Quadrotor Stabilization and Tracking Using Nonlinear Surface Sliding Mode Control and Observer

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Abstract: We propose a control method wherein the estimated angles converge to the desired value for quadrotor attitude stabilization and position tracking. To improve the performance of a quadrotor system, the unmeasured states of the quadrotor are estimated using a sliding mode observer (SMO). We set up a quadrotor dynamic model and augment the quadrotor dynamics by an SMO. We also derive the control inputs by sliding mode control (SMC) and calculate the desired angle of the quadrotor to reach the target position with the control inputs. For fast convergence speed and increased robustness of tracking performance, a nonlinear sliding surface is applied to SMC. The angle of the quadrotor converges to the desired value through the operation of SMC with a nonlinear sliding surface. The target tracking performance is improved by adaptively switching the deceleration curve of the sliding mode surface with a nonlinear curve. Using a tracking system based on a nonlinear surface sliding mode control (NSMC) and SMO, the quadrotor reaches the target position with a decreased settling time. The performance and effectiveness of the proposed system are proved through simulation results.

Keywords: sliding mode control; sliding mode observer; quadrotor; stabilization; tracking control

1. Introduction

Recently, unmanned aerial vehicles (UAVs) have attracted considerable research attention. A quadrotor that is one of the most widely used UAVs in various industrial fields is classified as a rotorcraft since it takes off vertically using four motors. Many researchers have studied quadrotors for various purposes, such as swarming, parcel delivery, exploration, and vehicles [1–6]. In various studies, the basic technology for quadrotor stabilization and movement is attitude control based on proportional-integral-derivative (PID) or sliding mode control (SMC) as general controllers. PID, which has the advantage of simple control structure, is a widely used maneuvering algorithm in controller design [7–12]. However, PID has a disadvantage against disturbances and is limited for precision control. Meanwhile, SMC shows strong, robust performance in the presence of disturbances and model uncertainties [13–19].

A quadrotor system is sensitive to parameter uncertainties and external disturbances. To solve this problem, we propose an augmented system that adopts correction variables (\( z_i \)). When the state converges to the sliding surface, the trajectory becomes less sensitive to disturbances during the sliding phase. The stability of the control system is proved by following the Lyapunov stability theorem. However, in spite of the robust characteristic to disturbances or model uncertainties, sliding mode control has drawbacks of chattering phenomena and time delay for attitude stabilization. Even if the state converges to the sliding surface, a discontinuous control value with high...
frequency switching action is induced so that it causes unnecessary noises and switching stress in the system. As a solution, we propose a nonlinear sliding surface in SMC to reduce the error between the desired value and the measured value. Moreover, the proposed nonlinear sliding surface reduces the response time of the controller. The control input is derived to satisfy the stability condition of the Lyapunov function with the sliding surface.

Sliding mode observer (SMO) is widely used to improve the state estimation performance [20–24]. In SMO, we estimate the unmeasured state and use a saturation function to set up the correction variable with the estimated and the measured data. To improve tracking performance, SMC has been applied in various controller design problems [25]. A quadrotor is a nonlinear system with four independent inputs and six coordinate outputs. A novel SMC based on a nonlinear disturbance observer was proposed in [26] wherein the disturbance estimation was included to attenuate the mismatched uncertainties. The control algorithm in [27] was a hybrid of SMC and backstepping to stabilize the attitude and was widely adapted in various control applications. To achieve sensorless control of a permanent magnet synchronous motor, a sliding mode observer was proposed in [28] according to the back electromotive force model. By estimating the capacitor voltages from measurement of the load current, an adaptive gain second-order sliding mode observer for a multi-cell power converter system was designed in [29]. The capacitor voltages were estimated under the input sequences using the observability concept.

In this paper, we propose an SMC- and SMO-based control method for the position tracking and attitude stabilization of a quadrotor system. We set up the quadrotor dynamic model and design the augmented quadrotor dynamics with correction variables obtained by SMO. In general, there are two approaches to derive the quadrotor motion dynamics: Newton–Euler and Euler–Lagrange methods. Newton–Euler formalism gives physical insights through the derivation. Euler–Lagrange formalism provides the linkage between the classical framework and the Lagrangian or Hamiltonian method [30]. In this paper, the dynamics of the quadrotor is formulated based on Newton–Euler approach. To derive the appropriate control input \( u(t) \) by SMC, we use the augmented quadrotor dynamics and the angles estimated by SMO. In SMC, the desired angle \( (\phi, \theta) \) is derived from the target position \( (x, y, z) \) and the control input \( u(t) \). With the desired angle \( (\phi, \theta) \), we operate the quadrotor system to reduce the error between the desired and measured angles. Moreover, to solve the time delay problem in the target tracking time due to the deceleration curve in SMC, we set up the nonlinear sliding surface to reduce the reaching time of a sliding mode controller. The design of a nonlinear sliding surface incorporates the deceleration curve of SMC into the sliding surface so that the delay of reaching time can be effectively reduced. As a result, the quadrotor follows the target position \( (x, y, z) \) by the SMO and control system.

This paper is structured as follows. In Section 2, we obtain the quadrotor dynamic model. In Section 3, the SMO estimates the unmeasured angle and augmented dynamics of the quadrotor. In Section 4, a nonlinear sliding surface is designed to reduce the influence of the deceleration curve. In Section 5, the desired angle \( (\phi, \theta) \) and control input \( u(t) \) are obtained by SMC with the estimated angle and augmented dynamics by SMO. In Section 6, we prove the proposed method by simulation. Finally, in Section 7, we present our conclusion.

2. Quadrotor Dynamic Modeling

The quadrotor, which can take off and land vertically, is composed of four motors. Figure 1 describes two pairs of propellers \( (\Omega_1, \Omega_2) \) and \( (\Omega_2, \Omega_1) \), which rotate in opposite directions. Two diagonal motors \( (\Omega_1, \Omega_2) \) rotate in a clockwise direction, while
the other two \((\Omega_2, \Omega_4)\) rotate in a counterclockwise direction. A quadrotor can change its direction rapidly by its two pairs of motors [31]. We can acquire different positions and attitudes of the quadrotor by adjusting the velocity of the four propellers. Figure 1 also shows the earth-fixed frame and rigid body model of a quadrotor. In this paper, the dynamic model of the quadrotor is represented based on the Newton–Euler equations. The Newton–Euler equations demonstrate the combined translational and rotational dynamics of a rigid body frame. The angles \((\phi, \theta, \psi)\), which represent roll, pitch, and yaw, respectively, are obtained from the rotation of the quadrotor’s body frame in the \(x\), \(y\), and \(z\) axes.

![Figure 1](image)

**Figure 1.** Configuration of a quadrotor.

\[
\begin{align*}
\ddot{\phi} &= \frac{I_x - I_z}{I_z} \dot{\theta} \dot{\psi} + \frac{J_z}{I_z} \dot{\Omega} \phi + \frac{l}{I_z} u_x + d_{\phi}, \\
\ddot{\theta} &= \frac{I_y - I_z}{I_z} \dot{\phi} \dot{\psi} + \frac{J_x}{I_z} \dot{\Omega} \theta + \frac{l}{I_z} u_y + d_{\theta}, \\
\ddot{\psi} &= \frac{I_z}{I_z} \dot{\phi} \dot{\theta} + \frac{l}{I_z} u_z + d_{\psi}, \\
\dddot{z} &= -g + \frac{1}{m} \cos \phi \cos \theta \dot{u}_z + d_z, \\
\dddot{x} &= \frac{1}{m} (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) u_x + d_x, \\
\dddot{y} &= \frac{1}{m} (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) u_y + d_y.
\end{align*}
\]

Here, \(I_{x,y,z}\) is the body inertia, \(J_r\) is the rotor inertia, \(\Omega\) is the overall rotor speed, \(l\) is the lever length, \(g\) is gravity, and \(m\) is the mass of the quadrotor. \(x, y,\) and \(z\) are the movement distance of the quadrotor in the direction of \(x, y,\) and \(z\) axes, respectively. As shown in Equation (1), the attitude and altitude of the quadrotor are decided by the gyroscopic effect resulted from the rigid body rotation in space and the rotor speed. The control inputs for quadrotor attitude are represented by \(u_x, u_y,\) and \(u_z\). The other control input, for altitude, is represented by \(u_z\). To consider the unknown disturbance in real environment, the dynamic model of Equation (1) contains the disturbance \((d_{\phi}, d_{\theta}, d_{\psi}, d_z, d_x, \text{ and } d_y)\) which causes the error between the measurement and the estimated value of the state. The control inputs can be represented as
where $\Omega_i$ is the rotational velocity of the $i$-th motor, $t$ is the thrust factor, and $d$ is the drag factor. We can obtain the whole motor velocity as represented in Equation (2). The dynamic model in Equation (1) can be rewritten in the state-space form of $\dot{x} = f(x, u)$ with the state vector $x = [x_1 \cdots x_12]^T$. The state vector is defined as follows:

\[
x_1 = \phi, \quad x_2 = \phi, \quad x_3 = \theta, \quad x_4 = \dot{\theta}, \quad x_5 = \psi, \quad x_6 = \dot{\psi},
\]
\[
x_7 = z, \quad x_8 = z, \quad x_9 = x, \quad x_{10} = x, \quad x_{11} = y, \quad x_{12} = y.
\]

In Equation (1), the mechanical structure can be represented as

\[
c_1 = \frac{I_y - I_z}{I_x}, \quad c_2 = \frac{J_x}{I_x}, \quad c_3 = \frac{I_y - I_z}{I_y}, \quad c_4 = \frac{J_y}{I_y},
\]
\[
c_5 = \frac{I_z - I_y}{I_z}, \quad c_6 = \frac{l}{I_x}, \quad c_7 = \frac{l}{I_y}, \quad c_8 = \frac{l}{I_z},
\]
\[
u_x = \cos x_1 \sin x_2 \cos x_3 + \sin x_1 \sin x_3,
\]
\[
u_y = \cos x_1 \sin x_2 \sin x_3 - \sin x_1 \cos x_3.
\]

In Equation (4), $c_1$ through $c_8$ are parameters comprising body inertia, lever length, and rotor inertia. $u_x$, $u_y$ are the control inputs of the quadrotor using roll, pitch, and yaw. From Equations (1) and (3), the dynamics of the quadrotor can be obtained as Equation (5).

\[
\dot{x} = f(x, u) = \begin{bmatrix}
x_2 \\
-G + \frac{1}{m}(\cos x_1 \cos x_3)u_4 + d_z \\
-x_10 \\
\frac{1}{m}u_4 + d_x \\
\frac{1}{m}u_4 + d_y
\end{bmatrix}
\]
3. Sliding Mode Observer-Based Augmented Dynamics

The purpose of an SMO is to estimate the states of a quadrotor system. Sliding mode observer works using a switching function to minimize the error between the real quadrotor’s state and the observer output [32–34]. In this section, we estimate the quadrotor rotational angle and angular velocity and design the augmented quadrotor dynamics by SMO. The super-twisting higher order sliding mode method is adopted as a state observer design for model uncertainties [34]. Using the super-twisting observer, we estimate the state values in finite time under the presence of disturbances. For the stabilization and tracking control of the quadrotor, the sliding mode controller is used in the observer design.

\[ \dot{x}_i = f(\dot{x}, u) + Lv, \]
\[ v = \zeta \text{sgn}(\dot{x}_i - \dot{o}_i), \]  

\[ \text{(6)} \]

where \( \zeta > 0 \), \( L > 0 \) and \( \text{sgn}(\cdot) \) means a signum function. The outputs of \( o_i \) \((i = 1, 3, 5)\) mean the measurements of attitude for roll, pitch, and yaw angles, respectively. \( \dot{x}_i \) is an estimated state of \( x_i \) and \( \dot{o}_i \) is an estimated output of \( o_i \). \( L \) is an observer gain that is large enough to guarantee the sliding mode. The correction variables are used in the adaptive SMO to estimate the system state in the presence of unknown external disturbances. To design the augmented model, we define the integral value \( (\omega_i) \) of the correction variables \( (z_i) \). These variables can be represented as follows:

\[
\begin{align*}
    z_1 &= k_1 |E_i|^\frac{3}{2} \text{sat}(E_i), \\
    z_2 &= k_1 |E_i|^\frac{3}{4} \text{sat}(E_i), \\
    z_3 &= k_1 |E_i|^\frac{3}{4} \text{sat}(E_i), \\
    z_4 &= k_1 |E_i|^\frac{3}{4} \text{sat}(E_i), \\
    z_5 &= k_1 |E_i|^\frac{3}{4} \text{sat}(E_i), \\
    z_6 &= k_1 |E_i|^\frac{3}{4} \text{sat}(E_i), \\
    z_7 &= k_1 |E_i|^\frac{3}{4} \text{sat}(E_i).
\end{align*}
\]

\[ \text{(7)} \]

In Equation (7), \( E_i \) is the error between the measured and estimated values, expressed as \( E_i = x_i - \dot{x}_i \), \((i = 1, 3, 5)\). We define a new state variable by applying SMO with the integral value and correction variables added to the quadrotor angles,

\[
    s = \begin{bmatrix}
    \dot{x}_1, \dot{x}_2, \omega_1, \dot{x}_3, \dot{x}_4, \omega_2, \dot{x}_5, \omega_3, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}
    \end{bmatrix}^T.
\]

We propose an observer based on differentiation of the state variables \( x_i \) through \( x_{12} \) of the quadrotor attitude dynamics as Equation (8).
The integral value and correction variables are applied to the roll, pitch, and yaw angles for quadrotor stabilization. Based on SMO with an augmented quadrotor model, the control procedure of the quadrotor is shown in Figure 2.

![Control scheme of quadrotor](image)

**Figure 2.** Control scheme of quadrotor.

### 4. Nonlinear Sliding Surface-Based SMC Design

For the case of a conventional SMC, a linear sliding surface can be designed using the state error and differential value of the error. In this section, we set up a nonlinear sliding surface for the controller to have a fast transient response. The general linear sliding surface is designed as follows:

\[
\sigma_i(x,t) = \dot{x}_i - \dot{x}_{i,d} + \lambda_i(x_i - x_{i,d}) \\
= \dot{e}_i + \lambda_i e_i, \quad (i = 1, 3, 5, 7, 9, 11).
\]  

(9)

In Equation (9), \( e_i \) represents the error between the desired value \( (x_{i,d}) \) and the state value \( (x_i) \), and \( \lambda_i \) is a positive constant. Figure 3 shows the system trajectory to reach the linear sliding surface. First, the system states move along the acceleration curve to reach the sliding surface and then switch to the deceleration curve. With the deceleration process, the settling time of the state error increases [35]. To reduce the effect of the deceleration curve, we propose a nonlinear sliding surface that compensates the deceleration curve with a sigmoid function. As a first step in designing the nonlinear sliding surface, we set up a new axis-based linear sliding surface as
\[ p_i(x,t) = \dot{x}_i - \dot{x}_{id} - \frac{1}{\lambda_i} (x_i - x_{id}) = \dot{\xi}_i - \frac{\dot{\xi}_i}{\lambda_i}, \quad (10) \]

where \( p_i(x,t) \) is the axis orthogonal to the linear sliding surface. The axes \( p_i(x,t) \) and \( \sigma_i(x,t) \) are reconstructed on a new \( \sigma_i - p_i \) plane to create the nonlinear sliding surface. The nonlinear sliding surface \( \hat{\sigma}_i(x,t) \) is modeled using a sigmoid function in the \( \sigma_i - p_i \) plane as

\[ \hat{\sigma}_i(x,t) = \sigma_i(x,t) + \left( \frac{\sigma^+ - \sigma^-}{1 + e^{-a_i(x,t)}} + \sigma^- \right), \quad (11) \]

where \( \sigma^+ \) and \( \sigma^- \) are the maximum and minimum values of the sigmoid function, respectively, and \( a_i > 0 \) is a scaling constant that determines the slope value. As the value of \( a_i \) increases, the shape of sliding surface becomes the step function. According to different values of \( a_i \), the nonlinear sliding surface will be adjusted to ensure the reduced reaching time. Figure 4 shows the proposed nonlinear sliding surface constructed by following the sigmoid function in the \( \sigma_i - p_i \) plane. We represent the sigmoid function of Equation (11) in the new \( \xi - \dot{\xi} \) plane. Using the angle of \( \theta_i = \tan^{-1}(1/\lambda_i) \), the \( \sigma_i - p_i \) plane can be transformed to the \( \xi - \dot{\xi} \) plane through the angular rotation. In this paper, \( \sigma^+ \) and \( \sigma^- \) have the same absolute value of \((q_i \cos \theta_i)/2\). The new nonlinear sliding surface defined in the \( \xi - \dot{\xi} \) plane is

\[ \hat{\sigma}_i(x,t) = p_i(x,t) + \left( \frac{\sigma^+ - \sigma^-}{\cos \theta_i} \right) \left[ \frac{1}{1 + e^{-a_i \sin \theta_i \sigma_i(x,t)}} + \frac{\sigma^-}{\sigma^+ - \sigma^-} \right] \]

\[ = \dot{\xi}_i - \frac{1}{\lambda_i} \dot{\xi}_i + q_i \left[ \frac{1}{1 + e^{-a_i \sin \theta_i \sigma_i(x,t)}} + \frac{\sigma^-}{\sigma^+ - \sigma^-} \right] \]

\[ = \dot{\xi}_i - \frac{1}{\lambda_i} \dot{\xi}_i + h(\sigma_i(x,t)). \quad (12) \]

Figure 3. State trajectory with a linear sliding surface.
5. Sliding Mode Control-Based Tracking System

5.1. Attitude and Altitude Control

The purpose of SMC is to make the measurement angle and tracking error converge to the desired value in finite time using a proper control input \( u \). In this section, we use SMC to stabilize the quadrotor attitude and to control the altitude tracking with the SMO-based quadrotor attitude dynamics of Section 3. We use the Lyapunov function to satisfy the stability condition \([36]\). The Lyapunov stability function for attitude is confirmed as follows:

\[
V(\dot{\sigma}) = \frac{1}{2} \dot{\sigma}^2, \quad \dot{V}(\dot{\sigma}) = \dot{\sigma} \ddot{\sigma} < 0. \tag{13}
\]

We derive the time derivative of the sliding surface as

\[
\dot{\sigma}(x,t) = \dot{x} - \dot{x}_{id} - \frac{1}{\lambda_1} (\dot{x}_x - \dot{x}_{id} + z) + h(\sigma(x,t))
\]

\[
= c_1 \dot{x} + c_2 \dot{x}_x + \alpha \dot{\omega} + z - \frac{1}{\lambda_1} (\dot{x}_x - \dot{x}_{id} + z)
\]

\[
- \alpha q (\ddot{x} + \dot{x}_x) \left[ \frac{1}{1 + e^{-\alpha \gamma(\sigma,x,t)}} - \frac{1}{(1 + e^{-\alpha \gamma(\sigma,x,t)})^2} \right]
\]

\[
= -\mu_{sat}(\dot{\sigma}) - \mu \dot{\sigma}, \tag{14}
\]

where \( \dot{x}_x \) is designed in Equation (8) and \( \alpha_1, \alpha_2 \) are positive constants. \( \alpha_1 \) means \( \alpha \sin \theta \) in Equation (12). In Equation (14), \( \dot{\sigma}(x,t) \) is defined as \(-\mu_{sat}(\dot{\sigma}) - \mu \dot{\sigma}\) to make \( \dot{\sigma}(x,t) \) and \( \dot{\sigma}(x,t) \) have the opposite sign for the satisfaction of Lyapunov stability condition. We can obtain the control input \( u \) from Equation (14) as

\[
u = -\frac{1}{c_2} \left[ c_1 \dot{x} + c_2 \dot{x}_x + \alpha \dot{\omega} + z - \frac{1}{\lambda_1} (\dot{x}_x - \dot{x}_{id} + z)
\]

\[
- \alpha q (\ddot{x} + \dot{x}_x) \times \{ \gamma(\sigma,x) - \gamma^2(\sigma,x) \} + \mu_{sat}(\dot{\sigma}) + \mu \dot{\sigma}, \tag{15}
\]

where \( \gamma(\sigma,x) \) is the sigmoid function \( \frac{1}{1 + \exp(-\alpha \sigma(x,t))} \). In Equations (14) and (15), we use the saturation function to replace the discontinuous signum function.
Following altitude control procedures, the rotational angles of a quadrotor must be influenced for stable, translational motions. For example, the input for position control is obtained by the following equations:

\[
\begin{align*}
    u_2 &= -\frac{1}{c_7} \left[ c_7 \hat{x}_d \hat{x}_d + c_7 \hat{\phi}_d + \hat{\phi}_d \right] - \frac{1}{\lambda_2} \left( \hat{x}_d - \dot{x}_d + z_d \right) \\
    u_3 &= -\frac{1}{c_8} \left[ c_8 \hat{\phi}_d \hat{\phi}_d + \hat{\phi}_d \right] - \frac{1}{\lambda_8} \left( \hat{x}_d - \dot{x}_d + z_d \right)
\end{align*}
\] (17)

As the system satisfies the Lyapunov stability condition, the sliding variable \( s_\phi - x_d \) converges to zero. Therefore, the control inputs \( u_2, u_3, \) and \( u_4 \) make the errors of quadrotor angles \( (\phi, \theta, \psi) \) converge to zero.

The altitude control input \( u_4 \) can also be obtained by SMC following the design procedures for the attitude control inputs. The sliding surface \( (\hat{s}_\phi) \) of the altitude control does not include the estimated states produced by SMO. In Section 3, we estimate only the quadrotor’s angle and angular velocity by SMO. Following the design procedures for the attitude control inputs, the control input \( u_4 \) can be derived as

\[
    u_4 = -\frac{m}{\cos \hat{x}_d \cos \hat{\phi}_d} \left[ g + \hat{x}_d - \frac{1}{\lambda_4} \left( \hat{x}_d - \dot{x}_d \right) - \alpha_4 \hat{\phi}_d \right]
\] (18)

5.2. Position Control

The control is performed successively by the time-scale separation between the rotational and translational movements. The altitude and attitude control is executed by \( u_1, u_2, u_3, \) and \( u_4 \) for the quadrotor stabilization. After the attitude of quadrotor becomes stable, the quadrotor is moved to the target position by \( u_1, u_3, \) and \( u_4. \) In Equation (8), we can see that motions in the \( x- \) and \( y- \) axes depend on \( u_4. \) For a quadrotor system, the control input \( u_4 \) also affects motion in the \( z- \)axis. As the desired angle \( (\phi_d, \theta_d) \) is influenced by the position control input, we also consider the control inputs \( u_1, u_3, \) and \( u_y \) for the tracking system. Position control inputs \( u_1, u_3, \) and \( u_y \) can be obtained by SMC from Equation (8), and these inputs are composed of the desired angle \( (\phi_d, \theta_d) \) which tracks the path in the \( x-y \) plane. The control algorithm calculates the desired angle \( (\phi_d, \theta_d) \) for quadrotor tracking to the desired \( x \) and \( y \) positions within minimal time. We can obtain the inputs \( u_1, u_3, \) and \( u_y \) following the design procedures of attitude and altitude control inputs for \( u_1 \) through \( u_4. \)
\[ u_x = \frac{m}{u_4} \left[ \dot{x}_{10d} - \frac{1}{\lambda_x}(x_{10d} - x_{10d}) - \alpha_y q_x (\ddot{e}_y + \hat{\lambda}_x) \right] \times \left[ \gamma_y(\sigma_y) - \gamma_y^*(\sigma_y) \right] + \mu \text{sat} (\ddot{\sigma}_x) + \mu_i \ddot{\sigma}_x, \]
\[ u_y = \frac{m}{u_4} \left[ \dot{x}_{12d} - \frac{1}{\lambda_y}(x_{12d} - x_{12d}) - \alpha_x q_x (\ddot{e}_x + \hat{\lambda}_y) \right] \times \left[ \gamma_x(\sigma_x) - \gamma_x^*(\sigma_x) \right] + \mu \text{sat} (\ddot{\sigma}_y) + \mu_i \ddot{\sigma}_y. \]  

(19)

From Equations (4) and (19), we can obtain the desired angle \( (\phi, \theta) \) as follows:

\[ \phi = \sin^{-1} \left( u_s \sin \psi - u_c \cos \psi \right), \]
\[ \theta = \sin^{-1} \left[ \frac{u_s \cos \psi + u_c \sin \psi}{\sqrt{1 - (u_s \sin \psi - u_c \cos \psi)^2}} \right]. \]  

(20)

6. Simulation Results

In this section, to verify the effectiveness of the proposed controller design method, simulations using SMC and SMO-based NSMC methods in the quadrotor system were demonstrated. In our simulation, the control system of a quadrotor is modeled using MATLAB/Simulink with ODE 45 as a solver. The disturbance model of the simulation is supposed to follow white Gaussian noise. Table 1 shows quadrotor dynamic system parameters used in the simulation. Moreover, we set the control parameters \( \lambda_{s_1}, \lambda_{s_2}, \lambda_{s_3}, \lambda_{s_4}, \lambda_{s_5}, \lambda_{s_6}, \mu_{s_{10}}, \mu_{s_{11}}, \) and \( \mu_{s_{12}} \) in the simulation as 2, 0.3, 30, 0.1, 0.08, and 0.05, respectively. First, we confirm the estimation performance to prove the improvement of the quadrotor control system by SMO. For the quadrotor attitude control, Figure 5 shows the stabilization of the quadrotor angle \( (\phi, \theta) \), which is initially slanted by 17 degrees. In Figure 5, the dotted line is a stabilization result by conventional SMC and the solid line is a stabilization result by SMO-based nonlinear surface sliding mode control (NSMC). We can confirm that the performance of NSMC combined with SMO is better than that of conventional SMC. The settling time for the angle \( (\phi, \theta) \) of a quadrotor is reduced with the use of SMO-based NSMC.

Figure 6 shows that the quadrotor reaches the target position \( (7,7,5) \) m from the initial position \( (3,3,0) \) m following the reference trajectory. In Figure 6, the SMO-based control shows stable tracking performance following the reference trajectory and has a small perturbation around the turning points. Figure 7 shows the trajectory projected onto the \( x - y \) plane. To compare tracking performance, we simulate the trajectory shown in Figure 6 using the SMO-based NSMC and conventional SMC without an observer. The quadrotor tracking starts at the point \( (3,3) \) m and reaches the final point \( (7,7) \) m by passing through several turning points. The SMO-based NSMC shows robustness against the effects of measurement noise. However, the conventional SMC trajectory shows SMC without an observer. The quadrotor tracking starts at the point \( (3,3) \) m and reaches the final point \( (7,7) \) m by passing through several turning points. The SMO-based NSMC shows robustness against the effects of measurement noise. However, the conventional SMC trajectory shows chattering caused by the measurement noise. We can confirm that the trajectory driven by the SMO-based NSMC reaches the target point with less fluctuation.
Table 1. Numerical parameters of quadrotor model.

| Parameter | Value      | Unit       |
|-----------|------------|------------|
| $m$       | 0.23       | [kg]       |
| $l$       | 0.215      | [m]        |
| $I_x$     | $5.136 \times 10^{-3}$ | [Ns²/rad] |
| $I_y$     | $5.136 \times 10^{-3}$ | [Ns²/rad] |
| $I_z$     | $1.0616 \times 10^{-2}$ | [Ns²/rad] |
| $J_r$     | 1.4193     | [Ns²/rad] |
| $g$       | 9.8        | [m/s²]     |

Figure 5. Stabilization of quadrotor with a slanted initial angle.

Figure 6. Quadrotor tracking system from initial position to target position.
Figure 7. Quadrotor’s trajectory on the $x-y$ plane.

Figure 8 shows the tracking performance of a quadrotor. The trajectory of the quadrotor with disturbances and the simulation results along the $x$, $y$, and $z$-axes are shown in this figure. While the conventional SMC shows low performance with slow convergence speed and large steady state error due to disturbance, the proposed NSMC shows more robust performance against disturbance since the proposed augmented dynamics with correction variables represents the quadrotor system precisely and NSMC reduces the delay of reaching time. This robustness against disturbance is possible owing to the state estimation precision of SMO. We can confirm the tracking performance on the $x$, $y$, and $z$-axes in Figure 8, wherein the nonlinear-sliding-surface-based SMC reduces bounces, settling time, and rising time more than conventional SMC. Moreover, chattering appears in the steady state of conventional SMC owing to the absence of SMO. We can confirm that the performance of NSMC is better than that of conventional SMC.

Figure 8. Quadrotor’s trajectory on each axis.
7. Conclusions

In this paper, we proposed a nonlinear-sliding-surface-based SMC for tracking and stabilization with an SMO. We set up the conventional quadrotor dynamic model based on Newton–Euler formalism. To improve the performance of the quadrotor system, we remodeled the augmented quadrotor dynamics and estimated the unmeasured states and their derivatives through the SMO. To improve the SMC control performance, we set up a nonlinear sliding surface to reduce the convergence time with a modified deceleration curve. We derived the proper control inputs \((u_1, u_2, u_3, u_4)\) using SMC and calculated the desired angle \((\phi_t, \theta_t)\) from the control input \(u_{4,5}\). Moreover, the augmented quadrotor dynamics with the estimated states and the correction variables using SMO was derived to improve the representation of quadrotor system. We confirmed the improved performance in attitude control by SMO, which estimated the angles of the quadrotor and augmented its dynamics. Moreover, we proved the performance of a tracking system driven by NSMC.

Author Contributions: Conceptualization, K.L. and K.Y.; methodology, K.L. and S.K. (Sangkyeum Kim); validation, S.K. (Sangkyeum Kim) and K.Y.; investigation, S.K. (Sangkyeum Kim) and K.Y.; writing—original draft preparation, K.L. and K.Y.; visualization, K.L. and S.K. (Sungwoo Kwak); writing—review and editing, K.L., S.K. (Sungwoo Kwak), and K.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (NRF-2019R1A2C1002343, NRF-2020R1I1A1A01061632) and the BK21 FOUR Project.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data sharing not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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