Coherent resonant tunneling in AC fields.

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Abstract

We have analyzed the tunneling transmission probability and electronic current density through resonant heterostructures in the presence of an external electromagnetic field. In this work, we compare two different models for a double barrier: In the first case the effect of the external field is taken into account by spatially dependent AC voltages and in the second one the electromagnetic field is described in terms of a photon field that irradiates homogeneously the whole sample. While in the first description the tunneling takes place mainly through photo sidebands in the case of homogeneous illumination the main effective tunneling channels correspond to the coupling between different electronic states due to photon absorption and emission. The difference of tunneling mechanisms between these configurations is strongly reflected in the transmission and current density which present very different features in both cases.

In order to analyze these effects we have obtained, within the Transfer Hamiltonian framework, a general expression for the transition probability for coherent resonant tunneling in terms of the Green’s function of the system.

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I. INTRODUCTION

In the last years several works have been devoted to the analysis of the effect of a time-dependent field on the transport properties of resonant heterostructures, i.e., double barriers, quantum wires and quantum dots. However there is not yet a systematic discussion of the different situations corresponding to an AC voltage applied between the left and right leads and which implies a position-dependent dephasing of the external field on the sample, and the case where the whole sample is homogeneously illuminated, where the electron-photon coupling depends on position just through the momentum matrix elements. In spite of the increasing interest in this field, most of the theoretical work has been done considering spatially time dependent voltages but the experimental information includes both configurations. Both situations, however, are physically different because the effective tunneling channels for the flowing of current are different in both cases.

The coherent tunneling in the presence of light applied homogeneously on a double barrier heterostructure has been recently treated in the scheme of the Transfer Matrix. An alternative way to obtain the Transmission coefficient is based in the Transfer Hamiltonian and considers a localized basis representation, i.e., approximated hamiltonians whose eigenstates are spatially localized. The extension of the TH to analyze coherent resonant tunneling (GTH) allows to analyze not only the sequential tunneling which consider the electrons tunneling through the single barriers, emitter and collector, sequentially, but the coherent one which includes the virtual transitions through the resonant states for electrons crossing coherently the heterostructure.

In this work we have extended the Generalized Transfer Hamiltonian formalism (GTH) to obtain the transition probability for the coherent tunneling in the presence of a time dependent potential.

In the first section of the paper we will develop the theory to obtain the transition probability for a double barrier structure in the presence of a spatially dependent AC modulation. In the next section the GTH is extended to describe the photoassisted tunneling process for a sample homogeneously illuminated. The different features obtained for the coherent tunneling current for the two different configurations are discussed in a further section.

II. TRANSITION PROBABILITY THROUGH A DOUBLE BARRIER IN AN AC FIELD

We are going to analyze the effect of an AC field applied just to the left and right leads with a dephasing of $\pi$. This configuration is schematically represented for the double barrier in fig. 1.

The Transfer Hamiltonian formalism developed by Bardeen, allows to describe in first order time dependent perturbation theory the transition probability in terms of the eigenstates of auxiliar hamiltonians spatially localized. This formalism has been extended to all orders in perturbation theory (Generalized Transfer Hamiltonian) to analyze as well tunneling through resonant states, i.e., to include virtual transitions through the localized states in the well (see fig.2a).
The quantum mechanical Hamiltonian for an electron in the presence of time dependent potential can be written:

\[ H(t) = H_L(t) + H_R(t) + H_T(t) + H_c \]  

(1)

Where \( H_L(t) \) and \( H_R(t) \) are the hamiltonians for the left and right sides respectively including the time dependent perturbation and \( H_T(t) \) is the coupling term which accounts for the transfer of electrons from the left to the right side. We are going to define now \( H_L(t) \) and \( H_R(t) \) as:

\[ H_L(t) \equiv \sum_k \epsilon_{kL}(t)c_{kL}^+c_{kL} = \sum_k [\epsilon_{kL} + \langle kL|H_2(t)|kL \rangle]c_{kL}^+c_{kL} \]

\[ H_R(t) \equiv \sum_p \epsilon_{pR}(t)c_{pR}^+c_{pR} = \sum_p [\epsilon_{pR} - \langle pR|H_2(t)|pR \rangle]c_{pR}^+c_{pR}. \]  

(2)

where \( c_{kL}^+, c_{kL}, c_{pR}^+, c_{pR} \) are creation and destruction operators for electron in the left- and right-hand side of the heterostructure, respectively; \( \epsilon_{kL}(\epsilon_{pR}) \) are the eigenenergies for \( H^0_{L} (H^0_{R}) \) and \( H_2(t) \) is the hamiltonian which describes the AC time dependent potential in the leads and can be written as:

\[ H_2(t) = V_{AC}\cos \omega_0 t \]  

(3)

In these expressions just diagonal terms are considered in the time dependent term. The reason is that the off-diagonal terms are zero in this case due to the orthogonality of the eigenstates of the auxilar hamiltonians \( H^0_{L}, H^0_{R} \). The retarded single electron Green’s function of these hamiltonians are:

\[
G_L^+(t,t') \equiv -\frac{i}{\hbar} \theta(t - t') \langle 0|c_{kL}(t)c_{kL}^+(t')|0 \rangle = -\frac{i}{\hbar} \theta(t - t') \exp[-\frac{i}{\hbar} \int_{t'}^t dt_1 \epsilon_{kL}(t_1)]
\]

\[ = -\frac{i}{\hbar} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n(\frac{V_{AC}}{\hbar \omega_0})J_m(\frac{V_{AC}}{\hbar \omega_0}) e^{-\frac{\pi}{\hbar}k_L(t-t')} e^{-im \omega_0 t} e^{i n \omega_0 t'} \]

\[
G_R^+(t,t') \equiv -\frac{i}{\hbar} \theta(t - t') \langle 0|c_{pR}(t)c_{pR}^+(t')|0 \rangle = -\frac{i}{\hbar} \theta(t - t') \exp[-\frac{i}{\hbar} \int_{t'}^t dt_1 \epsilon_{pR}(t_1)]
\]

\[ = -\frac{i}{\hbar} \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} J_r(\frac{V_{AC}}{\hbar \omega_0})J_s(\frac{V_{AC}}{\hbar \omega_0}) e^{-\frac{\pi}{\hbar}p_R(t-t')} e^{ir \omega_0 t} e^{-is \omega_0 t'} \]  

(4)

where \( |0 \rangle \) denotes the electron vacuum in the leads, \( c_{kL}^+, c_{kL}, c_{pR}^+, c_{pR} \) are the creation and destruction fermion operators in the leads evaluated in the Heisenberg representation and we have made use of the identity:

\[ e^{-i \frac{V_{AC}}{\hbar \omega_0} \sin \omega_0 t} \equiv \sum_{m=-\infty}^{\infty} J_m(\frac{V_{AC}}{\hbar \omega_0}) e^{-im \omega_0 t} \]  

(5)

In this equation \( J_m \) denotes the Bessel function of integer order \( m \).

We use an interaction picture to switch on adiabatically the required perturbations that allow us to recover the total hamiltonian \( H(t) \). In order to apply the Generalized Transfer Hamiltonian method we choose a hamiltonian that in its first quantized form is:

\[ H(t) = H_L(t) + V_L(t) = H_R(t) + V_R(t) \]  

(6)
Where $V_L(t)$ and $V_R(t)$ can be written as:

$$V_L(t) = V_{Le}^{\eta t} - V_{AC}\{\Theta(x-x_2) - \Theta(x-x_3)\}cos\omega_0te^{\eta t} - 2V_{AC}\Theta(x-x_3)cos\omega_0te^{\eta t}$$

$$V_R(t) = V_{Re}^{\eta t} + V_{AC}\{\Theta(x-x_2) - \Theta(x-x_3)\}cos\omega_0te^{\eta t} + 2V_{AC}\Theta(x_2-x)cos\omega_0te^{\eta t} \tag{7}$$

$V_L(t)$ and $V_R(t)$ will be considered of the same order in the perturbative procedure and $V_L, V_R$ are represented in fig 2a. The time evolution of the wave function for the total system can be written as:

$$|\Psi(t)\rangle = f(t) \sum_{m=-\infty}^{\infty} J_m(\frac{V_{AC}}{\hbar\omega_0})e^{-im\omega_0t}e^{-i\omega_kLt}|k_L\rangle + \sum_{n=-\infty}^{\infty} J_n(\frac{V_{AC}}{\hbar\omega_0})e^{in\omega_0t} \sum_{p_R} U_R(t,-\infty)e^{-ip\omega_Rt}|p_R\rangle \tag{8}$$

This wave function must describe a particle initially on the left side. This is satisfied by taking $f(-\infty) = 1$ and $U_R(-\infty,-\infty) = 0$. The electrons in a particular state $|k_L\rangle$ can in principle evolve to any state $|p_R\rangle$ in the right side so that a summation over right states is required in the expression of the wave function. The time evolution operator $U_R(t,-\infty)$ gives the evolution of an electron to a right state and is determined at every order from the time dependent Schrödinger equation by an expansion in a perturbative series. We take $f(t) = f^{(0)}$ and $U_R(t,-\infty) = \sum_{j=1}^{\infty} U_R^{(j)}(t,-\infty)$ where $j$ denotes the perturbation order. Applying the Schrödinger equation we obtain just to order $j$ the set of equations:

$$i\hbar \sum_{n=-\infty}^{\infty} J_n e^{in\omega_0t} \frac{\partial U_R^{(1)}}{\partial t} e^{-ip\omega_Rt}|p_R\rangle = \sum_{m=-\infty}^{\infty} J_m e^{-im\omega_0t} e^{-i\omega_kLt} V_L(t)|k_L\rangle.$$  

$$\vdots$$

$$i\hbar \sum_{n=-\infty}^{\infty} J_n e^{in\omega_0t} \frac{\partial U_R^{(j)}}{\partial t} e^{-ip\omega_Rt}|p_R\rangle = \sum_{n_1=-\infty}^{\infty} J_{n_1} e^{-in_1\omega_0t} \sum_{p_R_1} e^{-i\omega_R_1t} U_R^{(j-1)}(t)p_R_1|p_R_1\rangle (j \geq 2). \tag{9}$$

These iterative equations for $U_R^{(j)}$ are solved by projecting them on the state $|p_R\rangle$ giving:

$$U_R^{(1)} = -\frac{i}{\hbar} \{ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n J_m \langle p_R|V_L|k_L\rangle$$

$$- \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{1}{2} [J_{n+1} + J_{n-1} ] J_m V_{AC} \langle p_R|\Theta(x-x_2) - \Theta(x-x_3)\rangle|k_L\rangle$$

$$- \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} [J_{n+1} + J_{n-1} ] J_m V_{AC} \langle p_R|\Theta(x-x_3)\rangle|k_L\rangle \}$$

$$\times \int_{-\infty}^{t} dt_1 e^{i(\omega_Rt_1 - \omega_kL - m\omega_0 - m\omega_0 - in)t_1} \tag{10}$$

For the physical parameters considered experimentally the amplitude of the time dependent modulation is much smaller than the barrier height, $V_{AC} << V_0$ ($V_0$ is the height of the barriers) and then it is a good approximation to consider:
Then, we proceed considering just the static part of $V_L(t)$; the time evolution operator to $j$-order can be written:

$$U_R^{(j)} = (-\frac{\hbar}{i})^j \sum_{n,m} \sum_{n_1} \sum_{n_{j-1}} J_n J_m J_{n_1} \ldots J_{n_{j-1}} \langle p_R|V_R|p_R \rangle \ldots \langle p_{R_{j-1}}|V_R|p_{R_{j-1}} \rangle \langle p_{R_{j-1}}|V_L|k_L \rangle \times \int_{-\infty}^{t_n} dt_1 e^{i(\omega_{p_R} - \omega_{p_{R_1}} - n_1 \omega_0 - \omega_{k_L} - i\eta) t_1} \int_{-\infty}^{t_{n-1}} dt_2 e^{i(\omega_{p_R} - \omega_{p_{R_{j-1}}} - n_{j-1} \omega_0 - \omega_{k_L} - i\eta) t_2} \ldots \int_{-\infty}^{t_{j-1}} dt_j e^{i(\omega_{p_{R_{j-1}}} - \omega_{k_L} - n_{j-1} \omega_0 - \omega_{k_L} - i\eta) t_j}$$

(12)

The solution is:

$$U_R^{(j)}(t, -\infty) = \sum_{n,m} \sum_{n_1} \sum_{n_{j-1}} J_n J_m J_{n_1} \ldots J_{n_{j-1}} \frac{e^{i(\omega_{p_R} - \omega_{p_{R_1}} - n_1 \omega_0 - \omega_{k_L} - i\eta) t}}{(\omega_{p_R} - \omega_{k_L} - n_1 \omega_0 - \omega_{k_L} - i\eta) + nh\omega_0}$$

(13)

In this procedure we have assumed that the spectral density of the electrons in the leads is:

$$A_k(\epsilon) \equiv 2\pi \sum_{n=-\infty}^{\infty} J_n^2 \delta(\epsilon - \epsilon_k + nh\omega_0)$$

(14)

This assumption will be correct only in the nonadiabatic regime where the external frequency is much larger than the inverse resonant-tunneling time. This regime does not consider the ac components of the spectral densities and Green’s functions (see appendix).

This allows us to write the Fourier transform of the retarded single electron Green’s function of $H_R$ as:

$$G_R^+(\epsilon) = \sum_{n=-\infty}^{\infty} \sum_{p_R} J_n^2 \frac{\langle p_R \rangle \langle p_R \rangle}{\epsilon - \epsilon_{p_R} + nh\omega_0 + i\eta}$$

(15)

It can be found from:

$$A_k(\epsilon) = -2ImG_R^+(\epsilon)$$

(16)

And then, $U_R^{(j)}$ becomes in terms of the retarded Green’s function:

$$U_R^{(j)}(t, -\infty) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n J_m \frac{e^{i(\epsilon_{p_R} - \epsilon_{p_{R_1}} - n\omega_0 - nh\omega_0 - i\eta) t}}{(\epsilon_{p_R} - \epsilon_{p_{R_1}} - n\omega_0 - nh\omega_0 - i\eta)}$$

(17)

The transition probability from left to right per unit time can be expressed as:
\begin{align}
P_{RL} &= \lim_{\eta \to 0} 2\text{Re}[U_R^*(t, -\infty) \frac{dU_R(t, -\infty)}{dt}] \\
\end{align}

Where \( U_R(t, -\infty) = \sum_{j=1}^{\infty} U_R^{(j)}(t, -\infty) \) includes the sum over all orders in perturbation theory. In the case of a continuous spectrum there are no divergences in the analytic expression of the Green’s function so that the limit \( \eta \to 0 \) of the matrix elements in \( U_R \) can be taken independently from that of the fractions, and then, we can express the time evolution in terms of the total Green’s function of the system:

\[ G^+(\epsilon) = G^+_R(\epsilon) + G^+_R(\epsilon)V_RG^+_R(\epsilon) + \ldots \] (19)

In the case of a discrete spectrum some generalization is needed because the adiabatic limit \( \eta \to 0 \) would imply to have poles in the Green’s functions which coincide with those of the fractions that give the delta functions

The expression for the time dependent transition probability is:

\[
P_{RL} = \frac{i}{\hbar} \sum_{nmpq} J_n J_m J_p J_q e^{i(q\omega_0 + p\omega_0 - n\omega_0 - m\omega_0)t} \langle p_R|V_L + V_R G^+(\epsilon_{k_L} + m\hbar\omega_0) V_L|k_L\rangle \langle k_L|V_L G^+(\epsilon_{k_L} + q\hbar\omega_0) V_R + V_L|p_R\rangle^* \]

\[ PP \left( \frac{1}{\epsilon_{p_R} - \epsilon_{k_L} - p\hbar\omega_0 - q\hbar\omega_0} - i\pi \delta(\epsilon_{p_R} - \epsilon_{k_L} - p\hbar\omega_0 - q\hbar\omega_0) - \right. \]

\[ PP \left. \frac{1}{\epsilon_{p_R} - \epsilon_{k_L} - n\hbar\omega_0 - m\hbar\omega_0} - i\pi \delta(\epsilon_{p_R} - \epsilon_{k_L} - n\hbar\omega_0 - m\hbar\omega_0) \right] \] (20)

Where PP denotes the principal part and \( G^+ \) is the total Green’s function of the system.

In order to obtain the stationary transition probability we consider \( n=p \) and \( m=q \):

\[
P_{RL} = \frac{2\pi}{\hbar} \sum_{n,m=-\infty}^{\infty} J_n^2 \left( \frac{V_{AC}}{\hbar\omega_0} \right) J_m^2 \left( \frac{V_{AC}}{\hbar\omega_0} \right) \delta(\epsilon_{p_R} - \epsilon_{k_L} - n\hbar\omega_0 - m\hbar\omega_0) \]

\[ | \langle p_R|V_L + V_R G^+(\epsilon_{k_L} + m\hbar\omega_0) V_L|k_L\rangle |^2 \] (21)

This formula for the transition probability is a natural extension of the Fermi’s Golden Rule formula to analyze the coherent resonant tunneling in terms of the total Green’s function of the system. The term which does not contain the Green’s function corresponds to first order perturbation theory, and is the only one appearing in the Transfer Hamiltonian method, however the term containing the total Green’s function is the one which includes processes which involve real intermediate states and therefore describes correctly the coherent resonant tunneling.

From (21) one can obtain straightforward the Transmission probability and the coherent tunneling current density.

In many physical cases \( G^+ \) can be calculated but, in general, the main physics of the problem can be recovered by using approximations to \( G^+ \). This is the case of resonant tunneling in which the main physics is connected with well states so that a good approximation is to substitute \( G^+ \) by the Green’s function \( G_C^+ \) of an isolated quantum well (see fig 2b) with a
selfenergy which accounts for the coupling of the well with the continuum of states in the right and left leads. By means of a Dyson equation the selfenergy can be expressed as:

\[ \Sigma(\epsilon_{kL} + m\hbar\omega_0) = V_C + V_C G_C(\epsilon_{kL} + m\hbar\omega_0)V_C \quad (22) \]

The imaginary part of the selfenergy is related with the elastic coupling to the leads as:

\[ \Gamma(\epsilon_{kL} + m\hbar\omega_0) = -2\text{Im}\Sigma(\epsilon_{kL} + m\hbar\omega_0) \quad (23) \]

And the energy shift of the resonances:

\[ \text{Re}\Sigma(\epsilon_{kL} + m\hbar\omega_0) = PP \int_{-\infty}^{\infty} \frac{d\epsilon_{qL}}{2\pi} \frac{\Gamma(\epsilon_{qL})}{\epsilon_{kL} - \epsilon_{qL} + m\hbar\omega_0} \quad (24) \]

is the Hilbert Transform of the elastic coupling.

With this procedure the energy dependence of the broadening of the resonant levels for each bias voltage applied across the heterostructure (this dependence is included in the potentials \(V_C\)) is considered in a straightforward way.

Also its dependence on the external field is included through the dependence of \(\Sigma\) on the energy of the \(m\) photo sideband (ie., the broadening of each photo sideband is different) and through the spectral densities (see formula 14) that give the correct weight to each photo sideband. One should remark that usually the models based in the Transfer Hamiltonian define the elastic coupling to the leads as:

\[ \Gamma(\epsilon) = \Gamma_L(\epsilon) + \Gamma_R(\epsilon) \]

\[ = \sum_{kL} T_{kL}^2 A_{kL}(\epsilon) + \sum_{pR} T_{pR}^2 A_{pR}(\epsilon) \quad (25) \]

Here \(T_{kL}, T_{pR}\) are the matrix elements given by Bardeen and \(A_{kL}, A_{pR}\) are the spectral densities of the left and right leads respectively. This expression implies that the transmission probability is the sum of the transmission probabilities of each barrier separately and therefore that the tunnel is sequential. We consider coherent tunnel and this is the reason to define the broadening as in Eq(22-23). Therefore, by means of our model we do not have to treat the coupling with the leads as a constant as the usual Transfer Hamiltonian methods for sequential tunnel do.

### III. LASER IRRADITATING THE WHOLE HETEROSTRUCTURE

We consider now the case where the light illuminates homogeneously the whole heterostructure. In this case, there is no a position-dependent phase shift of the external field on the heterostructure and the spatial dependence, as we will see below, appears through the matrix elements of the electronic momentum operator.

The electromagnetic field is represented by a plane electromagnetic wave of wave vector \(\vec{k}\), parallel to the \(x\) direction and polarized in the tunnel direction (see fig.3) \(\vec{E} = (0,0,F)\).

The hamiltonian for this configuration has been solved within the framework of the Transfer Matrix formalism and time dependent perturbation theory. Our aim now is to obtain the transition probability and the tunneling current within the
scheme of the Transfer Hamiltonian, in the same way as it was done in the previous section for the AC case. The interest of doing that in this case is again to work in terms of localized states as basis and therefore it would allow to consider, as it was discussed previously, different kind of excitations in the different spatial regions, as well as systems with localized eigenstates.

Applying the same procedure as in the case of the oscillating voltages we can define the hamiltonians \( H_L(t) \) and \( H_R(t) \) including the time dependent part of the hamiltonian that includes only diagonal coupling between electronic states. In the Coulomb Gauge:

\[
H_L(t) = \sum_{k_L} [\epsilon_{k_L} + \frac{e}{m^*} \langle k_L | p_z | k_L \rangle \left( \frac{\hbar}{2e\omega_0} \right)^{1/2} (ae^{-i\omega_0 t} + a^+ e^{i\omega_0 t})] c_{k_L}^+ c_{k_L}
\]

\[
H_R(t) = \sum_{p_R} [\epsilon_{p_R} + \frac{e}{m^*} \langle p_R | p_z | p_R \rangle \left( \frac{\hbar}{2e\omega_0} \right)^{1/2} (ae^{-i\omega_0 t} + a^+ e^{i\omega_0 t})] c_{p_R}^+ c_{p_R}
\] \quad (26)

Here the vector potential operator is:

\[
\vec{A} = \left( \frac{\hbar}{2e\omega_0} \right)^{1/2} (ae^{-i\omega_0 t} + a^+ e^{i\omega_0 t}) \vec{e}_z
\] \quad (27)

\( a^+, a \) are the creation and destruction operators for photons and the wave vector of the electromagnetic field has been neglected. We will see below that the comparison with the previous configuration indicates that there are qualitative features in the transmission coefficient and current density which are very different for both cases. Therefore the configuration of the time dependent potential matters for analyzing the photoassisted transport properties.

The main reasons for those differences are, firstable, that in this case, the electron-photon coupling term contains the \( \vec{P} \) operator matrix elements, and secondly, that there are off-diagonal terms in the Hamiltonian, i.e., terms which describe how the electromagnetic field couples the different electronic states. These terms are those which produce a difference in the transition probability with respect to the case without a time dependent field applied to the sample. In the previous case, however, the transition probability difference comes from the photo sidebands which appear in the regions affected by the time dependent field and which behave as additional tunneling channels.

The left and right hamiltonians are exactly soluble, by means of a canonical transformation which decouples electrons and photons. We will not repit here this procedure but we give the expression for the retarded Green’s functions associated with the hamiltonians \( H_L(t) \) and \( H_R(t) \):

\[
G^+_{L}(t, t') = -\frac{i}{\hbar} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n(\beta_{k_L}) J_m(\beta_{k_L}) e^{-\frac{\pi}{\hbar}r_{k_L}(t-t')} e^{-i\omega_0 t} e^{i\omega_0 t'}
\]

\[
G^+_{R}(t, t') = -\frac{i}{\hbar} \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} J_r(\beta_{p_R}) J_s(\beta_{p_R}) e^{-\frac{\pi}{\hbar}r_{p_R}(t-t')} e^{i\omega_0 t} e^{-i\omega_0 t'}
\] \quad (28)

where:

\[
\beta_{k} = \frac{eF\langle k | p_z | k \rangle}{m^*\hbar\omega_0^2}
\] \quad (29)
Here, the argument of the Bessel function depends on the matrix element of the momentum operator and its value is very small. It is also a significative difference with respect to the previous case where the argument of the Bessel functions can be arbitrarily big (depending of the amplitude of the external field) and therefore the contribution of the terms which correspond to Bessel functions of higher order than zero and one, have to be considered.

In the present case we are discussing, as the argument of the Bessel functions is close to zero for typical intensities and frequencies of the field, without lose of generality it is usually a very good approximation to neglect terms of order higher than zero or one at most.

In order to obtain the transition probability for coherent tunneling through the double barrier structure, we proceed evaluating, as in the previous case, and in the framework of the Generalized Transfer Hamiltonian\[14\], the time evolution operator with:

\[
V_L(t) = V_L e^{i\eta t} \\
V_R(t) = V_R e^{i\eta t}
\] (30)

Its expression, obtained up to infinite order in time dependent perturbation theory for the barriers and exact for the diagonal terms in the electron-photon coupling, is the same as eq.(17) but now the argument of the Bessel functions depends on the electronic momentum matrix elements as was already discussed.

At this point we include the non-diagonal coupling terms in order to obtain the total time evolution operator. Its expression in interaction representation is:

\[
U(t, -\infty) = 1 - \frac{i}{\hbar} \int_{-\infty}^{t} ds \hat{H}_2(s) \\
+ \left( \frac{-i}{\hbar} \right)^2 \int_{-\infty}^{t} \int_{-\infty}^{s} d\tau \hat{H}_2(s) U(s, \tau) \hat{H}_2(\tau)
\] (31)

Where \( H_2 \) is:

\[
H_2(t) = \sum_{k_i'} \sum_{k_i} eF \frac{e^F}{m^*\omega_0} \langle k_i' P_z | k_i \rangle c_{k_i'}^* c_k \cos(\omega_0 t) \\
i = L, R
\] (32)

The caret on the \( H_2 \) operators indicates that they should be evaluated in interaction representation (here the ”non interacting” hamiltonian is just this one we have solved exactly by means of the GTH).

We consider only first order and we keep only the \( J_0 \) Bessel function terms because they are the only ones that give non negligible contributions due to the smallness of their arguments.

The new term appearing in the time evolution operator has the expression:

\[
U_2(t, -\infty) = \frac{1}{\hbar} \frac{J_0(\beta_{PR}) J_0(\beta_{KL})}{2} \frac{eF}{m^*\omega_0} \frac{e^{i(\omega_{PR} - \omega_{KL} + \omega_0 - i\eta)t}}{\omega_{PR} - \omega_{KL} + \omega_0 - i\eta} \\
\times \left\{ \langle p_R | V_R G_L^- (\epsilon_{PR}) P_z | k_L \rangle + \langle p_R | P_z G_R^+ (\epsilon_{KL}) V_L | k_L \rangle + \langle p_R | V_R G^- (\epsilon_{PR}) P_z G_L^+ (\epsilon_{KL}) V_L | k_L \rangle + \langle p_R | V_R G^- (\epsilon_{PR}) P_z G^+ (\epsilon_{KL}) V_L | k_L \rangle \right\} + (\omega_0 \rightarrow -\omega_0)
\] (33)
Where $G^\pm_R$ and $G^\pm_L$ are the Green’s functions of $H_R$ and $H_L$ respectively, $G^\pm$ includes the tunneling terms and we have used the property that the total Green’s function of the system can be expanded in terms of the two basis left and right:

$$G = G_L + G_LV_LG_L + ... = G_R + G_RV_RG_R + ...$$ (34)

The Green’s functions appearing in this expression are:

$$G^\pm_L(\epsilon) = \sum_{\kappa_L} J_0^2(\beta_{\kappa_L}) \frac{|k_{\kappa_L}'|}{\epsilon - \epsilon_{\kappa_L} \pm i\eta}$$

$$G^\pm_R(\epsilon) = \sum_{\kappa_R} J_0^2(\beta_{\kappa_R}') \frac{|p_{\kappa_R}'|}{\epsilon - \epsilon_{\kappa_R}' \pm i\eta}$$

$$G^\pm(\epsilon) = \sum_{\kappa_C} J_0^2(\beta_{\kappa_C}) \frac{|q_{\kappa_C}|}{\epsilon - \epsilon_{\kappa_C} - \frac{i\Gamma(\epsilon)}{2} \pm i\eta}$$ (35)

Following the same steps as in section II the transition probability can be written as a Fermi’s golden rule:

$$P_{RL} = \frac{2\pi}{\hbar} \{ |A_{RL}|^2 \delta(\epsilon_{p_R} - \epsilon_{k_L}) + |B_{RL}|^2 \delta(\epsilon_{p_R} - \epsilon_{k_L} + \hbar\omega_0) + (\omega_0 \rightarrow -\omega_0) \}$$ (36)

where $A_{RL}$ and $B_{RL}$ contain the matrix elements:

$$A_{RL} = J_0(\beta_{p_R})J_0(\beta_{k_L})\langle p_R|V_L + V_RG^+(\epsilon_{k_L})V_L|k_L \rangle$$

$$B_{RL} = \frac{J_0(\beta_{p_R})J_0(\beta_{k_L})}{2} \frac{eF}{m^*\omega_0} \{ \langle p_R|V_RG_L^-(\epsilon_{p_R})P_z|k_L \rangle + \langle p_R|P_zG_R^+(\epsilon_{k_L})V_L|k_L \rangle + \langle p_R|V_RG_R^-(\epsilon_{p_R})P_z|k_L \rangle + \langle p_R|P_zG_R^+(\epsilon_{k_L})V_L|k_L \rangle \}$$ (37)

In the previous expression (37) the different terms contain the possible transitions from the left to the right states assisted by the light. Both resonant transitions through the well states and non resonant ones, for coherent tunneling are included. This kind of matrix elements have been obtained in ref 7 in the context of sequential photoassisted tunneling.

**IV. RESULTS FOR THE TWO DIFFERENT CONFIGURATIONS**

In this section we have studied the two different configurations discussed in the previous section i.e., the transmission coefficient for electrons through a double barrier in the presence of a time dependent potential applied between the emitter and collector whose amplitude is position-dependent and an homogeneous electromagnetic field irradiating the whole sample. In fig.(4) the transmission coefficient for the first case has been drawn. The heterostructure consists in a double barrier of GaAs – Al$_x$Ga$_{1-x}$As with well and barrier thicknesses of 50 Å and 100 Å respectively. We have considered that the energy of the time-dependent field is 13.6 meV corresponding to a frequency of 3.3 THz. The important magnitude in this case
is the ratio between the amplitude and the energy of the field which enters in the argument of the Bessel functions (see formula 5). As we have already discussed depending of the value of this number higher order Bessel functions are non negligible and its value determines the number of them (i.e., the number of photo sidebands) which participate in the transmission probability and current density. In fig.(4) we consider that this ratio is one half and we have included in the calculation the Bessel functions up to fourth order. This fact is reflected in the four satellites which appear at both sides of the main central peak (which corresponds to the Bessel function of zero-order).

One should also remark that the broadening of the resonant states in the well is not a constant but depends on the photo sideband index m (see formula 23). In this case we observe that the contribution of the photo sideband of index m > 0 to the transmission coefficient is smaller than the one coming from the main peak (m=0) but of the same order of magnitude even for the higher order side-bands due to the finite value of the argument of the Bessel function \( \frac{V_{AC}}{\hbar \omega_0} \). Another interesting effect which was already discussed by M. Wagner is the quenching of the transmission probability in the presence of an AC field. This effect can be understood looking to formula (21) where the transition probability at the energy of the resonant state is weighted by \( \sum_{n=\infty}^{\infty} J_0^2 \). The quenching of the transmission takes place if the argument of the Bessel function \( \beta = \frac{V_{AC}}{\hbar \omega_0} \) is such that \( J_0(\beta) = 0 \). In fig (5) this effect is shown. We observed it for different values of \( \beta \) : as it approaches to the complete quenching limit the resonant transmission main peak is strongly reduced, therefore tuning the intensity and frequency of the time dependent modulation allows to reduce the transmission independently of the transparency of the barrier.

We have also plotted the current density for an AC potential through a double barrier (fig.6) for different amplitudes : As main features we observe that the threshold tunneling current moves to lower bias and also that for higher bias the current density is smaller in the presence of the AC field, and this effect increases as the ratio \( \frac{V_{AC}}{\hbar \omega_0} \) increases too. Also a step-like behaviour is observed in the current as a function of the external bias. We can explained these features in terms of the photo sidebands: the threshold, for this case is close to zero bias. That is due to the fact that there are photo sidebands associated to electronic states close to the Fermi energy which contribute to the resonant tunneling even when the resonant state \( E_r \) is higher in energy than the emitter Fermi energy \( E_F \). If \( E_r \) is higher than \( E_F \) in several photon energies the photo sidebands which allow the flow of current have a low spectral weight and the contribution to the current is small. As far as the resonant state energy closes into the Fermi energy, increasing the bias, the lower indexes photo sidebands, i.e., those which are more intense in the spectral function can be aligned with the resonant state, and therefore, their contribution to the current increases. Once the resonant state crosses the Fermi energy the current density increases but remains smaller than the current with no AC applied. That is due again to the fact that the spectral function has finite weight in all the photo sidebands and not only in the main one which is weighted by the zero order Bessel function and whose value is smaller than one. In this case , only a small number of them , for a fixed bias tunnels resonantly and the effect of the field is to reduce the current.

As the ratio between the field intensity and the frequency increases it does the coupling of the electron motion with the external field and the current density differs more from the case without this interaction and presents abrupt steps coming from the satellite bands.
The homogeneous illumination of the sample has been analyzed for the first time in ref 10. In order to compare this situation with the one discussed previously, we have plotted the transmission coefficient for a GaAs-AlGaAs double barrier for both configurations (fig.7). To make the two situations comparable we estimate the ratio $\frac{V_{AC}}{\hbar \omega_0}$ supposing the total potential drop due to the light to be $Fd$ where $d$ is the length of the heterostructure, for the values of intensity and energy of the electromagnetic field studied we have $V_{AC}/\hbar \omega_0 = 0.77$. The main difference between the two configurations is that for these heterostructures the momentum matrix elements are very small, therefore, in the case of homogeneous light irradiating the sample and for typical values of the intensities and frequencies of the electromagnetic field, the argument of the Bessel functions of higher order than zero are negligible and therefore the intensity of the corresponding photo sideband can be neglected. That is the reason why to consider just the main peak (the $m=0$ term) makes sense.

However we observe in fig.7b the presence of two satellites in the transmission coefficient. These side-peaks have another origin than the photo sidebands: They come from the mixing of electronic states due to the homogeneous light and which show up in the Hamiltonian as the off-diagonal matrix elements of the electronic momentum coupled by the light. In this case just processes involving absorption and emission of one photon are considered because they are the main ones.

Therefore, the tunneling channels for the two configurations are different: in the case of an AC field the off-diagonal terms cancel if the amplitude of the AC potential is considered constant within each region (left, center and right). In this case, the main tunneling channels (the only ones within this approximation) are the sidebands: those in the emitter align in energy with the sidebands in the well producing additional contributions to the transmission probability and the resonant current. Their contribution can be important even for high order photo sidebands if the ratio $V_{AC}/\hbar \omega_0$ is of the order or higher than one.

In the case of an homogeneous electromagnetic field, the situation is different. The off-diagonal electron-photon coupling terms in the Hamiltonian are those which modify the current density. Those channels, involving different electronic states (fig.7) contribute in principle also with all their photo sidebands, however, as the argument of the Bessel functions (i.e., the intensity of the $m$ photo sideband) is controlled by the momentum matrix elements and remains very small, just the zero index photo sideband (the main one) is non-negligible and gives a contribution to the current. Therefore the three peaks in the transmission coefficient come from the main bands (index zero) corresponding to three electronic states which differ in one photon energy and which are mixed by the field.

In fig. 8.a the current density through the double barrier in the presence of homogeneous light is represented. For the field intensity and frequency considered: $F = 4.10^5 \frac{V}{m}, \hbar \omega_0 = 13.6\text{meV} \ (3.3 \text{THz})$ the effect of the external field is very small due to the electron-photon coupling term involving the momentum matrix elements, and the effect of the light cannot be observed in the characteristic curve but in the current density difference with respect to the case of no light present in the sample (fig. 8.b).

This result is in very good agreement with the experimental results of ref 3 where a double barrier is irradiated with a laser in the far infrared regime.
V. CONCLUSIONS

In this work we have extended the Generalized Transfer Hamiltonian which allows to describe the coherent tunneling through a resonant heterostructure, to include the effect of a time dependent field. Two different configurations have been addressed: an homogeneous electromagnetic field applied to the heterostructure and a position-dependent AC field dropping through the heterostructure.

We have obtained the transition probability and the current density through a double barrier structure for both configurations, and the results have been discussed in terms of the main available tunneling channels for each situation.

In the first case the photoassisted tunneling is mainly due to the coupling between different electronic states due to photon absorption and emission processes. In this case, the satellite bands with index higher than zero have negligible contribution to the transmission probability.

In the second one the sidebands at the emitter region carry the electronic charge by aligning with those corresponding to well and collector electronic states. The intensity and frequency of the AC field determines the number of effective channels (sidebands) participating in the current.

We did not include the effect of the electron-electron interaction in this formalism. This effect is small in double barrier heterostructures but it has to be included for systems where the correlation is an important contribution. This is the purpose of a future work.

VI. APPENDIX

In this appendix we want to derive the expression for the spectral densities and to discuss in which limit is correct to apply the Tien and Gordon’s expression for the electronic density of states. We start with the retarded Green’s function which depends explicitly on two times $t$ and $t'$ but not on their difference because of the breakdown of time translational invariance due to the external field:

$$G_k(t, t') = -\frac{i}{\hbar} \theta(t - t') \langle 0 | c_k(t) e_k^+(t') | 0 \rangle = -\frac{i}{\hbar} \theta(t - t') \exp \left[ -\frac{i}{\hbar} \int_{t'}^{t} dt_1 \epsilon_k(t_1) \right]$$

$$= -\frac{i}{\hbar} e^{-\frac{i}{\hbar} \epsilon_k(t-t')} e^{-\frac{V_{AC}}{\hbar \omega_0} \sin \omega_0 t} e^{\frac{V_{AC}}{\hbar \omega_0} \sin \omega_0 t'}$$

$$= -\frac{i}{\hbar} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n \left( \frac{V_{AC}}{\hbar \omega_0} \right) J_m \left( \frac{V_{AC}}{\hbar \omega_0} \right) e^{-\frac{i}{\hbar} \epsilon_k(t-t')} e^{-i \omega_0 t} e^{i \omega_0 t'} \quad (38)$$

Let us now calculate the spectral density from the retarded Green’s function which in Wigner coordinates $\tau = t - t'$, $T = \frac{t + t'}{2}$ is defined as:

$$A_k(\omega, T) = \int_{-\infty}^{\infty} d\tau e^{i \omega \tau} G_k(T + \frac{\tau}{2}, T - \frac{\tau}{2}) \quad (39)$$

and:

$$G_k(T + \frac{\tau}{2}, T - \frac{\tau}{2}) = i[G_k^+(T + \frac{\tau}{2}, T - \frac{\tau}{2}) - G_k^-(T + \frac{\tau}{2}, T - \frac{\tau}{2})] \quad (40)$$
Performing the Fourier transform in the relative time we obtain:

\[
A_k(\omega, T) = \frac{2\pi}{\hbar} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n(\frac{V_{AC}}{\hbar \omega_0}) J_m(\frac{V_{AC}}{\hbar \omega_0}) \cos[(n-m)\omega_0 T] \delta(\omega - \omega_k + \frac{n+m}{2}\omega_0)
\] (41)

Only in the nonadiabatic regime when the frequency of the external field is larger than the inverse of the tunneling time for the electrons we can define an averaged spectral density as:

\[
A_k(\omega) = \frac{1}{T_0} \int_0^{T_0} dT A_k(\omega, T) = \frac{2\pi}{\hbar} \sum_{n=-\infty}^{\infty} J_n^2(\frac{V_{AC}}{\hbar \omega_0}) \delta(\omega - \omega_k + n\omega_0)
\] (42)

where the average is taken over one period \(T_0\) of the external field.

With this procedure we restrict ourselves to the dc component of the spectral density (and then of the Green’s functions) since the ac components (equation 41) are suppressed in the average.

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FIGURES

FIG. 1. Schematic drawing of a resonant-tunneling structure with AC voltages applied in the leads.

FIG. 2. Sketch of the different Hamiltonians used to study tunneling with the GTH method. (a) Left and right Hamiltonians, (b) center Hamiltonian.

FIG. 3. Schematic drawing of a resonant-tunneling structure in the presence of an electromagnetic field polarized in the tunnel direction.

FIG. 4. $\log_{10}$ of coherent transmission coefficient as a function of energy with and without an AC field of parameters $\frac{V_{AC}}{\hbar \omega_0} = 0.5, \frac{\hbar \omega_0}{V_{AC}} = 13.6 \text{meV}$ for a GaAs/AlGaAs double barrier 100-50-100 Å.

FIG. 5. Quenching of the transmission for $\frac{V_{AC}}{\hbar \omega_0} = 2.0, \frac{V_{AC}}{\hbar \omega_0} = 2.4, \frac{V_{AC}}{\hbar \omega_0} = 2.5, \hbar \omega_0 = 13.6 \text{meV}$ (sample fig4).

FIG. 6. Coherent tunneling current density as a function of voltage for the sample of fig4. (a) With and without an AC field of parameters $\frac{V_{AC}}{\hbar \omega_0} = 0.5, \frac{\hbar \omega_0}{V_{AC}} = 13.6 \text{meV}$. (b) For different ratios between the intensity and the energy of the AC field ($\frac{\hbar \omega_0}{V_{AC}} = 13.6 \text{meV}$).

FIG. 7. Comparison of $\log_{10}$ of coherent transmission coefficient as a function of energy for an AC field and an electromagnetic field. (a) $\frac{V_{AC}}{\hbar \omega_0} = 0.77$. (b) $F = 4.10^5 \frac{\text{Volt}}{m}, \hbar \omega_0 = 13.6 \text{meV}$

FIG. 8. (a) Current Density in the presence of an electromagnetic field for the same parameters as fig7.b (b) Current density difference.