Risk Mathematics and Quantum Games on Quantum Risk Structures - A Nuclear War Scenario Game

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Abstract

Quantum game theory is combined with risk mathematics' formalism to provide an approach to evolutionary scenario analysis. The formalism is addressed in its general form and is then applied to an extreme risks modelling case, to model a coevolving dynamical web of systemic situations representing the evolution of the regional tensions between two countries with nuclear weapons. The model’s results are addressed regarding the potential for regional nuclear conflict to take place, and how evolutionary scenario analysis may contribute to nuclear war threat assessment and dynamical risk analysis. A final discussion is provided in what regards risk mathematics based on the evolutionary approach to risk assessment resulting from the combination of quantum game theory, morphic web representations and scenario analysis.

Keywords: Risk mathematics, quantum game theory, morphic webs, evolutionary scenario analysis, extreme risk scenarios, nuclear war threat scenario.

1 Introduction

A key problem in risk assessment is the combination of scenario analysis tools with game theory to be applied to evolving unstable conditions and complex systems’ dynamics [2, 11, 14], quantum game theory has proven a successful tool in dealing with such dynamics, in particular, in what regards financial turbulence modelling [12].

In the present work, quantum game theory is combined with risk mathematics’ formalism to address evolutionary scenario dynamics. In section 2., a review is provided regarding the empirical effectiveness of quantum game theory and its conceptual and systemic foundations, integrating the current work within the broader background of quantum game theory and quantum computation.
In section 3., the formalism for the integration between risk mathematics and quantum game theory is provided, within a mathematical foundation that combines formal structures coming from risk mathematics with quantum computation theory (subsection 3.1) and modal systemics (subsection 3.2), to address quantum games on quantum risk structures.

The formalism is then exemplified, in section 4., through a geopolitical dynamics model between two countries with nuclear weapons, providing for an approach to nuclear war threat assessment. General conclusions are drawn, in section 5., regarding evolutionary scenario analysis and risk mathematics’ theory and applications.

2 Quantum Game Theory

Quantum game theory has been shown to hold with a good empirical matching in capturing social learning dynamics and in economic and financial decisional contexts [13]. A particularly effective application of quantum game theory regards risk contexts which include the analysis of financial crises [13] and financial turbulence modelling [12]. The good empirical matching of quantum game theory within quantum econophysics opens up the matter of other applications of quantum game theory, in particular, in areas such as political science and strategic studies.

Quantum game theory fits well in non-equilibrium dynamics of complex adaptive systems, since the probability structures may change with the system’s path-dependent quantum computation and result from the coevolving relational structure that links system with its environment.

In this way, one may have dynamical instability rather than stable fixed point strategies, and coevolving game conditions rather than a fixed framework to which players must conform. The rules of the game become more fluid and conditions may change suddenly and unexpectedly. This is possible due to the underlying framework of path-dependent quantum computation, which opens up the possibility of chaotic and stochastic quantum logical gate updating (an approach that has been followed in financial turbulence modelling [12]) and opens up the way to coevolving game conditions where the rules, themselves, are permanently enacted towards an adaptive effectiveness of game players.

In a systemic foundational framework, quantum game theory incorporates quantum computation bringing it to an adaptive setting [23, 5, 25] which expands an argument laid out by Everett [4] for a fundamental systemic accounting of the systems’ dynamics in the universe, an argument that was effectively expanded by Gell-Mann and Hartle in their reflection on quantum mechanics in light of quantum cosmology [7]. In this reflection, Gell-Mann and Hartle consider the strong dependence upon the quantum dynamical substratum of the regularities exploited by what they call the environmental sciences such as as-

\[\text{By modal systemics we mean an approach to modal logic based upon a systems science’s ontological foundation, sharing the same conceptual basis of risk mathematics, as developed in subsections 3.1 and 3.2.}\]
tronomy, geology and biology, which are ultimately traceable to the universe’s initial condition, involving correlations that stem from that initial condition.

Everett’s legacy, either in the form of the many worlds interpretation [3] or in the form of the possible histories’ interpretations [7, 19], offers a quantum physical basis for addressing a quantum game theory applied to an understanding of complex adaptive systems. A matter that is addressed within a quantum computational perspective on the universe, defended by Deutsch [3] and by Lloyd [20], and within a quantum theoretical approach to the adaptive cognition and information processing of complex adaptive systems, defended by Gell-Mann and Hartle [7] through the notion of information gathering and utilization systems (IGUSes), which expands Everett’s computational framework for the observer that is described by Everett [4] in terms of a computational system equipped with a physical interface with the universe and memory working that, in Everett’s theory, make the observer necessarily entangled with other systems in the universe.

In the current work we also take another step in addressing complex adaptive systems’ dynamics quantum theoretically, through the combination of quantum game theory with risk mathematics. This leads us to an evolutionary interpretation of the quantum state vector that links the quantum probability measures to a fitness measure defined in terms of a relative frequency of alternative possible configurations of the universe/world in act. Albeit following Everett’s framework, the mathematical results presented in the next section provide for what can be called a “weak” many worlds interpretation, in the sense that it works by reference to the possibilities from a world in act.

What we call here the “strong” many worlds interpretation assumes parallel alternatives all occurring in actualized branches, while, for the “weak” interpretation, the world in act, our universe, addressed quantum theoretically within the framework of complex quantum systems science, is considered to be in a permanent (quantum) computation, with the final resulting actualized configuration being systemically selected through an evolutionary process out of a systemic evaluation of alternatives, thus, out of the possible alternatives, one takes place in act, which allows the theory to approach the universe as an evolving complex quantum system, an approach that comes from both Deutsch and Lloyd [3, 20], as well as Gell-Mann and Hartle [7, 8] being rooted in Everett’s work [4].

The “strong” many worlds interpretation has been used in quantum game theory [25] as an effective way to consider how different decisions may lead to parallel realities in a relational universe. Within an evolutionary framework, Everett’s notion of measure [4], worked from the “strong” many worlds interpretation of quantum mechanics [3], can be addressed such that the occurrence and non-occurrence of the events leads to a statistical distribution over an ensemble.

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\[2\] It has also been called *decoherent histories interpretation* [7, 8], as well as *consistent histories interpretation* [19], we call it possible histories due to the linkages to modal systemics, since, in this interpretation, one assumes the *world in act* (our universe) and addresses the possible alternatives for that *world in act*, applying the formalism of quantum mechanics [4, 8, 19, 20].
of parallel worlds that reflects the systemic sustainability of each alternative in what can be considered a relative frequency fitness measure, such that, like a genetic type’s relative frequency in a population constitutes a measure of that genetic type’s fitness (relative frequency measure of fitness), so one may assume to take place in the “strong” many worlds interpretation, where the branches of a quantum state are interpreted as leading to a measure of each alternatives’ representativeness in the ensemble of parallel realities, thus alternatives with higher fitness in this ensemble become the ones that are more frequently observed and, simultaneously, the more likely paths for the systems.

A similar approach is followed here, only that the selection is for a single world in act between an ensemble of possible configurations of that world in act, with the fitness measure holding for such an ensemble.

One of the reasons for our choice to work with the “weak” many worlds interpretation is the problem that the “weak” interpretation might have to be invoked in addressing the “strong” interpretation, in particular, to address how each configuration takes place in parallel worlds in act, leading to a specific statistical distribution of actualized alternatives, so that the structure of the “weak” interpretation might have to be assumed for each parallel world in act, this would lead to a doubling of the formalism, becoming more parsimonious to assume and work with the “weak” interpretation right from the start.

However, this does not mean that a physical negation of parallel universes is being proposed, indeed, as stated, the “weak” interpretation is compatible with both the “strong” interpretation, that assumes parallel worlds in act, as well as with the view of those that choose not to assume parallel worlds in act. Since Physics is divided on this matter, we do not take a stand here, because taking a stand would lead us outside the scope of the present work.

The compatibility of the “weak” interpretation with the “strong”, as explained in the previous paragraphs, also makes the current work compatible with the other works in quantum game theory that assume the “strong” interpretation [25], however, by assuming the “weak” interpretation we are able to integrate in a single quantum game theory-based formal system, the three frameworks: risk mathematics, quantum game theory and a modal systemics.

3 Risk Mathematics, Quantum Games and Quantum Risk Structures

Risk mathematics takes risk, itself, as its object of research and addresses it systematically from its root in the Medieval Latin term resicum, which synthesized, in the context of Maritime Law, the three notions of periculum (peril, threat), fortuna (fortune, luck, destiny) and uncertainty: to play for one’s destiny in situations where there are threats and opportunities, with an outcome still open or unknown to the player [10, 11, 21]. Thus, when one addresses a risk situation (for instance, a nuclear war between two countries), one is addressing a systemic configuration that is ontologically contingent, and epistemologically non-determined, that is, it may or may not take place, so that it is not an
impossibility, and there is uncertainty with regards to its taking place.

The recognition of a risk situation, therefore, demands the recognition of the presence of the systemic ground for a threat. Thus, for instance, the presence of a gun in a place opens up immediately the possibility of that gun being used in a myriad of risk situations. The interconnectedness of risk situations has led authors to address risk scenarios in terms of network structures \([10, 11, 14]\), considering the influence webs between different risk situations, these network structures can be further formalized in terms of the notion of **morphic webs** \([9, 10, 11]\).

Formally, within mathematics, a **morphic web** is a weaker structure than a category, being defined as a mathematical structure \(\mathcal{W}\) of objects \(A, B, C, \ldots\) and morphisms, between objects \(A \xrightarrow{f} B\), with the identity morphism connecting each object to itself \(A \xrightarrow{id_A} A\). A mathematical category is a morphic web with further axioms regarding composition of morphisms\(^3\), however, not all morphic webs are categories \([9, 11]\).

A **morphism** synthesizes a directional relation from an origin object to a target object, expressing a systemic motion from the origin to the target with relational fundament \(f\) \([9]\).

**Morphic webs** can be applied as effective tools in a systemic analysis of risk, in particular in what regards scenario analysis built from morphic webs of systemic situations \([11]\), defined as morphic webs \(\mathcal{W}\) whose object collection, denoted by \(\text{ob}(\mathcal{W})\), is a collection of systemic situations and whose morphism collection, denoted by \(\text{morph}(\mathcal{W})\), is composed of morphisms of the kind \(A \xrightarrow{f_{AB}} B\) which are interpreted as \(A\) being a **systemic source** of \(B\), and \(f_{AB}\) is a number representing the coupling strength from \(A\) to \(B\). Scenarios can be built by addressing the different combinations of occurrences and influences, as explained in \([11]\).

However, to deal with evolutionary dynamics of scenarios, rather than a static scenario enumeration, one needs to expand the formal basis of **morphic webs**, so that one may address, in evolutionary terms, the occurrence of events and the dynamical linkages influenced by the morphic structure of the systemic situations’ web. It turns out that quantum game theory allows one to expand the **morphic webs’** formal basis and to address such a dynamical evolutionary setting, as we now show.

### 3.1 Quantum risk structures

To build a quantum game theoretical approach to evolutionary scenario analysis, within the framework of risk mathematics, we need to consider, first, a morphic web of systemic situations \(\mathcal{W}\) and introduce the Hilbert space \(\mathcal{H}\) such that \(\mathcal{H} := \bigotimes_{i=1}^{n} \mathcal{H}_i\), where \(\mathcal{H}_i\) is the Hilbert space associated with the \(i\)-th systemic situation \([\mathcal{W}]\) of \(\mathcal{W}\), for \(n = \#\text{ob}(\mathcal{W})\) (the number of systemic situations in \(\mathcal{W}\)), we

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\(^3\) In particular: closure, associativity and identity laws \((id_A \circ f = f \circ id_B = f, \text{ for } A \xrightarrow{f} B)\) with respect to the composition operation.

\(^4\) Formally, the \(i\)-th object.
also assume that each $H_i$ is spanned by the basis $\{|0\rangle, |1\rangle\}$, where $|0\rangle$ encodes the case in which the $i$-th systemic situation does not occur and $|1\rangle$ encodes the case in which the $i$-th systemic situation occurs.

Introducing, for each $H_i$, the basis projectors 
\[
\hat{P}^i_s = |s\rangle \langle s|,
\]
with $s = 0, 1$, we build the full projector space $P := \{\hat{P}^i_s = \bigotimes_{i=1}^{n} P^i_{s_i} : s_i = 0, 1\}$, with $s = (s_1, s_2, ..., s_n)$, so that each $\hat{P}^i_s$ projects onto a basis vector of $H$ and corresponds to a scenario of occurrences over $W$. By scenario we understand a description of a course of events or situations that may take place (are possible), thus a scenario is a complete description of systemic occurrences in a world configuration that is possible for the world in act (the universe), each scenario is, in this case, encoded by the syntax of a binary quantum basis alphabet with the semantics provided by $W$ which gives us the account of the scenario itself.

The general ket vector of $H$ expands as follows:
\[
|\Psi\rangle = \sum_s \hat{P}_s |\Psi\rangle = \sum_s \psi(s) |s\rangle
\]
with $\psi(s) = \langle s | \Psi \rangle$. Thus, each branch of $|\Psi\rangle$ corresponds to an alternative configuration of the universe that can take place in act, with an amplitude $\psi(s)$, different scenarios that can take place in act correspond to different possible configurations of the universe (different world configurations), the universe would have different information contents, for instance, if the South had won the American Civil War.

One must take care in addressing the formal syntax and the semantics underlying Eq.(1), in effect we have qubitized the information contents of the possible world configurations with respect to the description of whether or not each systemic situation in $W$ occurs, this description constitutes the semantics for the syntax, but it also allows us to formally assign a $n$-qubit state as per Eq.(1), with each branch from the state vector $|\Psi\rangle$ synthesizing a complete pattern of occurrences. This qubitization can only be done due to the fact that $|\Psi\rangle$ is an account of scenarios taking place in possible configurations for the universe, as stated in the previous paragraph.

The vector expansion in $|\Psi\rangle$, thus, shows a $n$-qubit state describing the alternative scenarios, the semantics for this quantum computational syntax is provided by the systemic situations’ morphic web $W$, which gives us how each alternative $\hat{P}_s$ can be interpreted, this allows us to introduce a complete (quantum) logical semantic structure that can be defined as:
\[
\mathfrak{A} (|\Psi\rangle) := (W, H, P, |\Psi\rangle)
\]
This last structure, introduced within the (quantum) model logical semantics, corresponds to what we call a quantum risk structure. A quantum risk structure is similar in many respects to Gell-Mann and Hartle’s proposal of addressing the quantum state of the universe in terms of an exhaustive set of projectors that project for yes or no questions [7], an approach that comes from Everett’s line of argument [4,7], linking the systems within the universe as systems in the
universe and therefore within a quantum cosmological setting, working from the universe’s quantum state with exhaustive sets of alternatives [7, 8, 19], which provides for a physical and systemic framework for general risk science.

To address risk, however, it becomes necessary to systematically interpret the quantum amplitudes \( \psi(s) \), so that we need to have a formalism that allows us to address the systemic possibility of occurrences of each alternative scenario connecting the amplitudes to this systemic possibility and, in turn, we need to extract from \( |\Psi\rangle \) information that allows us to introduce probabilities for how different scenarios can take place in act. This can only be done by addressing the notion of possibility within a modal systemics.

### 3.2 Modal systemics and quantum probabilities

Modal logics can be built from formal languages to address ontological modalities, including, in particular, necessity and possibility [1, 6], we need to address a specific formal structure that is simultaneously consistent with quantum game theory and with the systemic framework of the quantum risk structures introduced in the previous subsection. A solution is to introduce a quantum modal structure which includes the quantum risk structure \( \mathfrak{A}(|\Psi\rangle) \), defined as:

\[
\mathfrak{M}(|\Psi\rangle) := [\mathfrak{W}, \mathfrak{A}(|\Psi\rangle)]
\]

where \( \mathfrak{W} \) is a morphic web comprised of one object \( w_0 \) representing the world in act and where possibility is addressed from the world in act in terms of possible configurations of that world or possible world configurations that comprise the morphic structure of possibilities \( \mathfrak{W} \) and are formally built as:

\[
w_0 \xrightarrow{c} w_0
\]

where the fundament \( c \) corresponds to a specific possible world configuration of the world in act.

Given the morphic web of systemic situations \( \mathcal{W} \), a morphic structure of possibilities \( \mathfrak{W} \) consistent with \( \mathcal{W} \) is such that for each morphism \( w_0 \xrightarrow{c} w_0 \), there is one, and only one, scenario of occurrences \( s \) of the systemic situations of \( \mathcal{W} \) that occurs in \( c \), we, thus, write \( s \in c \).

While, for each configuration, there can only be a single occurring scenario, for different configurations the same scenario may take place. Since the projection of \( |\Psi\rangle \) over each basis element \( |s\rangle \) provides for quantum amplitudes \( \psi(s) = \langle s | \hat{P}_s | \Psi \rangle \) over each alternative scenario \( s \), these amplitudes must somehow be linked to the distribution of occurrences of \( s \) over \( \mathfrak{W} \). We, therefore, need to find, within quantum theory, a conceptual scheme to extract information from \( |\Psi\rangle \) on the distribution of configurations of \( \mathfrak{W} \) with respect to the

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5 Within formal mathematics, the construction of formal structures for concrete systems is sometimes non-univocal, in the sense that different paths to formalization can be built. In interdisciplinary applications, the choice becomes one of effectiveness and consistency with the different sciences involved. In the present case, we have systems science, risk mathematics and quantum game theory.
occurrences of the scenarios that come from $W$ and we need to connect such a distribution to a game theoretical evolutionary interpretation. Such a conceptual scheme can be introduced through the \textit{decoherence functional} \cite{7, 8, 19, 24} for the scenarios, defined as:

$$D(s, s') := \left| \Psi \right| \left| \hat{P}_s^\dagger \hat{P}_{s'} \Psi \right|$$ (5)

which satisfies:

$$D(s, s') = |\psi(s)|^2 \delta_{s,s'} \geq 0 \quad (6)$$

$$\sum_{s,s'} D(s, s') = \sum_s |\psi(s)|^2 = 1 \quad (7)$$

therefore, $D(s, s')$ behaves like an additive measure over the alternative scenarios. A further assumption is then made that, for $W$, the proportion of morphisms $w_0 \xrightarrow{c} w_0$ in which $s$ occurs is equal to $D(s, s) = |\psi(s)|^2$, within risk mathematics, this proportion is linked to the systemic tendency for $s$ to take place in the \textit{world in act}, so that we can measure from $|\psi(s)|^2$ the probability of the scenario to take place in the \textit{world in act} $w_0$.

Simultaneously, we can also provide for an evolutionary game theoretical interpretation of $|\psi(s)|^2$, since the relative frequency occurrences of $s$ in the morphisms of $W$ can be read as a measure of systemic sustainability underlying the occurrence of $s$, interpretable as a \textit{fitness} associated with each branch $\hat{P}_s$ of the quantum state $|\Psi\rangle$. Applying Eq.(5) we can obtain a \textit{fitness density operator} akin to evolutionary game theory’s version of a \textit{fitness} matrix:

$$\hat{D}(|\Psi\rangle) := \sum_{s',s} \left\langle \Psi \left| \hat{P}_s^\dagger \hat{P}_{s'} \right| \Psi \right\rangle |s\rangle \langle s'|$$ (8)

from Eq.(6) it follows immediately that this \textit{fitness density operator} is diagonal and given by:

$$\hat{D}(|\Psi\rangle) = \sum_s |\psi(s)|^2 \hat{P}_s \quad (9)$$

from this diagonal form, in a game theoretical sense, one can state that the scenarios are \textit{mixing} on the quantum strategy $|\Psi\rangle$, in a quantum theoretical sense, one can state that the alternatives $\hat{P}_s$ decohere \cite{7, 19, 24}. In the quantum game theoretical framework, $\hat{D}(|\Psi\rangle)$ has an evolutionary interpretation such that the selection of each alternative $s$ is systemically grounded in the \textit{fitness} or \textit{systemic sustainability} for $s$, leading to a probability measure of $s$ taking place in the \textit{world in act}.

For such a game theoretical framework, there can be no collapse of $\hat{D}(|\Psi\rangle)$ (nor of $|\Psi\rangle$), since the possibilities always remain as possibilities independently of what takes place in act (which is also necessarily possible) \cite{21}, so that the \textit{morphic web} $W$ always holds with its statistical configuration. Indeed, a potency is actualized by the act that determines it, one must not, however, mix the notion of potency with the notion possibility: possible is what can be, a potency
dynamis) is towards the act (energeia), these are different notions tracing back to Aristotle’s thinking, in which modal logics find their ground [1, 6, 21].

Under the current modal framework, the probability for each alternative to take place in act is numerically coincident with the fitness, measuring how the selection may take place in act, which means the probability with which an alternative scenario \( s \) is actualized is taken as numerically coincident with the relative frequency \( |\psi(s)|^2 \) of possible configurations of the world in act in which \( s \) occurs, after the actualization, on the other hand, the scenario \( s \) either took place in act or not, such that the probability is either 0 or 1, the fitness density \( \hat{D}(|\Psi\rangle) \) as well as the quantum state \( |\Psi\rangle \) are still, however, necessarily, unchanged, being related to the sustainability field associated with the systemic situation, leading to the ensemble distribution of possibilities for the world in act.

It is important to notice that the probability only holds if the fitness levels \( |\psi(s)|^2 \) remain unchanged, otherwise, if a change in the fitness of each alternative occurs at the very last moment of actualization, the probability changes and, thus, the system can change direction in its choice. Changes in the fitness density results from a quantum computation rule which is akin to a quantum replicator dynamics over possible configurations of the world in act, such that a quantum unitary operator encodes the system’s evolutionary computation. Indeed, the modal structure \( \mathcal{M}(|\Psi\rangle) \) changes for quantum computations that transform the state \( |\Psi\rangle \) and, thus, simultaneously, transform the configuration of the morphic web \( \mathcal{W} \) with respect to the statistical distribution of possible world configurations containing each scenario.

For a quantum game divided in rounds, indexing the rounds by \( t = 1, 2, ..., \) with a unitary quantum computing gate holding at each round, we obtain:

\[
|\Psi(t)\rangle = \hat{U}(t)|\Psi(t-1)\rangle
\]  

(10)

the unitary state transition changes the quantum game state in accordance with the nature of the systemic situations and the systems involved (in particular, it must reflect the morphic connections of \( \mathcal{W} \)). It is important to notice that \( t \) is a game round index, not a temporal physical clock frame, it just counts the number of quantum computing steps as per Eq.(10), thus, it matches the counting of quantum game rounds. In this case, there is a corresponding round indexing of the morphic web \( \mathcal{W}(t) \) matching the indexing of \( |\Psi(t)\rangle \), so that we have the quantum game round-dependent structure \( \mathcal{M}(|\Psi(t)\rangle) = [\mathcal{W}(t), \mathfrak{A}(|\Psi(t)\rangle)] \).

The quantum game conditions can be evolve from round to round so that the structure of unitary gates \( \hat{U}(t) \) can show a path-dependence, this path-dependence can be upon the sequence of the previous unitary gates \( \hat{U}(0), ..., \hat{U}(t-1) \), but it can also depend upon actualized alternatives. It is important to notice that the possible histories interpretation has shown that actualization is not synonymous with breakdown in unitary state transition from an initial condition, rather, the work done within decoherence theory has shown that unitary state transition can take place with an actualized history for the system.\(^6\)

\(^6\) A point made explicit for instance in Gell-Mann and Hartle’s recent work on an actualized
The fact that several decoherence theory proposals follow and indeed expand on Everett’s approach shows that the so-called breakdown in unitary state transition, with loss of quantum interference, takes place either through entanglement with a description in terms of local degrees of freedom monitoring, tracing out the correlated environment, or due to coarse-graining or even a mixture of both. In the first case, it becomes necessary to re-express the local state transition in terms of a quantum map reflecting the global unitary state transition with local loss of information due to interaction with a local environment. Hawking radiation is also another possible source of unitary state transition breakdown, even though Hawking has recently seemed to back out from this view on information loss. However, in no instance is there an unequivocal association between breakdown of unitarity and actualization, there is no physical causality link that can be made mathematically between the two, except by assuming it to be there as a postulate which is extra-theoretical.

Indeed, Everettian decoherence theory has shown us that we can have an actualized history of the world (or worlds in the case of the “strong” many worlds interpretation), with sequences of unitary state transitions without breakdown of quantum interference, the breakdown of quantum interference can be stated to be due to a tracing out of entangled states, or to coarse-graining, but this is an observer-related ignorance effect. In no way does the mathematical formalism support consistently the proposal that actualization is linked to breakdown of quantum interference effects due to observer-related local description tracing out the environment. Such a proposal is a meta-formalism postulate that is inconsistent with the notion of actualization, introducing to a conceptual confusion between possibility, potency and probability which are, indeed, three distinct notions within Philosophy and Mathematics as stated above. If one works with a notion of actualization, one cannot properly assume that actualization takes place due to tracing out degrees of freedom of an entangled system or due to coarse-graining, and physical experiments have shown this to be true.

Having addressed the main formalism and conceptual substratum we now provide an example of the current formalism applied to a nuclear war threat assessment problem.

4 A Nuclear War Quantum Game Model

Let us consider a geopolitical game between two countries, labelled A and B. The first step in building the game is to draw the morphic web of systemic situations, we consider the following eight situations, where the order of presentation and labelling has been arbitrarily assigned:

- $S_0$: growing political tensions between the two countries linked to conflicts of interest, possible diplomatic incidents, economic and political competition with possible military fallout.

fine-grained history present simultaneously with unitary state transition
• $S_1$: political turmoil in the two countries, which includes internal and external political turmoil involving the two countries’ tension issues, it can lead to both demonstrations and actions of supporters of each country in the streets, and political instability due to political statements and actions undertaken by both countries.

• $S_2$: failure of both countries to negotiate an agreement on key issues, which may prevent the countries from communicating and finding diplomatic solutions to their grievances.

• $S_3$: civil unrest, that is, when people take to the streets in protest on both internal problems and problems involving both countries, it also includes the possibility of social disorder and possible escalade in street violence linked to key problems that divide the two countries.

• $S_4$: conflict breakout between the two countries involving conventional non-nuclear arsenal.

• $S_5$: nuclear threat escalation, with the threat of both countries using their nuclear arsenal.

• $S_6$: Nuclear war, this corresponds to both countries using nuclear weapons on each other, triggering a nuclear war.

• $S_7$: United Nations (UN) intervention/mediation, corresponding to attempts of UN to influence both countries in finding common ground and mediating political negotiations between both countries, but it also entails possible deliberations and actions that can be taken by the UN Security Council in the case of military conflict between the two countries or of nuclear weapons being used.

The morphisms are the following:

• With origin $S_0$: $S_0 \xrightarrow{f_{0i}} S_i$, with $i = 1, 2, 3, 4, 6$;

• With origin $S_1$: $S_1 \xrightarrow{f_{1i}} S_i$, with $i = 0, 2, 3$;

• With origin $S_2$: $S_2 \xrightarrow{f_{2i}} S_i$, with $i = 0, 1, 3, 4, 7$;

• With origin $S_3$: $S_3 \xrightarrow{f_{3i}} S_i$, with $i = 0, 1, 4, 7$;

• With origin $S_4$: $S_4 \xrightarrow{f_{4i}} S_i$, with $i = 0, 1, 2, 5, 7$;

• With origin $S_5$: $S_5 \xrightarrow{f_{5i}} S_j$, with $j = 0, 1, 2, 6, 7$;

• With origin $S_6$: $S_6 \xrightarrow{f_{6j}} S_7$. 
Each connection strength is set to one of three levels: low, medium and high (with each level uniformly chosen to lie in the following ranges: 0 to 0.1 (low connection strength); 0.1 to 0.4 (medium connection strength); 0.4 to 0.9 (strong connection strength)).

Thus, given the links above, growing tensions between the two countries are set at the morphic origin of the systemic situations: political turmoil (medium connection strength); failure to negotiate (medium connection strength); civil unrest (low connection strength); conflict break out (low connection strength) and UN intervention/mediation (medium connection strength).

Political turmoil, on the other hand, is linked, with high connection strength, to: growing tensions; failure to negotiate and civil unrest. Failure to negotiate is linked, also with high connection strength to: growing tensions; political turmoil; civil unrest; conflict break out and UN intervention/mediation.

Civil unrest is linked with medium connection strength to conflict break out and with high connection strength to: growing tensions; political turmoil and UN intervention/mediation.

Conflict break out, on the other hand, is linked with high connection strength to growing tensions, political turmoil and failure to negotiate and it is linked with medium connection strength to nuclear threat escalation, in this case, the connection strength to UN intervention/mediation is non-randomly set to 1 (maximum connection strength).

Nuclear threat escalation and nuclear war are also set with connection strength of 1 to UN intervention/mediation, while nuclear threat escalation is set with high connection strength to: growing tensions; political turmoil; failure to negotiate and nuclear war.

This defines the morphic web. Now, the round-dependent quantum unitary gate for the game is, in the present case, set as:

\[ \hat{U}(t) = \hat{U}_+(t)\hat{U}_-(t) \]

\[ \hat{U}_+(t) = \bigotimes_{i=0}^{7} \hat{U}_i(r_i(t)) \quad \hat{U}_-(t) = \bigotimes_{i=0}^{7} \hat{U}_i(r_i(t-1))^\dagger \]

where \(r_i(t-1)\) and \(r_i(t) := F[r_i(t-1)]\) are local evolutionary dynamical parameters and \(F\) is a real-valued map, implemented by the local unitary operators \(\hat{U}_i\) which are defined by the quantum logical gate:

\[ \hat{U}_i(r) = \alpha_i(r) |0\rangle \langle 0| - |1\rangle \langle 1| + \beta_i(r) |0\rangle \langle 1| + |1\rangle \langle 0| \]

\[ \alpha_i(1-r) = \sqrt{1-r}, \quad \beta_i(r) = \sqrt{r} \]

where \(r\) is taken as a real number between 0 and 1 and, as in the previous section’s formalism, \(|0\rangle\) encodes the case in which the scenario \(S_i\) does not take

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7 For concrete countries, the web might be expanded to include tension triggering elements and other factors that are specific to the countries under analysis, as it stands the current morphic web constitutes a general work basis for a quantum game adaptable to different regional conflicts cases, being an example of how the formalism may be applied.
place and \(|1\rangle\) encodes the case in which the scenario takes place. This leads to a path-dependent quantum computation for each \(S_i\), with the quantum state for each round being given by:

\[
|\Psi(t)\rangle = \bigotimes_{i=0}^7 |\psi_i(t)\rangle = \bigotimes_{i=0}^7 \hat{U}_i (r_i) \hat{U}_i (r_i(t-1))^\dagger |\psi_i(t-1)\rangle \tag{15}
\]

with, as per Eqs. (11) to (14), \(r_i(t) = F[r_i(t-1)]\).

In the current quantum game, the map \(F\) is an actualization-contingent map, so that if \(S_7\) (UN intervention/mediation) does not take place in act, then \(F\) is given by

\[
F(r_i(t-1)) = G(r_i(t-1)) \tag{16}
\]

where \(G(r_i(t-1))\) is a lattice coupled nonlinear map to be specified shortly. On the other hand, if \(S_7\) takes place in act, then, \(F\) is, for all systemic situations except \(S_7\), given by:

\[
F(r_i(t-1)) = (1 - \theta_{UN}) G(r_i(t-1)) + \theta_{UN} (1 - d_{UN}) G(r_i(t-1)) \tag{17}
\]

where the parameters \(0 \leq \theta_{UN} \leq 1\) and \(0 \leq d_{UN} \leq 1\) correspond, respectively, to the effectiveness of UN intervention/mediation and \(d_{UN}\) is the impact of UN intervention/mediation. For the UN intervention/mediation the dynamics is, in this case, still given by Eq. (16). The nonlinear map \(G(\cdot)\), that appears in both Eqs. (17) and (16), is a coupled nonlinear map with the following structure:

\[
G(r_i(t-1)) = (1 - \varepsilon - \delta) M_i(t) + \varepsilon h_i(t) + \delta z_i(t) \tag{18}
\]

where \(M_i(t)\) is the result of the logistic map dynamics upon \(\beta_i(r_i(t-1))\) as follow:

\[
M_i(t) := b \cdot \beta_i (r_i(t-1)) (1 - \beta_i (r_i(t-1))) \tag{19}
\]

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8 If we were to consider here the “strong” many worlds interpretation one might assume a \(|\Psi(0)\rangle\) as holding for each parallel universe, then, for those universes/worlds in act in which the UN did not intervene, the state vector would evolve in accordance with the unitary gate \(U (F(r_i(1)))\), with \(F\) obeying Eq. (16), while, for those worlds in which UN did intervene, the state vector would evolve in accordance with the unitary state \(U (F(r_i(1)))\), with \(F\) obeying Eq. (17), the intervention and the non-intervention would thus become frozen events on each parallel universe holding from then on and leading to a divergence from \(|\Psi(0)\rangle\), with fitness amplitudes tracing back to the “ancestor” state \(|\Psi(0)\rangle\), in this case, actualization leads to a form of mixed state for the actualized multiverse description, since different worlds would be described by different pure states, evolving from an initial pure state that held for all, and one would have to account for this with an ensemble-based multiverse description where the initial ensemble is all in the same quantum state. This gives another perspective on branching, and shows how, in a multiverse description, one might have that unitary evolution path-dependent upon actualization would lead to an effective mixedness rather than a reduced mixed density operator due to a tracing out of a part of an entangled system’s degrees of freedom.

9 The logistic map expresses a growth limited by a carrying capacity which is useful in capturing a nonlinear dynamics of political resources that limit the decision to take a certain action. In the present case, the noisy chaotic dynamics leads to a noisy chaotic update of each qubit as per Eq. (15).
and $h_i(t)$ is the nonlinear mean field quantity:

$$h_i(t) := \frac{1}{\sum_j f_{ji}} \sum_j f_{ji} M_j(t) \cdot M_i(t)$$  \hspace{1cm} (20)

where the sum is over all the systemic situations $S_j$ that are at the morphic origin of $S_i$, that is, all the systemic situations that satisfy $S_j \xrightarrow{f_{ji}} S_i$, weighted by the respective morphic web couplings $f_{ji}$. The quantity $z_i(t)$ is a random uniform noise term between 0 and 1, thus, we have a noisy nonlinear coupled map lattice, emerging from the quantum computing dynamics due to the path-dependence and driving the quantum state transition.

At beginning of the game we set the state:

$$|\Psi(0)\rangle = \sqrt{1 - r_0(0)} |000...0\rangle + \sqrt{r_0(0)} |100...0\rangle$$  \hspace{1cm} (21)

where $r_0(0)$ is a real number uniformly chosen between 0 and 0.01, this means that the whole interconnected risk dynamics is driven by an initially small quantum amplitude for political tension between the two countries, apart from random factors integrated in the uniform noise coupling of Eq.(18). Thus, from the above equations, each unitary transition leads to a proportion of $|\alpha_i(r_i(t))|^2 = r_i(t)$ of morphisms of $W(t)$, where $S_i$ occurs and a proportion of $1 - r_i(t)$, where the event does not occur.

Taking into account Eq.(21), and from Eqs.(11) to (15) it follows that, for each qubit $|\psi_i(t)\rangle$, we can write the difference equation:

$$\triangle |\psi_i(t)\rangle =$$

$$= \{\alpha_i [F(r_i(t - 1))] - \alpha_i (r_i(t - 1))\} |0\rangle +$$

$$+ \{\beta_i [F(r_i(t - 1))] - \beta_i (r_i(t - 1))\} |1\rangle$$  \hspace{1cm} (22)

from the above equations it follows that the map $F$ reflects the morphic web connections as well as stochastic factors affecting each systemic situation $S_i$ and, therefore, the corresponding qubit $|\psi_i(t)\rangle$, which allows for an evolutionary dynamical framework to be incorporated in the path-dependent quantum computation.

The interconnected risk dynamics depends strongly upon the coupling parameter $\varepsilon$ in Eq.(18) to the local mean field $h_i(t)$ described by Eq.(20), without this coupling no coevolution would take place between the different systemic situations. The risk profile changes in a complex fashion with $\varepsilon$, thus, for instance, in 1,000 independent Netlogo simulations for $\varepsilon = 0.1, 0.3, 0.5$ and 0.7, with $\theta_{UN} = 0.9$ and $d_{UN} = 0.95$ (strong capability of the UN to influence the two countries), $b = 4$ and $\delta = 1.0E - 4$, we found that for each value of $\varepsilon$, in the corresponding 1,000 simulations, a nuclear war scenario eventually took place (actualization of a nuclear war) after a few quantum computation steps, which means that the model captures a dynamics of rising conflituality between two countries that, given the right conditions, are willing to use their nuclear arsenal, so that despite the very strong impact of the UN lowering the threat probability,
the two countries still go to war and for each value of $\varepsilon$, the corresponding 1,000 simulations always led to a nuclear war. The number of quantum computation steps that took for this to occur, in the simulations, however, differed critically with $\varepsilon$, as table 1’s statistics, presented in appendix, show.

On average, it took approximately eight quantum computation steps for these countries to reach nuclear war, however, the maximum number of steps it takes to reach the nuclear war scenario seems to depend upon the coupling, thus, for the strong coupling of $\varepsilon = 0.7$, for instance, the maximum is 28 times larger than the second largest maximum (that occurs for $\varepsilon = 0.5$). The minimum, for $\varepsilon = 0.7$, also falls below the previous couplings, with a minimum of 2 quantum computation steps to reach nuclear war. Half of the simulations have produced, for each coupling, less than 6 quantum computation steps to reach nuclear war (median), and while the most frequent number of steps is also 6 for $\varepsilon = 0.1$, it becomes 5 for the rest of the parameter values.

The simulation statistics, thus, show, in the central tendency statistics, some structural patterns that do not seem to vary much with the coupling, the only differences showing up at the extreme values evaluated in terms of the maxima. Indeed, it is at the extreme values that the coupling seems to lead to a break in pattern, which also explains the pattern of rise in heavy tails, especially in the highest coupling of $\varepsilon = 0.7$ with a kurtosis of near 29.

Higher coupling can lead to a higher kurtosis and a higher maximum, which, in this case, means that higher coupling can lead to some extreme cases in which it takes much longer for the countries to reach a nuclear war.

The histograms shown in the figure 1 (presented in appendix) for the same simulations complete this picture with some additional visible differences. Indeed, they show that, while as the coupling increases, in the simulations, there occur cases in which it takes a longer number of quantum computation steps to reach nuclear war with a greater relative frequency, the cases in which nuclear war is reached in a smaller number of steps also take place with a higher relative frequency.

Since the model is built from a morphic web that is worked from the linkages between risk situations that may lead to a nuclear war, it is not surprising that the model does eventually lead to a nuclear war scenario, through its path-dependent quantum unitary evolution, it was built exactly to capture such an extreme risk dynamics scenario, that is, it is modelling a dynamics of tensions between countries that possess nuclear weapons and that may use them, given the right conditions in terms of a very high conflictuality. This feature of the model may make it effective as a tool for constructing early warning systems that try to identify risk dynamics between two countries with nuclear weapons and a history of past political and social tensions, thus, the model is addressing the potential for regional nuclear conflict and the consequences that may arise from such conflicts.

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10 This is seen in the first bar of the histograms that rises steadily with rising coupling, becoming the modal class for $\varepsilon = 0.7$. 
5 Discussion

Scenario analysis and game theory are two major techniques central for risk analysis, but while classical game theory has focused on individuals and populations (in the case of classical evolutionary game theory), quantum game theory also allows for another type of application, in which one deals with an evolutionary scenario analysis. In this case, risk mathematics may benefit from the combination of its morphic webs-based formalism, used in the formulation of general scenarios, with quantum game theory, leading to a scenario dynamics that follow the structural systemics of the morphic webs of risk situations leading to coevolving dynamics in which one situation can trigger another.

A statistical analysis of the resulting dynamics, through repeated experiments, may be helpful in the different applications of risk mathematics as was shown in the present work, through the application of this modelling methodology to a geopolitical game with threat of nuclear war, in which a nuclear war scenario may result from the coevolution of the political tensions and conflictuality between two countries, thus addressing, through a quantum game on a quantum risk structure, political and social dynamics leading up to regional nuclear conflict, a methodology that can be expanded to other areas of application of risk mathematics that demand the employment of evolutionary scenario analysis in the context of interconnected risk situations’ dynamics.

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Appendix

| $\varepsilon$ | 0.1 | 0.3 | 0.5 | 0.7 |
|---------------|-----|-----|-----|-----|
| Number of simulations | 1,000 | 1,000 | 1,000 | 1,000 |
| Maximum | 49 | 43 | 52 | 80 |
| Minimum | 3 | 3 | 3 | 2 |
| Mode | 6 | 5 | 5 | 5 |
| Median | 6 | 6 | 6 | 6 |
| Mean | 8.043 | 7.911 | 8.089 | 7.79 |
| Standard-Deviation | 5.157 | 5.586 | 6.200 | 5.852 |
| Skewness | 3.618 | 2.920 | 3.088 | 3.948 |
| Kurtosis | 16.107 | 10.091 | 11.768 | 28.665 |

Tab. 1: Statistics from Netlogo experiments for the model, with parameters: $\theta_{UN} = 0.9$, $d_{UN} = 0.95$, $b = 4$ and $\delta = 1.0E - 4$, and $\varepsilon = 0.1, 0.3, 0.5$ and 0.7 with 1,000 simulations for each alternative value of $\varepsilon$.

Fig. 1: Histograms with 20 bins, estimated for table 1’s data.