Decoherence of Anyon Qubit

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I. INTRODUCTION

Quantum computation today is a fast and extensively developing field of investigations both theoretical and experimental. The main problem which stands on the way to implementation of effective qubit for a quantum computer is decoherence caused by interaction of the microscopic system with the rest of the world. It means that degrees of freedom of the qubit system in the process of its evolution entangle inevitably with huge number of other degrees of freedom leading to loss of information transferred to the system before. On the language of operators, density matrix of the world evolve by unitary time evolution operator, where as density matrix of microscopic system under consideration almost always does not \( \Pi \). Decoherence of a quantum system has a fundamental nature and actually the main reason of transition from quantum to classical mechanics. In spite of the chosen quantum system, this effect shall be taken into consideration. The question arises here: "What system to choose or to find so that small enough effect of decoherence could allow quantum computations"?

One mechanism of decoherence of anyon qubit due to interaction with edge states is considered. The calculations are made at low temperature in Markovian and "short-time" approximation. Two approximations are compared.

II. DESCRIPTION OF THE SYSTEM ANYON QUBIT AND EDGE STATES

In case of weak interaction the total hamiltonian of the system anyon qubit and edge states can be described in standard way:

\[ H = H_S + H_B + H_{int} \tag{1} \]

where \( H_S, H_B \) and \( H_{int} \) are hamiltonians of correspondingly the anyon qubit (Sistem), the edge states (Bath) and interaction between them.

Anyon qubit consists of two antidots separated by distance \( d \). Applying voltage to the gates near one or the other antidot it is possible to control location of anyon on one of the two antidots.

Tunnelling of anyons from one antidot to the other is similar to that of tunnelling of an electron from one well to the other one, which has in details reviewed in \( \mathbb{R} \). By analogy the Hamiltonian for the model of anyon localization on two antidots \( H_S \) can written as:

\[ H_S = \frac{1}{2} \varepsilon \sigma_z - \frac{1}{2} \Omega \sigma_x, \tag{2} \]

The first term in \( \mathbb{R} \) describes localization of the anyon either on the one or the other antidot with energy difference \( \varepsilon \). The second term describes tunnelling of the anyon between two antidots with energy splitting \( \Omega \). We choose units where \( \hbar = 1 \) and then return to ordinary units in final formulas.

The second term in \( \mathbb{R} \) describes bath modes or edge states. In the literature these states are also known as Luttinger liquid or 1D chiral modes. In our case we will use hydrodynamic model proposed by Wen in \( \mathbb{R} \). According to this model, 2DEG is considered to be as non-compressible liquid with 2D electron density \( n_e = \text{const.} \). Disturbance of this liquid causes emergence of edge excitations, or waves, described by \( \rho(x) = n_e h(x) \) - linear density along the edge \( (h(x) - \text{displacement of the excitation perpendicular to the edge}) \).
It is easy to show that hamiltonian for such excitations in Fourier representation can be written as:

$$H_B = \frac{2\pi v}{\nu} \sum_{k>0} \rho_k \rho_{-k}$$  \hspace{1cm} (3)$$

The final term $H_{int}$ in (1) is interaction between anyon qubit and edge states. It can be found as work produced by electric field of anyon qubit on the transfer of electron along the edge:

$$H_{int} = \sigma_z \int dx U(x) \rho(x)$$  \hspace{1cm} (4)$$

$U(x)$ - potential difference due to localization of anyon on one or the other antidot. For the simplicity reasons we consider edge excitation moving along the straight line perpendicular to anyon qubit and set the length of the edge as a unit. As it is easy to check the length is eliminated from final results.

Quantization of $\rho(x)$ can be made based on the following commutation relations:

$$[\rho_k, \rho_{k'}] = \frac{\nu k}{2\pi} \delta_{k, -k} \hspace{1cm} k, k' = \text{integer} \times 2\pi$$  \hspace{1cm} (5)$$

Based on the last commutation relation we can see that $\rho_k$ and $\rho_{-k}$ behave as boson operators of creation and destruction respectively:

$$\rho_k = \sqrt{\frac{\nu k}{2\pi}} b_k^+ \hspace{1cm} \rho_{-k} = \sqrt{\frac{\nu k}{2\pi}} b_k \hspace{1cm} k > 0$$  \hspace{1cm} (6)$$

The above hamiltonian now can be rewritten as

$$H = \frac{1}{2} \varepsilon \sigma_z - \frac{1}{2} \Omega \sigma_x + \sigma_z \sum_{k>0} \beta_k (b_k^+ b_k) + \sum_{k>0} \varepsilon_k b_k^+ b_k$$  \hspace{1cm} (7)$$

$$\beta_k = \sqrt{\frac{\nu k}{2\pi}} U_k, \hspace{1cm} \varepsilon_k = \nu k, \hspace{1cm} (8)$$

where $U_k$ is Fourier transformation of $U(x)$ and $\nu$ - filling factor of Landau level. As it is easy to see edge states have linear dispersion law, similar to that of the acoustic phonons in lattice. It means that edge states are gapless excitations.

**III. DECOHERENCE OF THE ANYON QUBIT IN MARKOVIAN APPROXIMATION**

Now let’s proceed to calculations of decoherence and consider the case when $\varepsilon = 0$. Master equation for density matrix $\rho = \rho_S \otimes \rho_B$ of the system "anyon qubit" and "edge states" is

$$\dot{\rho} = -i[H, \rho]$$  \hspace{1cm} (9)$$

Tracing over bath modes ($\rho_S = Tr_B \rho$) we can get equation for anyon qubit density matrix (see e.g. [8]):

$$\dot{\rho}_S = -i[H_S, \rho_S] - \int_0^\infty d\tau \{ \langle BB(-\tau) \rangle \cdot [S, S(-\tau) \rho_S] - \langle B(-\tau)B[S, \rho_S S(-\tau)] \}$$  \hspace{1cm} (10)$$

Here $\langle \ldots \rangle = Tr_B (\ldots \rho_B)$, $[O_1, O_2]$ - commutator between operators $O_1$ and $O_2$

$$H_S = -\frac{1}{2} \Omega \sigma_x$$

$$S = \sigma_z,$$

$$B = \sum_{k>0} \beta_k (b_k^+ b_k)$$

$$O(t) = e^{i t H_S} O e^{-i t H_S}$$

The Markovian approximation is also known as long-time approximation and the following assumptions are to be satisfied for the density matrix:

1. $\rho(t') = \rho(t)$ - It means that the system loses all memory of its past

2. $\rho(t) = \rho_S(t) \rho_B(0)$, where $\rho_B(0) = \exp(-\beta H_B)/Z$ that is obeys the Gibson distribution. It means that energy by the system never returns again to the system and any changes in it doesn’t effect on the "bath"

3. $\langle BB(-\tau) \rangle \rightarrow 0$ when $t \gg \tau_0$, where $\tau_0$ is often called correlation time for the bath. This assumption allows us to replace integral limits to infinity

After simple enough calculations the master equation for $\rho_S = \frac{1}{2}(1 + x\sigma_x + y\sigma_y + z\sigma_z)$ can be rewritten as

$$\dot{x} = -\Gamma x + \lambda$$

$$\dot{y} = (\Omega + \omega) z - \Gamma y$$

$$\dot{z} = -\Omega y$$  \hspace{1cm} (12)$$

Here

$$\Gamma = \frac{1}{v} \beta_k^2 \Omega \coth \left( \frac{\Omega}{2T} \right)$$

$$\lambda = \frac{1}{v} \beta_k^2 \Omega \coth \left( \frac{\varepsilon_k}{2T} \right)$$

$$\omega = \frac{2\Omega}{\pi} \int_0^\infty dk \beta_k^2 \coth \left( \frac{\varepsilon_k}{2T} \right) \frac{1}{\Omega^2 - \varepsilon_k^2}$$  \hspace{1cm} (13)$$
where $\Gamma$ - dissipation rate. Solving the system of differential equations it is easy to see that

$$
Tr_S(p_S^2(t)) = \frac{1}{2}[1 + x^2 + y^2 + z^2]
$$

$$
= \frac{1}{2}[1 + \tanh^2 \left( \frac{\Omega}{2T} \right) + C(T)e^{-\Gamma t}],
$$

where $C(T)$ - constant magnitude which depends only on initial conditions and temperature. The constant satisfies the following conditions: $|C(T)| < 1$ and $C(T) \to 0$ when $T \to 0$.

Specifically for the case of unscreened field of anyon qubit with $\nu = 1/m$ and $T = 0$

$$
U(x) = \frac{q^2d}{2\pi\epsilon_0 r^2}.
$$

$q$ is the charge of anyon qubit. According to the theory of anyons (see e.g. [11]) in FQHE $q = e\nu = e/m$. Then dissipation rate $\Gamma$ in conventional units will be equal to

$$
\Gamma = \left( \frac{d}{L} \right)^2 \left( \frac{e^2}{2\epsilon_0 hv} \right)^2 \frac{\Omega}{2\pi m^3\hbar} e^{-2\Omega L/hv}
$$

(15)

Here $L$ is the distance between the qubit and the edge, $d$ - distance between two antidots and $\epsilon$ - dielectric constant. This formula is different from that of obtained in [5]. For experimental values $\epsilon \simeq 10, v \simeq 10^5$ m/s, $\Omega \simeq 0.1K, d \simeq 100nm$ and $L \simeq 3\mu m$ we have $\hbar \Gamma/\Omega \simeq 10^{-3}$.

IV. DECOHERENCE OF THE QUBIT IN A SHORT-TIME APPROXIMATION

For the short-time approximation let’s use the formula given in [11].

$$
Tr_S(p_S^2(t)) = \frac{1}{2}[1 + e^{-2B^2(t)}]
$$

(16)

$$
B^2(t) = 8 \sum_{k > 0} \frac{\beta^2_k}{\xi^2_k} \sin \left( \frac{\xi_k t}{2} \right) \coth \left( \frac{\xi_k}{2T} \right) = AI(t),
$$

(17)

where

$$
A = \frac{2}{m^3} \left( \frac{d}{L} \right)^2 \left( \frac{e^2}{2\epsilon_0 hv} \right)^2
$$

(18)

For this particular system of anyon qubit we have case of Ohmic dissipation, for

$$
\beta^2(x) \sim x e^{-x/\omega_c}
$$

(19)

We are interested in the case when temperature of the system $T$ is lower than any energy scales in the system including characteristic cut-off frequency $\omega_c = v/4L$ that is $T \ll \omega_c$. Integral $I(t)$ has the form

$$
I(t) = \int_0^\infty \frac{dx}{x} e^{-x/\omega_c} \sin^2(xt) \coth \left( \frac{x}{T} \right)
$$

(20)

and doesn’t have analytical solution. But it can be estimated in asymptotic approximations.

1. The case $t \ll 1/\omega_c$ corresponds to short-times, when characteristic times of system evolution are much lower than inverse frequencies of the bath modes.

$$
I(t) \simeq \omega_c^2 t^2
$$

2. The case $1/\omega_c \ll t \ll 1/T$ corresponds to intermediate times. The integral can be approximated as

$$
I(t) \simeq \frac{1}{2} \text{Ei}(1/\omega_c t) \simeq \frac{1}{2} \ln(\omega_c t),
$$

where $\text{Ei}(x) -$ exponential integral function.

3. The case $1/T \ll t$ corresponds to long times much more than any inverse frequencies.

$$
I(t) \simeq \pi T t
$$

We see here that $B^2(t)$ increases quadratically for short times, logarithmically for intermediate times and linearly for long times.

Summarizing the asymptotics obtained above the equation (16) can be rewritten as

$$
Tr_S(p_S^2(t)) \simeq \begin{cases} 
\frac{1}{2} \left[ 1 + e^{-2\omega_c \xi^2 t^2} \right], & t \ll 1/\omega_c; \\
\frac{1}{2} \left[ 1 + (\omega_c t)^{-2} \right], & 1/\omega_c \ll t \ll 1/T; \\
\frac{1}{2} \left[ 1 + e^{-2\omega_c T/2} \right], & 1/T \ll t; 
\end{cases}
$$

(21)

According to the short-time approximation $Tr_S(p_S^2(t)) \gg 1/2$ at all times. Comparing the Markovian and the short time approximation one can see in both cases exponential decay of $Tr_S(p_S^2(t)) \to 1/2$ when $T \to 0$ and $t \to \infty$. The difference in behavior of decay can be explained by the fact that the short-time approximation is not valid at large times.

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