Relativistic Beaming as a Probe of Stellar and Planetary Masses

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Abstract. The primary method of extra-solar planet (exoplanet) detection and characterization is through planetary transits. These events occur when a planet is observed to pass in front of its host star with respect to the observer's line of sight, which causes a small dimming event. Transits alone yield information on the orbital properties such as period, inclination, semi-major axis as well as physical properties such as the planetary radius. With high-precision photometry, a new photometric effect has emerged as a probe of short-period exoplanet masses. This effect is known as relativistic Doppler beaming (or boosting), and has been used to estimate the masses and densities of numerous exoplanets and stars in binary systems. Here, this effect is discussed in detail along with the prospect of utilizing it with next generation space-based telescopes that will be devoted to the detection and characterization of exoplanets. Prospects for the characterization of binary systems will also be examined.

1. Introduction

The Kepler Space Telescope (Kepler) has revolutionized the study of extra-solar planets (exoplanets) — planets that orbit stars other than the Sun. Kepler was launched in 2009 with its primary function of monitoring the brightness of approximately 250,000 stars continuously over the course of its five year mission. The goal was to detect periodic dimming events caused by a planet transiting its host star and thus blocking a small portion of starlight. Perhaps the most revolutionary aspect of Kepler was the photometric precision that the telescope was able to achieve. With the ability to obtain photometric precisions down to $\sim 10$ parts per million, Kepler was not only well capable of detecting the transits of exoplanets, but also much more subtle photometric effects associated with a subset of exoplanetary systems.

To date, Kepler has discovered 2342 extrasolar planets [1] along with 2878 eclipsing binary systems [14]. A subset of the Kepler planets are large, short-period exoplanets and display out of transit photometric effects in addition to the primary transit. First is the reflection of starlight and thermal emission from the dayside of the planet, which results in a quasi-sinusoidal signal as the planet is observed to go through phases over the course of its orbital period. These mechanisms also give rise to what is known as the secondary eclipse, which occurs when the planet periodically disappears behind the star and any reflected/emitted light from the planet is blocked by the star. The secondary eclipse is often significantly ($>10\times$) shallower than the primary transit. The second effect is known as ellipsoidal variations and is a result of the planet inducing tides on the surface of the host star. This also results in a sinusoidal photometric signal in the photometric at twice the orbital frequency of the planet. The third photometric effect...
Figure 1. *Kepler* light curve of *Kepler*-76b. The relativistic beaming effect contributes 12ppm to the total observed variation.

is the relativistic Doppler beaming of light from the host star. Stars and planets collectively orbit about the system's barycenter. Therefore the host star will periodically advance toward and recede from an observer. Thus the brightness of the host star will vary sinusoidally at the orbital frequency of the planet. The resulting photometric variations due to these effects can be seen in Figure 1, which displays the *Kepler* light curve of the transiting hot Jupiter *Kepler*-76b. For this planet, the most dominant effect is reflection, which displays a peak to peak amplitude of $\sim 105$ ppm, followed by ellipsoidal variations with a peak to peak amplitude of $\sim 44$ ppm and finally Doppler boosting with an amplitude of $\sim 8$ ppm.

Due to the fact that the beaming effect is caused by the motion about the center of mass of an exoplanet system, the amplitude of the effect can be used to estimate the mass of any companions to the host star provided the host star mass is known. This can be used to vet possible false-positive scenarios that mimic exoplanets, such as eclipsing binary systems for which the beaming amplitude has the potential to be much greater than in planet-hosting systems. It also allows one to constrain the density of exoplanets when paired with transit observations.

2. The Relativistic Beaming Effect

First observed by [17] in 2000, a theoretical treatment of the relativistic beaming effect was first described by [11] with the application of rotating white dwarfs, and later in the context of ultra-compact eclipsing binaries [27]. [29] and [12] also give a detailed theoretical treatment. [16] were the first to consider this effect in the context of exoplanet characterization with the *Kepler* Space telescope, followed by [33] with the aim of characterizing so-called beaming binaries. Additionally, the beaming effect has been used extensively to characterize both planetary [24, 25, 30, 6, 7, 2, 8, 18, 20, 21, 23] and binary star systems [31, 33, 3, 32, 10, 4] and is in reality a result of combination of three effects. First, the beaming of light in the direction of motion, second the increase/decrease in observed rate of photons emitted from the host star, and third the Doppler shift of the host stars spectrum causing it to move in an out of the observational bandpass,
For completeness the following section gives a derivation of the bolometric flux observed from a moving light source. This follows closely from the derivation given by [19].

2.0.1. The Bolometric Flux  

We can begin to quantify such effects in the bolometric case (across all wavelengths) by considering a spherical star that radiates isotropically in its rest frame $S'$. It is assumed that the photons are emitted at a constant rate $\Gamma'$ as measured in the rest frame of the star. Due to the aberration effect, the angles at which the photons will be emitted in $S'$ will differ from those observed by someone moving relative to the star in $S$. This is given by

$$\cos \theta' = \frac{\cos \theta - \beta}{1 - \beta \cos \theta}$$

(1)

where $\beta$ is the velocity of the star in units of $c$. Therefore, the solid angles into which the photons are emitted will also differ such that

$$d\Omega = \sin \theta d\theta$$

(2)

and

$$d\Omega' = \sin \theta' d\theta'$$

(3)

By differentiating (1), it can be shown that

$$\frac{d\Omega}{d\Omega'} = \gamma^2 (1 - \beta \cos \theta)^2$$

(4)

where $\gamma$ is the Lorentz factor. Additionally, due to time dilation the ratio of the rates at which the photons are emitted as measured in the two frames is

$$\frac{d\Gamma'}{d\Gamma} = \gamma (1 - \beta \cos \theta).$$

(5)

The ratio of the observed flux between $S$ and the rest frame of the star, $S'$, is given by

$$\frac{dF'}{dF} = \frac{dE}{dE'} \frac{d\Gamma}{d\Gamma'} \frac{d\Omega'}{d\Omega}$$

(6)

where $dE$ is the energy associated with the radiated photons. Since energy and momentum form a four-vector, energy transforms as $dE' = \gamma(1 - \beta \cos \theta)dE$. This is simply a re-statement of the Doppler effect. Substituting (4) and (5) into (6) yields the boosted flux observed by $S$

$$F_b(t) = F_\star \left(1 - \frac{1}{\gamma(1 - \beta \cos \theta(t))}\right)^4$$

(7)

where $F_b(t)$ is the observed flux in $S$, $F_\star$ is the flux emitted by the star in its rest frame, and $\theta(t)$ is the angle between the stellar velocity and the line of sight. Since even in the most extreme cases, the velocities of planet-hosting stars about the center of mass doesn’t exceed $10^2 - 10^3$ ms$^{-1}$, we take the non-relativistic limit to calculate the stellar-normalized boosted flux

$$\frac{F_b(t)}{F_\star} = 1 + 4\beta_r$$

(8)

where $\beta_r = \beta \cos \theta(t)$ is the component of the velocity of the host star along the observers line of sight (i.e. the radial velocity of the host star).
2.0.2. The Bandpass-Integrated Flux  

Depending on the observational bandpass of the detector, only a portion of the bolometric flux will be observed. Therefore there is an additional bandpass-dependent component to the boosting effect, which is subject to the wavelength range over which one observes a beaming system. Due to the reflex motion of the host star, its spectrum will periodically Doppler shift causing portions of the spectrum to move in and out of the observational bandpass. If one assumed the flux to behave as a power law in frequency \( F_\nu \propto \nu^{-\alpha} \), the observed flux is given by

\[
\frac{F_B(t)}{F_*} = 1 + B \beta_r(t) \tag{9}
\]

where \( B \) is the photon-weighted bandpass-integrated beaming factor given by

\[
B = \frac{\int K(\lambda) (5 + \alpha) \lambda F_{\lambda,*} d\lambda}{\int K(\lambda) \lambda F_{\lambda,*} d\lambda}. \tag{10}
\]

Here, \( K(\lambda) \) is the Kepler response function, \( \lambda \) is the wavelength of observed light, \( F_{\lambda,*} \) is the stellar spectrum, and \( \alpha \) is the spectral index given by

\[
\alpha = \frac{d \ln F_{\lambda,*}}{d \ln \lambda}. \tag{11}
\]

[16] modeled the stellar spectrum, \( F_{\lambda,*} \), as a blackbody, which neglects absorption features that could substantially affect the beaming factor. Figure 2 shows how the beaming factor changes with effective temperature for a variety of values of stellar log-surface gravity. The stellar metallicity was set to solar metallicity as the effect of changing metallicity was found to have little affect on the beaming factor. The spectra were obtained by interpolating the PHOENIX library of stellar spectra.

If one is able to estimate the amplitude of the beaming effect, the companion mass, \( M_c \), is immediately obtained due to the fact that the effect is proportional to the radial velocity, which
Figure 3. Spectral response functions for *Kepler*, CHEOPS, and TESS. The peak of each response function was normalized to unity. *Kepler* and CHEOPS are sensitive to similar wavebands, whereas TESS is sensitive to red/Near-IR wavelengths.

has a semi-amplitude

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin i}{M_*^{2/3}} \frac{1}{\sqrt{1 - e^2}}$$

where $G$ is Newton’s gravitational constant, $P$ is the orbital period of the planet, $i$ is the orbital inclination, and $M_*$ and $e$ are the host star mass and orbital eccentricity, respectively.

3. Doppler Beaming and Future Space-Based Platforms

The Transiting Exoplanet Survey Satellite (TESS) [26] was recently launched in April 2018, and the CHaracterizing ExOPlanets Satellite (CHEOPS) [9, 5] is currently set to launch in early 2019. The primary objective of the TESS mission is to observe the entire celestial sphere over two years searching for exoplanet transits in much the same way as *Kepler*. The sky will be binned into 26 sectors, each of which will be observed for approximately 27 days. CHEOPS on the other hand will target bright stars that are already known to harbor exoplanets. Since the amplitude of the beaming effect depends on the observational bandpass, one may ask whether or not it will be detectable with the next generation of telescopes given their expected photometric sensitivity. Figure 3 displays the observational bandpasses for *Kepler*, CHEOPS, and TESS. Both the *Kepler* and CHEOPS bandpasses peak in the visible at $\sim 600$nm, while TESS is sensitive to redder wavelengths in general and peaks in the near-infrared at $\sim 900$nm. The expected photometric precision of TESS is 200ppm in 1 hour of observations [15]. The expected photometric precision for CHEOPS is an order of magnitude better at 20ppm for a bright ($6 < V < 9$) star per six hours of integration time, and 85ppm for a dimmer ($V = 12$) star over just 3 hours of integration time.

3.0.1. Prospects for Exoplanet Characterization

Figure 4 displays the expected beaming amplitudes for both CHEOPS (top) and TESS (bottom). Each pixel represents the expected beaming amplitude in parts-per-million, computed by varying the planetary mass $M_p = [0, 13]M_J$ and orbital period $P = [1, 10]$ days, and were produced for three different host star masses $M_* = [1, 1.5, 2]M_\odot$. Each of these assume circular, edge-on orbits with inclinations of $i = 90^\circ$. The effective temperatures were derived from the assumed host star masses by
interpolating the table of [22], which lists measurements of masses and effective temperatures for 90 stars ranging from spectral type M to O. For each combination, the photon-weighted bandpass integrated beaming factor was computed (eq.10) to account for the differing observational bandpasses of CHEOPS and TESS from which the amplitude of the beaming effect was calculated.

![Figure 4. Predicted beaming amplitudes (in ppm) observed through the CHEOPS (top) and TESS (bottom) bandpasses. Amplitudes were computed in both bandpasses for combinations of host star mass, $M_s$, planetary mass $M_p$, and orbital period $P$. Contours of 10ppm and 20ppm are overlaid for perspective.](image)

It seems that relativistic beaming will be difficult to detect in most situations with these two platforms. With CHEOPS, certain massive planets in short period configurations with a bright solar mass host star should be detectable with six hours integration time. If the photometric precision can be pushed to the 10ppm level, the beaming effect from planet-hosting stars with companions with masses as low as $\sim 4-7M_J$ should be detectable. Given TESS’ expected photometric precision of 200ppm over one hour of integration, it seem highly unlikely that the beaming effect will be detectable for most if not all planetary systems. However, the short 2-minute cadence of the TESS observations allows one to bin the phase curve to increase the SNR making it possible to detect beaming amplitudes on the order of 10-100ppm. The extent to which one can “Bin down” the observations will depend on the number of orbits observed.

Coincidentally, the first phase curve observation from TESS has recently been published for WASP-18b. This is a massive $10.4M_J$ planet in an ultra-short $\sim 0.94$ day orbit around a $1.46 \pm 0.29M_\odot$ star. A beaming amplitude was detected with an amplitude of $18 \pm 2$ ppm [28], which is consistent with our theoretical estimate of 16.6ppm.
3.0.2. Prospects for Eclipsing Binary Characterization

While CHEOPS will be devoted solely to characterizing exoplanets, TESS will inevitably find a large number of eclipsing binaries as it monitors the entire celestial sphere. In such systems, the beaming effect could play a significant role depending on the nature of the components.

When it comes to determining the masses of the components in a binary system, the beaming effect has the potential to bring with it a significant degeneracy. Since both components are stars, they will both contribute their own beaming effects to the overall observed flux. Additionally, since the components co-orbit the barycenter in opposite directions, it is possible for the beaming from one component to cancel out all or a portion of the beaming from the other as shown in Figure 5. The extent to which this degeneracy will be relevant will depend on the spectral type of the stars in the system. It should be noted that this would only impact systems for which there are no available radial velocity observations and more specifically systems for which the spectral lines of the components cannot be disentangled. In such cases, the radial velocities of both stars can be measured and therefore the degeneracy in the beaming effect (if observed) will not be present.

Figure 6 shows the amplitude of the beaming effect as a function of effective temperature of the primary (vertical axis) and secondary (horizontal axis) as observed through the TESS bandpass. These were generated assuming circular, edge-on orbits with orbital periods of 5, 10, and 15 days. Greater orbital periods were not considered since TESS will only observe each sector of the sky in 30d intervals. First, the masses of the components were derived from the effective temperatures by again interpolating the table from [22]. Next, theoretical spectra were obtained by interpolating the PHOENIX spectral database and the bandpass-integrated beaming factors were computed for both. Finally, the observed beaming amplitude was computed as the difference between that of the primary and secondary components. Overlayed with the beaming amplitudes is the 200ppm contour, which highlights the approximate limit of the TESS photometer. The diagonal represents binary components of equal effective temperature, which will in turn yield beaming components that cancel out. Any points under the diagonal represents configurations where the host star is less massive than the secondary and were therefore removed. It should be noted that these calculations do not take into account post-main sequence stars.
As the orbital period of the companions increases, the top of the 200ppm contour shifts to the right by approximately 200K.

Figure 6. Expected observed beaming amplitudes as measured by the TESS photometer and as a function of effective temperature for the primary and secondary in a beaming binary.

4. Summary
The relativistic beaming effect causes a periodic quasi-sinusoidal variation in the flux from planet-hosting stars due to their reflex motion about the center of mass. Measuring the amplitude of this effect allows the observer to estimate the mass of the companion if physical properties of the host star are known along with certain orbital characteristics. As this is a bandpass-dependent effect, the amplitude will vary across observational platforms.

By analyzing the beaming effect for a wide range of system configurations, across two platforms (CHEOPS & TESS), it was shown that CHEOPS will be able to detect the beaming effect for certain massive, short period exoplanets. The ability to detect this effect will rely on how many orbits are observed. Based on theoretical amplitudes and the expected precision of the photometer, TESS will only be able to detect the beaming effect for certain extreme (high planet mass, short orbital period) planetary systems. However, the amplitude should be large enough to be detected by TESS for a wide variety of combinations of short-period eclipsing binaries.

With the sensitivity of the CHEOPS photometer, beaming should be detectable for massive short period planets in the $\sim 4-7M_J$ range. However, the primary science goal of CHEOPS is
to constrain radii of planets in the range of 1 − 6R_E with known masses from radial velocity measurements. This class of planets will have beaming signals much smaller than 1ppm making it impossible to detect.

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