Temperature Dependence of Zero-Bias Resistances of a Single Resistance-Shunted Josephson Junction

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Zero-bias resistances of a single resistance-shunted Josephson junction are calculated as a function of the temperature by means of the path-integral Monte Carlo method in case a charging energy $E_C$ is comparable with a Josephson energy $E_J$. The low-temperature behavior of the zero-bias resistance changes around $\alpha = R_Q/R_S = 1$, where $R_S$ is a shunt resistance and $R_Q = h/(2e)^2$. The temperature dependence of the zero-bias resistance shows a power-law-like behavior whose exponent depends on $E_J/E_C$. These results are compared with the experiments on resistance-shunted Josephson junctions.

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Dissipative quantum systems have been studied for several decades in various systems. Among them, resistance-shunted Josephson junctions have been studied intensively as a typical system. Theoretically, it has been shown that a single resistance-shunted Josephson junction exhibits a dissipation-induced superconductor-insulator transition at $\alpha = R_Q/R_S = 1$, where $R_Q = h/(4e)^2$ and $R_S$ is a shunt resistance. This transition is studied for $E_J/E_C \ll 1$ and $E_J/E_C \gg 1$, and is also expected to occur at any ratio $E_J/E_C$, where $E_J$ and $E_C$ are a Josephson energy and charging energy of a junction, respectively. Recently, the phase diagram has been obtained experimentally for a single resistance-shunted Josephson junction in the $\alpha - (E_J/E_C)$ plane. Subsequently, the phase diagram has been obtained also for one-dimensional and two-dimensional arrays of resistance-shunted Josephson junctions. For these higher-dimensional systems, the corresponding phase diagrams are modified from the one for a single junction.

In this report, we study the temperature dependence of the zero-bias resistance $R_0$. Analytical calculation has shown that $R_0$ behaves as $T^{2\alpha-2}$ for $\alpha > 1$ and $E_J/E_C \gg 1$. The renormalization group approach have indicated that also for any ratio of $E_J/E_C$, below a crossover temperature $T_c$, the behavior $R_0 \propto T^{2\alpha-2}$ may be observed. However, there may be deviation from this result for $E_J \sim E_C$ above $T_c$. In order to clarify this deviation, we study the zero-bias resistance of means of the path-integral Monte Carlo method. Similar numerical study of a single resistance-shunted Josephson junction has been performed by Herrero et al. They have discussed phase fluctuations along the imaginary time, clarifying the superconductor-insulator phase boundary lies on $\alpha = 1$ even for $E_J \sim E_C$. They have also obtained the linear resistance on the Matsubara frequency, $R(\omega_n)$. They, however, have not obtained the zero-bias resistance on the real frequencies, which can be directly related to the experimental data. In this paper, we calculate $R(\omega_n)$ first, and obtain the zero-bias resistance by extrapolation as

$$R_0 = \lim_{\omega_n \to 0} R(\omega_n).$$

(1)

We also compare the numerical results with the experiments of resistance-shunted Josephson junctions.

The equation of motion of a single resistance-shunted Josephson junction is expressed by a phase difference $\phi$ as

$$\left(\frac{\Phi_0}{2\pi}\right)^2 C \ddot{\phi} + \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{R_S} \dot{\phi} + \frac{\partial V}{\partial \phi} = 0,$$

(2)

where $C$ is a capacitance, $R_S$ is a shunt resistance, and $\Phi_0 = h/(2e)$ is the flux quantum. The potential energy is given as

$$V(\phi) = -E_J \cos \phi - I \frac{\Phi_0}{2\pi} \phi,$$

(3)

where $I$ is an external current through the junction. This equation of motion is equivalent to a dissipative classical motion described by

$$M \dddot{\phi} + M \gamma \dot{\phi} + \frac{\partial V}{\partial \phi} = 0.$$

(4)

Here, a particle mass $M$ corresponds to $h^2/8E_C$, and a damping frequency $\gamma$ to $1/R_S C$, where $E_C = e^2/2C$. In order to consider the dissipation effect, we use the Caldeira-Leggett model, in which, a reservoir consisting of infinite harmonic oscillators is coupled to the system. In the path-integral formalism, the partition function $Z$ is calculated by integrating the degrees of freedom of a reservoir as

$$Z = \int D\phi(\tau)e^{-S_{\text{eff}}/\hbar},$$

(5)

$$S_{\text{eff}} = \int_0^{\beta\hbar} d\tau \left[ \frac{\hbar^2}{16E_C} \dot{\phi}(\tau)^2 + V(\phi(\tau)) \right] + \frac{1}{2} \int_0^{\beta\hbar} d\tau \int_0^{\beta\hbar} d\tau' \phi(\tau)K(\tau - \tau')\phi(\tau')$$

(6)

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where $K(\tau)$ is a damping kernel given by

$$K(\tau) = \sum_{n} \frac{|\nu_n|}{R SC} e^{-i\nu_n \tau}. \quad (7)$$

Here $\nu_n = 2\pi n/\beta h$ is the Matsubara frequency.

In order to apply the path-integral Monte Carlo method, we introduce a discrete imaginary time $\tau_j = h\beta j/P$, where $P$ is a Trotter number assumed to be even and $j = 0, 1, 2, \cdots, P-1$. Then, the path $\phi(\tau)$ is approximated by a discrete path $\phi_j = \phi(\tau_j)$. After the Fourier transformation $\phi_j = \bar{\phi} + 2\sum_{k=1}^{P/2-1} (a_k \cos \frac{2\pi kj}{P} + b_k \sin \frac{2\pi kj}{P}) + a_{P/2} (-1)^j$, the partition function is approximately given as

$$Z_P = A \int d\bar{\phi} \int \prod_{k=1}^{P/2-1} da_k db_k \int da_{P/2} e^{-S_{\text{eff},P}}, \quad (8)$$

$$S_{\text{eff},P} = -\sum_{k=1}^{P/2-1} \left( \frac{\beta E_C P^2}{2} \sin^2 \frac{\pi k}{P} + ak \right) \left( a_k^2 + b_k^2 \right) + \left( \frac{\beta E_C P^2}{2} + \frac{\alpha P}{2} \right) a_{P/2}^2 + \frac{\beta}{P} \sum_{l} V(\phi_l), \quad (9)$$

where $\alpha = R_Q/R_S$ and $A$ is a constant. By this modification, the dissipative quantum mechanics is reduced to a classical statistical problem. Applying the ordinary Monte Carlo method to this effective system, we can simulate the dissipative quantum system. Using the linear response theory, the zero-bias resistance can be related to the correlation function

$$\langle \phi(0) \phi(\tau) \rangle_{\omega_n} = \int_{0}^{h\beta} d\tau \ e^{i\omega_n \tau} \langle \phi(0) \phi(\tau) \rangle \quad (10)$$

as

$$R(\omega_n)/R_Q = \eta |\omega_n| \langle \phi(0) \phi(\tau) \rangle_{\omega_n}, \quad (11)$$

where $\eta = \alpha/2\pi$. The zero-bias resistance is then obtained by the extrapolation [11].

Generally, an analytic continuation of the correlation function to the real frequencies may have a large noise, and is ill-defined. Additionally in the present system, the correlation function is not analytic at $\omega = 0$ due to the presence of a characteristic factor $|\omega_n|$ of the ohmic damping. Therefore, the Padé approximation, which is one of the standard methods of the analytic continuation, does not work well except for moderate temperatures. Here, since we are interested only in $R(\omega_n = 0)$, we extrapolate the numerical data toward $\omega_n = 0$ by assuming a fitting function. At high temperatures, the numerical data of $R(\omega_n)$ can be fitted well to the form

$$R(\omega_n) = \frac{1}{A|\omega_n| + B}. \quad (12)$$

One example is shown in Fig. 1 (a). At low temperatures ($\beta > 5$), the deviation from the form is observed. Then, we fit the data to the form

$$R(\omega_n) = A' + B'|\omega_n| + C'|\omega_n|^2 \quad (13)$$

by using the lowest five frequencies. One example of this case is shown in Fig. 1 (b). For moderate temperatures, $R_0$ is consistent with the one obtained by the Padé approximation within 10 percent. We expect that the method adopted here for the extrapolation is accurate enough to discuss the temperature dependence of $R_0$.

Before showing the result of the zero-bias resistance $R_0$, we study a linear mobility defined as $\mu_1 = \mu_0 R_0/R_Q$, where $\mu_0 = 1/\eta$ is a linear mobility of a free particle. The temperature dependence of the linear mobility has been studied by Fisher and Zwerger [7] for $E_1/E_C \ll 1$. They have showed that for $\alpha < 1$, as the temperature decreases, the scaled linear mobility $\mu_1/\mu_0$ shows a reentrant behavior; above a temperature $T^*$ it once decreases from 1, and increases toward 1 below $T^*$. They have also claimed that for $\alpha > 1$ the linear mobility decreases monotonically as the temperature decreases, and becomes zero at the zero temperature. The temperature dependence of $\mu_1/\mu_0$ in the regime $E_1 \sim E_C$ has, however, not yet studied. The temperature dependence of $\mu_1/\mu_0$ obtained in our simulation is shown in Fig. 2 for $E_1/E_C = 1.0$. These results show that the linear mobility behaves as predicted by Fisher and Zwerger even for...
$E_1 \sim E_C$. The similar behaviors are observed also for other values of $E_1/E_C$.

Next, the superconductor-insulator phase transition is studied through the temperature dependence of the zero-bias resistance $R_0$. We show $R_0$ as a function of the temperature for various values of $\alpha$ in Fig. 3. Due to the difficulty of the extrapolation, the temperature region in which $R_0$ is properly obtained, is limited. Particularly, the lack of data at low temperatures makes it difficult to distinguish the phase of the ground state. The qualitative feature, however, can be observed in these figures; For $\alpha < 1$, the result shows the tendency of a reentrant behavior, while for $\alpha > 1$ the result shows a monotonic decrease as the temperature decreases. Hence, we expect that the phase boundary lies on $\alpha = R_Q/R_S = 1$ even for $E_1 \sim E_C$.

Finally, we discuss the temperature dependence of $R_0$ in the superconducting phase ($\alpha > 1$). For both $\alpha = 1.25$ and $\alpha = 1.5$, the zero-bias resistances show a power-law-like behavior ($R_0 \propto T^\nu$) in the low temperature region as shown in Fig. 4. This result disagrees with the analytic result ($R_0 \propto T^{2\alpha-2}$) expected in the low temperature limit: The exponent $\nu$ seems to depend also on $E_1/E_C$. This indicates that above a crossover temperature $T_{cr}$ there exists deviations from the result expected in the renormalization group analysis, and that $T_{cr}$ is much smaller than the temperature range studied in this report. Note that the exponent $\nu$ may change depending on the temperature range, and that $\nu$ should not be thought as a well-defined constant. Here, we use the exponent $\nu$ obtained in the temperature range under consideration only for a qualitative comparison with the experiments. In Fig. 5 the exponent $\nu$ is shown as a function of $E_1/E_C$. We observed the tendency that the exponent is approximately a linear function of $E_1/E_C$, and that the slope depend on $\alpha$.

Let us compare the results obtained here with the experiment. Experimentally, the temperature dependence of the zero-bias resistance $R_0$ has been measured to characterize the superconductor-insulator transition; In Ref. [6] the system is assigned to an insulator when $dR_0/dT < 0$ at low temperatures, and otherwise to a superconductor. Systematic analysis of zero-bias resistance, however, has not been performed for a single resistance-shunted Josephson junction [6, 7]. In the superconducting regime ($\alpha > 1$), the zero-bias resistances in these experiments weakly depend on the temperature, and do not show a clear power-law behavior. These experimental results disagree with the present theoretical calculation. The origin of this disagreement is not known. We speculate that the environment effect through the leads may be important to determine the temperature dependence of $R_0$. Finally, we point out that the zero-bias resistance observed in one-dimensional Josephson arrays [8] obeys the power-law behavior in the supercon-
FIG. 4: Power-law-like behavior of the zero-bias resistances for various $E_J/E_C$ values at (a) $\alpha = 1.25$ and (b) 1.5. From the top to the bottom: $E_J/E_C = 0.5, 0.8, 1.0, 1.2$ and 1.5.

FIG. 5: Plot of the ratio $E_J/E_C$ vs. the exponent $\nu$.

In summary, we have studied the temperature dependence of the zero-bias resistance of a single resistance-shunted Josephson junction by means of quantum path-integral Monte Carlo method. We reproduced the result predicted by Fisher and Zwerger even though $E_J/E_C \sim 1$. We have also shown superconductor-insulator transition around $\alpha = R_Q/R_S = 1$. The temperature dependence of the zero-bias resistance showed a power-law-like behavior in the temperature range under consideration. In conflict with the renormalization analysis, the exponent depends on $E_J/E_C$. These results have been compared with the experiment on one-dimensional Josephson arrays.

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