Digital Control with a Direct Torque Control Algorithm Applied to a Three-Phase Induction Motor using VHDL

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ABSTRACT
This work developed a simulation of a three-phase induction motor (MIT) with via the direct torque control (DTC) technique, which is executed in a field programmable gate array (FPGA) of the programmable logic type using the VHDL language. This simulation platform uses MATLAB/Simulink jointly with the DSP Builder software, on an Altera DE2 board. A specific methodology, using software configurations and a mixed simulation, allowed the verification of the behavior of the VHDL codes before their implementations on the FPGA device. The results of the simulations are presented and analyzed in the work.

Keywords: Induction motor, DTC control, FPGA, DSP

INTRODUCTION
Three-phase induction motors are heavily utilized in industrial applications due to their robustness, low cost, high operating speed, and low maintenance. For better efficiency of these engines, many direct torque control (DTC) techniques have been adopted in recent years [1, 2, 3]. In order to obtain an accurate control of the motor drive, several control algorithms have been widely investigated including field-oriented control, direct torque control, model predictive control, and generalized predictive control [4, 5, 6]. DTC of three-phase induction motors has gained popularity in industrial applications mainly due to its simple control structure. Several modifications and improvements have been made to the original control structure in order to overcome two major problems normally associated with DTC, namely the high electromagnetic torque ripple and variable switching frequency [2]. It is well established that these problems are mainly due to the use of hysteresis torque and flow controllers. For this reason, most of the methods used to overcome these problems were accomplished by replacing the hysteresis- with the non-hysteresis-based controllers [6, 7, 8].

The study of torque control strategies in three-phase induction motors has attracted a great deal of interest in researchers. Ibrahim et al [9] presented a new proposal to drive DTC using a fuzzy controller, and its results were simulated showing the potential of this strategy. Sandre-Hernandez et al [10] present the implementation of the DTC for a permanent magnet synchronous machine based on the technology of a field programmable gate array (FPGA). Pereira et al [11] propose a virtual teaching platform of DTC of induction motors to assist in the education of undergraduate students. Today, the rapid development in high-performance digital signal processors (DSP) not only replaced the analog technology of the conventional control method but also provides high computing capabilities. Recent developments in FPGA [10, 12, 13] have made it possible to combine complex analog and digital circuits. However, in motion control systems, FPGA technology is not that popular. Generally, VHDL codes are implemented in programmable logic devices, such as application-specific integrated circuits and especially in FPGAs. This methodology proposal allows one to verify the behavior of these codes before their implementations, reducing the risks of significant changes when implemented.
The contribution of this work is the development of a digital controller with a DTC algorithm applied to a three-phase induction motor using VHDL embedded in an FPGA, thanks to its parallel architecture for multiple-signal processing.

**THREE-PHASE INDUCTION MOTOR**

**Clarke Transformation**

The behavior of three-phase induction motors is usually described by their voltage and current equations. The coefficients of the differential equations that describe their behavior are time varying (except when the rotor is stationary). The mathematical modeling of such a system tends to be complex since the flow linkages, induced voltages, and currents change continuously as the electric circuit is in relative motion. For the analysis of such a complex electrical machine, mathematical transformations are often used to decouple variables and to solve equations involving time varying quantities by referring all variables to a common frame of reference [2, 10]. Among the various transformation methods available, the well-known ones are the Clarke transformation and Park transformation.

The Clarke transformation converts balanced three-phase quantities \((a, b, c)\) into balanced two-phase quadrature quantities \((\alpha, \beta)\). The Park transformation converts vectors in a balanced two-phase orthogonal stationary system into an orthogonal rotating reference frame \((d, q)\). Figure 1 shows the three reference frames.

![Reference frames involved in Clarke and Park transformation](image1)

**Figure 1.** Reference frames involved in Clarke and Park transformation

Clarke and Park transformations are mainly used in vector control architectures related to permanent magnet synchronous machines and asynchronous machines [10]. The three-phase quantities are translated from the three-phase reference frame to the two-axis orthogonal stationary reference frame using the Clarke transformation as shown in Figure 2.

![Clarke transformation from three phases to two phases](image2)

**Figure 2.** Clarke transformation from three phases to two phases
The Clarke transformation is expressed by Equations 1 and 2:

\[ I_\alpha = \frac{2}{3}(I_a) - \frac{1}{3}(I_b - I_c) \]  

and

\[ I_\beta = \frac{2}{\sqrt{3}}(I_b - I_c) \]  

Where \( I_a, I_b, \) and \( I_c \) are the three-phase quantities, and \( I_\alpha \) and \( I_\beta \) are the stationary orthogonal reference frame quantities. When \( I_a \) is superposed with \( I_a \) and when \( I_a + I_b + I_c = 0 \), the three-phase quantities \( I_a, I_b, \) and \( I_c \) can be transformed to \( I_\alpha \) and \( I_\beta \) such that:

\[ I_a = I_\alpha \]  

and

\[ I_\beta = \frac{1}{\sqrt{3}}(I_a + 2I_b) \]  

The transformation from a two-axis orthogonal stationary reference frame to a three-phase stationary reference frame is accomplished using an Inverse Clarke transformation as shown in Figure 3. The Inverse Clarke transformation is expressed by the following equations:

\[ V_a = V_\alpha \]  

\[ V_\beta = \frac{-V_a + \sqrt{3} * V_\beta}{2} \]  

And

\[ V_c = \frac{-V_a - \sqrt{3} * V_\beta}{2} \]  

Where:

\( V_a, V_b, V_c \) are the three-phase quantities, and \( V_\alpha, V_\beta \) are the stationary orthogonal reference frame quantities.
The matrix to transform the three vectors $V_a$, $V_b$, and $V_c$ into $V_\alpha$, $V_\beta$ is [10]:

$$
\begin{bmatrix}
V_\alpha \\
V_\beta
\end{bmatrix} = \frac{2}{3}
\begin{bmatrix}
1 & \frac{1}{2} & \frac{1}{2} \\
\sqrt{3} & \frac{-1}{2} & \frac{-1}{2} \\
0 & \frac{-\sqrt{3}}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
$$

(8)

Mathematical Model of the MIT

In order to obtain a simplification of the MIT model and a reduction of the number of variables, which describe the dynamic behavior of the motor, the three-phase coordinate system is projected to an orthogonal coordinate plane [2, 14]. In this methodology, the three-phase system (three axes with a lag of 120° between each other) is replaced by an orthogonal system. The MIT will be seen to consist of only two spatially lagged 90° windings on the stator and rotor. Based on Figure 3, the new axis system is called $\alpha\beta$ and has indices $\alpha$ and $\beta$.

The magnitudes involved in the fundamental expressions of motor voltage are related both to the reference of the stator and to the reference of the rotor. Thus, it is necessary to obtain a single and common reference for the stator and rotor. As the DTC technique controls stator quantities such as currents, voltages, and flow, to facilitate the mathematical modeling of MIT, we adopted the stationary reference. All the values evaluated are based on the mathematical model of the motor in a steady-state condition. Equations 9 – 13 are fundamental to the DTC strategy to calculate the stator flow linkage and torque.

Equation 9 presents the space vector of the stator voltage ($\vec{V}_{\alpha\beta}$) where $i_{\alpha\beta}$ is the space vector of the stator current, $\Psi_{\alpha\beta}$ is the concatenated stator space flow vector, $R_s$ is the winding resistance of a stator phase.

$$
\vec{V}_{\alpha\beta} = R_s \vec{i}_{\alpha\beta} + \frac{d}{dt} \vec{\Psi}_{\alpha\beta}
$$

(9)

Equation 10 shows the space vector of the rotor voltage ($\vec{V}_{\alpha\beta}$) where $R_r$ is the resistance of one rotor phase (equivalent winding) to the stator, $i_{\alpha\beta}$ is the space vector of the rotor current, $\Psi_{\alpha\beta}$ is the concatenated rotor flow space vector, $Z_p$ is the pole pair number, $\omega_{me}$ is the mechanical speed of the motor.

$$
\vec{V}_{\alpha\beta} = R_r \vec{i}_{\alpha\beta} + \frac{d}{dt} \vec{\Psi}_{\alpha\beta} - \frac{j}{Z_p} \omega_{me} \vec{\Psi}_{\alpha\beta}
$$

(10)

In Equation 11 $\vec{\Psi}_{\alpha\beta}$ is the concatenated stator space flow vector where $L_s$ is the stator inductance, $L_m$ is the mutual inductance between stator and rotor, $i_{\alpha\beta}$ is the space vector of the stator current, and $i_{\alpha\beta}$ is the space vector of the rotor current.

$$
\vec{\Psi}_{\alpha\beta} = L_s \vec{i}_{\alpha\beta} + L_m \vec{i}_{\alpha\beta}
$$

(11)
Equation 12 presents the concatenated rotor flow space vector \( \vec{\psi}_{r\alpha\beta} \) where \( L_m \) is the mutual inductance between stator and rotor, \( L_r \) is the rotor inductance, \( i_{s\alpha\beta} \) is the space vector of the stator current, and \( i_{r\alpha\beta} \) is the space vector of the rotor current.

\[
\vec{\psi}_{r\alpha\beta} = L_m \vec{i}_{r\alpha\beta} + L_r \vec{i}_{r\alpha\beta}
\]  

(12)

In Equation 13 \( m_d \) is the electromagnetic torque where \( Z_p \) is the pole pair number, \( \psi_{s\alpha} \) is the component \( \alpha \) of Stator flow, \( i_{s\beta} \) is the component \( \beta \) of stator current, \( \psi_{s\beta} \) is the component \( \beta \) of Stator flow, and \( i_{s\alpha} \) is the component \( \alpha \) of stator current.

\[
m_{d} = \frac{3}{2} Z_{p} (\psi_{s\alpha} i_{s\beta} - \psi_{s\beta} i_{s\alpha})
\]  

(13)

VOLTAGE SOURCE INVERTER

According to [10], the topology of a voltage source inverter (VSI) is as shown in Figure 4.

\[\text{Figure 4. Scheme of voltage source inverter}\]

It is comprised of six insulated gate bipolar transistors (IGBTs) and six freewheeling diodes. Assuming that in every instant the switching devices can accept only one of the two possible states on (1) or off (0), the VSI has only eight possible switching states, generating six active voltage space vectors (AVSVs) \( \vec{v}_{\alpha} \vec{v}_{\beta} \) and two zero voltage space vectors (ZVSVs) \( \vec{v}_{0} \vec{v}_{0} \). Using the Clarke transformation given by Equation 8, the states of the inverter can be mapped onto the \( \alpha-\beta \) complex plane, which are shown in Fig. 5.

The space sector \( N \) is obtained from the angle \( \delta_{\psi_s} \) between the stationary reference and the stator flow (Equation 14), and is limited by Equation 15:

\[
\delta_{\psi_s} = \tan^{-1} \left( \frac{\psi_{s\alpha}}{\psi_{s\beta}} \right) \quad (14)
\]

\[
\text{and}
\]
Where N is the space sector, and $\delta$ is the space angle.

![Image of voltage space vectors generated by the VSI](image)

**Figure 5. Voltage space vectors generated by the VSI**

**PRINCIPLE OF DIRECT TORQUE CONTROL**

In Figure 6, a summary block diagram of the DTC technique is presented using hysteresis comparators, a motor estimation model, and switching logic (vector table). The main objective of this technique is to control the torque and the flow of the stator using the comparators, ensuring a fast torque response. The vector table is used to select the voltage vector to be applied to the stator, determined the switches that must be activated in the inverter (space modulation). The choice of the voltage vector is made to maintain the stator torque and flow within the limits defined by the hysteresis comparators. There are six possible AVSVs $\overrightarrow{v_1} \text{ to } \overrightarrow{v_6}$ and two ZVSVs $\pm \overrightarrow{v_7}$ which are chosen based on the error between the torque ($\Gamma$) and flow ($\Phi$) reference values and the spatial sector of the stator flow ($t_g^{-1} \begin{pmatrix} \psi_{\alpha \beta} \\ \psi_{\alpha \beta} \end{pmatrix}$).

The selection of the optimal AVSV or ZVSV [10] is based on the optimal switching table listed in Table 1, where the estimated values of torque and flow are used to process the torque error ($\Gamma$) and flow error ($\Phi$).

**Table 1. Table of voltage vectors to be applied to the inverter**

| $\Phi$ | $\Gamma$ | Sector 1 | Sector 2 | Sector 3 | Sector 4 | Sector 5 | Sector 6 |
|-------|---------|----------|----------|----------|----------|----------|----------|
| 0     | 1       | $v_2$    | $v_3$    | $v_4$    | $v_5$    | $v_6$    | $v_1$    |
|       | 0       | $v_2$    | $v_0$    | $v_7$    | $v_6$    | $v_7$    | $v_0$    |
|       | -1      | $v_6$    | $v_1$    | $v_2$    | $v_3$    | $v_4$    | $v_5$    |
| 1     | 1       | $v_3$    | $v_4$    | $v_5$    | $v_6$    | $v_1$    | $v_2$    |
|       | 0       | $v_0$    | $v_7$    | $v_0$    | $v_2$    | $v_0$    | $v_7$    |
|       | -1      | $v_5$    | $v_0$    | $v_1$    | $v_7$    | $v_3$    | $v_4$    |

The currents measured at the motor input ($i_a, i_b$) are inputs from the “motor model” block, which estimates the torque and the stator flow at that instant, in addition to the space sector where the stator flow is located. The estimated stator torque ($m_\alpha$) and flow ($\psi_{\alpha \beta}$) are compared to their respective
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Comparison errors are inputs from the hysteresis comparators (usually two levels for the flow and three levels for the torque), which verify that the torque and flow are within their given limits. The outputs of these comparators (Γ and Φ), as well as the stator flow space sector, are inputs from the “Vector Table,” which determines the proper switching of the inverter to maintain stator flow and torque close to its reference values. Figure 6 shows the structure of the direct torque control system (DTC).

**SIMULATION PLATFORM**

The simulation platform consists of two software programs: MATLAB/Simulink and DSP Builder. MATLAB/Simulink is responsible for simulating the power part of the drive and control, whereas the DSP Builder software converts the MATLAB/Simulink codes to the VHDL language and allows the loading of the codes into the FPGA device (DA COSTA, 2014).

After installation, the DSP Builder software adds two sets of new functional block libraries to the Simulink software: (i) Altera DSP Builder Advanced Blockset and (ii) Altera DSP Builder Blockset. The DSP Builder software integrates the MATLAB/Simulink in a single environment, which allows simple and direct: (i) automatic implementation of the DTC application in VHDL; (ii) simulation of the created system; (iii) conversion of the algorithm into RTL code; (v) creation of the project in the Quartus II software and simulation of the generated RTL code using the same test vectors used in SIMULINK; (vi) compiling of the project; (vii) loading into the FPGA hardware; (viii) testing of the device with the created DTC algorithm. Figure 7 illustrates the design flow with the MATLAB/Simulink and DSP Builder software.

**DTC Strategy Simulation**

According to the DTC structure shown in Figure 6, the motor model block, estimation of flow and torque block, current acquisition, and inverter block are performed and simulated in the...
MATLAB/Simulink environment. Stator torque and flow controls are performed using the torque hysteresis comparator and flow hysteresis comparator block, which are, along with the vector table block, implemented and simulated in the DSP Builder software.

In the DTC control algorithm block, the motor model calculations and their respective comparisons with their reference values are performed, in addition to determining the space sector where the flow is. The outputs of the hysteresis comparators, in addition to the spatial flow sector, are inputs to the part of the program responsible for the inverter switching table. This table indicates the best alternative for the DTC at that instant and triggers the inverter keys to maintain the desired control.

The simulated inverter uses ideal switches, since the IGBT transistors are considered as ideal keys for the frequency level used in the simulation. The switching frequency chosen was 20 kHz. The instrumentation part (A/D converter) was performed and simulated in the MATLAB/Simulink environment. Stator currents enter as binary data in the DTC control algorithm. For resolution of the A/D converter, 16 bits were assumed. The sampling frequency used in the A/D converter was 40 kHz. The control algorithm that controls the motor based on the DTC strategy has been divided into two parts. The first part, responsible for calculations, operates at a frequency of 40 kHz. The second part, which contains the inverter switching table, operates at a frequency of 20 kHz. Parameters of induction motors used in the simulation are listed in Table 2. Figure 8 shows the DTC strategy simulation.

### Table 2. Parameter of Induction motors

| Parameter                  | Value           |
|----------------------------|-----------------|
| Rated Power                | 1.5 hp          |
| Pole Pairs                 | 2               |
| Stator Resistance          | 7.56Ω           |
| Stator Inductance          | 0.35085H        |
| Rotor Resistance           | 3.84Ω           |
| Rotor Inductance           | 0.35085H        |
| Rotor Inertia              | 0.027 kgm²      |
| Mutual Inductance          | 0.33615H        |
| Coefficient of Friction    | 0.000124        |
| Reference Torque           | 10 N·m          |
| Reference Flow             | 0.8 Wb          |
| Load Torque                | 6 N             |

**Figure 8. DTC strategy simulation**

*Test Results Simulation*

In order to verify the proposed DTC strategy, simulation results are presented. The Altera DE 2 board with a Cyclone III device is used to execute the DTC control algorithm. Fig. 9 (a) shows the simulation result of the DTC algorithm for positive and negative references of the magnitude of the estimated stator flow and its reference. Fig. 9 (b) shows the simulation result of the DTC algorithm for positive and negative references of the estimated electromagnetic torque and its reference. To perform the test of DTC, a positive reference torque is applied to the induction motor with a reference of 0 N·m during the initial time, and 10 N·m at 0.25 s.
Figure 9. (a) Simulation results of stator flow estimated, and (b) Electromagnetic torque estimated by the proposed DTC algorithm

Fig. 10 (a) shows the simulation result of the DTC algorithm for positive and negative reference of stator currents. In Figure 10 (a), a change can be observed between two phases at a time equal to 0.25 s due to the change in the torque direction of the motor. The period between the instants 0.3 s and 0.4 s shows the behavior of the phase currents when the velocity is reduced. In Figure 10 (b), the rate of growth of the motor speed is changed at 0.1 s when a load of 6 N-m is coupled to the shaft. At a time of 0.25 s, the torque reference changes direction, causing the motor speed to decrease and after a short time interval it change its direction of rotation.

Figure 10. (a) Simulation results of three-phase stator currents, and (b) Motor Speed by the proposed DTC algorithm

CONCLUSION

This paper has presented the digital design and simulation of the DTC strategy using VHDL language. A DSP design for motor control has been developed and simulated for the particular case of the DTC algorithm of a three-phase induction motor. A specific methodology, using software configurations and a mixed simulation, allowed the verification of the behavior of the VHDL codes before their implementations on the FPGA device. The methodology defined the digital adaptation of the DTC algorithm, taking account of the influences of digitization on computed values. An optimized DSP algorithm and a specific fixed-point format have been studied and defined. The implementation, which proved that the DTC strategy, was accomplished using the combination of a MATLAB/Simulink and DSP Builder software. This project addressed a way of structuring and simulating a DTC algorithm. The target technology was FPGAs, which have been growing steadily and has been taking up space in the industrial market worldwide.
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