Dark matter production from cosmic necklaces

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Abstract. Cosmic strings have gained a great interest, since they are formed in a large class of brane inflationary models. The most interesting story is that cosmic strings in brane models are distinguished from the standard cosmic strings in future cosmological observations. If the strings in brane models are branes or superstrings that can move along compactified space, and also if there are degenerated vacua along the compactified space, kinks interpolated between degenerated vacua become ‘beads’ on the strings. In this case, strings turn into necklaces. In the case where the compact manifold is not simply connected, a string loop that winds around a nontrivial circle is stable for topological reasons. Since the existence of the (quasi-)degenerated vacua and the nontrivial circle is a common feature of brane models, it is important to study cosmological constraints on the cosmic necklaces and the stable winding states. In this paper, we consider dark matter production from loops of the cosmic necklaces. Our result suggests that necklaces can put a stringent bound on certain kinds of brane models.

Keywords: dark matter, cosmology with extra dimensions, string theory and cosmology, cosmological applications of theories with extra dimensions

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1. Introduction

Cosmic strings have gained a great interest, since they are formed in a large class of brane inflationary models. In the context of the brane world scenario, cosmic strings are produced just after brane inflation [1,2]. It has been discussed that such strings lead to observational predictions that can be used to distinguish the brane world from conventional phenomenological models [3,13,22]. From phenomenological viewpoints, the idea of a large extra dimension [4] is important for higher-dimensional models, because it may solve or weaken the hierarchy problem. In the scenarios of large extra dimension, the fields in the standard model are localized on wall-like structures, while the graviton propagates in the bulk. In the context of string theory, a natural embedding of this picture is realized by brane construction. The brane models are therefore interesting from both the phenomenological and the cosmological viewpoints. In order to find cosmological signatures of branes, it is important to study the formation and the evolution of cosmological defects. Defects in brane models such as monopoles, strings, domain walls and Q-balls are discussed in [13]–[18], where it has been concluded that not only strings but also other defects can appear.

The purpose of this paper is to find distinguishable properties of the cosmic necklaces. We focus our attention on dark matter production from loops of cosmic necklaces. The

1 Inflation in models of low fundamental scale are discussed in [5]–[7]. Scenarios of baryogenesis in such models are discussed in [8]–[10], where defects play distinguishable roles. The curvatons might play significant roles in these models [11]. Moreover, in [12] it has been discussed that topological defects can play the role of the curvatons.

2 See figure 1.

3 See also [19].
Figure 1. If one considers only the conventional Kibble mechanism and the brane creation, the resultant cosmological defect should be cosmic strings. However, the branes (cosmic strings) may move along the direction of the internal space and may have kinks on their worldvolume, which look like ‘beads’ on the strings. Then the strings turn into necklaces, which are the hybrid of the brane creation and the brane deformation.

The evolution of the networks of cosmic necklaces was first discussed by Berezinsky and Vilenkin [20]. If one started with a low density of monopoles, one could approximate the evolution of the system by the standard evolution of a string network\(^4\). Then the authors found that if one could ignore monopole–antimonopole annihilation, the density of monopoles on strings would increase until the point where the conventional-string approximation breaks down. However, in [20] the authors leave the detailed analysis of the evolution of such systems to numerical simulations, in particular the effect of monopole–antimonopole annihilation. Later, in [21], numerical simulations of cosmic necklaces are performed, in which it has been found that the string motion is periodic when the total monopole energy is much smaller than the string energy, and that the monopoles travel along the string and annihilate with each other. In this paper, based on the results obtained in [21], we will assume that monopole–antimonopole annihilation is an efficient process. We can therefore approximate the evolution of the necklaces by the standard evolution given in [20], at least during the period between each annihilation\(^5\).

Let us consider the necklaces produced after or at the end of inflation [13]. In the case where the compact manifold is not simply connected, there is a possibility that loops wind around nontrivial circles\(^6\). If such winding states are stabilized, the simple statistical argument of a random walk indicates that the winding states on a long string loop should survive. If loops were chopped off from such strings, heavy winding states (coils or cycloops) would remain [13,23]. In this case, the nucleating rate of the winding states decreases, while the mass increases with time [22]. For example, one may simply assume that \(\chi\) always ‘increases’ with time due to the conventional expansion of the

\(^4\) In this paper, we use a dimensionless parameter \(r\), which denotes the ratio of the monopole energy density to the string energy density per unit length. The low density of monopoles corresponds to \(r \ll 1\).

\(^5\) See figure 1 in [22] and figure 2.

\(^6\) See the figure in [13].
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In this case, the evolution of the scale factor of the distance between kinks \( \chi(t) \), which is the step length between each random walk, is not assumed to be affected by the string dynamics. Then, one can easily find that the mass depends on time as \( m_{\text{coil}} \propto t^{1/4} \) during the radiation dominated epoch [22]. A similar argument has been discussed in [23]; however, the authors have assumed that \( \chi \) is a constant that does not depend on time. On the basis of this assumption, it has been concluded that the mass of cycloops depends on time as \( m_{\text{cycloops}} \propto t^{1/2} \). However, considering the result obtained in [20], it is obvious that one cannot simply ignore the possibility that \( \chi \) ‘decreases’ with time. Obviously, one cannot ignore this possibility even in the cases where the actual distance between monopoles increases with time\(^7\). In this paper, we assume that \( \chi \) depends on time as

\[
\chi(t) \propto t^{k-1},
\]

where \( k \approx 0 \) corresponds to the ‘natural’ solution \( \kappa_g - \kappa_s \approx 1 \) in [20], and \( k = 1 \) corresponds to the assumption that was made in [23]\(^8\). Then the typical mass of the winding state becomes

\[
m_{\text{coil}} \sim \left( \frac{l(t)}{\chi(t)} \right)^{1/2} m,
\]

where \( l(t) \) is the length of a loop that is chopped off from the long strings at \( t \), and \( m \) is the mass of a monopole.

Here we should note the following.

1. The necklace is similar to the standard strings when \( r < 1 \).
2. During the evolution, it is known that the networks of the strings emit loops. The typical size of the chopped strings (loops) does depend on time.
3. Then, the typical mass of the chopped loops is determined by \( r(t) \) and the length size of the loops. The mass of the coils must depend on time.
4. The winding number of the loops is conserved once it is chopped off from the long strings.

It has been claimed in [23] that cycloops pose a potential monopole problem because loops wound around a nontrivial circle behave like heavy matter at the radiation epoch. Then they have discussed that in order to avoid cycloop domination the strings must satisfy the severe constraint \( G\mu < 10^{-14} \). In their scenario, however, the authors assumed that cosmic strings can move freely in extra dimensions when dark matter is produced from their loops, and claimed that the mass of cycloops increases with time as \( m_{\text{cycloops}} \propto t^{1/2} \) if the strings obey a statistical model of random walk, and that \( m_{\text{cycloops}} \propto t \) if velocity correlation is considered. Obviously, their assumption of the free motion depends on the potential that lifts the moduli that parametrize extra dimensions. Cycloops turn into necklaces when the ‘lift’ becomes significant. Moreover, as we have stated above, the evolution of \( m(t) \) depends crucially on the value of \( k \) in equation (1.1). Therefore, the result must be re-examined if the ‘lift’ becomes significant before the dark-matter production, or the deviation from \( k = 1 \) becomes crucial. Moreover, if one wants

\(^7\) In any case, \( \chi \) is always much smaller than the actual distance between monopoles. See figure 2.

\(^8\) \( k = 3/2 \) corresponds to the ‘simplest’ (but not reliable due to the string dynamics) limit that we have mentioned above. Considering the result obtained in [20], it should be fair to say that \( k \geq 1 \) is unlikely in our set-ups.
to examine the relic density of superheavy dark matter that is produced from string loops, one cannot simply ignore frictional forces from the thermal plasma, which in some cases determines the string motion in the early Universe, as was already discussed for vortons [24]. Therefore, it should be important to consider string networks at the damped epoch if one wants to calculate the density of relic superheavy dark matter, such as coils and cycloops. Of course, one cannot simply ignore the effective potential that lifts the moduli, particularly the one that lifts the flat direction that corresponds to the string motion in the compactified space\textsuperscript{9}. If the potential stabilizes the vacuum at $t = t_p$, after this time one should consider cosmic necklaces/coils instead of cycloops. For example, if the potential is lifted by the effect of supersymmetry breaking, one can assume $t_p \sim m_{3/2}^{-1}$, where $m_{3/2}$ is the mass of the gravitino. Alternatively, in the case where the stabilization is induced by brane dynamics, one can assume that the stabilization occurs just after the string formation. Even in this case, it is natural to consider random distribution of the ‘vacua’ on the cosmic strings if brane annihilation or brane collision is so energetic that the strings have enough kinetic energy to climb up the potential hill at least just after they are produced. Moreover, in more generic models of necklaces, one can assume that monopoles are produced before string formation.

To understand the stability of the winding states, we consider necklaces whose loops are stabilized by their windings. The winding state could be a higher-dimensional object (brane) that winds around a nontrivial circle in the compactified space, or could be a non-Abelian string whose loop in the modulus space is stabilized by the potential barrier. In our previous paper [22] we have considered two concrete examples for the winding state. The first is an example of cosmic strings produced after brane inflation, and the second is an example of non-Abelian necklaces. Of course one can reconstruct the non-Abelian necklaces by using the brane language. We will comment on this issue in the appendix to clarify the origin of the frictional force acting on the necklaces.

In this paper, we consider dark matter (DM) production at the damped epoch. We have obtained distinguishable properties of the networks of cosmic necklaces.

2. Dark matter production from cosmic necklaces

2.1. Mass of the stable winding states

As we have discussed in our previous paper [22], it is appropriate to consider necklaces of $r \ll 1$ because of the efficient monopole–antimonopole annihilation. Then, the simple statistical argument of a random walk indicates that about $n^{1/2}$ of the initial $n$ monopoles on a long string could survive. Let us give a brief review of the result obtained in [22].

Here the important quantity for the necklace evolution is the dimensionless ratio $r = m/\mu d$. During the period between each annihilation, one can follow the discussions given in [20]. The equation for the evolution of $r$ is

$$\frac{\dot{r}}{r} = -\kappa_s t^{-1} + \kappa_g t^{-1},$$

where the first term on the right-hand side describes the string stretching that is due to the expansion of the Universe, while the second describes the effect of string shrinking due

\textsuperscript{9} The effective potential is supposed to be significant at $H \sim m_{\text{moduli}}$. 

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to gravitational radiation. Using the standard value from string simulations, it has been concluded that a reasonable assumption is $\kappa_g > \kappa_s$. The solution of equation (2.1) is

$$r(t) \propto t^\kappa.$$  \hspace{1cm} (2.2)

Considering the order-of-magnitude estimation, one can obtain $\kappa = \kappa_g - \kappa_s \sim 1$ [20]. Therefore, disregarding monopole–antimonopole annihilation, one can understand that the distance between monopoles ‘decreases’ as $d \propto t^{-\kappa} \sim t^{-1}$ until the conventional-string approximation is broken by dense monopoles. In our case, it should be reasonable to think that the ‘distance between monopoles’ obtained above is corresponding to $\chi$, the step length between each random walk\(^{10}\). To be precise, if $n$ monopoles are obtained disregarding monopole–antimonopole annihilation, one should obtain $n^{1/2}$ monopoles after annihilation. If the annihilation is an efficient process, $\chi$ can continue to decrease while the actual value of $r$ remains small\(^{11}\). In some cases, assuming efficient annihilation, the actual number density of the monopole may become a constant [22]\(^{12}\).

Then it is easy to obtain the mass of the stable relic [22];

$$M_{\text{coil}}(t) \sim n(t)^{1/2} m.$$  \hspace{1cm} (2.3)

Here the number of monopoles that are ‘initially’ contained in a loop is given by

$$n(t) \sim \frac{l(t)}{d(l_n) \times (t/t_n)^{k-1}}.$$  \hspace{1cm} (2.4)

In the scaling epoch one can obtain $M_{\text{coil}} \propto t$ for $k = 0 \ (\kappa = 1)$, which is similar to the result obtained from velocity correlation [23] despite the fact that we are not assuming free motion in extra dimensions.

### 2.2. Frictional force acting on strings and monopoles

A monopole moving through plasma in the early Universe experiences a frictional force due to its interaction with charged particles. The gauge and Higgs fields which have nonzero vacuum expectation value inside strings or monopoles can couple to various other fields. This results in effective interactions between the defects and the corresponding light particles. Naively, one might expect the typical length scale of the scattering cross-section to be comparable to the physical thickness of the defects, $\delta_s$ or $\delta_m$. Of course, this expectation is incorrect. The actual cross-section is determined by the wavelength of the incident particle, which means that the scattering cross-section becomes $\sigma_s \sim T^{-1}$ for strings and $\sigma_m \sim T^{-2}$ for monopoles. A rough estimate of this force is [25]

$$F_{m0} = \beta_m T^2 v,$$  \hspace{1cm} (2.5)

where $\beta_m$ is a numerical factor. Here we have assumed that a monopole is moving with a nonrelativistic velocity, $v$. A string moving through plasma in the early Universe experiences a frictional force from the background plasma. The force per unit length is [25]

$$F_s = \beta_s T^3 v,$$  \hspace{1cm} (2.6)

\(^{10}\) See figure 1 in [22].

\(^{11}\) See figure 2.

\(^{12}\) It has been suggested in [20] that the ratio $r$ might have an attractor point. Our result obtained in [22] supports this conjecture.
where $\beta_s$ is a numerical factor of order unity. The frictional force becomes negligible at
\[ t_s \sim (G\mu)^{-1}t_s, \]
where $t_s$ is the time of string formation.

Although the standard result that we have obtained above describes the essential properties of the necklaces, one should consider rather peculiar situations when the number of species of the monopoles becomes large, $N_n \gg 1$. In appendix A, we show how monopoles affect the frictional force acting on necklaces with $N_n \gg 1$.

### 2.3. Typical curvature radius $R$ and typical distance $L$ at the damped epoch

Since we are considering the efficient annihilation of monopole–antimonopole on the necklaces, we can assume $r \ll 1$. Then the frictional force acting on necklaces is given by (2.6). The damped epoch corresponds to the highest string densities and so should be important for baryogenesis, vorton formation [24] and other effects. In this paper, we consider production of superheavy states (coils) during this epoch and examine cosmological constraints. To start with, let us calculate the characteristic damping time for the necklaces. Denoting the kinetic energy per unit length and energy dispersion ratio by $\epsilon$ and $\dot{\epsilon}$, the characteristic damping time becomes
\[ t_d \sim \frac{\epsilon}{\dot{\epsilon}} \sim \frac{(r + 1)\mu v^2}{F_s v} \sim \frac{\mu}{T^3} \times \frac{r + 1}{\beta_s}. \]

Note that $t_d$ is much smaller than the Hubble time, $t \sim M_p/T^2$.

Now we can calculate the typical curvature radius $R(t)$ and the typical distance between the nearest string segments in the network, $L(t)$. The force induced by the tension of a string of curvature radius $R$ is
\[ F_t \sim \mu/R. \]

It is easy to find the approximate value of the corresponding acceleration,
\[ a_t \sim \frac{F_t}{(r + 1)\mu} \sim \frac{1}{(r + 1)R}. \]

At the damped epoch the string can be accelerated only for a time period $\sim t_d$, which suggests that the string moves with the typical velocity
\[ v \sim a_t t_d. \]

After the time period $t_d$, the force induced by the string tension is balanced by the frictional force from the background plasma. The balancing speed is obtained from the condition for the force balance;
\[ \frac{\mu}{R} \sim F_s. \]

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13 Here we disregard the ‘peculiar’ properties of the necklaces that will be discussed in appendix A.
The typical curvature radius will grow as $R(t) \sim vt \sim a(t) t$ [25]. We can therefore obtain the result

$$R(t) \sim \sqrt{\frac{t_{d} t}{r + 1}}.$$  \hspace{1cm} (2.13)

One can assume that $R(t)$ is of the same order as the typical coherence length $\xi$;

$$\xi(t) \sim R(t) \sim \left(\frac{\mu M_{P}}{T_{5}^{5} \beta_{s}}\right)^{1/2},$$  \hspace{1cm} (2.14)

which is always much smaller than the horizon size. As in the conventional scenario of the string network evolution, $R(t)$ depends on the Hubble time as $R(t) \sim t^{5/4}$. In this case, $R(t)$ grows faster than the horizon scale $t$. Therefore, as long as the evolution of the networks of necklaces is approximated by the evolution of conventional string networks, small-scale irregularities and loops of size smaller than $R$ should be damped out in less than a Hubble time. Since smaller wiggles are suppressed at the damped epoch, the typical size of the loops is $l(t) \sim R(t)$ [24].

### 2.4. Loops at the damped epoch

Our next task is to calculate the typical number density of the loops. In this case, we should take into account the low reconnection rate of the necklaces, $p \ll 1$. As we have discussed above, a necklace that has $N_{n}$ degenerated vacua on its worldvolume is macroscopically the same as a string with low reconnection rate $p \sim N_{n}^{-1}$. In this case, in order to have one reconnection per Hubble time, a necklace needs to have $\sim p^{-1}$ intersection per Hubble time. Then the number of such necklaces per volume $(vt)^{3}$ is $\sim p^{-1}$. Therefore, the mean number density of string loops becomes [24]

$$n_{l} \sim \frac{1}{p^{3} \xi^{3}}.$$  \hspace{1cm} (2.15)

For the loops to be stabilized, they must wind more than once around the nontrivial circle in their modulus space. Therefore, the loops are stabilized when loops of the length $l(t)$ contain at least $N_{n}$ monopoles after monopole–antimonopole annihilation. Then the condition for the stabilization becomes

$$l(t) > \frac{m N_{n}}{\mu r}.$$  \hspace{1cm} (2.16)

Since the typical length of the chopped loops grows with time, we may assume that the production of the stable loops starts at $t_f$. Then the mass of the coils that are produced from loops becomes

$$m_{\text{coil}}(t_f) \sim \mu r(t_f) l(t_f).$$  \hspace{1cm} (2.17)

From equations (2.15) and (2.17) one can obtain the energy density

$$\rho_{\text{coil}}(t_f) \sim \frac{\mu r(t_f) l(t_f)}{p^{3} \xi^{3}(t_f)^{3}} \sim \frac{T_{5}^{5} \beta_{s} r}{p M_{P}}.$$  \hspace{1cm} (2.18)
Since the coils behave as nonrelativistic matter, the evolution of their number density becomes

$$n_{\text{coil}}(T) \sim \frac{1}{p} \left( \frac{T_0^3 \beta_s}{\mu M_p} \right)^{3/2} T^3. \quad (2.19)$$

The situation that we are considering in this paper is quite similar to the one that has been considered to obtain the vorton relic density. Therefore, following the analysis in [24], it is easy to obtain the result

$$\rho_{\text{coil}}(T) \sim \frac{T_0^3 \beta_s r T^3}{p M_p}. \quad (2.20)$$

Obviously, the significant property of the above result is the disappearance of the string tension, $\mu$. Although the density $\rho_{\text{coil}}$ still depends on the formation time $t_f$, we should remember that $t_f$ is determined by the stabilization condition (2.16). As $t_f$ depends strongly on the initial value of $r$ and the numerical constant $k$ that controls the evolution of $r$, the cosmological constraints cannot put the direct bound on $\mu$ and $m$.

### 2.5. Cosmological constraints

Perhaps the most robust prediction of the standard cosmological particle model is the abundance of the light elements that were produced during primordial nucleosynthesis. Nucleosynthesis occurs at the temperature $T_N \sim 10^{-3}$ GeV. In order to preserve the well-established scenario of nucleosynthesis, it is necessary that the coil distribution should satisfy $\rho_{\text{coil}} < T^{-4}_N$. In our case, the condition becomes

$$T_f < 10^7 \text{ GeV} \times \left[ \frac{p}{10^{-2}} \right]^{1/2} \left[ \frac{1}{\beta_s} \right]^{1/2} \left[ \frac{10^{-3}}{r} \right]^{1/2}. \quad (2.21)$$

$\beta_s$ is a numerical factor of order unity [25]. The typical value of $p$ is $1 > p > 10^{-2}$ [26]. In the case where the number of the windings per loop is proportional to the length of the loop, $r$ is a constant. In this sense, $r$ has a fixed point in the scaling epoch, if the evolution of the necklaces is determined by the standard equation [15]. On the other hand, one can obtain $r \propto t^{-1/8}$ from (2.14) and (2.23), which suggests that $r$ is a slowly varying function during the damped epoch. The initial value of $r = m/\mu d$ is obtained if one assumes that initially $d$ is as large as the Hubble radius $H^{-1} \sim M_p/\mu$. Then, one can obtain $r_0 \sim m/M_p$. For $m \sim M_{GUT} \sim 10^{16}$ GeV, $r_0$ becomes $r_0 \sim 10^{-3}$. [16]

We now consider a stronger constraint. The stronger constraint is obtained if the winding states are sufficiently stable and can survive until the present epoch [17].

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14 In this paper, we have neglected the changes in the particle-number weighting factor $g^*$ in the temperature range under consideration.

15 See [20, 22] and figure 2.

16 The mass of the monopoles on the necklaces depends on the structure of the internal space, which is highly model dependent. If it winds around a large extra dimension, its mass becomes huge even if the fundamental scale is as low as O(TeV).

17 Coils and cycloops that wind around a nontrivial circle in the compactified space are stable for topological reasons. However, coils that are stabilized due to the potential barrier may decay by tunnelling. The lifetime of such unstable coils is determined by the potential that lifts the moduli. The peculiar cases of the unstable coils are interesting but highly model dependent.
The evolution of the necklaces between each annihilation is well described by the equation (2.1). During the period between each annihilation, the evolution of $d$ is therefore given by $d \propto t^{-\kappa} \sim t^{-1}$. Assuming that $\kappa$ is a constant during the evolution, one can understand that $\chi$ is a continuous decreasing function while the practical value of $d$ is discontinuous at each annihilation.

Following [24], one can easily obtain

$$T_f < 10^5 \text{ GeV} \times \left[ \frac{p}{10^{-2}} \right]^{1/2} \left[ \frac{1}{\beta_s} \right]^{1/2} \left[ \frac{10^{-3}}{r} \right]^{1/2}. \quad (2.22)$$

The above constraint seems already quite stringent. However, here we examine the above condition in more detail. Let us consider the case where $\chi$ evolves as $\chi \propto t^{-1}$. This assumption is appropriate both for the strings in free motion (velocity correlation has been discussed in [23]) and the necklaces (see figure 2). Then the expected winding number per loop $\langle n \rangle$ is given by

$$\langle n \rangle \sim \sqrt{\frac{\xi}{\chi}} \sim \sqrt{\frac{\mu^{1/2} M_p^{1/2} T^{-5/2} \beta_s^{1/2}}{\chi_0 \times (T^2/\mu)}} \quad (2.23)$$

where $\mu^{-1/2} < \chi_0 < M_p/\mu$ is the initial length of $\chi$ when strings are formed.$^{18}$

Having the modest assumption that $\chi_0 \sim M_p/\mu$, one can obtain the temperature $T_f$ from the equation $\langle n(T_f) \rangle \sim 1$,

$$T_f \sim \mu^{1/2} \left( \frac{\mu^{1/2}}{M_p} \right)^{1/9}, \quad (2.24)$$

$^{18} \chi_0 \sim \mu^{-1/2}$ is used in [23].
which suggests that the dark-matter production starts soon after the string formation. One can therefore understand that the effect of a small deviation from $k = 0$ is not significant for the obtained bound. Even in this case (where we have the modest assumption $\chi_0 \sim M_p/\mu$), the upper bound for $G\mu$ is about

$$G\mu < 10^{-23} \times \left[ \frac{p}{10^{-2}} \right]^{9/10} \left[ \frac{1}{\beta_s} \right]^{9/10} \left[ \frac{10^{-3}}{r} \right]^{9/10}.$$  

(2.25)

Therefore, our result (2.25) puts a severe bound on the inner structure of brane models in the case where stable coils are produced.

As we have mentioned in the previous section, the significant point is that the string tension $\mu$ has disappeared from the cosmological constraint. The obtained bound is for $T_f$, as we have discussed above. Of course, one may think that the bound is not significant because $T_f$ is determined by the dynamics of the cosmic necklaces. It is true that $T_f$ seems to depend crucially on the initial configuration and the numerical constant $k$ that controls the evolution of $r$. However, even in the case where the initial distance between monopoles is as large as the Hubble radius, and the evolution of the necklaces is determined by the standard equation (2.1), the bound we have obtained is equation (2.25), which is of course quite stringent. Moreover, the effect of a small deviation from $k = 0$ is not significant for the obtained bound, as we have discussed above.

Here we should make some comments about the discrepancy between our result and the result obtained for cycloops in [23]19. In [23], it has been claimed that cycloops poses a potential monopole problem because such loops behave like heavy matter at radiation epoch. Then they have shown that in order to avoid cycloop domination the strings must satisfy the severe constraint $G\mu < 10^{-14}$. However, in their analysis they have disregarded the damped epoch and also made a nontrivial assumption that the strings move freely in the internal space when the significant amount of the winding state is produced. In general, the damping term becomes negligible at temperatures [25]

$$T \ll T_s = G\mu M_p,$$  

(2.26)

which is always lower than $T_f$ that we have obtained in (2.24). Therefore, it is appropriate to consider the production of dark matter in the damped epoch rather than in the scaling epoch.

3. Conclusions and discussion

Cosmic strings have recently gained a great interest because they are formed in a large class of brane inflationary models. The most interesting story would be that cosmic strings in brane models are distinguished from conventional cosmic strings in future cosmological observations. It has already been discussed that such strings may lead to observational predictions that can be used to distinguish brane world from conventional phenomenological models [3,13,22]. If the strings in brane models are branes that can move along compactified space, and also if there are degenerated vacua along the compactified space, the strings turn into necklaces. Moreover, in the case where the compact manifold in not simply connected, a string loop that winds around a nontrivial

19 See also figure 3.
circle is stabilized. Since the existence of the (quasi-)degenerated vacua and the nontrivial circle is a common feature of brane models, it should be important to examine cosmological constraints on cosmic necklaces and their stable winding states. If the existence of a stable winding state is excluded, the necklace becomes a probe of the compactified space. In this paper, we have considered the production of dark matter from loops of cosmic necklaces. The bounds we have obtained are stringent. Our result suggests that necklaces may put stringent bound on brane models, as far as the standard scenario of the necklace evolution is applicable.

Finally, we will comment on the cosmological production of winding states in KKLT models, which seems far from obvious. In models that have been discussed so far in the literature, there is no winding state at the bottom of the inflation throat, because it is not required for successful inflation. Moreover, in some cases there is a (possibly large) potential barrier that blocks the strings from moving out of the throats to wind around the bulk. In models where the potential barrier is effective, one can see that nontrivial circle is hidden from the strings, which makes the production of winding states negligible. In this case, our analysis cannot put bounds on the scale. On the other hand, in the case when the potential barrier cannot block the string motion, strings can penetrate into the bulk and may produce winding states. We will leave the detailed argument about the KKLT models for future work, since the analyses in the KKLT models are highly model dependent due to the mechanisms of inflation and reheating that are still developing.

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Figure 3. This picture shows typical situations when the networks of necklaces and coils become significant. It is important to note that cycloops turn into necklaces/coils when their free motion in extra dimensions is stopped by the potential. In this sense, late-time production of PBH relics must be investigated in the framework of necklaces and coils [22].
Figure A.1. Frictional forces become important only when strings can interact with thermal plasma. In the case when thermal plasma is localized on a distant brane or a distant throat, interactions between cosmic strings and thermal plasma are suppressed by an exponential factor, thus the frictional forces should be negligible. In this case the strings must be produced somewhere at a distance while reheating occurs on the standard-model brane. One may think that the situation is peculiar; however, it is actually possible to construct models in which strings are located on a hypersurface (or throat) which is far-distant from the standard-model brane, on which reheating is induced after inflation [27]. Of course, one cannot ignore the possibility that reheating in bulk fields is not negligible at least just after inflation. One may consider another possibility that strings are produced after angled inflation [17]. In this case, cosmic strings are extended between branes, thus they can feel frictional forces from the plasma that is localized on the spacetime-filling branes. It should be noted that similar cosmic strings can be produced by later thermal phase transition in brane models, if the phase transition is explained by brane recombination.

Appendix A. Origin of the frictional force

Apart from KKLT multi-throat models\textsuperscript{20}, one may think that the thermal plasma is localized on spacetime-filling branes and cannot interact with cosmic strings. In the case where thermal plasma is localized on a distant brane or a distant throat, interactions between cosmic strings and thermal plasma are suppressed by an exponential factor, thus the frictional forces should be negligible. On the other hand, in this case one should assume that strings are produced somewhere at a distance while reheating occurs on the standard-model brane. One may think that the situation looks peculiar. Of course we know that it is possible to construct models in which reheating seems to occur only for the fields localized on branes. Thus, to understand the origin of the frictional forces,

\textsuperscript{20} If one considers KKLT multi-throat models in which standard model branes are localized at the bottom of the SM throat while inflation occurs at another throat (inflation throat), one may think that thermal plasma is localized at the bottom of the SM throat while cosmic strings are produced in the inflation throat. In this case, it seems obvious that strings cannot feel frictional forces from the plasma at a distance. However, what we are considering in this paper is the string networks just after the string production. Even if the reheating due to tunnelling is successful, the original decay products just after brane annihilation are produced in the inflation throat, where the cosmic strings are produced. In the KKLT scenarios, we need to understand more clearly the mechanism of inflation and reheating, which are highly model dependent. Thus, we will leave the detailed argument about the KKLT models for future work. See also figure A.1.
we think it is helpful to consider models in which the interactions between strings and thermal plasma on the spacetime-filling branes are obvious. For the cosmic strings that are produced at the last stage of brane inflation, we will consider strings produced after angled inflation. In angled inflation, cosmic strings are extended between branes, thus they can feel frictional forces from the plasma on the spacetime-filling branes. The same kind of cosmic strings can be produced at later thermal phase transition, if the phase transition is accompanied by brane recombination [15].

In the case where the lift of the potential is not important, one may use cycloops to obtain DM abundance. Our analysis on DM production is still useful in this case, if there are interactions between plasma. One can calculate DM production from cycloops in the damped epoch, which is consistent with our result because the evolution of the mass of the cycloops is given by \( m(t) \propto t \).

There may be models in which strings are produced at a distance from the standard-model branes. In these models, the damped epoch is highly model dependent and not obvious.

Appendix B. Non-Abelian strings in brane construction

As we have discussed in this paper, we think it is natural to consider necklaces in brane models. On the other hand, more explanations will be needed to understand whether one can construct necklaces and coils in four-dimensional gauge theory. To construct necklaces in four-dimensional gauge theory, the moduli that parametrizes the string motion in extra dimensions must be replaced by a flat direction that appears in the two-dimensional effective action on the strings. Let us consider the dynamics of cosmic strings living in a non-Abelian \( U(N_c) \) gauge theory that is coupled to \( N_f \) scalar fields \( q_i \), which transform in the fundamental representation [29];

\[
L = \frac{1}{4e^2} \text{Tr} F_{\mu\nu}F^{\mu\nu} + \sum_{i=1}^{N_f} D_\mu q_i D^\mu q_i^\dagger - \frac{\lambda e^2}{2} \left( \sum_{i=1}^{N_f} q_i \otimes q_i^\dagger - v^2 \right)^2. \tag{B.1}
\]

One can see that \( SU(N_f) \) flavour symmetry appears in the above Lagrangian that rotates the scalars. Then, it is possible to include explicit symmetry breaking terms in the Lagrangian, which breaks global flavour symmetry. The most obvious example is a small mass term for the scalars;

\[
V_{br1} \sim \sum_i m_i^2 q_i^\dagger q_i \tag{B.2}
\]

which shifts the vacuum expectation value to [29]

\[
q_i^a = \left( v^2 - \frac{m_i^2}{\lambda e^2} \right)^{1/2} \delta_i^a. \tag{B.3}
\]

Then, an Abelian vortex in the \( i \)th \( U(1) \) subgroup of \( U(N_c) \) can be embedded, whose tension becomes \( T_i \sim (v^2 - (m_i^2/\lambda e^2))^{1/2} \).

---

21 See figure 1 in [17] and figure 1 in the first paper in [15].
Figure B.1. This picture shows a typical brane construction of non-Abelian strings. String parts A and B in the right-hand side correspond to D2 on A and D2 on B. A kink that interpolates between A and B is a monopole on the necklaces. If one compactifies the $x^2$ direction on $S^1$ with radius $\rho$, one can take T-duality [28]. In this case, one can consider motion in the $x^2$ direction, which induces windings that are required to stabilize loops of the necklaces. In the four-dimensional effective action, the effective potential for $\rho$ will have a high barrier near the origin as expected in [22].

One may extend the above model to $N = 2$ supersymmetric QCD or simply include an additional adjoint scalar field $\phi$. Then, the typical potential for the adjoint scalar is given by

$$V_{br2} \sim \sum_i q_i^\dagger (\phi - m_i)^2 q_i. \quad (B.4)$$

Be sure that the potential breaks $U(1)_R$ symmetry, and the tensions of the strings degenerate in this case.

Alternatively, one can consider supersymmetry-breaking potential that could be induced by higher-dimensional effects,

$$V_{br3} \sim \sum_i q_i^\dagger (|\phi|^2 - m^2) q_i, \quad (B.5)$$

which preserves $U(1)_R$ symmetry. In this case, due to the D-flatness condition if supersymmetry is imposed, the vacuum expectation value of the adjoint field is placed on a circle and given by [15]

$$\phi = m \times \text{diag}(1, e^{2\pi/N_c}, e^{(2\pi/N_c)\times2}, \ldots, e^{(2\pi/N_c)\times(N_c-1)}). \quad (B.6)$$

One can break the remaining classical $U(1)_R$ symmetry by adding an explicit breaking term, or by anomaly [15, 29].

In any case, strings living in different $U(1)$ subgroups can transmute each other by kinks (walls on their 2D worldvolume) that interpolate between (quasi-)degenerated vacua.

Now our main concern is whether it is possible to construct ‘winding’ states from non-Abelian necklaces, which look like coils in brane models. A similar argument has already been discussed by Dvali et al [30] for the $Z_2$ domain wall in four-dimensional theory. The authors have discussed that similar ‘windings’ may stabilize the wall–antiwall
bound state if the potential is steep in the radial direction. Of course, one can apply a similar argument to non-Abelian necklaces. In our case, windings can be stabilized if (for example) the origin is lifted by an effective potential $\sim \phi^{-n}$. The important point here is whether the absolute value of the scalar field can vanish inside the bound state of walls (kinks). If ‘windings’ of such kinks cannot be resolved due to the potential barrier near the origin, the naive annihilation process is inhibited and the stable bound state will remain.

Of course, it is straightforward to construct the brane counterpart of the non-Abelian necklaces [29]. A typical brane construction is given in figure B.1.

Appendix C. More on frictional forces acting on necklaces

As long as the coefficient $\beta_m$ does not much exceed $\beta_s$, the drag force acting on monopoles does not play a crucial role. In general, the frictional force acting on necklaces is comparable to (2.6). However, in the case where the magnetic charge of neighbouring monopoles is originated from different $U(1)$ s, the frictional force acting on monopoles is induced by different species of particles. Then one should sum up the frictional force acting on monopoles. In this case, one should consider three phases that correspond to the following situations.

(1) The frictional force acting on necklaces is given by the formula (2.6). In this phase, one can neglect drag force acting on monopoles.

(2) The frictional force is dominated by the drag force acting on monopoles, but the force is still not saturated.

(3) The monopoles become dense and the cross-section of the monopoles of the same kind begins to overlap. In this case, the frictional force is saturated and looks qualitatively the same as (2.6).

Let us first consider the boundary between phases 1 and 2. As we have stated above, the frictional force acting on strings is given by equation (2.6). The drag force per unit length, which is induced by the frictional force acting on the monopoles, is

$$F_m = F_{m0} \times N_m = \beta_m T^2 v N_m,$$

where $N_m$ is the number density of the monopoles per unit length,

$$N_m = \frac{\mu r}{m}.$$  \hspace{1cm} (C.2)

Therefore, the condition $F_s < F_m$ that is required for the monopoles to dominate the frictional force becomes

$$T < T_{12} \equiv \frac{\mu r \beta_m}{m \beta_s}.$$  \hspace{1cm} (C.3)

Here $T_{12}$ is the boundary between phases 1 and 2.

Let us consider the boundary between phases 2 and 3. The cross-section between the monopoles of the same kind begins to overlap when

$$d < 1/(TN_n),$$  \hspace{1cm} (C.4)
where \(N_n\) is the number of species of monopoles\(^{22}\). Therefore, the boundary between phases 2 and 3 is given by

\[
T_{23} \equiv \frac{\mu r}{m N_n}. \tag{C.5}
\]

In phase 3, the frictional force is saturated and becomes

\[
F_n \sim \beta_n T^3 v N_n, \tag{C.6}
\]

which is qualitatively the same as equation (2.6). Obviously, phase 2 becomes important in models with \(N_n \gg 1\).

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\(^{22}\) The value of \(N_n\) becomes about \(N_n \sim 10^{2-3}\) in angled inflation.
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