Decay Rates of Fixed Planes and Closed-string Tachyons on Unstable Orbifolds

Shin NAKAMURA *

Theoretical Physics Laboratory
RIKEN (The Institute of Physical and Chemical Research)
2-1 Hirosawa, Wako, Saitama 351-0198, Japan

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Abstract

We consider closed-string tachyon condensation in the twisted sectors on the $C/Z_{2n+1} \times R^7$ orbifold. We calculate the localized energy density in the fixed plane on the orbifold at the one-loop level, and we obtain the decay rate per unit volume of the fixed plane to leading order. We show that the decay rate increases monotonically as a function of $n$. 
1 Introduction

It is generally believed that string theory, or its non-perturbative framework, has the capability of describing the dynamical transmutation of the spacetime structure. For example, a mechanism of dynamical reduction in spacetime dimensions has been proposed in the context of the IIB matrix model [1], and the dynamical transmutation of spacetime topology has also been investigated in the framework of string theory. Although we do not have a full description of the process, several possible mechanisms for the dynamical transmutation of spacetime topology have been proposed in the context of closed-string tachyon condensation on non-compact orbifolds [2, 3]. This feature of string theory is quite attractive, because we cannot describe the transmutation of the spacetime structure within the framework of ordinary particle field theories. With these matters in mind, we consider in this article closed-string tachyon condensation on unstable orbifolds.

Adams, Polchinski and Silverstein (APS) have pointed out that the fixed-point singularity in an unstable orbifold will disappear through the condensation of closed-string tachyons in the twisted sectors on the orbifold [2]. The closed-string tachyons in the twisted sectors are localized in the fixed plane,\(^1\) and this fixed plane disappears through tachyon condensation. This process resembles the decay of D-branes through open-string tachyon condensation on them. Thus, one way to study closed-string tachyon condensation in the twisted sectors is to formulate an analogy with open-string tachyon condensation; for example, Harvey, Kutasov, Martinec and Moore proposed the quantity \(g_{cl}\),\(^2\) which characterizes the process of closed-string tachyon condensation in the twisted sectors through such an analogy [3]. It is natural to inquire how far we can use an analogy between the tachyon condensation of open strings and that of closed strings in the twisted sectors. In the case of open-string tachyon condensation, the condensation is closely related to the decay of the D-brane; for example, the tension of the bosonic D-brane on which open-string tachyons exist is equal to the difference between the tree-level tachyon potentials of an unstable vacuum and a stable vacuum. The tension of the D-brane is essentially the localized energy density which is produced by open strings localized on the D-brane. In this sense, it is important to consider the energy density localized in the fixed plane in the present case, too. The tree-level localized energy density has been investigated in several works [5, 6, 7].

We consider the \(\mathbb{C}/\mathbb{Z}_{2n+1} \times \mathbb{R}^7\) orbifold for simplicity in this work, and the localized energy density in the fixed plane. However, we do not consider the tree-level localized energy density; we focus on the localized energy density at the one-loop level. We extract the decay

\(^1\)We refer to the set of fixed points as the ‘fixed plane’ in this article, because fixed points constitute a hyper-surface in general.

\(^2\)See Ref. [4] for another proposal for \(g_{cl}\).
rate per unit volume of the fixed plane at leading order from the localized energy density at the one-loop level by using the method proposed in Ref. [8]. The localized energy density diverges at the one-loop level because there exist tachyons in the twisted sectors. However, we can obtain a finite contribution after application of the appropriate regularization proposed in Ref. [8]. We then calculate the decay rate of the unstable fixed plane by using the regularized energy density and show that the decay rate of the fixed plane on the $C/Z_{2n+1} \times R^{7,1}$ orbifold grows monotonically as a function of $n$.

In the next section, we consider the one-loop amplitude of type II superstrings on the $C/Z_{2n+1} \times R^{7,1}$ orbifold. The one-loop amplitude corresponds to the energy induced by vacuum fluctuations on the orbifold. The induced energy can be divided into an unlocalized component and a localized component in the fixed plane. We consider the question of how to extract the localized component from the total one-loop amplitude. In section 3, we consider the one-loop correction to the energy density of the fixed plane on the orbifold, utilizing the method introduced in Ref. [8]. We also consider the decay rate of the fixed plane. In section 4, we explicitly evaluate the decay rate per unit volume of the fixed plane on the orbifold and show that this decay rate per unit volume of the fixed plane grows monotonically as a function of $n$. We present conclusions and some open problems in the last section.

## 2 Localized component of the partition function on a $C/Z_{2n+1} \times R^{7,1}$ orbifold

Let us start by reviewing some of the basic properties of type II superstrings on a $C/Z_{2n+1} \times R^{7,1}$ orbifold. We define the $C/Z_{N} \times R^{7,1}$ orbifold by identifying the 8-9 plane under the rotation $R$ given by

$$R = e^{2\pi i (1 + \frac{1}{N}) J_{89}},$$

(2.1)

where $J_{89}$ is the rotation generator. The 8-9 plane becomes a cone with deficit angle $2\pi (1 - \frac{1}{N})$ under this identification. The tip of the cone, the origin of the 8-9 plane ($x^8 = x^9 = 0$), is the fixed point that constitutes the $(7+1)$-dimensional fixed plane in the target space. Spacetime fermions exist if $N$ is odd, and the ground state of the untwisted sector is massless in this case [2].\(^3\) We therefore take $N$ to be a positive odd integer ($N = 2n + 1$) in this paper so that there are no tachyons in the untwisted sector.

Let us consider type II superstrings on such an orbifold. We use the RNS formalism with the following worldsheet action:

$$I = -\frac{1}{4\pi} \int d^2\sigma \left( \frac{1}{\alpha'} \partial_a X^\mu \partial^a X_\mu - i \bar{\psi}_\mu \rho^a \partial_a \psi_\mu \right).$$

(2.2)

\(^3\)The extra $2\pi$ rotation in $R$ is necessary for the existence of fermions in the untwisted sector [2].
It is convenient to represent $X^8, X^9, \psi^8$ and $\psi^9$ as complex fields according to the following:
\[
X = \frac{X^8 + iX^9}{\sqrt{2}}, \quad \bar{X} = \frac{X^8 - iX^9}{\sqrt{2}}, \quad \psi = \frac{\psi^8 + i\psi^9}{\sqrt{2}}, \quad \bar{\psi} = \frac{\psi^8 - i\psi^9}{\sqrt{2}}. \quad (2.3)
\]

The boundary conditions in the $m$-th twisted sector ($m = 0, \ldots, N - 1$)\(^4\) are
\[
\psi(\sigma_1 + 2\pi, \sigma_0) = e^{2\pi i(1 + \frac{1}{N})m}\psi(\sigma_1, \sigma_0), \quad \text{(R sector)}
\]
\[
\psi(\sigma_1 + 2\pi, \sigma_0) = -e^{2\pi i(1 + \frac{1}{N})m}\psi(\sigma_1, \sigma_0), \quad \text{(NS sector)}
\]
\[
X(\sigma_1 + 2\pi, \sigma_0) = e^{2\pi i(1 + \frac{1}{N})m}X(\sigma_1, \sigma_0). \quad (2.5)
\]

The one-loop amplitude of the strings on the orbifold is given by
\[
A_{\text{string}} \propto V_N \int_{\mathcal{F}} d\tau_1 d\tau_2 (4\pi^2 \alpha' \tau_2)^{-4} Z(\tau), \quad (2.6)
\]
\[
Z(\tau) = \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \frac{|\theta_1(\nu_{lm}|\tau)\theta_3(\tau)^3 - \theta_2(\nu_{lm}|\tau)\theta_2(\tau)^3 - \theta_4(\nu_{lm}|\tau)\theta_4(\tau)^3|}{4N|\eta(\tau)|^{18}|\theta_1(\nu_{lm}|\tau)|^2} = \frac{1}{N} \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} Z_{l,m}(\tau), \quad (2.7)
\]
where $\tau = \tau_1 + i\tau_2$ is the moduli of the torus, and $\nu_{lm} = \frac{N+1}{N}(l - m\tau)$ [9, 10, 11]. The quantity $V_N$ is the volume of the $\mathbf{R}^{2,1}$ part of the spacetime, which is equal to the volume of the fixed plane, and $\mathcal{F}$ is the fundamental region for $\tau$, which is typically chosen as that satisfying $-\frac{1}{2} \leq \tau_1 \leq \frac{1}{2}$ and $1 \leq |\tau|$. We define $Z_{l,m}(\tau)$ as
\[
Z_{l,m}(\tau) = \frac{|\theta_1(\nu_{lm}|\tau)|^8}{|\eta(\tau)|^9|\theta_1(\nu_{lm}|\tau)|^2}. \quad (2.8)
\]

The meaning of $m$ is that $Z_{l,m}(\tau)$ comes from the $m$-th twisted sector. The sum over $l$ should be regarded as the $Z_N$ projection $\frac{1}{N} \sum_{l=0}^{N-1} R^l$, which extracts the invariant states under the action of $R$.

Some comments regarding the untwisted part of the partition function are in order. The quantity $\frac{1}{N} \sum_{l=0}^{N-1} Z_{l,0}(\tau)$ is the contribution from the untwisted sector, although it should be noted that there is a slight difference between $Z_{0,0}(\tau)$ and $Z_{l\neq 0,0}(\tau)$. Let us first consider $Z_{0,0}(\tau)$. The denominator in Eq. (2.8) includes $|\theta_1(0|\tau)|^2 = 0$, which might cause a divergence. This divergence is due to the zero modes of the untwisted sector and represents the infinite volume of the 8-9 plane, which is perpendicular to the fixed plane [10, 11].\(^5\) Therefore, we
\(^4m = 0\) corresponds to the untwisted sector.
\(^5\)In the present model, however, $Z_{0,0}(\tau)$ eventually vanishes, due to the effect of numerator $|\theta_1(0|\tau)|^8 = 0$, which represents spacetime supersymmetry in the original type II superstring models.
have a ten-dimensional volume factor in the \( l = m = 0 \) part of \( A_{\text{string}} \). This can be easily understood, because the orbifold with \( N = 1 \), where \( Z(\tau) = Z_{0,0}(\tau) \), is not actually an orbifold but an ordinary flat spacetime in which all contributions of strings are extended into the ten-dimensional spacetime. By contrast, there is no such divergent component in \( Z_{l\neq 0,0}(\tau) \), and the volume factor is just \( V_8 \) in that case. This implies that \( Z_{l\neq 0,0}(\tau) \) represents a part of the quantity localized in the \((7 + 1)\)-dimensional subspace of the target space. We find that this \((7 + 1)\)-dimensional subspace is the fixed plane through the following consideration.

The one-loop amplitude corresponds to the vacuum energy induced by vacuum fluctuations at the one-loop level. Let us consider the vacuum energy induced by the twisted-sector closed strings at the one-loop level. Strings in the twisted sectors are localized in the fixed plane, because their center-of-mass coordinates are located in the fixed plane. Therefore, the vacuum energy induced by the twisted-sector closed strings is localized in the fixed plane. It may be thought that the contribution of the twisted sectors just gives the localized component of the induced vacuum energy. However, the contribution of the twisted sectors alone is not enough to construct the localized vacuum energy. This is because the contribution of the twisted sectors to \( Z(\tau) \) is given by 
\[
\frac{1}{N} \sum_{l=0}^{N-1} \sum_{m=1}^{N-1} Z_{l,m}(\tau) \equiv Z^{\text{tw}}(\tau),
\]
although \( \frac{d\tau_1 d\tau_2}{\tau^2} Z^{\text{tw}}(\tau) \) itself is not modular invariant. The only way to make it modular invariant is to incorporate \( \frac{1}{N} Z_{l\neq 0,0}(\tau) \) into \( Z^{\text{tw}}(\tau) \) and to consider
\[
Z^{\text{local}}(\tau) \equiv \frac{1}{N} \sum_{l=0}^{N-1} \sum_{m=1}^{N-1} Z_{l,m}(\tau) + \frac{1}{N} Z_{l\neq 0,0}(\tau)
= \frac{1}{N} \sum_{\{l,m\} \neq \{0,0\}} Z_{l,m}(\tau).
\]
It is thereby found that \( \frac{d\tau_1 d\tau_2}{\tau^2} Z^{\text{local}}(\tau) \) is modular invariant. Of course, the localized vacuum energy is a physical quantity and should be represented in a modular invariant way.

From the above considerations, it is found that the localized component of \( A_{\text{string}} \) in the fixed plane can be represented as
\[
A^{\text{local}}_{\text{string}} \propto V_8 \int_F \frac{d\tau_1 d\tau_2}{2\tau_2} (4\pi^2 \alpha' \tau_2)^{-4} Z^{\text{local}}(\tau).
\]
Therefore, \( \frac{1}{N} Z_{l\neq 0,0}(\tau) \) should also be regarded as a part of the localized component of the partition function in the fixed plane.

Let us consider the meaning of the word “localized” we have used here. It is known that closed strings in the twisted sectors can sweep the spacetime far from the fixed plane, although their center-of-mass coordinates are always in the fixed plane. However, twisted strings have to be stretched in order to reach points far from the fixed plane, and this costs energy. Therefore,
the main contribution of the twisted strings comes from the diagrams that sweep the vicinity of the fixed plane. This is the meaning of the “localized” contribution.

In the case of untwisted strings which contribute to $Z_{t\neq 0,0}(\tau)$, their center-of-mass coordinates are not restricted to the fixed plane. However, their one-loop diagrams are twisted around the fixed plane. The contribution of the one-loop diagrams that sweep points far from the fixed plane is therefore smaller than the contribution of those that sweep the vicinity of the fixed plane. Thus, the main contribution of $Z_{t\neq 0,0}(\tau)$ comes from the diagrams that sweep the vicinity of the fixed plane. With this realization, we find that it is natural to regard $Z_{\text{local}}(\tau)$ as the contribution to the induced vacuum energy localized in the fixed plane. The situation described here is schematically depicted in Figs. 1 and 2.

From the above discussion, we find that the vacuum energy density localized in the fixed plane at the one-loop level is given by

$$\rho_{\text{local}} = -\frac{1}{2} \int_{\mathcal{F}} \frac{d\tau_1}{\eta_2} \frac{d\tau_2}{\eta_2} (4\pi^2 \alpha' \tau_2)^{-4} Z_{\text{local}}(\tau).$$ (2.11)

We have fixed the overall normalization of the right-hand side of Eq. (2.11) so that the normalization is the same as that of the corresponding particle field theory in the low-energy limit.

It is easily found that $Z_{0,0}(\tau) = 0$ in the present model. Thus, $Z_{\text{local}}(\tau)$ is equal to $Z(\tau)$ in our model. This implies that the induced vacuum energy at the one-loop level in our model arises only in the vicinity of the fixed plane.

3 One-loop localized energy and decay rate of the fixed plane

APS have pointed out that the $C/Z_{2n+1} \times \mathbb{R}^{7,1}$ orbifold is dynamically transmuted into the $C/Z_{2n'+1} \times \mathbb{R}^{7,1}$ orbifold, where $n > n'$, through closed-string tachyon condensation in the twisted sectors on the original orbifold. The final result of this process is ordinary flat spacetime without orbifolding ($n' = 0$), which possesses supersymmetry and no tachyons. In other words, the fixed plane on the initial orbifold disappears through the condensation of the twisted-sector closed-string tachyons localized in the fixed plane. This is reminiscent of the decay process of bosonic D-branes, which is brought about by open-string tachyon condensation. It is therefore natural to formulate an analogy between a D-brane and the fixed plane.

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6The author thanks S. Sugimoto for discussions on this topic.
Figure 1: All one-loop diagrams of strings in the twisted sectors are twisted around the fixed plane, which is located at the tip of the cone. (The diagram corresponding to $Z_{l=0,m=1}$ on the $\mathbb{C}/\mathbb{Z}_3 \times \mathbb{R}^{7,1}$ orbifold is shown in this figure, where $t$ denotes the proper time.)

Figure 2: Some of the one-loop diagrams of untwisted strings are also twisted around the fixed plane. (Diagrams corresponding to $Z_{l=0,m=0}$ and $Z_{l=1,m=0}$ on the $\mathbb{C}/\mathbb{Z}_3 \times \mathbb{R}^{7,1}$ orbifold are shown in this figure. The $l \neq 0$ case is twisted. $t$ denotes the proper time.)
In the sense described above, consideration of the “tension” of the fixed plane on the orbifold is of interest. The tension of the D-brane is the energy density localized in the D-brane. Therefore, it is quite important to consider the energy density localized in the fixed plane. Several proposals to define such an energy density at the classical level have been made in Refs. [5, 6, 7].

We have seen that \( \rho_{\text{local}} \) can also be regarded as the energy density localized in the fixed plane induced by vacuum fluctuations. However, this is a quantity at the one-loop level of string theory and should be regarded as the one-loop correction to the classical energy density. Although the main part of the energy density of the fixed plane is given by the tree-level energy density, we consider the one-loop energy density \( \rho_{\text{local}} \) in this study.

We should point out that \( \rho_{\text{local}} \) defined by Eq. (2.11) has an IR divergence due to the twisted-sector tachyons, and we need an appropriate regularization in order to obtain a finite result. Fortunately, an appropriate regularization method with analytic continuation has been proposed in Ref. [8].

### 3.1 Analytic continuation of \( \rho_{\text{local}} \)

The presence of an IR divergence does not necessarily imply the failure of the theory. The IR divergence in \( \rho_{\text{local}} \) due to tachyons only indicates the instability of the vacuum, and we can extract physically sensible quantities from it. A method to regularize the IR divergence due to tachyons in the one-loop amplitude has been proposed in Ref. [8].

The IR divergence in \( \rho_{\text{local}} \) has the form

\[
\sim \sum_i \int_{\Lambda} \infty \frac{d\tau_2}{\tau_2^9} e^{-\pi \alpha' \tau_2 M_i^2}, \tag{3.1}
\]

where \( M_i^2 < 0 \) is the mass squared of the tachyon labeled \( i \) that propagates in the one-loop diagram. Here, \( \Lambda \) denotes the UV cutoff. This divergence can be regularized as follows by use of analytic continuation:

\[
\int_{\Lambda} \infty \frac{d\tau_2}{\tau_2^9} e^{-\pi \alpha' \tau_2 (M_i^2 - i\epsilon)} \equiv (\pi \alpha' M_i^2)^4 \int_{M_i^2 \Lambda - i\epsilon} \infty \frac{d\tau_2}{\tau_2^9} e^{-\tau_2}. \quad \text{(for } M_i^2 < 0) \tag{3.2}
\]

However, the regularized amplitude becomes complex in general.

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\(^7\)See also Ref. [12].

\(^8\)This method has also been applied to the regularization of the IR divergence due to open-string tachyons in the annulus diagram [13, 14, 15].
3.1.1 Imaginary part

The imaginary part of Eq. (3.2) can be obtained as

\[ \text{Im} \left\{ \int_{\Lambda}^\infty \frac{d\tau_2}{\tau_2^5} e^{-\pi\alpha'\tau_2(M_i^2 - i\epsilon)} \right\} = \frac{\pi (-\pi\alpha' M_i^2)^4}{\Gamma(1 + 4)}. \] (3.3)

Thus, the imaginary part of \( \rho_{\text{local}} \) after regularization is

\[ \text{Im}\{\rho_{\text{local}}\} = -\frac{1}{2} \frac{1}{4^4!\pi^3} \sum_i \frac{1}{4^4!\pi^3} \sum_i (-M_i^2)^4, \] (3.4)

where the sum over \( i \) runs for all the tachyonic states.

This imaginary part has a physical meaning [8, 16]: The quantity \( \rho_{\text{local}} V_8 \) can be regarded as the one-loop correction to the effective action of the string theory on the orbifold. Thus,

\[ \Gamma_n = -2\text{Im}\{\rho_{\text{local}}\} = \frac{1}{4^4!\pi^3} \sum_i (-M_i^2)^4 \] (3.5)

should correspond to the decay rate of the vacuum of the \( C/Z_{2n+1} \times R^7 \) orbifold. The subscript \( n \) of \( \Gamma_n \) indicates that the discrete group we have used in the construction of this orbifold is \( Z_{2n+1} \).

The precise meaning of \( \Gamma_n \) can be understood by employing the argument given in Ref. [16] in the framework of particle field theory. The decay rate per unit volume of an unstable vacuum is defined in Ref. [16] by considering the time evolution of the initial vacuum state; it is extracted from the time dependence of the overlap of the initial unstable state localized at the top of the tachyon potential with its time-developed state. The correspondence between \( \Gamma_n \) obtained in the framework of string theory and the decay rate given in Ref. [16] in particle field theory can be understood as follows. Let us consider as an example a \( d \)-dimensional bosonic scalar field theory. Then, the one-loop amplitude for the bosonic particles of mass \( M \) is given by

\[ A_{\text{field}} = -\frac{1}{2} V_d \int \frac{d^d p}{(2\pi)^d} \ln(p^2 + M^2 - i\epsilon). \] (3.6)

If these bosonic particles are tachyons and \( M^2 < 0 \), then \( \ln(p^2 + M^2 - i\epsilon) \) has an imaginary part, \(-i\pi\), when \( p^2 < -M^2 \). The imaginary part of the one-loop amplitude can be easily evaluated as

\[ \text{Im}\{A_{\text{field}}\} = \frac{1}{2} V_d \int \frac{d^d p}{(2\pi)^d} \theta(-M^2 - p^2) \frac{\pi (-M^2)^{d/2}}{(4\pi)^{d/2} \Gamma(1 + d/2)}. \] (3.7)
Here, $A_{\text{field}}$ is the one-loop correction to the effective action for the scalar field, and
\[ \Gamma_{\text{field}} \equiv \frac{2 \text{Im}\{A_{\text{field}}\}}{V_d} = \frac{\pi}{(4\pi)^{d/2}} \frac{(-M^2)^{d/2}}{\Gamma(1 + d/2)} \] (3.8)
gives the decay rate per unit volume of the unstable vacuum given in Ref. [16]. Note that if we have more than one scalar field in 8-dimensional spacetime, the generalized expression of Eq. (3.8) coincides with Eq. (3.5).

In the present model, there are no tachyons in the bulk, and the instability of the vacuum is localized only in the fixed plane. Furthermore, the canonical dimension of $\Gamma_n$ indicates that $\Gamma_n$ is the decay rate per unit volume of the 7-dimensional subspace. Therefore, it is natural to regard $\Gamma_n$ as the decay rate per unit volume of the fixed plane at the one-loop level of the perturbative string theory. The sum $\sum_i (-M_i^2)^4$ depends on discrete group $Z_{2n+1}$. We investigate the $n$ dependence of $\Gamma_n$ in the next section.

### 3.1.2 Real part

The real part of $\rho_{\text{local}}$ after the analytic continuation should be regarded as the one-loop correction to the localized energy density in the fixed plane. It has been pointed out in the framework of particle field theory that the real part of the vacuum energy after the analytic continuation depends on the choice of the wavepacket in which each Fourier mode of the field is placed [16]. It is also known that if we choose the minimum uncertainty wavepacket, the real part of the vacuum energy coincides with the real part of the regularized effective action. We assume here that we are considering the case in which we have chosen the minimum uncertainty wavepacket in the corresponding field theory, and we regard the real part of $\rho_{\text{local}}$ as the one-loop correction to the vacuum energy density.

Let us consider the real part of the regularized energy density of the fixed plane on the $C/Z_3 \times R^7,1$ orbifold as an example. We must carry out a numerical calculation to obtain the real part of $\rho_{\text{local}}$. The tachyonic states in this orbifold are only the ground states in the twisted sectors. $\rho_{\text{local}}$ can be approximately written as
\[ \rho_{\text{local}} \sim -\frac{1}{2 (4\pi^2\alpha')^4} \int_1^\infty \frac{d\tau_2}{d\tau_2^5} \{6e^{2\pi\tau_2\frac{1}{2}} + 108 + \cdots\}, \] (3.9)
where the contribution of the massive modes has been omitted. We have also ignored the integral in the region satisfying $\tau_2 < 1$ in $F$. The first term in the integrand is the contribution of the tachyonic modes, and it should be defined by using analytic continuation. After this analytic continuation, we obtain
\[ \text{Re}\{\rho_{\text{local}}\} \sim -\frac{O(10^{-6})}{(\alpha')^4}. \] (3.10)

\footnote{See also Ref. [17].}
Therefore, the one-loop contribution to the localized energy density in the fixed plane on the \( C/Z_3 \times R^{7,1} \) orbifold is negative. This suggests a positive energy density at the tree level whose magnitude is sufficiently large to overwhelm this negative one-loop contribution.

4 Decay rate per unit volume of the fixed plane on the \( C/Z_{2n+1} \times R^{7,1} \) orbifold

In this section, we consider the \( n \) dependence of \( \Gamma_n \) for the \( C/Z_{2n+1} \times R^{7,1} \) orbifold. We need to list all the physical tachyonic states in the twisted sectors, because \( \Gamma_n \) is essentially given by \( \sum_i (-M_i^2)^4 \), where the sum over \( i \) runs over all the tachyonic states.

The mass squared of the ground state in the \( m \)-th twisted sector is given as follows:

\[
M^2 = -\frac{2}{\alpha'} \left( 1 - \frac{m}{2n+1} \right), \quad \text{(for odd } m \text{)} \tag{4.1}
\]
\[
M^2 = -\frac{2}{\alpha' 2n+1}. \quad \text{(for even } m \text{)} \tag{4.2}
\]

Thus, every twisted sector has tachyons. Furthermore, some excited states can also be tachyonic. We find that the mass squared of such a \( k \)-th excited tachyonic state in the \( m \)-th twisted sector can be written as

\[
M^2 = -\frac{2}{\alpha'} \left( 1 - \frac{(2k+1)m}{2n+1} \right), \quad \text{(for odd } m \text{)} \tag{4.3}
\]
\[
M^2 = -\frac{2}{\alpha'} \left( \frac{(2k+1)m}{2n+1} - 2k \right), \quad \text{(for even } m \text{)} \tag{4.4}
\]

where \( k \) should be a non-negative integer satisfying \( M^2 < 0 \).\(^{10}\) An important property of Eqs. (4.3) and (4.4) is that the tachyonic spectrum for odd \( m \) is exactly the same as the tachyonic spectrum in the \( l \)-th twisted sector, where \( l \) is an even integer given by \( l = 2n+1 - m \). Therefore, we only need to consider the tachyonic spectrum in the \( m \)-th twisted sector with odd \( m \), that is, we have

\[
\sum_i (-M_i^2)^4 = 2 \sum_j (-M_j^2)^4, \tag{4.5}
\]

where \( j \) labels only the tachyonic states in the \( m \)-th twisted sector with odd \( m \). This makes the subsequent analysis simpler.

Let us consider the tachyonic spectrum in the \( m \)-th twisted sector with odd \( m \), which is given by Eq. (4.3). We note the following facts:

\(^{10}\)See Appendix A for details.
• Each tachyonic state becomes more tachyonic as \( n \) increases.

• The number of tachyonic states increases as \( n \) increases. This is because \( k \) for odd \( m \) is bounded as

\[
1 \leq 2k + 1 < \frac{2n + 1}{m}.
\] (4.6)

Of course, the number \( 2n \) of the twisted sectors also increases with \( n \). We can therefore conclude that \( \Gamma_n < \Gamma_{n'} \) if \( n < n' \). Thus, the decay rate per unit volume of the fixed plane grows monotonically as a function of \( n \). Note that \( \Gamma_n \) can be written explicitly as

\[
\Gamma_n = \frac{2}{4! \pi^3 \alpha'^4} \sum_{r=0}^{n-1} \sum_{k=0}^{\left\lfloor \frac{n-r}{2r+1} \right\rfloor} \left( 1 - \frac{(2k + 1)(2r + 1)}{(2n + 1)} \right)^4,
\] (4.7)

where we have defined \( r \) as \( m = 2r + 1 \). Here, \( \left\lfloor \frac{n-r}{2r+1} \right\rfloor \) denotes the integer part of \( \frac{n-r}{2r+1} \).

Let us now consider the behaviour of \( \Gamma_n \) in the large \( n \) limit. We find that

\[
\Gamma_n \sim \frac{2}{4! \pi^3 \alpha'^4} \frac{1}{20} (2n + 1) \ln(2n + 1), \quad (n \to \infty)
\] (4.8)

and the growth of \( \Gamma_n \) is not bounded. The derivation of Eq. (4.8) is given in Appendix B.

5 Conclusions and discussion

We have considered the energy density localized in the fixed plane on a \( C/Z_{2n+1} \times R^{7,1} \) orbifold at the one-loop level and have seen that the non-vanishing component of the one-loop amplitude on this orbifold gives the localized vacuum energy in the fixed plane at the one-loop level. This implies that a part of the contribution from the untwisted sector, as well as the contribution from the twisted sectors, should be regarded as the localized contribution in the fixed plane.

We used analytic continuation to regularize the IR divergence in the one-loop energy density and calculated the leading-order decay rate per unit volume of the fixed plane by using the imaginary part of the regularized energy density. We found that the decay rate per unit volume of the fixed plane on the \( C/Z_{2n+1} \times R^{7,1} \) orbifold decreases monotonically as a function of \( n \), and finally it reaches zero at \( n = 0 \) (note that \( \Gamma_0 = 0 \)).

According to the observation of APS, the \( C/Z_{2n+1} \times R^{7,1} \) orbifold decays into a flat spacetime through closed-string tachyon condensation in the twisted sectors. One of the decay paths that was investigated by APS is that in which \( n \) decreases monotonically through tachyon condensation until it finally reaches zero, which corresponds to a flat spacetime. The results of
this work indicate that $\Gamma_n$ decreases monotonically through this process and finally becomes zero. This property of $\Gamma_n$ tempts us to find a connection of $\Gamma_n$ to the total transition rate from the $C/Z_{2n+1} \times R^7,1$ orbifold to a flat spacetime through tachyon condensation. Unfortunately, there does not currently exist a theoretical framework which rigorously describes the transition of a spacetime structure. $\Gamma_n$ is defined in each perturbative string theory on each orbifold. Therefore we cannot immediately conclude that $\Gamma_n$ is the transition rate from the $C/Z_{2n+1} \times R^7,1$ orbifold to a flat spacetime. The relationship between $\Gamma_n$ presented here and the transition rate of the spacetime structure should be studied in future works.

The method we have utilized to calculate the regularized $\rho_{\text{local}}$ is also applicable to other orbifolds that have been considered in the context of closed-string tachyon condensation in the twisted sectors. It would be interesting to examine whether or not the decay rates of the fixed planes always decrease monotonically along the decay paths of the orbifolds. It would also be interesting to consider whether there is some connection between each tachyonic mode and each decay branch from the viewpoint of the decay rate considered in the present work.

The relationship between the transition rate from the initial orbifold to the flat spacetime and the decay rate we have calculated in this paper is not clear at this stage. However, further consideration of the results of the present work may yield some useful information concerning the construction of a formulation that describes the transmutation of the spacetime structure.

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Appendix

A Tachyonic spectrum

In this appendix, we show that the tachyonic spectrum in Eqs. (4.3) and (4.4) can be extracted from the partition function on the orbifold. To begin, let us rewrite \( Z_{t,m}(\tau) \) as

\[
Z_{t,m}(\tau) = \frac{|\theta_1(\frac{2m}{N}\tau)|^8}{|\eta(\tau)^9\theta_1(\nu_{tm}\tau)|^2} = e^{-2\pi i \tau (1 + \frac{k}{2})} F_1 F_2 F_3 F_4 F_5, \tag{A.1}
\]

where

\[
F_1 \equiv \prod_{k=1}^{\infty} (1 - q^k)^6(1 - \bar{q}^k)^6 = \prod_{k=1}^{\infty} \left( \sum_{j=0}^{\infty} q^{kj} \right)^6 \left( \sum_{j=0}^{\infty} \bar{q}^{kj} \right)^6, \tag{A.2}
\]

\[
F_2 \equiv G_1 G_2, \tag{A.3}
\]

\[
G_1 \equiv \prod_{k=1}^{k_*} \left| \left( 1 - (-1)^l e^{\pi i \frac{k}{N} \bar{q}^{k-1-(1+\frac{1}{N})\frac{m}{2}}} \right)^4 \right|^2, \tag{A.4}
\]

\[
G_2 \equiv \prod_{k=k_*+1}^{\infty} \left| \left( 1 - (-1)^l e^{\pi i \frac{k}{N} \bar{q}^{k-1-(1+\frac{1}{N})\frac{m}{2}}} \right)^4 \right|^2, \tag{A.5}
\]

\[
F_3 \equiv \prod_{k=1}^{\infty} \left| \left( 1 - (-1)^l e^{-\pi i \frac{k}{N} \bar{q}^{k-(1+\frac{1}{N})\frac{m}{2}}} \right)^4 \right|^2, \tag{A.6}
\]

\[
F_4 \equiv \prod_{k=1}^{\infty} \left| \left( 1 - e^{2\pi i \frac{k}{N} \bar{q}^{k-1-(1+\frac{1}{N})\frac{m}{2}}} \right)^4 \right|^2, \tag{A.7}
\]

\[
F_5 \equiv \prod_{k=1}^{m+1} \left| \left( 1 - e^{-2\pi i \frac{k}{N} \bar{q}^{k+m(1+\frac{1}{N})}} \right)^4 \right|^2. \tag{A.8}
\]

The quantity \( k_* \) in Eqs. (A.4) and (A.5) is \( \frac{m}{2} + 1 \) for even \( m \) and \( \frac{m+1}{2} \) for odd \( m \). We define \( q \) as \( q \equiv e^{2\pi i \tau} \).

\( Z_{t,m}(\tau) \) can then be rewritten in the form

\[
Z_{t,m}(\tau) = \sum_{i} e^{2\pi i \nu_{ih} f(\tau_1)}, \tag{A.11}
\]
where the sum over $i$ runs for all the states. The contribution of some states to the one-loop amplitude should vanish after integrating over $\tau_1$ in the amplitude when $\tau_2$ is sufficiently large; this gives the level-matching condition. All we then have to do is to list the states that survive the $\tau_1$ integration and choose positive $h_i$ that correspond to the tachyonic states.

Let us consider the most tachyonic state. The most tachyonic contribution that satisfies the level-matching condition can be extracted by picking up $1$ from $F_1$, $F_3$, $F_5$, $G_2$ and $G_4$, by picking up $\prod_{k=1}^{k_1}(q \bar{q})^{4(k-1-(1+\frac{1}{N})\frac{m}{2})}G_1$, and by picking up the terms with $j = 0$ from $G_3$. This yields the following as the most tachyonic term:

$$e^{-2\pi \tau_2 (1+\frac{1}{N})m}e^{-4\pi \tau_2 4\sum_{k=1}^{k_1} (k-1-(1+\frac{1}{N})\frac{m}{2})}e^{-4\pi \tau_2 (-1) \sum_{k=1}^{m+1} (k-m-1-\frac{m}{N})} = e^{2\pi \tau_2 h}.$$  \hfill (A.12)

Here, we have

$$h = \begin{cases} \frac{m}{N} & (m: \text{even}) \\ 1 - \frac{m}{N} & (m: \text{odd}) \end{cases},$$  \hfill (A.13)

which represents the correct ground states.

Excited states can also be tachyonic in general. The terms corresponding to excited tachyonic states can be extracted from $Z_{i,m}(\tau)$ by simply changing the terms picked up from $G_3$ and $G_4$. If we change the terms picked up from one of the other parts ($F_1$, $F_3$, $F_5$, $G_1$ or $G_2$), the terms corresponding to the excited states can give either massless or massive contributions. It can also be shown that we should pick up $e^{-2\pi \tau_2 \frac{2m}{N}(j+1)} \prod_{k=1}^{m} e^{4\pi \tau_2 (k-m-1-\frac{m}{N})}$ from $G_3$ and $1$ from $G_4$ if $m$ is odd, while we should pick up terms with $j = 0$ from $G_3$ and $e^{-2\pi \tau_2 (1-\frac{m}{N})j}$ from $G_4$ if $m$ is even. These considerations lead to the following possible values of $h_i$ for the tachyonic contribution:

$$h_i = \begin{cases} \frac{m}{N} - 2(1 - \frac{m}{N})j & (m: \text{even}) \\ 1 - \frac{m}{N} - 2\frac{m}{N}j & (m: \text{odd}) \end{cases}.$$  \hfill (A.14)

Here, $j$ is a non-negative integer. This gives the tachyonic spectrum presented in Eqs. (4.3) and (4.4). We can also show that the states corresponding to Eq. (A.14) satisfy the level-matching condition.

**B Derivation of Eq. (4.8)**

Here we evaluate the large $n$ behaviour of Eq. (4.7). We first decompose the sum over $r$ in Eq. (4.7) into two parts, the sum in the region $0 \leq r \leq \left\lfloor \frac{2m}{N+1} \right\rfloor$ and that in the region
\[\left\lfloor \frac{n-r}{2r+1} \right\rfloor + 1 \leq r \leq n-1.\] Explicitly, we write

\[
\sum_{r=0}^{n-1} \sum_{k=0}^{\left\lfloor \frac{n-r}{2r+1} \right\rfloor} \left( 1 - \frac{(2k+1)(2r+1)}{2n+1} \right)^4
= \sum_{r=0}^{n-1} \left( 1 - \frac{2r+1}{2n+1} \right)^4 + \sum_{r=0}^{n-1} \sum_{k=0}^{\left\lfloor \frac{n-r}{2r+1} \right\rfloor} \left( 1 - \frac{(2k+1)(2r+1)}{2n+1} \right)^4,
\]

where we have used the fact that \(\left\lfloor \frac{n-r}{2r+1} \right\rfloor = 0\) if \(r > \frac{n-1}{3}\).

Let us consider the first term on the right-hand side of Eq. (B.1). If \(s\) is a positive integer, we have

\[
\sum_{r=s}^{n-1} \left( 1 - \frac{2r+1}{2n+1} \right)^4
= \frac{8(1+2n-2s)(n-s)(1+n-s)(-1+3n+3n^2-3s-6ns+3s^2)}{15(2n+1)^4}
\equiv F_{(1)}(s,n).
\]

We use Eq. (B.2) as the formal definition of \(F_{(1)}(s,n)\), even for the case in which \(s\) is not an integer. We can show that

\[
F_{(1)} \left( \frac{n-1}{3}, n \right) > F_{(1)} \left( \left\lfloor \frac{n-1}{3} \right\rfloor + 1, n \right) \geq F_{(1)} \left( \frac{n-1}{3} + 1, n \right)
\]

for \(n \geq 2\). By using Eq. (B.3) and taking the limit \(n \to \infty\), we find that

\[
\sum_{r=\left\lfloor \frac{n-1}{3} \right\rfloor + 1}^{n-1} \left( 1 - \frac{2r+1}{2n+1} \right)^4 \sim \frac{16}{1215} N + G_{(1)}(N),
\]

where \(G_{(1)}(N)\) can be a non-trivial function of \(N = 2n+1\), although it is bounded as \(-\frac{8}{135} \leq G_{(1)}(N) \leq \frac{56}{405}\).

We next consider the second term on the right-hand side of Eq. (B.1). We define \(t \equiv \frac{n-r}{2r+1} - \left\lfloor \frac{n-r}{2r+1} \right\rfloor\). Then, we can write

\[
\sum_{k=0}^{\left\lfloor \frac{n-r}{2r+1} \right\rfloor} \left( 1 - \frac{(2k+1)(2r+1)}{2n+1} \right)^4 = \sum_{k=0}^{\frac{n-r}{2r+1} - t} \left( 1 - \frac{(2k+1)(2r+1)}{2n+1} \right)^4.
\]

Note that \(1 > t \geq 0\) and \(\frac{n-r}{2r+1} - t > 0\) if \(0 \leq r \leq \frac{n-1}{3}\). We can show that the following holds:

\[
\sum_{k=0}^{\frac{n-r}{2r+1} - t} \left( 1 - \frac{(2k+1)(2r+1)}{2n+1} \right)^4
\]
\[ \frac{8}{15(2n+1)^4(2r+1)} \{15n^4 + 6n^5 - 30n^2r(1+r) -10n^3(-1 + 2r + 2r^2) + n(-1 - 8r + 6r^2 + 28r^3 + 14r^4) + r(1 + r)(1 + 7r + 7r^2) \} - \frac{8(2r+1)^4t(-1 + t)(-1 + 2t)(-1 - 3t + 3t^2)}{15(2n+1)^4} \equiv F_2(t, r, n). \] (B.6)

We use Eq. (B.6) as the formal definition of \( F_2(t, r, n) \), even in the case that \( \frac{n-r}{2r+1} - t \) is not an integer. We can show that

\[ F_2(t_2, r, n) \geq F_2(t, r, n) \geq F_2(t_1, r, n), \quad (0 \leq t < 1) \] (B.7)

where

\[ t_1 = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{\sqrt{30}}}, \quad \text{(B.8)} \]

\[ t_2 = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{1}{\sqrt{30}}}. \quad \text{(B.9)} \]

Note that \( t_1 \) and \( t_2 \) depend on neither \( n \) nor \( r \). Thus we have

\[ \sum_{r=0}^{\left\lfloor \frac{n-1}{3} \right\rfloor} F_2(t_2, r, n) \geq \sum_{r=0}^{\left\lfloor \frac{n-1}{3} \right\rfloor} \sum_{k=0}^{\left\lfloor \frac{n-r}{2r+1} \right\rfloor} \left( 1 - \frac{(2k + 1)(2r + 1)}{2n+1} \right)^4 \geq \sum_{r=0}^{\left\lfloor \frac{n-1}{3} \right\rfloor} F_2(t_1, r, n). \] (B.10)

In order to evaluate \( \sum_{r=0}^{\left\lfloor \frac{n-1}{3} \right\rfloor} F_2(t, r, n) \), we define \( u \equiv \frac{n-1}{3} - \left\lfloor \frac{n-1}{3} \right\rfloor \). We can then show that

\[ \sum_{r=0}^{\left\lfloor \frac{n-1}{3} \right\rfloor} F_2(t, r, n) = \frac{1}{291600} \left\{ -27915 + 72900 \ln 4 -1280t(-1 + t)(-1 + 2t)(-1 - 3t + 3t^2) +120u(249 - 16t + 160t^3 - 240t^4 + 96t^5) +n \left( -5190 + 29160 \ln 4 - 128t(-1 + t)(-1 + 2t)(-1 - 3t + 3t^2) \right) \right\} +\frac{1}{20}(2n+1) \left\{ \gamma + \Psi \left( \frac{2n+1}{6} + 1 - u \right) \right\} + O(1/n) \equiv F_3(u, t, n) + O(1/n), \] (B.11)

where \( \gamma \) is Euler’s constant, and \( \Psi(z) = \frac{d}{dz} \ln \Gamma(z) \). We use Eq. (B.11) as the formal definition of \( F_3(u, t, n) \), even for the case in which \( \frac{n-1}{3} - u \) is not an integer. We note that \( F_3(0, t_2, n) \geq \)
$F_3(u, t, n)$ and $F_3(u, t_1, n) \geq F_3(1, t_2, n)$ for $0 \leq u < 1$ if $n$ is sufficiently large. It can thus be shown by using Eq. (B.10) that

$$F_3(0, t_2, n) \geq \sum_{r=0}^{[\frac{n-1}{2}]} \sum_{k=0}^{[\frac{r-1}{2}]} \left( 1 - \frac{(2k+1)(2r+1)}{2n+1} \right)^4 \geq F_3(1, t_1, n)$$  \hspace{1cm} (B.12)

in the limit that $n \to \infty$. By taking the large $n$ limit of $F_3(0, t_2, n)$ and $F_3(1, t_1, n)$, we find that

$$\sum_{r=0}^{[\frac{n-1}{2}]} \sum_{k=0}^{[\frac{r-1}{2}]} \left( 1 - \frac{(2k+1)(2r+1)}{2n+1} \right)^4 \sim \frac{1}{20} N \ln N + G_2(N)N + G_3(N), \hspace{1cm} (n \to \infty)$$  \hspace{1cm} (B.13)

where we have used the fact that $z\Psi(z) \sim z \ln z - \frac{1}{2}$ for $z \to \infty$. The quantities $G_2(N)$ and $G_3(N)$ can be non-trivial functions of $N$, although they are bounded as follows:

$$F_4(t_2) \geq G_2(N) \geq F_4(t_1),$$  \hspace{1cm} (B.14)

$$F_5(0, t_2) + \frac{3}{20} \geq G_3(N) \geq F_5(1, t_1) - \frac{3}{20}. \hspace{1cm} (B.15)$$

Here, $F_4(u, t)$ and $F_5(t)$ are defined as

$$F_4(t) \equiv \frac{-2595 + 14580 \ln 4 - 64t(-1 + t)(-1 + 2t)(-1 + 3t + 3t^2)}{291600} + \frac{\gamma}{20} - \frac{1}{20} \ln 6,$$  \hspace{1cm} (B.16)

$$F_5(u, t) \equiv \frac{1}{291600} \left\{ -25320 + 58320 \ln 4 - 1216t(-1 + t)(-1 + 2t)(-1 + 3t + 3t^2) + 120u(249 - 16t + 160t^3 - 240t^4 + 96t^5) \right\}, \hspace{1cm} (B.17)$$

and $F_3(u, t, n)$ is given by

$$F_3(u, t, n) = \frac{2n + 1}{20} \Psi \left( \frac{2n + 1}{6} + 1 - u \right) + (2n + 1)F_4(t) + F_5(u, t).$$  \hspace{1cm} (B.18)

Therefore Eqs. (B.3) and (B.13) lead to

$$\sum_{r=0}^{[\frac{n-1}{2}]} \sum_{k=0}^{[\frac{r-1}{2}]} \left( 1 - \frac{(2k+1)(2r+1)}{2n+1} \right)^4 \sim \frac{1}{20} N \ln N + G_2(N)N + G_3(N), \hspace{1cm} (n \to \infty)$$  \hspace{1cm} (B.19)
where $G_{(4)}(N)$ and $G_{(5)}(N)$ can be non-trivial functions of $N$ that are bounded as

$$ F_{(4)}(t_2) + \frac{56}{405} \simeq 0.14 \geq G_{(4)}(N) \geq F_{(4)}(t_1) - \frac{8}{135} \simeq -0.060, \quad (B.20) $$

$$ F_{(5)}(0, t_2) + \frac{3}{20} + \frac{16}{1215} \simeq 0.35 \geq G_{(5)}(N) \geq F_{(5)}(1, t_1) - \frac{3}{20} + \frac{16}{1215} \simeq 0.16. \quad (B.21) $$

The above calculation leads to the following:

$$ \Gamma_n \sim \frac{2}{4! \pi^3 \alpha'^4} \left\{ \frac{1}{20} N \ln N + G_{(4)}(N)N + G_{(5)}(N) \right\}, \quad (n \to \infty) \quad (B.22) $$

where

$$ 1.4 \times 10^{-1} \geq G_{(4)}(N) \geq -6.0 \times 10^{-2}, \quad (B.23) $$

$$ 3.5 \times 10^{-1} \geq G_{(5)}(N) \geq 1.6 \times 10^{-1}. \quad (B.24) $$

Therefore, the leading-order behaviour of $\Gamma_n$ is given by $\Gamma_n \propto (2n + 1) \ln(2n + 1)$ in the large $n$ region, and we can extract Eq. (4.8) from Eq. (B.22).

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