Numerical solution of flow in bypass for generalized Newtonian fluids

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Abstract. This work presents the numerical solution of generalized Newtonian fluids in the bypass geometry. The considered geometry consists of the narrowed host tube and the bypass graft with 30, 45, 60 degrees angle of the connection. Two values of the Reynolds number, 100, 200, are tested. The numerical results of non-Newtonian fluids are compared to the results of Newtonian fluids. The governing system of equations is based on the system of balance laws for mass and momentum. Generalized Newtonian fluids flow in the bypass tube is numerically simulated by using SIMPLE algorithm included in OpenFOAM.

1. Introduction
The cardiovascular diseases causes more than 30 % of deaths in the world. This category of diseases refers to the heart and blood vessels, e.g. hypertension, heart attack, atherosclerosis or stroke etc. Cardiovascular diseases are mainly caused by a formation of sediments on the inner wall of the vessel [15]. In the place where sediments are cumulated, the vessel is narrowed and the blood flow rate is restricted. One way how to solve this problem is to bridge the narrowing place by the graft - bypass.

The quality of the blood flow in the bypass can be influenced by a location of the narrowing or by the Reynolds number, see [7], or by the geometry. In a research work [6] the experimental study of bypass connection angles (30, 45 and 60 degrees) concluded that the optimal angle for the connecting of the bypass graft to the vessel is 45 degrees, since the lowest shear rates found in the primary separation region is higher than for other values (30 deg and 60 deg).

This work is mentioned on the numerical modelling of the varied mathematical models of the viscosity in the bypass tube. Two parameters were tested, different values of Reynolds numbers (Re = 100 and Re = 200) with corresponding to the different values of the inlet velocity, and various angles of the connection between narrowed channel and the bypass graft. Three values of the angle were used, 30, 45 and 60 degrees. Three mathematical models for viscosity are selected in this paper. Carreau-Yasuda model and modified Cross model were used for numerical testing. The obtained results were compared with corresponding results for Newtonian fluids.

In the idealized case the blood flow can be considered as the laminar flow and fluid is incompressible with the constant density. The governing system of equations describing the blood motion in the vessels is the generalized system of Navier-Stokes equations.
2. Mathematical model

Let us consider that the blood flow is the laminar incompressible generalized Newtonian fluid with constant density $\rho$ and shear dependent dynamic viscosity $\mu(\dot{\gamma})[12]$ where $\dot{\gamma}$ is the shear rate defined by

$$\dot{\gamma} = 2\sqrt{\frac{1}{2} \text{tr} \ D^2},$$

(1)

where $D = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$.

For the Newtonian fluid the dynamic viscosity reads (see [2], [14], [16])

$$\mu(\dot{\gamma}) = \mu_\infty,$$

(2)

and for the generalized Newtonian fluid the selected viscosity models are considered, see [1], [4], [14]:

- the modified Cross model [13]

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty) \left[1 + (\lambda \dot{\gamma})^b\right]^{-a},$$

(3)

- the Carreau-Yasuda model [3]

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty) \left[1 + (\lambda \dot{\gamma})^m\right]^{-\frac{1}{m}} ,$$

(4)

where $\mu_0$ and $\mu_\infty$ are the asymptotic viscosity values at zero and infinite shear rates. The symbol $\lambda$ denotes a relaxation time and $a, b, m, n$ are parameters of the generalized Newtonian viscosity models, see Table 1 (see [3], [8], [13]). In Fig. 1 the relationship between the viscosity $\mu$ and the shear rate $\dot{\gamma}$ for selected viscosity models is presented.

| viscosity model               | parameters                        |
|-------------------------------|-----------------------------------|
| Newtonian model               | $\mu_\infty = 3.5 \times 10^{-3}$ Pa s |
| modified Cross model          | $\mu_\infty = 3.5 \times 10^{-3}$ Pa s, $\mu_0 = 160 \times 10^{-3}$ Pa s, $\lambda = 8.2$ s, $a = 1.23$, $b = 0.64$ |
| Carreau-Yasuda model          | $\mu_\infty = 3.45 \times 10^{-3}$ Pa s, $\mu_0 = 56 \times 10^{-3}$ Pa s, $\lambda = 1.902$ s, $m = 1.25$, $n = 0.22$ |

Table 1. Characteristics of the presented viscosity models.

The governing system of equations is the system of Navier-Stokes equations. This system can be written in the form as

$$\text{div} \ \mathbf{u} = 0,$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \text{div} \ (2\mu(\dot{\gamma})\mathbf{D}),$$

(5)

where $P$ is the dynamic pressure, $\rho$ is the constant density, $\mathbf{u}$ is the velocity vector, $\mu(\dot{\gamma})$ denotes the dynamic viscosity of the generalized Newtonian fluid given by one of the Eqs. (2)-(4) and $\mathbf{D}$ is the symmetric part of the velocity gradient, see [5].
3. Numerical solution
For space discretization the cell-centered finite volume method is used [9]. In order to apply the finite volume method the computational domain is approximated by a set of closed non-overlapping hexahedral cells. This system of cells is called the finite volume mesh.

System of equations (5) is equipped with an initial condition \( u(x, 0) = u_0(x) \) and with the boundary conditions specified at the boundary of the computational domain. At the inlet Dirichlet boundary condition for velocity vector is used and for the pressure homogeneous Neumann boundary condition is used. At the outlet parts the pressure value is prescribed and for the velocity vector homogeneous Neumann boundary condition is used. The no-slip boundary condition for the velocity vector is used on the wall. For the pressure homogeneous Neumann boundary condition is considered.

OpenFOAM is a C++ toolbox for the development of customized numerical solvers. The SIMPLE algorithm is a pressure correction algorithm [10], [11] and it is the main method used for the numerical calculation of incompressible fluid flow problems.

4. Numerical Results
The computational domain as the model of the bypass geometry is shown in Fig. 2. It is described by the parameters \( R, R_s, L_R \). \( R \) denotes the radius of the main channel, \( R = 0.0031 \) m and \( L_R \) denotes the length of the channel with bypass, \( L_R = 33 R \). The radius of the narrow part of the channel is \( R_s, (R_s = 0.5 R) \). Three values of angle of the connection between the narrowed channel and the bypass were considered, 30, 45 and 60 degrees. The computational domain is discretized using an unstructured mesh composed of hexahedral cells, see Fig. 2.

The fluid is described by the constant density \( \rho = 1050 \) kg m\(^{-3}\) and the viscosity models specified by the parameters summarized in Table 1. At the inlet a fully developed flow is assumed. In the case of the Newtonian fluid, the parabolic velocity profile with the maximum velocity value \( U_0 \) is defined at the inlet. A constant pressure value is prescribed at the outlet. Two different values of Reynolds number are considered. Which corresponds to the different inlet velocity. For Reynolds number \( Re = 100 \) the velocity is \( U_0 = 0.053 \) m s\(^{-1}\) and for the second Reynolds number \( Re = 200 \) the velocity has the value \( U_0 = 0.106 \) m s\(^{-1}\).

Figs. 3, 5 and 7 show 2D slices of generalized Newtonian velocity profiles at selected cross-section for various angles of the connection, 30 degrees Fig. 3, 45 degrees Fig. 5 and 60 degrees Fig. 7. The numerical results shown up in these figures correspond the Reynolds number \( Re = 100 \). The sketched results shown bottom correspond to the Reynolds number \( Re = 200 \). The ratio of the diameters both parts of the channel, narrowed vessel and the bypass graft, is fixed with value 0.5 therefore the velocity for all tested fluids is maximal in the narrowed place. It
caused by the constricted channel. The fluid needs to produce significant force (pressure) for the going through this place. The maximum velocity is different for used viscosity models. The smallest one is for the Newtonian model while the highest is for modified Cross model. These differences can be given by the various asymptotic viscosity values at zero shear rate (shear thinning Newtonian fluids).

Figure 2. Unstructured hexahedral computational mesh.

In the Figs. 4, 6 and 8 the velocity distribution along the axis of the main channel for the various angle of the connection and for different Reynolds numbers are presented. From these figures it can be observed that main differences between selected rheology mathematical viscosity models and Newtonian fluid model are in the narrowed place. For smaller Reynolds number (Re = 100) the numerical results for Newtonian fluid and Carreau-Yasuda model are similar, whereas the numerical results for the modified Cross model is beyond them. In the case of the larger Reynolds number (Re = 200) the situation is reversed. The maximum

Figure 3. 2D slices of generalized Newtonian velocity profiles at selected cross-section, angle of connection 30 degrees, Re = 100 (up) and Re = 200 (bottom).
5. Conclusion
The problem of bypassing the stenosed human artery was modelled by incompressible laminar system of Navier-Stokes equations where different various rheology mathematical models for blood flow were considered, namely Newtonian, modified Cross and Carreau-Yasuda model. The numerical solution of fluid flow was obtained by the open-source solver OpenFOAM which is based on the finite volume method. Specially, the SIMPLE algorithm was used due to its good time performance for all considered viscosity models.
The tests were performed on the model of bypass geometry, where the different angles of the connection between the narrowed channel and the bypass graft were considered. Two values of the Reynolds number were used for numerical modelling, Re = 100 and Re 200.

The results show that for the considered angle of connection (45 degrees) the differences between Newtonian and non-Newtonian models are not significant. And thus the use of Newtonian model is reasonable there. On the other hand for other angles (less than 40 deg or higher than 50 deg) the influence of the non-Newtonian character of the fluid becomes more important and thus needs to be taken into account.
Figure 8. Velocity distribution along the axis of the main channel for the tested 30 degrees of angle of the connection and for different Reynolds numbers, Re = 100 (left) and Re = 200 (right).

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