Investigation of Doubly Heavy Tetraquark Systems using Lattice QCD

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**Experimental background**

- Experimentally observed states $Z_b(10610)^+$ and $Z_b(10650)^+$
- Mass suggests a bottomonium state $\bar{b}b$ but would be electrically neutral
  ⇒ Quantum numbers with four-quark structure possible to describe
Physical Motivation (1)

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Theoretical study

- We study similar but less challenging systems
- Quark content: $\bar{Q}Q'qq'$, here: $\bar{b}bud, \bar{b}bus, \bar{b}cud$
- In the limit $m_Q \to \infty$ stable tetraquark was shown

[J. Carlson, L. Heller and J. A. Tjon, Phys. Rev. D 37, 744 (1988)]
[A. V. Manohar and M. B. Wise, Nucl. Phys. B 399, 17 (1993)]
[E. J. Eichten and C. Quigg, Phys. Rev. Lett. 119, no. 20, 202002 (2017)]
Physical Motivation (2)

- **Born-Oppenheimer study** of $\bar{b}\bar{b}ud$, static $\bar{b}$-quarks:
  - Prediction of a bound tetraquark with $I(J^P) = 0(1^+)$ and a binding energy $M_{\bar{b}\bar{b}ud} - (M_B + M_{B^*}) \approx -90$ MeV → Talk by M. Wagner in Session 4B

  - [P. Bicudo et al. [European Twisted Mass Collaboration], Phys. Rev. D 87, no. 11, 114511 (2013)]
  - [Z. S. Brown and K. Orginos, Phys. Rev. D 86, 114506 (2012)]
  - [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach and M. Wagner, Phys. Rev. D 92, no. 1, 014507 (2015)]
  - [P. Bicudo, J. Scheunert and M. Wagner, Phys. Rev. D 95, no. 3, 034502 (2017)]
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- Resonance analysis applying methods of scattering theory predict a resonance in the $I(J^P) = 0(1^-)$ channel with $M_{\bar{b}b\bar{u}d} - (M_B + M_{B^*}) \approx +20$ MeV, $\Gamma \approx 100$ MeV
  - [P. Bicudo, M. Cardoso, A. Peters, M.P. and M. Wagner, Phys. Rev. D 96, no. 5, 054510 (2017)]
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- Investigate $\bar{b}b\bar{u}d$ bound state in the $I(J^P) = 0(1^+)$ channel with **Non-Relativistic QCD** i.e. non-static $\bar{b}$-quarks.
  - [A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. 118, no. 14, 142001 (2017)]
  - [P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D 99, no. 3, 034507 (2019)]
  - [A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D 99, no. 5, 054505 (2019)]
  - [L. Leskovec, S. Meinel, M.P. and M. Wagner, Phys. Rev. D 100, no. 1, 014503 (2019)]
Lattice Setup

- Use gauge link configuration generated by RBC and UKQCD collaboration
  
  [Y. Aoki et al. [RBC and UKQCD Collaborations], Phys. Rev. D 83, 074508 (2011)]  
  [T. Blum et al. [RBC and UKQCD Collaborations], Phys. Rev. D 93, no. 7, 074505 (2016)]

- 2 + 1 flavours domain-wall fermions and Iwasaki gauge action

- Five different ensembles which differ in
  
  lattice spacing \( a \approx 0.083 \text{ fm} \ldots 0.114 \text{ fm} \),
  
  lattice size \( L \approx 2.65 \text{ fm} \ldots 5.48 \text{ fm} \),
  
  pion mass \( m_\pi \approx 139 \text{ MeV} \ldots 431 \text{ MeV} \)

  \( \Rightarrow \) explore dependence on \( L, m_\pi \)

- Smeared point-to-all propagators for the up and down quarks

- Utilize all-mode-averaging technique
  
  [T. Blum, T. Izubuchi and E. Shintani, Phys. Rev. D 88, no. 9, 094503 (2013)]
  [E. Shintani, R. Arthur, T. Blum, T. Izubuchi, C. Jung and C. Lehner, Phys. Rev. D 91, no. 11, 114511 (2015)]
Relevant thresholds are $BB^*$ and $B^*B^*$ ($\approx 45$ MeV heavier)

Two types of interpolating operators:

- Local operators: four quarks at the same space-time position
- Non-local operators: two mesons separated in space-time position

Expectation:
- Local operators: good overlap to ground state (stable four-quark)
- Non-local operators: sizeable overlap to first excited state (2 meson state)

⇒ Isolate ground state from higher excitations, especially first excited state
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Energy Spectrum for the $\bar{b}budd$ system

- Due to point-to-all propagators, only $5 \times 3$ correlation matrix available (no scattering operator at source)
- Apply **multi-exponential matrix fitting**: employable also for non-symmetric matrices

$$C_{jk}(t) \approx \sum_{n=0}^{N-1} Z_n^j Z_n^k e^{-E_n t},$$

$E_n$ : $n$-th energy eigenvalue
$Z_n^j = \langle \Omega |\mathcal{O}_j|n\rangle$: overlap factor

Schematic representation of Wick contractions for different correlation matrix elements
Fit Results for Different Operator Bases

Results for the lowest two $\bar{b}b\bar{u}d$ energy levels relative to the $BB^*$ threshold. Black box: local operator included. Red box: scattering operator included.
Overlap Factors

For fixed $j$: $Z^n_j$ indicates relative importance of energy eigenstates $|n\rangle$

\[ O_j^{\dagger}|\Omega\rangle = \sum_{n=0}^{\infty} |n\rangle \langle n| O_j^{\dagger}|\Omega\rangle = \sum_{n=0}^{\infty} Z^n_j |n\rangle. \]

The normalized overlap factors $|\tilde{Z}^n_j|^2 = \frac{|Z^n_j|^2}{\max_m (|Z^m_j|^2)}$ as determined on ensemble C005.
Scattering Analysis

- Relate *finite volume* energy spectrum $E_n$ to *infinite volume* scattering amplitude for 2 energy levels in $T_1^+$ irrep.
- Use Lüscher’s formula and scattering momenta $k_n^2$ to determine phase shift.
- Apply effective-range-expansion (ERE)

$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + O(k^4).$$

Plot of the effective-range-expansion for C005.

- Blue curve: $ak \cot(\delta(k)) + |ak|$.
- Vertical green line: Inelastic $B^*B^*$ threshold.
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- Search bound state pole of scattering amplitude below threshold at

$$\cot \delta_0(k_{BS}) = i, \quad \text{so:} \quad -|k_{BS}| = \frac{1}{a_0} - \frac{1}{2} r_0 |k_{BS}|^2$$

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so:

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- Results essentially identical to the finite-volume energy levels
- Confirmation that ground state is stable tetraquark.
Fit of the pion-mass dependence of $E_{\text{binding}}$. The vertical dashed line indicates the physical pion mass.

$$E_{\text{binding}}(m_\pi,\text{phys}) = (-128 \pm 24 \pm 10) \text{ MeV}$$

$$m_{\text{tetraquark}}(m_\pi,\text{phys}) = (10476 \pm 24 \pm 10) \text{ MeV}$$
Expectations for $\bar{b}b\bar{u}s$ and $\bar{b}\bar{c}ud$

Subsequent promising candidates have heavier light or lighter heavy quarks:

- **$\bar{b}b\bar{u}s$:**
  - Similar quantum numbers to $\bar{b}b\bar{u}d$: $I(J^P) = \frac{1}{2}(1^+)$
  - Previous studies predict a bound state in this channel

- **$\bar{b}\bar{c}ud$:**
  - Due to different heavy quark structure: 2 promising channels:
    - $I(J^P) = 0(1^+)$ and $I(J^P) = 0(0^+)$
    - Supposed to have either a weakly bound state or no binding

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[R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, arXiv:2006.14294 [hep-lat]]
Expectations for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$

Subsequent promising candidates have heavier light or lighter heavy quarks:

- $\bar{b}\bar{b}us$:
  - Similar quantum numbers to $\bar{b}\bar{bud}$: $I(J^P) = \frac{1}{2}(1^+)$
  - Previous studies predict a bound state in this channel

- $\bar{b}\bar{c}ud$:
  - Due to different heavy quark structure: 2 promising channels: $I(J^P) = 0(1^+)$ and $I(J^P) = 0(0^+)$
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[1. A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. 118, no. 14, 142001 (2017)]
[2. P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D 99, no. 3, 034507 (2019)]
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[4. R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, arXiv:2006.14294 [hep-lat]]

Interpolating Operators

- Local operators: mesonic structure and diquark-antidiquark structure
- Nonlocal operators: relevant scattering states near threshold
Preliminary Results

- Strong indication of bound state in $\bar{b}bus$, stable after Lüscher analysis
- No evidence for bound state in $\bar{b}\bar{c}ud$

**top left:** $\bar{b}bus$. **bottom left:** $\bar{b}\bar{c}ud$, $J = 0$. **bottom right:** $\bar{b}\bar{c}ud$, $J = 1$. 
Summary

- Study bound states in doubly heavy tetraquarks
- Consider *local* and *nonlocal* interpolating operators
- Apply a *finite volume Lüscher analysis*

Preliminary studies show:
- Strong indication of bound state for $\bar{b}\bar{b}uds$, $I(J^P) = 1/2(1 + \frac{1}{2})$
- No evidence for bound tetraquark in $\bar{b}\bar{c}uds$, both $0(1 + \frac{1}{2})$ and $0(0 + \frac{1}{2})$

Outlook

- Perform calculation for $\bar{b}\bar{b}uds$ and $\bar{b}\bar{c}uds$ on all available ensembles
- Apply a rigorous Lüscher analysis

Thank You for Your Attention!
Summary

- Study bound states in doubly heavy tetraquarks
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- Predict a *bound state* in the $\bar{b}b\bar{u}d$ channel with $I(J^P) = 0(1^+)$; $E_{\text{binding}} = (-128 \pm 24 \pm 10)$ MeV
- Preliminary studies show:
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Comparison of Different Results for $\bar{b}\bar{b}ud$

Comparison of $\bar{b}\bar{b}ud$ tetraquark binding energies with $I(J^P) = 0(1^+)$ (black: this work; blue: lattice NRQCD; red: lattice QCD computations of static $\bar{b}\bar{b}$ potentials and solving the Schrödinger equation; green: effective field theories and potential models).