1. Introduction
The gravitational acceleration \( g \) at the surface of an approximately spherical planet is
\[
g = \frac{GM}{R^2},
\]
where \( G \) is the gravitational constant and \( M \) and \( R \) are, respectively, the mass and radius of the planet.

The time \( T \) required to fall from a height \( h \) due to only a gravitational acceleration \( g \) can be derived from the equation for distance travelled in time \( T \) at a fixed acceleration:
\[
h = \frac{1}{2} g T^2 \quad \rightarrow \quad T(h) = \sqrt{\frac{2h}{g}}. \tag{1}
\]

When the acceleration varies (due to air resistance), a computer simulation can be used to calculate the fall time \( T \).

We have designed an experiment on Earth to match the fall time of an object on Mars. The MarsSim row in table 1 gives the fall time results of a computer simulation using the parameters of that experiment. A drop height of 2 meters matches the fall time on Mars and produces a relatively good match to Martian dynamics.

2. Logistics
2.1. Theory
The net acceleration at time \( t \) on a falling object of total mass \( m_{\text{tot}} \) is a downward (negative) gravitational acceleration plus upward air resistance and buoyancy terms:
\[
a(t) = -g + \frac{1}{2} \rho_{\text{air}} C_{\text{drag}} A [v(t)]^2/m_{\text{tot}} + F_{\text{buoy}}/m_{\text{tot}} \tag{2}
\]
where the density of air is \( \rho_{\text{air}} \), the drag coefficient of the object is \( C_{\text{drag}} \), the horizontal cross-sectional area of the object is \( A \), and the object’s downward velocity is \( v(t) \). The buoyancy force is \( F_{\text{buoy}} = m_{\text{air}} g \), where \( m_{\text{air}} \) is the mass of air displaced by the object.

2.2. The simulation
For our simulation, the object is a hollow rectangular box or bag of vertical depth \( d \), length \( \ell \) and width \( w \) (so that \( A = \ell w \)), with sides of negligible thickness and with mass \( m_{\text{obj}} \). For such an object, the mass of air inside the box is \( m_{\text{air}} = \rho_{\text{air}} \ell w d \) and the total mass of the falling box is \( m_{\text{tot}} = m_{\text{obj}} + m_{\text{air}} \). The drag coefficient for a rectangular box will lie between \( C_{\text{drag}} = 1.05 \) (cube, \( \ell = w = d \)) and \( C_{\text{drag}} = 1.17 \) (thin rectangular plate with \( \ell/w < 5 \) and \( d \ll \ell \), Hoerner [1965]). We assume \( C_{\text{drag}} = 1.11 \) for our rectangular box with \( \ell \approx w \) and \( d/\ell \approx 0.5 \). Using the above quantities in the expression for \( a(t) \), the velocity \( v(t) \) and height fallen \( z(t) \) can be found numerically starting from \( a(0) = -g \), \( v(0) = 0 \).
and $z(0) = h$. The time $T$ required to fall a distance $h$ is found by stopping the calculation when $z(T) = 0$.

We use an empty, rectangular, vinyl bag (Richards Homewares Clear Vinyl Jumbo Blanket Bag No. 441W) with $l = 0.533$ m, $w = 0.635$ m and $d = 0.279$ m, so that with $\rho_{\text{air}} = 1.225$ kg m$^{-3}$, $m_{\text{air}} = 0.116$ kg. The bag itself weighs $m_{\text{obj}} = 0.140$ kg. When spray-painted to resemble a Martian rock using a can of orange or reddish paint the bag weighs $m_{\text{obj}} = 0.180$ kg.

With $\rho_{\text{air}} = 1.225$ kg m$^{-3}$, $C_{\text{drag}} = 1.11$, $A = 0.3385$ m$^2$, $m_{\text{tot}} = 0.2960$ kg, and $F_{\text{buoy}} = 1.1348$ N, we can then write (2) as

$$a(t) = -g + b[v(t)]^2 + a_{\text{buoy}}$$

(3)

where $b = 0.7774$ m$^{-1}$ and $a_{\text{buoy}} = 3.834$ m s$^{-2}$.

3. Results

Table 1 gives the calculated times for the spray-painted bag to fall distances of 1–3 meters on Earth. Those times match the times required for an object to fall from the same heights on Mars to within $\pm 11\%$. The fall time from 2 meters was experimentally verified to be $1.06 \pm 0.17$ s using 3 observers’ measurements of 3 separate drops. Since this drop was the best match to Martian gravity, we include its computer simulated dynamics as compared to real Martian gravity as figure 1. We also include an image of the bag as figure 2.

4. Conclusion

This simulation can be used as a straightforward demonstration of the dynamics in Martian gravity as compared to Earth gravity, by simultaneously dropping from the same height, an identical bag filled with textiles.

This simulation can also be used to illustrate the concept of air resistance. Rotating the bag can yield different values of $A$, resulting in different fall times. Furthermore, fall time measurement distributions, averages and root-mean-square uncertainties can be calculated by having students time one or more drops and record their measurements.

Alternatively, this simulation can be presented as a computational assignment. For example, given an object of fixed volume and mass (matching the spray-painted bag), find the horizontal surface area it must have so that the fall time is the same on Earth as on Mars for a given height.

Reference

Hoerner S F 1965 *Fluid Dynamic Drag* 2nd edn (Bakersfield, CA: Hoerner Fluid Dynamics) pp 3–16
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