Perturbative unitarity bounds for effective composite models

S. Biondini\textsuperscript{a}, R. Leonard\textsuperscript{b, d}, O. Panella\textsuperscript{b}, M. Presilla\textsuperscript{c, d}

\textsuperscript{a}Van Swinderen Institute, University of Groningen, Nijenborgh 4, NL-9747 AG Groningen, Netherlands
\textsuperscript{b}Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, Via A. Pascoli, I-06123 Perugia, Italy
\textsuperscript{c}Dipartimento di Fisica e Astronomia "Galileo Galilei", Università degli Studi di Padova, Via Marzolo, I-35131, Padova, Italy
\textsuperscript{d}Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Via Marzolo, I-35131, Padova, Italy

Abstract

In this paper we present the partial wave unitarity bound in the parameter space of dimension-5 and dimension-6 effective operators that arise in a compositeness scenario. These are routinely used in experimental searches at the LHC to constraint contact and gauge interactions between ordinary Standard Model fermions and excited (composite) states of mass $M$. After deducing the unitarity bound for the production process of a composite neutrino, we implement such bound and compare it with the recent experimental exclusion curves for Run 2, the High-Luminosity and High-Energy configurations of the LHC. Our results also applies to the searches where a generic single excited state is produced via contact interactions. We find that the unitarity bound, so far overlooked, is quite compelling and significant portions of the parameter space ($M, \Lambda$) become excluded in addition to the standard request $M \lesssim \Lambda$.

Keywords: perturbative unitarity, composite models, composite fermions, LHC Run 2, High Luminosity and High Energy LHC

1. Introduction

It is well known that partial wave unitarity is a powerful tool to estimate the perturbative validity of effective field theories (EFTs). It has been used in the past to provide useful insights both in strong and electroweak interactions\cite{1} as well as in quantum gravity\cite{2}. Perhaps the best known example is the bound on the Higgs mass derived from an analysis of $WW \rightarrow WW$ scattering within the Standard Model (SM)\cite{1,3}. On the other end, unitarity has also been applied to a number of approaches beyond the Standard Model (BSM). For instance in composite Higgs models\cite{4}, in searches of scalar di-boson resonances\cite{5,6}, searches for dark matter effective interactions\cite{7} and on generic dimension-6 operators\cite{8}.

One possible BSM alternative, widely discusses in literature and routinely pursued in high-energy experiments, is a composite-fermions scenario which offers a possible solution to the hierarchy pattern of fermion masses\cite{9–13}. In this context\cite{16–19}, SM quarks “$q$” and leptons “$\ell$” are assumed to be bound states of some as yet not observed fundamental constituents generically referred as preons. If quarks and leptons have an internal substructure, they are expected to be accompanied by heavy excited states $^{\ell'} q'$ of masses $M$ that should manifest themselves at an unknown energy scale, the compositeness scale $\Lambda$.

As customary in an EFT approach, the effects of the high-energy physics scale, here $\Lambda$, are captured in higher dimensional operators that describe processes within a lower energy domain, where the fundamental building blocks of the theory cannot show up. Hence, the heavy excited states may interact with the SM ordinary fermions via dimension-5 gauge interactions of the $SU(2)_L \otimes U(1)_Y$ SM gauge group of the magnetic-moment type (so that the electromagnetic current conservation is not spoiled by e.g. $\ell' \gamma$ processes\cite{17}). In addition, the exchange of preons and/or binding quanta of the unknown interactions between ordinary fermions and/or the excited states results in effective contact interactions (CI) that couple the SM fermions and heavy (excited) states\cite{18–21}. In the latter case, the dominant effect is expected to be given by the dimension-6 four-fermion interactions scaling with the inverse square of the compositeness scale $\Lambda$:

\begin{equation}
\mathcal{L}^6 = \frac{g^2}{\Lambda^2} \frac{1}{2} \bar{f}_j \gamma_\mu f_L, \tag{1a}
\end{equation}

\begin{equation}
\bar{f}_j = \bar{\eta}_L \bar{f}_L \gamma_\mu f_L + \bar{\eta}_R \bar{f}_R \gamma_\mu f_L + \bar{\eta}_L \bar{f}_L \gamma_\mu f_R + \text{h.c.} + (L \rightarrow R), \tag{1b}
\end{equation}

where $g^2 = 4\pi$ and the $\eta$’s factors are usually set equal to unity.

In this work the right-handed currents will be neglected for simplicity (this is also the setting adopted by the experimental collaborations).

As far as gauge interactions (GI) are concerned, let us consider the first lepton family and assume that the excited neutrino and the excited electron are grouped into left-handed singlets and a right-handed SU(2) doublet: $\ell^{(*)}_L, \nu^{(*)}_L, L^\alpha = (\nu_L, \ell^\alpha_L)^T$, so that a magnetic type coupling between the left-handed SM doublet and the right-handed excited doublet via the SU(2)$_L \otimes U(1)_Y$ gauge fields can be written down\cite{17,22}:

\begin{equation}
\mathcal{L}^5 = \frac{1}{2\Lambda} \bar{L}_\mu \sigma^{\mu\nu} \left( g f^{\tau}_{\mu\nu} \gamma_\tau - g' f' Y B_{\mu\nu} \right) L_L + \text{h.c.}. \tag{2}
\end{equation}

Here, $L^\tau = (\nu_L^\tau, \bar{\ell}_L^\tau)$ is the ordinary lepton doublet, $g$ and $g'$ are the SU(2)$_L$ and U(1)$_Y$ gauge couplings and $W_{\mu\nu}, B_{\mu\nu}$ are the
field strength tensor of the corresponding gauge fields respectively; \( \tau \) are the Pauli matrices and \( Y \) is the hypercharge, \( f \) and \( f' \) are dimensionless couplings and are expected (and assumed) to be of order unity.

Excited states interacting with the SM sector through the model Lagrangians (1a)-(1b) and (2) have been extensively searched for at high-energy collider facilities. The current strongest bounds are due to the recent LHC experiments. Charged leptons (\( e^+, \mu^+ \)) have been searched for in the channel \( pp \rightarrow \ell^\pm \rightarrow \ell\ell\gamma \) [23–29], i.e. produced via CI and then decay via GI, and in the channel \( pp \rightarrow \ell^\pm \rightarrow \ell\ell q\bar{q}' \) [30] where both production and decay proceed through CI. Neutral excited leptons have been also discussed in the literature and the corresponding phenomenology at LHC has been discussed in detail in the case of a heavy composite Majorana neutrino \( N^* \) [31].

A dedicated experimental analysis has been carried out by the CMS collaboration [32] on LHC data collected for \( \sqrt{s} = 13 \) TeV and looking for the process

\[
pp \rightarrow \ell N_i^* \rightarrow \ell\ell q\bar{q}'
\]  

(3)

with dilepton (dielectrons or dimuons) plus diquark final states. The existence of \( N_i^* \) is excluded for masses up to 4.6 (4.70) TeV at 95% confidence level, assuming \( M = \Lambda \). Moreover, the composite Majorana neutrinos of this model can be responsible for baryogenesis via leptogenesis [33, 34]. The phenomenology of other excited states has also been discussed in a series of recent papers [31, 35–41].

We emphasize that in all phenomenological studies referenced above as well as all experimental analyses that have searched for excited states at colliders, it is customary to impose the constraint \( M \leq \Lambda \) on the parameter space of the model. To the best of our knowledge unitarity has never been taken into account and/or discussed in connection with the effective interactions of the so-called excited states. The main goal of this work is to report instead that the unitarity bounds, as extracted from Eq. (1a)-(1b) and (2), are quite compelling and should be included in future studies of such effective composite models because they constrain rather strongly the parameter space. While we present an explicit calculation of the unitarity bound for heavy composite neutrino searches, we expect that similar bounds (i.e. equally compelling) would apply for excited electrons (\( e^+ \)), muons (\( \mu^+ \)) and quarks (\( q^+ \)). Indeed, the effective operators that describe the latter excited states have the very same structure of those referred to the composite neutrinos.

2. Unitarity in single-excited-fermion production

For the derivation of the unitarity bound, we adopt a standard method that makes use of the optical theorem and the expansion of the scattering amplitude in partial waves. In order to specify the CI and GI Lagrangians for a definite situation, we consider the production of the excited Majorana neutrino at the LHC. However, we shall highlight when the results apply to other composite fermion states in the following.

The central object that we shall derive from the operators in the effective Lagrangians (7) and (8) is the interacting part of the \( S \) matrix, indicated with \( T \) in this letter. It enters the partial wave decomposition of the scattering amplitude as follows

\[
M_{i \rightarrow f}(\theta) = 8\pi \sum_j (2j + 1) T_{i \rightarrow j}^{d_{i \rightarrow j}}(\theta) ,
\]  

(4)

where \( j \) is the eigenvalue of the total angular momentum \( J \) of the incoming (outgoing) pair, \( d_{i \rightarrow j}^{d_{i \rightarrow j}}(\theta) \) is the Wigner d-function and \( \lambda_i \) (\( \lambda_f \)) is the total helicity of the initial (final) state pair. Without loss of generality, we consider azimuthally symmetric processes and fix \( \phi = 0 \) accordingly. From the optical theorem and the decomposition in eq. (4), one can find the perturbative unitarity condition of an inelastic process for each \( j \) to be

\[
\sum_{i \neq j} |\beta_i \beta_j|^2 j^2 \leq 1 ,
\]  

(5)

where \( \beta_i \) (\( \beta_j \)) is the factor obtained from the two-body phase space and reads for two generic particles with masses \( m_1 \) and \( m_2 \)

\[
\beta = \sqrt{|s - (m_1 - m_2)^2| / s}.
\]  

(6)

It corresponds to the particle velocity when \( m_1 = m_2 \). It is important to notice that the unitarity bound is imposed on the subprocess involving the proton valence quarks as initial state, namely \( q\bar{q}' \rightarrow \ell N_i^* \) as shown in Figure 1. Then, for the process of interest, the relevant interaction(s) are as follows:

\[
L_{CI} = \frac{g^2}{\Lambda} q^\mu q'^\nu \gamma^\mu P_L q \gamma^\nu P_L f L + h.c. ,
\]  

(7)

\[
L_{GI} = \frac{g f}{\sqrt{2}\Lambda} N_i \sigma^{\mu\nu} (\partial_\mu W_\nu^\nu) P_L f L + h.c. .
\]  

(8)

Accordingly, in Eq. (6), \( s \) denotes the center-of-mass energy in each collision and it is obtained from the nominal collider energy and the parton momentum fractions as \( s = x_1 x_2 s \). As far as the kinematic is concerned, \( \beta_i \approx 1 \) can be used since the valence quark masses are negligible with respect to the center-of-mass energy. Instead, one finds \( \beta_i = 1 - M^2 / s \) for the final state, where the composite neutrino mass has to be kept.

The core of the method relies on the derivation of the amplitude for the process of interest induced by the contact and gauge-mediated effective Lagrangians (7) and (8). Then, one matches the so-obtained result for \( M_{i \rightarrow f} \) with the r.h.s of eq. (4) and extracts the corresponding \( T_{i \rightarrow f}^{d_{i \rightarrow f}} \) for each definite eigenvalue of the total angular momentum \( J \). The latter are inserted into
eq. (5) in order to derive the unitarity condition that the model parameters (\(\Lambda, M, g, \gamma\)) and the center-of-mass energy have to obey. To this end, the amplitude \(M_{VW}\) is decomposed in terms of definite helicity states and, therefore, we have to express the initial and final state particles spinors accordingly [42]. The helicity of each particle in the initial or final state is \(\lambda = \pm 1/2\), being all the involved particles fermions (also the composite states. We keep the boson mediates the scattering between the initial and final \(W\) way. A dimension-5 operator is involved and, in this case, the composite model and the helicity amplitudes are found to be much smaller than the typical \(\hat{s}\) values of the \(pp\) interactions [39]. In particular, it was shown that cross sections and the SM expectations was observed in the search, but the whole dataset of the Run 2 of the LHC still needs to be analysed. Therefore, the issue of the unitarity condition on the accessible parameter space \((M, \Lambda)\) urges to be assessed. As usual in BSM searches, the absence of a signal excess over the SM background is translated into an experimental bound on the parameter space \((M, \Lambda)\). Moreover, the sensitivity of this search was investigated for two future collider scenarios: the High-Luminosity LHC (HL-LHC), with a centre-of-mass energy of 14 TeV and an integrated luminosity of 3 ab\(^{-1}\), and the High-Energy LHC (HE-LHC), with a centre-of-mass energy of 27 TeV and an integrated luminosity of 15 ab\(^{-1}\) [43]. The projection studies, included in the recent Yellow Report CERN publication [44, 45], have shown the potential of such facilities in reaching much higher neutrino masses.

In this section, the perturbative unitarity bounds are applied to these searches in the dilepton and a large-radius jet channel with the CMS detector for the three different collider scenarios. As already clear from the rather different coupling values entering the Lagrangians (7) and (8), namely \(g/\sqrt{2} \approx 1\) versus \(g_2^2 = 4\pi\), the production mechanism of a heavy composite neutrino and other excited states is dominated by the contact interaction mechanism [39]. In particular, it was shown that cross sections in contact-mediated production are usually more than two orders of magnitude larger than the gauge mediated ones for all values of the \(\Lambda\) and \(M\) relevant in the analyses. This means that it is a reasonable approximation to consider only the bounds given in Eq. (11) to constraint the unitarity violation of the signal samples.

In order to estimate the effect of the unitarity condition on LHC searches, we need to implement the bounds in the case of a hadron collisions. Then, the square of the centre-of-mass energy of the colliding partons system, \(\hat{s}\), does not have a definite value, where \(x_1\) and \(x_2\) are the parton momentum fractions and \(\sqrt{s}\) is nominal energy of the colliding protons. To this aim, we have estimated \(\hat{s}\) in each event generated in the Monte Carlo (MC) samples, and we have plugged the result into Eq. (11) in order to obtain a level curve on the parameter space for which all the events satisfy the unitarity bound. The MC samples for the signal are generated at Leading Order (LO) with CalcHEP (v3.6) [46] for \(\sqrt{s} = 13, 14\) and 27 TeV proton-proton

\[
\begin{align*}
T_{j=1}^{\pm} (\pm, \pm) & = \frac{g^2}{12 \pi \Lambda^2} \left(1 - \frac{M^2}{s} \right)^{-\frac{1}{2}}, \\
T_{j=1}^{\pm} (\pm, \pm) & = \frac{\sqrt{2} g_2^2}{12 \sqrt{2} \pi \Lambda^2} \left(1 - \frac{M^2}{s} \right)^{-\frac{1}{2}}.
\end{align*}
\]

The production of heavy composite Majorana neutrinos has been studied by the CMS Collaboration by measuring the final state with two leptons and at least one large-radius jet, with data from \(pp\) collisions at \(\sqrt{s} = 13\) TeV and with an integrated luminosity of 2.3 fb\(^{-1}\) [32]. Good agreement between the data and the SM expectations was observed in the search, but the whole dataset of the Run 2 of the LHC still needs to be analysed. Therefore, the issue of the unitarity condition on the accessible parameter space \((M, \Lambda)\) urges to be assessed. As usual in BSM searches, the absence of a signal excess over the SM background is translated into an experimental bound on the parameter space \((M, \Lambda)\). Moreover, the sensitivity of this search was investigated for two future collider scenarios: the High-Luminosity LHC (HL-LHC), with a centre-of-mass energy of 14 TeV and an integrated luminosity of 3 ab\(^{-1}\), and the High-Energy LHC (HE-LHC), with a centre-of-mass energy of 27 TeV and an integrated luminosity of 15 ab\(^{-1}\) [43]. The projection studies, included in the recent Yellow Report CERN publication [44, 45], have shown the potential of such facilities in reaching much higher neutrino masses.
collisions, using the NNPDF3.0 LO parton distribution functions with the four-flavor scheme [47], spanning over the \((\Lambda, M)\) region covered by the experimental searches [43]. The information on the parton momenta is then retrieved from the LHE files of each signal process through the MadAnalysis framework [48].

We have explicitly checked that, in the mass range explored, the \(\hat{s}\)-distributions in our MC simulations are peaked at values around \(M^2\) almost irrespective of the nominal collider energy \(\sqrt{s} = 13, 14, 27\) TeV. This is somehow expected on general grounds since in the generated signal events the available energy, \(\hat{s}\), is mostly used to produce a heavy excited state (of mass \(M\)). Of course, the larger the collider energy the more prominent the distribution tails at high \(\hat{s}\) values. These expectations are corroborated by analytical expressions for the \(\hat{s}\)-distributions that can be retrieved from ref. [31], involving only the product of the parton luminosity functions and the (hard) production process cross section.

The results are presented in Figs. 2, 3 and 4 for the Run 2, HL-LHC and HE-LHC scenario respectively, where the solid violet lines represent the unitarity bound. Accordingly the violet shaded areas define the regions where the model should not be trusted because unitarity is violated for such \((M, \Lambda)\) values.

### 4. Discussion and Results

Let us elaborate on our findings and explain their impact on the experimental analyses carried out at the LHC. First of all, the experimental outcomes are summarized with exclusion regions in the \((M, \Lambda)\) plane, which are in turn set with the 95% C.L. observed (Run 2) [32] and expected limit (HL/HE-LHC) [44, 45], namely the dashed blue lines in Figure 2, 3 and 4 respectively. Above these lines the model is still viable, whereas below it is excluded. The largest excluded values of the composite neutrino mass is extracted by intersecting the 95% C.L. exclusion curves from the experimental analyses, LHC Run 2 or from the projection studies HL-LHC and HE-LHC, with the \(M \leq \Lambda\) constraint (dot-dashed gray line and gray shaded region in Figure 2, 3 and 4). This is the widely adopted condition imposed on the model validity and it originates from asking the heavy excited states to be at most as heavy as the new physics scale \(\Lambda\). Despite it is a reasonable constraint, it does not take into account the typical energy scale that enters the production process, i.e. \(\hat{s}\). The corresponding mass values are reported in the first raw of Table 1 for the three collider settings.

The unitarity condition in Eq. (11) is represented with the solid violet line in Figure 2, 3 and 4, and it defines the region of the parameter space \((M, \Lambda)\) where unitarity is satisfied (above the solid violet line) or violated (violet shaded area). It is clear that the unitarity bound is much more restrictive than \(M \leq \Lambda\) and it shrinks the available parameter space quite considerably. If one applies the unitarity bound to the experimental results by
Table 1: In the first line we quote the bounds quoted in the CMS analysis of like sign dileptons and diquark [32] and subsequent projections studies at HL-LHC and HE-LHC [43–45]. In second line, we quote instead the strongest mass bound obtained from Figures (2,3,4) when the line of the perturbative unitarity bound crosses the 95% C.L. exclusion curve from the experimental and/or projections studies.

|              | LHC Run 2 | HL-LHC | HE-LHC |
|--------------|-----------|--------|--------|
| $M = \Lambda$ | $M \leq 4.6$ TeV [32] | $M \leq 7.8$ TeV [43–45] | $M \leq 12$ TeV [43–45] |
| Unitarity    | $M \leq 2.6$ TeV ($\Lambda = 9.0$ TeV) | $M \leq 5.0$ TeV ($\Lambda = 20$ TeV) | $M \leq 7.0$ TeV ($\Lambda = 30$ TeV) |

In conclusion, we studied the perturbative unitarity bound extracted from the effective gauge and contact Lagrangians for a composite-fermion model. On general grounds, an effective theory is valid up to energy/momentum scales smaller than the large energy scale that sets the operator expansion. Since collider experiments are involving more and more energetic particle collisions, the usage and the applicability of effective operators can be questioned. In order to address this issue and to be on the safe side, one can impose the unitarity condition both on the EFT parameters ($M, \Lambda, g', g$) and the energy involved in a given process. To the best of our knowledge, such a constraint was not derived for the model Lagrangians in Eq. (7) and (8), and we have obtained the corresponding unitarity bounds, namely Eqs. (11) and (14). Thus, the applicability of the effective operators describing the production of composite neutrinos (and other excited states) has to be restricted accordingly.

It is the authors’ opinion that the findings here discussed will have a significant impact on ongoing and future experimental searches for excited states coupling to the SM fermions with the interactions given in (7) and (8). At the very least, the unitarity bounds play the role of a collider-driven theoretical tool for interpreting the experimental results for the considered composite models, more rigorous than the simple relation $M \leq \Lambda$.

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