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Stacked-origami mechanical metamaterial with tailored multistage stiffness

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Abstract:
Origami-based metamaterial has shown remarkable mechanical properties rarely found in natural materials, but achieving tailored multistage stiffness is still a challenge. This study proposes a novel zigzag-base stacked-origami (ZBSO) metamaterial with tailored multistage stiffness property based on crease customization and stacking strategies. A high precision finite element (FE) model to identify the stiffness characteristics of the ZBSO metamaterial has been established, and its accuracy is validated by quasi-static compression experiments. Using the verified FE model, we demonstrate that the multistage stiffness of the ZBSO metamaterial can be effectively tailored through two manners, i.e. varying the microstructures (through introducing new creases to the classical Miura origami unit cell) and altering the stacking way. Three strategies are utilized to vary the microstructure, i.e. adding new creases to the right, left, or both sides of the unit cell. We further reveal that the proposed ZBSO metamaterial has several outstanding advantages compared with traditional mechanical metamaterials, e.g. material independent, scale-invariant, lightweight, and excellent energy absorption capacity. The unravelled superior mechanical properties of the ZBSO metamaterials pave the way for the design of the next-generation cellular metamaterials with tailored stiffness properties.

Keywords: Zigzag-base stacked-origami metamaterial; tailored stiffness; tailored microstructures; compression molding; quasi-static compression experiment.
1. Introduction

Mechanical metamaterials are unique structures whose properties are determined not by their material composition as conventional structures but by the geometric configuration of the microstructure units [1-3]. Due to its unconventional mechanical properties, for instance, negative Poisson's ratio, negative effective mass, negative modulus, chirality, et al., it has broad application prospects [4-8]. The empirical method is the mainstream method of designing such mechanical metamaterials. Although this method has demonstrated powerful capabilities, it is difficult to exploit the superior properties fully. As a remedy, topology optimization methods have been further developed to design mechanical metamaterials with optimal (or locally optimal) mechanical properties [9-11]. Nevertheless, topology optimization may not always be possible obtaining the desired optimal solutions, thereby leading to the imperious demand for new design ideas for the mechanical metamaterial.

The origami technique may be one answer to the above question. Due to its simple concept, fascinating characteristic, and wide application prospects, origami has aroused great interest of mathematicians, scientists and engineers in recent year [12], leading to a series of innovative designs [13-35], for instance, foldable lithium-ion battery, origami robot, foldscope, bioinspired spring, active structures, energy absorbing-structures/materials sandwich panels. Furthermore, the recent inspiration for the design of mechanical metamaterials has made it even more compelling; especially recent studies reveal that origami-based mechanical metamaterials have unique properties that traditional mechanical materials do not have. Mechanical metamaterials based on Miura origami are the most widely studied among them. Miura origami itself is a mechanical metamaterial with two distinct Poisson’s ratio properties, i.e. a negative one for in-plane deformations and a positive one for out-of-plane bending, which is dominated by the kinematics of the folding [36]. Through expanding on the design space of Miura origami, its variants can achieve adjusting Poisson's ratio between positive and negative [37]. Tunable negative Poisson’s ratio can be realized by the cylindrical derivative of Miura origami, e.g. the Tachi Miura polyhedron [38]. Stiffness represents the ability to resist deformation, which is another key performance index of mechanical metamaterials. The operating environment of engineering equipment is generally complex and changeable, resulting in structures with different stiffness
characteristics for various load conditions that are significantly desired. Thus, the tailored stiffness characteristics of the mechanical metamaterial are essential for the engineering equipment, which is an uneasy task for traditional design methods. Cylindrical origami-inspired mechanical metamaterials were designed and analyzed via energy landscapes and strain variations; by controlling the stiffness, the required deployability and collapsibility can be realized [39]. By considering the mixed mode of deformations involving both rigid origami motion and facet bending, the deformation stiffness of mechanical metamaterials with the waterbomb bases can be tailored, forming a potential long-distance actuation mechanism with a single far-field force [40]. To further expand the design freedom of Miura origami, mechanical metamaterials based on curved crease origami can accomplish in situ stiffness manipulation by alteration of the curvature of the creases [41]. Although these studies have successfully achieved tailored stiffness characteristics, there is still room for further expansion of its design freedom.

Stacking origami mechanical metamaterials to structure novel mechanical metamaterials (SOMM) can gain more fruitful tailored stiffness characteristics due to introducing extra design freedoms, e.g. stacking order and sacking way [42]. For instance, the high-static-low-dynamic stiffness property can be harnessed by SOMM, which is extremely important in the field of low-frequency vibration isolation [43-46]. Our previous work revealed that stacking two Miura sheets to form a SOMM can achieve the realization of tailoring the dynamic stiffness [47]. By non-uniformly stacking Miura sheets, Chen and coworkers have designed a series of novel mechanical metamaterials, whose graded stiffness characteristics were investigated by combining kinematic analysis, numerical simulation and experimental method [48,49]. Interestingly, Li and coworkers combined the stacked Miura origami and rhombic honeycomb structure to develop new mechanical metamaterials with a two-stage programmable compressive strength [50]. The concept of this combination of different configurations to gain incredible performances has received considerable attention in many fields [51]. Alternatively, introducing additional microstructures to the mechanical metamaterial is another promising way to significantly improve the performance [52,53]. Nevertheless, studies on SOMM based on this idea are limited. In this study, we will show how to use the stacking and introducing additional microstructures strategies to tailor the stiffness of the mechanical metamaterial. The microstructures can be readily introduced by adding
extra creases [54]. To this end, we have proposed a novel zigzag-base stacked-origami (ZBSO) metamaterial based on the unique geometric construction and architecture. By adding new creases to the classical Miura sheet to introduce microstructures, the proposed ZBSO metamaterial can manifest beneficial tailored stiffness properties, systematically investigated by numerical and experimental methods.

2. Materials and methods

2.1. Geometric design

This section shows the geometric design for the ZBSO metamaterial based on strict mathematical geometric relationships. ZBSO metamaterial consists of $N_{\text{layer}}$ stacks in the $z$-direction, and each stack includes two basic zigzag-base sheets. The unit cell for constructing the zigzag-base sheet is the derivation of the Miura origami (DMO) unit cell with additional creases, as shown in FIG.1A. Miura origami is made of four parallelograms. We further introduce microstructures to the facet, i.e. equally divide the parallelogram into $n$ small ones, e.g. dividing into three small identical parallelograms, whose 2D crease pattern is depicted in the left panel of FIG.1A. Since we assume the DMO unit cell is rigid-foldable, only four parameters are needed to determine the geometry of the unit cell in space, i.e. the length, $a$, the width, $b$, and the acute angle, $\phi$, for the parallelogram, and the folding angle, $\theta$. The following geometric relationships should be carefully followed if one intends to obtain the 3D geometry as shown in the right of FIG.1A.

\[
\cos \gamma = \frac{\sin^2 \phi \cos^2 (\theta / 2) - \cos^2 \phi}{\sin^2 \phi \cos^2 (\theta / 2) + \cos^2 \phi} \\
\cos \eta = \sin^2 \phi \cos \theta + \cos^2 \phi \\
w = 2b \cdot \sin (\eta / 2) \\
h = a \cdot \cos (\gamma / 2) \\
l = 2a \cdot \sin (\gamma / 2) \\
v = b \cdot \cos (\eta / 2)
\]

We then repeat the DMO unit cell in the $x$-direction and $y$-direction, resulting in $n_x$ and $n_y$ unit cells in the $x$-direction and $y$-direction, respectively. For example, FIG.1B shows the case when
$n_x=3$ and $n_y=1$. We name this structure as the derivation of Miura origami sheet (DMOS) in subsequent sections to facilitate description. By assembling the upper-DMOS and the lower-DMOS at their open sides, one can obtain a stack for forming the ZBSO metamaterial. The upper-DMOS and the lower-DMOS can be identical or different but should be amenable to strict mathematical relationships, as follows,

$$a_{nl}^2 = \frac{a_{nl}^1 \cos \phi_{nl}^1}{\cos \phi_{nl}^2}$$  \hspace{1cm} (2A)

$$b_{nl}^2 = b_{nl}^1$$  \hspace{1cm} (2B)

$$\theta_{nl}^2 = \cos^{-1}\left(1 - \frac{\sin^2(\theta_{nl}^1/2) \sin^2 \phi_{nl}^1}{\sin^2 \phi_{nl}^2}\right)$$  \hspace{1cm} (2C)

where $(a_{nl}^1, b_{nl}^1, \phi_{nl}^1, \theta_{nl}^1)$ and $(a_{nl}^2, b_{nl}^2, \phi_{nl}^2, \theta_{nl}^2)$ are the parameters for dominating the geometric topology of lower-DMOS and upper-DMOS, respectively. A specific case, i.e. the first layer with $nl$ equals to 1, is depicted in FIG.1C.
FIG 1 Geometric design for the ZBSO metamaterial: (A) The unit cell; (B) DMOS with 3 and 1 units in x-direction and y-direction, respectively; (C) One stack (DMOS tube) for constructing ZBSO metamaterial; (D) and (E) Two Different ZBSO metamaterials by varying the acute angle \( \phi \). Left: Stereogram perspective; right: lateral view.

By duplicating the stack presented in FIG.1C in the z-direction, one finally obtains the ZBSO mechanical metamaterial with \( nl \) layers (\( nl \) is also the number of cells presented in FIG.1C).

Notice that each layer is connected through the flat regions of the upper-DMOS and the lower-DMOS, forming a DMOS tube. From Equations (2A)-(2C), it can be easily found that varying the acute angle, \( \phi \), of the DMOS in each stack will yield diverse ZBSO metamaterials. For ease of description, we utilize a pair of \( \phi \) to represent the DMOS tube, in which the first value refers to lower-DMOS and the second value the upper-DMOS. For instance, FIGs 1D-E depict two representative cases, as \([60^\circ,60^\circ], (60^\circ,60^\circ), (60^\circ,60^\circ)\) and \([58^\circ,60^\circ], (62^\circ,64^\circ), (66^\circ,68^\circ), (70^\circ,72^\circ)\] respectively.

It should be noted that we first identify the four geometric parameters for the lower-DMOS of the first layer, i.e. the length, \( a_1^l \), the width, \( b_1^l \), and the acute angle, \( \phi_1^l \), for the parallelogram, and the folding angle, \( \theta_1^l \), and the other geometric parameters of DMOS for constructing ZBSO metamaterial can be analytically calculated by using Equations (1A)-(1F) and Equations (2A)-(2C).

2.2 Tensile tests

Brass (H62) is utilized for constituting ZBSO metamaterial mainly considering its good ductility. To obtain the mechanical property of the brass, we use a machine CMT5105 (Type: SUST) with an electronic extensometer (Type: YYU-12.5/25) to conduct the tensile tests, finding that the brass with a thickness of 0.2 mm can be characterized by the parameters summarized in Table 1. These parameters will be utilized in the succeeding finite element (FE) simulations.

| Young’s modulus | Yield stress | Tensile strength | Elongation | Poisson’s ratio | Density |
|-----------------|--------------|-----------------|------------|----------------|---------|
| 103.6 GPa       | 363.0 MPa    | 622.5 MPa       | 18.8%      | 0.33           | 8.9 g·cm⁻³ |

2.3 FE modelling
To capture the tailored stiffness property of ZBSO metamaterial, numerical simulations are performed utilizing nonlinear finite element code Abaqus/Explicit. Four-node shell elements (S4) are employed to mesh the ZBSO metamaterial. Global element size is chosen as 1.2 mm, determined by the mesh sensitivity analysis as shown in subsection 3.1. ZBSO is rest on a rigid plate during the simulation. A rigid plate is initially on the top edge of ZBSO. All the nodes of the flat regions on the upper-DMOS of the top unit cell are coupled to a coupling point on the rigid plate, and the displacement of the coupling point is controlled to simulate the quasi-static compression process. Two contact properties are set in the simulation, i.e. penalty friction with the Coulomb friction coefficient as 0.3 and hard contact model to characterize the contact pressure between surfaces. Brass (H62) is used as the constituted material to construct ZBSO, whose mechanical properties are obtained from the tensile tests provided in subsection 2.2. FIG 2A shows the FE model's axonometric view and lateral view for the ZBSO metamaterial with four layers, respectively. The geometric parameters to determine the topology of the ZBSO metamaterial are presented in Table 2.
(A) Coupling point | Displacement | Rigid plate
Coupling point | Fixed | Rigid plate

(B) Female mould
\[ \phi = 58^\circ \]

Male mould
\[ \phi = 60^\circ \]

Cutting the flat sheet from the raw material
Putting the sheet between the moulds
Stamping the moulds by using a hydraulic stamping press
Getting the Miura sheet after the stamping

Obtaining the zigzag-base stacked-origami metamaterial.
Left: vertical view and right: right view
Obtaining one stack by gluing two zigzag-base sheets. Top: vertical view and bottom: right view

(C) Tesile machine
Data collector
ZBSO metamaterial sample
Loading plant
Fixed plant
FIG 2 FE modelling verification for the ZBSO metamaterial: (A) FE modelling (Left: Axonometric view and right: lateral view); (B) Fabrication process (Left: moulds for creating the DMOS and right: a representative manufacturing process); (C) Experimental setup for testing the tailored stiffness property of the ZBSO metamaterial.

Table 2 Design parameters of FE model for the ZBSO metamaterial.

| $\phi_{nl}$ (%) | $a_{nl}^1$, $a_{nl}^2$ (mm) | $b_{nl}^1$, $b_{nl}^2$ (mm) | $\theta_{nl}^1$, $\theta_{nl}^2$ (%) | $nl$ |
|----------------|-----------------------------|-----------------------------|----------------------------------|------|
| 58, 60         | 18.87, 20                   | 24                          | 135.49, 130.00                   | 1    |
| 58, 60         | 18.87, 20                   | 24                          | 135.49, 130.00                   | 2    |
| 58, 60         | 18.87, 20                   | 24                          | 135.49, 130.00                   | 3    |
| 58, 60         | 18.87, 20                   | 24                          | 135.49, 130.00                   | 4    |

It should be noted that, to ensure the accuracy of the simulations, the following two situations should be carefully considered [26]:

1. The ratio of artificial energy to internal energy is below 5% to make sure that the hour-glassing effect would not significantly affect the results;

2. The ratio of kinetic energy to internal energy is below 5% during most crushing processes to ensure that dynamic effects can be considered insignificant.

2.3. Fabrication of ZBSO

There are many manufacturing methods for making origami structure prototypes, e.g. manual-folding, cold grass-pressure folding process, compression molding, bending and welding, self-folding [22,47,56-58]. Here, we adopt a five-step fabrication strategy based on the compression molding method to make the real ZESO metamaterial samples, as shown in FIG 2B. To ensure the accuracy of the sample, two pairs of male and female moulds for fabricating DMOS are processed by a vertical machining center (Type: HS-1066H), and the material used is 45 steel; the surface of the mold is further plated with a layer of chromium metal to increase the hardness and wear resistance of the mold. A hydraulic stamping press (type: LY-WDQ20A4) is employed to conduct the compression moulding of DMOS. A commercial glue ergo 1690 is used to form the ZESO metamaterial samples.

2.4. Experimental setup

The identical tensile machine used in subsection 2.1 is employed to experimentally investigate the quasi-static compression process and to further reveal the tailored stiffness property of the ZBSO
metamaterial, as shown in FIG 3C. The ZBSO metamaterial sample is placed between the loading plant and the fixed plant. The lower end of the ZBSO metamaterial sample is fixed, and the upper end moves slowly down with the loading plant until the whole sample collapses. The data for the quasi-static compression process, i.e. the displacement and the load, is gathered by a data collector and then processed by a personal computer, finally obtaining the relationship between the displacement and the force.

3. Results and discussion

3.1 Mesh sensitivity analysis

Mesh sensitivity analysis is performed to determine the mesh size to weigh the simulation accuracy against efficiency. We test five different mesh sizes, i.e. 0.8mm×0.8mm, 1.0mm×1.0mm, 1.2mm×1.2mm, 1.4mm×1.4mm, and 1.6mm×1.6mm, determining that mesh size with 1.2mm×1.2mm has relatively high accuracy and good efficiency (see FIG 3A). Thus, the mesh size will be employed in the FE models through all the simulations in this work.

FIG 3 Comparative results between FE simulation and experimental study for the ZBSO metamaterial: (A) Mesh sensitivity analysis; (B) force VS displacement curve for Test 1 and (C)
3.2 FE modelling verification

To validate the effectiveness of the FE modelling, two sets of experiments are conducted using two different ZBSO prototypes, respectively. One ZBSO prototype is fabricated according to section 2.2 (corresponding to Test 1), and another is based on the parameters summarized in Table 3 (corresponding to Test 2). To make a full comparison, a comparison of the experiment and simulation results are conducted in terms of force-displacement curves and deformed shapes.

| \( \phi_1 \), \( \phi_2 \) (°) | \( a_1, a_2 \) (mm) | \( b_1, b_2 \) (mm) | \( \theta_1, \theta_2 \) (°) | \( nl \) |
|-----------------|-----------------|-----------------|-----------------|-----|
| 54, 54          | 17.01, 17.01    | 24, 24          | 151.94, 151.94  | 1   |
| 56, 56          | 17.89, 17.89    | 24, 24          | 142.43, 142.43  | 2   |
| 58, 58          | 18.87, 17.89    | 24, 24          | 135.49, 135.49  | 3   |
| 60, 60          | 20.00, 17.89    | 24, 24          | 130.00, 130.00  | 4   |

In FIGs 3B-C, the solid blue line and red dotted line refer to the force-displacement curve obtained from the experiments and simulations. For both Test 1 (see FIG 3B) and Test 2 (see FIG 3C), there are two apparent stages for both experiment and simulation, namely, the compression stage and the densification stage, in which the former tends to be our concern. In the compression stage, again, for both experiment and simulation, four prominent stiffness areas correspond to the four layers of the ZBSO metamaterial can be found. A closer observation of the force-displacement curves it can also be found that the trends of the two curves are basically the same. Overall, the experimental results are a bit larger than those from the numerical simulations, except that when the displacement is smaller than approximately 10 mm. This may be attributed to the use of the glue introducing extra damping, resulting in greater resistance. Comparing FIGs 3B-C, it can be found that the latter exhibits obvious gradient characteristics as presented in [48]. This is because the four layers of the former are uniformly stacked, while the latter is gradient stacking. Moreover, the ZBSO metamaterial of Test 2 outperforms that of Test 1 in the energy absorption capacity, proving from the side that the stacking can tailored the mechanical properties of the ZBSO metamaterial.
To further verify the accuracy of the FE model, close-up views during the deformation process for the simulations and experiments are grabbed, which are shown in FIGs 3D-E, respectively. Eight typical close-up views are selected according to the axial compressive strain, \( \varepsilon \), of the ZBSO metamaterial, i.e. \( \varepsilon \) is equal to 0, 0.17, 0.33, 0.38, 0.47, 0.53, 0.61 and 0.70 for Test 1 and 0, 0.09, 0.13, 0.23, 0.34, 0.40, 0.57 and 0.74 for Test 2. \( \varepsilon \) equals to 0 represents the initial state, and as \( \varepsilon \) increases, the ZBSO metamaterial is gradually compressed. For all close-up views, the simulation and experiment deformations are in excellent agreement. In short, the effectiveness of the FE modelling for the ZBSO metamaterials has been well verified. The validated FE model will be employed to comprehensively investigate the tailored stiffness property of the ZBSO metamaterial.

3.3 The tailored multistage stiffness characteristic of the ZBSO metamaterial

In this section, we will show how the stiffness of the ZBSO metamaterial can be tailored by varying the microstructures (three strategies to add new creases) and altering the stacked way by utilizing the validated FE model.

3.3.1 Varying the microstructures

We introduce three strategies to add new creases to the ZBSO metamaterial and investigate how they tailor the stiffness. The related geometric parameters are identical to that used in the FE modelling of Test 1, as shown in Table 2. FIG 4 shows how the multistage stiffness of the ZBSO metamaterial is tailored through adding new creases to the right sides of the unit cell, where five cases are investigated, i.e. \( n=1:1\), \( n=1:2\), \( n=1:3\), \( n=1:4\), and \( n=1:5\). As stated in section 2.1, \( n \) represents the number of microstructures introduced, e.g. \( n=1:5 \) means the right sides of the unit cell is evenly divided into five small microstructures with the left sides unchanged. At the top of FIG 4, the geometry of the unit cells for the five cases after introducing microstructures are presented. The snapshots of the deformation shapes of the three representative moments, i.e. the displacements (abbreviated as \( disp \) in the succeeding figures) are 15mm, 31mm, and 44mm, respectively, for the five ZBSO metamaterials are also given for ease of description. When \( n=1:1\), the ZBSO metamaterial is constructed by traditional Miura unit cells, which is geometrically similar to [48]. Look close to the force-displacement curve, it first slowly increases from zero,
then enters the platform area, and finally densification occurs, which is also consistent with that observed in [48]. It can be found that, in this case, multistage stiffness is not realized since identical DMOS tubes for four stacks are used and no additional microstructures are introduced. Now we keep the DMOS tubes identical but introduce new creases, e.g. \( n = 1:2, n = 1:3, n = 1:4, \) and \( n = 1:5 \). It can be found that multistage stiffness characteristics can be clearly observed. Moreover, the number of the creases introduced significantly influences the multistage stiffness characteristic. Roughly speaking, as decrease the value of \( n \) (with more microstructures), the multistage stiffness characteristic becomes increasingly apparent. Specifically, for the ZBSO metamaterial with \( n = 1:2 \), the first trough appears when the displacement is about 27mm, which lags other cases, e.g. it is roundly 15mm for the ZBSO metamaterial with \( n = 1:3 \). It can also be found that the more creases are introduced, the earlier the first trough will emerge, and the more obvious the multi-level stiffness characteristic will be. To further demonstrate the phenomena mentioned above, several snapshots are extracted from the deformation shapes of the ZBSO metamaterials of the representative moments, i.e. the displacement equals 15mm and 27mm. When \( disp = 15 \text{mm} \), the ZBSO metamaterials with \( n = 1:1 \) and \( n = 1:2 \) are in the platform area, and the deformation is mainly contributed by the rotation of the creases (served as plastic hinges); when \( n = 1:3 \), the first layer of the ZBSO metamaterial happens to be self-locking with the emergence of the first trough of the force-displacement curve [48]; for the cases \( n = 1:4 \) and \( n = 1:5 \), the first layer of the ZBSO metamaterial has been self-locked and the facets begin to deform, leading to the deformation of the ZBSO metamaterial is dominated by both the rotation of the creases and the bending of the facets. Let's turn our eyes back to the case when \( disp = 27 \text{mm} \), the deformation of the ZBSO metamaterial with \( n = 1:1 \) is also caused by the rotation of the creases; while the ZBSO metamaterial with \( n = 1:2 \) just enters the self-locking state for its first layer; for the ZBSO metamaterials with more microstructures, for instance, the ZBSO metamaterials with \( n = 1:3 \) and \( n = 1:4 \) undergo deformations contributed by layer 1 and layer 4, and layer 4 has not appeared self-locking; however, when \( n = 1:5 \), self-locking has occurred in two layers of the ZBSO metamaterial, i.e. layer 1 and layer 2, which deformation mode is different from the previous two. Therefore, it can be found that simply adding microstructures can significantly tailor the multistage stiffness of the ZBSO metamaterial.
We then investigate the capacity of the second strategy, i.e. adding new creases to the left sides of the unit cell, tailoring the multistage stiffness of the ZBSO metamaterial, as shown in FIG 5. Five sorts of ZBSO metamaterial are considered, namely, $n=1:1$, $n=2:1$, $n=3:1$, $n=4:1$, and $n=5:1$, whose geometry of the unit cell is depicted at the top of FIG 5. Below the geometry of the unit cell, we show representative deformation shapes when $dis=13\text{mm}, 27\text{mm}, 43\text{mm}$, respectively.

FIG 4 The tailored multistage stiffness of the ZBSO metamaterial realized by adding new creases to the right sides of the unit cell.
Through the force-displacement curve, it can be apparently detected that the multistage stiffness characteristic can be only carried out for ZBSO metamaterials with $n=3:1$, $n=4:1$, and $n=5:1$, which is slightly different from those in the first strategy. Thus, comparing FIG 4 and FIG 5, we can find a simple way to tailor the stiffness of the ZBSO metamaterial by altering the position of the introduced additional creases. When the axial deformation is 13mm, layer 1 of the ZBSO metamaterials with $n=3:1$, $n=4:1$, and $n=5:1$ reach the self-locking point almost simultaneously. Nevertheless, when $disp=27mm$, the ZBSO metamaterials with $n=3:1$ and $n=4:1$ enter the self-locking state and the ZBSO metamaterial with $n=5:1$ lags slightly. At the moment, the deformation of the ZBSO metamaterials is caused by both the rotation of the creases and bending of the facets of layer 1 and layer 4. Moreover, it can be found that the side where the microstructure is not introduced is more prone to self-locking, which is consistent with the phenomenon observed in FIG 4.
The last strategy for creating novel ZBSO metamaterials is through adding new creases to both sides of the unit cell. Note the number of the creases introduced for the two sides is kept identical without loss of generality, which is geometrically shown at the top of FIG 6. FIG 6 shows that multistage stiffness characteristic is hardly observed for all the ZBSO metamaterials with $n=1:1$, $n=2:2$, $n=3:3$, $n=4:4$, and $n=5:5$. All the force-displacement curves of the ZBSO metamaterials are
similar to that of when $n=1:1$ such that as the microstructure increases, the curves generally move up as a whole, except for some small fluctuations for $n=3:3$, $n=4:4$, and $n=5:5$. When $disp=28$ mm, the deformation of layer 1 is dominated by the rotation of the creases for all the five kinds of ZBSO metamaterials. When the displacement increases to 40 mm, it can be found that the deformation is almost evenly contributed by the rotation of the creases of four layers for ZBSO metamaterials with $n=1:1$ and $n=2:2$; while for the ZBSO metamaterials with $n=4:4$ and $n=5:5$, the deformation is mainly provided by layer 1 and layer 2, in which the former one is dominated by the rotation of the creases and the latter one by both the rotation of the creases and the bending of the facets; the notable difference is found for the ZBSO metamaterial with $n=3:3$ that the deformation of layer 2 mainly contributes the whole deformation. Hence, simply changing the number of additional creases can tailor the deformation mode and the multistage stiffness of the ZBSO metamaterial.

Comprehensively analyzing all the three strategies, it is interesting to find that when fewer creases are introduced, each layer of the ZBSO metamaterial will be compressed before the self-locking of the first layer occurs, i.e. simultaneous deformation of the four layers is more obvious. In comparison, sequence deformation of each layer of the ZBSO metamaterial is clearly observed when more creases are added. Besides, from the perspective of energy absorption, the ZBSO metamaterial with more microstructures introduced, the better the energy absorption performance will be. The peak force obtained by the first two strategies is not much different and larger than that of the third strategy. Analyzing the peak force and the area contained in the force and displacement curve before the densification area, the ZBSO metamaterials obtained by these three strategies all have good application potential in the field of energy absorption.
3.3.2 Altering the stacked order

FIG 7 shows how the stiffness of the ZBSO metamaterial can be tailored through altering the stacked order of different DMOS tubes with \( n=1:3 \). Four sorts of DMOS tubes are chosen, i.e. \((58°,60°), (62°,64°), (66°,68°), \) and \((70°,72°)\). The related geometric parameters are shown in Table 4. Adjusting the stacked order of these DMOS tubes leads to various ZBSO metamaterials; strictly
speaking, there are 24 kinds. Here we carefully select four ZBSO metamaterials without loss of
generality, i.e. (58°,60°)-(62°,64°)-(66°,68°)-(70°,72°), (58°,60°)-(70°,72°)-(66°,68°)-(62°,64°),
(66°,68°)-(62°,64°)-(58°,60°)-(70°,72°), and (62°,64°)-(70°,72°)-(58°,60°)-(66°,68°). FIG 7 reveals that
the multistage stiffness of the ZBSO metamaterial can be effectively tailored by simply changing
the stacking sequence of the DMOS tubes, particularly for the one
(66°,68°)-(62°,64°)-(58°,60°)-(70°,72°) that before the overall densification, there is a long section of
positive and negative stiffness regions (roughly between 60mm to 100mm). Before the overall
displacement reaches 32.5mm, the stiffness characteristics of the four ZBSO metamaterials are
basically identical, except for the moment of change of positive and negative stiffness is slightly
different. However, it is interesting to find that the deformation modes are significantly unlike.
When \( \text{disp}=12.5\text{mm} \), layer 1 of ZBSO metamaterials (58°,60°)-(62°,64°)-(66°,68°)-(70°,72°) and
(58°,60°)-(70°,72°)-(66°,68°)-(62°,64°) reaches the self-locking point almost simultaneously, and the
overall deformation is mainly caused by the rotation of the creases of layer 1; while for the ZBSO
metamaterials (66°,68°)-(62°,64°)-(58°,60°)-(70°,72°) and (62°,64°)-(70°,72°)-(58°,60°)-(66°,68°), it
becomes layer 2 to dominate the overall deformation. It is easy to understand that the DMOS tube
(58°,60°) possesses the most minor stiffness, causing it to deform first. Thus, the DMOS tube with
the smallest stiffness deforms first, and so on. This point of view can also be corroborated by other
deformed shapes when \( \text{disp}=32.5\text{mm}, 59.0\text{mm}, \) and \( 90.5\text{mm} \), as shown in FIG 12B. It can be seen
that for these four ZBSO metamaterials, the order of deformation of each layer is (layer 1→layer
2→layer 3→layer 4), (layer 1→layer 4→layer 3→layer 2), (layer 3→layer 2→layer 1→layer 4), and
(layer 3→layer 1→layer 4→layer 2), respectively. In addition, look close to FIG 7, it can be
found that the diverse stiffness properties can be observed after the overall displacement exceeds
32.5mm. For instance, there is a larger region with quasi-zero stiffness for the ZBSO metamaterial
(58°,60°)-(62°,64°)-(66°,68°)-(70°,72°), i.e. displacement from approximately 70mm to 110mm.
ZBSO metamaterial (66°,68°)-(62°,64°)-(58°,60°)-(70°,72°) appears local instability in the middle
layers earlier, leading to a considerable peak force occurs at roughly 70mm, which is often desired
to be avoided in the field of energy absorption. However, its total energy absorption capacity is
prominently superior to the other three. Furthermore, it is interesting to find that in the range of
about 33-50mm of the force-displacement curve, the stiffness of this ZBSO metamaterial is almost
opposite to that of other ZBSO metamaterials. Therefore, through the above discussion, we have
verified that by changing the stacking order of the DMOS tubes, one can tailor the stiffness characteristics of the ZBSO metamaterial to a large extent.

Table 4 Geometric parameters of the DMOS tube for varying the stacking order with $n=1:3$.

| DMOS tube | $\phi^1, \phi^2 (^\circ)$ | $a^1, a^2$ (mm) | $b^1, b^2$ (mm) | $\theta^1, \theta^2 (^\circ)$ |
|-----------|---------------------------|-----------------|-----------------|-----------------------------|
| (58°,60°) | 58, 60                    | 18.87, 20       | 24, 24          | 135.49, 130.00              |
| (62°,64°) | 62, 64                    | 21.30, 22.81    | 24, 24          | 125.48, 121.68              |
| (66°,68°) | 66, 68                    | 24.59, 26.69    | 24, 24          | 118.45, 115.67              |
| (70°,72°) | 70, 72                    | 29.24, 32.36    | 24, 24          | 113.29, 111.23              |
FIG 7 The tailored multistage stiffness of the ZBSO metamaterial realized by varying the stacked order.
4. Conclusions

A novel zigzag-base stacked-origami (ZBSO) metamaterial with desired multistage stiffness characteristics is developed in this study. The multistage stiffness properties of the proposed ZBSO metamaterial are geometrically achieved by simply introducing extra new creases to the Miura origami unit cell and altering the stacking sequence of the derivation of the Miura origami sheet (DMOS) tube, which is extensively investigated by the validated finite element models. Three different strategies are developed for introducing new creases, i.e. adding new creases to the right, left or both sides of the unit cell, which are highly effective in tailoring the stiffness of the ZBSO metamaterial. These three strategies have little impact on the manufacturing process, as shown in FIG 3B, further illustrating their effectiveness. Altering the stacking order of the DMOS tube results in manifold stiffness characteristics as well. For instance, the positive and negative changes in stiffness can be achieved in specific deformation regions. We also show that the proposed ZBSO metamaterial exhibits excellent energy absorption ability through force-displacement curves and deformation modes. It is worth emphasizing that although brass is used in this study, the proposed ZBSO metamaterial is inherently material independent, scale-invariant, and lightweight. In the future, the application of the proposed ZBSO metamaterial in other fields will be further explored, e.g. low-frequency vibration isolation load-bearing structures [43].

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Data availability:

The raw/processed data required to reproduce these findings cannot be shared at this time due to
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