Mass and width of unstable molecular state in quantum field theory

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Abstract

Applying resonance theory in the framework of relativistic quantum field theory, we investigate the temporal evolution of molecular state composed of two vector mesons as determined by the total Hamiltonian. Then exotic meson resonance $X(3915)$ is considered as a mixed state of two unstable molecular states $D^*0\bar{D}^*0$ and $D^{*+}D^{*-}$, and the corrected mass and width for resonance $X(3915)$ are calculated. In this actual calculation, we minutely show how to obtain the corrections for resonance and to exhibit the key features of dispersion relation in a new Feynman diagram. The numerical results are consistent with the experimental values.

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I. INTRODUCTION

Hadronic molecule structure has been proposed to interpret the internal structure of exotic meson resonance for many years [1, 2]. In the previous works [1–4], molecular states were considered as meson-meson bound states and Bethe-Salpeter (BS) equation was frequently used to investigate molecular states. In quantum field theory, BS equation is a nonperturbative method, which should be only applied to deal with two-body bound state in the strict sense. However, in experiments resonances are unstable states so that they can not be completely treated as stationary two-body bound states. In this work, we recognize that hadron resonance should be regarded as an unstable state created by two Heisenberg field operators and develop BS equation to deal with resonance in the framework of relativistic quantum field theory. We will comprehensively and systematically show the theoretical approach about unstable molecular state composed of two heavy vector mesons, and this approach is applied to investigate exotic meson resonance $X(3915)$, which is considered as a mixed state of two unstable molecular states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$.

Since resonance is an unstable state which decays spontaneously into other particles, the molecular state composed of two heavy vector mesons should not be a stationary vector-vector bound state. To investigate this unstable two-body system, we suppose that at some given time this unstable state has been prepared to decay and then study the temporal evolution of this system as determined by the total Hamiltonian. This prepared state can be described by the ground-state BS wave function for vector-vector bound state at the times $t_1 = 0$ and $t_2 = 0$. In our previous works [4, 5], the most general form of BS wave functions for the bound states created by two vector fields with arbitrary spin and definite parity has been given. According to the effective theory at low energy QCD, we have investigated the light meson interaction with quarks in heavy vector mesons and obtained the interaction kernel between two heavy vector mesons derived from one light meson $(\sigma, \omega, \rho, \phi)$ exchange [4, 6]. Solving BS equation with this interaction kernel, we have obtained the mass and BS wave function for bound state composed of two vector mesons [4, 7]. After providing the description for the prepared state, we can use Green’s function to study temporal evolution and obtain the relation between Green’s function and the scattering matrix element.

The crucial point of our resonance theory is that the scattering matrix element between bound states is calculated in the framework of relativistic quantum field theory. According
to dispersion relation, the total matrix element between a final state and an initial bound state should be calculated with respect to arbitrary energy. It is necessary to note that the total energy of the final state extends over the real interval while the initial state energy is specified. For the initial bound state composed of two heavy vector mesons, we have given the generalized Bethe-Salpeter (GBS) amplitude for four-quark state describing this meson-meson structure [5, 8], which should be specified. Because the total energy of the final state extends over the real interval, we may obtain several decay channels derived from the interaction Lagrangian and all decay channels should be considered. For exotic resonance $X(3915)$, we can obtain two decay channels $J/\psi\omega$ and $D^*\bar{D}^*$ from the effective interaction Lagrangian at low energy QCD, which are exhibited by new Feynman diagrams. Mandelstam’s approach is applied to calculate the matrix element between bound states with respect to arbitrary energy. Finally, we obtain the correction for energy level of resonance $X(3915)$ and the corrected mass is used to recalculate the decay width of resonance $X(3915)$.

The structure of this article is as follows. In Sec. II we give the revised general form of GBS wave functions for meson-meson bound states as four-quark states. GBS wave function for the mixed state of two bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ is obtained in instantaneous approximation. Section III gives the traditional technique to calculate the matrix element between a final state and an initial four-quark state, which is applied to investigate the decay mode $X(3915) \to J/\psi\omega$. Section IV gives the temporal evolution of meson-meson molecular state as determined by the total Hamiltonian. In Sec. V we emphatically introduce the matrix element between bound states with respect to arbitrary energy. Both decay channels $J/\psi\omega$ and $D^*\bar{D}^*$ are considered. In Sec. VI we obtain the corrected mass and width for unstable molecular state. Our numerical results are presented in Sec. VII and we make some concluding remarks in Sec. VIII.

II. GBS WAVE FUNCTION OF MESON-MESON BOUND STATE AS A FOUR-QUARK STATE

In this paper, we investigate the light meson interaction with the light quarks in heavy mesons. As in effective theory at low energy QCD, the interaction Lagrangian for the
coupling of light quark fields to light meson fields is

\[ \mathcal{L}_{\text{eff}} = i g_0 \left( \bar{u} \, \gamma_5 \, \frac{1}{\sqrt{2}} \eta \, d \right) \gamma^\mu \left( \begin{array}{ccc} \pi^0 + \frac{1}{\sqrt{2}} \eta & \sqrt{2} \, \pi^+ & \sqrt{2} \, K^- \\ -\pi^0 + \frac{1}{\sqrt{2}} \eta & \sqrt{2} \, K^0 & \sqrt{2} \, K^- \\ \sqrt{2} \, K^- & \sqrt{2} \, K^0 & -\frac{2}{\sqrt{2}} \eta \end{array} \right) \left( \begin{array}{c} u \\ d \\ s \end{array} \right) \]

\[ + i g_0 \left( \bar{u} \, \gamma_5 \, \frac{1}{\sqrt{2}} \eta \, d \right) \gamma^\mu \left( \begin{array}{ccc} \rho^0 + \omega & \sqrt{2} \, \rho^+ & \sqrt{2} \, K^{*-+} \\ \sqrt{2} \, \rho^- & -\rho^0 + \omega & \sqrt{2} \, K^{*0} \\ \sqrt{2} \, K^{*0} & \sqrt{2} \, K^0 & -\rho^0 \end{array} \right) \left( \begin{array}{c} u \\ d \\ s \end{array} \right) + g_\sigma \left( \bar{u} \, \gamma_5 \, \frac{1}{\sqrt{2}} \eta \, d \right) \left( \begin{array}{c} u \\ d \end{array} \right) \sigma. \]

The quark current \( J_\mu \) coupling with light vector meson and the quark scalar density \( J \) coupling with \( \sigma \) meson can be obtained. In this section, our attention is only focused on the bound state composed of two vector mesons and some errors in previous works are revised.

### A. BS wave function for bound state composed of two vector mesons

If a bound state with spin \( j \) and parity \( \eta_P \) is created by two Heisenberg vector fields with masses \( M_1 \) and \( M_2 \), respectively, its BS wave function is defined as

\[ \chi^j_{P(\lambda \tau)}(x_1', x_2') = \langle 0 | TA_1(x_1') A_2^\dagger(x_2') | P, j \rangle = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(P)}} e^{iP \cdot X} \chi^j_{P(\lambda \tau)}(X'), \]

where \( P \) is the momentum of the bound state, \( E(p) = \sqrt{p^2 + m^2} \), \( x_1' = (x_1', i\tau_1) \), \( x_2' = (x_2', i\tau_2) \), \( X = \eta_1 x_1' + \eta_2 x_2' \), \( X' = x_1' - x_2' \) and \( \eta_{1,2} = M_{1,2}/(M_1 + M_2) \). Making the Fourier transformation, we obtain the BS wave function in the momentum representation

\[ \chi^j_P(p_1', p_2')_{\lambda \tau} = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(P)}} (2\pi)^4 \delta^{(4)}(P - p_1' + p_2') \chi^j_{P(\lambda \tau)}(P, p), \]

where \( p \) is the relative momentum of two vector fields and we have \( P = p_1' - p_2' \), \( p = \eta_2 p_1' + \eta_1 p_2' \), \( p_1' \) and \( p_2' \) are the momenta of two vector fields, respectively. The polarization tensor of the bound state \( \eta_{\mu_1 \mu_2... \mu_j} \) can be separated,

\[ \chi^j_P(P, p) = \eta_{\mu_1 \mu_2... \mu_j} \chi_{\mu_1 \mu_2... \mu_j, \lambda \tau}(P, p), \]

where the subscripts \( \lambda \) and \( \tau \) are derived from these two vector fields. The polarization tensor \( \eta_{\mu_1 \mu_2... \mu_j} \) describes the spin of the bound state, which is totally symmetric, transverse and traceless:

\[ \eta_{\mu_1 \mu_2...} = \eta_{\mu_2 \mu_1...}, \quad P_{\mu_1} \eta_{\mu_1 \mu_2...} = 0, \quad \eta^\mu_{\mu_1 \mu_2...} = 0. \]
From Lorentz covariance, we have

\[
\chi_{\mu_1 \ldots \mu_j \lambda \tau} = p_{\mu_1} \ldots p_{\mu_j}[g_{\lambda \sigma} \mathcal{F}_1 + (P_\lambda p_\sigma + P_\sigma p_\lambda) f_2 + (P_\lambda p_\tau - P_\tau p_\lambda) f_3 + P_\lambda P_\tau f_4 + p_\lambda p_\tau f_5]
+ (p_{\mu_1} \ldots p_{\mu_j} g_{\mu_1}) \lambda P_\tau + p_{\mu_2} \ldots p_{\mu_j} g_{\mu_1} \tau P_\lambda f_6
+ (p_{\mu_2} \ldots p_{\mu_j} g_{\mu_1}) \lambda P_\tau - p_{\mu_2} \ldots p_{\mu_j} g_{\mu_1} \tau P_\lambda f_7
+ (p_{\mu_2} \ldots p_{\mu_j} g_{\mu_0}) \lambda P_\tau + p_{\mu_2} \ldots p_{\mu_j} g_{\mu_0} \tau P_\lambda f_8
+ (p_{\mu_2} \ldots p_{\mu_j} g_{\mu_0}) \lambda P_\tau - p_{\mu_2} \ldots p_{\mu_j} g_{\mu_0} \tau P_\lambda f_9
+ p_{\mu_1} \ldots p_{\mu_j} \epsilon_{\lambda \gamma \xi \zeta p_\xi P_\xi f_{10}} + p_{\mu_2} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \epsilon_{\gamma \xi \zeta p_\xi P_\xi f_{11}} + p_{\mu_2} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \tau \epsilon_{\xi \zeta p_\xi P_\xi f_{12}}
+ (p_{\mu_2} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \epsilon_{\xi \zeta p_\xi P_\xi f_{13}} + p_{\mu_2} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \tau \epsilon_{\xi \zeta p_\xi P_\xi f_{14}} + p_{\mu_2} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \xi \epsilon_{\zeta p_\xi P_\xi f_{15}}
+ (p_{\mu_2} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \xi \epsilon_{\zeta p_\xi P_\xi f_{16}} + p_{\mu_3} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \xi \epsilon_{\zeta p_\xi P_\xi f_{17}} + p_{\mu_3} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \tau \epsilon_{\zeta p_\xi P_\xi f_{18}}
+ (p_{\mu_3} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \tau \epsilon_{\zeta p_\xi P_\xi f_{19}} + p_{\mu_3} \ldots p_{\mu_j} \epsilon_{\mu_1} \lambda \xi \epsilon_{\tau p_\xi P_\xi f_{20}}),
\]

(6)

where \{\mu_1, \ldots, \mu_j\} represents symmetrization of the indices \mu_1, \ldots, \mu_j. In fact, the relative momenta \(p_{\mu_1}, \ldots, p_{\mu_j}, p_\lambda, p_\tau\) represent the orbital angular momenta. There should be 20 scalar functions \(f_i(P \cdot p, p^2)(i = 1, \ldots, 20)\) in Eq. (6). In Ref. [3], three tensor structures are omitted. In this paper, these missing terms are added as the last three terms in Eq. (6).

Using the transversality condition

\[
p_{1 \lambda} \chi_{\lambda \tau}^j(P, p) = p_{2 \tau} \chi_{\lambda \tau}^j(P, p) = 0
\]

(7)

and considering the properties of BS wave function under space reflection, we obtain the revised general form of BS wave functions for the bound states created by two massive vector fields with arbitrary spin and definite parity (see details in [4]), for \(\eta_P = (-1)^j\),

\[
\chi_{\lambda \tau}^j(P, p) = \frac{1}{N_j^2} \eta_{\mu_1 \ldots \mu_j} [p_{\mu_1} \ldots p_{\mu_j} (T_{1 \lambda} \Phi_1 + T_{2 \lambda} \Phi_2) + T_{3 \mu_1 \ldots \mu_j \lambda \tau} \Phi_3 + T_{4 \mu_1 \ldots \mu_j \lambda \tau} \Phi_4
+ T_{5 \mu_1 \ldots \mu_j \lambda \tau} \Phi_5 + T_{6 \mu_1 \ldots \mu_j \lambda \tau} \Phi_6],
\]

(8)

for \(\eta_P = (-1)^{j+1}\),

\[
\chi_{\lambda \tau}^j(P, p) = \frac{1}{N_j^2} \eta_{\mu_1 \ldots \mu_j} [p_{\mu_1} \ldots p_{\mu_j} \epsilon_{\lambda \gamma \xi \zeta p_\xi P_\xi \Phi'_1 + T_{7 \mu_1 \ldots \mu_j \lambda \tau} \Phi'_2 + T_{8 \mu_1 \ldots \mu_j \lambda \tau} \Phi'_3 + T_{9 \mu_1 \ldots \mu_j \lambda \tau} \Phi'_4
+ T_{10 \mu_1 \ldots \mu_j \lambda \tau} \Phi'_5 + T_{11 \mu_1 \ldots \mu_j \lambda \tau} \Phi'_6 + T_{12 \mu_1 \ldots \mu_j \lambda \tau} \Phi'_7),
\]

(9)
where $\mathcal{N}^j$ is normalization, the independent tensor structures $T_{\lambda\tau}^i$ are given in Appendix A. $\Phi_i(P \cdot p, p^2)$ and $\Phi'_i(P \cdot p, p^2)$ are independent scalar functions. Scalar functions $f_i$ in Eq. (6) are the linear combinations of $\Phi_i$ and $\Phi'_i$.

### B. Kernel between two heavy vector mesons

In this paper, we assume that resonance $X(3915)$ is a mixed state of two unstable molecular states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$. In this section, we only investigate the mixed state of two stable bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, and BS wave function for this system is a linear combination of two components as

$$
\chi_{\lambda\tau}^{D^{*}\bar{D}^{*},j}(P, p) = \frac{1}{\sqrt{2}} \chi_{\lambda\tau}^{D^{*0}\bar{D}^{*0},j}(P, p) + \frac{1}{\sqrt{2}} \chi_{\lambda\tau}^{D^{*+}D^{*-},j}(P, p),
$$

where

$$
\chi_{\lambda\tau}^{D^{*0}\bar{D}^{*0},j}(P, p) = \chi_{\lambda\tau}^j(P, p) \left( -\left| \frac{1}{2}, \frac{1}{2} \right> \right) \otimes \left| \frac{1}{2}, \frac{1}{2} \right>,
$$

$$
\chi_{\lambda\tau}^{D^{*+}D^{*-},j}(P, p) = \chi_{\lambda\tau}^j(P, p) \left( -\left| \frac{1}{2}, \frac{1}{2} \right> \right) \otimes \left| \frac{1}{2}, -\frac{1}{2} \right>,
$$

and $P$ becomes the total momentum for the mixed state of two meson-meson bound states, $\chi_{\lambda\tau}^j(P, p)$ is the component wave function in the momentum representation; $(-|\frac{1}{2}, -\frac{1}{2}|) \otimes |\frac{1}{2}, \frac{1}{2}|$ and $(-|\frac{1}{2}, \frac{1}{2}|) \otimes |\frac{1}{2}, -\frac{1}{2}|$ are the isospin wave functions of pure bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, respectively; $\chi_{\lambda\tau}^{D^{*0}\bar{D}^{*0},j}$ and $\chi_{\lambda\tau}^{D^{*+}D^{*-},j}$ represent BS wave functions for the bound states of two vector mesons, which are the eigenstates of Hamiltonian without considering the coupled-channel terms. These eigenstates have the same quantum numbers. The error in Ref. [5] has been revised. As usual the momentum for the mixed state of two bound states is set as $P = (0, 0, 0, iM_0)$ in the rest frame. For simplicity, we only consider that the spin-parity quantum numbers of $X(3915)$ are $J^P = 0^+$ in this work.

Let $D_l^*$ denote one of $D^{*0}$ and $D^{*+}$, and $l = u, d$ represents the $u$ or $d$ antiquark in heavy vector meson $D^{*0}$ or $D^{*+}$, respectively; $D_l^*$ denotes the antiparticle of $D_l^*$. From Eq. (8), we can obtain BS wave function describing pure bound state $D_l^*\bar{D}_l^*$

$$
\chi_{\lambda\tau}^{0^+}(P^{DD}, p) = \frac{1}{\mathcal{N}_{DD}^{0^+}} [T_{\lambda\tau}^1 \mathcal{F}_1 (P^{DD} \cdot p, p^2) + T_{\lambda\tau}^2 \mathcal{F}_2 (P^{DD} \cdot p, p^2)].
$$
$P^{D\bar{D}}$ represents the momentum of pure bound state in the rest frame, whose fourth component is different from the one of $P$. This BS wave function should satisfy the equation

$$\chi^{0+}_{\lambda\kappa}(P^{D\bar{D}},p) = -\int \frac{d^4p}{(2\pi)^4} \Delta_{F\lambda\theta}(p') \mathcal{V}_{\theta\theta',\kappa',\kappa}(p,p'; P^{D\bar{D}}) \chi^{0+}_{\theta',\kappa'}(P^{D\bar{D}},p') \Delta_{F\kappa\tau}(p'),$$

(13)

where $\mathcal{V}_{\theta\theta',\kappa',\kappa}$ is the interaction kernel, $P^{D\bar{D}} = (0,0,0,iM_{D\bar{D}})$, $p_1' = p + P^{D\bar{D}}/2$, $p_2' = p - P^{D\bar{D}}/2$, $\Delta_{F\lambda\theta}(p_1')$ and $\Delta_{F\kappa\tau}(p_2')$ are the propagators for the spin 1 fields, $\Delta_{F\lambda\theta}(p_1') = (\delta_{\lambda\theta} + \frac{p_1'\gamma_\mu}{M_1^2})p_1' - \frac{-i}{p_1' + M_1 - i\epsilon}$, $\Delta_{F\kappa\tau}(p_2') = (\delta_{\kappa\tau} + \frac{p_2'\gamma_\mu}{M_2^2})p_2' - \frac{-i}{p_2' + M_2 - i\epsilon}$, $M_1 = M_{D^*}$ and $M_2 = M_{\bar{D}^*}$. We emphasize that the kernel $\mathcal{V}$ is defined in two-body channel so $\mathcal{V}$ is not complete interaction. The kernel plays a central role for making two-body system to be a stable state, but it can not provide any motive for decay process.

To construct the interaction kernel between $D^*_1$ and $\bar{D}^*_1$, we consider that the effective interaction is derived from one light meson ($\sigma$, $\rho^0$, $V_1$ and $V_8$) exchange \cite{4,6,7}, shown as Fig. 1. The flavor-SU(3) singlet $V_1$ and octet $V_8$ states of vector mesons mix to form the physical $\omega$ and $\phi$ mesons as

$$\phi = -V_8 \cos \theta + V_1 \sin \theta, \quad \omega = V_8 \sin \theta + V_1 \cos \theta,$$

(14)

where the mixing angle $\theta = 38.58^\circ$ was obtained by KLOE \cite{9}. Then the exchanged mesons should be the octet $V_8$ and singlet $V_1$ states, and the relation of the octet-quark coupling constant $g_8$ and the singlet-quark coupling constant $g_1$ has the form

$$g_\phi = -g_8 \cos \theta + g_1 \sin \theta, \quad g_\omega = g_8 \sin \theta + g_1 \cos \theta,$$

(15)

where the meson-quark coupling constants $g_\omega^2 = 2.42/2$ and $g_\phi^2 = 13.0$ were obtained within QCD sum rules approach \cite{10}.

![Fig. 1: The light meson exchange between two heavy vector mesons.](image)
In Fig. 1, VM represents $D^*_L$ and $\bar{V}M^\prime$ represents $\bar{D}^*_L$. From the Lorentz-structure, the matrix elements of quark scalar density $J$ and quark current $J_\alpha$ can be expressed as

$$\langle VM^\varphi(p'_1)|J|VM^\varphi(q'_1)\rangle = \frac{1}{2\sqrt{E_D^*(p'_1)}} \frac{1}{E_D^*(q'_1)} \{[\varepsilon^\varphi(p'_1) \cdot \varepsilon^\varphi(q'_1)]h_1^{(s)}(w^2)
- h_2^{(s)}(w^2) \frac{1}{M^2} [\varepsilon^\varphi(p'_1) \cdot q'_1][\varepsilon^\varphi(q'_1) \cdot p'_1]\},$$

(16a)

$$\langle V\bar{M}^\varphi(-p'_2)|J|V\bar{M}^\varphi(-q'_2)\rangle = \frac{1}{2\sqrt{E_D^*(-p'_2)E_D^*(-q'_2)}} \{[\varepsilon^{\varphi'}(-p'_2) \cdot \varepsilon^{\varphi'}(-q'_2)]\bar{h}_1^{(s)}(w^2)
- \bar{h}_2^{(s)}(w^2) \frac{1}{M^2} [\varepsilon^{\varphi'}(-p'_2) \cdot (-q'_2)][\varepsilon^{\varphi'}(-q'_2) \cdot (-p'_2)]\},$$

(16b)

$$\langle V\bar{M}^\varphi(p'_1)|J_\alpha|V\bar{M}^\varphi(q'_1)\rangle = \frac{1}{2\sqrt{E_D^*(p'_1)E_D^*(q'_1)}} \{[\varepsilon^\varphi(p'_1) \cdot \varepsilon^\varphi(q'_1)]h_1^{(c)}(w^2)(p'_1 + q'_1)\alpha
- h_2^{(c)}(w^2) \frac{1}{M^2} [\varepsilon^\varphi(p'_1) \cdot q'_1][\varepsilon^\varphi(q'_1) \cdot p'_1](p'_1 + q'_1)\alpha\},$$

(16c)

$$\langle V\bar{M}^\varphi(-p'_2)|J_\beta|V\bar{M}^\varphi(-q'_2)\rangle = \frac{1}{2\sqrt{E_D^*(-p'_2)E_D^*(-q'_2)}} \{[\varepsilon^{\varphi'}(-p'_2) \cdot \varepsilon^{\varphi'}(-q'_2)]\bar{h}_1^{(c)}(w^2)(-p'_2 - q'_2)\beta
- \bar{h}_2^{(c)}(w^2) \frac{1}{M^2} [\varepsilon^{\varphi'}(-p'_2) \cdot (-q'_2)][\varepsilon^{\varphi'}(-q'_2) \cdot (-p'_2)](-p'_2 - q'_2)\beta\},$$

(16d)

where $p'_1 = (p, ip'_{10})$, $p'_2 = (p, ip'_{20})$, $q'_1 = (p', ip'_{10})$, $q'_2 = (p', ip'_{20})$, $w = q'_1 - p'_1 = q'_2 - p'_2$ is the momentum of the light meson and $w = p' - p$; $h(w^2)$ and $\bar{h}(w^2)$ are scalar functions, the four-vector $\varepsilon(p)$ is the polarization vector of heavy vector meson with momentum $p$ and $E_D^*(p) = \sqrt{p^2 + M^2_{D^*_L}}$. Taking away the external lines including normalizations and polarization vectors $\varepsilon^\varphi(p'_1)$, $\varepsilon^{\varphi'}(q'_1)$, $\varepsilon^{\varphi'}(-p'_2)$, $\varepsilon^{\varphi'}(-q'_2)$, we obtain the interaction kernel from
one light meson (σ, ρ, V₁ and V₈) exchange

\[ V_{θθ',κ',κ}(p, p'; P^{DD}) \]

\[ = h^{(s)}_1(w^2) \frac{-ig^2_σ}{w^2 + M^2_σ} h^{(s)}_1(w^2) δ_{θθ'} δ_{κ',κ} + \left( \frac{-ig^2_ρ}{w^2 + M^2_ρ} + \frac{-ig^2_κ}{w^2 + M^2_κ} + \frac{-ig^2_φ}{w^2 + M^2_φ} \right) \{ h^{(v)}_1(w^2) h^{(v)}_1(w^2) \} \]

\[ \times (p'_1 + q'_1) \cdot (-p'_2 - q'_2) δ_{θθ'} δ_{κ',κ} - h^{(v)}_2(w^2) h^{(v)}_2(w^2) δ_{θθ'} [-(p'_1 + q'_1)κ' q_{2κ} - p_{2κ} (p'_1 + q'_1)κ] \]

\[ - h^{(v)}_2(w^2) h^{(v)}_1(w^2) [q_{1θ} (-p'_2 - q'_2)θ' + (-p'_2 - q'_2)θ' p_{1θ}'] δ_{κ',κ} + h^{(v)}_2(w^2) h^{(v)}_2(w^2) [q_{1θ} δ_{θθ'} q_{2κ} + q_{1θ'} δ_{θθ'} p_{1θ'} q_{2κ} + δ_{θκ} p_{1θ'} (-p_{2κ}')], \]

where \( g_σ = \frac{B(M_σ)}{f_σ} = 3.15 \frac{f_σ}{m_σ} [11, 12], \)

\( g_ρ^2 = 2.42 \frac{f_σ}{m_σ}, \)

and these terms containing \( M_{1,2} \) are neglected because the masses of heavy mesons are large. Using the method above, we can obtain the interaction kernels from one-ρ⁻ exchange [4].

**C. Instantaneous approximation**

1. **The extended Bethe-Salpeter equation**

Substituting BS wave function given by Eq. (12) and the kernel (17) into BS equation (13), we find that the integral of one term on the right-hand side of (12) has contribution to the one of itself and the other term. Ignoring the cross terms, one can obtain two individual equations:

\[ \mathcal{F}_{λτ}(P^{DD} \cdot p, p^2) = - \int \frac{d^4p'}{(2π)^4} Δ_{Fλθ}(p'_1) V_{θθ',κ',κ}(p, p'; P^{DD}) \mathcal{F}_{θθ',κ',κ}(P^{DD} \cdot p', p^2) Δ_{Fκτ}(p'_2), \]  

(18)

\[ \mathcal{F}_{λτ}(P^{DD} \cdot p, p^2) = - \int \frac{d^4p'}{(2π)^4} Δ_{Fλθ}(p'_1) V_{θθ',κ',κ}(p, p'; P^{DD}) \mathcal{F}_{θθ',κ',κ}(P^{DD} \cdot p', p^2) Δ_{Fκτ}(p'_2), \]  

(19)

where \( \mathcal{F}_{λτ}(P^{DD} \cdot p, p^2) = T^{λ}_{κτ} \mathcal{F}_{1}(P^{DD} \cdot p, p^2) \) and \( \mathcal{F}_{λτ}(P^{DD} \cdot p, p^2) = T^{λ}_{κτ} \mathcal{F}_{2}(P^{DD} \cdot p, p^2) \). Comparing the tensor structures in both sides of Eqs. (18) and (19), respectively, we obtain

\[ \mathcal{F}_{1}(P^{DD} \cdot p, p^2) = \frac{1}{p'_1^2 + M^2_1 - iε} \frac{1}{p'^2_2 + M^2_2 - iε} \int \frac{d^4p'}{(2π)^4} V^{p+}_{1}(p, p'; P^{DD}) \mathcal{F}_{1}(P^{DD} \cdot p', p'^2), \]  

(20)

\[ p'^2_2 \mathcal{F}_{2}(P^{DD} \cdot p, p^2) = \frac{1}{p'_1^2 + M^2_1 - iε} \frac{1}{p'^2_2 + M^2_2 - iε} \int \frac{d^4p'}{(2π)^4} V^{p+}_{2}(p, p'; P^{DD}) q^2_{2} \mathcal{F}_{2}(P^{DD} \cdot p', p'^2), \]  

(21)
where \( V_1^{0+}(p, p'; P^{DD}) \) and \( V_2^{0+}(p, p'; P^{DD}) \) are derived from the interaction kernel between \( D^*_1 \) and \( D^*_1 \). In instantaneous approximation, we set the momentum of exchanged meson as \( w = (w, 0) \). Then Eqs. \((20)\) and \((21)\) become two relativistic Schrödinger-like equations (see details in Refs. \([4, 7]\))

\[
\left( \frac{b_1^2(M_{DD})}{2\mu_R} - \frac{\bar{p}^2}{2\mu_R} \right) \Psi_1^{0+}(p) = \int \frac{d^3w}{(2\pi)^3} V_1^{0+}(p, w) \bar{\Psi}_1^{0+}(p, w),
\]

\[
\left( \frac{b_2^2(M_{DD})}{2\mu_R} - \frac{\bar{p}^2}{2\mu_R} \right) \Psi_2^{0+}(p) = \int \frac{d^3w}{(2\pi)^3} V_2^{0+}(p, w) \bar{\Psi}_2^{0+}(p, w),
\]

where \( \Psi_1^{0+}(p) = \int dp_0\mathcal{F}_1(P^{DD} \cdot p, p^2), \bar{\Psi}_1^{0+}(p) = \int dp_0\mathcal{F}_2(P^{DD} \cdot p, p^2), \mu_R = E_1E_2/(E_1 + E_2) = [M_{DD}^4 - (M_1^2 - M_2^2)^2]/(4M_{DD}^4), b_1^2(M_{DD}) = [M_{DD}^2 - (M_1 + M_2)^2][M_{DD}^2 - (M_1 - M_2)^2]/(4M_{DD}^4), E_1 = (M_{DD} - M_2^2 + M_1^2)/(2M_{DD}) \) and \( E_2 = (M_{DD}^2 - M_1^2 + M_2^2)/(2M_{DD}) \).

The potentials between \( D^*_1 \) and \( D^*_1 \) up to the second order of the \( p/M_{DD} \) expansion are

\[
V_1^{0+}(p, w) = \frac{h_1^{(s)}(w^2)}{2E_1} \frac{g_2^1}{w^2 + M_2^2} \frac{\bar{h}_1^{(s)}(w^2)}{2E_2} + \frac{h_1^{(v)}(w^2)}{4E_1E_2} + \frac{g_1^2}{w^2 + M_2^2} \left( -1 - \frac{4p^2 + 5w^2}{E_1E_2} \right),
\]

\[
V_2^{0+}(p, w) = \frac{h_1^{(s)}(w^2)}{2E_1} \frac{g_2^1}{w^2 + M_2^2} \frac{\bar{h}_1^{(s)}(w^2)}{2E_2} \left( 1 - \frac{w^2}{M_1^2} \right) + \frac{h_1^{(v)}(w^2)}{4E_1E_2} + \frac{g_1^2}{w^2 + M_2^2} \left( -1 - \frac{2p^2 + 2w^2}{4E_1E_2} - \frac{2p^2 + 2w^2}{4E_1E_2} \right).
\]

Solving these two equations \((22)\) and \((23)\), respectively, one can obtain the eigenvalues \( b_1^2(M_{DD}) \) and \( b_2^2(M_{DD}) \) and the corresponding eigenfunctions \( \Psi_1^{0+}(p) \) and \( \Psi_2^{0+}(p) \). From \( \Psi_1^{0+} \) and \( \Psi_2^{0+} \), it is easy to obtain \( \mathcal{F}_1 \) and \( \mathcal{F}_2 \), respectively.

Because the cross terms are small, we can take the ground state BS wave function to be a linear combination of two eigenstates \( \mathcal{F}_{\lambda\tau}^{10}(P^{DD} \cdot p, p^2) \) and \( \mathcal{F}_{\lambda\tau}^{20}(P^{DD} \cdot p, p^2) \) corresponding to lowest energy in Eqs. \((18)\) and \((19)\). Then in the basis provided by \( \mathcal{F}_{\lambda\tau}^{10}(P^{DD} \cdot p, p^2) = \mathcal{T}_{\lambda\tau}^{10}\mathcal{F}_{10}(P^{DD} \cdot p, p^2) \) and \( \mathcal{F}_{\lambda\tau}^{20}(P^{DD} \cdot p, p^2) = \mathcal{T}_{\lambda\tau}^{20}\mathcal{F}_{20}(P^{DD} \cdot p, p^2) \), BS wave function \( \chi^{0+}_{\lambda\tau} \) is considered as

\[
\chi^{0+}_{\lambda\tau}(P^{DD}, p) = \frac{1}{\mathcal{N}_{DD}^{0+}} [C_1\mathcal{F}_{\lambda\tau}^{10}(P^{DD} \cdot p, p^2) + C_2\mathcal{F}_{\lambda\tau}^{20}(P^{DD} \cdot p, p^2)].
\]

Substituting \((26)\) into BS equation \((13)\) and comparing the tensor structures in both sides, we obtain an eigenvalue equation in instantaneous approximation \([4]\)

\[
\begin{pmatrix}
\frac{b_1^2(M_{DD})}{2\mu_R} - \lambda & H_{12} \\
H_{21} & \frac{b_2^2(M_{DD})}{2\mu_R} - \lambda
\end{pmatrix}
\begin{pmatrix}
C'_1 \\
C'_2
\end{pmatrix} = 0,
\]
where we have the matrix elements
\[
H_{12} = H_{21} = \int \frac{d^3p}{(2\pi)^3} \Psi_0^+(p)^* \int \frac{d^3w}{(2\pi)^3} \bar{h}^{(v)}_1(w^2) \\
\times \left( \frac{g_\rho^2}{w^2 + M_\rho^2} + \frac{g_i^2}{w^2 + M_i^2} + \frac{g_\phi^2}{w^2 + M_\phi^2} \right) h^{(v)}_1(w^2) \frac{w^2}{E_1 E_2} \Psi_0^0(p, w),
\]
and \(b_{10}^2(M_{D\bar{D}})/(2\mu_R)\) and \(b_{20}^2(M_{D\bar{D}})/(2\mu_R)\) are the eigenvalues corresponding to lowest energy in Eqs. (22) and (23), respectively; \(\Psi_0^+\) and \(\Psi_0^{10}\) are the corresponding eigenfunctions.

From this equation, we can obtain the eigenvalues and eigenfunctions which contain the contribution from the cross terms. Some errors in our previous works have been revised.

2. Form factors of heavy meson

To calculate these heavy vector meson form factors \(h(w^2)\) describing the heavy meson structure, we have to know the wave function of heavy vector meson \(D^*_l\) in instantaneous approximation. For heavy vector mesons, the authors of Refs. [13–16] have obtained their BS amplitudes in Euclidean space:

\[
\Gamma_{\lambda}^V(K, k) = \frac{1}{N^V} \left( \gamma_\lambda + K_\lambda \gamma \cdot \frac{K}{M_V^2} \right) \phi_V(k^2),
\]

where \(K\) is the momentum of heavy meson, \(k\) denotes the relative momentum between quark and antiquark in heavy meson, \(M_V\) is heavy vector meson mass, \(\Gamma_{\lambda}^V(K, k)\) is transverse \((K_\lambda \Gamma_{\lambda}^V(K, k) = 0)\), \(N^V\) is normalization, and \(\phi_V(k^2)\) is scalar function fixed by providing fits to observables. The charmed meson \(D^*_l\) is composed of \(c\)-quark and \(l\)-antiquark. As in heavy-quark effective theory (HQET) [17], we consider that the heaviest quark carries all the heavy-meson momentum and obtain BS wave function of \(D^*_l\) meson

\[
\chi_\lambda(K, k) = \frac{-1}{\gamma \cdot (k + K)} \frac{1}{N^D_{D^*_l}} \left( \gamma_\lambda + K_\lambda \gamma \cdot \frac{K}{M_{D^*_l}^2} \right) \phi_{D^*_l}(k^2) \frac{-1}{\gamma \cdot k - i m_l},
\]

where \(K\) is set as the momentum of heavy meson in the rest frame, \(k\) becomes the relative momentum between \(c\)-quark and \(l\)-antiquark, \(m_{c, l}\) are the constituent quark masses, \(\phi_{D^*_l}(k^2) = \phi_{D^*_l}(k^2) = \exp(-k^2/\omega_{D^*}^2)\) and \(\omega_{D^*}=1.50\) GeV [16]. The components of this BS wave function are 4 \(\times\) 4 matrices, which can be written as [18]

\[
\chi_\lambda(K, k) = \Psi_\lambda^S + \Psi_{\lambda, \mu}^V \gamma_\mu + \Psi_{\lambda, \mu, \nu}^T \sigma_{\mu \nu} + \Psi_{\lambda, \mu}^{AV} \gamma_\mu \gamma_5 + \Psi_{\lambda, \sigma, \epsilon}^{Pse} \gamma_5,
\]
and the coefficient corresponding to $\gamma_\mu$ is

$$\Psi^{V}_{\lambda,\mu} = \frac{1}{4} \text{Tr} [\gamma_\mu \chi_\lambda(K, k)].$$  \hspace{1cm} (32)

Substituting Eq. (30) into (32), we can obtain the heavy vector meson wave function in instantaneous approximation

$$\Psi^{D^*_i}_{ij}(k) = \int dk_i \frac{1}{N_{D^*_i}} \exp \left( -\frac{\mathbf{k}^2 - k_i^2}{\omega_{D^*_i}^2} \right) \frac{\mathbf{k}^2/3 + k_i^2 + m_c m_t}{(\mathbf{k}^2 + k_i^2 + m_c^2)(\mathbf{k}^2 + k_i^2 + m_t^2)} \delta_{ij} \quad i, j = 1, 2, 3.$$  \hspace{1cm} (33)

In the previous works [4, 6, 7], we have obtained the form factors for the vertices of heavy vector meson $D^*_i$ coupling to scalar meson ($\sigma$)

$$-\frac{h_1^{(s)}(w^2)}{2E_1} = \frac{\tilde{h}_1^{(s)}(w^2)}{2E_2} = F_1(w^2), \quad h_2^{(s)}(w^2) = \tilde{h}_2^{(s)}(w^2) = 0,$$  \hspace{1cm} (34)

$$F_1(w^2) = \int \frac{d^3k}{(2\pi)^3} \overline{\Psi}^{D^*_i}(k + \frac{2E_c(k)}{E_{D^*_i} + M_{D^*_i}} \mathbf{w}) \sqrt{\frac{E_i(k) + m_l}{E_i(k + w) + m_l}} \left\{ \frac{E_i(k + w) - E_i(k) + 2m_l}{2\sqrt{E_i(k + w)E_i(k)}} - \frac{k \cdot w}{2\sqrt{E_i(k + w)E_i(k)[E_i(k) + m_l]}} \right\} \Psi^{D^*_i}(k),$$

and to vector meson ($\rho^0, V_1$ and $V_8$)

$$h_1^{(v)}(w^2) = h_2^{(v)}(w^2) = h_3^{(v)}(w^2) = \tilde{h}_1^{(v)}(w^2) = \tilde{h}_2^{(v)}(w^2) = \tilde{h}_3^{(v)}(w^2) = 0,$$  \hspace{1cm} (35)

$$F_2(w^2) = \frac{2\sqrt{E_{D^*_i} M_{D^*_i}}}{E_{D^*_i} + M_{D^*_i}} \int \frac{d^3k}{(2\pi)^3} \overline{\Psi}^{D^*_i}(k + \frac{2E_c(k)}{E_{D^*_i} + M_{D^*_i}} \mathbf{w}) \sqrt{\frac{E_i(k) + m_l}{E_i(k + w) + m_l}} \left\{ \frac{E_i(k + w) + E_i(k)}{2\sqrt{E_i(k + w)E_i(k)}} + \frac{k \cdot w}{2\sqrt{E_i(k + w)E_i(k)[E_i(k) + m_l]}} \right\} \Psi^{D^*_i}(k),$$

where $E_{c,i}(p) = \sqrt{\mathbf{p}^2 + m_{c,i}^2}$ and $\Psi^{D^*_i}$ is the wave function of heavy vector meson expressed as Eq. (33). In this paper, some errors in our previous works have been revised. Equations (22) and (23) can be solved numerically with these form factors, and then the eigenvalue equation (27) can be solved. The masses $M_{D\bar{D}}$ and wave functions of pure bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*-}$ with $J^P = 0^+$ can be obtained.

Considering the interaction kernels from one-$\rho^\pm$ exchange and using the coupled-channel approach (see details in Ref. [2]), we can calculate the mass $M_0$ of the mixed state of two pure bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*-}$ with $J^P = 0^+$. Since the mixing of component wave
functions causes the change of energy, the fourth component of $P^{DD}$ in the original BS wave function becomes the total energy of mixed state, and $\chi^{0+}_{\lambda r}(P^{DD},p)$ in Eq. (20) becomes

$$\chi^{0+}_{\lambda r}(P,p) = \frac{1}{N_{0+}}[C_1 T_{\lambda r}^1 F_{10}(P \cdot p, p^2) + C_2 T_{\lambda r}^2 F_{20}(P \cdot p, p^2)].$$  \hspace{1cm} (36)$$

We emphasize that $M_0$ should not be the mass of resonance. Substituting Eq. (36) into (11), we obtain BS wave function $\chi^{D^0\bar{D}^0}_{\lambda r}(P,p)$ for the mixed state of two bound states $D^0\bar{D}^0$ and $D^+\bar{D}^-$ with $J^P = 0^+$. 

D. GBS wave function for four-quark state

The heavy meson is a bound state consisting of a quark and an antiquark and the meson-meson bound state is actually composed of four quarks. We have to give GBS wave function of meson-meson bound state as a four-quark state. If a bound state with spin $j$ and parity $\eta_P$ is composed of four quarks, its GBS wave function can be defined as

$$\chi^{j}_{P}(x_1, x_3, x_4, x_2) = \langle 0|TQ^C(x_1)\bar{Q}^A(x_3)Q^B(x_4)\bar{Q}^D(x_2)|P, j\rangle,$$ \hspace{1cm} (37)

where $P$ is the momentum of the four-quark bound state, $Q$ is the quark operator and its superscript is a flavor label. From translational invariance, this GBS wave function can be written as

$$\chi^{j}_{P}(x_1, x_3, x_4, x_2) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(P)}} e^{iP \cdot X} \chi^{j}_{P}(X', x, x'),$$ \hspace{1cm} (38)

where $X = \eta_1(\eta''_1 x_1 + \eta''_3 x_3) + \eta_2(\eta''_4 x_4 + \eta''_2 x_2)$, $X' = (\eta''_1 x_1 + \eta''_3 x_3) - (\eta''_4 x_4 + \eta''_2 x_2)$, $x = x_1 - x_3$, $x' = x_2 - x_4$, and $m_{a,b} = m_{C,A}/(m_{C} + m_{A})$, $\eta_{a,b}^0 = m_{D,B}/(m_{D} + m_{B})$ and $m_{A,B,C,D}$ are the quark masses. In the momentum representation, GBS wave function of four-quark bound state becomes

$$\chi^{j}_{P}(p_1, p_3, p_4, p_2) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(P)}} (2\pi)^4 \delta^{(4)}(P - p_1 + p_3 - p_4 + p_2) \chi^{j}(P, p, k, k'),$$ \hspace{1cm} (39)

where $p_1, p_3, p_4, p_2$ are the momenta carried by the fields $Q^C, Q^A, Q^B, Q^D$; $p, k, k'$ are the conjugate variables to $X'$, $x$, $x'$, respectively; and $p = \eta_2(p_1 - p_3) + \eta_1(p_2 - p_4)$, $k = \eta''_3 p_1 + \eta''_1 p_3$, $k' = \eta''_4 p_2 + \eta''_2 p_4$. In the hadronic molecule structure, $p$ is the relative momentum between two mesons in molecular state, $k$ and $k'$ are the relative momenta between quark and antiquark in these two mesons, respectively, shown as Fig. 2. This work is aimed to investigate the
bound state composed of two vector mesons. In Fig. 2, \( VM \) represents the vector meson with mass \( M_1 \), \( \overline{VM} \) represents the anti-particle of vector meson \( VM' \) with mass \( M_2 \), and \( MS \) represents the meson-meson bound state.

In Fig. 2, there are three two-body systems: a meson-meson bound state and two quark-antiquark bound states. We define BS wave functions of these two-body systems as \( \chi^j_P(p_1', p_2) \), \( \chi^j_p(p_1, p_3) \), \( \chi_{p_4}^j(p_4, p_2) \), respectively. BS wave function for the bound state of two vector mesons has been given by Eq. (39) and BS wave functions of two vector mesons are

\[
\chi^j_P(p_1, p_3) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(p_1)}} (2\pi)^4 \delta^{(4)}(p_1 - p_1 + p_3) \chi^j_P(p_1', k), \quad (40)
\]

\[
\chi_{p_4}^j(p_4, p_2) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(p_2)}} (2\pi)^4 \delta^{(4)}(p_4 + p_4 - p_2) \chi^j_{\tau}(p_2', k'), \quad (41)
\]

where \( p_1' \) and \( p_2' \) are the momenta of two vector mesons, respectively, \( p_1' = p + \eta_1 P \), \( p_2' = p - \eta_2 P \) and \( \eta_{1,2} = M_{1,2}/(M_1 + M_2) \). Applying the Feynman rules and comparing with Eq. (39), we obtain the revised GBS wave function for four-quark state describing the bound state composed of two vector mesons with arbitrary spin and definite parity

\[
\chi^j(P, p, k, k') = \chi^j_P(p_1', k) \chi^j_P(p_2', k') \chi^j_P(p_1, k) \chi^j_{\tau}(p_4, k') \chi^j_P(p_2, k'). \quad (42)
\]
From Eq. (30), we obtain BS wave functions of vector mesons

\[ \chi_\lambda(p'_1, k) = \frac{-1}{\gamma_c \cdot p_1 - i m_c \sqrt{V}} \left( \gamma_\lambda + p'_{1\lambda} \frac{\gamma \cdot p'}{M_V^2} \right) \varphi_V(k^2) \frac{-1}{\gamma_A \cdot p_3 - i m_A}, \]

\[ \chi_\tau(p'_2, k') = \frac{-1}{\gamma_B \cdot p_4 - i m_B \sqrt{V'}} \left( \gamma_\tau + p'_{2\tau} \frac{\gamma \cdot p'}{M_{V'}^2} \right) \varphi_{V'}(k'^2) \frac{-1}{\gamma_D \cdot p_2 - i m_D}. \] (43)

In this section, we consider a mixed state of two bound states \( D^{*0} \bar{D}^{*0} \) and \( D^{*+} D^{*-} \) with \( J^P = 0^+ \). In Fig. 2 \( VM \) and \( V M' \) become \( D'_1 \) and \( \bar{D}'_1 \), respectively, and in Eq. (37) the flavor labels \( C = D \) and \( A = B \) represent \( c \)-quark and \( l \)-quark, respectively. From Eqs. (10), (42) and (43), we obtain the GBS wave function for meson-meson bound state as a four-quark state

\[ \chi^{D^* \bar{D}^{*0}, 0^+}(P, p, k, k') = \frac{1}{\sqrt{2}} \chi^{D^{*0}}(p'_1, k) \chi^{D^{*0}, 0^+}(P, p) \chi^{\bar{D}^{*0}}(p'_2, k'), \]

\[ + \frac{1}{\sqrt{2}} \chi^{D^{*+}, 0^+}(p'_1, k) \chi^{D^{*+}, 0^+}(P, p) \chi^{D^{*-}}(p'_2, k'), \] (44)

where

\[ \chi^{D'_1}(p'_1, k) = \frac{-1}{\gamma \cdot p_1 - i m_c \sqrt{D'_1}} \left( \gamma_\lambda + p'_{1\lambda} \frac{\gamma \cdot p'}{M_{D'_1}^2} \right) \varphi_{D'_1}(k^2) \frac{-1}{\gamma \cdot p_3 - i m_c}, \]

\[ \chi^{D'_1}(p'_2, k') = \frac{-1}{\gamma \cdot p_4 - i m_c \sqrt{D'_1}} \left( \gamma_\tau + p'_{2\tau} \frac{\gamma \cdot p'}{M_{D'_1}^2} \right) \varphi_{D'_1}(k'^2) \frac{-1}{\gamma \cdot p_2 - i m_c}. \] (45)

**E. Normalization of BS wave function**

1. **Heavy vector meson**

Here, we determine normalizations \( \mathcal{N}^{D^{*0}} \) and \( \mathcal{N}^{D^{*+}} \). The authors of Refs. [15, 16] employed the ladder approximation to solve the BS equation for quark-antiquark state, and the reduced normalization condition for the BS wave function of \( D'_1 \) meson given by Eq. (30) is

\[ -i \frac{1}{(2\pi)^3} \int d^4k \chi_\lambda(K, k) \frac{\partial}{\partial K_0} [S_F(k + K)^{-1} S_F(k)^{-1} \chi_\lambda(K, k) = (2K_0)^2, \] (46)

where \( S_F(p)^{-1} \) is the inverse propagator for quark field and the factor 1/3 appears because of the sum of three transverse directions.
2. Molecular state

The reduced normalization condition for $\chi_{\lambda\tau}^0(P,p)$ expressed as Eq. (36) is

$$-i\left(\frac{2\pi}{4}\right)\int d^4p\bar{\chi}_{\lambda\tau}(P,p)\frac{\partial}{\partial P_0}[\Delta_{F\beta\alpha}(p + P/2)^{-1}\Delta_{F\beta\alpha}(p - P/2)^{-1}]\chi_{\lambda\tau}(P,p) = (2P_0)^2, \quad (47)$$

where $\Delta_{F\beta\alpha}(p)^{-1}$ is the inverse propagator for the vector field with mass $m$, $\Delta_{F\beta\alpha}(p)^{-1} = i(\delta_{\beta\alpha} - \frac{p_\beta p_\alpha}{p^2 + m^2})(p^2 + m^2)$ [8]. After determining normalization $N_0^+$, we automatically obtain the normalized BS wave function for the mixed state of two components $D^*\bar{D}^*$ and $D^*+D^*$ given by Eq. (10). Immediately, the normalized GBS wave function for meson-meson bound state as a four-quark state expressed as Eq. (44) is obtained.

III. SCATTERING MATRIX ELEMENT FROM FOUR-QUARK STATE TO FINAL STATE

In experiments only one strong decay mode of resonance $X(3915)$ has been observed: $X(3915) \rightarrow J/\psi\omega$ [19–21]. In this section, we present the traditional technique to calculate decay width for this process and revise some errors in previous works [5, 8]. Applying Mandelstam’s approach, we have obtained the scattering matrix element from a four-quark state to a heavy meson plus a light meson [8] in the momentum representation, as shown in Fig. 3. In present work, we retain only the lowest order term of the two-particle irreducible Green’s function.

![Fig. 3: The lowest order matrix element between bound states in the momentum representation.](image)

In Fig. 3 $VM$ and $VM\dagger$ still represent $D^*_l$ and $\bar{D}^*_l$, respectively; $HM$ represents $J/\psi$ with momentum $Q$ ($Q^2 = -M_{J/\psi}^2$) and $LM$ represents $\omega$ with momentum $Q'$ ($Q'^2 = -M_\omega^2$).
The momentum of the initial state is set as $P = (0, 0, 0, iM_0)$ in the rest frame, and $M_0$ is the mass of the mixed state of two pure bound states $D^{*0}D^{*0}$ and $D^{*+}D^{*-}$, which should not be the mass of resonance. It is necessary to emphasize that the momenta in the final state satisfy $Q + Q' = P$ in this section. Heavy vector meson $J/\psi$ in the final state is a bound state of $c\bar{c}$. From Eq. (43), we obtain the BS wave function of heavy vector meson $J/\psi$

$$
\chi_{\nu}(Q, q) = \frac{-1}{\gamma \cdot (q + Q/2) - im_{c}} \frac{1}{N^{J/\psi}_{\nu}} \left( \gamma_{\nu} + Q \nu \cdot Q \right) \varphi_{J/\psi}(q^2) \frac{-1}{\gamma \cdot (q - Q/2) - im_{c}},
$$

(48)

where $\varphi_{J/\psi}(q^2) = \exp(-q^2/\omega_{J/\psi}^2)$ and $\omega_{J/\psi}=0.826$ GeV was obtained from lattice QCD (see details in Ref. [8]). The reduced normalization condition for BS wave function of $J/\psi$ meson expressed as Eq. (48) is

$$
\frac{-i}{(2\pi)^4} \frac{1}{3} \int d^4q \tilde{\chi}_{\nu}(Q, q) \frac{\partial}{\partial Q_0} [S_F(q + Q/2)^{-1}S_F(q - Q/2)^{-1}] \chi_{\nu}(Q, q) = (2Q_0)^2, \quad (49)
$$

where the factor $1/3$ appears for the three transverse directions are summed. Normalization $N^{J/\psi}$ can be determined. These momenta in Fig. B become

$$
p_1 = (Q + Q')/2 + p + k, \quad p_2 = (Q + Q')/2 - Q + p + k, \quad p_3 = k, \quad p_4 = Q' + k,
$$

$$
q = Q'/2 + p + k, \quad q_1 = q + Q/2, \quad q_2 = q - Q/2, \quad Q + Q' = P, \quad (50)
$$

$$
k' = Q' + k, \quad p_1' = p + P/2, \quad p_2' = p - P/2.
$$

Using the Heisenberg picture, we obtain the total matrix element from the initial state $|P \text{ in}\rangle$ to a final state $|Q, Q' \text{ out}\rangle$

$$
-i R_{ba}(M_0) = \langle Q, Q' \text{ out}|P \text{ in}\rangle = -i(2\pi)^4 \delta^{(4)}(Q + Q' - P) T_{ba}(M_0), \quad (51)
$$

where $T_{ba}$ is the $T$-matrix element. According to Mandelstam’s approach, we obtain

$$
T_{ba}(M_0) = \frac{ig_\omega \varepsilon^{\nu}_{\nu}(Q) \varepsilon^{\nu}_{\nu}(Q)}{(2\pi)^{3/2} \sqrt{2E_{J/\psi}(Q) \sqrt{2E_\omega(Q)} \sqrt{2E(P)}} \left( \frac{1}{\sqrt{2}} M_{\nu\mu}^{D^{*0}D^{*0}} + \frac{1}{\sqrt{2}} M_{\nu\mu}^{D^{*+}D^{*-}} \right), \quad (52)
$$

where $\varepsilon^{\nu}_{\nu}=1,2,3(Q)$ and $\varepsilon^{\nu}_{\nu}=1,2,3(Q')$ are the polarization vectors of $J/\psi$ and $\omega$, respectively, $\varepsilon^{\nu}(Q) \cdot Q = \varepsilon^{\nu}(Q') \cdot Q' = 0$, and

$$
M_{\nu\mu}^{D^{*0}D^{*0}} = \int \frac{d^4k d^4p}{(2\pi)^8} \frac{1}{N^{J/\psi}} \frac{1}{N^{D^{*0}}_{\nu}} \frac{1}{N^{D^{*0}}_{\mu}} \frac{1}{N^{D^{*0}}_{\nu}} \frac{1}{N^{D^{*0}}_{\mu}} \frac{1}{N^{D^{*0}}_{\nu}} \frac{1}{N^{D^{*0}}_{\mu}} \frac{1}{N^{D^{*0}}_{\nu}} \frac{1}{N^{D^{*0}}_{\mu}} \frac{1}{N^{D^{*0}}_{\nu}} \frac{1}{N^{D^{*0}}_{\mu}}
$$

$$
\times \text{Tr}\{ (\gamma \cdot p_2 + im_\omega) \gamma_{\nu}(\gamma \cdot p_1 + im_\omega) \gamma_{\nu} \}
$$

$$
\times \{ (p_1' \cdot p_2') g_{\nu\rho} - p_{1\lambda} p_{2\rho}' - p_{1\lambda} p_{2\rho}' + p_{1\lambda} p_{2\rho}' - p_{1\lambda} p_{2\rho}' - p_{1\lambda} p_{2\rho}' \}
$$

$$
+ (p_1' \cdot p_2') g_{\nu\rho} - p_{1\lambda} p_{2\rho}' - p_{1\lambda} p_{2\rho}' - p_{1\lambda} p_{2\rho}' - p_{1\lambda} p_{2\rho}' - p_{1\lambda} p_{2\rho}' \}
$$

$$
\times (\gamma \cdot p_3 + im_\omega) \gamma_{\nu}(\gamma \cdot p_4 + im_\omega) \gamma_{\nu} \}, \quad (53)
$$

17
In Eq. \((53)\) the trace of the product of 8 \(\gamma\)-matrices contains 105 terms and the resulting expression has been given in Appendix B of Ref. \([8]\). In our approach, the \(p\) integral is computed in instantaneous approximation. To calculate this tensor \(M_{\nu\mu}^{D_i\bar{D}_i}\), we have given a simple method in Ref. \([8]\). It is obvious that the tensor \(M_{\nu\mu}^{D_i\bar{D}_i}\) only depends on \(Q\) and \(Q'\), so in Minkowski space \(M_{\nu\mu}^{D_i\bar{D}_i}\) can be expressed as
\[
M_{\nu\mu}^{D_i\bar{D}_i} = g_{\nu\mu}U_1(Q', Q) + Q'_{\nu}Q_{\mu}U_2(Q', Q) + Q'_{\nu}Q'_{\mu}U_3(Q', Q) + Q_{\nu}Q'_{\mu}U_4(Q', Q) + Q_{\nu}Q_{\mu}U_5(Q', Q),
\]
(54)
where \(U_i(Q', Q)(i = 1, \cdots, 5)\) are scalar functions. The above expression is multiplied by these tensor structures \(g_{\nu\mu}, Q'_{\nu}Q_{\mu}, Q'_{\nu}Q'_{\mu}, Q_{\nu}Q'_{\mu}, Q_{\nu}Q_{\mu}\), respectively; and a set of equations is obtained
\[
g_{\nu\mu}M_{\nu\mu}^{D_i\bar{D}_i} = U'_1 = 4U_1 + (Q' \cdot Q)U_2 + Q'^2U_3 + (Q' \cdot Q)U_4 + Q^2U_5,
Q'_{\nu}Q_{\mu}M_{\nu\mu}^{D_i\bar{D}_i} = U'_2 = (Q' \cdot Q)U_1 + Q'^2Q^2U_2 + Q'^2(Q' \cdot Q)U_3 + (Q' \cdot Q)^2U_4 + Q^2(Q' \cdot Q)U_5,
Q'_{\nu}Q'_{\mu}M_{\nu\mu}^{D_i\bar{D}_i} = U'_3 = Q'^2U_1 + Q'^2(Q' \cdot Q)U_2 + Q^2Q'^2U_3 + Q^2(Q' \cdot Q)U_4 + (Q' \cdot Q)^2U_5,
Q_{\nu}Q'_{\mu}M_{\nu\mu}^{D_i\bar{D}_i} = U'_4 = (Q' \cdot Q)U_1 + (Q' \cdot Q)^2U_2 + Q^2(Q' \cdot Q)U_3 + Q^2Q'^2U_4 + Q^2(Q' \cdot Q)U_5,
Q_{\nu}Q_{\mu}M_{\nu\mu}^{D_i\bar{D}_i} = U'_5 = Q^2U_1 + Q^2(Q' \cdot Q)U_2 + (Q' \cdot Q)^2U_3 + Q^2(Q' \cdot Q)U_4 + Q^2Q'^2U_5,
\]
(55)
where \(U'_i\) are numbers. Subsequently, we numerically calculate \(U'_i\) and solve this set of equations. The values of \(U_i\) can be obtained.

Then the decay width for this process in traditional calculation becomes
\[
\Gamma(M_0) = \int d^3Qd^3Q'2(2\pi)^4\delta^{(4)}(Q + Q' - P) \sum_{\varrho'=1}^{3} \sum_{\varrho=1}^{3} (2\pi)^3 |T_{ba}(M_0)|^2,
\]
(56)
where \(P = (0, 0, 0, iM_0)\), \(Q = (Q(M_0), i\sqrt{Q^2(M_0) + M_{J/\psi}^2})\), \(Q' = (-Q(M_0), i\sqrt{Q'^2(M_0) + M_{J/\psi}^2})\) and \(Q^2(M_0) = [M_0^2 - (M_{J/\psi} + M_{\omega})^2] [M_0^2 - (M_{J/\psi} - M_{\omega})^2]/(4M_0^2)\). To calculate the decay width \(\Gamma(M_0)\), we use the transverse condition \(\varepsilon^e(Q) \cdot Q = \varepsilon^{e'}(Q') \cdot Q' = 0\) and the completeness relation. It is necessary to emphasize that the decay width \(\Gamma(M_0)\) expressed as Eq. \((56)\) is not the final result for resonance.
IV. TEMPORAL EVOLUTION DETERMINED BY TOTAL HAMILTONIAN

Secs. II and III give the traditional technique to deal with molecular state in present particle physics. These masses of meson-meson bound states were regarded as masses of resonances [1–4] and used to calculate decay widths of resonances [5, 8], which should not be impeccable. To deal with resonance in the framework of relativistic quantum field theory, it is natural to seek a development of BS equation, which should consider the temporal evolution of molecular state as determined by the total Hamiltonian.

A. Prepared state

Because the temporal evolution of molecular state is determined by the total Hamiltonian, exotic meson resonance should be considered as an unstable meson-meson molecular state. Since resonance certainly decays, we can suppose that this unstable state has been prepared to decay at given time, and the prepared state can be regarded as a bound state with ground-state energy. Solving BS equation for arbitrary meson-meson bound state, one can obtain the mass $M_0$ and BS wave function $\chi_P(x'_1, x'_2)$ for this bound state. Setting $t_1 = 0$ and $t_2 = 0$ in the ground-state BS wave function, we obtain a description for the prepared state

$$\mathcal{X}_a = \chi_P(x'_1, t_1 = 0, x'_2, t_2 = 0) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(P)}} e^{iP \cdot \mathbf{X}} \chi_P(X').$$  \hspace{1cm} (57)

B. Temporal evolution of prepared state

Now it is necessary to consider the total Hamiltonian

$$H = K_I + V_I,$$ \hspace{1cm} (58)

where $K_I$ represents the interaction responsible for the formation of stationary bound state and $V_I$ stands for the interaction responsible for the decay of resonance. Then the time evolution of this system determined by the total Hamiltonian $H$ has the explicit form

$$\mathcal{X}(t) = e^{-iHt} \mathcal{X}_a = \frac{1}{2\pi i} \int_{C_2} d\epsilon e^{-iut} \frac{1}{\epsilon - H} \mathcal{X}_a,$$ \hspace{1cm} (59)
where $\mathcal{G}(\epsilon) = (\epsilon - H)^{-1}$ is the Green’s function and the contour $C_2$ runs from $ic_r + \infty$ to $ic_r - \infty$ in energy-plane ($\epsilon$-plane). The positive constant $c_r$ is sufficiently large that no singularity of $(\epsilon - H)^{-1}$ lies above $C_2$. The time-dependent wave function $\mathcal{X}'(t)$ provides a complete description of the system for $t > 0$. Since $H \neq K_I$, this system should not remain in the prepared state $\mathcal{X}_a$. The Green’s function can be represented by scattering matrix \[22\]

$$
G_a(\epsilon) = (\chi_a, G(\epsilon) \chi_a) = \frac{1}{\epsilon - M_0 - (2\pi)^3 T_a(\epsilon)},
$$

where $\chi_a$ represents $\chi_P(X')$ in Eq. \[57\]. The Green’s function $G(\epsilon)$ now operates on $\chi_a$, and we write as before $G(\epsilon) = (\epsilon - H)^{-1}$. The operator $T$ is just the scattering matrix in field theory, and $T_a(\epsilon)$ is the $T$-matrix element between two bound states, which is defined as

$$
\langle a \text{ out}|a \text{ in} \rangle = \langle a \text{ in}|a \text{ in} \rangle - i(2\pi)^4 \delta^{(4)}(P - P) T_a(\epsilon).
$$

This work will give $T_a(\epsilon)$ in the framework of relativistic quantum field theory.

V. T-MATRIX ELEMENT $T_a(\epsilon)$

Let $\epsilon_M$ denote the sum of all particle masses in the final state. It is easy to understand that $G_a(\epsilon)$ and $T_a(\epsilon)$ are analytic in the entire complex $\epsilon$-plane, except along a cut extending from $\epsilon_M$ to plus infinity on the real axis, defined by $\epsilon_M < \epsilon < \infty$. Considering that $\epsilon$ approaches the real axis from above, we define

$$
T_a(\epsilon) = D(\epsilon) - iI(\epsilon),
$$

where $D$ and $I$ are the real and imaginary parts of $T_a$, respectively. Using dispersion relation for the function $T_a(\epsilon)$, we obtain

$$
D(\epsilon) = \frac{\mathcal{P}}{\pi} \int_{\epsilon_M}^{\infty} \frac{I(\epsilon')}{\epsilon' - \epsilon} d\epsilon',
$$

where the symbol $\mathcal{P}$ means that this integral is a principal value integral. To calculate the real part, we need calculate the function $I(\epsilon')$ over the real interval $\epsilon_M < \epsilon' < \infty$. We suppose that the final state $b$ may contain $n$ composite particles and $n'$ elementary particles in decay channel $c$, and the function $I(\epsilon')$ has the form

$$
I(\epsilon') = \frac{1}{2} \int d^3Q' \cdots d^3Q_{n'} d^3Q_1 \cdots d^3Q_n (2\pi)^4 \delta^{(4)}(Q'_1 + \cdots + Q_n - P') \sum_{\text{spins}} |T_{(c;b)a}(\epsilon')|^2,
$$

\[64\]
where \( Q'_1 \ldots Q'_{n'}, Q_1 \ldots Q_n \) are the momenta of final particles, \( P^{e'} \) represents the total momentum of the final state, \( T_{(c:b)a}(e') \) is the T-matrix element with respect to \( e' \), and \( \sum_{\text{spins}} \) represents summing over final spins and averaging over initial spins. Since the mass \( M_0 \) and BS amplitude of initial bound state have been specified, the total energy \( e' \) of the final state extends from \( \epsilon_M \) to \(+\infty\) and the energy in scattering matrix \( T \) is equal to the final state energy. As usual the momentum of initial bound state is set as \( P = (0, 0, 0, iM_0) \) in the rest frame and \( P^{e'} = (0, 0, 0, ie') \). From Eq. (51), we obtain the total matrix element between the final state \( |J/\psi, \omega\rangle \) and the specified initial four-quark state

\[
-\imath R_{(c:b)a}(e') = \langle Q, Q' \text{ out} | P \text{ in} \rangle = -\imath (2\pi)^4 \delta^{(4)}(Q + Q' - P^{e'}) T_{(c:b)a}(e'),
\]

(66)

where the total energy \( e' \) of the final state extends from \( \epsilon_{c1,M} \) to \(+\infty\), and \( \epsilon_{c1,M} = M_{J/\psi} + M_\omega \). Though the T-matrix element \( T_{(c:b)a}(e') \) has the same form expressed as Eq. (52), these momenta should become

\[
p_1 = (Q + Q')/2 + p + k, \quad p_2 = (Q + Q')/2 - Q + p + k, \quad p_3 = k, \quad p_4 = Q' + k,
\]

\[
q = Q'/2 + p + k, \quad q_1 = q + Q'/2, \quad q_2 = q - Q/2, \quad Q + Q' = P^{e'},
\]

(67)

\[
k' = Q'(M_0) + k, \quad p'_1 = p + P/2, \quad p'_2 = p - P/2,
\]

where \( P = (0, 0, 0, iM_0) \), \( P^{e'} = (0, 0, 0, ie') \), \( Q'(M_0) = -Q(M_0), i\sqrt{Q^2(M_0) + M_\omega^2} \), \( Q = (Q(\epsilon'), i\sqrt{Q^2(\epsilon') + M_{J/\psi}^2}) \), \( Q' = (-Q(\epsilon'), i\sqrt{Q^2(\epsilon') + M_{J/\psi}^2}) \), \( Q^2(\epsilon') = [\epsilon'^2 - (M_{J/\psi}^2 + M_\omega^2)] \), and \( Q'(\epsilon') \).
where $\epsilon'$ depends on $P$. The energy in the two-particle irreducible Green’s function is equal to the final state energy $\epsilon'$, and the difference between the final and initial energies $\epsilon' \neq M_0$ is illustrated by crosses in new Feynman diagram. In Fig. 4, the momenta in left-hand side of crosses satisfy energy-momentum conservation and the momenta in right-hand side satisfy energy-momentum conservation, respectively, i.e., $p_1 - p_2 - p_3 + p_4 = P\epsilon'$ and $p_1' - p_2' = P$. When $\epsilon' = M_0$, the crosses in Fig. 4 disappear and then Fig. 4 becomes Fig. 3; $T_{(c1,b)a}(M_0)$ is regarded as the matrix element for this decay process in traditional calculation. Then we obtain the function $I_1(\epsilon')$ for decay channel $J/\psi \omega$

$$I_1(\epsilon') = \frac{1}{2} \int d^3Q d^3Q'(2\pi)^4 \delta^{(4)}(Q + Q' - P\epsilon') \sum_{\epsilon'=1}^{3} \sum_{\epsilon=1}^{3} |T_{(c1,b)a}(\epsilon')|^2.$$

FIG. 4: Matrix element between bound states in the momentum representation. The momenta in the final state satisfy $Q + Q' = P\epsilon'$ and the momentum of the initial state is $P$.

B. Decay channel $D^* \bar{D}^*$

The final state $|D^*, \bar{D}^*\rangle$ can be written as

$$|D^*, \bar{D}^*\rangle = \frac{1}{\sqrt{2}}|D^{*0}, \bar{D}^{*0}\rangle + \frac{1}{\sqrt{2}}|D^{*+}, D^{*-}\rangle.$$

(70)
The total energy $\epsilon'$ of the final state extends from $\epsilon_{c_2,M}$ to $+\infty$, and $\epsilon_{c_2,M} = M_{D_1^*} + M_{\bar{D}_1^*}$. Since bound state lies below the threshold, this decay channel can not occur inside the physical world. Considering the lowest order term of the two-particle irreducible Green’s function, we obtain the interaction between two heavy vector mesons derived from a light meson exchange.

Applying Mandelstam’s approach, we can calculate the $T$-matrix element $T_{(c_2:b)\alpha}(\epsilon')$ between the final state $|D_1^*\bar{D}_1^*\rangle$ and the initial four-quark state, which can be represented graphically by Fig. 5. In Fig. 5 $VM$ and $\overline{VM}$ still represent $D_1^*$ and $\bar{D}_1^*$, respectively; $Q_1$ and $Q_2$ represent the momenta of final particles, $p_1 - p_2 - p_3 + p_4 = p_1 - p_2 - q_3 + q_4 = P', p_1' - p_2' = P$, and the crosses mean that $\epsilon' \neq M_0$.

Using the Heisenberg picture, we obtain the reduced matrix element between the final state $|D^*,\bar{D}^*\rangle$ and the mixed state of two pure bound states $D_0^*\bar{D}_0^*$ and $D_+^*\bar{D}_-^*$

$$-iR_{(c_2:b)\alpha}(\epsilon') = \langle Q_1, Q_2 \text{ out} | P \text{ in} \rangle = -i(2\pi)^4 \delta^{(4)}(Q_1 + Q_2 - P') T_{(c_2:b)\alpha}(\epsilon'). \quad (71)$$
According to Mandelstam’s approach, the $T$-matrix element becomes

$$T_{\nu(\alpha)}(\epsilon') = \frac{1}{2} \sum_{i=u,d} \frac{-i\epsilon_{\mu}^{\nu}(Q_2)\epsilon_{\mu}^{\nu}(Q_1)}{(2\pi)^3} \left( \mathcal{M}_{c2,\nu\mu} + \mathcal{M}'_{c2,\nu\mu} \right),$$  \hspace{1cm} (72)

where $\epsilon_{\mu}^{\nu=1,2,3}(Q_1)$ and $\epsilon_{\mu}^{\nu'=1,2,3}(Q_2)$ become the polarization vectors of $D_1^*$ and $\bar{D}_1^*$, respectively, and

$$\mathcal{M}_{c2,\nu\mu} = \int \frac{d^4p}{(2\pi)^4} V_{\nu,\lambda,\tau\mu}(Q_1, Q_2, p) \frac{1}{N_{0^+}} \{ [(p_1' \cdot p_2')g_{\lambda\tau} - p_2'p_1'_{\lambda\tau}] C_1 F_{10}(P \cdot p, p^2) \\
+ [p_1^2p_2^2g_{\lambda\tau} + (p_1' \cdot p_2')p_1'_{\lambda\tau}p_2'_{\lambda\tau} - p_2^2p_1^2p_1'_{\lambda\tau}p_2'_{\lambda\tau} - p_1^2p_2^2p_1'_{\lambda\tau}p_2'_{\lambda\tau}] C_2 F_{20}(P \cdot p, p^2) \},$$  \hspace{1cm} (73a)

$$\mathcal{M}'_{c2,\nu\mu} = \int \frac{d^4p}{(2\pi)^4} V_{\nu,\lambda,\tau\mu}(Q_1, Q_2, p) \frac{1}{N_{0^+}} \{ [(p_1' \cdot p_2')g_{\lambda\tau} - p_2'p_1'_{\lambda\tau}] C_1 F_{10}(P \cdot p, p^2) \\
+ [p_1^2p_2^2g_{\lambda\tau} + (p_1' \cdot p_2')p_1'_{\lambda\tau}p_2'_{\lambda\tau} - p_2^2p_1^2p_1'_{\lambda\tau}p_2'_{\lambda\tau} - p_1^2p_2^2p_1'_{\lambda\tau}p_2'_{\lambda\tau}] C_2 F_{20}(P \cdot p, p^2) \}. $$  \hspace{1cm} (73b)

In Eq. (73), $V_{\nu,\lambda,\tau\mu}(Q_1, Q_2, p)$ and $V_{\nu,\lambda,\tau\mu}'(Q_1, Q_2, p)$ represent the kernels derived from one light meson ($\sigma, \rho^0, V_1$ and $V_8$) exchange and one-$\rho^\pm$ exchange, respectively.

Now, we determine the kernels $V_{\nu,\lambda,\tau\mu}(Q_1, Q_2, p)$ and $V_{\nu,\lambda,\tau\mu}'(Q_1, Q_2, p)$. Setting $\tilde{p}_1 = Q_1 + w$ and $\tilde{p}_2 = -Q_2 + w$, we obtain the vertices of the heavy vector meson interaction.
with light meson, structurally similar to Eq. (16)

\[
\langle VM^0(Q_1) | J | VM^α(\tilde{p}_1) \rangle = \frac{1}{2 \sqrt{E_{D_1}(Q_1) E_{D_1}(\tilde{p}_1)}} [\varepsilon^α(Q_1) \cdot \varepsilon^0(\tilde{p}_1)] h_1^{(s)}(w^2),
\]

(74a)

\[
\langle VM^0(Q_2) | J | VM^α(-\tilde{p}_2) \rangle = \frac{1}{2 \sqrt{E_{D_1}(Q_2) E_{D_1}(-\tilde{p}_2)}} [\varepsilon^α(Q_2) \cdot \varepsilon^0(-\tilde{p}_2)] h_1^{(s)}(w^2),
\]

(74b)

\[
\langle VM^0(Q_1) | J_α | VM^α(\tilde{p}_1) \rangle = \frac{1}{2 \sqrt{E_{D_1}(Q_1) E_{D_1}(\tilde{p}_1)}} \{[\varepsilon^α(Q_1) \cdot \varepsilon^α(\tilde{p}_1)] h_1^{(v)}(w^2)(Q_1 + \tilde{p}_1)_α \\
- h_2^{(v)}(w^2)\} \{[\varepsilon^α(Q_1) \cdot \varepsilon^α(\tilde{p}_1)] + [\varepsilon^{\alpha}(\tilde{p}_1) \cdot Q_1]\}
\]

(74c)

\[
\langle VM^0(Q_2) | J_β | VM^α(-\tilde{p}_2) \rangle = \frac{1}{2 \sqrt{E_{D_1}(Q_2) E_{D_1}(-\tilde{p}_2)}} \{[\varepsilon^α(Q_2) \cdot \varepsilon^0(-\tilde{p}_2)] h_1^{(v)}(w^2)(Q_2 - \tilde{p}_2)_β \\
- h_2^{(v)}(w^2)\} \{[\varepsilon^α(Q_2) \cdot (-\tilde{p}_2)] + [\varepsilon^{\alpha}(-\tilde{p}_2) \cdot Q_2]\}
\]

(74d)

where \( w = p + (Q_1 + Q_2)/2 - Q_1 \) is the momentum of the light meson, \( h(w^2) \) and \( \tilde{h}(w^2) \) are the heavy meson form factors with respect to \( \epsilon' \). Similarly, taking away the external lines including normalizations and polarization vectors \( \varepsilon^0_α(Q_1), \varepsilon^0_α(\tilde{p}_1), \varepsilon^α_μ(\tilde{p}_2), \varepsilon^α_τ(-\tilde{p}_2) \), we obtain the kernel from one light meson (\( \sigma, \rho^0, V_1 \) and \( V_8 \)) exchange

\[
\mathcal{V}_{νλ,τμ}(Q_1, Q_2, p)
\]

\[= -2E_1(\epsilon')F_1(w^2)\frac{-iɡ^2_σ}{w^2 + M_σ^2}2E_2(\epsilon')F_1(w^2)\delta_{νλ}\delta_{τμ} + F_2(w^2)\left(\frac{-iɡ^2_ρ}{w^2 + M_ρ^2} + \frac{-iɡ^2_τ}{w^2 + M_τ^2} + \frac{-iɡ^2_8}{w^2 + M_8^2}\right)\]

\[\times F_2(w^2)\{(Q_1 + \bar{p}_1) \cdot (Q_2 - \tilde{p}_2)\delta_{νλ}\delta_{τμ} - \delta_{νλ}[-(Q_1 + \bar{p}_1)_τ\bar{p}_{2μ} + Q_{2τ}(Q_1 + \bar{p}_1)_μ] \\
- [\bar{p}_{1ν}(Q_2 - \tilde{p}_2)_λ + (Q_2 - \tilde{p}_2)_νQ_{1λ}]\delta_{τμ} - \bar{p}_{1ν}\delta_{λ τ}\bar{p}_{2μ} + \bar{p}_{1ν}\delta_{λ μ}Q_{2τ} - \delta_{ντ}Q_{1λ}\bar{p}_{2μ} + \delta_{νμ}Q_{1λ}Q_{2τ}\}\]

(75)

where \( E_1(\epsilon') = E_2(\epsilon') = \epsilon'/2, \bar{p}_1 = p + (Q_1 + Q_2)/2, \tilde{p}_2 = p - (Q_1 + Q_2)/2 \) and \( w = (w, 0) \).

The kernel from one-\( ρ^± \) exchange becomes

\[
\mathcal{V}_{νλ,τμ}'(Q_1, Q_2, p)
\]

\[= F_2(w^2)\frac{-i2ɡ^2_ρ}{w^2 + M_ρ^2}F_2(w^2)\{(Q_1 + \bar{p}_1) \cdot (Q_2 - \tilde{p}_2)\delta_{νλ}\delta_{τμ} - \delta_{νλ}[-(Q_1 + \bar{p}_1)_τ\bar{p}_{2μ} + Q_{2τ}(Q_1 + \bar{p}_1)_μ] \\
- [\bar{p}_{1ν}(Q_2 - \tilde{p}_2)_λ + (Q_2 - \tilde{p}_2)_νQ_{1λ}]\delta_{τμ} - \bar{p}_{1ν}\delta_{λ τ}\bar{p}_{2μ} + \bar{p}_{1ν}\delta_{λ μ}Q_{2τ} - \delta_{ντ}Q_{1λ}\bar{p}_{2μ} + \delta_{νμ}Q_{1λ}Q_{2τ}\}\]

(76)
These momenta in Fig. 7 become

\[ w = (p - Q_D(e'), 0), \quad Q_1 + Q_2 = P^{e'}, \quad p'_1 = p + P/2, \quad p'_2 = p - P/2, \quad (77) \]

where \( Q_1 = (Q_D(e'), i\epsilon'/2), \quad Q_2 = (-Q_D(e'), i\epsilon'/2) \) and \( Q^2_D(e') = [\epsilon'^2 - (M_{D_t} + M_{D_t})^2]/(4\epsilon'^2) \).

Substituting Eqs. (75) and (76) into (73), we obtain the explicit forms for tensors \( \mathcal{M}_{c,2,\nu\mu} \) and \( \mathcal{M}'_{c,2,\nu\mu} \). The \( p \) integral is also computed in instantaneous approximation. \( \mathcal{M}_{c,2,\nu\mu} \) and \( \mathcal{M}'_{c,2,\nu\mu} \) only depend on \( Q_1 \) and \( Q_2 \), which can be calculated by means of the method given in Sec. III. Applying Eq. (64), we obtain the function \( I_2(e') \) for decay channel \( D^*\bar{D}^* \)

\[ I_2(e') = \frac{1}{2} \int d^3 Q_1 d^3 Q_2 (2\pi)^4 \delta^{(4)}(Q_1 + Q_2 - P^{e'}) \sum_{\nu' = 1}^3 \sum_{\mu' = 1}^3 |T_{(e,2)\mu'}(e')|^2. \quad (78) \]

VI. THE POLE OF GREEN'S FUNCTION FOR RESONANCE

Since \( G_{a}(\epsilon) \) and \( T_{a}(\epsilon) \) are analytic in the entire complex \( \epsilon \)-plane except for the cut on the real axis \( \epsilon_M < \epsilon < \infty \), the complex \( \epsilon \)-plane except this cut can be defined as the first Riemann sheet and Eq. (62) defines \( T_{a}(\epsilon) = T_{a}^0(\epsilon) \) on the first Riemann sheet. Then the function \( T_{a}(\epsilon) \) can be continued analytically onto the second Riemann sheet, where we write \( T_{a}(\epsilon) = T_{a}^{\Pi}(\epsilon) \). This continuation onto the second Riemann sheet permits us to obtain

\[ T_{a}^{\Pi}(\epsilon - i\eta) = T_{a}^0(\epsilon + i\eta) = D(\epsilon) - iI(\epsilon), \quad (79) \]

where \( \epsilon \) is real and \( \eta \) is small and positive. Moreover, Eq. (60) implies that \( G_{a}(\epsilon) = [\epsilon - M_0 - (2\pi)^3 T_{a}(\epsilon)]^{-1} \) is analytic on the first Riemann sheet. But \( G_{a}(\epsilon) \) is not in general analytic on the second sheet. In experiments, many exotic particles are narrow states and their decay widths are very small compared with their energy levels, i.e., \((2\pi)^3 I(M_0) \ll M_0\). This situation is ordinarily interpreted as implying that both \((2\pi)^3 D(\epsilon)|\) and \((2\pi)^3 I(\epsilon)\) are also very small quantities, as compared to \( M_0 \). Therefore, we can expect that \( G_{a}(\epsilon) \) has a pole on the second Riemann sheet

\[ \epsilon_0 \approx M_0 + (2\pi)^3 T_{a}^{\Pi}(M_0 - i\eta) = M_0 + (2\pi)^3[D(M_0) - iI(M_0)] = M_0 + \frac{\Gamma(M_0)}{2}, \quad (80) \]

where \((2\pi)^3 D(M_0)\) is the correction for energy level of resonance and \( M = M_0 + (2\pi)^3 D(M_0)\) is the corrected mass for resonance. This pole at \( \epsilon_0 \) describes the resonance. The mass \( M_0 \)
of two-body bound state is obtained by solving BS equation, which should not be the mass of resonance. For resonance \( X(3915) \), the dispersion relation (63) becomes

\[
D(M_0) = -\frac{P}{\pi} \int_{\epsilon_{c1,M}}^{\infty} \frac{I_1(\epsilon')}{\epsilon' - M_0} d\epsilon' - \frac{1}{\pi} \int_{\epsilon_{c2,M}}^{\infty} \frac{I_2(\epsilon')}{\epsilon' - M_0} d\epsilon',
\]

(81)

where \( \epsilon_{c1,M} = M_{J/\psi} + M_\omega \) and \( \epsilon_{c2,M} = M_{D^*_l} + M_{\bar{D}^*_l} \). Replacing \( M_0 \) by \( M \) in Eqs. (52) and (56), we recalculate the matrix element \( T_{ba}(M) \) and obtain the corrected decay width \( \Gamma \) for resonance \( X(3915) \).

VII. NUMERICAL RESULT

Since the isospin conservation, we have the constituent quark masses \( m_u = m_d = 0.33 \) GeV, \( m_c = 1.55 \) GeV [23] and the meson masses \( M_\sigma = 0.42 \) GeV, \( M_\omega = 0.782 \) GeV, \( M_{\rho^0} = M_{\rho^0} = 0.775 \) GeV, \( M_{\rho^+} = 1.019 \) GeV, \( M_{D^{*0}} = M_{D^{*+}} = 2.007 \) GeV, \( M_{J/\psi} = 3.097 \) GeV [24]. Numerically solving the eigenvalue equation (27), we obtain the masses and wave functions of pure bound states \( D^{*0} \bar{D}^{*0} \) and \( D^{*+} D^{*-} \) with \( J^P = 0^+ \). Considering the cross terms between these two pure bound states \( D^{*0} \bar{D}^{*0} \) and \( D^{*+} D^{*-} \) and using the coupled-channel approach, we obtain the mass \( M_0 \) of the mixed state with \( J^P = 0^+ \) without an adjustable parameter. Then \( M_0 \) and GBS wave function \( \chi^{D^{*} \bar{D}^{*},0^+}(P,p,k,k') \) given in Eq. (44) are used to evaluate the matrix element \( T_{ba}(M_0) \) and the traditional decay width \( \Gamma(M_0) \) for decay model \( X(3915) \rightarrow J/\psi \omega \), which should not be the final results for resonance. From Eqs. (52), (53) and (67), we calculate the \( T \)-matrix element \( T_{(c1;b)a}(\epsilon') \) for decay channel \( J/\psi \omega \). From Eqs. (72), (73), (75), (76) and (77), we calculate the \( T \)-matrix element \( T_{(c2;b)a}(\epsilon') \) for decay channel \( D^* \bar{D}^* \). From Eqs. (69) and (78), we calculate the functions \( I_1(\epsilon') \) over \( \epsilon_{c1,M} < \epsilon' < \infty \) and \( I_2(\epsilon') \) over \( \epsilon_{c2,M} < \epsilon' < \infty \), respectively. By doing the numerical calculation, we obtain the correction term \((2\pi)^3 D(M_0)\) and the corrected mass \( M \) for resonance \( X(3915) \). Finally, the corrected decay width \( \Gamma \) for this resonance can be recalculate. \( M \) and \( \Gamma \) should be the observed mass and decay width in experiments. Our numerical results for resonance \( X(3915) \) are in good agreement with the experimental data, which are presented in Table [I]

Up to now, a systematic and accurate theoretical approach from QCD to investigate resonance which is regarded as an unstable state created by two Heisenberg field operators has been established. In this paper, we only explore exotic meson resonance which
TABLE I: Corrected mass $M$ and width $\Gamma$ for resonance $X(3915)$. $M_0$ is the mass of the mixed state of two bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, and $\Gamma(M_0)$ is the traditional decay width. (Dimensioned quantities in MeV.)

| Quantity | $M_0$ | $\Gamma(M_0)$ | $(2\pi)^3 D(M_0)$ | $M$ | $\Gamma$ |
|----------|-------|---------------|------------------|-----|---------|
| this work | 3939  | 38            | $-27$            | 3912| 22      |
| PDG[24]  |       |               |                  | 3918.4±1.9 | 20±5 |

is considered as an unstable molecular state composed of two heavy vector mesons. The extension of our approach to more general resonances is straightforward, while the interaction Lagrangian may be modified. More importantly, it is most reasonable and fascinating to investigate resonance as far as possible from QCD. In the framework of quantum field theory, the nonperturbative contribution from the vacuum condensates can be introduced into BS wave function and the two-particle irreducible Green’s function, and then the calculated mass and decay width of resonance will contain more inspiration of QCD.

VIII. CONCLUSION

Exotic resonance $X(3915)$ is considered as a mixed state of two unstable molecular states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, and we investigate the temporal evolution of meson-meson molecular state as determined by the total Hamiltonian. According to dispersion relation, the total matrix elements for all decay channels should be calculated with respect to arbitrary energy. Because the total energy of the final state extends from $\epsilon_M$ to $+\infty$, we obtain two decay channels $J/\psi\omega$ and $D^*\bar{D}^*$ from the effective interaction Lagrangian at low energy QCD, which are exhibited by new Feynman diagrams. Using dispersion relation, the Heisenberg picture and Mandelstam’s approach, we obtain the corrected mass $M$ and decay width $\Gamma$ of resonance $X(3915)$, which are in good agreement with the experimental data. Obviously, our work can be extended to more general resonances.
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Appendix A: tensor structures in the general form of BS wave functions

The tensor structures in Eqs. (8) and (9) are given below [4, 5]

\[
\mathcal{T}^1_{\lambda\tau} = (p^2 + \eta_1 P \cdot p - \eta_2 P \cdot p - \eta_1 \eta_2 P^2) g_{\lambda\tau} - (p \lambda p_\tau + \eta_1 P_\tau p_\lambda - \eta_2 P_\lambda p_\tau - \eta_1 \eta_2 P_\lambda P_\tau),
\]

\[
\mathcal{T}^2_{\lambda\tau} = (p^2 + 2 \eta_1 P \cdot p + \eta_1^2 P^2) (p^2 - 2 \eta_2 P \cdot p + \eta_2^2 P^2) g_{\lambda\tau} + (p^2 + \eta_1 P \cdot p - \eta_2 P \cdot p - \eta_1 \eta_2 P^2)(p \lambda p_\tau + \eta_1 P_\lambda p_\tau - \eta_2 P_\lambda p_\tau - \eta_1 \eta_2 P_\lambda P_\tau) - (p^2 - 2 \eta_2 P \cdot p + \eta_2^2 P^2)(p \lambda p_\tau + \eta_1 P_\lambda p_\tau + \eta_1 P_\tau p_\lambda + \eta_1^2 P_\lambda P_\tau)
\]

\[
- (p^2 + 2 \eta_1 P \cdot p + \eta_1^2 P^2)(p \lambda p_\tau - \eta_2 P_\lambda p_\tau - \eta_2 P_\tau p_\lambda + \eta_2^2 P_\lambda P_\tau),
\]

\[
\mathcal{T}^3_{\mu_1 \ldots \mu_j \lambda \tau} = \frac{1}{j!} p_{\{\mu_2 \ldots \mu_j, g_{\mu_1}\}} (p^2 + \eta_1 P \cdot p + \eta_1^2 P^2) (p^2 - 2 \eta_2 P \cdot p + \eta_2^2 P^2)(p + \eta_1 P)\tau
\]

\[
- (p^2 + \eta_1 P \cdot p - \eta_2 P \cdot p - \eta_1 \eta_2 P^2)(p - \eta_2 P)\tau
\]

\[
- p_{\mu_1 \ldots \mu_j} [(p^2 - 2 \eta_2 P \cdot p + \eta_2^2 P^2)(p \lambda p_\tau + \eta_1 P_\lambda p_\tau + \eta_1 P_\tau p_\lambda + \eta_1^2 P_\lambda P_\tau)
\]

\[
- (p^2 + \eta_1 P \cdot p - \eta_2 P \cdot p - \eta_1 \eta_2 P^2)(p \lambda p_\tau - \eta_2 P_\lambda p_\tau - \eta_2 P_\tau p_\lambda - \eta_1 \eta_2 P_\lambda P_\tau)],
\]

\[
\mathcal{T}^4_{\mu_1 \ldots \mu_j \lambda \tau} = \frac{1}{j!} p_{\{\mu_2 \ldots \mu_j, g_{\mu_1}\}} (p^2 - 2 \eta_2 P \cdot p + \eta_2^2 P^2)(p^2 + \eta_1 P \cdot p
\]

\[
- \eta_2 P \cdot p - \eta_1 \eta_2 P^2)(p + \eta_1 P)\lambda
\]

\[
+ p_{\mu_1 \ldots \mu_j} [(p^2 + \eta_1 P \cdot p + \eta_1^2 P^2)(p \lambda p_\tau - \eta_2 P_\lambda p_\tau - \eta_2 P_\tau p_\lambda + \eta_2^2 P_\lambda P_\tau)
\]

\[
- (p^2 + \eta_1 P \cdot p - \eta_2 P \cdot p - \eta_1 \eta_2 P^2)(p \lambda p_\tau + \eta_1 P_\lambda p_\tau - \eta_2 P_\tau p_\lambda - \eta_1 \eta_2 P_\lambda P_\tau)],
\]

\[
\mathcal{T}^5_{\mu_1 \ldots \mu_j \lambda \tau} = \frac{1}{j!} (p^2 + 2 \eta_1 P \cdot p + \eta_1^2 P^2)(p^2 - 2 \eta_2 P \cdot p + \eta_2^2 P^2) p_{\{\mu_3 \ldots \mu_j, g_{\mu_1} P_{\mu_2}, g_{\mu_1} P_{\mu_2}\}}
\]

\[
- \frac{1}{j!} p_{\{\mu_2 \ldots \mu_j, g_{\mu_1}\}} (p^2 - 2 \eta_2 P \cdot p + \eta_2^2 P^2)(p + \eta_1 P)\lambda
\]

\[
- \frac{1}{j!} p_{\{\mu_2 \ldots \mu_j, g_{\mu_1}\}} (p^2 + 2 \eta_1 P \cdot p + \eta_1^2 P^2)(p - \eta_2 P)\tau
\]

\[
+ p_{\mu_1 \ldots \mu_j} (p \lambda p_\tau + \eta_1 P_\lambda p_\tau - \eta_2 P_\tau p_\lambda - \eta_1 \eta_2 P_\lambda P_\tau),
\]

29
\[ \mathcal{T}_{\mu_1 \cdots \mu_j, \lambda \tau}^6 = p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \zeta \xi \rho \zeta_1 p_\xi P_\xi \mathcal{T}_{\mu_2} \cdots P_\xi \mathcal{T}_{\mu_3} \cdots \]

\[ \mathcal{T}_{\mu_1 \cdots \mu_j, \lambda \tau}^7 = - (2p^2 + \eta_1 P \cdot p - \eta_2 P \cdot p) p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi \]

\[ + (2\eta_1 \eta_2 P \cdot p + \eta_2 P^2 - \eta_1 P^2) p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi + p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi p_\lambda, \]

\[ \mathcal{T}_{\mu_1 \cdots \mu_j, \lambda \tau}^8 = - (P \cdot p) p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi + P^2 p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi \]

\[ - p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi p_\lambda + p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi P_\lambda, \]

\[ \mathcal{T}_{\mu_1 \cdots \mu_j, \lambda \tau}^9 = - (2P \cdot p + \eta_1 P^2 - \eta_2 P^2) p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi \]

\[ + P \cdot (\eta_2 P - \eta_1 P + 2\eta_1 \eta_2 P) p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi \]

\[ + p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi p_\lambda + p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi P_\lambda, \]

\[ \mathcal{T}_{\mu_1 \cdots \mu_j, \lambda \tau}^{10} = - P^2 p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi + (P \cdot p) p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi \]

\[ - p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi P_\lambda + p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi P_\lambda, \]

\[ \mathcal{T}_{\mu_1 \cdots \mu_j, \lambda \tau}^{11} = (p^2 + \eta_1 P \cdot p - \eta_2 P \cdot p - \eta_1 \eta_2 P^2) p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi \]

\[ - p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi (p - \eta_2 P), \]

\[ \mathcal{T}_{\mu_1 \cdots \mu_j, \lambda \tau}^{12} = (p^2 + \eta_1 P \cdot p - \eta_2 P \cdot p - \eta_1 \eta_2 P^2) p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi \]

\[ - p_{1 \mu_2} \cdots p_{1 \mu_j} \epsilon_{\mu_1} \lambda \tau \xi \rho_\xi P_\xi (p + \eta_1 P). \]

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