Rare decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ via tensor unparticle mediation

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Abstract

The decay width of the rare decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ is strictly constrained from the LEP data. Tensor unparticles provide a tree-level contribution to this rare decay. We have calculated the tensor unparticle contribution to the rare decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$. The current experimental limit have been used to constrain unparticle couplings $\nu \bar{\nu} Z U^{\mu \nu}$ and $\gamma \gamma U^{\mu \nu}$.

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I. INTRODUCTION

Scale invariance plays a crucial role in theoretical physics. A possible scale invariant hidden sector that may interact weakly with the Standard Model (SM) fields is being discussed intensively in the literature. Based on a scale invariant theory by Banks-Zaks (BZ) \[1\], Georgi proposed a new scenario \[2, 3\] in which SM fields and a scale invariant sector described by (BZ) fields interact via the exchange of particles with a large mass scale \(M_U\). Below this large mass scale interactions between SM fields and BZ fields are described by non-renormalizable couplings suppressed by powers of \(M_U\) \[2, 4\]:

\[
\frac{1}{M_U^{d_{SM}+d_{BZ}-4}}O_{SM}O_{BZ}
\]

(1)

The renormalization effects in the scale invariant BZ sector then produce dimensional transmutation at an energy scale \(\Lambda_U\) \[5\]. In the effective theory below the scale \(\Lambda_U\), the BZ operators are embedded as unparticle operators. The operator (1) match onto the following form,

\[
C_{O_U} \frac{\Lambda_U^{d_{BZ}-d_{U}}}{M_U^{d_{SM}+d_{BZ}-4}}O_{SM}O_{U}
\]

(2)

here, \(d_U\) is the scale dimension of the unparticle operator \(O_U\) and the constant \(C_{O_U}\) is a coefficient function.

Phenomenological \[6\], astrophysical and cosmological \[7\] implications of unparticles have been intensively studied in the literature. In the some of these phenomenological researches several unparticle production processes have been considered. A possible evidence for this scale invariant sector might be a missing energy signature. It can be tested experimentally by examining missing energy distributions. Another evidence for unparticles can be explored by studying its virtual effects.

In this work we will present a detailed calculation of tensor unparticle contribution to the rare decay \(Z \rightarrow \nu \bar{\nu} \gamma \gamma\). Experimental results from LEP data \[8\] strictly constrains the decay width at 95\% confidence level:

\[
BR(Z \rightarrow \nu \bar{\nu} \gamma \gamma) \leq 3.1 \times 10^{-6}
\]

(3)
This experimental limit will be used to constrain unparticle couplings $\nu \bar{\nu} Z U^{\mu \nu}$ and $\gamma \gamma U^{\mu \nu}$.

II. THE RARE DECAY $Z \rightarrow \nu \bar{\nu} \gamma \gamma$

$Z \rightarrow \nu \bar{\nu} \gamma \gamma$ decay may occur via tensor unparticle exchange Fig.1. We consider the following effective interaction terms:

\[ -\frac{1}{4} \lambda \frac{\Lambda_{dU}}{\Lambda_{dU}} \bar{\psi} i(\gamma_\mu D_\nu + \gamma_\nu D_\mu) \psi \mathcal{O}_{dU}^{\mu \nu} \]  

\[ \frac{\kappa}{\Lambda_{dU}} G_{\mu \alpha} G^\alpha_\nu \mathcal{O}_{dU}^{\mu \nu} \]  

where $D_\mu = \partial_\mu + ig_2 W^a_\mu + ig' Y_2 B_\mu$ is the covariant derivative, $G^{\alpha \beta}$ denotes the gauge field strength, $\psi$ is the Standard Model fermion doublet or singlet, $\lambda$ and $\kappa$ are dimensionless effective couplings. Feynman rules for these operators have been given in ref.\[4\]. The vertex functions for $\nu \bar{\nu} Z U^{\mu \nu}$ and $\gamma(p_1) \gamma(p_2) U^{\mu \nu}$ generated from operators (4) and (5) are given by,

\[ \Gamma_{(\nu \bar{\nu} Z U)}^{\mu \nu} = \frac{ig}{4 \cos \theta_W} \frac{\lambda}{\Lambda_{dU}} [\gamma^\mu g^{\nu \alpha} + \gamma^\nu g^{\mu \alpha}] \]  

\[ \Gamma_{(\gamma \gamma U)}^{\mu \nu} = \frac{i\kappa}{\Lambda_{dU}} [K_{\mu \nu}^{\sigma \rho} - K_{\mu \sigma}^{\nu \rho}] \]  

\[ K_{\mu \nu}^{\sigma \rho} = -g^{\mu \rho} p_1^\sigma p_2^\nu - (p_1 \cdot p_2) g^{\rho \mu} g^{\sigma \nu} + p_1^\rho p_2^\sigma g^{\mu \nu} + p_1^\mu p_2^\rho g^{\sigma \nu} \]

respectively.

Spin-2 unparticle propagator is defined by [4]:

\[ \Delta(P^2)_{\mu \nu, \rho \sigma} = i \frac{A_{dU}}{2 \sin(d_U \pi)} (-P^2)^{d_U - 2} T_{\mu \nu, \rho \sigma}(P) \]  

where,

\[ A_{dU} = \frac{16 \pi^2}{(2 \pi)^{2d_U}} \frac{\Gamma(d_U + \frac{1}{2})}{\Gamma(d_U - 1) \Gamma(2d_U)} \]  

\[ \pi_{\mu \nu}(P) = -g_{\mu \nu} + \frac{P_\mu P_\nu}{P^2} \]
\[ T_{\mu\nu,\rho\sigma}(P) = \frac{1}{2} \left[ \pi_{\mu\rho}(P)\pi_{\nu\sigma}(P) + \pi_{\mu\sigma}(P)\pi_{\nu\rho}(P) - \frac{2}{3} \pi_{\mu\nu}(P)\pi_{\rho\sigma}(P) \right] \] (11)

The decay width can then be written as

\[ \Gamma(Z \to \nu\bar{\nu}\gamma\gamma) = \frac{S}{2^8 m_Z (2\pi)^8} \int |\mathcal{M}|^2 \delta^4(Q - p_1 - p_2 - k_1 - k_2) \frac{d^3p_1}{p_1^0} \frac{d^3p_2}{p_2^0} \frac{d^3k_1}{k_1^0} \frac{d^3k_2}{k_2^0} \] (12)

where \( S = 1/2! \) is the statistical factor for identical photons and momentum 4-vectors of the participating particles are denoted as \( Z(Q) \to \nu(p_1)\bar{\nu}(p_2)\gamma(k_1)\gamma(k_2) \). By applying Feynman rules we can obtain polarization summed squared amplitude. It is given by

\[ |\mathcal{M}|^2 = \frac{g^2 A_{4\nu}^2}{m_Z^2 \cos^2 \theta_W \sin^2(d_U \pi)} \left( \frac{\lambda}{\Lambda_{d\nu} U} \right)^2 \left( \frac{\kappa}{\Lambda_{d\nu} U} \right)^2 (k_1 + k_2)^2 |2d_{\nu\mu}^4 \Theta_{\mu\nu} p_1^\mu p_2^\nu| \] (13)

where we have introduced the definition

\[ \Theta_{\mu\nu} = [(k_1 \cdot k_2) m_Z^2 + 2(k_1 \cdot Q)(k_2 \cdot Q)] (k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu}) \] (14)

In amplitude (13) we have assumed the lepton universality and a factor of 3 has been taken into account for all of the known neutrino species. Integration over \( p_1 \) and \( p_2 \) can be provided with the aid of the following identity [9]:

\[ I^{\mu\nu} = \int \frac{d^3p_1}{p_1^0} \frac{d^3p_2}{p_2^0} \delta^4(V - p_1 - p_2) p_1^\mu p_2^\nu = \frac{\pi}{6} (V^2 g^{\mu\nu} + 2V^\mu V^\nu) \] (15)

Here \( V = Q - k_1 - k_2 \). Then the decay width is reduced to the following:

\[ \Gamma(Z \to \nu\bar{\nu}\gamma\gamma) = \frac{1}{3 \times 2^8 \pi^3 m_Z^4 \cos^2 \theta_W \sin^2(d_U \pi)} \left( \frac{\lambda}{\Lambda_{d\nu} U} \right)^2 \left( \frac{\kappa}{\Lambda_{d\nu} U} \right)^2 \times \int \frac{d^3k_1}{k_1^0} \frac{d^3k_2}{k_2^0} |(k_1 + k_2)^2 |2d_{\nu\mu}^4 \Theta_{\mu\nu} (V^2 g^{\mu\nu} + 2V^\mu V^\nu) | \] (16)

There still remains the integrations over \( k_1 \) and \( k_2 \). These integrations can be carried out numerically. We will work in the center of mass system of the Z boson. We choose the following integration variables:
\[ \xi = \frac{2Q \cdot k_1}{m_Z^2} = \frac{2k_1^0}{m_Z}, \quad \eta = \frac{2Q \cdot k_2}{m_Z^2} = \frac{2k_2^0}{m_Z}, \quad \omega = \frac{1 - \cos \theta}{2} \]  
(17)

where \( \theta \) is the angle between two outgoing photons. Then the decay width can be written as

\[ \Gamma(Z \rightarrow \nu \bar{\nu} \gamma \gamma) = \left( \frac{\lambda}{\Lambda_{d_U}} \right)^2 \left( \frac{\kappa}{\Lambda_{d_U}} \right)^2 F(d_U) \]  
(18)

where,

\[ F(d_U) = \frac{m_Z^{4d_U+1}}{3 \times 2^{16} \pi^5} \frac{g^2 A_{d_U}^2}{\cos^2 \theta_W \sin^2(d_U \pi)} \]  
(19)

\[ \times \int_{\Omega} \xi^2 \eta^2 |\xi \eta \omega|^{2d_U-4} \left( 1 + \omega \right) \left[ \omega(1 + \xi \eta \omega - \xi - \eta) + (1 - \eta \omega)(1 - \xi \omega) \right] d\xi d\eta d\omega \]

The integration region \( \Omega \) is given by [9, 10]:

\[ 0 \leq \omega \leq 1 \quad \text{when} \quad 0 \leq \xi \leq 1 - \eta \]  
(20)

\[ \frac{\xi + \eta - 1}{\xi \eta} \leq \omega \leq 1 \quad \text{when} \quad 1 - \eta \leq \xi \leq 1 \]  
(21)

together with \( 0 \leq \eta \leq 1 \). Numerical integrations have been performed by a Monte Carlo routine. During numerical integrations we impose a cut \(|\cos \theta| < 0.98\).

From current experimental limit (3) on the rare decay \( Z \rightarrow \nu \bar{\nu} \gamma \gamma \) we set the following bound:

\[ \left( \frac{\lambda}{\Lambda_{d_U}} \right)^2 \left( \frac{\kappa}{\Lambda_{d_U}} \right)^2 F(d_U) \leq 7.73512 \times 10^{-6} \]  
(22)

This expression gives us the allowed region in the \( \frac{\lambda^2}{\Lambda_{d_U}^2} \) versus \( \frac{\kappa^2}{\Lambda_{d_U}^2} \) plane. In Fig.2 we plot the boundary lines of scaled unparticle couplings \( \frac{\lambda^2}{\Lambda_{d_U}^2} \) and \( \frac{\kappa^2}{\Lambda_{d_U}^2} \) for various values of the scale dimension \( d_U \). Allowed regions are defined by the area restricted by these boundary lines.

In principle, unparticle couplings \( \kappa \) and \( \lambda \) can be different. If we assume \( \kappa = \lambda \), we can obtain a unique bound on \( \lambda \) for each value of the scale dimension. The bounds on scaled unparticle couplings \( |\frac{\lambda}{\Lambda_{d_U}}| \) are given on Table II.
III. CONCLUSION

One can see from Fig.2 that allowed area decreases in size as $d_U$ increases. This feature is reflected in the Table I also. For instance, bound on $|\frac{\Lambda}{M_U}|$ decreases by a factor of 11.9 as $d_U$ increases from 1.01 to 1.99 (Table I). This behavior is clear from factor $\frac{1}{\sin^2(d_U \pi)}$ in the squared amplitude (13).

Tensor unparticle interactions (4) and (5) also contribute to the rare decay $Z \to \ell \bar{\ell} \gamma \gamma$. Experimental limit on this rare decay is at the same order $O(10^{-6})$ as $Z \to \nu \bar{\nu} \gamma \gamma$ decay. Therefore this experimental limit can also be used to constrain tensor unparticle couplings. The difference is that the decay $Z \to \ell \bar{\ell} \gamma \gamma$ receives Standard Model tree-level contributions but $Z \to \nu \bar{\nu} \gamma \gamma$ decay does not.

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FIG. 1: Tensor unparticle contribution to the rare decay $Z \to \nu \bar{\nu} \gamma \gamma$.

FIG. 2: Boundary lines of scaled unparticle couplings $\lambda^2/\Lambda_U^{2d_U}$ and $\kappa^2/\Lambda_U^{2d_U}$. Legends are for various values of the scale dimension $d_U$. Allowed regions are defined by the area restricted by the boundary lines.
TABLE I: Bounds on tensor unparticle couplings.

| Scale dimension $d_U$ | Bounds on $|\frac{\lambda}{\Lambda_{dU}}|$ |
|-----------------------|----------------------------------|
| $d_U = 1.01$          | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00938$ |
| $d_U = 1.1$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00956$ |
| $d_U = 1.2$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00958$ |
| $d_U = 1.3$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00935$ |
| $d_U = 1.4$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00883$ |
| $d_U = 1.5$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00800$ |
| $d_U = 1.6$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00693$ |
| $d_U = 1.7$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00567$ |
| $d_U = 1.8$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00429$ |
| $d_U = 1.9$           | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00276$ |
| $d_U = 1.95$          | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00185$ |
| $d_U = 1.99$          | $|\frac{\lambda}{\Lambda_{dU}}| \leq 0.00079$ |