Anisotropic pressure and quark number susceptibility of strongly magnetized QCD medium

Bithika Karmakar$^{1,4}$, Ritesh Ghosh$^{1,4}$, Aritra Bandyopadhyay$^{2}$, Najmul Haque$^{3,4}$, and Munshi G Mustafa$^{1,4}$

1 Theory Division, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata 700064, India, bithika.karmakar@saha.ac.in
2 Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China,
3 School of Physical Sciences, National Institute of Science Education and Research, Jatni, Khurda 752050, India,
4 Homi Bhabha National Institute, Anushaktinagar, Mumbai, Maharashtra 400094, India.

Abstract. In this work, we compute the hard thermal loop pressure of quark-gluon plasma within strong magnetic field approximation at one-loop order. Magnetic field breaks the rotational symmetry of the system. As a result, the pressure of QGP becomes anisotropic and one finds two different pressures along the longitudinal (along the magnetic field direction) and transverse direction. Similarly, the second-order quark number susceptibility, which represents the fluctuation of the net quark number density, also becomes anisotropic. We compute the second order QNS of deconfined QCD matter in strong field approximation considering same chemical potential for two quark flavors.

Keywords: strong magnetic field, anisotropic pressure, quark number susceptibility

1 Introduction

Recent studies have shown that the magnetic field created by the spectator particles at heavy ion collisions can be as high as $eB \sim 10^{18}$ Gauss [12] at the time of collision. The energy of quarks get quantized in the presence of strong magnetic field. In strong field approximation, only the lowest Landau level of quark energy is considered. The EoS of the system in presence of strong magnetic field is particularly important as it is used to find the time evolution of the system using various hydrodynamic models. Besides, quark number susceptibility represents the fluctuation of the net quark number density over the average value. Fluctuation of the conserved quantities are used as probe of the hot and dense matter created in heavy ion collisions. Here, we calculate the pressure and second order diagonal QNS of the strongly magnetized medium using one loop HTL pt. theory within the scale hierarchy $gT < T < \sqrt{|qfB|}$. 
2 Formalism

In thermal medium the presence of the heat bath velocity $u^\mu$ breaks the boost symmetry of the system. Moreover, the presence of magnetic field breaks the rotational symmetry of the system. We consider the rest frame of the heat bath velocity $u^\mu = (1, 0, 0, 0)$ and the magnetic field along $z$ direction i.e., $n_\mu = (0, 0, 0, 1)$.

2.1 General structure of gluon effective propagator

Now, the gluon self energy $\Pi^{\mu\nu}$ can be expressed in terms of the seven available tensors $P^\mu P^\nu, u^\mu u^\nu, n^\mu n^\nu, P^\mu u^\nu + u^\mu P^\nu, P^\mu n^\nu + n^\mu P^\nu, u^\mu n^\nu + n^\mu u^\nu$. Using the transversality condition $P^\mu \Pi_{\mu\nu} = 0$, the gluon self energy can be written in terms of the four constituent tensors $B^{\mu\nu}, R^{\mu\nu}, Q^{\mu\nu}$ and $N^{\mu\nu}$ which are given in Ref. [3].

Using Dyson-Schwinger equation, the general structure of gluon effective propagator can be written as

$$D_{\mu\nu} = \frac{\xi P_{\mu} P_{\nu}}{P^2} + \frac{(P^2 - \gamma) B_{\mu\nu}}{(P^2 - \alpha)(P^2 - \gamma) - \delta^2} + \frac{R_{\mu\nu}}{P^2 - \beta} + \frac{(P^2 - \alpha) Q_{\mu\nu}}{(P^2 - \alpha)(P^2 - \gamma) - \delta^2}$$

(1)

2.2 General structure of quark effective propagator

Similarly, the general structure of fermion self energy in the presence of strong magnetic field can be written as [4]

$$\Sigma(P) = a\gamma^\mu + b\gamma^\nu + c\gamma^5 \gamma^\mu + d\gamma^5 \gamma^\nu.$$  

where the form factors in presence of strong magnetic field are calculated in Ref. [4]. The general structure of fermion effective propagator is given as

$$S_{eff}(P) = P_+ \frac{L(P)}{L^2} P_+ + P_- \frac{R(P)}{R^2} P_-$$

(2)

with

$$L^\mu = P^\mu + (a + c)u^\mu + (b + d)n^\mu,$$

$$R^\mu = P^\mu + (a - c)u^\mu + (b - d)n^\mu.$$  

(3)

(4)

3 Anisotropic pressure and second order QNS of quark gluon plasma in strongly magnetized medium

The pressure of quark gluon plasma in the presence of strong magnetic field becomes anisotropic i.e., the pressure along the magnetic field (longitudinal)
Anisotropic pressure and \( \cdots \)

and the pressure perpendicular (transverse) to it becomes different. In this case the free energy density in a finite spatial volume \( V \) is given by

\[
F = \frac{F}{V} = \epsilon^{\text{total}} - Ts - eB \cdot M,
\]

where \( \epsilon^{\text{total}} \) and \( s \) are total the energy density and entropy density. The magnetization per unit volume is given by

\[
M = -\frac{\partial F}{\partial (eB)}.
\]

Now, the longitudinal and transverse pressures are given as

\[
P_z = -F, \quad P_\perp = -F - eB \cdot M = P_z - eB \cdot M.
\]

The free energy of quarks and gluons in the presence of strong magnetic field can be calculated using HTL approximation as

\[
F_q = -N_c N_f \sum_f \sum_f \int \ln \left( \det \left[ S_{\mu \nu}^{-1} \right] \right)
\]

\[
= -N_c N_f \sum_f \sum_f \int \ln \left[ P_n^4 \left( 1 + \frac{4a^2 - 4b^2 + 4ap_0 + 4bp_3}{P_n^2} \right) \right],
\]

\[
F_g = (N_c^2 - 1) \left[ \frac{1}{2} \sum_f \ln \left| \det \left( D_{\mu \nu}^{-1} \right) \right| - \sum_f \ln \left| P^2 \right| \right].
\]

Similarly, the second-order longitudinal and transverse QNS are defined as

\[
\chi_z = \left. \frac{\partial^2 P_z}{\partial \mu^2} \right|_{\mu=0}, \quad \chi_\perp = \left. \frac{\partial^2 P_\perp}{\partial \mu^2} \right|_{\mu=0}.
\]

4 Results

In the left panel of Fig.\[1\] the longitudinal and transverse pressure of strongly magnetized QGP are shown with the variation of magnetic field strength at \( \mu = 0 \). It can be seen that the longitudinal and transverse pressure show different nature with magnetic field. The longitudinal pressure increases with the magnetic field whereas the transverse one decreases. This means that the system elongates more along the magnetic field direction and it can even shrink \[6\] along the transverse direction at very high magnetic field strength.

The variation of second order diagonal QNS of strongly magnetized QGP is shown with temperature in the right panel of Fig.\[1\]. Similar to the anisotropic pressure, the longitudinal and the transverse QNS also shows different nature.
Fig. 1. Variation of the longitudinal and transverse pressure at $\mu = 0$ with magnetic field is shown in left panel. Magnetization as a function of temperature at $\mu = 0$ is shown in right panel for $N_f = 3$.

At very high temperature, the QNS of the interacting system reaches the ideal value.

5 Summary

Using one loop HTL pt theory with strong magnetic field approximation we found that the QGP pressure becomes anisotropic. Due to the presence of magnetization, the transverse pressure decreases with magnetic field. The transverse pressure can even be negative at very high magnetic field strength indicating the shrink of the system along the transverse direction. It has been found that the system shows paramagnetic nature. On the other hand, second order diagonal QNS has been calculated using the longitudinal and the transverse pressure. We find qualitative match of our results with lattice studies [6]. However, perturbative studies can be improved by considering higher loop order calculations.

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