Topological Charge Analysis of Single Skyrmion Creation with a Nanosecond Current Pulse

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Magnetic skyrmions have been proposed for applications in future information storage because of their small size, their stability, and their facile movement with low current. For such purposes, the ability to create single skyrmions is required, and an understanding of the process of skyrmion creation and decay is highly desirable. Here we numerically show that the location and the moment of skyrmion creation or annihilation can be precisely controlled by a nanosecond unpolarized current pulse. To analyze the microscopic process, we employ a lattice version of the topological charge. It provides a clear picture of spin trajectories and orientations that locally trigger a topological transition, and it reveals the topological origin of a skyrmion’s stability at finite temperatures. The robustness and experimental feasibility of the proposed mechanism are numerically examined.

A two-dimensional skyrmion lattice may be formed under a uniform magnetic field [3, 4], however, the creation of single skyrmions is far more challenging [19–21]. Spintronic applications require the accurate control of both the position and the time of skyrmion creation. Recently, single skyrmion creation and annihilation has been achieved by injecting spin-polarized current from an STM tip into ultra-thin Pd/Fe/Ir(111) films at 4.2 K [22]. However, such skyrmions are created by the chance encounter with local defects, and the microscopic mechanism of the creation process is not clear. In this work, we theoretically investigate the topological transition of the microscopic spin texture during a dynamical skyrmion creation process. This microscopic picture of the topological transition provides insight into the critical condition to create isolated skyrmions and the robustness of this condition against small, random external perturbations. Based on this critical condition, we demonstrate that controlled skyrmion creation can be realized by applying an unpolarized nanosecond current pulse from a vertical metallic nanopillar.

The critical condition of the topological transition is determined by monitoring the topological charge during a micro-magnetic dynamical simulation. A skyrmion is distinguished from a ferromagnet or other trivial state by the topological charge $Q$, which is a nonvanishing integer [23, 24]. Each skyrmion contributes $\pm 1$ to the total topological charge. Usually $Q = \frac{1}{4\pi} \int d^2r \cdot (\partial_r S \times \partial_\theta S)$ is employed, but it is well defined only in the continuum limit where all the spins are almost parallel to their neighbors [24]. In this limit, magnetic dynamical processes can only distort the geometry of the spin texture, but cannot change the wrapping number in the spin space. Thus, the topological charge above is conserved [25], and it fails to capture the precise time evolution of the topological transition.

Here we employ the lattice version of the topological charge that provides a microscopic picture of the spin evolution and reveals the microscopic criteria for a topological transition to occur during any dynamical process [26]. This version of $Q$ is defined on a square lattice mesh illustrated in Fig. 1(b). The calculation of $Q$ starts by triangulating the entire lattice and then counting the solid angles $\Omega_\Delta$ for each triangle $\Delta(S_1, S_2, S_3)$ determined by

$$\exp(i \frac{\Omega_\Delta}{2}) = \rho^{-1} [1 + S_1 \cdot (S_2 + S_3)]$$

where $-2\pi < \Omega < 2\pi$, and $\rho = [2(1 + S_1 \cdot S_2)(1 + S_2 \cdot S_3) - 2]$.

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Helimagnetic film
Q=1
Vertical current
1+
thus must drive three spins an event crossing the branch cut, the dynamical process
the branch cut in the complex plane when
dynamical process causing of the complex plane in Eq. (1) is a discontinuous line,
must point ‘away’ from each other at the branch cut. For
ranges from −π/2 to π/2 [Fig. 2 (a)]. When Ωcir reaches a critical value in Fig. 2 (b), an integer topological charge is created indicating the creation of a single skyrmion. This new-born skyrmion is surrounded by an extended swirling spin texture. Once the skyrmion core region is formed, further increase of the circulating field does not bring any noticeable change. Once the topological charge is detected, Hcir is slowly turned off. After the circular magnetic field is removed, the extended swirling spin texture reverts to the ferromagnetic state leaving a perfect single skyrmion at the center [Fig. 2 (c)]. By reversing the nanopillar current and creating a circulating field with opposite swirling direction, this skyrmion can be annihilated. Movies of these dynamical processes can be found in [25].

The microscopic process of the skyrmion creation is the following. For simplicity of the initial discussion, we assume that the pillar center is exactly located on top of a magnetic site such that the generated swirling texture has rotational symmetry. At the center of the swirling texture, the central spin, S0, and its four nearest neighbors S1, S2, S3, and S4 form a configuration illustrated in Fig. 2(d). Due to the rotational symmetry of the applied field, these four spins relate to each other by successive rotations of π/2 about the ẑ axis. They thus share the same angle θ to the plane of the film, and the same azimuthal angle φ measured from x or y axis respectively. The effective field experienced by the central spin is along the z direction with an amplitude of

\[ H_{\text{eff}}^0 = 4J \sin \theta - 4D \cos \theta \sin \varphi + H_0, \]
where J and D are the strength of the Heisenberg and the DM interaction respectively. The direction of the electrical current is chosen so that the the swirling direction of the circulating field is the same as that of the in-plane

\( S_3) (1 + S_4 \cdot S_1)^{1/2} \) is the normalization factor [24]. The lattice version of the topological charge Q is then given by summing over all of the triangles.

\[ Q = \frac{1}{4\pi} \sum_{\Delta} \Omega_{\Delta} \]

From this definition, the directional solid angle \( \Omega_{\Delta} \) ranges from −2π to 2π so that the negative real axis of the complex plane in Eq. (1) is a discontinuous line, called branch cut mathematically. \( \Omega_{\Delta} \) is 2π immediately above, and −2π immediately below, the branch cut. Any dynamical process causing \( e^{i\Omega_{\Delta}/2} \) to cross the branch cut is accompanied by a change in the topological charge of ±1 as shown in Fig. 1(c). The exponential \( e^{i\Omega_{\Delta}/2} \) lies on the branch cut in the complex plane when \( S_1 \cdot (S_2 \times S_3) = 0, \) and \( 1 + S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1 < 0. \) To trigger an event crossing the branch cut, the dynamical process thus must drive three spins \( S_1, S_2, S_3 \) in one particular triangle coplanar from the condition \( S_1 \cdot (S_2 \times S_3) = 0. \) The other condition \( 1 + S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1 < 0 \) leads to the inequality \( (S_1 - S_2) \cdot (S_3 - S_2) > 0, \) so that \( z(S_1 - S_2) \) is an acute angle, and the same holds true for permutations of the three indices 1, 2, and 3. Consequently, three spins must point ‘away’ from each other at the branch cut. For fixed \( S_1 \) and \( S_2, S_3 \) must lie on the arc \( S_1, S_2', \) as shown in Fig. 1 (d). This coplanar but highly non-colinear critical state must be achieved during skyrmion creation or annihilation.

Our proposed scheme to control the precise location and the moment of the critical transition is to apply a circulating magnetic field \( H_{\text{cir}} \) in the plane generated by a vertical current pulse in a device geometry illustrated in Fig. 1(a). A metallic nanopillar electrode of radius \( R \) is deposited on top of a helimagnetic thin film, with a back contact on the bottom of the film which serves as the drain of the electron current. A uniform external magnetic field \( H_0 \) is always applied vertically to ensure a ferromagnetic ground state in which all spins are perpendicularly polarized. When a current is applied between the nanopillar electrode and the back contact, \( H_{\text{cir}}, \) the Oersted field, will be generated in the plane of the helimagnetic thin film, dragging the spins into a swirling spin texture. For simplicity, the current through the nanopillar is assumed to be uniform. The Oersted field and numerical details can be found in [25]. The topological charge is monitored throughout the whole process.

Starting from the ferromagnetic initial state, as the strength of the circulating field, \( H_{\text{cir}} \), increases linearly with time, the spins around the pillar electrode increase their in-plane components forming a swirling spin texture [Fig. 2(a)]. When \( \Omega_{\text{cir}} \) reaches a critical value in Fig. 2 (b), an integer topological charge is created indicating the creation of a single skyrmion. This new-born skyrmion is surrounded by an extended swirling spin texture. Once the skyrmion core region is formed, further increase of the circulating field does not bring any noticeable change. Once the topological charge is detected, \( H_{\text{cir}} \) is slowly turned off. After the circular magnetic field is removed, the extended swirling spin texture reverts to the ferromagnetic state leaving a perfect single skyrmion at the center [Fig. 2 (c)]. By reversing the nanopillar current and creating a circulating field with opposite swirling direction, this skyrmion can be annihilated. Movies of these dynamical processes can be found in [25].
spin component of a skyrmion; therefore \( \varphi \) is about \( \pi/2 \), and \( \sin \varphi \) is positive. Before a skyrmion can be created, the circulating field pulls spins \( \mathbf{S}_A, \mathbf{S}_B, \mathbf{S}_C, \) and \( \mathbf{S}_D \) downward towards the plane reducing the angle \( \theta \). \( H_{\text{eff}} \) therefore decreases accordingly, but still remains positive. When \( \theta \) reaches a critical threshold as the four spins rotate towards the plane, \( H_{\text{eff}} \) reverses its sign, and as a result spin \( \mathbf{S}_0 \) quickly flips down into the \( -z \) direction. This process changes the topological charge by an integer and creates a skyrmion.

To demonstrate this process, we draw \( \mathbf{S}_0 \) and its nearest neighbors, \( \mathbf{S}_A, \mathbf{S}_B, \mathbf{S}_C, \) and \( \mathbf{S}_D \), in a unit sphere at the critical state immediately before the reversal of the central spin [Fig. 2(e)]. \( \mathbf{S}_A, \mathbf{S}_B, \mathbf{S}_C, \) and \( \mathbf{S}_D \) are the mirror points of \( \mathbf{S}_A \), \( \mathbf{S}_B \), \( \mathbf{S}_C \), and \( \mathbf{S}_D \) with respect to the sphere center. Both planes \( \mathbf{S}_A\mathbf{S}_B\mathbf{S}_C\mathbf{S}_D \) and \( \mathbf{S}_A\mathbf{S}_B'\mathbf{S}_C'\mathbf{S}_D' \) are parallel with the equatorial plane, and the four points in each plane are equidistant. Through a fast process, \( \mathbf{S}_0 \) rapidly switches from the north pole \( (N) \) to the south pole \( (S) \) along a spherical arc. When \( \mathbf{S}_0 \) is located on the geodesic arc \( \mathbf{S}_A\mathbf{S}_B' \) shown as point \( \mathbf{P} \) in Fig. 2(f), the three spins \( \mathbf{S}_A, \mathbf{S}_B, \) and \( \mathbf{S}_0 \) are coplanar. As \( \mathbf{S}_0 \) crosses arc \( \mathbf{S}_A\mathbf{S}_B' \), the solid angle formed by these three spins changes sign resulting in a change in \( \Omega_0 \) and a change in the topological charge in Eq. 2 of 1. The same process applies to the other arcs \( \mathbf{S}_B\mathbf{S}_C', \mathbf{S}_C\mathbf{S}_D', \) and \( \mathbf{S}_D\mathbf{S}_A' \). Notice that these four arcs form a closed loop enclosing the south pole as shown by the red curve in Fig. 2(e)-(f). Therefore the trajectory of \( \mathbf{S}_0 \) must cross this closed loop once, and an integer change of the topological charge is guaranteed regardless of the actual geometry of the \( \mathbf{S}_0 \) trajectory. A single skyrmion is thus created in this process.

The previous discussion assumes a perfect rotational symmetry during the topological transition process. This is usually not exactly the case due to manufacturing tolerances or thermal fluctuations. Also, the center of the nanopillar can hardly coincide with a spin site in
real systems. These symmetry breaking effects distort the corresponding spherical quadrilateral $S_A S_B S_C S_D'$ from the symmetric case, and the reversal of $S_0$ does not, in general, start exactly from $N$ and end at $S$. However, as long as the perturbation is moderate, this distortion does not alter the fact that $S_A S_B S_C S_D'$ is a closed loop dividing the surface of the unit sphere into two parts, where the starting and ending points of $S_0$ locate respectively. Driven by $H_{\text{cir}}$, the spins in the swirling texture are forced downwards to the plane so that the closed loop $S_A S_B S_C S_D'$ is enlarged and approaches the equator. Thus, the trajectory of $S_0$ must cross the closed loop $S_A S_B S_C S_D'$ an odd number of times. The topological charge must change by one, and only by one, and a local skyrmion is thereby created.

Since the local topological transition is robust against symmetry-breaking effects as discussed above, thermal fluctuations cannot prevent the proposed skyrmion creation mechanism. The processes and trends discussed previously for the zero temperature case also apply at finite temperatures. Videos demonstrating the skyrmion creation and annihilation processes at finite temperature are attached to the Supplemental Material. At finite temperatures, thermal fluctuations work together with the magnetic stimuli given by $H_{\text{cir}}$ to overcome the potential barrier separating the skyrmion phase from the ferromagnetic phase\cite{27}. At even higher temperature, thermal fluctuations alone are capable of exciting skyrmions at uncontrollable locations. For application purposes, such high temperature regimes should be avoided.

After a single skyrmion has been created, it is important to ascertain its stability after the circulating field is turned off. Although the skyrmion has a higher energy $\Delta$ compared to the ferromagnet [as shown in the energy landscape Fig. 3 (a)], spontaneous decay from the skyrmion phase to the ferromagnetic phase will not occur since it is protected by the activation energy $E_a$. The height of the activation energy $E_a$ can be estimated by simple topological argument. As the topological charge changes, this dynamical process of decay must follow a trajectory that crosses the branch cut around the peak of $E_a$. The critical condition require that, there must exist three spins $S_1, S_2, S_3$ in one plaquette become coplanar and point away from each other such that $1 + S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1 < 0$. Consequently two pairs of neighboring spins $(S_1, S_2)$ and $(S_2, S_3)$ have the Heisenberg interaction $-J(S_1 \cdot S_2 + S_2 \cdot S_3) > J(1 + S_3 \cdot S_1) \geq 0$. In contrast, the neighboring spins of a stable skyrmion do not deviate much from each other, so that the exchange energy of a stable skyrmion is $-J(S_1 \cdot S_2 + S_2 \cdot S_3) \approx -2J$. Therefore, the energy of the intermediate coplanar state is higher than the skyrmion energy by an activation energy of $2J$.

This estimation is amazingly consistent with the value of $E_a \approx 1.7J$ determined from numerical simulations. The activation energy is extracted numerically by examining the lifetime of a single skyrmion as a function of temperature. A skyrmion is simulated at finite temperature until the topological charge switches from 1 to 0 due to the random thermal fluctuations. The time of the annihilation is recorded. This simulation is repeated 1000 times at each temperature, and the average lifetime $\tau$ is determined as a function of temperature. Plots of $\tau$ versus temperature for different background fields are shown in Fig. 3 (b). At low temperatures, a smaller $H_0$ results in a more stable single skyrmion with a longer lifetime. At higher temperatures such that $k_B T$ approaches $J$, all of the curves in Fig. 3 (b) converge and decay exponentially. For transition from skyrmion to ferromagnet, the transition rate $k$ well obeys the Arrhenius law $k \sim \exp(-E_a/k_B T)$. The lifetime $\tau$ is the inverse of $k$ so that $\tau \sim 1/k \sim \exp(E_a/k_B T)$. The plot of $\ln \tau$ in Fig. 3 (c) is linear in the inverse temperature $1/T$. Plots for various $H_0$ are nearly parallel with each other with an activation energy of $E_a \approx 1.7J$.

Further analysis shows that this activation energy is also insensitive to the DM interaction as shown in Fig. 3 (d). These results confirm the topological origin of the activation energy that stabilizes the single skyrmion.
To confirm the feasibility of our skyrmion creation method, a simulation is performed using physical parameters of an FeGe thin film \[5\, 28\]. The lattice constant of FeGe is \(a = 4.70\ \text{Å}\), and the helical period is \(\lambda = 70\ \text{nm}\). Utilizing the formula \(
abla = D/\sqrt{2}J\), \(D/J = 0.0597\), where \(J \sim k_BT_c \sim 24\ \text{meV}\). A temperature \(T \sim 0.1J/k_B \sim 28\text{K}\) with an effective easy-plane field of 0.7 T and external magnetic field of 1 T ensures a ferromagnetic ground state for FeGe/Si(111) epitaxial thin films \[28\]. A 0.85 ns square pulse of \(6 \times 10^8\ \text{A/cm}^2\) applied to a 75 nm radius pillar electrode generates a single skyrmion in the ferromagnetic FeGe thin film. The large Curie temperature for this material allows us to do experiments in a wide window of temperatures where thermal fluctuation further reduce the current density required. In experiments, current densities on the order of \(10^8\ \text{A/cm}^2\) are commonly employed for moving magnetic domain walls. In our case, the heating effect can be reduced by selecting materials with low resistivity for the nanopillar such as Cu and Au. Furthermore, the short nanosecond current pulse minimizes the switching energy and the switching time.

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\text{Appendix A: Conservation of Topological Charge in the Continuum Limit}

The time derivative of the topological charge is given by

\[
\frac{dQ}{dt} = \frac{1}{4\pi} \int d^2 r [\dot{S} \cdot (\partial_x S \times \partial_y S) + S \cdot (\partial_x \dot{S} \times \partial_y S) + S \cdot (\partial_x S \times \partial_y \dot{S})]
\]

where \(\dot{S}\) follows the Landau-Lifshitz-Gilbert dynamics given by

\[
\dot{S} = -\gamma S \times H_{\text{eff}} + \alpha S \times \dot{S}
\]

or equivalently

\[
\dot{S} = -\frac{\gamma}{1 + \alpha^2} S \times H_{\text{eff}} + \frac{\alpha \gamma}{1 + \alpha^2} (S \times H_{\text{eff}})
\]

\[
= -\frac{\gamma}{1 + \alpha^2} S \times H_{\text{eff}} + \frac{\alpha \gamma}{1 + \alpha^2} (S(S \cdot H_{\text{eff}}) - H_{\text{eff}})
\]

Therefore

\[
\frac{dQ}{dt} = -\frac{\gamma}{4\pi(1 + \alpha^2)} \int d^2 r [(S \times H_{\text{eff}}) \cdot (\partial_x S \times \partial_y S) + S \cdot ((\partial_x S \times H_{\text{eff}} + S \times \partial_x H_{\text{eff}}) \times \partial_y S) + S \cdot (\partial_x S \times (\partial_y S \times H_{\text{eff}} + S \times \partial_y H_{\text{eff}}))]
\]

\[
+ \frac{\alpha \gamma}{4\pi(1 + \alpha^2)} \int d^2 r [(S(S \cdot H_{\text{eff}}) - H_{\text{eff}}) \cdot (\partial_x S \times S \partial_y S) + S \cdot ((\partial_x S(S \cdot H_{\text{eff}}) - H_{\text{eff}}) \times \partial_y S) + S \cdot (\partial_x S \times (\partial_y S \cdot H_{\text{eff}} - \partial_y H_{\text{eff}}))] \]

Noticing the fact that \(S^2 = 1\), \(S \cdot \partial_x S = S \cdot \partial_y S = 0\), we get

\[
\frac{dQ}{dt} = -\frac{\gamma}{4\pi(1 + \alpha^2)} \int d^2 r [(S \cdot H_{\text{eff}})(\partial_x S \partial_y S) - (\partial_x H_{\text{eff}} \cdot \partial_y S) - (S \cdot H_{\text{eff}})\partial_y S \cdot \partial_x S + (\partial_x S \cdot \partial_y H_{\text{eff}})]
\]

\[
+ \frac{\alpha \gamma}{4\pi(1 + \alpha^2)} \int d^2 r [(S \cdot H_{\text{eff}})S \cdot (\partial_x S \times \partial_y S) - H_{\text{eff}} \cdot (\partial_x S \times \partial_y S) + S \cdot (\partial_x S \cdot \partial_y S) - S \cdot (\partial_x S \times \partial_y H_{\text{eff}})]
\]

\[
= -\frac{\gamma}{4\pi(1 + \alpha^2)} \int d^2 r [-\partial_y (S \cdot \partial_x H_{\text{eff}}) + \partial_x (S \cdot \partial_y H_{\text{eff}})]
\]

\[
+ \frac{\alpha \gamma}{4\pi(1 + \alpha^2)} \int d^2 r [(S \cdot H_{\text{eff}}) \times S] \cdot (\partial_x S \times \partial_y S) - \partial_x (S \cdot (H_{\text{eff}} \times \partial_y S)) - \partial_y (S \cdot (\partial_x S \times H_{\text{eff}}))]
\]
The integral of a total derivative vanishes due to the periodical boundary condition, therefore,

\[
\frac{dQ}{dt} = \frac{\alpha \gamma}{4\pi(1 + \alpha^2)} \int d^2r[(S \times H_{\text{eff}}) \times S] \cdot (\partial_x S \times \partial_y S)
\]

\[
= \frac{3\alpha \gamma}{4\pi(1 + \alpha^2)} \int d^2r(S \times H_{\text{eff}})[S \times (\partial_x S \times \partial_y S)]
\]

\[
= \frac{3\alpha \gamma}{4\pi(1 + \alpha^2)} \int d^2r(S \times H_{\text{eff}})[\partial_x S(S \cdot \partial_y S) - \partial_y S(S \cdot \partial_x S)]
\]

\[
= 0
\]

The total topological charge is unchanged.

In the above the specific LLG dynamics is taken into account, actually one can show that the continuum version of the topological charge is unchanged under any dynamics. The deviation of the topological charge can be written as

\[
\delta Q = \frac{1}{4\pi} \int d^2r[\delta S \cdot (\partial_x S \times \partial_y S) + S \cdot (\partial_x \delta S \times \partial_y S) + S \cdot (\partial_y S \times \partial_x \delta S)]
\]

\[
= \frac{1}{4\pi} \int d^2r[\delta S \cdot (\partial_x S \times \partial_y S) - S \cdot (\partial_x \delta S \times \partial_y S) - \partial_y S \cdot (\partial_x S \times \delta S) - S \cdot (\partial_y \delta S \times \delta S)]
\]

\[
= \frac{1}{4\pi} \int d^2r[\delta S \cdot (\partial_x S \times \partial_y S)]
\]

As long as the spin amplitude is unchanged, \(\delta S\) must be perpendicular to \(S\), or effectively one can write \(\delta S\) as \(\delta S = S \times \nu\). Therefore

\[
\delta Q = \frac{3}{4\pi} \int d^2r(S \times \nu) \cdot (\partial_x S \times \partial_y S)
\]

\[
= \frac{3}{4\pi} \int d^2r((\partial_x S \times \partial_y S) \times S) \cdot \nu
\]

\[
= \frac{3}{4\pi} \int d^2r[(\nu \cdot \partial_y S)(\partial_x S \cdot S) - (\nu \cdot \partial_x S)(\partial_y S \cdot S)]
\]

\[
= 0
\]

**Appendix B: Derivation of Solid Angle**

The solid angle \(\Omega\) expanded by three unit vectors \(\hat{s}_1\), \(\hat{s}_2\), and \(\hat{s}_3\) satisfy the law of spherical excess

\[
\Omega = \theta_1 + \theta_2 + \theta_3 - \pi \tag{B1}
\]

where \(\theta_1\) is the angle between two great circles \(O_{S_1S_2}\) and \(O_{S_1S_3}\). In order to calculate \(\theta_1\), a straightforward way is to find two unit vectors \(\hat{n}_{12}\) and \(\hat{n}_{13}\) perpendicular to the intersection line \(O_{S_1}\) but lying in the two planes \(O_{S_1S_2}\) and \(O_{S_1S_3}\) respectively. \(\cos \theta_1\) is then simply the inner product between these two vectors. Generally we can write \(\hat{n}_{12}\) as

\[
\hat{n}_{12} = a\hat{s}_1 + b\hat{s}_2 \tag{B2}
\]

From conditions \(\hat{n}_{12} \cdot \hat{s}_1 = 0\) and \(\hat{n}_{12} \cdot \hat{n}_{12} = 1\), one can easily get

\[
\hat{n}_{12} = b_{12}^{-1}(-r_{12}\hat{s}_1 + \hat{s}_2) \tag{B3}
\]

where \(r_{ij} = \hat{s}_i \cdot \hat{s}_j\), and \(b_{12} = \sqrt{1 - r_{12}^2}\). Similarly

\[
\hat{n}_{13} = b_{13}^{-1}(-r_{13}\hat{s}_1 + \hat{s}_3) \tag{B4}
\]

Consequently

\[
\cos \theta_1 = \hat{n}_{12} \cdot \hat{n}_{13} = b_{12}^{-1}b_{13}^{-1}t_{23} \tag{B5}
\]

where \(t_{23} \triangleq r_{23} - r_{12}r_{13}\). Define \(t_0\) as \(t_0 = \sqrt{1 + 2r_{12}r_{23}r_{13} - r_{12}^2 - r_{23}^2 - r_{13}^2}\), so that

\[
\sin \theta_1 = b_{12}^{-1}b_{13}^{-1}t_0 \tag{B6}
\]

One can similarly get the cosine and sine functions of \(\theta_2\) and \(\theta_3\). Consequently,

\[
\cos \Omega = \cos(\theta_1 + \theta_2 + \theta_3 - \pi)
\]

\[
= -\cos \theta_1 \cos \theta_2 \cos \theta_3 + \sin \theta_1 \sin \theta_2 \cos \theta_3
\]

\[
+ \sin \theta_1 \cos \theta_2 \sin \theta_3 + \cos \theta_1 \sin \theta_2 \sin \theta_3
\]

\[
= (b_{12}b_{23}b_{31})^{-1}[t_0(t_{12} + t_{23} + t_{31}) - t_{12}t_{23}t_{31}] \tag{B7}
\]

Utilizing the tangent half-angle formula

\[
\cos \alpha = \frac{1 - \tan^2(\alpha/2)}{1 + \tan^2(\alpha/2)} \tag{B8}
\]
one can get

$$\tan \frac{\Omega}{2} = \frac{t_0^2}{(1 + r_{12} + r_{23} + r_{31})^2} \quad (B9)$$

$$\tan \frac{\Omega}{2} = \pm \frac{t_0}{1 + r_{12} + r_{23} + r_{31}} \quad (B10)$$

The sign is determined by the sign of the scalar chirality $\hat{s}_1 \cdot (\hat{s}_2 \times \hat{s}_3)$.

Note that $t_0$ has a geometric meaning of the parallelepiped volume expanded by unit vectors $\hat{s}_1$, $\hat{s}_2$, and $\hat{s}_3$. To see this, let’s first get a unit vector $\hat{h}$ perpendicular to the plane $\hat{s}_2\hat{s}_3$. Let $\hat{h} = a\hat{s}_1 + b\hat{s}_2 + c\hat{s}_3$. The constrain $\hat{h} \cdot \hat{s}_2 = \hat{h} \cdot \hat{s}_3 = 0$ leads to the relations that $b = -b_{23}^2 t_{12} a$ and $c = -b_{23}^2 t_{13} a$. The normalization $\hat{h} \cdot \hat{h} = 1$ gives $a = b_{23} t_0^{-1}$. As a result,

$$\hat{h} = t_0^{-1}(b_{23}\hat{s}_1 - b_{23}^3 t_{12}\hat{s}_2 - b_{23}^- t_{13}\hat{s}_3) \quad (B11)$$

On the other hand, we need to get a unit vector $\hat{l}$ lying in the plane $\hat{s}_2\hat{s}_3$ but perpendicular to $\hat{s}_3$. Similar calculation gives

$$\hat{l} = b_{23}^{-1} (\hat{s}_2 - r_{23}\hat{s}_3) \quad (B12)$$

As a result, the volume $V = |\hat{s}_1 \cdot (\hat{s}_2 \times \hat{s}_3)|$ is given by

$$V = (\hat{s}_1 \cdot \hat{h})(\hat{s}_2 \cdot \hat{l}) = t_0 \quad (B13)$$

Therefore

$$\Omega = 2 \arctan \frac{\hat{s}_1 \cdot (\hat{s}_2 \times \hat{s}_3)}{1 + t_{12} + t_{23} + t_{31}} \quad (B14)$$

$\Omega$ is in the domain of $[-2\pi, 2\pi]$. Observing the fact that $t_0^2 + (1 + t_{12} + t_{23} + t_{31})^2 = 2(1 + r_{12})(1 + r_{23})(1 + r_{31}) \equiv \rho^2$, one can define $\Omega$ as

$$\exp(i\Omega/2) = \rho^{-1}[1 + \hat{s}_1 \cdot \hat{s}_2 + \hat{s}_2 \cdot \hat{s}_3 + \hat{s}_3 \cdot \hat{s}_1 + i\hat{s}_1 \cdot (\hat{s}_2 \times \hat{s}_3)] \quad (B15)$$

which is exactly Eq. 3 in the main content.

In the continuum limit, neighboring spins are almost parallel with each other. Let $\hat{s}_i = \hat{s}^0 + \delta \hat{s}_i$ so that $\Omega$ in Eq. (B14) is given by

$$\Omega = \frac{1}{2}\hat{s}^0 \cdot (\delta \hat{s}_1 \times \delta \hat{s}_2 + \delta \hat{s}_2 \times \delta \hat{s}_3 + \delta \hat{s}_3 \times \delta \hat{s}_1) + o(\delta^3) \quad (B16)$$

The total topological charge is then

$$Q = \frac{1}{4\pi} \sum_\triangle \Omega_\triangle$$

$$= \frac{1}{4\pi} \sum_\triangle \frac{1}{2} \hat{s}^0 \cdot (\delta \hat{s}_1 \times \delta \hat{s}_2 + \delta \hat{s}_2 \times \delta \hat{s}_3 + \delta \hat{s}_3 \times \delta \hat{s}_1) + o(\delta^3)$$

$$= \frac{1}{4\pi} \sum_r \frac{1}{2} \hat{s}_r^0 \cdot (\delta \hat{s}_{r+\hat{y}} \times \delta \hat{s}_r + \delta \hat{s}_r \times \delta \hat{s}_{r+\hat{y}} + \delta \hat{s}_{r+\hat{y}} \times \delta \hat{s}_{r+\hat{\hat{y}}})$$

$$+ \frac{1}{4\pi} \sum_r \frac{1}{2} \hat{s}_r^0 \cdot (\delta \hat{s}_{r-\hat{y}} \times \delta \hat{s}_r + \delta \hat{s}_r \times \delta \hat{s}_{r-\hat{y}} + \delta \hat{s}_{r-\hat{y}} \times \delta \hat{s}_{r-\hat{\hat{y}}} + o(\delta^3))$$

$$= \frac{1}{4\pi} \sum_r \frac{1}{2} \hat{s}_r^0 \cdot (\hat{s}_{r+\hat{y}} \times \hat{s}_r + \hat{s}_r \times \hat{s}_{r+\hat{y}} + \hat{s}_{r+\hat{y}} \times \hat{s}_{r+\hat{\hat{y}}})$$

$$+ \frac{1}{4\pi} \sum_r \frac{1}{2} \hat{s}_r^0 \cdot (\hat{s}_{r-\hat{y}} \times \hat{s}_r + \hat{s}_r \times \hat{s}_{r-\hat{y}} + \hat{s}_{r-\hat{y}} \times \hat{s}_{r-\hat{\hat{y}}} + o(\delta^3))$$

$$= \frac{1}{4\pi} \sum_r \frac{1}{2} \hat{s}_r^0 \cdot [(\hat{s}_{r+\hat{y}} - \hat{s}_r) \times (\hat{s}_{r+\hat{y}} - \hat{s}_r)]$$

$$+ \frac{1}{4\pi} \sum_r \frac{1}{2} \hat{s}_r^0 \cdot [(\hat{s}_r - \hat{s}_{r-\hat{y}}) \times (\hat{s}_r - \hat{s}_{r-\hat{y}})] + o(\delta^3)$$

$$= \frac{1}{4\pi} \int d^2 r \hat{s} \cdot (\partial_x \hat{s} \times \partial_y \hat{s}) \quad (B17)$$

This result is consistent with the definition of topological charge in the continuum limit.

**Appendix C: Numerical simulation details**

In order to numerically calculate the required condition and to estimate the feasibility of this skyrmion creation
mechanism, dynamical simulations of a spin system based on the LLG equation are performed. Assuming a uniform current is injected through the vertical pillar, the direction of the magnetic field generated by this current is tangential, and the magnitude is determined by Ampere’s law such that

\[ H(r) = h \left( \frac{r}{R} \right)^{\pm 1} \quad \text{for} \quad r \leq R \]  \hspace{1cm} (C1)

where \( R \) is the radius of the pillar and \( h \) is the field peak located at the pillar boundary \( r = R \). In the simulations, the pillar radius \( R \) is larger than the characteristic skyrmion radius, so that in the central regions of interest, Eq. (C1) is a good approximation. In addition to this circulating field, a static upward magnetic field \( H_0 \) is also applied perpendicular to the film to maintain a FM phase background. The full spin Hamiltonian is given by

\[ H = \sum_{i,j} [-JS_i \cdot S_j + D\tilde{r}_{ij}(\hat{S}_i \times \hat{S}_j)] - \mu_B \sum_i \tilde{S}_i (H_{\text{cir}} + H_0) \]  \hspace{1cm} (C2)

where the two terms in the square bracket are the Heisenberg and DM interactions respectively, \( \mu_B \) is the Bohr magneton, and the last term is the Zeeman coupling. \( \tilde{r}_{ij} \) is a unit vector pointing from \( \hat{S}_i \) to \( \hat{S}_j \).

Spin dynamics are simulated by numerically solving the Landau-Lifshitz-Gilbert (LLG) equation

\[ \dot{\mathbf{S}} = -\gamma \mathbf{S} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{S} \times \dot{\mathbf{S}} \]  \hspace{1cm} (C3)

where \( \gamma = g/h \) is the gyromagnetic ratio and \( \alpha \) is the Gilbert damping coefficient. \( \mathbf{H}_{\text{eff}} \) is the effective field given by \( \mathbf{H}_{\text{eff}} = -\partial H / \partial \mathbf{S} \). A fourth order Runge-Kutta algorithm is employed to integrate this first order differential equation. In our simulations, \( J = 1 \), \( D = 0.3 \), and the pillar radius is \( R = 12 \). Without loss of generality, the dynamic simulation of the spin system is carried out on a 2-dimensional square grid. Periodic boundary condition is applied such that a FM phase topological charge is zero. In order to rule out the Skyrmion-Skyrmion interaction across the periodic boundaries, the simulation plate is chosen as 100 \( \times \) 100. The periodic boundary length of 100 is \( \sim 10 \) times larger than the Skyrmion diameter, making the boundary effects negligible. The choice of \( H_0 \), e.g. \( H_0 = 0.09 \), sets the zero-temperature ground state to be ferromagnetic.

According to the LLG equation, the time evolution of each spin is determined by the on-site effective field. The effective field on site \( (i,j) \) is given as

\[ h_{i,j}^{\text{eff}} = -\frac{\partial H}{\partial S_{i,j}} = J \sum_{(j)} S_j - D (S_{i+1,j} - S_{i-1,j}) \times \mathbf{\hat{x}} - D (S_{i,j+1} - S_{i,j-1}) \times \mathbf{\hat{y}} + \left( h_{\text{cir}}^{i,j} + h_0^{i,j} \right) \]  \hspace{1cm} (C4)

where \( h_0^{i,j} \) and \( h_{\text{cir}}^{i,j} \) are the on-site background field and the circulating Oersted field, respectively. Since the topological charge creation only involves the spins close to the electrode center, we cut off the \( 1/r \) tail of the circulating field at \( r = 45 \). The corresponding step length, \( \Delta \tau \), is set as 0.1/J to assure the integral accuracy.

In order to find the critical field \( (H_c) \) at the adiabatic limit, we increase \( H_{\text{cir}} \) at a slow rate at zero temperature until a topological charge is detected. This increasing rate, \( \nu \) at \( r = R \), should be slow enough that the local spins and their effective fields are almost parallel during the dynamical process. Thus, \( H_c \) should not strongly depend on \( \nu \). To find this rate, we increase \( 1/\nu \) and choose the value when the corresponding \( H_c \) is stable. The relation of \( H_c \) when \( 1/\nu \to \infty \) is shown in Fig. 4. In this paper, \( \nu = 4 \times 10^{-5} \) J/s is chosen for all the critical field simulations.

![Hc_converging](image)

Figure 4: \( H_c \) converging with \( 1/\nu \). \( H_0 = 0.09 \) and \( D = 0.3 \) are applied in this simulation.

To simulate the dynamical process at a finite temperature, a stochastic field \( \mathbf{L} \) is added onto the effective field in Eq. (C3). The dissipation-fluctuation relation \( \langle L_\mu (r, t) L_\nu (r', t') \rangle = \xi \delta_{\mu \nu} \delta_{rr'} \delta_{tt'} \) is satisfied, where \( \xi = \alpha k_B T / \gamma \), and \( T \) is the temperature. The average \( \langle \_ \rangle \) is taken over the realizations of the fluctuation field. The deterministic Heun scheme is employed
to integrate out this stochastic LLG equation. In components, the stochastic LLG equation is written as

\[ \frac{dS^k}{dt} = A^k(S, t) + B^{k,l}(S, t)L^l \]  

(C5)

where \( k, l = x, y, z \). Summation over the same index is assumed. In Eq. (C5)

\[ A = -\frac{1}{1 + \alpha^2}S \times H_{\text{eff}} - \frac{\alpha}{1 + \alpha^2} [S \cdot \{H_{\text{eff}} - H_{\text{eff}}]\]  

(C6)

and

\[ B^{k,l}(S, t) \cdot L^l = -\frac{1}{1 + \alpha^2}S \times \frac{\alpha}{1 + \alpha^2} [S \cdot \{S \cdot L\} - L]. \]  

(C7)

Here,

\[ L^k = x\sqrt{24\lambda \Delta t} \]  

(C8)

where \( \lambda = \frac{\alpha}{4\Delta t}k_B T \), and \( x \) is a random number from \(-0.5\) to \(0.5\). To numerically solve Eq. (C5) the integral is taken to the first order as follows:

\[ S^k(t + \Delta t) = S^k(t) + \frac{1}{2} \left[ A^k(S(t), t + \Delta t) + A^k(S(t), t) \right] \Delta t + \frac{1}{2} \left[ B^{k,l}(S, t + \Delta t) + B^{k,l}(S, t) \right] L^l \]  

(C9)

where

\[ \tilde{S}^k = S^k(t) + A(S, t) \Delta t + B^{k,l}(S, t) L^l. \]  

(C10)

The time step in the stochastic integral is taken as \( \Delta t = 0.1/J \). According to our tests, the stochastic integral at zero temperature gives the same result as the Runge-Kutta algorithm. With the topological charge defined on a lattice, the creation of a skyrmion is detected by calculating the topological charge \( Q \) at each time step during the simulation.

**Appendix D: Examinations of Critical Fields**

To determine the external critical field \( H_c \) for skyrmion creation, \( H_{\text{crit}} \) is increased linearly at a slow enough rate such that the adiabatic limit is valid where the local spins follow the effective field, \( H_{\text{eff}} \). It is confirmed from Fig. S1 that \( H_c \) is almost unchanged by further reducing the rate at which the circulating field is turned on. In addition the time scale for skyrmion creation is on the order of nanoseconds which is slow compared to the spin dynamics timescale of picoseconds. The critical field magnitude, \( H_c \), is defined as the amplitude of the circulating field at the pillar electrode boundary \( R \) at the moment the topological charge is created. The results are shown in Fig. 5(a) for different values of background field \( H_0 \) and DM interaction strengths \( D \). In general, the critical field \( H_c \) is the same order of magnitude as the background field \( H_0 \), and the dependence of \( H_c \) on \( H_0 \) is superlinear. Although the above simulation places the electrode center exactly on a magnetic site, when the electrode center is scanned throughout the central

![Figure 6: \( H_c \) as a function of electrode radius, \( R \). The length unit is the tight-binding distance, \( a \). When \( H_0 = 0.09 \), the three curves with different values of \( \alpha \) overlap with each other.](image)

The trend in Fig. 5(a) can be explained by the effective field acting on the central spin, as derived in the main text. When the topological transition starts, the critical value \( \theta_c \) satisfies \( 4J \sin \theta_c - 4D \cos \theta_c \sin \varphi + H_0 = 0 \). \( \theta_c \approx (4D \sin \varphi - H_0)/4J \). Larger \( D \) or smaller \( H_0 \) leads to a larger \( \theta_c \), which can be achieved by a smaller critical circulating field \( H_c \). This is also consistent with the energetic considerations of the system. To create a skyrmion, a potential barrier must be overcome which consists of the energy difference \( \Delta = E_{\text{Skyr}} - E_{\text{Ferro}} \) between the skyrmion energy \( E_{\text{Skyr}} \) and ferromagnetic background \( E_{\text{Ferro}} \). The energy difference \( \Delta \) is sensitive.
to the strength of the DM interaction $D$ and the background field $H_0$ as shown in Fig. 5 (c). Smaller $D$ or larger $H_0$ correspond to a larger energy difference $\Delta$ and a larger potential barrier for the creation of a skyrmion. The critical field increases when $D$ decreases or $H_0$ increases, since more energy is required from the circulation field.

To further check the adiabatic limit, we consider the dependence of $H_c$ on the Gilbert damping factor, $\alpha$, and the electrode radius, $R$. When a local spin is subjected to an effective field, $H_{\text{eff}}$, the damping factor determines the time for the spin to relax into its equilibrium configuration, $t \sim 1/\alpha H_{\text{eff}}$. Thus, a larger $\alpha$ corresponds to a smaller relaxation time.

As shown in Fig. 6, $H_c$ does not vary significantly with the change of the damping factor indicating that the increasing rate of $H_{\text{cir}}$ is slow enough so that the adiabatic limit is satisfied.

$H_c$ also depends significantly on the pillar radius $R$. When the radius is large enough, a linear relation between $H_c$ and $R$ is observed. According to Ampere’s law, this linear relation indicates that the same current density is required, regardless of the size of the electrode. When $R$ is too small, the field outside of the electrode decays, and a higher current density is required.

The skyrmion creation is easier at finite temperature, since it requires smaller critical fields as shown in Fig. 7.

In this calculation, each value of $H_c$ is calculated as an average over 400 sampling runs.

Figure 5: The critical fields. (a), The $H_c$ variation with the change of the background field, $H_0$ and the DM interaction, $D$. $T = 0$, $\alpha = 0.1$ and $R = 12$ are applied. (b), A scan of the electrode center on the central plaquette at zero temperature. $H_c$ varies by only $\sim 0.7\%$ in this scan. (c), Dependence of $\Delta$ on $H_0$ and $D$. Downward sloping red curve plots $\Delta$ versus $D$ for a fixed $H_0 = 0.071$. Upward sloping blue curve plots $\Delta$ versus $H_0$ for a fixed $D = 0.30$. A negative $\Delta$ indicates a Skyrmion ground state.

Figure 7: Skyrmion creation at finite temperature. (a), Temperature assisted Skyrmion creation. $D = 0.3J$, $R = 12$ and $\alpha = 0.1$ are applied. Each data point is an average of 400 sampling runs. (b), A snapshot of a stable Skyrmion created on a FM background at $k_B T = 0.1 J$.

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