Elliptic flow of multi-strange particles: 
fragmentation, recombination and hydrodynamics

Chiho Nonaka\textsuperscript{a}, Rainer J. Fries\textsuperscript{a}, Steffen A. Bass\textsuperscript{a,b}

\textsuperscript{a}Department of Physics, Duke University, Durham, NC 27708, USA
\textsuperscript{b}RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

We study the elliptic flow $v_2$ of multi-strange hadrons such as the $\phi$, $\Xi$ and $\Omega$ as a function of transverse momentum in the recombination and fragmentation model and compare to a standard hydrodynamic calculation. We find that the measurement of $v_2$ for the $\phi$ and $\Omega$ will allow for the unambiguous distinction between parton recombination and statistical hadro-chemistry to be the dominant process in hadronization at intermediate transverse momenta.

Key words: Relativistic heavy-ion collisions, Elliptic flow
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1 Introduction

Elliptic flow of hadrons – more precisely defined as the second Fourier coefficient of their azimuthal momentum distribution – has been suggested as a sensitive probe for the buildup of pressure in the early reaction stage of relativistic heavy-ion collisions \cite{1}. Data from Au+Au collisions at the Relativistic Heavy Ion Collider (RHIC) has revealed unexpected features in the behavior of the elliptic flow $v_2$ as a function of transverse momentum $P_T$: for non-central collisions $v_2$ first rises and then saturates at higher $P_T$ \cite{2,3,4,5,6}, the saturation occurring at higher $P_T$ and at higher values for $v_2$ for baryons than for mesons.

Email address: nonaka@phy.duke.edu (Chiho Nonaka).
It was first pointed out by Voloshin that this surprising behavior can be explained in a naive parton recombination model for hadronization [7]. Several more detailed studies on elliptic flow based on recombination have been carried out subsequently [8,9,10,11,12,13]. In addition to elliptic flow, parton recombination has been very successful in explaining the behavior of particle spectra and ratios at intermediate transverse momenta. The general theory of recombination as a hadronization mechanism in dense parton systems is described in great detail in [8,9,13,14,15,16,17,18,19]. In this letter we present quantitative results for the elliptic flow of multi-strange particles in the recombination plus fragmentation approach – the theoretical foundation being based on our previous work in [9].

At low \( P_T \) the experimental data show that the elliptic flow of a heavy particle is smaller than that of a light particle. This mass-dependence can be understood in hydrodynamic models where particles with higher mass experience the collective movement of the medium less than lighter species [20,21]. However, the saturation of \( v_2 \) above \( P_T \approx 1.5 \text{ GeV}/c \) and its dependence on the hadron species in this region contradicts a purely hydrodynamic model.

At high \( P_T \), the energy loss of fast partons in the dense medium has been proposed as a mechanism that can translate the anisotropy in the initial Au+Au system into an momentum anisotropy in the final state as well [22,23,24]. The observation of jet quenching at RHIC energies is a strong indication for the existence of partonic energy loss which may lead to such an azimuthal anisotropy [25].

In the most interesting region of intermediate \( P_T \), between 2 and 5 GeV/c, the use of hydrodynamics becomes questionable, since the fundamental assumption of an infinitely small mean free path is not valid anymore. On the other hand, the use of perturbative QCD is not well justified either, since the relevant scale \( \sim P_T \) is rather small and may lead to uncontrollable higher twist corrections. In addition, the strong energy loss further suppresses the contribution from perturbative parton fragmentation. These considerations suggest recombination as the dominant hadronization mechanism in the intermediate \( P_T \) region.

To clarify the situation and to establish the dominant mechanism for hadronization and the creation of hadron \( v_2 \) in the intermediate \( P_T \) domain we propose the measurement of \( v_2 \) for multi-strange hadrons, in particular for the \( \phi \) and the \( \Omega \). In a hydrodynamic picture the differences between hadron species are predominantly driven by the mass differences. In the recombination formalism the mass only plays a role at low \( P_T \), while strong deviations appear at intermediate \( P_T \) due to the different valence structure of baryons and mesons. Since the mass of the \( \phi \) is very close to that of the \( \Lambda \), a statistical hadronization scheme as employed by hydrodynamics would predict the \( \phi \) to behave similar
to the Λ, while recombination implies that it behaves similar to other mesons, e.g. the kaon.

2 Recombination plus fragmentation approach

In [8,9] we found that hadron production at intermediate $P_T$ is governed by an interplay of fragmentation and recombination. Recombination is more efficient than fragmentation if the parton spectrum is exponential, while fragmentation dominates for the (perturbative) power law tail of the partonic $P_T$ spectrum. E.g. for mesons $M$ recombining on a hypersurface $\Sigma$, the momentum distribution is given by

$$E \frac{N_M}{d^3P} = C_M \int_{\Sigma} d\sigma_R \frac{P \cdot u(R)}{(2\pi)^3} \int_0^1 dx w_a(R; xP^+) |\phi_M(x)|^2 w_b(R; (1-x)P^+), \quad (1)$$

where $\phi_M(x)$ is a wave function for the meson on the light cone, $x$ is the momentum fraction of valence quark $a$, the $w$ are the momentum distributions of the quarks $a$ and $b$ and $C_M$ is a degeneracy factor. We refer the reader to [9] for more details.

The parton momentum distribution at hadronization is an input in Eq. (1). For quantitative predictions we assume a thermal distribution,

$$w_a(R; p) = \gamma_a e^{-p \cdot v(R)/T} e^{-\eta^2/2\Delta^2} f(\rho, \phi), \quad (2)$$

where $v(R)$ is a four velocity and $\gamma_a$ is fugacity factor for each parton species $a$. The longitudinal and transverse spatial distributions are described by the width $\Delta$ for the space-time rapidity $\eta$ and a profile function $f(\rho, \phi)$ for transverse size and azimuthal angle $\rho$ and $\phi$. We set $\Delta = 2$. $f(\rho, \phi)$ is impact parameter dependent and is scaled with the volume of the overlap zone of two nuclei in non-central collisions.

We assume that the hadronization process takes place at a temperature $T = 175$ MeV on a hypersurface $\Sigma$ given by $\tau = \sqrt{t^2 - z^2} = 5$ fm/c. The radial flow velocity $v_T$ is fixed at $0.55c$ for all values of $\rho$ and for all impact parameters. These input parameters were determined in [9] and are consistent with the observed hadron spectra and ratios from PHENIX and STAR. The recombination calculation has to be supplemented by a pQCD calculation using fragmentation functions. However, no information is available concerning the fragmentation functions of strange hadrons beyond kaons and Λs.
3 Elliptic flow

The azimuthal asymmetry \( v_2 \) is defined by

\[
v_2(P_T) = \langle \cos 2\Phi \rangle = \frac{\int d\Phi \cos 2\Phi d^2N/d^2P_T}{\int d\Phi d^2N/d^2P_T},
\]

where \( \Phi \) is the azimuthal angle in momentum space.

At low and intermediate \( P_T \) the initial geometric anisotropy results in an anisotropic pressure gradient that implies an elliptic flow profile. This is parametrized via an anisotropic parton momentum distribution at the time of hadronization at which partons then hadronize via recombination. We use Eq. (2) together with an elliptic modulation of the transverse rapidity

\[
\eta_T(\phi, p_T) = \eta^0_T (1 - F(p_T) \cos 2\phi)
\]

to determine \( v_2 \) for the partons at hadronization. \( \eta^0_T \) is the rapidity given by the radial flow velocity \( v_T = 0.55c \), so that \( \tanh \eta^0_T = 0.55 \). This is inspired by a hydrodynamical description of the parton phase at low \( p_T \). Hydrodynamics works well to describe the measured \( v_2 \) for hadrons up to \( P_T = 1.5 \text{ GeV}/c \) [4,26,27,28]. In order to accommodate the deviation from ideal hydrodynamics at higher \( P_T \) we assume that \( F(p_T) \) takes the from

\[
F(p_T) = \frac{\alpha}{1 + (p_T/p_0)^2},
\]

where for a given impact parameter \( b \)

\[
\alpha = \frac{w(b) - l(b)}{w(b) + l(b)}
\]

is determined by the collision geometry with width \( w(b) = R_A - b \) and length \( l(b) = \sqrt{R_A^2 - (b/2)^2} \) of the collision zone. \( R_A \) is the radius of a gold nucleus.

We assume that \( v_2^u = v_2^d = v_2^s = v_2^s \) and \( v_2^u = v_2^s \). The slight difference between light quarks and strange quarks originates from the mass difference. We use constituent masses \( m_{u,d} = 260 \text{ MeV} \) and \( m_s = 460 \text{ MeV} \). The parameter \( p_0 = 1.1 \text{ GeV}/c \) was obtained in [9] by a fit to the elliptic flow of pions obtained by PHENIX [2] and is also consistent with data on \( v_2 \) for charged hadrons, protons, kaons and \( \Lambda s \) [9].

In [9] we showed that for recombining mesons \( M \) and baryons \( B \) the anisotropy \( v_2 \) can be written as

\[
v_2^M(P_T) = \frac{\int dx|\phi_M(x)|^2[v_2^0(xP_T) + v_2^b((1-x)P_T)]k_M(x, P_T)}{\int dx|\phi_M(x)|^2[1 + 2v_2^0(xP_T)v_2^b((1-x)P_T)]k_M(x, P_T)},
\]

where...
\[ v_2^B(P_T) = \frac{\int D x_i |\phi_B(x_i)|^2 [\sum_{j=a,b,c} v_2^j(x_j P_T) + 3 \prod_{j=a,b,c} v_2^j(x_j P_T)] k_B(x_i, P_T)}{\int D x_i |\phi_B(x_i)|^2 [1 + 2 \sum_{j=a,b,c} \prod_{\mu \neq j} v_2^\mu(x_\mu P_T)] k_B(x_i, P_T)} \],

where \( k_M(x, P_T) \) and \( k_B(x, P_T) \) are given by

\[ k_M(x, P_T) = K_1 \left[ \frac{\cosh \eta_T}{T} \left( \sqrt{m^2_a + x^2 P_T^2} + \sqrt{m^2_b + (1-x)^2 P_T^2} \right) \right] \],

\[ k_B(x_i, P_T) = K_1 \left[ \frac{\cosh \eta_T}{T} \sum_{j=a,b,c} \sqrt{m^2_a + x^2 j P_T^2} \right] . \]

\( a, b \) and \( c \) stand for the valence partons respectively. We use the short notation \( \int D x_i = \int_0^1 dx_a dx_b dx_c (x_a + x_b + x_c - 1) \) in Eq. (8). Using infinitely narrow delta shaped wave functions \( |\phi|^2 \sim \delta \) that distribute the momentum of the hadron equally to all valence quarks and neglecting higher order terms one obtains the simple scaling law \( [5,6,7,9,11,12] \),

\[ v_2^B(P_T) = n v_2^a \left( \frac{1}{n} P_T \right) , \]

where \( n \) is the number of valence quarks in the hadron.

At high \( P_T \) the elliptic flow is dominated by the contribution of fragmentation and its value is determined by the azimuthal anisotropy due to jet energy loss. Since fragmentation functions of multi-strange particles are basically unknown, a precise calculation is not possible. However, the energy loss and hence the resulting anisotropy are partonic in nature and one can argue that in the ratio of Eq. (3) the effects of fragmentation functions tend to cancel. Indeed, we have shown in [9] that for the part of \( v_2 \) coming from fragmentation the difference between different hadron species is numerically negligible. Keeping this in mind we approximate the component of \( v_2 \) coming from fragmentation for baryons and mesons by that of \( \Lambda_s \) and kaons respectively.

The total elliptic flow can be written as

\[ v_2(P_T) = r(P_T) v_{2,R}(P_T) + (1 - r(P_T)) v_{2,F}(P_T) , \]

where \( r(P_T) \) is the ratio recombination / (recombination + fragmentation) for the hadron spectrum as a function of transverse momentum.

4 Numerical results

In the following we shall discuss our results for the elliptic flow of (multi-)strange particles. All results shown are for Au+Au collisions at \( \sqrt{s} = 200 \text{ GeV} \).
with an impact parameter $b$ of 8.0 fm. Starting point of our calculation is a system of constituent quarks prior to hadronization. Figure 1 shows $v_2$ of $u$ and $s$ quarks as a function of $p_T$ from Eqs. (3,4,5). The slight difference between $u$ and $s$ quarks stems from their different masses. The mass effect disappears at high $p_T$. The assumption of equal elliptic flow of $s$ and $u,d$ quarks is well justified given the nearly identical behavior of the kaon and pion elliptic flow as a function of $p_T$, as can be seen in [9]. Note that a different value for the $s$ quark elliptic flow will lead to a characteristic flavor ordering for the elliptic flow among multi-strange mesons and baryons respectively, depending on their strange quark content [10,11,12]. Such an effect has not been observed in the data so far.

Figure 2 shows $v_2$ for multi-strange mesons (top frame) and baryons (bottom frame) as a function of $p_T$. The experimental data are taken from the STAR collaboration [6,29]. Thick lines denote the full recombination+fragmentation calculation (labeled as R+F in the figure), whereas thin lines refer to recombination from thermal quarks alone (labeled as R in the figure). Under the assumption that the $s$ quark elliptic flow is identical to $u,d$ quark elliptic flow all mesons show the same behavior. The $v_2$ for $\phi$ is identical to that of the $K$. This is most prominent in the saturation region of $v_2$. In the recombination
approach the value of \( v_2 \) there is solely determined by the number of valence quarks of the hadron and the partonic \( v_2 \). The mass effect which drives the \( v_2 \) characteristics in hydrodynamical calculations is negligible for the saturation feature. The same systematics can be observed for baryons as well, leading to nearly identical elliptic flow values for \( \Lambda, \Xi \) and \( \Omega \). Experimental data confirm our calculation in the R+F approach for kaons, \( \Lambda \)s and \( \Xi \) between 1.5 and 6 GeV/c. We have shown in [9] that the dependence of elliptic flow on the shape of the wave function used in Eqs. (7,8) is negligible for transverse momenta larger than 2 GeV. For further studies on the influence of the wave function see e.g. [15].

At \( P_T \) lower than 1.5 GeV the data exhibit the well known splitting of the \( v_2 \) curves due to the mass of the hadrons. This is only poorly reproduced by the recombination formalism since binding energies are not properly taken into account, whereas hydrodynamic calculations perform very well in this domain.

The fragmentation contribution to \( v_2 \) and the ratio \( r(P_T) \) for kaons and \( \Lambda \)s are taken from [9]. As discussed above we use the values for kaons to estimate both \( v_2 \) from fragmentation and \( r(P_T) \) for the \( \phi \). Moreover we use the values for the \( \Lambda \) to estimate the corresponding quantities for the \( \Xi \) and the \( \Omega \) in the fragmentation process. The tendency of fragmentation functions to cancel in Eq. (3) makes this a good approximation and also makes the difference between mesons and baryons vanish. This implies a universal flat behavior for \( P_T > 6 \) GeV/c where recombination is suppressed and the azimuthal anisotropy is dominated by pQCD processes and parton fragmentation. The merging of meson and baryon \( v_2 \) into one universal curve at high \( P_T \) can be seen even better in Fig. 3. Multistrange hadrons with high masses might be disfavored by the fragmentation process. We expect the relative weight of recombination to be even larger for multi-strange particles: \( r_\phi(P_T) > r_K(P_T), r_\Omega(P_T) > r_\Lambda(P_T) > r_\Xi(P_T) \). Therefore hadrons like the \( \Xi \) and the \( \Omega \) might reach the universal curve for \( v_2 \) from fragmentation even later than indicated in Fig. 2.

Figure 3 elucidates the systematic differences between \( v_2 \) originating from statistical hadronization in a hydrodynamic scenario (blast wave model [21,30]) vs. hadron \( v_2 \) generated from a recombination+fragmentation scenario: thin lines represent a hydrodynamic calculation of the elliptic flow of \( \phi, \Lambda \) and \( \Omega \) as a function of \( P_T \). Since the masses of the \( \phi \) and the \( \Lambda \) are similar, their \( v_2 \) is identical as well in the hydrodynamic calculation. Due to its much larger mass the \( \Omega \) exhibits a smaller \( v_2 \), clearly distinct from that of the other two hadron species. The situation reverses in the region where recombination dominates: there the number of valence quarks in the hadron determines the resulting \( v_2 \), leading to a distinct difference in the \( v_2 \) between the \( \phi \) and the \( \Lambda \), which now coincides with the \( \Omega \). At highest \( P_T \) all curves merge to the universal \( v_2 \) from fragmentation.
Fig. 2. Elliptic flow for multi-strange mesons (top frame) and baryons (bottom frame) in the recombination+fragmentation approach compared to data from STAR (symbols) ($\Lambda + \bar{\Lambda}$ [6], $\Xi + \Xi^*$ [29]). Thick lines denote the full calculation, whereas thin lines refer to the recombination contribution.
Fig. 3. Elliptic flow for some strange hadrons in the recombination+fragmentation approach (thick lines) and in a hydrodynamic calculation (thin lines).

We assume a sudden freeze-out and neglect interactions in the hadronic phase in both descriptions, recombination plus fragmentation and hydrodynamics. This seems to be reasonable for particles with high and intermediate transverse momentum and is supported by the success of the scaling law Eq. (11). The study of resonance particles is particularly important in this case [17,31]. We will investigate this in a forthcoming publication.

5 Conclusion

In brief, we investigated the elliptic flow of multi-strange hadrons in the recombination plus fragmentation approach. We find that the behavior of $v_2$ as a function of $P_T$ in this approach is dominated by the number of valence quarks of the respective hadron, leading to very similar values of the elliptic flow for all mesons and likewise for all baryons, nearly independent of the hadron mass, as opposed to conventional statistical hadronization in hydrodynamic calculations. In particular we find that $v_2$ for $\phi$ mesons is nearly identical to that of kaons above 2 GeV/$c$. This uses a parton $v_2$ for strange quarks that is for all practical purposes equal to the $v_2$ of the light flavors, in accordance
with RHIC data. The measurement of $\phi$ and $\Omega$ elliptic flow will thus permit to distinguish between statistical hadronization as employed by standard hydrodynamic calculations vs. hadron production through parton recombination at intermediate $P_T$. At high $P_T$ we find the elliptic flow to be dominated by fragmentation, leading to a universal curve above 6 GeV/c.

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