Sharp transition FIR bandpass filter for processing bioelectric signals

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Abstract. The main objective in a filter module is to improve the signal-to-noise (SNR) ratio, intelligently extract certain frequency components or separation of frequency bands among other applications. These much needed filters can help in areas of speech or biomedical signal processing. A sharp transition (ST) finite impulse response (FIR) band pass filter (BPF) is proposed for any filter order (N). The FIR filter is implemented with least passband ripple and a satisfactory stopband attenuation. The proposed filter design permits to set the fiduciary edges for any filter order. The merits of this proposed ST FIR filter includes its linearity, sharpness of the transition band and mitigating the effects of the Gibb’s phenomenon at the fiduciary edges. The accuracy and failed detections were computed to evaluate the performance of the ST FIR BPF design using the Physionet database. It is observed that the designed ST FIR BPF was able to compute precise single fetus R-peaks and maternal R-peaks. The proposed ST filter can also be used for other ranged frequency signals other than biomedical electrical signals.

Keywords: Finite impulse response, band pass filter, sharp transition, MQRS, FQRS

1. Introduction
The choices of using digital filters for our research work was between using either FIR or infinite impulse response (IIR) filters [1]. The former was selected because of its advantages for our chosen biomedical application. The FIR filter stands taller for reasons of its linearity and stability. However higher coefficients maybe required for sharper transition bands [2]. This disadvantage in FIR filters is overlooked with today’s fast processing systems to implement such FIR filters. Whenever the FIR filter is truncated for designated fiduciary edges it is bound to generate oscillatory ripples at the stopband and the passband. This known behavior is commonly referred to as Gibb’s phenomenon [3]. This ripples tend to increase at higher filter orders [N] [4]. As per [5] the overshoots are permitted to about 18%.

2. Type of linear phase sharp FIR filters
In the past, linear phase FIR filters have been designed by various authors to achieve its goal. Principe et al., [6] presented the implemented FIR filter mainly for low frequency EEG signals which uses loose frequency response characteristics and good time resolution. Rabiner et al., [7] in his paper, discusses a novel implementation for narrow band FIR filter using the technique of decimation and interpolation. In another paper, Rabiner et al., [8] describes the FIR filters based on the Chebyshev approximation. Zhang et al., [9] uses a modified FRM process to implement ST FIR filters using a new technique called Interpolated FIR. Mintzer et al., [10] describes an algorithm by Park McClellan (PM) for designing an
optimal FIR BPF. The PM algorithm determines the filter order (lowest) to obtain maximum ripple. Rajan et al., [11] designed a sharp cut off wide band FIR filter where the interpolating factors L (optimal value) are derived. This reduces the multiplier and adders in the overall realization. Vaidyanathan et al., [12] describes another algorithm called as Remez for flat and equiripple bands for weighted Chebyshev filters. Sheikh et al., [13] describes the technique for implementing narrow band and wideband FIR filter on FRM method. Yang et al., [14] presents structural design of ST FIR filter using FRM method achieving a saving of 20% in hardware. Henzel et al., [15] presents a FIR filter based on the desired frequency response and the Chebyshev error. The proposed ST FIR filters can be used for any applications. However in this paper we look for using these FIR filters in signal processing of maternal and fetal ECG signals. The types of FIR filters as shown in Table 1 depend on whether (i) the filter coefficients are even or odd and (ii) if the impulse response is either symmetrical or anti-symmetrical. Based on the above conditions there will be four types of FIR filters, all having a linear phase.

### Table 1. Linear phase FIR filters types [1,2]

| Filter type (unit sample response) | Impulse response h(n) and number of filter coefficients | Frequency response H_r(ω) | Remark |
|-----------------------------------|-------------------------------------------------------|--------------------------|--------|
| Type 1 : Symmetric H_r(ω) = H_r(w) e^{-jω(N-1)/2} | h(n) = h(N-1-n) ; N = odd | \[ H_r(\omega) = \left(\frac{N-1}{2}\right)^{\frac{N-1}{2}} \sum_{n=0}^{N-1} h(n) \cos \left(\frac{N-1}{2}n\right) \] | - |
| Type 2 : Symmetric H_r(ω) = H_r(w) e^{-jω(N-1)/2} | h(n) = h(N-1-n) ; N/2 | \[ H_r(\omega) = \frac{N}{2} \sum_{n=0}^{N-1} h(n) \cos \left(\frac{N-1}{2}n\right) \] | H_r(0) gives maximum value, while H_r(π) = 0 |
| Type 3 : Anti Symmetric H_r(ω) = H_r(w) e^{jω(N-1)/2} | h(n) = -h(N-1-n) ; (N-1)/2 | \[ H_r(\omega) = \frac{N}{2} \sum_{n=0}^{N-1} h(n) \sin \left(\frac{N-1}{2}n\right) \] | H_r(0) = 0 and H_r(π) = 0 |
| Type 4 : Anti Symmetric H_r(ω) = H_r(w) e^{-jω(N-1)/2} | h(n) = -h(N-1-n) ; N/2 | \[ H_r(\omega) = \frac{N}{2} \sum_{n=0}^{N-1} h(n) \sin \left(\frac{N-1}{2}n\right) \] | H_r(0) = 0 and H_r(π) gives out a maximum value |

With reference to the listed four linear phase FIR filters as seen in Table 1, it can be analyzed that for filter Type 2, H_r(0) gives a maximum value while H_r(π) = 0 makes it suitable for low pass filter. In Type 3, since H_r(0) = 0 and H_r(π) = 0, this is unsuitable neither for low pass nor high pass, however it is most suitable for BPF. For Type 4 filter, H_r(0) and H_r(π) gives maximum value, thus is suitable for a high pass filter. Type 1 is the most preferred of the four when symmetric h(n) is concerned and the centre of symmetry for all types is given as (N-1)/2.

3. **Proposed sharp transition FIR filters**

A simple design of sharp transition FIR filters are proposed for a band pass filter comprised of a high pass filter (HPF) in tandem with a low pass filter (LPF). Each of the HPF and LPF are designed individually. The design does not use the centre frequency concept utilised by the author in [16]. The boon of this design customizes the usage of any fiduciary edges for any filter order.

3.1 **Design of ST FIR filters**
Following are the regions of the high pass filter using trigonometric functions as shown in Figure 1 and in equations (1). The design parameters as in equation (2) are derived from equation (1). The impulse response is then computed in equation (3).

Region 1 (stopband): \( H(\omega) = 1 - \delta_s \left[ 1 - \cos (ksh \omega) \right] \) \( 0 \leq \omega \leq \omega_{sh} \)

Region 2 (transition): \( H(\omega) = \delta_s + (1 - \delta_p - \delta_s) \sin \left[ ksh (\omega - \omega_{sh}) \right] \) \( \omega_{sh} \leq \omega \leq \omega_{ch} \)

Region 3 (passband): \( H(\omega) = (1 - \delta_p) + \delta_p \sin \left[ kph (\omega - \omega_{ch}) \right] \) \( \omega_{ch} \leq \omega \leq \pi \)

\[
\begin{align*}
  ksh &= \frac{\pi}{2\omega_{sh}} \\
  kph &= \frac{\pi}{2(\pi - \omega_{ch})} \\
  kph &= \frac{\pi}{2(\pi - \omega_{ch})}
\end{align*}
\]

\[
\begin{align*}
  h(\omega) &= \left( \frac{\delta_s}{k_l\pi} \right) (1 - \cos (ksh \omega)) + \left( \frac{\delta_s}{2\pi} \right) \left[ \frac{\cos ((k - ksh) \omega) - 1}{(k - ksh)} + \frac{\cos ((k + ksh) \omega) - 1}{(k + ksh)} \right] \\
  &+ \left( \frac{\delta_s}{k_l\pi} \right) (\cos (ksh \omega) - \cos (k\omega)) + \left( \frac{1 - \delta_p \cdot \delta_s}{2\pi} \right) \\
  &\left[ \sin ((ksh - k) \omega) \cdot (\omega - ksh) \omega + \sin (k\omega) \right] + \left( \frac{\delta_p}{\omega_{ch}} \right) \left[ \sin ((kph - k) \omega) \cdot (\omega - kph) \omega + \sin (k\omega) \right] \left( \omega_{ch} + k \right)
\end{align*}
\]
Similarly, following are the regions of the low pass filter using trigonometric functions as shown in Figure 2 and in equations (4). The design parameters as in equation (5) are derived from equation (4). The impulse response for the low pass filter is given in equation (6).

\[
\text{Region 1 (passband): } H(\omega) = (1 - \delta_p) + \delta_p \cos(kpl \omega) \quad 0 \leq \omega \leq \omega_{cl}
\]

\[
\text{Region 2 (transition): } H(\omega) = \delta_s + (1 - \delta_p - \delta_s) \cos[ksl (\omega - \omega_{sl})] \quad \omega_{cl} \leq \omega \leq \omega_{sl}
\]

\[
\text{Region 3 (stopband): } H(\omega) = \delta_s - \delta_s \sin[ksl (\omega - \omega_{sl})] \quad \omega_{sl} \leq \omega \leq \pi
\]

\[
kpl = \frac{\pi}{2\omega_{cl}}
\]

\[
kst = \frac{\pi}{2(\omega_{sl} - \omega_{cl})}
\]

\[
ksl = \frac{\pi}{2(\pi - \omega_{sl})}
\]

\[
H(v) = \left\{ \begin{array}{c} \frac{1}{k}\left[(1 - \delta_p - \delta_s) \sin(k\omega_{cl}) + \delta_s \sin(k\pi)\right] \\ + \left\{ \frac{\delta_p}{\pi(k_{sl}^2 - k^2)} \left[k_{pl} \sin(k_{pl} \omega_{cl}) \cos(k\omega_{cl}) - k \cos(k_{pl} \omega_{cl}) \sin(k\omega_{cl})\right] \\ + \left\{ \frac{(1 - \delta_p - \delta_s)}{\pi(k_{sl}^2 - k^2)} \left[k \sin(k\omega_{cl}) + k_{sl} \sin(k_{sl} \omega_{sl} - \omega_{cl}) \cos(k\omega_{sl}) - k \cos(k_{sl} \omega_{sl} - \omega_{cl}) \sin(k\omega_{sl})\right] \\ + \left\{ \frac{\delta_s}{\pi(k_{sl}^2 - k^2)} \left[k_{sl} \cos(k_{sl} (\pi - \omega_{sl})) \cos(k\pi) + k \sin(k\omega_{cl}) \sin(k\pi) - k_{sl} \cos(k\omega_{sl})\right] \end{array} \right. \right.
\]

where \( k \neq (k_{st}, k_{sl} \text{ and } k_{st}) \).

### 4. Synthesis results of composite ST FIR BPF

The ST HPF and LPF can be designed for the desired filter specifications for predefined frequency edges as shown in the Table 2 and 3.
With our proposed ST HPF and LPF, we observed from Tables 4 and 5 the following: i) Passband losses are at a minimum and ii) The Gibb’s ripple are mitigated at higher filter orders.

Table 2. ST HPF specifications along with measured magnitude response values.

| ST filter (filter order (N) = 1000) | Passband edge (ωp) rad/s | Stopband edge (ωb) rad/s | Transition bandwidth (ωb - ωp) rad/s | Max. passband loss (dB) | Min. stopband attenuation (dB) |
|------------------------------------|--------------------------|--------------------------|-------------------------------------|-------------------------|-------------------------------|
| Design specifications              | 56π                      | 54π                      | 2π                                  | ±0.873                  | 40                            |
| Measured specifications            | 57.3π                    | 52π                      | 5.3π                                | +0.284, -0.183          | 40                            |

Table 3. ST LPF specifications along with measured magnitude response values.

| ST filter (filter order (N) = 1000) | Passband edge (ωp) rad/s | Stopband edge (ωb) rad/s | Transition bandwidth (ωb - ωp) rad/s | Max. passband loss (dB) | Min. stopband attenuation (dB) |
|------------------------------------|--------------------------|--------------------------|-------------------------------------|-------------------------|-------------------------------|
| Design specifications              | 96π                      | 98π                      | 2π                                  | ±0.873                  | 40                            |
| Measured specifications            | 95.5π                    | 99.86π                   | 4.36π                               | -0.132                  | 40                            |

Table 4. Variations of passband loss and stopband attenuation for HPF with filter orders (N).

| Filter order (N) | 200  | 500  | 1000 | 1500 | 2000 |
|------------------|------|------|------|------|------|
| Passband loss (dB) | 0.534 | 0.164 | 0.284 | 0.044 | 0.076 |
| Stopband attenuation (dB) | 21.04 | 30.56 | 38   | 39.44 | 39.67 |

Table 5. Variations of passband loss and stopband attenuation for LPF with filter orders (N).

| Filter order (N) | 200  | 500  | 1000 | 1500 | 2000 |
|------------------|------|------|------|------|------|
| Passband loss (dB) | 0.559 | 0.442 | 0.132 | 0.1277 | 0.1264 |
| Stopband attenuation (dB) | 25.59 | 32.46 | 37.16 | 38.6 | 39.5 |

With our proposed ST HPF and LPF, we observed from Tables 4 and 5 the following: i) Passband losses are at a minimum and ii) The Gibb’s ripple are mitigated at higher filter orders.
Various filter orders ranging from 200 to 2000 were also simulated in Matlab to evaluate the filter performance as shown in Figure 3 and 4. The expected sharpness of the filter transition width was 1Hz or $2\pi$ rad/s. Further, filter responses of HPF and LPF with their linear passband views are shown in Figures 5(a) to (d) respectively.

![Figure 5(a) ST HPF response](image1)

![Figure 5(b) Linear plot of ST HPF](image2)

![Figure 5(c) ST LPF response](image3)

![Figure 5(d) Linear plot of ST LPF](image4)

5. **QRS detection using composite ST FIR BPF**

The ST BPF filter is followed by the modified Pan Tomkins QRS detector algorithm. This algorithm was used to compute the fetal and maternal R-peaks for the Physionet database [19]. Physionet adfecgdb database with five records was used to evaluate the results. The channel one exclusively gives out a raw fetal scalp ECG which is the most accurate fecg [20]. The parameters used to evaluate the BPF and the QRS detectors is the F1 (accuracy), positive predictive value (PPV) and Sensitivity (Se) [21,22]. The evaluation of the fetal QRS and maternal QRS detection are shown in Figure 6 and 7. The entire adfecgdb for all the 5 records and for each of the four channel are evaluated for both MQRS and FQRS.
Figure 6. Fetal QRS evaluation for Se, PPV and F1 using ST BPF for adfecgdb database.

Figure 7. Maternal QRS evaluation for Se, PPV and F1 using ST BPF for adfecgdb database.

6. Conclusion

This paper designed a sharp transition FIR BPF using standalone HP and LP filters in tandem which gave low passband losses and the decreased ripple for a higher filter order. The individual HP and LP filter designs are implemented with minimum passband ripple, large stopband attenuation for any filter order (N). The three regions of each of the filter response are approximated using trigonometric functions of frequency. The filter design lay stress on achieving a sharp transition. The filter models proposed, achieve a trade-off between the transition bandwidth and Gibb’s phenomenon. The filter design when evaluated with the five Physionet abfecgdb records gave satisfactory results. The 60 seconds record of r01 (ch 4) and r08 (ch 4) gave TP = 129, FN and FP equal to 0 giving 100% accuracy results. Most of the records obtained an average accuracy more than 92%. The worst accuracy of 79.28% was seen for record r10 (channel 3). It gave relatively better maternal R-peaks for nearly all records and mostly 100% accuracy for channel four. The sharp transition width obtained by the proposed design helped achieve the objective to extract the fetal R-peaks thereby monitor the fetal health status. The individual ST HP and LP FIR filters can also be applied for specific filtering purposes in the digital domain.
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