Prospects of mixing and CP violation in the D system at SuperB

Paolo Branchini
INFN Roma Tre
Via Della Vasca Navale 84
00146 Rome, ITALY

1 Abstract

The SuperB experiment hosted by the Cabibbo laboratory will have the possibility to study electron positron collision both at the \( \Upsilon(4S) \) center of mass and at the \( \psi(3770) \). Therefore the potential physics reach of the experiment will be greatly enriched. In this paper I present the implications such a variety of measurements can have on various new physics scenarios.

2 Introduction

The absence of flavor changing neutral currents (FCNCs) was considered as an indication of the existence of a fourth quark. The GIM mechanism was proposed in this framework and within this framework explained the suppression of some transitions \([1]\). FCNCs still represent a very important issue in flavor physics when looking for new physics. Another interesting aspect is that the Standard Model (SM) predicts small mixing and CP violation for charm mesons. While mixing has already been observed for \( D^0 \) mesons, CP violation in charm has still to be discovered and a time-dependent analysis may provide a tool to test the Cabibbo-Kobayashi-Maskawa mechanism \([2]\) and the SM itself through the measurement of the angle \( \beta_{c,\text{eff}} \) in the charm unitarity triangle. This work describes the potential reach of SuperB when studying rare decays, mixing and CP violation in charm mesons. It is useful to stress here the uniqueness of charm mesons in the understanding of the SM. \( D^0 \) mesons are the only mesons made from two up-type quarks \( (c \bar{u}) \). It is therefore necessary to study this system to improve our knowledge on the flavor changing structure of the SM. Moreover the interpretation we have nowadays of the CP violating mechanism in terms of the CKM matrix can be (dis)proved in the up-sector.
3 Notations for flavor mixing and CPV

Flavor mixing occurs when the Hamiltonian eigenstates $D_1, D_2$ differ from the flavor eigenstates $D^0, \bar{D}^0$. A neutral $D$ meson produced at time $t = 0$ in a definite flavor state $D^0$ will then evolve and oscillate into a state of opposite flavor $\bar{D}^0$ after a certain time. If we describe the time evolution of the flavor eigenstates in terms of a 2x2 effective Hamiltonian, $H = M - \frac{i}{2} \Gamma$, then, assuming $CPT$ is conserved, the mass eigenstates can be expressed in terms of the flavor eigenstates by:

$$|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$$ (1)

$$|D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$$ (2)

with the normalization $|q|^2 + |p|^2 = 1$ and

$$q = \frac{M_{12} - i/2\Gamma_{12}}{M_{12} - i/2\Gamma_{12}}$$ (3)

Assuming a phase convention such that $CP|D^0\rangle = -|\bar{D}^0\rangle$ and $CP|\bar{D}^0\rangle = -|D^0\rangle$ then, if CP is conserved, we have that $q = p = 1/\sqrt{2}$ and the mass eigenstates coincide with the CP eigenstates: $|D_1\rangle = |D_{CP-}\rangle$ ($CP$ - odd) and $|D_2\rangle = |D_{CP+}\rangle$ ($CP$ - even).

The mixing parameters $x, y$ can be expressed in terms of the difference of masses $(m_{1,2})$ and widths $(\Gamma_{1,2})$ of the Hamiltonian eigenstates,

$$x = \frac{m_2 - m_1}{\Gamma}; y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$$ (4)

where $\Gamma = (\Gamma_1 + \Gamma_2)/2$. $CP$ violation can be of three types:

1. $CPV$ in decay or direct $CPV$: this occurs when the decay amplitudes for $CP$ conjugate processes are different in modulus. If $\langle |H|D^0\rangle = A_f, \langle \bar{f}|H|\bar{D}^0\rangle = \bar{A}_{\bar{f}}$ are the $D^0$ and $\bar{D}^0$ decay amplitudes into the final $f$ and $CP$ conjugate $\bar{f}$, then

$$\frac{\bar{A}_{\bar{f}}}{A_f} \neq 1 \rightarrow CPV$$ (5)

2. $CPV$ in mixing or indirect $CPV$: it occurs when the Hamiltonian eigenstates do not coincide with the CP eigenstates. That is:

$$\frac{|q|}{p} \neq 1 \rightarrow CPV.$$ (6)

3. $CPV$ in the interference of mixing and decay: for neutral $D$ mesons there is a
third possibility to observe CP violation even when CP is conserved in mixing and also in decay. In this case, CP violation arises when, in a process with final state \( f \) that can be reached by neutral D mesons of both flavors (i.e. \( D^0 \) and \( \bar{D}^0 \)), there is a relative weak phase difference between the mixing and the decay amplitudes. The quantity of interest that is independent of phase conventions, and physically meaningful, is

\[
\lambda_f = \frac{q A_f}{\bar{p} A_f} \equiv \frac{|q|}{|\bar{p}|} |A_f| e^{i(\delta_f + \phi_f)}
\]

where \( \delta_f \) and \( \phi_f \) are the CP-conserving and CP-violating phases respectively. If CP is conserved in mixing and in decay, the signature of CP violation in the interference of mixing and decay is thus

\[
\sin(\phi_f) \neq 0 \rightarrow \text{CPV}.
\]

For CP eigenstates, CPV in either mixing or decay is indicated by

\[
|\lambda_f| \neq 1,
\]

while CPV in the interference of mixing and decay corresponds to

\[
\text{Im}(\lambda_f) \neq 1.
\]

If there is no weak phase in the decay amplitudes then \( \arg(q/p) = \phi \) and it is independent of the final state \( f \).

4 Experimental observables

Time dependent analyses provide the most precise tests on the mixing parameters. An excellent description of this approach is given in [3]. In this paper I'll quote the results. First of all when allowing for CPV to take place 10 parameters can be extracted from the \( \chi^2 \) fit: \( x, y, |q/p|, \phi, \delta, \delta_{k\pi\pi}, R_D, A_D, A_\pi, A_K \). The parameters \( \delta, \delta_{k\pi\pi} \) are the relative strong phases, \( R_D \) is the ratio \( \Gamma(D^0 \rightarrow K^+\pi^-)/\Gamma(\bar{D}^0 \rightarrow K^-\pi^+) \) and \( A_D, A_\pi, A_K \) are the direct CP-violation asymmetries for the \( \Gamma(D^0 \rightarrow K^+\pi^-), \Gamma(D^0 \rightarrow \pi^+\pi^-) \) and \( \Gamma(D^0 \rightarrow K^+K^-) \) modes. The relationships between these parameters and the measured observables are given below.

- **Semileptonic decays**: they search for mixing by reconstructing the “wrong-sign” (WS) decay chain, \( D^{*+} \rightarrow D^0 \pi^+, D^0 \rightarrow \bar{D}^0, \bar{D}^0 \rightarrow K^{(*)+}e^−\bar{\nu}_e \). In contrast to hadronic decays, the WS charge combinations can occur only through mixing. The measurement of \( R_M \) is related to the mixing parameters as follows

\[
R_M = \frac{1}{2}(x^2 + y^2)
\]
and can be obtained directly as the ratio of WS to the right sign (RS) signal events. The RS events correspond to the non-mixed process.

- **Decays to CP eigenstates**: They measure the mixing parameters \( y_{CP} \) and the CPV parameter \( A_{\Gamma} \) with a lifetime ratio analysis of the transitions to the CP eigenstates and the transitions to the CP-mixed state \( D^0 \to K^-\pi^+ \).

\[
2y_{CP} = (|\frac{q}{p}| + |\frac{p}{q}|)ycos\phi - (|\frac{q}{p}| - |\frac{p}{q}|)xsin\phi \quad (12)
\]

\[
2A_{\Gamma} = (|\frac{q}{p}| - |\frac{p}{q}|)ycos\phi - (|\frac{q}{p}| + |\frac{p}{q}|)xsin\phi \quad (13)
\]

The parameter \( A_{\Gamma} \) is the decay-rate asymmetry in CP-even eigenstates, e.g. in \( D^0 \to K^+K^- \) and \( D^0 \to \pi^+\pi^- \), provides constraints on the mixing and CPV parameters according to the relations:

\[
\frac{\Gamma(D^0 \to K^+K^-) - \Gamma(D^0 \to K^-K^+)}{\Gamma(D^0 \to K^+K^-) + \Gamma(D^0 \to K^-K^+)} = A_K + \frac{<t>}{\tau_D} A_{\Gamma}^{\text{indirect}} \quad (14)
\]

\[
\frac{\Gamma(D^0 \to \pi^+\pi^-) - \Gamma(D^0 \to \pi^-\pi^+)}{\Gamma(D^0 \to \pi^+\pi^-) + \Gamma(D^0 \to \pi^-\pi^+)} = A_{\pi} + \frac{<t>}{\tau_D} A_{\Gamma}^{\text{indirect}} \quad (15)
\]

and \( A_{\Gamma}^{\text{indirect}} \) is given by:

\[
2A_{\Gamma}^{\text{indirect}} = (|\frac{q}{p}| + |\frac{p}{q}|)xsin\phi - (|\frac{q}{p}| - |\frac{p}{q}|)ycos\phi \quad (16)
\]

\(<t>\) is the average reconstructed \( D^0 \) proper time and \( \tau_D \) is the nominal \( D^0 \) lifetime.

- **Three-body \( D^0 \to K^0_s\pi^+\pi^- \) and \( D^0 \to K^0_sK^+K^- \) decays**: They measure directly the mixing and CPV parameters \( x, y, |q/p| \) and \( \phi \) with a time dependent Dalitz plot analysis.

- **Wrong-sign decays to hadronic non-CP eigenstates**: They measure the parameters \( x'^\pm, y'^\pm \) and \( R_D \) and \( A_D \) in a time-dependent analysis of the WS events selected through the decay chain \( D^{*+} \to D^0 \pi^+, D^0 \to k^+\pi^- \). The parameters are defined in the following way:

\[
x'^\pm = \frac{(1 \pm A_M)^{1/4}}{1 \mp A_M} \quad (x'cos\phi \pm y'sin\phi) \quad (17)
\]

\[
y'^\pm = \frac{(1 \pm A_M)^{1/4}}{1 \mp A_M} \quad (y'cos\phi \mp x'sin\phi) \quad (18)
\]
\[ \frac{\Gamma(D^0 \to K^+\pi^-) + \Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^0 \to K^-\pi^+) + \Gamma(D^0 \to K^+\pi^-)} = R_D \]  

(19)

\[ \frac{\Gamma(D^0 \to K^+\pi^-) - \Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^0 \to K^-\pi^+) + \Gamma(D^0 \to K^+\pi^-)} = A_D \]  

(20)

and

\[ x' = x\cos\delta + y\sin\delta \]  

(21)

\[ y' = -x\sin\delta + y\cos\delta \]  

(22)

\[ A_M = \frac{|q/p|^2 - |p/q|^2}{|q/p|^2 + |p/q|^2}; \delta = \arg\left(\frac{A(D^0 \to K^+\rho^-)}{A(D^0 \to K^+\pi^-)}\right) \]  

(23)

In these equations \( x' \pm \) and \( y' \pm \) identify the flavor of the \( D \) sample, i.e. \( D^0(+) \parallel \overline{D}^0(-) \). The mixing parameters \( x'' \) and \( y'' \),

\[ x'' = x\cos\delta_{K\pi\pi} + y\sin\delta_{K\pi\pi} \]  

(24)

\[ y'' = -x\sin\delta_{K\pi\pi} + y\cos\delta_{K\pi\pi} \]  

(25)

may be measured by means of a time-dependent Dalitz plot analysis of the three-body WS event decay \( D^0 \to K^+\pi^-\pi^0 \). The strong phase that rotates the mixing parameters \( x \), \( y \) are defined here as \( \delta_{K\pi\pi} = \arg(A(D^0 \to K^+\rho^-)/A(D^0 \to K^+\pi^-)) \).

5 Mixing

Charm mixing was established by BABAR and Belle \[4\] \[5\] using the techniques previously described. However the precision with which the parameters \( x_D \) and \( y_D \) have been measured can still be improved. A better precision in the determination of the mixing parameters may allow an improved understanding of mixing in the up-sector and of whether the CP symmetry is broken in mixing. In fact the uncertainties on \( x_D \) and \( y_D \) are of the order of \( 2 \times 10^{-3} \), which is still too large to evaluate if there is any CP difference in between \( D^0 \) and \( \overline{D}^0 \). Different measurements of charm mixing may be combined and projected into \((x_D, y_D)\), as shown in Fig.[1] This would require:

- \( \chi^2 \) minimization technique.
- Correlation effects need to be taken into account.

\( (x', y') \) from WS \( D^0 \to K^+\pi^- \) decays.

\( (x'', y'') \) from time-dependent Dalitz plot (TDDP) analysis of \( D^0 \to K^+\pi^-\pi^0 \).

\( y_{CP} \) from tagged/untagged \( D^0 \to h^+h^- \).

\( (x_D, y_D) \) from the combined channels.
- CP conserving hypothesis.

The results of this analysis are given in table [1].

| Fit | $x \times 10^{-3}$ | $y \times 10^{-3}$ | $\delta_{k^+\pi^-}^0$ | $\delta_{k^+\pi^-\pi^0}^0$ |
|-----|--------------------|--------------------|---------------------|---------------------|
|     | $xxx \pm 0.19$    | $yyy \pm 0.11$    | $\delta\delta\delta \pm 0.71$ | $\delta\delta\delta \pm 0.83$ |

Table 1: Mixing parameters $(x_D, y_D)$ and strong phases $\delta_{K\pi}$ and $\delta_{K\pi\pi}$ from a $\chi^2$ fit to observables obtained from SuperB when 1.0 $ab^{-1}$ of simulated data at the charm threshold is considered. The central value is arbitrarily chosen, and for this reason it is not shown.

It is clear that the SuperB experiment not only will improve our knowledge of charm mixing using a large data sample collected at the $\Upsilon(4S)$, but with the machine running at the charm threshold it will be possible to increase the sensitivity to the mixing parameters. In fact, with a run at charm threshold, one can reduce the size of $D^0 \rightarrow K_s h^+ h^-$ ellipse. This reduces the area of the WS $D^0 \rightarrow K^+ \pi^-$ ellipse that combines strong phase measurement and $\Upsilon(4S)$ analysis of time-distribution of $k^+\pi^-$ decays, and reduces the area of the WS $D^0 \rightarrow K^+ \pi^- \pi^0$ decays. Another interesting aspect of these studies is the possibility to define the charm golden channels $D^0 \rightarrow K_s h^+ h^-$ which are self-conjugate multi-body final states and represent a combination of CP odd and even eigenstates. If the measurement of strong phases yields $\delta_f = 0, \pi$ then the parameters $x_D$ and $y_D$ are directly measurable with a TDDP analysis.

6 CP Violation

CP violation has been discovered in the Kaon system [6] long ago and more recently in the B meson one. In the charm sector CP violation is expected to be very small. Moreover the charm sector is the only up-type sector where we can expect to perform these measurements.

6.1 Indirect CP Violation

As shown in eq. [6] indirect CP violation may be manifest through asymmetries in a broad class of observables. Effective values of the mixing parameters are defined by measuring separately the $D^0$ and $\bar{D}^0$ mesons. In this case we have:

$$D^0 \rightarrow (x_D^+, y_D^+),$$  \hspace{1cm} (26)

$$\bar{D}^0 \rightarrow (x_D^-, y_D^-),$$  \hspace{1cm} (27)
Figure 1: Main charm mixing parameters combined into average values for $x_D$ and $y_D$ when considering a 1.0 $ab^{-1}$ of data collected at the charm threshold (including projections of strong phase measurements $\delta_{k\pi}$ and $\delta_{k\pi\pi}$) and 75 $ab^{-1}$ of data collected at the $\Upsilon(4S)$. The contours range from 1 to 4 standard deviations two-dimensional confidence regions from the $\chi^2$ fit to these results are shown as solid lines.

where the sign +/- depends on the electric charge of the charm quark ($Q_c=+2/3$ and $Q_c=-2/3$). Ignoring systematic uncertainties (which will be almost identical for both the $D$ and $D^0$ mesons and can be neglected) SuperB will be able to measure a difference of the order of $5 \times 10^{-4}$ in $x_D^+-x_D^-$ and $3 \times 10^{-4}$ in $y_D^+-y_D^-$ at 3$\sigma$ level. If these differences will be observed and interpreted as being due to $CP$ violation in mixing, then they would provide a measurement of $|\frac{Q_y}{p_D}|$. The following asymmetry can furthermore be measured:
\( a_z = \frac{(z^+ - z^-)}{(z^+ + z^-)} \approx \frac{(1 - |\frac{p_D}{q_D}|^2)}{(1 + |\frac{p_D}{q_D}|^2)} \quad (28) \)

\( z \) can be either \( x_D \) or \( y_D \). The same study may be applied assuming that \( z \) is \( y_{CP} \), \( y', x'', y'' \). Estimates of the expected uncertainties that \( SuperB \) may obtain by combining different modes are given in Table [2].

### 6.2 Direct \( CP \) Violation

The LHCB Collaboration and CDF recently reported the first hint of time integrated \( CP \) asymmetry by combining the measurement for \( D^0 \rightarrow k^+k^- \), respectively at 3.5 and 2.7 \( \sigma \) level [7] [8]. To evaluate this asymmetry the quantity:

\[
A_{CP}(h^+h^-) = \frac{\Gamma(D^0 \rightarrow h^+h^-) - \Gamma(\overline{D}^0 \rightarrow h^+h^-)}{\Gamma(D^0 \rightarrow h^+h^-) + \Gamma(\overline{D}^0 \rightarrow h^+h^-)} \approx A_{CP}^{dir}(h^+h^-) + \frac{<t(h^+h^-) >}{\tau} A_{CP}^{ind} \quad (29)
\]

has been defined. New measurements with better precision are needed in order to understand if this result is due to new physics. In any case following the same approach the \( SuperB \) experiment can reach a sensitivity of the order of \( \sigma = 3 \times 10^{-4} \).

### 6.3 Time-dependent \( CP \) violation analysis

In Ref. [9] time-dependent \( CP \) violation (\( TDCPV \)) studies have been proposed for charm using a very similar formalism to that adopted when studying a \( B_d \). The \( SuperB \) experiment will collect data at the charm threshold producing correlated \( D^0 \) mesons and at the \( \Upsilon(4S) \) producing un-correlated ones. It will be possible to perform measurements in the two different configurations of the machine. Observations of \( TDCPV \) in charm can be used to constrain the angle \( \beta_{c,eff} \) in the charm unitarity triangle and the time-dependent analysis in general can be used to measure mixing parameters when \( TDCPV \) is observed.

| Parameter | Mode | \( \sigma(|q_D/p_D| \times 10^2) \) |
|-----------|------|-----------------|
| \( x_D \) | all modes | \( \pm 1.8 \) |
| \( y_D \) | all modes | \( \pm 1.1 \) |

Table 2: Combination of estimated uncertainties in the CPV mixing parameter \( q_D/p_D \) that may be obtained at \( SuperB \) when considering the effective values of the mixing parameters from 75 \( ab^{-1} \) of data collected at the \( \Upsilon(4S) \)
6.4 Sensitivity to $\beta_{c,\text{eff}}, \phi_{\text{MIX}}, \phi_{\text{CP}}$ and $x_D$

Expected sensitivities on the parameters described in the previous section are reported here for un-correlated $D^0$ mesons production. Further results are given in [9]. Table [3] shows the expected uncertainties for $\phi_{\text{MIX}}$ and $\sigma(\beta_{c,\text{eff}})$ while Table [4] shows the expected sensitivities on $x_D$.

| Parameter | Un-correlated $D$'s ($\Upsilon(4S)$) |
|-----------|--------------------------------------|
| $\phi_{\pi\pi} = \arg(\lambda_{\pi\pi})$ | $2.2^\circ$ |
| $\phi_{kk} = \arg(\lambda_{KK}) = \beta_{c,\text{eff}}$ | $1.3^\circ$ |

Table 3: Summary of expected uncertainties from $75 \, ab^{-1}$ of data at $\Upsilon(4S)$.

| Mode | $\sigma_{x_D}(\phi = \pm 10^\circ)$ | $\sigma_{x_D}(\phi = \pm 20^\circ)$ |
|------|------------------------------------|------------------------------------|
| $D^0 \to \pi^+\pi^-$ | $0.12 \, \%$ | $0.06 \, \%$ |
| $D^0 \to K^+K^-$ | $0.08 \, \%$ | $0.04 \, \%$ |

Table 4: Estimates of the sensitivity on $x_D$ when $75 \, ab^{-1}$ of data collected at the $\Upsilon(4S)$ is considered for the decays $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ and $\phi = \phi_{\text{MIX}} - 2\beta_{c,\text{eff}}$.

This shows that a time-dependent analysis applied to charm not only may help us to better understand the flavor changing structure of the SM, but the observation of $CP$ violation in charm may allow us to study the charm triangle through the measurement of the angle $\beta_{c,\text{eff}}$. This will provide us with a consistency check of the CKM mechanism. It is important to mention that with current experimental sensitivity, any observation of a value of the $\beta_{c,\text{eff}}$ inconsistent with zero will be a clear signal of new physics. On the other hand, the time-dependent analysis can measure $x_D$ and $\phi_{\text{MIX}}$ using not only the decay modes $D^0 \to K^+K^-$ or $D^0 \to \pi^+\pi^-$, but additional decay modes are available to carry out time-dependent mixing-related measurements. Decay channels including neutral states, as it is in the case of $D^0 \to K_s^0\pi^0$, with branching ratios that are larger than those for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ may provide better constraints on the determination of mixing phase/parameters, and this makes an electron-positron machine unique when performing such a measurement.

7 Conclusions

The SuperB experiment with its run at the charm threshold will be able to perform strong phase measurements that will shrink the allowed parameter space, and it has
been shown that by using the effective values for the mixing parameters it is possible to perform a test of $CP$ violation (indirect). Direct $CP$ violation has been discussed, and it has been highlighted that it is not yet clear if the Standard Model may account for the current experimental central value, or if some new physics is showing up, and studies of additional modes help resolve this question. SuperB may play a leading role in the understanding of direct $CP$ violation repeating the analysis already carried out, and performing new ones that would otherwise be difficult for other experiments (final states including neutrals). Finally, time-dependent $CP$ violation has been introduced and it was shown that with that formalism it is possible to perform measurements of the quantities: $\phi_{MIX}$, $x_D$, and $\beta_{c,eff}$.

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