Calculation of parameters of power plants for autonomous power supply systems

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Abstract. Thanks to a number of achievements in the field of thermoelectricity interest in the use of thermoelectricity for the generation of electric energy was renewed. To solve a number of important problems, such as: generation and distribution of energy produced; optimization and rational distribution of capital costs in the construction sector, autonomous power supply systems are used, the core of which is a thermoelectric generator (TEG). Such interest is caused by the fact that such installations are the most economical due to additional generation of electric power and allow drastically reducing fuel consumption. However, the calculation of thermoelectric devices is accompanied by the difficulties, associated with the dependence of electrical and thermal parameters, little attention is paid in scientific publications to a detailed description of heat transfer processes through the nodes of a thermoelectric device, to estimates of the heat of cold / hot coolants and their effect on the operating mode of an autonomous power supply system. The article offers theoretical foundations for calculating the parameters of power plants for autonomous power supply systems and the method of automated calculation of parameters using automated systems for searching technical solutions at the stage of TEG search design.

1. Introduction

Interest in the use of thermoelectricity for the generation of electrical energy has increased in connection with the achievements in this field, the creation of new materials, the development of nanotechnology, etc. Advances in the field of thermoelectricity have made it possible to single out in a separate task the generation of electric power from autonomous energy systems. A successful solution to this problem finds its application in the construction industry for reliable power supply to the decentralized regions of the country.

In the works [1,2] the potential of renewable energy sources and directions of their practical application are revealed. Works [3] show ways to improve the systems of autonomous power supply. The works [4] reflect solutions of the general technical problem of harmonizing the production and consumption modes for the practical use of power plants for autonomous power supply systems: commensurability of generating power and consumer capacities in autonomous power supply systems reduces the overall level of reliability of power supply to consumers. In [5], solutions were proposed
for combining different types of power plants into a single autonomous energy system, the core of which is a thermoelectric generator (TEG).

TEG is used for large and small temperature drops, as thermoelectric conversion is universal and allows the use of virtually any heat flow sources, including for small temperature differences, where other methods of conversion are impossible.

However, despite the advantages of TEG, there are significant drawbacks, among which there is a lack of calculation of performance characteristics, in particular, the temperature of TEG junctions required at certain design stages, which affects the efficiency of using autonomous power supply systems.

Therefore, theoretical bases are needed for calculating the parameters of power plants for autonomous power supply systems and the method of automated calculation of parameters using automated systems for searching technical solutions at the stage of TEG search design.

2. Formulation of the problem

Suppose there is an installation with TEG (Figure 1).

![Figure 1. The scheme of installation: 1 – engine 1NVD24; 2 – TEG; 3 – loading device; 4 – the voltmeter; 5 – ammeter; 6, 7, 12, 13 – the thermometer; 8, 15 – the flowmeter; 9 – the pump; 10, 11 – the tank with water; 14 – an exhaust pipeline; 16 – branch pipe.](image)

Hot heat carriers are the exhaust gases from the engine 1. Cooling water refers to cold coolants, the supply of which to the TEG is carried out by the pump 9.

The principle of the installation is described in [6]. TEG is a removable structure that is built into the gas-slope system. The heating of the surface of the thermocouples occurs due to convection heat transfer of the hot coolant with the unit 1. Cooling of the cold junctions of the thermoelements occurs due to the flow of fresh water, which leads to a difference in the temperature between the cold and hot junctions of the thermoelements. Due to the Seebeck effect, a thermal EMF occurs on the junctions. Each module is a series-connected thermocouple in an insulating ceramic housing.

Using this installation to develop a method for calculating TEG parameters, which will allow at the design stage to determine:

1) the amount of heat that is given to the hot / cold heat carrier,
2) electrical parameters of the TEG,
3) the values of the temperature of the TEG junctions.

3. Research Methods

To calculate the thermal and electrical performance characteristics of the TEG, a mathematical apparatus for describing heat transfer processes [7-9]. For the sake of simplicity of calculation, circular cross sections of TEG nodes are adopted, the areas of which are equivalent to the areas of the corresponding hexagonal sections of the TEG nodes. Hot and cold knots are made of the same material (Figure 2).
Figure 2. The nature of the temperature change through the layers of the TEG: 1 – the wall of the hot unit, 3 – the wall of the cold unit, 2 – the thermoelectric module TGM-287.

To calculate the heat transfer of a hot coolant, the data in Table 1.

Table 1. Data for calculation.

| Parameter                                      | Equation                                      |
|------------------------------------------------|-----------------------------------------------|
| Cross-sectional area of the flue               | $F_h = \frac{a^2 \sqrt{3}}{2}, \text{ (m}^2\text{)}$ |
| Equivalent flue diameter                       | $d_e = \frac{4F_h}{6a}, \text{ (m)}$          |
| Gas velocity in the experimental section       | $\omega_h = \frac{G_h}{\rho_h \cdot F_h}, \text{ (m/s)}$ |
| $G_h$ – hot coolant flow rate (kg/s)           |                                               |
| $\rho_h$ – coolant density at temperature $t_{hw}$ (kg/m$^3$) |                                               |

Since the average water temperature in the TEG is calculated by the formula:

$$t_{wm} = (t_{w1} + t_{w2})/2, \text{ (}^\circ\text{C)}$$

where $t_{w1}$, $t_{w2}$ – water temperature at the inlet and outlet of the TEG, respectively, the average logarithmic temperature head is calculated:

$$\Delta t_m = (t_{h1} - t_{c1}) - (t_{h2} - t_{c2})/\ln \left(\frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}}\right), \text{ (}^\circ\text{C)}$$

The wall temperature of the hot / cold node is in the first approximation determined by the formulas:

$$t_{s1} = t_{hw} - \Delta t_m/2, \quad t_{s2} = t_{cw} + \Delta t_m/2$$

Therefore, the total heat transfer coefficient for a hot coolant, taking into account formulas (1 – 3) and the heat transfer coefficient for radiation from a hot coolant:

$$\alpha_{hl} = 5.67 \cdot 10^{-8} \cdot \varepsilon \cdot [(t_{hw} + 273)^4 - (t_{c1} + 273)^4]/(t_{hw} - t_{c1}), \quad (W/m^2 \cdot K)$$

has the form:

$$\alpha_h = \alpha_{hk} + \alpha_{hl}, \quad (W/m^2 \cdot K)$$
To calculate the heat transfer of cold coolant, the data from Table 2 are used.

**Table 2.** Data for calculation.

| Parameter                        | Equation                                      |
|----------------------------------|-----------------------------------------------|
| Water flow rate                  | $G_w = \frac{G_b \cdot c_p \cdot (t_{h1} - t_{h2})}{c_p \cdot (t_{w2} - t_{w1})}$ (kg/s) |
| Cross-sectional area             | $F_w = 3\sqrt{3} \left( c^2 - b^2 \right)/2$, (m²) |
| Equivalent cross-sectional area  | $d_{eqw} = \frac{4F_w}{\Omega}$, (m)          |
| Speed of water                   | $\omega_w = G_w / \rho_w \cdot F_w$, (m/s)    |
| Reynolds number for water        | $Re_w = \omega_w \cdot d_{eqw} / \nu_w$       |
| Grashof number                   | $Gr_w = \frac{9.8 \cdot \beta \cdot (t_{h1} - t_{w}) \cdot L}{\nu_w^2}$ |

Since the Nusselt number depends on the water flow regime, then the following cases are possible.

1. $Re_w \leq 2000$, The Nusselt number in the laminar regime is determined by the formula:

   $$Nu_w = 0.15 \cdot Re_w^{0.33} \cdot Pr_w^{0.43} \cdot Gr_w^{0.1} \cdot \left( \frac{Pr_w}{Pr_{16}} \right)^{0.25} \cdot \varepsilon_{ew}$$

   where $Pr_w / Pr_{16}$ – The Prandtl number of water at temperature $t_{w}$, $\varepsilon_{ew}$ – average correction factor, depending on the ratio $L / d_E$.

2. $Re_w \geq 104$, The Nusselt number in the turbulent regime is determined by the formula:

   $$Nu_w = 0.021 \cdot Re_w^{0.8} \cdot Pr_w^{0.43} \cdot \left( Pr_w / Pr_{16} \right)^{0.25} \cdot \varepsilon_{ew}$$

3. $2000 \leq Re_w \leq 104$, The Nusselt number in the transition mode is determined by formula:

   $$Nu_w = K_b \cdot Pr_w^{0.43} \cdot \left( Pr_w / Pr_{16} \right)^{0.25} \cdot \varepsilon_{ew}$$

Summarizing the three cases, the heat transfer by the cold heat carrier is calculated as follows:

$$\alpha_w = \frac{Nu_w \cdot \lambda_w}{d_{eqw}} \cdot \frac{t_{w}}{m^2 \cdot K}$$

where $\lambda_w$ – coefficient of thermal conductivity of water at temperature $t_{w}$.

For the calculation of electrical parameters, the data from Table 3.

**Table 3.** Data for calculation.

| Parameter                        | Equation                                      |
|----------------------------------|-----------------------------------------------|
| Hot / cold junction temperature  | $T_h = t_{h3} + 273$, $T_c = t_{c4} + 273$ (K) |
| Average temperature between junctions | $T_w = \frac{T_h + T_c}{2}$, (K)               |
| The Seebeck coefficient / of modules | $E_i = 2 \cdot N \cdot e$, $E = n \cdot E_i$ (V/K) | $N$ – number of thermoelectric pairs in the module; $e$ – the Seebeck coefficient of the thermoelement (V/K), $n$ – number of modules.
Since, the temperature difference between junctions (cold / hot):
\[ \Delta T = T_h - T_c \ (K) \]

taking into account the following values:
1. \( R_{mod} = 2 \cdot N \cdot \rho \cdot \gamma \) (Om) – module resistance (\( \rho \) – specific electrical resistance of the thermoelement),
2. \( \gamma = \delta / a^2 \) (m\(^{-1}\)) – geometrical factor of thermoelement (\( a_c \) – cross-section side of thermoelement),
3. \( R_m = n \cdot R_{mod} \) (Om) – resistance of modules, \( m = R_n / R_m \) – load factor (\( R_n \) – load resistance),
4. \( I = E \cdot \Delta T / R_n \cdot (1 + m) \), \( \ (A) \) – current strength in the circuit, \( U = E \cdot \Delta T \cdot m / (1 + m) \) – load voltage,
5. \( P = I \cdot U = [E^2 \cdot \Delta T^2 / R_n] \cdot [m / (1 + m)^2] \) – external circuit power,

We have the efficiency of a thermoelectric generator:
\[
\eta = [(T_h - T_c) / T_h] \cdot [1 / (1 + m) + 1 / 2 \cdot \frac{T_h - T_c}{T_h} \cdot \frac{1}{m}] 
\]

At the same time, the electric power given to the consumer: \( P_a = P - P_{H} \), where \( P_{H} \) – power used for cold coolant.

4. Results of the study

For automated calculation of the parameters of the thermoelectric unit as a whole and calculation of the thermal and electrical parameters of the TEG using automated systems for the search for technical solutions [10,11], a method based on the method of successive approximations was developed at the stage of the search design of the TEG. If the previous and subsequent approximations \( t_{st3} \) and \( t_{st4} \) coincide with the specified accuracy, then the calculation continues, if there is no such coincidence, then the calculations are repeated until the required coincidence is achieved.

We introduce the following concepts. The control object will be called TEG, the state of which we are interested. The state of the TEG can vary: \( X' = D_x(X) \), which is determined according to the methods of calculating the parameters of the operation of the TEG using the heat energy of cold coolants. The subject of management will be called the object, which is the source of the goals realized by the management: \( Y' = D_{y}(Y) \) (Figure 3). Figure 4 graphically shows the temperature distribution along the thickness of the module and the zone of realization of the accompanying effects.

Figure 3. Block diagram of the object management system.

Figure 4. Temperature distribution inside the thermoelectric module.
Here is denoted: $\Delta t^c$, $\Delta t^h$ – loss of temperature head on the structural part of the thermal module; energy characteristics of the thermo module (due to the presence of thermal resistance on its elements - ceramics, solder, commutation tires, contact junction) (Figure 5); $\Delta t_{rt}$ – working (useful) temperature difference, which determines the energy characteristics of the thermal module; D – Joule heat-realization zone; P – zone of Peltier heat; $l$ – length of thermoelement.

For the sake of simplicity, certain assumptions are introduced. The calculation of the thermoelectric unit is realized in block 1. The method for calculating the thermal and electrical parameters of the TEG is realized in block 2 and is carried out by successive approximations.

1. The calculation begins with the setting of the following parameters [12-14]:
   - $t_{ht1, t_{ct1}}$ – temperature of hot and cold coolants at the entrance to the thermoelectric generator;
   - $t_{ht2, t_{ct2}}$ – temperature of hot and cold coolants at the output of the thermoelectric generator;

2. According to formula (6), average temperatures of heat carriers:
   \[
   K_v = \Delta T \cdot \frac{F}{Q} = \frac{F}{(2c_p \cdot G)} \quad (m^2 K/VT)
   \]

3. The following parameters are determined by average temperatures of the heat-transfer media:
   - $c_{ht}$; $c_{ct}$ – specific heat of hot and cold coolants;
   - $\rho_h$, $\rho_c$ – density of hot and cold coolants;
   - $v_h$, $v_c$ – kinematic viscosity of hot and cold coolants;
   - $Pr_h$, $Pr_c$ – The Prandtl number at medium coolant temperatures;
   - $\lambda_h, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ – thermal conductivity coefficients of the hot coolant, hot unit walls, ceramic insulation layer on the side of the hot unit, thermocouples, ceramic insulation layer from the side of the cold unit, the wall of the cold unit and cooling water;
   - $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5$ – the thickness of the wall of the hot unit, the layer of ceramic insulation from the side of the hot unit, the height of the thermocouples, the layer of ceramic insulation from the side of the cold unit.

4. The amount of heat given off by the hot coolant, where $\tau$ – the Thomson coefficient:
   \[
   Q_\tau = I \cdot 10^{-6} \cdot \int_{t_h}^{T} \tau dT
   \]

The parameters of the operation of the TEG purposefully influence the increase in efficiency. The calculation results $D_1, D_2$ from blocks 1, 2 are sent to the control unit (УУ), which generates control commands U. These commands are processed by the actuator (ИМ) in order to change the state of the controlled input $U'$ of the object.

If the state of the object – TEG satisfies the subject's $Z^*$ needs – increasing the efficiency, interacting with this object and exploiting it, then no control is needed. If, for some reason, the state of the object does not satisfy the needs of the subject, then such an effect on the object $\phi$ (selection of parameters) is organized, which leads the object to a new state that satisfies the subject: $U = \phi(X^*, Y^*, Z^*)$.

The nature of the change in the exergy efficiency as a function of the heat carrier regimes is graphically presented in Figures 5 and 6.

In the process of working, the parameters of the heat carriers are clarified, and consequently their average temperatures and, as a consequence, the thermophysical properties of the heat carriers change. In addition, this takes into account the effect on the thermophysical and electrical characteristics of the thermoelectric material. Thus, the above parameters are constantly refined taking into account the temperature change.
5. Conclusion

The use of the developed method for calculating TEG parameters is due to the fact that the rational ratio of the powers of the main elements of the autonomous power supply system is not typical, but determined individually, taking into account the specific conditions of its operation. Therefore, the developed technique allows, by determining TEG parameters, to select the appropriate operating modes of autonomous power supply systems for their uninterrupted and reliable operation.

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