VARIATIONS IN STELLAR CLUSTERING WITH ENVIRONMENT: DISPERSED STAR FORMATION AND THE ORIGIN OF FAINT FUZZIES

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ABSTRACT

The observed increase in star formation efficiency with average cloud density, from several percent in whole giant molecular clouds to \( \sim 30\% \) or more in cluster-forming cores, can be understood as the result of hierarchical cloud structure if there is a characteristic density at which individual stars become well defined. Also in this case, the efficiency of star formation increases with the dispersion of the density probability distribution function (PDF). Models with lognormal PDFs illustrate these effects. The difference between star formation in bound clusters and star formation in loose groupings is attributed to a difference in cloud pressure, with higher pressures forming more tightly bound clusters. This correlation accounts for the observed increase in the clustering fraction with star formation rate and with the observation of scaled OB associations in low-pressure environments. “Faint fuzzies” star clusters, which are bound but have low densities, can form in regions with high Mach numbers and low background tidal forces. The proposal by Burkert, Brodie & Larsen that faint fuzzies form at large radii in galactic collisional rings satisfies these constraints.

Subject headings: ISM: structure — open clusters and associations: general — stars: formation

1. INTRODUCTION

Stars form in concentrations with a range of densities, from star complexes, OB associations, and T Tauri associations at the low end to compact clusters and super star clusters (SSC) at the high end. The overall structure is usually hierarchical (Scalo 1985; Feitzinger & Galinski 1987; Ivanov et al. 1992; Gomez et al. 1993; Battinelli et al. 1996; Bastian, et al. 2005, 2007; Elmegreen et al. 2006; see reviews in Efremov 1995; Elmegreen et al. 2000; Elmegreen 2006), and this hierarchy continues even inside the youngest clusters (Testi et al. 2000; Heydari-Malayeri et al. 2001; Nanda Kumar et al. 2004; Smith et al. 2005a; Gutermuth et al. 2005; Dahm & Simon 2005; Stanke et al. 2006; see review in Allen et al. 2007). Most likely, the hierarchy in stars comes from a hierarchy in the gas (Stutzki et al. 1998; Dickey et al. 2001), which is the result of turbulence compression and gravitational contraction that is self-similar over a wide range of scales (see review in Mac Low & Klessen 2004). Clusters form in the densest parts of this gas and lose their initial substructure as the stellar orbits mix (for simulations of star formation in clusters, see Klessen & Burkert 2000; Bonnell et al. 2003; Li et al. 2004; Tilley & Pudritz 2004; Klessen et al. 2005; Vázquez-Semadeni et al. 2005; Bonnell & Bate 2006; Li & Nakamura 2006).

Hunter (1999) and Maiz-Apellániz (2001) noted that some massive star-forming regions (which they called “scaled OB associations,” or SOBA’s) do not form dense clusters, while others with the same total mass do (e.g., the SSC’s). We would like to understand this difference. Obviously the density of the gas is involved, as dense clusters require dense gas, but the distinction between SSCs and SOBAs should also be related to the efficiency of star formation, because clusters forming at low efficiency disperse quickly when the gas leaves (Lada et al. 1984). At very low efficiency, stars form individually without passing through an embedded cluster phase. Recent observations of giant molecular clouds (GMCs) show star formation at both high and low densities, with some embedded stars in dense clusters and others more dispersed (Megeath et al. 2004; Jørgensen et al. 2006, 2007).

Larsen & Brodie (2000) discovered “faint fuzzies” at intermediate radii in the disks of the S0 galaxies NGC 1023 and NGC 3384, and suggested that they are old clusters with unusually large radii (7–15 pc) and low densities. They are gravitationally bound because of their large ages (8–13 Gyr; Brodie & Larsen 2002), and they are as massive as SSC’s and halo globular clusters. Such clusters appear to represent an intermediate stage between dispersed and bound star formation. They appear to be too low in average density to have had time for core collapse and envelope expansion as in models of globular clusters by Baumgardt et al. (2002). Burkert et al. (2005) suggested that they formed in a collisional ring interaction between two galaxies.

What determines the relative proportion of dispersed and clustered star formation? Larsen & Richtler (2000) showed that clustering on a galactic scale, measured as the fraction of UV light in the form of massive young clusters, increases as the star formation rate increases. This could be the result of a selection effect if starbursts are active for less than a cluster destruction time. On the other hand, the clustering fraction could depend on pressure. Higher pressure makes the cool phase of gas denser, which promotes more clustering, and locally high pressures trigger star formation on the periphery of GMCs, making clusters in the dense gas (e.g., Comerón et al. 2005; Zavagno et al. 2006). The Larsen & Richtler correlation could then follow from the mutual correlation between pressure and star formation rate with gas column density. Faint fuzzies are a counterexample, however: they formed bound but presumably at low pressure to have such low central column densities now. Can all of these clustering types be understood as a continuum of properties in a universal physical model?

There have been several attempts to explain the difference between clustered and distributed star formation based on numerical simulations. Klessen et al. (2004) suggested that clusters form in nonmagnetic gas when the turbulence driving scale is large. Heitsch et al. (2001) noted that nonmagnetic turbulence driven on large scales produces a clustered collapse, while magnetic turbulence in supercritical clouds produces a more distributed collapse. Mac Low (2002) suggested that stars form in clusters when there is no turbulent support, and form disbursed when...
there is. Vázquez-Semadeni et al. (2003) suggest that this transition from no global turbulent support to support corresponds to an increase in the Mach number and a decrease in the sonic scale, which is the length where the size-linewidth relation gives a Mach number of unity. Large sonic scale (low Mach number) corresponds to a sonic mass larger than the thermal Jeans mass, which means a lack of global support and the formation of a cluster. Low sonic scale corresponds to the dispersed formation of stars, one for each tiny compressed region where the mass exceeds the local thermal Jeans mass. Li et al. (2003) suggested that the equation of state determines the stellar clustering properties: soft equations produce dense clusters, while hard equations produce isolated stars. These suggestions all apply to initially uniform media. External compression of a cloud into a massive dense core can also make a cluster; most young clusters are in high-pressure regions like OB associations.

Simulations of turbulent media produce stars in compressed regions that act as seeds for the small-scale gravitational collapses that follow (Vázquez-Semadeni et al. 2003; Clark & Bonnell 2005). These simulations also have probability distribution functions (PDFs) for density that are either lognormal or lognormal with a power-law tail at high density, especially when self-gravity is important (e.g., Li et al. 2003). The efficiency of star formation is then proportional to the fraction of the gas in a dense form. Here we examine variations in this fraction as functions of average density and velocity dispersion, and as a function of the local density inside a cloud. The efficiency is taken to be the ratio of the stellar mass to the gas mass during a complete star-forming event. It generally increases with cloud density from a few percent in GMCs (Williams & McKee 1997) to several tens of percent in cluster-forming cores (e.g., Lada & Lada 2003). It may reach ~50% or more inside the densest star-forming cores. We explain this increase as a result of hierarchical structure, regardless of the dynamics and mechanisms of star formation, and we show that for lognormal or similar density PDFs, as expected in turbulent media, the mass fraction of regions with high efficiency increases with the Mach number and, independently, with the average density. This result may explain the Larsen & Richtler (2000) correlation, as well as the observed variations in clustering properties with pressure. We also show that at high Mach number, bound clusters can form with relatively low average densities, thereby explaining faint fuzzies. These are all consequences of star formation in the dense cores of clouds that are structured by turbulence. They result primarily from the geometry of the gas, which is somewhat universal, and should be nearly independent of the gas dynamics or the strength of the magnetic field.

We make an important assumption that gravitational contraction and star formation can occur in regions that are either larger or smaller than the sonic scale. This means we assume that contraction to one or more stars can occur in a supersonically turbulent region. Vázquez-Semadeni et al. (2003) suggest that if a cloud is supported by turbulence, then only regions smaller than a sonic scale and more massive than the thermal Jeans mass are unstable to form stars. Padoan (1995) was the first to consider this condition. However, clouds are probably not supported for any significant time by turbulence, and even if they were, it is only necessary that a clump mass exceed the turbulent Jeans mass for self-gravitational forces to exceed inertial forces. GMCs, for example, have comparable self-gravitational and turbulent energy densities and yet are much larger than the sonic length. Our assumption is contrary to that in Krumholz & McKee (2005), who assume the same as Vázquez-Semadeni et al. We are consistent with McKee & Tan (2003), however, as they consider the collapse of a highly turbulent core to make a massive star. Saito et al. (2006), for example, observe star formation in massive turbulent cores. Thus, the sonic length should not provide a threshold for star formation.

Our primary condition for star formation is that the gas density exceed some fixed value, taken here to be $10^7$ cm$^{-3}$. This is the density at which HCN gives a nearly constant star formation rate per unit gas mass (Gao & Solomon 2004a, 2004b; Wu et al. 2005) and at which a variety of microscopic processes conspire to shorten the magnetic diffusion time (Elmegreen 2007). Regions with this density should have a wide range of Mach numbers but a nearly universal efficiency, according to the Gao & Solomon and Wu et al. observations. The density of ~$10^5$ cm$^{-3}$, when converted to 5900 $M_\odot$ pc$^{-3}$, is also typical for star clusters, as most of those surveyed by Tan (2007), which span a factor of $10^3$ in mass, have about this average density. The cluster density equals the gas density times the efficiency, and the efficiency has to exceed ~10%–30% for a bound cluster to form. Thus, the gas density for both cluster formation and high efficiency appears to be around $10^7$ cm$^{-3}$ or, possibly, $10^8$ cm$^{-3}$. We note that the long timescale derived for HCN gas as the ratio of the total HCN mass divided by the total star formation rate (Gao & Solomon 2004b; Wu et al. 2005) is not the duration of star formation in any one place, but is the HCN consumption time. As long as a new HCN region is formed somewhere each time an old HCN region disperses, the HCN consumption time can be long even when each region of star formation lasts for a short time (see Elmegreen 2007 for a discussion of star formation timescales). As a result, the average efficiency of star formation in any one HCN region can be moderately large even if the average HCN consumption time is long. An average efficiency of ~5%–10% would be reasonable considering that most star-forming regions leave unbound clusters after the gas leaves (Lada & Lada 2003), most regions are observed at half their total ages, and star formation typically accelerates over time (Palla & Stahler 2000). With constant acceleration, only ~1/4 of the total stars form in the first 1/2 of the total time. The efficiency that determines whether a bound cluster will remain is taken here to be 14.4% (see below), and the peak efficiency in a single-star core is taken to be 50% at $10^7$ cm$^{-3}$ density. These values are uncertain and are used here only to illustrate how cluster formation might scale with the velocity dispersion and density of the cool component of the interstellar medium.

2. CLUSTER FORMATION IN A HIERARCHICAL ISM: BOUND AND UNBOUND CLUSTERS

The hierarchical structure of interstellar gas implies that the mass fraction of star-forming cores at high density $n_c$ increases in regions of the cloud with increasing average density. This density dependence may be illustrated with a simple model. Consider a local star formation rate proportional to $(\rho)\rho(Gp)^{1/2}$ for $p < \rho_c$ and efficiency $\epsilon$. In this paper, mass density will be denoted by $\rho$ and molecule density by $n$. If we denote the galactic average quantities by a subscript "0," then the Kennicutt-Schmidt star formation rate is analogous to $\epsilon_0 \rho_0 (G_0)^{1/2}$ for $\epsilon_0 \sim 0.012$ (Elmegreen 2002b). The same galactic rate would be obtained for observations at any other density, provided the efficiency and volume filling factor at that density are properly scaled. Thus, we write

$$\epsilon_0 \rho_0 (G_0)^{1/2} = \epsilon(\rho)\rho(Gp)^{1/2} f_\epsilon(\rho) = \epsilon_c \rho_c (G_0)^{1/2} f_\epsilon(\rho_0),$$

where $f_\epsilon(\rho)$ is the fraction of the volume having a density larger than $\rho$. This expression with the volume fraction can be converted
density of $1 \text{ cm}^{-3}$ is typical for normal galaxy disks. The second has a higher density, similar to the inner parts of galaxies, and the third has the same high density and a Mach number that is twice the effective value in Wada & Norman (2001), as determined from the relation between dispersion and Mach number in Nordlund & Padoan (1999), which is $\sigma = (\ln(1 + 0.25M^2))^{1/2}$ for Mach number $M$. These latter two cases represent moderately strong starburst regions. The fourth case has a Mach number that is half the Wada & Norman value, and a density that is also small. This case applies to quiescent regions with low-intensity star formation, such as the outer parts of galaxies, parts of dwarf galaxies, and low surface brightness galaxies. Wada & Norman (2007) simulate self-gravitating disks and find lognormal density PDFs with $\sigma$’s that increase from 2.4 to 3.0 as the initial density (within 10 pc of the midplane) increases from 5 to $50 M_\odot \text{ pc}^{-3}$. Our $\sigma$ are in this range. The equilibrium peak densities in the Wada & Norman simulations are all about $\sim 2 \text{ cm}^{-3}$, which is also comparable to the values used here.

The lognormal nature of the assumed density PDF is not critical to the conclusions of this paper. Any density PDF with an extended tail at high density and a breadth that increases with Mach number will give the same correlations between bound cluster fraction, Mach number, and pressure. Similarly, any PDF that has a smaller slope near the average density than at high density (like a lognormal) will give our additional dependence of the bound cluster fraction on the average density. The main point is that when the slope of the density PDF is relatively shallow near the critical density for star formation, bound clusters can form with a wide range of average densities. These types of PDFs will also produce faint fuzzies in the limit of high dispersion, as these clusters result from highly efficient star formation at a low average cloud core density. The absolute calibration of the density PDF is also not critical to our model. We could equally well consider PDFs for cluster-forming cores, where the average density is $\sim 10^2 \text{ cm}^{-3}$, and the critical density for “final” collapse is $\sim 10^3 \text{ cm}^{-3}$. The essential points are that (1) for all of these PDFs, the mass fraction of the “final” collapsed cores, and therefore the efficiency of star formation in a particular region, $\epsilon(\rho)$, increases with the average density of the region (an implication of hierarchical structure); and (2) the density PDF is more or less flat at the critical density of star formation, depending on some physical variable, taken here to be the Mach number and/or average density. Wada & Norman (2007) point out that there is no single Mach number in their simulations but a range of values, and still the density PDF is lognormal. They find the primary dependence of $\sigma$ to be on the initial disk density. The origin of this $\sigma$ is not important here, only the effect that variable $\sigma$ and $\rho_0$ have on the mass fraction of dense gas.

According to the equations that define $\epsilon(\rho)$ for the lognormal model, there is a minimum value in each $\epsilon(\rho)$ curve that occurs where $d\ln f_{\mu}/d\ln \rho = -0.5$. At lower density, the curve $f_{\mu}(\rho)$ flattens and $\epsilon(\rho)$ is dominated by $\rho^{-1/2}$, so it increases with decreasing $\rho$. At higher density, $\epsilon(\rho)$ is dominated by the Gaussian tail in $f_{\mu}(\rho)$, so it increases with increasing $\rho$. The low-density behavior of $\epsilon(\rho)$ is not realistic, because the low-density part of $f_{\mu}$ is not likely to remain a lognormal. This part corresponds to the low-density intercloud medium, and the physical processes there are different from those in dense gas. It may be controlled by supernova cavities, for example. In Wada & Norman (2001), the low-density part of $f_{\mu}$ was not a lognormal. Thus, we consider only the increase in $\epsilon(\rho)$ with $\rho$ to be characteristic of dense gas involved with star formation.

For each of the cases considered in Figure 1, the efficiency increases with density, because hierarchical structure gives $n_c$ cores.
a higher filling factor at higher average densities. The mass fraction decreases with density, because only a small fraction of the matter is dense. For the fiducial case (solid line), an average galactic efficiency, \(\epsilon(\rho) = 0.01\) (e.g., Kennicutt 1998), is indicated by the cross on the left, and a typical efficiency for a dense star-forming core, \(~50\%\) at \(\rho_c \sim 10^2 m(H_2) cm^{-3}\), is indicated by the cross on the right. The vertical lines show typical ranges for efficiencies and densities in OB associations (\(\epsilon \sim 1\%-5\%\) at \(n \sim 10^{3.3} \text{ cm}^{-3}\)) and in the cluster-forming cores of OB associations (\(\epsilon \sim 10\%-30\%\) at \(n \sim 10^{4.5} \text{ cm}^{-3}\)).

The approximate value of the efficiency separating clusters that end up mostly bound after gas dispersal from clusters that become mostly unbound (\(\epsilon_{\text{cluster}}\)) is placed in Figure 1 at \(\epsilon(\rho) \sim 0.144\), which is the value of \(\epsilon(\rho)\) in the fiducial case at a gas density of \(n = 10^{4.55} \text{ cm}^{-3}\). This efficiency is shown by the dotted horizontal line; its exact value is not important here. The efficiency for bound cluster formation depends on the rate of gas dispersal and the initial velocities of the stars. Slower gas dispersal rates and smaller initial speeds compared to virial require lower efficiencies for boundedness (e.g., Verschueren 1990; Lada & Lada 2003; Boily & Kroupa 2003; Goodwin & Bastian 2006).

The basic trends in Figure 1 illustrate the important differences between cluster formation in various environments. First, the threshold efficiency occurs at lower average densities for starburst PDFs and at higher average densities for quiescent PDFs. That is, the density \(\rho\) at which \(\epsilon(\rho) = 0.144\) in the figure is smaller than \(10^{4.55} m(H_2) cm^{-3}\) in the first case and larger than \(10^{4.55} m(H_2) cm^{-3}\) in the second. When the average density for \(\epsilon > \epsilon_{\text{cluster}}\) is much lower than the inner core density \(n_c\), a high fraction of the gas mass can form stars with \(\epsilon > \epsilon_{\text{cluster}}\). Long-lived clusters should be easier to form in this case. Because this is the starburst limit, the result is in qualitative agreement with the observations by Larsen & Richtler (2000).

Figure 2 shows this result by plotting as a solid line the density at which \(\epsilon(\rho) = 0.144\) versus the dispersion \(\sigma\) of the lognormal density PDF, for an average density of 1 cm\(^{-3}\). The threshold density value of \(10^{4.55} cm^{-3}\) when \(\sigma = 2.3\) from Figure 1 is one point on the curve. Higher dispersions correspond to lower threshold densities for bound cluster formation. Figure 2 also shows as a dashed line the fraction of the mass, \(f_M\), at densities greater than the threshold density as a function of the dispersion. Larger dispersions lead to a larger fraction of the mass in bound clusters, mostly because the threshold density decreases and more gas mass is above the threshold.

When the average density for \(\epsilon > \epsilon_{\text{cluster}}\) is comparable to \(n_c\), the mass fraction of gas with highly efficient star formation is very low. This case applies to low Mach numbers or low pressures and may explain why in some regions like the giant OB association NGC 604 in M33 there is a lot of star formation and thousands of young stars, but few bound clusters (Maiz-Appellanz 2001).

We propose that the difference between the formation of clusters and the formation of loose OB associations or stellar groupings of the same total mass and age depends mostly on the ratio of the characteristic density for stellar core formation, \(\rho_c\), to the density at which \(\epsilon(\rho) = \epsilon_{\text{cluster}}\). High-pressure gas has a high density ratio and should form a high proportion of stars in bound clusters. Low-pressure gas has a density ratio near unity and should form a low proportion of stars in bound clusters with most forming in loose OB associations.

Some GMCs in the solar neighborhood have substantial populations of protostars outside the dense clusters (Megeath et al. 2004). Jørgensen et al. (2006, 2007) found that 30\%–50\% of the protostars in Perseus are outside the two main clusters. This is to be expected in hierarchical clouds, because not all of the star-forming clumps are in the cluster-forming cores. In a high-pressure region, a hypothetical Perseus-like collection of protostars would form at a higher average density, putting the individual stars closer together and giving them a higher fraction of the total cloud mass where they form. As a result, they would be more tightly bound to each other when the gas leaves. In a low-pressure region, the cores would be farther apart with relatively more intercore gas, and they would be more likely to disperse into the field along with the gas. In both cases, a fixed fraction of the turbulence-compressed cores could form stars, but the mass fraction represented by these cores can be high or low, depending on the Mach number and density. The conditions in the Perseus cloud are apparently intermediate between these two limits.

Johnstone et al. (2004) found that 2.5\% of the \(\rho\) Ophiuchi cloud mass is in dense submillimeter cores, while Kirk et al. (2006) found that 0.4\% of the mass in the Perseus cloud is at \(A_V > 10\) mag. Enoch et al. (2006) observed <5\% of the Perseus cloud mass in 1.1 mm cores. These small fractions are consistent with the turbulent fragmentation model of Figure 1, as shown by the decreasing \(f_M(\rho)\) lines; recall that \(f_M\) is the fraction of the total mass at density larger than \(\rho\) [while \(\epsilon(\rho)\) is the fraction of the mass having a density greater than fixed \(\rho_c\) in regions of density \(\rho\)]. At high density, \(f_M(\rho)\) is low, while \(\epsilon(\rho)\) is high. If each submillimeter core evolves on its own dynamical time, and the surrounding cloud does the same as it forms new cores, then the cloud will have a number of core-forming events equal to the square root of the ratio of densities, which is \(~10\). Thus, the net efficiency of core formation can build up to several tens of percent over time, and the final efficiency of star formation might be \(~10\%\) or more, considering the efficiency inside each submillimeter core and the likelihood that some cores will disperse without forming a star (e.g., Vázquez-Semadeni et al. 2005b). A large number of core crossing times is not necessary for this to happen. The
increasing \( \epsilon(\rho) \) curves in Figure 1 illustrate qualitatively a second point made by these authors, and by Jorgensen et al. (2007) as well for Perseus, that the mass fraction of cores becomes high in the generally denser regions of the cloud; i.e., at high extinction. They find an \( A_V \) extinction threshold for the occurrence of submillimeter cores equal to 5–7 mag (15 mag in Johnstone et al. 2004). Figure 1 suggests that it is not the extinction, per se, which produces this correlation, but the hierarchical structure of clouds.

A second result from Figures 1 and 2 is that a bound cluster can have a lower average density when the Mach number or ambient density is higher, i.e., when the PDF is flatter or shifted to higher \( \rho \). This suggests an explanation for faint fuzzies: they are normal clusters that form at the limiting lower density for boundedness in high-PDF dispersion regions. The lowest density for a bound cluster is lower when the \( \epsilon(\rho) \) curve in Figure 1 is flatter at \( \rho_c \). This makes the density at \( \epsilon(\rho) = \epsilon_{\text{cluster}} \) lower and corresponds to a broader PDF dispersion \( \sigma \) in Figure 2. For an isothermal gas, a broader dispersion corresponds to a higher Mach number. Faint fuzzies also require low-background tidal forces, or they would have broken apart by now. Usually, high Mach numbers correspond to high critical tidal densities, because both occur in starburst regions. But faint fuzzies require an odd combination: high Mach numbers (or broad PDF dispersions) and low tidal densities. This is possible in highly shocked clouds that occur in remote regions of a galaxy. For example, during a galaxy collision, shocked clouds in the outer regions (or direct ISM impacts in the outer regions) can have Mach numbers approaching the pair orbital speed, but at these locations, the local tidal forces are fairly low. This result fits well with the model of faint fuzzies by Burkert et al. (2005), who suggested that they formed in galaxy collisional rings. Such rings are indeed highly shocked regions in the outer parts of galaxies, where the tidal density is fairly low. Burkert et al. did not explain how such conditions would produce faint fuzzies, however. Here, they result naturally in a turbulent ISM model as a consequence of the flattening of the density PDF at higher Mach number.

3. MULTIPLE STAR FORMATION EVENTS INSIDE THE HIERARCHIES

The observed hierarchies of star formation, such as subgroups inside OB associations, which, in turn, are inside star complexes, have the property that the duration of star formation increases approximately as the square root of size (Efremov & Elmegreen 1998) and is always in the range of 1–2 crossing times (Elmegreen 2000, 2007). The smaller regions come and go relatively quickly, while the larger regions form stars, and there are many small regions within the spatial bounds and during the total time span of the large region. When the duration of star formation is always about a crossing time, then the number of events at the density \( \rho \) is proportional to \( (G\rho)^{1/2} \). These small-scale events are not all at the same place, but they move around inside the large-scale event as clouds get dispersed and cloud envelopes get triggered.

For each large-scale event with cloud mass \( M \sim 10^7 M_\odot \) in main galaxy disks and average density \( \rho_0 \), the total mass of all the gas in star-forming events at any one time at a density greater than \( \rho \) is \( M_{\text{sd}}(\rho) \), according to the definition of \( f_{\text{sd}} \) in the previous section. The star formation efficiency in each of these high-density regions is \( \epsilon(\rho) \). The number of events of star formation at the density \( \rho \) during the lifetime of the larger scale cloud is \( (G\rho)^{1/2}/(G\rho_0)^{1/2} \), according to the previous paragraph. The product of these three terms is the total stellar mass formed in all the fragments of the large-scale cloud that ever had a density \( \rho \). This product is \( M_{\text{sd}}(\rho)\epsilon(\rho)(\rho/\rho_0)^{1/2} \). Substituting for \( \epsilon(\rho) \) from equation (2) and using the identity \( f_M(\rho_c) = f_\rho(\rho_c)\rho_c/\rho_0 \) from the discussion just before equation (2), all of the terms involving density cancel, and we simply get \( e_0M \). This result confirms the basic nature of hierarchical star formation: the mass of stars forming in each level of the hierarchy is the same and equal to the total mass of stars formed. This is a property of hierarchies, because stars that form in the densest level are the same stars as those that form in the lower density levels that contain the densest levels. The important point for star formation is that this hierarchy is in both space and time. The answer comes out correctly only when the multiple events of star formation at each density are considered.

4. STELLAR MIXING AND THE FORMATION OF CLUSTERS

If all star-forming regions last for about a crossing time before they disperse and reform into other star-forming regions inside the same larger scale regions (Elmegreen 2000, 2007), then there is an equal opportunity for stars on all scales to mix in their orbits and become smooth clusters. This means that the oldest stars in all groupings, ranging from embedded clusters to giant star complexes, should be fairly well mixed, while the youngest stars in these groupings should still be hierarchical. The scale dependence of the mixing fraction has never been observed systematically, but general observations suggest that it is approximately true. For example, the youngest objects in the Orion nebula, the proplyds, still cluster near \( t^\odot \). Ori C. while the slightly older stars are more dispersed (Smith et al. 2005b; Bally et al. 2005). Similarly, the youngest objects in Gould’s Belt, which could be the nearest star complex, are clustered into the local OB associations such as Orion, Perseus, and Sco-Cen, while the oldest objects, the Pleiades and Cas-Tau associations, are now dispersed into stellar streams (e.g., Elias et al. 2006). The large regions never get self-bound, unlike the small regions, because the efficiency of star formation is always small on large scales, as shown in § 2. Still, these large regions mix through the action of random stellar drift.

Galactic shear pulls apart the large-scale groupings over tens of millions of years, which is the crossing time. The shear time is independent of size and equal to the inverse of the Oort A constant, while the crossing time increases with size approximately as the square root. This means there is a sufficiently large size where the crossing time equals the shear time, and at larger sizes the crossing time exceeds the shear time. This critical size is always about the scale height in the galaxy for Toomre \( Q < 1 \) (Elmegreen & Efremov 1996). Larger regions look like star streams or flocculent spiral arms because shear dominates, while smaller regions look clumpy like star complexes and OB associations because the internal dynamics dominates.

5. DISCUSSION

Stars form bound clusters where the total efficiency is high. The efficiency should be proportional to the mass fraction of a cloud in the form of dense cores where the individual stars form. This mass fraction increases with the cloud density in hierarchical clouds. It also increases with the Mach number of the turbulence, because stronger shocks at higher Mach numbers compress the gas to a wider range of densities. As a result, a lognormal density PDF becomes flatter below the threshold density of star formation, and this means the mass fraction is higher at each density below this value. When the Mach number is high (really, when the dispersion of the density PDF is high), the efficiency is high even at a fairly low average cloud density, and so a high fraction of star formation ends up bound. An extreme example of this
trend is the type of cluster called a faint fuzzie. We propose that faint fuzzies form in moderately low density clouds with moderately high Mach numbers. The Galactic tidal ring environment proposed by Burkert et al. (2005) is an example of a region that would have such clouds.

The mass fraction of clumps at a particular high density of star formation also increases with the average density of the ISM, because then the whole density PDF shifts toward higher values. The combination of high densities and high Mach numbers, characteristic of starburst regions, makes for a high fraction of star formation in bound clusters.

On the other hand, relatively low Mach numbers and/or low densities should produce stars in a more dispersed way, as in the low-density regions of molecular clouds or in low surface brightness galaxies and regions of galaxies. This combination of parameters corresponds to a low ISM pressure, so we infer that low-pressure regions, which means those with a low gas column densities and low star formation rates per unit area, should produce relatively fewer bound clusters and relatively more unbound associations.

There is a physical explanation for the trends discussed here. Consider a moderately low density cloud with a low turbulent Mach number. The compressions inside that cloud will be modest, and most of them will not reach a level of density or enhanced magnetic diffusion rate that allows gravitational collapse before the compression ends. Then few stars will form and the efficiency will be low (unless the cloud continues to makes these weak compressions for a very large number of crossing times, which seems unlikely). Now consider this same cloud with the same size and average density but with a higher Mach number (this will require a higher ISM pressure). The stronger compressions will more easily produce high-density cores in which gravity is important and magnetic diffusion is fast. More stars will form, and the efficiency will be high. The only difference between these two examples is the Mach number, which affects the range of densities in the compressed regions. In terms of the density PDF, higher Mach numbers produce a broader PDF at the same average density. In terms of star formation in bound clusters, high-pressure regions having clouds with high Mach number produce a higher fraction of their stars in bound clusters.

The model predicts that young bound clusters in starburst regions, or in regions of high ISM pressures or Mach numbers, will have a wider range of densities than bound clusters in more quiescent, low-pressure regions. This is partly because the cloud densities should be higher in starburst regions, so the resulting clusters can be denser overall, but it is also because the lower density parts of starburst clouds can form stars with a high efficiency, leaving bound clusters when the gas leaves rather than dispersed OB associations. As there is generally more mass at low density than at high density, the net distribution of cluster density could shift toward lower values when the ISM density PDF is broad. According to the model, this density shift is relative to the mean ISM density, and it should be measured only before significant cluster expansion. Low surface brightness galaxies should make a preponderance of unbound OB associations rather than bound clusters, and the young clusters they form should have a relatively narrow range of densities compared to normal.

6. SUMMARY

Hierarchical structure in both gas and young stars is evident from fractal analysis, correlation functions, and power spectra (see references in § 1) and is probably the result of turbulence compression and self-gravity. Stars form in the densest part of the gas. In regions where the individual star-forming clumps represent a high fraction of the mass, the cluster that forms has a good chance of remaining self-bound after the gas leaves. Such regions typically have densities of \( \sim 10^4 \) cm\(^{-3}\) or more in the solar neighborhood. We suggest here that star formation at a high average density like this automatically has a high efficiency, because the individual star-forming clumps are only an order of magnitude or two denser. This follows entirely from the hierarchical ISM. In such a medium, clumps of any density cluster together with clumps of a similar density, and the mass fraction of these clumps increases as the average density of the surrounding region increases. As a result, low-density clouds like GMCs form stars with a low total efficiency, and high-density cloud parts like GMC cores form stars with a high total efficiency, leaving bound or marginally bound clusters.

The density where the efficiency first becomes high decreases as the Mach number of the turbulence increases and as the average density of the ISM increases. Consequently, high-pressure regions should place a higher fraction of their stars in bound clusters, while low-pressure regions should preferentially make unbound stellar groupings. Regions with moderately low density and moderately high Mach number should be able to make faint fuzzies, which are low-density bound clusters. Regions with high densities and low Mach numbers should make extremely dense clusters.

The masses of the high-density regions and of the star clusters that form in them have not been specified in this derivation of \( \epsilon (\rho) \). These masses should vary over a wide range and be distributed approximately as \( dN/dM \sim M^{-2} \) because of the hierarchical nature of the gas (Fleck 1996; Elmegreen & Efremov 1997; Elmegreen 2002a). Thus, bound and dispersed stellar groupings, as well as faint fuzzies, should have the usual \( M^{-2} \) mass distributions.

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