Mechanism of the high viscosity water shut –off baffle formation in vertical production wells

A M Il’yasov, G T Bulgakova\textsuperscript{1}, N F Kireev

Ufa State Aviation Technical University, Ufa, Russian Federation

\textsuperscript{1}E-mail: bulgakova.guzel@mail.ru

Abstract. In the paper, based on the model of perfect plastic body, the strength is analyzed for a two-layer water-shut-off baffle adjacent to the bore hole of a production well in a porous medium. This baffle is created by the synthetic resin injection (and its subsequent hardening) into the porous reservoir through the production well. The strength and yield areas for the outer and inner baffle layers were found in the problem parameter space.

Keywords: water shut-off (WSO) baffle, porous medium, differential self-coupling determination, perfect plastic body

1. Introduction

In the oil industry, synthetic resins are applied for the water shut-off (WSO) from the water-encroached zones of reservoirs. The resins, as a water shut-off agent, have advantages over the cement slurries. First, while injecting, the resins infiltrate into a porous medium, hardening and forming cylindrical baffle layers. Second, the resin shrinkage is much less than the cement material shrinkage. This paper describes a model for predicting the strength of a two-layer cylindrical baffle formed by a hardened synthetic resin, depending on the production well operation parameters.

2. Statement of the problem

“If block” the water-encroached zones in a productive reservoir interval, a water shut-off operation (WSO) is used. After the production well shutdown, the WSO material is injected into specified reservoir zones. After the hardening of a WSO agent, a production casing leak-off test (LOT) - pressure test - is carried out for the WSO interval. If the casing integrity is broken, the WSO operation is carried out repeatedly until the casing is completely leak-proof. After the water shut-off completion, the production well can be re-commissioned.

In most cases, the cement column between the production casing and the rock is damaged completely (incoherent) due to high drawdowns during hydrocarbon production. Therefore, it can be assumed that the WSO baffle is a two-layer cylinder consisting of an inner layer adjacent to the external wall of the production casing with the properties of a solidified resin, and also of the outer rock-resin layer adjacent to the inner layer. If the inner layer of the baffle is damaged, the well can be flooded from the overlying or underlying water zones. If the outer layer of the baffle is damaged, then the WSO operation for this reservoir zone can be described as a technological failure.
3. Pressure determination at inner boundary of the baffle

Modeling the strength of WSO baffles, we can neglect temperature stresses. We also assume that the WSO operation does not change the state of stress in the formation at the outer boundary of the rock-resin layer.

Let us denote the production casing parameters by subscript 1; parameters of the inner layer of WSO baffles (resin layer) by subscript 2; parameters of the outer rock-resin layer by subscript 3, and parameters related to the rock of the water shut-off interval - by subscript 4.

In addition, let us introduce the following notations: \( R_t \) – internal radius of the production casing; \( R_e \) – external radius of the production casing; \( R_w \) – external radius of the inner layer of water baffles (resin layer); \( R \) – external radius of the rock-resin outer layer.

Also, let us introduce the notation for well operation parameters after the WSO treatment:

- \( p_w \) – pressure inside the production casing;
- \( p_e \) – pressure on the external wall of the production casing (inner wall of the baffle inner layer);
- \( p_r \) – pressure on the outer boundary of the baffle inner layer (inner boundary of the baffle outer layer);
- \( p_p \) – pressure on the outer boundary of the baffe outer layer.

Let us introduce the averaged density and the averaged Poisson ratio for the overlying rocks:

\[
< \rho > = \frac{1}{H} \int_0^H \rho(z)dz, \quad < \nu > = \frac{1}{H} \int_0^H \nu(z)dz,
\]

where \( H \) – the midpoint depth for the considered water shut-off interval.

In [1], a solution was proposed for an elastic transversely-isotropic half-space with a cylindrical cavity. For an isotropic half-space the solution can be simplified and written as

\[
p_p = \frac{< \nu >}{1-< \nu >} < \rho > gH \left( 1 - \frac{R_w^2}{R^2} \right) + p_0 \frac{R_w^2}{R^2},
\]

where \( p_0 \) – pressure in the well against the water shut-off zone prior to the WSO treatment. If any zone within the water shut-off interval is water-saturated, then the pressure at the outer boundary of the outer layer can be established as

\[
p_p = \frac{< \nu >}{1-< \nu >} < \rho > gH \left( 1 - \frac{R_w^2}{R^2} \right) + p_0 \frac{R_w^2}{R^2} + \alpha p,
\]

where \( p \) – pore pressure in the water shut-off interval; \( \alpha = K_4 / K_{4m} \) – Biot’s constant, where \( K_4 \) and \( K_{4m} \) are, respectively, the bulk moduli of rock matrix and rock matrix grain material for the water shut-off interval of the reservoir.

It was also assumed that the bottomhole pressure \( p_w \) is established.

To determine the pressures on the external wall of the production casing \( p_e \) and the outer boundary for the baffle inner layer \( p_r \), we solve the Lame problem for a three-layer elastic cylinder with given pressures on its inner \( p_w \) and outer \( p_p \) boundaries.
As it was said, after the water shut-off of a specific reservoir zone, a leak-off test (LOT) is carried out for this interval. Therefore, we can assume that on the contact boundary between the two-layer baffle and the casing and rock and on the boundary between the baffle layers, the conditions of perfect contact are met [2].

Thus, at the contact boundaries, the conditions of continuity should be met for stress vector $\tilde{\sigma}$ and the radial component of displacement vector $w$

$$w(R_e + 0) = w(R_e - 0), \tag{3.4}$$

$$\tilde{\sigma}_n(R_e + 0) = \tilde{\sigma}_n(R_e - 0), \tag{3.5}$$

$$w(R_w + 0) = w(R_w - 0), \tag{3.6}$$

$$\tilde{\sigma}_n(R_w + 0) = \tilde{\sigma}_n(R_w - 0). \tag{3.7}$$

Then, we can write the boundary conditions for the internal wall of the production casing

$$\tilde{\sigma}_n(R) = -p_w \bar{n}, \tag{3.8}$$

and for the outer boundary of the WSO baffle

$$\tilde{\sigma}_n(R) = -p_w \bar{n}. \tag{3.9}$$

A detailed inscription of relations (3.4) - (3.9) leads to a system of equations for determining vector $\tilde{C}$ using the following notations

$$\tilde{C} = (A_1, B_1, A_2, B_2, A_3, B_3)^T, \quad \bar{b} = (0,0,0,0,-p_w,-p_w)^T \tag{3.10}$$

For the matrix $A$ entries, we introduced the following notations

$$\begin{align*}
a_{11} &= R_e, & a_{12} &= R_e^{-1}, & a_{13} &= -R_e, & a_{14} &= -R_e^{-1}, & a_{15} &= -a_{12}, \\
a_{21} &= \frac{E_1}{(1 + \nu_1)(1 - 2\nu_1)}, & a_{22} &= -\frac{E_1}{(1 + \nu_1)} R_e^{-2}, & a_{23} &= -\frac{E_2}{(1 + \nu_2)(1 - 2\nu_2)}, & a_{24} &= \frac{E_2}{(1 + \nu_2)} R_e^{-2}, \\
a_{33} &= R_w, & a_{34} &= R_w^{-1}, & a_{35} &= -R_w, & a_{36} &= -R_w^{-1}, & a_{37} &= -a_{34}, \\
a_{43} &= \frac{E_2}{(1 + \nu_2)(1 - 2\nu_2)}, & a_{44} &= -\frac{E_2}{(1 + \nu_2)} R_w^{-2}, & a_{45} &= -\frac{E_3}{(1 + \nu_3)(1 - 2\nu_3)}, \\
a_{46} &= \frac{E_3}{(1 + \nu_3)} R_w^{-2}, & a_{51} &= \frac{E_1}{(1 + \nu_1)(1 - 2\nu_1)} = a_{21}, & a_{52} &= -\frac{E_1}{(1 + \nu_1)} R_l^{-2},
\end{align*} \tag{3.13}$$

\begin{align*}
a_{53} &= -a_{23}, & a_{54} &= -\frac{E_2}{(1 + \nu_2)} R_w^{-2}, & a_{55} &= -a_{45}, & a_{56} &= -a_{46}, \\
a_{57} &= -a_{35}, & a_{58} &= -a_{36}, & a_{65} &= -a_{56}, & a_{66} &= -a_{55}.
\end{align*} \tag{3.12}
\[ a_{65} = \frac{E_3}{(1 + \nu_3)(1 - 2\nu_3)} = -a_{45}, \quad a_{66} = -\frac{E_3}{(1 + \nu_3)}R^{-2}. \]

Since the matrix (3.12) is not degenerate, the solution of system (3.10) has the following form

\[ \tilde{C} = A^{-1}\tilde{b}. \]  

After determining vector \( \tilde{C} \), we can find pressure \( p_e \) at the external wall of the production casing:

\[ p_e = -\frac{E_1}{(1 + \nu_1)(1 - 2\nu_1)}A_1 + \frac{E_1}{(1 + \nu_1)} \frac{1}{R_e^2}B_1, \]  

and pressure \( p_r \) at the outer boundary of the baffle inner layer

\[ p_r = -\frac{E_2}{(1 + \nu_2)(1 - 2\nu_2)}A_2 + \frac{E_2}{(1 + \nu_2)} \frac{1}{R_w^2}B_2. \]

4. Calculation of effective elastic moduli of the baffle outer layer

It was shown in [3] that the strength of cement column strongly depends on its Poisson's ratio. Since the outer layer of the baffle is a two-phase composite material, its elastic constants can be defined as parameters of effective homogeneous medium. To determine the effective elastic moduli, the differential self-consistency method was applied, which is described in the survey paper [4]. We simulate the pore space filled with synthetic resin as the chaotically arranged spherical inclusions. For the first time, the differential self-consistency method was applied for spherical inclusions in [5].

For an isotropic equivalent medium, this approach led to a system of two ordinary differential equations for determining two elastic constants - bulk modulus \( K_3 \) and shear modulus \( G_3 \). In the accepted notation, it can be written as

\[ \frac{dK_3}{dm} = \frac{(K_2 - K_3)(3K_3 + 4G_3)}{(3K_2 + 4G_3)(1 - m)} \]  

subject to the initial conditions

\[ K_3(0) = \frac{E_s}{3(1 - 2\nu_s)}, \quad G_3(0) = \frac{E_s}{2(1 + \nu_s)}, \]

where \( E_s, \nu_s \) – the Young's modulus and the Poisson's ratio of the dry rock matrix, respectively.

The technical constants of the effective medium were recalculated using the ordinary equations of the isotropic elasticity theory:

\[ E_3 = \frac{9K_3G_3}{3K_3 + G_3}, \quad \nu_3 = \frac{3K_3 - 2G_3}{6K_3 + 2G_3}. \]
5. Areas of strength and yield of water shut-off baffle layers in the parameter space of the problem

Let us calculate the strength and yield areas in the WSO baffle layers. According to [3], the dimensionless parameters can be applied for a single-layer isotropic perfect plastic pipe:

\[ x = p / P, \quad \Sigma = \sigma / P, \quad \rho = R / r_0, \]

where \( x \) – the pressure ratio, respectively, on internal \( p \) and external \( P \) walls of the pipe; \( \sigma \) – compression strength of the pipe material; \( R \) – external radius of the pipe; \( r_0 \) – internal radius of the pipe.

The introduced dimensionless parameters allow definition of the Huber-Mises strength condition for the pipe [3]:

\[
a_1 x^2 + b_1 x + c_1 < 0, \quad a_1 = 3 \rho^4 + (1 - 2 \gamma)^2 > 0, \quad b_1 = -2 [3 \rho^4 + \rho^2 (1 - 2 \gamma)^2] < 0,
\]

\[
c_1 = 4 (\gamma^2 - \nu + 1) \rho^4 + (\rho^2 - 1) \Sigma_3^2, \quad x_0 = -b_1 / 2 a_1 > 1, \quad D = b_1^2 - 4 a_1 c_1, \]

\[ x_{1,2} = (-b_1 \pm \sqrt{D}) / 2a_1. \]

Table 1 shows the areas of inequality (5.2) solutions.

| Number of the solution area for inequality (4.2) | \( D \) and \( c_1 \) ranges | \( x_0 \) ranges | Position of roots \( x_1, x_2 \) relative to unity | Strength area | Yielding area |
|-----------------------------------------------|-----------------------------|----------------|---------------------------------|--------------|--------------|
| 1                                             | \( D \leq 0 \)              | \( x_0 > 1 \)   | –                               | –            | \( x \in [0,1] \) |
| 2.1                                           | \( D > 0, c_1 \geq 0 \)     | \( x_0 > 1 \)   | \( x_2 \in [0,1], x_1 > 1 \)   | \( x \in [x_2,1] \) | \( x \in [0,x_1] \) |
| 2.2                                           | \( D > 0, c_1 \geq 0 \)     | \( x_0 > 1 \)   | \( x_1 > 1, x_2 > 1 \)          | –            | \( x \in [0,1] \) |
| 3                                             | \( D > 0, c_1 < 0 \)        | \( x_0 > 1 \)   | \( x_1 > 1, x_2 < 0 \)          | \( x \in [0,1] \) | –            |

Investigating the strength of the inner layer in the baffle, we must define:

\[ p = p_r, \quad P = p_r, \quad \Sigma = \sigma_2 / p_r, \quad \rho = R / R_s, \quad \gamma = \nu_2, \]

where \( \sigma_2 \) – compression strength of resin; \( \nu_2 \) – the Poisson’s ratio of resin.

Investigating the strength of the outer layer in the baffle, we define:

\[ p = p_r, \quad P = p_p, \quad \Sigma = \sigma_3 / p_p, \quad \rho = R / R_w, \quad \gamma = \nu_3, \]

where \( \sigma_3 \) – compression strength of resin-rock composite \( \nu_3 \) – the Poisson’s ratio of two-phase resin-rock medium, which is calculated by formulas (4.4) based on the solution of the ODE system (4.1) - (4.3).

When finding the strength and yield areas in the WSO baffle layers based on Table 1 and equations (5.2), the following parameters were used \( R_i = 0.066 \text{ m}; \quad R_c = 0.073 \text{ m}; \quad R_w = 0.108 \text{ m}; \quad \alpha = 0.85; \)

pore pressure in the water-saturated zone \( p_0 = p \) and the radius of the baffle outer layer \( R \) vary. Porosity varied from \( m = 0 \) to \( m = 0.5 \). The resin strength \( \sigma_2 \) and the strength of two-phase resin-rock baffle \( \sigma_3 \) vary also. The Young’s modulus of the steel pipe is \( E_1 = 204000 \text{ MPa}, \) and its Poisson’s ratio is \( \nu_1 = 0.25 \). The average rock density equals to \( < \rho >= 1500 \text{ kg/m}^3, \) and the average
Poisson ratio of the rock is $\nu = 0.25$. As an example, Figures 1 and 2 show the results of the strength area calculations for the inner and outer baffle layers at $R = 0.3$ m, porosity $m = 0.2$, pore pressure $p = 20$ MPa, and resin composite / rock compression strength $\sigma_{s3} = 15$ MPa. The green color corresponds to the yield areas, the blue color - to the strength areas.

Figure 1. The strength of the inner and outer baffle layers in dependence to the resin Poisson's ratio $\nu_2$, bottomhole pressure $p_w$ and strength $\sigma_{s2}$. The resin Young's modulus is $E_2 = 20$ GPa.

Figure 2. The strength of the inner and outer baffle layers in dependence to the resin Young's modulus $E_2$, bottomhole pressure $p_w$ and strength $\sigma_{s2}$. The resin Poisson's ratio is $\nu_2 = 0.2$.

6. Conclusion
A mathematical model was proposed for evaluating the strength of a two-layer water shut-off baffle, created from the injection of a hardening synthetic resin. The model is based on solution of the problem for an isotropic elastic half-space with a cavity, the Lame problem for a three-layer cylinder, and also on the Mises yield criterion for an isotropic perfect plastic material. To determine the effective moduli of the baffle outer layer in the rock, a differential method for the elastic field matching was applied. The areas of strength and yield were constructed in the space of the problem parameters.

Acknowledgments
This study was supported by the Russian Foundation for Basic Research (Project 17-41-020226 r_a).

References
[1] Lekhnitskii S G 1977 Teoriya uprugosti anizotropnogo tela (Moscow: Nauka) (in Russian)
[2] Dimitriyenko Yu.I 2009 Nelineynaya mekhanika sploshnoy sredy (Moscow: Fizmatlit) (in Russian)
[3] Ilyasov A M 2017 Journal of Applied Mechanics and Technical Physics 58 (1) 182–187
[4] Ustinov K B 2003 Uspekhi mekhaniki 2 126-168 (in Russian)
[5] Roscoe R A 1973 Rheol. Acta 12 404-411