Brane World Cosmology In Jordan-Brans-Dicke Theory

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Abstract

We consider the embedding of 3+1 dimensional cosmology in 4+1 dimensional Jordan-Brans-Dicke theory. We show that exponentially growing and power law scale factors are implied. Whereas the 4+1 dimensional scalar field is approximately constant for each, the effective 3+1 dimensional scalar field is constant for exponentially growing scale factor and time dependent for power law scale factor.

Introduction

In spite of the success of general relativity now called the standard theory of gravitation, there are many other alternative theories. Among them the scalar tensor theory is the most important one. The scalar-tensor theory was conceived originally by P. Jordan [1] who started to embed a four dimensional curved manifold in five dimensional flat space-time. He presented a general Lagrangian for the scalar field living in four-dimensional curved space-time:

\[ L_J = \sqrt{-g} \left[ \varphi_J^\gamma \left( R - \omega_J \frac{1}{\varphi_J} g^{\mu\nu} \partial_\mu \varphi_J \partial_\nu \varphi_J \right) + L_{\text{matter}} (\varphi_J, \Psi) \right], \tag{1} \]

where \( \varphi_J (x) \) is Jordan’s scalar field, \( \gamma \) and \( \omega_J \) are constants, and \( \Psi \) represents matter fields. \( \varphi_J^\gamma R \) is the nonminimal coupling term which marked the birth of scalar-tensor theory.

Jordan’s work was taken over particularly by C.Brans and R.H. Dicke [2]. They assumed that decoupling of scalar field from the matter part of the Lagrangian occurs. They defined their scalar field \( \varphi \) by

\[ \varphi = \varphi_J^\gamma \tag{2} \]

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and then Lagrangian will be
\[ L_{BD} = \sqrt{-g} \left( \varphi R - \frac{1}{\omega} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + L_{\text{matter}} (\Psi) \right). \] (3)

They demanded that the matter part of the Lagrangian \( \sqrt{-g} L_{\text{matter}} \) be decoupled from \( \varphi (x) \) as their requirement that the weak equivalence principle be respected, in contrast to Jordan’s model [3]. To remove the singularity from the second term on the right hand side we introduce a new field \( \phi \):
\[ \varphi = \frac{\phi^2}{8\omega}. \] (4)

Then the Brans-Dicke (B-D) action will be
\[ L_{BD} = \sqrt{-g} \left( \frac{\phi^2}{8\omega} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_{\text{matter}} \right). \] (5)

where in order to get cosmic acceleration, either the parameter \( \omega \) should be time dependent [4], or a potential term for the scalar field could be added [5] to the Lagrangian.

On the other hand, string theory predicts a new type of nonlinear structure, which is called a brane, a word created from ”membrane”. This also gives a new perspective to cosmology so that our universe is confined to a four dimensional space-time subspace or 3-brane. The extra dimension may have large compact toroidal topology [6] or be unbounded with a warp factor, depending on the distance from the brane [7, 8]. Additionally several works have studied higher dimensional B-D theory to combine the advantages of both the five dimensional cosmology and the B-D theory [9, 10]. Moreover, considering the scalar field in the five dimensional bulk with Einstein gravity was proposed by many works [11].

Our starting point is the paper of Bander who studied five dimensional bulk whose dynamics is governed by a scalar Liouville field coupled to gravity in the usual way [12]. Then he derived that the effective theory on the brane has a time dependent Planck mass and cosmological constant and also found expanding scale factors with no acceleration. In this paper we investigate the properties of the five dimensional bulk in Brans-Dicke theory. The layout of our paper is as follows. In section 2 we present the general framework for our five dimensional theory and compute the five dimensional Brans-Dicke equations. In section 3 we analyze the cosmological solutions. We find false vacuum energy \( p_B = -\rho_B \) for exponentially growing scale factors and radiation dominated universe \( p_B = \frac{1}{3} \rho_B \) for power law scale factors in the bulk. In section 4 we derive the effective four dimensional scalar field and obtain its time dependence. Finally, we sum up our results and conclusions in section 5.

1 The Action and Equations of Motion

In this work we look at the five dimensional Brans-Dicke action:
\[ S = \int d^5 x \sqrt{g} \left( \frac{\phi^2}{8\omega} R - \frac{1}{2} \partial_A \phi \partial_B \phi g^{AB} - V (\phi) \right), \] (6)

where \( \omega \) is the dimensionless Brans-Dicke parameter, \( \phi \) is the scalar field and \( V (\phi) \) is the scalar potential. The variation of the action with respect to \( g^{AB} \) gives
\[ \frac{1}{8\omega} \left( \phi^2 G_{AB} - \phi_{,A;B}^2 + g_{AB} \Box \phi^2 \right) - \frac{1}{2} \partial_A \phi \partial_B \phi + \frac{g_{AB}}{4} \partial_C \phi \partial^C \phi + \frac{1}{2} g_{AB} V (\phi) = T_{AB}. \] (7)
We choose a general five dimensional metric anzats which can be written in an orthonormal basis as [12]:

\[
    ds^2 = b(t)^2 dW^2 + f(W)^2 [-dt^2 + a(t)^2 \delta_{ij} dx^i dx^j],
\]

where \(i, j = 1, 2, 3\) and \(f(W)\) is the warp factor which depends on the fifth coordinate, \(a(t)\) is the cosmological scale factor and \(b(t)\) is the time dependent scale factor of the fifth dimension. More generally, this metric has been studied in papers [10, 13]. In Mendes’ work [10] the five dimensional brane cosmology with non-minimally coupled scalar field to gravity is interpreted in Jordan frame without a scalar potential. In our work we add a scalar potential to the action.

In the orthonormal basis \(e^0 = fdt, e^i = fadx^i\) and \(e^5 = bdW\), the stress-energy tensor can be considered as [13]

\[
    T_B^A = T_B^A |_{\text{bulk}} + T_B^A |_{\text{brane}},
\]

where \(T_B^A |_{\text{bulk}}\) is the energy momentum tensor of the bulk matter and

\[
    T_B^A |_{\text{brane}} = \frac{\delta(W)}{b} \text{diag}(-\rho, p, p, p, 0).
\]

The second term corresponds to the matter content in the brane \((W = 0)\),

\[
    T_B^A |_{\text{brane}} = \frac{\delta(W)}{b} \text{diag}(-\rho, p, p, p, 0).
\]

If we substitute the Einstein tensor components in eq(7) we obtain the B-D equations. In the coordinate basis, for component 00;

\[
    \frac{1}{8\omega} \left(3 \left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{ab} - \frac{3f^2}{b^2} \left(\frac{\dot{f}^2}{f^2} + \frac{\dot{t}^2}{f}\right) + \frac{3}{a} \frac{\partial_\phi \phi^2}{\phi^2} + \frac{b}{b} \frac{\partial_t \phi^2}{\phi^2} - \frac{f^2}{b^2} \left(\frac{3}{f} \frac{\partial_\phi \phi^2}{\phi^2} + \frac{\partial_{\phi} \phi^2}{\phi^2}\right)\right) - \frac{f^2}{4b^2} \frac{(\partial_W \phi)^2}{\phi^2} - \frac{1}{4} \frac{(\partial_t \phi)^2}{\phi^2} - \frac{f^2 V(\phi)}{2} \frac{\phi^2}{\phi^2} = \frac{T_{00}}{\phi^2}.
\]

For components \(ii\);

\[
    \frac{1}{8\omega} \left(- \left(\frac{2\ddot{a}}{a} + \frac{\ddot{a}}{a^2} + \frac{2\dot{a}b}{ab} + \frac{\ddot{b}}{b} \right) + \frac{3f^2}{b^2} \left(\frac{\dot{f}^2}{f^2} + \frac{\dot{t}^2}{f}\right) - \frac{\partial_t \phi^2}{\phi^2} - \frac{b}{b} \frac{\partial_t \phi^2}{\phi^2} - \frac{2\dot{a}}{a} \frac{\partial_\phi \phi^2}{\phi^2}\right) + \frac{1}{8\omega} \frac{f^2}{b^2} \left(\frac{\partial_W \phi^2}{\phi^2} + \frac{3}{f} \frac{\partial_W \phi^2}{\phi^2}\right) - \frac{1}{4} \frac{(\partial_t \phi)^2}{\phi^2} + \frac{f^2}{4b^2} \frac{(\partial_W \phi)^2}{\phi^2} + \frac{f^2 V(\phi)}{2} \frac{\phi^2}{\phi^2} = \frac{1}{a^2} \frac{1}{\phi^2} T_{ii}.
\]

For component 55;

\[
    \frac{1}{8\omega} \left(-3 \left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} \right) + 6 \frac{f^2}{b^2} \frac{\ddot{f}}{f^2} - \frac{\partial_t \phi^2}{\phi^2} - \frac{3}{a} \frac{\partial_\phi \phi^2}{\phi^2} + 4 \frac{f^2}{b^2} \frac{\dot{f}}{f} \frac{\partial_W \phi^2}{\phi^2}\right) - \frac{f^2}{4b^2} \frac{(\partial_W \phi)^2}{\phi^2} - \frac{1}{4} \frac{(\partial_t \phi)^2}{\phi^2} + \frac{f^2 V(\phi)}{2} \frac{\phi^2}{\phi^2} = \frac{f^2}{b^2} \frac{T_{55}}{\phi^2}.
\]
For component 05;

\[
\frac{1}{8\omega} \left( \frac{3b\dot{f}}{bf} - \frac{\partial_t \partial_W \phi^2}{\phi^2} + \frac{\dot{f} \partial_t \phi^2}{f^2 \phi^2} + \frac{\dot{b} \partial_W \phi^2}{b \phi^2} \right) - \frac{1}{2} \frac{\partial_t \phi \partial_W \phi}{\phi^2} = 0.
\] (13)

Assume that the 05 component of the energy-momentum tensor vanishes, which means that there is no flow of matter along the fifth dimension. Therefore the nonzero elements of the 5D stress-energy tensor are

\[
T_{00} = f^2 \rho_B + f^2 \frac{\delta (w)}{b} \rho
\]

\[
T_{ii} = a^2 f^2 \rho_B + a^2 f^2 \frac{\delta (w)}{b} p
\]

\[
T_{55} = b^2 q_B.
\]

Finally variation with respect to \( \phi \) gives,

\[
\frac{1}{4\omega} (\phi R) - \frac{\partial V (\phi)}{\partial \phi} + \square \phi = 0,
\]

which explicitly reads

\[
\frac{1}{4\omega} R - \frac{\partial^2 \phi}{f^2 \phi} + \frac{4}{b^2} \frac{\dot{f} \partial_W \phi}{f} - \frac{3}{a^2} \frac{\ddot{a} \partial_t \phi}{a} - \frac{\ddot{b} \partial_W \phi}{b f^2 \phi} + \frac{\partial_W \phi}{b^2 \phi} - \frac{1}{\phi} \frac{\partial V (\phi)}{\partial \phi} = 0,
\] (15)

where the Ricci scalar \( R \) is:

\[
R = \frac{1}{f^2} \left( \frac{6\dddot{a}}{a} + \frac{2\dddot{b}}{b} + \frac{6\ddot{a}^2}{a^2} + \frac{6\dot{a} \dot{b}}{ab} \right) - \frac{12 \dddot{f}}{f^2 b^2} - \frac{8 f'''}{f b^2}.
\]

The metric and the B-D field are continuous across the brane localized at \( W = 0 \). However their derivatives can be discontinuous at the brane. Since we have orbifold symmetry, second derivatives of scale factor and B-D field will contain Dirac delta function in the second derivatives of the metric with respect to fifth dimension. Therefore for a function \( f \), we have [10, 13]

\[
f'' = \hat{f}'' + [f'] \delta (W),
\]

where \( \hat{f}'' \) is the non-distributional part of the double derivative of \( f \), and \([f']\) is the jump in the first derivative of \( f \) across \( W = 0 \), it is defined as

\[
[f'] = f' (0^+) - f' (0^-).
\]

Matching the Dirac delta functions in equations (10), (11) and (15) we obtain that

\[
\left. \left[ \frac{f'}{f_0 b_0} \right] \right|_a = -\frac{8\omega^2}{(3\omega + 4) \phi^2 \rho}
\]

\[
\left. \left[ \frac{\phi'}{\phi_0 b_0} \right] \right|_a = -\frac{16\omega}{(3\omega + 4) \phi^2 \rho}
\] (16)
where the subscript ‘0’ stands for the brane at \( W = 0 \). Using eq(10) and eq(11) we get the remarkable result that the cosmological constant dominates on the brane i.e.

\[
\rho = -p = -\frac{\phi_0^2}{8\omega} \left( \frac{3[f']_0}{f_0 b_0} + \frac{2[\phi']_0}{\phi_0 b_0} \right).
\]

(17)

Here choice of the scalar factor \( a(t) \) does not make any differences on the equation of state \( \rho = -p \). Using eq(13) to evaluate the jump condition we get the equation for the matter on the brane

\[
4\dot{\rho} + 3\omega \frac{\dot{b}}{b} \rho + 6\omega \frac{\dot{\phi}}{\phi} \rho = 0
\]

(18)

where if \( 2\frac{\dot{b}}{b} = \frac{\dot{\phi}}{\phi} \) or in particular time derivatives of the \( b \) and \( \phi \) are zero, we obtain that \( \rho \) and \( p = -\rho \) are constant on the brane.

## 2 Solutions

Solutions of B-D equations restrict the scalar field to be in the form \( \phi(t, W) = B(t) C(W) \). Starting from this, to satisfy all of the B-D equations we make two possible ansatze for \( a(t) \). The first one is exponential growth in time and the other is power law expansion.

### 2.1 Exponential Expansion, \( a(t) = a_0 e^{\lambda t} \)

We see from eq(10-15) that \( b(t) \) must be constant, \( b(t) = b_0 \) and \( B(t) \) must be in the exponential form also \( B(t) = B_0 e^{\beta t} \) and than we can easily read that

\[
f(W) = \frac{W}{W_0}. \tag{19}
\]

For a brane at \( W = W_0 \), we introduce the coordinate \( W' \) such that \( \frac{W}{W_0} = 1 - \frac{W'}{W_0} \). The metric on both sides of brane can be written

\[
ds^2 = b_0^2 dW^2 + \left( 1 - \frac{|W|}{W_0} \right)^2 \left[ -dt^2 + e^{2\lambda t} d\vec{x}^2 \right],
\]

(20)

and the brane is at \( W = 0 \). Here we dropped the prime for simplicity. This warp factor is the same as in Bander’s work [12]. The brane we live in is embedded in the five-dimensional bulk space-time and the four dimensional part in the square parenthesis is the well known de-Sitter space-time. This metric is similar to a Randall- Sundrum type of model in the same sense. Instead of the exponential warp factor we obtain the linear warp factor. However for small \( W \) it is known that

\[
e^{-|W|} \simeq 1 - |W|, \tag{21}
\]

and two the models are similar.
From equations (10) and (11) it seems to be $p_B = -\rho_B$, which acts as a cosmological constant. In previous works [14, 15] this energy has been identified as the false vacuum energy density $\rho_f$. During the false vacuum phase the universe supercools. It is believed that as the universe expands it cools down and then it experiences a series of phase transitions. Since the cosmic expansion continues to drive the temperature downward, the universe enters a period of supercooling. As the universe supercools the energy density acts as an effective cosmological constant. Therefore we can consider this stage as the false phase.

For this condition we get the results

$$C(W) = c_0 \left(1 - \frac{|W|}{W_0}\right)^\alpha,$$

$$\phi = B_0 c_0 \left[e^{\beta t} \left(1 - \frac{|W|}{W_0}\right)^\alpha \right],$$

here $\phi$ depends only on the distance $W$. Where on the brane we live

$$(16\pi G)^{-1} = M_p^2 = \frac{\phi^2}{8\omega} = \frac{(B_0 c_0)^2 e^{2\alpha \beta}}{8\omega},$$

where $B_0 c_0$ is required to be within a few orders of magnitude of Planck mass [7]. We first discuss the solution where $T_{55} = 0$.

### $T_{55} = 0$:

From the B-D equations this condition causes $\rho_B = -p_B = 0$ (empty universe). Then solutions are very simple

| $\rho_B = 0$ | $p_B = 0$ | $V_0 = 0$ | $V_0 = -\frac{(3\omega+4)\lambda^2}{2\omega(1+\omega)} (B_0 c_0)^{2/\alpha}$ | $\beta = \lambda$ | $\alpha = \frac{1}{2(1+\omega)}$ |
|-------------|------------|-------------|-------------------------------------------------|----------------|-----------------|
| $\beta = 0$ | $\alpha = \frac{1}{1+\omega}$ |

where $b_W \lambda = \pm 1$. Here in the first row of the table the B-D equations give a scalar field which depends not only on time but also on the fifth coordinate. On the other hand in the second row the scalar field only depends on the fifth coordinate and there is a scalar potential $V_0 \neq 0$. Therefore the scalar potential is not depend on the time

$$V(\phi) = V_0 \phi^{2 - \frac{2}{\alpha}},$$

where $V_0$ is a constant has dimension $L^{-2 - \frac{4}{2\alpha}}$. From these results as $\omega \to \infty$, $\alpha, V_0 \to 0$. Therefore $V(\phi) \to 0$. This means that at the large values of the B-D parameter, the scalar field is constant

$$\frac{\phi^2}{8\omega} = M_p^2 = \frac{(B_0 c_0)^2}{8\omega}$$

with no scalar potential.

The $q = 0$ condition has been derived in [16] where it was found that empty and flat five dimensional universe where $R^{MN}_{\ PQ} = 0$ and $\Lambda_5 = 0$ gives rise to a four dimensional expanding universe with nonzero Riemann tensor and cosmological constant. This five dimensional space is a well known Minkowski universe

$$ds^2 = -dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2$$

(25)
transformed into
\[ ds^2 = b_0^2 dW^2 + \left(1 - \frac{|W|}{W_0}\right)^2 [-dt^2 + e^{2\lambda t} (dr^2 + r^2 d\Omega_2^2)] , \] (26)

by the following transformation
\[
\begin{align*}
    x_1 &= b_0 (W_0 - |W|) \left( \sinh (\lambda t) + \frac{\lambda^2 r^2}{2} e^{\lambda t} \right) \\
    x_2 &= b_0 (W_0 - |W|) \left( \cosh (\lambda t) - \frac{\lambda^2 r^2}{2} e^{\lambda t} \right) \\
    x_3 &= b_0 (W_0 - |W|) \lambda r e^{\lambda t} \cos \theta \\
    x_4 &= b_0 (W_0 - |W|) \lambda r e^{\lambda t} \sin \theta \cos \varphi \\
    x_5 &= b_0 (W_0 - |W|) \lambda r e^{\lambda t} \sin \theta \sin \varphi ,
\end{align*}
\]

after some calculations we get the factor \(b_0 W_0 \lambda\) in front of the four dimensional part. This was already found as unity. Therefore the four dimensional curved space time can be embedded in the five dimensional flat space time by these coordinate transformations.

2.1.2 \( T_{55} \neq 0 \)

From the B-D equations we obtain that \( p_B = -\rho_B \neq 0 \) and \( \beta = 0 \). As \( \omega \to \infty \),
\[
\begin{align*}
    \rho_B &= -p_B \to \frac{(B_0 c_0)^{2/\alpha}}{2 (b_0 w_0)^2} \frac{\alpha^2 (\alpha + 1)}{(\alpha - 1)} \phi^{2-2/\alpha} \\
    q_B &\to \frac{(B_0 c_0)^{2/\alpha}}{(b_0 W_0)^2} \frac{\alpha^2}{\alpha - 1} \phi^{2-2/\alpha} \\
    V_0 &\to \frac{(B_0 c_0)^{2/\alpha}}{(b_0 W_0)^2} \frac{\alpha^2 (3 + \alpha)}{2 (\alpha - 1)} ,
\end{align*}
\] (28)

for all of the results \( \beta = 0 \) and \( V(\phi) = V_0 \phi^{2-2/\alpha} \), therefore the scalar potential becomes again time independent for the exponentially expanding universe for \( q_B \neq 0 \).

If we suppose this phase as the false phase, the probability of a point remaining in the false phase during the bubble nucleation process is quite small as shown in [17]. Then the universe is dominated by the true vacuum and exits from the false vacuum.

In the true vacuum we can consider a power-law expansion.

2.2 Power-law Expansion:

The scale factors are:
\[
\begin{align*}
    a(t) &= a_0 (t/t_0)^\lambda , \\
    b(t) &= b_0 (t/t_0)^\gamma .
\end{align*}
\] (29) (30)
These power law solutions restrict us to choose $B(t) = B_0 \left(\frac{t}{t_0}\right)^\beta$. On the other hand B-D equations is satisfied only if $\gamma, \beta = 1$, and again we get same result for the warp factor, $f(W) = \left(1 - \frac{|W|}{W_0}\right)$. Then these results causes scalar field to be

$$\phi(t, W) = B_0 c_0 \left(\frac{t}{t_0}\right)^\beta \left(1 - \frac{|W|}{W_0}\right)^\alpha,$$  \hspace{1cm} (31)

where $B_0$ and $c_0$ are constants. Again to satisfy the B-D equations, we find the similar scalar potential;

$$V(\phi) = V_0 \phi^{\frac{2}{\alpha}(\alpha - 1)},$$  \hspace{1cm} (32)

and $B_0 c_0$ has dimension $L^{-3/2}$, therefore $V_0$ has dimension $L^{-3\alpha - 2}$. Here to make B-D equations simpler we set $\frac{b_0 W_0}{t_0} = 1$.

Now we want to find a general result so we consider the equation of state as:

$$p_B = \nu \rho_B.$$  \hspace{1cm} (33)

Putting all of these settings in the B-D equations we find a nice result: here the interesting thing is that there is no solution other than $\nu = \frac{1}{3}$ for $p_B \neq 0$ and $\rho_B \neq 0$ and solutions are valid only for $q_B = 0$. Different values of the variables in equations (10-15) are satisfied only for a single value of $\nu$ which is $\frac{1}{3}$. Then this ratio between the pressure and energy density corresponds to the radiation dominated universe; and $\omega$ dependence of $\lambda, \alpha$, and $V_0$ are:

$$\rho_B = 3p_B,$$  \hspace{1cm} (34)

$$\alpha_\pm = \frac{\pm \sqrt{3\omega + 4} + 1}{2(\omega + 1)},$$  \hspace{1cm} (35)

$$\lambda_\pm = \frac{\omega \mp \sqrt{3\omega + 4}}{4(\omega + 1)},$$  \hspace{1cm} (36)

and finally

$$V_{0\pm} = -\frac{3(B_0 c_0)^{2/\alpha} (3\omega + 4) \left(3\omega \pm \sqrt{3\omega + 4} + 5\right)}{32\omega (\omega + 1)^2}.$$  \hspace{1cm} (37)

All of these solutions do not give a specific value for $\omega$. From the time-delay measurements, experimentally $\omega > 500$ [18] and more recently $\omega > 3000$ [19]. As $\omega \to \infty$, $\alpha \to 0$, $\lambda_\pm \to \frac{1}{3}$ and $V_0 \to 0$. This means that at this limit, the scalar field becomes constant and the scalar potential vanishes.

For the power-law scale factor we obtain one more solution B-D equations give the empty universe, namely $\rho_B = 0$, $p_B = 0$ and $\lambda = 1$, $V_0 = 0$ and $\alpha_\pm = \frac{\pm \sqrt{3\omega + 4} + 1}{2(\omega + 1)}$ which are the same as previous value of $\alpha$ (35).

The solutions presented here represent decelerating cosmology for the radiation dominated universe and expanding cosmology with constant velocity for the empty universe. However astronomical observations show that the universe is not only expanding but also undergoing accelerated expansion [23, 24]. It may be possible to obtain power law acceleration in B-D theory if scale factors for external dimensions are time dependent. In string theory some cosmologies can achieve accelerating scale factors [20, 21, 22].
The metric which we found in this part may be related with the Kasner space-time [25]. It has a cosmological singularity at \( t = 0 \) where the square of Riemann tensor diverges. On the brane we live \((W = 0)\)

\[
R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{24\lambda^4}{t^4}.
\]

This is the physical singularity and it cannot be avoided by any coordinate transformation [26]. However, since the central part of the space-time is avoided in orbifold construction this has no importance for the brane world scenario [7].

3 The Effective Four Dimensional Gravitational Constant

Finally we calculate the four-dimensional effective gravitational constant on the brane and compare with the our previous results eq(24). On the left hand side of the action in eq (6) the first term is:

\[
\int d^5 x \sqrt{g} \frac{\phi^{(5)}_\phi^2}{8\omega} R^{(5)} = \int d^5 x \sqrt{g} M^{3}_{(5)} R^{(5)} = \int d^5 x \sqrt{g} \frac{1}{16\pi G_{(5)}} R^{(5)} .
\]

We can perform the \( W \) integral to obtain the effective gravitational constant. With the same manner in the [12] work, this equation reduces to

\[
\int d^5 x \sqrt{g} \frac{\phi^{(5)}_\phi^2}{8\omega} R^{(5)} = \int d^4 x \sqrt{g} \frac{\phi^{(4)}_\phi^2}{8\omega} \left( 1 - \frac{|W|}{W_0} \right)^2 b(t) R \left( g^{(4)}_{ij} (x) \right) \]

(40)

For the exponentially increasing scalar factor we have obtained time dependent scalar field that is \( \phi^{(5)}_\phi = B_0 c_0 \left[ e^{\beta t} \left( 1 - \frac{|W|}{W_0} \right) \right] ^\alpha \). Then eq(40) becomes

\[
\int d^5 x \sqrt{g} \frac{\phi^{(5)}_\phi^2}{8\omega} R^{(5)} = \int d^4 x 2 \frac{b_0 W_0 B_0^2 c_0^2 e^{2\alpha\beta t}}{(2\alpha + 3)} \sqrt{g^{(4)}} R \left( g^{(4)}_{ij} (x) \right)
\]

(41)

then since \( \alpha \simeq 0 \), the effective gravitational constant becomes

\[
\frac{1}{16\pi G_{eff}} = M_{p(\text{eff})} \frac{\phi^{(4)}_\phi^2}{8\omega} = \frac{b_0 W_0}{12\omega} (B_0 c_0)^2 ,
\]

(42)

which is independent of time. This is similar with the what we have discussed in eq(24) and here \( B_0 c_0 \) is within a few orders of Planck mass.

For the power law scalar factors the scalar field is \( \phi^{(5)}_\phi = B_0 c_0 \left( \frac{t}{t_0} \left( 1 - \frac{|W|}{W_0} \right) \right) ^\alpha \)

\[
\int d^5 x \sqrt{g} \frac{\phi^{(5)}_\phi^2}{8\omega} R^{(5)} = \int d^4 x \frac{b_0 W_0}{4\omega} \left( \frac{t}{t_0} \right)^{2\alpha + 1} \frac{B_0^2 c_0^2}{(2\alpha + 3)} \sqrt{g^{(4)}} R \left( g^{(4)}_{ij} (x) \right)
\]

(43)

here again for \( \alpha \simeq 0 \), the four dimensional Brans-Dicke scalar field (or the inverse of the effective gravitational constant) is

\[
\frac{1}{16\pi G_{eff}} = M_{p(\text{eff})} \frac{\phi^{(4)}_\phi^2}{8\omega} = \frac{W_0}{12\omega} (B_0 c_0)^2 b(t) .
\]

(44)

Hence the four dimensional effective gravitational constant depends on time [12].
4 Conclusion

In this work we introduced a five dimensional B-D action and studied the five dimensional metric with a warp factor. We showed that the field equations imply a linear warp factor. For an inflating scale factor we found that the energy density acts as an effective cosmological constant. For power law expansion of scale factor we obtained a radiation dominated universe. Additionally we have shown that the five dimensional scalar field is nearly but cannot be exactly constant. On the other hand the four dimensional effective scalar field is constant for exponentially growing scale factor and depends on time for the power law scale factors.

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