Gamow-Teller decay studies with 2p-2h configurations

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Abstract. Starting from a Skyrme interaction with tensor terms, the β-decay rates have been studied within a microscopic model including the 2p−2h configuration effects. As an application we present the evolution of the neutron-rich Ni isotopes near 78Ni that are important for stellar nucleosynthesis.

The study of spin-isospin excitations in neutron-rich nuclei is presently an important problem not only from the nuclear structure point of view but also because of the special role they play in many astrophysical processes. Many fundamental issues depend on our quantitative understanding of the β-decay of atomic nuclei. It is desirable to have theoretical models which can describe the data wherever they can be measured and which can predict the properties related to spin-isospin excitations in systems too short-lived to allow for experimental studies. One of the successful tools for studying charge-exchange nuclear modes is the quasiparticle random phase approximation (QRPA) with the self-consistent mean-field derived from a Skyrme-type energy-density functional (EDF), see e.g., [1, 2, 3, 4]. These QRPA calculations enable one to describe the properties of the ground states and excited charge-exchange states using the same EDF.

A comparison of such calculations with recent experimental data [5] demonstrates that the QRPA approach cannot reproduce correctly the strength distributions of the spin-isospin resonances. It is necessary to take into account the coupling with more complex configurations that result in shifting some strength upward to higher excitation energy [6, 7, 8]. Using the Skyrme EDF and the RPA, such attempts in the past [9, 10] have allowed one to understand the damping of charge-exchange resonances and their particle decay. Recently, the damping of the Gamow-Teller (GT) mode was investigated using the Skyrme-RPA plus particle-vibration coupling (PVC) [11]. However, the size of the configuration space increases very rapidly in the regions of interest of the nuclear chart, and one has to resort to severe space truncations.

It would be useful to study the effects of the 2p−2h configurations on the β−-decay rates of neutron-rich nuclei. It is somewhat simpler to include the PVC in QRPA calculations if one uses separable forces [7, 12]. Our tool is the QRPA with Skyrme interactions in the finite rank separable approximation (FRSA) [13, 14, 15, 16], allowing one to perform calculations in large configuration spaces. Successful applications of the method to study the electric ...
low-energy excitations and giant resonances within and beyond the QRPA can be found in Refs [13, 14, 15, 16, 17, 18, 19, 20]. Recently, the FRSA approach was extended to charge-exchange nuclear excitations [21] and also for accommodating the tensor correlations which mimic the Skyrme-type tensor interactions [22]. Also we have generalized the approach to the coupling between one- and two-phonon components in the wave functions [23]. In the present report we describe the FRSA model and discuss the $2p-2h$ configuration effect on the $\beta$-decay rates for neutron-rich Ni isotopes.

The FRSA model for charge-exchange excitations was already introduced in Refs. [21, 22, 23]. Taking into account the basic ideas of the quasiparticle-phonon model (QPM) [12], the Hamiltonian is then diagonalized in a space spanned by states composed of one and two QRPA phonons,

$$\Psi_{\nu}(JM) = \left( \sum_i R_i(J\nu)Q_{JM}^+ + \sum_{\lambda_1,\lambda_2} P_{\lambda_1\lambda_2}(J\nu) \left[ Q_{\lambda_1\mu_1i_1}^+ Q_{\lambda_2\mu_2i_2}^+ \right] \right) |0\rangle,$$

where $Q_{\lambda\mu}^+(0)$ ($\bar{Q}_{\lambda\mu}^+(0)$) is the GT (electric-type) excitation having the QRPA energy $\omega_{\lambda\mu}$ ($\bar{\omega}_{\lambda\mu}$). The normalization condition for the wave functions (1) is

$$\sum_i R_i^2(J\nu) + \sum_{\lambda_1,\lambda_2} (P_{\lambda_1\lambda_2}(J\nu))^2 = 1.$$

The amplitudes $R_i(J\nu)$ and $P_{\lambda_1\lambda_2}(J\nu)$ are determined from the variational principle which leads to a set of linear equations

$$(\omega_{\lambda\mu} - \Omega_\nu)R_i(J\nu) + \sum_{\lambda_1,\lambda_2} U_{\lambda_1\lambda_2}^{\lambda_1\mu_1i_1}(Ji) P_{\lambda_1\lambda_2}^{\lambda_1\mu_1i_1}(J\nu) = 0,$$

$$(\omega_{\lambda_1\lambda_2} + \bar{\omega}_{\lambda_2\lambda_2} - \Omega_\nu)P_{\lambda_1\lambda_2}(J\nu) + \sum_i U_{\lambda_1\lambda_2}^{\lambda_1\mu_1i_1}(Ji) R_i(J\nu) = 0.$$

The rank of the set of linear equations (3) and (4) is equal to the number of one- and two-phonon configurations included in the wave function of the excited states of the daughter nucleus. Its solution requires to compute the matrix elements coupling one- and two-phonon configurations

$$U_{\lambda_1\lambda_2}^{\lambda_1\mu_1i_1}(Ji) = \langle 0|Q_{ji}H\left[ Q_{\lambda_1\mu_1i_1}^+ Q_{\lambda_2\lambda_2}^+ \right] |0\rangle.$$

Equations (3) and (4) have the same form as the QPM equations [7, 12], but the single-particle spectrum and the residual interaction are derived from the same Skyrme EDF [23].

To calculate the half-lives, the same ansatz as Sec. II of Ref. [24] is used with the ratio of the weak axial-vector and vector coupling constants $G_A/G_V=1.25$ [25]. In the allowed GT approximation, the $\beta^-$-decay rate is expressed by summing the probabilities of the energetically allowed GT transitions (in units of $G_A^2/4\pi$) weighted with the integrated Fermi function

$$T_{1/2}^{-1} = \sum_m \lambda_{ij}^m = D^{-1} \left( \frac{G_A}{G_V} \right)^2 \sum_m f_0(Z, A, E_i - E_{1m}) B(GT)_m,$$

$$E_i - E_{1m} \approx \Delta M_{n-H} + \mu_n - \mu_p - E_m,$$

where $\lambda_{ij}^m$ is the partial $\beta^-$-decay rate, $D=6147$ [25] is used. $\Delta M_{n-H} = 0.782$ MeV is the mass difference between the neutron and the hydrogen atom, $\mu_n$ and $\mu_p$ are the neutron and proton chemical potentials respectively, $E_i$ is the ground state energy of the parent nucleus, and $E_{1m}$,
denotes a state of the daughter nucleus \((Z, A)\). \(E_m\) are the eigenvalues of the QRPA equations or Eqs. (3) and (4) taking into account the two-phonon configurations. The wave functions allow us to determine \(B(GT)_m\).

As for the parameter set in the particle-hole channel, we use the central Skyrme interaction SGII [26] and the same zero-range tensor interaction as that in Ref. [27]. Since the SGII parametrization gives reasonable values for the Landau parameters \(F'_0 = 0.73\) and \(G'_0 = 0.50\), one obtains a successful description of the spin-dependent properties and, in particular, experimental energies of the GT resonances of \(^{90}\)Zr [26]. For the studied region of nuclei we use the volume pairing force fixed in Ref. [18]. The single-particle continuum is discretized by diagonalizing the HF hamiltonian on a basis of 12 harmonic oscillator shells and cutting off the single-particle spectra at the energy of 100 MeV. This is sufficient to exhaust the Ikeda sum rule \(3(N − Z)\) [21, 22] as well as the sum rule for the electromagnetic excitation modes [16]. Since the tensor correlation effects within the \(1̕p−1̕h\) and \(2̕p−2̕h\) configuration spaces are treated, we do not need any quenching factor [6].

First, the properties of the low-lying \(1^+\) states in the daughter nuclei \(^{74,76,78,80}\)Cu are studied within the one-phonon approximation. As expected, the largest contribution (>88%) in the calculated \(β^−\)-decay half-life comes from the \([1^+_1]_{QRPA}\) state. To illustrate it, the \(β\)-transition rates \(\lambda_{ij}^m\) are shown in Fig. 1. The transition energies \(E_{1^+_m} − E_i\) refer to the ground state of the

**Figure 1.** The \(β\)-transition rates obtained within the QRPA.
Figure 2. The $\beta$-transition rates calculated with the effects of the $2p - 2h$ configurations.

parent nucleus. As shown in [23], the QRPA results indicate that the dominant configuration of the $[1^+_1]_{QRP A}$ states is $\{\pi 2p_3^+\nu 2p_2^+\}$ whose contribution decreases from 51% in $^{74}\text{Ni}$ to 39% in $^{80}\text{Ni}$.

To construct the wave functions (1) of the low-lying $1^+$ states we use only the $[1^+_i \otimes \lambda^+_i]_{QRP A}$ terms and all electric phonons with $\lambda > 2$ vanish. All one- and two-phonon configurations with the transition energies $|E_{1^+_m} - E_i|$ up to 10 MeV are included. We have checked that the inclusion of the high-energy configurations leads to minor effects on the half-life values [23]. The $[1^+_1 \otimes 2^+_1]_{QRP A}$ and $[1^+_1 \otimes 0^+_1]_{QRP A}$ configurations are the important ingredients for the half-life description [23] since they are the main two-phonon components of the $1^+_1$ wave function. Since there is a clear influence of the $2^+_1$ phonon on the half-life [23, 28], we examine the properties of the $2^+_1$ QRPA states of $^{74,76,78,80}\text{Ni}$. The neutron amplitudes are dominant in the $2^+_1$ structure and the contribution of the main neutron configuration $\{1g_{9/2}, 1g_{9/2}\}$ decreases from 79% for $^{74}\text{Ni}$ to 77% for $^{76}\text{Ni}$ when neutrons fill the subshell $1g_{9/2}$. There is a satisfactory description of $2^+_1$ energies in $^{74,76,80}\text{Ni}$ [23].

The inclusion of the two-phonon configurations results in an increase of the transition energies $|E_{1^+_m} - E_i|$ and partial rates of the main GT transitions (see Fig. 2). Thus, an additional constraint on the $\beta$-strength distribution is also given by delayed neutron emission probability ($P_n$-value) [29]. Since the FRSA model enables one to evaluate the coupling of QRPA phonons to
more complex configurations, such calculations that take into account the $2p-2h$ fragmentation of the QRPA excitations are now in progress.

In summary, starting from the Skyrme mean-field calculations the GT strength in the $Q_\beta$-window has been studied within the FRSA model including both the tensor interaction effect and $2p-2h$ configurations. The suggested approach enables one to perform the calculations in very large configurational spaces. We analyze the effects on the $\beta$-transition rates in the case of $^{74,76,80}\text{Ni}$, in comparison to the doubly-magic nucleus $^{78}\text{Ni}$. Including the $2p$-$2h$ configurations leads to the appearance of the weak fragmented satellites at low transition energy and the $1^+$ state is moved downwards in energy to the ground state of the daughter nucleus. As a result, the $\beta$-decay half-life is decreased. Using the strong tensor correlations [27] our estimation is rather the bottom limit of these half-lives [23].

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