Universality, Gravity, the enigmatic $\Lambda$ and Beyond

Naresh Dadhich *

Inter-University Centre for Astronomy & Astrophysics, Post Bag 4, Pune 411 007, India

Abstract

In this essay, I wish to share a novel perspective which envisions universalization as a guide from the classical world to relativistic and quantum world. It is the incorporation of zero mass particle in mechanics which leads to special relativity while its interaction with a universal field shared by all particles leads to general relativity. We also give a very simple classical argument to show that why the universal force has to be attractive. We try to envisage what sort of directions does this principle of universality point to for the world beyond general relativity?

*email: nkd@iucaa.ernet.in
In the classical Newtonian World, we have the absolute 3-space and the absolute 1-time with space having the Euclidean flat metric. Then there are the three laws of motion governing motion of particles. The laws however apply only to massive particles. That is massless particles cannot be accommodated in the Newtonian mechanics. If such particles exist in nature, they would ask for new mechanics. The characteristic of such a particle is that it can’t exist at rest in any frame. It should be moving relative to all observers and its speed must hence be the limiting speed for all. It must be the same for all observers. Existence of zero mass particles gives us a universal constant speed which needs to be accommodated in mechanics.

On the other hand we also know in the Newtonian framework that light always moves in a straight line in all directions. The straight line motion is indicative of constant speed. Not quite, there could occur accelerated motion in straight line for force acting along the line of motion. Since light moves straight in all directions, this possibility is also ruled out because there cannot exist a force acting in all directions. We are thus forced to conclude that either Newton’s laws of motion do not apply to light or its speed must be constant. This constant should be the same for all observers because light moves in a straight line for all observers. That is, light’s speed must also be a universal constant.

We know that zero mass particles do indeed exist in nature and they are nothing but the particles of light, photons. Though they are strictly speaking quantum objects but they do have classical manifestation as electromagnetic wave. They could however be used to universalize mechanics; i.e. it must apply to both massive as well as massless particles. That then asks for a new mechanics which is special relativity (SR).

The most fundamental universal entities we know are space and time and a universal statement is always expressed through geometry. For instance, motion of free particles is a straight line is a universal property which is expressed as a geometric statement. From
this we wish to propound a general guiding principle, *any universal physical property/force must be expressible as a property of universal entities, space and time, through a geometric statement.*

Thus the existence of universal speed, which we denote by $c$, must become a property of space and time and that is what precisely Minkowski did when he incorporated invariance of light’s speed in the geometry of 4-dimensional spacetime manifold. This bound space and time into one whole, spacetime.

What we have done is that in conformity with universality we have asked for incorporation of massless particles in the Newtonian mechanics, which was not allowed. Taking into account the characteristic property of motion of massless particles we are forced to enlarge the Newtonian framework to Minkowski framework which admitted both massive and massless particles. In our further exploration, this would be the guiding finger; i.e. *ask a question which is not admitted in the existing framework, then enlarge the framework as indicated/suggested by the question itself such that the question is answered.*

Let us consider that there exists a universal force which is shared by all particles. It must also act on massless particles which propagate with universal constant speed. Then how do we make it feel the universal force, and feel it must. We thus face the contradiction that massless particle must feel the force but its speed must not change. This is not possible in the existing framework. How do we enlarge the framework? Two suggestions come forward from our guiding principle, one since the force is universal it must be expressible as a property of spacetime and two, what do we really want massless particle to do? What we want the particle to do is that when it is skirting the source of the universal force it should acknowledge its presence by bending rather than going straight. To illustrate this point, let us consider a piece of wood floating in a river. It floats freely and bends as the river bends. No force really acts on it to bend its course, it simply follows the flow of the river. This
suggests that bend the *river* in which massless particle propagates freely. Thus we arrive at the profound insight into the nature of the universal force that it must bend/curve space - rather spacetime in which massive as well as massless particles propagate freely. That is the envisaged universal force can only be described by curvature of spacetime and in no other way.

Thus universality of the force does not let spacetime remain inert background but instead impregnates it with dynamics, a unique and distinguishing property. It then ceases to be an external force and its dynamics has to be fully governed by the spacetime curvature. This automatically incorporates the first suggestion mentioned above that universal force should become a property of spacetime. The equation of motion for the force does not have to be prescribed but must rather follow from the spacetime curvature all by itself. The curvature of spacetime is given by the Riemann curvature tensor for the metric $g_{ab}$ and it satisfies the Bianchi differential identity. The contraction of which yields the second rank symmetric tensor, constructed from the Ricci tensor, having vanishing divergence. That is,

\[
\nabla_b G^{ab} = 0 \tag{1}
\]

where

\[
G_{ab} = R_{ab} - \frac{1}{2} R g_{ab} \tag{2}
\]

The above equation implies

\[
G_{ab} = -\kappa T_{ab} - \Lambda g_{ab} \tag{3}
\]

with

\[
\nabla_b T^{ab} = 0 \tag{4}
\]

where $T_{ab}$ is a symmetric tensor, and $\kappa$ and $\Lambda$ are constants. On the left we have a differential expression involving the second derivative and square of the first derivative of the metric
which now acts as a potential for the universal force. Thus on the right should be the source of the force. What should be the source for a universal force? Something which is shared by all particles - mass/energy. That identifies $T_{ab}$ with the energy momentum tensor of matter distribution and vanishing of its divergence ensures conservation of energy and momentum.

We however know that in the Newtonian theory mass is the source for gravity and hence the above equation should agree with the Newtonian equation in the first approximation. This determines $\kappa = 8\pi G/c^2$ in terms of the Newtonian constant of gravitation, and the new constant $\Lambda$ should be small so as to have no effect over the stellar scale where the theory has been tested observationally. We have thus obtained the Einstein equation for gravitation - general relativity (GR) [1,2]. It is remarkable to note that the universal force can be nothing other than gravity. Thus gravity is the unique universal force.

Note that the constant $\Lambda$ enters into the equation naturally and is in fact at the same footing as the energy momentum tensor. It makes a very important statement that even when space is free of all non-gravitational matter/energy distribution, empty space has non-trivial dynamics. The above equation refers to spacetime in entirety which means whatever can be done in or to it is included in this equation. It is well known that vacuum can suffer quantum fluctuations which produce stress energy tensor precisely of the form $\Lambda g_{ab}$ and it must be included in the equation. Had Einstein followed this chain of arguments, he would have anticipated gravitational effect of quantum fluctuations of vacuum and would have made a profound prediction rather than a profound blunder.

Classically a constant scalar field has no dynamics while in GR it again generates precisely the stress tensor of the type $\Lambda g_{ab}$ which of course has non trivial dynamics. Thus $\Lambda$
is the potential to which vacuum has been raised. \(^1\) That means when \(\Lambda = 0\), the vacuum is at absolute zero potential. Since vacuum can suffer quantum fluctuations, it must have micro-structure which can fluctuate. So long as spacetime has micro-structure, which is necessarily required for all quantum phenomena in free space, the constant \(\Lambda\) cannot in general be made to zero without introducing some new feature like supersymmetry. Even then, I would wonder whether zero \(\Lambda\) vacuum could be stable? The big open question is what value should it have? It is in fact a new constant of the Einsteinian gravity which needs to be understood.

Another aspect of the equation is that it is valid in all dimensions, \(n \geq 2\), for which Riemann curvature can be defined. It is so because gravity is universal in the most general sense and hence it cannot by hand be restricted to a given dimension. All by itself gravity is thus inherently a higher dimensional interaction. One can restrict \(T_{ab}\) on the right to a given space dimension by confining matter field on it while \(\Lambda g_{ab}\) refers to dynamics of vacuum which cannot like matter field be confined. Gravity is described by curvature of spacetime and to realise its full dynamics the minimum number of dimensions required are 4. Gravitational charge is matter/energy which is always positive while gravitational field energy is negative. It is by taking the two together there can occur charge neutrality of the gravitational systems. Since the negative charge is spread over the entire space, the total charge will vanish only when you integrate over the whole space. That is in the local neighbourhood there will be net positive charge (energy distribution) which may be confined to 3-space, yet gravitational field will propagate off into the higher dimension. Though it

\(^1\)In the Schwarzschild field, we have the Newtonian potential, \(\phi = -M/r\), note that a non-zero constant, which is classically inert, cannot be added to it. The Einstein equation determines the potential of an isolated body absolutely with its zero fixed at infinity and nowhere else [3]. That is for the local situation and here we have constant potential in a global setting.
would not be able to penetrate deep into the higher dimension because globally there exists charge neutrality. If the matter fields are confined to $n$-space, the field will leak into the $(n + 1)$th dimension but will not propagate deep enough. That is the massless graviton will have ground state and hence will remain confined to $n$-space. This is exactly what is required to happen for the brane world gravity [4,5].

It is remarkable that we have here motivated the higher dimensional and the brane world nature of the gravitational field purely from classical standpoint. If the matter is confined to 3-brane, the bulk spacetime would be 5-dimensional with $\Lambda$ being its dynamical support. In particular we can have the Randall - Sundrum brane world model [5]. In that case, the confinement of gravity on the brane requires that bulk spacetime must be anti-deSitter (AdS) with negative $\Lambda$ in the bulk. It has been shown for the AdS bulk and flat brane system that the Newtonian gravity can be recovered on the brane with high energy $1/r^3$ correction to the potential [5,6]. The most interesting case is of Schwarzschild - AdS bulk harbouring FRW brane. In that case localization of gravity on the FRW brane requires non-negative effective $\Lambda$ on the brane [7,8].

We have established that universality of the force entirely determines its dynamics and it is nothing but the Einsteinian gravity [1]. We shall now argue that universality also determines its attractive character. Like electric charge for electric field, the charge for gravitational field is matter/energy distribution. It has only one polarity and calling it positive or negative is only a matter of convention. We do however know that stability of a system under any force demands charge neutrality. That is why all freely existing bodies like stars, planets down to atoms are all electrically charge neutral. So must also be the case for gravity. How could that be because it has only one kind of polarity? Charge neutrality has to be achieved for there exist large scale stable systems in the universe under gravity. Is there anything new which could be invoked to obtain neutrality? Yes, there is the gravitational field, which must now have gravitational charge of polarity opposite of the
matter/energy distribution. The field must therefore have negative energy and that is why it must be attractive. It is again the universality that determines the attractive character of the field. In the Newtonian theory, it is an assumption that gravity is attractive which has been taken over in the Einsteinian theory. Attractive character is however associated with the spin 2 character of the field. Here we have articulated a very simple and novel argument based purely on the classical considerations. That the field is non-abelian and further following this line of argument it is also possible to work up an independent and new derivation and motivation for general relativity [9].

It is however well known that charge neutrality is a necessary condition for stable equilibrium but not sufficient. The Earnshaw’s theorem states that it is impossible to attain stable equilibrium purely under electromagnetic force. This is because the field has two kinds of charges which are isolated and localized. On the other hand in the case of gravity the other (negative) charge is distributed all over the space and hence is not isolated and localized. A gravitational situation could be envisioned as follows: A positive charge (body) sitting in its own field which has negative charge spread around it in space like a net. This system is obviously stable. It is the distributed nature of the other charge that provides the stability. It is thus no surprise that systems bound by gravity are always stable.

Let us for a moment digress to quantum theory. What question should we ask which can lead us from classical to quantum mechanics? We have two kinds of motion, particle and wave. Like particle wave must also carry energy and momentum with it. So it must like particle have a 4-momentum vector while on the other hand its motion is completely determined by the 4-wave vector. Since both these vectors refer to the same wave, they must be proportional. This gives the basic quantum mechanical relations between energy and frequency, and momentum and 3-(wave)vector. From this it is easy to get to the uncertainty and commutation relations which form the basic quantum principle.
The quantum principle is universal. Going by the guiding principle of universality, it must be expressible as a property, like the speed of light, of spacetime. This has unfortunately not happened. What is required is exactly what Minkowski did to SR by synthesizing the speed of light into the spacetime structure. This is however very difficult because synthesis of quantum principle with the spacetime would ask for discrete structure which is in contradiction with the inherent continuum of spacetime. However so long as this doesn’t happen quantum theory will remain incomplete. Thus for completion of quantum theory it would be required that spacetime must have micro-structure which could accord to quantum principle. It is the geometry of that which would synthesize quantum principle with the spacetime. This is an open question of over 100 years standing.

The same question is also coming up from the gravity side as well. Unlike quantum theory, GR is complete like classical electrodynamics. At high energy we have quantum electrodynamics. At high energy we know matter attains quantum character, i.e. $T_{ab}$ on the right becomes quantum. On the left is the spacetime which would now have to become quantum - discrete. This is what being pursued in the canonical approach of loop quantum gravity [10]. In the string theory approach to quantum gravity, gravity is considered as a spin 2 gauge field in higher dimensions against a fixed flat spacetime background and GR appears as a low energy effective theory [11]. The two approaches appear to be complimentary, either catching some aspect of the problem. Asymptotically the two would have to converge when the complete theory comes about.

Both gravity at high energy as well as completion of quantum theory require discrete micro-structure for spacetime. This suggests that there may perhaps be one and the same answer to both the questions. In a sense the two approaches, string theory and loop quantum gravity anchor respectively to quantum and gravity.

Adhering to our guiding principle of universality, what question should we ask and how
should we enlarge the existing framework to answer the question? Since spacetime is the fundamental universal entity, let us ask do there still remain some properties of it which are yet untapped? One is dimensionality of space and the other is its (non) commutativity. The former is however essential for the string theory and is also quite in vogue in the brane world models while non commutativity of space has also attracted some attention recently and it is hoped that it might facilitate in building up discrete structure for space.

Returning to the enigmatic $\Lambda$, we would like to argue that it is really anchored on the micro-structure of vacuum and hence might hold the ultimate key to the problem. It may in a deep sense connect through some duality relation micro with macro structure. It defines a new scale given by the Einsteinian gravity. On the other hand we already have the Planck length which is not given by any theory but constructed by using three universal constants. I believe that it is not a wise thing to take the Planck length a priori fundamental but instead we should try to deduce it in a fundamental manner. $\Lambda$ on the other hand is a scale provided by a fundamental theory and hence should be respected. It is natural to expect that there should exist a relation between them which will perhaps encode a profound physical truth.

Let me come back to the guiding question one should ask? In the gravitational field equation, we have the curvature of spacetime on the left, and matter stress tensor and the vacuum energy $\Lambda$ on the right. If $T_{ab}$ lives only in 3-space/brane which becomes quantum at high energy, while vacuum energy can still support a continuum bulk spacetime. Taking the cue from what we have discussed so far what question should we ask and how should we enlarge the framework to admit and answer the question? The universality demands that the Einstein equation should remain valid whether the matter field source is classical or quantum. For the quantum case, the spacetime curvature will be required to have discrete quantum character. Should that mean that spacetime itself becomes quantum? This is what is being explored by the canonical loop quantum gravity approach [10]. It is then not an enlargement but rather adoption of a radically different framework. In this approach we
remain bound to the 4-dimensional spacetime and there is no background spacetime relative to which spacetime is being quantized. In the string theory approach, we are already in higher dimensions and gravity is being considered as massless spin 2 field and GR appears as the low energy effective theory in 4 dimensions along with plethora of other fields [11]. In either case it is adoption of totally new framework which cannot be seen as enlargement of the existing framework.

We believe and as we have argued above that gravity is a higher dimensional interaction. Perhaps impossibility of constructing a renormalizable theory of massless spin 2 particles in 4 dimensions is indication of this fact. The dynamics of the universal field should also determine the dimensionality of spacetime. One aspect of enlargement would therefore be lifting above 4 dimensions. And spacetime curvature has to become discrete. One possible enlargement of the framework could be for matter fields confined to 3-brane, and the 5-dimensional bulk spacetime with $\Lambda$ support could provide the continuum background for gravity (spacetime curvature) to be quantized on the 3-brane. This is the suggestion that crops up in the spirit of what has been done in going from Newton to Einstein. However the pertinent question is whether it is technically and conceptually workable? This needs to be investigated.

Let me reiterate that we have here enunciated a method of asking question which is motivated by the principle of universality, and the question also suggests enlargement of the framework such that it gets answered. And we arrive at a new framework. One of the remarkably interesting applications of this method is to establish why the universal force has to be attractive? This is perhaps the simplest and most direct demonstration of why gravity is attractive. Following this train of thought what we need to do is to ask the right kind of question which will show us the way beyond GR or quantum theory.

Finally, we have argued quite forcefully that $\Lambda$ cannot be zero and hence any quantiza-
tion scheme should have to address to it [12]. I think that there must exist some basic link connecting \( \Lambda \) with the Planck length which needs to be discovered. That may hold the key to the problem. Of course the problem is all encompassing and so involved that there may not be one key but we may have to integrate several of them together.

These are some of the rumbling thoughts which I wanted to share and the more of them could be read in [9].
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