Radiation fluid singular hypersurfaces with de Sitter interior as models of charged extended particles in general relativity

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Abstract. In present paper we construct the classical and minisuperspace quantum models of an extended charged particle. The modelling is based on the radiation fluid singular hypersurface filled with physical vacuum. We demonstrate that both at classical and quantum levels such a model can have equilibrium states at the radius equal to the classical radius of a charged particle. In the cosmological context the model could be considered also as the primary stationary state, having the huge internal energy being nonobservable for an external observer, from which the Universe was born by virtue of the quantum tunnelling.

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1. Introduction

About one hundred years ago, Lorentz suggested a model of an extended electron as the body having pure charge and no matter [1]. His model had severe problems with stability because the electric repulsion should eventually lead to explosion of the configuration proposed. Since that time many modifications of this model have been done. To provide stability of the Lorentz model, Poincaré introduced the stresses that greatly improved situation [2]. After appearance of general relativity Einstein raised the particle problem on the relativistic level. Subsequently, the Lorentz-Poincaré ideas obtained new sounding in terms of repulsive gravitation [3], and interest to such models has been renewed, especially in connection with the modelling of exotic extended particles. In spite of these relativistic models seem to be appropriate for a charged spinless particle (e.g., π-meson) rather then for an electron (except the works [4]), they nevertheless contain some consistent descriptions and concepts; it is implicitly assumed that the presence of spin entails only small deviations from spherical symmetry.

The main idea of the most poplar models is to consider the (external) Reissner-Nordström solution of the Einstein-Maxwell field equations,

$$ds^2 = - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

(1)
(we will work in terms of gravitational units), and to match it with some internal one; thereby this matching is performed across the boundary surface (which is considered as a particle surface), i.e., the first and second quadratic forms are continuous on it (it is the well-known Lichnerowicz-Darmois junction conditions) \[5\]. At the same time, more deep investigations of the electromagnetic mass theories, based on such a matching, elicit a lot of the features which point out some imperfection of the particle models based on the boundary surfaces (i.e., on the discontinuities of first kind).

The first of them is an accumulation of the electric charge on a core boundary. Let us illustrate it for the model \[6\]. Cohen and Cohen matched the external Reissner-Nordström solution \[1\], at \( r > R \) where \( R \) is a core radius, with the de Sitter one, at \( r \leq R \),

\[
ds^2 = -(1 - \lambda^2 r^2) dt^2 + (1 - \lambda^2 r^2)^{-1} dr^2 + r^2 d\Omega^2, \quad \lambda^2 = \frac{8\pi\varepsilon_v}{3}, \tag{2}
\]

thereby they obtained \( M = (4\pi\varepsilon_v/3)R^3 + Q^2/2R, \varepsilon_v \) is the physical vacuum energy density; the static chart describes interior at \( \lambda r < 1 \). The stress-energy tensor components for this solution look like those for the polarised vacuum \[7\]:

\[
T_{00} = \varepsilon_v = (3)\rho + Q^2(r)/8\pi r^4, \\
T_{11} = \varepsilon_v = -(3)p + Q^2(r)/8\pi r^4, \\
T_{22} = T_{33} = \varepsilon_v = -(3)p_\perp - Q^2(r)/8\pi r^4, \tag{3}
\]

where \( Q(r), (3)\rho, (3)p \) are the charge, energy density, and pressure inside \( r \), respectively. The generalisation of the Tolman-Oppenheimer-Volkov equation for such charged models yields \[8\]

\[
\frac{d}{dr} (3)p = \frac{1}{8\pi r^4} \frac{d}{dr} Q^2(r) \quad r \leq R, \tag{4}
\]

hence

\[
Q(r) = Q \theta(r - R), \tag{5}
\]

where \( \theta(r) \) is the Heaviside step function. Thus, there is no electric field inside the core, and charge is accumulated on the boundary only. However, the discontinuities of first kind cannot have surface charge distribution \textit{a priori}. The second feature is the energy of the false vacuum is transferred to a boundary as well \[9, 15\]. Nevertheless, the discontinuities of first kind cannot have surface energy density too. Such discordances of the boundary-surface models compel to seek for the more realistic models, e.g., having non-trivial surface properties.

2. Classical model

We can consider the system of the Einstein-Maxwell field equations plus singular (infinitely thin) shell instead of the simple boundary surface \[10, 11, 12\]. The singular surface has to be the discontinuity of second kind (the first quadratic form is continuous across it but the second one has a finite jump), and, unlike the boundary surface, can have proper surface charge and stress-energy tensor

\[
S_{ab} = \sigma u_a u_b + p(u_a u_b + (3)g_{ab}), \tag{6}
\]

where \( \sigma \) and \( p \) are respectively the surface energy density and pressure, \( u^a \) is the timelike unit tangent vector, \( (3)g_{ab} \) is the 3-metric on a shell.
Thus, let us study the spherical singular shell filled with de Sitter vacuum $\Sigma^-$ (2) and inducing the external Reissner-Nordström spacetime $\Sigma^+$ (1). The metric of the $(2+1)$-dimensional spacetime $\Sigma$ of the shell can be written in terms of the shell’s proper time $\tau$ as

$$d{s^2} = -d\tau^2 + R^2d\Omega^2,$$

where $R = R(\tau)$ turns to be the proper radius of the shell.

Then the foliated Einstein-Maxwell field equations give the two equation groups. The first one is the Einstein-Maxwell equations on the shell, integrability conditions of which yield both the conservation law of shell’s matter

$$d\left(\sigma^{(3)}g\right) + p\,d\left(\sigma^{(3)}g\right) + \sigma^{(3)}g\,\Delta T^\tau_\tau\,d\tau = 0,$$

where $\Delta T^\tau_\tau = (T^\tau_\tau)^+ - (T^\tau_\tau)^-$ is the projection of the stress-energy tensors in the $\Sigma^\pm$ spacetimes on the tangent and normal vectors, $\det \left(\sigma^{(3)}g_{ab}\right) = R^2 \sin \theta$ (it should be noted that $T^\tau_\tau \equiv 0$ for the spacetimes (1), (2)), and electric charge conservation law which in our case can be reduced to the relation $Q = \text{constant}$. The second group includes the equations of motion of a shell which are the Lichnerowicz-Darmois-Israel junction conditions

$$\left(K^a_b\right)^+ - \left(K^a_b\right)^- = 4\pi\sigma\left(2u^au^b + \delta^a_b\right),$$

where $(K^a_b)^\pm$ are the extrinsic curvatures for spacetimes $\Sigma^\pm$ respectively. Besides, the associated electromagnetic potential vector is discontinuous across the shell. Following aforesaid, the electromagnetic potential is zero inside the shell whereas outside it has to be

$$A_i = (-Q/r, 0, 0, 0).$$

Thus, taking into account (1), (2), (7), and (8), the $\theta\theta$ and $\tau\tau$ components of the equation (9) yield

$$\epsilon_+\sqrt{\dot{R}^2 + 1 - \frac{2M}{R} + \frac{Q^2}{R^2}} - \epsilon_-\sqrt{\dot{R}^2 + 1 - \lambda^2 R^2} = -4\pi\sigma R,$$

$$\epsilon_+\dot{R} - \epsilon_-\dot{R} - M\dot{R} - Q^2 - \lambda^2 R = -4\pi\frac{d(\sigma R)}{dR},$$

where $\epsilon_+ = \text{sign}\left[(K^\theta_\theta)^\pm\right]$, $\dot{R} = dR/d\tau$ etc. The root sign $\epsilon = +1$ if $R$ increases in the outward normal of the shell, and $\epsilon = -1$ if $R$ decreases. Thus, only under the additional condition $\epsilon_+ = \epsilon_- = 1$ we have the ordinary shell. Below we will deal with such shells only.

Besides, independently of the Einstein equations the equation of state for matter on the shell $p = p(\sigma)$ should be added as well. The simplest equation of state we choose is the linear one of a barotropic fluid [13],

$$p = \eta\sigma,$$

including as a private case, dust ($\eta = 0$), radiation fluid ($\eta = 1/2$, the reduction of shell’s spacetime dimensionality is taken into account), bubble matter ($\eta = -1$), and so on [13, 14, 15]. Then the constant $\eta$ remains to be arbitrary and will be specified
below for physical reasons. It can easily be seen that the equations (8), (10), and (12) form a complete system. Solving equations (8) and (12) together, we obtain
\[ \sigma = \frac{\alpha}{4\pi} R^{-2(\eta+1)}, \] (13)
where \( \alpha \) is an integration constant related to the surface energy density at some fixed \( R \), \( \alpha > 0 \) for ordinary shells. Then the equations (10) and (11) can be rewritten more strictly:
\[ \sqrt{R^2 + 1 - \frac{2M}{R} + \frac{Q^2}{R^2}} - \sqrt{R^2 + 1 - \lambda^2 R^2} = -\frac{\alpha}{R^{2\eta+1}}, \] (14)
\[ \frac{R^3 \dot{R} + MR - Q^2}{R^3 \sqrt{R^2 + 1 - \frac{2M}{R} + \frac{Q^2}{R^2}}} - \frac{\dot{R} - \lambda^2 R}{\sqrt{R^2 + 1 - \lambda^2 R^2}} = \frac{\alpha(2\eta + 1)}{R^{2(\eta+1)}}. \] (15)

Now we consider this system in the equilibrium \( \dot{R} = \ddot{R} = 0 \) at \( R = R_p \) that will correspond to a classical particle radius. Then these equations turn to be the equilibrium conditions \[16\]
\[ \sqrt{1 - \frac{2M}{R_p} + \frac{Q^2}{R_p^2}} - \sqrt{1 - \lambda^2 R_p^2} = -\frac{\alpha}{R_p^{2\eta+1}}, \] (16)
\[ \frac{M R_p - Q^2}{R_p^3 \sqrt{1 - \frac{2M}{R_p} + \frac{Q^2}{R_p^2}}} + \frac{\lambda^2 R_p}{\sqrt{1 - \lambda^2 R_p^2}} = \frac{\alpha(2\eta + 1)}{R_p^{2(\eta+1)}}. \] (17)

Further, it is well-known that the classical radius \( R_c \) of a charged particle can be defined as the radius at which the function \( 1 - \frac{2M}{R} + \frac{Q^2}{R^2} \) approaches a minimum. Therefore \( R_c = Q^2/M \); assuming \( R_p = R_c \) we have respectively
\[ \sqrt{1 - (M/Q)^2} - \sqrt{1 - (\lambda c Q^2/M)^2} = -\alpha_c (M/Q^2)^{2\eta+1}, \] (18)
\[ \lambda_c^2 = \alpha_c (2\eta + 1) (M/Q^2)^{2\eta+3} \sqrt{1 - (\lambda c Q^2/M)^2}. \] (19)

It should be noted that from the last equation the restriction for \( \eta, \eta > -1/2 \), follows which appears to be a necessary condition for existence of equilibrium at \( R = R_c \). Hence it is easy to see that our thin-wall model cannot be “made” from the bubble matter \( \eta = -1 \) (\( \sigma = \text{constant} \)). But the ultrarelativistic radiation fluid, on the contrary, seems to be a very suitable candidate for the role of two-dimensional matter on the particle surface. In this connection it is interesting to note that radiation fluid singular configurations can not be in equilibrium as a rule. However, in the present case the internal physical vacuum compensates both the radial pressure caused by the shell and repulsive electrical forces that stabilises the system.

Considering the choice \( \eta = 1/2 \) and eliminating \( \alpha_c \) from the system (18), (19), we obtain after rejecting a superfluous root the exact electromagnetic mass relation
\[ \lambda_c^2 = \frac{8\pi\varepsilon}{3} = \frac{2}{9} \left( \frac{M}{Q^2} \right)^2 \left[ 2 + \frac{M^2}{Q^2} - \sqrt{\left( \frac{M^2}{Q^2} \right) \left( 4 - \frac{M^2}{Q^2} \right)} \right], \] (20)
which determines the necessary stabilising energy of de Sitter vacuum inside a charged particle within frameworks of the classical model having a radiation fluid singular surface. We can simplify this relation using the fact that the value \( M/Q \) turns to
be very small for the known elementary particles. Indeed, performing the Taylor expansion of equation (20), we obtain

$$\varepsilon_v = \frac{3}{16\pi Q^6} M^4 + O\left(\frac{M^6}{Q^8}\right).$$  \hspace{1cm} (21)

Let us perform some numerical estimations. For instance, for an electron we have: $M = 0.511 \text{ MeV} = 6.76 \times 10^{-58} \text{ m}$, $Q = 1.38 \times 10^{-36} \text{ m}$, $R_c = 2.8 \times 10^{-15} \text{ m}$, hence $\varepsilon_v = 2.5 \text{ keV fm}^{-3}$. For $\pi^\pm$-mesons we get: $M = 139.57 \text{ MeV}$, $Q = 1.38 \times 10^{-36} \text{ m}$, $R_c = 1.02 \times 10^{-17} \text{ m}$, hence $\varepsilon_v = 14 \text{ TeV fm}^{-3}$. One can see that for particles of reasonably large mass the huge internal vacuum energy and electrical charge can be equilibrated by the surface gravitational forces providing the (classical) stability of the model. The gravitational defect of masses causes the property that the external (observable) mass of a particle appears to be much less than the internal energy. The more detailed analysis of the possible applicability of such a phenomenon will be given below at the consideration of quantum fluctuations, all the more so latter enforce this feature.

3. Minisuperspace quantization

Following the Wheeler-DeWitt’s approach in quantum cosmology the Universe is considered quantum mechanically and is described by a wave function $\chi$. This function is defined on the superspace which is the space of all admissible metrics and accompanying fields. The minisuperspace approach appears to be the direct application of Wheeler-DeWitt’s quantization procedure for $(2+1)$-dimensional singular hypersurfaces having own internal three-metric $(3)g^{ab}$. Thereby, in spherically symmetric case the world sheet of a singular hypersurface is determined by a single function, viz., proper radius $R(\tau)$. In the absence of a unified rigorous axiomatic approach this method has the following advantages in comparison with others: (i) it is simple and gives heuristic results in most cases in a nonperturbative way, which is very important for non-linear general relativity, (ii) there explicitly exists conformity with the correspondence principle that improves physical interpretation of all the concepts of the theory.

Let us consider a minisuperspace model initially described by the Lagrangian

$$L = \frac{m R^2}{2} - U,$$ \hspace{1cm} (22)

where

$$U = U_1 + U_2,$$ \hspace{1cm} (23)

$$U_1 = -\frac{\lambda^4 R^7}{8\alpha} + \frac{M \lambda^2 R^4}{2\alpha} - \frac{Q^2 \lambda^2 R^3}{4\alpha} - \frac{(\lambda^2 \alpha^2 + 2 M^2)R}{4\alpha},$$

$$U_2 = \frac{M Q^2}{2\alpha} + \frac{4\alpha^2 - Q^4}{8\alpha R} - \frac{\alpha M}{2 R^2} + \frac{\alpha Q^2}{4 R^3} - \frac{\alpha^3}{8 R^5},$$

and

$$m = 4\pi \sigma R^2 = \alpha / R$$ \hspace{1cm} (24)
is the mass of the radiation fluid shell in the static reference frame. The equation of motion is therefore
\[
\frac{d}{d\tau}(mR) = \frac{m R \dot{R}^2}{2} - (U_1 + U_2) R, 
\]
where "\( \dot{R} \)" means the derivative with respect to shell radius. Considering time symmetry, we can easily decrease an order of this differential equation and obtain
\[
\dot{R}^2 = \frac{2R}{\alpha}(H - U_1 - U_2),
\]
where \( H \) is the integration constant. It can be directly checked that if one supposes \( H \) to be vanishing,
\[
H = 0,
\]
then we obtain the (double squared) equation of motion of our radiation fluid shell (10) provided eqs. (13), (24) and \( \eta = 1/2 \). Thus, our Lagrangian indeed describes dynamics of the charged radiation fluid shell filled with physical vacuum up to topological features which were described by the signs \( \epsilon_\pm \). However, we always can take into account the topology \( \epsilon_\pm \) both at classical (rejecting superfluous roots) and quantum (considering boundary conditions for the corresponding Wheeler-DeWitt equation) levels.

Further, introducing the momentum \( \Pi = m \dot{R} \), from (27) we obtain that the (super) Hamiltonian,
\[
\mathcal{H} = \Pi \dot{R} - L \equiv H,
\]
has to be equal zero on the real trajectories (10). Therefore, we have obtained the constraint
\[
\frac{\Pi^2}{2\alpha/R} + U_1 + U_2 = 0, 
\]
which in quantum case (\( \Pi = -i\partial/\partial R \), here and below we assume Planckian units) yields the Wheeler-DeWitt equation for the wave function \( \Psi(R) \):
\[
\Psi'' + \Xi \Psi = 0, 
\]
where
\[
\Xi = \Xi_1 + \Xi_2, 
\]
\[
\Xi_1 = \frac{\lambda^4 R^6}{4} - M \lambda^2 R^3 + \frac{Q^2 \lambda^2 R^2}{2} + \frac{\lambda^2 \alpha^2}{2} + M^2, 
\]
\[
\Xi_2 = -\frac{MQ^2}{R} + \frac{Q^4 - 4\alpha^2}{4R^2} + \frac{M\alpha^2}{R^3} + \frac{Q^2 \alpha^2}{2R^4} + \frac{\alpha^4}{4R^6}. 
\]

Now we will consider small quantum oscillations of radius in a neighbourhood of the state of dynamical equilibrium represented by the minimum point at \( R = R_c \). In our theory the two types of equilibrium exist: kinematic
\[
\dot{R} = 0 \iff \Xi = 0, 
\]
and dynamical (all acting forces are equilibrated)
\[ \ddot{R} = 0 \iff \Xi' = 0. \]
(33)

In quantum theory the simultaneous exact satisfaction of the requirements \( R = R_c \)
and \( \dot{R} = 0 \) turns to be forbidden because of the uncertainty principle. The
quantum analogue of classical equilibrium is the (quantum) dynamical equilibrium
when a system passes to the bound state characterised by a discrete spectrum for
Corresponding parameters, first of all, the total mass (energy) \( M \). Therefore, we will
suppose that eq. (33), not (32), is valid at \( R = R_c \). Then eqs. (31), (33) give
\[ \alpha^2 = \frac{Q^6}{3M^6} [M^2(2Q^2 - M^2) + \zeta \beta], \]
where \( \zeta = \text{sign } [\beta] \) and
\[ \beta = \sqrt{4M^4Q^2(Q^2 - M^2) + (2M^4 - 3\lambda^2Q^6)^2}. \]

Then in the first-order \( \hbar \)-expansion approximation from eqs. (30), (31), (33)
and (34) we obtain the wave equation for some quantum harmonic oscillator of unit mass
\[ \frac{1}{2} \frac{d^2\Psi}{dx^2} + \left[ \varepsilon - \frac{\omega^2x^2}{2} \right] \Psi = 0, \]
(35)
where \( x = R - Q^2/M \),
\[ 2\varepsilon = \frac{1}{18M^6} \left[ 9\lambda^4Q^{12} - 6\lambda^2Q^6M^2(3M^2 - Q^2) \right. \]
\[ -4M^4(2Q^4 - 2M^2Q^2 - M^4) + (3\lambda^2Q^6 - 4M^2Q^2 + 2M^4)\zeta \beta \right], \]
\[ \omega^2 = \frac{1}{3M^2Q^4} \left[ 3\lambda^2Q^6(19M^4 - 18\lambda^2Q^6) - 2M^4(8Q^4 - 9M^2Q^2 + 7) \right. \]
\[ -M^2(8Q^2 - 5M^2)\zeta \beta \right]. \]
(36)

Further, for bound states the quantum boundary conditions, corresponding to
the singular Stourm-Liouville problem, require \( \Psi(-\infty) = \Psi(\infty) = 0 \) (that has to
be somewhere artificial but admissible \( \hbar \)-expansion condition if we suppose the wave
function to be localised near the stationary point of the effective potential \( (26) \)), and
the normalised solution of eq. (35), which approximate the exact solution in the
neighbourhood of the stationary point, can be expressed by means of the Hermite
polynomials \( H_n(y) \)
\[ \Psi(y) = \left(2^n\sqrt{\pi n!}\right)^{-1/2} \exp \left(\frac{-y^2}{2}\right) H_n(y), \]
(38)
where \( n = 0, 1, 2, \ldots \), \( y = \sqrt{\omega x} \). The spectrum of \( \varepsilon \) is known to be
\[ \varepsilon = \omega(n + 1/2), \]
(39)
which yields, together with eqs. (36) and (37), the restriction on the parameters \( \lambda \),
\( M, Q \) of the studied configuration, radiation fluid shell filled with physical vacuum,
in the bound state.
In general case equation (39) is hard to solve, however in squared (39) we can perform expansion in series with respect to the ratio \( \delta = M/Q \) which is expected to be small for non-macroscopical particles. Then we obtain \((N = 2n + 1)\):

\[
\zeta = +1 : -\frac{1}{2} \lambda^2 Q^2 \delta^{-4} + \frac{5}{9} \delta^{-3} + O(\delta^{-2}) = \left( \frac{3N}{3Q^3} \right)^2, \tag{40}
\]

\[
\zeta = -1 : -\frac{1}{2} \delta^{-2} + \frac{1 + 3\lambda^2 Q^2}{3\lambda^2 Q^2} \delta^{-1} + O(\delta^0) = \left( \frac{3N}{Q^2} \right)^2. \tag{41}
\]

Let us consider, e.g., the case \( \zeta = -1 \). Then from the last equation the total mass-energy spectrum follows:

\[
M = \frac{3\lambda^2 Q^3}{1 + 3\lambda^2 Q^2 \pm \sqrt{(1 + 3\lambda^2 Q^2)^2 - 162\lambda^3 N^2}}, \tag{42}
\]

in which the deSitter’s density \( \lambda^2 \) remains to be a free parameter. It appears to be the one of the most severe problems if we would seek to successively answer the questions: what are the particles our model describes? are they known or exotic particles? are they really microscopical objects? and so on. These questions are open yet.

Finally, let us perform analytical estimations of internal vacuum energy density. Equation (41) yields the expression (the case \( \zeta = 1 \) gives the same estimate)

\[
\varepsilon_v = \frac{1}{4\pi} \frac{M}{Q^3} + O \left( \frac{M^2}{Q^4} \right), \tag{43}
\]

which evidently does not depend on \( n \) at this order. Comparing expressions (41) and (43) one can see that in quantum case \( \varepsilon_v \) has to be much more (three orders with respect to \( \delta \)) than its classical counterpart. Thus, quantum fluctuations appeared to be the strong additive reason preserving the model from both the gravitational shrinking and electrical explosion. The gravitational defect of mass enforced by them leads to the internal vacuum energy confined inside such a shell† is much more than the external one. From the cosmological point of view it is an expected result: there exists the shell models of the birth of the Universe from a small region to infinite size by virtue of the quantum tunnelling from a stationary state (see [18, 21] and references therein). The released huge energy is nothing but the deconfined internal energy, the internal space turns to be the whole spacetime of the created Universe, and the surface electrical charge is non-observable for the internal observer. In this connection our radiation fluid shell indeed could be considered also as the primary stationary state because of the four necessary reasons: (i) it has a local stationary state, (ii) it has the states with infinite radius (see the effective potential defined by the equation (26)), (iii) the barrier between them and the stationary state has a finite height, (iv) it demonstrates the opportunity of the presence of huge confined energy inside itself.

### 4. Conclusion

Thus, in present paper we have constructed the classical model of an extended charged particle based on the equilibrium radiation fluid shell filled with physical vacuum.

† The internal energy density is estimated to be of order \( 10^{-21} - 10^{-19} \) of the Planckian one for the trial masses of the electron and pion.
Thereby we found the configuration in which the radiation fluid can be confined. Then we performed the minisuperspace quantization of this model and obtained the Wheeler-DeWitt equation. Resolving it in a neighbourhood of the point of dynamical equilibrium, we established that the configuration can have bound states near this point, obtained discrete spectra for them, and performed quantitative estimations for some trial masses. Thus it was shown that the radiation fluid singular surface filled with physical vacuum indeed can model some particle-like objects both at classical and quantum levels. Besides, in the cosmological context the model could be considered also as the primary stationary state from which the Universe can be born via the quantum tunnelling.

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