Ways of People’s Estimating Uncertain Quantitative Factors for Decision-Making in Economy and Business

Alexander G. Madera
Dr’Sci, Professor
National Research University Higher School of Economics
Department of Mathematics of the Faculty of Economics
National Research University Higher School of Economics
20 Myasnitskaya Str., Moscow, 101000, Russian Federation

Abstract
Decisions taken by a subject in economy and business are based on preliminary estimation of the uncertain factors. Along with this, there is still no unambiguous answer to the question about what methods subjects resort to when estimating the uncertain quantitative factors. This question is extremely topical for mathematical modeling of making the best decisions in economy, finance, business processes, and social structures under the conditions of uncertainty. This work is dedicated to research into the methods using which a subject estimates the uncertain quantitative factors. The researches have shown that the subjective estimation of the quantitative uncertain factors is an interval one, and the actual values of an estimated factor are equally distributed within the estimate interval. At the same time, the point estimations are used rather rarely and are not adequate to the subject’s psychology of estimation.

Keywords: psychology of estimation, uncertain factor, decision-making, interval estimation

1. Introduction
Decisions are taken, factors, objects and events are quantitatively and qualitatively assessed and prognosticated in various spheres of human activities (economy, finance, business processes, social structures, etc.) under the conditions of uncertainty. The uncertainty of the conditions under which a subject (an individual or a collective one) has to act and take decisions is in integral attribute of the reality. The uncertainty is conditioned by being exposed to a multitude of uncertain factors and their irreversible changeability: the subject them self, the object of their activities, the environment and its constituents (natural, technogenic, economic-financial, social-structural ones), the character of the subject’s interaction with the object and environment (Madera, 2014, a). This is why the mathematical description and mathematical modelling of decision-making in various spheres of activities will be adequate to the reality only if the uncertainty conditions are included therein. Since the uncertain conditions are caused by being exposed to a multitude of uncertain factors, for modelling them, it is necessary to, firstly, understand to what cognitive styles and methods resort subjects when estimating the uncertain quantitative factors, and, secondly, have a mathematical model that adequately reflects the subjective estimation of the uncertain factors.

Despite the fact that decisions taken by a subject are based on the preliminary assessment of the uncertain quantitative and qualitative factors, there is still no unambiguous answer to the question about what methods subjects resort to when assessing them. It is evident that before making a decision, for example, about surmounting an obstacle, a subject tries quantitatively estimating the parameters of this obstacle (its height, depth, width, etc.). At that, even if the estimated quantitative factors are accessible to the subject’s sensory perception, they remain uncertain due to the impossibility of measuring them directly (the height, depth, width, etc.). As it was shown in numerous researches on decision-making psychology, prediction and assessment (please see below in more detail), as a rule, people erroneously estimate the uncertain factors, which leads to incorrect, often inadequate assessments of the situation and, consequently, the decisions taken (Thurstone, 1954; Kozielecki, 1979; Kahneinan et al., 2005; Werth, 2004; Larichev, 2006; Madera a, b, 2014; Madera, 2016; Mirkin, 2014; Plous, 1993; Kahneinan, 2014; Hammond et al., 2017; Borcherding, et al., 1995, World Energy Outlook. IEA, 1994 – 2017; World Oil Outlook. OPEC, 2007—2017). Every human being has their own cognitive style, id est their individual ways of processing information about the environment conditioned by their individual differences in the perception, analysis, structuring, categorization, and assessment of the ongoing events. The individuals’ cognitive styles of the subjective estimation of the uncertain quantitative factors will also differ or group into various categorial clusters. On the basis of the results of researches into the psychology of subjective estimation of uncertain factors obtained both from an analysis of the existing literature on this issue and from the own researches by the author of the article, this work shows that in the majority of cases a subject estimates the uncertain factors in the form of some intervals with equally probable values within them.
A mathematical model in the form of an interval evenly distributed random value corresponds best to the interval method of estimation of an uncertain factor with the equally probable values inside the interval.

2. Subjective Estimation of the Uncertain Quantitative Factors

When taking a decision, a subject relies on their own assessments of the uncertain factors, which can be both directly accessible to the subject for perception at present (condition A) and inaccessible at the moment because they will actualize in future only (condition B). Hereinafter the estimation of the uncertain factors under the conditions A is called the estimation and the one under the conditions B is called the predictive estimation. When quantitatively estimating the factors (both under the conditions A and under the conditions B), subjects usually use the following estimation methods:

1. **Point estimation**, which consists in assignment of a single, exact value to an estimated uncertain factor.
2. **Interval estimation**: an uncertain factor is estimated in the form of some estimate interval, within which any particular values of the factor are equally possible.
3. **Probabilistically supported estimation**, which is the point or interval estimation that the subject supports with the value of the degree of their confidence in the correctness of their estimate using a value of the subjective probability.

Thus, when estimating the weight of a thing, a subject gives a single-value answer, for example, 250 g, if they use the point method of estimation; if they estimate it intervally, they specify a range wherein, in their opinion, will lie the possible weight values, for example, from 220 to 300 g, with implicating that the actual weight can lie in any point of the specified interval with equal probabilities (even distribution). But when using the probabilistic support, in addition to the estimate proper (the *point* one, for example, 250 g, or the *interval* one, 220–300 g) the subject indicates also the value of the subjective probability (for example, 0.8 or 80%) with which they support the degree of their confidence in the correctness of the estimate given by them. For example, when using the probabilistically supported estimation, a subject will say that they are 80% sure that the “weight of the thing is somewhere between 220 and 300 g” (if estimated intervally) or “80% sure that the exact weight of the thing is 250 g” (if estimated as a point).

Numerous researches (Thurstone, 1954; Kozielecki, 1979; Kahneman et al., 2005; Werth, 2004; Larichev, 2006; Madera, 2014; Mirkin, 2014; Plous, 1993; Gusev, Utochkin, 2011; Kahneman, 2014; Hammond et al., 2017; Borcherding et al., 1995; World Energy Outlook. IEA, 1994 – 2017; Yaniv, Foster, 1995,а; Yaniv, Foster, 1997,б; Speirs-Bridge, Fidler, McBride, et al. (2009); World Oil Outlook. OPEC. 2007—2017) and collation of subjective estimates with values actually observed in practice show that man is incapable of giving any exact single-value estimates of the quantitative factors under the conditions of both accessibility (A) and inaccessibility (B) of an estimated object for their perception at the moment of time. In other words, man is not a precision measuring instrument, which does not permit errors in quantitative measurements (Larichev, 2006).

Despite the fact that the actual daily practice of estimating uncertainties is invariably evidence of the erroneousness of estimations made by subjects, there are always individuals who give their point single-value estimates without a shadow of a doubt. When asked to estimate the distance between two visible objects (under the conditions A, id est if the objects are accessible for their direct perception), they peremptorily state that the distance equals exactly, for example, 138.3 m. The level of adequacy of such sort of the estimates is obviously paltry. They carry no reliable information about the estimated object and rather evidence that the minds of such people are subject to heuristics and mental traps (the trap of conceit in particular) described in detail by (Kahneman et al., 2005; Larichev, 2006; Plous, 1993; Hammond et al., 2017). In his works, L. Thurstone (Thurstone, 1954; Thurstone, 1974) showed that if a value to be estimated is submitted to a subject over and over again, the subject gives not the same fixed point value, but others that randomly take some values from an estimate interval implicated by them wherein the values are equally possible. So, it may be concluded that when assigning their point estimate to an uncertain factor, the subject in fact gives a value from some estimate interval implicated by them. In other words, the subject in fact estimates the uncertain factor in a natural interval form, which conforms to the psychology of making value judgements.

A subject who believes that they are capable of giving single-value point estimates often involuntarily performs the probabilistically supported estimation, which in fact converts their single-value point estimate into an interval one. Indeed, by quantitatively estimating the value of the factor A, in most cases a subject will not say that “the value A will exactly equal the value a”. On the contrary, their answer, as a rule, will be rather evasive and will be accompanied by various reservations: almost, somewhat, roughly, about, approximately, not less than, not more than, some quantity of percent sure, unlikely, rather probably, possibly, etc. Namely, the subject will say that A will be roughly (almost, somewhat, approximately) equal to a or that A will be not more (not less) than the value a or that they are 80% sure that the value A will approximately equal a, etc. With such introducing reservations, the estimating subject thus demonstrates their doubts in the reliability of the point estimate given by them and unconsciously converts it into an interval one.
For example, by saying that the value $A$ exceeds $a$ or, on the contrary, — does not exceed $a$ — the subject thus implicates some estimate interval wherein the value $a$ is its lower or upper boundary. In his researches into psychology of estimation and prediction, D. Kahneman (Kahneman et al., 2005) correctly remarked that a subject who gives, as they believe, an accurate estimate, will be rather flattered and will consider them self outstandingly precise, if their estimate will actually prove to be equal not to the value indicated by them but somewhat higher or lower, thus acknowledging that the point estimate given by them is not in fact a precise and single-value one but lies in some interval with unspecified boundaries. “Thus, a speaker, — concludes D. Kahneman, — who asserts a numerical prediction (point estimation — note by the author, A.G.M.) is committed to a range (interval estimate — note by the author, A.G.M.) rather than to a point.” D. Kahneman (Kahneman, 2014) gives a didactic example from his practice about flimsiness of the point estimation and adequacy of the interval estimation. Namely, he asked his colleagues to predict the amount of time that they would need to write a textbook and submit it to a publishing house. The point estimate given by them equaled 2 years, but the interval estimate given by an expert amounted to 7–10 years with his subjective 60% probability of his confidence in the correctness of his prediction. In reality the textbook was completed and submitted in 8 years, which favors the interval estimation.

Empirical researches show that the observed effect is rather general to affirm that it is the interval estimation, as opposed to the point one, that conforms to the psychology of a subject when they make value judgements and, as intuitively feels an estimating subject them self, reflects the actual value of the estimated quantity with a larger probability. A subject who estimates an uncertain factor in the form of an interval of the possible values implicates that their estimate is a random value that takes any values inside the estimate interval with the even distribution of the probabilities.

It is necessary to note that when quantitatively estimating the uncertain factors, a subject (except for experts to whom such an assignment is specially given) does not resort to building characteristic membership functions adopted in fuzzy sets theory. That is why this method of estimation is not examined herein.

An own research performed by the author of this work corroborates the aforesaid conclusions that, as a rule, a subject estimates an uncertainty in the form of an interval, not a point in fact. In order to find out the relative quantity of the respondents who prefer to give point, interval or probabilistically supported estimates, the author of the article carried out the research among a representative sample of 143 students of the 2nd year of one of Moscow universities. The research was performed in the form of an anonymous written survey wherein the respondents were proposed to answer the following question: “What methods of estimation do you prefer to use when quantitatively estimating objects directly accessible for perception at a moment of time?” The respondents were familiarized with the various estimation methods: point, interval and probabilistically supported ones (both for the purely point and purely interval estimates). In order to concretize the question, they were proposed to estimate the distance between two objects equally visible by all who were present in the lecture hall and to submit their estimates in writing in the form of the respondent’s most natural estimate — a particular number, interval or one of them, but with the probabilistic support. The results were distributed as follows: the groups of the respondents who preferred the unambiguously point ($p_1$), interval ($p_2$) or point ($p_3$) estimates but with the introducing reservations (almost, approximately, roughly, probably, etc.) amounted to: $p_1 = 16.08\%$, $p_2 = 51.75\%$ and $p_3 = 32.17\%$, respectively. As it has been shown above, the point estimates with the introducing reservations belong in fact to the interval estimates, that is why in reality the group of all the respondents who gave the interval estimate ($p_{int}$) numbered $p_{int} = p_2 + p_3 = 51.75 + 32.17 = 83.92\% \equiv 84\%$ and the group of the respondents who gave the exact point estimate in the form of a single number without any reservations and explanations equaled $p_{exact} \equiv 16\%$. Thus the number of the respondents who gave the interval estimate (84%) is 5.25 times bigger than the number of those who gave the point estimate (16%). Hence, the majority of the estimators (84%) give the interval estimates and the minority (16%) give the point ones; in other words, people prefer to perform the interval estimation. It is also necessary to note that none of the questioned respondents considered the possibility of the estimation using the membership function.

Several experimental researches (Kahneman et al., 2005; Hammond et al., 2017) also showed that the trap of conceit and overestimation of their own capabilities of giving exact single-value estimates shows itself not only during the point estimation but also during the interval one and results in the unjustifiable narrowing of the estimate interval. For example, the respondents were proposed to give an interval estimate of the Dow Jones index for the stocks of industrial companies with the error of setting the upper and lower boundaries of the interval that did not exceed 1%. Despite their conceited a priori statements concerning the high precision of their estimates, hundreds of the respondents specified too narrow intervals that stood away from the actual value of the Dow Jones index in average by 20–30%, and the extreme values of the interval boundaries differed from the average ones greatly in some cases. In other words, the intervals specified by the respondents were found to lie by 20–30% away from the actual value of the index and did not cover its actual value.
As (Hammond et al., 2017) correctly note, the presumptuous underestimation of the upper and overestimation of the lower boundaries of the estimate interval in business practice often leads to loss of profit, significant waste of material and financial resources — consequently, to a substantially greater risk. Thus, it can be concluded that subjects estimate the uncertain quantitative factors in the form of the intervals of the possible values, which can take any values inside the subjects’ specified intervals with an evenly distributed probability.

3. Conclusion

This work has researched into the methods for subjective estimation of the uncertain quantitative factors. On the basis of the analysis of researches existing in the scientific literature as well as the own experimental data obtained by the work author, it has been established that most people prefer to make their value judgements in the form of intervals wherein lie the equally possible actual values of the estimated quantitative factor. At the same time, the point estimates cannot be acknowledged as adequate to the subject’s psychology when they make their value judgements and, as the researches show, they are in fact interval ones, too. The fact that the interval method of the estimation with the values equally possible within the boundaries of the estimate interval corresponds to the subjective estimates of the uncertain factors permits suggesting a mathematical model adequate to the human psychology of estimation; the mathematical model of the uncertain factors has the form of an intervally stochastic evenly distributed random value that varies within a subjective estimate interval. Such an intervally stochastic mathematical model of uncertainty of the quantitative factors makes it possible to create adequate mathematical models for decision-making under uncertainty conditions in various spheres of human activities: economy, finance, business processes, and social structures, — in the spheres where a goal-setting subject is the main actor.

References

Kahneman, D., Slovic, P., Tversky, A. (2005). Judgement under uncertainty: Heuristics and biases. Transl. from Eng. Kharkiv: “Humanities Centre”.
Kahneman, D. (2014). Thinking, Fast and Slow. Transl. from Eng. M.: AST.
Kozielecki, J. (1979). Psychological Decision Theory. Transl. from Pol. M.: Progress.
Larichev, O.I. (2006). Verbal Analysis of Decisions. M.: Science.
Madera, A.G. (2014, a). Risks and Opportunities: Uncertainty, Prognostication and Assessment. M.: KRASAND.
Madera, A.G. (2014, b) Intervally Stochastic Uncertainty of Estimates in Multicriterial Problems of Decision-Making.
\[\text{Artificial Intelligence and Decision-Making, 3, 105–115.}\]
Madera, A.G. (2016). Estimating the probability of forecasted events. International Journal of Accounting and Economics Studies, 4, 1, 76–80. doi:10.14419/ijaes.v4i1.6146.
Mirkin, Ja.M., ed. (2014). International Practice of Prognostication of World Prices on Financial Markets (Raw Materials, Stocks, Currency Exchange Rates). M.: Magister.
Thurstone, L.L. (1974). Psychophysical Analysis. In book Psychophysics Problems and Methods [Ed. by A.G. Asmolov, M.B. Mikhailovskaya]. M.: Publishing House of Lomonosov Moscow State University, 33–55.
Hammond, J., Keeney, R., Raiffa, H. (2017). The Hidden Traps in Decision Making. Harvard Business Review. On making smart decisions. M.: Alpina Publisher.
Borcherding, K., Schmeer, S., Weber, M. (1995). Biases in multiattribute weight elicitation. Contributions to decision research [Ed. by J-P. Caverni at al.]. Amsterdam: Elsevier, 3–28.
Plous, S. (1993). The Psychology of Judgment and Decision Making. N.Y.: McGraw-Hill Inc., Wesleyan University.
Speirs-Bridge, A., Fidler, F., McBride, M., Flander, L., Cumming, G., Burgman, M. (2009) Reducing overconfidence in the interval judgments of experts. Risk Analysis, V. 30, No. 3, 512 - 523.
Thurstone, L.L. (1954). The measurement of values. Psychological Review, 61(1), 47–58.
Werth, L. (2004). Psychologie für die wirtschaft. Grundlagen und anwendungen. Berlin: Spectrum Akademischer Verlag.
World Energy Outlook. IEA. 1994—2017.
World Oil Outlook. OPEC. 2007—2017.
Yaniv, I, Foster D.P. (1995, a) Graininess of Judgment Under Uncertainty: An Accuracy-Informativeness Trade-Off. Journal of Experimental Psychology General, V. 124, No 4, 424-432.
Yaniv, I, Foster D.P. (1997, b) Precision and accuracy of judgmental estimation. Journal of Behavioral Decision Making, V. 10, No 1, 21-32.