Modeling of Harmonic Wave Propagation Through a Metasurface with Helmholtz Resonator Shaped Cells

A Y Ismail\(^1\) and B Y Koo\(^1\)
School of Mechanical Engineering, Kunsan National University, Gunsan, South Korea
yusuf@itats.ac.id

Abstract. Harmonic wave propagation through a novel metasurface design is presented in this paper. The metasurface is formed by using the Helmholtz resonator as the cells shape design since such resonator has uniqueness and advantageous performances. The study is conducted both numerically using the finite element method and experimentally using specific measurements to validate the numerical results. Parametric studies of the selected variables are also conducted to obtain broader information on the performance. From the result, it is found that the new proposed metasurface design has the potential to be implemented in future engineering practices.

1. Introduction
As a part of modern structural design, a metasurface (MS) has been playing an important role in the evolution of engineering design. It offers unique, specific, and broadband applications. Derived from the metamaterial concept [1]–[5], the well-known “negative index of refraction” which is completely not found in the common material behavior makes the MS flips something impossible becomes possible. In the bulk material, for instance, an MS opens a wider horizon of application that has never been existed before by showing capabilities of “negativity” performances.

This phenomenon has been employed in many cases. Zhu et al employ the MS for the vegetation sound absorption application [6]. Yu et al use the MS for ventilation windows in a building [7]. Still, in the building sector, Zhu et al use MS for carpet cloaking insertion structure [8]. These examples show that the performance of the applied MS should be given more attention and development. The MS is also interested for researchers in the field of fluid dynamics. One of the research is done by Pagliaroli et al where an ultra-thin MS is used to control a hypersonic flow [9] while Xia et al focused on the airy beam applications [10]. More specifically, Song et al [11] and Li et al [12] manages to use MS as a wave refractor controller. The aerospace and nautical industry are keen to explore this MS recently.

Thus, in terms of fluid dynamics and wave propagation, the MS development needs to be explored more deeply, particularly for the phenomena of wave refraction in conjunction with the pressure drop analysis. This paper then focuses on the investigation of wave propagation through an MS design to see the refraction performance and calculate the pressure drop between the incoming and out-coming waves. The MS, in this case, is made from the so-called Helmholtz resonator (HR) that is used as the core cells shape. Therefore, for the rest of the discussion, the proposed model is called the Helmholtz resonator-based metasurface (HRMS) in this paper. This is purposely investigated also due to the uniqueness of the HR performance in the wave-related applications [3], [12]–[14]. This study will become important as an opening gate of HR employment in a different way than before. The HR will not only be implemented as a single apparatus outside the system as it was, but instead, it will form a periodic structure of cores inside a system that will need fewer spaces. This is a great advantage for structural engineering applications.
The investigation is conducted in a numerical finite element model (FEM) using Matlab software, including the HRMS model and mesh generation, as well as the finite element formulation solution. Additionally, an experimental measurement is also conducted to validate the result. The FEM method is chosen since it offers simplicity and less time required over the other methods i.e. analytical method or finite difference method that requires a much complex mathematical derivation for the HRMS shape. Even though this method has a high computation requirement limitation, but it is still worth it because of its capabilities to solve any other cases such as very complex geometries as well as three-dimensional shape cases.

2. Methodology

2.1. Finite Element Model

Figure 1 shows the finite element model made in Matlab software. The size of the HRMS is 4.5 x 8 cm in width and height. For the slab length, the HRMS has an 8 cm length. The model has a triangular mesh of 208,092 nodes and 106,604 elements.

![Finite Element Model](image)

Figure 1. (a) The HRMS model, (b) HRMS Mesh

Since the analysis is based on the harmonic base, which also means a frequency-based analysis, therefore the Helmholtz equation is used:

\[
\nabla^2 p + k^2 p = 0 \quad \text{in} \quad \Omega \subseteq \mathbb{R}^2
\]  

(1)

And the equation is subjected to the boundary condition

\[
p = p_m \quad \text{on} \quad \Gamma_d
\]  

(2)

\[
p = 0 \quad \text{on} \quad \Gamma_n
\]  

(3)
Where $p$ is the sound pressure and $k$ is the wavenumber. The weak formulation and solution are made within the Matlab environment.

2.2. Parametric Study
To obtain a broad result, a parametric study for several variables is conducted. Table 1 summarizes all the scenarios. The cell numbers are varied from 6 to 28 cells, representing the geometrical size of the HR, which is important in terms of absorption. The thickness is varied into 3 different levels to represent the total impedance of the HRMS. And last, the effect of doubling the layer is also investigated to see its effect on wave propagation.

| Variables  | Variation          |
|------------|--------------------|
| Cell Number| 6 cells            |
|            | 15 cells           |
|            | 28 cells           |
| Thickness  | $t_1 = 1.5$ cm     |
|            | $t_2 = 3.0$ cm     |
|            | $t_3 = 4.5$ cm     |
| Layer      | Single             |
|            | Double             |

2.3. Sound Pressure Measurement
To validate the numerical result, an experimental measurement is conducted as seen in Figure 2. A sound source is used to generate the plane wave impinging from the left to the right-hand side of the HRMS. The incoming sound pressure is measured using an acoustic sensor and subtracted with the outcoming one. This is called the delta pressure or the insertion loss which is shown in Eqs. 4 and 5 below.

![Figure 2. The measurement of the delta pressure](image)

\[ \Delta p = \bar{p}_{in} - \bar{p}_{out} \] (4)

Where, \[ p = 20\log_{10} \left( \frac{p^*}{p_{ref}} \right) \] (5)

With $p^*$ is the experiment measured pressure and $p_{ref}$ is reference pressure given as $2 \times 10^6$ Pa.
3. Result and Discussion

3.1. Cell Size Variation

Figure 3 shows the wave propagation through the various size of HR cells. It varies from 6 cells in Figure (a), 15 cells in Figure (b), and 29 cells in Figure (c). Generally, each HR size has different simulation results from one another. The HRMS with 6 cells gives a high-pressure drop where the plane wave propagation seems to be dissipated at the outlet aperture of the HRMS. In terms of pressure drop, this can be said effective. However, from the aspect of wave refraction, this scenario does not show any effectiveness of refracting performance since no wave propagates at the outlet aperture. The wave refraction, somehow, is needed in some engineering applications.

Meanwhile, the HRMS with 15 cells shows good performance of refraction. At the outlet aperture, the wave is perfectly refracted to be a circular wave. This comes from the double performance given by the HRMS design which has upside-down HR cells. By referring again to the research from Li et al [12], this phenomenon can be explained as the superposition from two directions of refraction.

![Wave propagation through the HRMS with various cells size.](image)

(a) 6 cells (b) 15 cells (c) 28 cells

3.2. Thickness Variation

The thickness variation result is presented in Figure 4. The thickness is increased from 1.5, 3, and 4.5 cm depicted in Figures (a), (b), and (c), respectively. The thickness represents the impedance of the system. From Fahy and Gardonio [15] in their explanation of a single solid panel, the mass and stiffness of the material determine the impedance. The thicker the panel, the higher impedance is obtained.

Linear to that result, this HRMS thickness also behaves similarly. The thicker the design, the higher the system impedance and eventually increasing the pressure drop. It can be seen that the HRMS with \( t_3 \) gives the highest pressure drop. In terms of wave refraction, all the models give the same good wave refraction performance. This is due to the similarity of the outlet aperture of the 3 samples.
Figure 4. Wave propagation through the HRMS with various thicknesses,
(a) $t_1 = 1.5$ cm (b) $t_2 = 1.5$ cm (c) $t_3 = 1.5$ cm

3.3. Layer Variation
The effect of layers of the HRMS is depicted in Figure 5. Again, similar to Fahy and Gardonio [15], the double layer gives a higher value of pressure drop, shown by the color gradient in the simulation result. In terms of wave refraction, the double layer seems to have the same performance as the single layer only at the front layer of the double layer. The wave refraction can be observed similarly at the front layer aperture but then dissipated at the second layer. This is why the pressure drop of the double layer is obtained higher than the single one.

Figure 5. Wave propagation through the HRMS with different layers,
(a) single (b) double layer

3.4. Sound Pressure Measurement
As a validation result, Figure 6 shows the simple measurement setup of the pressure drop. The two attached microphones have been able to obtain the inlet and outlet pressure of the sound wave. As a tabulation, Table 1 shows all the results. When it comes to comparison, it is seen that the numerical result has a slightly higher value than the experimental around 4 to 20%. This might be due to the unavoidable small leakage from the 3D printing of the HRMS sample. This small leakage, however,
affects the pressure significantly. Another cause is the vibroacoustic effect from the tube structure that is not simulated in the numerical. In the future, this aspect may be considered.

![Figure 6. Measurement Setup](image)

### Table 2. Parametric Study

|          | 6 cells | 15 cells | 28 cells |
|----------|---------|----------|----------|
| Numerical| 6.02 dB | 4.4 dB   | 3.06 dB  |
| Experiment| 5.1 dB | 3.52 dB  | 2.92 dB  |
| Difference| 15.3%   | 20%      | 4.5%     |

4. Conclusions
The wave propagation through the HRMS model has been investigated numerically and validated experimentally. It is found that the HRMS design, with certain variable tuning, can provide both pressure drop and wave refraction performance. The pressure drop performance can be investigated expansively to be used as a noise barrier structure design. While the wave refraction performance can be further investigated to be implemented as a wave controller application.

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