Effects of the parallel acceleration on heavy impurity transport in turbulent tokamak plasmas

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Abstract
A process specific to the dynamics of the heavy impurities in turbulent tokamak plasmas is found and analyzed. We show that the parallel stochastic acceleration is strongly coupled to the perpendicular transport and generates a radial pinch velocity. The interaction is produced with the hidden drifts, a quasi-coherent component of the motion that consists of a pair of average radial velocities in opposite directions. The parallel acceleration breaks this symmetry and yields a radial average velocity that can be in the inward or outward direction. The pinch is generated in three-dimensional turbulence, in the presence of a poloidal average velocity. It is significant only for heavy, high \( Z \) ions. The transport of the tungsten ions is analyzed in the frame of the minimal test particle model that yields this pinch mechanism. We use a semi-analytical method and numerical simulations. The scaling laws of the pinch velocity and of the diffusion coefficient are found and analyzed in order to drive a clear physical image of these non-linear effects. We conclude that the pinch produced by the non-linear interaction of the parallel accelerated motion with the perpendicular transport is rather strong for the heavy impurities.

Keywords: tokamak plasma, turbulent transport, heavy impurities, radial pinch

(Some Figures may appear in colour only in the online journal)

1. Introduction

Tungsten (W) will be used for plasma facing components in ITER, because this material fulfils essential requirements such as low erosion rate, low tritium retention and good thermal properties [1]. The effects of the W wall on tokamak plasma performances have been intensively studied during the last decades in several tokamak devices (ASDEX Upgrade [2–4], JET [5–7], WEST [8], T-10 [9], JT-60U [10]). The main drawback is the large radiation emission of these high charge ions that can strongly affect the energy balance if they accumulate in the core plasma in concentrations higher than \( 10^{-5} \)–\( 10^{-4} \). It is, therefore, vital to acquire a good understanding of impurity transport and to develop methods for controlling the concentration of W in tokamak core plasmas. A large number of experimental [11–15], theoretical [16–20] and numerical [21–24] studies have provided important results on these complex processes, but the domain is still open as shown by very recent papers [25–29].

The dynamics of the W ions includes both neoclassical and turbulent effects. The large mass and charge determine strong inertial and electrostatic forces, with the result of specific phenomena that are not observed at light ions. Strong poloidal asymmetries, significant increase of the neoclassical transport and accumulation on the low field side of the plasma or even around the magnetic axis characterize W ions. Radial convectons with neoclassical or turbulence origins can determine W accumulation or decay.

The present paper deals with the turbulent transport of the heavy impurities. We analyze the effects of the parallel acceleration \( a_z \) on the diffusion coefficients and on the radial pinch velocity. The acceleration scales as \( a_z \sim Z/A \), where \( Z \) is the ionization rate and \( A \) is the mass number of the ions. Thus, it is smaller for the W impurities than for plasma ions. The
The factor $Z/A$ varies in the interval $(0.05, 0.33)$ for the W impurities, while it has the value 0.5 for the deuterium. Surprisingly, we have found that the effect of $a_r$ can be significant for W impurities, while it is negligible for deuterium ions.

The main effect of the non-linear interaction of the parallel accelerated motion with the perpendicular transport consists of the generation of a radial pinch velocity $V_r$. The aim of the paper is to understand and to characterize this new, rather unexpected phenomenon at the fundamental research level. Our study is focused on this particular process and consists of a detailed analysis in the frame of a simplified model that excludes elements that are not essential.

The parallel acceleration is expected to influence impurity transport through the modification of the parallel decorrelation time. We show that, beside this direct effect, a much stronger coupling of the parallel accelerated motion to the radial transport appears. It consists of the perturbation of hidden drifts (HDs). The HDs are a pair of opposite velocities in the radial direction that appears in the presence of a poloidal average velocity [30, 31]. This quasi-coherent motion has zero average and does not determine a convective velocity in the case of the $E\times B$ drift. The stochastic parallel acceleration perturbs the equilibrium of the HDs leading to a radial pinch.

The analysis is performed in the frame of a test particle stochastic model, which is shown to be the minimal model that yields this process. Therefore, the results contribute to the fundamental research in heavy impurity transport by clarifying important aspects of the non-linear interaction of the accelerated parallel motion with the radial transport. The model is presented in section 2.

We use two theoretical methods for determining the pinch velocity and the transport coefficients, the direct numerical simulations (DNS) [32] and the decorrelation trajectory method (DTM) [33]. A three-dimensional DNS code for ion trajectories and for the calculation of the statistical Lagrangian quantities was developed. It is described in section 3.1. The DTM is a semi-analytical approach that provides approximate evaluations of the transport characteristics. It is presented in section 3.2. The DTM is used for identifying and understanding qualitatively the new pinch mechanism, while the quantitative properties of the radial velocity $V_r$ are determined using the much more accurate results provided by DNS.

The effects of the parallel acceleration on the heavy impurity transport are identified in section 4 by comparing typical results of the transport model with those obtained for deuterium ions and for W ions in two-dimensional potentials.

The physical processes that determine the generation of the radial pinch are discussed in section 5. We use the DTM, which has the capability to provide physical pictures of complex non-linear transport processes [34–36]. We show that the acceleration can have a strong influence on the HDs that essentially consists of the attenuation of one of the HDs, which compensates only partially the other HD yielding an average velocity. Ion collisions and the gradient of the confining magnetic field are not essential ingredients in this mechanism and are not included in the minimal model analyzed here. However, they could interfere with the HDs by changing their amplitude and symmetry. We evaluate the effects of collisions and of the magnetic field gradient using the DTM. The physical image of the pinch generation mechanism is validated using DNS. A short discussion on the accuracy of the DTM is also presented in this section.

The properties of the pinch velocity $V_r$ and its dependence on the main parameters of the model are determined in section 6. They are obtained using the more accurate results of the DNS. The study is focused on the scaling of $V_r$ with the main parameters of the model. The results are analyzed and physical explanations are derived.

The relevance of the pinch generated by the parallel acceleration for the W ion transport is discussed in section 7. A summary of the results and the conclusions of this study are also included in this section.

2. The transport model

We study impurity transport in the trace limit, at the low field side of the plasma. The equations for the impurity ion trajectories in slab geometry are approximated by

$$\frac{dx}{dt} = -\frac{\nabla \phi \times e_z}{B} + V_d,$$  \hspace{1cm} (1)

$$\frac{dz}{dt} = v_z, \quad \frac{dv_z}{dt} = -\frac{q}{m} \partial_\phi \phi,$$  \hspace{1cm} (2)

where magnetic field $B$ is constant along $e_z$ axis and $x = (x, y)$ is in the perpendicular plane, with $x$ the radial and $y$ the poloidal coordinate. The first term in equation (1) is the stochastic drift determined by the electric field of the turbulence $-\nabla \phi(x,z,t)$ ($\phi(x,z,t)$ is the stochastic potential, $\nabla$ is the gradient in the perpendicular plane) and the second term $V_d = V_d e_z$ is a poloidal average velocity that can be produced by the magnetic drifts or plasma rotation. The parallel motion includes the variation of the velocity determined by the stochastic acceleration $a_r = -qلم\partial_\phi \phi$. The small scale trajectories (with displacements and times of the order of the potential correlation lengths and time) are necessary for determining the transport coefficients.

The potential $\phi(x,z,t)$ is modeled as a Gaussian random field with the Eulerian correlation (EC)

$$E(x,z,t) \equiv \langle \phi(0,0,0)\phi(x,z,t) \rangle$$

(3)

corresponding to drift type turbulence [34–36]

$$E(x,z,t) = A_{\phi} \partial_\phi \left[ \exp \left( -\frac{x^2}{2\lambda_x^2} - \frac{y^2}{2\lambda_y^2} - \frac{z^2}{2\lambda_z^2} \right) \sin \left( \frac{k_0 y}{\lambda_y} \right) \right] T(t),$$

(4)

where $A_{\phi}$ is the amplitude of the potential fluctuations, $\lambda_x$, $\lambda_y$, $\lambda_z$ are the correlation lengths along the radial, poloidal and parallel directions, and $k_0$ is the dominant wave number. The function $T(t)$ is the time correlation of the potential that is a decaying function with $\tau_d$ the decorrelation time

$$T(t) = \exp \left( -\frac{t^2}{4\tau_d^2} \right).$$

(5)
Dimensionless quantities are used, with the units: \( \rho_i = v_{thi}/\Omega_i \), the Larmor radius of the protons (for the perpendicular distances, for the correlation lengths \( \lambda_x \), \( \lambda_z \), and for \( 1/k_0 \)), \( a \), the small radius of the plasma (for the parallel distances and for the correlation length \( \lambda_z \), \( \tau_0 = a/v_{thi} \) (for time and for \( \tau_d \)), \( A \) (for the potential \( \phi \)), \( V_a = \rho_i v_{thi}/a \) (for the perpendicular velocities and \( V_{thi} \)) and \( v_{thi} = v_{thi}/\sqrt{A} \) (for the parallel velocity of the \( W \) ions), \( v_{thA} = \sqrt{T_i/m_p} \) is the thermal velocity of protons with temperature \( T_i \) and mass \( m_p \), and \( \Omega_i = eB/m_p \) is the cyclotron frequency of the protons. The notations are not changed for the dimensionless quantities, and equations (1) and (2) for ions with mass number \( A \) and ionization rate \( Z \) are

\[
\frac{dx}{dt} = -K_x \partial_x \phi(x,z,t), \quad \frac{dy}{dt} = K_x \partial_y \phi(x,z,t) + V_p,
\]

(6)

\[
\frac{dz}{dt} = \frac{1}{\sqrt{A}} v_\perp, \quad \frac{dv_\perp}{dt} = -P_a \partial_z \phi.
\]

(7)

The average velocity is equivalent with an average potential that contribute to the total potential \( \phi_t = \phi(x,z,t) + xV_p \).

The main characteristics of the model appear in three dimensionless parameters evidenced in the dimensionless equation. The parameter \( K_x \) is the dimensionless measure of turbulence amplitude

\[
K_x = \Phi \frac{a}{\rho_i} \Phi = \frac{eA_p}{T_i}.
\]

(8)

The parameter of the poloidal velocity \( V_p \) is

\[
V_p \equiv \frac{V_d}{V_a} = \frac{V_d}{v_{thi}} \frac{a}{\rho_i}.
\]

(9)

The parameter of the parallel acceleration \( a_r \) is

\[
P_a \equiv \Phi \frac{Z}{\sqrt{A}}.
\]

(10)

We note that the first two parameters that describe the perpendicular motion depend on plasma size factor \( \rho_x = \rho_i/a \).

The energy of the ions normalized with the temperature \( T_i \) is

\[
W = \frac{1}{2} v_\perp^2 + Z \Phi \phi_t
\]

(11)

when the impurities are in thermal equilibrium with plasma ions. It is the invariant of the motion in three-dimensional static potentials \( \phi(x,z) \). This constraint influences the transport for \( \tau_d \to \infty \), and its effects persist in the case of potentials with slow time variation (large \( \tau_d \)). We note that the energy is dominated by the potential energy for the \( W \) ions with large \( Z \), even at small turbulence amplitudes (\( \Phi \sim 10^{-2} \)).

The model considered here contains the essential components of ion motion that combine to yield a radial pinch velocity. It is a new mechanism of turbulent pinch that is determined by the non-linear effects of the parallel acceleration. The aim of our study is to characterize this new process and to achieve a clear physical understanding of the interaction of the parallel accelerated motion with the radial transport. This model is very simplified compared to the complex sophisticated approaches that are used in the present days numerical studies of the heavy impurities (see, for example, [25]). Such approaches are mandatory for predicting the \( W \) ion transport for experiment analyses or for the important issue of \( W \) accumulation control.

3. Theoretical methods

The model is analyzed using DNS [32] and the DTM [33].

3.1. DNS numerical methods and code

The numerical methods used in the DNS code are described and analyzed in [32]. A series of fast numerical generators of Gaussian random fields with given EC is proposed. In the present work, we have implemented the so called FRD representation

\[
\phi(X) = \sum_{i=1}^{N_t} \sqrt{\mathbb{S}(K_i)} \sin \left( K_i X + \frac{\pi}{4} \zeta_i \right),
\]

(12)

where \( X \equiv (x,z,t) \) is the four-dimensional space-time, \( K_i \equiv (k_1,k_2,k_3,k_4) \) are the \( N_t \) discrete values of the corresponding wave numbers and frequency and \( S(K) \) is the spectrum of the stochastic potential (the Fourier transform of the EC (4)). This representation is different of the usual discrete Fourier decomposition by the set of the values of \( K_i \), that are not the fixed points of a four-dimensional mesh, but random values with uniform distribution. Also, the random phases do not have continuous distributions, but discrete values \( \pm 1 \) (with equal probabilities). Each set of the \( N_t \) waves with random wave numbers and phases \( \zeta_i \) determines a realization of the potential. The statistical ensemble is constituted of a number of \( M \) realizations (has dimension \( M \)).

We have shown [32] that the representation (12) provides fast convergence of the Eulerian statistics of the generated fields, as well as of the Lagrangian statistics of trajectories. In particular, it was proven that a convergence level with a few percents error can be achieved with \( N_t \sim 10^d \) and \( M \sim 10^4 \), where \( d = 4 \) for time dependent potentials and \( d = 3 \) for \( \tau_d \to \infty \). Also, it is worth mentioning that such representations are able to reproduce with high accuracy the conservation laws of motion as well as certain Lagrangian statistical invariants.

The properties of the representation (12) enables to use commonly in the present simulations \( N_t \sim 1–5 \times 10^5 \) waves. The dimension of the statistical ensemble is usually set to \( M \sim 10^5 \) realizations (trajectories) which gives small statistical fluctuations. The numerical integration scheme used is a fourth order Runge-Kutta method which preserves well the energy with a minimal numerical effort. Typical number of steps is 600 for the integration time \( t = 30 \). Depending on the integration time and on the type of turbulence (frozen, or not), the usual CPU times on personal computer are \( t_{CPU} \sim 2–20 \) h per
run. Detailed analyses of the statistical and numerical precision of the DNS code and of the dependence of the errors on \( N_c, M \) are presented in [32].

### 3.2. DTM semi-analytical method

The DTM is a semi-analytical approach [33], which is able to describe both the random and the quasi-coherent components of the trajectories [38]. It is established by the finite correlation lengths of the stochastic potential and depends on the structure of the correlated zone that is described by the shape of the EC. The basic DTM is developed here by introducing the parallel acceleration.

The statistical ensemble of stochastic potentials is divided in subensembles \( S \) with given values of the potential and of its derivatives at the origin of the trajectories, \( x = 0, z = 0, t = 0 \)

\[
\phi(0,0,0) = \phi^0, \quad \partial_i \phi(0,0,0) = \phi_i^0,
\]

where \( i = x,y,z \). The potential and its derivatives, restricted at the realizations contained in a subensemble, are Gaussian fields with space-dependent averages

\[
\langle \phi_i(x,z,t) \rangle_S \equiv \Phi_i^S(x,z,t) = \phi^0 \frac{E(x,z,t)}{E(0,0,0)} - \sum_i \phi_i^0 \frac{E_i(x,z,t)}{E_{ii}(0,0,0)} + x \nu_p,
\]

\[
\langle \partial_i \phi(x,z,t) \rangle_S = \partial_i \Phi^S(x,z,t),
\]

where \( E_{ii} \) are derivatives of the EC, \( E_i = \partial_i E, \ E_{ii} = \partial_i^2 E \). The amplitudes of fluctuations in a subensemble vanish in \( x = 0, z = 0, \) and they reach the level corresponding to the whole set of realizations only at large distances compared to the correlation lengths.

Particle trajectories are studied separately in each subensemble \( S \). The average potential (14), determined by the EC, yields an average trajectory in each subensemble. It is obtained by averaging equations (6) and (7) over the realizations that belong to \( S \). Neglecting the fluctuations of the potential in \( S \) (see [33, 38] for the discussion of this approximation), we can obtain a system of subensemble average equations (S-eq). It has the same structure as equations (6) and (7), but with the stochastic potential \( \phi(x,z,t) \) replaced by the subensemble average potential \( \Phi^S(x,z,t) \). The solution of the (S-eq), \( X(t;\phi^0,\phi_i^0) \), is a smooth, simple trajectory, which is named decorrelation trajectory (DT) because it represents the average evolution of the particles through the correlated zone of the potential. An important feature of the DTs is that they obey any conservation law which characterize the real trajectories. In this case, the energy

\[
\langle W(i) \rangle_S = \frac{1}{2} \langle v_i^2(t) \rangle_S + Z \Phi \Phi^S(X(t;\phi^0,\phi_i^0),Z(t;\phi^0,\phi_i^0))
\]

is conserved along the DTs for static potentials.

The statistical characteristics of the stochastic trajectories are obtained as weighted averages along the DTS by summing the contributions of all subensembles. In particular, the time dependent diffusion coefficient and the average radial displacement are

\[
D_x(t) = \int d\phi^0 \int d\phi_i^0 d\phi_x^0 d\phi_z^0 P(\phi^0,\phi_i^0) \frac{\nu_x^0}{X(t;\phi^0,\phi_i^0)},
\]

\[
X(t) = \frac{\langle x(t) \rangle}{\tau_x^\infty}, \quad V_x^\infty = \frac{\langle x(t;\tau_x^\infty) \rangle}{\tau_x^\infty}.
\]

This is an increasing function of time that saturates at a finite value \( V_x^\infty \). The radial pinch velocity is obtained as

\[
V_x(t) = \frac{\langle x(t;\tau_x^\infty) \rangle}{\tau_x^\infty}, \quad V_x^\infty = \frac{\langle x(t;\tau_x^\infty) \rangle}{\tau_x^\infty}.
\]

In time dependent potentials, \( \tau_x^\infty \) combines with the time dependence of the EC \( T(t) \) in a modified function \( \tau_x^\infty(t) \) with a modified asymptotic value \( \tau_x^\infty \).

The time dependent functions \( D_x(t) \) and \( V_x(t) \) provide details of the transport process, while their asymptotic values \( D_x^\infty, V_x^\infty \) represent the diffusion coefficient and the pinch velocity that determine the impurity flux at the transport space-time scale.

The DTM is implemented in a code that calculates the DTS and the integrals (17)–(19). The number of DTS depends on the number of conditions (13), and it is of the order \( 2 \times 10^6 \) for this three-dimensional transport model. A predictor-corrector method with adaptable time step is used. The DTM is several times faster than the DNS code due to the potential (14), which is a simple analytical function. The errors result from the approximations of the DTM rather than from the numerical calculations.

### 4. Effects of the parallel acceleration

Typical results obtained with the DTM for W ions with \( Z = 40 \) in a turbulent plasma with the parameters \( \Phi = 0.03, \nu_p = 1, \lambda_x = 5, \lambda_y = 2, \lambda_z = 1, k_0 = 1, \tau_\parallel = \infty \) and \( a_\parallel = 500 \) are presented in figure 1. The time-dependent diffusion coefficient \( D_x(t) \) and the radial average velocity \( V_x(t) \) are shown (solid lines) compared to case of deuterium (D) ions (dashed-dotted lines) and to W ions with \( a_z = 0 \) (dashed lines).
The diffusion coefficients shown in figure 1 (left panel) have a similar time dependence for all three cases. The increase at small times corresponds to the quasilinear regime that is identical for all examples. It lasts for $t \ll \tau_\beta$, where $\tau_\beta$ is the time of flight defined as the ratio of $\lambda_\parallel$ and the amplitude of the stochastic radial velocity, which in this units is $\tau_\beta = \lambda_\parallel \lambda_\perp / K$. The maximum of $D_\parallel(t)$ appears at $\tau_\beta = 0.66$ and, at larger times, the eddying (trapping) determines the decay of $D_\parallel(t)$ that lasts until the decorrelation of the trajectories from the potential produces the saturation. One can see that the saturation is at a much smaller time for the D ions than for the W ions, which means that the parallel decorrelation time $\tau_\parallel^\infty$ is much smaller in the first case. The parallel acceleration does not change the diffusion of the D ions. The same result is obtained with/without $a_z$ (the dashed-dotted line). The parallel acceleration determines a modification of $D_\parallel(t)$ for the W ions (see the solid curve compared to the dashed one). It essentially consists, in this case, of a small increase of the time of flight.

The main effect of the parallel acceleration is the generation of a radial pinch. As seen in figure 1 (right panel), a negative (inward) average velocity $V_z(t)$ appears for the W ions due to $a_z$. It has a transitory large increase until $t \sim \tau_\beta$, then it decays and eventually saturates due to the parallel decorrelation. When $a_z$ is neglected, $V_z(t) = 0$ at any time. The radial pinch is much smaller for D ions than for the W ions. As seen in the figure, only multiplied by 100 the D pinch velocity (dashed-dotted curve) reaches values comparable to the W pinch velocity (solid curve).

These much larger effects of the parallel acceleration on the heavy impurity transport compared to the case of D ions are rather surprising, because the normalized parallel acceleration in equation (7) scales as $dv_z/dt \sim P_a \sim Z / \sqrt{A}$. It is smaller by a factor 0.3 for W compared to D ions, which strongly enables to predict very small effects on heavy impurity transport.

The interaction of the parallel motion with the perpendicular motion is the effect of the finite parallel correlation length $\lambda_z$, which makes the EC (3) a z-dependent function that decays with the increase of $z$. The average of the EC over the parallel motion $z(t)$, solution of equation (7), yields a time decaying function. This parallel decorrelation process has the characteristic time $\tau_z^\infty = \tau_\parallel^\infty$, the asymptotic value of equation (19). The values of $\tau_z^\infty$ and its scaling with the parameters of the parallel motion are different for the D and W ions, as shown below.

The variation range of the parallel velocity results from the energy conservation

$$v_z = \pm \sqrt{2(W - Z\Phi \phi_z)},$$

while its dynamics, reflected in the variation time, is determined by the parallel acceleration $a_z$.

At $Z = 1$, the potential energy is small, $\Phi \ll W$, and the velocity can be approximated by $v_z \approx \pm \sqrt{2W(1 - \Phi / 2W)}$. The acceleration determines for the D ions only a small fluctuation of $v_z(t)$ around the effective value $v_z^\parallel \approx \pm \sqrt{2W}$. The parallel displacements are $z = \pm \sqrt{2W / \lambda_z}$, and the decorrelation time is approximated by $\tau_z^\infty \approx \lambda_z / \sqrt{W}$ [39]. Thus, the parallel decorrelation time is practically not modified by $a_z$ at small $Z$, because $v_z^\infty$ has only a weak dependence on $\Phi$ and $\lambda_z$.

At large $Z$, the potential energy is large, of the order of the total energy $W, Z\Phi \sim W$. In these conditions, the trajectories cannot reach the regions with large, positive $\phi_z$ and the Lagrangian potential has an upper limit

$$\phi(x(t), z(t)) < \phi_{\text{max}} = \frac{W}{Z\Phi}.$$  (22)

In addition, the kinetic energy is larger than the total energy $W$ in the regions with negative potential. Both the average and the fluctuation amplitude of $v_z(t)$ are functions of $W, Z$ and $\Phi$. They also depend on $\lambda_z$ through the characteristic variation time of $v_z(t)$, which is determined by the acceleration $a_z \sim 1 / \lambda_z$. The analytical estimation of $v_z^\parallel(W, Z, \Phi, \lambda_z)$ and $\tau_z^\infty(W, Z, \Phi, \lambda_z)$ is not possible in this case, but only the general behavior with the parameters of the parallel motion. The range of variation of $v_z(t)$ is the interval $[0, W + Z\Phi]$, where the lower limit is determined by $\phi_{\text{max}}$ in equation (22) and the upper limit corresponds to the amplitude of the order $-\Phi$ of the negative potential. This shows that $v_z^\parallel$ increases with $W, Z$ and $\Phi$, and that $v_z^\parallel(W, Z, \Phi, \lambda_z) > \sqrt{2W}$. The dynamics of
\(v_c(t)\) that is determined by \(a_c \sim P_a/\lambda_z\) leads to the decrease of \(v_c^{ef}\) at the increase of \(\lambda_z\).

Thus, the parallel decorrelation time is modified by \(a_c\) at large \(Z\), and depends on all the parameters of the parallel motion. It can be approximated by

\[\tau_z^*(W, Z, \Phi, \lambda_z) \equiv \lambda_z \sqrt{A/v_c^{ef}}, \quad (23)\]

which is a decreasing function of \(W, Z, \Phi\) and an increasing function of \(\lambda_z\) (as \(\lambda_z^2 = \alpha > 1\).

The influence of the decorrelation time on the diffusion is different in the quasilinear \((\tau_z^* < \tau_p)\) and the trapping \((\tau_z^* > \tau_p)\) regimes. The asymptotic diffusion coefficients scale as

\[D_x^\infty \sim \left\{ \begin{array}{ll} \phi^2 \tau_z^*, & \tau_z^* < \tau_p \\ \phi \gamma \tau_z^{-1}, & \tau_z^* > \tau_p \end{array} \right., \quad (24)\]

where \(0 < \gamma < 1\).

The effects of the interaction of the parallel motion with the perpendicular transport through the parallel decorrelation explain the results obtained for the diffusion coefficients. The influence of the parallel acceleration is negligible for the D ions and noticeable for W ions. The strongest difference appears due to the dependence of \(\tau_z^*\) on the mass number, which leads for the case presented in figure 1 (left panel) to \(\tau_z^* = 1.15\) for the D ions (dashed-dotted line) and \(\tau_z^* = 11.25\) for the W ions (dashed line). The diffusion is in the trapping regime in both cases, which corresponds, according to equation (24), to much smaller \(D_x^\infty\) for the W ions compared to D ions, as seen in figure 1 (left panel).

The radial pinch velocity seen in figure 1 (right panel) cannot be explained by the parallel decorrelation process. A different interaction process provides the physical mechanism of pinch generation, as demonstrated in the next section.

5. The pinch mechanism

Particle trajectories described by equations (1) and (2) (or (6) and (7)) with \(\lambda_z \to \infty\) (two-dimensional potentials) have both stochastic and quasi-coherent aspects. The coherent motion is determined by the trapping or eddying in the structure of the potential, which yields small structures that produce a micro-confinement process [38]. Trapping hinders the diffusive transport by decreasing the diffusion coefficient. The quasi-coherent component of the motion can also yield flows [40–42].

We have shown [30, 31] that a special quasi-coherent effect, that is neither structure nor flow, appears in the stochastic transport in the presence of an average poloidal velocity \(V_p\). It consists of two average radial velocities in opposite directions, which exactly compensate. This pair of drifts are named in [30] HDs because they do not yield an average velocity in these conditions.

The HDs are essentially determined by the existence of average displacements of the trajectories that start from same values of the potential \(\phi^0\), and by the special property of these conditional averages \(\langle x(t) \rangle_{\phi^0}\) of having the sign correlated to the sign of \(\phi^0\). These quantities, evaluated by DTM from equation (18), are

\[\langle x(t) \rangle_{\phi^0} = \int_0^\infty d\phi^0 \langle x(t) \rangle_{\phi^0} P(\phi^0, \phi^0) X(t; \phi^0, \phi^0). \quad (25)\]

The conditional displacements \(\langle x(t) \rangle_{\phi^0}\) are zero in the case of the motion determined only by the electric drift \((V_p = 0)\), but they have finite values in the presence of an average poloidal velocity \(V_p\). In the absence of the parallel acceleration (two-dimensional potentials), \(\langle x(t) \rangle_{\phi^0}\) is an anti-symmetrical function of \(\phi^0\) and it leads, by integration over \(\phi^0\), to zero average displacement. A typical example is shown in figure 2 (left panel), which corresponds to the transport of the W ions with \(a_c = 0\) represented in figure 1 by the dashed lines.

This special type of quasi-coherent motion is generated by the average poloidal velocity \(V_p\), which determines strong modifications of the contour lines of the total potential \(\phi(x) = \phi(x) + xV_p\). At \(V_p = 0\), all contour lines are closed, nested curves, while at finite \(V_p\) a complex structure appears, with strips of open lines that oscillate between islands of closed lines. The trajectories are along the contour lines of \(\phi_0(x)\), and thus they are of two types: trapped (closed) and free (with unlimited displacements along \(V_p\)). The probability of finding the trajectory along a closed contour line is larger on the side on which \(V_p\) is opposite to the stochastic velocity than on the other side. This leads to average radial displacements on the contour line \(\phi^0\) that are positive for \(\phi^0 > 0\) and negative for \(\phi^0 < 0\). The free trajectories also contribute to the ordered conditional displacements. The Lagrangian invariance of \(\phi_0(x(t)) = \phi(x(t)) + x(t)V_p = \phi^0\) constrains these trajectories to oscillate around the line \(x = \phi^0/V_p\), which is the average for the trajectories that start from \(\phi^0\).

This physical image explains the importance of the average velocity \(V_p\) in the generation of quasi-coherent radial motion. The conditional average displacements \(\langle x(t) \rangle_{\phi^0}\) exist only for \(V_p \neq 0\). At \(V_p = 0\), all trajectories are closed and the velocity is statistically uniform along the contour lines, which corresponds to uniform probabilities of localization and to \(\langle x(t) \rangle_{\phi^0} = 0\) for any \(\phi^0\). Moreover, \(\langle x(t) \rangle_{\phi^0}\) are anti-symmetrical functions of \(V_p\), because the change of \(V_p\) in \(-V_p\) determines the increase of the localization probability on the opposite side of the closed paths and opposite average displacement of the free trajectories.

The average displacements conditioned by the sign of the initial potential

\[\langle x(t) \rangle_+ = \int_0^\infty d\phi^0 \langle x(t) \rangle_{\phi^0}, \quad \langle x(t) \rangle_- = \int_0^\infty d\phi^0 \langle x(t) \rangle_{\phi^0} \quad (26)\]

determine, using equation (20), two opposite radial velocities \(V_+\), \(V_-\) that exactly compensate \(V_+ + V_- = 0\) due to the anti-symmetry of \(\langle x(t) \rangle_{\phi^0}\) with respect to the initial potential \(\phi^0\) (see [30, 31] for details).

The HDs represent a reservoir for direct transport, because perturbations produced by other components of the motion can affect the equilibrium of the HDs leading to an average velocity. We have shown [31] that the polarization drift determines
a significant modification of the symmetry of the HDs and provides a mechanism of radial pinch generation for heavy impurities. Essentially, this pinch appears due to the compressibility effect of the polarization drift.

We show here that the three-dimensional stochastic motion described by equations (1) and (2) (or (6) and (7)) can influence the equilibrium of the HDs and generates an average radial velocity.

An important difference between the three-dimensional and two-dimensional motion is that the Lagrangian potential is not invariant. This means that the trajectories in the subensemble S do not evolve on the contour lines \( \phi_t(x) = \phi^0 \), but they move up and down \( \phi_t(x,z) \) according to the parallel acceleration and velocity. As a consequence, the conditional averages \( \langle x(t), \phi^0 \rangle \) undergo a complex averaging process that influences their anti-symmetrical dependence on \( \phi^0 \). The parallel acceleration moves the trajectories toward the minima of the stochastic potential, which favors the \( \langle x(t) \rangle_{\phi^0} \) with \( \phi^0 < 0 \). On the other hand, the parallel velocity increases in the regions with negative potential and decreases in the regions with positive potential. The ions spend smaller time at negative than at positive potential, which partly compensates the attraction toward the potential minima, and yields a small perturbation of the symmetry of the HDs. This explains the small pinch velocity observed for D ions.

The heavy ions with large ionization rates Z have smaller acceleration, but also a much higher potential energy (larger than for D ions by the factor \( Z \)). This explains the significant pinch velocity produced by the parallel acceleration for W ions.

Thus, the pinch velocity is essentially determined by the decrease of the HD that corresponds to \( \phi^0 > 0 \) due to energy conservation. \( V_\perp^\infty \) has significant values for high Z impurities, which have large potential energy \( Z\phi^0 \) \( > W \).

The mechanism of the pinch velocity relies on the HDs, a quasi-coherent component of the motion. Collisions usually contribute to the enhancement of the random characteristics at the expense of the coherent ones. Moreover, in the case of heavy ions with large Z, the collision frequency and diffusion coefficient are much larger than for plasma ions. It is thus important to evaluate the effect of collisions in order to validate the pinch mechanism.

Collisions are represented by time dependent stochastic velocities introduced in equations (6) and (7), and their effect is evaluated with the DTM following a procedure similar to those developed in other transport problems [36, 37]. The normalized collisional diffusion coefficients in the perpendicular and parallel directions are

\[
d_\perp = \sqrt{A} \frac{a}{\lambda_{mfp}}, \quad d_\parallel = \frac{1}{\sqrt{A}} \frac{\lambda_{mfp}}{\nu}
\]  

and the collision frequency is

\[

\nu = \frac{Z^2}{\sqrt{A}} \frac{a}{\lambda_{mfp}}
\]  

where \( \lambda_{mfp} = v_{thi}/\nu_i \) is the mean free path of the protons and \( \nu_i \) is their collision frequency. The perpendicular diffusion has a much stronger effect on transport since \( d_\perp/d_\parallel = Z^2 \lambda_{mfp}^2 >> 1 \) in the units used in equations (6) and (7). The parallel collisional diffusion is neglected for the simplicity of this estimation. The correlations of the perpendicular velocity components \( \eta_i(t) \) are

\[
C_i(t) \equiv \langle \eta_i(t) \eta_i(t) \rangle_c = d_\perp^2 \nu \exp(-\nu t),
\]  

where \( i = x, y \). They determine stochastic displacements \( \xi_i(t) \) that are Gaussian with the mean squares
\[
\langle \xi^2_i(t) \rangle_c = 2d^2_i [1 + 1/\nu (\exp(-\nu t) - 1)] .
\]

The change of variable from \(x\) to \(x' = x - \xi(t)\), permits to define an effective velocity that includes the effect of collisions and to determine its EC by averaging the EC of the potential over \(\xi(t)\). The perpendicular component of the EC (4) becomes

\[
E^c = (A^c_y)^2 \partial_x \left[ \exp \left( -\frac{x^2}{2\Lambda^2_r} - \frac{y^2}{2\Lambda^2_y} \right) \sin \left( K_0 y \right) \right].
\]

This effective EC has the same structure as the EC (4), but with modified parameters that become time dependent functions

\[
(A^c_y)^2 = \frac{\lambda_r \lambda_y}{\Lambda_r \Lambda_y} , \Lambda^2_r = \lambda^2 + \langle \xi^2_i(t) \rangle_c , K_0 = k_0 \left( \frac{\lambda_y}{\Lambda_y} \right)^2 .
\]

The correlation lengths increase in time, while the amplitude of the potential and the dominant wave number decrease due to collisions. The change of the EC determines the change of the average potential (14) and of the DTs.

The effect of collisions on the pinch mechanism is exemplified in figure 3, where the average radial displacement is presented together with its conditional components in equation (26) for several values of the perpendicular collisional diffusion coefficient. One can see that \(\langle x(t) \rangle\) decreases with the increase of \(d^c\), which determines the decrease of the pinch velocity \(V^\infty\). The collision effect depends on the sign of the initial potential \(\langle x(t) \rangle_c\) and \(\langle x(t) \rangle_{-}\) have different dependences on \(d^c\). The positive component \(\langle x(t) \rangle_+\) (that is affected by the energy cut) is not influenced by collisions for \(d^c \lesssim 7\), and it increases at larger values. The negative component \(\langle x(t) \rangle_-\) reacts at small collisionality by significant decrease of its amplitude and this behavior is maintained at large \(d^c\). Thus, ion collisions have significant attenuation effect on the coherent motion that produce the HDs, but the pinch mechanism survives even at large collisional diffusion. This estimation finds an attenuation factor of the order 2 for a rather large interval of \(d^c\).

Another process that could hinder the acceleration induced pinch is produced by the gradient of the magnetic field \(B\), \(dB/dx \equiv B/R\), where \(R\) is the major radius of the plasma. It generates a radial pinch [40] that could influence the symmetry breaking of the HDs. We have found that the effect appears, but it is negligible for large size plasmas with \(a/\rho_i \gtrsim 100\).

The physical mechanism for the generation of the radial pinch is validated using DNS. The results presented in figure 4 confirm the existence of the conditional displacements \(\langle x(t) \rangle_{\phi}\) and the perturbation produced by the parallel acceleration, which essentially consists of forbidding the trajectories to reach the maxima of the potential. It yields average displacements as function of the initial value of the potential that are similar to those obtained by DTM (shown in figure 2 (right panel)).

The time dependent pinch velocity and diffusion coefficient obtained from the numerical simulation of the stochastic trajectories are presented in figure 5. They correspond to the W ion transport for the set of parameters mentioned at the beginning of this section, which yield using DTM the results shown in figure 1 (continuous lines). One can see that the results of the simulation are similar to those of the DTM for both \(V^\infty\) and \(D_x(t)\). This shows that DTM is qualitatively adequate for the study of the three-dimensional model (1) and (2). This conclusion is in agreement with previous studies [43].

There are, however, important differences that result from the approximation used in the DTM. They consists of the overestimation of trajectory trapping, which leads to smaller \(D_x(t)\) and larger \(V^\infty\). Also, details in the time dependence (as the minimum of \(D_x(t)\) that is negative in figure 1 and positive in figure 4) are not reproduced by the semi-analytical DTM. The overestimation of the pinch velocity is stronger in the case of time dependent potentials.

In order to obtain an accurate characterization of the acceleration induced pinch, DNS is used in the next section.
6. Characterization of the pinch velocity

The impurity ion transport described by equations (6) and (7) depends on a large number of parameters. Taking the case of W ions in large size plasmas, we fix \( A = 184 \) and \( \rho_s = \rho_t / a = 1/500 \), and remain with nine dimensionless parameters: \( K_z, \lambda_z / a, V_p = V_d / V_s, \tau_d / \tau_0, W, \lambda_i / \rho_i, \lambda_t / \rho_t \) and \( 0.09, \). The first three parameters are essential because they describe the main ingredients of the pinch mechanism: a turbulent state of plasma \( K_z \) with three-dimensional stochastic potential \( \lambda_z \) and an average poloidal velocity \( V_p \). The characteristics of the impurity ions are represented by \( P_s \). The time variation of the stochastic potential is expected to damage the ordered component of the motion and to favor the random aspects. It is thus essential to investigate the dependence of W transport on the decorrelation time \( \tau_d \). The dependence of the pinch velocity on the energy of the ions W could be important as a control method. The other three parameters describe details of the shape of the turbulence EC (4), which are less important for the pinch mechanism produced by the parallel acceleration.

\[ K_z = \Phi / \rho_z \]

is the measure of turbulence amplitude \( \Phi = eA_0 / T_i \). The latter is also contained in the parameter of the parallel acceleration \( P_p \). For a more clear presentation of the results, we analyze the dependence of the W ion transport on the physical parameters \( \Phi \) and \( Z \) rather than on \( K_z \) and \( P_p \).

The physical range of the parameters is explored around a basic case with \( \Phi = 0.03, Z = 40 \) (corresponding to \( K_z = 15, P_a = 0.09, \lambda_z = 0.5, V_p = 1, \tau_d = \infty \) (static potential), \( W = 1, \lambda_i = 5, \lambda_t = 2, \) and \( k_0 = 1 \). The units in the figures are \( \rho_t V_s \) for the diffusion coefficient, \( \rho_t \) for the average trajectories, \( V_s \) for the pinch velocity and \( \tau_0 = a / v_{th} \) for the time.

The analysis is performed using numerical simulations (the DNS method and code described in section 3.1).

The existence of the radial pinch velocity can be clearly seen in figure 6, which presents examples of radial displacements \( X(t) = \langle x(t) \rangle \). A fast increase of \( |X(t)| \) appears in all cases at small times, followed by a transitory evolution (that depends on the parameters of the process), which eventually leads to the asymptotic regime. The latter is always linear in time and corresponds to the (asymptotic) radial pinch. We note that the poloidal average velocity \( \langle v_p \rangle \) is not invariant as in two-dimensional potentials. Starting from \( V_p \), it has a transitory variation that can end with a stabilized asymptotic value slightly different of \( V_p \). Examples of the paths of the average trajectories are presented in figure 7. They show that the average poloidal motion is not simply \( Y(t) = V_p t \), but it depends on the other parameters (especially on \( \lambda_z \), which controls the parallel acceleration).

We present below the results obtained for the dependence of the asymptotic values \( V_p^\infty \) and \( D_p^\infty \) on each parameter. Scaling laws are derived and physical explanations are deduced. The latter are based on the two interaction mechanisms between the parallel motion and the perpendicular transport: the symmetry breaking of the HDs that generates the pinch and the parallel decorrelation that influences both \( V_p^\infty \) and \( D_p^\infty \). The parallel decorrelation time \( \tau_d^\infty (W, Z, \Phi, \lambda_z) \), equation (23), is a decreasing function of \( W, \Phi \) and \( Z \) and an increasing function of \( \lambda_z \), as discussed in section 4.

- Turbulence amplitude \( \Phi \)

The amplitude of the turbulence influences the electric drift velocity, the parallel acceleration and the potential energy. It has a complex effect on the pinch velocity and on the diffusion coefficient.

The increase of the electric drift determines the decay of the time of flight as \( \Phi^{-1} \), and a stronger transient growth in the quasilinear regime \( t < \tau_d \) for both \( D_p(t) \sim \Phi^2 t^2 \) and \( V_p(t) \sim \Phi t \). Trajectory eddying combined with the increase of the potential energy and of the parallel acceleration modifies the dependence on \( \Phi \) in the non-linear regime, and, consequently, in the asymptotic \( V_p^\infty \) and \( D_p^\infty \).

The asymptotic radial velocity \( V_p^\infty \) is shown in figure 8 (left panel) as function of \( \Phi \). The pinch is negative (inward) for the whole range of \( \Phi \) and it increases with \( \Phi \). The dependence is approximately linear for \( \Phi \leq 0.04 \), and a tendency of saturation can be observed at larger \( \Phi \).

The saturation of \( V_p^\infty \) at large \( \Phi \) is determined by the energy conservation, which prevents the trajectories to reach the regions with positive values of \( \phi \) above the limit \( \phi_{max} \) defined in equation (22). This determines the cut of \( \langle x(t) \rangle \) seen in figure 2 (right panel), which destroys the equilibrium of the HDs. The maximum perturbation of the HDs corresponds to the limit \( Z\Phi \rightarrow \infty \), which eliminates the whole positive
The average radial displacement of the W ions for several values of $Z$ (left panel) and of $\lambda_z$ (right panel). The other parameters correspond to the basic case.

The asymptotic diffusion coefficient $D_{\infty}^z$ increases with the increase of $\Phi$ according to the law $D_{\infty}^z \sim \Phi^\gamma$ with $\gamma = 1.5$, as seen in figure 8 (right panel). The values $1 < \gamma < 2$ define the super-Bohm regime. Such regime is unusual in the presence of trajectory trapping or eddying, which yields the scaling (24) with $0 < \gamma < 1$. This stronger increase of $D_{\infty}^z$ is the effect of the parallel acceleration through the effective parallel decorrelation time $\tau_{\infty}^z$. As discussed in section 4, $\tau_{\infty}^z$ is a decreasing function of $\Phi$. The supplementary dependence on $\Phi$ through $\tau_{\infty}^z(\Phi)$ increases the exponent $\gamma$. Thus, the super-Bohm regime is the result of trajectory trapping coupled to the parallel accelerated motion.

- **Ionization rate $Z$**

The mechanism of generation of the radial pinch depends essentially on the product $Z\Phi$. Thus, the ionization rate has a similar effect with the amplitude $\Phi$ of the turbulence. As seen in figure 9 (left panel), the pinch velocity has an approximately linear increase followed by the tendency of saturation, a behavior that is similar to the dependence on $\Phi$ (figure 8 (left panel)). The diffusion coefficient shown in 9 (right panel) has a more complicated dependence on $Z$, but the variation of $D_{\infty}^z$ on the relevant range of $Z$ is small (of the order $\pm 20\%$ of the average). The influence of $Z$ on the diffusion is produced through the effective parallel decorrelation time $\tau_{\infty}^z$ that depends on $Z$.

- **Parallel correlation length $\lambda_z$**

The pinch mechanism analyzed here appears only in three-dimensional stochastic potentials. But, as discussed in section 5, $V_{\infty}^z$ essentially results from the symmetry breaking of the HDs determined by the energy conservation. The potential energy does not depend on $\lambda_z$, which means that a finite $\lambda_z$ is necessary, but its direct quantitative influence on the pinch mechanism is small.

However, $\lambda_z$ has a strong influence on the transport through the parallel decorrelation time $\tau_{\infty}^z$ in equation (23) that increases with $\lambda_z$ faster than linearly. It explains the large decrease rate of both $V_{\infty}^z$ and $D_{\infty}^z$ seen in figure 9, which shows that $|V_{\infty}^z| \sim \lambda_z^{-1.3}$ and $D_{\infty}^z \sim \lambda_z^{-1.2}$. Thus, $\lambda_z$ determines the decrease of $V_{\infty}^z$ and $D_{\infty}^z$ only through the modification of the $\tau_{\infty}^z$.

- **Poloidal velocity $V_p$**

The poloidal average velocity is the source of the HDs. It has a strong influence on both the pinch velocity and the diffusion coefficient, as seen in figure 11.
The equations of motion (6) and (7) are invariant at the change $V_p \rightarrow -V_p$ and $x \rightarrow -x$, which implies that the conditional displacements and the HDs change their sign when $V_p \rightarrow -V_p$. Thus, the pinch velocity $V_\infty$ is an anti-symmetrical function of $V_p$, as seen in figure 11 (left panel). $V_\infty$ is linear in $V_p$ at small $V_p$, it has a maximum at $V_p \approx 0.3$ and a long tail with $|V_\infty| \sim |V_p|^{1.2}$ at large $V_p$.

The direction of the pinch produced by the parallel acceleration can be changed from inward to outward by the inversion the orientation of the poloidal velocity.

The diffusion coefficient is strongly influenced by $V_p$, which determines a large decrease of $D_\infty$, as seen in figure 11 (right panel). $D_\infty$ is not dependent on the sign of $V_p$.

Thus, $V_p$ has a special effect on the transport, different compared to the other parameters. The physical image of the influence of $V_p$ on ion trajectories is sketched in section 5, where the essential role played by $V_p$ in the generation of the HDs is discussed. The pinch $V_\infty$ is modified because $V_p$ controls the amplitude of the HDs. The decay of $D_\infty$ is the result of
the structure of the contour lines of the total potential, which is changed by \( V_p \).

- **Time decorrelation \( \tau_d \)**

Our results have confirmed the idea that the time variation of the stochastic potential determines a process of elimination of the pinch velocity by strengthening the random aspects of the motion. In addition to this, the Lagrangian energy is not a constant, but a fluctuating function of time.

However, the pinch velocity survives in time dependent potentials \( \phi(x, z, t) \) if the decorrelation time \( \tau_d \) is not too small. As seen in figure 12 (left panel), \( V^\infty_\phi \) is weakly dependent on \( \tau_d \) for \( \tau_d > 1 \), and it has fast decrease as \( V^\infty_\phi \sim -\tau_d^2 \) for \( \tau_d < 0.5 \). The pinch velocity is eliminated for fast time variation with \( \tau_d \ll \tau_\parallel \).

A different behavior was obtained for \( D^\infty_s \). As seen in figure 12 (right panel), \( D^\infty_s \) increases at small \( \tau_d \) (in the quasilinear regime, \( \tau_d \ll \tau_\parallel \)), reaches a maximum and decreases due to trapping for \( \tau_d \gg \tau_\parallel \). At larger \( \tau_d \), of the order of the parallel decorrelation time, \( D^\infty_s \) saturates at the value corresponding to the static potential. This behavior results from the combination of the time decorrelation processes produced by the time variation of the potential (represented by the time dependence of the EC (5)) and by the parallel motion (19). This yields an effective decorrelation time \( \tau^\infty_s \) that is \( \tau^\infty_s \sim \tau_\parallel \tau^\infty \) and \( \tau^\infty_s \sim \tau^\infty_\parallel \tau^\infty \).

- **Energy of the W ions**

The energy is directly connected to the mechanism of pinch generation. The cut of the conditional average displacements \( \langle x(t) \rangle_{\phi, \rho} \), which has the dominant influence of the asymmetry of the HDs, appears at \( \phi_{\max} = W/(Z\Phi) \). It is expected that the change of \( W \) determines a variation of the pinch velocity of the order of those produced by \( Z \) or \( \Phi \), but inverse, in the sense that \( |V^\infty_x| \) is a decreasing function of \( W \).

We have obtained only a very weak decrease of the pinch velocity with the energy. The reason is the dependence on \( W \) of the parallel decorrelation time \( \tau^\infty_\parallel \), which is a decreasing function of \( W \). The saturation of \( V_x(t) \) at a smaller time determines the increase of \( |V^\infty_x| \) (because it is a decreasing function of \( t \)). The two effects of the parallel acceleration (symmetry breaking of the HDs and decorrelation) are opposite in this case of the energy dependence, and they partly compensate.

The increase of the energy also determines a weak increase of the diffusion coefficient.
7. Discussions and conclusions

The relevance of the results presented here in the context of numerous complex theoretical and numerical studies of the heavy impurity transport has to be clarified. These studies are based on complicated self-consistent models that include detailed descriptions of the experiments, and on massive numerical simulations that use sophisticated codes. Many aspects and processes combine and mix in the results of these studies. Some of them are well known as particular processes, but other ones have not yet been identified or completely understood. The non-linear effects of the parallel acceleration and the mechanism of pinch generation belong to the last category. Studies focused on particular processes that are based on simplified (minimal) models are important both for better understanding and for better control of the complex global transport of heavy impurities. The particular processes usually have specific dependences on parameters that have to be known for finding efficient control procedures. Synergistic effects between particular processes certainly appear in the complex models. Thus, the complex approaches yield reliable results that can be used for describing and predicting impurity behavior, while the studies of the particular processes search for deeper physical understanding.

The main finding of this work is a radial pinch that is generated by the stochastic parallel acceleration in turbulent plasmas. It is significant for high Z impurities and negligible for plasma ions. We have shown that the pinch is produced in three-dimensional turbulence by the interaction of the parallel motion with the HDs, a special type of quasi-coherent radial motion that appears due to a poloidal average velocity.

We have also shown that the influence of the parallel motion on the transport through the parallel decorrelation time \( \tau_2^\infty \) is much stronger for heavy impurities than for plasma ions. The fluctuations of the parallel velocity are very large for W ions, and they determine a complicated dependence on the parameters of the parallel motion \( \tau_2^\infty(W, Z, \Phi, \lambda_x) \). This complex decorrelation process influences both the pinch velocity and the diffusion coefficient. It leads to an unusual diffusion regime of super-Bohm type and modifies the scaling laws of \( V^\infty_x \) and \( D^\infty_x \).

The physical domains of the main parameters of the transport model were explored for evaluating the scaling laws and for obtaining the range of the normalized pinch velocity and diffusion coefficient. The typical values of \( |V^\infty_x| \) are in the interval (0.05, 0.25).

The relevance of the acceleration induced pinch velocity appears more clearly in physical units. The typical normalized values determine different velocities for present plasmas (ASDEX Upgrade, JET) and ITER conditions. The main difference (concerning \( V^\infty_x \)) is the electron temperature. Due to the time-scale separation of the atomic and transport processes, the W impurities are in coronal equilibrium. The fractional abundance of each ionization stage is a function of the electron temperature that is practically not influenced by the transport [44]. This determines different ranges of the ionization rates for the present plasmas and ITER. In the first case Z varies from boundary to the center in the interval (20, 48), while in the second case the interval is (45, 63). Typical values of Z in the core plasma are \( Z = [43, 44, 57] \), where the first value in this and the following triads corresponds to ASDEX Upgrade, the second to JET and the third to ITER. This determines normalized values of the pinch velocity of the order \( |V^\infty_x| \approx [0.10, 0.11, 0.15] \). Thus, the pinch velocity is larger in the ITER plasmas roughly by 50% at similar parameters of the turbulence and poloidal velocity. Using typical parameters of these plasmas, the pinch velocities are of the order \( |V^\infty_x| \approx [160, 110, 194] \) m sec\(^{-1}\). The convection time to plasma center is very small \( \Delta t_c = a/V^\infty_x \approx [4, 11, 10] \) ms. Convection dominates diffusion in all cases, because \( \Delta t_c \ll \Delta t_{df}, \) where \( \Delta t_{df} = a^2/(2D^\infty_x) \) is the diffusive time. The ratio \( r = \Delta t_c/\Delta t_{df} \) is \( r \approx [0.18, 0.10, 0.07] \).

These values of the pinch velocity are evaluated in the frame of the minimal model (1)–(2). They only show that the effect produced by the parallel acceleration is very large in these ideal conditions. According to the estimations in section 5, collisions determine the attenuation of \( V^\infty_x \) by a factor of the order 2. Also, interactions with other pinch mechanisms and neoclassical aspects can strongly modify these values. Compared to the results obtained from complex models and simulations, this pinch velocity is larger by an order of magnitude (see, for example, the very recent paper [25], where the central accumulation of the W ions seen in figure 3 appears in hundreds of ms, corresponding to pinch velocities of the order of 10 ms\(^{-1}\)).

We underline that the dependence of \( V^\infty_x \) on \( V_p \) (figure 11 (left panel)) could provide a very efficient control possibility. The change of \( V_p \) from the direction of the electron to the ion diamagnetic velocity determines the inversion of the pinch from inward to outward direction. A strong variation of \( V^\infty_x \) with \( V_p \) exists at small \( |V_p| \lesssim 0.5 \), which shows a high sensitivity of the pinch velocity to the poloidal velocity.

In conclusion, this study provides understanding of the complex processes of interaction of the parallel acceleration with the perpendicular transport. The main effect consists of the generation of a radial pinch by the perturbation of the HDs, a quasi-coherent component of the motion. The typical values of \( V^\infty_x \) obtained from the minimal model are large, which shows that the process found here is rather strong such that it could contribute, besides other pinch sources, to the W ion dynamics in the existing plasmas and in ITER conditions.

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