PARTIAL MYOPIA VS. FORWARD-LOOKING BEHAVIORS IN A DYNAMIC PRICING AND REPLENISHMENT MODEL FOR PERISHABLE ITEMS

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Abstract. This paper studies a dynamic pricing and replenishment problem for perishable items considering the behavior of decision-maker (partially myopic or forward-looking) and the dynamic effects of cumulative sales. A dynamic optimization model is presented to maximize the total profit per unit time and solved on the basis of Pontryagin’s maximum principle. The optimal pricing and replenishment strategies for partially myopic and forward-looking scenarios are obtained. By comparing the partially myopic and forward-looking strategies through numerical analysis, we find the main results: First, applying a skimming pricing strategy might be a good choice when the saturation effects are considered. Second, the decreasing rate of product sales, deterioration coefficient, and holding cost of perishable items per unit exhibit impact on the behavioral preference of decision-maker. Under certain conditions, partially myopic behavior can bring more profit than forward-looking behavior. These managerial implications provide useful guidelines for the decision-maker.

1. Introduction. In principle, a decision-maker should apply forward-looking strategies with the purpose of focusing on the long-term performance of an objective. However, previous studies, such as by [7] and [29], have argued that it is difficult to correctly accommodate the intertemporal impact of the decision-making. Sometimes, even for experienced decision-maker, applying the forward-looking approaches can be a challenge [24]. [16] suggest that employees (e.g., managers, service providers, etc.) are generally more focused on current strategies than a firm would like. As shown by [8], “shorter-horizon” games may explain firm behavior better than more complex “longer-horizon” games. Indeed, due to a range of factors, such as lack of information, high costs, etc., decision-maker is unable to consider the impact of their decisions on all future environments. Under this condition, the effects of some strategies on the future have to be neglected, which is known as partially myopic behavior [24]. Within the context of a dynamic pricing environment, myopic
behavior refers to situations in which the decision-maker ignores the future impact of decisions on the dynamics that describes the evolution of the corresponding environment [17, 4]. Decision-maker may prefer to adopt partially myopic strategies if this behavioral choice can bring higher profits. Several papers, such as by [13], [4], and [9], have investigated the effects of myopic behavior on operational decisions. However, little attention has been paid to the effects of partial myopia on operation decisions for perishable items, such as the pricing strategy and replenishment strategy. In this study, we try to fill this gap.

Perishable items, e.g., electronic components, fashion goods, etc., will always deteriorate, which results in quantitative changes during their normal storage period. The loss of these perishable items mainly stems from inappropriate inventory control [12]. The implementation of appropriate inventory and replenishment policies has a significant impact on a firm’s profits, especially for perishable products. From both a financial and an operational standpoint, pricing is one of the most effective ways for managers to influence demand. It is also a helpful tool to manage inventory and replenishment policies [5]. In addition, present pricing strategy can affect the future cumulative sales of a product. Pricing becomes more complicated when the decision-maker considers the dynamic effects of strategies.

Considering the behavior of the decision-maker (partially myopic or forward-looking) and the dynamic effects of cumulative sales, this study focuses on the following research questions: (1) What are the optimal pricing strategies for perishable items when the decision-maker is partially myopic or forward-looking; (2) How often, and in what quantities, should the inventory be replenished under the partially myopic and forward-looking scenarios; (3) Which behavior is better preference for the decision-maker, and how do the main parameters in the model affect this preference?

To answer these research questions, this paper develops an inventory control model for perishable items, which can help to determine the appropriate replenishment policy as well as the pricing strategy. A continuous time optimal control method is presented to provide an exact solution of the problem. We analytically derive the optimal strategies based on the partially myopic and farsighted behaviors considering the dynamic effects of cumulative sales. The optimal strategies obtained in this paper can provide guidelines for the decision-maker to set the optimal strategy. Particularly, our results suggest that applying a skimming pricing strategy might be a good choice when the sales’ saturation effects are considered. Our results also illustrate that the decreasing rate of product’s sales, the deterioration coefficient and the holding cost of perishable items per unit exhibit impact on the behavioral preference of the decision-maker. Particularly, under certain conditions, partially myopic behavior can bring more profit than forward-looking behavior although myopic behavior is commonly considered detrimental. This result may be a blessing in disguise for the decision-maker who is more concerned with current benefits or who is unable to look ahead.

The contributions of this study can be summarized as follows: First, the saturation effects are incorporated into a dynamic pricing and replenishment model for perishable items, which has received little attention previously. Second, the behavior of decision-maker, that is partially myopic behavior or forward-looking behavior, is incorporated into the dynamic pricing and replenishment model for perishable items. We present the joint pricing and replenishment policies for the partially myopic and forward-looking scenarios, respectively. The optimal dynamic
policies obtained by applying Pontryagin’s maximum principle can provide managerial hints to the decision-maker. Third, we illustrate that, under some conditions, the partially myopic strategy can bring more profit than the forward-looking strategy. Our results offer firms some guidelines on how to effectively manage behavioral choice.

The remainder of this paper is organized as follows: In Section 2, we review the existing literature related to our study. In Section 3, we provide a description of the joint pricing and replenishment model for perishables. In Section 4, we derive the optimal pricing strategy with partially myopic behavior. Also, an iterative algorithm is presented to find the optimal replenishment cycle. In Section 5, we give optimal strategies with forward-looking behavior. In Section 6, we present a numerical study to analyze the difference in strategies and profits between the partially myopic and forward-looking scenarios, and obtain some managerial insights into behavioral preference. Section 7 concludes this paper.

2. Literature review. Our work is mainly related to two streams of literature: 1) inventory control and pricing for perishable items; 2) myopic and forward-looking behaviors in the dynamic marketing literature. The inventory system for perishable items has been reported by [14] and [2]. Joint pricing and inventory control policies for the perishable items have gained much attention, such as [46], [52], [10], [45], [22], [32], [6], [33], [25], [43], [38], [34], [35]. In particular, [47] apply heuristics and search methods to derive the optimal replenishment and pricing strategies with considering the time-value of money in a deteriorating inventory system with price-dependent demand. [21] develop an integrated production-inventory model in order to determine the optimal production and replenishment strategies in the presence of multiple deliveries, partial back-ordering and time discounting. [48] study the optimal replenishment strategy considering the ameliorating and deteriorating effects, time value of money and finite planning horizon with the amelioration and deterioration rates following Weibull distributions. [40] study the pricing and lot-sizing problem for the perishable items where the demand is dependent on price, quality and on-display stock level. With allowing shortages and partially backlogged, [26] give the optimal selling price, replenishment schedule and order quantity with employing a price-dependent stochastic demand function. [41] consider the effect of changes in price on the product’s demand and investigate the dynamic pricing and inventory replenishment strategies for perishable items with instantaneous qualitative and noninstantaneous physical deterioration. [49] study a joint dynamic pricing and inventory control model for deteriorating items while considering the reference price effect. [11] presents an optimal replenishment model with dynamic pricing and quality investment for perishable products, in which both the quality and physical quantity deteriorate. Considering trade-credit policy, [36] present an integrated vendor-buyer model for deteriorating items, and [37] are concerned with a multi-item inventory model for deteriorating items.

For the myopic and forward-looking behaviors in the dynamic marketing literature, some researchers have focused on the marketing channel, such as [3], [13], [17], [39], [4], [15], [28], [9], and [23]. Particularly, [42] and [18] prove that the myopic behavior has negative effect on firms’ profits. [15] indicate that the manufacturer will be better off with a myopic retailer under some conditions. [9] finds that both the manufacturer and the retailer may be better off in the long run when they ignore the impact of current prices on the future demand and focus on immediate-term
profits. [24] study the effect of the partially myopic behavior of channel members on the performance of supply chain. They show that under decentralized scenario, partially myopic channel members can get higher profits than the forward-looking ones, while the opposite result occurs in centralized scenario.

The essential difference between these streams of literature and our work is that we incorporate the decision-maker’s behavior (partial myopia or forward-looking) into a dynamic pricing and replenishment model for perishable items, and present the optimal strategies when the decision-maker is partially myopic or forward-looking, respectively. In addition, we also show which behavior is better preference for the decision-maker, and how the main parameters in the model affect this preference.

3. Model formulation. In the section, a dynamic pricing model for perishable items with dynamic demand is presented. Our study focuses on a monopolistic market without direct competition. Consider a firm selling a single type of perishable products during the planning horizon. This assumption can be explained by the case that a firm releases a new product to the market. We consider the constant units of products arrive at the inventory system at the beginning of each replenishment cycle and drop to zero due to sales and deterioration. We also assume that shortages are not allowed to avoid lost sales.

We begin with explaining the dynamic demand. The demand for products depends not only on current price, but also on cumulative sales, which represents the diffusion and saturation effect [30], [9] presents a continuous time approach to characterize the products diffusion process. Let \( x(t) \) and \( p(t) \) denote the cumulative demand and the price at time \( t \), respectively. According to [9] and [30], the differential equation of demand dynamics can be expressed as

\[
\dot{x}(t) = \alpha - p(t) - \delta x(t), \quad x(0) = x_0,
\]

where \( \alpha > 0 \) is market potential; \( \delta > 0 \) is the decreasing rate of product’s sales. The parameter \( \delta \) represents the saturation effects (the shrinkage in potential demand with increasing market penetration). This parameter captures the proportion of market that will not buy the product at time \( t \) due to market’s saturation effects. Equation (1) can describe a new product diffusing in the market.

We consider a firm selling perishable products under a sales cycle \([0, T]\) with a given initial inventory level \( I_0 \) at the beginning. As the nature of perishable items, the serviceable inventory decays at rate \( f(I) \). When the perishable items are held in the serviceable inventory, the deterioration occurs immediately. For mathematical tractability, the \( f(I) \) is supposed to

\[
f(I) = \theta I,
\]

with a natural deterioration coefficient \( \theta > 0 \) [49]. Thus, the inventory level \( I(t) \) at time \( t \) can be expressed as

\[
\dot{I}(t) = -\dot{x}(t) - \theta I(t), \quad I(T) = 0.
\]

From Equation (3), it is found that the inventory level is non-negative throughout the whole planning horizon, i.e., \( I(t) \geq 0, t \in [0, T] \). Thus no shortage occurs in the planning horizon.

Following [49] and [43], it is assumed that the inventory holding cost \( C(I) \) is a linear function of the current inventory level as

\[
C(I) = hI,
\]
where $h > 0$ represents the holding cost of perishable items per unit.

The firm’s total profit consists of the income of product sales subtracting costs. The costs incurred for each replenishment cycle comprise: an inventory cost for holding the stocks, a replenishment cost denoted by $c$, and a constant cost denoted by $k$ which may represent fixed delivery and order-processing costs. Accordingly, the total profit per unit time in the planning horizon $[0, T]$ can be described as

$$J = \frac{1}{T} \int_{0}^{T} (p(t) \dot{x}(t) - hI(t)) \, dt - \frac{cI_0 + k}{T}. \quad (5)$$

The goal is to find the optimal pricing strategy $p(t)$ and replenishment policy that maximize the total profit per unit time $J$ over the selling horizon. By combining (1)-(5), the optimization problem for a monopolist, therefore, can be formulated as an optimal control model given by

$$\max_{p(\cdot), T} J = \frac{1}{T} \int_{0}^{T} (p(t) \dot{x}(t) - hI(t)) \, dt - \frac{cI_0 + k}{T}$$

s.t. $\dot{I}(t) = -\dot{x}(t) - \theta I(t)$, 

$\dot{x}(t) = \alpha - p(t) - \delta x(t)$, 

$I(T) = 0, \ x(0) = x_0$. \quad (6)

We should mention that the replenishment quantity $I_0$ is also a decision variable in the optimization problem (6). If the optimal strategies $p(t), T$ are obtained, the optimal inventory $I(t)$ is determined, then the replenishment quantity $I_0$ can be obtained by calculating $I_0 = I(0)$. So we do not explicitly treat $I_0$ as a decision variable.

4. **Partially myopic scenario.** In this section, we suppose that the decision-maker is partially myopic when designing the optimal strategy to maximize his profit. Partially myopic behavior indicates that the decision maker considers the effect of inventory dynamics and disregards sales evolution. As shown by [27] and [1], partially myopic behavior is common in both theory and practice. In practice, the inventory level is observable, thus the evolution of inventory is easily known. When the evolution of sales, however, is difficult to predict due to fund shortages, lack of information or other reasons, then decision-maker may act partially myopically [24]. Partially myopic decision maker tends to consider the instantaneous sales of a product while ignoring future results.

In the following, we first consider the dynamic pricing problem for a given replenishment period $T$, and characterize the optimal pricing policy with partially myopic behavior. Afterwards, an algorithm is designed to search the optimal replenishment period $T$. We employ Pontryagin’s maximum principle proposed in [19] to solve the optimal control problem (6). With partially myopic behavior, the decision-maker disregards sales evolution. We introduce the adjoint variable $\lambda$ associated with the state variable $I$, and form the Hamiltonian as follows

$$H_m(I, p, \lambda) = p(\alpha - p - \delta x) - hI + \lambda(-\alpha - p - \delta x - \theta I). \quad (7)$$

For the adjoint variable $\lambda$ which satisfies the adjoint equation $\dot{\lambda} = -\frac{\partial H_m}{\partial I}$, we can get

$$\dot{\lambda} = \theta \lambda + h, \ \lambda(0) = 0. \quad (8)$$

From the maximum principle, the optimal solution $p_{m}^*$ to optimal control model (6) has to maximize the Hamiltonian $H_m$ at each instant $t$, which is given as
with the optimal inventory $I^*$ following the differential equation (3).

From the necessary conditions for optimality above, we obtain the optimal pricing strategy with partially myopic behavior proposed in Proposition 1. All proofs are given in the Appendix.

**Proposition 1.** With a given replenishment cycle $T$, the optimal pricing strategy with partially myopic behavior is

$$p_m^*(t) = \frac{2h(\delta + \theta)e^{\theta t} + \theta(2h + (\alpha - \delta x_0)A)e^{-\frac{\theta}{2}t} - 2hA}{2\theta A},$$

and the corresponding inventory level and cumulative sales over time are

$$I_m^*(t) = \frac{h(\delta - 2\theta) e^{2\theta t} - 2h + 2\theta(2h + (\alpha - \delta x_0)A)(e^{-\frac{\theta}{2}t + (T-t)\theta} - e^{-\frac{\theta}{2}t})}{2(4\theta^3 - \delta^2\theta)},$$

$$x_m^*(t) = \frac{(h + \alpha \theta) A - h\delta e^{\theta t}}{\delta A} + x_0 - \frac{\delta A + 2h}{\delta A},$$

where $A = \delta + 2\theta$.

Using Equation (11), the replenishment quantity $I_0$ for the partially myopic case is obtained as follows:

$$I_m(0) = \frac{h(\delta - 2\theta) e^{2\theta t} + h(2\theta - \delta) e^{\theta t} + 2\theta(2h + (\alpha - \delta x_0)A)(e^{-\frac{\theta}{2}t + (T-t)\theta} - e^{-\frac{\theta}{2}t})}{2(4\theta^3 - \delta^2\theta)}.$$

Given the replenishment period $T$, the corresponding total profit per unit time $J_m(T)$ can be generated by substituting the optimal pricing strategy with partially myopic behavior (10), the inventory level (11) and sales level (12) into the objective function of (5). In order to obtain the optimal replenishment period $T$ that maximizes the total profit per unit time, we formulate the following optimization problem on the basis of the optimization problem (6).

$$\max_T \Pi_m = J_m(T)$$

s.t. $T \geq 0.$

It should be mentioned that the concavity of $\Pi_m$ in $T$ is hardly verified through a mathematically analytic approach although we can calculate the value of $\Pi_m$ for any given $T$. Thus, the optimal replenishment cycle cannot be obtained analytically. Inspired by [51] and [50], an iterative algorithm is designed to find the optimal replenishment cycle $T$.

**Algorithm 1**

**Step 1:** Start with $i = 0$ and initialize the value $T_i = 0$. Give a sufficiently small computational accuracy parameter $\varepsilon > 0$ and a sufficiently small iterative step-size $s > 0$.

**Step 2:** For given $T_i$, calculate the optimal price $p_i$ by solving the optimization problem (6) and the total profit per unit time denoted as $\Pi_i$. Set $T_{i+1} = T_i + s$. And calculate the optimal price $p_{i+1}$ as well as the total profit per unit time denoted as $\Pi_{i+1}$.

**Step 3:** If $\Pi_{i+1} - \Pi_i \leq \varepsilon$, set $T^* = T_i$, then output $T^*$ and stop. Otherwise, set $i = i + 1$ and go to Step 2.
Remark 1. We must mention that although the concavity of $\Pi_m$ with respect to $T$ cannot be proved by a mathematically analytic approach. We have performed a large number of numerical examples and found that the function $\Pi_m$ turns out to be concave for each given set of parameters we have tried. Thus, what we can ensure is that for each set of parameters we have tried, the functional to be optimized turns out to be concave and the algorithm could be used. Indeed, the above algorithm is one-dimensional search algorithm. This algorithm can be executed conveniently by the mathematical computing software such as Matlab.

5. Forward-looking scenario. In this section, we assume that the decision-maker is forward-looking when designing the pricing policy to maximize profit. With forward-looking behavior, the decision-maker is far-sighted and takes into account both the effects of inventory dynamics and the evolution of cumulative sales.

Similar to the partially myopic scenario, we first consider the dynamic pricing problem for a given replenishment period $T$. We also employ Pontryagin’s maximum principle to solve the optimal control problem (6). We introduce the adjoint variables $\lambda_1, \lambda_2$ associated with the state variables $I$ and $x$, respectively, and form the Hamiltonian as follows

$$H_f(I, x, p, \lambda_1, \lambda_2) = p(\alpha - p - \delta x) - hI + \lambda_1(-\alpha - p - \delta x - \theta I) + \lambda_2(\alpha - p - \delta x). \quad (15)$$

It can be seen that the Hamiltonian function involves two parts: the first part is the integrand of the objective function and the second part consists of adjoint variables multiplied by the right-hand sides of state equations (1) and (3). The first part explains the direct contribution to the objective function. The second part represents the indirect contribution to the objective function from the value of the changes in inventory level $I$ and sales $x$ resulted from decisions taken.

For the adjoint variables $\lambda_1, \lambda_2$ which satisfy the adjoint equations $\dot{\lambda}_1 = -\frac{\partial H_f}{\partial I}$ and $\dot{\lambda}_2 = -\frac{\partial H_f}{\partial x}$, respectively, we can get

$$\dot{\lambda}_1 = \theta \lambda_1 + h, \quad \lambda_1(0) = 0, \quad (16)$$

and

$$\dot{\lambda}_2 = \delta (p - \lambda_1 + \lambda_2), \quad \lambda_2(T) = 0. \quad (17)$$

From the maximum principle, the optimal pricing strategy $p_f^*$ has to maximize the Hamiltonian $H_f$ at each instant $t$, which is given as

$$H_f(I_f^*, x_f^*, p_f^*, \lambda_1, \lambda_2) \geq H_f(I_f^*, x_f^*, p_f, \lambda_1, \lambda_2), \quad (18)$$

with the optimal inventory $I^*$ and the optimal sales $x^*$ following the differential equations (1) and (3), respectively.

The following proposition presents the optimal pricing policy $p_f^*$ for dynamic optimization problem (6) with the decision-maker being forward-looking.

**Proposition 2.** With a given replenishment cycle $T$, the optimal pricing strategy with forward-looking behavior is

$$p_f^*(t) = \frac{-e^{\theta t}(2h + 2\theta(\alpha - \delta m_1) + (1 + \delta t)\theta m_2)}{2\theta^2}, \quad (19)$$

and the corresponding inventory trajectory $I_f^*$ and sales $x_f^*$ are respectively characterized as

$$I_f^*(t) = e^{-\theta t}m_3 - \frac{(e^{\theta t} - 2)h + 2\theta(\alpha - \delta m_1 + m_2)}{4\theta^2}, \quad (20)$$
and Algorithm 1, the optimal replenishment cycle can be obtained, which is a function of $T$. We also formulate the following optimization problem on the basis of the optimization problem \((\ref{eq:optimal_solution})\) to obtain the optimal replenishment period $T$ maximizing the total profit per unit time.

\[
\max_T \Pi_f = J_f(T) \\
\text{s.t. } T \geq 0. \tag{\ref{eq:optimal_solution}}
\]

Similar to the partially myopic case, although we can calculate the value of $\Pi_f$ for any given $T$, it is very difficult to verify the concavity of $\Pi_f$ in $T$ and get the optimum solution of the optimal replenishment period via an analytical approach.

### 6. Numerical analysis

While explicit pricing strategies and profits for partially myopic and forward-looking cases can be obtained, it is difficult to compare them directly and obtain an analytical solution due to their complexities. We resort to numerical analysis in order to analyze the difference in strategies and profits between partially myopic and forward-looking scenarios, and further obtain some managerial insights into the behavioral preferences (partially myopic or forward-looking) of the decision-maker.

#### 6.1. Numerical example

This subsection gives numerical examples to illustrate the effectiveness of the above theoretical results. Consider the following parameters: $\alpha = 30$, $x_0 = 5$, $h = 0.15$, $\theta = 0.25$, $\delta = 0.1$, $c = 3$ and $k = 25$. These parameters’ values are selected according to the previous studies in marketing and operations management, such as [44], [51], [24], [49] and [43]. As shown in Fig. 1, the total profits per unit time for partially myopic and forward-looking scenarios are both concave in the replenishment cycle $T$.

For the optimal strategies under the partially myopic scenario, by virtue of Proposition 1 and Algorithm 1, the optimal replenishment cycle $T$ can be obtained as $T^*_m = 1.24$, and the optimal price is

\[
p_m^0(t) = 15e^{-0.05t} + 0.35e^{0.25t} - 0.6.
\]

With the above optimal pricing policy, the optimal replenishment quantity per cycle can be calculated from Equation \((\ref{eq:optimal_solution})\) as $I_{m0} = 20.68$, and the optimal profit per unit time is obtained as $J_m = 132.9902$.\]
For the optimal strategies under the forward-looking scenario, by virtue of Proposition 2 and Algorithm 1, the optimal replenishment cycle $T$ can be obtained as $T^*_f = 1.23$, and the optimal price is

$$p^*_f(t) = 15.7816 - 0.18e^{0.25t} - 1.41984t. \tag{24}$$

With the optimal policy (24), the optimal replenishment quantity per cycle for the forward-looking scenario can be calculated from Equation (22) as $I_{f0} = 20.0926$, and the optimal profit per unit time is obtained as $J_f = 132.774$.

In order to investigate how decision behavior (partially myopic or forward-looking) affects pricing strategy, we depict the optimal pricing strategies of partially myopic and forward-looking scenarios in the following figure.
From Fig. 2, we can find that during most of the replenishment cycle, the price in the forward-looking case is higher than that in the partially myopic case. This means that forward-looking behavior, to some extent, induces a higher selling price, while partially myopic behavior decreases the selling price. When the replenishment cycle is near the end, the results are reverse, i.e., the price in the partially myopic case is higher than that in the forward-looking case. This indicates that partially myopic behavior may induce a higher selling price at the end of the replenishment cycle.

In addition, according to Fig. 2, the optimal selling price decreases over time during the optimal replenishment cycle. This result is different from that obtained by [31] which gives an optimal increasing pricing policy for deteriorating items with limited special order quantities. The results presented in this paper suggest that it is optimal to apply a skimming pricing strategy when the sales saturation effects are considered. Setting a higher selling price in the inception phase can compensate for a decline in product sales in the future, which is beneficial.

6.2. Behavioral choice (partially myopic or forward-looking). In this subsection, we compare the profits under the partially myopic scenario and the forward-looking scenario, and address the following question: which behavior (partially myopic or forward-looking) is better? There are three parameters in our model: a decreasing product sales rate $\delta$, the natural deterioration coefficient $\theta$, and the holding cost of perishable items per unit $h$. We will investigate how the changes in these main parameters affect the behavioral preference. We also study the effects of other parameters on the behavioral preference, and the results show that their effects are very small.

In the following, we investigate how the changes in decreasing rate of product sales $\delta$ affect the behavioral preference. When one parameter’s value is varied, the others’ values remain unchanged. Fig. 3 presents the optimal profits per unit time $J_m$ and $J_f$ with $\delta$ taking the values of 0.1 and 0.8, respectively.

![Figure 3](image_url)

**Figure 3.** The effect of $\delta$ on the unit time total profits $J_m$ and $J_f$

We observe from Fig. 3(a) that, when the decreasing rate of product sales $\delta$ is relatively small, $J_f^* < J_m^*$. This indicates, for a relatively small decreasing rate of product sales, it is optimal to take the partially myopic strategy. When the decreasing rate of product sales is relatively high, as shown in Fig. 3(b), $J_f^* > J_m^*$. 

...
which implies that the forward-looking strategy is a better choice for the decision-maker. These results may be explained by the following: When $\delta$ is relatively small, the saturation degree of the product’s sales is low. This means that, during the replenishment cycle, the product’s sales shrink slowly. For the decision-maker, the negative effect from sales accumulation of the product on profit is limited. Thus, the partially myopic strategy is the optimal choice, because it can induce a lower selling price, which stimulates sales and generates a higher profit. When $\delta$ is relatively high, the degree of saturation of the product’s sales is high. This implies that the decision-maker faces a relatively high decreasing rate of product’s sales during the replenishment cycle. Thus, it is optimal to adopt the forward-looking strategy with considering the effect of pricing strategy on product sales in the future. The forward-looking strategy can induce a higher selling price during most of the replenishment cycle. A higher selling price can decrease the speed of sales accumulation and thereby compensate for the decline in product sales in the future. This can weaken the negative effect induced by fast sales accumulation on profit and result in a higher overall profit.

Next, we investigate how the changes in the holding cost of perishable items per unit $h$ and the deterioration coefficient $\theta$ affect behavioral preference. In order to examine the impact of $h$ and $\theta$ on the optimal total profit per unit time $J_m$ and $J_f$, we set $h = 0.5$ and $h = 1.5$, respectively. Fig. 4 depicts the optimal profits per unit time $J_m$ and $J_f$ with $\theta$ taking the values 0.1, 0.1, respectively.

**Figure 4.** The effect of $\theta$ on the unit time total profits $J_m$ and $J_f$
We make some observations on Fig. 4. From Figs. 4(a) and 4(b), we find that when the holding cost per unit is relatively low (h = 0.5), $J_{f}^*$ is always larger than $J_{m}^*$, regardless of whether $\theta$ is relatively low or high. This means that the forward-looking strategy is always the optimal choice for the decision-maker with a relatively low holding cost. According to Figs. 4(c) and 4(d), we can see that when the holding cost per unit is relatively high (h = 1.5), the result is conditional on $\theta$. When the deterioration rate $\theta$ is relatively low, Fig. 4(c) shows that $J_{m}^* < J_{f}^*$. This means that the forward-looking strategy is the optimal choice for the decision-maker. When the deterioration rate is relatively high, from Fig. 4(d), we have $J_{m}^* > J_{f}^*$, which implies that it is optimal to choose the partially myopic strategy. Compared to the partially myopic strategy, the forward-looking strategy can effectively mitigate the loss caused by the declined product sales in the future because this strategy takes the dynamics of product sales into consideration. When the holding cost per unit (or the deterioration rate) is low, the negative effect of this low holding cost (or the deterioration rate) on the profit is limited. For the decision-maker, under this condition, more attention should be paid to the effect of pricing policy on the dynamics of product sales when designing the optimal pricing strategy. Thus, it is optimal to choose the forward-looking strategy, which takes into account the effects of inventory dynamics and cumulative sales evolution in order to balance current profits and future benefits. When both the holding cost per unit and the deterioration rate are high, the negative effect from the relatively high holding cost and deterioration rate on profit is great. Our results suggest that it is optimal to choose the partially myopic strategy. Under this condition, more attention should be paid to the dynamics of the inventory in order to save on the inventory cost and reduce deterioration losses. Thereby, the partially myopic strategy is the optimal choice, which tends to set a lower price to boost sales myopically and reduce the inventory level quickly. With partially myopic behavior, setting a lower selling price can lead to a sacrifice in the future benefits for the sake of current benefits.

7. Conclusion. This paper studies the problem of designing joint pricing and replenishment policies for a monopolistic firm which sells perishable items to customers, considering the behavior of the decision-maker (partially myopic or forward-looking) and the dynamic effects of cumulative sales. The deterioration rate of inventory is proportional to the physical quantity. To maximize the total profit per unit time, we establish a dynamic optimization model to set the optimal price and replenishment policies over time and decide how often, and in what quantities, the inventory should be replenished. The optimal policies for the partially myopic and forward-looking scenarios are obtained by applying Pontryagin’s maximum principle. By comparing the partially myopic and forward-looking strategies through numerical analysis, some managerial insights on pricing and behavioral preference are provided.

The theoretical contributions and important managerial implications of this paper are summarized as follows: First, considering the dynamic effects of cumulative sales, we present joint pricing and replenishment policies for the partially myopic and forward-looking scenarios, respectively. The analytical results of the optimal policies obtained by applying Pontryagin’s maximum principle can serve as useful tools for the decision-maker. Second, applying a skimming pricing strategy might be a good choice for the decision-maker when the sales saturation effects are considered. Third, our results illustrate that the decreasing rate of product sales, the
deterioration coefficient, and the holding cost of perishable items per unit exhibit impacts on the behavioral preference. Under certain conditions, partially myopic behavior can bring more profit than forward-looking behavior. Our results offer firms some guidelines about optimal pricing and replenishment decisions for perishable items considering the saturation effects, and provide insights to effective behavioral choice.

This research has a few limitations, although some useful managerial insights are presented. There are several extensions that can be investigated in the future. First, the impact of competition between different firms with different behavioral choices may be studied. Adding competition to the model may generate some interesting results. Second, adjusting the selling price may cause a cost. It may be interesting to consider the cost of price adjustments in the model.

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Appendix A. Proof of Proposition 1. Denote $A = \delta + 2\theta$. Note that the Hamiltonian $H_m$ is concave in $p$. Thus according to the maximum principle (9), the first-order condition for optimality is

$$\frac{\partial H_m}{\partial p} = 0,$$

which yields the optimal pricing policy as

$$p^*_m = \frac{1}{2}(\alpha - x\delta + \lambda). \quad (25)$$

With the transversality condition $\lambda(0) = 0$, solving the adjoint equation (8) gives rise to

$$\lambda = \frac{h}{\theta}(e^{\theta t} - 1). \quad (26)$$

Substituting Equations (25) and (26) into Equation (1), we obtain

$$\dot{x}(t) = \frac{h - he^{\theta t} + \alpha \theta - \delta \theta x}{2\theta} \quad (27).$$

Solving this differential equation (27) with $x(0) = x_0$ gives the optimal cumulative sales over time as

$$x^*_m(t) = \frac{(h + \alpha \theta)A - h\delta e^{\theta t}}{\theta\delta A} + \left(x_0 - \frac{\delta A + 2h}{\delta A}\right)e^{-\frac{h}{\theta}t}. \quad (28)$$

With substituting Equations (26) and (28) into Equation (25), we obtain the optimal pricing strategy with partially myopic behavior as

$$p^*_m(t) = \frac{2h(\delta + \theta)e^{\theta t} + \theta(2h + (\alpha - \delta x_0)A)e^{-\frac{h}{\theta}t} - 2hA}{2\theta A}. \quad (29)$$

Substituting Equations (28) and (29) into differential equation (3) and solving the differential equation with $I(T) = 0$, we can obtain the corresponding inventory level as

$$I^*_m(t) = \frac{h(\delta - 2\theta)e^{2T\theta - \theta t} + h(2\theta - \delta)e^{\theta t} + 2\theta(2h + (\alpha - \delta x_0)A)(e^{-\frac{h}{\theta}t}(T-t)\theta - e^{-\frac{h}{\theta}T})}{2(4\theta^3 - \delta^2 \theta)}. \quad (30)$$

The proof is complete.
Appendix B. Proof of Proposition 2. It can be found that the Hamiltonian $H_f$ is concave in $p$. According to the maximum principle (18), the first-order condition for optimality is

$$\frac{\partial H_f}{\partial p} = 0,$$

which yields the optimal pricing policy as

$$p_f^* = \frac{1}{2}(\alpha - x\delta + \lambda_1 - \lambda_2).$$

With the transversality condition $\lambda_1(0) = 0$, solving the adjoint equation (16) gives rise to

$$\lambda_1 = \frac{h}{\theta}(e^{\theta t} - 1).$$

Substituting Equations (30) and (31) into Equations (1) and (16), respectively, gives

$$\dot{x} = \frac{h - e^{\theta t}h + \theta(\alpha - \delta x + \lambda_2)}{2\theta},$$

and

$$\dot{\lambda}_2 = \frac{\delta(h - e^{\theta t}h + \theta(\alpha - \delta x + \lambda_2))}{2\theta}.$$ (33)

Solving the first-order linear differential equations (32) and (33) with $x(0) = x_0$ and $\lambda_2(T) = 0$, we can obtain the optimal cumulative sales over time as

$$x_f^*(t) = \frac{-e^{\theta t}h + h\theta t + t\alpha\theta^2 + 2\theta^2m_1 - \delta\theta^2m_1t + t\theta^2m_2}{2\theta^2},$$

and the corresponding inventory level as

$$I_f^*(t) = e^{-\theta t}m_3 - \frac{(-2 + e^{\theta t})h + 2\theta(\alpha - \delta m_1 + m_2)}{4\theta^2},$$

where $m_1 = \frac{h}{2\theta^2}$, $m_2 = \frac{\delta(2e^{\theta T}h + T(h(\delta - 2\theta)2(\alpha - x_0\delta)\theta^2))}{2(2+T\theta)\theta^2}$, $m_3 = \frac{e^{\theta T}((-2+e^{\theta T})h + 2\theta(\alpha - \delta m_1 + m_2))}{4\theta^2}$. (35)

By substituting Equations (34) and (35) into Equation (30), we can obtain the optimal pricing strategy for the forward-looking case as

$$p_f^*(t) = \frac{-e^{\theta t}(\theta - \delta) + \theta((1 - \delta t)(h + \theta(\alpha - \delta m_1)) - (1 + \delta t)\theta m_2)}{2\theta^2}.$$ (36)

The proof is complete.

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