Topological quantum state control through Floquet exceptional-point proximity

Maryam Abbasi, Weijian Chen, Mahdi Naghiloo, Yogesh N. Joglekar, and Kater W. Murch

1Department of Physics, Washington University, St. Louis, Missouri 63130
2Center for Quantum Sensors, Washington University, St. Louis, Missouri 63130
3Research Laboratory of Electronics, MIT, Cambridge, Massachusetts 02139
4Department of Physics, Indiana University Purdue University Indianapolis (IUPUI), Indianapolis, Indiana 46202

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We study the quantum evolution of a non-Hermitian qubit realized as a sub-manifold of a dissipative superconducting transmon circuit. Real-time tuning of the system parameters results in non-reciprocal quantum state transfer associated with proximity to the exceptional points of the effective Floquet Hamiltonian. We observe chiral geometric phases accumulated under state transport, verifying the quantum coherent nature of the evolution in the complex energy landscape and distinguishing between coherent and incoherent effects associated with exceptional point encircling. Our work demonstrates an entirely new method for control over quantum state vectors, highlighting new facets of quantum bath engineering enabled through time-periodic (Floquet) non-Hermitian control.

Small quantum systems that interact with an environment can be described by a Lindblad density matrix equation that encodes their approach to steady state. When the quantum trajectories of these decoherence-inducing dynamics are restricted to those with no quantum jumps, the resulting evolution is described by an effective non-Hermitian Hamiltonian. Such non-Hermitian quantum systems have complex energies, non-orthogonal eigenstates, and undergo a coherent, non-unitary evolution. The presence of special kinds of degeneracies known as exceptional points (EPs) play an important role in the unique characteristics of these non-Hermitian systems [3–9]. Such EPs occur when both the eigenvalues and eigenstates of the system coalesce. A plethora of phenomena associated with EPs have been revealed in classical platforms such as mechanical and optical systems [3–4]. In the vicinity of the EP, the shape of the Riemann manifold that describes the complex energies of a non-Hermitian system can lead to fundamentally new phenomena that are not present in their Hermitian counterparts with strictly real energies. For a second-order EP degeneracy, quasi-static tuning of the Hamiltonian parameters is expected to map one eigenstate, $|\psi_-\rangle$, to the other $e^{i\chi_+} |\psi_+\rangle$, modulo a global phase $\chi_+$. Furthermore, the geometric part of the global phase is expected to be chiral [10–15]. Such mode-switch behavior has been demonstrated in classical systems [16–19], yet the extension of such topological control to quantum systems—with no classical counterpart—has remained an outstanding goal in the field [20].

Here, we utilize the quantum energy levels of a superconducting circuit described by an effective non-Hermitian Hamiltonian to study quantum state control in the vicinity of the system’s EPs. While our previous work [21] characterized the static properties of this non-Hermitian system, we now employ dynamical control of the Hamiltonian parameters. We go beyond the slow driving limit demonstrated in previous works [16–19], where successful state transfer comes at the price of significant dissipation. Instead, we exploit quantum population transport in the limit of fast, closed-loop parameter variation predicted by Floquet theory. Finally, we use an auxiliary level of our quantum circuit to verify the coherent nature of this evolution by examining the geometric phases accumulated from quantum state transport. These reveal that a $\pi$ phase difference associated with the chirality of the transport [13–15]—expected from static experiments [10–12]—persists under non-Hermitian dynamical quantum evolution.

Setup—Our experiment comprises a superconducting Transmon circuit [22, 23] embedded inside a three-dimensional copper cavity (Fig. 1a) [24]. The circuit has anharmonic energy states and the first four energy levels are labeled by $|g\rangle, |e\rangle, |f\rangle$, and $|h\rangle$. The cavity mediates interaction with an environment that is set by the density of states in a microwave transmission line. We shape this density of states to enhance the dissipation of the $|e\rangle$ state while suppressing dissipation of the $|f\rangle$ state. While the evolution of the four-level quantum system can be described by a Lindblad master equation, the evolution within the excited (and lossy) manifold of states $\{|e\rangle, |f\rangle\}$ can be described by an effective non-Hermitian Hamiltonian [21, 25].

By introducing a microwave drive with detuning $\Delta = \omega_{ef} - \omega_{id}$, where $\omega_{ef}$ is the transition frequency between the $|e\rangle$ and $|f\rangle$ states, and $\omega_{id}$ is the microwave drive frequency, we produce the effective Hamiltonian:

$$H_{\text{eff}} = J (|e\rangle\langle f| + |f\rangle\langle e|) + (\Delta - i\gamma/2) |e\rangle\langle e|$$ (1)

where $J$ is the coupling rate between $|e\rangle$ and $|f\rangle$, and $\gamma$ is the decay rate of the $|e\rangle$ state. Quantum dynamics of the qubit are given by the (complex) eigenvalues $\lambda_{\pm}$ and (non-orthogonal) eigenstates $|\psi_{\pm}\rangle$ of the Hamiltonian:

$$\lambda_{\pm} = \Delta/2 - i\gamma/4 \pm \sqrt{J^2 + (\Delta/2 - i\gamma/4)^2},$$ (2)
FIG. 1: **Floquet engineering of a non-Hermitian qubit.** (a), The energy states of the transmon circuit with the non-Hermitian qubit submanifold |{e, f}| highlighted. The Hamiltonian parameters J and ∆ are tuned with a microwave drive. (b), The eigenstates and eigenvalues of H_{eff} are indicated for different values of J and ∆. (c), In the static limit, the eigenenergies are described by Riemann manifolds. (d), Floquet EP proximity is determined by plotting the overlap of Floquet eigenstates |⟨ϕ⁺|ϕ⁻⟩| in terms of J_{min} and T. These eigenstates can be visualized on the Bloch sphere (arrows).

\[ |ψ_±⟩ \propto \left( \begin{array}{c} \lambda_± J \\ 1 \end{array} \right), \]  

where \( \lambda_± \) and J are tuned with a microwave drive.

**Dynamics under Floquet Hamiltonian**—The quantum dynamics under quasistatic tuning of the system parameters is best understood as a walk through the complex energy landscape of a static Hamiltonian H_{eff} with two EPs (Fig. 1b). This understanding starts to fail once the system parameter tuning leaves the slow-driving limit. For a closed-loop in the parameter space (encircling EPs or not), since the initial and final parameter boundary conditions are the same, the Hamiltonian takes on a time-periodic—Floquet—structure.

In this case the picture of two Riemann sheets with periodic—Floquet—structure. For a closed-loop in the parameter space (encircling EPs or not), since the initial and final parameter boundary conditions are the same, the Hamiltonian takes on a time-periodic—Floquet—structure.

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In order to investigate the quantum nature of dissipation. Hence, gain/loss effects result in a chiral amplifies any population seeded by non-adiabaticity or to larger loss, and the relative gain of the other eigenstate sources of dissipation [29]. Along this parameter path, effects observed in previous works [16–19] as well as other this can be attributed to non-Hermitian gain/loss ef-
tion, the system does not evolve along the instantaneous eigenstates of $\rho_{\text{eff}}$. Two cuts from panels (b)\[(c)\] shown in blue\[red\] at $J_{\text{min}} = 6\,\text{rad/}\mu\text{s}$. (e,f), Quantum state tomography (solid lines) reveals the state evolution along the parameter path for the $\Delta_\phi (\Delta_\psi)$ direction. The dashed lines indicate the instantaneous eigenstates of $H_{\text{eff}}$.

We slowly vary the system parameters in a loop given by $T = 1.5\,\mu\text{s}$ and $J_{\text{min}} = 0.3\,\text{rad/}\mu\text{s}$. By choosing $\Delta_\phi = 10\pi\,\text{rad/}\mu\text{s}$ (Fig. 2), the system evolves from $\rho_-$, where $\text{tr} (\rho_- \sigma_z) \approx -1$, roughly following the instantaneous eigenstates of $H_{\text{eff}}$. After a complete loop that encircles the static EP, the system does not return to the initial state, instead the final state is close to $\rho_+$ which is nearly orthogonal to the initial state. This observation can be qualitatively understood by walking through the Riemann structure associated with the static EP. In addition, finite time evolution induces transitions between the two eigenstates of the system, leading to a small oscillation in the Pauli expectation values, with frequency given by the real part of the energy difference of the eigenstates.

In contrast, if we choose $\Delta_\phi = -10\pi\,\text{rad/}\mu\text{s}$, corresponding to encircling the static EP in a clockwise direction, the system does not evolve along the instantaneous eigenstates. As shown in Figure 2, the state significantly deviates from the eigenstate in the vicinity of the EP. This can be attributed to non-Hermitian gain/loss effects observed in previous works [10,11] as well as other sources of dissipation [29]. Along this parameter path, the imaginary component of the eigenenergy corresponds to larger loss, and the relative gain of the other eigenstate amplifies any population seeded by non-adiabaticity or dissipation. Hence, gain/loss effects result in a chiral population transfer in the slow driving limit.

Quantum State Transport and Geometric Phases—In order to investigate the quantum nature of state transport, as opposed to the population transfer noted in Figure 2 and prior work [10,11], we make use of the $|\text{h}\rangle$ level as a quantum phase reference, as shown in Figure 3. Resonant rotations are used to initialize the three state system in the state $\rho \propto (|\text{h}\rangle + |\text{f}\rangle)(|\text{h}\rangle + |\text{f}\rangle)$. The qubit then undergoes dynamical evolution under the Floquet Hamiltonian specified by $J_{\text{min}}, \Delta_\phi, \Delta_\psi = \pm 10\pi\,\text{rad/}\mu\text{s}$, and for $T = 800\,\text{ns}$. After this evolution, the three state system is in general in a mixed state, $\rho \propto e^{-i\Delta \chi_{\text{rad}}} (|\text{h}\rangle + e^{i\chi_+} |\text{f}\rangle)(|\text{h}\rangle + e^{-i\chi_+} |\text{f}\rangle) + e^{i\chi_-} (|\text{h}\rangle + e^{-i\chi_-} |\text{f}\rangle)$ involving both qubit eigenstates, where $\chi_{\pm}$ are phases accumulated on the states. We note that coherent terms such as $|\text{j}\rangle \langle \text{i}|$ remain negligible during the evolution. A second rotation is used to rotate either the $|\text{f}\rangle$ or $|\text{f}\rangle$ into the state $|\text{f}\rangle$ which then interferes with the $|\text{h}\rangle$ reference. We determine the contrast $c$ and total phase $\chi$ from the resulting interference (Fig. 3).

The interference contrast distinguishes between coherent state transport and incoherent population transfer between states; whereas the gain/loss effects arising from the imaginary energy components can favor population transfer between states, this process is not necessarily coherent. In Figure 3, we display the measured contrast for both final states. In the vicinity of $J_{\text{min}} = 0$, we observe higher contrast for the $|\text{f}\rangle$ final state for both the $\Delta_\phi$ and $\Delta_\psi$ parameter sweeps, indicating that the state transport that is expected from the static Riemann surfaces is quantum coherent. In comparison, we display in Figure 4 the relative population in the two states, as obtained in Figure 2,\[c\]. Near $J_{\text{min}} = 0$, our obser-
vation of larger populations in the \(|\psi_+\rangle\) and state \(|\psi_-\rangle\) states for \(\Delta_\uparrow(\Delta_\downarrow)\) sweeps is consistent with “chiral” features associated nonreciprocal population/energy transfer observed in previous work \([16–19]\). Here relative gain/loss of the two paths favors one or the other final states. This chiral effect, however, is comparatively incoherent, showing reduced contrast despite larger population.

Finally, we examine the total quantum phases accumulated for the two encircling directions, as displayed in Figure 3e,f. In general, the total quantum phase will be the sum of a dynamical phase arising from Hamiltonian evolution and a geometric phase. This is apparent in Figure 3a, where we observe significant dependence of the phase on the sweep parameter \(J_{\text{min}}\). However, for state transport that follows the Riemann surfaces, we expect the dynamical phase to cancel as the state spends equal time in either energy eigenstate. This results in the relative insensitivity of the total phase to the sweep parameter as shown in Figure 3f. Here we observe a \(\pi\) phase difference between the \(\Delta_\uparrow\) and \(\Delta_\downarrow\) sweeps as is anticipated from the static structure of EPs \([10–15]\).

Our investigation of state transport in the vicinity of exceptional point degeneracies reveals new methods of quantum coherent state control enabled through non-Hermitian (Floquet) Hamiltonian dynamics that are governed by the rich, Floquet EP landscape. This work, and the robustness with which we observe the predicted chiral geometric phases, opens new avenues to investigations of eigenvalue braiding in larger dimension non-Hermitian systems \([30, 31]\), allowing the study of exotic topological classes of these (knotted) systems. Future extensions to non-Hermiticities through non-reciprocity \([32]\) would enable scaling to quantum many-body systems where the study of topological edge-states and invariants \([33, 34]\) are expected to yield deviations from the paradigmatic bulk-boundary correspondence \([35, 36]\). Finally, the interplay of quantum measurement dynamics \([37–39]\) with the non-Hermitian dynamics explored here is expected to produce new fruitful avenues for quantum control.

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\[\Delta_\uparrow |\psi\rangle \rightarrow |\psi\rangle, \quad \Delta_\downarrow |\psi\rangle \rightarrow |\psi\rangle\]

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\[\phi_{fh}(t) = \frac{2\pi}{T} \left( \Delta(t) \right), \quad \Phi_{fh}(t) = \frac{\pi}{2} \left( \Delta(t) \right)\]

\[\psi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \psi_{\text{FH}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\]

\[\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}\]

\[\Delta_\downarrow |\psi\rangle \rightarrow |\psi\rangle, \quad \Delta_\uparrow |\psi\rangle \rightarrow |\psi\rangle\]

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(2008).
Supplemental Material for “Topological quantum state control through Floquet exceptional-point proximity”

Experimental setup—The Transmon transition frequencies are $\omega_e/2\pi = 5.7$ GHz, $\omega_f/2\pi = 5.41$ GHz, $\omega_{fh}/2\pi = 5.08$ GHz. We isolate the dynamics described by the non-Hermitian Hamiltonian by using high fidelity single shot readout to eliminate evolution that carries the system out of the excited manifold of states. The readout is achieved through standard dispersive measurement techniques [11], with coupling rates for the $|e\rangle$ and $|f\rangle$ states given by $\chi_e/2\pi = -2$ MHz, and $\chi_f/2\pi = -11$ MHz, respectively. The dispersive interaction results in a qutrit-state-dependent phase shift on a resonant cavity probe at frequency $\omega_e/2\pi = 6.634$ GHz which is subsequently amplified with a Josephson parametric amplifier [11][22] operating in phase sensitive mode. Accessing the populations of all three levels enables us to post-select on the quantum states where the evolution remains in the $\{|e\rangle, |f\rangle\}$ manifold. The final measurement fidelities are approximately 98% for $|f\rangle$, 80% for $|e\rangle$, and 82% for $|g\rangle$. For the tomography data displayed in Figure 2e, f we exclusively use measurement on the $|f\rangle$ state to improve the fidelity of the postselection.

Lindblad equation—To theoretically model the dynamics of the system, we utilize the Lindblad master equation:

$$\frac{\partial \rho(t)}{\partial t} = -i[H_e(t), \rho(t)] + \sum_{i=1}^{4} L_i \rho(t)L_i^\dagger - \frac{1}{2}[L_i^\dagger L_i, \rho(t)]$$

(4)

where $H_e = J(|e\rangle\langle f| + |f\rangle\langle e|) - \Delta/2 (|e\rangle\langle e| - |f\rangle\langle f|)$ and $L$ is the dissipation operator for relaxation and dephasing for 4-level quantum states. The relaxation operator includes $\sqrt{\gamma_e}|g\rangle\langle e|, \sqrt{\gamma_f}|e\rangle\langle f|$, and $\sqrt{\gamma_v}|h\rangle\langle f|$. The dephasing contribution to the lindblad operator includes $\sqrt{\gamma_{2e}/2}|e\rangle\langle e|, \sqrt{\gamma_{2f}/2}|f\rangle\langle f|$, and $\sqrt{\gamma_{2h}/2}|h\rangle\langle h|$. The dissipation rates are: $\gamma_e = 6.2/\mu$s, $\gamma_f = 0.32/\mu$s, $\gamma_h = 0.36/\mu$s, $\gamma_{2e} = 3.7/\mu$s, $\gamma_{2f} = 0.9/\mu$s, $\gamma_{2h} = 1.4/\mu$s. Since $\gamma_e \gg \gamma_f$ we are able to neglect the small effect of this additional decay channel and set $\gamma = \gamma_e$ in the effective Hamiltonian (Eq. 1).

Quantum state tomography—Quantum state tomography is achieved by pausing the evolution and performing measurements along the $X$, $Y$, and $Z$ axes of the qubit. Sampling 10^4 measurements along each axis yields the qubit Pauli expectation values $x \equiv \langle \sigma_x \rangle$, $y \equiv \langle \sigma_y \rangle$, and $z \equiv \langle \sigma_z \rangle$. Measurements about the different axes are obtained by abruptly setting $J_1 = 0$ and performing rotations about the $X$ and $Y$ axes, or no rotation, followed by a measurement along the $Z$ axis. For all measurements, we discard any experimental sequence where the final measurement finds the qubit has left the $\{|e\rangle, |f\rangle\}$ manifold.

Floquet Hamiltonian—The time-periodic Hamiltonian $H(t)$ allows us to obtain the system’s full evolution in terms of two Floquet eigenvalues $\epsilon_{\pm}$ and the family of periodic Floquet eigenstates $|\phi_{\pm}(t)\rangle = |\phi_{\pm}(t + T)\rangle$ by solving the eigenvalue problem for one-period propagator with different initial times $t \in [0, T]$,

$$G(T + t, t)|\phi_{\pm}(t)\rangle = \eta_{\pm}|\phi_{\pm}(t)\rangle,$$

(5)

where $G(T + t, t) = \exp \left[-i \int_{t}^{T+t} H(t') dt' \right]$ and $\epsilon_{\pm} = i \ln \eta_{\pm}/T$ are two Floquet quasienergies. The time-evolution of an arbitrary state $|\psi(0)\rangle = \sum_{\pm} c_{\pm}|\phi_{\pm}(0)\rangle$ to arbitrary time $t$ is then given by

$$|\psi(t)\rangle = \sum_{\pm} c_{\pm} e^{-i \epsilon_{\pm} t}|\phi_{\pm}(t)\rangle.$$

(6)

Floquet EP contours and EP proximity—For typical experimental parameters ($\gamma = 7/\mu$s, $J_{\text{max}} = 30$ rad/\mu s, and $\Delta = 5\pi$ rad/\mu s), Figure 3 shows normalized difference of Floquet exponents, $\Delta \eta \equiv (|\eta_+| - |\eta_-|)/(|\eta_+| + |\eta_-|)$. The boundary between $\Delta \eta = 0$ and $\Delta \eta > 0$ marks the Floquet EP contours; they only occur for $J_{\text{max}} < -\gamma/4$ and, at larger $\Delta$ are pushed further down. The non-orthogonality of the Floquet eigenvectors, $|\langle \phi_+ | \phi_- \rangle| \langle \psi_\pm \rangle$ (b) and the mode-switch probability $P(\psi_-)$ (c), on the other hand, track each other.

Floquet dynamics in the Hermitian limit—To study the effect of Floquet evolution in the Hermitian limit, we calculate the populations in the final state $|\psi_\mp \rangle$ for the two sweep directions for the parameters chosen in our experiment and in the Hermitian limit, by setting $\gamma_e = 0$. These calculations are displayed in Figure 3. We observe that several of the short-sweep-time features are preserved in the Hermitian limit. The robust state mapping, expected from adiabatic evolution on the Riemann manifolds, persists only in the non-Hermitian case for long evolution times. We further note that the geometric phases acquired in the non-adiabatic Hermitian limit where state mapping can occur also exhibit the chiral geometric phases we have observed in the non-Hermitian dynamical case.

Geometric phase—When the parameters of a Hamiltonian are tuned in a closed-loop fashion, the eigenstates acquire a combination of geometric and dynamical phases. In our experiment we prepare a superposition of the state $|\psi_- \rangle$ and the quantum phase reference state $|h \rangle$ using microwave pulses: we apply a resonant $\pi$ rotation to transfer all
population from $|g\rangle$ to $|e\rangle$; followed immediately by a second $\pi$ rotation from $|e\rangle$ to $|f\rangle$; a $\pi/2$ rotation on the $\{|h\rangle,|f\rangle\}$ manifold prepares the state $\propto (|h\rangle + |f\rangle)$; finally, a $\pi/2$ rotation along the $Y$ axis of the qubit $\{|e\rangle,|f\rangle\}$ manifold results in an equal superposition of $|h\rangle$ and $|\psi_-\rangle$. Following this state preparation, we apply a drive to the qubit to produce the Hamiltonian $H_{\text{eff}}$. The parameters $(\Delta(t), J(t))$ of $H_{\text{eff}}$ are tuned over a period of $T = 800$ ns.

Based on Figure 2b,c we anticipate that the final populations in $|\psi_{\pm}\rangle$ will depend on the sweep parameter $J_{\text{min}}$. Two primary effects contribute to this dependence: first, the Riemann surface for the real part of the eigenenergies of the static Hamiltonian predicts a state mapping behavior $|\psi_-\rangle \rightarrow |\psi_+\rangle$ for $|J_{\text{min}}| < J_{\text{EP}}$, independent of sweep direction, and second, the imaginary energy component yields a loop direction dependent effect, as displayed in Figure 2f.

After the parameter sweep is concluded we utilize a resonant qubit $\pi/2$ rotation to selectively rotate the $|\psi_-\rangle$ or $|\psi_+\rangle$ state into $|f\rangle$. A final $\pi/2$ rotation on the $\{|h\rangle,|f\rangle\}$ manifold completes the Ramsey measurement. By stepping the phase of this final rotation, we shift the interference by one complete “fringe”, allowing the measurement of the phase offset $\chi$ and contrast. The final projective measurement $\Pi_{g,e,f}$ is composed of a $\pi$ rotation on the qubit manifold to transfer population in $|f\rangle$ to the state $|e\rangle$ before readout.

This interference measurement allows us to determine the interference phase for four possible interference paths (two directions, two final states). The two paths with $|\psi_-\rangle \rightarrow |\psi_-\rangle$, for $\Delta_<$ and $\Delta_<$ accumulate total phases that are a combination of dynamical and geometric phase. In contrast, for the two paths $|\psi_-\rangle \rightarrow |\psi_+\rangle$, for $\Delta_<$ and $\Delta_<$, the state spends equal time in either eigenstate, causing the dynamical phase to cancel.
FIG. 5: **Comparing Floquet dynamics in the non-Hermitian and Hermitian limit.** The state mapping probability $P(\psi_-)$ with $\Delta_\gamma$ for the non-Hermitian case (a) remains robust, while its counterpart in the Hermitian limit (c) decreases with increasing sweep time $T$. The exchange asymmetry $\Delta_\gamma \leftrightarrow \Delta_\Theta$ in the non-Hermitian case (a,b) also disappears in the Hermitian limit (c,d).