Tensors and n-d Arrays: A Mathematics of Arrays (MoA), Psi Calculus and the Composition of Tensor and Array Operations

Our presentation will discuss the outer product/tensor product and a special case of the tensor product, the Kronecker Product: the algorithms, their origin, and optimal implementation when composed, and mapped to complex processor/memory hierarchies. We discuss how the use of MoA and the Psi Calculus, a calculus of indexing with shapes, provides optimal, verifiable, reproducible, scalable, and portable implementations of both hardware and software. This is due to the fact that we are using normal forms composed of multi-linear operations on Cartesian coordinates which are transformed into simple abstract machines: starts, stops, strides, count, up and down the processor/memory hierarchy.