Initial State Radiation in Majorana Dark Matter Annihilations

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Abstract

The cross section for a Majorana Dark Matter particle annihilating into light fermions is helicity suppressed. We show that, if the Dark Matter is the neutral Majorana component of a multiplet which is charged under the electroweak interactions of the Standard Model, the emission of gauge bosons from the initial state lifts the suppression and allows an $s$-wave annihilation. The resulting energy spectra of stable Standard Model particles are importantly affected. This has an impact on indirect searches for Dark Matter.
1 Introduction and setup

The fluxes of stable Standard Model particles that originate from the annihilation (or decay) of Dark Matter (DM) in the galactic halo are the primary observable for DM indirect searches. The radiation of ElectroWeak (EW) gauge bosons from the final state of the annihilation process turns out to have a great influence on the energy spectra of stable particles and hence on the predictions for fluxes to be measured at Earth [1, 2] (see also Refs. [3, 4] for related analyses). In particular, there are three situations where the effect of including the EW corrections is especially important:

1. when the low-energy regions of the spectra, which are largely populated by the decay products of the emitted gauge bosons, are the ones contributing the most to the observed fluxes of stable particles;

2. when some particle species are absent if EW corrections are not included, e.g. antiprotons from W/Z decays in an otherwise purely leptonic channel;

3. when the $2 \rightarrow 3$ annihilation cross section, with soft gauge boson emission, is comparable or even dominant with respect to the $2 \rightarrow 2$ cross section.

A possible realization of the latter condition has been studied in Ref. [2], where we considered the DM as a gauge-singlet Majorana particle $\chi$ of mass $M_\chi$ annihilating into light fermions $f$ of mass $m_f \ll M_\chi$; it is well known that in this case the $\chi\chi \rightarrow ff$ cross section is suppressed. In fact, one can perform the usual expansion of the cross section

$$v\sigma = a + bv^2 + O(v^4),$$  \hspace{1cm} (1.1)

where $v \sim 10^{-3}$ is the relative velocity (in units of $c$) of the DM particles in our galaxy today. The first term, which corresponds to the annihilation of particles in a state with $L = 0$ orbital momentum ($s$-wave), is constrained by helicity arguments to be proportional to $(m_f/M_\chi)^2$, and hence very small for light final state fermions. The second term, which corresponds to the annihilation in the $L = 1$ state ($p$-wave), suffers from the $v^2$ suppression. For a DM particle singlet under the SM gauge group, the radiation of EW gauge bosons from the final state and from the internal propagator of the annihilation process eludes the suppressions and opens up a potentially sizeable $s$-wave contribution to the cross section (see Ref. [5] for the case of photon radiation and Ref. [6] for gluon radiation).

In this paper we point out that there is another situation realizing the condition 3 above, where we expect therefore the EW corrections to have a great impact. Having in mind the Weakly Interacting Massive Particle candidates for DM, it is natural not to restrict oneself to $\chi$ being a gauge singlet, and to consider the possibility that the DM is part of a multiplet charged under the EW interactions. In this case, the DM annihilates predominantly in $s$-wave into $W^+W^-$, if kinematically allowed. However, now even the initial state of the channel $\chi\chi \rightarrow ff$ can radiate a gauge boson; we shall show that this process also lifts the helicity suppression and contributes to the $s$-wave cross section, becoming competitive with the di-boson channel.
For definiteness, we assume that the DM particle is the electrically-neutral Majorana component of a $SU(2)_L$ triplet $\chi^a$, with hypercharge $Y = 0$ (a wino-like particle). The coupling with the $Z$-boson is absent and this DM candidate is compatible with the direct detection limits. We neglect the mass splitting of the components of the multiplet, which is generated by loop effects [7, 10] and tends to make the charged components slightly heavier than the neutral one. The size of this splitting is typically of the order of 100 MeV for a TeV-scale DM mass.

In order to catch the relevance of initial state emission in a model-independent way, we work in an effective field theory setup and we restrict to consider the interactions of the triplet $\chi^a$ with the SM left-handed doublet $L = (f_1, f_2)^T$; the most general dimension-6 operators are

$$L_{\text{eff}} = \frac{C_D}{\Lambda^2} \delta_{ab} (\bar{L} \gamma_\mu P_L L) \left( \bar{\chi}^a \gamma^\mu \gamma_5 \chi^b \right) + i \frac{C_{\text{ND}}}{\Lambda^2} \epsilon_{abc} (\bar{L} \gamma_\mu P_L \sigma^c L) \left( \bar{\chi}^a \gamma^\mu \chi^b \right),$$

(1.2)

where $C_{D,\text{ND}}$ are real coefficients for diagonal and non-diagonal interactions in isospin space, $P_{R,L} = \frac{1}{2} (1 \pm \gamma_5)$ and $\sigma^i$ are the Pauli matrices. The assumed Majorana nature of the DM forbids some operators that would lead to an $s$-wave two-body annihilation. The initial state radiation lifts the helicity suppression already at the level of dimension-6 operators, unlike what happens for the final state radiation, where higher-dimensional operators are needed (see Ref. [2] for more details). A more general effective field theory analysis will be presented in Ref. [11]. The effective operators in Eq. (1.2) can be generated for instance as the low-energy limit of a simple toy model [2,12] where the DM interacts with the SM left-handed fermions through the exchange of a heavy scalar doublet $\Phi$

$$L_{\text{int}} = -y_L \bar{L} \sigma_a \chi^a \Phi + \text{h.c.};$$

(1.3)

integrating out the scalar sector $M_\Phi \gg M_\chi, m_W$, one obtains the operators in Eq. (1.2) with $C_{\text{ND}}/\Lambda^2 = -C_D/\Lambda^2 = |y_L|^2/(4M_\Phi^2)$.

In the next section we shall discuss the velocity dependence of the amplitudes describing the DM annihilation into light fermions, considering both the Final State Radiation (FSR) and the Initial State Radiation (ISR) contributions. Subsequently, in section 3 we shall present the results for the cross sections and the energy spectra of final particles. Our main results are summarized in Section 4, together with prospects for further research.

2 Velocity dependence of the amplitude

2.1 Two-body annihilation

When the DM is part of a multiplet charged under the EW gauge group, and $M_\chi > m_W$, the most important two-body annihilation channel is $\chi^0 \chi^0 \rightarrow W^+ W^-$, which proceeds through

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1 Other representations of the EW gauge group can be considered, see e.g. Ref. [7]. For instance, a simple possibility consists of two non-degenerate $SU(2)_L$ doublets with opposite hypercharge (higgsino-like particles). Direct detection constraints are avoided [7,8] and also an interesting LHC phenomenology can arise in the quasi-degenerate limit [9].
s-wave (see Fig. 1). We shall discuss the importance of this contribution with respect to the three-body ISR channel in Section 3 and more thoroughly in Ref. [11].

Let us consider instead the annihilation of the DM Majorana fermion into a pair of left-handed massless fermions:

$$\chi^0(k_1)\chi^0(k_2) \rightarrow f_i(p_1) \bar{f}_i(p_2). \quad (2.1)$$

Only the first term of the effective operators in Eq. (1.2) contributes to this process and the annihilation proceeds through the $p$-wave. The velocity dependence is manifest at the amplitude level. In fact, the matrix element for the two-body process is

$$M_{f\bar{f}} \sim \frac{1}{\Lambda^2} \bar{u}_f(p_1) \gamma_\alpha P_L v_f(p_2) \left[ \bar{v}_\chi(k_2) \gamma^\alpha \gamma_5 u_\chi(k_1) \right], \quad (2.2)$$

where $k_{1,2}^\alpha = (M_\chi/\sqrt{1-(v/2)^2}, 0, 0, \pm M_\chi(v/2)/\sqrt{1-(v/2)^2})$. The Majorana axial current in Eq. (2.2) can be manipulated using the Gordon identities into

$$\bar{v}_\chi(k_2) \gamma^\alpha \gamma_5 u_\chi(k_1) = -\frac{k_1^\alpha + k_2^\alpha}{2M_\chi} \bar{v}_\chi(k_2) \gamma_5 u_\chi(k_1) \gamma^\alpha - \frac{i}{2M_\chi} \bar{v}_\chi(k_2) \sigma^{\alpha\beta}(k_1 - k_2)_\beta \gamma_5 u_\chi(k_1). \quad (2.3)$$

For small $v$, the second term is proportional to $v$ because $(k_1 - k_2)_\beta \sim (0, 0, 0, v M_\chi)$; on the other hand, in the first term the vector $(k_1 + k_2)^\alpha = (p_1 + p_2)^\alpha$ saturates the final-state current in Eq. (2.2) and gives rise to terms proportional to the fermion mass, which are zero in our computation. We thus recovered the well-known fact that for Majorana spinors the scattering amplitude into massless fermions is proportional to the first power of the relative velocity of the incoming particles.

### 2.2 Gauge boson emission

The gauge interactions of the fermion triplet are described by the vector operator $\epsilon_{abc} \bar{\chi}^a W^b \chi^c$, from which it is evident that the neutral component of $\chi$ does not interact with $W^3$. Thus,
$W^3$ is emitted only from the final states, while $W^\pm$ can be emitted from either initial or final states. Let us study a $W$-bremsstrahlung process from either initial or final states, see diagrams in Fig. 1. We consider for definiteness the three-body annihilation

$$\chi^0(k_1)\chi^0(k_2) \rightarrow f_1(p_1) f_2(p_2) W^-(k).$$

(2.4)

For the process with FSR, the interaction vertex of two $\chi^0$'s and two SM fermions is described by the isospin-diagonal operator in Eq. (1.2); instead, since ISR changes the isospin state of one of the initial legs, the interaction operator is the non-diagonal one.

Considering the radiation from the final state, the amplitude is the product of the Majorana axial-vector current $\bar{v}_\chi \gamma_\mu \gamma_5 u_\chi$ and the fermionic current containing the gauge boson emission

$$\mathcal{M}_{\text{FSR}} \sim \frac{g}{\Lambda^2} \left[ \bar{v}_\chi \gamma_\mu \gamma_5 u_\chi \right] \left[ \bar{u}_f \left( \frac{f^\ast (\not{k}_1 + \not{k}_2 + M_\chi) \gamma^\mu}{2p_1 \cdot (k + m_W^2)} - \frac{\gamma^\mu (\not{k}_2 - \not{k} + M_\chi) f^\ast}{2p_2 \cdot k + m_W^2} \right) P_L v_f \right],$$

(2.5)

where $g$ is the $SU(2)_L$ gauge coupling. Using the Gordon identities the Majorana current can be simplified as in Eq. (2.3), where the second term is proportional to $v$, while the first term now contains the 4-vector $(p_1 + p_2 + k)_\mu$; saturating with the fermionic current in (2.5) gives terms proportional to $m_f$, which are zero in our case. Thus, at the lowest level in the expansion in $M_\chi^2/\Lambda^2$, which amounts to restricting to dimension-6 operators, FSR is not able to remove the helicity suppression. However, at higher orders (e.g. $\mathcal{O}(M_\chi^4/\Lambda^4)$ in the amplitude) $v$-independent terms can arise, for which the inclusion of Virtual Internal Bremsstrahlung (VIB) diagrams [5] is crucial. This point is extensively discussed in Ref. [2].

On the other hand, the amplitude describing the radiation from the initial state consists of the product of the fermionic current $\bar{u}_f \gamma_\mu P_L v_f$ and the Majorana current with the gauge boson emission

$$\mathcal{M}_{\text{ISR}} \sim \frac{1}{\Lambda^2} \left[ \bar{u}_f \gamma_\mu P_L v_f \right] \left[ \bar{v}_\chi \frac{\{f^\ast \not{k}, \gamma^\mu\}}{m^2 - 2k^0 M_\chi} u_\chi \right].$$

(2.6)

The effect of ISR is to alter the axial-vector structure of the initial state current, thus preventing the amplitude from vanishing in the $v \rightarrow 0$ limit

$$\mathcal{M}_{\text{ISR}}^{(v=0)} \sim \frac{g}{\Lambda^2} \left[ \bar{u}_f \gamma_\mu P_L v_f \right] \left[ \bar{v}_\chi \frac{\{f^\ast \not{k}, \gamma^\mu\}}{m^2 - 2k^0 M_\chi} u_\chi \right].$$

(2.7)

From the amplitudes we studied in this section one can deduce the behaviours of the cross sections for the two-body $f \bar{f}$ channel and the three-body channels with ISR, FSR and their interference; they can be schematically summarized as

$$v \sigma_{ff}(\chi \chi \rightarrow f \bar{f} \bar{f}) \sim \frac{1}{M_\chi^2} \mathcal{O} \left( \frac{M_\chi^4}{\Lambda^4} \right) \mathcal{O}(v^2),$$

(2.8)

$$v \sigma_{\text{FSR, ISR}/\text{FSR}}(\chi^0 \chi^0 \rightarrow f_1 \bar{f}_2 W^-) \sim \frac{g^2}{M_\chi^2} \mathcal{O} \left( \frac{M_\chi^4}{\Lambda^4} \right) \mathcal{O}(v^2),$$

(2.9)

$$v \sigma_{\text{ISR}}(\chi^0 \chi^0 \rightarrow f_1 \bar{f}_2 W^-) \sim \frac{g^2}{M_\chi^2} \mathcal{O} \left( \frac{M_\chi^4}{\Lambda^4} \right) \mathcal{O}(v^0),$$

(2.10)
and the precise expressions will be discussed in the next section. The importance of ISR is then clear: at the level of dimension-6 operators ISR already opens the s-wave annihilation while FSR is still in p-wave.

3 Results

3.1 Cross sections

The cross sections for the various processes are computed from the amplitudes in Eqs. (2.2), (2.5), (2.7), with the appropriate coefficients as dictated by the interaction lagrangian (1.2). For the two-body annihilation into massless fermions, the cross section reads

\[ v\sigma_{ff} = \frac{g^2 C_D^2 M_X^2}{12\pi \Lambda^4} v^2. \]  

(3.1)

For the three-body processes, it is convenient to define \( s \equiv (k_1 + k_2)^2 \) and \( z \equiv m_W/\sqrt{s} \). We are working in a situation where the DM mass is larger than the EW scale; so we report results as an expansion for small \( z \). Then, the cross sections for the processes with FSR and with ISR/FSR interference are

\[ v\sigma_{FSR} = \frac{g^2 C_D^2 M_X^2}{144\pi^3 \Lambda^4} \left[ 15 - \pi^2 + 6 \ln z (2 \ln z + 3) + \mathcal{O}(z^2) \right] v^2. \]  

(3.2)

\[ v\sigma_{ISR/FSR} = -\frac{g^2 C_D C_{ND} M_X^2}{16\pi^3 \Lambda^4} \left[ 9 + 4 \ln z + \mathcal{O}(z) \right] v^2, \]  

(3.3)

which are both in p-wave. As recalled already in the previous section, an s-wave term in the cross section also originates from FSR (together with VIB) but only at a higher order, \( \mathcal{O}(M_X^8/\Lambda^8) \) in the cross section. Nonetheless, this can have a great impact on the energy spectra of final particles [2], but we shall not consider it here. Instead the ISR opens up a large s-wave contribution to the total cross section

\[ v\sigma_{ISR} = \frac{g^2 C_{SD}^2 M_X^2}{9\pi^3 \Lambda^4}, \]  

(3.4)

which is mediated by the non-diagonal operator in Eq. (1.2) \(^2\).

As already anticipated, there is also an important two-body channel \( \chi^0\chi^0 \rightarrow W^+W^- \), whose cross section is not \( v \)-suppressed

\[ v\sigma_{WW} = \frac{g^4}{8\pi M_X^2} + \mathcal{O}(z^4). \]  

(3.5)

and can be comparable in size to \( \sigma_{ISR} \). The annihilation channel with ISR can even become the dominant one if for instance the \( W^+W^- \) final state is kinematically forbidden, or if the

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\(^2\) The isospin-diagonal operator leads to a spin-dependent elastic DM-nucleon cross section proportional to \( C_D^2 \). Experimental bounds from direct detection would therefore constrain \( C_D \), but would not preclude \textit{a priori} the possibility to have a large \( \sigma_{ISR} \).
coefficient $C_{\text{ND}}$ is large. Notice also that the additional $g^2$ factor in $\sigma_{\text{WW}}$ tends to reduce it with respect to $\sigma_{\text{ISR}}$, if $C_{\text{ND}} \sim 1$; furthermore, the number of colours or families of the final state fermions, which we do not consider here, also enhances $\sigma_{\text{ISR}}$ with respect to $\sigma_{\text{WW}}$.

### 3.2 Energy spectra

Let us now turn to the phenomenological implications of ISR, namely the effect of the $W$-bremsstrahlung on the energy spectra of stable particles resulting from the hadronization and decay of the annihilation products.

In this paper we are interested in extracting the features distinguishing ISR from other kind of processes, like the two-body $W^+W^-$ channel and the three-body FSR, although their cross section can be very different. To make the comparisons more immediate, we define the energy spectrum of the channel $i$ as

$$\frac{dN_i}{dE} \equiv \frac{1}{\sigma_i} \frac{d\sigma_i}{dE},$$

for $i = \text{FSR}, \text{ISR}, \text{ISR/FSR}, \text{WW}$, so that each contribution is normalized to 1. A more detailed analysis will be presented in Ref. [11], where we shall work out an explicit model and weigh the various channels appropriately.

Because the energy of the emitted $W$ boson is entirely distributed among the final particles, it is instructive to analyse its energy spectrum separating the contributions from ISR, FSR and their interference, see Fig. 2. Notice that the $W$ emission from an ultra-relativistic final state particle (FSR) has a characteristic soft/collinear behaviour $dN/dy \sim 1/y$, where $y = E_W/M_\chi$ (see Ref. [1] for further details); on the other hand, the $W$ emission from a non-relativistic initial state particle (ISR) shows a somehow peculiar energy spectrum, which turns out to be well approximated by the symmetric distribution $dN/dy \sim y(1-y)$. Due to the non-relativistic nature of the emitting DM particle and the failure of the factorization

![Figure 2: Energy spectrum of the emitted $W$ for the three contributions FSR, ISR, ISR/FSR. The parameters settings are $M_\chi = 1$ TeV, $v = 10^{-3}$. Each distribution is normalized to 1 according to Eq. (3.6).]
Figure 3: Energy spectra of final positrons (left panel) and antiprotons (right panel), for different annihilation channels defined in the text: $W^+W^-$ (dashed lines), $e\nu W$ (thick solid lines) and $udW$ (thin solid lines). The parameters are set as $M_\chi = 1$ TeV and $v = 10^{-3}$, and the normalization is chosen according to Eq. (3.6).

property for the three-body cross section, this contribution cannot be caught by the usual soft/collinear approximation technique [1].

The evolution of the particle species from the primary annihilation products to the final stable particles of the SM needs numerical tools. We have carried out this work using our own Monte Carlo code, for generating $2\to2$ and $2\to3$ annihilation events, then interfaced to PYTHIA 8.145 [13] for simulating the subsequent showering, hadronization and decay (see Ref. [2] for more details).

In Fig. 3, we compare the energy spectra of final positrons and antiprotons originating from ISR and from WW. We consider two different channels for the ISR case: the lepton channel “$e\nu W$”: $\chi^0\chi^0 \to e^+_L\nu e_LW^-$, $e^-_L\bar{\nu}e_LW^+$; and the quark channel “$udW$”: $\chi^0\chi^0 \to u_L\bar{d}_LW^-$, $\bar{u}_Ld_LW^+$. Interesting features can be extracted from this comparison. For the $e\nu W$ channel the very hard positrons can be much more abundant for ISR than for WW, due the contribution of the primary positrons. For the $udW$ channel, the antiprotons are copiously generated by the $W$ emitted in ISR, especially at low energies because the gauge boson is soft, and by the hadronization of the primary quarks, and they can easily overcome the antiprotons produced in the WW channel.

The fluxes of positrons and antiprotons received at Earth can be computed by integrating the energy spectra at the interaction point depicted in Fig. 3 over the diffused source constituted by the DM distribution in the galactic halo, and then propagating them through the halo itself. We do not enter here into any of the details of this process (see e.g. Ref. [14] and references therein) but we limit ourselves to a few qualitative considerations. There are irreducible uncertainties of astrophysical nature that originate from: i) the unknown distribution of DM in the halo; ii) the unknown values of the propagation parameters. For the positron fluxes at high energy, however, the propagation does not sensibly modify the shape and the normalization of the fluxes, for any reasonable choice of the uncertain variables. This is just because high energy positrons have anyway originated very close to the location of the Earth
and therefore have not been affected much during their travel. In turn, this implies that the spectral features at high energy introduced by ISR, apparent in Fig. 3, are preserved in the final positron fluxes, at least in the case of large DM mass. On the other hand, the fluxes of antiprotons are affected by large astrophysical uncertainties both in normalization and in shape all across the energy range, making it more difficult to disentangle the different spectral shapes.

4 Summary and outlook

The inclusion of EW corrections is an essential ingredient to be taken into account for indirect DM searches. In this paper we have assumed that the DM is the neutral Majorana component of a multiplet charged under the EW interactions and considered the effect of gauge boson radiation from the initial state of the DM annihilation process. We have restricted ourselves to the case where the multiplet containing the DM particle is a $SU(2)_L$ triplet, but it is straightforward to work out the cases of multiplets transforming under different representations of the EW gauge group.

The natural annihilation channel for such a candidate is of course through $s$-wave into $W^+W^-$, if kinematically allowed, while the annihilation into light SM fermions is helicity suppressed and proceeds through $p$-wave. However, we found that the $W$-bremsstrahlung from the initial state removes the suppression and adds a potentially sizeable contribution to the $s$-wave cross section. The gauge boson emission alters the energy spectra of final stable particles in a distinguishable way and cannot be ignored for reliable predictions to be used for indirect DM searches.

In a forthcoming paper [11] we shall expand the idea of this work in several directions: a complete effective field theory analysis, a full calculation in the context of an explicit model which will also allow to weigh precisely the different channels, a detailed computation of fluxes of stable particles with the inclusion of the propagation effects.

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