Asymptotic Freedom and Infrared Behavior in the Type 0 String Approach to Gauge Theory

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Abstract

In a recent paper we considered the type 0 string theories, obtained from the ten-dimensional closed NSR string by a GSO projection which excludes space-time fermions, and studied the low-energy dynamics of \( N \) coincident D-branes. This led us to conjecture that the four-dimensional \( SU(N) \) gauge theory coupled to 6 adjoint massless scalars is dual to a background of type 0 theory carrying \( N \) units of R-R 5-form flux and involving a tachyon condensate. The tachyon background leads to a “soft breaking” of conformal invariance, and we derived the corresponding renormalization group equation. Minahan has subsequently found its asymptotic solution for weak coupling and showed that the coupling exhibits logarithmic flow, as expected from the asymptotic freedom of the dual gauge theory. We study this solution in more detail and identify the effect of the 2-loop beta function. We also demonstrate the existence of a fixed point at infinite coupling. Just like the fixed point at zero coupling, it is characterized by the \( AdS_5 \times S^5 \) Einstein frame metric. We argue that there is a RG trajectory extending all the way from the zero coupling fixed point in the UV to the infinite coupling fixed point in the IR.

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1. Introduction

In a recent paper [1] we conjectured that 3 + 1 dimensional $SU(N)$ gauge theory coupled to 6 adjoint massless scalar fields is dual to a certain background of type 0 string theory [2] involving a non-vanishing tachyon field. This work was inspired by the recently discovered relations between type II strings and superconformal gauge theories on $N$ coincident D3-branes [3-11], as well as by Polyakov’s suggestion [8] (building on his earlier work [9]) that the type 0 string theory in dimensions $D \leq 10$ is a natural setting for extending this duality to non-supersymmetric non-conformal gauge theories.

The type 0 string has world sheet supersymmetry, but the GSO projection is non-chiral and breaks the space-time supersymmetry. Following the notation of [13], in $D = 10$ the spectra of the type 0A and type 0B theories are:

- type 0A: $(NS-, NS-) \oplus (NS+, NS+) \oplus (R+, R-) \oplus (R-, R+)$,
- type 0B: $(NS-, NS-) \oplus (NS+, NS+) \oplus (R+, R+) \oplus (R-, R-)$.

Both of these theories have no fermions in their spectra but produce modular invariant partition functions [2,13]. The massless bosonic fields are as in the corresponding type II theory (A or B), but with the doubled set of the Ramond-Ramond (R-R) fields. The type 0 theory also contains a tachyon from the $(NS-, NS-)$ sector, which is why it has not received much attention thus far. In [8,1] it was suggested, however, that the presence of the tachyon does not spoil its application to large $N$ gauge theories. A well-established route towards gauge theory is via the D-branes [14,15], which were first considered in the type 0 context in [16]. Large $N$ gauge theories, which are constructed on $N$ coincident D-branes of type 0 theory, may be shown to contain no open string tachyons [17]. Furthermore, the dual type 0 background necessarily includes $N$ units of R-R flux which has a stabilizing effect on the bulk tachyon [1]. In fact, in [1] the presence of a tachyon background was turned into an advantage because it gives rise to the renormalization group (RG) flow.

In [1] the 3 + 1 dimensional $SU(N)$ theory coupled to 6 adjoint massless scalars was constructed as the low-energy description of $N$ coincident electric D3-branes. The

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1. Investigation of some aspects of Polyakov’s proposal in the non-critical case appeared recently in [10].

2. Possible relevance of world sheet supersymmetry to string description of gauge theories was also advocated in [9,11,12].

3. The open string descendants of type 0B theory were originally constructed by orientifold projection in [17]. They, in general, have open-string tachyons in their spectra. A non-tachyonic model (which is anomaly-free and contains chiral fermions in its open-string spectrum) was found by a special Klein-bottle projection of the 0B theory in [18]. We will be concerned with a different tachyon-free theory which occurs on parallel like-charged D-branes.
conjectured dual type 0 background thus carries $N$ units of electric 5-form flux. In the Einstein frame the dilaton decouples from the $(F_5)^2$ terms in the effective action, and the only source for it originates from the tachyon mass term,

$$\nabla^2 \Phi = \frac{1}{8} m^2 e^{\frac{1}{2} \Phi} T^2 , \quad m^2 = -\frac{2}{\alpha'} .$$ (1.1)

Thus, the tachyon background induces a radial variation of $\Phi$. Since the radial coordinate is related to the energy scale of the gauge theory [5], the effective coupling decreases toward the ultraviolet (UV) [1], in agreement with the expected asymptotic freedom of the gauge theory [19].

Further progress was recently achieved by Minahan, who found the asymptotic UV (large radius) form of the solution to the equations for the type 0 background proposed in [1]. In addition to the “RG equation” (1.1), one has

$$R_{mn} - \frac{1}{4} g_{mn} R = \frac{1}{4} \nabla_m T \nabla_n T - \frac{1}{8} g_{mn} [ (\nabla T)^2 + m^2 e^{\frac{1}{2} \Phi} T^2 ] + \frac{1}{2} \nabla_m \Phi \nabla_n \Phi - \frac{1}{4} g_{mn} (\nabla \Phi)^2$$

$$+ \frac{1}{4 \cdot 4!} f(T) ( F_{mklpq} F_{n}^{klpq} - \frac{1}{10} g_{mn} F_{sklpq} F_{sklpq} ) ,$$ (1.2)

$$(-\nabla^2 + m^2 e^{\frac{1}{2} \Phi}) T + \frac{1}{2 \cdot 3!} f'(T) F_{sklpq} F_{sklpq} = 0 ,$$ (1.3)

$$\nabla_m [ f(T) F_{mklpq} ] = 0 ,$$ (1.4)

where $g_{mn}$ is the Einstein frame metric, and the tachyon–R-R field coupling function is

$$f(T) \equiv 1 + T + \frac{1}{2} T^2 .$$ (1.5)

Assuming as in [1] that the tachyon is approximately localized near the extremum of $f(T)$, i.e. $T = -1$, Minahan found [20] that $g_{mn}$ is asymptotic to $AdS_5 \times S^5$, while the effective string coupling is

$$\frac{1}{g_{st}} = e^{-\Phi} \sim N \ln^2 u .$$ (1.6)

Here $u$ is the radial coordinate of $AdS_5$ and $N \sim Q$ is the number of R-R flux units.

The logarithmic flow of the effective coupling is very encouraging. Furthermore, the calculation [20] of the quark-antiquark potential following the prescription of [21] gives

$$V \approx - \frac{k_1}{L \ln L_0^T} , \quad L \ll L_0 , \quad k_1 = \left[ \frac{4}{\Gamma(\frac{1}{4})} \right]^4 ,$$ (1.7)

Note that the semiclassical value of the Wilson loop is determined by the fundamental string action, which contains the string-frame metric $G_{mn} = e^{\frac{1}{2} \Phi} g_{mn}$. 
in agreement with the short-distance behavior of the gauge theory [19]:

\[ V \sim -\frac{N g_{YM}^2 (L)}{L}, \quad N g_{YM}^2 (L) \sim \frac{1}{\ln \frac{L_0}{L}}. \]  

(1.8)

The logarithmic term in the denominator appears as follows. In the AdS\(_5\) calculation of [21] the quark-antiquark potential determined from the Wilson loop factor (i.e. the semiclassical value of the fundamental string action) is proportional to \(\sqrt{g_{st}N}\), which is constant for \(\mathcal{N} = 4\) SYM theory. While the theory we are studying is non-conformal, \(\sqrt{g_{st}N} \sim \frac{1}{\ln u}\) varies slowly for large \(u\). If we consider a quark and an antiquark separated by distance \(L\), the string connecting them penetrates to \(u\) of order \(1/L\) due to the approximate conformal invariance. Introducing the “QCD scale” \(L_0\), we have \(u \sim L_0/L\), so that the potential is multiplied by the effective value of \(\sqrt{g_{st}N}\), which is \(1/\ln(L_0/L)\).

Finding the result (1.7) from the dual type 0 string description is striking, but it is important to study various corrections and show that they do not destroy it. In this paper we consider, in particular, the \(R^4\) correction to the effective action and show that it does not change the scaling found from the leading effective gravity solution, though it does change the coefficients. This conclusion actually applies to a whole class of possible \(\alpha'\) corrections. Thus, in order to compare the beta function coefficients, it seems necessary to know the exact string \(\sigma\)-model, but the effective gravity does capture the physics of the dual gauge theory.

Another important physical question is what happens to the theory in the infrared (IR) limit. One possibility is that the adjoint scalar fields become massive, so that the theory is in the same universality class as the pure glue \(SU(N)\) theory. On the dual string side this would manifest itself in the disappearance of the 5-sphere. In this paper we explore a different possibility: that in the IR limit the scalars remain massless and the 5-sphere remains macroscopic. We first observe that, as the theory starts flowing from the UV toward longer distance scales, the tachyon begins to shift from \(-1\) towards 0. In fact, we succeed in finding an asymptotic IR (small \(u\)) solution where the coupling increases logarithmically, the tachyon approaches zero, while the Einstein metric is \(AdS_5 \times S^5\). The physical interpretation of this is that the theory flows towards a fixed point at infinite coupling. Thus, instead of a confining theory in the infrared, we find a conformally invariant theory with infinite coupling!

The existence of an IR fixed point in \(SU(N)\) gauge theory with 6 adjoint scalars is not unexpected: as we shall discuss below, the one-loop coefficient in the beta-function is negative but the two-loop one is positive so that, in addition to the UV attractive fixed point at \(g_{YM}^2 = 0\), in the two-loop approximation there is an IR attractive fixed point [22]
at $g_{YM}^2 \sim 1/N$. However, since in this approximation the IR fixed point is found to be at rather large 't Hooft coupling, its position is expected to be changed by higher-order corrections. Our dual gravity description suggests that the fixed point is actually shifted to infinite coupling.

In Section 2 we study the $u \gg 1$ (UV) solution in detail and derive corrections to the asymptotic form of the solution of [20]. We also estimate the effect of the $\alpha'$ corrections. In section 3 we discuss the perturbative expression for the beta-function in the $SU(N)$ gauge theory with 6 adjoint scalars. We interpret the $\ln \ln u$ corrections to the leading UV solution as corresponding to the 2-loop terms in the running gauge coupling constant. In section 4 we derive the $u \ll 1$ (IR) asymptotic solution and discuss its physical interpretation. Section 5 contains some concluding remarks.

2. UV asymptotic solution: the asymptotic freedom

Parametrizing the 10-d string-frame and Einstein-frame metric as in [1] ($\mu = 0, 1, 2, 3$ are the 4-d indices)

$$ds^2 = e^{\frac{1}{2} \Phi} ds_E^2, \quad ds_E^2 = e^{\frac{1}{2} \xi - 5 \eta} d\rho^2 + e^{-\frac{1}{2} \xi} dx^\mu dx^\mu + e^{\frac{1}{2} \xi - \eta} d\Omega_5^2,$$

(2.1)

the radial effective action corresponding to (1.1)–(1.4) becomes [1]

$$S = \int d\rho \left[ \frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} \dot{\xi}^2 - 5 \dot{\eta}^2 + \frac{1}{4} \dot{T}^2 - V(\Phi, \xi, \eta, T) \right],$$

(2.2)

$$V = \frac{1}{2} T^2 e^{\frac{1}{2} \Phi + \frac{1}{2} \xi - 5 \eta} + 20 e^{-4 \eta} - Q^2 f^{-1}(T) e^{-2 \xi}.$$  

(2.3)

Here $\alpha' = 1$ and $\Phi, \xi, \eta$ and $T$ are functions of $\rho$. The constant $Q$ (the R-R charge) can be absorbed into the redefinition $\xi \to \xi + \ln Q$ and $\Phi \to \Phi - \ln Q$ and may be set equal to 1 in intermediate calculations. The resulting set of variational equations, the

$$\ddot{\Phi} + \frac{1}{4} T^2 e^{\frac{1}{2} \Phi + \frac{1}{2} \xi - 5 \eta} = 0,$$

(2.4)

$$\ddot{\xi} + \frac{1}{4} T^2 e^{\frac{1}{2} \Phi + \frac{1}{2} \xi - 5 \eta} + 2Q^2 f^{-1}(T)e^{-2 \xi} = 0,$$

(2.5)

$$\ddot{\eta} + 8 e^{-4 \eta} + \frac{1}{4} T^2 e^{\frac{1}{2} \Phi + \frac{1}{2} \xi - 5 \eta} = 0,$$

(2.6)

5 This is in contrast to what happens in theories with the number of matter fields of order $N$, where a similar IR fixed point is at $g_{YM}^2 \sim 1/N^2$ [23].
\[ \dot{T} + 2T e^{\frac{1}{2} \Phi + \frac{1}{2} \xi - 5 \eta} + 2 Q^2 \frac{f'(T)}{f^2(T)} e^{-2 \xi} = 0, \] 

(2.7)

should be supplemented by the ‘zero-energy’ constraint

\[ \frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} \dot{\xi}^2 - 5 \dot{\eta}^2 + \frac{1}{4} \dot{T}^2 + V(\Phi, \xi, \eta, T) = 0, \] 

(2.8)

which can be used instead of one of the second-order equations.

Let us recall that if one ignores the first term in (2.3), which originates from the tachyon mass, and takes the tachyon to be at the extremum of \( f(T) \) (\( T = -1, \ f(-1) = \frac{1}{2} \)), then one finds \[1\] the electric analogue of the standard R-R charged 3-brane solution \( (\rho = \frac{e^{2 \Phi_0}}{4\pi} = \frac{1}{u}) \)

\[ T = -1, \quad \Phi = \Phi_0, \quad e^\xi = e^{\Phi_0} + 2Q\rho, \quad e^\eta = 2\sqrt{\rho}. \] 

(2.9)

This becomes the \( AdS_5 \times S^5 \) space

\[ T = -1, \quad \Phi = \Phi_0, \quad \xi = \ln(2Q) + \ln \rho, \quad \eta = \ln 2 + \frac{1}{2} \ln \rho \] 

(2.10)

in the near-horizon \( (2Q\rho \gg e^{\Phi_0}) \) limit, i.e. \( (\rho = u^{-4}) \)

\[ ds_E^2 = R_0^2 \left( \frac{du^2}{u^2} + \frac{u^2}{2R_0^2} dx^\mu dx^\mu + d\Omega_5^2 \right), \quad R_0^2 = 2^{-1/2} Q^{1/2}. \] 

(2.11)

In \[20\] it was noted that, since \( e^\Phi \) becomes small in the UV region \[1\], the tachyon mass term acts as a small perturbation. We shall indeed confirm below that there is a systematic expansion giving a solution of the full equations (2.4) which is asymptotic to (2.11).

Let us study the full set of equations for small \( \rho \) by defining

\[ \rho \equiv e^{-y} \ll 1, \quad y \gg 1. \] 

(2.12)

By direct inspection of the system of equations we find the following asymptotic solution\[5\]

\[ T = -1 + \frac{8}{y} + \frac{4}{y^2} (39 \ln y - 20) + O\left( \frac{\ln^2 y}{y^3} \right), \] 

(2.13)

\[ \Phi = \ln(2^{15} Q^{-1}) - 2 \ln y + \frac{1}{y} 39 \ln y + O\left( \frac{\ln y}{y^2} \right), \] 

(2.14)

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\[6\] We thank J. Minahan for pointing out some wrong numerical coefficients in the original version of this paper.
The leading $O(\ln y)$ term in $\Phi$, and the $O(y)$ and $O(1/y)$ terms in $\xi$ and $\eta$ were found in [20]. What we have shown is that the leading-order solution of [20], which itself reduces to $AdS_5 \times S^5$ in the $\rho \to 0$ ($y \to \infty$) limit, can be systematically extended to a solution of the full set of equations, including the tachyon one. The evolution of the tachyon (which starts at the extremum $T = -1$ of $f(T)$ and grows at larger $\rho$ towards $T = 0$) is crucial for the consistency of the solution.

As a result, the inverse coupling is

$$e^{-\Phi} = 2^{-15}Q y^{2-\frac{39}{y}+O(\frac{1}{y^2})} = 2^{-15}Q (\ln \rho)^{2+\frac{39}{\ln 2 \rho}+O(\frac{1}{\ln 2 \rho})}, \quad (2.17)$$

and the 10-d Einstein-frame metric (2.1) is (cf. (2.11))

$$ds^2_E = R_0^2 \left[ \left( 1 - 9 \frac{2}{y} \ln y + \ldots \right) (\frac{1}{4} dy)^2 + \left( 1 - \frac{39}{4y^2} \ln y + \ldots \right) \frac{e^{\frac{1}{2} y}}{2R_0^2} dx^\mu dx^{\mu} + \left( 1 - \frac{1}{2y} - \frac{39}{4y^2} \ln y + \ldots \right) d\Omega_5^2 \right], \quad (2.18)$$

where

$$R_0^2 = 2^{-1/2} Q^{1/2}.$$ 

Noting that $y = 4 \ln u$, we see that this metric starts as $AdS_5 \times S^5$ (2.11) at $y = \infty$ and becomes a 10-d space with negative Ricci scalar at smaller $y$ (bigger $\rho$). The corrections to $\xi$ and $\eta$ cause the effective radius of $AdS_5$ to become smaller than that of $S^5$, leading to the negative deficit in the total curvature (cf. [20]).

While it is satisfying to find the asymptotic freedom from the large $u$ behavior of the leading effective gravity solution, the most important question is whether it survives the full string theoretic treatment. Naively, the string scale corrections should be large because $g_{st}N$ is becoming small for large $u$, but, remarkably, the solution is robust due to its special structure related to the approximate conformal invariance. One fact that is crucial for our purposes is that the Einstein metric is asymptotic to $AdS_5 \times S^5$. This geometry is conformal to flat space, so that the Weyl tensor vanishes in the large $u$ limit. Furthermore, both $\Phi$ and $T$ vary slowly for large $u$.
Let us consider, for instance, the leading $\alpha'^3$ correction to the effective action. In the Einstein frame, the dilaton equation becomes

$$\nabla^2 \Phi = -\frac{1}{4\alpha'} e^{\frac{1}{2}\Phi} T^2 + \frac{3}{16} \alpha'^3 \zeta(3) e^{-\frac{3}{2}\Phi} W, \quad (2.19)$$

where $W$ is built out of the fourth power of the Weyl tensor

$$W = C^{hmnk} C_{pmnq} C^{rspb} C_{qrsk} + \frac{1}{2} C^{hkmn} C_{pqmn} C^{rspb} C_{qrsk}. \quad (2.20)$$

Evaluating the Weyl tensor on the solution, we find that $|C| \sim \frac{1}{\ln u}$, so that

$$W \sim \frac{1}{\ln u}. \quad (2.21)$$

Since for large $u$ the inverse coupling grows as $e^{-\frac{3}{2}\Phi} \sim \ln u$, we see that the $\alpha'^3$ correction to the dilaton equation is of the same order as the leading order gravity contribution. We believe that this is a general feature of the solution. For instance, other admissible higher-order correction terms which are of the schematic form

$$e^{\frac{1}{2}\Phi} (e^{-\frac{3}{2}\Phi} C)^n,$$

also scale as $\frac{1}{\ln u}$. Another class of terms that could appear on the right-hand side of the Einstein-frame dilaton equation is $e^{-\frac{1}{2}(n-1)\Phi} (\nabla m \Phi \nabla n \Phi)^n$, which turns out to be subleading (it scales as $(\ln u)^{-n}$).

These considerations give us some confidence that the asymptotic freedom evident in the leading order gravity approach survives the full string $\sigma$-model treatment. As for the string loop corrections, they are further suppressed for $u \gg 1$ by powers of $e^{\Phi} \sim \frac{1}{Q \ln u}$.

3. Correspondence with the two-loop RG evolution of gauge coupling

We can further ask about the perturbative corrections to the RG flow. From the expression for the dilaton $(2.14),(2.17)$ we have ($y = 4 \ln u$, cf. (1.6))

$$\sqrt{g_{st} N} \sim Q^{1/2} e^{\frac{1}{2}\Phi} \sim \frac{1}{\ln u - \frac{39}{8} \ln \ln u + ...}. \quad (3.1)$$

7 As explained in [1] the tachyon-independent terms in the tree-level effective action of type 0 string theory are the same as in type II theory. For a discussion of the $R^4$-correction in a similar context in type II theory see [24] and references there.

8 Terms involving the Ricci tensor or $\nabla^2 \Phi$ can be traded for other terms by use of the equations of motion or, equivalently, by field redefinitions. For example, $e^{-\frac{3}{2}(n-1)\Phi} (\nabla^2 \Phi)^n$ turns out to be of the same order as $e^{\frac{1}{2}\Phi} T^2$. 
In calculating the quark-antiquark potential, the relevant value of \( u \) for the location of the string is of order \( L_0/L \). Thus, we estimate the correction to the potential (1.7) to be

\[
\mathcal{V} \approx -\frac{k_1}{L \left( \ln \frac{L_0}{L} - \frac{39}{8} \ln \ln \frac{L_0}{L} \right)} , \quad L \ll L_0 .
\] (3.2)

A more precise solution for the shape of the string in the metric (2.18), as well as the \( \alpha' \) corrections, can change the coefficients in (3.1). Nevertheless, it is satisfying that (3.2) does have the same structure (including the relative sign of the \( \ln(L_0/L) \) and the \( \ln \ln(L_0/L) \) terms) as in the dual gauge theory with the two-loop term in the beta function taken into account!

Let us demonstrate this in detail. The gauge theory in question can be thought of as a reduction of YM theory from 10 to 4 dimensions (or as a truncation of the \( \mathcal{N}=4 \) SYM theory which removes the fermions). The bare values of the gauge and quartic scalar couplings may be different since in the absence of supersymmetry their equality at the tree level does not survive renormalization. The quartic scalar coupling does not contribute to the 2-loop beta-function of the gauge coupling, so that the RG equation for the YM coupling is

\[
L \frac{dg_{YM}}{dL} = b_1 g_{YM}^3 \left( \frac{4\pi}{g_{YM}} \right)^2 + b_2 g_{YM}^5 \left( \frac{4\pi}{g_{YM}} \right)^4 + \ldots ,
\] (3.3)

where \( L \) is the coordinate scale \( L \ll L_0 \). The solution of this RG equation is (see, e.g., [25])

\[
g_{YM}^2(L) = \frac{(4\pi)^2}{2b_1 \ln \frac{L_0}{L}} \left[ 1 - b_2 \frac{\ln \ln \frac{L_0}{L}}{2b_1} \right] + \ldots = \frac{8\pi^2}{b_1 (\ln \frac{L_0}{L} + b_2 \ln \ln \frac{L_0}{L})} + \ldots .
\] (3.4)

The one- and two-loop coefficients \( b_1 \) and \( b_2 \) in a gauge theory with group \( G \) coupled to scalars transforming in a representation \( R \) are [26]

\[
b_1 = \frac{11}{3} C_2(G) - \frac{1}{6} T_2(R) ,
\] (3.5)

\[
b_2 = \frac{34}{3} [C_2(G)]^2 - [2C_2(R) + \frac{1}{3} C_2(G)] T_2(R) ,
\] (3.6)

where \( T_2 \) is the Dynkin index of the representation \( R \), \( \text{Tr}(T_AT_B) = T_2(R)\delta_{AB} \), and \( C_2(R) \) is the eigenvalue of the quadratic Casimir in this representation. We have \( d(G)T_2(R) = d(R)C_2(R) \) where \( d(R) \) and \( d(G) \) are the dimensions of the representation and the group. For the adjoint representation of \( SU(N) \)

\[
T_2(R) = C_2(R) = C_2(G) = N .
\]
For the case of the $SU(N)$ theory coupled to $N_s$ adjoint scalars the above formulae imply

\begin{align}
  b_1 &= \frac{1}{6}(22 - N_s)N , \\
  b_2 &= \frac{1}{3}(34 - 7N_s)N^2 ,
\end{align}

so that for $N_s = 6$ we finally have

\begin{align}
  b_1 &= \frac{8}{3}N , \\
  b_2 &= -\frac{8}{3}N^2 .
\end{align}

Thus, the relative coefficient between the $\ln \ln(L_0/L)$ term and the $\ln(L_0/L)$ is

\[ \frac{b_2}{2b_1^2} = -\frac{3}{16} . \]

This differs from the $-39/8$ found in the type 0 calculation (3.1), but the sign of the effect is correct.

Since $b_1 > 0$, this gauge theory has the usual asymptotically free UV attractive fixed point at $g_{YM} = 0$. Since $b_2 < 0$, in the 2-loop approximation there is also an IR attractive fixed point (cf. [22]) at

\[ g_{YM}^2 = -(4\pi)^2 \frac{b_1}{b_2} = \frac{(4\pi)^2}{N} \approx \frac{158}{N} . \]

However, this possible fixed point is located at a rather large value of the 't Hooft coupling $\lambda = g_{YM}^2 N$ where the perturbative analysis is obviously not reliable: it should be shifted by the higher-order $\lambda^n$ corrections in the large $N$ perturbation theory for the beta function. In fact, the discussion of the IR gravity solution in section 4 suggests that this fixed point is shifted to infinite coupling.

4. IR asymptotic solution: the fixed point at infinite coupling

In the previous sections we found the small $\rho$ expansion of a RG trajectory which originates from a UV fixed point at vanishing coupling. One interesting feature of the trajectory is that $T$ starts increasing from its critical value $T = -1$ determined from the condition $f'(T) = 0$. The precise form of the trajectory for finite $\rho$ is not known analytically, but we can extract some qualitative features from the RG equations (2.4)–(2.7).

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9 Note, in particular, that the theory obtained by reduction of the $D = 26$ YM theory ($N_s = 22$) which has a vanishing one-loop beta-function [27], has $b_2 < 0$ and thus is not asymptotically free.

10 Recently RG flows in gauge theories were studied from the type IIB perspective in [28,29,30]. There the context is somewhat different since the flow connects lines of fixed points.
We note that the fields $\Phi, \xi$ and $\eta$ have negative second derivatives. Thus, each of these fields may reach a maximum at some value of $\rho$. If for $\Phi$ this happens at a finite $\rho$, then we reach a peculiar conclusion that the coupling is decreasing far in the infrared. In fact, a different possibility seems to be realized: $\dot{\Phi}$ is positive for all $\rho$, asymptotically vanishing as $\rho \to \infty$. We have succeeded in constructing the asymptotic form of such trajectory. It is crucial that, as $\rho \to \infty$, $T$ approaches zero so that, as in the UV region, $T^2 e^{\frac{1}{2}\Phi}$ becomes small. For this reason the limiting Einstein-frame metric is again $AdS_5 \times S^5$. Thus, the theory flows to a conformally invariant point at infinite coupling.

Indeed, in the region
\begin{equation}
\rho \equiv e^y \gg 1, \quad y \gg 1, \quad (4.1)
\end{equation}
we find the following asymptotic large $\rho$ solution
\begin{align}
T &= -\frac{16}{y} - \frac{8}{y^2} (9 \ln y - 3) + O\left(\frac{\ln^2 y}{y^3}\right), \quad (4.2) \\
\Phi &= -\frac{1}{2} \ln(2Q^2) + 2 \ln y - \frac{1}{y} 9 \ln y + O\left(\frac{\ln y}{y^2}\right), \quad (4.3) \\
\xi &= \frac{1}{2} \ln(2Q^2) + y + \frac{9}{y} + \frac{9}{2y^2} (9 \ln y - \frac{20}{9}) + O\left(\frac{\ln^2 y}{y^3}\right), \quad (4.4) \\
\eta &= \ln 2 + \frac{1}{2} y + \frac{1}{y} + \frac{1}{2y^2} (9 \ln y - 2) + O\left(\frac{\ln^2 y}{y^3}\right). \quad (4.5)
\end{align}

Its Einstein-frame metric is asymptotic to $AdS_5 \times S^5$ at $\rho = \infty$ and evolves into a metric with negative Ricci scalar at smaller $\rho$. The structure of the asymptotic UV and IR solutions is similar (notice that the coordinates corresponding to the two regions are related by $y \to -y$, cf. (2.12)–(2.16)), suggesting that they can be smoothly connected into the full interpolating solution. This is indeed supported by our numerical analysis which shows, in particular, that the tachyon starts at $T = -1$ at $\rho = 0$, grows according to (2.13), then enters an oscillating regime and finally relaxes to zero according to (1.2).

The fact that $T$ goes to zero makes the details of the coupling function $f(T)$ largely irrelevant in this IR region, implying that the resulting picture is quite robust. In the IR the dilaton blows up, i.e. the coupling is
\begin{equation}
e^{\Phi} = 2^{-1/2} Q^{-1} y^{2-\frac{9}{2} + O\left(\frac{1}{y^2}\right)}, \quad (4.6)
\end{equation}
and the 10-d Einstein-frame metric (2.1) is (cf. (2.18))
\begin{equation}
ds^2_E = R_\infty^2 \left(1 - \frac{1}{2y} - \frac{9}{4y^2} \ln y + ... \right) \left(\frac{1}{4} dy\right)^2
\end{equation}
\[ + (1 - \frac{9}{2y} - \frac{81}{4y^2} \ln y + \ldots) \frac{e^{-\frac{1}{2}y}}{2R_\infty^2} \, dx^\mu dx^\mu + (1 + \frac{7}{2y} + \frac{63}{4y^2} \ln y + \ldots) d\Omega_5^2 \right), \quad (4.7) \]

where

\[ R_\infty^2 = 2^{-3/4} Q^{1/2}. \]

This metric starts again as \( AdS_5 \times S^5 \) \([2.11]\) \((y = -4 \ln u)\) at \( y = \infty \) and becomes a negative curvature 10-d space at smaller \( y \) (bigger \( u \)). As in the large \( u \) region, the corrections to \( \xi \) and \( \eta \) cause the effective radius of \( AdS_5 \) to become smaller than that of \( S^5 \), leading to the negative total scalar curvature.

Indeed, the scalar curvature in the Einstein frame is

\[ R = -\frac{5}{8} T^2 e^{\frac{1}{2} \Phi} + \frac{1}{2} g^{mn} \partial_m \Phi \partial_n \Phi, \quad (4.8) \]

where the first term gives the dominant \( O(1/y) \) contribution. We find that in the UV regime the \( 1/y \) corrections to the metric \([2.18]\) give \((\rho = e^{-y})\)

\[ R(\rho \to 0) = -\frac{80}{R_0^2 y} + O\left(\frac{\ln y}{y^2}\right), \quad (4.9) \]

while in the IR regime the metric \([4.7]\) gives \((\rho = e^y)\)

\[ R(\rho \to \infty) = -\frac{80}{R_\infty^2 y} + O\left(\frac{\ln y}{y^2}\right). \quad (4.10) \]

Thus, in both regimes the Ricci scalar becomes negative away from the critical point. The absolute value of the curvature is small in both asymptotic regions but grows in between.

Note also that for the IR solution all higher-order \( \alpha' \) corrections are suppressed. The string loop corrections may become important right at the fixed point. Away from the fixed point, \( g_{st} \) is of order \( 1/Q \) and is thus regarded as very small.

5. Discussion

We have presented new evidence for our conjecture \([1]\) that \( SU(N) \) gauge theory coupled to 6 massless adjoint scalars is dual to type 0 string theory. The type 0 formulation indicates the presence of two fixed points: one at vanishing coupling and the other at infinite coupling. The RG flow for weak coupling has a number of features expected from the asymptotic freedom of the dual gauge theory, including the effect of the two-loop correction to the beta function. We have further argued that, as the RG trajectory is continued towards the infrared, the coupling grows indefinitely, eventually reaching the
fixed point at infinite coupling. We constructed the asymptotic expansion near this IR
fixed point, but it would be of further interest to study the entire trajectory in detail.

In all our calculations we used a specific form (1.3) of the function $f(T)$ describing
the tachyon couplings to the 5-form gauge fields. One may be concerned that the form
of this function depends on the scheme adopted in the derivation of the effective action
(for example, it may be changed by field redefinitions). We believe that practically any
function $f(T)$ which has a minimum at $T \neq 0$ will lead to the same physical picture.

Another interesting issue is the type 0 definition of the scale-dependent Yang-Mills
coupling. The approach adopted in [20] and in this paper is to define the gauge coupling
using the static quark-antiquark potential computed from the Wilson loop as in [21]. This
is a standard definition of the coupling used in QCD and known as the $\mathcal{V}$-scheme (see, e.g.,
[31] and references therein). One may compare this definition with the expression for the
effective gauge coupling that follows from the structure of the D3-brane action. Careful
consideration of the D-brane effective action shows that in type 0 theory the effective
Yang-Mills coupling is determined not just by the dilaton alone, but also by the tachyon
field. Indeed, due to the existence of a tachyon tadpole on the D-brane, the Born-Infeld
part of the effective action of an electric D3-brane is of the form (here we set $2\pi \alpha' = 1$)
\begin{equation}
\int d^4x \ k(T) \ e^{-\Phi} \sqrt{-\det(G_{\alpha\beta} + G_{ij} \partial_\alpha X^i \partial_\beta X^j + F_{\alpha\beta})},
\end{equation}
where $k(T) = 1 + \frac{1}{4}T + O(T^2)$. The coefficient 1/4 of the tachyon tadpole was found in [1].

The reparametrization invariance and $T$-duality imply that the function $k(T)$ multiplies
$\sqrt{-\det(G + F)}$ in the BI part of the action. From the function that multiplies $F^2_{\alpha\beta}$ in
the D3-brane action we read off that
\begin{equation}
g_{\text{YM}}^2 \sim k(T) e^{-\Phi}.
\end{equation}
With this definition of the coupling (‘the D-scheme’) the tachyon field plays a role in
determining the $u$-dependence of the Yang-Mills coupling (in the UV one has $T \approx -1 + \frac{1}{\ln u}$).
If we retain only the linear term in $k(T)$ then we find, however, that the product on the
r.h.s. of (5.2) grows as $\ln^2 u$ in the UV which disagrees with the $\mathcal{V}$-scheme. One possible
source of this disagreement is that one needs to know the exact expressions for both $f(T)$
and $k(T)$. If a zero of $k(T)$ coincides with the minimum of $f(T)$, then we will find $\sim \ln u$ on the r.h.s. of (5.2). It is also possible that $u$ is not simply proportional to the energy
scale of the gauge theory and that $\ln^2 u$ is actually to be identified with $\ln(L_0/L)$. More
work is needed to resolve these issues.

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11 This form of the action was independently proposed in a recent paper [32]. Both we and
Garousi [32] have checked the $TF_{\alpha\beta}F^{\alpha\beta}$ coupling that follows from this action by calculating
the corresponding 3-point function on a disk. Ref. [32] also contains some further checks of the
consistency of this form of the action.

12 For example, with $f(T) = 1 + T + \frac{1}{2}T^2$ and $k(T) = 1 + T$, $k(T)e^{-\Phi}$ grows as $\ln u$, i.e. gives
$\ln u - \frac{39}{8} \ln \ln u + ...$, which is the same answer as the effective potential (3.2).
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References

[1] I.R. Klebanov and A.A. Tseytlin, “D-Branes and Dual Gauge Theories in Type 0 Strings,” [hep-th/9811035].
[2] L. Dixon and J. Harvey, “String theories in ten dimensions without space-time supersymmetry”, Nucl. Phys. B274 (1986) 93; N. Seiberg and E. Witten, “Spin structures in string theory”, Nucl. Phys. B276 (1986) 272; C. Thorn, unpublished.
[3] I.R. Klebanov, “World volume approach to absorption by nondilatonic branes,” Nucl. Phys. B496 (1997) 231, [hep-th/9702076]; S.S. Gubser, I.R. Klebanov, and A.A. Tseytlin, “String theory and classical absorption by three-branes,” Nucl. Phys. B499 (1997) 217, [hep-th/9703040].
[4] S.S. Gubser and I.R. Klebanov, “Absorption by branes and Schwinger terms in the world volume theory,” Phys. Lett. B413 (1997) 41, [hep-th/9708005].
[5] J. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231, [hep-th/9711200].
[6] S.S. Gubser, I.R. Klebanov, and A.M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B428 (1998) 105, [hep-th/9802109].
[7] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2 (1998) 253, [hep-th/9802150].
[8] A.M. Polyakov, “The Wall of the Cave,” [hep-th/9809057].
[9] A.M. Polyakov, “String theory and quark confinement,” Nucl. Phys. B (Proc. Suppl.) 68 (1998) 1, [hep-th/9711002].
[10] G. Ferretti and D. Martelli, “On the construction of gauge theories from non critical type 0 strings,” [hep-th/9811208].
[11] A.A. Migdal, “Hidden Symmetries of Large N QCD,” Prog. Theor. Phys. Suppl. 131 (1998) 269, [hep-th/9610129].
[12] E. Alvarez, C. Gomez and T. Ortin, “String representation of Wilson loops”, [hep-th/9806073].
[13] J. Polchinski, “String Theory,” vol. 2, Cambridge University Press, 1998.
[14] J. Polchinski, “Dirichlet Branes and Ramond-Ramond charges,” Phys. Rev. Lett. 75 (1995) 4724, [hep-th/9510017].
[15] E. Witten, “Bound states of strings and p-branes,” Nucl. Phys. B460 (1996) 335, [hep-th/9510135].
[16] O. Bergman and M. Gaberdiel, “A Non-supersymmetric Open String Theory and S-Duality,” Nucl. Phys. B499 (1997) 183, [hep-th/9701137].
[17] M. Bianchi and A. Sagnotti, “On the Systematics of Open String Theories”, Phys. Lett. B247 (1990) 517.
[18] A. Sagnotti, “Some Properties of Open - String Theories”, [hep-th/9509080]; “Surprises in Open-String Perturbation Theory”, Nucl. Phys. Proc. Suppl. B56 (1997)
332, hep-th/9702093; C. Angelantonj, “Nontachyonic Open Descendants of the 0B String Theory”, hep-th/9810214.
[19] D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343; H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346.
[20] J. Minahan, “Glueball Mass Spectra and Other Issues for Supergravity Duals of QCD Models,” hep-th/9811156.
[21] J. Maldacena, “Wilson loops in large \( N \) field theories,” Phys. Rev. Lett. 80 (1998) 4859, hep-th/9803002; S.-J. Rey and J. Yee, “Macroscopic strings as heavy quarks in large \( N \) gauge theory and anti-de Sitter supergravity”, hep-th/9803001.
[22] D.J. Gross and F. Wilczek, “Asymptotically free gauge theories 2,” Phys. Rev. D9 (1974) 980.
[23] T. Banks and A. Zaks, “On the phase structure of vector-like gauge theories with massless fermions,” Nucl. Phys. B196 (1982) 189.
[24] T. Banks and M.B. Green, “Nonperturbative effects in \( AdS_5 \times S^5 \) string theory and \( d = 4 \) SUSY Yang-Mills,” J. High Energy Phys. 05 (1998) 002, hep-th/9804170; S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, “Coupling constant dependence in the thermodynamics of \( N = 4 \) supersymmetric Yang-Mills theory”, Nucl. Phys. B534 (1998) 202, hep-th/9805156.
[25] S. Weiberg, The Quantum Theory of Fields, vol. 2 (Cambridge Univ. Press, 1996).
[26] D.R.T. Jones, “Asymptotic behavior of supersymmetric Yang-Mills theories in the two-loop approximation,” Nucl. Phys. B87 (1975) 127; M.E. Machacek and M.T. Vaughn, “Two-loop renormalization group equations in a general quantum field theory I: Wave function renormalization,” Nucl. Phys. B222 (1983) 83.
[27] E.S. Fradkin and A.A. Tseytlin, “Quantum properties of higher dimensional and dimensionally reduced supersymmetric theories,” Nucl. Phys. B227 (1983) 252.
[28] I.R. Klebanov and E. Witten, “Superconformal field theory on threebranes at a Calabi-Yau singularity,” hep-th/9807080; S.S. Gubser, “Einstein manifolds and conformal field theories,” hep-th/9807164.
[29] L. Girardello, M. Petrini, M. Porrati, and A. Zaffaroni, “Novel Local CFT and Exact Results on Perturbations of \( N=4 \) Super Yang Mills from \( AdS \) Dynamics”, hep-th/9810128; J. Distler and F. Zamora, “Non-Supersymmetric Conformal Field Theories from Stable Anti-de Sitter Spaces”, hep-th/9810206.
[30] S.S. Gubser, N. Nekrasov and S. Shatashvili, “Generalized conifolds and four dimensional \( N = 1 \) superconformal theories,” hep-th/9811230.
[31] Y. Schröder, “The static potential in QCD to two loops”, hep-ph/9812205
[32] M.R. Garousi, “String Scattering from D-branes in Type 0 Theories”, hep-th/9901089.