INFLATING DEFECTS

Patricio S. Letelier\footnote{Permanent Address: Departamento de Matemática Aplicada–IMECC, Universidade Estadual de Campinas, 13081 Campinas. S.P., Brazil} and Luís E. Mendes\footnote{email: m1480@beta.ist.utl.pt}

Departamento de Física
Instituto Superior Técnico
Av. Rovisco Pais 1
1096 Lisboa, Portugal

Abstract

A Kantowski-Sachs cosmological model with an O(3) global defect as a source is studied in the context of the topological inflation scenario.

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The Universe in which we live is, to a good approximation, flat and highly isotropic and homogeneous on the largest scales and it is well described by the standard Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model. The realization of this model is only possible with a very unnatural fine-tuning of the initial conditions thus making our universe improbable. The introduction of a phase of exponential expansion – inflation – short after the Big-Bang can solve these fine-tuning problems as well as other long standing puzzling features of our Universe [1] [2].

Another problem of a different nature is the monopole problem that arises when we take into account the predictions of Grand Unified Theories (GUT’s). Massive monopoles appear in such large numbers during the GUT phase transition that their sole contribution to the density of the universe would be enough to have caused the recollapse of the universe long ago. As such recollapse has not occurred and no monopole has been observed yet, we must find a mechanism to eliminate these monopoles. Once again inflation is the key: the exponential expansion of the universe will dilute the density of monopoles thereof solving this problem. In fact, it was this so called primordial monopole problem, one of the main motivations to introduce inflation [1].

Although inflation can solve the problems of the FLRW model, inflationary models also need some amount of fine tuning in the initial conditions. For instance, in the “new inflationary” models, the initial value of the scalar field must be localized in a narrow band of width $0.11m_{pl}$ around the top of the potential [3].

Recently, there have been some claims that inflation could occur without fine-tuning in the cores of topological defects where the scalar field $\phi$ is in the symmetric phase and $\phi = 0$ [4] [5]. If the defects are stable then this
topological inflation will never end. All the models of this type have been considered in the context of the FLRW models. Another possibility is to consider unstable defects like sphalerons [3] or unwinding textures [7].

In this note we extend these ideas by constructing an explicit model where inflation is obtained in a Kantowski-Sachs model of universe by means of a defect that in the unbroken phase has an O(3) global symmetry and whose vacuum manifold is O(2). We show (as in [7]) that even if the defects are not stable, an inflationary phase of the universe is also possible. Since the Kantowski-Sachs cosmological model is an anisotropic model we study also its isotropisation along the inflationary phase.

The Kantowski-Sachs metric is (c=1),

$$ds^2 = dt^2 - a^2(t)dr^2 - b^2(t)(d\theta^2 + \sin^2 \theta d\phi^2).$$  

(1)

The global O(3) defect that we shall consider is the one described by the Lagrangian density built with the triplet of scalar fields $\Phi^a$, $(a = 1, 2, 3)$,

$$L = \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu \Phi^a \partial_\nu \Phi^b \delta_{ab} - V(\Phi)\right]$$  

(2)

where $V(\Phi)$ is the usual Mexican hat potential,

$$V(\Phi) \equiv \frac{1}{4}\lambda(\delta_{ab}\Phi^a\Phi^b - \eta^2)^2.$$  

(3)

The O(3) field $\Phi$ has the form

$$\Phi \equiv (\Phi^a) = \eta\phi(t)[\sin \vartheta \cos \varphi, \sin \vartheta \sin \varphi, \cos \vartheta].$$  

(4)

The Einstein equations (in units such that $8\pi G = 1$), are

$$\begin{align*}
2\frac{\dot{a}}{a} \frac{\dot{b}}{b} + \left(\frac{\dot{b}}{b}\right)^2 + \frac{1}{b^2} &= \frac{\eta^2}{2}(\dot{\phi}^2 + 2\frac{\phi^2}{b^2} + \frac{\lambda\eta^2}{2}(\phi^2 - 1)^2) \\
2\frac{\dot{b}}{b} + \left(\frac{\dot{b}}{b}\right)^2 + \frac{1}{b^2} &= \frac{\eta^2}{2}(-\dot{\phi}^2 + 2\frac{\phi^2}{b^2} + \frac{\lambda\eta^2}{2}(\phi^2 - 1)^2) \\
\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a} \frac{\dot{b}}{b} &= \frac{\eta^2}{2}(-\dot{\phi}^2 + \frac{\lambda\eta^2}{2}(\phi^2 - 1)^2).
\end{align*}$$  

(5)
The equations for the scalar fields reduce to

\[ \ddot{\phi} + \left( \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \dot{\phi} + \frac{2}{b^2} \dot{\phi} + \lambda \eta^2 \phi (\phi^2 - 1) = 0 \] (6)

Note that this equations are the same as the ones of a single scalar field [8] except for the term \( \frac{2}{b^2} \dot{\phi} \) of (6) and \( \eta^2 \phi^2 / b^2 \) on the right hand side of the first two components of the Einstein equations (5). This last term alone enters equally in the components \( T_t^t \) and \( T_r^r \) and represents a cloud of Nambu strings with spherical symmetry [9][10]. The Einstein equations for the Kantowski-Sachs models for a cloud of spherically symmetric strings admit a first integral [11].

If we assume that the defect is stable like a O(3) monopole in our case, then forever \( \phi = 0 \) in the core of the defect, the extra term is 0 and the potential acts as a cosmological constant with \( p = -\rho \) driving the exponential expansion inside the defect. This is the analogous situation to the one analyzed by Vilenkin in the context of the FLRW model. Note that in this case there is no need for any type of fine tuning, because \( \phi = 0 \) in the core of the defects. However, when we take into account the fact that global defects are not necessarily stable, e.g., a global texture has dynamical instability, we can use the defect field itself to produce slow-roll inflation. If we assume that the defect vanishes very slowly inside the core, \( \dot{\phi} \approx 0 \), the field will roll-down the potential also very slowly. During this slow-roll, the interior of the defect expands quasi-exponentially. During this process the extra term from the defect has a negligible role. In fact, in the beginning of inflation \( \phi = 0 \) and this term vanishes; short after inflation has begun \( b \) starts an exponential growth and the extra term rapidly becomes negligible. The realization of this idea is depicted in Fig.1 in which we compare this inflating defect model with usual inflation obtained with the same potential (in the simulations we
used the variables $\alpha = \ln a$ and $\beta = \ln b$). We can see that the presence of this extra term can increase slightly the expansion during the inflationary epoch. This can be understood from the fact that the term $\eta^2 \phi^2 / b^2$ is always positive and whenever it appears it adds to the potential.

As in the models with this type of potentials, after reaching the bottom of the potential $\phi$ oscillates around the minimum of the potential giving rise to the reheating of the universe. In this model the slow-roll condition plays an important role, we need that the defect decays with a sufficiently large time scale to have enough time to produce inflation in the very early universe. In other words we need some fine tuning of the initial conditions. Maintaining all the initial conditions fixed, we can increase the time scale for the decay increasing the value of $\eta$. This is because the larger the value of $\eta$, the far from the origin the minimum of the potential will be.

A good measure of the anisotropy of the model is given by the ratio

$$\frac{\sigma}{\theta} = \frac{\dot{a}/a - \dot{b}/b}{\dot{a}/a + 2\dot{b}/b}$$  \hspace{1cm} (7)$$

where $\sigma$ is the shear scalar and $\theta$ the expansion rate. We find that in our model this quantity goes to zero very fast, as in the model with a single scalar field \cite{8}, thus indicating that once inflation has started the model rapidly isotropizes.

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FIGURE CAPTIONS

FIG. 1 The solid curve shows the evolution of $R = (ab^2)^{1/3}$ in our model during the last stages of inflation while the dashed curve represents the same quantity for the model with a single scalar field obtained with the same initial conditions. We see that the presence of the extra term in the model with the monopole can increase slightly the expansion during the inflationary phase. The initial conditions are $\alpha = 1$, $\dot{\alpha} = 2.25 \times 10^{11}$, $\beta = 0.17$, $\dot{\beta} = 5 \times 10^{-14}$, $\phi = 0$, and $\dot{\phi} = 10^{-43}$; we have set $\lambda = 1$ and $\eta = 1.3$.

FIG. 2 After reaching the bottom of the potential, the scalar field oscillates around the minimum. The initial conditions are the same as in Fig. 1
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