Exact magnetic field control of nitrogen-vacancy center spin for realizing fast quantum logic gates

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(Dated: September 25, 2015)

The negatively charged nitrogen-vacancy (NV) center spin in diamond can be used to realize quantum computation and to sense magnetic fields. Its spin triplet consists of three levels labeled with its spin z-components of +1, 0, and -1. Without external field, the +1 and -1 states are degenerate and higher than the 0 state due to the zero-field splitting. By taking the symmetrical and anti-symmetrical superpositions of the +1 and -1 states as our qubit basis, we obtain exact evolution operator of the NV center spin under time-dependent magnetic field by mapping the three-level system on time-dependent quantum two-level systems with exact analytical solutions. With our exact evolution operator of the NV center spin including three levels, we show that arbitrary qubits can be prepared from the starting 0 state and arbitrary rapid quantum logic gates of these qubits can be realized with magnetic fields. In addition, it is made clear that the typical quantum logic gates can be accomplished within a few nanoseconds and the fidelity can be very high because only magnetic field strength needs to be controlled in this approach. These results should be useful to realizing quantum computing with the NV center spin systems in diamond and exploring other effects and applications.

PACS numbers: 75.75.-c, 03.67.-a, 75.10.-b, 75.90.+w

I. INTRODUCTION

The negatively charged nitrogen-vacancy (NV) center in diamond has been intensively investigated because it can be used for realizing quantum computation and sensing weak magnetic field, electric field, strain etc. As quantum technology evolves, one can manipulate the NV center spin with electromagnetic field for the NV center spin (including three levels) under time-dependent magnetic fields. Then, we use these exact evolution operators to prepare arbitrary qubit states with the basis $|\pm\rangle$ from the starting state $|0\rangle$ and realize arbitrary quantum logic gates. All the typical quantum logic gates can be completed within a few nanoseconds. The fidelity can be made very high because one needs to control magnetic field strength only. More detailed results will be presented in the following.

The rest of the paper is organized as follows. In Set. II, we define the Hamiltonian and elucidate the new spin basis. In Sec. III, we construct exact evolution operators for the NV center spin under time-dependent magnetic fields. In Sec. IV, we show how to use the exact evolution operators to prepare arbitrary qubit states with the basis $|\pm\rangle$ from the starting state $|0\rangle$. In Sec. V, we construct arbitrary quantum logic gates for the NV center spin qubits by using the exact evolution operators. Finally, we make some necessary discussions and give our conclusion in Sec. VI.

II. NEW SPIN BASIS

The Hamiltonian of NV center spin $\vec{S}$, in the presence of time dependent magnetic field $\vec{B}(t)=(B_x(t), B_y(t), B_z(t))$, can be written as

$$H = D S_z^2 + \gamma \vec{S} \cdot \vec{B}(t),$$  \hspace{1cm} (1)
where \( h = 1 \) is used, \( D = 2.87 \text{GHz} \) is the zero-field splitting, and \( \gamma = 2.88 \text{MHz/G} \) is the electron gyromagnetic ratio. Accordingly, the Schrödinger equation for time-evolution operator \( U \) is given by \( i \frac{d}{dt} U = H U \).

In the \( S_z \) representation, the matrix form of the Hamiltonian \([1]\) can be written as:

\[
H = \left( \begin{array}{ccc}
D + \gamma B_z & \frac{\gamma (B_z - i B_x)}{\sqrt{2}} & 0 \\
\frac{\gamma (B_z + i B_x)}{\sqrt{2}} & 0 & \frac{\gamma (B_z - i B_x)}{\sqrt{2}} \\
\frac{\gamma (B_z + i B_x)}{\sqrt{2}} & 0 & D - \gamma B_z
\end{array} \right). 
\] (2)

Introducing the unitary transform

\[
S_1 = \left( \begin{array}{ccc}
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}
\end{array} \right),
\] (3)

we can transform the Hamiltonian \([2]\) into

\[
H' = \left( \begin{array}{ccc}
D & \gamma B_x & \gamma B_y \\
\gamma B_x & 0 & i \gamma B_y \\
\gamma B_y & -i \gamma B_y & D
\end{array} \right). 
\] (4)

It means that the basis is changed from \([|+\rangle, |0\rangle, |-\rangle\]) to \([|\uparrow\rangle, |0\rangle, |\downarrow\rangle\]), where \(|\uparrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \) and \(|\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \). When the magnetic field is turned off, the \( D \) term will produce the same overall phase for both \(|\uparrow\rangle \) and \(|\downarrow\rangle \). Therefore, there will be no relative phase between \(|\uparrow\rangle \) and \(|\downarrow\rangle \), and \(|\uparrow\rangle \) and \(|\downarrow\rangle \) can be used to make a stable qubit basis.

### III. Exact Evolution Operators

With a special magnetic field \( \vec{B}(t) = (\alpha B_1, \beta B_1, 0) \), the Hamiltonian in Eq. \([4]\) becomes

\[
H_o = \left( \begin{array}{ccc}
D & \alpha \gamma B_1 & 0 \\
\alpha \gamma B_1 & 0 & i \beta \gamma B_1 \\
0 & -i \beta \gamma B_1 & D
\end{array} \right), 
\] (5)

where \( \alpha \) and \( \beta \) are adjustable real parameters satisfying \( \alpha^2 + \beta^2 = 1 \). Because of this condition, \( \alpha \) and \( \beta \) can be parameterized as \( \alpha = \cos \theta \) and \( \beta = \sin \theta \), where \(-\pi \leq \theta \leq \pi \). Introducing another unitary transform

\[
S_2 = \left( \begin{array}{ccc}
\cos \theta & 0 & i \sin \theta \\
0 & 1 & 0 \\
i \sin \theta & 0 & \cos \theta
\end{array} \right),
\] (6)

we can transform \( H_o \) into a block-diagonal matrix \( H_t = S_2 H_o S_2^\dagger \),

\[
H_t = \left( \begin{array}{ccc}
D & J(t) & 0 \\
J(t) & 0 & 0 \\
0 & 0 & D
\end{array} \right).
\] (7)

where \( J(t) = \gamma B_1(t) \). Because \( H_t \) consists of a \( 2 \times 2 \) block

\[
H_{t2} = \left( \begin{array}{cc}
D & J(t) \\
J(t) & 0
\end{array} \right)
\] (8)

and a \( 1 \times 1 \) block \( H_{t1} = D \), we can focus on \( H_{t2} \) in Eq. \([8]\).

Introducing a \( 2 \times 2 \) unitary transform

\[
T_h = \frac{1}{\sqrt{2}} \left( \begin{array}{cc}
1 & 1 \\
1 & -1
\end{array} \right),
\] (9)

we can change \( H_{t2} \) into \( H_{k2} = H_2 + D/2 \), with \( H_2 \) given by

\[
H_2 = \left( \begin{array}{cc}
J(t) & D/2 \\
D/2 & -J(t)
\end{array} \right).
\] (10)

The constant term \( D/2 \) in \( H_{k2} \) contributes only an overall phase to the time evolution operator. For the time-dependent two-level Hamiltonian \( H_2 \), there are many exact solutions such as those \([19,21]\). Here, we choose a powerful method to construct its exact time evolution operator \([19,21]\).

The Schrödinger equation of \( H_2 \) can be exactly solved by the evolution operator \([19,21]\).

\[
U_2 = \left( \begin{array}{cc}
u_{11} & -u_{21} \\
u_{21} & u_{11}
\end{array} \right), \quad |u_{11}|^2 + |u_{21}|^2 = 1,
\] (11)

and the matrix elements \( u_{11} \) and \( u_{21} \) and the quantity \( J(t) \) can be expressed as \([19,21]\).

\[
\left\{ \begin{array}{l}
u_{11}(t) = \cos(\chi(t)) e^{i \xi_- (t)} \\
u_{21}(t) = i \eta \sin(\chi(t)) e^{i \xi_+ (t)} \\
J(t) = \frac{\dot{\chi}(t)}{\sqrt{D^2 - 4 \chi^2(t)^2}} - \frac{1}{2} \sqrt{D^2 - 4 \chi^2(t)^2} \cot(2 \chi(t))
\end{array} \right.
\] (12)

where \( \xi_\pm \) is defined as

\[
\xi_\pm = \int_0^t dt' \sqrt{D^2 - 4 \chi^2(t')} \csc(2 \chi(t')) \pm \frac{1}{2} \arcsin\left( \frac{2 \chi(t')}{D} \right) \pm \eta \frac{\pi}{4}.
\] (13)

Here, \( \eta \) can take either \( +1 \) or \( -1 \), and \( \chi \) must satisfies three conditions: \( |\dot{\chi}(0)| \leq \frac{D}{2} \), \( \dot{\chi}(0) = 0 \), and \( \dot{\chi}(0) = -\eta \frac{D}{2} \).

By choosing suitable \( \chi(t) \), we can exactly construct \( J(t) \) in \( H_2 \) and the evolution operator \( U_2 \) in this way.

After adding the phase factor due to the \( D/2 \) term and making the inverse unitary transformation with \( T_h^\dagger \), we obtain the evolution operator for \( H_{t2} \),

\[
U_{t2}(t) = \left( \begin{array}{cc}
u_{11} & -\bar{u}_{21} \\
\bar{u}_{21} & \bar{u}_{11}
\end{array} \right) e^{-i \frac{D}{2} t}
\] (14)

where the matrix elements \( \bar{u}_{11} \) and \( \bar{u}_{21} \) are expressed as

\[
\left\{ \begin{array}{l}
u_{11}(t) = \cos(\chi(t)) \cos(\xi_- (t)) + i \eta \cos(\xi_+ (t)) \sin(\chi(t)) \\
u_{21}(t) = \eta \sin(\chi(t)) \sin(\xi_+ (t)) + i \cos(\chi(t)) \sin(\xi_- (t))
\end{array} \right.
\]
Consequently, the whole time evolution operator of the Hamiltonian $H_o$ can be written as:

$$U_o(t) = \begin{pmatrix} u_{11} -u_{21} & 0 \\ u_{21} & u_{11} -u_{21} \\ 0 & 0 \end{pmatrix} e^{-i\frac{2}{\hbar}t}$$ (15)

After making the inverse transform $S^*_o$, we can get the time evolution operator of the starting Hamiltonian $H_o$ in the new basis of $|+\rangle$ and $|-\rangle$:

$$U_o(\theta, t) = d(t) \begin{pmatrix} u_{11}\alpha^2 + d(t)\beta^2 & -\bar{u}_{21}\alpha - i(d(t) - \bar{u}_{11})\alpha\beta \\ -\bar{u}_{11}\alpha - i(d(t) - \bar{u}_{11})\alpha\beta & \bar{u}_{11}\beta + d(t)\alpha^2 + \bar{u}_{21}\beta^2 \end{pmatrix}$$ (16)

where $\alpha = \cos \theta$, $\beta = \sin \theta$, and $d(t) = e^{-i\frac{2}{\hbar}t}$.

From above equations (12) and (13), we can see that if $d(t)$ is determined from equation (17), the time evolution operator $U_o(\theta, t)$ can be formally solved by using equation (18), reading

$$U_o(\pi - \theta, T_f) |0\rangle = -i \begin{pmatrix} \cos \theta_1 & 0 \\ 0 & i\sin \theta_1 \end{pmatrix} e^{-i\frac{2}{\hbar}T_f}.$$ (20)

Neglecting the overall factor, we obtain $\cos \theta_1 |+\rangle + i\sin \theta_1 |-\rangle$.

**IV. EXACT INITIALIZING OF ARBITRARY QUBITS**

Experimentally, the NV center spin can be easily prepared in state $|0\rangle$. We try to realize state transfer between $|0\rangle$ and $|\pm\rangle$. With the time evolution operator $U_o$ applied, the state $|0\rangle$ will become

$$U_o(\theta, t) |0\rangle = d(t) \begin{pmatrix} -\cos \theta \eta \sin \chi \sin \xi_+ - i \cos \chi \sin \xi_- & \sin \theta \eta \sin \chi \cos \xi_+ - i \cos \chi \sin \xi_- \\ \sin \theta \eta \sin \chi \cos \xi_+ - i \cos \chi \sin \xi_- & -\cos \theta \eta \sin \chi \sin \xi_+ - i \cos \chi \sin \xi_- \end{pmatrix}$$ (17)

We require that the function $\chi(t)$ is given by

$$\chi(t) = \chi_0 - \frac{2\chi_0^3}{3} + \frac{\chi_0^5}{5\beta_f} - \frac{2\chi_0^5}{5\beta_f}$$ (18)

where $\chi_0$ is as defined in $\frac{D}{\chi}$, $\kappa$ is an adjustable parameter, and $T_f$ describes the time duration. Using $\eta = -1$, we have $\chi_0 = (T_f) = \chi_0 (T_f)$ according to equation (18). Because the target state doesn’t contain state $|0\rangle$, we need to set $u_{11}(T_f) = 0$, i.e. $\cos \chi_0 (T_f) = \cos \chi_0 (T_f) = 0$. Then the quantity $\chi(T_f)$ contributes an overall phase in the state $U_o(\theta, T_f) |0\rangle$ in Eq. (17). In order to achieve a minimal time value $T_f$ and a finite field pulse in the time interval $t \in (0, T_f)$, we need two conditions: $0 < \chi(T_f) \leq \frac{\pi}{2}$ and

$$\int_0^{T_f} dt \left[ \frac{1}{2} \sqrt{D^2 - 4\chi^2} \csc(2\chi) \right] = \frac{\pi}{2}.$$ (19)

Once we set $t = T_f$ and choose a value for $\chi(T_f)$, $\kappa$ can be formally solved by using equation (18), reading $\kappa = 15(\Delta T_f - \chi(T_f))/\Delta T_f^3$. The time duration $T_f$ can be solved from equation (19), and then $\kappa$ can be calculated immediately. In the following, we shall show how to initialize three typical qubits from the spin state $|0\rangle$.

**Initialization the basis states** $|\pm\rangle$. Choosing $\alpha = 0$ (or $\beta = 0$) in Eq. (5), we can easily get the target state $U_o(\pi, T_f) |0\rangle = |-\rangle$ (or $U_o(0, T_f) |0\rangle = |+\rangle$) with an overall phase. In this way, we get $|\pm\rangle$ from $|0\rangle$.

**Initializing a superposed state** $\cos \theta_1 |+\rangle + i\sin \theta_1 |-\rangle$. In this case, we can assume $0 \leq \theta_1 \leq \pi$ without losing any effective information. Choosing $\alpha$ and $\beta$ in Eq. (5) to satisfy the equality $\arctan(\beta/\alpha) = \theta \geq 0$, we can let $\theta = \pi - \theta_1$ in Eq. (17) and thus obtain the final state

$$U_o(\pi - \theta_1, T_f) |0\rangle = -i \begin{pmatrix} \cos \theta_1 & 0 \\ 0 & i\sin \theta_1 \end{pmatrix} e^{-i\frac{2}{\hbar}T_f}.$$ (20)

Therefore, the whole procedure, achieved in two steps, can be represented as the evolution operator

$$U_I(\varphi, \theta_1) = P_f(\varphi - \frac{\pi}{2}) U_o(\pi - \theta_1, T_f)$$ (22)
The final magnetic field \( B_x(T_f) \) in unit of 10^4 G depending on parameter \( \chi(T_f) \).

For practical application, it is useful to adjust the value \( \chi(T_f) \) to connect the two magnetic fields \( B_1(t) \) and \( B_0 \) at the time \( T_f \) along either \( x \)-axis or \( y \)-axis. We show in Fig. 1 that magnetic field \( B_1(T_f) \) can vary from 450G to 740000G when \( \chi(T_f) \) changes within \( (0, \frac{\pi}{2}) \). Because of the large domain of \( B_1(T_f) \), we can likely realize continuous connection of magnetic field in either \( x \)-axis or \( y \)-axis.

V. REALIZATION OF QUANTUM LOGIC GATES

In last section, we show how to initiate the basis states \(|\pm\rangle\) and their superposed states from the state \(|0\rangle\). Arbitrary qubits are made from the basis states \(|\pm\rangle\). In the quantum circuit model of computation, a quantum gate is a basic quantum circuit operating on a small number of qubits. We show how to realize typical quantum gates on the qubit in the following. In some of the cases, the special state \(|0\rangle\) can be used as a auxiliary state, or a bridge.

\( \frac{\pi}{2} \) phase shift gate: \(|+\rangle + |-\rangle \rightarrow |+\rangle + e^{i\frac{\pi}{2}} |-\rangle \). In this case, we need only a phase factor \( e^{i\frac{\pi}{2}} \) for the \(|-\rangle\) term. This can be achieved by applying a unitary transformation \( P_f(\frac{\pi}{2}) \) on the starting state \(|+\rangle + |-\rangle \). As a result, we obtain the evolution operator for the \( \frac{\pi}{2} \) phase shift gate:

\[
U_{\frac{\pi}{2}} = P_f(\frac{\pi}{2})e^{-i\frac{\pi}{4}}.
\]

The time duration \( \tau_1 \) and the time-independent magnetic field \( h_1 \) can be given by \( \tau_1 = \frac{\pi}{B} \approx 1.1 \text{ns} \) and \( h_1 = \frac{\sqrt{3}B}{2\tau_1} \approx 888 \text{G} \).

\( \frac{\pi}{4} \) phase shift gate: \(|+\rangle + |-\rangle \rightarrow |+\rangle + e^{i\frac{\pi}{4}} |-\rangle \). It is similar to the \( \frac{\pi}{2} \) phase shift gate. The phase factor is \( e^{i\frac{\pi}{4}} \) in this case. Applying \( P_f(\frac{\pi}{4}) \) on \((|+\rangle + |-\rangle)\), we obtain the evolution operator for the \( \frac{\pi}{4} \) phase shift gate

\[
U_{\frac{\pi}{4}} = P_f(\frac{\pi}{4})e^{-i\frac{\pi}{8}}.
\]

The time duration \( \tau_2 \) and the time-independent magnetic field \( h_2 \) are given by \( \tau_2 = \frac{\pi}{2B} \approx 1.64 \text{ns} \) and \( h_2 = \frac{\sqrt{3}B}{2\tau_2} \approx 452 \text{G} \).

Pauli-X gate: \(|+\rangle \rightarrow |+\rangle \). This gate can be realized by applying \( U_f^X(0,t_1) \) on the initial state \(|+\rangle \) and then \( U_o(\frac{\pi}{2}, t_2) \) on the resulting intermediate state \(|0\rangle\):

\[
|0\rangle \propto U_o(\frac{\pi}{2}, t_2)U_f^X(0, t_1)|+\rangle.
\]

In this way the gate is realized in two steps. Letting \( \chi_1(t_1) = \chi_2(t_2) = \frac{\pi}{2} \), we have \( t_1 = t_2 = T \), and then obtain \( \kappa_1 = \kappa_2 = \frac{15(T-\frac{\pi}{2})}{\chi_1} \). Using the phase condition in (19), we can obtain \( T = \frac{3.93}{\kappa_1} \). For the first step, the time-dependent field function \( f(t)/\lambda \) and the probability evolution operator \( P_{\lambda} \) are shown in Fig. 2. For the second step, we have similar time dependence for the field and the probability. After these two steps, the evolution operator for the Pauli-X gate is given by

\[
V_X = U_o(\frac{\pi}{2}, T)U_f^X(0, T)e^{i\pi/2}.
\]

And the total time interval is equivalent to \( 2T \approx 2.7 \text{ns} \).

In addition, this state can be realized without applying \( U_o(\theta_2, T_1) \) on state \(|0\rangle\), where \( T_1 \) is pulse time duration. At this time, we must guarantee matrix element \( w_{21}(T_1) = 0 \). After applying operator \( U_o(\theta_2, T_1) \) on state \(|+\rangle\), we derive

\[
U_o(\theta_2, T_1)|+\rangle = d(T_1) \left( \begin{array}{cc} e^{i\frac{\pi}{2}T_1} \sin^2 \theta_2 - e^{-i\chi_{3}(T_1)} \cos^2 \theta_2 & 0 \\ 0 & e^{i\frac{\pi}{2}T_1} + e^{-i\chi_{3}(T_1)} \cos \theta_2 \sin \theta_2 \end{array} \right)
\]

with \( \chi_{\pm}(T_1) = 2m\pi \) and \( m \) is positive integer. The condition to achieve state \(|-\rangle\) is \( \sin^2(\theta_2) [1 + \cos(\frac{\pi}{2}T_1 - \chi_{3}(T_1))] = 2 \). It can be satisfied by setting \( \cos(\frac{\pi}{2}T_1 - \chi_{3}(T_1)) = 1 \) and \( \theta_2 = \frac{\pi}{4} \). A reasonable result is \( \frac{\pi}{2}T_1 - \chi_{3}(T_1) = 2\pi \) and then \( \chi_{3} = \frac{3\pi}{T_1} \). Using the phase condition about \( \chi_{\pm}(T_1) \) in equation (19) by replacing \( \frac{\pi}{4} \) with \( \pi + 2m\pi \), we can get a self-consistent value, \( T_1 \approx 5.3 \text{ns} \). Therefore, the evolution operator can be expressed as

\[
U'_{X} = U_o(\theta_2, T_1)e^{i\pi DT_1 + \pi/2}.
\]

The smooth pulse and probability evolution of state \(|+\rangle\) is shown in Fig. 3. Comparing it with Fig. 2, we can see that it is not as efficient as that using the intermediate state \(|0\rangle\) because the magnetic field as a function of time is irregular.

Hadamard gate: \(|\pm\rangle \rightarrow \frac{1}{\sqrt{2}} |+\rangle \pm \frac{1}{\sqrt{2}} |-\rangle \). In this situation, we can’t use operator (22) directly. So in first step, we change it to state \(|0\rangle\). It is the same step mentioned in dealing with Pauli-X gate: \( U_f^X(0, T_1)|+\rangle \). After this step, for case \(|0\rangle\) turning to \( \frac{1}{\sqrt{2}}(|+\rangle + |\rangle) \), we only have to set \( \theta_1 = \frac{\pi}{2} \) and \( \varphi = 0 \) in operator (22). The evolution operator for the Hadamard gate is

\[
U_H = U_I(0, \frac{\pi}{4})U_f^X(0, T).
\]
The time $t$ (in $1/\lambda$) dependence of $J(t)/\lambda$ (upper panel) and $P_{\downarrow\downarrow}$ (lower panel), with $\kappa = \frac{15(\lambda T)^2}{A}$ in Eq. (18).

\begin{align}
|\Phi(0)⟩ = |\uparrow⟩ + e^{i\theta_1} |\downarrow⟩ \\
|\Phi(t_f)⟩ = |\uparrow⟩ + e^{i\theta_2} |\downarrow⟩
\end{align}

where $J_z = \gamma B_z$. If the initial state is $|+\rangle$ and $J_z t_f = \frac{\pi}{2}$, where $t_f$ is time duration, $Q(t_f)$ makes the state $|+\rangle$ become $Q(t_f)|+\rangle = \frac{1}{\sqrt{2}}(|+\rangle - i|−\rangle)$. In order to get target state $\frac{1}{\sqrt{2}}(|+\rangle + |−\rangle)$, we need a phase factor $e^{i\pi/2}$ for the $|−\rangle$. It can be shown that the time value and magnetic field is the same as $\tau_1$ and $h_1$. With the phase shift operator $P_f(\frac{\pi}{2})$, the whole process can be represented as

\begin{align}
U_H = P_f(\frac{\pi}{2})Q(t_f) = \begin{pmatrix}
\frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & ie^{i\theta_1} & 0 \\
\frac{i}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}
\end{pmatrix} e^{-iDt_f}.
\end{align}

The total duration time is $t_f + \tau_1 = \frac{\pi}{4A} + \frac{\pi}{2} \approx 1.375$ns with $B_z \approx 1000$G. Furthermore, this gate can change $|−\rangle$ into $\frac{1}{\sqrt{2}}(|+\rangle - i|−\rangle)$.

**Arbitrary gating.** In this case, we need to realize $\cos \theta_2 |+\rangle + e^{i\varphi_2} \sin \theta_2 |−\rangle \rightarrow \cos \theta_2 |+\rangle + e^{i\varphi_2} \sin \theta_2 |−\rangle$. Assuming $0 \leq \theta_1, \theta_2 \leq \pi$ and $0 \leq \varphi_1, \varphi_2 \leq \pi$. If $\theta_1 = \theta_2$, we need only to modify the relative phase. According to the previous subsection, we can realize this control through the intermediate state $|0\rangle$. The first part is actually the inverse process of that in Sec. IV. As the operator $U_2$ is unitary, the first process can be achieved by the operator $U_2^\dagger(\varphi_2, \theta_1)$, with $T_2$ being the pulse duration. Then, the target state can be realized by $U_f(\varphi_2, \theta_2)$ with $T_3$ being the pulse time duration. The whole process can be represented by

\begin{align}
U_A(\varphi_2, \theta_2, \varphi_1, \theta_1) = U_f(\varphi_2, \theta_2)U_2^\dagger(\varphi_1, \theta_1)
\end{align}

This gate can be realized without the intermediate state $|0\rangle$. It is clear from operator $U_2$ that $J_z t$ can be seen as one variable because the overall phase factor does not play role in the gate. As a result, the larger the longitudinal field, the smaller the time duration. The initial relative phase $\varphi_1$ should be adjusted before the probability amplitude is changed. At first, we change the relative phase $\varphi_1$ into $\frac{\pi}{2}$ by $P_f^\dagger(\varphi_1 - \frac{\pi}{2})$, making a state $|\Phi(0)⟩ = \cos \theta_1 |+\rangle + i \sin \theta_1 |−\rangle$. Then, we apply the time evolution operator $Q(t_e)$ on $|\Phi(0)⟩$, where $t_e$ is the time duration, and obtain the state $|\Phi(t_e)⟩ = (\cos(\theta_1 - J_z t_e)|+\rangle + i \sin(\theta_1 - J_z t_e)|−\rangle)e^{-iDt_e}$. Here, $J_z t_e$ can be expressed as

\begin{align}
J_z t_e = \begin{cases}
\theta_1 - \theta_2 & \text{if } \theta_1 \geq \theta_2 \\
\theta_1 - \theta_2 + 2\pi & \text{if } \theta_1 < \theta_2
\end{cases}
\end{align}

Finally, we apply the phase regulation $P_f(\varphi_2 - \frac{\pi}{2})$ to get the phase $\varphi_2$. The whole procedure can be represented
as the unitary operator:

$$U'_A(\varphi_2, \theta_2, \varphi_1, \theta_1) = P_f(\varphi_2 - \frac{\pi}{2})Q(t_c)P_f(\varphi_1 - \frac{\pi}{2}) \quad (34)$$

| Gate | $\varphi$ phase | $\vartheta$ phase | Pauli-X | Hadamard |
|------|------------------|------------------|---------|----------|
| $T_C$ | 1.1 ns | 1.6 ns | 2.7 ns | 3.8 ns |
|       | 5.3 ns | 1.4 ns |         |          |

### VI. DISCUSSION AND CONCLUSION

The time durations $T_C$ of the typical gates are summarized in Table I. It is clear that the gates need at most a few nanoseconds. Furthermore, it can be estimated to take only nanoseconds to complete the initialization and gating of arbitrary qubits. Therefore, the qubits on the basis of $|+\rangle$ and $|-\rangle$ can be fast initialized and gated with magnetic fields, in comparison to those in terms of $|+1\rangle$ and $|-1\rangle$ with strong mechanical driving\cite{1}. When the magnetic field is applied along the $z$ axis, the effect of the time evolution operator (30) on our NV center qubit with the basis of $|+\rangle$ and $|-\rangle$ looks like that on spin-$\frac{1}{2}$ qubit in “bang-bang” approach \cite{22}, but they are different from each other. Because here one needs to control the magnetic field strength only, the theoretical fidelity can be very high for these gates.

In summary, by choosing $|\pm\rangle$, defined as $(|+1\rangle \pm |-1\rangle)/\sqrt{2}$, of the NV center spin as the qubit basis, we have obtained exact evolution operator of the NV center spin under time-dependent magnetic field by mapping the three-level system of the NV center spin on a two-level spin system under a time-dependent magnetic field and using the existing exact analytical results of the quantum two-level system. With the exact evolution operator of the NV center spin including three levels, we have shown how to prepare arbitrary qubits with the basis $|\pm\rangle$ from the starting $|0\rangle$ state and to realize arbitrary quantum logic gates for the qubits. It also has been estimated that the typical quantum logic gates can be accomplished within a few nanoseconds. We believe that the fidelity can be very high because only magnetic field strength needs to be controlled. These results are useful to realizing quantum computing with the NV center spin systems in diamond and exploring other quantum effects and applications.

### ACKNOWLEDGMENTS

This work is supported by Nature Science Foundation of China (Grant Nos. 11174359 and 11574366), by Chinese Department of Science and Technology (Grant No. 2012CB932302), and by the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant No. XDB07000000).

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