Effects of radiation on primordial non-Gaussianity

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Abstract. We have studied the non-Gaussian features in single-field slow-roll inflationary scenario, where inflation is preceded by a radiation era. In such a scenario, both bispectrum and trispectrum non-Gaussianities are enhanced. Interestingly, the trispectrum in this scenario does not depend up on the slow-roll parameters, and thus, $\tau_{NL}$ is larger than $f_{NL}$, which can be a signature of such a pre-inflationary radiation era.

1. Introduction
Study of non-Gaussian features in primordial perturbations generated during inflation has become a subject of great importance, as the precise determination of these primordial non-Gaussianities can quantify the dynamics of the early universe [1]. In generic single-field slow-roll inflationary scenario, the preferred initial vacuum chosen for the inflaton perturbations is that of Bunch-Davies. It is shown in [2] that if inflation is preceded by a radiation era then the inflaton fluctuations will have an initial thermal distribution where the initial vacuum will depart from the standard Bunch-Davies one. The presence of pre-inflationary radiation era enhances the power spectrum of scalar modes by an extra temperature dependant factor $\coth(k/2T)$. The enhanced power spectrum is in accordance with the observations if the comoving temperature $T$ of the primordial perturbations is less than $10^{-3} \text{ Mpc}^{-1}$ [2]. In this paper, we have shown that presence of pre-inflationary radiation era not only enhances the power spectrum, but also generates large bispectrum and trispectrum, and these non-Gaussianities will carry signatures of such pre-inflationary radiation era [3], which have been discussed in detail.

2. Bispectrum and trispectrum in single-field slow-roll inflation
The derivations shown in this paper are done in spatially flat gauge, which is preferred, as in this gauge, the comoving curvature perturbation $\mathcal{R}(t, x)$ is proportional to the inflaton fluctuations $\delta\phi(t, x)$ as:

$$\mathcal{R}(t, x) = \frac{H}{\dot{\phi}} \delta\phi(t, x),$$

(1)

where $H$ is the Hubble parameter and the overdot represents derivative with respect to cosmic time $t$. Thus, in this gauge, the comoving curvature power spectrum, i.e., the 2-point correlation function of $\mathcal{R}(t, k)$ in Fourier space, is directly related to inflaton’s power spectrum as:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \langle \mathcal{R}(k) \mathcal{R}(k) \rangle \leftrightarrow \left(\frac{H}{\dot{\phi}}\right)^2 \langle \delta\phi(k) \delta\phi(k) \rangle.$$ 

(2)
The comoving curvature power spectrum is measured through observations of the TT anisotropy spectrum of CMBR, which is nearly scale-invariant.

Bispectrum, the non-vanishing 3-point correlation function of primordial fluctuations, is the lowest order departure from Gaussianity of those primordial perturbations. The non-Gaussianity arising from bispectrum is quantified by a non-linear parameter $f_{NL}$, which is constrained by several experiments as: (i) the WMAP-5 data yields $-151 < f_{NL}^{eq} < 253$ (95% CL) [4], (ii) the PLANCK mission is sensitive to probe bispectrum up to $f_{NL} \sim 5$ [5], and (iii) in future experiments, if the primordial non-Gaussianities imprinted in 21-cm background is measured, then $f_{NL} < 0.1$ can be probed [6, 7]. In a free theory, as the primordial perturbations are Gaussian in nature, the 3-point correlation function vanishes, yielding no non-Gaussianity. It is shown in [8] that self-interactions of inflaton field of the kind $V(\phi) = \lambda \phi^5$ generates non-vanishing bispectrum proportional to $\lambda/H$, but the non-Gaussianity is too small ($\sim O(10^{-7})$) to be probed by any existing or future experiments. On the other hand, in a generic single-field slow-roll inflationary model, non-linearities in the evolution of primordial perturbations can also generate primordial non-Gaussianities in CMBR. In the non-linear limit, one can write:

$$R_{NL}(t, x) = \frac{H}{\dot{\phi}} \delta \phi_L(t, x) + \frac{1}{2} \frac{\partial}{\partial \phi} \left( \frac{H}{\dot{\phi}} \right) \delta \phi_L^2(t, x) + O(\delta \phi^3),$$

which yields a non-vanishing 3-point correlation function of $R_{NL}$ in terms of 4-point correlation function of inflaton perturbations $\delta \phi_L$ as:

$$\langle R_{NL} R_{NL} R_{NL} \rangle \approx \left( \frac{H}{\dot{\phi}} \right)^2 \frac{1}{2} \frac{\partial}{\partial \phi} \left( \frac{H}{\dot{\phi}} \right) \langle \delta \phi_L \delta \phi_L \delta \phi_L^2 \rangle,$$

even when the initial perturbations $\delta \phi_L$ are Gaussian in nature. The 4-point correlation function on the right hand side can be written in terms of product of two 2-point correlation functions, and defining $f_{NL}$ as:

$$\langle R(k_1) R(k_2) R(k_3) \rangle = (2\pi)^{-3} \delta^3(k_1 + k_2 + k_3) \frac{6}{5} f_{NL} \left( \frac{P_R(k_1)}{k_1^3} \frac{P_R(k_2)}{k_2^3} + 2 \text{ perms.} \right),$$

one can show that the non-linear parameter is of the order of slow-roll parameters $f_{NL} = \frac{5}{4}(\delta - \epsilon)$ [11], which is also too small ($O(10^{-2})$) to be detected by any present or forthcoming experiments. The delta-function in the above equation ensures that the three momenta form a triangle and $f_{NL}$ is determined in several such triangle configurations, and some of them which we have considered in this paper are: (i) Squeezed configuration ($|k_1| \approx |k_2| \gg |k_3|$), (ii) Equilateral configuration ($|k_1| = |k_2| = |k_3| = k$), and (iii) Folded configuration ($|k_1| = |k_3| = \frac{1}{2} |k_2| = k$).

The connected part of 4-point correlation function of primordial fluctuations is called the trispectrum, given as:

$$\langle R(k_1) R(k_2) R(k_3) R(k_4) \rangle_c \equiv \frac{\langle R(k_1) R(k_2) R(k_3) R(k_4) \rangle - \langle R_L(k_1) R_L(k_2) \rangle \langle R_L(k_3) R_L(k_4) \rangle + 2 \text{ perms.} }{\langle R_L(k_1) R_L(k_2) \rangle \langle R_L(k_3) R_L(k_4) \rangle}.$$ 

The non-linear parameter $\tau_{NL}$ quantifies the non-Gaussianity arising from trispectrum and it is constrained by observations as: (i) WMAP constraints trispectrum as $|\tau_{NL}| < 10^8$ [9], (ii) PLANCK is expected to reach the sensitivity up to $|\tau_{NL}| \sim 560$ [10], and (iii) future 21-cm experiments can probe trispectrum up to the level $\tau_{NL} \sim 10$ [7]. A free-scalar theory yields vanishing trispectrum like vanishing bispectrum. But non-linear evolution of $R$ as given in Eq. (3) yields a trispectrum, where $\tau_{NL} = \left( \frac{2 f_{NL}}{5} \right)^2$ [11], which being proportional to the square of slow-roll parameters is too small to be detected by any present or future experiments. It is to be noted that the generic single-field slow-roll inflation predicts a trispectrum, which is smaller than the bispectrum by orders of magnitude.
3. Inflation with prior radiation era and enhanced non-Gaussianity

If inflation is preceded by a radiation era, then the inflaton field will have an initial thermal distribution, where the thermal vacuum \(|\Omega\rangle\) will have finite occupation as \(N_k|\Omega\rangle = n_k|\Omega\rangle\) (the number operator \(N_k \equiv a_k^\dagger a_k\)). Also, there will be a probability of the system to be in an energy state \(\varepsilon_k \equiv n_k k\) as:

\[
p(\varepsilon_k) \equiv \frac{e^{-\beta n_k k}}{\sum_{n_k} e^{-\beta n_k k}} = \frac{e^{-\beta n_k k}}{z}.
\]

Due to this probability distribution, the correlation functions have to be thermal averaged. Taking into account the initial thermal distributions of the primordial fluctuations and the probability distribution due to pre-inflationary radiation era, the thermal averaged inflaton’s power spectrum will have an enhancement factor \(1 + 2f_B(k)\), where \(f_B(k)\) is the distribution function of primordial perturbations. For inflaton (scalar) perturbations, it will be Bose-Einstein distribution function \(f_B(k) \equiv \frac{1}{e^{\beta n_k k} - 1}\), and thus, the enhancement factor will be \(1 + 2f_B(k) = \cosh(\beta k/2)\), where \(\beta \equiv \frac{1}{k} [2, 3]\). This enhanced power spectrum is in accordance with the observations when \(T < 10^{-3}\) Mpc\(^{-1}\) [2].

As the 2-point correlation function is thermal averaged due to the effects of pre-inflationary radiation era, the other higher-point correlation functions have also to be thermal averaged in a similar way. For 3-point correlation function, this will be:

\[
\langle R_{NL} R_{NL} R_{NL} \rangle_\beta \simeq \left( \frac{H}{\dot{\phi}} \right)^2 \frac{1}{2} \frac{\partial}{\partial \phi} \left( \frac{H}{\dot{\phi}} \right) \langle \delta \phi \delta \phi \delta \phi \rangle^2 \beta,
\]

but now the probability of occupancy of a state with four energies \(\varepsilon\), will be:

\[
p(k_1, k_2, k_3, k_4) \equiv \frac{\prod_r e^{-\beta n_r k_r}}{\prod_r \sum_{n_r} e^{-\beta n_r k_r}}.
\]

Due to thermal averaging, \(f_{NL}\) is enhanced. We have computed the non-Gaussianity in this scenario arising from bispectrum in different triangle configurations: (i) **Squeezed configuration**: The enhanced non-Gaussianity is \(f_{NL}^{th} = f_{NL} \times 2 \left( 1 + 3.72 \cosh \left( \frac{\beta k}{T} \right) \right)\), where \(f_{NL}\) is enhanced by a factor of 64.82, (ii) **Equilateral configuration**: The enhanced non-Gaussianity is \(f_{NL}^{th} = f_{NL} \times \left( 3 + \frac{5}{4 \sinh^2(\frac{\beta k}{T})} \right)\), where \(f_{NL}\) is enhanced by a factor of 90.85, and (iii) **Folded configuration**: The enhanced non-Gaussianity is \(f_{NL}^{th} = f_{NL} \times \left( 3 + \frac{1}{\sinh^2(\frac{\beta k}{T})} \right)\), where \(f_{NL}\) is enhanced by a factor of 73.28. It can be seen that \(f_{NL}\), which is enhanced due to effects of pre-inflationary radiation era, is within the sensitivity of future 21-cm experiments, where primordial non-Gaussianities can be detected [6, 7]. It is also to be noted that the maximum non-Gaussianity can arise in the equilateral configuration when inflation is preceded by a radiation era. In a later work [12], similar analysis is done when perturbations are already present in the initial vacuum. It is found in [12] too that initial presence of quanta in the vacuum can significantly enhance non-Gaussianity arising from bispectrum, which is in agreement with the results presented here and in [3].

It is very interesting to note at this point that due to thermal averaging, the 4-point correlation function is not equal to the product of two 2-point correlation functions, which can yield a non-vanishing connected part as:

\[
\langle R(k_1) R(k_2) R(k_3) R(k_4) \rangle_\beta \neq \langle R(k_1) R(k_2) \rangle \langle R(k_3) R(k_4) \rangle_\beta - \langle \langle R_L(k_1) R_L(k_2) \rangle_\beta \langle R_L(k_3) R_L(k_4) \rangle_\beta + 2\text{perm} \rangle.
\]
Thus, defining the trispectrum in such a situation as:

$$\langle R(k_1)R(k_2)R(k_3)R(k_4) \rangle_c = \tau_{NL} \left[ \frac{P_R(k_1)}{k_1^3} \frac{P_R(k_2)}{k_2^3} \delta^3(k_1 + k_3)\delta^3(k_2 + k_4) + 2 \text{ perm.} \right], \quad (11)$$

and then, we have seen that as the linear perturbations can generate non-vanishing connected part due to thermal averaging, the non-linear parameter $|\tau_{NL}|$ will not depend up on slow-roll parameters and can be as large as 42.58 [3], which is within the detection range of future 21-cm background anisotropy experiments [7]. Hence, we have seen that the presence of pre-inflationary radiation era yields larger trispectrum non-Gaussianity than bispectrum.

4. Conclusion

In a generic single-field slow-roll model of super-cool inflation, non-linear evolution of primordial fluctuations generate bispectrum non-Gaussianity, which is proportional to the slow-roll parameters [11], and thus, too small to be detected by any present or future experiments. Non-linear evolution of primordial fluctuations also generates trispectrum non-Gaussianity, where $\tau_{NL}$ is proportional to the square of slow-roll parameters [11]. Thus, this generic inflationary scenario predicts trispectrum non-Gaussianity, which is much smaller than the bispectrum non-Gaussianity.

We have shown that if such a generic inflationary scenario is preceded by a radiation era, it can yield large bispectrum and trispectrum non-Gaussianities [3], which are within the range of detection of future 21-cm background anisotropy experiments [6, 7]. Due to the presence of pre-inflationary radiation era, the initial vacuum will contain thermal fluctuations, and also the energy states will have a probability distribution. Accordingly, the thermal averaged 3-point correlation function generates large non-Gaussianity, where the enhancement of $f_{NL}$ over the generic scenario is largest in the equilateral configuration. An interesting situation arises in the case of trispectrum as the thermal averaged 4-point correlation function is not equal to the product of two thermal-averaged 2-point correlation functions. Thus, the linear primordial perturbations can generate a non-vanishing connected part of 4-point correlation function due to thermal averaging, and $\tau_{NL}$ in such a case will not depend up on the slow-roll parameters. We have computed that in such a scenario, $|\tau_{NL}|$ can be as large as 43 [3]. Thus, a significant signature of such pre-inflationary radiation era is that it yields larger trispectrum than bispectrum. This signature can distinguish between an inflationary scenario preceded by a radiation era and the generic scenario of single-field slow-roll super-cool inflation.

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