3D image compression during ultrasound phased diagnostics based on wavelet subband coding planar scans

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Abstract. The proposed compression method is based on the application of a two-dimensional discrete fast wavelet transform (FWT) to planar scans of 3D ultrasound images in order to simultaneously reduce redundancy and suppress speckle at a fixed quota of bits. The main idea of the method is to fuse three rules for threshold processing the wavelet coefficients of the scans, uniform and non-uniform quantizers, and bit quota distributions over subbands of the scan FWT based on the proposed cost function. The simulation results have shown that at the encoding rate of up to 1 bit/pixel, the quantity of artefacts were decreased up to 5–7 % of the original quantity under a signal-to-speckle ratio more than 16 dB, and the structural similarity index (SSIM) increased to 0.94–0.97 for defects of rectangular, triangular and oval shapes. The paper also presents the results proving the effectiveness of the proposed method in comparison with some variants of the solution according to the scheme “pre-filtering + codec”.

1. Introduction
At the moment, the phased array transducers are widely used in non-destructive diagnostics of complex metal products due to the transducers’ capability to sweep the ultrasound beam through an angular range for a specific focal depth. This procedure known as sector scanning assumes data transmission and reception at different incidence angles. Using an image reconstruction algorithm such as synthetic aperture focusing technique there is a variety of options to display the results in the form of 3D images of the inspected product: A-scan, B-scan, C-scan, S-scan etc. Combining different scans, it becomes possible to obtain 3D ultrasound image of the product that allows conducting in-depth analysis of defects using personal computer (PC) options [1–3].

Unfortunately, two main problems arise: a large volume of 3D ultrasound images and speckle noise, which is a consequence of random phase deviations of the reflected ultrasonic beam [3–5]. The fully developed speckle introduces artifacts that can be interpreted as defects, and vice versa, the speckle can mask a defect by blurring its brightness and boundaries [5].

In this paper, we consider the multiplicative model of speckle [5–7]

\[ Y = X \cdot Z, \]

where \( Y \) is the observed scan, \( X \) is the unknown free-of-speckle scan, \( Z \) is the multiplicative noise with unity mean and different probability density functions (pdf). There is no any dependency between
random variables $X$ and $Z$ (it is theoretically and practically confirmed in the case of so-called fully developed speckle).

There are a lot of methods and algorithms to compress 3D images corrupted by speckle. In this paper, we suggest classifying the received compression methods by three groups. The first group of methods presented by the papers [8, 9] is based on using the Rissanen’s principle of minimal description length (MDL). Here, an end-user receives maximal quality of the decompressed image under the computed (not a given) compression ratio (CR) which is determined by the ratio of the number of bits of the input image to the number of its bits after processing. The second group of methods presented by the papers [10, 11] exploits the idea of existing an “optimal operation point” (OOP) which is such of CR or bit rate (BR) measured in bit per pixel (bpp) when the maximal peak-to-signal noise ratio (PSNR) is achieved. Existence of the OOP is explained by existence of a quantization interval located near zero (dead zone), the size of which is determined by the current BR. However, we can reach the desired quality of decompressed noisy scans due to the OOP of the codec but the obtained BR will not coincide with the given BR in many cases. The third group of methods presented by the papers [12, 13] including our works [14] is oriented to form a decompressed image with high quality under the given BR (or, the given quota of bits). Based mostly on wavelet transformation, these methods are characterized by huge computational and timing expenditures. Nevertheless, questions of the best choosing a thresholding rule, type and parameters of quantization and bit allocations were solved separately. It led to getting suboptimal estimators of threshold values and quantization intervals. In the paper [15] we firstly did an attempt to seek the optimal wavelet threshold values and the intervals of quantization within subbands of the fast wavelet transform (FWT) using one common criterion. The threshold values were found on the basis of Stein’s unbiased risk estimators (SURE) [16] and for the soft thresholding rule. In the paper [15] we considered the uniform quantizers only to encode the wavelet coefficients with the found individual quantization functions for each subband of the FWT. In this paper, we apply the method obtained in [15] to the problems of ultrasound diagnostics and expand the one with the hard- and Vidavievo thresholding rules [17] and different adaptive (uniform and non-uniform) quantizers; therefore, we encode planar scans of 3D ultrasound image by fusion of the obtained solutions within one framework of wavelet-based subband noisy image compression.

2. Problem definition
The FWT gives the $Q$-level image wavelet decomposition using one-dimensional filter bank in the two-dimensional case [17, 18], so wavelet decomposition $W$ of a noisy image (1) can be written as

$$W_y = W^{(q)}_X Z = W^{(q)}_X + W^{(q)}_X(Z-1) = W_X + W_z,$$

(2)

where $W_X = W^{(q)}_X$, $W_z = W^{(q)}_X(Zx)$. are centered and uncorrelated random processes. Therefore, the multiplicative model (1) is the additive one (2) in wavelet domain. We can write for any wavelet coefficient at the subband $j$ containing $I^{(j)}$ coefficients [17]

$$w^{(j)}_{X_i} = w^{(j)}_{X_i} + w^{(j)}_{Z_i}, \quad i = 1,\ldots, I^{(j)}, \quad j = 1,\ldots, J,$$

(3)

where $J$ is the number of subbands and $J = 3Q+1$ for the FWT. Let the noise in wavelet coefficients be represented as $w^{(j)}_{Z_i} = \sigma^{I^{(j)}}_{w} \cdot \epsilon_i$, where $|\epsilon_i| \leq 1, \quad \forall i \in I^{(j)}, \quad \sigma^{I^{(j)}}_{w} > 0$, and here we consider conditionally that $\epsilon_i$ is an additive Gaussian noise with zero mean and $\sigma^{I^{(j)}}_{w}$ stands for its variance, $j = 1, \ldots, J$. Therefore, the estimator of any wavelet coefficient $\hat{w}^{(j)}_{X_i}$ can be represented through a regression model along with quantization

$$\hat{w}^{(j)}_{X_i} = \text{round} \left( (w^{(j)}_{X_i} + \varphi(w^{(j)}_{X_i}, \sigma^{I^{(j)}}_{w}))/\Delta^{I^{(j)}}, \Delta^{I^{(j)}}, \right),$$

(4)
where $\Delta^j$ is an interval of quantization at the subband $j$, $\varphi(w_{ij}^j, \tau^j)$ is a function of thresholding with the threshold value $\tau^j$, $j = 1, \ldots, J$. The model (4) can be modified to the customized form

$$\hat{w}_{ij}^j = w_{ij}^j + \varphi(w_{ij}^j, \tau^j) + \sigma^j_{uq}, \quad \forall i \in I^j, \ j = 1, \ldots, J,$$

where $\sigma^j_{uq}$ is the error of uniform quantization of the wavelet coefficient with number $i$ from the subband with number $j$, $j = 1, \ldots, J$. The quality criterion to obtain the optimal estimators of the wavelet coefficients is similar to the Stein’s unbiased risk estimation (SURE) criterion [16]

$$D_j(\tau^j, \Delta^j) = \frac{1}{I^j} \sum_{i=1}^{I^j} (\hat{w}_{ij}^j - w_{ij}^j)^2 \to \min,$$

where two unknown parameters $\tau^j$ and $\Delta^j$ (what makes a difference from the original SURE criterion) determine the quality of image enhancement after compression. Let $a_j$ be the relative subband size; $B = (b_1, \ldots, b_J)$ contains the bit lengths allocated to the subbands; $\sigma^2_j$ be the subband variance calculated for all of wavelet coefficients, and

$$\sigma^2_j = \begin{cases} \frac{1}{I^j} \sum_{i=1}^{I^j} w_{ij}^j, & \hat{w}_{ij}^j = 1 \frac{1}{I^j} \sum_{i=1}^{I^j} \hat{w}_{ij}^j = 0, \quad \text{for details;} \\ \frac{1}{I^j} \sum_{i=1}^{I^j} (w_{ij}^j - \hat{w}_{ij}^j)^2, & \hat{w}_{ij}^j \neq 0, \quad \text{for approximation;} \end{cases}$$

$\hat{\sigma}^2_j$ be the subband variance of the thresholded wavelet coefficients; the dynamic range of the wavelet coefficients is changed after thresholding; and

$$\hat{\sigma}^2_j = \begin{cases} \frac{1}{M^j} \sum_{i=1}^{I^j} (\varphi(w_{ij}^j, \tau^j))^2, & \text{for details;} \\ \sigma^2_j, & \text{for approximation, if the one is kept up,} \end{cases}$$

Here $M$ is the number of the significant wavelet coefficients.

The uniform quantization error at the subband $j$ depends on bit allocation on subbands and the variance of thresholded wavelet coefficients but also depends on BR [12, 13]. Because the quantization interval $\Delta^j$ under uniform quantization comprises the quantization error as $\Delta^j = 12\sigma^2_{uq}$ [4] then the threshold value $\tau^j$ can be put equal to dead zone of a coder $\tau^j = 0 \Delta^j$ and it yields

$$\tau^j = 12 \theta \sigma^2_{uq}, \quad j = 1, \ldots, J,$$

where $\theta$ is the coefficient controlling the relation between the dead zone and the interval of quantization.

In this paper, we expand our results from [10] considering non-uniform (more exactly, quasi-uniform) quantization, too. Our quasi-uniform quantizer is being built on the base of non-linear approximation of wavelet coefficients in each subband $A(i)$, $i \in I^j$, $j = 1, \ldots, J$. (figure 1). As it is seen from the figure 1, values of numbers $t_i$, $l = 0,1, \ldots, L/2$, allow us to calculate the quantity of wavelet-coefficients $K = t_{j+1} - t_{j-1}$, belonging to the corresponding quantization intervals. The task of seeking the levels of quantization $Y_i$, $l = 0,1, \ldots, L$, is solved by minimizing the mean square error what yields
\[ \gamma_i = \frac{1}{K_i} \sum_{j=1}^{K_i} A(i). \]  \hspace{1cm} (10)

Below we consider the bit allocation problem for uniform quantizers only. The obtained results have been modified for non-uniform quantizers in similar way.

Summarizing all above and taking into consideration the results obtained by Stein for SURE [16], we get for distortions

\[ D_j(\tau^{(j)}, \Delta^{(j)}) = \hat{\sigma}_{w_j}^2 + \frac{1}{I^{(j)}} \sum_{i=1}^{I^{(j)}} \hat{\phi}(w_{k_i}^{(j)}, b_j) + \frac{2}{I^{(j)}} \sum_{i=1}^{I^{(j)}} \hat{\phi}(w_{k_i}^{(j)}, b_j) + \sigma_{w_j}^2(b_j) = D_j(b_j), \]  \hspace{1cm} (11)

where \( \hat{\sigma}_{w_j}^2 \) is the estimator of noise variance in the wavelet coefficients and \( \hat{\phi}(w_{k_i}^{(j)}, b_j) \) is the modified (residual) thresholding rule (function) after some algebra, \( j = 1, \ldots, J \).

After thresholding and quantization under the given parameter \( \theta \), the residual distortions at the subband \( j \) depend on the number of bits \( b_j \) of the given subband. If the bit rate (or the quota of available bits) \( R(B) = R_C \) is given then the problem definition of an optimal bit allocation along with de-noising is represented as follows

\[ D(B) = \sum_{j=1}^{J} D_j(b_j) \rightarrow \min_{B}, \]  \hspace{1cm} (12)

under constraints

\[ R(B) = \sum_{j=1}^{J} \alpha_j b_j. \]

The task of conditional optimization (12) is being resolved through Lagrange function as a task of unconditional optimization

\[ L(B) = [D(B) + \lambda R(B)] \rightarrow \min_{B}, \]  \hspace{1cm} (13)

where \( \lambda \) is an undetermined Lagrange multiplier [15].

Therefore, the task of joint de-noising and quantization of wavelet coefficients is described by the expression (13) in terms of the rate–distortion theory. However, the solution depends on the thresholding rule \( \phi(w_{k_i}^{(j)}, I^{(j)}) \), \( i = 1, \ldots, I^{(j)}, \) \( j = 1, \ldots, J \).

\[ \text{Figure 1. Non-linear approximation and allocations of intervals and levels of the quasi-uniform quantizer.} \]
3. Bit Allocations

3.1. Soft thresholding
Let the residual function of the soft thresholding rule be represented as
\[
\phi(w^{(j)}_i, \tau^{(j)}) = -\text{sign}(w^{(j)}_i)\tau^{(j)}\Theta(w^{(j)}_i \geq \tau^{(j)}) - w^{(j)}_i \Theta(w^{(j)}_i \geq \tau^{(j)}),
\]
where \(\Theta(\ldots)\) is the symbol of comparing to the threshold \(\tau^{(j)}\). Using the results obtained by Stein in [16], the expression (11) for distortions and letting symmetry of pdf for wavelet coefficients
\[
w^{(j)}_i \Theta(w^{(j)}_i \geq \tau^{(j)}) = w^{(j)}_i - w^{(j)}_i \Theta(w^{(j)}_i \geq \tau^{(j)}),
\]
we have [15]
\[
D_j(b_j) = (\sigma_j^2 - \epsilon_j \sigma_j^2 + \tilde{\sigma}_w^{(j)2} + \epsilon_j \tau^{(j)2}) + \epsilon_j \sum_{j=1}^{J} \alpha_j 2^{-2b_j} \sigma_j^2 + \lambda \sum_{j=1}^{J} \alpha_j b_j \rightarrow \min_{b_j}
\]
Because the two last members of the expression (16) depend on \(b_j\) then after derivation of them we obtain
\[
b_j = \frac{1}{2} \log_2 \left( \frac{2(2\ln 2)\epsilon_j \sigma_j^2}{\lambda} \right).
\]
After substitution (17) in constraints \(R(B)\) we can find the unknown Lagrange multiplier
\[
\lambda = 2 \left[ \sum_{j=1}^{J} \epsilon_j \log_2 ((2\ln 2)\epsilon_j \sigma_j^2 + 2R_c) \right].
\]
Finally, we have for \(b_j\)
\[
b_j = \frac{1}{2} \left[ \log_2 (2(2\ln 2)\epsilon_j \sigma_j^2) - \sum_{j=1}^{J} \alpha_j \log_2 ((2\ln 2)\epsilon_j \sigma_j^2 + 2R_c) \right].
\]
After substitution (18) in (16) we obtain that the total distortions \(D(B) = \sum_{j=1}^{J} D_j(b_j)\) depend on the variances of the thresholded wavelet coefficients \(\tilde{\sigma}_j^2\) [15].

3.2. Vidacovic thresholding
Analogically, the optimal bit allocation is determined for the case of Vidacovic thresholding rule [17] where the residual function of this rule looks as follows
\[
\phi(w^{(j)}_i, \tau^{(j)}) = \left( \text{sign}(w^{(j)}_i) \sqrt{w^{(j)2}_i - \tau^{(j)2}} - w^{(j)}_i \right) \Theta(w^{(j)}_i \geq \tau^{(j)}) - w^{(j)}_i \Theta(w^{(j)}_i \geq \tau^{(j)}).
\]
After substitution the residual function (19) in the expression (11) for the risk function, we can find its second and third members. Then, the expression determining distortions in each subband has the next representation

\[
D_j(b_j) = (\sigma_j^2 - \varepsilon \sigma_j^{(1/2)}) + \hat{\sigma}_w^{(1/2)} (2 \varepsilon - 1) + \varepsilon (\hat{\sigma}_j^{(2)} + \sigma_m^{(1/2)}) - \frac{2 \varepsilon}{M(1)} \sum_{i=m}^{M-1} \frac{1}{2} \ln \left( \sqrt{w_{i,j}^2 - r_{i,j}^2} + \frac{\sigma_m^{(1/2)} w_{i,j}^{(1/2)}}{w_{i,j}^{(1/2)} - r_{i,j}^2} \right) - \frac{2 \varepsilon \hat{\sigma}_w^{(1/2)} (2 \varepsilon - 1)}{M(1)} \sum_{i=m}^{M-1} \frac{1}{2} \ln \left( \sqrt{w_{i,j}^2 - r_{i,j}^2} + \frac{\sigma_m^{(1/2)} w_{i,j}^{(1/2)}}{w_{i,j}^{(1/2)} - r_{i,j}^2} \right). \tag{20}
\]

The Lagrange function is modified

\[
L(B) = \sum_{j=1}^{J} \left( \sigma_j^2 - \varepsilon \sigma_j^{(2)} \right) + \hat{\sigma}_w^{(1/2)} (2 \varepsilon - 1) + \varepsilon (\hat{\sigma}_j^{(2)} + \sigma_m^{(1/2)}) - \frac{2 \varepsilon}{M(1)} \sum_{i=m}^{M-1} \frac{1}{2} \ln \left( \sqrt{w_{i,j}^2 - r_{i,j}^2} + \frac{\sigma_m^{(1/2)} w_{i,j}^{(1/2)}}{w_{i,j}^{(1/2)} - r_{i,j}^2} \right) \\
+ \varepsilon \sum_{j=1}^{J} \left( \sigma_j^2 - \varepsilon \sigma_j^{(2)} \right) + \hat{\sigma}_w^{(1/2)} (2 \varepsilon - 1) + \varepsilon (\hat{\sigma}_j^{(2)} + \sigma_m^{(1/2)}) \rightarrow \min. \tag{21}
\]

To calculate the needed bit allocation, we use the same expression (16), because the two last members depend on \(b_j\) as in the case of soft thresholding rule.

3.3. Hard thresholding

Some standardized wavelet-based codec software allows to an end-user to regulate the dead zone. In this case, it is reasonable to tune the size of the dead zone using the hard thresholding rule. Then, the second member in (5) has the look as follows

\[
\Phi(w_{i,j}^{(1/2)}, \tau^{(1/2)}) = -w_{i,j}^{(1/2)} \text{Thr} \left( |w_{i,j}^{(1/2)}| < \tau^{(1/2)} \right). \tag{22}
\]

Taking into consideration the symmetry of the pdf for wavelet coefficients in subbands, distortions in each subband can be calculated as follows

\[
D_j(b_j) = (\sigma_j^2 - \varepsilon \sigma_j^{(2)}) + \hat{\sigma}_w^{(1/2)} (2 \varepsilon - 1) + \varepsilon \sigma_m^{(1/2)} . \tag{23}
\]

The Lagrange function has the next presentation

\[
L(B) = \sum_{j=1}^{J} \left( \sigma_j^2 - \varepsilon \sigma_j^{(2)} \right) + \hat{\sigma}_w^{(1/2)} (2 \varepsilon - 1) + \varepsilon \sigma_m^{(1/2)} + \varepsilon \sum_{j=1}^{J} \left( \sigma_j^2 - \varepsilon \sigma_j^{(2)} \right) + \hat{\sigma}_w^{(1/2)} (2 \varepsilon - 1) + \varepsilon \sigma_m^{(1/2)} \rightarrow \min , \tag{24}
\]

and search of the optimal threshold values which are absent in the obvious view in (24) is also conducted by using the selected variances \( \hat{\sigma}_j^{2} \), \( j = 1, \ldots, J \), of the significant wavelet coefficients.

3.4. Cost function

At this moment, three thresholding rules and two quantizers give us six possible combinations \( K = \{c_1, \ldots, c_6\} \) to choose the best combination from ones. Using results from [18] related to image restoration on the base of coherent structures, we consider the cost function which is similar to the Schur cost function [18]

\[
C(c) = \sum_{j=1}^{J} \Phi \left( \frac{\hat{\sigma}_j^{(1/2)} - \varepsilon \sigma_j^{(1/2)}}{\sigma_j^{(1/2)}}, \tau_j^{(1/2)} \right), \quad t = 1, \ldots, 6. \tag{25}
\]

Then, the best combination \( c^* \) minimizes the cost of processing

\[
C(c^*) = \min_i C(c_i) \tag{26}
\]

The choice of the best combination depends on a form of the concave function \( \Phi(u) \). To the current moment, we used the entropy function \( \Phi(u) = -u \ln u \), \( u \geq 0 \). The decision \( c^* \) needs to be
checked with respect to the constraints \( R(B) \). If these constraints are crossed then it is necessary to reconsider it with other “close” variant of bit allocation.

4. Simulation results
The proposed method has been checked by using the library of real ultrasound free-of-speckle test A-scans and B-scans distorting them by multiplicative noises with different probability density functions. It was discovered that the suggested method gives promising results under low bit rates of compression (less than 1 bit per pixel) in the sense of both objective (e.g., PSNR) and subjective (e.g., the structural similarity index, SSIM) criteria. The comparison has been done with different combinations of preliminary filtering and some wavelet-based coders like spatial partitioning in hierarchical trees (SPIHT) [19]. Our method does surely win up 14% for each criterion.

![Graphs](image1.png)

**Figure 2.** The threshold functions for high-frequency subbands; wavelet CDF 9.7, \( BR = 0.25 \) bpp (H, V, D are horizontal, vertical and diagonal subbands correspondingly; HTR, STR, VTR are hard, soft and Vidacovic thresholding correspondingly; UQ and non-UQ stand for uniform and non-uniform quantizing correspondingly).

Figure 1 demonstrates the results of fusing the thresholding rules and types of quantizing among subbands of the test gray-scaled (8 bpp) image sized 512×512 and distorted by multiplicative noise with exponential pdf and variance \( \sigma^2 = 25 \) for compression at 0.25 bpp. Figure 2 reflects statistical data
for PSNR in dependency on BR for the suggested method, the variant of soft-thresholding + uniform quantizing based on the solution from [16], and the combination of the Lee filter [6] + SPIHT.

In order to determine the effectiveness of the proposed method with respect to defect detection, there has been performed a special computer simulation in MATLAB. We modeled the parallelepiped as a sum of 8-bit gray scale A-scans forming 3D ultrasound image which contained defects (cavities) of different depths with rectangular, triangular and oval shapes. For each run of the simulation model (3780 runs in total), the position, shape, and size of the defects were selected using a random number generator (RNG). To simulate the speckle, another RNG was used which produced pseudo-random sequences of numbers distributed according to an exponential pdf with unity mean. The speckle variance $\sigma^2$ changed during the experiments.

Each scan of 3D ultrasound image has been processed independently. Based on the results of comparing the speckle-free A-scans containing defects and the processed ones we accumulated some statistics which reflected in the tables 1 and 2. As is seen from the table 1, at the encoding rate of up to 1 bpp, the quantity of artifacts that are stimulated by the action of speckle is decreased up to 5–7 % if a signal-to-speckle ratio is more than 16 dB. At the same time, the proposed method provides a reduction in redundancy almost twice (see the table 2).

**Figure 3.** The dependencies “PSNR – BR” for the suggested method (1), soft-thresholding+uniform quantizing (2), and Lee filter + SPIHT (3). Quantity of test images is equal to 56.

**Table 1.** Results of simulation modeling for detection and recognition of artifacts and defects.

| $\sigma^2$ | Changes of artifact quantities | Probability of type I error (loss of a defect) | Probability of type II error (an artifact is recognized as a defect) |
|-----------|--------------------------------|-----------------------------------------------|---------------------------------------------------------------|
|           | before processing, % | after processing, % | 0.002 | 0.003 |
| 10        | 5.65               | 0.11              | 0.002 | 0.003 |
| 20        | 14.55              | 3.63              | 0.017 | 0.08  |
| 30        | 36.27              | 10.58             | 0.076 | 0.27  |

**Table 2.** Statistical indicators of the proposed compression method.

| $\sigma^2$ | PSNR, dB (before processing) | PSNR, dB (after processing) | SSIM | CR |
|-----------|-----------------------------|-----------------------------|------|----|
|           |                             |                             |      |    |
| 10        | 21.37                       | 27.86                       | 0.984| ~27:1|
| 20        | 19.56                       | 24.67                       | 0.912| ~12:1|
| 30        | 15.88                       | 19.42                       | 0.873| ~2:1 |
5. Conclusion
The suggested method provides a more flexible framework for getting the strong de-noising effect during 3D ultrasound image compression with respect to well-known methods. Nevertheless, the questions of the cost function form and some non-stability when the constraints for the given quota of bits are crossed in some subbands are still under discussion. Future research will be concentrated to create procedures for quick and stable fusion of different adaptive quantizers and thresholding rules including the multi-parametrical ones.

The method can be implemented into the summation block which plays the role of interfacing between the phased array outputs and the PC [20]. In the case, this block should be expanded with FPGAs allowing all operations of compression to be realized in hardware independently of the PC environment. Fast hardware-network protocols such as Ethernet or the Serial Attachment (SATA) can be used to connect the PC to the data processing front end [2, 20].

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