A thermophysical and dynamical study of the Hildas (1162) Larissa and (1911) Schubart

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ABSTRACT

The Hilda asteroids are among the least studied populations in the asteroid belt, despite their potential importance as markers of Jupiter’s migration in the early Solar system. We present new mid-infrared observations of two notable Hildas, (1162) Larissa and (1911) Schubart, obtained using the Faint Object InfraRed CAmera for the SOFIA Telescope (FORCAST), and use these to characterise their thermal inertia and physical properties. For (1162) Larissa, we obtain an effective diameter of 46.5$^{+2.3}_{-1.7}$ km, an albedo of 0.12 ± 0.02, and a thermal inertia of $15^{+10}_{-2}$ J m$^{-2}$ s$^{1/2}$ K$^{-1}$. In addition, our Larissa thermal measurements are well matched with an ellipsoidal shape with an axis ratio a/b=1.2 for the most-likely spin properties. Our modelling of (1911) Schubart is not as refined, but the thermal data point towards a high-obliquity spin-pole, with a best-fit a/b=1.3 ellipsoidal shape. This spin-shape solution is yielding a diameter of 72$^{+3}_{-4}$ km, an albedo of 0.039± 0.02, and a thermal inertia below 30 J m$^{-2}$ s$^{1/2}$ K$^{-1}$ (or 10$^{+20}_{-3}$ J m$^{-2}$ s$^{1/2}$ K$^{-1}$). As with (1162) Larissa, our results suggest that (1911) Schubart is aspherical, and likely elongated in shape. Detailed dynamical simulations of the two Hildas reveal that both exhibit strong dynamical stability, behaviour that suggests that they are primordial, rather than captured objects. The differences in their albedos, along with their divergent taxonomical classification, suggests that despite their common origin, the two have experienced markedly different histories.

Key words: minor planets, asteroids: individual: (1162) Larissa, (1911) Schubart; radiation mechanisms: thermal; minor planets, asteroids: general; infrared: general; planets and satellites: formation

1 INTRODUCTION

Over the past few decades, there has been a growing consensus amongst astronomers and planetary scientists that the Solar system’s youth was a chaotic place. Rather than being the result of a sedate, in-situ formation, leading models now suggest that the terrestrial planets were shaped by cataclysmic collisions (e.g. Cameron & Benz 1991; Canup 2004; Benz et al. 2007; Canup 2012; Asphaug & Reufer 2014), whilst the giant planets underwent dramatic migration, potentially spanning hundreds of millions or even billions of kilometres (e.g. Tsiganis et al. 2005; Gomes et al. 2005; Levison et al. 2008; Lykawka et al. 2009, 2010, 2011; Walsh et al. 2011; Nesvorný 2018).

Much of the evidence for the chaotic evolution of the early Solar system has come, not from studies of the planets themselves, but from analysis of the vast numbers of small bodies littered throughout the system$^1$. The Solar system’s asteroidal bodies are of particular interest in determining the true narrative of the system’s past. The overwhelming majority of those objects move on orbits between those of Mars and Jupiter, within the Asteroid belt. Those asteroids are of great scientific value as natural records of the formation and evolution of our Solar system. The study of their orbits and physical properties are key research topics to understand the mechanics and evolution of the Solar system as a whole, and particularly the origin, hydration, and collisional history of the Earth (e.g. Morbidelli et al. 2000; Strom et al. 2005; Bottke et al. 2005; Gomes et al. 2005; Horner & Jones 2008; Horner et al. 2009; Minton & Malhotra 2011; Nesvorný 2018).

In recent years, studies of the Jovian Trojan population have brought fresh insights to the migration history of the giant planets (e.g. Morbidelli et al. 2005; Nesvorný et al. 2013; Roig & Nesvorný 2015; Nesvorný 2018; Nesvorný et al. 2018; Pirani et al. 2019). The degree to which the orbits of Jupiter’s Trojans are excited suggests that they must have been captured during the migration of the giant planets, rather than having formed in-situ – and as a result, the population has been used as a means to estimate the scale, speed, direction, and chaoticity of that migration (e.g. Morbidelli et al. 2005; Nesvorný et al. 2013; Pirani et al. 2019).

The Jovian Trojans are not, however, the only population of asteroidal bodies trapped in mean-motion resonance with Jupiter. Located

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$^1$ For a detailed recent review of the various theories of Solar system’s migration history, and the abundant evidence that supports them, we direct the interested reader to Horner et al. (2020), and references therein.

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between the outer edge of the Asteroid belt and Jupiter’s orbit lie the Hildas – a swarm of objects trapped in 3:2 Mean Motion Resonance with Jupiter, at a semi-major axis of around 4 au (Spratt 1989; Gil-Hutton & Brunini 2008; Milani et al. 2017). It is natural, therefore, to wonder whether this population, too, holds clues that may help us unveil the full narrative of the Solar system’s early days.

Following that logic, both Roig & Nesvorný (2015) and Pirani et al. (2019) considered the effect of Jupiter’s migration on the planet’s two main resonant populations. Roig & Nesvorný (2015) examined the “Jumping-Jupiter” model of Jovian migration, and find that “neither primordial Hildas, nor Trojans, survive the instability, confirming the idea that such populations must have been implanted from other sources”. Similarly, Pirani et al. (2019) investigated how a rapid inward migration of Jupiter from a more distant initial orbit (beyond ~ 15 au) would have affected the Jovian Trojans and the Hildas. They found that such migration would result in the capture of a massive and excited Hilda population – something that is strongly at odds with the modern observed distribution of Hildas, which move on remarkably dynamically cold orbits. Their model does, however, successfully replicate the observed asymmetry between the observed populations of the leading and trailing clouds of Jovian Trojans. These recent studies reveal the importance of the Hildas as a key test for models of planet formation, and as such, it is fair to consider the Hildas as a whole an interesting but underrated link to the Solar system’s formation and evolution.

The first Hilda asteroid to be discovered was that after which the family is named – (153) Hilda, which was found in November 1875 by Johann Palisa, some 30 years before the discovery of the first Jovian Trojans (e.g. Wolf 1907; Heinrich 1907; Strömgren 1908). Although the Hildas orbit markedly closer to the Sun than the Jovian Trojans, and their population is comparable to that of their more famous cousins, they remain relatively poorly studied.

Recent studies that have focused on the physical properties of Hildas have suggested that these asteroids may have had a common origin with Jovian Trojans. Studies of their near infrared spectra (Wong et al. 2017), size distributions (Terai & Yoshida 2018), size frequency distributions (Yoshida et al. 2019), and their optical and infrared colours (from the Sloan Digital Sky Survey; Gil-Hutton & Brunini 2008, and NEOWISE; De Prá et al. 2018), reveal that the overall properties of the two resonant populations are broadly similar.

Such studies have revealed the presence of at least two collisional/dynamical families in the Hilda population, both of which are named for their largest members (e.g. Brož & Vokrouhlický 2008). To date, 111 members of the (1911) Schubart family have been identified, whilst the (153) Hilda family has at least 216 confirmed members (Romanishin & Teğler 2018). It seems likely that future surveys will reveal both more members of these families, and new collisional families with different parent bodies.

Taxonomically, the detailed mapping of the asteroid belt carried out by DeMeo & Carry (2013, 2014) found the Hildas and Jovian Trojans to be distinct groups of objects, separate from their classification of the main belt asteroids. The Hildas and Trojans were typically labelled as P and D-types in contrast to the S and V-types that are common throughout the main belt, a result that has been verified by later studies (Terai & Yoshida 2018; Yoshida et al. 2019). In addition to their similar taxonomic classifications, the Hildas and Jovian Trojans both exhibit a clear bimodality in their spectral distribution (Terai & Yoshida 2018; De Prá et al. 2018), a fact that could be suggestive of a variety of source regions having contributed to the modern Hilda and Trojan populations.

To date, amongst the Hildas that have been classified on the basis of their spectral data, the majority of were classified under the low-albedo asteroid classifications C, P and D-type (Dymock 2010; Wong et al. 2017; Szabó et al. 2020) in the Tholen taxonomy (Kaasalainen & Torppa 2001; Warner et al. 2009; de Pater & Lissauer 2015). Of these, the D-type asteroids are undoubtedly the most numerous, followed by P-types and finally C-types (Grav et al. 2012).

Some studies state that the general outline for the surface composition of the Hildas is a mix of organics, anhydrous silicates, opaque material and ice (Dahlgren et al. 1998; Dahlgren et al. 1999; Gil-Hutton & Brunini 2008; De Prá et al. 2018). The surfaces of the Hildas have likely experienced significant space weathering, although it should be noted that they may well be considered to be more pristine than similar objects in the main asteroid belt, as a result of their larger heliocentric distances (Gil-Hutton & Brunini 2008; De Prá et al. 2018).

Moreover, De Prá et al. (2018) raised an interesting point on current knowledge of Hildas concluding that there is a population in the Jovian Trojans not present in the Hildas, supporting the idea that Trojans and Hildas may be celestial objects of different origins. Indeed, the stark contrast in mean albedo values amongst sub-groups in the same Hilda suggests that even within them we can appreciate a kind of intruder of another source (Romanishin & Teğler 2018). In the coming years, the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2014) will aid the identification of Hildas with different origins, by providing a wealth of high cadence light curves for these asteroids. As described by Szabó et al. (2020), that detailed photometric data will facilitate a direct comparison between the main belt asteroids, the Hildas, and the Jovian Trojans, allowing the similarities and differences between the populations to be studied in depth.

In this work, we focus on two of the largest and brightest members of the Hilda population – (1162) Larissa, and (1911) Schubart. (1162) Larissa was discovered in 1930 by German astronomer Karl Reinmuth, and is amongst the brightest Hildas due to its relatively high albedo (from 0.11 (Alí-Lagoa & Delbo 2017) to 0.18 (Grav et al. 2012) in the literature), which is markedly higher than the average of 0.055 for this group of asteroids (De Prá et al. 2018). Its taxonomy has been variously defined as M-type (Grav et al. 2012), P-type (Warner et al. 2009), and most recently as X-type (De Prá et al. 2018). It does not belong to either of the currently identified collisional families within the Hilda population (e.g. Romanishin & Teğler 2018), and as a result, it is plausible that it might be a primordial, undisrupted body.

(1911) Schubart was discovered in 1973 by Swiss astronomer Paul Wild, and is the largest member of a populous collisional family (the Schubart family), containing at least 111 members (Brož & Vokrouhlický 2008). The extremely low albedo calculated for it, under 0.05 in several studies (Grav et al. 2012; Warner et al. 2009; Romanishin & Teğler 2018), makes it one of the darkest asteroids in the Hilda group and even in the Solar system.

We describe the observations taken by the “Faint Object infraRed CAmera for the SOFIA Telescope” (FORCAST, Adams et al. 2010) instrument on the Stratospheric Observatory For Infrared Astronomy (SOFIA, Herter et al. 2012) used in this work in Section 2. The process by which flux values are extracted from the observations is then described in Section 3. In Section 4, we perform a detailed thermophysical analysis of (1162) Larissa and (1911) Schubart, before investigating the orbital stability of the two asteroids in Section 5. In Section 6, we discuss our results and present our

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2 Discovery details taken from the NASA/JPL Small-Body Database Browser at https://ssd.jpl.nasa.gov/sbdb.cgi, accessed on 2020 March 11.
conclusions.

2 OBSERVATIONS

We obtained SOFIA/FORCAST observations of the Hildas (1162) Larissa and (1911) Schubart, which we supplement with archival mid- and far-infrared data taken by IRAS (Tedesco 1986), AKARI (Usui et al. 2011) and WISE (Mainzer et al. 2011) projects. The SOFIA images collected for this research were part of a series of studies about small bodies and Solar system formation that were performed as part of the so-called “Basic Science” observation flights, under the proposal ID 82_0004, mission ID FO64, PI Dr. Thomas Müller, carried out during 2011 June.

Our data comprise a series of multi-wavelength mid-infrared imaging observations taken on 2011 June 4, for (1911) Schubart, and 2011 June 8 for (1162) Larissa. FORCAST is a dual-channel infrared camera and spectrometer, able to produce images with a field of view of approximately 3.4’ × 3.2’, with a plate scale of 0.768’. One channel is the short-wavelength camera (SWC), which operates at 5 to 26 μm, whilst the other, the long-wavelength camera (LWC), operates at 26 to 40 μm; both channels can be used either individually or simultaneously (Herter et al. 2013).

In order to take advantage of the dual channel nature of FORCAST, two observations were taken per target, in coupled in using first the 11.1 and 34.8 μm and then the 19.7 and 31.5 μm filters, under the symmetrical two-position chop-nod mode (C2N), to minimise the impact of a variety of sources of noise, including time variable sky backgrounds and the thermal emission from the telescope itself. That mode is recommended for observations of point-like sources such as (1162) Larissa and (1911) Schubart, as described in Herter et al. (2012).

The infrared images used in this work are level 3 coadded data products, which means that they were telluric and flux corrected by SOFIA team in units of Jy/pix (Herter et al. 2013). The files were stored and then downloaded from the SOFIA AIRBORNE OBSERVATORY PUBLIC ARCHIVE.

To complement our new data, we searched for archival observations of our two targets. Fortunately, both (1162) Larissa and (1911) Schubart have been observed multiple times at thermal infrared wavelengths by other surveys, so that in addition to the 12 SOFIA FORCAST images of (1162) Larissa, for the thermophysical analysis we used one observation from IRAS (at 25 μm), four from AKARI (at 18 μm), and 42 from WISE (21 images at 11.1 μm and 21 at 22.64 μm).

Moreover, for (1911) Schubart, in addition to the 23 SOFIA FORCAST images, we used 14 observations from IRAS (four at 12 μm, four at 25 μm, four at 60 μm and two at 100 μm), eight from AKARI (four at 9 μm and four at 18 μm) and also 39 from WISE (20 at 11.1 μm and 19 at 22.64 μm). All the data for both asteroids were taken from the SBNAF Infrared Database (Szakáts et al. 2020) at VizieR Online Data Catalog. Full details on the archival images used are given in Tables 1 and 2.

3 https://www.sofia.usra.edu/science/data/science-archive/ Accessed on 2019 October 10
4 VizieR Online Data Catalog: SBNAF Infrared Database Website at http://vizier.u-strasbg.fr/viz-bin/VizieR?–source=IAA/A/635/AS4, accessed most recently on 2020 July 14. Earlier in our analysis, we obtained data from the Konkoly Observatory Website at https://konkoly.hu/database.shtml, accessed on 2019 December 17, which was used for our preliminary work, but has since been superseded by the updated VizieR catalog.

3 ANALYSIS OF SOFIA DATA

In order to perform a thorough thermophysical analysis of the Hildas (1162) Larissa and (1911) Schubart, we first used the images taken of the asteroids at 11.1, 19.7, 31.5 and 34.8 μm to determine the flux from the asteroids in each waveband. A visual inspection of the images was carried out using SAOImageDS9 (Joye & Mandel 2003) to assess their quality. This revealed that in 12 of the 23 images of (1162) Larissa there was no clear presence of the asteroid, so those images were discarded after completing the process of checking their signal-to-noise values, and they were not analysed further. The images of (1162) Larissa were all of excellent quality and, as a result, all were used in our research.

The selected images were then analysed using a bespoke code that applied implementations of standard Python-based image analysis tools including the packages Astropy (Greenfield et al. 2013) and Photutils (Bradley et al. 2016), yielding the results shown in Tables 3 and 4.

Although the FORCAST Photometry Recipe reference document recommends aperture photometry radius and annulus dimensions of 12 pixels and 15 to 25 pixels, the same document suggests the use of smaller apertures to reduce the overall uncertainty in measurement of low S/N sources. In this sense, given the quite irregular backgrounds of several of the images and the diversity amongst them, we opted to individually optimise the values for the aperture photometry radii and annuli for each image.

We used the Aperture Photometry Tool (APT) (Lahe et al. 2012) and DS9 (Joye & Mandel 2003) as complementary tools to decide upon the best values for making the photometry measurements, making particular use of APT functions Aperture Slice, Radial Profile and Curve of Growth, which were combined with the DS9 horizontal and vertical graphs in order to set the correct values for the aperture and annulus extent as required.

We applied appropriate colour correction factors to the measured flux densities, assuming blackbody input spectra for our targets with a temperature close to 150 K, according to the Hildas’ distance from the Sun (Hinrichs et al. 1999). The magnitude of these corrections per each waveband filter, however, are values quite close to 1 (between 1.0005 and 1.009), implying a small correction for the raw fluxes calculated, and were taken from Table 3 of Herter et al. (2013).

The resulting aperture photometry measurements obtained from the SOFIA images at 11.1, 19.7, 31.5 and 34.8 μm are presented in Figures 1 and 2.

To determine the uncertainties in our results, we followed the FORCAST Photometry Recipe. The error propagation was done by...
In order to better understand the physical properties of (1162) Larissa and (1191) Schubart, we here present a detailed thermophysical analysis for both objects, based on the observational data we obtained using SOFIA, coupled with archival data taken by IRAS (Tedesco 1986), AKARI (Usui et al. 2011) and WISE (Mainzer et al. 2011), as detailed in Section 2.

The thermophysical analysis performed in this research is based on the thermophysical model (TPM) detailed in Lagerros (1996); Lagerros (1997, 1998) and Müller & Lagerros (1998, 2002). In the TPM, the surface temperature is calculated from the energy balance between absorbed Solar radiation, the thermal emission, and heat conduction into the surface material (here, 1-d heat conduction is considered). For a direct comparison with the observed fluxes, we use a given spin-shape solution to calculate the temperature of each object, based on the observational data we obtained adding the photometry, the calibration with the standard star and the calibration factor in quadrature. In that process, we used values from Table 5 of Herter et al. (2013) to provide the appropriate calibration factor, which was 0.055 for (1162) Larissa, and 0.102 for (1191) Schubart.

### 4 THERMOPHYSICAL MODELLING

Summary of the supplementary thermal observations of (1162) Larissa used in this work. The table shows the time at midpoint of the observation (in Julian Date), the reference wavelength (in μm), the monochromatic flux density (colour corrected in-band flux density) with its absolute error, the heliocentric distance ($r_{\text{Helio}}$ [au]), the observatory-centric distance ($\Delta$ [au]), the phase angle ($\alpha$ [°]), and the spacecraft telescope used for the observation.

| Date [JD] | Wavelength [μm] | Flux density [Jy] | $r_{\text{Helio}}$ [au] | $\Delta$ [au] | $\alpha$ [°] | Telescope/Instrument |
|-----------|-----------------|-------------------|--------------------------|---------------|-------------|---------------------|
| 24504154.4514 | 22.09 | 0.725 ± 0.045 | 3.51421 3.30036 -16.77 | WISE          |
| 24504154.3576 | 22.09 | 0.525 ± 0.033 | 3.51426 3.29854 -16.77 | WISE          |
| 24504154.3050 | 22.09 | 0.163 ± 0.008 | 3.51430 3.29671 -16.76 | WISE          |
| 24504154.2525 | 22.09 | 0.544 ± 0.033 | 3.51435 3.29489 -16.76 | WISE          |
| 24504154.2000 | 22.09 | 0.554 ± 0.033 | 3.51437 3.29398 -16.76 | WISE          |
| 24504154.1475 | 22.09 | 0.508 ± 0.033 | 3.51440 3.29034 -16.76 | WISE          |
| 24504154.1050 | 22.09 | 0.511 ± 0.033 | 3.51443 3.28640 -16.76 | WISE          |
| 24504154.0625 | 22.09 | 0.514 ± 0.033 | 3.51446 3.28306 -16.76 | WISE          |
| 24504154.0200 | 22.09 | 0.516 ± 0.033 | 3.51449 3.28070 -16.76 | WISE          |
| 24504153.9775 | 22.09 | 0.518 ± 0.033 | 3.51452 3.27852 -16.76 | WISE          |
| 24504153.9350 | 22.09 | 0.520 ± 0.033 | 3.51455 3.27630 -16.76 | WISE          |
| 24504153.8925 | 22.09 | 0.522 ± 0.033 | 3.51458 3.27412 -16.76 | WISE          |
| 24504153.8500 | 22.09 | 0.524 ± 0.033 | 3.51461 3.27204 -16.76 | WISE          |
| 24504153.8075 | 22.09 | 0.526 ± 0.033 | 3.51464 3.26996 -16.76 | WISE          |
| 24504153.7650 | 22.09 | 0.528 ± 0.033 | 3.51467 3.26788 -16.76 | WISE          |
| 24504153.7225 | 22.09 | 0.530 ± 0.033 | 3.51470 3.26580 -16.76 | WISE          |
| 24504153.6800 | 22.09 | 0.532 ± 0.033 | 3.51473 3.26372 -16.76 | WISE          |
| 24504153.6375 | 22.09 | 0.534 ± 0.033 | 3.51476 3.26164 -16.76 | WISE          |
| 24504153.5950 | 22.09 | 0.536 ± 0.033 | 3.51479 3.25956 -16.76 | WISE          |
| 24504153.5525 | 22.09 | 0.538 ± 0.033 | 3.51482 3.25748 -16.76 | WISE          |
| 24504153.5100 | 22.09 | 0.540 ± 0.033 | 3.51485 3.25540 -16.76 | WISE          |
| 24504153.4675 | 22.09 | 0.542 ± 0.033 | 3.51488 3.25332 -16.76 | WISE          |
| 24504153.4250 | 22.09 | 0.544 ± 0.033 | 3.51491 3.25124 -16.76 | WISE          |
| 24504153.3825 | 22.09 | 0.546 ± 0.033 | 3.51494 3.24916 -16.76 | WISE          |
| 24504153.3400 | 22.09 | 0.548 ± 0.033 | 3.51497 3.24708 -16.76 | WISE          |
Table 2. Summary of the supplementary thermal observations of (1162) Larissa and (1911) Schubart used in this work. The table shows the time at the midpoint of the observation (in Julian Date), the reference wavelength (in $\mu$m), the monochromatic flux density (colour corrected in-band flux density) with its absolute error, the heliocentric distance ($r_{\text{Helio}}$), the observatory-centric distance ($\Delta$ in au), the phase angle ($\alpha$ [$^\circ$]), and the spacecraft telescope used for the observation.
In Figure 3, which shows that, in general, the observational data can be relatively well fit by our final thermophysical model solutions, despite the lack of high-quality spin-shape information for both targets.

For (1162) Larissa, our radiometric tests were performed using spherical and ellipsoidal spin-shape solutions, with a rotation period of 6.519148 hours (Slyusarev et al. 2013; Warner & Stephens 2017, J. Durech, private communications). The recent, most likely, pole orientation (294°, -23°) in ecliptic longitude and latitude (J. Durech, private communications). The recent, most likely, pole orientation (294°, -23°) in ecliptic longitude and latitude (J. Durech, private communications). The recent, most likely, pole orientation (294°, -23°) in ecliptic longitude and latitude (J. Durech, private communications). The recent, most likely, pole orientation (294°, -23°) in ecliptic longitude and latitude (J. Durech, private communications). The recent, most likely, pole orientation (294°, -23°) in ecliptic longitude and latitude (J. Durech, private communications). The recent, most likely, pole orientation (294°, -23°) in ecliptic longitude and latitude (J. Durech, private communications). 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Figure 1. (1162) Larissa mid-infrared photometric data. The graph shows the flux densities derived from the four wavelength bands of FORCAST, as a function of the rotational phase of the asteroid, together with the given error bars. The obtained mid-infrared photometric light curves follow the trend of increments and decrements along the different wavelengths used in each chop nod shot.

Figure 2. (1911) Schubart mid-infrared photometric data. The graph shows the flux densities derived from the four wavelength bands of FORCAST, as a function of the rotational phase of the asteroid, together with the given error bars. The obtained mid-infrared photometric light curves follow the trend of increments and decrements along the different wavelengths used in each chop nod shot.

Cibulková et al. 2018\(^7\), equator-on prograde, equator-on retrograde, pole-on during the SOFIA observations. We assumed that the asteroid had a typical level of surface roughness (Müller & Lagerros 1998), and solved for its size, albedo and thermal inertia using all SOFIA-FORCAST data (12 points) plus all of the detailed archival data described in section 2.

Following this methodology, the radiometric range solution for a spherical object, independent of the spin-axis orientation, and assuming \(V_\text{r}=9.59\) and \(G=0.49\) (Oszkiewicz et al. 2011) is given by:

- Effective diameter: 41 to 48 km
- Geometric V-band albedo: 0.10 to 0.15
- Thermal inertia below 30 Jm\(^{-2}\)K\(^{-1}\)s\(^{-0.5}\)

The reduced \(\chi^2\) of the spherical solutions are poor (>4.0). Given an observed (visible) lightcurve amplitude up to 0.22 mag, the poor fit is likely related to our assumption of a simple spherical shape. This is confirmed by deriving radiometric sizes separately for the different data sets. Due to the different viewing geometries for IRAS, Akari, WISE, and SOFIA data, the corresponding dataset-specific sizes deviate substantially and can only be explained by a non-spherical body.

If we take the two recent spin-pole solutions, we can implement ellipsoidal shape solutions with varying \(a/b\) ratios from 1.0 to 1.5 to improve the radiometric analysis. For an \(a/b\) ratio of 1.2 we obtained lower \(\chi^2\) values (around 2.0 for the (114°, -23°) and 2.5 for the (294°, -28°) pole solution) with the following values:

- Effective diameter: 46.5\(^{+2.3}_{-1.7}\) km
- Geometric V-band albedo: 0.12\(^{+0.02}_{-0.02}\)
- Thermal inertia: \(15\^{+10}_{-8}\) Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\)
- Preference for the spin-pole solution with (114°, -23°), \(P_\text{sideral}=6.519148\) h, ellipsoidal shape with \(a/b=1.2\)

These two spin-poles with the specified ellipsoidal shape have the big advantage that they can reproduce reasonably well the observed thermal lightcurve (22 data points taking in Jan 2010, and another 20 data points taken in July 2010), and the overall flux changes with wavelengths and phase angles. But based on the thermal data alone, it is not possible to entirely exclude other spin-shape solutions. However, future observations of the asteroid will prove critical in order to mitigate the remaining spin-shape uncertainties and to improve the radiometric solution further.

In the case of (1911) Schubart, using the multiple thermal data described in section 2, in combination with a spherical shape, a rotation period of 11.9213 h (Stephens 2016), and pole solution of (320°, 20°) (J. Durech, private communication) leads to acceptable \(\chi^2\) values below 2.0 for all spin-axis orientations. The data are reasonably well fitted, only the IRAS 100 \(\mu\)m observations seem to be problematic. The derived radiometric size range is between about 64 km up to 78 km, the corresponding albedos are between 0.033 and 0.42, respectively. However, the analysis process gave a slight preference for a spin-axis with high-obliquity like the pole-on during SOFIA observations or one of the recently estimated solutions from lightcurve inversion techniques.

For completeness, we performed additional tests considering the alternative rotation period of 7.9121 hours rotation period proposed in Warell (2017). Changing the rotation period in this manner had no impact on the results, showing no influence on the spin-size-albedo-TI solution. However, we find that the 11.9 h rotation period seems to explain the WISE data in a more consistent way.

We also investigated if ellipsoidal shape solutions with varying \(a/b\) ratios (from 1.0 to 1.5) lead to better fits of all data. An improvement is seen only in the high-obliquity case (with the asteroid seen pole-on during SOFIA observations). Here, an axis ratio \(a/b=1.3\) pushes the \(\chi^2\) close to 1.0. However, the WISE lightcurve data set (36 data points taken within 3.6 days in April 2010) is still not very well matched. This points to a more complex shape solution. But if we accept this best-fit pole-on spin-shape solution as baseline, then our radiometric solution provides the following values:

\(^7\) Using entries from the database at https://www-n.oca.eu/delbo/astphys/astphys.html, accessed November 2020.
In Figure 4, we show (in the upper panels) the degree to which the thermophysical model agrees with our observations, as a function of wavelength. Aside from the 100 μm observations of (1911) Schubart made by IRAS, it is clear that, in general, the model and data are in good agreement. The observation-to-model ratios are well balanced over a wide range of wavelengths, phase angles, and rotational phases.

In the lower panels of Figure 4, we plot the agreement between our thermophysical model and the gathered observational data against rotational phase. For both targets, there appears to be a remaining small variation in goodness of fit between our model and the data as a function of rotational phase. In general, there is still a significant scatter in the observational-to-model ratios, which lends further support to the idea that the two asteroids may well have more complex shapes.

5 DYNAMICAL MODELLING

To complement our study of the physical properties of (1162) Larissa and (1911) Schubart, we performed two detailed suites of n-body simulations to examine the long-term dynamical evolution of the two asteroids. For each asteroid, we used the Hybrid integrator within the n-body dynamics package Mercury (Chambers 1999) to follow the evolution of a swarm of 61,875 test particles for a period of 10^6 years, under the gravitational influence of the Sun, Jupiter, Saturn, Uranus and Neptune. An integration time step of 60 days was used, and each individual particle was followed for the full 10^6 year duration of the simulations, unless it collided with one of the four giant planets, the Sun, or was ejected to a heliocentric distance of 10,000 au.

The test particles themselves were generated in a manner similar to that used in our earlier work (e.g., Horner & Lykawka 2010; Horner et al. 2012a; Horner et al. 2012b), with “clones” generated in a regular 4-dimensional grid spanning the full ±3σ uncertainties in the objects semi-major axis, a, eccentricity, e, inclination, i, and mean anomaly, M. In total, 25 unique values of semi-major axis were tested. For each of these values, 15 unique values of eccentricity were examined, creating a 25 × 15 grid in a – e space. For each a – e ordinate, 15 unique inclination values were tested - with 11 unique mean anomalies being considered for each individual a – e – i ordinate. In this way, we generated a four-dimensional hypercube of test particles, distributed on an even lattice, with dimensions (25 × 15 × 15 × 11) in (a, e, i, M) space. The initial orbital elements for the two Hildas used as the basis for our integrations, along with their associated uncertainties, are detailed in Table 6.

In stark contrast to our previous studies of resonant Solar system small bodies (Horner & Lykawka 2010; Horner et al. 2012a; Horner et al. 2012b), both (1162) Larissa and (1911) Schubart proved to exhibit staggering dynamical stability. Over the course of our simulations, not a single clone of either object escaped from the Hilda population. Over time, the test particles did diffuse to fill the entirety of the “Hildan Triangle” – the striking feature in plan view of the Solar system occupied by the Hildas (as shown in Figure 5) – but none became sufficiently excited to escape that population.

The extreme stability of the two Hildas argues towards them being primordial, rather than captured objects. It stands as an interesting contrast to the dynamical evolution of objects studied in a similar manner in the Jovian and Neptunian Trojan populations - all of which exhibit some degree of instability (e.g. Levison et al. 1997; Marzari & Scholl 2002; Horner & Lykawka 2010; Horner et al. 2012a; Horner et al. 2012b; Di Sisto et al. 2019; Holt et al. 2020). Given the extreme stability of these two objects, it seems reasonable to suggest that future studies of the migration of the giant planets and their impact on the Solar system’s young small body populations should examine whether these models can replicate such tightly captured objects moving on dynamically cold orbits in the Hilda region.

6 DISCUSSION AND CONCLUSIONS

In this work, we have presented a detailed thermophysical and dynamical analysis of the Hilda asteroids (1162) Larissa and (1911) Schubart, using observations made using the FORCAST instrument on the SOFIA airborne observatory on 2011 June 4 and June 8, supplemented by archival data from the IRAS, AKARI and WISE space observatories. Our dynamical simulations reveal both Hildas to exhibit extreme dynamical stability, with not a single one of the 61,875 clones generated of each object escaping from the Hilda region in the 10^6 years of our integrations. This finding is strongly suggestive that the two objects are primordial in nature, rather than having been captured, since objects in populations that are widely accepted to have been captured during the Solar system’s youth tend to exhibit dynamical instability to some degree (e.g. Levison et al. 1997; Marzari & Scholl 2002; Horner & Lykawka 2010; Horner et al. 2012a; Horner et al. 2012b; Di Sisto et al. 2019; Holt et al. 2020).

Our thermophysical analysis of the two Hildas reveals that both asteroids have non-spherical shapes, with the physical properties we derive on the basis of our analysis being in broad agreement with previously published values, albeit with markedly reduced uncertainties (See Table 5).

For (1162) Larissa, the geometric V-band albedo we obtained (0.12 ± 0.02) is in close agreement with the 0.127 ± 0.009 given by Usui et al. (2011). It does not, however, agree with the values of 0.169 ± 0.012 and 0.186 ± 0.03 given by De Prâ et al. (2018) and Grav et al. (2012). Despite this, our results confirm that (1162) Larissa is amongst the most reflective Hildas, whose mean albedo (0.055) is markedly darker (De Prâ et al. 2018). In this sense, it sounds reasonable to have slightly different values regarding surveys that use another infrared wavelengths to study asteroids and thus may under or over estimate the geometric albedo, making valuable the effort for making use of instrumentation such as FORCAST operating at 3 to 50 μm, which spans the wavelength range of peak thermal emission from asteroids (Harris & Lagerros 2002; Emery et al. 2006). Our results yield a refined value for the effective diameter of (1162) Larissa of 46.5+2.2−1.3 km, a measurement that sits close to the centre of the range of previously published diameters, spanning 40.38 to 48.59 km (Tedesco et al. 2004; Usui et al. 2011; Nugent et al. 2015).

Our thermophysical analysis of (1911) Schubart proved more challenging, as a result of having to discard a number of our images of the asteroid. When our observations are taken in concert with those obtained by WISE, we find better agreement with the longer rotation period of 11.915 hours proposed in (Warell 2017), rather than the short 7.9 hour period also presented in that work. By incorporating that longer period into our analysis, we obtain a geometric V-band albedo for (1911) Schubart of 0.039± 0.05, which is in agreement with the mean value of 0.035 presented by Grav et al. (2012) and Nugent et al. (2015). This confirms that (1911) Schubart is an unusually dark asteroid, amongst the least reflective objects known in the Solar system. Indeed, this outcome is in strong agreement with the work of Romanishin & Tegler (2018), who state that (1911) Schubart
Figure 3. Plot showing the both the observations of (1162) Larissa (top) and (1911) Schubart (bottom) and the best fit thermophysical model obtained from those observations. The left plots show the data in linear space, with those on the right showing the same data in a log-log space. The observations are coloured to denote their source - with SOFIA observations shown as red squares, IRAS as green stars, Akari as blue plus marks, and WISE as black circles. For each observation, the vertical lines denote the 1-σ uncertainties. The thermophysical model is plotted in grey. The spectral energy distribution was calculated for the SOFIA/FORCAST epoch at lightcurve mid-point, with all the other fluxes scaled to these SOFIA/FORCAST epochs.

Table 5. Results of thermophysical modelling for (1162) Larissa and for (1911) Schubart. This Table collates data from Tedesco et al. (2004), Usui et al. (2011), Grav et al. (2012), Nugent et al. (2015), (Alí-Lagoa & Delbo 2017) and De Prá et al. (2018).

| Variables                        | (1162) Larissa | (1911) Schubart |
|----------------------------------|----------------|-----------------|
| Effective diameter [km]          | 40.38-52.16    | 64.66-92.37     |
| Geometric albedo in V-band       | 0.102-0.186    | 0.022-0.04      |
| Thermal inertia [Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\)] | No value       | 10\(^{20}\)      |

Our thermophysical analysis also enabled us to determine the thermal inertias of the two Hilda asteroids we studied. That analysis yielded values of 15\(^{+10}_{-8}\) Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\) for (1162) Larissa and 10\(^{+20}_{-5}\) Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\) for (1911) Schubart. These results lie within the broad range of values given by Müller & Lagerros (1998) in their study of a number of the largest main-belt asteroids, whose thermal inertias were found to range between 5 and 25 Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\), and stand in stark contrast to the outliers found in previous studies of inner main belt asteroids, such as (306) Unitas, which yielded values as high as 260 Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\), and (277) Elvira, with values reaching 400 Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\) (Delbo et al. 2015).

Flux variations in the thermal measurements provide some evidence that (1911) Schubart has an elongated shape since the spherical one produces significant variations in the observation-divided-by-model plot which seem to correlate with the most-likely rotation
Figure 4. The degree to which the thermophysical models agree with the observations can be demonstrated by taking the observed flux and dividing it by the predicted values. Here, we show the degree to which the observations and model agree for (1162) Larissa (left) and (1911) Schubart (right). The upper panels show the agreement between the model and observations as a function of wavelength, whilst the lower panel plots the agreement instead as a function of rotational phase for the two asteroids. As in Figure 3, the observations are coloured to denote which instrument obtained them. In general, the observations and model are in good agreement, albeit with noticeable scatter at any given wavelength.

Table 6. Orbital elements, and their associated uncertainties, for (1162) Larissa and (1911) Schubart, as taken from the AstDys database on January 12, 2012.

| Variables             | 1162 Larissa | 1911 Schubart |
|-----------------------|--------------|---------------|
| Semi-major axis [au]  | 3.9234       | 3.97885       |
| Eccentricity          | 0.110524     | 0.165839      |
| Inclination [°]       | 1.888        | 1.651         |
| ω [°]                 | 39.791       | 285.447       |
| Ω [°]                 | 212.222      | 183.727       |
| Mean Anomaly [°]      | 59.024       | 246.572       |

It is interesting that the two asteroids studied, (1162) Larissa and (1911) Schubart, are similar in size and thermal inertia, but so different in terms of albedo. Indeed, they seem to mark extremes in the observed properties of the Hildas - with (1162) Larissa being unusually reflective, and (1911) Schubart particularly dark.

These albedo values motivate us to review the taxonomy of both asteroids; the P-type assigned to (1162) Larissa in the (LCDB) and JPL is not in tune with its low thermal inertia of \(15^{+10}_{-8}\) Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\), suggesting a clear difference in physical nature compared to the P-type low albedo (111 Schubart and its \(10^{+20}_{-5}\) Jm\(^{-2}\)s\(^{-0.5}\)K\(^{-1}\).

Romanishin & Tegler (2018) suggest that the (1911) Schubart family are interlopers to the Hilda region. The low albedo we obtain from our observations supports the idea that (1911) Schubart is physically somewhat different to the other Hildas – but the question remains of...
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Figure 5. The dispersion of a suite of 61,875 clones of (1162) Larissa (left two columns) and (1911) Schubart (right two columns). The top row shows the clones after 10 Myr of integration time have elapsed, the central row shows them after 100 Myr, and the lower row shows them after 1 Gyr, at the end of our simulations. For each object, the left-hand column shows the clones in semi-major axis-eccentricity space, whilst the right shows them in plan view, looking down at the Solar system from above. In the plan view plots, the yellow point at the centre denotes the location of the Sun, whilst the red dot shows the location of Jupiter at the instant the clones are plotted. Whilst the clones of both (1162) Larissa and (1911) Schubart take some time to disperse, it is noticeable that the clones of (1911) Schubart do so more slowly, and over a smaller distance in orbital element space.

whether those differences are the result of the collision that formed the (1911) Schubart family, or are instead indicative that the parent of the family is an interloper in the Hilda region. Our dynamical results reveal that (1911) Schubart is extremely firmly embedded in the Hilda population, trapped securely in 3:2 mean-motion resonance with Jupiter – a result that seems hard to reconcile with its having been captured to the population after the bulk had already formed. This result is potentially supported by the similarity between the thermal inertias of the two objects – suggesting that the divergent albedos are the result of processes that have occurred through the lifetime of the objects, rather than being indicative of their having markedly different origins.

It is clear that further study is needed before we can conclusively determine whether any or all of the Hildas are captured objects, rather than having formed in-situ. The origin of the Hildas is still an open and crucial question, and it seems likely that the answer to that question will shed new light on the formation and evolution of the Solar system as a whole.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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