Anisotropic Conductivity Measurements by Two Kinds of Multimode Rectangular Plate-Laminated Cavities

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Abstract—In this article, we present anisotropic conductivity measurements by using two kinds of multimode rectangular cavities. Plate-laminated waveguides are fabricated by diffusion bonding of etched thin metal plates and their sidewalls have anisotropic conductivity characteristics. The anisotropic conductivity is measured by two multimode rectangular cavities when polarization is vertical or horizontal to the lamination direction. Transmission characteristics of the cavities are measured and the conductivity is calculated by the measured resonant frequencies and unloaded quality factors. The anisotropic conductivities are derived by solving a linear equation. As an example, aluminum alloy (A6063) E-band (60.5–91.9 GHz) rectangular cavities are fabricated and the conductivity is characterized.

Index Terms—Anisotropic conductivity, millimeter-wave band, multimode rectangular cavity.

I. INTRODUCTION

MILLIMETER-WAVEBAND high-gain antennas realize high-speed fixed wireless communication using wide bandwidths [1]–[3]. Array antennas such as microstrip arrays or waveguide slot arrays enable high-gain characteristics with planar structures. The gain of array antennas is dominated by the losses in feeding circuits because the electrical length of the feeding line is proportional to the gain. Microstrip lines suffer from high losses in the millimeter-wave bands because of dielectric and conductor losses. Hollow waveguides have much lower losses and enable high-gain array antennas even in millimeter-wave bands. For example, the transmission loss of microstrip lines is 0.22 dB/cm at 24 GHz [4] and one of the waveguides is 0.04 dB/cm at 90 GHz [5].

Plate-laminated waveguides [6]–[9] realize a hollow multilayer structure such as horn array antennas and waveguide slot array antennas. These antennas realize high-gain characteristics in the 60- [7], 78.5- [8], and 120-GHz [9] bands. The plate-laminated waveguide is fabricated in two steps: etching of metal plates and bonding of the plates. Because the sidewalls of the waveguides are formed by etching, the sidewalls are not flat as suggested in Fig. 1. These nonflat walls result in degradation of the conductivity and also cause anisotropic properties in the conductivity, and the effective conductivity at the sidewalls varies with the direction of the current. For example, it may be expected that the waveguide shown in Fig. 1(a) has different effective conductivities in the longitudinal and tangential components on the broad walls because of the geometry of the cavity.

For an evaluation of the fabrication process and material and estimates of the antenna gain, the conductivity must be properly evaluated and much research on conductivity measurements has been conducted. Measurement methods of the conductivity are categorized into transmission line methods [10]–[12] and resonator methods [13]–[19]. The transmission line methods use transmission lines with different lengths and calculate attenuation constants based on the transmission characteristics. The methods make it simple to determine the attenuation constant of a transmission line; however, they cannot establish the conductivity for each of the walls or the anisotropic conductivity. Once the anisotropic conductivity is measured, it is possible to simulate a loss of waveguides or waveguide components with arbitrary kinds of shapes fabricated by the same process. This is impossible by measuring the effective conductivity or attenuation constant and neglecting the anisotropic property because electrical current distribution depends on the shape of waveguides. Therefore, it is worth to measure the anisotropic conductivity.

The resonator methods enable measurements of the conductivity by measuring the resonance characteristics. Various shapes of resonators have been used: rectangular cavities [13]–[15], circular cavities [16], [17], and circular dielectric resonators [18], [19]. The method can measure the conductivity...
directly, but none of the methods consider the anisotropic characteristics of the conductivity. Multimode metal cavities are also used for conductivity measurement [14]–[17], [20]–[22]; however, all of them treat the conductivity as isotropic. The nonflat walls could be considered as roughness effects as studied in [13]; however, uniform roughness is assumed and two orthogonal effective conductivities cannot be measured by the method in [13].

In this article, an anisotropic conductivity measurement method is proposed and verified by simulation and measurements. The proposed method uses two kinds of multimode rectangular cavities with the $E$-field direction parallel or perpendicular to the lamination direction. The conductivity is measured by solving linear equations. The methods in [13]–[15] use substrate integrated waveguide (SIW)-based multimode rectangular resonator which is the same as the proposed method; however, they do not consider anisotropic characteristics of the conductivity. As an example, E-band cavities are fabricated and the conductivities are characterized.

II. MEASUREMENT PRINCIPLES

A. Resonator Structures

Two cavities are used in these measurements as outlined in Fig. 2. One has coupling slots on the $E$-plane and is named an $E$-plane-coupled cavity and the other has the coupling slots on the $H$-plane and is named an $H$-plane-coupled cavity. The cavities are fed by two waveguides through the coupling slots and the transmission characteristics of the cavities are measured by a vector network analyzer (VNA).

Cross sections of the waveguides are shown in Fig. 1. The $E$-plane-coupled cavity has flat narrow and uneven broad walls. The $H$-plane-coupled cavity has flat broad and uneven narrow walls. Using the differences in resonant characteristics, the anisotropic conductivity is measured. The dimensions of the cavities are determined so that only the TE$_{10}$ modes resonate in the measurement frequency range. The conditions for only TE$_{10}$ mode excitation are

$$2b < a < \sqrt{3/((l_{\text{max}} - 1)/d)} \min(f_{\text{meas}}) < f_{10_{\text{max}}}$$

where $l_{\text{min}}$ and $l_{\text{max}}$ are the minimum order and maximum order $l$ of TE$_{10}$ modes used for the conductivity measurement and $f_{\text{meas}}$ is the measurable frequency range by a VNA. The derivation of this condition is detailed in Appendix A.

B. Measurement Principles

The measurement procedure consists of four steps as outlined in Fig. 3 and may be explained as follows.

Step 1: Measure the transmission characteristics $S_{21}$ of the $E$- and $H$-plane-coupled cavities.

Step 2: Extract the resonant frequencies $f_r$ and calculate the unloaded quality factors $Q_a$ from the measured transmission characteristics. The unloaded quality factors are calculated by the equations in the top right of Fig. 3.

Step 3: Calculate the dimensions of the cavities from the resonant frequency $f_r$ and mode index $l$ by solving the linear equations immediately below. The derivation is described further in Appendix B

$$A_d x_d = b_d$$

$$A_d x_d = b_d$$

where $(A)_{ij}$ is for an element at the $i$th row and $j$th column of the matrix $A$. $f_{10}$ is the measured resonant frequency of
mode TE_{10l} and c is the speed of light. The parameters a and d are the dimensions of either the E- or H-plane-coupled cavity shown in Fig. 2. Because there are two unknowns, the dimensions of a and d, two or more resonant frequencies are required. The dimensions of b cannot be calculated because the resonant frequencies of the TE_{10l} modes are independent of the dimension b though their unloaded quality factors depend on the dimension b. In this study, a designed value for b is used. The mode indices are calculated by the differences in the resonant frequencies [23], [24].

Step 4: Calculate the anisotropic conductivity by all derived parameters, resonant frequencies, mode indices, unloaded quality factors, and cavity dimensions by solving the following linear equations. The derivation is shown in Appendix C.

\[
A_{\sigma} x_{\sigma} = b_{\sigma}
\]

\[
A_{\sigma} = \begin{pmatrix} A_{\sigma E} & 0 \\ A_{\sigma H} & 0 \end{pmatrix}, \quad b_{\sigma} = \begin{pmatrix} b_{\sigma E} \\ b_{\sigma H} \end{pmatrix}, \quad x_{\sigma} = (x_1, x_2, x_3)^T
\]

where \(A_{\sigma}, b_{\sigma},\) and \(x_{\sigma}\) are the matrices of \((N_E + N_H) \times 3\), \((N_E + N_H) \times 1\), and \(3 \times 1\), respectively. Both the \(N_E\) and \(N_H\) values are a number for the resonant characteristics of the E- and H-plane-coupled cavities. The \(\sigma_r\) value is the conductivity on the flat surfaces, and \(\sigma_i\) and \(\sigma_t\) are the tangential and longitudinal components of the conductivity on the uneven surfaces as shown in Fig. 1. The superscript T indicates the transposed matrix. The matrices \(A_{\sigma}, b_{\sigma},\) and \(x_{\sigma}\) are defined as follows:

\[
(A_{\sigma E})_{i,1} = 2b_E d_E^3
\]

\[
(A_{\sigma E})_{i,2} = a_E d_E^3
\]

\[
(A_{\sigma E})_{i,3} = a_E^3 (2b_E + d_E) l_{E,i}^2
\]

\[
(A_{\sigma H})_{i,1} = l_{H,i}^2 a_H d_H + a_H d_H^2
\]

\[
(A_{\sigma H})_{i,2} = 2l_{H,i} a_H b_H + 2b_H d_H
\]

\[
(A_{\sigma H})_{i,3} = 0
\]

\[
(b_{\sigma E})_{i,1} = \frac{(k_{E,i} a_E d_E)^3 b_E \eta}{2 \pi^2 (Q_{E,i})_{1,1}^2 \sqrt{\pi} f_{E,i} \mu}
\]

\[
(b_{\sigma H})_{i,1} = \frac{(k_{H,i} a_H d_H)^3 b_H \eta}{2 \pi^2 (Q_{H,i})_{1,1}^2 \sqrt{\pi} f_{H,i} \mu}
\]

\[
Q = (Q_{E,i}) (Q_{H,i}) = Q_{E,i} (Q_{H,i})_{i,1} = Q_{H,i}
\]

where \(k_{E,i}, k_{H,i}, f_{E,i},\) and \(f_{H,i}\) elements are the \(i\)th resonant wavenumber and resonant frequency of the E- and H-plane-coupled cavities, respectively. The \(\eta\) and \(\mu\) are the wave impedance and permeability, respectively. The \(Q\) parameter is a vector for the unloaded quality factors of the E- and H-plane-coupled cavities.

Because the number of unknowns is three, three or more resonant characteristics are required to solve the linear equation. The number of resonant characteristics affects the sensitivity of the conductivity and will be discussed in Section III.

### III. Sensitivity Analysis

The sensitivity of the conductivity with respect to the unloaded quality factors is evaluated when the number of resonant characteristics used to solve the linear equation is changed. The sensitivity is explicitly expressed and its derivation is described in Appendix C.

The resonant frequency and the unloaded quality factor are analytically calculated by the dimensions shown in Table I and a conductivity of 3.1 \times 10^7 \text{ S/m} which is the bulk A6063 conductivity. The designed value of \(b\) is 0.8 mm. The equations are shown in Appendices B and C. The calculated resonant frequency is shown in Fig. 4. By using an extended E-band VNA from 56 to 94 GHz, TE_{103} to TE_{109} and TE_{103} to TE_{101} for the E- and H-plane-coupled cavities can be observed as shown in Fig. 4. The nearest higher order mode is TE_{201} with a resonant frequency of 107.3 GHz for both the E- and H-plane-coupled cavities.

The sensitivity analysis shows that increasing the number of resonant characteristics decreases the sensitivity, whereas the measurable frequency bandwidth decreases. Using the calculated resonant frequency and unloaded quality factor, the sensitivity of the conductivity can be evaluated. The number of resonant characteristics for solving the linear equations is changed from 2 to 8 for both the E- and H-plane-coupled cavities. The square root of the sum of the squares of the sensitivity is defined as below and shown in Fig. 5. The sensitivity decreases by increasing the number of mode

\[
c_i = \sqrt{\sum_j \left(\frac{\partial \sigma_i}{\partial (Q_j)}\right)^2}, \quad i = f, r, rt, rl.
\]

### IV. Measurements

#### A. Resonators

The E- and H-plane-coupled cavities were fabricated by chemical etching and diffusion bonding of an aluminum alloy (A6063) as shown in the photograph in Fig. 6. The two cavities...
and a $4 \times 4$ element slot array are integrated into a single part. It is composed of 21 aluminum alloy plates of 0.2-mm thickness. The dimensions of the cavity are shown in Table I and $b_E = b_H = 0.8$ mm. The dimensions of the coupling slot are $l = 1.5$ mm and $w = 0.2$ mm. They are determined so that the effect of resonance characteristics is small and transmission amount $S_{21}$ at a resonance frequency is in the measurable range ($-80$ to $0$ dB) by the VNA. The effect of the coupling slots is studied in Appendix D and it shows that the anisotropic conductivity is varied maximally 2.6%. The minimum simulated transmission amount $S_{21}$ at resonant frequencies is $-48.0$ dB, which is within the measurable range by the VNA. There are holes to fix the waveguide around the coupling slots. Its resonant frequencies are shown in Fig. 4.

### B. Conductivities

**Step 1:** $S_{21}$ was measured by directly connecting two waveguides (WR-12) to the cavities as shown in Fig. 7. The measured $S_{21}$ has multiple resonant characteristics as shown in Fig. 8; $S_{21}$ was measured six times and Fig. 8 shows the mean values.
Step 2: The resonant frequency and unloaded quality factor are extracted from the measured S21. The measured unloaded quality factors are shown in Fig. 9. The error bar in Fig. 9 expresses the measurement uncertainty. The maximum uncertainty is 2.41 and 1.46 for the E- and H-plane-coupled cavities, respectively. Because the coupling of the E-plane-coupled cavity at the lower frequency is low, the standard deviation of the unloaded quality factor becomes high.

Step 3: The calculated cavity dimensions and the expanded uncertainties are shown in Table I. The calculated dimensions are larger than the design. The differences between the calculated and designed dimensions result from minor perturbation by the coupling slots, the uneven walls, and fabrication tolerances.

The $\Delta f$ is analytically calculated from the calculated dimensions and mode indices and compared with the measured $\Delta f$ as shown in Fig. 10. The results confirm that $\Delta f$ agrees in the two calculations and the estimated mode indices are sufficiently accurate.

Step 4: The measured conductivity of the uneven walls is much lower than that of the flat walls as shown in Fig. 11. The conductivities are calculated using 5–8 modes of each of the resonant modes of the E- and H-plane-coupled cavities. When eight modes are used, the measured conductivities are $\sigma_f = 2.35 \times 10^7$ S/m, $\sigma_{rt} = 7.28 \times 10^5$ S/m, and $\sigma_{rl} = 6.79 \times 10^5$ S/m with the bulk A6063 conductivity $3.1 \times 10^7$ S/m. The differences between the conductivities on the uneven walls are not significant.

The expanded uncertainty of the conductivity is mainly dominated by uncertainties in the measured quality factors as shown in Fig. 12. The figure shows the product of the sensitivity coefficient and uncertainty by the unloaded quality factor, dimensions $a$ and $d$, and resonant frequencies $f_r$ in relation to each conductivity. The contribution from the unloaded quality factor is the largest and the other contributions are much lower.

V. VALIDATION

The measured conductivities are verified by a 4 × 4 element slot array antenna made in the same part as the cavities as in...
Fig. 12. Contribution of the combined standard uncertainty of the conductivities by unloaded quality factors, dimensions, and resonant frequencies. The values by the resonant frequency is less than 10^-2.

The antenna is composed of a 2 × 2 element subarray and a corporate feeding circuit as shown in Fig. 13(a) and (b). The lamination direction is the z-direction and the narrow walls of the waveguides, the cavities, and the slots have uneven walls.

Electric current distributions on nonflat walls are shown in Fig. 13(c). The loss of cavity #3 depends on all the anisotropic conductivity components because electric currents on the nonflat walls flow in both directions parallel and normal to the lamination direction. The loss of the other components depends on the two anisotropic conductivity components \( \sigma_f \) and \( \sigma_{rt} \) because electric currents on the nonflat wall flow only parallel to the lamination direction.

The measured and simulated gains with the measured conductivity show better agreement than the simulated gain with the bulk A6063 conductivity. Fig. 14(a) shows the measured and simulated frequency characteristics of the gain. The bulk A6063 and the measured conductivities are used in the simulation. The differences between the measured and simulated gains at 78.0 GHz are 0.1 and 0.8 dB for the measured conductivity and the A6063 bulk conductivity, respectively. Major fabrication tolerance is underetching/overetching [7] because all the components are made by chemical etching. Fig. 14(b) shows a simulated gain of 10-μm underetching/overetching antennas. The underetching/overetching causes a frequency shift of the gain and has less effect on the gain degradation than conductivity reduction.

VI. CONCLUSION

This article presents anisotropic conductivity measurements by using two kinds of multimode rectangular cavities. In these cavities (Figs. 1 and 2), one of the narrow or broad walls of the cavities is a one-dimensional uneven surface. Measuring the transmission characteristics of the cavities gives multiple resonant frequencies and unloaded quality factors. The conductivities are derived by these resonant characteristics and by solving a linear equation. For the validation of the conductivity of the proposed cavities, the E-band rectangular cavities were fabricated by an aluminum alloy (A6063). The measured conductivities were \( \sigma_f = 2.35 \times 10^7 \) S/m, \( \sigma_{rt} = 7.28 \times 10^5 \) S/m, and \( \sigma_{rl} = 6.79 \times 10^5 \) S/m with the bulk A6063 conductivity \( 3.1 \times 10^7 \) S/m. The difference between \( \sigma_{rt} \) and \( \sigma_{rl} \) is not significant practically.

APPENDIX A

CONDITION OF TE\(_{10l}\) MODE EXCITATION

The resonant frequency of a TE\(_{mnl}\) mode of a rectangular cavity is expressed as follows [25]:

\[
f_{mnl} = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}
\]

where \( c \) is the speed of light, \( a, b, d \) are dimensions of the cavity, and \( m, n, l \) are mode indices.

First, the dimension relation \( 2b < a < d \) is assumed to excite TE\(_{10l}\) modes only. \( l_{\text{min}} \) and \( l_{\text{max}} \) are defined as the lowest and the highest order \( l \) to be used in the measurement, respectively. The lowest resonant frequency of TE\(_{mnl}\) mode excepting TE\(_{10l}\) is that of TE\(_{201}\) because \( 2b < a \) gives \( f_{201} < \)
Fig. 14. Frequency characteristics of the gain with (a) conductivity degradation and (b) underetching/overetching (A6063 conductivity).

$\omega_{011}$ and $a < d$ gives $\omega_{011} < \omega_{110}$. Next, $\omega_{10\text{max}} < \omega_{20\text{max}}$ gives other dimension relation $a < (3/(l_2^{\text{max}} - 1))^{1/2} d$. Finally, considering a measurable frequency range from $\min(f_{\text{meas}})$ to $\max(f_{\text{meas}})$ by a VNA, the following condition for TE$_{10\text{max}}$ mode only excitation is obtained:

\[
2b < a < \sqrt{3/(l_2^{\text{max}} - 1)} d
\]

\[
\min(f_{\text{meas}}) < \omega_{10\text{min}} < \max(f_{\text{meas}}).
\]

**APPENDIX B**

**DERIVATION AND SENSITIVITY OF DIMENSIONS**

The resonant frequency of a TE$_{10l}$ mode of a rectangular cavity is expressed as follows:

\[
\omega_{10l} = \frac{c}{2\pi} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(l\pi/d\right)^2}.
\]

By substituting $x_{d,1} = 1/a^2$ and $x_{d,2} = 1/d^2$, the following linear equation is obtained:

\[
x_{d,1} + l^2 x_{d,2} = 4(\omega_{10l}/c)^2.
\]

Because there are two unknowns, two or more resonant characteristics are required to solve the simultaneous equations.

Sensitivities of the dimensions are explicitly expressed as follows:

\[
\frac{\partial a}{\partial f_{10l}} = \frac{\partial a}{\partial x_{d,1}} \frac{\partial (x_{d,1})}{\partial x_{d,2}} \frac{\partial \omega_{10l}}{\partial x_{d,2}},
\]

Fig. 15. Loading effect of the coupling slots to (a) resonant frequency, (b) unloaded quality factor, and (c) conductivity. These values are relative to ones without the coupling slots. $N_E = N_H = 5$ is used for conductivity calculation.

\[
\frac{\partial d}{\partial \omega_{10l}} = \frac{\partial d}{\partial x_{d,2}} \frac{\partial (x_{d,2})}{\partial x_{d,1}} \frac{\partial \omega_{10l}}{\partial x_{d,1}},
\]

\[
\frac{\partial a}{\partial \omega_{10l}} = \frac{\partial a}{\partial x_{d,1}} \frac{\partial (x_{d,1})}{\partial x_{d,2}} \frac{\partial \omega_{10l}}{\partial x_{d,2}}.
\]

\[
\frac{\partial \sigma_r}{\partial \omega_{10l}} = \left\{ (A_d^{-1})_{ij} A_d^T \right\}_{ij}, \quad N_d = 2
\]

\[
\frac{\partial \sigma_{el}}{\partial \omega_{10l}} = \left\{ (A_d^{-1})_{ij} A_d^T \right\}_{ij}, \quad N_d > 2
\]

\[
\frac{\partial \omega_{10l}}{\partial c^2} = 8 \omega_{10l} / c^2
\]

where $N_d$ is the number of resonant characteristics.
APPENDIX C

DERIVATION AND SENSITIVITY OF CONDUCTIVITIES

The unloaded quality factor of a TE_{10} mode of a rectangular cavity is expressed as follows [25]:

\[ Q_c = \frac{(kad)^2}{2\pi^2 (2R_x a^2 b + 2R_z bd^2 + R_x l^2 a^3 d + R_z x^2 d^3)} \]

\[ R_{ij} = \sqrt{\alpha_{ij}/2\sigma_{ij}} \]

where \( R_{ij} \) and \( \sigma_{ij} \) are the surface resistivity and conductivity of the \( i \)-direction current on the \( ij \) plane, respectively. For the \( E \)-plane-coupled cavity, \( \sigma_{yx} = \sigma_{zy} = \sigma_{xy} = \sigma_{yz} = \sigma_y \) and for the \( H \)-plane-coupled cavity, \( \sigma_{yx} = \sigma_{yz} = \sigma_{xy} = \sigma_{xz} = \sigma_x \). Substituting \( x_1 = 1/(\sigma_y)^{1/2} \), \( x_2 = 1/(\sigma_y)^{1/2} \), and \( x_3 = 1/(\sigma_y)^{1/2} \) gives the following linear equations:

\[ \left( \frac{l^2 a_x^2 d_E + a_x d_E^2}{(k_E a_x^2 b)^2} \right) x_1 + \left( \frac{2l^2 a_x^2 b_E + 2b_E d_E^2}{(k_E a_x^2 b)^2} \right) x_2 = \frac{2a_l d_E}{(k_E a_x^2 b)^2} x_3 \]

\[ \frac{2b_H d_H x_1 + a_y d_H x_2 + (2l^2 a_H b_H + l^2 a_H d_H) x_3}{(k_H a_H d_H)^2 b_H} = \frac{2a_l d_H}{(k_H a_H d_H)^2 b_H} \]

The sensitivity of the conductivities by the unloaded quality factors is explicitly expressed as follows:

\[ \frac{\partial(\bar{\sigma})}{\partial(Q)} = \frac{\partial(\bar{\sigma})}{\partial(x_i)} \frac{\partial(x_i)}{\partial(Q)} + \frac{\partial(\bar{\sigma})}{\partial(b_i)} \frac{\partial(b_i)}{\partial(Q)} \]

\[ \frac{\partial(x_i)}{\partial(b_j)} = \frac{a_i}{a_j} \]

\[ \frac{\partial(b_j)}{\partial(Q)} = -2 \frac{b_j}{(Q)^2} \]

\[ A_i = \begin{cases} A_i^{-1}, & N_E + N_H = 3 \\ (A_i^T A_i)^{-1} A_i^T, & N_E + N_H > 3 \end{cases} \]

\[ \frac{\partial(\bar{\sigma})}{\partial(Q)} = \frac{\partial(\bar{\sigma})}{\partial(x_i)} A_i \frac{\partial(x_i)}{\partial(Q)} + \frac{\partial(\bar{\sigma})}{\partial(b_i)} \frac{\partial(b_i)}{\partial(Q)} \]

\[ \frac{\partial(x_i)}{\partial(b_j)} = \frac{\partial(b_j)}{\partial(Q)} \]

The sensitivity of the conductivity by the dimensions is as follows:

\[ a = (a_E, a_H)^T \]

\[ d = (d_E, d_H)^T \]

\[ \frac{\partial(\bar{\sigma})}{\partial(a)} = \frac{\partial(\bar{\sigma})}{\partial(x_i)} A_i \frac{\partial(x_i)}{\partial(a)} + \frac{\partial(\bar{\sigma})}{\partial(b_i)} \frac{\partial(b_i)}{\partial(a)} \]

\[ \frac{\partial(x_i)}{\partial(b_j)} = \frac{\partial(b_j)}{\partial(a)} \]

\[ \frac{\partial(b_j)}{\partial(a)} = 3 \frac{b_j}{a} \]

\[ \frac{\partial(a)}{\partial(d)} = 3 \frac{a}{d} \]

where \( \partial(A)_{ki}/\partial(a) \) and \( \partial(A)_{ki}/\partial(d) \) are calculated numerically. The sensitivity of the conductivity by resonant frequencies is as follows:

\[ \frac{\partial(\bar{\sigma})}{\partial(f_{10})} = \frac{\partial(\bar{\sigma})}{\partial(x_i)} \sum_j \frac{\partial(x_j)}{\partial(b_j)} \frac{\partial(b_j)}{\partial(f_{10})} \]

\[ \frac{\partial(b_j)}{\partial(f_{10})} = 5 \frac{b_j}{f_{10}} \]

\[ \frac{\partial(f_{10})}{\partial(b_j)} = 2 \frac{f_{10}}{b_j} \]

APPENDIX D

LOADING EFFECT OF THE COUPLING SLOTS

The \( E \)- and \( H \)-plane-coupled multimode cavity with the coupling slots and the feeding waveguides is simulated by finite-element methods to obtain resonant frequencies and unloaded quality factors considering the loading effect of the coupling slots. The dimensions are the same as the fabricated ones and the bulk Al6063 conductivity of \( 3.1 \times 10^7 \) S/m is used. Fig. 15 shows the resonant frequencies, unloaded quality factors, and conductivities of the multimode cavity relative to the ones without the coupling slots and the feeding waveguides. The relative resonant frequency is less than 0.02% and the relative unloaded quality factor is less than 0.8%. The conductivity is calculated using the loaded resonant frequencies and unloaded quality factors by the proposed method and it has maximally 2.6% error as shown in Fig. 15(c).

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