Isospin Squeezed States, Disoriented Chiral condensates and Pion Production: A Dynamic Group Theoretical Approach

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We make a complete dynamical study of isotopic spin conservation effects on the multiplicity distributions of both hard and soft pions emitted in a quark gluon plasma undergoing a non-equilibrium phase transition.

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I. INTRODUCTION

Quantum optical analogies have long been useful tools in describing multipion production in hadronic collisions [1, 2, 3, 4, 5]. Quantities such as pion multiplicity distributions and correlations among identical and charged pions have been successfully explained on a phenomenological level by exploiting the similarity between light and bosonic particles such as pions. This has been very successful for systematizing data in high energy collisions since the "pre-QCD" days, when the concept of coherent pion production was introduced [6, 7, 8]. Since then, many distributions which have their origin in photon-counting experiments have found an application in particle physics [5]. In particular the negative binomial distribution [2], the Perina-McGill distribution [3], and the squeezed coherent distribution [4, 16] have proved very useful in various high energy applications. Recently these quantum optical states have been revived as possible states for the disoriented chiral condensate, which is conjectured to occur in relativistic heavy ion collisions [11, 12, 13].

The distinguishing factors between optical and pionic physics are the conservation laws and final state interactions between pions [14]. Thus, in translating the quantum optical analogy to pion production, care has to be taken to preserve the conservation laws such as isospin, charge etc. obeyed by the underlying interactions [17, 18, 19]. This is done by imposing constraints which exploit the group theoretic structure of the symmetries and involve projection operator techniques [20]. In particular for strong interactions where isospin is conserved, coherent states of pions are obtained by globally projecting out a particular isospin state from a general number operator coherent state. Such a global projection assumes that all pions are assigned the same momentum function and thus isospin dependence of event by event quantities like momentum dependence of pion correlations (HBT effect) and back-to-back particle-antiparticle correlations (PAC) cannot be described in this approach. One has to do a full quantum field theoretical treatment in momentum space and find momentum dependent isospin variables to give a full event by event description of the isospin coherent state.

Recently a number of papers in literature have addressed the question of isospin conservation in the context of pion production associated with the disoriented chiral condensate [20, 21, 22, 30]. These involve the construction of the isospin squeezed states. However, these papers do not make any proposal for the dynamical origin for the anomalous production of particles in relation to the squeezed states. This has motivated us to look deeper into the question of whether isospin squeezed states can be dynamically generated in models for pion production in certain high energy processes. Furthermore, only the isospin structure at zero relative momentum of pions has been investigated [21]. Our approach is the most general one which has the results of these papers as special cases, and is more suitable for...
the dynamical origin of pion distributions seen in physical processes. It can also be easily generalized to include disoriented chiral condensates, so that isospin invariance in DCC production does not have to be compromised.

II. THE PION HAMILTONIAN

The foundations for the model used are laid in our previous paper [23], hitherto referred to as I. In the next two sections, to make this paper complete and also to relieve the reader from the arduous task of continually referring to I, we outline the salient features and results derived in I. Since this is the basis for our subsequent discussion and results, the repetition is not redundant but useful.

In I, we used a background field analysis of the O(4) sigma model with symmetry breaking keeping one-loop quantum corrections to construct the most general Hamiltonian for the evolution, formation, decay of the DCC in an external, expanding metric.

The complete action for the pion-sigma part of the O(4) sigma model which preserves chiral symmetry is given by

$$S = \int d^4x \left[ \frac{1}{2} \left( \partial_\mu \pi^a \partial^\mu \pi^a + \partial_\mu \Sigma \partial^\mu \Sigma \right) - \frac{\lambda}{4} \left( \pi^a \pi_a + \Sigma^2 - f_\pi^2 \right)^2 \right]$$

(1)

where, the chiral field is

$$\Phi(r, t) = \Sigma(r, t) + i\vec{\pi}(r, t).$$

(2)

The true vacuum is \( \langle \Phi \rangle = \langle \Sigma \rangle + \langle \pi \rangle = f_\pi^2 \) and is taken traditionally as \( \langle \Sigma \rangle = f_\pi \) and \( \langle \pi \rangle = 0 \). The choice of this vacuum implies that chiral symmetry is spontaneously broken. Note that the fermionic part of the original sigma model Lagrangian has been neglected here. This is allowed since our focus is on the condensate formed in the symmetry broken phase where quark degrees of freedom are already confined. The explicit breaking of symmetry is implemented in the Lagrangian to give the pions mass through the presence of a term linear in the sigma field. The action becomes

$$S = \int d^4x \left[ \frac{1}{2} \left( \partial_\mu \phi^a \partial^\mu \phi_a - \frac{\lambda}{4} (\phi^a \phi_a - v^2)^2 + \epsilon \Sigma \right) \right].$$

(3)

where \( f_\pi \) is replaced by a general parameter \( v \), which in the limit \( \epsilon \to 0 \) goes to \( f_\pi \). Requiring the minimum to be still \( (0, f_\pi) \), to leading order we have

$$v^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda}$$

(4)

and the pion and sigma masses are \( m_\pi^2 = \frac{\epsilon}{f_\pi} \) and \( m_\sigma^2 = 2\lambda f_\pi^2 + m_\pi^2 \).

\( f_\pi = 92MeV \) is the pion decay constant and the meson masses are \( m_\pi = 138MeV/c^2 \) and \( m_\sigma = 600MeV/c^2 \).

Although, to give the pions mass, we have explicitly broken the symmetry, if the explicit symmetry breaking is very small compared to the relevant scale of QCD then it is still a good approximation to apply the notion of spontaneously broken symmetry. The symmetry breaking term tilts the double well potential given in (3). As long as the potential is tilted only slightly the pions are much softer than the sigma so the effect due to the spontaneous symmetry breakdown of chiral symmetry dominates the dynamics if \( \epsilon \) is small.

The formation of the DCC takes place when, in a rapidly cooling and expanding plasma, the expansion rate of plasma is greater than the rate at which the field evolves from a state of restored symmetry to an equilibrium state of broken symmetry [12,13]. This evolution of the field can be implemented by making the expectation value of \( \Phi \) a function of time. Such a study has been carried out in reference [23] in great detail. An additional feature in the study of the formation of the DCC is that of the expansion of the plasma - two scenarios exist, the first being that of a sudden quench, where \( \langle \Phi \rangle \) changes from 0 to \( f_\pi \) instantaneously and the second in which the system goes through a meta-stable, disordered vacuum typically represented by:

$$\Sigma = f_\pi Cos(\theta); \quad \pi = f_\pi Sin(\theta).$$

(5)

The system then relaxes by quantum fluctuations to an equilibrium configuration. Here \( \theta \)
measures the degree of disorientation of the condensate.

Some of the traditional signals of the DCC are the enhancement of low momentum pion modes \([13]\) and the anomalous charged to pion ratio of the soft pions \([15]\). The study of the DCC requires the further analysis of the pions produced by the decaying plasma and therefore it is imperative that the isospin structure, which is such an integral part of pion studies, has also to be incorporated. Studies carried out thus far have concentrated on classical isospin structure. We look at the isospin structure at the quantum level through the construction of an effective mean field Hamiltonian which allows us to examine the dynamical effects of isospin conservation. This provides us with a framework for examining the effects of isospin conservation on the pion multiplicities.

In \([23]\) we have studied the effects of an \(so(4)\) sigma model with spontaneous symmetry breaking in a spherically symmetric and homogeneously expanding plasma. The line element is the FRW metric:

\[
ds^2 = dt^2 - a(t)^2 d\vec{x}^2, \tag{6}\]

where \(a(t)\) is the expansion parameter. We treated the quantum field as a fluctuation around the general classical background field parameterized by three angles:

\[
< \Phi > = \begin{pmatrix} f_\pi \cos(\rho) \sin(\theta) \sin(\alpha) \\
 f_\pi (\rho) \sin(\theta) \cos(\alpha) \\
 f_\pi \sin(\rho) \sin(\theta) \\
 f_\pi \cos(\theta) \end{pmatrix} = \begin{pmatrix} v_+ \\
v_- \\
v_3 \\
\sigma \end{pmatrix} \tag{7}\]

where \(v_+, v_-\) are the vacuum expectation values of the charged pions and \(v_3\) of the neutral pion. Without DCC formation these would be zero. In order to consider all the special cases that are possible in a transparent way, we simplified the parameterizations of the possible form for the background field to two angles, \(\theta\) and \(\rho\) by letting \(\alpha = \frac{\pi}{4}\). Then, \(v_\pm = \frac{f_\pi}{\sqrt{2}} \cos(\rho) \sin(\theta)\)

\(v_3 = f_\pi \sin(\rho) \sin(\theta)\), and \(\sigma = f_\pi \cos(\theta)\).

The quantum Hamiltonian was derived using a Fourier mode decomposition of the fields and momenta through the definitions

\[
\pi_0(x, t) = \int \sqrt{\frac{1}{2\omega_\pi}} \frac{d^3k}{(2\pi)^3} \{a_\pm e^{ik\cdot x} + a_\pm^* e^{-ik\cdot x}\}
\]

\[
\pi_-(x, t) = \int \sqrt{\frac{1}{2\omega_\pi}} \frac{d^3k}{(2\pi)^3} \{b_\pm e^{ik\cdot x} + c_\pm^* e^{-ik\cdot x}\}
\]

\[
\pi_+(x, t) = \int \sqrt{\frac{1}{2\omega_\pi}} \frac{d^3k}{(2\pi)^3} \{c_\pm e^{ik\cdot x} + b_\pm^* e^{-ik\cdot x}\}
\]

\[
\Sigma(x, t) = \int \sqrt{\frac{1}{2\omega_\Sigma}} \frac{d^3k}{(2\pi)^3} \{d_\pm e^{ik\cdot x} + d_\pm^* e^{-ik\cdot x}\}
\]

where

\[
\frac{\omega_\pi^2(k)}{a^6} = \frac{\omega_{\pi0}^2(k)}{a^6} = \frac{\omega_{\pi\pm}^2(k)}{a^6} = (m_\pi^2 + \frac{k^2}{a^2}) \tag{9}\]

\[
\frac{\omega_\Sigma^2(k)}{a^6} = (m_\Sigma^2 + \frac{k^2}{a^2})
\]
and we also define

\[
\begin{align*}
\frac{\Omega_2^{\pi_+} - \omega_2^{\pi_+}}{a^6} &= \lambda [(< \Phi >^2 - v^2) + 2v_3^2] \\
\frac{\Omega_2^{\pi_-} - \omega_2^{\pi_+}}{a^6} &= \lambda [(< \Phi >^2 - v^2) + 2v_+v_-] \\
\frac{\Omega_2^{\sigma} - \omega_2^{\sigma}}{a^6} &= \lambda [(< \Phi >^2 - v^2) + 2\sigma^2]
\end{align*}
\]

and

\[2v_+v_- + v_3^2 + \sigma^2 = f^2_\pi.\]  

For the rest of the section, for brevity, we drop the k,t dependence of the \(\Omega\)'s and \(\omega\)'s. The Hamiltonian that describes the quantum evolution of a DCC where the corresponding classical condensate can be in any direction in isospin space is:

\[H = H_{\text{neutral}} + H_{\text{charged}} + H_{\text{mixed}}\]  

where,

\[H_{\text{neutral}} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \left( \frac{\omega_\pi}{a^3} (a_d^\dagger a_k + a_k a_d^\dagger) \right.\]
\[+ \frac{\omega_\pi}{a^3} \left( \frac{\Omega_\pi^2 - \omega_\pi^2}{\omega_\pi^2} - 1 \right) (a_d^\dagger a_k + a_k a_d^\dagger) + a_{-k} a_k + a_{-k}^\dagger a_k^\dagger) + \frac{\omega_{\Sigma}}{a^3} (d_d^\dagger d_k + d_k d_d^\dagger)\]
\[+ \frac{\omega_{\Sigma}}{a^3} \left( \frac{\Omega_{\Sigma}^2 - \omega_{\Sigma}^2}{\omega_{\Sigma}^2} - 1 \right) (d_d^\dagger d_k + d_k d_d^\dagger + d_{-k} d_k^\dagger + d_k^\dagger d_{-k}) \}

\[H_{\text{charged}} = \int \frac{d^3k}{(2\pi)^3} \left( \frac{\omega_\pi}{a^3} (b_d^\dagger b_k + c_k c_d^\dagger) + \frac{\omega_\pi}{a^3} \left( \frac{\Omega_{\pi_*}^2 - \omega_{\pi_*}^2}{\omega_{\pi_*}^2} - 1 \right) (b_d^\dagger b_k + c_k c_d^\dagger + b_{-k} c_k + c_{-k}^\dagger b_k) \right)\]

\[H_{\text{mixed}} = \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\lambda a^3 f_\omega^2 \cos^2(\rho) \sin^2(\theta)}{4\omega_\pi} \right\} (b_k b_{-k} + b_{-k} b_k^\dagger)
\[+ c_k^\dagger b_k + c_k c_{-k} + c_{-k}^\dagger c_{-k} + c_k b_k^\dagger + b_k^\dagger c_k + b_{-k}^\dagger b_{-k}) + \frac{\lambda a^3 f_\omega^2 \cos(\rho) \sin(\theta) \cos(\theta)}{\sqrt{\omega_\pi \omega_{\Sigma}}} \] \[\left( b_k a_{-k} + b_{-k} a_k^\dagger + c_k a_k + c_{-k} a_{-k} + c_{-k}^\dagger a_{-k} + c_k a_k^\dagger + b_{-k}^\dagger a_k + b_k^\dagger a_{-k} \right) + \frac{\lambda a^3 f_\omega^2 \sin(\rho) \sin(\theta) \cos(\theta)}{\sqrt{\omega_\pi \omega_{\Sigma}}} \]
\[\left( d_k a_{-k} + d_{-k} a_k^\dagger + d_k a_k^\dagger + d_{-k} a_{-k}^\dagger \right) + \frac{\lambda a^3 f_\omega^2 \cos(\rho) \sin(\theta) \cos(\theta)}{\sqrt{\omega_\pi \omega_{\Sigma}}} \]
\[\left( b_k d_{-k} + b_{-k} d_k^\dagger + c_k d_k^\dagger + c_{-k} d_{-k} + c_k d_k^\dagger + c_{-k} d_{-k}^\dagger \right) \}\]
In order to show the dynamical origin of squeezed isospin states we consider the simplest case \( \theta = 0 \). In subsequent communications the other cases will be considered. \( \theta = 0 \) implies the symmetry breaking takes place in the \( \Sigma \) direction and the mixed term Hamiltonian, \( H_{\text{mixed}} \), vanishes. The general problem of evolution of the quantum state of the DCC with arbitrary orientation in isospin space will be considered later [24]. The total Hamiltonian, \( H \) for this special case reduces to

\[
H = \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\omega_\pi}{2a^3}(a_k^\dagger a_k + a_k a_k^\dagger) + \frac{\omega_\pi}{4a^3} \left( \frac{\Omega_\pi^2}{\omega_\pi} - 1 \right)(a_k^\dagger a_k + a_k a_k^\dagger + a_{-k} a_k^\dagger) \right. \\
+ \frac{\omega_\Sigma}{2a^3} (d_k^\dagger d_k + d_k d_k^\dagger) \\
+ \frac{\omega_\Sigma}{4a^3} \left( \frac{\Omega_\Sigma^2}{\omega_\Sigma} - 1 \right)(d_k^\dagger d_k + d_k d_k^\dagger + d_{-k}^\dagger d_k^\dagger + d_k^\dagger d_{-k}^\dagger) \\
+ \left\{ \frac{\omega_\pi}{a^3} (b_k^\dagger b_k + c_k c_k^\dagger) + \frac{\omega_\pi}{2a^3} \left( \frac{\Omega_\pi^2}{\omega_\pi} - 1 \right)(b_k^\dagger b_k + c_k c_k^\dagger + b_{-k} c_k + c_{-k}^\dagger b_k^\dagger) \right\} \right. \\
\]

We notice that \( H \) has the form of a decoupled Hamiltonian. This is easy to understand from the \( S_0(4) \) parent. The \( S_0(4) \) vector has been decomposed into four fields: \( \pi_\pm, \pi_0 \) and \( \Sigma \) being respectively the charged pions, the neutral pions and the sigma fields. \( H \) has the characteristic quadratic \( su(1,1) \) structure that is associated with Hamiltonians that are diagonalised by a squeezing transformation, which was explicitly done in 1.

The parameters of the squeezing transformation in the pion sector are

\[
\mu = \text{Cosh}(r_\pi) = \sqrt{\frac{1}{2} \left[ \left( \frac{\Omega_\pi}{\omega_\pi} + \frac{\omega_\pi}{\Omega_\pi} \right) + 1 \right]}, \\
\nu = \text{Sinh}(r_\pi) = \sqrt{\frac{1}{2} \left[ \left( \frac{\Omega_\pi}{\omega_\pi} + \frac{\omega_\pi}{\Omega_\pi} \right) - 1 \right]},
\]

and in the \( \sigma \) sector are:

\[
\rho = \text{Cosh}(r_\Sigma) = \sqrt{\frac{1}{2} \left[ \left( \frac{\Omega_\Sigma}{\omega_\Sigma} + \frac{\omega_\Sigma}{\Omega_\Sigma} \right) + 1 \right]}, \\
\sigma = \text{Sinh}(r_\Sigma) = \sqrt{\frac{1}{2} \left[ \left( \frac{\Omega_\Sigma}{\omega_\Sigma} + \frac{\omega_\Sigma}{\Omega_\Sigma} \right) - 1 \right]}.
\]

\( r_\pi \) and \( r_\Sigma \) are the squeezing parameters, and \( \mu^2 - \nu^2 = 1 \) and \( \rho^2 - \sigma^2 = 1 \).

Since the sigma field decouples in this particular Hamiltonian, it can be analyzed inde-
pendently of the pion fields. The diagonalized Hamiltonian can be written as:

\[ H = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2a^3} \Omega_\pi \{ (A_k^\dagger A_k + \frac{1}{2}) + (C_k^\dagger C_k + B_k^\dagger B_k + 1) \} + \Omega_\Sigma (D_k^\dagger D_k + \frac{1}{2}) \}. \tag{20} \]

where

\[ A_k(t, r) = \mu(r, t) a_k + \nu(r, t) a_{-k}^\dagger \] \tag{21}
\[ D_k(t, r) = \rho(r, t) d_k + \sigma(r, t) d_{-k}^\dagger \]
\[ C_k(t, r) = \mu(r, t) c_k + \nu(r, t) b_{-k}^\dagger \]
\[ B_k(t, r) = \mu(r, t) c_{-k} + \nu(r, t) b_k^\dagger \]

Considering only the pion sector, the diagonalized Hamiltonian \( H \) can be converted into a Hamiltonian in terms of quantum fields corresponding to the operators \( A, B, C \) and their adjoints to obtain a purely quadratic Hamiltonian.

\[ H(t) = \int \frac{d^3k}{(2\pi)^3} \sum_{i=A,B,C} \frac{1}{2} (\frac{\Omega_\pi}{a^3})^2 \Pi_i^2(k, t) + P_i^2(k, t) \] \tag{22}

The Schroedinger equation for each momentum mode is simply:

\[ H_0(k, t) \psi(k, t) = i \frac{d}{dt} \psi(k, t). \tag{23} \]

If we use the \( \Pi \)-representation (coordinate space representation) for \( \psi(k, t) \), then, the \( su(1,1) \) symmetry of the Hamiltonian tells us that the solution for \( \psi(k, t) \) is just a Gaussian. The equation satisfied by the wave functions for each mode are then given by:

\[ \ddot{\psi}_A(k, t) + \frac{3\dot{a}}{a} \dot{\psi}_A + (\frac{\Omega_\pi}{a^3})^2(k, t) \psi_A(k, t) = 0. \tag{24} \]

(similar equations hold for fields B and C) where

\[ (\frac{\Omega_\phi}{a^3})^2(k, t) = (\frac{k^2}{a^2}) + \lambda (\langle \Phi \rangle^2(t) - v^2). \tag{25} \]

The expectation values of the number operator for the neutral pions for each momentum \( k \) is given by:

\[ < \psi_k(t)|a_k^\dagger a_k|\psi_k(t) > = \text{Sinh}^2(r) = < \psi_k|A_k^\dagger(t)A_k(t)|\psi_k >. \tag{26} \]

An identical expression holds for the charged scalar fields.

The non-equilibrium transition is carried out as described earlier by making \( \langle \Phi \rangle \) a function of time, where \( \langle \Phi \rangle \) changes from 0 to \( f_\pi \) either instantaneously (quench) or adiabatically. In ref [23], we have shown how the squeezing parameter \( r_k \) is related to the competing effects of the expansion rate of the plasma and the rolling down time of the system from a state of restored symmetry to that of broken symmetry. We found that for the quenched limit (fast expansion) the low momentum modes are enhanced due to the squeezing parameter being very large, whereas for the adiabatic limit (slow expansion) no such enhancement occurs and the squeezing parameter is small. This enhancement corresponds to DCC formation.

III. THE SU(1,1) DYNAMICAL SYMMETRY AND PION MULTIPLICITY DISTRIBUTIONS

The \( su(1,1) \) symmetry of this Hamiltonian was studied in ref [23]. In terms of \( su(1,1) \) generators the Hamiltonian can be written as
\[ H = H_\pi + H_\Sigma \]
\[ = \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^3} 2\Omega_\pi((\mu^2 + \nu^2)N + \mu\nu(D + D^\dagger)) + \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^3} 2\Omega_\Sigma((\rho^2 + \sigma^2)N_\Sigma + \sigma\rho(D_\Sigma + D_\Sigma^\dagger)). \] (27)

Where,

\[ \mathcal{D} = a_k a_{-k} + b_k c_{-k} + c_k b_{-k} = K_1^- + K_2^- + K_3^- \]
\[ \mathcal{D}^\dagger = a_{-k}^\dagger a_k^\dagger + c_{-k}^\dagger b_k^\dagger + b_{-k}^\dagger c_k^\dagger = K_1^+ + K_2^+ + K_3^+ \]
\[ N = \frac{1}{2}\{a_k^\dagger a_k + a_{-k}^\dagger a_{-k} + b_k^\dagger b_k + b_{-k}^\dagger b_{-k} + c_k^\dagger c_k + c_{-k}^\dagger c_{-k} + 3\} = K_1^0 + K_2^0 + K_3^0 \] (28)

\[ [N, \mathcal{D}] = -\mathcal{D}; \quad [N, \mathcal{D}^\dagger] = \mathcal{D}^\dagger; \quad [\mathcal{D}^\dagger, \mathcal{D}] = -2N \] (29)

We also define:

\[ \mathcal{D}_\Sigma = d_k d_{-k} \]
\[ \mathcal{D}_\Sigma^\dagger = d_{-k}^\dagger d_k^\dagger \]
\[ N_\Sigma = \frac{1}{2}\{d_k^\dagger d_k + d_{-k}^\dagger d_{-k} + 1\} \] (30)

which also satisfy an \( su(1,1) \) algebra.

Thus, the Hamiltonian is linear in the generators of the Lie Algebra of the \( SU(1,1) \) group and following [27] the solution of the time dependent Shrodinger equation

\[ i \frac{d\psi(t)}{dt} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^3} 2\Omega_\pi((\mu^2 + \nu^2)N + \mu\nu(D + D^\dagger))\psi(0) \] (31)

for each mode \( k \), we take \( tanh(r_k) \) for both charged and neutral modes. Then we have \( |\psi(t)\rangle = \Pi_k|\psi(k,t)\rangle \) and with discrete \( k \) we have for each mode \( k \):

\[ i \frac{d\psi_k(t)}{dt} = H_k\psi_k(t) \] (32)

, where

\[ H_k = \frac{1}{a^3} 2\Omega_\pi(k,t)((\mu^2 + \nu^2)N + \mu\nu(D + D^\dagger)). \] (33)

The solution \( \psi_k(t) \) is given by the coherent state relevant to the discrete series representation of the \( SU(1,1) \) group given by:

\[ \psi_k(t) = e^{i\Delta t}|\alpha_k(t)\rangle, \] (34)
where,
\[ |\alpha_k(t) > = e^{\alpha_k D_k^\dagger} e^{\eta N_k} e^{\alpha'(k) D_k} |\psi(0) > . \] (35)

Here, \( \alpha_k = \tanh(r(k)) \) and \( \eta_k = 2 \ln(Cosh(r(k))) = -ln(1 - |\alpha_k|^2), \alpha'(k) = -\alpha_k^* \).

\( r(k) \) is related to the frequencies \( \Omega_{\pi}(k,t) \) and \( \omega_{\pi}(k) \) by
\[ \tanh(2r_k) = \frac{(\Omega_{\pi}(k,t))^{2} - 1}{(\Omega_{\pi}(k,t))^{2} + 1} \] (36)

Using the above we get
\[ |\psi(k,t) > = (sech(r_k))^3 \sum_{n,m,p=0}^{\infty} \alpha_k^{n+m+p} |n_k, n_{-k} > |m_k, m_{-k} > |p_k, p_{-k} > \] (39)

\( n_0, n_+ \) and \( n_- \) the number of produced \( \pi_0^\pm, \pi_+^\pm \) and \( \pi_-^\pm \). At non-zero momentum, we have two such sets of number states, one for forward momentum \( (k) \) and one for backward momentum \( (-k) \).

The states contributing to (39) are only the states which have the same number of modes i.e
\[ n_k = n_{-k}, m_k = m_{-k} \text{ and } p_k = p_{-k}. \]

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From this function it is relatively easy to construct all the "p-th ordered moments" of the pion field through the formula
\[ C_1(\xi, \eta, p) = <0_{k}; 0_{-k} | \xi_{k,a_{-k}}^\dagger (r) e^{(\xi a_{-k}^\dagger - \xi^* a_k)} e^{(\eta m_{-k}^\dagger - \eta^* m_k)} S_{a_k,a_{-k}}(r) |0_{k}; 0_{-k} > e^{p(|\xi|^2 + |\eta|^2/2)} \] (40)
\[ < a_k^\dagger a_k a_{-k}^\dagger a_{-k}>_p = \left(\frac{\delta}{\delta \xi}\right)^q \left(\frac{\delta}{\delta \eta}\right)^l \left(\frac{\delta}{\delta \eta^*}\right)^m \left(\frac{\delta}{\delta \eta^*}\right)^n C_1(\xi, \eta, p)|_{\xi=\eta=0} \] (41)

Similarly for the charged pions the Characteristic function is

\[ C_2(\xi, \eta, p) = \langle 0_{b_k,0_{c_{-k}}} | S_{b_{-k},c_{-k}}(r)e^{(\xi b_{-k}^\dagger - \xi^* b_k)} e^{(\eta c_{-k}^\dagger - \eta^* c_{-k})} S_{b_k,c_{-k}} | 0_{b_k,0_{c_{-k}}} > e^{p(\xi^2 + |\eta|^2/2)} \] (42)

and the correlation function is

\[ < b_k^\dagger c_{-k} c_{-k}^\dagger b_k^n >_p = \left(\frac{\delta}{\delta \xi}\right)^q \left(\frac{\delta}{\delta \eta}\right)^l \left(-\frac{\delta}{\delta \eta^*}\right)^m \left(-\frac{\delta}{\delta \eta^*}\right)^n C_2(\xi, \eta, p) |_{\xi=\eta=0} \] (43)

Thus for example the forward backward correlation functions for neutral pions at non zero momenta are given by

\[ < a_k a_{-k} > = sinh(r)cosh(r) = < a_k^\dagger a_{-k}^\dagger > \] (44)

Thus, there is an entanglement of the forward and backward neutral pions .

\[ < b_k c_{-k} > = sinh(r)cosh(r) = < b_k^\dagger c_{-k}^\dagger > \] (45)

\[ < c_k b_{-k} > = sinh(r)cosh(r) = < b_{-k}^\dagger c_k^\dagger > \] (46)

This entanglement provides us with correlations between the pions with forward and backward momenta for both the charged and neutral sector, the forward momentum positively charged pions with the backward momentum negatively charged pions and the backward momentum positively charged pions and the forward momentum negatively charged pions.

In the non-zero k limit however the distributions of the neutral and charged pions are the same. This situation changes dramatically in the special case of soft pions \( k \rightarrow 0 \). We will now show that in this limit the correlations of the neutral pions as well as the number distributions are different in the limit of large squeezing. To see this observe that in this limit the probability distribution splits up into the convenient form

\[ | < n_0, n_+, n_- | \psi(t) >_{k=0} |^2 = | < n_0 | e^{r_0(a_0^\dagger)^2 - r_0 a_0^2} | 0 > < n_+, n_- | e^{2r_0 b_0^\dagger c_{-k}^\dagger - r_0 b_0 c_{-k}} | 0 > |^2 \] (47)

where \( r_0 = \lim_{k \rightarrow 0} r(k) \) and \( r_0^* \) is the complex conjugate of \( r_0 \). Defining \( S(r_0) \) as the one mode squeezing operator
\[ S(r_0) = <n_0|e^{r_0(a_b^0 - a_b^0)}|0> = S_{n_0,0} \]  
\[ S_{n_+,n_-0} \] is then the two mode squeezing operator

\[ <n_+,n_-|e^{(r_0 b^1 - r_0^* b^c)}|0> = S_{n_+,n_-0} \]  

The neutral and charged pion distribution is:

\[ P_{n_0,n_c} = <S_{n_0,0}^2 <S_{n_+,n_-0}^m^2> \]

which is just the product of squeezed distributions for charged and neutral pions and only even number of pions emerge. Writing \( n_+ = n_- = n_c \), we get the distribution of charged particles to be

\[ P_{n_c} = \frac{(tanh(r_0))^{2n_c}}{(cosh(r_0))^2} \]

Comparing the charged and neutral pion distributions for a squeezing parameter which corresponds to the total no. of pions \( <n> = 40 \)
(Fig. 3) and a squeezing parameter which corresponds to $<n> = 2400$ (Fig. 4) we see that there is a marked difference in the charged and neutral pion distributions for large values of the squeezing parameter for pions at zero relative momenta (soft pions).

![Graph showing comparison of $P_{m_0}(n)$ and $P_{m_c}(n)$ for the $<n> = 40$](image)

**FIG. 3:** Shows the comparison of $P_{m_0}(n)$ and $P_{m_c}(n)$ for the $<n> = 40$

In I we have found in the evolution of this Hamiltonian that in the quenched limit (fast expansion) the low momentum modes are enhanced significantly, whereas in the adiabatic (slow expansion) limit, no such enhancement occurs. This has been shown to be directly related to the value of the squeezing parameter, since for each mode $<n> = \text{Sinh}^2(r_0)$. The difference in the total charged and multiplicity distributions of pions produced at low momenta (soft pions) for large squeezing parameter is a direct mirror of this effect. Thus the measurable quantities like the charged and neutral total multiplicity distributions at low momenta can be used to determine whether a quench or an adiabatic expansion occurred in a heavy ion collision and whether a disoriented chiral condensate formed or not. This serves as a very useful signal for non-equilibrium phase transitions in Heavy ion collisions. We now proceed to examine whether this dramatic effect survives theoretically the imposition of isospin conservation.

### IV. THE ISOSPIN SYMMETRY OF THE HAMILTONIAN

We now examine the isospin structure of the pion Hamiltonian. Since isospin is conserved in strong interactions we need to extract states of fixed isospin from the squeezed number states which will be the eigenstates of the $\text{su}(1,1)$ Hamiltonian constructed above. Thus far, most studies have constructed such isospin states at zero momentum. However, for an event by event analysis we need to construct these states at non-zero momentum states which have not been considered before. We will do so by defining the isospin operators constructed from the Fock state basis of pions at non-zero momentum by constructing the vectors

$$\vec{a}(k) = \begin{pmatrix} \frac{b_k + c_k}{\sqrt{2}} \\ \frac{i(b_k - c_k)}{\sqrt{2}} \\ a_k \end{pmatrix} = \begin{pmatrix} a_1(k) \\ a_2(k) \\ a_3(k) \end{pmatrix} (53)$$

$$\vec{a}(k) = \begin{pmatrix} \frac{b_{-k} + c_{-k}}{\sqrt{2}} \\ \frac{i(b_{-k} - c_{-k})}{\sqrt{2}} \\ a_{-k} \end{pmatrix} = \begin{pmatrix} a_1(-k) \\ a_2(-k) \\ a_3(-k) \end{pmatrix} (54)$$

From the above, two sets of isospin operators can be constructed.
FIG. 4: Shows the comparison of $P_{n_0}(n)$ and $P_{n_e}(n)$ for the $< n > = 2400$

\[
I_i(k) = i\epsilon_{ijl}a_j(k)a_l(k)^\dagger \tag{55}
\]

\[
I_i(-k) = i\epsilon_{ijl}a_j(-k)a_l(-k)^\dagger.
\]

Generalizing the work of [19], we find that the two operators $I_i(k)$ and $I_i(-k)$ taken together define a direct sum algebra of two su(2) algebras

\[
[M_i, N_j] = i\epsilon_{ijl}M_l; \quad [N_i, N_j] = i\epsilon_{ijl}N_l; \quad [M_i, M_j] = i\epsilon_{ijl}M_l
\tag{57}
\]

characteristic of an so(4) algebra. The Casimir operators of this algebra are

\[
C_1 = M^2 + N^2 = I(k)^2 + I(-k)^2 \tag{58}
\]

\[
C_2 = M \cdot N = I(k)^2 - I(-k)^2
\]

From the vectors $\vec{a}(k)$ and $\vec{a}(\vec{k})$ we can form bilinear operators

\[
A(k) = \vec{a}(k) \cdot \vec{a}(\vec{k})
\]

\[
A^\dagger(k) = \vec{a}^\dagger(\vec{k}) \cdot \vec{a}^\dagger(k)
\]

\[
N(k) = \vec{a}^\dagger(k) \cdot \vec{a}(\vec{k})
\tag{59}
\]

\[
D = \vec{a}(k) \vec{a}(\vec{k}) \quad D^\dagger = \vec{a}^\dagger(\vec{k}) \vec{a}^\dagger(k) \quad N = \frac{1}{2} (\vec{a}^\dagger(k) \vec{a}(k) + \vec{a}^\dagger(-k) \vec{a}(-k) + 3) \tag{61}
\]

Since the two algebras are related, we can also relate the eigenstates of $H$ constructed through these two algebras.
cal harmonics $Y_m^l(a(k)^\dagger)Y_m^l(a(-k)^\dagger)|0> \) which are also eigenstates of $H$.

In terms of the Isospin operators the pion Hamiltonian can be written as:

$$H_\pi = \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^2} 2\Omega_{\pi k}(\mu^2 + \nu^2) \frac{1}{2} (a^\dagger(k)a(k) + \tilde{a}^\dagger(-k)\tilde{a}(-k) + 3) + 2\Omega_{\pi k}\mu(\tilde{a}(k)\tilde{a}(-k) + \tilde{a}^\dagger(-k)\tilde{a}^\dagger(k))$$  \hspace{1cm} (62)

V. ISOSPIN SQUEEZED STATES FOR $k \to 0$ PIONS

First, we construct the isospin squeezed states for the $k \to 0$ case. For this we need to relate the states

$$|r_0> = e^{r_0(D^\dagger - D)}|\psi_0(0)> = e^{r_0(a_0^2 + c_0^2\tilde{a}_0^\dagger + b_0^2\tilde{c}_0^\dagger - a_0b_0 - b_0c_0 + c_0b_0)}|\psi_0(0)>$$  \hspace{1cm} (63)

to the states of definite isospin. Note that we have added a subscript 0 to the operators to indicate the zero momentum limit.

For this purpose we note that for $k \to 0$ there exists only one isospin algebra constructed from the operator

$$\tilde{a}_0 = \left( \begin{array}{c} \frac{b_0 + c_0}{\sqrt{2}} \\ \frac{i(b_0 - c_0)}{\sqrt{2}} \\ a_0 \end{array} \right)$$  \hspace{1cm} (64)

This algebra is given in terms of

$$I_{0i} = i\epsilon_{ijj}a_{j0}a_{0j}^\dagger$$  \hspace{1cm} (65)

The squared Isospin vector is

$$I^2 = N^2 + \mathcal{A}_0^\dagger \mathcal{A}_0$$  \hspace{1cm} (66)

For further discussion, now, in the $k \to 0$ case we drop the subscript 0. The definitions are

$$\mathcal{A}^\dagger = \tilde{a}^\dagger \cdot \tilde{a}^\dagger$$  \hspace{1cm} (67)

$$\mathcal{A} = \tilde{a} \cdot \tilde{a}$$

We also have the relations

$$[\mathcal{A}, Y_m^l(\tilde{a}^\dagger)] = 0$$  \hspace{1cm} (68)

$$[N, \mathcal{A}] = -2\mathcal{A}$$

$$[N, \mathcal{A}^\dagger] = 2\mathcal{A}^\dagger$$

$$Y_m^l(\tilde{a}^\dagger)|0> = (-2)^{-m} \sqrt{(2l + 1)(l - m)!} (l + m)! \sum_{n=0}^{(l-m)/2} \frac{2^{m/2-n}}{(l - m - 2n)!n!(n + m)!} (a_0)^{l-m-2n} (a_+)^{n+m} (a_-)^n |0>$$  \hspace{1cm} (69)
are the eigenstates of $I^2$, $I_3$ and $N$ whose action on them is given by:

\[ |l, m, l + 2n > = N_{l,n}(A^l)^n Y_m^l (a^\dagger) |0 > \quad (70) \]
\[ I^2 |l, m, l + 2n > = l(l + 1) |l, m, l + 2n > \]
\[ I_3 |l, m, l + 2n > = m |l, m, l + 2n > \]
\[ N |l, m, l + 2n > = (l + 2n) |l, m, l + 2n > \]

and the relationship between the number states $|n_0, n_+, n_- >$ and $|l, m, l + 2n >$ is given by

\[ |l, m, l + 2n > = \sum_p c^{l,m,n}(p) |l + 2n - |m| - 2p, \frac{m + |m|}{2} + p, \frac{m - |m|}{2} + p > \quad (71) \]

\[ c^{l,m,n}_{p} = (-1)^m \sqrt{\frac{2^{l+1} p^2 (p + l + n) (l + m)! n! (l - m + 2n - 2p)! (m + p)!}{(l - m)! m!^2 (1 + 2l + 2n)! (n - p)!^2 p! \sum_j (p - j)! (n + j - p)! (n - 2j)! j! j!}} \quad (72) \]

The inverse relation is

\[ |n_0, n_+, n_- > = \sum_{l=0}^{n_+ + n_- + n_0} c^{l,n_+ - n_-, n_+ + n_- + n_0}_{n_+ + n_- - n_+ + n_-} |l, n_+ - n_-, n_+ + n_- + n_0 > \quad (73) \]

For strong interactions involving charged pions $n_+ - n_- = 0$ by charge conservation, hence the only values of $c^{l,m,n}_{p}$ contributing to physically measurable quantities will be

\[ c^{l,0,n}_{(n_+ + n_-)} = 2^{(n_+ + n_- + \frac{1}{2} l)} \frac{(n + l)! (2l + 1)! n! (l + 2n - 2(n_+ + n_-))! (n_+ + n_-)!}{(2n + 2l + 1)!} \frac{1}{2} \sum_{j=0}^{n_+ + n_-} \frac{(-1)^j}{(2^2 j)!} \frac{(-j + (n_+ + n_-))! (-2j + l)! (j)! (j - (n_+ + n_-) + n)!}{(n_+ + n_-)! n!} \quad (74) \]

In considering the probability distribution of pions with isospin conservation the squeezed state of definite isospin has to be projected from
the number state distribution.

\[ <n_0, n_+, n_-|S|l, m, 2n> = \sum_{p=0}^{n+|l-m|/2} c_p^{(l,m,n)} <n_0, n_+, n_-|S|l+2n-m-2p, m+p, +p> \]

(75)

The general expression for the one mode and two mode matrix elements are [26]:

\[ S_{\text{onemode}}(r)_{n_0,l+2n-m-2p} = (-1)^{\frac{l+2n-m-2p+n_0}{2}} \left( \frac{l+2n-m-2p+n_0}{\tanh(r)} \right) \frac{1}{\cosh(r)} \sum_{\lambda=0}^{\min\left[\frac{n_0}{2},\frac{l+2n-m-2p}{2}\right]} (-1)^\lambda \frac{(\sinh^2(r))^\lambda}{(2\lambda)! \left(\frac{l+2n-m-2p}{2} - \lambda\right)! (\frac{n_0}{2} - \lambda)!} \]

(76)

\[ n, m, \text{ even.} \]

\[ S_{\text{two-mode}}_{n_+,n_-,m+p,p}(r) = (-1)^{2n_++m+p} \sqrt{n_+!n_-!(m+p)!}(\tanh(r))^{n_++m+p} \left( \frac{\sinh^2(r)}{\cosh(r)} \right)^{n_-} \frac{1}{(\sinh^2(r)^\lambda)} (2\lambda)! (m+p-\lambda)! (n_- - n_+ + \lambda)! \]

(77)

Using these identities we have

\[ <n_0, n_-, n_+|S(r)|l, m, 2n> = \sum_{p=0}^{n+|l-m|/2} c_p^{(l,m,n)} S_{\text{onemode}}_{n_0,l+2n-m-2p}(r) S_{\text{two-mode}}_{n_+,n_-,m+p,p}(r) \]

(78)

For the special case of pion emission in strong interactions, the charge is conserved, hence \( m = n_+ - n_- = 0 \).

To show clearly the effect of isospin conservation, we focus on total isospin \( I = 0 \) as an example. In this case,

\[ <0|S|n_0, n_c> = \frac{(-\tanh(r))^{n_0+n_c}}{(\cosh(r))^{3/2}} \frac{n_0 + n_c!}{(2(n_0 + n_c) + 1)!} \frac{1}{2^{n_c}} \left( \frac{(2n_0)!}{(n_0)!} \right) \]

(79)

with \( n_+ + n_- = n_c \) and \( n = n_0 + n_c \). Thus, the probability of finding \( n_0 \) neutral pions and \( n_c \) charged pions in the state of \( I = 0 \).
is
\begin{equation}
P_{n_0,n_c} = \frac{(\tanh(r))^{(n_0+n_c)}}{(\cosh(r))^3} \left( \frac{(n_0+n_c)!}{(2(n_0+n_c)+1)!} \right) 4^{n_c} \frac{(2n_0)!}{(n_0!)^2}
\end{equation}

This gives the neutral pion distribution \( P_{n_0} = \sum_{n_c} P_{n_0,n_c} \)

\begin{equation}
P_{n_0} = 2^{-1-2n} e^{\tanh(r)^2} \sqrt{\pi} \text{Csch}(r)^2 \frac{(2n)!}{(\sinh(r)^2) n!} \frac{\Gamma(\frac{3}{2}+n) - \Gamma(\frac{1}{2}+n, \tanh(r)^2)}{\Gamma(\frac{1}{2}+n)} \left( 1+\frac{<n_0>}{1+<n_0>} <n_0> \right) \right)^{n_0} \text{Sech}(r)^2 \left( \sinh(r)^2 \right)^n \frac{\Gamma(\frac{3}{2}+n)}{2(\cosh(r)^2)^n \Gamma(\frac{3}{2}+n)}
\end{equation}

In terms of the average number \(<n_0>\) of neutral pions

\begin{equation}
P_{n_0} = 2^{-1-2n} e^{\tanh(r)^2} \sqrt{\pi} \text{Csch}(r)^2 \frac{(2n)!}{(\sinh(r)^2) n!} \frac{\Gamma(\frac{3}{2}+n) - \Gamma(\frac{1}{2}+n, \tanh(r)^2)}{\Gamma(\frac{1}{2}+n)} \left( 1+\frac{<n_0>}{1+<n_0>} <n_0> \right) \right)^{n_0} \text{Sech}(r)^2 \left( \sinh(r)^2 \right)^n \frac{\Gamma(\frac{3}{2}+n)}{2(\cosh(r)^2)^n \Gamma(\frac{3}{2}+n)}
\end{equation}

The corresponding distribution of charged pions is given by

\begin{equation}
P_{n_c} = \frac{\sqrt{\pi} \text{F}_1(\frac{1}{2}, \frac{3}{2} + n, \tanh(r)^2) \text{Sech}(r)^2 (\sinh(r)^2)^n}{2(\cosh(r)^2)^n \Gamma(\frac{3}{2}+n)}
\end{equation}

In terms of \(<n_c>\) we have

\begin{equation}
P_{n_c}(<n>) = \frac{<n>^{n_c} (1+<n>)^{-1-n_c} \sqrt{\pi} \text{F}_1(\frac{1}{2}, \frac{3}{2} + n_c, \frac{<n>}{1+<n>})}{2\Gamma(\frac{3}{2}+n_c)}
\end{equation}
FIG. 5: Shows the variation $P(n)$ with $n$ with isospin $I = 0$ for NEUTRAL PIONS for values of $< n > = 500, 1500, 2500$.

FIG. 6: Shows the variation $P(n)$ with $n$ for CHARGED PIONS for values of $< n > = 500, 1500, 2500$.

To compare neutral and charged pion distributions for large and small squeezing parameters the following figures are presented. Fig.[6] shows the variation of $P_{n_c}$ with $n_c$ for various values of $< n >$. Comparing the plots of the difference between charged and neutral pions distributions without isospin (fig.4) and those with isospin conservation (fig.7), we find that the dramatic difference in the two distributions is reduced by the imposition of isospin conservation for $I = 0$. However, still the charged pion distributions get broader with increasing squeezing and hence the distributions can still be used to determine whether a phase transition has occurred in Heavy ion collisions even when isospin conservation for soft pions is taken into account.

FIG. 7: Shows the variation $P(n)$ with $n$ for CHARGED PIONS (dashed line) and neutral pions (solid line) for values of $r_0 = 3.5$.

FIG. 8: Shows the variation $P(n)$ with $n$ for CHARGED PIONS (dashed line) and neutral pions (solid line) for values of $r_0 = 5$. 
VI. ISOSPIN SQUEEZED STATES FOR PIONS WITH ARBITRARY MOMENTUM

In this section we calculate the multiplicity distributions of squeezed pion states at non-zero momenta with the imposition of Isospin invariance. For arbitrary \( k \) we have to proceed slightly differently from above. We have seen that the states

\[ |\alpha> = Ne^{\alpha D^\dagger} |0> \]  

represent the dynamical wavefunctions of the pions at non zero momenta where, \( D^\dagger(k) = \alpha \bar{a}^\dagger(-k) \bar{a}^\dagger(k) \). Using the identity:

\[ e^{k \cdot x} = \sum_{l=0}^{\infty} \phi_l(k^2x^2) \sum_{-l}^{+l} Y_l^m(k)^* Y_m^l(x), \]  

where \( \phi_l \) is the spherical Bessel Function \( \phi_l(x) = j_l(-i\sqrt{x})/(\sqrt{x})^l \), and

\[ \phi_l(x) = 2^l \sum_{n=0}^{\infty} \frac{(n + l)!x^n}{(2n + 2l + 1)!n!}. \]

To get the isospin decomposition we use the fact that

\[ Y_l^m(a(\bar{k})^\dagger)|0, 0, 0> = \]  

\[ (-2)^{-m} \frac{\sqrt{(2l + 1)(l - m)!}(l + m)!}{\sum_{n=0}^{(l-m)/2} \frac{2^{m/2-n}}{(l - m - 2n)!n!(n + m)!}(a_{k,0})^{l-m-2n}(a_{k+})^{n+m}(a_{k-})^n|0, 0, 0>}. \]

\( Y_l^m(a(\bar{k})^\dagger)|0 > \) is an eigenstate of \( I_k^2 \) and \( I_{3k} \) and correspondingly \( Y_l^m(a(\bar{k})^\dagger)|0 > \) is an eigenstate of \( I(-k)^2 \) and \( I_{3-k} \).

The number operator eigenstates and the isospin eigenstates are related by the formula:

\[ |l, m, l + 2n(k) > = N_{l,n}(A(k)^\dagger)^n(k) Y_l^m(a_k^\dagger)|0 > \]  

where \( N_{l,n} = \left( \frac{2^{n(n+l)!}}{(2n+2l+1)!n!} \right)^{\frac{1}{2}} \).
Thus, the squeezed state wave function for fixed isospin is given by

$$e^{\alpha D^1}|0>_k|0>_{-k} = 4\pi \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=-l}^{l} \alpha^{l+2n} (A(k)^{\dagger})^n (A(-k)^{\dagger})^n (n+l)! Y_m^l (a(k)^{\dagger}) Y_m^l (a(-k)^{\dagger})|0>_k|0>_{-k}. \quad (91)$$

Because, $Y_m^l = (-1)^m Y_m^l (\bar{c})^* \text{ and } Y_m^l (c\bar{c}) = e^l Y_m^l (\bar{c})$ we get

$$|\alpha> = N_{l,n} \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{\alpha^{2n+l}}{2^l (n+l)!} (-1)^m |l,-m,l+2n>_{(k)} |l,m,l+2n>_{(-k)}. \quad (92)$$

We now have the necessary ammunition to calculate the multiplicity distribution:

$$P_{l,m,n}^{n_0,n_-,n_+} = <n_0,n_+,n_-|k<n_0,n_-,n_+|_{-k}|\alpha|>^2. \quad (93)$$

Defining

$$<n_0,n_+,n_-|k|l,-m,l+2n>_{(k)} = \xi_{p,m,n}^{l,m,n}(k) \quad (94)$$

and

$$<n_0,n_+,n_-|_{-k}|l,-m,l+2n>_{(-k)} = \xi_{p,m,n}^{l,m,n}(-k), \quad (95)$$

$$P_{l,m,n}^{n_0,n_-,n_+} (\alpha) = \frac{(tanh(r))^{n(k)+n(-k)+l}}{Sech(r)^{3/2}2^l(n(k)+n(-k)+l)!} \xi_{p,m,n}^{l,m,n}(k) \xi_{p,m,n}^{l,m,n}(-k)^2 \quad (96)$$

For $I=0$ and $m=0$ we have

$$P_{0,0,n}^{n_0,n_-,n_+} (\alpha) = \frac{(tanh(r))^{2(n(k)+n(-k)+l)}}{Sech(r)^3(n(k)+n(-k)+l)!} \frac{((n_0(-k)+n_0(k)+n_c(k)+n_c(-k)))!2^{n_0(-k)+n_0(k)+n_c(k)+n_c(-k)+1}}{(n_0(k)+n_0(-k)+n_c(k)+n_c(-k)+1)!} \quad (97)$$

In Figure 9 we show the effect of isospin conservation at non zero momentum on the total multiplicity distribution of pions and see that...
it has a significant effect of narrowing the multiplicity distribution. However as in the case without isospin conservation at non zero momentum there is no difference between the neutral and squeezed pion distributions.

![Graph](image)

**FIG. 9:** Shows the variation $P(n)$ with $n$ for pions with isospin conservation ($I=0$) (dashed line) and without isospin conservation (solid line) for values of $r_0 = 5$

**VII. CONCLUSION**

To conclude we have given a dynamical model for the production of pions in heavy ion collisions going though a non-equilibrium phase transition, and have shown how the charged and neutral pion distributions both globally and on an event by event basis can be used as a signal to measure the nature of the phase transitions. We have shown that the distributions of the charged and neutral zero momentum pions are conspicuously different when the system undergoes a quench as is thought to occur when a disoriented chiral condensate forms in a heavy ion collision. We have shown that this dramatic difference disappears when non zero momentum number distributions of pions are observed. However forward backard correlations are enhanced both in the neutral and charged pion sectors when the system undergoes a quench, which is parametrised by a squeezing parameter. We have shown how to incorporate isospin conservation on both the zero momentum and the non-zero momentum pion distributions and shown that the effect of the quench survives the imposition of this invariance on the pion multiplicity distributions. Thus we have presented a field theoretical dynamical description of the signals that can differentiate between the various ways in which a system can undergo a phase transition in heavy ion collisions. The distributions and correlations which we have derived in this paper should be useful in testing the data envisaged at the RHIC collider to discern not only whether a disoriented chiral condensate is formed by easily measurable pion distributions, but also the effect of conservation laws on the general distributions of pions emerging in the collision processs[31]. It is our hope that this work will prove useful to experimentalists when examining the data and we look forward to its application.

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