Quantum key distribution with asymmetric channel noise

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Abstract

We show that, one may take advantage of the asymmetry of channel noise. With appropriate modifications to the standard protocols, both the key rate and the tolerable total channel noise can be increased if the channel noise is asymmetric.
I. INTRODUCTION

Unlike classical key distribution, quantum key distribution (QKD) is built on the fact that measuring an unknown quantum state will almost surely disturb the state. The first published QKD protocol, proposed in 1984 by Bennett and Brassard, is called BB84. For a history of the subject, one may refer to, for example, Ref. [3]. Since then, studies on QKD are extensive. Strict mathematical proofs for the unconditional security have been given already [4, 8, 9, 10, 13]. It is greatly simplified if one connects this with the quantum entanglement purification protocol (EPP) [4, 11, 12, 13].

To all QKD protocols, the first important requirement is the unconditional security, i.e., using whatever type of attack, eavesdropper (Eve) can never have non-negligible information to the final key shared by Alice and Bob. A conditional secure protocol without a testable condition is not so useful, since it is essentially same to the classical protocol on its security. In particular, one cannot depend the security on certain specific properties of physical channels, since those specific properties may be changed by Eve at the time the QKD protocol is running. For example, based on certain properties of the physical channel, Alice and Bob may take some physical treatment to decrease the error rate of their physical channel. Given such a treatment, suppose the error rate of the improved physical channel is given by the function of $f(E_0)$, and $E_0$ is the error rate of the originally physical channel. Suppose in the QKD protocol Alice and Bob use the improved physical channel. It will be insecure if Alice and Bob in their QKD protocol only test the errors of the original physical and then use function $f$ to calculate the error rate for the improved channel and then use the calculated value as the error rate. This is because, Eve may change the properties of the original physical channel and the mapping $f$ could be incorrect then. However, it will be secure if Alice and Bob directly test the error rate of the improved physical channel and use the tested results as the input of error correction and privacy amplification.

Besides the security requirement, we also hope to improve the feasibility of a protocol in practical use, e.g., the larger tolerable channel error rate and the key rate. Different from the security issue, one can improve the feasibility of a protocol based on the properties of physical channel. Alice and Bob may first investigate certain properties of their physical channel and then modify the protocol according to those properties of their physical channel. Given the specific properties of the physical channel, the modified protocol will have certain
advantage, e.g., a lower error rate in their error test. Note that here they should first be sure that the modified QKD protocol is unconditionally secure even those assumed properties of physical channel don’t exist. An unconditional secure protocol with certain conditional advantage is allowed in QKD. That is to say, after taking the modification, we expect certain advantageous result of the error test. In running the modified protocol, after the error test step, if Alice and Bob find that the result is close to their expected result, the advantage holds; if Alice and Bob find that the result is quite different from the expected one, the advantage is seriously undermined or even totally lost, but the protocol is still secure. In case they find the error test result quite different from the expected, their only loss is in the issue of key rate. Conditional advantage is useful if we have substantial probability that the error test result in QKD is close to the expected one. Although Eve may in principle do whatever to change the original physical properties, if Eve wants to hide her presence, she must not significantly change the error test result of the QKD protocol, since Alice and Bob will judge that there must be an Eve if their error test result is too much different from the expected result. Moreover, the final key rate is only dependent on the error test results rather than the full properties of the channel. If the QKD protocol itself is unconditionally secure, two different channels are equivalent for the QKD purpose provided that they cause the same error test result. This is to say, given an unconditionally secure protocol, if Eve hides her presence, Eve’s action does not affect the error test result done by Alice and Bob, i.e., Eve’s channel is equivalent to the supposed physical channel. The conditional advantage always holds if Eve hides herself. If Eve does not hide herself, she allows her action to cause the error test result much different from the expected one. This may decrease the final key rate, or even destroy the protocol if the error test result is too far away from the expected one. Even in such a case Eve cannot obtain a nonnegligible amount of information to the final key, given an unconditionally secure protocol. In this work, in evaluating the feasibility of our protocol, we only consider an invisible Eve, i.e., Eve always hides her presence. Note that no protocol can work as efficiently as it is expected with a visible Eve. A visible Eve can always destroy any protocol.

We shall propose protocols with higher key rate and larger channel error rate threshold, given an asymmetric physical channel and invisible Eve. The key rate and channel error threshold of our protocol are dependent on the physical channel itself, but the security is independent of the physical channel, i.e., our protocols are unconditionally secure under
whatever attack outside Alice’s and Bob’s labs.

Most of the known prepare-and-measure protocols assume the symmetric channel to estimate the noise threshold for the protocol. Also, most of the protocols use the symmetrization method: Alice randomly chooses bases from certain set to prepare her initial state. All bases in the set is chosen with equal probability. In such a way, even the noise of the channel (Eve’s channel) is not symmetric, the symmetrization make the error rate to the key bits be always symmetric. In this work we show that actually we can let all key bits prepared in a single basis and we can have advantages in key rate or noise threshold provided that the channel noise is asymmetric.

II. CHANNEL ERROR, TESTED ERROR AND KEY-BITS ERROR

Normally, Alice will transmit qubits in different basis (e.g., Z basis and X basis) to Bob, Bob will also measure them in different basis. The Hadamard transformation $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ interchanges the Z-basis $\{|0\rangle, |1\rangle\}$ and X-basis $\{|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)\}$. We shall use the term key bits for those raw bits which are used to distill the final key and the term check bits for those bits whose values are announced publicly to test the error rates. Our purpose is to know the bit-flip rate and phase-flip rate to key-bits. We do it in this way: first test flipping rates of the check-bits, then deduce the channel flip rate and then determine the error rates of key-bits. As it was shown in Ref. 7, the flipping rates of qubits in different bases are in general different, due to the basis transformation. Here we give a more detailed study on this issue. We first consider the 4-state protocol with CSS code, where only two basis, Z-basis and X-basis are involved in operation. For such a case of 4-state protocol, we define asymmetric channel as the channel with its bit flip error rate being different from its phase flip error rate. The check bits will be discarded after the error test. We use the term Z-bits for those qubits which are prepared and measured in Z basis, the term $X$-bits for those bits which are prepared and measured in X basis. For clarity we shall regard Alice’s action of preparing a state in X basis as the joint actions of state preparation in Z basis followed by a Hadamard transform. We shall also regard Bob’s measurement in X basis as the joint action of first taking a Hadamard transform and then taking the measurement in Z basis. Therefore we shall also call those Z-bits as $I$-qubits.
and those $X$-bits as $H$-bits. To let the CSS code work properly, we need to know the value of average bit-flip rate and average phase-flip rate over all key bits. We define three Pauli matrices:

$$
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
$$

The matrix $\sigma_x$ applies a bit flip error but no phase flip error to a qubit, $\sigma_z$ applies a phase flip error but no bit flip error, $\sigma_y$ applies both errors. We assume the $\sigma_x, \sigma_y, \sigma_z$ rates of the physical channel are $q_{x0}, q_{y0}, q_{z0}$ respectively. Note that the phase flip rate or bit flip rate of the channel is the summation of $q_{z0}, q_{y0}$ or $q_{x0}, q_{y0}$, respectively. Explicitly we have

$$
\begin{align*}
p_{x0} &= q_{x0} + q_{y0} \\
p_{z0} &= q_{z0} + q_{y0} \\
p_{y0} &= q_{x0} + q_{z0}
\end{align*}
$$

and $q_{y0}$ is defined as the channel flipping rate to the qubits prepared in $Y$-basis ($\frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle))$. After the error test on check bits, Alice and Bob know the value of $p_{x0}, p_{z0}$ of the channel. Given the channel flipping rates $q_{x0}, q_{y0}, q_{z0}$, one can calculate the bit-flip rate and phase-flip rate for the remained qubits. For those $I$-bits, the bit-flip rate and phase-flip rate are just $q_{x0} + q_{y0}$ and $q_{z0} + q_{y0}$ respectively, which are just the channel bit flip rate and phase flip rate. Therefore the channel bit-flip rate $p_{x0}$ is identical to the tested flip rate of $I$-bits. The channel phase-flip rate $p_{z0}$ can be determined by testing the flip rate of those $H$-bits. An $H$-qubit is a qubit treated in the following order

prepared in $Z$ basis, Hadamard transform, transmitted over the noisy channel, Hadamard transform, measurement in $Z$ basis. If the channel noise offers a $\sigma_y$ error, the net effect is

$$
H\sigma_y H \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \sigma_y \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
$$

This shows, the channel $\sigma_y$ error will also cause a $\sigma_y$ error to an $H$-qubit. Similarly, due to the fact of

$$
\begin{align*}
H\sigma_x H &= \sigma_x \\
H\sigma_z H &= \sigma_z,
\end{align*}
$$

a channel $\sigma_x$ flip or a channel $\sigma_z$ flip will cause a net $\sigma_z$ error or $\sigma_x$ error, to an $H$-bit. Consequently, a channel phase flip causes a bit flip error to H-bit, a channel bit-flip causes
a phase flip error to H-bit. This is to say, *the measured error of H-bits is just the channel phase flipping rate.* Therefore the average bit-flip error rate and phase-flip error rate to each types of key bits will be

\[ p^I_z = p^H_x = p_{z0}, \]
\[ p^H_z = p^I_x = p_{x0} \]  \hspace{1cm} (5)

Here \( p^H_x \), \( p^I_z \) (\( p^H_z \), \( p^I_x \)) are for the bit flip (phase flip) error of H-bits and I-bits from those key-bits respectively. Suppose the key bits consist of \( \eta \) I-bits and \( 1 - \eta \) H-bits, the average flip error of the key bits is

\[ p_x = \eta p^I_x + (1 - \eta)p^H_x = \eta p_{x0} + (1 - \eta)p_{z0}; \]
\[ p_z = \eta p^I_z + (1 - \eta)p^H_z = \eta p_{z0} + (1 - \eta)p_{x0}. \]  \hspace{1cm} (6)

### III. KEY RATE OF QKD PROTOCOLS WITH ONE WAY COMMUNICATION.

We first consider an almost trivial application of our analysis of asymmetric channel above. In the standard BB84 protocol, since the preparation basis of key bits are symmetrized, the average bit flip error and phase flip error to those key bits are always equal no matter whether the channel noise itself is symmetric or not. That is to say, when half of the key-bits are X-bits and Half of them are Z-bits, the average flip rates over all key bits are always

\[ p_x = p_z = (p_{x0} + p_{z0})/2. \]  \hspace{1cm} (7)

Therefore the key rate for the standard BB84 protocol (Shor-Preskill protocol) [13] with whatever asymmetric channel is \( 1 - 2H(\frac{p_{x0} + p_{z0}}{2}) \) [13], where \( H(t) = -(t \log_2 t + (1-t) \log_2 (1-t)) \). (Note that in the 4-state protocol, asymmetric channel is simply defined by \( p_{x0} \neq p_{z0} \).) However, if all key bits had been prepared and measured in Z basis, then the bit flip and phase flip rates to key bits would be equal to those flipping values of the channel itself. In such a case the key rate is

\[ R = 1 - H(p_{x0}) - H(p_{z0}). \]  \hspace{1cm} (8)

Obviously, this is, except for the special case of \( p_{x0} = p_{z0} \), always larger than the key rate in standard BB84 protocol with CSS code (Shor-Preskill protocol), where the key-bits are
prepared in Z-basis and X-basis with equal probability. For a higher key rate, one should always use the above modified BB84 protocol with all key bits prepared and measured in a single basis, Z-basis.

In fact, this type of 4-state QKD protocol with one single basis for key bits had been proposed already in the past for different purposes, see e.g., ref [17].

Now we consider the case of 6-state protocol. In the standard protocol [5], the key bits are equally distributed over all 3 different bases. In distilling the final key, each type of flipping error used is the averaged value over 3 bases, i.e.,

\[
\bar{q}_x = \frac{q_{x0} + 2q_{z0}}{3}, \bar{q}_z = \frac{q_{x0} + 2q_{z0} + q_{y0}}{3}, \bar{q}_y = \frac{q_{x0} + 2q_{y0}}{3}.
\]

(9)

The key rate is

\[
r = 1 - H(\bar{q})
\]

\[
H(\bar{q}) = -\bar{q}_x \log_2 \bar{q}_x - \bar{q}_y \log_2 \bar{q}_y - \bar{q}_z \log_2 \bar{q}_z - q_{I0} \log_2 q_{I0}.
\]

(10)

This is the key rate for standard 6-state protocol where we mix all qubits in different bases together. However, such a mixing is unnecessary. We can choose to simply distill 3 batches of final keys from Z-bits, X-bits and Y-bits separately. If we do it in such a way, the key rate will be increased to

\[
r' = 1 - q_{x0} \log_2 q_{x0} - q_{y0} \log_2 q_{y0} - q_{z0} \log_2 q_{z0} - q_{I0} \log_2 q_{I0}.
\]

(11)

Obviously, \(r'\) is never less than \(r\) since the mixing operation never decreases the entropy. The only case where \(r' = r\) is \(q_{x0} = q_{y0} = q_{z0}\). Therefore for a higher key rate, we propose to always distill 3 batches of final key separately. The advantage in such a case is unconditional, there is no loss for whatever channel. Now we start to consider something more subtle: the advantage conditional on the prior information of the asymmetry property of noise of the physical channel.

IV. 6-STATE PROTOCOL WITH 2-WAY CLASSICAL COMMUNICATIONS.

The symmetric channel noise for a 6-state protocol is defined by \(q_{x0} = q_{y0} = q_{z0}\), if this condition is broken, we regard it as an asymmetric channel for 6-state protocols. In the standard 6-state protocol [5, 7, 16], symmetrization is used, i.e., the key-bits are equally
consisted by $X-, Y-, Z-$bits. When the channel noise itself is symmetric, i.e., $q_{x0} = q_{y0} = q_{z0}$, a 6-state protocol can have a higher noise threshold than that of a 4-state protocol. This is because in the 6-state protocol, the $\sigma_y$ type of channel error rate is also detected. In removing the bit flip error, $\sigma_y$ error is also reduced, therefore the phase-flip error is partially removed. However, in a 4-state protocol, $\sigma_y$ error is never tested therefore we have to assume the worst situation that $q_{y0} = 0$.

We shall show that one can have a higher tolerable channel errors if one modify the existing protocols, given the asymmetric channel (i.e., the channel with its $Y$-bits flipping error being different from that of $X$-bits or $Z$-bits.) For example, in the case that $q_{y0} = 0$ and $q_{x0} = q_{z0}$. The different types of error rates to the transmitted qubits are

$$ q_x = q_{x0}, q_y = 0, q_z = q_{z0} $$

for those $Z-$bits;

$$ q_x = q_{x0}, q_y = 0, q_z = q_{z0} $$

for those $X-$bits and

$$ q_x = q_{x0}, q_y = q_{z0}, q_z = 0 $$

for $Y-$bits. The average error rates over all transmitted bits are:

$$ \bar{q}_x = q, \bar{q}_y = q/3, \bar{q}_z = 2q/3, q = q_{x0} = q_{z0}. \quad (12) $$

With such a fact, the threshold of total channel noise $q_{t0} = (q_{x0} + q_{y0} + q_{z0})$ for the protocol is 41.4%, same with the case with symmetric noise. Actually, by our numerical calculation we find that the threshold of total channel noise for Chau protocol is almost unchanged with whatever value of $q_{y0}$. However, if all key bits were prepared in $Y-$basis (the basis of $\{|y\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle)$), there would be no $\sigma_z$ type of error therefore one only needs to correct the bit-flip error. To see this we can regard a $Y-$qubit as a qubit treated in the following order

*prepared in $Z$ basis, $T$ transform, transmitted over the noisy channel, $T^{-1}$ transform, measurement in $Z$ basis.*

Here $T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$, it changes states $|0, 1\rangle$ into $|y\pm\rangle$. The following facts

$$ T\sigma_x T^{-1} = \sigma_y, T\sigma_y T^{-1} = \sigma_z, T\sigma_z T^{-1} = \sigma_x $$

(13)
cause the consequence that a channel flip of the type $\sigma_x, \sigma_y, \sigma_z$ will cause an error to $Y$-bits in the type of $\sigma_y, \sigma_z, \sigma_x$ respectively. With this fact, if the channel error of $p_{y0}$ is 0, the $\sigma_z$ type of error to the $Y$-qubit is also 0. Using the iteration formula given in Ref. 16, once all bit-flip error is removed, all errors are removed. Therefore the error rate threshold is

$$Q_x = Q_z = 25\%,$$

(14)
i.e., a total error rate of 50%. In practice, it is not likely that $\sigma_y$ type of channel flip is exactly 0. Numerical calculation (Fig. 1) shows that, using $Y$-basis as the only basis for all key-bits always has an advantage provided that the channel flipping rate satisfies $q_{y0} < q_{x0} = q_{z0}$.

For the purpose of improving the noise threshold, we propose the following protocol:

1: Alice creates a random binary string $b$ with $(6 + \delta)n$ bits.

2: Alice generates $(6 + \delta)n$ quantum state according to each elements of $b$. For each bit in $b$, if it is 0, she produces a quantum state of either $|0\rangle$, or $|+\rangle$ or $|y+\rangle$ with probability $1/4, 1/4, 1/2$, respectively; if it is 1, she creates a quantum state $|1\rangle$, or $|-\rangle$ or $|y-\rangle$ with probability $1/4, 1/4, 1/2$, respectively.

3: Alice sends all qubits to Bob.

4: Bob receives the $(6 + \delta)n$ qubits, measuring each one in basis of either $\{|0\rangle, |1\rangle\}$ or $\{|\pm\rangle\}$ or $\{|y\pm\rangle\}$ randomly, with equal probability.

5: Alice announces basis information of each qubits.

6: Bob discards all those qubits he measured in a wrong basis. With high probability, there are at least $2n$ bits left (if not, abort the protocol). Alice randomly chooses $n$ $Y$-bits to use for the distillation of final key, and use the remained $n$ bits as check bits. (Among all the check bits, approximately $n/3$ are $X$-bits, $n/3$ are $Z$-bits, and $n/3$ are $Y$-bits.)

7: Alice and Bob announce the values of their check bits. If too many of them are different, they abort the protocol.

8: Alice randomly group the key bits with each group consisting 2 bits. Alice and Bob compare the parity values on each side to each group. If the values agree, they discard one bit and keep the other one. If the value disagree, they discard both bits of that group. They repeatedly do this for a number of rounds until they believe they can find a certain integer $k$ so that both bit-flip error and phase-flip error are less than 5% after the following step is done.
They randomly group the remained key bits with each group consisting $k$ bits. They use the parity value of each group as the new bits after this step.

Alice and Bob use classical CSS code to distill the final key.

Remark 1: Step 9 is to remove the phase flip error of the final key. Although in the extreme case that the phase flip error rate is always 0 if initially the $\sigma_y$ type of error is 0, however, in practice the initial $\sigma_y$ type of error rate is not exactly 0. Even in the case the tested error rate on check bits is zero, we still have to assume a small error rate on the key bits to increase the confidence level.

Remark 2: The above protocol is unconditionally secure. This means, under whatever type of intercept-and-resend attack, Eve’s information to the final key is exponentially small. The security proof can be done through the purification and reduction procedure given by Ref. [13]. The only thing that is a bit different here is that Alice and Bob will take measurement in $Y$ basis to make the final key, after the distillation.

Remark 3: We don’t recommend to use the above protocol blindly. Before doing the QKD, the users should test their physical channel and decide whether there is an advantage. Numerical calculation shows that, In the case of $q_{x0} = q_{z0}, q_{y0} < q_{x0}$, our protocol always has a higher error rate threshold than the corresponding 6-state protocol with key bits’ bases equally distributed in all 3 bases. This fact is shown in Fig.(1).

Remark 4: The conditional advantage require the users first test the properties of the physical channel before doing QKD. And we assume that the physical channel itself is stable. Note that we don’t require anything for Eve’s channel. As we have discussed in the beginning, physical channel is in general different from Eve’s channel since Eve may take over the whole channel only at the time Alice and Bob do the QKD. However, if Eve wants to hide her presence, she must respect the expected results of the error test in the protocol. In our protocol, Eve’s operation must not change the error rates of the physical channel, though she can change the error pattern. Since if these values are changed, Alice and Bob will find that their error test result is much different from the expected one therefore Eve cannot hide her presence.

One may still worry whether the conditional advantage in our protocol is really useful, especially for the conditional threshold part. To make it clear, we consider a specific game. Suppose now Alice and Bob are prisoned in two separate places. They are offered a chance to be freed immediately. The rule is set as this: If they can make an unconditionally secure
final key, they will be freed immediately. If any third party obtained a non-negligible amount of information of the final key, they will be shot immediately. Suppose both Alice and Bob want to be freed immediately, but they take the value that being alive is more important than freedom. The noise of the physical is known: the $p_y = 0; p_x = p_z = 22\%$. In such a case, those conditionally secure protocol with untestable conditions cannot be used, since those protocols will bring the risk to Alice and Bob of being shot. For example, protocol $T$ is conditionally secure with individual attack, but we don’t know how to see whether Eve has only used the individual attack. Even though $T$ has a very large tolerable channel noise, Alice and Bob cannot use this protocol because they have a risk to be shot immediately. Previously known unconditionally secure protocols will not bring the risk of being shot to Alice and Bob, but those protocols cannot bring liberty to Alice and Bob, since none of them can tolerate such a high channel flipping error. However, the protocol in this work can help Alice and Bob to be freed without any non-negligible risk of being shot. In such a case, our protocol is the only protocol that may help Alice and Bob while all previously known unconditionally secure protocols cannot.

Besides the advantage of a higher tolerable error rate, there are also advantages in the key rate of our protocol with asymmetric channel noise. Obviously, when the error rate is higher than other protocols’ threshold while lower than our protocol’s threshold, our protocol always has an advantage in key rate. More interestingly, even in the case that the error rate is significantly lower than the threshold of Shor-Preskill’s protocol, we may modify our protocol and the advantage in key rate may still holds. We modify our protocol in such a way: take one round bit-error-rejection with two way communication and then use CSS code to distill the final key. As it was shown in Ref. [16], the various flipping rates will
change by the following formulas after the bit-flip-error rejection

\[
\begin{align*}
q_I &= \frac{p_{I0}^2 + p_{z0}^2}{(q_{I0} + q_{z0})^2 + (q_{x0} + q_{y0})^2}, \\
q_x &= \frac{q_{x0}^2 + q_{y0}^2}{(q_{I0} + q_{z0})^2 + (p_{x0} + p_{y0})^2}, \\
q_y &= \frac{2q_{x0}q_{y0}}{(q_{I0} + q_{z0})^2 + (q_{x0} + q_{y0})^2}, \\
q_z &= \frac{2q_{I0}q_{z0}}{(q_{I0} + q_{z0})^2 + (q_{x0} + p_{y0})^2}.
\end{align*}
\] (15)

Also, it can be shown that the number of remained pairs is

\[ f = \frac{1}{2} \frac{1}{(q_{I0} + q_{z0})^2 + (q_{x0} + q_{y0})^2}. \] (16)

The key rate of our protocol is given by

\[ R = f \cdot (1 + q_x \log_2 q_x + q_y \log_2 q_y + q_z \log_2 q_z + q_t \log_2 q_t) \] (17)

We shall compare this with the key rate of our modified six-state protocol where key bits are equally distributed over 3 different bases but we distill 3 batches of final key, i.e.

\[ r' = 1 + q_{x0} \log_2 q_{x0} + q_{y0} \log_2 q_{y0} + q_{z0} \log_2 q_{z0} + q_{I0} \log_2 q_{I0}. \] (18)

We need not compare our results with the standard 6-state protocol since its key rate given by eq.(10) is superemed by eq.(11). The numerical results are given in Fig.(2).

V. SUMMARY

In summary, we have shown that, given the asymmetric channel flip rate one can have advantages in tolerable flip rates and efficiency, if one uses a single basis for the key-bits. We have demonstrated this point by both 4-state protocol with CSS-code and the 6-state protocol with 2-way communication. It should be interesting to investigate the most general case that \( p_{x0}, p_{y0}, p_{z0} \) are all different for the case of 6-state protocol.

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FIG. 1: Comparison of channel error threshold of different protocols. All values are in the unit of one percent. $Q_{t0}$ is the threshold value of total channel noise given certain value of $q_{y0}$. In calculating $Q_{t0}$, we assume $q_{x0} = q_{z0}$. The dashed line is the threshold value of Chau protocol, the circled curve is the threshold of the six state protocol given in this work.

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FIG. 2: Comparison of the key rate of our protocol (the solid line) and six-state Shor-Preskill protocol (the dashed line). The Y-axis is for the key rate, X axis is $q_x + q_y + q_z$; A, B, C, D are for the cases of $q_y = 0, 0.5\%, 1\%, 2\%$, respectively. In all cases we have assumed $q_x = q_y$. We can see that, if $q_y$ is small, our protocol can have a higher efficiency than six-state Shor-Preskill protocol even the total error rate is significantly lower than the threshold value of the six-state Shor-Preskill protocol.

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