REDUCING THE SPECTRAL INDEX IN SUPERNATURAL INFLATION

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Abstract

Supernatural inflation is an attractive model based just on a flat direction with soft SUSY breaking mass terms in the framework of supersymmetry. The beauty of the model is inferred from its name that the model needs no fine-tuning. However, the prediction of the spectral index is $n_s \gtrsim 1$, in contrast to experimental data. In this paper, we show that the beauty of supernatural inflation with the spectral index reduced to $n_s = 0.96$ without any fine-tuning, by considering the general feature that a flat direction is lifted by a non-renormalizable term with an A-term.

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1 Introduction

Inflation \textsuperscript{[1]-[3]} (for review, \textsuperscript{[4]-[6]}) is an vacuum-dominated epoch in the early Universe when the scale factor grew exponentially. This scenario is used to set the initial condition for the hot big bang model and to provide the primordial density perturbation as the seed of structure formation. In the framework of slow-roll inflation, the slow-roll parameters are defined by

$$
\epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2,  \\
\eta \equiv M_p^2 \frac{V''}{V},
$$

where $M_p = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The spectral index can be expressed in terms of the slow-roll parameters as

$$
n_s = 1 + 2\eta - 6\epsilon.
$$
The latest WMAP 5-year result prefers the spectral index around $n_s = 0.96$ [7]. The spectrum is given by

$$ P_R = \frac{1}{12\pi^2M_P^6} \frac{V^3}{V'}^2. \quad (4) $$

With the slow-roll approximation the value of the inflaton field $\phi$, in order to achieve $N$ e-folds inflation, is

$$ N = M_P^{-2} \int_{\phi_{\text{end}}}^{\phi(N)} \frac{V}{\sqrt{V'}} d\phi. \quad (5) $$

From observation [7] $P_R^{1/2} \simeq 5 \times 10^{-5}$ at $N \simeq 60$ (we call this CMB normalization).

In order to build a successful inflation model, we need a potential which is very flat when $N \simeq 60$ and becomes steep when inflation ends. It is difficult to achieve this form of potential by a single field without significant tuning of the coupling parameter [4]. The idea of hybrid inflation is more natural in achieving so, in which the jobs of ending the inflation and providing the scale of inflation is done by another scalar field (called waterfall field). The quadratic potential for the inflaton field $\phi$ is

$$ V = \frac{1}{2} m^2 \phi^2. \quad (6) $$

This potential is used in the case of chaotic inflation [8] where $\phi > M_P$ is required. It can be turned into a hybrid inflation [9] by adding a “false vacuum” to it [10]:

$$ V = V_0 + \frac{1}{2} m^2 \phi^2. \quad (7) $$

Consequently we can have an inflation model with $\phi < M_P$. In this case, the end of inflation is not due to the failing of slow-roll, but the tachyonic instability of the waterfall field.

The idea of supernatural inflation [11] is that the inflaton field $\phi$ is a flat direction in supersymmetric field theory and the mass term is provided by a soft SUSY breaking term and $V_0$ by another field coupled to the inflaton field. Therefore, the model is basically a tree-level hybrid inflation model. However, this model predicts a spectral index $n_s \gtrsim 1$. A method to reduce the spectral index in this tree-level hybrid inflation is by converting the model into “hilltop inflation” [12] via introduction of a negative quartic term to the potential as shown in [13]. The only problem here is how to introduce this quartic term and whether the coupling parameter needs fine tuning. The answer relies on the fact that the flat direction is generically affected not only by the soft SUSY breaking term but also the nonrenormalizable term and A-term. In this paper, we show that by considering these additional terms the spectral index can be reduced to $n_s = 0.96$ in a natural fashion.

This paper is organized as follows. In Sec. 2 we introduce the scalar potential of the model. In Sec. 3 the analytic solution base on the potential and the main result of this paper are provided. Finally, we present our conclusions in Sec. 4.

## 2 The Potential

Suppose we want to build an inflation model based on a SUSY flat direction [14], we have to know that generically a flat direction is lifted by supersymmetry breaking terms and non-renormalizable superpotential terms [15], which has the form

$$ W = \lambda_p \frac{\phi^p}{M_P^{p-3}} \quad (8) $$
where $\phi$ is the flat direction and $4 \leq p \leq 9$ \cite{14}. Therefore, the potential is \cite{17,18,19,20} (after minimizing the potential along the angular direction)

$$V = \frac{1}{2} m^2 \phi^2 - A \frac{\lambda_p \phi^p}{p M_P^{p-3}} + \frac{\lambda_p^2 \phi^{2(p-1)}}{M_P^{2(p-3)}}. \quad (9)$$

By spirit of hybrid inflation we can add a “false vacuum” to this potential via a coupling to a waterfall field similar to the case of supernatural inflation. Therefore, the potential we consider is of the following form

$$V(\phi) = V_0 + \frac{1}{2} m^2 \phi^2 - A \frac{\lambda_p A \phi^p}{p M_P} + \frac{\lambda_p^2 \phi^{2(p-1)}}{M_P^{2(p-3)}}. \quad (10)$$

If we just consider the first two terms, the result is the tree-level hybrid inflation, which is realized in supernatural inflation where the mass term comes from soft SUSY breaking. In this work, we focus on the case of $p = 4$ (the least of Planck-mass suppression), and neglect the last term (which is possible when $\phi \ll M_P$ and will be justified in the following section). Therefore, the potential is

$$V(\phi) = V_0 + \frac{1}{2} m^2 \phi^2 - \frac{\lambda_4 A \phi^4}{4 M_P} \quad (11)$$

$$\equiv V_0 \left( 1 + \frac{1}{2} \eta_0 \frac{\phi^2}{M_P^2} \right) - \lambda \phi^4 \quad (12)$$

with

$$\eta_0 \equiv \frac{m^2 M_P^2}{V_0} \quad \text{and} \quad \lambda \equiv \frac{\lambda_4 A}{4 M_P} \quad (13)$$

This form of potential has been considered in \cite{13}. The question is whether $\eta_0$ and $\lambda$ here need fine-tuning. The natural value of soft SUSY breaking terms, $m$ and $A$, are $m \sim A \sim O(\text{TeV}) \sim 10^{-15} M_P$. The coupling $\lambda_4$ is of $O(1)$, which makes $\lambda \sim O(10^{-15})$. As in the case of supernatural inflation, we choose $V_0 = M_I^4$ where $M_I \simeq 10^{11} \text{GeV} \simeq 10^{-7} M_P$ is the intermediate scale, therefore $\eta_0 = O(10^{-2})$. In the following section, we will apply those natural values without fine-tuning to achieve our goal of reducing the spectral index of supernatural inflation.

### 3 Analytical Solution

From Eq. (12), by using Eq. (3,5) we can obtain

$$\left( \frac{\phi}{M_P} \right)^2 = \left( \frac{V_0}{M_P^2} \right) \frac{\eta_0 e^{2N\eta_0}}{\eta_0 x + 4\lambda (e^{2N\eta_0} - 1)} \quad (14)$$

$$x \equiv \left( \frac{V_0}{M_P^2} \right) \left( \frac{M_P}{\phi_{end}} \right)^2, \quad (15)$$

and

$$P_R = \frac{1}{12 \pi^2} e^{-2N\eta_0} \frac{4\lambda (e^{2N\eta_0} - 1) + \eta_0 x} {\eta_0^3 (\eta_0 x - 4\lambda)^2} \quad (16)$$

$$n_s = 1 + 2\eta_0 \left[ 1 - \frac{12\lambda e^{2N\eta_0}}{\eta_0 x + 4\lambda (e^{2N\eta_0} - 1)} \right]. \quad (17)$$
By imposing $P_R^{1/2} \simeq 5 \times 10^{-5}$ and $n_s = 0.96$, we plot $\phi^2/M_P^2$ (at $N = 60$) and $\lambda$ as functions of $\eta_0$ in Fig. 1. The reason why we plot $\phi^2/M_P^2$ is to justify that we can ignore the last term in Eq. (10). By comparing the third and fourth terms in Eq. (10), $\lambda_p \simeq 1$ is the requirement for $\phi^2/M_P^2 \ll \lambda$.

![Figure 1: $\phi^2/M_P^2$ and $\lambda$ as a function of $\eta_0$ at $N = 60$.](image)

As we can see from Fig. 1, when $\lambda \simeq 5 \times 10^{-15}$, $\phi^2/M_P^2 \simeq 5 \times 10^{-17}$, which means the $\phi^6$ term is at least 100 times smaller than the $\phi^4$ term. Therefore, the last term can only contribute about 10% to $V'$ (compare $V'$ with $\phi^6$ to the original $V'$ obtained from CMB normalization), which results in less than 10% correction to $V_0$ (from CMB normalization). The contribution of the $\phi^6$ term to $V''$ is only about 1% (compare the original $V''$ from $\eta = -0.02$ with the value coming from the $\phi^6$ term). Nevertheless, we have a correction about 10% to $\eta$ because of the change of $V_0$, which only affects the spectral index at the level of $\Delta n_s \sim 0.001$. Thus, we still achieve $n_s \simeq 0.96$ even if we include the $\phi^6$ term in the potential. All these justify why we ignore the last in Eq. (12) from the beginning.

The gauge hierarchy problem requires the soft SUSY breaking parameters in the order of $TeV$, otherwise some level of fine-tuning still exists. The soft parameters involved in this study are $m$ and $A$, which we have set to be $O(TeV)$. With these soft parameters we have achieved a desirable spectral index. As long as $m$ and $A$ are in TeV scale the inflation with $N = 60$-fold can be obtained with reasonable $\phi$ and $\lambda$.

Since we are considering soft mass parameters in TeV scale, supersymmetric partners of SM fermions can be readily produced at the LHC. Once these soft mass parameters are determined, parameters for the inflaton field of the inflationary model can be constrained. We will pursue this possibility in future work.

4 Conclusions

In this paper, we have shown that the potential of supernatural inflation can be converted into a hilltop form by introducing a $A$-term (and a negligible non-renormalizable term). The natural value for the soft SUSY breaking parameters are of the order TeV, which is exactly the order
that we need to reduce the spectral index, without fine-tuning, into the latest result according to WMAP.

From Eq. (10), we are assuming that $V_0$ is at the scale of gravity mediated SUSY breaking (and for some versions of gauge mediation). But the SUSY breaking scale in the framework of gauge mediation can be right down to $10^3$ GeV [21]. With the hilltop form of our potential (but not with the original supernatural potential keeping only the quadratic term), we can accommodate these low scales. It is interesting to notice that even $V_0 = 0$, the model can still work (it is the MSSM inflation model). In that case we have a non-hybrid model. With lower $V_0$ there may be a model between hybrid and non-hybrid. We will consider these issues in future work.

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