Auxiliary Particle Filtering Over Sensor Networks Under Protocols of Amplify-and-Forward and Decode-and-Forward Relays

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Abstract—In this paper, the particle filtering problem is investigated for a class of stochastic systems with multiple sensors under signal relays. To improve the performance of signal transmissions, a relay is deployed between each sensor and the remote filter. Both amplify-and-forward (AF) and decode-and-forward (DF) relays are considered under certain transmission protocols. Stochastic series are employed to describe multiplicative channel gains and additive transmission noises. Novel likelihood functions are derived based on the AF/DF relay models under different protocols. With the measurements collected from all the sensor nodes, a new centralized auxiliary particle filter (APF) is designed by resorting to the statistical information of the channel gains and transmission noises. Next, a consensus-based distributed APF is further established at each node that requires only locally available information. Finally, the effectiveness of the proposed filtering approach is demonstrated through target tracking simulation examples in different situations.

Index Terms—Amplify-and-forward relay, decode-and-forward relay, multiple sensors, particle filter, stochastic systems.

I. INTRODUCTION

A S ONE of the fundamental issues in disciplines of control and signal processing, the filtering problem has been receiving an ever-growing research interest for a few decades, see e.g. [3], [8], [13], [16], [22], [25], [44], [52] for some recent references. Among various types of filters, the particle filter (PF) stands out as a representative in the Bayesian framework that has been extensively investigated [9], [50]. In a PF, a set of weighted particles is employed to approximate the posterior probability density function (PDF) of the system state conditioned on the measurement. The approximation-based PF method is particularly suitable to cope with nonlinear/uncertain/non-Gaussian systems for which the PDFs of the system states are difficult (or even impossible) to be accurately calculated. In practical systems, PFs have found many successful applications such as simultaneous localization and mapping [38], price prediction [29], corrosion quantification [1], polyp detection [33], and so on.

In conventional PF methods, the importance density for each particle is determined according to the compatibility of the particle to the newly obtained measurement [39]. So far, the APF has been exploited to solve the filtering problem for a variety of systems including economic systems [6] and Boolean dynamical systems [15]. It is worth mentioning that the APF is especially applicable to systems with hard/soft constraints [20], [26], where the auxiliary index has been calculated in consideration of both system dynamics and underlying constraints.

In communication systems, the transmission ability of a practical channel is often limited due to technical/physical constraints, and this is especially true when low-cost sensors are deployed and the measurements need to be transmitted through wireless links. To guarantee the quality of the long-distance signal transmissions, relays (also called relaying protocols) have been widely utilized to receive and process the signals from sources, and then send the signals to destinations [51]. Relays have found successful applications in practical situations such as multi-antenna systems and collaborative networks [28], [42]. There are different types of relays that are deployed in industry with examples including compress-and-forward relays [31], compute-and-forward relays [34], and filter-and-forward relays [41]. In particular, the amplify-and-forward (AP) relays and the decode-and-forward (DF) relays have aroused considerable attention for their simplicity and practicability.
In an AF relay, the received signal from the source is amplified and then retransmitted [36]. For AF relays, the corresponding relay selection and power allocation problems have been discussed in the context of performance specifications such as maximum information secrecy [45], minimum mean-square transmission error [12], and minimum total transmission power [18]. On the other hand, a DF relay demodulates and then decodes the received signal before retransmission. For DF relays, the performance of transmission channels has been analyzed in terms of the outage probability, average packet delay, system throughput, moments of signal to noise ratio, average bit-error-rate, ergodic sum rate, and so on [32], [37]. In [28], the transmission models for both AF and DF relays have been established, and the spatial diversity performances of the relays have been analyzed under various transmission protocols.

In the past few years, the filtering problem for systems subject to signal relays has started to gain some preliminary research attention. For a class of time-invariant systems with relays, some methodologies have been recently proposed based on the transfer functions of the addressed systems. For instance, in [46], the source (AF relay, respectively) has been formulated as a finite (infinite, respectively) impulse response filter, and the minimum mean-square error (MMSE) filter has been parameterized via quadratic programming. The MMSE filter has been designed for systems with full-duplex filter-and-forward relays under dynamic range constraints in [2], where the design problem has been transformed into a quadratic optimization problem with both equality and inequality constraints.

Pertaining to the time-varying systems with relays, some minimum-variance filtering methods have been proposed in the literature. For example, the Kalman filter has been constructed in [19] for systems with both relays and package dropouts, where the relay configuration problem has been thoroughly discussed. In [42], the filtering error covariance has been determined and then minimized for uncertain systems with AF relays, and sufficient conditions have been established to guarantee the boundedness of the error dynamics. Note that only linear systems with AF relays have been considered in [42]. When it comes to nonlinear systems with different relays and transmission protocols, the corresponding filtering problem has not been adequately addressed yet, and the main motivation of the paper is therefore to shorten such a gap.

It is worth mentioning that most existing results concerning relay-based filtering problems have been developed in a centralized manner, where a centralized filter has access to all the measurements and the filter gains are calculated by resorting to the global information. In practice, however, it is often the case that the underlying system is monitored by distributed sensor nodes. Under this circumstance, it is preferable to design a distributed filter for each sensor node with only local information and the information from its neighboring nodes, thereby effectively improving the flexibility and scalability of the filtering approach.

Among various distributed filtering schemes, the consensus-based filtering strategy has gained a particular research interest whose aim is to reach a consensus of the filtering performances at different nodes [4], [14], [27]. In the PF framework, the consensus can be achieved via the node-wise exchanges of likelihood functions [11], posterior PDFs [21], [23], particle weights [10], etc. It is noted that the existing consensus-based PF methodologies have been developed without the consideration of relays. Apparently, the introduction of relays would largely complicate the filter design in terms of both the structure and the algorithms, and this constitutes another motivation of the current investigation.

Based on the above discussions, in this paper, we endeavor to design an APF for a class of systems with relays in the transmission links. The addressed system is monitored by multiple sensors, and AF/DF relays are considered in the sensor-filter channels. In the APF, a novel auxiliary index is calculated to cater for the effects from relays on the received signals. A centralized APF is firstly established with the measurements collected from all the sensors, and then a consensus-based distributed APF is designed by resorting to the locally available information. The performance of the proposed approach is demonstrated with some target tracking examples.

The main contributions of this paper can be summarized as follows: 1) The system model is comprehensive that involves both AF and DF relays in the transmission links with random channel gains; 2) a new auxiliary index is proposed by incorporating the properties of relays and transmission channels; and 3) both centralized and distributed APFs are designed for a class of systems with AF/DF relays.

The rest of the paper is organized as follows. In Section II, the particle filtering problem is formulated for systems with AF/DF. The centralized and distributed APF design problems are solved in Section III in consideration of AF/DF relays. Simulation examples are presented in Section IV to illustrate the performance of the proposed approach, and Section V concludes the paper.

Notations: The notation used in the paper is fairly standard except where otherwise stated. \( \mathbb{R}^n \) denotes the \( n \)-dimensional Euclidean space. \( \mathbb{E}\{x\} \) is the expectation of the stochastic variable \( x \). \( p_x(\cdot) \) denotes the PDF of the random variable \( x \). \( p_x(\cdot|\mu) \) stands for the conditional PDF of a stochastic variable \( x \) given \( y \). \( x_{a:b} \) represents the trajectory of \( x \) from time step \( a \) to time step \( b \). \( N(\mu, \Sigma) \) denotes the Gaussian PDF with mean \( \mu \) and covariance \( \Sigma \).

II. PROBLEM FORMULATION

Consider the following nonlinear system with \( N \) sensor nodes:

\[
\begin{align*}
    x_{k+1} &= f(x_k) + w_k, \\
    y_{i,k} &= h_i(x_k) + v_{i,k}, \quad i = 1, \ldots, N
\end{align*}
\]

(1)

where \( x_k \in \mathbb{R}^n \) is the system state, \( y_{i,k} \in \mathbb{R}^{m_i} \) is the measurement of the \( i \)th sensor node, and \( w_k \in \mathbb{R}^n \) and \( v_{i,k} \in \mathbb{R}^{m_i} \) denote the plant noise and the measurement noise of the \( i \)th sensor node, respectively. Here, \( w_k, v_{i,k} \), and the initial condition \( x_0 \) are independent of each other with known \( p_{w_0}(\cdot), p_{v_{i,k}}(\cdot) \) and \( p_{x_0}(\cdot) \). Furthermore, \( f(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is the state transition function, and \( h_i(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^{m_i} \) is the measurement function of the \( i \)th sensor node.
In this protocol, the signal $E$ is provided in (5). So far, the transmission models have been established for AF/DF relays under two kinds of transmission protocols. Whether there is a direct link between the source and the destination, and Protocol 2 is especially suitable in the cases where the transmission links with relays, and the consideration of AF/DF relays under different protocols constitutes the main difference between the considered protocols lies in whether there is a direct link between the source and the destination, and Protocol 2 is especially suitable in the cases where the source is distant from the destination and the source-destination channel is noisy. In the APFs to be developed, novel likelihood functions will be calculated based on the properties of the transmission links with relays, and the consideration of AF/DF relays under different protocols constitutes the main difference between our approach and the conventional APF method.

The aim of this paper is to design APFs for systems with AF/DF relays under two kinds of protocols. With the global (local, respectively) information, the centralized (distributed, respectively) APFs will be established.

After receiving the signal $z_{i,k}$, the AF relay amplifies and retransmits it to the destination. At the remote filter, the measurement received from the AF relay can be formulated as

$$r_{a,i,k} = \sqrt{E_{RD,i}h_{RD,i,k}z_{i,k}} + m_{i,k},$$

(3)

where $E_{RD,i}$ is the average signal energy transmitted from the $i$th source to the destination, $h_{RD,i,k}$ is the random channel gain between the $i$th relay and the destination, and $m_{i,k}$ is an additive noise. Here, $p_{h_{RD,i,k}}(\cdot)$ and $p_{m_{i,k}}(\cdot)$ are assumed to be known.

The signal directly transmitted from the $i$th sensor to the filter is described as

$$s_{i,k} = \sqrt{E_{SD,i}h_{SD,i,k}y_{i,k}} + n_{i,k},$$

(4)

where $E_{SD,i}$ is the average signal energy forwarded by the $i$th source to the destination, $h_{SD,i,k}$ is the random channel gain between the $i$th source and the destination, and $n_{i,k}$ is an additive noise. $p_{h_{SD,i,k}}(\cdot)$ and $p_{n_{i,k}}(\cdot)$ are assumed to be available.

In the AFPI case, the signal from the $i$th source to the remote filter is $d_{a1,i,k} = [r_{a,i,k}, s_{i,k}^T]$.  

**AF under Protocol 2 (AFP2):** In this case, the $i$th source does not directly send the signal to the destination, and the signal received from the $i$th source is $d_{a2,i,k} = r_{a,i,k}$, where $r_{a,i,k}$ is defined in (3).

**DF under Protocol 1 (DFP1):** In a DF relay, the received signal is demodulated and then decoded. Assume that the signal can be decoded accurately and sent to the destination. The measurement transmitted from the DF relay to the filter can be expressed as

$$r_{d,i,k} = \sqrt{E_{RD,i}h_{RD,i,k}y_{i,k}} + m_{i,k}.$$

(5)

Then, the signal received by the remote filter is naturally $d_{d,i,k} = [r_{d,i,k}, s_{i,k}^T]^T$, where $s_{i,k}$ is given in (4).

**DF under Protocol 2 (DFP2):** Under this circumstance, the signal from the $i$th source to the filter is obviously $d_{d2,i,k} = r_{d,i,k}$, where $r_{d,i,k}$ is provided in (5).

**Remark 1:** So far, the transmission models have been established for AF/DF relays under two kinds of transmission protocols. In the following calculations, APFs in all these four situations will be designed. The relays in the transmission channels can effectively extend the transmission distance and improve the system performance. Unfortunately, the introduction of relays also poses extra challenges to the filtering problem due to the multiplicative channel gains and additive transmission noises. The main difference between the considered protocols lies in whether there is a direct link between the source and the destination, and Protocol 2 is especially suitable in the cases where the source is distant from the destination and the source-destination channel is noisy. In the APFs to be developed, novel likelihood functions will be calculated based on the properties of the transmission links with relays, and the consideration of AF/DF relays under different protocols constitutes the main difference between our approach and the conventional APF method.

As demonstrated in Fig. 1, in the centralized case, each sensor node in this paper sends the measurement to the remote filter through a relay. In this paper, only one relay is considered in each transmission channel, and the developed algorithms can be readily extended to deal with multi-relay channels.

In transmission channels with relays, the following two protocols are widely adopted:

- **Protocol 1:** the source communicates with the relay and the destination, and the relay communicates with the destination.
- **Protocol 2:** the source communicates with the relay, and the relay communicates with the destination.

The schematics for both of the protocols are depicted in Fig. 2. In this paper, the source of each link is the sensor, and the destination is the remote filter. The delays induced by data processing and signal transmission are neglected. Furthermore, in all the following cases, the additive noises and random gains of different channels are independent of each other.

Let us now discuss the transmission models for both AF and DF relays based on the analysis in [28].

**AF under Protocol 1 (AFP1):** In this protocol, the signal received by the $i$th relay can be written as

$$z_{i,k} = \sqrt{E_{SR,i}h_{SR,i,k}y_{i,k}} + n_{i,k},$$

(2)

where $E_{SR,i}$ is the average signal energy received at the $i$th relay, $h_{SR,i,k}$ is the random channel gain between the $i$th source and the $i$th relay, and $n_{i,k}$ is an additive noise. $p_{h_{SR,i,k}}(\cdot)$ and $p_{n_{i,k}}(\cdot)$ are supposed to be available.
III. ALGORITHM DESIGN AND DISCUSSION

A. Centralized APF Design

Denote the signal received at the remote filter as \(d_k\), which can be obtained via augmenting the collected information from every node. Because of the complexity induced by the AF/DF relays, it is difficult to accurately derive the marginal posterior PDF \(p(x_k|d_{1:k})\) for the addressed system, and the system states cannot be estimated in the sense of MMSE. To solve the problem, a set of weighted particles is used in the PF to approximate the PDF as follows:

\[
p(x_{0:k}|d_{1:k}) = \sum_{m=1}^{M} W_k^m \delta(x_{0:k} - x_k^m),
\]

where \(M\) is the number of the particles, and \(\delta(\cdot)\) is the Dirac delta function.

The particles \(\{x_k^m\}_{m=1}^M\) are generated based on an importance density \(q(x_{0:k}|d_{1:k})\), and the weight \(W_k^m\) satisfies the following recursion

\[
W_k^m \propto W_{k-1}^m p(d_k|x_k^m) p(x_k^m|x_{0:k-1}^m) q(x_k^m|x_{0:k-1}, d_k).
\]

A widely selected distribution is

\[
q(x_k|x_{0:k-1}^m, d_k) = p(x_k|x_{k-1}^m).
\]

It is worth mentioning that the above distribution is just dependent on the system dynamics, where the information contained in the measurements is neglected, and this may degrade the sampling and filtering performance. To tackle such a problem, the APF method has been developed via calculating an auxiliary particle index from a designed distribution to enhance the compatibility of the particles to the measurements. In most cases, the auxiliary index \(\lambda_k\) is determined based on the one-step prediction of the particle, i.e., \(\lambda_k^m \sim p(x_k|x_{k-1}^m)\). This index is also adopted in our APF with a DF relay.

Considering the transmission procedure in a channel with an AF relay, in our paper, we set

\[
\lambda_k^m \sim p(z_k|x_{k-1}^m),
\]

where

\[
z_k = [z_{1:k}^T, \ldots, z_{N,k}^T]^T.
\]

With the above preparation, the APF is implemented by the following steps.

1) Draw an index \(I\) with the probabilities proportional to \(W_k^m p(d_k|\lambda_k^m)\), where \(\lambda_k^m \sim p(z_k|x_{k-1}^m)\) with AF relays and \(\lambda_k^m \sim p(x_k|x_{k-1}^m)\) with DF relays.
2) Draw a particle \(x_k^m\) from \(p(x_k|x_{k-1}^m)\).
3) Calculate the weight with

\[
W_k^m = \frac{p(d_k|x_k^m)}{p(d_k|\lambda_k^m)}.
\]

Let us first consider the AFP1 situation in the centralized setting. Because the channel gain and the transmission noise of each channel are independent from each other, we have

\[
p(z_k|x_{k-1}^m) = \prod_{i=1}^{N} p(z_{i,k}|x_{k-1}^m),
\]

and \(\lambda_k^m = [(\lambda_{1,k}^m)^T, \ldots, (\lambda_{N,k}^m)^T]^T\) can be easily drawn according to \(\lambda_{i,k}^m \sim p(z_{i,k}|x_{k-1}^m)\) with available \(p_{SR,i,k}(\cdot)\) and \(p_{n_i,k}(\cdot)\).

Fig. 3. Block diagram of the \(i\)th node in the distributed case (no direct link between the sensor and the filter).

Fig. 4. True target trajectory and its centralized estimate in the AFP1 situation.

Fig. 5. True target trajectory and its centralized estimate in the AFP2 situation.
To deal with \( p(s_{i,k}|\lambda_{i,k}^m) \), we need the following relationships:

\[
p \left( s_{i,k}|\lambda_{i,k}^m \right) = \int p \left( s_{i,k}, y_{i,k}|\lambda_{i,k}^m \right) dy_{i,k}
= \int p \left( s_{i,k}|y_{i,k}, \lambda_{i,k}^m \right) p \left( y_{i,k}|\lambda_{i,k}^m \right) dy_{i,k}
= \int p \left( s_{i,k}|y_{i,k} \right) p \left( y_{i,k}|\lambda_{i,k}^m \right) dy_{i,k},
\]

(12)

where the last equality follows from the source-terminal transmission model (4). With the available information on the distribution of channel gains and disturbances, (12) can be used to calculate the \( p(s_{i,k}|\lambda_{i,k}^m) \).

Similar with \( p(r_{a,i,k}|\lambda_{i,k}^m) \), the accurate integral in (12) is difficult to be computed, and therefore we need to use the following formula to approximate \( p(s_{i,k}|\lambda_{i,k}^m) \):

\[
p \left( s_{i,k}|\lambda_{i,k}^m \right) \approx \frac{1}{C} \sum_{c=1}^{C} p_{i,k} \left( s_{i,k} - \sqrt{E_{SD,i} h_{SD,i,k}^c} \right)
\times \frac{\lambda_{i,k}^m - n_{i,k}^c}{\sqrt{E_{SR,i} h_{SR,i,k}^c}}.
\]

(13)

where \( n_{i,k}^c, h_{SD,i,k}^c, \) and \( h_{SR,i,k}^c \) are the realizations of \( n_{i,k}, h_{SD,i,k}, \) and \( h_{SR,i,k} \) in the \( c \)th particle, respectively. With (11) and (13), we can obtain \( p(d_{a,1,k}|\lambda_{1,k}^m) \) and then draw \( x_{k}^m \) from \( x_{k-1}^m \).

Now, we are in a position to determine \( p(d_{a,1,k}|x_{k}^m) \). Similar with \( p(d_{a,1,k}|\lambda_{1,k}^m) \), \( p(d_{a,1,k}|x_{k}^m) \) can be approximated as follows:

\[
p \left( d_{a,1,k}|x_{k}^m \right) = \prod_{i=1}^{N} p \left( r_{a,i,k}|x_{k}^m \right) p \left( s_{i,k}|x_{k}^m \right),
\]

(14)

where

\[
p \left( r_{a,i,k}|x_{k}^m \right) \approx \frac{1}{C} \sum_{c=1}^{C} p_{m,i,k} \left( r_{a,i,k} - \sqrt{E_{RD,i} h_{RD,i,k}^c} \right)
\times \left( \sqrt{E_{SR,i} h_{SR,i,k}^c} \left( h_{i}(x_{k}^m) \right) \right.
\left. + v_{i,k}^c + n_{i,k}^c \right),
\]

(15)

\[
p \left( s_{i,k}|x_{k}^m \right) \approx \frac{1}{C} \sum_{c=1}^{C} p_{i,k} \left( s_{i,k} - \sqrt{E_{SD,i} h_{SD,i,k}^c} \right)
\times \left( h_{i}(x_{k}^m) + v_{i,k}^c \right),
\]

(16)

and \( v_{i,k}^c \) is the realization of \( v_{i,k} \). With (8), (10), and (14), the particle weight \( W_{i,k}^m \) can be determined and the filter can be constructed. The implementation of the proposed APF design method in the AFPI situation is summarized in Algorithm 1.

In the AFPI situation, the source does not directly send information to the destination. Defining

\[
d_{a,2,k} = [d_{a,2,1,k}^T, \ldots, d_{a,2,N,k}^T]^T,
\]

we have

\[
p \left( d_{a,2,1,k}|x_{k}^m \right) = \prod_{i=1}^{N} p \left( r_{a,i,k}|x_{k}^m \right) p \left( s_{i,k}|x_{k}^m \right).
\]

(10)

Note that \( p(r_{a,i,k}|\lambda_{i,k}^m) \) can be determined based on the relay-terminal transmission model (3). However, the conditional probability \( p(r_{a,i,k}|\lambda_{i,k}^m) \) is difficult to be accurately calculated because the multiplicative gain \( h_{RD,i,k} \) and additive noise \( m_{i,k} \) are both random in the relay-destination channel. To solve the problem, we can approximate \( p(r_{a,i,k}|\lambda_{i,k}^m) \) as follows [24]:

\[
p \left( r_{a,i,k}|\lambda_{i,k}^m \right) \approx \frac{1}{C} \sum_{c=1}^{C} p_{m,i,k} \left( r_{a,i,k} - \sqrt{E_{RD,i} h_{RD,i,k}^c} \right) \lambda_{i,k}^m,
\]

(11)

where \( h_{RD,i,k}^c \) is the realizations of \( h_{RD,i,k} \) in the \( c \)th particle, and \( C \) is the number of the particles.

With the received signal at the remote sensor

\[d_{a,1,k} = [d_{a,1,1,k}^T, \ldots, d_{a,1,N,k}^T]^T,\]

we have

\[
p \left( d_{a,1,k}|\lambda_{1,k}^m \right) = \prod_{i=1}^{N} p \left( r_{a,i,k}|\lambda_{i,k}^m \right) p \left( s_{i,k}|\lambda_{i,k}^m \right).
\]

(10)
Algorithm 1: The Proposed Centralized Auxiliary Particle Filtering in The AFP1 Situation.

Step 1. Particle initialization

Draw $M$ particles $x_0^m$ from the prior PDF $p_{x_0}(\cdot)$ and the importance weights are all set as $\frac{1}{M}$. Set the maximum time instant $K$.

Step 2. Measurement collection

Collect the measurement $d_{a1,i,k}$ from every AF relay and sensor at time instant $k$.

Step 3. Particle selection and update

Determine the indices $I$ based on the probabilities proportional to $W_{m-1} = p(d_{a1,k}|x_{m-1}^m)$, where $\lambda_k^m \sim p(z_k|x_{m-1}^m)$, and $p(d_{a1,k}|x_{m-1}^m)$ is calculated by (10), (11) and (13). Generate particles $x_k^m$ from $p(x_k|x_{k-1}^m)$ for $m = 1, \ldots, M$, and determine $p(d_{a1,k}|x_k^m)$ with (14)–(16).

Step 4. Weight calculation

Calculate the weights with $W_k = p(d_{a1,k}|x_k^m)$ and normalize the weights as $W_k^m = \frac{W_k}{\sum_{t=1}^{M} W_t^m}$.

Step 5. State estimate update

Obtain the state estimate $\hat{x}_k = \sum_{m=1}^{M} W_k^m x_k^m$.

Step 6. Resampling

Resample a new set of particles with equal weights from $\sum_{m=1}^{M} W_k^m \delta(x_k - x_k^m)$.

Step 7. If $k < K$, then set $k = k + 1$ and go to Step 2; otherwise go to Step 8.

Step 8. Stop.

Then, we have

$$p(d_{a2,k}|x_k^m) = \prod_{i=1}^{N} p(r_{a,i,k}|x_k^m), \quad (17)$$

where $\lambda_k^m \sim p(z_k|x_{k-1}^m)$, and $p(r_{a,i,k}|x_k^m)$ can be approximated with (11). It also follows directly that

$$p(d_{a2,k}|x_k^m) = \prod_{i=1}^{N} p(r_{a,i,k}|x_k^m), \quad (18)$$

where $p(r_{a,i,k}|x_k^m)$ can be calculated with (15). After slightly revising the measurement collection step (there is no measurement that can be directly collected from the sensor in the AFP2 situation), and replacing $p(d_{a1,k}|x_k^m)$ by $p(d_{a1,k}|x_k^m)$ respectively with $p(d_{a2,k}|x_k^m)$ in (17) and $p(d_{a2,k}|x_k^m)$ in (18), we can apply Algorithm 1 to the AFP2 situation.

For the systems with DP relays, we have

$$\lambda_k^m \sim p(x_k|x_{k-1}^m),$$

and $\lambda_k^m$ can be drawn based on the system dynamics (1). In the DFP1 situation, define the received signal as

$$d_{1,k} = [d_{11,k}^T, \ldots, d_{1N,k}^T]^T,$$

where

$$d_{1,i,k} = [r_{d,i,k}^T, s_{i,k}^T]^T.$$

Algorithm 2: The Proposed Centralized Auxiliary Particle Filtering in The DFP1 Situation.

Step 1. Particle initialization

Draw $M$ particles $x_0^m$ from the prior PDF $p_{x_0}(\cdot)$ and the importance weights are all set as $\frac{1}{M}$. Set the maximum time instant $K$.

Step 2. Measurement collection

Collect the measurement $d_{d1,i,k}$ from every DF relay and sensor at time instant $k$.

Step 3. Particle selection and update

Determine the indices $I$ based on the probabilities proportional to $W_{m-1} = p(d_{d1,k}|x_{m-1}^m)$, where $\lambda_k^m \sim p(x_k|x_{k-1}^m)$, and $p(d_{d1,k}|x_{m-1}^m)$ is calculated by (19)–(21). Generate particles $x_k^m$ from $p(x_k|x_{k-1}^m)$ for $m = 1, \ldots, M$, and determine $p(d_{d1,k}|x_k^m)$ by replacing $\lambda_k^m$ with $x_k^m$ in (19)–(21).

Step 4. Weight calculation

Calculate the weights with $W_k = p(d_{d1,k}|x_k^m)$ and normalize the weights as $W_k^m = \frac{W_k}{\sum_{t=1}^{M} W_t^m}$.

Step 5. State estimate update

Obtain the state estimate $\hat{x}_k = \sum_{m=1}^{M} W_k^m x_k^m$.

Step 6. Resampling

Resample a new set of particles with equal weights from $\sum_{m=1}^{M} W_k^m \delta(x_k - x_k^m)$.

Step 7. If $k < K$, then set $k = k + 1$ and go to Step 2; otherwise go to Step 8.

Step 8. Stop.

Then, we have

$$p(d_{d1,k}|x_k^m) = \prod_{i=1}^{N} p(r_{d,i,k}|x_k^m) p(s_{i,k}|x_k^m). \quad (19)$$

Analogous to $p(s_{i,k}|x_k^m)$ in (16), $p(r_{d,i,k}|x_k^m)$ and $p(s_{i,k}|x_k^m)$ in (19) can be approximated as

$$p(r_{d,i,k}|x_k^m) \approx \frac{1}{C} \sum_{c=1}^{C} p_{RD,i,k} \left( r_{d,i,k} - \sqrt{E_{RD,i,k} h_{RD,i,k}^c} x_{RD,i,k} \right) \left( h_i(x_k^m) + v_{e,i,k} \right), \quad (20)$$

and

$$p(s_{i,k}|x_k^m) \approx \frac{1}{C} \sum_{c=1}^{C} p_{SD,i,k} \left( s_{i,k} - \sqrt{E_{SD,i,k} h_{SD,i,k}^c} x_{SD,i,k} \right) \left( h_i(x_k^m) + v_{e,i,k} \right). \quad (21)$$

Since $\lambda_k^m \sim p(x_k|x_{k-1}^m)$, $p(d_{d1,k}|x_k^m)$ can be determined readily via replacing $\lambda_k^m$ with $x_k^m$ in (19)–(21). Now we can outline the developed APF design algorithm in the DFP1 case in Algorithm 2.
based on the probabilities proportional to \( p_i \) and set \( M \) (the iteration number) times and determine the index \( \hat{i} \in \{1, \ldots, N\} \). 

\[
\lambda_k^m \sim p(z_k|x_{k-1}^m) \quad \text{and} \quad p(r_{d,i,k}^{\lambda_k^m}) \quad \text{can be approximated with (20)}. \quad \text{Through replacing } p(d_{a,i,k}) \quad \text{with } p(d_{a,i,k}|x_{k}^m) \quad \text{can also be calculated. After replacing } p(d_{a,i,k}|x_{k}^m) \quad \text{with } p(d_{a,i,k}|x_{k}^m) \quad \text{for the DFP2 situation.} \quad \text{We can apply Algorithm 2 to the DFP2 situation.}
\]

**Remark 2:** The centralized APFs have been constructed for systems with different relays and transmission protocols. APF has been selected with hope to improve the compatibility of the particles to the real-time measurements. Both AF and DF relays have been considered under two kinds of protocols. The effects of the relays have been reflected in the calculation of the likelihood function \( p(d_k|x_k) \). The consensus-based distributed APF has been adopted to approximate the likelihood function \( p(d_k|x_k) \). To cater for the transmission processes with AF relays, the auxiliary index has been calculated with \( \lambda_k^m \sim p(z_k|x_{k-1}^m) \). Due to the existences of both multiplicative random gains and additive disturbances in the transmission links, it is difficult to derive the analytical expressions of the likelihood functions. To circumvent this problem, a Monte-Carlo-based method has been adopted to approximate the likelihood functions.

**B. Distributed APF Design**

The algorithms proposed in Subsection III-A are in fact centralized, and the information from all the nodes is used in the filter. The implementation of a centralized APF depends on frequent transmissions between the sensor nodes and the remote filter, and it might be energy-costing or even impossible for the filter to collect measurement from every sensor node at each time step. For a system monitored by distributed sensor nodes, it is often more desirable to design a distributed filter for each node with only locally available information (the local measurement and the information from the neighboring nodes). The block diagram of the \( i \)th node with the distributed filter is shown in Fig. 3. In this paper, the consensus-based distributed filtering is considered, where the filtering results can reach agreement via information exchange between the sensor nodes. In the proposed distributed APF, the consensus procedure is executed twice: a likelihood-consensus method is adopted in the calculation of the auxiliary index, and a belief-consensus strategy is selected to obtain the association weights. To realize consensus, there is assumed to be at least one path between any two of sensor nodes.

Let us discuss the design of the distributed APF in the APFI case here. If the consensus is realized at time step \( k-1 \) then common particles can be generated for every node at time step \( k \) by providing the same samples for random number generation [35]. So, we can select \( \lambda_k^m \sim p(z_{k,i}^m|x_{k-1}^m) \) for some \( i \in \{1, \ldots, N\} \). Define \( N_i \) as the neighboring nodes of the \( i \)th sensor node, and the available information for the \( i \)th node is denoted as:

\[
r_{a,i,k}^{[i]} = \{r_{a,j,k}\}_{j \in N_i}, \quad s_k^{[i]} = \{s_{j,k}\}_{j \in N_i}, \quad d_k^{[i]} = \{d_{a,j,k}\}_{j \in N_i},
\]

Analogous to (10), the likelihood function with locally available information can be calculated as

\[
p(d_k^{[i]}|\lambda_k^m) = \prod_{j \in N_i} p(r_{a,j,k}|\lambda_k^m)p(s_{j,k}|\lambda_k^m),
\]

where \( p(r_{a,j,k}|\lambda_k^m) \) and \( p(s_{j,k}|\lambda_k^m) \) can be obtained by resorting to (11) and (12), respectively. Setting

\[
N_{i,k}^{(m,0)} = p(d_k^{[i]}|\lambda_k^m),
\]

the likelihood function is fused as

\[
N_{i,k}^{(m+1)} = N_{i,k}^{(m)} \prod_{j \in N_i} \left( \frac{N_{j,k}^{(m,0)}}{N_{i,k}^{(m,0)}} \right) ^{\xi},
\]

where

\[
\xi = \frac{1}{\max_i |G_i|},
\]

and \( |G_i| \) denotes the degree of a sensor node \( G_i \) [48]. Execute (27) for \( H_i \) (the iteration number) times and determine the index \( I_i \) based on the probabilities proportional to \( W_{i,k}^{(m)}N_{i,k}^{(m,H_i)} \).

After the calculation of the local index \( I_i \), arbitrarily choose an integer \( i \in \{1, 2, \ldots, N\} \) and set \( I = I_i \). The particles \( x_k^m \) can be drawn from \( p(x_k|x_{k-1}^m) \), and it follows naturally that

\[
p(d_k^{[i]}|x_k^m) = \prod_{j \in N_i} p(r_{a,j,k}|x_k^m)p(s_{j,k}|x_k^m),
\]

where \( p(r_{a,j,k}|x_k^m) \) and \( p(s_{j,k}|x_k^m) \) are determined with (15) and (16), respectively. Now, the importance weights for the \( i \)th node can be obtained with (8) and normalized. Denote the normalized weights as \( \hat{W}_{i,k}^{(m)} \) and set \( M_{i,k}^{(m,0)} = \hat{W}_{i,k}^{(m)} \) and the belief-consensus procedure is as follows:

\[
M_{i,k}^{(m+h+1)} = M_{i,k}^{(m,h)} \prod_{j \in N_i} \left( \frac{M_{j,k}^{(m,h)}}{M_{i,k}^{(m,h)}} \right) ^{\xi}.
\]

| Parameters | Values |
|------------|--------|
| \( p_{n}^{r} \) | 1 2 3 4 5 6 |
| \( p_{d}^{n} \) | 60 110 165 150 145 90 |
| \( p_{d}^{r} \) | -300 -320 -280 -250 -190 -160 |
| \( E_{SR,i} \) | 2.5 2.7 2.4 2.9 2.2 2.1 |
| \( E_{RD,i} \) | 3.1 3.4 3.1 3.2 3.5 3.3 |
| \( E_{SD,i} \) | 0.4 0.1 0.3 0.9 0.5 0.2 |

### TABLE II

**Centralized Estimation Results With 20 Monte Carlo Trials**

| Case | APF1 | APF2 | DFP1 | DFP2 |
|------|------|------|------|------|
| RMSE(m) | 1.9317 | 2.2353 | 1.8730 | 1.7899 |
Algorithm 3: The Distributed Auxiliary Particle Filtering in The AFP1 Situation (at the ith Node).

Step 1. Particle initialization

Draw $M$ particles $x_{i,m}^0$ from the prior PDF $p_{x_i}(\cdot)$ and the importance weights are all set as $\frac{1}{M}$. Set the maximum time instant $K$.

Step 2. Measurement collection

Collect the measurement $d_{a_1,k}^{[i]}$ from the local node and its neighboring nodes at time instant $k$.

Step 3. Indices calculation and consensus

Obtain the likelihood function $p(d_{a_1,k}^{[i]}|\lambda_{i,m}^0)$ with (26), where $\lambda_{i,m}^0 \sim p(z_{i,k}|x_{i,m}^{k-1})$. Run (27) for $H_1$ times and determine the index $I$ based on the probabilities proportional to $W_{k-1}^{m,i,k,m,h_i}$.

Step 4. Particle selection and update

Generate particles $x_{i,m}^0$ from $p(x_k|x_{k-1}^I)$ for $m = 1, \ldots, M$, and determine $p(x_{a_1,k}^{[i]}|x_{i,m}^0)$ with (28).

Step 5. Weight calculation

Calculate the weights with $W_{i,m}^{m,i,k} = \frac{p(d_{a_1,k}^{[i]}|x_{i,m}^0)}{p(d_{a_1,k}^{[i]}|\lambda_{i,m}^0)}$, and normalize the weights as $\tilde{W}_{i,m}^{m,i,k} = \frac{W_{i,m}^{m,i,k}}{\sum_{j=1}^{M} W_{i,j}^{m,i,k}}$.

Step 6. Weight consensus

Execute the iteration (29) and normalize the weights for $H_2$ times, and set $W_{i,m}^{m,H_2}$ as normalized $M_{i,k,m,h_i}$.

Step 7. State estimate update

Calculate the state estimate $\hat{x}_{i,k} = \sum_{m=1}^{M} W_{i,m}^{m,i,k} x_{i,m}^{m,i,k}$.

Step 8. Resampling

Resample a new set of particles with equal weights from $\sum_{m=1}^{M} W_{i,m}^{m,i,k} \delta(x_{i,k} - x_{i,m}^{m,i,k})$.

Step 9. If $k < K$, then set $k = k + 1$ and go to Step 2; otherwise go to Step 10.

Step 10. Stop.

Implement (29) and normalize $M_{i,k,m,h_i}$ for $H_2$ times, and the normalized $M_{i,k,m,H_2}$ is selected as the consensus importance weights. The overall algorithm is outlined in Algorithm 3.

Similar with the distributed filter in the AFP1 situation, the distributed APFs in the AFP2/DFP1/DFP2 cases can be established as well. Due to the page limitation, the detailed algorithms are not presented here.

Remark 3: In a centralized APF, every sensor node needs to send the local measurement to the remote filter through a relay, and the communication between the nodes and the remote filter may be time- and energy-consuming. By contrast, in the distributed APF, every node only needs to communicate with its neighboring nodes, and thus the scalability/flexibility of the distributed filtering method can be significantly improved. In the calculation of the auxiliary index and the association weights, the likelihood-consensus and belief-consensus strategies are put forward, respectively. (27) and (29) are the standard belief consensus calculations, which can guarantee the convergence of the consensus error [30]. Sufficient computational resources of the filters are hypothesized such that the developed filters can be implemented effectively. Furthermore, the values of $C$ and $M$ cannot be unduly large since the required $C \times M$ samples may bring in considerable calculation burden. The consensus steps $H_1$ and $H_2$ should be selected as no less than the diameter of the topology such that every filter can gather some information from every other node.

IV. NUMERICAL STUDY

In this section, the effectiveness of the proposed APF algorithm with multiple sensors and relays is demonstrated via a two-dimensional target tracking example. The system state is defined as $x_k = [p_{x,k}, v_{x,k}, p_{y,k}, v_{y,k}]^T$, where $(p_{x,k}, p_{y,k})$ and $(v_{x,k}, v_{y,k})$ are the position and velocity, respectively. The dynamics of the target is formulated by the widely adopted white noise acceleration model [5]:

$$x_{k+1} = Ax_k + w_k,$$

where

$$A = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

(31)
measurement of the $i$th sensor can be written as
\begin{equation}
    y_{i,k} = \arctan\left(\frac{p_{y,k} - p_{i,y}^{\text{sensor}}}{p_{x,k} - p_{i,x}^{\text{sensor}}}\right) + v_{i,k},
\end{equation}
where $v_{i,k}$ at different sensor nodes are independent of each other with $v_{i,k} \sim N(0, 0.01)$. In our simulation, six sensors are considered and connected through a given topology as demonstrated in Figs. 4–7. All the random channel gains and additive transmission noises obey the standard normal distribution. The parameters of the sensors and relays are provided in Table I.

In the proposed APF, the initial value is drawn from $N(x_0, Q)$, the particle number is selected to be 200, and the number of Monte Carlo trials in the likelihood approximation is 50.

The following root mean-square error (RMSE) is selected to evaluate the performance of the filtering scheme:
\begin{equation}
    \text{RMSE}_{k} = \sqrt{\frac{1}{L} \sum_{l=1}^{L} \left[ (p_{x,k}^{l} - p_{x,k}^{\text{sensor}})^2 + (p_{y,k}^{l} - p_{y,k}^{\text{sensor}})^2 \right]},
\end{equation}
where $L$ is the total number of the Monte Carlo runs, $(p_{x,k}^{l}, p_{y,k}^{l})$ is the realization of $(p_{x,k}, p_{y,k})$ in the $l$th Monte Carlo run, and the respective estimate is denoted by $(p_{x,k}^{l}, p_{y,k}^{l})$.

The tracking results obtained with the centralized APF in the four situations (AFP1, AFP2, DFP1 and DFP2), as well as the sensor positions are shown in Figs. 4–7, and the RMSEs obtained in the four situations are demonstrated in Table II. It can be seen that the target can be tracked effectively with the proposed method.

In the distributed APF, the iteration time steps $H_1$ and $H_2$ are both set to be 5. The initial value of each node is subject to a bias, where the position (velocity, respectively) bias is uniformly distributed over $[-5, 5] m$ ($[-0.05, 0.05] m/s$, respectively). The tracking errors from all the nodes are depicted in Figs. 8–11, and the RMSEs obtained in the four situations are presented in Table III (the last column means the standard deviation among different nodes). It is obvious that every sensor node can obtain satisfactory estimation results.

### V. Conclusion

In this paper, the APF has been designed for systems monitored by multiple sensors. AF and DF relays have been taken into account to improve the performance of the long-distance signal transmissions. Different protocols have been employed to govern the signal transmissions from the sensors to the filter. The likelihood functions have been determined according to the relay models and the transmission protocols. A new centralized APF has been established with a properly selected auxiliary index, and the consensus-based distributed APF has also been parameterized. The performances of the proposed
centralized/distributed filtering approaches have been presented via target tracking simulations in different cases. Further research directions would be the extension of our work to complex systems with more network-induced phenomena, such as transmission delays [7], communication protocols [17], [53], and cyber attacks [43], [47], [49].

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