Initial conditions for evolution of double parton distributions

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K. Golec-Biernat, E. Lewandowska, arXiv:1311.7392 [hep-ph]
Double parton distribution functions (DPDF)

- are used in the description of double parton scattering (DPS)
- evolve with hard scales through QCD evolution equations known in the leading logarithmic approximation (LLA)
- obey nontrivial momentum and valence quark number sum rules which are conserved by the evolution.

Problem of this talk: how to specify initial conditions for the evolution equations which obey these sum rules?
Outline

- Parton distribution functions
- Evolution equations
- Sum rules
- Initial conditions
- Summary
Parton distribution functions

- Single parton scattering
  - PDF: $D_f(x, Q)$

- Double parton scattering
  - DPDF: $D_{f_1 f_2}(x_1, x_2, Q_1, Q_2)$

- Two hard scales: $Q_1, Q_2$ and two flavours: $f_1, f_2$ (including gluon).

- Sum of partons’ momenta cannot exceed total nucleon momentum
  $$x_1 + x_2 \leq 1$$
QCD evolution equations for single PDF

- General form of evolution equations for single PDF \( t = \ln Q^2 \)

\[
\frac{\partial_t D_f(x, t)}{D_f(x, t)} = \sum_{f'} \int_0^1 du \, K_{f'f}(x, u, t) D_{f'}(u, t)
\]

- The integral kernels describe \textit{real} and \textit{virtual} parton emission

\[
K_{f'f}(x, u, t) = K_{f'R}(x, u, t) - \delta(u - x) \delta_{f'} \Delta_f V_f(x, t)
\]
The real emission kernels

\[ K^{R}_{f f'}(x, u, t) = \frac{1}{u} P_{f f'}(\frac{x}{u}, t) \theta(u - x) \]

Splitting functions

\[ P_{f f'}(z, t) = \frac{\alpha_s(t)}{2\pi} P^{(0)}_{f f'}(z) + \frac{\alpha_s^2(t)}{(2\pi)^2} P^{(1)}_{f f'}(z) + \ldots \]

Well known DGLAP evolution equations for single PDF

\[ \partial_t D_f(x, t) = \sum_{f'} \int_{x}^{1} \frac{dz}{z} P_{f f'}(z, t) D_{f'}(\frac{x}{z}, t) - D_f(x, t) \sum_{f'} \int_{0}^{1} dz \ z P_{f' f}(z, t) \]
Evolution equations for DPDF

- Evolution of DPDF with equal hard scales $Q_1 = Q_2 \equiv Q$:

$$D_{f_1 f_2}(x_1, x_2, Q_0, Q_0) \rightarrow D_{f_1 f_2}(x_1, x_2, Q, Q) \equiv D_{f_1 f_2}(x_1, x_2, t = \ln Q^2)$$

- Evolution equations with three terms:

$$\partial_t D_{f_1 f_2}(x_1, x_2, t) = \sum_{f'} \int_0^{1-x_2} du \, \mathcal{K}_{f_1 f'}(x_1, u, t) \, D_{f' f_2}(u, x_2, t)$$

$$+ \sum_{f'} \int_0^{1-x_1} du \, \mathcal{K}_{f_2 f'}(x_2, u, t) \, D_{f_1 f'}(x_1, u, t)$$

$$+ \sum_{f'} \mathcal{K}^{R}_{f' \rightarrow f_1 f_2}(x_1, x_1 + x_2, t) \, D_{f'}(x_1 + x_2, t)$$
The third splitting term contains single PDF

\[
\frac{\alpha_s(Q)}{2\pi} \sum_{f'} \frac{1}{x_1 + x_2} P_{f' \rightarrow f_1f_2} \left( \frac{x_1}{x_1 + x_2} \right) D_{f'}(x_1 + x_2, Q)
\]

The DPDF evolution equations need to be solved together with the DGLAP equations for single PDF.

Initial conditions for both DPDF and PDF have to be specified:

\[
D_{f_1f_2}(x_1, x_2, Q_0) \quad D_f(x, Q_0)
\]
Momentum sum rule

- Momentum sum rule for PDF (conserved by DGLAP eqs.)

\[
\sum_f \int_0^1 dx \cdot D_f(x, Q) = 1
\]

- By analogy: momentum sum rule for DPDF

\[
\sum_{f_1} \int_0^{1-x_2} dx_1 \cdot x_1 \frac{D_{f_1 f_2}(x_1, x_2, Q)}{D_{f_2}(x_2, Q)} = (1 - x_2)
\]

- The ratio in red is a conditional probability to find a parton with the momentum fraction \(x_1\) while the second parton fraction \(x_2\) is fixed.

- Momentum sum rule is a relation between DPDF and PDF

\[
\sum_{f_1} \int_0^{1-x_2} dx_1 \cdot x_1 \cdot D_{f_1 f_2}(x_1, x_2, Q) = (1 - x_2) \cdot D_{f_2}(x_2, Q) \quad (1)
\]
Valence number sum rule

- Valence number sum rule for PDF ($N_i =$ no. of valence quarks)

$$ \int_0^1 dx \{ D_{q_i}(x, Q) - D_{\bar{q}_i}(x, Q) \} = N_i $$

- Valence number sum rule for DPDF

$$ I_{q_i f_2} = \int_0^{1-x_2} dx_1 \{ D_{q_i f_2}(x_1, x_2, Q) - D_{\bar{q}_i f_2}(x_1, x_2, Q) \} $$

$$ = \begin{cases} 
N_i \, D_{f_2}(x_2, Q) & \text{for } f_2 \neq q_i, \bar{q}_i \\
(N_i - 1) \, D_{f_2}(x_2, Q) & \text{for } f_2 = q_i \\
(N_i + 1) \, D_{f_2}(x_2, Q) & \text{for } f_2 = \bar{q}_i
\end{cases} \quad (2) $$

- Relations (1) and (2) are conserved by the evolution equations once imposed at some initial scale $Q_0$.  

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Initial conditions

- In practice: initial DPDF are built out of the existing single PDFs, e.g. (Gaunt, Stirling, Korotkikh, Snigirev)

\[
D_{f_1 f_2}(x_1, x_2) = D_{f_1}(x_1) D_{f_2}(x_2) \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n_1}(1 - x_2)^{2+n_2}}
\]

- Symmetric input with respect to the parton interchange

\[
D_{f_1 f_2}(x_1, x_2) = D_{f_2 f_1}(x_2, x_1)
\]

and positive definite.
Sum rules with the symmetric input

\[
\text{Ratio}_{\text{Mom}}(x_2, f_2) = \frac{(1 - x_2) D_{f_2}(x_2)}{\sum_{f_1} \int_0^{1-x_2} dx_1 x_1 D_{f_1 f_2}(x_1, x_2)} = 1
\]

\[
\text{Ratio}_{\text{Val}}(x_2, f_2) = \frac{N_{uf_2} D_{f_2}(x_2)}{\int_0^{1-x_2} dx_1 \{D_{uf_2}(x_1, x_2) - D_{\bar{u}f_2}(x_1, x_2)\}} = 1
\]

Valence quark number sum rule is violated.
How to exactly fulfill the sum rules?

- Asymmetric ansatz to fulfill the momentum sum rule:

\[
D_{f_1f_2}(x_1, x_2) = \frac{1}{1-x_2} D_{f_1}(\frac{x_1}{1-x_2}) \cdot D_{f_2}(x_2)
\]

- Corrections for identical quark flavours/antiflavours to obey the valence number sum rule:

\[
D_{f_if_i}(x_1, x_2) = \frac{1}{1-x_2} \left\{ D_{f_1}(\frac{x_1}{1-x_2}) - \frac{1}{2} \right\} D_{f_1}(x_2)
\]

\[
D_{\bar{f}_if_i}(x_1, x_2) = \frac{1}{1-x_2} \left\{ D_{f_1}(\frac{x_1}{1-x_2}) + \frac{1}{2} \right\} D_{\bar{f}_1}(x_2)
\]

- DPDF for identical flavours, \( D_{f_if_i}(x_1, x_2) \), are not positive definite. This is the price to pay for the construction with single PDFs!
Symmetric vs. asymmetric input for $D_{uu}$

$D_{uu}(x_1, x_2=10^{-3})$

For $Q^2 = 2\text{ GeV}^2$

For $Q^2 = 100\text{ GeV}^2$

Initial conditions for evolution of double parton distributions
Symmetric vs. asymmetric input for $D_{u\bar{u}}$

$$D_{uubar}(x_1, x_2=10^{-3})$$

- $Q^2 = 2 \text{ GeV}^2$
- $Q^2 = 100 \text{ GeV}^2$

Initial conditions for evolution of double parton distributions
Symmetric vs. asymmetric input for $D_{gu}$

$D_{gu}(x_1, x_2=10^{-3})$

- $Q^2 = 2 \text{ GeV}^2$
- $Q^2 = 100 \text{ GeV}^2$

$D_{gu}$

$x_1$

$10^{-2}$ $10^{-1}$

$Q^2 = 2 \text{ GeV}^2$

$Q^2 = 100 \text{ GeV}^2$
If input DPDF are constructed from known single PDF:

|                  | symmetric input | asymmetric input |
|------------------|-----------------|------------------|
| Parton symmetry  | yes             | no               |
| Positivity       | yes             | no               |
| Sum rules        | no              | yes              |

Alternative: specify positive initial DPDF and generate initial PDF using sum rules. Unfortunately, no experimental knowledge on DPDF.

However, for $x_1, x_2 \to 0$ factorized form is a good approximation:

$$D_{f_1f_2}(x_1, x_2, Q) \approx D_{f_1}(x_1, Q) D_{f_2}(x_2, Q)$$
BACKUP
Evolution equations for DPDF

- For unequal hard scales, $Q_1 < Q_2$, two step evolution:

$$D_{f_1 f_2}(x_1, x_2, Q_0, Q_0) \rightarrow D_{f_1 f_2}(x_1, x_2, Q_1, Q_1) \rightarrow D_{f_1 f_2}(x_1^{fixed}, x_2, Q_1, Q_2)$$

- Single PDF evolution, $Q_1 \rightarrow Q_2$, with respect to the second variable $x_2$ in the second step:

$$\partial_{t_2} D_{f_1 f_2}(x_1, x_2, t_1, t_2) = \sum_{f'} \int_0^{1-x_1} du \ K_{f_2 f'}(x_2, u, t) \ D_{f_1 f'}(x_1, u, t, t_2)$$