Mansour, Toufik

**Counting peaks at height \( k \) in a Dyck path.** (English) J. Integer Seq. 5, No. 1, Art. 02.1.1, 10 p. (2002).

Summary: A Dyck path is a lattice path in the plane integer lattice \( \mathbb{Z} \times \mathbb{Z} \) consisting of steps \((1,1)\) and \((1,-1)\), which never passes below the \(x\)-axis. A peak at height \( k \) on a Dyck path is a point on the path with coordinate \( y = k \) that is immediately preceded by a \((1,1)\) step and immediately followed by a \((1,-1)\) step. In this paper we find an explicit expression for the generating function for the number of Dyck paths starting at \((0,0)\) and ending at \((2n,0)\) with exactly \( r \) peaks at height \( k \). This allows us to express this function via Chebyshev polynomials of the second kind and the generating function for the Catalan numbers.

**MSC:**

05A15 Exact enumeration problems, generating functions

**Keywords:**

Dyck paths; Catalan numbers; Chebyshev polynomials

**Software:**

OEIS

**Full Text:** arXiv EuDML EMIS