Abstract—We introduce a soft-detection variant of Guessing Random Additive Noise Decoding (GRAND) called Quantized GRAND (QGRAND) that can efficiently decode any moderate redundancy block-code of any length in an algorithm that is suitable for highly parallelized implementation in hardware. QGRAND can avail of any level of quantized soft information, is established to be almost capacity achieving, and is shown to provide near maximum likelihood decoding performance when provided with five or more bits of soft information per received bit.

Index Terms—GRAND, Soft Decision, Quantization

I. INTRODUCTION

As Maximum Likelihood (ML) error correcting decoding of linear codes is NP-complete [1], the engineering paradigm has been to co-design restricted classes of linear code-books with code-specific decoding methods that exploit the code-structure to enable computationally efficient approximate-ML decoding. For example, Bose-Chaudhuri-Hocquenghem (BCH) codes with hard detection Berlekamp-Massey decoding [2], [3], Low Density Parity Check codes [4] and belief propagation [5], CRC-Assisted Polar (CA-Polar) codes, which have been selected for all control channel communications in 5G New Radio (NR), and CRC-Assisted Successive Cancellation List (CA-SCL) decoding [6], [7], [8], [9].

Modern applications, including augmented and virtual reality, vehicle-to-vehicle communications, the Internet of Things, and machine-type communications, have driven demand for Ultra-Reliable Low-Latency Communication (URLLC) [10], [11], [12], [13], [14]. To enable these technologies requires the use of short, high-rate codes, revising the possibility of creating high-accuracy universal decoders that are suitable for hardware implementation. Accurate, practically realizable universal decoders offer the possibility of reduced hardware footprint, the provision of hard- or soft-detection decoding for codes that currently only have one class of decoder, and future-proofing devices against the introduction of new codes.

Guessing Random Additive Noise Decoding (GRAND) is a recently introduced universal decoder that was originally established for hard decision demodulation systems [15], [16]. GRAND algorithms operate by sequentially removing putative noise-effects from the demodulated received sequence and querying if what remains is in the code-book. The first instance where a code-book member is found is the decoding. Pseudocode for GRAND can be found in Fig. 1. If GRAND queries binary noise effects from most likely to least likely based on available hard or soft information, it identifies maximum likelihood (ML) decodings. In the hard detection setting, mathematical analysis of GRAND determines error exponents for a broad class of additive noise models [16].

For an \( [n, k] \) code, where \( k \) information bits are transformed into \( n \) coded bits for communication, all GRAND algorithms identify an erroneous decoding after approximately geometrically distributed number of code-book queries with mean \( 2^{n-k} \) [16, Theorem 2] and correctly decode if they identify a code-word beforehand. As a result, an upper bound on the complexity of all GRAND algorithms is determined by the number of redundant bits rather than the code length or rate directly, making them suitable for decoding any moderate redundancy code of any length. The performance difference between GRAND variants stems from their utilisation of statistical noise models or soft information to improve the targeting of their queries.

The evident parallelizability of hard detection GRAND’s code-book queries have already resulted in the proposal [17] and realization [18], [19] of efficient circuit implementations for binary symmetric channels. An algorithm has also been introduced for channels subject to bursty noise whose statistical characteristics are known to the receiver [20], which has also resulted in proposed circuit implementations [21], [22].

A natural question is how to make use of soft detection information, when it is available, in order to improve GRAND’s query order and several proposals have been made. Symbol Reliability GRAND (SRGRAND) [23], [24] avails of the most limited quantized soft information where one additional bit tags each demodulated symbol as being reliably or unreliably received. SRGRAND is mathematically analysable, implementable in hardware, and provides a 0.5 – 0.75 dB gain over hard-detection GRAND [24]. Soft GRAND (SGRAND) [25] uses real-valued soft information per demodulated bit to build a bespoke noise-effect query order for each received signal and provides a benchmark for optimal decoding accuracy performance. It is possible to create a semi-parallelizable implementation in software using dynamic max-heap structures, but the requirement for substantial dynamic memory does not
lend itself to efficient implementation in hardware.

Ordered Reliability Bits GRAND (ORBGRAND) [26] aims to bridge the gap between SRGRAND and SGRAND by obtaining the decoding accuracy close to the latter in an algorithm that is suitable for implementation in circuits. For a block code of length \( n \), it uses \( \lceil \log_2(n) \rceil \) bits of code-book-independent quantized soft detection information per received bit, the rank-order of each received bit’s reliability, to determine an accurate decoding. It retains the hard-detection algorithm’s suitability for a highly parallelized implementation in hardware and high throughput VLSI designs have been proposed [27], [28], [29], [30]. Moreover, theoretical results suggest that ORBGRAND is almost capacity achieving [31]. It has been observed that ORBGRAND provides near-ML decodings for block error rates greater than \( 10^{-4} \), but is less precise at higher SNR as ORBGRAND’s noise effect query order diverges from the optimal rank order in terms of decreasing likelihood. To rectify that, a list decoding approach to improve ORBGRAND’s performance at higher SNR has been suggested [32] as well as an improved statistical model [33] from which to determine an enhanced query order.

Here we introduce QGRAND, an alternative algorithm for efficient, practical soft-detection error correction decoding with GRAND that is an approximation to SGRAND and does not require received bits to be rank-ordered by their reliability. QGRAND can make use of any level of quantized soft detection information, with increasingly accurate performance as the number of soft information bits per received bit increases. With a few bits per bit, empirical results show it provides comparable block error rate performance to ORBGRAND in an algorithmically distinct package that is also suitable for hardware implementation.

II. GRAND AND SOFT DETECTION

Consider a system using a binary \([n, k]\) block code. Data is modulated, transmitted and subject to additive continuous noise. The modulated channel output, \( Y^n \), is then demodulated to provide the hard-detection output

\[
y^n = \text{demod}(Y^n) \in \mathbb{F}_2^n.
\]

In contrast to the continuous noise impacting the channel, the noise effect is the binary difference between the code-word and demodulated output in

\[
Z^n = c^n - \text{demod}(Y^n) = c^n - y^n \in \mathbb{F}_2^n.
\]

All GRAND [16] algorithms seek to identify the noise effect, \( Z^n \), by rank ordering putative noise effects, \( z^n \), from most likely to least likely based on the information available to them, and querying if what remains when a putative noise effect is removed from a demodulated signal is in the code-book. Let

\[
L(Y) = |\text{LLR}(Y)| = \left| \log \left( \frac{f_{\{c\}}(Y|1)}{f_{\{c\}}(Y|0)} \right) \right|
\]

denote the reliability of the signal \( Y \). Given \( L(Y) \), elementary manipulation reveals that the likelihood that the corresponding hard demodulated bit, \( y \), is in error is

\[
B(Y) = \frac{e^{-L(Y)}}{1 + e^{-L(Y)}},
\]

and so the a posteriori likelihood of the putative noise effect sequence \( z^n \) is

\[
P(Z^n = z^n) = \prod_{i=1}^{n} (1 - B(Y_i)) \prod_{i \neq j} B(Y_i) \frac{B(Y_j)}{1 - B(Y_j)} \propto \prod_{i \neq j} B(Y_i) \frac{B(Y_j)}{1 - B(Y_j)} = \exp \left( - \sum_{i=1}^{n} L(Y_i)z_i \right).
\]

Consequently, to create binary noise effect sequences, \( z^n \), rank-ordered by likelihood, it suffices to create them by increasing \( \sum_{i=1}^{n} L(Y_i)z_i \). Distinct GRAND algorithms use different approximations, \( L(Y_i) \), to \( L(Y) \) that depend on the information that is available to them.

In the absence of soft detection information, the approximation is that \( L(Y_i) \) is some unknown constant for all \( i \) and so noise effect sequences are rank ordered by their Hamming weight, \( w_H(z^n) = \sum_{i=1}^{n} z_i \). SRGRAND’s binary quantization sets \( L(Y_i) = \infty \) for bits above a threshold, tagging them as being perfectly reliable, and \( L(Y_i) \) to a constant for those below the threshold, resulting in noise effect sequences following increasing Hamming weight within the region of unreliable bits [24]. ORBGRAND considers the received bits rank-ordered in increasing reliability and their reliabilities are increasing linearly, i.e \( L(Y_i) = \alpha i \) then noise effect sequences follow increasing logistic weight \( w_L(z^n) = \sum_{i=1}^{n} i z_i \) regardless of the value of the slope \( \alpha > 0 \). An ORBGRAND variant is reported on in [34] that also assumes that bits are first rank-ordered by their reliability, but \( l_i \geq 1 \) bits are assigned to have weight \( \alpha i \) for each \( i \). Thus, with a pre-defined bit partition \( \{ l_i; \sum l_i = n \} \), which is empirically determined based on the code, the approximate log-likelihoods are given by
In contrast to other variants, QGRAND envisages a direct quantization of the real-valued reliabilities, \( L(Y) \), into a restricted number of categories determined by a quantization level without the need for received bit reliabilities to be rank-ordered, which can be energy expensive when implemented in circuits.

Consider the discretization of bit-reliability \( Y \) to one of \( Q \) distinct values, \( \{q_j : j \in \{1, \ldots, Q\}\} \), where each \( q_j \) is a natural number and \( \beta \geq 0 \) is a common real-valued base unit. For a given block of \( n \) bits and corresponding discretized reliabilities, let \( s(i,j) = 1 \) if \( L(Y_i) = \beta q_j \) and zero otherwise, let \( m_j = \sum_{i=1}^n s(i,j) \) be the number of bits in quantization level \( j \), and let \( z^{n,j} \) denote the subset of the the string \( z^n \) such that \( s(i,j) = 1 \). With this quantized approximation, QGRAND’s query order follows

\[
\sum_{i=1}^n L(Y_i) z_i = \beta \sum_{j=1}^Q q_j w_H(z^{n,j}) = \sum_{j=1}^Q q_j w_H(z^{n,j})
\]

in increasing order. That is, the query order would follow a weighted sum of Hamming weights of bit flips corresponding to indices in each discretized level, and the value \( \beta \) only impacts the quantization levels.

Thus for a received block of symbols with \( m^Q = (m_1, \ldots, m_Q) \) bits in each quantization level and a total weight \( W \), we wish to identify the set of noise effect sequences

\[
\{z^n : \sum_{j=1}^Q q_j w_H(z^{n,j}) = W, w_H(z^{n,j}) \leq m_j \text{ for all } j\}
\]

(2)

Defining \( w^Q = (w_1, \ldots, w_Q) \), where \( w_j \) is the number of bits at quantization level \( j \) to be flipped, and setting

\[
\omega(W, m^Q) = \left\{w^Q : \sum_{j=1}^Q q_j w_j = W, w_j \leq m_j \text{ for all } j\right\}
\]

(3)

to be all viable combinations of weights, \( w_j \), that give the desired overall weight, \( W \), the set \( \omega \) can be identified with

\[
\bigcup_{w^Q \in \omega(W, m^Q)} \left\{w_H(z^{n,1}) = w_1 \times \cdots \times \{w_H(z^{n,Q}) = w_Q\}\right\},
\]

(4)

where the product is Cartesian.

Consider the integer discretization \( q_j = 2j - 1 \), i.e., \( \{1, 3, 5, \ldots, 2Q - 1\} \). With a total weight \( W = 0 \), the only solution to the problem in eq. (3) is \( \omega(0, m^Q) = (0, \ldots, 0) \) and the most likely noise effect is no bit flips. With \( W = 1 \), \( \omega(1, m^Q) = (1, \ldots, 0) \) and eq. (4) says that all noise effect sequences that have any one of their least reliable bits, i.e., bits with \( s(i,1) = 1 \), are equally likely. With an overall weight of \( W = 2 \), if \( m_1 \geq 2 \), \( \omega(2, m^Q) \) consists of \( (2, 0, \ldots, 0) \) so that eq. (4) says that any sequence with two least-reliable bits flipped would be equally likely. When \( W = 3 \), putative noise sequences with 3 least reliable bits flipped or one second least reliable bit being flipped are equally likely, and so forth.

In practice, identifying the set \( \omega(W, m^Q) \) in eq. (3) amounts to an integer partition problem that can be solved efficiently. For a given \( w^Q \in \omega(W, m^Q) \), the generation of all noise effect sequences

\[
\left\{z^n : w_H(z^{n,1}) = w_1 \times \cdots \times \{w_H(z^{n,Q}) = w_Q\}\right\},
\]

can be achieved using minor modifications to the circuits developed for hard-detection GRAND algorithms [17], [18]. [19]. As are result, QGRAND is well suited to implementation in hardware. For a given number of quantization levels, \( Q \), what remains is to determine a set of quantization values, \( q_i \), and we turn to achievable rate considerations for guidance.

IV. QUANTIZATION AND THE ACHIEVABLE RATES OF QGRAND

Consider a memoryless binary input additive white Gaussian noise channel with codewords \( C^n \) uniformly distributed in a codebook \( \mathcal{C} \). The highest rate that can be supported, in the Shannon sense, is governed by mutual information, \( I(C; Y) \), which is achieved by an ML decoder, i.e.,

\[
\hat{c}^n = \arg \max_{c^n \in \mathcal{C}} I(c^n; Y^n | c^n) = \arg \min_{c^n \in \mathcal{C}} \sum_{i=1}^n L(Y_i) z_i.
\]

For a system with quantized channel output, the mutual information is given by \( I(C; \hat{Y}) \), where \( \hat{Y} \) denotes the quantized version of \( Y \). Replacing \( L(Y_i) \) with \( \hat{L}(Y_i) \) can be thought of as using a mismatched decoder [35]. A mismatched decoder uses a function \( \kappa(Y, C) \), called the auxiliary channel, as the decoding metric,

\[
\hat{c}^n = \arg \max_{c^n \in \mathcal{C}} \prod_{i=1}^n \kappa(Y_i, c_i).
\]

Defining

\[
R(Y, C, \kappa, s, r) = \log_2 \left( \frac{\kappa(Y, C)^s r(C)}{\kappa_{rs}(Y)} \right)
\]

where \( r(\cdot) \) is a real-valued function with finite expectation and \( \kappa_{rs}(\cdot) = \mathbb{E}[\kappa(\cdot, C)^s r(C)] \) is the corresponding auxiliary output distribution with \( s \geq 0 \), it is known that the block error probability of mismatched decoding approaches zero for \( n \) approaching infinity so long as \( k/n < R(Y, C, \kappa, s, r) \), e.g. [35]. In general, we have \( R(Y, C, \kappa, s, r) \leq I(C; Y) \) with equality if \( \kappa = f_{Y|C} \), \( s = 1 \) and \( r(\cdot) = 1 \). Fixing \( r(\cdot) = 1 \) and maximizing over \( s \) yields the generalized mutual information [36]. Maximizing over \( r, s \) yields the highest achievable rate, the LM-Rate [35], [37], in the system with quantized reliability:

\[
R_{LM}(Y, C, \kappa) = \max_{r,s} R(Y, C, \kappa, s, r).
\]

We evaluate the LM-Rate for the decoding metric

\[
\kappa(Y, C) = e^{-M(Y) / (1-2C)}
\]
where $M(Y)$ is the decoder input providing the soft-information about bit $C$. The LM-Rate is now

$$R_{LM}(Y,C,\kappa) = \max_{r,s} \mathbb{E} \left[ \log_2 \frac{e^{\frac{M(Y)}{2} (1-2c)} r(c)}{\sum_{c \in \{0,1\}} P_c(c) e^{\frac{M(Y)}{2} (1-2c)} r(c)} \right].$$

Note that we have $R_{LM}(Y,C,\kappa) = I(C;Y)$ with $M(Y) = LLR(Y)$ [38].

For QGRAND, we first quantize the reliability $L(Y)$ to $Q$ distinct levels as either

$$\Pi_1 = [0, \beta], [\beta, 2\beta], \ldots, [Q\beta - \beta, +\infty)$$

or $\Pi_2 = [0, \beta], [\beta, 3\beta], \ldots, [2Q\beta - \beta, +\infty)$. Note that the mutual information $I(C;Y)$ is achieved only if we use an optimal decoding metric. An example with $E_s/N_0 = 7$ dB, $Q = 4$ is shown in Fig. 2 for different bin size multiplier $\beta$, where it can be seen that the quantization $\Pi_2$ outperforms $\Pi_1$. Fig. 3 shows the quantization loss

$$E_s/N_0 - C_{BPSK}^{-1}(R_{LM}(Y,C,\kappa)),$$

where $C_{BPSK}$ is the Shannon channel capacity of the corresponding BPSK channel. As can be seen, this analysis suggests that $\Pi_2$ is preferred. Heuristically, this can be understood as the latter quantization provides greater resolution for the least reliable bits at the cost of lower resolution for more reliable bits. As a result, $\Pi_2$ is what we use in the simulations that follow.

V. PERFORMANCE EVALUATION

For simulated performance evaluation, we assume BPSK modulation and additive white Gaussian (AWGN) with variance $\sigma^2$. From eq. (1) we have that $L(Y) = 2|Y|/\sigma^2$, and we elect to set

$$\beta = \frac{2}{\sigma^2} \frac{1 - \sigma/2}{2Q - 1}.$$

The first term, $2/\sigma^2$, normalizes for the increase in reliability with SNR, while the second term, $(1 - \sigma/2)/(2Q - 1)$ ensures that approximately 30% of the least reliable bits are accurately assigned quantized reliabilities, while the 70% most reliable bits are grouped together. For comparison with QGRAND at distinct levels of quantization, we use GRAND [16] as an optimal ML hard detection decoder, SGRAND as an optimal ML soft detection decoder [25], and ORBGRAND as the state-of-the-art practical universal soft detection decoder.

Fig. 4 shows decoding results for a BCH code. BCH codes traditionally only have a hard detection decoder, Berlekamp-Massey, and previous results have shown that hard detection GRAND provides identical block error rate performance [39]. SGRAND, ORBGRAND and QGRAND enable soft detection decoding of BCH codes. ORBGRAND, which provides near soft-detection ML decoding using $8 = \lceil \log_2(255) \rceil$ bits of soft information per received bit, is seen to have a 1 dB gain over GRAND at a BLER of $10^{-4}$. As the number of quantised soft detection bits, $\log_2(Q)$, per demodulated bit increases, QGRAND is seen to give a gain of 1 dB with 2 bits of soft information, to providing near identical performance to ORBGRAND with 3 bits of soft information. With 5 or 6 bits of soft detection information per bit, QGRAND provides comparable performance to the optimally accurate SGRAND.

Fig. 4. BLER vs Eb/N0 for a rate 0.97 BCH[255,247] code in an AWGN BPSK channel. Decoding with hard detection GRAND, soft detection SGRAND and ORBGRAND, and QGRAND with different amounts of quantized soft information.
We also compare the performance of GRAND algorithms with CA-Polar codes, which will be used for control channel communications in 5G NR. CA-Polar codes are concatenated Polar inner codes with a CRC outer code. In their dedicated decoder, CA-SCL [6], [7], [8], [9], the Polar bits are used for soft detection list decoding, typically of length 8 [40], and the CRC bits are used to select a decoding from that list. For benchmarking, we also show results for CA-SCL as implemented in the AFF3CT toolbox [41]. As the product of a linear code with a linear code is a linear code, and all GRAND algorithms can decode any code, the GRAND algorithms use both the CRC and Polar bits of a CA-Polar code for error correction.

Fig. 5 shows the block error rate (BLER) performance for a CA-Polar[128,116] code. Again, QGRAND provides graded improved performance from GRAND to ORBGRAND with increasing levels of quantization. With \( \log_2(Q) = 1 \) bit of soft information per received bit, QGRAND sees a 0.8 dB gain over GRAND at a block error rate of \( 10^{-4} \). Each additional bit gives, approximately, an additional 0.25 dB gain. ORBGRAND, which uses \( 7 = \log_2(128) \) bits of soft information per received bit, sees a circa 2 dB gain over its hard detection equivalent. With 5 bits of soft information per bit, QGRAND performs similarly to ORBGRAND and with 7 bits it provides similar performance to SGRAND. CA-SCL gives similar performance to QGRAND with 1 or 2 bits of soft information per bit, as the error correcting power of the CRC is under-utilized for this code.

Fig. 6 provides analogous results for a longer, higher rate CA-Polar[512,500] code. Again, QGRAND sees a graduated improvement in performance from 0.75 dB with 1 bit quantization, to greater than 1.5 dB with 6 bits. With 5 bits of soft information, QGRAND provides similar performance to ORBGRAND, and 7 bits suffices to get SGRAND-like performance. Again, CA-SCL does not fully avail of the error correction capabilities of the CRC.

VI. DISCUSSION

As new applications drive demand for shorter, higher rate error correcting codes, computationally efficient universal decoding becomes a possibility. Universal decoders have many practical benefits, including the ability to support an astronomical number of distinct codes with one efficient piece of software or hardware, enabling the best choice of code for each application and future proofing devices to the introduction of new codes.

Even though it was only recently introduced, GRAND is one promising approach to realising this possibility. Hard detection GRAND algorithms enable accurate decoding of codes such as CA-Polar codes for which there is only a dedicated soft detection decoder. Moreover, they upgrade codes for which there are only hard detection decoders, such as BCH codes, or no error correcting decoder, such as CRCs [39], [42], to soft detection decoding. Consistent with theoretical predictions [43], results from GRAND algorithms show that decoding performance is largely driven by the quality of the decoder rather than the code, and that good CRCs and codes selected at random can offer comparable performance to highly structured ones [20], [39], [26], [18], [44].

The existing state-of-the-art soft detection version of GRAND that has been demonstrated to be suitable for hardware implementation is ORBGRAND. Here we introduce an alternative variant, QGRAND, that utilizes distinctly quantized soft detection information. It can avail of any level of...
quantization tailored to application need, it provides improved block error rate performance as quantization increases, and it does not require a reliability sorting step. QGRAND inherits all the desirable features of GRAND algorithms, including universality, parallelizability and reduced algorithmic effort as SNR increases. Results here establish that QGRAND can provide marginally better performance than ORBGRAND with comparable levels of soft information in an algorithmically distinct package that is suitable for efficient implementation in circuits.

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