Counting Topological Windings of Gauge Fields with Chiral Magnetic Effect

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Gauge fields provide the fundamental interactions in the Standard Model of particle physics. Gauge field configurations with nontrivial topological windings are known to play crucial roles in many important phenomena, from matter-anti-matter asymmetry of today’s universe to the permanent quark confinement. Their presence is however elusive for direct detection in experiments. Here we show that measurements of the chiral magnetic effect (CME) in heavy ion collisions can be used for counting the topological windings of the non-Abelian gauge fields in the Quantum Chromodynamics (QCD). To achieve this, we implemented a key ingredient, the stochastic dynamics of gauge field topological fluctuations, into a state-of-the-art framework for simulating the CME in these collisions. This tool has allowed us to quantitatively extract, for the first time, the initial topological windings $Q_w$ from the CME experimental data, revealing a universal scaling relation between $Q_w$ and the particle multiplicity produced in the corresponding collision events.

INTRODUCTION

The fundamental structures and interactions of all visible matter in our universe are well described by the Standard Model of particle physics together with gravitation. In the Standard Model, gauge fields arising from underlying gauge symmetry principles provide the strong, weak and electromagnetic forces that hold the physical world together. A fascinating aspect of gauge fields is related to gauge field configurations with nontrivial topological windings. They emerge in various physical systems of different dimensions and play crucial roles in many important phenomena. Well-known examples include magnetic vortices in superconductors, monopoles in the electron weak theory, as well as instantons and sphalerons in non-Abelian gauge theory. See reviews in e.g.

Let us focus on the instantons and sphalerons of non-Abelian gauge theories in four-dimensional space-time [5, 6]. These topological configurations constitute crucial ingredients for our very existence. On the very small scale of subatomic dynamics, they lead to the spontaneous chiral symmetry breaking and the quark confinement [2, 3, 7] and thus enable the making of the hadron and nuclear world that we know. On the very large scale of cosmic evolution, they allow the violation of baryon number conservation and thus possible emergence of the large matter-anti-matter asymmetry in today’s universe [8, 9]. The essence of such topological configurations is the tunneling transitions across energy barriers between the topologically distinct vacuum sectors of a non-Abelian gauge theory characterized by different Chern-Simons numbers. In doing so, they themselves “twist” topologically around spacetime boundary and can be characterized by their topological winding numbers, defined as:

$$ Q_w = \int d^4 x q(x) = \int d^4 x \left[ -\frac{g^2 \epsilon^{\mu\nu\rho\sigma}}{32\pi^2} \text{Tr} \{ G_{\mu\nu} G_{\rho\sigma} \} \right] $$(1)

where the integrand $q(x)$ is the local topological charge density of a given gauge field configuration described by gauge field strength tensor $G_{\mu\nu}(x)$ with $g$ being the corresponding coupling constant.

Despite their significance, the topological configurations are elusive experimentally. A direct detection of their presence and consequences in laboratories could substantially advance our understanding of the underlying tunneling mechanism that is at the heart of quark confinement and baryon asymmetry, but has not been achieved so far. A concrete proposal [10–13] toward this goal is to look for the parity-odd “bubbles” (i.e. local domains) arising from the topological transitions of gluon fields in Quantum Chromodynamics (QCD). Specifically, these bubbles could occur in the hot quark-gluon plasma (QGP) created by relativistic heavy ion collisions. The parity-odd nature of such a bubble can be quantified by the macroscopic chirality $N_5$ generated for the light quarks in the plasma. Indeed, this is enforced by the famous chiral anomaly relation for massless fermions (i.e. the light flavor quarks in the case of QCD):

$$ \partial_{\mu} J_{\mu}^5 = 2q(x) = -\frac{g^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \{ G_{\mu\nu} G_{\rho\sigma} \} $$(2)

$$ N_5 \equiv N_R - N_L = 2Q_w. $$ (3)
In the above $J^5_{\mu}$ is the local chiral or axial current for each quark flavor while the Eq. (3) is the spacetime-integrated version of Eq. (2), with $N_R$ and $N_L$ being the number of right-handed (RH) and left-handed (LH) quarks. This latter equation has its deep mathematical root in the celebrated Atiyah-Singer index theorem and physically means that each topological winding generates two units of net chirality per flavor of light quarks. Therefore, measuring the net chirality of QGP provides a unique way of counting the topological windings of QCD gluon fields in heavy ion collision experiments.

The quark net chirality, however, is also challenging to detect due to the fact that the QGP born from collisions would expand, cool down and eventually transition into a low temperature hadron phase where the spontaneous breaking of chiral symmetry makes the net chirality unobservable. Fortunately, there is a way out by virtue of the so-called chiral magnetic effect (CME) [14, 15]. The CME is an anomalous transport phenomenon where an electric current $J$ is induced along an external magnetic field $B$ under the presence of net chirality in a system with massless fermions of charge $Q_e$:

$$J = \frac{Q^2}{2\pi^2} \mu_5 B.$$  \hspace{1cm} (4)

The $\mu_5$ is a chiral chemical potential that quantifies the net chirality $N_5$. The study of CME has attracted significant interests and activities from a broad range of physics disciplines such as high energy physics, condensed matter physics, astrophysics, cold atomic gases, etc. See recent reviews in e.g. [16–20]. In the context of heavy ion collisions, the CME current (4) leads to a charge separation in the quark-gluon plasma that results in a specific hadron emission pattern and can be measured via charge-dependent azimuthal correlations [13]. Extensive experimental efforts have been carried out over the past decade to look for its signatures at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC): see more details in recent reviews [21–23].

In short, there is a promising pathway for experimental probe of gauge field topology in heavy ion collisions: the winding number $Q_w$ of gluon fields $\Rightarrow$ net charity of quarks $\Rightarrow$ CME current $\Rightarrow$ correlation observables. Here we demonstrate, for the first time, how this strategy actually works quantitatively for counting the topological winding numbers of gauge field with measurements of the CME. (To be precise, the $Q_w$ fluctuates from event to event with equal chance of being positive or negative and what one can hope for is to determine its average variance $\sqrt{\langle Q_{w \text{event}}^2 \rangle}$ which we would refer as $Q_w$ in the rest of this paper for simplicity.) This has become possible due to two developments. Experimentally, great progress has been made in data analysis methods to separate background contamination and extract the CME signal with much reduced uncertainty [22, 23]. Theoretically, we have developed a new framework for accurately computing the CME transport while taking into account the stochastic dynamics of gluon field topological fluctuations during the evolution in these collisions.

The various ingredients and flow chart of our framework is illustrated in Fig. 1. This is built upon a state-of-the-art modeling tool, the anomalous-viscous fluid dynamics (AVFD) [24–26], which provides the quantitative link by simulating the CME transport during the dynamical evolution of a heavy ion collision from initial topological winding $Q_w$ to final experimental signal. However, random topological fluctuations occur both at the very beginning of a collision and during the course of its evolution. They result in flipping of chirality and impose important impact on the CME transport, which was previously neglected. To precisely count the initial topological windings, one must account for such missing key ingredient. In this work we’ve successfully implemented both the event-by-event initial topological fluctuations and the stochastic dynamics of gluon field topological fluctuations into the AVFD simulations (as indicated by the green blocks in Fig. 1), thus paving the way for counting $Q_w$ with CME measurements. See Method section for a more detailed discussion about this framework.

![FIG. 1. An illustration of our framework: anomalous-viscous fluid dynamics (AVFD) with stochastic dynamics of gauge field topological fluctuations. See Method section for details.](image)

**METHOD**

Here we present details of the various components (as illustrated in Fig. 1) for the computational framework developed for this study.

Heavy ion collisions at RHIC and LHC energies are well described by relativistic viscous hydrodynamic simulations, which have been thoroughly vetted with extensive experimental data. The “backbone” of our framework for describing such bulk evolution is based on the MUSIC (2+1)D code package [27, 28] with initial time $\tau_0 = 0.4fm/c$ and shear viscosity parameter $\eta/s = 0.08$. The event-wise initial conditions (e.g. for entropy density profiles) of the bulk hydro are generated with Monte-Carlo Glauber simulations.

The anomalous-viscous fluid dynamics (AVFD) [24–26] is the key component for implementing the dynamical CME transport in the realistic environment of a rel-
ativistically expanding viscous fluid. This state-of-the-art tool numerically solves the anomalous hydrodynamic equations for the coupled evolution of quarks’ vector and axial currents together on top of the bulk collective flow. Starting from a given axial charge initial condition, the magnetic-field-induced CME currents eventually lead to a charge separation effect in the fireball, which turns into a dipole term \(2\zeta_1 \sin(\phi - \Psi_{RP})\) in the azimuthal angle distribution of positive/negative charged hadrons with respect to the reaction plane orientation \(\Psi_{RP}\). Such a dipole signal is experimentally measurable through a difference between same-sign (SS) and opposite-sign (OS) charged hadron pair correlations, \(H^{SS-OS} = \langle 2n^2 \rangle\). In short, the AVFD is a hydrodynamic realization of CME for quantifying its signal in heavy ion collisions. For more details, see refs. [24–26]. It may be noted that the small but finite mass of the topological charge \(Q\) is the temperature and \(\chi\) is the temperature of the medium along the line of studies in e.g. [29, 30]. The influence of this factor on CME signal is thoroughly investigated in [25]. We follow [25] to use a lifetime of 0.6 fm which is reasonable and supported by phenomenological analysis [31].

For the goal of this work, one crucial new ingredient has been introduced into our framework. During the hydrodynamic evolution, there exist random topological fluctuations of the gluon fields that would necessarily influence the axial current evolution. These fluctuations eventually amount to a relaxation effect toward equilibrium with vanishing topological charge on long time scale. To account for such effect, one needs to introduce the resulting relaxation term into the anomalous hydrodynamic equation for the axial current [32–35]:

\[
\partial_\mu J_{\mu,5} = -\frac{N_c Q^2_i}{2\pi^2} E \cdot B - \frac{n_i J_{\mu,5}}{\tau_{cs}}.\tag{5}
\]

In the above, \(J_{\mu,5}\) is the axial current of each quark flavor with electric charge \(Q_i\) and color number \(N_c = 3\), while \(n_i J_{\mu,5}\) is the corresponding axial charge density. The \(E \cdot B\) in Eq.(5) comes from the Abelian anomaly due to electromagnetic fields. The stochastic dynamics of gluon topological charge density \(q(x)\) gives rise to a new contribution i.e. the second term in Eq.(5), where the \(\tau_{cs}\) is an important relaxation time for the topological charge fluctuations. Its physical meaning is simple: over this time scale, the \(q\) approaches equilibrium value. The \(\tau_{cs}\) is controlled by the Chern–Simons diffusion rate \(\Gamma_{cs}\), i.e.

\[
\tau_{cs} = \frac{\chi T}{2N_c^2 \Gamma_{cs}}.\tag{6}
\]

In the above \(\chi\) is the total quark number susceptibility, \(T\) is the temperature and \(N_c = 3\) is the light flavor number. For \(\Gamma_{cs}\) we use the result \(\Gamma_{cs} \approx 30(\alpha_s T)^4\) (with \(\alpha_s = 0.3\)) from [36]. It may be noted that the small but finite mass for the light flavor quarks (i.e. u, d, and s quarks) would also contribute to the diffusion rate. However recent analysis [37] has shown convincingly that such mass contribution, even for the strange quarks with \(\sim 100\text{MeV}\) mass, is smaller than the above Chern–Simons diffusion rate by a few orders of magnitude and thus negligible.

Last but not least, it is important to properly generate the initial conditions for the axial charge density based on the initial topological charge density of the gluon fields. At the very early stage of high energy heavy ion collisions, many flux tubes of strong chromo electric and magnetic fields in parallel or anti-parallel configurations are formed. In such a glasma picture [38–41], these flux tubes extend along the collision beam axis and are localized on the transverse plane. The chromo field strength inside the tubes is on the order of \(Q_s^2\) while their transverse size is on the order of \(1/Q_s\), with \(Q_s\) being the saturation scale. Depending on whether the chromo electric and magnetic fields are in parallel or anti-parallel configurations, each flux tube possesses randomly positive or negative topological charge density. They seed the generation of initial axial charge density \(n_{5,i}\) in the collisions [11, 42, 43]. We develop the following procedure to sample the initial axial charge density. For each collision event, we randomly sample a total of \(N_{coll}\) glasma flux tubes on the transverse plane, where \(N_{coll}\) is the binary collision number for each event generated from Monte Carlo Glauber simulations. For the \(i\)-th tube located at position \(x_i\), the chromo fields generate the following local axial charge upon integrating the Eq.(2) up to the hydro initial proper time \(\tau_0:\)

\[
n_{5,i}(\tau_0, x, y) = (\pm) \cdot \lambda Q_s^2 8\pi^2 \Gamma_{cs} \int \frac{1}{2s^2} e^{-\left(\frac{x-x_i^2}{2s^2}\right)}\tag{7}
\]

where \(x, y\) are transverse spatial coordinates. In the above, the plus or minus sign is randomly sampled for each tube. For our computations we use an average value of \(Q_s^2 = 1.5\ \text{GeV}^2\) in the above for collisions at RHIC energy [38–41]. The Gaussian factor in the bracket represents a smearing of the generated axial charge over a spread size of \(\sigma\). In our calculations we vary \(\sigma\) in a reasonable range between \((0.5 \sim 1)\)fm as an estimator of systematic uncertainty due to the initial condition. The \(\lambda\) is a dimensionless strength parameter reflecting the fact that we only know these chromo fields up to the order of magnitude. Clearly \(\lambda\) controls the amount of initial topological windings and consequently the axial charges. As a last step, we superpose all the flux tubes in a given event together to obtain the overall axial charge initial profile: \(n_5(\tau_0) = \sum_{i=1}^{N_{coll}} n_{5,i}\). This can then be used as the initial condition for solving previously discussed evolution equation for the axial current in our framework. Finally for each initial profile, one can integrate the \(n_5\) over the fireball to obtain the total axial charge \(N_5\) as well as the initial topological winding number \(Q_w\).

Procedurally, one varies the value of control parameter \(\lambda\) and perform event-by-event simulations to generate the CME signal. This establishes a mapping between the initial topological winding number \(Q_w\) and the final CME signal \(H^{SS-OS}\). Then by comparison with experimentally extracted value for \(H^{SS-OS}\), one can deter-
mines the corresponding $Q_w$. It may be worth mentioning that the correlation measurements typically contain both CME signal and background contamination. Methods have been developed and demonstrated to be able to extract the signal part [44–51].

**RESULTS**

Using the above framework, we first show results for the AuAu collisions at 200GeV beam energy and (20 ∼ 50)% centrality in Fig. 2, where the dependence of CME-induced correlation signal $H^{SS−OS}$ on the initial topological winding number $Q_w$ is shown. The blue circles (along with statistical uncertainty bars and systematic uncertainty boxes) are obtained from numerical simulations by varying the control parameter $\lambda$ for initial chirality generation (— see Method section for details). A theoretically-expected quadratic dependence is clearly observed, with the dashed blue curve showing an excellent fitting: $H^{SS−OS} = 1.038 \times 10^{-9} Q_w^2$. The star symbol and grey band show the current experimental constraints from analysis of the STAR collaboration data [22, 23, 52, 53]. Comparison between calculations and measurements suggest an optimal value of $Q_w = 119$ (as indicated by the red dot), providing the first such counting.

Using the $Q_w$ determination for AuAu collisions at (20 ∼ 50)% centrality above as the benchmark, we next extend the calculations to all centrality classes as well as to other colliding systems at RHIC 200GeV beam energy, including the CuCu collisions as well as the isobaric RuRu and ZrZr collisions [54, 55]. The results are shown in Fig. 3. As these systems have very different size and multiplicity, the initial $Q_w$ is normalized by the corresponding system’s initial total entropy $S$ and plotted versus centrality for comparison across systems. We find that the normalized ratio $Q_w/S$ increases from central to peripheral collisions and also increases from larger to smaller colliding systems. This behavior implies that when the fireball created in the collisions becomes smaller (due to either centrality or nucleus size), the $Q_w$ decreases less sensitively than the entropy $S$. Our finding helps making theoretical predictions for the decisive isobar (RuRu and ZrZr) collision experiment [24] by suggesting a $Q_w/S$ ratio that is larger in isobar systems by about 20% as compared with that in AuAu collisions.

![FIG. 2. The dependence of CME-induced correlation signal $H^{SS−OS}$ on the initial topological winding number $Q_w$. The blue circles are from numerical simulations, along with statistical error bars and systematic error boxes. The dashed blue curve shows a quadratic fitting with $H^{SS−OS} = 1.038 \times 10^{-9} Q_w^2$. The star symbol and grey band show the current experimental constraints from analysis of the STAR collaboration data. The red dot at $Q_w = 119$ indicates agreement between the fitting curve and the experimental value.](image1)

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![FIG. 3. The initial topological winding number $Q_w$ normalized by the corresponding system’s initial total entropy $S$ is shown versus different centrality for four colliding systems at RHIC 200GeV beam energy: AuAu (red circle), RuRu (green square), ZrZr (orange cross) as well as CuCu (blue square).](image2)

**FIG. 3.** The initial topological winding number $Q_w$ normalized by the corresponding system’s initial total entropy $S$ is shown versus different centrality for four colliding systems at RHIC 200GeV beam energy: AuAu (red circle), RuRu (green square), ZrZr (orange cross) as well as CuCu (blue square).

Physically, the $Q_w$ is correlated with initial seeds of local gluon field flux tubes, the number of which roughly scales with the binary collision number $N_{binary}$. The $N_{binary}$ is not directly measurable but is closely correlated with measurable charged hadron multiplicity $N_{ch}$. To gain further insight into the counting of $Q_w$, we plot the dependence of $Q_w$ on $N_{ch}$ with all centrality and all colliding systems from our computations in Fig. 4. Remarkably all points demonstrate a universal behavior as is visible from the fitting curve (the black straight line on the log-log scale). A fitting analysis reveals the following universal scaling relation:

$$Q_w = \alpha \frac{N_{ch}^\beta}{S} , \quad \alpha = 2.56 \pm 0.17 \quad , \quad \beta = 0.648 \pm 0.013 .$$
relativistic heavy ion collisions. This link is established based on the newly developed framework that has implemented a key ingredient, the stochastic dynamics of gauge field topological fluctuations, into the EBE-AVFD simulations for CME transport in these collisions. By further applying this tool toward a variety of centrality class and colliding systems, a universal scaling relation between the initial gluon field topology $Q_w$ and the final charged hadron multiplicity $N_{ch}$ has been identified in Eq.(8), which can soon be tested with upcoming experimental analysis. With combined theoretical and experimental efforts in the near future, one can look forward to the exciting opportunity for precision detection of gauge field topological windings that would help extract the relevant topological transition rate and understand the mechanism underlying such phenomena as cosmic baryon asymmetry and permanent quark confinement.

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**SUMMARY AND DISCUSSION**

In summary this work reports the first quantitative determination of the topological winding number $Q_w$ of non-Abelian gauge fields (the QCD gluon fields) with experimental measurements of the chiral magnetic effect in......

**FIG. 4.** The initial topological winding number $Q_w$ is shown versus the charged hadron multiplicity $N_{ch}$ (on log-log scale) for various centrality and different colliding systems: AuAu (red circle), RuRu (green cross), ZrZr (orange cross) as well as CuCu (blue square). The black line with grey band demonstrates a universal power-law fitting $Q_w = \alpha N_{ch}^{\beta}$ with $\alpha = 2.56 \pm 0.17$ and $\beta = 0.648 \pm 0.013$.

It would be exciting to test this novel finding with future experimental analysis that would extract CME signals with further improved accuracy for existing AuAu and CuCu measurements as well as upcoming isobar data.
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