Provision of Controlled Motion Accuracy of Industrial Robots and Multiaxis Machines by the Method of Integrated Deviations Correction

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Abstract. There is a developed method of correction of the integrated motion deviations of industrial robots and multiaxis machines, which are caused by the primary geometrical deviations of their segments. This method can be used to develop a control system providing the motion correction for industrial robots and multiaxis machines.

1. Introduction
It is impossible to avoid deviations in the geometrical size and form, known as primary deviations, when producing and assembling parts and units of segments entering into the composition of such complex mechanical systems as industrial robots. These geometrical deviations result in a difference of the motion parameters of a real industrial robot from the motion parameters of its nominal model made in accordance with the design documents (3-D model). To improve the accuracy of controlled motions it is necessary to make corrections of the nominal model of an industrial robot taking into account measuring primary geometrical deviations of its segments [1].

2. A Mathematical Model of Primary Deviations
Primary deviations in the geometrical dimensions of manipulator system segments (Figure 1) can be taken into account by introduction of linear deviations on relevant coordinate axes $\delta_{x}^{(i)}, \delta_{y}^{(i)}, \delta_{z}^{(i)}$, and primary deviations in segment forms and joint cocking – by introduction of the angular deviations between relevant coordinate axes: $\alpha_{xx}^{(i)}, \alpha_{xy}^{(i)}, \alpha_{xz}^{(i)}, \theta_{xyz}^{(i)}, \alpha_{yy}^{(i)}, \alpha_{yz}^{(i)}, \alpha_{zx}^{(i)}, \alpha_{zy}^{(i)}, \alpha_{zz}^{(i)}$ (Figure 2).

Figure 1. Primary linear deviations.
Matrix $A_{i,i^*}$ that shows a transformation of homogeneous coordinates from coordinate system $S_{i^*}$ into coordinate system $S_i$, considering primary linear and angular deviations of the segment, will represent a mathematical model of primary deviations.

$$
\tilde{A}_{i,i'} = \begin{bmatrix}
\cos(\alpha_{ix}^{(i)}) & \cos(\alpha_{iy}^{(i)}) & \cos(\alpha_{iz}^{(i)}) & I_x^{(i)} + \delta_x^{(i)} \\
\cos(\alpha_{yx}^{(i)}) & \cos(\alpha_{yy}^{(i)}) & \cos(\alpha_{yz}^{(i)}) & I_y^{(i)} + \delta_y^{(i)} \\
\cos(\alpha_{zx}^{(i)}) & \cos(\alpha_{zy}^{(i)}) & \cos(\alpha_{zz}^{(i)}) & I_z^{(i)} + \delta_z^{(i)} \\
0 & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
\frac{\frac{i}{x}}{x} & \frac{\frac{i}{y}}{y} \\
\frac{\frac{x}{y}}{y} & \frac{\frac{x}{y}}{y} \\
\frac{\frac{z}{y}}{y} & \frac{\frac{z}{y}}{y} \\
0 & 1
\end{bmatrix}.
$$

(1)

Character $\sim$ in expression (1) denotes that the matrix contains primary geometrical deviations.

When drawing up the matrix it is unnecessarily to measure all nine angular deviations. It suffices to measure only three of them. The remaining six angular deviations can be calculated. For this it is necessary to solve six nonlinear equations. These equations represent orthonormal vectors constituting rows and columns of the matrix.

Besides, positioning deviations influence the accuracy of multilink mechanisms. Positioning deviations are associated with deviations of generalized coordinates (Figure 3).

![Figure 2. Primary angular deviations.](image)

The matrix taking into account the positioning deviations has the following view:
where \( q_i \) – generalized coordinate, \( \Delta q_i \) – positioning deviation for the \( i \)-th generalized coordinate, \( \beta_i \) – coefficient (\( \beta_i = 1 \) if \( q_i \)– angular; \( \beta_i = 0 \) if \( q_i \)– linear).

A mathematical model represents a matrix multiplication:

\[
A_{(i-1)\ldots j} = A_{(i-1)\ldots (i-1)} A_{(i-1)\ldots j},
\]

\[
A_{0\ldots k} = A_{0\ldots 1} A_{1\ldots 2} \ldots A_{(i-1)\ldots i} A_{i\ldots (i+1)} \ldots A_{k-1\ldots k} = \prod_{i=1}^{k} A_{(i-1)\ldots i}.
\]

3. An Alternative Model

Instead of the mathematical model described by matrix (1), an alternative model can be offered. Let us consider the model in Figure 4.

![An alternative model](image)

\[
\tilde{A}_{(i)} = A_{(i)} (\varphi, I) \tilde{A}_{(i)} (\gamma, \rho),
\]

\[
\tilde{A}_{(i)} = \begin{bmatrix}
\cos(\varphi_{xx}^{(i)}) & \cos(\varphi_{xy}^{(i)}) & \cos(\varphi_{xz}^{(i)}) & I_x^{(i)} \\
\cos(\varphi_{yx}^{(i)}) & \cos(\varphi_{yy}^{(i)}) & \cos(\varphi_{yz}^{(i)}) & I_y^{(i)} \\
\cos(\varphi_{zx}^{(i)}) & \cos(\varphi_{zy}^{(i)}) & \cos(\varphi_{zz}^{(i)}) & I_z^{(i)} \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos(\gamma_{xx}^{(i)}) & \cos(\gamma_{xy}^{(i)}) & \cos(\gamma_{xz}^{(i)}) & \rho_x^{(i)} \\
\cos(\gamma_{yx}^{(i)}) & \cos(\gamma_{yy}^{(i)}) & \cos(\gamma_{yz}^{(i)}) & \rho_y^{(i)} \\
\cos(\gamma_{zx}^{(i)}) & \cos(\gamma_{zy}^{(i)}) & \cos(\gamma_{zz}^{(i)}) & \rho_z^{(i)} \\
0 & 0 & 0 & 1
\end{bmatrix} =
\begin{bmatrix}
M_\varphi & L_x(I) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
M_\gamma & L_y(p) \\
0 & 1
\end{bmatrix} =
\begin{bmatrix}
\tilde{M}_\varphi & \tilde{L}_x \\
0 & 1
\end{bmatrix}. \tag{3}
\]

Angles \( \varphi(i) \) are set by the initial configuration of the multilink mechanism, and angles \( \gamma(i) \) reflect the primary deviation of the links form and distortions in the joints. Linear deviations \( \rho(i) \), used in model (3) are associated to previously entered linear deviations \( \delta(i) \) by relations:

\[
\rho_x^{(i)} \cos(\varphi_{xx}^{(i)}) + \rho_y^{(i)} \cos(\varphi_{xy}^{(i)}) + \rho_z^{(i)} \cos(\varphi_{xz}^{(i)}) = \delta_x^{(i)}.
\]
\[
\rho_x^{(i)} \cos(\phi_y^{(i)}) + \rho_y^{(i)} \cos(\phi_{yy}^{(i)}) + \rho_z^{(i)} \cos(\phi_{zy}^{(i)}) = \delta_y^{(i)},
\]

\[
\rho_x^{(i)} \cos(\phi_z^{(i)}) + \rho_y^{(i)} \cos(\phi_{yz}^{(i)}) + \rho_z^{(i)} \cos(\phi_{zy}^{(i)}) = \delta_z^{(i)}.
\]

As in case of (1) when drawing up matrix (3) is necessary to measure three angular deviations.

4. A Method of Correction of Integrated Deviations

Mathematical models considering primary deviations of segment geometrical parameters in complex mechanical systems with a daisy-chain structure can be used to determine the integrated deviations of segment positions arising because of the error accumulation in the positions of each previous segment of a kinematic chain. An integrated deviation of a complex mechanical system should be defined as the guidepath deviations of segment characteristic points and the segment orientation deviations from the specified orientation caused by primary geometrical deviations (Figure 5).

Figure 5. An illustration of integrated deviations

Taking into account primary geometrical deviations, guidepath integrated deviation \(\Delta r(q)\) can be defined from the expression which represents a difference between the actual position defined by radius vector \(\vec{r}\) and the program position defined by radius vector \(r\).

\[
\Delta r = \vec{r} - r, \quad \vec{r} = \tilde{A}_{0,n} r_o,
\]

\[
\Delta r(q) = (\tilde{A}_{0,n} - A_{0,n}) r_o = \begin{bmatrix}
    j_0^T (
    \tilde{A}_{0,n} - A_{0,n}
    ) r_o
    
    j_0^T (\tilde{A}_{0,n} - A_{0,n}) r_o
    
    k_0^T (\tilde{A}_{0,n} - A_{0,n}) r_o
\end{bmatrix}.
\]

\[
i_o^T = [1 \ 0 \ 0 \ 0], \quad j_o^T = [0 \ 1 \ 0 \ 0], \quad k_o^T = [0 \ 0 \ 1 \ 0],
\]

where \(A_{0,n}\) and \(\tilde{A}_{0,n}\) – matrices of nominal and corrected mathematical models, \(q\) – generalized coordinates. An integrated deviation, connected with the position defined by vector \(\Delta e(q)\) can be determined in a similar way. Integrated position deviations can be determined by complex vector \([\Delta r, \Delta e]^T\). the use of the nominal \(A_{0,k}\) and corrected \(\tilde{A}_{0,k}\) mathematical models allows expressing conditions of the integrated deviation compensation as follows:

\[
\begin{bmatrix}
    \Delta r(q) \\
    \Delta e(q)
\end{bmatrix} = 0 \rightarrow \tilde{q}.
\]

The solution of this set of nonlinear equations corresponding to zero value of the complex vector of integrated deviations, provides necessary laws of motion \(\tilde{q}(t)\) of a particular actuator of industrial robots or multiaxis machines (Figure 6).
Applying the mathematical support to realize this method of correction in the automatic control systems of industrial robots or multiaxis machines allows improving an enduring accuracy.

Figure 6. Correction of integrated deviations.

5. Conclusion
Mathematical models (1), (2) and (3) may be used in the parametrization methods of geometric models of multilink mechanisms. Applying mathematical support to realize the suggested method of correction in the systems of automatic control of industrial robots and multiaxis machines allows one to improve enduring accuracy. A detailed realization of the method of integrated deviations correction is given in the papers [2-7].

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