Inclusive production of $\chi_{cJ}$ in $e^+e^-$ annihilation is an excellent probe of the role of higher Fock states in the production of heavy quarkonia. Within the non-relativistic QCD approach, contributions from the short-distance production of colour-octet $c\bar{c}$ pairs are significantly larger than those from colour-singlet production. At the same time, $\chi_{cJ}$ production rates are significantly smaller than expected in the colour evaporation approach. Measurements of $\chi_{cJ}$ production at CLEO and future B-factories will thus constitute a major test of theoretical approaches to the production of heavy quarkonia.

PACS numbers: 13.65.+i, 12.38.Bx, 13.85.Ni
Calculations of heavy-quark production in $e^+e^-$ collisions are now approaching a level of high accuracy. Fifteen years after the calculation of the next-to-leading-order (NLO) perturbative QCD corrections to the total open heavy-quark cross section [1], we have now seen the completion of the NLO corrections to three-jet cross sections with massive quarks [2]. Despite continuous efforts over nearly 20 years, such a precision has not yet been reached in the calculation of cross sections of heavy bound states [3–12]. Predictive power depends on the understanding of the long-distance bound-state formation process, which is therefore at the heart of current quarkonium physics. In this letter we investigate the inclusive production of $P$-wave (orbital angular momentum $L = 1$) quarkonium states in $e^+e^-$ collisions. This is the first study of these processes since essentially [13] all previous work was concerned with the $S$-wave $J/\psi$ or $\Upsilon$ mesons.

For a sufficiently large quark mass $m$, a quarkonium bound state of a heavy quark $Q$ and its antiquark $\bar{Q}$ is a non-relativistic system. The spectroscopy of both charmonia and bottomonia is well described in non-relativistic potential models, where the quarkonia are considered to be pure $Q\bar{Q}$ states bound by an instantaneous potential. The question arises of whether the production of heavy quarkonia is also dominated by this leading Fock state. Is the $Q\bar{Q}$ pair produced, already at short distances, in a configuration that corresponds to the asymptotic valence Fock component of the quarkonium? If it is, then all non-perturbative information in the theoretical prediction reduces to a single number, namely the coordinate-space wave function at the origin, which can be extracted, e.g. from a potential-model calculation.

It is nowadays widely believed that higher Fock states are of decisive importance. Yet there exist differing approaches when it comes to relating observable cross sections to short-distance $Q\bar{Q}$ production amplitudes. In the colour evaporation model (CEM) [3], the quarkonium production cross section is prescribed to be a (process-independent) fraction of the $Q\bar{Q}$ production cross section, below the physical threshold for the production of a pair of heavy-light mesons. In the non-relativistic QCD (NRQCD) approach [14], the transition from $Q\bar{Q}$ to quarkonium is represented in terms of a multipole expansion, which leads to scaling rules.
for transition probabilities in terms of \( v \), the mean heavy-quark velocity in the meson. Any given cross section is a linear combination of several non-perturbative factors, multiplied by process-dependent hard factors. To leading order in \( \Lambda_{QCD}/\mu \) (\( \mu \gtrsim mv \)) the former are well-defined process-independent NRQCD matrix elements (MEs).

We have calculated the \( e^+e^-\rightarrow \chi_{cJ}+X \) cross sections within the NRQCD approach, using values of non-perturbative transition probabilities determined from measurements of other reactions. On the one hand, the results strongly violate the process-independence of cross-section ratios assumed in the CEM. Thus already an experimental upper limit on \( \chi_{cJ} \) production in \( e^+e^- \) annihilation can establish that non-perturbative effects play a decisive role in bound-state formation. On the other hand, the NRQCD results are dominated by colour-octet production channels. Observation of the predicted shapes and normalizations of the cross sections will therefore provide striking evidence of the importance of higher Fock states and the scaling of their contributions with \( v \).

In NRQCD, any cross section is given as a series expansion in both \( \alpha_s(m) \) and \( v \), to leading order in \( \Lambda_{QCD}/\mu \). Let us first consider the case of \( S \)-wave quarkonia, in particular the \( J/\psi \) \((J^{PC}=1^{--})\). The leading contribution in \( v \) is given by processes where the \( c\bar{c} \) pair is produced in the leading, colour-singlet Fock state \([13], \left| c\bar{c}_1(3S_1) \right> \): \[
\begin{align*}
e^+e^- & \rightarrow c\bar{c}_1(3S_1) + c\bar{c} , \\
e^+e^- & \rightarrow c\bar{c}_1(3S_1) + gg .
\end{align*}
\] The order \( v^2 \) correction to \( J/\psi \) production still involves only the leading Fock state and can be considered as a relativistic correction to the wave function. Subleading Fock states first contribute at relative order \( v^4 \), where the short-distance production of a colour-octet \( c\bar{c} \) pair is followed by dipole transitions to the final quarkonium. The chromo-electric (E1) and chromo-magnetic (M1) dipole transition scale as \( v^2 \) (E1) and \( v^4 \) (M1), at least for \( \alpha_s(mv)/\pi \ll 1 \) \([11]\). Dominant colour-octet \( J/\psi \) production processes at \( e^+e^- \) colliders are \[
\begin{align*}
e^+e^- & \rightarrow c\bar{c}_8(1S_0) + g ,
\end{align*}
\]
\[ e^+e^- \rightarrow c\bar{c}(3P_J) + g, \quad (4) \]
\[ e^+e^- \rightarrow c\bar{c}(3S_1) + q\bar{q} \quad (q = u, d, s). \quad (5) \]

Processes (3,4) are enhanced by \(1/\alpha_s(m_c)\) relative to (1), (2), and (5) is enhanced, at large energies, by a logarithm \(\ln(s/m_c^2)\) with respect to (1) (which, in turn, dominates (2) by a power \(s/m_c^2\) at large \(s\)). Current estimates of the long-distance MEs \(\langle O_{J/\psi}^{1,8} (2S+1L_J) \rangle\) [14], which parametrize the transition from a \(c\bar{c}_{1,8}(2S+1L_J)\) state to the \(J/\psi\), are listed, for example, in [11]. At the centre-of-mass energy studied by the CLEO experiment at the Cornell Electron Storage Ring, \(\sqrt{s} = 10.6\) GeV, the cross sections for the colour-singlet processes (1) and (2) are 0.20 pb and 0.35 pb, respectively, about one third of the experimental cross section \(\sigma = 1.65 \pm 0.25 \pm 0.33\) pb [16]. Hence there is room for colour-octet contributions, which we estimate as 0.50 pb for (3), 0.60 pb for (4), and 0.01 pb for (5).

Conclusions on the presence of colour-octet mechanisms would, however, be premature since estimates of the total \(J/\psi\) cross section suffer from large uncertainties. First, the colour-singlet MEs are known to 50% at best, and a factor of 2 uncertainty is certainly not exaggerated for the colour-octet MEs. Secondly, there could be large perturbative corrections. And finally, truly relativistic corrections of order \(v^2\) may also be large, cf. direct \(J/\psi\) production in hadronic collisions [17].

Therefore it has been suggested [11] to study the energy distribution \(d\sigma/dz\) of the \(J/\psi\), where \(z = 2E_{J/\psi}/\sqrt{s}\). The predicted large colour-octet processes (3) and (4) are concentrated near \(z = 1\) and should thus dominate the upper end point of the \(z\) spectrum. A dramatic change should be visible, notably in the polar angular distribution of the \(J/\psi\) w.r.t. the beam axis. Unfortunately, measurements at large \(z\) are plagued by a large background from radiative \(\psi'\) production [16].

Another potentially useful observable is the \(J/\psi\) polarization, which is measurable via the angular distributions of its decay leptons. Partial calculations already exist [6,12], but the measurement is definitely not easy.

We propose the study of \(\chi_{cJ}\) production as a means of investigating the importance of
colour-octet contributions in quarkonium production. Colour-octet mechanisms will leave distinct footprints in the total cross section, and in the energy and polar-angle distributions. Contrary to $S$-wave quarkonia, the $J^{++}$ $\chi_{cJ}$ mesons receive contributions from one of their higher Fock states $|c\bar{c}_8(3S_1)g\rangle$ already at leading order in $v$: the associated E1 transition suppresses the cross section by $v^2$, but this suppression is compensated because an $S$-wave operator scales as $1/v^2$ relative to a $P$-wave operator.

Colour-singlet production of $\chi_{cJ}$ proceeds through

$$e^+e^- \to c\bar{c}_1(3P_J) + \bar{c}c \ .$$

(6)

Note that the contribution from

$$e^+e^- \to c\bar{c}_1(3P_J) + gg$$

(7)

is zero for pure photon exchange in the $s$ channel. Also the Z-exchange contribution is negligible, at low energies because it is proportional to $s/M_Z^2$, and at high energies because (7) is suppressed by $m_c^2/s$ relative to (6).

Colour-octet contributions to the energy distribution of $\chi_{cJ}$ away from $z = 1$ arise from (5) and the processes

$$e^+e^- \to c\bar{c}_8(3S_1) + \bar{c}c \ ,$$

(8)

$$e^+e^- \to c\bar{c}_8(3S_1) + gg \ .$$

(9)

All three processes possess the same scaling in $v$ and $\alpha_s(m_c)$ as the colour-singlet process (6). Two further colour-octet processes dominate near the $z = 1$ end-point, namely (3) and (4). The process

$$e^+e^- \to c\bar{c}_8(3S_1) + g$$

(10)

is negligible for reasons similar to (7).

The calculation of processes (5,6,8,9) is standard but tedious. We calculate the distributions in both the energy and the polar angle $\theta$ of the $\chi_{cJ}$,
\[
\frac{d^2\sigma}{dz \, d\cos \theta} = S(z) \left[ 1 + \alpha(z) \right].
\] (11)

Details will be presented elsewhere. Here we comment on the relation of our results and previous calculations.

At high energies, \(m_c^2/s \to 0\), the cross section for (8) reduces to
\[
\frac{d\sigma}{dz} = 2 \sigma(e^+e^- \to \bar{c}c) \frac{1}{m_c^2} \langle O_1^{xJ}(3P_J) \rangle D_{c\to\bar{c}c(3P_J)}(z),
\] (12)
where \(D_{c\to\bar{c}c(3P_J)}^{(1)}\) are the partonic colour-singlet charm fragmentation functions for \(c \to \bar{c}c(3P_J) + c\). Our expressions for these functions agree with [18,19].

The fragmentation limit of (8) is more involved. Writing it as
\[
\frac{d\sigma}{dz} = 2 \sigma(e^+e^- \to \bar{c}c) \frac{1}{m_c^3} \langle \hat{O}_8^{xJ}(3S_1) \rangle \hat{D}_8(z),
\] (13)
we observe that \(\hat{D}_8(z)\) agrees with (3.6) in [19] if we take \(\mu^2 = z^2(1-z)s\) in the result of [19]. Note that (13) is not the sum of two fragmentation processes, \(e^+e^- \to \bar{c}c\) followed by \(c \to \chi_J\) or \(\bar{c} \to \chi_J\) and \(e^+e^- \to \bar{c}cg\) followed by \(g \to \chi_J\), as is the high-energy limit of (8).

We can decompose (13) as
\[
\frac{d\sigma}{dz} = \frac{1}{m_c^3} \langle \hat{O}_8^{xJ}(3S_1) \rangle \left\{ 2 \sigma_{\bar{c}c} \left[ D_{c\to\bar{c}c(3S_1)}^{(8)}(z) + R_8(z) \right] \right. \\
\left. + \int_z^1 \frac{d\hat{z}}{\hat{z}} \left. \frac{d\sigma_{\bar{c}cg}}{dy} \right|_{y=z/\hat{z}} D_{g\to\bar{c}c(3S_1)}^{(8)}(\hat{z}) \right\},
\] (14)
where \(\sigma_{\bar{c}c,\bar{c}cg}\) is the cross section for \(e^+e^- \to \bar{c}c, \bar{c}cg\); the gluon fragmentation function is simply \(D_{g\to\bar{c}c(3S_1)}^{(8)}(z) = \pi\alpha_s\delta(1-z)/24\), and \(D_{c\to\bar{c}c(3S_1)}^{(8)}\) is obtained by appropriately removing the NRQCD ME and changing the colour factor in the colour-singlet \(c \to J/\psi\) fragmentation function given in [3]. Also, \(D_{c\to\bar{c}c(3S_1)}^{(8)}\) coincides with the equal-flavour limit of the \(\bar{b} \to \bar{b}c_s(3S_1) + c\) fragmentation function of [18]. However, from (14) it is clear that \(D_{c\to\bar{c}c(3S_1)}^{(8)}\) is not the colour-octet part of the \(c \to \chi_\epsilon J\) fragmentation function, as conjectured in [18]. Interference terms present in the \(c\bar{c}s(3S_1) + c\bar{c}\) diagrams for the equal-flavour case (the non-zero remainder term \(R_8\)) forbid a simple rescaling of the colour-singlet result.
Our cross section for (9) agrees with [7] if we appropriately replace the colour-singlet NRQCD ME and overall colour factors by their colour-octet counterparts. Finally, our cross section for (5) reduces to a form that is analogous to the $\theta$-integrated expression given in [10].

For our numerical results we use $m_c = 1.48 \text{ GeV}$, $\alpha_s = 0.28$, $\alpha_{em} = 0.0075$, and values for the NRQCD MEs as given in [11]. Two MEs have not yet been determined from any experiment. We estimate their magnitude by scaling other known MEs:

$$\langle O_8^{\chi c J}(^1S_0) \rangle \approx v^4 \frac{1}{3} \langle O_8^{\chi c J}(^3S_1) \rangle = \frac{2J+1}{3} 2.8 \times 10^{-4} \text{ GeV}^3,$$

$$\langle O_8^{\chi c J}(^3P_0) \rangle \approx \frac{2J+1}{3} \frac{\langle O_8^{J/\psi}(^3S_1) \rangle}{\langle O_1^{J/\psi}(^3S_1) \rangle} \langle O_1^{\chi c J}(^3P_1) \rangle = \frac{2J+1}{3} 1.8 \times 10^{-3} \text{ GeV}^5. \quad (15)$$

$$\langle O_8^{\chi c J}(^3P_1) \rangle \approx \frac{2J+1}{3} \frac{\langle O_8^{J/\psi}(^3S_1) \rangle}{\langle O_1^{J/\psi}(^3S_1) \rangle} \langle O_1^{\chi c J}(^3P_1) \rangle = \frac{2J+1}{3} 1.8 \times 10^{-3} \text{ GeV}^5. \quad (16)$$

The total $\chi_{c1,2}$ cross sections are dominated by colour-octet processes, as shown in Table I. The (infrared-finite) colour-singlet parts are two orders of magnitude smaller than the $J/\psi$ cross sections. Adding in the colour-octet contributions leads to a marked increase. Still, the rates are only about 1/20–1/10 of the $J/\psi$ production rate. This confirms that $\chi_{cJ}$ production in $e^+e^-$ annihilation is indeed very sensitive to the power counting of long-distance effects in quarkonium formation. The non-observation of $\chi_c$'s at CLEO [16] (for integrated luminosities of about 3.1 fb$^{-1}$ on the $\Upsilon(4S)$ and about 1.6 fb$^{-1}$ off the resonance) already causes problems [20] for the CEM, where the $\chi_{cJ}:J/\psi$ ratio is predicted to be the same as was measured in fixed-target [17] and collider [21] hadroproduction, i.e. about 1 (5/3) for $J = 1$ ($J = 2$).

The rates in Table I are for direct production, i.e. excluding the production of $\chi_{cJ}$ in the decay of other hadrons. B-meson decay contributions can be removed experimentally, e.g. by a cut on $z$ at CLEO. There is also a contribution from $\psi(2S)$ decays, $\psi(2S) \rightarrow \gamma + \chi_{cJ}$. We estimate the $\chi_{c1}$ cross sections to be 11 fb for process (1), 19 fb for (2), 8.3 fb for (3), 19 fb for (4), and 0.64 fb for (5). In total we hence expect about 60 fb (10% less for $\chi_{c2}$), with
about one half due to colour-octet mechanisms. We conclude that direct $\chi_{cJ}$ production dominates over indirect one.

Figure 1 shows the energy distribution of direct $\chi_{cJ}$ production. A steep rise of the distribution at large $z$ signals the importance of the $e^+e^- \rightarrow c\bar{c}_8(3P_J)+g$ process. Signatures for the other colour-octet mechanisms are also clearly visible, e.g. a $\chi_{c1}:\chi_{c2}$ ratio close to 1 at $z < 1$, as opposed to the ratio $< 0.5$ obtained from colour-singlet processes alone.

With a separate measurement of $\chi_{c1,2}$ and with sufficient statistics to determine the double-differential cross section $d^2\sigma/dz d\cos \theta$, the angular distribution parameter $\alpha(z)$ will also serve to identify NRQCD production channels. As shown in Fig. 2, the colour-octet value of $\alpha(z)$ for $\chi_{c1}$ is significantly lower than the colour-singlet value, whereas for $\chi_{c2}$ the colour-octet value (at large $z$) is significantly higher than the colour-singlet one.

In summary, inclusive $\chi_{cJ}$ production is a powerful tool to establish the size of higher Fock state contributions in the formation of heavy bound states. Different theoretical approaches rely on different power-counting rules resulting in markedly different cross-section ratios. For example, the $\chi_{c2}:J/\psi$ ratio is as low as $1/100$ if only $c\bar{c}$ pairs in the leading Fock state contribute, while it is of order 1 if bound-state formation proceeds dominantly through (colour-singlet or -octet) $S$-wave $c\bar{c}$ pairs, as it does in the CEM. The velocity-scaling rules of NRQCD yield a ratio of about $1/12$, since the presence of colour-octet processes induced by higher Fock states is much more pronounced for $\chi_{cJ}$ production than for $J/\psi$ production. This is partly because colour-octet processes enter $\chi_{cJ}$ production at the same order as the colour-singlet ones, and partly because $C$-parity suppresses the process $e^+e^- \rightarrow c\bar{c}gg$, which dominates $J/\psi$ production.

Signatures for colour-octet contributions do not only show up in a dramatic increase in total $\chi_{cJ}$ production rates. Differences are also clearly visible in the energy and angular distributions. The expected rates could already be visible at CLEO with current statistics, and will definitely be measured in the near future at CLEO and at B-factories.

M.V. wishes to acknowledge financial support from Suomalainen Tiedeakatemia, Väisälän rahasto.
REFERENCES

* Heisenberg Fellow.

† Present address: Department of Physics, Technical University of Munich, 85747 Garching, Germany.

[1] J. Jersák, E. Laermann and P.M. Zerwas, Phys. Rev. D 25, 1218 (1982); D 36, 310 (E) (1987).

[2] W. Bernreuther, A. Brandenburg and P. Uwer, Phys. Rev. Lett. 79, 189 (1997); G. Rodrigo, A. Santamaria and M. Bilenky, Phys. Rev. Lett. 79, 193 (1997); P. Nason and C. Oleari, CERN-TH/97-219, hep-ph/9709360.

[3] G.P. Kane, J.P. Leveille and D.M. Scott, Phys. Lett. 85, 115 (1979); G.C. Branco, H.-P. Nilles and K.H. Streng, Phys. Lett. 85, 269 (1979); H. Fritzsch, Phys. Lett. 90, 164 (1980).

[4] W.Y. Keung, Phys. Rev. D 23, 2072 (1981); J.H. Kühn and H. Schneider, Z. Phys. C11, 263 (1981); L. Clavelli, Phys. Rev. D 26, 1610 (1982); J.H. Kühn and K.H. Streng, Z. Phys. C17, 175 (1983); V. Barger, K. Cheung and W.Y. Keung, Phys. Rev. D 41, 1541 (1990); K. Hagiwara et al., Phys. Lett. B267, 527 (1991); B316, 631 (E) (1993).

[5] E. Braaten, K. Cheung and T.C. Yuan, Phys. Rev. D 48, 4230 (1993).

[6] V.M. Driesen, J.H. Kühn and E. Mirkes, Phys. Rev. D 49, 3197 (1994).

[7] P. Cho and A.K. Leibovich, Phys. Rev. D 54, 6690 (1996). The reader should note a few misprints, notably overall factors 1/3 and 2 in the cross sections for (1) and (2), respectively.

[8] P. Cho, Phys. Lett. B368, 171 (1996); F. Yuan, C.-F. Qiao and K.-T. Chao, Phys. Rev. D 56, 1997 (321); C.-H. Chang, C.-F. Qiao and J.-X. Wang, Phys. Rev. D 56, 1997 (1363).
[9] E. Braaten and Y.-Q. Chen, Phys. Rev. Lett. 76, 730 (1996).

[10] K. Cheung, W.-Y. Keung and T.C. Yuan, Phys. Rev. Lett. 76, 877 (1996).

[11] G.A. Schuler, Int. J. Mod. Phys. A12, 3951 (1997).

[12] S. Baek, P. Ko, J. Lee and H.S. Song, Phys. Rev. D 55, 6839 (1997).

[13] Estimates exist only for fragmentation contributions, g or c \to \chi_{cJ} + X.

[14] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. D 51, 1125 (1995).

[15] We use the spectroscopic notation for angular momenta, and the subscript 1 (8) denotes a colour-singlet (colour-octet) state.

[16] K. Honscheid and A. Wolf, private communication.

[17] G.A. Schuler, preprint CERN-TH.7170/94, hep-ph/9403387.

[18] T.C. Yuan, Phys. Rev. D 50, 5664 (1994).

[19] J.P. Ma, Phys. Rev. D 53, 1185 (1996).

[20] The universality of \chi_{cJ} : J/\psi cross section ratios seems violated also in photoproduction, cf. the experimental upper limit on \sigma[\gamma p \to \chi_{cJ} + X] by NA14: R. Barate et al., Z. Phys. C33, 505 (1987).

[21] D0 Collaboration, S. Abachi et al., contributed to the International Europhysics Conference on High Energy Physics, Brussels, Belgium, July 1995; CDF Collaboration, F. Abe et al., contributed to the International Conference on High Energy Physics, Warsaw, Poland, July 1996.
TABLES

TABLE I. Integrated cross sections and angular coefficient $\alpha$ for $e^+e^- \rightarrow \chi_{c1,2} + X$ at $\sqrt{s} = 10.58$ GeV. The colour-octet $\chi_{c2}$ cross sections are a factor $5/3$ larger than the corresponding $\chi_{c1}$ ones. The cut $z > 0.693$ serves to exclude $\chi_{cJ}$’s originating from B-meson decays.

| $c\bar{c} + c\bar{c}(^3P_1) \rightarrow \chi_{c1}$ | $18.1$ | $0.44$ | $15.3$ | $0.49$ |
| $c\bar{c} + c\bar{c}(^3P_2) \rightarrow \chi_{c2}$ | $8.4$ | $0.10$ | $7.5$ | $0.09$ |
| $c\bar{c} + c\bar{c}(^3S_1) \rightarrow \chi_{c1}$ | $6.1$ | $0.24$ | $4.0$ | $0.35$ |
| $q\bar{q} + c\bar{c}(^3S_1) \rightarrow \chi_{c1}$ | $15.6$ | $0.26$ | $11.7$ | $0.34$ |
| $g + c\bar{c}(^3S_1) \rightarrow \chi_{c1}$ | $5.5$ | $-0.03$ | $4.2$ | $-0.05$ |
| $g + c\bar{c}(^1S_0) \rightarrow \chi_{c1}$ | $4.1$ | $1.00$ | $4.1$ | $1.00$ |
| $g + c\bar{c}(^3P_J) \rightarrow \chi_{c1}$ | $45.6$ | $0.64$ | $45.6$ | $0.64$ |
| Total $\chi_{c1}$ | $95.0$ | $0.47$ | $84.8$ | $0.53$ |
| Total $\chi_{c2}$ | $136.5$ | $0.46$ | $123.4$ | $0.51$ |
FIG. 1. Energy distribution of $\sigma(e^+e^- \rightarrow \chi_{c1,2} + X)$ at $\sqrt{s} = 10.58$ GeV: total, colour-singlet, and colour-octet $\chi_{c1}$ contributions, and total and colour-singlet $\chi_{c2}$ contributions. The peak from the $e^+e^- \rightarrow c\bar{c}s(1S_0, 3P_J) + g$ processes at large $z$ has been smeared over a range of width $\sim v^2$. In a physical process such a smearing follows from the energy transfer in the non-perturbative transition.
FIG. 2. Energy dependence of the angular coefficient $\alpha$ in $e^+e^- \rightarrow \chi_{c1,2} + X$ at $\sqrt{s} = 10.58$ GeV: total, colour-singlet, and colour-octet $\chi_{c1}$ contributions, and total and colour-singlet $\chi_{c2}$ contributions.