Active Disturbance Rejection Position Servo Control of PMSLM Based on Reduced-order Extended State Observer

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Abstract: To enhance the control accuracy of permanent magnet synchronous linear motor (PMSLM) servo systems affected by disturbances, such as time-varying parameters and abrupt load changes, an active disturbance rejection control (ADRC) algorithm based on a reduced-order extended state observer (ESO) is adopted to suppress the disturbances on the control system. First, the system’s ability to estimate disturbances is enhanced by linearizing the ESO. Second, the pole placement method is used for the construction of a reduced-order ESO that can reduce the influence of the number of adjustment parameters and the phase lag. The parameters of the reduced-order ADRC are adjusted, the optimal control parameters are selected, and the stability of controller is proved. Finally, practical experiments prove that the proposed method can improve control accuracy under multiple working conditions and features strong anti-interference ability. There is a smaller steady-state error, and no overshoot is observed.

Keywords: PMSLM, ADRC, reduced-order ESO, anti-interference

1 Introduction

Conventional servo systems mostly adopt the drive method of “roller screw + rotary motor” or “gear rack + rotary motor.” Transmission accuracy is restricted by return error and mechanical deformation, which limit the control precision of servo systems. Linear motor servo systems are widely used in laser processing, material addition manufacturing, and high-grade computer numerical control (CNC) machine tools because of their high speed, high dynamic response, high precision, and zero transmission\textsuperscript{[1-2]}. The permanent magnet synchronous linear motor (PMSLM) drive mode is an underdamped direct drive. Load and friction directly influence a motor’s mover and severely affect the control accuracy of servo systems. Therefore, research on the anti-interference control strategy of the servo system is of theoretical and practical importance.

Currently, typical control strategies such as model predictive control, conventional proportion integral derivative (PID) control, sliding mode control, and fuzzy control are used to reject disturbances. These control strategies have certain levels of anti-interference ability. However, they continue to suffer from certain shortcomings. Model predictive control has good tracking performance and strong anti-interference ability, but its sampling period is large in general, and overcoming random sudden disturbances in a timely manner is difficult\textsuperscript{[3]}. The conventional PID control method is characterized by its steady state and lack of static error, but the system cannot quickly restore its equilibrium state because of overshoot caused by time-varying signals or sudden disturbances\textsuperscript{[4]}. The sliding mode control method is robust to external disturbances and parameter perturbations. However, when the sliding mode trajectory approaches a switching surface, chattering and other phenomena occur. Moreover, the sliding mode trajectory cannot be controlled after entering the dead-time range owing to the dead-time effect of the switching function\textsuperscript{[5]}. Conversely, the fuzzy control method does not require a precise model and has strong fault-tolerant ability. It can solve problems related to strong coupling and time-varying disturbances in the control process, but
its fuzzy processing can affect control accuracy and reduce dynamic quality\(^6\).

Active disturbance rejection control (ADRC) is only possible when control is taken as an experimental science, instead of a mathematical one\(^7\). According to the input and output of a research object, ADRC can be achieved by total disturbance observation and compensation\(^8\-\^9\). The ADRC control strategy can also be applied to linear motor servo systems. High tracking accuracy, strong anti-interference ability, and fast response speed comprise the advantages of ADRC. Therefore, the ADRC control strategy is introduced to solve the disturbance rejection problem in PMSLM.

However, conventional ADRC consists of numerous adjustment parameters that are difficult to adjust, and obtaining extremely large gain parameters is difficult because of noise, thus, the ability of the observer to estimate disturbance is limited\(^10\-\^11\). In Ref. \(^12\), the adaptive particle swarm optimization algorithm (APSO) is introduced into the ADRC controller to reduce the dependence of the controller on parameters. To obtain a better optimization solution more efficiently, the aggregation degree and evolution speed are introduced into the APSO to modify the inertia weight dynamically based on the practical optimization process. The linear inertia weight avoids the premature convergence of the algorithm and local optima. However, this algorithm is characterized by low precision, and when there are sudden disturbances, the parameter adjustment is not so fast. In Ref. \(^13\), a phase-locked loop observer based on ADRC is added to improve the system’s ability to suppress disturbance. However, it is substantially affected by temperature. Hence, an additional temperature compensation circuit is required to enable the circuit to work normally, albeit at a high cost. In Ref. \(^14\), a simplified noise reduction disturbance observer based on ADRC is presented to handle system nonlinearities, parameter uncertainties, and disturbances. This method can reduce sensor noise while ensuring that the filtered sensor information retains disturbance information as much as possible. However, this method is mainly used to suppress sensor noise and improve rapidity, and it has difficulty handling sudden changes. Therefore, further research on ADRC is required.

This paper takes the PMSLM servo system as the research object. It introduces a linear ADRC strategy based on the reduced-order extended state observer (ESO) to achieve disturbance suppression. First, the adjustment range of the gain coefficient is increased by linearizing the nonlinear ESO, and the estimation ability of the disturbance is expanded. Second, owing to the high-order ESO, the ADRC has many parameters, strong parameter sensitivity, and phase lag\(^15\). Therefore, a reduced-order state observer is designed to replace the ESO in the original system, which reduces the number of adjustment parameters and the sensitivity of parameters and improves the anti-interference performance. The parameters of the linear reduced-order ADRC (ADRC-RESO) are then adjusted. Finally, the effectiveness and accuracy of the proposed method are demonstrated experimentally by comparing it with the methods proposed in Ref. \(^12\) and Ref. \(^14\), ADRC-NRDOB and ADRC-APSO, respectively.

2 PMSLM system structure

2.1 PMSLM structure

Fig. 1 depicts the topological structure of the double secondary PMSLM studied in this paper.

![Fig. 1 Structure of PMSLM](image)

The secondary PMSLM consists of primary and secondary stages. The primary stage consists of six centrally distributed winding coils, and the secondary stage consists of a stator yoke and a uniformly distributed permanent magnet.

Fig. 2 illustrates the structure of the design control system according to the precision requirements of laser processing—that is, additive manufacturing. \(S_{ref}\) is the reference value of displacement, and \(v_{ref}\) is the input value of velocity.

To realize high-precision control of the PMSLM, as the inner loop of the closed-loop system, the speed loop and the current loop must have certain anti-interference ability. The disturbance of the speed loop and the current loop will seriously affect the control
precision of the position servo control system. Therefore, a second-order speed ADRC controller is designed to track and compensate for the disturbance of the current and speed loops accurately. Thus, the control accuracy of the position servo system is improved.

![PMSLM position servo control system based on reduced-order observer](image)

**Fig. 2** PMSLM position servo control system based on reduced-order observer

### 2.2 PMSLM model

In the \(dq\) synchronous rotating coordinate system, the voltage equation of the PMSLM is\[^{[16]}\]

\[
\begin{align*}
u_d &= R_i i_d + \frac{d\psi_d}{dt} - \frac{\pi}{\tau} \psi_q \\
u_q &= R_i i_q + \frac{d\psi_q}{dt} + \frac{\pi}{\tau} \psi_d
\end{align*}
\]

where \(\psi_d = L_d i_d + \psi_f\) is a straight-axis flux linkage; \(\psi_q = L_q i_q\) is a quadrature-axis flux linkage; \(R_s\) represents the coil resistance of each phase winding; \(\tau\) is the permanent magnet pole pitch; \(u_d\) and \(u_q\) represent the direct-axis and quadrature-axis voltages, respectively; \(i_d\) and \(i_q\) are the direct- and quadrature-axis currents, respectively; \(L_d\) and \(L_q\) are the direct- and quadrature-axis inductances, respectively; \(v\) is the moving speed of the motor mover; and \(\psi_f\) is the permanent magnetic flux linkage.

The kinematic equation of the motor can be expressed as follows

\[
\begin{align*}
F_c &= F_i + B_v v + m\dot{v} \\
F_c &= \frac{1.5 p_s \pi}{\tau} i_q \psi_f
\end{align*}
\]

where \(F_c\) is the electromagnetic thrust; \(F_i\) is the load force of the motor; \(B_v\) is the viscous friction coefficient; \(p_s\) is the number of pole pairs; and \(m\) is the motor mover mass. According to Eq. (4), Eq. (3) can be transformed into

\[
\dot{v} = \frac{1.5 p_s \pi^2 \psi_f}{m \tau} i_q - \frac{F_i}{m} - \frac{B_v}{m} v
\]

Eq. (5) shows that the relationship between \(q\)-axis current and speed can be approximated by a first-order differential equation, whereas the additional disturbance of the current tracking error exists in the first-order ADRC of speed, which seriously affects the control accuracy of the servo system\[^{[17]}\]. Therefore, a second-order speed ADRC system is designed to realize speed and current composite control.

The second-order differential of velocity is calculated according to Eq. (5), and then Eq. (1) to Eq. (4) are combined to obtain Eq. (6)

\[
\dot{v} = \left[ \frac{1.5 p_s \pi^2 \psi_f}{m \tau^2 L_q} \right] v + \left[ \frac{1.5 p_s \pi \psi_f}{m \tau L_q} \right] i_q - \frac{B_v F_i}{m^2}
\]

### 3 Improved algorithm of second-order speed ADRC

ADRC is based on the development of conventional PID control, which retains the advantages of PID control and overcomes its shortcomings. The specific methods are as follows. (1) Arranging the appropriate transition process; (2) Discussing the method of disturbance estimation, “ESO”; (3) Discussing the appropriate combination method, “nonlinear feedback”\[^{[10]}\]. Thus, the second-order speed ADRC consists of a tracking differentiator, ESO, and state error feedback control law\[^{[7]}\].

The tracking differentiator is a transitional process that can be designed as\[^{[18]}\]

\[
\begin{align*}
\dot{v}_d &= \frac{a}{d} x_s - r \text{sgn}(a)(1 - s_s) \\
d &= r h^2, a_0 = h_0 v_s, y = (v_1 - v_0) + a_0 \\
a_1 &= \sqrt{[d + 8] y} \\
a_2 &= a_0 + \frac{\text{sgn}(y)(a_1 - d)}{2} \\
a &= (a_0 + y) s_x + a_2 (1 - s_x) \\
s_y &= (\text{sgn}(y + d) - \text{sgn}(y - d))/2 \\
s_s &= (\text{sgn}(a + d) - \text{sgn}(a - d))/2
\end{align*}
\]

where \(f_{st}\) is the fast optimal integrated control
function; \( r \) is the velocity factor; \( h \) is the integration step; \( h_0 \) is the filtering factor of the tracking differentiator; \( v_1 \) and \( v_2 \) are tracked by the velocity \( v_0 \) and its differential signal, respectively; and \( \phi_0 \) is the theoretical value of the velocity.

The ESO is designed as

\[
\begin{align*}
\hat{e} &= \hat{v}(k) - v(k) \\
\hat{\dot{e}} &= \hat{\dot{v}}(k) - \beta_0 \hat{e}(k) + \beta_1 \hat{e}(k) \\
\hat{z}_3 &= \hat{z}_3(1) - \beta_0 \hat{e}(k) + \beta_1 \hat{e}(k) \\
\hat{u} &= \phi_0 \hat{e}(k) + \phi_1 \hat{e}(k) + \phi_2 \hat{e}(k) + \phi_3 \hat{e}(k) + \phi_4 \hat{e}(k) + \phi_5 \hat{e}(k)
\end{align*}
\]

(8)

where \( \hat{e}(\alpha, \alpha, h) \) is a fast optimal control synthesis function; \( e \) is the speed tracking error; \( z_3 \) is the observation value of the total disturbance of the system; \( \beta_0, \beta_1, \beta_2, \beta_3 \) are the gain coefficients of the observer; and \( \alpha_1, \alpha_2, \alpha_3 \) are the nonlinear factors.

The state error feedback control law is designed as follows

\[
\begin{align*}
e_1 &= v(k) - \hat{v}(k) \\
e_2 &= v(k) - \hat{v}(k) \\
u_0 &= \phi_0 \hat{e}(k) + \phi_1 \hat{e}(k) + \phi_2 \hat{e}(k) + \phi_3 \hat{e}(k) + \phi_4 \hat{e}(k) + \phi_5 \hat{e}(k) \\
u(k) &= \phi_0 \hat{e}(k) + \phi_1 \hat{e}(k) + \phi_2 \hat{e}(k) + \phi_3 \hat{e}(k) + \phi_4 \hat{e}(k) + \phi_5 \hat{e}(k)
\end{align*}
\]

(9)

where \( e_1 \) is the tracking error of the speed signal; \( e_2 \) is the tracking error of the speed differential signal; \( \beta_0 \) and \( \beta_1 \) are the gain coefficients of the feedback controller; \( \alpha_0 \) and \( \alpha_1 \) are nonlinear factors; \( b \) is the compensation factor of the observer, and \( u_0 \) is the control amount.

Fig. 3 provides the structure block diagram of the second-order speed ADRC.

![Fig. 3 Second-order speed ADRC](image)

**3.1 Second-order speed ADRC based on reduced-order state observer**

Parameter tuning is complex because of the limited disturbance rejection range of the nonlinear ADRC, and the selection of the gain coefficient is limited by noise. Thus, it should be first linearized.

By replacing the nonlinear function \( fal() \) with error \( e \) approximation for linearization, the linear ESO and feedback control law can be obtained as

\[
\begin{align*}
e_1 &= v(k) - \hat{v}(k) \\
\dot{e}_1 &= v(k) - \hat{v}(k) \\
u_0 &= \phi_0 \hat{e}(k) + \phi_1 \hat{e}(k) + \phi_2 \hat{e}(k) + \phi_3 \hat{e}(k) + \phi_4 \hat{e}(k) + \phi_5 \hat{e}(k) \\
u(k) &= \phi_0 \hat{e}(k) + \phi_1 \hat{e}(k) + \phi_2 \hat{e}(k) + \phi_3 \hat{e}(k) + \phi_4 \hat{e}(k) + \phi_5 \hat{e}(k)
\end{align*}
\]

(10)

To overcome the shortcoming of large phase lag in ESO, linear ADRC based on reduced-order ESO is proposed, which can reduce the sensitivity of parameters and improve the efficiency of calculation and control accuracy under the same bandwidth. The detailed derivation process is shown below.

For the object shown in Eq. (6), let \( x = v \), and \( u = u_p \), the second-order ADRC system which can be obtained from Eq. (6) is designed as follows

\[
\dot{x} = a_1 x + a_2 x + f + bu
\]

(12)

where \( b = 1.5 p \pi \psi / L \varphi I \) is the control gain of the system; \( u \) is the control quantity of the system; \( a_1 = \frac{B^2}{m^2} \frac{3 \pi^2 \psi^2}{2 m^2 L_q} \); \( a_1 = 0 \); \( f = f \) \( v, v \dot{v} \) \( l_q \) \( \left[ \frac{1.5 p \pi \psi / R}{m^2 \varphi} \right] j_q + \frac{1.5 B \varphi E / m^2 \varphi} {j_q} \) is the sum of the disturbances inside and outside the system.

For the second-order speed system shown in Eq. (12), the state equation can be transformed into

\[
\begin{align*}
\dot{x} &= Ax + Bu + Eh \\
y &= Cx
\end{align*}
\]

(13)

where \( A = \begin{pmatrix} a_2 & 0 & 1 \end{pmatrix} \); \( B = \begin{pmatrix} b \end{pmatrix} \); \( C = (1 \ 0 \ 0) \);

\[
E = \begin{pmatrix} 0 & 0 & 0 \\
1 & 0 & 0 \\
x_1 & x_2 & x_3 
\end{pmatrix}.
\]

According to Eq. (13), \( h = \dot{f} \). The physical
The quantity that should be observed is reduced by one because \( x_1 = y = v \) can be measured\(^{[19]}\). Hence, a reduced-order ESO can be established, and \( x_2 \) and \( f \) are estimated by \( z_1 \) and \( z_2 \), respectively.

According to the pole assignment method, the reduced-order ESO is expressed as follows

\[
\begin{align*}
\dot{z} &= A_z z + B_z u + G_y + L(y - C_z z) \\
\dot{f} &= V
\end{align*}
\]

where \( A_z = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \); \( B_z = \begin{pmatrix} b \\ 0 \end{pmatrix} \); \( C_z = (1 \ 0) \); \( V = (0 \ 1) \);

\( G = \begin{pmatrix} a_z \\ 0 \end{pmatrix} \); \( L = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} = \begin{pmatrix} 2a_0 \\ \omega_0^2 \end{pmatrix} \), \( L \) is the observer gain; and \( \omega_0 \) is the bandwidth of the observer.

Then have

\[
\begin{align*}
z_1 &= v_1 + l_1 y \\
z_2 &= v_2 + l_2 y
\end{align*}
\]

By discretizing Eq. (11), it can be obtained as follow

\[
\begin{align*}
v(k + 1) &= Mv(k) + Nu(k) \\
y(k) &= Pv(k) + Qu(k)
\end{align*}
\]

where \( M = \begin{pmatrix} 1 - 2a_0 b t & t \\ 1 - a_0 b^2 t & 0 \end{pmatrix} \); \( P = E_2 \); \( Q = 0 \); and \( N = \begin{pmatrix} bt & -3a_0 b^2 t + a_2 \\ 0 & -2a_0 b^2 t \end{pmatrix} \), \( t \) is the sampling time.

According to Eq. (12) and Eq. (13), the state error feedback control law can be designed as follows

\[
u = \frac{a_z x - f + u_0}{b}
\]

where \( f = a_2 y \), and the design of \( u_0 \) is similar to that of the conventional PID control method. Hence, it can be designed as follows

\[
u_0 = k_p (v - \hat{v}) + k_d (\dot{v} - \dot{\hat{v}})
\]

where \( k_p \) and \( k_d \) are the proportional and differential gains, respectively.

The system adopts the vector control mode with \( i_d = 0 \). To optimize the control effect, Ref. \([11]\) proposes a bandwidth method for selecting parameters. According to

\[
G_e = \frac{k_p}{s^2 + k_d s + k_p}
\]

let \( \lambda(s) = s^2 + 2\zeta \omega_c s + \omega_c^2 = s^2 + k_d s + k_p \); then, \( k_d = 2\zeta \omega_c \), \( k_p = \omega_c^2 \), where \( \omega_c \) is the bandwidth of the controller.

The structure block diagram of the second-order speed ADRC based on reduced-order ESO is shown in Fig. 4.

In summation, as the ESO order decreases, the number of parameters is reduced. A reduction in the tuned parameters can reduce the noise sensitivity, so ADRC based on reduced-order ESO can improve the control accuracy.

### 3.2 Parameter tuning

In Ref. \([11]\), a bandwidth method is proposed to adjust the system and simplify the parameters of the ADRC controller into three parameters: controller bandwidth \( \omega_c \), observer bandwidth \( \omega_0 \), and damping coefficient \( \xi \). According to Eq. (19), the gain coefficient of the reduced-order ESO are \( l_1 = 2\omega_0 \), \( l_2 = \omega_0^2 \); and the parameters of the controller are \( k_d = 2\xi \omega_c \), \( k_p = \omega_c^2 \).

Generally, controller parameters \( \omega_c \) and \( \xi \) are first selected according to system requirements. When \( 0 < \xi < 1 \), the system is in an oscillating state. The oscillating amplitude of the system and overshoot decrease with increasing \( \xi \). When \( \xi = 1 \), the system is critically damped. At this time, the response rise time is shortest and its speed is fastest without overshoot. At \( 1 < \xi \), the system is in an overshoot state. Therefore, the greater the value of \( \xi \), the slower the response speed, and the response is not in overshoot. In general, take \( \xi = 1 \). A certain relationship exists between \( \omega_c \) and \( \omega_0 \), that is, \( \omega_c = 3 - 5\omega_0 \). According to Ref. \([18]\), \( \omega_c = 3000 \) and \( \omega_0 = 800 \) are selected.
To select the best PI parameters, differential evolution is used to identify the PI parameters. To prevent excessive control energy, the square term of the input signal is added to the objective function. The optimal index for parameter selection is as follows

\[ J = \int_0^\infty (\eta_1|e(t)| + \eta_2u^2(t))dt \]

where \( e(t) \) is the systematic error; \( u(t) \) is the input signal; and \( \eta_1 \) and \( \eta_2 \) are the weights.

To avoid overshoot, the overshoot is considered as one of the optimal indexes

\[ J = \int_0^\infty (\eta_1|e(t)| + \eta_2u^2(t) + \eta_3)dt \]

where \( \eta_3 \) is the weight. Fig. 5 depicts the process of PI parameter optimization and evaluation criteria.

![Fig. 5 PI parameter optimization process and evaluation criteria](image)

Fig. 5 shows that when \( J \) is the smallest, the PI parameters are considered to have the optimal value. At this point, \( k_p = 15.8 \) and \( k_i = 1.6 \) can be obtained.

### 3.3 Convergence proof of ESO

To prove the convergence of the ESO, the following assumptions are made.

(H1). Functions \( f \) and \( \omega \) are continuously differentiable for all their independent variables, and

\[ |u| + |f| + |\dot{\phi}| + \left| \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i} \right| \leq c_0 + \sum_{j=1}^n c_j |x_j| \]

where \( u \) is the input; \( f \) is the internal disturbance; \( \omega \) is the external disturbance; \( \phi \) is the gain coefficient; \( j = 0,1,\ldots,n \); and \( k \) is a positive integer.

(H2). The solution of \( \omega \) and the system satisfies

\[ |\omega| + |x(t)| \leq B, \text{ where } B > 0 \text{ is a constant; } i = 1,\ldots,n; \quad t \geq 0. \]

(H3). There are constant \( \lambda_i (i = 1,2,3,4) \alpha, \beta \) and continuous positive definite functions \( V, W : \mathbb{R}^{n+1} \rightarrow \mathbb{R} \), which yield

(a) \( \lambda_i \|y\|^2 \leq V(y(t)) \leq \lambda_i \|y\|^2 \)

(b) \( \sum_{i=1}^n \eta_1 (y_{i+1} - g_i (y_i)) - \frac{\partial V}{\partial y_{i+1}} g_{i+1} (y_i) \leq -W(y(t)) \)

(c) \( \left| \frac{\partial V}{\partial x_{i+1}} \right| \leq \beta \|y(t)\|^2 \)

where \( y = (y_1, y_2, \ldots, y_{n+1}) \); \( \|y\| \) refers to the Euclidean norm in \( \mathbb{R}^{n+1} \).

Considering the newly added system state \( x_{n+1} = f + \omega \), the system can be expressed as

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
&\quad \vdots \\
\dot{x}_n(t) &= x_{n+1}(t) + u(t) \\
\dot{x}_{n+1}(t) &= L(t) \\
y(t) &= x_1(t)
\end{align*}
\]

where \( L(t) = f(t, x_1(t), x_2(t), \ldots, x_n(t)) + \omega(t) \).

Therefore,

\[ \Delta(t) = \frac{d}{ds} f(s, x_1(s), \ldots, x_n(s))|_{s=t} + \dot{\omega} = \frac{\partial}{\partial t} (f(\epsilon t, x_1(\epsilon t), \ldots, x_n(\epsilon t))) + \sum_{i=1}^n x_{i+1}(\epsilon t) \frac{\partial}{\partial x_i} f(\epsilon t, x_1(\epsilon t), \ldots, x_n(\epsilon t)) + u(\epsilon t) \frac{\partial}{\partial x_n} f(\epsilon t, x_1(\epsilon t), \ldots, x_n(\epsilon t)) + \dot{\omega}(\epsilon t) \]

From assumptions (a)-(b), it can be seen that there is \( M > 0 \) which makes \( |\Delta(t)| \leq M \) hold in \( [0, \infty) \).

To make

\[ e_i(t) = x_i(t) - \dot{x}_i(t) \quad \eta_i(t) = \frac{e_i(\epsilon t)}{\epsilon^{n+1-t}} \]

where \( i = 1,2,\ldots,n \).

By derivation, \( \eta = (\eta_1, \eta_2, \ldots, \eta_n)^T \) can be
obtained to satisfy the following error differential equation
\[
\begin{aligned}
\eta_1 &= \eta_2 - g_1(\eta_1), \quad \eta_1(0) = e_1(0) / e^{\eta_1} \\
\eta_2 &= \eta_3 - g_2(\eta_1), \quad \eta_2(0) = e_2(0) / e^{\eta_2} \\
&\vdots \\
\eta_n &= \eta_{n+1} - g_n(\eta_1), \quad \eta_n(0) = e_n(0) / e^{\eta_n} \\
\dot{\eta}_{n+1} &= -g_n(\eta_1) + \Delta(t) \quad \eta_{n+1}(0) = e_{n+1}(0)
\end{aligned}
\]
(25)

According to assumption (c), and considering the derivative of \( V(\eta(t)) \) over \( t \), Eq. (26) can be obtained
\[
\frac{d}{dt} V(\eta(t)) = \sum_{i=1}^{n} \frac{\partial V}{\partial \eta_i} (\eta_i - g_i(\eta_i)) - \frac{\partial V}{\partial \eta_{n+1}} g_{n+1}(\eta_1) + \frac{\partial V}{\partial \eta_{n+1}} \Delta(t) \leq -W(\eta) + eM \beta \| \| - \frac{\lambda_1}{\lambda_2} \sqrt{V(\eta)} + \frac{\sqrt{\lambda_2} eM \beta}{2\lambda_2} \sqrt{V(\eta)}
\]
(26)

This means
\[
\frac{d}{dt} V(\eta(t)) \leq -\frac{\lambda_1}{2\lambda_2} \sqrt{V(\eta(t))} + \frac{\sqrt{\lambda_2} eM \beta}{2\lambda_2} \sqrt{V(\eta)}
\]
(27)

According to assumption (c), Eq. (28) can be obtained
\[
\| \| \leq \sqrt{V(\eta(t))} = \frac{\sqrt{\lambda_2} V(\eta(t))}{\lambda_1} \exp \left( -\frac{\lambda_2}{2\lambda_2} t \right) + \frac{eM \beta}{2\lambda_2} \int_0^t \exp \left( -\frac{\lambda_2}{2\lambda_2} (t-s) \right) ds
\]
(28)

By combining Eq. (28) with Eq. (25), Eq. (29) can be obtained
\[
\| e(0) \| = \| e^{\eta+1} \| \leq \| e^{\eta+1} \| \leq \| e^{\eta+1} \| \leq \| e^{\eta+1} \| \leq \| e^{\eta+1} \| \leq \| e^{\eta+1} \| \leq \| e^{\eta+1} \| \leq \| e^{\eta+1} \|
\]
(29)

It can be seen from the above equations that when \( \varepsilon \to 0 \), it holds in the interval \([0, \infty)\); that is, the linear ESO converges.

### 3.4 Stability analysis of controller

For the second-order system based on the reduced-order ESO, the system can be expressed as follows
\[
\begin{aligned}
\dot{x}_1 &= -a_1 x_1 - b_1 x_2 - 1 z_1 + \beta_{01} \omega \\
\dot{z}_1 &= 0 - b_1 x_2 + z_1 \\
\dot{z}_2 &= 0 - b_1 x_2 + z_1 \\
\omega &= -g(e)
\end{aligned}
\]
(30)

Let \( x_1 = -a_1 x_1 - z_2, x_2 = z_1 \), and \( z_1 = \xi / \beta_{02} \), Eq. (30) can then be expressed as follows
\[
\begin{aligned}
\dot{x}_1 &= -a_1 x_1 - b_1 x_2 + x_2 + \beta_{02} \xi \\
\dot{z}_2 &= -b_1 x_2 + z_1 \\
\xi &= u \\
\omega &= -g(y) \\
y &= x_1 / a_1 + x_2 + \beta_{02} / a_1 \xi
\end{aligned}
\]
(31)

The transfer function of Eq. (30) is
\[
f(s) = \frac{\beta_{01} s^2 + (a_1 \beta_{01} + b_1 \beta_{01} + \beta_{02}) s + b_1 \beta_{02}}{s^3 + (a_1 + b_1) s^2 + a_1 b_1 s}
\]
(32)

According to the Popov absolute stability frequency criterion, if two nonnegative real numbers \( \alpha \) and \( \beta \), which are not zero, are available, then \( T(s) = (2\alpha + \beta s) f(s) \) is a positive real function that satisfies the following: (i) \( T(s) \) has at least one negative real pole; (ii) when \( \alpha = 0 \) for any \( \epsilon > 0 \) and when \( g(y) = \epsilon y \), the system is asymptotically stable and absolutely stable, respectively.

When \( \alpha = 1/2 \rho \),
\[
T(s) = (1 + \beta s) \frac{\beta_{01} s^2 + (a_1 \beta_{01} + b_1 \beta_{01} + \beta_{02}) s + b_1 \beta_{02}}{s^3 + (a_1 + b_1) s^2 + a_1 b_1 s}
\]
(33)

Take \( \beta = 1 / a_1 \). When \( a_1 > 0 \) and \( b_1 > 0 \), \( T(s) \) is a positive real function with two negative real poles. Thus, the system is absolutely stable.

### 4 Experimental verification

To verify the feasibility and correctness of the ADRC-RESO, the position servo control system of PMSLM is tested experimentally. Tab. 1 provides the motor parameters used in the experiments.

In this paper, three methods are compared in terms of displacement and speed response performance: ADRC-APSO, ADRC-NRDOB, and ADRC-RESO. According to Tab. 1, \( a_1 = 0, a_2 = -3.25 \times 10^5 \), and \( b = 1.1 \times 10^4 \). The system sampling step size is 0.0001 s.
According to $\omega_0 = 800$ and $\omega_c = 3000$, the parameters of ADRC-RESO are $k_P = 9 \times 10^6$ and $k_D = 6 \times 10^3$, and the observer parameters are $l_1 = 1600$ and $l_2 = 6.4 \times 10^5$, respectively.

To verify the effectiveness of ADRC-RESO, an experimental PMSLM control platform based on the real-time simulation system AD5435 is built. The experimental platform comprises a communication interface, power source, driving board, grating encoder, real-time simulation system AD5435, and PMSLM. In the experiment, the frequency of the inverter is 10 kHz, and the position encoder is a grating encoder with a precision of 1 $\mu$m. All comparative experiments are carried out in the same experimental environment. Fig. 6 shows the experimental control platform.

| Parameter                  | Value       |
|----------------------------|-------------|
| Number of pole pairs $p_n$ | 4           |
| Mover mass $m$/kg          | 2           |
| $q$-frame inductance $L_q$/H | 0.008       |
| $d$-frame inductance $L_d$/H | 0.008       |
| Mover winding resistance $R$/Ω | 8.4        |
| PM flux linkage $\Psi_f$/Wb | 0.178       |
| Pole pitch $r$/m           | 0.019       |
| Viscous friction coefficient $B_v$/(N m s/rad) | 0.001       |

ADRC-APSO is the worst, with the largest fluctuation, and the error of ADRC-NRDOB is small. The tracking effect of the method proposed in this paper is the best, and the error is the smallest. By comparing the phase current response results of the three methods, it can be seen that the response of ADRC-APSO is the worst, and the method of this paper has the best response.

### 4.2 Steady-state performance verification

ADRC-NRDOB, ADRC-APSO, and ADRC-RESO are used to compare the speed and displacement under load conditions. The movement distance is $S = 0.228$ m, and the load is 10 N. Fig. 10 depicts the experimental results, and Tab. 2 shows the steady-state errors and dynamic response time of the three methods.
Fig. 10 and Tab. 2 show that all three methods can track the input signal effectively. Under the same conditions, all three methods can be implemented quickly and accurately with high accuracy. However, there are slight fluctuations in the three methods after reaching a stable state. According to Fig. 10b, the steady-state errors of ADRC-APSO and ADRC-NRDOB are 45 $\mu$m and 30 $\mu$m, respectively. Compared with the above methods, the steady-state error of ADRC-RESO is 20 $\mu$m, which indicates that the positioning accuracy of ADRC-RESO is higher than that of the two other methods. The dynamic response time of ADRC-NRDOB is approximately 0.35 s, that of ADRC-APSO is 0.33 s, and that of ADRC-RESO is approximately 0.50 s. Owing to the noise reduction disturbance observer, ADRC-NRDOB can accelerate the response speed, so the dynamic response time is smaller than that of ADRC-RESO; ADRC-RESO can improve the accuracy only and cannot accelerate the response speed.

As shown in Fig. 10a, in the same case, ADRC-NRDOB can respond at the fastest speed, but the method has overshoot when the speed becomes zero. Although the response speed of ADRC-RESO is slower than the other two methods, it can reach steady state at a smooth speed. The response speed of ADRC-APSO is similar to that of ADRC-NRDOB, but it is slightly slower than ADRC-NRDOB. However, by using these three methods, the motor will have perturbation when the system reaches stability. This is because the three methods can resist certain disturbances, but they cannot eliminate the influence of high-frequency harmonics, which cause motor chatter.

In terms of the displacement and speed response results of the three methods, the method proposed in this paper has smaller fluctuation, more uniform fluctuation, and smaller steady-state error.

### 4.3 Impact of load disturbance on system robustness

To verify the anti-interference ability of the controller designed in this paper further, the load is suddenly increased from 0 N to 10 N during the positioning process. Fig. 11 displays the experimental results and the dynamic response time of the three methods under the load disturbance shown in Tab. 3.
Tab. 3  Dynamic response times of three methods under load disturbance

| Method      | Dynamic response time/s |
|-------------|-------------------------|
| ADRC-APSO   | 0.47                    |
| ADRC-NRDOB  | 0.43                    |
| ADRC-RESO   | 0.55                    |

Fig. 11 and Tab. 3 show that although the displacement curve does not substantially change, the speed of the motor changes abruptly. These sudden changes in the speed of ADRC-APSO and ADRC-NRDOB are more obvious than that of ADRC-RESO, which implies that the anti-disturbance effect of ADRC-RESO is better. The reason for this effect is that as the order of the system decreases, the number of adjusting parameters and the sensitivity to noise decrease, and the controller obtains part of the system model through speed feedback and load observation and then partially compensates for the disturbance to the observer, which improves the anti-disturbance ability. Fig. 11b shows that ADRC-NRDOB and ADRC-APSO have poor ability to suppress sudden disturbances. The response time of ADRC-NRDOB is 0.43 s, and those of ADRC-RESO and ADRC-APSO are 0.55 s and 0.47 s, respectively. Fig. 11a shows that when disturbances are introduced into the system, ADRC-RESO can respond quickly, resist disturbances, and retain the original motion state. ADRC-NRDOB and ADRC-APSO are less resistant to sudden changes; they must spend more time responding, so their maximum speed is slower. It can be seen in Fig. 11c that when the load disturbance is introduced into the system, the disturbances observed in the three methods are approximately the same. However, the disturbance fluctuation observed in ADRC-APSO is the largest, the error of the disturbance is greater than the given value, and the disturbance observed in the method proposed in this paper is closest to the given value.

4.4  Speed estimation performance and positioning error in dynamic situation

To verify the tracking effect of the proposed method in the case of continuous motion, a velocity signal \(v(t) = 0.1\sin(20\pi t)\) is given, and an interference signal \(d(t) = 0.03\sin(\pi t)\) is injected at the same time. The experimental results are shown in Figs. 12-14, and the steady-state errors of the three methods in a dynamic situation are shown in Tab. 4.
It can be seen in Figs. 12-14 and Tab. 4 that when the system is injected with different frequency disturbances, ADRC-APSO has the worst speed tracking effect, ADRC-NRDOB can track the speed signal more accurately, and the method proposed in this paper can track the speed signal almost completely with only small deviation. At the same time, the positioning error of ADRC-APSO is large, approximately 30 μm, the average positioning error of ADRC-NRDOB is 20 μm, and the method used in this paper is approximately 15 μm. It can be observed that when the system introduces disturbances with different frequencies, the method used in this paper has better anti-interference performance, can accurately track signals, and has small errors.

5 Conclusions

Based on the mathematical model of PMSLM in the dq synchronous rotating coordinate system on a reduced-order ESO, the ADRC algorithm is used to reject disturbances. The research results and conclusions are as follows.

(1) The first-order ADRC has the additional disturbance of current tracking error. Hence, a second-order speed ADRC is constructed. Based on the conventional ADRC, the linearization of the ESO is used to expand the observer’s ability to estimate disturbances. The order of linear ESO is reduced to reduce the error, which is caused by phase lag. The control effect is then enhanced. The parameters of the controller are determined according to the bandwidth method.

(2) By comparing the positioning performance under steady-state conditions, it can be verified that the ADRC-RESO has smaller steady-state error than ADRC-NRDOB and ADRC-APSO. By introducing load disturbance, it can be proved that ADRC-RESO can improve not only the control accuracy of the linear motor but also the anti-disturbance ability of the system.

(3) When the motor is in a dynamic running state, by injecting an intentional disturbance with a different frequency, the speed tracking results and positioning errors of the three methods are compared to verify that ADRC-RESO has high speed tracking accuracy, small positioning errors, and strong anti-interference ability.

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