Multi-resonant piezoelectric shunting induced by digital controllers for subwavelength elastic wave attenuation in smart metamaterial

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Abstract

Instead of analog electronic circuits and components, digital controllers that are capable of active multi-resonant piezoelectric shunting are applied to elastic metamaterials integrated with piezoelectric patches. Thanks to recently introduced digital control techniques, shunting strategies are possible now with transfer functions that can hardly be realized with analog circuits. As an example, the ‘pole-zero’ method is developed to design single- or multi-resonant bandgaps by adjusting poles and zeros in the transfer function of piezoelectric shunting directly. Large simultaneous attenuations in up to three frequency bands at deep subwavelength scale (with normalized frequency as low as 0.077) are achieved. The underlying physical mechanism is attributable to the negative group velocity of the flexural wave within bandgaps. As digital controllers can be readily adapted via wireless broadcasting, the bandgaps can be tuned easily unlike the electric components in analog shunting circuits, which must be tuned one by one manually. The theoretical results are verified experimentally with the measured vibration transmission properties, where large insulations of up to 20 dB in low-frequency ranges are observed.

Keywords: elastic metamaterials, piezoelectric shunting, pole-zero design, digital controller, multi-resonant

(Some figures may appear in colour only in the online journal)

1. Introduction

In the last decade, acoustic/elastic metamaterials (EMMs) have gained increasing interest due to their unnatural elastic properties. As a kind of structural material, EMMs can be designed to have significant ability to suppress, absorb, or manipulate elastic waves by tailoring their metastructure at the subwavelength scale. Thus, a large structure is no longer necessary in order to gain a low-frequency bandgap [1], which is helpful for low-frequency isolation or attenuation of sound and vibration [2–7], negative refraction [8], acoustic imaging [9, 10], acoustic cloaking [11], etc.

Due to weak attenuations in locally resonant (LR) bandgaps [2] and the lack of real-time tuning, piezoelectric (PZT) shunting was introduced into the design of the EMMs [12]. The benefits of this are the lighter weight and tunability. Airoldi et al designed a tunable metamaterial beam with periodic arrays of resonant shunting (RS) of PZTs [13], and
also proposed bi-RS to generate two LR bandgaps in it [14]. We also performed some theoretical and experimental investigation of EMM beams with passive PZT shunting [15, 16]. This strategy has also been extended to a flat plate hosting periodic arrays of shunted PZT patches. The elastic wave in the metamaterial thin plates [17–21], as well as the sound transmission loss through it [22], were both researched. Moreover, Casadei et al implemented experimentally a tunable elastic wave waveguide by placing eight PZT disks in the L-shaped waveguide of a two-dimensional phononic plate, where all the PZT disks were shunted by passive inductive circuits [23]. Kwon et al also introduced passive PZT shunting to the design of vibroacoustic metamaterials that can exhibit acoustic waveguides with tunable bandgaps [24]. Nouh et al demonstrated a metamaterial plate with tunable local resonators composed of piezomembranes with adjustable stiffness controlled by external voltages [25].

The passive RS technique can barely produce large attenuation of vibration at deep subwavelength frequencies. As a revision of passive RS, negative-capacitance PZT shunting has been introduced into the EMM in order to enhance the coupling effect [26, 27]. It was also used in the design of cellular metamaterials that can exhibit reconfigurable, highly focused subwavelength wave patterns theoretically [28]. However, this technique requires the tuning of the circuit very close to the stability limit in order to get the desired performance [29]. We recently proposed two different kinds of enhanced RS, i.e., the ‘resonator-amplifier’ [30] and the ‘amplifier-resonator’ [31] active strategies, in order to intensify the resonant effect of the passive RS, and large attenuations within low frequency ranges were observed. All of the just-discussed techniques are based on analog electronic components such as inductors, resistors, and capacitors. Large inductors without internal resistance are required, which are usually realized using synthetic circuits composed of two (Antoniou circuit that needs to be partly grounded) or four (floating inductor) operational amplifiers and some other components. For the multi-resonant shunting technique [14], this situation becomes worse because \( N \times N \) floating inductors are needed to gain a \( N \)-resonant shunting. In order to achieve two combined or separate bandgaps, Zhou et al proposed a high-order resonant shunt circuit that can exhibit two local resonances, and demonstrated the effects via simulations [32]. In their high-order resonant shunt circuit, the additional resonance was introduced by adding a capacitor, resistor, and floating inductor to the traditional RL shunt. The floating inductor will be the main complexity in practice, too. Besides their complexity and attendant inconvenient tunability, another shortcoming of the analog shunting circuits is the difficulty in design. Ordinarily, they are built by trial and error.

When linearity is assumed, all the dynamic behavior of the shunted PZT can be described as a single input and single output (SISO) linear system, so it will be much more convenient if it is possible to bypass complex procedures related to analog circuits and design its transfer function directly. This paper makes the first attempt, to our knowledge, in this area. For the first time, digital controllers (active shunting units) are designed and applied to an EMM structure composed of an aluminum alloy beam and arrays of PZT patches glued on it.
2. Theoretical model and experimental setup

2.1. Theoretical model of the beam with digital shunting controllers

Figure 1(a) illustrates the mechanical structure of a six-period specimen of the proposed smart EMM beam, while figures 1(c) and (d) show the close-up views of it from a different angle. The specimen is composed of an aluminum alloy beam and six pairs of PZT patches periodically glued on both sides of it. In each period, the beam’s segment with the PZT patch is denoted by A, while the other is denoted by B. For the specimen with finite size, the segment on both ends of the beam are denoted by C. Each pair of PZT patches is placed with opposite polarizing directions along the z-axis and connected with a digital controller for active piezoelectric shunting, as shown in figure 1(b). All active shunting units are the same and work independently under the same mechanism in order to guarantee the periodicity of the smart EMM structure. Each active shunting unit is composed of a charge amplifier, analog-to-digital converter (ADC), microprocessor, digital-to-analog converter (DAC), and proportional amplifier. The mechanical strain on PZT patch $a$ (the sensor patch) is converted into a voltage signal $V_1$ by the charge amplifier and then digitalized by the ADC. Afterwards, the microprocessor uses these data to calculate in real time the value of output signal $V_2$ based on the desired transfer function. Then the digital value is transferred into the analog voltage signal $V_2$ by the DAC. Finally, the voltage signal $V_2$ is magnified $\beta$ times with a proportional amplifier circuit and applied to the PZT patch $b$ in order to achieve the active PZT shunting based on the designed transfer function. The geometric and material parameters of the proposed smart EMM beam are listed in tables 1 and 2.

Figure 1(d) shows the photo of the low-cost circuit board that contains two independent digital controllers used for active PZT shunting in this paper. When transfer functions of different controllers are same, the multi-physics mechanism in each unit cell are the same and thus the periodicity is guaranteed. Two kinds of precise operational amplifiers (LMV612 and OPA454) from Texas Instruments are chosen to make each circuit board. One LMV612 unit is used as a two-channel charge amplifier that transfers the mechanical strain (in form of electric charge) of PZT patch $a$ into a voltage signal, and two OPA454 units capable of operating up to $\pm 50 \text{V}$ are chosen as two $\beta$-times amplifiers in order to provide high driving voltages applied to PZT patches $b$. The STM32F103RCT6 from STMmicroelectronics, which has two channels of DACs and several channels of ADCs on-chip, is used as the microprocessor and ADC/DAC. ZigBee wireless

### Table 1. Geometric and material parameters of the hosting beam (aluminum alloy 6061-T6).

| Parameter       | Symbol | Value  |
|-----------------|--------|--------|
| Density         | $\rho_{\text{beam}}$ | 2700 kg m$^{-3}$ |
| Young’s modulus | $E_{\text{beam}}$       | 69.5 GPa          |
| Width           | $b_1$  | 0.04 m |
| Thickness       | $h_1$  | 0.01 m |
| Length of segment A | $l_A$       | 0.04 m          |
| Length of segment B | $l_B$       | 0.06 m          |
| Mass of sensor  | $m$    | 8 g    |
| Number of periods | $N$       | 6      |
| Length of segment C | $l_C$       | 0.13 m         |

### Table 2. Geometric and material parameters of the piezoelectric patch (P-42).

| Parameter       | Symbol | Value  |
|-----------------|--------|--------|
| Density         | $\rho_{p}$ | 7600 kg m$^{-3}$ |
| Compliance coefficient | $s_i^P$ | $11 \times 10^{-12}$ m$^2$ N$^{-1}$ |
| Piezoelectric strain constant | $d_{33}$ | $-101 \times 10^{-12}$ C N$^{-1}$ |
| Dielectric constant | $\varepsilon_{33}$ | $1.239 \times 10^{-9}$ F m$^{-1}$ |
| Width           | $b_p$  | 0.04 m |
| Thickness       | $h_p$  | $1 \times 10^{-3}$ m |

"pole-zero" method is developed to design single- or multi-resonant transfer functions related to the equivalent Young’s modulus of the shunted piezoelectric patches. The transfer function is transformed into recurrence equations in order to be realized by digital controllers. Finally, both the single- and multi-resonant bandgap behavior of the proposed smart EMM are analyzed theoretically and validated experimentally.

Compared with the aforementioned works (including ours [30, 31]) that are based on analog shunting techniques, the main novelty and original contribution of this paper is as follows: Thanks to the introduced digital control technique, all the restraints that relate to analog electronic circuits can be cast off. New shunting strategies with transfer functions that can hardly be realized with analog circuits are possible now, which provide new possibilities for the design of smart EMM. As an example, the proposed “pole-zero” method can be used to design the multi-resonant bandgaps directly by simply adjusting poles and zeros in the transfer function that relate to certain equivalent Young’s modules instead of entwining them into a confusing mass of analog circuits. Moreover, the proposed digital controllers can be readily adapted via wireless broadcasting, and thus the bandgaps can be tuned easily as opposed to tuning the electric components manually, one by one. This advantage will be helpful in the design of self-adaptive smart metamaterials.
units are connected with each circuit board and the computer in order to modify the parameters of the controllers synchronously.

As PZT patch $a$ in figure 1(b) is equivalently grounded, the piezoelectric equations in it can be reduced [31]. Thus, the relationship between the mechanical strain $S_1^a$ and voltage $V_1$ can be deduced as

$$V_1 = \frac{A_1 d_{31}}{s_1^a} S_1^a = FS_1^a$$  \hspace{1cm} (1)$$

where $F = A_1 d_{31}/s_1^a C_1$; $A_1$ is the area of the PZT electrode; $d_{31}$ is the piezoelectric strain constant; $s_1^a$ is the compliance coefficient of the PZT material at the constant electric field intensity; $C_1 = 100\text{mF}$ is the capacitor in the charge amplifier circuit; and $S_1^a$ is the mechanical strain in PZT patch $a$.

As the voltage $V_1$ is in a constant ratio with the strain $S_1^a$, it can be used as a perfect sensor of the strain inside the PZT patch $a$. Furthermore, the effective elastic modulus $E_1^a$ of PZT patch $a$ is a constant [31], which is the mechanical stress $T_1^a$ divided by the strain $S_1^a$ as

$$E_1^a = \frac{T_1^a}{S_1^a} = \frac{1}{s_1^a}$$  \hspace{1cm} (2)$$

The voltages on both ends of the $\beta$-times amplifier have the relation as

$$V_{\text{out}} = \beta V_2$$  \hspace{1cm} (3)$$

where $\beta$ is the amplifying ratio of the proportional amplifier circuit, which is used to amplify the output voltage of the on-chip DAC to the output voltage of the OPA454 (the operational amplifiers). As the maximum output voltage ranges of the on-chip DAC and the OPA454 are $\pm1.65\text{V}$ and $\pm50\text{V}$ respectively, and the OPA454’s maximum voltage range has a little shrinking of up to 5% based on different current, the amplifying ratio $\beta$ is chosen as $\beta = 100/3.3 \times 95\% \approx 28.8$ in this paper.

Assuming that the analog transfer function of the microprocessor along with ADC and DAC is $H_{\text{DP}}(s) = V_{\text{out}}/V_1$, we can have the following relationship by combining equations (1) with (3):

$$\frac{V_{\text{out}}}{S_1^a} = \beta H_{\text{DP}}(s)$$  \hspace{1cm} (4)$$

where $s$ is the complex frequency variable in the Laplace transform.

As the electric field in PZT patch $b$ is $E_2^b = V_{\text{out}}/h_b$, along with equation (4), the piezoelectric equations [31] in patch $b$ can be transformed as

$$S_2^b = s_1^b T_1^b + \frac{d_{31} \beta F H_{\text{DP}}(s)}{h_b^b} S_1^a$$  \hspace{1cm} (5)$$

where $h_b$ is the thickness of PZT patch $b$; $S_1^b$ and $T_1^b$ are the mechanical strain and stress of PZT patch $b$, respectively.

When deep subwavelength flexural waves are considered, the strains of PZT patches $a$ and $b$ in one unit cell can be approximately regarded as $S_1^a = -S_1^b$. Thus, the equivalent elastic modulus of PZT patch $b$ can be derived as mechanical stress $T_1^b$ divided by strain $S_1^b$:

$$E_2^b(s) = \frac{T_1^b}{S_1^b} = h_b^b + d_{31} \beta F H_{\text{DP}}(s)$$  \hspace{1cm} (6)$$

Assuming $s = j\omega$, PZT patches $b$ can be considered as ordinary elastic materials with equivalent complex stiffness determined by equation (6) at certain frequencies.

If each segment A or B of the beam is considered as a Timoshenko beam, the flexural elastic waves in it are governed by the differential equation

$$\frac{EI}{\rho S} \frac{\partial^4 u}{\partial x^4} - \frac{I}{S} \left( 1 + \frac{E}{\kappa G} \right) \frac{\partial^2 u}{\partial x^2 \partial t^2} + \frac{\partial^2 u}{\partial t^2} + \frac{\rho l}{\kappa GS} \frac{\partial^4 u}{\partial t^4} = 0$$  \hspace{1cm} (7)$$

where $u(x, t)$ is the transverse displacement of the beam along the z-axis; $E$ and $G$ are the Young’s and shear modulus, respectively; $\rho$ is the density; $S$ is the cross-section area; $\kappa$ is the Timoshenko shear coefficient; and $l$ is the moment of inertia with respect to the axis perpendicular to the beam axis.

If harmonic vibrations are assumed as $u(x, t) = U(x) e^{i\omega t}$, where $\omega = 2\pi f$ (f being the frequency), the solution of Timoshenko equation (7) can be written as [33]

$$U(x) = \left[ k_1^{-3} e^{k_1 x} + k_3^{-3} e^{k_3 x} + k_5^{-3} e^{k_5 x} + k_4^{-3} e^{k_4 x} \right] \psi$$  \hspace{1cm} (8)$$

where $k_j = (-1)^{(j/2)} \sqrt{(\alpha j^2 + 4\beta)}$ for $j \geq 1$, and $\alpha$ is the mechanical and geometrical properties of the beam through $\alpha = -\rho \omega^2/E - \rho \omega^2/\kappa G$ and $\beta = \rho \omega^2/\kappa G E$, $j/2$ is the largest integer less than $j/2$, and $\psi = \{A, B, C, D\}^T$ are factors needing to be determined.

In period $n$, the continuity of displacement, rotating angle, bending moment, and shearing force at the boundary of beam segments A and B can be described as

$$H_{\Lambda}(l_n) \psi_{A,n} = H_{B}(l_n) \psi_{B,n}$$  \hspace{1cm} (9)$$

while the continuity at the boundary of beam segment B in period $n$ and beam segment A in period $n + 1$ ensures

$$H_{B}(l_n) \psi_{B,n} = H_{\Lambda}(l_{n+1}) \psi_{A,n+1}$$  \hspace{1cm} (10)$$

where

$$H_{A,B}(l) = \begin{bmatrix} k_1^{-3} e^{k_1 x} & k_2^{-3} e^{k_2 x} & k_3^{-3} e^{k_3 x} & k_4^{-3} e^{k_4 x} \\
 k_1^{-2} e^{k_1 x} & k_2^{-2} e^{k_2 x} & k_3^{-2} e^{k_3 x} & k_4^{-2} e^{k_4 x} \\
 E k_1^{-1} e^{k_1 x} & E k_2^{-1} e^{k_2 x} & E k_3^{-1} e^{k_3 x} & E k_4^{-1} e^{k_4 x} \\
 E e^{k_1 x} & E e^{k_2 x} & E e^{k_3 x} & E e^{k_4 x} \end{bmatrix}_{A,B}$$  \hspace{1cm} (11)$$

Combining equations (9) and (10), we can get

$$\psi_{A,n+1} = T \psi_{A,n}$$  \hspace{1cm} (12)$$

where $T = [H_{\Lambda}(l_0)^{-1} H_{B}(l_n) H_{\Lambda}(l_0)^{-1}]$ is the transfer matrix between adjacent periods of the beam.

For the infinite period structure illustrated in figure 1, the Bloch boundary condition [34] on the beam can be described as

$$\psi_{A,n+1} = e^{i\omega n} \psi_{A,n}$$  \hspace{1cm} (13)$$
where \( qa \) is the wave propagation constant in the infinite periodic structure and \( \alpha = l_A + l_B \) is the lattice constant of the 1D metamaterial. Here, \( l_A \) and \( l_B \) are the lengths of beam segments A and B.

Combining equations (12) and (13), an eigenvalue equation as below can be derived [31]:

\[
| \mathbf{T} - e^{i\omega \mathbf{I}} | = 0
\]  

(14)

For a given \( \omega \), we can calculate the corresponding propagation constant \( qa \) with equation (14). The real part of \( q \) is the wave vector (also called the wave number in one-dimensional cases). The graph of wave vector versus frequency is called the dispersion curve. When the propagation constant \( qa \) is a real number, a flexural wave can propagate freely in the infinite metamaterial structure. On the contrary, a non-zero imaginary part of \( qa \) (named the attenuation constant \( \mu = \text{imag} \)).

Figure 2. Designed poles and zeros of the transfer function \( E^b(s) \) for (a) single-resonant and (b) multi-resonant control sets. The crosses and circles represent the poles and zeros, respectively.

Figure 3. Experimental setup.
The analog transfer function $f$ is determined, the analog transfer function $Y$ is the vibration on the free edge in the frequency domain. Based on the harmonic assumption, the FRF is measured by two accelerometers LC0101-2 and a data acquisition board USB-1616FS. Post-processing of all the phase and amplitude data measured with accelerations is equivalent to that calculated with displacements.

2.2. The ‘pole-zero’ designing method

In coordination with the new digital shunting controllers, the expression of $E_p^b(s)$ in equation (6) is regarded as a transfer function where the strain is the input ‘signal’ and the stress is the output one. Thus, it can also be described with the typical zero-pole-gain model

$$E_p^b(s) = k \prod_{n=1}^{m} \left( s - z_m \right) \prod_{n=1}^{b} \left( s - p_n \right)$$  \hspace{1cm} (16)$$

where $p_n$, $z_m$, and $k$ are system poles, zeros, and gain, respectively.

Equation (16) can be used to design the equivalent dynamic elastic modulus of PZT patches $b$ and generate LR bandgaps afterwards. A conjugate pair of poles represents a resonant mode. Their imaginary parts are positive/negative angular frequency $\omega_{osc} = 2\pi f_{osc}$ of it, while their real parts $R$ correspond to the damping factor. When the form of $E_p^b(s)$ is determined, the analog transfer function $H_{DP}(s)$ of the controller can be derived from equation (6). Then the $z$-transformations are applied to convert $H_{DP}(s)$ into a recurrence equation over discrete times, which is necessary for the programming in microprocessors to fulfill the required $E_p^b(s)$.

All the designed poles, zeros, and gains are listed as seven control sets in table 3 and illustrated in figure 2 (the Pole-Zero map). Control sets 1–4 correspond to a single-resonant controller while the others are related to multi-resonant ones (with two or three resonant modes). As the Young’s modulus of the shorted PZT patches used in this paper is about $9.1 \times 10^{11}$ Pa [31], the gain is chosen as $k = 1 \times 10^{11}$ Pa in all the control sets. Each control set of the controller corresponds to one (single-resonant controller) or more (multi-resonant controller) pairs of poles. Each pair of poles corresponds to one pair of zeros with the same real parts $R$ and smaller imaginary parts $\pm \eta \omega_{osc}$, where $\eta$ is regarded as a parameter in the control sets.

2.3. Experimental setups

Figure 3 shows the vibration experimental setup as well as the smart EMM specimen. The specimen is made up of an aluminum alloy (6061-T6) beam and six arrays of pairs of PZT (P-42) patches glued on it. The lattice constant of the EMM structure $a = 0.1$ m. All the geometric and material parameters of the specimen can be found in tables 1 and 2. The specimen is hung up by thin and soft strings in order to simulate a free boundary condition. A filtered (1.2 kHz low-pass) white noise signal is generated by the computer and amplified by the power amplifier (HEAS-20). It is then transformed into vibration and applied on left edge of the specimen through the exciter (HEV-20). Vibration accelerations at both the left and right sides of the specimen are measured by two accelerometers (LC0101), a signal conditioner (LC0201-2), and a data acquisition board (USB-1616FS). Post-processing of all the phase and amplitude data.
The vibration responses is conducted in the computer, and thus the vibration FRF from the left to right sides of the specimen is measured. The circuit board illustrated in figure 1(c) has two independent digital controllers. Three circuit boards are used in the experiments in order to provide the active shunting for six pairs of PZT patches. The circuit boards are powered by a DC power supply (Zhaoxin RXN-6050D), which provides two voltage sources (±60 Volts and +5 Volt).

| Modal Frequency $f_n$ (Hz) | 57.3 | 183.4 | 379.1 | 644.0 |
|----------------------------|------|-------|-------|-------|
| **Calculated** average attenuation near modal frequency (dB) $0.9f_n \sim 1.1f_n \pm 10\%$ bandwidth |
| Control set 1 | 0.41 | 0.73 | 1.75 | **9.17** |
| Control set 2 | 0.50 | 1.38 | **7.55** | 0.63 |
| Control set 3 | 0.62 | **11.01** | 0.23 | 0.00 |
| Control set 4 | **9.29** | 0.04 | −0.01 | −0.03 |
| Control set 5 | 0.76 | 1.64 | **7.00** | **5.55** |
| Control set 6 | 0.74 | **5.29** | 0.73 | 0.55 |
| Control set 7 | 0.90 | **5.71** | 5.20 | **3.87** |

| **Measured** average attenuation near modal frequency (dB) $0.9f_n \sim 1.1f_n \pm 10\%$ bandwidth |
| Control set 1 | 1.68 | **5.46** | 4.34 | **0.15** |
| Control set 2 | 0.81 | **1.13** | **4.34** | **2.99** |
| Control set 3 | 0.69 | **7.40** | −0.05 | −0.13 |
| Control set 4 | **3.89** | −0.09 | −0.17 | −0.11 |
| Control set 5 | 1.57 | 1.66 | **4.45** | **2.99** |
| Control set 6 | 0.40 | **5.46** | **4.34** | **2.99** |
| Control set 7 | 1.93 | **5.29** | 3.29 | **1.97** |

| Average attenuation over 0–800 Hz (dB) |
| Control set 1 | 1.42(Calculated)/0.97(Measured) |
| Control set 2 | 0.72(Calculated)/0.44(Measured) |
| Control set 3 | 0.26(Calculated)/0.15(Measured) |
| Control set 4 | 0.01(Calculated)/−0.04(Measured) |
| Control set 5 | 1.35(Calculated)/0.89(Measured) |
| Control set 6 | 0.77(Calculated)/0.51(Measured) |
| Control set 7 | 1.08(Calculated)/0.82(Measured) |

![Figure 5](image-url) (a) Calculated dispersion curve/attenuation factors and (b) the equivalent elastic modulus of beam segments A when the control set 3 is applied. The subfigures on the right illustrate the corresponding zoom views of (a) and (b).
3. Results and discussion

In figure 4, the calculated attenuation factors of the smart EMM structure governed by single-resonant controllers (control sets 1–4) are compared with both the calculated and measured vibration FRF of its specimen. The ticks on the upper horizontal axis in each sub-figure denote the normalized frequencies \( f_a/\omega_{beam} \), where \( f \) is the actual frequency, \( a \) is the lattice constant, and \( \omega_{beam} \) is the velocity of the flexural wave on the hosting beam. The normalized frequency of subwavelength scale should be obviously below 0.5, about where the lowest Bragg bandgap exists. For the hosting beam with rectangle cross section in this paper, \( \omega_{beam} \) can be calculated with [35]

\[
c_{\text{beam}} = \sqrt{\frac{h \omega}{2 \sqrt{3} \sqrt{E_{Al} \rho_{Al}}}}
\]

(17)

where \( E_{Al} \) is the Young’s modulus of the aluminum alloy, \( h \) is the thickness of the beam, and \( \omega \) is the angular frequency.

All the theoretical and experimental results in figure 4 basically match. Large peaks of attenuation factors corresponding to the designed frequencies are observed in figure 4(a).

The normalized frequencies of these attenuation peaks are 0.077, 0.144, 0.206, and 0.267, respectively, which certifies the LR mechanism of these bandgaps. Since attenuation factors reflect the weakening effects of wave propagation per period in an ideal infinite periodic structure, these peaks strongly weaken the corresponding modal peaks in the FRF curves illustrated in figures 4(b) and 4(c). In the experimental results shown in figure 4(c), the lowest four modal peaks drop by 10.9, 20.9, 19.8, and 18.5 dB, respectively. In order to evaluate the attenuation abilities more objectively, table 4 illustrates both the calculated and measured average vibration attenuations that are induced by the proposed smart EMM for different control sets around different modal frequencies \( f_n \). The averaging window in the frequency domain is chosen as 90\% \times f_n to 110\% \times f_n, and those modal frequencies \( f_n \) are obtained from the peaks of the FRF curve in the short circuit case. The average attenuation over the studied frequency range (0–800 Hz) are also illustrated in table 4. These measured attenuations are significant even though they are not as large as the predictions, which is mainly due to the signal–noise limitation of the measurement system where the strongly weakened vibration signal is covered up by noise.
Figure 7. (a) Calculated attenuation factors of the proposed smart EMM governed by multi-resonant controller (control sets 5–7). (b) Calculated and (c) measured FRF of the specimen of the smart EMM structure. The thin solid lines represent the results in which all the PZT patches are shorted.

Figure 8. Measured FRF of the smart EMM specimen under control set 5 when only two, four, or six smart shunting units near the vibration exciter are active. All the PZT patches corresponding to the inactive shunting units are shorted. The thin solid lines represent the results in which all the PZT patches are shorted.

of the attenuation peak, the half-peak bandwidth narrows visibly with \( R \) and the maximum attenuation factor increases exponentially with \( R \). This means that the parameter \( R \) has only the damping effect as previously discussed. Afterwards, when the parameter \( \eta \) increases towards 1, the attenuation peak value decreases dramatically while its frequency increases slightly towards \( f_{osc} \). Thus, the parameter \( \eta \) can be used to adjust the intensity of the attenuation. Finally, when the resonant frequency \( f_{osc} \) of the controller increases, both the frequency and value of the attenuation peak increases. Therefore, \( f_{osc} \) should be chosen first to adjust the frequency of attenuation roughly, and then \( \eta \) and \( R \) can be used to adjust the intensity and bandwidth of the attenuation peaks. During the adjusting process, negative values of the equivalent Young’s modulus of beam segments \( A \) should be avoided in order to insure the stability of the active control system.

When more pairs of poles and zeros are involved in equation (16), multi-resonant controllers can be realized easily instead of the usage of numerous analog electronic components [14]. In figure 7, the calculated attenuation factors of the smart EMM structure governed by dual- and tri-resonant controllers (control sets 5–7) are compared with the corresponding calculated and measured vibration FRF. The use of dual- and tri-resonant controllers allows additional low-frequency bandgaps, which is due to the enhanced shunting effect that corresponds to the resonances of \( E_s(s) \). The 3rd–4th, 2nd–3rd, or 2nd–4th modal peaks in FRF are strongly and simultaneously attenuated when control sets 5–7 are applied respectively.

In order to demonstrate how the number of active-shunted unit cells affects the ability of attenuations within bandgaps, figure 8 illustrates the measured FRF of the smart EMM structure under control set 5 when only two, four, or six smart shunting units near the vibration exciter are active. All the PZT patches corresponding to the inactive shunting units are shorted. The thin solid lines represent the results in which all the PZT patches are shorted. We can see that vibration attenuations within the bandgaps are larger when more cells (or periods) are engaged, which matches with the common sense of metamaterials. When more cells (or periods) are
engaged, the properties of the metamaterials are more close

to the dispersion analysis based on one unit cell with the
Bloch boundary condition. As for the vibration attenuation,
more cells ordinarily produce larger attenuations within the
bandgaps.

4. Conclusions

In this paper, we have proposed digital controllers for active
piezoelectric shunting and applied them to an elastic meta-
material beam integrated with arrays of piezoelectric patches.
The corresponding ‘pole-zero’ design method has been
developed, which allows us to design the frequencies, band-
widths, and attenuations of single- or even multi-resonant
bandgaps in the deep subwavelength region simply by
adjusting poles and zeros in the transfer function of Young’s
modulus. Theoretical and experimental results have shown
that the proposed smart metamaterial can achieve a large
insulation of up to 20 dB in low-frequency ranges. Negative
group velocity of the flexural wave has been found in the
bandgaps, and thus the locally resonant mechanism has been
confirmed. Furthermore, as parameters in digital controllers
can be readily adapted synchronously via wireless broad-
casting, the bandgaps of the metamaterial can be tuned easily,
without modifying the structure or the shunting circuits.

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