Energy Loss in Nuclear Drell-Yan Process
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Abstract
By means of the nuclear parton distributions which can be used to provide a good explanation for the EMC effect in the whole x range, we investigate the energy loss effect in nuclear Drell-Yan process. When the cross section of lepton pair production is considered varying with the center-of-mass energy of the nucleon-nucleon collision, we find that the nuclear Drell-Yan(DY) ratio is suppressed due to the energy loss, which balances the overestimate of the DY ratio only in consideration of the effect of nuclear parton distributions.

1 Introduction
In 1983, the European Muon Collaboration(EMC)\textsuperscript{[1]} surprisingly found that the nucleon structure function, as measured in deep-inelastic lepton-nucleus scattering (DIS), varies with the target nucleus. This phenomenon has been known as the EMC effect. Since the discovery of the EMC effect, various models have been proposed to investigate the nuclear effect\textsuperscript{[2, 3, 4]}. Among them, the $x-$rescaling\textsuperscript{[2]} or $Q^2$-rescaling mechanism\textsuperscript{[3]} is commonly accepted to adequately explain the EMC effect. In addition, continuum dimuon production in high-energy hadron collisions, known as the Drell-Yan(DY) process\textsuperscript{[5]}, provides an independent measure of the modification of the quark structure of nuclei. Recently, many investigations on the EMC and nuclear Drell-Yan effects are still going on with great progress\textsuperscript{[6, 7]}. Several years ago, the E772 Collaboration\textsuperscript{[8]} at Fermilab published the data of high-mass dilepton production measured in the nuclear Drell-Yan (DY) process. These data aroused special attention in clarifying the various explanations of the nuclear effect on the parton distributions. Their results show that the ratio of DY dimuon yield per nucleon on a nuclear target to that on a free nucleon (from now on, shorten as the nuclear DY ratio) is slightly less than unity.
if the momentum fraction $x$ carried by a target quark is less than 0.1. The ratios over the range $0.10 \leq x \leq 0.30$, however, do not reveal distinct nuclear dependence. Bickerstaff et al. [9] found that although most of the theoretical models provide good explanations for the EMC effect, they do not give a good description of the nuclear Drell-Yan ratio. Most of the theoretical models of the EMC effect overestimate the nuclear DY ratio.

Several years ago, we put forward an extended $x-$rescaling model [10] in which different $x-$rescaling parameters for the valence quarks and sea quarks (gluons) in the nucleon structure function are employed in consideration of the nuclear momentum conservation. With a simultaneous consideration of the nuclear shadowing and nuclear momentum conservation, the experimental data of the EMC effect can be well explained in the whole $x$ region. However, similar to the prediction for nuclear DY ratio in pion-excess and quark-cluster models [9], by using the obtained nuclear parton distributions in the extended $x$-rescaling model, the nuclear DY ratio is also overestimated. The difference of the nuclear effects between the nuclear DY and DIS processes is not clear yet. In this paper, we suggest an additional nuclear effect due to the energy loss in DY process. We find that the nuclear DY ratio is suppressed significantly as a consequence of continuous energy loss of the projectile nucleon to the target nucleon in their successive binary nucleon-nucleon collisions. This suppression balances the overestimate of the DY ratio only in consideration of the nuclear effect on the parton distributions. Therefore, a combination of these two types of nuclear effects can give a good explanation of the experimental data of the nuclear DY ratio.

2 Nuclear Parton Distributions in the Extended $x-$rescaling Model

To provide the nuclear parton distributions which can be used to explain the experimental data of the EMC effect in the whole $x$ region, we work in our familiar extended $x-$rescaling model. Let $I_{A(N)}(x, Q^2)$, $I=V, S, G$ be the probability distributions of valence quarks(V), sea quarks(S), and gluons(G) in the nucleus A(or nucleon N), respectively. Then $K_A^I(x, Q^2) = xI_{A(N)}(x, Q^2)$, $I=V, S, G$, are the momentum distributions of valence quarks(V), sea quarks(S), and gluons(G) in the nucleus A (or nucleon N), respectively. In Ref. [10], we
pointed out that, the nuclear binding effect together with the $x$-rescaling mechanism further introduced does not affect the valence quark number conservation. However, the nuclear momentum is no longer conserved in the $x$-rescaling model. In order to keep nuclear momentum conservation, we extended the $x$-rescaling model and employ different $x$-rescaling parameters for the momentum distributions of valence quarks and sea quarks (gluons) in the nucleon structure function, i.e.,

$$K_A^{V(S)}(x, Q^2) = K_N^{V(S)}(\delta V(x), Q^2),$$  

(1)

Because the momentum distributions of sea quarks and gluons have similar forms, we take the same $x$-rescaling parameter for the momentum distributions of sea quarks and gluons in the nucleus. The numerical result shows that, by properly choosing these parameters (one of them is determined according to the nuclear momentum conservation condition), one can well explain the experimental data of the EMC effect. Because of its simple form and also for giving a good explanation of the EMC effect, the extended $x$-rescaling model has been adopted by EMC \cite{11} to fit their experimental data. Naturally, one hope that the nuclear DY ratio can also be well predicted by using the obtained nuclear parton distributions. Unfortunately, the calculation results show that the nuclear DY ratio is overestimated only if the nuclear effect on the parton distributions is considered (see the dotted-line in Fig. 1). This indicates that some other mechanisms of the nuclear effect should be further taken into account.

### 3 Energy Loss in Nuclear DY Process

The Drell-Yan(DY) model \cite{5} gives a good description of the continuum of massive dimuon pair production in the collision of proton with the nucleus $A$:

$$p + A \rightarrow \mu^+ \mu^- + X.$$  

(2)

This process is described as an electromagnetic annihilation of a quark (antiquark) in the proton $p$ and an antiquark (quark) in the nucleon embedded in the nucleus $A$ into a dimuon pair. The parton-model cross section for the DY process is given by
\[
\frac{d^2\sigma}{dM^2dx_F} = \frac{4\pi\alpha^2}{9sM^2} \sum_i e_i^2 \left[ q_i^{p(A)}(x_1)\bar{q}_i^{A}(x_2) + \bar{q}_i^{P}\bar{q}_i(x_1)q_i^{A}(x_2) \right] / \sqrt{x_F^2 + 4M^2/s},
\]

where \(K\)-factor, with \(K \sim 2\), is due to next-to leading order QCD calculations \[12\], and \(\alpha\) is the fine-structure constant, \(e_i\) is the fractional charge of the quark of flavor \(i\), and \(q_i^{p(A)}(x)\) and \(\bar{q}_i^{p(A)}(x)\) are, respectively, the quark and anti-quark distributions in the proton (nucleon embedded in the nucleus \(A\)).

The Feynman scaling variable \(x_F\) is defined as

\[x_F = \frac{2p_l}{s},\]

where \(\sqrt{s}\) is the nucleon-nucleon center-of-mass system (cms) energy and \(p_l\) is the longitudinal momentum of the virtual photon of mass \(M\). The quantities \(x_{1,2}\) are related to \(x_F\) and \(M^2\) by

\[x_{1,2} = \frac{1}{2} \left( \sqrt{x_F^2 + \frac{4M^2}{s}} \pm x_F \right),\]

\[x_1 - x_2 = x_F,\]

and

\[x_1x_2 = \frac{M^2}{s}.\]

The above relations are easily extended to account for the evolution of quark structure functions with \(Q^2\).

Now let us turn to the case in which the effect of energy loss in the initial states is taken into account. For a nucleon-nucleus collision, the probability of having \(n\) collisions at an impact parameter \(\vec{b}\) can be expressed as

\[P(\vec{b}, n) = \frac{A!}{n!(A - n)!} [T(\vec{b})\sigma_{in}]^n [1 - T(\vec{b})\sigma_{in}]^{A - n},\]

where \(\sigma_{in} \sim 30mb\) is the non-diffractive cross section for inelastic nucleon-nucleon collision, and \(T(\vec{b})\) is the thickness function of the impact parameter \(\vec{b}\). The basic thickness function \(T(\vec{b})\) can be well approximated by a Gaussian function with a standard deviation \(\beta_p\). If the collided nuclei are
small \((A \leq 32)\), their density function \(\rho\) can also be taken to be a Gaussian function of the spatial coordinates. Consequently, the thickness function can be conveniently written as

\[
T(\vec{b}) = \exp\left(-\vec{b}^2/2\beta_A^2\right)/2\pi\beta_A^2.
\]

In terms of the standard root-mean-squared-radius parameter \(r'_0\) for the nucleus \(A\), the standard deviation \(\beta_A\) is given by

\[
\beta_A = r'_0 A^{1/3}/\sqrt{3},
\]

here \(r'_0\) is found to be 1.05 fm in Ref. [13], and therefore

\[
\beta_A = 0.606A^{1/3}.
\]

For the nucleus with larger mass number \(A(A > 32)\), the thickness function can be approximated by using a sharp-cutoff density distribution of the form

\[
T(\vec{b}) = \frac{3}{2\pi R_A^3} \sqrt{R_A^2 - \vec{b}^2} \theta(R_A - |\vec{b}|),
\]

where \(R_A = r_0 A^{1/3}\) is the radius of colliding nucleus with \(r_0 = 1.2\) fm.

In Eq. (8), the first factor on the right-hand side represents the number of combinations for finding \(n\) collisions out of \(A\) possible nucleon-nucleon encounters, the second factor gives the probability of exactly \(n\) collisions and the third factor gives the probability of having exactly \(A - n\) misses. The total probability for the occurrence of an inelastic event in the collision of proton with the nucleus \(A\) at an impact parameter \(\vec{b}\) is the sum of Eq. (8) from \(n = 1\) to \(n = A\):

\[
d\sigma_{in}^{p-A}/db = \sum_{n=1}^{A} P(n, \vec{b}) = 1 - [1 - T(\vec{b})\sigma_{in}]^A.
\]

Therefore, from Eq. (13), the total inelastic cross section \(\sigma_{in}^{p-A}\) for the collisions of protons with the nucleus \(A\) is

\[
\sigma_{in}^{p-A} = \int d\vec{b}\{1 - [1 - T(\vec{b})\sigma_{in}]^A\}.
\]
In an inelastic nucleon-nucleus collision without impact parameter selection, the number of nucleon-nucleon collisions \( n \) (for \( n = 1 \) to \( A \)) has a probability distribution \( P(n) \). This is obtained by integrating \( P(n, \vec{b}) \) over all impact parameters:

\[
P(n) = \frac{\int d\vec{b} P(n, \vec{b})}{\sum_{n=1}^{A} \int d\vec{b} P(n, \vec{b})},
\]

where the denominator is to ensure that \( P(n) \) is properly normalized as

\[
\sum_{n=1}^{A} P(n) = 1.
\]

From Eq. (13), the denominator of the right hand side of Eq. (15) can be replaced by

\[
\sum_{n=1}^{A} \int d\vec{b} P(n, \vec{b}) = \int d\vec{b} \{1 - [1 - T(\vec{b})\sigma_{in}]^A\}.
\]

(17)

To describe the energy loss in the collision of a proton with the nucleus \( A \), we start with remarks on the relative role of "soft" and "hard" interactions in nuclear collisions at very high energy: The incident proton interacts with spectator nucleon and makes soft (nonperturbative) minimum bias collisions before making the high \( Q^2 \) dimuon pair; During the "soft" collisions, the projectile proton imparts energy to the struck nucleon and therefore must loose energy; Thus energy loss must affect the cross sections of producing dimuon pair. After the projectile proton has had additional \( n \) collisions with nucleons embedded in the nucleus, the cms energy of the colliding nucleons with "hard" DY collisions can be expressed as

\[
\sqrt{s'} = \sqrt{s} - (n - 1)\frac{d\sqrt{s}}{dn},
\]

(18)

where \( \frac{d\sqrt{s}}{dn} \), generally taken as 0.2–0.4\, GeV, is the cms energy loss per collision in the initial state. Therefore, the cross section for the DY process can be re-written as

\[
\frac{d^2\sigma}{dM^2dx_F} = K\sqrt{s} \frac{4\pi\alpha^2}{\sqrt{s'9s'}} \sum_i e_i^2 \frac{[q_i^p(x'_1)\bar{q}_i^A(x'_2) + \bar{q}_i^p(x'_1)q_i^A(x'_2)]}{\sqrt{x'_2^2 + 4M^2/s'}}.
\]

(19)
where the rescaled quantities are defined as

$$x'_F = \frac{2p_t}{\sqrt{s'}} = r_s x_F,$$

(20)

and

$$x'_{1,2} = r_s x_{1,2},$$

(21)

with the cms energy ratio:

$$r_s = \sqrt{s}/\sqrt{s'}.$$

(22)

The average cross section for the dimuon production in nuclear DY process can be expressed as

$$\langle \frac{d^2\sigma}{dM^2 dx_F} \rangle = \sum_{n=1}^{A} P(n) \frac{d^2\sigma}{dM^2 dx_F}.$$

(23)

To make a comparison between the theoretical prediction for the nuclear DY ratio and the experimental data with respect to the variables $x_1$ and $x_2$, Eq. (19) and Eq. (23) can be re-expressed as

$$\frac{d^2\sigma}{dx_1 dx_2} = K \frac{4\pi\alpha^2}{9M^2} \sum_i e_i^2 [q_i^p(r_s x_1)\bar{q}_i^A(r_s x_2) + \bar{q}_i^p(r_s x_1)q_i^A(r_s x_2)],$$

(24)

and

$$\langle \frac{d^2\sigma}{dx_1 dx_2} \rangle = \sum_{n=1}^{A} P(n) \frac{d^2\sigma}{dx_1 dx_2},$$

(25)

respectively. It is noteworthy that $r_s$ in (22) is always greater than one if there exists an energy loss in the collisions of protons with the nucleus $A$.

### 4 Numerical Results

To compare the theoretical prediction for nuclear Drell-Yan process with the experimental data of E772 collaboration [8], we introduce the nuclear DY ratio as

$$T^{A/D}(x_2) = \frac{\int dx_1 \frac{d^2\sigma_{p^-A}}{dx_1 dx_2}}{\int dx_1 \frac{d^2\sigma_{p^-p}}{dx_1 dx_2}},$$

(26)
where \( \frac{d^2 \sigma^{p-d}}{dx_1 dx_2} \) is the differential cross section for the dimuon pair production in the proton-deuteron collision. The integral range for \( x_1 \) in Eq. (26) is determined according to the kinematic region of the experiment in Ref. [8], i.e. \( x_1 - x_2 > 0 \), and \( 0.025 \leq x_2 \leq 0.30 \). By using the free parton distributions of GRV [14] and taking the \( x \)-rescaling parameters \( \delta_V = 1.026 \) and \( \delta_S = 0.945 \) for valence quarks and sea quarks, respectively, we calculate the nuclear Drell-Yan ratios for \(^{56}\text{Fe}\). In addition, we assume that the gluon and the sea quark structure functions have similar shadowing effects at small \( x_2 \), and according to Ref. [15], we introduce the shadowing factor \( R_{A_{sh}} \) both for gluons and sea quarks as

\[
R_{A_{sh}}(x_2) = \begin{cases} 
1 + a \ln A \ln(x_2/0.08), & x_2 < 0.08, \\
1 + b \ln A \ln(x_2/0.08) \ln(x_2/0.24), & 0.08 < x_2 < 0.3
\end{cases}
\]

where the parameters \( a, b \) are taken as 0.025 and -0.02, respectively. The calculation results with \( \frac{d\sqrt{s}}{dx} = 0.0, 0.2, 0.4 \text{GeV} \) shown in Fig. 1 indicate that the energy loss effect is essential in the explanation of the nuclear DY ratio.

5 Discussion and Summary

To sum up, by means of the nuclear parton distributions in the extended \( x \)-rescaling model on which the experimental data of the EMC effect can be well explained, we investigate the nuclear DY process focusing on the continuous energy loss of the projectile nucleon to target nucleons in their successive nucleon-nucleon collisions. We find that the nuclear DY ratio will be overestimated if the nuclear effect on the parton distributions is the only factor considered. The calculation results show that the nuclear DY ratio is sensitive to the change of sea quark distributions. On one hand, to adequately explain the antishadowing effect and the EMC effect, there should exist an enhancement of the sea quarks in the nuclear quark distributions in the range of \( 0.1 \leq x \leq 0.3 \), resulting in the overestimation of the nuclear DY ratio; On the other hand, the energy loss causes a suppression of the nuclear DY ratio, which balances the overestimate of the nuclear DY ratio due to the nuclear effect on the parton distributions. So, in the nuclear DY process, there is no distinct nuclear effect as observed in DIS process due to the combination of the nuclear effect on the parton distributions and the energy loss effect.
Similarly, the J/ψ suppression is also partially due to the energy loss in the initial states.

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**Figure Caption**

Fig. 1. The nuclear Drell-Yan ratios $T_{Fe/D}(x_2)$ predicted with the energy loss (solid curve for $d\sqrt{s}/dn=0.2$ GeV, dashed curve for $d\sqrt{s}/dn=0.4$ GeV) and without the energy loss (dotted curve for $d\sqrt{s}/dn=0.0$ GeV). The experimental data are taken from the E772 Collaboration [8].
Fig. 1