Decoding the \(X(5568)\) as a fully open-flavor \(\bar{s}d\) tetraquark state
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We investigate the recent evidence for a charged \(X(5568)\) meson as an exotic open-flavor tetraquark state \(\bar{s}d\) with \(J^P = 0^+/1^+\) in the framework of QCD sum rules. We use the color antisymmetric \([\bar{s}d]_{sym} \otimes [\bar{s}d]_{as}\) tetraquark currents in both scalar and axial-vector channels to perform evaluations and numerical analyses. Our results imply that the \(X(5568)\) can be interpreted as both the scalar \(\bar{s}d\) tetraquark state and the axial-vector one, which are in good agreement with the experimental measurement. We also discuss the possible decay patterns of the \(X(5568)\) and suggest to search for its neutral partner in the radiative decay into \(B_s^0\gamma\) and \(B_s^+\gamma\), which can be used to determine its spin-parity quantum numbers. Moreover, we predict its charmed partner state around 2.55 GeV with the quark content \(\bar{s}c\bar{u}d\) and \(J^P = 0^+/1^+\).

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Introduction.—As a long-standing puzzle, understanding the nonperturbative QCD behavior quantitatively is one of the most important and intriguing research topics of the hadron physics. Thoroughly figuring out the map of hadron spectrum is a key step to achieve this goal. The possible hadron configurations not only include conventional mesons and baryons, but also contain exotic states like glueball, hybrid, and multiquark states, etc. However, exotic states have not yet been well established, which is the reason why experimentalists and theorists have already paid many and are still paying more attentions to them. With significant experimental progress in the past decade, more and more charmonium-like and bottomonium-like states (also named as \(XYZ\) particles) \([1]\), and the hidden-charm pentaquarks \(P_c(4380)\) and \(P_c(4450)\) \([4]\) were observed, which provide good chance for identifying exotic states (see Refs. \([2, 3]\) for review). It is obvious that this story still continues with recent evidence for a \(X(5568)\) state \([5]\).

The \(X(5568)\) is a narrow structure seen by the DØ Collaboration \([5]\), which appears in the \(B^0\pi^\pm\) invariant mass spectrum with 5.1\(\sigma\) significance. Its measured mass and width of \(M = 5567.8 \pm 2.9\) (stat) \(\pm 10\) (syst) MeV and \(\Gamma = 21.9 \pm 6.4\) (stat) \(\pm 2.9\) (syst) MeV, respectively. Its decay final state \(B^0\pi^\pm\) requires the valence quark component of the \(X(5568)\) to be \(\bar{s}d\) (or \(\bar{s}b\)). Hence, the reported \(X(5568)\) state, if exist, cannot be categorized into the conventional meson family, and is a good candidate of exotic tetraquark state with valence quarks of four different flavors.

With the mass of \(D_s^+\) (2632) \([6]\) as input, the scalar isoscalar \(b\bar{s}q\bar{q}\) tetraquark mass was estimated to be around 5832 MeV in Ref. \([7]\), while the mass of the isovector \(b\bar{s}u\bar{d}\) tetraquark state can be similarly estimated to be around 5700 MeV. In Ref. \([8]\), the heavy-light meson resonances with \(J^P = 0^+\) and \(J^P = 1^+\) and similar flavor configurations as the \(X(5568)\) were studied in terms of the non-linear chiral SU(3) Lagrangian and their masses were estimated around 5750-5790 MeV, which is 200 MeV higher than the mass of the \(X(5568)\). The \(X(5568)\) was studied as a \(\bar{s}d\) tetraquark state with \(J^P = 0^+\) in Ref. \([9]\). Inspired by the charm-strange state \(D_{s0}(2317)\) \([9]\), various exotic pictures were proposed such as the \(cq\bar{s}q\) tetraquark state \([10-14]\).

In this work, we investigate the \(X(5568)\) as a fully open-flavor \(\bar{s}d\) tetraquark state with \(J^P = 0^+/1^+\) in the framework of QCD sum rules. Our results suggest that the interpretations of the \(X(5568)\) as the scalar \(\bar{s}d\) tetraquark state and the axial-vector one are both possible. To differentiate them and determine its spin-parity quantum numbers, we further investigate its possible decay patterns, from which we propose to observe the radiative decay of the neutral partner of the \(X(5568)\) into \(B_s^0\gamma\) and \(B_s^+\gamma\). The charmed partner state of the \(X(5568)\), with the fully open-flavor quark content \(\bar{s}c\bar{u}d\), can be searched for in many current experiments. We predict the mass of this charmed-partner to be around 2.55 GeV for both the cases of \(J^P = 0^+\) and \(1^+\).

Interpretation of the \(X(5568)\) state.—We shall first construct the diquark-antidiquark type of tetraquark interpolating currents with quark content \(\bar{s}d\). There are five independent diquark fields: \(q_a^T C q_b, q_a^T C \gamma_5 q_b, q_a^T C \gamma_\mu q_b, q_a^T C \gamma_\mu \gamma_5 q_b\) and \(q_a^T C \sigma_\mu \nu q_b\), where \(a, b\) are color indices. In general, one can use all these diquarks and the corresponding antidiquarks to compose tetraquark operators. However, the \(P\)-wave diquarks \(q_a^T C q_b, q_a^T C \gamma_\mu q_b\) and \(q_a^T C \sigma_\mu \nu q_b\) are not favored configurations \([15]\). In QCD sum rules, they lead to unstable sum rules and thus unreliable mass predictions \([16, 17]\). Following our previous works in Refs. \([18, 19]\), we use only the \(S\)-wave diquark fields \(q_a^T C \gamma_\mu q_b\) and \(q_a^T C \gamma_\mu \gamma_5 q_b\) to compose the tetraquark currents coupling to the lowest lying hadron states with \(J^P = 0^+/1^+\). Considering the Lorentz and color struc-
catures, we finally obtain the interpolating currents with \( J^p = 0^+ \)
\[
\begin{align*}
J_1 &= s_c^T C \gamma_5 u_b (\bar{b}_a \gamma_5 C \bar{d}_b^T + \bar{b}_b \gamma_5 C \bar{d}_a^T), \\
J_2 &= s_c^T C \gamma_\mu u_b (\bar{b}_a \gamma_\mu C \bar{d}_b^T + \bar{b}_b \gamma_\mu C \bar{d}_a^T), \\
J_3 &= s_c^T C \gamma_5 u_b (\bar{b}_a \gamma_5 C \bar{d}_b^T - \bar{b}_b \gamma_5 C \bar{d}_a^T), \\
J_4 &= s_c^T C \gamma_\mu u_b (\bar{b}_a \gamma_\mu C \bar{d}_b^T - \bar{b}_b \gamma_\mu C \bar{d}_a^T),
\end{align*}
\] (1)
in which \( J_1 \) and \( J_2 \) belong to the symmetric color structure \([\bar{6}_c]_u \otimes [\bar{6}_c]_d\), and \( J_3 \) and \( J_4 \) belong to the antisymmetric color structure \([\bar{3}_c]_u \otimes [\bar{3}_c]_d\). The interpolating currents with \( J^p = 1^+ \) are
\[
\begin{align*}
J_{\mu 1} &= s_c^T C \gamma_\mu u_b (\bar{b}_a \gamma_\mu C \bar{d}_b^T + \bar{b}_b \gamma_\mu C \bar{d}_a^T), \\
J_{\mu 2} &= s_c^T C \gamma_\mu u_b (\bar{b}_a \gamma_\mu C \bar{d}_b^T + \bar{b}_b \gamma_\mu C \bar{d}_a^T), \\
J_{\mu 3} &= s_c^T C \gamma_5 u_b (\bar{b}_a \gamma_5 C \bar{d}_b^T - \bar{b}_b \gamma_5 C \bar{d}_a^T), \\
J_{\mu 4} &= s_c^T C \gamma_\mu u_b (\bar{b}_a \gamma_\mu C \bar{d}_b^T - \bar{b}_b \gamma_\mu C \bar{d}_a^T),
\end{align*}
\] (2)
in which \( J_{\mu 1} \) and \( J_{\mu 2} \) belong to the symmetric color structure \([\bar{6}_c]_u \otimes [\bar{6}_c]_d\), and \( J_{\mu 1} \) and \( J_{\mu 4} \) belong to the antisymmetric color structure \([\bar{3}_c]_u \otimes [\bar{3}_c]_d\). We note that these currents can be related to those \( c \bar{q} b \bar{q} \) currents in Ref. [18] by simply replacing the charm quark \( c \) by the strange quark \( s \).

In the following, we use these currents to study the fully open-flavor \( s \bar{b} d \bar{b} \) tetraquark states in the framework of QCD sum rules. Arising from QCD itself, QCD sum rule techniques provide a model-independent method to study nonperturbative problems in strong interaction physics. In the past several decades, QCD sum rules have been used to study a vast number of hadronic properties for conventional mesons and baryons as reviewed in Refs. [20–24]. Recently, QCD sum-rules have been also extended to studies of exotic hadron states and the results are encouraging [23]. However, the accuracy of the method is limited by the truncation of the OPE series and the complicated structure of the hadronic dispersion integrals.

We start from the two-point correlation functions
\[
\Pi(p^2) = i \int d^4 x e^{i p \cdot x} \left\langle 0 \mid T [J(x) J(0)] \mid 0 \right\rangle,
\]
\[
\Pi_{\mu \nu}(p^2) = i \int d^4 x e^{i p \cdot x} \left\langle 0 \mid T [J(x) J^\dagger(0)] \mid 0 \right\rangle
\]
\[
= \left( \frac{p_\mu p_\nu}{p^2} - g_{\mu \nu} \right) \Pi_0(p^2) + \frac{p_\mu p_\nu}{p^2} \Pi_0(p^2),
\]
\[\text{(3)}\]
for scalar and axial-vector tetraquark systems, respectively. The imaginary parts of the invariant functions \( \Pi_1(p^2) \) and \( \Pi_0(p^2) \) receive contributions from the pure spin-1 and spin-0 intermediate states, respectively. In this paper we shall use \( \Pi_1(p^2) \) and \( \Pi_0(p^2) \) to investigate the scalar and axial-vector channels, respectively.

One can build QCD sum rules on the hypothesis that the two-point correlation function can be evaluated at the quark-gluonic level, which is then equated to that obtained at the hadronic level. The correlation function at the hadronic level can be described in the form of the dispersion relation
\[
\Pi(p^2) = \frac{(p^2)^N}{\pi} \int_0^\infty \frac{\text{Im} \Pi(s)}{s^{N-2}} ds + \sum_{n=0}^{N-1} b_n (p^2)^n,
\]
\[\text{(5)}\]
where \( b_n \) are subtraction constants. These unknown constants can be removed later by performing the Borel transform. The imaginary part of the correlation function is defined as the spectral density \( \rho(s) \equiv \text{Im} \Pi(s)/\pi \), which can be written as a sum over \( \delta \) functions by inserting intermediate hadronic states
\[
\rho(s) = \sum_n \delta(s - m_n^2) \langle 0 | J(n) J(n) | 0 \rangle + \text{continuum},
\]
\[\text{(6)}\]
in which a narrow resonance approximation is adopted in the second step. The interpolating current \( J(x) \) can couple to all intermediate hadrons \( | n \rangle \) with the same quantum numbers. As usual we only investigate the lowest lying resonance \( | X \rangle \), and \( m_X \) and \( f_X \) are its mass and coupling constant, respectively. The coupling constants are defined as
\[
\langle 0 | J_X \rangle = f_X,
\]
\[\text{(7)}\]
\[
\langle 0 | J_p \rangle = f_X e_p,
\]
\[\text{(8)}\]
for the scalar and axial-vector interpolating currents \( J_X(x) \) and \( J_p(x) \), respectively. \( e_p \) is a polarization vector \((e \cdot p = 0)\).

At the quark-gluonic level, the correlation functions \( \Pi(p^2) \) and \( \Pi_1(p^2) \) can be evaluated using the method of the operator product expansion (OPE). In this paper, we shall use the interpolating currents \( J_3(x) \) with \( J^p = 0^+ \) and \( J_{\mu 4}(x) \) with \( J^p = 1^+ \) to investigate the \( X(5568) \) as a \( s \bar{b} d \) tetraquark state. The spectral density for the current \( J_3(x) \) with \( J^p = 0^+ \) is
\[
\rho_0(s) = \frac{1}{512 \pi^6} \int_0^{\alpha_{\text{max}}} d\alpha \int_0^{\beta_{\text{max}}} d\beta \frac{(1 - \alpha - \beta)^2}{\alpha \beta^3} \left( m_3^2 \beta + m_3^2 \alpha - 3 \alpha \beta s (m_3^2 \beta + m_3^2 \alpha - \alpha \beta s)^3 \right),
\]
\[\text{(9)}\]
\[
\rho_3(s) = -\frac{\langle \bar{q} q \rangle}{16 \pi^3} \int_0^{\alpha_{\text{max}}} d\alpha \int_0^{\beta_{\text{max}}} d\beta \frac{(m_3^2 \beta + m_3^2 \alpha - \alpha \beta \bar{s})}{\alpha \beta} \left( m_b^2 + m_s^2 \alpha - 2 m_b^2 \beta - 2 m_s^2 \beta \right) + m_b^2 \langle \bar{s} s \rangle \int_0^{1-m_b^2/s} \frac{d\alpha}{1 - \alpha} \left[ m_b^2 - s(1 - \alpha) \right] \left[ m_s^2 - 2s(1 - \alpha) \right],
\]
\[\text{(10)}\]
\[
\rho_4(s) = \frac{\langle \bar{q}^2 G G \rangle}{1024 \pi^6} \int_0^{\alpha_{\text{max}}} d\alpha \int_0^{\beta_{\text{max}}} d\beta \frac{(1 - \alpha - \beta)^2}{3} \left( 2 m_3^2 \beta + 2 m_3^2 \alpha - 3 \alpha \beta s \left( m_3^2 \beta + m_3^2 \alpha \right)^2 \right) \left( m_b^2 + m_b^2 \beta + m_s^2 \bar{s} \right) + \frac{1}{\alpha \beta} \left( m_b^2 + m_s^2 \alpha - 2 \alpha \beta s \right) \left( m_b^2 + m_s^2 \alpha - 2 \alpha \beta s \right),
\]
\[\text{(11)}\]
\[ \rho_0(s) = \frac{1}{1024\pi^6} \int_{\Delta_{\alpha\beta}} d\alpha \int_{\beta_{\max}}^{\alpha_{\max}} d\beta (1 - \alpha - \beta)^2 \]

\[ \rho_3(s) = \frac{\langle \bar{q}q \rangle}{16\pi^2} \int_{\Delta_{\alpha\beta}} d\alpha \int_{\beta_{\max}}^{\alpha_{\max}} d\beta (m_b^2 + m_s^2 - \alpha \beta) \]

\[ \rho_4(s) = \frac{\langle g^2 GG \rangle}{3072\pi^6} \int_{\Delta_{\alpha\beta}} d\alpha \int_{\beta_{\max}}^{\alpha_{\max}} d\beta (1 - \alpha - \beta)^2 \]

\[ \rho_5(s) = \frac{\langle \bar{q}g_s^s Gq \rangle}{64\pi^4} \int_{\Delta_{\alpha\beta}} d\alpha \int_{\beta_{\max}}^{\alpha_{\max}} d\beta \]

The spectral density for the current \( J_{3q}(x) \) with \( J^6 = 1^+ \) is

\[ \rho_0(s) = \frac{1}{1024\pi^6} \int_{\Delta_{\alpha\beta}} d\alpha \int_{\beta_{\max}}^{\alpha_{\max}} d\beta (1 - \alpha - \beta)^2 \]

\[ \rho_3(s) = \frac{\langle \bar{q}q \rangle}{16\pi^2} \int_{\Delta_{\alpha\beta}} d\alpha \int_{\beta_{\max}}^{\alpha_{\max}} d\beta (m_b^2 + m_s^2 - \alpha \beta) \]

\[ \rho_4(s) = \frac{\langle g^2 GG \rangle}{3072\pi^6} \int_{\Delta_{\alpha\beta}} d\alpha \int_{\beta_{\max}}^{\alpha_{\max}} d\beta (1 - \alpha - \beta)^2 \]

\[ \rho_5(s) = \frac{\langle \bar{q}g_s^s Gq \rangle}{64\pi^4} \int_{\Delta_{\alpha\beta}} d\alpha \int_{\beta_{\max}}^{\alpha_{\max}} d\beta \]

\[ \alpha_{\min} = \frac{1}{2} \left\{ 1 + \frac{m_b^2 - m_t^2}{s} - \left( 1 + \frac{m_b^2 - m_t^2}{s} \right)^2 - 4m_b^2 \right\}^{1/2} \]

\[ \alpha_{\max} = \frac{1}{2} \left\{ 1 + \frac{m_s^2 - m_t^2}{s} + \left( 1 + \frac{m_s^2 - m_t^2}{s} \right)^2 - 4m_s^2 \right\}^{1/2} \]

\[ \beta_{\min} = \frac{\alpha m_b^2}{3s - m_b^2}, \]

\[ \beta_{\max} = 1 - \alpha. \]

We adopt the following parameter values to perform QCD sum rule analysis \[1, 21, 25-28\] in the chiral limit \( m_u = m_d = 0 \):

\[ m_s(2 \text{ GeV}) = (95 \pm 5) \text{ MeV}, \]

\[ m_t(m_c) = \overline{m}_c = (1.275 \pm 0.025) \text{ GeV}, \]

\[ m_b(m_b) = \overline{m}_b = (4.18 \pm 0.03) \text{ GeV}, \]

\[ \langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3, \]

\[ \langle \bar{s}s \rangle = \frac{(0.8 \pm 0.1)}{(1.0 \pm 0.1)}^3 \text{ GeV}^3, \]

\[ \langle \bar{q}g_s^s Gq \rangle = -M_0^2 \langle \bar{q}q \rangle, \]

\[ \langle \bar{q}g_s^s Gq \rangle = -M_0^2 \langle \bar{s}s \rangle, \]

\[ M_0^2 = (0.8 \pm 0.2) \text{ GeV}^2, \]

\[ \langle g_s^s GG \rangle = (0.48 \pm 0.14) \text{ GeV}^4, \]

in which \( \overline{m}_c \) and \( \overline{m}_b \) are the “running masses” of the heavy quarks in the \( \overline{MS} \) scheme. There is an additional minus sign in mixed condensates due to the different definition of the coupling constant \( g_s \), compared to that in Ref. \[21\].
In the following, we use $J^0(x)$ and $J^0_\mu(x)$ to perform numerical analyses. To establish the QCD sum rules, the Borel transform is usually performed on the correlation functions to pick out the lowest lying state and remove the unknown subtraction constants $b_n$ in Eq. (5). Comparing $\Pi(p^2)$ at both phenomenological and OPE sides, one can obtain the following QCD sum rules via the quark-hadron duality

$$\mathcal{L}_i(s_0, M_B^2) = f_i^2 m_X^{2k} e^{-m_i^2/M_B^2} \int_{s_0}^{\infty} dse^{-s/M_B^2}\rho(s)s^k,$$

(12)

where $s_0$ and $M_B$ are the continuum threshold and Borel parameter. The hadron mass is then extracted as

$$m_X(s_0, M_B^2) = \frac{\mathcal{L}_i(s_0, M_B^2)}{\mathcal{L}_0(s_0, M_B^2)}.$$

(13)

One notes that the hadron mass in Eq. (13) is a function of $s_0$ and $M_B$, which are two essential parameters in our following analysis. The QCD sum rule prediction of the hadron mass is very stable in the Borel window $6 \leq s_0 \leq 17$ GeV, which is thus a reasonable working region. Accordingly, we fix $s_0 = 34$ GeV$^2$, which is then used to fix the Borel window $6.0 \text{ GeV}^2 \leq M_B^2 \leq 7.4 \text{ GeV}^2$ using the two criteria discussed above.

Within the above parameter regions, we show the variation of the hadron mass $m_X$ with respect to the Borel parameter in the right panel of Fig. 1. We find that the $M_B$ dependence of the hadron mass becomes weak for $32 \text{ GeV}^2 \leq s_0 \leq 36 \text{ GeV}^2$, which is thus a reasonable working region. Accordingly, we fix $s_0 = 34$ GeV$^2$, which is then used to fix the Borel window $6.0 \text{ GeV}^2 \leq M_B^2 \leq 7.4 \text{ GeV}^2$, and the hadron mass is extracted as

$$m_{X,0^+} = 5.58 \pm 0.14 \text{ GeV},$$

(15)

where the error comes from the uncertainties of the bottom quark mass $m_b$, the condensates $\langle \bar{q}q \rangle$, $\langle \bar{s}s \rangle$, $(g_s^2 G G)$, $(\bar{q}g_s \sigma \cdot G \bar{s})$, the parameter $M_{X}^{2}$ in Eq. (11) and the continuum threshold $s_0$. This mass value is in good agreement with the observed mass of the $X(5568)$ state in Ref. [5], and implies a possible $J^P = 0^+ \text{ subd}$ tetraquark interpretation for this exotic state.

For the axial-vector current $J^0_\mu(x)$ with $J^P = 1^+$, the OPE series has a similar behavior to that of the scalar current $J^0(x)$. After performing the QCD sum rule analyses, we find the working regions for the continuum threshold $32 \text{ GeV}^2 \leq s_0 \leq 36 \text{ GeV}^2$ and Borel parameter $6.3 \text{ GeV}^2 \leq M_B^2 \leq 7.4 \text{ GeV}^2$. We show the hadron mass $m_X$ as a function of $s_0$ and $M_B$ in the above working regions in Fig. 2. These mass curves have similar behavior to those in Fig. 1, and the hadron mass is extracted as

$$m_{X,1^+} = 5.59 \pm 0.15 \text{ GeV}.$$

This mass value is also in good agreement with the measured mass of the $X(5568)$ state in Ref. [5], and supports the axial-vector subd tetraquark interpretation for this meson.

For the other interpolating currents $J_1(x), J_2(x), J_3(x)$ with $J^P = 0^+$ and $J_{1\mu}(x), J_{2\mu}(x), J_{3\mu}(x)$ with $J^P = 1^+$ in Eqs. (1)-(2), we have also performed QCD sum rule analyses. We shall detailly discuss them in our future studies, but just note that the hadron masses extracted by using these currents are higher than the mass of the $X(5568)$. Especially, the scalar currents $J_1(x), J_2(x)$ and axial-vector currents $J_{1\mu}(x), J_{2\mu}(x)$ lead to hadron masses above 6 GeV, which may be due to that the color structures of these tetraquark currents are symmetric $\{6_{1,0} \otimes \{6_{1,0}\}_{ij},$ while those of $J_3(x)$ and $J_{3\mu}(x)$ used above are antisymmetric $\{3_{1,0} \otimes \{3_{1,0}\}_{ij}$, and the former color symmetric diquarks and antidiquarks probably have larger masses than the latter antisymmetric ones.
Decay patterns of the $X(5568)$ state.—As discussed in Refs. [28, 29], the diquark-antidiquark tetraquark currents can be transformed to be the mesonic-meson currents through the Fierz transformation, from which we can study the decay patterns of the $X(5568)$:

1. The current $J_3(x)$ can be transformed into

$$J_3(x) \rightarrow \bar{b}_{u} s_{u} d_{b} \gamma_{5} u_{b} \equiv 0 + \bar{b}_{u} s_{u} d_{b} \gamma_{5} u_{b}$$

$$\oplus \bar{b}_{u} \gamma_{5} s_{u} d_{b} \gamma_{5} u_{b} \equiv \bar{b}_{u} \gamma_{5} s_{u} d_{b} \gamma_{5} u_{b}$$

$$\oplus \{ s \leftrightarrow u \} .$$ (17)

This suggests that if the $X(5568)$ is a $sub\bar{d}$ tetraquark state of $J^P = 0^+$, its kinematically allowed decay channel would be only the $S$-wave $B^0\pi^+$, which is just its observed channel [5]. Besides this, its neutral partner may also decay into $B_s^0\gamma$ through $\bar{q}\gamma_{5} u \rightarrow \gamma$.

2. The current $J_{3b}(x)$ can be transformed into

$$J_{3b}(x) \rightarrow \bar{b}_{u} s_{u} d_{b} \gamma_{5} u_{b} \equiv 0 + \bar{b}_{u} s_{u} d_{b} \gamma_{5} u_{b}$$

$$\oplus \bar{b}_{u} \gamma_{5} s_{u} d_{b} \gamma_{5} u_{b} \equiv \bar{b}_{u} \gamma_{5} s_{u} d_{b} \gamma_{5} u_{b}$$

$$\oplus \{ s \leftrightarrow u \} \oplus \{ \bar{b} \leftrightarrow \bar{d} \} \oplus \{ s \leftrightarrow u \} .$$ (18)

This suggests that if the $X(5568)$ is a $sub\bar{d}$ tetraquark state of $J^P = 1^+$, its kinematically allowed decay channel would be only the $S$-wave $B_s^0\pi^+$. This channel was also suggested by the DØ Collaboration in the case that the low-energy photon was not detected [5]. Besides this, its neutral partner may decay into $B_s^0\gamma$.

We notice that the radiative decay of the neutral partner of the $X(5568)$, into $B^0\gamma$ and $B_s^0\gamma$, can be used to determine its spin-parity quantum numbers.

The prediction of the charged partner state of the $X(5568)$.—The charged partner state of the $X(5568)$, with the fully open-flavor quark content $sub\bar{d}$, can be searched for in many current experiments, such as the Belle, BESIII, LHCh, etc. We can simply replace the bottom quark $b$ by the charm quark $c$ in the interpolating currents $J_3(x)$ with $J^P = 0^+$ and $J_{3b}(x)$ with $J^P = 1^+$, and use them to investigate these charmed partner states employing the previously obtained formalism. Similar to $m_b$, we use the "running mass" of the charm quark in the $\overline{\text{MS}}$ scheme, as shown in Eq. (11). If the $X(5568)$ is a $sub\bar{d}$ tetraquark state of $J^P = 0^+$, its charged partner state is predicted to have the mass

$$m_{X,0^+} = 2.55 \pm 0.09 \text{ GeV},$$ (19)

and its possible decay patterns are $D_s\pi$ and $DK$, etc. While, if the $X(5568)$ is of $J^P = 1^+$, its charged partner state has the mass

$$m_{X,1^+} = 2.55 \pm 0.10 \text{ GeV},$$ (20)

and its possible decay patterns are $D_s^*\pi$ and $D_s^*K$, etc.

Conclusion.—In conclusion, we have investigated the recently reported $X(5568)$ resonance as a fully open-flavor $sub\bar{d}$ tetraquark state with $J^P = 0^+/1^+$. In the framework of QCD sum rules, we use the scalar $J_1(x)$ and axial-vector $J_{3b}(x)$ tetraquark currents of the color anti-symmetric $[\bar{3}_c]_{10} \otimes [3_c]_{1\bar{d}}$ to perform QCD sum rule analyses, and extract hadron masses to be $5.58 \pm 0.14 \text{ GeV}$ and $5.59 \pm 0.15 \text{ GeV}$, respectively. These two values are both in good agreement with the experimental mass of the $X(5568)$. Hence, our results suggest that the $X(5568)$ can be interpreted as either a scalar or an axial-vector $sub\bar{d}$ tetraquark state. We also investigate its possible decay patterns, and propose to observe the radiative decay of the neutral partner of the $X(5568)$ into $B^0\gamma$ and $B_s^0\gamma$ to determine its spin-parity quantum numbers. Moreover, we predict the mass of the possible charmed partner state of the $X(5568)$, if it exists, to be around $2.55 \text{ GeV}$ for both cases of $J^P = 0^+$ and $1^+$.

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