MATHEMATICAL ANALYSIS OF THE ENDEMIC EQUILIBRIUM OF MALARIA-HYGIENE MODEL

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Abstract - In this study, we analyzed the endemic equilibrium point of a malaria-hygiene mathematical model. We prove that the mathematical model is biological and meaningfully well-posed. We also compute the basic reproduction number using the next generation method. Stability analysis of the endemic equilibrium point show that the point is locally stable if reproduction number is greater that unity and globally stable by the Lasalle’s invariant principle. Numerical simulation to show the dynamics of the compartment at various hygiene rate was carried out.

Keywords- Malaria, Hygiene, Endemic Equilibrium, Mathematical Model.

I. INTRODUCTION

One of the serious public health problem affecting the wealth and health of individuals and nations in Africa is Malaria [1, 2]. From 2015-2017, no progress in reducing the global case of malaria has been made [3]. Preventive and symptomatic treatment of malaria, use of long-lasting insecticidal mosquito nets (LLINs) and spraying are efforts employed in malaria prevention, these have reduced the incidence and mortality of malaria [4, 5].

Poor Sanitation system- stagnant water and streams, Socio-economic factors are the ideal location for the development of malaria transmission vector (Anopheles mosquitoes) [6]. The link of the water, sanitation and hygiene (WASH) efforts with malaria transmission has been neglected. Regular cleaning of the surrounding has been associated with malaria infection prevalence [7].

Mathematical modelling has been an essential tool for understanding disease transmission dynamics [8]. [9] proposed a mathematical model of typhoid fever assuming budget allocation for protection against the disease as a variable. The model analysis revealed that sanitation and awareness program has capacity to control the spread of the infection. [10] Formulated a mathematical model of cholera using hygiene consciousness as a control strategy. Using the next generation, the basic reproduction was computed and they showed that the disease free equilibrium is locally stable. The numerical simulation revealed that hygiene consciousness is effective in controlling cholera. [11] model the transmission dynamics of cholera establishing the effects of hygiene, famine, climate and environment. Numerical simulations was carried out to show the evolution of cholera spread. [12] proposed a system of non-linear ordinary differential equation of TB to study the effects of hygiene as a control strategy. The equilibrium points was analyzed and established. The local and global stability of the DFE is stable when unity is less than one. The result of the simulation shows that hygiene consciousness can help control TB disease. Many mathematical models have been formulated to study malaria transmission but to the knowledge of the author none has studied transmission of malaria and hygiene model.

II. MODEL FORMULATION

In this model, the total human population denoted by \( N_H \) is subdivided into Unhygienic susceptible human population \( S_u \), Hygienic Susceptible Human population \( S_h \), Unhygienic infected human population \( I_u \), hygienic infected human population \( I_h \) and the Recovered Human population \( R_h \). The mosquito population denoted by \( N_V \) is subdivided into Non-disease carrier mosquitoes \( S_v \) and disease carrier mosquitoes \( I_v \). Therefore, we have the following sub populations:

\[
N_H = S_u + S_h + I_u + I_h + R_h.
\] (1)
Let $\Lambda_H$ be the recruitment rate of the human population. A fraction $(1 - \alpha)\Lambda_H$ enters unhygienic susceptible human class while the remaining fraction $\alpha \Lambda_H$ enters the hygienic susceptible human class. The unhygienic susceptible class is increased by the rate at which unhygienic human class lose immunity after recovery given as $\omega$, and reduced by the rate of progression to hygienic class $\tau_1$, the force of infection for the unhygienic class $\lambda_u$ and natural human death rate $\mu_H$. The hygienic susceptible human compartment is increased by $\tau_1$, while the compartment is reduced by natural human death rate $\mu_H$ and the force of infection for the hygienic class $(1 - \zeta)\lambda_h$. The unhygienic infected human class $I_u$ is increased by $\lambda_u$ and reduced by natural human death rate $\mu_H$, rate of progression from $I_u$ to $I_h$ given as $\tau_2$, malaria induced death for unhygienic human class $\delta_u$ and recovery for unhygienic human class $\theta_u$. The hygienic infected class $I_h$ is increased by $(1 - \zeta)\lambda_h$ and $\tau_2$, then reduced by the recovery rate for a hygienic human class given as $\theta_h$, malaria induced death for hygienic human class $\delta_h$ and natural death rate $\mu_H$. The Human recovery class $R$ is increased by $\theta_h$ and $\theta_u$, then reduced by $\mu_H$, $\omega_h$ and $\omega_u$. The susceptible mosquito class $S_v$ is increased by the Mosquito recruitment rate given as $\Lambda_v$, reduced by the mosquitoes death rate $\mu_v$, and force of infection for mosquito given as $\lambda_v$. The infected mosquito class $I_v$ is increased by $\lambda_v$ and $\mu_v$.

**Figure 1. Model Schematic Diagram**

Given the above description and definitions of variables and parameters in Table 1 and 2, the following are the model equations:

$$\frac{dS_u}{dt} = (1 - \alpha)\Lambda_H - (\tau_1 + \lambda_u + \mu_H)S_u + \omega R$$

$$\frac{dS_h}{dt} = \alpha\Lambda_H + \omega R + \tau_1 S_u - ((1 - \zeta)\lambda_h + \mu_H)S_h,$$

$$\frac{dI_u}{dt} = \lambda_u S_u - (\tau_2 + \delta_u + \theta_u + \mu_H)I_u,$$

$$\frac{dI_h}{dt} = (1 - \zeta)\lambda_h S_h + \tau_2 I_u - (\delta_h + \theta_h + \mu_H)I_h$$

$$\frac{dR}{dt} = \theta_u I_u + \theta_h I_h - (\omega + \mu_H)R,$$

$$\frac{dS_v}{dt} = \Lambda_v - \lambda_v S_v - \mu_v S_v,$$

$$\frac{dI_v}{dt} = \lambda_v S_v - \mu_v I_v$$

where

$$\lambda_u = \frac{b_2 b_h b_v}{N_H}, \quad \lambda_h = \frac{b_2 b_h b_v}{N_H}, \quad b_1 > b_2, \quad \lambda_v = \frac{b_3 b_u (b_1 + 2 b_u)}{N_H}, \quad \delta_u > \delta_h, \quad \theta_h > \theta_u.$$  

**Table 1. Variables**

| Symbols | Description                  |
|---------|-----------------------------|
| $S_u$   | Unhygienic Susceptible Human|
| $S_h$   | Hygienic Susceptible Human  |
| $I_u$   | Unhygienic Infected Human   |
| $I_h$   | Hygienic Infected Human     |
| $R$     | Recovered Human             |
| $S_v$   | Non-disease carrier Mosquito|
| $I_v$   | Disease carrier Mosquito    |

**Table 2. Model Parameters**

| Parameters | Definitions                                |
|------------|--------------------------------------------|
| $\Lambda_H$| Recruitment rate of Human Population       |
| $\Lambda_v$| Recruitment rate of Mosquitoes             |
| $\tau_1$  | Progression from $S_u$ to $S_h$            |
| $\tau_2$  | Progression from $I_u$ to $I_h$            |
| $\delta_u$| Disease-Induced death for the unhygienic human class |
| Symbol | Description |
|--------|-------------|
| δ_h   | Disease-Induced death for the hygienic human class |
| b_1   | Biting rate of mosquito for unhygienic human class |
| b_2   | Biting rate of mosquito for hygienic human class |
| β_νh   | Transmission probability of infection from human to mosquito |
| β_νv   | Transmission probability of infection from mosquito to human |
| λ_u   | The force of infection for unhygienic human class |
| λ_v   | The force of infection for hygienic human class |
| λ_ν   | Force of infection for mosquitoes |
| ζ     | Rate of reduction of infection for hygienic class |
| ρ     | Modification Parameter |
| θ_u   | Rate of recovery for unhygienic human class |
| θ_v   | Rate of recovery for hygienic human class |
| ω     | Rate at which recovered human become susceptible |
| α     | Hygienic rate |
| μ_H   | Natural human death rate |
| μ_ν   | Natural death rate of mosquitoes |
| N_H   | Total Human Population |

### 3.1 Invariant Region

The invariant region can be obtained by the following theorem.

**Theorem 3.1**

The solutions of the model are feasible for all \( t > 0 \) if they enter the invariant region

\[
\Omega = \Omega_H \times \Omega_\nu. \tag{12}
\]

**Proof:**

Let

\[
\Omega = (S_u, S_h, I_u, I_h, R, S_v, I_v) \in \mathbb{R}_+^7,
\]

\[
\delta_h \text{ solution of the system with non-negative initial conditions.}
\]

Hence, all feasible solution set of the human population of the malaria model enters the region

\[
\Omega = \{ (S_u, S_h, I_u, I_h, R) \in \mathbb{R}_+^5 \}
\]

Similarly, the feasible solution set of the vector population enter the region

\[
\Omega_\nu = \{ (S_v, I_v) \in \mathbb{R}_+^2 \}
\]

Therefore, the region \( \Omega \) is positively invariant i.e. the solution remains positive for all initial values.

Thus, the model is biologically meaningful and mathematically well-posed in the domain \( \Omega \).

### 3.2 Disease Free Equilibrium (DFE)

The DFE of the model equations (3 - 9) is given as

\[
E_0 = \left( S_u^0, S_h^0, I_u^0, I_h^0, R^0, S_v^0, I_v^0 \right) = \left( \frac{(1-a)\Lambda_H}{\mu_H(\tau_1+\alpha\mu_H)}, \frac{\Lambda_H(\tau_1+\alpha\mu_H)}{\mu_H(\tau_1+\mu_H)}, 0, 0, 0, \frac{\Lambda_H}{\mu_\nu}, 0 \right) \tag{16}
\]

### 3.3 Reproduction Number (\( R_0 \))

The basic reproduction number \( (R_0) \) is defined as the number of secondary malaria infections produced by one infected individual in a completely susceptible community. The next-generation method will be employed to compute \( R_0 \). \( F(x) \) is the rate of new infection appearance while \( V(x) \) is the rate of transfer of individuals into compartments. So we have

\[
FV^{-1} = R_0 = \sqrt{\frac{b_3\beta_\nu\rho\beta_u\lambda_\nu\mu_H(1-a)(\tau_2+\tau_2\rho)+b_3\beta_\nu\rho(\alpha\mu_H+\tau_2)(1-\zeta)}{\Lambda_H(\tau_1+\alpha\mu_H)}} \tag{17}
\]

### 3.4 Endemic Equilibrium (EE)

The EE is when the disease continues in the community. It is computed by equating all the model equations to zero. It is denoted by

\[
E_* = (S_u^*, S_h^*, I_u^*, I_h^*, R^*, S_v^*, I_v^*) \tag{18}
\]

So,

\[
S_u^* = \frac{\Lambda_H(1-a)\Lambda_H+\alpha R^*}{\Lambda_H(\tau_1+\alpha\mu_H)+b_1\beta_\nu\rho} \tag{19}
\]
\[ S'_h = \frac{\Lambda_H (a\Lambda_H + \omega R^* + \tau_s S'_u)}{(1 - \zeta) b_d \Lambda_H + \Lambda_H \mu_H} \quad \text{(20)} \]

\[ I'_u = \frac{\mu_H \bar{\beta}_u S'_u}{\Lambda_H k_1 k_2} \quad \text{(21)} \]

\[ I'_h = \frac{\mu_H \bar{\beta}_w S'_h (1 - \zeta) k_1 b_2 S'_h + \delta_2 S'_u}{\Lambda_H k_1 k_2} \quad \text{(22)} \]

\[ R' = \frac{\theta_u \bar{\mu}_u + \theta_h \bar{\mu}_h}{k_3} \quad \text{(23)} \]

\[ S'_v = \frac{\Lambda_v \Lambda_H}{\rho \bar{\beta}_u \bar{\mu}_u (\bar{\mu}_u + \rho \bar{\mu}_u)} + \Lambda_H \mu_v \quad \text{(24)} \]

\[ I'_v = \frac{\lambda_v S'_v}{\mu_v} \quad \text{(25)} \]

Where

\[ k_1 = \left( \tau_2 + \delta_u + \theta_u + \mu_H \right); \quad k_2 = \left( \delta_h + \theta_h + \mu_H \right); \quad k_3 = \left( \omega + \mu_H \right) \]

Substituting (24) for \( S'_v \) (21) and (22) for \( I'_u \) and \( I'_h \) respectively, (25) becomes

\[ Z_1 I'_v^2 - Z_2 I'_v = 0 \quad \text{(26)} \]

This gives solutions of

\[ I'_v = 0 \] which is the DFE point,

or \( I'_v = \frac{Z_2}{Z_1} \quad \text{(27)} \)

Where

\[ Z_1 = \frac{b_3 \bar{\beta}_w \bar{\mu}_h \mu_v (b_2 \mu_H (1 - \zeta) \delta_2 \bar{\mu}_u + \delta_2 \bar{\mu}_u (1 - \zeta) \mu_H)}{k_1 k_2 (\tau_1 + \mu_H)} \quad \text{(28)} \]

\[ Z_2 = R_0 - 1 \quad \text{(29)} \]

It therefore shows that there exist a unique Endemic Equilibrium point at \( R_0 > 1 \).

\[ \text{3.5 Local Stability of EEP} \]

**Theorem 3.2**: The EE of the model is locally asymptotically stable whenever \( R_0 > 1 \).

Proof:

At the Endemic Equilibrium point, we have a Jacobian Matrix given as:

\[ J(E_e) = \]

\[ \begin{bmatrix}
-\Delta & 0 & 0 & 0 & \omega & 0 & -A_1 \\
-\tau_1 - A_2 & 0 & 0 & \omega & 0 & -A_3 \\
A_4 & 0 & -P_1 & 0 & 0 & 0 & A_1 \\
0 & A_5 & \tau_2 & -P_2 & 0 & 0 & A_3 \\
0 & 0 & \theta_u & \theta_h & -P_1 & 0 & 0 \\
0 & 0 & -A_6 & -A_7 & 0 & -(\lambda'_v + \mu_v) & 0 \\
0 & 0 & A_6 & A_7 & 0 & 0 & -\mu_v \\
\end{bmatrix} \quad \text{(30)} \]

Where

\[ A = \tau_1 + \lambda'_u + \mu_H; A_1 = \frac{b_1 \bar{\beta}_w \mu_H S'_u}{\Lambda_H}; A_2 = (1 - \zeta) \lambda'_h + \mu_H; \]

\[ \begin{align*}
A_3 &= \frac{\Lambda_H H}{b_3 \bar{\beta}_w \mu_H \delta_2 S'_u}; A_4 &= \frac{\Lambda_H H}{b_3 \bar{\beta}_w \mu_H \delta_2 S'_u}; \\
A_5 &= \frac{\Lambda_H H}{b_3 \bar{\beta}_w \mu_H \delta_2 S'_u}; A_6 &= \frac{\Lambda_H H}{b_3 \bar{\beta}_w \mu_H \delta_2 S'_u}; \\
\end{align*} \]

\[ \begin{align*}
P_1 &= \tau_2 + \delta_u + \theta_u + \mu_H; \\
P_2 &= \delta_h + \theta_h + \mu_H; \\
P_3 &= \omega + \mu_H \\
\end{align*} \quad \text{(31)} \]

Applying elementary row operation to (30) we have

\[ J(E_e) = \]

\[ \begin{bmatrix}
-\Delta & 0 & 0 & 0 & \omega & 0 & -A_1 \\
0 & -\Delta A_2 & 0 & 0 & A\omega & 0 & -A_3 \\
0 & 0 & -\Delta P_1 & 0 & A_4 \omega & 0 & A_9 \\
0 & 0 & 0 & -A_{12} & A_{12} & 0 & -A_{14} \\
0 & 0 & 0 & 0 & -A_{17} & 0 & A_{18} \\
0 & 0 & 0 & 0 & 0 & -(\lambda'_v + \mu_v) & -\mu_v \\
0 & 0 & 0 & 0 & 0 & 0 & -A_{19} \\
\end{bmatrix} \quad \text{(32)} \]

Where

\[ \begin{align*}
A_9 &= AA_2 + \tau_1 A_1; A_9 = A_4 A_1 + AA_3; A_{10} = A_9 - \\
A A_2 A_3; A_{11} &= A_6 \theta_h + \theta_u A_7; A_{12} = A k_1 (A A_2 k_2 + \\
A A_4 \omega); A_{13} &= AA_2 A_4 T_2 \omega; A_{14} = AA_{10} k_1 + \\
A A_2 A_5 \tau_2; A_{15} &= A_1 A_{13} - A_3 A_6 A_{12}; A_{16} = A_{11} A_{14} - \\
A_{12} \theta_u \mu_v; A_{17} &= A A_{13} k_1 \theta_h - A_{12} (A k_1 k_3 - \\
\theta_1 A_4 \omega); A_{18} &= A k_1 (\theta_k A_{14} - A_9 A_{12}); A_{19} = A_{15} A_{18} + \\
A_{16} A_{17} \end{align*} \quad \text{(33)} \]

\[ |J - \lambda I| = 0 \]

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It is observed that all the eigenvalues of $J(E_*)$ are negative. Hence, it is concluded that the endemic equilibrium $E^*$ of the model is locally asymptotically stable if $R_0 > 1$.

### 3.6 Global stability of EEP.

**Theorem 3.3:** The Endemic Equilibrium $E_*$ is globally asymptotically stable if $R_0 > 1$.

**Proof:**

We define the Lyapunov function $U$ as:

$$U = (S_u - S_u' - S_u'\log S_u' S_u) + (S_h - S_h' - S_h'\log S_h' S_h) + (I_u - I_u' - I_u'\log I_u' I_u) + (I_h - I_h' - I_h'\log I_h' I_h) + (R - R' - R'\log R' R) + (S_v - S_v' - S_v'\log S_v' S_v) + (I_v - I_v' - I_v'\log I_v' I_v).$$

$$\frac{dU}{dt} = (1 - \frac{S_u}{S_u'}) \frac{dS_u}{dt} + (1 - \frac{S_h}{S_h'}) \frac{dS_h}{dt} + (1 - \frac{I_u}{I_u'}) \frac{dI_u}{dt} + (1 - \frac{I_h}{I_h'}) \frac{dI_h}{dt} + (1 - \frac{R}{R'}) \frac{dR}{dt} + (1 - \frac{S_v}{S_v'}) \frac{dS_v}{dt} + (1 - \frac{I_v}{I_v'}) \frac{dI_v}{dt}$$

$$\frac{dV}{dt} = G_1 - G_2$$

Substituting the expression for the derivatives and separating positive and negative terms as $G_1$ and $G_2$, we have

$$G_1 = \Lambda_H + 2\omega R + (\tau_1 + \lambda_u)S_u + (1 - \xi)\lambda_hS_h + (\tau_2 + \theta_u)I_u + \theta_h I_h + \Lambda_v + \lambda_v S_v + \omega R'S_u + \omega R'S_h + \tau_1 S_u' I_u' S_h + \tau_1 S_v' I_v' S_h + \tau_2 S_u' I_u' S_h + \theta_u I_u' R' S_h + \lambda_v S_u' S_h$$

$$G_2 = \tau_1 S_u' + (1 - \xi)\lambda_h S_h' + (\tau_2 + \theta_u)I_u' + \theta_h I_h' + 2\omega R' + \lambda_v S_v' + (1 - \xi)\lambda_h S_h' + \lambda_v S_v' + \omega R'S_u + \omega R'S_h + \tau_1 I_u' S_u + \tau_2 I_u' S_h + \theta_u I_u' R' S_h + \lambda_v S_u' S_h$$

Then $G_1 > 0$ and $G_2 > 0$ and $G_1 - G_2 = 0$. If $G_1 < G_2$, then $\frac{dV}{dt} < 0$ if and if $S_u = S_u', S_h = S_h', I_u = I_u', I_h = I_h', R = R', S_v = S_v', I_v = I_v'$. The largest invariant set is $\{S_u', S_h', I_u', I_h', R', S_v', I_v'\} \in \Omega$ if $\frac{dU}{dt} = 0$ is a singleton of $E_*$, and $E_*$ as the endemic equilibrium.

Therefore by the Lasalle’s invariance principle, $E_*$ is globally asymptotically stable in $\Omega$ if $G_1 < G_2$.

### 3.7 Numerical Simulation

In this section, we carry out numerical simulations for the model equations using the parameter values in table 3 and initial conditions $S_u(0) = 55, S_h = 45, I_u(0) = 45, I_h(0) = 30, R(0) = 50, S_v(0) = 1000, I_v(0) = 50$.

| Symbols | Values | Source |
|---------|--------|--------|
| $\Lambda_H$ | 100 | [13] |
| $\Lambda_v$ | 1000 | [14] |
| $\tau_1$ | 0.25 | (Assumed) |
| $\tau_2$ | 0.5 | (Assumed) |
| $\theta_u$ | 0.13 | (Assumed) |
| $\theta_h$ | 0.06 | (Assumed) |
| $b_1$ | 0.17 | (Assumed) |
| $b_2$ | 0.1 | (Assumed) |
| $\beta_{th}$ | 0.03 | [15] |
| $\beta_{hv}$ | 0.09 | [15] |
| $\nu$ | 0.08 | (Assumed) |
| $\rho$ | 0.5 | (Assumed) |
| $\theta_{\mu}$ | 0.05 | (Assumed) |
| $\alpha$ | 0.15 | (Assumed) |
| $\omega$ | 0.7902 | [14] |
| $\mu_H$ | 0.000004 | [13] |
| $\mu_v$ | 0.0000569 | [14] |
IV. CONCLUSION
In this study, we proposed and analyzed the endemic equilibrium point of a malaria hygiene mathematical model. We solve the mathematical model showing the endemic equilibrium points. The analysis show that the endemic equilibrium point is locally stable if $R_0 > 1$ given that the eigenvalues of the Jacobian matrices are negative, also by the Lasalle’s invariant principle defining a Lyapunov function we show that the endemic equilibrium is globally in the set $\Omega$.

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