Parton-model calculation of a nonstandard decay process in isotropic modified Maxwell theory

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Abstract

We have performed a calculation in the parton-model approximation of a nonstandard decay process in isotropic modified Maxwell theory coupled to standard Dirac theory, with a single Lorentz-violating parameter $\kappa$ in the photonic sector. Previous calculations of this process (vacuum Cherenkov radiation by a proton for $\kappa > 0$) were performed for point-like particles and an upper bound on $\kappa$ at the $10^{-19}$ level was obtained from data on ultra-high-energy cosmic rays. The parton-model results change the decay rate by about an order of magnitude but give essentially the same upper bound on $\kappa$ because of the large experimental errors in the energy determination of the cosmic primary. The previous point-particle calculation of photon decay into an electron-positron pair for the theory with $\kappa < 0$ remains valid and data on cosmic gamma rays provide a lower bound on $\kappa$ at the $-10^{-15}$ level.

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I. INTRODUCTION

Possible small deviations of Lorentz invariance in the photonic sector are best bounded by direct experiments in the laboratory (references will be given later). But it is also possible to obtain tight indirect bounds on the Lorentz violation by detecting charged or neutral particles with ultrahigh-energies in the Earth atmosphere, possibly coming from distant astronomical sources.

For these indirect but terrestrial bounds it is of crucial importance to determine the theoretical decay rates reliably. Up till now, most of these calculations were performed for pointlike particles. The pointlike calculations are expected to be quite reliable for certain decay processes (e.g., photon decay into an electron-positron pair) but not for other decay processes (e.g., vacuum Cherenkov radiation by protons or heavy nuclei). These latter processes are better calculated using the parton-model approximation and the present article sets out to perform such a calculation for vacuum Cherenkov radiation by a realistic proton.

II. BACKGROUND

Throughout this article, we use natural units with $\hbar = 1 = c$ and the Minkowski metric $g_{\mu\nu}(x) = \eta_{\mu\nu} \equiv \text{diag}(+1, -1, -1, -1)$. As the theory considered will be seen to violate Lorentz invariance, we must state clearly what is meant by $c$. For the particular theory considered, $c$ is the maximum attainable velocity of the fermionic particles.

The Lagrange density for modified electrodynamics in the presence of CPT-even Lorentz violation in the photon sector can be written as follows:

$$\mathcal{L}_{\text{modMaxwell}} = -\frac{1}{4} \eta_{\mu\rho} \eta_{\nu\sigma} F^{\mu\nu} F^{\rho\sigma} - \frac{1}{4} (k_F)_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma},$$

(2.1)

in terms of the conventional field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. The first term on the right-hand side of (2.1) describes standard Maxwell electrodynamics and the second term corresponds to a dimension-four operator that breaks Lorentz invariance [1–3]. The size of the deviation from exact Lorentz invariance is controlled by the dimensionless coefficient $(k_F)_{\mu\nu\rho\sigma}$. The Lagrange density (2.1) describes a theory which preserves CPT, gauge, and coordinate invariance, whereas particle Lorentz invariance is broken. The coefficient $(k_F)_{\mu\nu\rho\sigma}$ has the symmetries of the Riemann tensor and has 20 independent components. Its double trace can be taken to be zero because it simply modifies the normalization of the Lorentz-invariant part of the Lagrange density (2.1). In other words, the coefficient $k_F$ is assumed to satisfy $(k_F)_{\mu\nu\rho\sigma} = 0$, which reduces the total number of independent components to 19.

The modified field equations are given by

$$\partial^\alpha F_{\alpha\beta} + (k_F)_{\alpha\beta\mu\nu} \partial^\alpha F^{\mu\nu} = 0.$$  

(2.2)

From (2.2), it can be seen that ten of the 19 independent components of the coefficient $(k_F)_{\alpha\beta\mu\nu}$ produce birefringence in the photon sector, which can be bounded with remarkable
precision using cosmological observations [4–6]. For this reason, we focus in the present article on the remaining nine components that produce nonbirefringent effects. If the ten components generating birefringence are neglected, the coefficient \((k_F)_{\alpha\beta\mu\nu}\) can be expressed as follows [3]

\[(k_F)_{\alpha\beta\mu\nu} = \frac{1}{2} \left[ \eta_{\alpha\mu} (k_F)^{\lambda}_{\beta\lambda\nu} - \eta_{\alpha\nu} (k_F)^{\lambda}_{\beta\lambda\mu} 
- \eta_{\beta\mu} (k_F)^{\lambda}_{\alpha\lambda\nu} + \eta_{\beta\nu} (k_F)^{\lambda}_{\alpha\lambda\mu} \right], \tag{2.3}\]

where \((k_F)^{\lambda}_{\beta\lambda\nu}\) is symmetric and traceless. A simple form for this nonbirefringent coefficient can be obtained if we restrict the modified Maxwell theory to describe an isotropic deviation from exact Lorentz invariance. For this isotropic case, the relevant coefficient in the last expression of (2.1) can be written as (2.3) with

\[(k_F)^{\lambda}_{\mu\lambda\nu} = \frac{\kappa}{2} \text{diag}(3, 1, 1, 1), \tag{2.4}\]

where \(\kappa\) is a short-hand notation of \(\tilde{\kappa}\text{tr}\) used in most of the recent literature.

The isotropic breakdown of Lorentz invariance in the photonic sector is then controlled by the single coupling constant \(\kappa\) which physically parameterizes a modification of the photon phase velocity. Modern versions of the Ives–Stilwell experiment [7] allow for direct laboratory bounds on this parameter down to the \(10^{-10}\) level [8–10].

The parameter \(\kappa\) endows the vacuum with an effective index of refraction

\[n = \sqrt{(1 + \kappa)/(1 - \kappa)}. \tag{2.5}\]

Writing the photon momentum in the form \(q^\mu = (\omega(q), q, 0, 0)\) for \(q \equiv |q|\), its energy and momentum are related by

\[\omega(q) = \frac{1}{n} q = \sqrt{\frac{1 - \kappa}{1 + \kappa}} q. \tag{2.6}\]

It will be seen that this modified photon dispersion relation allows for processes which are kinematically forbidden in the conventional Lorentz-invariant theory.

Lorentz-violating quantum electrodynamics can be constructed by coupling the free photon described by the Lagrange density (2.1) to a standard spin-\(\frac{1}{2}\) particle \(f\) with electric charge \(e_f\) and inertial mass \(M_f\),

\[\mathcal{L}_{\text{modQED}} = \mathcal{L}_{\text{modMaxwell}} + \mathcal{L}_{\text{Dirac}}, \tag{2.7}\]

\[\mathcal{L}_{\text{Dirac}} = \bar{\psi} \left[ \gamma^\mu (i \partial_\mu - e_f A_\mu) - M_f \right] \psi. \tag{2.8}\]

The Lorentz-violating operators with the symmetry structure used for the isotropic modified Maxwell theory can also be moved into the fermion sector by an appropriate change of spacetime coordinates [3], but, for definiteness, we keep the theory as defined by (2.7). The quantum theory corresponding to (2.7) has been studied in Ref. [11], in particular as regards microcausality and unitarity.
For negative values of \( \kappa \), the photon becomes unstable and can produce other particles by the decay process \( \tilde{\gamma} \rightarrow f + \bar{f} \), where \( f \bar{f} \) corresponds to a generic pair of electrically charged fermions and \( \tilde{\gamma} \) denotes a photon described by the isotropic modified Maxwell theory. This process is called photon decay (PhD). For positive values of \( \kappa \), photons can be emitted by an electrically charged fermion \( f \) and the process has been called vacuum Cherenkov (VCh) radiation, \( f \rightarrow f + \tilde{\gamma} \).

The exact tree-level calculations of the PhD and VCh decay processes have been presented in Ref. [12]. The corresponding effects in a CPT-odd theory with both scalar and spinor electrodynamics have also been studied [13, 14]. The absence of experimental evidence for these two processes has been used in the past to put constraints on \( \kappa \). Precise measurements of the energy loss of electrons in particle colliders have been considered for determining indirect laboratory bounds on \( \kappa \) of order \( 10^{-11} \) [15] and the precise knowledge of the synchrotron loss rate can improve this bound to the \( 5 \times 10^{-15} \) level [16]. But the most stringent two-sided bound has been obtained by using the observation of high-energy gamma rays and cosmic rays in the Earth atmosphere [12],

\[
-9 \times 10^{-16} < \kappa < 6 \times 10^{-20},
\]

which holds at the two-\( \sigma \) level (98\% CL).

In most of the literature, the particles involved in these reactions have been considered to be structureless (pointlike). The description of photon decay into Dirac fermion-antifermion pairs as well as vacuum Cherenkov radiation by a Dirac fermion is appropriate for elementary particles such as electrons. But, for bounds obtained from particle showers in the Earth atmosphere, we need a careful treatment of the internal structure of the primary particle.

In this work, we extend the analysis of Ref. [12] by considering vacuum Cherenkov radiation emitted by a realistic proton at high energies, relevant for high-energy cosmic rays. For completeness, we also discuss the decay of an energetic astrophysical photon into electron-positron pairs, relevant for observations made with gamma-ray telescopes.

### III. VACUUM CHERENKOV RADIATION

For a positive Lorentz-violating parameter \( \kappa \) in the theory (2.7), the maximum attainable velocity of a charged particle \( (c = 1) \) can be larger than the photon phase velocity from (2.6). The Cherenkov-like emission of a photon by a proton of charge \( e_p \equiv e \) is then already possible in the vacuum (Fig. 1). The modifications introduced by \( \kappa \) to the photon polarization vectors \( \tilde{\varepsilon}_\mu^{(\lambda)} \) are obtained by solving the equation of motion (2.2) for the gauge field \( A_\mu \), with proper normalization factors from the quantum theory [11]. The averaged squared amplitude for a photon of energy \( \omega \) emitted by a Dirac fermion with proton mass \( M_p \equiv M \) and energy \( E \)
has the form

$$\left| \mathcal{M}_{\text{VCh}} \right|^2 = \frac{e^2}{2} \sum_{s} \sum_{s'} \left| \bar{\pi}_{s'}(p') \gamma^\mu u_s(p) \bar{\varepsilon}_{\mu}^{(\lambda)s} \right|^2$$

$$= \frac{32 \pi \alpha |\kappa|}{\left(1 + \kappa \right)^2} \left[ E(E - \omega) - \left(1 + \kappa \right) \frac{M^2}{2 \kappa} + \frac{\omega^2}{2 \left(1 - \kappa \right)} \right], \quad (3.1)$$

where $\alpha \equiv e^2/(4\pi)$ is the fine-structure constant. The total emission rate of Cherenkov photons by a structureless charged fermion is then given by

$$\hat{\Gamma}_{\text{VCh}} = \frac{1}{4\pi^2} \frac{1}{2E} \int \frac{d^3 q}{2\omega} \frac{d^3 p'}{2E'} \frac{\left| \mathcal{M}_{\text{VCh}} \right|^2}{\delta^4(p - p' - q)}, \quad (3.2)$$

where, from now on, the ‘hat’ signifies that we are considering pointlike particles. Inserting a factor $\omega$ into the integrand and performing the necessary phase-space integrals produces the Cherenkov power radiated by a pointlike “proton” of energy $E$,

$$\hat{P}(E) = \frac{\alpha}{12 \kappa^3 E \sqrt{E^2 - M^2}} \left( \sqrt{E^2 - M^2} - E/n \right)^2$$

$$\times \left[ 2 E^2 (2\kappa^2 + 4\kappa + 3) - 3 M^2 (1 + \kappa)(1 + 2\kappa) - 2 n E \sqrt{E^2 - M^2} \left(1 - \kappa \right) (4\kappa + 3) \right]. \quad (3.3)$$

Expression (3.3) reduces to Eq. (8) of Ref. [12].

If the fermion producing the Cherenkov emission is a realistic proton, the internal structure becomes relevant at high energies, which is the case for proton primaries in energetic cosmic rays. Considering the proton as a composite particle, the Cherenkov photon can be taken as emitted by the charged partons (quarks) in the proton rather than by the proton as a whole. Kinematically, the Cherenkov emission can occur when a proton of mass $M$ has an energy above the threshold

$$E_{\text{th}} = M \sqrt{\frac{1 + \kappa}{2\kappa}}. \quad (3.4)$$

This threshold energy arises from energy-momentum conservation and is independent of the internal structure of the proton.

For a proton with energy $E > E_{\text{th}}$, the Cherenkov emission rate produced by a charged parton carrying a fraction $x$ of the proton momentum takes the following form at tree level:

$$\frac{d^2 \hat{\Gamma}_{i \text{VCh}}}{dx \, d\omega} = \frac{\alpha e_i^2}{E \sqrt{E^2 - M^2}} \left[ \frac{2\kappa E}{1 - \kappa^2} \left( E - \omega \right) x \right]$$

$$- \frac{M^2}{1 - \kappa}$$

$$+ \frac{\kappa}{(1 - \kappa^2)(1 - \kappa)} \frac{\omega^2}{x^2}, \quad (3.5)$$

where the index $i$ denotes the parton flavor and $e_i$ is the parton charge in units of the electron charge.
Notice that expression (3.5) allows for the identification of the Cherenkov angle. The obtained Cherenkov angle then includes the classical Huygens term ($\propto \omega^0$) [17–20], the linear quantum correction ($\propto \omega^1$) [21–24], and the quadratic quantum correction ($\propto \omega^2$) which arises due to the fermionic nature of the radiating particle [14]. Direct calculation shows that, even though the index of refraction is nondispersive, the energy of the radiated photon has a cutoff given by

$$\omega_{\text{max}} = x \left( \frac{1 - \kappa}{\kappa} \right) \left[ \sqrt{\frac{1 + \kappa}{1 - \kappa}} \sqrt{E^2 - M^2 - E} \right].$$  \hspace{1cm} (3.6)

Classically, the radiated photon energy is unbounded and dimensional analysis is typically used to introduce a cutoff in order to avoid a divergent radiated power [25]. In contrast, the expression for $\omega_{\text{max}}$ in Eq. (3.6) is a consequence of the quantum-mechanical treatment of the problem [14, 21–24].

The expression of the cutoff energy for the composite proton implies that the Cherenkov emission becomes suppressed for small $x$. This suggests that the quark sea is irrelevant for Cherenkov radiation and that the process is dominated by the valence quarks. Remark also that, even though the emitted photon can have ultrahigh energies, the momentum transfer from the primary proton is suppressed by $\kappa$,

$$Q^2 = -q^2 = \frac{2 \kappa \omega^2}{1 - \kappa} \leq 2 \kappa \omega_{\text{max}}^2 = O(\kappa x^2 E^2).$$  \hspace{1cm} (3.7)

The quantity (3.7) corresponds to the so-called effective mass square, discussed extensively in Secs. 2 and 4 of Ref. [13] for various Lorentz-violating decays.

The total power radiated by the composite proton is now obtained by folding the parton distribution function $f_i(x)$ with the power radiated by the corresponding parton,

$$P(E) = \sum_i \int_0^1 dx \int_0^{\omega_{\text{max}}} d\omega \, f_i(x) \omega \frac{d^2 \hat{\Gamma}^{\text{VCh}}_i}{dx \, d\omega},$$  \hspace{1cm} (3.8)

where (3.5) is to be used for the integrand. Recall that $f_i(x)$ corresponds to the probability density that a charged parton of flavor $i$ carries a fraction $x$ of the longitudinal momentum. The characteristic Cherenkov radiation length is then given by

$$l_{\text{VCh}}(E) \equiv c E / P(E),$$  \hspace{1cm} (3.9)

with the velocity $c$ temporarily restored.

Libraries with global fits from experimental data as well as the evolution to different energy scales of the parton distribution functions (PDFs) of the proton are publicly available [26] and can be used to numerically perform the integrations in (3.8). With the PDFs from the CTEQ collaboration [27], the Cherenkov power radiated by a proton is shown in Fig. 2. For completeness, we also show the power radiated by a neutron, which can also be obtained approximately by the isospin transformation $u \leftrightarrow d$ on the proton PDFs. The “structureless proton” in Fig. 2 corresponds to the limit in which the proton is approximated...
by a pointlike Dirac fermion [12]. The corresponding Cherenkov radiation lengths are shown in Fig. 3.

Figure 2 shows that the energy loss characterized by the radiated power is lower for the proton than the structureless fermion. This reduction can be understood as the combination of several effects. On average, charged partons carry only half of the proton energy, while the other half is carried by gluons. Furthermore, the functional form of the power radiated by the proton is similar to that of the structureless fermion folded with the PDFs of each charged parton and with an overall factor $x^2$, where the momentum fraction $x f_u(x)$ peaks around $x = 0.2$ for relatively low momentum transfer. These effects give a total suppression factor of approximately 10, in agreement with the numerical result shown in Fig. 2. This suppression factor slowly increases for higher energies as the $u$-quark sea spreads the momentum fraction to lower values of $x$.

The detection of a cosmic-ray primary with energy $E_{\text{prim}}$ implies the condition

$$E_{\text{prim}} < E_{\text{th}},$$

with $E_{\text{th}}$ from (3.4) for $M = M_{\text{prim}}$. Namely, if (3.10) would not hold, the primary would have lost most of its energy on the journey through space and the Earth’s atmosphere (with a height of order 100 km). The validity of the condition $E_{\text{prim}} < E_{\text{th}}$ is independent of the astrophysical processes involved in the creation and acceleration of the cosmic-ray primary. In fact, we can focus on the path through the Earth’s atmosphere, where the particle track can be observed directly (for example, by the fluorescence detectors of the Pierre Auger Observatory [28] or by gamma-ray telescopes [29]). The only assumption is that the proton propagation distance $d$ is significantly larger than the characteristic Cherenkov radiation length (3.9). The upper bound (2.9) on the parameter $\kappa$ was obtained by using the detection of a 212-EeV cosmic ray by the Pierre Auger Observatory [30] and by assuming a structureless iron nucleus with $M_{\text{prim}} = 52$ GeV [12]. For a primary structureless proton with mass $M = 0.94$ GeV, the upper bound in (2.9) is reduced by a factor $(0.94/52)^2$ to a value of $2 \times 10^{-23}$.

The reduced power radiated by a realistic proton or nucleus with respect to the structureless fermion indicates that this realistic particle will travel over a somewhat larger distance before efficiently losing energy in the form of vacuum Cherenkov radiation. As discussed above, this extra distance is about one order of magnitude more than the structureless case. Since the characteristic distance for the photon emission by a structureless fermion is only a fraction of a meter, the assumption of travel distances being larger than the decay length ($d \gg l_{\text{VC}}$) is completely justified, even for the realistic proton primary described in this work (for a nucleus $N$, we simply scale the proton results with $e \to Z_N e$ and $M \to M_N$). These remarks imply that the upper bound on the parameter $\kappa$ as given in Ref. [12], based on the threshold condition (3.10), also holds for a realistic proton or nucleus with internal structure taken into account.
IV. PHOTON DECAY

For a negative Lorentz-violating parameter \( \kappa \) in the theory (2.7), the photon becomes unstable and the production of a pair of electrically charged fermions in vacuum is kinematically allowed (Fig. 4). The averaged squared amplitude for photon decay equals the averaged squared amplitude for vacuum Cherenkov radiation (3.1),

\[
|\mathcal{M}_{\text{PhD}}|^2 = \frac{e^2}{2} \sum_{\lambda} \sum_{s, \bar{s}} \left| \tau_s(p) \gamma^\mu v_{s'}(p') \tilde{\varepsilon}^{(\lambda)} \varepsilon_\mu \right|^2 = |\mathcal{M}_{\text{VCh}}|^2 .
\]  

(4.1)

The photon decay rate into a fermion-antifermion pair then takes the form

\[
\hat{\Gamma}_{\text{PhD}} = \frac{1}{4\pi^2} \frac{1}{2\omega} \int \frac{d^3 p}{2E} \frac{d^3 p'}{2E'} |\mathcal{M}_{\text{PhD}}|^2 \delta^4(q - p - p') .
\]  

(4.2)

The energy-momentum condition implies that vacuum pair production occurs for photon energies above the threshold

\[
\omega_{\text{th}} = 2M_f \sqrt{\frac{1 - \kappa}{-2\kappa}} ,
\]  

(4.3)

where \( M_f \) is the equal mass of the fermion and the antifermion (note that CPT and C are still exact symmetries of the theory considered). Above this threshold, the decay of a photon of energy \( \omega \) produces a fermion with energy \( E \) in the range \([E_-, E_+]\), with

\[
E_\pm = \frac{\omega}{2} \left[ 1 \pm n \sqrt{1 - \frac{\omega_{\text{th}}^2}{\omega^2}} \right] ,
\]  

(4.4)

where \( n \) is the index of refraction (2.5). The decay constant is obtained by integrating over the fermion energies,

\[
\hat{\Gamma}_{\text{PhD}}(\omega) = \int_{E_-}^{E_+} dE \frac{d\hat{\Gamma}_{\text{PhD}}}{dE} .
\]  

(4.5)

Specializing to the case of an electron-positron pair (with charges \( e_f = \pm e \) and mass \( M_f = M_e \equiv m \)) and performing the energy integral in (4.5) then gives

\[
\hat{\Gamma}_{\text{PhD}} = \frac{\alpha}{3} \frac{-\kappa}{1 - \kappa^2} \omega \sqrt{1 - \omega_{\text{th}}^2/\omega^2} \left[ 2 + \omega_{\text{th}}^2/\omega^2 \right] ,
\]  

(4.6)

where \( \alpha \equiv e^2/(4\pi) \) is the fine-structure constant. Expression (4.6) is equivalent to Eq. (11) of Ref. [12]. The characteristic decay length determined by the photon lifetime is defined as follows:

\[
\hat{l}_{\text{PhD}}(\omega) \equiv c/\hat{\Gamma}_{\text{PhD}}(\omega) ,
\]  

(4.7)

with the velocity \( c \) temporarily restored.
Similar to the case of vacuum Cherenkov radiation, the photon momentum transfer is suppressed by the parameter $\kappa$, which implies that $Q^2 = q^2$ is significant only for large values of the photon energy. In fact, expression (3.7) gives

$$Q^2 = \left( \frac{2m\omega}{\omega_{\text{th}}} \right)^2,$$

(4.8)

with $\omega_{\text{th}}$ given by (4.3) in terms of the electron mass $m$. Again, the quantity (4.8) plays the role of the effective mass square entering a Lorentz-violating decay [13].

The energy scale $Q^2$ from (4.8) is given by the mass $2m$ of the electron-positron pair for photon energies close to the threshold. At these low momentum transfers, the electron is a point particle to very high precision: compositeness bounds for the electron are at multi–TeV energy scales [31]. In short, the point-particle calculation for photon decay into an electron-positron pair is completely reliable. The results are shown in Figs. 5 and 6.

Following the same reasoning as for the case of vacuum Cherenkov radiation, the observation of an astrophysical photon with energy $\omega_{\gamma,\text{prim}}$ implies the condition

$$\omega_{\gamma,\text{prim}} < \omega_{\text{th}},$$

(4.9)

with $\omega_{\text{th}}$ given by (4.3) in terms of the electron mass $m = 511$ keV. Namely, if (4.9) would not hold, the photon would have decayed along the journey through space and the Earth’s atmosphere. Once again, the validity of this condition is independent of the astrophysical processes involved in the creation of these gamma-ray photons and the only assumption is that the photons have traveled a distance $d$ larger than the characteristic decay length (4.7). The lower bound (2.9) on the parameter $\kappa$ was obtained by using the detection of 30–TeV gamma rays by the High Energy Stereoscopic System (HESS) [32] and by assuming the decay of photons into electron-positron pairs [12]. In this case, the decay length of a 30–TeV photon would be a few millimeters, so that the assumption $d \gg \hat{l}_{\text{PhD}}$ is unquestionably valid.

V. SUMMARY

In this article, we have studied two nonstandard decay process in CPT-even Lorentz-violating quantum electrodynamics. In particular, we have considered the effects of an isotropic nonbirefringent modification leading to vacuum Cherenkov radiation by a proton and photon decay into an electron-positron pair. Previous studies [12], assumed structureless particles for both processes.

For the first process, vacuum Cherenkov radiation by a proton in the theory (2.7) with $\kappa > 0$, we have now also performed a calculation in the framework of the parton model (higher-order QCD corrections have not been considered). The Cherenkov power is found to be reduced by approximately one order of magnitude but the threshold energy remains unchanged compared to the point-particle calculation. For the second process, photon decay into an electron-positron pair in the theory (2.7) with $\kappa < 0$, the point-particle calculation
still holds and we have given somewhat more details than available up till now, in particular the momentum transfer $Q^2$ and the photon decay length. The upshot is that the previous two-sided bound (2.9) from Ref. [12] remains unchanged.

The multi-messenger astrophysics program studying high-energy phenomena with ultra-high-energy cosmic rays [33–35], cosmic gamma rays [36], and cosmic neutrinos [37] has developed the field of astroparticle physics over the last years and now serves as a powerful tool to test fundamental physics symmetries.

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FIG. 1. Tree-level Feynman diagram for vacuum Cherenkov radiation by a proton, \( p \to p + \gamma \). Lorentz-violating effects are contained in the modified polarization vector of the outgoing photon. The proton is a composite object, which can be described by the parton model.

FIG. 2. Cherenkov power \( P \) radiated as a function of the hadron energy \( E \) for a proton, a neutron, and a structureless charged fermion. The Lorentz-violating parameter is \( \kappa = 6 \times 10^{-20} \) and the threshold energy is given by Eq. (3.4).

FIG. 3. Cherenkov radiation length \( l_{\text{VCh}} \) in meters for a proton, a neutron, and a structureless charged fermion with \( \kappa = 6 \times 10^{-20} \) as in Fig. 2.
FIG. 4. Tree-level Feynman diagram for photon decay into an electron-positron pair, $\bar{\gamma} \rightarrow e^- + e^+$. Lorentz-violating effects are contained in the modified polarization vector of the incoming photon.

FIG. 5. Photon decay constant $\hat{\Gamma}$ for $\bar{\gamma} \rightarrow e^- + e^+$ as a function of the photon energy $\omega$. The Lorentz-violating parameter is $\kappa = -9 \times 10^{-16}$ and the threshold energy is given by Eq. (4.3).

FIG. 6. Photon decay length $\hat{l}_{\text{PHD}}$ in meters for $\bar{\gamma} \rightarrow e^- + e^+$ with $\kappa = -9 \times 10^{-16}$ as in Fig. 5.