Blasius–Rayleigh–Stokes flow of nanofluid past an isothermal magnetized surface

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Abstract
The present article investigated the unsteady flow of a nanofluid past an isothermal magnetized plate emanating from a moving slot. This unique form of unsteady boundary layer flow is analogous to stretching/shrinking sheet problems subject to the direction of motion of the slot. Governing partial differential equation can be reduced into a similar form using the Blasius–Rayleigh–Stokes variable. The consequences of the movable slot and magnetic field on flow and heat transfer of nanofluid are examined by solving the problem numerically. The behavior of the magnetic field in the presence of nanoparticles is also examined. Effects of the magnetic field upon the existence of dual solutions for the specific range of moving slot parameter are also studied in detail.

Keywords
Unsteady flow, magnetized plate, nanofluid, Blasius–Rayleigh–Stokes variable, numerical solution, dual solution

Date received: 15 May 2021; accepted: 14 September 2021

Handling Editor: Chenhui Liang

Introduction
Precise and efficient control of heat transfer and fluid flow under extreme conditions is important for future science and technology. Optimal flow and heat transfer control can significantly enhance the energy efficiency of processes in many industries. Several methods have been proposed for heat and drag reduction in physical systems like the addition of polymers and nanoparticles in base fluid, magnetic fields, and flexible walls.

Nanofluids, an accomplishment of researchers and scientists of nanotechnology have improved the thermal conductivity of a fluid by exploiting the thermal conductivity of solids when tiny solid particles are added to the carrier fluid. Masuda et al. reported the thermal conductivity enhancement in liquid dispersions of nanofluid. Pure fluid equations are directly extended to nanofluids in a homogenous flow model proposed by Choi. The dispersion model is another model based on the surveillance, that the nanofluid has enhanced heat transfer due to high thermal conductivity and dispersion of nanofluids. The homogenous flow model is conflicted with experimental observations. Buongiorno rejected the dispersion model, clearly mentioned that the nanofluids have negligible effects of heat transfer for nanoparticle dispersion. He proposed a unique model to eradicate the shortcomings of the above-mentioned models. Extensive uses of nanofluid in different fields of science and industry and the Buongiorno model of nanofluid have motivated the experimentalists and researchers to study nanofluids in the last few years.

The electromagnetic field is one of the useful agents for flow and heat transfer control for electrically
conducting fluids. In the unsteady flow of electrically conducting fluids, an induced magnetic field has an important role while describing flow stability and drag reduction. The subject of thermo-MHD flow and heat transfer is an exigent field of research due to the additional coupling of Maxwell and Navier–Stoke equations and due to Lorentz force’s action. Greenspan and Carrier\(^{10}\) initiated the investigation of the flow of incompressible electrically conducting viscous fluid past a semi-infinite plate in the presence of a magnetic field coincident with the ambient fluid velocity. Pavlov\(^{11}\) provided the exact solution for the fluid flow in the presence of the uniform magnetic field. Davies\(^{12}\) investigated the flow of electrically conducting fluid past a magnetized surface. For the boundary layer to have developed, he considered the large values of the magnetic Reynolds and Reynolds number. Some recent development related to MHD flow can be seen in articles\(^{13–15}\) and the references given therein.

In 1997, Todd\(^{16}\) presented a family of unsteady boundary layer flow past a movable surface evolving from a moving slot. A unique set of transformations encompassing the Blasius–Rayleigh–Stoke the variable was proposed by him to reduce the governing partial differential equations into a similar form. Heat transfer analysis was conducted by Fang et al.\(^{17}\) for this boundary layer flow. Lu et al.\(^{18}\) conducted a detailed analysis of heat transfer of unsteady flow of viscous-based nano-fluids past a plate evolving from a movable slot using Blasius–Rayleigh–Stoke variable. For recent work, related to Blasius–Rayleigh–Stoke variable see the articles\(^{19–22}\) and references given therein.

Keeping the fact in the view, that the unsteady flows are more generalized and the applications of nanofluids and magnetic field to minimize drag and heat loss, the present article analyzes the unsteady flow of nanofluid past an isothermal magnetized plate. Unsteady governing equations are transformed into similar form considering Blasius–Rayleigh–Stokes variable. Effects of different considered parameters are also observed on velocity, temperature, magnetic, and concentration fields. Dual solutions are also observed.

**Mathematical formulation**

The problem of unsteady MHD flow and heat transfer of nanofluid past a magnetized plate is discussed in present paper. The surface is emerging out horizontally from a moving slit (see Figure 1). The fluid is initially at rest. In this section, the fundamental equations governing the flow and heat transfer of nanofluid fluid with magnetic field are discussed. The equations of conservation mass and momentum are:

\[ \nabla \cdot \mathbf{u} = 0 \quad (1) \]

\[ \rho \frac{D\mathbf{u}}{Dt} = \nu \nabla^2 \mathbf{u} + \mathbf{J} \times \mathbf{B} \quad (2) \]

where \( \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \), \( \rho \) is density of fluid, \( \nu \) is kinematic viscosity, \( \mathbf{u} = (u, v) \) is the velocity vector, \( \mathbf{J} \) is electric current density, \( \mathbf{B} = (B_x, B_y) \) represent the magnetic induction vector, \( \tau \) is the stress tensor for the fluid and \( \mathbf{J} \times \mathbf{B} \) is the Lorentz force. Maxwell’s equation in reduced form may be written as:

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (i) \]
\[ \nabla \cdot \mathbf{B} = 0, \quad (ii) \]
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad (iii) \]

Where (i) is Faraday’s law (ii) is Gauss’s law for Magnetism (iii) is Ampere’s Law along with Ohm’s law:

\[ \mathbf{J} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad (3) \]

Here \( \mathbf{E} \) is applied electric field, and \( \sigma \) is the electrical conductivity. The energy equation of nanofluid and continuity equation for nanoparticles with magnetic field are:

\[ \rho c_f \frac{DT}{Dr} = k \nabla^2 T + \rho c_p \Phi + \frac{1}{\sigma} \mathbf{j} \cdot \mathbf{j} \quad (4) \]
\[ \frac{D\mathbf{b}}{Dr} = \nabla \left( D \Phi \nabla \phi + D_T \frac{\nabla T}{T} \right) \quad (5) \]

The corresponding governing boundary layer equations may be written as:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6) \]
The associated boundary conditions are:

\[ u = U_w, \quad v = 0, \quad T = T_w, \quad \phi = \phi_w, \quad B_x = B_{\alpha}, \]

\[ B_y = 0 \quad \text{at} \quad y = 0, \]

\[ u \to 0, \quad T \to T_{\infty}, \quad \phi \to \phi_{\infty}, \quad B_x \to 0 \quad \text{as} \quad y \to \infty. \]

(12)

where \( u, v \) are velocity components and \( B_x, B_y \) are magnetic field components in the \( x \) and \( y \) direction respectively and \( \beta \) represents magnetic diffusivity of the fluid. \( D_B \) is Brownian motion where \( D_T \) is thermodifficent diffusion. \( T \) is used for fluid temperature, \( \alpha \) is thermal diffusivity of the fluid, \( \phi \) is nanoparticles volume fraction, \( \varepsilon \) is ratio of heat capacities of nanoparticles and base fluid.

A modified form of Blasius and Rayleigh–Stokes variables is generalized by Todd\textsuperscript{16} to obtain similar form of equations for unsteady boundary layer flow and heat transfer, named as Blasius–Rayleigh–Stokes variable, is given as:

\[ \eta = \frac{y}{\sqrt{\cos(\alpha)\nu t + \sin(\alpha)\frac{\nu x}{U_w}}}. \]

(13)

To obtain similar form of equations (6)–(12) we have following similarity variable:

\[ \psi = U_w \sqrt{\cos(\alpha)\nu t + \sin(\alpha)\frac{\nu x}{U_w}} \sqrt{f(\eta)}, \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad s(\eta) = \frac{\phi - \phi_{\infty}}{\phi_w - \phi_{\infty}}. \]

(14)

Here \( \psi \) is stream function and defined as \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \). Thus velocity components are:

\[ u = U_w \psi'(\eta), \quad v = \frac{\nu}{2} (\eta \psi'(\eta) - f(\eta)) \frac{\sin(\alpha)}{\sqrt{\cos(\alpha)\nu t + \sin(\alpha)\frac{\nu x}{U_w}}}. \]

(15)

Magnetic field components are:

\[ B_x = B_{\alpha} \psi'(\eta), \quad B_y = \frac{B_{\alpha} \nu}{2U_w} (\eta \psi'(\eta) - f(\eta)) \frac{\sin(\alpha)}{\sqrt{\cos(\alpha)\nu t + \sin(\alpha)\frac{\nu x}{U_w}}}. \]

(16)

Satisfying the continuity equations (6) and (8), the similar form of governing equations is given as:

\[ f'''' - \frac{1}{2} \text{Sc} \sin(\alpha)g'''' + \frac{1}{2} \eta \cos(\alpha)f'''' + \frac{1}{2} \sin(\alpha) ff'''' = 0 \]

\[ \frac{1}{Pm} g'''' + \frac{1}{2} \eta \cos(\alpha)g'''' + \sin(\alpha)g''''f - \sin(\alpha) f''''g = 0 \]

\[ \frac{1}{Pr} \theta'''' + \frac{1}{2} \cos(\alpha)\eta \theta'''' + \frac{1}{2} \eta \theta''''f' + \sin(\alpha)(\eta f'' - f)\theta'' - N_k s' \theta' - N_r \theta'' + \frac{1}{4} \text{ScBr} \sin^2(\alpha)(f'2(\eta g' - g)) + g'2(\eta f'' - f) = 0 \]

\[ s'' + S \left( \frac{1}{2} \cos(\alpha) s''' + \frac{1}{2} \sin(\alpha) f''' \right) + \frac{N_l}{N_b} \theta'' = 0 \]

(19)

subject to the following boundary conditions:

\[ f(\eta) = 0, f'(\eta) = 1, g(\eta) = 0, g'(\eta) = 1, \theta(\eta) = 1, s(\eta) = 0 \quad \text{at} \quad \eta = 0, \]

\[ f'(\eta) = 0, g'(\eta) = 0, \theta(\eta) = 0, s(\eta) = 0 \quad \text{as} \quad \eta \to \infty, \]

(20)

(21)

(22)

where \( \text{Sc} \) is the magnetic interaction parameter, \( \text{Pm} \) is magnetic Prandtl number, \( \text{Pr} \) is Prandtl number, \( N_k \) is thermophoretic parameter, \( N_B \) is Brownian diffusion parameter, \( S \) is Schmidt number, \( B_r \) is Brinkman number, \( E_c \) is Eckert number with following expressions:

\[ \text{Sc} = \frac{\mu B_{\alpha}^2}{\rho U_w}, \quad \text{Pm} = \frac{\nu}{\gamma}, \quad \text{Pr} = \frac{\nu}{\alpha}, \quad N_k = \frac{\varepsilon D_B \nabla \phi}{\alpha}, \]

\[ N_B = \frac{\varepsilon D_T \nabla T}{\alpha T_{\infty}}, \quad S = \frac{\nu}{D_B}, \quad B_r = \text{Ec Pr}, \quad E_c = \frac{U_w^2 / \text{VT}_{cp}}{\text{VT}_{cp}}. \]
Result and discussion

In this section, numerical results are discussed graphically. Discussion is made for different values of $\alpha$, since in the case of this unique time dependent flow, a movable slot having constant speed $-U_w \cot(\alpha)$ is considered. For $\alpha = \pi/2$ the velocity will behave like Sakiadis flow. When $\alpha \to 0$, it is observed that the speed of the moving slot has infinite value in opposing direction of stretchable surface similar to Rayleigh starting plate problem. For $0 < \alpha < \frac{\pi}{2}$, constant speed of moving slot in the opposing direction of stretchable surface give rise the situation named as leading edge accretion. Moving slot and stretching sheet has same direction for the situation named as leading edge ablation. Numerical domain $\alpha_L < \alpha < \alpha_U$ is specified for skin friction. Non-linear shooting technique is used to obtain the numerical solution of the boundary value problem (17–22), since for generalized value of $\alpha$ exact analytical solutions is difficult to find.

The crux of is to reduce the boundary value problem to an initial value problem and then use a shooting numerical technique to guess the missing initial conditions until the boundary conditions are satisfied. The resulting differential equations can then be integrated using initial value solver like Runge–Kutta technique.

Skin friction coefficient is one of the physical quantities of scientific value and is defined as:

$$ C_f = \frac{\tau_w}{\rho U_w^2} $$

where $\tau_w$ is shear stress and can be represented as:

$$ \tau_w = \mu \frac{\partial u}{\partial y} |_{y=0} $$

Using expression of $\tau_w$ in equation (23) reduced skin friction coefficient is obtained as:

$$ \text{Re}C_f = f''(0) $$

In Figure 2 reduced skin friction coefficient is plotted against the moving slot parameter $\alpha$. Fang et al.\textsuperscript{17} stated that the lower limit is $\alpha_L = -53.55^\circ$ and the upper limit is $\alpha_U = 92^\circ$ for the occurrence of the solution. This range of $\alpha$ is also observed by Ahmed and Razi\textsuperscript{19} for magnetic interaction parameter $Sc = 0.0$. It is noticed that interval for the existence of dual solution is strongly affected by magnetic field. Dual solution exists for $\alpha$ in the interval $(\alpha_L, -\pi/4)$ when magnetic field is absent. A unique phenomenon is observed that the range of dual solution decreases by increasing $Sc$. For $Sc = 0.1$, dual solution does not exist. For $Sc = 0.01$ the maximum skin friction is observed about $\alpha = 31^\circ$. This maximum value move toward left for increasing values of $Sc$ and for $Sc = 1.0$, this maxima is noticed at $\alpha = -20^\circ$. At $\alpha = 0^\circ$ it is noticed that skin friction does not depend upon $Sc$ and all curves have common point at $\alpha = 0^\circ$. Moreover for $\alpha > 0$, skin friction is decreasing function of the magnetic parameter $Sc$ while for $\alpha < 0$ skin friction is increasing function of $Sc$.

In Figure 3(a) and (b), the velocity profile is plotted to elaborate leading edge accretion and ablation for increasing values of magnetic field parameter $Sc$. It is observed that for leading edge ablation, fluid velocity reduces for increasing values of $Sc$. An opposite behavior is noticed for leading edge accretion. In Figure 4(a) and (b), velocity dual solutions and also of shear stress are shown graphically for $\alpha = -51^\circ$ and different values of the magnetic parameter. For boundary layer thickness of upper and lower solution branch opposite behavior is noticed. For increasing values of $Sc$, interval of existence of dual solution vanishes and for $Sc = 0.1$, no dual solutions are observed. Also, shear stress is shown graphically as for increasing values of magnetic parameter shear stress increases and duality vanishes for higher values of $Sc$.

Next, we will make discussion about different involved parameters effects on the magnetic flux at the surface. The coefficient of magnetic flux is defined as:

$$ C_M = \beta \frac{\partial g}{\partial y} |_{y=0} B(x)^2, $$

Using value of $\frac{\partial g}{\partial y}$ and $B(x)^2$ in above expression, the magnetic flux coefficient at the surface can be written as:

$$ M_B = \frac{\text{Re}_{M_H}}{\text{Re}_{s}} C_M = g''(0) $$
In Figure 5(a) and (b) graphs for magnetic flux at the surface to moving slot parameter $\alpha$ are shown for varying $Sc$ (magnetic field parameter) and $Pm$ (magnetic Prandtl number). It is noticed that the lower solution curve disappear with an increase in $Sc$ (magnetic field parameter) and $Pm$. Increment in magnetic flux on the surface is noticed for increasing values of $Pm$. At $\alpha = 0$, magnetic flux is maximum, that is, for Rayleigh starting plate problem. A flow with strong magnetic diffusion corresponds to smaller $Pm$. For decreasing magnetic Prandtl number, decrease in magnetic diffusion of the flow became stronger, which results decrement in magnetic flux at the surface. The effects of leading edge accretion and ablation on magnetic field are shown in Figure 6(a) and (b). For $\alpha = 0$, minimum value of magnetic boundary layer is noticed. Magnetic boundary layer thickness increases for increasing $|\alpha|$. In Figure 7(a) and (b) the dual solutions of magnetic field and magnetic flux are shown graphically for different values of $Pm$. For increasing values of $Pm$, decrement in boundary layer thickness is noticed due to dominant effects of viscous forces to magnetic forces. Magnetic flux is maximum at the surface while it is zero away from the surface. Thickness of magnetic boundary layer for upper branch is smaller as compared to lower solution curve.

In Figure 8(a) and (b), Nusselt number is plotted by considering effects of Brownian $Nb$ and thermophoretic

Figure 3. Velocity profile for different value of magnetic interaction parameter $Sc$ with $Pm = 0.1$, $Nb = 0.1$, $Nt = 0.1$, and $S = 1.0$: (a) $\alpha = 30$ and (b) $\alpha = -30$.

Figure 4. Different curves of: (a) velocity and (b) shear stress for different values of magnetic interaction parameter $Sc$ for $\alpha = -51$, $Pmt = 0.1$, $Nb = 0.1$, $Nt = 0.1$, and $S = 1.0$.
diffusion parameters $N_t$. It is noticed that for increasing values of $N_b$ and $N_t$, Nusselt number reduces. Since maximum Brownian and thermophoretic diffusion corresponds to maximum temperature as a result of which reduction in surface heat flux is also observed. Moreover, dual solutions appear for specific range of moving slot parameter $\alpha$ and this specific range can be seen in Figure 8. A significant observation is made here, that increment in $N_b$ and $N_t$ dramatically reduces range of $\alpha$ and also the duality of solution vanishes as value of these parameters reach to 0.1. In Figure 9, temperature profile is plotted for increasing values of Brownian and thermophoretic diffusion parameter for specific value of movable slot parameter $\alpha = -51^\circ$. For increasing values of $N_b$ and $N_t$, increased value of temperature is noticed while thickness of boundary layer of upper solution curve is smaller as compared to lower solution curve. It is interesting to notice a visible change in temperature profile, close to the surface for upper solution curve. In Figure 10, temperature profile and Nusselt number is plotted for different values of magnetic interaction parameter $Sc$. When magnetic field is absent, then dual solution exists for $\alpha \in (\alpha_L, -\pi/4)$.$^{15}$ The range of dual solution start shrinking for increasing values of $Sc$. For $\alpha = 0.01$ the range of dual solution is observed to be $(-53.6, -51.2)$ and dual solutions diminish for $Sc = 0.05$. Effects of magnetic field for the flow in leading edge accretion are negligible as compare
to leading edge ablation. In Figure 11, temperature profile and Nusselt number is plotted for different values of Prandtl number. Upper solution decreases for increasing values of Prandtl number as by increasing Prandtl number thermal diffusivity decreases results decrement in temperature profile. Where lower solution curve increases and then starts decreasing. It can be observed that for increasing values of Prandtl number dual solutions start disappearing. Nusselt number increases for increasing values of Prandtl number.

Local Nusselt number is defined as;

$$\text{Nu} = \frac{hL}{k}$$  \hspace{1cm} (27)

Here $h$ is heat flux and can be written as;

$$h = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}$$

By using value of $h$ in equation (25) we have following expression;

$$\text{Re}^{-1}\text{Nu} = -\theta'(0)$$  \hspace{1cm} (28)
which is defined as reduced Nusselt number.

Figure 12 is the graphical representation of thermophoretic and Brownian diffusion effects for nanoparticles concentration flux at the surface. Graphical representation of Sherwood number to moving slot parameter $a$ is also shown. The range of dual solution become shrink for increasing values of $Nb$ and $Nt$. Sherwood number is non-decreasing function of $a$ for specific range ($\frac{2}{54}$, $30^\circ$), and reducing function for the range ($30^\circ$, $\alpha_c$). Concentration flux at surface decreases for increasing values of $Nb$ and $Nt$. In contrast to thermophoretic diffusion, negligible effect of Brownian diffusion parameter is observed on lower solution curve of Sherwood number. Figure 13 is graphical representation of nanoparticles concentration profile for increasing values of (a) Brownian motion parameter $N_b$ and also (b) thermophoretic diffusion parameters $N_t$. Boundary layer thickness of nanoparticle concentration decreases for upper solution and increases for lower solution curves for increasing values Brownian and thermophoretic diffusion parameters. In Figure 14, nanoparticle concentration profile and also Sherwood number is plotted for increasing values of magnetic interaction parameter $Sc$. Concentration flux
at surface is increasing function of $Sc$. Also the length of the interval of dual solution decreases with an increase in $Sc$ and ultimately vanished for $Sc = 0.05$.

Local Sherwood number can be written as:

$$Sh = \frac{h_{\text{mass}}L}{D} \quad (29)$$

And $h_{\text{mass}}$ is mass flux and it is defined as:

$$h_{\text{mass}} = -D \frac{\partial \phi}{\partial y} \bigg|_{y=0}$$

Using above, the expression of reduced Sherwood number may be written as:

$$\text{Re}^{-1}Sh = -s'(0) \quad (30)$$

Figure 15 is portrayed to elaborate the effects of magnetic field on heat and concentration flux at the surface in the presence of nanoparticles. Effects of Brownian and thermophoretic diffusion parameters have already been discussed but it is important to note that the heat and nanoparticles concentration fluxes can be manipulated with magnetic field by keeping the quality and quantity of nanoparticles fixed. It is further observed that the magnitude of variation in Nusselt number with respect to magnetic interaction parameter $Sc$ remain constant with an increment in Brownian and thermophoretic diffusion parameters. On the contrary, the magnitude of change in Sherwood number enhances
with an increment in thermophoretic diffusion parameter and it decreases for increasing values of Brownian diffusion parameter.

**Conclusion**

Present article is a discussion about unsteady boundary layer flow of nanofluid past a moving magnetized plate, evolving from a movable slot having constant speed $-U_w \cot(\alpha)$. This problem is similar to stretching/shrinking sheet problem and it depends upon direction of motion. Effects of the magnetic field on skin friction, velocity, and temperature are discussed in detail.

Effects of different involved parameters on temperature and nanofluid concentration are shown graphically and discussed in detail. Moreover, detailed discussion is made about effects of Brownian motion parameter, Lewis number and thermophoretic diffusion parameter upon reduced Nusselt number and Sherwood number.

Results are concluded as follow,

- For $\alpha < 0$ skin friction becomes reducing function of $Sc$ but opposite behavior is observed for $\alpha > 0$. For $\alpha = 0$ skin friction is constant. For leading edge accretion and ablation velocity
profile shows opposite results for different values of magnetic interaction parameter \( \text{Sc} \).

- Magnetic flux is increasing function of magnetic Prandtl number. Duality vanishes for increasing values of magnetic Prandtl number.
- Nusselt number decreases for higher values of \( N_t \). Duality in solution appears for small values of \( N_t \) and disappears for increasing values of \( N_t \). For Nusselt number, \( N_t \) showed same behavior as of \( N_t \). For increasing values of \( N_t \) and \( N_b \), temperature increases and increasing behavior of temperature reduces surface heat flux.
- Increment in Nusselt number for increasing values of Prandtl number is noticed. Thickness of Thermal boundary layer has maximum value for lower solution while minimum for upper solution curve.
- Duality in solutions is observed for Sherwood number for specific range of movable slot parameter and also when Lewis number is minimized.
- For increasing thermophoretic diffusion, Sherwood number reduces while opposite behavior is examined for Brownian motion parameter.
- For lower solution curve, thickness of concentration boundary layer is greater while opposite behavior is noticed for upper solution curve.
- Constant change is observed in Nusselt number with respect to magnetic interaction parameter \( \text{Sc} \) for increasing Brownian and thermophoretic diffusion. While the magnitude of change in Sherwood number is enhanced for increment in thermophoretic diffusion whereas an opposite behavior is observed for increasing values of Brownian diffusion.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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