Elaboration and research of planetary precessional multiplier type K-H-V

I Bostan1, V Dulgheru1 and R Ciobanu1

1Fundamentals of Machines Design Department, Technical University Moldova, Chişinău, Rep. Moldova
E-mail: dulgheru@mail.utm.md

Abstract. The multiplier is an indispensable part of the micro hydropower plant and high power wind turbine. It helps to increase rotor low speeds limited by the water flow small velocity and by the relative big placement diameter of the blades that participate in the energy conversion. For example, the microhydrostation rotor’s speed is \((2 – 3) \text{ min}^{-1}\) for water flow velocity \(V = (1...1.6) \text{ m/s}\) and for blade placement diameter \(D = 4 \text{ m}\). Diversity of requirements forwarded by the beneficiaries of mechanical transmissions consists, in particular, in increasing reliability, efficiency and lifting capacity, and in reducing the mass and dimensions. It becomes more and more difficult to satisfy the mentioned demands by partial updating of traditional transmissions. The target problem can be solved with special effects by developing new types of multipliers based on precessional planetary transmissions with multiple gear, that were developed by the authors. Absolute multiplicity of precessional gear (up to 100% pairs of teeth simultaneously involved in gearing, compared to 5%-7% - in classical gearings) provides increased lifting capacity and small mass and dimensions. To mention that until now precessional planetary transmissions have been researched and applied mainly in reducers. Therefore it was necessary to carry out theoretical research to determine the geometrical parameters of the precessional gear that operates in multiplier mode. Also, it was necessary to develop new conceptual diagrams of precessional transmissions that function under multiplier regime. The majority of precessional planetary transmissions diagrams developed previously operate efficiently in reducer’s regime. Depending on the structural diagram, precessional transmissions fall into two main types – \(K-H-V\) and \(2K-H\), from which a wide range of constructive solutions with wide kinematical and functional options that operate in multiplier regime.

1. Introduction
The majority of precessional planetary transmissions diagrams developed previously operate efficiently in reducer’s regime [1]. Depending on the structural diagram, precessional transmissions fall into two main types – \(K-H-V\) and \(2K-H\), from which a wide range of constructive solutions with wide kinematical and functional options that operate in multiplier regime. The kinematical diagram of the precessional transmission \(K-H-V\) (figure 1) comprises five basic elements: planet career \(H\), satellite gear \(g\), two central wheels \(b\) with the same number of teeth, controlling mechanism \(W\) and the body (frame). The roller rim of the satellite gear \(g\) gears internally with the sun wheels \(b\), and their teeth generators cross in a point, so-called the centre of precession. The satellite gear \(g\) is mounted on the planet (wheel) career \(H\), designed in the form of a sloped crank, which axis forms some angle with the central wheel axis \(\theta\).
Revolving, the sloped crank \( H \) transmits sphero-spatial motion to the satellite wheel regarding the ball hinge installed in the centre of precession. For the transmission with the controlling mechanism designed as clutch coupling (figure1), the gear ratio (gear reduction rate) varies in the limits:

\[
i_{HV}^{g} = -\frac{z_g \cos \Theta - z_b}{z_b}, \quad i_{HV}^{b} = -\frac{z_g \cos \Theta - z_b}{z_b \cos \Theta},
\]

reaching the extreme values of 4 times for each revolution of the crank \( H \). If necessary, this shortcoming can be eliminated using as a controlling mechanism the constant Cardan joint (Hooke’s joint), the ball synchronous couplings, etc.

\[
i_{HVmed}^{g} = -\frac{z_g - z_b}{z_b}.
\]

For \( z_g = z_b + 1 \), \( i_{HV}^{g} = \frac{1}{z_b} \), the driving and driven shafts have opposite directions.

For \( z_g = z_b - 1 \), \( i_{HV}^{g} = \frac{1}{z_b} \), the shafts revolve in the same direction.
This kinematical diagram of the precessional transmission ensures a range of gear ratios $i = 8...60$, but in the multiplication regime it operates efficiently only for the range of gear ratios $i = 8...25$. As well, in the controlling mechanism $W$, that operates with pitch angles of the semi couplings up to $3^\circ$, power losses occur reducing the efficiency of the multiplier on the whole.

To avoid power losses in the multiplier and to widen the kinematical options (figure 2) [2] the conceptual diagram of the precessional multiplier with wide kinematical options was designed.

The planetary precessional multiplier comprises the following units: the housing 1, inside which the fixed sun wheel 2 is placed and connected rigidly to the housing cover 3, exterior satellite wheel 4 with the teeth in the shape of rollers, movable sun wheel 5, linked rigidly to the input shaft 6. The satellite wheel 3 is connected kinematically with the sloped flange of the disk 7, connected rigidly with the sun wheel 8, that gears with the interior satellite wheel 9 mounted unbound on the output crank shaft 10, and linked rigidly to the rotor generator 11. The exterior satellite wheel 4 is mounted void on gear bodies on exterior spherical surfaces of the interior wheel 9. The pitch angle of the output crank shaft 10 axis and of the sloped flange is equal to $\theta$. The rotational motion of the input shaft 6 is transmitted to the movable sun wheel 5. Due to the difference in the number of teeth of wheel 5 and of the exterior satellite wheel (gear) 4 ($Z_5 = Z_4 \pm 1$), the last will have to carry out a precession motion around the fixed point O (centre of precession). Precessional motion around its axis is excluded as the number of teeth of the sun wheel 2 is equal to the number of rollers of the satellite gear 4 ($Z_2 = Z_4$). The precessional motion of the exterior satellite gear 4 is transformed, by means of an inclined flange of the disc 7, into rotational motion around the disc axis 7 that will revolve by the degree of multiplication:

$$i_7 = -\frac{Z_5}{Z_4 - Z_5}, \quad (3)$$

where $Z_4$ is the number of rollers of the exterior satellite gear 4;
$Z_5$ – is the number of teeth of the movable sun wheel 5.

The multiplied rotational motion of the disc 7 is transformed into multiplied precessional motion of the interior satellite gear 9, due to the difference in the number of teeth $Z_9 = Z_8 \pm 1$. At the rotation of disc 7 of the sun wheel 8 at an angle equal to the teeth angular pitch, the interior satellite gear will perform a complete precession cycle around point „O”. The precessional motion of the interior satellite gear 9 is transformed by means of output crank shaft 10 into rotational motion, multiplied to the degree of multiplication

$$i_{10} = -\frac{Z_9}{Z_9 - Z_8}, \quad (4)$$

where $Z_8$ is the number of teeth of the sun wheel 8,
$Z_9$ – is the number of rollers of the satellite gear 9.

The multiplied rotational motion of the output crank shaft 10 is transmitted to the electrical generator rotor 11.

2. Analytic description of teeth profile and justification of precessional gear parameters selection

Teeth profiles have an important role in the efficient transformation of motion in the precessional transmissions that operate as multiplier. Multiple precessional gear theory, previously developed, did not take into consideration the influence of the diagram error of the linking mechanism in the processing device for gear wheel on the teeth profile.

Functioning under the multiplication regime, these errors have major influence, which can lead to instant blocking of gear and to power losses. With this purpose, a thorough analysis was conducted on the motion development mechanism under multiplication, and on the teeth profile error generating source. On the basis of fundamental theory of multiple precessional gear, previously developed, a new
gear with modified teeth profile and the technology for its industrial manufacturing was proposed and patented [3].

To ensure continuity of the transfer function and to improve the performances of precessional transmission under multiplication it is necessary to modify teeth profile with the diagram error value \( \Delta \phi \) by communicating supplementary motion to the tool. In this case the momentary transmission ratio of the manufactured gear will be constant. Usually, in theoretical mechanics the position of the body making spherical-spatial motion is described by Euler angles. The mobile coordinate system \( OX_1Y_1Z_1 \) is connected rigidly with the satellite wheel, which origin coincides with the centre of precession \( \theta \) (figure 3) and performs spherical-spatial motion together with the satellite wheel relative to the motionless coordinate system \( OX_1Y_1Z_1 \).

![Figure 3. Tooth profile in normal section.](image1)

![Figure 4. Teeth generating surface.](image2)

The elaboration of the mathematic model of the modified teeth profile is based integrally on the mathematic model of teeth profile, previously developed by the authors. With this purpose it is necessary to present the detailed description of teeth profile without modification and, then, to present of the description of modified profile peculiarities.

3. Description of teeth profile designed on sphere.

An arbitrary point \( D \) of the tool axis describes a trajectory relative to the fixed system according to the equations:

\[
X_D^m = -\sin \delta \sin [Y_C^m \sin \theta + Z_C^m (1 - \cos \theta) \cos \psi] ;
Y_D^m = -Y_C^m \cos \delta + Z_C^m \sin \delta \left[ \cos \psi + \cos \theta \sin \psi \right] ;
Z_D^m = -Y_C^m \sin \delta \left( \cos \psi + \cos \theta \sin \psi \right) - Z_C^m \cos \delta .
\]

Index \( m \) means „modified”.

The motion of point \( D^m \) compared to the movable system connected rigidly to the semi product is described by formulas:
The projections of point $D^m$ velocities are expressed by formulas:

\[
\begin{align*}
\dot{X}_{1d}^m &= X_{1d}^m \cos \Psi Z_i - Y_{1d}^m \sin \Psi Z_i; \\
\dot{Y}_{1d}^m &= X_{1d}^m \sin \Psi Z_i + Y_{1d}^m \cos \Psi Z_i; \\
\dot{Z}_{1d}^m &= Z_{1d}^m.
\end{align*}
\] (6)

The projections of point $D^m$ velocities are expressed by formulas:

\[
\begin{align*}
\dot{X}_d^m &= -\sin \delta \cos \Psi \left[ Y_{E}^m \sin \theta + Z_{E}^m \left(1 - \cos \theta \right) \cos \Psi \right] - \sin \Psi \left[ Y_{E}^m \sin \theta + Z_{E}^m \left(1 - \cos \theta \right) \cos \Psi \right] - \sin \Psi \cdot \dot{\psi} \left[ Y_{E}^m \sin \theta + Z_{E}^m \left(1 - \cos \theta \right) \cos \Psi \right]; \\
\dot{Y}_d^m &= -Y_{E}^m \cos \delta + \dot{Z}_{E}^m \sin \delta \left[ \cos \Psi \cos \theta \sin \Psi \dot{\psi} \right] + \\
&+ Z_{E}^m \sin \delta \left[ -2 \cos \Psi \sin \Psi + 2 \cos \theta \sin \Psi \cos \Psi \right] \dot{\psi}; \\
\dot{X}_{1d}^m &= X_{1d}^m \cos \Psi Z_i - \frac{\dot{Y}_{1d}^m \sin \Psi Z_i}{Z_i} - \frac{\dot{Y}_{1d}^m \sin \Psi Z_i - \frac{\dot{Y}_{1d}^m \cos \Psi Z_i}{Z_i} \frac{\dot{Y}_{1d}^m \cos \Psi Z_i}{Z_i}}; \\
\dot{Y}_{1d}^m &= X_{1d}^m \sin \Psi Z_i + \frac{\dot{Y}_{1d}^m \cos \Psi Z_i}{Z_i} + \frac{\dot{Y}_{1d}^m \cos \Psi Z_i - \frac{\dot{Y}_{1d}^m \sin \Psi Z_i}{Z_i}}.
\end{align*}
\] (7)

The coordinates of point $E^m$ on the sphere is calculated by formulas:

\[
\begin{align*}
X_{1E}^m &= k_1^m Z_{1E}^m + d_{1E}^m; \\
Y_{1E}^m &= k_1^m Z_{1E}^m - d_{1E}^m; \\
Z_{1E}^m &= (k_1^m d_{1E}^m - k_{1E}^m d_{1E}^m) - \left(\sqrt{(k_1^m d_{1E}^m - k_{1E}^m d_{1E}^m)^2 + (k_2^m + k_{1E}^m)^2 + 1 \cdot (R_{1E}^m - d_{1E}^m - d_{1E}^m)}ight) k_1^m + k_{1E}^m + 1.
\end{align*}
\] (8)

where:

\[
\begin{align*}
k_1^m &= \frac{X_{1d}^m \left( X_{1d}^m X_{1d}^m + Y_{1d}^m Y_{1d}^m \right) + Y_{1d}^m Y_{1d}^m Y_{1d}^m}{Z_{1d}^m \left( X_{1d}^m Y_{1d}^m - Y_{1d}^m X_{1d}^m \right)}, \quad k_1^m = -\frac{Y_{1d}^m Y_{1d}^m Y_{1d}^m}{X_{1d}^m}; \\
d_{1E}^m &= \frac{R_{1E}^m \cos \beta X_{1d}^m}{X_{1d}^m Y_{1d}^m - X_{1d}^m Y_{1d}^m}, \quad d_{1E}^m = \frac{R_{1E}^m \cos \beta + d_{1E}^m Y_{1d}^m}{X_{1d}^m}.
\end{align*}
\]

According to the obtained analytical relations a soft for the calculation and generation of teeth was developed in CATIA V5R7 modelling system that allowed obtaining the modified trajectories of points $E^m$, and $E^n$, on the spherical front surfaces, both exterior and interior ones, by which the teeth surface was generated (figure 4).

Projection of point $E^m$ on the tooth transversal plane has the following coordinates:

\[
\begin{align*}
X_E^m &= e^m \cdot X_{1E}^m, \quad Y_E^m &= e^m \cdot Y_{1E}^m, \quad Z_E^m &= e^m \cdot Z_{1E}^m,
\end{align*}
\] (9)

where $e^m = \frac{D}{AX_{1E}^m + BY_{1E}^m + CZ_{1E}^m}$.

The modified teeth profile in plane is described by the equations:
\[
\zeta^{x_1} = X^{x_1} \cos \frac{\pi}{Z_1} + \left[ R_x \cos (\delta + \theta + \beta) + Y^{x_1}_E \right] \sin \frac{\pi}{Z_1};
\]
\[
\zeta^{x_2} = X^{x_2} \sin \gamma \sin \frac{\pi}{Z_1} \left[ R_x \cos (\delta + \theta + \beta) + Y^{x_2}_E \right] \sin \gamma \cos \frac{\pi}{Z_1} +
\]
\[
+ \left[ R_x \sin (\delta + \theta + \beta) + Z^{x_2}_E \right] \cos \gamma.
\]

(10)

A wide range of modified teeth profiles with different geometrical parameters were generated in MathCAD 2010 Professional software (figure 5). The solid model of a gear wheel is shown in figure 6.

4. Conclusions

In conclusion, it was elaborated new conceptual diagrams of precessional transmissions that function under multiplier regime. As a result of research and observations it was established the equations of modified teeth profile in plane and were generated a wide range of modified teeth profiles with different geometrical parameters.
5. References

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