Design Model for Determination of the Strength Fiber-Reinforced Concrete

Viktor Sylovanyuk¹, Andriy Lisnichuk¹
Kostiantyn Kolesnyk² and Roman Panchak²

¹Karpenko Physico-Mechanical Institute of the National Academy of Sciences of Ukraine,
79053, Naukova str. 5, Lviv, Ukraine
²Lviv Polytechnic National University, CAD Systems Department,
79013, S. Bandera St. 12, Lviv, UKRAINE

*Email: kostiantyn.k.kolesnyk@lpnu.ua

Abstract. The model of fiber-reinforced concrete for determination of strength under tensile was developed. Formula for assessment of the strength of fiber-reinforced concrete, which takes into account micro-cracks and pores in the materials structure and the presence of reinforcing fibers, was established on the basis of the model. Experimental study for establishment of strength of cement reinforced with fiber of different nature and different volume content were conducted to confirm the received formula. It was shown that theoretical prediction of fiber-reinforced concrete strength correlates with experimental data.

1. Introduction

The structural material that is able to solve a set of the most complex problems in the construction industry is fiber-reinforced concrete (fiber concrete). It is a composite based on a cement matrix, reinforced with different fibers, (steel, glass, basalt, carbon, polymer and some others). Today, three types of fiber reinforced materials are chiefly widely used: fibers in the form of short pieces of thin steel wire, glass fibers and fibers based on polypropylene (Fig. 1). A broad use of reinforced concrete is due to higher strength, crack resistance, toughness, less microcracking during hardening compared to conventional concrete [1].

Considering concrete as a conglomerate of cement stone, sand and gravel, three levels of its structure are distinguished:

- microstructure – structure of cement stone;
- mesostructure – the structure of cement-sand mortar;
- macrostructure – the structure of gravel and mortar.

Each of these structures contributes to the strength of concrete, but it is largely determined by the strength of cement stone. The modelling of such materials is a topical task [2-7].

The work is devoted to modelling the processes of cracking, deformation and fracture of cement-matrix composites reinforced with various types of microfibres. Computational models are developed for prediction of tensile strength, compressive strength and fracture (behaviour) of these composites. The models are based on fracture mechanics approaches, in particular on the $\delta_c$-model and the power criterion of the critical stress intensity factors [8].
The analytical dependences of strength and fracture toughness of the material on the mechanical properties of the phases and their volume content, the matrix porosity characteristics, etc. are obtained. The influence of the material porosity, presence of cracks in it and the volume content of the reinforcing fibers in the composites on their service characteristics are analyzed.

![Carbon fiber](image1.jpg) ![Glass fiber](image2.jpg) ![Basalt fiber](image3.jpg)

**Figure 1.** Different types of reinforcing fibers.

2. Main results and their discussions

The aim of the work is to develop a mathematical model of composites based on a cement matrix, which will make it possible to predict the strength of such materials, i.e. to create materials with predetermined strength properties.

There are two groups of defects in the structure of cement stone that affect its strength characteristics: rounded like cavities (pores) and pointed like cracks. The pointed ones are decisive in the conditions of material tension, because they cause a high concentration of stresses in the vicinity of vertices, resulting in destruction. Round defects, on the contrary, are quite safe in tension, and under compression conditions initiate tensile stress zones, where cracks appear already under 50% loads of the destructive ones [1]. Cracks and pores with cracks on the material contour are usually randomly oriented relative to the direction of tensile forces. Let's consider the option of placing defects, as given in Fig. 2, which will make it possible to obtain a lower estimate of the material strength, because such a crack orientation is more unfavorable in terms of strength than the stochastic one.

The finite element method is used to calculate stress intensity coefficients (SIC) in a plate with a two-period system of cracks in the holes contour. The results of the calculation are shown in the form of SIC dependence on the parameter which characterizes the distance between the defects in the vertical plane (Fig. 3). As one can see, the two-period system of holes with cracks can be replaced by a collinear one-period system without a significant loss of accuracy in the calculations within the framework of parameter change \( g \geq 0.3 \text{ mm} \).

![Figure 2](image4.jpg)

**Figure 2.** Schematic representation of reinforced cement stone with cracks and pores.
The influence of the size of the holes on SIC in the plate with a two-period system of holes with cracks was also determined by the finite element method (Fig. 4) [9]. It was found that the hole will have little effect on the SIC, if \( r/l < 0.7 \). Thus, the results of calculations by the finite element method of the problem of a two-period system of holes with cracks on their contour (Fig. 2) give grounds for replacing these defects with a system of collinear cracks with a slight error, which will be assigned to the safety margin. Due to the small size of the defects (0.1…1 mm), the problem of the periodic system of collinear cracks is considered within the \( \delta \)-model framework (Fig. 5) as more appropriate in this case.

As it is known, Irwin criterion and the concept of stress intensity coefficients are not applicable to minor defects.

![Figure 3. Defects influence on Stress Intensity Factor in parallel planes.](image)

![Figure 4. SIC dependence on pores radius.](image)

![Figure 5. Design diagram of the material with cracks tension.](image)

We use the well-known solution of this problem within the Leonov – Panasyuk model [8, 10], where we obtain a relation that connects the intensity of external load \( p \) with the size of the fracture process zone.

\[
\cos \left( \frac{\pi p}{2\sigma_0} \right) = \frac{\sin \left( \frac{\pi}{2d} \right)}{\sin \left( \frac{\pi p}{2d} \right)}.
\]

The value of \( \sigma_0 \) is equivalent to the strength limit of cement stone volumes between pores and cracks.

Taking the condition of merging the fracture process zones of adjacent cracks as a criterion for destruction of the material, we obtain a formula that allows determining the tensile strength of cement stone under tension.

\[
R_{bt}^c = p_\sigma = \sigma_0 (1 - \frac{a}{d}).
\]

(2)
We should note that the ratio of the crack size to the distance between them $a/d$ reflects the content of defects in the material, that is, its damage. Let’s consider now the cement stone in terms of tension reinforced with microfibers of arbitrary stiffness. The thickness of these fibers will be assumed as much smaller than the characteristic size of the dominant defects – pores and cracks. This assumption makes it possible to consider the material between the pores as homogeneous with certain effective characteristics. Reinforcement with continuous or discrete fibers can be both directional and chaotic. The restraining effect of microfibers in the fracture process zones will be expressed by evenly distributed forces $\sigma_f^0$. Their intensity is described by the known Kelly – Tyson dependence \[ \sigma_f^0 = \lambda \sigma_j V_f \left( 1 - \frac{l}{2l} \right) + \sigma_0 (1 - V_f) , \] where $\sigma_j$ – the fibers strength; $l_c$ – boundary inefficient fiber length; $l$ – the fiber length; $V_f$ – volume content of fiber; $\lambda$ – the coefficient of chaotic reinforcement reduction to directional one [12] ($\lambda = 1$ – for directional reinforcement along the tensile axis; $\lambda = 0.33$ – for chaotic reinforcement).

By accepting into account the dependences (2), (3), we obtain a calculation formula for predicting the composite strength (fiber concrete) based on the cement matrix.

\[
R_{bt}^{f} = \left( 1 - \frac{a}{d} \right) \left( \lambda \sigma_j V_f \left( 1 - \frac{l}{2l} \right) + \sigma_0 (1 - V_f) \right) .
\]

Figure 6 shows the graphical dependences of the materials $R_{bt}^{f}$ strength with different volumetric contents of basalt, polypropylene, glass and carbon fibers and damage to the cement matrix. The mechanical characteristics of the fibers are provided in table 1.

**Table 1. Fiber mechanical properties**

| Fiber name | Density, g/cm³ | Tensile strength, MPa | Elasticity modulus, GPa |
|------------|----------------|-----------------------|------------------------|
| Basalt     | 2.65           | 1200                  | 12                     |
| Carbon     | 2              | 2000                  | 245                    |
| Glass      | 2.6            | 1050                  | 75                     |
| Steel      | 7.8            | 900                   | 200                    |
| Polypropylene | 0.9          | 500                   | 5                      |

In order to confirm the validity of the obtained analytical dependence (4), experiments were performed to determine the strength of the cement matrix-based composite for different volumetric content of basalt fiber. The following materials were used to prepare the solutions: basalt fiber; Portland cement PC II/A-3-500; superplasticizer.

Based on the above components, prismatic samples with a size of $40 \times 40 \times 160$ mm were made for tensile testing under bending.

The water-cement ratio was assumed as 0.4 with plasticizer Berament TB-1 in the amount of 0.04% by weight of cement. 12 mm long fiber (roving) was taken for studies, the fiber was in the unused condition and has the form of threads bundles. The solution was mixed for 5 min until a homogeneous mixture was obtained, after which it was compacted on a vibrating table to remove air bubbles to reduce the number of pores. All series of testing samples were kept in the laboratory for 27 days.

In fig. 7 a solid line corresponds to the design data, the dots indicate the experimental data. As can be seen, the results correlate well enough. From the above data it follows that the tensile strength of the composite increases more than twice at 2% of the fiber content compared to the base non-reinforced material.
Figure 6. Dependence of the predicted composite strength $R_{bt}$ on the volumetric content of reinforcing fibers: carbon (1); basalt (2); fiberglass (3); steel (4) and polypropylene fiber (5); in the absence of pores (a) and the porosity parameter a/d = 0.2 (b).

Figure 7. Tensile strength of cement stone reinforced with different basalt fiber content.

Conclusions
Dependence to predict the strength of the cement-based composite matrix tensile fracture mechanics concepts including was obtained. The parameters in the calculation formula are fiber volume content and crack defects, ultimate tensile strength of the composite components, geometric characteristics fiber and others. Calculation on the impact of defects on the strength of cement matrix composite is proposed. Theoretical prediction and experimental strength data correlate very well with each other. This gives reason to use the dependencies when creating composites with desired characteristics on the basis of the concrete matrix.

References
[1] Marukha V Panasyuk V and Sylovanyuk V 2014 Injection Technologies for Repair of Damaged Concrete Structures (Springer Dordrecht) p 230 https://doi.org/10.1007/978-94-007-7908-2
[2] Jaworski N Andrushchak N Lobur M Iwaniec M and Sitek W 2019 Composite material microlevel cellular model data transfer and verification by fem analysis software systems IEEE 15th International Conference on the Perspective Technologies and Methods in MEMS Design (MEMSTECH) 17-22 doi: 10.1109/MEMSTECH.2019.8817376
[3] Jaworski N Iwaniec M and Lobur M 2019 Implementation features of composite materials effective mechanical characteristics finding method based on microlevel cellular structural
models 15th International Conference on the Experience of Designing and Application of CAD Systems (CADSM) 1-5 doi: 10.1109/CADSM.2019.8779273

[4] Jaworski N Lobur M and Iwaniec M 2018 Implementation features of cellular composites microlevel structural models construction based on Voronoi tessellation and OpenCL technology usage 14th International Conference on Perspective Technologies and Methods in MEMS Design (MEMSTECH) 109-13 doi: 10.1109/MEMSTECH.2018.8365713

[5] Jaworski N and Andrushchak N 2017 A method of nanoporous anodic aluminum oxide structure modeling based on Bezier curves generation 14th International Conference The Experience of Designing and Application of CAD Systems in Microelectronics (CADSM) 63-6 doi: 10.1109/CADSM.2017.7916085

[6] Jaworski N Iwaniec M and Lobur M 2016 Composite materials microlevel structure models visualization distributed subsystem based on WebGL XII International Conference on Perspective Technologies and Methods in MEMS Design (MEMSTECH) 22-4 doi: 10.1109/MEMSTECH.2016.7507511

[7] Jaworski N Farmaga I Matviyikiv O Lobur M Słpiewak P Ciupiński Ł and Kurzydłowski K J 2014 Thermal analysis methods for design of composite materials with complex structure ECS Transactions 59(1) 513-23 doi: 10.1149/05901.0513ecst

[8] Cherepanov G 1977 Mechanics of fragile destruction (Moscow: Nauka) p 640

[9] Nevlyudov I Yevsieiev V Kolesnyk K and Saliieva V 2018 Finite element type and the features of the i-beam simulation influence on the calculation results CAD in Machinery Design: Implementation and Educational Issues (CADMD) 57-60

[10] Vytvyzkyj P 1970 Elastic-plastic equilibrium of a plate with a periodic system of slits AN URSR Ser A 6 524-7

[11] Kelly A and Tyson W 1965 Fiber Strengthened Materials in High Strength Materials (New York: John Wiley & Sons Inc) 578-83

[12] Rabinovych F 2004 Composites based on dispersed reinforced concrete Problems of theory and design, technologies, structures 560