Six-Photon Amplitudes

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We present analytical results for all six-photon helicity amplitudes. For the computation of this loop induced process two recently developed methods, based on form factor decomposition and on multiple cuts, have been used. We obtain compact results, demonstrating the applicability of both methods to one-loop amplitudes relevant to precision collider phenomenology.

The self-interaction of photons (light-by-light scattering) mediated through a virtual charged fermion loop is a fundamental, although yet unobserved, prediction of quantum electrodynamics. The corresponding multi-photon scattering amplitudes are of outstanding theoretical interest, since they exhibit a high degree of symmetry. They can be used to establish and further develop methods for the calculation of virtual corrections to multi-leg processes, and to study symmetry patterns in the results, thus providing further insight into the analytical structure of quantum field theories at the loop level.

In the past, the four-photon amplitudes were derived at one loop for massless and massive fermions [1], and at two loops for massless fermions [2]. At two loops, four-photon scattering in supersymmetric Yang-Mills theories [3] was studied as well, yielding first evidence for a leading transcendentality behaviour, which was uncovered subsequently also in multi-gluon amplitudes [4], and has sparked many new developments (see [5] and references therein) in the study of multi-loop scattering amplitudes in super-Yang-Mills theories and perturbative gravity.

The computation of one-loop multi-particle amplitudes is currently among the most pressing issues in the preparation of precision next-to-leading order (NLO) calculations for the upcoming CERN LHC experiments. Given the large variety of potentially interesting multi-particle final states, automated methods for one-loop corrections would be very desirable, and are currently under intense development [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. These methods range from purely analytical schemes to completely numerical approaches.

Compact analytical expressions for amplitudes involving $n > 4$ external photons were obtained only for specific helicity configurations. All amplitudes with odd $n$ vanish due to parity conservation; amplitudes with even $n > 4$ vanish if all or all but one photons have the same helicity [28]. For $n = 6$ external photons, one finds therefore only two non-vanishing amplitudes, which we denote by $A_6(-++++)$ and $A_6(-+---)$. An analytical expression for $A_6(-+++)$ was computed already long ago [29], using the method described in [28]. $A_6(-+---)$ was obtained recently using a purely numerical method for the loop integration. In this paper, we use two completely different recently developed methods (based either on form factor decomposition [6] or on multiple cuts [14, 15, 16, 17]) to compute $A_6(-+++)$ and $A_6(-+---)$, obtaining compact results which respect the symmetry properties of the process under consideration. Besides allowing a prediction for the process $\gamma\gamma \to 4\gamma$, our results serve as a highly non-trivial proof of applicability of both methods used, and illustrate how form factor-based and cut-based techniques can be matched onto each other in detail.

1. Structure of the Amplitudes

Despite the absence of a corresponding tree-level process and the (Feynman) diagram-by-diagram UV-finiteness in four dimensions, the cut-constructibility of the 6-photon amplitudes is not guaranteed, in accordance with the power counting argument in [34]. The fact that the rational parts of the six-photon amplitudes actually do evaluate to zero was shown in [30]. Consequently, the amplitude can be written as a linear combination of poly-logarithms and transcendental constants, associated to a known basis of functions, the master integrals (MI), formed by box-, triangle- and bubble-type integrals.
The result for the amplitude \( A_6(-++++) \) has the following structure

\[
A_6(-++++) = \frac{e^6}{(4\pi)^2} \sum_{\sigma \in S_4/(\mathbb{Z}_2 \times \mathbb{Z}_2)} \left[ \mathcal{F}_1(s_{1\sigma_7}, s_{2\sigma_4}, s_{1\sigma_3}) + \mathcal{F}_1(s_{2\sigma_3}, s_{2\sigma_4}, s_{2\sigma_3}) - \mathcal{F}_{2B}(s_{1\sigma_7}, s_{1\sigma_3}, s_{1\sigma_3}, s_{2\sigma_3}) - \mathcal{F}_{2B}(s_{2\sigma_3}, s_{2\sigma_4}, s_{2\sigma_3}, s_{1\sigma_3}) \right],
\]

where

\[
\mathcal{F}_i = d_i \times F_i \quad (i = 1, 2B).
\]

\( F_1, F_{2B} \) are the finite parts of the one-mass and two-mass easy box functions \([32, 33]\), and \( d_1 \) and \( d_{2B} \) are their coefficients, that can be obtained from \([10] \) and \([17] \) below. The sum in Eq. (1) runs over the discrete quotient group \( S_4/(\mathbb{Z}_2 \times \mathbb{Z}_2) \) which has 6 elements.

The quotient space structure can be inferred from the symmetry properties of the combination of basis functions. It is invariant under interchanging the indices \( \sigma_3 \leftrightarrow \sigma_4 \) and \( \sigma_5 \leftrightarrow \sigma_6 \). The permutations generate exactly those functions which are allowed by the cutting rules.

The result for the amplitude \( A_6(-+-+--) \) has the following structure

\[
A_6(-+-+--) = \frac{e^6}{(4\pi)^2} \left[ \sum_{(\sigma, \tau) \in \mathbb{S}_2/(\mathbb{Z}_2 \times \mathbb{Z}_2)} \mathcal{F}_1(s_{1\sigma_7}, s_{2\tau_3}, s_{2\tau_3}, s_{1\tau_3}) \right]
\]

for the first term, \( S_3(1, 3, 5)/\mathbb{Z}_2(3, 5) \times S_4(2, 4, 6)/\mathbb{Z}_2(2, 6) \) for the second term, \( S_3(1, 3, 5) \times S_4(2, 4, 6) \) for the third one and \( S_3(2, 4, 6) \) for the last term, contain 9, 9, 36 and 6 elements respectively. They generate all possible different basis functions \( F_1, F_{2A} \) and \( I_3 \) which are allowed by cutting rules. Note that each cut is corresponding to a certain Madelstam variable. Compatibility with cutting rules means that no Mandelstam variables \( s_{ij} \) or \( s_{ijk} \) appear as a function argument such that the corresponding legs \( i, j, k \) have like-sign helicities. This is certainly only true for amplitudes with massless particles.

2. Cut-Construction

As the six-photon amplitudes are cut-constructible, the computational effort is reduced to the computation of the rational coefficients of a linear combination of master integrals as described in section I. According to the principle of unitarity-based methods \([24]\), the exploitation of the unitarity-cuts of each master integral enables the extraction of the corresponding coefficient from the amplitude.

To that aim, we employ the quadruple-cut technique \([14]\) for box-coefficients, the triple-cut integration \([17]\) for triangle-coefficients, and the double-cut integration \([15, 16]\) for bubble-coefficients, by sewing in the multiple cuts the QED tree-level amplitudes given in \([35]\). Spinor algebra and numerical evaluation of spinor products has been implemented in a Mathematica package \([36]\). In the following we use by now standard spinor notation, with \( \langle a|P_{\tau \cdots j}|b \rangle = \langle a|(-1)^{j-\tau}P_{\tau \cdots j}|b \rangle \), and multi-particle momenta defined as \( P_{\tau \cdots j} = p_i + \ldots + p_j \), with \( p_i \) being the momentum of the \( k \)-th external photon, considered incoming.

2.1. Construction of \( A_6(-++++) \)

The analytic expression for \( A_6(-++++) \) was computed by Mahlon \([29]\). It can be expressed in terms of the two classes of box functions \( F_1 \) and \( F_{2B} \) as shown in eq. (1). Using the symmetry of the amplitude, it is sufficient to compute the coefficients of a representative box-function for each of the two classes (and their parity-conjugates).

One-mass Box \((5^+6^+1^-|2^+3^-|4^+)\)

Defining the prefactor

\[
\gamma_1 = \frac{8}{(24)^2(25)(26)(27)(28)},
\]

the result of the quadruple-cut \( d_1^{(1)} \) for the configuration
displayed in Fig.1 reads
\[ d_1^{(1)} = r_1 (23)^2 (14)^2. \] (4)
By reversing the internal helicities one gets
\[ d_1^{(2)} = r_1 (12)^2 (34)^2. \] (5)
The coefficient of \( F_1(s_{23}, s_{34}, s_{561}) \), the finite part of the one-mass box, reads
\[ d_1(s_{23}, s_{34}, s_{561}) = 2 \times \frac{(-2)}{s_{23}s_{34}} \times \frac{d_1^{(1)} + d_1^{(2)}}{2}, \] (6)
where the prefactor 2 accounts for the contribution coming from a fermion looping in the opposite direction; the second factor is the standard coefficient of the finite part of the one-mass box; and the last one is the average of the solutions of the quadruple-cut. The same structure is understood for the forthcoming four-point coefficients.

**Two-mass easy Box** \((6^+1^-|2^+|3^-5^+|4^+)\)
The coefficient of \( F_{2B}(s_{612}, s_{235}, s_{61}, s_{35}) \), the finite part of the considered two-mass-easy box is defined as
\[ d_{2B}(s_{612}, s_{235}, s_{61}, s_{35}) = 2 \times \frac{(-2)}{s_{612}s_{235} - s_{61}s_{35}} \times \frac{d_{2B}^{(1)} + d_{2B}^{(2)}}{2}, \] (7)
with \( d_{2B}^{(1)} \) being the quadruple-cut for the configuration displayed in Fig.1 and \( d_{2B}^{(2)} \) the complementary contribution coming from a fermion circulating around the loop in the opposite direction.

Although the quadruple-cuts \( d_{2B}^{(1)} \) and \( d_{2B}^{(2)} \) are respectively different from the quadruple-cuts \( d_1^{(1)} \) and \( d_1^{(2)} \) in Eqs. (4, 5), the coefficients of \( F_{2B}(s_{612}, s_{235}, s_{61}, s_{35}) \) in (7) and of \( F_1(s_{23}, s_{34}, s_{561}) \) in (6) are not independent, and one finds the relation,
\[ d_{2B}(s_{612}, s_{235}, s_{61}, s_{35}) = -d_1(s_{23}, s_{34}, s_{561}). \] (8)

**2.2. Construction of \( A_6(--+-++) \)**
The analytic expression for \( A_6(--+-++) \) is the main result of this letter. There appear three classes of functions: one-mass and two-mass-hard box-functions \( F_1 \) and \( F_{2A} \), and three-mass triangle-function \( I_3 \). Bubble-functions are absent. As before, we compute the coefficients of a representative function for each of the three classes (and their parity-conjugates), and obtain the whole amplitude by summing over all non-identical permutations of external particles.

![Diagram](image1)

**Two-mass hard Box** \((1^-2^+|3^-4^+|5^-6^+)\)
We define the prefactor
\[ r_{2A} = \frac{s_{345}s_{345}^2}{(26)[35][6]P_{12}[3][2]P_{34}[5][6]P_{34}P_{12}[6][5]P_{34}P_{12}[5]}, \] (9)
thus the result of the quadruple-cut \( d_{2A}^{(1)} \) for the configuration displayed in Fig.2 is
\[ d_{2A}^{(1)} = r_{2A}s_{345}^2(61)^2[45]^2. \] (10)
By reversing the internal helicities one gets
\[ d_{2A}^{(2)} = r_{2A}(1|P_{34}[5]|2|P_{12}|4)^2 \] (11)
The coefficient of \( F_{2A}(s_{56}, s_{345}, s_{12}, s_{34}) \), the finite part of the two-mass-hard box, reads
\[ d_{2A}(s_{56}, s_{345}, s_{12}, s_{34}) = 2 \times \frac{(-2)}{s_{345}s_{56}} \times \frac{d_{2A}^{(1)} + d_{2A}^{(2)}}{2}, \] (12)

**One-mass Box** \((1^-2^+3^-|4^+|5^-|6^+)\)
With the prefactor
\[ r_1 = \frac{s_{45}s_{56}s_{456}}{(46)^2[4]P_{12}[1][4]P_{23}[3][6]P_{12}[1][6]P_{23}[3]}, \] (13)
the result of the quadruple-cut $\hat{d}_1^{(1)}$ for the configuration displayed in Fig[2] is

$$\hat{d}_1^{(1)} = r_1 \langle 45 \rangle^2 \langle 6|P_{123}|2\rangle^2. \quad (14)$$

By reversing the internal helicities one gets

$$\hat{d}_1^{(2)} = r_1 \langle 56 \rangle^2 \langle 4|P_{123}|2\rangle^2. \quad (15)$$

The coefficient of $F_1(s_{45}, s_{56}, s_{123})$, the finite part of the one-mass box, reads

$$d_1(s_{45}, s_{56}, s_{123}) = 2 \times \frac{(-2)}{s_{45}s_{56}} \times \frac{\hat{d}_1^{(1)} + \hat{d}_1^{(2)}}{2}. \quad (16)$$

### Three-mass Triangle $(1^-2^+3^-4^+5^-6^+)$

From the triple-cut of the configuration displayed in Fig[2] the triangle coefficient reads

$$c_3^{(1)} = 4 \frac{(1\ 2\ (1|P_{34}|2|^2 e_1^2)}{(2\ 4\ (3\ 4)} \frac{N_{e_1}}{D_{e_1}} + \{e_1 \rightarrow e_2\}, \quad (17)$$

with

$$N_{e_1} = \left[ -s_{12}(1\ 3) + (s_{12} - s_{34})(1\ 3) + (1|P_{34}|P_{12}|3) \right] e_1 \right]$$

$$\times \left[ s_{12}s_{34}(1\ 5)e_1 + (1|P_{12}|P_{34}|5) - s_{12} + (s_{12} - s_{34})e_1 \right] \right)^2$$

$$\times \left[ -s_{12}(1\ 3)(2\ 4) + (s_{12} - s_{34})(1\ 3)(2\ 4) +$$

$$+ (3\ 4)(1|P_{34}|P_{12}|2) - (3\ 2) (1|P_{34}|P_{12}|4) \right] e_1 \right], \quad (18)$$

$$D_{e_1} = \left[ -s_{12}(1\ 2) + (s_{12} - s_{34})(1\ 2) + (1|P_{34}|P_{12}|2) \right] e_1 \right]$$

$$\times \left[ s_{12}s_{34}(1\ 6)e_1 + (1|P_{12}|P_{34}|6) - s_{12} + (s_{12} - s_{34})e_1 \right] \right)^2$$

$$\times \left[ -s_{12}(1\ 4) + (s_{12} - s_{34})(1\ 4) + (1|P_{34}|P_{12}|4) \right] e_1 \right]$$

$$\times \left[ -s_{12}(1\ 6) + (s_{12} - s_{34})(1\ 6) + (1|P_{34}|P_{12}|6) \right] e_1 \right]$$

$$\times \left[ s_{12}(1|P_{34}|3)e_1 + (1|P_{12}|3) - s_{12} + (s_{12} - s_{34})e_1 \right] \right] \right)^2$$

$$\times \left[ -s_{12} + (s_{12} - s_{34} + (1|P_{34}|1) \right] e_1 \right], \quad (19)$$

where

$$e_{1,2} = \frac{s_{12}}{2} - \frac{3s_{34} - s_{56} - s_{12} \pm \sqrt{\Delta_{12,34,56}}}{s_{12}(s_{34} - s_{56}) + s_{34}(-2s_{34} + s_{56})}, \quad (20)$$

with

$$\Delta_{12,34,56} = s_{12}^2 + s_{34}^2 + s_{56}^2 - 2s_{12}s_{34} - 2s_{12}s_{56} - 2s_{34}s_{56}. \quad (21)$$

The contribution coming from reversing the inner helicities, $c_3^{(2)}$, amounts to the same value, $\hat{c}_3^{(2)} = c_3^{(1)}$, therefore the coefficient of $I_3(s_{12}, s_{34}, s_{56})$, the three-mass triangle within the amplitude is,

$$c_3(s_{12}, s_{34}, s_{56}) = 2 \times (\hat{c}_3^{(1)} + \hat{c}_3^{(2)}) \quad (22)$$

where the factor 2 accounts for the contribution coming from a fermion looping in the opposite-direction. Although not manifest in Eqs. [18,19], one can see analytically that in [17] the dependence on $\sqrt{\Delta}$ drops out, because $e_1$ and $e_2$ only differ in the sign of $\sqrt{\Delta}$. Therefore $c_3$ is a rational function of spinors.

### 3. Form factor approach

In the Feynman-diagrammatic approach the six-photon amplitude is represented by 120 one-loop diagrams which differ only by permutations of the external photons.

As the corresponding integrals are IR/UV finite, the Dirac algebra can be performed in $D = 4$ dimensions. However, using algebraic tensor reduction, one has to work with a $(4 - 2\epsilon)$-dimensional loop momentum, since at intermediate steps scalar integrals are generated which are formally divergent in $D = 4$ dimensions. The respective coefficients will drop out in the end, which serves as a check of the computation. We use the spinor helicity method and define projectors on the helicity amplitudes $A_6(- + + + +)$ and $A_6(- + + --)$ in such a way that – by choosing convenient reference momenta for the polarisation vectors – we obtain global spinorial factors for each amplitude. The resulting expressions contain integrals with scalar products of external vectors and loop momenta in the numerator. For the six-point integrals all scalar products between the loop momentum and external momenta are reducible, i.e. they can be written as differences of inverse propagators. As a result, at most rank-one six-point functions have to be evaluated. For the $(N < 6)$-point functions at most three loop momenta remain in the numerator. As was shown in [30], this can be related to the cut-constructibility argument of [31]. To reduce the irreducible tensor integrals to scalar integrals we use the algebraic approach outlined in [10], which leads to a representation of the amplitude in terms of the basis functions $I_3, F_1, F_{2A}, F_{2B}$. The formalism is implemented using FORM [35]. The coefficients of the basis functions are then stored and simplified further with Maple and/or Mathematica. The same setup was already used for simpler 4- and 5-point loop amplitudes [37].
Although only a restricted set of functions actually needs to be evaluated due to the symmetry properties of the amplitude, the symmetry relations serve as a stringent check on the implementation. Therefore, and in order to test our setup in view of future applications, all 120 diagrams have been calculated.

We stress that the resulting expressions for the coefficients, although they are not evaluated in terms of spinor products, allow for a fast numerical evaluation. We note that in all expressions at most one power of inverse Gram determinants survives, which is intrinsic to the chosen function basis.

We have cross-checked the result obtained by the form factor approach with the one achieved by using the cutting rules and find perfect agreement. We have further compared to the recent numerical result of Nagy and Soper [23]: mapping our helicity configuration onto their one, according to $A_{6}(2^{\mp}, 1^{\pm}, 3^{-}, 4^{\pm}, 6^{-}, 5^{\pm}) = A_{6}(1^{\mp}, 2^{-}, 3^{\pm}, 4^{\pm}, 5^{\pm}, 6^{-})$, and using the same kinematics as in Fig. 5 of [23], we find the result shown in Fig. 3, which agrees with Nagy and Soper within their plot range. We note that our results are produced by simply evaluating the analytical expressions obtained by the form factor approach with the one achieved by using the cutting rules and find perfect agreement. We have further compared to the recent numerical result of Nagy and Soper [23]: mapping our helicity configuration onto their one, according to $A_{6}(2^{\mp}, 1^{\pm}, 3^{-}, 4^{\pm}, 6^{-}, 5^{\pm}) = A_{6}(1^{\mp}, 2^{-}, 3^{\pm}, 4^{\pm}, 5^{\pm}, 6^{-})$, and using the same kinematics as in Fig. 5 of [23], we find the result shown in Fig. 3, which agrees with Nagy and Soper within their plot range. We note that our results are produced by simply evaluating the analytical expressions obtained by the form factor approach, therefore we do not have numerical errors. The peak structures in the plots stem from complicated phase patterns in the amplitudes.

4. Conclusions

The six-photon amplitudes can be fully expressed as linear combinations of known one-loop master integrals with three and four external momenta. Using the techniques of form factor decomposition [6] and of multiple cuts [14, 15, 16, 17], we have derived analytic expressions for the coefficients of the master integrals. The expressions obtained from the multiple-cut method are typically more compact than the form factor results, such that we decided to quote only the former. All coefficients agree numerically. We fully confirm earlier purely numerical results [23] for the six-photon amplitudes. Our calculation demonstrates the applicability of the form factor decomposition and multiple-cut methods to non-trivial multi-leg processes at one-loop, and illustrates that both methods can be formulated in the same integral basis. In future applications, one could therefore envisage to combine both methods. Both approaches used here can also be applied to amplitudes containing massive particles. Our result shows that the analytical evaluation of one-loop amplitudes with similar kinematics relevant for the LHC is feasible.

Note added: Shortly after the appearance of this Letter, numerical results on the same subject were presented by Ossola, Papadopoulos and Pittau using a reduction method at the integrand level [39]. Full consistency was found where applicable. Moreover, the triangle coefficient (17) in $A_{6}(- + - + - +)$ was rederived recently by Forde [40] using an independent method. Full agreement was found with our result.

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