From Binary Error Correcting Codes
to a Relation Between Maximal D=4 and D=3 Supergravities

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Abstract

This short note provides (TensorFlow-based) numerical evidence for the embedability (in the limit of a scalar parameter going to infinity) of the scalar potential of dyonic $N=8$, $D=4$ $SO(8)$ supergravity into the scalar potential of $N=16$, $D=3$ $SO(8)\times SO(8)$ supergravity. One finds that the dyonic $\omega$-rotation gets identified with the compact $U(1)$ part of the $SL(2)$ factor of the $SL(2)\times E_{7(+7)}$ subgroup of $E_{8(+8)}$.

1 Claims and Insights

This short note is accompanied by a Google Colab notebook\footnote{Available at [https://github.com/google-research/google-research/tree/master/m_theory/colab/hamming78.ipynb] and also alongside the arXiv source code of this article. The reader can launch this via web browser by navigating to [https://colab.research.google.com/] selecting ‘GitHub’ as source for a new notebook, and pasting the above url.} (based on TensorFlow \cite{tensorflow}) that numerically demonstrates the validity of each of these claims:

1. (From \cite{2}, Eq. (7.5)): One can embed $M_{14} := (SU(1,1)/U(1))^7$ in such a way into $E_{7(+7)}$ that the holomorphic superpotential is in 1:1 correspondence with the code words of the 1-bit error correcting (7,4,3) Hamming code \cite{3}:

$$W_7 := +\zeta_1\zeta_2\zeta_3\zeta_4\zeta_5\zeta_6\zeta_7$$
$$+\zeta_3\zeta_5\zeta_6\zeta_7 + \zeta_2\zeta_4\zeta_5\zeta_7 + \zeta_2\zeta_3\zeta_4\zeta_6 + \zeta_1\zeta_3\zeta_4\zeta_5 + \zeta_1\zeta_4\zeta_6\zeta_7 + \zeta_1\zeta_2\zeta_5\zeta_6 + \zeta_1\zeta_2\zeta_3\zeta_7$$
$$+\zeta_1\zeta_2\zeta_4 + \zeta_1\zeta_3\zeta_6 + \zeta_1\zeta_5\zeta_7 + \zeta_2\zeta_6\zeta_7 + \zeta_2\zeta_3\zeta_5 + \zeta_3\zeta_4\zeta_7 + \zeta_4\zeta_5\zeta_6$$
$$+1.$$ (1)

2. Expanding the (7,4,3) Hamming code with a parity bit to the self-dual (8,4,4) Hamming code, we can define a corresponding hypothesized holomorphic superpotential as follows, by adding a factor $\zeta_8$ to those summands that have an odd number of $\zeta$-factors:

$$W_8 := +\zeta_1\zeta_2\zeta_3\zeta_4\zeta_5\zeta_6\zeta_7\zeta_8$$
$$+\zeta_3\zeta_5\zeta_6\zeta_7 + \zeta_2\zeta_4\zeta_5\zeta_7 + \zeta_2\zeta_3\zeta_4\zeta_6 + \zeta_1\zeta_3\zeta_4\zeta_5 + \zeta_1\zeta_4\zeta_6\zeta_7 + \zeta_1\zeta_2\zeta_5\zeta_6 + \zeta_1\zeta_2\zeta_3\zeta_7$$
$$+\zeta_1\zeta_2\zeta_4\zeta_8 + \zeta_1\zeta_3\zeta_6\zeta_8 + \zeta_1\zeta_5\zeta_7\zeta_8 + \zeta_2\zeta_6\zeta_7\zeta_8 + \zeta_2\zeta_3\zeta_5\zeta_8 + \zeta_3\zeta_4\zeta_7\zeta_8 + \zeta_4\zeta_5\zeta_6\zeta_8$$
$$+1.$$ (2)
Observing that the scalar potential corresponding to such a holomorphic superpotential on \((SU(1,1)/U(1))^{\times 8}\) indeed does have many equilibria that align nicely (after rescaling the cosmological constant) with equilibria reported in \cite{4} for \(\mathcal{N} = 16\), \(D = 3\) \(SO(8) \times SO(8)\) supergravity, one may conjecture that one can indeed obtain this \(“(8,4,4)\) Hamming code holomorphic superpotential” from the \(A_1\)-tensor of maximal \(D = 3\) supergravity. This indeed holds – the details can be found in appendix \cite{B}.

3. Starting from the commonly used roots for the \(e_{8(+8)}\) algebra, where the \(120 - 8 = 112\) roots of the compact \(\text{spin}(16)\) subalgebra are given by \((\pm 1; \pm 1; 0; 0; 0; 0; 0; 0) + \{\text{permutations}\}\), and the \(128 \text{“spin}(16)\text{-spinor}\) roots corresponding to the generators used to define the scalar manifold of \(SO(8) \times SO(8)\) supergravity \cite{5,6}, \((\pm \frac{1}{2}; \pm \frac{1}{2}; \pm \frac{1}{2}; \pm \frac{1}{2}; \pm \frac{1}{2}; \pm \frac{1}{2}; \pm \frac{1}{2}; \pm \frac{1}{2})\) (where the total number of \((-)\) signs is \(\text{even}\)), it is possible to choose eight positive roots from the 128 such that when adding the corresponding eight negative roots to the set, no pair taken from these 16 roots have the same sign in exactly two positions\cite{3}. For any such choice, adding the corresponding negative roots, and encoding a \((+)-\)sign as 1 and a \((-)\)-sign as 0 (or vice versa) gives us sixteen eight-bit code words that correspond to a self-dual \((8,4,4)\) Hamming code\cite{3}. These sixteen roots then correspond to a \(\mathfrak{s}(2)^{\times 8}\) subalgebra of \(e_8\).

4. Performing \(\omega\)-deformation \cite{9,10,11,12} of \(\mathcal{N} = 8\), \(D = 4\) \(SO(8)\) supergravity, \(E_7(2)\), the superpotential in Eq. \ref{1} acquires phase factors \(\phi := \exp(-i\omega)\) on summands with an \(\text{odd}\) number of \(\zeta\)-factors and \(\bar{\phi} = \exp(+i\omega)\) on summands with an \(\text{even}\) number of \(\zeta\)-factors:

\[
W_{7c} := +\zeta_1 \zeta_2 \zeta_3 \zeta_4 \zeta_5 \zeta_6 \zeta_7 \bar{\phi} + \zeta_2 \zeta_4 \zeta_5 \zeta_6 \zeta_7 \bar{\phi} + \zeta_1 \zeta_3 \zeta_4 \zeta_5 \zeta_6 \zeta_7 \bar{\phi} + \zeta_1 \zeta_2 \zeta_5 \zeta_6 \zeta_7 \bar{\phi} + \zeta_1 \zeta_2 \zeta_3 \zeta_6 \zeta_7 \bar{\phi} + \zeta_1 \zeta_2 \zeta_3 \zeta_4 \zeta_7 \bar{\phi} + \zeta_1 \zeta_2 \zeta_3 \zeta_4 \zeta_5 \zeta_7 \bar{\phi} + \zeta_1 \zeta_2 \zeta_3 \zeta_4 \zeta_5 \zeta_6 \zeta_7 \bar{\phi} + \bar{\phi}.
\]

(3)

Observe that the scalar potential does not change if the superpotential gets multiplied by a complex number of magnitude 1, and multiplying the above expression with \(\phi\) shows \(\bar{\phi}W_{7c} = W_{8\zeta=\phi^2}\). Indeed, one finds that for \(\omega = \pi/8\), the corresponding scalar potential on \((SU(1,1)/U(1))^{\times 7}\) has equilibria for which the cosmological constants closely correspond to known solutions of the ‘dyonic \(SO(8)\)’ gauging with \(\omega = \pi/8\) \cite{14}. The relation between the scalar potentials and superpotentials is given in appendix \cite{B}.

5. The above properties suggest that, at least on \((SL(2)/U(1))^{\times 7} \sim (SU(1,1)/(2)/U(1))^{\times 7}\), we might be able to retrieve the scalar potential of \(D = 4\) \(SO(8)\) supergravity from that of \(D = 3\) \(SO(8) \times SO(8)\) supergravity by taking some suitable \(\zeta_8 \rightarrow 1\) limit\cite{L} and correspondingly, get the scalar potential of dyonic \(D = 4\) \(SO(8)\), supergravity by taking some \(\zeta_8 \rightarrow \exp(i\omega)\) limit. Hence, it seems natural to expect that a corresponding limit may exist for the full scalar potential: Using the \(E_7(+7) \times SL(2) \subset E_{8(+8)}\) embedding for which we have \(248 \rightarrow (133, 1) + (56, 2) + (1, 3)\), the \(SL(2)\) becomes the eighth \(SL(2)\) in \(E_8\) that commutes with the seven \(SL(2)\)s whose noncompact directions yield \(M_{14}\). Considering the triality-symmetric constructions of \(e_7 = \text{spin}(8) + 35_v + 35_s + 35_c\) and \(e_8 = \text{spin}(8)^L + \text{spin}(8)^R + (8^L_v, 8^R_v) + (8^L_s, 8^R_s) + (8^L_c, 8^R_c)\), it is

\[\text{This would be a requirement for the commutator of the associated ladder operators to belong to \text{spin}(16), but not the u(1)^8 generated by the commutators of the ladder operators for each positive root and its associated negative root.}\]

\[\text{A related well-known observation is that scaling the self-dual} \ E_8 \text{ lattice to integer coordinates and then taking coordinates modulo 2 yields the \(8,4,4\) self-dual Hamming code. Doing the same for the} \ E_7 \text{ root lattice yields the \((7,3,4)\) ‘dyonic SO(8)’ gauging, while doing this for the dual} \ E_7 \text{ weight lattice} \ (E_7^*) \text{ yields the \(7,4,3\) Hamming code, see e.g.} \cite{7,8}.\]

\[\text{Given that the} \ \zeta \text{ parameters are coordinates in the Poincaré disc model of the hyperbolic plane, this is at infinite distance from the origin.}\]
clear how $\mathfrak{e}_7 + \mathfrak{sl}(2)$ is obtained from the ‘symmetric’ pieces of the decomposition of $\mathfrak{e}_8$ with respect to the diagonal $\text{spin}(8)$ subalgebra of $\text{spin}(8)^L + \text{spin}(8)^R$.

Using the corresponding embedding of the $\mathfrak{e}_{7(7)} + \mathfrak{sl}(2)$ $D = 4$ scalar manifold coset generators $35_s + 35_c + 1_s + 1_c$ (‘symmetric traceless $8 \times 8$ matrices over the spinors and co-spinors from $\mathfrak{e}_7$ plus multiples-of-the-identity trace-parts from $\mathfrak{sl}(2)$’) into the space of $D = 4$ scalar manifold coset generators $(8^L_s, 8^R_s) + (8^L_c, 8^R_c)$ via a linear function $E(v_{70}, s, c) : \mathbb{R}^{70+2} \to \mathbb{R}^{128}$, one finds for the $D = 3$ scalar potential of $SO(8) \times SO(8)$ supergravity: $g_{D=3}^2 V_{D=3}(E(0, s, 0)) < 0$, and for $s > 0$: $|\nabla V| > 0$. These are non-equilibrium points with negative cosmological constant. If we now introduce an auxiliary (helper) function $H : \mathbb{R}^{70+1} \to \mathbb{R}$ as:

$$H(\vec{v}, s) := (-6) \cdot \frac{V_{D=3}(E(\vec{v}, s, 0))}{V_{D=3}(E(0, s, 0))}.$$  

then we may conjecture that $H$ is related to the $D = 4$ scalar potential of $SO(8)$ supergravity $g_{D=4}^2 V_{D=4} : \mathbb{R}^{70} \to \mathbb{R}$ via:

$$V_{D=4}(\vec{v}) = \lim_{s \to \infty} H(\vec{v}, s).$$  

Numerical evidence strongly supports that this hypothesis holds on the full 70-dimensional scalar manifold of $N = 8$, $D = 4$ $SO(8)$ Supergravity!

6. The generalization to dyonic-$SO(8)$ also holds. Specifically, with

$$H_c(\vec{v}, s, \omega) := (-6) \cdot \frac{V_{D=3}(E(\vec{v}, s \cos(2\omega), s \sin(2\omega)))}{V_{D=3}(E(0, s \cos(2\omega), s \sin(2\omega)))},$$

we find:

$$V_{D=4}(\vec{v}, \omega) = \lim_{s \to \infty} H_c(\vec{v}, s, \omega).$$

(As one would expect from the $\omega$-invariance of the $SO(8)$-symmetric vacuum of $SO(8)$ supergravity, one actually finds $V_{D=3}(E(0, s \cos(2\omega), s \sin(2\omega))) = V_{D=3}(E(0, s, 0))$, so the above expression, presented ‘in symmetric form’, can be simplified.) Appendix A shows the numerical evidence, verifiable by running the accompanying Google Colab notebook.

2 Discussion

The maximal (32 supercharges) gauged $D = 2 + 1$ supergravity of Nicolai and Samtleben [5, 6] so far has been mostly regarded as an exotic curiosity, as to this date there is no known way to embed it into M theory. Correspondingly, it has perhaps not yet received as much attention as this note suggests it should have – given that we observe that it indeed seems to be closely related to the $S^7$-compactification of 11-dimensional supergravity, i.e. the de Wit-Nicolai model – as well as the dyonic deformations of that model [10], for which there currently is no known way to embed these into M theory, either [15, 16]. Naturally, this then means that taking the limit in a different way will also allow us to retrieve scalar potentials of other gaugings with already-known M theory embeddings, such as that of ‘dyonic ISO(7) supergravity’ [17, 18].

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5 Given that we can apply a triality relabeling on one of the $\text{spin}(8)$ algebras, there is more than one way to take a diagonal. The relevant diagonal here does not involve a triality rotation.

6 The factor $-6$ is for alignment with the usual normalization of the $D = 4$ scalar potential.
2.1 Early Clues

As it is often useful to understand the intuition that underlies an idea, it may be appropriate to list some major clues that contributed to generating the idea of exploring the final claim in the list presented above. In chronological order, these clues were:

- The (stable and also unstable) equilibria of maximal supergravities often have remarkable similarities across different dimensions. Notably, this also holds in particular for $D = 4$ and $D = 3$. For example, whereas maximal $D = 5$ supergravity has a $SU(2) \times U(1)$ $\mathcal{N} = 2$ vacuum, maximal $D = 4$ supergravity has a $SU(3) \times U(1)$ $\mathcal{N} = 2$ vacuum; in $D = 4$, we see a $G_2$ $\mathcal{N} = 1$ vacuum, whereas in $D = 3$, we find $G_2 \times G_2$ $\mathcal{N} = (1, 1)$, etc. – see [19, 20, 4].

- As the problem of finding equilibria can be expressed entirely in terms of geometric invariants, the relevant properties of the equilibria can be expressed in terms of algebraic numbers. There is a general tendency for the $D = 3$ expressions to often have remarkably low algebraic complexity (see e.g. [21]), just as if $D = 4$ had to rebalance terms to make up for some loss of a more fundamental symmetry.

- John Baez’s article about triality and the exceptional groups [22] clearly was inspirational for structuring the code that does calculations in $E_7$ in such a way that it emphasizes the role of triality, despite virtually all of the other literature only using (anti)self-dual four-form language for $E_7$.

- Closely studying the long list of equilibria of $SO(8)$ supergravity [23] reveals some remarkable coincidences, such as the existence of a triplet of equilibria with residual symmetry $SO(4)$ where embeddings of $SO(4)$ into $SO(8)$ are related by triality. Likewise, there are closely-related-via-triality pairs of solutions, such as $\text{S0668704} - \text{S0698771}$, $\text{S0869596} - \text{S0983994}$, $\text{S1068971} - \text{S1301601}$, etc., that are related by triality (see also [24], as well as [25]).

- There have been various earlier indications that the 7-bit Hamming code is useful to understand some nontrivial aspects of M theory [26, 27].

2.2 Outlook

It certainly is bemusing to observe how intuition related to binary error correcting codes did provide a relevant clue here towards uncovering a relation between $D = 4$ and $D = 3$ supergravities – especially with a view on Wheeler’s “it from bit” essay [28] which proposes an agenda that includes “[Translating] the quantum versions of string theory and of Einstein’s geometrodynamics from the language of continuum to the language of bits”. One may wonder whether there are more interesting insights that could be obtained by focusing on the relation between remarkable lattices and binary codes – noting however that the (even unimodular Lorentzian) $E_{10}$ root lattice [29] does not directly correspond to an error correcting binary code – likely due to the implicit notion of ‘Euclidean distance’ in the definition of error-correcting codes. This might, however, be fixable, and suggests that a study of the relation between $E_{10}$ and generalized binary codes might bear fruit.

While our focus here was exclusively on the scalar potential, this is of course closely linked to the entire structure of the model supersymmetry. Nominally, we are here observing a correspondence between $D = 3$ and $D = 4$ supergravity in some “AdS radius goes to zero” (i.e. $g^{-2}V \to -\infty$) limit. To do this, we had to ad-hoc fix one scalar parameter and move it towards infinity without Supergravity offering a mechanism to stabilize this configuration. We may, at this point, only speculate whether M theory also in this setting “fights against being squeezed” by growing new spatial dimensions via some tower of massive excitations (which would mean: degrees
of freedom not present in the supergravity truncation) collapsing to zero mass. Given our current understanding of M theory, this speculation is however too outlandish to be taken seriously.

More tangibly, the observation that there is a \( SO(8) \) subgroup of \( E_8(8) \) that rotates the eight commuting \( SL(2) \)s may provide useful to extract additional information about the structure of the \( D = 4 \) potential, given that this \( SO(8) \) cannot be a subgroup of \( E_7 \) (since it mixes the seven \( U(1) \)s sitting inside \( E_7 \) with the one outside). This might lead to an explanation for some observations about the equilibria of the \( D = 4 \) scalar potential that are currently hard to explain, such as the high degeneracies in the mass spectra of the equilibrium \( \text{S1800000} \) despite complete breaking of \( SO(8) \) with zero residual symmetry – neither Lie nor discrete. Signs of a hidden \( E_{8(+8)} \) symmetry in maximal \( D = 4 \) supergravity are, of course, not new (e.g. [30]), and so the hope is that the rather concrete new puzzle piece explained in this work will lead to new angles of attack to resolve the question about the underlying symmetries of M theory.

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**Appendix**

**A Notebook transcript from checking claim 6**

It certainly is gratifying to look at the numbers that substantiate the claim in Eq. (6). Below, we see what happens when one randomly picks ten (generic, non-equilibrium) points on the \( E_7(+) / (SU(8)/\mathbb{Z}_2) \) manifold and, in \( E_8(8) \), rotates outwards using \( SL(2) \), both for \( \omega = 0 \) and some generic \( \omega \).

```plaintext
>>> check_so8c_limit (omega=0, r=3, num_spot_checks=10)
V_so8c = -10.4047666737, V_so8xso8 = -10.4528652474, rel_delta = 0.000083
V_so8c = -9.4520901976, V_so8xso8 = -9.0556327510, rel_delta = 0.000082
V_so8c = -5.0839620894, V_so8xso8 = -5.0848796316, rel_delta = 0.000180
V_so8c = -10.8347526006, V_so8xso8 = -10.834767696, rel_delta = 0.000118
V_so8c = -10.6441227948, V_so8xso8 = -10.6448297935, rel_delta = 0.000066
V_so8c = -7.1489366176, V_so8xso8 = -7.1496894995, rel_delta = 0.0000104
V_so8c = +1.3503469257, V_so8xso8 = +1.3499888355, rel_delta = 0.000106
V_so8c = +26.0683609298, V_so8xso8 = +26.065829595, rel_delta = 0.000095
V_so8c = +9.4490606160, V_so8xso8 = +9.4480316002, rel_delta = 0.000109
V_so8c = +9.7993167162, V_so8xso8 = +9.7976288261, rel_delta = 0.000172

>>> check_so8c_limit (omega=0, r=3.5, num_spot_checks=10)
V_so8c = -10.4047666737, V_so8xso8 = -10.4048838868, rel_delta = 0.000011
V_so8c = -9.4520901976, V_so8xso8 = -9.4521950911, rel_delta = 0.000011
V_so8c = -5.0839620894, V_so8xso8 = -5.0848796316, rel_delta = 0.000011
V_so8c = -10.8347526006, V_so8xso8 = -10.834767696, rel_delta = 0.000011
V_so8c = -10.6441227948, V_so8xso8 = -10.6448297935, rel_delta = 0.000024
V_so8c = -7.1489366176, V_so8xso8 = -7.1496894995, rel_delta = 0.0000104
V_so8c = +1.3503469257, V_so8xso8 = +1.3499888355, rel_delta = 0.000106
V_so8c = +26.0683609298, V_so8xso8 = +26.065829595, rel_delta = 0.000095
V_so8c = +9.4490606160, V_so8xso8 = +9.4480316002, rel_delta = 0.000109
V_so8c = +9.7993167162, V_so8xso8 = +9.7976288261, rel_delta = 0.000172
```
B Scalar Potentials from Superpotentials

While numerics currently often appears to be the most powerful tool to study the scalar potentials of maximal $D = 4, 5, 6$ supergravities on the full coset manifolds $E_{d(+d)}/\mathcal{K}(E_{d(+d)})$, consistent truncation to maximal sets of commuting $SU(1,1) \approx SL(2)$ subgroups yields analytically rather manageable expressions on these low-dimensional subspaces\footnote{This likely may be a useful starting point for explorations of larger subspaces, observing that the Fano plane also shows in the decomposition of $E_7$, respectively $E_8$, into irreducible representations of $SL(2) \times 7, 8$.}

Following the conventions of [2], we start from the Kähler potentials for the product manifold of seven, respectively eight, Poincare discs:

$$
\mathcal{K}(7, 8) = - \sum_{j=1}^{7, \text{ resp. } 8} \log(1 - \zeta_j \bar{\zeta}_j). \tag{B.8}
$$

From this, we obtain the Kähler metric and its inverse:

$$
\mathcal{K}^{ab} := \partial_a \partial_b \mathcal{K}, \quad \mathcal{K}^{-1} = (\mathcal{K}^{ab})^{-1}. \tag{B.9}
$$

With the covariant derivative being given by

$$
\nabla_a (\cdot) = \partial_a (\cdot) + (\cdot) \partial_a \mathcal{K}, \tag{B.10}
$$
the scalar potential of $\mathcal{N} = 8$, $D = 4$ SO(8) on $\mathcal{M}_{14} = (SU(1,1)/U(1))^7$ is given by

$$V_{D=4;\mathcal{M}_{14}} = 2 \exp(\mathcal{K}) \left( K_{ij}^{ab} \nabla_a W_7 \nabla_b W_7 - 3 W_7 \overline{W}_7 \right),$$  \hspace{1cm} (B.11)$$

while the scalar potential of $\mathcal{N} = 16$, $D = 3$ SO(8) supergravity on $\mathcal{M}_{16} := (SU(1,1)/U(1))^8$ is found to match

$$V_{D=3;\mathcal{M}_{16}} = 2 \exp(\mathcal{K}) \left( K_{ij}^{ab} \nabla_a W_8 \nabla_b W_8 - 4 W_8 \overline{W}_8 \right).$$  \hspace{1cm} (B.12)$$

In both cases, the superpotential can be read off from the $A_1$-tensor of the model: For $D = 4$, there is a 8-vector $X^i$ such that $A_{ij}^{12} X^i X^j \cdot \prod_k (1 - \zeta_k \overline{\zeta}_k)^{1/2} = W_7$, and in $D = 3$, there is a 16-vector $Y^I$ such that $A_{IJ}^{12} Y^I Y^J \cdot \prod_k (1 - \zeta_k \overline{\zeta}_k)^{1/2} = W_8$. 

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