Deconfined quantum criticality and Neel order via dimer disorder

Michael Levin\textsuperscript{1} and T. Senthil\textsuperscript{1}

\textsuperscript{1}Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Recent results on the nature of the quantum critical point between Neel and valence bond solid(VBS) ordered phases of two dimensional quantum magnets are examined by an attack from the VBS side. This approach leads to an appealingly simple physical description, and further insight into the properties of the transition.

Recent theoretical work\cite{1,2} on quantum phase transitions in two dimensional spin-1/2 quantum antiferromagnets has unearthed some interesting phenomena dubbed ‘deconfined quantum criticality’. The theory of such deconfined quantum critical points is described in terms of excitations that carry fractionalized quantum numbers which interact through an emergent gauge field. A precise characterization of the deconfinement is provided by the emergence of an extra global topological $U(1)$ symmetry not present at a microscopic level. This symmetry leads to an extra conservation law at the critical fixed point that is conveniently interpreted as the conservation of a gauge flux.

The most prominent example of such a deconfined quantum critical point arises at the transition between Neel and valence bond solid(VBS) ordered phases of spin-1/2 magnets on a square lattice. A direct second order transition is possible between these two phases despite their very different broken symmetries, and in contrast to naive expectations based on the Landau paradigm for phase transitions. Previous results\cite{1,2} on this transition have been based primarily on an attack starting from the Neel ordered side. Here we will take the alternate approach of attacking from the VBS side. This new approach provides for an appealingly simple physical description of the transition.

The Neel ordered state is described by an $O(3)$ vector order parameter. The VBS state, on the other hand, is described by a $Z_4$ clock order parameter. The four degenerate ground states associated with the $Z_4$ order parameter are illustrated in Fig. 1 for a specific VBS state in which the valence bonds have lined up in columns. A naive approach to the transition from the Neel side would associate the critical fixed point with the usual $O(3)$ fixed point in $D = 2 + 1$ dimensions. This expectation is incorrect. Similarly a naive approach to the transition from the VBS side would lead one to expect a critical fixed point in the $Z_4$ universality class in $D = 2 + 1$. This expectation is again incorrect. Similarly a naive approach to the transition from the VBS side would lead one to expect a critical fixed point in the $Z_4$ universality class in $D = 2 + 1$.

Why do these naive expectations fail? The answer is rooted in the observation that the topological defects in either order parameter carry non-trivial quantum numbers. When the defects in one order parameter, say the Neel vector, proliferate and condense they kill long range Neel order. At the same time the quantum numbers they carry induces a different broken symmetry. This non-trivial structure of the defects is inherently quantum mechanical and is not captured in naive macroscopic treatments of the broken symmetry state. For the Neel ordered states, the structure of the defects (known as hedgehogs)\cite{4} and their role in producing the VBS ordered paramagnet\cite{5} was elaborated many years ago. This provided the basis for the theory of the transition developed in Ref. \cite{1,2}. Here we will expose this physics starting from the VBS side.

As the VBS order is described by a discrete $Z_4$ clock order parameter, the natural topological defects are domain walls. Various kinds of walls between the four different broken symmetry states are possible. It is convenient to consider an ‘elementary’ domain wall across which the clock angle shifts by $\pi/2$, and to assign an orientation to such a wall. An example is shown in Fig. 2. All other walls, where the clock angle shifts by higher
multiples of $\pi/2$, may be built up from the elementary wall.

A key point is that 4 such elementary walls can come together and terminate at a point. In a macroscopic description focusing only on the order parameter this is illustrated in Fig. 3. It is clear that such termination points may be associated with $Z_4$ vortices - the clock angle winds by $2\pi$ upon encircling such a termination point. $Z_4$ antivortices may also be similarly defined.

What do such $Z_4$ vortices correspond to in terms of the underlying VBS configurations? An example is illustrated in Fig. 4. A remarkable property of this cartoon is that at the ‘core’ of such a vortex there is a site with an unpaired spin - i.e. a spin that is not part of any valence bond. It is easy to see that this is a general property of any such vortex pattern of the VBS order parameter. Furthermore, translating the entire valence bond pattern by one lattice spacing reverses the direction of the winding - thus the $Z_4$ vortices are associated with one sublattice, say the $A$ sublattice, and the $Z_4$ antivortices with the $B$ sublattice.

Thus in this particular quantum problem, the $Z_4$ vortices (and antivortices) carry an uncompensated spin-$1/2$ moment. They may therefore be identified with ‘spinons’. In the VBS ordered phase the energy cost of two such $Z_4$ vortices increases linearly with their separation at long distances. Thus the spinons are confined and do not exist as free excitations.

It is the non-trivial structure of the $Z_4$ vortex in this problem that distinguishes the VBS state from a more ordinary state with a $Z_4$ order parameter. Such an ordinary state obtains for instance in a simple lattice quantum $O(2)$ rotor model with a four-fold anisotropy. In this case the $Z_4$ vortices in the ordered state have featureless cores. The disordering transition in this simple model may be described by the usual three dimensional classical $Z_4$ model and is hence in the $3D$ $XY$ universality class (since the clock anisotropy is irrelevant). In contrast, disordering transitions out of the VBS phase must necessarily take into account the presence of the spin-$1/2$ moment in the cores of the $Z_4$ vortices. Any mapping to a classical $3D$ $Z_4$ model is then complicated by the need to incorporate this vortex structure.

Consider moving out of the VBS phase by proliferating and condensing the $Z_4$ vortices. Clearly once the vortices proliferate long ranged $Z_4$ order cannot be sustained. Furthermore, as these vortices carry spin, the resulting state will break spin symmetry, and as argued below may be identified as the Neel state.

These simple considerations therefore provide a mechanism for a direct second order transition between the VBS and Neel phases. As for the usual $Z_4$ model, it is reasonable to expect that the clock anisotropy will be
irrelevant at this transition as well. Indeed we will argue later this is strongly supported by the evidence from Ref. \[1, 2\]. For the present let us explore the consequences of the expected irrelevance of the clock anisotropy.

The critical theory will then be that of a (quantum) $XY$ model in $D = 2 + 1$ but with vortices that carry spin-1/2 \cite{Note1} (See Fig. 5). The spinon nature of these vortices will change the universality class from $D = 3XY$ to something different. Clearly to expose this difference and to obtain a description of the resulting new universality class, it will be most convenient to go to a dual basis in terms of the vortices and their interactions (analogous to the familiar Coulomb gas description of classical 2DXY models).

The structure of such a dual vortex reformulation is well-known. The basic idea is to regard the phase mode of the $XY$ model as the ‘photon’ associated with a fictitious non-compact $U(1)$ gauge field. The vortices then correspond to gauge charges that are minimally coupled to this photon field. At the critical point, the vortices are gapless: the critical theory may be constructed as a theory of gapless vortex fields minimally coupled to a fluctuating non-compact $U(1)$ gauge field. For the problem at hand, the spinon nature of the vortices is readily incorporated by introducing a two-component spinor field $z_a$ to represent the vortices ($a = 1, 2$ is the spin index). The transition out of the VBS phase to the Neel phase will then be described by a theory of gapless spinon-vortices coupled minimally to a fluctuating non-compact $U(1)$ gauge field (which is the dual of the $XY$ phase mode).

The critical theory is readily written down. The most general theory consistent with the $U(1)$ gauge structure, $SU(2)$ symmetry, and vortex/antivortex exchange symmetry of the microscopic model (the latter required by the symmetry under sublattice exchange $A \leftrightarrow B$), is described by the action $S_z = \int d^2 x d\tau \mathcal{L}_z$, with

$$
\mathcal{L}_z = \frac{2}{\kappa} \left[ (\partial_\mu - ia_\mu) z_a \right]^2 + s|z|^2 + u \left( |z|^2 \right)^2
$$

The transition occurs as the parameter $s$ is tuned. The $a_\mu$ represent the components of a fluctuating gauge field.

Remarkably this is exactly the same field theory as the one proposed in Ref. \[1, 2\] for the Neel-VBS transition based on an approach that attacked from the Neel side. We have thus shown how to recover that field theory in an approach from the VBS side. These considerations may be formalized as follows.

First we note that $z_a$ represents a $Z_4$ spinon-vortex, and hence must transform as a spinor under physical $SU(2)$ spin rotations. The antivortex must also transform as a spinor - we must therefore represent antivortices by $-i\sigma^y_{ab} z^*_{ab}$ where $\sigma^y$ is the usual Pauli matrix. As discussed pictorially above, elementary lattice translations take vortices to antivortices so that $z_a \rightarrow -i\sigma^y_{ab} z^*_{ab}$. It follows that the vector $\vec{N} = z^* \delta_{ab} z_b$ changes sign under an elementary lattice translation. We may therefore identify it with the Neel order parameter. Thus for instance a uniform condensate of $z_a$ corresponds to the Neel state.

We may formally justify the critical theory in Eqn. \[1\] above as follows. The arguments developed above show that the critical theory is that of an $XY$ model where the vortices are spinons. Consider the conserved current $J_\mu$ of this $XY$ model. In the ordered phase this may be expressed in terms of the $XY$ phase field $\chi$ through

$$
J_\mu = K \partial_\mu \chi
$$

where $K$ is the stiffness of the $XY$ model. To access the $XY$ disordered phase, it is necessary to include vortex configurations and account for the periodicity of the phase $\chi$. The vortex current $j_\mu$ is given by

$$
\hat{j}_\mu = \frac{1}{2\pi} \epsilon_{\mu\nu\lambda} \partial_\nu \partial_\lambda \chi
$$

Note that $\hat{j}_\mu$ must be invariant under physical spin rotations even though it is carried by spinons. The conservation condition on $J_\mu$ may be implemented by expressing it as

$$
J_\mu = \frac{1}{2\pi} \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda
$$

This equation defines the field $a_\mu$ which may be interpreted as a non-compact $U(1)$ gauge field. Clearly it is
defined only up to a gauge transformation $a_\mu \rightarrow a_\mu + \partial_\mu \theta$. The vortex current may now be re-expressed in terms of $a_\mu$:

$$j_\mu = \frac{1}{4\pi^2 K} \epsilon_{\mu\nu\lambda\rho} \partial_\nu b_\rho$$  \hspace{1cm} (5)

where $b_\lambda = \epsilon_{\lambda\alpha\beta} \partial_\alpha a_\beta$ is the gauge-invariant field strength associated with the $a_\mu$ field. This equation now takes the form of the familiar Ampere law. The duality is completed by requiring a continuum field theory of the spinon-vortices $z_a$ whose equations of motion reduce to this Ampere law equation. The action in Eqn. 1 above has precisely this property as is readily checked.

Note that as usual the conserved density $J_0$ of the XY model is simply the magnetic flux density in the dual description. As the phase $\chi$ is the conjugate operator, the operator $e^{4i\chi}$ simply increases the total gauge flux by $8\pi$. We may therefore identify it with a quadrupled monopole operator of the dual gauge theory. Thus the quartic anisotropy in the XY model corresponds precisely to the quadrupled monopole operator. Strong evidence for the irrelevancy of this operator at the critical fixed point of Eqn. 1 was presented in Ref. [2].

Is it possible for quantum fluctuations to destroy the VBS order without inducing Neel order? Clearly the answer is yes. One possibility is a transition to a topologically ordered $Z_2$ spin liquid state. To obtain a topologically ordered state from the VBS state it is as usual necessary to condense paired vortices in the VBS order parameter - but here these vortices are spinons. To get a spin singlet state it is necessary to form a singlet pair of these spinons. As discussed above, vortices live on one sublattice and antivortices on another. Consequently we need to condense a spin-singlet pair of spinons living on the same sublattice to obtain the $Z_2$ spin liquid. All of this is completely consistent with existing gauge theoretic descriptions of $Z_2$ spin liquids [7].

To conclude, we have examined the nature of the Neel-VBS transition by an attack from the VBS side. This approach leads to a simple physical description of the transition and is completely consistent with the alternate approach of attacking from the Neel side. All of the physics associated with the transitions out of the VBS phase may be fruitfully understood from the perspective of this paper.

This research is supported by the National Science Foundation grant DMR-0308945 (T.S.). T.S also acknowledges funding from the NEC Corporation, the Alfred P. Sloan Foundation, and an award from The Research Corporation.

[1] T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science 303, 1490 (2004).
[2] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, cond-mat/0312617.
[3] J. M. Carmona, A. Pelissetto, and E. Vicari, Phys. Rev. B 61, 15136 (2000).
[4] F. D. M. Haldane, Phys. Rev. Lett. 61, 1029 (1988).
[5] N. Read and S. Sachdev, Phys. Rev. Lett. 62, 1694 (1989).
[6] L. Balents, M. P. Fisher, and C. Nayak, Phys. Rev. B 62, 1654 (1999).
[7] N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991).
[8] Gauge theory aficionados will recognize that the domain walls may be understood as electric field lines of the dual $Z_4$ gauge theory.
[9] Readers familiar with the gauge theoretic description of the $Z_2$ spin liquid will recognize that this is the same as the usual procedure of condensing a gauge charge-2 spinon pair to obtain the $Z_2$ phase.