TWO-ECHELON SUPPLY CHAIN MODEL WITH MANUFACTURING QUALITY IMPROVEMENT AND SETUP COST REDUCTION

BISWAJIT SARKAR
Department of Industrial & Management Engineering
Hanyang University
Ansan Gyeonggi-do, 426 791, South Korea

ARUNAVA MAJUMDER
Department of Basic Science And Humanities
Techno India College of Technology
Rajarhat, Kolkata 700 156, India

MITALI SARKAR∗
Department of Industrial & Management Engineering
Hanyang University
Ansan Gyeonggi-do, 426 791, South Korea

BIKASH KOLI DEY
Department of Mathematics & Statistics
Banasthali Vidyapith
Banasthali, Rajasthan, 304 022, India

GARGI ROY
New Jersey Institute of Technology University Heights
Newark, NJ 07102-1982, USA

(Communicated by Wing-Kuen Ling)

Abstract. For quality improvement purposes often times, a manufacturing unit has to change certain parts of equipment. Any such changes in the assembly line manufacturing system or production process involves a cost known as the setup cost. Minimizing the setup cost and improving the product quality is of prime importance in today’s competitive business arena. This paper develops the effects of setup cost reduction and quality improvement in a two-echelon supply chain model with deterioration. The objective is to minimize the total cost of the entire supply chain model (SCM) by simultaneously optimizing setup cost, process quality, number of deliveries, and lot size. Numerical examples are provided to illustrate the model.

2010 Mathematics Subject Classification. Primary: 58F15, 58F17; Secondary: 53C35.

Key words and phrases. Supply chain management, manufacturing quality improvement, manufacturing setup cost reduction, deterioration, SSMD policy.

∗ Corresponding author: mitalisarkar.ms@gmail.com(Mitali Sarkar), Phone Number-010-7490-1981, Fax No +82-31-436-8146.

1085
1. Introduction. In an attempt to refurbish manufacturing operations it is important to improve supply chain and logistics associated with it. Before venturing into any business investments, manufacturers must delve into the matters pertaining to supply chain management and distribution models and the successive impacts. The main objective of this is to reduce the inventory, increase the speed of transactions with real-time data exchange and to maximize revenue by satisfying customer demands more efficiently. Admittedly, an integrated inventory control policy is a matter of concern when dealing with SCM. The following papers [46, 50, 47, 4] are based on an inventory control policy. Goyal [10] presented the first research work on an integrated vendor-buyer problem which was further extended by Banerjee [2] with a joint economic lot size model for the purchaser and vendor.

When dealing with both parties (vendor-buyer or manufacturer-retailer) there are various policies that confirms how a product will be delivered, namely the SSMD (Single-setup-multi-delivery) and the SSSD (Single-setup-single-delivery). Choosing a suitable delivery policy is an important criteria. A single-vendor single-buyer integrated production inventory model was extended by Hill [14] as a generalized policy. Cárdenas-Barrón [5] presented a note on optimizing inventory decisions in a multi-stage multi-customer supply chain. Cárdenas-Barrón [6] discussed the variation of inventory models with two backorder costs using analytical geometry and algebra. Teng et al. [45] considered the economic lot size of the integrated vendor-buyer inventory model without derivatives and with a closed form optimal solution. Asghari et al. [4] reversed a logistic network design with incentive-dependent return. Watanable and Kusukawa [48] evaluated an optimal ordering policy in dual-sourcing supply chain considering supply disruptions and demand information. Wisittipanich and Hengmeechai [49] discussed about a multi-objective differential evolution for just-in-time door assignment and truck scheduling in multi-door cross docking problems. Park [26] invented a partial backordering inventory model where purchasing of products have an important role. Kusukawa and Alozawa [19] approached an optimal operation for green supply chain with quality of recyclable parts and contract for recycling activity. Sarkar and Moon [36] developed an improved quality and reduced set up cost and variable backorder costs in an imperfect production process.

The effect of degradation of items in the inventory model was first studied by authors Ghare and Schrader [8]. An EOQ model for deteriorating items with Weibull distribution was later discussed by Covert and Philip [7]. Misra [21] and Shah [40] respectively proposed an optimal production lotsize model and an order-level lot size model for a system with deteriorating inventory. Economic ordering policy for deteriorating items over an infinite time horizon was developed by Goyal [11]. A literature survey on continuously deteriorating inventory model was done by Raafat [29]. The inventory models with different types of deteriorating rates was extended in this direction by some researchers like Goyal [12], Goswami and Chaudhuri [9], Skouri and Papachristos [43], Skouri et al. [44], Sarkar [31], Sarkar et al. [37], Sarkar and Sarkar [38], Sarkar and Sarkar [39], Sarkar [32].

Supply chain management (SCM) and manufacturing models help the industries to generate low-cost ideas which ensure business productivity and success. The focus is to make valuable decisions regarding system design as it appears in manufacturing systems and supply chains. The economic production quantity (EPQ) model, one of the oldest classical production scheduling models applies only when demand for a product is constant over the year and each new order is delivered in full when
the inventory reaches zero, the setup cost is constant and lead time is negligible, and quality is assumed to be fixed. However, this may not be realistic as it is seen in many practical situations that quality can be improved and setup cost can be reduced. Such an issue was named by Silver et al. [42] as hanging the given where the given considered a parameters can be assumed as fixed.

In order to improve customer service level and to reduce safety stock, delivery lead time should be shortened. A probabilistic inventory model concerning lead time reduction was first presented by Liao and Shyu [20] in which the order quantity is predetermined and lead time is a unique variable. Liao and Shyu [20] model was extended by Ben-Daya and Raouf [3] by considering both lead time and order quantity as decision variables. Ben-Daya and Raouf’s [3] model was generalized by Ouyang et al. [21] by allowing shortages with partial backorders which was further revised by Moon and Choi [23] and Hariga and Ben-Daya [13] to include the reorder point as one of the decision variables. Benefits from lead time reduction was the main focus in all these aforesaid papers where the quality-related issues are not taken into account and the setup cost is treated as a fixed constant.

In the long run it is probable for a production process to produce defective items. As a result, a cost is to be incurred in order to ensure that the product quality is improved. The production of defective items in a real production environment often leads to the incurring of substantial costs in the ordering policy. Therefore, it is pertinent to take quality-related costs into account while determining the optimal ordering policy. Porteus [27] initially presented the concept and framework of setup cost reduction in another paper. There are several papers that discuss setup cost reduction like Keller and Noori [17], Paknejad et al. [25], Sarker and Coates [34]. The significant relationship between quality imperfection and lot size was explicitly elaborated by Porteus [28] and Rosenblatt and Lee [30] who were among the first to do so.

The effects of investment in quality improvement and setup cost reduction was studied specifically by Porteus. Many researchers like Keller, Noori [18], Hwang et al. [16], Moon [22], Hong and Hayya [15] were encouraged by his work on modeling the quality improvement issue. From the above literature review it is evident that there is no shortage of those studying lead time reduction, quality improvement, and setup cost reduction, but little work has been done on considering them simultaneously. Silver [41] said that “if mathematical models are to be more useful as aids for managerial decision making, then they must represent more realistic problem formulations, particularly permitting some of the aforementioned given to be treated as decision variables”. He illustrated the effect of changing a given by using two numerical examples. Sarkar et al. [35] discussed about the quality improvement and backorder price-discount under controllable lead time in an inventory model. Sarkar et al. [33] reduced the manufacturing cost and improved the quality for the distribution free continuous-review inventory model with a service level constraint.

This article optimizes the setup cost, process quality, lot size, and the integral number of deliveries per production batch cycle, simultaneously with the objective to minimize the total relevant cost. The design of this paper is as follows: Section 1 contains introduction, in Section 2, notation and assumptions are given. Section 3 contains model formulation. Numerical examples are provided in Section 4. See Table 1 for contribution of authors.
| Author(s) Name | Setup cost reduction | Quality improvement | Lot size | Deterioration | SSMD |
|----------------|----------------------|---------------------|----------|---------------|------|
| Goyal [10]     | √                    |                     |          |               |      |
| Sarkar and Sarkar [48] |                     |                     |          |               |      |
| Goswami and Chaudhuri [9] |                     |                     |          |               |      |
| Sarker and Coates [44] |                     |                     |          |               |      |
| Skouri and Papachristos [31] |               |                     |          |               |      |
| Sarkar [37] |                      |                     |          |               |      |
| Sarkar et al. [38] |                     |                     |          |               |      |
| Sarkar and Sarkar [39] |                     |                     |          |               |      |
| Porteus [27] |                      |                     |          |               |      |
| Paknejad et al. [25] |                     |                     |          |               |      |
| Hong and Hayya [34] |                     |                     |          |               |      |
| Rosenblatt and Lee [30] |                     |                     |          |               |      |
| This paper              |                      |                     |          |               |      |

2. **Notation and assumptions.** The model is developed on the basis of the following notation

2.1. **Notation.**

2.2. **Decision variables.**

- $q$ delivery lot size (units)
- $N$ number of deliveries per production batch, $N \geq 1$
- $C$ setup cost for a production batch ($/\text{setup}$)
- $\theta$ probability that the process may go to ‘out-of-control’ state.

2.3. **Parameters.**

- $C_o$ original setup cost for a production batch ($/\text{setup}$)
- $A$ ordering cost for the buyer ($/\text{order}$)
- $A_b$ area under the buyer’s inventory level
- $A_s$ area under the supplier’s inventory level time
- $s$ cost of replacing a defective item ($/\text{item}$)
- $D$ demand (units/unit time)
- $F$ transportation cost per delivery ($/\text{delivery}$)
- $d$ deterioration rate
- $C_d$ deterioration cost per unit ($/\text{unit}$)
- $H_s$ holding cost for the supplier ($/\text{unit/unit time}$)
- $H_b$ holding cost for the buyer ($/\text{unit/unit time}$)
- $Q$ production lot size per batch-cycle (units)
- $P$ production rate (unit/unit time)
- $V$ unit variable cost for order handling and receiving ($/\text{unit}$)
- $T$ duration of inventory cycle (time unit)
- $T_1$ production time duration for the supplier (time unit)
- $T_2$ non-production time duration for the supplier (time unit)
- $T_3$ duration between the two successive deliveries (time unit)
- $TC$ total cost of the system ($/\text{unit time}$)
2.4. **Assumptions.** The following assumptions are considered to develop the model.

1. Two-echelon supply chain model (SCM) is considered with a buyer and a supplier for single-type of product.
2. To save buyer’s holding cost, the single-setup-multi-delivery (SSMD) policy is assumed for transportation of products between vendor and buyer.
3. As SSMD policy is used to save the holding cost of buyer, thus buyer pays transportation costs. For SSMD policy, it is assumed that there are some constant transportation costs and some variable costs. These both constant and variable transportation cost are paid by buyer.
4. Information regarding the inventory position and demand of the buyer are given to the supplier. Production rate is always greater than demand, i.e., $P > D$ such that there are no shortages.
5. The model assumes SSMD policy which indicates that the products are produced within a single-setup which is generally in long-run. Thus, during long-run, any time the production of defective items may come.
6. The vendor uses automation policy (automatically detects the defective item by machine, no human inspector is needed to inspect the defectiveness of items) to detect the imperfect production. As a result, if the system moves to out-of-control state from in-control state, it will continue production of defective items until the whole lot is produced.
7. Two investments are considered to reduce setup cost and to improve quality of products.
8. A constant rate of deterioration is considered for products.

3. **Mathematical model.** A single-setup-multi-delivery (SSMD) policy for a supplier is developed in this model and the average total cost of the production-inventory model is developed for the buyer’s and the supplier’s which is then minimized. In the proposed model the buyer’s ordered quantity is manufactured at a time and the manufactured products are delivered in an equal amount over multiple deliveries after a fixed interval of time. The splitting of the ordered quantity into multiple lots is in accordance with JIT implementation. Without any loss of generality, it is considered that the products ordered arrives at the exact time when the items from the previous delivery has just been depleted. The total time span $T$ is divided into two components: $T_1$, the production time duration for the supplier and $T_2$, the non-production time duration for the supplier. $T_3$ is considered as the time duration between the two successive deliveries. Now separately the buyer’s and the supplier’s inventory cost are calculated as follows:

3.1. **Buyer’s cost function.** The buyer’s cost function is comprised of the following relevant costs.

1. Ordering cost = $A$ 
2. Holding cost = $H_b A_b$, where $A_b$ is the area over which the inventory holds for buyer.
3. Deterioration cost = $C_d d A_b$
4. Transportation cost and handling cost = $\frac{(NF + VNq)}{T}$

From Figure 1 and Figure 2, we obtain

$$\frac{1}{T} = \frac{D}{Nq} + \frac{d}{2N},$$

(1)
Using the above equations, the buyer’s total cost function is obtained as

\[ TC_b = \left( \frac{D}{Nq} + \frac{d}{2N} \right) (A + NF + VNq) + \frac{q}{2} [H_b + C_d d] \] (3)

3.2. **Supplier’s cost function.** Now the supplier’s cost function is comprised of the following relevant costs.
1. Setup cost = \( \frac{C}{T} \)

2. Holding cost = \( \frac{H_s A_s}{T} \), where \( A_s \) is the area over which the inventory holds for supplier.

3. Deterioration cost = \( \frac{C_d d A_s}{T} \)

Let \( y \) be the number of deteriorating units for the supplier. \( y \) can be expressed as \( y = d A_s \). \( y + \frac{dqT}{2} \) denotes the total number of deteriorating items for the entire SCM. With the following expressions \( Q = Nq + y \) and \( t_1 = \frac{Q}{T} \) and considering the initial and the total inventory for the entire SCM, we obtain

\[
y + \frac{dqT}{2} = \frac{dT}{2P} \{2Dq + (Nq + y)(P - D)\}.
\]

Hence,

\[
A_s = \frac{y}{d} = \frac{qT}{P} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) . \tag{4}
\]

Considering an investment for quality improvement and setup cost reduction, the capital investment function is assumed as a logarithmic function as suggested by Porteus [14]. \( I_\theta(\theta) \), investment to reduce the ‘out-of-control’ probability \( \theta \) is given as

\[
I_\theta(\theta) = b \ln \left( \frac{\theta_0}{\theta} \right) \text{ for } 0 < \theta \leq \theta_0
\]

It is to be noted that lower value of the probability \( \theta \) gives higher value of quality level, where \( \theta_0 \) is the initial probability that the production process may go to ‘out-of-control’ state and \( b = 1/\delta \), where \( \delta \) being the percentage decrease in \( \theta \) per dollar increase in \( I_\theta(\theta) \).

Now \( I_C(C) \), the investment for setup cost reduction is expressed as

\[
I_C(C) = B \ln \left( \frac{C_0}{C} \right) \text{ for } 0 < C \leq C_0
\]

where \( C_0 \) is the initial setup cost, \( B = 1/\Delta \), \( \Delta \) being the percentage decrease in \( C \) per dollar increase in \( I_C(C) \).

Thus the total investment in quality improvement and setup cost reduction is obtained as

\[
I(\theta, C) = I_\theta(\theta) + I_C(C) = G - b \ln \theta - B \ln C,
\]

where \( G = b \ln \theta_0 + B \ln C_0 \) [see for instance Sarkar and Moon [36], Sarkar et al. [35], [33]]

This model considers a possible relationship between lot size and quality by incorporating a quality-related cost. In an imperfect production process there is a certain probability \( \theta \) that a system may go to ‘out-of-control’ state. \( \theta \) is provided and considered to be very small and close to zero. Once the process goes to ‘out-of-control’ state it starts producing defective items and continues to do so unless the entire lot is produced. In such a situation, the expected number of defective items in a production lot size \( Q \) is approximated to be \( \frac{Q^2 \theta}{2} \) (for more details [See Appendix A]). Again it is considered \( s \) as the cost of replacing a defective item.

Thus, the expected annual defective cost is \( \frac{sD^2}{2} \left[ Nq + \frac{2Ndq^2}{2D + dq} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) \right] \) (for more details [See Appendix B]).
Therefore, the supplier’s total cost function is obtained as
\[ TC_s = \left( \frac{D}{Nq} + \frac{d}{2N} \right)C + q(H_s + C_d)\left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) + \alpha(G - b \ln \theta - B \ln C) + \frac{sD\theta}{2} \left[ Nq + \frac{2Ndq}{2D + dq} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) \right] \quad (5) \]

The integrated inventory cost for the entire SCM is obtained as \( TC_b + TC_s \)

\[ TC(\theta, N, q, C) = \left( \frac{D}{Nq} + \frac{d}{2N} \right)(A + C + NF + VNq) + q \left( B \right) + (H_s + C_d)\left( \frac{(2 - N)D}{P} + N - 1 \right) + \alpha(G - b \ln \theta - B \ln C) + \frac{sD\theta}{2} \left[ Nq + \frac{2Ndq}{2D + dq} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) \right] \quad (6) \]

for \( 0 < \theta \leq \theta_0 \) and \( 0 < C \leq C_0 \), \( \alpha \) being the fractional cost of capital investment (e.g., the rate of interest).

3.3. Lemma. If \( \theta^*, N^*, q^*, C^* \) are the optimal values of \( \theta, N, q, C \), then \( TC(\theta, N, q, C) \) is global minimum solution at \( \theta^*, N^*, q^*, C^* \) if the constraints \( 0 < \theta \leq \theta_0 \) and \( 0 < C \leq C_0 \) are relaxed.

3.4. Proof. For the global minimum solution of the total cost function the principal minor should be positive definite. For that purpose, the principal minor has to be greater than zero at the point where the first order partial derivatives with respect to \( \theta, N, q, C \) equal to zero.

Differentiating (6) partially with respect to \( \theta, N, q, C \), respectively, one can obtain

\[ \frac{\partial TC}{\partial \theta} = -\frac{ab}{2\theta} + \frac{sD}{2}\left[ Nq + \frac{2Ndq}{2D + dq} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) \right] \quad (7) \]

\[ \frac{\partial TC}{\partial N} = -\left( \frac{D}{Nq^2} + \frac{d}{2N^2} \right)(A + C + NF + VNq) + \left( \frac{D}{Nq} + \frac{d}{2N} \right)(F + Vq) + \frac{q}{2} \left( H_s + C_d \right) \left( 1 - \frac{D}{P} \right) + \frac{sD\theta}{2} \left[ q + \frac{2dq^2}{2D + dq} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) + \frac{2Ndq^2}{2D + dq} \left( 1 - \frac{D}{P} \right) \right] \quad (8) \]

\[ \frac{\partial TC}{\partial q} = -\frac{D}{Nq^2}(A + C + NF + VNq) + \left( \frac{D}{Nq} + \frac{d}{2N} \right)VN + \frac{1}{2} \left( H_B + C_d \right) + (H_s + C_d)\left( \frac{(2 - N)D}{P} + N - 1 \right) + \frac{sD\theta}{2} \left[ N + \frac{4Ndq}{2D + dq} - \frac{2Ndq^2}{(2D + dq)^2} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) \right] \quad (9) \]

\[ \frac{\partial TC}{\partial C} = \left( \frac{D}{Nq} + \frac{d}{2N} \right) - \alpha \left( \frac{B}{C} \right) \quad (10) \]
To find optimal value, the values of $\frac{\partial TC}{\partial \theta}$, $\frac{\partial TC}{\partial N}$, $\frac{\partial TC}{\partial q}$, $\frac{\partial TC}{\partial C}$ equal to zero, one has from (7)

$$-\alpha b + sDq^2 = 0$$

$$\Rightarrow \theta = \frac{2b\alpha}{sD}$$

from (8)

$$-\left(\frac{D}{N^2q} + \frac{d}{2N^3}\right)(A + C + NF + VNq) + \left(\frac{D}{Nq} + \frac{d}{2N}\right)(F + Vq) + \frac{q}{2} \left[H_s + C_d\left(1 - \frac{D}{P}\right)\right] + \frac{sD\theta}{2} \left[q + \frac{2dq^2}{2D + dq} \left(\frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P}\right) + \frac{2Ndq^2}{2D + dq} \left(1 - \frac{D}{P}\right)\right] = 0$$

$$\Rightarrow N = \sqrt{\frac{\phi_2 - 3\phi_3}{\phi_1}}$$

where,

$$\phi_1 = \frac{q}{2} \left[H_s + C_d\left(1 - \frac{D}{P}\right)\right] + \frac{sD\theta q}{2D + dq} (1 + \frac{dq}{P})$$

$$\phi_2 = \frac{(2D + dq)(A + C)}{2q}$$

$$\phi_3 = \frac{3}{2} \left(1 - \frac{D}{P}\right) \left(\frac{sD\theta q^2}{2D + dq}\right)$$

from (9)

$$-\left(\frac{D}{N^2q^2}(A + C + NF + VNq) + \left(\frac{D}{Nq} + \frac{d}{2N}\right)VN\right) + \frac{1}{2} \left[H_B + C_d\{\frac{(2 - N)D}{P} + N - 1\}\right] + \frac{sD\theta}{2} \left[N + \left(\frac{4Ndq}{2D + dq} - \frac{2Ndq^2}{(2D + dq)^2}\right) \left(\frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P}\right)\right] = 0$$

$$\Rightarrow q = \sqrt{\frac{\rho_1}{\rho_4 + N\rho_2 \{1 + \frac{2dq}{(2D + dq)^2}\}}}$$

where,

$$\rho_1 = \frac{D}{N}(A + C + NF)$$

$$\rho_2 = \frac{sD\theta}{2}$$

$$\rho_3 = \left(\frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P}\right)$$

$$\rho_4 = \frac{1}{2} \left[H_B + C_d\{\frac{(2 - N)D}{P} + N - 1\}\right] + \frac{dV}{2}$$
from (10)
\[
\left( \frac{D}{Nq} + \frac{d}{2N} \right) - \alpha \left( \frac{B}{C} \right) = 0
\]
\[
\Rightarrow C = \frac{2\alpha BNq}{2D + dq}
\] (14)

Now, the Hessian matrix \( H \) are calculated as follows:
\[
H = \begin{bmatrix}
\frac{\partial^2 TC}{\partial \theta^2} & \frac{\partial^2 TC}{\partial \theta \partial N} & \frac{\partial^2 TC}{\partial \theta \partial q} & \frac{\partial^2 TC}{\partial \theta \partial C} \\
\frac{\partial^2 TC}{\partial N \partial \theta} & \frac{\partial^2 TC}{\partial N^2} & \frac{\partial^2 TC}{\partial N \partial q} & \frac{\partial^2 TC}{\partial N \partial C} \\
\frac{\partial^2 TC}{\partial q \partial \theta} & \frac{\partial^2 TC}{\partial q \partial N} & \frac{\partial^2 TC}{\partial q^2} & \frac{\partial^2 TC}{\partial q \partial C} \\
\frac{\partial^2 TC}{\partial C \partial \theta} & \frac{\partial^2 TC}{\partial C \partial N} & \frac{\partial^2 TC}{\partial C \partial q} & \frac{\partial^2 TC}{\partial C^2}
\end{bmatrix}
\] (15)

Where \( TC = TC(\theta, N, q, C) \).

Now we proceed by evaluating the principal minor of \( H \).

The first principal minor of \( H \) is
\[
|H_{11}| = \left| \frac{\alpha b}{q^2} \right| > 0
\]

The second principal minor of \( H \) is
\[
|H_{22}| = \frac{\alpha b}{x_3} \frac{2}{N^3} \left( \frac{D}{q} + \frac{d}{2} \right) (A + C) + \frac{sD\theta dq(1 - \frac{D}{P})}{2(2D + dq)}
\]
\[
= \frac{\alpha b}{q^2} \left[ \frac{2}{N^3} \left( \frac{D}{q} + \frac{d}{2} \right) (A + C) + \frac{sD\theta dq(1 - \frac{D}{P})}{2(2D + dq)} \right] - x_3^2 > 0
\] (16)
as \( \theta \) is very small and it is in the denominator with square power. Thus, the 1st term of the expression is a large positive quantity than the 2nd term of the expression, even though it is in power two. Hence, for small value of \( \theta \), i.e, the small value of \textit{out-of-control} probability, \( H_{22} > 0 \).

The third principal minor of \( H \) is
\[
|H_{33}| = \left| \frac{\alpha b}{q^2} x_3 x_4 \xi_1 \xi_2 \right|
\]
\[
= \frac{\alpha b}{q^2} (x_4 x_1 - \xi_1^2) - x_3(x_3 x_1 - x_2 \xi_1) + x_2(x_3 \xi_1 - x_2 x_4)
\]
\[
H_{33} = x_2(x_3 \xi_1 - x_2 x_4) - \xi_1 \left( \frac{\alpha b}{q^2} \xi_1 + H_{22} \right)
\]
\[
> 2x_2x_3 \xi_1 - x_2^2x_4 - \frac{\alpha b}{q^2} \xi_1^2
\]
\[
= [(x_2x_3)^2 + 2x_2x_3 \xi_1 + \xi_1^2] - \left[ x_2^2(x_3^2 + x_4) + (1 + \frac{\alpha b}{q^2}) \xi_1^2 \right]
\]
\[
= (x_2x_3 + \xi_1)^2 - \left[ x_2^2(x_3^2 + x_4) + (1 + \frac{\alpha b}{q^2}) \xi_1^2 \right]
\]
\[
> 0
\]
[See Appendix E]
The forth principal minor of $H$ is $|H_{44}| = \begin{vmatrix} \frac{ab}{\theta^2} & x_3 & x_2 & 0 \\ x_3 & x_4 & \xi_1 & x_5 \\ x_2 & \xi_1 & x_6 & 0 \\ 0 & x_5 & x_6 & \frac{AB}{C^2} \end{vmatrix}$

$$= x_5 \begin{vmatrix} \frac{ab}{\theta^2} & x_3 & x_2 \\ x_3 & x_4 & \xi_1 \\ 0 & x_5 & x_6 \end{vmatrix} - x_6 \begin{vmatrix} \frac{ab}{\theta^2} & x_3 & x_2 \\ x_3 & x_4 & \xi_1 \\ 0 & x_5 & x_6 \end{vmatrix} + \frac{AB}{C^2} H_{33}$$

$$= \frac{ab}{\theta^2} [2\xi_1 x_5 x_6 - x_1 x_5^2 - x_4 x_6^2] + \frac{ab}{\theta^2} [x_2 x_5^2 - 2 x_3 x_2 x_6 x_5 + x_3 x_6^2] + \frac{AB}{C^2} H_{33}$$

$$> (x_2 x_5 - x_3 x_6)^2 - \frac{ab}{\theta^2} [\xi_1 x_5^2 + \xi_1 x_6^2 - 2\xi_1 x_5 x_6]$$

$$> (x_2 x_5 - x_3 x_6)^2 - \frac{ab}{\theta^2} \xi_1 [x_5^2 + x_6^2 - 2x_5 x_6]$$

$$> (x_2 x_5 - x_3 x_6)^2 - \frac{ab}{\theta^2} \xi_1 (x_5 - x_6)^2$$

$$> 0$$ (See Appendix F) \hspace{1cm} (17)

Thus, the total cost function has the global optimum solution at the optimum values of the decision variables if the conditions are satisfied.

4. **Numerical examples.** The buyer is staying in urban areas whereas the supplier is staying in rural areas, thus the holding cost of the buyer is huge comparing with supplier. This is the reason to use more holding cost for numerical experiment. This type of model can be managed easily by SSMD policy. The numerical data is taken as follows: $A = $10/ order, $P = 100$ units/year, $C_o = $100/batch, $H_b = $4000/unit/year, $H_s = $6/unit/year, $D = 40$ units, $d = 0.02$, $F = 50$/delivery, $V = $1/unit, $C_d = $50/unit and we consider the numerical data $b = 10$, $B = 4800$, $\theta_0 = 0.02$, $\alpha = 0.01$/year, $s = $10/defective unit. Then, the optimal solution is $\{TC = $6297.31/year, $N = 2,C =$ 28.18/order, $q = 1.17$ units, $\theta = 0.0021$.

4.1. **Special case 1.** A special case arises when the rate of deterioration is considered to be zero. From the following Table 2, we see that the single-supplier-single delivery (SSSD) policy is less favorable over SSMD policy as the cost increases for SSSD policy. The results are given in Table 2.

**Table 2** Study for non-deterioration case

| Total cost | Lot size | Number of deliveries | Setup cost | $\theta$ |
|------------|---------|---------------------|------------|--------|
| 6297.31    | 1.17    | 2                   | 28.17      | 0.0021 |

4.2. **Special case 2.** A special case arises when the model follows a single-supplier-single delivery (SSSD) policy instead of SSMD policy then the total cost of the whole supply chain model is increased and it is given in Table 3. The values indicate that SSMD is more beneficial than SSSD for this model.

**Table 3** Study for SSSD case

| Total cost | Lot size | Number of deliveries | Setup cost | $\theta$ |
|------------|---------|---------------------|------------|--------|
| 6342.83    | 1.22    | 1                   | 14.65      | 0.0041 |
5. **Sensitivity analysis.** This section performs sensitivity analysis of this model. This analysis gives clear idea about the behavior of parameters over the cost function. It can also be found out which parameter is more sensitive to the cost. The analysis is given in Table 4.

**Table 4** Sensitivity analysis for key parameters

| Parameters | Changes of parameters (in %) | TC (in %) | Parameters | Changes of parameters (in %) | TC (in %) |
|------------|-------------------------------|----------|------------|-------------------------------|----------|
| $C_o$      | -50%  -2.19                   | -5.29    | $d$        | -50%  -0.01                  | -0.007   |
|            | -25%  1.70                    |          |            | -25%  0.007                  |          |
|            | +50%  3.09                    |          |            | +50%  0.01                   |          |
| $s$        | -50%  -7.47                   |          | $C_d$      | -50%  -0.02                  | -0.007   |
|            | -25%  -3.31                   |          |            | -25%  -0.01                  |          |
|            | +25%  3.73                    |          |            | +25%  0.003                  |          |
|            | +50%  7.47                    |          | $H_s$      | -50%  0.02                   | 0.01     |
|            |                               |          |            | +50%  0.02                   |          |
| $A$        | -50%  -1.67                   |          | $V$        | -50%  -0.32                  | -0.16    |
|            | -25%  -0.83                   |          |            | -25%  -0.16                  |          |
|            | +25%  0.82                    |          |            | +25%  0.16                   |          |
|            | +50%  1.64                    |          |            | +50%  0.32                   |          |
| $F$        | -50%  -18.73                  |          |            |                               |          |
|            | -25%  -8.74                   |          |            |                               |          |
|            | +25%  7.88                    |          |            |                               |          |
|            | +50%  15.12                   |          |            |                               |          |

From Table 4, the sensitivity of the key parameters can be observed easily.

- The negative sensitiveness of setup cost parameter $C_o$ is more sensitive than the positive sensitiveness parameters. But it is more sensitive in negative percentage change than positive percentage change. From this observation, it can be found that if setup cost is reduced, total cost is reduced.
- The parameter $s$ is almost same sensitive with respect to the total cost. Negative percentage change and positive percentage change are almost same. Decreasing this cost, total cost can be reduced effectively. In reverse, if this cost is very high, the total cost will be increased gradually.
- The parameter $A$ which stands for ordering cost for buyer is less sensitive with respect to other parameters within the cost function. The positive and negative percentage change are almost same. If ordering cost is increased, then total cost is increased and vice versa.
- Transportation cost parameter $F$ is more sensitive in negative percentage change than positive percentage change. This parameter is most sensitive parameter comparing to others.
- Sensitivity of this parameter is almost same in negative percentage change than the positive percentage change. When $d$ increases, total cost increases and when $d$ decreases, total cost decreases.
- The effect of this parameter $C_d$, namely, deterioration cost, is like deterioration rate $d$. The negative percentage change and positive percentage change is exactly same. Increased deterioration cost implies decreased total cost and decreased deterioration cost imply increased total cost.
- This parameter $H_s$ is less sensitive with respect to the other parameters. The negative percentage change is much effective for total cost compare to other parameters. If holding cost for supplier reduces, total cost increased reasonably, but when supplier’s holding cost is increased, total cost reduction is not reasonably decreased.
The parameter \( V \) is not rationally sensitive for total cost. The negative percentage change and positive percentage change are same for the unit variable cost for order handling and receiving. When this cost increases, total cost increases and vice versa.

5. Concluding remarks. The objective of this paper was to minimize the total cost of the entire SCM while simultaneously optimizing lot size, number of deliveries, setup cost, and process quality. Two logarithmic investment functions for quality improvement and setup cost reduction, respectively were incorporated in this model. Quality improvement and setup cost reduction played a very significant role in improving efficiency of businesses and organizations from every sphere by reducing redundancy in costs and enhancing productivity thereby accounting for the flexibility of today’s diverse business environment. Any adverse event would have a direct consequence on the business and customers leading to wastage of time and resource. An accurate expertise on the approaches of industries and organizations to implement these changes for a sustainable quality improvement is therefore critical. This model proved the global optimization solution of the decision variables. The model saved a large enough for any business industry to adopt the policies suggested by this model. A constant demand rate is one of the limitations of this model. Further research could be done by considering a general investment function with variable demand. Sarkar’s [31] inventory model with delay-in-payments and time-varying deterioration rate idea could be the extension of this model with a multi-item supply chain model.

Acknowledgments. This work was support by the research fund of Hanyang University (HY - 2016 - P) (Project Number 20160000000282).

Appendix A. From Porteus [43], Sarkar et al. [35] the expected number of defective items in a lot size \( Q \) is

\[
Q - \frac{\hat{\theta}(1 - \hat{\theta}Q)}{\theta}
\]

As \( \hat{\theta} = 1 - \theta \) is approximately 1, we use the Taylor series expansion of \( \hat{\theta}Q \) and obtain

\[
\hat{\theta}Q = e^{(\ln \hat{\theta})Q} \approx 1 + (\ln \hat{\theta})Q + \frac{[(\ln \hat{\theta})Q]^2}{2}
\]

Hence we have the number of defective items

\[
= Q - \frac{\hat{\theta}(1 - \hat{\theta}Q)}{\theta}
\]

\[
= Q - 1 - 1 - (\ln \hat{\theta})Q - \frac{(\ln \hat{\theta})^2Q^2}{2}
\]

\[
= Q - \frac{\theta}{\hat{\theta}Q - \frac{\theta^2Q^2}{2}}
\]

\[
= Q - \frac{\thetaQ^2 - \frac{\theta^2Q^2}{2}}{\theta}
\]

\[
= \frac{\thetaQ^2}{2}
\]
Appendix B. The expected annual defective cost is

\[ \text{Appendix B.} \quad \text{The expected annual defective cost is} \]

\[ sD - \frac{sD\hat{\theta}(1 - \hat{\theta}Q)}{\theta Q} \]

\[ = sD - \frac{sD \left( 1 - (\ln \hat{\theta})Q - \frac{(\ln \hat{\theta})^2 Q^2}{2} \right)}{\theta Q} \]

\[ \text{since} \quad \hat{\theta} = 1 - \theta \equiv 1 \quad \text{and} \quad \hat{\theta}Q = e^{(\ln \hat{\theta})Q} \equiv 1 + (\ln \hat{\theta})Q + \frac{[(\ln \hat{\theta})Q]^2}{2} \]

\[ = sD - \frac{sD \left( \frac{\theta}{\theta} Q - \frac{\theta^2 Q^2}{2} \right)}{\theta Q} \]

\[ = sD - \frac{sD \left( \theta Q - \frac{\theta^2 Q^2}{2} \right)}{\theta Q} \]

\[ = sD - sD \left( 1 - \frac{\theta Q}{2} \right) \]

\[ = \frac{sD \theta Q}{2} \]

\[ = \frac{sD\theta Q}{2} \left[ Nq + 2Ndq^2 \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) \right] \]

Appendix C. The delivery lot size \( q \), expressed as \( q = x + DT_3 \) is divided into two components: \( x \) and \( DT_3 \), \( x \) being the number of deteriorating items during the time span \( T_3 \) and \( DT_3 \) for consumption.

For smaller rate of deterioration square and higher powers of it can be neglected. Hence, during time interval \( T_3 \)

\[ x = \frac{T_3dq}{2} \]

and

\[ q = \frac{T}{N} \left( D + \frac{dq}{2} \right) \quad \text{as} \quad \frac{T}{N} = T_3 \]

Now

\[ q = x + DT_3 \]

implies

\[ \frac{1}{T} = \frac{D}{Nq} + \frac{d}{2N} \]

i.e.,

\[ \frac{q}{2} = \frac{Nq - D}{dq} - \frac{D}{d} \]

Again the total deterioration for the buyer is obtained as

\[ dA_b = Nq - DT \]

which implies

\[ A_b = \frac{(Nq - DT)}{d} \]
i.e.,
\[ \frac{A_b}{T} = \frac{q}{2} \]

**Appendix D.** Differentiating (3) partially with respect to \( \theta, N, q, C \), respectively, one can obtain

\[
\begin{align*}
\frac{\partial^2 TC}{\partial C \partial \theta} &= 0 \\
\frac{\partial^2 TC}{\partial C \partial N} &= -\left( \frac{D}{N^2 q} + \frac{d}{2N^2} \right) \\
\frac{\partial^2 TC}{\partial C \partial q} &= -\frac{D}{Nq^2} \\
\frac{\partial^2 TC}{\partial C^2} &= \frac{\alpha B}{C^2}
\end{align*}
\]

Differentiating (4) partially with respect to \( \theta, N, q, C \), respectively, one has

\[
\begin{align*}
\frac{\partial^2 TC}{\partial \theta^2} &= \frac{\alpha b}{b^2} \\
\frac{\partial^2 TC}{\partial \theta \partial N} &= \frac{sD}{2} \left[ q + \frac{2dq^2}{2D + dq} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) + \frac{2Ndq^2 (1 - \frac{D}{P})}{2D + dq} \right] = x_3 \\
\frac{\partial^2 TC}{\partial \theta \partial q} &= \frac{sD}{2} \left[ N + \left( \frac{4Ndq}{2D + dq} - \frac{2Ndq^2}{(2D + dq)^2} \right) \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) \right] = x_2 \\
\frac{\partial^2 TC}{\partial \theta \partial C} &= 0
\end{align*}
\]

Differentiating (5) partially with respect to \( \theta, N, q, C \), respectively, the partial derivatives are obtained as follows:

\[
\begin{align*}
\frac{\partial^2 TC}{\partial N \partial \theta} &= \frac{sD}{2} \left[ q + \frac{2dq^2}{2D + dq} \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) + \frac{2Ndq^2 (1 - \frac{D}{P})}{2D + dq} \right] = x_3 \\
\frac{\partial^2 TC}{\partial N^2} &= \frac{2}{N^3} \left( \frac{D}{q} + \frac{d}{2} \right) (A + C) + \frac{sDdq^2 (1 - \frac{D}{P})}{2(2D + dq)} = x_4 \\
\frac{\partial^2 TC}{\partial N \partial q} &= \frac{D}{N^2 q^2} (A + C + NF + VN) + \frac{1}{2} \left( 1 - \frac{D}{P} \right) \left( Hs + C_ad \right) \\
&\quad + \frac{sDq}{2} \left[ 1 + \left( \frac{4dq}{2D + dq} - \frac{2dq^2}{(2D + dq)^2} \right) \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) + \frac{2Ndq (1 - \frac{D}{P})}{2D + dq} \right] \\
&= \xi_1 \\
\frac{\partial^2 TC}{\partial N \partial C} &= -\left( \frac{D}{N^2 q} + \frac{d}{2N^2} \right)
\end{align*}
\]

Differentiating (6) partially with respect to \( \theta, N, q, C \), respectively, the partial derivatives are obtained as follows:

\[
\begin{align*}
\frac{\partial^2 TC}{\partial q \partial \theta} &= \frac{sD}{2} \left[ N + \left( \frac{4Ndq}{2D + dq} - \frac{2Ndq^2}{(2D + dq)^2} \right) \left( \frac{D}{P} + \frac{N - 1}{2} - \frac{DN}{2P} \right) \right] = x_2 \\
\frac{\partial^2 TC}{\partial q \partial N} &= \frac{D}{N^2 q^2} (A + C + NF + VN) + \frac{1}{2} \left( 1 - \frac{D}{P} \right) \left( Hs + C_ad \right)
\end{align*}
\]
The third principal minor of Appendix E.

First we show that

is of the form

\[ x \]

which can be written in the form

\[ \frac{\partial^2 TC}{\partial q \partial C} = \frac{2D}{Nq^2} (A + C + NF) + \frac{8sD^3 \theta N d}{(2D + dq)^3} = x_1 \]

\[ \frac{\partial^2 TC}{\partial q \partial C} = -\frac{D}{Nq^2} \]

Appendix E. The third principal minor of \( H \) is

\[ |H_{33}| = \left| \begin{array}{ccc} \alpha \frac{D}{\theta^2} & 0 & 0 \\ \alpha \frac{D}{\theta^2} N N q^2 (A + C) & \frac{2D}{Nq^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) \\ \frac{sD}{2} & \frac{2D}{Nq^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) & \frac{2D}{Nq^2} (A + C + NF) \end{array} \right| \]

\[ = \frac{\alpha \theta}{\theta^2} \left\{ \frac{2D}{Nq^2} (A + C + NF) + \frac{sD}{2} (A + C) \right\} \left\{ \frac{2D}{Nq^2} (A + C + NF) - \left( \frac{sD}{2} \right)^2 \right\} \]

\[ = \frac{\alpha \theta}{\theta^2} \left( \frac{sD}{N^2 q^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) \right)^2 \]

is of the form \( x \ast y - z^2 \). In order to prove \( x \ast y - z^2 > 0 \), we show that \( x > z \) and \( y > z \), where

\[ x = \frac{2}{N^3} \left( \frac{D}{q} + \frac{d}{2} \right) (A + C) \]

\[ y = \frac{2}{I^3} \left( \frac{D}{q} + \frac{d}{2} \right) (A + C) \]

\[ z = \frac{\sqrt{\alpha \theta}}{\theta} \left( \frac{D}{N^2 q^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) \right) \]

First we show that \( x - z > 0 \).

\[ \frac{\alpha \theta}{\theta^2} \left( \frac{D}{N^2 q^2} (A + C + NF) \right) \left( \frac{2D}{N^3 q^2} (A + C) \right) \]

\[ \left( \frac{2D}{N^2 q^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) \right)^2 \]

\[ \left( \frac{SD}{2} \right)^2 \frac{2}{N^3} \left( \frac{D}{q} + \frac{d}{2} \right) (A + C) \]

\[ = \frac{2}{N^3} \left( \frac{D}{q} + \frac{d}{2} \right) (A + C) \left( \frac{2sD}{N^3 q^2 \theta^2} (A + C + NF) - \frac{s^2 D^2}{4} \right) \]

\[ - \frac{\alpha \theta}{\theta^2} \left( \frac{D}{N^2 q^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) \right)^2 \]
Substituting the value of $N$ it is always true for any positive optimum value of decision variable.

Now we show that $y - z > 0$.

\[
\left\{ \frac{\sqrt{ab}}{\theta^2} \frac{2D}{Nq^3} (A + C + NF) \right\} - \frac{SD}{4} - \left\{ \frac{\sqrt{ab}}{\theta} \left( \frac{D}{Nq^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) \right) \right\} > 0
\]

Substituting the value of $N^2$ the above expression takes the form

\[
\frac{\sqrt{ab}}{\theta} \sqrt{\frac{(H_s + C_d)(P - D)}{P(2D + qd)}} \left( \frac{\sqrt{ab} 2D}{\theta^2} (A + C + NF) - \frac{4D + qd}{2} \sqrt{\frac{(H_s + C_d)(P - D)}{P(2D + qd)}} \right)
\]

Then $y - z > 0$ if and only if \( \frac{\sqrt{ab}}{\theta^2} \frac{2D}{Nq^3} (A + C + NF) - \frac{4D + qd}{2} \sqrt{\frac{(H_s + C_d)(P - D)}{P(2D + qd)}} > 0. \) (ii)

**Appendix F.** The forth principal minor of $H$ is

\[
[H_{44}] = \alpha \frac{b}{\theta} \begin{vmatrix}
\frac{2}{Nq^2} (A + C) + \frac{d}{2} (A + C) & \frac{D}{Nq^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) \\
\frac{D}{Nq^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) & \frac{D}{Nq^2} (A + C + NF)
\end{vmatrix}
\]

To show that the fourth principal minor of $H$ is greater than 0, we consider parts of it and solve them separately. Let us first show that \( \frac{2BDa}{Nq^3} (A + C + NF) - \frac{D^2}{Nq^4} > 0 \).

\[
\frac{D}{Nq^3} \left\{ \frac{2Ba(A + C + NF)}{C^2} - \frac{D}{Nq} \right\}
\]

which is greater than zero if and only if \( \left\{ \frac{2Ba(A + C + NF)}{C^2} - \frac{D}{Nq} \right\} > 0 \)...........(iii)

Now we show that \( \left\{ -\frac{b}{\theta} \frac{D}{Nq^2} (A + C) + \frac{P - D}{2P} (H_s + C_d) \right\} + \frac{D}{Nq^2} \left\{ \frac{D}{Nq^2} + \frac{d}{2Nq} \right\} \) > 0.

Substituting the value of $N^2$ in the above expression, one can obtain

\[
\frac{D(H_s + C_d)(P - D)}{NP(A + C)2q} - \frac{\alpha B(P - D)(H_s + C_d)}{C^2P} \left\{ \frac{D}{2D + qd} + \frac{1}{2} \right\}
\]

which is greater than 0 if and only if \( \left\{ \frac{D}{Nq^2(A + C)} - \frac{\alpha B}{C^2(4D + 2qd)} \right\} > 0 \)...........(iv)

Now, we show that \( \frac{D}{Nq^2} \left\{ \frac{(A + C)}{Nq^2} + \frac{(P - D)(H_s + C_d)}{4P} \right\} - \frac{1}{4} \left\{ \left( \frac{D}{q} + \frac{d}{2} \right) \left( \frac{2D(A + C + NF)}{Nq^2} \right) \right\} > 0. \)
Substituting the value of $N^2$ the above expression takes form
\[
\frac{(H_s + C_d)(P - D)D}{PNq^2} \left[ \frac{D}{2D + dq} - \frac{1}{2} \frac{NQ}{A + C} \right]
\]
which is greater than 0 if and only if \(D \frac{D}{N^2q} + \frac{d}{2N^2} > \left( \frac{1}{2} + \frac{NQ}{A + C} \right) \).

Now we show that \(D \frac{D}{N^2q} + \frac{d}{2N^2} - \alpha B(A + C) > 0\)

\[
\left( D \frac{D}{N^2q} + \frac{d}{2N^2} \right) \left\{ \frac{D}{N^2q} + \frac{d}{2N^2} - \frac{2\alpha B(A + C)}{C^2N} \right\}
\]

which is greater than 0 if and only if \(\frac{D}{N^2q} + \frac{d}{2N^2} > \frac{2\alpha B}{C^2N} (A + C) \).

Thus, if we can show
\[
\frac{\alpha B}{C^2N} \left( \frac{D}{N^2q} + \frac{d}{2N^2} \right) - \frac{2\alpha B}{C^2N} (A + C) \]

Thus, if we can show
\[
\frac{\alpha B}{C^2N} \left( \frac{D}{N^2q} + \frac{d}{2N^2} \right) - \frac{2\alpha B}{C^2N} (A + C) > 0.
\]

Now we show that \(\frac{\alpha B}{C^2N} \left( \frac{D}{N^2q} + \frac{d}{2N^2} \right) - \frac{2\alpha B}{C^2N} (A + C) > 0\).

Thus, we can show
\[
\frac{\alpha B}{C^2N} \left( \frac{D}{N^2q} + \frac{d}{2N^2} \right) - \frac{2\alpha B}{C^2N} (A + C) > 0.
\]

thus, \(H_{44} > 0\)

REFERENCES

[1] M. Asghari, S. J. Abrishami and F. Mahdavi, Reverse logistic network design with incentive-dependent return, *Industrial Engineering & Management Systems, 13* (2014), 383–397.

[2] A. Banerjee, A joint economic-lot-size model for purchaser and vendor, *Decision Sciences, 17* (1986), 292–311.

[3] M. Ben-Daya and A. Raouf, Inventory models involving lead time as decision variable, *The Journal of the Operational Research Society, 45* (1994), 579–582.

[4] S. Byika, Competitive and cooperative policies for the vendor-buyer system, *International Journal of Production Economics, 81/82* (2003), 533–544.

[5] L. E. Cárcenas-Barrón, Optimizing inventory decisions in a multi-stage multi-customer supply chain: A note, *Transportation Research Part E: Logistics and Transportation Review, 43* (2007), 647–654.
L. E. Cárdenas-Barrón, The derivation of EOQ/EPQ inventory models with two backorders costs using analytic geometry and algebra, Applied Mathematical Modelling, 35 (2011), 2394–2407.

R. P. Covert and G. C. Philip, An EOQ model for items with Weibull distribution deterioration, A I I E Transactions, 5 (1973), 323–326.

P. M. Ghare and G. F. Schrader, A model for exponentially decaying inventory, Journal of Industrial Engineering, 14 (1963), 238–243.

A. Goswami and K. S. Chaudhuri, An EOQ model for deteriorating items with shortages and a linear trend in demand, The Journal of the Operational Research Society, 42 (1991), 1105–1110.

S. K. Goyal, An integrated inventory model for a single supplier-single customer problem International Journal of Production Research, 15 (2007), 107–111.

S. K. Goyal, Economic ordering policy for deteriorating items over an infinite time horizon European Journal of Operational Research, 28 (1987), 298–301.

S. K. Goyal, A joint economic-lot-size model for purchaser and vendor: A comment, Decision Sciences, 19 (1988), 236–241.

M. Hariga and M. Ben-Daya, Some stochastic inventory models with deterministic variable lead time, European Journal of Operational Research, 113 (1999), 42–51.

R. M. Hill, The single-vendor single-buyer integrated production-inventory model with a generalised policy, European Journal of Operational Research, 97 (1997), 493–499.

J. D. Hng and J. C. Hayya, Joint investment in quality improvement and setup reduction, Computers & Operations Research, 22 (1995), 567–574.

H. Hwang, D. B. Kim and Y. D. Kim, Multiproduct economic lot size models with investments costs for setup reduction and quality improvement International Journal of Production Research, 31 (1993), 691–703.

G. Keller and H. Noori, Impact of investing in quality improvement on the lot size model OMEGA, 16 (1988), 595–601.

G. Keller and H. Noori, Justifying new technology acquisition through its impact on the cost of running an inventory policy, IIE Transactions, 20 (1988), 284–291.

E. Kusakawa and S. Aozawa, Optimal operation for green supply chain with quality of recyclable parts and contract for recycling activity, Industrial Engineering & Management Systems, 14 (2015), 248–274.

C. J. Liao and C. H. Shyu, An analytical determination of lead time with normal demand inventory models Computers and Operations Research, 25 (1998), 1007–1012.

L. Y. Ouyang, N. C. Yeh and K. S. Wu, Mixture inventory model with backorders and lost sales for variable lead time, The Journal of the Operational Research Society, 47 (1996), 829–832.

M. J. Paknejad, F. Nasri and J. F. Affisco, Defective units in a continuous review $(s, Q)$ system International Journal of Production Research, 33 (1995), 2767–2777.

C. Park, Partial backordering inventory model under purchase dependence, Industrial Engineering & Management Systems, 14 (2015), 275–288.

E. L. Porteus, Optimal lot sizing, process quality improvement and setup cost reduction Operations Research, 34 (1986), 137–144.

E. L. Porteus, Investing in reduced setups in the EOQ model Management Science, 31 (1985), 998–1010.

F. Raafat, Survey of literature on continuously deteriorating inventory model, The Journal of the Operational Research Society, 42 (1991), 27–37.

M. J. Rosenblatt and H. L. Lee, Economic production cycles with imperfect production processors IIE Transactions, 18 (1986), 48–55.

B. Sarkar, An EOQ model with delay in payments and time varying deterioration rate Mathematical and Computer Modelling, 55 (2012), 367–377.
[32] B. Sarkar, A production-inventory model with probabilistic deterioration in two-echelon supply chain management, *Applied Mathematical Modelling*, 37 (2013), 3138–3151.

[33] B. Sarkar, K. Chaudhuri and I. Moon, Manufacturing setup cost reduction and quality improvement for the distribution free continuous-review inventory model with a service level constraint, *Journal of Manufacturing Systems*, 34 (2015), 74–82.

[34] B. R. Sarkar and E. R. Coates, Manufacturing setup cost reduction under variable lead times and finite opportunities for investment, *International Journal of Production Economics*, 49 (1997), 237–247.

[35] B. Sarkar, B. Mandal and S. Sarkar, Quality improvement and backorder price discount under controllable lead time in an inventory model *Journal of Manufacturing Systems*, 35 (2015), 26–36.

[36] B. Sarkar and I. Moon, Improved quality, set up cost reduction and variable backorder costs in an imperfect production process, *International Journal of Production Economics*, 155 (2014), 204–213.

[37] B. Sarkar, S. Saren and H. M. Wee, An inventory model with variable demand, component cost and selling price for deteriorating items *Economic Modelling*, 30 (2013), 306–310.

[38] B. Sarkar and S. Sarkar, An improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand *Economic Modelling*, 31 (2013), 548–556.

[39] Y. K. Shah, An order-level lot-size inventory model for deteriorating items, *A I I E Transactions*, 9 (1977), 108–112.

[40] E. A. Silver, Changing the givens in modelling inventory problems: The example of just-in-time systems *International Journal of Production Economics*, 26 (1992), 347–351.

[41] E. A. Silver, D. F. Pyke and R. Peterson, *Inventory Management and Production Planning and Scheduling*, Wiley, New York, 1998.

[42] K. Skouri and S. Papachristos, Four inventory models for deteriorating items with time varying demand and partial backlogging: A cost comparison *Optimal Control Applications and Methods*, 24 (2003), 315–330.

[43] K. Skouri, I. Konstantaras, S. Papachristos and I. Ganas, Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate *European Journal of Operational Research*, 192 (2009), 79–92.

[44] J. T. Teng, L. E. Cárdenas-Barrón, K. R. Lou and H. M. Wee, Optimal economic order quantity for buyer-distributor-vendor supply chain with backlogging without derivatives *International Journal of Systems Science*, 44 (2013), 986–994.

[45] A. Villa, Introducing some supply chain management problems *International Journal of Production Economics*, 73 (2001), 1–4.

[46] S. Viswanathan, Optimal strategy for the integrated vendor-buyer inventory model *European Journal of Operational Research*, 105 (1998), 38–42.

[47] N. Watanabe and E. Kusukawa, Optimal ordering policy in dual-sourcing supply chain considering supply disruptions and demand information, *Industrial Engineering & Management Systems*, 14 (2015), 129–158.

[48] W. Wisittipanich and P. Hengmeechai, A multi-objective differential evolution for just-in-time door assignment and truck scheduling in multi-door cross docking problems, *Industrial Engineering & Management Systems*, 14 (2015), 299–311.

[49] P. C. Yang and H. M. Wee, An arborescent inventory model in a supply chain system *Production Planning & Control: The Management of Operations*, 12 (2001), 728–735.

Received September 2015; 1st revision January 2016; 2nd revision August 2016.

E-mail address: bsbiswajitsarkar@gmail.com
E-mail address: am.arunavamajumder@gmail.com
E-mail address: mitalisarkar.ms@gmail.com
E-mail address: bikashkolidey@gmail.com
E-mail address: gargi.roy37@gmail.com