Metamagnets in uniform and random fields

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Abstract

We study a two-sublattice Ising metamagnet with nearest and next-nearest-neighbor interactions, in both uniform and random fields. Using a mean-field approximation, we show that the qualitative features of the phase diagrams are significantly dependent on the distribution of the random fields. In particular, for a Gaussian distribution of random fields, the behavior of the model is qualitatively similar to a dilute Ising metamagnet in a uniform field.

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I. INTRODUCTION

The random-field Ising model has been a considerable source of research over the last twenty years. Systems with quenched random fields are experimentally realized in antiferromagnets with bond mixing or site dilution. A large variety of these systems have been subjected to detailed experimental studies.

Although most theoretical problems associated with the ferromagnetic Ising model in a random field (as the lower critical dimension, the pinning effects, and the existence of a static phase transition) have been solved, some questions are still open. In particular, there is still room to investigate the existence of a tricritical point and the exact relation to the dilute antiferromagnet in a uniform field. Depending on the choice of the random-field distribution, the mean-field approximation gives rise to a tricritical point (which is present for a symmetric double-delta distribution, but does not occur in the case of a Gaussian form). On the basis of the central limit theorem, some hand-wave arguments can be used to support the physical relevance of the Gaussian distribution (the tricritical point produced by the double-delta functions being a mere artifact of the mean-field approximation).

The proof of the equivalence between an Ising ferromagnet in a random field and a dilute antiferromagnet in a uniform field is based on renormalization-group arguments that can be applied for weak fields. In the mean-field approximation (or in the equivalent and exactly soluble model with infinite-range interactions), it is possible to establish a complete mapping between the parameters of the Ising ferromagnet in a random field and the dilute Ising antiferromagnet or metamagnet in a uniform field. In particular, it is known that the random fields should be associated with a symmetric double-delta distribution for arbitrary dilution, including the pure case where there is no dilution! This peculiar result suggests that, instead of describing the random fields generated by dilution, the mean-field approximation is just referring to the two-sublattice structure of the antiferromagnet (which is reflected in the symmetric double-delta distribution of the random fields). It should be mentioned that the mean-field approximation for the dilute Ising metamagnet in a uniform...
field suffers from other difficulties when confronted with Monte Carlo calculations\cite{20,21} and experimental results\cite{22,23}. Whereas numerical simulations and experiments indicate that the first-order transition is destroyed when the dilution is increased, no such effect is predicted in a mean-field calculation.

In this paper we use a mean-field approximation to consider an Ising metamagnet with nearest and next-nearest-neighbor interactions, in a uniform field and a random field. This model is equivalent to a dilute Ising metamagnet in a field for an appropriate choice of the random field distribution. Since the exact mapping of the dilution to the random fields is unknown, only a qualitative comparison can be made. We consider double-delta and Gaussian random-field distributions. The behavior of the model and the phase diagrams depend very much on these random-field distributions. The Gaussian form seems to be more appropriate for a description of the diluted system.

II. DEFINITION OF THE MODEL

We consider a regular lattice of $N$ sites, with Ising spins $S_i = \pm 1$ at each site, that can be divided into two equivalent interpenetrating sublattices, A and B. The $z$ nearest neighbor (nn) spins of a given spin are on the other sublattice, while the $z'$ next-nearest neighbor (nnn) spins are all on the same sublattice. The Hamiltonian of the system is given by

$$
\mathcal{H} = J \sum_{nn} S_i S_j - J' \sum_{nnn} S_i S_j - \sum_i (H + H_i) S_i,
$$

where $J$ is the nn exchange parameter, the sum $\sum_{nn}$ is over all pairs of nn spins, $J'$ is the nnn exchange parameter, the sum $\sum_{nnn}$ is over all nnn spins, $H$ is the strength of the external uniform magnetic field, and $H_i$ is the strength of the local random field. We assume that the nn interactions are antiferromagnetic ($J > 0$), the nnn interactions are ferromagnetic ($J' \geq 0$), and the local random fields $H_i$ are uncorrelated. Even though it is possible to consider sublattice-dependent probability distributions, in this paper we use the same probability distribution at every site.
III. MEAN-FIELD EQUATIONS

We derive the mean-field equations from Bogoliubov’s variational principle

\[
\langle F \rangle_{\text{av}} \leq \langle F_t \rangle_{\text{av}} + \langle (\mathcal{H} - \mathcal{H}_t) \rangle_{\text{av}}, \tag{3.1}
\]

where \(\langle \cdots \rangle_{\text{av}}\) denotes averaging over the random-field distribution and \(\langle \cdots \rangle_t\) the thermal averaging with respect to the trial Hamiltonian \(\mathcal{H}_t\). Choosing the non-interacting trial Hamiltonian

\[
\mathcal{H}_t = -\sum_i (H + H_i)S_i - \eta_A \sum_{i \in A} S_i - \eta_B \sum_{i \in B} S_i, \tag{3.2}
\]

where \(\eta_A\) and \(\eta_B\) are the variational parameters, we obtain

\[
\langle F \rangle_{\text{av}} \leq -\frac{N}{2\beta} \langle \ln 2 \cosh \beta (H + H_i + \eta_A) \rangle_{\text{av}} - \frac{N}{2\beta} \langle \ln 2 \cosh \beta (H + H_i + \eta_B) \rangle_{\text{av}} + \frac{J N z}{2} m_A m_B - \frac{J' N z'}{4} (m_A^2 + m_B^2) + \frac{N}{2} \eta_A m_A + \frac{N}{2} \eta_B m_B, \tag{3.3}
\]

with

\[
m_A = \langle \tanh \beta (H + H_i + \eta_A) \rangle_{\text{av}}, \tag{3.4a}
\]

\[
m_B = \langle \tanh \beta (H + H_i + \eta_B) \rangle_{\text{av}}. \tag{3.4b}
\]

The condition that the right-hand side of Eq. (3.3) is stationary determines the variational parameters,

\[
\eta_A = -J z m_B + J' z' m_A, \tag{3.5a}
\]

\[
\eta_B = -J z m_A + J' z' m_B. \tag{3.5b}
\]

Inserting Eqs. (3.5a)-(3.5b) into Eqs. (3.4a)-(3.4b), we arrive at the mean-field equations,

\[
m_A = \langle \tanh \beta (H + H_i - J z m_B + J' z' m_A) \rangle_{\text{av}}, \tag{3.6a}
\]

\[
m_B = \langle \tanh \beta (H + H_i - J z m_A + J' z' m_B) \rangle_{\text{av}}. \tag{3.6b}
\]
The right-hand side of Eq. (3.3) at the stationary point gives the mean-field free energy per spin,

\[ f = -\frac{1}{2}\beta \langle \ln 2 \cosh \beta (H + H_i - J z m_B + J' z' m_A) \rangle_{av} \]

\[ - \frac{1}{2}\beta \langle \ln 2 \cosh \beta (H + H_i - J z m_A + J' z' m_B) \rangle_{av} \]

\[ - \frac{J_z}{2} m_A m_B + \frac{J' z'}{4} (m_A^2 + m_B^2). \]

(3.7)

IV. LANDAU EXPANSION

In this Section we develop the Landau expansion along the same steps used for the pure case\[3.\]. It is convenient to introduce the reduced quantities

\[ t = \frac{1}{\beta (J_z + J' z')}, \quad h = \frac{H}{J_z + J' z'}, \quad h_i = \frac{H_i}{J_z + J' z'}, \]

(4.1)

and the parameters

\[ \epsilon = \frac{J' z'}{J_z} \geq 0, \quad \gamma = -\frac{J_z + J' z'}{J_z + J' z'} = \frac{\epsilon - 1}{\epsilon + 1}. \]

(4.2)

In terms of the uniform and staggered magnetizations,

\[ M = \frac{m_A + m_B}{2}, \quad m_s = \frac{m_A - m_B}{2}, \]

(4.3)

the mean-field equations (3.6a) and (3.6b) can be written as

\[ M = \frac{1}{2} \left[ \langle \tanh \frac{1}{t} (h + h_i + \gamma M + m_s) \rangle_{av} + \langle \tanh \frac{1}{t} (h + h_i + \gamma M - m_s) \rangle_{av} \right], \]

(4.4)

and

\[ m_s = \frac{1}{2} \left[ \langle \tanh \frac{1}{t} (h + h_i + \gamma M + m_s) \rangle_{av} - \langle \tanh \frac{1}{t} (h + h_i + \gamma M - m_s) \rangle_{av} \right]. \]

(4.5)

Also, the free energy per spin, given by Eq. (3.7), may be written in the form

\[ f = -\frac{t}{2} \langle \ln 2 \cosh \frac{1}{t} (h + h_i + \gamma M + m_s) \rangle_{av} \]

\[ - \frac{t}{2} \langle \ln 2 \cosh \frac{1}{t} (h + h_i + \gamma M - m_s) \rangle_{av} - \frac{\gamma}{2} M^2 + \frac{1}{2} m_s^2. \]

(4.6)
Let us now write the uniform magnetization as $M = M_0 + m$, where $M_0$ is the paramagnetic solution, given by equation

$$M_0 = \langle \tanh \frac{1}{t} (h + h_i + \gamma M_0) \rangle_{av}. \quad (4.7)$$

The expansion of the right-hand side of Eqs. (4.4) and (4.5) in powers of $(\gamma m \pm m_s)$ gives the expressions

$$m = \frac{1}{2} \sum_{n=1}^{\infty} A_n [(\gamma m + m_s)^n + (\gamma m - m_s)^n], \quad (4.8a)$$

$$m_s = \frac{1}{2} \sum_{n=1}^{\infty} A_n [(\gamma m + m_s)^n - (\gamma m - m_s)^n], \quad (4.8b)$$

where the coefficients $A_n$ are given by

$$A_1 = -\frac{1}{t} (T_2 - 1), \quad (4.9a)$$

$$A_2 = \frac{1}{t^2} (T_3 - T_1), \quad (4.9b)$$

$$A_3 = -\frac{1}{3t^3} (3T_4 - 4T_2 + 1), \quad (4.9c)$$

$$A_4 = \frac{1}{3t^4} (3T_5 - 5T_3 + 2T_1), \quad (4.9d)$$

$$A_5 = -\frac{1}{15t^5} (15T_6 - 30T_4 + 17T_2 - 2). \quad (4.9e)$$

with

$$T_k = \langle \tanh^k \frac{1}{t} (h + h_i + \gamma M_0) \rangle_{av}. \quad (4.10)$$

We now determine $m$ in terms of $m_s$ in the form

$$m = B_1 m_s^2 + B_2 m_s^4 + B_3 m_s^6 + \ldots. \quad (4.11)$$

Inserting this expansion into Eq. (4.8a), and equating the coefficients of same degree in $m_s$, we find the coefficients $B_n$ in terms of $A_n$. Finally, substituting $m$, given by Eq. (4.11), into Eq. (4.8b) we obtain the expansion

$$a m_s + b m_s^3 + c m_s^5 + \cdots = 0, \quad (4.12)$$
where

\[
\begin{align*}
a &= 1 - A_1, \\
b &= \frac{2\gamma A_2^2}{\gamma A_1 - 1} - A_3, \\
c &= \frac{2\gamma^3 A_2^4}{(\gamma A_1 - 1)^3} - \frac{9\gamma^2 A_2^2 A_3}{(\gamma A_1 - 1)^2} + \frac{6\gamma A_2 A_4}{\gamma A_1 - 1} - A_5.
\end{align*}
\] (4.13)

The second order transition is found at \( a = 0 \) with \( b > 0 \). The tricritical point occurs for \( a = b = 0 \) with \( c > 0 \).

In the absence of random fields the model exhibits a tricritical point in the \( h - t \) phase diagram for \( \epsilon > 3/5 \). In the numerical calculations of the next sections, we just consider the case \( \epsilon = 1 \), which is typical for the range of values \( \epsilon > 3/5 \).

\section*{V. PHASE DIAGRAMS FOR THE DOUBLE-DELTA DISTRIBUTION}

In this Section we study the phase diagrams for the case of a double-delta distribution,

\[
P(h_i) = \frac{1}{2} [\delta(h_i - \sigma) + \delta(h_i + \sigma)].
\] (5.1)

Fig. 1 shows the phase diagrams in the \( h - t \) plane for various values of the randomness \( \sigma \).

The case of no randomness (\( \sigma = 0 \)) corresponds to the pure Ising metamagnet. The phase diagram comprises a metamagnetic phase (\( m_s \neq 0 \)) at low fields and a paramagnetic phase (\( m_s = 0 \)) at high fields. The transitions between these phases are first-order for low temperatures and second-order for high temperatures, being separated by a tricritical point at \( t = 2/3 \), as shown in Fig. 1(a).

For \( \sigma > (2/3) \tanh^{-1}1/\sqrt{3} = 0.438 \ldots \), there is a second tricritical point at lower fields, as illustrated by Figs. 1(d)–(e). Also, for the randomness in the interval 0 < \( \sigma < 0.5 \), there is a first-order transition line inside the metamagnetic phase at low temperatures. Through this transition line the staggered magnetization decreases discontinuously as the field is increased. This internal first-order transition line ends at a critical point. Finally, for \( \sigma > 0.5 \), the internal and lower first-order transition lines merge into a single first-order transition line ending at the tricritical point, as illustrated in Fig. 1(f).
For the particular case of the double-delta distribution and $\epsilon = 1$ or $\gamma = 0$ that we are considering, the phase diagrams in the $\sigma - t$ plane are exactly the same as in the $h - t$ plane. This comes from the invariance of Eqs. (4.5) and (4.6), for the staggered magnetization and the free energy, respectively, under the interchange between $h$ and $\sigma$ (and from the independence of the free energy on the uniform magnetization $M$). Therefore, Fig. 4 also represents the phase diagrams in the $\sigma - t$ plane if we interchange $h$ and $\sigma$ throughout this figure and in its caption. The phase diagram comprises a metamagnetic phase ($m_s \neq 0$) for small $\sigma$ and a paramagnetic phase ($m_s = 0$) for high values of $\sigma$. In particular, the phase diagram for $h = 0$ is equivalent (after flipping all the spins on a sublattice) to the diagram of a ferromagnetic Ising model in a double-delta random field.

**VI. PHASE DIAGRAMS FOR THE GAUSSIAN DISTRIBUTION**

Now we study the phase diagrams for the Gaussian distribution,

$$
P(h_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{h_i^2}{2\sigma^2}\right).
$$

(6.1)

In Fig. 2, we show the $h - t$ plane for various values of the randomness $\sigma$. Again, the case of no randomness ($\sigma = 0$) corresponds to the pure Ising metamagnet, with a first-order separated from a second-order transition line by a tricritical point at $t = 2/3$. The tricritical temperature decreases as the randomness is increased until $\sigma = 0.5$, when the transition between the metamagnetic and paramagnetic phases becomes everywhere of second-order. The similarity of these phase diagrams as a function of $\sigma$ with those of a dilute metamagnet as a function of dilution is quite striking. It suggests that a Gaussian random field gives, at least qualitatively, a good description of the random fields generated by dilution in a metamagnet.

In Fig. 3, we show the phase diagram in the $\sigma - t$ plane for various values of the uniform field $h$. The case $h = 0$ is equivalent (after flipping all the spins on a sublattice) to the ferromagnetic Ising model in a Gaussian random field. The transition line is of second-order for all temperatures and it crosses the $\sigma$ axis at $\sigma = \sqrt{2/\pi} = 0.79\ldots$ For large $\sigma$ the
transition at low fields becomes first-order and a tricritical point separates the first-order and the second-order lines. For still larger randomness, the transition becomes first-order always.

VII. CONCLUSIONS

We have used the mean-field approximation to show that the phase diagrams of an Ising metamagnet in the presence of a uniform and of random fields are strongly dependent on the form of the distribution of probabilities of the random fields. In particular, if the model exhibits a first-order transition in zero random field, then a double-delta distribution never destroys this first-order transition, in contradistinction to the case of a Gaussian distribution. In this respect, there is a striking similarity in the qualitative behavior of the metamagnet in a Gaussian random field and a dilute metamagnet. This suggests that, by keeping the two-sublattice structure and choosing an appropriate random field distribution, we can give a better description of the dilute metamagnet than the previous mean-field studies that map dilute Ising metamagnets in a uniform field into Ising ferromagnets in a double-delta distribution of random fields.

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FIG. 1. Phase diagrams in $h - t$ plane in the case of a double-delta distribution for (a) $\sigma = 0$, (b) $\sigma = 0.3$, (c) $\sigma = 0.4$, (d) $\sigma = 0.45$, (e) $\sigma = 0.49$ and (f) $\sigma = 0.65$. The solid lines represent continuous transitions. The dashed lines are first-order transitions. The filled circles are tricritical points, and the empty circles are critical points.
FIG. 2. Phase diagrams in $h - t$ plane in the case of a Gaussian distribution for (a) $\sigma = 0$, (b) $\sigma = 0.4$, (c) $\sigma = 0.6$, (d) $\sigma = 0.7$ and (e) $\sigma = 0.75$. The solid lines represent continuous transitions. The dashed lines are first-order transitions. The filled circles are tricritical points.
FIG. 3. Phase diagrams in $\sigma - t$ plane in the case of a Gaussian distribution for (a) $h = 0$, (b) $h = 0.3$, (c) $h = 0.4$, (d) $h = 0.47$ and (e) $h = 0.49$. The solid lines represent continuous transitions. The dashed lines are first-order transitions. The filled circle is a tricritical point.