A remark about the Mermin-Squires Music Hall’s inteludium

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Abstract

The Mermin-Squires Music Hall inteludium on the Einstein-Podolsky-Rosen affair is analyzed by showing the fallacity of the One-Borel-Normality Criterion and the necessity of replacing it with the more restrictive Algorithmic-Randomness Criterion.

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Let us consider the Squires’ Music Hall’s inteludium \(1\) based on Mermin’s analysis of the Einstein-Podolski-Rosen experiment \(2\):

Alice and Bob are able of executing a show at first sight astonishing:
they are closed in two separated boxes of a theatre’s stage so that they can’t communicate each other.

At regular intervals two public’s people, let’s say one from the right side and the other from the left side of the parterre, give respectively to Alice and Bob a ticket on which it is written one of the numbers 1, 2 or 3.

The choice of the numbers appearing on the tickets the two spectators take to the artists is absolutely random.

At this point Alice and Bob are asked to write YES or NO on the received ticket.
Obviously Alice can’t know which number is written on Bob’s ticket and viceversa.

The show’s exceptionality is that all the times Alice and Bob receive a couple of tickets on which it is written the same number, they answer in the same way.

Let us now suppose to be in the public and to look for a criterion in order to establish if Alice and Bob are really telepathic or they are simply swindlers.

Indeed anyone would think that there is a very simplest explanation of Alice and Bob’s performance: they agreed before the show on how to answer.

Is there, then, a way of certifying if this is the case?

Let us adopt the following notation:

- \(t_A(n), t_B(n)\) denote the tickets given, respectively, to Alice and Bob at the \(n^{th}\) repetition of the performance
- \(a_A(n), a_B(n)\) denote the answers written, respectively, by Alice and Bob on the \(n^{th}\) ticket

By hypothesis \(\{t_A(n)\}\) and \(\{t_B(n)\}\) are two independent sequence of i.i.d. uniform random variables over \(\{1, 2, 3\}\):

\[
Prob(t_A(n) = i) = \frac{1}{3} \quad \forall i \in \{1, 2, 3\}, \forall n \in \mathbb{N}
\] \hspace{1cm} (1)

\[
Prob(t_B(n) = i) = \frac{1}{3} \quad \forall i \in \{1, 2, 3\}, \forall n \in \mathbb{N}
\] \hspace{1cm} (2)
Let us then introduce the \textbf{concordance sequence} $\bar{C} \in \{0, 1\}^\infty$ whose $n^{th}$ digit $C_n$ is defined as:

$$C_n := \begin{cases} 1 & \text{if } a_A(n) = a_B(n), \\ 0 & \text{otherwise}. \end{cases}$$

(S3)

Squires introduced the following:

\textbf{Theorem 1.1}

\textbf{CRITERION OF 1-BOREL NORMALITY}

\textbf{HP}:

$$\exists \phi \in \text{RECURSIVE - MAPS}([1, 2, 3], \{\text{YES, NO}\}) :$$

$$a_A(n) = \phi(t_A(n)) \text{ and } a_B(n) = \phi(t_B(n))$$

(4)

\textbf{TH}:

$$p_{1-\text{Borel}}(\bar{C}) \text{ doesn’t hold}$$

(5)

\textbf{PROOF}:

We have clearly that:

$$N_0(\phi(1)\phi(2)\phi(3)) \neq N_1(\phi(1)\phi(2)\phi(3))$$

(6)

By the eq\ref{eq:6} and the Law of Large Numbers it immediately follows that $\bar{C}$ can’t be 1-normal.

Anyway Squires doesn’t consider the case in which Alice and Bob adopt a more clever way of cheating, i.e. they use a previously concorded answering-algorithm depending also from $n$; in this way they may easily to scoff at Squires’ criterion of Borel-1-normality.

Let us suppose, for example, that each of them answers according to the following rule:

$$a(n)(t) := \begin{cases} \text{YES} & \text{if } t = 1, \\ \text{NO} & \text{if } t = 2, \\ \text{YES} & \text{if } t = 3 \text{ and } n \text{ is even,} \\ \text{NO} & \text{if } t = 3 \text{ and } n \text{ is odd,} \end{cases}$$

(7)
Since in this case the according-rule doesn’t break the balance among YES and NO, by the eq.(11) and the Law of Large Numbers it follows immediately that $\bar{C}$ is Borel-1-normal.

So if Alice and Bob answer according to this rule, they scoff at Squires according to which they appear as really telepathic.

There exist, anyway, a way to unmask the deception; this, anyway, requires the adoption of a stronger criterion:

**Theorem 2**

**CRITERION OF ALGORITHMIC RANDOMNESS**

**HP:**

$$\exists \phi \in RECURSIVE - MAPS(\{1, 2, 3\} \times \mathbb{N}, \{YES, NO\}) :$$

$$a_A(n) = \phi(t_A(n), n) \text{ and } a_B(n) = \phi(t_B(n), n) \quad (8)$$

**TH:**

$$\bar{C} \notin RANDOM(\{0, 1\}^{\infty}) \quad (9)$$

**PROOF:**

The Chaitin-Schnorr Theorem states the equivalence of Martin-Löf statistical characterization of algorithmic-randomness and Chaitin’s one as algorithmic incompressibility:

$$\bar{x} \in RANDOM(\{0, 1\}^{\infty}) \iff \exists c > 0 : I(\bar{x}(n)) \geq n - c \forall n \geq 1 \quad (10)$$

where:

$$I(\bar{x}) := \min\{|\bar{u}| : U(\bar{u}, \lambda) = \bar{x}\} \quad (11)$$

and where $U$ is a fixed Chaitin universal computer.

But by hypothesis Alice and Bob’s answer is algorithmically-compressible through the rule $\phi \in RECURSIVE - MAPS(\{1, 2, 3\} \times \mathbb{N}, \{YES, NO\})$ they concorded before the show, implying the thesis ■
[1] E. Squires. *The mister of the quantum world*. Institute of Physics Publishing, Bristol and Philadelphia, 1994.

[2] N.D. Mermin. *Boojums All the Way Through: Communicating Science in a Prosaic Age*. Cambridge University Press, Cambridge, 1990.

[3] C. Calude. *Information and Randomness. An Algorithmic Perspective*. Springer Verlag, Berlin, 2002.