Action for the eleven dimensional multiple M-wave system

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ABSTRACT: We present the covariant supersymmetric and $\kappa$–symmetric action for a system of $N$ nearly coincident M-waves (multiple M0-brane system) in flat eleven dimensional superspace.

KEYWORDS: String/M-theory, supersymmetry, p-branes, spinor moving frame.
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Introduction

When speaking about Mp-branes, eleven dimensional (11D) supersymmetric extended objects playing an important role in String/M-theory, one usually mentions M2–brane, also known as 11D supermembrane (p=2) [1] and M5-brane [2, 3, 4]. However, as it was noticed in [5], one more supersymmetric 11D object exists. This is 11D massless superparticle (p = 0), which can be called M0–brane or M–wave. The last name looks natural as far as 'M' refers to M-theory and hence to 11D, while 'wave' is an appropriate name for a massless particle moving along a light–like line in spacetime. The other name, M0-brane, comes from the observation that (see [5]) the dimensional reduction of this 11D superparticle down to 10D produces type IIA massive superparticle which is called D0-brane because it belongs to the numerous family of Dp–branes (Dirichlet p–branes) of the type II superstring theories (see [5] and refs therein).

Being the simplest representative of the family of M-branes, the M-wave provides a natural starting point for studying their properties and the toy model to attack difficult problems related to them. One of such problems is the search for an effective action for the system of multiple M2 (mM2) and multiple M5–branes (mM5).

The dimensional reduction of these hypothetical mMp' actions should produce the actions for mDP (multiple Dp–brane) systems. For these a (very) low energy limit is provided [6] by the maximal $d = p + 1$ dimensional supersymmetric Yang–Mills (SYM) theory with the gauge group $U(N)$. This includes $(9 – p)$ Hermitian matrices of scalar fields, $\tilde{X}^i$, the diagonal elements of which describe the positions of different Dp-branes while the off–diagonal elements account for the strings stretched between different Dp-branes. The SYM description provided the basis for the search for
the complete (a more complete) nonlinear description of mDp–system, similar to the description of single Dp–brane in e.g. [5]; see [7, 8, 9] and refs. therein for the progress in this direction.

For the case of mM5 even the question on what should be a counterpart of the very low energy approximate SYM description of mDp is still obscure (see e.g. [10] for a relevant result and references). For the case of very low energy mM2 system such a problem was unsolved many years, but recently two related models were proposed in [11, 12] and [13].

As far as multiple M0–brane (mM0) action is considered, a purely bosonic candidate was constructed in [14] as the 11D generalization of the Myers ’dielectric D0-brane’ action in [7]. On the other hand, an approximate but supersymmetric and Lorentz covariant equations of motion for mM0–system were obtained in [15] in the frame of superembedding approach (see [16, 2] as well as [17, 18] and refs. therein). The generalization of these equations for the case of mM0–system in curved 11D supergravity superspace, which describes the generalization of the M(atrix) theory [19] for the case of its interaction with arbitrary supergravity background, were presented in [20] and studied in [21]. In [22] it was shown that, when specialized for the case of 11D pp-wave superspace, these equations reproduce (in an approximation) the so–called BMN matrix model proposed for this background by Berenstein, Maldacena and Nastase in [23] (see [24] for the derivation of the bosonic limit of the BMN action). This result has confirmed that the equations of [20, 21] describe the Matrix theory interacting with supergravity background, but its derivation has shown that, due to their superspace origin, application of these equations for the case of some definite, even purely bosonic supergravity backgrounds are technically complicated: it requires the lifting of the bosonic supersymmetric solution of 11D supergravity to the complete superfield solution of the 11D superspace supergravity constraints [25] and to use them to specify the induced geometry on the center of energy worldline.

Hence, for applications it is desirable to find an action which reproduces the Matrix model equations of [20] or their generalizations. In this paper we present such action for multiple M0–brane system, for the case of flat target 11D superspace. This action is essentially based on the spinor moving frame formalism for the 11D massless superparticle [26] (see [27, 28] for D=4 and D=6,10 cases and [18] for refs. on related studies) which we briefly describe in sec. 2 below.

1. M0–brane, 11D massless superparticle

As single M0–brane (M-wave) is just the D=11 massless superparticle, it can be described by the 11D version of the Brink–Schwarz action [5] \( S_{BS} \). In the first order formalism

\[
S_{BS} = \int_{W^1} \left( p_a \hat{E}^a - \frac{e}{2} p_a p^a d\tau \right),
\]

where \( a = 0, 1, ..., 10 \) is the SO(1,10) vector index, \( e(\tau) \) is the Lagrange multiplier the variation of which imposes the mass shell condition \( p_a p^a = 0 \), \( \hat{E}^a = E^a(\hat{Z}) = d\hat{Z}^M(\tau) E^a_M(\hat{Z}) \) is the pull–back of the bosonic supervielbein of the 11D target superspace, \( E^a = E^a(Z) = dZ^M E^a_M(Z) \), to the worldline \( W^1 \) parametrized by proper time \( \tau \). The pull–back is obtained by substituting the bosonic and fermionic coordinate functions \( \hat{Z}^M(\tau) = (\hat{x}^\mu(\tau), \hat{\theta}^{\dot{\alpha}}(\tau)) \) (\( \mu = 0, ..., 9, 10, \dot{\alpha} = 1, ..., 32 \)) for the superspace coordinates \( Z^M = (x^\mu, \theta^{\alpha}) \), so that

\[
\hat{E}^a = d\tau \hat{E}^a_\tau, \quad \hat{E}^a_\tau = \partial_\tau \hat{Z}^M E^a_M(\hat{Z}^M(\tau)).
\]
The supervielbein $E^a = dZ^M E^a_M(Z)$ should obey the 11D superspace supergravity constraints [25]. In this paper we will mainly restrict ourselves by the case of flat target superspace for which

$$E^a = \Pi^a = dx^a - i d\theta^a \Gamma^a \theta , \quad E^\alpha = d\theta^\alpha .$$

(1.3)

Here and below we use the real 32$\times$32 matrices $\Gamma^a_{\alpha\beta} = \Gamma^a_{\beta\alpha} = \Gamma^\alpha_{\gamma\beta}$ and $\tilde{\Gamma}^a_\alpha = \tilde{\Gamma}^a_\alpha = C^{\alpha\gamma} \Gamma^\gamma_{\alpha\beta}$ constructed as a product of the 11D Dirac matrices $\Gamma^a_{\alpha\beta}$ (pure imaginary in our mostly minus conventions $\eta^{ab} = diag(1,-1,\ldots,-1)$ and obeying $\Gamma^a \Gamma^b + \Gamma^b \Gamma^a = \eta^{ab} I_{32 \times 32}$) with 11D charge conjugation matrix $C_{\gamma\beta} = - C_{\beta\gamma}$ and its inverse $C^{\alpha\beta} = - C^{\beta\alpha}$.

The action (1.1) is invariant under the local fermionic $\kappa$–symmetry [30, 31]

$$\delta_\kappa \hat{x}^a = -i \dot{\theta}^a \delta_\kappa \hat{\theta} , \quad \delta_\kappa \hat{\theta}^\alpha = p_\alpha \tilde{\Gamma}_{\alpha\beta} \kappa^\beta (s) , \quad \delta_\kappa z = 4 i \kappa \beta J_\beta \hat{\theta}.$$  

(1.4)

This symmetry is important because it reflects the supersymmetry preserved by ground state of the superparticle. This fact is not manifest due to the infinite reducibility of the $\kappa$–symmetry [31] (which results in that the 32 parameters in the 11D Majorana spinor $\kappa_\alpha (s)$ can be used to remove only 16 component of the fermionic coordinate function $\hat{\theta}^\alpha (s)$).

2. M0–brane in spinor moving frame formulation

The $\kappa$–symmetry appears in its irreducible form in the so–called spinor moving frame formulation of superparticle [27, 28, 26]. The spinor moving frame action of M0–brane reads

$$S_{\text{M0}} = \int_{W^1} \rho_t \hat{E}^t = \int_{W^1} \rho^t u^a E^a (\hat{Z})$$

$$= \frac{1}{16} \int_{W^1} \rho^t (v_q^{-\alpha} \Gamma^\alpha_\beta v_q^{-\beta}) \hat{E}^a ,$$

(2.1)

(2.2)

where $\rho^t(r)$ is a Lagrange multiplier and $u^- = u^- (r)$ is a light–like 11D vector, $u^- = 0$. This can be considered as a kind of square of any of the 16 spinors $v_q^{-\alpha} = v_q^{-\alpha} (r)$ (which have appeared in (2.2)) provided these are constrained by

$$v_q^{-\alpha} (\Gamma^\alpha_\beta v_q^{-\beta}) = \delta_{qp} u_q^-$$

$$2v_q^{-\alpha} v_q^{-\beta} = u^- \Gamma^\alpha_\beta$$

(2.3)

With the use of these constrained spinors, the $\kappa$–symmetry of the spinor moving frame action can be written in the following irreducible form

$$\delta_\kappa \hat{x}^a = -i \dot{\theta}^a \delta_\kappa \hat{\theta} , \quad \delta_\kappa \hat{\theta}^\alpha = e^{+q} (r) v_q^{-\alpha} , \quad \delta_\kappa \rho^t = 0 = \delta_\kappa u^- .$$

(2.4)

We have denoted the parameter of the irreducible $\kappa$–symmetry by $e^{+q} = e^{+q} (r)$ to stress its relation with the extended worldline supersymmetry.

The transformations (2.4) can be obtained from the infinitely reducible (1.4) by substituting for $p_a$ the solution $p_a = \rho^t u^a$ of the constraint $p_a p^a = 0$. It is easy to see that, with this substitution, the action (1.1) acquires the form of (2.1). Furthermore, using (2.3), we find

$$e^{+q} = 2 \rho^t v_q^{-\alpha} \kappa_\alpha .$$

(2.5)
However, one might still find the origin of our $v_q^{-\alpha}$ a bit mysterious. To clarify this, it is useful to consider the null–vector $u_a^\pm$ as an element of the *moving frame* matrix,

$$U_b^{(a)} = \left( \frac{u_b^+ + u_b^-}{2}, u_b^+ - u_b^- \right) \in SO(1, 10) \quad (2.6)$$

$(i = 1, \ldots, 9)$. The statement that this matrix belongs to the $SO(1, 10)$ is tantamount to saying that the moving frame vectors obey the constraints [29]

$$u_a^- u_a^+ = 0, \quad u_a^- u_a^# = 2, \quad u_a^- u_a^i = 0, \quad (2.7)$$

$$u_a^# u_a^+ = 0, \quad u_a^# u_a^i = 0, \quad u_a^i u_a^j = -\delta^{ij}. \quad (2.8)$$

Then $v_q^{-\alpha}$ can be defined as an $16 \times 32$ block of $Spin(1, 10)$ valued *spinor moving frame* matrix

$$V_\alpha^\beta = \left( \begin{array}{c} v_q^{+\alpha} \\ v_q^{-\alpha} \end{array} \right) \in Spin(1, 10). \quad (2.10)$$

This is double covering of the moving frame matrix (2.6) in the sense of that

$$VT_b V^T = U_b^{(a)} \Gamma_{(a)}, \quad (a) \quad V^T \tilde{\Gamma}_b V = \tilde{U}_b^{(a)}, \quad (b) \quad VCV^T = C. \quad (c) \quad (2.11)$$

The two seemingly mysterious constraints (2.3) appear as a block of the relation (2.11a) and as the $= 0$ component, $V^T \tilde{\Gamma} = V = \tilde{U}_b^{(a)}$, of (2.11b), with an appropriate representation of the 11D Gamma matrices. The other blocks/components of these constraints involve the second set of constrained spinors, $v_q^{+\alpha}$,

$$v_q^+ \Gamma_a v_q^+ = u_a^i \delta_{qp}, \quad v_q^- \Gamma_a v_q^+ = -u_a^i \gamma_{qp}^i, \quad v_{qp}^{i\alpha} = \delta_{qp}^i \gamma_{qp}^i, \quad (2.12)$$

$$v_q^{+\alpha} v_q^{+\beta} = \tilde{\Gamma}_{\alpha\beta} u_a^i, \quad 2v_q^{+\alpha} v_q^{+\beta} = -\tilde{\Gamma}_{\alpha\beta} u_a^i. \quad (2.13)$$

Here $\gamma_{qp}^i$ are the 9d Dirac matrices; they are real, symmetric $\gamma_{qp}^i = \gamma_{qp}^i$ and obey $\gamma^i \gamma^j + \gamma^j \gamma^i = 2\delta^{ij}I_{16 \times 16}$.

The third constraint, (2.11c), implies that the inverse spinor moving frame matrix

$$V^{(a)}_\alpha = (v_{aq}^+, v_{aq}^-) \in Spin(1, 10) \quad (2.14)$$

can be constructed from the same $v_q^{+\alpha}$ as in (2.10),

$$v_{aq}^+ = \pm iC_{\alpha\beta} v_q^{+\beta} = \left\{ \begin{array}{l} v_{aq}^- v_{aq}^+ = \delta_{aq} v_{aq}^+ v_{aq}^- , \\
\frac{v_{aq}^- v_{aq}^+ - \delta_{aq} v_{aq}^+ v_{aq}^- }{2} = 0 = v_{aq}^+ v_{aq}^+. \end{array} \right\} \quad (2.15)$$

The moving frame vectors can be used to split the pull–back of the supervielbein in a Lorentz covariant manner,

$$\hat{E}^{(a)}_b \rightarrow \hat{E}^{(a)}_b U^{(a)}_b = \left( \hat{E}^+, \hat{E}^#, \hat{E}^i \right). \quad (2.16)$$

One can show that the equations of motion for the Lagrange multiplier $\rho^#$ and for the moving frame vector $u_a^\pm$ (or for $v_q^{-\alpha}$) result in $\hat{E}^+ = 0$ and $\hat{E}^i = 0$, respectively [see 26, 18, 15] for details on varying the moving frame and spinor moving frame fields), so that on the mass shell

$$\hat{E}^+ := \hat{E}^a u_a^- = 0 \quad \hat{E}^i := \hat{E}^a u_a^i = 0 \quad \Leftrightarrow \quad \hat{E}^a := \frac{1}{2} \hat{E}^# u_a^- \quad (2.17)$$
This implies that the M0–brane worldline \( W^1 \) is a light–like line in target (super)space, which is in agreement with the statement that M0–brane is the massless 11D superparticle.

Furthermore (2.17) suggests to consider \( \hat{E}^\# = d\tau \hat{E}^\#_\tau \) as an einbein on \( W^1 \). Its gravitino–like companion is given by the covariant projections \( \hat{E}^{+q} = \hat{E}^{\alpha}\nu^q_{\alpha} \) of the pull–back of the fermionic 1–form \( E^\alpha \). One can show \([26]\) that the other projection, \( \hat{E}^{-q} = \hat{E}^\alpha v^-_{\alpha} \), vanishes due to the fermionic equation of motion of the M0-brane, so that, on the mass shell,

\[
\hat{E}^{-q} := \hat{E}^\alpha v^-_{\alpha} = 0 \quad \Leftrightarrow \quad \hat{E}^\alpha := \hat{E}^{+q} v^+_{\alpha}.
\]  

(2.18)

The suggestion to treat \( \hat{E}^{+q} = \hat{E}^\alpha v^+_{\alpha} = d\tau \hat{E}^{\#}_\tau \) as composed gravitino and \( (\hat{E}^\#, \hat{E}^{+q}) \) as composed supergravity multiplet induced by embedding of \( W^1 \) into the 11D target superspace may be taken from the observation that under the irreducible \( \kappa \)--symmetry (2.4)

\[
\delta_\kappa \hat{E}^{+q} = D\epsilon^{+q}(\tau), \quad \delta_\kappa \hat{E}^\# = -2i\hat{E}^{+q}\epsilon^{+q} \tag{2.19}
\]

(\( D = d\tau D\tau \) is defined below, in (3.6)). Our action for the mM0 system, which we are going to present, contains the coupling of these induced 1d supergravity to the matter describing the relative motion of the mM0 constituents.

### 3. Covariant action for multiple M0–brane (mM0) system

The study of \([15, 20]\) suggests that, describing the system of \( N \) nearly coincident M0–branes (mM0 system), it is convenient to separate the coordinate functions \( \hat{Z}^M(\tau) \) describing the center of energy motion (with the same properties as the ones describing single M0–brane) and the variables describing the relative motion of the mM0 constituents. That are the bosonic and fermionic hermitian traceless \( N \times N \) matrix fields \( \hat{X}^i(\tau) \) and \( \Psi_q(\tau) \) depending on a proper time variable \( \tau \) parametrizing the center of energy worldline \( W^1 \). The bosonic \( \hat{X}^i(\tau) \) carries the index \( i = 1, \ldots, 9 \) of the vector representation of \( SO(9) \), while the fermionic \( \Psi_q \) transforms as a spinor under \( SO(9) \), so that \( q = 1, \ldots, 16 \). The \( SO(1,1) \) weight of the fields are 2 and 3, respectively, so that in a more explicit notation \( \hat{X}^i = \hat{X}^i_{\#} := \hat{X}^i_{++} \) and \( \Psi_q = \Psi_{++q} := \Psi_{++q} \).

We propose to describe the system of \( N \) nearly coincident M0–branes by the following action

\[
S_{mM0} = \int_{W^1} \rho \hat{E}^\alpha + \int_{W^1} \rho^3 \left( \left[ \hat{X}^i, \hat{X}^j \right] \psi_q + \hat{E}^\alpha \psi_q \right) + \int_{W^1} \rho^3 \hat{E}^{\#}\psi_q \gamma^i \psi_q \left[ \hat{X}^i, \hat{X}^j \right]. \tag{3.1}
\]

In it the measure factor \( d\tau \) is hidden in \( D = d\tau D\tau \) (described below) and inside the bosonic and fermionic one forms\(^1\)

\[
\hat{E}^\# = \hat{E}^\alpha u^-_{\alpha} = d\tau \hat{E}^{-\#} \quad \text{and} \quad \hat{E}^{\#} = \hat{E}^\alpha u^+_{\alpha} = d\tau \hat{E}^{+\#}, \quad \hat{E}^{+q} = \hat{E}^\alpha v^+_{\alpha} = d\tau \hat{E}_{\tau}^{+q}. \tag{3.2}
\]

These, as well as the Lagrange multiplier \( \rho = \rho^\#(\tau) \), have been described above, in sec. 2, when discussing the single mM0-brane case. But now the moving frame vectors \( u^-_{\alpha} \) and \( u^+_{\alpha} \), obeying

\(^1\)Let us recall that we consider the case of flat target 11D superspace, in which \( \hat{E}^\alpha = d\tau(\partial_\alpha \hat{x}^\alpha - i\partial_\alpha \hat{\theta}^\alpha \hat{\theta}) \), and \( \hat{E}^\alpha = d\hat{\theta}^\alpha = d\tau \partial_\alpha \hat{\theta}^\alpha \), see Eqs. (1.2) and (1.3).
(2.7) and (2.8), and spinor moving frame variable $v^q_{\alpha}$, obeying (2.12) and (2.13), are related to the center of energy motion of the mM0 system. Further, $P^i := P^i_{\#}$ are 9 auxiliary bosonic matrix fields having the meaning of the momentum of $X^i$ and

$$H := H_{\#}(X, \mathbb{P}, \Psi) = \frac{1}{2} tr \left( \mathbb{P}^i \mathbb{P}^i \right) + \mathcal{V}(X) - 2 tr \left( X^i \Psi \gamma^i \Psi \right)$$

(3.3)

is the relative motion Hamiltonian. Besides the kinetic term $tr(P^i)^2$, this includes the Yukawa coupling $tr \left( X^i \Psi \gamma^i \Psi \right)$ and the scalar potential

$$\mathcal{V} := \mathcal{V}_{\#}(X) = -\frac{1}{64} tr \left[ X^i, X^j \right]^2 .$$

(3.4)

The covariant derivatives $D = d\tau D_\tau$ are defined by

$$D X^i := dX^i + 2 \Omega^{(0)} X^i - \Omega^{ij} X^j + [A, X^i] ,$$

(3.5)

$$D \Psi_q := d\Psi_q + 3 \Omega^{(0)} \Psi_q - \frac{1}{4} \Omega^{ij} \gamma_{ij} \Psi_p + [A, \Psi_q] .$$

(3.6)

Here $A = d\tau A_\tau(\tau)$ is the $SU(N)$ connection on $W^1$, $A_\tau(\tau)$ is an anti-Hermitian traceless $N \times N$ matrix gauge field in 1d, which is an independent variable in our model. In contrast, $\Omega^{(0)} = d\tau \Omega^{(0)}_\tau$ and $\Omega^{ij} = d\tau \Omega^{ij}_\tau$ are the composed (induced) $SO(1, 1)$ and $SO(9)$ connections on $W^1$. They are constructed from the moving frame vector fields (2.6) corresponding to the center of energy motion as

$$\Omega^{(0)} = \frac{1}{4} u^{-a} du_a^# , \quad \Omega^{ij} = u^a du_a^j .$$

(3.7)

The action (3.1) is invariant under the transformations of the $\mathcal{N} = 16$ local worldline supersymmetry ‘parameterized’ by fermionic $SO(9)$ spinor function $\epsilon^+ q = \epsilon^+ q(\tau)$. This acts on the matrix fields describing the relative motion of the mM0 constituents as

$$\delta_{\epsilon} X^i = 4 i \epsilon^+ \gamma^i \Psi , \quad \delta_{\epsilon} P^i = \left[ (\epsilon^+ \gamma^{ij}) \Psi, X^j \right] ,$$

(3.8)

$$\delta_{\epsilon} \Psi_q = \frac{1}{2} \left( \epsilon^+ \gamma^i \right) q_{\#} P^i - \frac{i}{16} \left( \epsilon^+ \gamma^{ij} \right) q \left[ X^i, X^j \right] ,$$

(3.9)

$$\delta_{\epsilon} A = - \hat{E}^# + \epsilon^+ q \Psi + (\hat{E}^+ \gamma^i \epsilon^+) X^i ,$$

(3.10)

and on the center of energy variables as

$$\delta_{\epsilon} x^a = - i \partial^a \delta_{\epsilon} \theta + 3 \rho^2 u^a # tr \left( i(\epsilon^+ \gamma^i \Psi) P^i - (\epsilon^+ \gamma^{ij} \Psi) \left[ X^i, X^j \right] / 8 \right) ,$$

(3.11)

$$\delta_{\epsilon} \theta^a = \epsilon^+ q v_{-a} ,$$

(3.12)

$$\delta_{\epsilon} \rho = 0 = \delta_{\epsilon} u_{-a} .$$

(3.13)

Eqs. (3.11), (3.12) and (3.13) describe a deformation of the irreducible $\kappa$–symmetry (2.4) of the free massless superparticle. Actually the only deformed relation is $\delta_{\epsilon} x^a$, (3.11), which acquires an additional (with respect to (2.4)) contribution constructed from $X^i, P^i$ and $\Psi_q$.

The local supersymmetry (3.8)– (3.13) guaranties that the ground state of the dynamical system described by the action (3.1) preserves 1/2 of 32 11D supersymmetries (is a 1/2 BPS state), the fact which allows to identify (3.1) with the action of multiple M0-brane system.

\footnote{To prove the invariance of the action (3.1) under these supersymmetry transformations the following identity for 9d gamma matrices is useful: $\gamma^{ij}_{q(p)} \gamma^{k}_{p(q)} \gamma^{l}_{p(r)} \gamma^{j}_{q(r)} = \gamma_{q(p)} \delta_{qp} - \delta_{q(p)} \gamma_{q(p)}$.}
4. Effective mass of the center of energy motion of the mM0 system

The fact that the worldline supersymmetry transformations of the mM0 center of energy variables are so close to the $\kappa$–symmetry transformations of single M0-brane can be traced to the fact that the first term in (3.1) coincides with the action (2.1) of the single M0–brane. However, due to the presence of the Lagrange multiplier $\rho(\tau) = \rho^\#(\tau)$ and of the moving frame variables, $u^\#_a$ and $v^{+q}_a$, also in the second part of the action (containing $\hat{E}^\# = \hat{E}^\alpha u^\#_a$ and $\hat{E}^{+q} = \hat{E}^\alpha v^{+q}_a$), the equations for the auxiliary fields differ from (2.17) in such a way that, in contrast with the case of a single M0-brane, generic motion of the center of energy of the mM0 system is not light-like; it is characterized by a nonvanishing effective mass constructed from the relativ motion fields $X^i$ and $\Psi_q$.

4.1 Effective mass of the center of energy motion

To see this in a simple way, let us calculate the canonical momentum conjugate to the center of energy coordinate function $\hat{x}^a(\tau)$. This reads

$$p_a(\tau) = \frac{\partial \mathcal{L}_{mM0}}{\partial \dot{x}^a(\tau)} = \rho u^+_a + (\rho)^3 u^\#_a \mathcal{H}(X^i, \Pi^i, \Psi_q),$$

(4.1)

where $\mathcal{L}_{mM0}$ is the Lagrangian density of the action (3.1), $S_{mM0} = \int d\tau \mathcal{L}_{mM0}$, and $\mathcal{H} = \mathcal{H}^\#^\#^\#^\#$ is defined in Eq. (3.3). Now, using Eqs. (2.7) and (2.8) one easily finds that

$$M^2 := p^a p_a(\tau) = 4 \rho^4 \mathcal{H}(X^i, \Pi^i, \Psi_q).$$

(4.2)

In purely bosonic limit it is easy to see that $M^2$ is a nonnegative constant. Indeed, when $\Psi = 0$ the relevant equations of the relative motion which follow from the action (3.1) read

$$\hat{E}^\# \Pi^i = D X^i \implies \Pi^i = D^\# X^i,$$

(4.3)

$$D^\# D^\# X^i = \frac{1}{16} [X^i, X^j] X^j,$$

(4.4)

where $D^\#$ is defined by $D^\# := D_\tau / \hat{E}^\#$ or, equivalently, by $D = E^\# D^\# = d\tau D_\tau$ where $D$ is the covariant derivative defined in (3.5), (3.6). As a consequence of Eqs. (4.3) and (4.4), the relative motion Hamiltonian is covariantly constant, $D\mathcal{H} = 0$. As far as $\mathcal{H} = \mathcal{H}^\#^\#^\#^\#$ carries the weight 8 with respect to SO(1,1), the covariant derivative involves the induced SO(1,1) connection (3.7) so that a more explicit form of this equation is $d\mathcal{H} + 8 \Omega^{(0)} \mathcal{H} = 0$. Now we shall notice that the set of equations of motion following from the action (3.1) also includes $D \rho := d\rho - 2 \Omega^{(0)} \rho = 0$ which can be solved for SO(1,1) connection $\Omega^{(0)} = d\rho / 2\rho$. Using this solution we see that $D\mathcal{H} = 0$ can be written in the form of $\frac{d(\rho^4 \mathcal{H})}{\rho^4} = 0$ which makes manifest that $M^2$ in (4.2) is a constant. This constant is nonnegative just due to the explicit form of (the bosonic limit of) the relative motion Hamiltonian $\mathcal{H}$, Eqs. (3.3), (3.4).

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\[3\] See [26] and forthcoming [34] for details on variation of 11D spinor moving frame variables; in this paper we would like to omit such type technicalities. Remember that $\rho = \rho^\#(\tau)$ has SO(1,1) weight -2, opposite to the weight of $X^i = X^i_\#(\tau)$. 
4.2 On relation with the results of [15, 20]

This is the place to comment on relation of our model with [15, 20]. The reader who is not interested
in superembedding approach can omit this subsection and pass directly to the next sec. 4.3.

The complete equations of motion following from the action (3.1) are close to, but not identical
with, the flat superspace case of the mM0 equations obtained in [15, 20] in the frame of the
superembedding approach. The difference is due to the contribution of the center of energy fields
into the equations of relative motion and vice versa (see [34] for details). For instance, the complete
form of the the bosonic equation of the relative motion which follows from the action (3.1) reads

\[ D_\# D_\# X^i = \frac{1}{16} [[X^i, X^j], X^j] + 2i \Psi \gamma^i \Psi + 4i D_\# (\hat{E}_\#^+ \gamma^i \Psi) + i (\hat{E}_\#^+ \gamma^{ij})_q [\Psi_q, X^j]. \]  \hspace{0.5cm} (4.5)

The last two terms in the r.h.s. of (4.5), which contain contributions of the center of energy
Goldstone fermion \( \hat{\theta}^\alpha \) (through \( \hat{E}_\#^+ := D_\# \hat{\theta}^\alpha v^{\alpha q} = \partial_\tau \hat{\theta}^\alpha v^{\alpha q} / \hat{E}_\#^\tau \)) are absent, in the flat target
superspace case of equations from [15, 20]. The reason for such a difference is that the center of
energy motion is considered as a (worldline superspace) background in [15, 20] while in our present
action (3.1) it is treated on the same footing as the relative motion.

Due to the same reason, the approach of [15, 20] did not catch the backreaction of the relative
motion on the center of energy motion (which is also characteristic for the purely bosonic Myers
action [7]), one of the manifestation of which is the appearance of generically nonvanishing effective
mass of the mM0 system, Eq. (4.2), which we discussed above in the frame of purely bosonic
approximation.

To make the approach of [15, 20] accounting for the mutual influence of the center of energy and
relative motion, the basic center of energy superembedding equation, which was taken in [15, 20]
to be the same as in the single M0 case, should be modified by the terms constructed with the use
of the relative motion superfields. In the next section we will comment on one of the possible ways
to find such a modification.

4.3 \( M^2 = 0 \) as a BPS equation. Vanishing effective mass of all supersymmetric bosonic
solutions

Interestingly enough, all the supersymmetric bosonic solutions of the equations which follow from
the action (3.1) are characterized by the vanishing effective mass, \( M^2 = 0 \). Indeed, setting \( \Psi_q = 0 \)
we find from (3.9) the following Killing spinor equation

\[ \epsilon^p \mathcal{K}_{pq} := (\epsilon^q \gamma^i)_q \Psi^i - \frac{i}{8} (\epsilon^q \gamma^{ij})_q [X^i, X^j] = 0. \] \hspace{0.5cm} (4.6)

Its consistency condition \( \epsilon^p tr((\mathcal{K} \gamma^j)_{pq} \Psi^p) + \frac{i}{8} (\mathcal{K} \gamma^{jk})_{pq} [X^j, X^k] = 0 \) can be presented in the form
\( \epsilon^q \mathcal{H} = 0 \). Taking into account Eq. (4.2), this can be written as \( \epsilon^q M^2 = 0 \). Hence (one of) the
BPS equation(s) for supersymmetric bosonic solutions of mM0 equations is

\[ M^2 = 0 \quad \leftrightarrow \quad \mathcal{H}|_{\Psi=0} = \frac{1}{2} tr (P^i \Psi^i) - \frac{1}{64} tr [X^i, X^j]^2 = 0. \] \hspace{0.5cm} (4.7)

This fact is very important. It implies that a supersymmetric solution of the 11D supergravity
describing our mM0 system has the same property as the single M0–brane (M-wave) solution. In

\footnote{To this end one have to use Jacobi identities for commutators and the properties of 9d gamma matrices.}
other words, our result does not imply the existence of a new exotic supersymmetric solution of the 11D supergravity.

Furthermore, the form of the Hamiltonian of the relative motion $\mathcal{H}$, which essentially coincides with $M^2$, Eq. (4.2), indicates that all supersymmetric bosonic solutions have $P^i = 0$ and $[X^i, X^j] = 0$, i.e. that their relative motion sector is in its ground state.

5. Conclusion and discussion

In this paper we have presented the complete action for the multiple M0–brane (multiple M-waves or mM0) system in flat target 11D superspace. This action is Lorentz covariant, invariant under the global 11D target space supersymmetry and under the local 1d $\mathcal{N} = 16$ supersymmetry which acts on the center of energy variables like a deformed version of the $\kappa$-symmetry of the massless superparticle. We have also shown that the generic motion of the mM0 system is characterized by a nonnegative center of energy mass constructed from the relative motion variables so that the center of energy worldline is generically not light–like. On the other hand, we have shown that all the supersymmetric bosonic solutions of mM0 equations are characterized by vanishing effective mass, $M^2 = 0$.

The importance of this observation is related to the fact that supersymmetric extended objects, for instance M2 and M5-branes, can be described not only by worldvolume actions, but also by supersymmetric solutions of the appropriate (in this case 11D) supergravity equations. Indeed, if it were found that there existed a supersymmetric solution of mM0 equations (mM0 BPS state) with $M^2 \neq 0$, this would imply the existence of new exotic supersymmetric solution of the 11D supergravity equations. In contrast, our result that all the supersymmetric mM0 BPS states are massless, $M^2 = 0$, imply that the 11D supergravity solutions describing them are similar to a simple M-wave describing the single M0-brane (see [38] for discussion on this solution).

The detailed study of the equations of motion which follow from the multiple M-wave action (3.1) and search for their solutions will be the subject of the forthcoming paper [34]. Here we restrict ourself by commenting on the relation with [15, 20] in sec. 4.2 and on relation with [8] below.

The dimensional reduction of our action on a circle should be related to (the moving frame reformulation of) the mD0 action from [8]. The generalized mass term of the 10D model of [8] is defined by an arbitrary function $M_{10D} = M_{10D}(\bar{p}^i, \bar{X}^i, \bar{\Psi})$, while the dimensional reduction of our action gives more definite expression,

$$M_{10D}^2 = p_0^2 - p_1^2 - \ldots - p_9^2 = p_{10}^2 + 4\rho^4 \mathcal{H}.$$ (5.1)

Here $\mathcal{H} = \mathcal{H}_{\#\#\#}(\bar{p}^i, \bar{X}^i, \bar{\Psi}_q)$ is the Hamiltonian of the multiple M0 system, Eq. (3.3), and, as we discussed above, $4\rho^4 \mathcal{H} = M^2$ is a constant 11D effective mass of the mM0 system. Notice however that in our expression for $M_{10D}^2$ some freedom is still present: we have to choose the form of the momentum $p_{10}$ corresponding to the compactified direction. Thus the possibility to reproduce the counterpart of the mD0 model from [8] starting from our mM0 action is related with an exotic dimensional reduction defined with the use of the relative motion variables.

The study of dimensional reductions and the search for possible reformulation of our model without spinor moving frame variables are interesting problems for future study. On the other
hand, we intend to use our spinor moving frame action as a basis of the generalized moving frame principle [33] and to study the superfield equations which can be derived from it. This will produce such a deformation of the basic superembedding equation that will result in a modification of the dynamical equations from [20, 21] by terms describing the influence of the relative motion fields on the center of energy motion and vice versa.

The other important direction for future study is to elaborate the generalization of the action (3.1) for the mM0 system in curved 11D superspace. The form of equations in [20] suggests that, besides understanding the $E^a$ and $E^\alpha$, included in (3.1) inside of $E^\#, \hat{E}^\#$, and $\hat{E}^{+q}$, to be supervielbein of the 11D supergravity superspace (instead of (1.3)), one should also add an explicit interaction with the field strengths ("fluxes") of the 11D supergravity, $F_{abcd} = F_{[abcd]}(Z)$, $R_{abcd}(Z)$ and $T_{\alpha}^{\beta}(Z)$. The terms suggested by the equations from [20, 21] read $\Delta^{fluxes}S_{mM0} = \int_W \hat{E}^a (\hat{\rho}^\#)^3 L_{fluxes}^a$ with

$$L_{fluxes}^a = \frac{1}{4!} \hat{F}^{aijk} tr (X^i[X^j, X^k] + 4i \Psi^{ijk} \Psi) + \frac{1}{8} \hat{R}^{ai=j} tr (X^iX^j) + 2i \hat{T}^{ai=q} tr (X^i\Psi_q), \quad (5.2)$$

and $\hat{F}^{aijk} = F^{abcd}(\hat{Z}) u^i_c u^j_d u^k_a$, $\hat{R}^{ai=j} = R^{abcd}(\hat{Z}) u^i_c u^j_d$, $\hat{T}^{ai=q} = T^{\alpha\beta}(\hat{Z}) u^i_\alpha u^j_\beta$. The explicit presence of the flux superfields creates some difficulties in the calculations so that the question of whether $S_{mM0}[\text{Eq.(3.1)}] + \Delta^{fluxes}S_{mM0}$ gives a supersymmetric action for mM0 in the supergravity superspace, or some additional terms are needed to provide the local worldline supersymmetry (i.e. $\kappa$-symmetry), is still open. It is natural to begin with the cases of mM0 in vacuum superspaces where the fluxes acquire constant values; notice that the $AdS \times S$ and $pp$-wave superspaces are of this type.

Actually, the analogy with the bosonic multiple Dp-brane actions of [7, 36] suggests to expect the background superfields to depend on matrix coordinates. To do this in a covariant manner one must study a model involving, schematically, something like

$$E^a_M \left( \hat{Z}^N + \hat{X}^u u^a E^N_a (\hat{Z} + ...) + \hat{\Psi} q v^+ a E^N_a (\hat{Z} + ...) \right). \quad (5.3)$$

Although our moving frame and spinor moving frame variables seems to be useful in writing (5.3) and similar expressions, to deal with them is quite a difficult problem, see [37] for relevant studies. However, for the case of nearly coincident branes one can use the series decomposition of the background superfields near the center of energy of multiple brane system. After such a decomposition, instead of function of non-commuting coordinates, like (5.3), we will have the sum of polynomials in these matrix coordinates multiplied by derivatives of the background gauge (super)fields depending on the center of energy variables only. In an appropriate gauge these can be expressed in terms of the field strength (fluxes) and their covariant derivatives calculated at the center of energy 'position' in superspace. The straightforward search for curved superspace generalization of the action (3.1) corresponds to such a decomposition with a hope that probably this generically infinite series can be consistently truncated to a polynomial in matrix field with preservation of local supersymmetry. If this truncation does not occur, the above line might be stopped 'by hand'.

\[^5]\text{Notice that in our case the problem of coupling to 11D supergravity seems to be pure technical (although probably difficult). This differs from the problem of whether the Matrix model provides the description of the complete M-theory in some particular frame, discussed in [35].}
at some power ($\geq 4$) in a relative motion fields $X^i$ and $\Psi^i$, thus giving a weak field approximation to the action for mM0 system in 11D supergravity background.

Finally, probably the most intriguing question is whether it is possible to find a generalization of our mM0 action for the case of multiple M2–brane system. We should note that, although the search for the answer for this question does not promise to be simple, in particular in the light of the recent results in [39], however, neither it looks hopeless.

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