Non-perturbative electron dynamics in crossed fields

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Intense AC electric fields on semiconductor structures have been studied in photon-assisted tunneling experiments with magnetic field applied either parallel ($B_{||}$) or perpendicular ($B_{\perp}$) to the interfaces. We examine here the electron dynamics in a double quantum well when intense AC electric fields $F$, and tilted magnetic fields are applied simultaneously. The problem is treated non-perturbatively by a time-dependent Hamiltonian in the effective mass approximation, and using a Floquet-Fourier formalism. For $B_{||} = 0$, the quasi-energy spectra show two types of crossings: those related to different Landau levels, and those associated to dynamic localization (DL), where the electron is confined to one of the wells, despite the non-negligible tunneling between wells. $B_{||}$ couples parallel and in-plane motions producing anti-crossings in the spectrum. However, since our approach is non-perturbative, we are able to explore the entire frequency range. For high frequencies $\omega$, we reproduce the well known results of perfect DL given by zeroes of a Bessel function. We find also that the system exhibits DL at the same values of the field $F$, even as $B_{||} \neq 0$, suggesting a hidden dynamical symmetry in the system which we identify with different parity operations. Symmetries under general parity operations explain many of the features in the spectra, and their overall behavior under magnetic field. The return times for the electron at various values of field exhibit interesting and complex behavior which is also studied in detail. We find that smaller $\omega$ shifts the DL points to lower $eF/d/\hbar\omega$ ratios, and more importantly, yields poorer (less effective) localization by the field, while other states change also physical character. We analyze the explicit time evolution of the system, monitoring the elapsed time to return to a given well for each Landau level, and find non-monotonic behavior for decreasing frequencies.

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I. INTRODUCTION

The dynamics of charged particles in semiconductor quantum well structures subject to a time-dependent external electric field has been a subject of intense research.\cite{1} The progress of techniques in nanoscale lithography and the development of the free-electron lasers (FEL) which can be continuously tuned in the terahertz (THz) range, have made possible the systematic study of effects only present in intense alternating fields in this domain. Examples of these effects are the coherent suppression of tunneling (or dynamic localization) despite interwell tunneling in the structure\cite{2}, the collapse of minibands in superlattices\cite{3} and absolute negative conductance\cite{4} and photon-assisted-tunneling (PAT) in resonant tunneling diodes\cite{5} the AC Stark Effect\cite{6} and many others. These effects have been predicted and partially experimentally verified\cite{7}.

Intense AC fields on semiconductor structures have been also studied in connection with photon-assisted tunneling experiments with magnetic field applied either parallel ($B_{||}$)\cite{8} or perpendicular ($B_{\perp}$)\cite{9} to the interfaces. Quantum wells in tilted magnetic fields are also of great current interest as experimental probes of the transition to quantum chaos in a mesoscopic system, in the presence of a static electric field. In this work, we study the combined influence of tilted magnetic field and strong AC fields and find quite a rich behavior for different parameters in the problem.

A double quantum well (DQW) can be treated to lowest approximation by considering only the first level in each well, resulting in a two level system with a splitting $\Delta$ due to the inter-well tunneling. This two-level dynamics, moreover, plays an important role in understanding the behavior of more elaborate dynamical systems, which explains why this subject has been so intensely studied (for a comprehensive review see Ref.\cite{10}).

In this paper we address the problem of a double quantum well in simultaneous tilted magnetic field and an intense AC drive in a non-perturbative approach. To do so we make use of the Floquet-Fourier formalism developed by Shirley.\cite{11} For strong oscillating fields, the approach based on the Floquet theory,\cite{12} has proved most useful in treating the dynamics of such systems, although some analytical solutions have been proposed.\cite{13} Shirley’s approach is very convenient since a time dependent problem can be mapped onto a time-independent infinite matrix eigenvalue problem (eventually truncated at the desired accuracy), which yields a series of quasieigenvalues describing the time evolution of the system. This formalism is also useful as the Floquet states possess either odd or even dynamical symmetry. As a result, to explore the effects of magnetic fields, we use a generalized parity operator and its eigenfunctions to provide understanding about crossings in the quasienergy spectra.
We are specially interested in the dynamics of the system in the presence of the intense AC electric field and how it changes with the applied tilted magnetic field. Unlike most of the previous studies of AC-driven systems, where the main attention was paid to the high frequency regime, our approach includes the full range of frequency values (within the single quantum well level approximation).

We begin in section II with a complete description of the model used, discussing some important properties like the parity of the system. We find in fact that this system can be understood using two kinds of parity operators, one when there is no parallel magnetic field applied (S_p) and a full parity operator in the other case (S_p). Based in the two level approximation, we also develop an analytical description that yields the degeneracy in the quasienergy for either the high frequency (Δ/ℏω ≪ 1) or weak electric field (eFd/ℏω ≪ 1) regimes, and it is in complete agreement with our numerical calculations.

In section III we make a systematic study of the system exploring both the quasienergy spectra and the explicit time evolution. We find that in the absence of a parallel magnetic field, as the Landau levels are conserved, the problem can be understood in terms of two level systems for each Landau level. When the parallel magnetic field is turned on, however, the crossing levels which originated from the Landau level conservation (for B∥ = 0), start mixing and develop anticrossings. Thus, the symmetry changes from S_p → S_p due to B∥, lending a transition from crossing to anticrossing of levels as the AC field amplitude is varied. For all fields, however, we find a number of crossings, normally associated to dynamic localization, which show no crossing-anticrossing transition and little change in dynamics when B∥ is turned on. We have also performed a complete study of the change in the field condition for dynamic localization, as function of the ratio Δ/ℏω. We find a monotonic drop in the field values needed to achieve localization, as the ratio Δ/ℏω increases (smaller frequency), with successive low field DL points disappearing as ω decreases. We are able to provide an empirical equation that fits surprisingly well the numerical results, although an analytical description valid in the different limits is also presented. An analysis of the ‘quality’ of DL for various AC fields is provided by a study of the minima in the probability of a particle initially in the left well to remain there. This result shows a surprising and non-understood behavior. As expected for the high frequency limit, the dynamic localization is very well defined (fully localizing the particle in a well) and at fields given by the zeros of the Bessel function. However, for low frequency, the dynamical localization becomes poorer, although the particle never fully leaves the well, and presents small ‘revivals’ for the first and third dynamic localization points. This behavior is not intuitive and not reported previously. The full range of complex behavior is however experimentally relevant, as we discuss in section IIV.

II. THEORY

The Hamiltonian for an electron in a double quantum well structure under a tilted magnetic field and a strong AC field can be written as

\[ H = H_0 + eFz \cos(\omega t), \]

where \( H_0 = (p^2 + m^* \omega_\perp^2 x'^2 + x'^2 + V_0 \]

\[ + \frac{e^2 \omega_\perp^2}{2m^*} x'^2 - \frac{e^2 \omega_\perp^2}{m^*} x x', \]

where \( \omega_\perp = eB_\perp/m^* \) and \( x' = x + k_y/eB_\perp \), with \( k_y \) conserved throughout. In the absence of \( B_\parallel \), and allowing for only one state in each well of the DQW, Eq. (3) is exactly soluble and its eigenenergies are \( \hbar \omega_\perp (n + 1/2) \pm \Delta/2 \) (harmonic oscillator in plane with the symmetrical and antisymmetrical solutions of the DQW), where \( \Delta \) is the splitting due to the tunneling. Inclusion of additional levels in each quantum well is straightforward, and it does not add to the qualitative behavior and conclusions described here (except for possible multi-photon resonances at high field, the AC Stark effect). Moreover, thin wells will likely have others levels at much higher energies, and our treatment here will be quantitatively accurate.

The dynamics of the system is governed by the time-dependent Schrödinger equation

\[ i\hbar \frac{\partial \psi}{\partial t} = H|\psi\rangle. \]

Since \( H \) is periodic in time (\( H(t) = H(t + \tau) \), where \( \tau = 2\pi/\omega \) is the period) we can make use of the standard Floquet theory and write the eigenfunction \( |\psi\rangle \) as

\[ |\psi\rangle = \exp(-i\epsilon t/\hbar)|u\rangle, \]

which allows us to rewrite Eq. (3) as

\[ (H - i\hbar \partial_t)|u\rangle = \epsilon|u\rangle, \]

where \( |u(t)\rangle = |u(t + \tau)\rangle \) is also periodic in time, with the same period \( \tau \), and \( \epsilon \) is a real-valued parameter termed the Floquet characteristic exponent, or the quasienergy. The term quasienergy reflects the formal analogy with the quasi-momentum, \( k \), characterizing Bloch eigenstates in a periodic solid. Following this analogy we can see that \( |u(m)\rangle = \exp(im\omega t)|u\rangle \) is also solution of Eq. (3), with eigenvalues \( \epsilon_m = \epsilon + m\hbar\omega \), if \( m \) is an integer number \( m = 0, \pm 1, \pm 2, \ldots \). Then, by a subtraction of a suitable integral multiple of \( \hbar\omega \), the
quasienergy $\varepsilon$ can be mapped into a first ‘Brillouin zone’ obeying: $\hbar \omega/2 \leq \varepsilon \leq \hbar \omega/2$. For vanishing intensity of the AC field $F$, the Floquet states are connected with the stationary states $|\alpha(0)\rangle$ of $H_0$ by $|u(l)\rangle = \exp(il\omega t)|\alpha(0)\rangle$ and the quasienergies with the unperturbed states by $\varepsilon_l = E_\alpha^{(0)} + il\hbar \omega$, where the index $l$ counts how many AC energy quanta have to be subtracted from the unperturbed energy $E_\alpha^{(0)}$ in order to arrive at the first Brillouin zone (the ‘photon index’).

Following the procedure developed by Shirley, we solve Eq. (3) by expanding the function $|u\rangle$ in a Fourier series for the temporal component, and linear combinations of the $B_{\|}=0$ solutions of $H_0$ for the spatial component,

$$|u\rangle = \sum_{s,n,m} C_{s,n,m}|s,n,m\rangle,$$  

where $|s\rangle$ refers to the symmetrical ($|S\rangle$) and antisymmetrical ($|A\rangle$) eigenfunctions of the DQW, $|n\rangle$ is the Landau level function, and $|m\rangle$ is a Fourier expansion ‘state’ ($|t(m)\rangle = \exp(\im \omega t n)$). This leads to the time-independent infinite matrix eigenvalue equation

$$\sum_{s,n,m} \left\{ E_s + \hbar \omega \frac{1}{2} \right\} \delta_{s,s'} \delta_{n,n'} \delta_{m,m'} + \left[ \frac{e^2 B_z^2}{2m^*} \right] \frac{z_{s',s} z_{n',n}}{m^*} \delta_{m,m'} + \frac{e F z_{s',s} + z_{n',n}}{2} \left[ \delta_{m,m'}^{\delta_{m,m'}-1} + \delta_{m,m'} \right] \delta_{n,n'} C_{s,n,m} = 0,$$

where $z_{s',s} = \langle s|z|^s\rangle$, $z_{n',n} = \langle s|z|^n\rangle$, $x_{n,n'} = \langle n|x'|n'\rangle$, and $E_s = \pm \Delta/2$.

### A. Floquet states and parity

The quasienergies in Eq. (8) can be regarded as the eigenvalues of a stationary problem analogous to a time-independent Schrödinger equation, with $|u\rangle$ playing the role of stationary states of the operator $\hat{\mathcal{H}} = \hat{H} - \im \hbar \partial_t$. As a consequence of $[\hat{\mathcal{H}}, S_p] = 0$, where $S_p$ is the parity operator defined by $S_p : (x \rightarrow -x', z \rightarrow -z, t \rightarrow t + \pi/2)$, the Floquet states $|u\rangle$ have even or odd parity under $S_p$. Since our expansion basis (Eq. (4)) has also a definite parity under $S_p$, the matrix (8) can be separated into two blocks for subspaces of different symmetries, one for $s+n+m = \text{even}$ and another for $s+n+m = \text{odd}$. This result is very useful when interpreting the quasienergy spectrum, which contains multiple crossings (that come from different symmetries), as well as avoided crossings associated with mixing between same-parity states.

It is important to note also that for vanishing $B_{\|}$, the parallel and perpendicular motions are completely decoupled, and in this case the problem reduces to solving a DQW system under intense AC field for each in-plane Landau level separately. In this $B_{\perp}$-only case, the parity operator can be defined as $S_p'^n : (z \rightarrow -z, t \rightarrow t + \pi/2)$, and as $[\hat{\mathcal{H}}, S_p'^n] = 0$, this yields separate parities under the condition $s+m = \text{even}$ or $s+m = \text{odd}$.

### B. Evolution operator and degeneracies

In the Floquet representation, the time-evolution operator $U(t,t_0)$, defined by $|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$, with $U(t_0,t_0) = I$, where $I$ denotes the identity operator, can be expressed as

$$U(t,t_0) = \sum_i |u_i(t)\rangle\langle u_i(t_0)| \exp(-\im \varepsilon_i (t - t_0)/\hbar),$$

as it is easily checked with the help of Eq. (5). A full period $U(\tau,0)$ operator is all we need to construct a discrete quantum map, propagating an initial state over multiple periods of the fundamental period. An evolution operator formulation allows one to explore the condition of quasienergy degeneracy, which we will see is an important necessary condition for dynamic localization (although not sufficient).

As mentioned above, without the in-plane magnetic field, the problem reduces to a two-level system for each Landau level driven by an external time-dependent electric field, which has been extensively studied in the literature (for a review, see Ref. 13). Although this system looks simple, the physics is in rich, and a complete analytical solution is either lacking or too complicated to use in the different parameter regimes. An analytical extension to the solution in the high frequency limit was recently found providing a perturbative connection to this case. Here we give an analytical solution applicable in both the high frequency and weak field limits. For simplicity, let us consider the system in the absence of a magnetic field, so that the Hamiltonian can be written in a two level basis as

$$H = H_1 + H_2 = \frac{\Delta}{2} \sigma_z + e F d/2 \sigma_x \cos(\omega t),$$

where $\sigma_z$, $\sigma_x$ are the usual Pauli matrices.

The time evolution operator $U$ satisfies the Schrödinger equation in the operator form

$$i\hbar \frac{dU}{dt} = HU.$$  

The difficulty in solving the problem lies in the non-commutation between $H_1$ and $H_2$. For only $H_1$ or $H_2$, the solution is $U_1 = \exp(-i\Delta \sigma_z t/2\hbar)$, or $U_2 = \exp(-i\frac{F d}{2} \sigma_x \cos(\omega t) dt/2\hbar)$, respectively. We write the time evolution operator in the form $U = U_0 U'$, with $U'$ satisfying the equation

$$i\hbar \frac{dU'}{dt} = H_{\text{eff}} U',$$

where $H_{\text{eff}} = U_0^{-1} H_2 U_0 - H_2$. In the high frequency limit ($\Delta/\hbar \omega \ll 1$) or weak electric field regime ($e F d/\hbar \omega \ll 1$),
we can write
\[ U' = \exp \left( -\frac{i}{\hbar} \int_0^t H_{\text{eff}}(t')dt' \right). \] (12)
By diagonalizing \( U(\tau) \), we can obtain the quasienergies. After some tedious but straightforward algebra, we find that the condition for degeneracy of quasienergy levels, \( (\Delta_{\text{eff}}/\hbar \omega = 0 \mod (2\pi)) \) is given by the condition
\[
\begin{align*}
\Delta \hbar/\omega \int_0^{2\pi} \cos \left( \frac{\Delta}{\hbar \omega} t' \right) \cos \left( \frac{eFd}{\hbar \omega} \sin(t') \right) dt' \\
+ \frac{\Delta}{\hbar \omega} 2\pi - \sin \left( \frac{\Delta}{\hbar \omega} 2\pi \right) = 0 \pmod{2\pi}.
\end{align*}
\] (13)
Since the energy splitting \( \Delta_{\text{eff}} \) is due to tunneling and level mixing by the AC field, this quasienergy degeneracy means a suppression of tunneling, or in other words, this level mixing by the AC field, this quasienergy degeneracy (\( \Delta_{\text{eff}}/\hbar \omega \approx 0 \)) yields the well-known condition for the dynamic localization to occur. It means a suppression of tunneling, or in other words, this quasienergy degeneracy (\( \Delta_{\text{eff}}/\hbar \omega \approx 0 \mod (2\pi) \)) is given by the condition
\[
\begin{align*}
\Delta \hbar/\omega \int_0^{2\pi} \cos \left( \frac{\Delta}{\hbar \omega} t' \right) \cos \left( \frac{eFd}{\hbar \omega} \sin(t') \right) dt' \\
+ \frac{\Delta}{\hbar \omega} 2\pi - \sin \left( \frac{\Delta}{\hbar \omega} 2\pi \right) = 0 \pmod{2\pi}.
\end{align*}
\] (13)

Since the energy splitting \( \Delta_{\text{eff}} \) is due to tunneling and level mixing by the AC field, this quasienergy degeneracy means a suppression of tunneling, or in other words, this condition for the dynamic localization to occur. It is easy to see that in the high frequency limit, Eq. (13) yields the well-known condition \( \Delta_{\text{eff}}/\hbar \omega = 0 \). In the weak field limit, one obtains the result \( \Delta_{\text{eff}}/\hbar \omega = n, n \) being an even integer number (for \( n \) odd there is only solution for \( F = 0 \) and does not lead to dynamic localization). As we will see later, these limiting degeneracy conditions are in fact realized in our numerical work, and allow one to understand the physics of dynamic localization.

### III. RESULTS AND DISCUSSIONS

In order to perform numerical calculations, we choose a symmetric double quantum well (50 - 40 - 50 Å GaAs/Al0.3Ga0.7As/GaAs) with effective electron mass \( m^* = 0.067 m_0 \) (\( m_0 \) is the electron mass), \( d = 2(S|z|A) \approx 90 \) Å is the mean separation between the left and right wells, and \( \Delta = 8.87 \) meV. We have performed a systematic study varying the external parameters (\( B_{\parallel}, B_{\perp}, \omega \) and \( F \)) and analyze the results for different ranges of the ratio \( \Delta/\hbar \omega \). It is important to emphasize that our numerical model is valid for all values of the ratio \( \Delta/\hbar \omega \). However, as will become clear in the discussion below, the behavior of the system is markedly different for \( \Delta/\hbar \omega \lesssim 1 \) and for \( \Delta/\hbar \omega > 1 \). We first discuss the first regime, \( \Delta/\hbar \omega \lesssim 1 \), as it is perhaps more intuitive, to later look at lower frequencies.

#### A. High frequency, \( \Delta/\hbar \omega < 1 \)

We start our analysis by considering the quasienergy spectrum for two Landau levels and varying \( B_{\parallel} \). In Fig. (a) we show the first Brillouin zone of quasienergy in units of photon energy, for \( \Delta/\hbar \omega = 0.5 \) and \( B_{\perp} = 1.0 \) T. The labels \( E_{s,n,m} \) for each level, correspond to the \( z \)-symmetry (\( S \) or \( A \)), the Landau level index \( n \), and the ‘photon index’ \( m \), all at \( F = 0 \). For \( B_{\parallel} = 0 \) the parallel and perpendicular motions are completely decoupled and the Landau level number is conserved (only transitions with the same Landau level index are allowed). This explains the level crossings at points such as \( eFd/\hbar \omega \approx 1.96 \) and all others not explicitly labeled, according to \( S_p \) symmetry (that is, levels crossing have different symmetry under \( S_p \)). When we turn on the parallel magnetic field, however, the parallel and in-plane motions are coupled, so that the Landau level index is no longer a good quantum number. This produces anticrossings in the quasienergy spectrum at points such as \( eFd/\hbar \omega \approx 1.96 \), while still keeping crossings at the points denoted by \( l_i \) (localization points), according to their \( S_p \) symmetry. Here we see explicitly that \( B_{\parallel} \) produces a change of symmetry \( S_{p'} \rightarrow S_p \). Thus the system for \( eFd/\hbar \omega \approx 1.96 \) shows a transition from level crossing (the states are odd and even in \( S_{p'} \)) to level anticrossing (the states are both odd under \( S_p \)) due to \( B_{\parallel} \). It is easy to see that there is no such transition for \( l_i \) points (dynamic localization points), since at \( l_i \) states have opposite parity under both \( S_{p'} \) and \( S_p \) symmetry operations. This is the essential difference between the point \( eFd/\hbar \omega \approx 1.96 \) and \( l_i \).

In order to better understand the physical behavior of the system at the various crossings, we calculate the time evolution of the system for some particular points (points indicated by arrows in Fig. (a)), using Eqs. (3) and (4). Here, the system is prepared in the first Landau level of the left well at \( t_0 \), \( \psi(t_0) = |L,0) \), where \( |L,0 \rangle = |L \rangle \otimes |0 \rangle \) with \( |L \rangle = 1/\sqrt{2(|S| - |A|)} \), and \( |0 \rangle \) refers to the lowest Landau level for the in-plane motion. We investigate how the probability of finding the particle in some state \( |\beta) = |l,n) \) \( (l = L, R \) and \( n = 0, 1, 2, \ldots) \) changes with time. We also investigate the total probability of the system to remain in the left well
\[
P_L(t) = \sum_n |\langle L, n|\psi(t)\rangle|^2.
\] (14)
The probability for staying in the left well at the crossing/anticrossing point \( eFd/\hbar \omega \approx 1.96 \) can be seen in Fig. (b). We note a large change in its behavior with \( B_{\parallel} \). For \( B_{\parallel} = 0 \), the time evolution shows that the system can be found in both left and right wells with the same probability and with a rather simple periodic behavior. The particle is effectively tunneling back and forth with a period \( \approx 10\tau = 20\pi/\omega \), and with fast oscillations of period \( \tau \). For \( B_{\parallel} \neq 0 \) the probability presents a more complex behavior, with quasiperiodic oscillations. Notice in Fig. (b) that the system does not have short-term periods, and a long-time plot of \( P_L(t) \) in this case yields periods \( \approx 1000\tau \) and quasi-periodic motion at \( \approx 46\tau \). We should also point out that the detailed frequency content will become even richer if one includes higher Landau levels (ignored here for simplicity).
the AC field despite the interwell tunneling allowed in the structure. It is important to note that the parallel magnetic field has no effect on the position of this dynamic localization point. However, as can be seen in the right inset, there is a small probability for the system to be found in the second Landau level, $n = 1$. At this point, the system exhibits small transitions between Landau levels, but in the same well, so that the localization is basically invariant under the presence of the parallel magnetic field (although its return period to the $n = 0$ level is now longer, as seen in the right inset in (c)).

For the crossing point at $eFd/h\omega \simeq 2.33$, Fig. (b) shows similar results, except for the fact that the time evolution presents better dynamic localization (notic minima in $P_L \gtrsim 0.92$). We in fact find that in general, dynamic localization points are ‘better localized’ for higher $F$ values, resulting in higher minima for $P_L$ and/or weaker $P_R$ maxima, although there is a few exceptional examples, as we will see in Fig. (d).

Another approach to monitor the dynamic localization is to look for the oscillation period in $P_L$, defined naturally as the time for the electron with initial state in the left well to go to the right well (with amplitude of 100%) and fully back (with amplitude of 100%). In Fig. (b), for $B_{||} = 0$, it is easy to see that the oscillation period is about $10\tau$ as mentioned before. Figure (d) displays the return periods obtained from direct analysis of $P_L$ for different values of the AC field amplitude.
good, even with a tilted magnetic field, as we would expect from the different parity of the crossing states under $S_p$, and as we can see in Fig. 3 for $\Delta/h\omega = 0.1$. The dynamic localization regime here is also much better than at $\Delta/h\omega = 0.5$, as shown in the inset (with $P_{t_{\text{min}}} \gtrsim 0.98$ at $l_1$, $eFd/h\omega \approx 2.40$, for example). As we will see below, the degree (or ‘quality’) of dynamic localization decreases for large $\Delta/h\omega$, even as the quasienergy levels still cross.

B. Low frequency, $\Delta/h\omega > 1$

In Fig. 4 we show the quasienergy spectrum for $\Delta/h\omega = 2.2$. Up to now, all the spectra analyzed could have been described without the inclusion of the photon index, since it is zero in all cases. In the present case of large $\Delta/h\omega$, however, we need to be more careful, since the first Brillouin zone of quasienergies involves multiple ‘photon replicas’ and various levels could be involved in crossings for different photon index. To understand crossing and anticrossing we need to explore the full symmetry of the states under the parity operator $S_p$.

In Fig. 5 we choose some quasienergy crossing points (indicated by arrows in Fig. 4) and carry out the time evolution. For the point at $eFd/h\omega \approx 3.46$, the time evolution shows that it is a ‘normal’ point, namely with the particle fully jumping to the right well, so that the minimum $P_{t_{\text{min}}} = 0$. This behavior suggests that the states crossing have different symmetry which the tilted magnetic field seems not to affect. On the other hand, the crossing point $eFd/h\omega \approx 4.63$ ($l_2$) is similar to the dynamic localization points discussed before, in the sense that the probability of staying in the left well never reaches zero (although here the localization is somewhat poor, with minimum $P_{t_{\text{min}}} \approx 0.2$). More-
FIG. 4: First Brillouin zone of quasienergies (in units of $\hbar \omega$) as function of $eFd/\hbar \omega$ (intensity of AC field), for $\Delta/\hbar \omega = 0.9$ and $B_\perp = 1.0$ T. The $l_2$ label refers to a localization point and it presents no change with $B_\parallel$. Others crossing/anticrossing points are also indicated (see text). Notice labels $E_{s,n,m}$ include different photon index $m$. 

FIG. 5: Time evolution of the probability $P_l(t)$ of an electron prepared in the first Landau level in the left well, and to be found projected in the same well: a) for $eFd/\hbar \omega \simeq 3.46$; b) for $eFd/\hbar \omega \simeq 4.63$; c) for $eFd/\hbar \omega \simeq 5.54$. Only (b) exhibits dynamic localization, although incomplete.

over, the point $eFd/\hbar \omega \simeq 5.54$ is also a ‘normal’ case of crossing/anticrossing produced by the inclusion of parallel magnetic field, so that the probability of staying in the left well changes when we include $B_\parallel$, and presents complex time-dependent behavior. For any value of $B_\parallel$ however, the minimum $P_l \simeq 0$, and the particle fully tunnels back and forth between the wells.

It is important to point out that the crossing of levels at $eFd/\hbar \omega \simeq 3.46$, and similar others, is associated with their different parity under $S_p$. For example, at the point $\simeq 3.46$, the levels crossing are those with $E_{A,1,-2}$ and $E_{S,0,+1}$ at $F = 0$. These states have then even and odd parity under $S_p$, respectively (as given by $s + n + m$), which remains true for $F \neq 0$. On the other hand, the crossing (anticrossing) at $\simeq 5.54$, and others, correspond to Landau levels conservatism at $B_\parallel = 0$ (or mixing for $B_\parallel \neq 0$), and the associated parity under $S'_p$.

Another region of interest is $\Delta/\hbar \omega \simeq \text{odd}$ integer. In that case, different photon replicas are nearly degenerate but bring together states with the same symmetry under $S_p$ (or $S'_p$). The same parity allows mixing of the levels, producing an interesting anticrossing behavior for $F \simeq 0$, as shown in Fig. 4 for $\Delta/\hbar \omega = 0.9$ and $\Delta/\hbar \omega = 1.1$. There we see that in (a), the states labeled $s = S$ (or $A$) have an upward (downward) dispersion for $F \simeq 0$ (similar to all cases with $\Delta/\hbar \omega < 1$, such as Figs. 1 and 3). However, for $\Delta/\hbar \omega = 1.1$ in Fig. 4(b), the $S$ and $A$ states have an apparent exchange of behavior in the $F$ field, which is also exhibited for all $\Delta/\hbar \omega > 1$ (as in Fig. 4). This apparent switching is due to the mixing (and level repulsion) seen near $F \simeq 0$, and the fact that the labels are always obtained for the $F = 0$ limit, where the AC field is non-existent, and the spatial symmetry is well defined Notice, interestingly, that the related AC Stark effect in atoms and superlattice systems occurs for $\Delta > \hbar \omega$, but this effect is somewhat different.

C. Incomplete dynamic localization

Comparing the cases analyzed so far, one promptly notices that the ‘degree’ of dynamic location changes with the ratio $\Delta/\hbar \omega$. In the limit $\Delta/\hbar \omega \ll 1$, we saw that the $l_i$ values tend to the zeros of the Bessel function, and that the probability of staying indefinitely in the left well is approximately 1 at all times (full localization). For $\Delta/\hbar \omega = 0.5$, Fig. 4, the dynamic localization points occur for somewhat smaller values of the AC field, and the probability of staying fully in the left well is smaller (poorer localization). In fact, the particle is able to increasingly leak out to the right well for lower frequencies, even for strong AC field amplitudes.

A more complete analysis of this behavior is shown in Fig. 5. The full line in 5(a) shows how the dynamic localization points change with the ratio $\Delta/\hbar \omega$. We have numerically computed this function by looking for the crossing points in the first quasienergy Brillouin zone that produce localization behavior ($l_i$). An interesting result, as described above, is that the tilted magnetic field has no discernible effect on these points, so that this result is identically the same as for a simple two level problem. Notice that increasing the $\Delta/\hbar \omega$ ratio not only decreases the $F$ values needed for localization, but eventually suppress a given $l_j$, at even values $\Delta/\hbar \omega = 2j$, where the unperturbed photon replicas would have degeneracy points (see discussion after Eq. (13)). The odd ratios do not yield level crossing and localization, as the $S_p$ parity symmetry of nearby states is the same ($E_{S,0,0}$ and $E_{A,0,-1}$, for example, for $\Delta \geq \hbar \omega$), and states in fact mix (anticross) as we discuss above in reference to Fig. 4. We should emphasize that the high frequency results...
discussed in the literature and yielding dynamic localization at the zeroes of the Bessel functions, agree identically with our numerical results. Similarly, the weak field limit yields \( l_1 \) points at integer values of \( \Delta/h\omega \), as given by the degeneracy condition, Eq. (13). That equation, however, does yield an appropriate estimate of the \( \Delta/h\omega \) also changes the \( \omega \) of localisation at the zeroes of the Bessel function, \( J_0(x_1) = 0 \). This simple fit to the end points describes surprisingly well the curves over the whole range, as one can see in the dotted lines in Fig. 7(a). The fit is so good, that it is basically indistinguishable for the first localization point \( l_1 \), and it only departs noticeably for \( l_3 \) and \( l_4 \).

In addition to the shift in position of the localization points \( l_j \), and as noticed in the discussion before, we find that the value of \( \Delta/h\omega \) also changes the degree of localization of the particle. To further study this point, we have arrived at a quantitative measure of how good the localization is, by looking at the minimal probability of remaining in the left well during a periodic oscillation of \( P_L(t) \). For long times, a large minimal value of \( P_L \) illustrates that the localization is ‘good’. A small value of this quantity indicates that the particle is able to jump or tunnel to the right well with a large probability, even if the particle eventually returns fully to the original the localization is then ‘poor’. Figure 7(b) presents the resulting presents the resulting minimal \( P_L \) for different dynamical localization points \( l_j \). These curves have some surprising behavior: (a) Increasing the ratio \( \Delta/h\omega \) produces a rapid deterioration of the dynamic localization points for all \( l_j \). In fact, for \( \Delta/h\omega > 5 \), all localization points shown exhibit such poor localization (minimal \( P_L \simeq 0 \), that the time-dependent oscillations are basically non-localized (even though the \( F \) field values are still somewhat large, Fig. 7(a)). (b) For a range of \( \Delta/h\omega \), the second crossing point \( l_2 \) is in fact better localized than \( l_3 \), unlike the situation at high frequency.
FIG. 8: Minimal probability to remain in the left well for the first crossing points \(l_1\), for several values of magnetic field. The legend indicates the two magnetic field components; the first number (left) refers to the intensity of \(B\perp\) and the second (right) to \(B\parallel\) in Tesla. Insets show magnification of different curve regions, for clarity.

(c) For \(l_1\) and \(l_3\) (but not for \(l_2\) and \(l_4\)), there is a sudden ‘revival’ of localization, as the ‘bumps’ in the those curves indicate. This ‘revival’ behavior is not intuitive and its source not yet understood. We have attempted to link this to the details of the quasienergy spectra (such as possible degeneracies or level anticrossings). We have not been able to identify a clear correlation.

We have also analyzed to what extent the degree of localization is affected by other parameters, such as the in-plane field, \(B\parallel\). Figure 8 illustrates the minimal \(P_{l_1}\) for different values of the magnetic field. We find that to a great extent, the field (both in-plane and perpendicular) produces no change in the main drop of the \(l_1\) curve. However, the height and peak position of the ‘revival feature’ are more noticeably changed by the magnetic field. Although we have computed these curves for a wide variety of total magnetic field amplitudes and/or angles, we find that the curve is not drastically modified, and at most yields a suppression of this revival feature by a factor of nearly two. Notice, however, that the changes appear only when both components of the field are present, and are larger for the larger total field attempted. Again, the quasienergies show no evident feature that one can associate with these changes, and its precise elucidation remains a challenge for future work.

IV. CONCLUSIONS

We have analyzed a double quantum well system in the presence of a tilted magnetic field, and a strong oscillating electric field. We find, as other authors, that for high frequency, the otherwise tunneling particle is localized in one of the two wells at certain values of the electric field amplitude. We have however, studied this system over the entire regime of frequencies and field amplitudes. We find, in agreement with recent analytical work, that the electric field at which localization occurs changes monotonically down with lower frequency and that this trend continues far away from the perturbative regime of \(\Delta/\hbar\omega \ll 1\). We find however, that the degree of localization does not change in a monotonic fashion, and even shows a certain degree of recovery or revival, for lower frequencies. This rather involved behavior with the different system parameters is likely to produce a number of experimentally observable consequences. For example, the varying degree of charge localization for different frequency values in a given structure would yield variations in the generation power of THz radiation. The tunability with \(B\parallel\), on the other hand, would produce entirely different frequencies in the generation of radiation, away from the anticipated driving harmonics. This behavior would then contribute significantly to the already rich assortment of different phenomena observed in these systems.

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Notice that for $\Delta/\hbar \omega = odd$, the quasienergies show no gap at $F = 0$, but split quadratically away, similar to an anticrossing behavior. This however, does not yield dynamic localization and agree with a set of solutions to Eq. 13.