Research on Speed Sensorless Control of Induction Motor Based on Back EMF

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Abstract: This article mainly uses the reduced-order model of induction motor (IM) to study the speed identification. Since the traditional voltage model (VM) and current model (CM) each have certain limitations, a general reduced-order model obtained by combining VM and CM is proposed for flux observation. This method is more conducive to analyzing system stability and determining gain, so that the entire induction motor vector control system reaches complete stability. In addition, for the full-order observer, according to the rapidity of current convergence, the back EMF can also be used to convert the full-order observer into an equivalent reduced-order observer. In this way, state feedback matrix can be avoided, which can greatly reduce the complexity of calculation and be more convenient to analyzing the stability of the system. Through the simulation of this novel back EMF reduced-order model, the feasibility of the system can be verified.

1. Introduction

The IM itself has the advantages of simple structure, low price, stable operation and strong applicability. Coupled with the proposal and development of vector control, the excellent performance of induction motors stands out among AC motors [1]. In practical industrial applications, it is difficult and costly to simply rely on the speed sensor to obtain the speed. Therefore, the research on speed identification has become a research hotspot of IM speed sensorless vector control technology [2].

In order to obtain a highly reliable estimated speed and good dynamic performance, many experts and scholars use ideal motor models for speed estimation. Common methods include model reference adaptation systems (MRAS), full-order adaptive observers (AFO), and reduced-order observers. The MRAS algorithm calculates the motor speed according to the error adaptive law by comparing the same state variables output of reference model and the adjustable model, which can improve speed recognition accuracy and system stability through PI adjustment [3-4]. AFO usually combine stator current or flux and rotor flux as a state variable, and correct it according to the error which is also used to adaptively identify the speed between the measured stator current and its estimated value. It is also essentially MRAS, while the design of feedback matrix is a little difficult, the quality of the design will affect the dynamic response of the system and the sensitivity of the system to the motor parameters [5].

Both of the methods mentioned above use the CM which exits a certain coupling that can affect each other between the flux and the speed [6]. In order to eliminate the influence and realize decoupling, this paper proposes the rotor field-oriented control based on back EMF, and uses the combination of VM and CM to get a universal reduced-order model [7]. Finally, the observer equation does not contain the speed information that can be obtained by direct calculation or adaptive
identification, which can reduce the sensitivity of the parameters, improve the anti-disturbance and the stability. This paper proves the feasibility and effectiveness of this method through the simulation analysis of MATLAB/Simulink.

2. Induction motor model

In the static coordinate system (denoted by superscript s), according to the “inverse τ ” equivalent circuit diagram as shown in Figure 1, the VM and CM of the rotor flux are obtained as follows:

\[
\frac{di^s}{dt} = \frac{1}{L_s} \left( v^s - R_s i^s - \frac{d\Psi^s_R}{dt} \right)
\]

(1)

\[
\frac{d\Psi^s_R}{dt} = R_s i^s - a\Psi^s_R = E^s
\]

(2)

\( v^s \) is stator voltage; \( i^s \) is stator current; \( R_s \) is stator resistance; \( \Psi^s_R \) is rotor flux; \( \omega \) is the rotor speed; \( L_s, L_s \) is stator and rotor inductance; \( L_s = L_s - L_M \) is leakage inductance; \( a = -j\omega, a = R_s / L_M \) is inverse rotor time constant; “ˆ” indicates an estimated quantity; “~” indicates an error.

Taking (2) into (1), you can get the deformation of VM and CM.

\[
\frac{di^s}{dt} = \frac{1}{L_s} \left( v^s - R_s i^s + a\Psi^s_R \right)
\]

(3)

3. Reduced-order observer

As we all know, the classic VM, CM is a special reduced-order observer, and the flux can be calculated by integrating the back EMF in VM and CM, i.e. \( E^s = d\Psi^s_R / dt \), \( \hat{E} = d\hat{\Psi}^s_R / dt \). The actual back EMF can be obtained from the VM of (1), and the estimated back EMF can be obtained from CM of (2).

3.1. Synchronous implementation of reduced-order model

In order to achieve complete decoupling of current, and more convenient control of flux and torque, this paper is implemented in a synchronous coordinate system.

Article [8] mentioned that VM and CM can be merged into a universal reduced-order observer.

\[
\frac{d\hat{\Psi}_R}{dt} + j\omega \hat{\Psi}_R = kE + (1-k)\hat{E} = \hat{E} + k\hat{E}
\]

(4)

where \( \hat{E} = E^s - \hat{E}^s \) is error back EMF, \( k= k_d + jk_q \) is the observer gain. When \( k=1 \), the equivalent is VM, and when \( k=0 \), the equivalent is CM.
From (1) (2), we can respectively get the back EMF in synchronous coordinates
\[ \hat{\psi}_R = \psi_R e^{j\theta}, \]
\[ d\theta / dt = \omega. \]

\[ E = v_s - (R_s + j\omega_L) i_s - L_s \frac{di_s}{dt} \quad (5) \]
\[ \hat{E} = R_s i_s - \hat{a}\hat{\psi}_R \quad (6) \]

and \( \hat{E} = E - \hat{E} = \hat{E}_d + j\hat{E}_q \) where the corresponding dq axis components are

\[ E_d = v_d - R_i i_d + \omega_L i_q - L_s \frac{di_q}{dt} \quad (7) \]
\[ E_q = v_q - R_i i_q - \omega_L i_d - L_s \frac{di_d}{dt} \quad (8) \]
\[ \hat{E}_d = E_d - a(L_M i_d - \hat{\psi}_R) \quad (9) \]
\[ \hat{E}_q = E_q i_d - R_i i_q - \hat{\omega}\hat{\psi}_R \quad (10) \]

The general reduced-order model (4) is decomposed into real and imaginary parts to obtain \( d\psi_R / dt \) and \( \omega \). For speed sensorless control, the speed \( \omega \) can be obtained by adaptive

\[ \frac{d\hat{\psi}_R}{dt} = k_d \hat{E}_d - k_q \hat{E}_q + a(L_M i_d - \hat{\psi}_R) \quad (11) \]
\[ \omega = \frac{E_q + k_q \hat{E}_d - (1 - k_d) \hat{E}_q}{\hat{\psi}_R} \quad (12) \]
\[ \frac{d\hat{\omega}}{dt} = \text{Im}\{k_w \hat{\psi}_R \hat{E}\} \quad (13) \]

where \( k_w = k_w e^{-j\Omega} \), adaptive gain \( k_w > 0 \). (11), (12), (13) are the reduced-order observers based on back EMF. In addition to using (13) to estimate speed adaptively, it can also be obtained from the slip \( \omega_s = R_i i_q \hat{\psi}_R \), i.e. \( \hat{\omega} = \omega - \omega_s \). The above algorithm shows that there are many parameters, including \( k_d, k_q, k_w, \Omega \) and \( k_p, k_i \), which makes the stability analysis relatively complicated.

Since the convergence rate of \( \omega \) is much faster than \( \psi_R \), the partial derivative of (13) gives

\[ \frac{\partial \text{Im}\{k_w \hat{\psi}_R \hat{E}\}}{\partial \hat{\omega}} = -\text{Re}\{k_w \hat{\psi}_R \hat{E}\} = -k_w \hat{\psi}_R \cos \Omega \quad (14) \]

When \( \hat{\psi}_R \neq 0 \), \( k_w = \infty \) and \( \cos \Omega > 0 \), \( \hat{\omega} \) in (13) will be asymptotically stable, then \( d\hat{\omega} / dt = \text{Re}\{k_w \hat{\psi}_R \hat{E}\} + \text{Im}\{k_w \hat{\psi}_R \hat{E}\} = 0 \), and get

\[ \hat{\xi} = \frac{\hat{E}_q}{\hat{E}_d} = -\frac{\text{Im}\{k_w \hat{\psi}_R \hat{E}\}}{\text{Re}\{k_w \hat{\psi}_R \hat{E}\}} = \tan \Omega \quad (15) \]
From (10), it can be seen that $q_E$ contains in $\hat{\omega}$, which leads to the coupling between $\hat{\psi}_R$ and $\hat{\omega}$. After analysis, we can substitute $q_d = \xi q_E$ into (11)(12), and get (16)(17).

$$\omega_i = \frac{E_q - \lambda_i \hat{E}_d}{\hat{\phi}_R} \quad (16)$$

$$\frac{d \hat{\phi}_R}{dt} = \gamma \hat{E}_d + a(L_m i_d - \hat{\phi}_R) \quad (17)$$

$$\hat{\lambda}_i = \xi(I - k_d) - k_q \quad \gamma = k_d - \xi k_q \quad (18)$$

It can be seen that (16) (17) only have gain $\lambda_i$ and $\gamma$, and (11), (12), (13) have six gains, which reduces the difficulty and calculation in gain selection.

### 3.2. Stability analysis
The characteristic polynomial derived from the flux error equation of the reduced-order observer is

$$\det(sI - A) s^2 + c_1 s + c_0, \quad \text{where}$$

$$c_i = (1 - \gamma) a + \lambda_i \omega_j, \quad c_0 = \omega_j [\omega_j + \lambda_i a - (1 - \gamma) \omega_j] \quad (19)$$

According to the Routh criterion, $c_i, c_0 > 0$. When $\omega_j = 0$, for zero stator frequency, the system can only remain stable at one point. In order to find a parameter selection of completely stable under all operating conditions, we assume $\omega_j \neq 0$ and bring $\gamma = 1 + (\lambda_i \omega_j - l_1)$ into (19) to get $c_i = l_1 > 0$

$$c_0 = \omega_j [\omega_j + \lambda_i a (\omega_j^2 + \omega_j) - \frac{l_1 \omega_j}{a}] = l_1 \omega_j > 0 \quad (20)$$

By combining (18) and (20), the observer gain limit $l_1, l_2$ can be derived

$$l_1 = (1 - k_d - \xi k_q)a + (\xi - \xi k_d - k_q) \hat{\omega}_j$$

$$l_2 = \frac{\omega_j + (\xi - \xi k_d - k_q)a - (1 - k_d - \xi k_q) \hat{\omega}_j}{\omega_j} \quad (21)$$

and $l_1, l_2 > 0$. This is the parameter determination when the system is completely stable.

### 4. Full-order observer
Through VM, CM and current error for feedback correction, a full-order model can be established in a stationary coordinate system, and the speed is still obtained by adaptive estimation.

$$\frac{d \hat{i}_s^t}{dt} = \frac{1}{L_a} \left( \psi_s^t - R_i \hat{i}_s^t + \hat{\dot{\psi}}_R \right) + k_i \hat{i}_s^t \quad (22)$$

$$\frac{d \hat{\psi}_R^t}{dt} = R_{\psi} \hat{\psi}_R^t - \hat{\dot{\psi}}_R + k_2 \hat{i}_s^t \quad (23)$$

$$\hat{\omega}_j = k_e \varepsilon + k_1 \int \varepsilon \ v \ dt \ \varepsilon = - \text{Im} \left\{ e^{-j\beta} (\hat{\psi}_R^t) \hat{i}_s^t \right\} \quad (24)$$
where \( k_1 = k_{1d} + k_{1q} \) and \( k_2 = k_{2d} + k_{2q} \). In order to equate the full-order observer to a reduced-order observer, combining (3), we can transform (22), (23) into

\[
\frac{d\hat{i}_s^s}{dt} = \frac{1}{L_\sigma}(v_s^s - R\hat{i}_s^s - \hat{E}^s) + (k_1 + \frac{R}{L_\sigma})\hat{i}_s^s
\]

(25)

\[
\frac{d\hat{\Psi}_R^s}{dt} = \hat{E}^s + (k_2 - R_\sigma)\hat{i}_s^s
\]

(26)

By combining (25) and (5), you can get the error back EMF

\[
\tilde{E} = E^s - \hat{E}^s = -(k_1L_\sigma + R)\hat{i}_s^s - L_\sigma\frac{d\hat{i}_s^s}{dt}
\]

(27)

Because the convergence rate of \( \hat{i}_s^s \) is much faster than \( \hat{\Psi}_R^s \), from the point of \( \hat{\Psi}_R^s \), \( \hat{i}_s^s \) is equivalent to being stable and can be obtained

\[
\frac{d\hat{i}_s^s}{dt} \to j\omega \hat{i}_s^s \Rightarrow \frac{\tilde{E}}{(k_1 + j\omega)\sigma + R}
\]

(28)

At this time, we can substitute (28) into (24) and (26), so that the current correction term of the full-order observer is completely eliminated, and obtain an equivalent reduced-order observer. The transformed (29) and (30) are essentially the same as the reduced-order model in the third part, except that the specific values of the parameters are different.

\[
\frac{d\hat{\Psi}_R^s}{dt} = \hat{E}^s + k\tilde{E}^s, \quad k = -\frac{k_2 - R_\sigma}{(k_1 + j\omega)\sigma + R}
\]

(29)

\[
\hat{\omega} = k_\epsilon + k\int \epsilon dt, \quad \epsilon = \text{Im} \left\{ \frac{e^{-j\theta}}{(k_1 + j\omega)\sigma + R} (\hat{\Psi}_R^s)^* \hat{E}^s \right\}
\]

(30)

5. System simulation and result analysis

This section builds a simulation model under MATLAB/Simulink environment, and performs speed identification simulation on the reduced-order model. The block diagram of the IM speed sensorless vector control system is shown in Figure 2. The actual parameters of the motor used in the simulation are number of pole pairs 4, rated power 2.2kW, rated voltage 380V, rated frequency 50Hz, rated speed 1500r/min, stator resistance 2.804Ω, rotor resistance 2.178Ω, stator inductance 0.3303H, rotor inductance 0.3303H, stator and rotor mutual inductance 0.3197H, moment of inertia 0.03kg·m², damping coefficient 0kg·m²/s, rated torque 14.7N·m.

In order for the system to perform accurate magnetic field orientation, at 1s, motor speed is given with 1200 r/min and standardized as 0.8pu. The speed response diagrams of the reduced-order model and the full-order equivalent reduced-order model based on back EMF obtained under zero load are shown in Figure 3 and Figure 4, respectively.
Figure 3. Reduced-order model $\omega_r = 0.8$

It can be seen from Figure 3 that the speed follow ability of the reduced-order model is better. Given the speed at 1s, the speed can reach a steady state after 0.4s. However, the glitch is obvious and the fluctuation is not uniform. Relative to the reference value of 0.8, $\hat{\omega}_r$ has a fluctuation of $\pm 0.024$ up and down, which shows the rationality of the system.

Figure 4. Equivalent reduced-order model $\omega_r = 0.8$

It can be seen from Figure 4 that the estimated speed in the full-order equivalent reduced-order model can also follow the actual speed of the motor well. Besides, the burr is smaller and the fluctuation is more uniform. Relative to the reference value, $\hat{\omega}_r$ has a fluctuation of $\pm 0.021$, which is reduced by 0.003pu compared to Figure 3, and the convergence is more superior.

Figure 5 and Figure 6 are the dynamic response of switching speed under two models. The given speed drop is reduced from 0.8pu to 0.5pu (i.e. 750 r/min). Before 2s, it is consistent with the above analysis. However, after 2s, the waveform of Figure 6 at the turning point of 2.14s rises slightly faster than that of Figure 5. And the speed fluctuation in the 0.5pu segment is also smaller, which is reduced by 0.002. In general, in the case of sudden changes in speed, it follows very well, so that the estimated speed and the actual speed completely overlap.

Figure 5. Reduced-order speed change response  Figure 6. Equivalent model speed change response

6. Conclusion

This article is based on the back EMF according to the general reduced-order model of VM and CM combination, and uses the back EMF idea to transform the full-order model into an equivalent reduced-order model. This method simplifies the full-order model and reduces the amount of calculation. Furthermore, it is helpful to realize the complete stability and enhance the robustness of the system. In terms of implementation, this article simply uses the back EMF, without making some
optimizations on this basis. From the simulation results, this method can realize the speed sensorless control of the IM and has good stability.

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