I. INTRODUCTION

Creation of large multipartite entangled states is a fundamental scientific interest [1] and the enabling technology for quantum information and quantum networking [2]. Due to the large spread speed of photon, multi-photon entanglement attracts widespread attention. Up to now, the most widely used method of generating multi-photon entanglement is that prepare several pairs entangled photons based on the spontaneous parametric down-conversion in a nonlinear crystal [3], and use interferometer to combine them into the multi-photon entangled state, such as the Schrödinger cat state. Experimentally, six spatially separated entangled photons have been created in the experiment [4, 5]. However, in trapped ions, eight spatially separated single photons have been created [6, 7]. So far, up to eight entangled photons by transferring the multi-particle entangled state, such as cavity modes, the multi-quantum-dot entanglement in our scheme can be protected from the decoherence induced by the noise. Thus, it is possible to generate more than eight spatially separated entangled photons in the realistic experimental conditions.

Finally, input photons to swap the multi-photon entanglement with the multi-quantum-dot entanglement. Recently, direct mapping of the spin quantum state of Ce$^{3+}$ ion onto the polarization state of the emitted photon was completed, and the degree of spin polarization was higher than 99% [8]. Those experiment establishes the foundation for creating multi-photon entanglement.

The rest of this article is arranged as follows. In the sections II, the multi-quantum-dot entanglement is generated by superconductors. In the section III, utilize the cavity modes to protect the multi-quantum-dot entanglement state. The multi-photon entanglement is created by the input photons in the section IV. We draw up the conclusion in the section V.

II. MULTI-QUANTUM-DOT ENTANGLEMENT

To form the multi-quantum-dot entanglement, we consider following two kinds of interaction Hamiltonian between quantum dot and superconductor.

The first kind: two quantum dots $(a,b)$, each of which contains one electron and is connected to two superconducting leads $(L,R)$ by tunnel junctions [10], as shown in Fig. 1. The effective interaction Hamiltonian is given by $(\hbar = c = 1)$:

$$H_{int}^1 = J_1 \vec{\sigma}_a \cdot \vec{\sigma}_b,$$

(1)

where $J_1$ is the coupling constant, and $\vec{\sigma}_a$ is the Pauli operator of electronic spin in the quantum dot a.

The second kind: two quantum dots $(a,b)$ are connected by a superconductor [11], as shown in Fig. 1. Via crossed Andreev reflection, two quantum dots form a strong long-range interaction. The effective interaction
result, the above state is mapped into

\|\Psi\rangle_{0011} = \frac{1}{\sqrt{2}} \left( |0\rangle + |1\rangle \right) \otimes \left( |0\rangle + |1\rangle \right).

(3)

Then, use a \pi/2 pulse to act on one quantum dot. As a result, the above state is mapped into \|\frac{00}{\sqrt{2}} + \frac{11}{\sqrt{2}}\rangle. The two quantum dots are in the maximum entangled state

\|\Psi\rangle = \frac{1}{2} \left( |0\rangle (|0\rangle + e^{i\pi/4} |1\rangle) + |1\rangle (|0\rangle + e^{-i\pi/4} |1\rangle) \right).

(4)

For arriving at the GHZ state, firstly, use the \pi pulse to get the state

\|\Psi\rangle_{1234} = \frac{1}{2} \left( |0\rangle |0\rangle (|0\rangle + |1\rangle) |1\rangle + |1\rangle |0\rangle (|0\rangle - |1\rangle) |1\rangle \right).

(5)

Secondly, utilize the first interaction Hamiltonian \(H_{int}^1\) between two quantum dots 3 and 4. Finally, we get the four qubits GHZ state. The corresponding interaction time during the first interaction Hamiltonian is given by \(t = \frac{\pi}{8J_1}\), and \(\Delta = \frac{\pi}{2}\), which is the necessary condition for generating the GHZ state.

To get the GHZ state for six quantum dots, as the above way combine four quantum dots entanglement and two quantum dots entanglement. By that analogy, one can get the multi-quantum-dots entanglement by just controlling the interaction time, not requiring the complex technique. So it is operative in the experiment.

The total time that is required to create \(n\) quantum dots entanglement is given by (omitting the time of pulse)

\[T_{\text{total}} = \frac{n+1}{2}, \frac{\pi}{4J_2} + \frac{n-1}{2}, \frac{\pi}{8J_1},\]

(6)

where we define that \([n/2] = (n-1)/2\) when \(n\) is even, otherwise \([n/2] = n/2\). For strong coupling, \(J_1 \approx 10^8\) Hz [13], the total time \(T_{\text{total}} \approx n \times 10^{-8}\) s.

III. PROTECTION OF THE COHERENCE

With the increasing of entangled particles, the decoherence effects become more and more strong due to new decoherence channels. The harmonic oscillators can be chosen to protect the coherence, because it is an infinite system and provides a vast Hilbert space without adding new dechoherence channels. For the cavity mode, the main decoherence channel comes from the photon damping. It can be corrected by a quantum non demolition parity measurement [12].

For protecting the coherence, it is necessary to create the entangled state between cavity modes and quantum dots by using the unitary operation,

\[U_{\text{encode}}(|n\rangle + |0\rangle) \otimes |0\rangle_{\text{cavity}} = |0\rangle \otimes \left( |0 C_\alpha^+ \rangle + |1 C_\alpha^+ \rangle \right) + O(e^{-|\alpha|^2}),\]

(7)

where \(|C_\alpha^+ \rangle = \mathcal{N}(|\alpha\rangle \pm i |\alpha\rangle\), and \(|C_{\alpha}^- \rangle = \mathcal{N}(|\alpha\rangle \pm i |\alpha\rangle\). The normalizing factor \(\mathcal{N} \approx 1/\sqrt{2}\), and \(|\alpha\rangle\) denotes a coherent state of complex amplitude \(\alpha\). The error term \(O(e^{-|\alpha|^2})\) is generated by the fact that the two states \(|C_\alpha^+ \rangle\) and \(|C_{\alpha}^- \rangle\) are not exactly orthogonal. For large photon number \(|\alpha|^2\), the error term is approximate to 0. Next, utilizing two kinds of interaction Hamiltonian as the way in the above section to combine the state

\[\pi/2\] pulse to move away the superconductor. (2) Two quantum dots are connected by a superconductor.

Hamiltonian is described by

\[H_{int}^2 = J_2 \sigma_a^Z \otimes \sigma_b^Z.\]

(2)

It deserves to note that the quantum dots are defined in a semiconducting nanowire (e.g., InSb or InAs), which is an experimentally potential system for spin qubits [17, 18] and can form good interfaces with a superconductor [19, 20].

Use the second interaction to create two quantum dots entanglement. Let the initial state of quantum dots to be \(|\langle 00 | \rangle\otimes |\langle 00 | \rangle\rangle \). Control the interaction time (for example, move away the superconductor) to be \(t = \pi/4J_2\). The two quantum dots are in the maximum entangled state

\[\langle 00 | \rangle \otimes |\langle 00 | \rangle\rangle \]

(5)

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\(\pi/2\) pulse to move away the superconductor. (2) Two quantum dots are connected by a superconductor. (3) Two quantum dots are connected by a superconductor.
\[ |0C^+_a \rangle + |1C^+_a \rangle \over \sqrt{2} \] and \[ |00C^+_a \rangle + |11C^+_a \rangle \over \sqrt{2} \]. Use the unitary operations to arrive at \[ |0C^+_a C^+_b C^+_c \rangle + |1C^+_a C^+_b C^+_c \rangle \]. In this way, the decoherence channels mainly come from the cavity modes. 

This process realizes the swap: \(|0⟩|1⟩\rightarrow |1⟩|0⟩\).

Input a single photon to excite the state |0⟩. The initial state vector of input photon is given by
\[ |Ψ⟩_{in} = \int dk f(k)a^\dagger_k |0⟩. \]

The output state can be written as
\[ |Ψ⟩_{out} = \int dk (g_1(k)a^\dagger_k |0⟩_{ph}|0⟩ + g_2(k)a^\dagger_k |0⟩_{ph}|1⟩ + g_3 |0⟩_{ph}|ρ⟩). \]

Utilize the Schrödinger equation to obtain
\[ \dot{g}_1(k) = -\sqrt{\frac{Γ_1}{2π}}g_3 e^{-iδ_k}, \]
\[ \dot{g}_2(k) = -\sqrt{\frac{Γ_2}{2π}}g_3 e^{-iδ'_k}, \]
\[ \dot{g}_3 = \int dk \left( \sqrt{\frac{Γ_1}{2π}}g_1 e^{iδ_k} + \sqrt{\frac{Γ_2}{2π}}g_2 e^{iδ'_k} \right), \]
in which, \(δ_k = w_1 - k\) and \(δ'_k = w_1 - w_2 - k\).
If the input photon has a Gaussian mode with bandwidth $d$, the mode function can be described as

$$f(k) = \left(\frac{2}{\pi d^2}\right)^{1/4} \exp[-\frac{(k - w_1)^2}{d^2}].$$

(15)

As a result, we can solve the Eq.(14) and obtain

$$g_2(k) = \int_0^t dt' \int_0^t dt'' - \frac{\sqrt{\Gamma_1 \Gamma_2 d}}{(2\pi)^{3/4}} e^{-it\delta_k - \frac{d^2 t'^2}{4} + \frac{\Gamma_1 + \Gamma_2}{2} t''}.$$  

(16)

So the probability for generating the photon with central frequency $(w_1 - w_2)$ is given by

$$P(t) = \int dk |g_2(k)|^2 = \int_0^t dt' \frac{\Gamma_1 \Gamma_2 d}{(2\pi)^{3/2}} \left| \int_0^t dt'' e^{-\frac{d^2 t'^2}{4} + \frac{\Gamma_1 + \Gamma_2}{2} t''} \right|^2.$$  

(17)

From Fig. 4, we can find that the maximum probability $P$ is close to 1. And one can input other photons to ensure the maximum probability $P$ to be 1. It means that swapping the multi-quantum-dot entanglement with multi-photon can be with the high efficiency by this process.

Next, we try to create the polarization entanglement of multi-photon due to its advantage in the experiment. As shown in Fig. 5, the input photon state $|10\rangle + |01\rangle$ can be converted into a polarization state $|HV\rangle + |VH\rangle$ (photon 1). It can be treated as a single photon with polarization state $|H\rangle + |V\rangle$. The small frequency difference between two polarization directions can be removed by the ultrafast downconversion technique [12]. In the way, the multi-photon entanglement $|10\rangle \otimes 2^n + |01\rangle \otimes 2^n$ can be converted to the polarization state $|H\rangle \otimes n + |V\rangle \otimes n$.

FIG. 5: The input photons in the initial state $|10\rangle$ transmit through two β-barium borate (BBO) crystals, generating the state $|HV\rangle + |VH\rangle$. Then, pass through four polarization beam splitters (PBS) to generate the polarization entanglement photon 1, which is ensured by the detection. Namely, if the detector finds a photon, then it means that the photon 1 is created.

V. CONCLUSION

We have proposed a systematic scheme to create the multi-quantum-dot entanglement, and generate the multi-photon entanglement. Controlling the coupling between superconductor and quantum dots can effectively form the interaction for generating the entanglement, which is operable in the experiment. For the robust against the decoherence noise, the cavity modes are used to store the entanglement temporally. A error correction protocol can restore the coherence from the photons loss. It will assist the more qubits entanglement. Finally, use Gaussian photons to map the multi-quantum-dot entanglement into the multi-photon entanglement. And we find that the transform efficiency can be high with a few input photons.

The scheme will help to create more photons entanglement beyond the present 8 photons entanglement, which is useful in the quantum computation [24], simulation [25, 26], and communication [27]. This scheme can be performed with present-day technology. It stimulates the further study about the continuous-variable entanglement of photons, for example in Hilbert space of the angular momentum of photons [25] and multi-quantum-dot entanglement will also play an important role in the quantum computation.

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