Wavelet Filtering in Shock Stochastic Systems with High Availability

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Abstract: For filtering problems in StSHA under nonGaussian ShD methodological and algorithmically WL support is developed. 3 types of filters are considered: KBF (WLKBF), LPF (WLLPF) and SOLF (WLSOLF). These filters have the following advantages: on-line regime, high accuracy and possibility of algorithmically description of complex ShD. Wavelet filter modifications are based on Galerkin method and Haar wavelet expansions. WLF unlike KBF, LPF and SOLF do not need to integrate system of ordinary differential Eqs. These filters must solve system of linear algebraic Eqs with constant coefficients. KBF (WLKBF) and SOLF (WLSOLF) are recommended for StSHA with additive ShD whereas LPF (WLLPF) are recommended for StSHA with parametric and additive ShD. Basic applications are: on-line identification and calibration of nonstationary processes in StSHA of ShD. Methods are illustrated by example of 3 dimensional differential linear information control system at complex ShD. Basic algorithms and error analysis for KBF (WLKBF) and LPF (WLLPF) are presented and illustrated filters peculiarities for small and finite damping. These filters allow to estimate the accumulation effects for systematic and random errors. Results may be generalized for filtration, extrapolation with interpolation problems in StSHA and multiple ShD.

Keywords: Kalman-Bucy WLF, Linear Pugachev WLF, Shock Disturbances, Stochastic Systems with High Availability (StSHA), Suboptimal Linear WLF, Wavelet Filtering (WLF)

1. Introduction

In series [1–6] methodological support for on-line express analysis of stochastic systems with high availability (StSHA) functioning at shock disturbances (ShD) was presented. Special attention was paid to wavelet methods and software tools. Wavelet modifications of Kalan-Bucy filters (WLKBF) for nonstationary linear StSHA at complex ShD were given and illustrated. Wavelet modifications of linear mean square (m.s.) conditionally optimal (Pugachev) filter (WLLPF) for StSHA with parametric ShD are presented and illustrated. Comparative computer results were described. Instrumental accuracy of WLKBF and WLLPF was considered.

Let generalize [1] for KBF, LPF and suboptimal linearized filters (SOLF) in case of non Gaussian ShD. Section 2 is dedicated to KBF and WLKBF. LPF and WLLPF are described in Section 3. In Section 4 SOLF based on linearization by known exact shock distributions are considered. Basic Propositions 1-5 are illustrated by 3 dimensional information control system at deterministic and stochastic ShD.

2. Kalman-Bucy Filters at Shock Disturbances

Kalman-Bucy Filter (KBF) for linear nonstationary StSHA is widely used for on-line analysis and synthesis problems. KBF is based on the following proposition [7–11].

Proposition 1. Let nonstationary differential StSHA being described by the following Eqs:
\[
\dot{X}_i = a_i^{(j)} + a_i^{(b)} X_i + V_i^{(b)}, \quad (1)
\]
\[
Z_t = Y_t = b_i X_i + V_2. \quad (2)
\]

Here \( \dot{X}_i, Y_t \) are states and observation vectors, \( V_i^{(b)} = V_i \) and \( V_2 \) are independent white noises (in strict sense) and nonGaussian in general case with intensity matrices \( V_i^{(b)} = v_i \) and \( v_2 \). Then at nonsingular observation noise \( \phi(t) \) and \( \beta_i \):

\[
\dot{X}_i = a_i^{(j)} + a_i^{(b)} \dot{X}_i + \beta_i (Z_t - h_i \dot{X}_i), \quad \dot{X}(0) = \dot{X}_0, \quad (3)
\]
\[
\beta_i = R_i h_i v_i^{-1}, \quad (4)
\]
\[
\dot{R}_i = a_i R_i + R_i a_i^T + v_i^{(b)} - \beta_i v_i^{(b)} \beta_i^T, \quad R(0) = R_0. \quad (5)
\]

where \( \dot{X}_i \) being mean error estimate of \( X_i \); \( R_i \) being mean covariance matrix; \( \beta_i \) being matrix amplifier.

**Remark 1.** Calculation of \( R_i \) and \( \beta_i \) does not need current observation and may be calculated a priori.

For getting Eqs for WLKBF let us exchange variable \( \phi(t) \) according to the following Eqs:

\[
T = \frac{t - t_0}{T - t_0}, \quad T \in [0,1], \quad (6)
\]
\[
\dot{\tilde{X}}(T) = \tilde{X}((T-t_0)T + t_0), \quad \dot{\tilde{X}}(0) = \dot{X}_0, \quad (7)
\]
\[
\dot{\tilde{X}}(T) = (T-t_0)[a_i^{(j)}((T-t_0)T + t_0) + 
+ \beta_i ((T-t_0)T + t_0)Z_t ((T-t_0)T + t_0)] \dot{\tilde{X}}(0) = \dot{X}_0, \quad (8)
\]

Define integrals \( p_i \) by formulae

\[
p_i(t) = \int_0^t \omega_i(s)ds, \quad (i = 1,2,...,L), \quad (15)
\]

\[
\omega_i(t) = \frac{t}{\sqrt{2^i}} \psi(t), \quad \psi(t) = \frac{t}{\sqrt{2^i}} \phi(t), \quad \phi(t) \text{ being scale function}; \quad \psi(t) \text{ mother wavelet} \quad (14)
\]

\[
c_{h i}(T) = \int_0^T \tilde{X}_h w_i d\tau. \quad (18)
\]

\[
\dot{\tilde{X}}_h(T) = \tilde{X}_{h0} + \sum_{k=1}^p \tilde{A}_{h k} \dot{\tilde{X}}_k. \quad (16)
\]

Let us expand the derivative \( \dot{\tilde{X}}_h \) into Haar series:

\[
\dot{\tilde{X}}_h(T) = \sum_{i=1}^L c_{h i} w_i, \quad (17)
\]

As a result, we have

\[
\dot{\tilde{X}}(T) = \tilde{A}_{hT} \dot{\tilde{X}}(T), \quad \dot{\tilde{X}}(0) = \dot{X}_0. \quad (10)
\]

**Remark 2.** Further we put \( t = T \).

Following [10–14] we introduce Haar wavelets \( w_i \) and integral of \( w_i \) using formulae

\[
w_i(t) = \phi(t) = \phi_{00}(t) = \begin{cases} 
1 & \text{at } t \in [0,1), \\
0 & \text{at } t \not\in [0,1), 
\end{cases} \quad (11)
\]
\[
w_2(t) = \psi(t) = \psi_{00}(t) = \begin{cases} 
-1 & \text{at } t \in [0,1/2), \\
0 & \text{at } t \not\in [0,1), 
\end{cases} \quad (12)
\]
\[
w_j(t) = \psi_{j k}(t) = \begin{cases} 
\sqrt{2^j} & \text{at } t \in [k/l,(k+0.5)/l), \\
-\sqrt{2^j} & \text{at } t \in [(k+0.5)/l,(k+1)/l), \\
0 & \text{at } t \not\in [k/l,(k+1)/l), 
\end{cases} \quad (13)
\]

Here \( \phi = \phi(t) \) being scale function; \( \psi = \psi(t) \) mother wavelet

\[
\psi_{j k} = \psi_{j k}(t) = \sqrt{2^j} \psi(2^j t - k), \quad (14)
\]

\[
k = 0,1,..,l-1; \quad l = 2^j; \quad j = 1,2,...,J; \quad L = 2 \times 2^j ; \quad i = l+k+1; \quad i = 3,4,...,L; \quad J \text{ being maximal level of wavelet resolution}.
\]

For every component \( \dot{\tilde{X}}_h \) \( (h = 1,2,...,p) \) for \( \dot{\tilde{X}}(T) \) Eq (10) gives the following expression:

\[
\dot{\tilde{X}}_h(T) = \dot{X}_{h0} + \sum_{k=1}^p A_{hk} \dot{\tilde{X}}_k. \quad (16)
\]
After projecting Eq (20) on basis $w_i$, taking into consideration $w_i$ orthonormality we come to $(L \times p)$ dimensional system of linear algebraic equations (SLAEq):

$$c_\alpha = \sum_{k=1}^{L} \sum_{j=1}^{k} c_{\alpha j} (\hat{A}_{\alpha j}, w_j) + \hat{X}_{\alpha 0} \sum_{j=1}^{k} (\hat{A}_{\alpha j}, w_j) \quad (s = 1, 2, ..., L).$$

(Putting $\rho_j = (\hat{A}_{\alpha j}, w_j) = \int \hat{A}_{\alpha j} w_j d\tau$,)

we rewrite Eqs (22) in the final form:

$$c_\alpha = \sum_{k=1}^{L} \sum_{j=1}^{k} c_{\alpha j} g_{\alpha j} + \hat{X}_{\alpha 0} \sum_{j=1}^{k} q_{\alpha j} + \rho_j \quad (s = 1, 2, ..., L).$$

Thus we have Proposition 2 [6].

**Proposition 2.** At conditions

(i) scalar functions $\hat{X}_h, \hat{X}_k, \hat{A}_{hk}, \hat{A}_{00}$ ($h, k = 1, 2, ..., p$) belong to space $L^2[0,1]$.

(ii) Haar functions $w_i, p_i$ are defined in space $L^2[0,1]$.

Then Eqs for WLKBF being (19) at conditions (24).

Note that Proposition 2 is the basis of corresponding algorithm for calculating m.s. estimation of state $\hat{X}_h$ vector described by Eqs (1), (2):

1) $R_t$ and $\beta_t$ off-line calculation according to Eqs (4), (5).

2) Off-line definition of Haar $w_i, p_i$ in space $L^2[0,1]$ according to Eqs (11)-(14) with maximal level of wavelet resolution $J$.

3) Off-line calculation of $p_i$ according to Eq (15).

4) Off-line reduction of Eq (3) to Eq (10).

5) Assign values of observation $Z(t)$ ($t \in [t_0, T]$) at points $t_j = t_0 + (j-1)\Delta t$ for $j = 1, 2, ..., L; \Delta t = (T-t_0)/(L-1)$.

6) On-line composition and solution SLAEq (24) for coefficients $c_\alpha$.

7) On-line calculation of m.s. estimate $\hat{X}_h$ for every component $\hat{X}_h$.

8) On-line transition from $\hat{X} \in [1,0], t \in [1,0]$ and calculation of m.s. estimate $\hat{X}(t)$ according to formula $\hat{X}(\tau) = \hat{X}((\tau-t_0)\tau+t_0)$.

### 3. Linear Pugachev Filter at Shock Disturbances

Let us consider the following StSHA described by Eqs with shock parametric noises:

\begin{align*}
\dot{X}_t &= a_{00} + a_1 X_t + a_2 Y_t + \left(c_{10} + \sum_{k=1}^{N} c_{k1} Y_t + \sum_{k=1}^{N} c_{n_1, n_2 r_1 r_2} X_t \right) V, \quad X(t_0) = X_0, \\
\dot{Y}_t &= b_{00} + b_1 Y_t + b_2 X_t + \left(c_{20} + \sum_{k=1}^{N} c_{21} Y_t + \sum_{k=1}^{N} c_{2n_1, n_2 r_1 r_2} X_t \right) V, \quad Y(t_0) = Y_0,
\end{align*}

where $V$ being vector white noise. In this case LPF is defined by the following proposition [6].

**Proposition 3.** Let StSHA at ShD is describe by Eqs:

$$\dot{\hat{X}}_t = a_{00} + a_1 \hat{X}_t + a_2 Y_t + \beta_t \left[Z_t - (b_{00} + b_1 Y_t + b_2 \hat{X}_t) \right], \quad \hat{X}(t_0) = \hat{X}_0.$$  

Probabilistic moments of first and second order of $[Y_t, X_t, X_0, X_n, X_{n_k}]^T$ satisfy the following Eqs:

$$m_t = a_{00} + a_1 m_t, \quad m_0 = m_0,$$

\begin{align*}
\dot{K}_t &= \alpha_t K_t + K_t \alpha_t^T + \sigma_{00} \sigma_{00}^T + \sum_{j=1}^{N} (\sigma_{0j} \sigma_{0j}^T + \sigma_{1j} \sigma_{1j}^T) m_{0j} + \sum_{j=1}^{N} \sigma_{2j} \sigma_{2j}^T (m_{1j} m_{2j} + K), \quad K_{t_0} = K_0,
\end{align*}
where

$$\varpi_t = \begin{bmatrix} h_t \\ a_t \\ b_t \end{bmatrix}, \quad \varpi_{0t} = \begin{bmatrix} b_{0t} \\ a_{0t} \end{bmatrix}, \quad \varpi_{ret} = \begin{bmatrix} c_{20t} \\ c_{10t} \end{bmatrix}$$  \hspace{1cm} (30)$$

Error covariance matrix $R_t$ satisfy Eq

$$R_t = \begin{bmatrix} a_t R_t + R_t a_t - \left[ R_t b_t \right]^2 \left\{ c_{20t} + \sum_{r=1}^{n_r} c_{2rt} \right\} \times \begin{bmatrix} c_{20t} + \sum_{r=1}^{n_r} c_{2rt} \end{bmatrix} + \sum_{r,s=1}^{n_r} c_{10r} c_{10s} K_{rs} \right\} \times \begin{bmatrix} c_{20t} + \sum_{r=1}^{n_r} c_{2rt} \end{bmatrix} + \sum_{r,s=1}^{n_r} c_{10r} c_{10s} K_{rs},$$  \hspace{1cm} (31)$$

where $a_{11t}$ and $\beta_t$ as follows

$$a_{11t} = \begin{bmatrix} c_{20t} + \sum_{r=1}^{n_r} c_{2rt} \end{bmatrix} \times \begin{bmatrix} c_{20t} + \sum_{r=1}^{n_r} c_{2rt} \end{bmatrix} + \sum_{r,s=1}^{n_r} c_{20r} c_{20s} K_{rs},$$  \hspace{1cm} (32)$$

$$\beta_t = \begin{bmatrix} R_t b_t + \sum_{r=1}^{n_r} c_{2rt} \end{bmatrix} \times \begin{bmatrix} c_{20t} + \sum_{r=1}^{n_r} c_{2rt} \end{bmatrix} + \sum_{r,s=1}^{n_r} c_{10r} c_{10s} K_{rs},$$  \hspace{1cm} (33)$$

Remark. 3. LPF as KBF does not depend on current observations and the basic calculations may be performed a priori. For WLLPF we have the following Eqs [6]:

$$\tilde{X}(T) = \tilde{X}_0 + \tilde{A}_T \tilde{X}(T), \quad \tilde{X}(T) = \tilde{X}_0(T),$$  \hspace{1cm} (34)$$

$$\tilde{X}_h = \tilde{A}_h + \sum_{k=1}^{L} \tilde{A}_{hk} \tilde{X}_k,$$  \hspace{1cm} (35)$$

$$\tilde{X}_h = \sum_{j=1}^{L} \tilde{c}_{hi} \tilde{p}_j + \tilde{X}_{oh},$$  \hspace{1cm} (36)$$

$$\tilde{c}_{hi} = \sum_{k=1}^{L} \tilde{c}_{hi} \tilde{k}_{is} + \tilde{X}_{oh} \sum_{j=1}^{L} \tilde{q}_{ks} \tilde{b}_{s} (h = 1, 2, ..., n_x, s = 1, 2, ..., L),$$  \hspace{1cm} (37)$$

$$\tilde{A}_{hk} \tilde{p}_j = \sum_{j=1}^{L} \tilde{g}_{kj} \tilde{w}_j, \quad \tilde{A}_{h0} + \sum_{j=1}^{L} \tilde{g}_{kj} \tilde{w}_j, \quad \tilde{A}_{h0} = \int_0^T \tilde{A}_{h0} \tilde{w}_j d\tau, \quad \tilde{A}_{h0} = \int_0^T \tilde{A}_{h0} \tilde{w}_j d\tau,$$  \hspace{1cm} (38)$$

$$\tilde{\rho}_j = \int_0^T \tilde{\rho}_j \tilde{w}_j d\tau, \quad \tilde{\rho}_j = \int_0^T \tilde{\rho}_j \tilde{w}_j d\tau, \quad \tilde{\rho}_j = \int_0^T \tilde{\rho}_j \tilde{w}_j d\tau,$$  \hspace{1cm} (39)$$

$$\tilde{A}_0(T) = (T - t_0) [a_{0t} (T - t_0) + t_0] + \tilde{B}_t ((T - t_0) + t_0) \tilde{Z}_t ((T - t_0) + t_0), \quad \tilde{A}(T) = (T - t_0) [a_t - \tilde{B}_t ((T - t_0) + t_0) b_t]$$  \hspace{1cm} (40)$$
\[
\hat{R}_T = a_1 R_T + R_T a_1^T - \left[ R_T b_1^T + \left( c_{10} + \sum_{r=1}^{n_r+n_s} c_{1r}m_r \right) \right] \times \hat{v}_T \left[ c_{20}^T + \sum_{r=1}^{n_r+n_s} c_{2r}^T m_r + c_{2r} \hat{v}_T c_{2r}^T K_{r} \right] \times \hat{a}_{11}^{-1} 
\]
\[
\times \left[ R_T b_1^T + \left( c_{20} + \sum_{r=1}^{n_r+n_s} c_{2r}m_r \right) \right] \hat{v}_T \left[ c_{10}^T + \sum_{r=1}^{n_r+n_s} c_{1r}^T m_r + \sum_{r,s=1}^{n_r+n_s} c_{1s} \hat{v}_T c_{2r}^T K_{r} \right] 
\]

\[
\tilde{\beta}_r = R_T b_1^T + \left( c_{10} + \sum_{r=1}^{n_r+n_s} c_{1r}m_r \right) \hat{v}_T \left[ c_{20}^T + \sum_{r=1}^{n_r+n_s} c_{2r}^T m_r + \sum_{r,s=1}^{n_r+n_s} c_{1s} \hat{v}_T c_{2r}^T K_{r} \right] \times \hat{a}_{11}^{-1}, \quad \tilde{\alpha}_{11}^{-1} = \left( c_{20} + \sum_{r=1}^{n_r+n_s} c_{2r}m_r \right) \hat{v}_T \left[ c_{20}^T + \sum_{r=1}^{n_r+n_s} c_{2r}^T m_r + \sum_{r,s=1}^{n_r+n_s} \hat{c}_{1s} \hat{v}_T c_{2r}^T K_{r} \right] 
\]
\[
\hat{m}_T = a_0 \hat{m}_T, \quad \hat{m}(0) = m_0, \quad \hat{m}(t) = \hat{m}_T, \quad t \in [t_0, T]. \quad \hat{m}_T = \hat{a} \hat{K}_T + \hat{K}_T a_T^T + \sum_{r=1}^{n_r+n_s} \left( \hat{c}_{2r} \hat{v}_T c_{2r}^T + \hat{c}_r \hat{v}_T c_{0r}^T \right) \hat{m}_T + \sum_{r,s=1}^{n_r+n_s} \hat{c}_{rs} \hat{v}_T c_{rs}^T \left( \hat{m}_T \hat{m}_s + \hat{K}_{rs} \right), \quad K(0) = K_0, \quad (45) \]

Here we use wave for functions depending on dimensionless time \( \hat{t} \).

Using Proposition 3 in case \( a_1 = 0, b_1 = 0, b_0 = 0 \) for Eqs (25)-(27) we get the following proposal [6].

**Proposition 4.** At conditions of Proposition 3 and conditions:

(iii) scalar functions \( \hat{X}_h, \hat{X}_h, \hat{A}_h, \hat{P}_l, \hat{A}_h^{00} \) belong to space \( L^2[0,1] \); 

(iv) Haar functions \( w_j, p_i \) belling to space \( L^2[0,1] \), WLLPF is defined by Eqs (36), (37).

From Proposition 4 the corresponding algorithm follow.

1) Off-line reduction of Eqs (25), (2) to dimensionless form.

2) Off-line calculation of probability moments \( \hat{m}_T \) and \( \hat{K}_T \) of random vector \( \left[ \hat{Y}_1 \ldots \hat{Y}_n \hat{X}_1 \ldots \hat{X}_n \right]^T \) by integrating Eqs (44), (45).

3) Off-line calculation of error covariance matrix \( \hat{R}_T \) by Eq (41) integrating.

4) Off-line calculation of parameter \( \hat{\alpha}_{11}^{-1} \) and optimal amplifier \( \hat{\beta}_r \) according to formulae (42) and (43).

5) Off-line introduction in space \( L^2[0,1] \) Haar wavelet defined by Eqs (11)-(14) with maximal level of wavelet resolution \( J \).

6) Off-line calculation \( p_i \) for Haar wavelets according to formulae (15).

7) On-line assign values \( Z_i \) (\( t \in [t_0, T] \)) in points \( t_j = t_0 + (j-1) \Delta t \) for \( j = 1,2,\ldots,L \); \( L = 2^J \); \( t \in [t_0, T] \).

8) On-line composition and solution SLAEq (37) for determination of coefficients \( \xi_{ih} \).

9) On-line computation of error m.s. estimation \( \hat{X}_h \) for every component \( \hat{X}_i \) according to formula (36).

10) On-line transition from \( \hat{t} \in [0,1] \) to \( t \in [t_0, T] \)

4. **Suboptimal Linearized Filters at Shock Disturbances**

If the noise \( V \) in Eqs (25)-(26) is autocorrelated and connected with white noise \( V^{sh} \) by linear Eqs of shape filter such StSHA with parametric ShD during small sock time comparable with time of StSHA inertia are reduced to bilinear StSHA of the form [7–10]:
For given exact distribution (ED) and using equivalent linearization of bilinear functions \( E_{ED} X_h N_{0h} \), we reduce Eqs (47) to Eqs for KBF (Section 2) and for WLKBF for the composed vector \( U_{i} = [X^T Y^T N_{i}^{wh} T] \) (Section 3). So we have the following Proposition.

Proposition 5. Let bilinear SiSHA (47) at known exact distribution of ShD may be reduced to linear autoregressive stochastic system with additive noises. Then KBF and WLKBF are defined by Eqs of Proposition 1 and 2 for the corresponding state vector \( U_{i} \).

5. Example

At first let us consider KBF for the following system:

\[
\begin{align*}
\dot{X}_i &= a_{0i} + a_{1i} X_i + a_{2i} Y_i + \left( c_{0i} + \sum_{r=1}^{n_w} c_{r1} Y_r + \sum_{r=1}^{n_v} c_{1,r+1} X_r \right) N_{ih}, \quad X(t_0) = X_0, \\
\dot{Y}_i &= b_{0i} + b_{1i} Y_i + b_{2i} \dot{X}_i + \left( c_{20i} + \sum_{r=1}^{n_w} c_{2r1} Y_r + \sum_{r=1}^{n_v} c_{2,r+1} X_r \right) N_{ih}, \quad Y(t_0) = Y_0,
\end{align*}
\]

where \( X = [X_1 X_2 X_3]^T \) and \( Y = [0 Y_2 0]^T \) are state and observation vectors, \( N_{ih} \) and \( N_{ih} \) are scalar independent Gaussian noises with intensities \( \nu_{ih} \) and \( \nu_{ih} \). Note that in case of Eq (48) KBF and LPF coincide. So using Proposition 1 we have the following vector Eqs for KBF:

\[
\dot{X}_i = a_{0ih} + \beta_i Z_i + (a_{ih} - \beta_i b_i) \dot{X}_i, \quad \dot{X}_i(t_0) = \dot{X}_0. \tag{49}
\]

So we get the final Eqs for KBF:

\[
\begin{align*}
\dot{X}_1 &= [1+(1/v_2) R_{12}] \tilde{X}_2 - (1/v_2) R_{12} Z_2, \\
\dot{X}_2 &= \tilde{X}_1 + (1/v_2) R_{22} \tilde{X}_2 - (1/v_2) R_{23} Z_2, \\
\dot{X}_3 &= S + n_{ih} - \omega_v^2 \left( \tilde{X}_1 - (1/v_2) \tilde{X}_2 \right) \times R_{22} \tilde{X}_2 - (1/v_2) R_{23} Z_2.
\end{align*}
\]

Secondly we use Proposition 2 and notations

\[
\begin{align*}
\hat{X}_i &= A_0 + A X_i, \quad \hat{X}(t_0) = \hat{X}_0, \\
A_0 &= [A_{00}]_{h,k} = [-1/(1/v_2) R_{12} Z_2, \\
A &= [A_{h,k}] = \begin{bmatrix}
0 & 1/(1/v_2) R_{12} & 0 \\
0 & -(1/v_2) R_{22} & 0 \\
0 & -(1/v_2) R_{23} & 0
\end{bmatrix}, \quad \hat{X}_0 = \hat{A}_0 + \hat{A} \hat{X}, \quad \hat{X}(0) = \hat{X}_0,
\end{align*}
\]

Eq (52) may be written in scalar form
\[ A = \begin{bmatrix} 0 & 1 + (1/v_2)R_{12} & 0 \\ 1 & 0 & (1/v_2)R_{23} \end{bmatrix} \]  \hspace{1cm} (60)

So we get the final WLKBF Eqs:

\[
\begin{align*}
R_1 &= (T - t_0)(2R_{12} - (1/v_2)R_{21}) + R_{10}; \\
R_{12} &= (T - t_0)(R_{12} - (1/v_2)R_{21} - 2 + (1/v_2)R_{22}) - (1/v_2)R_{23} = R_{120}; \\
R_3 &= (T - t_0)(R_{31} + R_{23} - (1/v_2)R_{23}R_{31}) = R_{120}; \\
R_{23} &= (T - t_0)(R_{12} - 2 + (1/v_2)R_{23}) - (1/v_2)R_{23} = R_{230}; \\
R_{33} &= (T - t_0)(2R_{31} - (1/v_2)R_{23}) = R_{330}; \\
R_{13} &= (T - t_0) = R_{130};
\end{align*}
\]  \hspace{1cm} (61)

\[ \hat{x}_k = \sum_{k=1}^{n} \sum_{s=1}^{L} c_{ks}R_{ks}, \]  \hspace{1cm} (62)

\[ c_{ks} = \sum_{l=1}^{n} \sum_{s=1}^{L} c_{kl}R_{kl} + \hat{x}_k \sum_{s=1}^{L} q_{ks} + p_{ks}, \]  \hspace{1cm} (63)

Computer experiments “Figures 1-15” were realized for the following values of parameters: \[ \omega = 1; \ S = 1; \ v_2 = 1, \ j = 5 \]
and \[ \varepsilon = 0.7; \ \varepsilon = 0.1. \]
The following variants of ShD were considered.

1) Deterministic ShD with \[ \nu_{t}^{sh} \] and stochastic ShD with \[ \nu_{t}^{sh} \] at \[ t_{sh} = 2.1875, \ \Delta n = 3, \ \Delta t = 3. \]

\[ \nu_{t}^{sh} = \begin{cases} \Delta n, & \text{at } t \in [t_{sh}; t_{sh} + \Delta t], \\
0, & \text{at } t \not\in [t_{sh}; t_{sh} + \Delta t] \end{cases}, \]

\[ \nu_{t}^{sh} = \begin{cases} \Delta n, & \text{at } t \in [t_{sh}; t_{sh} + \Delta t], \\
0, & \text{at } t \not\in [t_{sh}; t_{sh} + \Delta t] \end{cases}. \]

2) Deterministic ShD \[ \nu_{t}^{sh} \] and stochastic ShD \[ \nu_{t}^{sh} \] at \[ t_{sh} = 2.1875 \]

\[ \nu_{t}^{sh} = \begin{cases} 10, & \text{at } t \in [t_{sh}; t_{sh} + 5], \\
0, & \text{at } t \not\in [t_{sh}; t_{sh} + 5] \end{cases}, \]

\[ \nu_{t}^{sh} = \begin{cases} 6, & \text{at } t \in [t_{sh}; t_{sh} + 7], \\
0, & \text{at } t \not\in [t_{sh}; t_{sh} + 7] \end{cases}. \]

3) Deterministic ShD \[ \nu_{t}^{sh} \] and stochastic ShD \[ \nu_{t}^{sh} \] at \[ t_{sh} = 0.135, \]

\[ \nu_{t}^{sh} = \begin{cases} 10, & \text{at } t \in [t_{sh}; t_{sh} + 1], \\
0, & \text{at } t \not\in [t_{sh}; t_{sh} + 1] \end{cases}, \]

\[ \nu_{t}^{sh} = \begin{cases} 6, & \text{at } t \in [t_{sh}; t_{sh} + 1], \\
0, & \text{at } t \not\in [t_{sh}; t_{sh} + 1] \end{cases}. \]
Figure 3. Plot realization of $X_3$ and its estimation (variant 1) for $\varepsilon = 0.7$ (a) and $\varepsilon = 0.1$ (b).

Figure 4. Filter error variance plots $R_{11}, R_{22}, R_{33}$ (variant 1) for $\varepsilon = 0.7$ (a) and $\varepsilon = 0.1$ (b).

Figure 5. Filter error variance plots $R_{12}, R_{13}, R_{23}$ (variant 1) for $\varepsilon = 0.7$ (a) and $\varepsilon = 0.1$ (b).

Figure 6. Plot realization of $X_1$ and its estimation (variant 2) for $\varepsilon = 0.7$ (a) and $\varepsilon = 0.1$ (b).
Figure 7. Plot realization of $X_2$ and its estimation (variant 2) for $\epsilon = 0.7$ (a) and $\epsilon = 0.1$ (b).

Figure 8. Plot realization of $X_3$ and its estimation (variant 2) for $\epsilon = 0.7$ (a) and $\epsilon = 0.1$ (b).

Figure 9. Filter error variance plots $R_{11}, R_{22}, R_{33}$ (variant 2) for $\epsilon = 0.7$ (a) and $\epsilon = 0.1$ (b).

Figure 10. Filter error variance plots $R_{12}, R_{13}, R_{23}$ (variant 2) for $\epsilon = 0.7$ (a) and $\epsilon = 0.1$ (b).
Figure 11. Plot realization of $X_1$ and its estimation (variant 3) for $\epsilon = 0.7$ (a) and $\epsilon = 0.1$ (b).

Figure 12. Plot realization of $X_2$ and its estimation (variant 3) for $\epsilon = 0.7$ (a) and $\epsilon = 0.1$ (b).

Figure 13. Plot realization of $X_3$ and its estimation (variant 3) for $\epsilon = 0.7$ (a) and $\epsilon = 0.1$ (b).

Figure 14. Filter error variance plots $R_{11}, R_{22}, R_{33}$ (variant 3) for $\epsilon = 0.7$ (a) and $\epsilon = 0.1$ (b).
6. Conclusion

For filtering problems in StSHA under nonGaussian ShD methodological and algorithmically WL support is developed. 3 types of filters are considered: KBF (WLKBF), LPF (WLLPF) and SOLF (WLSOLF). These filters have the following advantages: on-line regime, high accuracy and possibility of algorithmically description of complex ShD. Wavelet filter modifications are based Galerkin method and Haar wavelet expansions. WLF unlike KBF, LPF and SOLF do not need to integrate system of ordinary differential Eqs. These filters must solve system of linear algebraic Eqs with constant coefficients. KBF (WLKBF) and SOLF (WLSOLF) are recommended for StSHA with additive ShD whereas LPF (WLLPF) are recommended for StSHA with parametric and additive ShD.

Basic applications are on-line identification and calibration of nonstationary processes in StSHA at ShD. Methods are illustrated by example of 3 dimensional differential linear information control system at complex ShD. Basic algorithms and error analysis for KBF (WLKBF) and LPF (WLLPF) are presented on 15 figures illustrate filters popularities for small and big damping. These filters allow to estimate the accumulation effects for systematic and random errors.

Results may be generalized for filtration, extrapolation and interpolation problems in StSHA with multiple ShD.

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References

[1] Sinitsyn I. N., Sergeev I. V., Korepanov E. R., Konashenkova T. D. Software tools for analysis and synthesis of stochastic systems with high availability (IV). Highly Available Systems. 2017. V. 13. № 3. P. 55–69.

[2] Sinitsyn I. N., Sergeev I. V., Korepanov E. R., Konashenkova T. D. Software tools for analysis and synthesis of stochastic systems with high availability (V). Highly Available Systems. 2018. V. 14. № 1. P. 59–70.

[3] Sinitsyn I. N., Sergeev I. V., Korepanov E. R., Konashenkova T. D. Software tools for analysis and synthesis of stochastic systems with high availability (VI). Highly Available Systems. 2017. V. 14. № 2. P. 40–56.

[4] Sinitsyn I. N., Zykov D. V., Korepanov E. R., Konashenkova T. D. Software tools for analysis and synthesis of stochastic systems with high availability (VII). Highly Available Systems. 2019. V. 15. № 1. P. 47–61.

[5] Sinitsyn I. N., Zykov D. V., Korepanov E. R., Konashenkova T. D. Software tools for analysis and synthesis of stochastic systems with high availability (VIII). Highly Available Systems. 2019. V. 15. № 1. P. 62–69.

[6] Sinitsyn I. N., Sinitsyn V. I., Korepanov E. R., Konashenkova T. D. Software tools for analysis and synthesis of stochastic systems with high availability (X). Highly Available Systems. 2020. V. 16. № 4. P. 24–39. DOI: 10.18127/j20729472-202004-02 (In Russian).

[7] Pugachev V. S., Sinitsyn I. N. Stochastic Differential Systems. Analysis and Filtering, John Wiley & Sons, Chichester, 1987.

[8] Pugachev V. S., Sinitsyn I. N. Stochastic Systems. Theory and Applications, World Scientific, Singapore, 2001.

[9] Socha L. Linearization Methods for Stochastic Dynamic Systems. Lect. Notes Phys. 730. Springer, Berlin Heidelberg, 2008.

[10] Sinitsyn I. N. Kalman and Pugachev Filtering, Torus Press, Moscow, 2005, 1st ed; 2007, 2nd ed.

[11] Heil C., Walnut D. F. Fundamental Papers on Wavelet Theory. Princeton University Press, Princeton, New-Jersey, 2006.

[12] Percival D. B., Walden A. T. Wavelet Methods for Time Series Analysis. Cambridge University Press, Cambridge, 2000.

[13] Gagnon L., Lina J. M. (1994). Symmetric Daubechies' wavelets and numerical solutions of NLS2 equations. J. Phys. A: Math. Gen. 27: 8207–8230.

[14] Xu J., Shann W. (1992). Galerkin-wavelet methods for two point value problems. Number. Math. 63: 123–144.
[15] Lepik U. (2005). Numerical solution of differential equations using Haar wavelets. Mathematics and Computers in Simulation, 68: 127–143.

[16] Nason G. P. Wavelet Methods in Statistics with R, 2008, Springer Science Business Media, LLC.