Nucleon Decay in Non-Minimal Supersymmetric SO(10)

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Abstract

Evaluation of nucleon decay modes and branching ratios in a non-minimal supersymmetric SO(10) grand unified theory is presented. The non-minimal GUT considered is the supersymmetrised version of the ‘realistic’ SO(10) model originally proposed by Harvey, Reiss, and Ramond, which is realistic in that it gives acceptable charged fermion and neutrino masses within the context of a phenomenological fit to the low energy standard model inputs. Despite a complicated Higgs sector, the SO(10) $\mathbf{10}$ Higgs superfield mass insertion is found to be the sole contribution to the tree level F-term governing nucleon decay. The resulting dimension 5 operators that mediate nucleon decay give branching ratio predictions parameterised by a single parameter, the ratio of the Yukawa couplings of the $\mathbf{10}$ to the fermion generations. For parameter values corresponding to a lack of dominance of the third family self coupling, the dominant nucleon decay modes are $p \to K^+ + \bar{\nu}_\mu$ and $n \to K^0 + \bar{\nu}_\mu$, as expected. Further, the charged muon decay modes are enhanced by two orders of magnitude over the standard minimal SUSY SU(5) predictions, thus predicting a distinct spectrum of ‘visible’ modes. These charged muon decay modes, along with $p \to \pi^+ + \bar{\nu}_\mu$ and $n \to \pi^0 + \bar{\nu}_\mu$, which are moderately enhanced over the SUSY SU(5) prediction, suggest a distinguishing fingerprint of this particular GUT model, and if nucleon decay is observed at Super-KAMIOKANDE the predicted branching ratio spectrum can be used to determine the validity of this ‘realistic’ SO(10) SUSY GUT model.
1 Introduction

Nucleon decay is by definition a baryon number violating process, and within the context of the standard model (SM) of particle physics is forbidden \(^1\). Yet there is a strong motivation for assuming that baryon number violation occurs; particularly the fact that there is no baryonic analog of the electromagnetic gauge invariance (which guarantees the conservation of electric charge), the presence of a baryonic asymmetry in the Universe \(\[3\] \), and the violation of baryon number conservation by black holes \(\[4\] \). Allowing baryon number violation then suggests that the SM is only a low energy effective theory, and as such the stability of the nucleon is brought into question. This view was further reinforced when the adoption of Grand Unified Theories (GUTs) as a SM extension appeared to explain a large number of questions left unanswered by the SM \(\[5\] \). The GUT scheme was introduced to attempt to unify the SM interactions under a single simple gauge group. Imposition of such an underlying GUT structure then provided a new mechanism by which baryon number violation could occur, and nucleon decay induced \(\[6\] \), \(\[7\] \).

This nucleon decay mechanism is due to the fact that for conventional GUTs, quarks and leptons are placed in the same multiplets of the GUT gauge group. The coupling of these multiplets to either gauge or Higgs boson representations then gives interactions that couple quarks to leptons, and below the GUT scale, produce effective operators that induce nucleon decay. These tree level operators are four fermion dimension 6 operators \(\[8\] \), \(\[9\] \) built from two fermion-fermion-boson vertices by means of a gauge or Higgs boson exchange. As the low energy limit of the internal boson propagator is \(1/M_G^2\) \((M_G\) is the mass scale at which the GUT is spontaneously broken\), the four fermion interaction reduces at low energy to an effective four fermion vertex scaled by two inverse powers of \(M_G\). It is this class of effective vertex that would mediate nucleon decay in non-supersymmetric GUT models.

In the archetypal GUT - minimal non-supersymmetric SU(5) - first proposed by Georgi and Glashow \(\[6\] \), the unification scale is \(M_G \sim 5 \times 10^{14}\) GeV \(\[10\] \), \(\[11\] \), and predicts the most dominant decay mode to be \(p \to \pi^0 + e^+\) with a partial lifetime of \(\tau_p \sim 4.5 \times 10^{29\pm1.7}\) yrs \(\[11\] \). This is to be

\(^1\)Baryon number is not conserved in the SM, as violation occurs in weak interactions via instanton effects and the triangle anomaly, but the rate is suppressed and also involves violation of 3 units of baryon number for 3 standard model generations, thus making it irrelevant to nucleon decay \(\[\] \).
contrasted with the experimental lower bound obtained from IMB-3 Collaboration, of \( \tau_p > 5.5 \times 10^{32} \) yrs [12, 13]. Clearly the minimal SU(5) model predicts proton decay at too rapid a rate, thereby ruling it out as a realistic GUT candidate. Nucleon decay channels and partial lifetime predictions have been calculated for a variety of GUT models [14], including non-minimal SU(5) (which includes a 45 Higgs rep in an attempt to predict the fermion masses), minimal and non-minimal SO(10), and an E_6 GUT model. Unfortunately, all these models tend to fail on the basis of a unification scale \( M_G \sim (2 - 7) \times 10^{14} \) GeV, which implies an overly rapid nucleon decay rate as well as a prediction for \( \sin^2 \theta_W \) that is inconsistent with the high precision LEP measurements [15].

As conventional GUTs are essentially condemned by these failings, attention has turned to the supersymmetric GUT models (SUSY GUTs). Imposing supersymmetry - a symmetry that relates bosons and fermions - has the effect of doubling the particle content below the GUT scale, which results in the slowing of the SM gauge coupling running, and consequently predicts a consistent gauge coupling unification at a higher scale. Thus, a SUSY GUT model not only addresses the matter of the consistency of the \( \sin^2 \theta_W \) prediction, but it also predicts a unification scale that is typically two orders of magnitude larger than that of conventional GUTs. This increase in the unification scale induces a suppression factor of order \( 10^{-8} \) in the decay rates of four fermion dimension 6 operators generated by boson exchange, placing the dimension 6 mediated nucleon partial lifetime predictions well beyond the experimental lower bound. However, with the advent of Super-KAMIOKANDE, even the decay mediated by the dimension 6 operators may be observable.

Yet the extension to a SUSY GUT model permits a new operator, capable of being the dominant contribution to nucleon decay. This operator is a dimension 5 fermion-fermion-sfermion-sfermion effective operator [16, 17] constructed from either two fermion-sfermion-Higgsino vertices or a fermion-fermion-Higgs and a sfermion-sfermion-Higgs vertex by means of a heavy colour triplet Higgino or Higgs exchange below the GUT scale. Such an operator then evolves down to the SUSY breaking scale, at which point the sfermions are ‘dressed’ by gaugino exchange to give an effective four fermion vertex that mediates nucleon decay. As the low energy limit of the dimension 5 operator is scaled by \( \frac{1}{M_G} \), nucleon decay via this operator generally dominates over those mediated by the conventional dimension 6 operators\(^2\).

\(^2\)It is assumed that R-parity is invoked so to rule out dangerous dimension 4 operators.
Investigations of nucleon decay in a number of SUSY GUT models have been carried out, beginning with the supersymmetrised version of minimal SU(5) [18,19]. Unlike its non-SUSY cousin, this model predicts the dominant nucleon decay modes to be $p \rightarrow K^+ + \bar{\nu}_\mu$ and $n \rightarrow K^0 + \bar{\nu}_\mu$, and as the unification scale is $M_G \sim 2.5 \times 10^{16}$ GeV the partial lifetime prediction is $\tau_{p \rightarrow K^+ + \bar{\nu}_\mu} \sim 10^{29 \pm 4}$ yrs [20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38] (with $M_{H_3}$ set to $M_G$). This prediction does not disagree with the experimental bound of $\tau_{p \rightarrow K^+ + \bar{\nu}_\mu} > 10^{32}$ yrs (obtained from the water Čerenkov detector of the KAMIOKANDE Collaboration [39,13]) due mainly to the large uncertainty resulting from the value of the Higgs/Higgsino colour triplet mass. Likewise, predictions for non-minimal SUSY SU(5), minimal SUSY SO(10) result in a marginal degree of compatibility with the experimental lower bounds on the partial lifetimes of the various nucleon decay channels [14]. This marginal consistency suggests that an improvement on the experimental lower bounds could lead to either a rejection of nucleon decay via SUSY GUT generated dimension 5 operators, or an observation of nucleon decay. Yet the uncertainties in nucleon partial lifetime predictions preclude model discrimination by rate. In order to distinguish the underlying SUSY GUT structure, the relative decay rate predictions within a model should be determined, and then used to identify the SUSY GUT candidate, once nucleon decay has been observed.

With this strategy in mind, this paper presents the branching ratios for nucleon decay in a particular ‘realistic’ SUSY GUT model. The model chosen is a supersymmetrised version [40] of the non-minimal SO(10) GUT proposed by Harvey, Reiss, and Ramond [41], which was constructed primarily to reproduce a consistent phenomenological fit to the observed SM fermion masses and mixing angles. This realistic non-minimal SUSY SO(10) model, like its non-SUSY counterpart, can be viewed as a sophisticated phenomenological one, as it supports a rather expansive Higgs sector that is responsible for the required Yukawa coupling texture. It will be shown that analysis of the various nucleon decay channels mediated by the dimension 5 operators of this model results in branching ratio predictions depending on a single parameter, with the branching ratios for some observable modes enhanced by factors of order 100 over the minimal SUSY SU(5) predictions. This in turn suggests that if nucleon decay is observed at Super-KAMIOKANDE, the $p \rightarrow K^0 + \mu^+$, $p \rightarrow \pi^0 + \mu^+$, and $n \rightarrow \pi^- + \mu^+$ decay channel may play a significant role in identifying the structure of the underlying SUSY GUT.

In this paper, section 2 presents the non-minimal SUSY SO(10) model to be used, section 3
examines in detail the low energy quark-level effective lagrangian, while section 4 discusses the effective lagrangian at the hadronic level and presents the branching ratio predictions. Finally, in section 5 a discussion of these predictions and the conclusions that can be drawn from them is given.

2 The Non-Minimal SUSY SO(10) Model

As mentioned, this analysis is based on the non-minimal SO(10) GUT model of Harvey, Reiss, and Ramond [41], which has been explicitly constructed to generate a mass spectrum (including mixing angles) of the SM fermions from the GUT. An advantage of the choice of SO(10) as the gauge group is that the lowest dimensional chiral representation that accommodates the observed SM fermions is the $16$, which allows for the assignment of one family of SM fermions plus a right handed neutrino, and does not include any mirror fermions. This in turn places constraints on the possible Higgs sector representations, since the fermion masses transform under SO(10) as $16 \times 16 = (10 + 126)_S + 120_A$, (where $S$ and $A$ refer to the symmetric and antisymmetric parts respectively), implying that the allowed Higgs reps that couple to fermions to form SO(10) invariant Yukawa terms are the $10, 120$, and the $126$. The ‘realistic’ model of Harvey et al. [41] is then constructed from the representations in such a way that SO(10) GUT is broken directly to the SM gauge structure of $SU(3)_C \times SU(2) \times U(1)$, and the GUT scale texture of Yukawa couplings incorporates the up quark mass matrix ansatze of Fritzsch [42] and the down quark and charged lepton mass matrix ansatze of Georgi and Jarlskog [43] in such a way that the Oakes relation [44] results. The cost of such a model is the expansion of the Higgs sector well beyond that of most minimal models. The particle content of this model is given in terms of a $45$ that is the adjoint of vector bosons, three families of fermions ($16_1, 16_2, 16_3$), and a scalar sector composed of a $54$, a complex $10$, and three families of $126$ ($126_1, 126_2, 126_3$) - all of which are required for a viable spectrum of fermion masses. Note that the phenomenologically observed mass spectrum can be produced without requiring the presence of the $120$ rep, which has a Yukawa coupling to the fermions that is antisymmetric in generation indices (as the SM fermions are expressed in terms of a single chirality, and the spin 0 fields occur in a product that is symmetric in Lorentz indices).

The extension [40] of this model to that of a SUSY SO(10) model is straightforward, as the
gauge, fermion and conjugate Higgs fields are converted into vector and chiral superfields, giving a superfield content of:

- Vector superfields: 45
- Chiral superfields: 10, 16, 16, 16, 54, 126, 126, 126,

This SUSY SO(10) model, like its non-SUSY counterpart, is distinguished by its sophisticated Yukawa texture, composed of the 10, 16, and 126 chiral superfield reps. Specifically, the model is defined in terms of its superpotential, and for the purposes of nucleon decay, the relevant terms of the superpotential for this SO(10) model are

\[
W = (A 16 \times 16 + B 16 \times 16) \times 126 + (a 16 \times 16 + b 16 \times 16) \times 10 + c (16 \times 16) \times 126 + d (16 \times 16) \times 126 + M_G 10 \times 10
\]

with the superpotential expressed in terms of the SO(10) representations, and \(A, B, a, b, c, d\) as the undetermined GUT scale Yukawa couplings.

The beauty of this globally supersymmetric model is that as \(10 \times 10 \supset 1\) and \(126 \times 126 \supset 1\), the only SO(10) invariant F-term that contributes to nucleon decay below the spontaneously broken SO(10) GUT scale is given by Figure 1.

![Figure 1: The only F-term supergraph that contributes to nucleon decay.](image)

The key point here is that this superfield diagram has a Higgs/Higgsino mass insertion that involves only the 10 (the 120 reps are absent!), which implies that only the GUT scale Yukawa couplings of the 16’s to the 10 are of relevance to the predictions of nucleon decay (i.e. \(a\) and \(b\) in
equation (1) are the only relevant couplings). In terms of the particle diagrams, the only tree level diagrams of concern are

\[ \text{Figure 2: The two particle diagrams generated by the F-term supergraph of Figure 1.} \]

Here the first dimension 5 diagram exhibits the exchange of a Higgsino of GUT scale mass \( M_G \), and so for momentum below the GUT scale the Higgsino propagator reduces to a factor of \( \frac{1}{M_G} \). The second diagram in Figure 2 involves the exchange of a GUT scale Higgs scalar whose propagator reduces to \( \frac{1}{M_G^2} \), but due to the weighting of the sfermion-sfermion-Higgs trilinear coupling by one power of \( M_G \), the resulting diagram also contributes to the dimension 5 operator with weight \( \frac{1}{M_G} \). Thus the effective dimension 5 operator, valid between the SUSY GUT scale and the SUSY breaking scale (assumed to be of order the electroweak scale) is a combination of both diagrams, and is represented by the effective vertex in Figure 2.

### 3 The Dimension 5 Operators

In order to evaluate these dimension 5 operator contributions to nucleon decay, the superpotential must be re-expressed in terms of the superfields corresponding to the SM content. This may be done in a two step process, which first involves the re-expression of the superpotential in a compact SU(5) notation, followed by a decomposition of the SU(5) superfields into their SM components. Such a decomposition can be used as the F-term of Figure 1 is the only dimension 5 contribution to nucleon decay, and it relies only on the Higgs 10 of SO(10) which has the SU(5) decomposition 10 → 5 + 5. (Note that the SU(5) decomposition of the 16 is 16 → 10 + 5 + 1.) Thus the superpotential terms that contribute to nucleon decay can be written as

\[
W_{SU(5)} = \sqrt{2} \chi^\alpha \chi^\beta M^D_{a b} \psi_{b a} H_{2 \beta} - \frac{1}{4} \epsilon_{\alpha \beta \gamma \delta} \chi^\alpha \chi^\beta M^U_{c d} \chi^\gamma \chi^\delta H' + M_G H_1^\alpha H_{2 \alpha}
\]  

(2)
with \( W_{SU(5)} \) being valid at the GUT scale. Here \( M^U \) and \( M^D \) are \( 3 \times 3 \) matrices in generation space that express in a compact form the Yukawa coupling texture expressed in equation (1). Below the SO(10) scale, the heavy Higgs superfield can be integrated out to give an effective superpotential (that is appropriate below the GUT scale but above the SUSY breaking scale). This effective superpotential is

\[
W_{SU(5)}^{\text{eff}} = \sqrt{2} \frac{\epsilon_{\alpha \beta \gamma \delta i} \chi^\alpha_\beta M^{U}_{\alpha b} \chi^\gamma_\delta \epsilon_{\delta i} M^{D}_{\epsilon d} \psi_{\epsilon d}}{4 M_G}
\]  

(3)

Here the Greek indices \( \alpha, \beta, \ldots \) are SU(5) indices, the family indices are \( (a, b, c, d) \), and the index \( i \) runs from 1 to 3. Also, the Lorentz structure is suppressed, so to focus on the generation and SU(5) structure. Restriction to the tree level diagrams relevant to nucleon decay (Figure 1) then implies the Yukawa texture matrices for this effective superpotential are of the form

\[
M^U = \begin{bmatrix}
0 & a & 0 \\
0 & a & 0 \\
0 & 0 & b
\end{bmatrix} = M^D
\]  

(4)

As it is assumed that this SUSY SO(10) breaks straight to the minimally supersymmetric standard model (MSSM), the superpotential can then be further decomposed into the SM quark and lepton superfields. Typically the decomposition for one family of left-handed SU(5) matter superfields are

\[
\psi_{\alpha} = \begin{bmatrix}
D_1 \\
D_2 \\
D_3 \\
l
\end{bmatrix}
\quad \chi^{\alpha \beta} = \frac{1}{\sqrt{2}} \begin{bmatrix}
0 & U_3 & -U_2 & -u^1 & -d^1 \\
-U_3 & 0 & U_1 & -u^2 & -d^2 \\
U_2 & -U_1 & 0 & -u^3 & -d^3 \\
u^1 & u^2 & u^3 & 0 & -L \\
d^1 & d^2 & d^3 & L & 0
\end{bmatrix}
\]  

(5)

where \( U_i, D_i \) and \( L_i \) are the charge conjugations of the right handed SU(2) singlet up, down, and charged lepton fields. Substitution of this decomposition into the superpotential (equation (3)), results in an effective superpotential relevant to nucleon decay that is expressed in chiral superfields associated with the SM. The form of the effective superpotential in question is

\[
W_{\text{SM}}^{\text{eff}} = -\frac{1}{2 M_G} \left[ \epsilon_{ijk} L_a M^U_{ab} U_b U_{ck} M^D_{cd} D_{dj} + \frac{\epsilon_{ijk}}{4} U_{ai} M^U_{ab} L_b U_{ck} M^D_{cd} D_{dj} ight.
\]

\[
- \epsilon_{ijk} (u^a_i M^U_{ab} d^j_b - d^a_i M^U_{ab} u^j_b) (u^k_c M^D_{cd} L_d - d^k_c M^D_{cd} u^d_c)]
\]  

(6)

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From this superpotential it is clear that as a result of the SU(2) content, there are two classes of F-terms; the \((LLLL)_F\) and the \((RRRR)_F\) terms (here the notation of reference \[L7\] is used to emphasise the SU(2) weak content of the operators). However as the \((RRRR)_F\) terms are antisymmetric in generation indices \((a,b,c)\) - due to the Bose statistics of superfields in a superpotential - their composition is such that they must contain either a charm or a top SU(2) singlet superfield. This superfield generation remains, to a first approximation, unchanged on the dressing of the operator by gluino or bino exchange at the SUSY breaking scale (SU(2) gaugino exchange is forbidden for these singlet superfields), and so the low energy four fermion operator contains either a charm or a top quark. This implies that the \((RRRR)_F\) term contribution to nucleon decay is suppressed, leaving only the \((LLLL)_F\) terms. The effective lagrangian relevant to nucleon decay is then obtained from the \((LLLL)_F\) term of the superpotential by the usual method

\[
\mathcal{L}_{\text{Int}} = \frac{1}{2} \sum_{i,j} \left( \frac{\partial W}{\partial \Phi_i} \frac{\partial W}{\partial \Phi_j} \right|_{\Phi=\phi} \psi_i \psi_j + \text{h.c.} \right) - \sum_i |\frac{\partial W}{\partial \Phi_i}|^2_{\Phi=\phi}
\]

with \(\Phi\) representing a chiral superfield, and \(\phi\) and \(\psi\) the scalar and fermionic parts), which results in vertices composed of two particles and two sparticles. These vertices are then renormalisation group evolved down to the SUSY breaking scale \((\sim O(M_W))\) at which point the dimension 5 operator is converted to a dimension 6 operator via gaugino and Higgsino exchanges. This dressing is schematically shown in Figure 3.

![Figure 3: The particle diagram for the nucleon decay operators after being dressed by the wino or charged higgsino exchange.](image)

Of all the gaugino and higgsino exchanges associated with the dressing of the \((LLLL)_F\) dimension 5 operators, the dominant contribution comes from the charged wino. The gluino, neutral gaugino, and neutral Higgsino exchange contributions to the dressed operator are suppressed as
their exchange is approximately generation diagonal and their contribution is thereby suppressed
due to small Yukawa couplings of the first and second generation fields present in the dimension 5
operators \[20, 21\]. The charged-Higgsino exchanges are also suppressed, due to their Higgs strength
Yukawa couplings to the first and second generation fermions, thereby leaving the charged wino
as the dominant contribution to the loop integral. Complete calculations of the loop dressing by
chargino eigenstates have been performed by Sakai \[19\], and their implications for nucleon decay
rates have been explored for minimal and non-minimal versions of SU(5) \[29\]. Here however, the
simplifying assumption of wino dominance of the decay amplitude is invoked. Preforming the loop
integration, results in a triangle diagram factor, that although it depends on mass eigen-values and
mixing angles of the sparticles in the loop, can be approximated (in the pure charged wino exchange
limit) by \[19\]

\[
\frac{\alpha_2}{2\pi} f(\tilde{u}, \tilde{d}, \tilde{W}) = g_2^2 \int \frac{d^4k}{i(2\pi)^4 m_\tilde{u}^2 - k^2 m_\tilde{d}^2 - k^2 m_\tilde{W}^2 - k^2} \approx \frac{\alpha_2}{2\pi} \frac{m_\tilde{W}}{m_\tilde{u}^2 - m_\tilde{d}^2} \ln \frac{m_\tilde{u}^2}{m_\tilde{W}^2} - \frac{m_\tilde{d}^2}{m_\tilde{W}^2} \ln \frac{m_\tilde{d}^2}{m_\tilde{W}^2}
\]

and so becomes a multiplicative factor of the dressed four fermion operator.

From the superpotential (equation (7)) the effective lagrangian for the dressed quark level op-
erators of Figure (3) can be obtained, and with the use of equation (8) it has the form

\[
\mathcal{L} = \frac{\alpha_2}{2\pi M_G} R_S R_L M_{ab}^U M_{cd}^D \varepsilon_{ijk} \left[ (u_a^i d_b^j)(d_c^k u_d^l)\{f(u_c, l_d, m_\tilde{W}) + f(u_a, d_b, m_\tilde{W})\} \\
+ (d_a^i u_b^j)(u_c^k l_d^l)\{f(d_c, v_d, m_\tilde{W}) + f(d_a, u_b, m_\tilde{W})\} \\
+ (u_b^i d_c^j)(u_a^k l_d^l)\{f(u_c, d_b, m_\tilde{W}) + f(d_a, v_d, m_\tilde{W})\} \\
+ (d_a^i v_d^j)(d_b^k u_c^l)(f(u_a, l_d, m_\tilde{W}) + f(u_b, d_c, m_\tilde{W})) \right] + h.c.
\]

Here the \( R_S \) and \( R_L \) are the short and long range renormalisation factors. The short range renor-
malisation accounts for the renormalisation effects from the SO(10) to the SUSY breaking scale,
while the long range factor is from the SUSY breaking scale to a low energy scale (assumed here to
be 1 GeV). \( R_S \) can be shown to be generation independent, and can be taken to be \[20, 21\]

\[
R_S = \left[ \frac{\alpha_3(m_S)}{\alpha_G} \right]^{\frac{\pi^4}{2}} \left[ \frac{\alpha_2(m_S)}{\alpha_G} \right]^{\frac{\pi^2}{2}} \left[ \frac{\alpha_1(m_S)}{\alpha_G} \right]^{\frac{\pi^2}{2}} \approx 0.91
\]

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where $m_S$ is the SUSY breaking scale (which here, has been set to the electroweak scale $m_W$). The long range renormalisation is predominantly a result of QCD interactions between the SUSY scale and 1 GeV, and encompasses the renormalisation of the Yukawa couplings and anomalous dimension corrections to the four fermion operators. Again, following reference [21],

$$R_L \simeq \left[ \frac{\alpha_3(1\text{GeV})}{\alpha_3(m_c)} \right]^{2/3} \left[ \frac{\alpha_3(m_c)}{\alpha_3(m_b)} \right]^{1/3} \left[ \frac{\alpha_3(m_b)}{\alpha_3(m_Z)} \right]^{1/3} \simeq 0.22$$ (10)

This effective lagrangian, as written, is for four fermion operators with the quarks and leptons expressed in their gauge interaction eigenstates; this however is easily remedied by rotating from a gauge interaction to a mass eigenstate basis. Due to the mismatch in the rotations of the charge $\frac{2}{3}, -\frac{1}{3}$, and $-1$ fields, the operators incur additional generation mixing. The rotation matrices appropriate to this model are obtained from the diagonalisation of the mass matrices which are defined in terms of the Yukawa coupling texture (as specified in equation (1)) and the vevs of the various Higgs reps. For these $\Delta I_W = \frac{1}{2}$ Dirac masses ($I_W$ denotes weak isospin), it is convenient to consider the SO(10) vev contributions in terms of their SU(5) content. The contribution of SU(5) vevs to the quark and charged lepton masses is as follows [41]:

- $\langle ... \rangle \sim 5$ gives a contribution to the charge $\frac{2}{3}$ mass
- $\langle ... \rangle \sim \overline{5}$ gives an equal weight contribution to the charge $\frac{-1}{3}$ and $-1$ masses
- $\langle ... \rangle \sim 45$ gives a contribution of relative weight $1: -3$ to the charge $\frac{-1}{3}$ and $-1$ masses

The SO(10) Higgs vevs structure is then decomposed as

$$\langle 10 \rangle = r(\text{along } \overline{5}) + p(\text{along } 5)$$

$$\langle 126_1 \rangle = t(\text{along } 5)$$

$$\langle 126_2 \rangle = s(\text{along } 45)$$

$$\langle 126_3 \rangle = q(\text{along } 5)$$ (11)

where $p, q, r, s, t$ are taken as complex vevs. The assumption of complex vevs allows for the generation of soft CP violation through the process of symmetry breaking. Yet it is assumed that soft CP violation is not the sole source of CP violation in the model. Hard CP violation is also permitted
due to the fact that, unlike the non-SUSY model of reference [41], the Yukawa couplings of equation (1) are taken to be complex [40].

As the masses in the low energy effective SUSY theory arise from the Yukawa couplings of the quarks and charged leptons to a single light Higgs doublet of $\Delta I_W = \frac{1}{2}$, the mass matrices can be formulated in terms of the vev of this light Higgs. From the SU(5) decomposition of the SO(10) Higgs vevs, this light Higgs vev is a linear combination of the doublets in the $10, \overline{126}, \overline{126}_2$, and $\overline{126}_3$ (in the ratio $|r + p| : t : s : q$), and so the GUT scale couplings appearing in the mass matrices can be read off. Yet as it is the mass texture at the SUSY breaking scale that must be diagonalised, the entries in these Yukawa coupling texture matrices at the GUT scale must be evolved down to the SUSY scale via the renormalisation group equations, as done in Dimopoulos, Hall, and Raby [45] for ‘realistic’ Yukawa matrices of this form. The quark and charged lepton Yukawa matrices specified at the SUSY breaking scale are then the mass matrices that are diagonalised. From equations (1) and (12) the GUT and SUSY scale mass matrix textures of the quarks and charged leptons are:

\[
\begin{align*}
U &= \begin{bmatrix} 0 & P_G & 0 \\ P_G & 0 & Q_G \\ 0 & Q_G & V_G \end{bmatrix} \quad \rightarrow \quad U = \begin{bmatrix} 0 & P & 0 \\ P & \delta_u & Q \\ 0 & Q & V \end{bmatrix} \\
D &= \begin{bmatrix} 0 & R_G e^{i\varphi_G} & 0 \\ R_G e^{-i\varphi_G} & S_G & 0 \\ 0 & 0 & T_G \end{bmatrix} \quad \rightarrow \quad D = \begin{bmatrix} 0 & R e^{i\varphi} & 0 \\ Re^{-i\varphi} & S & \delta_d \\ 0 & 0 & T \end{bmatrix} \\
L &= \begin{bmatrix} 0 & R_G & 0 \\ R_G & -3S_G & 0 \\ 0 & 0 & T_G \end{bmatrix} \quad \rightarrow \quad L = \begin{bmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{bmatrix}
\end{align*}
\]

with the assignments

\[
P = ap + At \quad V = bp + Bt \\
R = ar \quad T = br \quad S = cs \quad Q = dq
\]

and the subscript $G$ indicating entries defined at the GUT scale. Here, the zero entries in the mass textures are the result of accidental discrete symmetries, which if broken, allow the generation of non-zero entries by means of the renormalisation group equations as the mass matrices are
renormalised down to lower energies. This is indeed the case for the entries $\delta_u$ and $\delta_d$ which occur due to the violation of a discrete symmetry at the GUT scale.

Although both the Yukawa couplings and the SO(10) Higgs vevs are complex, thereby permitting both hard and soft CP violation, the entries in the mass matrix textures, as given in (12), have been rendered explicitly real by means of quark and charged lepton field redefinitions. It is then these (SUSY scale) matrices, with 8 real parameters and one phase, that are diagonalised and the mass eigenvalues fitted to the low energy data, following Dimopoulos, Hall, and Raby [45]. The diagonalisation proceeds by means of unitary and biunitary transformations of the form $U^{\text{diag}} = V_u U_u^\dagger$, $D^{\text{diag}} = V_d^L D V_d^R$, and $L^{\text{diag}} = V_L^L L V_L^R$, and in following the assumptions of reference [45], that $V \gg Q \approx \delta_u \gg P$ and $T \gg S \approx \delta_d \gg R$, the approximate mixing matrices are of the form

\begin{align}
V_u &= \begin{bmatrix}
  c_2 & s_2 & 0 \\
  -s_2 & c_2 & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  1 & 0 & 0 \\
  c_3 & s_3 & 0 \\
  -s_3 & c_3 & 0
\end{bmatrix}
\end{align}

\begin{align}
V_d^L &= \begin{bmatrix}
  c_1 & -s_1 & 0 \\
  s_1 & c_1 & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  1 & 0 & 0 \\
  1 & 0 & 0 \\
  -s_4 & c_4 & 0
\end{bmatrix} \begin{bmatrix}
  0 & e^{i\varphi} & 0 \\
  0 & 0 & e^{i\varphi}
\end{bmatrix}
\end{align}

\begin{align}
V_l &= \begin{bmatrix}
  c_5 & s_5 & 0 \\
  -s_5 & c_5 & 0 \\
  0 & 0 & 1
\end{bmatrix}
\end{align}

with $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$. The angles define in these rotation matrices can then be determined by fitting the mass eigenvalues to the low energy data. Using the low energy input data of [45], the resulting phenomenological fit specifies the angles as

\begin{align}
s_1 &\approx 0.196 \\
 s_2 &\approx 0.05 \\
 s_3 &\approx 0.046 \\
 s_4 &\approx 0.0066 \\
 s_5 &\approx 0.070 \\
 \cos \varphi &\approx 0.41^{+0.22}_{-0.15}
\end{align}

This phenomenological fit may need some revision in view of the subsequent and more precise low energy data (especially in light of the recent improvement to the bounds on the CKM matrix entry $V_{cb}$ [40]) but it is expected that any revisions will have small effects on our results, and so we continue to use the original fit.
4 The Hadronic Lagrangian and Branching Ratio Predictions

With the mass eigenstate rotations defined by equations (15) and (16), the low energy effective lagrangian of equation (9) can be explicitly evaluated in terms of the dimension 6 four fermion quark level operators. In focusing on nucleon decay, these quark level \((qqql)\) operators can be restricted by energy conservation, to have a quark composition of only the \(u, d,\) and \(s\) quarks. This in turn results in only five distinct operators, namely

\[
O^q(dud\nu_a) = \epsilon_{ijk}(d^i u^j)(d^k \nu_a) \\
O^q(sud\nu_a) = \epsilon_{ijk}(s^i u^j)(d^k \nu_a) \\
O^q(uds\nu_a) = \epsilon_{ijk}(s^i u^j)(s^k \nu_a) \\
O^q(duul) = \epsilon_{ijk}(d^i u^j)(u^k l_a) \\
O^q(suul) = \epsilon_{ijk}(s^i u^j)(u^k l_a)
\] (16)

Thus, the effective Lagrangian, expressed at the quark level, can then be written as \(\mathcal{L}_{\text{nucleon}} = \sum C(qqql)O^q(qqql)\). Here the \(C(qqql)\)'s are the coefficients of the distinct quark level operators \(O^q(qqql)\), and are determined from equation (9) by summing the coefficients of the equivalent four fermion effective operators, modulo Fierz transformations. By classifying nucleon decay in terms of its various allowed channels, the effective lagrangian for nucleon decay can be written as

\[
\begin{align*}
\mathcal{L}(n, p \rightarrow \pi + \bar{\nu}_i) &= C(dud\nu_i)O^q(dud\nu_i) \\
\mathcal{L}(n, p \rightarrow \pi + l^+_i) &= C(duul_i)O^q(duul_i) \\
\mathcal{L}(n, p \rightarrow K^0 + l^+_i) &= C(suul_i)O^q(suul_i) \\
\mathcal{L}(n, p \rightarrow K^+ + \bar{\nu}_i) &= C(sud\nu_i)O^q(sud\nu_i) + C(dus\nu_i)O^q(dus\nu_i)
\end{align*}
\] (17)

Yet these effective lagrangian contributions are in terms of quark level operators, and so inappropriate for hadronic decay rate calculations. Instead, they must be converted to effective lagrangian contributions at the hadronic level, thereby permitting evaluation of the nucleon decay rates, which although calculated at the hadronic level, are expressed in terms of the coefficients of the quark level four fermion effective operators specified by equations (3) and (18). This conversion may be performed using the chiral lagrangian techniques developed in references [47], [24], and [25], which express general hadronic level decay rates in terms of coefficients of generic four fermion quark level
operators. The results of these decay rate calculations, in the notation of \[34\], are as follows:

\[
\begin{align*}
\Gamma(p \to K^+ + \bar{\nu}_i) & = \frac{(m_p^2 - m_K^2)^2}{32\pi m_3^3 f_\pi^2} \left[ 2m_p D C(sud\nu_i) + \left[ 1 + \frac{m_p}{3m_B}(D + 3F) \right] C(dus\nu_i) \right]^2 \\
\Gamma(p \to \pi^+ + \bar{\nu}_i) & = \frac{m_p}{32\pi f_\pi^2} \left[ 1 + D + F \right] C(dud\nu_i)^2 \\
\Gamma(p \to K^0 + l_i^+) & = \frac{(m_p^2 - m_K^2)^2}{32\pi m_3^3 f_\pi^2} \left[ 1 - \frac{m_p}{m_B}(D - F) \right] C(suu\nu_i)^2 \\
\Gamma(p \to \pi^0 + l_i^+) & = \frac{m_p}{64\pi f_\pi^2} \left[ 1 + D + F \right] C(duu\nu_i)^2 \\
\Gamma(n \to K^0 + \bar{\nu}_i) & = \frac{m_n}{32\pi m_3^3 f_\pi^2} \left[ 1 + \frac{m_n}{m_B}(D - 3F) \right] C(sud\nu_i) \\
& \quad \quad + \left[ 1 + \frac{m_n}{3m_B}(D + 3F) \right] C(dus\nu_i)^2 \\
\Gamma(n \to \pi^0 + \bar{\nu}_i) & = \frac{m_n}{64\pi f_\pi^2} \left[ 1 + D + F \right] C(dud\nu_i)^2 \\
\Gamma(n \to \pi^- + l_i^+) & = \frac{m_n}{32\pi f_\pi^2} \left[ 1 + D + F \right] C(duu\nu_i)^2 \\
\Gamma(n \to \eta + \bar{\nu}_i) & = \frac{3(m_n^2 - m_\eta^2)^2}{64\pi m_3^3 f_\pi^2} \left[ 1 - \frac{1}{3}(D - 3F) \right] C(\nu\nu\nu\nu_i)^2 
\end{align*}
\]

Here \(m_B \equiv m_\Sigma = m_\Lambda = 1150\) MeV is the mass to be associated with the virtual baryon exchange, \(m_n = m_p\) is the nucleon mass, and \(D = 0.81\) and \(F = 0.44\) are numerical factors.

From these decay rates, it is then very simple to construct branching ratios, which have the advantage over decay rates in that most of the as yet unspecified factors hidden in the quark level operators \(C(qqql)\) divide out, leaving the branching ratios parameterised by the ratio of the GUT scale Yukawa couplings of the complex \(10\). The numerical predictions for the branching ratios of the most dominant proton and neutron decay channels, for a large range of this parameter, \(\frac{a}{b}\), are presented in Figures \[4\] and \[5\] respectively. For this numerical evaluation, the branching ratios are defined as

\[
Br(N \to x + y) = \frac{\Gamma(N \to x + y)}{\Gamma(N \to \text{anything})}
\]
where $N$ represents either the nucleon, and the decay rate for $N \rightarrow \text{anything}$ has been taken as the sum of all the relevant decay rates listed in (19).

5 Conclusions

With the results of the analysis of nucleon decay in this non-minimal SUSY SO(10) model presented in Figures 4 and 5, a number of important conclusions can be drawn. The first and most significant point is that this model gives one-parameter predictions for all the relevant nucleon decay branching ratios. Once nucleon decay is observed through any two channels, the ratio $\frac{a}{b}$ is determined, and all the remaining partial lifetimes of the proton and the neutron then have a definite prediction. As with the SUSY SU(5) models, this model predicts that for a large region of $\frac{a}{b}$ parameter space, $p \rightarrow K^+ + \bar{\nu}_\mu$ and $n \rightarrow K^0 + \bar{\nu}_\mu$ are the most dominant proton and neutron decay modes. This prediction could only be altered by a strong suppression of the GUT scale Yukawa coupling $a$ relative

Figure 4: The branching ratios of the most dominant proton decay channels.
to the third family self-coupling $b$, as shown by the prominence of the $p \rightarrow K^+ + \bar{\nu}_\tau$, $p \rightarrow \pi^+ + \bar{\nu}_\tau$, $n \rightarrow K^0 + \bar{\nu}_\tau$, and $n \rightarrow \pi^0 + \bar{\nu}_\tau$ decay modes for $\frac{a}{b} < 3 \times 10^{-2}$. Another striking feature is that for $\frac{a}{b} > 10^{-2}$ the branching ratio predictions are insensitive to the actual value of the parameter, thereby implying a degree of robustness to the predictions, regardless of the relative importance of the $10$ of SO(10) in the assumed form of the GUT scale texture.

However, it is the relative strengths of some of the individual branching ratios that serve to identify this model, and in particular, it is the nucleon decay channels involving the $\mu^+$ and the $\bar{\nu}_\mu$ that are the distinctive fingerprints of this model. For both the proton and the neutron, the branching ratio predictions for channels involving the charged muon show a marked enhancement over corresponding predictions of minimal SUSY SU(5). Specifically, the branching ratio predictions for the $p \rightarrow K^0 + \mu^+$, $p \rightarrow \pi^0 + \mu^+$, and $n \rightarrow \pi^- + \mu^+$ relative to the dominant proton and neutron decay channels are enhanced over the minimal SUSY SU(5) predictions by factors of 50-500, 10-100, and 20-200 respectively (the ranges given in these enhancement factors are due to the uncertainty of

Figure 5: The branching ratios of the most dominant neutron decay channels.
the minimal SUSY SU(5) predictions as quoted by [29, 25, 34]. To a lesser extent, the \( p \to \pi^+ + \bar{\nu}_\mu \) and \( n \to \pi^0 + \bar{\nu}_\mu \) decay channels show a similar enhancement, but only by factors of 3.6 and 2.6 respectively. Thus, these enhancements in the decay rate predictions result in branching ratios for this non-minimal SUSY SO(10) model that are both qualitative and quantitatively different from that of the SUSY SU(5) nucleon decay spectrum, thereby making this ‘realistic’ non-minimal model a testable candidate for a SUSY GUT extension to the standard model. The issue of testing the predictions of this model could be addressed at Super-KAMIOKANDE, provided that Super-KAMIOKANDE in fact observes nucleon decay.

In sum, the distinctive tests of this realistic supersymmetric SO(10) GUT which arise from the consideration of nucleon decay come not from the actual decay rates or partial lifetimes of the nucleon, as the nature of the Higgs sector and the uncertainty of the Higgs and Higgsino colour triplet masses make the SUSY dimension 5 operator decay rate predictions uncertain. Rather, they come from the calculation of nucleon decay branching ratios. The fact that this realistic model predicts ratios of branching ratios \( \frac{Br(p \to K^0 + \mu^+)}{Br(p \to K^+ + \bar{\nu}_\mu)} \), \( \frac{Br(p \to \pi^0 + \mu^+)}{Br(p \to K^+ + \bar{\nu}_\mu)} \), and \( \frac{Br(n \to \pi^+ + \mu^+)}{Br(n \to K^0 + \bar{\nu}_\mu)} \) of order 20%, shows the relevance of ‘observable’ channels such as \( p \to K^0 + \mu^+ \), \( p \to \pi^0 + \mu^+ \), and \( n \to \pi^+ + \mu^+ \) to the testing of models of GUT unification. (For related considerations involving mass textures induced by higher dimensional operators see [18].) These enhanced branching ratio predictions are instead simply a result of the composition of the Higgs superfield sector, which is such that the GUT scale Yukawa couplings relevant to nucleon decay are not the full set of couplings that contribute to SM fermion mass generation.

The results presented here may be seen as some of the possible implications of a viable SUSY GUT model, and any observation of \( p \to K^0 + \mu^+ \), \( p \to \pi^0 + \mu^+ \) or \( n \to \pi^+ + \mu^+ \) at a level significantly enhanced above the expected SUSY SU(5) predictions is an indication that the underlying structure of a realistic extension to the standard model is best described in terms of a SUSY GUT model with a non-minimal Higgs sector. Unfortunately, because only a partial set of the GUT scale Yukawa couplings is directly involved in the analysis of nucleon decay, whereas the light Higgs is a linear combination of contributions from the various SO(10) Higgs reps, the actual values of the GUT scale couplings remain undetermined and the texture unexplained, at least in this model.

Acknowledgements
The author wishes to thank B.A. Campbell for suggesting the problem and his subsequent supervision
and encouragement, as well as N. Rodning for the useful discussions that occurred over the course of the work.

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