The Universal Composite Fermion Hall Conductance at $\nu = 1/2$

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We show that at electronic filling factor $\nu = 1/2$, the lowest Landau level constraint implies that at any temperature, and in the presence of any amount of particle-hole symmetric disorder, the composite fermion Hall conductivity is precisely $-e^2/2h$. (The electronic Hall conductivity is $e^2/2h$.) This is inconsistent with the response of a composite Fermi liquid in zero effective magnetic field. We also examine perturbatively the nature of the putative composite Fermi liquid in the case in which the bare particles are anyons with “nearly” Fermi statistics.

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The observation of a seemingly metallic DC magneto-transport and the subsequent discovery of an acoustic wave anomaly near $\nu = 1/2$, opened a new chapter in the studies of quantum Hall effects. (Here $\nu \equiv \phi_0 \rho/B$, where $\rho$ is the mean electron density, $\phi_0 = \hbar c/e$, and $B$ is the externally applied magnetic field.) A very intriguing idea, the composite fermion theory, has been put forward to explain these phenomena. In this theory, each electron is represented as a composite-fermion carrying two quanta of fictitious magnetic flux which pierce the physical plane in the direction opposite to that of the real magnetic flux. Formally, this transformation maps the system of electrons in a strong magnetic field onto a system of “composite fermions” moving in the same external field while interacting with a fluctuating “statistical” gauge field governed by a Chern-Simons action. At the mean-field level, the averaged statistical magnetic field, $\tilde{b} = 2 \phi_0 \rho = |B|$, cancels the external one, and the composite-fermions see no net field. It has been argued that the transport properties of the electrons near $\nu = 1/2$ simply reflect the underlying Fermi liquid (or, possibly, the marginal Fermi liquid) behavior of the composite fermions in zero magnetic field. This intriguing picture acquired further support when Fermi-surface-like features were observed in recent experiments.

Despite these successes, there are concerns about the composite Fermi liquid theory. In part, these stem from the fact that when one tries to improve upon the mean-field theory (MFT) by including the fluctuations of the statistical magnetic field, one encounters divergences. Attempts to sum these divergences have led to suggestive, but so far inconclusive results. The purpose of the present paper is to reexamine the Fermi liquid picture when the cyclotron frequency, $\omega_c = eB/mc$, is so large that the low energy states of the electrons lie primarily in the lowest Landau level. Our findings are as follows:

1. At $\nu = 1/2$, and in the presence of particle-hole symmetric external potentials, the projected Hamiltonian in the lowest Landau level has p-h symmetry. In the limit $\omega_c \to \infty$, and assuming that this symmetry is not spontaneously broken, we have shown that the electron Hall conductivity is temperature independent, with value $\sigma_{xy} = e^2/2h$. So long as $\rho_{xx} \neq 0$, this result further implies that the composite fermion Hall conductivity is

\[ \sigma_{xy}^f = -e^2/2h. \] (1)

We take Eq.(1) as implying the inadequacy of a zero field composite Fermi liquid in describing the system.

For finite $\omega_c$ and in the absence of disorder, we consider the more general problem in which the bare particles have fractional statistics, $\theta$, defined so that they can be viewed as composite fermions with $\theta$ fictitious flux quanta tied to each. (See Eqs.(3) and (4)) Within this class of models, the problem of physical interest corresponds to $\theta = 2$, while the problem is simple in the limit $\theta \to 0$ with $\alpha \equiv \hbar \omega_c/E_c$ fixed. (Here $E_c = e^2/2c\sqrt{\pi}/\alpha$, is the typical strength of the Coulomb interaction.) In that limit, the composite fermions are the bare electrons, and MFT is exact. Here, we consider only the case in which the magnetic field satisfies the commensurability condition $B = \tilde{b}/\alpha \phi_0$, so that the net effective magnetic field seen by the composite fermions is zero. Therefore, for finite $\omega_c$, the lowest Landau-level projection means taking the limit $\alpha \to \infty$. For all $\alpha > 0$, and $\theta \to 0$, we have computed corrections to the MFT, and the results are as follows.

2. To the lowest order in $\theta$ the perturbative contributions (see Fig.1) to $\lim_{q \to 0} \sigma_{xy}^f(q)$ is zero. Thus the fact that the composite fermion system lacks time-reversal symmetry (Eq.(1)) is not manifested to $O(\theta)$.

3. It is well known that to lowest order in $\theta$, the correction to the composite fermion self-energy due to longitudinal gauge fluctuations diverges logarithmically with the size of the system. It was pointed out that this arises from virtual excitations across the energy gap $\hbar \omega_c$. We have shown that to the same order, and for any values of $\tilde{k}$ and $\omega$, there are no divergent contributions to the density-density correlation function. We tentatively interpret this result as implying a confinement phenomenon: only excitations which are “statistical charge
neutral”, such as particle-hole excitations, are physical, while single composite fermion excitations are not part of the physical spectrum!

4. It is also well known that there are singular corrections to the composite fermion self-energy in the limit $\omega \to 0$ and $|\vec{k}| \to k_F$ due to transverse gauge fluctuations, and these have been the focus of much of the work in the field. However, if the single-composite fermion is unphysical, the question arises whether this singularity appears in any physical correlation function. Strikingly, Kim et al showed that in an $1/N$ expansion, there is no singular corrections to the density-density and current-current correlation in the long wave-length limit. There do, however, remain singular corrections to the $|\vec{q}| = 2k_F$ density-density correlation function due to the transverse gauge fluctuations. To the lowest order in $\alpha\theta$, we have found that

$$\Delta \Pi(\omega, 2k_F)/\Delta \Pi_0(\omega, 2k_F) = 1 + \alpha\theta C_1 \ln \left( \frac{E_F}{\omega} \right). \quad (2)$$

Here $\Pi(\omega, q) \equiv \int d^3x dt e^{i(\omega t - \vec{q} \cdot \vec{x})} \rho(x, t)\rho(0, 0)$ with $\Pi_0$ being the density-density correlation function of free electrons, $\Delta \Pi(\omega, q) = \Pi(\omega, q) - \Pi(0, 0)$, and $C_1 = -\frac{1}{4}(1 + \ln(\frac{\omega}{\theta}^2))$. The above result shows that a small $\alpha$, hence strong Landau level mixing, suppresses the amplitude of the singular corrections.

In sum, at finite $\omega_c$, while the perturbative results cast doubts on the validity of a composite Fermi liquid description, they also suggest that observation of this breakdown may be difficult not only because i) the lack of time reversal symmetry of composite fermions does not show up in low order perturbation theory, but also due to: ii) The density-density and current-current correlation functions do not show any anomalous behavior for $q \neq 2k_F$. iii) At $q = 2k_F$ the anomaly in the density-density correlation function is a very weak one (see Eq. (3)). For $\theta = 2$ (i.e., the case of physical interest), Eq. (1) implies that the composite fermions can not form a Fermi liquid at $\alpha \to \infty$. If the system is a composite Fermi liquid for small $\alpha$, there must be a quantum phase transition at a critical $\alpha_c$. In the following we elucidate on some technical details, and expand on several of the results:

The Model: In units where $\hbar = k_B = 1$, the Euclidean Lagrangian for the composite fermion is given by

$$L[\tilde{\psi}, \psi, a] = \int d^2x \tilde{\psi}(\partial_0 - ieA_0 + ia_0)\psi - \frac{i}{2m} \int d^2x \tilde{\psi}i \gamma^0 \nabla - i\frac{e}{c} \vec{A} \gamma^\mu + ia\vec{A})^2 \psi + L_\alpha[a], \quad (3)$$

where

$$L_\alpha = -\frac{i}{4m \theta} \int d^2x d^2t \lambda_{\alpha\mu} \partial_\mu a_\lambda + \frac{1}{8\pi^2 \theta^2} \int d^2xd^2x' [b(x, t) - \tilde{b}](x - x') [b(x', t) - \tilde{b}]. \quad (4)$$

In the above $\tilde{\psi}$ and $\psi$ are the Grassmann fields associated with the composite fermions; $A_\mu$ and $a_\mu$ are the external and statistical gauge fields respectively; $m$ is the electron bare effective mass; $b = \nabla \times \vec{a}$; $V(x - x')$ is the bare interaction between electrons; $\tilde{b} = 2\pi\theta \rho$ is the averaged statistical magnetic field. Moreover, we have made use of the Chern-Simons constraint that $b(x, t) = 2\pi\theta \rho(x, t)$. By rescaling space, time, and the fermion fields, so that $x \to k_F x, t \to tk_F^2/m$, and $\psi, \tilde{\psi} \to k_F^{-1} \psi, k_F^{-1} \tilde{\psi}$, (here $k_F \equiv \sqrt{\rho/\pi}$) one can easily prove that in Eq. (3) and (4) the only dimensionless parameters are $\theta$ and $\alpha$.

An important ingredient of the above Chern-Simons formulation is the relation between the bare particle, and composite fermion correlation functions. It is the nature of the mapping that the density of composite fermions equals that of the bare particles, but the relation between current operators is more complicated. To compute the bare particle current-current correlation function, we need to string together the composite fermion “irreducible bubbles” (10) using the bare gauge propagator $<\alpha_\rho \alpha_\nu >$ computed from Eq. (4). As shown in Refs. this results in the following relations between the resistivity tensor of bare particle $\rho_{\alpha\beta}$ and that of the composite fermion $\rho^f_{\alpha\beta}$:

$$\rho_{xx} = \rho^f_{xx} \frac{\hbar}{\epsilon^2} + \rho^f_{yx} \frac{\hbar}{\epsilon^2}. \quad (5)$$

Physically, this expresses the fact that associated with the composite fermion current, there is a statistical flux current, which produces a corresponding EMF proportional to $\theta$ times the electrical current.

In Ref. Stern and Halperin defined the “(marginal) composite Fermi liquid” by requiring the irreducible bubbles to be that of a (marginal) Fermi liquid in zero magnetic field. Among other things, this assertion implies that $\sigma_{xy}^f = 0$. For subsequent discussions, we take this as a working definition of a (marginal) composite Fermi liquid. This is a reasonable definition since, after all, the idea of the composite fermion approach is to describe the behavior of electrons at $\nu = 1/2$ in terms of those of composite fermions in zero field.

Lowest Landau Level Projection: For the case of physical interest, $\theta = 2$, and in the limit $\alpha \to \infty$, the low energy eigenstates are solely made up of states in the lowest Landau level. The projection of the electron Hamiltonian into the lowest Landau level gives $H_{LL}^0 = \mu \int d^2x \rho_L(x) + \frac{1}{2} \int d^2x d^2x' V(x - x') \rho_L(x) \rho_L(x')$ where $\rho_L(x) = \psi_L^\dagger(x) \psi_L(x)$, with $\psi_L = \sum_k \psi_k(x) c_k$, where $\psi_k$ is the lowest Landau level basis and $c_k$ is the associated annihilation operator. The p-h transformation is implemented via $\psi_L(x) \to \psi_L^\dagger(x) = \sum_k \psi_k^\dagger(x) c_k^\dagger$.

At $\nu = 1/2$, the value of $\mu$ is such that $H_L$ is invariant under the p-h transformation. In the presence of disorder potential $H_L = H_L^0 + \int d^2x U(x) \psi_L^\dagger(x) \psi_L(x)$, Thus, $H_L[U(x)] \to H_L[U'(x)]$ under p-h transformation. To prove that $\sigma_{xy} = e^2/2\hbar$ when $\omega_c \to \infty$, we start with the
Kubo formula \( \sigma_{xy} (\omega) = \frac{1}{\pi} \int dt e^{i\omega t} \langle [j_x(t), j_y(0)] \rangle \), \( \omega < ... > \) stands for the quantum, thermal, and impurity averages, \( j_\alpha = \frac{i}{\hbar} \int d^2 x j_\alpha (x) \) (\( A \) is the total area). Then we let \( \omega_c \to \infty \) and keep all terms to order \( (1/\omega_c)^0 \). Finally we perform the \( p-h \) transformation. The details will be reported elsewhere but the result is self-evident. If there is no spontaneous particle-hole symmetry breaking,

\[
\sigma_{xy} \equiv \lim_{\omega \to 0} \sigma_{xy}(\omega) = -\sigma_{xy} + e^2/\hbar, \quad (6)
\]

In the above, \( e^2/\hbar \) arises as the Hall conductivity of the hole vacuum, i.e., the filled lowest Landau level. Eq. (6) implies that \( \sigma_{xy} = e^2/2\hbar \). So long as \( \rho_{xx} \neq 0 \), it can be seen directly from Eq. (5) that \( \sigma_{xy} = e^2/2\hbar \) implies \( \sigma_{\alpha y} = -e^2/2\hbar \). In the literature it is noted that potential disorder for the electrons induces both potential and magnetic flux disorder for the composite fermions. [3,4] According to this picture, when the potential disorder is \( p-h \) symmetric, the associated random magnetic field is also symmetrically distributed about zero. Under that condition, the impurity-averaged Hall conductivity \( \sigma_{\alpha y} \) should vanish, in contradiction with the requirements imposed by the projection onto the lowest Landau level.

**Perturbative Results:** When the flux and particle density are related according to \( B = \theta_0 \phi \), the average statistical magnetic field seen by the composite fermions exactly cancels the external one. The question remains, what are the fluctuation corrections to this mean-field picture? As mentioned above, in the limit \( \theta \to 0 \) with non-zero \( \alpha \), fluctuations of the statistical gauge field trivially vanish, and this limit provides a reference point where MFT is exact. [3] In carrying out these fluctuation calculations, we choose to work in Coulomb gauge, in which the gauge-field propagator, \( D_{ij} \), is a \( 2 \times 2 \) matrix, with \( j = 0, 1 \) representing the time and space component, respectively.

First, we comment on our result concerning \( \lim_{\bar{q} \to 0} \sigma_{\alpha y}(\bar{q}) \). We calculated \( \sigma_{\alpha y} \) perturbatively by evaluating the Feynman diagrams shown in Fig.1. In that figure the wavy line represents the mixed gauge propagator \( D_{01} \) and \( D_{10} \). The open, solid triangles, and the square represent the density, current and the diamagnetic vertices respectively. To the lowest order in \( \theta \) and \( \alpha \) we used the bare gauge propagator. In this case, since \( D_{01} \) does not depend on frequency, the integration can be easily done and the contributions to \( \sigma_{\alpha y} \) from Figs.1(a), 1(b), and 1(c),1(d) are \( \pm \theta \theta / (e^2/\hbar) \) respectively, thus the net result is zero. In this calculation we found that the characteristic momentum carried by the gauge line is of order \( k_F \).

Next, we summarize our results for the density-density correlation function. The bare gauge propagator in the Coulomb gauge has the property that \( D_{11} = 0 \) and \( D_{00}(q_0, \bar{q}) = V(\bar{q}) \). Thus the effects of \( D_{00} \) are identical to those of a static Coulomb interaction. As is customary in this case, a RPA resummation is performed to screen \( D_{00} \) and \( D_{11} \). If one uses the renormalized \( D_{00} \) and \( D_{11} \) to compute the 1-loop corrections to the composite fermion self-energy, \( \Sigma(q_0, \bar{q}) \), the contribution from longitudinal fluctuations, i.e. those which involve \( D_{00} \), diverges logarithmically with the size of the system for fixed \( q_0 \) and \( \bar{q} \), and the contribution from transverse fluctuations, i.e. those involving \( D_{11} \), are regular in the system size, but contribute a logarithmically diverging correction to the effective mass. [3] However, at the same level of approximation in computing the density-density correlation function (see Fig.2), we find that the singular self-energy correction caused by \( D_{00} \) is canceled by the corresponding vertex correction for all \( \bar{q} \) and \( \omega \). To prove this we have used the fact that the divergent part of the vertex \( \Gamma \) and the self-energy are related by the following identity

\[
\Gamma(p, p + q) = \frac{\Sigma(p) - \Sigma(p + q)}{i q_0 - \epsilon(p + q) + \epsilon(p)}, \quad (7)
\]

where \( \bar{q} \) and \( q_0 \) are the incoming gauge field momentum and frequency. After summing a)-c) in Fig.2 the divergent part of the density-density correlation function is given by

\[
\int \frac{d^3 p}{(2\pi)^3} \left\{ G(p + q) \Sigma(p + q) - [G(p)]^2 \Sigma(p) \right\} \frac{i q_0 - \epsilon(p + q) + \epsilon(p)}{i q_0 - \epsilon(p + q) + \epsilon(p)}, \quad (8)
\]

which vanishes after integration over \( q_0 \) because the poles of \( G \) (the composite fermion propagator) and \( \Sigma \) lie on the same side from the real axis. The same thing can not be said for the corrections caused by \( D_{11} \). In that case the singular contribution from the self-energy and vertex corrections do not cancel. [4] They do cancel at other \( | \bar{q} | \). [4] The graphs used in that calculation are summarized in Fig.2. The result for the \( 2k_F \) density-density correlation function is given by Eq.(2).

The fact that the divergent self-energy correction from \( D_{00} \) is canceled by the vertex correction for all external momenta sheds light on the following fundamental issue. The divergent self-energy correction caused by \( D_{00} \) stems from the pole of \( D_{00} \) at \( \omega = \omega_c \). It is eliminated if one suppresses all intermediate states that are not in the lowest Landau level. This has been taken as an indication that the Landau level projection is, somehow, a necessary step in constructing a meaningful theory for composite fermions. [3] Our result suggests an alternative view. We believe that single composite fermion excitations are not part of the physical spectrum. Instead, the physical excitations are statistical charge-neutral \( p-h \) excitations. [4] Thus to describe physical excitations, the Landau level projection is not necessary. Of course, we have proven the consistency of this viewpoint only to lowest order in perturbation theory, so at this point we can only conjecture that it remains valid more generally.

Our result for \( \sigma_{\alpha y} \) is strictly perturbative in \( \theta \). The same is not true of the result for \( \Pi(\omega, q) \), where a RPA resummation has been performed. If we take Eq.(2) at face value, there will be a crossover temperature/frequency.
below which the singular corrections to $\Pi$ at $|\tilde{q}| = 2k_F$ becomes significant.

**Possible relevance to experiment:** Our most important conclusion is that when a Hall sample exhibits $p$-$h$ symmetry at long wavelength, i.e. $\sigma_{xy} = e^2/2h$, it is not describable as a composite Fermi liquid. In a recent study of gated GaAs heterojunctions with mobility $\mu \leq 2 \times 10^6 cm^2/Vs$, a line in the density-magnetic field plane has been identified at which $\nu \approx 1/2$, and $\sigma_{xy} = e^2/2h$ independent of temperature from 50mK to 1.5K. (Meanwhile $\rho_{xx}$ varies with temperature and density, taking values between 0.02 to 1h/e^2.) We interpret this result as indicating that $p$-$h$ symmetry is respected in real systems near $\nu = 1/2$. This line merges with the phase boundary between the $\nu = 1$ and the insulating phase (on which $\sigma_{xx} = \sigma_{xy} = e^2/2h$ at low temperatures). In the same experiments, a line is also observed on which $\rho_{xy} \approx 2h/e^2$, and is approximately temperature independent. (These two lines converge as $\rho_{xx} \to 0$.) If the composite Fermi liquid exists, it must be along this latter line. Finally, our perturbative analyses suggest that there can exist a crossover temperature which, according to Eq. (2), is exponentially small in the limit of large Landau level mixing, which separates a high temperature regime in which the mean-field Fermi liquid theory is valid, from a low temperature regime in which more subtle fluctuation physics pertains. (A similar observation was made previously in Ref. [1].)

The present considerations do not directly address the nature of the true composite fermion ground state at $\nu = 1/2$ in the limit of $\alpha \to \infty$. It is possible that this state possesses a Fermi surface, and has $\sigma_{xy} = -e^2/2h$.

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[1] H.W. Jiang et al., Phys.Rev.Lett. 65, 633 (1990).
[2] R.L. Willet et al., Phys.Rev.Lett. 71, 3846 (1993).
[3] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B 47, 7312 (1993).
[4] V. Kalmeyer and S. C. Zhang, Phys. Rev. B 46, 9889 (1992).
[5] J.K. Jain, Phys.Rev.Lett. 63, 199 (1989).
[6] S.C. Zhang, H. Hanson and S. Kivelson, Phys. Rev. Lett. 62, 82 (1989); 62, 980 (E) (1989). A. Lopez and E. Fradkin, Phys. Rev. B 44, 5246 (1991).
[7] W. Kang et al., Phys.Rev.Lett. 71, 3846 (1993); V.J. Goldman, B.Su and J.K. Jain, Phys. Rev. Lett. 72, 2065 (1994); R.L. Willet, K.W. West and L.N. Pfeiffer, Phys.Rev.Lett. 75, 2988 (1995).
[8] A. Stern and B.I. Halperin, Phys. Rev. B 52, 5890 (1995).
[9] C. Nayak and F. Wilczek, Nucl.Phys.B 417, 359 (1994).
[10] H-J Kwon, A. Houghton and J.B. Marston, Phys. Rev. B 52, 8002 (1995).
[11] B.L. Altshuler, L.B. Ioffe and A.J. Millis, Phys. Rev. B 50, 14048 (1994).
[12] D.V. Khveshchenko and P.C.E. Stamp, Phys.Rev.Lett. 71, 2118 (1993); Phys.Rev.B 49, 5227 (1994).
[13] In the presence of disorder potential $U(x)$, we speak of $p$-$h$ symmetry if the disorder ensemble has the property that $P[U(x)] = P[-U(x)]$, where $P[U]$ is the probability that a particular $U(x)$ is realized. Here, without loss of generality, we have set $\int d^2x P(x) = 0$.
[14] This is not the canonical order of limits, but for the purpose of illustrating the lack of time reversal symmetry it is sufficient.
[15] Y.B. Kim, A. Furusaki, X-G Wen and P.A. Lee, Phys. Rev. B 50, 17917 (1994).
[16] S.Kivelson, D.H. Lee and S.C. Zhang, Phys.Rev.B 46, 2223 (1992).
[17] R.Rajaraman and S.L. Sondhi, Int. J. Mod. Phys. B 8, 1065 (1994).
[18] L. W. Wong and H.W. Jiang, unpublished.

**Figure Captions**

Fig. 1. Feynman diagrams for $\lim_{q \to 0} \sigma_{xy}^f(q)$. Note that the diagrams a), b) cancels diagram c), d). Moreover, the diagrams corresponding to self-energy insertions vanish due to symmetry.

Fig. 2. Feynman diagrams for $\Pi(q_0, \tilde{q})$. For longitudinal gauge fluctuations, diagrams d) and e) are absent.
Fig. 1
Fig. 2