MATHEMATICAL INPUT-OUTPUT RELATIONSHIPS & ANALYSIS OF FUZZY PD CONTROLLERS

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Abstract

The present paper deals with the comparison of output performance of fuzzy Proportional plus Derivative controller with conventional controllers. Fuzzy PD controller having 2 fuzzy sets for every i/p variable and 3 fuzzy sets for o/p variable in the universe of discourse. Mathematical input-output relationships of simple fuzzy Proportional plus Derivative controller is developed via arbitrary membership functions for fuzzification, Zadeh AND operation for the evaluation of antecedent part of the rules and centroid method for defuzzification. Computer simulations show the effectiveness of Fuzzy Proportional plus Derivative controllers over the conventional controllers for time delay and nonlinear systems by choosing Triangular and Trapezoidal membership functions as input and output fuzzy sets. As a case study P.M.D.C. Servo system with saturation nonlinearity is considered with load disturbance and without load disturbance. MATLAB environment developed results are added to show the importance of the fuzzy controllers for P.M.D.C. Servo system with saturation nonlinearity with and without load disturbance using Triangular and Trapezoidal membership functions as input and output fuzzy sets.
Keywords: Analytical structures, fuzzification, defuzzification, fuzzy Proportional plus Derivative controllers, membership functions, MATLAB/simulink.

I. Introduction

In present paper it is an attempt to analyze in detail the derivation of mathematical input-output relationships of fuzzy controllers using triangular MFs and arbitrary MFs, and to show the effectiveness of fuzzy controllers over conventional controllers for time delay and nonlinear systems. The most commonly used industrial controllers are the Proportional-Integral-Derivative controllers because of their low cost and suitable for linear systems [I].

Despite their effectiveness for linear systems, conventional PID controllers are not used if the controlled plants are higher order, time delay systems, non-linearity and uncertainties [I], [II]. Therefore, to improve the performance of above complex systems, there is a need to design controllers which work well in absence of exact mathematical models.

Analytical structures for fuzzy proportional–derivative (PD) controllers are derived by using triangular membership functions for inputs, singletons or triangular membership functions for output, minimum triangular norm, maximum or drastic sum triangular conorm, Mamdani minimum, drastic or Larsen product inference, nonlinear control rules and center of sum defuzzification [IV].

As a case study of servo mechanism along with considering saturation nonlinearity, simulations have done by using different membership sets both input and output. The performance of Proportional plus Derivative controller is considered to be quite adequate for linear first order system with the small time delay [III]. However, its performance for a system with large time delay and also for a nonlinear system is sluggish due to expense overshooting and excessive oscillation. To improve the performance of a conventional Proportional plus Derivative controller, engineers have tried to use fuzzy Proportional plus Derivative controller instead of classical ones.

II. Mathematical Modelling of Controllers

Various configurations of controllers using the basic controllers are available in literature [IX]. The possible combinations of the various controllers are given below.

(1) For PI, Input, \( u(t) = K_p e(t) + K_I \int_0^t e(\tau) d\tau \)

(2) For PD, Input, \( u(t) = K_p e(t) + K_D \frac{d}{dt} e(t) \)

(3) For PID, Input, \( u(t) = K_p e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d}{dt} e(t) \)

PD and PI control systems are shown in fig.1. and in fig.2.
Fig. 1: PD control system

Fig. 2: PI control system
Fuzzification

Fuzzy controller membership functions structures are shown in below figures 3 and 4.

Fig. 3: FLC input MF’s

Fig. 4: The FLC output MF’s

Knowledge Rule Base

Mamdani [VI] type has both i/p and o/p variables fuzzified by fuzzy MF’s. It is considered the most popular method that is used to design FLC, because it is simple to implement and has fewer variables to specify than Takagi-Sugeno method [VII]. Let us now take FLC with two i/p’s and o/p to show the form of Mamdani fuzzy rules.

Rule1: If $x_1$ and $x_2$ both are positive means ‘u’ is positive.

Rule2: If $x_1$ is ‘+’ and $x_2$ is ‘-’ means ‘u’ is zero.

Where $x_1$ and $x_2$ are linguistic variables known as plant state variables.

The number of rules can be increased by increasing the number of linguistic variables [XI].
Defuzzification

The commonly used COA [VIII] method gives the COG of the possibility distribution of a control action

$$u = \frac{\sum_i \mu(x_i) x_i}{\sum_i \mu(x_i)}$$

Here $x_i$ is a running point in the universe of discourse, and $\mu(x_i)$ is its membership value in the M.F. The expression can be interpreted as weighted average of the elements in the support set. For the continuous case, replace the summation by integrals. This method is most commonly used although its computational complexity is relatively high. This is also known as center of gravity method.

The MOM [VII] method gives a control action which represents the mean value of all local control actions whose membership functions reach the maximum. The control action is given by

$$Z_0 = \frac{\sum_{j=1}^k w_j}{k}$$

Where $w_j$ is the support value at which the membership function reaches the maximum value $\mu_x(w_j)$, and $k$ is the number of such support values.

III. Simulation Results

The Step responses of Fuzzy PD controller for time delay and nonlinear systems [VIII] by using the Triangular and Trapezoidal membership functions are shown.

Fuzzy Proportional Plus Derivative (PD) Controller

Example 1: For a second order time delay system with $G(s) = \frac{e^{-3s}}{(100s + 1)^2}$
Observations

It is observed from the graphs fuzzy PD controller is giving a better response for all the membership functions, when compared with the conventional controller. Peak overshoot is less in case of Fuzzy controller for all the membership functions considered.

Example 2:

The second nonlinear system is $y(t) = -y(t) + 0.5\dot{y}(t) + u(t)$ using the set point $sp=2.0$. 

Fig. 7: Response for Triangular membership function.
Fig. 8: Response for Trapezoidal membership function

Observations

Fuzzy PD controller gives a better response for the nonlinear system, when compared with the conventional controller. By considering the $t_r$ and $t_s$. For all membership functions, fuzzy PD controller is giving better performance.

Example 3:

Fig. 9: Schematic diagram of P.M.D.C. Servo motor having PID Controller with saturation non-linearity
Fuzzy PD Controller

Without load disturbances:

Fig. 10: Response for conventional and fuzzy controllers for Triangular M.F.

Fig. 11: Response for conventional and fuzzy controllers for Trapezoidal membership function.
With load disturbances:

![Graph showing response for conventional and fuzzy controllers]

**Fig. 12:** Response for conventional and fuzzy controllers for Triangular M.F.

![Graph showing response for conventional and fuzzy controllers]

**Fig. 13:** Response for conventional and fuzzy controllers for Trapezoidal membership function.

**Observations**

Fuzzy PD controller giving a better response for the servo system with and without load disturbance, when compared with the conventional controller for all the membership functions considered.

**IV. Conclusion**

In this paper after discussing the fundamental concepts of Fuzzy logic and Fuzzy logic controllers, analytical input-output relation using arbitrary membership functions are derived. The analytical structures derived for arbitrary membership functions.
function clearly indicate that the fuzzy controller is a nonlinear controller with
different control algorithms in different ICs (i.e. in different input combination
regions). Time delay and nonlinear systems are considered for simulation, to show
the efficiency of the Fuzzy Proportional plus Derivative controllers over their
counterpart conventional controllers. Membership functions i.e. Triangular,
Trapezoidal membership functions are used for simulations. PMDC motor having
saturation nonlinearity with and without load disturbances is considered as a case
study, to show the efficiency of the Fuzzy Proportional plus Derivative controllers
over their conventional counterpart controllers. The effect of different membership
functions i.e. Triangular, Trapezoidal membership functions is observed. MATLAB
simulation results shows the importance of the fuzzy Proportional plus Derivative
controller over their counterpart conventional controller, especially when the systems
to be controlled are time delay and nonlinear. In case of fuzzy controllers the effect of
different membership functions i.e. ‘Triangular’, and ‘Trapezoidal’ was studied.

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