Modified black hole solution with a background Kalb-Ramond field

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Abstract

We study the gravitation effects on a static and spherically symmetric spacetime due to the vacuum expectation value (VEV) of Kalb-Ramond field. The Kalb-Ramond VEV is a background tensor field which produces a local Lorentz symmetry breaking (LSB) of spacetime. Considering a non-minimal coupling between the Kalb-Ramond (vev) and the Ricci tensor we obtain an exact parameter-dependent power-law hairy black hole. For $s = 1$, the Lorentz violation produces a solution similar to the Reissner-Nordstrom, despite the absence of charge. The near horizon geometry is modified by including a new inner horizon and shifting the Schwarzschild horizon. Asymptotically, the usual Minkowski spacetime with a background tensor field is recover. By means of the mercury perihelion test, an upper bound to the local Lorentz violation (LV) is obtained and its corresponding effects on the black hole temperature is investigated.

Keywords: Lorentz symmetry breaking, modified black hole

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I. INTRODUCTION

The search for reminiscent quantum gravity effects at low energy regime has attracted attention over the last decades. Some models in string theory [1], Very special relativity [2], Doubly special relativity [3], noncommutative spacetime [4], Horava gravity [5] and Loop quantum gravity [6] among other, assume that the Lorentz symmetry might be broken in the gravitational UV regime.

A mechanism for the local Lorentz violating is provided by a spontaneously symmetry breaking potential due to self-interacting tensor fields [1, 7]. The vacuum expectation value (VEV) of these tensor fields yields to background tensor fields which by coupling to the Standard Model (SM) fields violate the particle local Lorentz symmetry [7].

The simplest self-interacting tensor field is a vector field, the so-called bumblebee [7, 8] whose VEV defines a privileged direction in spacetime. In flat spacetime, the bumblebee fluctuations over the VEV have two massless modes, known as the Nambu-Goldstone modes and one massive or Higgs mode [8, 9]. In cosmology, the effects of the bumblebee field on the universe expansion were analysed [10]. In addition, static and spherically solutions were obtained considering a localized source and non-minimal couplings between the Ricci tensor and the VEV vector. Imposing a constancy on the squared norm of the VEV vector, the authors found a modified black hole solution keeping invariant the event horizon [11]. By assuming a covariant constant VEV, the authors found black hole solution with an interesting modified event horizon [12]. The bumblebee vacuum vector also allows exotic solutions, such as the wormhole solutions [13]. The effects of the modified geometry due to the bumblebee VEV on the black hole thermodynamics properties were considered in Ref. [19].

In this work we consider that the Lorentz symmetry breaking (LSB) is driven by a self-interacting antisymmetric 2-tensor, $B_{\mu\nu}$, the so-called Kalb-Ramond (KR) field [14]. Likewise the graviton and dilaton, the Kalb-Ramond field arises in the spectrum of the bosonic string theory [14]. Assuming that the potential $V$ has a nonzero vacuum expectation value $b_{\mu\nu}$, such antisymmetric background tensor can be decomposed in two spacelike vectors and one timelike vector, much like the electromagnetic tensor $F_{\mu\nu}$ decomposition [15]. We are interested in modifications on spherically symmetric black holes driven by the Lorentz violating KR VEV. In a Lorentz invariant theory, the Kalb-Ramond field minimally coupled to gravity yields to an axion hairy black hole which deforms the event horizon into a naked
singularity \[17\].

Here we assume a spacelike Kalb-Ramond with constant squared norm VEV and non-minimally coupled with the Ricci tensor. As a result we found an exact spherically symmetric and static modified black hole solution. In addition to the Schwarzschild \(1/r\) solution, we obtained a power-law correction of form \(\Upsilon/r^{2s}\), where \(s\) and \(\Upsilon\) are Lorentz violating (LV) parameters.

The work is organized as the following. In section II we present a short review on the spontaneous symmetry breaking mechanism for the Lorentz symmetry driven by the Kalb-Ramond field. Moreover, we define the non-minimal coupling between the KR VEV and the Ricci tensor and we choose the vacuum configuration. In section III we obtain an exact solution for the modified Einstein equation found in II. By varying the parameter \(s\) we analyse the modifications on the event horizon, relating them to known solutions such as the charged black hole and the Schwarzschild-De Sitter solution. In addition, the effects of this modified geometry on the black hole thermodynamics is obtained by means of the tunnelling method. In section IV the modified gravitational potential is obtained and an upper bound for the Lorentz violating parameter \(\Upsilon\) is obtained using the precession period of Mercury as a test. Finally, in section V final comments and perspectives are outlined. Throughout the text we adopt the metric signature \((-,+,+,+).\)

II. THE KALB-RAMOND MODEL FOR SPONTANEOUS LORENTZ SYMMETRY BREAKING

In this section we present the Kalb-Ramond vev and specify the coupling of this background field with gravity.

The Kalb-Ramond field is a tensorial field arising from the bosonic spectrum of string theory \[14\]. It can be represented by a 2-form potential \(B_2 = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu\) whose fields strength is given by \(H_3 = dB_2\), or \(H_{\lambda\mu\nu} \equiv \partial_\lambda B_{\mu\nu}\) in coordinates.

Inspired in the gravitational sector of the SME, we consider a self-interacting potential for the Kalb-Ramond field \[15\]. Assuming a potential of form \(V = V(B_{\mu\nu}B^{\mu\nu} \pm b_{\mu\nu}b^{\mu\nu})\) with a non-vanishing (VEV) \(<B_{\mu\nu}>=b_{\mu\nu}\) which defines a background tensor field, the Lorentz symmetry is spontaneously broken by the Kalb-Ramond self-interaction. Note that the dependence of the potential on \(B_{\mu\nu}B^{\mu\nu}\) is required in order to maintain the theory
invariant upon observer local Lorentz transformations. Further, the potential breaks the gauge invariance $B_2 \rightarrow B_2 + d\Lambda_1$, where $\Lambda_1$ is an arbitrary 1-form [15].

Let us consider the action for a self-interacting Kalb-Ramond field non-minimal coupled with gravity in the form [15]

$$S_{KR}^{nonmin} = \int e \, d^4x \left[ \frac{R}{2\kappa} - \frac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - V(B_{\mu\nu} B^{\mu\nu} \pm b_{\mu\nu} b^{\mu\nu}) + \frac{1}{2\kappa} \left( \xi_2 B^{\lambda\nu} B^{\mu}_\nu R_{\lambda\mu} + \xi_3 B^{\mu\nu} B_{\mu\nu} R \right) \right],$$

where $\xi_2$ and $\xi_3$ are non-minimal coupling constants (with dimensions $[\xi]=L^2$), $e$ is the metric determinant and $\kappa = 8\pi G$ is gravitational coupling constant.

Since we are interested in the effects of the background Kalb-Ramond VEV on the gravitational field, we consider $B_{\mu\nu} B^{\mu\nu} = b_{\mu\nu} b^{\mu\nu}$. In order to be well defined, the vev $b_{\mu\nu}$ ought to have constant norm $b^2 = b_{\mu\nu} b^{\mu\nu}$ and vanishing field strength $H_3 = db_2$. These conditions ensure the vanishing of the Hamiltonian.

The antisymmetry of $b_{\mu\nu}$ enables us to rewrite it as $b_{\mu\nu} = E_\mu [\nu_\alpha] + \epsilon^{\alpha\beta} \nu_\alpha B_\beta$, where $E_\mu$ and $B_\mu$ can be interpreted as pseudo electric and magnetic fields, respectively, and $\nu^\mu$ is a timelike 4-vector. The pseudo-fields $E_\mu$ and $B_\mu$ are spacelike, i.e., $E_\mu u^\mu = B_\mu u^\mu = 0$. Thus, the KB VEV yields two background vector instead of only one produced by the bumblebee vev [11].

In this work we consider an pseudo-electric configuration of form

$$b_2 = -\tilde{E}(x^1) \, dx^0 \wedge dx^1. \quad (2)$$

The ansatz in eq.(2) satisfies the condition $H_3 = db_2 = 0$. The function $E(x^1)$ will be determined by the condition $b^2$ constant in the next section.

The constancy of $b^2$ turns the Lagrangian term $\xi_3 B^{\mu\nu} B_{\mu\nu} R$ into $\xi_3 b^2 R$ which can be absorbed into a redefinition of variables. Thus, by varying Eq. 1 with respect to the metric, the modified Einstein equations are

$$G_{\mu\nu} = \kappa T^{\xi_2}_{\mu\nu}. \quad (3)$$
with $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ and

$$T^{\xi_2}_{\mu\nu} = \frac{\xi_2}{\kappa} \left[ \frac{1}{2} g_{\mu\nu} B^{\alpha\gamma} B^{\beta\delta} \gamma R_{\alpha\beta} - B^{\alpha\mu} B^{\beta\nu} R_{\alpha\beta} - B^{\alpha\beta} B_{\mu\beta} R_{\nu\alpha} + \frac{1}{2} D_{\alpha} D_{\mu} (B_{\nu\beta} B^{\alpha\beta}) + \frac{1}{2} D_{\alpha} D_{\nu} (B_{\mu\beta} B^{\alpha\beta}) - \frac{1}{2} D^2 (B^{\alpha\mu} B_{\alpha\nu}) - \frac{1}{2} g_{\mu\nu} D_{\alpha} D_{\beta} (B^{\alpha\gamma} B^{\beta\gamma}) \right].$$

Therefore, the non-minimal coupling yields to a source modifying the field equation by new derivative terms. In the next section, we seek for modifications of the spherically symmetric spacetime.

**III. SPHERICALLY SYMMETRIC SOLUTIONS OF KALB-RAMOND BLACK-HOLE**

We consider a static and a spherically symmetric vacuum spacetime solution. One thus adopts the metric, as given by the line element,

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (5)$$

Since $b^2 = g^{\mu\alpha} g^{\nu\beta} b_{\mu\nu} b_{\alpha\beta}$, the Kalb-Ramond VEV ansatz given by Eq.(2) has a constant norm $b^2$ with the metric (5) provided that

$$\tilde{E}(r) = |b| \sqrt{\frac{A(r) B(r)}{2}} \quad (6)$$

where $b$ is a constant. Note that the function $E(r)$ in Eq.(6) defines a radial pseudo-electric static field $\tilde{E} = \tilde{E}(r) \hat{r}$, consistent with the spheric and static spacetime symmetry. Indeed, the background vector $\tilde{E}$ is orthogonal to both the timelike and spacelike Killing vectors responsible for the static and spheric symmetries. The constancy of the VEV was also proposed in a bumblebee modified black hole [11]. From Eq.(6) is clear that $b_{\mu\nu}$ depends on the spacetime position keeping only $b^2$ constant.
Rewriting the modified Einstein equation (3) as

\[ R_{\mu\nu} = \xi^2 \left[ g_{\mu\nu} b^{\alpha\gamma} b^{\beta} \gamma R_{\alpha\beta} - b^{\alpha}_{\mu} b^{\beta}_{\nu} R_{\alpha\beta} ight. \]

\[ - b^{\alpha\beta} b_{\mu\beta} R_{\nu\alpha} - b^{\alpha\beta} b_{\nu\beta} R_{\mu\alpha} + \frac{1}{2} D_\alpha D_\mu (b_{\nu\beta} b^{\alpha\beta}) \]

\[ + \frac{1}{2} D_\alpha D_\nu (b_{\mu\beta} b^{\alpha\beta}) - \frac{1}{4} D^2 (b^{\alpha}_{\mu} b_{\alpha\nu}) \right], \tag{7} \]

and using the metric ansatz (5) we obtain the system of equations

\[ \left( 1 - \frac{s}{2} \right) R_{tt} = 0 \tag{8} \]

\[ \left( 1 - \frac{s}{2} \right) R_{rr} = 0 \tag{9} \]

\[ R_{\theta\theta} = \frac{sr^2}{2} \left( \frac{R_{tt}}{A[r]} - \frac{R_{rr}}{B[r]} \right), \tag{10} \]

\[ R_{\phi\phi} = sen^2 \theta R_{\theta\theta} \tag{11} \]

where \( s = |b|^2 \xi^2 \). Since the components of Ricci tensor are

\[ R_{tt} = \frac{A''}{2B} - \frac{A'}{4B} \left( \frac{A'}{A} + \frac{B'}{B} \right) + \frac{A'}{rB}, \tag{12} \]

\[ R_{rr} = -\frac{A''}{2A} + \frac{A'}{4A} \left( \frac{A'}{A} + \frac{B'}{B} \right) + \frac{B'}{rB}, \tag{13} \]

\[ R_{\theta\theta} = 1 - \frac{1}{B} - \frac{r}{2B} \left( \frac{A'}{A} - \frac{B'}{B} \right), \tag{14} \]

then Eq.(8) and Eq.(9) yields to

\[ A(r) = \frac{1}{B(r)}. \tag{15} \]

for \( s \neq 2 \). Substituting the constrain Eq.(13) in Eq.(10) yields to the equation

\[ \frac{r^2}{2} A'' + (s + 1)r A' + A - 1 = 0, \tag{16} \]

whose solution is

\[ A(r) = 1 - \frac{R_{Sch}}{r} + \frac{\Upsilon}{r^2}, \tag{17} \]
where $R_{Sch} = 2GM$ is the usual Schwarzschild radius and $\Upsilon$ is a constant (with dimensions $[\Upsilon] = L^2$) that controls the Lorentz violation effects upon the Schwarzschild solution.

Therefore, the Lorentz violation trigged by the Kalb-Ramond VEV produces a power-law hairy black hole of form

$$ ds^2 = - \left[ 1 - \frac{R_{Sch}}{r} + \frac{\Upsilon}{r^2} \right] dt^2 + \left[ 1 - \frac{R_{Sch}}{r} + \frac{\Upsilon}{r^2} \right]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. $$

(18)

(18)

It the limit $s \to 0$ ($|b|^2 \to 0$) we recover the usual Schwarzschild metric, as expected. Moreover, the Kretschmann scalar has the form

$$ R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} = \frac{12 r^4 R_{Sch}^2 - 8 r^2 + 1 R_{Sch} \Upsilon (1 + \frac{3}{s} + \frac{2}{s^2}) + 4 r^2 \Upsilon^2 (1 + \frac{5}{s} + \frac{4}{s^2} + \frac{4}{s^3})}{r^2 s^6}, $$

(19)

(19)

and then, the LV modifications can not vanish by a coordinate change. In fact, the LBS solution Eq. (18) differs from the Lorentz invariant KR solution [17] and from the LSB bumblebee black hole solutions in Ref. [11] and Ref. [12].

An interesting result occurs for $s = |b|^2 \xi_2 = -1$. In this case, $A(r) = 1 - \frac{R_{Sch}}{r} + \Upsilon r^2$, which is similar to the Schwarzschild-de Sitter (SdS) solution. Thus, the background LV Kalb-Ramond vev can be interpreted as a source for the cosmological constant $\Lambda$. The small value of the LV coefficient $\Upsilon$ provides a tiny cosmological constant, as observed.

For $s = 1$, the black hole geometry (18) resembles the charged Reissner-Nordstrom solution. However, using solution (18), the pseudo-electric field is actually constant $E(r) = \frac{|b|}{\sqrt{2}}$ and radial. This result is consistent with the asymptotic flat spacetime with a spacelike LV background field. Nevertheless, a constant electric field is inconsistent with a field created by a localized charge. Therefore, $\Upsilon$ can not be identified with a charge and it represents a LV hair of the black hole.

A. Horizons

Once we have obtained the black hole solution (18) we can find the event horizons by assuming $A(r) = 0$ in (17), leading to

$$ r^2 - R_{Sch} r^{2-1} + \Upsilon = 0. $$

(20)
For \( s = 1 \) there are two horizons given by

\[
\begin{align*}
    r_\pm &= \frac{R_{Sch}}{2} \left( 1 \pm \sqrt{1 - \frac{4\Upsilon}{R_{Sch}^2}} \right). \tag{21}
\end{align*}
\]

Assuming \( \Upsilon \ll R_{Sch}^2 \), the two horizons have the form

\[
\begin{align*}
    r_+ &\approx R_{Sch} - \frac{\Upsilon}{R_{Sch}} - \frac{\Upsilon^2}{R_{Sch}^3}, \tag{22} \\
    r_- &\approx \frac{\Upsilon}{R_{Sch}} + \frac{\Upsilon^2}{R_{Sch}^3}, \tag{23}
\end{align*}
\]

where \( \lim_{R_{Sch} \to \infty} r_+ = \infty \) and \( \lim_{R_{Sch} \to \infty} r_- = 0 \). Therefore, the LV produces a new inner horizon \( r_- \) and reduces the outer (Schwarzschild) horizon by \( r_+ = R_{Sch} - r_- \). That property differs the LSB KR black hole from the bumblebee black hole found in Ref. [11] whose (Schwarzschild) horizon is kept unchanged. In Ref. [12] the authors found a LBS bumblebee black hole with only one modified horizon.

The structure of the LBS KR event horizon is rather different from the Lorentz invariant axion-KR solution [17]. In fact, the KR field turns the event horizon into a naked singularity [17], whereas the LBS KR modifies and produces new horizons.

By increasing the power which the LV term decays, the correction of the horizons decreases. For \( s = \frac{2}{3} \) there are three roots in Eq.20 but only one real root. For the physical solution the changes in the usual Schwarzschild radius has the form

\[
    r_h \approx R_{Sch} + \mathcal{O} \left( \frac{\Upsilon^2}{R_{Sch}^6} \right). \tag{24}
\]

The LV parameter \( \Upsilon \) is supposed to be small compared to a power of \( R_{Sch} \). In the next section we establish an upper bound to \( \Upsilon \) by considering the effects of the modified geometry of this LSB black hole on test particles.

**B. Temperature**

In order to obtain the Hawking temperature for the black hole characterized by the metric [18], we will employ the Hamilton-Jacobi formalism to the tunneling approach [18]. In this method, the event horizon is treated as a potential barrier such that the particles created near the horizon can escape from the black hole through quantum tunneling. The method
consists in computing the probability of the tunneling. For this, we will consider only events near the horizon and radial trajectories, such that we can solve this in $t - r$ plane.

We consider the scalar perturbation from a massive scalar field $\phi$ around a black hole background. The equation of motion of this perturbation is the Klein-Gordon equation

$$\hbar^2 g^{\mu\nu} \nabla_\mu \nabla_\nu \phi - m^2 \phi = 0,$$

(25)

where $m$ is the mass associated with the field $\phi$.

For a metric form Eq.(5) we obtain the following result after spherical harmonics decomposition

$$- \partial_t^2 \phi + A^2(r) \partial_r^2 \phi + \frac{1}{2} \partial_r A^2(r) \partial_r \phi - \frac{m^2}{\hbar^2} A(r) \phi = 0.$$

(26)

If we interpret the field $\phi$ as a semi-classical wave function associated with the particles created in the black hole, we can solve the equation (26) through the WKB method which consists of using the following ansatz

$$\phi(t, r) = \exp \left[ -\frac{i}{\hbar} I(t, r) \right].$$

(27)

Expanding (27) for the lowest order in $\hbar$, one has

$$(\partial_t I)^2 - f^2(r)(\partial_r I)^2 - m^2 f(r) = 0,$$

(28)

such that the equation (28) is the Hamilton-Jacobi equation with $I$ playing the role of relativistic action. Using the separation of variables, (28) has the solution:

$$I(t, r) = -\omega t + W(r),$$

(29)

where the $\omega$ is a constant of motion which can be interpreted as the energy of the emitted radiation. Putting the solution (29) in the equation (28) we obtain the spatial part of the action:

$$W(r) = \int \frac{dr}{A(r)} \sqrt{\omega^2 - m^2 A(r)}.$$

(30)

Now, we take the approximation of the function $f(r)$ near the event horizon $r_+$,

$$A(r) = A(r_+) + A'(r_+)(r - r_+) + \cdots,$$

(31)

and the Eq. (30) takes the form

$$W(r) = \int \frac{dr}{A'(r_+)} \frac{\sqrt{\omega^2 - m^2 A'(r_+)(r - r_+)}}{(r - r_+)},$$

(32)
where the prime denotes derivative with respect to the radial coordinate. The last integral can be made using the residue theorem such that

\[ W = \frac{2\pi i \omega}{A'(r_+)} . \]  

(33)

The tunnelling probability of a particle escape of the black hole is given by

\[ \Gamma \sim \exp(-2 \text{Im}(\mathcal{I})) = \exp\left[-\frac{4\pi \omega}{A'(r_+)}\right] , \]  

(34)

where we note that \( \text{Im}\mathcal{I} = \text{Im}W \).

Comparing Eq. (34) with the Boltzmann factor \( e^{-\omega/T} \), we obtain the Hawking temperature of the black hole:

\[ T_H = \frac{\omega}{2 \text{Im}(\mathcal{I})} = \frac{A'(r_+)}{4\pi} . \]  

(35)

In our case, the radius of the horizon is given by (21) and the above results provide

\[ T_H = \frac{R_s \left( \sqrt{R_s^2 - 4\Upsilon + R_s} \right) - 4\Upsilon}{\pi \left( \sqrt{R_s^2 - 4\Upsilon + R_s} \right)^3} , \]  

(36)

where we assume \( s = 1 \). It is convenient to write the Hawking temperature for small values of \( \Upsilon \) (\( \Upsilon \ll R_s^2 \)). This approximation leads to a temperature

\[ T_H \approx \frac{1}{4\pi R_s} - \frac{\Upsilon^2}{4\pi R_s^5} . \]  

(37)

Note that the first term on the right in Eq. (37) is the Hawking temperature for Schwarzschild black hole. The second term represents the leading order correction due to LSB and implies that the LSB KR black hole obtained in this work is colder than the Schwarzschild black hole. This interesting feature could be considered an observational discrepancy between LSB and LI black holes and had been obtained for the bumblebee \(^{19}\) and the regular \(^{20}\) black holes.

IV. CLASSICAL EFFECTS

In this section we study the effects of the Kalb-Ramond LBS black hole solution Eq. (18) on massive classical test particle. We consider only the gravitational effects upon the particle and neglect the coupling between the particle and the background KR VEV.
For a massive particle, the 4-velocity $u^\mu = \frac{dx^\mu}{d\tau}$ satisfies

$$g_{\mu\nu}(x)u^\mu u^\nu = -1.$$  \tag{38}$$

Despite the presence of a LSB KB field, the static and radial VEV configuration in Eq.(2) preserves the time and isotropic symmetries of the black hole. Thus, the timelike Killing vector $t^\mu$ and the spacelike Killing vector $\psi^\mu$ provides two constants of motions, namely

$$E = \left(1 - \frac{R_s}{r} + \frac{\Upsilon}{r^2}\right) \frac{dt}{d\lambda},$$ \tag{39}

$$L = r^2 \frac{d\phi}{d\lambda},$$ \tag{40}

which for massive particles they are the conserved energy and angular momentum per unit mass of the particle, respectively.

In terms of the energy in (39) and of the angular momentum (40), the constrain (38) can be rewritten as

$$\frac{E}{2} = \frac{1}{2} \left(\frac{dr}{dr}\right)^2 + V_{\text{eff}}(r),$$ \tag{41}$$

where the effective potential $V_{\text{eff}}$ is given by

$$V(r) = \frac{1}{2} - \frac{R_s}{2r} + \frac{L^2}{2r^2} - \frac{L^2 R_s}{2r^3} + \frac{\Upsilon}{2r^2} + \frac{L^2 \Upsilon}{2r^4 + 2}.$$ \tag{42}$$

Accordingly, the LSB geometry induces power-law short-range forces whose strength depends on the LV parameter.

1. Perihelion Precession

Let us now study the modifications driven by the LSB geometry on the bound orbits. In special, we consider the effects upon the advance of the perihelion and find an upper bound for the LV parameter. We concern ourselves to the $s = 1$ case for simplicity.

By considering $\frac{dr}{dr} = \frac{dr}{dr} \frac{L}{r^2}$ and setting $x = r^{-1}$, the constrain in Eq.(41) leads to the following equation

$$\frac{d^2 x}{d\phi^2} + \left(1 + \frac{\Upsilon}{L^2}\right) x = \frac{R_s}{2L^2} + \frac{3R_s}{2} x^2 - 2\Upsilon x^3.$$ \tag{43}$$

The LBS geometry provides a linear and a cubic new terms. Defining the parameter $\chi = \frac{3R_s^2}{4L^2}$ and neglecting the cubic term, the Eq.(43) takes the form

$$\frac{d^2 x}{d\phi^2} + \left(1 + \frac{\Upsilon}{L^2}\right) x = \frac{R_s}{2L^2} + \chi \left(\frac{2L^2}{R_s}\right) x^2.$$ \tag{44}$$
Let us look for a perturbed solution of form \( x = x_0 + \chi x_1 \) where \( x_0 \) is the Newtonian solution and \( \chi x_1 \) is a small deviation. Neglecting the second order terms for the parameter \( \chi \), we obtain the following equation

\[
\frac{d^2 x_0}{d\phi^2} + \left(1 + \frac{\Upsilon}{L^2}\right) x_0 - \frac{R_s}{2L^2} + \chi \left(\frac{d^2 x_1}{d\phi^2} + \left(1 + \frac{\Upsilon}{L^2}\right) x_1 - \frac{2L^2}{R_s} x_0^2\right) = 0. \tag{45}
\]

The zeroth-order solution \( x_0 \) is given by

\[
x_0 = \frac{R_s}{2L^2(1 + \frac{\Upsilon}{L^2})} \left(1 + e \cos \left(\sqrt{1 + \frac{\Upsilon}{L^2}} \phi\right)\right), \tag{46}
\]

where \( e \) is the eccentricity of an ellipse and the first-order solution is

\[
x_1 = \frac{R_s}{2L^2(1 + \frac{\Upsilon}{L^2})^2} \left[1 + e^2 \frac{\phi}{2} \sin \left(\sqrt{1 + \frac{\Upsilon}{L^2}} \phi\right) - \frac{e^2}{6(1 + \frac{\Upsilon}{L^2})} \cos \left(2\sqrt{1 + \frac{\Upsilon}{L^2}} \phi\right)\right]. \tag{47}
\]

Thus, the general solution has the form

\[
x \approx \frac{R_s}{2L^2(1 + \frac{\Upsilon}{L^2})} \left[1 + \cos \left[(1 - \beta) \sqrt{1 + \frac{\Upsilon}{L^2}} \phi\right]\right], \tag{48}
\]

where

\[
\beta = \frac{3R_s^2}{4L^2(1 + \frac{\Upsilon}{L^2})^2}. \tag{49}
\]

Therefore, the orbit period with the LV correction that will be given by

\[
\Phi = 2\pi + \Delta \Phi_{GR} + \Delta \Phi_{LV}, \tag{50}
\]

where \( \Delta \Phi_{GR} = \frac{3\pi R_s}{(1 - e^2)a} \) is GR correction with \( a \) being the semi-major axis of the orbital ellipse.

The contribution of the Lorentz violation of the model is given by

\[
\Delta \Phi_{LV}^{s=1} = -\frac{2\pi \Upsilon}{R_s(1 - e^2)a}. \tag{51}
\]

Similarly, we can make the same account for \( s = 2/3 \) and find that the correction in the period is given by

\[
\Delta \Phi_{LV}^{s=2/3} = -\frac{6\pi \Upsilon}{R_s(1 - e^2)^2 a^2}. \tag{52}
\]

that is, smaller than the previous one, so we can see that the higher is \( \frac{2}{3} \), the lower its contribution to the gravitational corrections.
From the results we can get an estimate for the Lorentz violation parameter $\Upsilon$ from the perihelion shifts for the orbit of the inner planets consistent with the GR. Here we consider just the case of the planet Mercury, where it has as predicted (by GR) perihelion advance $\Delta \Phi_{GR} = 42.9814''/C$ (in arcseconds per century) and the observational error is $e = 0.003''/C$, according to the most up to date measurement found in Ref. \[21\]. Then, it is possible to calculate a upper-bound for $\Upsilon$, if it considers that the contribution of constant arising from the Lorentz violation is less than the observational error. Thus, we obtained $\Upsilon_{s=1} < 2.8 \times 10^{-3} km^2$. For a small astrophysical black hole with ten times the solar mass, the shift in the event horizon is of order

$$\frac{\Upsilon^s_{s=1}}{(R_{Sch})^2} \approx 0.20 \times 10^{-5}.$$  \hspace{1cm} (53)

V. FINAL REMARKS AND PERSPECTIVES

In this work we obtained a spherically symmetric and static solution of gravity non-minimally coupled to a VEV of the Kalb-Ramond field. The self-interaction potential breaks the KR gauge invariance and produces a VEV background tensor field which violates the local Lorentz symmetry.

The vacuum background tensor was chosen in order to vanish the self-interaction potential and the Kalb-Ramond hamiltonian. Further, the VEV is perpendicular the timelike and spacelike Killing vectors and then, the Lorentz violation preserves the static and spheric symmetries of the gravitational vacuum. By assuming a non-minimal coupling between the KR VEV and the Ricci tensor, we found a power-law correction to the Schwarzschild solution. The black hole solution have two parameters controlling the Lorentz violation, $s$ and $\Upsilon$. For $s = 1$, the LSB solution exhibits two horizons, likewise a charged black hole. Since no charge or angular momentum are present, the solution represents a Lorentz violating hairy black hole.

The radius of the outer horizon is the Schwarzschild radius minus the inner horizon. For $s = -2$ the LSB solution resembles a Schwarzschild-De Sitter black hole, with the Lorentz symmetry parameter $\Upsilon$ being proportional to the cosmological constant. The more is the $s$ the less is the correction to the horizon radius and then, we focus on the $s = 1$ configuration. It is worthwhile to mention that, unlike the LSB black holes generated by the bumblebee field, the KR LSB solution found not only modifies the usual Schwarzschild event horizon.
but also produces additional horizons.

The Lorentz violation also modifies the black hole temperature. We employed the tunnelling method to derive the Berkenstein-Hawking (BH) temperature. It turns out that the Lorentz violating parameter $\Upsilon$ reduces the BH temperature by a term proportional to $\Upsilon^2$. Similar results were also obtained in other modified black holes, such as the LSB bumblebee \[19\] and the regular black hole \[20\].

In the classical level, the LSB solution found yields to an additional gravitational potential whose power depends on the Lorentz violating parameter $s$. For $s = 1$, we obtained an upper bound for the LV parameter $\Upsilon$ by studying the correction of the precession period of the planet Mercury. From this bound we can estimate that, for a peculiar black hole with ten times the solar mass the shift in the outer event horizon is about 20$mm$ compared to 30$Km$ of the Schwarzschild radius.

As possible extensions of the present work, we point out the stability analysis of the solution found by considering the corrections due to fluctuations of the KR field around the VEV. Moreover, wormhole and regular solutions can also be found. The effects of a coupling between the KR field and the Riemann tensor is another important development.

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[1] V. A. Kostelecký and S. Samuel, Phys. Rev. D 39, 683 (1989); Phys. Rev. Lett. 63, 224 (1989). Phys. Rev. Lett. 66, 1811 (1991).

[2] A. G. Cohen and S. L. Glashow, Phys. Rev. Lett. 97, 021601 (2006). R. V. Maluf, J. E. G. Silva, W. T. Cruz and C. A. S. Almeida, Phys. Lett. B 738, 341 (2014).

[3] J. Magueijo and L. Smolin, Phys. Rev. Lett. 88, 190403 (2002).

[4] S. M. Carroll, J. A. Harvey, V. A. Kostelecký, C. D. Lane, and T. Okamoto, Phys. Rev. Lett. 87, 141601 (2001).

[5] P. Hořava, Phys. Rev. D 79, 084008 (2009).
[6] J. Alfaro, H. A. Morales-Tecotl and L. F. Urrutia, Phys. Rev. Lett. 84, 2318 (2000). Phys. Rev. D 65, 103509 (2002).

[7] V. Alan Kosteleck Phys. Rev. D 69, 105009 (2004).

[8] R. Bluhm and V. A. Kostelecký, Phys. Rev. D 71, 065008(2005).

[9] R. V. Maluf, J. E. G. Silva and C. A. S. Almeida, Phys. Lett. B 749, 304 (2015).

[10] D. Capelo and J. Pramos, Phys. Rev. D 91, no. 10, 104007 (2015).

[11] R. Casana, A. Cavalcante, F.P. Poulis, and E.B. Santos Phys. Rev. D 97, 104001 (2018)

[12] O. Bertolami and J. Páramos, Phys. Rev. D 72, 04400 (2005).

[13] A. Övgün, K. Jusufi and . Sakall, Phys. Rev. D 99, no. 2, 024042 (2019).

[14] M. Kalb and P. Ramond, Phys. Rev. D 9, 2273(1974).

[15] Brett Altschul, Quentin G. Bailey, and V. Alan Kostelecký Phys. Rev. D 81, 065028 (2010).

[16] R. V. Maluf, A. A. Arajo Filho, W. T. Cruz and C. A. S. Almeida, EPL 124, no. 6, 61001 (2018).

[17] W. F. Kao, W. B. Dai, S. Y. Wang, T. K. Chyi and S. Y. Lin, Phys. Rev. D 53, 2244 (1996).

[18] K. Srinivasan, and T. Padmanabhan, Phys. Rev. D 60, 024007 (1999).

[19] S. Kanzi and . Sakall, Nucl. Phys. B 946, 114703 (2019).

[20] R. V. Maluf and J. C. S. Neves, Phys. Rev. D 97, no. 10, 104015 (2018).

[21] N. P. Pitjev and E. V. Pitjeva, Astron. Lett. 39, 141 (2013); Mon. Not. R. Astron. Soc. 432, 3431 (2013).