Recently, we extracted the strong coupling, $\alpha_s(m_\tau^2)$, from the revised ALEPH data for non-strange hadronic tau decays. Our analysis is based on a method previously used for the determination of the strong coupling from OPAL data. In our strategy, we employ different moments of the spectral functions both with and without pinching, including Duality Violations, in order to obtain fully self-consistent analyses that do not rely on untested assumptions (such as the smallness of higher dimension contributions in the OPE). Here we discuss the $\alpha_s$ values obtained from the ALEPH and the OPAL data, the robustness of the analysis, as well as non-perturbative contributions from DVs and the OPE. We show that, although the $\alpha_s$ determination is sound, non-perturbative effects limit the accuracy with which one can extract the strong coupling from tau decay data. Finally, we discuss the compatibility of the data sets and the possibility of a combined analysis.

Keywords: strong coupling; tau decays; QCD.

*Speaker
1. Introduction

Since the 90s, hadronic tau decays have been used to extract the QCD coupling, $\alpha_s$. One of the appealing features of this determination is that it represents a non-trivial test of the $\alpha_s$ evolution as predicted by the celebrated QCD $\beta$-function. The scale set by the $\tau$ mass is rather low, $m_\tau \approx 1.78$ GeV, but it still allows for a meaningful perturbative treatment, provided also non-perturbative contributions are taken into account. It has become standard to organize the QCD description of these decays in the form of Wilson’s operator product expansion (OPE).\(^1\) In this OPE, apart from the purely perturbative contribution and quark-mass corrections, non-perturbative QCD condensates also occur.

The extraction of $\alpha_s$ is performed through the use of finite energy sum rules (FESR). Observables such as the ratio

$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau \text{hadrons}(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^-\bar{\nu}_e(\gamma)]},$$

(1)
can be written as weighted integrals over the experimentally accessible QCD spectral functions, that can be reconstructed from the measurement of the dominant exclusive channels in the decays $\tau \to (\text{hadrons}) + \nu_\tau$.\(^2\) The integrals of experimental data are performed over the total hadronic momentum $s$ and run from zero to $m_\tau^2$. Clearly, the OPE description is not valid in the low energy part of this interval. One then resorts to the analyticity properties of the QCD correlators to write the theoretical counterpart of the weighted spectral integral as an integral along a closed circle of radius $|s| = m_\tau^2$ in the complex plane. Additional sum rules, apart from the one giving $R_\tau$, can be constructed by using different weight functions. This freedom is exploited in order to constrain additional parameters of the OPE, such as QCD condensates, and extract them in combination with $\alpha_s$.

The QCD spectral functions (in the vector and axial-vector channels) were determined originally from hadronic tau decays by the LEP collaborations ALEPH and OPAL, in the 90s.\(^3,4\) Recently, a re-analysis of the ALEPH data was published.\(^5\) This analysis was performed with a different binning and employing a new unfolding method. The new analysis corrects a problem in the older version of the correlation matrices.\(^6\) Since this correction, ALEPH’s can be considered as the best data set, since it has smaller uncertainties.

On the theory side, two aspects of these $\alpha_s$ determinations have received special attention, recently. The first regards the use of the Renormalisation Group in the improvement of the perturbative series. Several prescriptions are advocated in the literature, among which two stand out the most: Contour Improved Perturbation Theory (CIPT)\(^7,8\) and Fixed Order Perturbation Theory (FOPT).\(^9\) The two prescriptions lead to different perturbative series and, hence, to different values of $\alpha_s$ when used in $\alpha_s$ extractions. The differences have not diminished with the computation of the NNNLO term, $\mathcal{O}(\alpha_s^4)$, in the perturbative expansion.\(^10\) This discrepancy remains one of the main sources of theoretical error associated with $\alpha_s$ from $\tau$ decays. Arguments in favour of FOPT have been put forward recently,\(^11,12,13\) but the
issue is still being debated.\textsuperscript{14,15} We prefer to remain conservative and always quote two values of $\alpha_s$ from our analyses.

The second aspect that has received attention in the past few years is related to the non-perturbative contributions. Since the work of Ref. 16, it has become clear that the treatment of the higher order condensates advocated in some of the recent $\alpha_s$ analyses is inconsistent. The OPE alone, moreover, cannot account for all non-perturbative effects in the vicinity of the Minkowski axis; duality violations (DV) are present and should be included in the theoretical description. However, since the kinematic weight function related to $R_\tau$ possesses a double zero on the positive axis at $s = m^2_\tau$ that suppresses contributions from this region, the DV part of the correlators was often disregarded. For this reason, older analyses were restricted to the so-called pinched moments — those moments that also exhibit a zero on the axis. The problem with that strategy is that more pinching enhances higher-dimension contributions in the OPE, which augments the number of parameters to be determined from the experimental data. One way around this complication, pursued in a number of analyses, was to simply assume that contributions of dimension higher than 8 could be neglected. However, it was shown in Ref. 16 that this leads to results that do not survive self consistency checks — they provide poor matches to the corresponding spectral integrals when $s_0$ is lowered below $m^2_\tau$. In conclusion, the assumption is too strong and results based on it carry an unquantified systematic uncertainty.\textsuperscript{a}

Recently, thanks to progress in modelling the DV contributions,\textsuperscript{17,18,19,20} it has become possible to include them in a fully self-consistent analysis of $\alpha_s$.\textsuperscript{21,22,23} In this new framework, no additional assumptions regarding the OPE are required: at each order in the OPE, the leading contribution is taken into account and, as a consequence, we find that the results thus obtained pass all consistency checks. After a brief recollection of the theoretical framework in Sec. 2, we discuss, in Sec. 3, results from analyses following this new strategy obtained from ALEPH and OPAL spectral function data. An issue that is still open is to what extent one can combine results obtained from the two data sets, and whether or not a combined analysis is justified. This question will be touched on in Sec. 4. In Sec. 5 we present our conclusions.

2. Analysis framework

The purpose of this section is to provide a brief review of the framework of our analysis. The details can be found in the original publications, Refs. 21, 22 and 23.

We employ FESRs of the following form\textsuperscript{1,25}

$$I^{V/A}_{V/A}(s_0) \equiv \int_0^{s_0} \frac{ds}{s_0} \frac{w(s)}{s_0} \rho^{(1+0)}_{V/A}(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s_0} w(s) \Pi^{(1+0)}_{V/A}(s), \quad (2)$$

\textsuperscript{a}We refer to S. Peris’ contribution to these proceedings for a more detailed discussion about this issue.\textsuperscript{24} See also in Section VII of Ref. 23.
where $\rho_{V/A}^{(1+0)}$ is the experimentally accessible spectral function and the weight-functions $w(s)$ are polynomials in $s$. The correlators $\Pi_{V/A}^{(1+0)}(s)$ are given by

$$i \int d^4 x e^{i q x} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle$$

$$= (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_{V/A}^{(1+0)}(s) + q^2 g_{\mu\nu} \Pi_{V/A}^{(0)}(s),$$

\hspace{1cm} (3)

with $s = q^2 = -Q^2$. The current $J_\mu$ is one of the non-strange $V$ or $A$ currents, namely, $\bar{u} \gamma_\mu d$ or $\bar{u} \gamma_\mu \gamma_5 d$. The superscripts $(0)$ and $(1)$ refer to spin. For a given weight function, we construct FESRs at several values of $s_0 \leq m^2$.

The correlators $\Pi_{V/A}^{(1+0)}(s)$ admit a decomposition into three parts

$$\Pi_{V/A}^{(1+0)}(s) = \Pi_{\text{pert}}^{(1+0)}(s) + \Pi_{\text{OPE}}^{(1+0)}(s) + \Pi_{\text{DV}}^{(1+0)}(s).$$

\hspace{1cm} (4)

In the above, “pert” denotes perturbative (which can be regarded as the dimension zero contribution to the OPE), “OPE” refers to OPE corrections of dimension larger than zero (including quark-mass corrections), whereas “DV” denotes the DV contributions to $\Pi_{V/A}^{(1+0)}(s)$.

The ambiguity related to the use of the renormalisation group to which we alluded affects (mainly) the perturbative part of the correlators. When treating the contour integration in the FESR, one must adopt a prescription for the renormalisation scale. As discussed above, we perform our analysis using CIPT or FOPT and quote results for both on an equal footing.

Contributions from higher dimensions in the OPE are parametrised with effective condensates $C_D$ as

$$\Pi_{\text{OPE}}^{(1+0)}(s) = \sum_{k=1}^{\infty} \frac{C_{2k}(s)}{(-s)^k}.$$ \hspace{1cm} (5)

Since we work with non-strange spectral functions, the dimension-two quark-mass corrections can safely be neglected.$^b$ Therefore, in our analysis, the dimension two contribution is absent. In principle, the first non-negligible contribution is dimension 4, encoded in $C_4$, that can be related to the gluon and quark condensates. However, the weight functions employed in our analysis (see below) are polynomials constructed from combinations of unity, $s^2$, and $s^3$. As a result, in our FESRs, the leading contributions from the OPE arise solely from $C_6$ and $C_8$. Subleading logarithmic corrections to these coefficients are neglected and all $C_D$ are treated as constants.

Through the use of analyticity, the DV contribution to the sum rules can be cast as$^{20}$

$$D_w(s_0) = - \int_{s_0}^{\infty} \frac{ds}{s_0} w(s) \rho^{\text{DV}}(s),$$ \hspace{1cm} (6)

\hspace{1cm} $^b$This has been checked explicitly.
where \( \rho^{DV}(s) \) is the DV part of the spectral function in a given channel

\[
\rho^{DV}(s) = \frac{1}{\pi} \text{Im} \Pi_{DV}^{(1+0)}(s). \tag{7}
\]

For \( s \) large enough, the DV part of the spectral function can be parametrised with the Ansatz of Refs. 18, 19

\[
\rho_{V/A}^{DV}(s) = \exp \left( -\delta_{V/A} - \gamma_{V/A} s \right) \sin \left( \alpha_{V/A} + \beta_{V/A} s \right). \tag{8}
\]

The parameters associated with DV are, in principle, channel dependent. There is no a priori reason to treat them as equal in \( V \) and \( A \) since they depend, ultimately, on the large-\( s \) behaviour of the QCD resonances that appear in each channel. Therefore, we keep them different in \( V \) and \( A \), thus avoiding any additional assumptions. The parametrisation of Eq. (8) adds four new parameters per channel.

Since we include the DV contribution, our analysis does not have to be restricted to pinched weight functions. Rather, a weight function without pinching is included in order to constrain the DV parameters better. After extensive explorations in Refs. 21 and 22 we concluded that it is sufficient to work with three weight functions, namely,

\[
w_1 = 1, \quad w_2 = 1 - x^2, \quad w_3 = (1 - x)^2 (1 + 2x), \tag{9}
\]

where \( x \equiv s/s_0 \). The weight function \( w_3 \) appears multiplying the \( J = 0 + 1 \) spectral function in the expression of \( R_\tau \), of Eq. (1), and is hence often called the “kinematical weight.” The main motivation behind this choice is the fact that we want to perform a self-consistent analysis, including all leading order contributions in the OPE, without making any untested assumption about higher order condensates. The explorations of Refs. 21 and 22 established that this set of weight functions fulfils these requirements and allows for a good determination of \( \alpha_s \). As a final remark, we observe that these three weight functions have good perturbative behaviour, in the sense of the analysis performed in Ref. 12.

3. Fits and main results

3.1. Fits

In our analyses we have performed several different fits: to individual channels, \( V \) or \( A \), or combining \( V \) and \( A \) together; fits with a single weight function or combining different subsets of the weight functions given in Eq. (9). (In the combined \( V \) and \( A \) fits the equality of \( \alpha_s \) in the two channels is, of course, always imposed.) Also, we always employ a window of \( s_0 \) values [\( s_{\text{min}}, s_{\text{max}} \)] and we make sure to test the stability of the results under variations of this window. The window must be chosen such as to maintain the validity of the description of the QCD correlator through Eq. (4).
Fits that involve a single weight function are performed minimising a standard $\chi^2$, including all correlations among the different moments. Simultaneous fits to more than one weight function, on the other hand, are too strongly correlated to allow for the use of a fit quality of this type. Alternative fit qualities must be employed in these cases. The error propagation (including all correlations) is done according to a standard procedure, described in detail in the appendix of Ref. 21.

Several consistency checks are performed on the fit results to assess their robustness. On the statistical side, we have performed a study of the posterior probability in the parameter space using Markov-chain Monte Carlo simulations. This was particularly helpful in the case of OPAL data, since this data set has larger uncertainties that produce a shallower fit quality, which oftentimes displays multiple minima. On the physical side, the results must be as immune as possible to changes in the $s_0$ window employed in the fits. To maximize the use of the data, we always integrated the experimental results up to the last bin, $s_{\max} = m_\tau^2$. The lower edge of the $s_0$ window, $s_{\min}$, was varied between 1.3 and 1.7 GeV$^2$ to check for the stability of the results. Other important tests that we implemented are the Weinberg sum rules. The results of our fits including both $V$ and $A$ channels fulfil these sum rules within the whole fit window employed in our analyses.

3.2. Results for $\alpha_s$ from the OPAL- and ALEPH-based analyses

Our fits determine simultaneously $\alpha_s$, and non-perturbative contributions, such as OPE condensates as well as the DV parameters that were introduced in Eq. (8). Here, we choose to focus mainly on $\alpha_s$ results. The results are given in full detail in the original publications.$^{22,23}$

At first, we restricted our attention to OPAL data.$^{21}$ The reason why we focussed, at the time, on OPAL’s data is the fact that it was discovered that the correlation matrix of the then publicly available ALEPH spectral functions missed a contribution from the unfolding procedure.$^6$ Therefore, we decided to perform an update of the original OPAL spectral functions to reflect modern values of branching fractions and constants used for normalisation.$^6$ The results described here are based on the analysis of these updated OPAL spectral functions, first reported in Ref. 22.

The $\alpha_s$ values that we obtained from the different fit set-ups described above are consistent within their somewhat large error bars (dominated by statistical errors of the OPAL data). However, fits including the $A$ channel do require an additional assumption, related to the larger mass of the $a_1$ resonance, as compared to the $\rho$. Since the parametrisation of DV that we employed is based on the asymptotic regime of the QCD resonances, one must assume that this regime is already reached

---

$^a$The exact values used in this variation depend on the data set, since the binning is different.

$^b$We checked that our results fulfil both the first and second Weinberg sum rules,$^{26}$ as well as the sum rule for the pion electromagnetic mass splitting.$^{27}$

$^c$The updated version of the OPAL spectral functions can be provided upon request.
close to the tail of the $\alpha_1$. The overall consistency of our results seems to indicate that this assumption is fulfilled within uncertainties. However, to avoid having an unquantified systematic from this extra assumption, we prefer to quote as final values those that arise from fits to the $V$ channel only.

In the case of the analysis based on OPAL data, our final values come from the analysis of the $V$ channel with the weight function $w_1 = 1$. This choice is motivated by the wider range of stability of these results against variations of $s_{\text{min}}$ although, again, all our results are consistent among themselves. In the $\overline{\text{MS}}$ scheme and with $N_f = 3$ we found

$$
\alpha_s(m^2_\tau) = 0.325 \pm 0.018 \quad (\text{OPAL, FOPT}),
$$
$$
\alpha_s(m^2_\tau) = 0.347 \pm 0.025 \quad (\text{OPAL, CIPT}).
$$

The errors that we quote are dominated by statistics, but they include an estimate of the error due to variations of $s_{\text{min}}$ and due to the truncation of the perturbative series. When evolved to $m^2_Z$ these results read (with $N_f = 5$)

$$
\alpha_s(m^2_Z) = 0.1191 \pm 0.0022 \quad (\text{OPAL, FOPT}),
$$
$$
\alpha_s(m^2_Z) = 0.1216 \pm 0.0027 \quad (\text{OPAL, CIPT}).
$$

The recently published re-analysis of the ALEPH data\(^5\) corrects the correlation matrices and provides us with reliable spectral functions that have errors significantly smaller than those of OPAL. At present, ALEPH’s can be considered as the best data set.

The analysis within our framework based on the new version of the ALEPH spectral functions was first presented in Ref. 23. The main difference with respect to results derived from OPAL data is in the uncertainties. The significantly smaller uncertainties of ALEPH spectra not only translate into smaller uncertainties for the parameters of the fit, they also resolve any possible ambiguity due to multiple minima in the fit quality. In general, $\alpha_s$ and the $V$ and $A$ channel DV parameters are much better determined. The analysis based on the Markov-chain Monte Carlo as well as the physical tests of the outcome of the fits indicate that the data are sufficient to constrain the parameters of the fits with reasonable accuracy, contrary to the speculation that “one has a too large number of free parameters to be fitted” made in Ref. 28. For the same reasons, there is nothing that indicates that our uncertainties are underestimated.

The final value of $\alpha_s$ in the ALEPH-based analysis is obtained from a fit to the vector channel combining the three weight functions of Eq. (9). The choice for this fit is based on the wider stability range against variations of $s_{\text{min}}$, but other results

\(^5\)The number of parameters varies from 5 to 13, depending of the specific fit set-up one considers.
are compatible within errors. We find for $\alpha_s$ in the $\overline{\text{MS}}$ and with $N_f = 3$
\begin{align}
\alpha_s(m^2_\tau) &= 0.296 \pm 0.010 \quad \text{(ALEPH, FOPT),} \\
\alpha_s(m^2_\tau) &= 0.310 \pm 0.014 \quad \text{(ALEPH, CIPT).} 
\end{align}
(12)

As before, errors are dominated by statistics but include an estimate of the error due to varying the $s_0$ window and the truncation of the perturbative series. Evolving these results to $m^2_Z$ we find (with $N_f = 5$)
\begin{align}
\alpha_s(m^2_Z) &= 0.1155 \pm 0.0014 \quad \text{(ALEPH, FOPT),} \\
\alpha_s(m^2_Z) &= 0.1174 \pm 0.0019 \quad \text{(ALEPH, CIPT).} 
\end{align}
(13)

Uncertainties in $\alpha_s$ are smaller when using the ALEPH data. However, the improvements in other parameters such as, for example, $\delta_V$ of Eq. (8), are more significant. As an illustration, Fig. 1 shows the allowed intervals for $\alpha_s$ and $\delta_V$ within 68% and 95% confidence levels from OPAL- and ALEPH-based analyses. The improvement in the vertical spread of Fig. 1(b) is impressive (note the different scales).

3.3. Final values

The results for $\alpha_s$ obtained from ALEPH data tend to be lower than those from the analysis of OPAL data. However, within uncertainties, they are compatible. Since the spectral functions are virtually uncorrelated it is legitimate to perform a weighted average of our results. The weighted average, which we consider our final result for $\alpha_s$, gives ($\overline{\text{MS}}$, $N_f = 3$)
\begin{align}
\alpha_s(m^2_\tau) &= 0.303 \pm 0.009 \quad \text{(ALEPH and OPAL, FOPT),} \\
\alpha_s(m^2_\tau) &= 0.319 \pm 0.012 \quad \text{(ALEPH and OPAL, CIPT),} 
\end{align}
(14)

and at the $Z$ boson mass scale ($\overline{\text{MS}}$, $N_f = 5$)
\begin{align}
\alpha_s(m^2_Z) &= 0.1165 \pm 0.0012 \quad \text{(ALEPH and OPAL, FOPT),} \\
\alpha_s(m^2_Z) &= 0.1185 \pm 0.0015 \quad \text{(ALEPH and OPAL, CIPT).} 
\end{align}
(15)

A visual account of the individual results at the $\tau$ mass scale as well as the FOPT and CIPT averaged values is given in Fig. 2. The averages are compared in Fig. 2(b). This comparison shows an overall consistent picture, with results compatible with each other and with the average. The residual difference between CIPT and FOPT remains at the same order of the individual uncertainties.

\footnote{We should remark that the fit set-up discussed in Sec. 7 of Ref. 29 does not correspond to the fit that gives our $\alpha_s$ value.}
3.4. A (failed) attempt to fit the spectral functions

A criticism that has been raised against our analysis strategy regards the use of moments of $w_1 = 1$ integrated up to several different $s_0$ values inside the window $[s_{\text{min}}, s_{\text{max}}]$. Besides the integral of the spectral function up to a certain $s_0$, such a fit includes information about the shape of the spectral functions which, of course, plays a role in the extraction of the DV parameters. Clearly, because we use a FESR of the type shown in Eq. (2), the experimental data below $s_{\text{min}}$ also enter the fit. In view of this fact, it may be legitimate to question whether the integral over the data is putting constraints on our value of $\alpha_s$ and to what extent these constraints come from the shape of the spectral function itself. In fact, it is the
integrated spectral function that makes a crucial difference in the $\alpha_s$ extraction. For this reason, equating our procedure to a mere fit of the spectral functions would be very misleading.\(^{30}\)

An exercise that can be helpful in understanding what is constraining the $\alpha_s$ values that we obtain is to perform a direct fit to the actual spectral functions in the interval $s_{\text{min}} < s < s_{\text{max}}$. This fit excludes all experimental information for $s < s_{\text{min}}$. Obviously, we do not advocate the use of such a fit in an $\alpha_s$ extraction, since one would be ignoring an important part of the spectral functions — a part which plays a key role in obtaining the total weighted spectral integral. However, the inclusion of DVs in our description of the QCD correlators allows, at least in principle, for a fit of this type. A direct fit to the ALEPH vector spectral function in the window $1.575 \text{ GeV}^2 \leq s \leq m^2_\tau$ produces an acceptable fit ($\chi^2/\text{dof} = 1.62$), but this fit can barely put any constraint on the strong coupling: it gives $\alpha_s(m^2_\tau) = 0.3 \pm 0.1$ (for FOPT). Moreover, the results are essentially meaningless since the correlations between the fit parameters are huge. For instance, $\alpha_s$ is 96\% correlated with $\delta V$.\(^{30}\)

We also observe a rather poor agreement between theory and experiment in FESR obtained from the results of this fit.

What can be learnt from this exercise is that the experimental information from regions below $s_{\text{min}}$, that enter our fits through the sum rule of Eq. (2), is absolutely

\(^{30}\)In all $\alpha_s$ analysis from $\tau$ decays (including ours) the value of $\alpha_s$ turns out to be correlated with non-perturbative parameters. However, these correlations reach at most $\sim 65\%$.

Fig. 2. Comparison between the $\alpha_s(m^2_\tau)$ values obtained from the OPAL\(^{22}\) and ALEPH-based\(^{23}\) analysis. FOPT results are shown in the left-hand panel, CIPT results in the right-hand panel. Weighted averages given in Eq. (14) are also shown for comparison.
crucial to our $\alpha_s$ determination. Given the present statistical errors, the shape of the spectral function only very weakly constrains the value of $\alpha_s$. This happens because the spectral function itself in the larger $s$ region has a weak dependence on $\alpha_s$ and, hence, more strongly constrains the DV parameters. Therefore, the dominant source of constraint on $\alpha_s$ is the one imposed, through analyticity, by the large contribution in the weighted integral from the low-$s$ region. The inclusion of the integral over the data reduces the uncertainty by an order of magnitude and leads to a good match in the FESR. In conclusion, what constrains the value of $\alpha_s$ is the integral over the data — as it should be — and not the shape of the spectral functions.

4. Preliminary combined analysis

It is reasonable to assume that the spectral functions obtained by OPAL and ALEPH are uncorrelated. Given the fact that the outcome of our analysis is fully compatible in the two cases, the averages of Eqs. (14) and (15) are justified, as a first approximation. However, they do not guarantee that a set of parameters that gives a good description of both data sets exist. The most rigorous way to combine all the experimental information is to fit to both data sets simultaneously. Although it is expected that the results will be closer to ALEPH ones, given the smaller uncertainties, it remains a non-trivial test to check that a single parameter set can describe both data sets reasonably well.

Here we briefly discuss preliminary results of such a combined analysis. In this first exploration we fit only to the $V$ channel, with moments of $w_1 = 1$. The $\chi^2$ to be minimised is simply given by

$$\chi^2 = \chi^2_{\text{ALEPH}} + \chi^2_{\text{OPAL}},$$

(16)

since the correlations between different data sets are assumed negligible. In this type of fit, one has, in principle, two different $s_0$ windows to consider: one for the OPAL data and one for ALEPH’s. Fit results should be independent of variations in any of the fit windows.

Results for three different fits, performed in different $s_0$ windows, are shown in Table 1. A representative two-dimensional contour plot on the $\delta V-\alpha_s(m^2)$ plane is given in Figure 3. The results show three main features:

- It is possible to find parameter values that give a good description of the two data sets simultaneously. The $\chi^2$ values obtained are acceptable (if a bit too small) and $p$ values are close to unity.
- A comparison with the results of Refs. 22 and 23 shows that $\alpha_s$ values are more stable in this combined analysis than in fits to a single data set.
- $\alpha_s$ values tend to be slightly larger than the weighted average, but there is good agreement within errors.
- Values of DV parameters, on the hand, are very close to values obtained in the analysis of ALEPH data.
Table 1. Preliminary results for combined fits to ALEPH and OPAL data with \( w(x) = 1 \), vector channel only, FOPT. The ALEPH and OPAL data are assumed to be uncorrelated. All correlations inside the individual data sets are taken into account. The values \( s_{\text{min}}^{\text{AL}} \) and \( s_{\text{min}}^{\text{OP}} \) show the choice for the minimum \( s_0 \) in the ALEPH and OPAL data respectively. Uncertainties are solely statistical.

| \( s^{\text{OP}}_{\text{min}} \) [GeV\(^2\)] | \( s^{\text{AL}}_{\text{min}} \) [GeV\(^2\)] | \( \chi^2/\text{dof} \) | \( \alpha_s(m_\tau^2) \) | \( \delta_V \) | \( \gamma_V \) | \( \alpha_V \) | \( \beta_V \) |
|---|---|---|---|---|---|---|---|
| 1.5 | 1.50 | 46.4/70 | 0.308(10) | 3.43(37) | 0.63(23) | -0.98(66) | 3.60(34) |
| 1.5 | 1.55 | 43.9/68 | 0.307(09) | 3.56(38) | 0.57(23) | -1.19(64) | 3.71(33) |
| 1.6 | 1.60 | 41.0/63 | 0.308(10) | 3.50(40) | 0.60(23) | -1.10(85) | 3.66(43) |

Fig. 3. Two-dimensional contour plots for 68% and 95% confidence levels in the \( \alpha_s(m_\tau^2) - \delta_V \) plane for the combined fit shown in the second row of Table 1. The star marks the central values of the fit, \( A \) gives the values of a fit to ALEPH data whereas \( O \) gives the central values corresponding to a fit to OPAL data.

5. Conclusions

We have discussed a new strategy for the QCD analysis of hadronic \( \tau \) decay data. The main advantage of this strategy is that it allows for a fully self-consistent analysis, without relying on any untested assumption. This can be achieved thanks to the introduction of a physically motivated parametrisation for the DV contributions. Our analysis does not rely only on pinched moments, avoiding contamination by higher-order OPE condensates.

The strategy was applied to OPAL data at first\(^{22}\) and, more recently, to ALEPH data as well.\(^{23}\) In both cases one can extract \( \alpha_s \) together with non-perturbative contributions such as the dimension 6 and 8 OPE condensates and DV parameters. The results from the two data sets are compatible. Since the data are uncorrelated we can perform weighted averages and, at present, our recommended value of \( \alpha_s \) is obtained as an average from the ALEPH- and OPAL-based determinations. These values can be found in Eqs. (14) and (15).
We have performed extensive statistical and physical tests on the results of our fits. They indicate that the results are reliable and satisfy various self-consistency checks. In addition, the uncertainties obtained are realistic and the number of parameters that we fit is manageable. Among the self-consistency checks passed by our results are those given by the Weinberg sum rules.\textsuperscript{21,22,23,24}

Finally, we have performed preliminary simultaneous fits to ALEPH and OPAL data. The results obtained are encouraging because they are stable and give a good representation of both data sets. This lends support to the validity of the theoretical description and to the compatibility of the two data sets.

Further progress would require a better theoretical understanding of DVs and/or better spectral functions. In principle, with a dedicated effort, the latter could be extracted from Belle and BaBar data and could lead to significant progress in the field.

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