Theoretical study of the differential equation concept of tsunami waves using the leapfrog scheme

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Abstract. In this research work, tsunami wave propagation models and the underlying theory are examined through solving numerical wave-based partial differential equations using the leapfrog scheme. Following the wave physics theory, shallow water equations are explored for their basics and influential parameters; then, numerical methods are employed to solve said equations. As a result, the derivatives of shallow water equations for tsunami propagation modeling have been successfully formulated.

1. Introduction
Tsunamis are natural disasters caused by earthquakes taking place in seabeds. Until today, this type of disaster remains a mystery in that its occurrences (when and where) and strength can not be predicted. More often than not, it causes severe damages and thousands of casualties \(^1\). Relevant risk reduction should be implemented efficiently to minimize detrimental impacts \(^2\), and this measure originates from an in-depth understanding of earthquake-generated tsunami hazards to which modeling can provide answers.

Crustal deformation and plate movement caused by tectonic earthquakes in the subduction zone are always offset by vertical motion (removal). When these events occur in large magnitude (> 6.5 Mg) at a shallow hypocenter (<50 kilometers below sea level), the vertical motion uplifts the seabed above the focus. This movement can either raise or drop seafloors suddenly, resulting in disturbance to the water balance above it. The greater the earthquake in the subduction zone, the wider the raised seabed area—i.e., a known cause of a tsunami.

Several models have been generated to simulate tsunami wave propagation characteristics from an earthquake hypocenter to the coastline and then the mainland. Some examples include TUNAMI, COMCOT, and Delft3D-Flow. Based on \(^3\), \(^4\), \(^5\), \(^6\), propagation models can estimate wave velocity, arrival time at shorelines, run-up height, and tsunami inundation.

Tsunami wave propagation can be modeled numerically using differential equations \(^7\). Shallow water equations (SWEs) based on water wave propagation theory are developed from fluid mechanics theory, including the law of conservation of mass and Newton’s second law. SWEs can be derived from the law of conservation of mass and the conservation of momentum expressed by the Navier-Stokes equation. SWEs are characterized by waves with relatively larger length compared to their depth \(^8\). In the case of earthquake-generated tsunamis, the wave propagation from the hypocenter to the mainland is predicted using a numerical approach. This model generally employs the principle of conservation of mass and hydrodynamic equations involving many variables, such as acceleration, advection, pressure gradients, surface/surface friction, and eddy viscosity \(^3\), \(^8\).
In wave propagation modeling, differential equations can be modeled using various numerical approaches, e.g., the Lax method, to solve hyperbolic partial differential equations. The Lax method consists of several schemes, including the leapfrog. Before applying it, the tsunami wave propagation equation is first discretized with the finite difference method.

In this research work, the basic theory of SWEs is examined, and the differential equations for tsunami wave propagation modeling are derived using the fluid mechanics theory. Also, the modeling equation is solved by creating numerical procedures using the volume method, i.e., the leapfrog scheme.

2. Basic theory
2.1. Partial differential equation
A partial differential equation (PDE) is a mathematical equation that relates the functions of several variables and their partial derivatives of these functions. The general form of a partial differential equation is:

$$A\phi_{xx} + B\phi_{xy} + C\phi_{yy} = f(x, y, \phi, \phi_x, \phi_y)$$

(1)

where \(A, B,\) and \(C\) are constants called quasilinear. Table 1 shows three categories of quasilinear equations.

| \(B^2 - 4AC\) | Category | Equation | Name |
|---------------|----------|----------|------|
| \(< 0\) | Elliptic | \(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0\) | Laplace equation |
| \(= 0\) | Parabolic | \(k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}\) | Heat conduction equation |
| \(> 0\) | Hyperbolic | \(\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}\) | Wave equation |

2.2. Tsunami wave propagation equation
2.2.1. Shallow water theory
The following equations are those of conservation of mass and momentum in three dimensions:

$$\frac{\partial \eta}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(2)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right) = 0$$

(3)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \left( \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right) = 0$$

(4)

$$g + \frac{1}{\rho} \frac{\partial p}{\partial z} = 0$$

(5)

where \(x\) and \(y\) are the horizontal axes, \(z\) is the vertical axis, \(t\) is time, \(h\) is water depth, \(\eta\) is vertical displacement above the water surface, \(u, v,\) and \(w\) are the velocities of water particles on the \(x, y,\) and \(z\) axis, \(g\) is gravitational acceleration, and \(\tau_{ij}\) is the normal voltage or tangential shear direction \(i\) on the normal plane \(j\).

The momentum equation in the direction of the vertical axis (\(z\)) with dynamic boundary conditions on surfaces where the pressure is zero \((p = 0)\) results in a hydrostatic equation \(p = \rho g (\eta - z)\). To
solve equations (1) - (4), dynamic and kinetic boundary conditions on the surface and at the bed are integrated using the Leibniz rule, leading to

\[ p = 0 \quad \text{on} \quad z = \eta \]  
\[ w = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y} \quad \text{on} \quad z = \eta \]  
\[ w = -u \frac{\partial \eta}{\partial x} - v \frac{\partial \eta}{\partial y} \quad \text{on} \quad z = -h \]  

The first form of the momentum equation in the \( x \)-direction can be written as follows

\[ \int_{-h}^{\eta} \frac{\partial u}{\partial t} \, dz = \frac{\partial}{\partial t} \int_{-h}^{\eta} u \, dz - u \left. \frac{\partial \eta}{\partial t} \right|_{z=h} + u \left. \frac{\partial (-h)}{\partial t} \right|_{z=-h} \]  

with the dynamic and kinetic conditions expressed in equations (5) - (7), a three-dimensional equation is obtained from the tsunami wave propagation concept (shallow water theory):

\[ \frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \]  
\[ \frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left( \frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{\tau_x}{\rho} = A \left( \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \right) \]  
\[ \frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left( \frac{MN}{D} \right) + \frac{\partial}{\partial y} \left( \frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial y} + \frac{\tau_y}{\rho} = A \left( \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right) \]  

where \( D \) is the total depth of water given by \( h + \eta \), \( \tau_x \) and \( \tau_y \) are the seabed frictions in the \( x \) and \( y \) directions, and \( A \) is the viscosity of a constant horizontal range in space; the shear stress of surface waves is negligible. \( M \) and \( N \) are the discharge fluxes in the \( x \) and \( y \)-directions, as expressed below

\[ M = \int_{-h}^{\eta} u \, dz = u(h + \eta) = uD \]  
\[ N = \int_{-h}^{\eta} v \, dz = v(h + \eta) = vD \]  

\[ 2.2.2 \text{ Sea floor friction} \]

In analogy with the uniform flow, the seabed section is generally stated as follows

\[ \frac{\tau_x}{\rho} = \frac{1}{2g} \frac{f}{D^2} M\sqrt{M^2 + N^2} \]  
\[ \frac{\tau_y}{\rho} = \frac{1}{2g} \frac{f}{D^2} N\sqrt{M^2 + N^2} \]  

where \( f \) is the coefficient of friction, and the Manning's roughness coefficient \( n \) is

\[ n = \frac{f D^{1/3}}{\sqrt{2g}} \]  

Equation (15) means that if the depth \( D \) is narrow, then \( f \) becomes large, and \( n \) remains almost constant; thus, the seabed friction is expressed as

\[ \frac{\tau_x}{\rho} = \frac{fn^2}{D^{1/3}} M\sqrt{M^2 + N^2} \]
In this tsunami wave propagation model, the Manning’s roughness coefficient \(n\) is chosen based on the average condition of the sea surface. Compared with the seabed friction, the range of horizontal turbulence during tsunami propagation in shallow waters is negligible, except for run-ups on the surface. Therefore, the basic equation of the model is as follows.

\[
\frac{\tau_y}{\rho} = \frac{f n^2}{D^{7/3}} N\sqrt{M^2 + N^2}
\]  

(19)

3. Results and discussion

3.1. Research procedure

The research described in this article focused on exploring the basic theory of SWEs as the input to numerical tsunami wave propagation models and searching for the solution to the modeling equation with the leapfrog scheme.

The stages of the research were:

1. Observation and collection of research data
   - Introduction to the basic equation of tsunami propagation and observations of relevant parameters and research variables were carried out. Furthermore, data and references related to the propagation model were collected.

2. Identification and classification of variables
   - The data and references obtained through observation formed the basis for the formulation and derivation of the SWE for the propagation model.

3. Formulation of the tsunami propagation equation
   - The tsunami propagation equation’s theory was studied using the basic theories underlying SWEs formulation. It includes the fluid mechanics theory that obeys the law of conservation of mass and Newton’s second law. In this case, the equation is an SWE derived from the laws of conservation of mass and conservation of momentum expressed by the Navier-Stokes equation. The methods employed were exploring the basic equations and deriving the tsunami wave propagation equation.

4. Discretization of the tsunami propagation equation
   - Formulating the tsunami propagation equation using the fluid mechanics theory results in a tsunami wave propagation model. The model is in the form of differential equations, which are analytically very difficult to solve.

5. Model interpretation
   - In this final stage of the model development process, the solution to the tsunami wave propagation equation was searched using numerical methods (i.e., the finite volume method) designed to solve the leapfrog scheme.

3.2. Differentiation of continuity equation

The differentiation of the continuity equation to an SWE refers to [10]. A differential continuity equation is a differential equation with four unknown variables. Then, it is integrated into \(z\), i.e.

\[
\int_{-h}^{\eta} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \right) \, dz
\]
Using Leibniz’s rule for differentiation under the integral sign, the derivative of the above integral is expressible as

\[
\int_{-h}^{\eta} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \, dz = \frac{\partial}{\partial x} \int_{-h}^{\eta} u \, dz - (u|_{z=-h}) \frac{\partial h}{\partial x} - (u|_{z=\eta}) \frac{\partial h}{\partial x} \\
\frac{\partial}{\partial y} \int_{-h}^{\eta} v \, dz - (v|_{z=\eta}) \frac{\partial h}{\partial y} - (v|_{z=-h}) \frac{\partial h}{\partial y} \\
\frac{\partial}{\partial z} \int_{-h}^{\eta} w \, dz - (w|_{z=\eta}) \frac{\partial h}{\partial z} - (w|_{z=-h}) \frac{\partial h}{\partial z}
\]

\(\eta\) and \(b\) are not functions of \(z\), resulting in

\[
\int_{-h}^{\eta} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \, dz = \frac{\partial}{\partial x} \int_{-h}^{\eta} u \, dz + \frac{\partial}{\partial y} \int_{-h}^{\eta} v \, dz \\
+ \left[ -(u|_{z=\eta}) \frac{\partial h}{\partial x} - (v|_{z=\eta}) \frac{\partial h}{\partial y} + w|_{z=\eta} \right] \\
- \left[ -(u|_{z=-h}) \frac{\partial h}{\partial x} - (v|_{z=-h}) \frac{\partial h}{\partial y} + w|_{z=-h} \right]
\]

Then, taking into account the boundary conditions leads to

\[
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \int_{-h}^{\eta} u \, dz + \frac{\partial}{\partial y} \int_{-h}^{\eta} v \, dz = 0 \\
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} [u(\eta + h)] + \frac{\partial}{\partial y} [v(\eta + h)] = 0
\]

Let \(M = u(\eta + h)\) and \(N = v(\eta + h)\), and using the rule of substitution, the continuity equation can be written as

\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0
\]  \hspace{1cm} (23)

3.3. Differentiation of momentum equation

The differentiation of the momentum equation to an SWE is performed in two directions: \(x\) and \(y\). Just like the continuity equation, this process also uses Leibniz’s rule for differentiation under an integral sign.

3.3.1. Momentum equation in \(x\)-direction

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -g \frac{\partial}{\partial x} (\eta + h)
\]

\[
\int_{-h}^{\eta} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \, dz = -\int_{-h}^{\eta} \left( g \frac{\partial}{\partial x} (\eta + h) \right) \, dz
\]

\[
\frac{\partial}{\partial t} \int_{-h}^{\eta} u \, dz + \frac{\partial}{\partial x} \int_{-h}^{\eta} (u^2) \, dz + \frac{\partial}{\partial y} \int_{-h}^{\eta} (uv) \, dz = -\frac{1}{2} g \frac{\partial}{\partial x} (\eta + h)^2
\]

\[
\frac{\partial}{\partial t} \left[ u(\eta + h) \right] + \frac{\partial}{\partial x} \left[ u^2 (\eta + h) \right] + \frac{\partial}{\partial y} \left[ uv (\eta + h) \right] = -\frac{1}{2} g \frac{\partial}{\partial x} (\eta + h)^2
\]
Let \( M = u(\eta + h) = uD \) and \( N = v(\eta + h) = vD \) where \( D \) is the total depth of water given by \( \eta + h \), \( \tau_x \) and \( \tau_y \) are the seabed frictions in \( x \)- and \( y \)-directions, \( A \) is the viscosity of a constant horizontal range in space; the shear stresses of the surface waves are negligible. Thus, the momentum equation in the \( x \)-direction is expressible as

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left( \frac{MN}{D} \right) = - \frac{1}{2} g \frac{\partial}{\partial x}(\eta + h)^2
\]

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left( \frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{\tau_x}{\rho} = A \left( \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \right)
\]

(24)

3.3.2. Momentum equation in \( y \)-direction

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -g \frac{\partial}{\partial y}(\eta + h)
\]

\[
\int_{-h}^{\eta} \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) dz = - \int_{-h}^{\eta} \left( g \frac{\partial}{\partial y}(\eta + h) \right) dz
\]

\[
\frac{\partial}{\partial t} \int_{-h}^{\eta} v dz + \frac{\partial}{\partial x} \int_{-h}^{\eta} (uv) dz + \frac{\partial}{\partial y} \int_{-h}^{\eta} (v^2) dz = - \frac{1}{2} g \frac{\partial}{\partial y}(\eta + h)^2
\]

\[
\frac{\partial}{\partial t} \left[ v(\eta + h) \right] + \frac{\partial}{\partial x} \left[ uv(\eta + h) \right] + \frac{\partial}{\partial y} \left[ v^2(\eta + h) \right] = - \frac{1}{2} g \frac{\partial}{\partial y}(\eta + h)^2
\]

3.3. Numerical schemes for shallow water equations

Based on the differentiation of continuity and momentum equations, the basic equation of the tsunami wave propagation model is written as follows:

\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0
\]

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left( \frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{\tau_x}{\rho} = A \left( \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \right)
\]

\[
\frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left( \frac{MN}{D} \right) + \frac{\partial}{\partial y} \left( \frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial y} + \frac{\tau_y}{\rho} = A \left( \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right)
\]
where \( \frac{\tau_x}{\rho} = \frac{1}{2gD^2} M \sqrt{M^2 + N^2} \) and \( \frac{\tau_y}{\rho} = \frac{1}{2gD^2} N \sqrt{M^2 + N^2} \)

where \( f \) is the coefficient of friction, and the Manning’s roughness efficient \( n \) is

\[ n = \left( \frac{fD^{1/3}}{2g} \right) \]

The seabed frictions in \( x \)- and \( y \)-directions are expressible as

\[ \frac{\tau_x}{\rho} = \frac{fn^2}{D^{7/3}} M \sqrt{M^2 + N^2} \]

\[ \frac{\tau_y}{\rho} = \frac{fn^2}{D^{7/3}} N \sqrt{M^2 + N^2} \]

As a result, the basic equation of the tsunami wave propagation model is as follows

\[ \frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \]  \hspace{1cm} (26)

\[ \frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left( \frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{gn^2}{D^{7/3}} M \sqrt{M^2 + N^2} = 0 \]  \hspace{1cm} (27)

\[ \frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left( \frac{MN}{D} \right) + \frac{\partial}{\partial y} \left( \frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial y} + \frac{gn^2}{D^{7/3}} N \sqrt{M^2 + N^2} = 0 \]  \hspace{1cm} (28)

To illustrate the numerical scheme for a tsunami propagation model, one needs to discretize the three dimensions of SWEs. First, continuity equations are approximated by finite difference equations with three forms of central differencing schemes, namely

\[ \frac{\partial \eta}{\partial t} = \frac{1}{\Delta t} \left[ \eta_{i,j}^{k+1} - \eta_{i,j}^{k} \right] \]

\[ \frac{\partial M}{\partial x} = \frac{1}{\Delta x} \left[ M_{i+rac{1}{2},j}^{k+1} - M_{i-rac{1}{2},j}^{k+1} \right] \]

\[ \frac{\partial N}{\partial y} = \frac{1}{\Delta y} \left[ N_{i,j+rac{1}{2}}^{k+1} - N_{i,j-rac{1}{2}}^{k+1} \right] \]
Assuming that the values of $k$ and $k + 1/2$ at the time step are known, the only unknown variable to solve is $\eta (i, j, k + 1)$ to be solved

$$\eta_{i,j}^{k+1} = \eta_{i,j}^{k} - \frac{\Delta t}{\Delta x} \left[ M_{i+\frac{1}{2},j}^{k+\frac{1}{2}} - M_{i-\frac{1}{2},j}^{k+\frac{1}{2}} \right] - \frac{\Delta t}{\Delta y} \left[ N_{i,j+\frac{1}{2}}^{k+\frac{1}{2}} - N_{i,j-\frac{1}{2}}^{k+\frac{1}{2}} \right]$$

Second, the momentum equation is approximated by finite difference equations with convection and upwind schemes, as presented below

$$\frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) = \frac{1}{\Delta x} \left[ \lambda_{11} \left( \frac{M_{i+\frac{1}{2},j}^{k-\frac{1}{2}}}{D} \right)^2 + \lambda_{21} \left( \frac{M_{i+\frac{1}{2},j}^{k+\frac{1}{2}}}{D} \right)^2 + \lambda_{31} \left( \frac{M_{i-\frac{1}{2},j}^{k+\frac{1}{2}}}{D} \right)^2 \right]$$

$$\frac{\partial}{\partial y} \left( \frac{N^2}{D} \right) = \frac{1}{\Delta y} \left[ v_{12} \left( \frac{N_{i,j+\frac{1}{2}}^{k-\frac{1}{2}}}{D} \right)^2 + v_{22} \left( \frac{N_{i,j+\frac{1}{2}}^{k+\frac{1}{2}}}{D} \right)^2 + v_{32} \left( \frac{N_{i,j-\frac{1}{2}}^{k+\frac{1}{2}}}{D} \right)^2 \right]$$

$$\frac{\partial}{\partial y} \left( \frac{MN}{D} \right) = \frac{1}{\Delta y} \left[ v_{11} \left( \frac{M_{i+\frac{1}{2},j+\frac{1}{2}}^{k-\frac{1}{2}}}{D} \right) + v_{21} \left( \frac{M_{i+\frac{1}{2},j+\frac{1}{2}}^{k+\frac{1}{2}}}{D} \right) + v_{31} \left( \frac{M_{i-\frac{1}{2},j+\frac{1}{2}}^{k+\frac{1}{2}}}{D} \right) \right]$$

$$\frac{\partial}{\partial x} \left( \frac{MN}{D} \right) = \frac{1}{\Delta x} \left[ \lambda_{12} \left( \frac{M_{i+\frac{1}{2},j+\frac{1}{2}}^{k-\frac{1}{2}}}{D} \right) + \lambda_{22} \left( \frac{M_{i+\frac{1}{2},j+\frac{1}{2}}^{k+\frac{1}{2}}}{D} \right) + \lambda_{32} \left( \frac{M_{i-\frac{1}{2},j+\frac{1}{2}}^{k+\frac{1}{2}}}{D} \right) \right]$$

where
Based on the numerical description of the leapfrog scheme of continuity and momentum equations, a numerical equation can be derived from SWEs

\[
\frac{\partial \eta}{\partial t} + \frac{1}{\Delta x} \left[ \lambda_{11} \left( \frac{M_{k+\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 + \lambda_{21} \left( \frac{M_{k-\frac{1}{2}}^{i-\frac{1}{2}}}{D_{i-\frac{1}{2}}} \right)^2 + \lambda_{31} \left( \frac{N_{k-\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 \right] + \frac{1}{\Delta y} \left[ v_{11} \left( \frac{N_{k+\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 + v_{21} \left( \frac{N_{k-\frac{1}{2}}^{i-\frac{1}{2}}}{D_{i-\frac{1}{2}}} \right)^2 \right] = 0
\]

\[
\frac{\partial M}{\partial t} + \frac{1}{\Delta x} \left[ \lambda_{12} \left( \frac{M_{k+\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 + \lambda_{22} \left( \frac{M_{k-\frac{1}{2}}^{i-\frac{1}{2}}}{D_{i-\frac{1}{2}}} \right)^2 + \lambda_{32} \left( \frac{N_{k-\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 \right] + \frac{1}{\Delta y} \left[ v_{12} \left( \frac{N_{k+\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 + v_{32} \left( \frac{N_{k-\frac{1}{2}}^{i-\frac{1}{2}}}{D_{i-\frac{1}{2}}} \right)^2 \right] = 0
\]

\[
\frac{\partial N}{\partial t} + \frac{1}{\Delta x} \left[ \lambda_{12} \left( \frac{M_{k+\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 + \lambda_{22} \left( \frac{M_{k-\frac{1}{2}}^{i-\frac{1}{2}}}{D_{i-\frac{1}{2}}} \right)^2 + \lambda_{32} \left( \frac{N_{k-\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 \right] + \frac{1}{\Delta y} \left[ v_{12} \left( \frac{N_{k+\frac{1}{2}}^{i+\frac{1}{2}}}{D_{i+\frac{1}{2}}} \right)^2 + v_{32} \left( \frac{N_{k-\frac{1}{2}}^{i-\frac{1}{2}}}{D_{i-\frac{1}{2}}} \right)^2 \right] = 0
\]
4. Conclusion

The application of mathematics, especially partial differential equations and the numerical difference method approach designed in this study, has succeeded in differentiation SWEs. Here, SWEs are used to model tsunami wave propagation from an earthquake hypocenter to the mainland. The numerical equation can simulate wave dynamics in shallow waters well; thus, it is applicable for modeling waves in coastal areas. Further developments need to incorporate a laboratory model into SWE-based equations and modeling.

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