Almost three years have passed since the publication of the first Special Issue on three-dimensional underwater acoustics in 2019 [1], and some new ideas have since emerged in this field, while others have been developed to the extent that a new paper collection focused on the modelling of sound propagation has become necessary.

When planning this Special Issue, we intentionally extended the scope (as compared to [1]) and attempted to cover two-dimensional propagation alongside with three-dimensional models and effects. The paper collection is relatively small but well-focused. It consists of thirteen research works of three different kinds.

Papers of the first kind [2–7] are related to new or insufficiently investigated physical effects related to sound propagation in complex media with various inhomogeneities, including undulating bottom [3], variations in acoustical properties of the bottom across the propagation path [2], the presence of seamounts [4], and bubbles [6] or internal waves [7] in the water column. It is remarkable that both new 2D [3,7] and 3D [2,4] effects are reported in these papers. In most of them, new interesting features of sound fields are investigated both theoretically and experimentally, and the modeling results are compared with the measurement data. The spectrum of theoretical approaches used by the authors covers almost all techniques existing in theoretical underwater acoustics, including parabolic equations theory [3], normal modes [2], and ray-theoretical considerations.

Papers of the second type are related to applications of underwater acoustics where the models of sound propagation play a significant role [8–11]. These include geoacoustic inversion [9], source-bearing estimation [8], a technique that allows one to estimate both the source position and the media properties [11], and the problem of estimating the acoustic noise levels over some water areas neighbouring the source [10]. For example, the novel geoacoustic inversion technique from [9] requires the precise calculation of ray paths (which is performed using the BELLHOP code) that are necessary to estimate travel times of head waves. Source image methods are used in [8] to analyze the effect of horizontal refraction onto the source bearing estimation. This study once again highlights the need in accurate and efficient 3D models for the solution of everyday practical problems of ocean acoustics. Similar conclusion can be drawn from the paper [10], where the authors show that 3D effects are important for the estimation of noise levels in shallow-water environments.

There are also three papers in which some advances in mathematical approaches to the modelling of sound propagation are reported [12–14]. In particular, the study of [13] describes a versatile and robust finite-element-based method for solving sound propagation problems. In the paper [14], which is dedicated to the anniversary of the pioneering work on the invariant imbedding in wave propagation problems [15], the results of the latter study are generalized to the case of a vector-valued unknown function. This generalization allows one to handle mode coupling equations without neglecting the coupling effects (and also without using staircase approximation). Finally, in [12], the mode perturbation theory developed by the authors is used to improve the performance of 3D propagation.
codes that require multiple solutions of the acoustic spectral problem. In particular, the perturbative formulae from [12] are important for computationally efficient implementation of the numerical techniques based on the mode parabolic equations [10].

Currently, the mathematical aspect of underwater acoustics is steadily gaining importance for their applications. The increased performance of modern computers and the development in sound propagation modelling approaches is demonstrated in the growth in the publication output in this research field throughout the past 20 years. This trend will likely persist in the near future, and it is our hope that our Special Issue may become some tiny milestone of this long road.

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**References**

1. Lin, Y.T.; Porter, M.B.; Sturm, F.; Isakson, M.J.; Chiu, C.S. Introduction to the special issue on three-dimensional underwater acoustics. *J. Acoust. Soc. Am.* 2019, 146, 1855–1857. [CrossRef] [PubMed]
2. Lunkov, A.; Sidorov, D.; Petnikov, V. Horizontal refraction of acoustic waves in shallow-water waveguides due to an inhomogeneous bottom structure. *J. Mar. Sci. Eng.* 2021, 9, 1269. [CrossRef]
3. Liu, D.; Li, Z.; Wang, G.; Liu, Y. Sound propagation with undulating bottom in shallow water. *J. Mar. Sci. Eng.* 2021, 9, 1010. [CrossRef]
4. Li, S.; Li, Z.; Li, W.; Yu, Y. Three-dimensional sound propagation in the south china sea with the presence of seamount. *J. Mar. Sci. Eng.* 2021, 9, 1078. [CrossRef]
5. Wu, S.; Li, Z.; Qin, J.; Wang, M.; Li, W. The effects of sound speed profile to the convergence zone in deep water. *J. Mar. Sci. Eng.* 2022, 10, 424. [CrossRef]
6. Liu, R.; Li, Z. The effects of bubble scattering on sound propagation in shallow water. *J. Mar. Sci. Eng.* 2021, 9, 1441. [CrossRef]
7. Jiang, Y.; Grigorev, V.; Katsnelson, B. Sound field fluctuations in shallow water in the presence of moving nonlinear internal waves. *J. Mar. Sci. Eng.* 2022, 10, 119. [CrossRef]
8. Zhou, J.; Tang, J.; Yang, Y. A study on the estimation of source bearing in an asa wedge: Diminishing the estimation error caused by horizontal refraction. *J. Mar. Sci. Eng.* 2021, 9, 1449. [CrossRef]
9. Uzhansky, E.; Gadol, O.; Lang, G.; Katsnelson, B.; Copel, S.; Kazaz, T.; Makovsky, Y. Geoaoustic estimation of the seafloor sound speed profile in deep passive margin setting using standard multichannel seismic data. *J. Mar. Sci. Eng.* 2021, 9, 1423. [CrossRef]
10. Manul’chev, D.; Tsyshchenko, A.; Fershalov, M.; Petrov, P. Estimating sound exposure levels due to a broadband source over large areas of shallow sea. *J. Mar. Sci. Eng.* 2022, 10, 82. [CrossRef]
11. Dai, M.; Li, Y.; Ye, J.; Yang, K. Joint tracking of source and environment using improved particle filtering in shallow water. *J. Mar. Sci. Eng.* 2021, 9, 1203. [CrossRef]
12. Zakharenko, A.; Trofimov, M.; Petrov, P. Improving the performance of mode-based sound propagation models by using perturbation formulae for eigenvalues and eigenfunctions. *J. Mar. Sci. Eng.* 2021, 9, 934. [CrossRef]
13. Zhou, Y.Q.; Luo, W.Y. A finite element model for underwater sound propagation in 2-D environment. *J. Mar. Sci. Eng.* 2021, 9, 956. [CrossRef]
14. Kazak, M.; Koshel, K.; Petrov, P. Generalized form of the invariant imbedding method and its application to the study of back-scattering in shallow-water acoustics. *J. Mar. Sci. Eng.* 2021, 9, 1033. [CrossRef]
15. Babkin, G.; Klyatskin, V. Invariant imbedding method for wave problems. *Wave Motion* 1982, 4, 195–207. [CrossRef]