Galaxy Formation and Evolution. II. 
Energy Balance, Star Formation and Feed-Back

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ABSTRACT
In this paper we present a critical discussion of the algorithms commonly used in N–body simulations of Galaxy Formation to deal with the energy equation governing heating and cooling, to model star formation and the star formation rate, and to account for the energy feedback from stars.

First, we propose our technique for solving the energy equation in presence of heating and cooling, which includes some difference with respect to the standard semi-implicit techniques.

Second, we examine the current criteria for the onset of the star formation activity. We suggest a new approach, in which star formation is let depend on the total mass density - baryonic (gas and stars) and dark matter - of the system and on the metal-dependent cooling efficiency.

Third, we check and discuss the separated effects of energy (and mass) feedback from several sources - namely supernovae, stellar winds from massive stars, and UV flux from the same objects.

All the simulations are performed in the framework of the formation and evolution of a disk galaxy.

We show that the inclusion of these physical phenomena has a significant impact on the evolution of the galaxy models.

Key words: Numerical methods: SPH; Galaxy: formation, evolution; Stars: formation, feedback

1 INTRODUCTION
Numerical hydrodynamics and N–body simulations are nowadays the fundamental tool to investigate galaxy formation and evolution. The current status of numerical and semi-analytical modeling of various galaxy properties has been recently summarized by Frenk et al. (1997).

One of the poorly understood processes is the formation of stars. Since the pioneering study by Katz (1992), the onset of star formation (SF) is usually empirically parameterized (see also Gerritsen 1997). The key idea is that an element of fluid must satisfy some conditions in order to be eligible to SF and turn part of its gas content into stars according to a suitable star formation rate (SFR), which is customarily a reminiscence of the Schmidt (1959) law. The reader is referred to the exhaustive discussion of this topic by Mihos & Hernquist (1994) for more details.

The basic criteria to select the fluid elements prone to SF are (i) the gas particle must be in a convergent flow, and (ii) the gas particle must be Jeans unstable. The conditions are met if the velocity divergence is negative and if the sound crossing time-scale is shorter than the dynamical time-scale.

Schmidt–like laws imply that $SFR \propto \rho^n$, where the power $n$ ranges between 1 and 2. Conservation arguments suggest that the most probable value of $n$ is $\frac{3}{2}$ (see Katz 1992). The Schmidt law written for the volume density is

$$\frac{dp_*}{dt} = -\frac{dp_g}{dt} = \frac{c_*\rho_g}{t_g}$$ (1)

where $c_*$ is the so-called dimensionless efficiency of star formation, and $t_g$ is the characteristic time for the gas to flow. This is chosen to be the maximum between the cooling time and the free-fall time $t_{ff} = (4\pi G\rho_g)^{-\frac{1}{2}}$.

Over the past few years, starting from this general background a number of refinements have been brought forth. These mainly concern the criteria to decide whether a gas particle is suitable to form stars, and the way stars emerge from gas, i.e. how gas clouds undergo fragmentation.

Navarro & White (1993) and Katz, Weinberg & Hern-
quist (1996) introduce an additional condition for the onset of SF, i.e. the so-called over-density criterion, which secures that a collapsing cloud remains cool. The threshold density implies that the cooling time-scale is much shorter than the dynamical time-scale, and this always occurs for temperatures greater than the cut-off temperature of the cooling curve \((T = 10^4 K)\) for the typical mass resolution of this type of problem. In addition to this, Katz, Weinberg & Hernquist (1996) impose that the gas-particles can form stars only if a minimum physical density corresponding to 0.1 hydrogen atoms per \(cm^2\) is met.

In Katz et al. (1996) and Navarro & White (1993), the fragmentation of a gas cloud is usually modeled in the following way. When in a gas-particle SF switches on, the gaseous mass decreases until it reaches some minimum value, say 5\% of the original value. At this stage, the gas-particle is turned into a collision-less star-particle, while the remaining gas is distributed among the surrounding gas-particles. In the meantime the gas-particle is considered as a dual entity, whose mass is partly collision-less and partly collisional. This approach is a good compromise in terms of computational effort because it is not necessary to add a collision-less particle every time a new star is formed (Katz, Weinberg & Hernquist 1996). Then the choice of the star forming regions is made by means of suitable probability arguments. Every time a gas-particle forms stars, its mass is reduced by 1/3 (free parameter). A different value is adopted by Navarro & White (1993), who suppose that at each star forming event the gas mass is halved, and that each gas-particle can split into four star-particles at most (this limit is set by computational limitations).

A somewhat different scheme is followed by Steinmetz (1995) who calculates the mass of the newly born star-particle using equation (1) in the form:

\[
m_\star = m_\odot (1 - \exp(-\frac{c_s \Delta t}{t_{ff}})),
\]

where \(\Delta t\) is the particle time step. The mass of the gas-particle is accordingly reduced. Although this latter approach appears to be more physically sounded, computational problems can arise when many star-particles are expected to form, so that the number of star formation episodes must be artificially limited.

Once stars are present, they are expected to return to the interstellar medium (ISM) part of their mass (in form of chemically processed gas) and energy via Supernovae (SNe) explosions, stellar winds, and UV flux. These latter contributions are significant only in the case of massive stars. All this is known as the stellar energy Feed-Back (FB).

The key problem here is to know how the energy released by stars is given to the ISM because the limited resolution of N-body simulations does not allow one to describe the ISM as a multi-phase medium. Basically two different schemes exist:

(i) all the energy (from SNe, stellar winds, and UV flux) is given to the thermal budget of the fluid element (Katz 1992, Steinmetz & Müller 1994);
(ii) only a fraction of this energy is let kinetically affect the surrounding fluid elements. This fraction \((f_\nu \text{ or } \epsilon_{\text{kin}})\) is a free parameter (Navarro & White 1993, Mihos & Hernquist 1994) chosen in such a way that some macroscopic properties of the galaxy, like for instance the metallicity distribution (Groom 1997) or the shape of the disk (Mihos & Hernquist 1994), are reproduced.

The deposit of all the energy in the thermal budget of the fluid (first alternative) has little effect on a galaxy’s structure and evolution. In fact, since the thermal energy is given to a medium of high density and short cooling time-scale in turn, it is almost immediately radiated away. The difficulty cannot be cured by supposing that the thermal energy is injected and shared over an e-folding time scale (Summers 1993).

In all the above schemes the fraction of energy from stellar FB which is not transformed into kinetic energy is added to the energy budget of the fluid element, and the fluid element is let cool. The integration of the energy equation with respect of time is generally made according to a semi-implicit scheme (see the discussion in Hernquist & Katz 1989).

In this paper we present in detail our prescriptions for computing energy balance, SF and FB. The organization of the paper is as follows. Section 2 describes the numerical code and the method used to integrate the energy equation, whereas Section 3 discusses in details the adopted initial conditions. In section 4 we test our energy integration scheme following the formation of a galactic disk. Section 5 presents tests a discussion on various SF criteria, our new approach to SF and the corresponding models. Section 6 examines the effects of different sources of FB, while, finally, some concluding remarks are presented in Section 7.

2 THE BASIC NUMERICAL TOOL

2.1 The Tree-SPH code

All the simulations presented here have been performed using the Tree-SPH code developed by Carraro (1996) and Lia (1996), and described by Carraro et al. (1998a), to whom the reader is referred for all details.

The code, which can follow the evolution of a mix of Dark Matter (DM) and Baryons (gas and stars), has been carefully checked against several classical tests with satisfactory results, as reported in Carraro et al. (1998a).

In this code, the properties of the gas component are followed by means of the Smoothed Particle Hydrodynamics (SPH) technique (Lucy 1977, Gingold & Monaghan 1997, Benz 1990), whereas the gravitational forces are computed by means of the hierarchical tree algorithm of Barnes & Hut (1986) using a typical tolerance parameter \(\theta = 0.8\) and expanding tree nodes to quadrupole order. We adopt a Plummer softening parameter.

In SPH each particle represents a fluid element whose position, velocity, energy, density etc. are followed in time and space. The properties of the fluid are locally estimated by an interpolation which involves the smoothing length \(h_\star\). In our code each particle possesses its own time and space variable smoothing length \(h_\star\), and evolves with its own time-step. This renders the code highly adaptive and flexible, and suited to speed-up the numerical calculations.

Radiative cooling is described by means of numerical...
tabulations as a function of temperature and metallicity taken from Sutherland & Dopita (1994). This allows us to account for the effects of variations in the metallicity among the fluid elements and for each of these as a function of time and position.

The chemical enrichment of the gas-particles caused by SF and stellar ejecta (Portinari et al 1998) is described by means of the closed-box model applied to each gas-particle (cf. Carraro et al. 1998a,b for more details).

Star formation and Feed-back are discussed below.

Finally, all the calculations presented here have been carried out on a DIGITAL ALPHA-2000 (330 Mhz of Clock, 512 Mbyte of RAM) workstation hosted by the Padua Observatory & Astronomy Department.

### 2.2 On the integration of the energy equation

The usual form of the energy equation in SPH formalism is

\[
\frac{du_i}{dt} = \sum_{j=1}^{N} m_j \left( \sqrt{\frac{P_i}{\rho_i}} + \frac{1}{2} \Pi_{ij} \right) v_{ij} \times \left( \nabla_i W(r_{ij}, h_i) + \nabla_j W(r_{ij}, h_j) \right) + \frac{\Gamma - \Lambda_C}{\rho},
\]

(3)

(Benz 1990; Hernquist & Katz 1989). The first term represents the heating or cooling rate of mechanical nature, whereas the second term \(\Pi\) is the total heating rate from all sources apart from the mechanical ones, and the third term \(\Lambda_C/\rho\) is the total cooling rate by many physical agents (see Carraro et al. 1998a for details).

In absence of explicit sources or sinks of energy the energy equation is adequately integrated using an explicit scheme and the Courant condition for time-stepping (Hernquist & Katz 1989).

The situation is much more complicated when considering cooling. In fact, in real situations the cooling time-scale becomes much shorter than any other relevant time-scale (Katz & Gunn 1991), and the time-step becomes considerably shorter than the Courant time-step, even using the fastest computers at disposal. This fact makes it impossible to integrate the complete system of equations (cf. Carraro et al. 1998a) adopting as time-step the cooling time-scale.

To cope with this difficulty, Katz & Gunn (1991) damp the cooling rate to avoid too short time-steps allowing gas particles to loose only half of their thermal energy per time-step. However according to modern gravitational instability theory (White & Rees 1978), protogalaxies comprise a mixture of dissipation-less dark matter (DM), whose nature is not clear, and dissipational baryonic material, roughly in the mass ratio 1:10. After a violent relaxation process, which follows the separation from the Hubble expansion, a DM halo becomes isothermal, and acquires baryonic material which heats up at the halo virial temperature. Gas then cools and collapses, and through fragmentation turns into the stars we see today in disks and spheroidal systems.

Our scheme to update energy is conceptually the same, but differs in the predictor stage and in the iteration scheme adopted to solve equation (3).

In brief, at the first time-step the quantity \(u^{n-1/2}\) is calculated and for all subsequent time steps, the leap-frog technique, as in Steinmetz & Müller (1993), is used:

(i) We start with \(u^n\) at \(t^n\);

(ii) compute \(\tilde{u}^n\) as

\[
\tilde{u}^n = u^{n-1/2} + \frac{1}{2} \frac{\partial u}{\partial t} \times e^n.
\]

This predicted energy, together with the predicted velocity is used to evaluated the viscous and adiabatic contribution to \(e_i^{n+1}\). In other words the predictor phase is calculated explicitly because all the necessary quantities are available from the previous time step \(t^n\).

(iii) finally, derive \(u^{n+1}\) solving the equation (3) iteratively (corrector phase) for both the predicted and old adiabatic and viscous terms;

In the corrector stage the integration of the equation (3) is performed using the Brent method (Press et al 1989) instead of the Newton-Raphson, the accuracy being fixed to a part in \(10^{-5}\). The Brent method has been adopted because it is better suited as root-finder for functions in tabular form (Press et al. 1989).

### 3 INITIAL CONDITIONS

In the following we are going to check our treatment of the energy equation and our implementation of SF and FB simulating the formation of a disk galaxy. Since our code is not a cosmological code we shall consider an ad hoc initial configuration for a protogalaxy. This approach is justified by the fact they we are going to focus on processes occurring at scales much lower than the cosmological ones.

#### 3.1 Theoretical overview

According to modern gravitational instability theory (White & Rees 1978), protogalaxies comprise a mixture of dissipation-less dark matter (DM), whose nature is not clear, and dissipationless baryonic material, roughly in the mass ratio 1:10. After a violent relaxation process, which follows the separation from the Hubble expansion, a DM halo becomes isothermal, and acquires baryonic material which heats up at the halo virial temperature. Gas then cools and collapses, and through fragmentation turns into the stars we see today in disks and spheroidal systems.

In this scenario disks form as a consequence of the angular momentum that DM halos acquire due to the tidal torque felt by surrounding halos, while spheroidal systems are thought to be produced by merging of disks.

Instead of selecting halos from cosmological N–body simulations (see for instance Weil et al 1998), in our code we
We consider a triaxial DM halo whose density profile is

\[ \rho(r) \propto \frac{1}{r}. \]  

(5)

set up a protogalaxy as an isolated rotating DM halo with baryonic material inside, proceeding as follows.

### 3.2 Initial configuration

We consider a triaxial DM halo whose density profile is

\[ \rho(r) \propto \frac{1}{r}. \]

(5)

Although rather arbitrary, this choice seems to be quite reasonable. Indeed DM halos emerging from cosmological N-body simulations are not King or isothermal spheres, but show, independently from cosmological models, initial fluctuations spectra and total mass, an universal profile (Navarro et al 1996; Huss et al 1998). This profile is not a power law, but has a slope \( \alpha = d\ln \rho / d\ln r \) with \( \alpha = -1 \) close to the halo center, and \( \alpha = -3 \) at larger radii. Thus in the inner part the adopted profile matches the universal one. Moreover this profile describes a situation which is reminiscent of a collapse within an expanding universe, being the local free fall time a function of the radius (see the discussion in Curir et al 1993; Aguilar & Merritt 1990). Moreover triaxiality is quite natural for galactic DM halos (Becquert & Combes 1997; Olling & Merrifield 1998).

DM particles are distributed by means of an acceptance-rejection criterium, following step by step the procedure developed by Curir et al (1993). Specifically, we consider a triaxial ellipsoid whose radial scale \( r \) is given by

\[ r = (x^2 + \frac{y^2}{(b/a)^2} + \frac{z^2}{(c/a)^2})^{1/2}. \]

(6)

![Figure 1. The initial specific angular momentum \( j \) as a function of the radial distance \( r \) from the center of a 1/r dark matter halo.](image)

Table 1. Initial conditions of the simulated Dark Matter halo:

| \( \lambda \) | \( \beta \) | \( \delta_1 \) | \( \delta_2 \) |
|---|---|---|---|
| 0.09 | 0.20 | 0.05 | 0.025 |

where \( a > b > c \) are the axial ratios (see Table 1). The velocity field has been chosen to produce an angular momentum which depends linearly on the distance, \( j(r) \propto r \) (Barnes & Efstatiou 1987). It has been built up sampling the moduli \( V \) of the particles velocity by using a Maxwellian distribution and imposing that the virial ratio \( \beta = 2T/|W| \) is equal to some fixed value ranging between 0.05 and 0.20. Here \( T \) is the kinetic energy, while \( W \) is the potential energy. The final cartesian components of the particle velocity can be derived from the following formulas

\[ V_x = -V \times \sin(\theta + \alpha) \]

(7)

\[ V_y = V \times \cos(\theta + \alpha) \]

(8)

\[ V_z = \xi \]

(9)

where \( \theta \) is the angle between the position vector of a particle and the \( x \) coordinate axis. \( \alpha \) is an angle varying between \(-\delta_1 \pi \) and \(+\delta_1 \pi \), whereas \( \xi \) is a random parameter ranging from \(-\delta_2 V \) and \(+\delta_2 V \). \( \delta_1 \) and \( \delta_2 \) are chosen so that the kinetic energy \( T \) changes by less than 1% of the value giving the initial virial ratio \( \beta \).

Table 1 summarizes the adopted initial values for the set of parameters introduced above.

Due to the assigned velocity field, the halo acquires an amount of angular momentum, which is conventionally described by means of the dimensionless spin parameter \( \lambda \):

\[ \lambda = \frac{|E|^{1/2}}{GM^{5/2}} \]

Here \( G \) is the gravitational constant, \( J \) the system angular momentum, \( M \) the total mass and \( E \) the total system energy. In our case the \( \lambda \) parameter has chosen to be 0.09, significantly greater than the mean values of cosmological halos (Steinmetz & Bartelmann 1995). This choice is motivated by the comparison we are going to make with similar initial conditions (Navarro & White 1993; Thacker et al 1998; Raiteri et al 1996).

The softening parameter \( \epsilon \) is computed as follows. After plotting the inter-particles separation as a function of the distance to the model center, we compute \( \epsilon \) as the mean inter-particles separation at the center of the sphere, taking care to have at least one hundred particles inside the softening radius (Romeo A. G., Pearce F. R., private communications). We consider a Plummer softening parameter, equal for both DM and gas particles (see below), and keep it constant along the simulation. For this particular choice of the initial configuration the softening parameter \( \epsilon \) turns out to be 3.6 kpc.

Total energy and angular momentum are conserved within 1% and 0.1% level, respectively.

We consider a dark matter halo with mass \( 10^{12}M_\odot \) and radius \( a \) 120 kpc in order to simulate a galaxy with a size.
Figure 2. The formation process of a galactic disk in the $x-z$ plane. In the bottom-right corner of any snapshot time is reported in billion years.

similar to the Milky Way. Axial ratios are $a = 3.00$, $b = 2.25$, $c = 1.50$.

4 THE FORMATION OF A GASEOUS DISK

The formation of a galactic disk is simulated distributing gas inside the halo described above and switching cooling on. To mimic the in-fall of gas inside the potential well of the halo, we distribute gas particles (10,000 in number) on the top of DM particles. The baryonic fraction adopted is $f_b = 0.1$, and gas particles are Plummer-softened in the same way as the DM particles.

Under cooling and the velocity field of the halo, the gas is expected to settle down in a rotating thin structure. For this purpose we need to specify for the baryonic component a temperature and a metal content. We assume that the gas has a temperature of $\approx 1.0 \times 10^4$ °K (Navarro & White 1993), and an almost primordial metal content amounting to $Z \approx 10^{-4}$, which translates into $[Fe/H] \approx -3$ (Bertelli et al 1994).

This simulation is meant to verify that our energy integration scheme works properly when cooling is switched on. For this reason SF, FB and chemical enrichment are turned off.

The formation of the disk is shown in Fig. 3. Left panels follow from the top to the bottom the evolution of the gas component in the $x-z$ plane. Time is in Gyr. Starting from an ellipsoidal initial configuration ($t = 0.0$),
baryons due to cooling and angular momentum settle down in a thin rotating structure \((t = 1.75)\), eventually developing a bulge like structure in the model center \((t = 2.5)\). At this time the gaseous disk is about \(5 \text{kpc}\) high and \(40 \text{kpc}\) wide. Comparing our results with similar simulation by Raiteri et al (1996) we see that in our case the disk forms more slowly. This is due to the lower efficiency of our cooling with respect to the more widely adopted Katz & Gunn (1991) cooling.

Figure 3. The Star Formation history (in solar masses per year) for four models in which star formation criteria are defined as in Section 5. See text for details.

5 STAR FORMATION

As already recalled, SF in N-body simulations is let occur if suitable criteria meant to simulate the real behaviour of the ISM are satisfied.

Current prescriptions for SF stand on three time-scales, i.e. the crossing, cooling, and free-fall time-scales, which ultimately depend (although in a different fashion) on the density of the fluid. According to the most popular prescription (Katz 1992; Navarro & White 1993), SF is let occur if

- \(\nabla \cdot \vec{v}_i < 0\)
- \(t_{\text{sound}} > t_{\text{ff}}\)
- \(t_{\text{cooling}} < t_{\text{ff}}\)

Aim of this section is to closely scrutiny the effects of three conditions on the model results. We start with some general considerations.

Most likely, the divergence criterion is not strictly required. Firstly the typical mass resolution \((10^6 - 10^7 M_\odot)\) of this kind of N-body simulation is clearly not sufficient to follow the kinematical behaviour of the gas at the molecular clouds scale. Then in principle a fluid element can form stars...
without being in a convergent flow (let us think about shock induced or stochastic SF). For instance Mihos & Hernquist (1994) adopt in the special case of an already formed spiral galaxy a different dynamical criterium, based upon the Toomre instability criterium.

The free-fall time scale is usually computed as

\[ t_{ff} = \left( \frac{4\pi G \rho}{1} \right)^{-1/2} \] (10)

where \( \rho \) is the gas density of the fluid element. We have replaced this condition with the more general one

\[ t_{ff} = \left( \frac{4\pi G \rho_{tot}}{1} \right)^{-1/2} \] (11)

where \( \rho_{\text{tot}} = \rho_{\text{gas}} + \rho_{\text{DM}} \). In principle, one should consider also \( \rho_{\text{star}} \). In fact the dynamical behaviour of the fluid element under consideration depends on all the mass inside the volume with radius \( 2 \times h \), where \( h \) is the smoothing length. Therefore at the typical resolution of these simulations (about 2.5 kpc) the presence of DM accelerates the collapse of a fluid element by lowering the free-fall time-scale. DM density is evaluated in the same manner as gas density, defining a smoothing length for any DM particle, although this scheme results computationally expensive.

The importance of DM on SF has been recently studied from an analytical point of view by Caimmi & Secco (1997). This stems from suitable arguments about the cooling time-scale is customarily replaced by a condition on the density, i.e. a constant density threshold (Navarro & White 1993). This results from suitable arguments about the cooling time-scale based on the notion that the gas metallicity has no effect. However, as the cooling rate does increase with the metal content of the ISM with consequent increase of the threshold density, the more general condition on the cooling time-scale ought to be preferred.

### 5.1 Testing the SF recipes

To this aim we perform the same simulation as in Section 4, say the formation of a disk galaxy, but letting stars form according to different set of criteria. We follow the model till the end to the major star formation episode.

The simulations have been made using 10,000 DM and 10,000 baryonic particles, neglecting in this particular set of models any source of FB. The following four cases are examined.

| Case | SF peak | Peak time | Stars formed |
|------|---------|-----------|--------------|
| A    | 85      | 2.25      | 3381         |
| B    | 73      | 2.10      | 3771         |
| C    | 71      | 2.03      | 6580         |
| D    | 77      | 1.81      | 8543         |

The results for the SFR as a function of time are shown in Fig. 3, whereas a few relevant quantities of the models are given in Table 2.

In our models, fragmentation of gas-particles undergoing SF is let occur and new star-particles are created when the gas content of the fluid element has fallen below 20% of the initial value, being the remaining gas spread out over the surrounding particles. This limits the number of star particles produced. Moreover the gas particle is allowed to experiment up to ten SF episodes. Afterwards it cannot make stars anymore.

In all simulations the efficiency parameter \( \epsilon \) is fixed to 0.1. Finally, when more than two star-particles form inside a sphere with radius equal to the softening parameter \( \epsilon = 3.6 \text{ kpc} \), they are merged together to form a single object, in order to keep the total number of particles small.

The results for the SFR as a function of time are shown in Fig. 3, whereas a few relevant quantities of the models are given in Table 2.

Comparing either case A to case B or case C to case D, we notice that the criterion on the velocity divergence has in practice no effect, as argued above.

In contrast, there is a significant difference passing from the cases A and B based on the over-density condition to the cases C and D based on the cooling time-scale. In fact the models of the first group start sensibly later to form stars and after the burst, which occurs somewhat later, SF maintains higher that in the second group models, and show some secondary peaks. Dropping the divergence criterium significantly increases the number of spawned stars (model C and D).

In these models after the peak SF proceeds with a rate around \( 4 - 6 \leftarrow M_\odot/\text{yr} \), quite usual for disk galaxies. Putting together all these differences and the discussion above we are led to prefer the combination of criteria of the model D, which we are going to use in all the other simulations presented in this paper.

| Case | \( \nabla \cdot \vec{v} \nabla \rho \) | \( t_{\text{sound}} > t_{ff} \) | \( \rho > \rho_{\text{crit}} \) |
|------|----------------------|----------------|----------------|
| A    | \( \nabla \cdot \vec{v} \nabla \rho \) < 0 | \( t_{\text{sound}} > t_{ff} \) | \( \rho > \rho_{\text{crit}} \) |
| B    | \( t_{\text{sound}} > t_{ff} \) | \( \rho > \rho_{\text{crit}} \) |
| C    | \( \nabla \cdot \vec{v} \nabla \rho \) < 0 | \( t_{\text{sound}} > t_{ff} \) | \( t_{\text{cooling}} << t_{ff} \) |
| D    | \( t_{\text{sound}} > t_{ff} \) | \( t_{\text{cooling}} << t_{ff} \) |
All simulations exhibit the SF peak at about 2 Gyr, much later than, for instance, the Raiteri et al. (1996) simulation. This is clearly due to the different cooling functions adopted. Katz & Gunn (1991) analytical formulas provide a much more efficient cooling (see below and Carraro et al 1998a).

6 STELLAR FEED-BACK

In this section, we test separately the effect of FB of different nature making use of the case D for the recipe of star formation. Furthermore, we include the effect of chemical enrichment.

The energy FB originates from SNe explosions of both type Ia and II (Greggio & Renzini 1983), the stellar winds from massive stars (Chiosi & Maeder 1986) and the UV flux from the same objects (Chiosi et al. 1998).

In order to evaluate the amount of energy injected into ISM by the above sources we suppose that each newly formed star-particle, which for the mass resolution of our simulations has a total mass of the order of \(10^7 M_\odot\), is actually made of a larger number of smaller subunits (the real stars) lumped together and distributed in mass according to a given initial mass function (IMF). For the purposes of the present study we adopt the IMF by Miller & Scalo (1979) over the mass interval from 0.1 to 120 \(M_\odot\).

Chemical evolution is followed by means of the closed-box approximation (Tinsley 1980; Portinari et al. 1998) and sharing of the metals among the gas-particles is described by means of a diffusive scheme (Groom 1997; Carraro et al. 1998a)

Details on implementation of FB in our Tree-SPH code has been already reported in Carraro et al. (1998a), to whom the reader should refer. With respect to the previous study, only two major changes have been made. First, of the UV flux emitted by massive stars only a small fraction (0.01) is actually used. All the remaining UV flux is supposed to be re-processed by dust into the far-infrared and lost by the galaxy (Silva et al. 1998). Second, the energy released to the ISM by a SN event amounts only to \(10^{50}\)ergs, roughly 10 times less than in older evaluations (see Thornton et al. 1998 for an exhaustive discussion of this topic). The heating rates from the three sources of energy are displayed in Fig. 4 for the sake of illustration. A notable feature to remark, is that the energy injection by stellar winds may parallel that from SNe explosions, and the contribution from the UV flux plays a significant role although a strong dumping factor has been applied. All the FB mechanisms have almost the same trend, since they derive basically from the same stellar sources.

Finally, we stress that all the energy from FB is given to the thermal budget of the particles, because the resolution we are working with makes it impossible to describe this process in more detail. In fact, the space resolution of these simulations is about 2.5 kpc, much larger than the typical distance over which the effects of SNe explosions and stellar winds are visible (Mkée & Ostriker 1977). See also the arguments given by Carraro et al. (1998a). Possible interactions of kinetic nature among gas particles occur only via the pressure gradients.

6.1 Testing the sources of Feed-Back

In order to understand the role played by each source of FB we perform the following experiments using the same model as in the previous section. Three cases are considered:

Case **D1**: FB from SNe;
Case **D2**: FB from SNe and stellar winds;
Case **D3**: FB from SNe, stellar winds, and UV flux.

First of all in Fig. 5, for the case **D1**, we show the SF history, and the rate of SNe of both types, Ia and II. Due to the different stellar progenitors, SNe Ia and II rates show different trends. SNe II explode immediately, and their rate strictly follow the SF history. On the contrary, SNe Ia start to appear later, being the progenitor less massive. The relative abundance of SNe events clearly depends on the adopted IMF.

The results of these simulations are shown in the various panels of Fig. 6. A few relevant quantities of the models are given in Table 3. Namely we consider the mean gas temperature, the SF peak and the time at which it occurs, the final maximum metal abundance, and the radial and vertical dimensions of the final stellar disk.

Looking at these results the following conclusions can be drawn. The models **D1** and **D2** do not differ significantly, simply turning on FB from stellar winds the final models becomes a bit hotter, less stars are formed and the final maximum metal abundance achieved is slightly lower. Since almost the same amount of stars are generated, the final stellar disk morphology is basically identical.

When the combined effect of all the FB sources are considered (model **D3**) the situation changes more significantly (see also Fig. 4). The model becomes hotter more rapidly when SF starts, SF peak occurs before, and less stars are formed. As a consequence the final mean temperature is somewhat lower and the disk morphology is not well resolved as in the previous models. The final maximum metallicity turns out somewhat greater due to the effect of the diffusive mixing. Metals are spread and smooth around the gas neighbors. Since SF in this model stops before, no metal poor stars are produced, and the mean metallicity does increase.

In all the models a number of gas particles do not experience SF and keep the primordial metal content.

7 SUMMARY AND CONCLUSIONS

In the framework of the formation of a spiral-like galaxy, we have examined in detail the effect of different prescriptions for Star Formation and Feed-Back on numerical simulations of galaxy formation and evolution.

First of all we described in details how we integrate energy equation when cooling is switched on. We adopted cooling functions which depend on metal abundance, and use the Brent method as root finder instead of the more widely used Newton-Raphson scheme. Brent scheme performs better when using functions in tabular form.

As far as the physical conditions at which Star Formation is likely to start, we argue that the criterion on the
Figure 5. SF history and SNe rates for the model D3. SNe II closely follow SF history, whereas SNe Ia exhibit a peak somewhat later, as expected from the different progenitors lifetime.

Table 3. Results for models with different Feed-Back.

| Case | $T_{ave}$ | SF peak | Peak time | $[Fe/H]_{f,_{in}}$ | $r_D$ | $h_D$ |
|------|---------|---------|-----------|----------------|------|------|
|      | $10^6$ $^o$K | $M_\odot$/yr | Gyr | dex | kpc | kpc |
| D1   | 1.15    | 39      | 1.72      | 0.446 | 30  | 5    |
| D2   | 1.46    | 38      | 1.72      | 0.406 | 30  | 5    |
| D3   | 1.33    | 32      | 1.67      | 0.513 | 20  | 5    |

velocity divergence has no sizable effect on the model results.
Moreover the over-density and cooling time criteria are equivalent only if the effect of metallicity on the cooling time-scale is neglected. However, as in real galaxies the metallicity varies as a function of time and space, the criterion based on the cooling time-scale ought to be preferred. This finding is a step further toward better understanding under which conditions Star Formation takes place in real galaxies.
In addition we introduce in the gas free-fall time computation the contribution of DM and eventually stars which lie
within the neighbors sphere. This has the effect to decrease the free-fall time, accelerating the cloud collapse.

Finally we have quantitatively shown the separated and cumulated effects of different sources of Feed-Back (SNe, stellar wind and UV flux from massive stars) on the global properties (SFR, metallicity and morphology) of galaxy models.

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