after the initial perturbative times and before the large-time steady-state oscillations), where \( \mathcal{L}(0) - \mathcal{L}(t) = 1 - \mathcal{L}(t) \) follows a new power-law behavior. If \( p(\omega) \sim \omega^{-b} \) with \( b \in (1, 3) \) in the energy range \( [\Delta, \Lambda] \) and the
contributions outside this energy window can be neglected, then in the time window between \( t \sim \Lambda^{-1} \) and \( t \sim \Delta^{-1} \) one sees the behavior \( I - \mathcal{L}(t) \sim t^{b-1} \).

The same phenomenon is also found in some observables: \(|\hat{O}(t) - \hat{O}(0)|\) can have an extended region after the initial perturbative \( t^{\delta_0} \) region with a new exponent \( t^{b_0} \). When \( \hat{O} \) has the form of a rank-1 projector, \( \hat{O} = |\chi\rangle\langle\chi| \), we can write the time evolution as \( \hat{O}(0) = \int d\omega p_\omega(\omega) e^{i\omega t/\hbar} \), where

\[
p_\omega(\omega) = \sum_m \delta(\omega - \epsilon_m) \langle \phi_{m}^{(l)} | \phi_{n}^{(f)} \rangle \langle \phi_{n}^{(f)} | \chi \rangle
\]

differs from the work distribution \([2]\) in that one factor of the overlap is replaced by \( \langle \phi_{m}^{(l)} | \chi \rangle \). If \( p_\omega(\omega) \) has a power-law singularity structure \( \omega^{-\delta_0} \), the time evolution of \( \hat{O}(t) \) away from \( \hat{O}(0) \) will show the intermediate-time region \( \sim t^{b_0} \), with \( \delta_0 = b_0 - 1 \). When the operator \( \hat{O} \) does not have the form \( \hat{O} = |\chi\rangle\langle\chi| \), it is not simple to formulate an analogous expression. A generic operator for a many-body system will not have this form, but the site occupancies for systems with single itinerant particles (e.g., \( n_j \) for our \( N_c = 1 \) system) have the forms of rank-1 projectors, as does the Loschmidt echo for any system. Currently, little is known about \( p_\omega(\omega) \) behaviors for projector-type observables in different quantum quenches, or about the conditions necessary for having an intermediate-time regime in generic observables.

The intermediate-time regime for low-density systems is illustrated in Fig. \(4\). For the model \([1]\) with \( N_c = 1 \) particle, this regime is present in the Loschmidt echo, for both \( A \rightarrow A \) and \( A \rightarrow C \) quenches. In the occupancy \( n_0(t) \), the intermediate-exponent regime can be seen for \( A \rightarrow A \) quenches (with form \( \sim t^{1/2} \)), but not for the \( A \rightarrow C \) quenches, for which case the eigenstate dependence of \( \langle \phi_{m}^{(f)} | \chi \rangle \) does not favor a large enough exponent in \( p_{n_0}(\omega) \) \([8]\). Fig. \(4\)g displays the intermediate-time region for a Bose-Hubbard chain with interaction quenches, and Fig. \(4\)h shows the same for the Kondo-like model with \( N_c = 3 \) itinerant fermions.

With hard-core bosons on a ladder-shaped lattice, considering time evolution after various local and global quenches of hopping strengths and on-site potentials, we find extended intermediate-time regions \( \sim t^{b} \) in \( 1 - \mathcal{L}(t) \), with exponents matching \( \beta = b - 1 \) where \( b \) is the singularity exponent in \( p(\omega) \), calculated with gaussian regularization \([8]\). With quenches of a trapping potential, we find quench parameter combinations where \( p(\omega) \) shows super-linear edge singularities \( (b > 1) \) but no intermediate-time regime shows up in the \( \mathcal{L}(t) \) dynamics because the singularity exponents are too large, \( b > 3 \) \([8]\). We have also found an example (Aubry-André system) where there are well-defined \( p(\omega) \sim \omega^{-b} \) regions but the contributions from outside the power-law region are so large that the dynamical intermediate-time signature is washed out \([8]\).

**Extent of the intermediate-time region.** If the power-law window for \( p(\omega) \) is \( \omega \in [\Lambda, \Lambda] \), the \( \sim t^\beta \) region with \( \beta \in (0,2) \) extends from \( t \sim \Lambda^{-1} \) to \( t \sim \Delta^{-1} \). The scale \( \Lambda \) is generally of the order of the bandwidth, and so is set by the hopping strength. Since the finite-size gap \( \Delta \) vanishes with increasing system size \( L \), the intermediate-time region gets more and more extended in time for larger \( L \). This is shown in Fig. \(4\)g through a comparison of two different \( L \) values.

**Discussion.** For systems that are not well-described by the traditional thermodynamic limit but instead have a fixed number of particles in a large size, as is common in setups relevant for large classes of non-equilibrium experiments, we have presented universal features of quantum quenches. These include edge singularities with large exponents not possible in ‘regular-limit’ systems, a loss of the usual distinctions between local and global quenches, and a novel intermediate-time region in the dynamics.

Universal behaviors in quantum quenches are generally sought and discussed in asymptotic times. A new universality at intermediate times, visible in widely different systems, is of obvious distinction and interest.

Our results open up various questions and research avenues. One issue is to bridge the gap between the regime considered here and the regular thermodynamic limit. A systematic study of overlap/work distributions and associated quench dynamics with varying density and system sizes is currently lacking. A related issue is that of experimental accessibility. Since a measurement requires finite density, it is important to demarcate which densities in a real finite-size system show features of our fixed-number large-size limit, from those corresponding to the regular thermodynamic limit. It may also be interesting to supplement our results on the Loschmidt echo with time-evolution studies of traditionally measurable observables such as densities and correlation functions. Finally, Eq. \([6]\) highlights the general lack of knowledge about expectation values of observables in (and overlaps of states with) general eigenstates of many-body Hamiltonians. Current activity on the Eigenstate Thermalization Hypothesis is addressing eigenstate expectation values of some observables \([7,10]\), but clearly further investigations are warranted.

**Acknowledgments.** MH thanks M. Vojta for discussions on equilibrium properties of the Hamiltonian \([1]\).

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In these Supplementary Materials,

- We provide a derivation of the existence of an extended intermediate-time power-law region in the Loschmidt echo \( \mathcal{E}(t) \), when the work distribution \( p(\omega) \) has a power-law behavior \( \omega^{-b} \) with exponent \( b \in (1, 3) \). (Section S.II)

- Considering the case where the density of states has power-law behavior, we derive a relation between various exponents. (Section S.III)

- We give some details for quantum quenches in four low-density systems. (Interaction quench in Bose Hubbard chain, trap quench in Bose Hubbard chain, several quenches in hard-core bosons on a ladder geometry, and quasiperiodic potential quench in an Aubry-André lattice.) We show various examples of superlinear \( p(\omega) \sim \omega^{-b} \) behaviors, and examples of the intermediate-time power-law region in \( 1 - \mathcal{E}(t) \). (Section S.IV)

- We show time evolution for the Kondo-like model with \( N_c = 3 \) fermions, to show that there is no equilibration. (Section S.V) (Only \( N_c = 1 \) is shown in the main text.)

- We provide additional details and derivations for local quenches in the Kondo-like model for \( N_c = 1 \). (Section S.VI)

S.II. DERIVATION OF THE INTERMEDIATE REGIME FOR \( \mathcal{E}(t) \)

In this Section, we prove that an edge singularity \( p(\omega) \sim \omega^{-b} \) with \( b \in (1, 3) \) leads to the intermediate time behavior \( \sim t^{b-1} \) for \( \mathcal{E}(0) - \mathcal{E}(t) \), provided that contributions of parts of \( p(\omega) \) outside this power-law region can be neglected.

We consider \( p(\omega) \) to have power-law behavior \( p(\omega) \sim \omega^{-b} \) in the energy window \( \omega \in [\Delta, \Lambda] \). Here \( \Delta \) is an energy scale of the order of the finite-size gap, which should vanish in the \( L \to \infty \) limit. The energy \( \Lambda \) up to which the power law holds is system-dependent, generally of the order of the bandwidth. There are two time scales \( \Lambda^{-1} \) and \( \Delta^{-1} \) corresponding to these energy scales. We show below that, when the exponent \( b \) in the appropriate range, \( b \in (1, 3) \), the novel intermediate-time regime \( 1 - \mathcal{E}(t) \sim t^{b-1} \) appears in the time window between these two timescales, when contributions from \( \omega \notin [\Delta, \Lambda] \) are neglected.

The Loschmidt echo \( \mathcal{E}(t) \) is related to the Fourier transform of the work distribution:

\[
G(t) = \int p(\omega)e^{it\omega}d\omega \quad \text{(S.1)}
\]

as \( \mathcal{E}(t) = G^*(t)G(t) \).
Neglecting the contributions from energies outside the power-law region, we obtain
\[
G(t) \approx \int_{\Delta}^{\Lambda} e^{-i t \omega} d\omega = \Lambda^{1-b} \mathcal{E}_b(-it\Lambda) - \Delta^{1-b} \mathcal{E}_b(-it\Delta),
\] (S.2)
where \( \mathcal{E}_b(z) = \int_{-\infty}^{\infty} e^{-i z \omega} d\omega \) is the generalized exponential integral \((b, z \in \mathbb{C})\). When \( 1 - \mathcal{L}(t) \) is plotted using the above expression (S.2), we see a clear intermediate-time \( \sim t^{b-1} \) region between a \( \sim t^2 \) and a \( \sim t^3 \), as shown in Fig. S1 for \( b = 2.5 \).

Fig. S1. Time evolution of the Loschmidt echo \( \mathcal{L}(t) = |G(t)G^*(t)| \) obtained from expression (S.2) with \( b = 2.5, \Delta = 10^{-4}, \Lambda = 4. \)

Below we derive the behaviors of \( \mathcal{L}(t) \) at short and intermediate times using expansions of the exponential integral \( \mathcal{E}_b(z) \).

S.II.-1. Short time regime.

When \( t\Lambda \ll 1 \), both terms in (S.2) can be expanded in Taylor series:
\[
\Delta^{1-b} \mathcal{E}_b(-it\Lambda) = (-it)^{b-1} \Gamma(1-b) + \frac{\Delta^{1-b}}{b-1} + \frac{\Delta^{2-b}}{b-2} t - \frac{\Delta^{3-b}}{2(b-3)} t^2 + O(t^3),
\] (S.3)
and
\[
\Lambda^{1-b} \mathcal{E}_b(-it\Lambda) = (-it)^{b-1} \Gamma(1-b) + \frac{\Lambda^{1-b}}{b-1} + \frac{\Lambda^{2-b}}{b-2} t - \frac{\Lambda^{3-b}}{2(b-3)} t^2 + O(t^3).
\] (S.4)

Using (S.3), (S.4) in (S.2), and assuming \( b \in (1,3) \) and \( \Lambda^{-1} \ll \Delta^{-1} \), we get
\[
\mathcal{L}(t) = |G(t)G^*(t)| = 1 + \frac{(b-1)}{(b-3)} \Delta^{b-1} \Lambda^{3-b} t^2 + O(t^3).
\] (S.5)

Here \( G(t) \) was normalized using the criterion \( \mathcal{L}(0) = 1 \).

S.II.-2. Intermediate-time regime.

In the intermediate time interval \( \Lambda^{-1} \ll t \ll \Delta^{-1} \) the expression for \( \Lambda^{1-b} \mathcal{E}_b(-it\Delta) \) is still given by the Taylor expansion (S.3), but we cannot use (S.4) because \( t\Lambda \) is not small. Instead we use the large-time asymptotics for \( \mathcal{E}_b(-it\Lambda) \):
\[
\Lambda^{1-b} \mathcal{E}_b(-it\Lambda) = e^{it\Lambda} \left( \frac{i\Lambda^{1-b}}{t} + O(1) \right).
\] (S.6)

Case 1: \( 1 < b < 3 \). When \( b \) is in this range, using the expression (S.3) for small \( t\Delta \) and (S.6) for large \( t\Lambda \), we obtain
\[
G(t) \approx -\frac{\Delta^{1-b}}{b-1} - \Gamma(1-b) \cos \left[ \frac{\pi}{2} (1-b) \right] t^{b-1} - \frac{i \Delta^{2-b}}{b-2} t - i \Gamma(1-b) \sin \left[ \frac{\pi}{2} (1-b) \right] t^{b-1}.
\] (S.7)

As before, \( G(t) \) has to be normalized such that \( \mathcal{L}(0) = 1 \). Then the Loschmidt echo at leading order is
\[
\mathcal{L}(t) = 1 + 2(1-b) \Gamma(1-b) \cos \left[ \frac{\pi}{2} (1-b) \right] \Delta^{b-1} t^{b-1}.
\] (S.8)

Case 2: \( b \geq 3 \). In this case the leading term is \( (t\Delta)^{b-1} \) rather than \( (t\Delta)^2 \):
\[
G(t) \approx -\frac{\Delta^{1-b}}{b-1} - \frac{\Delta^{3-b}}{2(b-3)} t^2 - \frac{b-1}{2(b-3)} t - \frac{\Delta^{2-b}}{b-2} t.
\] (S.9)

This gives, for the Loschmidt echo at leading order,
\[
\mathcal{L}(t) = 1 - \frac{b-1}{(b-3)(b-2)} \Delta^{2} t^2.
\] (S.10)

Therefore there is no observable intermediate-time region with exponent different from 2.

S.III. Relationship between power-law exponents in overlap distribution, work distribution, and density of states

In cases where the density of states has a power-law behavior, \( p(\omega) \sim \omega^\gamma \), the eigenstate index scales with the energy (relative to ground state) as
\[
m \sim \int_{-\infty}^\omega d\omega' p(\omega') \sim \omega^{\gamma+1}.
\] (S.11)

Therefore for an overlap distribution with \( m^{-\alpha} \) behavior we will have the dependence
\[
\left| \langle \phi_{ij}^0 | \phi_{im}^\beta \rangle \right|^2 \sim \omega^{-2\alpha(\gamma+1)}.
\] (S.12)

The definition of the work distribution \( p(\omega) \) contains a factor of the density of states in addition to this overlap squared; thus the exponent \( b \) of the edge singularity \( p(\omega) \sim \omega^{-b} \) is
\[
b = 2\gamma \alpha + 2\alpha - \gamma.
\] (S.13)
In the case of the \( N_c = 1 \) Kondo-like model in the main text, we had \( \gamma = -1/2 \) (single-particle density of states in one dimension), and \( \alpha = 2 \) (1) for an A to A (C) quench. This leads to super-linear singularities in the work distribution: \( b = \alpha + \frac{1}{2} = 5/2 \) (3/2) for the two quench regimes.

A consequence of (S.13) is that, if the exponent for the overlap happens to be \( \alpha = 1/2 \), the work distribution \( \rho(\omega) \) will have linear singularity (\( b = 1 \)) for any power-law form of the density of states.

Note that, in a generic multi-particle situation, we have no \emph{a priori} reason to expect a clean power-law behavior of \( \rho(\omega) \) at small \( \omega \). In Section S.IVA, we will see a case where, even though \( \alpha = 1/2 \), it can combine with a \( \rho(\omega) \) with some more complicated behavior to give a \( \rho(\omega) \) with super-linear singularity (\( b > 1 \)).

**S.IV. VARIOUS MODELS: SUPER-LINEAR EDGE SINGULARITIES AND INTERMEDIATE-TIME REGIME**

In this section, we present numerical results on local and global quenches in several different systems in the low-density limit. This sampling of low-density systems shows that power-law behaviors in the overlap distribution, and super-linear power-law behaviors in the work distribution \( \{ \rho(\omega) \sim \omega^{-b} \) with \( b > 1 \), are generic in this important limit. The signature in real-time dynamics (an extended intermediate-time power-law regime), requires the additional condition \( b < 3 \) and that contributions outside a single power-law regime in \( \rho(\omega) \) can be neglected. Therefore, the intermediate-time regime appears in some but not all cases.

In S.IVA we describe results for a global quench of the interaction strength, for a Bose-Hubbard chain in the dilute limit. In S.IVB we consider Bose-Hubbard systems in a weak harmonic trap, and consider quenches of the trap strength and the trap position. In S.IVC we present results for a dilute system of hard-core bosons in a ladder geometry, considering both local and global quenches. Finally, in S.IVD we consider a single particle in a quasi-periodic ( Aubry-André) potential, and present results for quenches of the strength of the quasi-periodic ( ‘disorder’) potential. For the systems of S.IVC and S.IVD we also present some examination of the effect of increasing system size.

**S.IVA. Bose Hubbard, interaction quench**

We consider the celebrated Bose-Hubbard Hamiltonian

\[
H_{BH} = - \sum_{j=1}^{L-1} b_j^\dagger b_{j+1} + U \sum_{j=1}^{L} n_j (n_j - 1) \tag{S.14}
\]

in one dimension. Here \( b_j, b_j^\dagger \) are bosonic operators for site \( j \), and \( n_j = b_j^\dagger b_j \) are site occupancies. Energies (times) are measured in units of the hopping (inverse hopping) strength. We use open boundary conditions and consider global quenches of the interaction parameter \( U \), form \( U_i \) to \( U_f \).

Figure S2 shows numerical results for \( N_b = 3 \) bosons in \( L = 30 \) sites, which we expect to represent the “fixed \( N_b \) in large \( L \)” limit. We show results for initial and final values of \( U \) in both the small \( \langle U \rangle \lesssim 3 \) and large \( \langle U \rangle \gtrsim 3 \) regimes. The phenomena of large-exponent power-law singularity in the work distribution and intermediate region in the time evolution of \( Z(t) \) appear for both large and small quenches, and for final parameter in either the large-\( U \) or the small-\( U \) regime.

The overlap magnitude \( \langle \phi_0^{(i)} | \phi_{(f)}^{(f)} \rangle \), plotted in the top panels, naturally shows more scatter and more structures compared to the \( N_c = 1 \) Kondo-like model treated in the main text. However, in each case there is a dominant set of data points which behave as \( \sim \sqrt{m^{-1/2}} \). In the large \( U \) cases, where there are several “bands”, there are also large contributions from higher bands. The extra features (the large-energy contributions, and the contribution of the many low-energy states for which the overlap is nonzero but falls below the dominant \( m^{-1/2} \) states) will of course leave signatures in the temporal dynamics. However, our signature phenomenon (extended intermediate regime between perturbative and final regimes) appears in all these cases.

In the second and third rows, we show continuous approximations for the density of states \( \rho(\omega) \) for the final \( U = U_f \), and the work distribution \( \rho(\omega) \). This is done by replacing the delta functions by gaussians of energy width \( \sigma \),

\[
\delta(\omega - \epsilon_m) \longrightarrow \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(\omega - \epsilon_m)^2}{2\sigma^2} \right], \tag{S.15}
\]

in the definitions

\[
\rho(\omega) = \sum_m \delta(\omega - \epsilon_m) \tag{S.16}
\]

\[
\rho(\omega) = \sum_m \delta(\omega - \epsilon_m) \left| \langle \phi_0^{(i)} | \phi_f^{(f)} \rangle \right|^2. \tag{S.17}
\]

Here \( \epsilon_m = E_m^{(f)} - E_0^{(f)} \) are the final eigenenergies measured from the final ground state energy.

In Figure S2 we show continuous curves with \( \sigma = L^{-1} \) for \( \rho(\omega) \) and with \( \sigma = 3L^{-1} \) for \( \rho(\omega) \). The choice of \( \sigma \) is a compromise for visualization; with smaller \( \sigma \) one sees more oscillatory behavior associated with the discreteness of the spectrum, while choosing larger \( \sigma \) washes out features near the beginning of the spectrum.

The density of states \( \rho(\omega) \) has a maximum in the central region of each band. For present purposes, the relevant part of the spectrum is where \( \rho(\omega) \) is increasing toward its first maximum; this is naturally highlighted in the log-log plots of the second row.

In the smoothed work distributions (third row), the region \( \omega \lesssim \sigma \) shows a broad plateau which is an artifact of our smoothing procedure, and looks artificially broad on a logarithmic scale. We have mostly omitted this part from the plot range. After this part (\( \omega \gtrsim \sigma \)), there is an extended spectral region where \( \rho(\omega) \) follows an approximate power law. The
exponent $b$ of this “edge singularity” $p(\omega) \sim \omega^{-b}$ is super-linear, with $b \approx 2$ for small $U_f$ and $b$ slightly smaller for large $U_f$.

The approximate power-law behavior of $p(\omega)$ emerges in a more complicated manner than in the $N_e = 1$ system of the main text. In this case, if the relevant part of the density of states followed a power law $\rho(\omega) \sim \omega^{\alpha}$, it would not be possible to have a super-linear edge singularity because $\alpha = 1/2$ would imply $b = 1$ for any $\gamma$ (see Section S.III). Remarkably, the non-powerlaw form of $p(\omega)$ conspires with the $m^{-1/2}$ overlap distribution to cause an approximate power law in $p(\omega)$, with exponent $b > 1$. At present it is not clear whether the power-law behavior of $p(\omega)$, or the exponent values, becomes exact in some limit.

The $p(\omega) \sim \omega^{-b}$ behavior results in the dynamical feature that we have highlighted in this work: in the bottom row, we see clear intermediate regimes between the perturbative and the steady-state regimes. The behavior in this regime is approximately $\sim t^{2\beta}$, with $\beta \approx 1$ for small $U_f$ and $\beta$ slightly smaller for large $U_f$. This is consistent with our prediction of $\beta = b - 1$ (Section S.III). Note that, in the large $U_f$ cases, there are oscillations in the $\sim t^{b_2}$ region, which result from the contribution of the higher bands.

It would be interesting to find out if an extended intermediate power-law region is visible in the time evolution of observables that are more commonly studied (or more likely to be experimentally measurable) than the Loschmidt echo.

As the density (filling) is increased, the features we have presented gradually disappear. Fig. S3 shows the case of unit filling (8 bosons in 8 sites). The density of states and work distributions are significantly different; we do not analyze them here.

Curiously, in weak quenches between small values of $U$ (leftmost panels), there is a sequence of dominant overlaps that seems to follow a $m^{-1/2}$ behavior, and a small intermediate-time region does appear in $1 - L(t)$. In the corresponding work distribution (not shown), there is no obvious $p(\omega) \sim \omega^{-b}$ behavior. It is somewhat surprising that the intermediate-time regime appears at unit filling. To explain this, one could consider two different types of $L \to \infty$ limits. Presumably, the small intermediate-time region will disappear in the large-$L$ limit taken with filling held constant (usual thermodynamic limit), but would get more extended in the large-$L$ limit taken with fixed $N_h = 8$. A systematic study of the

FIG. S2. $N_e = 3$ bosons in $L = 30$ sites. The interaction $U$ is quenched; the $(U_i, U_f)$ pair is indicated on top of each column. Top row: overlap distributions. The dashed line in second panel is $\sim m^{-1/2}$; the dominant overlaps follow same exponent in every case. For large $U_f$ (middle two panels) significant contributions are visible from higher bands. Second row: many-body density of states $\rho(\omega)$ at $U = U_f$. Insets show same with frequencies in linear scale. At large $U_f$, there are several bands. Third row: Work distributions $p(\omega)$. Dashed lines are $\sim b^{-3/2}$ and $\sim b^{-2}$. Approximate power-law behavior of $p(\omega)$ clearly extends into $\omega$ values near the top of the (first) band, where $\rho(\omega)$ definitely cannot be approximated with a power-law. For large $U_f$ (middle two panels), the contributions from higher bands are not visible in the energy range shown. Bottom row: Evolution of Loschmidt echo away from its initial value. A region with intermediate exponent between the initial $\sim t^3$ and the final oscillatory $\sim t^b$ is visible in all cases.
\( \text{FIG. S3.} \) \( N_b = 8 \) bosons in \( L = 8 \) sites. The interaction \( U \) is quenched; the \((U_i, U_f)\) pair is indicated on top of each column. Top row: overlap distributions. The dashed line in first panel is \( \sim m^{-1/2} \). There seems to be a power-law behavior in this weak quench, but not in any of the other quenches. Bottom row: Loschmidt echo. A small intermediate-exponent region is seen in the weak-quench between small \( U \) values, but not in any of the other quenches.

shape of the overlap distribution, work distribution and density of states as a function of filling and size is clearly called for, but is beyond the scope of this work. Some \( L \)-dependence is explored in Sections S.IV.C and S.IV.D.

No intermediate-time regime is seen in the dynamics for the other quenches in Fig. S3; nor is there any power law like behavior in the overlap distributions. This is expected because the super-linear edge-singularities and the intermediate-time regime in \( L(t) \) are novel features of low-density systems.

**S.IV.B. Bose Hubbard in harmonic trap; trap quench**

We now consider bosons on an open-boundary chain subject to a harmonic confining trap in addition to the Bose-Hubbard Hamiltonian (S.14):

\[
H_{BH+\text{trap}} = H_{BH} + \frac{1}{2} k_{\text{tr}} \sum_{j=1}^{L} (j - j_0)^2 n_j \quad (S.18)
\]

Harmonic traps are fundamental to considerations of cold-atom experiments, the most prominent experimental setting for non-equilibrium dynamics in isolated systems.

We consider quenches of both the trap strength \( k_{\text{tr}} \) and the trap center \( j_0 \). The Hamiltonian (S.18) has various regimes of possible interest, such as large and small interaction \( U \), strong and weak trapping potential, etc. For present purposes, we will confine ourselves to small \( U \) and weak trapping, \( k_{\text{tr}} \sim \mathcal{O}(10^{-3}) \). The weak and strong trapping regimes are loosely analogous to the A and C regimes of the Kondo-like model detailed in the main text.

In Figure S4, we show data for a trap strength quench \( (k_{\text{tr}}^{(i)} = 0.001 \text{ to } k_{\text{tr}}^{(f)} = 0.0012) \) and for a trap position strength quench \( (j_0^{(i)} = 7.3 \text{ to } j_0^{(f)} = 8.3) \). In both cases the overlap distribution \( \left| \langle \phi_0^{(i)}|\phi_m^{(f)} \rangle \right| \) shows a dominant series which follows an approximate power law; the exponent \( \alpha \) is between 3/2 or 2. As in the case without a trap (Section S.IV.A), we show approximations to the density of states \( \rho(\omega) \) obtained by replacing delta functions with gaussians.

The density of states is similar to the case without a trap.
but the small-$\omega$ behavior of $\rho(\omega)$ seems closer to a power-law form in the presence of a trap. This exponent ($\gamma$) is close to 1. The argument of Section S.III then predicts the work distribution $p(\omega) \sim \omega^{-b}$ with $b = 5$ (for $\alpha = 3/2$) or $b = 7$ (for $\alpha = 2$). Indeed, $p(\omega)$ does have an approximate power-law decrease with exponent around the range 5 to 7, in the energy window where $p(\omega)$ increases roughly linearly. It is difficult to be more certain about the exact values of the exponents, because of the scatter in the overlap distribution and because of the uncertainties associated with replacing delta components, because of the scatter in the overlap distribution and difficult to be more certain about the exact values of the exponents, the relation (S.II) is at least approximately valid. Despite the difficulties with rigorous determination of exponents, the relation (S.III) is at least approximately valid.

In this low-density system and these types of quenches, we thus have edge singularities in $p(\omega)$ with exponents far larger than that allowed in systems having the usual thermodynamic limit. However, as explained in Section S.II an exponent in $p(\omega) \sim \omega^{-b}$ with $b > 3$ does not lead to a distinguishable intermediate-time region in the evolution of the Loschmidt echo. Indeed the bottom panels of Figure S4 show the $-t^2$ region directly followed by the oscillatory region. The singularity exponent is too large to see an intermediate-time signature in $L(t)$. Of course, an extended intermediate region might be visible in some observable other than the Loschmidt echo. An exploration of the evolution of physical observables in such quenches remains an open task for future work.

S.IV.C. Hard-core bosons in a ladder

We next consider a system of $N_b$ hard-core bosons on a ladder of $N_v$ vertices with vertex-dependent on-site potentials $V_i$ and bond-dependent hopping $J_{ij}$:

$$H_{HCB} = \sum_{\langle i,j \rangle} J_{ij} b_i^\dagger b_j + \sum_i V_i b_i^\dagger b_i$$  \hspace{1cm} (S.19)

where $b_i^\dagger$ and $b_i$ are hard-core bosonic operators for site $i$ with $[b_i, b_j^\dagger] = 1$ and $[b_i^\dagger, b_j^\dagger] = 0$. One leg of the ladder is taken to have one site more than the other, in order to avoid spurious symmetries.

In the initial Hamiltonian, the hopping terms are set to $J_{ij} = -1$ and the on-site potentials are set to $V_i = 0$. At $t = 0$ some of the hopping terms or on-site potentials are changed abruptly according to the color coding of Fig. S5(a) where $V_i = 0$ is colored gray and $J_{ij} = -1$ yellow. We consider a local quench of a rung hopping strength ($J_{ij}$) (first column of Fig. S5), a global quench of the rung hoppings (second column), and a local quench of an on-site potential $V_i$ (third column).

Fig. S5(b) shows the overlaps $|\langle \phi_m^{(1)} | \phi_m^{(1)} \rangle|$ as a function of the eigenenergies $\varepsilon_m$. The overlaps have a maximum magnitude at $m = 0$ for all the quenches considered. The density of states $\rho(\omega)$ of the final Hamiltonian, given in Fig. S5(c), was obtained by Gaussian regularization of the delta function (as in Sections S.IVA and S.IVB) with width $\sigma = 0.1$. For the system sizes considered here, $\rho(\omega)$ is noisy near the band edges as there are not enough states in this regions to obtain a smooth curve. The same phenomenon appears in the work distribution $p(\omega)$ shown in Fig. S5(d). This quantity is sharply peaked around the bottom of the spectrum. The power-law lines in Fig. S5(d) are only guides to the eye.

Even though it is difficult to assign unambiguous power-law exponents for $p(\omega)$, the existence of an intermediate-time regime in the temporal dynamics is clear, as seen in Fig. S5(e) for $1 - L(t)$. The exponents in $p(\omega) \sim \omega^{-b}$, Fig. S5(d), are consistent with the exponents for the intermediate-region ($\sim \omega^\beta$, Fig. S5(e), following the results of Section S.II $\beta = b - 1$.

Fig. S5(e) also shows the perturbative region for small $t$ and the oscillatory behavior obtained at large times. The time at which the large-time oscillatory behavior starts (corresponding to $\sim \Delta^{-1}$ of Section S.II clearly grows with system size. The extent of the intermediate-time regime gets progressively wider as the system size increases at fixed particle number.

Finally Fig. S5(g), shows how $L(t)$ itself (not subtracted from $L(0) = 1$) looks like in the intermediate-time region.

S.IV.D. Single particle in Aubry-André potential

We consider a single particle on an L-site chain subject to a quasiperiodic potential, i.e., a cosine potential with incommensurate period:

$$H = -J \sum_{j=0}^{L-2} \left( c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j \right) + \sum_j V_j c_j^\dagger c_j$$  \hspace{1cm} (S.20)

with $V_j = V \cos(2\pi q_{1j})$ having irrational wave vector $q_1$, here taken to be $q_1 = \frac{\sqrt{15} - 1}{2}$. This is known as the Aubry-André model. There is a localization transition at $V/J = 2$; single-particle eigenstates are exponentially localized in space for $V/J > 2$. We will consider quenches within the delocalized regime, from $V = 0.1J$ to $V = 0.6J$.

As $V$ is changed from 0 toward 2$J$, the tight-binding band splits up into sub-bands and continues splitting further until it becomes fractal at $V = 2J$. Fig. S6(a) shows the density of states for $V = 0.6J$, where the (sub)band edges are seen through $|\omega - \omega_{edge}|^{-1/2}$ cusps characteristic of single-particle bands in 1D. (The energy gaps between (sub)bands are sometimes too small to be seen.) Fig. S6(b) shows the work distribution $p(\omega)$. The work distribution shows edge singularities at every band edge and is thus quite intricate. Fig. S6(c) shows that the work distribution near the bottom of the first band ($\omega \to 0$) has a clear power law behavior; $p(\omega) \propto \omega^{-3/2}$. As expected, in order to fulfill the normalization condition $\int d\omega p(\omega) = 1$, the magnitude of $p(\omega)$ decreases with $L$. The pre-factor $p_0$ decreases as $L^{-4}$ overall [Fig. S6(d)]. There are some fluctuations because of the quasi-random nature of the system. The gap between the ground and the first excited state, $\Delta$, shown in Fig. S6(e), shows a characteristics $L^{-2}$ decay with $L$. 

(S.IVA),
FIG. S5. Three different quenches for a system of \( N_b = 3 \) hard-core bosons in a ladder geometry with different ladder sizes. (a) Geometry of the system and quench type. Yellow bonds correspond to \( J_{i,j} = -1 \) and gray vertices to \( V_i = 0 \). Blue bonds and black vertices show parameters changed in the quench. (b) Eigenstate overlap as function of energy relative to final ground state energy. (c) Density of states normalized to the total number of states. (d) Work distribution. Black dashed lines are \( \sim \omega^{-b} \) with exponents \( b \) chosen by eye. (e-f) Loschmidt echo. In (e), the perturbative (\( \sim t^2 \)) and intermediate \( \sim t^{b-1} \) regions are highlighted with dashed black power-law lines.

Because \( p(\omega) \) has large contributions in several regions outside the small-\( \omega \) power-law region, no intermediate-time region is visible in the time evolution of the Loschmidt echo.

S.V. LACK OF EQUILIBRATION IN KONDO-LIKE MODEL WITH \( N_c = 3 \) FERMIONS

In the main text we presented the absence of equilibration to the final equilibrium value for \( N_c = 1 \) fermion, in contrast to the expectation for local quenches in finite-density systems. This is not a single-particle curiosity, but a generic feature of low-density systems. In Fig. S7 we show an example of the time evolution of the local occupancy at the impurity-coupled site, \( n_0(t) \), after a quench from \( J_i = 10 \) to \( J_f = 100 \). The long-time average \( \langle n_0 \rangle_{DE} \) is the black solid horizontal line around which \( n_0(t) \) oscillates. This is markedly different from the final equilibrium value, shown as the red dot-dashed horizontal line.

S.VI. DETAILS FOR \( N_c = 1 \) KONDO-LIKE MODEL

In this Section we provide details for results presented in the main text for the Kondo-like model with a single mobile fermion (\( N_c = 1 \)). In S.VI.A we provide derivations of the
FIG. S6. Single particle in Aubry-André potential. (a) Density of states \( \rho(\omega) \). (b) Work distribution \( p(\omega) \) for different system sizes \( L \). (c) Rescaled work distributions at small \( \omega \). Values of \( p_0 \) are chosen such that \( p_0^{-1}p(\omega) \) for different \( L \) lies on the same curve. A power-law dependence, \( p(\omega) \sim \omega^{-b} \) with \( b = 3/2 \), is very clear. (d) Scaling of the pre-factor \( p_0 \) as a function of \( L \). (e) Dependence of the gap \( \Delta \) on \( L \).

FIG. S7. Blue solid curve: Time evolution of the local density \( n_0(t) \) in the Kondo-like model, with \( N_c = 3 \) fermions, \( L = 20 \), \( J_i = 10 \), \( J_f = 100 \). Solid black line: the long-time average value of \( n_0(t) \), i.e., the diagonal average value \( \langle n_0 \rangle_{DE} \). The gray dashed and red dot-dashed lines indicate the ground-state values of \( n_0 \) corresponding to \( J = J_i \) and to \( J = J_f \).

Spatial index as basis label. Since we are restricted to the singlet sector, for \( N_c = 1 \), we can use the position of the mobile fermion as the label for a complete set of states spanning the singlet sector:

\[
|j\rangle = \frac{1}{\sqrt{2}} \left( c_{j\downarrow}^\dagger |\Omega\rangle |\uparrow\rangle - c_{j\uparrow}^\dagger |\Omega\rangle |\downarrow\rangle \right) \tag{S.21}
\]

where \( j \) is the site index and \( |\Omega\rangle \) is the fermionic vacuum.

S.V.I.A. Derivation of the overlap behaviors

We summarize below analytic calculations for the overlaps \( |\langle \phi^{(i)}_m | \phi^{(f)}_m \rangle | \) for quench cases starting and ending in A and C regimes.

We start with A→A quenches, \( J_{i,f} \ll 1 \). We derive the overlap behavior (power law, \( \sim m^{-2} \)) by treating \( J_{i,f} \) perturbatively. Next, we derive the overlap behaviors for quenches between A and C regimes when \( J_{i(f)} \ll 1 \) and \( J_{f(i)} \gg 1 \), by using zeroth-order expressions for the eigenfunctions in the two limits. For C→C quenches, we derive the sine behavior of the overlap by using the zeroth-order \( (J \gg 1) \) expression for the final eigenstates.

In the main text, the index \( m \) was used to label the eigenstates within the symmetry sector where the z-component of the total spin is zero. Within this symmetry sector, there are also triplet states and states with odd spatial reflection parity, which have zero overlap with the initial state. Here, we restrict further to eigenstates with nonzero overlap, which are about one-fourth of the states spanned by the \( m \) index. We use \( \mu \) to index this restricted set of eigenstates.
S.VIA.1. A→A quench

For \( J_{i,f} \ll 1 \), we write both initial and final Hamiltonians as 
\[ \hat{H}_{i} = \hat{H}_{0} + J_{i,f} \hat{V} \] 
and use first-order perturbation theory. The reflection-symmetric eigenfunctions of the unperturbed Hamiltonian \( \hat{H}_{0} \) are the plane waves:

\[ \phi_{\mu}^{A,(0)} = \sum_{i=0}^{L-1} \sqrt{2L-1} \cos \left( \frac{\pi}{L} 2\mu i \right) |j \rangle \]  
(S.22)

and the energies are \( E_{\mu}^{(0)} = -2 \cos \left( \frac{2\pi \mu}{L} \right) \). At first order, the eigenfunctions are

\[ \phi_{\mu}^{A,(1)} = \phi_{\mu}^{A,(0)} + \sum_{k=0,k \neq \mu}^{L/2} \frac{\langle \phi_{\mu}^{A,(0)} | \hat{J}_{f} \hat{V} | \phi_{k}^{A,(0)} \rangle}{E_{\mu}^{(0)} - E_{k}^{(0)}} |\phi_{k}^{A,(0)} \rangle \]  
(S.23)

Therefore, the overlaps of the ground state \( |\phi_{0}^{A,(0)} \rangle \) of \( \hat{H}_{f} \) with the excited states \( |\phi_{\mu}^{A,(0)} \rangle \) of \( \hat{H}_{f} \) are, at first order,

\[ \langle \phi_{0}^{A,(0)} | \phi_{\mu}^{A,(1)} \rangle = \langle \phi_{0}^{A,(0)} | \phi_{\mu}^{A,(0)} \rangle + \langle \phi_{0}^{A,(1)} | \phi_{\mu}^{A,(0)} \rangle 
= \frac{J_{f} - J_{i}}{E_{\mu}^{(0)} - E_{0}^{(0)}} V_{\mu,0}^{(0)}. \]  
(S.24)

The matrix elements \( V_{\mu,0} = \langle \phi_{\mu}^{A,(0)} | \hat{V} | \phi_{0}^{A,(0)} \rangle \) are found to be constant, \( V_{\mu,0} = 3\sqrt{2} \left( 4L \right)^{-1} \). Thus the overlaps as function of \( \mu \) are

\[ \left| \langle \phi_{0}^{A,(1)} | \phi_{\mu}^{A,(1)} \rangle \right| = \frac{3\sqrt{2}}{8L} \frac{|J_{f} - J_{i}|}{1 - \cos \left[ 2\pi L^{-1} \mu \right]}. \]  
(S.25)

This expression is compared to exact numerical values in Fig. S8(a).

S.VIA.2. A→C quench

The final ground state is not included in the power-law regime for A to C quenches, reflecting the fact that the ground state in the C regime is well-separated and quite different from the other eigenstates. In the calculations below, we are therefore only interested in \( \mu > 0 \). For very large \( J \) (extreme C regime), the particle is localized at site \( i = 0 \) in the ground state, and excluded from this site in the other eigenstates. Thus the excited eigenstates ("band states") are the single-particle wave functions in a chain of length \( L - 1 \) with hard-wall boundary conditions:

\[ \left| \phi_{\mu \neq 0}^{C,(0)} \right\rangle = \sum_{i=1}^{L-1} \sqrt{2L-1} \sin \left( \frac{\pi}{L} (2\mu + 1) i \right) |j \rangle \].  
(S.26)

The initial \( (J \ll 1) \) state is \( |\phi_{0}^{A,(0)} \rangle = L^{-1/2} \sum_{x} |x \rangle \) at lowest order. Therefore the overlaps are

\[ \langle \phi_{0}^{A,(0)} | \phi_{\mu \neq 0}^{C,(0)} \rangle = \frac{2\sqrt{2}}{\pi \left( 2\mu + 1 \right)} \].  
(S.27)

In the limit \( L \to \infty \) one can replace summation with integration, leading to

\[ \langle \phi_{0}^{A,(0)} | \phi_{\mu \neq 0}^{C,(0)} \rangle = \frac{2\sqrt{2}}{\pi \left( 2\mu + 1 \right)}. \]  
(S.28)

This expression is compared to exact numerical values in Fig. S8(b).

S.VIA.3. C→A quench

For \( J_{i} \gg 1 \) and \( J_{f} \ll 1 \), we use the zeroth-order expressions in the two limits:

\[ |\phi_{0}^{C,(0)} \rangle = \sum_{x=0}^{L-1} \delta_{x,0} |x \rangle \]

and \( |\phi_{\mu}^{f,(0)} \rangle = |\phi_{\mu}^{A,(0)} \rangle \) given by Eq. (S.22). The resulting overlap distribution is the constant function

\[ \langle \phi_{0}^{C,(0)} | \phi_{\mu}^{A,(0)} \rangle = \sqrt{\frac{L}{2}}. \]  
(S.29)

This expression is compared to exact numerical values in Fig. S8(c).

S.VIA.4. C→C quench

We prove the sine behavior for quenches from anywhere in B or C regimes to the extreme C case \((J_{f} = \infty)\). Describing the sine behavior more generally for anywhere in C to anywhere in C involves the same physical ideas, but is too clumsy in notation to write out.
For the final excited states we use the zeroth order expression S.26. For the initial ground state, we can formally use the exact form \( \phi^{(i)}_0 = N^{−1/2} \exp[−x/ξ(J_i)] \), where \( x = \min(x, L−x) \) is the distance from the impurity coupled site. Here \( ξ(J) \) is a decreasing function of \( J \) and \( N(J) \) is a normalization factor. This gives for the overlaps

\[
\langle \phi^{(i)}_0 | \phi^{(j)}_n \rangle = \frac{2\sqrt{2}}{\sqrt{L N(J_i)}} \sum_{x=1}^{L/2} \sin \left[ \frac{\pi}{L} (2\mu + 1) x \right] e^{−[x]/ξ(J_i)}. \tag{S.30}
\]

By definition of the C regime, \( ξ(J_i) < 1 \) and \( \exp{−[x]/ξ(J_i)} \) is a fast decaying function. Hence the main contribution to the sum is given by the first term \( (x = 1) \):

\[
\langle \phi^{(i)}_0 | \phi^{C, (0)}_n \rangle = \frac{2\sqrt{2}}{\sqrt{L N(J_i)}} \sin \left[ \frac{\pi}{L} (2\mu + 1) \right] e^{−1/ξ(J_i)}. \tag{S.31}
\]

This expression is compared to exact numerical values in Fig. S8.b).

S.VLB. Scaling demonstration of the power law in A to A quench

\[
\text{FIG. S9. The overlap distribution } \langle \phi^{(i)}_0 | \phi^{(j)}_n \rangle \text{ and the numerically determined power-law exponent (absolute value of the logarithmic derivative), plotted for different sizes. Sizes } L = 100, 200, 400, 800 \text{ are used with the } J_i, J_f \text{ adjusted so as to keep } ξ(J_i, J_f)/L \text{ fixed. For } L = 100 \text{ we used } J_i = 10^{−3}, J_f = 10^{−2}.
\]

The A→A quench overlap data in Figure 3 of the main text clearly shows a power-law behavior \( \langle \phi^{(i)}_0 | \phi^{(j)}_n \rangle \sim m^{−2} \), for a particular system size \( L \). However, since the definition of the A region is itself \( L \)-dependent, the extension of this behavior with increasing \( L \) is not immediately obvious.

In Figure S9 we plot A→A overlap distributions for several system sizes. As \( L \) is varied, we adjust \( J_i, J_f \) such that the ratio of \( ξ(J) \) to \( L \) stays fixed. Here \( ξ(J) \) is the localization length that the mobile fermion would have in case of an infinite system. In the A regime, \( ξ(J) > L \). Adjusting \( J_i, J_f \) in this manner ensures that we do not get into the B regime as \( L \) is increased. In addition, it turns out that increasing \( L \) and decreasing \( J_i, J_f \) in this coordinated manner yields a series of curves that systematically extends the power-law behavior, as seen in Figure S9.a).

Figure S9(b) shows the logarithmic derivative of the \( \langle \phi^{(i)}_0 | \phi^{(j)}_n \rangle \) versus \( m \) data, which extracts the power-law exponent. We note that the region where this quantity is numerically \( \alpha = 2 \) (dashed horizontal line), also gets systematically extended through this procedure.

S.VLC. Absence of intermediate-time regime in \( n_j(t) \) for A to C quench

For A→C quenches, an extended intermediate-exponent regime in the time evolution is seen in the Loschmidt echo \( \mathcal{L}(t) \) but not in the local density \( n_j(t) \). To clarify this situation, we note that \( \hat{n}_j \) is a rank-1 projection operator: \( \hat{n}_j = |j\rangle \langle j| \). Using the language introduced in the main text for such observables, instead of the work distribution we should consider

\[
p_{\hat{n}_j}(ω) = \sum_m δ(ω − E_m) \langle \phi^{(i)}_m | \phi^{(j)}_m \rangle \langle \phi^{(j)}_m | j \rangle \tag{S.32}
\]

One factor of the overlap distribution is now replaced by \( \langle \phi^{(j)}_m | j \rangle \). In the C region, for the excited (‘band’) eigenstates \( m > 0 \), this quantity is approximately constant except when it vanishes for symmetry reasons. In the excited eigenstates \( m > 0 \), the quantity is very small for the impurity-coupled site \( j = 0 \), and \( O(1/L) \) for the other \( (j > 0) \) sites. In either case, this leads to the power-law behavior \( p_{\hat{n}_j}(ω) \propto ω^{−b_{\hat{n}_j}} \), with \( b_{\hat{n}_j} = 1 \). This exponent is not large enough to cause a distinct intermediate-time regime in \( |n_j(t) − n_j(0)| \).

In contrast, the exponent relevant for the Loschmidt echo is \( b = 3/2 \), appearing in the work distribution. This leads to an intermediate-exponent regime \( 1 − \mathcal{L}(t) \sim t^{1/2} \) between the initial \( \sim t^2 \) and the large-time oscillatory \( \sim t^0 \) regimes. (Figure 4 in main text.)

Note that, in addition to having an unsuitable exponent, the quantity \( p_{\hat{n}_j}(ω) \) corresponding to site \( j = 0 \) has large contribution from \( m = 0 \), which is outside the power-law region. The corresponding quantities for other sites, say \( p_{\hat{n}_{i,j}}(ω) \) for \( j = L/2 \), would have larger spectral weight in the power-law region. However, since \( b_{\hat{n}_j} = 1 \) for all sites \( j \), there is still no intermediate-time region for \( j \neq 0 \).