Optimal Control Method for Reconstruction of Geometric Optical Transmission Characteristic Data

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Abstract. Geometrical optical projection three-dimensional profile measurement has the advantages of non-contact, non-destructive, and fast data acquisition. Its measurement system is the most promising kind of macro-optical profiler. The thesis analyzes the laser light source image acquisition and image processing, designs the tomographic image reconstruction and three-dimensional image display experimental system of the imaging system, and conducts theoretical analysis of its imaging theory, imaging quality and imaging error. At the same time, the paper uses compressed sensing algorithm (CS) to reconstruct and optimize the transmission data, and realize the analysis of the transmission characteristics of geometric optics.

1. Introduction

Many research and application fields involve processing a special type of measurement data-scan line measurement data surface reconstruction problem, the data comes from three coordinate measuring machines, laser line structured light scanning, medical computed tomography imaging, magnetic resonance imaging, industrial computed tomography Measurement methods such as scanning imaging and layer-to-image method, the measurement data point storage form is an open or closed linear structure. Aiming at the disadvantage of the large amount of data required for the above-mentioned spectrum collection, this paper applies the compressed sensing (CS) algorithm to spectrum collection and reconstruction [1]. The number of observations required to reconstruct the original signal using CS theory is not related to the highest frequency of the signal, but is closely related to the sparsity of the signal. As long as the signal is compressible or sparse, the compressed sensing theory can be used, which greatly saves the cost of collecting optical signals.

2. Geometrical optics principle and reconstruction algorithm

2.1. Principles of geometric optics technology

Traditional X-ray imaging is to directly project the internal organs and tissues of the human body onto the film in the order of overlapping back and forth, showing things with a certain resolution but still not clear enough, while CT technology uses sections at different depths. Above, from each different angle, the detector receives the X-ray beam emitted from the rotating X-ray tube and passes through the human body but attenuates the intensity of the radiation [2]. After measurement and calculation, the image of human organs and tissues is reconstructed. It is called image reconstruction. X-ray intensity attenuation
and the mathematical principle of image reconstruction. When X-rays pass through a homogeneous material, the attenuation rate of its intensity is proportional to the intensity itself, that is

\[ \frac{dI}{dl} = -pl \]  

(1)

Among them, \( I \) is the ray intensity, \( l \) is the thickness of the substance in the ray direction, and \( p \) is the attenuation coefficient of the substance to the rays. Therefore

\[ I = I_0 e^{-pl} \]  

(2)

Among them is the incident intensity. When the energy of the X-ray is constant, the attenuation coefficient changes with the material through which the rays pass. For example, bones are larger than soft tissues, and the intensity of X-rays attenuates faster in bones. (2) The formula is called Beer-Lambert law. When X-rays pass through a non-uniform object composed of materials with different attenuation coefficients, such as a certain section inside the human body. (1) In the formula is the function \( p(x, y) \) of a plane coordinate \( x, y \). When the ray travels along a straight line in the \( xy \) plane, the formula (2) becomes

\[ I = I_0 e^{-\int p(x, y) dl} \]  

(3)

\[ \int_p(x, y) dl = \ln \frac{I}{I_0} \]  

(4)

Define the line integral of function \( f(x, y) \) along the straight-line \( L \) on the plane as

\[ P_f(L) = \int_L f(x, y) dl \]  

(5)

For any point \( Q(x, y) \), take the line integral \( P_f(L) \) of the straight line \( L \) with a distance of \( q > 0 \) from \( Q \), take the average value of \( P_f(L) \) for all \( q \), and record it as \( F_f(q) \), then the function value \( f \) of \( Q \) is

\[ f(Q) = \frac{1}{\pi} \int_0^\infty \frac{dF_f(q)}{q} \]  

(6)

2.2. Introduction to the principle

It is a new three-dimensional microscopic imaging technology, which is a combination of microscopic technology and technology. The principle is similar to that of X-CT. First, the projection data of the sample is obtained, and the three-dimensional structure of the sample is obtained by computer reconstruction [3]. The difference is: is a straight-line projection, but is an approximate straight-line projection. The principle of is shown in Figure 1. Let \( I_0 \) be the light intensity entering the cylinder, \( I \) am the light intensity emitted from the cylinder, and \( p \) is the attenuation coefficient, then there is

\[ \ln \frac{I_0}{I} = \int_L p(x, y) dl \]  

(7)
Therefore, in the case of satisfying such an approximation, the projection \( \int_{L} p(x, y) dl \) of the sample can be obtained from the image of the sample passing through the optical system.

![OPT schematic](image_url)

**Figure 1.** OPT schematic

### 3. Optical projection tomography experimental system

#### 3.1. Compressed Sensing Back Projection Algorithm

Different from the optical image acquisition grating spectrum, the CS algorithm requires the measured signal to be sparse or compressible. If the coefficients of a signal under a certain set of basis vectors contain a large number of zero elements, then the signal can be called sparse. The following takes the spectral data collected by SM125 as an example to illustrate that the grating spectrum is sparse. SM125 obtains the grating spectra by collecting 1,601 discrete grating spectral data points and performing Gaussian nonlinear fitting. Among them, only a few hundred discrete data points are needed to invert the spectrogram of a single grating, that is, there are a large number of useless data points in the 1,601 discrete data points collected by SM125. After a certain transformation, such as coordinate axis transformation, the useless data points in the discrete spectrum data collected by SM125 can be set to zero. In summary, the grating spectrum data collected by the optical image is used as the original data, which has sparseness [4]. Furthermore, in theory, the CS algorithm can realize the collection and inversion of the grating spectrum. Two-dimensional imaging of objects is the basis of three-dimensional imaging. The coordinate system used for FPB is shown in Figure 2. The algorithm is based on Radon’s inverse transformation and the central slice theorem, and its form is
In the formula, \( f(x, y) \) and \( f(r, \theta) \) represent the reconstructed image under Cartesian coordinates \((x, y)\) and polar coordinates \((r, \theta)\) respectively, \( x_r \) is the coordinate value of the pixel point \((x, y)\) perpendicular to the projection direction \( \varphi \), \( P(x_r, \varphi) \) is the projection value measured in the projection direction \( \varphi \), \( H(\rho) \) is the compressed sensing function. In formula (8), the internal integral represents the compressed sensing of projection, which can eliminate the image distortion caused by simple projection operation \([5]\). The outer integral means the back-projection calculation, which specifically includes the beam calculation and the accumulation of the integral direction. The beam calculation is performed as follows

\[
x_r = r \cos(\theta - \varphi) = x \cos(\varphi) + y \sin(\varphi)
\]  

Through beam calculation, the coordinate value \( x_r \) corresponding to the pixel point \((x, y)\) in each projection direction \( \varphi \) can be determined, so that the projection value \( g(x_r, \varphi) \) after compressed sensing can be determined. Integrate the \( g(x_r, \varphi) \) passing through the pixel points \((x, y)\) in all the projection directions as follows to obtain the image reconstruction value of the pixel point

\[
f(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\rho) P(x_r, \varphi) e^{j2\rho \cos(\varphi - \theta)} dx_r
\]  

In the back-projection calculation, the beam calculation is the most time-consuming. For this reason, equation (9) is not used for beam calculation point by point, but the symmetry of the pixel coordinates is used to reduce the amount of calculation. The pixel point that is symmetrical to the pixel point \((x, y)\) about the coordinate axis and the origin is \((-x, y), (x, -y), (-x, -y)\), and the pixel point that is symmetrical to the above four points about the \(45^\circ\) and \(135^\circ\) axes is \((y, x), (-y, x), (y, -x), (-y, -x)\). Make
Table 1 shows the reduction rate $r$ of the reconstruction time $t$ and the original reconstruction time $t_0$ under different projection angles $M$ after using this method. The table shows that the calculation time has been reduced by approximately 15%.

**Table 1.** Reconstruction time reduction rate under different number of projection angles

| $M$ | 240  | 180  | 120  | 60  |
|-----|------|------|------|-----|
| $r = \frac{t-t_0}{t_0}$ | 15.52% | 15.86% | 14.56% | 14.46% |

The calculation of the sine function and the cosine function by the computer is very slow, and there are a lot of such calculations in the beam calculation. Therefore, the sine function and cosine function array table can be established first, and then the corresponding value can be directly called by the lookup table method, which can further improve the reconstruction speed.

### 3.2. Analysis of factors affecting the quality of image reconstruction

In order to improve imaging efficiency and ensure imaging quality, a reasonable number of projection angles must be selected. The mean square error of the paper is used to evaluate the quality of the reconstructed image, which is defined as follows

$$
\sigma = \sqrt{\frac{\sum_{ij}(\text{spec}[i,j]-\text{recon}[i,j])^2}{\text{recon}^2[i,j]}}
$$

When $M=120$, the gray value comparison of the 320th line of the original image and the reconstructed image is shown in Figure 3. At this time, $\sigma$ is close to 0.044, and the image reconstruction error is already noticeable; and when $M=160$, $\sigma$ is less than 0.03, and the reconstruction quality is very good. However, the increase in the number of projection angles will increase the number of calculations for image reconstruction and increase the requirements for subdivision and accuracy of the sample rotating table. When the system has a lot of noise, it will reduce the reconstruction quality. Therefore, the number of projection angles should be controlled between 160 and 200.

**Figure 3.** The gray value comparison between the original image and the reconstructed image
3.3. Comparative analysis of data spectrum

The data centre wavelength used in the spectral comparison experiment is 1550.178nm, the grating length is 5.5mm, and the 3dB bandwidth is 5.33nm. Assuming that the data spectrum data is a discrete signal $\hat{\lambda}$, the data spectrum data collected by SM125 is $\hat{\lambda}_{SM}$, and the CS algorithm reconstructed data is $\hat{\lambda}_{CS}$. In this spectrum comparison experiment, the maximum relative error (MRE) in the 3dB bandwidth of the spectrum and at the centre wavelength is selected as the evaluation index of the experiment [6]. Under the same experimental conditions, the smaller the root mean square error and the maximum relative error between the spectral data reconstructed by the CS algorithm and the optical image data, the stronger the ability of the CS algorithm to reconstruct the data spectrum. The MR calculation formula can be expressed as

$$\text{EMR} = \max\left(\frac{|\hat{\lambda}_{SM}(i) - \hat{\lambda}_{CS}(i)|}{\hat{\lambda}_{SM}(i)}\right)$$

Figure 4 shows the comparison between the optical image and the CS spectral data. It can be seen from Figure 4 that the CS algorithm can reconstruct the data spectrum perfectly, and the spectrum data at certain wavelengths has certain flaws [7]. Figure 4 shows the grating spectra corresponding to different wavelengths within the data “3dB bandwidth. The maximum relative error between the optical image and the CS within the 3dB bandwidth of the data spectrum is 1.03%, and the relative error at the data centre wavelength is 0.69%, that is, the CS algorithm is much lower than the amount of optical image data collection. The relative error within the 3dB bandwidth of the reconstructed data spectrum does not exceed 1.03%. The above-mentioned errors have little effect on the analysis of data spectrum characteristics under external stress, that is, the CS algorithm has certain application value in data spectrum acquisition and reconstruction.
4. Conclusion
The tomographic image reconstruction and three-dimensional image display of the geometrical optical projection imaging system can be realized by using the functions in MATLAB, without too much image processing knowledge, and the imaging system and image acquisition can be established by using common laboratory equipment. It is theoretically analysed that the application of the CS algorithm can realize the collection of a small amount of experimental data and the reconstruction of the high-precision grating spectrum. Choosing the grating spectra obtained from the optical image as the reference standard and using the FBG and data as the research objects, the application prospects of the CS algorithm in the collection and reconstruction of the grating spectra are verified.

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