Untangling tectonic slip from the potentially misleading effects of landform geometry

David Mackenzie and Austin Elliott
Department of Earth Sciences, University of Oxford, South Parks Road, Oxford OX1 3AN, UK

ABSTRACT

We present a new three-dimensional (3D) approach to the analysis of fault scarps using high-resolution elevation models. Advances in topographic measurement techniques [e.g., lidar (light detection and ranging) and photogrammetric techniques] have allowed extensive measurement of single earthquake and cumulative scarps to draw conclusions about along-strike slip variation and fault slip history. The resulting slip distributions are almost always variable and noisy, but the cause is often unclear. We first present the results of sensitivity analysis to demonstrate significant apparent noise due to varying terrain and fault and measurement geometry (topographic slope attitude, fault dip and slip obliquity). We show, with a case study on the Hoshab fault, Pakistan, that oblique slip can have a significant effect on the measured apparent slip.

Individual planar geomorphic markers only constrain one component of the full 3D slip vector. We use the variation in apparent offset with marker geometry to constrain the slip vector in 3D. Combining multiple offset measurements along strike, we show that determining the slip vector is reduced to a simple linear formulation. We test our method using a terrestrial lidar data set from the ruptures on the Borrego fault from the 2010 El Mayor–Cucapah earthquake (Baja California, Mexico). Combining 22 observations, we estimate a throw of 1.56 ± 0.02 m and a lateral slip of 1.9 ± 0.3 m. The vertical slip estimate agrees well with previous studies, but the lateral slip is significantly smaller. In regions of steep varied topography or with oblique slip, our method will give enhanced slip resolution while standard methods will give biased estimates.

INTRODUCTION

Fault displacement estimates from landform offsets are used in a number of applications, e.g., estimating fault slip rates in combination with Quaternary dating (e.g., McCalpin, 2009), determining earthquake repeatability or fault slip history (e.g., Lindvall et al., 1989; Kondo et al., 2010; Klinger et al., 2011), and estimating maximum earthquake magnitudes through application of scaling laws (e.g., Scholz et al., 1993; Scholz, 1994; Wells and Coppersmith, 1994; Manighetti et al., 2007; Leonard, 2010; Stirling et al., 2013). In addition, repeat surveys have been used to study temporal variation in fault morphology, e.g., scarp degradation (Elliott et al., 2012a) and postseismic tectonic deformation (Wilkinson et al., 2010; Lienkaemper et al., 2016; Mackenzie et al., 2016).
Figure 1. (A) A map view diagram for oblique right-reverse slip demonstrates the very different apparent lateral offsets that can be measured on 2 fences crossing the fault at 45°, even with only a small component of shortening. See text for explanation of variables. (B) Cross-section view showing the effect of slope angle on the vertical separation measured by a profile. For the same slip vector and therefore throw \( s_z \), the vertical separation measured will differ. (C) Three-dimensional block diagram of a topographic slope displaced by a left-normal fault, resulting in a surface fissure across which the linear stream marker is projected. \( S \) is the lateral component of slip, correctly measured by projecting the end of the channel across to the free face, in the slope-parallel direction; \( \varepsilon \) is the extra component of apparent lateral slip introduced by the horizontal advection of topography; \( \omega \) is the vertical component unaccounted for by a profile, due to the advection of topography. The true fault throw is therefore \( s_z = \Delta z + \omega \) (see text). FW—footwall; HW—hanging wall.
the information provided by the full topography field. The resulting slip distributions are almost always variable and noisy and it is often unclear whether this reflects real slip variation or artifacts of measurement noise, erosion, and slip geometry (e.g., Haeussler et al., 2004; Klinger et al., 2006; Elliott et al., 2012b; Schwartz et al., 2012; Rockwell and Klinger, 2013; Billant et al., 2016).

We aim to lay out the assumptions associated with present techniques for calculating fault slip distributions and slip vectors using profiles whether measured directly by differential global positioning system (GPS) or total station, or from a high-resolution digital elevation model (DEM). We derive an analytical expression linking the vertical separation a profile measures to fault slip under a generalized surface geometry. We use it to test the validity of the geometric assumptions and quantify the errors associated with breaking them. Based on the analytical expressions for fault offset, we suggest a Monte Carlo approach to uncertainty analysis of fault offset measurements similar to that of Thompson et al. (2002), but taking into account the 3D topography-fault intersection geometry.

Given the variation of apparent landform offset caused by fault-topography intersection geometry, we derive a new, inherently 3D method to solve for the full slip vector. Utilizing the variation of apparent offset with topographic attitude and slip geometry allows us to fully exploit high-resolution topographic data sets. We test the method on the terrestrial lidar data from the 2010 El Mayor–Cucapah (Baja California, Mexico) earthquake ruptures (Gold et al., 2013) and the satellite-photogrammetry elevation model of the scarps on the Hoshab fault (Pakistan; Zhou et al., 2015a).

### OFFSET LANDFORM GEOMETRY

Conventional methods will not always provide an accurate measure of fault slip. In general, measured separations are sensitive to the orientation of the displaced feature, so without a priori information or assumptions about the slip orientation, an individual displaced landform will often not be a good proxy for total fault slip (examples in the following).

#### Geomorphic Markers

Fault offsets are measured using geomorphic markers, i.e., features of known or estimatable prior geometry, displaced by surface faulting. These can generally be divided into two classes; quasi-linear features such as ridge crests or stream channels, and correlative surfaces such as alluvial fan surfaces or river terraces. Linear features provide the tighter constraint on fault slip because there are fewer degrees of freedom. They are only ambiguous along the line of the feature; e.g., a horizontal line perpendicular to a dip-slip fault cannot constrain the horizontal shortening or extension across the fault. Correlated planar surfaces are ambiguous along two directions within the plane and therefore give a slightly weaker constraint on fault slip; e.g., a planar slope with an aspect perpendicular to a pure strike-slip fault will not record fault offset.

In both cases, the separation vector in the direction orthogonal to the marker is the only measurement that can be uniquely made with no additional information or geometric assumptions. This is conventionally estimated with profiles in the case of marker surfaces, and with more sophisticated 3D line fitting for line markers. In order to relate this separation to the slip vector, the geometry of the fault plane relative to the separation vector must be known. In the case of nonparallel markers (lines or surfaces), the marker separation is measured at the estimated location of the fault surface trace, between the projected markers (e.g., Thompson et al., 2002).

#### Geometric Effects

Markers forming a simple geometry relative to the fault can provide a good measure of fault displacement; e.g., a line feature perpendicular to a fault trace will provide an accurate measurement of lateral slip without being affected by any dip-slip motion. However, for less perfect geometries the orientation of the fault relative to the topography affects the apparent offset measurement, disguising the slip vector.

The effect of marker geometry on the apparent offset for a given slip vector can be illustrated by several simple cases. Figure 1A shows an example map of two perpendicular fences intersecting an oblique right-lateral thrust fault, each at an orientation of 45° to the fault. The fences give apparent lateral offsets that depart from the true value of lateral slip ($S_L$) substantially as fault-perpendicular motion increases (in this case shortening). Similarly, Figure 1B shows a profile view example of a pure dip-slip thrust fault intersecting two different topographic slopes. The vertical separation ($\Delta z$) measured with a profile is the sum of two components: the fault throw ($s_f$) and a component contributed by the horizontal advection of the topography. Unless the second component is accounted for, two different vertical throw estimates will be made for an identical slip vector. The slope angle and the fault dip must therefore be known in order to accurately estimate the throw from a profile across a pure dip-slip scarp (e.g., Baljinnyam et al., 1993; Thompson et al., 2002).

Figure 1C shows a perspective diagram of a left-normal fault scarp cutting a slope with fissuring at the base of the scarp, demonstrating the effects of slope angle, fault dip, and oblique slip on the slip measurement. A profile across this scarp would measure a vertical separation of $\Delta z = s_f - \omega$, understimating the throw because the horizontal component of slip contributes its own apparent vertical separation by the advection of topography. Projecting the channel thalweg along its axis to the scarp free-face results in an overestimate of the lateral component of slip ($S_L + \omega$). Projecting the end of the channel to the free face in the fault-perpendicular direction gives the correct lateral measurement, but relies on a precise identification of the end of the channel, a measure that is often difficult to identify in practice and very sensitive to erosion and scarp degradation. Particularly with dipping faults, the strike-perpendicular component of
slip will distort the apparent offsets of fault-oblique landforms, which should therefore be avoided unless further constraints are available. This example shows a slope with an aspect perpendicular to the fault strike; nonperpendicular aspects lead to further complication.

In the general case, a marker can only constrain slip in the direction orthogonal to the marker; slip parallel to the marker is not recorded. Thus an individual marker separation is only a proxy for total fault slip in ideal or geometrically constrained cases.

2D Profile Assumptions

In using profiles to measure lateral slip or throw, several important assumptions are inherently made, although not always stated and often overlooked. Slope and fault geometry is usually assumed to be simple, with fault strike perpendicular to slope aspect. Slip is usually taken to be pure dip slip or strike slip unless oblique motion is evident. The azimuth of the profile is chosen by the operator and assumed to be either perpendicular to the fault strike, or parallel to the slope aspect. In addition, the geometry of the fault (strike and dip) is usually taken as moderately well known. Not all of these assumptions are made every time, but usually at least one is. Plots of separation against profile azimuth are sometimes employed to check for geometric variation (e.g., Middleton et al., 2016). We form analytical expressions to test the effect of breaking the methodological assumptions (see discussion of Analytical Fault Throw). However, in addition to the methodological assumptions, it is also generally assumed that the surfaces measured are well preserved and linear or planar; markers are usually carefully chosen to avoid areas of erosion and deposition around the scarps, adding a degree of subjectivity and potential operator bias (e.g., Scharer et al., 2014; Salisbury et al., 2015). We present an alternative to measuring profiles that may reduce the sensitivity to scarp degradation using a more 3D approach (see the discussion of Slip Geometry: 3D Analysis).

Nomenclature

In this paper we use a number of terms related to slip geometry that have different meanings and interpretations between communities. Our usage is based on Ragan (2009), extended to incorporate further mathematical terms and described here to avoid ambiguity. The fault slip vector is the slip in the fault plane, composed of three components: lateral slip ($s_L$, horizontal, along strike), heave ($s_h$, horizontal, strike perpendicular), and throw ($s_z$, vertical). The vector sum of heave and throw is the dip-slip component of slip, $S = \sqrt{s_h^2 + s_z^2}$, in the updip direction on the fault plane (Fig. 2).

A planar marker displaced by a fault can be described by its attitude: the dip ($\delta$) and aspect ($\psi$) of the surface. The apparent offset of the marker in a specified direction is termed the separation and can be measured in a number of directions (Hill, 1959): strike separation and dip separation in fault-plane space ($P$ and $Q$ in Fig. 2B) and so-called stratigraphic separation when measured orthogonal to bedding planes. In our geomorphic focus on offset fluvial and alluvial surfaces we primarily refer to the latter, which for generality we term the orthogonal separation, the orthogonal distance, $d$, between the displaced planes (Figs. 2A, 2B). Similarly, a displaced line marker is described by its attitude and the orthogonal separation. Where the fault geometry is well known, linear and planar markers are commonly projected to the fault plane intersection, defining piercing points and piercing lines, respectively (e.g., Gold et al., 2013).

Geometry Definitions

The simplest test case for fault geometry is a planar topographic slope displaced by a discrete fault, giving two parallel surfaces offset by fault motion (Fig. 2). We define the geometry using the slope unit normal vector $\hat{n}$ and choose the fault to strike east, in the positive $x$ direction (Fig. 2C). We follow the convention that fault dip is to the right of strike, i.e., in the negative $y$ direction (south). The slope unit normal, $\hat{n}$, and the slip vector, $s$, are:

$$\hat{n} = \begin{pmatrix} \sin \psi \sin \theta \\ \cos \psi \sin \theta \\ \cos \theta \end{pmatrix} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix},$$

$$s = \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} S_L \\ S_h \cos \delta \\ S_h \sin \delta \end{pmatrix}$$

where $\psi$ = slope aspect relative to the fault, the angle between the topographic gradient and the normal to the fault trace measured in the horizontal ($-90^\circ$–$90^\circ$), $\theta$ = slope angle ($-90^\circ$–$90^\circ$), $\delta$ = fault dip ($0^\circ$–$90^\circ$) and $S_L$, $S_h$ and $S_z$ are the lateral and dip-slip components of $s$. Under this geometry left-lateral and reverse slip are positive. For a left-lateral and/or reverse fault, a positive slope angle ($\theta > 0$) gives a typical downhill-facing scarp and a negative slope angle ($\theta < 0$) describes an uphill-facing scarp. For a right-lateral and/or normal fault, the opposite is true (diagram in Fig. 3).

For a point $r$ on a plane, the plane is described by $(r - a) \cdot \hat{n} = 0$ where the plane passes through the point $a$. Fixing the footwall plane to pass through the origin ($a = 0$), we write the footwall surface as:

$$r_{fw} \cdot \hat{n} = 0.$$ (3)

The hanging wall is the same plane, displaced by slip vector $s$:

$$r_{hw} \cdot \hat{n} = s \cdot \hat{n}.$$ (4)

These two planes, $r_{fw}$ and $r_{hw}$, are a simplified representation of a natural topographic slope offset by discrete fault slip (Figs. 2A, 2B).
Figure 2. (A) Slip vector $\mathbf{s}$ displaces the hanging wall along the fault plane. The orthogonal separation between planes is $\mathbf{s} \cdot \hat{n} = d$. (B) Example of a marker plane displaced by $\mathbf{s}$. The observed strike separation ($P$) and dip separation ($Q$) are much larger than the lateral and dip-slip components of the slip vector. The dip-slip component of $\mathbf{s}$ ($S_D$) can be further divided into two components, the heave ($s_y$) and throw ($s_z$) (after Ragan, 2009). (C) Geometry definitions: $\theta$ = slope angle, $\psi$ = slope aspect, $\delta$ = fault dip, $\hat{n}$ = slope unit normal vector, and $-\nabla h$ = downslope direction.
ANALYTICAL FAULT THROW

Using the idealized geometry defined here, we find an expression for the vertical separation that a profile would measure. Considering a profile in the vertical plane, the basic measurement is the vertical separation between the two planes (lines on the projected profile). Considering the point at \( x = 0, y = 0 \), we write the vertical separation, \( \Delta z \), between the hanging-wall and footwall planes as:

\[
\Delta z = \frac{s \cdot n}{n_0}.
\]

Expanding, and dividing by the vertical component of slip \( (s = S_0 \cos \delta) \), we write this as:

\[
\frac{\Delta z}{s_0} = 1 + \tan \left( \frac{S_0 \sin \psi + \cos \psi}{S_0 \sin \delta + \tan \delta} \right). \tag{6}
\]

In the case of pure dip slip with the slope aspect perpendicular to fault strike \((\psi = 0)\), this reduces to:

\[
\frac{\Delta z}{s_0} = 1 + \frac{\tan \theta}{\tan \delta}. \tag{7}
\]

Equivalently, the total slip on the fault is:

\[
s = \frac{\Delta z \cos \delta}{s_0 \sin \delta + \theta}. \tag{8}
\]

Equation (8) is true for \( \theta > 0 \). It is also commonly assumed that the fault dip is well known. In the next section we test the effects of breaking these assumptions.

If a profile (in a vertical plane) is measured across these two slopes, we expect to measure the same vertical separation between them regardless of the azimuth of the profile. However, such a profile will generally give a biased estimate of the slope angle \( \theta \); a profile at an azimuth of \( \beta \) relative to aspect will give an apparent slope angle \( \theta' \):

\[
\frac{\tan \theta'}{\tan \theta} = \cos \beta. \tag{9}
\]

Thus the slope angle will be systematically underestimated by a profile, except in the ideal case where the profile is in the direction of the slope aspect.

Assumption: Dip Slip and Fault Perpendicular Aspect

A typical set of assumptions is that the slip vector is purely dip slip \((S_0/S_\delta = 0)\) on a fault of known dip and that the slope aspect is perpendicular to fault strike \((\psi = 0)\). We derive analytical expressions linking the estimated throw under these assumptions \( (s_\text{profile} = 0) \) to the true throw \( (s_\text{true}) \). Plotting this function for several different cases of fault geometry and fault slip, we investigate where such profiles can reasonably be used with these assumptions and where they should not be used directly. The measured fault throw can be written as:

\[
s_\text{profile} = \frac{\Delta z}{1 + \tan \delta \cot \theta}. \tag{10}
\]

Taking the ratio \( s_\text{profile}/s_\text{true}, this becomes:

\[
\frac{s_\text{profile}}{s_\text{true}} = 1 + \frac{\tan \theta}{\frac{S_0 \sin \psi + \cos \psi}{S_0 \sin \delta + \tan \delta}} \times \frac{1 + \tan \delta \cot \theta}{1 + \tan \delta \cot \theta}. \tag{11}
\]

Where this expression is close to 1, a profile will give a good estimate for the fault throw (still assuming the fault dip is known well). As this expression departs from 1, we get a systematic measurement error.

Figure 4 shows plots of the ratio \( s_\text{profile}/s_\text{true} \) for several different cases. Under these assumptions, with fault-perpendicular aspects \((\psi = 0)\), a profile can perfectly recover the throw, provided that the fault dip is known and the profile gives a good estimate of slope angle. However, the measurement error increases with aspect, \( \psi, as it moves away from fault perpendicular, becoming larger with steeper slope angles and shallower fault dip. The dashed lines demonstrate that even a small degree of lateral slip significantly exacerbates this discrepancy. If we are willing to accept 5\% errors in throw, then for steeply dipping (\( \pm60^\circ \)) dip-slip faults, a slope aspect varying by as much as \( \pm40^\circ \), even with reasonably steep slope angles, does not significantly affect the throw estimate. However, for shallower faults \((e.g., \sim 30^\circ)\) this is reduced to \(20^\circ–30^\circ\), or even less if there is a small component of lateral slip \((e.g., Zhou et al., 2015a; see following).
Full Parameter Space

The plots of Figure 4 show several specific cases, but it is worth considering the full parameter space of Equation 11. Figure 5 shows a series of density plots for different cases of fault dip (30°, 45°, 60°) and different degrees of oblique slip ($S_L/S_D = 0, 0.5, 1$). For steeply dipping dip-slip faults, the effect of the slope attitude is relatively small; with $\delta = 60°$ with slope angles to $\theta$ ~ 20°, varying the slope aspect relative to global fault strike by ~40° does not affect the measured value at the 5% level.

However, as we might expect from the conceptual models, introducing a small component of lateral slip significantly reduces this range, an effect that is enhanced by shallower fault dips. Shallow fault dips and increased obliqueness of slip result in very narrow geometric tolerance; e.g., with $S_L/S_D = 1$ on a 30° dipping fault, and slope angles of $>5°$, a slope aspect deviating by as little as $-10°$ yields a >5% systematic error. Therefore, with gentle slopes, measurements will be quite tolerant of breaking the assumption of simple geometry ($\psi = 0$), but even a small component of lateral slip can cause significant errors in the throw estimate. Importantly, these errors are systematic, and so cannot be characterized by a simple Gaussian uncertainty.

2D: Monte Carlo Error Analysis

In cases where slopes are close to flat ($\theta = 0$) and/or fault-perpendicular ($\psi = 0$), profiles can still provide a good constraint on fault slip, but we need a rigorous way to characterize the uncertainty in the measurements due to the fault, topography, and slip geometries. The geometric formalism laid out here (Equations 5–9) can provide this framework.

Thompson et al. (2002), Davis et al. (2005), Amos et al. (2010), and Gold et al. (2012) used Monte Carlo simulations on scarp profiles to model the errors associated with estimating fault offsets in 2D; assigning probability distribution functions (PDFs) to each of the important 2D geometric parameters, they sampled the PDFs 100,000 times to estimate the probability distribution for the fault throw. Using the geometry defined herein, we extend this method to a full 3D approach, providing a means to quantify the effect of imperfectly known 3D geometry on estimates of landform offsets.

In order to estimate the PDF for the fault throw (or total slip), each component of the fault scarp geometry must be attributed a PDF (Fig. 6). In the case of surfaces fitted with a least squares regression (e.g., slope angle and intercept), a normal distribution with standard error can generally be used. Similarly, parameters known from direct measurement (e.g., dip from fault exposure) are generally also assigned a measurement PDF. Parameters with asymmetric or otherwise constrained distributions (e.g., fault dip if unknown) are generally given uniform, triangular, or trapezoidal distributions depending on the individual case, although Scharrer et al. (2014) noted that nonuniform PDFs must be carefully justified. Generalizing to 3D, we incorporate two further parameters (compared to Thompson et al., 2002): the slope aspect ($\psi$) and the strike-slip to dip-slip ratio ($S_L/S_D$).

We can relate vertical separations ($\Delta z$) to throw ($s_z$) with Equation 6; we could also write this in terms of total fault slip, or lateral slip in the case of a strike-slip fault. Thus, provided that we can constrain the PDFs of all the geo-
Figure 5. Plots of the relative error ($\delta_{\text{rel}}/\delta_{\text{true}} - 1$) for expected ranges in slope aspect ($\psi$) and slope angle ($\theta$) for three different fault dips (30°, 45°, 60°) and three different degrees of lateral slip ($S_L/S_D = 0.1, 0.5, 1.0$), with the dip slip held fixed at 1 m. The described rake values are an angular measure of $S_L/S_D$, indicating the direction of slip in the fault plane ($\text{rake} = \arctan S_L/S_D$). Each plot is colored by the error in the measurement due to the assumptions of pure dip slip and simple measurement geometry for slope angle and aspect. The contours represent values of 0.05, 0.1, and 0.2, i.e., 5%, 10%, and 20% departure from the true value. Dashed lines are the transects shown in Figure 4. Following the convention in Figure 3, a downhill-facing reverse scarp will have $\theta > 0$ while a normal scarp will have $\theta < 0$. The converse is true for uphill-facing scarps. Rad—radians.
metric parameters, we can estimate the PDF of the fault throw. Figure 6 shows a hypothetical case for profile measurements across a reverse fault scarp. In this case the fault dip is unknown but a possible range is estimated, which we represent with a uniform PDF. The slip is taken to be dip slip, but the rake is allowed to vary, introducing as much as 10% lateral slip, as we might expect such small amounts of lateral slip to be missed on inspection. We take the slope aspect and the strike-slip to dip-slip ratio to be normally distributed with zero mean (though other distributions could be used). The resulting throw PDF is asymmetric as a result of the trigonometric relations between the geometric parameters. The median and 95% confidence intervals are shown as dashed lines in Figure 6. The MATLAB (https://www.mathworks.com/) script for generating the throw PDF is provided as Supplemental Item 1. We suggest that this full error analysis should be carried out for any slip estimate from profiles in this case the fault dip is unknown but a possible range is estimated, which we represent with a uniform PDF. The slip is taken to be dip slip, but the rake is allowed to vary, introducing as much as 10% lateral slip, as we might expect such small amounts of lateral slip to be missed on inspection. We take the slope aspect and the strike-slip to dip-slip ratio to be normally distributed with zero mean (though other distributions could be used). The resulting throw PDF is asymmetric as a result of the trigonometric relations between the geometric parameters. The median and 95% confidence intervals are shown as dashed lines in Figure 6. The MATLAB (https://www.mathworks.com/) script for generating the throw PDF is provided as Supplemental Item 1. We suggest that this full error analysis should be carried out for any slip estimate from profiles in order to accurately characterize the geometric error.

2D Profiles. Case Study: 2013 Balochistan Ruptures

An example of the errors associated with varied geometry can be seen from the fault scarps on the Hoshab fault in Pakistan, the source of the 2013 Mw 7.7 Balochistan earthquake (e.g., Avouac et al., 2014; Jolivet et al., 2014). The 2013 earthquake generated ~200 km of surface ruptures, with as much as 11–14 m lateral slip and 1–4 m vertical slip (Zinke et al., 2014; Gold et al., 2015; Zhou et al., 2015a). Zhou et al. (2015a) used postearthquake stereo imagery to generate a high-resolution (1 m) DEM of the entire fault zone; they used planform projection to measure lateral offsets and profiles to measure vertical offsets, both for the 2013 event and cumulative scarps formed over multiple earthquake cycles.

We reassessed the fault displacement at the site described in Zhou et al. (2015a, Fig. 2 therein), focusing on a set of cumulative scarps in an old alluvial fan (Fig. 7A). Using the elevation point cloud (subset provided as Supplemental Item 2; full data set to be made available through OpenTopography; www.opentopography.org), we took swath profiles along the profile lines P1 and P2, and fit parallel lines above and below the scarp (Fig. 7A), estimating vertical separations of 12.1 ± 1.5 m and 7.3 ± 0.3 m, respectively. Our estimate for P1 is much larger than that of Zhou et al. (2015a), highlighting both the subjectivity of profile measurements and the tendency toward confirmation bias in expecting neighboring profiles to have similar apparent offsets.

Fitting a plane to the hanging-wall slope in each profile (yellow polygons, Fig. 7), we estimated the slope attitude for the surface represented by each. At P1, the hanging-wall surface has an aspect close to fault perpendicular (ψ = −3°) with a slope angle of −7°, so we expect measured separations to be relatively insensitive to lateral motion. The hanging-wall surface at P2 has an aspect of −32° and a slope angle of −7°, so we must consider the contribution of lateral slip to the vertical separation. Using these slope attitudes and the profile separations, we apply the Monte Carlo method to investigate the site with a rigorous treatment of the errors, comparing them to estimates made under simple geometry assumptions.

In Figure 7 we plot estimated PDFs for the fault throw under three different cases. We represent the measurement uncertainties on the vertical profile separations as Gaussian distributions with 1σ errors. Figure 7B shows the PDF for the approach of Zhou et al. (2015a), where it is implicitly assumed that the geometry is simple, i.e., 0 = 0, ψ = 0, so the throw is exactly the vertical separation measurement. The two estimates of throw disagree at the 95% confidence level.
Assuming $\psi=0$:

- Hillslope (Gaussian): $\theta_s = 6.7 \pm 0.5^\circ$
  $\theta_t = 7.9 \pm 0.5^\circ$

- Fault dip (uniform): $\delta = 50 - 70^\circ$

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As in Zhou et al. 2015 ($\psi=0$, $\theta=0$)

Fault dip $\delta$ irrelevant

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Assuming $\psi=0$:

- Hillslope (Gaussian): $\theta_s = 6.7 \pm 0.5^\circ$
  $\theta_t = 7.9 \pm 0.5^\circ$

- Slope Aspect (Gaussian): $\psi = 2.5 \pm 5^\circ$
  $\psi = -32 \pm 5^\circ$

- Fault dip (uniform): $\delta = 50 - 70^\circ$
  A priori (Gaussian) $S/S_0 = 4.5 \pm 0.5$

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A priori (Gaussian) $S/S_0 = 4.5 \pm 0.5$

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B

As in Zhou et al. 2015 ($\psi=0$, $\theta=0$)

Fault dip $\delta$ irrelevant

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C

Assuming $\psi=0$:

- Hillslope (Gaussian): $\theta_s = 6.7 \pm 0.5^\circ$
  $\theta_t = 7.9 \pm 0.5^\circ$

- Fault dip (uniform) $\delta = 50 - 70^\circ$

---

D

Full geometry from point cloud:

- Hillslope (Gaussian): $\theta_s = 6.7 \pm 0.5^\circ$
  $\theta_t = 7.9 \pm 0.5^\circ$

- Slope Aspect (Gaussian): $\psi = 2.5 \pm 5^\circ$
  $\psi = -32 \pm 5^\circ$

- Fault dip (uniform) $\delta = 50 - 70^\circ$
  A priori (Gaussian) $S/S_0 = 4.5 \pm 0.5$

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P1: $\Delta z = 12.1 \pm 1.5$

- Throw: $12.2 +3.6 -3.0$

P2: $\Delta z = 7.3 \pm 0.3$

- Throw: $7.3 +0.6 -0.6$

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Figure 7. Effects of geometry on profile measurements for an oblique slip case study, the Hoshab fault, Pakistan. (A) Adapted from Zhou et al. (2015a, Fig. 3 therein). We took swath profiles through the photogrammetry derived point cloud along the lines P1-P1' and P2-P2'. Yellow and orange polygons outline the sections of hanging wall and footwall used for estimating slope attitudes and three-dimensional separations. We measured vertical separations of $12.1 \pm 1.5$ m from profile P1 and $7.3 \pm 0.3$ m from P2. (B) Throw probability distribution functions (PDFs) for the two profiles P1 and P2 under the assumptions of Zhou et al. (2015a). In each case the solid line represents a Gaussian distribution with the same mean and standard deviation; dashed lines show the median and 95% bounds. (C) PDFs for P1 and P2 correcting for the slope angle (estimate from point cloud), but assuming $\psi = 0$, with a fault dip of 50°–70° (see text). (D) Estimated PDFs for the throw at P1 and P2 incorporating the slope aspect, slope angle, and an a priori $S/S_0$ estimate along with their associated uncertainties (see text). Both profiles are consistent at the 1σ level, suggesting a throw of ~10–11 m.
Figure 7C shows the PDF for the case of assuming \( v = 0 \), but accounting for the slope angle \( \theta \). The fault dip is assumed to be 50°–70°, covering the values estimated by Barnhart et al. (2014) and Zhou et al. (2016) for this reach of the fault; lacking any further information, we assume a uniform probability distribution across this range. The estimated throw is systematically smaller than the measured separation (in this case by \(-0.5\) m), and the resultant PDFs still disagree at the 95% confidence level between the two profiles.

Figure 7D shows the PDFs for the fault throw in both profiles taking account of the full orientation of the slopes measured. We assume a value of \( S_1/S_2 = 4.5 \pm 0.5 \) based on the estimate of Barnhart et al. (2014) for this stretch of fault in the most recent event, which is consistent within error to the average ratio based on measurements of Gold et al. (2015) and Zhou et al. (2015a). The orientation of the topographic surface at P2 is such that lateral slip reduces the vertical separation, so a larger \((10.3\pm1.7\) m) throw would be consistent with the measured vertical separation, while P1 is largely unaffected by lateral slip \((11.2\pm3.7\) m), but the measurement error is larger. These estimates for the throw agree well within the 1σ error bound, confirming that the change in vertical separation is primarily due to the change in orientation of the surface topography.

We conclude that in locations where slope aspect is approximately perpendicular to fault strike, profiles are a reasonable tool to estimate fault throw, with a mild (<10%) overestimate if the fault dip is unaccounted for. However, where slope aspects are not fault perpendicular, profiles do not constrain the throw; further geometric information is necessary. We describe an alternative 3D method to solve for the components of slip using the two slopes described and an offset river channel (see discussion: 3D Example: Hoshab Fault).

### 2D Profiles: Summary

In general, for shallow slope angles and steep fault dips, variation in the expected vertical separation is relatively small with varying slope geometry (equivalent to moving along strike over different landforms). Therefore profiles can recover fault throw well and Monte Carlo simulations can give a reasonable estimate of the associated errors. However, with steeper slope angles and shallower faults, there is significant variation in vertical separation with change in slope aspect for a given fault slip, so individual 2D-projected profiles are insufficient to constrain fault motion. Similarly, adding even a small degree of oblique slip further increases the discrepancy. While this analysis was carried out for the case of throw estimates from profiles, a similar analysis can be performed for estimating lateral slip from planform projections.

To resolve statistically significant slip variation along strike, a rigorous assessment of these errors should be completed. Monte Carlo simulations based on our geometric framework provide a much needed method to rigorously characterize the uncertainty arising from poorly constrained 3D geometry in fault offset estimates from profiles or planform projection.

### SLIP GEOMETRY: 3D ANALYSIS

Instead of taking profiles, we can exploit the availability of high-resolution DEMs by approaching the analysis of surface faulting in an inherently 3D fashion. Elliott et al. (2012b) combined pairs of nearby planform markers across the Darfield fault to calculate the magnitude and orientation of the horizontal component of the slip vector, assuming that both markers were subjected to the same slip vector. Billant et al. (2016) amassed suites of apparent topographic offsets to define both the horizontal and vertical components of a single slip vector represented by adjacent contour separations. We extend these concepts to exploit the change in apparent landform offset with varying marker geometry. We build an inherently 3D framework to constrain the full slip vector by combining multiple along-strike measurements starting from the base assumption of a constant slip vector over the region of interest. This assumption is unlikely to be valid at the entire fault scale, but may be applicable at the scale of individual and/or neighboring geomorphic features (e.g., alluvial fans, river channels, and moraines). This will not always be applicable, as there are good examples of short wavelength variations in fault offsets that are not due to geometric effects (e.g., Rockwell et al., 2002; Rockwell and Klinger, 2013). However, even where there is minor variation over the region of interest, an average slip vector is still a useful constraint, e.g., in estimating paleo-earthquake magnitudes. The absence of precisely defined piercing lines often warrants reconstructions of broader features to obtain a representative slip estimate.

Instead of profiles, we use correlative surfaces on either side of the fault because these provide a natural estimate of the topographic slope attitude. The simplest case is to approximate the surfaces on either side of the scarp as two parallel planar surfaces. Nonplanar (e.g., conical fan surfaces) and nonparallel surfaces could similarly be used, but they require more free parameters, so we illustrate the concept with the simplest case. The incorporation of line markers in this model is also discussed in the following.

### General Solution

With parallel planar surfaces, the unique separation measurement at any point along the fault is the orthogonal separation vector, \( d\hat{n} \), between the hanging-wall and footwall planes. A well-oriented 2D profile (in the downslope direction) through the same planar surfaces could measure the same orthogonal separation \( d \), but in general a 2D profile is an inappropriate tool to constrain the slope normal vector \( \hat{n} \), because it requires the user to choose the profile azimuth. Projecting the slip onto the normal direction, the component of slip orthogonal to the planes (Fig. 2) is described by:

\[
\mathbf{s} \cdot \hat{n} = d.
\]  

(12)

Assuming a constant slip vector over a region of interest, several measurements can be combined along strike, which can be written as a linear problem (matrix \( \mathbf{N} \):
\[ \mathbf{N} \cdot \mathbf{s} = \mathbf{d}. \]  

(13)

where \( m \) is the number of observations. Provided the problem is well constrained (see following), this can be solved by least squares minimization. Generally each observation will have an associated uncertainty, which can be incorporated using weighted least squares. Thus we have a mathematical formalism to resolve the full 3D slip vector by combining along-strike measurements.

**Validity and Geometric Interpretation**

Each row of Equation 13 represents a single observation, independent of the measurement location, a hanging-wall plane displaced by \( \mathbf{d} \mathbf{n} \) from the footwall. A point from the footwall plane translated by the slip vector (\( \mathbf{s} \)) from the origin by (\( \mathbf{s} \)) must be in that hanging-wall plane. Introducing a second observation (\( \mathbf{d} \mathbf{n} \), from a common origin), the end of the slip vector must be along the line of intersection between the two hanging-wall planes; with a third observation, the slip vector is uniquely constrained.

Thus, the problem is equivalent to solving for the intersection between all of the hanging-wall planes superimposed, where each plane is displaced from the origin by (\( \mathbf{s} \cdot \mathbf{n} \)\( \hat{\mathbf{n}} \), (Fig. 8A). For Equation 13 to have a unique solution, there must be at least three independent rows in the matrix \( \mathbf{N} \). There must be significant geometric variation between the measurements, i.e., at least three different slope aspects and/or dips. If all of the observation planes are parallel, the problem is completely unconstrained because the planes will not intersect. Similarly, if all the observed planes share an axis (or equivalently their normals are all within a single plane), then the slip vector is only constrained to be on that axis.

We showed that changing various geometric parameters results in different apparent vertical separations, confounding slip estimates; this same variation is necessary for solving for the slip vector in 3D. In areas with very shallow slope angles, slope aspects entirely perpendicular to fault strike, or steep fault dips and no lateral slip, the problem will likely be underdetermined. In these cases, with additional assumptions, or geometric knowledge (e.g., direct measurement of fault dip), we may still be able to solve a constrained problem, as we demonstrate herein for the Hoshab fault, Pakistan.

**Line Markers**

Alongside planar surfaces, line features such as ridge crests, moraine crests, or terrace risers are often displaced by faulting. These can provide a tighter constraint on the slip vector than a planar surface; they provide information equivalent to two intersecting planar surfaces (Fig. 8B). A line feature (fully captured in 3D) can be incorporated into the formalism of Equation 13 as two rows of \( \mathbf{N} \), i.e., two planes intersecting along the line of the marker.

For a simple straight line feature, the choice of intersecting plane pairs is arbitrary. Two parallel lines must be within a plane; in the following case studies, we choose the first pair of planes such that the line feature in both the hanging wall and footwall is within the same plane, i.e., two planes with a separation of zero. The second pair of planes is then perpendicular to the first, with the maximum orthogonal separation. Many anthropogenic or geomorphic line markers may often be better defined by the intersection of two planes on either side than as a line directly, because this is less sensitive to erosion and/or modification (e.g., ridge crests or curbstone crests).

Figure 8. (A) Solving for the three-dimensional slip vector is equivalent to finding the intersection between a collection of planes. Each plane represents the hanging-wall plane displaced from the origin by (\( \mathbf{s} \cdot \mathbf{n} \)\( \hat{\mathbf{n}} \), i.e., each plane actually represents one pair of planes describing an offset planar feature. (B) The intersection of two planes is a line, which can be displaced by some amount \( d \) (orthogonal to the line). Thus a line feature can be incorporated as two planes intersecting along the line (Equation 13). In many cases, this can give a less subjective characterization of a linear feature than other methods relying on a user picking points on the line. See Figure 2 for symbol explanations.
Because a 3D line marker provides a tighter constraint than a planar marker, only one more offset feature is required to fully constrain the slip vector, provided that it is not a plane in which the line marker lies. However, it is important that a single line feature does not constrain any component of slip along the line. Knowing the precise orientation of the fault plane can provide this third constraint as the slip vector must be in the fault plane (\( s \cdot f = 0 \), for fault plane unit normal \( f \)).

### 3D EXAMPLE: EL MAYOR–CUCAPAH

The scarps on the Borrego fault from the 4 April 2010 El Mayor–Cucapah earthquake provide an excellent test case for the methods described here and verify the method as a means of assessing paleo-earthquake scarps. Gold et al. (2013) studied the Borrego fault using terrestrial laser scans of two ~200 m sections of the fault cutting a variety of different terrain geometries, and made very precise slip measurements throughout. We used the same terrestrial lidar data available through OpenTopography (http://dx.doi.org/10.5069/G9BB86Z2, http://dx.doi.org/10.5069/G9SH4FBB) for two of their sites, applying our method directly to the bare-earth point cloud. We attempt to measure an average slip vector on this stretch of the fault, with the assumption that it remains approximately constant. This is an inherently different approach from that of Gold et al. (2013), who looked for the change in slip across the sites, but we expect to measure similar slip to their average.

Using the KeckCAVES (W.M. Keck Center for Active Visualization in the Earth Sciences, University of California, Davis) open-source virtual reality software, LidarViewer (Kreylos et al., 2013), we selected sets of points on the raw point cloud in 3D, from correlated, approximately planar geomorphic surfaces in the footwall and hanging wall (Fig. 9A). We then fit parallel planes to each correlative pair of point sets, and determine the orthogonal distance between them. We discard pairs for which a parallel model does not give a good fit, i.e., apparently correlative surfaces with different attitude in the hanging wall and footwall. The varied slope angle and aspect along strike provide sufficient independent rows of \( N \), so we solved for the slip vector \( s \) with a simple least squares fit. The MATLAB script used to fit parallel planes and solve for the slip vector is provided as Supplemental Item 3. We find a vertical slip (throw) of 1.56 ± 0.02 m, lateral slip of 1.9 ± 0.3 m, and fault-perpendicular horizontal component (heave) of 0.8 ± 0.2 m. The errors quoted are the statistical error on the mean; the individual measurements vary by much more, but this represents the error in mean estimated from all the observations together. This captures the aleatoric uncertainties and highlights those components well or poorly constrained, but does not account for any epistemic uncertainty, such as the suitability of the landscape for reconstruction using planar features.

Figure 9B shows a plot of the surface-orthogonal separations measured and those predicted for the best-fit slip vector applied to the surface orientation at each location. Although the data are noisy, the model provides a reasonable fit, with a root mean square (rms) misfit of 20 cm. The black line shows the expected separations based on the average slip vector for each section measured by Gold et al. (2013) (site 1: vertical 1.4 m, lateral 2.5 m, site 2: vertical 1.2 m, lateral 2.1 m), giving a larger rms misfit of 25 cm.

Examining the variance of our three slip vector components, we find the vertical component very well constrained and in agreement with the previous estimate at the 2\( \sigma \) level. The lateral and extensional components are more poorly constrained but differ from those estimated by Gold et al. (2013) at the 3\( \sigma \) level, with less lateral motion but a larger heave.

In particular, the main difference in fit between the prior slip vector and our estimate occurs at site 1, where there was significant scarps collapse and secondary faulting in the hanging wall. The average slip vector of Gold et al. (2013) suggests a consistently smaller separation than we measure, likely arising from the wavelength of the problem. Projection of channel thalwegs to the fault plane measures the offsets at tens of centimeters away from the fault, whereas our estimate measures the offset of surfaces 1–10 m from the fault plane, capturing any distributed slip in the region closest to the fault.

A further reason for the discrepancy may be the absence of a measurement of the extensional component (heave) by Gold et al. (2013), who projected linear features on either side of the fault to the exposed nearly vertical footwall fault plane. This approach assumes a slip vector in the near-surface fault plane. Along the rupture, fissures as wide as ~30 cm were observed, so significant extension did occur and the free face at the surface does not necessarily reflect the fault plane on which slip occurred. Therefore, in projecting non-fault-orthogonal linear features (e.g., channel thalwegs) across the fissure to the scarp-face plane, some of the heave can contaminate the apparent lateral and dip-slip measurements (Fig. 1C).

### 3D EXAMPLE: HOSHAB FAULT

For the site on the Hoshab fault described herein (Fig. 7A), we can carry out a similar analysis, demonstrating the solution to the constrained problem with a known fault dip. Previous studies estimated a fault dip of 50°–70° on this part of the fault (Barnhart et al., 2014; Zhou et al., 2016), so we assume a dip of 60° to demonstrate the method.

The geometric interpretation of the known dip case is very similar to that of the full 3D case; the fault plane forms an extra plane in which the slip vector must be. We can therefore project each observation plane to the fault plane. Each hanging-wall plane is displaced from the footwall plane by \( \hat{n} \), and intersects with the fault plane along a line (Fig. 10A). Figure 10B shows the intersection lines of each hanging-wall and footwall surface in fault geometry and in their correct spatial locations. Shifting each pair to a common origin (Fig. 10C), the slip vector is measured from the origin to the intersection point, in analogy to the 3D case in Figure 8A. Solving for the best-fit slip vector required to give the observed separations is therefore equivalent to finding the intersection between the projected lines. We solve for the intersection (Equation 13) using weighted least squares, requiring the slip vector \( s \) to be in the fault plane (\( \hat{f} \), by

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*Supplemental Item 3. MATLAB script which performs two stages to estimate a 3D slip vector. First, parallel planes are fitted to a series of pairs of point sets, to measure orthogonal separations (diagnostic plots generated to assess quality of fit). Weighted or unweighted least squares minimization is then used to estimate a 3D slip vector, based on the orthogonal separations. Data is assumed to have the same units in all three dimensions (e.g., UTM meters). Please visit [http://doi.org/10.1130/GES01386.S3](http://doi.org/10.1130/GES01386.S3) or the full-text article on www.gsapubs.org to view Supplemental Item 3. \( \hat{n} \), \( \hat{f} \)."
setting $\mathbf{f} \cdot \mathbf{s} = 0$ with a large weight (though a dip uncertainty could be used to weight the constraint relative to the observations).

At the site on the Hoshab fault (see Balochistan ruptures case study; point cloud provided as Supplemental Item 2 [footnote 2]), we consider three markers; two pairs of parallel planes measuring the separation of the slopes at P1 and P2 and the displaced channel, which forms a line feature (Fig. 7A). At profiles P1 and P2, we select sets of points from the raw point cloud in the hanging wall and footwall of the correlated alluvial fan surface (yellow and orange polygons, respectively, Fig. 7A), fitting parallel planes through the points. We fit parallel lines to points picked in the river channel from which the lateral offset was estimated by Zhou et al. (2015a). The river channel has incised since the abandonment of the P1-P2 fan surface, so the vertical offset in the channel is arbitrary.

Figure 9. (Top) Perspective view of the site 2 terrestrial lidar (light detection and ranging) data set of Gold et al. (2013) generated in LidarViewer (see Kreylos et al., 2013) showing the points selected for fitting planes. Footwall selections are red, hanging-wall selections are blue. (Bottom) Comparison of our measured perpendicular displacements to those predicted by our average slip vector and the average slip vectors of Gold et al. (2013), for both site 1 and site 2. The data points have been sorted by location along strike, but the x-axis is arbitrary.
only represents slip in the most recent event. However, the position provides a measure of horizontal slip as the river channel location was locked in after the abandonment of the fan surface. We therefore represent the channel thalweg as a pair of vertical planes in the hanging wall and footwall (assuming the river has not migrated laterally since the abandonment of the fan surface). The river therefore constrains a single component of offset (one row of $N$). If each surface or line for each observation pair is fitted independently, the slope normals change by $<2^\circ$, justifying the use of parallel planar markers in all three cases.

We project each marker to the fault plane (Fig. 11), and solve for the slip vector using a weighted least squares fit (weighted by plane separation uncertainties), estimating a lateral slip of $39 \pm 4$ m and $14.2 \pm 2.0$ m of updip slip ($12.3 \pm 1.7$ m throw). This throw matches our earlier best estimates from the two profiles at the 1σ level. Incorporating the assumed fault dip, this gives a ratio of lateral to vertical slip of $3.1 \pm 0.6$ (or equivalently $2.7 \pm 0.5$ strike slip to dip slip), significantly lower than that estimated by Zhou et al. (2015a) and that of the most recent event (Barnhart et al., 2014; Gold et al., 2015; Zhou et al., 2015a).

The extra constraint of the fault geometry can resolve the slip vector with fewer observations necessary compared to the unconstrained case. However, given that we only have three observations, the uncertainty is not well constrained. We also note that the slip estimate is strongly dependent on the fault dip; we have chosen a fixed dip of 60° to demonstrate the dip-constrained method, but allowing a range of dips would further quantify the uncertainty.

### DISCUSSION AND IMPLICATIONS

Salisbury et al. (2015) discussed the influence of operator decisions on the measurement of offsets on strike-slip faults, and concluded that discrepancies generally arise from misinterpretation of features and their pre-event geometries. They also found that standardizing remote measurement methods is crucial in obtaining useful and repeatable offset measurements. In this paper we have provided solutions to standardize two elements of fault offset measurement: the rigorous characterization of errors in 2D profiles, and a method to characterize fault slip in the case of nonoptimal slope geometry and/or oblique slip when the assumptions are violated for conventional methods. Both approaches still require the user to identify correlated surfaces on either side of the fault scarp and address the associated epistemic uncertainty in feature correlation, but we hope that they provide a framework under which measurements can be standardized.

### A 3D Framework

In cases where profiles do not perform well, slip measurements have generally been very limited. In some cases, with fresh ruptures, distinct point to point vector displacements can be measured (e.g., Treiman, 2002), or where the entire geometry is well constrained, features may be projected appropriately to estimate offsets (e.g., Gold et al., 2013). In an increasing num-

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**Figure 10.** (A) Illustrative diagram of the geometry for projecting two planar surfaces onto the fault plane. $P$ illustrates a topographic surface, and $Q$ represents a vertical plane through a river channel (river is shown as a blue line). Black arrows mark the plane perpendicular separations. (B) Fault plane view of each of the two intersection lines (footwall, FW, is dashed), with each plane separation in the correct spatial location. (C) The intersections are shifted to a common origin in the footwalls. (D) The slip vector ($S$) is then the distance between the footwall intersection point (the origin) and the intersection of the hanging-wall lines.
A detailed understanding of the 3D surface geometry highlights a number of common pitfalls associated with landform offset measurement. We have shown that in the case of relatively simple geometries, the common assumptions hold well. For dip-slip faults of known dip and surfaces that have fault-perpendicular aspect, 2D topographic profiles provide an easy and accurate method by which to estimate fault throw. Similarly, for pure strike-slip faults, planform projection of linear markers can provide an excellent measure of fault slip. Numerous previous studies have used these methods to good effect, using careful selection parameters to choose sites with good geomorphic preservation and simple geometries (Scharer et al., 2014). Where the assumptions hold, profiles and planform projection can give a good estimate of fault displacement, justifying their continued use provided that the assumptions and limitations are recognized.

In general, however, profile surveys do not characterize the errors associated with these assumptions. We provide a best-practice recommendation for rigorously characterizing fault slip and associated aleatoric uncertainties using a Monte Carlo simulation method. Using analytical geometric relations, a PDF can be generated for the component of fault slip based on the uncertainties in all the geometric parameters. Previous studies (e.g., Baljinnyam et al., 1993; Thompson et al., 2002) performed this analysis in an inherently 2D framework, but our sensitivity tests show that incorporating uncertainty in the full 3D geometry is also important. The resultant fault displacement PDFs will generally be asymmetric, so uncertainties should generally be reported either as a PDF or a range covering the possible values (or 95% confidence bounds). Use of this Monte Carlo method will naturally highlight cases where a profile or planform projection based approach is inappropriate, presenting as large uncertainty (broad PDF), e.g., in cases of nonzero slope aspects ($) or signficant oblique slip ($\beta > 0$).

**Measurement of Slip Distributions**

Many studies compile offset measurements along strike to estimate a fault slip distribution (e.g., Haussler et al., 2004; Klinger et al., 2011; Haddon et al., 2016). This treats measured marker offsets as independent proxies to the local fault slip at the surface. However, any marker only constrains the component of slip in directions perpendicular to the marker (e.g., Figs. 1A–1C). The method...
we present combines multiple markers to constrain more components of the slip vector but still requires operator judgement in feature selection.

Scharer et al. (2014) and Salisbury et al. (2015) discussed the effects of operator biases and experience in the quality of the measurements made; they found that the quality of the geomorphic reconstruction is often the largest source of uncertainty and introduced a quality measure whereby each observation is assigned a quality rating (1–3, 1 being highest quality). Salisbury et al. (2015) noted that in general, the more experienced researchers can make more consistent estimates from a given feature, but the preservation and shape of the geomorphic marker often limit the reconstruction quality. Thus, both the quality of the reconstruction and the measurement error must be taken into account when comparing landform offset measurements. The methods we described reduce or characterize the aleatoric uncertainties associated with the measurement geometry but do not account for epistemic uncertainty arising from poor-quality geomorphic markers. In practice, the operator must still identify suitably correlated surfaces on either side of the fault, with the uncertainty that introduces.

Scharer et al. (2014) also noted that a significant trade-off exists in building a slip distribution from multiple observations along strike, between making as many observations as possible and the quality of those observations. Our methods allow the inclusion of sites that would traditionally be excluded due to poor marker geometry, enabling more measurements along strike. In this framework, markers that are not fault perpendicular or slopes that are cut obliquely are incorporated in a mathematically rigorous manner to provide additional constraints on fault slip.

With the availability of high-resolution topographic data with point uncertainties of ≲5–10 cm (e.g., Gold et al., 2013), the formal measurement uncertainties are typically very low (often <10 cm), excluding any errors due to the geometric assumptions. However, in slip distributions compiled along strike, we typically see much greater variation over short wavelengths (hundreds of meters), and it is unclear whether this reflects slip variation or reconstruction quality, measurement errors, or geometric errors. Our sensitivity tests show that significant noise can arise from the breaking of the geometric assumptions and therefore be removed by correctly accounting for the geometry. However, noise arising from the marker reconstruction must still be evaluated on a site by site basis (Scharer et al., 2014). Using our methods with careful identification of high-quality markers should allow slip variation to be resolved over a variety of length scales.

### CONCLUSIONS

The recent explosion in availability of high-resolution topographic data presents great potential for the measurement of fault slip at the surface. In general, for shallow slope angles and steep fault dips, conventional analysis tools can capture fault offsets accurately. However, with steeper terrain and shallower fault dips, we get significant change in apparent offset as the slope attitude changes; this is worsened by even a small (e.g., 10%) component of oblique slip. The rigorous 3D framework described here can be used to characterize fault slip uncertainties through Monte Carlo methods, incorporating geometric uncertainty into the resultant slip estimate. A case study for this method on the obliquely slipping Hoshab fault demonstrates that profile measurements can recover the fault throw well when the slope aspect is fault perpendicular, but give biased estimates when this assumption is violated.

Where the assumptions of conventional profile based methods are violated, we propose to exploit new high-resolution topographic data sets, combining multiple measurements to solve for a 3D slip vector. With 3D marker surfaces (here planar) we use the variation in apparent offset with terrain and measurement geometry to determine the slip vector under a simple linear formulation. We have demonstrated the method in estimating a slip vector from the El Mayor–Cucapah rupture scars on the Borrego fault, where we estimate a throw similar to previous estimates, but less lateral slip and greater (though poorly constrained) heave. A further example from the Hoshab fault demonstrates the constrained case where the fault dip is relatively well known. The geomorphic marker surfaces can be projected to the fault plane and an analogous 2D problem solved.

In some regions of significant topography or oblique slip, 2D-projected measurements will generally give biased estimates of fault slip. In contrast, our inherently three-dimensional approach can resolve the true slip vector and is enhanced by oblique slip and topographic variation.

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