Cherenkov radio pulses from EeV neutrino interactions: the LPM effect

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Abstract

We study the implications of the LPM effect for the Cherenkov radiation of EeV electromagnetic showers in the coherent radiowave regime for ice. We show that for showers above 100 PeV the electric field scales with shower energy but has a markedly narrower angular distribution than for lower energy showers. We give an electric field frequency and angular spectrum parameterization valid for showers having energy up to the EeV regime and discuss the implications for neutrino detectors based on arrays of radio antennas. Implications of the LPM effect for underwater neutrino detectors in project are also briefly addressed.

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1. INTRODUCTION

Neutrino detection is one of the most exciting challenges in particle astrophysics. Low energy neutrinos from astrophysical environments have already been detected from the sun and from the supernova explosion SN 1987A, opening up a new window to the Universe. High energy neutrinos have to be produced in the interactions of cosmic rays with both matter or radiation and are also very likely to be produced by whatever mechanism that accelerates protons or nuclei to cosmic ray energies. Gamma Ray Bursts, Active Galactic
Nuclei and topological defect annihilation are sites where hadrons may be accelerated to energies above $10^{19}$ eV, but this is still a matter of speculation and intense research [1]. Many of the proposed models for acceleration in these sites also predict distinct neutrino fluxes which extend above the EeV range [2]. In any case cosmic rays above few $10^{19}$ eV (believed to be extragalactic) must produce high energy neutrinos in their interactions with the cosmic microwave background, whatever their origin, at flux levels that are challenging low [3]. Neutrinos of energies as high as observed in cosmic rays can be expected at flux levels which will be uncertain until the cosmic ray acceleration mechanism is identified and well understood [4]. Their detection would provide extremely relevant information for the establishment of the origin of the highest energy cosmic rays.

There are several “conventional” projects to instrument large volumes of water or ice with photodetectors [5] (two in operation [6]). They can detect Cherenkov light exploiting the long range of upcoming muons produced in charged current neutrino interactions. The search of EeV neutrinos with these detectors implies the detection of showers. As the earth is opaque to EeV neutrinos only vertical downgoing to horizontal neutrinos can be observed. The atmospheric muon background must be eliminated to identify neutrino induced muons, looking for energetic showers developed along the muon track. Alternatively the Cherenkov light can be also detected from the showers directly produced in the neutrino interactions. This is the Cherenkov light emitted incoherently by shower particles in the optical band.

The search for coherent radio pulses generated in ice by the charge imbalance in neutrino induced electromagnetic showers provides an interesting alternative, [7] known since the 1960’s [8], that may turn out to be more cost effective at EeV energies [9]. The Cherenkov radiation of shower particles is coherent when the wavelength of the radiation is greater than the physical dimensions in the shower. The radiated electric field becomes proportional to the square of the excess charge that develops in the shower and the power in the radio emission scales with the square of the shower energy [8]. Simulations of radio pulses from these showers have been made up to 10 PeV [10] which indicate that only neutrinos above few PeV can be detected by a single antenna at distances in the km range. In spite of experimental
difficulties still unexplored [11], the technique is very attractive to detect neutrinos of energy in the EeV range and above and there are efforts under way to test its viability [12]. The mechanism also suggests that it may be possible to detect neutrino radio pulses produced under the moon surface using radiotelescopes on earth [13].

The development of electromagnetic showers in dense media is of utmost importance for the detection of EeV neutrinos from both conventional underground and radio pulse detectors. The highest energy showers generated along a muon track are mostly due to bremsstrahlung because it has the hardest cross section and hence they will be electromagnetic. In charged current electron neutrino interactions the electron at the lepton vertex carries on average most of the energy [14] and the induced shower will also have a dominant electromagnetic character. Electromagnetic EeV showers in water or ice are dramatically affected by the Landau Pomeranchuck Migdal effect (LPM) [15,16] which is a manifestation of the collective electric field of matter at large distances [17,19]. In this work we consider the development of EeV electromagnetic showers in water and the implications for Cherenkov radiation both in the incoherent (optical) and coherent (radio pulses) regions which are respectively relevant for the conventional underground muon detectors and the radio technique for EeV neutrino detection. Hadronic showers will be less affected by the LPM effect and will be addressed elsewhere.

II. THE LPM EFFECT

The LPM effect is due to the peculiar bremsstrahlung and pair production kinematics for which the average interaction distance is proportional to interaction energy [20]. When the energy of the incoming photon or electron particle gets sufficiently high, the interaction distance becomes comparable to the interatomic spacing and collective atomic and molecular effects affect the static electric field responsible for the interaction. This naturally introduces an energy scale above which these effects become significative, \( E_{LPM} \) [18] which is highly dependent on the density of the medium. The result is a dramatic reduction in both total
cross sections with lab energy \((E)\) which drop like \(E^{-0.5}\) above \(\sim E_{LPM}\). It also suppresses the central part of the differential cross section for pair production (where the electron and positron carry similar fractions of the incoming photon energy) and cuts the cross section off for bremsstrahlung of very low energy photons \([17,18]\).

As a result the mean free path of an EeV photon or electron is considerably larger than for PeV energies and the shower develops further away from injection. This is irrelevant for neutrino detection because the low cross sections involved make most points around the detector equally probable for a neutrino interaction. Secondary interactions will also be separated by depth intervals that are considerably longer than the radiation length as long as they are induced by particles of energy above \(E_{LPM}\). This elongates the shower’s depth distribution with respect to lower energy showers. At EeV energies there is also a tendency for photons to pair produce electron-positron pairs with a markedly leading particle (more likely to have energy above \(E_{LPM}\)), which contributes to elongate the shower even further. The resulting showers are markedly different from standard showers obtained in shower theory \([20]\). As the interaction length of high energy particles is larger than in standard showers, showers affected by LPM fluctuate more than conventional ones; this can compromise the possibility of shower energy reconstruction.

III. SHOWER SIMULATIONS

The brute force simulation of radio pulses from EeV showers involves following particles through at least twelve orders of magnitude in energy because half of the tracklength in an electromagnetic shower in water is due to particles below 3 MeV, which produce interference effects that affect the overall angular and frequency distribution of the pulse \([10]\). We have developed a fast 3D Monte Carlo for electromagnetic showers in ice to run at the highest energies based on the one described in \([10]\). The Monte Carlo is combined with shower parameterizations to achieve reasonable computing times avoiding thinning, because of possible biasing of the results due to interference effects. We follow in detail particles
of energy above a threshold $E_{th}$, which is always chosen to be well below $E_{LPM}$, typically 100 TeV to run at the highest energies. We use the Greisen’s parameterization for the subthreshold showers to obtain the particle depth distributions. Lateral distributions and tracklengths are calculated using the NKG parameterization \cite{21} and the tracklength results obtained in \cite{10}.

Intermediate shower results have been checked for consistency and compared to full simulations when the primary energy is not impractically large. The full simulations are consistent with simulations using $E_{th}$ up to energies below $E_{LPM}$, which constitutes a test of the parameterizations used. Results are not significantly affected by changes in $E_{th}$. Moreover they are in agreement with other work on LPM showers \cite{18,19}.

The main differences with ordinary showers can be summarized as follows:

1) As discussed in section II, showers of energy above $E_{LPM}$ display an elongated depth distribution.

2) The total tracklength and the difference of electron and positron tracklengths projected onto the shower axis (weighted projected tracklength) are very little affected by the LPM effect, that is they continue to scale with primary energy to very high accuracy. This is not surprising since only high energy particles suffer LPM effect and most of the tracklength is due to low energy particles in agreement with earlier discussions \cite{22,13}.

3) The depth distribution around shower maximum becomes considerably longer than the Greisen parameterization but the maximum number of particles $N_{\text{max}}$ becomes lower. The integral in depth of the (excess) number of particles can be related to the (weighted) tracklength. The area under the depth distribution curve remains proportional to shower energy to a very good approximation. We conveniently introduce shower length as depth over which a shower has more particles than a given fraction of $N_{\text{max}}$. The results for the average shower length are displayed in Fig. 1. Significant deviations from Greisen showers are observed when the energy exceeds $\sim 10 E_{LPM}$. Beyond this point the rise in length with shower energy $E$ can be approximately parameterized as $E^{1/3}$.

4) The showers display a multi-peak longitudinal structure which is due to the
change in the LPM differential pair production cross section with energy, as well as to the total cross section decrease for both pair production and bremsstrahlung [17].

5) The showers show a lateral distribution which is consistent with the NKG parameterization for values of the age parameter $s$ close to $s = 1$.

6) Fluctuations are in general a great deal larger than in regular showers and they get enhanced with energy. Shower fluctuations to $N_{\text{max}}$ larger than the average value correlate with shorter cascades. These are strong correlations that can be related to point 3). Fig. 2 displays the distributions of shower length as defined in 3) for a sample of 200 showers illustrating the enhancement in the large length tail as the shower energy rises.

IV. CHERENKOV RADIATION

The implications for optical Cherenkov emission (incoherent regime) are straightforward. The total light output is proportional to the tracklength and continues to scale with shower energy also for showers where the LPM effect is very important. When this behavior is combined with the shower length increase discussed in the previous section, the light output per unit area is expected to rise as $E^{-2/3}$. As the length becomes comparable or even longer than the typical attenuation length of light in water or ice, the effective volume for shower detection of an underground neutrino telescope is increased accordingly. Reconstruction of shower energy will however require that the shower is contained in the detector so that the total light output can be sampled. For showers totally or partially outside the instrumented volume the fluctuations in $N_{\text{max}}$ will make energy reconstructions much more uncertain.

The consequences for radio detection are more subtle. The radio pulse spectrum has a complicated shape which can be interpreted as a Fraunhofer diffraction pattern of the charge excess in shower as discussed in ref. [10]. Basically the electric field spectrum rises linearly with frequency until the destructing interference effects start to take place inducing a maximum at $\nu_{\text{max}}$. For pulses emitted at the Cherenkov angle with respect to shower axis, this interference is governed by the lateral distribution of the shower, the narrower
the shower lateral distribution the higher $\nu_{max}$. This frequency plays the role of an upper limit of the frequency integral for the total energy of the pulse. The angular distribution displays a diffraction pattern around the Cherenkov angle with a half width of the central peak of $\lambda/L$ where $L$ is shower length (using $\sim 0.7 N_{max}$, see Fig. 2). As the shower length rises only logarithmically with shower energies below $E_{LPM}$, the angular spread of the pulse practically only depends on frequency for such energies. Finally the normalization of the spectrum is determined by the weighted projected tracklength: the difference of tracklength of negative and positive charges after projection onto the shower axis, because they interfere destructively.

Several cross checks have been implemented to further confirm these interpretations. Firstly the Monte Carlo threshold is raised to energies orders of magnitude above the nominal 3 MeV. The resulting showers have shorter tracklengths (in agreement with the tracklength-threshold relation provided in Fig. 5 of ref. [14]) and much steeper lateral distributions but the depth distributions still display the typical multi peaked and elongated shapes and the same enhancement associated to LPM showers [17]. The radiopulses generated have a higher $\nu_{max}$ than those from showers with thresholds in the MeV range because of the steepness of the lateral distribution. The electric field at the Cherenkov angle remains proportional to the weighted tracklength reduction. The width of the pulse angular distribution around the Cherenkov angle becomes smaller than those obtained in the full simulation of lower energy showers because of the enhancement in shower length, this effect is truly due to the LPM effect and is observed not to depend on the $E_{th}$.

As an alternative check we have artificially enhanced the LPM effect in the medium, lowering $E_{LPM}$. This allows simulations with a threshold in the nominal MeV range, for lower energy showers displaying (fictitious) LPM characteristics. Fig. 1 also illustrates the rise in shower length for energies above $\sim 10 E_{LPM}$ which we have chosen to be $\sim 10$ TeV in this case. At the Cherenkov peak the generated radio pulse scales with weighted tracklength and the angular width of the diffraction peak is reduced in proportion to the increase in shower length. The value of $\nu_{max}$ remains at a similar value to a real shower of
the same energy (not affected by the LPM). All the results confirm the diffraction pattern interpretation. Moreover the field spectrum at angles close to the Cherenkov angle can be reproduced with a Fourier transform of the depth distribution. Results will be presented elsewhere.

The implications of the LPM effect for relevant shower parameters have been discussed in section III and can be used to infer with confidence the pulse characteristics for large electromagnetic showers that are strongly affected by the LPM effect. In summary the electric field value at the Cherenkov peak for EeV showers continues to scale with the shower energy as for ordinary showers below $E_{LPM}$ and the electric field can be parameterized as [10]:

$$R|\vec{E}(\omega, R, \theta_C)| = 1.1 \times 10^{-7} \frac{E_0}{1 \text{ TeV}} \frac{\nu}{\nu_0} \frac{1}{1 + 0.4\left(\frac{\nu}{\nu_0}\right)^2} \text{ V MHz}^{-1}$$

where $\vec{E}$ is the electric field, $R$ is the observation distance and $E_0$ is the shower energy, with $\nu_0 \simeq 500$ MHz. The angular distribution width at the Cherenkov peak is reduced in proportion to $E_0^{1/3}$ for energies above $\sim 10 \times E_{LPM}$:

$$E(\omega, R, \theta) = E(\omega, R, \theta_C) e^{-\ln^2\left[\frac{\theta - \theta_C}{\Delta \theta}\right]^2} \text{ with } \Delta \theta \simeq \begin{cases} 2.7^\circ \frac{\nu}{\nu_0} E_0^{-0.03} & \text{for } E_0 < 1 \text{ PeV} \\ 2.7^\circ \frac{\nu}{\nu_0} \left[\frac{E_{LPM}}{0.14 E_0 + E_{LPM}}\right]^{0.3} & \text{otherwise} \end{cases}$$

(2)

Fluctuations in the angular spread can be easily deduced from fluctuations in shower length.

In the absence of attenuation of the pulses, a simple relation can be obtained for the maximum distance from which a shower can be detected. This distance scales with shower energy for given antenna parameters [10]. The relation can be obtained dividing the power in the signal at the Cherenkov angle where it is maximum by the noise power. If we characterize an antenna reception system by its operating frequency $\nu_{op}$, its bandwidth $\Delta \nu$ and its equivalent white noise temperature $T$ (in K) and we demand a signal-to-noise ratio given by $s/n$, the following distance to shower energy relation can be obtained [10]:

$$E_0(\text{PeV}) \simeq 5 f \sqrt{\frac{s/n}{T}} \frac{1 + 1.6 \times 10^{-6}[\nu_{op}(\text{MHz})]^2}{\sqrt{\Delta \nu(\text{MHz})}} R(\text{km})$$

(3)
We have assumed that the bandwidth is small compared to the frequency of observation and that the effective area of the antenna is $\lambda^2/8$ as for a half wave dipole \[11\]. Somewhat larger ranges can be obtained with a ”TEM horn” \[23\] or biconical antennas \[24\]. The factor $f$ is a fractional reduction of the electric field due to observation away from the Cherenkov direction, detection within half width corresponds to $f = 2$.

A signal-to-noise ratio of 10:1 in a single antenna operating at 1 GHz with bandwidth $\sim 100$ MHz and equivalent noise temperature of 300 K (consistent with measurements \[25\]) would detect showers above 70 PeV at distances below 1 km. Corresponding energy thresholds for lower frequencies and bandwidths are given in table I. At lower frequencies the loss in the relation comes because of the reduced bandwidth implied by lower operation frequencies.

For a power reduction by a factor $\eta = e^{2\alpha R}$ with $\alpha$ the attenuation coefficient, Eq. (3) has an extra factor of $\sqrt{\eta}$. Measurements of the attenuation distance in ice cores \[26\] are tabulated in table II as the distance at which the power spectrum is halved for several frequencies. This attenuation length increases as the temperature drops. Final results however should be corrected when measurements are performed ”in situ”. Absorption in the ice prevents detection of pulses from EeV showers at distances where otherwise the signal-to-noise ratio requirement would be well satisfied. In any case it is reasonable to assume that the range of one of these receivers could cover practically the whole Antarctic sheet depth for EeV showers.

V. IMPLICATIONS FOR A LARGE RADIO ANTENNA COMPLEX

A large effective volume for neutrino detection can be obtained with an array of such receiver systems. The separation between antennas is a critical parameter for estimating the viability of the idea and it is directly related to the angular spread of the pulse at the Cherenkov peak \[27\]. The most important implication of the LPM effect for these showers is precisely the reduction in angular width. The pulses will become significantly
narrower for energies above $\sim 20$ PeV and consequently the antenna separation should become smaller. The constraint becomes most severe at the operation frequencies used in [10]. A 20 EeV shower will emit a pulse with an angular half-width reduced by about a factor of 10 with respect to PeV showers, i.e. about 2 mrad at 1 GHz. Fortunately the width rises as the operating frequency decreases, making lower frequencies more appropriate for detection of radio pulses from EeV showers. The significant reduction in the power received by each antenna when considering low frequencies, associated to the frequency dependence of Cherenkov radiation, is compensated by the fact that the pulse power scales with the square of shower energy. Also there is an extra benefit in going to lower operational frequencies because the lower frequency component is less attenuated and is less sensitive to temperature [11].

For an actual design the array parameters such as antenna separation, noise level, operating frequency and bandwidth of each antenna are to be optimized for largest effective volume and minimal cost. The optimization will depend strongly on the shape of the neutrino flux to be measured, but it is clear that EeV neutrino sources can allow sparse arrays of antennas in the 100 MHz range and below, which may turn out to be cheaper. Multichannel measurements may be advisable for several reasons. Firstly they will reduce background noise (mostly man made) and secondly they will allow a better reconstruction of the pulse angular-frequency spectrum. The distributions of the electromagnetic pulses in the array can be fitted to theoretical predictions, what should allow energy reconstruction in a clear analogy to extensive air shower arrays.

In summary EeV showers produce electric pulses that scale with energy in the Cherenkov direction but have a significantly narrower angular distribution. The technique continues to be promising from the point of view of EeV neutrinos if lower frequencies are used, and may allow the detection of the low flux expected from the interactions of cosmic rays with the cosmic microwave background. Such a detector could also improve bounds on different models for cosmic ray acceleration and open the possibility to further explore the Universe in this energy range. It should be stressed however that the technique has not yet been proven
and that difficulties are anticipated [11]. The attractive prospects for neutrino detection however justify the experimental effort that is required to test these ideas.

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**FIGURE CAPTIONS**

**Figure 1:** Average shower length (defined in text) versus shower energy for showers above $E_{LPM} = 2$ PeV in ice. The dotted lines represent the effect of introducing showers with $E_{LPM} = 10$ TeV. From top to bottom curves correspond to shower length defined with $\sim 0.1 \ N_{max}$, $\sim 0.5 \ N_{max}$ and $\sim 0.7 \ N_{max}$. Dashed lines are corresponding shower lengths using Greisen’s parameterization.

**Figure 2:** Shower length distributions illustrating the increased fluctuations with energy using fractions of 0.1 (solid), 0.5 (dashed) and 0.7 (dot-dashed).

**Figure 3:** Angular distribution of radiopulse around the Cherenkov angle ($56^\circ$ for radiofrequencies) at 300 MHz and 1 GHz, for three different primary energies $E_0$.

**TABLES**

| $\nu_{op}$ | 10 MHz | 100 MHz | 1 GHz |
|------------|--------|--------|------|
| $\Delta \nu$ | 1 MHz | 10 MHz | 100 MHz |
| 265 PeV | 85 PeV | 70 PeV |

**Table I:** Energy thresholds for detection of radiopulses for different operation frequencies and bandwidths.

| $T$ | $\nu$ | 10 MHz | 100 MHz | 1 GHz |
|-----|-------|--------|--------|------|
| $-60^\circ$C | 4.4 km | 2.6 km | 440 m |
| $-40^\circ$C | 660 m | 450 m | 265 m |
| $-20^\circ$C | 260 m | 130 m | 70 m |

**Table II:** Distance at which power spectrum is halved in ice for different frequencies and ice temperatures.
Shower Length (Rad. Lengths)

Figure 1

Energy (E/TeV) vs. Length (l/TeV)

- Bethe-Heitler
- E_{\gamma\gamma} = 2 PeV
- E_{\gamma\gamma} = 10 TeV
- E_{\gamma\gamma} = 100 TeV
- E_{\gamma\gamma} = 1000 TeV
Figure 3

$\text{Mod}(E(\omega))$ (arbitrary units)

$E_0 = 1 \text{ EeV}$
$E_0 = 100 \text{ PeV}$
$E_0 = 1 \text{ GHz}$

Thick $\rightarrow \nu = 300 \text{ MHz}$
Thin $\rightarrow \nu = 1 \text{ GHz}$