Stiffness Modeling of Cylindrical Hydrostatic Guideways

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Abstract. Static stiffness is an important performance index of the cylindrical hydrostatic guideway. In this paper, the static stiffness model of the cylindrical hydrostatic guideway is studied. Considering the guideway is a system with a rail and a hydrostatic bearing in series, firstly, geometric relationship between the displacement of the table and the deformation of the rail and the displacement of the hydrostatic bearing was derived, the mathematical expression of the stiffness was given. Secondly, based on the beam bending deformation theory, the linear superposition principle was used to derive the deformation equation of the rail under the force of the oil film. Finally, the static stiffness test rig was developed. The static stiffness test was carried out and the theoretical model was verified. The results show that the static stiffness model of the cylindrical hydrostatic guideway established in this paper has high accuracy, which can lay a theoretical foundation for the static characteristics analysis of the cylindrical hydrostatic guideway.

Keywords: cylindrical hydrostatic guideway; static stiffness; hydrostatic lubrication theory.

1. Introduction
The hydrostatic guideways is a typical type of linear motion support component, which has been used in precision machinery such as CNC machine tools. Static stiffness is an important performance index of the hydrostatic guideways.

Scholars at home and abroad have conducted research on the static stiffness of hydrostatic guideways. In general, the methods can be divided into following:

(1) Theoretical calculation method: Mainly based on the Reynolds equation and flow continuity equation of lubrication theory, the mathematical model of the guideway system is established, and then the model is modified or simplified appropriately, and the stiffness and damping of the guideway are solved [1-3], and some scholars use the finite difference method [4, 5] or finite element method [6, 7] directly solve the mathematical model to obtain the stiffness and damping of the guide rail;

(2) Experimental analysis method: Based on modal analysis, the transfer function model of the system is established, the excitation signal and corresponding signal are collected through experiments, or the excitation signal and response signal are obtained directly through simulation, and then the parameter identification is used to obtain the system stiffness, damping. [8, 9].

However, the cylindrical hydrostatic guideway is a series system of rail and cylindrical linear bearing. Under the action of external force, the influence of the deflection of the rail on the stiffness of the cylindrical hydrostatic guideway is non-negligible.
Based on the static pressure theory, in this paper, the displacement of the hydrostatic linear bearing considering the bending effect of the rail under the action of external force is derived; according to the series relationship between the rail and the hydrostatic bearing, the rail deflection and the bearing displacement calculated above are synthesized, the static stiffness model of the guideway is established.

2. Theoretical Model

2.1. Expression of the stiffness $k_z$

Fig. 1 shows the deformation of the guideway under vertical load. Assuming that the worktable and bed are rigid bodies, ignore the deformation of the slider itself under external force. In this paper, slider 1 and slider 2 are studied. For the slider 1, the initial position of the geometric center is $O_1$, the load $F_z$ will produce the rail deflection $\delta_{11}$ and a displacement of the bearing bush relative to the guide $\delta_{12}$. The center of the slider 1 will move to $O_1'$ and the displacement is $\delta_{1}$. Similarly, the rail deflection at the slider 2 is $\delta_{21}$ and the displacement of the bearing bush relative to the guide is $\delta_{23}$, the geometric center will move from $O_2$ to $O_2'$ and the displacement is $\delta_{2}$. The geometric center of the worktable will move from $P$ to $P'$ and the displacement is $\delta_z$.

![Figure 1. Deformation of the guideway under vertical force](image)

Since the displacement of the slider is much smaller than the length of the worktable, it can be considered that the displacement direction of the slider is vertically downward. Therefore, the displacement at the geometric center of the worktable can be expressed as:

$$\delta_z = \frac{1}{2}(\delta_{11} + \delta_{21}) + \frac{1}{2}(\delta_{12} + \delta_{22})$$  \hspace{1cm} (1)

Stiffness of the guideway can be expressed as:

$$k_z = \frac{2F_z}{\delta_{11} + \delta_{12} + \delta_{21} + \delta_{22}}$$  \hspace{1cm} (2)

2.2. Calculation of Rail Deformation $\delta_{11}$

In order to analyze the deflection of the guide when the worktable is under load, the coordinate system is established as shown in Fig. 2. Assuming that the length of the oil pad is $l$ and the pressure in the oil recess is evenly distributed, the distance between the left end face of the slider 1 and the coordinate origin is $s$, the distance between the sliders is $L_1$, and the rail span is $L'$. According to the principle of force interaction, the load on the slider on the rail is the same as the supporting force of the rail on the worktable in Fig. 1, the magnitude is equal, and the direction is opposite, so the magnitude of the uniformly distributed force acting on the rail is $q_1 = F_{11}/l$, $q_2 = F_{12}/l$, respectively.
According to the principle of superposition, the deformation caused by the resultant force acting on the same object is equal to the sum of the deformations generated by the respective component forces in the same direction. Combined with the beam bending deformation theory in material mechanics, the deflection of the rail under load can be obtained:

\[ w^*(x) = -\frac{1}{EI} M(x) \]  

(3)

Where \( E \) is elastic modulus of material, and \( I \) is moment of inertia of the rail section.

Once the axial position of the worktable is determined, the deformation of the rail at the slider under the load can be obtained from (3).

2.3. Calculation of the displacement \( \delta_{ij} \)

After the rail is deformed, the thickness of the oil film in the bearing variety as shown in Fig. 3. In the figure, \( h_0 \) is the initial oil film gap, the displacement at the center of the slider No.\( i \) is \( \delta_i \). Suppose there is a section \( j \) in the bearing, the rail deflection on this section is \( w_j \), the eccentricity of the bearing is \( \varepsilon_j \) and \( h_j \) is the thickness of the oil film on the cross section. From the geometric relationship in the figure, the eccentricity on section \( j \) can be expressed as:

\[ \varepsilon_j = \frac{\delta_i + \delta_{ij} - w_j}{h_0} \]  

(4)

Therefore, the thickness equation of oil film on section \( j \) can be expressed as:

\[ h_j = h_0(1 + \varepsilon_j \cos \theta) = h_0(1 + \frac{\delta_i + \delta_{ij} - w_j}{h_0} \cos \theta) \]  

(5)
Function \( f_k(\theta) = (1 + \varepsilon_k \cos \theta)^3 \) and the coordinate of the center angle of the oil recess \( \theta_i (i=1,2,3,4) \) are defined. According to [10], using Gauss-Legend integral formula, the outlet flow can be simplified as follows:

\[
Q = h_0^3 \frac{p_r R}{12\eta l_i}(\Delta_1 + \Delta_3) + h_0^3 \frac{p_r}{6\eta h_1} \Delta_2
\]

Where \( \Delta_k = \alpha_k \left[ \frac{8}{9} f_k \left( -\frac{\sqrt{5}}{5} \theta_c + \theta_c \right) + \frac{8}{9} f_k (\theta_c) + \frac{2}{9} f_k \left( \frac{\sqrt{5}}{5} \theta_c + \theta_c \right) \right] \).

The flow continuity equation is expressed as follows:

\[
Q = C_g (p_s - p_r)
\]

Where \( p_s \) is supply pressure; \( C_g = 128\eta l_d / \pi d^4 \), \( l_i \) is the capillary length, and \( d \) is the capillary diameter. By substituting (7) into (6), expression of recess pressure can be derived as:

\[
p_r = \frac{p_s C_g}{h_0^3 \frac{R}{12\eta l_i} (\Delta_1 + \Delta_3) + h_0^3 \frac{1}{6\eta h_1} \Delta_2 + C_g}
\]

Assuming that the slider is evenly loaded, the vertical force equilibrium equation of the worktable is given as follows:

\[
A_s (p_{r1} - p_{r3}) = \frac{F_z}{4}
\]

Where \( A_s \) is effective bearing area.

For opposite oil recess on the same section, the sum of both thickness is constant, the following formulation can be given:

\[
h_j + h_j'' = 2h_0
\]

Where \( h_j, h_j'' \) represents the oil film thickness on the upper and lower boundary respectively.

Substituting (5),(8),(10) into (9), the relationship between displacement \( \delta_2 \) and load \( F_z \) is established, and it can be solved with the iteration method.

3. Results and Discussion

Taking a cylindrical hydrostatic guideway as an example, the theoretical model established in this paper is used to calculate its static stiffness parameters, and the influence of the axial position of the worktable on the stiffness coefficient is analyzed. The parameters of the guideway and the oil recess are listed in Tab. 1 and Tab. 2.
Table 1. Parameters of the Guideway

| Parameter                  | Value |
|----------------------------|-------|
| Supply pressure, $p_s$/Mpa | 3.5   |
| Rail span, $L$/mm          | 980   |
| Outer diameter of rail, $D$/mm | 100  |
| Inner diameter of rail, $d$/mm | 35   |
| Geometric dimension, $L_1$/mm | 190  |
| Geometric dimension, $L_2$/mm | 420  |
| Geometric dimension, $H_1$/mm | 420  |
| Young’s modulus, $E$/Gpa   | 206   |

Table 2. Structural Parameters of the Bearing

| Parameter                  | Value  |
|----------------------------|--------|
| Length of the pad, $l$/mm  | 140    |
| Length of the pad, $b$/mm  | 64.3   |
| The radial clearance, $h_0$/μm | 30   |
| Axial land width, $l_1$/mm | 10     |
| Circumferential land width, $l_2$/mm | 10.4  |
| Angle of the land width in Circumferential direction, $\theta_1$/° | 15     |
| Angle between circumferential line and axis line, $\theta_2$/° | 40     |
| Dynamic viscosity, $\eta$/Pa·s | 0.0276 |

Fig. 4 shows the variation curve of the stiffness of the cylindrical hydrostatic guideway with the axial position of its worktable. It can be seen that when the center of the worktable is located at the midpoint of the guide pillar, the stiffness is the smallest; as the worktable moves along the guide pillar to both ends, its vertical stiffness gradually increases. Specifically, when the center of the worktable is located at the midpoint of the guide post, the minimum value is 165.2 N/μm; when the worktable
moves from the midpoint of the guide post to both ends, the maximum value is 221.4 N/μm, and the stiffness value is increased by 25%. The reason is mainly due to the fixed beam structure. According to the principle of material mechanics, when the center of the worktable is located at the midpoint of the guideway, the vertical deformation caused by the unit vertical force on the table is the largest, as the table moves toward the ends of the guide, the vertical deformation of the table gradually decreases.

4. Numerical method

![Stiffness calculation flowchart](image)

The stiffness calculation flowchart is shown Fig. 5. The detailed calculation procedure is described as follows:

1) Calculate the value of deformation $\delta_1$ using (3).
2) Based on Gauss-Legend integral formula, calculate the value of deformation $\delta_2$.
3) For external load $F_z$, iterations are continued until $W$ equals $F_z$.
4) Calculate the value of stiffness $k_z$.

5. Conclusion

1) Considering the series relationship between the rail and journal bearing, according to the definition of the static stiffness of cylindrical hydrostatic guideway, the mathematical expression of the vertical stiffness of the guideway is derived;
2) As the worktable moves from the midpoint of the rail to the two installation ends, the stiffness of the guideway gradually increases.

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