Topography optimization of truss support systems for a point-supported glass curtain wall based on truss-like continua

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Abstract. Steel truss structures for point-supported glass curtain walls with a web member configuration are presented using topology optimization based on truss-like continua. The initial structure comprises chords and partial web members. The initial design domain, surrounded by the members of the initial structure, is filled with a truss-like continuum, which is used to simulate the distributive web members of a steel truss. The structure members, together with the truss-like continuum, are analysed and calculated under horizontal point forces. The densities and orientations of the truss-like members at every node are taken as design variables and optimized. The members in the truss-like continuum are optimized using the fully stressed criterion. The topology optimization problem of minimum compliance with volume constraints is solved. Thus, an optimal web member configuration that suitably reinforces the lateral stiffness of the structure is obtained.

1. Introduction
As a type of building envelope system, point-supported glass curtain walls, which provide the advantages of transparency, simplicity and lightness, have been widely adopted in all types of large public buildings [1]. According to the material, the support system for point-supported glass curtain walls can be classified as glass rib support systems, cable net and cable truss support systems and steel structure support systems, etc. The most common structural style of support systems is a steel truss. With the extensive use of high-strength steel, the lateral stiffness rather than the strength of a truss tends to be the dominating factor, and maximizing the lateral stiffness of a steel truss is regarded as the most important design objective. To obtain superior structural performance, geometry and cross-sectional size optimization methods can be adopted to optimize the structural geometries and the sectional sizes of members. However, the structural performance will not be improved greatly by these optimization techniques unless the structural topology is determined.

The extensive research that exists on optimizing truss structures focuses primarily on an integrated layout and a topology-optimized design [2, 3, 4, 5]. However, in practical engineering, the intended function always determines the form of a building. Accordingly, the actual outer contour (initial structure) of a truss for a glass curtain wall tends to be prescribed by architects in advance. Therefore, the challenge arises of how to obtain a web member configuration that imparts the whole truss structure with satisfactory performance (e.g., a greater lateral stiffness). Similarly, many researches have been conducted on the effect of bracing systems on a frame. Bracing systems usually tend to be used to reduce the lateral deformation of both steel frames [6, 7] and RC frames [8, 9]. The artificial bee colony optimization technique was applied to determine the optimal distribution of diagonal steel braces by Aydin et al [10]. A new bracing system of RC frames was proposed by Qu et al [11]. Just as bracing systems have a significant effect on the lateral stiffness of a frame [12, 13, 14], web members...
play a similar role for a truss structure. However, little research has been conducted on the effect of web members on a truss. In practical engineering, the location, quantity, and arrangement of web members are often determined intuitively. Fortunately, the theory and method of topology optimization offer a powerful tool to address this challenge. Zhou and Li [15] presented a new method for continuum topology optimization based on a truss-like material continuum. In this paper, a web member configuration is explored using topology optimization based on a truss-like material continuum with volume constraints. In addition, the common structural styles of the steel truss and the optimal configuration obtained by the method above are analyzed and compared with SAP2000 software. The results demonstrate that with an identical amount of web member material, the optimal steel truss possesses a higher lateral stiffness and a lower and balanced stress ratio.

2. Finite element analysis
In a planar truss-like material model with two families of orthotropic members, the relation between stress and strain can be derived as

$$\begin{bmatrix} \sigma_x & \sigma_y & \tau_{xy} \end{bmatrix}^T = D(t_1, t_2, \alpha) \begin{bmatrix} \varepsilon_x & \varepsilon_y & \gamma_{xy} \end{bmatrix}^T$$

where $A_r$ denotes constant matrices and $s_{br}$ and $g_{br}$ are the element values of the matrices $s$ and $g$, respectively [15].

The elastic matrix at any point $(\xi, \eta)$ within an element $e$ can be obtained by the interpolation of the elastic matrices at the nodes belonging to the element.

$$D_{ij}(\xi, \eta) = \sum_{j \in S_e} N_j(\xi, \eta) D_j = E \sum_{j \in S_e} N_j(\xi, \eta) \sum_{b=1}^{3} t_{bj} \sum_{r=1}^{3} s_{br} g_{br}(\alpha_j) A_r$$

where $D_j$ is the elastic matrix at node $j$, $S_e$ the set of nodes belonging to element $e$, $N_j$ the shape function.

According to the basic theory of the finite element method, we obtain the element stiffness matrix

$$k_e = \int_{V_e} B^T D B dV = \sum_{j \in S_e} t_{bj} \sum_{r} g_{br}(\alpha_j) H_{ejr}$$

where $B$ represents a geometric matrix and $H_{ejr}$ is a constant matrix for rectangular elements [15].

The stiffness matrix for planar beam elements is

$$k_e = \begin{bmatrix}
\frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\
0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\
0 & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} \\
-\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\
0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} \\
0 & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{4EI}{l}
\end{bmatrix}$$

The nodes belonging to both beam elements and plane rectangular elements share an identical displacement along two global coordinate axes. Each node that only belongs to plane rectangular elements has two degrees of freedom (two translational degrees of freedom), while those that are shared by beam elements and plane rectangular elements have three degrees of freedom (a rotational
degree of freedom and two translational degrees of freedom). Then the structural stiffness matrix \( K \) can be assembled from the element stiffness matrices as follows

\[
K = \sum_k k_e
\]  

(6)

Thus, a calculation model for the structure using the finite element method is established.

3. The optimization method and procedure

The rectangular area surrounded by the members of the initial structure is taken as the initial design domain. Further, the initial design domain (which is divided into four-node plane rectangular elements) is filled with a truss-like continuum, while the members of the initial structure are divided into two-node beam elements. The members in the truss-like continuum are involved in optimization rather than that of the initial structure. The densities and orientations of the members in the truss-like continuum at every node are regarded as design variables.

The compliance \( c \) of truss structure is regarded as an object function of the optimization problem. The optimization problem can be described as

\[
\begin{align*}
\text{find} & \quad t_{b_j}, \alpha_j, \\
\text{min} & \quad c = F^T U_j \quad j = 1, 2, ..., J \\
\text{s.t.} & \quad V \leq \bar{V}
\end{align*}
\]  

(7)

where the volume of the continuum \( V \) can be derived as follows

\[
V = \sum_e \sum_{b=1}^2 \sum_{j=1}^N N_e f_{b_j} dV
\]  

(8)

According to the optimization method and procedure presented by Zhou [16], the fully stressed criterion is adopted to optimize the structure to maximize the lateral stiffness of a steel truss.

4. Numerical examples

The initial structure is demonstrated in figure 1(a). A hinged support and a roller support are used to attach the structure to its foundation. Members of the initial structure are classified as three groups (marked in figure 1(a)) with circular steel tube sections, as listed in table 1. In addition, the Young’s modulus and allowable stress of the steel are \( E = 206 \) GPa and \( \sigma_p = 235 \) MPa, respectively. Horizontal point force applied to the structure is 17.82kN (for a point-supported curtain wall, only horizontal point forces are applied to the structure).

Table 1. Member section of the initial structure.

| Group number | Sectional model(mm) | Sectional area(cm²) | Sectional inertia moment(cm⁴) |
|--------------|---------------------|---------------------|-------------------------------|
| 1            | Pipe95x6            | 16.78               | 166.86                        |
| 2            | Pipe89x6            | 15.65               | 135.43                        |
| 3            | Pipe76x6            | 13.20               | 81.41                         |

The initial structure is composed of members connected by rigid joints. The truss-like continuum filled in the initial design domain is meshed by 360 plane rectangular elements, and the initial structure is divided into 110 beam elements, as demonstrated in figure 1(b). Since the glass curtain wall is subjected to wind loading, which creates both a uniform normal pressure and a suction pressure, a symmetrical structure (also for the sake of aesthetics) is generally adopted in curtain wall engineering. To obtain a symmetric structure, the loads are applied symmetrically. The volume of the initial structure is \( V_0 = 2.25 \times 10^2 \) m³. The volume (upper limit value) of truss-like material filled in the initial design domain is taken as 0.2\( V_0 \). The optimal distribution is acquired after 13 iterations, as illustrated in figure 2(a). The lengths and directions of the straight lines represent the densities and orientations, respectively, of the members in the truss-like continuum. The optimal configuration of web members is established in figure 2(b). Furthermore, the configuration is modified appropriately for the convenience of construction in practical engineering, as demonstrated in figure 2(c). The two
most commonly used configurations in curtain wall engineering are shown in figure 3 with spans identical to that of the initial structure. With the same amount of web member material, the compliances of different truss support systems are calculated and compared in table 2.

Table 2. Calculation results.

| Structure number | Structure styles               | Figure     | Compliance (kN·m) | Volume of web members (m³) |
|------------------|--------------------------------|------------|-------------------|----------------------------|
| 1                | Initial structure              | figure 1(a)| 1.839             | --                         |
| 2                | Truss-like continuous          | figure 2(a)| 6.779×10⁻²         | 4.50×10⁻³                   |
| 3                | Truss-like discrete            | figure 2(c)| 1.147×10⁻¹         | 4.50×10⁻³                   |
| 4                | Ordinary truss                 | figure 3(a)| 2.964×10⁻¹         | 4.50×10⁻³                   |
| 5                | Vierendeel truss               | figure 3(b)| 5.122×10⁻¹         | 4.50×10⁻³                   |

Table 2 shows that the compliance of the truss with the truss-like continuous configuration, as illustrated in figure 3(a), drops dramatically to 3.69% of the initial structure, while the volume of the material increases by no more than 20%. The compliance of the structure with a truss-like discrete configuration (which is derived from the truss-like continuous configuration) shown in figure 2(c) is 38.70% of that of the ordinary truss shown in figure 3(a) and 22.40% of that of the vierendeel truss shown in figure 3(b). It can be seen that the optimal steel truss possesses a lateral stiffness larger than that of the others.

Members of the initial structure are calculated as compression-bending (tension-bending) members whose allowable slenderness ratio is taken as 150 (350). Unbraced length ratios (minor) of the left chord and the right chord are prescribed as 0.25 and 0.33, respectively. The bending moments at both ends of web members, exclusive of those of the initial structure, are released. Under these conditions, the common structural styles of the steel truss and the optimized one obtained by the method above are analysed and compared with SAP2000 software under the same loads. With the same amount of web member material, as shown in table 3, figure 4 indicates that two diagonal web members in the middle of the ordinary truss span have a relatively low stress ratio (the value is 0.005). Relative to common structural styles, the optimal steel truss possesses a lower and well-balanced stress ratio overall. It can be concluded that the optimized support system is more reasonable.
Table 3. Web member section.

| Structure number | Structure styles          | Figure  | Total length of web members(m) | Sectional model(mm) |
|------------------|---------------------------|---------|--------------------------------|---------------------|
| 1                | Truss-like discrete configuration | figure 2(c) | 8.994 | Pipe40×4.50 |
| 2                | Ordinary truss            | figure 3(a) | 8.179 | Pipe42×4.70 |
| 3                | Vierendeel truss          | figure 3(b) | 3.200 | Pipe76×6.45 |

5. Conclusions
By means of optimizing the web member configuration of a truss using topology optimization based on a truss-like continuum, the topology optimization problem of minimum compliance with volume constraints was solved. The results demonstrate that with the same amount of web member material,
the optimal steel truss possesses a relatively higher lateral stiffness and a lower and balanced stress ratio. The optimized truss structures along with the common structural styles were applied to demonstrate the feasibility and validity of the optimization method. It can be concluded that the optimized support system of point-supported glass curtain walls is more reasonable.

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