Conserving T-matrix theory of superconductivity

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Abstract. We remove a self-interaction from the Galitskii-Feynman T-matrix approximation. This correction has no effect in the normal state but makes the theory applicable to the superconducting state. It is shown that identical theory is obtained by removing nonphysical repeated collisions in the spirit of the Fadeev-Watson-Lovelace multiple scattering expansion. Our correction does not violate the two-particle symmetry of the T-matrix, therefore the present theory is conserving in the Baym-Kadanoff sense. The theory is developed for retarded interactions leading to the Eliashberg theory in the approximation of a single pairing channel.

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1. Introduction

The family of superconducting materials collects so diverse systems as conventional metals and metallic alloys [1], high-$T_c$ ceramics [2], fullerenes [3], organic superconductors [4], doped diamond [5], heavy fermion metals [6], $^3$He [7], symmetric nuclear matter in nuclei [8, 9, 10] and very non-symmetric nuclear matter in neutron stars [11], Fermi gases [12], not talking about hypothetical condensates like the color superconductivity of quarks [13]. It is rather surprising how many features of these systems have been successfully explained within the mean-field Bardeen-Cooper-Schrieffer (BCS) theory and its Green function extension due to Eliashberg. On the other hand, there is a growing list of experimental facts which require to employ more elaborate theories.

Recent theoretical approaches to superconductivity range from trial wave functions of Gutzwiller type [14], over improved Eliashberg theories [15, 16], renormalization group approaches within path integrals [17], exact diagonalizations and quantum Monte Carlo studies [18] on simple models having small size or infinite dimensions [19], to the two-particle T-matrix approximation [20, 21, 22, 23].

The majority of studies focus on high-order correlations leaving aside details of the pairing interaction. Among the most popular approaches, there are the Hubbard model and the Fermi gas model with point interaction. Much less attention is payed to the retardation and finite range of interaction which are studied only by Eliashberg-type theories. For many of systems listed above, however, the retarded nature or finite range of interaction are among the most important ingredients of a theoretical model if one wants to achieve a quantitative agreement.

In this paper we derive a T-matrix approximation which combines advantages of the Galitskii-Feynman (GF) and the Kadanoff-Martin (KM) approximation. It applies to Fermi systems interacting via boson fields and provides the superconducting gap which depends on the energy reflecting the retardation of the interaction line.

2. Self-interaction in the T-matrix approach

The GF and the KM approximation are compared in a diagrammatic representation in Fig. 1. Both approximations are based on the two-particle T-matrix in the ladder approximation. As one can see, the KM approximation is nothing more but a simplified version of the GF approximation neglecting the exchange and possessing a bare line in the closed loop of selfenergy. Nevertheless, these two approximations are very different with very disjunct fields of applications.

The GF approximation is used in nuclear physics for both equilibrium [24, 25, 26] and non-equilibrium [27, 28] problems, in the theory of moderately dense gases [29] and liquid $^3$He [30], and in studies of electron-electron correlations in molecules and solids [31, 32, 33, 34, 35]. The KM approximation was never adopted to these problems by many reasons. The most important is that the conservation laws are guaranteed only
Conserving T-matrix theory of superconductivity

Galitskii-Feynman

\[
\begin{align*}
\text{Figure 1. T-matrix approximations in diagrams.} & \quad \text{Both approximations have a selfenergy constructed from the two-particle T-matrix. The interaction carried by boson propagators (wavy lines) is included in the ladder approximation. The GF approximation has all internal lines of the selfenergy dressed (thick arrows). In the KM approximation the loop is made of bare propagators (thin arrows). The GF approximation includes the exchange channel in which the lower line matches the upper one so that the diagram has no loop. The exchange channel contributes only if both particles have parallel spins.}
\end{align*}
\]

if the T-matrix is symmetric with respect to the interchange of the upper and lower line \[36\]. The non-symmetry of the KM approximation is viewed as unjustified and unacceptable.

The KM approximation is used exclusively in the theory of superconductivity \[21, 22, 37, 38, 39, 40, 41, 42, 43, 44\]. It describes the superconducting gap on the level of the mean-field theory and covers the lowest order fluctuations. The GF approximation cannot be employed for superconductors in spite of its superiority in other fields. Although it becomes unstable at the critical temperature \[45\] and the T-matrix diverges there, the GF selfenergy constructed from the T-matrix fails to describe the superconducting gap \[46\].

The paradox that the worse approximation (KM) works well while the better one (GF) fails was first noticed by Prange \[47\] and Wilde \[46\] prior to the work of Kadanoff and Martin. The Prange paradox is not a common knowledge and some authors, see e.g. \[48\], report a superconducting gap obtained within the GF approximation. By closer inspection one finds that they simplify formulas using the bare propagator to close the loop. This additional simplification turns the GF into the KM approximation.

Apparently, the GF approximation includes diagrams which block a formation of the gap, while the KM approximation is free of them. One of such contributions is sketched in figure 2. The diagram shows two kinds of the nonphysical self-interaction; the simple self-interaction in the Hartree approximation and the indirect self-interaction which is enhanced by the resonant collisions when the superconducting condensate appears. The self-interaction via the Hartree field becomes the important mistake when a particle is bounded in the real space. The indirect self-interaction becomes crucial when particles are bounded in the momentum space as it is the case of Cooper pairs.

In Feynman’s diagrammatic expansion the self-interaction is canceled by corresponding exchange diagrams. Such compensating exchange diagrams are beyond
Figure 2. **a)** Schematic picture of four interacting particles of initial momenta (and spins) $k$, $p$, $q$, and $m$. Due to the Pauli principle all particles are in different states, e.g. $m \neq k$ and $q \neq k$. **b)** Corresponding Feynman diagrams for the Green function of the particle $k$. The other lines are closed into loops and momenta $p$, $q$, and $m$ are summed over with no restriction. For $m = k$ the diagram yields the self-interaction, the artifact appearing already in the Hartree approximation. For $q = k$ the nonphysical self-interaction is more complex. In the superconducting state collisions are enhanced for $p = -k$. Similarly, the enhanced processes couple $p$ with $q = -p$, which leads to $q = -p = k$. This indirect resonant self-interaction blocks the formation of the gap. The $q$-loop is included in the GF approximation while it is absent in the KM approximation.

the T-matrix approximation and it is not clear how many and how complicated diagrams ought to be included. Such cancellation can be demonstrated on the simpler self-interaction included in the Hartree field. This self-interaction is canceled by the Fock selfenergy – one step more complicated approximation. Apparently, the original simple problem is cured only on cost of an increase of complexity of theory.

One can eliminate the self-interaction restricting sums in the Hartree term in specified real space representation. Such correction is not fully equivalent to inclusion of the Fock term, but it removes gross mistakes, e.g. the potential binding electrons to neutral atom has Coulomb long range asymptotics while in the Hartree approximation it is fully screened. Similarly, one can eliminate the self-interaction restricting sums in the T-matrix approximation. With respect to Copper pairs, this restriction has to be done in the momentum space. This approach we follow here.

### 3. Restricted T-matrix approximation

When a pair of particles interact in stationary and homogeneous systems, its energy and momentum $Q \equiv (\omega, Q)$ conserve. Dressing of a particle of four-momentum $k$ is given by the selfenergy $\Sigma_\uparrow(k)$ which is a sum over interacting pairs, $\Sigma_\uparrow(k) = \sum_Q \sigma_{Q\uparrow}(k)$. Here $\sigma_{Q\uparrow}(k)$ is a single $Q$ contribution and $\sum_Q \ldots \equiv \sum_\omega \sum_Q \ldots$ denotes sums over bosonic Matsubara frequencies and discrete momenta in the quantization volume $\Omega$.

In the spirit of the multiple scattering expansion [49] we introduce a reduced
The reduced propagator \( G_{Q\downarrow}(p) \) of the particle of four-momentum \( p \) and spin \( \downarrow \), briefly \( (p; \downarrow) \)-particle, is dressed by all binary interactions except for the one in which the sum four-momentum of it and its interaction partner equals \( Q \). By this we exclude the interaction of the \( (Q-k; \downarrow) \)-particle with the \( (k; \uparrow) \)-particle.

The contribution of a collision of pair four-momentum \( Q \) to the self-energy

\[
\sigma_Q^\uparrow(k) = \frac{k_B T}{\Omega} T_{\uparrow\downarrow}(k,Q-k;k,Q-k) G_{Q\downarrow}(Q-k)
\]

we will call a \( Q \)-channel of selfenergy for brevity. The dressed propagator \( G \) is given by the Dyson equation with the unrestricted sum over channels

\[
G^\uparrow(k) = G^0(k) + G^0(k) \sum_Q \sigma_Q^\uparrow(k) G^\uparrow(k).
\]

To avoid the self-interaction in Eq. (2) we use the reduced propagator \( G_{Q\downarrow}(Q-k) \) to close the loop. To guarantee conservation laws, all propagators in the loop have to be in the same approximation. Accordingly, the T-matrix has to be constructed as

\[
T_{\uparrow\downarrow}(k,Q-k;p,Q-p) = D(k,Q-k;p,Q-p) - \frac{k_B T}{\Omega} \sum_{k'} D(k,Q-k;k',Q-k') G^\uparrow(k') G_{Q\downarrow}(Q-k') T_{\uparrow\downarrow}(k',Q-k';p,Q-p),
\]

where \( D \) is a bosonic interaction line with interaction vertices included. The prime on sum denotes that it runs over fermionic Matsubara frequencies. We follow the sign convention of Ref. [50] Sec. 14.2. with \( D \) going to the interaction potential in the non-retarded limit.

The set of equations (1)-(4) is closed. For the sake of simplicity, we write only equations for a selected spin orientation, the complementary equations are obtained by flipping all spins, \( \uparrow \leftrightarrow \downarrow \). These equations do not have diagrammatic representation in the strict sense. This is because the Feynman diagrammatic rules are based on unrestricted sums over states.

4. Asymptotic properties

The theory is complete. Now we discuss its properties.

In the normal state, the single-channel contribution vanishes in the thermodynamic limit, \( \sigma_Q^\uparrow \propto 1/\Omega \to 0 \). The reduced propagator then equals to the dressed propagator, \( G_{Q\uparrow} \to G^\uparrow \), and the present theory is identical to the GF approximation.

In the well developed superconducting state a single channel becomes singular. In the absence of currents and in equilibrium, it is the zero energy and zero momentum
Conserving T-matrix theory of superconductivity

channel \( Q = (0, \mathbf{0}) \). Its T-matrix is separable \([51, 45]\) and diverges being proportional to the volume

\[
\mathcal{T}_{\uparrow \downarrow}(k, -k; p, -p) = -\frac{\Omega}{k_B T} \bar{\Delta}(k) \Delta(p).
\]  

(5)

It is advantageous to split the singular part off the remaining regular terms

\[
\Sigma_{\uparrow}(k) = -\bar{\Delta}(k) G_{\uparrow \downarrow}(-k) \Delta(k) + \Sigma_{\not{\uparrow}}(k).
\]  

(6)

The reminder

\[
\Sigma_{\not{\uparrow}}(k) = \sum_{Q \neq 0} \sigma_{Q\uparrow}(k)
\]  

(7)

covers normal processes in the background of superconductivity. From equations (1) and (5) one can see that \( \Delta \) does not enter the reduced propagator

\[
G_{\not{\uparrow}}(k) = G^0_{\not{\uparrow}}(k) + G^0_{\not{\uparrow}}(k) \Sigma_{\not{\uparrow}}(k) G_{\not{\uparrow}}(k).
\]  

(8)

With reservations one can say that \( G_{\not{\uparrow}} \) is a propagator of the normal metal.

Using the selfenergy (6), the dressed Green function (3) can be expressed via the reduced propagator

\[
G_{\uparrow}(k) = G_{\not{\uparrow}}(k) - G_{\not{\uparrow}}(k) \bar{\Delta}(k) G_{\not{\uparrow}}(-k) \Delta(k) G_{\uparrow}(k).
\]  

(9)

This equation shows that \( \Delta(k) \) equals to the energy and momentum dependent anomalous selfenergy giving the superconducting gap in the Eliashberg theory \([1]\). The anomalous selfenergy itself follows from the equation for the T-matrix (4) and the separability (5)

\[
\bar{\Delta}(k) = -\frac{k_B T}{\Omega} \sum_{k'} D(k, -k; k', -k') G_{\uparrow}(k') G_{\not{\uparrow}}(-k') \bar{\Delta}(k').
\]  

(10)

Deriving (10) we have used that \( D/\Omega \to 0 \) in the thermodynamic limit.

Apparently, in the well developed superconducting state the present theory asymptotically approaches the Eliashberg theory. There are some differences, however. In the present theory all processes, normal and pairing, are treated within the same T-matrix approximation. In the Eliashberg theory the normal processes are in the Migdal (or Born) approximation while the pairing covered by equations for anomalous functions is described by the approximation corresponding to the T-matrix.

Detail discussion of a region near the critical temperature is beyond the scope of this paper. We merely state that the single-channel approximation (10) can be simply modified to non-zero pair momentum giving again the familiar equation for the anomalous functions. It implies that phase fluctuations of the gap function are the same as those following from the time dependent Ginzburg-Landau theory. There are, however, differences in excitation spectrum of noncondensed bound pairs. The mean-field theories of Eliashberg type ignore these excitations. The KM approximation overestimates their density \([52]\) having quadratic dispersion relation for bound pairs. In a next paper it will be shown that the present theory yields an energy gap between condensed and noncondensed pairs.
5. Two-particle symmetry

Now we show that the present theory is conserving in the Baym-Kadanoff sense, i.e., that it satisfies conditions (A) and (B) from Ref. [36]. The condition (A) links the single-particle Green function \( G \) with the two particle function \( G \). For the present approximation it is satisfied if the two-particle function relates to the T-matrix as

\[
G_{\uparrow \downarrow}(k, Q-k; p, Q-p) = G_{\uparrow}(k)G_{\downarrow}(Q-k) \delta(k-p) - G_{\uparrow}(k)G_{\downarrow}(Q-k) \mathcal{T}_{\uparrow \downarrow}(k, Q-k; p, Q-p) G_{\uparrow}(p)G_{\downarrow}(Q-p).
\]

(11)

The condition (B) demands that the two-particle function is symmetric with respect to interchange of the upper and lower lines. This symmetry is satisfied although it is not obvious from expression (11).

First we show that the T-matrix (11) is symmetric with respect to the interchange of the upper and lower lines

\[
\mathcal{T}_{\uparrow \downarrow}(k, Q-k; p, Q-p) = \mathcal{T}_{\downarrow \uparrow}(Q-k, k; Q-p, p),
\]

(12)

in spite of the selfconsistency restricted only in the upper line. Apparently, symmetry (12) is satisfied for the lowest order \( T \approx D \).

Now we assume that the T-matrix is symmetric in the \( n \)-th order and show that the symmetry of \( (n+1) \)-th order follows. For a general order obtained by iteration of Eq. (11) it is sufficient to show that

\[
G_{\downarrow}(k)G_{\downarrow}(Q-k) = G_{\uparrow}(k)G_{\downarrow}(Q-k).
\]

(13)

In the iteration process we assume \( n \)-th order in the selfenergy of Green functions \( G_{\bar{Q}}G \). From Eq. (11) we then have

\[
\sigma_{\uparrow}(k)G_{\downarrow}(k) = \mathcal{T}_{\uparrow \downarrow}(k, Q-k; k, Q-k)G_{\downarrow}(Q-k)\sigma_{\downarrow}(k) = \mathcal{T}_{\downarrow \uparrow}(Q-k, k; Q-k, k)G_{\downarrow}(Q-k)\sigma_{\downarrow}(k) = G_{\downarrow}(Q-k)\sigma_{\downarrow}(Q-k),
\]

(14)

where we have used symmetry of the \( n \)-th order. Now we are ready to proof (13),

\[
G_{\downarrow}(k)G_{\downarrow}(Q-k) = G_{\uparrow}(k)(1 - \sigma_{\downarrow}(k)G_{\downarrow}(k)) \times (1 + G_{\downarrow}(Q-k)\sigma_{\downarrow}(Q-k)) G_{\downarrow}(Q-k) = G_{\uparrow}(k)G_{\downarrow}(Q-k)(1 - \sigma_{\downarrow}(Q-k)G_{\downarrow}(Q-k)) \times (1 + G_{\downarrow}(Q-k)\sigma_{\downarrow}(Q-k)).
\]

(15)

In the first step we have used

\[
G_{\downarrow}(Q-k) = G_{\downarrow}(Q-k) + G_{\downarrow}(Q-k)\sigma_{\downarrow}(Q-k)G_{\downarrow}(Q-k)
\]

(16)

which follows from Eqs. (11) and (13), and a similar relation for \( G_{\uparrow}(k) \). In the second step we have substituted from Eq. (14). From equation (16) follows that the product of the two brackets in Eq. (15) equals to unity. The relation (13) is proved. Therefore, the symmetry (12) of the T-matrix is proved.
From the symmetry of the T-matrix (12) and relation (13) follows the symmetry of the two-particle Green function
\[ G_{↑↓}(k, Q-k; p, Q-p) = G_{↓↑}(Q-k, k; Q-p, p). \] (17)
The present theory thus also satisfies the condition (B) and is conserving in the Baym-Kadanoff sense.

We note that in the KM approximation the two-particle Green function satisfying the condition (A) is made of one dressed and one bare line. Such two-particle function does not satisfy the condition (B).

6. Multiple scattering approach

The nonphysical resonant self-interaction is naturally avoided in the Fadeev-Watson-Lovelace multiple scattering expansion [53, 54, 55, 56, 57, 58, 49]. In Ref. [23] the method of the multiple scattering theory was implemented to the theory of superconductivity following a rather different approach. It was argued that the GF approximation includes nonphysical contributions in which two particles interact again after they have accomplished a collision. Indeed, the T-matrix sums the interaction potential to the infinite order so that the next interaction is possible only with a next particle. When one eliminates such repeated collisions from the GF approximation, the resulting theory goes to the GF approximation in the normal state while it yields the gap in the superconducting state. Here we show that the present approach and the approach of Ref. [23] modified to four-momentum yield identical results.

The nonphysical repeated collisions can be eliminated with the help of Soven’s concept of effective medium modified for binary collisions. In this spirit one selects a Q-channel which will be described explicitly while all other channels are described by the effective medium represented by the selfenergy. We thus eliminate the Q-channel from the selfenergy of the \((k; ↑)-particle. This channel is included explicitly into the full propagator with a single collision being allowed,
\[ G_{↑}(k) = G_{Q↑}(k) + G_{Q↑}(k) \frac{k_B T}{\Omega} \mathcal{T}_{↓↑}(k, Q-k; k, Q-k) G_{↓}(Q-k) G_{Q↑}(k). \] (18)
Note that the loop is closed by the full Green function. This is because we have eliminated \(\sigma_{Q↑}\) which does not enter \(G_{↓}\) in the loop.

The scattering equation (18) defines the selfenergy indirectly. Comparing (18) with
\[ G_{↑}(k) = G_{Q↑}(k) + G_{Q↑}(k) \sigma_{Q↑}(k) G_{↑}(k) \] we find that Q-channel selfenergy is given by
\[ \frac{\sigma_{Q↑}(k)}{1 - \sigma_{Q↑}(k) G_{Q↑}(k)} = \frac{k_B T}{\Omega} \mathcal{T}_{↓↑}(k, Q-k; k, Q-k) G_{↓}(Q-k). \] (19)
This relation complicates a straightforward implementation of the theory obtained from Soven’s scheme. Fortunately, it can be simplified to the set (1)-(4).

Using the symmetry (13) one can readily see that equation (18) is equivalent to the approximation introduced in this paper. Alternatively, one can multiply Eq. (19) by the denominator of the left hand side and use Eq. (14) to turn relation (19) into Eq. (2).
Finally, we have to take into account that inside the T-matrix the reduced propagator appears for spin $\uparrow$, i.e., in the lower line. According to (12) this is equivalent to the T-matrix with the reduced upper line. The present derivation is thus equivalent to the application of Soven’s scheme.

We note that here we identify the channel via energy and momentum $Q \equiv (\omega, Q)$. In Ref. [23] the channel was identified only via momentum $Q$ what applies only to non-retarded interactions and leads to slightly different results. In particular, the identification of a channel via momentum does not provide a two-particle Green function symmetric with respect to interchange of the upper and lower lines. The theory in Ref. [23] thus does not satisfy the condition (B) and it is not conserving in the Baym-Kadanoff sense.

7. Effect of self-consistency

The present theory can be implemented to the same family of problems as the KM approximation which was recently successfully used to explain many properties of such systems as the high-$T_c$ materials [37, 38, 21, 39], organic superconductors [4] and the ultra-cold gasses [40, 22, 41, 42, 43, 44].

We do not expect that all results will be identical. As already mentioned, the major difference between the present theory and the KM approximation rests in the self-consistency with which one includes normal processes in the loop. While the KM approximation covers only the Hartree field, the present approximation keeps all normal processes described at the level of the T-matrix.

In this section we demonstrate the effect of the self-consistency on the phase diagram of the Fermi gas. For simplicity we use a non-retarded attractive interaction of Yamaguchi type

$$D(k, Q-k; p, Q-p) = -\frac{\lambda}{\Omega} \sum_{k,p,Q} \psi^\dagger_{Q+p\uparrow} \psi^\dagger_{Q-p\downarrow} \frac{\gamma^2}{\gamma^2 + p^2} \frac{\gamma^2}{\gamma^2 + k^2} \psi_{Q-k\uparrow} \psi_{Q+k\downarrow}$$

with a soft cutoff at momentum $\gamma$. In figure 3 all quantities are dimensionless: the temperature is scaled with the cutoff energy $\epsilon_\gamma = \hbar^2 \gamma^2 / 2m$, the momentum with cutoff momentum, $k \rightarrow k/\gamma$, the density with the cutoff cube, $n \rightarrow n/\gamma^3$. The used interaction strength is $\lambda \rightarrow \lambda/(8\pi \hbar^2 / m \gamma) = 10$.

As one can see in figure 3 at high densities and temperatures the system is a Fermi gas or liquid. At lower densities, the Pauli blocking of internal states is not effective and diatomic molecules become stable. Region with and without molecules are separated by the dash-dot line. The stable molecules appear as a pole of the T-matrix at a negative energy – the bound state energy. The grey region at low temperatures has non-zero gap. At high densities it corresponds to the BCS condensation of Cooper pairs, at low densities to the Bose-Einstein Condensation (BEC) of molecules. The dashed line which separates these two regions is the crossover line given by the zero value of the chemical potential. Since the chemical potential is fixed to the bound-state energy, it can be also interpreted as a line separating stable molecules from Cooper
Conserving $T$-matrix theory of superconductivity

Figure 3. Phase diagram of the Fermi system with attractive interaction. Compared to the present theory (thick lines), the Kadanoff-Martin approximation (thin lines) reduces the region of the BCS condensate shifting the superconducting phase transition towards lower densities. This suppression follows from unequal masses of particles in a Cooper pair – an artifact of the KM approximation.

The value of the chemical potential is evaluated from the fixed particle density,

$$n = \frac{k_B T}{\Omega} \sum_k (G_{\uparrow}(k) + G_{\downarrow}(k)).$$

The present theory (thick lines) and the KM approximation (thin lines) yield similar phase diagrams, but the transition lines visibly differ. In the KM approximation the region of BCS condensate is slimmer. This is a consequence of unequal masses of spin $\uparrow$ and $\downarrow$ particles in this approximation – one mass is dressed while the other is bare. These unequal masses complicate a formation of Bogoliubov-Valutin excitations and reduce an energy gain due to the gap and also the critical temperature at higher densities.

In spite of the destructive effect of unequal masses, the KM approximation predicts higher critical temperature at low densities. This follows from the employed Yamaguchi-type of attractive potential, which reduces the density of states at the Fermi level. In the KM theory, the two-particle density of states is only partly renormalized and thus higher than in the fully renormalized present theory.

8. Conclusions and discussion

In summary, removing the self-interaction from the Galitskii-Feynman approximation we have derived a theory which describes the gap including the gap fluctuations. Its structure reminds a renormalized Kadanoff-Martin approximation, however, it has four major improvements compared to it. First, in the normal state it recovers the well-tested original Galitskii-Feynman approximation. Second, the two-particle propagator is symmetric with respect to interchange of both lines. This symmetry implies that the theory is conserving in the Baym-Kadanoff sense. Third, the removed self-interaction is
Conserving $T$-matrix theory of superconductivity

a correction of the $T$-matrix which depends on the momentum of the Cooper pair. Non-condensed Cooper pairs thus satisfy a different equation than condensed pairs what explains stability of the super-currents with respect to excitations of non-condensed pairs. Fourth, the present theory is rooted in the standing many-body approach known as the multiple scattering expansion.

We have discussed only singlet channels covered by $T_{\uparrow\downarrow}$. The triplet interaction of equal spin components is described by $T_{\uparrow\uparrow}$. As long as triplet pairs do not form a condensate and no triplet channel is singular, the triplet $T$-matrix can be handled in the GF approximation including the exchange term. We note that the triplet pairing with the exchange included is a formidable problem which has not been treated yet within the $T$-matrix approaches to superconductivity.

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Conserving T-matrix theory of superconductivity

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