An air cavity above a complex vortex: an experimental and analytical study of the features of its lower part

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Abstract. The description of the experiments with the water vortex generated in a variety of facilities by means of different activators is presented. The changes in the form and size of the air cavity above such a vortex are studied for different regimes of rotations. An analytical description for the free surface form of the water vortex is given, which is in good agreement with both experimental data and previously obtained relations based on fundamental hydrodynamics equations. The problem of the “spout’s” volume of the air cavity is also considered. It is shown that “spout’s” volume can be used for calculating of the radius of the water “solid-state” rotation. Estimated calculations of the “spout's” volume based on experimental data are made.

1. Introduction

Vortex and wave flows are hereditarily studied in fluid mechanics. The purport to go on in this study has grown significantly in recent years owing to increasing importance of flow structure features, which allow to detect specific objects automatically. The structure features of vortex flows – closed or spiral trajectories of certain particles, – allow to identify single vortices over complex processes background and improve vortex theory. Vortex flows are observed in natural air and water flows (tornadoes, typhoons, etc.) and are widely used in various technical applications such as aeronautics, heat exchange, spray drying, separation, enrichment, combustion and so forth.

Information delivered by satellites may reveal vortex flow pattern in the Ocean. Recording from space is performed in visible, infrared, and radio ranges and demonstrated the existence of wide variety of different-scale vortex formations with a lifetime of several days to several weeks (Fig. 1) alongside with large-scale long-lived quasi-stationary vortices in the Ocean. It became possible to refine notion of the space-time scales, mechanisms of formation, evolution, and dissipation of vortex structures.

The regular study of vortex motion in fluids, started in the fundamental work of Helmholtz [1] and continued by outstanding scientists of the XIX and XX centuries Kelvin [2, 3], Prandtl [4], Poincare [5], Zhukovsky [6] and others, remains relevant to the present time, as evidenced by a large number of monographs and papers [7–10].

Vortex flows have different structures in the depth and on the free surface of the liquid, where co-exist and interact with waves of different provenance. Although the existence of various types of vortices in contact with free surface (such as vertical Rankin vortex or Helmholtz semiloop) [11] are known for a long time, important issues of vortex dynamics and influence on the transport of matter require thoughtful analysis.
Figure 1. Intense vortices: a) Maelstrom Whirlpool in the Norwegian sea [12]; b) Salstraumen Whirlpool [13]; c) marine environment structure offshore the Gotland Island in Baltic Sea [14]

The study of the vortex flows dynamics in such complex systems as natural reservoirs is accompanied by many methodic and fundamental features: the extreme complexity of full-scale experiments, intricacy and variability of hydrodynamic fields and meteorological conditions during research, as well as in some cases – the complexity and variability of the properties of the transferred substance. In this regard, the study of vortex flows that can be generated in laboratory facilities with stable external conditions are of particular interest. In case of laboratory modelling it is possible to avoid problems related to the spatial and temporal variability of natural sources of vortex formation and directly observe and log the characteristic flow parameters.

This paper presents the results of experimental and analytical studies of the structure and features of vortex flow with free surface generated in laboratory facilities in a wide range of determining parameters.

2. Experimental modelling of vortex flows: facilities, methods and parameters of the studied flows in laboratory

Now several centers for geophysical flows modeling are actively operating in the world. In particular cases stationary vortex flow studies are performed in facilities that provide global rotation of the liquid, when the liquid volume is located on rotating platform or in containers with uniformly rotating activators.

The implementation of stationary vortex flows in laboratory conditions is also described in [15] for container with rotating activators within finite volume of liquid, and in [16], where a flow with a free surface is studied in container with various rotating activators. The study of the vortex flow that occurs when a liquid flows out of a rotating cylinder is described in [17]. A series of experiments are carried out using cylindrical containers, in which the liquid flow is initiated by rotation of both endwalls at once [18–20], showed results similar to those of experimental studies, where the source of vorticity is only one rotating endwall of the cylindrical chamber.

In this paper, laboratory experiments are performed in two facilities that allow inducing stationary vortex flow and controlling its parameters:

1. Facility Vortex Flow with Torsion (VFT) created in the Laboratory of Fluid Mechanics of IPMech RAS to study the swirling flow, and is a part of unique facilities complex UNU GFK IPMech RAS. The vortex flow is created by a rotating disc mounted on the bottom of transparent cylindrical container immersed in rectangular tank to reduce optical distortion. The complete description of VFT facility and flow diagrams are given in [21].

2. Part of the experiments are carried out on facilities for flows simulation, where the vortex is generated using magnetic anchor set in motion by magnetic field Vortex Flows with Magnetic Inductor (VFMI, Fig. 2). Magnetic stirrers used as a vortex flow activators: Intllab MS-500 and ES-6120 (with heating). The necessary substance in container (pool) made of glass or special plastic is
placed on the platform of the magnetic stirrer. An anchor (a magnet with an inert fluorine plastic coating) is placed directly inside the container, and is driven into rotation.

The containers utilized in experiments are of different shapes and sizes (rectangular, square, triangular, and cylindrical), the free surface kept open to simplify the introduction of admixtures and registration of the fine structure of the flow.

![Figure 2. Vortex Flows with Magnetic Inductor (VFMI) with a test sample of liquid (a) and the implied containers (b) (base edge of the triangle and square containers is 30 cm, the cylinder diameter is 25 cm)](image)

The complex flow that occurs in this liquid volume with elementary geometry includes both vortex and wave components both in the depth and on the free surface of the liquid and is composed of two vortices superposition – one with vertical axis and toroidal other one.

Observations show that on the surface of a rotating liquid there is always a deflection region – an air cavity, the depth of which depends on combination of all flow parameters. According to the results of the experiments, three types of surface caverns are consistently registered: smooth; bumpy – with relatively large spiral perturbations running along the surface; full form cavern – with developed wave perturbations of two types, systems of inertial (larger) and spiral (Fig. 3, a – b) surface waves.

In the case of a smooth surface of the air cavity, its depth grows monotonously towards the center, the lower part has parabolic form, typical for solid-state liquid rotation. The system of wave perturbations detected on a free surface (for bumpy and full form caverns) does not change its position relative to the accompanied to the vortex coordinate system and is steadily reproduced in a wide range of flow parameters (Fig. 3, d).

Implementation of disks with radial ribs of any height significantly increases the radial component of the velocity, the frequency of rotation of the liquid in the complex vortex, and, consequently, the depth of the cavern. The generalized dependence of the depth of a surface of the air cavity in a deep liquid on the rotational velocity for the case of smooth (d) or ribbed (e) disk with radius $R = 5.00\text{ cm}$ with radial ribs $h_r = 1\text{ mm}$ high is shown in Fig. 3 (d, e) on a double logarithmic scale. In a wide range of parameters, experimental data are approximated by a function $h = \exp(A\Omega^B)$.

In experiments with the flow induced by rotating magnetic anchor, similar patterns are observed (the determining parameters are the ratio of the depth and radius of the container, the radii of the rotating activator and the container, as well as the frequency of rotation of the activator, see Fig. 4). The experiments carried out have shown that changes in the density and viscosity of the basic fluid have very little effect on the form and depth of the free surface deflection above the induced vortex flow. The range of density changes explored is from 0.950 to 1.090 kg/m$^3$, the viscosity varied from 1 to 4 cSt.
Figure 3. Typical surfaces of the pure liquid above the rotating disk: a) smooth \((H = 40 \text{ cm}, \ R_d = 7.5 \text{ cm}, \ \Omega = 460 \text{ RPM})\), b) bumpy \((H = 30 \text{ cm}, \ R_d = 5.0 \text{ cm}, \ \Omega = 480 \text{ RPM})\), c) ripened \((H = 20 \text{ cm}, \ R_d = 5.0 \text{ cm}, \ \Omega = 720 \text{ RPM})\); g) wave perturbations system on the free surface; d, e) the depth of surface cavern in the deep fluid vs. the inductor rotation rate (for smooth and ribbed disks, respectively) in double-logarithm scale.

Figure 4. Characteristic forms of the free surface over vortex flow \((H = 15.5 \text{ cm}, \ R_d = 5.3 \text{ cm})\) for clear water and with salt or glycerol added: a, c) pure water, frequency of inductor rotation 460 and 720 RPM, b) water with 12\% of glycerol 460 RPM, d) water with 6\% of glycerol 720 RPM.

Synthesis of all the experimental data results in the conclusion that there is no explicit difference between fresh and salt water, and when supplementing glycerol, neither the shape of the cavity, nor its depth does not change prominently.

All air cavities that occur when a liquid is put into rotation in a cylindrical container in VFMI facility at the same rotation rates of the same activator are similar. Constancy is also specific for the dynamics of increase the air cavity depth with growth of activator rotation velocity. Central cross-sections of the air cavity over rotating activator in different fluids under similar conditions are presented in Fig. 5.
Figure 5. Central section profiles of the complex vortex free surface (diameter of the cylindrical container is 11.8 cm, depth of the initial liquid layer is 15.5 cm, anchor linear size 53 mm, the activator rotation velocity 444 RPM) to compare with air cavity profile in clear water and: a – with supplemented glycerol in an amount of 100 or 200 ml, b – with supplemented salt to the density values of 1.05 and 1.11 g/m³

An air cavity in water with the addition of glycerol (Fig. 5, a) grows more narrow as the viscosity/surface tension of the base liquid decreases. An increase in the base liquid density does not change the form of the cross-section of the free surface over complex vortex (Fig. 5, b). The profile in base liquid with supplemented ethyl alcohol results in slight increase in the depth of the air cavity surface, while maintaining its width – therefore, this additive affects the most on the surface tension, and as a result – the form of the lower part of the air cavity, which becomes more acute.

3. Analytical description of the flow

The analytical description of the flow characteristics in a vortex cavity presented below develops the results of the articles [21, 22], the mathematical model of which is based on the fundamental equations of hydrodynamics. The vortex flow is induced in the cylindrical container with radius $R_0$, the initial depth of water in the container is $H$. The flow is induced by the rotating central part of the bottom endwall of cylinder (which is disk) of radius $R$. The angular rotation velocity of disk is $\Omega$. The liquid itself is characterized by the kinematic viscosity coefficient $\nu$, and surface tension coefficient $\alpha$ [21]. Accordingly to results received in [22], if the air cavity does not touch the disk that induces the flow, the deviation of the surface from the unperturbed level is described as (1)

$$
\zeta = b(1 - \mu f(\tilde{r})) ,
$$

$$
\mu^{-1} = p^2 \left(3 - 2 \ln p + \frac{2\tilde{r}^2}{\kappa^4} \left(\frac{I_0(\tilde{r})}{I_1(\tilde{r})} - \frac{\tilde{r}^2}{4} - 2\right)\right) ,
$$

$$
f(\tilde{r}) = \left[\frac{2\tilde{r} (I_0(\tilde{r}) - I_0(\tilde{\tilde{r}}))}{\tilde{r}^2 I_1(\tilde{r})} + \tilde{r}^2 - \tilde{\tilde{r}}^2\right] \vartheta(\tilde{r} - \tilde{\tilde{r}}) + \left(2 - \frac{\tilde{r}^2}{\tilde{\tilde{r}}^2}\right) \vartheta(\tilde{\tilde{r}} - \tilde{r}) + \frac{\tilde{r}^2}{\tilde{\tilde{r}}^2} \vartheta(\tilde{\tilde{r}} - \tilde{r}) ,
$$

where $\tilde{r} = kr$, $\tilde{\tilde{r}} = kr$, $\tilde{\tilde{r}} = kr$, $p = r / R_0$, $k^2 = g / \alpha$, $I_n(x) -$ modified Bessel function of the first kind.

The expression (1) includes three parameters: $b$, $\tilde{r}$, and $r$, that are generally determined by geometry ($R_0$, $R$ and $H$), other properties and characteristics of the system, which are calculated from the experimental data (in particular, based on the form of vertical cross-sectional contour of the air cavity, as it is shown in Fig. 6).
In the case where the air cavity is deep and reaches bottom wall of the container, and the central part of rotating disk is dried, the shape of the air cavity is described by another expression (2), which contains only one empirical parameter $r$ – radius of the part of disk that is free from water [22].

$$\zeta \approx -H \left\{ \frac{1}{1 + \frac{p^2}{(2 \ln p - 1)} \left( \frac{\rho_r^2}{\rho_f} \right)} \right\}$$

Comparison of analytically obtained and experimentally registered air cavity form is carried out for the value $R_0 = 14.7$ cm. The results of the comparison are shown in Fig. 6 which consists of the registered water surface view and the overlapped calculated zero-order vortex cavity form [22] above it for different experimental parameters.

**Figure 6.** Flow scheme (a) with notation and registered pattern (b) with zero-order approximation overlapped [22]

In the regime of high-velocity rotation of activator ($\Omega > 500 \text{ min}^{-1}$) the feature of air cavity form is observed. The volume of air in the “spout” of the air cavity (the air region bounded by the upper level of “solid-state” rotation) is the constant value and does not depend on the sizes and geometry of container. Besides, the volume of air in the “spout” does not depend the physical properties of the medium. Experiments have shown that in the intensive flow, the volume of air in the air cavity is the same in both fresh and salt water, as well as in water with the addition of different amounts of alcohol and glycerol. The surface tension coefficients of the fresh and salt water ($\alpha = 73 \text{ g/s}^2$) and weak solutions of glycerol (5% solution, $\alpha = 65 \text{ g/s}^2$) and alcohol (10% solution, $\alpha = 50 \text{ g/s}^2$) in it at temperature 20°C vary quite widely. At the same time, the volume of the air in the air cavity remains unchanged. This means that this volume does not depend on the surface tension coefficient of the rotating liquid medium and is determined primarily by its inertial properties, that is, the density and angular speed of rotation, as well as the external gravitational force.

One of the obvious properties of the water vortex is the conservation of rotating water mass. In assumption of the surface tension absence the form of free vortex surface in zero-order approximation is described by the relation

$$\zeta(r) = \zeta_0 + \frac{\alpha^2}{2g} \left[ r^2 \theta(a - r) + a^2 (2 - a^2/r^2) \theta(r - a) \right]$$

where $a$ is the radius of “solid-state” rotation, and the mass conservation low has the form

$$\int_0^{R_0} \zeta(r) r \, dr = 0 \tag{4}$$

The deviation of the free surface from its undisturbed state $z = 0$ is $\zeta(r)$. The volume $V_s$ of water in the region of positive deviations over the undisturbed level, as it follows from (4), is equal to the
volume of the air $V_0$ in the deep part of the air cavity (see Fig. 7). The integration of (4) gives the result defining the depth of cavity’s deep part in the center

$$\zeta_0 = -\frac{a^2 \omega^2}{gR_0^2} \left( R_0^2 - a^2 \left( \frac{3}{4} + \ln \frac{R_0}{a} \right) \right) < 0$$  \hspace{1cm} (5)

The inequality in (5) follows from the fact that $R_0^2 - a^2 (0.75 + \ln(R_0/a)) > 0$ when $R_0 \geq a$ as it is registered in experiments. The angular frequency $\omega$ in the relations (3) – (5) is the frequency of “solid-state” rotation of the water.

The substitution of (5) into (3) gives the expression for the air cavity shape in that the only unknown parameter, namely the “solid-state” rotation frequency, is presented. The height of the water rise on the container’s wall, i.e. when $r = R_0$, is defined by relation

$$\zeta(R_0) = -\frac{a^2 \omega^2}{4gR_0^2} \left( 1 - 4\ln \frac{a}{R_0} \right) > 0$$  \hspace{1cm} (6)

For the goal of the further investigations it is convenient to present (5) in the form

$$\zeta_0 = -\frac{a^2 \omega^2}{g} \left( 1 - \frac{a^2}{R_0^2} \left( \frac{3}{4} - \ln \frac{a}{R_0} \right) \right)$$  \hspace{1cm} (7)

The experimental results show that for high-velocity rotations of activator the radius of “solid-state” rotation is decreased and it is sufficiently less then radius of container, so the condition is valid

$$\frac{a}{R_0} \ll 1$$  \hspace{1cm} (8)

Simultaneously with this fact the experiments show that the depth of the drop in the center of air cavity is proportional to the frequency of activator rotation, i.e.

$$|\zeta| \sim \Omega$$  \hspace{1cm} (9)

If the hypothesis that $\omega \sim \Omega$ is taken as the basis, then regarding (7) – (9) it follows

$$|\zeta_0| \sim \Omega, \quad \zeta_0 \sim \omega; \quad a \sim \Omega^{-1/2}, \quad a \sim \omega^{-1/2}$$  \hspace{1cm} (10)

and from (9) the expression for the height of water rise at the container wall can be obtained

$$\zeta(R_0) \sim A + B \ln \Omega, \quad \zeta(R_0) \sim A + B \ln \omega$$  \hspace{1cm} (11)
where \( A \) and \( B \) are some values defined by gravity acceleration and geometric sizes of container but with no relation to the activator rotation frequency. In accordance with (10) the following relation is valid

\[
a = \frac{a_0}{\sqrt{\Omega}}
\]  

(12)

The \( a_0 \) constant has the dimension \([a_0] = \text{cm/}\sqrt{\text{s}}\) and is fixed for containers with given geometric sizes and given type of activator. Then it is possible to define the \( a_0 \) value on the basis of experimental results it is also likely to predict the radius of “solid-state” rotation for all the range of rotation frequencies regardless the experiments are not yet performed.

In cases when \( \Omega > 500 \text{ min}^{-1} \) the expression for the depth of the air cavity center has an approximate form

\[
\zeta_0 \approx \frac{a_0^2 \omega_0}{g}
\]  

(13)

On the basis of (3) and results above the volume of air “spout” can be calculated

\[
V = \frac{2\pi \omega_0^2}{2g} \int_0^a (a^2 - r^2) r \, dr = \frac{\pi a^4 \omega_0^2}{4g} \Omega > 500 \text{ min}^{-1}
\]  

(14)

It is found that for the high-intensity rotation the volume of air “spout“ does not depend on frequency, with the expression (12) for “spout” volume this brings up the following

\[
a_0 = \left(\frac{4gV}{\pi}\right)^{1/4}
\]  

(15)

With regard of (13) it means that the “solid-state” rotation radius under conditions mentioned above is defined by the relation

\[
a = \left(\frac{4gV}{\pi \Omega^2}\right)^{1/4}
\]  

(16)

This is quite an engineering formula that can be used for practical calculations. For example, by measuring the radius of “solid-state” rotation, the volume of the “spout” can immediately be calculated.

Experimentally obtained flow patterns (the series of experiments with liquid depth 200 mm, activator radius 50 mm and three values of rotation frequencies 60, 70, 90 rad/s) are processed to find out volumes of the “spout“ (forms of “spouts“ are presented in Fig. 8).

\[\text{Figure 8.} \ \text{The form of the “spout” (scale in mm) for} \ H = 200 \text{ mm,} \ R_d = 50 \text{ mm} \quad (R_0 = 300 \text{ mm}): \ \text{curves} \ 1 - 4 \ \text{frequencies of inductor rotation 60, 68, 72, 90 rad/s}\]
The “solid-state” rotation radii are measured and calculated on the basis of the (16). The results are provided in Table 1. Unfortunately the experimental results are interfered by many factors for accurate measurements, so the inaccuracy rate reaches 15%.

| Frequency of inductor rotation, rad/s | Radius (experiment), mm | Spout volume, mm$^3$ | Radius (calculated), mm |
|--------------------------------------|-------------------------|----------------------|-------------------------|
| 60                                   | 9.0                     | 356.33               | 8.48                    |
| 70                                   | 7.0                     | 385.09               | 8.03                    |
| 70                                   | 7.0                     | 314.71               | 7.63                    |
| 90                                   | 6.3                     | 424.89               | 7.11                    |

### 4. Conclusion

Experimental studies of vortex flow of homogeneous liquid in containers of various geometries, as well as at various physical parameters of experiments at original laboratory facilities for the research of the formation and structure of vortex flow in wide range of determining parameters were done.

It is found that the form of the free surface vortex cavity cross section depends in different measure on all the physical parameters of the problem (depth of the liquid layer, radius and shape of flow activator, velocity of rotation, liquid properties). The depth of the surface air cavity increases monotonically with growth of velocity of the activator rotation.

An analytical relation depicting the universal geometry of air cavities that occur in cylindrical containers under action of the coaxial rotating disk is obtained. This relation coincides with both experimental data and the previously obtained model of vortex flow [22].

The overlapping of the free surface forms acquired through analytical relations on those experimentally observed gives good coincidence that indicates the applicability of the developed simplified theoretical description of this flow type presented here. The calculated and measured “solid-state” rotation radii correspond each other well.

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