Nonrelativistic one-hadron-exchange k`p interaction model

H Fadhlurahman*, A Salam, I Fachruddin

Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok 16424, Indonesia
E-mail: hamzah.fadhlurahman@ui.ac.id

Abstract. We have derived a nonrelativistic K`p interaction model as one-hadron-exchanges. The exchanged particles are the scalar meson σ, the vector mesons ω, ρ, the hyperons Λ, Σ, and the resonances Λ*(1600), Σ*(1385). Parameters in this model are determined by fitting to experimental data of spin-averaged differential cross section for kaon laboratory energy range of 51.27 MeV up to 900.64 MeV. The K-p scattering is calculated using a 3D technique without partial wave expansion.

1. Introduction
One of interesting topics in the field of nuclear and particle physics is how hadrons interact with each other. The purpose of this work is to derive an interaction model for K`p to be used in calculations of nonrelativistic processes. The interaction is formulated as one-hadron-exchange potential, since it is more practical in the application. The exchanged hadrons are the scalar meson σ, the vector mesons ω, ρ, the hyperons Λ, Σ, and the resonances Λ*(1600), Σ*(1385). The model is derived from Feynman diagrams describing K`p scattering by referring to the work in Reference [1]. Note, we also add the Σ* exchange from K`p model to the calculation [7]. To make it applicable to nonrelativistic calculations, we perform some reduction following Blankenbecler-Sugar reduction [2]. The parameter of the potential are determined by fitting to the data of the spin-averaged differential cross section for kaon laboratory energy range of 51.27 MeV up to 900.64 MeV.

As nowadays experiments can be done at higher energies, suitable theoretical calculation technique is needed. This technique does not use the standard partial wave expansion and is usually called a three-dimensional (3D) technique. We implement a 3D technique described in Reference [3] to calculate K`p scattering.

2. Kp scattering
The potential model is derived from Feynman diagrams describing scattering and is modified using Blankenbecler-Sugar reduction, so it can be used in nonrelativistic process. Parameters of this model are obtained by fitting to the experiment data using MINUIT subroutine. The experiment data are the spin-averaged differential cross section for Kaon laboratory energy of 51.27 MeV to 900.64 MeV [4, 5].

Note: The original text contains references [1], [7], [2], [3], [4], and [5], which should be included in the final version of the document.
2.1. Basic state

According to the 3D technique [3], we define the basis state \( |\mathbf{p}\lambda\tau\nu\rangle \) as a combination of the free state \( |\mathbf{p}\rangle \), the spin state \( |\lambda\rangle \), and the total-isospin state \( |\tau\nu\rangle \) of the system:

\[
|\mathbf{p}\lambda\tau\nu\rangle \equiv |\mathbf{p}\rangle |\lambda\rangle |\tau\nu\rangle,
\]

with \( \mathbf{p} \) being the relative momentum, \( \lambda = \pm \frac{1}{2} \) the spin projection in an arbitrary fixed z axis, \( \tau \) the total isospin, and \( \nu \) the z-component of the total isospin. The normalization of the basis state of Equation (1) is

\[
\langle \mathbf{p}'\lambda'\tau'\nu'|\mathbf{p}\lambda\tau\nu\rangle = \delta(\mathbf{p}' - \mathbf{p})\delta_{\lambda\lambda'}\delta_{\tau\tau'}\delta_{\nu\nu'},
\]

and the completeness relation is as follows

\[
\sum_{\lambda\tau\nu} \int d\mathbf{p}|\mathbf{p}\lambda\tau\nu\rangle\langle \mathbf{p}\lambda\tau\nu| = 1.
\]

2.2. Potential matrix elements

In the 3D basis state given in Equation (1), we define the potential matrix elements \( V_{\lambda\lambda'}^{TV}(\mathbf{p}', \mathbf{p}) \) as

\[
V_{\lambda\lambda'}^{TV}(\mathbf{p}', \mathbf{p}) \equiv \langle \mathbf{p}'\lambda'\tau'\nu'|\mathbf{p}\lambda\tau\nu\rangle = \langle \lambda'|(\tau\nu|\mathbf{p}|\mathbf{p})|\lambda\rangle = \langle \lambda'|(\tau\nu|\mathbf{p}^\prime)|\lambda\rangle,
\]

with \( V^{TV}(\mathbf{p}', \mathbf{p}) \) is the potential matrix elements in the momentum space for the given total isospin. The general form of \( V^{TV}(\mathbf{p}', \mathbf{p}) \) is

\[
V^{TV}(\mathbf{p}', \mathbf{p}) = f_0^{TV}(\mathbf{p}', \mathbf{p}, \mathbf{p}' \cdot \mathbf{p}) + f_1^{TV}(\mathbf{p}', \mathbf{p}, \mathbf{p}' \cdot \mathbf{p})(s \cdot \mathbf{p}')(s \cdot \mathbf{p}),
\]

with \( f_i^{TV}(i = 0,1) \) being spin-independent functions and \( s = \frac{1}{2} \sigma \) is a spin \( \frac{1}{2} \) operator, where \( \sigma \) is the Pauli matrix. Substituting Equation (5) into Equation (4) we get the potential matrix elements \( V^{TV}(\mathbf{p}', \mathbf{p}) \) as follows

\[
V_{\lambda\lambda'}^{TV}(\mathbf{p}', \mathbf{p}) = \delta_{\lambda\lambda'}(f_0^{TV}(\mathbf{p}', \mathbf{p}, \mathbf{p}' \cdot \mathbf{p}) + \frac{1}{4} f_1^{TV}(\mathbf{p}', \mathbf{p}, \mathbf{p}' \cdot \mathbf{p})
\]

\[
\times \{\cos \theta'\cos \theta + e^{-2i\lambda(\phi' - \phi)}\sin \theta'\sin \theta\}
\]

\[
+ \delta_{\lambda', -\lambda} \frac{\lambda}{2} e^{2i\lambda\phi'} f_1^{TV}(\mathbf{p}', \mathbf{p}, \mathbf{p}' \cdot \mathbf{p})
\]

\[
\times \{\sin \theta'\cos \theta - e^{-2i\lambda(\phi' - \phi)}\cos \theta'\sin \theta\}.
\]

For \( \mathbf{p} = \mathbf{2} \), we get

\[
V_{\lambda\lambda'}^{TV}(\mathbf{p}', \mathbf{2}) = \delta_{\lambda\lambda'}(f_0^{TV}(\mathbf{p}', \mathbf{p}, \cos \theta') + \frac{1}{4} f_1^{TV}(\mathbf{p}', \mathbf{p}, \cos \theta')\cos \theta')
\]

\[
+ \delta_{\lambda', -\lambda} \frac{\lambda}{2} e^{2i\lambda\phi'} f_1^{TV}(\mathbf{p}', \mathbf{p}, \cos \theta')\sin \theta'.
\]

Because of the Kronecker delta, we can put \( e^{-i(\lambda' - \lambda)\phi'} \) as an overall factor in Equation (7). Thus, the potential matrix elements \( V_{\lambda\lambda'}^{TV}(\mathbf{p}', \mathbf{2}) \) depends on the azimuthal angle \( \phi' \)

\[
V_{\lambda\lambda'}^{TV}(\mathbf{p}', \mathbf{2}) = e^{-i(\lambda' - \lambda)\phi'} V_{\lambda\lambda}^{TV}(\mathbf{p}', \mathbf{0}, \mathbf{p}),
\]

with
The potential matrix elements $V_{\lambda'\lambda}^T(p', \theta', p)$ have symmetry relations as follows

$$V_{\lambda'\lambda}^T(p', \theta', p) = (-)^{\lambda' - \lambda} e^{-2i(\lambda' - \lambda)\phi'} V_{-\lambda'\lambda}^T(p', p')$$  \hspace{1cm} (10)

$$V_{\lambda'\lambda}^T(p', \lambda', p) = (-)^{\lambda' - \lambda} V_{-\lambda'\lambda}^T(p', \theta', p).$$  \hspace{1cm} (11)

2.3. T-Matrix Elements and Spin-Averaged Differential Cross Section

We define the T-matrix elements in the 3D basis state similarly as

$$T_{\lambda'\lambda}^T(p', p) \equiv \langle p' | T | p \rangle_{\lambda'\lambda}.$$  \hspace{1cm} (12)

The T-matrix elements defined in Equation (12) satisfies the Lipmann-Schwinger equation for T-matrix in the 3D basis state is written as

$$T_{\lambda'\lambda}^T(p', p) = V_{\lambda'\lambda}^T(p', p) + \lim_{\epsilon \to 0} \sum_{\lambda}^{1/2} \int dp \sum_{\lambda'=-1/2}^{1/2} \frac{V_{\lambda'\lambda}^T(p', p')}{E_p - E_{p'} + i\epsilon} T_{\lambda'\lambda}^T(p''', p),$$  \hspace{1cm} (13)

with $E_p = \frac{p^2}{2\mu}$ and $E_{p'} = \frac{p'^2}{2\mu}$. Being rotational invariant, the T-matrix elements have the same property as the potential matrix elements given in Equation (8), thus

$$T_{\lambda'\lambda}^T(p', p) = e^{-i(\lambda' - \lambda)\phi'} T_{\lambda'\lambda}^T(p', \theta', p).$$  \hspace{1cm} (14)

The matrix elements $T_{\lambda'\lambda}^T(p', \theta', p)$ satisfy the following integral equation

$$T_{\lambda'\lambda}^T(p', \theta', p) = V_{\lambda'\lambda}^T(p', \theta', p) + 2\mu \lim_{\epsilon \to 0} \sum_{\lambda}^{1/2} \int_{-1/2}^{1/2} dp'' \frac{p''^2}{p^2 + i\epsilon - p''^2}$$

$$\times \int_{-1}^{1} d \cos \theta'' V_{\lambda'\lambda}^{\lambda',\lambda}(p', \theta', p', \theta'', p) \},$$  \hspace{1cm} (15)

with $V_{\lambda'\lambda}^{\lambda',\lambda}(p', \theta', p', \theta'')$ being defined as

$$V_{\lambda'\lambda}^{\lambda',\lambda}(p', \theta', p', \theta'') = \int_{0}^{2\pi} d \phi'' V_{\lambda'\lambda}^{\lambda',\lambda}(p', \phi'') e^{i(\lambda' \phi'' - \lambda \phi')} e^{-i(\phi'' - \phi')}.\hspace{1cm} (16)$$

We obtain a symmetry relation for $V_{\lambda'\lambda}^{\lambda',\lambda}(p', \theta', p', \theta'')$ as

$$V_{\lambda'\lambda}^{\lambda',\lambda}(p', \theta', p', \theta'') = (-)^{\lambda' - \lambda} V_{-\lambda'\lambda}^{\lambda',\lambda}(p', \theta', p', \theta'').$$  \hspace{1cm} (17)

Applying Equation (17), we obtain a symmetry relation for $T_{\lambda'\lambda}^T(p', \theta', p)$ being similar to that for $V_{\lambda'\lambda}^T(p', \theta', p)$ given in Equation (11). Thus,

$$T_{\lambda'\lambda}^T(p', \theta', p) = (-)^{\lambda' - \lambda} T_{-\lambda'\lambda}^T(p', \theta', p).$$  \hspace{1cm} (18)

As usual in scattering calculations, the incident momentum $p$ is chosen as the z-axis, thus $\phi' = 0$ and
Applying the symmetry relation in Equation (18), we obtain the spin-averaged differential cross section as

$$\frac{d\sigma}{d\hat{p}'} = (2\pi)^4 \mu^2 \left( |T_{11}^{\text{c}}(p, \theta', p)|^2 + |T_{11}^{\text{c}}(p, \theta', p)|^2 \right).$$  \hfill (20)

In Equation (20), all possible total isospin states are already calculated in $T_{2\Lambda}(p, \theta', p)$ as

$$T_{2\Lambda}(p, \theta', p) = \sum_{\tau} C^2 (\tau_1 \tau_2 \tau; \nu_1 \nu_2 \nu) T_{1\Lambda}(p, \theta', p).$$  \hfill (21)

### 2.4. KP Interaction Model

We construct the Kp interaction model as a one-hadron-exchange potential. The Feynman diagram of the interaction is shown in Figure. 1. Note that we also include the resonance of the hyperon ($\Lambda^*$ and $\Sigma^*$). After applying the Blankenbecler-Sugar reduction [2], we obtain the interaction model in operator form as

$$V_\sigma(p', p) = \frac{-g_{KK\sigma}g_{NN\sigma}}{32\pi^2 m_N m_K \sqrt{E_N + \omega_K} \sqrt{E_N + \omega_K}} \frac{(m_N + m_K)}{E_N + \omega_K} \times \frac{F_{NN\sigma}[\hat{(p'-p)}^2] F_{K\sigma}[\hat{(p'-p)}^2]}{(p'-p)^2 + m_\sigma^2} \frac{\partial_\sigma(p', p)}{(\omega_K \omega_K W'W')^{1/2}}, \hfill (22)$$

$$V_\nu(p', p) = \frac{g_{KK\nu}g_{NN\nu}}{64\pi^2 m_N m_K \sqrt{E_N + \omega_K} \sqrt{E_N + \omega_K}} \frac{(m_N + m_K)}{E_N + \omega_K} \times \left(1 + \frac{f_{NN\nu}}{g_{NN\nu}} \right) \frac{F_{NN\nu}[\hat{(p'-p)}^2] F_{K\nu}[\hat{(p'-p)}^2]}{(p'-p)^2 + m_\nu^2} \frac{\partial_\nu_{1}(p', p) + \frac{f_{NN\nu}}{g_{NN\nu}} \partial_\nu_{2}(p', p)}{(\omega_K \omega_K W'W')^{1/2}}, \hfill (23)$$
\[
V_Y(p', p) = \frac{-g_{\pi KK}^2}{128 \pi^3 m_n m_K} \frac{(m_N + m_K)}{\sqrt{E_n' + \omega_{KK}} \sqrt{E_n + \omega_K}} \frac{F_{\pi KK}^2[(p' + p)^2]}{E_n + \omega_K} \frac{\partial Y(p', p)}{\omega_{KK} W W} \frac{1}{1/2}.
\]

(24)

\[
V_{\Lambda^*}(p', p) = \frac{g_{\Lambda K}^2}{128 \pi^3 m_n m_K} \frac{(m_N + m_K)}{\sqrt{E_n' + \omega_{KK}} \sqrt{E_n + \omega_K}} \frac{F_{\Lambda K}^2[(p' + p)^2]}{E_n + \omega_K} \frac{1}{1/2} \frac{1}{\omega_{KK} W W} \frac{1}{1/2} \frac{\partial \Lambda^*(p', p)}{\omega_{KK} W W}.
\]

(25)

\[
V_{\Sigma}(p', p) = \frac{g_{\Sigma KK}^2}{256 \pi^3 m_n m_K} \frac{(m_N + m_K)}{\sqrt{E_n' + \omega_{KK}} \sqrt{E_n + \omega_K}} \frac{F_{\Sigma KK}^2[(p' + p)^2]}{E_n + \omega_K} \frac{1}{1/2} \frac{1}{\omega_{KK} W W} \frac{1}{1/2} \frac{\partial \Sigma^*(p', p)}{\omega_{KK} W W}.
\]

(26)

With \(\partial_\sigma(p', p), \partial_{\nu 1}(p', p), \partial_{\nu 2}(p', p), \partial_T(p', p), \partial_{\Lambda^*}(p', p), \) and \(\partial_{\Sigma^*}(p', p)\) being given as

\[
\partial_\sigma(p', p) = m_K(W W - \sigma \cdot p' \sigma \cdot p),
\]

(27)

\[
\partial_{\nu 1}(p', p) = W p'^2 + W p^2 + W W (\omega_K + \omega_K + (W' + W + \omega_K + \omega_K)) \sigma \cdot p' \sigma \cdot p,
\]

(28)

\[
\partial_{\nu 2}(p', p) = \frac{1}{2 m_n}(E_n' + E_n)(\omega_K + \omega_K) + (p' + p)^{1/2} \sigma \cdot p' \sigma \cdot p - W W,
\]

(29)

\[
\partial_T(p', p) = (2m_N - m_N - \omega_K - E_N)W W + (m_N - 2m_N - \omega_K - E_N) \sigma \cdot p' \sigma \cdot p,
\]

(30)

\[
\partial_{\Lambda^*}(p', p) = \sigma \cdot p' \sigma \cdot p,
\]

(31)

\[
\partial_{\Sigma^*}(p', p) = \omega_K(E_n' - 3m_K + m_{\Sigma}) + W p'^2 \omega_K(2m_K - \omega_K - E_n' + m_{\Sigma}) \]

\[
+W W (3m_K - E_n' + m_{\Sigma}) + 3W p'^2 + W p^2 \]

\[
+[2 \omega_K (E_n' - \omega_K + m_{\Sigma}) + \omega_K p'^2 - \omega_K p^2] W W
\]

\[
+(W' - W - E_n' - \omega_K + m_{\Sigma}) p'^2 + m_{\Sigma} \]

\[
+W (E_n' - 3m_K - W' - W - m_{\Sigma}) \]

\[
+2 \omega_K (E_n' - 3m_K - W' - W - m_{\Sigma}) + \omega_K p' \]

\[
+p'^2 (W + \omega_K - p'' (W + \omega_K) - (E_n' + \omega_K + m_{\Sigma}) W W
\]

\[
+(3m_K - E_n' + W + 3W + m_{\Sigma}) p' \sigma \cdot p' \sigma \cdot p.
\]

(32)

In Equations (22) – (26) the form factors are given as

\[
F_\sigma(q^2) = \left(\frac{\Lambda_\sigma^2 - \frac{m_{\pi}^2}{4}}{\Lambda_\sigma^2 - q^2}\right)^{n_\pi},
\]

\[
F_\rho(q^2) = \left(\frac{\Lambda_\rho^2 - \frac{m_{\pi}^2}{4}}{\Lambda_\rho^2 - (q^2)}\right)^{n_\rho}.
\]

(33)

with \(q^2_r = \sigma, \omega, \rho\) and \(q^\mu_r\) being the momentum transfers or momenta of the particles being exchanged:

\[
q^2_r = p' - p,
\]

\[
q^\mu_r = (p' - p_K)^\mu
\]

\[
= (E_n' - \omega_K, p' - p_K),
\]

\[
= (E_n' - \omega_K, p' + p),
\]

(34)
and additional isospin factor of $\tau_1 \cdot \tau_2$ for $\rho$ exchange, $\frac{1}{2} (1 + \tau_1 \cdot \tau_2)$ or $\frac{1}{2} (1 - \tau_1 \cdot \tau_2)$ for $\Lambda$ exchange, and $\frac{1}{2} (3 - \tau_1 \cdot \tau_2)$ or $\frac{1}{2} (3 + \tau_1 \cdot \tau_2)$ for $\Sigma$ exchange.

3. Results and discussion

We fit to the experiment data of the differential cross section for Kaon laboratory energy of 51.27 MeV to 900.64 MeV [4]-[5] to get the parameters used in this model. First, we give the initial values of the parameters that are to be fitted with the experiment data to obtain the new values of the parameters. The initial values of the parameters are taken from Reference [1], but for the nucleon-nucleon-meson vertices we refer to Reference [6]. The unfitted parameters are shown in Table 1. Note that $\sigma$ mass is not a constant, thus it also has to be fitted with the initial value of 720 MeV and 550 MeV for $\sigma_0$ and $\sigma_1$, respectively. The indices 0 and 1 for the $\sigma$ particle are the total isospin of the $Kp$ system. The initial value for the parameters are shown in Table 2.

| Particle | Mass (MeV) | $f/g$ | Exponent |
|----------|------------|-------|-----------|
| $\sigma_0$ | -         | 0     | 1         |
| $\sigma_1$ | -         | 0     | 1         |
| $\rho$ | 769       | 6.1   | 2         |
| $\omega$ | 782.6     | 0     | 2         |
| $\Lambda$ | 1116      | 0     | 1         |
| $\Sigma$ | 1193      | 0     | 1         |
| $\Lambda^*$ | 1600      | 0     | 1         |
| $\Sigma^*$ | 1385      | 0     | 1         |

Table 1. Unfitted parameters

| Particle | $g_N/\sqrt{4\pi}$ | $g_K/\sqrt{4\pi}$ | $\Lambda_N (MeV)$ | $\Lambda_K (MeV)$ |
|----------|-------------------|-------------------|-------------------|-------------------|
| $\sigma_0$ | 4.2869           | 0.3009            | 2000              | 1400              |
| $\sigma_1$ | 2.9906           | 0.3009            | 1900              | 1400              |
| $\rho$ | 0.9487            | 0.8285            | 1850              | 1550              |
| $\omega$ | 4.9497            | -0.4762           | 1850              | 1600              |
| $\Lambda$ | -3.944            | -3.944            | 1400              | 1400              |
| $\Sigma$ | 0.759             | 0.759             | 1400              | 1400              |
| $\Lambda^*$ | 0.2               | 0.2               | 2000              | 2000              |

Table 2. Initial value of the parameters.
The fitting process gives $\chi^2/N$ value of 10.39 and new values of the parameters as shown in Table 3. New parameter value of $\sigma_0$ and $\sigma_1$ mass respectively 900 MeV and 700 MeV. The $\chi^2/N$ value is obtained using Equation (35).

$$
\frac{\chi^2}{N} = \frac{1}{N_{data} - N_{par}} \sum_{i=1}^{N_{data}} \left( \frac{z_{th} - z_{exp}}{\Delta z} \right)_i^2 
$$

Parameters value obtained from the fitting process are to be used in the differential cross sections calculation. We choose 3 energy to represent the low energy, medium energy, and high energy respectively 60.79 MeV, 500.54 MeV, and 900.64 MeV. The differential cross sections for those 3 energy shown in Figure 2. Figure 2. shows that at higher energies the cross sections obtained from the calculations match better the experiment data.

To see the significance of the exchanged particles, we eliminate the contribution of each exchanged particle by setting the coupling constant equals zero ($g_N = 0$ and $g_K = 0$). After that, we recompute the differential cross sections and compare it with the previous calculations. If those two calculations show a big differences, it means the contribution of the particle is large.
Figure 2. Differential cross sections for energy range (a) low, (b) medium, and (c) high.
Figure 3. Differential cross sections for energy range (a) low, (b) medium, and (c) high without mesons $\sigma$, $\omega$, and $\rho$ exchange.
Figure 4. Differential cross sections for energy range (a) low, (b) medium, and (c) high without baryons $\Lambda$ and $\Sigma$ exchange.
Figure 5. Differential cross sections for energy range (a) low, (b) medium, and (c) high without $\Lambda^*$ and $\Sigma^*$ exchange.
In Figures 3 – 4 it is clearly shown that the exchanged particles $\sigma$, $\rho$, $\omega$, $\Lambda$, and $\Sigma$ have large contribution. On the contrary, shown in Figure 5 the exchanged particles $\Sigma^*$ and $\Lambda^*$ show little contributions. Furthermore, the $\sigma$ particle has a significant contribution at forward angles in medium and high energy ranges, while in low energy range it contributes significantly at forward and backward angle. The $\rho$ particle has significant contribution at forward angle in the whole energy range. The $\omega$ particle contributes significantly at all angle in all energy ranges. $\Lambda$ particle has a significant contribution at the backward angle in all energies, but contributes largely at all angle in low energy ranges. $\Sigma$ particle has a small contribution in medium and high energy ranges. While Figure 5 shows that $\Sigma^*$ and $\Lambda^*$ have a very small contribution in this model. This small contribution also shown in Table 3 that $\Sigma^*$ and $\Lambda^*$ have an almost-zero coupling constant value.

4. Conclusion
We have derived a $Kp$ interaction model as one-hadron-exchanges for nonrelativistic calculations. The potential parameters are determined by means of fitting processes to experimental data of differential cross section for kaon’s laboratory energy up to around 900 MeV. We obtain a $\chi^2/N$ value of 10.39, which is a large number. The model, Thus still needs to be improved. We check the contributions from each particle exchange and found that $\Sigma^*$ and $\Lambda^*$ give a very small contribution.

5. References
[1] Muller-Groeling A, Holinde K and Speth J 1990 $K\bar{N}$ Interaction in Meson Exchange Framework. Nucl. Phys. 513 pp 557-583
[2] Blankenbecler R and Sugar R 1966 Linear Integral Equations for Relativistic Multichannel Scattering. Phys. Rev. 142 pp 1051
[3] Salam A and Fachruddin I 2013 $KN$ Scattering in 3D Formulation. Few Body System 54
[4] Adams C J, Davies J D, Dowell J D, G H Grayer, Hattersley P M, Homer R J, Howells R J, McLeod C, McMahon T J, Van der Raay H B, Rob L, Darnell C J S and Motchkiss M J 1975 Measurement of $Kp$ Elastic Differential Cross Sections between 610 and 943 MeV/c. Nucl. Phys. 96 pp 54-66
[5] Mast T S, Alston-Garnjost M, Bangerter R O, Barbaro-Galtieri A S, Solmitz F T and Tripp R D 1976 Total $Kp$ Cross-Sections in the Momentum Range 220 to 470 MeV/c. Phys. Rev. D. 14 pp 13
[6] Machleidt R 1989 The Meson Theory of Nuclear Forces and Nuclear Structure. Adv. Nucl. Phys. 19 p 189
[7] Buttgen R, Holinde K, Muller-Groeling A, Speth J and Wyborny P 1990 Meson Exchange for $KN$ Interaction. Nucl. Phys. A. 506 pp 585-614

Acknowledgments
This research also supported by research grant PUTI of Universitas Indonesia