FlexibleDecay: An automated calculator of scalar decay widths

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Abstract

We present FlexibleDecay, a tool to calculate decays of scalars in a broad class of BSM models. The tool aims for high precision particularly in the case of Higgs boson decays. In the case of scalar and pseudoscalar Higgs boson decays the known higher order SM QED, QCD and EW effects are taken into account where possible. The program works in a modified $\overline{\text{MS}}$ scheme that exhibits a decoupling property with respect to heavy BSM physics, with BSM parameters themselves treated in the $\overline{\text{MS}}/\overline{\text{DR}}$-scheme allowing for an easy connection to high scale tests for, e.g., perturbativity and vacuum stability, and the many observable calculations readily available in $\overline{\text{MS}}/\overline{\text{DR}}$ programs. Pure BSM effects are taken into account at the leading order, including all one-loop contributions to loop-induced processes. The program is implemented as an extension to FlexibleSUSY, which determines the mass spectrum for arbitrary BSM models, and does not require any extra configuration from the user. We compare our predictions for Higgs decays in the SM, singlet extended SM, type II THDM, CMSSM and MRSSM, as well as for squark decays in the CMSSM against a selection of publicly available tools. The numerical differences between our and other programs are explained. The release of FlexibleDecay officially deprecates the old effective couplings routines in FlexibleSUSY.

Keywords: decays, Higgs boson, BSM

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PROGRAM SUMMARY

Program Title: FlexibleDecay

CPC Library link to program files: (to be added by Technical Editor)

Developer’s repository link: https://github.com/FlexibleSUSY/FlexibleSUSY

Code Ocean capsule: (to be added by Technical Editor)

Licensing provisions: GPLv3

Programming language: C++, Wolfram Language, Fortran, Bourne shell

Supplementary material: none

Nature of problem: Calculation of decay widths of scalar bosons in extensions of the Standard Model of particle physics. For the calculation of Higgs boson decay widths known higher-order contributions are taken into account to achieve a high precision.

Solution method: The decay widths of the scalar bosons are expressed in terms of Feynman diagrams in a general renormalizable quantum field theory and specialized to the considered model. The resulting expressions are numerically evaluated using special functions and numerical integration.

Additional comments including restrictions and unusual features: FlexibleDecay is an addon to FlexibleSUSY [1, 2]. The decay widths can only be calculated in models that can be treated with FlexibleSUSY. Decay widths of fermions or vector bosons are not calculated. For non-Higgs scalar bosons the calculation is restricted to the leading order in the perturbation series.

References

[1] P. Athron, J. h. Park, D. Stöckinger and A. Voigt, Comput. Phys. Commun. 190 (2015), 139-172 doi:10.1016/j.cpc.2014.12.020 [arXiv:1406.2319 [hep-ph]].

[2] P. Athron, M. Bach, D. Harries, T. Kwasnitza, J. h. Park, D. Stöckinger, A. Voigt and J. Ziebell, Comput. Phys. Commun. 230 (2018), 145-217 doi:10.1016/j.cpc.2018.04.016 [arXiv:1710.03760 [hep-ph]].
1. Introduction

Higgs boson properties are rapidly transforming into high-precision observables, less than a decade after the discovery of the Higgs boson at the LHC [1, 2]. The Higgs decay branching fractions are being determined with increasing accuracy [3, 4] and combined in global fits (see e.g. Ref. [5]). Potential deviations from predictions of the Standard Model (SM) can provide important fingerprints of effects from physics beyond the SM (BSM). The latest results show that the Higgs boson is mostly SM-like—without any visible BSM deviations. This provides a major constraint on any BSM physics that modifies the Higgs sector or Higgs decay modes, making it essential to rigorously test these models against this data.

This requires precise predictions of Higgs boson decay widths and branching ratios. Many software packages that calculate Higgs decays have been developed over the years, including HDECAY [6, 7], 2HDECAY [8–11], FeynHiggs [12, 13], 2HDMC [14], H–Coup [15, 16], eHDECAY [17], sHDECAY [18], HFOLD [19], SPheno [20, 21], SOFTSUSY [22], NMSSMCALC [23], and N2HDECAY [24]. Most of these codes work only for specific models such as the Minimal Supersymmetric Standard Model (MSSM) or the two-Higgs doublet model (THDM), their slight extensions or arbitrary models through higher dimensional operator parametrization. However this represents just a tiny fraction of the models that are viable candidates for physics beyond the standard model and have been proposed in the literature.

In this paper we present an alternative FlexibleDecay, which is an extension of FlexibleSUSY [25–27], that can work for a very broad class of models. FlexibleSUSY is a spectrum generator generator: once a BSM model is specified via model files, it generates a spectrum generator for the desired model, which computes the pole masses after integrating renormalization-group equations and solving the electroweak symmetry breaking conditions. This makes it much easier to study the phenomenology of BSM scenarios for which no model-specific codes exist and also allows comparisons of well-known models on an equal footing. The only other code with this capability is SARAH/SPheno [20, 21, 28–31, 34–36].

FlexibleSUSY also implements many model-specific precision corrections, so that it provides the same—or higher—precision as the model specific codes. For example in the MSSM FlexibleSUSY has state-of-the-art Higgs mass calculations with up to three-loop fixed-order, effective field theory and hybrid calculations [27, 37–40], see also Ref. [41] for a review. For arbitrary models FlexibleSUSY implements the FlexibleEFTHiggs hybrid calculation [26], providing the most precise predictions for the Higgs mass for BSM models without model-specific Higgs mass calculators. FlexibleSUSY has also been extended to compute other important observables, such as the anomalous magnetic moment of the muon, lepton electric dipole moments and $\mu \rightarrow e\gamma$.

For decays, generic calculations of loop-induced effective vertices for the important $h \rightarrow \gamma\gamma$ and $h \rightarrow gg$ processes were implemented in both FlexibleSUSY and SARAH/SPheno in Ref. [42]. SARAH/SPheno is also able to compute decay widths in two different ways: one takes into account full BSM one-loop effects [43], and the other takes into account SM-like higher-order effects partially beyond one-loop order [21, 31].

FlexibleDecay extends FlexibleSUSY with an automated calculation of decays of scalars (and Higgs bosons in particular) in a broad class of BSM models. FlexibleDecay specifically aims for the highest level of precision and reliability possible in Higgs decays. This is important since no BSM effects in the Higgs sector have been observed so far, and any potential BSM effects amount to small corrections to the SM predictions. An important observation is therefore that taking into account higher-order SM corrections even at the multi-loop level is indispensable, while purely BSM loop effects are typically very small in appropriate renormalization schemes.

The precision goal of FlexibleDecay is achieved in two ways. First, for Higgs decays into Standard Model states we implement all known higher order corrections that are applicable to general models,
ensuring state-of-the-art precision. Second, the BSM effects are treated at leading order, but in a special, decoupling renormalization scheme. This scheme minimizes potential pure BSM loop effects from heavy states and thus maximizes the accuracy of the leading order treatment. This implementation supersedes the previous effective coupling routines available in FlexibleSUSY. While the emphasis of the current release is on the Higgs boson decays, the code is capable of automatically evaluating leading-order contributions to the partial width of any scalar.

Among the differences between FlexibleDecay and other codes are not only the selection of included higher-order corrections and the choice of renormalization scheme but also the treatment of subtleties related to potential numerical violations of Ward identities. The present paper describes all choices made in FlexibleDecay in detail and presents an extensive comparison to numerical results of other codes, including those that are model-specific and to the decay calculations of SARAH/SPheno.

The paper is structured as follows. In Section 2 we give a quick start guide to FlexibleDecay. Section 3 describes the special renormalization scheme used in the code. In Section 4 we describe the implementation in detail, particularly focusing on specifying the higher order corrections in Higgs decays. Here we also explain the treatment of potential violations of the Ward identities in loop-induced decays. Section 5 compares our results with existing codes for a wide selection of models. The comparison includes supersymmetric and non-supersymmetric models. The limitations of the current version of FlexibleDecay are discussed in Section 6, and our conclusions are presented in Section 7.

2. Quick start guide

FlexibleDecay is an extension to the FlexibleSUSY package and is distributed with FlexibleSUSY from version 2.6.0. In the following we briefly describe the initial steps to download, setup and run the program to calculate decay widths.

2.1. Downloading

FlexibleSUSY 2.6.2 (current version) can be downloaded from [http://flexiblesusy.hepforge.org](http://flexiblesusy.hepforge.org) by running

```
$ wget https://www.hepforge.org/archive/flexiblesusy/FlexibleSUSY-2.6.2.tar.gz
$ tar -xf FlexibleSUSY-2.6.2.tar.gz
$ cd FlexibleSUSY-2.6.2
```

Alternatively, FlexibleSUSY 2.6.2 can be downloaded via the version control system git from [https://github.com/FlexibleSUSY/FlexibleSUSY](https://github.com/FlexibleSUSY/FlexibleSUSY) by running

```
$ git clone https://github.com/FlexibleSUSY/FlexibleSUSY
$ cd FlexibleSUSY
```

2.2. Prerequisites

FlexibleSUSY requires the following programs/libraries to be available:

- Mathematica/Wolfram Engine, version 11.0 or higher [48]
- SARAH, version 4.11.0 or higher [28–31, 34–36] [http://sarah.hepforge.org](http://sarah.hepforge.org)
- C++14 compatible compiler (g++ 5.0.0 or higher, clang++ 3.8.1 or higher, icpc 17.0.0 or higher)

There are also optional dependencies that are not required for FlexibleDecay: TSIL [44], GM2Calc [45] and Himalaya [37–39] (using the three-loop $\mathcal{O}(\alpha_s\alpha^2)$ corrections from Refs. [46, 47]), which are only needed for specific models.
• FORTRAN compiler (gfortran, ifort, etc.)
• GNU Scientific Library [49] [http://www.gnu.org/software/gsl]
• Eigen library, version 3.1 or higher [50] [http://eigen.tuxfamily.org]
• Boost library, version 1.37.0 or higher [51] [http://www.boost.org]
• LoopTools version 2.8 or higher [52] [http://www.feynarts.de/looptools] and/or Collier [53–56] [http://collier.hepforge.org]

SARAH can be installed automatically by running

```
./install-sarah
```

### 2.3. Running FlexibleDecay

To generate a spectrum generator that calculates decays for a given model, the FSCalculateDecays variable must be set to True in the corresponding model file `model_files/<model>/FlexibleSUSY.m.in`:

```
FSCalculateDecays = True;
```

If the FSCalculateDecays variable is not set in the model file, it is assumed to be False, so decays are not calculated by default. However, for many commonly studied models that are distributed with FlexibleSUSY, such as the SM, MSSM, THDMII, UMSSM, NMSSM, HRSSM, muuSSM, E6SSM, and others, the FSCalculateDecays variable is set to True in the model file. Note that by default only decays of neutral scalar, pseudoscalar and charged Higgs bosons are calculated. See Appendix A for further configuration options.

To generate a spectrum generator for the singlet-extended Standard Model (SSM) the following commands must be run:

```
$ ./createmodel --name=SSM
$ ./configure --with-models=SSM --with-loop-libraries=collier,looptools
$ make
```

Note that either LoopTools [52] or Collier [53–56] are required for the decays. Once the spectrum generator has been compiled, the code can then be run using an SLHA-like input file [57, 58], for example:

```
$ cd models/SSM
$ ./run_SSM.x --slha-input-file=LesHouches.in.SSM
```

For a detailed description of the build- and run-time configuration options for FlexibleSUSY and FlexibleDecay see Appendix A and Ref. [59].

FlexibleSUSY also generates a standalone program that accepts the SLHA spectrum file as input and calculates the decays. Using the SSM model from above, for example, one can calculate the decays by running:

```
$ cd models/SSM
$ ./run_SSM.x --slha-input-file=LesHouches.in.SSM \
   --slha-output-file=LesHouches.out.SSM
$ ./run_decays_SSM.x LesHouches.out.SSM
```
For the variant of the SSM model the output of FlexibleDecay may read

| Block DCINFO | FlexibleSUSY | 2.6.2 | SSMHInput2 | 4.14.3 |
|--------------|-------------|-------|------------|--------|
| DECAY | 25 | 3.20822225E-03 # hh(1) decays |
| | 5.82132981E-01 | 2 | -5 | 5 | BR(hh(1) → barFd(3) Fd(3)) |
| | 2.10420263E-01 | 2 | -24 | 24 | BR(hh(1) → conjVWp VWp) |
| | 8.56749173E-02 | 2 | 21 | 21 | BR(hh(1) → VQ VG) |
| | 6.19478919E-02 | 2 | -15 | 15 | BR(hh(1) → barFe(3) Fe(3)) |
| | 2.87695050E-02 | 2 | -4 | 4 | BR(hh(1) → barFu(2) Fu(2)) |
| | 2.67970867E-02 | 2 | 23 | 23 | BR(hh(1) → VZ VZ) |
| | 2.92077094E-03 | 2 | 22 | 22 | BR(hh(1) → VP VP) |
| | 1.48184338E-03 | 2 | 22 | 22 | BR(hh(1) → VP VZ) |
| | 2.64746094E-04 | 2 | -3 | 3 | BR(hh(1) → barFe(2) Fe(2)) |
| DECAY | 35 | 8.56617667E-01 # hh(2) decays |
| | 6.81973282E-01 | 2 | -24 | 24 | BR(hh(2) → conjVWp VWp) |
| | 3.04006836E-01 | 2 | 23 | 23 | BR(hh(2) → VZ VZ) |
| | 1.21750789E-02 | 2 | 25 | 25 | BR(hh(2) → hh(1) hh(1)) |
| | 8.72302442E-04 | 2 | -5 | 5 | BR(hh(2) → barFd(3) Fd(3)) |
| | 7.25578847E-04 | 2 | 21 | 21 | BR(hh(2) → VG VG) |
| | 1.06692644E-04 | 2 | -15 | 15 | BR(hh(2) → barFe(3) Fe(3)) |
| | 7.66832048E-05 | 2 | 22 | 22 | BR(hh(2) → VP VP) |
| | 4.35647959E-05 | 2 | -4 | 4 | BR(hh(2) → barFu(2) Fu(2)) |
| | 1.89931168E-05 | 2 | 22 | 22 | BR(hh(2) → VP VZ) |

The DCINFO block contains information about the calculation. The decay output conforms to the SLHA standard [57].

3. Renormalization scheme

FlexibleSUSY is a tool that allows one to create spectrum generators for large classes of BSM models of an a priori unknown phenomenology. Due to the large degree of automation required, the $\overline{\text{MS}}$ and $\overline{\text{DR}}$ renormalization schemes are FlexibleSUSY’s default choice to perform the necessary loop calculations. While these schemes are well suited for the calculation of properties of pure BSM observables, these schemes are not always the optimal choice when it comes to studying properties of some low-energy SM observables, such as the decays of the SM-like Higgs boson. That is because in the presence of a heavy BSM state those schemes tend to introduce large higher-order corrections, resulting in a large theoretical uncertainty in the calculated low-energy observables. This can for instance lead to a non-decoupling behavior of the low-energy observables when the BSM states become heavy.

To circumvent this issue, the decay calculation in FlexibleDecay is performed in a specific “de-coupling” renormalization scheme. This scheme is designed in a way that can be automatically applied to all BSM models that can be studied with FlexibleSUSY. The scheme is similar to the familiar decoupling scheme for the strong coupling $\alpha_3$ [60], which has been applied to a non-minimal SUSY model e.g. in Ref. [61], and recovers SM results in case when all BSM particles are heavy.

In the decoupling scheme of FlexibleDecay all SM-like $\overline{\text{MS}}/\overline{\text{DR}}$ couplings of the BSM model (the electroweak gauge couplings $g_1$ and $g_2$, the strong gauge coupling $g_3$, the Yukawa couplings $Y_u$, $Y_d$ and $Y_e$ and the SM-like vacuum expectation value $v$) are set equal to the corresponding SM $\overline{\text{MS}}$ couplings. Thus, formally, the decoupling scheme of FlexibleDecay is defined by the following set of renormalization conditions:

\begin{align}
    g_i(m_X) &= \hat{g}_i(m_X), \\
    Y_f(m_X) &= \hat{Y}_f(m_X),
\end{align}
\[ v(m_X) = \hat{v}(m_X), \]  

(1c)

where \( i = 1, 2, 3 \) and \( f = u, d, e \). On the r.h.s. of Eqs. (1) the \( \hat{g}_i, \hat{Y}_f \) and \( \hat{v} \) denote the SM gauge couplings, Yukawa couplings and vacuum expectation value, defined in the \( \overline{\text{MS}} \) scheme with 6 quark flavors. The conditions (1) are imposed individually for every decay calculation of a particle \( X \) with mass \( m_X \) at the respective renormalization scale \( m_X \). In the decoupling scheme, the non-SM-like BSM parameters remain defined in the \( \overline{\text{MS/DR}} \) scheme by the boundary conditions specified in the \texttt{FlexibleSUSY} model file. Note that the renormalization conditions (1) are imposed as relations on the finite parameters; they could equivalently be imposed via relations on renormalization constants, however the values of renormalization constants are not needed for this work.

A nontrivial point is that relations of the form (1) can always be identified by \texttt{FlexibleSUSY} for any supported BSM model, even in models that have a more complicated structure of Yukawa couplings or vacuum expectation values. For a given model the relations (1) are extracted from the mandatory \texttt{LowScaleInput} variable from the corresponding \texttt{FlexibleSUSY} model file. Consider the MSSM as an example: If the model file contains the lines

\[
\texttt{LowScaleInput} = \{ \\
\quad \{g1, \text{Sqrt}[5/3] \text{EDRbar} / \text{Cos[\text{ThetaWDRbar}]]}, \\
\quad \{g2, \text{EDRbar} / \text{Sin[\text{ThetaWDRbar}]]}, \\
\quad \{g3, \text{Sqrt}[4\pi \text{AlphaS}]]}, \\
\quad \{Yu, \text{Sqrt}[2] \text{Tp[upQuarksDRbar] / vu}], \\
\quad \{Yd, \text{Sqrt}[2] \text{Tp[downQuarksDRbar] / vd}], \\
\quad \{Ye, \text{Sqrt}[2] \text{Tp[downLeptonsDRbar] / vd}], \\
\quad \{vd, 2 \text{MZDRbar} / \text{Sqrt}[3/5 g1^2 + g2^2] \text{Cos[\text{ArcTan[\text{TanBeta}]]}}}, \\
\quad \{vu, 2 \text{MZDRbar} / \text{Sqrt}[3/5 g1^2 + g2^2] \text{Sin[\text{ArcTan[\text{TanBeta}]]}} \\
\};
\]

then the corresponding relations (1) take the specific form

\[
g_1(m_X) = \frac{\hat{e}(m_X)}{\sqrt{\frac{5}{3} \text{Cos[\text{ThetaW}(m_X)]}}},
\]

(2a)

\[
g_2(m_X) = \frac{\hat{e}(m_X)}{\text{Sin[\text{ThetaW}(m_X)]}},
\]

(2b)

\[
g_3(m_X) = \sqrt{4\pi \text{AlphaS}(m_X)},
\]

(2c)

\[
Y_u(m_X) = \frac{\sqrt{2}[\hat{m}_u(m_X)]^T}{v_u(m_X)},
\]

(2d)

\[
Y_d(m_X) = \frac{\sqrt{2}[\hat{m}_d(m_X)]^T}{v_d(m_X)},
\]

(2e)

\[
Y_e(m_X) = \frac{\sqrt{2}[\hat{m}_e(m_X)]^T}{v_d(m_X)},
\]

(2f)

\[
v_d(m_X) = \frac{2\hat{m}_Z(m_X) \text{Cos(\beta)}}{\sqrt{\frac{2}{5}[g_1(m_X)]^2 + [g_2(m_X)]^2}},
\]

(2g)

\[
v_u(m_X) = \frac{2\hat{m}_Z(m_X) \text{Sin(\beta)}}{\sqrt{\frac{2}{5}[g_1(m_X)]^2 + [g_2(m_X)]^2}},
\]

(2h)

\footnote{Previously the \texttt{LowScaleInput} boundary condition was not required in models with \texttt{FlexibleEFTHiggs} = \text{True} because the \texttt{FlexibleEFTHiggs} algorithm for the spectrum generation does not require this. However this block is required if decays are enabled in the model file via \texttt{FSCalculateDecays} = \text{True}.}
Note that in many BSM models **FlexibleSUSY** can determine the SM-like gauge and Yukawa couplings automatically from corresponding SM parameters. In this case the definitions in the **LowScaleInput** variable can be simplified as follows:

\[
\text{LowScaleInput} = \{ \\
\{Y_u, \text{Automatic}\}, \\
\{Y_d, \text{Automatic}\}, \\
\{Y_e, \text{Automatic}\}, \\
\{v_d, 2 \frac{\text{MZDRbar}}{\sqrt{\frac{3}{5} g_1^2 + g_2^2}} \cos[\text{ArcTan}[\text{TanBeta}]]\}, \\
\{v_u, 2 \frac{\text{MZDRbar}}{\sqrt{\frac{3}{5} g_1^2 + g_2^2}} \sin[\text{ArcTan}[\text{TanBeta}]]\} \\
\};
\]

Furthermore, **FlexibleSUSY** provides the symbols **EDRbar**, **ThetaWDRbar**, **MZDRbar** and many more that can be used to define relations between couplings and masses in **LowScaleInput**, see Table 3.1. Note further, that in Eqs. (1) the expression

\[
\frac{2 \hat{m}_Z(m_X)}{\sqrt{\frac{3}{5} g_1(m_X)^2 + g_2(m_X)^2}} =: \hat{v}(m_X)
\]

(3)

corresponds to the Higgs vacuum expectation value \(\hat{v}(m_X)\) in the SM in the \(\overline{\text{MS}}\) scheme at the scale \(m_X\). Thus, Eqs. (2g)–(2h) correspond to

\[
\begin{align*}
\nu_d(m_X) &= \hat{v}(m_X) \cos(\beta), \\
\nu_u(m_X) &= \hat{v}(m_X) \sin(\beta),
\end{align*}
\]

(4a)(4b)

which implicitly fixes the combination

\[
\nu(m_X) := \sqrt{[\nu_u(m_X)]^2 + [\nu_d(m_X)]^2} = \hat{v}(m_X),
\]

(5)

but leaves the ratio of the up- and down-type vacuum expectation values as

\[
\frac{\nu_u(m_X)}{\nu_d(m_X)} = \tan \beta.
\]

(6)

After imposing the decoupling conditions (1), the dependent parameters that are fixed by the electroweak symmetry breaking conditions are calculated using the tree-level tadpole equations with the SM-like decoupling scheme parameters defined in (1). For example, in the MSSM these dependent

| Symbol               | Description                                      |
|----------------------|--------------------------------------------------|
| **EDRbar**           | running electromagnetic gauge coupling           |
| **MWDRbar**          | running \(W\) boson mass                         |
| **MZDRbar**          | running \(Z\) boson mass                         |
| **ThetaWDRbar**      | running weak mixing angle \(\theta_W\)           |
| **upQuarksDRbar**    | running diagonal up-quark \(3 \times 3\) matrix  |
| **downQuarksDRbar**  | running diagonal down-quark \(3 \times 3\) matrix |
| **downLeptonsDRbar** | running diagonal down-lepton \(3 \times 3\) matrix |
| **neutrinoDRbar**    | running diagonal up-lepton \(3 \times 3\) matrix  |
| **VEV**              | running SM-like vacuum expectation value         |
| **CKM**              | running CKM matrix                               |
| **PMNS**             | running PMNS matrix                              |

Table 3.1: **FlexibleSUSY** model file symbols to define relations between BSM and SM quantities.
parameters may be soft-breaking Higgs doublet mass parameters. Following this, all tree-level masses are calculated in the decoupling scheme. The BSM pole masses are not calculated in the decoupling scheme, but in the traditional (non-decoupling) \( \text{MS}/\text{DR} \) scheme as in standard \texttt{FlexibleSUSY} \cite{25–27}.

The advantage of this decoupling scheme is that it does not suffer from artificial large quantum corrections in the presence of only heavy BSM particles. Instead, in the limit of infinitely heavy BSM physics one recovers SM results. Furthermore, equating SM-like parameters between the SM and the BSM, as done in (1), also makes it possible to take over known higher-order SM corrections to light Higgs decays from the literature, which are necessary for a meaningful comparison between theory and experimental results.\(^4\)

In Figure 1 we show this decoupling property for the MSSM, using \texttt{FlexibleSUSY}'s \texttt{MSSMEFTHiggs} model file, with all the new BSM masses set to a common \( m_{\text{SUSY}} \) mass (for details see Eq. (10) of Ref. [26]) and \( \tan \beta(m_{\text{SUSY}}) = 10 \). This allows us to test the decoupling property by increasing \( m_{\text{SUSY}} \) so that the new MSSM particles get heavier and seeing if their effects really decouple by plotting them against the Higgs decay partial widths in the SM. Therefore we show color coded partial widths for our decoupling scheme (solid lines), the regular \( \text{DR} \) scheme (dotted lines), and the SM (dashed lines).

Our focus is on the differences between the SM and MSSM results and the scheme differences. Since increasing \( m_{\text{SUSY}} \) also increases the Higgs mass, we show \( m_h \) on the lower horizontal and \( m_{\text{SUSY}} \) on the upper horizontal axis. Please note that it is the changes in \( m_{\text{SUSY}} \) which are driving the differences

\(^4\)We note that despite this desirable decoupling property, the tree-level matching does mean that the computation can miss important genuine BSM higher-order effects. For example it is well known in the MSSM that SUSY corrections to the bottom quark Yukawa coupling, the so-called \( \Delta \beta \) corrections, can be sizeable at large \( \tan \beta \). These contributions are particularly important in the case where the additional non-SM-like Higgs states remain light, i.e. where the set of low energy states is that of the two-Higgs Doublet Model (2HDM), see e.g. Ref. [62].
between the results and not the change in the Higgs mass, which is not essential for the differences between the results and is simply a feature of the MSSM where the Higgs mass is a prediction and increases logarithmically with $m_{\text{SUSY}}$. We have chosen the MSSMEFTHiggs model here because it also calculates the Higgs mass precisely at large $m_{\text{SUSY}}$, however we have also checked that we get the same behavior for the fixed-order Higgs mass calculation in the MSSM.

As can be seen in the figure, in most of the partial widths the decoupling scheme result interpolates between the $\overline{\text{DR}}$ result at low $m_{\text{SUSY}}$ to the SM result at high $m_{\text{SUSY}}$. The agreement with the SM result at high $m_{\text{SUSY}}$ indicates the decoupling property of this scheme. Only for $h \rightarrow \gamma\gamma$ we see a very mild deviation from the SM at high $m_{\text{SUSY}}$. We have traced this to a violation of the QED Ward identity at higher orders, which is an interesting issue we discuss in Section 4.3.5. The deviation increases with the splitting between the Higgs pole mass and the $\overline{\text{DR}}$ mass, so that it is maximized at the largest $m_{\text{SUSY}}$, growing to about 10% on the right hand side of the plot. This deviation comes from higher order corrections and is therefore part of the theory uncertainty of the calculation and due to the small size of this partial width it does not appreciably affect the BRs for other channels. This also shows up in the $h \rightarrow \gamma Z$ decay, and the same remarks apply there, but in that case the effect is barely visible on the plot. At low $m_{\text{SUSY}}$ the two different renormalization schemes give very similar results in the MSSM. The reason is that at low scales the two schemes differ only by non-enhanced higher-order corrections. The largest scheme differences occur in $h \rightarrow gg$ and $h \rightarrow bb$, which are affected by strong interactions. In total, the results confirm that the FlexibleDecay predictions in the decoupling scheme show the expected properties and the associated theory uncertainty is under control. The significant deviations between the MSSM and SM results at low $m_{\text{SUSY}}$ are therefore an example physics prediction which allows one to test and constrain a BSM scenario against experimental data.

4. Implementation

In this section we describe the quantum field theoretical results that are used in the implementation of the decay processes in FlexibleDecay. Since SM-like higher-order corrections, even at the multi-loop level, are very important for Higgs boson decays into SM particles we take special care of the treatment of these decay modes and almost always include appropriate higher-order corrections. In particular, we include a large set of higher-order corrections that can be incorporated in a model-independent way, making our calculation of Higgs decay modes close to state-of-the-art already in the standard model and actually state-of-the-art in most standard model extensions. In the case of a decay into a gauge boson pair we moreover include single and double off-shell decays to SM fermion pairs. Regarding decays of (i) Higgs bosons into one or more BSM final states, (ii) decays of non-Higgs BSM scalars, however, we restrict ourselves to a pure leading-order calculation in the decoupling scheme, which already gives a good approximation of the true result.

In Subsection 4.1 we describe the implementation of generic, tree-level or loop-induced, 2-body decays of a scalar $S$ used in the latter cases. In the subsequent Subsection 4.2 we then go through the important SM decay modes of Higgs bosons which exist already at the tree level, while in Subsection 4.3 we discuss relevant loop-induced decay modes. In the case of CP-conservation we use the symbols $H$ and $A$ for generic CP-even and CP-odd Higgs bosons, respectively, also collectively denoted as $\Phi$. We reserve the symbol $S$ for generic scalars which may or may not be Higgs bosons. Some corrections are applicable irrespective of the CP properties of Higgs bosons and are applied in all cases, as stated in the text. A special Subsection 4.3.5 is devoted to a discussion of the QED Ward identity that is relevant for the processes of $H \rightarrow \gamma\gamma$ and $H \rightarrow \gamma Z$, and which can be numerically violated in a generic spectrum generator.
4.1. Generic result for $S \rightarrow AB$

The generic formula for the leading order decay of a scalar particle $S$ into particles $A$ and $B$ (where $A$ or $B$ can be of type scalar, fermion or vector) reads

$$\Gamma(S \rightarrow AB) = \frac{1}{16\pi m_S} \lambda^{1/2} \left(1, \frac{m_A^2}{m_S^2}, \frac{m_B^2}{m_S^2}\right) |A_{S\rightarrow AB}|^2,$$

(7)

where $\lambda$ is the Källén function

$$\lambda(x, y, z) \equiv (x - y - z)^2 - 4yz$$

(8)

and $A_{S\rightarrow AB}$ are the matrix elements, computed at the leading order, be it at tree or a one-loop level. The $|A_{S\rightarrow AB}|^2$ are the squared matrix elements (if applicable summed over spins or polarizations) listed in Appendix B with potential extra symmetry and/or color factors. The automatized process of obtaining one-loop amplitudes is described in more detail in Subsection 4.3.1. The formulas are applied in all those cases that are not listed in the remainder of this section, in particular in the case of obtaining one-loop amplitudes. Running parameters are used in the calculation of the vertices and, if present, inside any loops.

All processes for which we don’t describe a special treatment below are treated in the manner described in this subsection.

4.2. Decays allowed at tree-level

4.2.1. Special treatment of $\Phi \rightarrow q\bar{q}$

In the case of a Higgs decay into a quark–anti-quark pair of the same flavor\(^5\) in the final state we include several higher-order effects as this mode is particularly important since the SM-like Higgs and many BSM Higgs bosons have dominant decay modes into $b\bar{b}$ or $t\bar{t}$ pairs. It is therefore of utmost importance to calculate such decay widths as precisely as possible. Due to the similarities in the calculation of the decay widths of scalar and pseudoscalar Higgs bosons we present them here together. Following HDECAY, we implement a scheme which allows us to use Higgs boson masses calculated in an arbitrary renormalization scheme. In this way Higgs boson masses calculated for instance in an MS or an on-shell (OS) scheme can be used, whichever is appropriate in the given BSM model.

We start with the description of the implementation of the NLO QCD corrections to the $\Phi \rightarrow q\bar{q}$ decays in the OS scheme. Similarly to Eq. (7) the loop corrected decay width can be written as

$$\Gamma(\Phi \rightarrow q\bar{q}) = \frac{1}{16\pi m_\Phi} \lambda^{1/2} \left(1, \frac{m_q^2}{m_\Phi^2}, \frac{m_\Phi^2}{m_\Phi^2}\right) |A_{OS-Yukawa}^{\Phi\rightarrow q\bar{q}}|^2 \left(1 + C_F \frac{\alpha_3}{\pi} \Delta_\Phi(\beta)\right),$$

(9)

where $\alpha_3 \equiv g_3^2/(4\pi)$, $C_F = 4/3$, and $\beta = (1 - 4m_q^2/m_\Phi^2)^{1/2}$. The squared matrix element $|A_{OS-Yukawa}^{\Phi\rightarrow q\bar{q}}|^2$ is defined similarly as in Eq. (9) but with an on-shell Yukawa coupling. In the OS scheme for quark masses and Yukawa couplings the NLO correction coefficients $\Delta_\Phi$ are given by [63]

$$\Delta_H(\beta) = \frac{1}{\beta} A(\beta) + \frac{1}{16\beta^3} \left(3 + 34\beta^2 - 13\beta^4\right) \ln \left(\frac{1 + \beta}{1 - \beta}\right) + \frac{3}{8\beta^2} (7\beta^2 - 1),$$

(10)

$$\Delta_A(\beta) = \frac{1}{\beta} A(\beta) + \frac{1}{16\beta} \left(19 + 2\beta^2 + 3\beta^4\right) \ln \left(\frac{1 + \beta}{1 - \beta}\right) + \frac{3}{8} (7 - \beta^2),$$

(11)

\(^5\)Flavor violating decays, if present in a given model, are computed according to Section 4.1, i.e., without any higher order corrections.
where
\[
A(\beta) = (1 + \beta^2) \left[ 4 \text{Li}_2 \left( \frac{1 - \beta}{1 + \beta} \right) + 2 \text{Li}_2 \left( \frac{1 - \beta}{1 + \beta} \right) - 3 \ln \left( \frac{1 + \beta}{1 - \beta} \right) \ln \left( \frac{2}{1 + \beta} \right) \right. \\
- 2 \ln \left( \frac{1 + \beta}{1 - \beta} \right) \ln \beta \left. - 3 \beta \ln \left( \frac{4}{1 - \beta^2} \right) - 4 \beta \ln \beta \right].
\] (12)

Eq. (9) is applicable as long as \( m_q \) is of the order of \( m_\Phi \). Otherwise, in the limit \( m_q^2/m_\Phi^2 \to 0 \), the term \( \Delta_\Phi \) contains a large logarithmic contribution
\[
\Delta_H \approx \Delta_A \approx \frac{9}{4} \frac{3 \ln m_\Phi^2}{m_q^2},
\] (13)
which spoils the perturbative expansion. Hence, for light quarks a different approximation is required.

Consider the one-loop relation between the pole mass \( m_q \) and the corresponding running mass \( \hat{m}_q \) in pure QCD,
\[
m_q = \hat{m}_q \left\{ 1 + C_F \frac{\alpha_3}{\pi} \left( 1 + \frac{3}{4} \ln \frac{\mu^2}{m_q^2} \right) \right\}.
\] (14)

Since at the order \( \alpha_3 \) there are no contributions to the vacuum expectation value, the relation (14) also holds for the quark Yukawa couplings. The mass-singular logarithms in Eqs. (10)–(11) can therefore be resumed by using the running Yukawa coupling at the scale of the Higgs mass in \( A_{\Phi \to q\bar{q}} \) [64]. A similar argument holds for the \( \mathcal{O}(\alpha) \) QED correction. For small ratios of \( m_q^2/m_H^2 \) the more appropriate expression for the partial width is therefore
\[
\Gamma_{\text{light,massless}}(\Phi \to q\bar{q}) = \frac{1}{16\pi m_H} \lambda^{1/2} \left( 1, \frac{m_q^2}{m_\Phi^2}, \frac{m_q^2}{m_\Phi^2} \right) |\mathcal{A}_{\Phi \to q\bar{q}}| \left( 1 + \Delta_{qq}^{\text{QCD}} + \Delta_{qq}^{\text{QED}} + \Delta_{qq}^{\text{QED} \times \text{QCD}} + \Delta_\Phi^3 \right),
\] (15)
where
\[
\Delta_{qq}^{\text{QCD}} = \frac{17}{3} \frac{\alpha_3}{\pi} \\
+ \left( \frac{10801}{144} - \frac{19}{12} \pi^2 - \frac{39}{2} \zeta_3 - N_f \left[ \frac{65}{24} + \frac{1}{18} \pi^2 + \frac{2}{3} \zeta_3 \right] \right) \frac{\alpha_3^2}{\pi^2} \\
+ \left( \frac{6163613}{5184} - \frac{3535}{72} \pi^2 - \frac{109735}{216} \zeta_3 + \frac{815}{12} \zeta_5 \right) \\
+ N_f \left[ \frac{-46147}{486} + \frac{277}{72} \pi^2 + \frac{262}{9} \zeta_3 - \frac{5}{6} \zeta_4 - \frac{25}{9} \zeta_5 \right] \\
+ N_f^2 \left[ \frac{15511}{11664} - \frac{11}{162} \pi^2 - \frac{1}{3} \zeta_3 \right] \frac{\alpha_3^3}{\pi^3} \\
+ \left( 39.34 - 220.9 N_f + 9.685 N_f^2 - 0.0205 N_f^3 \right) \frac{\alpha_3^4}{\pi^4},
\] (16)
\[
\Delta_{qq}^{\text{QED}} = \frac{17}{4} Q_2^2 \frac{\alpha}{\pi},
\] (17)
\[
\Delta_{qq}^{\text{QED} \times \text{QCD}} = \left( \frac{691}{24} - 6 \zeta_3 - \pi^2 \right) Q_2^2 \frac{\alpha \alpha_3}{\pi^2},
\] (18)
\footnote{Note that while there are no \( \mathcal{O}(\alpha) \) QED corrections to the vacuum expectation value, there are electroweak corrections, which cannot be resummed in this way.}
\[
\Delta_H^2 = g^H \frac{g}{g_\Phi^H} \left( \frac{8}{3} - \frac{\pi^2}{9} - \frac{2}{3} \ln \frac{m_H^2}{m_t^2} + \frac{1}{9} \ln^2 \frac{\hat{m}_q}{m_H^2} \right) \frac{\alpha_3^2}{\pi^2}, \tag{19}
\]
\[
\Delta_A^2 = g^A \frac{g}{g_\Phi^A} \left( \frac{23}{6} - \ln \frac{m_A^2}{m_t^2} + \frac{1}{6} \ln^2 \frac{\hat{m}_q}{m_A^2} \right) \frac{\alpha_3^2}{\pi^2}, \tag{20}
\]

where \( \zeta_n = \zeta(n) \) is the Riemann Zeta function with \( \zeta_2 = \pi^2/6 \), \( \zeta_3 \approx 1.202 \) etc. and \( N_f \) is the number of quark flavors lighter than \( \Phi \). The Yukawa couplings in Eq. (15), the couplings \( \alpha \) and \( \alpha_3 \) as well as light quark masses \( \hat{m}_q \) in Eqs. (19)–(20) are SM parameters defined in the SM with \( N_f \) active flavors.\(^7\) The term \( \Delta_{\text{QCD}} \) contains the higher-order corrections in massless QCD up to the four-loop level [65, 66]. Similarly, \( \Delta_{\text{QED}} \) contains the one-loop QED correction for quarks with electric charge \( Q \), while \( \Delta_{\text{QED} \times \text{QCD}} \) contains two-loop mixed QED–QCD correction [67]. The contributions \( \Delta_{\Phi}^2 \) of \( O(\alpha_3^2) \) are only added in the case of a CP-conserving Higgs sector. They originate from a flavor singlet diagram with both bottom and top quark loops (see Fig. 1a of Ref. [68]). This explains the occurrence of the two weight factors \( g^\Phi_t \) and \( g^\Phi_q \), defined as

\[
g^\Phi_q = \frac{v C_{\Phi q \bar{q}}}{\hat{m}_q}, \tag{21}
\]

where \( C_{\Phi q \bar{q}} \) is the \( \Phi q \bar{q} \) coupling. Note that the weight factors \( g^\Phi_t \) and \( g^\Phi_q \) reflect the BSM nature of the model, whereas \( g^\Phi_q = g^\Phi_t = 1 \) in the SM. Furthermore, since the \( \Delta_{\Phi}^2 \) contain corrections from a top loop, they are added only for decays into quarks lighter than the top quark. The loop contributions from lighter quarks to \( \Phi \to q \bar{q} \) are suppressed by the small masses of those quarks and are therefore neglected.

To calculate decays of particles with an arbitrary ratio \( m_q/m_\Phi \) in a uniform fashion we follow the approach of HDECAY and implement a linear interpolation in \( m_q^2/m_\Phi^2 \) between the two regimes:

\[
\Gamma(\Phi \to q \bar{q}) = \left( 1 - \frac{4m_q^2}{m_\Phi^2} \right) \Gamma^\text{light}(\Phi \to q \bar{q}) + \frac{4m_q^2}{m_\Phi^2} \Gamma^\text{heavy}(\Phi \to q \bar{q}). \tag{22}
\]

Moreover, we replace \( \Gamma^\text{light, massless} \) by \( \Gamma^\text{light} \) to include the full mass effects at NLO through

\[
\Gamma^\text{light}(\Phi \to q \bar{q}) = \frac{1}{16\pi m_\Phi} \chi^{1/2} \left( 1, \frac{\hat{m}_q^2}{m_\Phi^2}, \frac{\hat{m}_q^2}{m_\Phi^2} \right) |A_{\Phi \to q \bar{q}}^\text{MS Yukawa}|^2 \times \left( 1 + C_{\Phi} \frac{\alpha_3}{\pi} \left[ \Delta_{\Phi}(\hat{\beta}) + \Delta_{\Phi}^\text{mass} \right] + \Delta_{\Phi}^{\text{QED}} + \Delta_{\Phi}^{\text{QCD} \geq 2\text{-loop}} + \Delta_{\Phi}^{\text{QED} \times \text{QCD}} + \Delta_{\Phi}^2 \right), \tag{23}
\]

where \( \hat{\beta} = (1 - 4\hat{m}_q^2/m_\Phi^2)^{1/2} \) and

\[
\Delta_H^\text{mass} = 2 \left( 1 + \frac{3}{4} \ln \frac{m_H^2}{\hat{m}_q^2} \right) \frac{1 - 10m_q^2/m_\Phi^2}{1 - 4m_q^2/m_\Phi^2}, \tag{24}
\]

\[
\Delta_A^\text{mass} = 2 \left( 1 + \frac{3}{4} \ln \frac{m_A^2}{\hat{m}_q^2} \right) \frac{1 - 6m_q^2/m_\Phi^2}{1 - 4m_q^2/m_\Phi^2}. \tag{25}
\]

The contribution \( \Delta_{\Phi}^\text{mass} \) originates from expressing the tree-level decay in terms of the running quark mass up to \( O(\alpha_3) \). One also has \( \lim_{\hat{m}_q^2/m_\Phi^2 \to 0} (\Delta_{\Phi} + \Delta_{\Phi}^\text{mass}) = 17/4 \), as expected from Eq. (16). The

\(^7\) In FlexibleDecay we assume that \( \Phi \) is always heavier than the $t$-quark, such that \( N_f \geq 5 \).
term $\Delta_{qq}^{\text{QCD} \geq 2\text{-loop}}$ is obtained from Eq. (16) by dropping only the $O(\alpha_3)$ term.\footnote{Since HDECAY does not include QED effects in the running of the quark Yukawa coupling, it uses $\Delta_\Phi$ instead of $\Delta_{qq}^{\text{QED}}$ in the analog of Eq. (23).} Furthermore, Eq. (23) differs from Eq. (15) by terms of $O(\alpha_3^2 \bar{m}_q / m_\Phi^2)$ and higher. Apart from the corrections from Eq. (9), $\Gamma^{\text{heavy}}$ also contains one-loop QED corrections in the OS scheme

$$
\Gamma^{\text{heavy}}(\Phi \to q\bar{q}) = \frac{1}{16\pi m_\Phi} \lambda^{1/2} \left( 1 - \frac{m_q^2}{m_\Phi^2} \right) |A_{\Phi \to q\bar{q}}| \Delta_{\Phi}^{\text{OS-Yukawa}} \left[ 1 + \left( \frac{C_F \alpha_3 + \alpha}{\pi} \right) \Delta_\Phi(\beta) \right] .
$$

(26)

Since all couplings in FlexibleSUSY are running couplings, we obtain the OS Yukawa coupling needed in the above equation from the relation

$$
|A_{\Phi \to q\bar{q}}| = \frac{m_q^2}{\bar{m}_q^2} |A_{\Phi \to q\bar{q}}|^2 .
$$

(27)

As there is no interference between scalar and pseudoscalar parts, in models with CP-violation we add both contributions in the final result.

Finally, we emphasise, that no higher-order weak corrections are applied in Eqs. (15) and (23).

4.2.2. Special treatment of $\Phi \to l^+ l^-$

For the case of a pair of charged leptons in the final state we follow the generic approach from Eq. (7) but include universal one-loop QED corrections in massless limit as

$$
\Gamma(\Phi \to l^+ l^-) = \frac{1}{16\pi m_\Phi} \lambda^{1/2} \left( 1 - \frac{m_l^2}{m_\Phi^2} \right) |A_{\Phi \to l^+ l^-}| \Delta_{\Phi}^{\text{OS-Yukawa}} \left[ 1 + \frac{17 \alpha}{4 \pi} \right] ,
$$

(28)

where $\alpha$ denotes the SM $\overline{\text{MS}}$ electromagnetic coupling. In contrast to the $\Phi \to q\bar{q}$ case we always choose an $\alpha$ with 6 active quark flavors and ignore 2-loop contributions.

Eq. (28) can be obtained immediately by adapting Eq. (15) to the color singlet case at the one-loop level. This is sufficient, because QED corrections to $\Phi \to l^+ l^-$ are small and the leptonic decay rates of the Higgs are much smaller than decay rates to quarks.

4.2.3. Special treatment of $H \to ZZ$ and $H \to W^+ W^-$

For the SM-like Higgs boson, 3-body decays through off-shell $W$ and $Z$ bosons are phenomenologically very relevant. We therefore include both single and double off-shell diagrams as well as two body on-shell decays in this case.

The single off-shell decay width, assuming the SM decay pattern and neglecting fermion masses, is given by [69–72]

$$
\Gamma(H \to VV^*) = \frac{3|C_{HVV}|^2 R_T(m_H^2/m_V^2)}{256\pi^3 m_H^3} \frac{g_V^2}{2} \delta_V ,
$$

(29)

where

$$
R_T(x) \equiv \frac{3(1 - 8x + 20x^2)}{4x - 1} \text{arccos} \left( \frac{3x - 1}{2x^{3/2}} \right) - \frac{1 - x}{2x} \left( 2 - 13x + 47x^2 \right) - \frac{3}{2} \left( 1 - 6x + 4x^2 \right) \ln(x) ,
$$

(30)

$C_{HVV}$ is the coupling coefficient in front of the metric tensor, and

$$
\delta_V \equiv 1 ,
$$

(31)
\[ \delta Z \equiv \frac{1}{\hat{c}_w^2} \left( \frac{7}{12} - \frac{10}{9} \hat{s}_w^2 + \frac{40}{21} \hat{s}_w^4 \right), \]  

(32)

with \( \hat{c}_w \) and \( \hat{s}_w \) being the cosine and sine of the running weak mixing angle, respectively.

The double off-shell decay width is given by [72–74]

\[ \Gamma(H \rightarrow V^*V^*) = \frac{1}{\pi^2} \int_0^{m_H^2} ds_1 \frac{m_V \Gamma_V}{(s_1 - m_V^2)^2 + m_V^2 \Gamma_V^2} \int_0^{(m_H - s_1)^2} ds_2 \frac{m_V \Gamma_V}{(s_2 - m_V^2)^2 + m_V^2 \Gamma_V^2} \Gamma_0, \]  

(33)

where

\[ \Gamma_0 = \frac{|C_{HHV}|^2 m_H^3}{128\pi m_V^4} S_V \lambda^{1/2} \left( 1, \frac{s_1}{m_H^2}, \frac{s_2}{m_H^2} \right) \left( \lambda \left( 1, \frac{s_1}{m_H^2}, \frac{s_2}{m_H^2} \right) + \frac{12 s_1 s_2}{m_H^4} \right). \]  

(34)

while \( \Gamma_V \) is the width of the final state vector boson \( V \). In Eq. (34) the coupling \( C_{HHV} \) is the same as in the case of the 3-body decay and

\[ S_V = \begin{cases} 2 & \text{for } V = W, \\ 1 & \text{for } V = Z. \end{cases} \]  

(35)

We implicitly assume, that an off-shell boson can only “decay” to SM leptons or quarks other than the top quark. This assumption enters either by directly considering only SM decays in Eq. (29) or by using the SM \( W \) and \( Z \) widths in Eq. (33). This assumption is no longer valid if \( V \) couples to two BSM electroweak particles or to one BSM and one SM particle, whose sum of masses is smaller than \( m_H - m_V \). FlexibleDecay automatically detects such cases and disables the calculation of those off-shell decay widths, while still allowing all other decay calculations to proceed as normal. In that case the off-shell partial width is effectively set to zero. As a result, all the branching ratios printed by FlexibleDecay in such a case will be incorrect, though the partial widths themselves are correct.

By default, FlexibleDecay uses double off-shell decays both for \( m_H < m_V \) as well as for \( m_V < m_H < 2m_V \) and two-body decays, according to Eq. (7), if \( m_H > 2m_V \). The user is free to select which off-shell decays are used for which mass ordering, though. This is controlled by the flag 4 in block FlexibleDecay as described in Table A.2. Technically, Eq. (33) is integrated numerically in FlexibleDecay using an algorithm from the GNU Scientific Library [49] with a relative error goal of \(< 0.1\%\).

4.3. Loop-induced decays

Several important decay channels of Higgs and BSM scalars do not arise at tree level but instead are loop-induced. These loop-induced decays are computed by FlexibleDecay to at least the full one-loop level. As in the case of tree-level decays, loop-induced decays of high phenomenological importance are augmented by higher order SM-like corrections. This concerns the decays of \( \Phi \rightarrow \gamma \gamma \) and \( \Phi \rightarrow gg \). Regarding the loop-induced decay of \( H \rightarrow \gamma Z \), the SM QCD corrections are small [75–77], so the calculation is restricted to the leading order, which is sufficient for current and near future experiments [78]. In the following we first describe the general one-loop calculation and afterwards focus on the details of the implementation of \( \Phi \rightarrow \gamma \gamma \) and \( \Phi \rightarrow gg \).

4.3.1. General one-loop computation of loop-induced decays

Whenever some decay amplitude with two final-state particles vanishes analytically at tree level, FlexibleDecay switches to computing the amplitude at full one-loop level.\(^9\) In general, the ten different diagram topologies shown in Figure 2 can contribute. FlexibleDecay has implemented all

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\(^9\) The check is performed analytically at the level of particle multiplets in the mass eigenstate basis, as defined in the SARAH model file. This means that currently no loop-induced flavor-violating decays are generated.
these topologies as well as the analytic expressions for the corresponding amplitudes after inserting all possible concrete particle types (scalars, fermions, vectors). These generic analytic expressions were pre-generated using *FeynArts* and *FormCalc* [52, 79] and are distributed with *FlexibleSUSY*. For a given concrete model the diagram topologies are filled with concrete particle insertions during the C++ code generation of *FlexibleSUSY*.

### 4.3.2. Special treatment of $\Phi \rightarrow \gamma \gamma$

Following the outlined procedure, we first evaluate all one-loop diagrams for $\Phi \rightarrow \gamma \gamma$ in a BSM model. The amplitude can be split into loops with fermions, scalars and vector bosons as

$$A = \sum_f A_f + \sum_{S,V} A_{S,V}. \quad (36)$$

Some of those individual terms are then augmented by SM-like higher-order corrections, which is described in the following.

First, fermion triangle loops (topology $T1$ from Figure 2) for $\Phi \rightarrow \gamma \gamma$ consisting of a closed loop of quarks of the same flavor are multiplied by a correction factor that implements two-loop QCD corrections. Specifically, every triangle loop with a color triplet fermion $f$, which can be written as

$$A_{T1,f} = A_{T1,S}^{(f)} \left[ k^2 (\epsilon_1^* \epsilon_2^*) - (k_1 \epsilon_2^*) (k_2 \epsilon_1^*) \right] + A_{T1,P}^{(f)} \det(\epsilon_1, \epsilon_2, k_1, k_2), \quad (37)$$

where $k_i$ and $\epsilon_i$ are the momentum and polarization vectors of photon $i$, respectively, is replaced by

$$A_{f}^{T1,X} \rightarrow A_{f}^{T1,X} \left[ 1 + \frac{\alpha_3}{\pi} \left( C_1^X (\tau_q) + C_2^X (\tau_q) \ln \frac{\mu^2}{m_f^2} \right) \right], \quad (38)$$
where $\hat{m}_f$ is the $\overline{\text{MS}}$ fermion mass, $\alpha_3$ is the $\overline{\text{MS}}$ strong coupling in the SM, $X \in \{S,P\}$, and $\tau_f \equiv m_\Phi^2/(4\hat{m}_f^2)$. Similarly to Sec. 4.2.2 we do not apply NNLO corrections and always choose the 6-flavour scheme. The correction coefficients $C_i^S$ were first evaluated numerically for intermediate $\tau_f$s in [80], and $C_i^N$ were given analytically in the limit of $\tau_f \gg 1$ and $\tau_f \ll 1$ in [81]. Here we implement fully analytic result without any approximations as given in Eqs. (2.7)–(2.8) ($C_i^S$) and in Eqs. (2.19)–(2.20) ($C_i^N$) of Ref. [82]. In the case of the SM top-quark loop it would be sufficiently accurate to implement only the first few terms of the Taylor expansion around $\tau = 0$. However we implement the full expressions in order to improve the precision for decays of arbitrary Higgs bosons and BSM quark-like fermion loops with arbitrary Higgs/fermion mass ratios.

In the limit of $\tau_f \to 1$, $C_i^P$ exhibits a Coulomb singularity [82] which is regulated by a finite fermion $f$ width and should be resummed [83]. Here we resort to a solution where we simply do not apply higher order corrections when $|1 - \tau_f| \leq 10^{-2}$. This case is also flagged, with an appropriate warning printed when it occurs.

The two-loop corrections to closed scalar loops with T1 and T4 topologies are implemented according to Ref. [84]. Similar as before, closed scalar loops are multiplied by the correction factor

$$A_S^{T1+T4} \to A_S^{T1+T4} \left[ 1 + \frac{\alpha_3}{\pi} C(R) \frac{\mathcal{F}_0^{(2l,a)}(r) + \left( \mathcal{F}_0^{(2l,b)}(r) + \mathcal{F}_0^{(2l,c)}(r) \right) \ln \frac{\hat{m}_f^2}{\mu^2}}{\mathcal{F}_0^{(1l)}}(r) \right],$$

(39)

where $\hat{m}_0$ is a running scalar mass in the loop, $\alpha_3$ is the $\overline{\text{MS}}$ strong coupling in the SM with 6 active quark flavors, $r = m_\Phi^2/\hat{m}_0^2$ and $C(R)$ is the Casimir of the color representation of the scalar. Currently, FlexibleDecay handles color triplets with $C_F = (N_c^2 - 1)/(2N_c)$ and octets with $C_A = N_c$. The functions $\mathcal{F}_0$ are available analytically as a series expansion. For $r \ll 1$ one has

$$\mathcal{F}_0^{(1l)}(r) = -\frac{1}{3} - \frac{2}{45}r - \frac{1}{140}r^2 - \frac{2}{1575}r^3 - \frac{1}{4158}r^4 + O(r^5),$$

(40)

$$\mathcal{F}_0^{(2l,a)}(r) = -\frac{3}{4} \left( \frac{216}{15}r - \frac{4973}{15} r^2 - \frac{3137}{4} r^3 - \frac{1180367}{209563200} r^4 + O(r^5) \right),$$

(41)

$$\mathcal{F}_0^{(2l,b)}(r) = -\frac{1}{4} \left( \frac{9}{5} r - \frac{560}{252} r^2 - \frac{5}{5544} r^3 + O(r^5) \right),$$

(42)

while for $r \gg 1$

$$\mathcal{F}_0^{(1l)}(r) = \frac{4}{r} + 4 \ln^2(-r) \frac{1}{r^2} + O\left(\frac{1}{r^3}\right),$$

(43)

$$\mathcal{F}_0^{(2l,a)}(r) = [14 - 3 \ln(-r)] \frac{1}{r} + \left[ \frac{6}{5} - \frac{1}{5} \frac{72}{5} \zeta_2 - 24 \zeta_3 - 8(1 - \zeta_2 - 4 \zeta_3) \ln(-r) \right] \frac{1}{r^2} + O\left(\frac{1}{r^3}\right),$$

(44)

$$\mathcal{F}_0^{(2l,b)}(r) = \frac{6}{5} \ln(-r) - 3 \ln^2(-r) \frac{1}{r^2} + O\left(\frac{1}{r^3}\right).$$

(45)

Throughout the calculation we choose a renormalization scale $\mu = m_\Phi$, as opposed, for example, to a $\mu = m_\Phi/2$ choice of Ref. [81]. Finally, the total amplitude for $\Phi \to \gamma\gamma$ is multiplied by $\alpha(0)/\alpha(m_\Phi)$. This effectively incorporates certain two-loop QED corrections and minimizes the remaining theoretical uncertainty.\textsuperscript{10}

For the SM Higgs boson it is known that these implemented two-loop QCD corrections increase the partial width by around 2\%. In addition to these corrections, also the complete SM three-loop QCD [85] and the complete SM two-loop electroweak corrections [86, 87] are known. For the SM Higgs, the

\textsuperscript{10} This is controlled by flag 3 in block FlexibleDecay (see Table A.2).
three-loop QCD part increases the partial with by $< 0.1\%$; therefore we opt to not include them in the current version of FlexibleDecay. The two-loop order $\tilde{y}_t^2$ corrections are known in an analytic form in the large $m_t$ limit and for $m_W < m_H < 2m_W$ [88]. These corrections turn out to not be dominant, being on par with the one from the pure Yang–Mills sector. For the SM-like Higgs the total electroweak correction is negative and accounts for around $-2.5\%$. Since those corrections cannot be easily incorporated into our calculation we neglect them. The precision of $\Gamma(h \rightarrow \gamma\gamma)/\Gamma(h \rightarrow \gamma\gamma)_{SM}$ ratio at the high luminosity LHC is estimated to be $3.8\%$ [78], so neglecting them will not be the biggest source of uncertainty when confronting our calculation with experimental data.

This implementation supersedes the previously implemented calculation of effective $\Phi \rightarrow \gamma\gamma$ coupling [42]. Compared to the old case we now compute branching ratios rather than effective couplings, include two-loop QCD corrections to fermion and scalar loops and improve handling of CP-violating models.

4.3.3. Special treatment of $\Phi \rightarrow \gamma Z$

In the double limit of a heavy color triplet fermion in the loop and $m_Z/m_\Phi \rightarrow 0$, the NLO QCD correction to $\Phi \rightarrow \gamma Z$ approaches the same value as in the $\Phi \rightarrow \gamma\gamma$ case [72, 75]. For a generic scalar $\Phi$, the correction can be applied by the replacement

$$A_{\gamma\gamma}^{T1,S} \rightarrow A_f^{T1,S} \left( 1 - \frac{\alpha_3}{\pi} \right),$$

$$A_f^{T1,P} \rightarrow A_f^{T1,P},$$

i.e. the correction to the CP-odd part of the amplitude vanishes in this limit. In FlexibleDecay we implement this correction for $m_\Phi/m_f < 0.8$ and $m_Z/m_\Phi < 0.75$, where $m_f$ is the running mass of the colored fermion running in the loop.

Finally, similarly as in the $\Phi \rightarrow \gamma\gamma$ case, we multiply the amplitude by the $(\alpha(0)/\alpha(m_\Phi))^{1/2}$ factor related to a single on-shell photon.$^{10}$

4.3.4. Special treatment of $\Phi \rightarrow gg$

The higher-order corrections for the $\Phi \rightarrow gg$ decay are implemented differently from the $\Phi \rightarrow \gamma\gamma$ case. The reason is that a calculation of higher-order corrections involves not only virtual, but also real amplitudes. The known higher order corrections can therefore be only implemented at the level of the partial width, not the amplitude.

The $\Phi \rightarrow gg$ decay width, including higher order SM QCD corrections to the top quark loop, can be written as

$$\Gamma(H \rightarrow gg) = \Gamma^{\text{full}}_{\text{LO}}(H \rightarrow gg)$$

$$+ \Gamma_H \left[ 1 - \left( \frac{\alpha_3^{(6)}}{\alpha_3^{(5)}} \right)^2 + \frac{1}{\Gamma_H} \left( \frac{\alpha_3^{(5)}}{\pi} \right)^2 \delta_{\text{NLO}}^H + \left( \frac{\alpha_3^{(5)}}{\pi} \right)^2 \delta_{\text{NNLO}}^H + \left( \frac{\alpha_3^{(5)}}{\pi} \right)^3 \delta_{\text{NNNLO}}^H \right],$$

$$\Gamma(A \rightarrow gg) = \Gamma^{\text{full}}_{\text{LO}}(A \rightarrow gg) + \Gamma_A \left[ 1 - \left( \frac{\alpha_3^{(6)}}{\alpha_3^{(5)}} \right)^2 + \frac{1}{\Gamma_A} \left( \frac{\alpha_3^{(5)}}{\pi} \right)^2 \delta_{\text{NLO}}^A + \left( \frac{\alpha_3^{(5)}}{\pi} \right)^2 \delta_{\text{NNLO}}^A \right],$$

where $\Gamma^{\text{full}}_{\text{LO}}$ is the full, model specific, one-loop BSM contribution to $\Phi \rightarrow gg$ and

$$\Gamma^0_H = \left[ \frac{3}{2\tau} \left[ 1 + \left( 1 - \frac{1}{\tau} \right) \arcsin^2(\sqrt{\tau}) \right] \right]^2,$$

$$\Gamma_H = \frac{m_H}{18\pi} \left( \frac{\alpha_3^{(5)}}{\pi} \right)^2 |C_{Hii} \sqrt{\tau}|^2 \Gamma^0_H,$$
\[ \Gamma_0^A = \left| \frac{1}{\tau} \arcsin^2(\sqrt{\tau}) \right|^2, \quad (52) \]

\[ \Gamma_A = \frac{m_A}{8\pi} \left( \frac{\alpha_3^{(5)}}{\pi} \right)^2 \left| C_{A\ell t} \sqrt{\tau} \right|^2 \Gamma_0^A, \quad (53) \]

where \( \tau = m_3^2/(4\tilde{m}_t^2) \). In Eqs. \((48)\)–\((49)\) \(\alpha_3^{(n)}\) denotes the \(\overline{\text{MS}}\) strong coupling in the SM with \(n\) active quark flavors.

For the decay of a CP-even scalar \(H\), two-, three- and four-loop QCD contributions are taken into account. For the decay of a CP-odd scalar \(A\), two- and three-loop QCD contributions are taken into account. The term \(1 - (\alpha_3^{(6)}/\alpha_3^{(5)})^2\) converts the squared top quark contribution in \(\Gamma_{\text{LO}}^{4\ell}(\Phi \rightarrow gg)\) from 6 to 5 flavors.

The NLO, NNLO and NNNLO QCD corrections in Eqs. \((48)\)–\((49)\) were obtained analytically in Refs. \([89\text{--}94]\) by employing a heavy quark limit. For a CP-even Higgs \(H\) we implement \(\delta^H_{\text{NLO}}\) at \(\mathcal{O}(\tau^3)\) \([89, 90, 95]\), \(\delta^H_{\text{NNLO}}\) at \(\mathcal{O}(\tau^5)\) \([90, 91]\) and \(\delta^H_{\text{NNNLO}}\) at \(\mathcal{O}(\tau^9)\) \([92]\):

\[ \delta^H_{\text{NLO}} = \frac{95}{4} - \frac{7}{6} N_f + \frac{33 - 2N_f}{6} L_H \]

\[ + \left( \frac{5803}{540} L_H + \frac{14}{15} L_t + N_f \left[ \frac{-29}{60} - \frac{7}{45} L_H \right] \right) \tau \]

\[ + \left( \frac{1029839}{189000} \frac{16973}{12600} \frac{1543}{1575} L_t + N_f \left[ \frac{-89533}{378000} - \frac{1543}{18900} L_H \right] \right) \tau^2 \]

\[ + \left( \frac{9075763}{2976750} \frac{1243}{1575} L_H - \frac{452}{525} L_t + N_f \left[ \frac{-3763}{28350} - \frac{226}{4725} L_H \right] \right) \tau^3 \]

\[ + \left( \frac{50854463}{27783000} \frac{27677 L_H}{606375} - \frac{442832}{4729725} L_t + N_f \left[ \frac{-10426231}{127338750} - \frac{55354}{1819125} L_H \right] \right) \tau^4 \]

\[ + \left( \frac{252432553361}{218513295000} \frac{730612}{2149875} \frac{2922448}{4729725} L_t + N_f \left[ \frac{-403722799}{7449316875} - \frac{1461224}{70945875} L_H \right] \right) \tau^5 \]

\[ \delta^H_{\text{NNLO}} = \frac{149533}{288} - \frac{363}{8} L_2 - \frac{495}{16} L_H + \frac{3301}{16} L_t + \frac{363}{16} L_H^2 + \frac{19}{8} L_t \]

\[ + N_f \left( \frac{-4157}{72} + \frac{11}{2} L_2 + \frac{5}{4} L_3 + \frac{95}{4} L_H - \frac{11}{4} L_H^2 + \frac{2}{3} L_t \right) \]

\[ + N_f^2 \left( \frac{127}{108} - \frac{1}{6} L_2 + \frac{7}{12} L_H \right) \]

\[ + \left( \frac{104358341}{1555200} \frac{847}{240} \frac{7560817}{69120} \frac{203257}{2160} \frac{77}{15} L_3 + \left[ \frac{9124273}{3888000} \frac{77}{180} \frac{7}{12} L_3 + \left( \frac{-67717}{6480} + \frac{14}{45} L_t \right) \right] \right) \tau \]

\[ + \left( \frac{-1279790053883}{121927680000} \frac{186703}{100800} \frac{39540255113}{232243200} \frac{29}{120} L_H + \frac{7}{180} L_H^2 \right) \]

\[ + \left( \frac{-1279790053883}{121927680000} \frac{186703}{100800} \frac{39540255113}{232243200} \frac{29}{120} L_H + \frac{7}{180} L_H^2 \right) \]

\[ \frac{5597}{12960} - \frac{7}{540} \frac{29}{120} L_H + \frac{7}{180} L_H^2 \right) \tau \]

\[ + \left( \frac{-1279790053883}{121927680000} \frac{186703}{100800} \frac{39540255113}{232243200} \frac{29}{120} L_H + \frac{7}{180} L_H^2 \right) \]

\[ + \left( \frac{186703}{33600} \frac{10980293}{453600} L_t + \frac{20059}{37800} L_t + N_f \left[ \frac{-6466142939}{5715360000} \left( \frac{-10306537}{19440000} + \frac{1543}{4725} L_t \right) \right] \right) \tau \]

\[ + \left( \frac{186703}{33600} \frac{10980293}{453600} L_t + \frac{20059}{37800} L_t + N_f \left[ \frac{-6466142939}{5715360000} \left( \frac{-10306537}{19440000} + \frac{1543}{4725} L_t \right) \right] \right) \tau \]
For physical vector bosons, since composing the coupling of the CP-mixture state discovery channel and so the effect of those corrections gets significantly diluted.

\[ \Phi \]

precision of our program. Although viable squark masses the correction to the SM Higgs boson decays can be neglected within the target scheme using Eq. (14). The expansions in Eqs. (54) and (55) provide a good approximation of the full dependence for \( \tau \lesssim 0.7 \), and are applied in FlexibleDecay in this regime. For the CP-odd Higgs \( A \) we only implement the leading terms from Refs. [93, 94] in the large mass expansion:

\[ \delta_{\text{NNLO}}^A = \frac{97}{4} - \frac{7}{6} N_f + \frac{33 - 2N_f}{6} \ln \frac{\mu^2}{m_A^2}, \quad (57) \]

\[ \delta_{\text{NNLO}}^A = \frac{237311}{864} - \frac{529}{24} \zeta_2 - \frac{445}{8} \zeta_3 + 5 \ln \frac{m_A^2}{m_t^2}. \quad (58) \]

where \( m_t \) is the top quark pole mass and, as before, we set \( N_f = 5 \). Moreover, in the expression for \( \delta_{\text{NNLO}}^A \), we have also set \( \mu = m_A \).

From the point of view of Higgs decays, where \( \Phi \to gg \) is neither the main discovery channel nor does it influence the total width very significantly, the implemented corrections offer a good compromise between accuracy and complexity. Beyond the included QCD corrections, further model-specific corrections are also known in the literature. Two-loop EW corrections for the SM-like Higgs were found to be around a few percent [87], comparable with the four-loop QCD corrections. As they are small and model-dependent, we opt to not include them in the current version of FlexibleDecay. The NLO QCD corrections to squark contributions are also known by Ref. [96]. For phenomenologically viable squark masses the correction to the SM Higgs boson decays can be neglected within the target precision of our program. Although \( \Phi \to gg \) is important channel for the total width, it is not a discovery channel and so the effect of those corrections gets significantly diluted.

In models with CP violation we include both the CP-even and the CP-odd corrections by decomposing the coupling of the CP-mixture state \( \Phi \) as \( C_{\Phi i\bar{i}} = C_{Hi\bar{i}} + \gamma^5 C_{Ai\bar{i}} \) and applying the corrections appropriately.

As mentioned at the end of Section 4.3.2, the above described implementation is an improvement over the previously available effective couplings calculation [42]—it offers increased precision by including higher order terms in \( \tau \) expansion for the CP-even part and generalizes handling of CP-violating models.

4.3.5. Discussion of the Ward identity

The general amplitude for a decay of a scalar \( S \) into two vectors \( V_1 \) and \( V_2 \) has the form

\[ A_{S \to V_1 V_2} = \epsilon^*_{1\mu} \epsilon^*_{2\nu} \left( F_{0} p_{\mu} p_{\nu} + F_{11} p_{1\mu} p_{1\nu} + F_{12} p_{1\mu} p_{2\nu} + F_{21} p_{2\mu} p_{1\nu} + F_{22} p_{2\mu} p_{2\nu} + F_{c} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \right). \quad (59) \]

For physical vector bosons, since \( \epsilon_i p_i = 0 \), the terms \( F_{11}, F_{22} \) and \( F_{12} \) do not contribute and only the three coefficients \( F_{\eta}, F_{21} \) and \( F_{c} \) coefficients are relevant. For at least one massless vector in the final state, two of these, \( F_{\eta} \) and \( F_{21} \), are related by the Ward identity

\[ F_{\eta} = -p_{1\mu} p_{2\nu} F_{21}, \quad (60) \]

reflecting the gauge invariance under the \( \epsilon_i^n \to \epsilon_i^n + \lambda p_i^n \) transformation. A technical consequence of the Ward identity is that squared and spin-summed matrix elements \( |A_{S \to V_1 V_2}|^2 \) in Eqs. (B.13), (B.14) are automatically non-negative.
In gauges other than the unitary gauge where loops with Goldstone bosons have to be included, the fulfilment of the identity in Eq. (60) relies on the relation between $hG^+G^-$ coupling and the external, physical Higgs boson momentum. For example in the case of the SM, the $hG^+G^-$ coupling is determined by $\lambda = m_h^2/\nu^2$, and the Ward identity holds only if the same value for $m_h$ is used both for evaluating the coupling and the external Higgs boson momentum $p_h^2 = m_h^2$. Otherwise the one-loop diagram with a mixed charged Goldstone boson and W-boson loop contributes a term to the $F_\eta$ coefficient that violates the Ward identity.

This situation leads to a potential problem in calculations which work in Feynman–’t Hooft gauge and do not use the OS renormalization scheme for the coupling constants: the external Higgs boson momentum is always set to the squared Higgs pole mass, while internal couplings may be evaluated as running couplings in some scheme, and the Ward identity at some fixed order is numerically violated.\textsuperscript{11} Since internally \textit{FlexibleDecay} and \textit{FlexibleSUSY} work in $\overline{\text{MS}}/\text{DR}$ renormalization scheme and employ Feynman–’t Hooft gauge, our calculation of $H \to \gamma V$, where $V = \gamma, Z$, suffers from this problem, including the possibility of a negative decay width.

Numerical violation of Ward identity happens both in SM as well as in BSM models. Our solution to this problem is as follows. \textit{FlexibleDecay} uses the Ward identity to eliminate $F_\eta$ in favor of $F_{21}$ before squaring the matrix element. This ensures that the squared matrix element is non-negative. Still, \textit{FlexibleDecay} calculates all form factors $F_i$ in order to test the validity of the Ward identity. Specifically the squared matrix elements obtained by eliminating $F_\eta$ or by eliminating $F_{21}$ in favor of $F_\eta$ are compared; if the difference is larger than 10% a warning is printed.\textsuperscript{12}

5. Comparison with existing tools

In this section we investigate the performance, generality and accuracy of \textit{FlexibleDecay} and compare it with other established decay calculators. Since higher order corrections to Higgs decays are not small, it is reasonable to expect moderate differences between results, depending on the details of the implementation. This issue is also sometimes exacerbated by differences in the renormalization schemes in which input parameters are defined.

To discuss all the differences we apply \textit{FlexibleDecay} to a variety of models representing qualitatively distinct features: the SM, the singlet extended SM, the type II THDM, the constrained MSSM (CMSSM), the CP-violating MSSM, and the MRSSM. In each model we compute Higgs-boson decays, and in the CMSSM we additionally compute squark decays. The \textit{FlexibleDecay} results are compared with results from existing public programs: \textit{HDECAY} 6.53, \textit{sHDECAY}, \textit{2HDECAY} 1.1.4, \textit{SUSY-HIT} 1.5a, \textit{SARAH} 4.14.3/\textit{SPheno} 4.0.4 and \textit{SOFTSUSY} 4.1.9.

Among these programs, \textit{SARAH/SPheno} is a generic code, like \textit{FlexibleDecay}, based on \textit{SARAH} and able to treat large classes of BSM models. The other programs are dedicated to specific models. Before discussing the detailed comparisons we remark that the comparisons require scheme translations because the different programs utilize different renormalization schemes and different selections of input parameters. Generally we adopt the following procedure. In non-supersymmetric models we will start from MS parameters, use \textit{FlexibleSUSY} to calculate physical masses and use them, depending on the model, as inputs to \textit{HDECAY} or \textit{sHDECAY} (the exception is \textit{2HDECAY} were we use directly $\overline{\text{MS}}$ parameters). In the case of the CMSSM comparison, due to the incompatibility between \textit{HDECAY} and \textit{FlexibleSUSY} generated SLHA output, we use \textit{SUSY-HIT} and its interface to \textit{HDECAY}.\textsuperscript{13} The technical details of the \textit{SARAH/SPheno} setup are given in Appendix C.

\textsuperscript{11}The violation is of course formally of higher order but can be numerically relevant.
\textsuperscript{12}We regard violation of Ward identity as an indication of an existing theory uncertainty from unknown higher order contributions.
\textsuperscript{13}Internally, \textit{SUSY-HIT} 1.5a uses \textit{HDECAY} 3.4.
In the following comparisons the following SM parameters are used in almost all programs:

\[
\begin{align*}
    m_t &= 172.76 \text{ GeV}, \\
m_\tau &= 1.77686 \text{ GeV}, \\
m_Z &= 91.1876 \text{ GeV}, \\
m_b^{\overline{\text{MS}}} (m_b^{\overline{\text{MS}}}) &= 4.18 \text{ GeV}, \\
m_c^{\overline{\text{MS}}} (m_c^{\overline{\text{MS}}}) &= 1.27 \text{ GeV}, \\
\alpha^{-1}(m_Z) &= 127.934, \\
\alpha_3^{(5)}(m_Z) &= 0.1179, \\
V_{\text{CKM}} &= 1, \\
\Gamma_W &= 2.085 \text{ GeV}, \\
\Gamma_Z &= 2.4952 \text{ GeV}.
\end{align*}
\] (61)

The exceptions are: HDECAY, which takes \( m_c^{\overline{\text{MS}}} (3 \text{ GeV}) \) and which we leave set to its default value of 0.986 GeV [7], and SUSY-HIT which expects \( m_c^{\text{pole}} \) that we set to 1.67 GeV [97]. The \( m_W \) is a derived quantity in FlexibleSUSY, SARAH/SPheno and SOFTSUSY while for other codes we leave it to their respective defaults.

As various codes organize their calculations in slightly different ways, we take special care to compare like with like for a fair and uniform comparison. To that end we set certain publicly available options as will be described throughout this section and in Appendix C. However since the purpose is to provide a fair comparison of the results of public codes as they have been written, we do not make any changes in the source code of external programs. We did however create custom steering files for the SSM in SARAH/SPheno and FlexibleSUSY and MRSSM in SARAH/SPheno. In the case of the MRSSM this was necessary to make a meaningful comparison with the FlexibleSUSY extension of FlexibleSUSY, while in the case of the SSM we did it to match the sHDECAY.

For reproducibility, input and output files from all programs used in this comparison, as well as the model steering files mentioned above, are attached to the arXiv version of this work.

5.1. Standard Model

In Table 5.1 we show the comparison between different programs for the SM Higgs boson decays, together with partial widths recommended for this Higgs boson mass by the LHC Higgs Cross Section Working Group (LHCXSWG) [98]. We fix the physical Higgs mass to \( m_h = 125.1 \text{ GeV} \) [97] in all programs, although this corresponds to different values of the quartic Higgs coupling \( \lambda \) in the different programs. Dashes denote channels which cannot be computed by the used version of SARAH/SPheno. We also do not report the total width from the DECAY1L block, as SARAH/SPheno cannot compute some channels which are numerically important for the total width.

Generally there is very good agreement between FlexibleDecay and the LHCXSWG recommendations and HDECAY, which validates the results of FlexibleDecay. The largest relative deviation between FlexibleDecay and HDECAY of below 5% occurs for the decay into \( W^+W^- \) and \( \gamma Z \). The relative deviations for the other channels are smaller than 2%. The results from the SARAH/SPheno...
DECAY block also agree well with the LHCXSWG recommendations, except for the $h \rightarrow b\bar{b}$ and $h \rightarrow \gamma\gamma$ channels, where deviations of around 10% can be observed. As expected, the results from the DECAY1L block of SARAH/SPheno show larger deviations due to missing Higgs-specific higher-order corrections. Note that several decay modes cannot be computed by SARAH/SPheno in the given setup or lead to negative outputs, as indicated in the table.\(^{14}\)

Systematic small differences exist between the various computations of the $h \rightarrow VV$ ($V = W, Z$) decay modes. For HDECAY and FlexibleDecay the table presents calculations including double off-shell decays of the Higgs boson, even if $2m_V > m_h > m_V$, as it matches more closely what is actually reported as the $\Gamma(h \rightarrow VV)$ width in the literature (corresponding to the default in both programs). In contrast, SPheno only includes single off-shell decays to gauge bosons.\(^{15}\)

Figure 3: SM results for Higgs branching ratios with those from the LHC Higgs Cross Section Working Group in the low mass range [99].

\(^{14}\)Except for 2HDECAY, all programs discussed in this entire section output branching ratios and total Higgs decay widths rather than partial widths. For these programs we define the entries of Table 5.1 and following tables by multiplying the total width and branching ratios; negative entries in the tables thus correspond to negative outputs for branching ratios.

\(^{15}\)As explained in Appendix A.2, the behavior of SARAH can be reproduced in FlexibleDecay by setting the flag FlexibleDecay[4] = 1.
In Figure 3 we look at the SM Higgs branching ratios over the Higgs mass range \( m_h \in [80, 200] \) GeV and compare to the results of the Higgs Cross Section Working Group, which were presented in Ref. [99]. As can be seen in the figure our results are in good agreement with Ref. [99] over most masses. However a notable difference is that our results contain noticeable kinks at thresholds associated with twice the \( W \) and twice the \( Z \) masses. This reflects our treatment of the off-shell Higgs decays. The Higgs XSWG uses Prophecy4f [100–102] which computes the \( W/Z \to 4f \) process therefore not exhibiting any thresholds.

### 5.2. Real singlet extension of the Standard Model

In this section we consider the comparison to the \( Z_2 \) real scalar singlet extension of the SM with the potential of Eq. (2.11) in Ref. [18], where the real-singlet gets a non-zero VEV. In this model, the SM-like mass eigenstate has reduced couplings due to the admixture of the scalar singlet, which couples neither to gauge bosons nor to quarks or leptons. The mixing is parametrized by an angle \( \alpha_h \). We use a benchmark point inspired by the RxSM.B1 reference point of Ref. [18]: \( \lambda \approx 0.420, \lambda_S = 0.710, \lambda_{H_S} \approx -0.594, \) and \( v_s = 140.3 \) GeV at the scale \( \mu = 125.1 \). In FlexibleSUSY, one obtains at the one-loop level \( m_h = 125.1 \) GeV, the heavier Higgs mass \( m_{h_2} \approx 265.3 \) GeV and a one-loop mixing angle \( \alpha_h \approx -0.4284 \). The results of the comparison are gathered in Table 5.2. As expected, all partial widths are reduced compared to the case of the SM in Table 5.1, and the relative agreement between FlexibleDecay and shDECAY and SARAH/SPheno is similar to the one between FlexibleDecay and HDECAY and SARAH/SPheno in case of the SM. Even with the large value of \( \lambda_S \), which enhances the BSM effects, the comparison is still as good as in the case of the SM implying good control over pure-BSM corrections.

### 5.3. Two-Higgs Doublet Model of Type II

The THDM is an important extension of the SM. Due to the second Higgs doublet its Higgs and Yukawa sectors are rich and, especially in the most general case, involve a large number of free parameters. To be specific we focus here on the THDM of type II, which postulates a discrete symmetry to restrict the Higgs potential and Yukawa couplings.

In this model, the SM-like Higgs state arises from mixing of the two doublets described by two mixing angles \( \alpha_h \) and \( \beta \). Compared to the SM, its tree-level couplings to gauge bosons are reduced by a factor \( \sin(\beta - \alpha_h) \) and the couplings to down-type and up-type fermions are governed by \( (-\sin \alpha_h / \cos \beta) \)

\[ \lambda = \lambda_S = 8.52 \text{ and } \lambda = 0.84 \text{ in Ref. [18] because of the difference in normalization of the Higgs quartic coupling.} \]

| channel       | shDECAY | SARAH/SPheno (DECAY) | SARAH/SPheno (DECAY1L) | FlexibleDecay |
|---------------|---------|----------------------|------------------------|---------------|
| \( h \to bb \) | 1.994   | 1.786                | 1.532                  | 1.868         |
| \( h \to W^+W^- \) | \( 7.063 \cdot 10^{-1} \) | \( 7.486 \cdot 10^{-1} \) | —                   | \( 6.753 \cdot 10^{-1} \) |
| \( h \to \tau\bar{\tau} \) | \( 2.142 \cdot 10^{-1} \) | \( 2.072 \cdot 10^{-1} \) | \( 2.115 \cdot 10^{-1} \) | \( 1.987 \cdot 10^{-1} \) |
| \( h \to c\bar{c} \) | \( 9.762 \cdot 10^{-2} \) | \( 8.423 \cdot 10^{-2} \) | \( 7.789 \cdot 10^{-2} \) | \( 9.230 \cdot 10^{-2} \) |
| \( h \to ZZ \) | \( 8.786 \cdot 10^{-2} \) | \( 7.441 \cdot 10^{-2} \) | —                   | \( 8.595 \cdot 10^{-2} \) |
| \( h \to gg \) | \( 2.636 \cdot 10^{-1} \) | \( 2.763 \cdot 10^{-1} \) | \( 1.412 \cdot 10^{-1} \) | \( 2.749 \cdot 10^{-1} \) |
| \( h \to \gamma\gamma \) | \( 7.830 \cdot 10^{-3} \) | \( 8.914 \cdot 10^{-3} \) | \( 1.322 \cdot 10^{-2} \) | \( 7.349 \cdot 10^{-3} \) |
| \( h \to \gamma Z \) | \( 5.212 \cdot 10^{-3} \) | —                   | \<0  \)             | \( 4.754 \cdot 10^{-3} \) |
| total width   | 3.378   | 3.187                | —                      | 3.208         |

Table 5.2: Comparison of Higgs boson decay widths in the real singlet extended SM as calculated by FlexibleDecay, shDECAY and SARAH/SPheno. All widths are given in MeV.
and \((\cos \alpha_h/\sin \beta)\), respectively. In the following we consider the parameter point given by
\[
\begin{align*}
\lambda_1 &= 0.05, & \lambda_2 &= 0.13155, & \lambda_3 &= 0.13, & \lambda_4 &= 0.14, & \lambda_5 &= 0.2, \\
\lambda_6 &= 0, & \lambda_7 &= 0, & m_{12}^2 &= 2 \cdot 10^4 \text{GeV}^2, & \tan \beta &= 13
\end{align*}
\] (62)
in the potential notation of\(^{17}\)
\[
V = m_1^2 \Phi_1^2 + m_2^2 \Phi_2^2 + \lambda_1 \Phi_1^4 + \lambda_2 \Phi_2^4 + \lambda_3 |\Phi_2|^2 |\Phi_1|^2 + \lambda_4 |\Phi_1 \Phi_2|^2 \\
+ \left( -m_{12}^2 \Phi_1 \Phi_2 + \frac{\lambda_5}{2} (\Phi_2 \Phi_1)^2 + \text{h.c.} \right).
\] (63)

In the following we compare the calculations of FlexibleSUSY, SARAH/SPheno and 2HDECAY (with RENSCHEM = 16) in the \(\overline{\text{MS}}\) scheme. The parameter point (62) corresponds to the on-shell quantities
\[
\begin{align*}
m_h &\approx 125.1 \text{GeV}, & m_{h^2} &\approx 511.4 \text{GeV}, & m_A &\approx 499.1 \text{GeV}, & m_{H^+} &\approx 501.3 \text{GeV}, \\
\alpha_h &\approx -7.307 \cdot 10^{-2}.
\end{align*}
\] (64)

Table 5.3 shows the Higgs decay channels in the THDM-II as calculated by different programs. As 2HDECAY also optionally includes EW corrections, which are not present in other codes, for the sake of comparison we report results only with QCD ones. The behavior of FlexibleDecay, 2HDECAY and SARAH/SPheno is similar to the previous cases, and there is very good agreement between all these programs with the expected, slightly worse behavior of the DECAY1L block output of SARAH/SPheno. The THDM-specific program 2HDECAY shows similarly good agreement, but there is an \(\sim 5\%\) upward shift in its \(h \to bb\) prediction compared to the other programs.

| channel | 2HDECAY | SARAH/SPheno (DECAY) | SARAH/SPheno (DECAY1L) | FlexibleDecay |
|---------|---------|-----------------------|------------------------|---------------|
| \(h \to bb\) | 2.237 | 2.110 | 1.759 | 2.121 |
| \(h \to W^+W^-\) | 8.889 \cdot 10^{-1} | 8.321 \cdot 10^{-1} | --- | 8.504 \cdot 10^{-1} |
| \(h \to \tau\bar{\tau}\) | 2.406 \cdot 10^{-1} | 2.445 \cdot 10^{-1} | 2.483 \cdot 10^{-1} | 2.256 \cdot 10^{-1} |
| \(h \to c\bar{c}\) | 1.210 \cdot 10^{-1} | 1.014 \cdot 10^{-1} | 8.894 \cdot 10^{-2} | 1.164 \cdot 10^{-1} |
| \(h \to ZZ\) | 1.114 \cdot 10^{-1} | 8.124 \cdot 10^{-2} | --- | 1.084 \cdot 10^{-1} |
| \(h \to gg\) | 3.262 \cdot 10^{-1} | 3.339 \cdot 10^{-1} | 1.785 \cdot 10^{-1} | 3.472 \cdot 10^{-1} |
| \(h \to b\bar{b}\) | 1.005 \cdot 10^{-2} | 1.049 \cdot 10^{-2} | 1.572 \cdot 10^{-2} | 9.130 \cdot 10^{-3} |
| \(h \to c\bar{t}\) | 6.814 \cdot 10^{-3} | --- | < 0 | 5.961 \cdot 10^{-3} |
| total width | 3.944 | 3.715 | --- | 3.786 |

Table 5.3: Comparison of Higgs boson decay widths in the type II THDM as calculated by 2HDECAY, SARAH/SPheno and FlexibleDecay. All widths are given in MeV.

5.4. Constrained Minimal Supersymmetric Standard Model

The MSSM is a very popular extension of the SM. The CMSSM imposes simple universality relations on the fundamental MSSM input parameters. Since this is implemented in many public programs, it is a good choice that allows a well-defined comparison. The tree-level Higgs sector of the MSSM is that of the type II THDM, with additional constraints from supersymmetry. At the loop level, non-type II-like contributions arise via quantum corrections.

We follow the standard notation given in, e.g., Ref. [103]. For the following comparison we use the parameter point \(m_0 = 1.4 \text{ TeV}, m_{1/2} = 3.5 \text{ TeV}, \tan \beta = 10, \text{sign}(\mu) = +1, A_0 = -1.4 \text{ TeV}\).

\(^{17}\)The default type II THDM models in FlexibleSUSY and SARAH use different normalizations of \(\lambda_{1,2}\) and sign convention for \(m_{12}^2\) than 2HDECAY: \((\lambda_{1,2})_{\text{FlexibleSUSY,SARAH}} = \frac{1}{2} (\lambda_{1,2})_{2HDECAY}, - (m_{12}^2)_{\text{SARAH}} = (m_{12}^2)_{\text{FlexibleSUSY,2HDECAY}}\)
Table 5.4 shows a comparison of FlexibleDecay and a selection of well-known public programs that work in the \( \overline{\text{DR}} \) scheme (for the status of further public programs we refer to the literature [13, 104]). Note that SUSY-HIT 1.5a uses HDECAY 3.4 internally and implements QCD corrections up to the two-loop level to \( h \to gg \).

The difference in the \( h \to c\bar{c} \) comes from the difference in the running charm mass. SUSY-HIT accepts as input a pole charm mass, which we set to 1.67 GeV and which is internally converted by SUSY-HIT to \( m_c(m_h) = 0.69 \) GeV. Meanwhile FlexibleSUSY takes as input \( m_c^{\text{MS}}(m_c^{\overline{\text{MS}}}) \), which we set to 1.27 GeV which then corresponds to \( m_c(m_h) = 0.62 \) GeV.

5.5. Minimal R-symmetric Supersymmetric Standard Model

The MRSSM is a non-minimal supersymmetric model with an unbroken U(1) R-symmetry, which has received considerable attention in recent years. Its Higgs sector is particularly rich, containing two Higgs doublets, one complex singlet and one complex triplet (which arise as double superpartners of the SU(2)×U(1) gauge bosons). There are no dedicated MRSSM public programs, but both FlexibleSUSY/FlexibleDecay and SARAH/SPheno are distributed with model files that generate MRSSM-specific program for determining its mass spectrum and decay widths.

For the following numerical comparison, we choose the parameter point BMP2 from Ref. [105]. The comparison between the results of the programs is shown in Table 5.5 for the SM-like Higgs boson \( h \) and in Table 5.6 for the next heavier state \( H_2 \). Their masses as computed by FlexibleSUSY are 124.5 GeV and 921.5 GeV, respectively. Overall we see similar agreement between FlexibleDecay and SARAH/SPheno (DECAY block) and a similar behavior from SARAH/SPheno (DECAY1L block) as in the previous cases in the case of light Higgs.

In the case of heavy Higgs decays we observe much worse agreement. The disagreement of some of the important channels can be tracked down

- disagreement in \( H_2 \to gg \) comes from SARAH including in the DECAY column higher order corrections that are not appropriate, since they were derived in the limit \( m_t \gg m_H \), whereas in this scenario we actually have \( m_t \ll m_{H_2} \),

- disagreement between FlexibleDecay and DECAY in \( H_2 \to \gamma\gamma \) comes from the NLO QCD correction. This correction in SARAH is negative and around < 5% while above the 2 top threshold it should be positive and \( \gtrsim 20\% \) [81, 106],

- SARAH result for \( H_2 \to h\gamma \) cannot be correct as this process is forbidden by CP-symmetry,

\[ \text{Table 5.4: Comparison of Higgs boson decay widths in the MSSM as calculated by FlexibleDecay, HDECAY via SUSY-HIT, SOFTSUSY and SARAH/SPheno (for CMSSM SPS1a slope with } m_0 = 1.4 \text{ TeV). All widths in MeV.} \]

\[
\begin{array}{|l|lllll|}
\hline
\text{channel} & \text{SUSY-HIT} & \text{SOFTSUSY} & \text{SARAH/SPheno (DECAY)} & \text{SARAH/SPheno (DECAY1L)} & \text{FlexibleDecay} \\
\hline
h \to bb & 2.662 & 3.843 & 2.403 & 1.541 & 2.348 \\
h \to W^+W^- & 8.342 \cdot 10^{-1} & 6.751 \cdot 10^{-1} & 5.887 \cdot 10^{-1} & - & 8.141 \cdot 10^{-1} \\
h \to \tau\tau & 2.595 \cdot 10^{-1} & 2.726 \cdot 10^{-1} & 2.778 \cdot 10^{-1} & 2.355 \cdot 10^{-1} & 2.499 \cdot 10^{-1} \\
h \to c\bar{c} & 1.183 \cdot 10^{-1} & 2.235 \cdot 10^{-1} & 1.031 \cdot 10^{-1} & 1.073 \cdot 10^{-1} & 1.160 \cdot 10^{-1} \\
h \to ZZ & 1.060 \cdot 10^{-1} & 7.606 \cdot 10^{-2} & 5.882 \cdot 10^{-2} & - & 1.032 \cdot 10^{-1} \\
h \to gg & 2.731 \cdot 10^{-1} & 2.760 \cdot 10^{-1} & 2.993 \cdot 10^{-1} & 9.555 \cdot 10^{-2} & 3.434 \cdot 10^{-1} \\
 h \to \gamma\gamma & 9.439 \cdot 10^{-3} & 1.052 \cdot 10^{-2} & 8.580 \cdot 10^{-3} & 1.024 \cdot 10^{-2} & 9.940 \cdot 10^{-3} \\
h \to Z\gamma & 6.316 \cdot 10^{-3} & 6.779 \cdot 10^{-3} & 4.303 \cdot 10^{-1} & 6.098 \cdot 10^{-3} \\
\hline
\text{total width} & 4.272 & 5.386 & 3.741 & - & 3.993 \\
\hline
\end{array}
\]

\[ ^{18} \text{The corresponding masses in SARAH/SPheno are 123.8 GeV and 937.4 GeV, respectively.} \]
result for $H_2 \to A_2 \gamma$ (where $A_2$ is the lightest, physical pseudoscalar Higgs boson) should be 0 as the amplitude for physical (i.e. transversal) photons that also fulfils the Ward identity is identically 0.

5.6. Squark decays in the Minimal Supersymmetric Standard Model

The FlexibleDecay module is also capable for automatically generating $1 \to 2$ decays for BSM particles. As an example, in Tab. 5.7 and 5.8 we present results for decays of the first and second sbottom squarks in the CMSSM parameter point discussed in Section 5.4. For a uniform comparison with other codes, we disable higher-order corrections in SDECAY [107].

For this parameter point, the pole masses as calculated by FlexibleDecay are as follows: $m_{\tilde{b}_2} = 6.074$ TeV, $m_{\tilde{b}_1} = 5.874$ TeV, $m_{\tilde{t}_1} = 4.937$ TeV, $m_{\tilde{t}_2} = 5.882$ TeV, $m_{\tilde{\chi}_i^0} = \{1.571, 2.889, 3.715, 3.719\}$ TeV and $m_{\tilde{\chi}_i^\pm} = \{2.889, 3.720\}$ TeV.

Overall one sees a rather good agreement between most of the codes. Some of the bigger differences in Table 5.8, e.g. for $\tilde{b}_2 \to \tilde{t}_2 W^-$, originate from differences in sub-leading, off-diagonal entries in mass matrices. The $\tilde{b}_2$ is mostly a right handed squark with only a percent level admixture of a left-handed component. Therefore all of its weak decays suffer from a large uncertainty as an exact value of this mixing will vary substantially from code to code.

6. Limitations and future extensions

Although FlexibleDecay is designed with generality in mind, the current version still has some limitations on the decays or models we support. These are:

- Decays of fermions and vector bosons are not supported.
- Decays of a color octet into color octets are not supported. Other combinations, like for example $8 \to 3 \otimes 3$ or $3 \to 8 \otimes 3$ are supported.
- Decays containing vertices which cannot be decomposed into a single product of Lorentz and color structure, e.g. the quartic-gluon vertex, are not supported.
- In general only $1 \to 2$ decays are supported. The exception are the decays of scalar and pseudoscalar Higgs bosons to $ZZ$ and $W^+W^-$ pairs, where we include single and double off-shell decays, assuming SM decays of $W^\pm$ and $Z$ bosons.
- Models with forced fermion flavor-conservation (where leptons and quarks are not combined into their respective multiplets — NoFV models in FlexibleSUSY nomenclature) are not supported.
### Table 5.6: Comparison of heavy Higgs boson decay widths in the MRSSM as calculated by FlexibleDecay and SARAH/SPheno. All widths are given in GeV.

| channel        | SARAH/SPheno (DECAY) | SARAH/SPheno (DECAYIL) | FlexibleDecay |
|----------------|-----------------------|-------------------------|---------------|
| \( H_2 \to bb \) | 1.274                 | 7.950 \cdot 10^{-1}    | 1.253         |
| \( H_2 \to t\bar{t} \) | 4.163 \cdot 10^{-1} | 3.714 \cdot 10^{-1}    | 4.178 \cdot 10^{-1} |
| \( H_2 \to \tau^+\tau^- \) | 1.974 \cdot 10^{-1} | 1.770 \cdot 10^{-1}    | 1.900 \cdot 10^{-1} |
| \( H_2 \to \chi^0_2\chi^0_1 \) | 1.560 \cdot 10^{-1} | 2.342 \cdot 10^{-4}    | 1.557 \cdot 10^{-1} |
| \( H_2 \to \chi^0_1\chi^0_1 \) | 1.560 \cdot 10^{-1} | 1.692 \cdot 10^{-1}    | 1.557 \cdot 10^{-1} |
| \( H_2 \to hh \) | 3.743 \cdot 10^{-3} | 5.624 \cdot 10^{-3}    | 3.420 \cdot 10^{-3} |
| \( H_2 \to \mu^+\mu^- \) | 6.829 \cdot 10^{-4} | 6.079 \cdot 10^{-4}    | 6.719 \cdot 10^{-4} |
| \( H_2 \to W^+W^- \) | 1.454 \cdot 10^{-3} | —                       | 5.782 \cdot 10^{-4} |
| \( H_2 \to s\bar{s} \) | 4.746 \cdot 10^{-4} | 5.744 \cdot 10^{-3}    | 5.756 \cdot 10^{-4} |
| \( H_2 \to ZZ \) | 7.229 \cdot 10^{-4} | —                       | 3.014 \cdot 10^{-4} |
| \( H_2 \to \chi^+_1\chi^-_1 \) | 3.390 \cdot 10^{-4} | 3.209 \cdot 10^{-4}    | 2.592 \cdot 10^{-4} |
| \( H_2 \to \rho_1\rho_1 \) | 2.747 \cdot 10^{-4} | 2.343 \cdot 10^{-4}    | 1.746 \cdot 10^{-4} |
| \( H_2 \to \chi^0_1\rho_1 \) | 2.480 \cdot 10^{-7} | 1.149 \cdot 10^{-6}    | 4.793 \cdot 10^{-5} |
| \( H_2 \to \chi^0_2\rho_2 \) | 1.678 \cdot 10^{-8} | 3.437 \cdot 10^{-7}    | 2.880 \cdot 10^{-5} |
| \( H_2 \to gg \) | 4.339 \cdot 10^{-4} | 2.499 \cdot 10^{-4}    | 2.599 \cdot 10^{-4} |
| \( H_2 \to \gamma\gamma \) | 1.520 \cdot 10^{-6} | 1.847 \cdot 10^{-6}    | 1.845 \cdot 10^{-6} |
| \( H_2 \to \gamma Z \) | —                     | 7.762 \cdot 10^{-7}    | 2.735 \cdot 10^{-7} |
| \( H_2 \to h\gamma \) | —                     | 3.759 \cdot 10^{-4}    | 0             |
| \( H_2 \to A_2\gamma \) | —                     | 6.157 \cdot 10^{-10}   | 0             |
| total width    | 2.208                 | —                       | 2.179         |

- It is assumed that the BSM model contains the SM as a subset. For example, BSM models where a SM-like weak mixing angle cannot be defined are not supported.

Future extensions removing these limitations and widening the scope of FlexibleDecay are planned. In particular adding decays for fermions and vectors is a priority. In the near future we will also add an interface to HiggsBounds and HiggsSignals to make it easier to use the output of FlexibleDecay in those programs. The program is actively developed and carefully tested for correctness with nightly units tests, minimising the possibility of bugs being introduced during development. Users who find them new features or nonetheless still encounter bugs can report these to the FlexibleSUSY developers at [https://github.com/FlexibleSUSY/FlexibleSUSY/issues](https://github.com/FlexibleSUSY/FlexibleSUSY/issues) or by contacting one of the developers directly.

### 7. Conclusions

We have presented FlexibleDecay, a computer program for the calculation of scalar decays in a broad class of BSM models with a special emphasis on precise predictions of Higgs boson decays. FlexibleDecay is fully integrated into the FlexibleSUSY framework. It enables decay calculations for existing FlexibleSUSY models with minimal modifications to the setup, and is switched on by default for many models with new installations of FlexibleSUSY.

To achieve high precision of scalar and pseudoscalar Higgs boson decays for the following final states, we have included:

- \( q\bar{q} \): full mass dependent, one-loop QCD corrections; up to four-loop corrections in massless QCD; one-loop corrections in massless QED; massless, two-loop, mixed QCD–QED correction; leading order top-mass corrections for decays into \( q \neq t \),

- \( l^+l^- \): one-loop QED corrections in the massless limit,
Table 5.7: Comparison of $b_1$ decay widths in the MSSM as calculated by SUSY-HIT, SOFTSUSY and SARAH/SPheno and FlexibleDecay. All widths are given in GeV.

| channel      | SUSY-HIT | SOFTSUSY | SARAH/SPheno (DECAY) | FlexibleDecay |
|--------------|----------|----------|-----------------------|---------------|
| $b_1 \rightarrow \tilde{\chi}_1^1 t$ | 26.931   | 26.569   | 27.061                | 26.380        |
| $b_1 \rightarrow \tilde{\chi}_2^0 t$ | 26.690   | 33.160   | 25.931                | 26.371        |
| $b_1 \rightarrow t_1 W^-$           | 23.434   | 23.906   | 23.903                | 23.635        |
| $b_1 \rightarrow \tilde{\chi}_1^0 b$ | 13.389   | 13.318   | 13.419                | 13.239        |
| $b_1 \rightarrow \tilde{\chi}_1^0 b$ | 7.617 · 10^{-1} | 7.635 · 10^{-1} | 6.807 · 10^{-1} | 7.650 · 10^{-1} |
| $b_1 \rightarrow \tilde{\chi}_3^0 b$ | 3.420 · 10^{-1} | 4.308 · 10^{-1} | 3.927 · 10^{-1} | 3.575 · 10^{-1} |
| $b_1 \rightarrow \tilde{\chi}_3^0 b$ | 3.078 · 10^{-1} | 4.010 · 10^{-1} | 3.404 · 10^{-1} | 3.311 · 10^{-1} |
| total width  | 91.856   | 98.548   | 91.728                | 91.079        |

Table 5.8: Comparison of $b_2$ decay widths in the MSSM as calculated by SUSY-HIT, SOFTSUSY and SARAH/SPheno and FlexibleDecay. All widths are given in GeV.

| channel      | SUSY-HIT | SOFTSUSY | SARAH/SPheno (DECAY) | FlexibleDecay |
|--------------|----------|----------|-----------------------|---------------|
| $b_2 \rightarrow \tilde{\chi}_1^0 b$ | 3.126   | 3.115   | 3.152                | 3.119        |
| $b_2 \rightarrow \tilde{\chi}_2^0 t$ | 7.730 · 10^{-1} | 1.046  | 8.021 · 10^{-1} | 8.702 · 10^{-1} |
| $b_2 \rightarrow \tilde{\chi}_3^0 b$ | 3.560 · 10^{-1} | 4.657 · 10^{-1} | 3.821 · 10^{-1} | 3.849 · 10^{-1} |
| $b_2 \rightarrow t_1 W^-$ | 3.487 · 10^{-1} | 4.550 · 10^{-1} | 3.719 · 10^{-1} | 3.747 · 10^{-1} |
| $b_2 \rightarrow t_2 W^-$ | 6.164 · 10^{-2} | 9.891 · 10^{-2} | 7.273 · 10^{-2} | 1.099 · 10^{-1} |
| $b_2 \rightarrow h_1 t$ | 3.790 · 10^{-2} | 5.942 · 10^{-2} | 3.489 · 10^{-2} | 6.608 · 10^{-2} |
| $b_2 \rightarrow h_1 Z$ | 1.671 · 10^{-2} | 2.906 · 10^{-2} | 1.916 · 10^{-2} | 3.939 · 10^{-2} |
| $b_2 \rightarrow h_1 h_1$ | 2.104 · 10^{-2} | 3.297 · 10^{-2} | 1.837 · 10^{-2} | 3.715 · 10^{-2} |
| $b_2 \rightarrow h_2 h_1$ | 2.553 · 10^{-2} | 3.319 · 10^{-2} | 2.009 · 10^{-2} | 2.633 · 10^{-2} |
| $b_2 \rightarrow h_3 h_3$ | 8.435 · 10^{-3} | 1.461 · 10^{-2} | 9.808 · 10^{-3} | 1.972 · 10^{-2} |
| total width  | 4.775   | 5.349   | 4.883                | 5.048        |

- $VV$: single and double off-shell decays,
- $\gamma\gamma$: two-loop QCD corrections to decay through fermion and scalar loops; use of $\alpha$ in the Thomson limit to minimize impact of two-loop QED/EW corrections,
- $H \rightarrow gg$: up to four-loop QCD corrections through top-loop, including $m_t$ suppressed terms,
- $A \rightarrow gg$: up to three-loop QCD corrections in the leading $m_t$ approximation,
- $\gamma Z$: two-loop QCD corrections to quark loop in the heavy quark limit; $\alpha$ treatment as in the $\gamma\gamma$ case.

The corrections in CP-violating models are implemented by combining scalar and pseudoscalar limits. No weak corrections are applied in the current version.

We have carried out detailed numerical tests in many models and comparisons with other existing codes wherever possible, demonstrating the accuracy, generality and reliability of FlexibleDecay. In the comparisons we found particularly good agreement with HDECAY in models which HDECAY can handle. We also found mostly good agreement with the SARAH/SPheno DECAY block results. As various programs organize their calculations in different ways discrepancies between all of them are expected. Their sources are understood and explained throughout the comparison section.

A noteworthy property of our approach is the use of a decoupling variant of the $\overline{\text{MS}}/\overline{\text{DR}}$ renormalization scheme: pure BSM parameters are $\overline{\text{MS}}/\overline{\text{DR}}$ renormalized, but SM-like parameters are defined
in a decoupling scheme. The separation between the two kinds of parameters is automatically derived from information provided in the model files. This has two advantages. On the one hand it leads to a correct decoupling limit, i.e. the BSM results approach the SM ones for heavy BSM masses within 10%. On the other hand, it allows seamless integration into the \( \overline{\text{MS}}/\text{DR} \) framework of FlexibleSUSY with the possibility for connection to studies of, e.g., evolution under the renormalization group equations or the effective scalar potential.

As illustrated by Figure 1 the decoupling scheme leads to reliable predictions not only at high BSM scales, where the SM prediction is recovered. Importantly, it also enables accurate predictions of characteristic BSM signatures, allowing tests of BSM scenarios against data. In this case the prediction of the decoupling scheme is almost indistinguishable from a standard DR calculation. This allows for a smooth interpolation between regions of heavy and light BSM physics.

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Appendix A. Configuration of FlexibleDecay

Appendix A.1. FlexibleDecay build options

The generation of the decays in FlexibleSUSY is controlled by the FSCalculateDecays variable in the FlexibleSUSY model file. For a given model <model> this file is located at model_files/<model>/FlexibleSUSY.m.in. The model file is read when calling createmodel. The FSCalculateDecays variable can take on 2 values: True (calculate decays, default) or False. The particles for which the decays shall be calculated must be given in the FSDecayParticles variables in the model file. Allowed values are: Automatic (default) or a list of particles. Currently, Automatic expands to a list containing the neutral scalar, pseudoscalar and charged Higgs bosons of the model. In a future version the meaning of Automatic might change to include also other particles as well.

As an example, one might add decays to the FlexibleSUSY version of the THDMII by adding the following lines to the model file model_files/THDMII/FlexibleSUSY.m.in:

FSCalculateDecays = True;
FSDecayParticles = Automatic; (* expands to {hh, Ah, Hm} *)

In the example the value of FSDecayParticles expands to the list {hh, Ah, Hm}, where hh, Ah and Hm are the names of the neutral scalar, pseudoscalar and charged Higgs bosons in the model.

To compute decay width, FlexibleSUSY must be configured with at least one of the dedicated libraries for evaluation of one-loop integrals. Up to FlexibleSUSY 2.4.2 only to LoopTools was supported. In FlexibleSUSY 2.5.0 support for Collier was added. FlexibleSUSY can be configured with both libraries at the same time as

./configure --with-loop-libraries=collier,looptools [...]

where the ellipsis stand for potential further configuration options. If needed, the location of LoopTools and Collier can be specified through the following parameters:

```
--with-looptools-libdir= Path to search for LoopTools libraries
--with-looptools-incdir= Path to search for LoopTools headers
--with-collier-libdir= Path to search for COLLIER libraries
--with-collier-incdir= Path to search for COLLIER modules
```

Note that for usage with FlexibleSUSY, Collier has to compiled as a static library, in position independent mode (see Ref. [108] for detailed instructions). See ./configure -h for all available configuration options.

Appendix A.2. FlexibleDecay runtime options

To calculate the decays for a given parameter point with a spectrum generator generated by FlexibleSUSY, the following flags must be set (see also Table A.1):

- The calculation of the SM and BSM particle pole masses has to be enabled (FlexibleSUSY[3] = 1 and FlexibleSUSY[23] = 1).
- LoopTools or Collier has to be selected as loop library (FlexibleSUSY[31] = 1 or 2).
- The calculation of the decays has to be enabled (FlexibleSUSY[32] = 1).

The behaviour of decay module is controlled by flags in the FlexibleDecay block as documented in Table A.2.
## Table A.1: Entries for the FlexibleSUSY SLHA input block and corresponding Mathematica symbols to specify the runtime configuration options of FlexibleSUSY which are relevant for FlexibleDecay. Flags 3 and 23 are automatically set to 1 if decays are enabled (see flag 0 in Table A.2). Flag 31 must be set to 1 or 2 to calculate the decays.

| Index | Mathematica symbol                        | Default | Description                                      |
|-------|-------------------------------------------|---------|--------------------------------------------------|
| 3     | calculateStandardModelMasses              | 0       | calculate SM pole masses (0 = no, 1 = yes)       |
| 23    | calculateBSMMasses                        | 0       | calculate BSM pole masses (0 = no, 1 = yes)      |
| 31    | loopLibrary                               | 0       | loop library (0 = SOFTSUSY, 1 = Collier, 2 = LoopTools, 3 = FFLite) |

## Table A.2: Entries for the FlexibleDecay SLHA input block and corresponding Mathematica symbols to specify the runtime configuration options for FlexibleDecay.

| Index | Mathematica symbol                        | Default | Description                                      |
|-------|-------------------------------------------|---------|--------------------------------------------------|
| 0     | –                                         | 1       | calculate decays (0 = no, 1 = yes)               |
| 1     | minBRtoPrint                              | $10^{-5}$| minimum BR to print                               |
| 2     | maxHigherOrderCorrections                 | 4       | include higher order corrections in decays (0 = LO, 1 = NLO, 2 = NNLO, 3 = NNNLO, 4 = N^4LO) |
| 3     | alphaThomson                              | 1       | use $\hat{\alpha}(m)$ or Thomson $\alpha(0)$ in decays to $\gamma\gamma$ and $\gamma Z$ (0 = $\hat{\alpha}(m)$, 1 = $\alpha(0)$) |
| 4     | offShellVV                                | 2       | decays into off-shell $VV$ pair (0 = no off-shell decays, 1 = single off-shell decays if $m_V < m_H < 2m_H$, double off-shell if $m_H < m_V$, 2 = double off-shell decays if $m_H < 2m_V$) |

## Appendix B. Matrix elements

The matrix elements required in the calculation of the two-body partial widths are parametrized in terms of a set of standard matrix elements and associated form-factors [109, 110]. For a particular two-body decay $X \rightarrow AB$, external leg wavefunctions and polarizations are factored out to write the matrix element $A_{X \rightarrow AB}$ in the form

$$A_{X \rightarrow AB} = A^{IJK} x_I x_J x_K,$$  

where the indices $I$, $J$, and $K$ collectively denote the Lorentz and Dirac indices for the initial and final states, and $x_I$, $x_J$, and $x_K$ the appropriate external leg factors. In turn, the quantity $A^{IJK}$ is expressed in terms of a set of Lorentz scalar form-factors and covariant operators,

$$A^{IJK} = \sum_i F_i A_i^{IJK},$$  

where the operators $A_i^{IJK}$ are chosen to be the same for all processes of a given type (i.e., $S \rightarrow FF$, $S \rightarrow SV$, and so on). The standard matrix elements

$$A_i = A_i^{IJK} x_I x_J x_K$$

then depend solely on the kinematic variables and polarization factors and may be calculated independently of the particular model at hand. The matrix element for a particular process may then be written generically in terms of the model-dependent form-factors as

$$A_{X \rightarrow AB} = \sum_i F_i A_i,$$  

33
Similarly, the unpolarized squared matrix element reads

\[ \sum |A_{X \rightarrow AB}|^2 = \sum_{i,j} F_i F_j \sum A_i A_j^*, \quad (B.5) \]

where \(\sum\) denotes the summation over final helicities and polarizations and averaging over initial helicities and polarizations; note that, in general, additional summations over gauge indices not shown above are also carried out to derive appropriate multiplicity factors. To evaluate the squared amplitude for a process in a given model, FlexibleSUSY computes the contributions in that model to the form-factors \(F_i\), which are then substituted into the appropriate generic expression for the squared amplitude. In the following, we summarize the conventions used by FlexibleSUSY for the standard matrix elements \(A_i\) for each class of two-body decays and the resulting formulas for the squared amplitudes.

**Appendix B.1. \(S \rightarrow SS\)**

The simplest case is the decay of a scalar to a pair of scalars, for which the Lorentz structure of the matrix element is trivial. A single form-factor \(F\) is used to parametrize the matrix element,

\[ A_{S \rightarrow SS} = F, \quad (B.6) \]

and the unpolarized squared amplitude is simply

\[ \sum |A_{S \rightarrow SS}|^2 = |F|^2. \quad (B.7) \]

**Appendix B.2. \(S \rightarrow FF\)**

The decay of a scalar \(S\) to a pair of fermions \(F_1, F_2\) is described by the form-factors \(F_L\) and \(F_R\), with the general matrix element taking the form

\[ A_{S \rightarrow FF} = F_L \bar{u}(p_{F_1}, s_{F_1}) P_L v(p_{F_2}, s_{F_2}) + F_R \bar{u}(p_{F_1}, s_{F_1}) P_R v(p_{F_2}, s_{F_2}), \quad (B.8) \]

where \(P_L, P_R\) are standard left- and right-handed projection operators, and \(p_{F_1}, s_{F_1}\) denote the 4-momentum and spin of the final state fermion \(F_i\), respectively. The corresponding unpolarized squared amplitude is

\[ \sum |A_{S \rightarrow FF}|^2 = (m_S^2 - m_{F_1}^2 - m_{F_2}^2) \left( |F_L|^2 + |F_R|^2 \right) - 2 m_{F_1} m_{F_2} (F_L F_R + F_R^* F_L^*), \quad (B.9) \]

where \(m_S, m_{F_1}\), and \(m_{F_2}\) are the masses of the decaying scalar and the final state fermions.

**Appendix B.3. \(S \rightarrow SV\)**

The matrix element for the decay of a scalar \(S_1\) into a scalar \(S_2\) and a massive vector \(V\) is parametrized using a single form-factor \(F\) according to

\[ A_{S_1 \rightarrow S_2 V} = F \epsilon_{\mu}^{\gamma \nu} (p_V) \left( p_{S_1}^\mu + p_{S_2}^\nu \right), \quad (B.10) \]

where \(\epsilon_{\mu}^{\gamma \nu}(p)\) is a polarization vector for the final state vector boson, and \(p_{S_1}, p_{S_2},\) and \(p_V\) are the 4-momenta of the initial and final states. The squared amplitude reads

\[ \sum |A_{S_1 \rightarrow S_2 V}|^2 = \frac{|F|^2}{m_V^4} \left[ m_{S_1}^4 + (m_V^2 - m_{S_2}^2)^2 - 2 m_{S_2}^2 \left( m_{S_2}^2 + m_V^2 \right) \right]. \quad (B.11) \]

with \(m_{S_1}\) and \(m_{S_2}\) the masses of the initial and final state scalars, respectively, and \(m_V\) the mass of the vector.

For massless vector, like the photon, the amplitude that fulfils the Ward identity and where the external photon is physical (i.e. transversal) is identically \(0\).\(^{19}\)

\(^{19}\)Eq. B.36 of Ref. [110] gives a non-zero, and negative, result for the squared amplitude. This can only be valid if one considers the \(1 \rightarrow 2\) process as a part of a larger amplitude, which is not the case we consider here.
Appendix B.4. \( S \rightarrow VV \)

To also take into account the possibility of loop-induced decays, the decomposition of the matrix element for the decay of a scalar into two vectors \( V_1 \) and \( V_2 \) is chosen to be

\[
\mathcal{A}_{S \rightarrow V_1 V_2} = \varepsilon_\mu^{V_1} (p_{V_1}) \varepsilon_\nu^{V_2} (p_{V_2}) \left( F_\eta \eta^{\mu\nu} + F_{11} p_1^\mu p_1^\nu + F_{12} p_2^\mu p_2^\nu + F_{21} p_1^\mu p_2^\nu \right) + F_{22} p_1^\mu p_2^\nu + F_\epsilon^{\mu\nu\alpha\beta} p_{V_1\alpha} p_{V_2\beta},
\]

where \( p_{V_1} \) and \( p_{V_2} \) are the 4-momenta of the final state vectors, and \( \varepsilon_\mu^{V_1} (p_{V_1}) \) the corresponding polarizations. There are three cases for the resulting squared amplitude, depending on whether both vector bosons are massless, only one is, or both have non-zero masses. If both of the vector boson masses \( m_{V_1} = m_{V_2} = 0 \), then the generic unpolarized squared amplitude is given by

\[
\sum |\mathcal{A}_{S \rightarrow VV}|^2 = 4 |F_\eta|^2 + m_S^2 \Re(F_\eta F_\eta^*) + \frac{1}{2} m_S^4 |F_\epsilon|^2, \quad (m_{V_1} = m_{V_2} = 0).
\]  

If \( m_{V_1} = 0 \) and \( m_{V_2} \neq 0 \), then the squared amplitude is given by

\[
\sum |\mathcal{A}_{S \rightarrow VV}|^2 = 3 |F_\eta|^2 + \frac{1}{4} (m_S^2 - m_{V_2}^2)^2 (2 |F_\epsilon|^2 - |F_{21}|^2), \quad (m_{V_1} = 0, m_{V_2} \neq 0).
\]  

If \( m_{V_1} \neq 0 \) and \( m_{V_2} \neq 0 \) instead, then the squared amplitude takes the same form as above with the replacements \( m_{V_1} \rightarrow m_{V_2} \) and \( F_{11} \rightarrow F_{22} \). Finally, if both vector bosons are massive, then the generic squared amplitude is given by

\[
\sum |\mathcal{A}_{S \rightarrow VV}|^2 = \frac{|F_\eta|^2}{4 m_{V_1}^2 m_{V_2}^2} \left[ m_{V_1}^4 + m_{V_1}^4 + m_{V_2}^4 + 10 m_{V_1}^2 m_{V_2}^2 - 2 m_S^2 \left( m_{V_1}^2 + m_{V_2}^2 \right) \right]
+ \frac{|F_{21}|^2}{16 m_{V_1} m_{V_2}^2} \left[ m_{V_1}^4 + \left( m_{V_1}^2 - m_{V_2}^2 \right)^2 - 2 m_S^2 \left( m_{V_1}^2 + m_{V_2}^2 \right) \right]
+ \frac{|F_\epsilon|^2}{2} \left[ (m_S^2 - m_{V_1}^2 - m_{V_2}^2)^2 - 4 m_{V_1}^2 m_{V_2}^2 \right]
+ \frac{1}{8 m_{V_1}^2 m_{V_2}^2} \left( F_\eta F_{21}^* + F_\eta^* F_{21} \right) \left[ m_S^4 - 3 m_{V_1}^4 \left( m_{V_1}^2 + m_{V_2}^2 \right) \right]
- \left( m_{V_1}^2 - m_{V_2}^2 \right)^2 \left( m_{V_1}^2 + m_{V_2}^2 \right) + m_S^4 \left( 3 m_{V_1}^2 + 2 m_{V_1}^2 m_{V_2}^2 + 3 m_{V_2}^4 \right), \quad (m_{V_1}, m_{V_2} > 0).
\]  

The color algebra is performed using a custom version of the \texttt{ColorMath} package [111] distributed together with \texttt{FlexibleSUSY}.

Appendix C. SPheno setup

In the comparison to \texttt{SPheno}, we can directly use the same \texttt{MS/DR} Lagrangian parameters from \texttt{FlexibleSUSY} as inputs, since the two programs have a very similar setup. For every model we use the default version of the model file distributed with \texttt{SPheno} and \texttt{FlexibleSUSY}. There are small differences between the \texttt{FlexibleSUSY} and \texttt{SPheno} setups, which we discuss on a model-by-model basis. The generic technical setup for \texttt{SPheno} and \texttt{FlexibleSUSY}/\texttt{FlexibleDecay} is chosen as similarly as possible, to focus on differences in the actual decay computations:

- We switch on the running to the scale of a decaying particle: flag 14 in block \texttt{SPhenoInput} is set to 1 (default is 0); in \texttt{FlexibleDecay} the \texttt{MS/DR} parameters are always run to the scale of the decaying particle.

\footnote{Note that in \texttt{SPheno} this running is automatically turned off if the mass of the decaying particle is above the renormalization scale of the model (e.g. above 1 TeV in SUSY models in case SPA convention [112] is used).}
• The mass of the decaying Higgs boson is determined at the one-loop level and without the supersymmetry-specific effective field theory methods of Refs. [26, 27, 40, 113] (flags 66 and 67 in block SPhenoInput are set to 0).

• Tree-level running masses are used in internal lines in the computation of loop-induced decays (this is always the case in the DECAY block in SPheno and in FlexibleDecay; for the DECAY1L block in SPheno we set flag 1118 in block DECAYOPTIONS to 0).

SARAH/SPheno has two separate modules for decay calculations, whose results are provided in the output blocks DECAY and DECAY1L, respectively. The first is tailored to the calculation of Higgs decays [30, 114], contains SM-like higher-order corrections similar to the ones in HDECAY and FlexibleDecay, however it does not compute the loop-induced $h \rightarrow \gamma Z$ partial width. The second decay computation (DECAY1L output block) is more general and based on the general full one-loop computation of Ref. [43], but contains no specific higher-order corrections and is thus expected to be less precise in case of Higgs boson decays.
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