A NEW INSIGHT INTO NEUTRINO ENERGY LOSS BY ELECTRON CAPTURE OF IRON GROUP NUCLEI IN MAGNETAR SURFACES

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Received 2015 June 18; accepted 2016 February 28; published 2016 June 8

ABSTRACT

Based on the relativistic mean-field effective interactions theory, and the Lai dong model, we discuss the influences of superstrong magnetic fields (SMFs) on electron Fermi energy, nuclear blinding energy, and single-particle level structure in magnetar surfaces. Using the Shell-Model Monte Carlo method and the Random Phase Approximation theory, we analyze the neutrino energy loss rates (NELRs) by electron capture for iron group nuclei in SMFs. First, when $B_{12} < 100$, we find that the SMFs have a slight influence on the NELRs for most nuclides at relativistic low temperatures (e.g., $T_9 = 0.23$); nevertheless, the NELRs increase by more than four orders of magnitude at relativistic high temperatures (e.g., $T_9 = 15.53$). When $B_{12} > 100$, the NELRs decrease by more than three orders of magnitude (e.g., $T_9 = 15.53$ for $^{52-61}$Fe, $^{55-60}$Co, and $^{56-63}$Ni). Second, for a certain value of magnetic field and temperature, the NELRs increase by more than four orders of magnitude when $\rho_f < 10^5$, but as the density increases (i.e., when $\rho_f > 10^3$), there is almost no influence on the density of NELRs. For the density around $\rho_f = 10^5$, there is an abrupt increase in NELRs when $B_{12} \geq 10^{3.5}$. Such jumps are an indication that the underlying shell structure has changed due to single-particle behavior by SMFs. Finally, we compare our NELRs with those of Fuller et al. (FFN) and Nabi & Klapdor-Kleingrothaus (NKK). For the case without SMFs, one finds that our rates for certain nuclei are close to about one order of magnitude lower than FFN and NKK at relativistic low temperatures (e.g., $T_9 = 1$). However, at a relativistic high temperature (e.g., $T_9 = 3$), our results are in good agreement with NKK, but about one order of magnitude lower than FFN. For the case with SMFs, our NELRs for some iron group nuclei can be about five orders of magnitude higher than those of FFN and NKK. (Note that $B_{12}$, $T_9$, and $\rho_f$ are in units of $10^{12}$ G, $10^9$ K, and $10^7$ g cm$^{-3}$, respectively.)

Key words: magnetic fields – nuclear reactions, nucleosynthesis, abundances – stars: neutron

1. INTRODUCTION

In core-collapse supernova explosions, neutrino processes and weak interaction (e.g., electron capture (hereafter EC) and beta decay) play pivotal roles. At the late stages of evolution, a large amount of energy from a star is lost mainly through neutrinos and this process is fairly independent of the mass of star. For instance, white dwarfs and supernovae both have cooling rates that are largely dominated by neutrino production. At high temperatures and densities, the EC and the accompanying neutrino energy loss rates (hereafter NELRs) are of prime importance for determining the equation of state of a supernova. An accurate determination of neutrino emission rates is very necessary in order to perform a careful analysis of the final branches of stellar evolutionary tracks. The neutrino cooling rates can strongly influence the evolutionary timescale and the configuration of the iron core at the onset of the supernova explosion.

Some research on the cooling of white dwarfs, neutron stars, and magnetars is necessary for us to give an accurate estimate of some weak interaction rates and NELRs. Due to the importance of the EC and NELRs in astrophysical environments, Beaudet et al. (1967), Braaten & Segel (1993), Ahmad et al. (2010), Esposito et al. (2003), Fuller et al. (1980, 1982, 1985), Itoh et al. (1996), Juodagalvis et al. (2010), Langanke et al. (2001), Langanke & Martinez-Pinedo (1998), Liu (2013c, 2013d, 2014, 2015, 2016), and Ray et al. (1984) have done some pioneering works on the weak interaction reactions and NELRs for some iron group nuclei. Recently, by using the proton–neutron quasi-particle random phase approximation (pn-QRPA) theory, Nabi et al. (2016) and Nabi & Klapdor-Kleingrothaus (1999, 2004) have investigated neutrinos and anti-NELRs. However, their works seemed to pay no attention to the influence of superstrong magnetic fields (hereafter SMFs) on NELRs.

The neutrino processes will play a crucial role in magnetars and some neutron stars due to EC and beta decay. A great deal of energy is set free with the escape of the neutrino. Thus, the works on neutrinos and NELRs have been a hotspot and somewhat controversial regarding magnetars and some neutron stars. In order to understand the nature of the weak interaction process and NELRs in magnetars, the study of matter in SMFs is an important component of magnetars in astrophysics research. Some works show that the strengths of magnetic fields at the crusts of neutron stars are in the range of $10^8$–$10^{13}$ G (Baym et al. 1975; Dai et al. 1993; Broderik et al. 2000). There has recently been growing evidence for the existence of neutron stars with magnetic fields with strengths that exceed the quantum critical field strength of $4.4 \times 10^{13}$ G. Such evidence has been provided by new discoveries of radio pulsars with very high spin-down rates and by observations of bursting gamma-ray sources termed magnetars. In addition, some research shows that magnetars possess magnetic fields with strengths from $10^{13}$ to $10^{15}$ G (Lai & Shapiro 1991; Lai 2001, 2015; Harding & Lai 2006; Peng & Tong 2007; Gao et al. 2011, 2015; Peng et al. 2012; Zhu et al. 2016). Although it is not clear just how strong magnetic fields in magnetars could be, some calculations indicate that fields of $10^{15}$–$10^{16}$ G are not impossible. Theoretical models show that these magnetic fields might reach up to $10^{18}$ G, and perhaps even
larger values when one considers the limit imposed by the virial theorem (Ormand et al. 1994). For such SMFs, the classical description of the trajectory of a free electron is longer valid and quantum effects must be considered.

Previous works (Liu 2013a, 2013b) show that SMFs greatly influence EC rates and NELRs and decrease with increasing magnetic field strength. Recent studies (Lai & Shapiro 1991; Lai 2001, 2015) have found that the magnetic field will make the Fermi surface elongate from a spherical surface to a Landau surface along the magnetic field direction and its level is perpendicular to the magnetic field direction and strongly quantized. The properties of matter (such as atoms, molecules, and condensed matter/plasma) are dramatically changed by the SMFs in magnetars. Due to the influence of SMFs, the electron cyclotron kinetic energy will be greater than electron-static energy in magnetars. The microscopic state number of electron cyclotron kinetic energy will be greater than electron-static energy. The present paper is organized as follows. In Section 2, we discuss the influence of SMFs on the electron properties and nuclear energy in magnetars. Our studies of the EC process and NELRs in SMFs are given in Section 3. Some numerical results and discussion are given in Section 4. In the final section, we summarize our conclusions. (Note that in this paper, B12, T9, and ρf are in units of 1012 G, 109 K, and 109 g cm−3, respectively.)

2. THE INFLUENCE OF SMFS IN MAGNETARS

2.1. How Will the SMFs Influence the Electron Properties of Magnetar Surfaces?

The properties of matter, such as the states equation, electron energy, the outer crust structure, and the composition of neutron stars, are significantly modified by strong magnetic fields. Many works (Canuto & Ventura 1977; Lai & Lifshitz 1991) have presented the quantum mechanics of charged particles in SMFs. When we consider the non-relativistic motion of a particle (charge e, and mass m0) in a uniform magnetic field, which is assumed to be along the z-axis, the circular orbit radius and (angular) frequency in the process of particle gyrate are given by r = m0cν/[e][B] and ωc = |e|B/m0c, respectively; here, ν⊥ is the velocity perpendicular to the magnetic field. The kinetic energy for an electron (m0 → mν, e → −e) of the transverse motion is quantized in Landau levels in non-relativistic quantum mechanics, and is written as

\[ E_{\text{kin}} = \frac{1}{2}m\nu^2 = \left(n_l + \frac{1}{2}\right)\hbar\omega_c, \]

where n_l = 0, 1, 2... are the Landau level numbers. The cyclotron energy for a electron, which is the basic energy quantum is given by

\[ E_{\text{cycl}} = \hbar\omega_c = \hbar \frac{eB}{m_0c} = 11.577B_{12}\text{ keV}, \]

where \( B_{12} = B/10^{12}\text{ G} \) is the magnetic field strength in units of \( 10^{12}\text{ G} \). The total electron energy, which is included the kinetic energy associated with the z-momentum (p_z) and the spin energy in non-relativistic quantum surroundings, can be written as (Canuto & Ventura 1977; Lai & Lifshitz 1991)

\[ E_n = \nu\hbar\omega_c + \frac{p_z^2}{2m_0}, \]

where \( \nu = n_l + (1 + \sigma_z)/2 \). Here, \( \sigma_z = -1 \) and \( \sigma_z = \pm 1 \) are the spin degeneracy for the ground Landau level (\( n_l = 0 \)) and excited levels, respectively.

We define a critical magnetic field strength \( B_{\text{cr}} \) using the relation \( \hbar\omega_c = m_0c^2 \) (i.e., \( B_{\text{cr}} = m_0c^2/e\hbar = 4.414 \times 10^8\text{ G} \)). The transverse motion of the electron becomes relativistic when \( \hbar\omega_c \geq m_0c^2 \) (i.e., \( B \geq B_{\text{cr}} \)) for extremely strong magnetic fields. The energy eigenstates of an electron must obey the relativistic Dirac equation and are given by (Johnson & Lippmann 1949; Canuto & Ventura 1977)

\[ E_n = \left[c^2p_z^2/m_0^2 + m_0^2c^4\left(1 + 2\nu B/B_{\text{cr}}\right)^{3/2}\right]^{1/2}, \]

where the shape of the Landau wavefunction in the relativistic theory is the same as in the non-relativistic theory due to the fact the cyclotron radius is independent of the particle mass (Johnson & Lippmann 1949).
In SMFs the number density \( n_e \) of electrons is related to the chemical potential \( \mu_F \) by (Lai & Shapiro 1991; Lai 2001, 2015; Harding & Lai 2006)

\[
n_e^B = \frac{1}{(2\pi \hbar)^2} \sum \frac{g_{a0}}{\rho_{a0}} \int_{-\infty}^{\infty} f dp_e, \tag{5}
\]

where \( \rho = (\hbar c/eB)^{1/2} = 2.5656 \times 10^{-10} B_{12}^{1/2} \text{cm} \) is the cyclotron radius (the characteristic size of the wave packet), and \( g_{a0} \) is the spin degeneracy of the Landau level, \( g_{a0} = 1 \) and \( g_{a0} = 2 \) for \( n \geq 1 \), and \( f = [1 + \exp((\mu_F - U_F)/kT)]^{-1} \) is the Fermi–Dirac distribution.

According to the relation of the usual relativistic energy and momentum from Equation (4), the interaction energy term, which is proportional to the quantum number \( \nu \), and cannot exceed the electron chemical potential, will appear to be due to the electron interaction with the magnetic field. Thus, the maximum number of Landau levels \( \nu_{\text{max}} \), related to the highest value of the allowed interaction energy, should be satisfied with \( E_n(\nu_{\text{max}}, p_z = 0) = U_F \). So we have

\[
\nu_{\text{max}} = \frac{B \sigma}{2B} \left( \frac{U_F^2}{m_e^2 c^4} - 1 \right). \tag{6}
\]

However, in the general case (i.e., \( 0 \leq \nu \leq \nu_{\text{max}} \)), when the maximum electron momentum is equal to the Fermi momentum \( p_F \) for different Landau levels \( \nu \), the electron chemical potential from Equation (4) can be computed as follows:

\[
E_n(\nu) = \left[ c^2 p_F^2 + m_e^2 c^4 \left( 1 + 2\nu B / B_{\text{cr}} \right) \right]^{1/2} = U_F. \tag{7}
\]

When we define a non-dimensional Fermi momentum \( x_e(\nu) = p_F / m_e c^2 \), and Fermi energy \( \gamma_e = U_F / m_e c^2 \), at the condition of zero temperature the electronic density, electron energy, and pressure can be written as (Lai & Shapiro 1991)

\[
\begin{align*}
n_e = & \frac{B}{2\pi B_{\text{cr}}^3} \sum x_e(\nu), \\
\varepsilon_e = & \frac{B m_e c^2}{2\pi^2 B_{\text{cr}}^3} \sum x_e(\nu) \left[ 1 + 2\nu B / B_{\text{cr}} \right] \frac{x_e(\nu)}{\left( 1 + 2\nu B / B_{\text{cr}} \right)^{1/2}}, \\
P_e = & \frac{B m_e c^2}{2\pi^2 B_{\text{cr}}^3} \sum x_e(\nu) \left[ 1 + 2\nu B / B_{\text{cr}} \right] \frac{x_e(\nu)}{\left( 1 + 2\nu B / B_{\text{cr}} \right)^{1/2}}.
\end{align*} \tag{8, 9, 10}
\]

where \( \partial_x(x) = \frac{1}{x} \sqrt{1 + x^2} \pm \frac{1}{2} \ln(x + \sqrt{x^2 + 1}) \), \( \lambda_e = \hbar / m_e c \) is the electron Compton wavelength.

2.2. How Will the SMFs Influence the Energy of Nuclei in Magnetar Surfaces?

The matter in the outer crust of a cold (\( T = 0 \text{ K} \)) magnetar consists of a Coulomb lattice of completely ionized atoms and a uniform Fermi gas of relativistic electrons. The Gibbs free energy per baryon \( g(A, Z, P) \) at a constant pressure and zero temperature can be given by

\[
g(A, Z, P) = \frac{E(A, Z, P) + PV}{A} = \varepsilon + \frac{P}{n}, \tag{11}
\]

where \( \varepsilon \) is the corresponding energy per nucleon, \( n = A / V \) is the baryon density in a cell, and \( V \) is the volume occupied by a unit cell of the Coulomb lattice. The energy per nucleon \( \varepsilon \), which consists of three different contributions from nuclear, electronic, and lattice sources is given by

\[
\varepsilon = \varepsilon_n(A, Z) + \varepsilon_e(A, Z, P) + \varepsilon_l(A, Z, n), \tag{12}
\]

where the nuclear contribution to the total energy per nucleon is simple and independent of the density and is given by (Gamblin et al. 1990; Vretenar et al. 2005)

\[
L = L_N + L_m + L_{\text{int}} + L_{\text{BO}} + L_{\text{BM}}, \tag{14}
\]

where \( L_N, L_m, \) and \( L_{\text{int}} \) are the Lagrangians of the free nucleon, the free meson fields and the electromagnetic field generated by the proton, and the Lagrangian describing the interactions, respectively. These Lagrangians are represented by (Boguta & Bodmer 1977; Boguta 1981; Boguta & Stocker 1983)

\[
L_N = \bar{\psi} (i\gamma^\mu \partial_\mu - m_n) \psi, \tag{15}
\]

\[
L_m = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma \sigma^2 - \frac{1}{4} \Omega_{\mu\nu\sigma} \Omega^{\mu\nu\sigma} + \frac{1}{2} m_{\omega}^2 \omega^2 - 2 \frac{1}{4} \Omega_{\mu\nu\omega} \Omega^{\mu\nu\omega} - \frac{1}{4} R_{\mu\nu\sigma\tau} R^{\mu\nu\sigma\tau} - \frac{1}{4} m_\rho^2 \rho^2 \rho^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - U(\sigma), \tag{16}
\]

\[
L_{\text{int}} = -g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma^\mu \omega_\mu \psi \sigma - g_\rho \bar{\psi} \gamma^\mu \tau^\rho \psi \sigma - e_\gamma A_\mu \bar{\psi} \gamma^\mu \psi \sigma, \tag{17}
\]

where \( \psi \) is the Dirac spinor. Here, \( m_n, m_\sigma, m_\omega, m_\rho, m_\sigma, g_\omega, g_\rho, e_\gamma \) are the nucleons and meson masses, respectively. \( U(\sigma) = (g_0/3)\sigma^3 + (g_3/4)\sigma^4 \) is the standard form for the nonlinear coupling of the \( \sigma \) meson field. \( g_\sigma, g_\omega, g_\rho, e_\gamma \) are the coupling constants for the \( \sigma, \omega, \rho \), and the electric charge for protons, respectively.

The coupling of the proton orbital motion with the external magnetic field, and the coupling of protons’ and neutrons’ intrinsic dipole magnetic moments with the external magnetic field can be, respectively, expressed as (Bjorken & Drell 1965)

\[
L_{\text{BO}} = e_\nu \bar{\psi} \gamma^\mu A_\mu^{(\text{ext})} \psi, \tag{18}
\]

\[
L_{\text{BM}} = -\bar{\psi} (i\gamma_\mu \psi) A_\mu, \tag{19}
\]
3. THE NELRs DUE TO EC IN MAGNETARS

3.1. The SMMC Method and GT Response Functions

The GT properties of nuclei in the medium-mass region of the periodic table are crucial determinants in the process of EC. Some works (Fassio-Canuto 1969; Canuto & Ventura 1977) have demonstrated that the GT transition matrix elements for EC and beta decay do not depend on the magnetic fields. Thus we will neglect the effect of SMFs on the GT properties of nuclei in this paper. The SMMC method is based on a statistical formulation of the nuclear many-body problem. We use the SMMC approach to find the GT strength distributions. SMMC has the added advantage that it treats nuclear temperatures exactly. Based on a statistical formulation of the nuclear many-body problem, in the finite temperature version of this approach, an observable is calculated as the canonical expectation value of a corresponding operator \( \hat{A} \) by the SMMC method at a given temperature \( T \), and is written by (Johnson et al. 1992; Alhassid et al. 1994; Ormand et al. 1994; Koonin et al. 1997)

\[
\hat{A} = \frac{\text{Tr}_A[\hat{A} e^{-\beta \hat{H}}]}{\text{Tr}_A[e^{-\beta \hat{H}}]].
\]

The problem of a shell model Hamiltonian \( \hat{H} \) has been investigated in detail by Alhassid et al. (1994). When a many-body Hamiltonian \( \hat{H} \) is given, a tractable expression for the imaginary time evolution operator is given by

\[
\hat{U} = \exp^{-\beta \hat{H}},
\]

where \( \beta = 1/T_N \), and \( T_N \) is the nuclear temperature in units of MeV. \( \text{Tr}_A \hat{U} \) is the canonical partition function for \( A \) nucleons. In terms of a spectral expansion, the total strength of a transition operator \( \hat{A} \) is then given by the following expectation value:

\[
B(A) = \langle \hat{A}^\dagger \hat{A} \rangle = \frac{\sum_i e^{-\beta E_i} |\langle f|\hat{A}|i\rangle|^2}{\sum_i e^{-\beta E_i}}.
\]

where \( |i\rangle \), \( |f\rangle \) are the many-body states of the initial (final) nucleus with energies \( E_i \), \( E_f \), respectively.

The SMMC method is used to calculate the response function \( R_A(\tau) \) of an operator \( \hat{A} \) at an imaginary time \( \tau \). By using a spectral distribution of initial and final states \( |i\rangle \) and \( |f\rangle \) with energies \( E_i \) and \( E_f \), \( R_A(\tau) \) is given by (Dean et al. 1998; Langanke & Martinez-Pinedo 1998; Langanke et al. 2001)

\[
R_A(\tau) = \frac{\text{Tr}_A[\hat{A} e^{-\beta \hat{H}}]}{\text{Tr}_A[e^{-\beta \hat{H}}]]}
\]

Note that the total strength for the operator is given by \( R_A(\tau = 0) = S_{\text{GT}} \), the total amount of GT strength available for an initial state that is given by summing over a complete set of final states in GT transition matrix elements \( |M_{\text{GT}if}|^2 \). The strength distribution is given by (Dean et al. 1998)

\[
S_{\text{GT}}(E) = \sum_i e^{-\beta E_i} |\langle f|\hat{A}|i\rangle|^2
\]

which is related to \( R_A(\tau) \) by a Laplace Transform, \( R_A(\tau) = \int_{-\infty}^{\infty} S_A(E) e^{-\beta E} dE \). Note that here \( E \) is the energy transfer within the parent nucleus, and that the strength distribution \( S_{\text{GT}}(E) \) has units of MeV^{-1}.

3.2. The NELRs and EC Process in the Case without SMFs

Based on the RPA theory with a global parameterization of the single-particle numbers, the stellar EC rates that are related to the EC cross-section for the \( A \)th nucleus in thermal equilibrium at temperature \( T \) are given by a sum over the initial parent states \( i \) and the final daughter states \( f \) in the case without SMFs (Dean et al. 1998; Langanke & Martinez-Pinedo 1998; Langanke et al. 2001):

\[
\chi_{\nu e}^0 = \frac{1}{\pi \hbar^2} \sum_q \int_{e_0}^{\infty} p^2 \sigma_{\nu e}(\varepsilon_e, \varepsilon_i, \varepsilon_f) f(\varepsilon_e, U_F, T) d\varepsilon_e,
\]

where \( \varepsilon_0 = \max(Q_{if}, m_e c^2) \). \( p = \sqrt{\varepsilon_e^2 - m_e^2 c^4} \) is the momenta of the incoming electron, \( \varepsilon_e \) the total rest mass and kinetic energies of the incoming electron, \( U_F \) is the electron chemical potential, and \( T \) is the electron temperature. The electron Fermi–Dirac distribution is defined as

\[
f = f(\varepsilon_e, U_F, T) = \left[ 1 + \exp\left(\frac{\varepsilon_e - U_F}{kT}\right)\right]^{-1}.
\]

Due to the energy conservation, the electron, proton, and neutron energies are related to the neutrino energy, and the \( Q \)-value for the capture reaction (Cooperstein & Wambach 1984; Juodagalvis et al. 2010),

\[
Q_{if} = \varepsilon_e - \varepsilon_\nu = \varepsilon_n - \varepsilon_\nu = \varepsilon_f - \varepsilon_\nu,
\]

and

\[
\varepsilon_f - \varepsilon_\nu^p = \varepsilon_f^p + \hat{\mu} + \Delta_{np},
\]
where $\mu = \mu_n - \mu_p$, the difference between neutron and proton chemical potentials in the nucleus and $\Delta m = m_p c^2 - m_e c^2 = 1.293$ Mev, the neutron and the proton mass difference. $Q_{00} = M_e c^2 - M_p c^2 = \mu + \Delta m$, with $M_e$ and $M_p$ being the masses of the parent nucleus and the daughter nucleus, respectively; $\xi_{ef}$ corresponds to the excitation energies in the daughter nucleus at the states of the zero temperature.

The electron chemical potential is found by inverting the expression for the lepton number density (Fuller et al. 1980, 1982, 1985; Aufderheide et al. 1990)

$$n_e = \frac{8\pi}{(2\pi)^3} \int_0^{\infty} p_e^2 (f_{-e} - f_{+e}) dp_e,$$

where $f_{-e} = [1 + \exp((\varepsilon_e - U_F)/kT)]^{-1}$ and $f_{+e} = [1 + \exp((\varepsilon_e + U_F)/kT)]^{-1}$ are the electron and positron distribution functions, respectively, and $k$ is the Boltzmann constant.

According to the SMMC method, which discussed the GT strength distributions, the total cross-section by EC is given by (Dean et al. 1998; Juodagalvis et al. 2010)

$$\sigma_{ec} = \sigma_{ec}(E_e) = \sum_{\ell} \frac{(2\ell + 1)\exp(-\beta E_l)}{Z_A} S_{GT}(E_e),$$

where $\beta = 1/T_N$ is the inverse temperature, $T_N$ is the nuclear temperature, in units of MeV, and $E_e = \varepsilon_e - E_F$ is the electron energy. $S_{GT}$ is the GT strength distribution, which is as a function of the transition energy $\xi$. $g_{nk} = 1.1661 \times 10^{-5}$ GeV$^{-2}$ is the weak coupling constant and $G_A$ is the axial-vector form-factor, which, at zero momentum is $G_A = 1.25$. $F(Z, \varepsilon_e) = \exp(-e z_e)$ is the Coulomb wave function, which is the ratio of the square of the electron wavefunction distorted by the Coulomb scattering potential to the square of the wavefunction of the free electron.

By folding the total cross-section with the flux of a degenerate relativistic electron gas, the NELRs due to EC in the case without SMFs are given by

$$\lambda_{\text{NEL}}^0 = \frac{\ln 2}{6163} \int_0^{\infty} d\xi S_{\text{GT}} \frac{e^3}{(m_e c^2)^3} \int_0^{\infty} dp_e p_e^2 (-\xi + \varepsilon_e)^3 F(Z, \varepsilon_e) (s^{-1}),$$

where $\xi$ is the transition energy of the nucleus, and $f(\varepsilon_e, U_F, T)$ is the electron distribution function. The $p_0$ is defined as

$$p_0 = \begin{cases} \sqrt{Q_{f}^{2} - m_{e}^{2} c^{4}} - (Q_{f} < m_{e} c^{2}) \\ 0, \text{ (otherwise).} \end{cases}$$

3.3. The NELRs due to the EC process in the Case with SMFs

The NELRs due to EC in SMFs from one of the initial states to all possible final states is given by

$$\lambda_{\text{NEL}}^B = \frac{\ln 2}{6163} \int_0^{\infty} d\xi S_{\text{GT}} \frac{e^3}{(m_e c^2)^3} f_{fB}.$$
field at a given density is, the more available phase space there is for electrons. On the other hand, elections are very relativistic in the crusts of magnetars, where the matter density is higher and magnetic field strength may greatly exceed the surface value. The mean Fermi energy of an electron will exceed its rest-mass energy at sufficiently high density. Elections are also very relativistic when the cyclotron energy of an electron is higher than its rest-mass energy at a sufficiently high magnetic field. The EC will occur rapidly when the electron energy becomes larger than the difference between the neutron and proton rest-mass energy (about 1.3 MeV). This EC process will destroy electrons and emit massive neutrinos, thereby changing the composition of matter and softening the state equation of the magnetar surface.

The electron capture cross-sections (hereafter ECCSs) are very important parameters in the EC process. We find that the influence of SMFs on ECCSs at different temperatures is very significant for some nuclides due to the difference in Q-values. Given the significant energy dependence of cross-sections for a process like EC on nuclei, it is clear that in some cases the rates will be increased at the same temperature and density as the magnetic field increases. With this increasing of electron energy, the ECCS increases, according to our investigations. The higher the temperature, the faster the changes to ECCSs become. This is because the higher the temperature, the larger the electron energy becomes. Thus even more electrons will join in the EC process due to their energy being greater than the Q-values. Furthermore, the GT transition may be dominated at high surrounding temperatures. On the other hand, the trigger mechanism of the EC process requires a minimum electron energy given by the mass splitting between a parent and a daughter (i.e., $Q_{ij}$). The EC threshold energy is lowered by the internal excitation energy at a finite temperature. The GT strength for even–even parent nuclei is centered at daughter excitation energies of the order of 2 MeV at low temperatures. Therefore, the ECCSs for these parent nuclei increase drastically within the first couple of MeV of electron energies above the threshold, which reflect the GT distribution. But the GT distribution for odd-A nuclei will peak at noticeably higher daughter excitation energies at low temperatures. So the ECCSs are shifted to higher electron energies for odd-A nuclei in comparison to even–even parent nuclei by about 3 MeV.

However, in Figures 1 through 4 one finds that there is a systematic decrease in NELR when $B_{12} > 100$. We know that the Landau energy level spacing will become a very small fraction for the Fermi energy when $\nu_{\text{max}} \gg 1$. The continuous integral may be taken in lieu of the discrete sum over all Landau levels. The thermodynamic relation will reduce to the case of non-magnetic fields. We have $\nu_{\text{max}} \to \frac{U_{\text{Fermi}}}{\hbar \omega}$, when the electron gas is of a mildly relativistic state. For relativistic

![Figure 1](image1.png)  
![Figure 2](image2.png)
electron gas, the $\nu_{\text{max}} \geq 100/B_{12}$ when the $U_e \geq 1$ MeV. Therefore, as magnetic field strength increases (i.e., when $B_{12} \geq 10–100$), the $\nu_{\text{max}}$ will tend to the order of unity (or zero), and the field will be termed as strongly quantizing (Lai & Shapiro 1991). On the other hand, the structures at the outer crusts of magnetars are fundamentally determined by the energies of isolated nuclei, the kinetic energy of electrons, and the lattice energy. Thus its composition is strongly dependent on the binding energy per particle of stable and unstable nuclei in the outer crusts of magnetars below the neutron drip density.

Based on the relativistic mean-field effective interactions NL3 (Lalazissis et al. 1997) and DD-ME2 (Lalazissis et al. 2005), the influence of SMFs on the binding energy of nuclei has been investigated following the works of Peña Arteaga et al. (2011) and Basilico et al. (2015). We find that, with increasing the magnetic field, the binding energy per particle will have a mean parabolic increasing trend. For example, the binding energy increases by 0.311 MeV, 0.632 MeV, and 0.445 MeV for $^{56}$Fe, $^{78}$Ni, and $^{56}$Co, respectively, when the magnetic field strength increases from $10^{17}$ to $10^{18}$ G. Due to the increase of the nuclear binding energy, the nuclei will be more stable. This is equivalent to significantly raising the threshold energy of the EC reaction. Thus, the NELRs and EC rates will be decreased in SMFs. Meanwhile, as the magnetic field strength increases, the electron Fermi energy will greatly decrease due to interactions between the electrons and SMFs. This actually discourages the EC reaction, so the the NELRs and EC rates decrease.

The magnetic field strongly affects the electron phase space. Only axial symmetry is preserved, and it breaks the spherical symmetry for the Dirac and Klein–Gordon equations (Peña Arteaga et al. 2011). For a certain value of magnetic field, Figures 5 and 6 present the NELRs of some typical iron group nuclei versus the density $\rho_7$ at temperatures of $T_9 = 0.133$, 11.33. One finds that the NELRs increase greatly and even exceed four orders of magnitude for certain values of magnetic field and temperature. With an increase in density, there is almost no influence on the densities of NELRs. On the other hand, for the density around $\rho_7 = 10^7$, there is an abrupt increase in NELRs when $10^{3.5} \lesssim B_{12} \lesssim 10^5$. Such jumps are an indication that the underlying shell structure has changed in a fundamental way. These jumps in nuclear properties can be traced to the single-particle behavior due to SMFs. As the magnetic field increases, a particle will be removed from a level going upward and brought to a level going downward with increasing spin. Furthermore, the nucleus becomes spin-polarized due to these two levels having opposite angular momenta along the symmetry axis.

The SMFs influence the single-particle structure of nuclei for protons and neutrons. First, the interaction between the
magnetic field and the neutron (proton) magnetic dipole moment will cause nucleon paramagnetism. Second, the coupling of the orbital motion of protons with the magnetic field will also cause proton orbital magnetism. Due to the interaction between the nucleus and SMFs, all degeneracies in the single-particle spectrum may be removed, and the formerly degenerate levels with opposing signs of angular momentum projection will also tend to break (as an example, for $^{56}$Fe, detailed discussions can be seen in Figures 3 and 5 of Peña Arteaga et al. 2011). Such single-particle energy splitting will produce a reduction of the neutron and proton pairing gaps, with increasing magnetic fields, and eventually result in their disappearance.

According to the discussion of the influence of SMFs on single-particle levels, one will find that the Kramer’s degeneracy in the angular momentum projection of proton levels is removed by the orbital magnetism associated with proton ballistic dynamics, which can bring those aligned with the magnetic field down in energy. On the other hand, the paramagnetic response (Pauli magnetism) also removes the angular momentum projection degeneracy for both protons and neutrons.

Summarizing the above analysis, one can conclude that the last occupied single-particle levels (e.g., for $^{56}$Fe) for neutrons, including the influence of the proton orbital coupling and the anomalous magnetic moments coupling, decrease as the magnetic field strength increases. However, the last occupied single-particle levels for protons will increase. As is well-known, the EC is actually the process in which protons turn into neutrons and discharge a neutrino when a nuclei captures an electron. Therefore, to a certain extent, this influence of SMFs on single-particle levels for proton and neutron states ultimately makes the EC reaction become more active and increases the NELRs.

The SMFs may not directly influence the lattice energy. Nevertheless, some indirect influence on the lattice configuration will be caused by Coulomb screening. We find it to be energetically favorable to arrange the ionized nuclei in a Coulomb lattice in the typical density range of the magnetar surface (i.e., $10^{3} \text{ g cm}^{-3} \sim 4 \times 10^{11} \text{ g cm}^{-3}$). In a relativistic lower range of density, the electron energy and the Coulomb crystal do not play a relevant role in magnetar crusts. As the density increases, the electron energy raises greatly compared to the total energy, which is very advantageous for electronic capture processes. However, the lattice energy’s influence remains negligible. So, we ignore the influence of SMFs on the lattice energy, the EC, and NELRs. On the other hand, the cyclotron energy $\hbar \omega_{c}$ is much larger than the typical Coulomb energy. Therefore, the properties of atoms, molecules, and condensed matter are qualitatively changed by the magnetic

Figure 5. NELRs for some typical iron group nuclei as a function of $\rho_{7}$ at $B_{12} = 10^{2}, 10^{3}, 10^{4}, 10^{5}, 10^{6}, 10^{7}$, and $T_{9} = 0.133$.

Figure 6. NELRs for some typical iron group nuclei as a function of $\rho_{7}$ at $B_{12} = 10^{2}, 10^{3}, 10^{4}, 10^{5}, 10^{6}, 10^{7}$, and $T_{9} = 11.33$. 

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field when $B \gg 2.3505 \times 10^9$ G. The usual perturbative treatment from the magnetic influence on Zeeman splitting of atomic energy levels does not apply in such a regime of SMFs due to the Coulomb forces acting as a perturbation to the magnetic forces (Garstang 1977). The Coulomb force becomes much more effective at binding the electrons along the magnetic field direction due to the extreme confinement of the electrons in the transverse direction (i.e., perpendicular to the magnetic field). The atom attains a cylindrical structure. Moreover, it is possible for these elongated atoms to form molecular chains by covalent bonding along the field direction.

One can also see that as SMFs increase, the change to NELRs will reflect some different due to strong quantum effects in SMFs from Figures 5 to 6. As the density increases, when $\rho \gg \rho_B$ and $T \leq T_B$, where $\rho_B = 7.04 \times 10^5 Y_e^{-1} B_{12}^{3/2} \ g\ cm^{-3}$, $T_B = 1.34 \times 10^8 B_{12}(1 + x_e^2(\nu))^{-1/2}$ K, the electrons will be strongly degenerate, and populate many Landau levels (Lai 2001, 2015). The magnetic field is termed weakly quantizing. Thus the chemical potential, EC rates, and NELRs are only slightly affected by SMFs. With increasing $T$, the oscillations become weaker because of the thermal broadening of the Landau levels. When $T \geq T_B$ or $\rho \gg \rho_B$, more electrons will populate many Landau levels and the thermal widths of the Landau levels ($\sim kT$) are higher than the level spacing. The magnetic fields have almost no influence on the EC and NELRs.

GT strength distributions play an important role in EC processes in the astrophysical context. Fassio-Canuto (1969) and Canuto & Ventura (1977) demonstrated that the GT transition matrix elements for EC do not depend on the magnetic fields. Thus we will neglect the effect of SMFs on the GT properties of nuclei in this paper. A strong phase space dependence makes the EC rates more sensitive to GT distributions than to total strengths. We present GT strength distributions from shell model Monte Carlo studies of some typical fp-shell iron group nuclei in Figures 7 and 8. We also display the experimental data for GT distributions (Rapaport et al. 1984; Alford et al. 1993; El-Kateb et al. 1994; Rapaport et al. 1984) for some typical iron group nuclei as a function of excitation energy in the corresponding daughter nuclei at temperature $T = 0.8$ MeV.

The structure and composition of the crust are important for the thermal and magnetic evolution of neutron stars. The SMFs not only strongly influence the weak interaction rates and NELRs, but also influence the late evolution and determine the core entropy and electron-to-baryon ratio of magnetars.

Figure 7. Comparison of calculated $B(GT_+)$ strength distributions against experiment results (Alford et al. 1993; Williams et al. 1995; El-Kateb et al. 1994; Rapaport et al. 1984) for some typical iron group nuclei as a function of excitation energy in the corresponding daughter nuclei at temperature $T = 0.8$ MeV.

Figure 8. Comparison of calculated $B(GT_+)$ strength distributions against experiment results (Alford et al. 1993; Williams et al. 1995; El-Kateb et al. 1994; Rapaport et al. 1984) for some typical iron group nuclei as a function of excitation energy in the corresponding daughter nuclei at temperature $T = 0.8$ MeV.
Note. All of the NELRs are in units of \( m_e c^2 \text{s}^{-1} \).

Table 1

The Maximum Values of NELRs \( (\lambda_{\text{NEL(max)}}) \) at the Relativistic Low Temperature \( T_0 = 0.233 \) for Different Densities when \( 10 \leq B_{12} \leq 1000 \)

| Nuclide  | \( B_{12} \) | \( \lambda_{\text{NEL(max)}} \) | \( B_{12} \) | \( \lambda_{\text{NEL(max)}} \) | \( B_{12} \) | \( \lambda_{\text{NEL(max)}} \) | \( B_{12} \) | \( \lambda_{\text{NEL(max)}} \) |
|----------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|
| \(^{52}\text{Fe}\) | 61.36 | 3.482e6 | 141.7 | 8.07e6 | 497.7 | 2.834e7 | 1.0e3 | 5.694e7 |
| \(^{53}\text{Fe}\) | 61.36 | 3.319e6 | 141.7 | 8.070e6 | 497.7 | 2.832e7 | 1.0e3 | 5.477e7 |
| \(^{54}\text{Fe}\) | 53.37 | 3.039e6 | 141.7 | 8.069e6 | 497.7 | 2.726e7 | 1.0e3 | 5.476e7 |
| \(^{55}\text{Fe}\) | 53.37 | 3.037e6 | 141.7 | 7.763e6 | 497.9 | 2.725e7 | 1.0e3 | 5.694e7 |
| \(^{56}\text{Fe}\) | 53.37 | 2.333e6 | 141.7 | 6.198e6 | 497.7 | 2.124e7 | 1.0e3 | 4.269e7 |
| \(^{57}\text{Fe}\) | 53.37 | 2.332e6 | 141.7 | 6.196e6 | 497.7 | 1.893e7 | 1.0e3 | 4.373e7 |
| \(^{58}\text{Fe}\) | 53.37 | 1.645e6 | 141.7 | 3.318e6 | 432.9 | 1.213e7 | 1.0e3 | 2.803e7 |
| \(^{59}\text{Fe}\) | 53.37 | 1.642e6 | 141.7 | 3.317e6 | 432.9 | 1.335e7 | 1.0e3 | 3.083e7 |
| \(^{60}\text{Fe}\) | 53.37 | 9.998e5 | 141.7 | 1.697e6 | 432.9 | 8.110e6 | 1.0e3 | 1.645e7 |
| \(^{61}\text{Fe}\) | 53.37 | 7.254e5 | 141.7 | 1.059e6 | 432.9 | 5.884e6 | 1.0e3 | 1.032e7 |

According to the above discussion, we can conclude that the SMFs have a significant influence on the NELRs for a given temperature-density point. Generally, the stronger the density and the lower the SMFs, the larger the effect on the NELRs becomes. One also finds that when \( 10^{14} \text{G} \leq B \leq 10^{16} \text{G} \), for most iron group nuclei, the rates decrease greatly for a given temperature-density point. The reason is that the Fermi energy of electrons decreases, but the binding energy of the nucleus will increase with the increase of SMFs when the temperature and density are constant. Therefore, these lead to more and more electrons whose energies will be less than the threshold for the EC process.
Tables 3 and 4 display comparisons of our results with those of FFN (Fuller et al. 1982, 1985) ($\lambda_{\text{NEL}}^F$ (FFN)), and NKK ($\lambda_{\text{NEL}}^F$ (NKK); Nabi & Klapdor-Kleingrothaus 1999) at $\rho/\mu_p = 10^7$, $T_9 = 1, 3$. For the case without SMFs, at relativistic low temperatures $T_9 = 1$. One finds that our rates are close to about five orders of magnitude lower than those of FFN (e.g., for $^{59}$Fe, $^{60}$Co), and NKK (e.g., $^{57}$Mn, $^{55}$, $^{56}$Cr). However, at the relativistic high temperature $T_9 = 3$, our numerical results are in good agreement with those of NKK, but are about one order of magnitude lower than those of FFN. For the case with SMFs, due to SMFs our rates at $T_9 = 1$ can increase by more than four orders of magnitude when $B_{12} < 10^2$, and then decrease by more than three orders of magnitude as the magnetic fields increase to $B_{12} = 10^7$. On the other hand, our NELRs for some iron group nuclei can be about five orders of magnitude higher than those of FFN and NKK.

Due to the fact that the EC $Q$-value for the neutron-rich nuclide (e.g., $^{60}$Fe) has not been measured, FFN had to use the semiempirical atomic mass formula to estimate them. Thus, the $Q$-values used in the effective rates are quite different. For instance, for odd-A nuclei (e.g., $^{59}$Fe), FFN places the centroid of the GT strength at excitation energies that are too low (see the detailed discussed in Fuller et al. 1982, 1985). The method for truncating a state-density
Comparisons of Calculations log 10(λ^0_{NEL}) (LJ) in SMFs for Some Typical Iron Group Nuclides with Those of FFN (log 10(λ^0_{NEL}(FFN))) (Fuller et al. 1982, 1985) and NKK (log 10(λ^0_{NEL}(NKK))) (Nabi & Klappdor-Kleingrothaus 1999), and Ours (log 10(λ^0_{NEL}(LJ))), which Are For the Case without SMFs at ρ/μn = 10^3 g cm^{-3}, T = 1

| Nuclide | log 10(λ^0_{NEL}(FFN)) | log 10(λ^0_{NEL}(NKK)) | log 10(λ^0_{NEL}(LJ)) | B_{12} = 10 | B_{12} = 10^2 | B_{12} = 10^3 | B_{12} = 10^4 |
|---------|------------------------|------------------------|------------------------|-------------|-------------|-------------|-------------|
| Fe^{58} | -2.266 | -2.207 | -2.408 | 5.4623 | 1.3769 | 1.9555 | 2.9406 |
| Fe^{59} | -1.533 | -1.367 | -1.408 | 5.4454 | 0.8495 | 1.4371 | 2.4227 |
| Fe^{60} | -9.439 | -8.710 | -8.807 | 5.4470 | -3.2184 | -4.5502 | -3.5769 |
| Fe^{57} | -4.988 | -4.222 | -4.817 | 5.4639 | -2.1618 | -2.1086 | -1.1235 |
| Fe^{58} | -19.733 | -21.613 | -21.782 | 5.3235 | -17.8704 | -19.3726 | -18.3976 |
| Fe^{56} | -15.352 | -15.800 | -15.907 | 5.2719 | -15.5220 | -16.9372 | -15.9614 |
| Fe^{56} | -32.165 | -32.041 | -32.197 | 5.0051 | -30.7358 | -32.0685 | -31.0918 |
| Co^{58} | -27.288 | -21.174 | -28.299 | 4.9952 | -28.0682 | -29.3225 | -28.3450 |
| Co^{59} | -49.560 | -43.170 | -50.508 | 4.6955 | -42.1232 | -43.3029 | -42.3247 |
| Co^{57} | -2.466 | -1.853 | -1.909 | 5.6083 | 0.7675 | 1.3545 | 2.3401 |
| Co^{58} | -2.316 | -2.774 | -2.882 | 5.5983 | 1.1178 | 1.7169 | 2.7030 |
| Co^{57} | -4.385 | -4.596 | -4.656 | 5.5130 | -0.9916 | -0.4964 | 0.4900 |
| Co^{58} | -2.511 | -4.520 | -4.623 | 5.5124 | 0.2398 | 0.8292 | 1.8162 |
| Co^{58} | -11.977 | -11.121 | -11.856 | 5.2615 | -7.4566 | -8.8876 | -7.9119 |
| Co^{59} | -7.682 | -12.430 | -12.823 | 5.1847 | -2.1135 | -1.8874 | -0.9006 |
| Ni^{56} | -3.074 | -3.060 | -3.103 | 5.5258 | 0.1892 | 0.7522 | 1.7372 |
| Ni^{57} | -2.830 | -1.412 | -1.606 | 5.5346 | 0.8998 | 1.4911 | 2.4767 |
| Ni^{58} | -10.608 | -6.577 | -9.001 | 5.3955 | -2.3616 | -2.7028 | -1.7202 |
| Ni^{58} | -3.935 | -3.763 | -3.972 | 5.4534 | -0.2020 | 0.3629 | 1.3494 |
| Ni^{59} | -16.769 | -18.503 | -21.697 | 5.2962 | -13.5209 | -15.0489 | -14.0742 |
| Mn^{55} | -11.289 | -10.253 | -11.462 | 5.2746 | -7.850 | -9.1641 | -8.1873 |
| Mn^{57} | -32.154 | -27.400 | -32.902 | 5.0488 | -25.0826 | -26.3156 | -25.3380 |
| Mn^{58} | -25.530 | -33.658 | -33.774 | 4.8537 | -19.5938 | -20.7505 | -19.77213 |
| Mn^{59} | -44.333 | -38.534 | -41.612 | 4.6066 | -37.8604 | -38.9445 | -37.9654 |
| Mn^{56} | -29.206 | -35.262 | -36.124 | 4.5672 | -30.2670 | -31.2823 | -30.3024 |
| Cr^{55} | -18.601 | -18.708 | -18.899 | 5.2452 | -16.7240 | -18.1043 | -17.1281 |
| Cr^{56} | -35.781 | -35.287 | -35.814 | 5.5851 | 1.7351 | 2.3534 | 3.3416 |
| Cr^{58} | -34.262 | -30.927 | -35.169 | 4.9785 | -29.4122 | -30.6225 | -29.6464 |
| Cr^{57} | -53.637 | -46.164 | -53.944 | 4.7316 | -44.9487 | -46.0808 | -45.1021 |
| Cr^{58} | -41.403 | -40.338 | -42.526 | 4.8297 | -39.8564 | -40.9143 | -39.9349 |
| Cr^{59} | -63.730 | -57.779 | -64.983 | 4.5370 | -57.6312 | -58.6188 | -57.6386 |
| Cr^{56} | -50.130 | -50.187 | -51.356 | 4.4363 | -49.6191 | -50.5389 | -49.5590 |
| Cr^{57} | -73.126 | -68.563 | -72.963 | 3.8418 | -68.8443 | -69.7016 | -68.7202 |
| V^{50} | -1.816 | -1.835 | -1.932 | 5.7858 | 2.1492 | 2.7438 | 3.7295 |
| V^{51} | -3.214 | -3.024 | -3.564 | 5.4915 | 0.7222 | 2.2959 | 2.9596 |
| V^{52} | -4.081 | -3.415 | -4.355 | 5.3836 | -1.4139 | -0.9871 | -6.6199 |
| V^{50} | -5.011 | -8.005 | -8.213 | 5.3945 | 0.1058 | 0.6896 | 1.6769 |
| V^{56} | -39.223 | -35.798 | -39.314 | 5.0146 | 2.0691 | 2.7072 | 3.6971 |

Note. All of the NELRs are in units of MeV s^{-1}.
Table 4
Comparisons of Calculations log 10(\(\lambda_{\text{NEL}}^B\) (LJ)) in SMFs for Some Typical Iron Group Nuclei with those of FFN (log 10(\(\lambda_{\text{NEL}}^B\) (FFN))) (Fuller et al. 1982, 1985), NKK (log 10(\(\lambda_{\text{NEL}}^B\) (NKK))) (Nabi & Klapisch-Kleingrothaus 1999), and Ours (log 10(\(\lambda_{\text{NEL}}^B\) (LJ))), which Are For the Case without SMFs at \(\rho/\mu_0 = 10^7\) g cm\(^{-3}\), \(T_0 = 3\)

| Nuclide | \(\log 10(\lambda_{\text{NEL}}^B)\) (FFN) | \(\log 10(\lambda_{\text{NEL}}^B)\) (NKK) | \(\log 10(\lambda_{\text{NEL}}^B)\) (LJ) |
|---------|-----------------------------------|-----------------------------------|-----------------------------------|
| \(^{52}\text{Fe}\) | 6.637 | 5.664 | 5.558 |
| \(^{53}\text{Fe}\) | 6.019 | 5.598 | 5.578 |
| \(^{54}\text{Fe}\) | 6.125 | 5.458 | 5.447 |
| \(^{55}\text{Fe}\) | 6.283 | 5.357 | 5.347 |
| \(^{56}\text{Fe}\) | 5.836 | 5.335 | 5.328 |
| \(^{57}\text{Fe}\) | 6.110 | 5.120 | 5.075 |
| \(^{58}\text{Fe}\) | 5.760 | 5.360 | 5.347 |
| \(^{59}\text{Fe}\) | 5.781 | 5.458 | 5.443 |
| \(^{60}\text{Fe}\) | 5.156 | 4.025 | 4.018 |
| \(^{53}\text{Co}\) | 6.498 | 5.903 | 5.779 |
| \(^{56}\text{Co}\) | 6.469 | 5.851 | 5.814 |
| \(^{57}\text{Co}\) | 6.313 | 5.669 | 5.556 |
| \(^{56}\text{Cr}\) | 6.377 | 5.717 | 5.696 |
| \(^{58}\text{Cr}\) | 6.265 | 5.544 | 5.531 |
| \(^{59}\text{Cr}\) | 6.289 | 7.390 | 7.234 |
| \(^{58}\text{Ni}\) | 6.492 | 6.000 | 5.863 |
| \(^{70}\text{Fe}\) | 6.453 | 5.826 | 5.786 |
| \(^{57}\text{Ni}\) | 6.200 | 5.760 | 5.731 |
| \(^{56}\text{Ni}\) | 6.286 | 5.751 | 5.682 |
| \(^{60}\text{Ni}\) | 6.234 | 5.590 | 5.560 |
| \(^{58}\text{Mn}\) | 5.807 | 5.451 | 5.412 |
| \(^{57}\text{Mn}\) | 5.686 | 5.466 | 5.432 |
| \(^{56}\text{Mn}\) | 5.757 | 5.731 | 5.710 |
| \(^{59}\text{Mn}\) | 5.236 | 5.004 | 4.997 |
| \(^{60}\text{Mn}\) | 5.486 | 5.677 | 5.642 |
| \(^{53}\text{Cr}\) | 6.534 | 5.459 | 5.411 |
| \(^{54}\text{Cr}\) | 5.276 | 5.518 | 5.484 |
| \(^{55}\text{Cr}\) | 5.120 | 5.504 | 5.499 |
| \(^{56}\text{Cr}\) | 5.002 | 4.775 | 4.714 |
| \(^{57}\text{Cr}\) | 5.287 | 4.468 | 4.456 |
| \(^{58}\text{Cr}\) | 4.533 | 4.314 | 4.243 |
| \(^{59}\text{Cr}\) | 4.859 | 4.257 | 4.158 |
| \(^{60}\text{Cr}\) | 3.826 | 3.803 | 3.793 |
| \(^{49}\text{V}\) | 5.832 | 5.587 | 5.577 |
| \(^{50}\text{V}\) | 5.706 | 5.575 | 5.565 |
| \(^{51}\text{V}\) | 5.543 | 5.496 | 5.423 |
| \(^{52}\text{V}\) | 5.560 | 5.559 | 5.516 |
| \(^{53}\text{V}\) | 5.356 | 4.173 | 4.063 |

Note. All of the NELRs are in units of MeV s\(^{-1}\).

The process of weak interaction, especially for information regarding the total GT strength distribution in nuclei. The experimental information is particularly rich for some iron group nuclei and it is the availability of both GT\(^{+}\) and GT\(^^{-}\) that makes it possible to study in detail the problem of renormalization of \(\sigma\tau\) operators. We have calculated the total GT strength in a full \(p-f\) shell calculation, resulting in \(B(\text{GT}) = g_A^2(\sigma\tau_+)^2\), where \(g_A^2\) is the axial-vector coupling constant. For example, in magnetars the EC on \(^{58}\text{Fe}\) is dominated by the wavefunctions of the parent and daughter states. The total GT strength for \(^{58}\text{Fe}\) in a full \(p-f\) shell calculation results in \(B(\text{GT}) = 10.1g_A^2\) (Langanke & Martinez-Pinedo 1998; Langanke et al. 2001). For instance, the total GT strength of the other important nuclides \(^{56}\text{Fe}\) and \(^{56}\text{Ni}\) in a full \(p-f\) shell calculation can be found Dean et al. (1998). An average of the GT strength distribution is in fact obtained by the SMMC method. A reliable replication of the GT distribution in the nucleus is can be found using an amplification of the electronic shell model. Thus the method is relatively accurate.

In summary, by analyzing the influence of SMFs on NELRs in the surfaces of magnetars, we find that SMFs have a significant effect on NELRs for different nuclides, particularly for some heavier nuclides, whose thresholds are negative at higher density. According to the above calculations and discussion, we conclude that the NELRs can increase by more than four orders of magnitude. As the magnetic fields increase, the NELRs decrease greatly, by more than three orders of magnitude. On the other hand, we compared our results for the SMFs with those of FFN and NKK. For the case without SMFs, we find that our rates are close to about four orders of magnitude lower than those for FFN and NKK. However, in the relativistic low temperature \(T_0 = 1\), our results are in good agreement with those of NKK, but about one order of magnitude lower than those of FFN. For the case with SMFs, our NELRs for some...
iron group nuclei can be about five orders of magnitude higher
than those of FFN and NKK.

5. CONCLUSIONS AND OUTLOOKS

The properties of matter in magnetar surfaces with SMFs have always been an interesting and challenging subject for physicists. These properties are an important component of neutron star research of matter in strong magnetic fields. In particular, some thermal and magnetic evolutions from the cooling of neutron stars requires a detailed theoretical understanding of the physical properties of highly magnetized atoms, molecules, and condensed matter. In this paper, we have focused on electronic structures and the properties of matter in SMFs in magnetars. We have also discussed the influence of SMFs on electron Fermi energy, binding energy per nucleon, and single-particle level structure in magnetar surfaces based on the relativistic mean-field effective interactions theory. By using the SMMC method and the RPA theory, we analyze in detail the NELRs using the EC process of iron group nuclei. We also compare our results for SMFs with those of FFN and NKK, which did not consider SMFs.

First, we analyze the influence of the SMFs on NELRs when temperature and density are constant in the EC process. We find that the influence of SMFs on NELRs is very obvious and significant. At $T_9 = 0.233$, when $B_{12} < 100$, the SMFs have a slight influence on the NELRs for most nuclides. Nevertheless, the NELRs increase by more than four orders of magnitude at $T_9 = 15.53$ when $B_{12} < 100$. And then the NELRs rates decrease by more than three orders of magnitude when $B_{12} > 100$ at relatively high temperatures (e.g., at $T_9 = 15.53$ for $^{52,61}$Fe, $^{55,66}$Co and $^{56,63}$Ni).

Second, we discuss the influence of density on NELRs at different temperature and magnetic field points in the EC process. We find that for certain values of magnetic field and temperature, the NELRs increase greatly and even exceed four orders of magnitude when $B_{12} > 100$. Such jumps are an indication that the underlying shell structure has changed in a fundamental way due to single-particle behavior by SMFs.

Finally, we compare our results with those of FFN and NKK due to their different methods for calculating the NELRs. For cases without SMFs, one finds that our rates are close to about five orders of magnitude lower than FFN and NKK at the relativistic low temperature $T_9 = 1$. However, at the relativistic high temperature $T_9 = 3$, our results are in good agreement with those of NKK, but about one order magnitude lower than those of FFN. For the case with SMFs, our NELRs for some iron group nuclei can be about five orders of magnitude higher than those of FFN and NKK.

On the other hand, the composition structure at the outer crusts of magnetars are fundamentally determined by the energies of isolated nuclei, such as the binding energy, the kinetic energy of electrons, and the lattice energy. We discuss the influence of SMFs on the binding energy of the nuclei, the single-particle level structure, and the electron Fermi energy. One finds that the NELRs increase due to an increase of the electron Fermi energy, and the change to the single-particle level structures by SMFs. On the contrary, the NELRs decrease due to an increase of the binding energy of the nuclei by SMFs.

As is well-known, the NELRs of the EC play an important role in the dynamics processes and cooling mechanisms of magnetars. NELRs also are a main parameter, which leads to the thermal evolution and magnetic evolution of magnetars. Recent studies have shown that observations of magnetars suggest that the luminosities of persistent X-rays radiated from magnetars are likely of thermal origin, such as heating by a magnetospheric current, or by EC in the outer crust. The heat released due to EC for some iron group nuclei on magnetar surfaces may balance both the surface and inner temperatures of a magnetar to different degrees. However, the considerable mechanism of the X-ray source is not clear. How can we determine the influence of SMFs on soft X-ray emission, which is the possible origin of the NELRs in magnetars? How can we understand the nature of the cooling from NELRs in magnetar? How can we determine the influence of SMFs on the EC process in NELRs when the magnetic pressure decreases and the crust shrinks, and the density and electron Fermi energy increase? These problem with SMFs in magnetars have always been an interesting and challenging issue. Our conclusions may be helpful for investigating the thermal evolution, the nucleosyntheses of heavy elements, the numerical calculations, and simulations of neutron stars and magnetars.

We thank the anonymous referee for carefully reading the manuscript and providing valuable comments that substantially improved this paper. This work was supported in part by the National Basic Research Program of China (973 Program) under grant 2014CB845800, by the National Natural Science Foundation of China under grants 11565020, 11573023, and 1122238 and by the Natural Science Foundation of Hainan province under grant 114012.

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