We describe an application of the linear $\delta$-expansion to the calculation of correlation functions in SU(2)-Higgs lattice gauge theory. A significant advantage of the technique is that an infinite volume lattice may be used, allowing the non-analyticity in certain observables at a phase transition to be observed directly. We illustrate the approach with a preliminary application to the 3D SU(2)-Higgs model, as the dimensionally reduced effective theory for the electroweak standard model at high temperature, and calculate certain gauge invariant observables near the phase boundaries.

1 Introduction

When considering the static equilibrium properties of a thermal field theory, careful Green function matching has shown that many weakly coupled gauge theories at high temperature may be reliably approximated by a super-renormalizable effective 3D theory, generically a bosonic gauge-Higgs model. The dynamical degrees of freedom of this effective theory then correspond to the Matsubara zero-modes of the full theory, thus retaining all the infrared divergences and necessitating non-perturbative analysis. However, one has the advantages of having integrated out the thermally massive fermions. Furthermore, super-renormalizability, and non-triviality of the scalar sector in 3D, allow perturbatively exact renormalization group trajectories to be determined with a two-loop calculation, thus allowing control over the continuum extrapolation of lattice results. With the added benefit of simulations being less numerically expensive in 3D, recent Monte Carlo studies of the dimensionally reduced electroweak standard model, and MSSM, have led to a reasonably complete knowledge of the phase structure and related static properties, including the insight that the first order symmetry breaking transition ceases to be of first or second order for Higgs masses above approximately 80 GeV.

Nevertheless, study of the model near the (possibly second order) endpoint of the transition line and, in particular, determination of the spectrum in this regime require large lattices due to the diverging correlation lengths. Therefore analytic approaches are still desirable, and such approaches may still take


---

Presented by A. Ritz. To appear in the Proceedings of 'Strong and Electroweak Matter (SEWM'97)', Eger, Hungary, May 1997.
advantage of many of the features of dimensional reduction discussed above. In this talk we present an analytic approach, the linear $\delta$-expansion, and apply it to the 3D $SU(2)$-Higgs model discretized on an infinite lattice. We present some preliminary results for certain gauge invariant ‘order parameter like’ variables near the phase boundaries.

2 The Linear $\delta$-Expansion

The basis of this approach is that when one is interested in calculating Greens functions for a theory with classical action $S$, one can instead consider the following extended action

$$S_\delta = S_0(J) + \delta(S - S_0(J)), \quad (1)$$

where $\delta$ is an artificial parameter. $S_{\delta=1} = S$, the theory under study, and $S_{\delta=0} = S_0$ is a new ‘free’ action, chosen for convenience of calculation, but also to mimic the dominant degrees of freedom as closely as possible. The generating functional for Green functions may then be expanded to an appropriate order in the parameter $\delta$, which is then set to unity.

Calculating an observable quantity $P$ at each order in $\delta$ one then obtains a sequence of approximants $\{P_1, P_2, \ldots, P_N\}$. An essential feature of this approach is the ability to optimize the convergence of this sequence, independent of the existence of a small coupling constant, by carefully fixing additional parameters $J$ appearing in the extended action. As the power series in $\delta$ is only calculated to a finite order, it retains some dependence on $J$ which would be absent in a full summation. A well motivated criterion for fixing $J$ is the principle of minimal sensitivity (PMS), whereby $J$ is chosen at a local minimum of an observable quantity. i.e. if $P_N(J)$ denotes the $N$th approximation to $P$, then we impose

$$\frac{\partial P_N(J)}{\partial J} = 0. \quad (2)$$

This or a similar criterion is intrinsic to the success of the $\delta$-expansion, providing a generically non-perturbative dependence on the coupling constant, and there exist rigorous proofs for convergence of the sequence of approximants for the energy levels $\{E_N\}$, and the partition function, in 0- and 1-D field theories where the corresponding perturbative series are asymptotic and eventually diverge factorially or in some cases are even non-Borel-summable.

Motivation for the present application to gauge-Higgs models has come from previous successful studies of pure gauge theories on the lattice. In particular order parameters, and the glueball mass gap have been well reproduced,
while 4D considerations of the phase structure of the SU(2)-Higgs model\[7\] have also shown encouraging results.

3 Application to the 3D SU(2)-Higgs Model

Dimensional reduction of a general class of 4D gauge theories at finite temperature by static Green function matching has been studied in detail by Kajantie et al.\[1\] Perturbative Green function matching to order $\delta G/G \sim O(g^3)$ is possible with a super-renormalizable effective 3D theory. For the standard model with Weinberg angle $\theta_W = 0$, this is the 3D SU(2)-Higgs model.

In the present case we can make use of the advantages of dimensional reduction discussed earlier, and for reasons of gauge invariance it is also convenient to use a lattice regularization. Therefore we consider the standard 3D lattice action

$$ S = \beta \sum_P \frac{1}{2} \text{Tr} U_P + \frac{1}{2} \beta_h \sum_{l_{ij}} \rho_i \rho_j \text{Tr} U_{ij} - \sum_i \left[ \rho_i^2 + \beta_R (\rho_i^2 - 1)^2 \right], \quad (3) $$

where we represent the Higgs doublet in the form $\Phi = \rho V$ with $\rho \in \mathbb{R}_+$ and $V \in \text{Fund}[SU(2)]$. We have performed a gauge transformation of the form $U(x, y) \rightarrow V(x) U(x, y) V(y)^\dagger$ to write the action in unitary gauge. The relation between the dimensionless lattice parameters $\beta, \beta_h, \beta_R$ and the continuum physics, which may be described in terms of two dimensionless ratios with the scale set by the gauge coupling, is given in Ref. 2. Super-renormalizability implies that the necessary renormalization group trajectories may be obtained exactly with a two-loop perturbative calculation.

For calculation using the linear $\delta$-expansion we use the following “free” action,

$$ S_0 = \sum_{l_{ij}} (J + \beta_R \rho_i \rho_j) \frac{1}{2} \text{Tr} U_{ij} - \sum_i \left[ \rho_i^2 + \beta_R (\rho_i^2 - 1)^2 \right], \quad (4) $$

which differs from the full action only in the pure gauge sector, where a single link action is used which has been successfully applied in the analysis of pure gauge theories\[5\]. Note that while formally gauge variant, only gauge invariant expectation values are nonzero when one allows the parameter $J$ to be site dependent\[8\].

Expectation values are calculated using the extended action up to a given order in $\delta$. For an observable $P$, the contribution to $N^{th}$ order has the form

$$ < P >_N = \left[ \sum_{n=0}^\infty \frac{\delta^n}{n!} \int [dU] [d\rho \rho^3] P(S - S_0) \rho^n e^{S_0} \right]_{\delta N}, \quad (5) $$

3
where the notation implies expansion to $O(\delta^N)$. The expectation value then assumes the form of a cumulant since the expansion of the denominator naturally subtracts the disconnected pieces. We present here some preliminary results for the average plaquette, $E_P \equiv \sum_P \text{Tr} U_p / 2 N_P$, and the hopping term, $E_L \equiv \sum_{<ij>} \rho_i \rho_j \text{Tr} U_{ij} / 2 N_L$, standard local observables which exhibit discontinuities at a transition point. We assume that the modulus of the Higgs field varies slowly over the lattice, i.e. $\rho_i \rho_j = \rho_i^2 + \rho_i \Delta \rho_i \approx \rho_i^2 + O(\delta^2)$, and then calculations are simplified by noting that all terms in the action depend either on the real variable $\rho$ or are $SU(2)$ characters. Using character expansion techniques, one may reduce all expectation values to polynomials in the factors $A_n^r / A_0^r$ where

$$ (A_n^r)^d = \int_0^\infty d\rho \rho^{n+3} e^{-V_H} \left( r + 1 \frac{I_{r+1}(J + \beta_R \rho^2)}{J + \beta_R \rho^2} \right)^d, \quad (6) $$

in which $V_H = \rho^2 + \beta_R (\rho^2 - 1)^2$ is the Higgs potential, $I_{r+1}$ is a modified Bessel function, and $d$ is the dimension of the lattice.

In Fig. 1 results in the transition/crossover regions for the average plaquette and hopping term calculated to first order in $\delta$ are shown. For each observable the parameter $J$ was fixed using (3). A PMS extremum solving
exists for $<E_P>$ on both sides of the deconfinement crossover, with the strong coupling solution corresponding to $J = 0$ and a weak coupling PMS point appearing only for $\beta > 2.4$. Comparison with Monte Carlo data available in 4D at $\beta_h = 0$ indicates very good agreement in the crossover region even at this low order. For non-zero $\beta_h$ the transition shifts slightly to lower $\beta$, again in agreement with Monte Carlo studies.

For the hopping term we plot the results with parameters appropriate to an approximate continuum 4D Higgs mass of $m_H^* = 70$ GeV (see Ref. 2). Particular branches of the solution to (2) in each phase are lost as one approaches the transition region, leading to difficulties in pinpointing the precise critical parameters for the transition. It is expected that the branches will persist closer to the true transition point as one evaluates the results to higher order in $\delta$, and such a higher order calculation is currently in progress.

Acknowledgements

The financial support of T.S.E by the Royal Society, and A.R. by the Commonwealth Scholarship Commission is gratefully acknowledged. This work was supported in part by the European Commission under the Human Capital and Mobility programme, contract number CHRX-CT94-0423.

References

1. K. Kajantie, M. Laine, K. Rummukainen, and M. Shaposhnikov. *Nucl. Phys. B*, 458:90–136, 1996.
2. K. Kajantie, M. Laine, K. Rummukainen, and M. Shaposhnikov. *Nucl. Phys. B*, 466:189–258, 1996; *Phys. Rev. Lett.*, 77:2887–2890, 1996; M. Laine, these proceedings.
3. P. M. Stevenson, *Phys. Rev. D*, 23:2916, 1981.
4. I. R. C. Buckley, A. Duncan, and H. F. Jones. *Phys. Rev. D*, 47:2554, 1993; A. Duncan and H. F. Jones. *Phys. Rev. D*, 47:2560, 1993. R. Guida, K. Konishi, and H. Suzuki. *Ann. Phys.*, 249:109, 1996.
5. X.-T. Zheng, Z. G. Tan, and J. Wang. *Nucl. Phys. B*, 287:171–188, 1987; J. O. Akeyo and H. F. Jones. *Phys. Rev. D*, 47:1668, 1993.
6. J. O. Akeyo, C. S. Parker, and H. F. Jones. *Phys. Rev. D*, 51:1298, 1995.
7. X.-T. Zheng and B.-S. Liu. *Int. J. Mod. Phys. A*, 6(1):103–118, 1991.
8. C.-I. Tan and X.-T. Zheng. *Phys. Rev. D*, 39(2):623–635, 1989.
9. H. G. Evertz, J. Jersák and K. Kanaya. *Nucl. Phys. B*, 285:229–252, 1987; P. H. Damgaard and U. M. Heller. *Nucl. Phys. B*, 304:63-76, 1988.