An effect of the uniaxial strain on the temperature of Bose-Einstein condensation of the intersite bipolarons

B. Ya. Yavidov

Phase Transitions Physics Laboratory, Institute of Nuclear Physics, 100214 Ulughbek, Tashkent, Uzbekistan
Department of Polymer Physics, National University of Uzbekistan named after Mirzo Ulughbek, 100174 Tashkent, Uzbekistan

Abstract

We have studied an effect of uniaxial strain to the temperature of Bose-Einstein condensation of intersite bipolarons within the framework of Extended Holstein-Hubbard model. Uniaxial lattice strains are taken into an account by introducing a generalized density-displacement type force for electron-lattice interaction. Associating the superconducting critical temperature \( T_{\text{c}} \) with the temperature of Bose-Einstein condensation \( T_{\text{BEC}} \) of intersite bipolarons we have calculated strain derivatives of \( T_{\text{BEC}} \) and satisfactorily explained the results of the experiments on La-based high-\( T_{\text{c}} \) films.

Key words: intersite bipolaron, uniaxial strain, temperature of Bose-Einstein Condensation, La-based High-\( T_{\text{c}} \) films

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1. Introduction

The first observation of high-\( T_{\text{c}} \) superconductivity in cuprates (copper oxide La-Ba-Cu-O compound) was reported almost quarter century ago [1]. Since then an intensive both theoretical and experimental research studies have been carried out on revealing the microscopic origin of the high-\( T_{\text{c}} \) superconductivity. In despite of decades of investigations at present there is no consent about the microscopic mechanism that bounds electrons (holes) into pairs, and consequently leads to high-\( T_{\text{c}} \) superconductivity. Normal state of cuprates also exhibit a variety of phase states. The early and subsequent studies show that a number of physical parameters of the cuprates as well as the critical temperature (and role of the electron-lattice interactions are significant [2–4]. It has been understood that a number of physical parameters of the cuprates as well as the critical temperature (\( T_{\text{c}} \)) strongly depend on a variety of factors as oxygen content, doping level, applied pressure or strain for films, and etc. Similar to some low-\( T_{\text{c}} \) superconductors an applied external pressure increases the value of \( T_{\text{c}} \) of copper based oxides. The highest critical temperature \( T_{\text{c}} \approx 164 \) K was obtained for HgBa\(_2\)Ca\(_2\)Cu\(_3\)O\(_{8+\delta}\) under hydrostatic pressure [5] (\( T_{\text{c}} \approx 133 \) K at ambient pressure [6]). So far an external pressure remains as powerful tool that affects all physical parameters of a sample simultaneously and influences the value of critical temperature. However, the influence of an external pressure to the cuprates is not a straightforward. In particular, it is the case when considering the dependence of a critical temperature on applied external pressure. The same is true for the films grown on various substrates. In the latter case pressure originates from lattice mismatch between substrate and film. The lattice mismatch causes a microscopic strain which in its turn, on the analogy of pressure, vary the critical temperature. Due to quasi two dimensional (2D) lattice structure and strong anisotropy of the properties of cuprates the dependence of the critical temperature on uniaxial strain (pressure) is not simple. For many cuprate compounds compressive uniaxial strain (pressure) in the CuO\(_2\) plane (along \( a \)- or \( b \)- axes) enhances \( T_{\text{c}} \), while compressive uniaxial strain (pressure) along \( c \)-axis reduces \( T_{\text{c}} \) [7]. There are contradictorily reports in the cases of YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) and GdBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) where compressive strain (pressure) in the CuO\(_2\) plane gives rise to increase [14, 15] or decrease [16, 17] of the critical temperature. Moreover, the sign of the pressure derivatives of \( T_{\text{c}} \) along \( a \) and \( b \)- axes are different for YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) [18]. Uniaxial pressure study of electron-doped cuprate Nd\(_{1.84}\)Ce\(_{0.16}\)CuO\(_4\) has shown that dependence of the critical temperature of the electron doped compound are the same with those of hole doped counterparts [19].

There are a few theoretical studies of the effect of an uniaxial strain (pressure) on the critical temperature [20–24]. Goddard estimated the effect of the pressure on the critical temperature of the optimally doped La\(_2\)CuO\(_4\) within the magnon pairing model [20]. The analytical formula for \( T_{\text{c}} \) was obtained in the weak coupling BCS approach. The coupling parameters of the model enter to the formula of \( T_{\text{c}} \) and depend on the orbital overlaps. An applied pressure decreases the distance between the centers of orbits, and this in its turn leads to the exponential increase of coupling parameters. In this way \( T_{\text{c}} \) is affected by the pressure (strain). The uniaxial pressure results of Goddard along \( a \)- and \( b \)- axes are in good agreement with the results of Ref. [8]. Meanwhile, the prediction of the model with regard to the effect of the pressure along \( c \)-axis is smaller than those results of Ref. [8] by order. The effect of the uniaxial stress to the critical temperature \( T_{\text{c}} \) of YBa\(_2\)Cu\(_3\)O\(_7\) cuprate was studied by Li in Ref. [21] in terms of the anisotropy of the density of states at the Fermi energy. Using a simple picture of van Hove singularity
The experimental data obtained so far and tested models for different cuprate compounds suggest that the phenomenon is complex and differ for various compounds. Nevertheless there are common features too. As a consequence at present one experiences a need of a model that able to explain all experimental data from universal points of view or to point out, at least, the most relevant contributions. From theoretical point of view the works [20–22, 24] study the effect of uniaxial strain on \( T_c \) totally ignoring quasi two dimensionality of the structure of cuprates. In addition, they ignore the most relevant electron-phonon interaction. The crucial role of electron-phonon interaction in cuprates comes from different experiments [3]. It is well established that electron-phonon interaction in cuprates is strong and (bi)polaron is formed in the cuprates [26, 27]. These issues are not discussed in the Refs. [20–24]. In present paper we will try to fill this blank. We study an effect of uniaxial strain to \( T_c \) within the framework of the extended Holstein-Hubbard model (EHHM). EHHM is believed appropriate model for study strongly correlated electron-phonon systems. An estimation of \( T_c \) within the model will be given for a chain model of cuprates at strong coupling limit and nonadiabatic regime, and discussion of the obtained results with regard to La-based high-\( T_c \) films will be done. Here we associate \( T_c \) with the Bose-Einstein condensation temperature of intersite bipolarons which form due to strong electron-phonon interaction.

2. Hamiltonian for a strained lattice

The systems that have strongly correlated electron and strongly coupled electron-phonon subsystems can be described by means of extended Holstein-Hubbard model which proved to be a more realistic model for cuprates. Hamiltonian of the

\[ H_{f,\text{str}} = \sum_{\langle n,m \rangle} J_f c_n^\dagger c_m \delta_{n,m} + \sum_{n} \varepsilon_f c_n^\dagger c_n + \sum_{\langle n,m \rangle} \delta_{n,m} c_n^\dagger c_m \]

where \( c_n^\dagger \) is the creation operator of \( f \)-electron in the \( n \)-th site, \( \varepsilon_f \) is the energy of \( f \)-electron, \( J_f \) is the hopping integral, and \( \delta_{n,m} \) is the Kronecker delta function.
system is given by [28]

\[ H = H_e + H_{ph} + H_V + H_{e-ph}, \]  

(1)

where

\[ H_e = \sum_{n \neq n'} T(n - n')c_n^\dagger c_{n'}. \]  

(2)

describes hopping of electrons between adjacent sites,

\[ H_{ph} = \sum_{q \sigma} \hbar \omega_{q \sigma} (d_{q \sigma}^\dagger d_{q \sigma} + 1/2), \]  

(3)

is Hamiltonian of phonon system,

\[ H_V = \sum_{n \neq n'} V_c(n - n')c_n^\dagger c_{n'}, \]  

(4)

is Hamiltonian of interacting particles on sites \( n \) and \( n' \) via Coulomb forces,

\[ H_{e-ph} = \sum_{nmr} f_{nm} \epsilon_{nm} \]  

(5)

is Hamiltonian of electron-phonon interaction. Here \( T(n - n') \) is transfer integral of electron form site \( n \) to site \( n' \), \( c_n^\dagger (c_n) \) is creation (annihilation) operator of electron on site \( n \), \( d_{q \sigma}^\dagger (d_{q \sigma}) \) is creation (annihilation) operator of phonon with \( \alpha (\alpha = x, y, z) \) polarization and wave vector \( \mathbf{q} \), \( \omega_{q \sigma} \) is the phonon’s frequency, \( V_c(n - n') \) is Coulomb potential energy of two electrons located at sites \( n \) and \( n' \), \( f_{nm} \) is “density-displacement” type coupling force of an electron on site \( n \) with the ions on site \( m \), \( \xi_{nm} \) is normal coordinate of ion vibrations on site \( m \) which expresses through phonon creation and destruction operators as

\[ \xi_{nm} = \sum_{q} \left( \frac{\hbar}{2NM\omega_{q \sigma}} e^{i\mathbf{q} \cdot \mathbf{r}_m} + h.c. \right). \]  

(6)

Here \( N \) is number of sites and \( M \) is ion mass.

EHHM enables one to take into account electron correlations and long range feature of electron-phonon interaction. Many physical properties of cuprates were satisfactorily explained within the EHHM (see for example [29]) as well as the possibility of formation of intersite bipolarons with \( s-, p- \) and \( d- \) type wave functions [50]. The possibility of formation of magnetic bipolaron with \( d- \) type wave function is also shown in Ref. [31] within the framework of \( t-J \) model.

Though model Hamiltonian Eq.(1) enables one to study polaron and bipolaron formation in a discrete lattice, here for the sake of simplicity estimation of polaron’s mass will be done without the term Eq.(4) (i.e. within extended Holstein model) and for the dispersionless phonons. In strong electron-phonon coupling limit and nonadiabatic regime use of the standard procedures such as Lang-Firsov transformation [32] eliminates electron-phonon interaction term (5). Subsequent perturbation expansion of the transformed Hamiltonian \( H_e \) with respect to parameter \( \lambda = E_p/2T(\alpha) \) (\( E_p \) is polaronic shift) and estimation of polaron’s renormalized mass for a lattice in Fig.1 yields \( m_p/m^* = \exp [\lambda^2/2] \) [33] (see also [14]), where

\[ \lambda^2 = \frac{1}{2M\hbar^2} \sum_{m} [f_{nm}^2(n) - f_{mn}(n)f_{nm}(n + a)]. \]  

(7)

and \( m^* = \hbar^2/2T(\alpha)a^2 \) is the bare band mass. Lattice of Figure 1 was introduced by Alexandrov and Kornilovitch in Ref. [33] in order to mimic an interaction of a hole on CuO₂ plane with the vibrations of apical ions in the cuprates. Convincing evidence for such coupling of in-plane holes with the \( c- \) axis polarized vibrations of apical oxygen vibrations from many experiments (see for example [55]). Therefore from here we will omit index \( \alpha \) and consider only that component of the electron-lattice force which represents an interaction of a hole on CuO₂ plane with the \( c- \) axis polarized apical oxygen vibrations. In addition, in order to consider stress of a lattice and its influence to the (bi)polaron mass, and consequently to the temperature of Bose-Einstein condensation of intersite bipolarons an analytical expression

\[ f_{m}(n) = \frac{\kappa c(1 - \epsilon_c)}{\sqrt{[(n - m)(1 - \epsilon_c)]^2 + (\kappa(1 - \epsilon_c))^2}} \]  

(8)

will be accepted for the density-displacement type force. Here \( \kappa \) is some coefficient, \( \epsilon_c \) and \( \epsilon_e \) are lattice strains along \( a \) and \( c \) axes, respectively, \( [n - m] \) is measured in units of lattice constant \( |a| \ = 1 \). The lattice strains defined as \( \epsilon_c = (\delta_{unstr} - \delta_{unstr})/\delta_{unstr} \) and \( \epsilon_e = (\delta_{unstr} - \delta_{unstr})/\delta_{unstr} \), where subscripts \( unstr \) and \( str \) stand for unstrained and strained, respectively. Eq.(7) is generalization of the force considered in Ref. [33] (see Eq.(9) there) and allows one to interrelate the temperature of Bose-Einstein condensation of the intersite bipolarons with the lattice strains through the mass of intersite bipolaron. Indeed, it has been shown that within the model Eq.(1) intersite bipolaron tunnel in the first order of polaron tunneling and its mass has the same order as polaron mass [28]. For the sake of simplicity we suppose that intersite bipolarons form an ideal gas of charged carriers and mass of bipolaron is \( m_{bp} = 2m_p \) (this point does not lead to loose of generality). Then the temperature of Bose-Einstein condensation of the intersite bipolarons defines as

\[ T_{BEC} = \frac{3.31\hbar^3 a^{2/3}}{2k_B m^*\Gamma e^{-\Gamma}}. \]  

(9)

Here \( k_B \) is Boltzmann constant and \( n \) is density of intersite bipolarons. The Eqs.(6), (7) and (8) are the main analytical results of the paper, according which discussion of experimental data for La-based high-\( T_c \) thin films will be done in the next section.

3. Results and discussion

The Eq.(8) expresses \( T_{BEC} \) through two basic parameters of a system: (i) the density of intersite bipolarons \( n \) and (ii) the exponent \( \Gamma \) of the polaron mass enhancement. Here we discuss the possibility of application of the Eq.(8) to thin films of La-based high-\( T_c \) cuprates grown on different substrates. This choice is quite simple and reasonable. In contrast with other cuprates La_{2-x}Sr_{x}CuO_{4} has somewhat a simplest crystal structure and serves as a test material for a variety of theoretical models. In addition, it is commonly believed that pressure induced charge transfer model (PICTM) [25] does not work in La_{2-x}Sr_{x}CuO_{4} compound due to lack of chain structures. In these circumstances Eq.(8) allows one to study the dependence
of $T_{\text{BEC}}$ on lattice strains $\varepsilon_a$ or $\varepsilon_c$ at constant $n$. This dependence is, of course, originated obviously from polaronic effects. We have calculated the values of $T_{\text{BEC}}$ as a function of the strains along $a$ axis $\epsilon_a$ and $c$ axis $\epsilon_c$ for the model lattice given in Fig. 1. Here we put $n = 1 \cdot 10^{21}$ sm$^{-3}$ and $k^2/(2\hbar^2 a^2) = 8.51$ in order to coincide $T_{\text{BEC}}$ at $\varepsilon_i = 0$ with the bulk value of $T_c = 25$ K of La$_{1.9}$Sr$_{0.1}$CuO$_4$ [13]. The strain derivatives of $T_{\text{BEC}}$ of the model lattice (Fig. 1) are: $\partial T_{\text{BEC}}/\partial \varepsilon_a \approx 112$ K and $\partial T_{\text{BEC}}/\partial \varepsilon_c \approx -450$ K.

The strain derivatives of $T_c$, $\partial T_c/\partial \varepsilon_i$ ($i = a, b, c$) were reported for a series of high-quality single crystals of La$_{2-x}$Sr$_x$CuO$_4$ from high-resolution dilatometry experiments in Ref. [8]. According to this report $\partial T_c/\partial \varepsilon_b$ depends on doping level and lies in range of $70 \div 470$ K, $280 \div 580$ K and $-2440 \div -1090$ K for $i = a, i = b$ and $i = c$, respectively. Comparison of these data show that the value of $\partial T_{\text{BEC}}/\partial \varepsilon_a \approx 112$ K of our model lattice lies in the range of $\partial T_c/\partial \varepsilon_a$. However, $\partial T_{\text{BEC}}/\partial \varepsilon_b$ lies well below of the range of $\partial T_c/\partial \varepsilon_b$ and absolute value $\partial T_{\text{BEC}}/\partial \varepsilon_c$ is approximately four times smaller than mean absolute value of $\partial T_c/\partial \varepsilon_c$. Another experiment reports about uniaxial strain derivatives of $T_c$ of La$_{1.91}$Sr$_{0.09}$CuO$_4$ [7] in which $\partial T_c/\partial \varepsilon_{ab} = 284$ K and $\partial T_c/\partial \varepsilon_c = -851$ K. One can easily extract the value of $\partial T_c/\partial \varepsilon_c$ if $\partial T_c/\partial \varepsilon_{ab}$ if one assumes that CuO$_2$ composed of isotropic square lattice, and the strains along $a$ and $b$ axes are approximately the same $\varepsilon_a \approx \varepsilon_b$ and $\varepsilon_{ab} = \varepsilon_a + \varepsilon_b$. Then one finds $\partial T_c/\partial \varepsilon_c = 142$ K ($i = a, b$) which is close to our $\partial T_{\text{BEC}}/\partial \varepsilon_c$. Meanwhile our $\partial T_{\text{BEC}}/\partial \varepsilon_b$ is still less than $\partial T_c/\partial \varepsilon_c$ of Ref. [7], now by two times. The reasons for a such discrepancies will be discussed below. Though strain derivatives of $T_{\text{BEC}}$ are differ from those of bulk samples we will see below that they can explain the strain effects to $T_c$ of La-based high-$T_c$ films.

Let us now turn our attention to the results of two particular experiments of Refs. [12, 13]. The experiments report about the growth of thin La-based high-$T_c$ films on various substrates and an influence of lattice mismatch between film and substrate to the $T_c$.

As reported in [12] Sato and Naito have grown of (001) oriented La$_{1.85}$Sr$_{0.15}$CuO$_4$ (LSCO) thin films on LaSrAlO$_4$ (LSAO) and SrTiO$_3$ (STO) substrates by reactive co-evaporation method. Unstrained in-plane lattice parameters of these LSCO, LSAO and STO are 3.777 Å, 3.756 Å and 3.905 Å, respectively. They were able to obtain LSCO film on LSAO substrate with $T_c$ as high as 44 K, which is approximately 7 K higher than that for single LSCO crystal under ambient pressure. For the films on STO substrates they obtained $T_c = 29$ K. X-ray diffraction measurements of LSCO thin films grown on LSAO (STO) substrate clearly showed that the lattice parameter of CuO$_2$- plane is compressed by 0.4% (expanded by 1.6%), while $c$-axis is expanded by 0.5% (compressed by 0.4%). As a matter of recorded results from structural analysis Sato and Naito concluded that increase (decrease) of $T_c$ is directly related to both the compression (tension) of CuO$_2$- plane by the strain generated by lattice mismatch and the expansion (shortening) of the $c$- axis due to the Poisson effect. The ratio of the strain along $c$- axis to the strain of CuO$_2$-plane lattice parameter were: $|\varepsilon_c|/|\varepsilon_a| = 1.25$ and $|\varepsilon_c|/|\varepsilon_a| = 0.25$ for the films grown on LSAO and STO substrates, respectively. We have performed calculation of $T_{\text{BEC}}$ according to the Eqs.(6)-(8) taking into account the above relations $|\varepsilon_i|/|\varepsilon_a|$. The results are presented graphically in Fig.3(a). For the compressive strain $\varepsilon_a = 0.4\%$ our model yields $T_{\text{BEC}} = 44.1$ K which is very close to the...
observed value of $T_c = 44$ K for LSCO film grown on LSAO substrate. In the meantime for the tensile strain $\varepsilon_a = -1.6\%$ our model yields $T_{BEC} = 28.9$ K which is also very close to the observed value of $T_c = 29$ K for LSCO film grown on STO substrate. One should notice that there is an excellent agreement between our results and the experimental results of Ref. [12].

Locquet et al reported growth of La$_{1.6}$Sr$_{0.4}$CuO$_4$ (LSCO) thin films on LaSrAlO$_3$ (LSAO) and SrTiO$_3$ (STO) substrates by the block-by-block molecular epitaxial method in Ref. [13]. The $\theta - 2\theta$ X-ray measurements revealed that the grown films strongly strained both in CuO$_2$-plane and out of the plane. The film grown on LSAO substrate was found strained with parameters $\varepsilon_{ab} = 0.63\%$ and $\varepsilon_c = -0.76\%$. The same parameters for the film grown on STO substrate were found as $\varepsilon_{ab} = -0.54\%$ and $\varepsilon_c = 0.35\%$. The ratio of the strains $|\varepsilon_a|/|\varepsilon_d|$ for the films grown on LSAO and STO substrates are 1.2 and 0.65, respectively. Taking into account that $\varepsilon_{ab} \approx 2\varepsilon_a$ one rewrites the same relations as $|\varepsilon_a|/|\varepsilon_d| = 2.4$ and $|\varepsilon_c|/|\varepsilon_d| = 1.3$ for the films grown on LSAO and STO substrates, respectively. The results of the calculation of $T_{BEC}$ for our lattice with the same parameters are given in Fig.3(b). For the compressive lattice strain $\varepsilon_a = 0.63\%$ and the relation $|\varepsilon_a|/|\varepsilon_d| = 2.4$ our model yields the value of $T_{BEC} = 46$ K, while for the tensile strain $\varepsilon_a = -0.54\%$ and the relation $|\varepsilon_a|/|\varepsilon_d| = 1.3$ one obtains $T_{BEC} \approx 18$ K. As one can see our result $T_{BEC} = 46$ K is in reasonable agreement with the value of $T_c = 49$ K for the film on LSAO substrate. However, there is some discrepancy of the value of $T_{BEC} \approx 18$ K from observed value of $T_c = 10$ K for the film on STO substrate.

Let’s now briefly discuss some aspects of our model and its results. As pointed out above our model takes into account the most relevant interaction in cuprates which is electron-phonon interaction. We were able satisfactorily explain the $T_c$ of La-based high-$T_c$ films within the framework of EHHM and in the assumption of formation of ideal Bose-gas of intersite bipolarons. Some discrepancies of our results from the experimental results might be as result of several factors: (i) the simplest of our model lattice under study. In reality one should consider more complex structures; (ii) the choice of the analytical formula for the density-displacement type electron-lattice force; (iii) an assumption that intersite bipolarons form an ideal Bose-gas. In reality due to other factors there are may be deviation from an ideality thus forming nonideal Bose-gas or Bose-liquid; (iv) superconductivity of the La-based films may be due to not the only electron-phonon interaction, but has contributions from other interactions as well. These factors suggest performance of more comprehensive research on the studied problem. Even though we have considered a very simple lattice model its common features are consistent qualitatively with many experimental observations. Indeed, in our model compressive strain in $ab$-plane of high-$T_c$ cuprates gives rise an increase of $T_c$, while that of along $c$-axis acts contrary. Such a type variations of $T_c$ with respect to the strains were observed in many La-based cuprates [36-42]. This feature is common both to films [36-39] and bulk samples [40-42]. Though our model allows one estimate $T_{BEC}$ at any $\varepsilon _i$ there are might be technological limits in obtaining of films with a large value of strains induced by lattice mismatch. It is also worthwhile to notice that our consideration are limited to the films of optimal thickness. Because of generality of our approach one might want to extend the model to other cuprates. In general, there is no restrictions for that. However, in each case specific features of a compound (film or bulk sample) must be taken into an account.

4. Conclusion

In this paper we have considered an effect of uniaxial strain (pressure or stress) to the superconducting critical temperature of cuprates. A brief survey of both experimental and theoretical works on the effect of the uniaxial pressure (strain or stress) to $T_c$ of cuprates (bulk and film samples) reveal that the phenomenon is complex and there are contradictory reports too. However, it is emphasized that there are common features relevant to all of them. In particular, in the experiments with La-based high-$T_c$ cuprates compressive pressure (strain) both along $a$ and $b$ axes gives rise to increase of $T_c$, while that along $c$ axis gives rise to decrease of $T_c$. On the other hand, there are convincing evidences of importance of electron correlations, electron-lattice interactions and bipolaron formation in the cuprates. Within the framework of extended Holstein-Hubbard model we introduced a generalized density-displacement type force for the electron-lattice interaction which takes into account lattice strain. We were able express bipolaron mass through the lattice strains. In the assumption that intersite bipolarons form an ideal Bose-gas, and associating the superconducting critical temperature $T_c$ with the temperature of Bose-Einstein condensation of intersite bipolarons we directly relate $T_{BEC}$ with lattice strain $\varepsilon_i$ ($i = a, b, c$). Operating on a simple model lattice introduced early in Ref. [33] we have calculated strain derivatives of $T_{BEC}$. Further we have tried to explain the effect of the uniaxial strain to the superconducting critical temperature of La-based high-$T_c$ films. Here we limit ourselves to two particular experiments of Refs. [12,13]. Our calculated results for $T_{BEC}$ are in close agreement with the results of experiments of Refs. [12,13] for $T_c$. A brief discussion is made of the reasons of small discrepancies of our results from experimental ones.

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References

[1] J.G. Bednorz, K.A. Müller, Z.Phys. B 64 (1986) 189.
[2] Y. Bar-Yam, T. Egami, J.M. de Leon, A.R. Bishop, Lattice Effects in High-$T_c$ Superconductors, World Scientific, Singapore, 1992.
[3] O. Gunnarsson and O. Rösch, J.Phys.: Condens. Matter 20 (2008) 043201.
[4] A.S. Mishchenko, Phys. Usp. 52 (2009) 1193.
[5] L. Gao et al., Phys. Rev. B 50 (1994) 4260.
[6] A. Schilling, M. Cantoni, J.D. Guo and H.R. Ott, Nature 363 (1993) 56.
[7] M. Nohara et al., Phys. Rev. B 52 (1995) 570.
[8] F. Gugenberger, Ch. Meingast, G. Roth, K. Grube, V. Breit, T. Weber, H. Wühl, S. Uchida, Y. Nakamura, Phys. Rev. B 49 (1994) 13137.
[9] C. Meingast, A. Junod and E. Walker, Physica C 272 (1996) 106.
[10] K. Fukamachi et al., in: K. Yamafuji and T. Morishita (Eds.), Advances in Superconductivity, vol. VII, Springer, Tokyo, 1995, p. 225.
[11] X. Chen, G.X. Tessema and M.J. Skove, Physica C 181 (1991) 340.
[12] H. Sato and M. Naito Physica C 274 (1997) 221.
[13] J.P. Locquet, J. Perret, J. Fompeyrine, E. Mächtler, J.W. Seo and G. Van Tendeloo, 1998 Nature 394 (1998) 453.
[14] G.L. Belenky, S.M. Green, A. Royburd, C.J. Lobb, S.J. Hagen, R.L. Greene, M.G. Forrestor, J. Talvacchio, Phys. Rev. B 44 (1991) 10117.
[15] C. Meingast, O. Kraut, T. Wolf, H. Wühl, A. Erb and G. Müller-Vogt, Phys. Rev. Lett. 67 (1991) 1634.
[16] S.L. Bud’ko, J. Guimpel, O. Nakamura, M.B. Maple and Ivan K. Schuller, Phys. Rev. B 46 (1992) 1257.
[17] U. Welp, M. Grimsditch, S. Flesher, W. Nessler, J. Downey, G.W. Crabtree and J. Guimpel, Phys. Rev. Lett. 69 (1992) 2130.
[18] O. Kraut, C. Meingast, G. Bräuchle, H. Claus, A. Erb, G. Müller-Vogt and H. Wühl, Physica C 205 (1993) 139.
[19] Y. Kaga, T. Sasagawa, S. Takahashi, K. Unosawa, H. Takagi, Physica B 359-361 (2005) 442.
[20] W.A. Goddard III, in: D.P. Tunstall and W. Barfold (Eds.), High Temperature Superconductivity (Proceedings of the 39th Scottish Universities Summer School in Physics), Hilger, Bristol, 1991, p. 351.
[21] Q.P. Li, Physica C 209 (1993) 513.
[22] Michael W. Klein and Sergey B. Simanovskiy, Phys. Rev. Lett. 78 (1997) 3927.
[23] X.J. Chen, H.Q. Lin, W.G. Yin, C.D. Gong, and H.-U. Habermeier, Phys. Rev. B 64 (2001) 212501.
[24] I.A. Ovid’ko, J.Phys.: Condens. Matter 13 (2001) L97.
[25] A.J. Millis, K.M. Rabe, Phys. Rev. B 38 (1988) 8908.
[26] A.S. Alexandrov, N.F. Mott, Rep. Prog. Phys. 57 (1994) 1197.
[27] E.K.H. Salje, A.S. Alexandrov, W.Y. Liang (Eds.), Polaron and Bipolaron in High- $T_c$ Superconductors and Related Materials, Cambridge University Press, Cambridge, England, 1995.
[28] A.S. Alexandrov, P.E. Kornilovitch, J.Phys.: Condens. Matter 14 (2002) 5337.
[29] A.S. Alexandrov, Theory of Superconductivity: From Weak to Strong Coupling, IOP Publishing, Bristol and Philadelphia, 2003.
[30] Yavidov Bakhrum, Physica C 470 (2010) 85.
[31] L. Vidmar, J. Bonca, S. Maekawa and T. Tohyama, Phys. Rev. Lett. 103 (2009) 186401.
[32] I.G. Lang, Yu.A. Firsov, Zh. Eksp. Teor. Fiz. 43 (1962) 1843, Sov. Phys.-JETP 16 (1963) 1301.
[33] A.S. Alexandrov, P.E. Kornilovitch, Phys. Rev. Lett. 82 (1999) 807.
[34] A.S. Alexandrov, B.Ya. Yavidov, Phys. Rev. B 69 (2004) 073101.
[35] T. Timusk, C.C. Homes and W. Reichardt, in: D. Mihailović et al. (Eds.), Anharmonic Properties of High-$T_c$ Cuprates, World Scientific, Singapore, 1995, p. 171.
[36] M. Suzuki, Phys. Rev. B 39 (1989) 2312.
[37] M.Z. Cieplak, M. Berkowski, S. Guha, E. Cheng, A.S. Vagelos, D.J. Rabinowitz, B. Wu, I.E. Troumiov and P. Lindenfeld, Appl. Phys. Lett. 65 (1994) 3383.
[38] H.L. Kao, J. Kwo, R.M. Fleming, M. Hong and J.P. Mannaerts, Appl. Phys. Lett. 59 (1991) 2748.
[39] H. Tabata, T. Kawai and S. Kawai, Appl. Phys. Lett. 58 (1991) 1443.
[40] M. Nohara, T. Suzuki, Y. Maeno, T. Fujita, I. Tanaka and H. Kojima, Physica C 185-189 (1991) 1397.
[41] T. Hanaguri, R. Toda, F. Fukase, I. Tanaka and H. Kojima, Physica C 185-189 (1991) 1395.
[42] Y. Motoi, K. Fujimoto, H. Uwe and T. Sakudo, J. Phys. Soc. Jpn. 60 (1991) 384.