A Novel Approach Algorithm for Determining the Initial Basic Feasible Solution for Transportation Problems

E.M.D.B. Ekanayake¹, E. M. U. S. B. Ekanayake¹
¹Department of Physical Sciences, Faculty of Applied Sciences, Rajarata University of Sri Lanka, Sri Lanka

Corresponding Author: Ekanayake; Email: dananjayaekanayake96@gmail.com

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In optimization, the transportation problem is of utmost significance. The fundamental idea for solving the transportation problem is to develop methods that lower the overall cost between the source and the destination. Numerous research techniques are given in the literature as solutions to transportation problems. The majority of strategies focus on locating the initial basic, feasible solution to the transportation problem, while some techniques concentrate on locating an optimal solution. To find an initial, basic, feasible solution, Vogel's approximation, the least-cost method, the northwest method, and other methods are used. The Modified Distribution Method and the Stepping Stone Method are gaining acceptance as the optimal solution to the transportation problem. In this research study, a model approach to figuring out the basic, feasible solution for transportation problems with balanced and unbalanced components is proposed based on the average unit cost value of columns and rows. A new table is created using the unit cost values and average unit cost values in the columns and rows and compared to the transportation problem to solve the problem by balancing demand and supply. The proposed methodology is effortless, easy to understand, and simple to use. Comparatively speaking, the algorithmic technique suggested by this work is less complex than the well-known meta-heuristic algorithms in the literature. Finally, provide a case study to demonstrate the proposed approach.

INTRODUCTION

Transportation problems can be expressed and resolved as a linear equation also with the expectation of minimizing the total cost of transportation problems. F. L. Hitchcock (1941) proposed the first mode of transportation. Afterward, the algorithm is implemented by T.C. Koopmans (1949) and G.B. Dantzig (1951). Dantzig became the first to propose a transportation problem-solving method. He invented the phrase “North West Coner Method (NWCM)” to describe his approach. This methodology depends on the specified location. Later, the Column Minimum Method (CAM) and Row Minimum Method (RAM) were also introduced. The primary goal of these was to find an initial basic feasible solution. These methods can provide a basic feasible solution to the transportation problem. Dantzig later developed an algorithm called the least-cost method in 1963. Reinfeld and Vogel (1958) pioneered the VAM method. In addition, Goyal (1984) enhanced the VAM for the unbalanced transportation problem by including the dummy’s maximum cost. The methods described above can be used to identify the most basic feasible solution to the transportation problem. After obtaining the initial basic feasible solution using these procedures, there are two main methods for determining the optimal solution. “Stepping Stone Method” (SSM) was the first optimal solution search method to be introduced. Charnes and Cooper created this. The other optimal solution is the Modified Distribution Method (MODI), introduced in 1955.

Moreover, there are other models for evaluating the initial basic, feasible solution to the transportation problem. For example, Priyanka and
Sushma devised the Average Transportation Cost Method (ATCM). The modified ATCM was developed by Priyanka and Sushma, which calculated a penalty equivalent two the average of two minimum costs per row and column. In 2002, Sabawi and Hayawi proposed a new solution to the transport model problem by using the difference within cost in the highest and lowest cost. Several new methods have recently been introduced, including Ekanayake at el. An Effective Alternative New Approach to Solving Transportation Problems (2021) and A Modified Ant Colony Optimization Algorithm for Solving Transportation Problems. Babu et al. (2020) developed the lowest allocation method (LAM), which began with the smallest supply (2013). Introduced To find a basic feasible solution, Das et al. (2014) proposed the Advanced Vogel's Approximation Method (AVAM).

In 2015, Alkubaisi calculated a median penalty cost, while VAM calculated the difference between two minimum penalty costs. In 2017, Hessain introduced the Average Row Penalty (ARP) and Average Column Penalty (ACP) methods to find the basic feasible solution value. Kaur et al. upgraded the Maximum Method (MDM) in 2018. Moreover, several new approaches for determining initial basic feasible solutions to unbalanced problems have recently been introduced. There is Ahmad (2020), A New method establishing the IBFS to the Unbalanced Transportation Problem. Gill et al (2020), An Improved Algorithm for Optimal Solution of Unbalanced Transportation Problems (2020), and a new approach to solving unbalanced transportation problems using the least cost method (LCM), A. Sridhar et al. (2018), and also introduced a solving method for unbalanced fuzzy transportation problems by Dr. Muruganandam (2016).

Meanwhile, a new algorithm has been presented to solve the problems of real life by using the transportation model. A study on applications of transportation problems to minimize cost in various pharmacies (K. Subhikshaa et al, 2019), Application of Operations Research in the Steel Industry (Mollah et al, 2021), Solution of a Transportation Problem using Bipartite Graph (Ekanayake et al, 2021), aggregate planning using transportation method: a case study in cable industry (Sultana et al, 2014) and Solving A Transportation Problem Actual Problem Using Excel Solver (Abdelwali et al, 2019). Such research methods can be pointed out as examples of that. Case Studies of Market Research for Three Transportation Communication Products (Parish, 1994), minimization of transport costs in an industrial company through linear programming (Prifti et al, 2020), optimal feasible solutions to a road freight transportation problem (Latunde et al,2020) and Analysis of Transportation Method in Optimization of Distribution Cost Using Stepping Stone Method and Modified Distribution (Febriani et al., 2021). Meanwhile, through the above-mentioned research methods, the research has been improved to an initial solution as well as to its optimal solution.

All of the research papers mentioned above have attempted to solve the problem in such a way that the cost of the transportation problem is minimized. Although this model is a method based on cost minimization, its various uses are carried out in operational research. It is used to carry out various tasks in daily life efficiently and effectively. Here, a mathematical approach is given to these problems, and the problem is guided so that the objective considered initially is a minimum value. Thus, obtaining an optimal solution is the primary objective of a transportation problem. It is best to get a basic solution to a transportation problem before getting the optimal one. It requires driving the solution so that it is close to the optimal solution or the optimal solution in the least number of steps. For that purpose, a new algorithm was introduced for this research, and it can be shown by the results obtained as a successful method that was guided by the abovementioned objectives. Several definitions are commonly used in transportation problems, and it is important to discuss them.

**METHODS**

**Preliminaries**

1. **Source:** Where the commodities are located, a transportation problem is a supply. This is frequently where the factory started. The transport table shows that \((S_1, S_2, ..., S_n)\).
2. **Destination:** The demand for a transportation problem is the storage of goods supplied from sources. This is often referred to as “destination storage”. The transport table represents this as \((D_1, D_2, ..., D_n)\).
3. **Supply limit:** The supply limit is the number of goods available at a source to meet demand.
4. Demand requirement: The quantity of goods required to meet demand is referred to as the demand requirement.

5. Initial Basic Feasible Solution (IBFS): If the basic parameters are \( m + n - 1 \), then the initial basic feasible solution applies to a situation with \( m \) sources and \( n \) destinations.

6. Optimal Solution: When the initial basic feasible solution is the optimum, it is called the optimum solution.

Mathematical formulation
When solving a transportation problem, it is mathematically derived. The equations can be presented as follows: A single product must be shipped from the warehouses to the outlets. Each warehouse has a specific supply \( m \) source denoted by \( S_1, S_2, \ldots, S_m \) with respective capacities \( a_1, a_2, \ldots, a_m \) and each outlet has a specific demand \( n \) denoted by \( D_1, D_2, \ldots, D_n \) with respective demands \( b_1, b_2, \ldots, b_n \).

Additionally, consider the cost of transportation from \( i^{th} \) source to \( j^{th} \) sink is \( C_{ij} \) and the amount shipped is \( X_{ij} \), where \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

Mathematical Model
The total transportation cost is

Minimize \( \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij} \)

Subject to the constraints

1. \( \sum_{j=1}^{n} X_{ij} = a_i, \quad i=1,2,\ldots,m \)
2. \( \sum_{i=1}^{m} X_{ij} = b_j, \quad j=1,2,\ldots,n \)

and

3. \( X_{ij} \geq 0 \) for all \( i=1, 2, \ldots, m \) and \( j=1,2,\ldots,n \)

Note that in this case, the sum of the supplies and demands equal the overall. i.e., \( \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \). Such problems are called balanced transportation problems and otherwise, i.e., \( \sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \), known as unbalanced transportation problems.

1. \( \sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j \)
2. \( \sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j \)

Add a dummy origin to the transportation table, and set the cost for such an origin to be zero.

The availability of this source.

\[ \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 0. \]

Transportation tableau
The transportation problem can be described using a mathematical model based on linear programming, and it is typically displayed in a transportation tableau.

| To destination \( \rightarrow \) | \( D_1 \) | \( D_2 \) | \( \ldots \) | \( D_n \) | Supply |
|-------------------------------|-----------|-----------|-------------|-----------|--------|
| \( \downarrow \) From source   | \( C_{11} \) | \( C_{12} \) | \( \ldots \) | \( C_{1n} \) | \( a_1 \) |
| \( S_1 \)                      | \( X_{11} \) | \( X_{12} \) | \( \ldots \) | \( X_{1n} \) |        |
| \( S_2 \)                      | \( C_{21} \) | \( C_{22} \) | \( \ldots \) | \( C_{2n} \) | \( a_2 \) |
| \( \vdots \)                   | \( \vdots \) | \( \vdots \) | \( \ldots \) | \( \vdots \) |        |
| \( S_m \)                      | \( C_{m1} \) | \( C_{m2} \) | \( \ldots \) | \( C_{mn} \) | \( a_m \) |
| \( \downarrow \) Demand \( b_j \) | \( b_1 \) | \( b_2 \) | \( \ldots \) | \( b_n \) | \( \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \) |
After the transportation problem is formulated as mentioned above, researchers have developed various algorithms to find solutions to it, and some of the popular methods used to find basic solutions are shown below.

**Northwest Corner Method (NWCM)**

The following steps are used to determine the transportation cost by the northwest corner method.
1. Step 1 - Begin with each upper-left cell of the transportation table's northwest corner.
2. Step 2 - For the first row and the first column, allocate an item equal to the needed value. If the allocation is the same as the supply from the first source, move it down vertically, that is, repeat the above steps for the second row and the first column.
3. Step 3 - Allocate the same sources as the supply on the second row as well. Finish the problem by the demand number of items in a row from left to right horizontally and column downwards in this way.
4. Step 4 - Then calculate the relevant transportation cost.

**Least cost method (LCM)**

The least cost method is another useful strategy for identifying the initial basic feasible solution to the transportation problem. The next step should be taken for that method.
1. Find the cell's position in the transportation problem table that has the lowest unit cost. Remove the row or column after completing as much of the supply or demand for this cell as you can. Cross that row and that column if both the row and the column are satisfied simultaneously.
2. For all uncrossed rows and columns, it will choose the cell with the lowest unit cost to equalize supply and demand. Complete the supply or demand, and remove that row or column.
3. To solve the problem, pick the cell with the lowest unit cost, delete every row and column, so that all the supply and demand are met, and finish the problem.
4. Then calculate the relevant transportation cost.

**Vogel's approximation method (VAM)**

The efficient answer can be found using the VAM, which is close to it. It follows the steps below to find the basic feasible solution values.
1. Frisk, we will use the difference between the smallest unit cost and the next smallest unit cost in the transportation table to calculate penalties for each row and column.
2. Then choose the column or row with the highest value of penalties. Then choose the cell that belongs to that column or row and has the lowest unit cost. Remove the row or column after completing as much of the supply or demand for this cell as you can. Cross that line if both the row and the column are satisfied.
3. Repeat the above steps until all supplies and demands are met.
4. All the supplies and demand are met, complete the process and calculate the transportation cost.

The new algorithm that we introduce, which is a very important part of this research solution, can be introduced. It is developed based on the average value of the unit costs of the column and rows of the transportation problem, and it can be described as follows:

**Proposed Algorithm**

Step 1. If the transportation problem is unbalancing a dummy column or dummy row should be added to make it balanced.
Step 2. For each column and row the average unit transportation cost value. (If the problem is unbalanced, do not take the dummy row or dummy column into account when calculating the average value).
Step 3. Create a new table (called the “maximum value table”) using new values for each cell. By adding the corresponding row and column average unit costs and subtracting the unit cost of each cell, new cell values are calculated.
Step 4. Select the value in the table of transportation problems corresponding to the first highest value in the first column of the above table. (If they have the same highest value, consider the minimum value of the corresponding supply column to find the cell and give it a priority. Also, in the first round of the next reconsideration, the other cell should be given priority.
Step 5. Thus, select the maximum value from the first highest value in the first round and the second highest value from the second round in the maximum value table. Complete the supply...
and demand values for the transportation problem table's left-to-right move.

Step 6. Change the supply and demand for the chosen cell. To indicate that no further assignment can be made in that row or column, cross out the row or column with zero supply or demand.

Step 7. Repeat steps 5 and 6 for the other columns from left to right.

Numerical Example
Balance Transportation Problem

Problem 1.

Table 2. Table representation of balance transportation problem

|        | D₁  | D₂  | D₃  | D₄  | Supply |
|--------|-----|-----|-----|-----|--------|
| S₁     | 11  | 13  | 17  | 14  | 250    |
| S₂     | 16  | 18  | 14  | 10  | 300    |
| S₃     | 21  | 24  | 13  | 10  | 400    |

Demand 200 225 275 250

Step 2
Table 3. The average values of each column and row are given in the following table

| Column 1 | D₁  | D₂  | D₃  | D₄  | Supply | Average of Row |
|----------|-----|-----|-----|-----|--------|----------------|
| S₁       | 11  | 13  | 17  | 14  | 250    | 13.75         |
| S₂       | 16  | 18  | 14  | 10  | 300    | 14.5          |
| S₃       | 21  | 24  | 13  | 10  | 400    | 17            |

Average of column 16 18.33333 14.66667 11.33333

Step 3
Table 4. Maximum value table

|       |      |      |      |      |        |        |
|-------|------|------|------|------|--------|--------|
|       | 18.75| 19.08333| 11.41667| 11.08333|        |        |
|       | 14.5 | 14.83333| 15.16667| 15.83333|        |        |
|       | 12   | 11.33333| 18.66667| 18.33333|        |        |

Following steps 4, 5 and 6
Table 5. Iteration 1- For the first highest value corresponding to the 1st column of the maximum value table, give the cell the first allocation of the corresponding transportation problem

|        | D₁  | D₂  | D₃  | D₄  | Supply |
|--------|-----|-----|-----|-----|--------|
| S₁     | 11  | 13  | 17  | 14  | 250    |
| S₂     | 16  | 18  | 14  | 10  | 300    |
| S₃     | 21  | 24  | 13  | 10  | 400    |

Demand 200 225 275 250

Step 8. Calculate the transportation problem’s initial basic feasible solution.

The solutions obtained using the algorithm presented in this research paper can be shown by the following transportation problems. It has been shown in detail how the final answer was reached according to the relevant steps.
Table 6. Iteration 2 - For the first highest value corresponding to the 2nd column of the maximum value table, give the cell the next allocation of the corresponding transportation problem.

|      | D₁     | D₂     | D₃     | D₄     | Supply     |
|------|--------|--------|--------|--------|------------|
| S₁   | 11 (200) | 13 (50) | 17    | 14    | 250 (50) (0) |
| S₂   | 16    | 18    | 14    | 10    | 300        |
| S₃   | 21    | 24    | 13    | 10    | 400        |
| Demand | 200 (0) | 225 (175) | 275   | 250   |

Table 7. Iteration 3 - For the first highest value corresponding to the 3rd column of the maximum value table, give the cell the next allocation of the corresponding transportation problem.

|      | D₁     | D₂     | D₃     | D₄     | Supply     |
|------|--------|--------|--------|--------|------------|
| S₁   | 11 (200) | 13 (50) | 17    | 14    | 250 (50) (0) |
| S₂   | 16    | 18    | 14    | 10    | 300        |
| S₃   | 21    | 24    | 13 (275) | 10    | 400 (125) |
| Demand | 200 (0) | 225 (175) | 275 (0) | 250 (125) |

Table 8. Iteration 4 - For the first highest value corresponding to the 4th column of the maximum value table, give the cell the next allocation of the corresponding transportation problem.

|      | D₁     | D₂     | D₃     | D₄     | Supply     |
|------|--------|--------|--------|--------|------------|
| S₁   | 11 (200) | 13 (50) | 17    | 14    | 250 (50) (0) |
| S₂   | 16    | 18    | 14    | 10    | 300        |
| S₃   | 21    | 24    | 13 (275) | 10 (125) | 400 (125) (0) |
| Demand | 200 (0) | 225 (175) | 275 (0) | 250 (125) |

For the second round, check the second highest value from the maximum value table, as all columns relevant to the first round have been checked.

Table 9. Iteration 5 - For the second highest value corresponding to the 2nd column of the maximum value table, give the cell the next allocation of the corresponding transportation problem.

|      | D₁     | D₂     | D₃     | D₄     | Supply     |
|------|--------|--------|--------|--------|------------|
| S₁   | 11 (200) | 13 (50) | 17    | 14    | 250 (50) (0) |
| S₂   | 16    | 18 (175) | 14    | 10    | 300 (125) |
| S₃   | 21    | 24    | 13 (275) | 10 (125) | 400 (125) (0) |
| Demand | 200 (0) | 225 (175) (0) | 275 (0) | 250 (125) |

Table 10. The table represents the final allocation for each unit cost value.

|      | D₁     | D₂     | D₃     | D₄     | Supply     |
|------|--------|--------|--------|--------|------------|
| S₁   | 11 (200) | 13 (50) | 17    | 14    | 250 (50) (0) |
| S₂   | 16    | 18 (175) | 14    | 10 (125) | 300 (125) (0) |
| S₃   | 21    | 24    | 13 (275) | 10 (125) | 400 (125) (0) |
| Demand | 200 (0) | 225 (175) (0) | 275 (0) | 250 (125) (0) |
Transportation cost = \((11 \times 200) + (13 \times 50) + (18 \times 175) + (13 \times 275) + (10 \times 125) + (10 \times 125) = 12075\)

**Unbalance Transportation Problem**

**Problem 2**

**Table 11. Table representation of unbalance transportation problem**

|       | \(D_1\) | \(D_2\) | \(D_3\) | \(D_4\) | \(D_5\) | Supply |
|-------|---------|---------|---------|---------|---------|--------|
| \(S_1\) | 5       | 1       | 8       | 7       | 5       | 15     |
| \(S_2\) | 3       | 9       | 6       | 7       | 8       | 25     |
| \(S_3\) | 4       | 2       | 7       | 6       | 5       | 42     |
| \(S_4\) | 7       | 11      | 10      | 4       | 9       | 35     |
| Demand  | 30      | 20      | 15      | 10      | 20      | 22     |

**Step 1**

**Table 12. Added a dummy column and balance the problem**

|       | \(D_1\) | \(D_2\) | \(D_3\) | \(D_4\) | \(D_5\) | Dummy column | Supply |
|-------|---------|---------|---------|---------|---------|--------------|--------|
| \(S_1\) | 5       | 1       | 8       | 7       | 5       | 0            | 15     |
| \(S_2\) | 3       | 9       | 6       | 7       | 8       | 0            | 25     |
| \(S_3\) | 4       | 2       | 7       | 6       | 5       | 0            | 42     |
| \(S_4\) | 7       | 11      | 10      | 4       | 9       | 0            | 35     |
| Demand  | 30      | 20      | 15      | 10      | 20      | 22           |        |

**Table 13. average values of each column and row are given in the following table. (Do not take the dummy column values to calculate the average values.)**

| Column 1 | \(D_1\) | \(D_2\) | \(D_3\) | \(D_4\) | \(D_5\) | Dummy column | Supply | Average of row |
|----------|---------|---------|---------|---------|---------|--------------|--------|---------------|
| \(S_1\)  | 5       | 1       | 8       | 7       | 5       | 0            | 15     | 5.2           |
| \(S_2\)  | 3       | 9       | 6       | 7       | 8       | 0            | 25     | 6.6           |
| \(S_3\)  | 4       | 2       | 7       | 6       | 5       | 0            | 42     | 4.8           |
| \(S_4\)  | 7       | 11      | 10      | 4       | 9       | 0            | 35     | 8.2           |
| Demand   | 30      | 20      | 15      | 10      | 20      | 22           |        |               |

**Step 3**

**Table 14. Maximum value table**

|        | 4.95   | 8.35   | 5.55   | 5.95   |
|--------|--------|--------|--------|--------|
| \(D_1\)| 9.95   | 3.35   | 8.55   | 2.95   |
| \(D_2\)| 4.95   | 8.35   | 5.55   | 5.95   |
| \(D_3\)| 4.2    | 5.6    | 4.8    | 10.2   |
| \(D_4\)| 6.95   | 5.35   | 6.55   | 5.95   |
| \(D_5\)| 5.2    | 6.6    | 4.8    | 8.2    |

**Following steps 4, 5 and 6**

**Table 15. Iteration 1- For the first highest value corresponding to the 1st column of the maximum value table, give the cell the first allocation of the corresponding transportation problem**

|       | \(D_1\) | \(D_2\) | \(D_3\) | \(D_4\) | \(D_5\) | Dummy column | Supply |
|-------|---------|---------|---------|---------|---------|--------------|--------|
| \(S_1\) | 5       | 1       | 8       | 7       | 5       | 0            | 15     |
| \(S_2\) | 3 (25)  | 9       | 6       | 7       | 8       | 0            | 25 (0) |
| \(S_3\) | 4       | 2       | 7       | 6       | 5       | 0            | 42     |
| \(S_4\) | 7       | 11      | 10      | 4       | 9       | 0            | 35     |
| Demand  | 30 (25) | 20      | 15      | 10      | 20      | 22           |        |
Table 16. Iteration 2 - For the first highest value corresponding to the 2\textsuperscript{nd} column of the maximum value table, give the cell the next allocation of the corresponding transportation problem

|       | D\textsubscript{1} | D\textsubscript{2} | D\textsubscript{3} | D\textsubscript{4} | Dummy column | Supply |
|-------|-------------------|-------------------|-------------------|-------------------|--------------|--------|
| S\textsubscript{1} | 5                 | 1 (15)            | 8                 | 7                 | 5            | 0      | 15 (0) |
| S\textsubscript{2} | 3 (25)            | 9                 | 6                 | 7                 | 8            | 0      | 25 (0) |
| S\textsubscript{3} | 4                 | 2                 | 7                 | 6                 | 5            | 0      | 42     |
| S\textsubscript{4} | 7                 | 11                | 10                | 4                 | 9            | 0      | 35     |
| Demand | 30 (25)           | 20 (5)            | 15                | 10                | 20           | 22     |

Table 17. Iteration 3 - All supply values are satisfied in the transport table of the first highest value corresponding to the 3\textsuperscript{rd} column. Therefore, the allocation next to the highest value in the fourth column is assigned to the corresponding transportation table cell

|       | D\textsubscript{1} | D\textsubscript{2} | D\textsubscript{3} | D\textsubscript{4} | Dummy column | Supply |
|-------|-------------------|-------------------|-------------------|-------------------|--------------|--------|
| S\textsubscript{1} | 5                 | 1 (15)            | 8                 | 7                 | 5            | 0      | 15 (0) |
| S\textsubscript{2} | 3 (25)            | 9                 | 6                 | 7                 | 8            | 0      | 25 (0) |
| S\textsubscript{3} | 4                 | 2                 | 7                 | 6                 | 5            | 0      | 42     |
| S\textsubscript{4} | 7                 | 11                | 10                | 4                 | 9            | 0      | 35 (25) |
| Demand | 30 (25)           | 20 (5)            | 15                | 10 (0)            | 20           | 22     |

Table 18. Iteration 4 - Considering the 5\textsuperscript{th} column, due to the reasons mentioned above, it has been left out, and the dummy column has been considered and allocation has been given accordingly

|       | D\textsubscript{1} | D\textsubscript{2} | D\textsubscript{3} | D\textsubscript{4} | Dummy column | Supply |
|-------|-------------------|-------------------|-------------------|-------------------|--------------|--------|
| S\textsubscript{1} | 5                 | 1 (15)            | 8                 | 7                 | 5            | 0      | 15 (0) |
| S\textsubscript{2} | 3 (25)            | 9                 | 6                 | 7                 | 8            | 0      | 25 (0) |
| S\textsubscript{3} | 4                 | 2                 | 7                 | 6                 | 5            | 0      | 42     |
| S\textsubscript{4} | 7                 | 11                | 10                | 4 (10)            | 9            | 0      | 22     |
| Demand | 30 (25)           | 20 (5)            | 15                | 10 (0)            | 20           | 22     |

For the second round, check the second highest value from the maximum value table, as all columns relevant to the first round have been checked.

Table 19. Iteration 5 - For the second highest value corresponding to the 1\textsuperscript{st} column of the maximum value table, give the cell the next allocation of the corresponding transportation problem

|       | D\textsubscript{1} | D\textsubscript{2} | D\textsubscript{3} | D\textsubscript{4} | Dummy column | Supply |
|-------|-------------------|-------------------|-------------------|-------------------|--------------|--------|
| S\textsubscript{1} | 5                 | 1 (15)            | 8                 | 7                 | 5            | 0      | 15 (0) |
| S\textsubscript{2} | 3 (25)            | 9                 | 6                 | 7                 | 8            | 0      | 25 (0) |
| S\textsubscript{3} | 4                 | 2                 | 7                 | 6                 | 5            | 0      | 42     |
| S\textsubscript{4} | 7 (3)             | 11                | 10                | 4 (10)            | 9            | 0      | 22     |
| Demand | 30 (5) (2)        | 20 (5)            | 15                | 10 (0)            | 20           | 22 (0) |

Table 20. Iteration 6 -For the second highest value corresponding to the 2\textsuperscript{nd} column of the maximum value table, give the cell the next allocation of the corresponding transportation problem

|       | D\textsubscript{1} | D\textsubscript{2} | D\textsubscript{3} | D\textsubscript{4} | Dummy column | Supply |
|-------|-------------------|-------------------|-------------------|-------------------|--------------|--------|
| S\textsubscript{1} | 5                 | 1 (15)            | 8                 | 7                 | 5            | 0      | 15 (0) |
| S\textsubscript{2} | 3 (25)            | 9                 | 6                 | 7                 | 8            | 0      | 25 (0) |
| S\textsubscript{3} | 4                 | 2 (5)             | 7                 | 6                 | 5            | 0      | 42 (37)|
| S\textsubscript{4} | 7 (3)             | 11                | 10                | 4 (10)            | 9            | 0      | 22     |
| Demand | 30 (5) (2)        | 20 (5) (0)        | 15                | 10 (0)            | 20           | 22 (0) |
Table 21. Iteration 7 - For the second highest value corresponding to the 5th column of the maximum value table, give the cell the next allocation of the corresponding transportation problem

|      | D_1 | D_2 | D_3 | D_4 | D_5 | Dummy column | Supply |
|------|-----|-----|-----|-----|-----|--------------|--------|
| S_1  | 5   | 1   | 8   | 7   | 5   | 0            | 15 (0) |
| S_2  | 3 (25)| 9   | 6   | 7   | 8   | 0            | 25 (0) |
| S_3  | 4   | 2 (5)| 7   | 6   | 5 (20)| 0           | 42 (37) (17) |
| S_4  | 7 (3)| 11  | 10  | 4 (10)| 9   | 0 (22)       | 35 (25) (3) (0) |
| Demand | 30 (5) (2) | 20 (5) (0) | 15 | 10 (0) | 20 (0) | 22 (0) |

For the third round, check the third highest value from the maximum value table, as all columns relevant to the second round have been checked.

Table 22. Iteration 8 - For the third highest value corresponding to the 1st column of the maximum value table, give the cell the next allocation of the corresponding transportation problem

|      | D_1 | D_2 | D_3 | D_4 | D_5 | Dummy column | Supply |
|------|-----|-----|-----|-----|-----|--------------|--------|
| S_1  | 5   | 1 (15)| 8   | 7   | 5   | 0            | 15 (0) |
| S_2  | 3 (25)| 9   | 6   | 7   | 8   | 0            | 25 (0) |
| S_3  | 4 (2) | 2 (5)| 7   | 6   | 5 (20)| 0           | 42 (37) (17) (15) |
| S_4  | 7 (3)| 11  | 10  | 4 (10)| 9   | 0 (22)       | 35 (25) (3) (0) |
| Demand | 30 (5) (2) | 20 (5) (0) | 15 | 10 (0) | 20 (0) | 22 (0) |

Table 23. Iteration 9 - For the third highest value corresponding to the 3rd column of the maximum value table, give the cell the next allocation of the corresponding transportation problem

|      | D_1 | D_2 | D_3 | D_4 | D_5 | Dummy column | Supply |
|------|-----|-----|-----|-----|-----|--------------|--------|
| S_1  | 5   | 1 (15)| 8   | 7   | 5   | 0            | 15 (0) |
| S_2  | 3 (25)| 9   | 6   | 7   | 8   | 0            | 25 (0) |
| S_3  | 4 (2) | 2 (5)| 7   | 6   | 5 (20)| 0           | 42 (37) (17) (15) |
| S_4  | 7 (3)| 11  | 10  | 4 (10)| 9   | 0 (22)       | 35 (25) (3) (0) |
| Demand | 30 (5) (2) | 20 (5) (0) | 15 | 10 (0) | 20 (0) | 22 (0) |

Following steps 7 and 8

Table 24. The table represents the final allocation for each unit cost value

|      | D_1 | D_2 | D_3 | D_4 | D_5 | Dummy column | Supply |
|------|-----|-----|-----|-----|-----|--------------|--------|
| S_1  | 5   | 1 (15)| 8   | 7   | 5   | 0            | 15 (0) |
| S_2  | 3 (25)| 9   | 6   | 7   | 8   | 0            | 25 (0) |
| S_3  | 4 (2) | 2 (5)| 7   | 6   | 5 (20)| 0           | 42 (37) (17) (15) |
| S_4  | 7 (3)| 11  | 10  | 4 (10)| 9   | 0 (22)       | 35 (25) (3) (0) |
| Demand | 30 (5) (2) | 20 (5) (0) | 15 | 10 (0) | 20 (0) | 22 (0) |

Transportation cost = (25×3)+(4×2)+(7×3)+(1×15)+(2×15)+(7×15)+(4×10)+(5×20)+(0×22)=374

RESULTS AND DISCUSSION

Comparison of the Numerical Example with the New Method

In this section, the effectiveness of several well-known methods including the least cost method, northwest corner method, Vogel's approximation method, row minimum method, column minimum method, and some other well-liked methods are compared using results from various problems. Comparative evaluation is carried out and displayed. The detailed representation of the numerical data of tables 25 and 26.
### Table 25. Balance transportation problems

| Method | Problem 1 \[11\] | Problem 3 \[13\] | Problem 4 \[2\] | Problem 5 \[14\] | Problem 6 \[15\] | Problem 7 \[16\] | Problem 8 \[13\] | Problem 9 \[13\] |
|--------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|        | \( C_i = [11, 13, 17, 14, 16, 18, 14, 10, 21, 24, 13, 10] \) | \( C_j = [4, 3, 5] \) | \( C_{ij} = [6, 3, 5, 4, 5, 9, 6, 5, 4, 8, 2, 7, 7, 10, 7, 12, 17, 9] \) | \( C_{ij} = [20, 22, 17, 4, 24, 37, 9, 3, 4, 6, 3, 4, 3] \) | \( C_{ij} = [5, 4, 6, 9, 5, 3, 4, 12] \) | \( C_{ij} = [4, 6, 9, 5, 3, 4, 12] \) | \( C_{ij} = [4, 6, 9, 5, 3, 4, 12] \) | \( C_{ij} = [19, 8, 14, 20, 2, 2, 9] \) |
|        | \( S_i = [250, 300, 400] \) | \( S_i = [90, 80, 100] \) | \( S_i = [120, 70, 50] \) | \( S_i = [7, 3, 5] \) | \( S_i = [100, 300, 300] \) | \( S_i = [16, 12, 15] \) | \( S_i = [500, 400, 300] \) | \( S_i = [500, 400, 300] \) |
|        | \( D_i = [200, 225, 275, 300] \) | \( D_i = [90, 80, 100] \) | \( D_i = [7, 12, 17, 9] \) | \( D_i = [60, 40, 30, 110] \) | \( D_i = [4, 5, 6] \) | \( D_i = [300, 200, 200] \) | \( D_i = [12, 14, 9, 8] \) | \( D_i = [250, 350, 500, 100] \) |
| NWCM   | 12200            | 1500             | 176              | 3680            | 53              | 4100            | 226             | 21250           |
| LCM    | 12200            | 1450             | 150              | 3670            | 51              | 4100            | 156             | 7100            |
| VAM    | 12075            | 1390             | 149              | 3520            | 50              | 3900            | 156             | 7100            |
| RAM    | 13175            | 1450             | 150              | 3790            | 51              | 4100            | 156             | 8100            |
| CAM    | 12075            | 1500             | 184              | 3460            | 51              | 4100            | 204             | 12350           |
| Russell’s Approximation Method (RuAM) | 12075 | 1390 | 149 | 3460 | 50 | 3900 | 156 | 7100 |
| Proposed Method (RuAM) | 12075 | 1390 | 149 | 3460 | 50 | 3900 | 156 | 7100 |
| Optimum Solution | 12075 | 1390 | 149 | 3460 | 50 | 3900 | 156 | 7100 |

### Table 26. Unbalance transportation problems

| Method | Problem 2 \[12\] | Problem 10 \[2\] | Problem 11 \[17\] | Problem 12 \[17\] | Problem 13 \[18\] | Problem 14 \[19\] | Problem 15 \[20\] |
|--------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|        | \( C_{ij} = [5, 1, 8, 7, 5, 3, 9, 6, 7, 8, 4, 2, 7, 6, 5, 7, 11, 10, 4, 9] \) | \( C_{ij} = [4, 8, 16, 8, 11, 10, 24, 16, 8, 10, 12, 5, 16, 24, 4, 6, 11, 10, 9] \) | \( C_{ij} = [7, 9, 3, 11, 5, 2, 8, 10, 12, 4, 7, 6, 5, 4, 7, 3] \) | \( C_{ij} = [6, 1, 10, 14; 3, 2, 6, 5, 4, 5, 2, 1; 12, 19, 21, 15, 4, 17] \) | \( C_{ij} = [3, 4, 6, 7, 3, 8, 6, 4, 5, 7, 5, 2] \) | \( C_{ij} = [4, 2, 3, 6, 5, 8, 4, 2, 7, 6, 5, 4, 7, 3] \) | \( C_{ij} = [5, 4, 6, 7, 3, 8, 6, 4, 5, 7, 5, 2] \) |
|        | \( S_i = [15, 25, 42, 35] \) | \( S_i = [76, 82, 77] \) | \( S_i = [30, 45, 35] \) | \( S_i = [70, 55, 70] \) | \( S_i = [8, 12, 14] \) | \( S_i = [50, 50, 50] \) | \( S_i = [170, 250, 130, 350] \) |
|        | \( D_j = [30, 20, 15, 10, 20] \) | \( D_j = [72, 102, 41] \) | \( D_j = [20, 28, 19, 33] \) | \( D_j = [85, 35, 50, 45] \) | \( D_j = [4, 6, 8, 8] \) | \( D_j = [30, 40, 55] \) | \( D_j = [200, 300, 500] \) |
| NWCM   | 547              | 3528             | 788              | 1125            | 104             | 1815            | 1010            |
| LCM    | 374              | 2712             | 606              | 965             | 88              | 1695            | 840             |
| VAM    | 374              | 2424             | 630              | 965             | 80              | 1745            | 880             |
| RAM    | 472              | 2968             | 710              | 965             | 100             | 1625            | 1410            |

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The analysis of six different methods for getting an initial basic feasible solution, including the proposed approach method, is the focus of this part. It will also be compared with the optimum solution. This analysis is expected to confirm the justification of the new methodology. The bar graph prepared according to the data values represented in tables 25 and 26 is shown above.

**Figure 1.** Bar chart of comparison the balance transportation problem initial basic feasible solution with different methods.

**Figure 2.** Bar chart of comparison the unbalance transportation problem initial basic feasible solution with different methods.

![Graphs showing the comparison of transportation problem solutions](image)

It is demonstrated from such graphs that there exist solutions in the new methodology that are competitively close to the initial basic feasible solution found through other methods. Along with the optimal answer, the proposed method's solutions were evaluated. The proposed methodology produces similar results for thirteen of the fifteen problems as the numerically optimal method. As a result, these studies back up the effectiveness of the proposed approach.

**CONCLUSION**

The main goal of this article was to present a novel method for estimating the cost (basic feasible solution) of a transportation problem. A unique approach based on the mean value of rows and columns was presented here. This method offered an algorithm for finding an abasic feasible solution to transportation problems that are both balanced and unbalanced. The primary goals of solving a transportation problem are to reduce the time it takes to solve the problem, improve the proposed algorithm to make it easier to understand, and show that the initial basic feasible solution is closer to or near to the optimum solution. The article's proposed model looked at eight problems of balance and seven problems of unbalanced transportation problems. The solutions to those problems were able to reach the initial basic feasible solution and also competitive approximated answers derived.
from other main transport problem-solving methods. Furthermore, thirteen out of the fifteen problems examined obtained the optimum solution. This comparative analysis also leads to the reasonable conclusion that this method is effective when compared to other methods. It has also been shown that effective solutions can be obtained by solving problems using this method. This can be demonstrated as a model that can be successfully applied to real-world problems. That solves the problem in such a way as to minimize a specific objective such as cost, time, and distance.

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