Interruption flows for reliability evaluation of power distribution networks

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Abstract

Energy networks should strive for reliability. How can it be assessed, measured, and improved? What are the best trade-offs between investments and their worth? The flow-based framework for the reliability assessment of energy networks proposed in this paper addresses these questions with a focus on power distribution networks. The framework introduces the concept of iflows, or interruption flows, which translate the analytical reliability evaluation into solving a series of node balance equations computable in linear time. The iflows permeate the network, providing relevant information to support linear formulations of reliability optimization problems. Numerical examples showcase the evaluation process obtained through iflows in illustrative distribution networks with distributed generation. A new visual representation of the reliability state, called iflow diagram, provides insights into the most vulnerable regions of the network. The methodology was validated by a practical application of the iflows on the optimal allocation of switches in power distribution systems. Computational experiments were conducted using a benchmark of distribution networks, having up to 881 nodes. The results confirm the effectiveness of the approach in terms of providing high-quality information and optimal trade-offs to aid reliability decisions for energy networks.

Keywords Reliability · Power distribution · Network flows · Switch allocation · Integer programming

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Extended author information available on the last page of the article
1 Introduction

The reliability assessment and management of power distribution systems have attracted the attention of governments, utility providers, and the scientific community (Escalera et al. 2020). There are many reasons for this: approximately 70% of the duration of power supply interruptions originate in distribution networks (Billinton and Allan 1996); outages affect the revenues of utilities and shareholders because of the cost of unsupplied energy; continuity standards are established by regulatory agencies, imposing low interruption frequency and duration indices; and mathematical models and efficient methodologies for the planning, operation, and maintenance of smart grids are under development.

The likelihood of consumers being disconnected from their power supply because of outages can be reduced by investing in many areas, e.g., the maintenance of electrical components, location of response teams, signaling devices, allocation of distributed, energy storage devices, and switches. Computation of the incremental cost of reliability, i.e., the ratio of reliability cost and reliability worth, is an effective tool for determining the economic viability of an investment. Ensuring that limited capital resources can achieve the best possible outcome requires computationally efficient methodologies that can determine each investment’s reliability worth.

Chismant (1998) discussed the lack of accurate and efficient methods for evaluating reliability. He proposed a simulation approach for determining the reliability of a utility system. The methodology, despite allowing accurate computation of time-between-failure (TBF) and time-to-repair (TTR) distributions, was computer-time intensive. Heydt and Graf (2010) proposed a Monte Carlo approach to determine the reliability of distribution systems. Computational experiments in which their methodology was compared with an analytical reliability procedure showed that the accuracy of the simulation was relatively high. Rocha et al. (2017) evaluated the effect of distributed generation (DG) on the reliability of distribution networks. The effects of component failures and islanding operations with respect to voltage and frequency variations are considered in their model, which uses simulation to evaluate the reliability and assess the dynamics of the islanding process. In the same context, Conti et al. (2012) and Adefarati and Bansal (2017) proposed probabilistic models to address the stochastic behavior of renewable DG resources and their impact on reliability.

A non-simulated approach was proposed by Muñoz-Delgado et al. (2018) formulating the reliability assessment of a distribution network as a linear programming optimization problem solving a set of shortest paths in the network. This model was later improved by Tabares et al. (2019), who proposed a set of linear expressions which can be solved without requiring optimization. Li et al. (2020b) included in their analytical reliability evaluation methodology the assessment of post-fault network reconfiguration. In their experiments, the authors observed that optimal network reconfiguration allows load transfer between feeders and significantly improves the reliability of the network. Another facet explored by Jooshaki et al. (2020) is the switching interruption times, which derive from the isolation
of a faulty portion of the network. This is formulated as a mixed-integer programming model, and the computational tests have shown that their model improved the accuracy of the reliability evaluation by considering switching interruptions.

An assumption of the methodologies proposed by Muñoz-Delgado et al. (2018), Tabares et al. (2019), Li et al. (2020b), and Jooshaki et al. (2020) is that all branches of the network are equipped with a switch; this could preclude applications in which the positions of the switches are decision variables. This requirement was lifted by Li et al. (2020a) in their optimization-based reliability evaluation methodology. They use the concept of fictitious flows, representing the path taken by a fault in the network, calculated by solving an integer linear programming model.

Juanwei et al. (2019) assess the reliability of integrated energy systems. They propose an analytical method considering the interdependencies between power distribution and gas distribution subsystems, which was shown to be more efficient than simulation approaches. Comprehensive surveys on computational methodologies for the reliability assessment of distribution systems are provided by Borges (2012) and Lin et al. (2014).

The demand for fast and accurate reliability evaluation methodologies has been increasing in smart distribution networks (Ghiani et al. 2018). These networks, commonly referred to as smart grids, use information and communication technologies, and state estimations of the network to perform automated actions to improve reliability, efficiency, and sustainability of the use of electricity. For example, the reliability can be enhanced with the so-called self-healing of the network, provided by automatic reconfigurations and the capability to perform islanding operations.

**Overview and contributions** This research introduces the concept of interruption flows, iflows for short, to address the growing demand for a fast analytical reliability evaluation methodology. The iflows express reliability states and unfold new insights into how the interruption circulates throughout a distribution network. To grasp this new concept, we first provide the terminology and fundamentals on distribution networks, reliability indices, and the standard computation of interruption duration (Billinton and Allan 1996) in Sect. 2.

The iflows are formally defined in Sect. 3 with a parallel to the idea of fictitious flows by Li et al. (2020a). The iflow computation algorithm is provided in Sect. 3.3 and it was shown to run in linear-time on the size of the distribution network.

In Sect. 4 we provide mathematical proofs showing the equivalence between iflows and the standard reliability evaluation approach (Lemmas 1, 2, and Theorem 1). From this mathematical equivalence, we have determined lower and upper bounds (Lemma 3), which describe the worst and best reliability scenarios for a distribution network (discussed in Sect. 4.2).

In Sect. 5, we propose iflow diagrams, an original way to represent the reliability state of a distribution network. Through the iflow diagram, one can rapidly identify the most vulnerable areas of the network. Moreover, we use iflow diagrams to illustrate the benefits of a suitable allocation of switches.

The application of iflows to meshed networks is discussed in Sect. 6, which includes an illustrative example of a network with a normally-opened switch and DG.
To validate our methodology as an effective way to model reliability, in Sect. 7 we employ iflows on a practical application, which is to find the optimal locations of sectionalizers (normally-closed switches) to minimize the expected energy not supplied. Computational experiments are conducted on a benchmark of distribution networks with up to 881 nodes. The results show our model’s effectiveness to find optimal and near-optimal solutions for all networks, considering different numbers of switches. The model was also capable of finding optimal trade-offs to support decisions on the best compromises between budgets and reliability.

In summary, this paper provides new mathematical and computational concepts that push on the current body of knowledge regarding exact reliability evaluation and optimization methodologies for energy systems. The iflows compare favorably with the literature in what follows:

- They provide meaningful knowledge of the reliability state of the network which can be visually represented by iflow diagrams;
- The iflows generalize the fictitious flows proposed by Li et al. (2020b), expressing local reliability states and indicating the most vulnerable areas of the network.
- To the best of our knowledge, it is the first time that reliability lower and upper bounds describe the worst and best reliability scenarios for a distribution network.
- The iflows lift the requirement of having a switch in every branch of the network (an assumption adopted by Muñoz-Delgado et al. (2018), Tabares et al. (2019), Li et al. (2020b), and Jooshaki et al. (2020)), which could preclude some applications;
- A straightforward evaluation algorithm was developed, with a time complexity that increases linearly with the size of the network; for instance, this is asymptotically better than solving a set of linear systems of equations, as required by previous analytical approaches (Muñoz-Delgado et al. 2018; Tabares et al. 2019; Li et al. 2020b; Jooshaki et al. 2020).
- Rigorous mathematical proofs ensure the correctness of the iflows being translated to reliability indices;
- The linear descriptions of reliability indices, provided by the iflows, allow strengthening the mathematical formulations of reliability optimization problems; for instance, our model for the switch allocation problem was able to tackle instances of size up to 881 nodes, while Abiri-Jahromi et al. (2012) and Farajollahi et al. (2019) could solve networks up to 144 nodes.

2 Terminology and definitions

2.1 Network representation and terminology

A radially operated distribution system can be modeled as a directed tree (arborescence) $G(V, A)$, rooted at the substation (Node 0) (Ahuja et al. 1993). Each node $i \in V \setminus \{0\}$ denotes a load point with power load $l_i$ (kW), failure rate $\lambda_i$ (failures/year),
and number of customers \(n_i\). An arc, \((i, j) \in A, i, j \in V\), is oriented in the same manner as the power flow, i.e., from the root to the customers. Each node \(j \in V \setminus \{0\}\) has a predecessor node \(i\), or simply, \(i = \text{pred}(j)\). The set of arcs that contain switches is denoted by \(A_s\).

A unique directed path connects the root to every node in the tree. The sequence of nodes representing the directed path connecting two nodes \(i\) and \(j\) is given by \(\text{path}(i, j) = \{i, \ldots, j\}\). If no such path exists, then \(\text{path}(i, j) = \emptyset\); also, \(\text{path}(i, i) = \{i\}\). For every pair of nodes \(i\) and \(j\), if \(\text{path}(i, j) \neq \emptyset\), then \(j\) is downstream of \(i\); otherwise, \(j\) is upstream of \(i\). The set of downstream nodes of \(i\) is represented by \(V_i\). If \(V_i = \{i\}\), node \(i\) is a leaf. The downstream power load of a node \(i\), \(\bar{L}_i\), can be computed without distributed generation by

\[
\bar{L}_i = \sum_{j \in V_i} l_j
\]  

(1)

2.2 Reliability indices

Regulatory agencies adopt reliability indices to define the minimum levels of reliability, the violation of which can trigger the imposition of fines on utilities. Moreover, reliability indices can also be used to (i) identify areas of the network that require additional investment, (ii) determine the reliability shifts and trends over time, (iii) compare historical values with the values of current network state, and (iv) compute the benefit/loss of proposed change to the network (Brown 2008). Several indices are presently used to evaluate the reliability of a distribution system quantitatively (Billinton and Allan 1996), such as the system average interruption frequency index (SAIFI), system average interruption duration index (SAIDI), and energy not supplied (ENS).

Without loss of generality, the ENS [Eq. (2)] is employed in the mathematical developments of the proposed evaluation methodology.

\[
ENS = \sum_{i \in V} l_i u_i
\]  

(2)

Variable \(u_i\) represents the duration of interruptions a node \(i\) is expected to suffer in a one-year period. A procedure to determine this variable is discussed in Sect. 2.4.

2.3 Assumptions

The following assumptions are considered.

- The network is radially operated.
- All failures are non-transient short circuits that propagate upstream until reaching a switch.
- The failure rate is a stochastic parameter, and its value represents the expected amount of failures that should occur in a one-year period.
• All switches are automatic sectionalizers with negligible failure rates and operation times.

### 2.4 Computation of interruption duration

This section summarizes the methodology, discussed in depth by Billinton and Allan (1996), to determine the expected duration of power interruptions that follow an outage.

A switch is a normally closed device that opens when a short circuit flows through it. This event disconnects all downstream load points but prevents power interruption of upstream loads. Consider the illustrative network shown in Fig. 1, in which the substation is represented by Node 0. If a fault occurs at Node 2, the switch in arc (1, 2) opens, interrupting the power supply of Node 2. However, if the fault occurs at Node 3, the short circuit propagates up to the substation switch (circuit breaker), causing the interruption of all the load points.

The *expected restoration time* $t_i$ is the expected time required to restore power supply to all the customers affected by a fault in node $i$. This parameter includes the identification of the failure location, the organization of the maintenance team, the repair of all defective network components, and the reclosure of any switch that opened because of the outage. The IEEE Gold (1998) (IEEE Std 493-1997) provides a standard procedure for estimating the restoration time $t_i$ and failure rate $\lambda_i$ for each node $i$ of a distribution network.

The *self-interruption* $\theta_i$ [Eq. (3)] represents the duration of interruptions that a node $i$ is expected to suffer in a one-year period as a result of local faults (occurring at node $i$).

$$\theta_i = \lambda_i t_i$$

(3)

The *downstream interruption* $\tilde{\theta}_i$, determined by Eq. (4), is a variable representing the time node $i$ is expected to be interrupted because of downstream faults in a one-year period.

$$\tilde{\theta}_i = \theta_i + \sum_{(i,j) \in A \setminus A_s} \tilde{\theta}_j$$

(4)

As shown by Eq. (4), the downstream interruption variables depend on the location of the switches (given by set $A_s$). They can be computed in a bottom-up fashion,
starting at the leaves of the network. The downstream interruption of a leaf is simply its self-interruption. In any other node, the downstream interruption comprises the node self-interruption added to the downstream interruptions of every node adjacent to $i$ not isolated by a switch.

The full interruption $u_i$ is the duration of interruptions a node $i$ is expected to suffer in a one-year period as a result of all the faults occurring in the network. This variable can be calculated by

$$u_0 = \tilde{\theta}_0, \quad u_j - u_i = \begin{cases} 0 & (i, j) \in A\setminus A_s \\ \tilde{\theta}_j & (i, j) \in A_s \end{cases}$$  \hspace{1cm} (5)

Equation (5) shows that the full interruption of the root is equal to its downstream interruption. Moreover, for any arc $(i, j)$ not isolated by a switch, the full interruptions of both its nodes are equal; otherwise, the switch prevents the downstream interruption of $j$ from affecting its predecessor $i$.

To compute the values of full interruptions, a double-sweep procedure can be employed, starting from the bottom-up calculation of the downstream interruptions. Then, a top-down procedure starts from the root, using Eq. (5) to determine the full interruption of every node.

3 Interruption flows (If flows)

3.1 Definition

The interruption flow, or iflow, $f_{ij}$, from node $j$ to node $i$ (reverse oriented from the power flow), is the expected duration of interruptions at node $i$ in a one-year period as a result of faults originating at nodes downstream of $j$. Formally, the iflow is defined by

$$f_{ij} = \begin{cases} 0 & (i, j) \in A_s \\ \tilde{\theta}_j & (i, j) \in A\setminus A_s \end{cases}$$  \hspace{1cm} (6a)

Equation (6a) shows that an iflow $f_{ij}$ is zero if there is a switch in arc $(i, j)$, which asserts the switch’s role in preventing faults downstream of $j$ from affecting node $i$. In the absence of a switch in arc $(i, j)$, Eq. (6b) states that the iflow $f_{ij}$ is equal to the downstream interruption of node $j$, which is consistent with the definition of an iflow.

An alternative identity of an iflow can be obtained by replacing the downstream interruption in Eq. (6b) according to Eq. (4).

$$f_{ij} = \theta_j + \sum_{(j, k) \in A\setminus A_s} \tilde{\theta}_k \quad (i, j) \in A\setminus A_s$$  \hspace{1cm} (6b)

The resulting sum of downstream interruptions can be represented as the sum of iflows, according to Eq. (6b):
Because the iflow of an arc containing a switch is zero, the following representation emerges.

\[
\begin{align*}
   f_{ij} &= \theta_j + \sum_{(j,k) \in A \setminus A_s} f_{jk} & (i,j) \in A \setminus A_s \\
   f_{ij} &= 0 & (i,j) \in A_s \\
   f_{ij} + F_j &= \theta_j + \sum_{(j,k) \in A} f_{jk} 
\end{align*}
\]

Li et al. (2020a) proposed the idea of fictitious flows which represent the path of an interruption from the fault’s origin up to the first upstream switch. These flows are defined as binary variables, assuming a value equal to one if an interruption flows through the corresponding arc, or zero otherwise. There is a connection between the iflows and Li et al. (2020a)’s fictitious flows in the sense that they are both equal to zero when a switch is present in an arc, meaning both of them are blocked by the presence of a switch. By capturing the bulk of interruptions in motion, the iflows generalize the fictitious flows, expressing local reliability states and indicating the most vulnerable areas of the network. The iflows also have the advantage of being directly translated into reliability indices, from which lower and upper bounds can be obtained. A theoretical analysis of the iflows is presented in Sect. 4.

### 3.2 Iflow node balance

The presence of a switch obstructs the iflow streaming through an arc, and to capture this event within a balance equation, an interruption slack, or islack, \( F_j \) is defined for each node \( j \). Equation (8) gives the node balance, which is depicted in Fig. 2.

\[
f_{ij} + F_j = \theta_j + \sum_{(j,k) \in A} f_{jk}
\]
The node balance shows that the values of an iflow $f_{ij}$ and its corresponding islack $F_j$ are complementary. In the presence of a switch, the iflow is zero, while the islack assumes the value that the iflow would take in the absence of the switch.

### 3.3 Iflow computation

The Algorithm 1 summarizes the method for computing the iflows.

```
Algorithm 1 iflowEval(node i)

Input: network $G(V,E)$, self-interruption $\theta_u$ for every node $u \in V$, and the root $i$ passed as argument.
Output: downstream interruption $\bar{\theta}_u$ for every node $u \in V$, iflow $f_{uv}$ for every arc $(u,v) \in A$.

begin
  $\bar{\theta}_i \leftarrow \theta_i$;
  forall $(i,j) \in A$ do
    iflowEval(j)
    if $(i,j) \in A \setminus A_u$ then
      $\bar{\theta}_i \leftarrow \bar{\theta}_i + \bar{\theta}_j$
      $f_{ij} \leftarrow \bar{\theta}_j$
    else
      $f_{ij} \leftarrow 0$
  end
end
```

The algorithm implements a recursive depth-first search starting at the root. Once a leaf is reached, the method backtracks carrying the downstream interruption to determine the iflow for the corresponding arc. The time complexity of Algorithm 1 is bounded by $\Theta(V + E)$ since each node and edge is visited once. This is an improvement over the previous algebraic methods which require solving a set of linear equations (Muñoz-Delgado et al. 2018; Tabares et al. 2019; Li et al. 2020b; Jooshaki et al. 2020).

Figure 3 shows the execution steps of Algorithm 1 applied to the illustrative network depicted in Fig. 1. It is assumed that the self-interruptions of Nodes 0, 1, 2, and 3 ($\theta_0, \theta_1, \theta_2, \theta_3$, respectively) are known. Step 1 starts by calling the method at the root, Node 0. The execution then recursively dives into the network until a leaf is reached. In Step 7 it arrives at Node 2, a leaf. From there, it backtracks from Arc (1, 2), setting the value of the corresponding iflow to zero ($f_{12} = 0$), since the iflow was blocked by a switch. Similarly, in Step 12 the execution reaches Node 3, another leaf. It then backtracks from Arc (1, 3), setting the value of the corresponding iflow in accordance with the value of the downstream interruption ($f_{13} = \bar{\theta}_3$). Finally, the execution backtracks one last time from Arc (0, 1), setting the iflow to zero ($f_{01} = 0$).

The following section describes the extraction of reliability measures for a network once the iflows have been obtained.
4 Flow-based reliability evaluation

This section revisits the ENS reliability index under the perspective of iflows, which allows the inference of lower and upper bounds for the ENS. The evaluation of other reliability indices with iflows is briefly discussed later in this section.

4.1 Formulating ENS with iflows

Lemma 1 shows that the interruption times $u$ can be expressed in terms of the islacks.

**Lemma 1** The full interruption $u_j$ of a node $j$ can be expressed as

\[ u_0 = F_0 \]  
\[ u_j - u_i = F_j \quad (i, j) \in A \]  

**Proof** Proof of Eq. (9a) (root):

\[ F_0 = \sum_{(0,i) \in A} f_{0i}^{\text{Eq.} (6b)} = \sum_{(0,i) \in A \setminus A_s} \tilde{\theta}_i^{\text{Eq.} (4)} \tilde{\theta}_0^{\text{Eq.} (5)} = u_0 \]

Proof of Eq. (9b) when $(i,j) \in A \setminus A_s$:

\[ F_j = \sum_{(j,k) \in A} f_{jk} - f_{ij}^{\text{Eq.} (6b)} = \sum_{(j,k) \in A \setminus A_s} \tilde{\theta}_k - \tilde{\theta}_j \]

\[ \tilde{\theta}_j - \tilde{\theta}_j = 0 = u_j - u_i \]

Proof of Eq. (9b) when $(i,j) \in A_s$:
Equation (9b) is a recursive expression that links full interruptions and islacks. Lemma 2 removes this recursion to attain a more straightforward expression.

**Lemma 2** The full interruption \( u_j \) is the sum of the islacks on the path from the root to node \( j \):

\[
\begin{align*}
  u_j &= \sum_{i \in \text{path}(0,j)} F_i \quad j \in V \\
  \text{Proof.} & \quad \text{The proof is through induction on the path from the root to node } j. \text{ First, the base case, where node } j \text{ is the root (} j = 0 \text{), is proven.}
  \\
  & \quad u_0 = F_0 = \sum_{j \in \text{path}(0,0)} F_j
\end{align*}
\]

As the induction hypothesis, it is assumed that Eq. (10) holds for any node in the path from the root to node \( a \). Now, we prove that Eq. (10) also holds for node \( b \), such that \((a, b) \in A\), starting with the expression given by Eq. (9b):

\[
\begin{align*}
  u_b - u_a &= F_b \Rightarrow u_b - \sum_{i \in \text{path}(0,a)} F_i = F_b \\
  & \Rightarrow u_b = \sum_{i \in \text{path}(0,b)} F_i
\end{align*}
\]

Theorem 1 presents the core of the reliability evaluation methodology. It shows the ENS expressed in terms of the iflows.

**Theorem 1** The ENS can be expressed in terms of the iflows, as follows.

\[
\begin{align*}
  \text{ENS} &= \sum_{(i,j) \in A} (\bar{l}_i - l_j) f_{ij} + \sum_{i \in V} l_i \theta_i \\
  \text{Proof.} & \quad \sum_{i \in V} l_i u_i = \sum_{i \in V} \left( l_i \sum_{j \in \text{path}(0,i)} F_j \right)
\end{align*}
\]
For the proof of Theorem 1, first the definition of ENS is utilized, as stated in Eq. (2). Equation (12a) comes from the equivalence between full interruptions and islacks, given by Eq. (10). Equation (12b) is derived by the fact that node \( j \) belongs to the path from the root to node \( i \) (\( j \in \text{path}(0, i) \)) if and only if node \( i \) is downstream of node \( j \) (\( i \in V_j \)). Equation (12c) comes from the definition of the downstream power load [Eq. (1)]. By using the iflow node balance given by Eqs. (8), (12d) is obtained. Finally, Eq. (12e) can be achieved through the algebraic rearrangement of terms.

4.2 Lower and upper bounds

It is worth mentioning that the term \( \sum_{i \in V} \bar{l}_i \theta_i \) [Eq. (11)] is a constant, independent of the iflows, and represents a lower bound for the ENS. Thus, the minimum ENS that can be expected in a network is obtained by taking the self-interruption of each node multiplied by its downstream load. This observation is extended in Lemma 3 by showing that the ENS value has a feasible interval, expressed by a lower and an upper bound. The relative reliability state of a network can be established in terms of the distance from these bounds.

**Lemma 3** A lower bound \( E_{lb} \) and upper bound \( E_{ub} \) for the ENS are described by the following inequalities.

\[
\sum_{i \in V} \bar{l}_i \theta_i = E_{lb} \leq ENS \leq E_{ub} = \bar{l}_0 \sum_{i \in V} \theta_i
\]

**Proof** As discussed previously, the ENS lower bound \( E_{lb} \) is trivially inferred as the constant in Eq. (11). The scenario in which \( ENS = E_{lb} \) implies that all arcs contain a switch (\( A_s = A \)), and thus, \( f_{ij} = 0 \) for every arc \( (i, j) \).
With respect to the ENS upper bound $E_{ub}$, an expression can be derived by assuming that the value of the iflow of every arc $(i, j)$ is maximum, which occurs when there is no switch in the network ($A_s = \emptyset$). Under this assumption, the maximum iflow of an arc $(i, j)$, $f_{ij}^{\text{max}}$, is given by

$$f_{ij}^{\text{max}} = \tilde{\theta}_j \tilde{\delta}_j + \sum_{(i,j) \in A \setminus A_s} \tilde{\delta}_j \sum_{k \in V_j} \theta_k \quad (i,j) \in A$$  \hspace{1cm} (14)$$

The maximum iflow of an arc $(i, j)$ is thus the sum of self-interruptions from all nodes downstream from $j$. Using this knowledge, the $E_{ub}$ can be derived

$$E_{ub} = \sum_{(i,j) \in A} (\tilde{l}_i - l^\text{max}_{ij}) + \sum_{i \in V} \tilde{l}_i \theta_i$$  \hspace{1cm} (15a)$$

$$= \sum_{(i,j) \in A} \left( \tilde{l}_i \sum_{k \in V_j} \theta_k \right) - \sum_{(i,j) \in A \setminus A_s} \left( \tilde{l}_j \sum_{k \in V_j} \theta_k \right) + \sum_{i \in V} \tilde{l}_i \theta_i$$  \hspace{1cm} (15b)$$

$$= \sum_{i \in V} \left( \tilde{l}_i \left( \sum_{j \in V_i} \theta_j - \theta_i \right) \right) - \sum_{i \in V \setminus 0} \left( \tilde{l}_i \sum_{j \in V_i} \theta_j \right) + \sum_{i \in V} \tilde{l}_i \theta_i$$  \hspace{1cm} (15c)$$

$$= \tilde{l}_0 \left( \sum_{i \in V} \theta_i - \theta_0 \right) + \sum_{i \in V \setminus 0} \tilde{l}_i \left( \sum_{j \in V_i} \theta_j - \theta_i \right)$$

$$- \sum_{i \in V \setminus 0} \left( \tilde{l}_i \sum_{j \in V_i} \theta_j \right) + \sum_{i \in V} \tilde{l}_i \theta_i$$

$$= \tilde{l}_0 \sum_{i \in V} \theta_i - \tilde{l}_0 \theta_0 - \sum_{i \in V \setminus 0} \tilde{l}_i \theta_i + \sum_{i \in V} \tilde{l}_i \theta_i = \tilde{l}_0 \sum_{i \in V} \theta_i$$  \hspace{1cm} (15d)$$

$$= \tilde{l}_0 \sum_{i \in V} \theta_i - \tilde{l}_0 \theta_0 - \sum_{i \in V \setminus 0} \tilde{l}_i \theta_i + \sum_{i \in V} \tilde{l}_i \theta_i = \tilde{l}_0 \sum_{i \in V} \theta_i$$  \hspace{1cm} (15e)$$

Equation (15a) is derived by entering the maximum iflows in Eq. (11). Equation (15b) follows by using the maximum iflow expression given in Eq. (14). By rewriting the summations over the arcs in the summations over the nodes, Eq. (15c) is obtained. To obtain Eq. (15d), the root is detached from the summation over the nodes. Finally, Eq. (15e) can be achieved through the algebraic rearrangement of terms.

The $E_{ub}$ expression provides the worst possible reliability for any network, which in terms of ENS represents the sum of all loads multiplied by the sum of all self-interruptions. Conversely, the $E_{lb}$ value gives the reliability that a network can attain if every interruption is contained in the best possible manner. This can be achieved by locating a switch in every arc, a scenario considered by Tabares et al. (2019)
their analytical reliability evaluation methodology. The $E_{lb}$ expression encapsulates the solution of the linear system of equations that appears in Tabares et al. (2019)’s methodology.

The iflows do not have the prior requirement of a switch in every arc to compute the network reliability. This attribute is an advantage to model problems in which the switches locations are decision variables, as will be shown in Sect. 7.

### 4.3 Other reliability indices

The formal analysis described in this section can also be extended to other reliability indices. In fact, any index tied to interruption times $u$, such as SAIDI, can be expressed in terms of iflows by replacing $u$ with the islacks [Eq. (2)] and then using the iflow node balance [Eq. (8)]. Also, lower and upper bounds can be inferred by the same analysis. With respect to indices that rely solely on interruption frequencies, such as SAIFI, it should be noticed that the frequency and duration of interruptions are correlated by the restoration time $t$ [Eq. (3)]. By using this correlation, the iflow magnitude can be translated from units of time to units of frequency, allowing the computation of indices such as SAIFI.

### 5 Iflow diagrams

Iflow diagrams are snapshots of a network’s reliability state, represented by the values of the iflows for each edge. These diagrams retain information that can be relevant to the planning, operation, and maintenance of distribution networks. The distribution networks shown in Fig. 4, proposed by Billinton and Allan (1996), is used to showcase iflow diagrams for three different configurations with respect to the location of switches. The iflows were obtained by Algorithm 1, and all other parameters are the same for the three configurations, and they are presented in Table 1.

Configuration 1 (Fig. 4a) contains only the substation switch, which means that any fault disconnects all the load points. The iflow diagram for this configuration is shown in Fig. 4b where local reliability states are readily accessible through the iflows intensity (depicted in red for the most critical areas and blue for the most reliable areas). For Configuration 1, the iflow diagram shows that the most vulnerable area will endure a 4.8 h/y interruption time. The iflow diagram also shows, for an informative purpose, the balance among iflows, islacks, and self-interruptions, as predicted by Eq. (8).

Represented by Fig. 4c, Configuration 2 contains four lateral switches. Any fault on Nodes 5, 6, 7, or 8 is contained by their corresponding switch. The iflow diagram depicted in Fig. 4d shows a distinct improvement in the overall values of the iflows; compared to Configuration 1, the maximum iflow is reduced by half, to 2.4 h/y.

With the iflow diagrams of Configurations 1 and 2, their ENS can now be calculated using Eq. (11), resulting in the following expression:
By filling the values of the iflows from Configurations 1 and 2 in Eq. (16), an ENS of 84.0 and 54.8 MWh/y are obtained, respectively. It should be noticed that these results match the predictions from Billinton and Allan (1996).

\[
\text{ENS} = (\bar{l}_1 - \bar{l}_2)f_{12} + (\bar{l}_2 - \bar{l}_3)f_{23} + (\bar{l}_3 - \bar{l}_4)f_{34} + (\bar{l}_1 - \bar{l}_5)f_{15} + (\bar{l}_2 - \bar{l}_6)f_{26} + (\bar{l}_3 - \bar{l}_7)f_{37} + (\bar{l}_4 - \bar{l}_8)f_{48} + \bar{l}_1\theta_1 + \bar{l}_2\theta_2 + \bar{l}_3\theta_3 + \bar{l}_4\theta_4 + \bar{l}_5\theta_5 + \bar{l}_6\theta_6 + \bar{l}_7\theta_7 + \bar{l}_8\theta_8
\]  

(16)

By filling the values of the iflows from Configurations 1 and 2 in Eq. (16), an ENS of 84.0 and 54.8 MWh/y are obtained, respectively. It should be noticed that these results match the predictions from Billinton and Allan (1996).

Table 1 Parameters used in the numerical examples

| Node i | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|
| \(l_i\) | 0.0 | 0.0 | 0.0 | 0.0 | 5.0 | 4.0 | 3.0 | 2.0 |
| \(\bar{l}_i\) | 14.0 | 9.0 | 5.0 | 2.0 | 5.0 | 4.0 | 3.0 | 2.0 |
| \(k_i\) | 0.2  | 0.1  | 0.3  | 0.2  | 0.2  | 0.6  | 0.4  | 0.2  |
| \(t_i\) | 4.0  | 4.0  | 4.0  | 4.0  | 2.0  | 2.0  | 2.0  | 2.0  |
| \(\theta_i\) | 0.8  | 0.8  | 1.2  | 0.8  | 0.4  | 1.2  | 0.8  | 0.4  |

\(l_i\), node i power load (MW); \(\bar{l}_i\), node i downstream power load (MW); \(k_i\), node i failure rate (failures/year); \(t_i\), average interruption time (h); \(\theta_i\), node i self-interruption (h/y)
If the lateral switches of Configuration 2 were moved to their best possible locations, the result would be Configuration 3, represented by Fig. 4e. Once again, using Eq. (16), the ENS of Configuration 3 will result in 37.6 MWh/y, which represents an improvement of more than 30% compared to Configuration 2 presented by Billinton and Allan (1996). We discuss the general problem of finding the optimal allocation of switches more thoroughly in Sect. 7.

6 A note on meshed networks

The mathematical underpinning of iflows, layered in Sect. 3, assumes radially operated networks, which include meshed networks with normally opened switches and DG.

Represented by Fig. 5, Configuration 4 considers a meshed network, with a DG power source located at Node 9 with a capacity of 9.0 MW. Provided that Configuration 4 retains the same nominal operation as Configuration 3 (the DG acts only during a contingency), its iflow diagram also remains the same and it is depicted in Fig. 4f.

To determine the ENS of Configuration 4, the effect of the DG must be accounted for. For example, it should be noticed that a failure at Node 1 would no longer affect the bulk of downstream load $\hat{l}_1$. By opening switch (1, 2) and closing switch (4, 9), the downstream load $\hat{l}_2$ would not be affected by the failure at Node 1. Therefore, the downstream load of Node 1 can be adjusted to consider DG by discounting from it the downstream load of Node 2. Similarly, the same rationale is entailed for Nodes 2 and 3. To this end, Eqs. (17)–(19) show the adjusted downstream loads for Nodes 1, 2, and 3, respectively:

$$\hat{l}_{1dg} = \hat{l}_1 - \hat{l}_2$$  (17)

$$\hat{l}_{2dg} = \hat{l}_2 - \hat{l}_3$$  (18)

$$\hat{l}_{3dg} = \hat{l}_3 - \hat{l}_4$$  (19)
Equation (20) is the ENS expression for Configuration 4. It uses the adjusted power loads of Nodes 1, 2, and 3 to correctly account for the DG on Node 9. The ENS for this configuration results in 20.8 MWh/y, which is an improvement of more than 44% compared to Configuration 3, without the DG.

\[ ENS = \tilde{\ell}_{1} f_{12} + \tilde{\ell}_{2} f_{23} + \tilde{\ell}_{3} f_{34} + (\tilde{\ell}_{1} - \tilde{\ell}_{5}) f_{15} + (\tilde{\ell}_{2} - \tilde{\ell}_{6}) f_{26} + (\tilde{\ell}_{3} - \tilde{\ell}_{7}) f_{37} + (\tilde{\ell}_{4} - \tilde{\ell}_{8}) f_{48} \]

Configuration 4 illustrates the application of iflows on meshed networks that are radially operated. The fact that a contingency can modify the nominal configuration by setting new paths for the power flows, and even reversing the orientation of some of them, does not change the network’s iflow diagram. The power flows of the post-contingency configuration are duly regarded by the adjusted downstream power loads computed in a preprocessing stage.

As for non-radially operated networks, such as transmission systems and special cases of distribution networks, there are known network reduction techniques that transform a network with serial and parallel components into a radial equivalent (Billinton and Allan 1996; Chowdhury and Koval 2009).

### 7 Application of iflows for switch allocation

The switch allocation problem (SAP) aims toward the best locations of switches on a distribution network considering cost-reliability trade-offs. The potential benefits of switch allocation include a reduction in the average duration of failures, an improvement in the quality of the power supply, and the avoidance of fines related to the violation of reliability standards. Here, we consider the allocation of switches to minimize the ENS.

Many heuristics have been proposed to solve the SAP. Levitin et al. (1994) introduced this optimization problem and proposed a genetic algorithm to allocate sectionalizers in a radial distribution network. Several other researchers followed the ideas of Levitin et al. (1994) in their proposals of metaheuristics for the SAP, such as simulated annealing (Billinton and Jonnavithula 1996), iterated sample construction with path relinking (Benavides et al. 2013), memetic algorithm (Assis et al. 2015), and bee colony algorithm (Aman et al. 2016). Abiri-Jahromi et al. (2012) proposed a mixed-integer linear model with an explicit enumeration of the locations of switches, resulting in an exponential number of variables and constraints. Farajollahi et al. (2019) extended the previous model also to consider the allocation of fault indicators. The authors conclude that the allocation of switches is more effective cost-wise.
As shown in the following section, by using the iflows, a polynomial-size mixed-integer programming model with a better scaling capability is attained.

### 7.1 Mixed-integer programming model

A fixed number of $N$ switches is considered in the optimization problem to minimize the ENS. A switch is on arc $(i, j)$ if and only if $x_{ij} = 1$. Variable $f_{ij}$ gives the iflow (Eq. 7) on arc $(i, j)$. Parameters $\bar{l}_i$, $\theta_i$, and $E_{lb}$ are described by Eqs. (1), (3), and (13), respectively. A sufficiently large constant $M_i$ is defined for each node $i$.

\[
\text{(SAP)} \quad \text{MIN} \quad \sum_{(i,j)\in A} (\bar{l}_i - \bar{l}_j)f_{ij} + E_{lb} \tag{21}
\]

s.t.

\[
\text{(number of switches)} \quad \sum_{(i,j)\in A} x_{ij} \leq N \tag{22}
\]

\[
\text{(iflow node balance)} \quad F_j + f_{ij} = \theta_j + \sum_{(j,k)\in A} f_{jk} \quad (i, j) \in A \tag{23}
\]

\[
\text{(islack coupling with switch allocation)} \quad F_j \leq M_j x_{ij} \quad (i, j) \in A \tag{24}
\]

\[
\text{(variables bounds and integrality)} \quad f_{ij}, F_j \geq 0, \quad x_{ij} \in \{0, 1\} \quad (i, j) \in A \tag{25}
\]

The objective function (21) represents the solution ENS. The number of switches is constrained by (22). The iflow node balance is expressed in (23). Constraints (24) couple the decision of allocating a switch in an arc with the value of the corresponding islack. If an arc $(i, j)$ does not contain a switch ($x_{ij} = 0$), the islack $F_j$ is forced to zero, while the iflow assumes the value predicted by the node balance. Conversely, if arc $(i, j)$ does contain a switch, the value of $F_j$ must be allowed to assume a sufficiently large value to absorb all downstream interruptions, thus allowing $f_{ij}$ to be zero. In the worst case, the amount of downstream interruptions that the islack should absorb is equal to the maximum iflow of arc $(i, j)$ (Eq. (14)). Thus, $M_i = \sum_{j \in V_i} \theta_j$ was set in the computational experiments described in the next section.
7.2 Computational experiments

The SAP is solved for a benchmark of radially-operated distribution networks,\textsuperscript{1} the attributes of which are described in Table 2 (Kavasseri and Ababei 2020). Solutions are obtained with Gurobi 8.1 under a time limit of ten minutes, on an Intel i7 3930k with 16 GB of RAM, and Ubuntu 18.04.

The maximum number of switches in a solution is set to $N = |P| |A|$. Five networks and four values of $P$ result in a total of 20 instances. Table 3 gives the objective function (ENS) of the best feasible solutions ($UB$), the optimality gap ($Gap$), and the execution times ($CPU$).

\textsuperscript{1} Data available at the address (last accessed in May 2021): http://www.dejazzer.com/reds.html.
The SAP model obtains optimal solutions for 11 instances (in bold) with up to 205 nodes, taking less than one second of computation. The model was unable to prove optimality for nine instances, with an average gap of 2.01% and the worst gap of 8.21%. The results indicate that the model is sensitive to the size of the instance, as well as to the number of switches allowed. The solution gap decreases as the number of switches increases, with only one exception (network R5, \( P = 40\% \)).

Now consider a greenfield scenario, in which a planner must decide on economic grounds the best number and locations of switches on an empty network. The annual investment to acquire and maintain a sectionalizer and the cost of interrupted energy are estimated as \( C_s = \text{US$1,358.00/year} \) and \( C_e = \text{US$1.53/kWh} \), respectively (Brown 2008). Figure 6 shows the ENS cost savings \((C_e \cdot (E_{ub} - E_N))\) and the switch investment \((C_s \cdot N)\) for solutions of Network R5 containing \( N = \{0, \ldots, 100\} \) switches. By subtracting the switch investment from the ENS cost savings, the annual returns enabled by the allocation of switches are obtained. For Network R5, solutions with up to 83 switches have positive returns, which means the reliability investment pays for itself in these cases. The best solution is to allocate 21 switches, giving a maximum return of \( \text{US$66,247.41/y} \).

These numerical experiments support the model’s strength in enabling exact solutions to large instances with more than 800 nodes. Previous approaches either rely on heuristic methods or solve instances of modest sizes—smaller than the problem with 144 vertices addressed by Farajollahi et al. (2019). The strength of the model lies in the relationship between interruptions and network flow, which the concept of iflows renders straightforward.

![Fig. 6 Cost and returns pertaining to the switch allocation in Network R5](image-url)
8 Final Remarks

The paper advocates the concept of iflows, which supports a new reliability evaluation framework for energy networks. These flows can be computed efficiently in linear time, and they readily provide information about reliability indices. This aspect may be crucial for modern energy networks such as smart grids, demanding frequent and expeditious reliability evaluations.

The iflows embody essential reliability states allowing an analytic evaluation through a series of flow balance computations. The framework was employed in a mixed-integer linear model for a switch allocation problem. The results of case studies using a benchmark of distribution networks show that high-quality solutions, most of them optimal, can be obtained within short computational times. This result sets a new standard for solving the switch allocation problem, opening paths for research on polyhedral aspects of the model, and its facet-defining inequalities. Moreover, reducing the reliability evaluation as a network flow problem suggests the design of customized algorithms for the switch allocation problem.

The use of iflows in energy networks should be developed in future studies to encompass other reliability optimization problems, which include contingency planning, maintenance scheduling, self-healing, fault indicator allocation, and the effects of switching interruptions, non-radially operated meshed networks, islanding operation, and component failure. Also, a promising line of research is to extend the proposed framework to other service networks such as sensor networks, and smart metering.

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