Numerical simulation the influence of bubbles on the structure and friction of turbulent gas–liquid flow in vertical pipe

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Abstract. The paper the results of a computational study of the local structure of the ascending gas-liquid flow in a vertical pipe are presented. The mathematical model is based on the use of two-fluid Eulerian approach taking into account the inverse influence of bubbles on averaged characteristics and turbulence of carrying phase. The equations conservation of mass and momentum quantity of motion in the form of Navier-Stokes equations averaged over Reynolds for each phase are written down. For turbulent stresses the relations under the assumption of the Boussinesq hypothesis are written. Turbulent viscosity for the carrier liquid phase is determined using a two-parameter turbulence model modified for two-phase media. Investigation is performed of the impact made by the variation of the degree of dispersion of the gas phase, volumetric flow ratio of gas and velocity of the liquid phase on the local structure and skin friction in two-phase flow.

1. Introduction

Multiphase flows are found in a large variety of industrial applications, such as nuclear reactors, chemical and petrochemical processes, boilers and heat exchange devices amongst many others, and in a multitude of natural phenomena as well. The presence of multiple phases and the discontinuity of properties at the interface between the phases complicate the physics of these kinds of flows and poses great challenges to our ability to predict them. The complexity of modeling such flows is associated with a large number of phenomena of different nature, since such flows characterized by strong mutual influence of the carrier and dispersed phase, accompanied by the processes of heat exchange, phase transitions, coalescence, crushing, etc. Therefore, the simulation of turbulent bubble flows represents a large practical interest and a lot of publications have been devoted to it [1-3]. Despite the large number of works in this field, the questions of detailed turbulent structure for two-phase flows, the polydispersity of the distribution of bubbles, regularities of the dynamics of a two-phase flow still exist. This is due to the fact that the flow structure and hydraulic resistance in gas-liquid flows are affected a large number of hydrodynamic and geometric parameters, which is especially important in engineering applications. At the present time in the literature there is a large amount of experimental data and computational recommendations on the hydrodynamic resistance and heat transfer during the flow of gas-liquid flows in pipes of different diameters (see, for example, the monograph [4]). The papers [5-10] to the experimental and numerical study of turbulent bubble flows are devoted.
Important information is contained in [6], which presents the results of an experimental study of hydrodynamic resistance during the flow of steam-water mixture in a heated pipe.

The aim of this work is a detailed numerical study of the characteristics of turbulent upstream flow in the pipe when changing the mass rates and gas flow media. The main attention is paid to the analysis of turbulent flow structure, polydispersity of gas-liquid flow, as well as surface friction of two-phase flow.

2. Governing equations

The three-dimensional two-fluid model solves the ensemble averaged of mass and momentum transport equations governing each phase [1-3,11]. Denoting the liquid as the continuum phase (α_l) and the gas (i.e. bubbles) as disperse phase (α_g), these equations can be written as:

\[
\frac{\partial}{\partial t}(\rho_m \alpha_m) + \nabla \cdot (\rho_m \alpha_m \mathbf{u}_m V_m) = 0, \quad \alpha_l + \alpha_g = 1, \quad m = l, g, \quad i, j = 1,2,3, \tag{1}
\]

\[
\frac{\partial}{\partial t}(\rho_m \alpha_m V_m) + \nabla \cdot (\rho_m \alpha_m \mathbf{u}_m V_m) = -\alpha_m \frac{\partial P}{\partial x_j} + \alpha_m \rho_m g_i + M_{mi} + \tag{2}
\]

\[
\frac{\partial}{\partial x_j} \left[ \alpha_m \mu_{\text{eff,m}} \left( \frac{\partial V_m}{\partial x_j} + \frac{\partial V_m}{\partial x_i} \right) \right] - \frac{2}{3} \frac{\partial}{\partial x_j} \left[ \alpha_m \mu_{\text{eff,m}} \frac{\partial V_m}{\partial x_i} \delta_{ij} \right].
\]

Here \( x_j \) - cartesian coordinates, \( u_i \) - components of the average component velocity of phase \( m \), \( t \) - time, \( p \) - pressure, \( \rho_l \), \( \rho_g \) - density of continuous and dispersed phases respectively, \( \alpha_m \) - is the volume fraction of phase \( m \) in mixture, \( \delta_{ij} \) - is the Kronecker symbol, \( \mu_i, \mu_g \) - dynamic coefficient of viscosity of water and air, \( \mu_{\text{eff,m}} \) - is the coefficient of turbulent viscosity, \( g_i \) is the body force vector which comprises of gravity and \( M_{mi} (m = l, g) \) is the momentum interfacial exchange between phases. To account for the effect of bubbles on liquid turbulence, the Sato’s bubble-induced turbulent viscosity model is employed [1-2]. The turbulent viscosity of liquid phase is thus given by

\[
\mu_{\text{eff,l}} = \mu_l + \mu_{\text{ij}} + \mu_{\text{bi,l}}. \tag{3}
\]

The shear-induced turbulence \( \mu_{\text{ij}} \) and the bubble-induced turbulence \( \mu_{\text{bi,l}} \) can be expressed as

\[
\mu_{\text{ij}} = \frac{C_{\text{ij}1}}{\varepsilon_i}, \quad \mu_{\text{bi,l}} = C_{\text{bl,1}} \rho_l \alpha_l d_i |V_{\text{ij}} - V_{\text{li}}|, \quad \mu_{\text{eff,g}} = \frac{\rho_g}{\rho_l} \mu_{\text{eff,l}}, \quad C_{\text{bl,1}} = 0.6. \tag{4}
\]

To evaluate shear-induced turbulent viscosity in the liquid phase, the \( k-\varepsilon \) turbulence model is used. Conservation equations for \( k \) and \( \varepsilon \) are modeled through the following equations:

\[
\frac{\partial}{\partial t} \left( \alpha_i \rho_i k_i \right) + \frac{\partial}{\partial x_j} \left( \alpha_i \rho_i V_{ij} k_i \right) = \frac{\partial}{\partial x_j} \left( \alpha_i \frac{\mu_{\text{eff}}}{\sigma_i} \frac{\partial k_i}{\partial x_j} \right) + \alpha_i (G_i - \rho_i \varepsilon), \tag{5}
\]

\[
\frac{\partial}{\partial t} \left( \alpha_i \rho_i \varepsilon_i \right) + \frac{\partial}{\partial x_j} \left( \alpha_i \rho_i V_{ij} \varepsilon_i \right) = \frac{\partial}{\partial x_j} \left( \alpha_i \frac{\mu_{\text{eff}}}{\sigma_i} \frac{\partial \varepsilon_i}{\partial x_j} \right) + \frac{\varepsilon_i}{k_i} \left( C_{\text{e,k}} G_{ij} - C_{\text{e,k}} \rho_i \varepsilon \right), \tag{6}
\]

\[
G_i = \frac{1}{2} \mu_{\text{eff}} \left[ \frac{\partial V_{\text{ii}}}{\partial x_i} + \left( \frac{\partial V_{\text{ij}}}{\partial x_i} \right)^2 \right]. \tag{7}
\]

Coefficients in \( k-\varepsilon \) model are as follows:

\[
C_{\text{e,k}} = 0.09, \quad C_{\text{e,t}} = 1.44, \quad C_{\text{e,\varepsilon}} = 1.92, \quad \sigma_i = 1.0.
\]

The momentum interfacial exchange between bubbles (gas phase) and liquid is modeled by considering interfacial momentum source. The total interfacial force \( M_{mi} \) appearing in equation (2) is formulated according to the appropriate consideration of different sub-forces affecting the interface.
between each phase. Drag force $M_{\rho}^{D}$ takes into account interaction forces between liquid and bubble in a uniform flow field with non-accelerating conditions. If, however, bubbles are accelerated relatively to the liquid, a part of the surrounding liquid would be accelerated as well. This additional contribution is called virtual mass $M_{\rho}^{VM}$ or added mass force. To account for the rotation of bubbles induced by the flow field the lift force $M_{\rho}^{L}$ is used. For the liquid phase, the total interfacial force is given by

$$M_{\rho} = -M_{\rho}^{D} - (M_{\rho}^{L} + M_{\rho}^{VM}).$$  

These forces are calculated by the following equations:

$$M_{\rho}^{D} = \frac{3}{4} \alpha_s \rho_l \frac{C_d}{d_s} |V_{\rho} - V_\rho| (V_{\rho} - V_\rho) \times (\nabla \times V_\rho),$$

$$M_{\rho}^{L} = C_L \alpha_s \rho_l (\frac{\partial V_\rho}{\partial t} - \frac{\partial V_{\rho}}{\partial t}),$$

$$M_{\rho}^{VM} = C_{VM} \alpha_s \rho_l \left( \frac{\partial V_\rho}{\partial t} - \frac{\partial V_{\rho}}{\partial t} \right), \quad C_{VM} = 0.5.$$  

Here $C_D$ is the drag coefficient which can be evaluated by

$$C_D = \begin{cases} 
24(1+0.15 \text{Re}^{0.87}) / \text{Re}, & \text{Re} \leq 1000, \\
0.44, & \text{Re} > 1000,
\end{cases}$$

Where $\text{Re} = 2 \rho_l |V_{\rho} - V_\rho| d_s / \mu_l$ - the Reynolds number, $d_s$ - mean diameter of bubbles. For the lift coefficient, $C_L$, the Eotvos number dependent correlation is adopted and the lift coefficient can then be expressed as

$$C_L = \begin{cases} 
\min[0.288 \tanh (0.121 \text{Re}) ; f \left( E_{\phi} \right)], & E_{\phi} < 4, \\
f \left( E_{\phi} \right), & 4 \leq E_{\phi} < 10, \\
-0.29, & E_{\phi} \geq 10,
\end{cases}$$

$$f \left( E_{\phi} \right) = 0.00105 E_{\phi}^3 - 0.0159 E_{\phi}^2 - 0.0204 E_{\phi} + 0.474, \quad E_{\phi} = g \left( \rho_l - \rho_g \right) d_s / \sigma.$$  

To account for non-uniform bubble size distribution, the model that employs multiple discrete bubble size groups to represent the population balance of bubbles are used. Assuming each bubble class travel at the same mean algebraic velocity, individual number density of bubble class $i$ represents the net change in the number density distribution due to coalescence and break-up processes. Bubbles are divided into finite number of classes $i (i=1,M)$, therefore it is assumed that no break-up of the smallest bubble class, ($i=1$) and no coalescence in the largest class ($i=M$). Integrating the specific density of particles $n(v,\tau)$ within the limits of $[v_i,v_{i+1}]$, we obtain the number of particles whose volume is between $v_i$ and $v_{i+1}$, $N_{i} = \int_{v_i}^{v_{i+1}} n(v,\tau) dv$. The volume fraction $\phi_i$ of bubble class $i$ is related with the gas volume fraction by $\phi = \alpha_g / \alpha_s$. The total volume $\alpha_i$ of bubble class $i$ is the product of the total number of bubble in class $i$ and the volume of single bubble of that class $\alpha_i = N_{i} v_i$. The population balance equation can be expressed for $\phi_i$ as:

$$\frac{\partial}{\partial t} (\alpha_s \phi_i) + \frac{\partial}{\partial x_j} (\alpha_s \phi_i V_{\phi}) = B_i^e - D_i^e + B_i^d - D_i^d, \quad i=1,M,$$

$$B_i^e = \int_{v_i}^{v_{i+1}} B_e d v, \quad D_i^e = \int_{v_i}^{v_{i+1}} D_e d v, \quad B_i^d = \int_{v_i}^{v_{i+1}} B_d d v, \quad D_i^d = \int_{v_i}^{v_{i+1}} D_d d v,$$
where \( S^i = B^i_a - D^i_a + B^i_b - D^i_b \) is the source term of bubble class \( i \) due to break-up and coalescence and \( B^i_a, D^i_a \) and \( B^i_b, D^i_b \) are birth and death due to break-up and coalescence, respectively. Here, the break-up rate of bubbles \( B^i_a, D^i_a \), of volume \( v_i \) into volume \( v_j \) is calculated according to the model developed by [12]. The coalescence rate \( B^i_b, D^i_b \), considering turbulent collision taken from [13]. Mean bubble size of the distribution is usually determined by Sauter diameter \( d_s = d_{32} \). Change in \( d_s \) produces a change in the drag force, so this causes a change in the overall flow field. Mean Sauter diameter \( d_{32} \) is determined as

\[
d_s = \frac{\sum_{i=1}^{M} f_i / \sum_{i=1}^{M} f_i / d_i}{f_i / \sum_{i=1}^{M} f_i / d_i}.
\]

(14)

The motion of the gas–liquid flow is described by a system of conservation equations for the carrier and gas phases (1) – (2), supplemented by a modified two-parameter turbulence model for a two-phase medium (5) – (7). The forces of interphase interaction (9) – (12) that act on the elementary volume of the medium are determined through the parameters of the flow, the fraction of the volume gas concentration \( \alpha_i \) and the average diameter of the bubbles \( d_s \) in it. Knowing the flow parameters at the current time, we solve the system of equations (13) and find the distribution \( \phi_i \) for each group of bubbles, which enables us to determine \( d_s \) in each elementary infinite volume of the grid. The distribution \( \phi_i \) yields additional information on the particle size distribution in the flow volume of a two-phase medium. Within the framework of this approach, the motion of a polydisperse two-phase gas–liquid flow is described by a closed system of equations (1) – (2), (5) – (7), (13) – (14).

The friction pressure losses calculated in this paper with known calculated relations obtained as a result of processing a large number of experimental data for different flow regimes of a two-phase flow are compared. To calculate the pressure loss for friction in a two-phase flow \( \Delta P^P_T \), two types of models are distinguished: homogeneous and models with phase separation [4,14]. Pressure losses due to friction are determined using values of the gradient two-phase flow pressures \( (dp/dz)_T^P \). For a homogeneous model, a two-phase medium is considered as a mixture with variable density \( \rho_T \), viscosity \( \mu_T \). The pressure gradient is calculated using a similar formula for a single-phase fluid as

\[
(dp/dz)_T^P = \frac{C^P_T}{2D_{\rho_T} f},
\]

(15)

where \( C^P_T = \rho_i <J_i> + \rho_g <J_g> \) – total flow rate of the mixture, and for the density \( \rho_T \) and viscosity of the mixture \( \mu_T \) the relations are applied

\[
\rho_T = \left( \frac{x \rho_g + (1 - x) \rho_i}{\rho_i} \right)^{-1}, \quad \mu_T = x \mu_g + (1 - x) \mu_i.
\]

(16)

In (15) - (16) \( x \) - mass fraction of the gas phase \( x = \rho_g <J_g>/G_T \). To determine the friction coefficient \( f \), a similar relation is applied for a single-phase flow in the following form

\[
f = \begin{cases} 
16 \text{Re}_T^{-0.25}, & \text{Re}_T \leq 2000, \\
0.079 \times \text{Re}_T^{-0.25}, & \text{Re}_T > 2000.
\end{cases}
\]

(17)

For comparison, pressure losses are calculated using the empirical relation obtained in [14], where the authors proceed from the known formula for the pressure drop in the flow of a homogeneous mixture. The friction pressure loss gradient for the two-phase flow \( (dp/dz)_T^P \) is expressed as a function, dependent on pressure gradients for single-phase flows \( (dp/dz)_T^P \), \( (dp/dz)_T^P \), homogeneous media
with liquid or gas at the flow through the pipe have the same flow rate \( G_{fr} \). In [14] the approximation for the definition of \((dp/dz)_{fr}^{TP}\) is proposed as

\[
(dp/dz)_{fr}^{TP} = F(1 - x)^{1/3} + (dp/dz)_{fr}^l x^{1/3},
\]

(18)

\[
F = (dp/dz)_{fr}^l + 2\left( (dp/dz)_{fr}^g - (dp/dz)_{fr}^l \right) x.
\]

(19)

In relation (19) pressure gradients for single-phase flows \((dp/dz)_{fr}^l, (dp/dz)_{fr}^g\) are calculated by formulas

\[
(dp/dz)_{fr}^l = \frac{G_{fr}^2}{2D \rho_l f_{lo}}, \quad (dp/dz)_{fr}^g = \frac{G_{fr}^2}{2D \rho_g f_{go}},
\]

(20)

where \( f_{lo}, f_{go} \) are friction coefficients for the carrier phase and the dispersed phases, calculated similarly (17).

3. Results
For comparison, the experimental data [15], carried out for the upward turbulent flow regime are selected. The measurements were carried out in an acrylic pipe with a diameter of D = 50.8 mm and a length of L = 3061 mm at a distance of \( x/D = 6, 30.3, 53.5 \) from the cross section of the air bubbles. Measurements of fluid velocity were conducted using a thermo-anemometer, and the local gas content and bubble size with using a special double sensor. Table presents the main parameters for different flow modes of a two-phase bubble medium in a vertical pipe (\( \{ J_1 \} \)), is the reduced velocity of the liquid phase at the pipe inlet, \( \{ J_1 \} \) is the reduced velocity of the gas phase, and \( \alpha_g \) is the volume fraction, \( \theta \) is volume flow gas concentration. The Reynolds number for the liquid phase flow varies within \( Re, = \rho_l \langle J \rangle D/ \mu_l = (2.5-10.2) \times 10^4 \). The following parameters of the components of the two-phase water–air medium are set: \( \rho_l = 998.2 \text{ kg/m}^3, \rho_g = 1.2 \text{ kg/m}^3, \mu_l = 1.1 \times 10^{-3} \text{ kg/(m s)} \).

Table. The parameters of the experimental measurements \( d_s = 2.5 \text{ mm} \).

|        | \( \langle J_1 \rangle \), m/s | \( \langle J_\alpha \rangle \), m/s | \( \alpha_g \) | \( \theta \) |
|--------|-------------------------------|-------------------------------|----------------|---------|
| L1     | 0.491                         | 0.028                         | 0.049          | 0.054   |
| L2     | 0.491                         | 0.190                         | 0.259          | 0.279   |
| H1     | 0.986                         | 0.047                         | 0.051          | 0.045   |
| H2     | 0.986                         | 0.321                         | 0.231          | 0.245   |
| W1     | 2.01                          | 0.103                         | 0.056          | 0.048   |
| W2     | 2.01                          | 0.471                         | 0.183          | 0.189   |

and \( \mu_g = 1.7 \times 10^{-5} \text{ kg/(m s)} \). The particle-size distribution is presented in the form of six groups of bubbles with the minimum diameter of \( d_s^{\text{min}} = 1 \times 10^{-3} \) m and the maximum diameter of \( d_s^{\text{max}} = 1.2 \times 10^{-2} \) m. The calculated and experimental data of the distribution the characteristics of the carrier and polydisperse flows in the developed turbulent flow in the section of \( x/D = 53.6 \) along the pipe radius are presented (\( r \) is the distance from the pipe axis). On Figure 1 the flow patterns of the gas-liquid flow for \( \langle J_1 \rangle = 2.01 \text{ m/s}, \langle J_\alpha \rangle = 0.471 \text{ m/s} \) in the form of isolines of the axial velocity of the carrier phase \( V_1 \), the contour of the gas volume \( \alpha_g \), of the average bubble diameter \( d_s \) in the cross-section by the plane \( o x_1 x_2 \) are shown. There is an increase in the speed of the bubbles in the
central part of the channel and a slowdown in the speed of the bubbles that are closer to the walls are observed. Two zones of uniform velocity distribution are separated by a strip characterized by the sharpest change both directions and speed values. It can be seen that on the input section there is a triangular region of the velocity field rearrangement from the uniform of the velocity profile of the bubbles at the entrance to the parabolic, as they move up the tube. The presence of this zone in the distribution of the volume gas content in the inlet part of the pipe is most pronounced, as shown in Figure 1b. Clearly visible area with a uniform distribution of the volume of gas content, this is a consequence of the flow of bubbles of the same diameter in the inlet part of the pipe. Because the input in the experiment [15] are fed bubbles of constant size with a diameter of \( d = 2.5 \) mm, that is clearly observed narrowing in the width of the pipe zone with a uniform distribution \( \alpha_g \).

![Figure 1](image_url)

**Figure 1.** The flow pattern of a gas-liquid flow (W2) as isolines of the axial velocity \( V_x \)-m/s \((a)\); volume gas fraction \( \alpha_g \) \((b)\); average diameter of bubbles \( d \)-mm \((c)\).

In article six modes of flow of a mixture of water and air bubbles, characterized by the increase in the proportion of bubbles and input velocity at the entrance from \( \alpha_g = 0.049 \) to 0.259 are described. The mode of flows L1, L2 can be described as the modes of stream with the maximum concentration of bubbles near the wall, when the bulk of the bubbles is in the wall region. This is evidenced by the Figure 2a, which shows the profiles of the volume concentration of bubbles (gas content) in the selected section. With an increase in the proportion of bubbles and their initial velocity at the inlet (W2 mode), a new type of gas-liquid mixture flow is formed, when most of the bubbles are concentrated in the center of the pipe, which is illustrated in Figure 1b and Figure 2b (curve -2). The considered regimes cover the most characteristic features of hydrodynamics of bubble regimes. Of course, the transition between these two modes does not occur immediately, but there are transitional bubble modes, when some of the bubbles are in the near-axis flow region, and the other part of the bubbles is near the wall. The difference between the modes of bubble flows is mainly due to the change in the action of the lifting force in the transverse direction, which leads to a different nature of the distribution of bubbles over the pipe section depending on their size. The Figure 2 presents comparative data of the averaged profile of the disperse void fraction \( \alpha_g \) of the gas–liquid flow for
\( \langle J_i \rangle = 0.49 \text{ m/s} \) и \( \langle J_s \rangle = 0.028, 0.190 \text{ m/s} \) (flow types L1, L2) on the left, and for \( \langle J_i \rangle = 2.01 \text{ m/s} \) и \( \langle J_s \rangle = 0.103, 0.471 \text{ m/s} \) (W1, W2) on the right. At low flow velocities, a peak bubble concentration in the near-wall region and a uniformly low bubble concentration in the flow core at different gas concentrations are observed (Figure 2a). Similar results in the distribution diagram of \( \alpha_g \) along the pipe radius are confirmed by the experimental data of many researchers. As an example, the saddle profiles of the gas concentration along the pipe section were found experimentally in the flow of an ascending gas-liquid flow [7-9]. In these works, the peak distribution of the volume gas concentration in the near-wall region and the uniform flow in the core (called the “skin effect”) were obtained in the flow of a bubble mixture in the pipe at \( Re = 6 \times 10^3 - 6 \times 10^4 \). The difference in the effect of interphase forces on bubbles of different sizes leads to the fact that the bubbles with a diameter that is smaller than the original one are displaced into the near-wall region. Note that the considered flow mode can be characterized as a flow in that the bulk of the bubbles move in the near-wall region.

**Figure 2.** Comparison of (curves 1 and 2) calculated results and (symbols) experimental data obtained in the L1 and L2 modes for \( \alpha_g \) (a); modes W1, W2 (b).

Figure 3 presents the results of calculations of the pressure loss due to friction in a two-phase flow \( \Delta P_{\text{fpt}} / \Delta P_f \) with respect to the pressure loss in a single-phase flow under identical conditions with increasing carrier phase velocity at low (Figure 3a) and high gas content values (Figure 3b). The results of calculations of the pressure loss in a homogeneous model (Figure 3, line 2) give an underestimate, and calculations for the relations [14] (Figure 3, line 3) are close to the results of this paper. The increase in friction for a two-phase flow compared with a single-phase flow is confirmed in experiments [4-7]. From the figures it can be seen that at low gas content the friction pressure losses for the gas-phase flow are increased to about 10% percent as compared to the single-phase. The increase in the gas content leads to a significant increase in the friction resistance, for low speeds, the increase is doubled, and with the growth of speed decreases to 1.5 times. This is explained by the increase in the velocity gradient in the liquid in wall zone due to the more shallow form in the central part for two-phase, in comparison with single-phase. An increase in the flow rate of the gas phase leads to an increase in the hydraulic friction resistance, however, as noted in the works [4-7] with an increase in the total flow rate of the mixture, the relative pressure \( \Delta P_{\text{fpt}} / \Delta P_f \) decreases. This influence increases with increasing gas content. An increase in the rate of a two-phase flow leads to decrease in the relative pressure due to friction due to a stronger increase in friction in a single-phase flow in compared with the bubble flow. The results of the calculations are in satisfactory agreement with the dependences of the work [14].
4. Conclusions

A numerical model for the calculation of gas-liquid polydisperse flow using the Eulerian description is developed. The turbulence of the liquid was calculated using a model of transport of Reynolds stress components modified to the case of the presence of bubbles. The model takes into account the polydispersity of the bubbles in the flow region, their breakup and coalescence. The turbulence model takes into account the additional generation of turbulence in the wake behind the bubbles during their flow. The paper presents the results of a comparative analysis of the upward polydisperse turbulent bubble flow in the pipe. The comparisons were carried out over the averaged phase velocities and the bubble concentration distribution over the cross section of the tube. At low reduced velocities of the liquid and gas phases in the near-wall region, the volume content of the gas is higher than the corresponding value for the main part of the two-phase flow. This is explained by the fact that in the flow under the given conditions there are mainly small bubbles, the distribution of which has a maximum near the wall. An increase in the number of particles in the vicinity of the wall leads to an increase in the role of coalescence. However, with increasing carrier and dispersed phase rates the structure of the flow is completely changed. The volumetric gas content in the central part becomes maximum, and in regions close to the walls of the channel monotonically decreases. Due to the action of interphase forces there is a displacement of small bubbles to the near-wall region, and large bubbles accumulate in the center of the channel. The flow structure at low concentrations and velocities of the gas phase, characterized by the accumulation of bubbles at the walls, is replaced by the opposite one. At high velocities of the gas-liquid flow, the bulk of the bubbles moves in the central region, and near the walls their concentration is negligible. The paper shows the importance of taking into account the processes of crushing and coalescence of bubbles and interfacial forces arising during the transfer of bubbles of different diameters over the cross section of the pipe in the modeling of gas-liquid flows.

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