Backscattering-immune one-way surface magnetoplasmons at terahertz frequencies

Linfang Shen,1 Yun You,1 Zhuoyuan Wang,2 and Xiaohua Deng1,*

1Institute of Space Science and Technology, Nanchang University, Nanchang 330031, China
2Ningbo Institute of Technology, Zhejiang University, Ningbo 315010, China
*dengxhncu@gmail.com

Abstract: Surface magnetoplasmons (SMPs) in a basic physical model for the terahertz regime, which consists of a semi-infinite magnetized semiconductor with dielectric cladding terminated by a metal slab, are theoretically investigated. The dispersion properties of such SMPs are analyzed and examined in detail. It is shown that SMPs may follow three different kinds of dispersion diagrams, depending on the applied dc magnetic field intensity. Complete one-way propagation that operates within the band gap of the semiconductor is available for SMPs, and the one-way bandwidth reaches a maximum at a certain magnetic field intensity. Regular modes guided by the dielectric layer are also analyzed. These modes may cause the (complete) SMP one-way region to be compressed or even removed, but they can be suppressed by reducing the dielectric layer thickness. Owing to the mirror effect of the metal slab, one-way propagating and backscattering-immune basic SMPs can exhibit a larger propagation length than those sustained by a single dielectric–semiconductor interface.

© 2015 Optical Society of America

OCIS codes: (240.6680) Surface plasmons; (230.3810) Magneto-optic systems; (230.7380) Waveguides, channelled; (040.2235) Far infrared or terahertz.

References and links
1. F. D. M. Haldane and S. Raghu, “Possible realization of directional optical waveguides in photonic crystals with broken time-reversal symmetry,” Phys. Rev. Lett. 100(1), 013904 (2008).
2. S. Raghu and F. D. M. Haldane, “Analogs of quantum-Hall-effect edge states in photonic crystals,” Phys. Rev. A 78(3), 033834 (2008).
3. The Quantum Hall Effect, edited by R. E. Prange and S. M. Girvin (Springer, 1987).
4. J. D. Joannopoulos, R. D. Meade, and J. N. Winn, Photonic Crystals: Molding the Flow of Light, 2nd ed. (Princeton University, 2008).
5. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljacic, “Reflection-free one-way edge modes in a gyromagnetic photonic crystal,” Phys. Rev. Lett. 100(1), 013905 (2008).
6. X. Ao, Z. Lin, and C. T. Chan, “One-way edge mode in a magneto-optical honeycomb photonic crystal,” Phys. Rev. B 80(3), 033105 (2009).
7. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljacic, “Observation of unidirectional backscattering-immune topological electromagnetic states,” Nature 461(7265), 772-775 (2009).
8. Z. Yu, G. Veronis, Z. Wang, and S. Fan, “One-way electromagnetic waveguide formed at the interface between a plasmonic metal under a static magnetic field and a photonic crystal,” Phys. Rev. Lett. 100(2), 23902 (2008).
9. A. B. Khanikaev, A. V. Baryshev, M. Ionue, and Y. S. Kivshar, “One-way electromagnetic Tamm states in magnetophotonic structures,” Appl. Phys. Lett. 95(1), 011101 (2009).
10. L. Feng, M. Ayache, J. Huang, Y. L. Xu, M. H. Lu, Y. F. Chen, Y. Fainman, and A. Scherer, “Nonreciprocal light propagation in a silicon photonic circuit,” Science 333(6043), 729-733 (2011).
11. K. Fang, Z. Yu, and S. Fan, “Realizing effective magnetic field for photons by controlling the phase of dynamic modulation,” Nature Photon. 6(11), 782-787 (2012).
12. A. B. Khanikaev, S. H. Mousavi, W. K. Tse, M. Kargarian, A. H. MacDonald, and G. Shvets, “Photonic topological insulators,” Nature Mater. 12(3), 233-239 (2013).
13. M. C. Rechtsman, J. M. Zeuner, Y. Plotnik, Y. Lumer, D. Podolsky, F. Dreisow, S. Nolte, M. Segev, and A. Szameit, “Photonic Floquet topological insulators,” Nature 496(7444), 196-200 (2013).
14. H. B. Zhu and C. Jiang, “Broadband unidirectional electromagnetic mode at interface of anti-parallel magnetized media,” Opt. Express 18(7), 6914-6921 (2010).
15. B. Hu, Q. J. Wang, and Y. Zhang, “Broadly tunable one-way terahertz plasmonic waveguide based on nonreciprocal surface magneto plasmons,” Opt. Lett. 37(11), 1895-1897 (2012).
16. V. Kuzmiak, S. Eyderman, and M. Vanwolleghem, “Controlling surface plasmon polaritons by a static and/or time-dependent external magnetic field,” Phys. Rev. B 86(4), 045403 (2012).
17. X. G. Zhang, W. Li, and X. Y. Jiang, “Confined one-way mode at magnetic domain wall for broadband high-efficiency one-way waveguide, splitter and bender,” Appl. Phys. Lett. 100(4), 041108 (2012).
18. J. J. Brion, R. F. Wallis, A. Hartstein, and E. Burstein, “Theory of surface magnetoplasmons in semiconductors,” Phys. Rev. Lett. 28(22), 1455-1458 (1972).
19. R. F. Wallis, J. J. Brion, E. Burstein, and A. Hartstein, “Theory of surface polaritons in anisotropic dielectric media with application to surface magnetoplasmons in semiconductors,” Phys. Rev. B 9(8), 3424-3437 (1974).
20. M. S. Kushwaha and P. Halevi, “Magnetoplasmons in thin films in the Voigt configuration,” Phys. Rev. B 36(11), 5960-5967 (1987).
21. M. S. Kushwaha and B. D. Rouhani, “Theory of magnetoplasmons in semiconductor superlattices in the Voigt geometry: A Green-function approach,” Phys. Rev. B 43(11), 9021-9032 (1991).
22. T. H. Isaac, W. L. Barnes, and E. Hendry, “Determining the terahertz optical properties of subwavelength films using semiconductor surface plasmons,” Appl. Phys. Lett. 93(24), 241115 (2008).
23. Y. S. Lee, Principles of Terahertz Science and Technology (Springer, 2009).
24. L. D. Landau, L. P. Pitaevskii, and E. M. Lifshitz, Electrodynamics of Continuous Media, Second Edition: Volume 8 (Course of Theoretical Physics) (Butterworth-Heinemann, 1984).

1. Introduction

One-way propagating electromagnetic (EM) modes, which were proposed by Raghu and Hal dan [1, 2] as analogues of quantum Hall edge states [3] in photonic crystals (PhCs) [4], are not only interesting for fundamental science but also important for practical applications. One-way EM modes are such that they are allowed to propagate in only one direction, and because of the absence of a back-propagating mode in the waveguide system, they are immune to backscattering at imperfections or bends [2, 5], which is of particular interest for slow light systems. It was predicted that one-way EM modes can be confined at the edges of certain two-dimensional (2D) PhCs made of magnetic-optical (MO) materials, where time-reversal symmetry is broken by applying a dc magnetic field [1, 2, 5, 6]. The existence of such modes was first experimentally demonstrated by Wang and his colleagues using MO PhCs in the microwave regime [7], and since then one-way EM modes have received increasing attention. Different schemes for realizing EM one-way propagation have also been proposed [8–13]; among them, the one based on surface plasmons seems to be most attractive owing to its robust mechanism and simple configuration [8,14–17].

Surface plasmons (SPs) sustained by the interface between a dielectric and a metal will become nonreciprocal under an external dc magnetic field [8]. It has been shown that, when the electron cyclotron frequency is comparable to the plasma frequency of the metal, the SP asymptotic frequency will differ significantly in the forward and backward directions, causing SPs to propagate in only one direction within the interval between the two different asymptotic frequencies. To make one-way SPs immune to backscattering, Yu et al. introduced a PhC structure into the dielectric to remove the coupling between SP and bulk modes, where the latter is completely suppressed by the PhC’s band gap [8]. Though such one-way SPs seem unrealistic because of the required magnetic field strength ($B_0 \sim 10^4$ T), the scheme can be extended to the terahertz regime and they may become realizable with the use of semiconductors. A semiconductor generally has a plasma frequency in the terahertz regime, and meanwhile its electron
effective mass is far less than the electron mass. As a result, the electron cyclotron frequency is often comparable to the plasma frequency for a semiconductor under normal magnetic field intensities. It is interesting to note that the study of SPs in magnetized semiconductors, referred to as surface magnetoplasmons (SMPs), began forty years ago [18–21]. We refer the reader to the work of Brion et al. for unidirectional propagation of SMPs in certain frequency regions. However, the SMPs studied in [18] are not robust against imperfections on the semiconductor surface, as the dielectric cladding is semi-infinite and bulk modes are allowed to propagate within it. Certainly, the bulk modes can be suppressed by introducing a PhC structure into the dielectric as proposed in [8]. For the terahertz regime, because metal resembles a perfect electric conductor, bulk mode suppression can be achieved by simpler means, i.e., by terminating the dielectric with a metal slab. One-way SMPs in such a basic physical model have recently been studied by Hu et al., and some interesting results were reported [15].

It is well known that SMP dispersion is characterized by its asymptotic frequencies, at which the propagation constant approaches infinity while the depth of field penetration into the dielectric approaches zero. Accordingly, in the basic model for the terahertz regime, the influence of the metal slab on the SMPs vanishes at the asymptotic frequencies. However, the SMP dispersion behaviour near one asymptotic frequency in Hu’s work [15] is found to be quite different from that in Brion’s [18]. Moreover, the dielectric layer sandwiched by a metal and a semiconductor (with a band gap) in the basic SMP model may support regular modes, which are guided based on total internal reflection. Evidently, regular modes are generally allowed to propagate in both the forward and backward directions, and their existence may make SMPs lose their robustness of one-way propagation. Unfortunately, attention has yet to be paid to the regular modes in the basic model. Furthermore, the loss of the semiconductor cannot be neglected generally for terahertz frequencies. Therefore, for basic one-way SMPs, it is necessary to examine whether the introduction of the metal slab will significantly increase the EM energy fraction in the semiconductor and cause serious propagation loss. In this paper, we will carefully investigate the characteristics of SMPs in the basic one-way propagation model and then physically clarify all the questions mentioned above.

2. Basic physical model of one-way SMPs

The basic physical model of SMPs consists of a semi-infinite semiconductor ($x \leq 0$) with dielectric cladding ($x > 0$) terminated by a metal slab (see the upper panel of Fig. 1). The metal slab is used to suppress bulk modes in the dielectric with relative permittivity $\varepsilon_r$, and the thickness of the dielectric layer is denoted by $d$. The semiconductor is uniformly magnetized by an external dc magnetic field $B_0$ in the $-y$ direction, and SMPs propagate along the $z$ direction. Gyroelectric anisotropy is induced in the semiconductor, with the permittivity tensor taking the form [18]

$$\begin{bmatrix}
\varepsilon_1 & 0 & i\varepsilon_2 \\
0 & \varepsilon_3 & 0 \\
-i\varepsilon_2 & 0 & \varepsilon_1
\end{bmatrix},$$

with

$$\varepsilon_1 = \varepsilon_\infty \left( 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} \right),$$

$$\varepsilon_2 = \frac{\omega_c^2}{\omega(\omega^2 - \omega_c^2)}.$$

---

#224928 - $15.00 USD

Received 15 Oct 2014; accepted 23 Dec 2014; published 15 Jan 2015
(C) 2015 OSA
26 Jan 2015 | Vol. 23, No. 2 | DOI:10.1364/OE.23.000950 | OPTICS EXPRESS 952
where $\omega$ is the angular frequency, $\omega_p$ is the plasma frequency of the semiconductor, $\omega_e = eB_0/m^*$ (where $e$ and $m^*$ are, respectively, the charge and effective mass of the electron) is the electron cyclotron frequency, and $\varepsilon_\infty$ is the high-frequency (relative) permittivity of the semiconductor. The semiconductor is assumed to be lossless herein, and its loss will be later taken into account when we consider the SMP propagation loss. Moreover, we assume the metal slab to be a perfect electric conductor, which is a good approximation for the terahertz regime. It should be noted that the metal slab in this system also has a mirror effect, making the system equivalent to a more complicated system, as shown in the lower panel of Fig. 1, where the latter requires the application of two oppositely directed magnetic fields.

![Schematic of the basic physical model (upper panel) of terahertz one-way SMPs. Owing to the mirror effect of the metal slab, this waveguide system is equivalent to that in the lower panel, where two opposite magnetic fields are applied.](image)

SMPs are transverse-electric-polarized in the (2D) basic model, and the nonzero component of the magnetic field ($\mathbf{H}$) can be written as

$$H_y(x,z) = [A_1 \exp(-\alpha dx) + A_2 \exp(\alpha dx)] \exp[i(kz - \omega t)]$$

(2)

in the dielectric layer for $0 < x \leq d$ and

$$H_y(x,z) = B \exp(\alpha x) \exp[i(kz - \omega t)]$$

(3)

in the semiconductor for $x \leq 0$, where $k$ is the propagation constant, $\alpha_d = \sqrt{k^2 - \varepsilon_r k_0^2}$ with $k_0 = \omega/c$ (where $c$ is the light speed in vacuum), and $\alpha = \sqrt{k^2 - \varepsilon_v k_0^2}$ with $\varepsilon_v = \varepsilon_1 - \varepsilon_2^2/\varepsilon_1$ being the Voigt permittivity. The nonzero components ($E_x$ and $E_z$) of the electric field ($\mathbf{E}$) can be obtained straightforwardly from $H_y$. The tangential component $E_z$ must vanish at the metal boundary $x = d$, yielding $A_2 = A_1 \exp(-2\alpha_d d)$. The boundary conditions further require the field components $E_x$ and $H_y$ to be continuous at the interface $x = 0$; based on this we can obtain

$$\alpha + \frac{\varepsilon_2}{\varepsilon_1} k + \frac{\varepsilon_v}{\varepsilon_r} \alpha_d \tanh(\alpha_d d) = 0,$$

(4)

which is the SMP dispersion relation. The second term in the dispersion equation is linear with respect to $k$, meaning that the SMP dispersion relation is asymmetric about $k = 0$. It should be pointed out that the SMP dispersion relation in the guiding system as shown in the lower panel of Fig. 1 is completely identical to Eq. (4).
3. SMP dispersion characteristics

SMP dispersion is characterized by its asymptotic frequencies, at which $k \to \pm \infty$. Owing to the nonreciprocity of SMPs, we analyze their dispersion behaviour for the cases of $k > 0$ and $k < 0$ separately. We first consider the case of $k > 0$. As $k \to +\infty$, $\alpha \approx k \to +\infty$ and $\alpha_d \approx k \to +\infty$, and from Eq. (4) two solutions can be found for $\omega$:

$$\omega_a = \frac{1}{2} \left( \sqrt{\omega_c^2 + 4 \omega_p^2} - \omega_c \right),$$  \hspace{1cm} (5)$$

$$\omega_{sp} = \frac{1}{2} \left( \sqrt{\omega_c^2 + 4 \omega_p^2 \frac{\varepsilon_{\infty}}{\varepsilon_{\infty} + \varepsilon_r} + \omega_c} \right).$$  \hspace{1cm} (6)

The second solution, Eq. (6), is surely the SMP asymptotic frequency for $k > 0$, as it reduces to $\omega_{sp} = \sqrt{\varepsilon_\infty/(\varepsilon_\infty + \varepsilon_r)} \omega_p$ for $\omega_a = 0$, which is just the result for SPs without magnetization. The first solution, Eq. (5), is found to correspond to a zero point of $\varepsilon_v$. Thus, for $\omega = \omega_a$, $\alpha = k$ since $\varepsilon_v = 0$, and the dispersion equation (4) reduces to $(1 + \varepsilon_2/\varepsilon_1)k = 0$. This equation seems to hold for any (positive) $k$ value, since the factor $(1 + \varepsilon_2/\varepsilon_1)$ vanishes in this case. To obtain a physical solution for $\omega = \omega_a$, Eq. (4) should be solved in a proper way. Let us consider the limit case of $\omega \to \omega_a$, for which $\varepsilon_v \to 0$ and $\alpha \approx k - \varepsilon_v k_0^2/(2k)$. In this situation, by deleting the factor $(1 + \varepsilon_2/\varepsilon_1)$, Eq. (4) can be rewritten as

$$k + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_r} \left[ \alpha_d \tanh(\alpha_d d) - \frac{\varepsilon_v k_0^2}{2k} \right] = 0,$$  \hspace{1cm} (7)

which is the actual equation that can be used to directly determine $k$ for the special case of $\omega = \omega_a$. Obviously, $k$ is generally finite at $\omega = \omega_a$; thus, $\omega_a$ is not an asymptotic frequency.

Because SMPs are a nonradiative mode, their dispersion curve lies outside the zone of bulk modes in the semiconductor, whose boundaries are described by $\omega = c|k|/\sqrt{\varepsilon_r}$. For a magnetized semiconductor, there exist two bulk-mode zones where $\varepsilon_v \geq 0$. The lower zone is located in the frequency region of $\omega_a \leq \omega \leq \omega_b$, where $\omega_b = \sqrt{\omega_c^2 + \omega_p^2}$, and $\varepsilon_v$ increases from 0 at $\omega = \omega_a$ to $+\infty$ at $\omega = \omega_b$. The upper zone is located in the region of $\omega \geq \omega_b$, where $\omega_b$ is the second zero point of $\varepsilon_v$ and is given by

$$\omega_b = \frac{1}{2} \left( \sqrt{\omega_c^2 + 4 \omega_p^2} + \omega_c \right).$$  \hspace{1cm} (8)

Correspondingly, there exist two band gaps for the semiconductor: one for $\omega < \omega_a$ and the other for $\omega_a < \omega < \omega_b$. By using the bulk-mode zones as the position reference, the SMP dispersion diagram can be classified into three types for $k > 0$, depending on the applied magnetic field intensity, and we refer to them as dispersion diagrams I, II, and III, for which $\omega_{sp}$ is, respectively, located in the regions $\omega < \omega_a$, $\omega_a \leq \omega \leq \omega_b$, and $\omega > \omega_b$. Dispersion diagram I with $\omega_{sp} < \omega_a$ pertains when $\omega_a < \omega_a^{(1)}$, where the critical value $\omega_a^{(1)}$ is given by

$$\omega_a^{(1)} = \frac{\varepsilon_r}{\sqrt{2(\varepsilon_{\infty} + \varepsilon_r)/(2\varepsilon_{\infty} + \varepsilon_r)}} \omega_p.$$  \hspace{1cm} (9)

In this case, the SMP dispersion relation has only a single band and lies completely within the lower semiconductor band gap. The dispersion band starts from the origin, rises to the right of the light line in the dielectric, which is described by $\omega = c|k|/\sqrt{\varepsilon_r}$, and finally approaches $\omega_{sp}$ as $k \to +\infty$. Dispersion diagram II with $\omega_a \leq \omega_{sp} \leq \omega_b$ pertains when $\omega_a^{(1)} \leq \omega_a \leq \omega_a^{(2)}$, where $\omega_a^{(2)}$ is given by

$$\omega_a^{(2)} = \frac{\varepsilon_r}{\sqrt{\varepsilon_{\infty}^2 - \varepsilon_r^2}} \omega_p.$$  \hspace{1cm} (10)
Similar to dispersion diagram I, the SMP dispersion relation in this case has only a single dispersion band, but the dispersion band extends outside the lower semiconductor band gap.

If the applied magnetic field is so large that \( \omega_k > \omega_k^{(2)} \), the SMPs follow dispersion diagram III with \( \omega_k^{(2)} > \omega_k \). In this case, the dispersion relation is split into two branches by the lower bulk-mode zone of the semiconductor. The lower branch is just terminated by the boundary of the bulk-mode zone. The upper branch starts from \( \omega_k \) at \( k = k^+_{cf} \) and approaches \( \omega_p^{(+)} \) as \( k \rightarrow +\infty \). As \( \omega \rightarrow \omega_k \), \( \epsilon_1 \rightarrow 0 \) and \( \epsilon_r \rightarrow \infty \), and from Eq. (4) we derive

\[
\left( k^+_{cf} \right)^2 = \frac{\tanh^2(\alpha_d d)}{\tanh^2(\alpha_d d) - \epsilon_r^2 \omega_s^2 / \epsilon_\infty^2 \alpha^2} k_s^2,
\]

where \( k_s = \sqrt{\epsilon_\epsilon_\epsilon_\epsilon / c} \) and \( \alpha_d = \sqrt{(k^+_{cf})^2 - k_s^2} \). If the upper branch starts from the light line of the dielectric, it can be derived from Eq. (4) that \( \alpha + k^+_{cf} \epsilon_2 / \epsilon_1 = 0 \). This equation cannot hold for positive \( k^+_{cf} \), since both \( \epsilon_1 \) and \( \epsilon_2 \) are positive for \( \omega > \omega_k \), so the upper branch cannot start from the light line. In the case when \( \omega_k \) is close to \( \omega_k^{(2)} \) (but \( \omega_k > \omega_k^{(2)} \)), \( k^+_{cf} \) should depart much farther from the light line, i.e., \( k^+_{cf} \gg k_s \), so \( \tanh(\alpha_d d) \approx 1 \) and Eq. (11) can be rewritten as

\[
k^+_{cf} = \frac{1}{\sqrt{1 - \left( \frac{\omega_k^{(2)}}{\omega_k} \right)^2 \left( 1 - \frac{\epsilon_r^2}{\epsilon_\infty^2} \right)}} k_s.
\]

Clearly, the value of \( k^+_{cf} \) approaches \( +\infty \) as \( \omega_k \rightarrow \omega_k^{(2)} \) (as expected), and it decreases when \( \omega_k \) increases from \( \omega_k^{(2)} \).

We now consider the case of \( k < 0 \). Similar to the case of \( k > 0 \), as \( k \rightarrow -\infty \), two solutions are found from Eq. (4). One solution is \( \omega = \omega_0 \) and the other is \( \omega = \omega_{sp}^{-} \) with

\[
\omega_{sp}^{-} = \frac{1}{2} \left( \sqrt{\omega_k^2 + 4 \omega_p^2 \frac{\epsilon_\infty}{\epsilon_\infty + \epsilon_r} - \omega_k} \right).
\]

Evidently, \( \omega_{sp}^{-} \) is really the SMP asymptotic frequency for \( k < 0 \). The first solution \( \omega_0 \) is just the second zero of \( \epsilon_r \), at which Eq. (4) holds for any negative \( k \) value, similar to the case of \( k > 0 \) at \( \omega = \omega_k \). For \( \omega = \omega_k \), we can also similarly derive

\[
-k + \frac{\epsilon_1 + \epsilon_2}{\epsilon_r} \left[ \alpha_d \tanh(\alpha_d d) + \frac{\epsilon_r k_s^2}{2k} \right] = 0,
\]

which is the actual equation that can be used to directly determine \( k \) for this special frequency. For the case of \( k < 0 \), \( \omega_{sp}^{-} \) is smaller than \( \omega_k \) for any \( \omega_k \), so there always exists a complete SP dispersion band, which starts from the origin, rises, and approaches \( \omega_{sp}^{-} \) as \( k \rightarrow -\infty \). However, for \( k < 0 \), an additional dispersion branch can always be found in the region where \( \omega > \omega_k \). This branch starts from the light line of the dielectric and ends at the boundary of the upper bulk-mode zone of the semiconductor. At the starting point of this additional branch, \( \omega_d = 0 \); thus, from Eq. (4) we have \( \alpha + k \epsilon_2 / \epsilon_1 = 0 \) with \( k = \sqrt{\epsilon_r} k_0 \), from which we find that \( \epsilon_1 \) is equal to \( \epsilon_r \) and the cutoff frequency is

\[
\omega_{cf}^{-} = \sqrt{\omega_k^2 + \frac{\epsilon_\infty}{\epsilon_\infty - \epsilon_r} \omega_p^2}
\]

for the additional branch. Evidently, the existence of the additional branch only requires \( \epsilon_\infty > \epsilon_r \), and this condition is generally met by transparent dielectrics in the terahertz regime.
To validate our analysis above, we numerically calculate the SMP dispersion relation with Eq. (4) for various \( \omega_c \) values. As an example, the basic parameters of the system are taken as follows: \( \varepsilon_\infty = 15.6 \) (InSb) [22], \( \varepsilon_r = 2.28 \) (polymer) [23], and \( d = 0.16 \lambda_p \) (where \( \lambda_p \) is the vacuum wavelength for the plasma frequency \( \omega_p \)). From now on, the semiconductor is assumed to be InSb in all numerical calculations. For the present system, we find that \( \omega_c^{(1)} = 0.066\omega_p \) and \( \omega_c^{(2)} = 0.148\omega_p \). Figures 2(a)–2(c) display the SMP dispersion relations for the cases of \( \omega_c^{(1)} = 0.05\omega_p, 0.1\omega_p, \) and \( 0.2\omega_p \), respectively. As can be seen from Figs. 2(a)–2(c), SMPs always follow two dispersion branches for \( k < 0 \), and the upper one (i.e., the additional branch) occurs in a narrow range compared with the lower one. Figure 2(d) shows a magnified view of the upper branch (with \( k < 0 \)) in Fig. 2(a); it clearly indicates that this branch starts from the light line and ends at the boundary of the bulk-mode zone. For SMPs with \( k > 0 \), the second dispersion branch is observed only in Fig. 2(c). Our numerical analysis also shows that the second dispersion branch can be found only when \( \omega_c > 0.148\omega_p \). All the numerical results agree well with our analytical analysis.

4. Complete SMP one-way propagation

It is evident that, for each kind of dispersion diagram, there always exist certain frequency ranges in which SMPs are allowed to propagate unidirectionally. For dispersion diagrams I and II, as shown in Figs. 2(a) and 2(b), SMPs only propagate forward in the frequency range from
for $\omega_c \leq \omega_c^{(1)}$, or

$$\Delta \omega_m = \frac{2\omega_p^2 \varepsilon_r / (\varepsilon_\infty + \varepsilon_r)}{\sqrt{\omega_c^2 + 4\omega_p^2 + \sqrt{\omega_c^2 + 4\omega_p^2 \varepsilon_\infty / (\varepsilon_\infty + \varepsilon_r)}}}$$

for $\omega_c > \omega_c^{(1)}$. Obviously, $\Delta \omega_m$ first increases linearly with $\omega_c$, then reaches a maximum of $\omega_c^{(1)}$ at $\omega_c = \omega_c^{(1)}$, and finally decreases as $\omega_c$ grows further.

Fig. 3. (a) Complete one-way bandwidth of SMPs as a function of $\omega_c$. Solid, dashed, and dotted lines correspond to the dielectrics of air, polymer ($\varepsilon_r = 2.28$), and silica ($\varepsilon_r = 11.68$), respectively. (b) Complete one-way range versus $\omega_c$ for the case of air. The other parameters are the same as in Fig. 2.

The dependence of $\Delta \omega_m$ on $\omega_c$ is plotted in Fig. 3(a). Three different dielectrics are analyzed: air, polymer ($\varepsilon_r = 2.28$), and silica ($\varepsilon_r = 11.68$), all of which are transparent media commonly used in the terahertz regime [23]. As seen from Fig. 3(a), $\Delta \omega_m$ is independent of $\varepsilon_r$ when $\omega_c < 0.031 \omega_m$, which just corresponds to the value of $\omega_c^{(1)}$ for air. Therefore, under a small magnetic field, it is impossible to increase $\Delta \omega_m$ by choosing the dielectric. However, the maximal value of $\Delta \omega_m$, which is equal to the value of $\omega_c^{(1)}$ and occurs at $\omega_c = \omega_c^{(1)}$, closely depends on $\varepsilon_r$. The larger $\varepsilon_r$ is, the larger the maximal $\Delta \omega_m$ is. It is found that the maximal values of $\Delta \omega_m$ are 0.031 $\omega_m$, 0.066 $\omega_m$, and 0.240 $\omega_m$ for air, polymer, and silica, respectively. These values also represent the peak positions $\omega_c^{(1)}$ for the corresponding dielectrics. Figure
increases from dc valued, and it has the form as d one-way region is compressed, and its upper limit is just determined by the cutoff frequency.

region is preserved for SMPs. Obviously, $\alpha_0$. In the region below $\omega_c$, the cutoff of the regular mode shifts downward. As a result, the complete one-way region is compressed, and its upper limit is just determined by the cutoff frequency. Figure 4(d) indicates that, when $d = 0.29\lambda_p$, the cutoff drops to $\omega_c$, and consequently the complete one-way region vanishes completely.

5. Regular mode guided by the dielectric layer

In the basic one-way SMP model, the dielectric layer can still support (transverse-electric-polarised) regular modes, which are guided by a means of zigzag reflections at the surfaces of the metal and the semiconductor (with a band gap). As regular modes are generally allowed to propagate in both forward and backward directions, their existence causes SMPs to lose their robustness of one-way propagation against imperfections. The dispersion equation for regular modes can be obtained from Eq. (4) by replacing $\alpha_d$ with $-i\omega$, where $p = \sqrt{\varepsilon c k_0^2 - k^2}$ is real-valued, and it has the form

$$\alpha + \frac{\varepsilon_2}{\varepsilon_1}k - \frac{\varepsilon_\varepsilon}{\varepsilon_p}p\tan(pd) = 0. \quad (16)$$

Obviously, the dispersion relation for regular modes is also asymmetric about $k = 0$. Because regular modes have a tangential electric field component ($E_\parallel$) that vanishes at the metal surface, they should exhibit transverse resonances and related cutoff frequencies. These cutoffs increase as $d$ decreases; hence, it is possible to remove the regular modes from the complete one-way region by reducing the dielectric layer thickness.

Owing to the nonreciprocity, the cutoff point of a regular mode may deviate from $k = 0$. However, this deviation should be very small under a normal magnetic field, as $\omega_c$ is generally much smaller than $\omega_0$ (e.g., for InSb at room temperature, $\omega_0$ is equal to 0.1$\omega_p$ for $B_0 = 0.1$ T). If we let $k \approx 0$, then $p \approx \sqrt{\varepsilon c k_0^2}$, and from Eq. (16) we have $\alpha = \varepsilon_c k_0 \tan(\sqrt{\varepsilon c k_0 d})/\sqrt{\varepsilon c}$. In the region below $\omega_c$, $\varepsilon_c$ is always negative, so Eq. (17) has no solution if $\sqrt{\varepsilon c k_0 d} < \pi/2$. This means that even the fundamental regular mode with the lowest cutoff will vanish at this frequency. By setting $k_0 = \omega_0/c$, the critical thickness ($d_c$) of the dielectric layer is found to be

$$d_c = \frac{1}{8\sqrt{\varepsilon_c}} \left( \sqrt{4 + \omega_c^2/\omega_p^2} + \omega_0/\omega_p \right) \lambda_p, \quad (17)$$

below which no regular mode appears in the region for $\omega < \omega_0$; i.e., the complete one-way region is preserved for SMPs. Obviously, $d_c$ is inversely proportional to $\sqrt{\varepsilon_c}$. For $\omega_0 = 0.1\omega_p$, we find that $d_c = 0.26\lambda_p$ for $\varepsilon_r = 1$, $d_c = 0.17\lambda_p$ for $\varepsilon_r = 2.28$, and $d_c = 0.077\lambda_p$ for $\varepsilon_r = 11.68$.

To illustrate the regular mode guided by the dielectric layer, we numerically calculate its dispersion relations using Eq. (16) for various $d$ values, and the results are plotted in Fig. 4, where $\varepsilon_r = 2.28$ and $\omega_c = 0.1\omega_p$ (corresponding to $d_c = 0.17\lambda_p$) for all cases. Figures 4(a)–4(d) show the dispersion relations (lines with filled circles) of the (fundamental) regular mode for $d = d_c$, $0.2\lambda_p$, $0.25\lambda_p$, and $0.29\lambda_p$, respectively. As expected, in the case of $d = d_c$, the fundamental regular mode has a cutoff of $\omega_0$, and the complete one-way SMP region is preserved. When $d$ increases from $d_c$, the cutoff of the regular mode shifts downward. As a result, the complete one-way region is compressed, and its upper limit is just determined by the cutoff frequency. Figure 4(d) indicates that, when $d = 0.29\lambda_p$, the cutoff drops to $\omega_0$, and consequently the complete one-way region vanishes completely.
Fig. 4. Dispersion relations of all modes in the guiding system. Solid lines with and without filled circles correspond to the dispersion curves for regular and SMP modes, respectively. Dashed lines represent the light line in the dielectric. (a) $d = d_c = 0.17\lambda_p$, (b) $d = 0.2\lambda_p$, (c) $d = 0.25\lambda_p$, and (d) $d = 0.29\lambda_p$. In (a)–(c), the lower shaded area represents the complete one-way region, and the upper one represents the zone of bulk modes in the semiconductor. The other parameters are $\varepsilon_{\infty} = 15.6$, $\varepsilon_r = 2.28$, and $\omega_c = 0.1\omega_p$ for all cases.

Using the finite element method (FEM), we performed a simulation to demonstrate the regular mode in the basic SMP model. For this purpose, the parameters of the system are taken to be the same as in Fig. 4(c). In the simulation, a magnetic current line source is used to excite SMPs and two operation frequencies of $f_A = 0.905 f_p$ and $f_B = 0.94 f_p$ (where $f_p = \omega_p / 2\pi$) are considered, as marked with points A and B in Fig. 4(c). The plasma frequency is set to be $f_p = 2$ THz, so $f_A = 1.81$ THz and $f_B = 1.88$ THz. To examine the robustness of one-way propagating SMPs, we consider both cases with and without a metal rod obstacle. The metal rod with a radius of 8 $\mu$m is inserted at the polymer–semiconductor interface at a horizontal distance of 150 $\mu$m from the source. All simulated results are displayed in Fig. 5. The left panels show the SMP transmission at $f_A = 1.81$ THz, where the source is placed at the polymer–semiconductor interface at a horizontal direction. Furthermore, SMPs at this frequency can go around the obstacle without any power loss, as no backward-propagating wave emanates from the left side of the source (see the lower left panel). For $f_B = 1.88$ THz, to avoid strong direct excitation of the regular mode, the source is placed inside the semiconductor at 30 $\mu$m below the surface. Nevertheless, a weak backward-propagating wave, which corresponds to the regular mode and is directly excited by the source, is still observed in the upper right panel of Fig. 5. As shown in the lower right panel, the backward-propagating wave is enhanced in the presence of the obstacle. Evidently, in this case, coupling between SMPs and the backward-propagating regular mode occurs via...
scattering at the obstacle. The results shown in the right panels indicate that SMPs at \( f_B = 1.88 \) THz are no longer immune to backscattering owing to the existence of the regular mode in the dielectric layer. The simulated results agree well with our previous analysis.

Fig. 5. Simulated electric field amplitudes. For \( f = 1.81 \) THz, the source is placed at the polymer–semiconductor interface; for \( f = 1.88 \) THz, the source is placed inside the semiconductor, 30 \( \mu m \) below the surface. In the lower panels, a metal rod with a radius of 8 \( \mu m \) is inserted at the polymer–semiconductor interface, which lies at a horizontal distance of 150 \( \mu m \) from the source. The other parameters are the same as in Fig. 4(c).

6. Mirror effect of the metal slab

As a dispersive medium, the semiconductor is inherently lossy, and, consequently, SMPs suffer a propagation loss in it. For a lossy semiconductor, the expressions for \( \varepsilon_1 \) and \( \varepsilon_2 \) in Eq. (1) become

\[
\varepsilon_1 = \varepsilon_\infty \left( 1 - \frac{(\omega + i\nu)\omega_p^2}{\omega[(\omega + i\nu)^2 - \omega_c^2]} \right),
\]

\[
\varepsilon_2 = \varepsilon_\infty \frac{\omega_k \omega_p^2}{\omega[(\omega + i\nu)^2 - \omega_c^2]},
\]

where \( \nu \) is the electron scattering frequency. In the lossy case, the propagation constant \( k \) in Eq. (4) becomes complex, i.e., \( k = k_r + ik_i \) (where both \( k_r \) and \( k_i \) are real-valued), and the SMP propagation length is determined by \( L = 1/(2k_i) \). Evidently, in the case of \( \nu \ll \omega_p \), the SMP loss is proportional to the fraction of EM energy in the semiconductor. Correspondingly, the larger the EM energy fraction of SMPs is distributed within the dielectric layer, the larger the propagation length is. In the basic one-way SMP model, the metal slab has two opposing effects on the SMP energy distribution: 1. As the metal slab compresses the dielectric region, the energy fraction in the dielectric layer is decreased, and 2. the metal slab introduces the mirror effect, which will enhance the EM energy density and thus increase that energy fraction. Owing to the mirror effect, a SMP in the present model may be viewed as a symmetric combination of a SMP associated with a single (dielectric–semiconductor) interface and its image produced by the metal slab; the field in the dielectric layer is therefore enhanced compared with the case of a single interface. However, the mirror effect is effective only when \( d \leq 1/\alpha_d \), which is the field penetration depth into the dielectric. Thus, for robust one-way SMPs, an interesting question arises as to whether the propagation loss is aggravated or lessened (compared with SMPs at a single interface) by the metal slab or whether the mirror effect of the metal slab is the dominant effect on the SMP energy distribution.

To clarify this question, we numerically calculate the propagation length \( L \) using Eq. (4) for a lossy semiconductor. As a numerical example, we take \( \nu = 0.0025\omega_p \) for the semiconductor,
aggravates the SMP loss owing to the compression of the dielectric region. This frequency lies at the centre of the complete one-way region for \( d \leq d_c \), i.e., \( \omega = (\omega_{sp} + \omega_0)/2 \). With the above parameters, the critical thickness is \( d_c = 0.17\lambda_p \). For comparison, we also calculate the propagation length (\( L_0 \)) for SMPs at a single interface, and we find that \( L_0 = 7.7\lambda (\lambda = 2\pi c/\omega) \) for \( \omega = 0.918\omega_p \), which is indicated in Fig. 6(a) by the horizontal dashed line. From Fig. 6(a), it is seen that \( L < L_0 \) for small \( d \), meaning that the metal slab aggravates the SMP loss owing to the compression of the dielectric region. \( L \) rapidly increases with \( d \), becoming larger than \( L_0 \) at \( d = d_c \), meaning that the mirror effect becomes the dominant effect on the energy distribution. Our calculation gives \( L = 8.3\lambda \) for \( d = d_c \) (\( L \) is inversely proportional to \( v \), and for this case \( L = 20.75\lambda \) if \( v = 0.001\omega_p \)). \( L \) reaches a maximal value of 8.4\( \lambda \) at \( d = 0.20\lambda_p \), but the cutoff of the fundamental regular mode in this case is found to be below \( \omega \). The inset of Fig. 6(a) shows the distribution of \( d \) versus \( \omega \) for the frequency \( \omega = 0.918\omega_p \). The other parameters are \( \varepsilon_\infty = 15.6 \), \( \varepsilon_r = 2.28 \), and \( \omega_0 = 0.1\omega_p \).

To gain a deeper insight into the mirror effect, we further calculate the SMP EM energy in the dielectric layer and semiconductor in the lossless case \((v = 0)\). The other parameters are kept the same as in Fig. 6(a). As the semiconductor is strongly dispersive, the EM energy density in it can be expressed as [24]

\[
U_s = \frac{1}{4} E^* \frac{d(\omega E_s)}{d\omega} E + \frac{1}{4} \mu_0 |H|^2.
\]

The SMP EM energy fraction in the dielectric layer is

\[
\eta_d = \frac{\int_0^L U_d dx}{\int_0^L U_s dx + \int_0^d U_d dx} , \tag{18}
\]

where \( U_d = (1/4)(\varepsilon_0 \varepsilon_r |E|^2 + \mu_0 |H|^2) \), which is the energy density in the dielectric layer. The calculated results of \( \eta_d \) for various \( d \) values are plotted in Fig. 6(b). As expected, the dependence of \( \eta_d \) on \( d \) is similar to that of \( L \), as shown in Fig. 6(a). \( \eta_d \) first increases with \( d \) and then
reaches a maximum at \( d \approx 0.2 \lambda_p \). As \( d \) continues to grow, it decreases and finally approaches a limit, which corresponds to that for SMPs at a single dielectric–semiconductor interface. The behaviour of \( \eta_d \) clearly indicates that the mirror effect of the metal slab enhances the EM energy density in the dielectric layer and thus may increase the energy fraction.

7. Conclusions

In summary, we have theoretically studied terahertz SMPs in a basic physical model comprising a semi-infinite magnetized semiconductor with dielectric cladding terminated by a metal slab. It has been shown that, depending on the applied magnetic field intensity, such SMPs may follow three different kinds of dispersion diagrams. For each kind of dispersion diagram, there always exists a complete one-way SMP region, which lies within the lower band gap of the semiconductor. If the magnetic field is strong enough, another complete one-way band appears in the upper band gap of the semiconductor, but it generally has a bandwidth significantly smaller than that in the lower band gap. The bandwidth of the (complete) one-way propagation closely depends on the magnetic field intensity, and it reaches a maximum at a finite magnetic field. Regular modes guided by the dielectric layer have also been investigated. The regular modes are allowed to propagate in both the forward and backward directions, and when they are present, the complete one-way region is often compressed or even removed. However, regular modes can be suppressed by reducing the dielectric layer thickness. Moreover, the mirror effect of the metal slab in this basic one-way SMP model has been analyzed. It has been shown that, because the mirror effect can significantly increase the EM energy fraction in the dielectric layer, the SMPs immune to backscattering in the basic physical model can exhibit a larger propagation length than those at a single dielectric–semiconductor interface.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant No. 61372005), the National Natural Science Foundation of China under a key project (Grant No. 41331070), the Science Foundation of Zhejiang Province (Grant No. LY14F030013), and the Natural Science Foundation of Ningbo (Grant No. 2013A610004).