Gravitational Wave Signatures of Lepton Universality Violation

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We analyze the prospects for using gravitational waves produced in early universe phase transitions as a complementary probe of the flavor anomalies in $B$ meson decays. We focus on the Left-Right SU(4) Model, for which the strength of the observed lepton universality violation and consistency with other experiments impose a vast hierarchy between the symmetry breaking scales. This leads to a multi-peaked gravitational wave signature within the reach of upcoming gravitational wave detectors.

I. INTRODUCTION

Although the Standard Model (SM) does not provide all the answers to fundamental questions in particle physics and needs to be augmented by new physics, nearing half a century since its formulation $^{13}$ it has certainly stood the test of time with respect to its predictive power. A huge number of models beyond the SM have been constructed proposing solutions to the outstanding problems, however, it is not certain which of them, if any, is realized in nature. At this time, guidance from experiment is especially important in order to achieve further progress on the theory side.

So far, among the strongest experimental hints of new physics are the indications of lepton universality violation in $B$ meson decays, the so-called $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies. Although the $R_{D^{(*)}}$ anomalies (reported by BaBar $^{6}$, Belle $^{7}$ and LHCb $^{8}$) have not been confirmed in the most recent set of Belle data $^{9}$, and the $R_{K^{(*)}}$ anomaly reported by LHCb $^{10}$ has become less significant $^{11}$, the $R_{K^{(*)}}$ has persisted with new LHCb data $^{13}$.

From an effective theory point of view, the observed signals of lepton universality violation are best accounted for by either the vector leptoquark $(3,1)_{2/3}$ or $(3,3)_{2/3}$ $^{14}$-$^{16}$. A natural origin of the former in the context of flavor anomalies has been proposed in $^{17}$, where it was suggested that this leptoquark can be the gauge boson of a Pati-Salam-type unified model. This has been followed by several model-building efforts aimed at explaining either both the $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies $^{18}$-$^{24}$ or just $R_{K^{(*)}}$ $^{25}$-$^{26}$ using the $(3,1)_{2/3}$ vector leptoquark. Apart from those models, there exist also other explanations of the anomalies including $Z'$ bosons $^{27}$-$^{31}$ and scalar leptoquarks $^{32}$-$^{35}$.

Since the latest results on $R_{D^{(*)}}$ from Belle $^{9}$ are consistent with the SM, lowering the overall significance of those anomalies, in this paper we focus on the solution to just the $R_{K^{(*)}}$ anomalies offered by the Left-Right SU(4) Model $^{26}$. This is the only model for the flavor anomalies proposed so far which does not require any mixing between quarks and new vector-like fermions. Apart from the existing experimental searches for lepton universality violation, the only other conventional way of looking for signatures of this model is to produce the vector leptoquark in particle colliders. However, given its large mass of $\sim 10$ TeV, this would require using the 100 TeV Future Circular Collider, whose construction has not yet been approved.

A new window of opportunities for probing particle physics models has recently been opened by gravitational wave experiments. The gravitational wave detectors LIGO $^{36}$ and Virgo $^{37}$, in addition to observing signals from astrophysical phenomena like black hole and neutron star mergers, have unique capabilities of detecting the imprints of cosmic events in the early universe, providing access to regions of parameter space unexplored so far in various extensions of the SM. This will be even more promising with future experiments like the Laser Interferometer Space Antenna (LISA) $^{38}$, Cosmic Explorer (CE) $^{39}$, Einstein Telescope (ET) $^{40}$, DECIGO $^{41}$ or Big Bang Observer (BBO) $^{42}$.

One class of particle physics signals that gravitational wave detectors are sensitive to arises from early universe phase transitions. If the scalar potential has a nontrivial vacuum structure, the universe could have settled in a state which, as the temperature dropped, became metastable. The universe would then undergo a tunneling from the false vacuum to the true vacuum. During such a first order phase transition, bubbles of true vacuum would form in different patches of the universe and start expanding. Gravitational waves would be generated from bubble wall collisions, magnetohydrodynamic turbulence and sound shock waves of the early universe plasma generated by the bubble’s violent expansion. At the Lagrangian level of a theory, a phase transition is triggered by spontaneous symmetry breaking. In models with a rich gauge structure, multiple steps of symmetry breaking can occur, resulting in a chain of phase transitions, each generating gravitational waves.

First order phase transitions from symmetry breaking have been studied with respect to their predictions regarding the gravitational wave signals in various models of new physics (see, e.g., $^{43}$-$^{58}$). Here we investigate the complementarity between gravitational wave experiments and direct searches for lepton universality violation. Such a connection has recently been made in $^{56}$, in the context of the Pati-Salam Cubed Model $^{20}$, which consists of three copies of the Pati-Salam gauge group, each for a different family of particles. In the Left-Right SU(4) Model which we are considering, the gauge group is common to all the families. The symmetry breaking pattern consists of three steps, each leading to a distinct peak in the gravitational wave spectrum. The position of the two lower-frequency peaks in the three-peaked gravitational wave spectrum is determined by the magnitude of the flavor anomalies, offering a way to discriminate the model.
II. LEFT-RIGHT SU(4) MODEL

In this section we provide a summary of the most important properties of the model (for further details, see [23]).

The model is based on the gauge group

\[ G = SU(4)_L \times SU(4)_R \times SU(2)_L \times U(1)' . \]  

(1)

The fermion, scalar and vector particle contents are provided in Table I. The gauge group \( G \) is broken by the vacuum expectation values (vevs) of the scalar fields \( \hat{\Sigma}_R, \hat{\Sigma}_L \) and \( \hat{\Sigma} \). The parameters of the scalar potential can be chosen such that the following vev structure is obtained,

\[ \langle \hat{\Sigma}_R \rangle = \frac{v_R}{\sqrt{2}} \delta^{i4}, \quad \langle \hat{\Sigma}_L \rangle = \frac{v_L}{\sqrt{2}} \delta^{i4}, \]

\[ \langle \hat{\Sigma} \rangle = \frac{v_{\Sigma}}{\sqrt{2}} \text{diag}(1, 1, 1, z) , \]  

(2)

where \( z > 0 \). The \( SU(4)_R \) symmetry is broken at a high scale \( v_R \) in order to suppress right-handed lepton flavor changing currents and comply with the stringent experimental bounds, whereas the other scales, \( v_L \) and \( v_{\Sigma} \), are constrained by the size of the \( R_{iY'\nu} \) anomalies to be much lower than \( v_R \). We make an additional assumption that there is also a hierarchy between the scales \( v_L \) and \( v_{\Sigma} \), i.e.,

\[ v_R \gg v_L \gg v_{\Sigma} . \]  

(3)

This implies the following symmetry breaking pattern (with the numerical choice for the vevs explained below):

\[ \begin{align*}
SU(4)_L \times SU(4)_R \times SU(2)_L \times U(1)' \\
\downarrow v_R \sim 5000 \text{ TeV} \\
SU(4)_L \times SU(3)_R \times SU(2)_L \times U(1)'' \\
\downarrow v_L \sim 40 \text{ TeV} \\
SU(3)_L \times SU(3)_R \times SU(2)_L \times U(1)' \\
\downarrow v_{\Sigma} \sim 2 \text{ TeV} \\
SU(3)_c \times SU(2)_L \times U(1)' \nonumber
\end{align*} \]

The \( U(1)' \) charge \( Y' \), the \( U(1)'' \) charge \( Y'' \) and the SM hypercharge \( Y \) are related via

\[ \begin{align*}
Y'' &= Y' + \frac{1}{3} \text{diag}(1, 1, 1, -3) , \\
Y &= Y'' + \frac{1}{6} \text{diag}(1, 1, 1, -3) .
\end{align*} \]  

(4)

The covariant derivative can be written as

\[ D_{\mu} = \partial_{\mu} + ig_L G^A_{\mu \nu} T^A + \frac{ig_R G^{A}_{R \mu}}{2} T^{A} \]

\[ + ig_2 W^a_{\mu} T^a + ig'_1 Y'_\mu Y' , \]  

(5)

TABLE I. The fermion, scalar and vector particle content of the model. The masses were calculated assuming the hierarchical vev structure \( v_{10} \gg M \gg v_R \gg v_L \gg v_{\Sigma} \).
generators, respectively. At the low scale, the gauge couplings $g_L, g_R, g_1'$ are related to the SM gauge couplings $g_\ast, g_1$ via

$$g_\ast = \frac{g_L g_R}{\sqrt{g_L^2 + g_R^2}}, \quad g_1 = \frac{g_1' g_L g_R}{\sqrt{g_1' g_L^2 (g_L + g_R)^2} + g_1'^2 g_R^2}. \quad (6)$$

The Lagrangian terms describing the fermion masses are

$$L_f = \left[ y_{ij}^d \bar{\Psi}^d_L H \Psi^{d^c}_R + y_{ij}^e \bar{\Psi}^e_L H \Psi^{e^c}_R + Y_{ij} \bar{\Sigma}_L \Sigma_R^c \right] + h.c. + y_{ij}^{u'} (\hat{\Psi}^u_i D^c \hat{\Psi}^u_j), \quad (7)$$

where the scalar field $\hat{\Phi}_10 = (1, \bar{T}_1, 1, -1)$ develops a higher-scalar vev $v_{10} \sim 10^{15} \text{ GeV}$ and provides a seesaw mechanism for the neutrino masses, $m_\nu \sim v^2 / v_{10}$ with $v$ being the SM Higgs vev. After symmetry breaking down to the SM gauge group, the fermion mass terms become

$$L_f^\text{SM} \supset \left[ y_{ij}^d \bar{\Sigma}^d_L S_i d_j + y_{ij}^d Q^d_i S_2 d_j + y_{ij}^{u'} \bar{\Sigma}^u_i D^c \bar{\Sigma}^u_j \right] + h.c. + y_{ij}^{u'} v_{10} (\nu_R^c) e_p \nu_R^c. \quad (8)$$

The scalar sector is described by the Lagrangian

$$\mathcal{L}_s = \left| \Delta_{u} \Sigma u^c \right|^2 + \left| \Delta_{d} \Sigma d^c \right|^2 + \left| \Delta_{e} \Sigma e^c \right|^2 + \left| \Delta_{u} \hat{H} u^c \right|^2 + \left| \Delta_{d} \hat{H} d^c \right|^2 + \left| \Delta_{e} \hat{H} e^c \right|^2 + \left| \Delta_{\tilde{u}} \tilde{H} u^c \right|^2 + \left| \Delta_{\tilde{d}} \tilde{H} d^c \right|^2 + \left| \Delta_{\tilde{e}} \tilde{H} e^c \right|^2 \times \left( \Sigma^c, \Sigma, \hat{H}, \tilde{H}, \Phi_10 \right), \quad (9)$$

where the scalar potential consists of all possible contributions of the scalar fields (its full form is provided in [26]). The terms relevant for our analysis are (traces are implicit)

$$V \supset - \mu_R^2 |\Sigma_R|^2 + \lambda_R |\Sigma_R|^4 - \mu_L^2 |\Sigma_L|^2 + \lambda_L |\Sigma_L|^4 \quad \text{and} \quad - \mu_{\Sigma}^2 |\Sigma|^2 + \lambda_{\Sigma} |\Sigma|^4 + \lambda_{\Sigma}^u |\Sigma|^4 \quad \text{and} \quad - \lambda_{\Sigma}^L |\Sigma_R|^2 |\Sigma_L|^2 + \kappa |\Sigma_R|^2 |\Sigma_L|^2. \quad (10)$$

As discussed in [26], it is possible to tune the parameters of the scalar potential such that only one linear combination of the fields $S_1, S_2, S_3, S_4$ is light, reproducing the SM scalar sector at low energies ($S_1^T \equiv H$). The remaining fields $S_{2,3,4}$ and all other components of $H_R^1$ and $H_R^2$ have masses set by the hard mass parameter $M$, which we take to be $M \gg v_R$. The relative mass hierarchies between the SM down-type quarks and charged leptons are reproduced reasonably well within this minimal setup. One can also introduce into the model the scalar representation $\tilde{\Phi}_{15} = (15, 1, 1, 0)$ that develops a vev at a high scale and leads to terms $\bar{\Psi}^d_L H_R \psi^{d^c}_R \tilde{\Phi}_{15} / \Lambda$ providing distinct contributions to the quark and lepton masses.

The Lagrangian terms involving the fermion and vector fields are given by

$$\mathcal{L}_v = \bar{\Psi}^d L^\dagger \hat{\Psi}^d L + \bar{\Psi}^u R^\dagger \hat{\Psi}^u R + \bar{\Psi}^e L^\dagger \hat{\Psi}^e L + \bar{\Sigma}^u R^\dagger \hat{\Sigma}^u R + \bar{\Sigma}^d L^\dagger \hat{\Sigma}^d L + \bar{\Sigma}^e L^\dagger \hat{\Sigma}^e L, \quad (11)$$

which, at the low scale, result in the following interactions between quarks, leptons and gauge leptquarks,

$$L_v^\text{SM} \supset \frac{g_L}{\sqrt{2}} X \mu_L^u \left[ \bar{L}^{(u)}_i (\bar{u}^c \gamma^\mu P_L \nu^\mu) + \bar{L}^{(d)}_i (\bar{d}^c \gamma^\mu P_L e^\mu) \right] + \frac{g_R}{\sqrt{2}} X \mu_R \left[ \bar{R}^{(u)}_i (\bar{u}^c \gamma^\mu P_R \nu^\mu) + \bar{R}^{(d)}_i (\bar{d}^c \gamma^\mu P_R e^\mu) \right] + h.c., \quad (12)$$

where $L^u, L^d, R^u$ and $R^d$ are mixing matrices. They are all unitary and related to the Cabibbo-Kobayashi-Maskawa matrix and the Pontecorvo-Maki-Nakagawa-Sakata matrix via $L^u = V_{\text{CKM}} L^d U_{\text{PMNS}}$ and $R^u = V_{\text{CKM}} R^d U_{\text{PMNS}}$.

To circumvent the stringent experimental constraints on lepton universality violation [58, 79], the scale of SU(4)$_R$ breaking needs to be $v_R \gtrsim 5000 \text{ TeV}$ for a generic unitary matrix $R^d$. At the same time, in order for the vector leptoquark $X_L$ to explain the $R_{K^{(*)}}$ anomalies, one requires [26]

$$M_{X_L} = g_L V \Re \left( L^d_2 L^{d^*}_3 - L^d_3 L^{d^*}_2 \right) \approx 23 \text{ TeV}. \quad (13)$$

Because of the unitarity of the matrix $L^d$, this relation can only be fulfilled if $M_{X_L} \lesssim (23 \text{ TeV}) g_L$. The experimental constraints then force $L^d$ to be of the form

$$L^d \approx e^{i\phi} \begin{pmatrix} \delta_1 & 1 \\ \frac{\delta_2}{e^{i\phi}} \sin \theta & \frac{\delta_3}{e^{-i\phi}} \cos \theta \end{pmatrix}, \quad (14)$$

where $\delta_i \lesssim 0.02$. The allowed leptoquark mass in Eq. (13) is maximized for $\theta = \pi / 4$ and $\phi_1 + \phi_2 = 0$, which implies

$$\sqrt{v_R^2 + v_R^2 (1 + z^2)} \lesssim 46 \text{ TeV}. \quad (15)$$

As mentioned above, we consider the scenario $v_{12} \gg v_3$, since with this additional hierarchy of scales the gravitational wave signal has a richer structure.

There is a lower bound on $v_3$, coming from experimental searches for the color octet vectors $G^\prime$. The analysis of LHC di-jet data yields $M_{G^\prime} \gtrsim 2 \text{ TeV}$ [80]. To be consistent with this constraint, we take $M_{G^\prime} \approx 3 \text{ TeV}$. Assuming the gauge couplings at the low scale $g_L = g_R \sim \sqrt{2}$ for a proper match to the SM strong gauge coupling, this leads to $v_{12} \approx 2 \text{ TeV}$. The only particles other than $G^\prime$ with masses governed by $v_{12}$ are the radial modes of $\Sigma$ and the vector-like fermions $Q'$ and $L'$. The former do not couple to SM quarks, and our choice $v_{12} \approx 2 \text{ TeV}$ is consistent with all experimental bounds, even for $\lambda_{\Sigma}^{(t)}$ as small as $10^{-2}$. The latter do not mix with SM quarks, and our choice $v_{12} \approx 2 \text{ TeV}$ is also consistent with collider searches, even for a relatively small $z$.

The vev structure takes the form as in Eq. (2) if the parameters of the scalar potential satisfy the relations: $\lambda_{13}^{(t)} > 4 \lambda_{23}^{(t)} v_{12}^2 / v_3^2, \lambda_{23}^{(t)} > 4 \lambda_{23}^{(t)} v_{12} v_3 / v_3^2, \lambda_{13}^{(t)} > 0$ and $\kappa < 0$. In order to maximize the allowed value of $v_{12}$, it is then sufficient to arrange for a relatively small $z$. In our further analysis, we consider the case $z \approx 1 / 4$, so that the cross terms in the scalar potential are small. This is realized, e.g., with the following choice of parameter values: $\lambda_{13}^{(t)} = \lambda_{23}^{(t)} = 2 \times 10^{-2}, \lambda_{13}^{(t)} = 2.5 \times 10^{-4}, \lambda_{23}^{(t)} = 1.6 \times 10^{-8}, \kappa = -1.8 \times 10^{-6} \text{ TeV}$ and the hierarchical symmetry breaking pattern

$$v_R \approx 5000 \text{ TeV}, \quad v_L \approx 40 \text{ TeV}, \quad v_3 \approx 2 \text{ TeV}. \quad (16)$$

This is the benchmark scenario we adopt in the subsequent gravitational wave analysis.


III. EFFECTIVE POTENTIAL

Because of the vast hierarchy of scales in the model and small cross terms in the scalar potential, the three steps of symmetry breaking can be considered independently from one another. Denoting the background fields as

\[ \phi_R \equiv \text{Re}(\bar{\Sigma}_R) \sqrt{2}, \quad \phi_L \equiv \text{Re}(\bar{\Sigma}_L) \sqrt{2}, \quad \phi_{\Sigma} \equiv \text{Re}(\bar{\Sigma}) \sqrt{2}, \]

the effective potential splits into three pieces,

\[ V_{\text{eff}} = V_{\text{eff}}^{(R)}(\phi_R) + V_{\text{eff}}^{(L)}(\phi_L) + V_{\text{eff}}^{(\Sigma)}(\phi_{\Sigma}). \]

Before analyzing the phase transitions, we first discuss the effective potential in the general case. We adopt the collective notation for the background fields \( \phi = \phi_R, \phi_L, \phi_{\Sigma} \), the vevs \( v = v_R, v_L, v_{\Sigma} \) and the quartic couplings \( \lambda = \lambda_R, \lambda_L, \lambda_{\Sigma}, \lambda_{\Sigma}' \). Each piece of the effective potential consists of a tree-level part, a one-loop Coleman-Weinberg correction and a finite temperature contribution,

\[ V_{\text{eff}}(\phi, T) = V_{\text{tree}}(\phi) + V_{\text{loop}}(\phi) + V_{\text{temp}}(\phi, T). \]

Using the fact that the minimum of the tree-level potential for \( \phi = \phi_R, \phi_L \) is at \( v = \mu/\sqrt{\lambda} \), one can write

\[ V_{\text{tree}}(\phi) = -\frac{1}{2}\lambda v^2 \phi^2 + \frac{1}{4} \lambda \phi^4. \]

The tree-level potential for \( \phi = \phi_{\Sigma} \) contains terms involving \( \lambda_{\Sigma} \) and \( \lambda_{\Sigma}' \) with different \( z \)-dependence.

To obtain the Coleman-Weinberg term, we implement the cutoff regularization scheme and assume that the minimum of the one-loop potential and the mass of \( \phi \) are the same as their tree-level values \([81]\). In this scheme, the one-loop zero temperature correction is

\[ V_{\text{loop}}(\phi) = \sum_{\text{particles}} n_i \frac{m_i^2(\phi)}{64\pi^2} \left\{ m_i^2(\phi) \left[ \log \left( \frac{m_i^2(\phi)}{m_i^2(v)} \right) - \frac{3}{2} \right] + 2 m_i^2(\phi) m_i^2(v) \right\}, \]

where the sum is over all particles charged under the gauge group that undergoes symmetry breaking, including the Goldstone bosons, \( n_i \) is the number of degrees of freedom with an extra minus sign for fermions, and \( m_i(\phi) \) are the background field-dependent masses. For the contribution of the Goldstone bosons one needs to replace \( m_x(v) \rightarrow m_x(v) \).

The temperature-dependent part of the potential consists of the one-loop finite temperature contribution \( V_{\text{temp}}^{(1)}(\phi, T) \) and, in case of bosonic degrees of freedom, the Daisy diagrams contribution \( V_{\text{temp}}^{(2)}(\phi, T) \). The corresponding formulae are given by \([82]\)

\[ V_{\text{temp}}^{(1)}(\phi, T) = \frac{T^4}{2\pi^2} \sum_{\text{particles}} n_i \int_0^\infty dy y^2 \times \log \left( 1 + e^{-\sqrt{m_i^2(\phi)/T^2 + y^2}} \right), \]

\[ V_{\text{temp}}^{(2)}(\phi, T) = \frac{T}{12\pi} \sum_{\text{bosons}} n_i \left\{ m_i^2(\phi) - [m_i^2(\phi) + \Pi_i(T)]^{1/2} \right\}. \]

The thermal masses \( \Pi_i(T) \) can be calculated following the prescription provided in \([83]\).

IV. PHASE TRANSITIONS

A strong first order phase transition is required to produce a gravitational wave signal. This occurs when the effective potential develops a barrier separating the false vacuum from the true vacuum. In the subsequent analysis we choose the quartic couplings to be: \( \lambda_R(v_R) = \frac{3}{2}\lambda_L(v_L) = \frac{1}{2}\lambda_{\Sigma}'(v_{\Sigma}) = 10^{-2} \). For such small quartics the only relevant contributions to the field-dependent masses and thermal masses are those involving the gauge couplings \( g_R/L \).

To properly estimate those contributions, we first analyze the running of the gauge couplings. We match \( g_R \) and \( g_L \) to the SM strong coupling \( g_s \) at the scale \( v_{\Sigma} = 2 \text{ TeV} \) via Eq. (6) and choose \( g_R(v_{\Sigma}) = g_L(v_{\Sigma}) \). This implies that \( g_R(v_{\Sigma}) = g_L(v_{\Sigma}) \approx 1.44 \). We then perform the running using the renormalization group equations

\[ \frac{dg_R/L(\mu)}{d\log \mu} = -\left( 11 - \frac{n_s}{6} - \frac{2n_f}{3} \right) \frac{g_{R/L}^3(\mu)}{16\pi^2}, \]

where \( n_s \) is the number of complex scalars and \( n_f \) is the number of Dirac fermions in the fundamental representation of the gauge group SU(4)_R/SU(4)_L with masses below the scale \( \mu \). We find that \( g_R(v_R) \approx 0.98 \) and \( g_L(v_{\Sigma}) \approx 1.23 \).
(1) First phase transition: SU(4)\(_{R}\) \(\rightarrow\) SU(3)\(_{R}\)

This transition is triggered when the field \(\Sigma_{R}\) develops the vev as in Eq. (2) with \(v_{R} \approx 5000\) TeV. The relevant background field-dependent masses are

\[
m_{X_{R}}(\phi_{R}) = \frac{1}{2} g_{R} \phi_{R}, \quad m_{Z_{R}}(\phi_{R}) = \frac{M_{Z_{R}}}{v_{R}} \phi_{R}.
\]

The numbers of degrees of freedom corresponding to the gauge bosons \(X_{R}\) and \(Z_{R}\) are: \(n_{X_{R}} = 18\) and \(n_{Z_{R}} = 3\). The thermal masses are given by

\[
\Pi_{X_{R}}(T) = \Pi_{Z_{R}}(T) = \frac{1}{3} g_{R} T^2,
\]

\[
\Pi_{\phi_{R}}(T) = \Pi_{X_{GB}}(T) \approx \frac{1}{3} \left( 3 g_{R}^2 + \frac{M_{Z_{R}}^2}{v_{R}^2} \right) T^2,
\]

where we dropped terms involving the small quartic coupling. The superscript \(L\) for the gauge boson thermal masses denotes longitudinal components, \(\phi_{R}\) is the radial mode and \(\chi_{GB}\) are the Goldstone bosons. The corresponding numbers of degrees of freedom are: \(n_{X_{R}}^{L} = 6\), \(n_{Z_{R}}^{L} = 1\), \(n_{\phi_{R}} = 1\) and \(n_{X_{GB}} = 7\).

Figure [1] shows the full \(\phi_{R}\)-dependent part of the effective potential, \(V_{\text{eff}}(\phi_{R}, T) - V_{\text{eff}}(0, T)\), for the parameter values discussed above and for three different temperatures: \(T = 0\), \(T_{c} = 1.1\) PeV and \(T = 1.4\) PeV. At the critical temperature \(T_{c}\), the two vacua become degenerate. The order parameter is equal to \(\xi(\phi_{R}) = (\phi_{R}/v_{R}) T_{c} \approx 4.4\), indicating a strong first order phase transition.

(2) Second phase transition: SU(4)\(_{L}\) \(\rightarrow\) SU(3)\(_{L}\)

This transition happens when the field \(\Sigma_{L}\) develops the vev \(v_{L} \approx 40\) TeV. The corresponding background field-dependent masses and thermal masses are obtained from Eqs. (25) and (26) upon substituting \(R \rightarrow L\). The critical temperature is \(T_{c} \approx 12\) TeV and the order parameter \(\xi(\Sigma_{L}) \approx 3.3\).

(3) Third phase transition: SU(3)\(_{R}\) \(\times\) SU(3)\(_{L}\) \(\rightarrow\) SU(3)\(_{c}\)

This symmetry breaking is triggered when \(\Sigma\) develops the vev as in Eq. (3) with \(v_{\Sigma} \approx 2\) TeV. Given our choice \(z = 1/4\), the contribution of the cross terms to the effective potential is small, as are those of the vector-like fermions \(Q'\) and \(L'\), even with Yukawas \(Y_{i j} \sim 1\) (see, e.g., [2] [3] [4] for the corresponding formulae). Therefore, the only relevant background field-dependent mass is that of \(G'\). For transverse components

\[
m_{G'}(\phi_{\Sigma}) = \sqrt{\frac{1}{3}} \left( g_{R}^2 + g_{L}^2 \right) \phi_{\Sigma},
\]

with the number of degrees of freedom \(n_{G'}^{L} = 16\). For the longitudinal modes of \(G'\) and the SM gluon, the masses \(m_{G'}(\phi_{\Sigma}) + \Pi_{G'}(T)\) are given by the eigenvalues of the matrix

\[
\mathcal{M}_{G'}(\phi_{\Sigma}, T) = \left( \begin{array}{cc} g_{R}^2 (\phi_{\Sigma}^2 + 4 T^2) & -g_{R}g_{L} \phi_{\Sigma}^2 \\ -g_{R}g_{L} \phi_{\Sigma}^2 & g_{L}^2 (\phi_{\Sigma}^2 + 4 T^2) \end{array} \right).
\]

The numbers of degrees of freedom are \(n_{G'}^{L} = n_{G'} = 8\). The thermal masses for the radial modes and Goldstone bosons are

\[
\Pi_{G'}(T) \approx (g_{R}^2 + g_{L}^2) T^2
\]

with \(n_{G'} = 32\). A strong first order phase transition occurs since \(\xi(\Sigma) \approx 2.5\) for the critical temperature \(T_{c} \approx 290\) GeV.

V. GRAVITATIONAL WAVE SIGNALS

As a result of a first order phase transition, bubbles of true vacuum are nucleated, they expand (with velocity \(v_{\mu}\)) and eventually fill up the entire universe. The bubble nucleation rate per unit volume is given by the expression [83]

\[
\Gamma(T) \approx \left( \frac{S_{3}(T)}{2 \pi T} \right)^{3/2} e^{s_{3}(T)/4},
\]

where \(S_{3}(T)\) is the Euclidean action

\[
S_{3}(T) = 4 \pi \int dr r^2 \left[ \left( \frac{d\phi_{b}}{dr} \right)^2 + V_{\text{eff}}(\phi_{b}, T) \right].
\]

Here \(\phi_{b}(r)\) is the SO(3) symmetric bounce solution describing the profile of the expanding bubble, i.e., the solution of the equation

\[
\frac{d^2 \phi_{b}}{dr^2} + \frac{2 d \phi_{b}}{r} \frac{dV_{\text{eff}}(\phi, T)}{d\phi} \bigg|_{\phi=\phi_{b}} = 0
\]

with the boundary conditions

\[
\frac{d \phi_{b}}{dr} \bigg|_{r=0} = 0, \quad \phi_{b}(\infty) = \phi_{\text{true}},
\]

where \(\phi_{\text{true}}\) is the field value of the true vacuum.

The phase transition begins at the temperature \(T_{s}\), called the nucleation temperature, at which \(\Gamma(T_{s}) \approx H^4\), where \(H\) is the Hubble value at that time. This is equivalent to

\[
4 \log \left( \frac{M_{P}}{T_{s}} \right) \approx \frac{S_{3}(T_{s})}{T_{s}},
\]

where \(M_{P} = 1.22 \times 10^{19}\) GeV is the Planck mass. The nucleation temperatures for the phase transitions in our benchmark scenario are: \(T_{s}^{(1)} \approx 430\) TeV, \(T_{s}^{(2)} \approx 5.3\) TeV and \(T_{s}^{(3)} \approx 210\) GeV.

Each phase transition continues until most of the universe is filled with bubbles of true vacuum. The inverse of the duration of this process, the so-called \(\beta\) parameter, is given by

\[
\tilde{\beta} \equiv T_{s} \frac{d}{dT} \left( \frac{S_{3}(T)}{T} \right) \bigg|_{T=T_{s}}.
\]

The strength of a phase transition, denoted by \(\alpha\), is defined as

\[
\alpha \equiv \frac{\rho_{\text{vac}}(T_{s})}{\rho_{\text{rad}}(T_{s})},
\]

where \(\rho_{\text{vac}}(T_{s})\) is the energy density of the false vacuum (i.e., the latent heat released during the phase transition),

\[
\rho_{\text{vac}}(T_{s}) = \Delta V_{\text{eff}}(T_{s}) - T_{s} \frac{\partial \Delta V_{\text{eff}}(T)}{\partial T} \bigg|_{T=T_{s}},
\]

with \(\Delta V_{\text{eff}}(T) = V_{\text{eff}}(\phi_{\text{false}}, T) - V_{\text{eff}}(\phi_{\text{true}}, T)\), and \(\rho_{\text{rad}}(T_{s})\) is the energy density of radiation at nucleation temperature,

\[
\rho_{\text{rad}}(T_{s}) = \frac{\pi^2}{30} g_{s} T_{s}^4.
\]
In the expressions above $\phi_{\text{false}}$ is the field value of the false vacuum, whereas $g_*$ is the number of relativistic degrees of freedom at the time of the transition. In our benchmark scenario: $g_*^{(1)} \approx 274$, $g_*^{(2)} \approx 252$ and $g_*^{(3)} \approx 228$. The four parameters: $\alpha$, $\beta$, $v_w$ and $T_*$ determine the size and peak frequency of the stochastic gravitational wave signal. In our analysis we set the bubble wall velocity to $v_w = 0.6$ (for an extensive discussion of the bubble expansion, see [86]).

There are three sources of gravitational waves generated from phase transitions: sound waves, bubble collisions and magnetohydrodynamic turbulence. Those three contributions combine linearly to give the total gravitational wave signal

$$h^2\Omega_{GW} \approx h^2\Omega_{\text{sound}} + h^2\Omega_{\text{collision}} + h^2\Omega_{\text{turbulence}} .$$

The contribution from sound waves is [87, 88]

$$h^2\Omega_{\text{sound}}(\nu) \approx (1.86 \times 10^{-5}) \frac{v_w}{\beta} \left( \frac{\kappa_s \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{\frac{1}{2}} \times \left[ 1 + 0.75 \left( \frac{\nu}{v_w} \right)^2 \right]^{\frac{1}{2}} ,$$

where the model-dependent parameter $\kappa_s$ is the fraction of the latent heat that is transformed into the bulk motion of the plasma, approximated by [86]

$$\kappa_s \approx \frac{\alpha}{0.73 + 0.083\sqrt{\alpha + \alpha}} ,$$

and $\nu_s$ is the peak frequency given by

$$\nu_s = (0.019 \text{ Hz}) \frac{\tilde{\beta}}{v_w} \left( \frac{g_*}{100} \right) \left( \frac{T_*}{100 \text{ TeV}} \right) .$$

The contribution from bubble collisions is [88-90]

$$h^2\Omega_{\text{collision}}(\nu) \approx (1.66 \times 10^{-5}) \frac{1}{\beta^2} \left( \frac{\kappa_e \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{\frac{1}{2}} \times \left( \frac{v_w}{1 + 2.4v_w} \right) \left( \frac{\nu}{v_w} \right)^{2.8} \left[ 1 + 2.8 \left( \frac{\nu}{v_w} \right)^{3.8} \right] ,$$

where $\kappa_e$ is the fraction of the latent heat that is deposited into a thin shell close to the bubble front [91],

$$\kappa_e \approx \frac{0.715 \alpha + \frac{4}{27} \sqrt{\frac{3\alpha}{\alpha}}}{1 + 0.715 \alpha} ,$$

and the peak frequency $\nu_c$ is

$$\nu_c = (0.010 \text{ Hz}) \frac{\tilde{\beta}}{v_w} \left( \frac{g_*}{100} \right) \left( \frac{T_*}{100 \text{ TeV}} \right) \times \left( \frac{1}{1.8 - 0.1v_w + v_w^2} \right) .$$

The contribution from turbulence is [92, 93]

$$h^2\Omega_{\text{turbulence}}(\nu) \approx (3.35 \times 10^{-4}) \frac{v_w}{\beta} \left( \frac{\kappa_t \alpha}{1 + \alpha} \right)^{\frac{1}{2}} \left( \frac{100}{g_*} \right)^{\frac{1}{2}} \times \left( \frac{\nu}{v_w} \right)^{3} \left[ 1 + \frac{8\pi\nu^2}{h_*} \right] \left( 1 + \frac{\nu}{M_*} \right)^{\frac{1}{2}} ,$$

where $\kappa_t = c_\kappa \kappa_e$ denotes the fraction of the latent heat transformed into magnetohydrodynamic turbulence (following [88], we take $c_\kappa = 0.05$), the peak frequency $\nu_t$ is

$$\nu_t = (0.027 \text{ Hz}) \frac{\tilde{\beta}}{v_w} \left( \frac{g_*}{100} \right)^{\frac{1}{2}} \left( \frac{T_*}{100 \text{ TeV}} \right) ,$$

and the parameter $h_*$ [88]

$$h_* = (0.0165 \text{ Hz}) \left( \frac{g_*}{100} \right)^{\frac{1}{2}} \left( \frac{T_*}{100 \text{ TeV}} \right) .$$

To find the gravitational wave signal from the three phase transitions, we analyzed separately the $\phi_R$, $\phi_L$, $\phi_S$: dependent pieces of the potential in Eq. (18). In each case we determined numerically the temperature at which the shape of the potential yields the Euclidean action $S(T_*)$ satisfying Eq. (14). For this nucleation temperature, we calculated the values of the parameters $\alpha$, $\beta$, and used them to derive the gravitational wave spectrum via Eqs. (39)-(48).

Our calculation revealed that in the Left-Right SU(4) Model the sound wave contribution dominates over the contributions from bubble collisions and magnetohydrodynamic turbulence in the majority of the peak region, thus the shape of the signal is well approximated by Eq. (40) and the peak frequency by Eq. (42). This is illustrated in Fig. 2 for the phase transition associated with SU(4)$_R$ → SU(3)$_R$.

Figure 3 shows the combined spectrum of gravitational waves from all three phase transitions in our benchmark scenario. As expected, each phase transition produces a distinct peak in the spectrum. As seen from Eq. (42), the position of individual peaks depends linearly on the nucleation temperature $T_*$, thus signals from phase transitions corresponding to higher symmetry breaking scales appear at higher frequencies. The peak frequency depends also linearly on $\tilde{\beta}$. The height of the peak is governed by $\alpha$ and $\tilde{\beta}$: it increases with bigger $\alpha$ and decreases with larger $\tilde{\beta}$. Of course all those parameters depend on the values of the vevs, quartic couplings and gauge couplings in the model.
FIG. 3. Gravitational wave signature of the Left-Right SU(4) Model (black line) for the benchmark scenario described in Eq. (16). Overplotted are the sensitivities of the future gravitational wave experiments: LISA in the C1 configuration [88] (red), Big Bang Observer [94] (light green), DECIGO [94] (dark green), Einstein Telescope [95] (blue) and Cosmic Explorer [39] (purple). The three peaks correspond to the phase transitions: (1) SU(4)\textsubscript{R} \rightarrow SU(3)\textsubscript{R}, (2) SU(4)\textsubscript{L} \rightarrow SU(3)\textsubscript{L} and (3) SU(3)\textsubscript{R} \times SU(3)\textsubscript{L} \rightarrow SU(3)\textsubscript{c} discussed in the text.

Within the benchmark scenario, the gravitational wave signal generated by the symmetry breaking SU(4)\textsubscript{R} \rightarrow SU(3)\textsubscript{R} at the scale $v_R \approx 5$ PeV (peak (1)) falls within the sensitivity of the future gravitational wave detectors Cosmic Explorer and Einstein Telescope. The signal resulting from the second phase transition SU(4)\textsubscript{L} \rightarrow SU(3)\textsubscript{L} at the scale $v_L \approx 40$ TeV (peak (2)) is well within the reach of the Big Bang Observer and DECIGO. The third phase transition SU(3)\textsubscript{R} \times SU(3)\textsubscript{L} \rightarrow SU(3)\textsubscript{c} occurring at the scale $v_\Sigma \approx 2$ TeV (peak (3)) can also be probed by the Big Bang Observer and DECIGO. In addition, it can be searched for by LISA, but only if the C1 configuration [88] is implemented.

A unique property of the Left-Right SU(4) Model is that the range of peak frequencies for the phase transitions SU(4)\textsubscript{L} \rightarrow SU(3)\textsubscript{L} and SU(3)\textsubscript{R} \times SU(3)\textsubscript{L} \rightarrow SU(3)\textsubscript{c} is constrained by the size of the $R_{K^{(*)}}$ anomalies, as shown in Eq. (15). In our benchmark scenario we assumed that there is a maximal hierarchy between the scales $v_L$ and $v_\Sigma$, which leads to two well-separated peaks in the spectrum. However, if the two scales are comparable, then the size of the flavor anomalies sets the symmetry breaking scale at $v_L \approx v_\Sigma \lesssim 25$ TeV, resulting in a single peak shifted towards lower frequencies compared to peak (2) in Fig. 3. This is still within the reach of the Big Bang Observer and DECIGO.

Finally, we point out that the scale for the symmetry breaking SU(4)\textsubscript{R} \rightarrow SU(3)\textsubscript{R} is not bounded from above. In particular, it can be larger than $\sim 100$ PeV, shifting peak (1) to higher frequencies and escaping the detection at the Cosmic Explorer and Einstein Telescope. A gravitational wave experiment sensitive to such high frequencies would be necessary to probe this scenario.

VI. CONCLUSIONS

Gravitational wave experiments have recently emerged as a powerful tool for testing particle physics models. One class of signatures which those experiments are sensitive to arises from first order phase transitions in the early universe, making them valuable probes of the scalar sector in models with spontaneous symmetry breaking.

In this paper we demonstrated, in the context of the Left-Right SU(4) Model, that gravitational wave detectors can be used to look for signatures specific to the flavor anomalies recently observed at the LHCb, BaBar and Belle experiments. The measured magnitude of lepton universality violation implies that there can be two peaks in the gravitational wave spectrum within the sensitivity of the upcoming LISA, Big Bang Observer and DECIGO experiments. There is also a possibility of a third peak which could be observed by the Cosmic Explorer and Einstein Telescope.

If the hints of lepton universality violation are confirmed and a gravitational wave signal with features similar to those of the Left-Right SU(4) Model is discovered, this would be a strong motivation for building the 100 TeV collider, which could provide a complementary direct detection method of testing the model.

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