CRITICAL DEPTH MODEL (MODIFIED SVERDRUP EQUATION) —DESIGN METHOD FOR BUBBLE CIRCULATION—

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A new critical depth ($z_{cr}$) model was developed by adding light saturation effects during photosynthesis to a Sverdrup $z_{cr}$ model in order to design an artificial circulation system as a countermeasure to eutrophication. A Blackman-type equation was selected from various P-I equations for its accuracy and relationship to the original Sverdrup model. Quantitative differences between the $z_{cr}$ values of some new equations were found to be remarkable in specific high light conditions. Furthermore, the $z_{cr}$ calculated by the Sverdrup model overestimated the $z_{cr}$ compared to the modified Sverdrup equation proposed by this research. We discuss methods for the design of a practical air diffuser depth that prevents cyanobacteria bloom from the point of view of design and management of bubble circulation countermeasures.

Key Words : air diffuser depth, photosynthetic rate, mixing depth, harmful cyanobacteria

1. INTRODUCTION

Bubble circulation using bubble plume is an important option for the management of reservoir water quality$^{1,2,3}$). In many reservoirs and lakes, the system has been installed and successful results have been reported in certain water bodies$^{3,4})$. On the other hand, even now, empirical applications are used in technical guidelines$^{1,5})$. Further researches have to be done for constructing reasonable engineering design and management methods. An important factor for design methods is diffuser depth. Although the diffuser depth of bubble circulation system installed in almost all Japanese reservoirs is from 15m to 20 m$^3$), the quantitative theory for this design is not clear. In this paper, we propose a new equation for critical depth$^{6})$ as scientific basis for the rational design method of bubble diffuser depth.

Several and various processes have been pointed out for preventing the growth of harmful cyanobacteria by bubble circulation$^{1,2,3})$. One of the most basic/essential and efficient processes for preventing harmful cyanobacteria is light limitation by increase in mixing depth. In discussing the light limitation processes, critical depth ($z_{cr})$ mainly used in oceanography play an important role. The critical depth is a special mixing depth equal to the mixing depth in which depth integral net photosynthetic rate
becomes 0. After Gran and Braarud\(^7\) proposed their qualitative theory, Sverdrup\(^5\) constructed a quantitative equation based on several ideal conditions (SCDM; Sverdrup Critical Depth Model). Recently there have been some controversies on the concept of the critical depth\(^8\). However, if the mixing depth can be estimated with enough accuracy as hydraulics, the critical depth concept can be established\(^9\). Therefore, recently the importance of critical depth has been increasing in oceanography for estimating phytoplankton dynamics. As a result, the number of citations per year has also increased over the past 50 years\(^9\). Unlike that in the ocean, mixing depth can be regulated artificially by bubble circulation. Therefore, the critical depth is remarkably important as an index for reservoir water quality management.

The SCDM model depends on many ideal conditions\(^5\) including unrealistic assumptions being linear relationship between photosynthesis–light intensity (PI). Therefore, there is possible existence of problem regarding the accuracy of the \(z_{cr}\) value itself estimated by SCDM. Thus, some researchers (Platt et al.\(^10\) and Huisman et al.\(^11\)) reconsidered the \(z_{cr}\) equations. Even now, it is difficult to find discussions for the \(z_{cr}\) based on the consideration of PI equation being basic and important factor for the \(z_{cr}\) concept in spite of the many applications of classical SCDM\(^9\). For the design of bubble circulation countermeasure facilities in practical conditions, the accuracy of bubbler depth of around 5 m is required. Therefore high-accuracy equation for estimating the critical depth is important for civil engineering knowledge.

This paper intends to construct the scientific basis for the design of a bubble circulation system diffuser depth as an engineering method for bubble circulation countermeasure to prevent harmful cyanobacteria. Furthermore, we developed a new critical depth equation as modified Sverdrup critical depth model as scientific basis for design methods for bubble circulation. In particular, we considered the differences in critical depth between some phytoplankton ecological trait groups although previous researches have not focused on it. Generally, a detailed numerical model is used for estimating the effects of this countermeasure in not only scientific research but also in practical engineering. On the other hand, a simple model, such as a critical depth equation that extracts the essence of the phenomenon with as few parameters as possible, is useful for obtaining prospect of design conditions. In order to verify the critical depth, from the hydraulic point of view it is necessary to estimate the mixing depth with high precision. However, at present, there are various arguments for this evaluation method\(^9\). In this study, the accuracy of the critical depth model is based on the consistency with the experimental value of the PI equation to be used.

2. MATERIALS AND METHODS

Figure 1 shows the conceptual figures used in this research. Because only light condition is treated as a limitation factor for phytoplankton dynamics, several other factors, such as water temperature, nutrient, loss by settlement and grazing, are neglected similar to the SCDM theory\(^6\) assumed by Sverdrup.

### (1) Photosynthetic rate and light intensity

#### a) Model

In the past, many researchers have proposed P-I curve equations representing the relationship between Photosynthetic rate (\(P\)) and light intensity (\(I\)) (Fig.1(a) and Eq. (1)). The first P-I curve model was proposed by Blackman in 1905\(^12\) following several researchers\(^3\)-\(^19\) (Table 1). In this research, we focus on only light saturation-type equations and neglect optimum light intensity\(^18\) and photoinhibition types\(^19\). The reason for this assumption is that since we discuss the critical depth corresponding to the conditions where the mixing depth is deeper than the euphotic depth, photoinhibition near the surface layer is considered to be unremarkable.

![Diagrammatic figures for the concepts used in this research.](image)

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**Fig.1** Diagrammatic figures for the concepts used in this research. (a): the relationships between photosynthetic rates and light intensity, (b): Vertical profiles of underwater light intensity, photosynthetic rates and respiration (loss) rate, (c): Variation in DINP (Depth Integral Net Photosynthetic rate) with (\(z_{cr}\)) (mixing depth). Detailed values in the figures are selected as typical. The saturation-type shapes of the PI curve are selected as of general. Also, \(I_0\)\(>I_s\) conditions are presented. \(P\): Gross photosynthetic rate (\(d^{-1}\)), \(R\): Respiration (loss) rate (\(d^{-1}\)), \(P_{max}\): Maximum photosynthetic rate, \(I, I_0, I_s\): Cell surroundings, just below the water surface and photosynthesis saturation light intensity (\(\mu\) mol-Photon m\(^{-2}\)s\(^{-1}\)), \(x\): Initial slope of PI curve (\(=P_{max} / I_s\)), \(\varepsilon\): attenuation coefficient (m\(^{-1}\)), \(z\): depth (m).
b) Experimental values

We used various observation data shown in Table 2 for selecting the appropriate P-I curve equation model for the modified Sverdrup equation from representative PI curves proposed in the past researches (Table 1). These include a wide range of phytoplankton data as picoplankton20), green algae21) and diatoms22) as well as harmful cyanobacteria algae23),24). The target water area and experimental method are diverse. Compared with the PI formula, the photosynthetic rate and the light intensity of the experimental values were used in a dimensionless manner with the maximum photosynthetic rate (Pm) and saturated light intensity (Is), respectively.

(2) Vertical profiles of underwater light intensity and photosynthesis

Light intensity attenuate under water surface exponentially with depth by attenuation coefficient (ε) according to Lembert-Beer low (Fig.1(b), Eq. (2)). The value ε is affected by not only the amount and quality of various materials and elements in water but also by the wave length of underwater23). Therefore, the values of ε vary temporally and spatially even in the same water body. However, we treated this value as constant similar to Sverdrup’s original article assumption. The value used in this paper was selected as constant value in this paper for simplification.

Light saturation effect on photosynthesis above the depth corresponds to the zIs according to underwater light intensity being Is (Fig.1(b)). In this research, we newly define the light saturation layer (LS) above the zIs and the light limitation layer (LL) below it. The water depth at which P = R in LL is the compensation depth or the euphotic depth (zcr) and the maximum depth at which the net photosynthetic rate (P - R) becomes positive. This euphotic depth is generally defined as 1% light intensity depth of Is or about three times the depth of transparency23). Respiration rate should not be treated as constant value from strict physiological theory and observations/experiments. Although the effects of growth rate on this parameter have been pointed out23), we treated this as constant value in this paper for simplification.

(3) DINP (Depth Integral Net Photosynthetic rate ) and zcr

DINP means depth integral net photosynthetic rate in arbitrary mixing depth (zm)23),10) (Fig.1(c), Eq. (3)). zm is the depth unrelated to the flow of the lake, zcr is a special condition concerning zm related to photosynthesis. DINP increases up to the compensation depth (zsm) as zm increases. However, when zm becomes deeper than zsm, it decreases, DINP = 0 at a certain depth and critical depth occurs (Fig.1 (c), Eq. (4)). The Eq. (5) for zcr is a critical depth model formulated by Sverdrup based on many ideal assumptions in 1953 (SCDM) 8).

\[
\frac{z_{cr}}{1 - e^{-\frac{zm}{\varepsilon R}}} = \frac{a I_a}{\varepsilon R} (5)
\]

Even though Eq. (5) is based on simple assumptions of a linear PI equation (Table 1, Sv53 type) and a constant respiration rate, it has been used in many oceanographic studies in recent years. The importance of zcr has been increasing in various research fields recently9). Huismen et al.11) and Furusato et al.2) proposed the zcr formula using the BI35 type by changing the PI formula used. (Eq. (6)).

\[
z_{cr} = \frac{I_z}{I_a} \ln \left[ \frac{1 + I_a / I_z}{1 + (I_a / I_z) \, e^{-z_{cr}/\varepsilon R}} \right] = \frac{a I_a}{\varepsilon R} (6)
\]
The topography of the reservoir has a characteristic that the capacity per unit depth decreases with depth. In order to ensure accuracy, it is necessary to include this effect in DINP (Eq. (3)). As the first trial in this study, we obtained the modified Sverdrup model as a general critical depth model by the definition of Eq. (3).

(4) Analytical conditions
a) Comparison of \( z_{cr} \) based on different PI curves
Non-dimensional values of critical depth equations are used for the comparison between SCDM and the modified Sverdrup model. The rightside term is the same in both \( z_{cr} \) equations in spite of different PI equation used for the equations (Eqs. (5) and (6)).

Because we used \( I^* (= I_0 / I_S) \) and \( P_m^* (= P_m / R) \) as non-dimensional values representing the characteristics of \( z_{cr} \) equations, the rightside terms of Eqs. (5) and (6) can become Eq. (7):

\[
\frac{I_0 \alpha}{eR} = \frac{I_0 P_m}{I_S R e} = I^* P_m^* \frac{1}{e} \tag{7}
\]

From Eq. (7), it is clear that critical depth depends on non-dimensional light intensity (\( I^* \)), non-dimensional maximum photosynthetic rate (\( P_m^* \)), and optical parameter of the water (\( e \)). These two non-dimensional values were used in Fig.2 for \( x \) and \( y \) axes. Here, in addition to not only that \( P_m^* \) exists only on the right side of the critical depth formula (Eqs. (5) and (6)), \( e \) exists also on the left side but can be regarded as \( e^{z_{cr}R} < 1 \); it can only be present on the right side \( ^{28} \). Therefore, they do not influence the ratio of different critical depth equations. Based on above conditions, only \( P_m^* \) is selected for the comparison of \( z_{cr} \) values among several \( z_{cr} \) equations. The value of \( P_m^* \) was set to 1 to 20 as the width of \( I^* \), which can actually occur in the field shown in Table 3. In the comparison, Eq. (6) is also used as the existing \( z_{cr} \) model.

b) Analysis for critical depth in reservoirs
By using the modified Sverdrup equation proposed in this paper, we analyzed critical depth for cyanobacteria and other algae in reservoir conditions. Table 4 shows the parameter values used in the analysis. The subjects of the analysis were cyanobacteria algae, diatoms, and green algae, and their characteristics were expressed by the difference in \( P_m \) and \( I_S \) among three phytoplankton groups. The parameters of each phytoplankton group were set from the results of the study obtained by Reynolds as values in fields instead of laboratory culture experiments \(^{29} \). The colony size of bloom forming cyanobacteria such as Microcystis is large, so it has a property of a slow growth rate and intense light adaptive PI curve. In this paper, values consistent with these characteristics were used. In addition, \( e \) and \( I_0 \) related to the ambient light conditions were common regardless of phytoplankton. \( I_0 \) set two cases of low light conditions (200 \( \mu \)E \( m^2 \) \( s^{-1} \)) assuming cloudy weather, early morning, evening, etc., in addition to the clear sky weather condition (1000 \( \mu \)E \( m^2 \) \( s^{-1} \)). \( e \) was 1.5 \( m^{-1} \) corresponding to 1 m of transparency (\( z_{SD} \)) as the value in the eutrophication water area (\( z_{SD}=1.5/e^{0.5} \)).

3. RESULTS AND DISCUSSIONS
(1) Selection of PI equation
Figure 2 shows the comparison between several
PI relationships proposed by past researchers (Table 1) and experimental values (Table 2). We used BK05-type PI equation for obtaining modified Sverdrup equation based on several reasons in addition to the agreements with many experimental values. The Bl35-type equation used by Furusato et al., for DINP equation and linear equation (Sv53) do not agree with the experimental values (Fig. 2). The model with high conformity with measured values is the tanh equation (JP76) and the bilinear equation (Bk05). Previous studies pointed out that conformity was high for the JP76 type and low for the Bl35 type based on comparison of various PI formulas. However, the JP76-type PI curve function types are not theoretically derived, but are approximations-based empirical formulas. Since the objective of this research is to add a light saturation effect to SCDM, we decided to obtain a modified Sverdrup equation using the PI model of the BK05 type.

(2) Modified Sverdrup equation

The PI equation of the BK05 type consists of two different lines depending on the light intensity (Table 1). Therefore, the modified Sverdrup equation is obtained from the sum of DINP of the light saturation layer (LS layer) and the light limitation layer (LL layer) (Fig. 1 (b)). Since we discuss \( z_{cr} \), we can consider only the \( z_0 > z_{IS} \) condition. Furthermore, in the case of \( I_0 < I_S \), there is no LS layer, thus the PI equation is the same as the Sverdup type. For this reason, we assume the condition of \( I_0 > I_S \) in this paper.

The \( z_{IS} \) (Fig. 1 (b)), which is the boundary between the LS layer and the LL layer, has the formula (8) when \( I(z) \) and \( z \) in Eq. (2) are \( I_S \) and \( z_{IS} \), respectively.

\[
z_{IS} = \frac{\ln(I_0/I_s)}{\epsilon}, \quad z_0 > 0 \quad (8)
\]

DINP under the \( z_0 > z_{IS} \) condition is the sum of DINP in LS and LL layers as DINP_{LS} and DINP_{LL}.

\[
DINP = DINP_{LS} + DINP_{LL} = (P_m - R)z_{IS} + \int_{z_{IS}}^{z_0} (P - R) dz = (\alpha I_s - R)z_{IS} + \frac{\alpha I_0}{\epsilon} \left( \frac{I_0}{I_s} - e^{-\epsilon z_{IS}} \right) - R(z_m - z_{IS})
\]

Using Eq. (8) for deleting \( z_{IS} \), we can derive new DINP equation (Eq. (9)) based on the BK05 PI equation.

\[
DINP = \frac{\alpha I_0}{\epsilon} \left[ \frac{I_s}{I_0} \left( \ln \frac{I_0}{I_s} + 1 \right) - e^{-\epsilon z_{IS}} \right] - Rz_m \quad (9)
\]

Since the critical depth is \( z_m \) where DINP from the surface layer to \( z_m \) becomes 0 (Fig. 1 (c)), we can obtain a new critical depth equation (Eq. (10)) by assuming the same depth of \( z_m \) and \( z_{cr} \).

\[
\frac{I_s/I_0 [\ln(I_0/I_s) + 1] - e^{-\epsilon z_{IS}}}{\alpha R} = z_{cr} \quad (10)
\]

Compared to the SCDM estimated by Eq. (5), the new critical depth equation includes light-saturation effect in the denominator on the left side (Eq. (10)). Therefore, although an axiom from the difference in PI equations is qualitative, the critical depth estimated by the traditional/classical SCDM (Eq. (5)) overestimates it, compared to the modified Sverdup model (Eq. (10)) proposed by this paper.

(3) The effects of the differences in some PI equations on the critical depth

Figure 3 shows the relationship between \( z_{cr}^* \) and \( I^* \) calculated by Eqs. (6) and (10) for estimating the differences in the \( z_{cr} \) obtained by the original Sverdup model and the modified model proposed by this paper. In the relationship, \( I^* = 10 \) is a standard condition. Needless to say, based on the properties of PI equations, regardless of \( I^* \), the \( z_{cr} \) estimated by the \( z_{cr} \) expressions of BK05 and Bl35 types (Eqs. (10) and (6)) is smaller than the \( z_{cr} \) of SCDM. Focusing on the relationship with \( I^* \), the ratio of both models to SCDM is close to 1 under conditions where \( I^* \) is small. On the other hand, in the condition of \( I^* \) larger than the value corresponding to the standard value, the ratio gradually approaches a fixed value. The ratio to SCDM under the standard \( I^* \) condition is 0.33 for BK05 and 0.25 for Bl35. As can be seen from Eqs. (5), (6), and (10), this ratio is equal even if the values of parameters other than \( I^* \) are changed. Based on the above, in the condition that light limitation occurs in the surface layer in photosynthesis, it is not possible to ignore the influence on \( z_{cr} \) of the PI formula. For example, with \( I^* = 10 \), when SCDM is used, the critical depth is about three times overestimated from the proposed \( z_{cr} \) equation (Eq. (10)). When using the Bl35-type PI curve equation, the critical depth will be underestimated by approximately 25% from the abovementioned ratio \( I^* = 10 \) as compared with the modified Sverdup formula. These differences are directly related to air diffuser depth in the design of bubble circulation counter-
measures. Therefore, in designing a bubble circulation countermeasure facility, it is necessary to evaluate the critical depth with high accuracy. For this purpose, it is desirable to use the modified Sverdrup formula proposed in this paper.

(4) Application to engineering

Figure 4 shows the critical depth of some typical phytoplankton groups estimated by the modified Sverdrup model (Eq. (10)) using the general / representative conditions (Table 4) with consideration for the design condition of bubble circulation countermeasure (critical depth of harmful cyanobacteria). In this figure, transparency ($z_{SD}$) is used instead of attenuation coefficient ($\varepsilon$) in consideration of engineering use. As shown in this figure, the $z_{cr}$ values of harmful cyanobacteria are shallower than those of other phytoplankton. This tendency is derived from the small maximum photosynthetic rate and large $I_{S}^{29}$ (Table 4) as adaptation to high light intensity as the physiological and ecological characteristics of harmful cyanobacteria. The figure also shows the range of $z_{SD}$ in the eutrophicated reservoir and the width of the air diffuser depth of a general bubble circulation facility. Even in the high light intensity condition where the $z_{SD}$ is deep (Fig. 4 (a)), the $z_{cr}$ of harmful cyanobacteria is shallower than the general air diffuser depth conditions in general optical condition of eutrophicated reservoirs ($z_{SD} = 0.5$ to $2$ m). On the other hand, the $z_{cr}$ values of green algae and diatoms are deeper than the air diffuser depth. This tendency is consistent with the fact that the dominant species changed from cyanobacteria to diatoms and green algae after implementing this countermeasure in many water bodies. However, various mechanisms have been pointed out to have prevented the growth of cyanobacteria by this countermeasure 1)-3). Here, based on the fact that the relationship between mixing depth and critical depth is a fundamental mechanism concerning the utilization of light energy itself in the photosynthetic reaction, mixing depth management considering the critical depth is important for water quality management in eutrophicated reservoirs. Especially, if mixing depth is controlled by considering different critical depths depending on the ecotype 29) of phytoplankton, there is a possibility of dominant species management in eutrophic reservoirs. This is one of the important conclusions obtained in this research.

In the future, some problems should be solved when applying the results obtained by this research to practical use. First, only light saturation-type PI equation is used for modified Sverdrup model proposed by this paper (Table 1). On the other hand, from the viewpoint of rigor, the conclusion obtained in this study may be different from the analysis result using the photoinhibition-type PI formula used in a general eutrophication model. In the future, research on the critical depth taking this effect into consideration is necessary. Next, in this study, we analyzed based on general parameter values (Table 4) as an example, but when applying in the field, it will be necessary to design and manage countermeasures based on the parameter values specific to each water area. In this paper, to simplify the problem, vertical one-dimensional analysis was performed without considering the topography of the reservoir. On the other hand, in actual reservoirs, the area of the deeper layer is smaller than that of the surface layer, thus, there is a possibility that the influence of respiration loss in the aphotic layer becomes small. In this case, the critical depth becomes deeper than the result of this research. In particular, this effect is greater for phytoplankton with a deep critical depth such as diatoms. In the future, other factors neglected in this
paper, such as the characteristics of water body topography, should be considered and a more practical model for estimating critical mode is required.

4. CONCLUSIONS

The conclusions of this study are as follows:

• We modified the Sverdrup Critical Depth Model (SCDM) and proposed a modified Sverdrup equation considering the photosynthesis light saturation effect.

• As a photosynthetic light curve, a bilinear equation was used as a formula that was highly compatible with various experimental values and observed values.

• Compared to the modified Sverdrup formula, the SCDM overestimates the critical depth calculated by SCDM by two to three times.

• As a result of analyzing the critical depth for each phytoplankton taxon group using the modified Sverdrup equation, the critical depth of harmful cyanobacteria algae is remarkably shallower than those of green algae and diatoms.

• Furthermore, the general bubble circulation air discharge depth is deeper than the critical depth of cyanobacteria.

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