W Pair Production at the LHC
II. One-loop Squared Corrections in the High Energy Limit

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Abstract

We present the result for the one-loop squared virtual QCD corrections to the W boson pair production in the quark-anti-quark-annihilation channel in the limit where all kinematical invariants are large compared to the mass of the W boson. The infrared pole structure is in agreement with the prediction of Catani’s general formalism for the singularities of QCD amplitudes.
1 Introduction

The Large Hadron Collider (LHC) is expected to have a huge impact on particle physics phenomenology. Most of the processes which will be studied at the LHC need to be calculated at least to next-to-leading order (NLO) in QCD whereas there are some for which a theoretical prediction is needed to next-next-to-leading order (NNLO). Electroweak gauge boson pair production falls into the latter category. One of the reasons is that the increase of the centre-of-mass energy at the LHC with respect to the Tevatron from 1.96 TeV to 14 TeV will result in a huge boost of the available data.

The importance of hadronic W-pair production is two-fold. Firstly, it is a process which allows the measurement of the vector boson trilinear couplings and therefore a comparison with the Standard Model (SM) predictions. Most attempts to model New Physics, such as Supersymmetry and Extra-dimensions in all variations, should be able to explain any deviations by consistently adjusting the anomalous couplings and/or by incorporating decays of new particles into vector boson pairs [1].

Secondly, hadronic W pair production is important for investigations of the nature of the Electroweak symmetry mechanism by contributing the dominant background for the Higgs boson mediated process (see Refs. [2–14]),

\[ pp \rightarrow H \rightarrow W^+W^* \rightarrow l\bar{\nu}l'\bar{\nu}', \]

in the Higgs mass range between 140 GeV < \( M_H \) < 180 GeV [15].

The interest in hadronic W pair production is well displayed by the fact that the Born cross section was calculated some thirty years ago [16]. The NLO QCD corrections were computed in the 90’s and seen to contribute a 30% [17–21]. Next, soft gluon resummation effects were considered in Ref. [22] whereas massless fermion-boson scattering was studied at NNLO in Ref. [23]. The first step towards a complete NNLO study is the computation of the NNLO two-loop virtual corrections in a high energy expansion, \( M_W^2 \ll s, t, u \) [24]. The purpose of the present paper is to complete this study by providing the one-loop squared contributions in the same limit.

The method used in [24], and already sketched in [26–30], is somehow different to the one used for the present work. In fact this time we have used the helicity matrix formalism to reduce the problem to a small set of integrals. We treat these again with Mellin-Barnes representations [31,32] which are constructed by means of the MBrepresentation package [33] and then analytically continued in the number of space-time dimensions \( D = 4 - 2\varepsilon \) using the MB package [34]. After the asymptotic expansion in the mass parameter, contours are closed and integrals finally resummed either with the help of XSummer [35] or the PSLQ algorithm [36].

In Section 2 we introduce our notation, in Section 3 we verify the correctness of the infrared pole structure by comparing with the Catani prediction [37]. We present our results in Section 4 after which we conclude in Section 5.
2 Notation

Although the notation adopted here is identical to that of Ref. [24], we shall recapitulate it for completeness. The charged vector-boson production in the leading partonic scattering process corresponds to

\[ q_j(p_1) + \bar{q}_j(p_2) \rightarrow W^-(p_3, m) + W^+(p_4, m), \]  

where \( p_i \) denote the quark and W momenta, \( m \) is the mass of the W boson and \( j \) is a flavour index. We are considering down-type quark scattering in our paper. Obtaining the corresponding result for up-type quark scattering is actually trivial as we will show in the following. Energy-momentum conservation implies

\[ p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu. \]

We consider the scattering amplitude \( \mathcal{M} \) for the process (1) at fixed values of the external parton momenta \( p_i \), thus \( p_1^2 = p_2^2 = 0 \) and \( p_3^2 = p_4^2 = m^2 \). The amplitude \( \mathcal{M} \) may be written as a series expansion in the strong coupling \( \alpha_s \),

\[ |\mathcal{M}\rangle = \left[ |\mathcal{M}^{(0)}\rangle + \left( \frac{\alpha_s}{2\pi} \right) |\mathcal{M}^{(1)}\rangle + \left( \frac{\alpha_s}{2\pi} \right)^2 |\mathcal{M}^{(2)}\rangle + O(\alpha_s^3) \right], \]

and we define the expansion parameter in powers of \( \alpha_s(\mu^2)/(2\pi) \) with \( \mu \) being the renormalisation scale. We work in conventional dimensional regularisation, \( d = 4 - 2\varepsilon \), in the \( \overline{\text{MS}} \)-scheme for the coupling constant renormalisation.

We explicitly relate the bare (unrenormalised) coupling \( \alpha_s^b \) to the renormalised coupling \( \alpha_s \) by

\[ \alpha_s^b S_\varepsilon = \alpha_s \left[ 1 - \frac{\beta_0}{\varepsilon} \left( \frac{\alpha_s}{2\pi} \right) + \left( \frac{\beta_0}{\varepsilon^2} - \frac{1}{2} \frac{\beta_1}{\varepsilon} \right) \left( \frac{\alpha_s}{2\pi} \right)^2 + O(\alpha_s^3) \right], \]

where we set the factor \( S_\varepsilon = (4\pi)^\varepsilon \exp(-\varepsilon \gamma_E) = 1 \) for simplicity and \( \beta \) is the QCD \( \beta \)-function known at present up to the four-loop level [38, 39]

\[ \beta_0 = \frac{11}{6} C_A - \frac{2}{3} T_F n_f, \quad \beta_1 = \frac{17}{6} C_A^2 - \frac{5}{3} C_A T_F n_f - C_F T_F n_f. \]

The color factors in a non-Abelian SU(\( N \))-gauge theory are \( C_A = N, C_F = (N^2 - 1)/2N \) and \( T_F = 1/2 \). Throughout this paper, \( N \) denotes the number of colors and \( n_f \) the total number of flavors.

For convenience, we define the function \( \mathcal{A}(\varepsilon, m, s, t, \mu) \) for the squared amplitudes summed over spins and colors as

\[ \sum |\mathcal{M} (q_j + \bar{q}_j \rightarrow W^+ + W^-)|^2 = \mathcal{A}(\varepsilon, m, s, t, \mu). \]

\( \mathcal{A} \) is a function of the Mandelstam variables \( s, t \) and \( u \) given by

\[ s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2 - m^2, \quad u = (p_1 - p_4)^2 - m^2, \]

and has a perturbative expansion similar to Eq. (3),

\[ \mathcal{A}(\varepsilon, m, s, t, \mu) = \left[ \mathcal{A}^{(0)}(\varepsilon) + \left( \frac{\alpha_s}{2\pi} \right) \mathcal{A}^{(1)}(\varepsilon) + \left( \frac{\alpha_s}{2\pi} \right)^2 \mathcal{A}^{(2)}(\varepsilon) + O(\alpha_s^3) \right]. \]
In terms of the amplitudes the expansion coefficients in Eq. (8) may be expressed as

\[ \mathcal{A}^{(0)} = \langle M^{(0)} | M^{(0)} \rangle, \]  
(9)

\[ \mathcal{A}^{(1)} = \left( \langle M^{(0)} | M^{(1)} \rangle + \langle M^{(1)} | M^{(0)} \rangle \right), \]  
(10)

\[ \mathcal{A}^{(2)} = \left( \langle M^{(1)} | M^{(1)} \rangle + \langle M^{(0)} | M^{(2)} \rangle + \langle M^{(2)} | M^{(0)} \rangle \right). \]  
(11)

Note that in the above equations, only the two-loop amplitude needs to be renormalised.

\[ \mathcal{A}^{(0)} \] is given by

\[ \mathcal{A}^{(0)} = N \left\{ c_1 \left[ 16(1-\varepsilon)^2 \frac{x}{1-x} + 4(3-4\varepsilon) \frac{1}{m_s} + \frac{4x(1-x)}{m_s^2} \right] + c_2 \left[ -24 + 16x + 16\varepsilon(2-x) + 4 \frac{(3-4\varepsilon) - 2x(1-x)}{m_s} + \frac{4x(1-x)}{m_s^2} \right] + c_3 \left[ -24(1-x)(1-x) + 16\varepsilon(2-x(1-x)) + \frac{6 - 8\varepsilon - 8x(1-x)}{m_s} + \frac{2x(1-x)}{m_s^2} \right] \right\}, \]

(12)

where we have defined \( x = -\frac{\lambda_3}{s}, m_s = \frac{m_2}{s} \) and only the leading physical powers (i.e. down to the constant) in the \( m_s \)-expansion are retained. Notice that, once the actual values of the \( c_i \) are substituted, the terms singular in \( m_s \) cancel as required by unitarity. This will be the case for the one-loop squared expression as well. The coefficients \( c_1, c_2 \) and \( c_3 \) are in their essence combinations of EW coupling constants defined as

\[ c_1 = \frac{g_{WL}^4}{4}, \]

\[ c_2 = \frac{1}{4s_w^2} \left( Q_q + 2g_{ZL}^q \frac{c_w}{s_w \left( 1 - \frac{m_2^2}{s} \right)} \right), \]

\[ c_3 = \frac{c_w^2}{s_w^2 \left( 1 - \frac{m_2^2}{s} \right)^2} \left( (g_{ZA}^q)^2 + (s_{ZV}^q + Q_q \frac{s_w \left( 1 - \frac{m_2^2}{s} \right)}{c_w}) \right)^2. \]  
(13)

The expressions for \( \mathcal{A}^{(1)} \) have been presented e.g. in Refs. [17, 18], whereas the real part of the full two-loop contributions, namely the result for the last two terms in Eq. (11) in the high energy limit, was presented in Ref. [24] (the leading color coefficient of \( \langle M^{(0)} | M^{(2)} \rangle \) was discussed in Ref. [25]). Here we provide the result for the one-loop\(\otimes\)one-loop result in the high energy limit, namely the NNLO contribution \( \langle M^{(1)} | M^{(1)} \rangle \) in \( \mathcal{A}^{(2)} \).

In order to compute the \( \langle M^{(1)} | M^{(1)} \rangle \) we will use the helicity matrix formalism, namely we will express the result in terms of helicity amplitudes, \( M^{g(h_1, h_2, s, t)} \). The quark and anti-quark have opposite helicities in the centre-of-mass system so one helicity label above, \( g = \pm 1 \), suffices. \( h_1 \) and \( h_2 \) stand for the helicities of the \( W^+ \) and \( W^- \) respectively.
Starting from the one-loop amplitude, the initial expression can be rearranged as

$$|M^{(1)}| = \sum_{i,j,k} C_i(s,t,u) I^j_i(s,t,u;\mu^2) M_j(\{p_k\},g),$$  \hspace{1cm} (14)$$

where the $C_i$ are coefficients, the $I^j_i$ are one-loop dimensionally regularized scalar integrals, $M_j$ are helicity matrix elements, $g = \pm$ and $k = 1,\ldots,4$. The ten helicity matrix elements $M_j(p_k,g) = M^g_j$ have been taken as defined in Ref. [40] (see also [41]):

$$
\begin{align*}
M^g_0 &= \bar{v}(p_2) \epsilon_1 (p^2_3 - p^2_1) \epsilon_2 p_g u(p_1), \\
M^g_1 &= \bar{v}(p_2) p_3 \epsilon_2 p_g u(p_1) \epsilon_1 \cdot \epsilon_2, \\
M^g_2 &= \bar{v}(p_2) \epsilon_1 p_g u(p_1) \epsilon_2 \cdot p_3, \\
M^g_3 &= -\bar{v}(p_2) \epsilon_2 p_g u(p_1) \epsilon_1 \cdot p_4, \\
M^g_4 &= \bar{v}(p_2) \epsilon_1 p_g u(p_1) \epsilon_2 \cdot p_1, \\
M^g_5 &= -\bar{v}(p_2) \epsilon_2 p_g u(p_1) \epsilon_1 \cdot p_2, \\
M^g_6 &= \bar{v}(p_2) p_3 \epsilon_2 p_g u(p_1) \epsilon_1 \cdot p_2 \epsilon_2 \cdot p_1, \\
M^g_7 &= \bar{v}(p_2) p_3 \epsilon_2 p_g u(p_1) \epsilon_1 \cdot p_2 \epsilon_2 \cdot p_3, \\
M^g_8 &= \bar{v}(p_2) p_3 \epsilon_2 p_g u(p_1) \epsilon_1 \cdot p_4 \epsilon_2 \cdot p_1, \\
M^g_9 &= \bar{v}(p_2) p_3 \epsilon_2 p_g u(p_1) \epsilon_1 \cdot p_4 \epsilon_2 \cdot p_3,
\end{align*}$$  \hspace{1cm} (15)$$

where $p_g = p_\pm = \frac{1+\gamma}{2}$. All colour indices as well as the arguments of the polarization vectors, $\epsilon_1(p_3,\lambda_1)$ and $\epsilon_2(p_4,\lambda_2)$, have been suppressed.

Even though the representations in Eq. (15) have been used internally, we present our result only for the amplitude squared and summed over helicities.

## 3 Infrared Pole Structure

In the case of one-loop QCD amplitudes, their poles in $\epsilon$ can be expressed as a universal combination of the tree amplitude and a colour-charge operator $I^{(1)}(\epsilon)$. The generic form of $I^{(1)}(\epsilon)$ was found by Catani and Seymour [42] and it was derived for the general one-loop QCD amplitude by integrating the real radiation graphs of the same order in perturbation series in the one-particle unresolved limit.

The pole structure of our one-loop expression is given, according to the prediction by Catani, by acting with the operator $I^{(1)}(\epsilon)$ onto the tree-level result:

$$|M^{(1)}| = I^{(1)}(\epsilon) |M^{(0)}| + |M_{finite}^{(1)}|,$$  \hspace{1cm} (16)$$

where $I^{(1)}(\epsilon)$ is defined as

$$I^{(1)}(\epsilon) = -C_F \frac{\epsilon^2}{\Gamma(1-\epsilon)} \left( \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right) \left( \frac{\mu^2}{s} \right)^\epsilon.$$  \hspace{1cm} (17)$$
The expression for the one-loop squared result then takes the form:

\[
A_{\text{NNLO}}^{(1\times1)} = \langle M^{(1)} | M^{(1)} \rangle = |I_1^{(1)}(s)|^2 \langle M^{(0)} | M^{(0)} \rangle + 2\text{Re} \left[ I_1^{(1)}(s) | M^{(0)} | M^{(1)}_{\text{finite}} \right] + \langle M^{(1)}_{\text{finite}} | M^{(1)}_{\text{finite}} \rangle.
\] (18)

We checked that our result is in agreement with the Catani prediction.

4 Results

In this section, we give explicit expressions for the finite remainder of the one-loop squared contribution \( g_{\text{finite}}^{(1\times1)} \) defined as

\[
g_{\text{finite}}^{(1\times1)}(s,t,u,m,\mu) = A_{\text{NNLO}}^{(1\times1)}(s,t,u,m,\mu) - C_{\text{atani}}^{(1\times1)}(s,t,u,m,\mu),
\] (19)

where \( C_{\text{atani}}^{(1\times1)}(s,t,u,m,\mu) \) is given by the first two terms of Eq. (18) and is expanded through to \( O(1) \), which means that it contains finite contributions as well.

The EW structure of the finite remainder for a down-type quark can be factorised as

\[
g_{\text{finite, down}}^{(1\times1)} = N C_F^2 \sum_{i=1,3} c_i g_{i,\text{down}}^{(1\times1)}(m_s, x, \frac{s}{\mu^2}),
\] (20)

This decomposition allows one to easily obtain the result for the up-type quark scattering. The latter is then given by

\[
g_{\text{finite, up}}^{(1\times1)} = N C_F^2 \sum_{i=1,3} c_i g_{i,\text{up}}^{(1\times1)}(m_s, x, \frac{s}{\mu^2}),
\] (21)

where one needs to use the following formulae \( (y = -\frac{u}{s}) \)

\[
g_{1,\text{up}}^{(1\times1)}(m_s, x, \frac{s}{\mu^2}) = g_{1,\text{down}}^{(1\times1)}(m_s, y, \frac{s}{\mu^2}),
\] (22)

\[
g_{2,\text{up}}^{(1\times1)}(m_s, x, \frac{s}{\mu^2}) = -g_{2,\text{down}}^{(1\times1)}(m_s, y, \frac{s}{\mu^2}),
\] (23)

\[
g_{3,\text{up}}^{(1\times1)}(m_s, x, \frac{s}{\mu^2}) = g_{3,\text{down}}^{(1\times1)}(m_s, y, \frac{s}{\mu^2}),
\] (24)

and naturally to make the corresponding changes in the definitions of the couplings \( c_1, c_2 \) and \( c_3 \) namely to use the up-type quark charge and isospin. In the following we will suppress all indices that indicate the type of scattered quark.

Our result reads:

\[
g_{1}^{(1\times1)} = \frac{64 (1-x) x}{m_s^2} + \frac{192}{m_s} + \left[ -4 \left( \frac{2}{x} - \frac{1}{x^2} - \frac{1}{x^3} + 1 - \frac{1}{1-x} \right) L_y^4 + 8 \left( \frac{3}{x} + \frac{2}{x^2} + \frac{3}{1-x} \right) L_y^3 \right]
\]
\[ \begin{align*}
+ \left( 4 \left( \frac{6}{x} + 15 - \frac{5}{1-x} \right) - 16 \left( -\frac{2}{x} - \frac{1}{x^2} - \frac{1}{x^3} + 1 - \frac{1}{1-x} \right) \pi^2 \right) L_y^2 \\
+ \left( 16 \left( \frac{3}{x} + \frac{2}{x^2} + \frac{3}{1-x} \right) \pi^2 + 8 \left( 7 - \frac{5}{1-x} \right) \right) L_y - 4 \left( \frac{4}{x} + 1 - \frac{9}{1-x} \right) \pi^2 \\
-4 \left( -32x - \frac{83}{1-x} + 83 \right) + 128 \left( x - \frac{1}{1-x} + 2 \right) L_m, \\
\end{align*} \]

\[ j_2^{(1 \times 1)} = \frac{64(1-x)x}{m_s^2} + \frac{1}{m_s} \left[ 64 \left( 2x^2 - 2x + 3 \right) \right] + \left[ -32(x-2)L_y^2 - 32 \left( x - \frac{2}{1-x} + 2 \right) L_y \right] \\
-32 \left( -9x - \frac{2}{1-x} + 14 \right) + 64 \left( x - \frac{1}{1-x} + 2 \right) L_m, \]

\[ j_3^{(1 \times 1)} = \frac{32(1-x)x}{m_s^2} + \frac{1}{m_s} \left[ 32 \left( 4x^2 - 4x + 3 \right) \right] + \left[ -384 \left( x^2 - x + 1 \right) \right], \quad (25) \]

where \( L_m \) and \( L_y \) are defined as

\[ L_m = \log (m_s), \quad L_y = \log (1-x). \quad (26) \]

### 5 Conclusions

We have computed the one-loop squared \( O(\alpha_s^2) \) corrections to the process \( q\bar{q} \rightarrow W^+W^- \) in a high energy expansion through to the zeroth-order in \( \frac{M_W^2}{s} \). We checked that the infrared structure of our result agrees with the prediction of Catani’s formalism for the infrared structure of QCD amplitudes.

The present result, given as the finite remainder of the NNLO one-loop squared virtual corrections after subtraction of the structure predicted by Catani’s formalism, in combination with the result in Ref [24] for the two-loop amplitude, completes the calculation of the virtual corrections to the process in the high energy limit. In a forthcoming publication, we will derive a series expansion in the mass and integrate both results numerically similarly to what has been done for top quark pair production [43].

To complete the NNLO project one still needs to consider \( 2 \rightarrow 3 \) real-virtual contributions and \( 2 \rightarrow 4 \) real ones. The real-virtual corrections are known from the NLO studies on \( WW + jet \) production in Refs. [44, 45].

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Appendix: \( \langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)}_{\text{finite}} \rangle \) to order \( \epsilon^2 \)

Here we present the expression for the one-loop result, \( \langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)}_{\text{finite}} \rangle \), up to order \( \epsilon^2 \) for down-type quarks. This result completes the list of the elements needed in Eq. (8) in order to have the perturbative expansion of the amplitude up to order \( \alpha_s^2 \) in the high energy limit.

\[
\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)}_{\text{finite}} \rangle = N_C F \sum_{i=1,3} c_i \mathcal{F}^{(0\times1)}_i .
\] (27)

\[
\mathcal{F}^{(0\times1)}_i = \left\{ \frac{1}{m_s^2} \left[ -16(1-x)x + \frac{1}{m_s} [-48] + \left[ -8 \left( 1 - \frac{1}{1-x} \right) L_y^2 - 8 \left( 1 - \frac{1}{1-x} \right) L_y \right. \right. \\
+ 8 \left( -2x - \frac{9}{1-x} + 9 \right) - 16 \left( x - \frac{1}{1-x} + 2 \right) L_m \right] \right\} \\
+ i\pi \left\{ -16 L_y \left( 1 - \frac{1}{1-x} \right) - 8 \left( 1 - \frac{1}{1-x} \right) \right\} \\
+ \epsilon \left\{ \frac{1}{m_s^2} \left[ -32(1-x)x + 8(1-x)\xi_3 x + 16(1-x)\xi_3 x \right] + \frac{1}{m_s} [24\xi_3 + 48L_s - 32] \\
+ \left[ \frac{16}{3} \left( 1 - \frac{1}{1-x} \right) L_3^3 + 4 \left( 3 - \frac{5}{1-x} \right) L_y^2 + 8 \left( 1 - \frac{1}{1-x} \right) L_s L_y^2 + \right. \\
\left. \left( 8 \left( 1 - \frac{1}{1-x} \right) \pi^2 - 8 \left( 1 + \frac{1}{1-x} \right) \right) L_y + 8 \left( 1 - \frac{1}{1-x} \right) L_y L_s + 4 \left( 1 - \frac{1}{1-x} \right) \pi^2 \right. \\
\left. + 8 \left( x - \frac{1}{1-x} + 2 \right) L_m^2 - 32 \left( x + \left( 1 - \frac{1}{1-x} \right) \xi_3 \right) - 16 \left( x - \frac{2}{1-x} + 4 \right) L_m \\
\left. - 8 \left( -2x - \frac{9}{1-x} + 9 \right) L_s + 16 \left( x - \frac{1}{1-x} + 2 \right) L_m L_s - 16 \left( 1 - \frac{1}{1-x} \right) S_{1,2}(x) \right] \right\} \\
+ \epsilon i\pi \left\{ \frac{1}{m_s^2} \left[ -16(1-x)x + \frac{1}{m_s} [-48] + \left[ 8 \left( 1 - \frac{1}{1-x} \right) L_y^2 + 16 \left( 1 - \frac{2}{1-x} \right) L_y \right. \right. \\
\left. + 16 \left( 1 - \frac{1}{1-x} \right) L_y L_m + 16 \left( -x - 5 \frac{1}{1-x} + 4 \right) + 16 \left( 1 - \frac{1}{1-x} \right) L_2(x) \right. \\
\left. - 16 \left( x - \frac{1}{1-x} + 2 \right) L_y + 8 \left( 1 - \frac{1}{1-x} \right) L_s \right] \right\} \\
+ \epsilon^2 \left\{ \frac{1}{m_s^2} \left[ \frac{2}{15} (1-x) x \pi^4 + \frac{28}{3} (1-x) x \pi^2 - 8(1-x) x L_y^2 - 64(1-x) x + 32(1-x) x L_s \right. \right. \\
\left. \left. + \xi_3(12(1-x) x - 8(1-x) x L_s) \right] + \frac{1}{m_s} \left[ \frac{2\pi^4}{5} + 28\pi^2 - 24L_y^2 + \xi_3(4 - 24L_s) + 32L_s - 64 \right] \right. \\
\left. + \left[ -\frac{8}{15} \left( 1 - \frac{1}{1-x} \right) \pi^4 - \frac{2}{3} \left( -14x - \frac{69}{1-x} + 57 \right) \pi^2 - 8 \left( 1 - \frac{1}{1-x} \right) L_2(x) \pi^2 \right. \right. \\
\left. - 2 \left( 1 - \frac{1}{1-x} \right) L_y^2 \left( 8 \left( x - \frac{1}{1-x} + 2 \right) L_m^2 + 4 \left( 5 - \frac{9}{1-x} \right) L_y^2 + 16 \left( 1 - \frac{1}{1-x} \right) L_y L_s \right. \right. \\
\left. + 8 \left( x - \frac{2}{1-x} + 4 \right) L_m^2 + 4 \left( -2x - \frac{9}{1-x} + 9 \right) L_y^2 - 8 \left( x - \frac{1}{1-x} + 2 \right) L_m L_y \right] \right\}.
\]
\[-4 \left(1 - \frac{1}{1-x} \right) L^2_{\alpha} L^2_{\beta} + \left(12 \left(1 + \frac{1}{1-x} \right) - \frac{10}{3} \left(1 - \frac{1}{1-x} \right) \pi^2 \right) L^2_{\gamma} - 4 \left(3 - \frac{5}{1-x} \right) L^2_{\delta} \]
\[+ 8 \left(-8x + \left(2 - \frac{2}{1-x} \right) \xi_3 - \frac{9}{1-x} + 9 \right) + \left(\frac{28}{3} \left(x - \frac{1}{1-x} + 2 \right) \pi^2 - 16 \left(2x - \frac{3}{1-x} + 6 \right) \right) L_m \]
\[-8 \left(x - \frac{1}{1-x} + 2 \right) L^2_m L_{\alpha} + \left(-4 \left(1 - \frac{1}{1-x} \right) \pi^2 + 32x + \left(32 - \frac{32}{1-x} \right) \xi_3 \right) L_{\alpha} \]
\[+ 16 \left(x - \frac{2}{1-x} + 4 \right) L_m L_{\alpha} - 4 \left(1 - \frac{1}{1-x} \right) L_{\beta} L_{\alpha} + \left(-\frac{2}{5} \left(11 - \frac{23}{1-x} \right) \pi^2 - 24 \left(1 - \frac{1}{1-x} \right) \right) L_{\gamma} \]
\[+ \left(8 \left(1 + \frac{1}{1-x} \right) - 8 \left(1 - \frac{1}{1-x} \right) \pi^2 \right) L_m L_{\gamma} + 16 \left(1 - \frac{2}{1-x} \right) S_{1,2}(x) + 16 \left(1 - \frac{1}{1-x} \right) L_{m} S_{1,2}(x) \]
\[+ 16 \left(1 - \frac{1}{1-x} \right) L_{\gamma} S_{1,2}(x) + 16 \left(1 - \frac{1}{1-x} \right) S_{1,3}(x) - 16 \left(1 - \frac{1}{1-x} \right) S_{2,2}(x) \right] \RIGHT \}
\[+ e^2 i \pi \left\{ \frac{1}{m^2_s} \left[-32 (1-x)x + 8 (1-x) \xi_3 x + 16 (1-x) L_{\alpha} x \right] + \frac{1}{m_s} \left[4 \xi_3 + 48 L_{\alpha} - 32 \right] \]
\[+ \left[ -\frac{8}{3} \left(1 - \frac{1}{1-x} \right) L^3_{\gamma} - 8 \left(1 - \frac{2}{1-x} \right) L^2_{\gamma} - 8 \left(1 - \frac{1}{1-x} \right) L_{\gamma} L^2_{\alpha} \right] \]
\[+ \left(4 \left(1 - \frac{1}{1-x} \right) \pi^2 + 16 \left(1 + \frac{1}{1-x} \right) \right) L_{\gamma} - 16 \left(1 - \frac{1}{1-x} \right) L_{m} L_{\gamma} - 16 \left(1 - \frac{2}{1-x} \right) L_{m} L_{\gamma} \]
\[+ 2 \left(1 - \frac{1}{1-x} \right) \pi^2 + 8 \left(x - \frac{1}{1-x} + 2 \right) L^2_{m} - 4 \left(1 - \frac{1}{1-x} \right) L^2_{\alpha} \]
\[+ 8 \left(4x + \left(4 - \frac{4}{1-x} \right) \xi_3 + \frac{3}{1-x} + 3 \right) - 16 \left(1 - \frac{2}{1-x} \right) L_{2}(x) + 16 \left(1 - \frac{1}{1-x} \right) \right] L_{3}(x) \]
\[+ 16 \left(x - \frac{2}{1-x} + 4 \right) L_{m} L_{\alpha} - 16 \left(x - \frac{5}{1-x} + 4 \right) L_{\alpha} - 16 \left(1 - \frac{1}{1-x} \right) L_{2}(x) \]
\[+ 16 \left(x - \frac{1}{1-x} + 2 \right) L_{m} L_{\alpha} - 16 \left(1 - \frac{1}{1-x} \right) S_{1,2}(x) \right] \}
\]

(28)

\[j_2^{(0\times1)} = \left\{ \frac{1}{m^2_s} \left[-16 (1-x)x \right] + \frac{1}{m_s} \left[-16 \left(2x^2 - 2x + 3 \right) \right] + \left[4(x-2) L^2_{\alpha} + 4 \left(x - \frac{2}{1-x} + 2 \right) L_{\gamma} \right] \]
\[+ 4 \left(-17x - \frac{2}{1-x} + 26 \right) - 8 \left(x - \frac{1}{1-x} + 2 \right) L_m \left\{ \right. \]
\[+ i \pi \left\{ \right. \]
\[+ \left. \frac{1}{m^2_s} \left[-32 (1-x)x + 8 (1-x) \xi_3 x + 16 (1-x) L_{\alpha} x \right] + \frac{1}{m_s} \left[4 \xi_3 + 48 L_{\alpha} - 32 \right] \]
\[+ 8 \left(2x^2 - 2x + 3 \right) \xi_3 + 16 \left(2x^2 - 2x + 3 \right) L_{\alpha} \right] + \left[ -\frac{8}{3} \left(x - 2 \right) L^3_{\alpha} + 2 \left(\frac{2}{1-x} - 5x \right) L^2_{\alpha} \right] \]
\[+ 4(x-2) L_{\alpha} L^2_{\beta} + \left(-4(x-2) \pi^2 - 8 \left(\frac{1}{1-x} - 2 \right) \right) L_{\gamma} - 4 \left(x - \frac{2}{1-x} + 2 \right) \right] L_{m} L_{\gamma} \]
\[+ 2 \left(x - \frac{2}{1-x} + 2 \right) \pi^2 + 4 \left(x - \frac{1}{1-x} + 2 \right) L^2_{m} + 4 \left(-17x + (8x - 12) \xi_3 - \frac{2}{1-x} + 18 \right) \]
\[+ 8 \left(x - \frac{2}{1-x} + 4 \right) L_{m} - 4 \left(-17x - \frac{2}{1-x} + 26 \right) L_{\alpha} + 8 \left(x - \frac{1}{1-x} + 2 \right) L_{m} L_{\alpha} + 8(x-2) S_{1,2}(x) \right] \}

8
\[
+\varepsilon i\pi \left\{ \frac{1}{m_s} \left[ -16(1-x)x + \frac{1}{m_s} \left[ -16 \left( 2x^2 - 2x + 3 \right) \right] + \left[ -4(x-2)L_x - 8(2x-1)L_y \right. \right. \\
-8(x-2)L_yL_x + 4 \left( -13x - \frac{4}{1-x} + 26 \right) - 8(x-2)L_{i2}(x) - 8 \left( x - \frac{1}{1-x} + 2 \right) L_m \right. \\
\left. -4 \left( x - \frac{2}{1-x} + 2 \right) L_s \right] \right\} \\
+\varepsilon^2 \left\{ \frac{1}{m_s} \left[ \frac{2}{15} \pi^4 + \frac{28}{3} \pi^2 - 8 \left( 1-x \right) x L_x^2 - 64 \left( 1-x \right) x + 32 \left( 1-x \right) x L_x \\
+\zeta_3 \left( 12 \left( 1-x \right) x - 8 \left( 1-x \right) x L_x \right) \right] + \frac{1}{m_s} \left[ \frac{2}{15} \left( 2x^2 - 2x + 3 \right) \right. \\
\pi^4 + \frac{28}{3} \left( 2x^2 - 2x + 3 \right) \pi^2 \\
-8 \left( 2x^2 - 2x + 3 \right) L_x^2 - 64 \left( 2x^2 - 2x + 1 \right) + 32 \left( 2x^2 - 2x + 1 \right) L_x + \zeta_3 \left( 4 \left( 6x^2 - 6x + 1 \right) \right. \\
-8 \left( 2x^2 - 2x + 3 \right) L_x \right] + \left[ \frac{4}{15} \left( 2x - 3 \right) \pi^4 + \frac{1}{3} \left( 95x + \frac{26}{1-x} - 182 \right) \pi^2 \\
+4(x-2)L_{i2}(x) \pi^2 + (x-2)L_y^4 \left( \frac{4}{3} \left( x - \frac{1}{1-x} + 2 \right) L_m^3 - \frac{2}{3} \left( -9x + \frac{2}{1-x} + 2 \right) L_y^3 \\
+\frac{8}{3} (x-2)L_y L_y^3 + 4 \left( x - \frac{2}{1-x} + 4 \right) L_m^2 + 2 \left( -17x - \frac{2}{1-x} + 26 \right) L_s \\
-4 \left( x - \frac{1}{1-x} + 2 \right) L_m L_y^2 + 2(x-2)L_y L_y^2 + \left( \frac{5}{3} \left( x - \frac{1}{1-x} + 1 \right) \right) L_y^2 \\
-2 \left( \frac{2}{1-x} - 5x \right) L_y L_y^2 + 8 \left( -17x + (2x-1) \zeta_3 - \frac{2}{1-x} + 18 \right) + \left( \frac{14}{3} \left( x - \frac{1}{1-x} + 2 \right) \pi^2 \\
-8 \left( 2x - \frac{3}{1-x} + 6 \right) \right) L_m - 4 \left( x - \frac{1}{1-x} + 2 \right) L_m L_s + \left( 2 \left( x - \frac{2}{1-x} + 2 \right) \pi^2 + 68x \\
+(48 - 32x) L_3 + \frac{8}{1-x} - 72 \right) L_s + 8 \left( x - \frac{2}{1-x} + 4 \right) L_m L_s + 2 \left( x - \frac{2}{1-x} + 2 \right) L_s L_y \\
+ \left( \frac{1}{3} \left( 23x + \frac{2}{1-x} - 14 \right) \pi^2 - 4 \left( -5x + \frac{4}{1-x} + 2 \right) \right) L_y \\
+ \left( \left( 4(x-2) \pi^2 + 8 \left( \frac{1}{1-x} - 2x \right) \right) \right) L_y L_y - 8(2x-1) S_{1,2}(x) - 8(x-2) L_x S_{1,2}(x) \\
-8(x-2) L_y S_{1,2}(x) - 8(x-2) S_{1,3}(x) + 8(x-2) S_{2,2}(x) \right] \right\} \\
+\varepsilon^2 i\pi \left\{ \frac{1}{m_s} \left[ -32(1-x)x + 8(1-x) \zeta_3 x + 16(1-x) L_x x \right] + \frac{1}{m_s} \left[ -32 \left( 2x^2 - 2x + 1 \right) \right. \\
+8 \left( 2x^2 - 2x + 3 \right) \zeta_3 + 16 \left( 2x^2 - 2x + 3 \right) L_x \right] + \left[ \frac{4}{3} \left( x - 2 \right) L_y^3 + 4(2x-1) L_y^2 + 4(x-2) L_y \right. \\
+4(x-2) L_y L_y + \left( 8(x-1) - 2 \left( x - 2 \right) \pi^2 \right) L_y + 8(x-2) L_{i2}(x) L_y + 8(2x-1) L_x L_y \\
+ \left( -x + \frac{2}{1-x} - 2 \right) \pi^2 + 4 \left( x - \frac{1}{1-x} + 2 \right) L_m^2 + 2 \left( x - \frac{2}{1-x} + 2 \right) L_s^2 \\
+8 \left( -6x + (4x - 6) \zeta_3 - \frac{3}{1-x} + 8 \right) + 8(2x-1)L_{i2}(x) - 8(x-2)L_{i3}(x) \\
-8 \left( x - \frac{2}{1-x} + 4 \right) L_m - 4 \left( -13x - \frac{4}{1-x} + 26 \right) L_s + 8(x-2)L_{i2}(x) L_s \\
+8 \left( x - \frac{1}{1-x} + 2 \right) L_m L_s + 8(x-2) S_{1,2}(x) \right] \right\} ,
\end{align}
\]
\begin{equation}
J_s^{(0 \times 1)} = \left\{ \frac{1}{m_s} \left[ -8 (1 - x)x + \frac{1}{m_s} \left[ -8 \left( 4x^2 - 4x + 3 \right) \right] + \left[ 96 \left( x^2 - x + 1 \right) \right] \right] \right\} \\
+ \varepsilon \left\{ \frac{1}{m_s} \left[ -16 (1 - x)x + 4 (1 - x)\zeta_3 x + 8 (1 - x)L_s x \right] + \frac{1}{m_s} \left[ -16 \left( 4x^2 - 4x + 1 \right) \right] \\
+ 4 \left( 4x^2 - 4x + 3 \right) \zeta_3 + 8 \left( 4x^2 - 4x + 3 \right) L_s \right] + \left[ -16 \left( -8x^2 + 8x + \left( 3x^2 - 3x + 3 \right) \zeta_3 - 4 \right) \right] \\
- 96 \left( x^2 - x + 1 \right) L_s \right] \right\} \\
+ \varepsilon^2 \left\{ \frac{1}{m_s} \left[ \frac{1}{15} (1 - x)\pi^4 + \frac{14}{3} (1 - x)\pi^4 - 4(1 - x)\pi^4 + 16 (1 - x)\pi^4 \right] \\
+ \zeta_3 \left( 6 (1 - x)x - 4 (1 - x)L_s \right) \right] + \frac{1}{m_s} \left[ \frac{1}{15} \left( 4x^2 - 4x + 3 \right) \pi^4 + \frac{14}{3} \left( 4x^2 - 4x + 3 \right) \pi^2 \right] \\
- 4 \left( 4x^2 - 4x + 3 \right) L_s^2 - 32 \left( 4x^2 - 4x + 1 \right) L_s + \zeta_3 \left( 2 \left( 12x^2 - 12x + 1 \right) \right) \\
- 4 \left( 4x^2 - 4x + 3 \right) L_s \right] + \left[ -\frac{4}{3} (x^2 - x + 1) \pi^4 - 56 (x^2 - x + 1) \pi^2 + 48 (x^2 - x + 1) L_s^2 \right] \\
- 8 \left( -32x^2 + 32x + \left( 5x^2 - 5x + 1 \right) \zeta_3 - 16 \right) + 16 \left( -8x^2 + 8x + \left( 3x^2 - 3x + 3 \right) \zeta_3 - 4 \right) L_s \right] \right\} \\
+ \varepsilon^2 i\pi \left\{ \frac{\zeta_3}{m_s} \left[ -16 (1 - x)x + 4 (1 - x)\zeta_3 x + 8 (1 - x)L_s x \right] + \frac{1}{m_s} \left[ -16 \left( 4x^2 - 4x + 1 \right) \right] \\
+ 4 \left( 4x^2 - 4x + 3 \right) \zeta_3 + 8 \left( 4x^2 - 4x + 3 \right) L_s \right] + \left[ -16 \left( -8x^2 + 8x + \left( 3x^2 - 3x + 3 \right) \zeta_3 - 4 \right) \right] \\
- 96 \left( x^2 - x + 1 \right) L_s \right] \right\}, \tag{30}
\end{equation}

where $L_m$ and $L_y$ are defined in Eq. (26) and

\begin{equation}
L_s = \log \left( \frac{s}{\mu^2} \right). \tag{31}
\end{equation}

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