STUDIES ON REINFORCED CONCRETE INVERTED UMBRELLA HYPERBOLIC PARABOLOID SHELLS

Prabhakaran Veerasamy1*, Yuvaraj Subramaniyan1, Ashwathi Rajendran1

1 Assistant Professor, Department of Civil Engineering, Bannari Amman Institute of Technology, Erode.

Corresponding author’s email address: prabhakaranv@bitsathy.ac.in

Abstract. Hyperbolic paraboloid shell roofs have been extensively used in construction because of impressive appearance and economy in cost. For wide spans, the cost of form work is generally high. By reusing this form work we can reduce the cost of construction. These shells require special care in laying reinforcement, concrete pouring and compaction and hence the labour required for this type of construction is highly skilled. In this study, the design of inverted umbrella type hyperbolic paraboloid shells have been done using IS 2210 - 1988 code and IS 456-2000 code. Concrete of grade M30 and Steel of grade Fe500 are used throughout the investigation. The central dip is taken as variable and it will be taken as a/3, a/4 and a/5. In this study, 21 inverted umbrella type reinforced concrete hyperbolic paraboloid shells have been designed and the estimation of cost has been done. The effect of central dip on the cost per sq.m covered investigated for the inverted umbrella shells.

1. Introduction

Hyperbolic paraboloids are anticlastic shells which are formed by the translation of parabolic curve with downward curvature on another parabola with upward curvature. It is also known as hypar shell. Several types of hyperbolic paraboloids are there such as inverted umbrella, hipped hypar, umbrella and saddle.

The hyperbolic paraboloid umbrella shape is formed by abutting four paraboloids in rectangular shape. All the stresses acting on the body concentrated at one point (at the point of intersection of paraboloids) and resolved along the edge beam in conformity of statics law. Hypar shell is widely used for variety of structures like swimming pools, cycle stands in modern monumental structures. An added advantage of hypar shell being that it can be used for roofs as well as foundations where the safe bearing capacity of the soil is very low[3].

Analysis and design of inverted umbrella hyperbolic paraboloid shells have been carried out with the help of Excel software. The cost of construction of inverted umbrella type hyperbolic paraboloid shells, 21 numbers are designed and the estimation of cost done by using Excel software.

2. Membrane Forces

2.1 Geometry of hypar shell

Figure 2.1 – Geometry of shell
The generation of hypar shell surface is shown in Figure 2.1. Consider a plane rectangle OXBY. If B is now elevated by an amount BB'=h, the warped surface OXB’Y is a hyperbolic paraboloid. Each pair of opposite sides is divided into equal parts and the corresponding points joined by straight lines to obtain the hypar surface. The surface between X and Y is parabola (Convex Parabola) and is well suited to resist compressive forces similar to that of arch elements. The surface between O and B’ is an inverted parabola (Concave Parabola) and is well suited to take tensile forces much similar to that of a catenary’s.

The hypar surface combining these shapes has a great stiffness and resistance to buckling and except for secondary bending effects, normal applied loads are carried by membrane or direct forces within the thickness of shell. The shear forces within the shell gather along the four edges transferring the applied loads to the positions where support is provided.

2.2 Analysis of Membrane forces

The equation of the hypar shell in Cartesian co-ordinates is given by

\[ z = \frac{h}{ab}xy = Kxy \]

In the above equation, the constant \( K = \frac{h}{ab} \) determines the intensity of warping of the surface and is referred as the co-efficient of specific distortions. Under vertical loads in the direction z, only element with edges parallel to the generatrix (boundaries of shell) is subjected to only shear forces along the edges.

The state of stress is defined as,

\( Nx = 0, Ny = 0, Q = \frac{Wab}{2h} \)

where,

\( Q = \text{Shear force}/\text{metre} \)

The principal stresses are also of same magnitude as shear stress and occur along lines at 450 to the shell boundaries. There will be compression \( Fc \) along XY and tension \( Ft \) along OB’. The shears which collect along OX and OY are resisted by these edges acting in compression and shears along XB’ and YB’ are resisted by these edges acting in tension.

\( Q = \frac{Wab}{2h} \) kN/m

Therefore shear stress,

\( q = Q/(1000t) \) N/mm²

3. Inverted Umbrella Hyperbolic Paraboloid Shells

3.1 Introduction

The inverted umbrella type of roof consisting of hypar units are widely used for cycle and bus stands, race tracks and market halls. The umbrella roof consists of four hypar units joined together at the centre where the main supporting column is provided. The structural components of typical inverted umbrella roof shown in Fig.3.1. are as follows.

![Figure 3.1 - Components of inverted umbrella shell](image-url)
The edge beams are subjected to tensile force (CA, AD, DE, EF, FG, GH, HB and BC). The tensile forces are zero at the corners (C, D, F and H) and maximum at intermediate points. The sloping ribs OG, OE, OA and OB are subjected to compression, varying from zero at the edges to a maximum at the junction O.

3.2. Design example

Given

| Parameter                          | Value          |
|------------------------------------|----------------|
| Plan area                          | 10mX10m        |
| Grade of concrete, $f_{ck}$         | 30 N/mm²       |
| Grade of steel, $f_{y}$             | 500 N/mm²      |
| Unit Weight of concrete            | 24 N/mm²       |
| Allowable tensile stress in concrete | 3.6 N/mm²    |
| Allowable compressive stress in concrete | 10 N/mm²    |
| Allowable tensile stress in steel  | 275 N/mm²      |
| Allowable compressive stress in steel | 190 N/mm²   |
| SBC of soil                        | 200 kN/m²      |
| $X_{umax} / d$                      | 0.46           |
| Modular ratio, $m$                  | 9.33           |

Solution

Step 1:-
Calculation of Central dip:-
a= 5 m, b= 5 m
Central dip, $h = a / 5 = 1$ m

Step 2:-
Load calculations:-
Let the thickness, $t = 40$ mm
Dead Load = 0.96 kN/m²
Live Load = 0.50 kN/m²
Load due to self weight of edge beams, water proofing etc. = 0.22 kN/m²
Total Load, $w = 1.68$ kN/m²

Step 3:-
Calculation of Membrane shear force:-
Membrane shear force= $(W_{ab} / 2h) = 21$ kN/m
Shear stress = 0.53 N/mm²

Step 4:-
Calculation of nominal reinforcement:-
Use nominal reinforcement for temperature and shrinkage (0.2 % of gross cross section)
$A_{st} = 80$ mm² per m
But
$A_{st} = 76.36$ mm² per m
Using 8 mm diameter @ 300 mm c/c both ways
Tensile stress = 0.52 N/mm² < 3.6 N/mm² , Hence safe

Step 5:-
Calculation of compression on sloping ribs:-
Length of member $OF = OE = OH = OG = 5.10$ m
Maximum compression in sloping rib at O = 214 kN
Assuming Least Lateral Dimension = 200 mm
$b = 0.2$ m
Since $(Lef/b) = 25.50 > 12$
Therefore the compression member should be designed as a long column
Reduction co-efficient
$Cr = 0.719$
Using 1% steel in compression member
\[ P = \sigma_c A_c + \sigma_s A_{sc} \]
\[ A_c = 25247.27 \text{ mm}^2 \]
Width of beam = 126.24 mm
Provide 130 mm wide x 200 mm deep compression ribs
\[ A_{st} = 252.47 \text{ mm}^2 \]
Using 16 mm diameter bars
No. of bars = 2
Provide 2 numbers of 16 mm diameter bars as main reinforcement
Spacing of ties:
Provide 8 mm diameter ties
1. Least lateral dimension = 130 mm
2. 16 x diameter of longitudinal bar = 256 mm
3. 48 x diameter of ties = 384 mm
Provide 8 mm diameter ties @ 130 mm c/c
Step 6:-
Design of Edge Beams (Tension Members):-
At the centre of the beam
Maximum tension developed = 105 kN
\[ A_{st} = 381.81 \text{ mm}^2 \]
\[ A_{st \ prov} = 628 \text{ mm}^2 \]
If \( A_c \) = area of concrete
\[ A_c = 25984.85 \text{ mm}^2 \]
Adopt Width = 200 mm
Depth of beam = 129.92 mm
Therefore we can provide edge beams of size = 200 mm x 130 mm
Design of columns:-
Load from edge beam = 24.95 kN
Load from sloping rib = 12.36 kN
Load from self-weight of shell = 97.90 kN
Live load = 195.80 kN
Total load = 331.01 kN
Let the overall area of the column section be \( A \)
The column is designed as hollow to accommodate the drainage pipe
Providing 3% of steel
\[ A_{sc} = 0.03A \]
Area of concrete = 0.97A
\[ P_u = 0.4*f_{ck}*A_c + 0.67*f_y*A_{sc} \]
\[ A_c = 22891.26 \text{ mm}^2 \]
Provide size of column as 150 mm x 150 mm
Provide 4 numbers of 16 mm diameter bars
\[ A_{st} = 804.25 \text{ mm}^2 \]
Lateral ties:-
1. (1/4) diameter of longitudinal bars = 6 mm
2. 5 mm
Provide 6 mm diameter ties
Spacing of ties:-
Least lateral dimension =150 mm
16 x 16 = 256 mm
48 x 6 = 288 mm
Provide 6 mm diameter ties @150 mm c/c
Design of foundation:-
Load calculation:-
Total factored load from column = 496.51 kN
Area of footing = 2.48 m²
Side of footing = 1.58 m and Provide 1.58 m x 1.58 m
Net upward pressure intensity = 200000 N/m²
Depth of footing:-
B.M at the critical section, \( \mu_u = 3606.778 \text{ Nm} \)
Equating the limiting moment of resistance to the ultimate bending moment
\[ 0.133 \cdot f_{ck} \cdot b \cdot d^2 = \mu_u \]
\( d = 92.94 \text{ mm} \)
Pulim = 1.14 %
\( A_{st} = 160.67 \text{ mm}² \)
Provide 2 numbers of 16 mm diameter bars

![Figure 3.2 - Reinforcement details in inverted umbrella shell](image)

![Figure 3.3 - Footing Reinforcement details](image)
4. Estimation of Cost of Reinforced Concrete Inverted Umbrella Hyperbolic Paraboloid Shells

4.1. Cost Variations of inverted umbrella hyperbolic paraboloid shells

| Area covered (Sq.m) | Total cost (Rs) per Sq.m |
|---------------------|--------------------------|
| 100                 | 585                      |
| 225                 | 732                      |
| 400                 | 991                      |
| 625                 | 1231                     |
| 900                 | 1563                     |
| 1225                | 2089                     |
| 1600                | 2486                     |

Figure 4.1 - Total cost per sq.m for given area

Figure 4.2 - Cost variations of inverted umbrella type shells
4.2. Effect of variation of central dip on total cost

From the above design results, it has been found that the increase in central dip increases the cost of construction.

5. Conclusion

Hyperbolic paraboloid shell roofs are preferable to cover large floor spaces with economical use of materials. The usage of curved space requires 25% to 35% less quantity of materials than that of the plane elements. Structurally hyperbolic paraboloid shells are superior because of the whole cross section is uniformly stressed due to the direct forces with negligible or zero bending effects. It has been found that, for inverted umbrella type shells, to cover a given area, as the area increases cost per unit area covered also increases. Also the increase in central dip increases the cost of construction. So it is recommended for the design of roofs in economical cost.

References

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