Uncollapsing of a quantum state in a superconducting phase qubit

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We demonstrate in a superconducting qubit the conditional recovery (“uncollapsing”) of a quantum state after a partial-collapse measurement. A weak measurement extracts information and results in a non-unitary transformation of the qubit state. However, by adding a rotation and a second partial measurement with the same strength, we erase the extracted information, effectively canceling the effect of both measurements. The fidelity of the state recovery is measured using quantum process tomography and found to be above 70% for partial-collapse strength less than 0.6.

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The observation of a quantum system necessarily perturbs the state of the system. For a strong measurement, the quantum state is understood to collapse irrevocably to one of the eigenstates of the measurement operator; this concept of projective measurement is a central paradigm of modern physics [1]. For weak measurements, however, the collapse is now understood to be partial, with correspondingly partial information drawn from the measurement yielding a non-unitary transformation of the quantum state. It has been predicted that after such a weak measurement, the initial quantum state of the system can be recovered by essentially undoing the effect of the measurement [2, 3] and causing a quantum “uncollapsing”.

Superconducting phase qubits provide an excellent system for testing this concept of “uncollapsing”. Our experimental implementation [4] uses a controlled measurement process whose projective strength can be tuned continuously from a weak partial measurement to a full projective one [5]. Using this system, we can experimentally test reversing the partial, measurement-induced collapse of a quantum state. Similar tests of partial or continuous weak measurements should also be possible for other types of solid-state qubits [6].

In our experiment, the superconducting phase qubit is prepared in a combination of its ground |0⟩ and first excited |1⟩ states. A partial measurement of the qubit then yields a “detection” event, which occurs with probability p when the qubit is in the |1⟩ state, while it never occurs for the qubit in the |0⟩ state. If the measurement yields a null result (i.e. no event detected), this leads to the partial collapse of the qubit state towards |0⟩. This evolution towards the |0⟩ state is driven by the extracted information, and does not involve any energy exchange. We then employ the following method (proposed by Jordan and one of the authors [3]) to “uncollapse” the result of the measurement [see pulse sequence in Fig. 1(d)]: After the preparation of an arbitrary initial state of the qubit and (i) partial collapse due to null-result measurement with strength p, we (ii) apply a π-pulse, coherently swapping the amplitudes of the qubit states |0⟩ and |1⟩, and (iii) partially measure the qubit state again, with the same measurement strength p [2]. The combination of steps (ii) and (iii) “anti-symmetrizes” the information extracted from the first measurement of the qubit, and with an overall probability 1 − p of two null results [8], regardless of the initial state, the qubit state is coherently recoherent.

In our experiment, we use the microwave pulses, which cause coherent transitions between states |0⟩ and |1⟩. The phase of the Josephson junction is δ. (b) During a partial measurement the state |1⟩ tunnels out of the well with probability p. (c) The tunneling probability p (solid line) is determined by the amplitude of the measurement pulse which lowers the barrier; we use sufficiently small amplitude to avoid tunneling from the state |0⟩ (dotted line). The maximal measurement visibility (the difference between the solid and dotted lines) is about 90%. (d) Pulse sequences (including state tomography) for the partial-collapse experiment (upper trace, as in 4) and for the quantum uncollapsing (lower trace). The effect of the partial measurement [step (i)] is undone by applying the π-pulse [step (ii)] and additional (uncollapsing) partial measurement [step (iii)] with the same strength p.

FIG. 1: (a) The qubit potential during application of the microwave pulses, which cause coherent transitions between states |0⟩ and |1⟩. The phase of the Josephson junction is δ. (b) During a partial measurement the state |1⟩ tunnels out of the well with probability p. (c) The tunneling probability p (solid line) is determined by the amplitude of the measurement pulse which lowers the barrier; we use sufficiently small amplitude to avoid tunneling from the state |0⟩ (dotted line). The maximal measurement visibility (the difference between the solid and dotted lines) is about 90%. (d) Pulse sequences (including state tomography) for the partial-collapse experiment (upper trace, as in 4) and for the quantum uncollapsing (lower trace). The effect of the partial measurement [step (i)] is undone by applying the π-pulse [step (ii)] and additional (uncollapsing) partial measurement [step (iii)] with the same strength p.

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ently restored to its initial, pre-measurement state (here including a π-rotation).  

In order for the uncollapsing procedure to work, we have to erase the information that was already extracted classically. This distinguishes this measurement-induced uncollapsing from a “quantum eraser” [11], in which only potentially extractable information is erased. Note also that the result of the second measurement is stochastic, and only a particular result will succeed in erasing the information, thus leading to a less than unity probability of success for uncollapsing; however, in the case of this desired result, the qubit’s initial state is fully recovered.

In this Letter, we report the first experimental demonstration of quantum uncollapsing by implementing the above described protocol, where we obtain fidelities well over 70%, quantified by quantum process tomography [11]. Besides confirming the ability to undo a partial quantum measurement, this result confirms the high quantum efficiency of our measurement.

Our superconducting phase qubit has been described in detail previously [2]. We briefly review the relevant details and modifications here. The qubit is fabricated as a superconducting loop interrupted by a ~1 µm² sized Josephson junction of critical current 2µA, shunted by a low loss-tangent parallel plate capacitor (1pF) formed with a-Si:H dielectric. We initialize the system in the ground state of the qubit’s cubic-shaped potential well [see Fig. 1(a)]. The logic qubit is formed by the ground state of the qubit’s cubic-shaped potential well (partially collapses) to tunneling occurs (null result), the initial state |ψ⟩ changes (partially collapses) to

\[ |\psi_M⟩ = \cos(\theta_0/2)|0⟩ + e^{-i\phi_0} \sin(\theta_0/2)|1⟩, \]  

\[ \theta_M = 2 \tan^{-1}[\sqrt{1-p} \tan(\theta_0/2)], \]  

where \(\phi_M\) is an accumulated phase due to an adiabatic change in the energy level spacing during the measurement. This information-related non-unitary transformation (confirmed in the experiment [2]) is precise only in the ideal case. It neglects energy and phase relaxation within the qubit well, which is an acceptable approximation since the corresponding relaxation times \(T_1 = 450\ ns\) and \(T_2 = 350\ ns\) (and \(2 = 120\ ns\)) are significantly longer than the experiment duration 13. Eqs. 2-3 also neglect incoherence and noise in the process of virtual tunneling; however, the theoretical analysis [10] of the density matrix evolution confirms that the simple result (2-3) is a good approximation.

The qubit state after the partial collapse is analyzed by state tomography (as in 2, 17, 18, 19), consisting of 3 types of tomographic rotations (either a π/2-pulse rotating about the Y-axis of the Bloch sphere, a π/2-pulse rotating about the X-axis, or no rotation) followed by a full measurement (with \(p \approx 1\) – see the upper trace in Fig. 1(d)). In this way we measure the qubit tunneling probabilities \(P_X, P_Y, P_Z\), which correspond to the qubit state components \(X, Y, Z\) on the Bloch sphere (in the rotating frame). Since \(P_X, P_Y, P_Z\) include both the probability of tunneling during the tomography measurement and the background probability \(\bar{P}_B = p \sin^2(\theta_0/2)\) due to the partial measurement, the qubit state components are given by \(\{X, -Y, Z\} = 2(P_{X,Y,Z}−\bar{P}_B)/(1−\bar{P}_B)−1\) (the minus signs on \(Y\) and \(Z\) come from following the convention setting the \(|0⟩\) at \(Z = +1\)). The measured tunneling probabilities \(P_X, P_Y, P_Z\) for the initial state (\(|0⟩+|1⟩\)/√2) are shown in Fig. 2(a), as functions of the pulse amplitude for partial measurement (which is in a one-to-one correspondence with \(p\)); in this case \(P_B = p/2\) [also shown in Fig. 2(a)]. Note the large oscillations in \(P_X\) and \(P_Y\), indicating that the partial measurement is accumulating a significant phase \(\phi_M\), as was seen in 2. The qubit state components \(X, Y, Z\) calculated from the data in Fig. 2(a) are shown on the Bloch sphere in Fig. 3(c).

In order to recover the initial quantum state, we now add steps (ii) and (iii) of the uncollapsing protocol – see the lower trace in Fig. 1(d). The π-pulse about the X-axis [step (ii)] after the partial collapse exchanges the basis states in Eq. 2, creating the qubit state \(|\psi_π⟩ = \sin(\theta_M/2)|0⟩ + e^{i(\phi_0+\phi_M)} \cos(\theta_M/2)|1⟩\). The second partial measurement with the same strength [step (iii)] can either result in a tunneling event, or not. In the case of no tunneling (null result again) the partial-collapse evolution \(|\psi_π⟩ → |ψ_F⟩\) is described by the same transformation as \(|\psi_0⟩ → |ψ_M⟩\) [see Eqs. 1-3], and therefore produces the state \(|ψ_F⟩ = \sin(\theta_0/2)|0⟩ + e^{i\phi_0} \cos(\theta_0/2)|1⟩\). As expected, \(|ψ_F⟩\) coincides with the initial state \(|ψ_0⟩\) up to a π-rotation about the X-axis. Notice that not only the polar angle \(θ_0\) is restored (which is essentially the uncol-
lasing), but the azimuthal angle shift $\phi_M$ is also canceled (due to the usual spin echo effect).

The state tomography of the uncollapsed state $|\psi_f\rangle$ is done in the same way as for the partially collapsed state $|\psi_M\rangle$. The only difference is that now the background probability $P_B$ is due to both partial measurements, and therefore $P_B = 1 - [1-p\sin^2(\theta_0/2)][1-p\cos^2(\theta_M/2)] = p$, independent of the initial state. The measured probabilities $P_X$, $P_Y$, and $P_Z$ for the initial state $(|0\rangle + |1\rangle)/\sqrt{2}$ are shown in Fig. 2(b) as functions of the measurement pulse amplitude, and the corresponding qubit states on the Bloch sphere are shown in Fig. 3(g). Notice that compared to the partial-collapse results, the oscillations in $P_X$ and $P_Y$ are clearly suppressed (spin echo) and the qubit state is restored to the equatorial plane. The measured state is quite close to the ideal result of uncollapsing $(|0\rangle + |1\rangle)/\sqrt{2}$ for $p \leq 0.6$ (see below). If we purposefully change the $\pi$ pulse of the step (ii) to a 0.9$\pi$ pulse, the oscillations of $P_Y$ and $P_Z$ are somewhat recovered (see Fig. 2(c)), and the qubit state moves significantly out of the equatorial plane (not shown), indicating that the uncollapsing procedure performance is degraded.

So far we discussed experimental uncollapsing of the initial state $(|0\rangle + |1\rangle)/\sqrt{2}$. However, any initial state should be restored by the same procedure. Instead of examining all initial states to check this fact, it is sufficient to choose 4 initial states with linearly independent density matrices and use the linearity of quantum operations [11]. We choose initial states $|1\rangle$, $(|0\rangle - i|1\rangle)/\sqrt{2}$, $(|0\rangle + |1\rangle)/\sqrt{2}$ and $|0\rangle$. The corresponding qubit states on the Bloch sphere after the partial collapse and after uncollapsing are shown in Fig. 3 for measurements with a range of measurement strength $p$. For clarity we only show the results for $p \leq 0.7$, since beyond this range our simple theory becomes too inaccurate. The main reason why the protocol begins to fail for large $p$ is a noticeable probability $p_r \sim 0.1$ of energy relaxation to the ground state during our 44 ns long sequence ($T_1 = 450$ ns). The relative contribution of such cases increases with $p$ and becomes very significant when the selection probability $1 - p$ becomes comparable to $p_r$, thus ruining the fidelity of uncollapsing. Also notice that we use the experimentally determined $p$, as shown in Fig. 1(c), which contains an approximate 5% error due to state preparation and measurement infidelities. The data is not re-scaled to correct for this error.

Uncollapsing of the states $|0\rangle$ and $|1\rangle$ is straightforward (they do not change in null-result measurements), so the small deviations from the ideal results on the Bloch spheres in the left and right columns of Fig. 3 characterize the imperfections of our experiment. Uncollapsing of the states $(|0\rangle - i|1\rangle)/\sqrt{2}$ and $(|0\rangle + |1\rangle)/\sqrt{2}$ is non-trivial; however, we see that the small deviations in Figs. 3(f) and 3(g) from the theoretical result are approximately the same as for the trivial cases, thus indicating that the uncollapsing itself is nearly ideal. Besides the Bloch spheres, in Figs. 3(a-h) we also show the dependence of the corresponding polar angles $\theta$ on the measurement strength $p$. The small discrepancy with the theory (with no fit parameters) is mainly due to intrinsic decoherence of the qubit and measurement error. As discussed above, the discrepancy becomes significant when $p$ approaches
The state tomography for these four initial states is sufficient for full characterization of the quantum process tomography (QPT) \cite{11, 21, 22}. In Fig. 3(a) we show the QPT matrix $\chi$ in the standard Pauli-matrix basis ($I, \sigma_X, \sigma_Y, \sigma_Z$), generated by applying the conventional linear algebra formalism \cite{11} to our results \cite{22} for the uncollapsing protocol with $p = 0.47$. As expected, we see a clear peak at the $(X, X)$ location, indicating that the process is mainly that of a $\pi$ rotation about the $X$ axis. The uncollapsing fidelity is defined as the overlap of the $\chi$-matrix with the ideal one (of a perfect $\pi$ pulse), i.e. the fidelity is simply $\text{Re}\chi(X, X)$. The dependence of this fidelity on the measurement strength $p$ is presented in Fig. 3(b), which shows that the uncollapsing fidelity remains above 70% until the degradation of the state recovery at $p \gtrsim 0.6$ (because of the reason discussed above).

In conclusion, we demonstrate a conditional uncollapsing of a partially measured quantum state, and quantify this process by quantum process tomography. While our protocol has apparent similarity with the spin echo sequence (and includes the azimuth angle recovery due to the echo effect), we emphasize the clear difference between the two effects: the spin echo is the undoing of an unknown unitary transformation, while uncollapsing is the undoing of a known but non-unitary transformation.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{(a) The quantum process tomography matrix $\chi$ for the uncollapsing with $p = 0.47$. (b) The fidelity of the quantum uncollapsing as a function of the partial measurement probability $p$.}
\end{figure}

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8. This probability coincides with the upper bound for the success probability obtained in the general theory of uncollapsing \cite{3}. Obviously, the success probability for uncollapsing decreases with increasing strength $p$ of the partial measurement, and reaches zero for the projective collapse ($p = 1$).

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