Conservation, Dissipation, and Ballistics: Mesoscopic Physics beyond the Landauer-Büttiker Theory

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The standard physical model of contemporary mesoscopic noise and transport consists in a phenomenologically based approach, proposed originally by Landauer and since continued and amplified by Büttiker and others. Throughout all the years of its gestation and growth, it is surprising that the Landauer-Büttiker approach to mesoscopics has matured with scant attention to the conservation properties lying at its roots: that is, at the level of actual microscopic principles. We systematically apply the conserving sum rules for the electron gas to clarify this fundamental issue within the standard phenomenology of mesoscopic conduction. Noise, as observed in quantum point contacts, provides the vital clue.

I. INTRODUCTION

A. Background

If someone were to declare that a successful kinetic theory is one that explains experiment without any need to obey Newton’s laws, such an assertion would not be likely to be received unreservedly – even by most philosophers of science. In this paper we ask what it would mean if, in some sense, an analogous proposition were to have been made to a segment of the mesoscopics community, and implicitly accepted. It would be a curious situation indeed; one with intriguing implications for the pursuit of mesoscopic physics and, in particular, for noise theory.

Here, the conservation laws of charge and particle number stand at stage center. The principles of conservation are in every sense as cardinal as Newton’s laws. In transport physics, their validity is neither compliant to any quantitative revision nor contingent upon any set of intuitive preconceptions. Conservation is the inalienable heart of every rational model of conduction. There is no exception.

Let us revisit the essentials of transport in mesoscopic conductors. Typically, structures have lengths comparable to, or smaller than, the mean free paths for electron scattering in the bulk. Charge transport can then be regarded as ballistic (collision-free) within the confines of such a metallic channel. 1

![Figure 1. An ideal, uniform ballistic wire, or quantum point contact. Its diffusive and dissipative leads (S and D, stippled) are permanently neutral and permanently in equilibrium. A paired source and sink of current \( I \) at the boundaries drive the transport. Local charge clouds (shaded), induced by the imposed influx and efflux of \( I \), set up the dipole voltage \( V = E(I)L \) between D and S. The ballistic wire manifests its optimal (Landauer) conductance \( I/V \) if, and only if, the mean free paths for elastic and inelastic scattering are equally matched and straddle the whole wire, from its strongly dissipative source terminal to the corresponding drain. See Section 3 for a detailed proof.

1The term “metallic channel” has the specific sense of a device with a band of extended conducting states that can be freely populated and depopulated by controlling the carrier density. Carriers have free access to and from a set of (large) bounding leads which are themselves strongly metallic.
Figure 1 illustrates this for a so-called “quantum point contact” (QPC). We are dealing with small – potentially quasi-molecular – structures. Consequently they experience a high degree of openness to their macroscopic environment. The restricted dimensionality also accentuates the inter-particle correlations. It is no secret [1,2] that these are the very attributes that hold the hardest set of challenges to be met in developing mesoscopic electronics.

Among the challenges, the foremost is how to reconcile microscopic conservation with both openness at the device interfaces and strong quantum correlations just about everywhere else. Without a firm anchoring in the conservation laws, nothing prevents an unconstrained model from “creating” or “destroying” charges and currents willy-nilly. Do we need such a model?

Even if seemingly able to explain an experiment, a theory that is nonconserving must be regarded as fundamentally incoherent, for we have learned that Nature does not work in this way. That is the case, anyway, in the realm of condensed matter. The absolute centrality of conservation in mesoscopic transport needs no further comment.

### B. Plan of the Paper

The paper is set up as follows. In the next Section we look at the prime relation of mesoscopic transport; that is, the immediate link between finite conductance and finite energy dissipation. This leads into the third Section and our first cardinal result: the strictly kinetic derivation of Landauer’s quantized conductance formula for an ideal metallic channel [2–4], using only the axioms of microscopic theory and free of any recourse to the standard model’s very special assumptions. The message is that a mesoscopic model (one which is faithful to orthodox quantum kinetics) does not need phenomenology for a backbone.

Our result shows that the Landauer-Büttiker (LB) project is already subsumed in a more mature, fundamentally established, and truly first-principles framework: quantum kinetics. This means that the phenomenological assumptions, on which the unique mesoscopic character of LB has been predicated [2], are not foundational.

Later in Sec. 3, then in 4, we go on to demonstrate how the logic of the conservation laws generates a completely natural microscopic description of a major exemplar of mesoscopic fluctuations: QPC noise. We account for data that went unexplained for nearly a decade. In the process it becomes clearer that the standard Landauer-Büttiker approach to fluctuations does not respect conservation. Because of that, it fails to predict the directly observed consequences of conservation.

Finally we sum up the origin, motivation, and cogency of our microscopically conserving analysis. We also retrace the implications for setting up a novel perspective on mesoscopic processes. Interested readers will find an Appendix containing the formal proof that the Landauer-Büttiker noise theory does not satisfy the perfect-screening and compressibility sum rules. These basic rules govern the dynamics of mesoscopic noise, as experimentally demonstrated for QPCs.

### II. THE PHYSICAL ISSUE

At the outset, we remark that our analysis applies specifically to metallic conduction and fluctuations. We examine open mesoscopic structures where the carrier states form a quasi-continuum, are spatially extended, and have considerable overlap. This is territory that the LB approach is supposed to cover extremely well [2]. We are not considering – here – systems in which quite different, nonmetallic mechanisms of transport (e.g. hopping and tunneling) involve electronic orbitals that are discrete, highly localized, and with little overlap.

A simple intuitive understanding of mesoscopic transport and noise was established, in the last fifteen years or so, through the insights of Landauer, Büttiker, Imry, and many similarly inclined contributors [2–5]. (The sheer proliferation of such works precludes citing all but a few outstanding examples.) In that succinct perspective the physical basis of current flow, mesoscopic and otherwise, is identified as a mismatch of carrier density between two or more metallic reservoirs – terminals – across which the QPC or other low-dimensional conductor is connected.

The hallmark of metallic transport in quantum point contacts, the quantization of conductance as “Landauer steps” in units of $2e^2/h \approx 0.078 \text{ mS}$, appears to be adequately explained in terms of coherent transmission of electron waves through a perfectly loss-free barrier. However, a picture of quantum-coherent scattering as simple as this cannot address the central theoretical issue of metallic conduction:

*What causes dissipation in a ballistic quantum point contact?*

Such a question is more than academic. In the near future, reliable and effective nano-electronic design will demand models that are credible; credible not just as physical theory but as engineering practice. Therefore a comprehensive theory has to confront the unavoidable presence, and action, of power dissipation.
For conduction, the central issue is plain. Any finite conductance $G$ must dissipate electrical energy (inelastically) at the rate $P = IV = GV^2$, where $I = GV$ is the current and $V$ the potential difference across the terminals of the driven channel. This is mandated by the underlying physics of charge conservation in externally driven (open) conduction [6,7]; as such it cannot be hostage to any transport philosophy insisting on coherent propagation as the exclusive origin of conductance.

Our question has its definitive answer in many-body quantum kinetics [9], free of all “supplementary hand-waving” [2]. The application of well tested microscopic methods leads not only to conductance quantization precisely by the direct inclusion of inelastic energy loss [8–11], but, as we show in Sec. 3 to follow, it also resolves a long-standing experimental enigma [12] in the noise spectrum of a quantum point contact (QPC) [13]. The same developments foreshadow a systematic pathway to truly predictive design of novel structures.

In their dissipative action, inelastic collisions are beyond the reach of transport models that rely on coherent quantum scattering alone to explain the origin of $G$. Coherence implies elasticity, and elastic scattering is always loss-free: it conserves the energy of the scattered particle. The recent review of mesoscopic phenomenology by Agrait and others [1] expresses a similar conclusion on the insufficiency of coherence-based arguments for actual experimental systems.

There is no choice but to explicitly admit the dissipative mechanisms (e.g. phonon emission) by which the energy gained by carriers, when transported from source to drain, is returned incoherently to the surroundings. A very detailed microscopic demonstration of this has been given by Sorée et al. [8]. Thus, alongside elastic and coherent scattering, inelastic processes are always in place. Together, they fix $G$; yet it is only the energy-dissipating mechanisms that secure thermodynamic stability in steady-state conduction. Coherence on its own cannot bring this about.

A serious deficiency underlies purely elastic approaches to transmission. They cannot handle dissipation. We now review the well-defined microscopic resolution of that problem.

III. THE PHYSICAL SOLUTION

The complete microscopic understanding of the ubiquitous power-loss formula $P = GV^2$ [6,7,15] is couched in terms of the fluctuation-dissipation theorem (details are standard in the kinetic literature [15,16]). It holds for all resistive devices at all scales without exception. The theorem expresses the condition of thermodynamic stability. With it comes the conclusion that [9–11]

- inelasticity is necessary and sufficient to stabilize current flow at finite conductance;
- ballistic quantum point contacts have finite $G \propto 2e^2/h$; therefore
- processes of energy loss are indispensable to any rational theory of ballistic transport.

To allow for the energy dissipation vital to any microscopic description of ballistic transport, we recall that open-boundary conditions imply the intimate coupling of the QPC channel to its interfaces with the reservoirs. The interface regions must be treated as an integral part of the device model. They are the actual sites for strong scattering effects: dissipative many-body events as the current enters and leaves the ballistic channel, and elastic one-body events as the carriers interact with background impurities, the potential barriers that confine and funnel the current, and so on. See Fig. 1 for an illustration.

A key idea in our treatment is to subsume the interfaces within the total kinetic description of the ballistic channel. At the same time, strict charge conservation in an open device requires the direct supply and removal of charge flux by a generator outside the system of interest [6]. With this canonical precondition – that an open channel must be energized by a strictly external driving agent – the current is completely independent of the physics locally governed by the reservoirs.

That fundamental condition sets the quantum kinetic approach totally apart from Landauer-like treatments [2–5]. Instead of the gauge-invariant open-system requirement [6,7] that any current flow is perforce supplied independently

\[P = IV = GV^2\] 

There is an intimate structural correspondence between the dissipation formula $P = IV$, originating from the canonical, many-body Hamiltonian description of a driven conductor [7], and the form of the steady-state solution to the quantum kinetic equation for its carrier distribution. In the latter case, the fluctuation-dissipation theorem follows directly when the limit of a weak driving field is taken. The formalities are in Ref. [11], which also shows the fundamental role played by electron-hole polarization in determining the current-current correlator and the conductance.
from outside, \(^3\) they rely on the intuitive assumption that the current depends, passively, on a notional density “mismatch” between reservoirs. It is nothing other than an *ad hoc* extrapolation, to strongly quantum systems, of purely classical diffusion [4].

### A. Ballistic Conductance

One can straightforwardly write the algebra for the conductance in our model system. A uniform, one-dimensional ballistic QPC, of operational length \(L\), will be associated with two mean free paths determined by \(v_F\), the Fermi velocity of the electrons, and a pair of characteristic scattering times \(\tau_{el}, \tau_{in}\). Thus

\[
\lambda_{el} = v_F \tau_{el}; \quad \lambda_{in} = v_F \tau_{in}. \tag{1}
\]

Respectively, these are the scattering lengths set by the elastic and inelastic processes active at both interfaces. The device (i.e. the QPC with its interfaces) has a conductive core that is collisionless. It follows that

\[
\lambda_{el} = L = \lambda_{in}. \tag{2}
\]

Finally, the channel’s conductance is given by the familiar formula (deducible straight from Kubo’s microscopic prescription [9,15])

\[
G = \frac{n e^2 \tau_{tot}}{m^* L} = \frac{2k_F}{\pi} \frac{e^2}{m^* L} \left( \frac{\tau_{in}\tau_{el}}{\tau_{el} + \tau_{in}} \right); \tag{3}
\]

the effective mass of the carriers is \(m^*\). The first factor of the rightmost expression for \(G\) has the density \(n\) recast in terms of the Fermi momentum \(k_F\); in the final factor, Matthiessen’s rule \(\tau_{tot}^{-1} = \tau_{el}^{-1} + \tau_{in}^{-1}\) fixes the total scattering rate.

On applying Equations (1)–(3), the conductance reduces to

\[
G = 2 \frac{e^2 h k_F}{\pi h m^* L} \left( \frac{(L/v_F)^2}{2L/v_F} \right) = \frac{2e^2}{h} \equiv G_0. \tag{4}
\]

This is no more – and no less – than the Landauer conductance of a single, one-dimensional, ideal metallic channel.

What is the most important lesson that one can draw from this microscopic result? It is that Occam’s Razor can always be used to advantage. Of two contending explanations of a phenomenon, choose the one with fewer hypotheses.

The microscopic interpretation of the Landauer formula *does not need* any of the adventitious assumptions otherwise invoked to explain conductance quantization [2–5]. The tight derivation of Eq. (4) follows from completely standard quantum kinetics for open systems, within which total phase coherence is not even a physical possibility let alone a theoretical necessity. Nor are the remaining, singular assumptions of the Landauer-Büttiker approach relevant to actual ballistic transport, any more than coherence is. See Ref. [9] for further discussion.

In Figure 2 we plot the results of our model for a QPC made up of two one-dimensional conduction bands at energies 5\(k_B T\) and 17\(k_B T\), in thermal units at temperature \(T\). We use the natural extension of Eq. (4) to cases where one or more channels may be open to conduction [9], depending on \(T\) and the size of the chemical potential \(\mu\). As the role of inelastic scattering is enhanced \((\tau_{in} < \tau_{el})\) the conductance deviates from the ideally ballistic Landauer limit.

At the core of the quantum-kinetic Landauer formula is the clear and pivotal influence of inelastic energy loss. It is one of the underpinnings of quantum transport. Charge conservation, the complementary underpinning, is guaranteed by the use of microscopically consistent open-boundary conditions at the interfaces [6,7].

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\(^3\)Transport at fixed current and at fixed voltage are completely equivalent. The electrodynamics of conduction in an open-boundary device driven by an external current generator [6] is isomorphic to that of a closed conducting loop, comprising the “open” device and a voltage source (a battery) [7]. Discussion of the underlying gauge invariance is beyond our scope here. We urge the interested reader to consult, in parallel, both Sols [6] and Magnus & Schoenmaker [7].
Figure 2. Conductance quantization in a two-band ballistic point contact, as a function of chemical potential $\mu$, calculated from Eq. (3) within our conserving kinetic theory; see Ref. [9]. Full curve: ideal ballistic channels. Broken curves: non-ideal behavior increases with the onset of inelastic phonon emission inside the contact, which preserves the Landauer steps while suppressing their height.

The twin requirements of open-system dissipation and open-system gauge invariance are neither adequately considered nor, as it turns out, respected by any of the phenomenological derivations of Eq. (4) [2–5]. If this is so at the plain level of conduction, such approaches will be far more problematic for current fluctuations, a decisive test of microscopic transport theories. We discuss that very point now.

B. Nonequilibrium Noise

The noise response of a quantum point contact is a fascinating aspect of mesoscopic transport. It is a much more demanding one both experimentally and theoretically. In 1995, a landmark measurement of nonequilibrium noise was performed by the Weizmann group [12], which yielded a tantalizing result. While conventional models [5] predict no fine structure whatsoever in the noise of a QPC driven at constant current levels, the measured data (Fig. 3, left-hand panel) show a prominent and robust set of peaks at threshold, just where the carrier density in the QPC starts to enter the degenerate metallic regime.

Figure 3. Nonequilibrium low-frequency current noise of a QPC at constant source-drain current, as a function of gate bias. Left: the data from Reznikov et al., Ref. [12], display a dramatic series of peaks coincident with the energy threshold of the lowest subband, where carriers first populate the states for conduction. Right: calculation from Green et al., Ref. [13]. In each case the dotted line at 100nA shows the shot-noise prediction of the Landauer-Büttiker (LB) formula [5], using as its required phenomenological inputs the measured and calculated data for $G$. The LB formula falls well short, qualitatively and quantitatively; compare the dash-dotted lines also at $I = 100nA$. 

5
We have accounted for the Reznikov et al. measurements within a strictly conserving, kinetic formulation of nonequilibrium noise in a quantum point contact [13]. Figure 3 displays, side by side with the experimental data, our computation of excess QPC noise under the same operating conditions.

In contrast to the outcome of popular mesoscopic phenomenology [5] one notes the close affinity between the measurements and the quantum kinetic calculation, as the carrier density is swept across the first conduction-band threshold, where the conductance exhibits its lowest step. At fixed values of source-drain current, the accepted noise account [5] predicts no peaks at all, but rather a featureless monotonic drop in the noise strength as the carrier density passes through threshold.

Remarkable as they are to this day, the Weizmann measurements have remained absolutely unexplained for a decade. Moreover their obvious message, namely that established theories are inadequate to the experiment, was simply ignored by the mesoscopic community. To the contrary, the folklore seemed to spread that Ref. [12] amply confirmed accepted understandings [5].

IV. THE POWER OF THE CONSERVATION LAWS

The key to all quantum kinetic descriptions of conductance is the fluctuation-dissipation theorem [16], whose practical implementation is Eq. (3) (where the overall relaxation time \( \tau_{\text{tot}} \) encodes all the electron-fluctuation dynamics via the Kubo formula [15]). This universal relation is one of the electron-gas sum rules [17]. In this instance, it expresses the conservation of energy, dissipatively transferred from an external source to the thermal surroundings, for any process that involves resistive transport – including that in a ballistic quantum point contact.

A second, and equally fundamental, sum rule concerns the compressibility of an electron fluid in a conductive channel. This sum rule turns out to have an immediate link with the nonequilibrium noise behavior reviewed above. Here we give a brief explanation of that crucial link. For all of the formal details, see Refs. [13,14,18,19].

Recall that the carriers in a metallic quantum point contact are stabilized by the presence of the large leads, which pin the electron density to fixed values on the outer boundaries of the interfaces (recall too that the interfaces and the channel together define the open system). No matter what the transport processes within the QPC may be, or how extreme, the system’s global neutrality is guaranteed by the stability of the large and charge-neutral reservoirs. It follows that the mean total number \( N \) of active electrons in the device remains independent of any current that is forced through the channel, for \( N \) is always neutralized by the rigid ionic background in its neighborhood as well as the stabilizing leads. The presence of the latter means that any and all fringing fields are screened out beyond the device boundaries; hence the global neutrality.

It is readily seen that, if the mean occupation number \( N(\mu) \) is independent of any external current as a result of global neutrality (i.e. the perfect-screening sum rule [17]), so is the total mean-square number fluctuation \( \Delta N = k_B T \partial N/\partial \mu \) [18]. The carriers’ compressibility in the QPC is specified in terms of \( N \) and \( \Delta N \) by [17]

\[
\kappa = \kappa_{\text{cl}} \frac{\Delta N}{N} \quad \text{where} \quad \kappa_{\text{cl}} \equiv \frac{L}{N k_B T},
\]

which, in consequence, remains strictly unaffected by any transport process.

This a surprising corollary of global neutrality. It asserts that, in an open conductor, the system’s equilibrium compressibility completely determines the compressibility of the electrons even away from equilibrium, regardless of the strength of the driving field.

The compressibility sum rule expresses the unconditional conservation of carriers in a nonequilibrium conductor. Previously unexamined in mesoscopics, this principle has an immediate importance and applicability. It means that a new, independent, and strong criterion has become available to test the consistency of any model for ballistic transport.

How does \( \kappa \) determine the noise in a QPC? The strength of the current fluctuations is, in broad physical terms, a convolution of two competing phenomena:

\[
S(I,t) \sim \langle I(t)I(0) \rangle \frac{\Delta N}{N} = \langle I(t)I(0) \rangle \frac{\kappa}{\kappa_{\text{cl}}},
\]

\[4\] Independently of Sols’ theorem [6] and its consequences, the importance of global neutrality and metallic screening by the reservoirs was driven home long ago, in great microscopic detail, by Fenton [20]. His analysis shows that it is confinement of the electric field within the QPC (via perfect screening), and not perfect elastic transmission, that is a crucial prerequisite for conductance quantization.
The leading factor represents the self-correlation of the dynamical electron current $I(t)$ evaluated as a trace over the nonequilibrium distribution of excited electrons throughout the device region. The second factor – evidently a basic characteristic of the electron gas in the channel – is independent of $I$, meaning that the invariant compressibility, Eq. (5), dictates the overall scale of the noise spectrum.

Let us analyze the noise curves of Fig. 3 in light of this microscopic result [13,14].

- At large negative bias $V_g$, the channel is depleted. The remnant carriers are classical, so $\Delta N/N \to 1$. The noise is then dominated by strong inelastic processes at high driving fields, represented within the structure $\langle I(t)I(0) \rangle$.

- In the opposite bias limit (right-hand sector of each panel in Fig. 3), the channel is richly populated and thus highly degenerate, with a large Fermi energy $E_F$. Then $\Delta N/N \to k_B T/2E_F \ll 1$. The noise spectrum falls off according to Eq. (6), since the current-correlation factor – now comfortably within the regime of ballistic operation – reaches a fixed ideal value.

- In the mid-range of $V_g$ there is a point where the carriers’ chemical potential matches the energy threshold for populating the first conduction sub-band. Here there is robust competition. As inelastic scattering becomes less effective, the correlation $\langle I(t)I(0) \rangle$ grows while the onset of degeneracy starts to cut down the other factor, the compressibility. Where this interplay is strongest, peaks appear.

The outworking of the compressibility rule is clear: it is, quite directly, the “inexplicable” emergence of the noise peak structures. The striking case of QPC noise gives an insight into the central importance of the conserving sum rules in the physics of metallic transport at meso- and nanoscopic dimensions.

As detailed in the Appendix, the more phenomenological treatments of noise fail to address the explicit action of microscopic conservation in ballistic phenomena, to the point that control over the sum-rule violations does not exist for them. Therefore they cannot offer a rational understanding of the real nature of ballistic conduction – much less its noise.

V. SUMMARY

The kinetic analysis of transport provides a detailed, fully microscopic account of conductance and noise. This applies to the specific case of quantum point contacts. We accurately reproduce the entire current response of mesoscopic conductors, in particular conductance quantization. The keys to this newly fruitful picture are open-system charge conservation and the physical reality of dissipative scattering.

Our unified theory yields a comprehensive understanding of the nonequilibrium fluctuations fundamental to a QPC. We have successfully tested this comprehension by fully explaining the long-standing puzzle posed by the noise measurements of Reznikov et al. [12]. The theoretical impact of noise and fluctuation physics is that it carries much more information on the internal dynamics of mesoscopic systems – knowledge that is not accessible through the $I$-$V$ characteristics alone.

The capacity to advance a self-contained microscopic explanation of processes in mesoscopic transport underwrites a matching ability to build up a program for device design that is inherently rational. This goes deeper than assembling, ad hoc, any clutch of ideas that happen to fall easily to hand. The reward for more workmanlike efforts is a huge potential to improve the practical engineering of novel generations of devices. These can – and evidently should – be built on a sound and non-speculative knowledge base. Many-body physics is at the heart of the program.

A well grounded mesoscopic theory, at its simplest, will not be simplistic. To go so far as to forget or even ignore the conservation laws, merely to fit subjective notions of “simplicity”, would scarcely be science in the tradition of Boltzmann, Sommerfeld, and Landau. At any rate, it would not do for serious investigations of co-ordinated behavior in matter at small scales. For, a nonconserving model is incoherent; and an incoherent model is unusable. Nor does the continued shirking of honest and public debate on the issue serve the credibility of mesoscopic theory.

It is the universal principles of conservation that are the gatekeepers to a real mesoscopic understanding. We have revisited the conserving sum rules in their specific application to small-scale metallic transport. Given its prime role in carrier dynamics, a truly conservative quantum kinetic approach holds opportunities for both traditional many-particle theory applied to mesoscopics, and for expanding noise research. More than ever, the spotlight falls on the fluctuation properties of active mesoscopic devices. Noise physics is vital not only for its own sake, but as a robust and trustworthy diagnostic means for electronic concepts to come.

Now is a good time indeed to introduce genuine many-body methods into mesoscopics: a field in no small need of a microscopically cohesive, and more far-reaching, vision. Some of the tools to make a start on this are freely available [6–11,13–20].
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APPENDIX A: SUM RULES IN THE LANDAUER-BÜTTIKER THEORY

“To lose one parent, Mr Worthing, may be regarded as a misfortune; to lose both looks like carelessness.” Oscar Wilde

In this Appendix we provide, for the record, the formal demonstration that the Landauer-Büttiker model violates both crucial sum rules for perfect screening (charge conservation) and compressibility (particle conservation) [17–19]. We are not by any means the first to examine the defective gauge invariance of this phenomenology. The problem is already well understood and discussed, for example, by Blanter and Büttiker, Ref. [5]. See their Eq. (51) et seq.

1. Perfect Screening Sum Rule

As we saw in Sec. 4, the overall neutrality of a quantum point contact (indeed, of any bounded conductor) is maintained by the strong screening property of its large conductive leads. Suppose the QPC sustains a carrier density profile \( n(x) \). When \( n \) is integrated for the total carrier number, the contact’s global neutrality implies

\[
\int_0^L n(x)dx \equiv N = \text{constant}; \quad \text{that is, } \frac{\partial N}{\partial I} = 0 \quad \text{for all } I.
\]

Regardless of external current \( I \) forced through the structure, and the voltage \( V \) measured across it in response, the total of mobile charges dwelling in the QPC (including the active interface regions themselves) cannot depend on \( I \) or \( V \). The entire structure, though open, stays unconditionally neutral. This is the practical meaning of perfect screening.

Now consider the same situation from the perspective of Landauer and Büttiker [2–5]. In place of the gauge-invariant (and thus microscopically correct) prescription of open, externally driven transport that leads to perfect screening [6,7,19], we posit that the current in a narrow conducting wire is sustained purely by a difference of chemical potentials. \(^5\) between an upstream electron reservoir (chemical potential \( \mu = \mu_{\text{up}} \)) and a different downstream one \( (\mu = \mu_{\text{dn}} = \mu_{\text{up}} - eV) \).

The density along the conductor would then be some function \( n(\mu_{\text{up}} - eV(x)) \) that took the boundary values \( n(\mu_{\text{up}}) \) and \( n(\mu_{\text{dn}}) \) at the ends of the sample. According to such an account, in the linear-response limit the total number of carriers in the active structure of length \( L \) would change, with the field, by

\[
N(V) - N(0) = \int_0^L [n(\mu_{\text{up}} - eV(x)) - n(\mu_{\text{up}})]dx
\]

\[
\approx -\Delta N(0)eV \frac{2k_BT}{e} \neq 0,
\]

a result manifestly counter to the perfect-screening sum rule, Eq. (A1), and thus to gauge invariance.

\(^5\)One of the central phenomenological assumptions of LB is that there must be a difference of chemical potentials between the leads of a driven device. Otherwise, in LB, the current cannot flow “diffusively” [4] This phenomenology has absolutely no basis in the quantum kinetic description of mesoscopic transport. For, it would mean that \( \mu \) – a thermodynamic property which is always locally invariant and guarantees the individual stability of each lead’s neutral state – is subject to arbitrary change. In reality it is the external driving source, and not any “difference” in \( \mu \), that directly sustains the current. Thus in kinetic theory the leads’ chemical potentials are left alone to do their proper job: to stabilize the system and maintain its overall neutrality.
We remind ourselves that the corollary to perfect screening is the nonequilibrium compressibility sum rule (recall Eq. (5)):

\[
\frac{\kappa}{\kappa_{cl}} \equiv \frac{k_B T}{N} \frac{\partial N}{\partial \mu} = \text{const.; that is, } \frac{\partial \kappa}{\partial I} = 0 \text{ for all } I.
\] (A3)

Note in particular that \(\kappa\) is calculable as a strictly equilibrium, and not a linear-response, quantity [17]. It cannot depend on any transport parameter.

How does compressibility relate to the so-called Landauer-Büttiker noise formula? The formula emerges from quite a variety of different arguments, which nevertheless all converge to the same final expression (specializing to one subband will not alter our own argument):

\[
S(V) = 4G_0 k_B T \left[ T^2(v_F) + T(v_F) \left(1 - T(v_F)\right) \frac{\mu_{up} - \mu_{dn}}{2k_BT} \coth \left(\frac{\mu_{up} - \mu_{dn}}{2k_BT}\right) \right].
\] (A4)

The coefficient \(T(v_F)\) for coherent transmission is the probability that an incoming carrier at Fermi velocity \(v_F\) will propagate coherently from source to drain, right through the subband in the QPC.

The noise formula is the sum of separate contributions [5]. With these same components, one may also reconstruct the LB version of the mean-square number fluctuation. After constructing the would-be “single-carrier” fluctuation, \(^6\) say \(dN\) (refer to Martin and Landauer [21] for a clear account of this methodology), we obtain the LB mean-square expectation \(\Delta N\) over the quantum point contact:

\[
\Delta N \equiv \langle (dN)^2 - \langle dN \rangle^2 \rangle = L \frac{n(\mu_{up})}{2E_F} \left[ T^2 k_B T + T \left(1 - T\right) \frac{\mu_{up} - \mu_{dn}}{2k_BT} \coth \left(\frac{\mu_{up} - \mu_{dn}}{2k_BT}\right) \right],
\] (A5)

where \(n(\mu_{up})\) is the density in the uniform wire at equilibrium. Note that LB models work only in the degenerate limit of large Fermi energy: \(E_F \gg k_B T\).

Now let the driving field go to zero; the term that depends on \(eV = \mu_{up} - \mu_{dn}\) goes to \(k_B T\). Using \(Ln(\mu) = N\), we are left with the ratio

\[
\frac{\Delta N}{N} = T \left[ \frac{\Delta N}{N} \right]_{eq}, \text{ where } \left[ \frac{\Delta N}{N} \right]_{eq} \equiv \frac{k_B T}{2E_F} = \frac{\kappa}{\kappa_{cl}}.
\] (A6)

Equation (A6) very clearly fails to recover the physical compressibility, Eq. (A3), \(\text{even at equilibrium}\). It depends manifestly on the transport parameter \(T\).

Momentary reflection shows that Eq. (A5), the LB form for \(\Delta N\) – intimately related to the noise formula – depends explicitly on the voltage. Therefore it visits further violence upon the invariance of \(\kappa\) under transport.

It follows that the Landauer-Büttiker noise formula violates number conservation. The formula is inconsistent with the compressibility sum rule not only at equilibrium, which by itself is fatal, but in every transport situation.

\(^6\)In a charged Fermi gas the fluctuation that should be averaged for \(\Delta N\) is an \textit{electron-hole pair} which is correlated, internally and irreducibly, via conservation of energy, momentum – and charge [17]. It \textit{cannot} be expressed as a product of two stochastically independent single-particle terms.

\(^7\)Since in LB the active states are limited to lie within a (thin) shell \(k_B T\) centered on the Fermi surface, the velocity of all carriers contributing to \(S(V)\) always has magnitude \(\langle |v| \rangle = v_F \gg \langle v \rangle\). Therefore the mean-square LB fluctuation in one dimension simply scales as

\[
\Delta N \propto \frac{(I^2) - \langle I \rangle^2}{e^2 (\langle v^2 \rangle - \langle v \rangle^2)} \propto \frac{S(V)}{e^2 v_F^2}.
\]
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