Supernova Precursor Emission and the Origin of Pre-explosion Stellar Mass Loss

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Abstract

A growing number of core-collapse supernovae (SNe) that show evidence for interaction with dense circumstellar medium (CSM) are accompanied by "precursor" optical emission rising weeks to months prior to the explosion. The precursor luminosities greatly exceed the Eddington limit of the progenitor star, implying that they are accompanied by substantial mass loss. Here, we present a semi-analytic model for SN precursor light curves, which we apply to constrain the properties and mechanisms of the pre-explosion mass loss. We explore two limiting mass-loss scenarios: (1) an "eruption" arising from shock breakout following impulsive energy deposition below the stellar surface; and (2) a steady "wind," due to sustained heating of the progenitor envelope. The eruption model, which resembles a scaled-down version of Type IIP SNe, can explain the luminosities and timescales of well-sampled precursors, for ejecta masses \( \sim 0.1-1 \, M_\odot \) and velocities \( \sim 100-1000 \, \text{km} \, \text{s}^{-1} \). By contrast, the steady wind scenario cannot explain the highest precursor luminosities \( \gtrsim 10^{41} \, \text{erg} \, \text{s}^{-1} \), under the constraint that the total ejecta mass does not exceed the entire progenitor mass (though the less luminous SN 2020tlf precursor can be explained by a mass-loss rate \( \sim 1 \, M_\odot \, \text{yr}^{-1} \)). However, shock interaction between the wind and pre-existing (earlier ejected) CSM may boost its radiative efficiency and mitigate this constraint. In both the eruption and wind scenarios, the precursor ejecta forms compact (\( \lesssim 10^{13} \, \text{cm} \)) optically thick CSM at the time of core collapse; though only directly observable via rapid post-explosion spectroscopy (\( \lesssim \) a few days before being overtaken by the SN ejecta), this material can boost the SN luminosity via shock interaction.

Unified Astronomy Thesaurus concepts: Supernovae (1668); Core-collapse supernovae (304); Stellar mass loss (1613)

1. Introduction

A fraction of massive stars undergo strongly enhanced mass loss near the very ends of their lives, forming a dense circumstellar medium (CSM) around themselves (e.g., Smith 2014). When the stars explode as supernovae (SNe), the CSM manifests through narrow emission lines in the optical spectra generated by the as-yet-unshocked slowly expanding CSM, and photoionized by the SN light (e.g., SNe Type IIn, Ibn, or Icn, depending on whether the spectrum is hydrogen-rich, hydrogen-poor but helium-rich, or both hydrogen- and helium-poor, respectively; Schlegel 1990; Filippenko 1997; Foley et al. 2007; Pastorello et al. 2008; Nyholm et al. 2020; Fraser et al. 2021; Gal-Yam et al. 2022). The related technique of "flash spectroscopy" (e.g., Gal-Yam et al. 2014; Khazov et al. 2016) demonstrates that the progenitor’s mass-loss rate is elevated just before core collapse, among a significant fraction of even nominally CSM-free SNe (e.g., Yaron et al. 2017; Bruch et al. 2021).

The light curves of some of the most luminous SNe are powered at least in part by shock interaction between the SN ejecta and the dense CSM released from the progenitor star in the days to weeks to years prior to its terminal core collapse (e.g., Smith & McCray 2007; Chevalier & Irwin 2011; Ginzburg & Balberg 2012; Svirski et al. 2012; McDowell et al. 2018; Suzuki et al. 2019). At typically larger radii and lower densities, CSM shock interaction can also produce thermal and nonthermal X-ray and radio emission (e.g., Chandra et al. 2012; Margutti et al. 2014; Chakraborti et al. 2016; Dwarkadas et al. 2016; Margutti et al. 2017; Chiba et al. 2020).

If the pre-SN stellar mass loss is modeled as a steady wind, the inferred mass-loss rates in Type IIn SNe, \( \dot{M} \sim 10^{-4}-0.1 \, M_\odot \, \text{yr}^{-1} \) (Fox et al. 2011; Kiewe et al. 2012; Moriya et al. 2014), are significantly larger than can be explained by line-driven winds \( \dot{M} \lesssim 10^{-5} \, M_\odot \, \text{yr}^{-1} \) (Vink et al. 2001; Smith et al. 2014). Instead, several alternative physical processes have been proposed as giving rise to enhanced mass loss just prior to core collapse. Mass loss can occur due to the intense heating of the stellar envelope by the damping of waves excited by vigorous convection in the stellar core (e.g., Quataert & Shiode 2012; Shioide & Quataert 2014; Fuller 2017; Fuller & Ro 2018; Leung & Fuller 2020; Wu & Fuller 2022). In this scenario, the timing of the mass loss should correspond to late stages of nuclear burning, which occurs on timescales of days to weeks prior to explosion for silicon burning and days to several years for oxygen and neon burning (e.g., Woosley et al. 2002). Another mechanism for generating pre-SN mass loss invokes a sudden energy release deep inside the star, due to the instabilities associated with the late stages of nuclear shell burning (e.g., Meakin & Arnett 2007; Smith & Arnett 2014; Fields & Couch 2021; Varma & Muller 2021; Yoshida et al. 2021). Mass loss due to interaction with a close binary companion may also play a role in some events (e.g., Chevalier 2012; Meley & Soker 2014; Sun et al. 2020). In very massive rotating stars that experience efficient mixing, centrifugally induced mass loss may accompany contraction during the star’s final burning stages (e.g., Aguiler-Dena et al. 2018).

“Precursor” outbursts prior to the main SN explosion (e.g., Ofek et al. 2014) offer a clue to distinguishing these various mass-loss mechanisms. Several precursor events accompanying interacting SNe have been observed in recent years, typically weeks to
months prior to the main explosion, the most well studied of which accompanied SN 2009ip (Fraser et al. 2013; Mauerhan et al. 2013; Pastorello et al. 2013; Prieto et al. 2013; Graham et al. 2014; Levesque et al. 2014; Margutti et al. 2014; Mauerhan et al. 2014; Martin et al. 2015; Graham et al. 2017; Reilly et al. 2017; Smith et al. 2022), 2010mc (Ofek et al. 2013), 2015bh (Elias-Rosa et al. 2016; Ofek et al. 2016; Thöne et al. 2017; Jencon et al. 2022), 2016bhu (Pastorello et al. 2018), LSQ13zm (Tartaglia et al. 2016), and 2020tlf (Jacobson-Galán et al. 2022), as well as there being implications of pre-SN activity in SN 2010bt (Ofek et al. 2013; Pastorello et al. 2019, 2019zrk, Fransson et al. 2022), 2013gc (Reguitti et al. 2019), 2018cnf (Pastorello et al. 2019), 2019zk (Fransson et al. 2022), 2021foa (Reguitti et al. 2022), and 19 events compiled in Strotjohann et al. (2021). Efforts to monitor a large number of massive stars in nearby galaxies, to determine which explode as SNe (Kochanek et al. 2008), and systematic analyses of interacting SNe (Ofek et al. 2014; Bilinski et al. 2015; Strotjohann et al. 2021) rule out luminous precursors accompanying all SNe (Kochanek et al. 2017; Johnson et al. 2018). However, Strotjohann et al. (2021) found that ∼ 25% of Type II SNe (themselves accounting for ∼ 10% of core-collapse SNe; e.g., Perley et al. 2020) exhibit precursors brighter than −13 mag (or luminosity > 5 × 10^40 erg s^{-1}) three months before the SN explosion.

Figure 1 and Table 1 summarize the bolometric light curves and other observable properties of a set of well-sampled SN precursors. Most of these SNe are classified as Type II, so we focus in this work on precursors from hydrogen-rich stars (though precursor outbursts have also been observed in some hydrogen-poor SNe; e.g., Foley et al. 2007; Pastorello et al. 2008). The luminosities of the observed precursors ~ 10^{40} – 10^{41} erg s^{-1} are typically well above the Eddington limit for typical SN progenitor stars of mass ≃ 10 – 20 M_☉. indicating that this emission phase is accompanied by substantial mass loss, which should then contribute to the CSM interaction with the SN light or ejecta in the post-explosion phase.

In this paper, we present a model for the light curves of SN precursors from hydrogen-rich stars, which we apply to the observed precursor sample in order to constrain the pre-SN mass-loss phase. With the exception of the radiation hydrodynamical simulations by Dessart et al. (2010), few previous theoretical works have attempted to model the precursor emission phase. Instead, most constraints on the mass and radial distribution of the CSM around SN progenitors have come from the post-explosion interaction (e.g., Smith & McCray 2007; Chevalier & Irwin 2011; Chatzopoulos et al. 2013; McDowell et al. 2018; Suzuki et al. 2019). Though our light-curve model makes several simplifying assumptions, these same features make it flexible and applicable to a wide parameter space of potential pre-explosion mass-loss behavior. As we shall discuss, we find that the bulk of the CSM generated during the observed precursor outbursts will usually be located so close to the progenitor star at the time of the explosion as to be quickly overtaken by the SN ejecta, making this material challenging to directly observe by other means.

This paper is organized as follows. In Section 2, we introduce two idealized models for the nature of the pre-SN mass-loss phase (dynamical “eruption” versus steady “wind”) and describe our model for calculating the precursor light curve from each case. In Section 3, we present the results of our light-curve calculations and interpret them analytically. We also assess the ability of the eruption and wind models to explain the observed SN precursor events and determine the ejecta parameters required in each case. We discuss the resulting CSM from precursor events in Section 4, and finally summarize our results and conclude in Section 5.

2. Precursor Emission Model

We calculate SN precursor light curves largely following the semi-analytical model developed in Matsumoto & Metzger (2022) and Metzger et al. (2021). The general approach is as follows. The pre-SN mass loss is divided into multiple ejecta shells, ordered based on their velocity, and for each shell we calculate its thermal evolution and resulting emission by means of a one-zone model. The total light curve is then obtained by summing the luminosity contributions from each shell. For simplicity, we assume the ejecta to be spherically symmetric, even though in some mass-loss scenarios it could possess a nonspherical (e.g., equatorial disk-like) geometry.

A key input into the model is the velocity distribution of the pre-SN ejecta. We explore two physically motivated scenarios for its form, which bracket the extremes along a continuum of possible behaviors.

1. An “eruption,” in which mass loss occurs at a single event on a timescale comparable to or less than the dynamical time at the ejection radius. Physically, a temporally concentrated injection of energy deep inside the star generates a shock wave, which propagates radially outward and accelerates the stellar envelope,

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3 Some massive stars also generate bright eruptions—so-called “giant eruptions” in luminous blue variable phase or “SN imposters” (e.g., Pastorello et al. 2010; Pastorello & Fraser 2019)—which are potentially related to instabilities in massive star envelopes (e.g., Humphreys & Davidson 1994; Owocki 2015; Jiang et al. 2018) and accompanied by mass loss (e.g., Gal-Yam et al. 2007; Gal-Yam & Leonard 2009), but typically are not coincident with the terminal core-collapse event. Here, we focus on SN precursors that precede the terminal explosion by at most a few years, and thus are likely to be causally connected to the late stages of nuclear burning.

4 The progenitors of SN 2009ip and 2015bh may be more massive > 5–50 M_☉ (Smith et al. 2010; Foley et al. 2011; Boian & Groh 2018).
unbinding a portion of its mass (Dessart et al. 2010; Kuriyama & Shigeyama 2020; Ko et al. 2022; Linial et al. 2021). Although we remain indifferent to the origin of the sudden energy injection, one possible physical realization would be a dynamical instability associated with unstable nuclear shell burning.

2. A continuous “wind,” in which mass loss occurs from the star at a roughly constant rate, over timescales much longer than the dynamical time. Physically, this is expected to occur when the stellar envelope is heated well above the Eddington luminosity at a roughly constant rate (e.g., Smith & Owocki 2006; Quataert et al. 2016). This scenario may be approximately realized in wave heating scenarios (e.g., Quataert & Shiode 2012; Fuller 2017).

The following subsections describe the eruption and wind scenarios individually. Throughout this work, we assume that the precursor emission arises directly from a single mass-loss event or mass-loss phase prior to the terminal explosion; however, we caution that some events may arise from more complicated circumstances (e.g., multiple episodes of mass loss giving rise to collisions between ejecta shells, dim SNe followed by bright CSM interaction, or binary star mergers), in particular for SN 2009ip (e.g., Pastorello et al. 2013; Soker & Kashi 2013; Smith 2014).

2.1. Eruption Scenario

In the eruption scenario, energy is suddenly injected into the stellar envelope, driving a shock wave toward its surface (e.g., Dessart et al. 2010). As a result, we assume that the ejecta achieve a homologous density profile \( \rho(\vec{r}) = \rho(r/R_{0}) \), defined such that \( M_{\text{ej}}(\vec{r}) = 4\pi \int_{0}^{\infty} \rho(r) r^2 dr \) is the ejecta mass above a given velocity \( \vec{v} \).

Depending on the magnitude of the injected energy, the ejecta can be accelerated to an arbitrarily high velocity, as long as the latter exceeds the escape speed at the radius \( R_{0} \) where matter is ejected (e.g., Linial et al. 2021):

\[
\vec{v}_{\text{esc}} = \sqrt{\frac{2GM_{*}}{R_0}}
\]

Table 1: Observed Properties of SN Precursor Emission

| Event       | \( L_{\text{pre}} \) (erg/s) | \( t_{\text{pre}} \) (d) | \( E_{\text{pre}} \) (erg) | \( V_{\text{obs}} \) (10^3 km s^{-1}) | Reference |
|-------------|-------------------------------|--------------------------|-----------------------------|----------------------------------------|-----------|
| SN 2009ip   | \( 3.6 \times 10^{11} \)     | 44\(^a\)                 | \( 1.4 \times 10^{57} \) \(^a\) | 0.8–1.4, 8–9, 14–15\(^a\)             | 1,2,3     |
| SN 2010mc   | \( 1.6 \times 10^{11} \)     | 31                       | \( 4.4 \times 10^{47} \)    | 1–3 (6 days\(^f\))                    | 4         |
| SN 2015bh   | \( 9.3 \times 10^{10} \)     | 95\(^d\)                 | \( 7.6 \times 10^{47} \) \(^d\) | 0.6–1, 0.9–1.5, 2.6–6\(^a\)          | 5         |
| SN 2016bdu  | \( 1.2 \times 10^{11} \)     | 97\(^d\)                 | \( 1.0 \times 10^{46} \) \(^d\) | 0.4 (≈10 days\(^f\))                 | 6         |
| SN 2020llf  | \( 8.8 \times 10^{9} \)      | 127                      | \( 9.7 \times 10^{46} \)    | 0.05–0.2 (10 days\(^f\))              | 7         |

Notes.
\(^a\) Average luminosity of precursor emission, defined as \( L_{\text{pre}} = E_{\text{pre}}/t_{\text{pre}} \).
\(^b\) Duration from the first precursor detection to the time of the SN explosion.
\(^c\) Total radiated energy of the precursor emission.
\(^d\) Lower limit due to the lack of detection or flux upper limit.
\(^e\) Obtained via spectroscopy during the precursor emission itself. The \( H\alpha \) line profile is fitted by multiple velocity components.
\(^f\) Obtained via flash spectroscopy \( t_{\text{搁}} \) (shown in parentheses) after the SN explosion. Note that these velocities are sometimes too low to be associated with the mass ejection responsible for the precursor emission, because the latter would have been overtaken by the SN ejecta by the time of the flash spectroscopy \( t_{\text{搁}} \).

References: 1: Maehara et al. (2013); 2: Pastorello et al. (2013); 3: Margutti et al. (2014); 4: Ofek et al. (2013); 5: Elias-Rosa et al. (2016); 6: Pastorello et al. (2018); 7: Jacobson-Galán et al. (2022).

\[ \approx 200 \text{ km s}^{-1} \left( \frac{M_{*}}{10M_{\odot}} \right)^{1/2} \left( \frac{R_{0}}{10^2R_{\odot}} \right)^{-1/2}, \]

where \( G \) is the gravitational constant and \( M_{*} \) is the progenitor mass. Spectroscopic observations of SN precursors, in the few cases available, indicate high ejecta speeds \( \approx 10^{2}–10^{3} \text{ km s}^{-1} \) (Table 1). This suggests that the deposited energy is comparable to the stellar binding energy, if released at radii \( R_{0} \sim 1–100 R_{\odot} \) below the surface \( R_{\odot} \sim 10^{3} R_{\odot} \) of a red supergiant (RSG) progenitor (or with energy greatly exceeding the local binding energy, if deposited closer to the surface).

We now describe our method for calculating the emission from a single shell of mass \( M \) and velocity \( v \) ejected at \( t = 0 \) from radius \( R_{0} \). The radial distance of the shell from the progenitor’s center is given by \( v \cdot t + R_{0} \) and its volume is given by \( V = 4\pi R^2 \Delta R \), where \( \Delta R \) is the width of the shell. For a homologously expanding ejecta, the width is given by \( \Delta R = dv \cdot t + \Delta R_{0} \), where \( dv \) is the velocity difference from the next shell and \( \Delta R_{0} \) is the initial width of the shell. The initial internal energy of the shell is assumed to equal its kinetic energy \( E_{0} = Mv^2/2 \). The internal energy is comprised of ideal gas, radiation, and ionization energy:

\[ E = \frac{3}{2} (1 + \bar{x}) N k_{B} T + a T^{4} V + \sum_{i} N_{A_{i}} x_{i} \varepsilon_{i}, \]

where \( \bar{x} = \sum_{i} A_{i} x_{i}, A_{i}, x_{i}, \varepsilon_{i} \) are the mean ionization degree, abundance fraction, degree of ionization, and ionization energy of species \( i \), respectively. We calculate \( \bar{x} \) and \( x_{i} \) by solving the Saha equation, taking into account singly ionized hydrogen and helium. The other quantities are the total number of nuclei \( N \), the Boltzmann constant \( k_{B} \), temperature \( T \), and radiation constant \( a \).

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\(^5\) In detail, for this initial condition, the shell will accelerate after the ejection by \( PdV \) within several dynamical times \( \approx R_{0}/v \); however, since the increase of the velocity is by at most a factor of \( \sqrt{2} \approx 1.4 \), we neglect this acceleration phase and assume that the shells expand at constant velocity from \( t = 0 \).
The thermal evolution of the ejecta is described by the first law of thermodynamics:
\[ \frac{dE}{dt} = -(\gamma_3 - 1) \frac{E}{V} \frac{dV}{dt} - L, \]  
(4)
where \( \gamma_3 \) and \( L \) are the adiabatic index and radiated luminosity, respectively. In the adiabatic loss term, we have used the fact that pressure is given by \( P = (\gamma_3 - 1)E/V. \) The adiabatic index is calculated from the density, temperature, and ionization state of the shell, as in Matsumoto & Metzger (2022) and Kasen & Ramirez-Ruiz (2010). The radiative loss term is approximated by the photon diffusion luminosity,
\[ L = \frac{E_{\text{rad}}}{S(t) t_d + t_c}, \]  
(5)
where \( E_{\text{rad}} = aT^4V \) is the radiation’s internal energy, \( t_d \) is the photon diffusion time, and \( S(t) \) is a suppression factor discussed below. The diffusion time is that over which photons escape radially through the ejecta,
\[ t_d = \frac{R \tau}{c} \quad \text{and} \quad \tau = \int_R^\infty K \rho dr, \]  
(6)
where \( c \) is the speed of light and \( K(\rho, T) \) is the Rossland mean opacity, which we approximate using the analytic expression provided in Matsumoto & Metzger (2022) for material of solar metallicity composition. The dominant form of opacity near the peak light is electron scattering (under conditions of partial or full ionization), though Kramer’s opacity can become relevant at high densities and low temperatures. The suppression factor,
\[ S(t) \equiv \frac{e^{t_d/t} - 1}{t_d/t}, \]  
(7)
acts to reduce the radiative losses exponentially for \( t \ll t_d \). Without this correction, the luminosities of all radiation-pressure–dominated shells reach a value \( L \propto R_0 \rho/\kappa \), starting immediately after their ejection, and contribute almost equally (up to a factor of \( v \)) regardless of their optical depth; this behavior is unphysical, however, because at \( t \lesssim t_d \), most photons are still trapped within ejecta and only the tiny fraction in the “diffusion tail” can escape (see also Piro & Nakar 2013). The light-crossing time \( t_c = R/c \) in the denominator of Equation (5) limits the photon escape timescale from the system at late times.

The mass profile of the ejecta depends on the details of the ejection process. The process of shock breakout motivates a power-law profile (Nakar & Sari 2010),
\[ M_{\text{ej}}(>v) = M_{\text{ej}}(v/v_\text{ej})^{-\beta}, \quad v > v_\text{ej}, \]  
(8)
where \( M_{\text{ej}} \) is the total ejecta mass and \( v_\text{ej} \) is the minimum velocity. Assuming that the stellar envelope can be described as a polytrope of index \( n \), and that the energy injection occurs close to the stellar surface, then we can set the initial radius \( R_0 \sim R_* \) for all shells, and the process of shock breakout will impart a self-similar velocity profile of the form \( v \propto \rho^{-\mu} \) (Gandel’man & Frank-Kamenetskii 1956; Sakurai 1960), where \( \mu \simeq 0.22 \) (for \( n = 3/2 \)) and \( \mu = 0.19 \) (for \( n = 3 \)). Since the density profile and external mass in this scenario obey \( \rho \propto x^n \) and \( M(>r) \propto \rho x R_*^2 \), where \( x = R_*/r \) is the depth measured from the surface, the power-law index entering Equation (8) in this scenario becomes \( \beta = (n + 1)/\mu \simeq 7.6 \) for \( n = 3/2 \) and 7.0 for \( n = 3 \). The shock breakout solution also gives a relation between the velocity and initial depth of each shell \( v = v_\text{ej}(x/R_*)^{-\mu/\beta} \), where the depth of the innermost slowest shell \( v = v_\text{ej} \) obeys \( x_\text{ej}/R_* \sim (M_{\text{ej}}/M_*)^{-1/\beta} \). The initial width of each shell is likewise given by
\[ \Delta R_0(v) = \frac{dx}{dv} = \frac{x_\text{ej}}{\mu v_\text{ej}^2} \left( \frac{v}{v_\text{ej}} \right)^{-\mu/\beta} dv. \]  
(9)
Note that the above prescription holds only for energy injected into layers of the star close to the surface \( (R_0 \sim R_*) \), and \( M_{\text{ej}} \ll M_* \). Nevertheless, in what follows, we shall also apply \( R_0 = R_* \) to the eruptions with greater ejecta masses, seeded by energy deposition at a deeper layer \( \ll R_* \). Realistically, after the shock passage, each mass shell is quickly imparted comparable internal and kinetic energies, and hence the initial radius \( R_0 \) should be smaller than \( R_* \), and different for each shell. While this complicated hydrodynamics should be studied by numerical approaches with more specific stellar models to set more accurate initial conditions, we assume for simplicity that all shells are ejected at \( t = 0 \) from \( R_* \) with equal internal and kinetic energies, as adopted for the analytical light-curve modeling of SNe (Arnett 1980, 1982; Popov 1993). Physically, heating near the surface will occur as the ejecta from deeper layers collides with and shocks material closer to the surface.

In summary, the eruption scenario is described by four main parameters: total ejecta mass \( M_{\text{ej}} \), initial radius \( R_0 \) (which we canonically set to \( R_0 = R_* \), motivated by the shock breakout prescription), minimum ejecta speed \( v_\text{ej} \) (which must exceed \( v_\text{esc} \); Equation (2)), and the time of the eruption prior to the SN explosion, \( t_{\text{erupt}} \). The power-law index of the mass profile \( \beta \) is also a free parameter, though we shall take \( \beta = 7.6 \) as fiducial (corresponding to RSG progenitors), motivated by the above discussion. Although the self-similar shock breakout solution described above only holds near the stellar surface, we find that the light-curve properties are not sensitive to the precise value of \( \beta \) for otherwise fixed values of \( M_{\text{ej}} \) and \( v_\text{ej} \) (see Matsumoto & Metzger 2022; their Figure 6). We assume the initial shell widths given by Equation (9); however, the calculated light-curve properties are also not sensitive to this choice (assuming that a constant width for all shells, \( \Delta R_0 = \text{const} \), gives a similar result). Relative to Matsumoto & Metzger (2022), the updates to our current model include: (1) the diffusion time calculated as a full radial integral (Equation (6)), instead of using a local estimate; and (2) the inclusion of the early-time suppression factor (Equation (7)).

For the assumed mass profile, the diffusion time is
\[ t_d \simeq \frac{\beta}{\beta + 2} \frac{\kappa M(>v)}{4 \pi c R}, \]  
\[ \simeq \frac{240 \text{ days}}{\beta = 7.6} \left( \frac{M(>v)}{M_*} \right)^{1/\beta} \left( \frac{R}{10R_*} \right)^{-1}, \]  
(10)
where the second equality assumes a constant electron scattering opacity \( \kappa = \kappa_{\text{esc}} \simeq 0.32 \text{ cm}^2 \text{ g}^{-1} \) (hereafter adopted for other analytic estimates, unless otherwise specified). The
The diffusion timescale is calculated by Equation 6, its estimate is given by
\[ t_d \simeq \frac{\nu M}{4\pi \rho v_w^2} \simeq 32 \text{ days} \left( \frac{M}{M_\odot} \text{ yr}^{-1} \right) \times \left( \frac{v_w}{v_{esc}} \right)^2 \left( \frac{M}{10M_\odot} \right)^{-1/2} \left( \frac{R_s}{10^2 R_\odot} \right)^{1/2}, \]  
(14)

which follows by taking the density as \( \rho = \frac{M}{4\pi r^3} \) (for a wind density profile \( \rho \propto r^{-2} \), the diffusion time receives roughly equal contributions from all decades in radius; e.g., Chevalier & Irwin 2011).

In summary, the wind scenario is described by three parameters: wind mass-loss rate \( \dot{M} \), wind velocity \( v_w \) (equivalently, sonic point \( R_s \) for a given progenitor mass \( M_\odot \)), and the duration of the wind prior to the SN explosion \( t_w \). Rather than define the latter as the entire duration of the mass loss prior to the explosion, \( t_{active} \), we define \( t_w \) as the more limited duration over which the luminosity has reached its roughly constant value, i.e., \( t_w = t_{active} - t_{rise} \), where \( t_{rise} \) is the light-curve rise time (see below, Section 3.2).

What range of wind properties is expected physically? Quataert et al. (2016) show that an energy injection at some radius \( R_{in} \) below the stellar surface at a super-Eddington rate \( \dot{E} \gg L_{Edd} \) drives a continuum radiation pressure–driven outflow. For sufficiently high values of \( \dot{E} \gg \dot{E}_Q \), most of the injected power is eventually converted into the kinetic energy of the wind, i.e., \( \dot{E}_w \simeq \dot{E} \), where the threshold power is given by
\[ \dot{E}_Q \equiv f^{5/2} \frac{M_{env} v_{esc}^5}{M_* \ G} \simeq 2 \times 10^{40} \text{ erg s}^{-1} \]
\[ \times \left( \frac{f}{0.3} \right)^{5/2} \left( \frac{M_{env}}{10^{-3} M_*} \right) \left( \frac{M_*}{10 M_\odot} \right)^{5/2} \left( \frac{R_{in}}{10^2 R_\odot} \right)^{-5/2}. \]
(15)

Here, \( M_{env} \simeq 10^{-3} - 10^{-2} M_* \) is the mass of the stellar envelope above the energy injection point up to the sonic radius and \( f \sim 0.1-1 \) is a dimensionless parameter that depends on the envelope structure (Quataert et al. 2016; their Figure 11). Thus, we typically have \( \dot{E}_Q \simeq 10^{39} - 10^{41} \text{ erg s}^{-1} \sim 1 - 100 L_{Edd} \) where the Eddington luminosity
\[ L_{Edd} = \frac{4\pi GM_*c}{\kappa_{es}} \simeq 1.6 \times 10^{39} \left( \frac{M_*}{10 M_\odot} \right) \text{ erg s}^{-1}. \]
(16)

In the limit \( \dot{E} \gg \dot{E}_Q \), Quataert et al. (2016) show that the terminal velocity and mass-loss rate of the wind are related according to (their Equations (20) and (26)):
\[ v_w \simeq 1.7 \left( \frac{M_*}{M_{env}} \right)^{1/5} \left( \frac{G \dot{E}}{10^{-3} M_*} \right)^{1/5} \left( \frac{\dot{E}}{100L_{Edd}} \right)^{1/5}, \]
(17)
\[ \dot{M} \simeq 0.15 \left( \frac{v_w}{G M_*} \right)^3 \left( \frac{M_{env}}{10^{-3} M_*} \right) \left( \frac{v_w}{300 \text{ km s}^{-1}} \right)^3. \]
(18)

The lower range of the ejecta velocities inferred from SN precursor observations, \( v_w \sim 100 - 500 \text{ km s}^{-1} \) (Table 1), can be obtained for \( \dot{E} \sim 1-100 L_{Edd} \gg \dot{E}_Q \) and \( M_{env} \sim 10^{-4} - 10^{-2} M_* \), corresponding to a range of \( \dot{M} \sim 1-100 M_\odot \text{ yr}^{-1} \). On the other hand, the higher end of the observed precursor velocity \( \gtrsim 10^3 \text{ km s}^{-1} \) would require both a small envelope mass \( M_{env} \lesssim 10^{-5} M_* \) and a large injection luminosity \( \dot{E} \gtrsim 10^5 L_{Edd} \).

Although \( v_w \) represents the final velocity of the wind, Quataert et al. (2016) found that the initial expansion rate of the overlying envelope material, after the onset of heating, is significantly slower than \( v_w \). This rising velocity inevitably leads to shock interaction between the wind and the overlying envelope material. Additional heating from such shocks may boost the transient luminosity compared to the constant-velocity wind modeled here (see Section 3.2.1).
The luminosity is calculated by excluding radiation of frequency $\nu > 10^{15}$ Hz from the bolometric light curve (the gray curve). The colored solid lines show the luminosity contributed by single shells with velocities $v = 1000$ to $1750$ km s$^{-1}$ (separated by $250$ km s$^{-1}$) and a shell width of $dv = 2$ km s$^{-1}$.

The dashed curves show an otherwise identical calculation, calculated following the original model of Matsumoto & Metzger (2022), which neglects the early-time suppression factor (Equation (7)) and uses a local estimate for the radial optical depth.

3. Results

We now summarize the light-curve results, for the eruption and wind scenarios separately.

3.1. Eruption Scenario

Figure 2 shows example light curves in the eruption scenario, calculated for: $M_{ej} = 1 M_{\odot}$, $M_*= 10 M_{\odot}$, $R_0 = R_e = 10^3 R_\odot$, and $v_{ej} = 1000$ km s$^{-1}$, for an assumed stellar mass $M_*= 10 M_{\odot}$. The optical luminosity is calculated by excluding radiation of frequency $\nu > 10^{15}$ Hz from the bolometric light curve (the gray curve). The colored solid lines show the luminosity contributed by single shells with velocities $v = 1000$ to $1750$ km s$^{-1}$ (separated by $250$ km s$^{-1}$) and a shell width of $dv = 2$ km s$^{-1}$.

The same as Figure 2, but showing light curves in the eruption scenario for different ejecta parameters, and compared with observed SN precursors (Figure 1). The blue, black, and red curves show models with a common ejecta mass $M_{ej} = M_*$, but different ejecta velocities $v_{ej} = 200, 500$, and $1000$ km s$^{-1}$, respectively. The yellow curve shows a model with $v_{ej} = 200$ km s$^{-1}$, but larger $M_{*} = 3 M_{\odot}$. The lighter shaded curves denote the bolometric luminosity. The stellar mass $M_*= 10 M_{\odot}$ and radius $R_0 = 10^3 R_\odot$ are fixed for all models. The time of the eruption prior to the SN is set to $t_{erupt} = 140$ days (except for the high-velocity models shown with the blue and red curves, for which $t_{erupt} = 100$ and 50 days), to roughly match the onset of the observed precursor emission. The dependence of the predicted plateau luminosity and the duration of the ejecta properties broadly agree with the Popov analytic scalings (Equations (19) and (20)).

Below the eruption, the effective temperature is high and the peak of the assumed blackbody spectrum is in the UV range. We approximate the optical wavelength light curve (the black curve) by subtracting the UV luminosity (defined as radiation with frequency $\nu > 10^{15}$ Hz) from the bolometric luminosity (the gray curve). The colored solid lines denote the single-shell light curves for different velocities ($v = 1000$–1750 km s$^{-1}$, corresponding to the colors from purple to red).

As a result of the suppression factor (Equation (7)), low-velocity shells do not contribute significantly to the total luminosity at early times, until their diffusion times become shorter than the expansion times.

In order to quantify the impacts of the new features of the light-curve model introduced in this paper, a dashed line shows the calculation performed with the original model of Matsumoto & Metzger (2022), which does not include the early-time suppression effect (Equation (7)) and used a local estimate for the radial optical depth rather than a full integral taken over the external shells. The newly calculated light curve exhibits a shallower decay, but its overall luminosity and duration do not change appreciably from the Matsumoto & Metzger (2022) model.

Due to the close similarity between the physical conditions in the precursor ejecta and those of Type IIP SNe, the characteristic luminosity and duration of the light curve can be estimated using the analytic expressions from Popov (1993):

$$L_{Popov} = 2.0 \times 10^{44} \text{ erg s}^{-1} \left( \frac{M_{ej}}{M_\odot} \right)^{1/3} \times \left( \frac{R_0}{10^3 R_\odot} \right)^{2/3} \left( \frac{v_{ej}}{10^3 \text{ km s}^{-1}} \right)^{5/3},$$

(19)

$$t_{Popov} = 74 \text{ days} \left( \frac{M_{ej}}{M_\odot} \right)^{1/3} \left( \frac{R_0}{10^3 R_\odot} \right)^{1/6} \left( \frac{v_{ej}}{10^3 \text{ km s}^{-1}} \right)^{-1/3},$$

(20)

where we have approximated the mean ejecta velocity as $v_{ej}$ and adopted a normalization calibrated from the radiative transfer
41 simulations of Sukhbold et al. (2016; see also Blagorodnova et al. 2021).

Figure 3 shows light curves in the eruption scenario, calculated for different variations of the ejecta parameters relative to the fiducial case in Figure 2. The resulting changes in the plateau emission properties roughly agree with the analytic scaling relations in Equations (19) and (20). For example, increasing the ejecta mass increases the duration and luminosity of the plateau, while increasing the minimum velocity increases the plateau luminosity, but shortens its duration.

For comparison, in Figure 3, we show the light curves of observed SN precursors (Figure 1). In order to roughly match the observations, we set the time of the eruption prior to core collapse to \( t_{\text{erupt}} = 140, 100, \) and 50 days for \( v_{\text{ej}} = 200 \text{ km s}^{-1} \) (black and yellow), 500 km s\(^{-1}\) (blue), and 1000 km s\(^{-1}\) (red), respectively. Before exploring the detailed model parameters required to fit the observations, we first note that—broadly speaking—the luminosities predicted in the outburst model, \( L_{\text{pre}} \sim 10^{41} \text{ erg s}^{-1} \), match those of observed precursors. In detail, our outburst model predicts light curves that decay monotonically following the initial peak, in conflict with the observed precursor emission (in particular, SN 2015bb and 2016bdu), which instead rises for a few months leading up to the SN. However, this discrepancy may at least in part arise from some of the simplifying assumptions of our model, such as the velocity profile and initial shell radii; indeed, the radiation hydrodynamic simulations of precursor outbursts by Dessart et al. (2010) predict rising light curves.\(^6\)

Figure 4 shows an estimate of the plateau luminosity \( L_{\text{pl}} \) and duration \( \tau_{\text{pl}} \), calculated from a large grid of models in the space of ejecta mass \( M_{\text{ej}} \) and ejecta velocity \( v_{\text{ej}} \), for different assumptions about the initial (and progenitor) radii \( R_0 = R_\ast = 10, 10^2, \) and \( 10^3 \text{ R}_\odot \). Here, we define the duration \( \tau_{\text{pl}} \) as that over which 90\% of the total energy is radiated, \( 0.9E_{\text{pl}} = \int L_{\text{pl}} \text{d}t \), where \( E_{\text{pl}} = \int L_{\text{pl}} \text{d}t \). The average luminosity is then defined as \( L_{\text{pl}} = E_{\text{pl}}/\tau_{\text{pl}} \). Note that the model-predicted plateau duration \( \tau_{\text{pl}} \) only represents an upper limit on the observed precursor duration if the SN explosion occurs before the end of the eruption emission phase. However, due to the relatively flat shape of the predicted light curve, the model-predicted value of \( L_{\text{pl}} \) will still be comparable to the time-averaged observed luminosity \( L_{\text{pl}} \), even when the eruption emission is prematurely terminated by the SN.

The distributions of plateau luminosity and duration roughly follow those predicted by Equations (19) and (20), namely \( M_{\text{ej}} \propto v_{\text{ej}}^{-3} \) and \( M_{\text{ej}} \propto v_{\text{ej}}^{-2} \), respectively. For lower ejecta velocities, comparable to the RSG surface escape speed \( \sim 10–100 \text{ km s}^{-1} \), gas pressure instead of radiation pressure dominates in the ejecta, and other assumptions of the Popov analytic estimates (e.g., the neglect of hydrogen recombination energy) are violated (Matsumoto & Metzger 2022). On the other hand, for ejecta masses \( \sim M_\odot \), velocities \( \sim 10^3 \text{ km s}^{-1} \), and radii close to the surface of the RSG progenitors \((R_0 \sim R_\ast \sim 10^3 \text{ R}_\odot)\), the predicted transient properties are consistent with the observed precursors. The high required ejecta velocity, exceeding the surface escape speed by factors \( \gtrsim 10 \), may point to energy deposition deep within the star \( \sim 10 \text{ R}_\odot \). To reproduce the observed precursors, with \( \sim 10^{41} \text{ erg s}^{-1} \) and \( \sim 10^3 \text{ km s}^{-1} \), from more compact progenitor stars, such as blue supergiants \((R_\ast \sim 10 \text{ R}_\odot)\); see the bottom panel in Figure 4), would require more massive ejecta \( \gtrsim 3 M_\odot \).

3.2. Wind Scenario

Figure 5 shows examples of the light-curve and effective temperature evolution in the wind mass-loss scenario, for a fixed progenitor mass \( M_\ast = 10 M_\odot \). A thick black curve depicts the chosen fiducial model, with wind mass-loss rate \( \dot{M} = M_\ast \text{ yr}^{-1} \) and speed \( v_{\text{w}} = 200 \text{ km s}^{-1} \) (corresponding to \( R_\ast = 100 \text{ R}_\odot \); Equation (2)). This mass-loss rate is broadly motivated by observed Type IIn SNe, though it is somewhat higher than the values typically inferred (albeit usually on larger radial scales; e.g., Smith et al. 2014). As described in Section 2.2, the total light curve is obtained by adding those from successive single shells (an example of which is shown by the thin black curve in Figure 5). Since the total luminosity approaches a steady state on a timescale roughly set by the characteristic duration of the single-shell emission, we denote the latter \( \tau_{\text{rise}} \) and calculate its value in the same way as the eruption model at the stationary state (i.e., it is defined by the timescale over which 90\% of the energy is radiated). The rise time is roughly equal to that over which photons diffuse out from the shell, \( \tau_{\text{r}} \) (Equation (14)).

The single-shell light curve exhibits a gradual rise, followed by a sharp peak, powered by the escape of energy that accompanies a rapid decrease in the photon diffusion timescale at hydrogen recombination. For the fiducial model, radiation pressure dominates (marginally) over gas pressure during the recombination phase, rendering the contribution of recombination energy to the radiated energy small. The luminosity reached during the steady plateau phase is roughly given by the product of the single shell’s characteristic luminosity, calculated in the same way as in the eruption model (dividing the radiated energy by \( \tau_{\text{rise}} \)), by the number of shells ejected during the rise phase \( \tau_{\text{rise}} \sim \tau_{\text{rise}}/\Delta t \). We remark that the rise time \( \tau_{\text{rise}} \) is independent of the choice of shell thickness \( v_{\Delta t} \), because it does not enter into Equation (14).

The colored curves in Figure 5 show additional light-curve calculations varying the wind mass-loss rates and velocities from the fiducial model. Because the rise time is roughly set by the diffusion timescale \( \tau_{\text{r}} \propto \dot{M}/v_{\text{w}} \) (Equation (14)), the steady-state luminosity is more quickly achieved for higher wind velocities. Similarly, winds with higher mass-loss rates take longer to reach a steady state.

As an aside, we remark that for winds with low velocities and/or high \( \dot{M} \), our calculation overestimates the photosphere radius and hence underestimates (by a factor of \( \lesssim 2 \)) the effective temperature of the emission. As a result of the shallow wind density profile \( \rho \propto r^{-2} \), the material ahead of the recombination front (which quickly cools to \( \lesssim 10^3 \text{ K} \)) can contribute significantly to the optical depth through molecular absorption. However, at such low temperatures, our analytic opacity \( \kappa_{\text{law}} \) (which adopts a constant value \( \kappa_{\text{mol}} = 0.1 \text{ Z} \times 10^{-3} \text{ cm}^2 \text{ g}^{-1} \) overestimates the true molecular opacity. Unless significant dust formation occurs, to boost the opacity further (Freedman et al. 2008; Pejcha et al. 2016), the true photosphere location should therefore coincide with the recombination front, at somewhat smaller radii than our model predicts.

The top panels of Figure 6 show the rise time \( \tau_{\text{rise}} \) (upper right panel) and steady-state luminosity \( L_{\text{pl}} \) (upper left panel) as
a function of the wind velocity $v_w$ and mass-loss rate $\dot{M}$, for fixed progenitor mass $M_* = 10 M_\odot$. Regions of the phase space for which the wind kinetic energy is below the Eddington luminosity of the star are blacked out, based on the expectation that the star would respond to sub-Eddington energy deposition by driving convection or expanding, rather than giving rise to significant mass loss (e.g., Quataert et al. 2016). Likewise, regions for which the total mass loss in wind ejecta experienced over the light-curve rise time exceeds the total stellar mass (i.e., $M_{ej} > M_{\star}$) are shaded gray. As shown in Figure 7, this condition imposes severe limits on the wind scenario. The white shaded regions denote the mass-loss rate $\dot{M}(v_w)$ predicted...
the ejecta at solar metallicity. Unlike in the cases of most stellar merger transients (Metzger et al. 2021; Matsumoto & Metzger 2022), the radiated luminosity generally exceeds the recombination luminosity, insofar as radiation pressure dominates over gas pressure (as satisfied to the right of the solid white line in Figure 6).

For \( M < M_{\text{ion}} \), we invert Equations (14) and (A3) to obtain the wind mass-loss rate and velocity in terms of the observed precursor luminosity and rise time:

\[
\frac{L_{\text{pl}}}{10^{40} \text{ erg s}^{-1}} = 1.5 M_{\odot} \text{ yr}^{-1} \left( \frac{M_*}{10 M_{\odot}} \right)^{-2/3} \left( \frac{t_{\text{rise}}}{10 \text{ days}} \right)^{2/3}
\]

These expressions reveal that very large mass-loss rates \( M \gtrsim 10 M_{\odot} \text{ yr}^{-1} \) are required to explain the SN precursor luminosities, \( L_{\text{pre}} \sim 10^{40} - 10^{41} \text{ erg s}^{-1} \). In fact, a conservative upper limit can be placed on the steady-state plateau luminosity in the wind scenario under the condition that the total wind ejecta mass \( M_{\text{ej}} \) does not exceed the progenitor star mass. This translates into an upper limit on the wind mass-loss rate: \( M_{\text{ej}} = \text{factorM} < M_{\odot} \), where \( t_{\text{rise}} \) is the duration of the mass-loss episode (from the wind launch to the SN explosion). This timescale may be reasonably approximated by the longer of the (theoretical) light-curve rise times, \( t_{\text{rise}} \), or by the observed precursor plateau duration, \( t_{\text{pre}} \). For \( t_{\text{rise}} \), the maximal \( M \) and corresponding maximum steady-state luminosity \( L_{\text{max}} \) are derived in the Appendix (Equations (A10) and (A11)). For \( t_{\text{pre}} \), the maximal luminosity follows by substituting \( M = M_{\text{max}} = M_*/t_{\text{pre}} \) into Equations (A3) and (A7). We set the maximal luminosity by taking the smaller one calculated for \( t_{\text{rise}} \) and \( t_{\text{pre}} \).

This theoretical maximum luminosity is shown with the solid black line in Figure 7, as a function of \( v_w \) for \( M_* = 10 M_{\odot} \) and a plateau duration \( t_{\text{pre}} = 100 \text{ days} \) close to those observed (Figure 1). For lower wind speeds \( v_w \lesssim 1000 \text{ km s}^{-1} \), the light-curve rise time exceeds the observed plateau duration \( t_{\text{rise}} > t_{\text{pre}} \), placing a tighter limit on the maximal luminosity. For comparison, the colored symbols show the time-averaged luminosities and spectroscopically measured outflow velocities for individual SN precursors (Table 1). The precursor luminosities typically exceed the theoretical limit by at least a factor of a few, thus challenging the wind scenario.

In precursor events for which flash spectroscopy observations were conducted soon after the SN explosion, the measured CSM velocities from these observations (\( V_{\text{obs}} \) in
Table 1 are in two cases (SN 2016bdu and SN 2020tlf) so low that the SN ejecta would have swept up any precursor-ejected material before the observations were taken (hence, the CSM velocities measured must arise from an earlier mass-loss episode, prior to the precursor rise). This hypothetical requirement for the measurable CSM velocity can be written as:

$$\frac{v_{\text{obs}}}{v_{\text{engulf}} \equiv v_{\text{SN}} \frac{t_{\text{flash}}}{t_{\text{pre}}}} \approx 1000 \text{ km s}^{-1} \left(\frac{v_{\text{SN}}}{10^4 \text{ km s}^{-1}}\right) \left(\frac{t_{\text{flash}}}{10 \text{ days}}\right) \left(\frac{t_{\text{pre}}}{100 \text{ days}}\right)^{-1},$$

(24)
source of shock interaction constitutes internal collisions within the wind itself, due to the time variability in the wind speed.

Consider two shells with the same mass $M$, but different velocities. The slower shell, with velocity $v$, is ejected at time $t$ before the faster one, with velocity $(1 + \delta)v$. The latter one catches up with former one in a time $\delta t/\delta v$, and they merge to form a single shell. The collision radius is estimated by $R_{\text{col}} \sim (2 + 1/\delta_v)R_v$, where we have assumed that both shells originate from the same radius $\sim R_v$, separated by a time $\delta t \sim R_v/v$ roughly equal to the dynamical time at the launching surface. From momentum and energy conservation, the final velocity of the merged shell is $(1 + \delta_v/2)v$ and the energy dissipated by the collision is $M(v^2/\delta_v^2)/2$

The velocity difference $\delta_v$ has two competing effects on the radiated luminosity of the shocked wind. While smaller values of $\delta_v$ increase the collision radius and thus reduce the adiabatic losses, such collisions also dissipate less energy than for larger $\delta_v$. Adiabatic losses exterior to $R_{\text{col}}$ reduce the observed luminosity by an amount $L \propto R_{\text{col}}^{2/3} \propto (2 + 1/\delta_v)^{-2/3}$ (Equation (A2)), while the boost in thermal energy $\propto (\delta_v)^2$; cases with the largest, order-unity fluctuations $\delta_v \sim 1$ therefore give the biggest luminosity boost.

To demonstrate the potential effects of shock heating, we recalculate the fiducial wind scenario light curve for the (most luminous) case $\delta_v = 1$, by artificially increasing the “sonic” radius $R_v$ at which the wind thermal energy is initialized in our calculation, by a factor of 3 compared to the fiducial case. As shown in Figure 5, the radiated luminosity roughly doubles compared to the case without shock interaction. Our simplified treatment of the effects of shock heating results in a smooth plateau-shaped light curve similar to the case without shocks. In reality, however, the light curve should exhibit variability, due to the stochastic nature of the heating, though this could be smoothed out by photon diffusion effects. Most SN precursor light curves are not perfectly smooth (Figure 1), with the exception of SN 2020tf (for which shock heating is anyway not strictly required in the wind scenario; Figure 7).

Another source of shock interaction is between the wind and pre-existing CSM earlier ejected from the star, or, equivalently, if the wind velocity rises in time. In fact, Quataert et al. (2016) found that shock interaction inevitably happens before the wind settles into a steady state. After the onset of energy injection below the stellar surface, the stellar envelope is initially only weakly accelerated, to form a slowly expanding atmosphere. Continuous heating decreases the density at the heating site, accelerating the material there to higher velocity (Equation (17)). The high-speed wind then collides with the slowly expanding atmosphere, generating shocks that can dominate the radiated luminosity, at least initially. Although we defer a detailed study to future work, shock interaction due to an accelerating wind provides another way of increasing the wind’s radiative efficiency, particularly at early times.

4. Precursor-generated CSM

The mass ejected from the progenitor star during the precursor phase becomes a source of CSM for shock interaction with the SN ejecta following the explosion. The application of our precursor light-curve models to individual events enables us to constrain their ejecta masses and velocities, which we can

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8 Note that we here define the critical velocity not by the entire wind active period $t_{\text{active}}$, but rather by the observed duration $t_{\text{obs}}$; this is justified because the emission from the maximal CSM velocity material typically rises quickly $t_{\text{rise}} \sim t_{\text{obs}}$.

9 Hypothetical dense clumps bound to and accumulating around the progenitor may also play a role in the dense CSM (Soker 2021).
then check for consistency with the CSM properties inferred by modeling interaction signatures during the SN phase.

### 4.1. Eruption Scenario

We have found (Section 3.1) that the eruption scenario can account for the luminosities and timescales of SN precursor emissions for physically reasonable ejecta properties. The ejecta mass and initial radius in this scenario are obtained by inverting Equations (19) and (20) in terms of the precursor luminosity $L_{\text{pl}}$ and duration $t_{\text{pl}}$:

$$M_{\text{ej}} \approx 6.7 M_{\odot}\left(\frac{v_{\text{ej}}}{10^3 \text{ km s}^{-1}}\right)^3\left(\frac{t_{\text{pl}}}{100 \text{ days}}\right)^4\left(\frac{L_{\text{pl}}}{10^{41} \text{ erg s}^{-1}}\right)^{-1},$$

(25)

$$R_{0} \approx 140 R_{\odot}\left(\frac{v_{\text{ej}}}{10^3 \text{ km s}^{-1}}\right)^{-4}\left(\frac{t_{\text{pl}}}{100 \text{ days}}\right)^{-2}\left(\frac{L_{\text{pl}}}{10^{41} \text{ erg s}^{-1}}\right)^{2}.$$  

(26)

Unfortunately, these estimates depend sensitively on the ejecta speed $v_{\text{ej}}$, which is challenging to measure and comes with large uncertainties. Furthermore, the observed precursor duration $t_{\text{pre}}$ provides only a lower limit on the total duration ($t_{\text{pl}}$), which enters these formulae if the SN prematurely terminates the precursor emission.

Table 2 provides the ejecta mass, kinetic energy ($E_{\text{kin}} = M_{\text{ej}}v_{\text{ej}}^2/2$), and progenitor radius for our precursor sample (Table 1), as obtained from the inverted Popov formulae (Equations (25) and (26)). Again, the observed precursor duration is only a lower limit on the total plateau duration, and hence we obtain only a lower (upper) limit on the ejecta mass (progenitor radius). All values are calculated assuming the lowest spectroscopically observed velocity range. However, we again caution that our estimates are highly dependent on the adopted ejecta speed; a factor of 2 difference in the velocity changes the results by an order of magnitude or more.

At the time of the SN explosion, the characteristic outer radius and density of the precursor ejecta (hereafter CSM) are thus given by, respectively,

$$R_{\text{CSM}} \sim v_{\text{ej}} t_{\text{pre}} \approx 8.6 \times 10^{14} \text{ cm}\left(\frac{v_{\text{ej}}}{10^3 \text{ km s}^{-1}}\right)\left(\frac{t_{\text{pre}}}{100 \text{ days}}\right),$$

(27)

$$\rho_{\text{CSM}} \sim \frac{M_{\text{ej}}}{4\pi R_{\text{CSM}}^3} \geq 5.0 \times 10^{-12} \text{ g cm}^{-3} \times \left(\frac{t_{\text{pre}}}{100 \text{ days}}\right)^{-1}\left(\frac{L_{\text{pre}}}{10^{41} \text{ erg s}^{-1}}\right)^{-1},$$

(28)

where the second equality in Equation (28) makes use of Equation (25). The upper limit on $\rho_{\text{CSM}}$ results from $t_{\text{pl}} > t_{\text{pre}}$ and is notably independent of the (poorly constrained) ejecta velocity.

The left panel of Figure 8 shows the estimated CSM density $\rho_{\text{CSM}}$ and radius $R_{\text{CSM}}$ in the eruption scenario for the observed precursors (Figure 1, Table 1). As discussed in Section 3.1, and shown in Figure 3, the required CSM masses are typically in the range $\sim 0.1\--1 M_{\odot}$. Since the light-curve properties depend only weakly on the ejecta velocity profile $M_{\text{ej}} \propto v^{-3}$ (Equation (8)), the slope of the resulting CSM radial density profile $\rho \propto r^{-\beta-1}$ (at $r > R_{\text{CSM}}$ or, equivalently, $v > v_{\text{ej}}$) is not well constrained by the precursor observations. However, as long as $\beta \geq 0$, the CSM mass will be concentrated in the $\sim v_{\text{ej}}$ shell on the radial scale $\sim R_{\text{CSM}}$.

Several implications follow from the implied CSM properties. The inferred CSM is sufficiently massive and compact as to be optically thick ($\rho \gtrsim 10^{-14} \text{ g cm}^{-3} (R_{\text{CSM}}/10^{14} \text{ cm})^{-1}$), thus implying that (1) the SN shock breakout will occur from the precursor-generated CSM shell, instead of the original stellar surface, and (2) flash spectroscopy cannot readily probe the mean velocity of the precursor ejecta (though it may probe its high-velocity tail).

Shock heating of the CSM shell by the SN ejecta will dissipate the kinetic energy of the latter and boost the luminosity of the SN relative to the CSM-free case. The luminosity of this shock interaction can be crudely estimated as:

$$L_{\text{sh}} \sim \frac{1}{4} M_{\text{ej}} v_{\text{SN}} R_{\text{CSM}} \frac{\rho_{\text{CSM}}}{\rho_{\text{SN}}} \frac{R_{\text{CSM}}}{t_{\text{diff}}} \approx 6 \times 10^{42} \text{ erg s}^{-1}\left(\frac{v_{\text{SN}}}{10^4 \text{ km s}^{-1}}\right)^2,$$

(29)

where $t_{\text{diff}} = \frac{1}{4} M_{\text{ej}} v_{\text{SN}} R_{\text{CSM}}$ is the characteristic photon diffusion time through the shocked CSM and $\varepsilon$ is the radiative efficiency. Interestingly, the luminosity depends only on the CSM radius. For the CSM radii $R_{\text{CSM}} \sim 3 \times 10^{14} \text{ cm}$ implied by the precursor emission (Figure 4), the predicted luminosities $L_{\text{sh}} \sim 10^{49} \text{ erg s}^{-1}$ broadly agree with those of the SNe following the precursor events (Figure 1). However, for SN 2020tlf with $R_{\text{CSM}} \sim 10^{14} \text{ cm}$, the shock-boosted SN luminosity would be too low, and another CSM component—for example, optically thick CSM extending to large radii $\sim 10^{15} \text{ cm}$—is required to power the early SN light curve via CSM interaction.

Though constraints from the literature on the CSM properties of our precursor sample are limited, we comment briefly on

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\[^{10}\] This scaling differs from that for an SN interacting with a wind-like CSM because of the steeper density profile.
those available. For SN 2009ip, Margutti et al. (2014) and Moriya (2015) interpret the SN emission as arising due to the shock breakout emission from a dense shell produced by the precursor outburst; they estimate a precursor mass of $\sim 0.1 M_\odot$ and a radius $\sim 10^{15}$ cm at explosion, by equating the SN rise time to the photon diffusion time through the dense shell. Their result is essentially consistent with theirs for this event, for an assumed ejecta velocity equal to the lowest value measured spectroscopically, $v_{ej} \approx 800 - 1400 \text{ km s}^{-1}$. For SN 2020tlf, Chugai & Utrobin (2022) model the precursor emission as arising from an ejecta mass of $M_{ej} \sim 0.1 M_\odot$, also consistent with our result. They find that subsequent SN emission is not affected by this compact CSM, but that an additional wind-like CSM extending up to $\sim 10^{15}$ cm is required (we return to this event in the context of the wind scenario in the next section).

4.2. Wind Scenario

Detecting the rising phase of the light curve in the wind scenario is typically challenging (see Figure 5). We therefore assume that the emission has already reached its stationary state by the time of the observations. Inverting Equation (A3), we derive the required wind mass-loss rate in terms of the observed precursor properties:

$$\dot{M} \simeq 1.5 M_\odot \text{ yr}^{-1} \left( \frac{M_{ej}}{10 M_\odot} \right)^{-2} \times \left( \frac{v_w}{10^3 \text{ km s}^{-1}} \right)^{-2} \left( \frac{L_{pre}}{10^{40} \text{ erg s}^{-1}} \right)^{3} \left( \frac{t_{pre}}{100 \text{ days}} \right)^{-2}.$$

(30)

Insofar as the outer radius of the wind ejecta at the time of the SN explosion is given by $R_{\text{CSM}} \sim v_w t_{\text{active}} \gtrsim v_w t_{\text{pre}}$, Equation (27), with $v_{ej}$ replaced by $v_w$, gives a lower limit on $R_{\text{CSM}}$ while the rise time is typically shorter than $t_{\text{pre}}$ for relevant wind velocities. This in turn implies an upper limit on the CSM density at $R_{\text{CSM}}$:

$$\rho_{\text{CSM}} = \frac{M}{4\pi R_{\text{CSM}}^2 v_w},$$

$$\lesssim 1.0 \times 10^{-13} \text{ g cm}^{-3} \left( \frac{M_{ej}}{10 M_\odot} \right)^{-2} \times \left( \frac{v_w}{10^3 \text{ km s}^{-1}} \right)^{-5} \left( \frac{L_{pre}}{10^{40} \text{ erg s}^{-1}} \right)^{3} \left( \frac{t_{pre}}{100 \text{ days}} \right)^{-2},$$

(32)

which depends sensitively on the wind velocity. Assuming that the wind mass-loss rate is roughly constant prior to the SN, the CSM density at $r < R_{\text{CSM}}$ will follow a wind-like $\rho \propto r^{-2}$ profile.

The right panel of Figure 8 shows the estimated CSM density of the observed precursors in the wind mass-loss scenario. As discussed in Section 3.2, the required CSM density, or, equivalently, the mass-loss rate $\dot{M}$, is enormous, exceeding that allowed by the total mass budget of the star in most events.

Only in SN 2020tlf can the observed precursor be reasonably accommodated, for the maximally allowed velocity $v_w \sim v_{\text{engulf}} \sim 10^3 \text{ km s}^{-1}$ (and we note that $\dot{M} \approx \text{const mass loss may indeed be suggested for this event by the flat shape of the precursor light curve}). The implied radial extent of the CSM $\sim 10^{15}$ cm in the wind scenario is consistent with an upper limit based on X-ray nondetections (Jacobson-Galán et al. 2022) and light-curve modeling (Chugai & Utrobin 2022; though we note that the density required in our scenario is $3 - 10$ times greater than that found by these authors). Given the uncertainties on parameters, such as the progenitor mass and wind velocity, and the potential for internal shock heating (Section 3.2.1), this level of discrepancy may nevertheless be tolerable. We note that the CSM radius obtained for SN 2020tlf in the eruption scenario (Table 2) is also consistent with the modeling of Chugai & Utrobin (2022).
As in the eruption case, the inferred CSM density (the wind mass-loss rate) should not be so large that it overproduces the SN luminosity via CSM shock interaction. For freely expanding SN ejecta into a wind-like density profile, the SN luminosity can be estimated as (e.g., Chevalier & Irwin 2011, but here ignoring deceleration of the shocked CSM shell):

\[
L_{\text{sh}} \sim \frac{1}{2} \rho \mathcal{M}_{\text{swpt}} v_{\text{SN}}^2 \frac{r_{\text{diff}}}{t_{\text{diff}}},
\]

\[
\simeq 3 \times 10^{43} \text{erg s}^{-1} \left(\frac{v_{\text{SN}}}{10^{4} \text{km s}^{-1}}\right)^{3} \left(\frac{M}{10^{1} \text{M}_{\odot}}\right)^{-1} \left(\frac{v_{w}}{10^{3} \text{km s}^{-1}}\right)^{-1},
\]

where \(M_{\text{swpt}} \sim \rho R_{\text{diff}}^3\) is the CSM wind mass swept up by SN ejecta on the wind diffusion timescale \(t_{\text{diff}} = \frac{s^{2} M_{\odot}}{4 \pi v_{w}}\) (Equation (6)), and we assumed the radius \(R_{\text{diff}} \sim v_{\text{SN}} t_{\text{diff}} \sim 6 \times 10^{14} \text{cm} (M/M_{\odot} \text{ yr}^{-1})(v_{\text{SN}}/10^{4} \text{ km s}^{-1})(v_{w}/10^{3} \text{ km s}^{-1})^{-1}\) is smaller than the outer edge of the CSM, \(R_{\text{CSM}}\).

The estimate for \(L_{\text{sh}}\) in the case of SN 2020lff is roughly consistent with its SN luminosity, while we find that \(L_{\text{sh}} \gg L_{\text{SN}}\), given the much higher values \(M\) required to explain the other precursors. This again favors the eruption scenario over the steady wind scenario as the origin of most SN precursors, at least neglecting the shock heating within the wind (Section 3.2.1).

5. Summary

We have modeled the precursor optical emission detected from a growing sample of core-collapse SNe using an extension of the semi-analytical light-curve model described in Matsumoto & Metzger (2022). The observed precursors can be regarded as extreme cases of mass-loss events increasingly inferred to occur just before the terminal collapse, whose direct emission can provide clues to the nature of the mass-loss mechanisms at work and more generally about the final stages of massive star evolution. We develop light-curve models in the context of two scenarios (“eruption” and “wind”) for the nature of the pre-SN mass-loss phase.

In the eruption model, energy is deposited near the base of stellar envelope on a timescale shorter than the dynamical time. While our model is indifferent to the mechanism responsible for this energy injection, it could in principle be caused by a violent outburst associated with late stages of nuclear shell burning. This sudden energy deposition creates a shock wave, which propagates outward toward the surface of the star, ejecting a portion of its envelope. For the eruption scenario, our findings are summarized as follows.

1. The eruption light curve is characterized by a recombination-driven plateau similar to that of Type IIP SNe (Figure 3). The luminosity and duration of the plateau are reasonably estimated by the so-called “Popov formulae” (Popov 1993), which relate these observables to the ejecta mass and progenitor radius (Equations (19) and (20)). However, because the entirety of the precursor light curve is generally not observable (due to interruption by the SN explosion), we can only obtain lower limits on the ejecta mass \(M_{\text{ej}}\) and kinetic energy \(E_{\text{kin}}\) for individual events. Adopting an ejecta velocity \(v_{\text{ej}} \sim 100–1000 \text{ km s}^{-1}\), consistent with those inferred from spectroscopy, the observed precursor luminosities are reproduced for \(M_{\text{ej}} \geq 0.1–1 M_{\odot}\) and \(E_{\text{kin}} \geq 10^{48–50} \text{ erg}\) (Table 2). We caution that the latter estimate depends sensitively on the ejecta velocity, which is not available in all events, and could in principle be confused with CSM from other, earlier phases of mass loss.

2. A wide range of precursor luminosities can be produced in this model, \(L_{\text{pre}} \sim 10^{39–42} \text{ erg s}^{-1}\) (Figure 4; see also Dessart et al. 2010). The luminosity mainly depends on the progenitor radius and ejecta velocity. Bright precursors of \(L_{\text{pre}} \sim 10^{41} \text{ erg s}^{-1}\) require large progenitor radii \(R_{\star} \sim 10^{2} R_{\odot}\), such as RSGs. For these progenitors, super-Eddington luminosities \(L_{\text{pre}} \sim L_{\text{Edd}}\sim 10^{39} \text{ erg s}^{-1}(M_{\odot}/10 M_{\odot})\) are achieved only for ejecta speeds larger than the escape speed of the stellar surface, supporting the source of energy deposition occurring deep inside the stellar envelope.

The second scenario that we considered is one of steady wind-like mass loss, which results from a continuous energy deposition at the base of the stellar envelope (mainly motivated by the wave heating scenario; e.g., Quataert & Shiode 2012; Fuller 2017). For the wind scenario, our findings are summarized as follows.

1. The light curve undergoes a gradual rise, before settling into a stationary state. The rise time is essentially given by the diffusion timescale of the wind and the steady-state luminosity as \(L \sim L_{\text{Edd}} \left(\frac{1}{2} M v_{w}^{2}/L_{\text{Edd}}\right)^{1/3}\) (Equations (A3) and (A7); see also Quataert et al. 2016; Shen et al. 2016). For typical values of the observed precursor ejecta velocity, very large mass-loss rates \(M \gtrsim 10 M_{\odot} \text{ yr}^{-1}\) are required to achieve the observed precursor luminosities \(L_{\text{pre}} \sim 10^{41} \text{ erg s}^{-1}\) (Figure 6). Such large mass-loss rates would also increase the light-curve rise time, which (in conjunction with the large required \(M\)) would exceed the mass budget of the star. This constraint can be formalized by defining a theoretical maximum precursor luminosity for a given wind velocity (Figure 7; Appendix).

Among the five well-observed precursors in our sample, only SN 2020lff, with its relatively low luminosity, can be explained by the wind model, for a mass-loss rate of \(M \gtrsim 1 M_{\odot} \text{ yr}^{-1}\) and velocity of \(v_{w} \sim 10^{3} \text{ km s}^{-1}\). The corresponding sonic radius is again deep inside the star, \(R_{\star} \sim 10 R_{\odot} \sim 10^{5}–10^{6} R_{\odot}\), requiring energy deposition at the base of stellar envelope.

2. The luminosity of the wind can be boosted due to internal shocks—for instance, due to a time-variable wind speed—by a modest factor \(\lesssim 2\). However, the relatively smooth observed shape of the precursor light curves may disfavor shock interaction.

While we do not study them in detail, external shocks between the progenitor ejecta and pre-existing CSM may also take place and contribute to the luminosity (see also Quataert et al. 2016; Strotjohann et al. 2021; Jacobson-Galán et al. 2022). The shock luminosity can be crudely estimated by energy and momentum conservation. Neglecting any losses of efficiency due to the deceleration of wind, the shock-dissipated energy is given...
by \( E_{sh} \sim \frac{1}{2} \frac{M_{\text{w}} M_{\text{CSM}}}{M_{\text{w}} + M_{\text{CSM}}} v_r^2 \), where \( M_{\text{w}} = \dot{M}_{\text{pre}} \) is the wind mass ejected during the precursor and \( M_{\text{pCSM}} \) is the pre-existing CSM mass swept up by the wind. For pre-existing CSM with a wind-like profile, the swept-up mass is given by \( M_{\text{pCSM}} = \int_{r_{\text{ion}}}^{r_{\text{t}}} 4\pi r^2 \rho_{\text{pCSM}} dr = M_{\text{pCSM}} v_{\text{pre}} t_{\text{pre}} / v_{r,\text{pCSM}} \), where \( M_{\text{pCSM}} \) and \( v_{r,\text{pCSM}} \) are the mass-loss rate and the velocity of the pre-existing CSM, respectively. Combining results, the radiated shock luminosity is given by

\[
L_{\text{sh}} \sim \frac{E_{sh}}{t_{\text{pre}}} \sim \left( \frac{1}{1 + r_p} \right) E_w, \tag{35}
\]

where \( r_p = \rho_{\text{w}} / \rho_{\text{pCSM}} \) is independent of \( r \) for wind-like profiles. While \( r_p > 1 \) is required to neglect the deceleration of the wind, and hence \( L_{\text{sh}} \lesssim E_w \), the stronger dependence of the radiated luminosity on \( E_w \) in Equation (35) than Equation (A3) suggests that shock interaction can still dominate over the intrinsic wind luminosity. Future work is needed to explore the external shock scenario in greater detail.

3. Absent external shock interaction, the fact that the observed SN precursors have violated the maximal luminosity in the wind scenario may favor eruptive events (e.g., those driven by unstable nuclear shell burning) over a more continuous heating source (e.g., due to wave heating).

In both the eruption and wind models, the precursor ejecta forms an optically thick compact CSM shell surrounding the progenitor star that will be present at the time of collapse and could boost the luminosity of the SN light curve substantially through shock interaction. Our main finding is:

1. The radial extent of the CSM is typically so small \( R_{\text{CSM}} \lesssim 10^{15} \text{ cm} \) (see Equation (27)) that it is engulfed by SN ejecta less than ten days after SN explosion. Therefore, flash spectroscopy carried out a few days after the SN explosion may miss the signature of the precursor ejecta and probe only the CSM released before the precursor mass-loss episode (see also Strotjohann et al. 2021). In the eruption model, the density profile of the CSM is steeper than the steady wind profile \( (\rho \propto r^{-2}) \), forming a dense CSM core.

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**Appendix**

**Analytic Estimates of Precursor Emission in the Wind Mass-loss Scenario**

The rise time of the precursor light curve (equivalently, the duration of the single-shell light curve; Figure 5) is determined by the diffusion timescale through the shell, \( t_d \) (Equation (14)). Radiation thus escapes the wind ejecta from the characteristic radius

\[
R_d \approx v_w t_d = \frac{\kappa \dot{M}}{4\pi c} \approx 5.4 \times 10^{13} \text{ cm} \left( \frac{\dot{M}}{M_{\odot} \text{ yr}^{-1}} \right), \tag{A1}
\]

In this Appendix, we normalize \( \kappa \) to the fully ionized electron scattering opacity, \( \kappa_{\text{es}} \approx 0.32 \text{ cm}^2 \text{ g}^{-1} \). Because photons are trapped, the wind material expands adiabatically between the sonic and diffusion radii \((R_s < r < R_d)\), such that \( T \propto \rho^{1/3} r^{-2/3} \) under a radiation pressure–dominated condition and \( \rho \propto r^{-2} \) for a steady wind. The thermal and kinetic luminosities of the wind are comparable at \( R_d \); thus, the photon (advection) luminosity \( L \approx 4\pi r^2 v_w a T^{5/2} \propto r^{-2/3} \) evolves as

\[
L \approx \dot{E}_w \left( \frac{T}{T_{R_d}} \right)^{-2/3}, \tag{A2}
\]

until the diffusion becomes important. Here, \( \dot{E}_w \) is the wind kinetic power (Equation (12)).

There are two cases to consider, depending on whether the wind material remains fully ionized or has begun to recombine below \( R_d \). When \( \dot{M} \) is sufficiently low (with the precise threshold, \( M_{\text{ion}} \), to be defined below), the ejecta is still fully ionized \( (T \lesssim T_{\text{ion}} \approx 10^4 \text{ K}) \) when photons start to diffuse out of the wind. Inserting Equation (A1) into Equation (A2), we find

\[
L = L_{\text{Edd}} \left( \frac{\dot{E}_w}{L_{\text{Edd}}} \right)^{1/3} \approx 3.2 \times 10^{39} \text{ erg s}^{-1} \dot{M}_{14}^{2/3},
\]

\[
\times \left( \frac{\dot{M}}{M_{\odot} \text{ yr}^{-1}} \left( \frac{v_w}{200 \text{ km s}^{-1}} \right)^{2/3} \right), \tag{A3}
\]

where \( \dot{M}_{14} = \dot{M} / (10 M_{\odot}) \). This expression agrees with previous works (e.g., Quataert et al. 2016; Shen et al. 2016).

On the other hand, when \( \dot{M} \) is sufficiently large, hydrogen recombination begins below \( R_d \), which triggers photon diffusion by reducing the opacity \( \kappa \ll \kappa_{\text{es}} \) and effectively shrinking the diffusion radius. Recombination occurs at the temperature \( T = T_{\text{ion}} = 10^4 T_{\text{ion},4} \text{ K} \), as is achieved at the radius

\[
R_{\text{ion}} = R_d \left( \frac{T}{T_{\text{ion}}} \right)^{3/2} = \left( \frac{\dot{M}^2 (2GM_c)^2}{8\pi a T_{\text{ion}}^4 v_w^4} \right)^{1/8}
\]

\[
\approx 2.5 \times 10^{14} \text{ cm} \dot{M}_{14}^{1/4} T_{\text{ion},4}^{3/2} \left( \frac{\dot{M}}{M_{\odot} \text{ yr}^{-1}} \right)^{3/8} \left( \frac{v_w}{200 \text{ km s}^{-1}} \right)^{-9/8}, \tag{A4}
\]

on the timescale

\[
t_{\text{ion}} = \frac{R_{\text{ion}}}{v_w} \approx 140 \text{ days} \dot{M}_{14}^{1/4} T_{\text{ion},4}^{-3/2}
\]

\[
\times \left( \frac{\dot{M}}{M_{\odot} \text{ yr}^{-1}} \right)^{3/8} \left( \frac{v_w}{200 \text{ km s}^{-1}} \right)^{-9/8}, \tag{A5}
\]

where

\[
T_s = \left( \frac{M v_w}{8\pi a R_d^2} \right)^{1/4} \approx \left( \frac{M v_w^2}{8\pi a (2GM_c)^2} \right)^{1/4}
\]

\[
\approx 1.1 \times 10^5 \text{ K} \dot{M}_{14}^{-1/2} \left( \frac{M}{M_{\odot}} \text{ yr}^{-1} \right)^{1/4} \left( \frac{v_w}{200 \text{ km s}^{-1}} \right)^{5/4}, \tag{A6}
\]
is the wind temperature at the sonic radius. The wind luminosity in the high-$M$ case (Equation (A2)) is then given by

\[ L = \frac{E_w}{R_{\text{son}}} - \frac{2}{3} = L_{\text{edd}} \left( \frac{E_w}{L_{\text{edd}}} \right)^{1/3} \left( \frac{M}{M_{\text{ion}}} \right)^{5/12} \]

\[ = \left( \frac{\pi a T_{\text{ion}}^{4}(2GM_{\text{w}})^{3/2} M_{\text{ion}}^{1/4}}{2} \right)^{1/4} \]

\[ \approx 1.2 \times 10^{39} \text{ erg} M_{\odot}^{1/4} T_{\text{ion},4}^{1/4} \times \left( \frac{\rho v_{\text{w}}}{200 \text{ km s}^{-1}} \right)^{3/4}, \quad M > M_{\text{ion}}. \quad (A7) \]

The critical mass-loss rate above which recombination defines the transient luminosity can be estimated by equating Equations (A3) and (A7):

\[ M_{\text{ion}} = \left( \frac{2 \pi^{5} (2GM_{\text{w}})^{3/2}}{a^{3} \rho^{1/2} T_{\text{ion},4} v_{\text{w}}} \right)^{1/5} \approx 11 M_{\odot} \text{ yr}^{-1} M_{\odot}^{2/5} T_{\text{ion},4}^{-12/5} \]

\[ \times \left( \frac{\rho v_{\text{w}}}{200 \text{ km s}^{-1}} \right)^{-1/5}. \quad (A8) \]

For precursor emission that exhibits a steady-state plateau-like curve, the mass loss must be active for a timescale $\lesssim t_{\text{ion}}$. For a given allowed ejecta mass $M_{\text{ej}}$, this implies that a maximum on the wind mass-loss rate can be obtained. For the low-$M$ case, demanding $t_{\text{ion}} \dot{M} < M_{\text{ej}}$ gives

\[ \dot{M}_{\text{max}} = \left( \frac{4\pi cv_{\text{w}} M_{\text{ej}}}{\kappa} \right)^{1/2} \]

\[ \approx 11 M_{\odot} \text{ yr}^{-1} M_{\odot}^{1/2} \left( \frac{M_{\text{ej}}}{M_{\odot}} \right)^{1/2} \left( \frac{v_{\text{w}}}{200 \text{ km s}^{-1}} \right)^{1/2}, \quad (A9) \]

and the corresponding maximal steady-state luminosity is given by

\[ L_{\text{max}} = 7.0 \times 10^{39} \text{ erg s}^{-1} M_{\odot}^{5/6} \left( \frac{M_{\text{ej}}}{M_{\odot}} \right)^{1/6} \left( \frac{v_{\text{w}}}{200 \text{ km s}^{-1}} \right)^{5/6}. \quad (A10) \]

On the other hand, for the low-$\dot{M}$ case, the maximal mass-loss rate and luminosity are given by

\[ M_{\text{max}} \approx \left( \frac{8\pi a T_{\text{ion}}^{4}(2GM_{\text{w}})^{3/2} M_{\text{ion}}^{1/4}}{2} \right)^{1/11} \]

\[ \approx 11 M_{\odot} \text{ yr}^{-1} M_{\odot}^{1/11} T_{\text{ion},4}^{12/11} \left( \frac{M_{\text{ej}}}{M_{\odot}} \right)^{8/11} \]

\[ \times \left( \frac{v_{\text{w}}}{200 \text{ km s}^{-1}} \right)^{9/11}. \quad (A11) \]

\[ L_{\text{max}} = 6.8 \times 10^{39} \text{ erg s}^{-1} M_{\odot}^{10/11} T_{\text{ion},4}^{20/11} \]

\[ \times \left( \frac{M_{\text{ej}}}{M_{\odot}} \right)^{6/11} \left( \frac{v_{\text{w}}}{200 \text{ km s}^{-1}} \right)^{15/11}. \]
