Quantum correlations between identical and unidentical atoms in a dissipative environment

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Abstract
We have studied the dynamics of quantum correlations such as entanglement, Bell nonlocality and quantum discord between identical and unidentical atoms interacting with a single-mode cavity field and subject to cavity decay. The effect of single-atom detuning, cavity decay rate and initial preparation of the atoms on the corresponding correlation measures have been investigated. It is found that even under strong dissipation, time evolution can create high quantum discord while entanglement and Bell nonlocality stay zero for an initially separable state. Quantum discord increases while entanglement decreases in a certain time period under dissipation for the initial state that both atoms are in the excited state if the qubits are identical. For some types of initial states, cavity decay is shown to drive the system to a stationary state with high entanglement and quantum discord.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Entanglement, nonlocality and quantum discord (QD) are all different but somehow related aspects of the quantumness of the correlated composite quantum systems. Entanglement is a kind of quantum correlation and determines whether the given state is separable or not [1]. On the other hand, the quantum systems may contain quantum correlations other than entanglement. QD, which is defined as the difference between the quantum versions of two analogous classically equivalent definitions of mutual information, captures all types of nonclassical correlations including entanglement [2]. Although entanglement and QD are equivalent for pure states [3], QD is found to be nonzero for some separable states [3, 4]. Such states were shown to be useful in the deterministic quantum computations with one pure qubit context [5, 6]. Therefore, QD is believed a new resource for quantum computation. The nonlocality as measured by the violation of Bell’s inequalities is also a signature of the inseparability of a quantum state and signifies entangled states whose correlations cannot be reproduced by a classical local hidden variable model [7]. However, it must be emphasized here that the presence of entanglement does not always imply violation of Bell’s inequalities, while the violation of Bell’s inequalities implies entanglement. Such entangled states violating Bell’s inequalities play a central role in some applications in quantum information science [8], such as to guarantee the safety of device-independent key distribution protocols in quantum cryptography [9, 10].

The relations among different measures of quantum correlations and their dynamical behaviour have been an active area of research with an aim to understand the fundamental question of what quantumness is as well as to characterize these correlations as useful resources for various quantum tasks [11–36]. One of the most important obstacles in realizing quantum operations is the interaction between the qubits and the environment which tends to wipe out the quantumness of the system [1]. Characterization of the effect of environment on the system such as decoherence and dissipation is important. The dynamics of quantum correlations such as entanglement, QD and Bell nonlocality for a two-qubit system subject to various environments have been studied by many groups recently [11–36]. Entanglement sudden death (ESD) is one of the new concepts that arise within the context of these studies [37]. ESD refers to death of entanglement between two qubits in a finite time while the coherence of the single-qubit decays.

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only exponentially. In contrast to entanglement, QD is more robust in a decohering environment; actually it was shown that almost all states have non-zero QD [38]. QD was shown to decay exponentially in Markovian environments, even at finite temperatures [14, 16] and can become zero momentarily under non-Markovian dynamics [15]. QD was also experimentally investigated in an all-optical setup recently [39]. Furthermore, QD and entanglement were analysed for a Heisenberg spin chain with quantum phase transitions and QD was found to detect the critical points of the transition at finite temperatures while entanglement could not [17]. Very recently, Sun et al investigated the induced quantum correlations between two qubits coupled to a common environment modelled by the Ising spin chain in a transverse field and during the time evolution, non-zero QD is found to be created for specific initial states, whereas entanglement cannot be induced [12]. In [21] QD is found to be totally unaffected by non-dissipative noise for a long-time interval under certain conditions, while entanglement decays to zero quickly. These remarkable properties of QD make it more practical than the entanglement to characterize quantum correlations.

As mentioned, the nonlocality is crucial for some applications in quantum information. Therefore, many theoretical studies have been devoted to the comparison of entanglement and the violations of Bell’s inequalities [29–36]. It was demonstrated by several authors that the survival time for entanglement is much larger than that of the Bell-inequality violation [29–31, 35]. In [34], the nonlocal entanglement identified by the violations of a Bell inequality is found to be preserved during the time evolution of a system consisting of two qubits which are embedded independently in zero-temperature bosonic reservoirs and are initially in an entangled mixed state. Bellomo et al showed that Bell’s inequality might not be violated for a state with high entanglement for a two-qubit system subject to amplitude damping [35]. Moreover, the nonlocal entanglement between two qubits subject to classical dephasing environment modelled as Ornstein–Uhlenbeck noise has been investigated in [30, 31], very recently, and it was demonstrated that the strong non-Markovian effect can protect the nonlocal entanglement [30, 31]. Additionally, the nonlocal entanglement induced by the qubit–qubit interaction is found to be protected against the sudden death despite the strong influence of the environmental noise for a certain class of initial states regardless of whether the environment is Markovian or non-Markovian [31].

Although the dynamical relation between the entanglement and QD as well as entanglement and Bell nonlocality for a bipartite system subject to different environments have been investigated by a large number of groups as mentioned above, further investigations would help in clarifying the relation of these three correlation measures. Along these lines, in this paper, we have investigated the dynamics of quantum correlations such as QD, entanglement and Bell nonlocality between two qubits in a lossy cavity. Very recently, entanglement, Bell nonlocality and QD have been studied in cavity QED [15, 16, 18, 20, 22, 24, 26, 32, 34, 35, 40]. Most of them assume identical atoms, or the atom–cavity resonant case, or only one excitation.

Here, we have investigated the effect of atom–cavity detuning and the cavity decay rate on the corresponding correlation measures for a system which is initially prepared in a separable or maximally entangled state. The most important results of the present study are as follows: depending on the initial preparation of the system and the type of the detunings between the frequency of the cavity mode and the transition frequency of the qubits, the induced QD is found to increase for a long time under a cavity decay loss mechanism for certain conditions, while there is no increase in entanglement and Bell nonlocality. In some situations, the QD is induced even while the atoms remain separable at all times. Also, the preservation of induced entanglement and QD was observed under strong dissipation for certain types of initial states.

The paper is organized as follows. In section 2, the model is introduced and the exact differential equations governing the dynamics of the two atoms and the single-cavity mode are derived. The correlation measures, concurrence, CHSH-Bell inequality and QD, are also introduced briefly. In sections 3.1 and 3.2, the effect of cavity damping on the corresponding correlation measures is revealed for different detunings for the atoms initially prepared in a separable or maximally entangled state. A summary of the important results is given in section 4.

2. The model and correlation measures

Here, we consider two two-level atoms with excited states |eA⟩, |eB⟩ and ground states |gA⟩, |gB⟩ interacting with a single-mode cavity field of frequency ω. The Hamiltonian of this system is given as [41]

\[ H = \frac{\hbar \Omega_A}{2} (|eA⟩⟨eA| - |gA⟩⟨gA|) + \frac{\hbar \Omega_B}{2} (|eB⟩⟨eB| - |gB⟩⟨gB|) + \omega a_a^† a + \hbar g_a (a |eA⟩⟨gA| + g_A^† |eA⟩⟨eA|) + \hbar g_b (a^† |gB⟩⟨eB| + g_b^† |gB⟩⟨gB|). \] (1)

where \(\omega_a\) and \(\omega_B\) are the transition frequencies for the atoms A and B, a, a^† are the annihilation and creation operators for the field and g_a and g_b are the coupling constants to the cavity mode for the atoms A and B, respectively.

When maximum two photons are considered to be present in the cavity (i.e. the total excitation is equal to 2) and if there is no dissipation in the cavity modes, the closed form of the total state vector at any time may be written as [41, 42]

\[
|\Psi(t)\rangle = C_1|e_A, e_B, 0\rangle + C_2|e_A, g_B, 1\rangle + C_3|g_A, e_B, 1\rangle + C_4|g_A, g_B, 2\rangle.
\] (2)

where C_i (i = 1, 2, 3, 4) are the time-dependent probability amplitudes. For example, |g_A, g_B, 2⟩ represents that the atoms are in their ground states while there are two photons inside the cavity. For this case, the dynamics of the atoms and the single-mode cavity field can be solved by using a time-dependent Schrödinger equation and recently many studies have been reported for this case [41–43]. On the other hand, when cavity decay is taken into account, since the cavity decay only changes the number of photons inside the cavity, the atoms and the cavity mode can be found in any of the states [40, 41, 44]: |e_A, e_B, 0⟩, |e_A, g_B, 1⟩, |e_A, g_B, 0⟩, |g_A, e_B, 1⟩,
\[ |g_A, e_B, 0 \rangle, |g_A, g_B, 2 \rangle, |g_A, g_B, 1 \rangle, |g_A, g_B, 0 \rangle. \] Then, the density matrix of the system of two atoms interacting with a single-mode cavity field will evolve according to the master equation [40, 41, 45]:

\[ \dot{\rho} = -i \frac{\hbar}{\kappa} [H, \rho] - \frac{\kappa}{2} (a^\dagger a \rho - 2 \rho a a^\dagger + \rho a^\dagger a), \tag{3} \]

where \( \sigma_i^+ = |e_i \rangle \langle g_i | \) and \( \sigma_i^- = |g_i \rangle \langle e_i | \) \((i = A, B)\) are the spin inversion operators. Here, the first term in equation (3) is generated by the Hamiltonian given in equation (1) and represents the coherent evolution, while the other term denotes the incoherent dynamics; \( \kappa \) is the rate of loss of photons from the cavity mode. By setting \(|1\rangle = |e_A, e_B, 0\rangle, |2\rangle = |e_A, g_B, 1\rangle, |3\rangle = |e_A, g_B, 0\rangle, |4\rangle = |g_A, e_B, 1\rangle, |5\rangle = |g_A, e_B, 0\rangle, |6\rangle = |g_A, g_B, 2\rangle, \) \(7\rangle = |g_A, g_B, 1\rangle, \) \(8\rangle = |g_A, g_B, 0\rangle\) as a basis and considering equations (1) and (3), the differential equations governing the dynamics of two atoms and the single-mode cavity field can be obtained easily. In our analysis we will restrict ourselves for the initial states \( \rho(0) = |e_A, e_B, 0\rangle \langle e_A, e_B, 0|, \) \( \rho(0) = |e_A, g_B, 1\rangle \langle e_A, g_B, 1|, \) \( \rho(0) = |g_A, e_B, 1\rangle \langle g_A, e_B, 1|, \) \( \rho(0) = |g_A, e_B, 0\rangle \langle g_A, e_B, 0|, \) \( \rho(0) = |g_A, g_B, 2\rangle \langle g_A, g_B, 2|, \) \( \rho(0) = |g_A, g_B, 1\rangle \langle g_A, g_B, 1|, \) \( \rho(0) = |g_A, g_B, 0\rangle \langle g_A, g_B, 0| \) \(|\rangle \langle \rangle\) for the dynamics preserve its form. The considered initial states. The density matrix of the two atoms is obtained by taking a partial trace over the cavity degrees of freedom: \( \rho_{AB}(t) = Tr_c(\rho) = \sum_{i=0}^{2} \langle i | \rho | i \rangle \), and used to calculate the quantum correlations as measured by the CHSH–Bell inequality, concurrence and QD between the two atoms interacting with a single-mode cavity field. For the considered initial states whose dynamics is given by equation (4), the reduced density matrix of the atoms in the two-qubit standard basis \(|1\rangle = |e_A, e_B\rangle, |2\rangle = |e_A, g_B\rangle, |3\rangle = |g_A, e_B\rangle, |4\rangle = |g_A, g_B\rangle\) can be calculated as

\[ \rho_{AB}(t) = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & e & 0 \\ 0 & e^* & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} \tag{5} \]

where \(a = \rho_{11} + \rho_{22} + \rho_{33} + \rho_{44}, b = \rho_{11} - \rho_{22} - \rho_{33} + \rho_{44}, \) \(c = \rho_{11} + \rho_{22} - \rho_{33} - \rho_{44}, d = \rho_{11} - \rho_{22} + \rho_{33} - \rho_{44}. \) It should be noted that the density matrix in equation (5) has an X-form and the dynamics preserve its form. The considered correlation measures for this type of state can be calculated easily [3, 11, 23, 24, 32]. The concurrence [46] as an entanglement measure is given by [11]

\[ C(t) = 2 \max\{0, |e| - \sqrt{ad}\}. \tag{6} \]

The Bell nonlocality measured by the CHSH inequality [47] can be found as [32]

\[ B(t) = \max\{B_1(t), B_2(t)\}, \]

\[ B_1(t) = 2 \sqrt{u1 + u2}, \quad B_2(t) = 2 \sqrt{u1 + u3}, \tag{7} \]

where \(u1 = u3 = 4|e|^2\) and \(u2 = (a + d - b - c)^2\). QD is defined as [2]

\[ D(t) = I(t) - J(t), \tag{8} \]

where \(I(t) = S(\rho_{A}) + S(\rho_{B}) - S(\rho_{AB})\) measures the total correlation between the atoms; \(S(\rho) = -\text{Tr}(\rho \log_2 \rho)\) is the von-Neumann entropy and \(\rho_A(\rho_B)\) is the reduced density matrix obtained by tracing \(\rho_{AB}\) over the subsystem \(B(A)\). The other quantity \(J(t)\) is the amount of classical correlations between the atoms defined as the maximum information one
Figure 1. Quantum discord versus $g t$ for unidentical atoms ($b_1 = -b_2 = 5g$) and the $\rho(0) = |e_A, e_B, 0\rangle \langle e_A, e_B, 0|$ initial state for the decay rates: (a) $\kappa = 0$, (b) $\kappa = 0.02g$, (c) $\kappa = 0.2g$, (d) $\kappa = 2.0g$ and (e) $\kappa = 20g$. Note that under these conditions entanglement and Bell nonlocality are not induced; thus, these are not plotted here and the inset in (c) represents the total $I(t)$ and classical $J(t)$ correlations versus $g t$ for $\kappa = 2.0g$.

can get about the atom $A$ by performing measurements on the atom $B$, or vice versa. Its calculation requires maximization over one of the subsystems and generally obtaining an analytic result is not an easy task [3]. Recently, for the density matrix having X-form, the explicit expressions of QD and classical correlation are reported [3, 23, 24]. We have used the expressions given in [24]. The classical correlation $J(t)$ for the density matrix (equation (5)) is given as

$$J(t) = \max\{C_1, C_2\}, \tag{9}$$

where $C_j = M(a + b) - P_j$. And QD is given as

$$D(t) = \min\{Q_1, Q_2\}, \tag{10}$$

where $Q_j = M(a + c) + \sum_{i=1}^{4} \lambda^{ij}_{AB} \log_2 \lambda^{ij}_{AB} + P_j$, with $\lambda^{ij}_{AB}$ being the eigenvalues of $\rho_{AB}$, $P_1 = M(\tau)$, $P_2 =$
Figure 2. Quantum discord and concurrence (insets in (a), (b) and (c)) versus $Gt$ for identical atoms ($\delta_1 = \delta_2 = 5g$) and the $\rho(0) = \left| e_{A,e_{B}},0 \right> \left< e_{A,e_{B}},0 \right|$ initial state for the decay rates: (a) $\kappa = 0$, (b) $\kappa = 0.02g$, (c) $\kappa = 0.2g$, (d) $\kappa = 2.0g$ and (e) $\kappa = 20g$. Note that under these conditions the Bell nonlocality is not induced, and for $\kappa = 2g$ and $\kappa = 20g$, $C(t)$ has no dynamics as well; thus, they are not plotted here. The insets are plotted under the same conditions as $D(t)$.

\[-(a \log_2 a + b \log_2 b + c \log_2 c + d \log_2 d) - M(a + c), \tau = (1+\sqrt{(1-2(c + d))^2 + 4|e|^2})/2\text{ and }M(\alpha) = -\alpha \log_2 \alpha -(1-\alpha) \log_2 (1-\alpha).\]

We have also calculated QD numerically by considering projective measurements \cite{2, 14} and observed that the results obtained by using equation (10) and the numerical results agree perfectly.

As is well known, concurrence is equal to 1 (0) for a maximally entangled (separable) state. Although QD is equal to 1 for a maximally entangled state, it may or may not be equal to zero for a separable state because it was shown that even some separable states can carry non-zero QD \cite{3, 4}. For pure states, QD is found to be equal to the entanglement of formation \cite{3}. On the other hand, when $B(t) > 2$ the CHSH–Bell inequality is violated which signifies that the correlations cannot be accessible by any classical local model \cite{36}. 

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Figure 3. CHSH inequality (left plots), concurrence (middle plots) and quantum discord (right plots) versus $gt$ for the $\rho(0) = |g_A, g_B, 1\rangle \langle g_A, g_B, 1|$ initial state and $\kappa = 0$ ((a), (b), (c)), $\kappa = 0.02g$ ((d), (e), (f)), $\kappa = 0.2g$ ((g), (h), (k)), $\kappa = 2g$ ((l), (m), (n)) and $\kappa = 20g$ ((o), (p), (r)). Here the main subfigures are for identical atoms with $\delta_1 = \delta_2 = 5g$, while the insets are for unidentical atoms with $\delta_1 = -\delta_2 = 5g$ under the same decay rate parameters. Note that for unidentical atoms $B(t)$ is not violated, so they are not reported here.
In this section, we will examine the effect of the cavity damping-separable states.

3.1. Effects of cavity damping-separable states

In this section, we will examine the effect of the cavity damping-separable states. For an initially separable state \( \rho(0) = |e_A, e_B, 0 \rangle \langle e_A, e_B, 0 | \) and \( \rho(0) = |e_A, g_B, 1 \rangle \langle e_A, g_B, 1 | \) and \( \rho(0) = |\Psi(0) \rangle \langle \Psi(0) | \), where \( |\Psi(0) \rangle = \frac{1}{\sqrt{2}} (|g_A, e_B, 1 \rangle + |e_A, g_B, 1 \rangle) \) and we will set \( g_1 = g_2 = g \) (symmetric coupling), \( \delta_1 = \delta_2 = 5g \) for identical atoms and \( \delta_1 = -\delta_2 = 5g \) for unidentical atoms. One should note that the detunings are chosen somewhat large because higher detunings the qubit–cavity energy exchange is low [41, 42, 48] and a high degree of cavity-induced quantum correlations can be created [48].

We first consider the dynamics of concurrence and QD for an initially separable state \( \rho(0) = |e_A, e_B, 0 \rangle \langle e_A, e_B, 0 | \) and display \( C \) and \( D \) versus dimensionless time \( gt \) in figures 1 and 2 for unidentical and identical atoms, respectively. For this initial state neither non-zero concurrence nor violation in the Bell inequality ever occurs for unidentical qubits, so their dynamics are not plotted in figure 1. However, this does not indicate the loss of quantum correlations; QD induced by the atom–field interaction oscillates nearly between 0 and 0.18 for \( \kappa = 0 \) (figure 1(a)). A similar result is found by Sun et al. in [12] where they studied the entanglement and QD between two qubits which are independently coupled to a common environment modelled as an Ising spin chain. It can be seen from the analysis of figures 1(b)–(e) that QD is created independently of the cavity decay rate, but its maximum value depends on the decay rate \( \kappa \) inversely and the effect of cavity decay on the dynamics of QD is to create a damped oscillatory behaviour; damping in QD is proportional to the cavity decay rate. Also, in order to understand the nature of non-zero QD, we have plotted the total correlation, \( I(t) \), and classical correlation, \( J(t) \), for \( \kappa = 0.2g \) in the inset of figure 1(c). It should be noted that for this initial state, the total as well as classical correlations are also induced and \( I(t) \) is always greater than \( J(t) \); as a consequence non-zero QD is present. This inset also demonstrates that \( I(t) \) and \( J(t) \) are also damped oscillatorily as \( D(t) \). Correlations in the system are gradually lost to the reservoir because of the cavity decay as found in the system studied in [18].

Figure 4. The populations \( \rho_{ij}(t) \) \((i, j = 1, 2, \ldots, 8) \) and the absolute value of the coherences \( \rho_{13}(t), \rho_{15}(t) \) given by equation (4) versus \( gt \) for the initial state \( \rho(0) = |e_A, g_B, 1 \rangle \langle e_A, g_B, 1 | \) and \( \kappa = 20g \) for identical \((\delta_1 = \delta_2 = 5g) \) (a) and unidentical \((\delta_1 = -\delta_2 = 5g) \) (b) atoms. (c) The atomic purity (\( \text{Tr} \rho_A^2 \)) for the density matrix (equation (5)) versus \( gt \) for identical and unidentical atoms. Here the insets are for small time regions.
We have calculated the dynamics of the CHSH inequality, concurrence and QD for the same initial state \( \rho(0) = \frac{1}{\sqrt{2}} (|e_A, g_B, 0\rangle + |e_A, e_B, 1\rangle) \otimes |1\rangle \langle 1| \) for the identical atom \((\delta_1 = \delta_2 = 5g)\) case to understand the effect of the type of detuning for the same leaky cavity and plot the resulting \( D \) and \( C \) as a function of \( gt \) in figures 2(a)–(e) and the insets of figures 2(a)–(c), respectively. As can be seen from the insets of figures 2(a)–(c), non-zero entanglement, although small, is created which decays to zero as the cavity decay rate is increased and the induced entanglement has no dynamics for higher values of \( \kappa \) \((\kappa = 2g \text{ and } \kappa = 20g)\). The Bell inequality is found not to be violated for any value of \( \kappa \), which is expected because entanglement is low and generally Bell inequality violation is shown to be a hallmark of high \( C(t) \) \([31, 34, 35]\), so \( B(t) \) is not displayed in figure 2. One should note the difference in the dynamics of QD in the leaky cavity for unidentical and identical atom cases (figures 1 and 2). While QD has an oscillatory decay-type dynamics for the unidentical atoms (figure 1), it increases until the time \( t = t_{\text{max}} \) and then decays for the identical atoms. Also, for the same decay parameters, QD is found to have a much longer lifetime for the identical atoms. The higher leakage in the cavity decreases \( t_{\text{max}} \) and the lifetime of QD after the time \( t = t_{\text{max}} \). On the other hand, \( \kappa \) has no appreciable effect on the maximum value of \( D(t) \). Also note that the fast oscillations in QD are gradually smoothed for larger values of \( \kappa \) and longer times as can be seen from figures 2(b)–(e).

Now we consider another initial separable state of the form \( \rho(0) = |e_A, g_B\rangle \langle e_A, g_B| \otimes |1\rangle \langle 1| \) which has one excitation in the field and the other excitation in one of the atoms to see the effect of initial partition of the excitation on the dynamics of the quantum correlations. The time dependences of the CHSH inequality, concurrence and QD are displayed in the first, second and third columns of figure 3, respectively. In these figures, the insets show the dynamics for unidentical atoms \((\delta_1 = -\delta_2 = 5g)\) while the main subfigures are for the identical atoms \((\delta_1 = \delta_2 = 5g)\). Inspection of the subfigures demonstrates the pronounced effect of qubits being identical or unidentical. For identical atoms, the atom–field interaction induces high values of \( C(t) \) and \( D(t) \) and the Bell inequality is also violated at the high values of entanglement for a high-quality cavity (figure 3(a), (b) and (c)). In the case of unidentical atoms, \( C(t) \) and \( D(t) \) are also created, but their maximum are small, 0.018 and 0.13 respectively, and \( B(t) \) is not violated. When unidentical atoms interact with a low-quality cavity, entanglement can cease to exist very quickly and QD shows a fast oscillatory decaying behaviour (see the insets in figures 3(e)–(r)) which is very similar to what
we have found for the $\rho(0) = |e_A, e_B, 0\rangle \langle e_A, e_B, 0|$ initial state. On the other hand, this situation is completely and significantly different in the case of identical atoms. Although the CHSH–Bell inequality goes below 2 very quickly for $0 < \kappa \leq 2.0g$ and is no longar violated for $\kappa > 2.0g$, the dynamics of entanglement and QD are very similar that both approach constant values in an oscillatory manner; the oscillations die down as the cavity decay rate is increased. In effect, cavity decay entangles the initially separable state and the resulting entanglement as well as QD is robust. In many studies, the dissipation of cavity fields was found to play a constructive role in the generation of quantum correlations as measured by concurrence and QD [22, 25, 27, 28, 48–50]. This situation can be understood by looking at the steady state of the dynamics given in equation (3). For $g_1 = g_2 = 1$, $\kappa > 0$ and $\delta_1 = \delta_2 = \delta$, the $\dot{\rho} = 0$ equation has a solution independent of $\kappa$ and $\delta$ and the atom–atom-reduced density matrix is given as $\rho_{SS} = \frac{1}{2} (|e_A, g B\rangle \langle e_A, g B| + |g_A, e B\rangle \langle g_A, e B|) - \frac{1}{2} (|e_A, g B\rangle \langle g_A, e B| + |g_A, e B\rangle \langle e_A, g B|) + \frac{1}{2} |g_A, g B\rangle \langle g_A, g B|$. The concurrence for this state is 0.50 and QD of it can be calculated from equation (10) to be 0.412 which is seen clearly from figures 3(p) and (t). Also, we want to emphasize that the dynamics of $B$, $C$ and $D$ between identical atoms for the initial state $|\Psi(0)\rangle = |e_A, g B, 1\rangle$ is the same as that for $|\Psi(0)\rangle = |g_A, e B, 1\rangle$, since we assume symmetric coupling (i.e. $g_1 = g_2$) between the atoms and the single-mode cavity field.

To further elucidate the asymptotic constant entanglement and QD for the initial state $\rho(0) = |e_A, g B, 1\rangle \langle e_A, g B, 1|$, we display the time dependence of the density matrix elements of the atom–field system given by equation (4) and the purity of the atomic subsystem for the density matrix (equation (5)) for identical and unidentical detuning cases in figures 4(a)–(c) for $\kappa = 20g$. As can be seen from figure 4(a), all populations, except $\rho_{3,3}$, $\rho_{5,5}$ and $\rho_{8,8}$, become zero for a short time while $\rho_{8,8}$ goes to 1/2 and $\rho_{3,3} = \rho_{5,5}$ go to 1/4 asymptotically. The key mechanism for the creation of the entanglement and QD in this case seems to be the coherence $\rho_{3,5}$ generated by the interaction of the atom with the dissipative cavity. The absolute value of $\rho_{3,5}$ also reaches 1/4 asymptotically as can be seen from figure 4(a). The evolved state is a trapping state which has the property that the probabilities of the different decay channels interfere destructively and the state is robust against the cavity loss mechanism [51, 52]. Generation or protecting entanglement by population trapping has been discussed by a number of groups before [48, 49, 51–53]. The magnitude of $\kappa$ has no effect on the long-time limit of $\rho_{3,3}$, $\rho_{5,5}$ and $\rho_{8,8}$; it only affects how long other population components can be nonzero. The dynamics of atomic purity (which is equal to 1 for a pure state and 1/4 for a maximally mixed state in a four-dimensional Hilbert space) displayed in figure 4(c) as the solid line for $\delta_1 = \delta_2 = 5g$ indicates that the state approaches a partially mixed state which has high entanglement and QD as indicated before in figures 3(p) and (t). For unidentical atoms, the short-time dynamics is similar to the identical detuning case that all states are populated for a short time, but the asymptotic state is now $\rho(t \rightarrow \infty) = |g_A, g B, 0\rangle \langle g_A, g B, 0|$ as can be seen from figure 4(b). The atomic purity (dotted line in figure 4(c)) indicates that the state of the atoms first goes to a mixed state in a short time and then approaches the pure state $\rho = |g_A, g B\rangle \langle g_A, g B|$ which is the reason for the loss of correlations between unidentical atoms in the asymptotic limit.
3.2. Effects of a cavity decay-maximally entangled initial state

In this subsection, we will investigate the effect of initial correlations in the atom–atom part of the state on the dynamics of quantum correlations.

We take $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|g_A, e_B\rangle + |e_A, g_B\rangle) \otimes |1\rangle$ as the initial state which is the maximally entangled Bell state for the atomic system. QD and Bell inequality violation are also maximal for this initial state. We display $B$, $C$ and $D$ as a function of $g_t$ for several cavity decay rates for unidentical (figure 5) and identical (figure 6) atoms. At $\kappa = 0$, all correlations show oscillatory behaviour; for the unidentical atoms $B(t)$, $C(t)$ and $D(t)$ oscillate nearly in the ranges $[2\sqrt{2}$, 2.2], $[1.0, 0.63]$ and $[1.0, 0.58]$, respectively (figure 5). On the other hand, $B(t)$ and $C(t)$ for $\kappa = 0$ have periodic sudden deaths and births for the identical detunings of the atoms (figures 6(a) and (b)), while $D(t)$ oscillates nearly between 0.3 and 1 (figure 6(c)). For small values of $\kappa$ ($\kappa = 0.2g$), the effect of cavity decay on the dynamics of correlations can be described as underdamped oscillations for both identical and unidentical detuning cases. As $\kappa$ is increased to $2g$, the oscillation becomes overdamped and finally at large $\kappa$ ($\kappa = 20g$) damping becomes critical, i.e. $C(t)$ and $D(t)$ show exponential decay for the unidentical atom cases. For the identical atoms at $\kappa = 2g$ and $\kappa = 20g$ concurrence has sudden death, while QD decays exponentially. One should note that Bell nonlocality is the most fragile quantum correlation considered here; it suffers sudden death in a short time as long as $\kappa > 0$.

4. Conclusion

We have investigated the dynamics of quantum correlations as quantified by entanglement, Bell nonlocality and quantum discord for two qubits embedded in a common leaky cavity for three different initial states with double excitations. The qubits were considered as identical or unidentical based on the detuning of their transition frequency with respect to the frequency of the field mode. The main findings of the study can be summarized as follows. Qubits being identical or not have profound influence on the dynamics of the considered correlations; starting from the same initial state, the correlations show very different behaviour depending on whether the detunings of the individual qubits satisfy $\delta_1 = \delta_2 = \delta$ or $\delta_1 = -\delta_2 = \delta$. For unidentical qubits initially in their excited states and interacting with a high- or low-quality cavity, quantum discord is found to be induced for all considered cavity decay rates and it decays oscillatorily for a leaky cavity although the qubits remain unentangled for all times. For the same initial state and the leaky cavity, QD is found to increase in the long time for identical qubits, while entanglement is either ‘never created’ or decays to zero in a short time.

One of the most interesting findings is the constant high entanglement and the quantum discord state which is obtained as the long-time limit of the dynamics of the $\rho(0) = |e_A, g_B\rangle \langle e_A, g_B| \otimes |1\rangle$ initial state for the identical qubit case. Here, the final state is independent of the cavity decay rate (if $\kappa > 0$) and the magnitude of the detuning, and has high concurrence ($C = 0.5$) and quantum discord ($D = 0.412$). From an analysis of the time dependence of the density matrix elements it is seen that the considered initial state leads to a trapping state which happens to have a high degree of quantum correlations. On the other hand, for the same initial state, the correlations between the unidentical atoms are found to be destroyed quickly even for a small cavity decay rate.

For the atomic Bell-type initial state we have found ESD for the identical qubits interacting with a low-quality cavity, while concurrence decays exponentially for the unidentical qubits. Quantum discord is found to have asymptotically decay behaviour independent of the type of atoms for this initial state and leaky cavity. On the other hand, CHSH–Bell inequality violations survive only for a finite time for $\kappa > 0$ for both identical and unidentical qubits. It is interesting to note that the steady-state nonzero correlation state of $|\Psi(0)\rangle = |g_A, e_B\rangle \otimes |1\rangle$ (or $|\Psi(0)\rangle = |e_A, g_B\rangle$) is not observed for $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|g_A, e_B\rangle + |e_A, g_B\rangle) \otimes |1\rangle$. This shows that the stationary states depend strongly on the initial conditions [49]. We have also demonstrated that Bell nonlocality as manifested by CHSH inequality is the most fragile correlation measure under dissipative environment and is only violated at high values of entanglement.

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