The frictional contact of coated bodies. Part I – The sliding contact

S Spinu\textsuperscript{1,2}, D Cerlinca\textsuperscript{1,2} and I Musca\textsuperscript{1,2}

\textsuperscript{1}“Stefan cel Mare” University of Suceava, Department of Mechanics and Technologies, 13\textsuperscript{th} University Street, 720229, Romania
\textsuperscript{2}Integrated Center for Research, Development and Innovation in Advanced Materials, Nanotechnologies, and Distributed Systems for Fabrication and Control (MANSiD), Stefan cel Mare University, Suceava, Romania

E-mail: sergiu.spinu@fim.usv.ro

Abstract. Hard coatings provide low friction, high wear resistance and corrosion protection that improve the tribological performances of the machine elements undergoing contact load. In lack of analytical solutions, a numerical study is performed in this paper for a better understanding of the engineering applications of hard coatings with consideration of friction. A previous study on the frictionless contact of bi-layered materials is extended by considering the effect of friction on the contact stress state. The assumption of a sliding contact in which the shear tractions do not affect the pressure distribution is adopted. This simplification, often present in the literature, decouples the contact problems in the normal and in the tangential direction and allows for a pressure distribution solution that is independent of the frictional regime. The shear tractions are subsequently obtained from a kinetic Coulomb-type friction law, under the assumption that the normal load is kept constant. The stress state in the coated system results by superposition of stresses due to the normal and the shear tractions. The frequency response functions needed for the calculation of the six stress tensor components induced by the shear tractions are presented for clarity and completeness. The convolution between these functions and the contact stresses is performed in the frequency domain for improved algorithmic efficiency. The performed contact analysis proves the ability of the numerical method to provide insight on the critical contact condition for plasticity failure, and to assist the optimal design of hard coatings.

1. Introduction

Various materials used in gears, bearings, dental crowns, hip prostheses, hard disks or electronic parts may benefit from substrate protection by means of a hard coating layer, providing low friction, high wear resistance and long fatigue life to the tribological element undergoing contact loads. Assessment of the contact tractions and stresses arising in the coated system under contact load is important for the competent design of the protective layer. However, the closed-form solutions to the contact problems are limited to a few cases, e.g. the Hertz contact, involving prescribed contact geometry, homogenous materials, and the neglect of friction. The understanding of the frictional contact of coated bodies may be advanced by numerical simulation.

The starting point in the numerical treatment of the contact problem is the fundamental displacement and stress solutions (i.e., the Green’s functions) derived for the elastic, homogenous and isotropic half-space subjected to concentrated load. Such formulas have been obtained for the multi-
layered half-space only in the Fourier transform domain, i.e. frequency response functions (FRF). This encourages the development of methods for the treatment of the contact problem in the spectral domain. Considering that calculation of both displacement and stresses reduces to convolution products between the imposed load and the Green’s functions, the convolution theorem, stating that a convolution product reduces to element-wise multiplication in the frequency domain, provides grounds for increased algorithmic efficiency.

The spectral treatment of the contact problems by means of the fast Fourier transform (FFT) was initiated by Ju and Farris [1]. Based on the framework developed by Burmister [2], O’Sullivian and King [3] derived the FRFs for stresses in the sliding contact of bi-layered materials and obtained the distributions of the von Mises stress in the coated system. The contact solution was obtained using the least-squares approach. The coated contact of rough surfaces was subsequently studied by Nogi and Kato [4] using the FFT technique and the conjugate gradient (CGM) method to obtain the contact pressure. They pointed out that an error is introduced when displacement is computed in the frequency domain, due to the implicit problem periodization associated with the application of the FFT to discrete series. Basically, the computed result is perturbed by spurious pressure periods non-existent in the original contact problem. Polonsky and Keer [5] further discussed this periodicity error and proposed corrective measures. A more in-depth study accomplished by Liu et al. [6] concluded in the development of a method of discrete convolution and FFT (DCFFT) that circumvents completely the periodicity error. The latter method was applied by Liu and Wang [7] in the study of contact stress fields caused by surface tractions, and by Wang et al. [8] to the partial slip contact of coated bodies. Following a similar approach, Zhang et al. [9] obtained the local yield maps for the identification of the yield initiation positions of hard coatings in the three-dimensional sliding contact. Yu et al. [10] calculated the maximum von Mises stress and its location in trilayer materials in contact, whereas Yu et al. [11] investigated the contact of multilayered bodies.

This paper continues a previous work [12, 13] of the same authors by considering the frictional effects in the coated contact of elastic materials. The sliding contact is investigated first and the combined effect of the coating thickness and of the frictional coefficient value on the maximum von Mises equivalent stress is assessed by computing numerous contact cases. To limit the number of independent parameters, the ratio of the coating elastic modulus to that of the substrate is kept constant in all simulations. The solution of the contact state is achieved via CGM, and the shear stresses are computed from a Coulombian friction hypothesis.

2. Contact model

The concentrated contact model is conveniently reported to a Cartesian coordinate system with the \( x \) and \( y \) axes laying in the plane that separates best the bounding surfaces of the contacting bodies. A normal force is first transmitted, under which the bodies deform elastically, so that the initial point of contact progresses into a contact region accommodating a mutual contact pressure balancing the applied load, equation (1):

\[
W = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x, y) dx dy
\]

The equation (2) of the separation \( h(x, y) \) between the deformed bodies results by equating the parameters of the contact geometry before and after the deformation:

\[
h(x, y) = h_i(x, y) + u(x, y) - \omega,
\]

where \( h_i \) denotes the gap in unloaded state, \( u \) the composite (i.e., relative) displacement induced by the pressure \( p(x, y) \) along the \( z \) -axis , and \( \omega \) the rigid-body approach, i.e., the approach of points within the elastic bodies distant to the contact region. In the frictional contact, the normal
displacement \( u \) comprises contribution of both normal and shear tractions, i.e. \( u = u^{(p)} + u^{(q)} \), whereas in the frictionless contact only the contribution of pressure is retained. A tangential force, large enough to generate the relative macro-sliding of the contacting surfaces, is subsequently applied while keeping the normal force constant. The Coulomb kinetic friction equation yields, equation (3):

\[
q(x, y) = \mu p(x, y),
\]

where \( q \) is the shear contact traction along the direction of the tangential force, and \( \mu \) the frictional coefficient, assumed uniform on the contact area. Without losing generality, the tangential force is applied along the \( x \)-axis.

The numerical treatment of the contact model (1) - (3) normally involves two levels of iteration. A guess computational domain \( \Gamma \) is meshed into cuboidal elements of the same size, and model parameters computed at the centre of each element is assumed representative for each elementary cell. In this manner, discrete series of nodal values are considered instead of the continuous distributions. In the discrete model, problem parameters are denoted by using discrete indexes instead of continuous coordinates. To achieve a computational form with unique solution [14], the model of the normal contact problem is completed with the boundary conditions (6) - (7), expressing the non-negativity of pressure and the impenetrability of the contacting bodies [15, 16]:

\[
W = \sum_{(i,j)\notin \Gamma} p(i, j)
\]

\[
h(i, j) = h(i, j) + u(i, j) - \delta \quad \text{on} \ \Gamma
\]

\[
p(i, j) > 0, h(i, j) = 0 \quad \text{on the contact area}
\]

\[
p(i, j) = 0; \quad h(i, j) > 0 \quad \text{outside the contact area}
\]

\[
q(i, j) = \mu p(i, j)
\]

The discretization circumvents continuous integration and allows for computation of displacement and stresses induced by prescribed, but otherwise arbitrary contact tractions. A first level of iteration based on the CGM may be employed to obtain the pressure distribution and the contact area simultaneously, as described in [16], whereas an outer level assures that the pressure and the shear tractions are stabilised. A common assumption [3, 7, 9] employed in sliding contact models eliminates the outer loop by neglecting the displacement induced by the shear tractions. Taking advantage of the fact that \( u^{(q)} \) is small compared to \( u^{(p)} \), only the contribution of pressure is retained. As a consequence, the solution of the sliding contact model (4) - (8) can be divided into two independent steps: (I) the solution of the frictionless normal contact problem (4) - (7) with the displacement \( u = u^{(p)} \), and (II) the subsequent computation of the frictional shear stresses according to equation (8). The contact problem in the normal direction is thus decoupled from the frictional effects, and pressure can be obtained in one iterative loop only, by solving a frictionless contact scenario. Finally, the subsurface stress state results by superposing stresses due to the normal and the shear tractions. The computation of the stress state in the coated contact undergoing normal load was discussed in [13].

3. Displacement and stresses in coated bodies due to shear tractions

The Green’s functions express the elastic response of a half-space to a point force, and their counterparts in the Fourier domain are the frequency response function (FRF). In case of homogenous materials, both the Green’s functions and the FRFs were calculated in closed-form. However, for coated bodies, only the FRFs were derived. The existence of the analytical solutions for the FRFs of
coated bodies encourages the computation of the coated half-space response directly in the frequency domain. This also results in increased computational efficiency, as the convolution calculation can be performed in \(O(N \log N)\) operations in the frequency domain, whereas in the space domain the computational cost amounts to \(O(N^3)\). The discrete convolution theorem is the source of this reduction, which becomes critical for series with \(N > 10^6\) terms.

The method proposed [12] for the calculation of the displacement and stresses induced by the normal tractions can be applied to the shear tractions as well. The periodicity error is eliminated by a domain extension in which the pressure is zero-padded, and the aliasing phenomena is diminished by decreasing the sampling interval in the frequency domain. A detailed description of the algorithm can be found elsewhere [12].

The FRFs are an essential part of the computation of the layered material elastic response. The stresses and displacements in the layered system were expressed in the literature as functions of the Papkovich-Neuber potentials, by taking their double Fourier transform with respect to the \(x\) and \(y\) and by imposing the boundary conditions and the continuity condition of tractions and displacements at the interface. The explicit forms of the FRFs in bilayered materials were derived by Nogi and Kato [4] for frictionless normal loading, and by Liu and Wang [7] and Wang et al. [8] for the shear stresses. Yu et al. [10] derived the FRFs for trilayer materials, whereas Yu et al. [11] obtained the FRFs of multilayered materials in a recurrence format.

The FRFs for the shear stresses are given below for completeness. In the space domain, stresses in the coated system are expressed in the Cartesian coordinates \(x, y, z\), with the origin of the \(z\)-axis for each layer located on its top. In equations (9) - (30), \(i = 1\) stands for the coating and \(i = 2\) for the substrate. The shear moduli and the Poisson’s ratios for the layer and the substrate are denoted by \(G_i\) and \(\nu_i\), and the coating thickness by \(h\). The FRFs \(\tilde{f}_{mn}^{(i)}\) of the six stress tensor components \(\sigma_{mn}^{(i)}\), \(m, n = x, y, z\) induced by a unit shear traction acting on the direction of the \(x\)-axis, are functions of \(\zeta, \xi, \zeta_1, \xi_1\) in the coating, and of \(\zeta, \xi, \zeta_2, \xi_2\) in the substrate, with \(\zeta, \xi\) the coordinates in the frequency domain corresponding to \(x\) and \(y\), respectively:

\[
\tilde{f}_{xx}^{(i)} = -\zeta \zeta_1 \zeta_{1,2} \left( D(i) e^{-\alpha z_1} + \tilde{D}(i) e^{\alpha z_1} \right) - 2\sqrt{-1} \zeta \zeta_1 \zeta_2 \zeta_2 \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) + 2\alpha \nu_1 \times \\
\left( C(i) e^{-\alpha z_1} + \tilde{C}(i) e^{\alpha z_1} \right) + \sqrt{-1} \zeta \zeta_2 \zeta_2 \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) - \zeta \zeta_1 \zeta_2 \zeta_2 \left( C(i) e^{-\alpha z_1} + \tilde{C}(i) e^{\alpha z_1} \right) 
\]

\[
\tilde{f}_{yy}^{(i)} = -\xi \xi_1 \xi_{1,2} \left( D(i) e^{-\alpha z_1} + \tilde{D}(i) e^{\alpha z_1} \right) - 2\sqrt{-1} \xi \xi_1 \xi_2 \xi_2 \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) + 2\alpha \nu_1 \times \\
\left( C(i) e^{-\alpha z_1} + \tilde{C}(i) e^{\alpha z_1} \right) + \sqrt{-1} \xi \xi_2 \xi_2 \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) - \xi \xi_1 \xi_2 \xi_2 \left( C(i) e^{-\alpha z_1} + \tilde{C}(i) e^{\alpha z_1} \right) 
\]

\[
\tilde{f}_{zz}^{(i)} = -\zeta \xi \xi_1 \xi_{1,2} \left( D(i) e^{-\alpha z_1} + \tilde{D}(i) e^{\alpha z_1} \right) - 2\sqrt{-1} \zeta \xi_1 \xi_2 \zeta_1 \xi_2 \alpha \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) + z_1 \alpha^2 \left( C(i) e^{-\alpha z_1} + \tilde{C}(i) e^{\alpha z_1} \right) 
\]

\[
\tilde{f}_{xy}^{(i)} = -\zeta \xi \xi_1 \xi_{1,2} \left( D(i) e^{-\alpha z_1} + \tilde{D}(i) e^{\alpha z_1} \right) - 2\sqrt{-1} \zeta \xi_1 \xi_2 \zeta_1 \xi_2 \alpha \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) + \sqrt{-1} \times \\
\xi_2 \xi \xi_2 \xi_2 \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) - \xi \xi_1 \xi_2 \xi_2 \left( C(i) e^{-\alpha z_1} + \tilde{C}(i) e^{\alpha z_1} \right) 
\]

\[
\tilde{f}_{yz}^{(i)} = -\zeta \xi \xi_1 \xi_{1,2} \left( D(i) e^{-\alpha z_1} + \tilde{D}(i) e^{\alpha z_1} \right) - 2\sqrt{-1} \zeta \xi_1 \xi_2 \zeta_1 \xi_2 \alpha \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) - \zeta \xi_2 \xi_2 \xi_2 \alpha \left( C(i) e^{-\alpha z_1} + \tilde{C}(i) e^{\alpha z_1} \right) - \sqrt{-1} \zeta \xi_1 \xi_2 \zeta_1 \xi_2 \alpha \left( B(i) e^{-\alpha z_1} + \tilde{B}(i) e^{\alpha z_1} \right) - \sqrt{-1} \zeta \xi_2 \xi_2 \xi_2 \alpha \left( C(i) e^{-\alpha z_1} + \tilde{C}(i) e^{\alpha z_1} \right) 
\]
\[
\tilde{f}^{(i)}_{z_1} = -\sqrt{1-\zeta^2} \alpha (D^{(i)} e^{-\alpha z_1} - \bar{D}^{(i)} e^{\alpha z_1}) + \sqrt{1-\zeta^2} \alpha (1-\nu_1) ((B^{(i)} e^{-\alpha z_1} - \bar{B}^{(i)} e^{\alpha z_1}) - (C^{(i)} e^{\alpha z_1} - \bar{C}^{(i)} e^{-\alpha z_1})).
\]

The FRFs were defined with the aid of the following variables:

\[
\alpha = \sqrt{\zeta^2 + \zeta^2} ; \quad \mu = G_z / G_z ; \quad \nu = (1-\nu_2)/(1-\nu_1) ; \quad \kappa = (\mu - 1)/ (\mu + 3 - 4\nu_1)
\]

\[
\beta_1 = -1/[2\alpha (1-\nu_1) [(1+\mu) + (1-\mu) e^{-2\alpha h}]]; \quad \beta_2 = 2\sqrt{-1} \zeta \beta_1 / (\nu \alpha)
\]

\[
B^{(i)} = \beta_1 (\mu + 1) ; \quad \bar{B}^{(i)} = \beta_2 (\mu - 1) e^{-2\alpha h} ; \quad B^{(2)} = 2\beta_1 e^{-\alpha h} / \nu
\]

\[
\lambda_0 = -4(1-\nu_1) / [1+\mu (3-4\nu_2)] ; \quad \lambda_1 = -\sqrt{1-\zeta^2} (B^{(i)} - \bar{B}^{(i)})
\]

\[
\lambda_2 = -2\sqrt{-1} \zeta (1-\nu_1) (B^{(i)} + \bar{B}^{(i)}) / \alpha ; \quad \lambda_3 = 2\sqrt{-1} \zeta \beta_1 [(1-\nu)/(\nu \alpha) + h \mu]
\]

\[
\lambda_4 = 2\sqrt{-1} \zeta \beta_2 [2(1-\nu_2)^2 (1-\mu) / (\nu \alpha) + h]; \quad \lambda_5 = 2\sqrt{-1} \zeta \beta_2 h
\]

\[
\lambda_6 = 2\sqrt{-1} \zeta \beta_2 [(\mu - \nu)/(\nu \alpha) + h \mu] ; \quad \gamma_1 = (2\nu_2 - 1) \beta_2 / [1+\mu (3-4\nu_2)]
\]

\[
\gamma_2 = [(\lambda_2 - \lambda_1) (\lambda_0 + 2\alpha h) + \lambda_4 + \lambda_2 - 2\lambda_4 + 4(1-\nu_2) (\mu \gamma_1 + \beta_2)] e^{-2\alpha h} \times [2(\lambda_0 e^{-2\alpha h} - 1 + 2\alpha h e^{-2\alpha h})]^{-1}
\]

\[
\gamma_3 = [\lambda_4 + \lambda_2 - \lambda_2 - \lambda_4 - (3-2\nu_2) \beta_2 + 2\alpha h (2\gamma_2 - \lambda_2 + \lambda_1)] \times [1+\lambda_4 \kappa e^{-2\alpha h} -(\lambda_0 + \kappa + 4\alpha^2 h^2 \kappa) e^{-2\alpha h}]^{-1}
\]

\[
C^{(i)} = \gamma_3 e^{-2\alpha h} (1 - \kappa e^{-2\alpha h} - 2\alpha h \kappa) - 2\gamma_2 + \lambda_2 - \lambda_1
\]

\[
\tilde{C}^{(i)} = \gamma_3 e^{-2\alpha h} \kappa(\lambda_0 e^{-2\alpha h} + 2\alpha h e^{-2\alpha h} - 1)
\]

\[
C^{(2)} = e^{-\alpha h} [C^{(i)} (1 - \lambda_0) + (1 - \mu) \gamma_1]
\]

\[
D^{(i)} = [\lambda_1 + \lambda_2 - (3-4\nu_1) C^{(i)} + \tilde{C}^{(i)}] / (2\alpha)
\]

\[
\bar{D}^{(i)} = [\lambda_2 - \lambda_1 - C^{(i)} + (3-4\nu_1) \tilde{C}^{(i)}] / (2\alpha)
\]

\[
D^{(2)} = -e^{-\alpha h} [C^{(i)} (3-4\nu_1) (1-\lambda_0) + \gamma_1 (\lambda_0 e^{-2\alpha h} + 2\alpha h e^{-2\alpha h} - 1) (\kappa - 1) + 4(1-\nu_2) \times (\beta_2 + \gamma_1)(2\alpha)^{-1}]
\]

\[
\bar{D}^{(2)} = \tilde{C}^{(2)} = \bar{D}^{(2)} = 0
\]
4. Results and discussions

The indentation of a coated half-space by a rigid sphere is simulated numerically. A reference contact scenario (i.e., a Hertz contact) involving a homogenous half-space with the Young modulus $E_2 = 210$ GPa, the Poisson’s ratio $\nu_2 = 0.3$, and a normal load $W = 1000$ N, is first considered. The maximum (central) pressure $p_{H}$ and the contact radius $a_{H}$ calculated with the proposed numerical program are in good agreement with the Hertz theory. In the subsequent simulations, the maximum pressure is used as normalizer for stresses, and the contact radius for lengths and coordinates. The magnitude of the normal load and the Young moduli of the coating, $E_1$, and of the substrate, $E_2$, are kept constant in all simulations, whereas the coating thickness $h$ and the frictional coefficient $\mu$ are varied. In all cases, $E_1 = 2E_2$. The contour plots of $\sqrt{J_2/p_{H}}$ in the plane $y=0$, with $J_2$ the second invariant of the stress tensor, are depicted in figures 1-4 for various frictional regimes. The location of the maximum is depicted with an X mark and its dimensionless magnitude is indicated on each plot.

The numerical simulations predict significant stress discontinuities at the interface between the coating and the substrate. When the friction coefficient is small (figure 1), the maximum stress is located in the thick coating at a certain depth, or on the free surface when the coating is thin. For a specific coating thickness, the maximum is placed near the interface (figure 1, $h/a_{H} = 0.5$) and its intensity is increased. This scenario may be particularly dangerous for the service life of the contacting element, considering that cracks are more likely to be initiated as a result of the mismatch in the elastic moduli of the coating and of the substrate, when the location of maximum shear stress is at the coating-substrate interface.

Figure 1. Iso-contours of $\sqrt{J_2/p_{H}}$ in the plane $y=0$, $\mu = 0.2$. 

.png
Figure 2. Iso-contours of $\sqrt{J_z}/p_H$ in the plane $y=0$, $\mu=0.4$.

Figure 3. Iso-contours of $\sqrt{J_z}/p_H$ in the plane $y=0$, $\mu=0.6$. 
Figure 4. Iso-contours of $\sqrt{J_z/p_n}$ in the plane $y = 0$, $\mu = 0.8$.

More intense frictional regimes ($\mu > 0.3$, figures 2-4) always place the maximum on the contact area, toward the trailing edge of the contact (i.e., negative $x$ coordinates). For very thin coatings, the $x$ coordinate of the maximum nearly reaches the contact radius in the direction opposite to that of the tangential force. Generally, the higher the value of the frictional coefficient, the greater the intensity of the maximum stress. When $\mu = 0.8$, the maximum stress is nearly 3.5 times higher than that (of 0.358, according to [3]) for the frictionless and uncoated contact.

5. Conclusions
The contact problem involving friction and coated bodies lacks analytical solution. A numerical approach is employed in this paper to investigate the combined influence of the coating thickness and of the frictional coefficient value on the stress state developed in the coated system. The simulations focus on location and intensity of the maximum von Mises equivalent stress, which is an indicator of the plastic flow susceptibility in the elastic body.

The calculation of stresses is performed in the frequency domain, based on closed-form solutions for displacement and stresses induced in a coated half-space by unit point forces. The contact process is simulated using a simplified approach that decouples the contact problems in the normal and in the tangential directions. Essentially, the pressure distribution is obtained from a frictionless contact process, whereas shear tractions results from a Coulombian friction law. This assumption is well adapted to full-sliding considering that the normal displacement induced by the shear tractions is small compared to its counterpart induced by pressure. The strong point of the proposed method consists in its computational efficiency, allowing for increased resolution in stress calculation.
The simulations suggest that increasing the frictional coefficient beyond 0.3 shifts the maximum stress position from depth to the contact surface and its coordinate can reach the trailing edge of the contact. Specific coating configurations place the maximum stress at the interface between the coating and the substrate and therefore should be avoided to minimize the probability of crack initiation and propagation.

6. References
[1] Ju Y and Farris T N 1996 J. Tribol. – Trans. ASME 118 320
[2] Burmister D M 1945 J. Appl. Phys. 16 126
[3] O’Sullivan T C and King R B 1988 J. Tribol. – Trans. ASME 110 235
[4] Nogi T and Kato T 1997 J. Tribol. – Trans. ASME 119 493
[5] Polonsky I A and Keer L M 1999 J. Tribol. – Trans. ASME 122 30
[6] Liu S B, Wang Q and Liu G 2000 Wear 243 101
[7] Liu S B and Wang Q 2002 J. Tribol. – Trans. ASME 124 36
[8] Wang Z J, Wang W-Z, Wang H, Zhu D and Hu Y-Z 2010 J. Tribol. – Trans. ASME 132 021403
[9] Zhang P Y, Diao D F and Wang Z J 2012 J. Tribol. – Trans. ASME 134 021301
[10] Yu C, Wang Z, Liu G, Keer L M and Wang Q J 2016 J. Tribol. – Trans. ASME 138 041402
[11] Yu C, Wang Z and Wang Q J 2014 Mech. Mater. 76 102
[12] Spinu S and Cerlinca D 2018 IOP Conf. Ser.: Mater. Sci. Eng. 400 042054
[13] Spinu S and Cerlinca D 2018 IOP Conf. Ser.: Mater. Sci. Eng. 400 042055
[14] Kalker J J and Van Randen Y 1972 J Eng Math 6 193
[15] Johnson K L 1985 Contact Mechanics (Cambridge: University Press)
[16] Polonsky I A and Keer L M 1999 Wear 231 206

Acknowledgement
This work was partially supported from the project “Integrated Center for Research, Development and Innovation in Advanced Materials, Nanotechnologies, and Distributed Systems for Fabrication and Control”, Contract No. 671/09.04.2015, Sectoral Operational Program for Increase of the Economic Competitiveness co-funded from the European Regional Development Fund.