Position Estimation Using Trilateration based on ToA/RSS and AoA Measurement

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Abstract— with the increasing demand for recent lifestyle new achievements have been developed in various fields of science and technology, target localization is considered one of the most important subjects especially in indoor conditions since most of the social activities done in the indoor environment. The presence of large buildings such as hospitals, malls, schools and other facilities of modern life increases the need for accurate target positioning. In this paper, a proposed system included simple equipment on three transmitters and some computer packages are used to apply our experiment. The case study chosen in this work is simulated using Wireless InSite (WI) software to model the campus with its all building. Three transmitters (TXs) are installed in selected position while twenty receivers (RXs) are deployed randomly as a target to estimate their position utilizing Time of Arrival (ToA) and Received Signal Strength (RSS) to estimate distance then trilateration is applied to determine target position with Angle of Arrival (AoA). The achieved result is addressed using specific equations to estimate the target positions.

1. Introduction
With the revolutionary development of wireless communications networks during the last few years, it becomes an important part of wireless communication devices such as smartphones and tablets...etc. [1].These devices allow easy communication with their surroundings all over the world [2]. This development leads to the emergence of Location-Based-Services (LBS) which considered from the most important application involving tracking applications such as animal, vehicle and mobile, civil engineers mapping, tracking and navigation systems, …etc. All these applications aiming to the same goal which is target coordinates determination [3]. Techniques used to achieve that goal is varied between indoor and outdoor environments.

The localization process at outdoor scenario adapting Global Positioning System (GPS) as the main localization technique which can achieve an accuracy of around 5 meters. However, in the case of campus or large office, more accuracy is needed, this is not the case in the indoor environment since the signal severely declined inside buildings, in addition to the high cost of GPS receiver [4-6].

Trilateration is one of the localization technique which is a geometric method for object position determination requiring distance estimation between target and three reference points at least [7]. The distance measured indirectly utilizing Time of Arrival (ToA) or Received Signal Strength (RSS) parameters.

Many researchers investigate this subject as in [8] where two different localization algorithms are used: Trilateration and Multilateration techniques based on available anchor nodes and found that accuracy can be enhanced by increasing the number of anchor nodes. In [9], outdoor localization is studied through simulation study via Wireless InSite (WI) tool looking for optimal transmitter (TX) location with reasonable RSS to provide better coverage, there was a significant effect of obstacles presence and its related material properties on a location determination. Also, in [10] the researcher used
the trilateration technique with a single uncertain reference point and two certain reference points, this method required costly calculations. On the other hand, [11] suggested a Time-based Positioning Scheme (TPS) in the outdoor environment relies on Time Difference of Arrival (TDoA) of Radio Frequency (RF) signals measured locally at a sensor to estimate range differences from the sensor to three base stations through trilateration. They found that TPS is effective for outdoor sensor self-positioning. Moreover, [12] proposed an improved RSS-trilateration based method for Wi-Fi indoor localization where trilateration is implemented for target position determination and then improve the result using a specific reference point, average error found to be 2 meters reduced to 1 meter with the improved scheme.

Our research proposes an algorithm for outdoor AoA measurement based on ToA/RSS-trilateration technique utilizing three reference points represented by TXs with certain coordinates. Distance between TX and Receiver (RX) which represent target can be calculated based on ToA/RSS value for each reference point. The target position (RX coordinates) can be determined through the trilateration method.

2. Mathematical calculation
   A. RSS Based Distance Measurements
   RSS is the strength of a received signal measured at the receiver’s antenna [13] and can be calculated according to the Friis equation [14]:

   \[ P_r = \frac{P_t G_r G_t \lambda^2}{(4\pi)^2 d^2} \]  

   Where \( P_t \) is the transmitted power, \( G_r \) and \( G_t \) are the antenna gain of the transmitter and receiver respectively, \( d \) is the distance between TX and RX and the \( \lambda \) is the carrier wavelength of the signal.

   The propagated signals are subjected to various types of attenuations called path loss parameter which can be calculated by [15], [16]:

   \[ \text{FSPL} = 20 \log_{10}(d) + 20 \log_{10}(f) + 20 \log_{10}\left(\frac{4\pi}{c}\right) - G_t - G_r \]  

   To estimate the distance \( d \) between the TX and RX coordinator can be based on the following equation:

   \[ d = 10^{-\frac{(\text{RSS} - P_t + P_{10} - \sigma)}{(10 \gamma)}} \]  

   Where \( P_{10} \) is the path loss at a reference distance \( d_0 \) (1 meter) and it's equal to (8.044) dB, the path loss exponent \( \gamma \) depends on the environment and which is in a range of 2 – 6, according to [17]. In this paper, \( \gamma \) is considered as 4. Also, according to [17-19], the standard deviation \( \sigma \) depended on the environments of case study in the range of 2 – 14 which assumed to be 7 in this paper.

   B. ToA Based Distance Measurements
   ToA based distance was estimated using WI software according to the speed of electromagnetic wave propagation, the distance \( d \) is obtained by:

   \[ d = c \times t \]  

   Where \( c \) is speed of light \((3 \times 10^8) \text{ ms}^{-1}\), \( t \) is the propagation time in one way signal with maximum power received with multipath propagation; the time is taken as \( \bar{t} \) that can be calculated as:

   \[ \bar{t} = \frac{\sum_{i=1}^{N_p} P_{t_i}}{P_R} \]
Where $N_p$ is the number of the paths and $P_i$ is the time-averaged power in watts of the $i^{th}$ path, $P_R$ is total power received, $t_i$ is the ToA for each propagation path which can be calculated from:

$$t_i = \frac{L_i}{c} \quad (6)$$

Where $L_i$ is the total geometrical path length [20].

C. Trilateration Method

The trilateration method utilizes measured distance between TX and RX which is calculated based on ToA/RSS value for RX coordinates determination. This method needs the presence of three nonlinear reference points that represented by three TXs in this paper. The distance from each TX to RX form a three intersected circles. Distance between each TX and target is equal to the radius ($r$) of the corresponding circle. The point at which all the three circles intersect with each other represents our target which is the RX position in figure 1. Target coordinates $(x_t, y_t)$ can be estimated based on equation (7) [3], [12]:

$$r_1^2 = (x_t - x_1)^2 + (y_t - y_1)^2$$
$$r_2^2 = (x_t - x_2)^2 + (y_t - y_2)^2$$
$$r_3^2 = (x_t - x_3)^2 + (y_t - y_3)^2 \quad (7)$$

Where $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ represent coordinates of TX1, TX2, and TX3 consecutively.

Thus, AoA ($\theta_i$) can be determined to utilize target coordinates according to the following equation:

$$\theta_i = \tan^{-1} \frac{y_t - y_i}{x_t - x_i} \quad (8)$$

Where $(x_i, y_i)$ represents reference points (TXs) coordinates.

3. Case Study
A simulated case study has been designed using WI software, which represents the campus of the Electrical Engineering Technical College composed of multiple buildings. All of these buildings simulated based on real dimensions with multiple floors and 3.5m height for each one.

Three TXs were deployed in preplanned locations with 2.5m height and highlighted with blue color while fifty RXs scattered randomly over the work area at 1.3m height coded with red color. The distribution of TXs and RXs are illustrated in figure 2. All properties of both TX and RX are listed in table 1.

Also, the impact of serious effects on wave propagation causes by different building materials (Concrete, Wood, Brick, Glass,) with a frequency of 2.4GHz band and 1MHz bandwidth. Such impacts were taken into consideration for the entire investigation, where each material thickness, conductivity ($\sigma$) and relative permittivity ($\varepsilon'$) was determined based on the recommendation of the International Telecommunication Union (ITU) [21]. The results of ($\sigma$ & $\varepsilon'$) calculations at a frequency of 2.4 GHz are listed in Table 2. Twenty out of fifty RXs were involved in calculations.

![Simulation model of case study](image1)

**Figure 2.** Simulation model of case study; (a) 3D view, (b) Distribution of TXs and RXs

| Table 1. X and RX Antenna properties |
| Antenna properties | TX Antenna          | RX Antenna          |
|--------------------|---------------------|---------------------|
| Antenna Type       | Directional 120°    | Omni-Directional 360° |
| Gain (dBi)         | 5                   | 2                   |
| Input Power (dBm)  | 30                  | -                   |
| Polarization       | V                   | V                   |
| Waveform           | Sinusoid            | Sinusoid            |
| (VSWR)             | 1                   | 1                   |

Where (VSWR) is the Value for the antenna’s voltage standing wave ratio.

Table 2. Material thickness, conductivity and permittivity values

| Material  | Thickness (m) | σ    | ε′   |
|-----------|---------------|------|------|
| Wood      | 0.030         | 0.0120 | 1.99 |
| Brick     | 0.125         | 0.0380 | 3.75 |
| Glass     | 0.003         | 0.0122 | 6.27 |
| Concrete  | 0.125         | 0.0660 | 5.31 |

4. RESULT AND DISCUSSION

A. Estimated Distance Based ToA/RSS
As mentioned in sections 3, a simulated case study via the WI tool utilizes mean ToA for estimating distance depending on equation (4). RSS measured through the Wi-Fi meter application is used for estimating real distance using equation (3). Trilateration method adapting mean ToA based distance used for Rx position determination according to equation (7). All these parameters obtained from TX1 (295, 2), TX2 (169, 169) and TX3 (29, 70) are listed in Table 3, 4 and Table 5 respectively. Ranging error is the difference between estimated distance based ToA (virtual measurement) and distance-based RSS (real measurement), the result of ranging error is listed in the 6th column in Table 3, 4 and Table 5, and it was considered for distance measurements obtained from all the three TXs. The RSS can be shown in figure 3, 4 and Figure 5 respectability.

Figure 3. Receive Power V. Distance in TX1

Table 3. The estimated distance with RX position and error obtained from TX1
| Receiver | Distance/ToA | RSS | Distance/RSS | Position/Trilateration | Error     |
|----------|-------------|-----|--------------|-------------------------|-----------|
| RX1      | 159         | -85 | 177.3781006  | (108, 53)               | 18.37810057 |
| RX2      | 125         | -81 | 140.8964335  | (146, 57)               | 15.8964335  |
| RX3      | 126         | -81 | 140.8964335  | (161, 81)               | 14.8964335  |
| RX4      | 161         | -85 | 177.3781006  | (155, 125)              | 16.37810057 |
| Rx5      | 122         | -80 | 133.0148105  | (170, 87)               | 11.01481048 |
| RX6      | 150         | -84 | 167.4557251  | (177, 128)              | 17.45572506 |
| RX7      | 166         | -86 | 187.8884138  | (173, 144)              | 21.88841379 |
| RX8      | 163         | -86 | 187.8884138  | (196, 153)              | 24.88841379 |
| RX9      | 103         | -77 | 111.9180153  | (177, 165)              | 8.918015304 |
| RX10     | 137         | -83 | 158.0883985  | (162, 32)               | 21.08839854 |
| RX11     | 79          | -72 | 83.9266716   | (185, 31)               | 4.926671597 |
| RX12     | 118         | -79 | 125.5740785  | (142, 25)               | 7.57407854  |
| RX13     | 123         | -80 | 133.0148105  | (141, 38)               | 10.01481048 |
| RX14     | 111         | -79 | 125.5740785  | (209, 101)              | 14.57407854 |
| RX15     | 160         | -85 | 177.3781006  | (124, 89)               | 17.37810057 |
| RX16     | 150         | -84 | 167.4557251  | (139, 93)               | 17.45572506 |
| RX17     | 169         | -86 | 187.8884138  | (128, 110)              | 18.88841379 |
| RX18     | 131         | -82 | 149.2450721  | (176, 104)              | 18.24507205 |
| RX19     | 140         | -83 | 158.0883985  | (132, 64)               | 18.08839854 |
| RX20     | 142         | -83 | 158.0883985  | (138, 76)               | 16.08839854 |

Figure 4. Receive Power V. Distance in TX2
Table 4. The estimated distance with RX position and error obtained from TX2

| Receiver | Distance /ToA | RSS  | Distance/RSS | Position/Trilateration | Error     |
|----------|---------------|------|--------------|------------------------|-----------|
| RX1      | 155.68        | -85  | 177.3781006  | (108, 53)              | 21.69810057 |
| RX2      | 114.22        | -80  | 133.0148105  | (146, 57)              | 18.79481048 |
| RX3      | 88.72         | -76  | 105.6574196  | (161, 81)              | 16.9374196  |
| RX4      | 46.61         | -63  | 49.9919411   | (155, 125)             | 3.381941103 |
| RX5      | 82            | -73  | 88.89963953  | (170, 87)              | 6.89963953  |
| RX6      | 41.41         | -61  | 44.55536441  | (177, 128)             | 3.145364407 |
| RX7      | 25.32         | -52  | 26.53994387  | (173, 144)             | 1.219943868 |
| RX8      | 30.82         | -56  | 33.41180976  | (196, 153)             | 2.591809763 |
| RX9      | 104           | -78  | 118.5495746  | (177, 165)             | 14.54957463 |
| RX10     | 137.45        | -83  | 158.0883985  | (162, 32)              | 20.63839854 |
| RX11     | 138.86        | -81  | 140.864335   | (185, 31)              | 2.036433504 |
| RX12     | 147.89        | -84  | 167.4557251  | (142, 25)              | 19.56572506 |
| RX13     | 133.87        | -82  | 149.2450721  | (141, 38)              | 15.37507205 |
| RX14     | 78.62         | -73  | 88.89963953  | (209, 101)             | 10.27963953 |
| RX15     | 92            | -76  | 105.6574196  | (124, 89)              | 13.6574196  |
| RX16     | 82.18         | -74  | 94.16727434  | (139, 93)              | 11.98727434 |
| RX17     | 72.57         | -71  | 79.23188713  | (128, 110)             | 6.661887131 |
| RX18     | 65.66         | -69  | 70.61549374  | (176, 104)             | 4.955493736 |
| RX19     | 126.01        | -81  | 140.864335   | (132, 64)              | 14.8864335  |
| RX20     | 97.976        | -77  | 111.9180153  | (138, 76)              | 13.9420153  |

Figure 5. Receive Power V. Distance in TX3
Table 5. The estimated distance with RX position and error obtained from TX3

| Receiver | Distance/ToA | RSS | Distance/RSS | Position/Trilateration | Error        |
|----------|--------------|-----|--------------|-------------------------|--------------|
| RX1      | 80.8         | -72 | 83.926672    | (108, 53)               | 3.126671597 |
| RX2      | 118.1        | -80 | 133.01481    | (146, 57)               | 14.91481048 |
| RX3      | 132.04       | -82 | 149.24507    | (161, 81)               | 17.20507205 |
| RX4      | 137.12       | -83 | 158.0884     | (155, 125)              | 20.96839854 |
| Rx5      | 166.36       | -86 | 187.88841    | (170, 87)               | 21.52841379 |
| RX6      | 158.79       | -85 | 177.3781     | (177, 128)              | 18.58810057 |
| RX7      | 162.15       | -85 | 177.3781     | (173, 144)              | 15.22810057 |
| RX8      | 186.07       | -88 | 210.81427    | (196, 153)              | 24.74426763 |
| RX9      | 148.13       | -84 | 167.45573    | (177, 165)              | 19.32572506 |
| RX10     | 138.32       | -83 | 158.0884     | (162, 32)               | 19.76839854 |
| RX11     | 160.28       | -85 | 177.3781     | (185, 31)               | 17.09810057 |
| RX12     | 121.68       | -80 | 133.01481    | (142, 25)               | 11.33481048 |
| RX13     | 116.13       | -79 | 125.57408    | (141, 38)               | 9.44407854  |
| RX14     | 182.78       | -87 | 199.0215     | (209, 101)              | 16.24150223 |
| RX15     | 96.97        | -76 | 105.65742    | (124, 89)               | 8.6874196  |
| RX16     | 112.23       | -79 | 125.57408    | (139, 93)               | 13.34407854 |
| RX17     | 106.3        | -78 | 118.54957    | (128, 110)              | 12.24957463 |
| RX18     | 150.87       | -84 | 167.45573    | (176, 104)              | 16.58572506 |
| RX19     | 103.43       | -77 | 111.91802    | (132, 64)               | 8.488015304 |
| RX20     | 108.8        | -78 | 118.54957    | (138, 76)               | 9.749574625 |

Also, the resultant position adapting trilateration algorithm based on equation (7) are listed in the 5th column corresponding to each RX for all previous result.

B. Estimated AoA

Finally, AoA can be estimated using RX coordinates with equation (8). All results are summarized in Table 6 with an estimated AoA related to TX1. Figure 6 illustrated the different distances estimated between ToA and RSS.

![Figure 6. Distance ToA/RSS Estimated](image-url)
Table 6. TX1 Related AoA measurement

| Receiver | Distance/ToA | RSS  | Distance/RSS | Position/Trilateration | Theta | Theta related TX1 |
|----------|--------------|------|--------------|------------------------|-------|-------------------|
| RX1      | 159          | -85  | 177.3781     | (108, 53)              | -18   | 162               |
| RX2      | 125          | -81  | 140.8964     | (146, 57)              | -25   | 155               |
| RX3      | 126          | -81  | 140.8964     | (161, 81)              | -38   | 142               |
| RX4      | 161          | -85  | 177.3781     | (155, 125)             | -49   | 131               |
| Rx5      | 122          | -80  | 133.0148     | (170, 87)              | -47   | 133               |
| RX6      | 150          | -84  | 167.4557     | (177, 128)             | -56   | 124               |
| RX7      | 166          | -86  | 187.8884     | (173, 144)             | -58   | 122               |
| RX8      | 163          | -86  | 187.8884     | (196, 153)             | -67   | 113               |
| RX9      | 103          | -77  | 111.9180     | (177, 165)             | -63   | 117               |
| RX10     | 137          | -83  | 158.0883     | (162, 32)              | -17   | 163               |
| RX11     | 79           | -72  | 83.92667     | (185, 31)              | -21   | 159               |
| RX12     | 118          | -79  | 125.5740     | (142, 25)              | -11   | 169               |
| RX13     | 123          | -80  | 133.0148     | (141, 38)              | -16   | 164               |
| RX14     | 111          | -79  | 125.5740     | (209, 101)             | -63   | 117               |
| RX15     | 160          | -85  | 177.3781     | (124, 89)              | -32   | 148               |
| RX16     | 150          | -84  | 167.4557     | (139, 93)              | -37   | 143               |
| RX17     | 169          | -86  | 187.8884     | (128, 110)             | -39   | 141               |
| RX18     | 131          | -82  | 149.2450     | (176, 104)             | -50   | 130               |
| RX19     | 140          | -83  | 158.0883     | (132, 64)              | -26   | 154               |
| RX20     | 142          | -83  | 158.0883     | (138, 76)              | -31   | 149               |

5. Conclusions

In this paper, a positioning location has been tested for the case study of campus for Electrical Engineering Technical College (Baghdad, Al-Dorra). The results show that estimated distance based ToA and RSS to determine the position localization, has a minimum error using RSS in Tx1 (4.926671597), TX2 (1.219943868) and TX3 (3.126671597) that can be obtained using ToA algorithm by RSS algorithm. The result of the current work shows that better position estimation can be obtained when the position target near (less than) the TX and barrier between them were known. There was some variation in RSS measurement between simulated and real results that was about (-1 - -3) dB, this variation can be explained by the fact that presence of obstacles, barriers, electric cables, other electric and electronic devices which are affected the strength of the received signal. For all obtained result it can be concluded that the proposed system can achieve a good result in corresponding to the cost and its requirements.

6. References

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