Simulating Supersonic Turbulence in Galaxy Outflows

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ABSTRACT

We present three-dimensional, adaptive mesh simulations of dwarf galaxy outflows driven by supersonic turbulence. Here we develop a subgrid model to track not only the thermal and bulk velocities of the gas, but also its turbulent velocities and length scales. This allows us to deposit energy from supernovae directly into supersonic turbulence, which acts on scales much larger than a particle mean free path, but much smaller than resolved large-scale flows. Unlike previous approaches, we are able to simulate a starbursting galaxy modeled after NGC 1569, with realistic radiative cooling throughout the simulation. Pockets of hot, diffuse gas around individual OB associations sweep up thick shells of material that persist for long times due to the cooling instability. The overlapping of high-pressure, rarefied regions leads to a collective central outflow that escapes the galaxy by eating away at the exterior gas through turbulent mixing, rather than gathering it into a thin, unstable shell. Supersonic, turbulent gas naturally avoids dense regions where turbulence decays quickly and cooling times are short, and this further enhances density contrasts throughout the galaxy—leading to a complex, chaotic distribution of bubbles, loops and filaments as observed in NGC 1569 and other outflowing starbursts.

Key words: galaxies:starburst–hydrodynamics–galaxies:dwarf

1 INTRODUCTION

Theoretical work has long shown that supernovae in low-mass galaxies should produce energetic outflows that heat and enrich their environments (Larson 1974; Dekel & Silk 1986; Vader 1986). Since then such outflows have been observed in starbursting galaxies of all masses (e.g. Axon & Taylor 1978; Heckman 1990; Brüggen & Martin 1998; Martin 1999; Heckman et al. 2000; Schwartz & Martin 2004; Veilleux et al. 2005) and at all cosmological epochs (e.g. Frax et al. 1997; Pettini et al. 1998; 2001; Frye, Broadhurst, & Benitez 2002; Rupke et al. 2005). The existence of these ubiquitous galaxy outflows has several important implications for galaxy formation. The ejection of heavy elements has been invoked to explain the strong correlation between mass and metallicity observed in low-mass galaxies (e.g. Dekel & Silk 1986; Richer & McCall 1995; Mateo 1998; Thacker et al. 2000; Tremonti et al. 2004; Erb et al. 2006; Kewley & Ellison 2008). The suppression of gas accretion onto starbursting galaxies (Benson et al. 2003) and onto neighbouring density perturbations (Scannapieco et al. 2000; Scannapieco et al. 2002) has been shown to be crucial to reconcile the small number of observed dwarf galaxies with the large number of low mass dark-matter halos in the favored cosmological model (e.g. Somerville & Primack 1999; Cole et al. 2000; Benson et al. 2003; Dekel & Woo 2003). The ratio of the baryonic mass to the gravitating mass of galaxies has been found to be several times less than the cosmic ratio (Hoekstra et al. 2005; Mandelbaum 2006), meaning that either the baryons never fell into galaxies or that powerful galactic winds removed them. And, widespread galaxy outflows have proven to be essential to understanding the history of the intergalactic medium (IGM), which is observed to be widely enriched with heavy elements (Tytler et al. 1995; Songaila & Cowie 1996; Rauch et al. 1997; Chen et al. 2001; Simcoe et al. 2002; Schaye et al. 2003; Aracil et al. 2004; Adelberger et al. 2005; Scannapieco et al. 2006), that drastically change its cooling properties (e.g. Sutherland & Dopita 1993; Wiersma et al. 2008).

It has also become clear that most of the outflows that play a key role in each of these processes are driven by core-collapse SN ejecta and winds from massive stars. These create hot, metal-enriched bubbles that expand into the interstellar medium (ISM) and eventually break out of the host galaxy in the form of bipolar outflows that can reach velocities of hundreds of km/s (Veilleux et al. 2005). During this process the bubbles sweep up cooler ambient gas that can also be blown out of the galactic disk. In fact, all cur-
rent velocity measurements made in starburst galaxies are of the entrained cooler material, which is measured from UV or optical emission or via absorption lines (e.g. Rupke et al. 2005; Martin 2005). However, theoretical models predict that around 90% of the energy and metal content of the winds exist in the hot ($T \gtrsim 10^6$ K) phase, which has only recently been detected in X-ray emission (Strickland & Heckman 2007).

The fact that the majority of the most important material remains invisible has led to a wide range of assumptions regarding the efficiency with which starburst-driven winds can eject metal-enriched gas. Some authors claim that only winds from dwarf galaxies can reach the IGM (Ferrara & Tolstoy 2000) while other claim that winds can also escape from massive galaxies (Strickland et al. 2004). Compounding this observational uncertainty is the theoretical problem that the starforming disk itself is not well modelled as a single-temperature medium in hydrostatic balance. Rather the gas is both constantly cooling and condensing into molecular clouds and being stirred and heated by ionization fronts, stellar winds, and supernovae (e.g. McKee & Ostriker 1977). This combination of extremely short cooling times and constant driving leads to a supersonic medium in which the turbulent velocities exceed the thermal velocities, and turbulent eddies act to support the disk even as they compress a fraction of gas, driving star formation (e.g. Mac Low & Klessen 2004).

To approximate this configuration, previous simulations have been forced to both artificially increase the mass-averaged ISM temperature and suppress its cooling through a variety of relatively crude techniques. The two-dimensional outflow simulations in Mac Low et al. (1989) and Mac Low & Ferrara (1999), for example, included an empirical heating function that was tuned to balance cooling in their initial configuration, but taken to be linearly proportional to the density, such that it was overwhelmed by cooling within dense regions that developed during the simulation. Strickland & Stevens (2000) following Tomisaka & Bregman (1993), Tenorio-Tagle & Muñoz-Tuñón (1998), and Suchkov et al. (1994) imposed a minimum temperature of $6.5 \times 10^3$K in their two-dimensional simulations to account for turbulent support of the disk. D’Ercole & Brighenti (1999) imposed a temperature floor in their simulations, equal to the $4.5 \times 10^3$K temperature of the initial ISM, and Fujita et al. (2003; 2004) employed a similar hard cutoff at $10^4$K. Mori et al. (2002) studied supernova feedback with full atomic cooling in a spherical “galactic halo” system with a $2 \times 10^4$K virial temperature that was low-enough that catastrophic cooling of the initial was not a problem. Finally Cooper et al. (2008) carried out simulations with an inhomogeneous ISM model made up of dense clouds of $T \lesssim 3 \times 10^4$K material surrounded by much more tenuous $5 \times 10^3$K, “halo gas” such that very little gas in their simulations was located near the peak of the cooling function.

While the extremely short cooling times within the ISM make it impossible to model supernova feedback by simply adding thermal energy to the gas, at the same time, the lack of feedback between the gas and the disk leads to delayed cooling (Gerritsen & Icke 1997), to implementing momentum kicks rather than heating (Navarro & White 1993; Mihos & Hernquist 1994; Scannapieco et al. 2001; Springel & Hernquist 2003). These theoretical uncertainties have also lead to suggestions that direct driving by supernovae and stellar winds (e.g. Silk 1997) may not be effective at all, but rather the primary driver might involve additional physics such as radiation pressure on dust (Thompson et al. 2005), or non-thermal pressure caused by cosmic rays (Socrates et al. 2008). In fact some of the scaling relations from such alternative models have been shown to reproduce many of the observed trends seen in the intergalactic medium (Oppenheimer & Davé 2006).

In this paper, we present simulations of starburst-driven outflows from a dwarf galaxy using an entirely new approach. Building on a method developed by Dimonte & Tipton (2006), we develop a subgrid turbulence model that accounts both for the turbulent support of the disk and the extra turbulent energy input that drives a global outflow. Properly accounting for turbulence leads to a model for the galaxy that has a realistic temperature and pressure distribution, while at the same time including cooling throughout the simulation. Furthermore, adding supernova input as turbulent energy lets us include this contribution without tracking supernova remnants directly or adopting arbitrary approximations to avoid the energy from being immediately radiated away. This then allows us to make observational predictions for the structure of the highly-disturbed ISM as well as assess the role of turbulent mixing on the escape fractions of gas, kinetic energy, and metals.

The structure of the paper is as follows: In §2 we describe our galaxy model, feedback model, and subgrid model for supersonic turbulence. Whenever possible we tune our parameters to approximate the starforming dwarf galaxy NGC 1569, whose outflow has been well-studied at a variety of wavelengths. In §3 we present the results of our simulations, examine their dependencies on run parameters, and assess their observational consequences. Conclusions are given in §4.

2 METHOD

2.1 Simulation and Model Galaxy

All simulations were performed with FLASH version 3.0, a multidimensional adaptive mesh refinement hydrodynamics code (Fryxell et al. 2000) that solves the Riemann problem on a Cartesian grid using a directionally-split Piecewise-Parabolic Method (PPM) solver (Colella & Woodward 1984; Colella & Glaz 1985; Fryxell, Müller, & Arnett 1989). In all runs we simulated a galaxy made up of a gas+stellar disk, contained within a dark matter halo. As a prototypical dwarf starburst, we chose parameters to approximate the nearby galaxy NGC 1569, which has been well observed and analyzed in a wide variety of wavelengths (e.g. Rea 1980; Israel 1988; González Delgado et al. 1997; Martin 1998; Greggio et al. 1998; Heckman et al. 2001; Stil & Israel 2002; Martin, Kobulnicky, & Heckman 2002) Consistent with observations of this and other starbursting galaxies, we considered a gas distribution with nearly exponential radial and vertical profiles (e.g. Barazza et al. 2006). However, as in
Roediger & Brüggen (2006) we softened the distribution in both directions, in order to prevent steep density gradients in the galactic plane and centre. This gave a gas distribution of

$$\rho(r, z) = \frac{M_{\text{gas}}}{2 \pi a_{\text{gas}} b_{\text{gas}}} \frac{\text{sech}(\frac{r}{a_{\text{gas}}}) \text{sech}(\frac{|z|}{b_{\text{gas}}})}{2\zeta \pi/2},$$

where \((r, z)\) are the radius and distance from the plane in galactic cylindrical coordinates, \(M_{\text{gas}}\) is the total gas mass, the vertical scale lengths are \(a_{\text{gas}}\) and \(b_{\text{gas}}\), respectively, and \(\zeta = 0.9159\) is Catalan’s constant. For \(r \gtrsim a_{\text{gas}}\) and \(|z| \gtrsim b_{\text{gas}}\), this density distribution converges towards the usual exponential disk

$$\rho(r, z) = \frac{M_{\text{gas}}}{2 \pi a_{\text{gas}} b_{\text{gas}}} \exp(-r/a_{\text{gas}}) \exp(-|z|/b_{\text{gas}}),$$

and we fix \(M_{\text{gas}} = 2 \times 10^8 M_\odot\) (Israel 1988), \(a_{\text{gas}} = 0.7\) kpc, and \(b_{\text{gas}} = 0.2\) kpc (Reakes 1980).

We did not calculate the self-gravity of the gas explicitly, but rather as in Roediger & Brüggen (2006) we modeled the gravitational potential of the gas and stars as a Plummer-Kuzmin disk (Miyamoto & Nagai 1975; Binney & Tremaine 1987), which approximates the exponential distribution in disk galaxies, but is much more manageable analytically. The gravitational potential in this case is

$$\Phi_{\text{disk}}(r) = -\frac{GM_{\text{disk}}}{\sqrt{r^2 + [a_{\text{disk}} + (z^2 + b_{\text{disk}}^2)^{1/2}]^2}},$$

where \(a_{\text{disk}} = a_{\text{gas}}, b_{\text{disk}} = b_{\text{gas}},\) and \(M_{\text{disk}} = 3 \times 10^8 M_\odot\) are the radial scale length vertical scale length, and total mass, respectively, and \(G\) is the gravitational constant. We also added a second contribution to this potential due to the dark matter halo in which the galaxy is contained. In this case we assumed a Burkert (1995) model (see also Mori & Burkert 2000), which is given by

$$\Phi_{\text{DM}}(R) = -\pi G \rho_{\text{DM}} R^2 \{ -2 \left( 1 + \frac{R}{R_0} \right) \arctan \left( \frac{R}{R_0} \right) \}
+ 2 \left( 1 + \frac{R}{R_0} \right) \ln \left( 1 + \frac{R}{R_0} \right) - \left( 1 - \frac{R_0}{R} \right) \ln \left( 1 + \left( \frac{R_0}{R} \right)^2 \right) + \pi \},$$

where \(R\) is the radius from the centre of the galaxy in spherical coordinates, \(R_0\) is the core radius of the halo, and \(\rho_{\text{DM}} = 3.08 \times 10^{-24} \text{g cm}^{-3} (R_0/\text{kpc})^{-2/3}\) is the central density of the halo. For this potential, the maximum circular velocity of the halo as a function of \(R_0\) is

$$v_{\text{c,max}} = 23.1 \text{ km s}^{-1} \left( \frac{R_0}{\text{kpc}} \right)^{2/3},$$

and for our model dwarf we assumed \(v_{\text{c,max}} = 35 \text{ km/s as in NGC 1569 (Martin 1998).}\)

Outside of the disk, the gas was assumed to consist of a uniform medium, with a mean density of \(\rho_{\text{ambient}} = 10^{-28} \text{ g cm}^{-3}\), which is \(\approx 200\) times the mean \(z = 0\) cosmological density. This gas was taken to be non-rotating, and in hydrostatic balance with the assumed gravitational potential asymptotically approaching \(T_{\text{ambient}} = 2 \times 10^5 \text{K}\) at large distances. Finally, we assumed that metals were smoothly distributed within the galaxy, such that all cells within the galaxy were initially at a fixed metallicity, \(Z_{\text{init}} = 0.25Z_\odot\) (González Delgado et al. 1997). For simplicity, we defined the boundary of the galaxy at \(\rho = 10 \rho_{\text{ambient}},\) taking \(Z = 0.01Z_\odot\) outside of this region.

All our simulations were performed in a three-dimensional 25 kpc \(\times\) 25 kpc \(\times\) 30 kpc region, with outflow boundaries on all sides. For our grid, we chose a block size of \(8^3\) zones and an unrefined root grid with \(10 \times 10 \times 12\) blocks, for a native resolution of 313 pc. The refinement criteria were the standard density and pressure criteria, and for our fiducial runs we allowed for 3 levels of refinement beyond the base grid, corresponding to an effective resolution of \(640 \times 640 \times 728\) zones, 39 parsecs on a side.

Having fixed the density distribution in the disk, its pressure, turbulence, and temperature distribution were set such that: (i) hydrostatic balance was maintained in the direction perpendicular to the disk plane; and (ii) throughout the galaxy \(T \leq 10^5\) K as atomic cooling quickly radiates thermal energy away above this temperature. In radial direction, the circular velocity was set so that the centrifugal force balanced the gravitational force and pressure gradients. Figure 1 shows the resulting radial and vertical profiles of density, pressure and circular velocity for our fiducial dwarf-starburst model. The choice of parameters for this model are summarized in Table I.
Table 1. Fixed model parameters.

| Component | Parameter | Value |
|-----------|-----------|-------|
| gas       | \(a_{gas}\) | 0.7 kpc |
|           | \(b_{gas}\) | 0.2 kpc |
|           | \(M_{gas}\) | \(2 \times 10^3 \, M_\odot\) |
|           | SFR       | 0.25 \(M_\odot/yr\) |
|           | \(Z_{gas}\) | 0.25 \(Z_\odot\) |
| gravitational potential | \(a_{disk}\) | 0.7 kpc |
|           | \(b_{disk}\) | 0.2 kpc |
|           | \(M_{disk}\) | \(3 \times 10^8 \, M_\odot\) |
| DM halo   | \(\tau_{DM}\) | 2 kpc |
|           | \(v_c\) | 35 km/s |

2.2 Star Formation and Feedback

For each galaxy, we computed a star-formation rate per unit area, \(\Sigma_{SFR}\) from the surface density of gas, \(\Sigma_{gas}\) according to the Kennicutt-Schmidt law

\[
\Sigma_{SFR} = 2.5 \times 10^{-4} \frac{M_\odot}{yr \, kpc^2} \left( \frac{\Sigma_{gas}}{10^6 M_\odot kpc^{-2}} \right)^{1.5},
\]

which provides an accurate fit over \(\approx 6\) orders of magnitude in \(\Sigma_{gas}\) (Schmidt 1959; Kennicutt 1998). In this case the total star formation rate is given by

\[
SFR = 6.3 \times 10^{-5} \frac{M_\odot}{yr} \left( \frac{M_{gas}}{10^6 M_\odot} \right)^{1.5} \left( \frac{a_{gas}}{10^6 M_\odot kpc^{-2}} \right)^{-1},
\]

which we assumed to last for 50 Myrs in all of our simulations. For our choice of parameters this gave an overall star-formation rate of 0.25 \(M_\odot \, yr^{-1}\), which is comparable to that observed in NGC 1560 (Israel 1998; Greggio et al. 1998). This corresponds to a total mass of \(1.3 \times 10^7 M_\odot\) stars formed over the course of each simulation.

Each starburst was accompanied by energy and metal input from supernovae. One estimate of this contribution comes from comparing the cosmic SN rate per comoving Mpc\(^3\) as measured by Dahlén et al. (2004) with the cosmic star formation rate density as measured by Giavalisco et al. (2004), which gives a core-collapse supernova rate of \((7.5 \pm 2.5) \times 10^{-3}\) SNe per solar mass of stars formed (Scannapieco & Bildsten 2005). An alternative estimate is to count the number of \(M \geq 8 \, M_\odot\) stars, by assuming a Salpeter initial mass function with upper and lower mass cutoffs equal to 120 \(M_\odot\) and 0.1 \(M_\odot\), respectively (Scannapieco et al. 2002) which gives 1 SNe per 136 \(M_\odot\) of stars formed. We compromised between these two values and assumed that 1 SNe was generated per 150 \(M_\odot\) of stars formed, releasing an energy of \(10^{51}\) ergs. Furthermore, we assumed that a fraction \(f_w\) of this energy was instantaneously deposited into the galaxy, such that during the starburst the mechanical energy input was

\[
L_{\text{mech}} = f_w \times 2.2 \times 10^{41} \, \text{ergs} \, s^{-1} \left( \frac{SFR}{M_\odot yr^{-1}} \right),
\]

where in our fiducial models \(f_w = 0.7\). As described in more detail below, this energy was added as turbulent kinetic energy, rather than as gas thermal energy, vastly changing the rate at which it was lost to cooling.

As in a real galaxy, supernova energy was deposited into the gas stochastically, approximating the patchy distribution of OB associations within which stars and SNe are formed. In nearby galaxies, the luminosity function of OB associations is well-approximated by a power law of the form

\[
\frac{dN_{OB}}{dN} = AN^{-\beta},
\]

where \(N_{OB}\) is the number of OB associations containing \(N\) many OB stars, and \(\beta \approx 2\) (McKee & Williams 1997; Oey & Clarke 1997). To approximate this distribution in our simulations, we drew random numbers \(a_i \in [0, 1]\) such that for each forming OB association \(i\), the number of OB stars was given by

\[
N_i = (a_i N_{\text{max}}^{(1-\beta)} + (1 - a_i) N_{\text{min}}^{(1-\beta)})^{1/(1-\beta)},
\]

where we took \(N_{\text{min}} = 20\) and \(N_{\text{max}} = 1000\). By drawing two other random numbers \(b_i, c_i \in [0, 1]\) we assigned each OB association a random azimuthal angle given by \(\phi_i = 2\pi b_i\) and a random radial position given by the transcendental equation

\[
r_i = \frac{\tau}{v_c \, a_{gas}/1.5} \text{ where } \tau = \ln[(1 + \tau)/(1 - \beta)].
\]

All associations were assumed to be centered around the mid-plane of the galaxy, and we paused for a time \(150 N_i/SFR\) between bursts to maintain the overall star formation rate of the simulation.

For each OB association, we injected \(f_w \times 10^{51} N_i\) ergs of turbulent kinetic energy into the simulation in a region of size radius \(r_{\text{bubble}}\), which was at least the size of the region containing twice the mass of gas converted into stars, but no smaller than 60 pc so as to avoid extremely high pressure, largely-unresolved regions that excessively slow down the computational time step. The turbulent length scale for each OB association, discussed in more detail below, was taken to be \(r_{\text{bubble}}\). Each SN was also taken to deposit additional gas and metals into this region. Here we assumed that the average ejected total mass of 8 \(M_\odot\) per SN, 2 \(M_\odot\) of which is made up of heavy elements. This is consistent with the average stellar yields from a range of SNe II simulations (e.g. Maeder 1992; Woosley & Weaver 1995; Arnett 1996; Tsujimoto et al. 1995; Nagataki 1998), although there are significant theoretical uncertainties between various estimates.

Metallicity-dependent radiative cooling was calculated in the optically thin-limit, assuming local thermodynamic equilibrium:

\[
E_{\text{cool}} = - (1 - Y) (1 - Y/2) \frac{\rho \Lambda(T, Z)}{\mu m_p} c_s^2 \left( 1 + 0.25 v_t^2/c_s^2 \right),
\]

where \(E_{\text{cool}}\) is the radiated energy per unit mass, \(\rho\) is the density in the cell, \(m_p\) is the proton mass, \(Y\) is the helium mass fraction, \(\mu\) the mean atomic mass, and \(\Lambda(T, Z)\) is the cooling rate as a function of temperature and metallicity, \(V_t\) is the turbulent velocity, discussed below, and \(c_s\) is the local sound speed. Here we made use of the tables compiled by Wiersma, Schaye, & Smith (2009) from the code CLOUDY (Ferland et al. 1998), making the simplifying approximation that the abundance ratios of the metals both within the galaxy and ejected by the supernovae occurred in the solar proportions. Furthermore, we also account for unresolved substructure in highly turbulent regions, assuming that within each cell the averaged density squared, \(\langle \rho^2 \rangle\) is given by the bulk density squared \(\rho^2\) times an enhancement factor that increases with the Mach number of supersonic turbulence as \(1 + 0.25(v_t/c_s)^2\) as measured by Padoan, Nordlund, & Jones (1997).
2.3 Turbulence Modeling

While the direct simulation of turbulence is extremely challenging, computationally expensive, and dependent on resolution (e.g., Glimm et al. 2001), its behavior can be approximated to a good degree of accuracy by adopting a subgrid approach. Recently, Dimonte & Tipton (2006) described a subgrid model that is especially suited to capturing the buoyancy-driven turbulent evolution of subsonic bubbles (Scannapieco & Brüggen 2008; Brüggen & Scannapieco 2009), which we have modified heavily to apply to supersonic galaxy-scale outflows. Other recent efforts at the subgrid modeling of turbulence on galactic and extragalactic scales are described in Maier et al. (2009) and Shen et al. (2009).

Our model is based on the Navier-Stokes equations extended to include a turbulent viscosity $\mu_t$ that depends on the eddy size $L$ and kinetic energy per unit mass $K$. The interaction between the turbulence and the mean flow, is modeled by decomposing the flow into average components and fluctuating components, for example $\mathbf{u}_{tot} = \mathbf{u} + \mathbf{u}'$ the sum of the mass-averaged mean-flow velocity and the fluctuating component of the velocity, and $\rho_{tot} = \rho + \rho'$ the sum of the mean-flow density and the fluctuating component of the density. The sum of these two components is substituted back into Navier-Stokes equations, which are then averaged to obtain separate evolution equations for the mean and fluctuating components. For compressible flows, the averages are weighted by the density such that

$$\bar{\rho}_{tot}\mathbf{u}' \equiv 0,$$  (11)

and

$$\mathbf{u} \equiv \bar{\rho}_{tot}\mathbf{u}_{tot}/\rho.$$  (12)

The resultant equations constitute an expansion about the mean flow that must be terminated with a simplifying set of closure assumptions.

To leading order, the mean flow fluid equations in this case are given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \mathbf{u}_j)}{\partial x_j} = 0,$$  (13)

$$\frac{\partial \rho \mathbf{u}_i}{\partial t} + \frac{\partial (\rho \mathbf{u}_j \mathbf{u}_i)}{\partial x_j} + \frac{\partial P}{\partial x_i} = 0,$$  (14)

$$\frac{\partial \rho E}{\partial t} + \frac{\partial (\rho \mathbf{E}u_j)}{\partial x_j} + \frac{\partial (\mu_t \partial E_{\partial x_j})}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{N_E} \frac{\partial E}{\partial x_j} \right) + \rho E_{\text{mech}} + \rho E_{\text{cool}},$$  (15)

where $t$ and $\mathbf{x}$ are time and position variables, $\rho(\mathbf{x}, t)$ is the average density field, $u_i(\mathbf{x}, t)$ is the mass-averaged mean-flow velocity field in the $i$ direction, $P(\mathbf{x}, t)$ is the total pressure, both turbulent and kinetic, and $E(\mathbf{x}, t)$ is the mean internal energy per unit mass, also including both turbulent and thermal motions, and $N_E = 1$. Note that by modeling the impact of turbulence on the momentum equation by using on $\mathbf{y}$ a pressure, we are neglecting off-diagonal terms $(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$, where $i \neq j$. This is because in the presence of shocks, such strain terms become unphysically large (e.g., Gauthier & Bonnet 1990; Klem 2004). Although these shortcomings may be overcome with appropriate use of limiters (e.g., Wilcox 1994; Sinha et al. 2003), we set this aside as a future refinement to the basic method presented here.

In eq. (13), the first term on the right captures the effects of turbulent mixing, which is modeled as a turbulent viscosity $\mu_t$, the second, $E_{\text{mech}}$ term is an explicit source term that is determined by the mechanical luminosity as given by eq. (7) and the size of the region into which the energy is being added, and the third, $E_{\text{cool}}$, term is given by eq. (10). Note that $\mu_t$ does not appear in the continuity equation due to eq. (11) and does not appear in the momentum equation due to eq. (12).

In any type of subgrid model a number of fit parameters such as $N_E$ arise, whose values are expected to be $\approx 1$, but must ultimately be fine tuned versus experiments to achieve the most accurate results. Here we take the same fit parameters as used in Dimonte & Tipton (2006), and in Table 2 we summarize how they have been determined. However, there are several important differences in our approach in this paper, and thus it is likely our model will eventually be able to be further improved by re-adjusting these values to experiments and observations. Unlike in Scannapieco & Brüggen (2009), $E$ is now the total internal energy, including both turbulent and thermal contributions. Likewise the pressure is the sum of both thermal and turbulent component, computed as $P = \frac{2\mu}{3m_p}E$, with $m_p$ the mass of the proton and $\mu$ the mean atomic weight of the gas. This redefinition allows us to apply the PPM solver to capture both the effects of turbulent and thermal pressures, yielding accurate solutions even in cases in which most of the internal energy is in the subgrid turbulent flow. An implicit and simplifying assumption associated with this approach is that turbulent and thermal velocities provide pressure support through the same equation of state. To track the metals ejected by supernovae, eqs. (13)-(15) are supplemented by a mass-fraction equation:

$$\frac{\partial \rho F_r}{\partial t} + \frac{\partial (\rho F_r u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{N_F} \frac{\partial F_r}{\partial x_j} \right),$$  (16)

where $F_r$ is the mass fraction of species $r$ in a given zone, and $N_F = 1.0$ is a scale factor.

The turbulence quantities that appear in these equations are calculated from evolution equations for $L$, the scale of the largest turbulent eddies, and $K$, the turbulent kinetic energy. Simple equations for the evolution of these quantities are given by

$$\frac{\partial \rho L}{\partial t} + \frac{\partial (\rho L u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{N_L} \frac{\partial L}{\partial x_j} \right) + C_{\text{cool}} \rho L \frac{\partial u_i}{\partial x_i},$$  (17)

and

$$\frac{\partial \rho K}{\partial t} + \frac{\partial (\rho K u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{N_K} \frac{\partial K}{\partial x_j} \right) + \rho E_{\text{mech}}$$

$$- \frac{\kappa}{E} \frac{\partial u_j}{\partial x_j} - \rho V_t C_D \frac{\max(V_{\text{rel}} - V_{t,0})^2}{L},$$  (18)

where $V_t \equiv \sqrt{2K}$ is the average turbulent velocity, $V_{t,0}$ is the...
average turbulent velocity at the start for the simulation, and $C_C = 1/3$ is given by mass conservation, $N_L = 0.5$, $N_K = 1.0$, and $C_D = 1.25$ are experimental fit constants. In the $L$ equation the two terms on the right hand side represent, respectively: turbulent diffusion and the growth of turbulent motions due to the expansion of the mean flow. In the $K$ equation the three terms on the right hand side represent, respectively: turbulent diffusion, the energy input from SNe, and a term that causes turbulence to decay away at a characteristic time scale of $\approx L/(V_t - V_{t,0})$ in the absence of external driving. This energy goes directly into gas heating, because of energy conservation $E = K + E_{\text{thermal}}$. The $V_t - V_{t,0}$ in this term means that after the starburst episode, the gas remaining in the galaxy will eventually settle back into a turbulently-supported disk, rather than dissipate all its turbulence into heat. Finally, the turbulent viscosity was calculated as

$$\mu_t = \rho L \max(V_t - V_{t,0}, 0),$$

where again the $V_t - V_{t,0}$ term assures that in its initial configuration the galaxy remains static and does not diffuse into the intergalactic medium. Note that including the $V_{t,0}$ term in eqs. (19) and (19) is akin to assuming a low level of persistent turbulence in addition to the much larger contribution by the SNe that are added to the galaxy explicitly during the simulation.

Note that unlike in Dimonte & Tipton (2006) and our previous modeling, we do not include terms that attempt to track the growth of the Rayleigh-Taylor or Richtmyer-Meshkov instabilities on subgrid scales. This is because we are no longer working in the regime in which turbulence generated by these instabilities is dominant. Rather, we are interested in approximating the evolution of the ejecta associated with a random collection of SNe that move supersonically within overpressured bubbles surrounding OB associations. Our approach assumes that for each OB association, the initial maximum turbulent length scale is approximately equal to the radius of the overpressured region, and the initial turbulent kinetic energy per unit mass is approximately equal to mechanical energy input from SNe divided by the total mass in the region. Although such overpressured regions will eventually expand and become Rayleigh-Taylor unstable, the length scale and velocities generated by this instability will initially be much smaller than the motions within the bubble. This means that the turbulent regions will not grow according to the linear equations of growth of the Rayleigh-Taylor or Richtmyer-Meshkov instabilities. In particular, one would not expect $L$ and $K$ in eqs. (19) and (19), which represent SNe driven random motions, to grow as quickly as $\propto t^2$ as they would in the Rayleigh-Taylor case. So our approach is to conservatively assume that $L$ expands along with the mean flow, and that $K$ is driven only by SNe input, and decays to thermal energy on the $L/V_t$ timescale expected from a self-similar scaling analysis of incompressible turbulence (Kolmogorov 1941), which has also been confirmed in the compressible (magnetohydrodynamic) case (Stone et al. 1998; Mac Low et al. 1998; Padoan & Nordlund 1999). Thus only resolved Rayleigh-Taylor and Richtmyer-Meshkov instabilities are captured in our simulations, while eqs. (17) and (19) track the much more important subgrid turbulent motions driven by SNe.

As in our previous modeling, our numerical implementation of these equations was divided into three steps, which were carried out after the main hydro step in FLASH3, which advects all the variables above. In the first step, we implement the $\partial u_i/\partial x_i$ terms in eq. (17) explicitly. In the second step, we: (i) compute $V_t$ as $\sqrt{2K}$, (ii) use a leapfrog technique to add the source term to $V_t$ as $S_K/\rho V_t$, and then (iii) write $V$ back to the $K$ array as $K = V_t^2/2$. Finally, in the third step, we calculate the turbulent viscosity and use this to implement the diffusive mixing terms in eqs. (15)–(19) explicitly. This final step requires us to impose an additional constraint on the minimum times step of $dt \leq (\Delta x/p/\mu_t)/4$ where $\mu_t$ is the minimum of $dx$, $dy$, and $dz$ in any given zone. This diffusive constraint must be satisfied for all zones in the simulation, but as discussed in SB08 this explicit approach works well in concert with the AMR hydrodynamic solver, because as $\mu_t$ increases near the interface, density and pressure fluctuations are smoothed, allowing the code to dереfine in these regions. Thus the diffusive time step remains greater than or comparable to the one required by the Courant condition for most of the evolution in our simulations.

In all zones within the galaxy, $K$ is initialized such that the galaxy is in hydrostatic balance, even though the initial thermal energy is chosen such that $T \leq 10^4$ everywhere. Outside the galaxy, the internal energy was taken to be solely thermal such that $K$ was initialized to be to $10^{-10}E$. Throughout the simulation, $L$ was initialized to 2 parsec, 1/100th of the initial gas scale height of the galaxy.

3 RESULTS

3.1 Fiducial Model

In Figs. 2 - 4 we show results from our fiducial run, which has $f_w = 0.7$ and 4 total refinement levels, including the base
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Figure 2. Horizontal slices through the central 8 × 8 kpc in our fiducial simulation (7-4D) at four representative times: 10, 20, 30, & 40 Myrs (arranged from left to right in each row). Top Row: Contours of log density from $\rho = 10^{-29}$ to $10^{-23}$ g cm$^{-3}$. Second Row: Contours of log temperature from $T = 10^{3}$ to $10^{8.5}$ K. Third Row: Contours of log turbulent velocity, $V_t$, from 1 to 3000 km s$^{-1}$. Bottom Row: Contours of log turbulent length scale, $L$, from 30 to 1000 parsecs.

grid. A summary of all the runs carried out for this study is given in Table 3. Each run is labelled m-nD for cases with subgrid diffusive mixing and m-nN for cases without subgrid mixing, where $m = 10 \times f_w$ and $n$ is the number of refinement levels. Thus our fiducial run is referred to as 7-4D.

Fig. 2 shows horizontal slices through the central 8 × 8 kpc for this run at four representative times, and Fig. 3 shows vertical slices through the central 8 × 12 kpc. These plots contain numerous “superbubbles” driven by SNe from individual OB associations, as have been studied in several classic theoretical papers (Weaver et al. 1977; McCray & Snow 1979; Tomisaka & Ikeuchi 1986; Mac Low & McCray 1988; Tenorio-Tagle & Bodenheimer 1988; Mac Low et al. 1989). Note that the bubbles are somewhat larger at greater distances from the galactic centre as the ambient pressure is lower there. Regardless of galactocentric distance, however, the bubbles are significantly less empty than those described in the papers mentioned above. This primarily because we assume that SNe form and deposit energy into a region containing twice as much gas mass as is converted into stars. This means that for every $10^{51}$ ergs added in SN driven turbulent energy, the affected region initially contains at least $2 \times 150 M_\odot$ of gas. In this case, the increase in internal energy per unit mass is $\approx f_w 500$ times that of the gas within the midplane, which corresponds to an initial turbulent velocity of $\approx f_w^{-1/2} 500$ km/s.

Within these regions three time scales are almost equal: (i) the time scale for turbulent energy to dissipate into thermal energy, $t_{\text{diss}} \approx r_{\text{bubble}} / V_t$; (ii) the time scale for bubble expansion, $t_{\text{dyn}} \approx r_{\text{bubble}} / c_{\text{eff}} \approx r_{\text{bubble}} / V_t$, where the effective sound speed within the heated regions is dominated by the turbulent motions, and (iii) the time scale.
Figure 3. Vertical slices through the central $8 \times 12$ kpc in our fiducial simulation (7-4D) at four representative times of 10, 20, 30, & 40 Myr (arranged from left to right in each row). Top Row: Contours of log density from $\rho = 10^{-29}$ to $10^{-23}$ g cm$^{-3}$. Second Row: Contours of log temperature from $T = 10^3$ to $10^{8.5}$ K. Third Row: Contours of log pressure from $p = 10^{-17}$ ergs cm$^{-3}$ to $10^{-10}$ ergs cm$^{-3}$. Bottom Row: Contours of log turbulent length scale, $L$ from 30 to 1000 parsecs.
for mixing of the bubble interior with the surrounding matter, \( t_{\text{mix}} \approx r_{\text{bubble}}^2 \rho / \mu \approx r_{\text{bubble}}^2 (\rho / \Sigma) \approx r_{\text{bubble}} / V_t \).

Physically, this similarity of time scales occurs because the driving of gas into the exterior medium and the mixing of the exterior medium with the bubble interior both occur roughly at the time at which SNe kinetic energy is converted into thermal energy through a turbulent cascade. Furthermore, all these time scales are short, on the order of 10 pc / (\( f_w^{-1/2} 500 \, \text{km s}^{-1} \)) \( \approx 0.2 \) Myrs, which is much less than their interior cooling time and the dynamical time of the surrounding medium.

Thus the bubbles quickly expand to the point that they are in pressure equilibrium with their surroundings, with only moderate heating of their interiors through turbulent decay, and moderate mixing of shells with the interior gas through turbulent “diffusion.” As \( \rho \propto n^{5/3} \propto 1^{-1/5} \) during this approximately adiabatic expansion, the result is the formation of \( \approx (f_w 500)^{1/5} \times 60 \) pc \( \approx f_w^{1/5} 200 \) pc superbubbles whose exterior shells are relatively thick, and whose interiors are roughly \( (f_w 100)^{1/5} \approx f_w^{1/5} 40 \) times underdense and somewhat hotter and more turbulent than their surroundings.

As the turbulent length scale expands along with the flow, \( L \) also rises to \( \approx 200 \) pc within these regions, but remains well below the grid scale throughout the rest of the simulation. Many such regions can be seen in Figs. 2 and 3 particularly at later times and larger galactocentric radii. Note however, that turbulence will tend to be strongest in-
tion of the starburst. After blow-out, the collective outflow remains overpressured with respect to the surrounding intergalactic medium, even at large distances. While the highly rarefied free wind does eventually collect up a denser shell of intergalactic gas during this expansion, mass loading arises mainly from dense clumps of interstellar material that are gradually mixed into the diffuse gas. Much of this material comes from the conical shear interface between the free wind and the surrounding galaxy, but there is also a contribution from clumps of gas being evaporated directly in the path.
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Figure 5. Rendered images of log density of metals from $10^{-30}$ to $10^{-26}$ g cm$^{-3}$, separated into metals from the IGM (upper panels) and from the SNe that go off during the simulations (lower panels), at times of 20, 30, and 40 Myrs. The galaxy is being observed at an inclination of $\approx 30$ degrees, and the galaxy disk visible in the upper panels has a radius of $\approx 6$ kpc. Metals from SNe driving the outflow are ejected to large distances while the ISM is largely retained by the galaxy. The plotting artifacts at large radii in the top panels illustrate the $8^3$ cell “blocks” in our adaptive mesh approach, which has a resolution of 39 parsecs in the central regions where superbubbles develop, but only 312 parsec at large radii, where density and pressure contrasts are small.

Metals can escape from the galaxy either by being entrained in the wind or by being directly ejected in the SNe remnants driving the outflow. To distinguish between these two contributions, in Fig. 5 we show rendered images of log $\rho_{\text{metals}}$ separated into the component arising from the metals initially in the ISM, and metals that arise from SNe occurring during the simulation. Here we see that despite the fact that turbulence is driving the outflows in our galaxies, the mixing of supernova ejecta into the ISM is minimal, and each of the two metal components evolves completely differently. Consistent with the blow-out picture of outflow generation, the ISM metals are swept up into dense shells of gas that remain bound to the galaxy. On the other hand, SNe ejecta are only weakly mixed into the shells, such that the metallicity changes very little within dense regions during the starburst, as observed in the HII regions in NGC 1569 (Kobulnicky & Skillman 1997). Instead SNe metals are mostly found within the rarefied high-pressure regions, and are able to escape to large distance following bubble overlap.

To quantify the kinematics of the ejected gas further, we plot in Fig. 5 the evolution of the ejected mass, energy, momentum, and metals. In the upper panel of this figure, we show the gas mass ejected by the galaxy as a function of time, compared to the total initial gas mass. Here we define escaping gas as material that is at least a scale height from the midplane of the galaxy and traveling outwards such that the component of velocity in the direction of the gravitational acceleration exceeds the local escape velocity.

The wind is able to effectively blow out $\approx 3 \times 10^6 M_\odot$ of gas from the dwarf starburst, which is comparable to the total mass of stars formed, as observed in local starbursts (Martin 1999). However, the ejected mass is relatively small as compared to the total $2 \times 10^8 M_\odot$ gas mass of the galaxy, and thus the majority of the ISM remains bound. This is true even though the total kinetic energy from SNe added to
In the third panel of Fig. 6 we compare the total momentum of the ejected gas, which is at least an order of magnitude smaller than the total galaxy gas mass times the escape velocity. At late times $KE_{\text{Ejected}}/P_{\text{Ejected}} \approx 200 \text{ km/s}$ indicating that the majority of the outflow escapes at a few times the escape velocity, although again most of this mass is in the form of the largely invisible free wind rather than the much better observed entrained material.

In the bottom panel of this figure we compare the metal mass produced during the starburst to the total ejected metal mass and the ejected metal mass coming from SNe that occur during the simulation. As seen in Fig. 5 the majority of metals ejected from the galaxy come from the supernovae that are driving the wind itself, while only a small fraction comes from metals already contained within the ISM. However, most of the metals produced by the starburst are retained by the galaxy, confined to hot regions that mix into the dense ISM on time scales much longer than the starburst itself.

### 3.2 Parameter Dependencies

#### 3.2.1 Wind Efficiency

In Figs. 7 and 8 we study the impact of reducing the efficiency with which SNe energy is converted into turbulent energy ($f_w$) and the efficiency of turbulent mixing ($C_w$). In the top two rows of Fig. 7 we show vertical slices of density for runs 4-4D and 2.5-4D, for which $f_w$ has been reduced to 0.4 and 0.25 respectively. Here we see that varying $f_w$ leads to drastic differences in outflow strengths and morphologies. The galaxy in our fiducial (7-4D) run achieves blow-out within 30 Myrs from the start of the simulation, rapidly expanding to fill most of the simulation volume by 50 Myrs, but reducing $f_w$ delays blow-out to $\approx 40-50$ Myrs in the $f_w = 0.4$ and $f_w = 0.25$ cases, and substantially reduces the volume enriched by the outflowing gas. This can be understood in the context of a collective wind that arises from overlapping superbubbles. From eq. 23, we see directly that at a fixed radius, overlap occurs much later in models with low $f_w$ as the individual superbubbles are much more compact. Note also that the opening angle of the outflow is reduced as $f_w$ decreases, and the central collective outflow is only able to punch its way through a smaller region of the ISM. Again this is consistent with eq. 23 which shows that for a given time, it is only the most central regions that can achieve superbubble overlap, followed by “blow-out” of the gas almost vertically. Thus in the $f_w = 0.25$ case, not only does a small fraction of the hot gas escape from the galaxy, this is concentrated into a plume traveling almost directly perpendicular to the disk.

In the lower panel of this figure, we examine the effect of reducing turbulent mixing by setting $f_w = 0.4$ and $C_w = 0$ in a case we label as 4-4N, where $N$ indicates that no subgrid diffusive mixing has been modeled. In this run, the mixing between bubble interiors and the shells is severely reduced, which in turn reduces radiative losses, and results in somewhat larger superbubbles. In general, this run behaves similarly to our fiducial case in which we assume more efficient mixing.
SNe driving of turbulence, but mix a significant fraction of the energy into dense, rapidly radiating regions.

In Fig. 8, we plot the ejected gas mass, energy, momentum, and metal mass for each of the runs shown in Fig. 7. As apparent from the density slices, the ejected mass and momentum depend extremely sensitively on $f_w$. After 60 Myr the galaxy with $f_w = 0.4$ has ejected $\approx 10\%$ as much mass as in the fiducial $f_w = 0.7$ case, while the $f_w = 0.25$ galaxy has ejected only $\approx 3\%$. The differences in ejected energy and momentum between the runs are even more dramatic. The energy ejected in the $f_w = 0.25$ run, for example, is less than 1% that of the fiducial, $f_w = 0.7$ run. This large difference also translates into a large difference in the ejected metal fraction, which is almost negligible in the $f_w = 0.25$ case. Finally, in the no mixing case with $f_w = 0.4$, ejection of mass, energy, momentum, and metals are all significantly enhanced, reaching values similar those in our fiducial run.

3.2.2 Resolution

Next we carried out a test of the impact of resolution on our results. Leaving the base grid spacing fixed at 313 pc, we resimulated our fiducial $f_w = 0.7$ galaxy with different maximum levels of refinement. In the first of these runs, labeled 7-3D, we only allowed 2 levels of refinement above the base (level 1) grid, for an effective resolution of 78 pc. In the second run, labeled 7-2D, we allowed only 1 additional level, for an effective resolution of 156 pc. In this very low-
resolution case, the minimum value of $r_{\text{bubble}}$ was taken to be 120 pc instead of 60 pc, so that each bubble region would encompass more than a single zone.

Vertical slices at various times through the simulation volumes from these runs are shown in Fig. 9. Note when comparing these runs that the turbulent length scale can grow larger than the grid scale, as shown in Figures 2 and 3, and thus turbulent diffusion can smooth features on scales larger than the effective resolution of each of the runs. With this limitation in mind, we see that in general, both low-resolution runs display the same evolution as in the fiducial run. In all cases, a low density cavity builds up near the centre of the galaxy, pushing its way through the lowest-density regions until it makes its way into the surrounding intergalactic medium. While blow-out occurs at slightly different times in each of the runs, it is always an abrupt transition that is quickly followed by gas ejection out to very large distances. At late times, in all runs, the base of the outflow widens and the flow becomes less collimated as bubbles overlap at larger radii and significant mass entrainment occurs at the interface between the free wind and the surrounding galaxy. However, it is only in the highest resolution run that clouds of dense ISM are resolved within the outflow itself.

In Fig. 10 we compare the evolution of the ejected quantities as a function of resolution. At early times, before the initial blow-out occurs, there are notable differences between the runs, and in general, the higher resolution cases achieve higher ejected fractions earlier. As the simulations progress, however, the evolution becomes much more similar between the runs, and by the final time of 60 Myrs, all ejected quantities are consistent to within less than a factor of two. This is true even though the maximum volume resolution, and hence the mass resolution, varies by a factor of $2^5$ between these runs. This means that our implementation of supersonic turbulence-driven outflows is only weakly dependent on the extent to which turbulence is directly resolved, as opposed to approximated by subgrid modeling. Instead, the qualitative and quantitative evolution of the starburst is largely independent of resolution.

### 3.2.3 ISM Structure

Next, we examined the effects of pre-existing ISM structure on the evolution of the outflow. As it lies outside of the scope of this paper, we did not attempt to model a realistic density distribution of molecular clouds. Rather we simply altered the gas distribution so as to add a regular series of dense $\approx 0.5$ kpc knots, which interact with the outflow as it develops. Specifically, we adopted a perturbed density distribution within the galaxy, given by:

$$
\rho_{\text{perturbed}}(x, y, z) = \rho_{\text{average}}(x, y, z)[1 - 0.8 \times \cos(\pi x/\lambda)^2 \cos(\pi y/\lambda)^2 \cos(\pi z/\lambda)],
$$

where $\lambda = 0.5$ kpc and $\rho_{\text{average}}$ is the smooth distribution given by eq. (1). At the same time we rescaled the temperature and turbulent kinetic energy per unit mass through-
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Figure 9. Vertical slices through the central $8 \times 12$ kpc in simulations with varying resolutions at four representative times of 10, 20, 30, & 40 Myrs (arranged from left to right in each row). The lowest resolution run, 7-2D, is shown in the top row, the intermediate resolution run, 7-3D, in shown in the centre row, and the highest resolution, fiducial run, 7-4D is shown in the bottom row. All panels give contours of log $\rho$ from $10^{-29}$ to $10^{-23}$ g cm$^{-3}$.

out the galaxy as $[\rho_{\text{perturbed}}/\rho_{\text{average}}]^{-1}$ such that the overall pressure profile remained the same as in the fiducial run, with each of the dense clouds in pressure equilibrium with its surroundings. All other parameters for this run were identical to the fiducial 7-4D run, and we refer to it as 7-DC, where the C indicates the presence of a clumpy ISM.

In Fig. [11] we show vertical slices of the density, temperature, and pressure from this simulation at several representative times. These illustrate the strong tendency for the outflow to avoid dense pockets of gas. Again, this is both because at high densities radiative cooling, which goes as $\rho^2$, is much more efficient and because the time scale for turbulence to decay to thermal energy is much shorter.

As in the fiducial run, superbubbles in the 7-4DC run begin to overlap into a large rarefied region near the centre of the galaxy, but the asymmetry in this run is much stronger than in the fiducial case. Here, the gas moves primarily laterally in the plane of the disk, so as to avoid passing directly through the pair dense clumps located directly above and below the centre. This is an extreme example of the cooling instability, which preserves dense clumps even in the direct path of the hot, high-pressure gas. The hot region pushes its way around these clumps, forming two distinct chim-
necks of hot material that punch out into the intergalactic medium on either side of the central axis. The free wind then streams outwards on both sides of dense regions, enveloping this cold gas and eventually entraining it into the outflow. Only at late times does the gas within the clumps eventually begin to move out of the galaxy, as it gradually shears and mixes into the wind much like the ISM along the edges of the outflow.

Note that this evolution is markedly different than what would occur if SN heating were modeled purely as thermal energy input and radiative cooling were neglected. In this case, the shocks from the developing outflow would raise the pressure in the cold clouds, causing them to expand and smoothing out the overall density distribution. A tendency to preserve a clumpy medium near the base of the wind, as observed in NGC 1569 (Hunter, Hawley & Gallagher 1993; Tomita, Ohta & Saito 1994; Heckman et al. 1995; Martin 1998; Westmoquette, Smith, & Gallagher 2008), is one of the hallmarks of outflows driven by supersonic turbulence.

3.3 Observational Consequences

Finally, we carried out a preliminary comparison of our models with observations of NGC 1569. Here we focus on Hα imaging as an optical tracer of warm, dense, ionized gas and X-ray imaging as a tracer of the hot, rarefied gas.

3.3.1 Hα Emission

As a tracer of the shocked and ionized interstellar medium we computed the Hα emissivity of the gas, $j_{H\alpha}$, in each grid zone throughout the simulation. Note that for reasons of scope we neglect photoionization for this calculation, although this can be very important source in the presence of large numbers of O and B stars as would naturally occur during a starburst. For this reason the images below should be considered as only tracing the rough morphology of the ionized gas, while more detailed studies, such as comparisons between the velocity structure of Hα emitting gas and the coldest outflowing gas as measured though NaI absorption (e.g. Heckman et al. 2000; Fujita et al. 2009), will require more complete calculations including radiative transfer effects.

With these limitations in mind $H\alpha$ can be calculated as function of temperature and density from the $H\beta$ emissivity of the gas from the theoretical fit (Ferland 1980)

$$
\frac{4\pi j_{H\alpha}}{n_e n_p} = \begin{cases} 
2.53 \times 10^{-22} T_e^{-0.833} \text{ergs s}^{-1} & \text{for } T_e \leq 2.6 \times 10^4 K \\
1.12 \times 10^{-20} T_e^{-1.2} \text{ergs s}^{-1} & \text{for } T_e > 2.6 \times 10^4 K,
\end{cases}
$$

combined with an assumed Balmer decrement, $j_{H\alpha}/j_{H\beta} = 2.9$, ignoring the small ≈ 10% changes that occur over the range of temperatures encountered in starbursts. Again neglecting any photoioniz-
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Figure 11. Vertical slices through the central 8 × 12 kpc in our perturbed simulations (7-4DC) at four representative times of 10, 20, 30, & 40 Myrs (arranged from left to right in each row). Top Row: Contours of log $\rho$ from $\rho = 10^{-29}$ to $10^{-23}$ g cm$^{-3}$. Centre Row: Contours of log $T$ from $T = 10^3$ to $10^8.5$ K. Bottom Row: Contours of log $p$ from $p = 10^{-17}$ ergs cm$^{-3}$ to $10^{-10}$ ergs cm$^{-3}$.

ing background and assuming local thermodynamic equilibrium, we calculated $n_e = n_p$ directly from the Saha equation as

$$\frac{n_e n_p}{n_h} = \left(\frac{2\pi m_e k T}{\hbar}\right)^{3/2} \exp \left(\frac{-13.6eV}{k T}\right),$$

where $n_h$ is the neutral hydrogen density, $m_e$ is the mass of the electron, and $\hbar$ is Planck’s constant. To account for unresolved gas inhomogeneities we assumed this emission was enhanced by a factor of $1 + 0.25(V_t/c_s)^2$, as in eq. (10).

Finally we projected the total Hα emissivity in the $z$ and $x$ directions, to produce vertical and horizontal surface brightness maps of our fiducial (7-4D) starbursting galaxy.

Plots of the logarithm of the surface brightness at times of 20, 30, and 40 Myrs are given in Fig. 12. As in NGC 1569, the simulated vertical Hα profiles shown in the upper panels of this figure exhibit a complex and chaotic structure, which is brightest near the plane of the galaxy. Also as seen in NGC 1569, which is observed almost edge-on, our images display bubbles and loops of strong Hα emission, which correspond to the shells of material swept up by the superbubbles generated by each OB association (e.g. Hunter, Hawley, & Gallagher 1993; Martin 1998; Westmoquette, Smith, & Gallagher 2008).

Furthermore, and especially at late times, outgoing filaments of heated gas are apparent near the central axis.
**Figure 12.** $H\alpha$ luminosity (arbitrary scale) of our fiducial simulations (7-4D) at three representative times of 20, 30, & 40 Myrs (arranged from left to right in each row). **Top Row:** Central $6 \times 6$ kpc$^2$, projected in the $z$ direction. **Bottom Row:** Central $6 \times 6$ kpc$^2$, projected in the $y$ direction.

(Tomita, Ohta & Saito 1994; Heckman et al. 1995; Martin 1998). This is also consistent with observations, and a comparison of these plots with the density contours in Fig. 3 shows that these features arise largely from ISM material that is being entrained by the hot wind, either because the dense gas is on the edges of the blow-out region, or because it is contained in cold clumps that are left directly within the path of the collective central outflow. Again, these clumps form naturally in our models from the material swept up between superbubbles, and they are enhanced by the cooling instability and the tendency for supersonic turbulent gas to avoid density perturbations rather than disrupt them.

3.3.2 X-ray Emission

Next we turn our attention to the X-ray properties of our simulated galaxy. Such observations provide the most direct picture of outflowing starbursts as they reveal diffuse emission not only from the disk and the halo, but from the hottest and highest pressure regions of the flow. Martin et al. (2002), for example, have taken Chandra observations of NGC 1569 and found that its X-ray luminosity is dominated by diffuse, thermal emission from the disk ($0.7$ keV) and a bipolar $0.3$ keV halo. After subtracting hard point sources, they found a luminosity from the thermal component in the band of $0.3-6$ keV of $\approx 8 \times 10^{38}$ erg s$^{-1}$. Moreover, Chandra observations of galactic winds have shown that the X-rays are often spatially-correlated with $H\alpha$ emission (Cecil et al. 2002; Strickland et al. 2004; Grimes et al. 2005). A prominent example is the edge-on spiral NGC 3079 (Cecil et al. 2002), which shows towers of intertwined $H\alpha$ filaments that line its central outflow and emit in the X-ray, as discussed by Strickland et al. (2002).

From a theory point of view, Suchkov et al. (1994) computed the X-ray emission from two-dimensional simulations of galactic superwinds that interacted with a two-component ISM. They found that the bulk of the soft X-ray emission originated from shocked material in the disk and halo, and the bulk of the hard X-ray emission arose from the free wind itself. Consequently, they concluded that soft X-ray spectra need not show abundances enhanced in metals. Strickland

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Figure 13. False colour maps of the logarithmic surface brightness in the soft (0.5-2 keV; red) and hard (2-10 keV; blue) X-ray bands in our fiducial simulation (7-4D) at 30 (left panels) & 40 Myrs (right panels). The top row shows a projection in the y-direction and the bottom row shows a projection in the z-direction, spanning the central 12 × 12 kpc.

& Stevens (2000) also performed two-dimensional starburst simulations, computed the X-ray emission, and found that most of the soft X-rays come from shock-heated ambient gas and from the interface between the hot gas and the ISM. However, the symmetry of these simulations did not allow them to form filamentary structures to the same degree as in three-dimensional models, and this limited their predictions.

For this study, we computed the soft (0.5-2 keV) and hard (2-10 keV) X-ray emissivities of our fiducial simulation using the ATOMDB code\(^1\) which includes the Astrophysical Plasma Emission Database (APED) and the spectral models output from the Astrophysical Plasma Emission Code (APEC). The APED files contain information such as wavelengths, radiative transition rates, and electron collisional excitation rate coefficients, and APEC uses these data to calculate model spectra, for optically-thin plasmas in collisional ionization equilibrium. We also included sub-grid enhancement as in eq. (10) and projected these metal-dependent emissivities in each grid cell to give the surface brightness maps shown in Fig. 13 which shows the distribution at 30 and 40 Myrs. In these composite colour images, the red colours correspond to the soft band, the blue colours correspond to the hard band, and both colour scales cover six orders of magnitude.

This figure shows that most of the soft X-ray emission comes from the the winds that are blown out of the disk. Also, we find towers of X-ray emission that rise more than 4 kpc above the disk and follow the channels through

\(^1\) http://cxc.harvard.edu/atomdb/
which the hot wind escapes (cf. Fig.3). The hard X-rays, on the other hand, come mainly from the starbursting bubbles, most of which are in the centre of the galaxy. Comparing Figs.12 and 13, it appears that the sites of X-ray emission are correlated, similar to the observations of NGC 3079. This is because the outflow is directly responsible for accelerating filaments of cold gas out of the disk.

The X-ray emission is very temperature dependent and thus the emission changes over time. At 30 Myrs the soft X-ray emission dominates the emission from the wind and the regions that are emitting hard X-rays are confined to the starbursting bubbles. At this time, the overall X-ray luminosity is \( \approx 6 \times 10^{38} \) erg s\(^{-1}\) in the 0.5-2 keV band and \( \approx 3 \times 10^{35} \) erg s\(^{-1}\) in the 2-10 keV band. The mean X-ray weighted temperature of the soft X-ray emitting gas is 1.6 \( \times 10^6 \) K and of the hard band 7.6 \( \times 10^7 \) K. The soft X-ray emission is so luminous because most of the emission measure occurs at temperatures at which emission is strong. Thus the X-ray luminosity is very close to the value measured for NGC 1569.

At 40 Myrs, the wind has moved much of the gas to higher temperatures with the effect that the emissivity in the soft band drops, while the emissivity in the hard band remains roughly constant. The overall X-ray luminosity is \( \approx 9 \times 10^{36} \) erg s\(^{-1}\) in the 0.5-2 keV band and \( \approx 4.5 \times 10^{35} \) erg s\(^{-1}\) in the 2-10 keV band. The mean X-ray weighted temperature of the soft X-ray emitting gas is 5.6 \( \times 10^6 \) K and of the hard band 3.7 \( \times 10^7 \) K. At 40 Myrs, emission from the free wind can be seen in the hard X-ray band, moving out to large distances and escaping from the potential well of the galaxy (seen as blue patches in Fig.13).

However, one should note that we do not include a hot X-ray emitting halo, and have calculated only the thermal emission from the shocked disk and the supernovae. Especially in the hard X-ray band non-thermal emission becomes important. Real spectra show a combination of thermal emission and non-thermal components due to X-ray binaries, young supernova remnants, low-luminosity AGN, and Compton scattering of relativistic electrons by the ambient far-IR and CMB radiation fields (see e.g. Persic & Rephaeli 2002). Therefore, a detailed comparison to observations is not straightforward, especially in the hard band, and for reasons of scope we postpone this to a future publication.

Finally, we note that a major driver of future X-ray instruments such as the X-ray Microcalorimeter System on board of the International X-ray Observatory (IXO)\(^2\) is to take high-resolution spectra of galactic winds in order to measure their velocity, temperature and abundance structure. While models such as ours that are able to capture turbulent broadening of lines will be essential for this mission, we do not attempt to show spectra in this study as a thorough simulation of the X-ray emission would need to include a number of additional important processes such as absorption. Again, this is left for future work.

4 SUMMARY AND CONCLUSIONS

Starburst-driven outflows play a key role in structure formation, impacting issues ranging from the gas and metallicity evolution of galaxies to the chemical and thermal history of the intergalactic medium. While much observational and theoretical progress has been made in understanding these objects, formidable challenges remain. Many local outflowing starbursts have been observed in great detail, but the phase that contains 90% of the ejected energy and metals has gone largely undetected. Many theoretical studies of galaxy outflows have been conducted, but these have been faced with large uncertainties arising from rapid radiative cooling and a complex turbulent gas distribution that contains structures over a wide range of scales. In fact, the medium within starbursting galaxies is disturbed so strongly, and the cooling times within the gas are so short, that the turbulent velocities far exceed the thermal velocities.

Here we have explored a completely new theoretical approach that addresses these issues by tracking not only the thermal and bulk velocities of the gas, but also its turbulent velocities, pressures, and length scales. By adding an intermediate class of gas motions that operates on scales much larger than the particle mean free path, but much smaller than the resolved motions in the simulation, we are able to carry out starburst simulations that overcome many of the problems seen in previous models. In particular, our approach allows us for the first time to model starbursting galaxies such as NGC 1569 without imposing a two-phase medium by hand, but still including realistic radiative cooling throughout the simulation.

The resulting three-dimensional AMR simulations reproduce a number of key observational features of nearby starbursts, some of which have been previously captured in hydrodynamic simulations without gas cooling and some of which are unique to simulations that include supersonic turbulence. Thus, in accord with previous studies, we find that:

- With realistic choices of star formation rates and energy input, our simulations lead to large, bipolar outflows that drive substantial amounts of gas, metals, momentum, and energy into the intergalactic medium. These exhibit a “blow-out” morphology, in which the majority of the ISM is retained, but a large fraction of the extremely hot SN-driven gas escapes in a diffuse and rapidly-expanding “free wind.” The properties of these outflows are only very weakly dependent on simulation resolution, and they occur without invoking any additional physical mechanisms such as cosmic ray pressure or radiation pressure on dust.
- Most mass entrainment from the ISM occurs in the shear interface between the free wind and the denser ISM medium. Unlike other simulations with an initially homogeneous medium, however, this shearing occurs both along the edge of the wind and from cool clouds directly in the path of the wind. This interacting gas leads to the majority of the \( \text{H}_\alpha \) emission from the galaxy.
- X-ray images of the galaxy show that most of the soft X-rays originate from the shocked material in the disk and from gas interactions between the hot winds and the dense ISM gas. Regions of X-ray emission roughly correlate with regions of \( \text{H}_\alpha \) emission, but in hard X-ray images, weaker emission from the free wind can be seen directly, moving

\(^2\) http://ixo.gsfc.nasa.gov/
out to large distances and escaping from the potential well of the galaxy.

At the same time, in contrast with previous studies, we find that:

- Structures in our simulations arise primarily from the interaction of shells around individual OB associations, which sweep up thick shells of material around more rarified pockets of hot gas. Unlike in simulations without ISM cooling, these dense regions persist for long times due to the cooling instability and the tendency for turbulence to decay away quickly in dense regions of gas. These effects lead to inhomogeneous structures throughout the starburst, which are far more important than the Rayleigh-Taylor instability in determining the outflow morphology. The result is a complex, chaotic Hα distribution, full of bubbles, loops and filaments, as observed in NGC 1569 and other outflowing starbursts.

- Outflows develop not from a single large superbubble, but rather from the collective action of series of smaller bubbles that overlap near the centre of the simulated galaxy, where star formation occurs most vigorously. These repeated outbursts open an expanding, rarefied region near the galaxy centre, which eats away at the denser external gas through turbulent mixing, rather than gathering it into a thin, fragile shell. As a result, the rarefied region drills its way outwards almost directly vertically, following the path along which the minimum amount of material separates the bubble interior from the intergalactic medium.

- Blow-out occurs when the overpressurised bubble regions from different OB associations overlap and push their way out into the intergalactic medium, rather than when the material surrounding a single superbubble becomes Rayleigh-Taylor unstable. This means that the strength of the outflow is strongly dependent on the strength of SN driving. Weaker outflows escape the galaxy later, are much more collimated, and carry less mass, momentum, and energy, than outflows from stronger starbursts.

While each of these features are in excellent agreement with the Hα and X-ray morphology of starburst galaxies, such observations represent only a small fraction of the many detailed multi-wavelength constraints currently available. In particular, a large body of emission and absorption line spectral data can be brought to bear in understanding the physics of starbursting galaxies with varying masses and outflow strengths. We expect future comparisons with these observations to not only confirm aspects of our models, but to lead to significant refinements.

After all, the subgrid turbulence model presented here represents only a first pass at a complex and multifaceted problem. Our key point, then, is not that our models are complete, but rather that they point towards a new direction for future research. Due to the extremely short cooling times in starbursting galaxies, it will be ultimately impossible to accurately simulate them without tracking the essential cascade of random velocities that takes place between the bulk motions and the thermal scale. A full understanding of galaxy outflows will only be achieved when we have first understood and simulated the evolution of supernova-driven, supersonic turbulence.

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