Parity Violation in Low-Energy $np \rightarrow d\gamma$ and the Deuteron Anapole Moment

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Abstract

Parity violation in low-energy nuclear observables is included in the pionless effective field theory. The model-independent relation between the parity-violating asymmetry in $\bar{n}p \rightarrow d\gamma$ and the non-nucleon part of the deuteron anapole moment is discussed. The asymmetry in $\bar{n}p \rightarrow d\gamma$ computed with KSW power-counting, and recently criticized by Desplanques, is discussed.

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I. INTRODUCTION

Parity violation continues to be a focus of the nuclear physics community. It is being used both to uncover the structure of the nucleon in electron-scattering experiments such as SAMPLE [1], and to determine parity-violating (PV) but flavour-conserving couplings between pions and nucleons [2,3]. While the PV interaction between leptons and nucleons is well understood, the purely hadronic PV sector is presently somewhat confused. Recently it has been pointed out that measurements of PV observables in the single nucleon sector would significantly improve this situation [4,5].

The conventional description of nuclear parity-violation is formulated in terms of a PV potential generated through single-meson exchange (SME), such as the $\pi$, $\rho$ and $\omega$ [6]. The experimental measurements of parity-violation in light nuclei [3] do not yet indicate that the SME picture is consistent. In addition, in the framework of SME, the data from $^{18}$F suggest that the $\Delta I = 1$ pion-nucleon weak coupling constant, $h^{(1)}_{\pi NN}$, is much smaller than naive dimensional analysis (NDA) would suggest [3]. If, in fact, $h^{(1)}_{\pi NN}$ is much smaller than the NDA estimate then weak one-pion-exchange (OPE) may not provide the dominant long-distance component of the PV potential. It becomes necessary to include parity-violating two-pion [7,8] and multi-pion interactions consistent with the chiral symmetries of QCD as detailed in Ref. [8] and further developed in Ref. [9]. For very low-energy PV observables, the underlying mechanism generating parity-violation does not need to be known in order to make model-independent relations between different observables. This approach has been applied to PV in the NN-sector for many years [10–12] and we continue to develop this approach using the pionless effective field theory defined with dibaryon fields [13], dEFT(π) [14]. The difference between dEFT(π) and the more familiar EFT(π) [15] is that effective range contributions are summed to all orders with dEFT(π).

Low-energy PV observables such as the deuteron anapole moment and the forward-backward asymmetry, $A_\gamma$, in polarized neutron capture $\vec{n}p \rightarrow d\gamma$, can be described by an effective field theory (EFT) involving only nucleons and photons [13]. The strong interactions between nucleons in an s-wave are most economically described by dibaryon fields [13,14], with a Lagrange density

$$L_t = N^\dagger \left[ i\partial_0 + \frac{\nabla^2}{2M_N} \right] N - t_1^a \left[ i\partial_0 + \frac{\nabla^2}{4M_N} - \Delta \right] t^a - y \left[ t_j^a N^T P^j N + h.c. \right], \quad (1)$$

where $t_i$ is the $^3S_1$-dibaryon, $P^j$ is the spin-isospin projector for the $^3S_1$ channel, and $y$ is the coupling between $t_i$ and two nucleons in the $^3S_1$ channel. A similar Lagrange density exists

$^1$ The motivation for performing this re-ordering of the perturbative expansion is the relatively large size of the product $\gamma r^{(\chi)} \sim 0.4$, where $\gamma$ is the deuteron binding momentum and $r^{(\chi)}$ is the effective range in the $^3S_1$ channel. For processes involving the deuteron, the convergence is substantially improved by resumming such terms, as the normalization of the deuteron s-state plays a central role. Empirically, it has been found that these terms make the largest contribution at each order in the EFT(π) expansion. After resumming such terms, the remaining contributions from the NN scattering amplitude involve the shape-parameter $r_1$ and higher order terms in the effective range expansion.
for nucleons in the $^1S_0$-channel. This Lagrange density alone reproduces the NN scattering

$$y^2 = \frac{8\pi}{M_N^2 r^{(3S_1)}}$$

$$\Delta = \frac{2}{M_N r^{(3S_1)}} \left( \frac{1}{a^{(3S_1)}} - \mu \right)$$

where $\mu$ is the renormalization scale. A complete discussion of the power-counting and implementation of the dibaryon fields in the two-nucleon sector can be found in Refs. [13,14].

Weak interactions will be described by a Lagrange density of four-nucleon operators in EFT ($\pi / \Delta$), which are written in terms of s-wave dibaryon fields in dEFT ($\pi / 4\Delta$). For both the deuteron anapole moment and $A_\gamma$ we need only consider PV interactions of the $^3S_1$ channel. Restricting ourselves to $\Delta I = 1$ parity-violation means that we consider only a $^3S_1 - ^3P_1$ coupling, described by a Lagrange density of the form

$$\mathcal{L}_{wk} = i h^{(1)}_{33} \frac{\epsilon^{ijk}}{8M_N r^{(3S_1)}} t_i^T \sigma_j \tau_2 \tau_3 \frac{1}{2} \left( i \mathbf{\nabla} - i \mathbf{\nabla} \right)_k N + \text{h.c.} + \ldots$$

where $h^{(1)}_{33}$ is an unknown weak coupling constant that must be fit to data or predicted from the standard model of electroweak interactions. The ellipses denote higher dimension operators involving more powers of the center of mass energy, insertions of the electromagnetic gauge field or isospin breaking effects, each suppressed by inverse powers of the pion mass (or the appropriate high scale).

A similar approach to PV interactions has been recently explored by Khriplovich and Korkin [16], where they use effective short range interactions to describe the asymmetry in $\vec{\gamma}d \to np$ between the photon circular polarization states. This process depends only upon the $\Delta I = 0, 2$ PV interactions, and does not receive a contribution from OPE in the SME description.

\section*{II. ASYMMETRY IN \(\vec{N}P \to D\gamma\)}

The forward-backward asymmetry in $\vec{n}p \to d\gamma$ is defined by the coefficient $A_\gamma$ in

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta} = 1 + A_\gamma \cos \theta$$

where $\Gamma$ is the width for $np \to d\gamma$, and where $\theta$ is the angle between the emitted photon momentum and the polarization direction of the incident neutron. An upcoming measurement [17] of $A_\gamma$ will determine the coupling $h^{(1)}_{33}$ appearing in eq. (3). The asymmetry is
generated by an interference between neutron capture from the $^1S_0$ channel through a strong interaction emitting an M1 photon, and capture from the $^3S_1$ channel via the weak interaction emitting an E1 photon. The matrix element for $np \to d\gamma$ at threshold is [18] (retaining only the parity conserving M1 amplitude $Y$ and the PV E1 amplitude $W$)

$$\mathcal{M} = i \epsilon_{ijkl} \epsilon^{*}_{ij} k^j \epsilon^{*}_{(\gamma)} N^T \tau_2 \tau_3 \sigma_2 N + i \epsilon_{ijkl} \epsilon^{*}_{ij} \epsilon^{*}_{(\gamma)} N^T \tau_3 \sigma_2 \sigma_3 N \ , \quad (5)$$

where $k$ is the outgoing photon three-momentum and $N$ is a nucleon iso-spinor. In terms of $Y$ and $W$, the PV asymmetry is [18]

$$A_\gamma = -\frac{2 M_N \Re[Y^* W]}{|Y|^2} \ , \quad (6)$$

where $\gamma = \sqrt{M_N B}$ is the deuteron binding momentum, with $B$ the deuteron binding energy. The amplitudes $Y$ and $W$ are

$$Y = \frac{1}{M_N} \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{1 - \gamma r^{(S_1)}}} \left[ \kappa_1 \left(1 - \gamma a^{(S_0)} \right) - \frac{1}{2} \gamma^2 a^{(S_0)} \gamma L_1 \right]$$

$$W = \frac{h^{(1)}_{33}}{4 \sqrt{2}} \sqrt{\frac{\gamma}{M_N}} \sqrt{\frac{1}{1 - \gamma r^{(S_1)}}} \left(1 - \gamma a^{(S_1)} \right) \gamma \ , \quad (7)$$

where $\kappa_1$ is the isovector magnetic moment of the nucleon. These expressions for $W$, and $Y$ with $L_1 = 0$ are directly related to amplitudes determined in previous zero-range interaction computations, e.g. [10,19]. We have retained the formally higher order contribution from the gauge-invariant four-nucleon-one-photon interaction proportional to $L_1$, that couples the $^3S_1$ and $^1S_0$ dibaryons and the magnetic field, described by the Lagrange density

$$\mathcal{L}^B = \frac{e}{2 M_N} N^T (\kappa_0 + \kappa_1 \tau_3) \sigma \cdot B \ N + e \frac{L_1}{M_N \sqrt{r^{(S_0)} r^{(S_1)}}} t^\dagger s_3 B_j + \text{h.c.} \ , \quad (8)$$

where $s_a$ is the $^1S_0$ dibaryon. This interaction has not been included in previous zero-range computations, with the exception of those recently performed with EFT’s. The well measured cross section for cold neutron capture $np \to d\gamma$ fixes $L_1$ [14,15,18].

The determination of $h^{(1)}_{33}$ from this process is model independent and can then be used to compute other low-energy PV observables that depend only on this channel (otherwise additional information will be required). Corrections to this result are suppressed by powers of $Q \sim \gamma/m_\pi \sim 0.3$, where one particular contribution is of the form

$$\mathcal{L}^E = ie \frac{R_1}{M_N \sqrt{r^{(S_0)} r^{(S_1)}}} \epsilon_{ijkl} t^\dagger t^j E^k \ + \text{h.c.} \ , \quad (9)$$

that has not been included in previous PV discussions. $E_j$ is the electric field operator and $R_1$ is an unknown counterterm that must be determined experimentally.
III. DEUTERON ANAPOLE FORM FACTOR

The anapole moment is the PV coupling between a particle with non-zero spin and the electromagnetic field. As the operator vanishes for on-shell photons, it manifests itself as local interactions between the particle and electrically charged particles. For the deuteron, the anapole interaction is described by a Lagrange density of the form

$$\mathcal{L} = iA_d \frac{1}{M_N^2} \epsilon_{ijk} \, d^i \, d^j \partial_\mu \, F^{\mu k},$$  \hspace{1cm} (10)

where $d^j$ is the deuteron annihilation operator. $A_d$ receives two contributions, one from the anapole interactions of the nucleon $A_N$, and one from the PV interaction between nucleons, $A_h$, defined by $h^{(1)}_{33}$. The single nucleon contribution, is defined to be $A_N$ [20,21],

$$A_N = (A_n + A_p) \frac{1}{1 - \gamma r^{(S_1)}} \frac{4\gamma}{|\mathbf{k}|} \tan^{-1} \left( \frac{|\mathbf{k}|}{4\gamma} \right),$$ \hspace{1cm} (11)

arising from diagrams of the form shown in fig. 2, where $A_{n,p}$ are the neutron and proton anapole moments, respectively. If the weak one-pion nucleon interaction dominates the long-distance PV interaction, then at leading order in the chiral expansion [22]

$$A_n = A_p = - \frac{egA h^{(1)}_{\pi NN} M_N^2}{48\pi f_\pi m_\pi},$$ \hspace{1cm} (12)

in the low-energy regime [20,23]. $f_\pi$ is the pion decay constant. For the purposes of this discussion it is $A_h$ that is more interesting. The nucleon magnetic moment and the minimally coupled electric interaction give rise to a contribution of the form

$$A_h = - \frac{eh^{(1)}_{33} M_N^{3/2}}{8\sqrt{2}\pi} \left( \kappa \frac{4\gamma}{|\mathbf{k}|} \tan^{-1} \left( \frac{|\mathbf{k}|}{4\gamma} \right) \right.$$ \hspace{1cm} (13)
$$\left. + \frac{4}{|\mathbf{k}|^2} \left[ \gamma^2 - \left( \gamma^2 + \frac{|\mathbf{k}|^2}{16} \right) \frac{4\gamma}{|\mathbf{k}|} \tan^{-1} \left( \frac{|\mathbf{k}|}{4\gamma} \right) \right] \right),$$

due to the diagrams shown in fig. 3. In the zero-momentum transfer limit $A_h$ reduces to

\[2\] I expect, though have not checked, that this result can be obtained from Ref. [24] when effective range theory wavefunctions are inserted into eq.(2) of Ref. [24].
FIG. 3. Non-nucleon contributions to the deuteron anapole moment. The solid square denotes an insertion of the parity violating $^3S_1 - ^3P_1$ coupling, while the small solid circle denotes an insertion of the nucleon magnetic moment or a minimally coupled interaction.

$$A_\gamma \rightarrow \frac{e h^{(1)}_{\pi NN} M_N^{3/2}}{8 \sqrt{2} \pi} \frac{1}{1 - \gamma r^{(S_1)}} \left( \kappa_1 - \frac{1}{6} \right),$$

consistent with limiting values determined previously [20,21,25].

Therefore, it is clear that there is a model-independent relation between $A_\gamma$ and the non-nucleon contribution to the deuteron anapole moment at leading order in the effective field theory expansion. In the theory with perturbative pions and KSW power-counting [26], both $A_\gamma$ [18] and the deuteron anapole form factor [20,21] have been computed at leading order in $h^{(1)}_{\pi NN}$ and this relation is found to exist. However, we see that it is more general than leading order in $h^{(1)}_{\pi NN}$. Corrections to this relation arise from not only operators involving insertions of time-derivatives in PV operators but also from insertions of the shape parameter (and higher NN interactions terms) and from the anapole moment of the dibaryon itself,

$$\mathcal{L} = i A_t \frac{1}{M_N^{1/2} (S_1)} \epsilon_{ijk} t^i t^j \partial_{\mu} F^{\mu k}.$$  

This gives a contribution to $A_d$ of

$$\delta A_d = \frac{\gamma}{M_N (1 - \gamma r^{(S_1)})} A_t,$$

suppressed by just one power of $Q$ in the power-counting. Thus, the relation between $A_\gamma$ and $A_d$ is expected to hold only up to order $Q$.

IV. SIGNS, EFT AND ALL THAT

It is appropriate to address some concerns recently expressed about EFT calculations of PV processes [27]. In Ref. [27] much was made of the difference in both the sign and magnitude of the calculation of $A_\gamma$ in Ref. [18] using an EFT with perturbative pions and
KSW power-counting. The relative sign between the PV asymmetry $A_\gamma$ arising from weak OPE alone and the PV OPE NN potential computed in Ref. [18] agrees with previous calculations, as detailed in Ref. [27]. It appears that the overall sign discrepancy is due to convention alone, and so we will detail the interaction terms that were used in determining the sign of the asymmetry. Firstly, the strong interaction Lagrange density is
\[
L^{st} = g_A N^\dagger \sigma \cdot A N = \frac{g_A}{f_\pi} N^\dagger \sigma \cdot \nabla \Pi N + \mathcal{O}(\Pi^3),
\]
where $A = \frac{i}{2} \left( \xi \nabla \xi^\dagger - \xi^\dagger \nabla \xi \right)$ transforms as $A \rightarrow U A U^\dagger$ under $SU(2)_L \otimes SU(2)_R$ chiral transformations. The $\xi$ field is defined as
\[
\Sigma = \xi^2 = \exp \left( \frac{2i}{f_\pi} \Pi \right), \quad \Pi = \left( \frac{\pi^0/\sqrt{2}}{\pi^-}, -\frac{\pi^+}{\sqrt{2}} \right),
\]
and under chiral transformations $\Sigma \rightarrow L \Sigma R^t$. It is straightforward to derive the Noether current associated with axial transformations which, when compared to the $\beta$-asymmetry in $n \rightarrow p e^- \nu_e$, fixes $g_A \sim +1.25$ (for instance, see Refs. [28,29]). Secondly, the leading order isovector PV interactions are defined by the Lagrange density
\[
L^{\text{wk}} = -\frac{h^{(1)}_{\pi NN}}{4} f_\pi N^\dagger \left( X_L^3 - X_R^3 \right) N = -\frac{h^{(1)}_{\pi NN}}{\sqrt{2}} N^\dagger [\tau, \Pi]^3 N + \mathcal{O}(\Pi^3),
\]
where $X_L^a = \xi^\dagger \tau^a \xi$, $X_R = \xi \tau^a \xi^\dagger$, transform as $X_{L,R} \rightarrow U X_{L,R} U^\dagger$ under chiral transformations. These definitions lead to an opposite sign for the PV pion-exchange potential compared to that of Ref. [6]. To reconcile this difference one finds $h^{(1)}_{\pi NN}$ used in these definitions is of opposite sign to that used in Ref. [27], despite the fact that the weak Lagrange densities appear to be identical. This set of sign conventions will only matter when there is a rigorous theoretical prediction for the sign of $h^{(1)}_{\pi NN}$. At this point in time no such calculation exists. A comparison with existing hadronic model estimates will provide information about the validity of such models for nonleptonic matrix elements [27], and we will have to be satisfied with this somewhat primitive level of understanding until a result from lattice-QCD is computed.

In the context of KSW power-counting, where a unified theory of pion-nucleon and nucleon-nucleon interactions was proposed, the estimated $\sim 30\%$ in $A_\gamma$ uncertainty reflects not only the presence of higher dimension pion-nucleon parity violating interactions, but higher order contributions to the strong nucleon-nucleon interaction. Given the present convergence problems [30] of KSW power-counting for certain quantities one cannot read too much into this uncertainty. However, it remains true that until a converging expansion

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Ref. [27] uses a $\gamma_5$ sign convention such that the left-handed chirality projector is $1 + \gamma_5$ (e.g. eq. (29) in Ref. [11]) and defines the strong coupling $g_{\pi NN}$ through $\tilde{\mathcal{L}} = -ig_{\pi NN} N \gamma_5 \pi N$ (eq. (113) in Ref. [6]). This requires that $g_{\pi NN}$ is negative for the pion field we have defined in the text. Ref. [27] assumes that $g_{\pi NN}$ is positive.

Absolute signs determined in hadronic models are not necessarily consistent with QCD.
for the NN interaction, consistent with chiral symmetry is constructed, it is unrealistic to claim precision predictions for any observables of this type within either the potential model approach or using an EFT where pions are included as dynamical degrees of freedom. At present, claims such as those of Ref. [27] are unfounded.

V. DISCUSSION

We have shown how to describe the low-energy PV interactions in the two-nucleon sector by local operators in dEFT(π). Given the possibility that \( h_{\pi NN}^{(1)} \) may be much smaller than NDA estimates, it is fruitful to consider the most general structure of the PV interactions. This attitude has been taken recently in Ref. [16] to describe the circular polarization asymmetry in \( \vec{\gamma}d \rightarrow np \). We have shown that there is a model independent relation between \( A_{\gamma} \), PV asymmetry in \( \vec{n}p \rightarrow d\gamma \), and \( A_h \), the non-nucleon part of the deuteron anapole moment at leading order in the dEFT(π) expansion. It is important to test this relation if possible but it would require the measurement of \( A_{\gamma} \), the nucleon anapole moment, and the deuteron anapole moment.

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5 In the relevant potential model calculations, local gauge invariant operators involving one-photon and four nucleons have not been included. Such operators are certainly present, and may or may not be saturated by meson-exchange currents. In the case of the strong width for \( np \rightarrow d\gamma \), the pion-exchange current saturates the local counterterm that appears in EFT(π) [15]. In contrast, the local counterterm for the deuteron quadrupole moment is not saturated by pion-exchange currents [15], nor is the local counterterm that contributes at NLO to νd break up [31]. If \( h_{\pi NN}^{(1)} \) is much smaller than NDA estimates, such formally higher dimension operators will make an enhanced contribution to electromagnetic observables.
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