Superconducting phase transition of Sr$_2$RuO$_4$ in magnetic field

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The superconducting state formed due to direct intra-orbital pairing in tetragonal multi-band superconductor Sr$_2$RuO$_4$ has different properties than the state formed due to intra-band pairing. In particular, the theory operating with direct intra-orbital pairing successfully explains the Kerr rotation of reflected light polarization observed several years ago. Here we apply intra-orbital approach to the problem of Ginzburg-Landau description of basal plane upper critical field in this material. It is shown that typical for two component superconducting state additional phase transition in the vortex state at $H < H_{c2}$ for all four crystallographic directions of magnetic field in the basal plane and the basal plane upper critical field anisotropy still are inevitable properties even in case of direct intra-orbital pairing.

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I. INTRODUCTION

During about two decades the tetragonal metal Sr$_2$RuO$_4$ attracts a lot of attention (for the reviews see\textsuperscript{1–2}). In particular, the measurements of the finite Kerr rotation in the superconducting phase of this material causes a great interest as a decisive proof for the time reversal symmetry breaking, that is ferromagnetism, spontaneously arising in superconducting state. A superconducting state possessing spontaneous magnetization is described by multicomponent order parameters. In a tetragonal crystal the superconducting states with two-component order parameters $(\eta_x, \eta_y)$ corresponding either to singlet or to triplet pairing are admissible. In application to Sr$_2$RuO$_4$ the triplet pairing state with time reversal symmetry breaking form of the order parameter $(\eta_x, \eta_y) \propto (1, i)$ has been proposed first in the paper\textsuperscript{3}.

The specific properties for the superconducting state with two-component order parameter in a tetragonal crystal under magnetic field in basal plane are (i) the existence of an additional phase transition in the vortex state at $H < H_{c2}$ for all four crystallographic directions of magnetic fields, and (ii) the anisotropy of the upper critical field. Both of these properties should manifest themselves starting from the Ginzburg-Landau temperature region $T \approx T_c$ but till now there is no experimental evidence for that. The in-plane anisotropy of the upper critical field has been observed only at low temperatures,\textsuperscript{4} where it is quite well known phenomenon for any type of superconductivity originating from the Fermi surface anisotropy.

Theoretically in application to Sr$_2$RuO$_4$ the phase transition splitting and the upper critical field anisotropy have been investigated by Agterberg and co-workers.\textsuperscript{5,6} They have found that one particular choice of the basis functions of two-dimensional irreducible representation for a tetragonal point group symmetry is appropriate for decreasing of basal plane upper critical field anisotropy but at the same time the considerable phase transition splitting occurs. Vice versa, another particular choice of the basis functions almost eliminates the phase transition splitting for the particular field directions but keeps the basal plane upper critical field anisotropy. Thus the basal plane upper critical field properties look as incompatible with multicomponent order parameter structure dictated by the experimental observations manifesting the spontaneous time-reversal breaking.

The theoretical treatment\textsuperscript{11} of $H_{c2}$ problem have been undertaken for the two component superconducting state in a single band superconductor. In Sr$_2$RuO$_4$ we deal with three bands of charge carriers. The investigation performed by the present author\textsuperscript{14} has demonstrated that problems with phase transition splitting and basal plane anisotropy still exist even in multi-band case.

A new opportunity to resolve this inconsistency between the theory and experiments was appeared in relation with new approach to multiband superconductivity in strontium ruthenate developed in attempts to explain the Kerr effect observations.\textsuperscript{8} The Kerr rotation of polarization of light reflected from the surface of a ferromagnet is expressed through the anomalous Hall conductivity. However, there was clearly demonstrated that the Hall conductivity in a single band translationally invariant chiral superconductor is equal to zero\textsuperscript{13,14}. In view of this general statement several theoretical explanations for the Kerr effect in clean Sr$_2$RuO$_4$ were proved to be incorrect. Then there were proposed explanations based on skew impurity scattering\textsuperscript{15,16}, such that the Kerr effect has obtained sort of extrinsic explanation. Valuable theoretically these approaches are seemed to be inappropriate to quite clean strontium ruthenate superconductor.

Recently two groups\textsuperscript{17,18} showed that in some particular multiorbital superconducting models an intrinsic anomalous Hall conductivity does not vanish. Soon after that these results were criticized by the present author\textsuperscript{19}, who argued, on the basis of symmetry considerations and traditional approach to the description of multi-band superconductivity,\textsuperscript{20} that the intrinsic Kerr effect has to vanish even in a multi-band case. The criticism was mostly addressed to the paper\textsuperscript{18} containing the proper analytic calculation of the Hall conductivity but at the same time based on the model which does not
possess general symmetry properties of a superconducting state in crystal with tetragonal symmetry. However, the following research\textsuperscript{21} has demonstrated that working with direct intra-orbital pairing one can prove the existence of anomalous Hall conductivity in multi-band chiral superconductor.

So, in frame of direct intra-orbital pairing approach one can describe some physical properties which don’t give in to an explanation in terms of traditional description of multi-band superconductivity. Here we develop the Ginzburg-Landau theory of multi-band unconventional superconductivity based entirely on the intra-orbital pairing. It is shown that typical for two component superconducting state phase transition splitting under basal plane magnetic field in all four crystallographic directions is obligatory even in case of direct intra-orbital pairing. The basal plane upper critical field anisotropy takes place as well.

II. INTRA-BAND VERSUS INTRA-ORBITAL PAIRING APPROACHES TO MULTI-BAND SUPERCONDUCTIVITY

The conducting bands in Sr$_2$RuO$_4$ are formed by three Ru-4$d$ orbitals $d_{xz}, d_{yz}, d_{xy}$, denoted as $a,b,c$ respectively. The dispersion $\varepsilon_c(k)$ of $c$ band has tetragonal symmetry, but the dispersion of $a$ and $b$ bands $\varepsilon_a(k)$ and $\varepsilon_b(k)$ is orthorhombic. The full tetragonal symmetry of real normal state is recreated after including the interband $\varepsilon_{ab}(k), \varepsilon_{ac}(k), \varepsilon_{bc}(k)$ coupling matrix elements and diagonalization of total Hamiltonian. As result, from initial $\varepsilon_a(k), \varepsilon_b(k), \varepsilon_c(k)$ bands dispersion we come to dispersion laws $E_{\alpha}(k), E_{\beta}(k), E_{\gamma}(k)$ of Sr$_2$RuO$_4$ $\alpha, \beta, \gamma$ bands\textsuperscript{21} possessing full tetragonal symmetry.

The regular procedure accepted in multi-band superconductivity theory is the following. First, one must perform the normal state band Hamiltonian diagonalisation and then introduce pairing between the electrons filling the states in the bands with full tetragonal symmetry. It is correct not only from the symmetry point of view but also because usually the normal state band splitting is much larger than the thickness of the layer near the Fermi surface where a pairing interaction is effective. Applying then the Bogohlovbov transformation to the normal state band Hamiltonians for bands $\alpha, \beta, \gamma$, intraband pairing and interband pairing is the only description of multi-band superconducting state\textsuperscript{21}. Following this procedure and then calculating current response one can prove that Hall conductivity in a multi-band superconducting state vanishes completely.

Another approach is to introduce direct pairing between the electrons filling the initial orbital bands, and then, taking into account inter-band normal state hopping amplitudes like $\varepsilon_{bc}$, to make diagonalisation of total Hamiltonian including all the superconducting and normal parts. So, the diagonalisation of normal orbital parts and the pairing terms is produced simultaneously. This approach has a sense when at any band splitting still there is an attraction between the electrons with opposite momenta filling initial orbital states. The procedure is resulted in formation of multi-orbital superconducting state possessing nonzero Hall conductivity\textsuperscript{21}.

In what follows we compare the upper critical field problem description in $\alpha, \beta, \gamma$ band representation and in $a, b, c$ band representation. We shall discuss triplet unitary superconducting state having only one spin component $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ corresponding to the equal spin pairing with spins perpendicular to the spin quantization axis chosen parallel to the tetragonal axis $\hat{z}$.

For mathematical simplicity as in Ref.11 we limit ourselves by the description of two bands $\alpha, \beta$ and $a,b$ situation. One can demonstrate that addition of the third band making the treatment much more cumbersome does not change results qualitatively. Also we shall ignore inter-orbital spin-orbital coupling (it can be included following the paper\textsuperscript{22}) that introduces nothing important in the description of triplet superconducting state put forward here.

Following Ref.18,21 for the description of the normal state electronic band structure we consider:

\begin{equation}
\varepsilon_{\alpha k} + \mu = -2t \cos k_x - 2t^\perp \cos k_y, \tag{1}
\end{equation}
\begin{equation}
\varepsilon_{\beta k} + \mu = -2t \cos k_y - 2t^\perp \cos k_x, \tag{2}
\end{equation}
\begin{equation}
\varepsilon_{\alpha \beta k} = -2t'' \sin k_x \sin k_y. \tag{3}
\end{equation}

Corresponding $\alpha, \beta$ bands dispersion laws are

\begin{equation}
E_{\alpha,\beta k} = \frac{1}{2}(\varepsilon_{\alpha k} + \varepsilon_{\beta k}) \pm \frac{1}{2} \sqrt{(\varepsilon_{\alpha k} - \varepsilon_{\beta k})^2 + 4\varepsilon_{\alpha \beta k}^2}. \tag{3}
\end{equation}

A. H$_{c2}$ problem in superconducting state with $\alpha, \beta$ intra-band pairing

The general form of two component order parameter corresponding to each band is

\begin{equation}
\Delta_\lambda(k, q) = \eta_{\lambda x}(q) \varphi_{\lambda x}(k) + \eta_{\lambda y}(q) \varphi_{\lambda y}(k), \tag{4}
\end{equation}

where $\lambda$ runs the band labels $\alpha, \beta$, ($\varphi_{\lambda x}, \varphi_{\lambda y}$) are basis functions of two dimensional representation of tetragonal group transforming as $(k_x, k_y)$. In general they are different for the different bands.

The upper critical field is determined as the eigen value of linear equation for the order parameter

\begin{equation}
\Delta_\lambda(k, k') = T \sum_n \int \frac{d^3k'}{(2\pi)^3} \sum_\mu V_{\mu \lambda}(k, k') \times G_\mu(k', \omega_n) G_\mu(-k' + q_+, -\omega_n) \Delta_\mu(k', q). \tag{5}
\end{equation}

Here

\begin{equation}
V_{\lambda \mu}(k, k') = V_{\lambda \mu} \sum_{i=x,y} \varphi_{\lambda i}(k) \varphi_{\mu i}(k'), \tag{6}
\end{equation}
is the pairing interaction matrix, and
\[ G_\mu(k, \omega_n) = \frac{1}{i\omega_n - E^\mu_{ik}} \]  
(7)

is the normal state band \( \mu \) Green function.

Performing the Taylor expansion of equation \( 5 \) in powers of \( \mathbf{q} \) up to the second order and transforming to the coordinate representation, that means simple substitution

\[ \mathbf{q} \rightarrow \mathbf{D} = -i\nabla + 2eA(r), \]

we obtain Ginzburg-Landau equations

\[ \eta_\lambda(x) = \sum_{\mu j} V_{\lambda\mu}(L^\mu_{ij} + M^\mu_{ijlm}D_lD_m)\eta_{\mu j}(x), \]

(9)

where

\[ L^\mu_{ij} = T \sum_n \int \frac{d^3k}{(2\pi)^3} \phi_{\mu i}(k)\phi_{\mu j}(k)G_\mu(k, \omega_n)G_\mu(-k, -\omega_n), \]

(10)

\[ M^\mu_{ijlm} = \frac{T}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} \phi_{\mu i}(k)\phi_{\mu l}(k) \times G_\mu(k, \omega_n) \frac{\partial^2 G_\mu(-k, -\omega_n)}{\partial k_i \partial k_m}. \]

(11)

(12)

In absence of magnetic field \( \mathbf{D} = 0 \) and taking into account that

\[ L^\mu_{ij} = L^\mu_{xx}\delta_{ij} = L^\mu_{yy}\delta_{ij} \]

(13)

we come to two separate systems of homogeneous equations

\[ \eta_\lambda(x) = \sum_\mu V_{\lambda\mu}L^\mu_{xx}\eta_{\mu x}(x). \]

for \( x \) and \( y \) components of the order parameter for both bands. Determinant of each of them determines the critical temperature of phase transition. These systems are completely equivalent each other. Hence, the phase transition to superconducting state occurs at the same critical temperature for all the component of the order parameter in all the bands. Simple BCS-like formula for \( T_c \) was pointed out in Ref.12 where it was found in assumption that logarithmically divergent terms \( L^\mu_{xx} \) have the same energy cutoff in different bands. In general it is not true and an expression for critical temperature is more cumbersome.

Relative value of \( x \) and \( y \) components of the order parameter is fixed by the nonlinear terms in GL equations. For single band superconductors this problem was solved by Volovik and Gor’kov.\(^{12} \) There was shown that the complex superconducting state arising directly from the normal state by means of phase transition of second order always has form \( \eta = \eta(1, \nu) \). The equality of modulus of \( x \) and \( y \) components of the order parameter guarantees the minimum of GL free energy.

Unlike single band superconductivity in two band case the complex superconducting state with order parameter \( \eta_\alpha = (\eta_{\alpha x}, \eta_{\alpha y}) \), \( \eta_\beta = (\eta_{\beta x}, \eta_{\beta y}) \) in bands \( \alpha \) and \( \beta \) does not oblige to have equal modulus of \( x \) and \( y \) components (see Appendix).

In presence of a magnetic field in the basal plane \( \mathbf{H} = H(\cos \varphi, \sin \varphi, 0) \), choosing the vector potential as

\[ A = H(0, 0, y \cos \varphi - x \sin \varphi), \]

(15)

such that

\[ D_x = -i\frac{\partial}{\partial x}, \quad D_y = -i\frac{\partial}{\partial y}, \]

\[ D_z = -i\frac{\partial}{\partial z} + 2eH(y \cos \varphi - x \sin \varphi) \]

(16)

the equations determining the upper critical field acquire the form

\[ \begin{pmatrix} \eta_{\alpha x} \\ \eta_{\alpha y} \end{pmatrix} = \sum_\mu V_{\lambda\mu} \begin{pmatrix} L^\mu_{xx} + M^\mu_{xxxx}D_y^2 + M^\mu_{xxyy}D_y^2 + M^\mu_{xxxx}D_z^2 \\ 2M^\mu_{xxyy}D_xD_y + L^\mu_{yy} + M^\mu_{yyyy}D_y^2 + M^\mu_{yyyy}D_z^2 \end{pmatrix} \begin{pmatrix} \eta_{\mu x} \\ \eta_{\mu y} \end{pmatrix}. \]

(17)

For arbitrary field direction in the basal plane the system of equations for the \( x \) components of the order parameters is entangled with the system of equation for \( y \) components. At \( \mathbf{H} \parallel \mathbf{\hat{x}} \), the dependence from \( x \) coordinate drops out and the system of equations is split into independent systems of equations for \( x \) and \( y \) components of the order parameter.

\[ \begin{pmatrix} \eta_{\alpha x} \\ \eta_{\alpha y} \end{pmatrix} = \sum_\mu V_{\lambda\mu} \begin{pmatrix} L^\mu_{xx} + M^\mu_{xxxx}D_y^2 + M^\mu_{xxyy}D_y^2 + M^\mu_{xxxx}D_z^2 \\ 2M^\mu_{xxyy}D_xD_y + L^\mu_{yy} + M^\mu_{yyyy}D_y^2 + M^\mu_{yyyy}D_z^2 \end{pmatrix} \begin{pmatrix} \eta_{\mu x} \\ \eta_{\mu y} \end{pmatrix}. \]

(18)

Let us assume that the maximum critical field is determined by system of equations for the \( y \) components of
the order parameter. Its solution is given by functions independent of \(x\) coordinate

\[
\eta_{\lambda y} = \eta_{\lambda y}(y, z), \quad \lambda = \alpha, \beta. \tag{19}
\]

Hence, the order parameters in both bands

\[
\eta_{\lambda y}(y, z)\varphi_{\lambda y}(k), \quad \lambda = \alpha, \beta. \tag{20}
\]

are invariant under reflection \(\sigma_x\) about \(\hat{x}\) direction. At the same time, two component zero field order parameter for both bands

\[
\eta_{\lambda x}(k) + \eta_{\lambda y}(k), \quad \lambda = \alpha, \beta \tag{21}
\]

does not possess the \(\sigma_x\) symmetry. Hence, exactly as in single band case, there must exist a second transition in the finite field \(H < H_{c2}\) at which \(\eta_{\lambda x}\) and \(\eta_{\lambda y}\) become nonzero. Similar arguments hold for the field along any of the other three crystallographic directions in the basal plane. The existence of two transitions for all four crystallographic axes in the basal plane is a consequence of two component structure of the order parameter in each band, which can be either real or complex.

For the arbitrary direction of magnetic field in the basal plane one can diagonalize the system directly demonstrating the anisotropy of \(H_{c2}(\varphi)\). However, at arbitrary field direction the order parameter does not obey the symmetry in respect to reflection in the plane perpendicular to field direction, so the second phase transition is not obliged to be present.

### B. \(H_{c2}\) problem in superconducting state with \(a, b\) intra-band pairing

When we deal with direct pairing of electrons filling the states in non hybridized bands \(a\) and \(b\) the two component order parameter corresponding to each band keeps the same form

\[
\Delta_u(k, q) = \eta_{u a}(q)\varphi_x(k) + \eta_{u b}(q)\varphi_y(k), \tag{22}
\]

where index \(u\) runs the band labels \(a, b\), \((\varphi_x, \varphi_y)\) are basis functions of two dimensional representation of tetragonal group transforming as \((k_x, k_y)\). Unlike situation discussed in previous subsection they are the same for the different bands.

The linear equation for order parameter acquire the following form

\[
\begin{align*}
\Delta_u(k, q) = & \frac{1}{T} \sum_n \int \frac{d^3k'}{(2\pi)^3} \left[ \sum_w V_{uw}(k, k') G_w(k', \omega_n) G_w(-k' + q, -\omega_n) \Delta_w(k', q) \\
+ & \sum_{\{vw\}} V_{uw}(k, k') G_{vw}(k', \omega_n) G_{vw}(-k' + q, -\omega_n) \Delta_w(k', q) \right],
\end{align*}
\]

where sign \(\sum_{\{vw\}}\) denotes summation over \(v\) and \(w\) at \(v \neq w\). Here

\[
V_{uw}(k, k') = \sum_{i=x,y} \varphi_i(k)\varphi_i(k') \tag{24}
\]

is the pairing interaction matrix with properties

\[
V_{aa} = V_{bb}, \quad V_{ab} = V_{ba}, \tag{25}
\]

and

\[
G_a(k, \omega_n) = \frac{i\omega_n - \epsilon_{0k}}{(i\omega_n - E_{\alpha k})(i\omega_n - E_{\beta k})} \tag{26}
\]

\[
G_b(k, \omega_n) = \frac{i\omega_n - \epsilon_{0k}}{(i\omega_n - E_{\alpha k})(i\omega_n - E_{\beta k})} \tag{27}
\]

\[
G_{ab}(k, \omega_n) = G_{ba}(k, \omega_n) = \frac{\epsilon_{0k}}{(i\omega_n - E_{\alpha k})(i\omega_n - E_{\beta k})} \tag{28}
\]

are the normal state bands \(a, b\) and the interbands \(ab, ba\) Green functions.

Transforming order parameter equation to the coordinate representation and keeping only terms up to the second order in gradients we obtain Ginzburg-Landau equations

\[
\eta_{ai}(r) = \sum_{w_j} V_{uw}(L_{ij}^w + M_{ij}^w D_l D_m)\eta_{wj}(r) + \sum_{\{vw\}j} V_{uw}(L_{ij}^{vw} + M_{ij}^{vw} D_l D_m)\eta_{wj}(r), \tag{29}
\]

where

\[
L_{ij}^w = T \sum_n \int d^3k \varphi_i(k)\varphi_j(k) G_w(k, \omega_n) G_w(-k, -\omega_n), \tag{30}
\]
\[ M_{ijlm}^{w} = \frac{T}{2} \sum_{n} \int d^3k \ \varphi_i(k) \varphi_j(k) \times G_w(k, \omega_n) \frac{\partial^2 G_w(\mathbf{k}, \mathbf{-k}, -\omega_n)}{\partial k_i \partial k_m}, \] (31)

\[ L_{ij}^{vw} = T \sum_{n} \int d^3k \ \varphi_i(k) \varphi_j(k)G_{vw}(k, \omega_n)G_{vw}(-k, -\omega_n), \] (33)

\[ M_{ijlm}^{vw} = \frac{T}{2} \sum_{n} \int d^3k \ \varphi_i(k) \varphi_j(k) \times G_{vw}(k, \omega_n) \frac{\partial^2 G_{vw}(\mathbf{k}, \mathbf{-k}, -\omega_n)}{\partial k_i \partial k_m}, \] (35)

In absence of magnetic field we have two separate systems of homogeneous equations

\[ \eta_{ui}(r) = \sum_{w} V_{uw} L_{ij}^{w} \eta_{uj}(r) + \sum_{\{vw\}} V_{vw} L_{ij}^{vw} \eta_{uj}(r) \] (36)

for \( x \) and \( y \) components of the order parameter for both bands. Determinant of each of them determines the critical temperature of phase transition. Using explicit expressions for the Green functions, bands dispersion laws, and pairing interaction properties one can be convinced in following symmetry relations

\[ L_{xx}^a = L_{yy}^b, \quad L_{xy}^a = L_{yx}^b, \quad L_{ab}^{a,b} = L_{ba}^{a,b} = 0 \]
\[ L_{xx}^{ab} = L_{yy}^{ab} = L_{xx}^{ba} = L_{yy}^{ba} = 0. \] (37)

\[
\begin{pmatrix}
\eta_{ux} \\
\eta_{uy}
\end{pmatrix}
= \sum_w V_{uw} \begin{pmatrix}
L_{xx}^w + M_{xx}^w D_x^2 + M_{xy}^w D_x D_y + M_{yy}^w D_y^2 & 2M_{xy}^w D_x D_y + M_{xzz}^w D_z^2 + M_{yz}^w D_z D_y \\
M_{xy}^w D_x D_y & L_{yy}^w + M_{yy}^w D_y^2 + M_{yz}^w D_z D_y + M_{yzz}^w D_z^2
\end{pmatrix}
\begin{pmatrix}
\eta_{ux} \\
\eta_{uy}
\end{pmatrix}
+ \sum_{\{vw\}} V_{vw} \begin{pmatrix}
L_{xx}^{vw} + M_{xx}^{vw} D_x^2 + M_{xy}^{vw} D_x D_y + M_{yy}^{vw} D_y^2 & 2M_{xy}^{vw} D_x D_y + M_{xzz}^{vw} D_z^2 + M_{yz}^{vw} D_z D_y \\
M_{xy}^{vw} D_x D_y & L_{yy}^{vw} + M_{yy}^{vw} D_y^2 + M_{yz}^{vw} D_z D_y + M_{yzz}^{vw} D_z^2
\end{pmatrix}
\begin{pmatrix}
\eta_{ux} \\
\eta_{uy}
\end{pmatrix}, \tag{39}
\]

At \( \mathbf{H} \parallel \hat{x} \) the dependence of \( x \) coordinate drops out and the system of equations is split into two independent systems of equations for \( x \) and \( y \) components of the order parameter

\[ \eta_{ux} = \sum_w V_{uw} (L_{xx}^w + M_{xxy}^w D_y^2 + M_{xzz}^w D_z^2) \eta_{ux} + \sum_{\{vw\}} V_{vw} (L_{xx}^{vw} + M_{xxy}^{vw} D_y^2 + M_{xzz}^{vw} D_z^2) \eta_{ux}, \]

\[ \eta_{uy} = \sum_w V_{uw} (L_{yy}^w + M_{yy}^w D_y^2 + M_{yzz}^w D_z^2) \eta_{uy} + \sum_{\{vw\}} V_{vw} (L_{yy}^{vw} + M_{yy}^{vw} D_y^2 + M_{yzz}^{vw} D_z^2) \eta_{uy}. \] (40)

Argumentation about the symmetry of the order parameter given at the end of previous section is applicable here as well. Solution of Eq. (40) for \( y \) order parameter components is given by functions independent of \( x \) coordinate

\[ \eta_{uy} = \eta_{uy}(y, z), \quad u = a, b. \] (41)

Hence, the order parameters in both bands

\[ \eta_{uy}(y, z) \varphi_{uy}(k), \quad u = a, b. \] (42)

are invariant under reflection \( \sigma_x \) about \( \hat{x} \) direction. At the same time, two component zero field order parameter for both bands

\[ \eta_{ux} \varphi_{ux}(k) + \eta_{uy} \varphi_{uy}(k), \quad u = a, b \] (43)
does not possess the $\sigma_x$ symmetry.

Hence, even in case of direct intra-orbital pairing, there must exist a second transition in the finite field $H < H_{c2}$ at which $\eta_{ax}$ and $\eta_{bx}$ become nonzero. Similar arguments hold for the field along any of the other three crystallographic directions in the basal plane. The existence of two transitions for all four crystallographic axes in the basal plane is a consequence of two component structure of the order parameter in each band, which can be either real or complex. At arbitrary direction field the order parameter does not obey the symmetry in respect to reflection in the plane perpendicular to field direction, so the second phase transition is not obliged to be present.

For the arbitrary direction of magnetic field in the basal plane one cannot diagonalize the system [10], but obviously the anisotropy of $H_{c2}(\varphi)$ still takes place.

### III. CONCLUSION

We have demonstrated that there is an additional phase transition in the vortex state at $H < H_{c2}$ for four $\pm x, \pm y$ magnetic field directions in the basal plane in a multicomponent tetragonal superconductor. Being independent of intra-band or intra-orbital type of pairing this quality and also the upper critical field anisotropy in the basal plane are the inherent properties of multicomponent superconducting state with tetragonal symmetry.

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### Appendix A: Two component two-band superconductivity

#### 1. One band superconductivity

The GL free energy functional for the two component superconducting state with order parameter

$$
(\vec{\eta} \cdot \vec{\varphi}(k)) = \eta_x \varphi_x(k) + \eta_y \varphi_y(k)
$$

in tetragonal superconductor is

$$
F = \alpha_0(T - T_c)\vec{\eta}^2 + \beta_1(\vec{\eta}^2)^2 + \beta_2|\vec{\eta}|^2 + \beta_3(|\eta_x|^4 + |\eta_y|^4)
$$

(A2)

Here $(\varphi_x, \varphi_y)$ are basis functions of two dimensional representation of tetragonal group transforming as $(k_x, k_y)$.

There was shown [22] that at $\beta_2 > 0$, and $\beta_3 > -2\beta_2$ the state with two component complex order parameter $\vec{\eta} = \eta(1, i)$ arising by the second order phase transition directly from the normal state realizes the absolute minimum of the free energy density [A2]. This state belongs to the superconducting class

$$
D_4(E) = \left\{ \text{exp}\left(\frac{i\pi n}{2}\right) C_n, \text{exp}\left(-\frac{i\pi n}{2}\right) RU_n \right\},
$$

(A3)

which is the symmetry group of the order parameter. Here $n = 0, 1, 2, 3$, and $C_n$ are rotation around $z$-axis on angles $\pi n/2$, $U_n$ are rotations on angle $\pi$ around axes $\hat{x}, \hat{x} + \hat{y}, \hat{y}, -\hat{x} + \hat{y}$ correspondingly. The group $D_4(E)$ is the subgroup of maximal symmetry of the group of symmetry $G$ of the normal state given by direct product of tetragonal point group, group of gauge transformations and the group of time inversion

$$
G = D_{4h} \times U(1) \times R.
$$

The symmetry $D_4(E)$ of the order parameter guarantees invariance of GL free energy [A2] in respect to all transformations of normal state symmetry group $G$.

The superconducting state with order parameter

$$
\vec{\eta} = \eta(1, ir), \ r \neq 1
$$

(A5)

where $r$ is real number, can arise from the superconducting state with tetragonal symmetry $D_4(E)$ at some lower temperature $\tilde{T}_c < T_c$. The corresponding free energy term responsible for this phase transition

$$
\tilde{\alpha}_0(T - \tilde{T}_c)(\eta_x + i\eta_y)(\eta_x^* - i\eta_y^*)
$$

(A6)

is not invariant in respect of symmetry group $G$ of the normal state. But it is invariant in respect to all transformations of the group $D_4(E)$ of the superconducting state with order parameter $\vec{\eta} = \eta(1, i)$. The symmetry of state $\vec{\eta} = \eta(1, ir), \ r \neq 1$ is given by orthorhombic group

$$
D_2(E) = \left\{ \text{exp}\left(\frac{i\pi n}{2}\right) C_n, \text{exp}\left(-\frac{i\pi n}{2}\right) RU_n \right\}
$$

(A7)

Here, unlike to Eq. [A3] index $n = 0, 2$ only. The group $D_2(E)$ is a subgroup of the group $D_4(E)$.

#### 2. Two band superconductivity

Unlike single band superconductivity a two band state with band order parameters such that

$$
\eta_{ax} = \eta_{ay}, \ u = a, b
$$

(A8)

is not in general the absolute minimum of free energy. It is because along with the invariant $\beta_2^3(|\eta_x\tilde{\eta}_b|^2 + |\eta_b\tilde{\eta}_b|^2)$ vanishing at fulfillment [A5] decreasing free energy at $\beta_2 > 0$ there are several mixing terms in free energy expansion. For instance, the term $\beta(|\eta_x\tilde{\eta}_b|^2 + |\eta_b\tilde{\eta}_b|^2)$ obviously works to decrease free energy at $\beta > 0$ when modulus of $x$ and $y$ components of the order parameter are quite different.

Authors of paper [21] consider the state with order parameters with following relationship between components

$$
\eta_{ay} = i\eta_{ax}, \ \eta_{by} = i\eta_{ax}.
$$

(A9)

This state with symmetry $D_2(E)$ can arise directly from the normal state by the second order phase transition.
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24. Note that unlike to the tetragonal crystal for the hexagonal superconductor the additional phase transition must exist at arbitrary basal plane field direction.