ARE GALAXIES OPTICALLY THIN
TO THEIR OWN LYMAN CONTINUUM RADIATION?
I. M33

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ABSTRACT

Previously published $H\alpha$ data and UBV photometry of blue stars in the inner kpc of M33 are used to study the distribution of OB stars and HII regions in the galaxy and to determine whether individual regions of the galaxy are separately and/or collectively in a state of ionization balance. Based on the surface brightness of the $H\alpha$ emission, we identify three distinct ionized gas environments (bright, halo and field). We find that $\sim$50\% of the OB stars are located in the field, so that 1/2 of the lifetime of OB stars must be spent outside recognizable HII regions. If OB stars escape from bright HII regions by destroying their parent molecular clouds, this result would imply that molecular cloud lifetimes after forming OB stars could be as low as $\sim$5$\times$10$^6$ yrs or 1/2 the typical lifetime of OB stars. We show that a possible origin for the large field OB population is that they were born in and subsequently percolated out of the $\sim$10$^3$ giant molecular clouds with masses $\geq$10$^3$M$_\odot$ predicted to exist within the inner kpc of the galaxy. Using ionization models, we predict $H\alpha$ fluxes in the bright, halo and field regions and compare them to those observed to find that the regions, separately as well as collectively, are not in ionization balance: predicted fluxes are a factor of $\sim$3-7 greater than observed. The heaviest loss of ionizing photons appears to be taking place in the field. Observed and predicted $H\alpha$ luminosities in the field are in best agreement when Case A recombination is assumed. Therefore, our findings suggest that star formation rates obtained from $H\alpha$ luminosities must underestimate the true star formation rate in these regions of M33. We have performed a similar analysis of an individual, isolated region with bright and halo $H\alpha$ emission to find that comparable results apply and that the region, as a whole, is also not in ionization balance.
1. INTRODUCTION

Although massive OB stars have long been recognized as the source of the ionizing radiation that defines optically luminous HII regions, only recently has it become clear that OB associations and HII regions do not always coincide spatially. The Local Group galaxy M33 contains many examples of extragalactic OB associations that do not contain bright HII regions (Wilson 1990) while the edge-on spiral NGC 891 contains faint, filamentary $H_\alpha$ structures high above the galactic disk with no obvious ionizing sources (Rand et al. 1990). In addition, the luminous young stars in M33 are not restricted to the spiral arms, traditionally believed to be the primary site of star formation (Madore 1978). Studies of the OB associations in the Magellanic Clouds show that there is an entire population of massive young stars that are located far enough away from neighbouring OB associations that they must have been born in the “field” and not in the associations (Massey et al. 1994). These observations suggest that massive stars must spend some portion of their lives far from dense gas clouds and hence outside bright HII regions. The presence of significant numbers of field O stars has important implications for a number of key issues in star formation and the interstellar medium, including the evolution of molecular clouds, the ionization of diffuse interstellar gas and the calculation of extragalactic star formation rates (SFRs).

The length of time that an O star spends in the HII region phase, and thus at least partially embedded in its parent gas cloud, is important for estimating molecular cloud lifetimes and, therefore, understanding the evolution of the molecular interstellar medium in galaxies. Estimates of molecular cloud lifetimes require an assessment of the destructive influence of massive stars and thus the time such stars spend in their natal environment is very important. The short lifetimes of $\sim 10^7$ yrs obtained by some authors (cf. Blitz & Shu 1980) assume this embedding time is a large fraction of the total lifetime of the star so that the first O stars that form will disrupt their parent cloud. Longer lifetime estimates ($\geq 10^8$ yrs, Solomon & Sanders 1980) are based on clouds being formed in a stable environment, the spiral interarm region of the Galaxy, where the overall SFR is low. Molecular clouds undergoing periods of intense star formation followed by quiescent star formation would result in long molecular cloud lifetimes so that each cloud could produce several OB associations of different ages (Elmegreen 1991). The fraction of self-gravitating clouds currently experiencing star formation ($\sim 0.5$) can be combined with the duration of the process ($\sim 20 \times 10^6$ yrs in OB associations) to obtain molecular cloud lifetimes of $\sim 4 \times 10^7$ yrs (Elmegreen 1991). If OB stars destroy their natal environments, then the fraction of O stars seen in the field determines the time required to destroy the cloud. However, if these stars are seen in the field because they simply moved out of the clouds, then cloud lifetimes could be considerably longer.

The presence of OB stars in the field may be important to understanding the origin of the diffuse ionized gas (DIG) observed in our own Galaxy as well as extragalactic systems. This gas is characterized by its low density ($n_e \sim 0.2$ cm$^{-3}$), cool temperatures ($< 10,000$ K) and high [SII]/$H_\alpha$ line intensities (Walterbos 1991). Observations of edge-on galaxies indicate scale heights of the DIG can be of the order of a few kpc (Walterbos 1991). Diffuse ionized gas in external galaxies is believed to account for a substantial fraction of the total $H_\alpha$ emission from a galaxy (20-30% in the Large Magellanic Cloud, Kennicutt & Hodge 1986; 20-40% in M31, Walterbos & Braun 1994). Currently the most likely sources of ionizing photons for the DIG are OB stars. One problem with this scenario is that there is no clear understanding of how ionizing photons can traverse the large distances above the plane. If significant numbers of OB stars are found in field regions (where $H_\alpha$ emission is low and optical depths are likely to be smaller than in bright HII regions), a potentially large reservoir of ionizing photons would be available to power the DIG.
Star formation rates that are calculated from observed \( H\alpha \) fluxes rely on the assumption that the region in question is ionization bounded (Kennicutt 1983) i.e. all Lyman continuum photons produced by massive stars are absorbed by the gas in the region (or galaxy). (The alternative scenario is for a region (or galaxy) to be density bounded, i.e. the Lyman photons are more than sufficient to ionize all the (dense) gas.) The observed \( H\alpha \) luminosity is then directly proportional to the Lyman continuum luminosity which can be converted into a SFR for massive stars via an assumed initial mass function (IMF) and theoretical models for the Lyman continuum luminosity as a function of stellar mass. The total SFR of a region can be obtained by extending the IMF over the full range of stellar masses. In these calculations, the assumption that the region is ionization bounded is crucial: since the SFR is directly proportional to the Lyman continuum, and therefore \( H\alpha \) luminosity, a region out of which continuum photons are escaping will have a smaller estimated SFR than one that is in ionization balance. The assumption of ionization boundedness may not be valid if large portions of the OB population lie outside bright H II regions. Considering the wide spread use of this technique for obtaining SFRs in extragalactic systems, it is vital to verify this underlying assumption of SFR measurements from \( H\alpha \) luminosities.

Ionization balance calculations in extragalactic OB associations and HII regions have concentrated on single HII regions in the Large and Small Magellanic Clouds (LMC and SMC). Massey et al. (1989a) find the LMC OB associations NGC 2122 and LH 118 to be density rather than ionization bounded, since the predicted fluxes from stars in the association are a factor of two greater than what is observed. A similar study of NGC 346 in the SMC (Massey et al. 1989b) found the opposite to be true: the models predict less flux (up to a factor of \( \sim 2 \)) than what is observed and so the region is most likely ionization bounded. Finally, Parker et al. (1992) found that the LMC OB associations LH 9 and LH 10 were at the limit of being ionization bounded (observed fluxes were at most a factor of 1.5 greater than those predicted). To date there have been no such studies of systems more distant than the Magellanic Clouds and no attempt to measure the ionization boundedness of a large area of a galaxy.

At a distance of 0.79 Mpc (van den Bergh 1991), the Local Group galaxy M33 is an ideal system in which to conduct such an investigation. This Sc type galaxy is undergoing vigorous high mass star formation. The inner kpc region of the galaxy has recently received much attention: molecular gas maps in CO have identified a population of molecular clouds (Wilson & Scoville 1990), optical imaging at \( H\alpha \) has provided quantitative photometry of the HII regions (Wilson & Scoville 1991), and a UBV photometric survey of the region has identified luminous blue stars and OB associations (Wilson 1991). Diffuse ionized gas within the galaxy has been investigated by Hester & Kulkarni (1990).

In this paper, we use \( H\alpha \) and UBV photometry of the inner kpc region of M33 to study the distribution of the HII regions and luminous blue stars (hereafter OB stars) so as to calculate the fraction of an OB star’s lifetime that is spent outside an HII region and to address the issues of molecular cloud lifetimes, ionization balance, and the implications for the calculation of star formation rates and powering the diffuse ionized gas. The analysis of the \( H\alpha \) data and UBV photometry and the theoretical ionization models used are outlined in section §2. A discussion of the OB star distribution is given in §3 and the results of the ionization balance calculations appear in §4. The results of our investigation are summarized in §5.

2. SELECTION AND ANALYSIS OF DATA

2.1. Photometric Data
We use the photometric UBV data from Wilson (1991), who used it to study the OB associations in the nucleus of M33. The observations were made at the Palomar 60-inch and Canada-France-Hawaii telescope (CFHT), using the blue sensitive Tektronix chip, CCD 6 (scale 0.235′′ pixel$^{-1}$) at Palomar and the RCA2 double density CCD chip (scale 0.205′′ pixel$^{-1}$) at the CFHT. The seeing was 1.2′′-1.9′′ at Palomar and ~0.8′′ at the CFHT. Additional details of the observations can be found in Wilson (1991). Systematic uncertainties in the photometric zero points were estimated to be ±0.05 mag in B, ±0.04 mag in V and ±0.15 mag in U. The data were estimated to be incomplete by 30% for 19.5<V<20, 55% for 20<V<20.5 and 65% for 20.5<V<21. Subsequent comparison of the data with independent photometry of M33 obtained by P. Massey (private communication) showed that there is a systematic offset (Massey-Wilson) of 0.14 mag in (U-B), while the B and V magnitudes agreed to better than 0.04 mag. Hence we have applied a +0.14 mag correction to the (U-B) colours, i.e. the colours are shifted to the red relative to the values published in Wilson (1991).

This data set is particularly suitable for our use as it provides the most complete (to 21 mag in V) large-area census of the blue star population in the inner regions of the galaxy. The average reddening (foreground and internal) for the surveyed region is $E(B-V)=0.3±0.1$ mag (Wilson 1991). We selected only those stars with $B-V\leq0.4$ mag. This criterion, which is based on the estimated average reddening for the galaxy, the intrinsic colour of O stars on the the zero age main sequence (ZAMS), $B-V=-0.3$ mag (Flower 1977), and the observed width of the main sequence, assures us that most potential OB stars will be selected.

2.2. $H\alpha$ Emission

We used a red-continuum subtracted $H\alpha$ image of M33 from Wilson & Scoville (1991) to study the $H\alpha$ emission in the galaxy. The image was obtained on the Palomar 60-inch telescope (rebinned chip scale 1.3″ pixel$^{-1}$). Visual inspection of the continuum subtracted $H\alpha$ image (henceforth the $H\alpha$ image) strongly suggested that three types of HII regions, bright, halo and field, are present in M33. The different regions were defined by their $H\alpha$ surface brightness, $I$, such that $I\geq1.37\times10^{-15}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$ for the bright regions, $3.61\times10^{-16}\leq I<1.37\times10^{-15}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$ for the halo regions and $I<3.61\times10^{-16}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$ for the field regions (see Figure 1a). The divisions into the three regions are motivated purely by visual inspection of the total image; the adopted dividing lines in surface brightness between each type of region are not based on any intrinsic physical quantity. The surface brightnesses, which have already been corrected for extinction in $H\alpha$ ($A_{H\alpha}=2.59\times E(B-V)=0.78$ mag (Schild 1977)) were measured above an observed average sky surface brightness over the galaxy of $I_{bg}=5.43±0.10\times10^{-16}$ erg s$^{-1}$ arcsec$^{-2}$. The latter was obtained by averaging the observed surface brightness of four broad regions of the galaxy that appeared free of any $H\alpha$ emission. Because our $H\alpha$ image of M33 is restricted only to the inner kpc of the galaxy and does not cover any areas far from the galaxy, we cannot be certain that the background emission is in fact “sky”, and not a very smooth emission component in M33. We will be in a better position to determine the true source of this background emission in our upcoming study of NGC 6822, for which we have $H\alpha$ images that extend past the optical limits of the galaxy. Finally, we note that these $H\alpha$ surface brightnesses as well as all measures of the $H\alpha$ luminosity have been corrected for contamination by the [NII] 6583Å line that was also observed through the filter used to obtain the $H\alpha$ image. To correct for this contamination, we used the data of Vilchez et al. (1988) to determine the average [NII]/$H\alpha$ line ratio of 21% ±1% in three HII regions that lie within the surveyed region of M33. Scaling down all counts in the $H\alpha$ image by a factor of 0.79 then corrects for this contamination in the data.
The uncertainties associated with the observed H\(\alpha\) luminosity include those in the calibration of the H\(\alpha\) emission and the extinction at H\(\alpha\). Wilson and Scoville (1991) estimate the calibration to be accurate to \pm 5\%. The uncertainty in A\(_{H\alpha}\) was determined to be A\(_{H\alpha}\)=0.78\pm0.26 mag from the assumed mean reddening for the galaxy E(B−V)=0.3\pm0.1 mag and the uncertainty in the conversion factor between A\(_{H\alpha}\) and E(B−V), which we estimated at 0.07 mag or \sim10\% of A\(_{H\alpha}\). We note that a radio continuum survey of the HII regions in M33 found an average A\(_{H\alpha}\) of 0.91 mag (Viallefond & Goss 1986) which is comparable to our estimate. The total uncertainty in the observed H\(\alpha\) luminosities is thus \sim30\%.

2.3. Predicted H\(\alpha\) Luminosity from OB stars

In order to compare the observed H\(\alpha\) fluxes with those expected from the associated stellar population, we estimate the Lyman continuum flux, and hence the H\(\alpha\) luminosity, for stars with (B−V)\leq0.4 by applying a number of stellar ionization models: blackbody, Auer & Mihalas (1972) (hereafter A&M), Kurucz (1979) and Panagia (1973). Both A&M and Kurucz calculate the expected number of Lyman continuum photons as a function of the effective temperature, T\(_{\text{eff}}\), for various values of the effective gravity, g. The A&M results are based on nonblanketed NLTE model atmospheres composed of hydrogen and helium only. The Kurucz models, which include the effects of line blanketing, have LTE atmospheres of solar composition. The Panagia models are based on a combination of the NLTE A&M models and the LTE models due to Bradley and Morton (1969), Morton (1969) and Van Citters & Morton (1970), and appropriate values of g are used to calculate the flux of Lyman continuum photons as a function of T\(_{\text{eff}}\) for stars of luminosity class I, III, V as well as the ZAMS.

Effective temperatures (T\(_{\text{eff}}\)) and bolometric corrections (BC) for stars with M\(_V\)\leq21 and B−V\leq0.4 were obtained using the calibration equations given in Parker & Garmany (1993). The six equations (4a to 4f) and Panagia (1973) give T\(_{\text{eff}}\) in terms of the reddening free index Q=(U-B)−0.72(B-V) and the dereddened colour, (B−V)$_o$, for various luminosity classes. These equations which are, in large part, taken from Massey et al. (1989a), are based on the data of Flower (1977), FitzGerald (1970) and Humphreys & McElroy (1984). The BC of the star was determined from T\(_{\text{eff}}\) using one of equations 5a to 5d in Parker & Garmany (1993). These calibrations are from Chelbowski & Garmany (1991) and Flower (1977). In applying these calibration equations, we followed the basic procedure outlined in Parker & Garmany, with a few modifications. We first selected all stars with M\(_V\)\leq-6 as potential supergiants. If the star had a (U-B) colour, we used equation (4f) to calculate T\(_{\text{eff}}\). For -1.1<Q\leq-0.9 or Q>0.0, we used equation (4a) (which is valid for main sequence stars) and subtracted 4000 K from the calculated T\(_{\text{eff}}\) (as per Parker & Garmany 1993; see their section 3.1 for a discussion of this point). For stars with Q<-1.1 or -0.3<Q\leq0.0, we used (i) equation (4b) -4000 K if (B−V)$_o$\leq0.0. (ii) equation (4c) if 0.0\leq(B−V)$_o$\leq0.2 or (iii) equation (4d) if 0.2\leq(B−V)$_o$\leq0.5. Effective temperatures for all remaining, non-supergiant stars were calculated using one of equations (4a) to (4e) depending on the value of Q (if available) and (B−V)$_o$.

For the blackbody, A&M and Kurucz models, we applied Table XIV of Massey et al. (1989), which neatly summarizes the expected Lyman flux at the surface of the star for T\(_{\text{eff}}\) between 20,000 K and 50,000 K. For the Kurucz models, we have used log(g)=3.5 for stars with T\(_{\text{eff}}\) \leq 35,500 K, and log(g)=5.0 for stars with T\(_{\text{eff}}\) >35,500 K while for the A&M models we take log(g)=4.0. The total flux in Lyman continuum photons is then found by integrating over the surface area of the star. For each star for which M\(_{\text{bol}}\) and T\(_{\text{eff}}\) were determined (as outlined above) and 18,000 K \leq T\(_{\text{eff}}\) \leq60,000 K, we calculate the stellar radius, R\(_\ast\), using the luminosity-effective temperature relation, L=4\pi R\(_\ast\)^2 \sigma T\(_{\text{eff}}^4\).
To apply the Panagia models, we need to know the luminosity class of the star as well as its $T_{\text{eff}}$. Panagia gives visual magnitudes ($M^p_v$) as a function of effective temperature ($16,100 \text{ K} \leq T_{\text{eff}} \leq 50,000 \text{ K}$) for ZAMS, supergiant, class III and main sequence stars. We estimate the luminosity class of the star by comparing the observed visual magnitude corrected for foreground and internal extinction, $M_{V_o}$, with $M^p_v$. Only stars with $15,100 \text{ K} \leq T_{\text{eff}} \leq T_{\text{eff, U}} \equiv 60,000 \text{ K}$ are considered. We take all stars with $M_{V_o} \leq -6.0$ to be supergiants (class I). For the remainder of the stars, we compare the observed $M_{V_o}$ to the values of the three $M^p_v$'s (for class III, V and ZAMS) corresponding to the star’s $T_{\text{eff}}$. The $M^p_v$ that is closest to the observed $M_{V_o}$ determines the luminosity class. In turn, the luminosity class and $T_{\text{eff}}$ gives the total flux of Lyman continuum photons from the entire star.

To convert Lyman continuum photons to $H\alpha$, we refer to Osterbrock (1989) to find $N(\text{LyC})/N(\text{H}\alpha)=2.22$ assuming Case B recombination (appropriate for optically thick gas at 10,000 K and densities of $10^2$-$10^4 \text{ cm}^{-3}$). Although Case B recombination is the most commonly used approximation in the study of HII regions, we also consider Case A recombination for which $N(\text{LyC})/N(\text{H}\alpha)=5.36$ (Brockelhurst 1971), for stars in the “field” where the optical depth of the surrounding gas may be smaller. Clearly, Lyman continuum photons in optically thin gas produce fewer $H\alpha$ photons than those in optically thick gas.

### 3. IONIZED GAS AND OB STAR DISTRIBUTION

O and B stars in galaxies are commonly assumed to be located in bright HII regions. To study this assumption quantitatively, we identified the ionized gas environment of a star as either bright, halo, or field according to the average $H\alpha$ surface brightness inside a circle of radius of 2 pixels ($\sim 9.8 \text{ pc}$) centered on the star. Table 1 summarizes the total numbers of stars as a function of magnitude found in the three different ionized gas environments.

Inspection of these tables reveals a number of surprising results. First and foremost, we find that all O stars are not in bright HII regions: there are a significant number of O stars in the field (see Figure 1b). From our analysis, roughly 50% of all blue ($B-V \leq 0.4$) stars brighter than $V=21$ are located in the field. In fact, the number of field stars outnumbers those in bright HII regions by almost a factor of three. This result is reflected also in the brightest stars ($V \leq 18$), where almost a third of the total are field stars. The halo component of the $H\alpha$ emission also contains a significant percentage ($\sim 30\%$) of the stellar population.

Additionally, the bright regions are predominantly populated by faint stars: 50% of all stars in the bright regions have $21<V\leq20$ compared to 6% with $V\leq18$. Although it appears that fainter stars form a larger fraction of the population in the field than in the bright HII regions ($\sim 64\%$ in the field compared to $\sim 50\%$ in the bright regions), this may be due to a higher incompleteness level in the brighter regions where the crowding is larger (see Table 1).

From these results, we can estimate the fraction of the main sequence lifetime of an O star that is spent within a recognizable HII region ($f_{\text{HII}}$). We consider such an object to be composed of either a bright or halo emission region so as to obtain the most conservative estimate of this important quantity. With this assumption, we obtain $f_{\text{HII}} \equiv (N_B + N_H)/N_{TOT}=0.5$ where $N_{TOT}$ is the total number of stars in the sample and $N_B$ and $N_H$ are the numbers of stars in, respectively, the bright and halo regions of the galaxy. Thus roughly 50% of the main sequence lifetime of an O star is spent outside of a recognizable HII region. If OB stars escape from bright HII regions by destroying their parent molecular clouds, this result would
imply that molecular cloud lifetimes after forming OB stars could be as low as $\sim 1-4 \times 10^6$ yrs (or 1/2 the typical MS lifetimes of 2.6-8.1 $\times 10^6$ yrs for 120-20 $M_\odot$ stars assuming $Z=0.020$, Schaller et al. 1992). Since 50% of the molecular clouds in M33 contain optically visible HII regions (Wilson & Scoville 1991), under this scenario the lifetimes of the molecular clouds would be $\leq 10^7$ yrs. This result, however, is inconsistent with the evidence that some OB associations in the galaxy have undergone at least two episodes of star formation separated by $10^7$ yrs (Regan & Wilson 1993).

An alternative scenario is that the field O stars have percolated out of existing bright HII regions. In Table 2 we summarize a few key properties of the O star distribution and ionized gas environments. We see that although the field region occupies an area about 10 times that of the bright region, the surface density of stars in bright regions is only about 4 times that in the field. Assuming an average O star lives for 10 Myrs and has a velocity relative to the parental molecular cloud of 3 km s$^{-1}$ (Churchwell 1991), it would travel only $\sim 30$ pc over its lifetime. Typical dimensions of large field regions are 300 $\times$ 600 pc (Figure 1b).

Clearly, the field stars cannot all have simply percolated out of the HII regions we currently see. Yet another scenario is one where the field stars originated in molecular clouds located in the field that have since been destroyed so as to leave the field O stars in a low density gas environment. The Owens Valley Millimeter-Wave Interferometer (OVRO) has been used to detect 38 giant molecular clouds (masses $\geq 5 \times 10^4 M_\odot$) lying within an area of 1.5 to 2.0 kpc$^2$ in M33 (Wilson & Scoville 1990). The average surface density of molecular hydrogen in the OVRO fields is $\Sigma_{H_2} (OVRO)=7.8 M_\odot$ kpc$^{-2}$ (from Wilson & Scoville 1990, corrected by a factor of 1.44; see Thornley & Wilson 1994) while the average surface density in the inner kpc of the galaxy is $\Sigma_{H_2}(1kpc)=6 M_\odot$ kpc$^{-2}$ (Wilson & Scoville 1989, again corrected by a factor of 1.44). From the numbers of giant molecular clouds (GMCs) observed, the gas surface density, and the area covered in the OVRO survey (1.5-2 kpc), we estimate the expected number of GMCs (with masses $\geq 5 \times 10^4 M_\odot$) within the inner kpc of M33 to be of the order of $\sim 50$. If the clouds are spread uniformly across a circular region of radius 1 kpc, the average distance between these clouds would be 130-150 pc. Clearly O stars that formed and percolated out from these large GMCs would not be able to fill the entire area of the field.

It is important to note, however, that the OVRO interferometer detected only $\sim 40\%$ of the flux measured in single dish observations of the same regions of the galaxy (Wilson & Scoville 1990). The missing flux was hypothesized to be distributed in a smooth component or dense molecular clouds less massive than $0.5 \times 10^5 M_\odot$ (Wilson & Scoville 1990). If we assume that half of the missing flux (30\% of the single dish flux) is due to small, dense molecular clouds, that the molecular cloud mass spectrum follows a power law of the form $N(m) \propto m^{-1.5}$ (as determined from the more massive molecular clouds in M33), and that the minimum mass of a molecular cloud of interest is $10^3 M_\odot$ (a typical Taurus-type “dark cloud”, Goldsmith 1987), then the estimated number of low-mass ($10^4-0.5 \times 10^5 M_\odot$) molecular clouds in the inner kpc of M33 is $\sim 10^3$. Since the average separation between these clouds is 30 pc, O stars could easily disperse from these clouds to fill the entire field. Therefore, if even half the missing flux is in low-mass molecular clouds, most of the field O stars could have been born in and percolated out of (or destroyed) low-mass molecular clouds. Observational support for massive star formation in low-mass molecular clouds can be found within our own Galaxy. A survey of small Galactic HII regions ($\sim 1/5$ the $H_\alpha$ luminosity of Orion) and their associated molecular clouds (masses $\sim 1-60 \times 10^3 M_\odot$) by Hunter & Massey (1990) showed that OB star formation has occurred in all of the molecular clouds. Finally, we point out that these energetic stars may be more effective at dispersing their lower-mass parental molecular clouds (compared to the more massive clouds associated with the bright HII regions) which could also account for the low $H_\alpha$ emission in field regions.
From Table 2 we see that the relative total $H\alpha$ luminosity per field star is $\sim 1/10^{th}$ that of a star in a bright HII region. This result raises the question of whether the field is “leaking” Lyman continuum photons, which presumably escape from the galaxy. Alternatively, the field population may be older than that in bright HII regions so that fewer early type stars with large ionizing fluxes remain. To address these questions, we have determined the stellar luminosity functions for the three ionized gas environments as well as the entire sample to see if there is any indication of age segregation (Figure 2). Weighted least-squares fits to the data to a limiting magnitude of 19 for the bright HII region (to reduce incompleteness effects) and 19.5 for all others yield slopes of 0.44 $\pm$ 0.09 (bright), 0.49 $\pm$ 0.08 (halo), 0.47 $\pm$ 0.07 (faint) and 0.52 $\pm$ 0.04 (entire image). The uniformity of the luminosity function slopes does not suggest that the stellar populations of the three HII environments have different average ages. Although including stars up to $V=19.5$ in the bright HII regions decreases the slope to 0.40 $\pm$ 0.07, the difference is still not significant enough to suggest a younger population. As age differences could affect the distribution of the bright stars most severely, we also examined the distribution of stars that are brighter than $V=18$, normalized to the total number brighter than $V=19$. The percentage of stars with $V \leq 18$ in the bright, halo and field regions are respectively, 30 $\pm$ 10%, 18 $\pm$ 7% and 15 $\pm$ 5%. Clearly again, no region appears to have an over-abundance of luminous stars and therefore there is no evidence for age segregation between the different $H\alpha$ emission regions. It appears therefore that the difference in the $H\alpha$ luminosity per O star in the field compared to that in the bright regions is not due to a difference in age but is perhaps indicative that ionizing radiation from field stars may be escaping out of the surveyed region.

4. IONIZATION BALANCE IN THE INNER 1 KPC

Star formation rates calculated from $H\alpha$ luminosities assume that all the Lyman continuum flux emitted by the stars remains in the galaxy (or region) so that the galaxy (or region) is ionization bounded (eg. Kennicutt 1983). To test this hypothesis, we have estimated the Lyman continuum and hence $H\alpha$ flux emitted by the stars and compared it to that observed in the $H\alpha$ image. In carrying out these ionization balance calculations, we have relied heavily on the models of Panagia (1973) which are applicable to zero age main sequence stars (ZAMS) as well as main sequence (class V), luminosity class III and supergiant stars. As an estimate of the uncertainties in the theoretical models, we have also used the blackbody, Kurucz, and A&M models.

In all applications of the ionizing radiation models, we have imposed an upper and lower limit to $T_{\text{eff}}$, respectively $T_{\text{eff},U}$ and $T_{\text{eff},L}$. For comparative purposes, we have set $T_{\text{eff},U} =$60,000 K and $T_{\text{eff},L}$=30,000 K. The lower limit $T_{\text{eff},L}$ is chosen as the minimum effective temperature for which all ionization models are defined. Although all four models are defined up to $T_{\text{eff}}$=50,000 K (for an O4, class V star), we have chosen $T_{\text{eff},U} =$60,000 K so as to provide a small margin of error at the high $T_{\text{eff}}$ end as well as to roughly account for the Lyman continuum flux for stars earlier than O4. Stars that had calculated effective temperatures between 50,000 K and 60,000 K were all assigned Lyman fluxes corresponding to 50,000 K from the models. By doing this, we are most likely under estimating the true Lyman continuum flux from those stars earlier than O4. Stars with effective temperatures outside the limits set by $T_{\text{eff},U}$ and $T_{\text{eff},L}$ have no ionizing flux assigned to them. We have not attempted to account for stars with $T_{\text{eff}}>T_{\text{eff},U}$ as these effective temperatures may most likely be the result of photometric errors (for example, Massey et al. (1989a, 1989b) have found many stars with unrealistic (B-V) or (U-B) colours the cause of which they attributed to photometric errors). Similarly, stars with $T_{\text{eff}}<T_{\text{eff},L}$ are also not accounted for as these
stars are most likely to be late type stars (with correspondingly small ionizing fluxes) or more evolved stars. We note that the presence of Wolf-Rayet stars is also not taken into account. While the ionizing Lyman continuum flux from these stars is estimated to be significant, the exact values are highly uncertain (Massey et al. 1989b). Therefore not accounting for the presence of Wolf-Rayet stars in the surveyed regions of M33 serves to underestimate the predicted theoretical flux. Finally, studies in the LMC and SMC have shown that effective temperatures calculated from photometry are in general lower than those determined from spectroscopy (Massey et al. 1989b; Parker et al. 1992) so that here again, predicted ionizing fluxes from photometry are most likely to be underestimated.

In terms of the $H\alpha$ fluxes (assuming Case B recombination), the total flux in $H\alpha$ from all stars with $T_{\text{eff},L}\leq T_{\text{eff}}\leq T_{\text{eff},U}$ calculated from the four ionization models are (in units of $10^{39}$ erg s$^{-1}$), $F_{H\alpha}=14.46$ (Panagia), 15.15 (blackbody), 11.05 (Kurucz) and 13.49 (A&M). The agreement amongst the different models is quite good: the lowest (Kurucz) and highest (blackbody) estimates of $F_{H\alpha}$ differ by a factor of 1.4. At low $T_{\text{eff}}$, the blackbody model predicts more Lyman continuum photons than the A&M and Kurucz models so that it is not surprising that the total flux from all stars (many of which have low $T_{\text{eff}}$) for the blackbody model is the highest estimate. In the remainder of the section we will discuss only the $H\alpha$ flux estimated using the Panagia models. As the minimum effective temperature for which these models are defined depends on the luminosity class of the star, i.e. $T_{\text{eff}}=15,100$ K (for class I), $T_{\text{eff}}=16,900$ K (for class V and ZAMS) and $T_{\text{eff}}=16,000$ K (for class III), we have defined for each luminosity class the corresponding $T_{\text{eff},L}$ inferred from the models. As such, the percentage of stars with $(B-V)\leq0.4$ that have $T_{\text{eff}}$ between $T_{\text{eff},L}$ and $T_{\text{eff},U}$ is then 54% ($V<18$), 60% ($18<V<19$), 74% ($19<V<20$) and 75% ($20<V<21$).

To obtain the most complete and accurate estimate of the Lyman continuum, and hence the $H\alpha$ luminosity, we have applied two corrections to the data. The first concerns incompleteness of the photometric survey at the faint end of the luminosity function ($V\leq21$). Wilson (1991) determined that the data were incomplete by 30% for $19.5<V\leq20$, 55% for $20<V\leq20.5$ and 65% for $21<V\leq21$ by comparing the observed luminosity function with a power law with slope of 0.65. Thus the corrected fluxes are obtained by scaling the uncorrected fluxes by 1.4 for stars with $19.5<V\leq20$, 2.2 for stars with $20<V\leq20.5$ and 2.9 for stars with $20.5<V\leq21$. Secondly, we must account for flux from stars that are below our faint magnitude limit. We have calibrated a relation between main sequence mass and $M_V$ using theoretical evolutionary tracks between the ZAMS and the terminal age main sequence (TAMS) for metallicity $Z=0.020$ from Schaller et al. (1989). The tracks were interpolated in steps of 1 $M_\odot$ and then transformed to the observational plane using the equations outlined in section three of Massey et al. (1989a). As was done in Wilson (1992), we determined the total range of stellar masses possible for each range in $M_V$ and assigned to each magnitude bin the average of the maximum and the minimum possible stellar mass. At the distance and reddening of M33, $V=21$ corresponds to 24 $M_\odot$. Linearly interpolating the spectral type-main sequence mass calibrations given by Popper (1980) and Schmidt-Kaler (1982), we find that 24 $M_\odot$ corresponds to roughly an O7.5-O8 star. We determined the total number of stars for each spectral (sub)class from O7.5 to B3 (the latest class for which the Panagia models give theoretical fluxes) using our spectral type-main sequence mass calibration and the Salpeter initial mass function (IMF) $N(m)dm(M)=Am^\alpha dm$ (Salpeter 1955), where $\alpha=2.35$. The normalization constant $A$ is determined by integrating the initial mass function between 24 $M_\odot$ and 65.5 $M_\odot$ and equating this to the total number of stars for which we could determine $T_{\text{eff},f}$, either uncorrected ($N_\star\sim240$ (bright), $\sim400$ (halo) and $\sim640$ (field)) for the conservative flux estimate, or corrected for incompleteness brighter than $V=21$ ($N_\star\sim540$ (bright), $\sim1020$ (halo) and $\sim1280$ (field)) for the best flux estimate. We determined the total flux in each spectral class (or subclass) bin between O7.5 and B3 due to main sequence stars fainter than $V=21$ by multiply the average theoretical flux for that bin (class V
assumed) by the total predicted number of stars in the bin.

In Table 3 we summarize the observed and predicted \( H\alpha \) fluxes, with theoretical values given in several steps correcting in turn for the effects discussed above. Included are the theoretical fluxes predicted from all stars with \( T_{\text{eff},L} \leq T_{\text{eff,U}} \) \( \leq T_{\text{eff,V}} \) (1) for the stars actually observed ("minimum flux"); (2) corrected for stars fainter than \( V=21 \) ("conservative flux"); (3) corrected for incompleteness for both stars brighter and fainter than \( V=21 \). Based on the limited reliability of the Panagia model in assigning ionizing fluxes from photometric data alone (Patel & Wilson 1995), we will consider any agreement between the observed and predicted \( H\alpha \) fluxes that are within a factor of two to be acceptable.

If we make no corrections to the data, we find that the predicted fluxes are in very good agreement with what is observed: over the total image, the two differ by less than a factor of 2. The best agreement between theory and observation is in the bright region (~1.3:1 correspondence) followed by the halo region, where the predicted flux is ~1.6 times the observed flux. The field may be losing some ionizing photons as the predicted flux is a factor of ~2.4 (Case A) and ~5.7 (Case B) greater than that observed.

Including the flux from stars below the faintness limit of the survey increases the predicted fluxes by a factor of ~2 over both the individual ionized gas regions as well as the entire survey area. The total theoretical flux exceeds that observed by factors of 2.1 (bright), 3.6 (halo), 5.4 (field: Case A), 12.9 (field: Case B), 3.1 (total image: Case A) and 4.1 (total image: Case B). Clearly adopting Case B recombination for the field results in predicted ionizing fluxes that are substantially larger than observed. Including stars below the faintness limit of our survey is the minimum correction we must make to our data and thus these fluxes represent a lower limit to the theoretically predicted \( H\alpha \) fluxes.

Correcting the data for incompleteness (\( V\leq21 \)) increases the predicted fluxes by a factor of less than 2 over the different ionized gas environments. The most complete estimate of the predicted flux includes both incompleteness corrections for \( V\leq21 \) and main sequence stars with \( V>21 \). From Table 3 we see that over the entire surveyed region, these final fluxes are a factor of ~6.7 (Case A) to ~9.3 (Case B) greater than observed. The best agreement is again found in the bright regions where the predicted flux is ~4 times that observed and the worst agreement is in the field where the predicted flux exceeds the observed by factors of ~13 (Case A) to ~31 (Case B). This most generous estimate of the theoretical fluxes after corrections and the most conservative estimate differ by less than a factor of 3 from each other.

The best agreement between observations and theory in the field, as well as for the entire surveyed region, is obtained when we assume Case A (optically thin) recombination. Bright HII regions are likely to have larger opacities than faint regions where gas densities are likely to be smaller. Although we have no direct measure of the density of gas in the various regions, we can place limits on this quantity by comparing the observed \( H\alpha \) surface brightness with that expected for HII regions of various different gas densities. If we consider a B0 star \( (L_{H\alpha}=5.82\times10^{35} \text{ erg sec}^{-1} \text{ and } N_{L_{\gamma}C}=4.27\times10^{47} \text{ photons sec}^{-1}; \text{ Panagia 1973} ) \) located in the field, then the maximum surface brightness observed in the field \( (I_F=3.61\times10^{-16} \text{ erg s}^{-1}\text{cm}^{-2}\text{arcsec}^{-2}) \) limits the gas density to be \( \leq5 \text{ cm}^{-3} \). We have used the Stromgren radius (eg. Spitzer 1978) to estimate the radius of the HII region and assumed the gas within the Stromgren sphere of the star is completely ionized. For the HII region of a field O5 star to have a surface brightness equal to \( I_F \), the density of gas would have to be \( \leq0.5 \text{ cm}^{-3} \). In contrast, if the surface brightnesses of HII regions formed by B0 and O5 stars are to equal the average surface brightness observed in bright HII regions \( (I_G=3.44\times10^{-15} \text{ erg s}^{-1}\text{cm}^{-2}\text{arcsec}^{-2}), \text{ the gas density must be } \sim15 \text{ cm}^{-3} \) (B0) and \( \sim1 \text{ cm}^{-3} \) (O5). Thus the density of gas in the field is likely to be lower than that in the bright HII regions which lends some support to our assumption of Case A recombination in the field and Case B elsewhere in the galaxy.
Comparing the corrected final theoretical fluxes with the observed fluxes when the uniform background has not been removed reveals that the two still differ by up to a factor $\sim 2.5$. Clearly the field region is most sensitive to the inclusion or exclusion of the uniform background: the field could be in ionization balance if Case A recombination is assumed and the uniform background is not removed. However, including the background and assuming Case A recombination where appropriate, comparison of the theoretical best estimate fluxes with the observed fluxes suggests that over the total, as well as the bright and halo regions surveyed, the galaxy is not in a state of ionization balance.

With so much ionizing flux missing we are left with the question: where does it go? One possible fate for leaked photons is in powering the halo ionized gas. The low density and expected small opacities in the field combined with the enormous reservoir of leaked flux provided by the OB stars would clearly increase the chances for the photons to reach the large distances above the plane before recombining and producing the halo ionized gas. Additionally, the large pool of ionizing photons available would be advantageous in producing a halo ionized gas with a total luminosity that is a sizable fraction of the total $H\alpha$ emission measured in the galaxy.

Star formation rates from $H\alpha$ luminosities are based on the assumption that all the ionizing radiation from stars remains in the galaxy. If a significant portion of the ionizing flux is leaking out of the galaxy, SFRs calculated from observed $H\alpha$ fluxes would underestimate the true SFR. Our results suggest that this is the case in M33. We find, in fact, that SFRs calculated from observed $H\alpha$ luminosities alone underestimate those calculated from the most realistic predicted fluxes (best estimate scenario and assuming Case A recombination in the field) by a factor of $\sim 4$ (bright), $\sim 8$ (halo), $\sim 13$ (field) and $\sim 7$ (total image). The implications of this result for SFR calculations in other systems could be serious. Clearly much will depend on how ionizing photon leakage correlates with other characteristics of a galaxy such as its morphology, age and the state of the interstellar medium. If M33 is typical of its class, then applying this technique in late-type spiral galaxies could severely underestimate the actual SFR. The outcome for irregular galaxies (due to their patchy gas distribution) and elliptical galaxies (due to their low gas densities) may be comparable to that found here for M33 but would certainly need to be verified by a similar type of investigation before any conclusions can be drawn.

### 4.1. IONIZATION BALANCE IN AN ISOLATED REGION

Having looked at the question of ionization balance on the large scale, we consider now the small scale and focus on one isolated region of the galaxy containing both bright and halo HII emission. The chosen region, denoted Region A, lies in the south-east portion of the galaxy, and covers an area of $\sim 300 \times 500$ pc ($90'' \times 130''$; Figure 3). We have photometry for 19 stars located in the region with bright $H\alpha$ emission and 17 stars in the halo emission region. Table 4 summarizes the observed and theoretical $H\alpha$ fluxes in the bright, halo and combined (bright and halo) regions. We find that observations and model predictions are best matched for the simplest situation where no corrections are made: the predicted fluxes in the bright and halo regions are, respectively, a factor 1.8 and 0.1 times that observed so that the region as a whole is in balance. Correcting for stars fainter than $V=21$ increases the predicted fluxes by factors of $\sim 2$ in the bright, $\sim 14$ in the halo region and $\sim 1.4$ over the combined region. This result can be considered to be the most conservative corrected estimate of the theoretical flux in Region A. Correcting for incompleteness ($V \leq 21$) increases the minimum theoretical flux by a factor of $\sim 3$ over both the bright and halo regions. The best estimate of the predicted fluxes are a factor of 8 (bright), 5 (halo) and 7 (combined bright+halo) greater...
than that observed. Therefore, only in the simplest situation of the uncorrected data is Region A close to being ionization bounded. In the more realistic case where the data has been corrected for incompleteness in stars brighter than \( V=21 \) and flux from stars fainter than \( V=21 \), this region is clearly not ionization bounded. Even the conservative estimate of the corrected flux indicates this region is leaking photons to its surroundings. Furthermore, including the uniform background cannot bring the combined bright and halo emission regions into ionization balance. Thus our results for a small, individual region confirm those obtained for the larger surveyed area: ionizing flux from OB stars is found to be missing on both large and small scales in M33.

5. CONCLUSIONS

Using \( H\alpha \) data and UBV photometry of blue stars, we have investigated the distribution of HII regions and OB stars and tested the hypothesis of ionization balance within the inner kpc of M33. The main results are summarized below.

(1) The \( H\alpha \) emission appears to be distributed in three distinct components, denoted bright, halo and field, that are spread out over the galaxy. Roughly one third of the brightest blue stars (\( V \leq 18 \)) are found in the field region. Including all stars within the photometric limit \( V=21 \), we find that \( \sim 50\% \) of the OB stars (luminous blue stars with \( B-V \leq 0.4 \)) are located in the field. From this result we estimate \( 50\% \) of the main sequence lifetime of an OB star is spent outside a recognizable HII region. The implication of this for molecular cloud lifetimes is that if the field stars escape from bright HII regions by destroying their parent molecular clouds, then molecular cloud lifetimes must be shorter than \( \sim 10^7 \) yrs. On the other hand, if OB stars simply move out of the clouds, molecular cloud lifetimes could be considerably longer.

(2) Assuming typical OB star velocities are \( \sim 3 \) km s\(^{-1} \) we find field stars could not all have originated in and percolated out of existing HII regions or for that matter, a strictly massive (\( \geq 0.5 \times 10^5 M_\odot \)) GMC population. If the galaxy contains GMCs with masses down to \( \sim 10^3 M_\odot \) (typical Taurus-type dark clouds), there would exist sufficient numbers of GMCs to make the average separation between them comparable to the maximum distance an OB star can travel over its lifetime. Therefore, the field star population could have originated from and percolated out of a GMC population with masses \( \geq 10^3 M_\odot \).

(3) The slopes of the luminosity functions for stars in the bright, halo and field regions are comparable to one another, which suggests that the stellar populations in the different ionized gas environments must have similar ages. This result is supported by the brightest (\( V \leq 18 \)) blue stars which show no evidence of being concentrated in any one type of ionized gas environment.

(4) We have estimated the theoretical ionizing flux from the observed OB stars using the Panagia (1973) models and converted these to theoretical \( H\alpha \) fluxes assuming Case B recombination for the bright and halo regions and Case A and B recombination for the field region. Ionization balance calculations show that the inner region of M33 is not in ionization balance: the observed fluxes are a factor of \( \sim 7-9 \) (Case A-Case B) less than the predicted best estimate fluxes and a factor of \( \sim 3-4 \) (Case A-Case B) less than the predicted conservative estimate fluxes. The greatest loss of ionizing flux occurs in the field region: even in the conservative case, the predicted \( H\alpha \) flux is a factor of 5 (Case A recombination) greater than that observed. The discrepancy between the two increases to a factor of 13 (Case A recombination) for the best estimate case of the predicted flux. We find Case A recombination provides the best agreement between observed and predicted \( H\alpha \) fluxes in the field. Of the three ionized gas environments, the bright regions are
closest to being ionization bounded. Investigation of an individual, isolated region containing both bright and halo emission produces comparable results: the region is unbalanced by a factor of $\sim 3$ and $\sim 7$ for respectively, the conservative and best estimate cases.

(5) Based on our result that the separate ionized gas environments and individual regions of the galaxy are leaking ionizing flux, SFRs calculated from $H\alpha$ luminosity must underestimate the true SFR. Over the entire surveyed region, the difference between the true SFR and that calculated from $H\alpha$ could differ by a factor of $\sim 3$-17.
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FIGURE LEGENDS

Figure 1a. The continuum subtracted $H\alpha$ image of the central 8.7$'$×8.7$'$ of M33 illustrating the distribution of the bright, halo and field $H\alpha$ emission regions. North is at the top and east is to the left. The contours, corresponding to surface brightnesses of $I=1.74\times10^{-15}$ and $4.57\times10^{-16}$ erg s$^{-1}$cm$^{-2}$arcsec$^{-2}$, separate, respectively, the bright regions from the halo and the halo regions from the field. For orientation, the giant HII region NGC 595 is located on the top right-hand side of the image and the center of the galaxy coincides roughly with that of the image.

Figure 1b. The distribution of all field OB stars ($V\leq21$ and $(B-V)\leq0.4$) in the inner kpc of M33. As photometric coverage of the two vertical strips at the far left and right edges of the image was not available, these regions were excluded from the analysis.

Figure 2. The luminosity functions for stars in the entire image as well as the bright, halo and field HII regions. An arbitrary offset in the ordinate has been applied for the purposes of clarity. Overlaid on the data are the weighted least squares fits (to a limiting magnitude of $V=19$ for the bright region and $V=19.5$ elsewhere).

Figure 3. Expanded View of Region A which was selected for individual investigation. Overlaid are the stars in the bright and halo $H\alpha$ emission areas within the region.
| V MAG * | BRIGHT | DIFFUSE | FIELD | ENTIRE IMAGE |
|---------|--------|---------|-------|--------------|
|         | # of stars | % in region | # of stars | % in region | # of stars | % in region | # of stars | % in region |
| V <18   | 22 ±5 | 5.6 ±1.2% | 15 ±4 | 2.2 ±0.6% | 18 ±4 | 1.6 ±0.4% | 55 ±7 | 2.5 ±0.3% |
| 18 <V<19 | 50 ±7 | 13 ±2% | 66 ±8 | 10 ±1% | 100 ±10 | 9 ±1% | 216 ±15 | 10 ±1% |
| 19< V<20 | 123 ±11 | 32 ±3% | 173 ±13 | 25 ±2% | 276 ±17 | 25 ±1% | 572 ±24 | 26 ±1% |
| 20< V<21 | 195 ±14 | 50 ±4% | 434 ±21 | 63 ±4% | 708 ±27 | 64 ±3% | 1337 ±37 | 61 ±2% |
| V≤21(Total) | 390 ±20 | 688 ±26 | 1102 ±33 | 2180 |   |

Table 1: The Distribution of Blue Stars in Three Ionized Gas Environments in M33.

*Note: Only stars with (B-V) ≤ 0.4
\[^{a}\] In units of $10^{-15}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$. This is measured above the average background.

\[^{b}\] In units of $L_{39}$, which is defined as $10^{39}$ erg s$^{-1}$. $L_{ob}$ is measured above the average background luminosity $L_{bg} = 4.0 \pm 0.08 \times 10^{34}$ erg s$^{-1}$ arcsec$^{-2}$. The total observed luminosity $L_{tot} = L_{ob} + L_{bg}$, for the bright, halo, field and entire image are, respectively, $3.9 \times 10^{39}$, $4.6 \times 10^{39}$, $7.0 \times 10^{39}$ and $15.6 \times 10^{39}$ erg s$^{-1}$.

\[^{c}\] In units of $0.97 \times 10^{37}$ erg s$^{-1}$ star$^{-1}$. In the case where the average background was not subtracted off, the corresponding numbers are: 1.0, 0.67, 0.64 and 0.72 (in units of $1.0 \times 10^{37}$ erg s$^{-1}$) for, respectively, the bright, halo, field and collective regions.

|                  | BRIGHT | HALO  | FIELD | ENTIRE IMAGE |
|------------------|--------|-------|-------|--------------|
| Total \# of Stars $M_V \leq 21$ | 390    | 688   | 1102  | 2180         |
| Area Covered (kpc$^2$)    | 0.22   | 0.83  | 2.28  | 3.33         |
| Surface Density of Stars (kpc$^{-2}$) | 1770   | 829   | 483   | 655          |
| Average Surface Brightness $[I]$ \[^{a}\] | 3.44   | 0.62  | 0.09  | 4.15         |
| Total Luminosity $[L_{ob}]$ \[^{b}\] | 3.78   | 2.57  | 1.01  | 7.36         |
| Relative Luminosity per Star \[^{c}\] | 1.0    | 0.38  | 0.10  | 0.35         |

Table 2: Average Properties of the Three Ionized Gas Environments.
Minimum: no corrections applied. All fluxes in units of $L_{39} = 10^{39} \text{ erg s}^{-1} \text{ arcsec}^{-2}$.

Incompleteness: corrected for incompleteness for stars brighter than $V=21$.

Best estimate: corrected for incompleteness at bright ($V \leq 21$) and faint ($V > 21$) magnitudes.

Conservative estimate: corrected only for stars with $V > 21$.

Observed fluxes do not include the average sky background. If the average background is included, the relevant numbers are: 3.9 (bright), 4.6 (halo), 7.0 (field) and 15.6 (entire image).

|                | BRIGHT | HALO  | FIELD (Case A/ B) | TOTAL (Case A/ B) |
|----------------|--------|-------|-------------------|-------------------|
| Minimum Flux$^a$ | 4.90   | 4.07  | 2.40 / 5.75       | 11.48 / 14.79     |
| Incompleteness ($V \leq 21$)$^b$ | 8.13   | 8.32  | 4.75 / 11.48      | 21.20 / 27.93     |
| Best Estimate$^c$ | 15.03  | 21.47 | 13.23/31.88       | 49.73 / 68.38     |
| Conservative Estimate$^d$ | 7.92   | 9.32  | 5.42 / 13.00      | 22.66 / 30.24     |
| Observed Flux$^e$ | 3.78   | 2.57  | 1.01              | 7.36              |

Table 3: Observed and Predicted H\textalpha Flux in the Three Ionized Gas Environments.
Predicted Hα luminosity accounting for incompleteness (as in Table 3). All fluxes in units of $L_{39} = 10^{39}$ erg s$^{-1}$ arcsec$^{-2}$.

Observed luminosities have been background subtracted. If the background is included, the numbers are: 0.18 (bright) and 0.14 (halo) and 0.32 (combined bright and halo).

In units of $6.8 \times 10^{36}$ erg s$^{-1}$ arcsec$^{-2}$ star$^{-1}$.

|                        | BRIGHT | HALO | (BRIGHT + HALO) |
|------------------------|--------|------|-----------------|
| Minimum Flux$^a$       | 0.24   | 0.01 | 0.25            |
| Incompleteness (V≤21)$^a$ | 0.69   | 0.03 | 0.72            |
| Best Estimate $^a$     | 1.00   | 0.37 | 1.37            |
| Conservative Estimate $^a$ | 0.40   | 0.14 | 0.54            |
| Observed Luminosity $[L_{ob}]^b$ | 0.13   | 0.07 | 0.20            |
| # Stars Observed $[S_{ob}]$ | 19   | 17   | 36              |
| # Stars with $T_{eff,L} < t_{eff} < T_{eff,U}$ | 11 | 14 | 25 |
| $L_{ob}/S_{ob}$ $^c$ | 1      | 0.64 | 0.82            |

Table 4: The Observed and Predicted Hα Fluxes and OB Star Content in Region A.