TESTING COSMOLOGICAL MODELS WITH TYPE Ic SUPER LUMINOUS SUPERNOVAE

JUN-JIE WEI1,2, XUE-FENG WU1,4,5, AND FULVIO MELIA1,3

1 Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
2 University of Chinese Academy of Sciences, Beijing 100049, China; jjwei@pmo.ac.cn
3 Department of Physics, The Applied Mathematics Program, and Department of Astronomy, The University of Arizona, AZ 85721, USA; fmelia@email.arizona.edu
4 Chinese Center for Antarctic Astronomy, Nanjing 210008, China; xfwu@pmo.ac.cn
5 Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing University–Purple Mountain Observatory, Nanjing 210008, China

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ABSTRACT

The use of type Ic super luminous supernovae (SLSNe Ic) to examine the cosmological expansion introduces a new standard ruler with which to test theoretical models. The sample suitable for this kind of work now includes 11 SLSNe Ic, which have thus far been used solely in tests involving the Λ cold dark matter (ΛCDM) model. In this paper, we broaden the base of support for this new, important cosmic probe by using these observations to carry out a one-on-one comparison between the $R_0 = ct$ and ΛCDM cosmologies. We individually optimize the parameters in each cosmological model by minimizing the χ² statistic. We also carry out Monte Carlo simulations based on these current SLSNe Ic measurements to estimate how large the sample would have to be in order to rule out either model at a ~99.7% confidence level. The currently available sample indicates a likelihood of ~70–80% that the $R_0 = ct$ universe is the correct cosmology versus ~20–30% for the standard model. These results are suggestive, though not yet compelling, given the current limited number of SLSNe Ic. We find that if the real cosmology is ΛCDM, a sample of ~240 SLSNe Ic would be sufficient to rule out $R_0 = ct$ at this level of confidence, while ~480 SLSNe Ic would be required to rule out ΛCDM if the real universe is instead $R_0 = ct$. This difference in required sample size reflects the greater number of free parameters available to fit the data with ΛCDM. If such SLSNe Ic are commonly detected in the future, they could be a powerful tool for constraining the dark-energy equation of state in ΛCDM, and differentiating between this model and the $R_0 = ct$ universe.

Key words: cosmic background radiation – cosmological parameters – cosmology: observations – cosmology: theory – distance scale – supernovae: general

1. INTRODUCTION

A new type of luminous transient has been identified in recent years through the use of deep, wide surveys searching for supernovae in the local universe (Quimby et al. 2005, 2007; Quimby 2006; Smith et al. 2007; Drake et al. 2009; Rau et al. 2009; Kaiser et al. 2010; Gal-Yam 2012; Balty et al. 2013). These super luminous supernovae (SLSNe) have peak magnitudes $M_{AB} < -21$ mag and integrated burst energies $\sim 10^{51}$ erg. They are therefore much brighter than both Type Ia SNe (SNe Ia) and the majority of core-collapse events. Their unusually high peak luminosities, hot blackbody temperatures, and bright rest frame ultraviolet emission (which renders their continuum easily detectable at optical and near-infrared wavelengths at high redshifts; Cooke et al. 2012) allow them to be studied in concert with possible gamma-ray burst (GRB) associations (Cano & Jakobsson 2014; Li et al. 2014) and, more importantly, make them viable standardizable candles and distance indicators for use as cosmological probes (Inserra & Smartt 2014).

Inserra et al. (2013) and Nicholl et al. (2013) used the classification term SLSNe Ic to refer to all the hydrogen poor SNIe, though there appear to be at least two observational groups in this category. These are distinguished via the terms SN2005ap-like and SN2007bi-like events, since these are the prototypes with the faster and slower evolving light curves. SLSNe Ic have now been discovered out to redshifts $z \sim 4$ (Chomiuk et al. 2011; Berger et al. 2012; Cooke et al. 2012; Howell et al. 2013), and appear to be rather homogeneous in their spectroscopic and photometric properties.

The brighter events decline more slowly, not unlike the Phillips relation for SNe Ia, which raises the possibility of correlating their peak magnitudes to their decline over a fixed number of days to reduce the scatter (Rust 1974; Pskovskii 1977; Phillips 1993; Hamuy et al. 1996). Without the use of such a relation, the uncorrected raw mean magnitudes show a dispersion of ~0.4 (e.g., Inserra & Smartt 2014). Correlating the peak magnitude to the decline over 30 days reduces the scatter in standardized peak magnitudes to ±0.22 mag. And apparently using a magnitude–color evolution reduces this scatter even more, to a low value somewhere between ±0.08 and ±0.13 mag.

It is therefore quite evident that SLSNe Ic may be useful cosmological probes, perhaps even out to redshifts much greater ($z \gg 2$) than those accessible using SNe Ia. The currently available sample, however, is still quite small; adequate data to extract correlations between empirical, observable quantities, such as light curve shape, color evolution and peak luminosity, are available only for tens of events. Our focus in this paper is specifically to study whether SLSNe Ic can be used—not only to optimize the parameters in the Λ cold dark matter (ΛCDM) model (Cano & Jakobsson 2014; Inserra & Smartt 2014; Li et al. 2014), e.g., to refine the dark-energy equation of state but, also—to carry out comparative studies between competing cosmologies, such as ΛCDM versus the $R_0 = ct$ universe (Melia 2007, 2013a; Melia & Abdelqader 2009; Melia & Shevchuk 2012).

Like ΛCDM, the $R_0 = ct$ universe is a Friedmann–Robertson–Walker cosmology that assumes the presence of dark energy, as well as matter and radiation. The principle difference between them is that the latter is also constrained by
Table 1
Sample of SLSNe Ic

| SN          | \(z\)  | \(E(B - V)\) | \(m_V\) | Filter | \(A_V\) | \(K^{(400)}_{\text{abs,600}}\) | \(m(400)\) | \(\Delta M_{30}(400)\) | \(\Delta M_{30}(400 - 520)\) | References |
|-------------|--------|--------------|---------|--------|---------|------------------|------------|------------------|------------------|------------|
| SN2011ke    | 0.143  | 0.01         | 17.70   | \(g \rightarrow 400\) | 0.05     | −0.18            | 17.83 ± 0.20 | 2.47 ± 0.16   | 0.54 ± 0.17         | 1, 8       |
| SN2012li    | 0.175  | 0.02         | 18.00   | \(g \rightarrow 400\) | 0.09     | −0.18            | 18.09 ± 0.21 | 2.19 ± 0.16   | 0.49 ± 0.17         | 1, 8       |
| PTF10xks    | 0.190  | 0.04         | 19.13   | \(g \rightarrow 400\) | 0.17     | −0.19            | 19.15 ± 0.20 | 2.62 ± 0.14   | 0.95 ± 0.14         | 1, 8       |
| SN2010gx    | 0.250  | 0.04         | 18.43   | \(g \rightarrow 400\) | 0.13     | −0.23            | 18.53 ± 0.18 | 2.00 ± 0.19   | 0.34 ± 0.16         | 2, 8       |
| SN2011kd    | 0.245  | 0.02         | 18.60   | \(g \rightarrow 400\) | 0.09     | −0.13            | 18.64 ± 0.18 | 1.49 ± 0.16   | ...               | 1, 8       |
| LSO12df     | 0.255  | 0.01         | 18.76   | \(V \rightarrow 400\) | 0.03     | −0.27            | 19.02 ± 0.12 | 1.00 ± 0.10   | 0.29 ± 0.12         | 3, 8       |
| PTF09end    | 0.258  | 0.03         | 18.29   | \(R \rightarrow 400\) | 0.03     | −0.34            | 18.58 ± 0.22 | 1.09 ± 0.14   | ...               | 4, 8       |
| SN2013dg    | 0.265  | 0.01         | 19.06   | \(g \rightarrow 400\) | 0.03     | −0.30            | 19.33 ± 0.20 | 2.08 ± 0.20   | 0.51 ± 0.17         | 3, 8       |
| PS1-10bzj   | 0.650  | 0.01         | 21.23   | \(i \rightarrow 400\) | 0.02     | −0.58            | 21.79 ± 0.20 | 1.70 ± 0.14   | 0.60 ± 0.15         | 5, 8       |
| PS1-10ky    | 0.956  | 0.03         | 21.15   | \(i \rightarrow 400\) | 0.06     | −0.73            | 21.82 ± 0.18 | 1.31 ± 0.15   | 0.30 ± 0.18         | 6, 8       |
| SCP-06F6    | 1.189  | 0.01         | 21.04   | \(z \rightarrow 400\) | 0.01     | −1.35            | 22.38 ± 0.20 | 0.89 ± 0.15   | ...               | 7, 8       |

Notes. All values are from Inserra & Smartt (2014), except for \(m(400)\), which is calculated here. The error bars are directly from Figures 5 and 6 in Inserra & Smartt (2014). Columns: SN name; measured redshift; extinction; observed peak magnitude (AB system) and filter; Galactic extinction in the observed filter; calculation of the synthetic 400 nm magnitude from the observed filter; apparent magnitude mapped into the 400 nm passband, and its dispersion; magnitude difference in 30 restframe days and its dispersion; the color change between the 400 and 520 nm synthetic bands at peak and 30 days later, and its dispersion.

References. (1) Inserra et al. (2013), (2) Pastorello et al. (2010), (3) Nicholl et al. (2014), (4) Quimby et al. (2011), (5) Lunnan et al. (2013), (6) Chomiuk et al. (2011), (7) Barbary et al. (2009), (8) Inserra & Smartt (2014).

The equation of state \(p + 3\rho = 0\), in terms of the total pressure \(p\) and energy density \(\rho\). In recent years, this model has generated some discussion concerning its fundamental basis, including claims that it is actually a vacuum solution, even though \(p\neq 0\). However, all criticisms leveled against \(R_0 = ct\) thus far appear to be based on incorrect assumptions and theoretical errors. A full accounting of this discussion may be found in Melia (2015) and references cited therein.

In fact, the application of model selection tools in one-on-one comparisons between these two cosmologies has shown that the data tend to favor \(R_0 = ct\) over \(\Lambda CDM\). Tests completed thus far include high-z quasars (Melia 2013b, 2014), GRBs (Wei et al. 2013), the use of cosmic chronometers (Melia & Maier 2013) and, most recently, the SNe Ia themselves (Wei et al. 2015). In all of these tests, information criteria show that \(R_0 = ct\), with the important additional constraint \(p + 3\rho = 0\) on its equation of state, is favored over \(\Lambda CDM\) with a likelihood of ~90% versus only ~10%.

Here, we broaden the comparison between \(R_0 = ct\) and \(\Lambda CDM\) by now including SLSNe Ic in this study. In Section 2 we briefly describe the currently available sample and our method of analysis. Our results are presented in Section 3. We will find that the current catalog of SLSNe Ic suitable for this study already confirms the tendencies discussed above, though the statistics are not yet good enough to strongly differentiate between these two competing models. We show in Section 4 how large the source catalog needs to be in order to rule out one or the other expansion scenario at a \(3\sigma\) confidence level, and we present our conclusions in Section 5.

2. METHODOLOGY

Eleven of the SLSNe Ic identified by Inserra & Smartt (2014) are appropriate for this work, and we base our analysis on the methodology described in their paper and in Li et al. (2014). Briefly, the chosen SLSNe Ic must have well sampled light-curves around peak luminosity, and photometric coverage from several days pre-maximum to 30 days (in the rest frame) after the peak. This time delay appears to be optimal for use in the Phillips-like peak magnitude–decline relation (Inserra & Smartt 2014).

All 11 of these events appear to be similar to the well-observed SN2010gx, and these decay rapidly after peak brightness. They belong to the group of 2005ap-like events, the first such SN discovered in this category. Other SLSNe Ic could not be included simply because of lack of sufficient temporal coverage, even though their identification has been securely classified in previous work (see, e.g., Chomiuk et al. 2011; Quimby et al. 2011; Berger et al. 2012; Leloudas et al. 2012; Inserra et al. 2013; Nicholl et al. 2014).

The SNe listed in Table 1 were located in faint, dwarf galaxies, and were unlikely to have suffered significant extinction beyond the reddening induced by interstellar dust in our Galaxy. We here adopt the reddening corrections from Tables 1 and 2 of Inserra & Smartt (2014), who assumed a standard reddening curve with \(R_V = A_V / E(B - V) = 3.1\). We also adopt their \(K\)-corrections and time dilation effects in order to obtain the absolute rest-frame peak magnitudes. This step is necessary due to the large \((0.143 < z < 1.206)\) redshift coverage of the sample. These are listed in the table, along with the source names, their redshifts, apparent peak magnitudes (and filters), the magnitude decrease over 30 days, and the color change between the 400 and 520 nm synthetic (restframe) bands at peak and 30 days later.

In order to provide consistent, standarized comparative rest frame properties, the observed apparent magnitudes \(m(400)\) in Table 1 have been converted into defined, synthetic magnitudes. Since the SLSNe Ic spectrum around 400 nm is continuum dominated, Inserra & Smartt (2014) defined a synthetic passband with an effective width of 80 nm, centered at wavelength 400 nm, having steep wings and a flat top. In Table 1, this is referred to as the 400 nm band. All of the chosen events in this table have sufficient photometric coverage to allow the \(K\)-correction to uniformly map the observed filter’s wavelength range to this 400 nm bandpass in the rest frame. The absolute magnitudes are then formally defined by the
relation
\[ M_{\text{peak}}(400) = m(400) - \mu = m_f - K_{\text{peak}400} - A_f - \mu, \]

where \( m(400) \) is the apparent magnitude mapped into the restframe 400 nm band, \( \mu \) is the distance modulus calculated from the luminosity distance, \( m_f \) is the AB magnitude in the observed filter \( f (g, V, R, r, z, \text{or } i, \text{as indicated in Table 1}) \), \( A_f \) is the Galactic extinction in the observed filter, and \( K_{\text{peak}400} \) is the \( K \)-correction from the observed filter in Table 1 to the synthetic 400 nm bandpass. The values of \( A_f \) and \( m(400) \) are listed in Table 1. Note that \( m(400) \) is a cosmology-independent apparent magnitude.

The peak magnitude–decline relation for SLSNe Ic in the rest frame 400 nm band is (Inserro & Smartt 2014)
\[ M_{\text{peak}}(400) = M_0 + \alpha \Delta M_{30}(400), \]

where \( \alpha \) is the slope, \( \Delta M_{30}(400) \) is the decline at 30 days, and \( M_0 \) is a constant representing the absolute peak magnitude at \( \Delta M_{30}(400) = 0 \). Inserro & Smartt (2014) also found that \( M_{\text{peak}}(400) \) appears to have a strong color dependence. Redder objects are fainter and also become redder faster. This peak magnitude–color evolution relation is given by
\[ M_{\text{peak}}(400) = M_0 + \alpha \Delta M_{30}(400 - 520), \]

where \( \Delta M_{30}(400-520) \) is the difference in color \( M(400)-M(520) \) between the peak and 30 days later. (Note that \( M_0 \) and \( \alpha \) need not be the same in these two expressions.)

With Equations (2) and (3), an effective (standardized) apparent magnitude \( m_{\text{eff}} \) may be obtained as follows:
\[ m_{\text{eff}} \equiv m(400) - \alpha \Delta M_{30}. \]

The term \( \alpha \Delta M_{30} \) represents an adjustment due to the peak magnitude–decline relation, in terms of \( \alpha \Delta M_{30}(400) \), or the peak magnitude–color evolution relation, in terms of \( \alpha \Delta M_{30}(400 - 520) \), as the case may be. The effective apparent magnitude may also be expressed as (Perlmutter et al. 1997, 1999)
\[ m_{\text{eff}} = Y + 5 \log_{10}(H_0 \ d_L(z)), \]

where \( H_0 \) is the Hubble constant in units of \( \text{km} \ \text{s}^{-1} \ \text{Mpc}^{-1} \) and \( d_L(z) \) is the luminosity distance in units of Mpc. Here \( Y \) is the “\( H_0 \)-free” 400 nm absolute peak magnitude, represented in terms of the standardizable absolute magnitude \( M_0 \), according to the definition
\[ Y \equiv M_0 - 5 \log_{10}(H_0) + 25 \]

(see Li et al. 2014 and references cited therein).

The apparent correlations suggest the use of two “nuisance” parameters \( (\alpha \) and \( Y \) whose optimization along with the model parameters decreases the overall scatter in the distance modulus. For each model, we therefore find the best fit by minimizing the \( \chi^2 \) statistic, defined as follows:
\[ \chi^2 = \sum_i \left[ \frac{(m_i(400) - \alpha \Delta M_{30}) - Y - 5 \log_{10}(H_0 \ d_L(z_i))}{\sigma_{m_i(400)}^2 + \sigma_{\Delta M_{30}}^2} \right]^2. \]

In \( \Lambda \)CDM, the luminosity distance \( d_L^{\Lambda \text{CDM}} \) depends on several parameters, including \( H_0 \) and the mass fractions \( \Omega_m \equiv \rho_m/\rho_c \), \( \Omega_\Lambda \equiv \rho_\Lambda/\rho_c \), and \( \Omega_{de} \equiv \rho_{de}/\rho_c \), defined in terms of the current matter \( (\rho_m) \), radiation \( (\rho_r) \), and dark-energy \( (\rho_{de}) \) densities, and the critical density \( \rho_c \equiv 3c^2H_0^2/8\pi G \). Assuming zero spatial curvature, so that \( \Omega_m + \Omega_\Lambda + \Omega_{de} = 1 \), the luminosity distance to redshift \( z \) is given by the expression
\[ d_L^{\Lambda \text{CDM}}(z) = \frac{c}{H_0}(1 + z) \int_0^z \left[ \Omega_m(1 + z)^3 + \Omega_\Lambda(1 + z)^\Lambda + \Omega_{de}(1 + z)^{\Lambda+\sigma} \right]^{-\frac{1}{2}} \ dz, \]

where \( \rho_{de} = \omega_{de} \rho_c \) is the dark-energy equation of state. The Hubble constant \( H_0 \) cancels out in Equation (7) when we multiply \( d_L \) by \( H_0 \), so the essential remaining parameters in flat \( \Lambda \)CDM are \( \Omega_m \) and \( \omega_{de} \). If we further assume that dark energy is a cosmological constant with \( \omega_{de} = -1 \), then only the parameter \( \Omega_m \) is available to fit the data.

In the \( R_h = ct \) universe (Melia 2007; Melia & Abdelqader 2009; Melia & Shevchuk 2012), the luminosity distance depends only on \( H_0 \), but since here too the Hubble constant cancels out in the product \( H_0 d_L \), there are actually no free (model) parameters left to fit the SLSNe Ic data. In this cosmology,
\[ d_L^{R_h=ct}(z) = \frac{c}{H_0}(1 + z)\ln(1 + z). \]

3. RESULTS

We have assumed that SLSNe Ic can be used as standardizable candles and applied the \( \Delta M_{30} \) decline relation (with 11 objects) and the peak magnitude–color evolution relation (with eight objects) to compare the standard (\( \Lambda \)CDM) model with the \( R_h = ct \) universe. In this section, we discuss how the fits have been optimized, first for \( \Lambda \)CDM, and then for \( R_h = ct \). The outcome for each model is more fully described and discussed in subsequent sections.

3.1. \( \Lambda \)CDM

In the most basic \( \Lambda \)CDM model, the dark-energy equation of state parameter, \( \omega_{de} \), is exactly \(-1\). The Hubble constant \( H_0 \) cancels out in Equation (7) when we multiply \( d_L \) by \( H_0 \), so the essential remaining parameter is \( \Omega_m \). SN Ia measurements (see, e.g., Perlmutter et al. 1998, 1999; Riess et al. 1998; Schmidt et al. 1998), cosmic microwave background anisotropy data (e.g., Hinshaw et al. 2013), and baryon acoustic oscillation peak length scale estimates (e.g., Samushia & Rafie 2009), strongly suggest that we live in a spatially flat, dark energy-dominated universe with concordance parameter values \( \Omega_m \approx 0.27 \) and \( H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

We will therefore first attempt to fit the data with this concordance model, using prior values for all the model parameters, but not the two “nuisance” parameters \( \alpha \) and \( Y \). For the \( \Delta M_{30} \) decline relation (with 11 objects), the resulting constraints on \( \alpha \) and \( Y \) are shown in Figure 1(a). For this fit, we obtain \( \alpha = 0.57^{+0.19}_{-0.17} \) (1σ), \( Y = -6.75^{+0.28}_{-0.32} \) (1σ), and a \( \chi^2 \) per degree of freedom of \( \chi^2_{dof} = 27.04/9 = 3.00 \), remembering that all of the \( \Lambda \)CDM parameters are assumed to have prior values, except for the “nuisance” parameters \( \alpha \) and \( Y \). (The corresponding data and best fit are shown in the top left-hand
Figure 1. 1–3σ constraints on α and Υ for the concordance model, using the ΔM_{90} decline relation (panel (a)) and the peak magnitude–color evolution relation (panel (b)).

For the peak magnitude–color evolution relation (with eight objects), the resulting constraints on α and Υ are shown in Figure 1(b). The best-fit parameters are α = 2.00^{+1.19}_{−0.79} (1σ), Υ = −6.65^{+0.45}_{−0.97} (1σ). The χ^2 per degree of freedom for the concordance model with these optimized “nuisance” parameters is χ^2_{df} = 2.48/6 = 0.41 (see also the top right-hand panel in Figure 4).

If we relax some of the priors, and allow Ω_m to be a free parameter, we obtain the 1–3σ constraint contours shown in the top panel of Figure 2 for the optimized ΔM_{90} decline relation, and the bottom panel for the peak magnitude–color evolution relation. These contours show that at the 1σ level, the optimized parameter values are α = 0.46^{+0.18}_{−0.18}, Υ = −6.41^{+0.34}_{−0.33}, and Ω_m = 0.62^{+0.35}_{−0.35}. We find that the χ^2 per degree of freedom for the optimized flat ΛCDM model is χ^2_{df} = 24.89/8 = 3.11. The bottom panel of Figure 2 shows the 1–3σ constraints on α, Υ, and Ω_m for the flat ΛCDM model, using the peak magnitude–color evolution relation. The contours show that at the 1σ level, α = 2.05^{+1.19}_{−0.84}, Υ = −6.65^{+0.57}_{−0.60}, but that Ω_m is poorly constrained; only a lower limit of ~0.05 can be set at this confidence level and the best-fit Ω_m is 0.59. The χ^2 per degree of freedom is χ^2_{df} = 2.15/5 = 0.43.

3.2. The R_h = ct Universe

The R_h = ct universe has only one free parameter, H_0, but since the Hubble constant cancels out in the product H_0d_L, there are actually no free (model) parameters left to fit the SLSNe Ic data. The results of fitting the ΔM_{90} decline relation with this cosmology are shown in Figure 3(a). We see here that the best fit corresponds to α = 0.50^{+0.19}_{−0.17} (1σ) and Υ = −6.48^{+0.28}_{−0.31} (1σ). With 11 − 2 = 9 degrees of freedom, the reduced χ^2 is χ^2_{df} = 25.79/9 = 2.87. The results of fitting the peak magnitude–color evolution relation with this cosmology are shown in Figure 3(b). We see here that the best fit corresponds to α = 1.88^{+0.84}_{−0.78} (1σ) and Υ = −6.44^{+0.54}_{−0.94} (1σ). With 8 − 2 = 6 degrees of freedom, the reduced χ^2 is χ^2_{df} = 2.19/6 = 0.37.

To facilitate a direct comparison between ΛCDM and R_h = ct, we show in Figure 4 the Hubble diagrams for SLSNe Ic. In the left panel of Figure 4, the effective magnitudes m_{eff} of 11 SLSNe Ic are plotted as solid points, together with the best-fit theoretical curves (from top to bottom) for the concordance model (with prior values of the parameters, and with α = 0.57 and Υ = −6.75), for the optimized flat ΛCDM model (with Ω_m = 0.62, and with α = 0.46, and Υ = −6.41), and for the R_h = ct universe (with α = 0.50 and Υ = −6.48). The Hubble diagrams derived using the peak magnitude–color evolution relation are shown in the right-hand panels of Figure 4. Here, the effective magnitudes m_{eff} of 8 SLSNe Ic are plotted as solid points, together with the best-fit theoretical curves for the concordance model (with prior values of the parameters, and with α = 2.00 and Υ = −6.65), for the optimized flat ΛCDM model (with Ω_m = 0.59, and with α = 2.05, and Υ = −6.52), and for the R_h = ct universe (with α = 1.88 and Υ = −6.44). Strictly based on their χ^2 values, the optimized ΛCDM model and the R_h = ct universe appear to fit the SLSNe Ic data comparably well. However, because these models formulate their observables (such as the luminosity distances in Equations (8) and (9)) differently, and because they do not have the same number of free parameters, a comparison of the likelihoods for either being closer to the “true” model must be based on model selection tools.

3.3. Model Selection Tools

Several information criteria commonly used in cosmology (see, e.g., Melia & Maier 2013, and references cited therein) include the Akaike Information Criterion, AIC = χ^2 + 2n, where n is the number of free parameters (Akaike 1973; Takeuchi 2000; Liddle 2004, 2007; Tan & Biswas 2012), the Kullback Information Criterion, KIC = χ^2 + 3n (Bhansali & Downham 1977; Cavanaugh 1999, 2004), and the Bayes Information Criterion, BIC = χ^2 + (lnN)n, where N is the number of data points (Schwarz 1978; Liddle et al. 2006; Liddle 2007). With AIC_α characterizing model M_α, the unnormalized confidence that this model is true is the Akaike weight exp(−AIC_α/2). Model M_α has a likelihood of

\[ P(M_α) = \frac{\exp(-AIC_α/2)}{\exp(-AIC_1/2) + \exp(-AIC_2/2)} \]  

of being the correct choice in this one-on-one comparison. The difference ΔAIC ≡ AIC_2 − AIC_1 determines the extent to
Figure 2. Top panel: constraints on $\alpha$, $\Upsilon$, and $\Omega_m$ for the flat $\Lambda$CDM model, using the $\Delta M_{30}$ decline relation. Bottom panel: same as the top panel, but using the peak magnitude–color evolution relation.
which $\mathcal{M}_1$ is favored over $\mathcal{M}_2$. For Kullback and Bayes, the likelihoods are defined analogously.

With the optimized fits we have reported in this paper, our analysis of the $\Delta M_{30}$ decline relation (with 11 objects) shows that $R_h = ct$ is favored over the flat $\Lambda$CDM model with a likelihood of $\approx 63.4\%$ versus $36.6\%$ using AIC, $\approx 74.1\%$ versus $\approx 25.9\%$ using KIC, and $\approx 67.9\%$ versus $\approx 32.1\%$ using BIC. In our one-on-one comparison using the peak magnitude–color evolution relation (with eight objects), the $R_h = ct$ universe is preferred over $\Lambda$CDM with a likelihood of $\approx 72.7\%$ versus $27.3\%$ using AIC, $\approx 81.5\%$ versus $\approx 18.5\%$ using KIC, and $\approx 73.5\%$ versus $\approx 26.5\%$ using BIC.

4. MONTE CARLO SIMULATIONS
WITH A MOCK SAMPLE

These results are interesting, though the current sample of SLSNe Ic is clearly too small for either model to be ruled out just yet. However, this situation will change with the discovery of new SNe, particularly at redshifts $z > 2$. To anticipate how well the SLSNe Ic catalog obeying the peak magnitude–color evolution relation may be used to constrain the dark-energy equation of state in $\Lambda$CDM, and to differentiate between this standard model and the $R_h = ct$ universe, we will here produce mock samples of SLSNe Ic based on the current measurement accuracy. In using the model selection tools, the outcome $\Delta \equiv$
AIC$_1$–AIC$_2$ (and analogously for KIC and BIC) is judged “positive” in the range $\Delta = 2 - 6$, “strong” for $\Delta = 6 - 10$, and “very strong” for $\Delta > 10$. In this section, we will estimate the sample size required to significantly strengthen the evidence in favor of $R_h = ct$ or $\Lambda$CDM, by conservatively seeking an outcome even beyond $\Delta \approx 11.62$, i.e., we will see what is required to produce a likelihood $\approx 99.7\%$ versus $\sim 0.3\%$, corresponding to a $3\sigma$ confidence level.

We will consider two cases: one in which the background cosmology is assumed to be $\Lambda$CDM, and a second in which it is $R_h = ct$, and we will attempt to estimate the number of SLSNe Ic required in each case to rule out the alternative (presumably incorrect) model at a $\sim 99.7\%$ confidence level. The synthetic SLSNe Ic are each characterized by a set of parameters denoted as $(z, m[400], \Delta M_{30}[400-520])$. We generate the synthetic sample using the following procedure.

1. Since a subclass of broad-lined SNe Ic are observed to be associated with GRBs (Hjorth et al. 2003; Stanek et al. 2003), we simulate the $z$-distribution of our sample based on the observed $z$-distribution of GRBs. Shao et al. (2011) found that the $z$-distribution of GRBs appears to be asymptotic to a parameter-free probability density function, $f(z) = ze^{-z}$. The redshift $z$ of our SLSNe Ic events is generated randomly from this function. Since they can be discovered out to redshifts $z \sim 4$ (e.g., Chomiuk et al. 2011; Berger et al. 2012; Cooke et al. 2012; Howell et al. 2013), the range of redshifts for our analysis is $[0, 4]$. We assign the absolute peak magnitude $M(400)$ uniformly between $-23.0$ and $-21.0$ mag, based on the current SLSNe Ic measurements.

2. The synthetic apparent peak magnitude $m(400)$ is calculated using the relation $m(400) = M(400) + \mu(z)$, where $\mu(z)$ is the distance modulus at $z$, for a flat $\Lambda$CDM cosmology with $\Omega_m = 0.27$ and $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ (Section 4.1), or the $R_h = ct$ universe with $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ (Section 4.2).

3. With the selected $z$ and $m(400)$ values, we first infer a $\Delta M_{30}(400 - 520)$ from the peak magnitude–color evolution relation in a flat $\Lambda$CDM cosmology with $\Omega_m = 0.27$ and $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ (Section 4.1), or the $R_h = ct$ universe with $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ (Section 4.2). And then we add scatter to this relation by assigning a dispersion to the $\Delta M_{30}(400 - 520)$ value; i.e., we randomly map this quantity to the new value $\Delta M_{30}(400 - 520)$ assuming a normal distribution with a center at $\Delta M_{30}(400 - 520)$ and a dispersion $\sigma = 0.1$ mag. This value of $\sigma$ is typical for the current (observed) peak magnitude–color evolution relation, which yields a standard deviation $\sigma = 0.08$ mag for the linear fit (Inserra & Smartt 2014). If $\Delta M_{30}(400 - 520) > 0.0$ mag, this SLSNe Ic is included in our sample. Otherwise, it is excluded.

4. We next assign “observational” errors to $m(400)$ and $\Delta M_{30}(400 - 520)$. We will assign a dispersion $\sigma_i = 0.05 + 0.05k_i$ to each event $i$, where $k_i$ is a random number between 0 and 1.

This sequence of steps is repeated for each new SLSNe Ic in the sample, until we reach the likelihood criterion discussed above. As with the real 11-SLSNe Ic sample, we optimize the model fits by minimizing the $\chi^2$ function in Equation (7).

### 4.1. Assuming $\Lambda$CDM as the Background Cosmology

We have found that a sample of at least 240 SLSNe Ic is required in order to rule out $R_h = ct$ at the $99.7\%$ confidence level. The optimized parameters corresponding to the best-fit $\Lambda$CDM model for these simulated data are displayed in Figure 5. To allow for the greatest flexibility in this fit, we relax the assumption that dark energy is a cosmological constant with $w_{de} = -1$, and allow $w_{de}$ to be a free parameter, along with $\Omega_m$. Figure 5 shows the 1D probability distribution for each parameter ($\Omega_m$, $w_{de}$, $\alpha$, $\gamma$), and 2D plots of the $1\sigma$ confidence regions for two-parameter combinations. The best-fit values for $\Lambda$CDM using the simulated sample with 240 SNe Ic in the $\Lambda$CDM model are $\Omega_m = 0.28^{+0.03}_{-0.04}$ (1$\sigma$), $w_{de} = -1.14^{+0.28}_{-0.35}$ (1$\sigma$), $\alpha = 2.21^{+0.10}_{-0.09}$ (1$\sigma$), and $\gamma = -6.80^{+0.03}_{-0.13}$ (1$\sigma$).

To gauge the impact of these constraints more clearly, we show in Figure 6 the confidence regions (shaded, with red contours) for $\Omega_m$ and $w_{de}$ using these 240 simulated SLSNe Ic (the same as the bottom left-hand panel of Figure 5), and compare these to the constraint contours for the 580 Union2.1 Type Ia SN data (Suzuki et al. 2012) (represented by the blue contours in Figure 6). It is straightforward to see how effectively the SLSNe Ic could be used as a cosmological tool, because the confidence regions resulting from their analysis are smaller (and narrower for $\Omega_m$) than those corresponding to the Type Ia events. The better constraints are mainly due to the fact that SLSNe Ic are distributed over a much wider redshift range, extending toward high-$z$, where tighter constraints on the model can be achieved.

In Figure 7, we show the corresponding 2D contours in the $\alpha$–$\gamma$ plane for the $R_h = ct$ universe. The best-fit values for the simulated sample are $\alpha = 2.22^{+0.09}_{-0.08}$ (1$\sigma$) and $\gamma = -6.53^{+0.03}_{-0.04}$ (1$\sigma$).

Since the number $N$ of data points in the sample is now much greater than one, the most appropriate information criterion to use is the BIC. The logarithmic penalty in this model selection tool strongly suppresses overfitting if $N$ is large (the situation we have here, which is deep in the asymptotic regime). With $N = 240$, our analysis of the simulated sample shows that the BIC would favor the $\Lambda$CDM model over $R_h = ct$ by an overwhelming likelihood of $99.7\%$ versus only $0.3\%$ (i.e., the prescribed $3\sigma$ confidence limit).

### 4.2. Assuming $R_h = ct$ as the Background Cosmology

In this case, we assume that the background cosmology is the $R_h = ct$ universe, and seek the minimum sample size to rule out $\Lambda$CDM at the $3\sigma$ confidence level. We have found that a minimum of 480 SLSNe Ic are required to achieve this goal. To allow for the greatest flexibility in the $\Lambda$CDM fit, here too we relax the assumption of dark energy as a cosmological constant with $w_{de} = -1$, and allow $w_{de}$ to be a free parameter, along with $\Omega_m$. In Figure 8, we show the 1D probability distribution for each parameter ($\Omega_m$, $w_{de}$, $\alpha$, $\gamma$), and 2D plots of the $1\sigma$ confidence regions for two-parameter combinations. The best-fit values for $\Lambda$CDM using this simulated sample with 480 SLSNe Ic are $\Omega_m = 0.0$, $w_{de} = -0.33^{+0.01}_{-0.06}$ (1$\sigma$), $\alpha = 2.05^{+0.07}_{-0.07}$ (1$\sigma$), and $\gamma = -6.52^{+0.04}_{-0.05}$ (1$\sigma$). Note that the simulated SLSNe Ic give a good constraint on $w_{de}$, but a weak constraint on $\Omega_m$; only an upper limit of 0.09 can be set at the $1\sigma$ confidence level.

The corresponding 2D contours in the $\alpha$–$\gamma$ plane for the $R_h = ct$ universe are shown in Figure 9. The best-fit values for
the simulated sample are $\alpha = 2.05^{+0.07}_{-0.06}$ (1$\sigma$) and $\Upsilon = -6.52^{+0.02}_{-0.02}$ (1$\sigma$). These are similar to those in the standard model, but not exactly the same, reaffirming the importance of reducing the data separately for each model being tested. With $N = 480$, our analysis of the simulated sample shows that in this case the BIC would favor $R_0 = ct$ universe, using $\Lambda$CDM by an overwhelming likelihood of 99.7% versus only 0.3% (i.e., the prescribed 3$\sigma$ confidence limit).

5. CONCLUSIONS

It is quite evident that SLSNe Ic may be useful cosmological probes, perhaps even out to redshifts much greater ($z \gg 2$) than those accessible using SNe Ia. The currently available sample, however, is still quite small; adequate data to extract correlations between empirical, observable quantities, such as light curve shape, color evolution and peak luminosity, are available only for tens of events. In this paper, we have
proposed to use SLSNe Ic for an actual one-on-one comparison between competing cosmological models. This must be done because the results we have presented here already indicate a strong likelihood of being able to discriminate between models such as $\Lambda$CDM and $\omega$CDM. Such comparisons have already been made using, e.g., cosmic chronometers (Melia & Maier 2013), GRBs (Wei et al. 2013), and SNe Ia (Wei et al. 2015).

We have individually optimized the parameters in each model by minimizing the $\chi^2$ statistic. With the optimized fits we have reported above, our analysis of the $\Delta M_{30}$ decline relation (with 11 objects) shows that $R_h = ct$ is favored over the flat $\Lambda$CDM model with a likelihood of $\approx 63–74\%$ versus $\approx 26–37\%$ (depending on the information criterion). In our one-on-one comparison using the peak magnitude–color evolution relation (with eight objects), the $R_h = ct$ universe is preferred over $\Lambda$CDM with a likelihood of $\approx 73–82\%$ versus $\approx 18–27\%$.

But though SLSNe Ic observations currently tend to favor $R_h = ct$ over $\Lambda$CDM, the known sample of such measurements is still too small for us to completely rule out either model. We have therefore considered two synthetic samples with characteristics similar to those of the eight known SLSNe Ic measurements, one based on a $\Lambda$CDM background cosmology, the other on $R_h = ct$. From the analysis of these simulated SLSNe Ic, we have estimated that a sample of about 240 such events are necessary to rule out $\omega$CDM at a $\sim 99.7\%$ confidence level if the real cosmology is in fact $\Lambda$CDM, while a sample of at least 480 SNe would be needed to similarly rule out $\omega$CDM if the background cosmology were instead $R_h = ct$. The difference in required sample size results from $\omega$CDM’s greater flexibility in fitting the data, since it has a larger number of free parameters.

Our simulations have also shown that a moderate sample size of $\sim 250$ events could reach much tighter constraints on the dark-energy equation of state $w_{de}$ and on the matter density fraction $\Omega_m$ than are currently available with the 580 Union2.1 Type Ia SNe. If SLSNe Ic can be commonly detected in the future, they have the potential of greatly refining the measurement of cosmological parameters, particularly the dark-energy equation of state $w_{de}$.

Figure 8. Same as Figure 5, except now with $R_h = ct$ as the (assumed) background cosmology. The simulated model parameter was $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$.

Figure 9. Same as Figure 7, except now with $R_h = ct$ as the (assumed) background cosmology.
Assembling samples of this size may be feasible with upcoming surveys. For example, the planned survey SUDSS (Survey Using Decam for Super-luminous Supernovae) using the Dark Energy Camera on the CTIO Blanco 4 m telescope (Inserra & Smartt 2014), and the Subaru/Hyper Suprime-Cam deep survey (Tanaka et al. 2012), have a goal of discovering several hundred SLSNe out to $z \sim 4$ over 3 yr by imaging tens of square degrees to a limiting magnitude $\sim 25$ every 2 weeks. These numbers are based on an estimated production in the local universe by Quimby et al. (2013), who reported a rate of $32^{+27}_{-16}$ SNe Gpc$^{-3}$ yr$^{-1}$ at a weighted redshift of $\langle z \rangle = 0.17$. This number is low (0.01%) compared to that of SNe Ia. McCrum et al. (2014) estimate a SN Ic rate $\sim 10^{-4}$ of the overall core-collapse SN rate within $0.3 < z < 1.4$, though this number could be higher at $z > 1.5$ (Cooke et al. 2012) due, perhaps, to a decreasing metallicity. Moreover, the increase in cosmic star formation rate would boost the absolute numbers of SLSNe. As far as observations from the ground are concerned, assembling a sample of several hundred SLSNe Ic over 3–5 yr therefore looks quite promising, particularly with the surveying capability of LSST (see, e.g., Lien & Fields 2009), which should cover the whole sky every 2 nights, down to a limiting magnitude $\sim 24$. The expected rate of discovery of core-collapse SNe with this survey is expected to be $\sim 1$–2 S$^{-1}$ out to a redshift $z \sim 2$. Given that $\sim 10^{-4}$ of these are expected to be SNe Ic in this redshift range, one should expect LSST to assemble a $\sim 500$ SLSN sample in less than a year. The situation from space is even more exciting. Detecting SLSNe Ic out to redshifts $z \sim 10$ via their restframe 400 and 520 nm bands is plausible with, e.g., EUCLID (Laureijs et al. 2011), WFIRST and the James Webb Space Telescope (specifically the NIRCam). 6

A major problem with this approach right now, however, is that one must rely on the use of a luminosity-limited sample of supernovae, i.e., those selected to be super-luminous with peak magnitudes $M_{AB} < -21$ mag and integrated burst energies $\sim 10^{51}$ erg (Inserra & Smartt 2014), in order to test the luminosity distances. It is clear that the luminosity-limited sample may turn out to be different with different background cosmologies, given that their inferred luminosities are themselves dependent on the models. This situation is unlikely to change as the sample grows, so it would be necessary to find a way of identifying these SNe, other than simply through their magnitudes. Fortunately, in the case of $R_{0} = ct$ versus $\Lambda$CDM, even though their luminosity distances are formulated differently, it turns out that the ensuing distance measures derived from these are quite similar all the way out to $z \sim 6$. As such, SLSNe Ic that make the cut for one model and not the other are the exception rather than the rule. In other words, the luminosity-limit applied to the sample examined here does not bias either model very much. But this may not be true in general, and an alternative method of selection is highly desirable.

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REFERENCES

Akaïke, H. 1973, in 2nd Int. Symp. on Information Theory, Information Theory and an Extension of the Maximum Likelihood Principle, ed. B. N. Petrov, & F. Csáki (Budapest: Akadémiai Kiadó), 267
Baluy, C., Rabowitç, D., Hadjiyska, E., et al. 2013, PASP, 125, 685
Barbary, K., Dawson, K. S., Tokita, K., et al. 2009, ApJ, 690, 1358
Berger, E., Chornock, R., Lunnan, R., et al. 2012, ApJL, 755, L29
Bhansali, R. J., & Downham, D. Y. 1977, Biometrika, 64, 547
Canio, Z., & Jakobsson, P. 2014, arXiv:1409.3570
Cavanaugh, J. E. 1999, Statist. Probab. Lett., 42, 333
Cavanaugh, J. E. 2004, Aust. N. Z. J. Stat., 46, 257
Chomiuk, L., Chornock, R., Soderberg, A. M., et al. 2011, ApJ, 743, 114
Cooke, J., Sullivan, M., Gal-Yam, A., et al. 2012, Natur, 491, 228
Drake, A. J., Djorgovski, S. G., Mahabal, A. A., et al. 2009, ApJ, 696, 870
Gal-Yam, A. 2012, Sci, 337, 927
Hamuy, M., Phillips, M. M., Suntzeff, N. B., et al. 1996, AJ, 112, 2438
Hinshaw, G., Larson, D., Komatsu, E., et al. 2013, ApJS, 208, 19
Hjorth, J., Sollerman, J., Møller, P., et al. 2003, Natur, 423, 847
Howell, D. A., Kasen, D., Lidman, C., et al. 2013, ApJS, 79, 98
Inserra, C., Smartt, S. J., Jerkstrand, A., et al. 2013, ApJ, 770, 128
Inserra, C., & Smartt, S. J. 2014, ApJ, 796, 87
Kaiser, N., Burgett, W., Chambers, K., et al. 2010, Proc. SPIE, 7733, 1277330E
Leloudas, G., Chatzopoulos, E., Dilday, B., et al. 2012, A&A, 541, A129
Lien, A., & Fields, B. D. 2009, JCAP, 1, 047
Lunnan, R., Chornock, R., Berger, E., et al. 2013, ApJL, 771, 97
McCrum, M., Smartt, S. J., Rest, A., et al. 2014, MNRAS, 448, 1206
Melia, F. 2007, MNRAS, 382, 1917
Melia, F. 2013a, A&A, 553, A76
Melia, F. 2013b, ApJ, 764, 72
Melia, F. 2014, JCAP, 1, 27
Melia, F. 2015, MNRAS, 446, 1191
Melia, F., & Abdelqader, M. 2009, IUMPD, 18, 1889
Melia, F., & Shevchuk, A. S. H. 2012, MNRAS, 419, 2579
Melia, F., & Maier, R. S. 2013, MNRAS, 432, 2669
Nicholl, M., Smartt, S. J., Jerkstrand, A., et al. 2013, Natur, 502, 346
Nicholl, M., Smartt, S. J., Jerkstrand, A., et al. 2013, MNRAS, 444, 2096
Pastorello, A., Smartt, S. J., Botticella, M. T., et al. 2010, ApJL, 724, L16
Perlmutter, S., Aldering, G., della Valle, M., et al. 1998, Natur, 391, 51
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Perlmutter, S., Gabr, S., Goldhaber, G., et al. 1997, ApJ, 483, 565
Phillips, M. M. 1993, ApJ, 413, L105
Pskovskii, I. P. 1977, SVA, 21, 675
Quimby, R. 2006, CBET, 644, 1
Quimby, R. M., Aldering, G., Wheeler, J. C., et al. 2007, ApJL, 668, L99
Quimby, R. M., Castro, F., Gerardy, C. L., et al. 2005, BAAS, 37, #171.02
Quimby, R. M., Kulkarni, S. R., Kasliwal, M. M., et al. 2011, Natur, 474, 487
Quimby, R. M., Werner, M. C., Oguri, M., et al. 2013, ApJL, 768, L20
Rau, A., Kulkarni, S. R., Law, N. M., et al. 2009, PASP, 121, 1334
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Rust, B. W. 1974, BAAS, 6, 309
Samushia, L., & Ratra, B. 2009, ApJ, 703, 1904
Schmidt, B. P., Suntzeff, N. B., Phillips, M. M., et al. 1998, ApJ, 507, 46
Schwarz, G. 1978, AnSta, 6, 461
Shao, L., Dai, Z.-G., Fan, Y.-Z., et al. 2011, ApJ, 738, 19
Smith, N., Li, W., Foley, R. J., et al. 2007, ApJ, 666, 1116

Stanek, K. Z., Matheson, T., Garnavich, P. M., et al. 2003, ApJL, 591, L17
Suzuki, N., Rubin, D., Lidman, C., et al. 2012, ApJ, 746, 85
Tan, M. Y. J., & Biswas, R. 2012, MNRAS, 419, 3292
Tanaka, M., Moriya, T. J., Yoshida, N., & Nomoto, K. 2012, MNRAS, 422, 2675
Takeuchi, T. T. 2000, Ap&SS, 271, 213
Wei, J.-J., Wu, X.-F., & Melia, F. 2013, ApJ, 772, 43
Wei, J.-J., Wu, X.-F., Melia, F., & Maier, R. S. 2015, AJ, 149, 102