Modulated Periodic Activations for Generalizable Local Functional Representations

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Abstract

Multi-Layer Perceptrons (MLPs) make powerful functional representations for sampling and reconstruction problems involving low-dimensional signals like images, shapes and light fields. Recent works have significantly improved their ability to represent high-frequency content by using periodic activations or positional encodings. This often came at the expense of generalization: modern methods are typically optimized for a single signal. We present a new representation that generalizes to multiple instances and achieves state-of-the-art fidelity. We use a dual-MLP architecture to encode the signals. A synthesis network creates a functional mapping from a low-dimensional input (e.g., pixel-position) to the output domain (e.g., RGB color). A modulation network maps a latent code corresponding to the target signal to parameters that modulate the periodic activations of the synthesis network. We also propose a local-functional representation which enables generalization. The signal’s domain is partitioned into a regular grid, with each tile represented by a latent code. At test time, the signal is encoded with high-fidelity by inferring (or directly optimizing) the latent code-book. Our approach produces generalizable functional representations of images, videos and shapes, and achieves higher reconstruction quality than prior works that are optimized for a single signal.

1. Introduction

Functional neural representations using Multi-Layer Perceptrons (MLPs) have garnered renewed interest for their conceptual simplicity and ability to approximate complex signals like images, videos, audio recordings \cite{37}, lightfields \cite{25} and implicitly-defined 3D shapes \cite{7, 31, 3}. They have shown to be more compact and efficient than their discrete counterparts \cite{22, 38}. While recent contributions have focused on improving the accuracy of these representations, in particular to model complex signals with high-frequency details \cite{37, 42, 25}, it is still challenging to generalize them to unseen signals. Recent approaches typically require training a separate MLP for each signal \cite{25, 9}. Previous efforts sought to improve generalization by imposing priors on the functional space spanned by the MLP parameterization \cite{31, 34}, using hypernetworks \cite{16, 37}, or via meta-learning \cite{36}. But multi-instance generalization still causes significant degradations in quality.

We introduce a neural functional representation that simultaneously achieves high-reconstruction quality and generalizes to multiple instances. Our approach can encode functional representations for multiple discrete signals using a single model. Unlike previous works, which train a model for each signal, it can do so in a single feed-forward pass (Fig. 1). We represent each signal using a low-dimensional latent code. These codes serve as conditioning variables in a
functional mapping that uses two MLPs: a modulator and a synthesis network. The synthesis network implements a mapping from coordinates (e.g. spatial position) to signal values (e.g. RGB colors). It uses the sine function as activation, which enables accurate reconstructions of high-frequency content [37], but also makes naive conditioning strategies ineffective (§ 3.2). The modulator is the key to generalization. It consumes latent code and outputs, at each layer, parameters that modulate the amplitude, phase and frequency of periodic activations in the synthesis network. The modulator uses ReLU activations. Our model can either be used as an autoencoder, where the latent codes are produced by a third network (the encoder); or as an auto-decoder, where the latent codes are optimized jointly with all the network parameters.

As we show in Figure 2, the quality of functional representations that fit images as-a-whole, degrades as we increase the target resolution. High-resolution images typically have a broad power spectrum, thereby requiring more expensive models to represent them functionally. But images are usually much simpler locally: simple edges and textures re-occur commonly across images that are otherwise quite distinct at the global level. This motivates our strategy to exploit locality. We partition the signal domain into a regular tiling, and assign each tile a latent code (Fig. 1). By “zooming in” on the local structure, computing functional approximations that generalize becomes more tractable [24], because simple parts exhibit fewer variations than complete objects [15]. Some recent work has explored locality, but they focus on relatively simple, low-frequency signals like signed distance fields [5, 15], which can locally be well approximated using a single linear decision boundary—well in the purview of ReLU-based MLPs. For more complex signals like images and videos, even local patterns contain high-frequency components that a standard ReLU-MLP fails to reconstruct (Figure 3). We show that locality, together with our model architecture, makes it possible to obtain functional representations of large, complex signals.

Compared to previous methods, ours produces qualitatively and quantitatively superior functional representations, with improved generalization capabilities. In summary, our contributions are as follows:

- A local neural-functional representation that enables generalization and achieves high fidelity. We use a set of local functions defined on a tiling of the input domain that combine to reconstruct the target signal.
- A new network architecture, which uses modulation and synthesis sub-networks for high-fidelity functional neural representations of images, shapes and videos.
- A novel conditioning mechanism in which a ReLU-MLP modulates the amplitude, phase, and frequency of periodic activations in the synthesis sub-network.

Figure 2: **SIREN struggles with high-res signals.** Obtaining a neural representation of an image using SIREN [37] requires overfitting a new MLP to every new image instance. This works well in practice for low-resolution images, but we observe a sharp drop in reconstruction accuracy for high-resolution images. Our method can be used to encode multiple images at high-resolution with only feedforward passes without gradient computation.

2. Related Work

**Continuous Representations of Visual Signals** Our work builds upon the extensive use of Multi Layer Perceptrons (MLPs) to encode images [33, 41], videos [37], shapes [31, 7] and 3D scenes [39, 22, 4, 25]. Once trained, these models yield continuous representations that can be queried at arbitrary locations in the signal’s input domain. They have had significant impact in view-synthesis [22, 25] and other interpolation problems [26, 43]. Similar approaches have been used for end-to-end differentiable texture mapping [27] and volumetric rendering [28].

**Periodic Activations** Lapedes and Farber [18] show the earliest use of periodic activations in neural networks. They observe that networks with more than one layer with periodic activations are difficult to train, and often converge to undesired local minima. This problem is formalized further in [30]. For small datasets, Sopena et al. [40] show compelling results using sine activations in the first layer and monotonic functions in the others. This is similar to preconditioning the input using a Fourier basis, which was shown to be useful in feature visualization [29] and image synthesis [27]. More recently, Tancik et al. [42] proposed Fourier Feature Networks (FFN), where they encode the MLP’s input into a high-dimensional space using a random sampling of Fourier basis functions. Concurrently, Sitzmann et al. [37] showed that, with careful network initializations, sine activations can be used in all layers. They demonstrate regressions of small, single images and videos, as well as more complex shapes. However, as we show in Section 4, these networks struggle with larger datasets, or individual instances when the complexity is increased. In Figure 2,
we show how quality degrades with these networks as we regress a progressively higher-resolution image, or a longer video. Our work lifts these limitations by exploiting locality, and introduces an effective modulation mechanism to enable generalization.

**Instance-conditioned Implicit Functions** A major limitation of current implicit representations, is that they need to be optimized for each test signal individually, unlike more established models than only require a forward pass at test time, having been trained on large datasets. Building implicit models with similar generalization properties is typically done via a conditioning mechanism, using latent variables. Conditioning by concatenating the latent code with an MLPs spatial input coordinates has been successful in signed-distance fields regression tasks [7, 31]. Schwarz et al. [34] use the same strategy for MLPs encoding radiance fields, although with limited resolution. In Section 3.2, we show why this conditioning-by-concatenation approach is inadequate for MLPs with sine activations, and limits reconstruction quality. Conditional hypernetworks [13] achieves similar goals. A hypernetwork estimates all the parameters of a hyponetwork [37] from the latent code. Hypernetworks are prohibitively expensive, in both compute and memory, thereby limiting the resolution of the reconstructed signal in practice. We propose a new approach to modulate the implicit function based on the conditioning variable, which is inspired from attention mechanisms [44].

**Applications of Local Models** Local methods have been used largely to process complex systems in the form of KD-trees for real-time fluid simulation [8, 47], as regular grids for photon mapping [12] and popularly for fast ray tracing [2]. Local representations have also been used to compress surface light fields [45] and for pre-computed radiance transfer [24]. Our method also relates to recent work on voxelized implicit models for 3D representation [5, 10, 38, 15]. We show that our approach is general, and can be used for a variety of applications.

**3. Method**

We introduce a novel parameterization of neural functions approximating signals defined on a Euclidean input domain $\mathbb{R}^n$. Our pipeline is illustrated in Figure 1. Our model can simultaneously encode a large number of functions, each of which is summarized into a latent code. The latent codes are processed by a modulation network, which conditionally modulates the activations of a synthesis network, that acts as a template for the functional mapping (§ 3.1). We show that this new architecture is crucial to generate functional representation of multiple signals with a single model, a task where our approach significantly outperforms previous work that use a single MLP (§ 3.2). This ability to generalize lets use compute functional representations for signals with much higher-resolution than previously possible, and with much higher fidelity. For this, we decompose the input signals into local tiles, each represented with a latent code (§ 3.3). The latent codes can either be estimated from the discrete input by a convolutional encoder (§ 3.4.1), or they can be optimized simultaneously with the model parameters (§ 3.4.2), as in [5].

**3.1. Modulated Periodic Activations**

Concretely, we define our functional representation as a continuous conditional mapping.

$$f_\theta : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}^m.$$  

Equation (1)

$f_\theta$ is a neural network, with parameters $\theta$, $d = 256$ is the dimension of the latent space. In the case of images, we use $n = 2$ for pixel coordinates, $m = 3$ for the RGB color values. We find the optimal model parameters $\theta$ by minimizing a domain-specific reconstruction loss $\mathcal{L}(f_\theta(x_i; z), y_i)$ on a dataset of signal values $y_i \in \mathbb{R}^m$, sampled at coordinate $x_i \in \mathbb{R}^n$. The signal is encoded into a latent code $z$. Our network architecture has two components—a synthesis network (§ 3.1.1), and a modulation network (§ 3.1.2).

**3.1.1 Synthesis Network**

The synthesis network defines a continuous function from the spatial coordinates of a discrete signal like an image to its output domain (e.g. color). It is a composition of $K$ hidden layers, with hidden features $h_1, \ldots, h_K$. Each layer uses a periodic-nonlinear activation function and is defined recursively as:

$$h_i = \alpha_i \odot \sin(w_i h_{i-1} + b_i),$$  

Equation (2)

with $w_i \in \mathbb{R}^{d_i \times d_{i-1}}$ and $b_i \in \mathbb{R}^{d_i}$, the learnable weights and biases for layer $i$, and $\alpha_i \in \mathbb{R}^{d_i}$ a modulation variable
discussed in Section 3.2. We set \( h_0 := x \in \mathbb{R}^n \) to be the input coordinates. The sine function is applied pointwise and \( \odot \) denotes element-wise multiplication. Sine activations have proven to be beneficial in modeling high-frequency signals [37]. We confirm this by comparing our synthesis network to an alternative that uses ReLU activations in Figure 3.

### 3.1.2 Modulation Network

We modulate the activations of the synthesis network with a second MLP using ReLU activations, which acts on the latent code \( z \) corresponding to the target signal. It is defined recursively as:

\[
\begin{align*}
    h'_0 &= \text{ReLU}(w'_0 z + b'_0), \\
    \alpha_{i+1} &= h'_{i+1} = \text{ReLU}(w'_{i+1} \left[ h'_{i} z^T + b'_{i+1} \right]),
\end{align*}
\]

(3) (4)

where \( \text{ReLU}(\cdot) = \max(0, \cdot) \) and \( w'_i, b'_i \) are the weights and biases of the MLP. We establish an explicit relationship between \( w' \) and \( z \), by feeding in \( z \) at every layer in the modulation network in the form of a skip connection. As can be seen from Equations 4 and 2, the latent codes \( z \) can modulate the amplitude of the sine activations of each hidden layer in the synthesis network, through the modulation parameters \( \alpha_i \). Furthermore, expanding Equation 2, we get

\[
h_i = \alpha_i \odot \sin(w_i \cdot (\alpha_{i-1} \odot \sin(\ldots)) + b_i),
\]

(5)

which shows the latent codes also indirectly control the frequency and phase shift of the sinusoids in subsequent layers. Figure 4 illustrates the expressivity of our modulation mechanism visually.

### 3.2. Expressivity of Modulation

A simpler alternative to using a separate modulation network would be to concatenate the latent codes and the input coordinates, and use the resultant vector as a single input to the synthesis network. This strategy has shown to be fruitful for ReLU-based synthesis networks for encoding signed distance fields [31]. However, we find it consistently fails with sine activations (see Section 4.1 for details).

In this alternative conditioning mechanism, the network takes the concatenation \([x, z]\) as input, so the first layer can be rewritten as:

\[
h_1 = \sin(w_{1,x} x + w_{1,z} z + b_1),
\]

(6)

where \( w_{1,x} \) and \( w_{1,z} \) are submatrices of \( w_1 \), corresponding to \( x \) and \( z \) respectively. The latent code \( z \) can therefore only act as a phase shift, \( w_{1,z} z \), on the first layer. This severely limits expressivity, in contrast to our model, where the latent code \( z \) modulates the amplitude, frequency and phase-shift of the functional representation at all layers, via the \( \alpha_i \). Figure 4 illustrates the difference in expressivity between our model and a concatenation-based MLP.

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**Figure 4:** Our modulator improves expressivity. Conditioning a sine-synthesizer by concatenation does not yield much control over the MLP’s internal feature maps, thereby limiting the expressivity of the latent space. Here, we show feature maps obtained at the second layer of a randomly initialized synthesis network conditionally modulated on four different latent vectors \( z_i \). Concatenating \( z_i \) with the input (left) only changes the phase of the signals at the second layer. Using our modulator sub-network (right) provides much more control over the internal feature maps, with variations in phase, amplitude and frequency.

### 3.3. Local Functional Representations

The ability to generalize to more than one signal gives us an additional opportunity. Rather than computing a single neural function for the entire signal, we decompose the domain into a regular grid, and calculate a local continuous representation for each tile (illustrated in Figure 5). Concretely, we assign each tile a latent code \( z_i \), so that the entire signal is represented by a codebook \( \{ z_i \} \), and the corresponding neural functions \( \{ f_\theta(\cdot, z_i) \} \), whose first argument is the normalized local coordinates in the tile \( x \in [0, 1]^n \).

**Continuity at tile boundaries.** In practice, to eliminate visual discontinuities at the tile boundaries, the images are split into a set of overlapping tiles. When evaluating the continuous representation, the contribution of overlapping tiles is weighted \( n \)-linearly according to the distance between

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**Figure 5:** Continuity at tile boundaries. We partition the target signals into uniform grids of overlapping tiles. Here, an image (left) is partitioned into 4 tiles (middle) with overlapping areas. During inference the tiles are blended using an \( n \)-linear (bilinear here) weighting scheme (right). The colors represent the resultant contribution of each tile at a given position. (Best viewed in color)
the point and the tile centers (Fig. 5).

3.4. Training procedure

We present two modes of training of our model. In the auto-encoder setting, (§ 3.4.1), the latent codes are estimated using a discrete encoder. In the auto-decoder configuration (§ 3.4.2), the latent codes are randomly initialized and optimized with the network parameters as in [31].

3.4.1 Auto-encoder

Unless otherwise specified, we use our model in an auto-encoder configuration. Auto-encoding lets us to build a continuous representation, from discrete input signals, using an auxiliary encoder network (shown in Figure 1). This could be useful in spatial super-resolution (images, videos), frame interpolation (videos), or reconstruction problems from sparse samples (lightfields, compression).

3.4.2 Auto-decoder configuration

In the auto-decoder configuration, we jointly optimize the network parameters θ, and the latent codes for all the training signals. That is, we do not use the optional encoder of Figure 1. We use this configuration in our shape reconstruction experiments, as proposed by [31]. After training, we obtain a functional representation for new, unseen test signals by sampling a new latent code z for the unseen signal, and optimizing it with the same objective used during training, but this time keeping the network parameters θ constant. We initialize all latent codes $z \sim \mathcal{N}(0, s^2)$ as Gaussian random vectors with $s = 10^{-2}$.

4. Experiments

We demonstrate two classes of experiments, on three domains (images, videos, 3D shapes). First, we demonstrate the generalization capabilities of the proposed model (§ 4.1) in a global setting. That is, we compute functional representations for many discrete signals, each of which represented (as a whole) by a latent code (i.e., without the tiling procedure described in Section 3.3). Second, we show how our model can be used to learn local functional representations of discrete signals (§ 4.2), with high reconstruction quality. In this set of experiments, each signal is defined using a latent codebook (one code per tile of the input signal). We also show our model can be applied to other multi-domain tasks, such as image relighting (§ 4.3), where the function’s input is a 2D pixel coordinate and a 3D lighting direction.

We compare to state-of-the-art MLP-based functional baselines, illustrated in Figure 6, together with our model. These are:

- **ReLU/FFN**: a standard MLP with two inputs—latent code and sample coordinates. In case of FFN [42], the sample coordinates are transformed using a random fourier gaussian matrix with scale σ.

- **SIREN+**: A single MLP with sine activations adapted from [37], with an additional input for the latent code z, concatenated with the coordinates x.

- **HyperNet-SIREN**: A SIREN whose weights are conditionally predicted using a hypernetwork [13], as described in [37]. Instead of a modulator sub-network, a hyper-network takes the latent code z as input, and predicts all the parameters of the synthesis network (the $w_i, b_i$).

4.1. Global Functional Representation

**Images** We run our image experiments on the CelebA [21] and CIFAR-10 [17] datasets separately. We use our model in an auto-encoder setting (§ 3.4.1). A convolutional encoder estimates a latent code for each image. From the latent codes, we decode a functional representation for each image. All the images are resampled to $32 \times 32$ resolution for training. The parameters of the modulator, synthesizer and encoder are trained simultaneously to minimize the sum of a reconstruction loss,

$$\mathcal{L}(f_\theta(x; z_i), y) = \|f_\theta(x; z_i) - y\|_2^2,$$  

We train all the models for 1000 epochs. Our train/test splits contain 167K/33K and 60K/10K images respectively. We use $128 \times 128$ center-crops for CelebA and entire image for CIFAR-10 as ground truth. The images are resampled to $64 \times 64$ and $32 \times 32$ using bicubic sampling.

We evaluate generalization by sampling $f_\theta$ at pixel centers, at the input image resolution $32 \times 32$ (1×) and computing the PSNR. Additionally, we evaluate continuity by sampling $f_\theta$ more finely, at $64 \times 64$ (2×) resolution, and comparing to ground-truth resampled to the same resolution; we do not train the models with these higher resolution targets. Table 1, summarizes our result on the CelebA dataset, and Table 2 on CIFAR-10 [17]. SIREN+ struggles
We train conditional functional on 200K 32×32 images from the CelebA [21] dataset, with a conditional latent code per image. We show PSNR on the test set at 1× resolution to show reconstruction accuracy, and at 2× resolution to show the interpolation behaviors of the functions. We mark the best result in yellow. Without careful tuning of $\sigma$, FFN produces discontinuous functions (red). HyperNetwork requires $\sim 60 \times$ more parameters (orange).

| Method          | PSNR ↑ | PSNR-2× ↑ | Params. |
|-----------------|--------|-----------|---------|
| ReLU            | 28.31  | 24.91     | 1.1M    |
| SIREN+ [37]     | 20.15  | 19.19     | 1.1M    |
| FFN ($\sigma = 10$) [42] | 26.48  | 4.22      | 1.2M    |
| FFN ($\sigma = 1$) [42]  | 27.87  | 24.37     | 1.2M    |
| HyperNet-SIREN [13, 37] | 26.70  | 24.54     | 72M     |
| Ours            | 29.42  | 25.49     | 1.5M    |
| Ours - Large Conv. | 29.64  | 26.05     | 17M     |

Table 1: Comparison of generalization ability on CelebA. We train conditional functional on 200K 32×32 images from the CelebA [21] dataset, with a conditional latent code per image. We show PSNR on the test set at 1× resolution to show reconstruction accuracy, and at 2× resolution to show the interpolation behaviors of the functions. We mark the best result in yellow. Without careful tuning of $\sigma$, FFN produces discontinuous functions (red). HyperNetwork requires $\sim 60 \times$ more parameters (orange).

| Method          | PSNR ↑ | PSNR-2× ↑ |
|-----------------|--------|-----------|
| ReLU            | 23.65  | 25.18     |
| SIREN+ [37]     | n/a    | n/a       |
| FFN ($\sigma = 10$) [42] | 22.75  | 5.75      |
| FFN ($\sigma = 1$) [42]  | 24.97  | 25.51     |
| HyperNet-SIREN [13, 37] | 18.07  | 19.16     |
| Ours            | 25.73  | 27.05     |

Table 2: Comparison of generalization ability on CIFAR-10. We also show results on CIFAR-10 [17] dataset, which contains more diverse images than the CelebA dataset. The numbers are reported for the test set with 10K images. PSNR values for 2× resolution are computed against ground-truth images at 1× resolution upsampled using bilinear interpolation. SIREN+ fails to converge on this dataset.

with generalization, and in the case of CIFAR-10, does not even converge. We hypothesize this is due to the higher image variability in CIFAR-10, in comparison to CelebA where faces are aligned. We observe a similar behavior with HyperNet-SIREN. As shown in Tables 1 and 2, the scaling parameter $\sigma$ of FFN [42] is critical for continuity: PSNR 2× drops with the recommended $\sigma = 10$ value. Since in the case of HyperNet-SIREN, the last layer predicts all the parameters of the hyp network, it makes the last layer highly over-parameterized. This leads to slow training, unstable convergence and inefficient memory usage. We show reconstructions on CelebA test images in Figure 7.

Shapes Generative modeling of 3D shapes has recently been driven by implicit neural representations [31, 7] trained to regress a shape’s signed distance field (SDF), by sampling discrete locations in the 3D space. The shape can be reconstructed from the learned SDF using sphere tracing [20] or marching cubes. We show that our model is a powerful replacement for the conditional ReLU-MLPs typically used for this application; it can encode SDFs more accurately. For this experiment, we sample 500K points for each shape in the cars category of ShapeNet [6]. Half these points are sampled close to the surface, the remaining are randomly sampled inside the unit sphere encompassing the shapes [14]. We use a similar training objective as in case of images (Eq. 7), but we replace the $L_2$ loss with an $L_1$ penalty in the fidelity term. Following [31], all conditional models are trained in an auto-decoder for this experiment (§ 3.4.2). Table 3 shows quantitative comparisons in terms of bi-directional Chamfer distance, computed between the ground-truth shapes and the reconstructions. We show renderings in Figure 8. Compared to DeepSDF [31], we produce higher-quality reconstructions, with finer details. As for images, we found SIREN+ does not converge. In this comparison, we do not include recent improvements that are orthogonal to our contribution, e.g., improvements to the spatial sampling [9], training procedure [36] or loss functions [11] These improvements would benefit our method as well as the baselines.

Videos For videos, we train our model on $\sim$90K videos from the Vimeo-90k septuplet dataset [46]. Each video is 7 frames long and has a spatial resolution of $448 \times 256$. During training, we randomly crop $32 \times 32 \times 7$ tiles from the videos. We use a 3D convolutional encoder to predict the latent codes. At test time, the videos are structured in a grid and each tile is reconstructed with the estimated latent code. The train-test split is used as provided in the dataset. We show quantitative comparisons in Table 4.
DeepSDF [31] 0.00284 0.00363 0.00559
DeepSDF + SIREN [31, 37] n/a n/a n/a
DeepSDF + FFN [31, 42] 0.00399 0.00519 0.00757
Ours 0.00230 0.00273 0.00285

Table 3: Comparison of generalization ability on ShapeNet. Our model can be used in an auto-decoder setting, where the latent codes for each shape are optimized simultaneously with the network parameters. We implicitly represent 3514 car shapes from the ShapeNet [6] dataset and report the bi-directional Chamfer distance to the ground truth shapes.

| Method            | Local PSNR ↑ | Overfit | Time ↓ |
|-------------------|--------------|---------|--------|
| ReLU              | 18.94        | ✓       | ~60m   |
| SIREN [37]        | 22.88        | ✓       | ~60m   |
| FFN [42]          | 28.48        | ✓       | ~60m   |
| Ours-ReLU         | 34.73        | ✓       | 13s    |
| Ours              | 38.03        | ✓       | 15s    |

Table 5: Encode high-res images faster. We use our model as a functional auto-encoder to encode images from Div2K [1] dataset. The global methods are overfit on each image separately and do not generalize. Our local methods are pre-trained and numbers are reported on a test set.

| Method            | Local PSNR ↑ | Overfit | Time ↓ |
|-------------------|--------------|---------|--------|
| ReLU              | 2.16         |         | 3.61   |
| SIREN [31, 37]    | 2.00         |         | 7.36   |
| FFN [31, 42]      | 1.93         |         | 3.54   |
| Ours-ReLU         | 1.37         | ✓       | 8.37   |
| Ours              | 1.32         | ✓       | 2.40   |

Table 6: Implicit encoding of large 3D scenes. Our method can be used to approximate a signed-distance-field for large scenes. The global methods are overfit on each scene separately. Our local models are trained on Scene A and then generalized to Scene B by freezing the weights of the model and optimizing the latent codes.

4.2. Local Functional Representations

Images Our dual-MLP model can also be used to generalize to high-resolution implicit functions. We train our model on 100 images from Div2K [1]. Each image has a long-side resolution of 2K and split into 32 × 32 overlapping tiles. We found 32 × 32 tile size to provide good reconstruction accuracy as well as good interpolation properties. The total number of tiles in the training set is ∼ 2M. Each tile is encoded using our method as an auto-encoder. At test time we sample 50 unseen images, and encode them using the trained model. Since other baselines do not generalize, we train a separate MLP for each of the images individually for global methods (i.e. ReLU, SIREN, FFN). Reconstruction PSNR at 1× resolution is reported in Table 5. Additionally, we perform an ablation on our model by using a standard ReLU MLP with our local parameterization.

Shape For this experiment, we collect 18 high-resolution (∼2M triangles) shapes from the the ThreedScans project [19]. These shapes are split into two Scenes A and B, each of which have 9 shapes. We compute a ground truth signed-distance-field (SDF) as in the global shapes ex-
Figure 10: **Encoding large 3D scenes.** Our method can be used to encode signed-distance-fields (SDFs) for large scenes (example shown on the left). The local representation enables the MLP $f_\theta$ to distribute its capacity sparsely in the scene. As a result, the estimated SDF is not overly smooth like we observe in the case of global methods.

Table 7: **Encoding high-res videos faster.** We use our per-trained model on Vimeo-90k [46] to encode high-res videos faster. To achieve a similar reconstruction accuracy, overfitting other models take up to 15 hours.

| Method          | Local | PSNR ↑ | Overfit | Time ↓ |
|-----------------|-------|--------|---------|--------|
| ReLU            | Local | 19.28  | ✓        | ~15hr  |
| FFN [42]        | Local | 20.87  | ✓        | ~15hr  |
| SIREN [37]     | Local | 25.19  | ✓        | ~15hr  |
| Ours-Generalized| Local | 25.21  | ✓        | ~1m    |

Figure 11: **Image-based relighting using MLPs.** Using light direction and image coordinates concatenated as input to a SIREN fails to reconstruct cast-shadows and specularities faithfully (insets). Our method of using conditional modulation reconstructs at higher fidelity with smoother interpolation in the light domain.

5. Conclusion

We propose a novel method for representing signals using multi-layer perceptrons (MLPs). We show that partitioning
the signal domain into tiles simplifies the signal locally. This leads to representing images, videos and shapes using MLPs with high-quality reconstructions. MLPs with ReLU activations fail to reconstruct high-frequency components of the signals. Instead, we use sine activations which we show to work with a wider frequency spectrum. Using local models requires MLPs to be conditioned on latent codes. We show that concatenating latent codes with the input hinders expressivity. Our method uses a dual-MLP architecture instead. The proposed model also generalizes to multiple instances of these signals. Our local parameterization is general enough to be applied in other applications that use implicit neural functions [25]. We merge local functions using n-linear blending which mitigates perceptual discontinuities for the tasks that we explored; however, it is still unclear if that strategy can be applied to other function domains.

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