Reduced modified Chaplygin gas cosmology

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In this paper, we study a cosmology with including reduced modified Chaplygin gas (RMCG) fluid. For this fluid, it has an equation of state, \( p = A\rho - B\rho^{\frac{1}{2}} \) that is reduced from the modified Chaplygin gas \( p = A\rho - B\rho^{-\alpha} \). Taking the different values of model parameter \( A \), it is found that the dark-cosmology models can be realized in RMCG fluid. By discussing the evolutions of cosmological quantities and comparing with the recent observational data, we investigate the rationality of dark models in RMCG. It is shown that the RMCG \((A = 0 \text{ or } A = 1)\) unified model of dark energy (DE) and dark matter (DM) should be abandoned. The RMCG \((A = \frac{1}{3})\) unified model of DE and dark radiation (DR) and the dynamical dark-energy RMCG \((A < 0)\) model can be considered as the candidates to interpret the accelerating universe. Both of them(180,750),(930,984) have some attractive features.

For examples, RMCG \((A = \frac{1}{3})\) is a good model to provide the origin of DR and DE, and it is favored by the objective Akaike information criteria; RMCG \((A < 0)\) can achieve the dynamical quintessence and phantom models, where the evolutional universe is not sensitive to the variation of model-parameters values, etc.

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I. Introduction

Current observations present some questions to the standard Big Bang model of cosmology. Some examples of these unsolved questions are that some unknown components in universe are indicated. For examples, observations on rotation curve of galaxy \(^{1}\) are directly related to the amount of pressureless matter, where dark matter (DM) is proposed; observations of supernovae of type Ia \( ^{2,3} \) point out an accelerating universe at late time, which is usually interpreted as the existence of a new ingredient, called dark energy (DE); Cosmic Microwave Background (CMB) results from the 9-year Wilkinson Microwave Anisotropy Probe (WMAP) combined with baryon acoustic oscillation (BAO) and Hubble constant measurements from the Hubble Space Telescope, provide a constraint on the effective number of relativistic degrees of freedom \( N_{\text{eff}} = 3.84 \pm 0.4 \), whose bounds can be an indication for the presence of extra dark radiation (DR) component at 95% confidence level \( ^{4,5} \). So, it is valuable to search for origins of these dark sectors. Plenty of studies on these dark sectors, comprising DM, DE and DR, have been done, such as seeking

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for candidates of cold and warm DM \cite{6,7}, discussing on cosmological constant and other dynamical DE \cite{8–20} including quintessence, phantom, holographic, agegraphic and quintom, and exploring origins of DR coming from decayed particle \cite{21,22}, interacting DM \cite{23}, Horava-Lifshitz gravity \cite{24,25}, extra dimensions \cite{26}, etc.

Besides dark sectors deduced from cosmic observations, visible constituents of baryon and radiation naturally exist in universe. Observations \cite{27} suggest that in universe about 70\% energy density are negative-pressure DE, about 30\% are pressureless matter (or called dust) including DM and baryon, and there are a small quantity of radiation components including photon, neutrino as well as additional relativistic species. Someone propose that unified models composed by DE and DM can be expressed in one cosmic fluid, such as generalized Chaplygin gas \cite{28–30}, modified Chaplygin gas (MCG) \cite{31,32} and so on. In this paper we perform a new search for dark sector from reduced MCG model. Here the unified model of dark sectors and the interested properties of dark energy can be given, such as the DE and DR can be unified in one fluid, and the evolutions of dynamical dark-energy model are not sensitive to the variation of model-parameter values, etc.

II. Dark models in RMCG cosmology

MCG model has been widely studied. In this model, MCG fluid can be used to explain cosmic inflation \cite{33–36} or seen as a unified model of DE and DM \cite{37–40}. We consider an equation of state (EOS)

\[ p = A \rho - B \rho^{1/2} \]  

(1)

that is reduced from the modified Chaplygin gas \( p = A \rho - B \rho^{-\alpha} \), with constant model parameter \( \alpha = -\frac{1}{2} \). We call this model as reduced modified Chaplygin gas (RMCG). Using Eq. (1) to study a no time-singularity emergent universe has been done in Refs.\cite{41–44}. Here RMCG fluid is studied by regarding it as the dark components.

Using the energy conservation equation \( \frac{dp}{dt} = -3H(\rho + p) \), we have the energy density of RMCG fluid,

\[ \rho_{RMCG}(a) = \left[ \frac{B}{(1 + A)} + \frac{C}{1 + A} \right] a^{-3(1 + A)/2} \]

\[ = \rho_0RMCG \left[ A_s^2 + (1 - A_s)^2 a^{-3(1 + A)} + 2A_s(1 - A_s)a^{-3(1 + A)} \right] \]

\[ = \rho_1 + \rho_2 a^{-3(1 + A)} + \rho_3 a^{-3(1 + A)/2} , \]

(2)

where \( C \) is an integration constant, \( A_s = \frac{B}{1 + A} \rho_{RMCG}^{-1/2} \rho_0 \), \( \rho_1, \rho_2 \) and \( \rho_3 \) are current values of three energy densities of RMCG fluid, respectively. From Eq. (2), we can get some unified models by taking different values of parameter \( A \). For examples, fixing \( A = 0 \), we have a unified model of DE, DM and a cosmic component \( (w = \frac{p}{\rho} = -1/2) \); for \( A = 1 \), RMCG unified fluid includes DE, DM and stiff matter \( (w = 1) \); for \( A = 1/3 \), a unified model of DE, DR and exotic component \( (w = -1/3) \) is given; taking \( A \) as a free parameter with \( A > 0 \) \( (A \neq 0, 1, \frac{1}{3}) \), we read a unified model of DE and other unknown components; and for \( A \) as a free parameter with \( A < 0 \), RMCG fluid plays a role of dynamical DE \( (A < -1, \text{ its behavior is a phantom}) \); for \( 0 > A > -1 \), it is a quintessence.). Considering that there are the RMCG fluid and other known components (baryon, radiation) in universe, one has the Friedmann equation

\[ H^2(a)/H_0^2 = \Omega_0 a^{-3(1 + w_i)} + \Omega_{RMCG}(a) \]

\[ = \Omega_0 a^{-3(1 + w_i)} + (1 - \Omega_0) \left[ A_s^2 + (1 - A_s)^2 a^{-3(1 + A)} + 2A_s(1 - A_s)a^{-3(1 + A)} \right] \]

\[ = \Omega_0 a^{-3(1 + w_i)} + \Omega_0 + \Omega_0 a^{-3(1 + A)} + \Omega_0 a^{-3(1 + A)/2} , \]

(3)
where \( \Omega_{0i} \) is the current dimensionless energy density of anyone known ingredient in universe, \( \Omega_{01}, \Omega_{02} \) and \( \Omega_{03} \) correspond to three current dimensionless energy densities given by RMCG fluid. \( a \) is the scale factor that is related to cosmic redshift by \( a = \frac{1}{1+z} \). In the following, we show expressions of some basic cosmological parameters in RMCG model.

1. Adiabatic sound speed for RMCG fluid, \( c_s^2 = \frac{\delta p}{\delta \rho} = A - \frac{(1+A)A_\delta}{A_\rho + (1-A_\rho)a^{-2(1+A)}} \). A small non-negative sound speed for matter component is necessary for forming the structure of universe.

2. Equation of state for RMCG fluid, \( w = \frac{\rho}{\rho_\delta} = A - \frac{(1+A)A_\delta}{A_\rho + (1-A_\rho)a^{-2(1+A)}} \). To obtain an accelerating expanded universe at present, the current value of EOS for RMCG fluid \( w_0 < -\frac{1}{3} \) should be appeared. Assuming \( w_0 \) in quintessence region or in phantom region, the theoretical constraint on model parameter \( A_\delta \) in RMCG cosmology is listed in table I following taking the different values or intervals for model parameter \( A \).

3. Deceleration parameter \( q(a) = -\frac{\delta^2 a}{a^2} \). An expanded universe from deceleration to acceleration is consistent with the current cosmic observations.

4. Dimensionless density parameter \( \Omega_j = \rho_j/\rho_c = \frac{3H^2}{8\pi G} \) is the critical density, and \( j \) denotes anyone component in universe.

| \( A \) | \( -1 < w_0 < -\frac{1}{3} \) | \( 1 > A_\delta > \frac{1}{3} \) | \( 1 > A_\delta > \frac{1}{1+3A} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) |
|-------|-----------------|-----------------|-----------------|-----------------|
| \( A = 1 \) | \( 1 > A_\delta > \frac{1}{3} \) | \( 1 > A_\delta > \frac{1}{1+3A} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) |
| \( A = 0 \) | \( 1 > A_\delta > \frac{1}{3} \) | \( 1 > A_\delta > \frac{1}{1+3A} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) |
| \( -1 < A_\delta < 0 \) | \( 1 > A_\delta > \frac{1}{3} \) | \( 1 > A_\delta > \frac{1}{1+3A} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) |
| \( A < -1 \) | \( 1 > A_\delta > \frac{1}{3} \) | \( 1 > A_\delta > \frac{1}{1+3A} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) | \( 1 > A_\delta > \frac{1+3A}{2} \) |

TABLE I: Theoretical constraint on RMCG model parameter \( A_\delta \) by assuming \( -1 < w_0 < -\frac{1}{3} \) (quintessence) and \( w_0 < -1 \) (phantom), for taking the different values or intervals for parameter \( A \) at prior.

### A. Should the unified model of DE and DM be ruled out in RMCG cosmology

![Evolutions of adiabatic sound speed, EOS w, deceleration parameter q, and values of dimensionless density parameter, for fixing A = 0 in RMCG model. Dot lines represent the RMCG (A=0) model, and solid lines correspond to the_LCDM.](image)

FIG. 1: Evolutions of adiabatic sound speed \( c_s^2 \), EOS \( w \), deceleration parameter \( q \), and values of dimensionless density parameter, for fixing \( A = 0 \) in RMCG model. Dot lines represent the RMCG \( (A=0) \) model, and solid lines correspond to the \( \Lambda \_CDM \).

For \( A = 0 \) and \( A = 1 \) in RMCG fluid, unified models of DE and DM can be given. RMCG fluid \( (A = 0) \) plays a role of unified model including DE, DM and a new hinted dark ingredient \( (w = -1/2) \). We have the Friedmann equation

\[
H^2(a)/H_0^2 = \Omega_{0b}a^{-3} + \Omega_{0r}a^{-4} + (1 - \Omega_{0b} - \Omega_{0r})[A_\delta^2 + (1 - A_\delta)^2a^{-3} + 2A_\delta(1 - A_\delta)a^{-3/2}]
= \Omega_{0b}a^{-3} + \Omega_{0r}a^{-4} + \Omega_{01} + \Omega_{02}a^{-3} + \Omega_{03}a^{-3/2}.
\]  

(4)
\(\Omega_{m0}\) and \(\Omega_{r0}\) represent the fractional energy densities for baryon and radiation (including all relativistic particles, such as CMB photon \(\Omega_{r0}\), neutrino \(\Omega_{r0}\), etc.). From Eq. (4), obviously it has current dimensionless dark-energy density \(\Omega_{\Lambda} = \Omega_{01} = (1 - \Omega_{m0} - \Omega_{r0})A_s^2\), dark-matter density \(\Omega_{0dm} = \Omega_{02} = (1 - \Omega_{m0} - \Omega_{r0})(1 - A_s)^2\), and an unfound component \(\Omega_{0u} = \Omega_{03} = 2(1 - \Omega_{m0} - \Omega_{r0})A_s(1 - A_s)\).

Adopting current values \(\Omega_{r0} \sim 0\), \(\Omega_{m0} = 0.05\) and \(\Omega_{0m} \in (0.2, 0.4)\), after calculation one has \(A_s \in (0.39, 0.6)\), \(\Omega_\Lambda \in (0.15, 0.34)\) and \(\Omega_{0u} \in (0.45, 0.46)\). It is obvious that, for existence of \(\Omega_{0u}\), the value of DE density is smaller than observations. Taking \(a = 1\) in Eq. (4), one has \(\sqrt{\Omega_{\Lambda}} = \sqrt{1 - \Omega_{m0} - \Omega_{r0} - \Omega_{0dm}}\). According to this relation, the values of \(\Omega_{\Lambda}\) and \(\Omega_{0m}\) are illustrated in Fig. (1), where one can read the deviation of density-parameter values in RMCG model from \(\Lambda\)CDM case. Furthermore, when we take \(\Omega_{r0} = 0.3\), \(A_s \sim 0.49\) and \(\Omega_{\Lambda} \sim 0.23\) are solved.

| Density parameter | Explicit form | Parameter value | EOS |
|-------------------|---------------|-----------------|-----|
| \(\Omega_{m0}\)   | \(\Omega_{m0}\) | ---             | \(w = 1/3\) |
| \(\Omega_{r0}\)   | \(\Omega_{r0}\) | 0.05            | \(w = 0\) |
| \(\Omega_{0dm}\)  | \((1 - \Omega_{m0} - \Omega_{r0})(1 - A_s)^2\) | (0.15, 0.35) | \(w = 0\) |
| \(\Omega_{\Lambda}\) | \((1 - \Omega_{m0} - \Omega_{r0})A_s^2\) | (0.15, 0.34) | \(w = -1\) |
| \(1 - \Omega_{\Lambda} - \Omega_{0dm} - \Omega_{m0} - \Omega_{r0}\)(1 + \(A_s\)) | (0.45, 0.46) | \(w = -1/2\) |

TABLE II: Values of dimensionless density parameters in RMCG \((A = 0)\) cosmology.

Analyzing the evolution of deceleration parameter in RMCG \((A = 0)\) model, a universe from decelerated expansion to accelerated expansion can be emerged, with the larger current value \(q_0 \in (-0.355, -0.056)\) than in the standard \(\Lambda\)CDM cosmology \(q_0 \in (-0.7, -0.4)\), as plotted in Fig. (1). In Fig. (1) the parameter values \(A_s \in (0.39, 0.49, 0.6)\) are respectively used, corresponding to \(\Omega_{0m} \in (0.2, 0.3, 0.4)\). For plotting EOS \(w\), a negative-pressure RMCG fluid is obtained. In addition, according to the shapes of \(c_s^2\) in Fig. (1) a negative sound speed is appeared in this RMCG fluid. Since this unified fluid include dust component, the negative sound speed will induce the classical instability to the system at structure form, where the perturbations on small scales will increase quickly with time and the late time history of the structure formations will be significantly modified \([43]\). Then it seems that, this RMCG \((A = 0)\) model is not a good one.

FIG. 2: Evolutions of \(c_s^2\), \(w\), \(q\) and \(\Omega_i\) versus redshift \(z\) by taking \(A = 1\) in RMCG model.

For \(A = 1\), the RMCG fluid is comprised of DE, DM and stiff matter \((w = 1)\). Friedmann equation is written as,

\[
H^2(a)/H^2_0 = \Omega_{0b}a^{-3} + \Omega_{0r}a^{-4} + (1 - \Omega_{m0} - \Omega_{r0})(A_s^2 + 2A_s(1 - A_s)a^{-3} + (1 - A_s)^2a^{-6}) \\
= \Omega_{0b}a^{-3} + \Omega_{0r}a^{-4} + \Omega_{01} + \Omega_{02}a^{-3} + \Omega_{03}a^{-6}.
\]  

(5)
From Eq. (5), one gets $\Omega = \Omega_{01} = (1 - \Omega_{0b} - \Omega_{0v})A_1^2$, $\Omega_{0dm} = \Omega_{02} = 2(1 - \Omega_{0b} - \Omega_{0v})A_s (1 - A_s)$ and $\Omega_{0s} = \Omega_{03} = (1 - \Omega_{0b} - \Omega_{0v})(1 - A_s)^2$. Taking $\Omega_{0m} \in (0.2, 0.4)$, we receive $A_s \in (0.76, 0.91)$, $\Omega_{0} \in (0.55, 0.79)$ and $\Omega_{0s} \in (0.01, 0.05)$, as listed in Table. For this case, $\Omega_{0m} = 0.3$ gives $\Omega_{0} = 0.67$.

| Density parameter | Explicit form | Parameter value | EOS |
|-------------------|--------------|-----------------|-----|
| $\Omega_{0v}$    | $\Omega_{0v}$ | $\sim$          | $w = 1/3$ |
| $\Omega_{0b}$    | $\Omega_{0b}$ | 0.05            | $w = 0$ |
| $\Omega_{0dm}$   | $2(1 - \Omega_{0b} - \Omega_{0v})A_s (1 - A_s)$ | (0.15,0.35) | $w = 0$ |
| $\Omega_{0s}$    | $(1 - \Omega_{0b} - \Omega_{0v})A_s^2$ | (0.55,0.79) | $w = -1$ |
| $1 - \Omega - \Omega_{0dm} - \Omega_{0b} - \Omega_{0v}$ | $(1 - \Omega_{0b} - \Omega_{0v})(1 - A_s)^2$ | (0.01,0.05) | $w = 1$ |

TABLE III: Values of dimensionless density parameters in RMCG ($A = 1$) cosmology.

Fig. 2 illustrates the evolutions of adiabatic sound speed, EOS, deceleration parameter and dimensionless energy density in RMCG ($A = 1$). As we can see, the EOS of RMCG fluid can realize a transition from positive value to negative value. Correspondingly, a transition from deceleration-expansion universe to acceleration-expansion universe can be satisfied. In addition, the RMCG unified fluid do not have a negative adiabatic sound speed. But it has other problems we have to face, such as (1) deceleration parameter is $q > \frac{1}{2}$ at high redshift, which is not satisfied with the matter dominated universe $q \leq \frac{1}{2}$. Matter dominated universe is necessary for requirement of structure formation. (2) Radiation dominated universe will not appear in this RMCG universe, for the existence of stiff matter. From these points, it seems that this model is not consistent with our understanding on current observational universe.

B. A unified model of dark energy and dark radiation in RMCG cosmology

Combined analysis of recent cosmological data hint the existence of an extra relativistic energy component (called dark radiation) in the early universe, in addition to the well-known three neutrino species predicted by the standard model of particle physics. The total amount of this extra DR component is often related to the parameter $N_{eff}$. $N_{eff}$ denotes the effective number of relativistic degrees of freedom, which is related to the energy density of relativistic particles by $\rho_{\nu} = \rho_{\gamma} \frac{(4/3)}{3} N_{eff}$, where $\rho_{\nu}$ and $\rho_{\gamma}$ represent the fractional energy densities for neutrino and CMB photon. The inclusion of entropy transfer between neutrinos and the thermal bath modifies this number to about $N_{eff} = 3.046 \pm 0.46$ [46, 47]. However, larger values of $N_{eff}$ are reported by the recent cosmic observations. Depending on the datasets used, constraint results on $N_{eff}$ are qualitatively changed. For instance, it is pointed out that the observed deuterium abundance D/H favors the presence of extra radiation [52, 53]: $N_{eff} = 3.90 \pm 0.44$. The analysis of Refs. [48, 49] that combined CMB data from 7-year WMAP with Atacama Cosmology Telescope (ACT) reports an excess $N_{eff} = 5.3 \pm 1.3$, and the addition of BAO and $H_0$ data improves this constraint, $N_{eff} = 4.56 \pm 0.75$ [48, 49]. CMB data from the 9-year WMAP combining with the South Pole Telescope (SPT) and the 3-year Supernova Legacy Survey (SNLS3) gives a non-standard value of $N_{eff} = 3.96 \pm 0.69$ [50, 51]. Recently, Planck Collaboration finds $N_{eff} = 3.52^{+0.48}_{-0.45}$ [27] for using Planck+BAO+$H_0$. And a combination of the Planck-satellite data combining with previous CMB data and Hubble constant measurements, Ref. [54] shows $N_{eff} = 3.62^{+0.50}_{-0.48}$, whose analysis is clearly suggesting the presence for dark radiation at 95% confidence level, here previous CMB data include low multipole polarization measurements from the 9-year WMAP data and high multipole CMB data from both the SPT and the
As shown in Eq. (6), dimensionless density parameters are related to the RMCG model parameter \( \Omega \) as \( \Omega_{\text{tot}} \) particle including photon, neutrino and dark radiation component, the total dimensionless density parameter is written as \( \Omega_{\text{eff}} \) curvature density \( \Omega_{\text{dr}} \) effective curvature density. Thus, in a non-flat universe the current curvature density is modified as \( \Omega_{\text{dr}} \) dilutes as \( a \to 0 \) just like the curvature density in a non-flat geometry, called effective curvature density. Thus, in a non-flat universe the current curvature density is modified as \( \Omega_{\text{dr}} + \Omega_{\text{eff}} \) besides the RMCG fluid, in Eq. (6) we supplement matter and radiation components, according to the current view of cosmic ingredients.

We note that this RMCG fluid is composed of CC dark energy, dark radiation and effective curvature component. As shown in Eq. (6), dimensionless density parameters are related to the RMCG model parameter \( A \). For values of these density parameters, they should be self-consistent with observed allowable values. Considering relativistic particle including photon, neutrino and dark radiation component, the total dimensionless density parameter is written as \( \Omega_{\text{tot}} = \Omega_{\text{ph}} + \Omega_{\text{neu}} + \Omega_{\text{dr}} = \Omega_{\text{ph}} [1 + \Omega_{\text{SM}}^0] / 1.0057 \) and \( \Omega_{\text{dr}} = \Omega_{\text{ph}} [1 + \Omega_{\text{SM}}^0] / 1.0057 \) [58].

Writing \( N_{\text{eff}} = N_{\text{eff}}^0 + \Delta N_{\text{eff}} \) (the standard value \( N_{\text{eff}}^0 = 3.04 \)), one sees \( \Omega_{\text{dr}} = \Omega_{\text{ph}} [1 + \Omega_{\text{SM}}^0] / 1.0057 \) \( \Delta N_{\text{eff}} \). On the other hand, for this RMCG model as a unification of DE and DR, it has \( \Omega_{\text{dr}} = \Omega_{\text{ph}} [1 + \Omega_{\text{SM}}^0] / 1.0057 \) \( (1 - A) \).

Taking \( \Omega_{\text{ph}} = 0.3 \) and \( \Delta N_{\text{eff}} = (0.5, 1, 2) \), the values of model parameter \( A \) and dimensionless density parameters can be calculated. The results are listed in Table IV. From this table, it is found that these values of density parameters including dark energy and curvature density are compatible to the current observations of [4, 27], where \( \Omega_{\text{de}} \) is about 0.7 and \( \Omega_{\text{dr}} \) is around zero. And for model parameter \( A \) = 1 (or \( A \) = 0.1), correspondingly, it has the effective curvature density \( \Omega_{\text{eff}}^0 \) > 0 (or \( \Omega_{\text{we}}^0 < 0 \)).

| \( \Delta N_{\text{eff}} \) | \( A \) | \( \Omega_{\text{ph}} \) | \( \Omega_{\text{we}}^0 \) |
|-----------------|--------|-------------|-------------|
| 0.5             | 0.9972 or 1.0028 | 0.6929 or 0.7080 | 0.0079 or -0.0080 |
| 1               | 0.9960 or 1.0040 | 0.6944 or 0.7056 | 0.0056 or -0.0056 |
| 2               | 0.9943 or 1.0057 | 0.6961 or 0.7039 | 0.0039 or -0.0039 |

Table IV: The values of parameter \( A \), \( \Omega_{\text{ph}} \) and \( \Omega_{\text{we}}^0 \) calculated by using \( \Delta N_{\text{eff}} \) values, here \( \Omega_{\text{ph}} = 0.3 \) is fixed.

Fixing different values of parameter \( A \), the behaviors of dimensionless density parameters and deceleration parameter are plotted in Fig. 8. A universe from decelerated expansion to accelerated expansion is obtained. Since the variable region of parameter \( A \) is small, bounded by the constraint values of \( \Delta N_{\text{eff}} \), the evolutions of deceleration parameter are almost the same for taking different values of model parameter \( A \). Here, the values of current deceleration parameter and transition redshift are, \( q_0 = -0.546_{-0.004}^{+0.004} \) and \( z_T = 0.668_{-0.003}^{+0.003} \), a narrow variable range. Fig.8 also illustrates the evolutions of adiabatic sound speed versus \( z \) for this RMCG unified fluid. Positive values of \( c_s^2 \) and \( w \) are converted to negative values with the evolution of universe. Since for this case at hand the RMCG
unified fluid do not include matter, the negative value of $c_s^2$ will not destroy the structure formation. Just as for cosmological constant dark energy, it has $c_s^2 = -1$. With requiring the negative pressure to produce an accelerating universe, a negative value of $c_s^2$ for dark energy is necessary, which is not inconsistent with the structure formation. For EOS $w$ of RMCG fluid in Fig.3 at late time we have $w < 0$ which can be responsibility to an accelerating universe, and at early time $w$ approaches to a value of radiation component $w = \frac{1}{3}$. According to above analysis the behaviors of these cosmological quantities in RMCG ($A = \frac{1}{3}$) model are accordant with the current understanding on our universe, which is a candidate for dark energy and dark radiation. At last, we note the values of $A_s > 0$ is not discussed for RMCG model at hand, since adiabatic sound speed and EOS will be divergent at some points (when $A_s = -(1 - A_s)a^{-\frac{1}{2}(1+A)}$).

C. RMCG fluid as dark energy

The unification of dark energy and dark matter (or dark radiation) have been discussed in above parts. In the following, we investigate other possible properties of RMCG fluid by taking other values of constant model parameter $A$ (except $A = 0, 1$ and $\frac{1}{3}$). For $A > 0$ ($A \neq 0, 1, \frac{1}{3}$), Eq. (2) states that RMCG fluid is composed of CC and other positive-pressure or negative-pressure components (depending on the concrete values of parameter $A$). We know nothing about these indeterminate components, such as their function in universe or their responsibility to observations. So, here we do not discuss this case. For $A < 0$, RMCG fluid plays a role of dynamical phantom or dynamical quintessence dark energy, as shown in follows. Assuming there are matter, radiation and RMCG fluid in universe, the Friedmann equation has forms

$$H^2(a)/H_0^2 = \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{0RMCG}[A_s^2 + (1 - A_s)^2 a^{-3(1+A)} + 2A_s(1 - A_s) a^{-3(1+A)}]$$

$$= \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{01} + \Omega_{02} a^{-3(1+w_2)} + \Omega_{03} a^{-3(1+w_3)}, \quad (7)$$

where $\Omega_{0RMCG} = 1 - \Omega_m - \Omega_r$, $\Omega_{01} = \Omega_{0RMCG}A_s^2$, $\Omega_{02} = \Omega_{0RMCG}(1 - A_s)^2$ and $\Omega_{03} = 2\Omega_{0RMCG}A_s(1 - A_s)$. Obviously, for taking $A < -1$ one has $w_2 = A < -1$ and $w_3 = \frac{A+1}{2} < -1$, so RMCG fluid is composed of CC and phantom dark energy, i.e. it plays a role of phantom-type dark energy; for taking $0 > A > -1$, RMCG is composed of CC and quintessence dark energy. For taking $A_s = 1$ or $A = -1$, a CC model is obtained from RMCG fluid. Since the theoretical constraint on current dimensionless density parameter is $0 < \Omega_{0j} < 1$, we have $0 \leq A_s \leq 1$ for RMCG.
As we can see from four upper figures in Fig. 4, the value of more near to the quintessence-type DE (\( \frac{1+3A}{3(1+A)} < A_s < 1 \)), in which EOS of RMCG locates at the quintessence region. For \( A < -1 \) and \( 0 < A_s < 1 \), EOS is the phantom type. For \( A_s > 1 \) in Table II the phantom-type DE (\(-1 < A < 0\)) and the quintessence-type DE (\(1 < A_s < \frac{1+3A}{3(1+A)}\), equivalently \(1 < A_s < \infty\)) are non-physical, which should be ruled out.

We discuss the evolutions of EOS of RMCG fluid. The dependence of EOS on model parameters are illustrated in Fig. 4. From this figure, we can read several properties for EOS of RMCG dark energy, such as (1) CC, quintessence and phantom dark energy can be realized in this RMCG fluid by taking different values of model parameters \( A \) and \( A_s \); (2) According to four upper figures in Fig. 4 for phantom (two upper-right figures) it has the result that the less values of parameters \( A \) and \( A_s \), the less influence on EOS from parameter \( A_s \), the less value of parameters \( A \) and \( A_s \); (3) As we can see from four upper figures in Fig. 4 the value of more near to \( A = -1 \) is taken (such as two middle figures relative to two-side figures), the less influence on EOS from parameter \( A_s \) is happened. Also, from four lower figures in Fig. 4 we have the results that the value of more near to \( A_s = 1 \) is taken, the less influence on EOS from parameter \( A \) is happened (it has the smaller varied region of \( w \) for using different values of \( A \)).

Trajectories of deceleration parameter in RMCG dark energy model are drew in Fig. 5 with describing an universe from deaccelerated expansion to accelerated expansion. Here an interested property for \( q \) can be found. When we take the value being more near to \( A = -1 \) (or \( A_s = 1 \)), such as for \( A = -0.9 \) or \( A = -1.1 \) (two upper-middle figures in Fig. 5), for \( A_s = 0.9 \) or \( A_s = 0.95 \) (two lower-back figures in Fig. 5), behaviors of deceleration parameter are almost the same for using the different values of another model parameter \( A_s \) (or \( A \)), i.e. \( q \) is not sensitive to the change of another parameter value \( A_s \) (or \( A \)). By the way, Fig. 6 illustrates the evolution of adiabatic sound speed \( c_s^2 \) of RMCG dark-energy fluid, where the negative values of \( c_s^2 \) are obtained.

FIG. 4: EOS of RMCG (\( A < 0 \)) fluid with taking different values of model parameters.
III. Evolutions of growth factor and Hubble parameter in RMC model with comparing with cosmic data

Peoples obtain some values of growth factor $f$ and Hubble parameter $H$ according to the cosmic observations. Comparing with these observational values listed in table V and VI we apply parameters $f$ and $H$ to

| Number | 1  | 2  | 3   | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|--------|----|----|-----|----|----|----|----|----|----|----|
| $z$    | 0.15 | 0.22 | 0.32 | 0.35 | 0.41 | 0.55 | 0.60 | 0.77 | 0.78 | 1.4 |
| $f$    | 0.51 | 0.6 | 0.654 | 0.7 | 0.7 | 0.75 | 0.73 | 0.91 | 0.7 | 0.9 |
| $\sigma$ | 0.11 | 0.1 | 0.18 | 0.18 | 0.07 | 0.18 | 0.07 | 0.36 | 0.08 | 0.24 |

Ref. [59, 60] [61] [62] [63] [61] [64] [61] [65] [61] [66]

TABLE V: Data of growth factor $f$ with their errors at different redshifts.
| Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|
| $z$    | 0.09 | 0.17 | 0.27 | 0.40 | 0.90 | 1.30 | 1.43 | 1.53 | 1.75 | 0.44 | 0.60 | 0.73 |
| $H$    | 69 | 83 | 77 | 95 | 117 | 168 | 177 | 140 | 102 | 82.6 | 87.9 | 97.3 |
| $\sigma$ | 12 | 8 | 14 | 17 | 23 | 17 | 18 | 14 | 40 | 7.8 | 6.1 | 7.0 |
| Ref.   | [67] | [67] | [67] | [67] | [67] | [67] | [67] | [67] | [67] | [68] | [68] | [68] |

| Number | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|
| $z$    | 0.48 | 0.88 | 0.179 | 0.199 | 0.352 | 0.593 | 0.68 | 0.781 | 0.875 | 1.037 | 0.24 | 0.43 |
| $H$    | 97 | 90 | 75 | 75 | 83 | 104 | 92 | 105 | 125 | 154 | 79.69 | 86.45 |
| $\sigma$ | 62 | 40 | 4 | 5 | 14 | 13 | 8 | 12 | 17 | 20 | 3.32 | 3.27 |
| Ref.   | [69] | [69] | [70] | [70] | [70] | [70] | [70] | [70] | [70] | [70] | [71] | [71] |

**TABLE VI:** $H(z)$ data with their errors at different redshifts (in units $\text{km s}^{-1} \text{Mpc}^{-1}$).

![Graph](image1)

**FIG. 7:** Evolutions of $\Omega_m^\gamma$ and $H$ versus $z$ for RMCG and $\Lambda$CDM model.

test the RMCG cosmology. Parameter $f$ is defined as $f \equiv \frac{d\ln \delta}{d\ln a} = \frac{\delta}{a} \frac{d\delta}{da}$. It complies with the following equation

$$\frac{df}{da} + \frac{f^2}{a} + \frac{2}{a} + \left(\frac{d\ln H}{da}\right)f - \frac{3\Omega_m(a)}{2} = 0,$$

(8)
derived by the perturbation equation $\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m \delta = 0$. $\delta \equiv \frac{\delta\rho_m}{\rho_m}$ is the matter density contrast, and "dot" denotes the derivative with respect to cosmic time $t$. Analytical solutions to Eq. (8) are usually hard to find. An approximation to $f \simeq \Omega_m^\gamma$ has been used in many papers, which provides an excellent fit to the numerically obtained form of $f(z)$ for various cosmological models 72–77. Growth index $\gamma$ can be given by considering the zeroth order and the first order terms in the expansion for $\gamma$ 78,

$$\gamma = \frac{3(1-w)}{5-6w} + \frac{3(1-w)(1-\frac{1}{2}w)(1-\Omega_m)}{125(1-\frac{3}{2}w)}.$$  

Taking values of RMCG model parameters, evolutions of $\Omega_m^\gamma$ versus redshift $z$ are plotted in Fig. 7 where $\Omega_m = 0.3$ and $A_s = 0.49$ are used for RMCG ($A = 0$); $\Omega_m = 0.3$ and $A_s = 0.84$ are used for RMCG ($A = 1$); $\Omega_m = 0.3$ and $A_s = 0.997$ are taken for RMCG ($A = 1/3$); $\Omega_m = 0.3$, $A_s = 0.95$ and $A = -1.1$ are fixed for RMCG ($A < 0$). It can be seen from Fig. 7 for RMCG ($A = 1/3$) and RMCG ($A < 0$) model, their behaviors of $\Omega_m^\gamma$ are almost the same to the popular $\Lambda$CDM model (solid line in Fig. 7), where an increasing function versus $z$ is consistent with the current observations. But for RMCG ($A = 1$) model, $\Omega_m^\gamma$ much deviates from $\Lambda$CDM model at higher redshift. Also, for trajectory of $H$ the difference between RMCG ($A = 1$) model and $\Lambda$CDM is obvious at high redshift. Furthermore, in RMCG ($A = 1$) model the deviation of $\Omega_m^\gamma$ (and $H$) relative to observed data, is large at high redshift. From above, it is shown that the RMCG
(A = 1) fluid as the unification of dark matter and dark energy is not well accordant with the f data and Hubble data. But for cases of \( A = \frac{1}{3} \) and \( A < 0 \), RMCG model are well consistent with these two cosmic datasets.

IV. Parameter evaluation and model comparison

\[ A_s = 0.998 \pm 0.0018 \pm 0.0030, \quad \Omega_{0m} = 0.286 \pm 0.012 \pm 0.024 \quad \text{and} \quad H_0 = 69.69 \pm 1.26 \pm 2.50 \text{ with 68% and 95% confidence levels.} \]

For RMCG1, we have \( \Omega_{0m} = 0.286 \pm 0.012 \pm 0.024 \) and \( H_0 = 69.69 \pm 1.26 \pm 2.50 \text{ with 68% and 95% confidence levels.} \]

In this part, we investigate the parameter space of RMCG model. One knows that the RMCG unified model of DE and DM are not favored by above analysis. For example, they have some questions on structure formation. For RMCG (A=0) unified model, a negative sound speed will introduce the instability at structure formation. For RMCG (A=1) unified model, perturbation quantity \( f \) is not compatible to cosmic data, and a super-deceleration \((q > \frac{1}{2})\) expanded universe is not satisfied with the matter dominated universe. So, these two cases are not studied in this part. We discuss the cosmic constraint on RMCG models with \( A = \frac{1}{3} \) (RMCG1) and \( A < 0 \) (RMCG2). The used data includes: baryon acoustic oscillation (BAO) data from WiggleZ [79], 2dfGRs [80] and SDSS [81] survey, X-ray cluster gas mass fraction [82], Union2 dataset of type supernovae Ia (SNIa) [83] and 24 Hubble data shown in table VI. The constraint methods are described in appendix. For RMCG1, we have \( A_s = 0.9995 \pm 0.0018 \pm 0.0030 \), \( \Omega_{0m} = 0.286 \pm 0.012 \pm 0.024 \) and \( H_0 = 69.69 \pm 1.26 \pm 2.50 \) with 68% and 95% confidence levels. The value of parameter \( A_s \) is near to 1, with the small confidence level. This calculation result on \( A_s \) is approximatively equal to the cosmic constraint on \( \Delta N_{eff} \in (0, 1) \), which is consistent with the other combined constraints on \( N_{eff} \) [27, 54]. By using error-propagation formula, the dark energy density are calculated, \( \Omega_{\Lambda} = 0.713 \pm 0.012 \pm 0.024 \). For RMCG2, it is found \( \Omega_{0m} = 0.296 \pm 0.016 \pm 0.034 \) and \( H_0 = 69.31 \pm 1.45 \pm 2.86 \), while the model parameters \( A \) and \( A_s \) are not convergent. The results are illustrated in Fig. 8. From Eq. (7), we notice that for fixing \( A = -1 \) (or \( A_s = 1 \)), RMCG2 dark energy model are reduced to the popular cosmological constant model, whatever values of \( A_s \) (or \( A \)) are taken. This non-convergent results on model parameters \( A \) and \( A_s \) may be interpreted that this RMCG2 dark energy model can not be distinguished with cosmological constant model according to the used cosmic data in this paper.

Next we use the objective information criteria (IC) to estimate the quality of above RMCG models. Akaike information criteria (AIC) is defined as [84, 85]

\[ AIC = -2 \ln L_{max} + 2K, \quad (9) \]
where $L_{\text{max}}$ is the highest likelihood in the model with $-2 \ln L_{\text{max}} = \chi^2_{\text{min}}$, $K$ is the number of free parameters that interprets model complexity. For candidate models, the one that minimizes the AIC is usually considered the best. Comparing with the best one, the difference for other model is expressed as $\Delta \text{AIC} = \Delta \chi^2_{\text{min}} + 2 \Delta K$. The rules for judging the strength of models are as follows. When $0 \leq \Delta \text{AIC}_i \leq 2$ model $i$ has almost the same support from the data as the best model, for $2 < \Delta \text{AIC}_i \leq 4$, model $i$ is supported considerably less, and with $\Delta \text{AIC}_i > 10$ model $i$ is practically irrelevant [84].

Since the current observations indicate the existence of dark radiation, we take dark radiation density $\Omega_{0\text{dr}}$ as an additional free parameter in $\Lambda$CDM, RMCG2 and MCG models. But in RMCG1 model, the DR density is naturally included by relating to model parameter $A_s$ and $\Omega_{0m}$. According to the calculation results in table VII one reads that the best model is the RMCG1, while the $\Lambda$CDM model almost has the same support as RMCG1, since they almost have the same AIC values. Comparing with the best RMCG1 model, the $\Delta \text{AIC}$ values of RMCG2 and associated MCG model are calculated and listed in table VII too. From table VII it is easy to see that for compared models, RMCG2 model is supported considerably less by the AIC model-selection method, since $\Delta \text{AIC} = 2.840$ at the range $2 \leq \Delta \text{AIC}_{\text{RMCG2}} \leq 4$. In addition, though the MCG model has the minimum value of $\chi^2$, it is not favored by analysis of the AIC, as it has more large value $\Delta \text{AIC} = 4.529$ resulted from the more model parameter. Corresponding to the $\chi^2_{\text{min}}$ value, the constraint results on free parameters are, $\Omega_{0m} = 0.284^{+0.012+0.024}_{-0.012-0.022}$ and $H_0 = 69.58^{+1.27+2.49}_{-1.25-2.45}$ for $\Lambda$CDM model; $A_s = 0.788^{+0.026+0.051}_{-0.028-0.060}$, $\alpha = 0.156^{+0.104+0.200}_{-0.126-0.193}$, $A = -0.0022^{+0.0074+0.0104}_{-0.0075-0.0170}$, $\Omega_{0b} = 0.0543^{+0.0085+0.0144}_{-0.0153-0.0210}$ and $H_0 = 69.97^{+1.64+3.20}_{-1.64-3.13}$ for MCG model.

In addition, one can notice that the other criticism mechanism—Bayesian information criteria (BIC) that is defined as $\text{BIC} = -2 \ln L_{\text{max}} + K \ln n$, is not studied in this paper. $n$ is the number of data points in the fit. As we can see from the BIC definition, the BIC value not only depends on the numbers of free parameters $K$ and $\chi^2$, but also depends on the numbers of data points $n$. So for the same models, the different evaluation results could be given by the BIC method (induced by the different value of $\ln n$) when one use the different data points. For instance, especially for combined constraint including or not including SNIa data, the value of $\ln n$ is obviously different, since the SNIa data have the large number. Considering that the data points are always increasing, it seems that the calculation result from BIC is not "fair" for more-parameter model when the more data points are appeared. Quantitatively, for $\ln n = 2$ ($n \approx 7.4$), AIC and BIC can give the same result. For the used data in our analysis, it has $\ln n = 6.444$. Seeing that the BIC is not "absolutely objective", i.e. its value much depends on the number of used data points, here we do not discuss the BIC criticism method to above RMCG models.

| Case model          | Free parameters | $\chi^2_{\text{min}}$ | $K$ | $\Delta \text{AIC}$ |
|---------------------|-----------------|------------------------|-----|----------------------|
| RMCG1 ($A = \frac{3}{5}$) | $\Omega_{0m}, A_s, H_0, \Omega_{0\text{dr}}$ | 603.705 | 3   | 0                    |
| $\Lambda$CDM        | $\Omega_{0m}, H_0, \Omega_{0\text{dr}}$ | 603.710 | 3   | 0.005                |
| RMCG2 ($A < 0$)     | $\Omega_{0m}, A_s, A, H_0, \Omega_{0b}, \Omega_{0\text{dr}}$ | 602.545 | 5   | 2.840                |
| MCG                 | $\Omega_{0b}, A_s, A, \alpha, H_0, \Omega_{0\text{dr}}, \Omega_{0\text{dr}}$ | 602.234 | 6   | 4.529                |

TABLE VII: Information criteria results.
V. Conclusions

RMCG models are from a subclass of the famous MCG model that has been studied in great detail over the years. But, most of them were studied as a unification of DM and DE in the past. In this paper studies on RMCG cosmology are performed from different angles, where the RMCG fluid as dark energy or as unified model of dark components are discussed. New interesting physical results are given in RMCG dark models. Studies show, (1) the RMCG unified model of dark energy and dark matter (with model parameter $A = 0$ or $A = 1$) tends to be ruled out, according to the behaviors of some cosmological quantities. For example, for RMCG ($A = 0$) unified model, a negative sound speed appears, which will introduce the instability at structure formation. For RMCG ($A = 1$) unified model, growth factor $f$ is not consistent with cosmic data, and a super-deceleration ($q > \frac{1}{2}$) expanded universe is not satisfied with the matter dominated epoch. Also, a radiation dominated universe will not appear in RMCG ($A = 1$) model, for existence of the stiff matter. (2) the RMCG ($A = \frac{1}{3}$) unified model of dark energy and dark radiation, is a candidate for interpreting the accelerating universe, which is well accordant with the current understanding on our universe, for examples, the good behaviors of cosmological quantities and good fits to the current observational data: growth factor and Hubble parameter. In addition, it provide an origin of dark radiation and dark energy. And energy densities of these two dark components are self-consistent; (3) the RMCG ($A < 0$) fluid as dark energy also has some attractive features, such as the CC, the quintessence and the phantom dark energy can be realized in this RMCG fluid, at some situations the evolutions of cosmological quantities are not much sensitive to the variation of model-parameters values.

At last, for RMCG ($A = \frac{1}{3}$) and RMCG ($A < 0$) model, we investigate their parameter space by using the recent cosmic data. Fitting the observational data to the RMCG ($A = \frac{1}{3}$) model, it is found that the constraint result on RMCG ($A = \frac{1}{3}$) model parameter is $A_s = 0.9995^{+0.0018}_{-0.0018} + 0.0030_{-0.0030}$ at 68% and 95% confidence levels, which is consistent with other cosmic constraint result on the effective number of relativistic degrees of freedom with $\Delta N_{eff} \in (0, 1)$. By the AIC calculation, it is shown that RMCG ($A = \frac{1}{3}$) model almost has the same support as the most popular $\Lambda$CDM model. For RMCG ($A < 0$) model, the model parameters $A$ and $A_s$ are not convergent by comparing this model with the combined observational data. The theoretical constraint on RMCG ($A < 0$) model parameters are $0 < A_s < 1$ with $-1 < A < -\frac{1}{3}$ for EOS at quintessence region, and $0 < A_s < 1$ with $A < -1$ for EOS at phantom region. But by the analysis of AIC, the RMCG ($A < 0$) model has the less support from the observational data.

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VI. Appendix

Here we describe the used cosmic data including the BAO, the $f_{gas}$, the SNIa, and the H(z) data. Firstly, we introduce three distance parameter in the following. $D_A(z)$ is the proper angular diameter distance

$$D_A(z) = \frac{c}{(1 + z)\sqrt{\Omega_k}} \sinh\left[\sqrt{\Omega_k} \int_0^z \frac{dz'}{H(z')}\right].$$

(10)

It relates with other two distance quantities $D_L$ and $D_V$ by

$$D_L(z) = \frac{H_0}{c}(1 + z)^2 D_A(z)$$

(11)
\[ D_V(z) = [(1 + z)^2 D_A^2(z) \frac{c_s}{H(z; p_s)}]^{1/3} = H_0 \left[ \frac{z}{E(z; p_s)} \left( \int_0^z \frac{dz'}{E(z'; p_s)} \right)^2 \right]^{1/2}. \]

\( p_s \) denotes the theoretical model parameters, \( \sin(n(\sqrt{|\Omega_k|}x)) \) respectively denotes \( \sin(\sqrt{|\Omega_k|}x) \), \( \sqrt{|\Omega_k|}x \) and \( \sinh(\sqrt{|\Omega_k|}x) \) for \( \Omega_k < 0 \), \( \Omega_k = 0 \) and \( \Omega_k > 0 \).

### A. BAO

BAO data are extracted from the WiggleZ Dark Energy Survey (WDWS) \([79]\), the Two Degree Field Galaxy Redshift Survey (2dFGRS) \([80]\) and the Sloan Digital Sky Survey (SDSS) \([81]\). The \( \chi^2_{BAO}(p_s) \) is given by

\[ \chi^2_{BAO}(p_s) = X^t V^{-1} X, \]

with

\[ V^{-1} = \begin{pmatrix} 4444 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30318 & -17312 & 0 & 0 & 0 \\ 0 & -17312 & 87046 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23857 & -22747 & 10586 \\ 0 & 0 & 0 & -22747 & 128729 & -59907 \\ 0 & 0 & 0 & 10586 & -59907 & 125536 \end{pmatrix}, \quad X = \begin{pmatrix} r_s(z_d) \\ \frac{r_s(z)}{D_V(0.106)} \\ \frac{r_s(z)}{D_V(0.2)} \\ \frac{r_s(z)}{D_V(0.38)} \\ \frac{r_s(z)}{D_V(0.44)} \\ \frac{r_s(z)}{D_V(0.73)} \end{pmatrix}. \]

\( V^{-1} \) is the inverse covariance matrix \([81, 82]\). \( X \) is a column vector formed from theoretical values minus observational values, and \( X^t \) denotes its transpose. \( r_s(z) \) is the comoving sound horizon size \( r_s(z) = c \int_0^z \frac{c_s dt}{a} = \frac{c_s}{\sqrt{\Omega}} \int_0^{1/(1+z)} \frac{da}{a^2 \Omega_s (1 + 3 \Omega_b h^2) / (4 \Omega_s)} \). \( c_s \) is the sound speed of the photon–baryon fluid, \( c_s^2 = 3 + \frac{\gamma}{2} \times (\Omega_{\gamma} / H_0) \) with \( \Omega_{\gamma} = 2.469 \times 10^{-5} h^{-2} \). \( z_d \) denotes the drag epoch (where baryons were released from photons), \( z_d = \frac{1291(\Omega_{\text{om}} h^2)^{-0.419}}{1 + b_1(\Omega_{\text{om}} h^2)^{0.223}} \) with \( b_1 = 0.31(\Omega_{\text{om}} h^2)^{-0.419}[1 + 0.607(\Omega_{\text{om}} h^2)^{0.674}] \) and \( b_2 = 0.238(\Omega_{\text{om}} h^2)^{0.223} \). \( h \) is a re-normalized quantity defined by the Hubble constant \( H_0 = 100 h \) km s\(^{-1}\) Mpc\(^{-1}\).

### B. X-ray gas mass fraction

In observation of the X-ray gas mass fraction, for the reference model \( \Lambda \)CDM, parameter \( f_{\text{gas}} \) is presented as \([82]\)

\[ f_{\text{gas}}^{\Lambda \text{CDM}}(z) = \frac{K A \gamma b(z)}{1 + s(z)} \left( \frac{\Omega_b}{\Omega_{\text{om}}} \right) \left[ \frac{D_{\Lambda \text{CDM}}(z)}{D_A(z)} \right]^{1.5}. \]

\( A \) is the angular correction factor, \( A = \left( \frac{H(z)D_A(z)}{H(z)D_A(z)} \right)^{\eta} \). Index \( \eta \) is the slope of the \( f_{\text{gas}}(r/r_{2500}) \) data with \( \eta = 0.214 \pm 0.022 \). \( \gamma \) denotes permissible departures from the assumption of hydrostatic equilibrium, due to non-thermal pressure support; bias factor \( b(z) = b_0(1 + \alpha_b z) \) accounts for uncertainties in the cluster depletion factor. \( s(z) = s_0(1 + \alpha_s z) \) accounts for uncertainties of the baryonic mass fraction in stars and a Gaussian prior for \( s_0 \) is employed, with \( s_0 = (0.16 \pm 0.05) h_0^{0.5} \). \( K \) is used to describe the combined effects of the residual uncertainties, such as the instrumental calibration and certain X-ray modelling issues, and a Gaussian prior for the 'calibration' factor is considered by \( K = 1.0 \pm 0.1 \). Adopting the datapoints published in Ref. \([82]\) and following
the method in Refs. [82], $\chi^2$ for the X-ray gas mass fraction analysis is expressed as

$$
\chi^2_{f_{gas}} = \sum_{i=1}^{42} \left[ \frac{f_{\Lambda CDM}^{gas}(z_i) - f_{gas}(z_i)}{\sigma_{f_{gas}}(z_i)} \right]^2 + \frac{(s_0 - 0.16)^2}{0.0016^2} + \frac{(K - 1.0)^2}{0.01^2} + \frac{(\eta - 0.214)^2}{0.022^2},
$$

(16)

where $\sigma_{f_{gas}}(z_i)$ is the statistical uncertainties. As pointed out in [82], the acquiescent systematic uncertainties have been considered according to the parameters $\eta, b(z), s(z)$ and $K$.

C. SNIa

Cosmic constraint from SNIa observation can be determined by a calculation on the likelihood [88–98]

$$
\chi^2_{SNIa}(p_s) \equiv \sum_{i=1}^{557} \left[ \frac{\mu_{th}(p_s, z_i) - \mu_{obs}(z_i)}{\sigma_{\mu_i}} \right]^2.
$$

(17)

$\mu_{th}(z)$ is the theoretical distance modulus $\mu_{th}(z) = 5 \log_{10}[D_L(z)] + \mu_0$ with the Hubble-free luminosity distance $D_L(z)$ and $\mu_0 = 5 \log_{10} \left( \frac{H_0}{100} \right) + 25 = 42.38 - 5 \log_{10} h$. $\mu_{obs}(z_i)$ is the observed distance moduli at different redshift $z_i$, which can be given by SNIa observation datasets [83].

D. H(z) data

Using the H(z) data listed in table VI, the model parameters are determined by minimizing [99–108]

$$
\chi^2_H(H_0,p_s) = \sum_{i=1}^{24} \left[ \frac{H_{th}(H_0,p_s; z_i) - H_{obs}(z_i)}{\sigma_H(z_i)} \right]^2,
$$

(18)

where $H_{th}$ is the predicted value for the Hubble parameter and $H_{obs}$ is the observed value.

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