Spin Environment Engineering for a Persistent Current Qubit

Jacek Dziarmaga¹,² *

¹) Los Alamos National Laboratory, Theory Division T-6, MS B288, Los Alamos, NM87545, USA
²) Institut Fizyki Uniwersytetu Jagiellońskiego, Kraków, Poland

(January 30, 2001)

A persistent current qubit has two quantum states with opposite currents flowing in a superconducting loop. Their magnetic field couple to nuclear spins. The qubit state is not only perturbed by the spins but it also gets entangled with the spins’ state on a very short timescale. However, when the same spins are exposed to a strong but less than critical external magnetic field, then the qubit field is just a small perturbation on top of the external field and the entanglement with each spin is negligible. For a qubit which is more microscopic than certain threshold this partial entanglement results in negligible partial decoherence.

I. INTRODUCTION

The idea to perform computation with the help of quantum mechanics dates back to Feynman [1]. Research in this direction got a considerable acceleration after the realisation [2] that quantum computers could solve in a polynomial time certain problems which require nonpolynomial time on classical computers. Two main obstacles to a universal quantum computer are decoherence and scalability. Decoherence, if sufficiently weak, can be dealt with by quantum error correcting codes, see Ref. [3] for an experimental implementation. Scalability requires that a given technology can be upscaled to a quantum coherent circuit with hundreds or thousands of quantum gates, see Ref. [4] for reviews on various proposals of scalable implementations. At the present stage solid state implementations have most to offer from the point of view of scalability but at the same time they are most likely to suffer from decoherence. The aim of this note is to reduce decoherence in the persistent current qubit (PCQ) proposal [3].

Two remarkable experiments [6] took place last year where a superconducting loop broken by Josephson junctions was forced into states which are quantum superpositions of two states with opposite macroscopic persistent currents: |↑⟩ and |↓⟩. The two considered combinations |±⟩ = |↑⟩ ± |↓⟩ were eigenstates separated by a gap of 0.1K so it is not surprising that each of them does not suffer from decoherence at the temperature of 50mK. A more general superposition α|+⟩ + β|−⟩ would quickly decohere into a mixed state |α|²|+⟩⟨+| + |β|²|−⟩⟨−|. To use a persistent current qubit for quantum computation any superposition must be free from decoherence. A design of a more microscopic scalable PCQ was proposed in Ref. [5]. These authors went on to estimate decoherence times from different sources relevant for their design [7] and found out that the most dangerous are nuclear spins which couple to the magnetic field induced by persistent currents. Similar conclusion is reached in Ref. [8]. In the calculations of Ref. [7] the spins are treated as random static background fields as may be justified by the long relaxation time of nuclear spins which is of the order of minutes. Such a static background can be measured at the beginning of the quantum computation and compensated for by an adjustable counterterm in the qubit Hamiltonian. In this note we re-address this problem and find out that spins cannot be treated as a mere random background because the qubit state quickly gets entangled with the spins state. This observation is in agreement with Ref. [8]. What is more, because of the large number of involved spins, even for the long relaxation time the spins background performs a random walk which leads to dephasing in the qubit state. The former more acute entanglement problem can be treated by exposing the nuclear spins to a strong external magnetic field B parallel to the plane of the superconducting loop and penetrating through its thin layer, see Fig.1.

FIG. 1. The idea of how the external magnetic field B can be introduced into a superconducting loop. Only one Josephson junction (JJ) is marked.

B has to be less than the critical field in a type I superconducting film. If the N nuclear spins are exposed to a magnetic field b from the persistent current which is much weaker than B, as can be characterized by a quality factor

\[ Q = \frac{B^2}{NB^2} , \]  

then the entanglement between the qubit state and the



*e-mail address: dziarmaga@t6-serv.lanl.gov
spins state is negligible and leads to negligible partial decoherence. The means which treat entanglement incidentally also reduce the latter random walk problem, which can be further eliminated by increasing the spin relaxation time $T_r$ which depends exponentially on temperature.

II. SINGLE SPIN ENVIRONMENT

We begin with an elementary example where the qubit is coupled to just one environmental $1/2$ spin. For the purpose of this paper the spin-1/2 is a sufficient representation of the nuclear spin-9/2 of niobium. The spin is coupled to the external magnetic field $B$; its Hamiltonian is

$$H_E = B\sigma^E_z.$$

The current of the qubit induces a magnetic field which at the location of the environmental spin has strength $b$. For the time being we assume that the field is perpendicular to the external field in, say, $x$-direction. The Hamiltonian of interaction between the qubit and the spin is

$$H_{Q-E} = b\sigma_x\sigma^E_z.$$

The resulting system state has a flipped sign of $B$ due to spin relaxation. We note that for $\tau/T \gg 2/b B^2$, the up transition at the time $t$ can be described as a projection of $|\psi(t)\rangle$ on $|\downarrow_E\rangle$ followed by a spin flip $|\downarrow_E\rangle \rightarrow |\uparrow_E\rangle$. As a result $|\psi(t)\rangle$ jumps to

$$|\psi(t^+)\rangle = (|\uparrow\rangle + |\downarrow\rangle)|\uparrow_E\rangle.$$

The resulting system state has a flipped sign of $\beta$ as compared to the initial system state $|\beta\rangle$. This sign flip also flips the sign of the off-diagonal elements in the reduced qubit density matrix, compare Eqs. (6) and (9).

The probability of the down transition in the interval $(t, t + dt)$ is $dt/T_r$ times the probability that $|\psi(t)\rangle$ is in the $|\uparrow_E\rangle$ state which is $|\langle\uparrow_E|\psi(t)\rangle|^2 \approx (b_2^2/B^2)^2 Bt \approx b_2^2/2B^2$, where we assume $b_2 \ll B^2$. The probability is $dt b_2^2/2B^2T_r$. The down transition is a projection on $|\downarrow_E\rangle$ followed by a spin flip $|\uparrow_E\rangle \rightarrow |\downarrow_E\rangle$. $|\psi(t)\rangle$ jumps to

$$|\psi(t^-)\rangle = (|\uparrow\rangle - |\downarrow\rangle)|\downarrow_E\rangle.$$

The resulting state of the system is the same as its initial state: this up transition causes no decoherence.
III. N-SPIN ENVIRONMENT

Decoherence becomes more efficient when there are many copies of the environmental spin numbered by \(i = 1, \ldots, N\). Interaction Hamiltonians are \(H_{Q-E_i} = b_i \sigma_z \sigma_z^{E_i}\). The initial product state

\[
|\psi(0)\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \prod_{i=1}^N |\downarrow_{E_i}\rangle
\]

(13)
evolves into |\psi(t)\rangle such that the reduced state of the qubit has the form of Eq.(7) but with \(\Omega_i = \sqrt{B^2 + b_i^2}\). The early time overlap is

\[
O(t) = \prod_{i=1}^N \left( 1 - \frac{2b_i^2}{\Omega_i^2} \sin^2 \Omega_i t \right)
\]

(14)

where \(\Omega_i = \sqrt{B^2 + b_i^2}\), \(T\) is a timescale of \(\tau = 1/\sqrt{N}\). For \(b_i^2 \ll B^2\) different factors in Eq.(14) oscillate with different frequencies dispersed in the range \(B \pm b_i^2/B\). For \(t \gg 2\pi B/b^2\) the phases of oscillating terms in different factors become effectively random and the overlap averages to

\[
\tilde{O} = 1 - \frac{NB^2}{B^2} = 1 - \frac{1}{Q}.
\]

(15)

For \(Q = B^2/Nb^2 \gg 1\) the system state remains pure; there is only a negligible partial decoherence. Fig.2 shows \(O(t)\) for a sample of \(b_i\)'s at two different values of \(B\).

![Fig. 2. The overlap \(O(t)\) from Eq.(14) for \(N = 10^3\) spins with \(b_i^2 = 1\) at two different values of \(B\). \(b_i\)'s were chosen at random with a uniform probability distribution between 0 and \(\sqrt{3}\). The spin relaxation is not included in this plot or, in other words, \(T_r\) is assumed to be infinite.](image)

Let us include spin relaxation. To get full decoherence it is enough that only one out of \(N\) spins makes the down transition so the decoherence time is \(N\) times shorter than for a single spin. \(\tau_{(2)} = (2T_r B^2/b^2)/N = 2Q T_r\). \(Q \gg 1\) makes this decoherence time is much longer than \(T_r\).

It is worthwhile to compare the large \(Q\) limit with the case of \(B = 0\) when Eq.(23) gives

\[
O(t) = \prod_{i=1}^N \cos 2b_i t
\]

(16)

The different factors, which oscillate with frequencies in a range proportional to \(b\), go out of phase after a time of \(2\pi/b\) when the overlap averages to zero. For \(B = 0\) the time \(\tau = 1/\sqrt{N} b\) when \(O(t)\) decays away from 1 is a time of full decoherence.

IV. THERMALIZED ENVIRONMENT

So far we considered the single spin environment and the multiparticle environment with a fully polarized initial state. A thermal initial state is a weighted average over different partially polarized states, like e.g. \(|\uparrow_{E_1}\rangle|\downarrow_{E_2}\rangle \cdots |\uparrow_{E_N}\rangle\). The overlap \(\tilde{O}\) is the same for any such partially polarized state as for the fully polarized state so the estimates of decoherence time for the polarized state go through unchanged for the thermal density matrix.

The mixedness of the initial environmental state does matter when we finally take into account the \(z\)-component of the qubit magnetic field at the location of spins which gives an extra term in the interaction Hamiltonian

\[
H_{Q-E}^z = \sigma_z \sum_{i=1}^N b_i \sigma_z^{E_i}.
\]

(17)

This Hamiltonian, when applied to a partially polarized spins state, gives \(H_{Q-E}^z = \sigma_z \sum_{i=1}^N b_i |p_i\rangle\), where \(p_i = \pm 1\) is a polarization of the \(i\)-th spin. If \(p_i\)'s were static, then this \(H_{Q-E}^z\) could be measured at the beginning of the quantum computation and balanced by a \(\sigma_z\)-counterterm in the qubit Hamiltonian, as suggested in Ref. [4]. The problem is that \(p_i\)'s are not static because they flip at the rate of one spin per \(T_{r}/N\). The total polarization \(p(t) = \sum_{i=1}^N p_i(t)\) performs a random walk with one \(\pm 2\) step every \(T_r/N\) so that \(p(t) - p(0)\) is random so the qubit states \(|\uparrow\rangle\) and \(|\downarrow\rangle\) accumulate opposite random phases \(\phi(t)\), which grow as \(\phi(t)^2 = 4N b^2 t^2/3T_r\). They lead to full dephasing after a dephasing time \(\tau_{(3)} = (3\pi^2 T_r/Nb^2)^{1/3}\). In this discussion we assume \(Q \gg 1\) so that entanglement is negligible.
V. NUMERICAL ESTIMATES

The external magnetic field \( B \) is bounded from above by the critical field of the type I superconducting material \( H_c \). This parameter is the highest \((0.206 \, T)\) for niobium. In this respect niobium is much better than alluminium which has much lower \( H_c = 0.01 \, T \). We choose niobium and set \( B = 0.1 \, T \). There are materials like lead or tin that have abundant isotopes with zero nuclear spin. We do not consider them here because niobium and alluminium microtechnology is more advanced and, what is even more important, even lead or tin or their substrates would have spin impurities requiring environment engineering.

In the qubit design of Ref. \( ^{[5]} \) the \( 1 \mu m \times 1 \mu m \) superconducting loop is made of an alluminium \( 0.5 \mu m \times 0.5 \mu m \) wire with persistent current of \( 100 \mu A \). We suggest niobium instead of alluminium and the loop to be just \( 0.1 \mu m \) thick. \( 0.1 \mu m \) is roughly twice the penetration depth so that \( B \) can penetrate through the superconductor. Note that the critical field is higher in such thin films than in bulk superconductor \( ^{[9]} \). Our superconducting loop will produce a flux of \( 10^{-4} \Phi_0 \) which is an order of magnitude less than in the design of Ref. \( ^{[5]} \). The loop is also thinner so it contains fewer spins. The superconductor volume of \( 10^{-19} m^3 \) contains \( 10^{10} \) of niobium nuclear spins. They are subject to an average \( b = 10^{-15} T^2 \). The quality factor \( Q = B^2 / Nb^2 = 10^5 \) gives the overlap \( \bar{O} = 0.999 \).

If the external field were \( B = 0 \), then full decoherence would take place after

\[
\tau^{(1)} = \frac{1}{\sqrt{N b}} \approx 10^{-5} s ,
\]

(18)

where we assume the nuclear spin magnetic moment of \( 10^7 H z / T \). For our \( B = 0.1 \mu m \) the \( \tau^{(1)} \) gets only negligible partial decoherence with \( \bar{O} = 0.999 \). The full decoherence due to spin relaxation could happen after

\[
\tau^{(2)} = 2QT \tau \approx 10^5 s ,
\]

(19)

where we assume \( T_r \) to be of the order of minutes, but the dephasing time

\[
\tau^{(3)} = \left( \frac{3 \pi^2 T_r}{Nb^2} \right)^{1/3} \approx 10^{-3} s ,
\]

(20)

is much shorter. Even this \( 1 ms \) time can be significantly increased by decreasing temperature because \( T_r \sim \exp (\Delta / k_B T) \), where \( \Delta \) is the superconducting gap. Reduction of temperature from \( 50 mK \) down to \( 5 mK \) increases \( \tau^{(3)} \) \( 25 \) times.

VI. CONCLUSION

Maximalization of the quality factor \( Q = B^2 / Nb^2 \) so that \( Q \gg 1 \) makes entanglement with spin environment negligible. The same technique can be used for any qubit design which involves superposition of states with different magnetic fields. According to our estimates, the present day persistent current qubit proposal \( ^{[5]} \) is not far from achieving a \( 1 ms \) dephasing time due to the finite spin relaxation time \( T_r \). \( 1 ms \) is sufficient for the planned quantum computation at a \( GHz \) rate. To achieve this desired decoherence time it is important to replace alluminium by niobium and make the circuit more microscopic than in Ref. \( ^{[5]} \). It is a common wisdom that more microscopic systems are better from the point of view of decoherence, compare e.g. our formula for \( \tau^{(1)} \).

We showed that in addition to this much expected behaviour, there is a threshold below which a sufficiently microscopic qubit can be made free from entanglement with the spin bath.

Acknowledgements. I would like to thank Diego Dalvit and Nikolay Prokof'ev for their comments on the manuscript. This work was supported in part by NSA.

[1] R.P. Feynman, Found. Phys. 16, 507 (1986).
[2] P.W. Shor, in Proceedings of the 35th Annual Symposium on Foundations of Computer Science (IEEE Press, Los Alamitos, CA, 1994), pp.124-134; L.K. Grover, Phys. Rev. Lett. 79, 325 (1997).
[3] D.G. Cory, M.D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek, T.F. Havel, and S.S. Somaroo, Phys. Rev. Lett. 81, 2152 (1998).
[4] Special issue Fortschr. Phys. 48 (2000).
[5] J.E. Mooij, T.P. Orlando, L. Levitov, L. Tian, C.H. van der Wal, and S. Lloyd, Science 285, 1036 (1999); T.P. Orlando, J.E. Mooij, L. Tian, C.H. van der Wal, L. Levitov, S. Lloyd, and J.J. Mazo, Phys. Rev. B 60, 15398 (1999).
[6] J.R. Friedman, V. Patel, W. Chen, S.K. Tolpygo, and J.E. Lukens, Nature 406, 43 (2000); C.H. van der Wal, A.C.J. ter Haar, F.K. Wilhelm, R.N. Schouten, C.J.P.M. Harmans, T.P. Orlando, S. Lloyd, and J.E. Mooij, Science 290, 773 (2000).
[7] L. Tian, L. S. Levitov, C. H. van der Wal, J. E. Mooij, T. P. Orlando, S. Lloyd, C. J. P. M. Harmans, and J. J. Mazo, proceedings of the NATO-ASI on Quantum Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics, cond-mat/9910062.
[8] N.V. Prokof'ev and P.C.E. Stamp, cond-mat/000605 for more general theory see N.V. Prokof'ev and P.C.E. Stamp, Rep. Prog. Phys. 63, 669 (2000).
[9] T. Van Duzer and C.W. Turner, Principles of Superconductive Devices and Circuits, Elsevier North Holland Inc., New York, 1981.