Investigation of impulse load attenuation in closed-cell foam in the framework of couple stress elasticity

O Mikulich, V Shvabyuk

Applied Mathematics and Mechanics Department, Lutsk National Technical University, 75 Lvivska Street, Lutsk, Ukraine

shypra@gmail.com

Abstract. The article analyzes the processes of attenuation of impulse loads in foam materials with closed-cell. In the framework of the couple stress elasticity, the analysis of the distribution of dynamic radial stresses in foam media with tunnel cavities of constant cross-section under the action of an impulse load of different duration applied to the cavity boundary in the radial direction is performed. An analytical-numerical approach is used for calculating of the dynamic stresses, which is based on the application of the Fourier transform over time and the modified boundary integral equation method. Numerical calculations of the dynamic stresses were performed using the method of mechanical quadrature and collocation method. The modified algorithm for discrete inverse Fourier transform was used for the calculation of the originals of the dynamic radial stresses. Based on the analysis of numerical calculations, the influence of the impulse duration on the attenuation rate of the pulse load in foam media is established.

1. Introduction

Foam materials are now becoming increasingly popular in construction, as they have good heat-retaining, noise and vibration-absorbing properties. Polyurethane-based polymer foams are characterized by the lightness of the obtained foam material, the absence of the effect of temperature and humidity, water resistance.

Therefore, for selecting the optimal parameters of such foam materials for ensuring vibration and noise protection of structural elements, it is necessary to have information on the rate of attenuation of non-stationary influences in the respective foam materials.

The use of the model of the classical theory of elasticity, which is suitable for the study of elastic deformations in structural materials: steel, concrete, aluminum, gives quite large errors for the case of the study of mechanical effects in porous, granular, fibrous, etc. materials, which is confirmed by the results of experiments. This is due to the need of accounting for the structural heterogeneity of their structure, and therefore it is necessary to use refined models of the continuous mechanics, which accounts for the discrete structure of these materials [1, 2].

However, this consideration leads to significant complications of the equations describing the relationship between stresses and strains in such media, and, at the same time, to a significant complication of the methods of constructing solutions of the corresponding classes of problems. Since within the classical mechanics, there are a large number of problems that need to be solved, much less attention is paid to refined models and, together with them, to the development of analytical and
numerical methods for constructing solutions of the corresponding problems. But the relevance and importance of the described problems necessitate the search for approaches to their solution in the framework of refined models of continuous mechanics.

2. Statements of the problem
To study the processes of attenuation of the impulse load in foam materials, we investigate the distributions of dynamic radial stresses in foam media under the action of according load applied to the boundary of the tunnel cavity. Based on the analysis of the values of the maximum normalized dynamic stresses at different internal points of the medium, we analyze the influence of the impulse duration on the rate of its attenuation.

Consider the foam medium with the tunnel cavity of constant cross-section (Fig. 1, a). To the boundary of the cavity the non-stationary load in the normal direction is applied (Fig. 1, b).

Denote by L the boundary of a cross-section of the cavity. The centre of gravity is placed at the origin of a Cartesian coordinate system $Ox_1x_2$.

![Figure 1, a. 3D full geometry of the problem.](image1)

![Figure 1, b. 2D plane geometry of the problem.](image2)

The problem consists in the determination and comparing of the radial dynamic stresses in the different internal points of foam medium with tunnel cavity under the action of impulse load different duration, which is applied in normal to the boundary of cavity direction.

For accounting for the influence of the microstructure of the foam material on the stress distribution in the structurally inhomogeneous material, we use the apparatus of couple stress elasticity theory [3], which makes it possible the accounting for the influence of shear-rotational deformations of microparticles with some simplification. In numerical calculations, the values of the elastic characteristics of the microstructure of the material are chosen according to the works of R.S. Lakes [4,5] obtained in Eringen's notation [1]. For the case of micropolar stress elasticity in Novatsky's notation [2], these elastic characteristics are given in S. Hassanpour [6].

The boundary conditions of the problem are written:

$$
\sigma_{nl}|_L = \sigma_0 P(t), \quad \tau_m|_L = 0;
$$

(1)

where $\sigma_0$ is a constant characterized the value of the maximum intensity of applied load, $P(t)$ is known a function describes the change of load intensity over time. In the case of the action of impulse load the change of the load intensity over time is chosen in the form [7]:

$$
P(t) = p, T^{-n} e^{-\alpha t}, \quad T > 0, \quad n > 0,
$$

(2)

where $\alpha$, $n$, $p$ are the constants, $T = \frac{t \cdot c_i}{a}$ is the time parameter, $c_i = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ is the speed of
the expansion wave, \( a \) is character size (for the case of tunnel cavity of circular cross-section \( a = R \), where \( R \) is the radius of cavity), \( \lambda \) and \( \mu \) are Lame parameters.

3. Solution of the problem
To ensure the accuracy of numerical calculations under the action of non-stationary load for the determination of dynamic radial stresses, we use the analytical-numerical approach [8], which is based on the use of Fourier transform for time variable and modified boundary integral equation method. The use of such an approach makes it possible to obtain analytical dependences for the determination of dynamic stresses in integral form. Numerical calculation occurs only at the last stage of determining the dynamic stresses.

3.1. Analytical approach for solving of the problem
For the solving of the non-stationary problem in the framework of couple stress elasticity we use the equation of the motion in displacement form \([8, 9]\):

\[
\left( \lambda + 2\mu \right) \text{grad} \, \text{div} \, \vec{u} + \text{rot} \, \text{rot} \, \vec{u} = \rho \frac{\partial^2 \vec{u}}{\partial t^2},
\]  
(3)

where \( \vec{u} = \vec{u}(x,t) = \left\{ u_j(x,t) \right\}, j = 1,2 \) is a vector of displacements, \( B \) is the constant of microstructure of a material, \( \Delta \) is Laplace operator, \( \lambda \) is Lame parameter, \( \mu \) is shear modulus.

Applying the Fourier transform \([10]\):

\[
\tilde{f}(x, \omega) = \int_{-\infty}^{\infty} f(x,t) e^{-i\omega t} dt,
\]
(4)
to the motion equation of couple stress elasticity (3) we obtained the equations, which are equivalent to the equations of time-harmonic motion with cyclic frequency \( \omega \) [8]. Further solving of the problem we realized for Fourier transforms of displacements and stresses, which we denote with symbol ‘\(~\)’.

In the framework of using of modified boundary integral equation method [8] we wrote the representation of displacements for the second exterior problem as [8]:

\[
\tilde{u}_i = \int_{L} p_j U^*_j(x, x'^0) ds + \int_{L} Q_j U^*_j(x, x'^0) d\Omega
\]
(5)

where \( p_j, j = 1,2 \) are unknown potential functions. The fundamental functions \( U^*_j \) for couple stress elasticity were built regarding Sommerfield radiation condition in form:

\[
U^*_j = U_{j}^{*(CTE)} + U_{j}^{*(CSE)}
\]
(6)

where elements with index (CTE) correspond to the case of classical theory of elasticity, and elements with index (CSE) correspond to the case of couple stress elasticity [8].

For the determination of the unknown functions \( p_j, j = 1,2 \) we satisfy the Fourier transforms of the boundary condition (1) and apply Plemelj-Sokhotski formulas for the limits, when internal point tends to the boundary. We obtain the system of integral equations in form:

\[
\frac{\Re (q)}{2} + PV \left\{ (f_2(x,x'^0) \zeta + f_2(x,x'^0) \bar{\zeta}) d\langle q \rangle \right\} = \sigma_0 \tilde{P}(\omega);
\]
(7)
where $p_{ds} = -iqd\zeta$, $p = p_1 + ip_2$ is the unknown complex function, $\zeta = x^0 + ix^0$, $q$ is the constants:

$$q = 1 - \frac{B\rho}{4E}(1+\nu) \omega^2$$ for the case of plane deformation. Here the integrals are understood in the sense of Cauchy principal value.

The representation for determination of Fourier transforms of radial stresses in the internal points of medium were built by substituting the dependencies (5) to the stress formulas [2] in form:

$$\tilde{\sigma}_r = PV \int_L (h_1(x, x^0)q d\zeta + h_2(x, x^0)p d\zeta);$$

where $h_j(x, x^0), j = 1, 2$ are known functions [8].

3.2. Numerical approach for stress calculations

The system of integral equations (7) is solved numerically using the methods of mechanical quadrature and collocation method. After discretization of the boundary of the cross-section of the tunnel cavity, the system of integral equations is reduced to a system of linear algebraic equations. A refined Gaussian method was used to solve the resulting system of equations.

After determining the unknown functions from the system of equations (7), the Fourier images of dynamic radial stresses are calculated numerically by formula (8).

For calculation of originals of the dynamic radial stresses the modified discrete Fourier transform is used:

$$\sigma_r(t_k) = \frac{2}{T} \sum_{n=0}^{K-1} \left( Re \tilde{\sigma}_r(\omega_n) \exp\left( \frac{2\pi i nk}{K} \right) - \tilde{\sigma}_r(\omega_n) \right)$$

where $K$ is the number of elements of selection, $\omega_n = \frac{2\pi n}{t}$ are frequencies, $t_k = \frac{k \cdot t}{K}, k = 0, K - 1$ is time moments.

4. Analysis of numerical calculation results

Let’s study the distribution of dynamic radial stress in internal points of foam media under the action of the impulse load applied to the boundary of tunnel cavity (Fig. 2).

![Cross-section of tunnel cavity](image)

**Figure 2.** Cross-section of tunnel cavity.

Numerical calculations are performed for the case of WF300 and WF110 foam media. Numerical calculations are performed for the case when the radius of the tunnel cavity is $R = 3\ell$, where

$$\ell = \sqrt{\frac{\gamma + \rho}{4\mu}}$$ is scale effect in couple stress elasticity.

Micropolar properties of these materials are present in [6] based on the works of R.S. Lakes [4, 5]. The values of elastic classical and micropolar properties of these foams are chosen according to [6]. For the foam WF300 the values of the physics characteristics of were chosen: the density of foam was
\( \rho = 380 \, \text{kg/m}^3 \), Young’s modulus was \( E = 655.5 \, \text{MPa} \), Poisson’s ratio \( \nu = 0.13 \), Cosserat Twist coefficients \( \varepsilon = 502.2 \, \text{N}, \gamma = 185.6 \, \text{N} \). For the foam WF110 the values of the physics characteristics of were chosen: the density of foam was \( \rho = 110 \, \text{kg/m}^3 \), Young’s modulus was \( E = 215 \, \text{MPa} \), Poisson’s ratio \( \nu = 0.43 \), Cosserat Twist coefficients \( \varepsilon = 16.47 \, \text{N}, \gamma = 20.28 \, \text{N} \).

The distributions of normalized dynamic radial stresses are performed for seven internal points of foam medium for different distance from the boundary of the cavity and different duration of impulse. For numerical calculations values of constants of the impulse load in (2) are chosen as: \( \alpha_r = 5, \, \alpha_s = 47, \, n_s = 2 \) for the case of \( \tilde{r} = 2 \), and \( \alpha_r = 2.5, \, \alpha_s = 11.7, \, n_s = 2 \) for the case of \( \tilde{r} = 4 \), and \( \alpha_r = 1.25, \, \alpha_s = 2.89, \, n_s = 2 \) for the case of \( \tilde{r} = 8 \), and \( \alpha_r = 0.9375, \, \alpha_s = 1.64, \, n_s = 2 \) for the case of \( \tilde{r} = 12 \). Here \( \tilde{r} \) is dimensionless parameter, which according to the impulse duration.

The distribution of normalized dynamic radial stress in WF300 foam for the case of different durations of impulse load is present in Fig. 3. Here the curve 1 according to the case of \( \delta = R \) (on the boundary of the cavity), curve 2-7 according to the case of \( \delta = \{2.5; 3; 5; 7; 10; 13\}R \) (in the internal points of the medium).

![Figure 3](image)

**Figure 3.** The distribution of normalized dynamic radial stress in WF300 foam for the case of different durations of impulse load.

The values of the maximum normalized stresses at different points of the medium for WF300 foam are presented in Table 1.

| Distance | Impulse duration | \( \tilde{r} = 2 \) | \( \tilde{r} = 4 \) | \( \tilde{r} = 8 \) | \( \tilde{r} = 12 \) |
|----------|------------------|------------------|------------------|------------------|------------------|
| \( \delta = R \) | 1                | 1                | 1                | 1                |
| \( \delta = 2.5R \) | 0.8017           | 0.7734           | 0.6989           | 0.6978           |
| \( \delta = 3R \) | 0.6806           | 0.6500           | 0.5640           | 0.5611           |
| \( \delta = 5R \) | 0.4719           | 0.4375           | 0.3620           | 0.3581           |
| \( \delta = 7R \) | 0.3797           | 0.3541           | 0.2874           | 0.2843           |
| \( \delta = 10R \) | 0.3053           | 0.2866           | 0.2270           | 0.2282           |
| \( \delta = 13R \) | 0.2626           | 0.2469           | 0.1989           | 0.1959           |

Similar results were obtained for the case of foam WF 110. The results are shown in Table 2.
Table 2. Maximum values of normalized radial stresses $\sigma_r / \sigma_0$ for the case of WF110.

| Distance | Impulse duration | $t_0 = 2$ | $t_0 = 4$ | $t_0 = 8$ | $t_0 = 12$ |
|----------|------------------|-----------|-----------|-----------|-----------|
| $\delta = R$ | 1 | 1 | 1 | 1 | |
| $\delta = 1.5R$ | 0.4519 | 0.4503 | 0.0442 | 0.4489 | |
| $\delta = 2R$ | 0.2542 | 0.2533 | 0.2426 | 0.2415 | |
| $\delta = 4R$ | 0.06354 | 0.0633 | 0.06321 | 0.06312 | |
| $\delta = 6R$ | 0.02823 | 0.02814 | 0.02809 | 0.02801 | |
| $\delta = 9R$ | 0.01254 | 0.01251 | 0.0125 | 0.01247 | |
| $\delta = 12R$ | 0.00705 | 0.007036 | 0.007023 | 0.007013 | |

For analyzing the attenuation processes of the impulse load in foam depending on impulse duration, the corresponding curves are constructed, which are shown in Fig. 4.

5. Conclusion

Analysis of the numerical results shown in Fig. 3 confirms the congruence of the obtained results with the basic principles of wave mechanics - until the impulse does not reach the appropriate internal points the radial stresses are equal to zero.

Analysis of numerical data of Tables 2 and 3 and curves in Fig. 4 confirm the vibration-absorbing properties of foam materials. For medium stiffness foam (WF110) there is no dependence of the impulse duration on its attenuation. Numerical calculations show that a layer of material with a thickness of $30\ell$ is sufficient for almost complete attenuation of the impulse.

For high stiffness foam (WF300) there is a little dependence of the impulse damping on its duration. For an impulse load with a long pulse duration ($\tilde{t} \geq 8$), the damping processes are almost the same for these types of foams. For impulse loads with a short impulse duration, the attenuation rate is 7-10% lower than in the case of a pulse load with a long impulse duration. Numerical calculation shows that in a layer of material with a thickness of $30\ell$ the impulse attenuates on 70-80%.

These numerical results are explained by the fact that in foam materials of medium stiffness the shear-rotational deformations of the microparticles of the medium are higher than in foam of high stiffness.

References

[1] Eringen A C 1999 Microcontinuum Field Theory. I. Foundations and Solids (Springer, New York)
[2] Nowacki W 1986 Theory of Asymmetric Elasticity (Pergamon-Press, Oxford) p 126
[3] Hadjesfandiari A R and Dargush G F 2011 Couple stress theory for solids International Journal of Solids and Structures vol 48(18) pp 2496-2510
[4] Anderson W B and Lakes R S 1994 Size effects due to Cosserat elasticity and surface damage in closed-cell polymethacrylimide foam *Journal of Materials Science* vol 29(24) pp 6413–6419

[5] Anderson W B, Lakes R S and Smith M C 1995 Holographic evaluation of warp in the torsion of a bar of cellular solid *Cellular Polymers* vol 14(1) pp 1–13

[6] Hassanpour S, Heppler G 2015 Micropolar elasticity theory: A survey of linear isotropic equations, representative notations, and experimental investigations *Mathematics and Mechanics of Solids* vol 22(2) pp 224-242

[7] Sulym H, Mikulich O and Shvabyuk V 2020 Modelling of impulse load influence on stress state of foam materials with positive and negative Poisson’s ratio *Acta Mechanica et Automatica* vol 14 no 2(52) pp 79-83

[8] Mikulich O, Shvabyuk V, Pasternak Ia and Andriichuk O 2018 Modification of boundary integral equation method for investigation of dynamic stresses for couple stress elasticity *Mechanics Research Communications* vol 91 pp 107-111

[9] Savin G N and Shulga N A 1967 Dynamic plane problem of the moment theory of elasticity *Applied mechanics* vol 3(6) pp 216-221

[10] Ramamohan K, Kim D and Hwang J 2010 *Fast Fourier Transform: Algorithms and Applications* (Springer, New York)