LSND, solar and atmospheric neutrino oscillation experiments
and \( R \)-parity violating Supersymmetry

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Abstract

With only three flavors it is possible to account for various neutrino oscillation experiments. The masses and mixing angles for three neutrinos can be determined from the available experimental data on neutrino oscillation and from the astrophysical arguments. We have shown here that such masses and mixing angles which can explain atmospheric neutrino anomaly, LSND result and the solar neutrino experimental data, can be reconciled with the \( R \)-parity violating Supersymmetric Models through lepton number violating interactions. We have estimated the order of magnitude for some lepton number violating couplings. Our analysis indicates that the lepton number violation is likely to be observed in near future experiments. From the data on neutrino oscillation and the electric dipole moment of electron, under some

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circumstances it is possible to obtain constraint on the complex phase of some
supersymmetry breaking parameters in $R$-parity violating Supersymmetric
models.

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Although in the Standard Model of electroweak and strong interactions the neutrinos are massless to all orders in perturbation theory, in its extension, the neutrinos may acquire small masses with see-saw type mechanism in presence of sterile neutrinos. Also such masses can be present in the minimal supersymmetric model with the renormalizable lepton number violating terms in the lagrangian. On the other hand, the astrophysical and cosmological considerations also strongly suggest the existence of massive neutrinos. Presently, there are some possible evidences \[1\] of massive neutrinos and the mixing of different flavor of neutrinos particularly coming from the anomalies observed in the solar neutrino flux \[2\], in the atmospheric neutrino production \[3,4\] and in the neutrino beams from accelerators and reactors \[3\]. Although some of the evidences like those coming from solar neutrinos and accelerator data has been explained \[3\] considering one massive and two nearly massless neutrinos but it is in general difficult to fit various neutrino data considering three neutrinos as particularly the first three evidences are best fitted by three different mass gaps for neutrinos. However, the conventional approach to analyse various observed neutrino anomalies in the experiments is to parametrise those in terms of oscillation of two neutrino states only. This assumption may not hold good while fitting several observed anomalies simultaneously and consistent three flavor mixing scheme \[7\] for three neutrinos to analyse various data is essential. Several authors \[8–10\] have tried to fit various experimental data on neutrino oscillation in the three flavor mixing scheme. It is interesting to note that including the recent CHOOZ \[11\] and the SuperKamiokande \[3\] result on neutrino oscillation alongwith other experiments in this direction, it is possible to find the mass square differences and the mixing angles for three neutrinos almost uniquely \[9,10\]. Furthermore, analysis in three flavor mixing scheme indicates sizeable oscillations of electron neutrinos to tau neutrinos that should be observed by the long baseline neutrino experiments such as those utilizing a muon storage ring at Fermilab \[12\]. These analysis \[8,10\] also indicate that the solar neutrinos observed on earth should show no MSW effect \[1\] as the large mass squared differences
has been considered in those analysis. Precise measurement of the multi-GeV, ‘overhead’ 
(cos $\theta_z \sim 1$) events at SuperKamiokande will also be able to verify the three flavor mixing 
scheme [9,10] as the double ratio $R = (N_\mu/N_e)_{\text{measured}}/(N_\mu/N_e)_{\text{no oscillation}}$
for electron and muon for those events is somewhat less than 1 in the three flavor mixing scheme but this 
ratio is 1 in the analysis with a single oscillation process with small mass square differences
for neutrinos. However present SuperKamiokande data are inconclusive in this low $L/E$
region. Three flavor mixing schemes [9,10] give very good fit to the SuperKamiokande data
for the double ratio for upward going events (cos $\theta_z < -0.6$) but do not give very good
fit to the data on individual ratio for electron and muon. However, the double ratios are less
sensitive to systematic errors than the individual ratios. In these analysis [9,10] the LSND
result has been considered as an oscillation effect rather than an unexplained background.
In near future the BooNE experiment [13] will test the same channel of neutrino oscillation
as LSND with higher sensitivity and statistics. Particularly the solutions for the mass square
differences and the mixing angles in the three flavor mixing scheme as obtained in reference
[9] are not significantly contradicted by any existing experimental result and the conflicting
evidences are below two sigma level. Future various experiments on neutrino oscillation and
some of those experiments with higher statistics and lesser systematic errors will be able to
verify the three flavor mixing scheme [9,10] and it will be certain whether we really need a
fourth sterile neutrino [14]. At present, we feel that three flavor mixing scheme for neutrinos
are very interesting as it has some specific predictions as mentioned before which can be
verified by experiments and it tells about the mass squared differences and the mixing angles
almost uniquely.

The uniqueness of the mass square differences and the mixing angles [9,10] for three neu-
trinos may have strong impact on physics beyond standard model in the way those constrain
the parameters of other theories. We like to study such impact on the minimal $R$-parity vo-
lating Supersymmetric model where neutrinos can acquire mass. In supersymmetric models,
$R$-parity was introduced as a matter of convenience to prevent fast proton decay. It is now
realised that the proton lifetime can be made consistent with experiment without invoking
discrete $R$-parity symmetry. If we do not impose conservation of $R$-parity in the model, the minimal supersymmetric standard model allows the following $B$ and $L$ violating terms in the superpotenial

$$W = \lambda_{ijk} L^i L^j (E^k)^c + \lambda'_{ijk} L^i Q^j (D^k)^c + \lambda''_{ijk} (U^i)^c (D^j)^c (D^k)^c$$

(1.1)

Here $L$ and $Q$ are the lepton and quark doublet superfields, $E^c$ is the lepton singlet superfield, and $U^c$ and $D^c$ are the quark singlet superfields and $i, j, k$ are the generation indices. In the above, the first two terms are lepton number violating while the third term violates baryon number. For the stability of the proton, we assume that only the $L$ - violating first two terms in the superpotential is non-zero. One may consider some $Z_n$ symmetry to remove $B$ -violating term in the superpotential. As discussed later, $L$ -violating couplings give rise to masses for Majorana neutrinos through one loop diagrams as shown in figure 1, which lead to neutrino oscillation phenomena. In this work we like to show that in the $R$ - parity violating Supersymmetric models, it is possible to obtain the required mass squared differences and the mixing angles for such massive neutrinos to explain LSND, solar and atmospheric neutrino oscillation experiments. In our analysis, it is possible to satisfy the bound on the effective mass for the Majorana neutrinos obtained from the neutrinoless double beta decay experiment. We have estimated the magnitude of some of the lepton number violating couplings $\lambda_{ijk}$ and $\lambda'_{ijk}$ which is required to obtain the appropriate mass square differences and the mixing angles for neutrino oscillation. This kind of study was made earlier in the two flavor mixing scheme with the lesser available neutrino data.

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1 One may consider another term $\mu_\alpha L^\alpha H_2$ in the superpotential. However in general this lepton number violating term can be rotated to the first two terms in the superpotential in (1.1) unless a symmetry of $W$ does not commute with the $SU(4)$ symmetry of $L^\alpha$ rotations in the field space.

2 See for other alternative approaches to forbid dimension four as well as dimension five $B$ violating operators but keeping $L$ violating operators in the Lagrangian.
Very recently some other studies [17] also has been made to analyse solar and atmospheric neutrino data in the context of Supersymmetric Models. However in our work, unlike other works, we have considered solar, atmospheric neutrino oscillation experiments as well as LSND data to reconcile with $R$ parity violating Supersymmetric Model. We have also discussed the case for which neutrinos may be considered as dark matter candidate.

There are stringent bounds on different $\lambda_{ijk}$ and $\lambda'_{ijk}$ [18] from low energy processes [19] and very recently the product of two of such couplings has been constrained significantly from the neutrinoless double beta decay [21] and from rare leptonic decays of the long-lived neutral kaon, the muon and the tau as well as from the mixing of neutral $K$ and $B$ meson [22]. In most cases it is found that the upper bound on $\lambda'_{ijk}$ and $\lambda_{ijk}$ varies from $10^{-1}$ to $10^{-2}$ for the sfermion mass of order 100 GeV. For higher sfermion masses these values are even higher. In our analysis, it seems that for sfermion mass of the order of 100 GeV various $L$ violating couplings are less than $10^{-2}$. Considering the values of $L$-violating $\lambda'$ couplings as obtained from our analysis and also considering the constraint obtained from the electric dipole moment of electron it is possible to obtain constraint on the complex phase of some Supersymmetry parameters. In section II, we briefly discuss the constraints on masses and mixing for three neutrinos obtained from LSND, solar and atmospheric neutrino oscillation experiments. In section III, we discuss about masses of three neutrinos in $R$ violating Supersymmetric Model and show how it is possible to reconcile the masses and the mixing angles as obtained in the three flavor mixing scheme with the $R$ violating Supersymmetric Model. We present the required values of some $L$ violating couplings which satisfy particularly various neutrino oscillation experimental data and also satisfy the constraint on the effective mass for Majorana neutrinos in the neutrinoless double beta decay experiment. We compare these values with the earlier constraint on such couplings. In section IV, we discuss that under some circumstances it is possible to get constraint on the complex phase of some Supersymmetry parameters like $A$ - parameter. In section V, as concluding remarks we mention the possible implications of the obtained values of $L$ violating couplings in collider physics and cosmology.
II. CONSTRAINT ON NEUTRINO MASSES AND MIXING

We first mention here the necessary parameters for the three flavor neutrino oscillation. After that following references [9,10] we shall consider some specific values for the the masses and mixing as solutions to satisfy various available experimental data. The neutrino flavor eigenstate are related to the mass eigenstate by

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i$$

(2.1)

where $U_{\alpha i}$ are the elements of a unitary mixing matrix $U$, $\nu_\alpha = \nu_{e,\mu,\tau}$ and $\nu_i = \nu_{1,2,3}$. According to the standard parametrization [23] of the unitary matrix

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\delta \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\delta & c_{12}c_{23} - s_{12}s_{23}s_{13}\delta & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\delta & -s_{12}s_{23} - c_{12}c_{23}s_{13}\delta & c_{23}c_{13} \end{pmatrix}.$$  

(2.2)

where $\delta = e^{i\delta_{13}}$ corresponds to the CP-violating phase (which will be neglected here) and $c$ and $s$ stand for sine and cosine of the associated angle placed as subscript. The non-diagonal neutrino mass matrix $M_\nu$ in the flavor basis is diagonalised by the unitary matrix $U_\nu$ as

$$U_\nu^T M_\nu U_\nu = D_\nu$$

(2.3)

where $D_\nu$ is the diagonal mass matrix with the real eigenvalues. In the three generation neutrino mixing scheme, there are two independent mass square differences. These may be considered as $\Delta_{21}$ and $\Delta_{32}$ where

$$\Delta_{ij} = |m_i^2 - m_j^2|$$

(2.4)

and $m_i$ and $m_j$ are the neutrino mass eigenvalues. From the solar neutrino deficit one may consider [9]

$$10^{-4} eV^2 \leq \Delta_{21} \leq 10^{-3} eV^2$$

(2.5)
in which the lower limit is obtained from SuperKamiokande data [3] and the upper limit 
is obtained from the CHOOZ experiment [11]. Keeping in mind both the atmospheric and 
LSND data, another mass square difference $\Delta_{32}$ can be considered as [9]

$$\Delta_{32} \approx 0.3eV^2$$ (2.6)
in which the lower limit from the Bugey reactor constraint [24] and the upper limit from the 
CDHSW [25] have also been considered. In the one mass square difference dominating the 
other, the three flavor mixing scheme greatly simplifies and one can write the probability of 
the observation of neutrino oscillation in LSND and SuperKamiokande in terms of $\Delta_{32}$ and 
the elements of $U$. For the solar neutrino experiment the sine squared terms containing 
two mass square differences in the expression for the survival probability of solar electron 
neutrino, can be averaged for the flight length and the energy of the neutrinos observed on 
earth. In this case the probability can be written in terms of the elements of $U$ only. As 
the probability of oscillation in LSND and Superkamiokande and the survival probability for 
solar electron neutrinos are provided by the experiments, one can solve for the three angles 
by which matrix $U$ in (2.2) is defined. The results obtained by Barenboim et al [9] show 
that four set of solutions for three angles are possible. However, two set of solutions can 
be discarded by considering the SuperKamikande zenith angle ($\cos \theta_z < -0.6$) behavior of 
atmospheric neutrino data for upward going events. The other two allowed set of solutions 
for the three angles as obtained in reference [9] are

$$\theta_{12} = 54.5; \quad \theta_{13} = 13.1; \quad \theta_{23} = 27.3$$ (2.7)

$$\theta_{12} = 35.5; \quad \theta_{13} = 13.1; \quad \theta_{23} = 27.3$$ (2.8)
The result obtained by Thun et al [10] to satisfy solar, atmospheric and LSND data matches 
almost with the second set of solutions for three angles as mentioned in (2.8). In our analysis 
we shall consider either (2.7) or (2.8) for the three angles which specify the unitary matrix 
$U$ in (2.2).
The neutrino oscillation experiments give us information about the mass squared differences of three neutrinos in the three flavor mixing scheme as discussed above. However, to know the mass of different neutrinos we have to consider some other experiment. The masses are generated for Majorana neutrinos in $R$ violating Minimal Supersymmetric Model, so we have to consider the constraint coming from neutrinoless double beta decay. This gives us an estimate for the masses of neutrinos. The contribution of Majorana neutrinos to the amplitude of the neutrinoless double beta decay [32] is

$$|\mathcal{M}| = |\sum_{i=1,2,3} U_{ei}^2 m_{\nu_i}| < m(0\nu\beta\beta) = 0.68 \text{ eV}$$

To satisfy this constraint and keeping in mind that there are some uncertainties in the calculation of the nuclear matrix elements one may consider different masses of the Majorana neutrinos of the order of eV or less [29]. Considering (2.5) and (2.6) alongwith this constraint it is found that there are two interesting possibilities for the masses of neutrinos. In one case, all three neutrinos have almost degenerate mass and we may consider

$$m_2 \approx 1\text{eV}$$  \hspace{1cm} (2.9)

then the masses for other two neutrinos are

$$m_1 \approx 1\text{eV}; \quad m_3 \approx 1.14\text{eV}$$  \hspace{1cm} (2.10)

In another case, the masses of two neutrinos are nearly degenerate whereas the third one is heavier and we may consider

$$m_2 \approx 3 \times 10^{-2}\text{eV}$$  \hspace{1cm} (2.11)

then the masses for other two neutrinos are

$$m_1 \approx 2 \times 10^{-2}\text{eV}; \quad m_3 \approx 0.55\text{eV}$$  \hspace{1cm} (2.12)

One may consider the neutrinos as candidate for the dark matter solutions also. In that case, if one assumes $\Omega = 1$ and the energy density of the neutrinos $\rho_\nu = 0.2 \rho_c$ where $\rho_c$ is the
critical density in the Big-Bang Model, it is desirable to have the sum of the neutrino masses around 5 eV and one may consider the nearly degenerate three masses of neutrinos given by (2.9) and (2.10).

In our analysis we shall consider the above mentioned four interesting possible solutions for masses and mixing angles - one set of solutions from (2.7), (2.9) and (2.10), one set of solutions from (2.8), (2.9) and (2.10), one set from (2.7), (2.11) and (2.12) and the other set from (2.8), (2.11) and (2.12). We shall discuss in the next section how all these solutions can be reconciled with $R$ parity violating Supersymmetric Model.

III. NEUTRINO MASS MATRIX IN $R$-VIOLATING SUPERSYMMETRIC MODEL AND CONSTRAINT ON $L$ VIOLATING COUPLINGS

The trilinear lepton number violating renormalizable term in the superpotential in (1.1) generates Majorana neutrino masses through the generic one loop diagram as shown in figure 1 in which $s$ and $\tilde{s}$ stand for either lepton and slepton or quark and squark respectively. The helicity flip on the internal fermion line is necessary and that requires the mixing of $\tilde{s}$ and $\tilde{s}^c$. The contribution to the mass insertion as shown in figure 1, is proportional to the mass $m_k$ of the fermionic superpartner $s_k$ of $\tilde{s}_k$ and is also proportional to $\tilde{m}$ ($\sim A \sim \mu$, where $A$ and $\mu$ are the susy--breaking mass parameter) . The single diagram in figure 1 that contributes to the Majorana neutrino mass matrix $m_{\nu_i,\nu_j}$ is

$$m_{\nu_i,\nu_j} \approx \frac{\lambda_{jkn} \lambda_{ink} m_n m_k \tilde{m}}{16\pi^2 \tilde{m}_k^2}$$

(3.1)

when one considers the lepton and slepton for $s$ and $\tilde{s}$ in the diagram in figure 1. Both the diagrams in (a) and (b) are to be considered together and summed to evaluate the neutrino mass matrix element. However for $i = j$ and $k = n$, the two diagrams coincide and for that only one is to be considered. For quark and squark in the diagram the similar contribution will be obtained. However, in that case, the above contribution is to be multiplied by a color factor 3 and $\tilde{m}_k$ in the above equation is to be considered as the squark mass instead of slepton mass and the $\lambda$ couplings in (3.1) is to be replaced by $\lambda'$ couplings.
In constructing neutrino mass matrix we shall consider the following things. Firstly we shall relate squark and slepton mass as

$$\tilde{m}_{\text{slepton}}^2 = \tilde{m}_{\text{squark}}^2 / K$$  \hspace{1cm} (3.2)$$

where $K$ is a number depending on the various choices of Supersymmetry parameters. Different squarks have almost degenerate mass and different sleptons also have almost degenerate mass as otherwise there is severe constraint from the flavor changing neutral current. There are $9 \lambda$ couplings and $27 \lambda'$ couplings entering the neutrino mass matrix. However, in writing each element of the neutrino mass matrix we shall consider only the leading term in terms of the magnitude of mass obtained from (3.1). We shall consider the diagram with lepton and slepton in figure 1 and shall also consider the diagram with squark and quark in figure 1 in each element of the neutrino mass matrix for which two different types of $L$ violating couplings appear in each element. Under this consideration only the following $L$ violating couplings appear in the neutrino mass matrix. These are $\lambda'_{133}, \lambda'_{233}, \lambda'_{333}, \lambda_{133}, \lambda_{233}, \lambda_{232}, \lambda_{132}$ couplings. The notations for the first five couplings in later discussion will be $\lambda_q^e$, $\lambda_q^\mu$, $\lambda_q^\tau$, $\lambda_l^e$, $\lambda_l^\mu$ respectively. We are ignoring the effect of other couplings in our analysis and we are assuming that the $\lambda$ couplings are not much hierarchical among themselves and $\lambda'$ couplings are also not much hierarchical among themselves. At the end of this section we shall make a few qualitative comments about considering other $L$ violating couplings in the mass matrix.

We write the neutrino mass matrix as

$$N = a \begin{pmatrix}
K m_r^2 \lambda_e^l \lambda_e^l + 3 m_b^2 \lambda_q^q & 2 K m_r^2 \lambda_e^l \lambda_q^q + 6 m_b^2 \lambda_q^q \lambda_q^q & -2 K m_r m_\tau \lambda_q^q \lambda_{132} + 6 m_b^2 \lambda_q^q \lambda_q^q \\
2 K m_r^2 \lambda_e^l \lambda_e^l + 6 m_b^2 \lambda_q^q \lambda_q^q & K m_r^2 \lambda_e^l \lambda_e^l + 3 m_b^2 \lambda_q^q \lambda_q^q & -2 K m_r m_\mu \lambda_e^l \lambda_{232} + 6 m_b^2 \lambda_q^q \lambda_q^q \\
-2 K m_r m_\tau \lambda_q^q \lambda_{132} + 6 m_b^2 \lambda_q^q \lambda_q^q & -2 K m_r m_\mu \lambda_q^q \lambda_{232} + 6 m_b^2 \lambda_q^q \lambda_q^q & K m_r^2 \lambda_{132}^2 + 3 m_b^2 \lambda_q^q \lambda_q^q
\end{pmatrix}$$

\hspace{1cm} (3.3)$$

where

$$a = \frac{\tilde{m}}{16 \pi^2 \tilde{m}_s^2}$$  \hspace{1cm} (3.4)$$
and $\tilde{m}_s$ is the almost degenerate squark mass. The eigenvalues for this matrix correspond to three masses $m_1$, $m_2$ and $m_3$ for three Majorana neutrinos. We can write the diagonal mass matrix as

$$D_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

(3.5)

All the elements of this diagonal mass matrix can be written by considering particular set of solutions for the masses from the earlier section. The unitary matrix $U_\nu$ in (2.2) diagonalising the non-diagonal neutrino mass matrix in the flavor basis is also known to us if we consider particular set of solutions for the three angles from the earlier section. As both $D_\nu$ and $U_\nu$ are known we can obtain the non-diagonal mass matrix $M_\nu$ in the flavor basis using the relation

$$U_\nu D_\nu U_\nu^T = M_\nu.$$ 

(3.6)

So for particular set of solutions for the masses and the mixing angles discussed in the earlier section, all the elements of $M_\nu$ are known. However, this $M_\nu$ is equal to $N$ which is also the non-diagonal mass matrix expressed in terms of different $L$ violating couplings. So we write

$$N = M_\nu$$

(3.7)

From (3.7) we get six equations for the $L$ violating couplings :

$$K m_\tau^2 \lambda_{le}^2 + 3m_b^2 \lambda_{le}^2 = M_\nu(1, 1)/a$$

(3.8)

$$2K m_\tau^2 \lambda_{le}^l \lambda_{\mu}^l + 6m_b^2 \lambda_{le}^l \lambda_{\mu}^l = M_\nu(1, 2)/a$$

(3.9)

$$-2K m_\mu m_\tau \lambda_{l}^l \lambda_{132}^l + 6m_b^2 \lambda_{le}^l \lambda_{\mu}^l = M_\nu(1, 3)/a$$

(3.10)

$$K m_\tau^2 \lambda_{\mu}^2 + 3m_b^2 \lambda_{\mu}^2 = M_\nu(2, 2)/a$$

(3.11)
\[ -2K m_{\tau} m_{\mu} \lambda_{232}^\ell + 6m_\mu^2 \lambda_\mu^q \lambda_\tau^q = M_\nu(2,3)/a \]  
(3.12)

\[ K m_{\mu}^2 \lambda_{232}^2 + 3m_\mu^2 \lambda_{\tau}^q = M_\nu(3,3)/a \]  
(3.13)

after comparing the elements (1,1), (1,2), (1,3), (2,2), (2,3) and (3,3) respectively. However, there are seven \( L \) violating couplings involved in these six equations. So we shall consider the possible value of one of the \( L \) violating couplings from some other experiment instead of neutrino oscillation experiments for solving above six equations to find six \( L \) violating couplings. We shall consider particularly some value of \( \lambda_{232} \) lower than 0.0068 which is allowed after considering the constraint from lepton universality [18,19]. From these six equations we can determine the values of six \( L \) violating couplings for which it is possible to reconcile LSND, solar and atomic neutrino oscillation experimental data with the \( R \) parity violating Supersymmetric Model.

To determine \( M_\nu \) we first consider (2.7), (2.9) and (2.10) for the masses and the mixing angles. From (2.9) and (2.10) we get a specific \( D_\nu \) in (3.5), and from (2.7) we get a specific \( U_\nu \) in (2.2). Using relation (3.6) we obtain the following form of \( M_\nu \) (Here and in later discussions to obtain \( M_\nu \) we shall consider \( m_1, m_2 \) and \( m_3 \) in (3.5) in eV unit):

\[
M_\nu = \begin{pmatrix}
1.00712 & 0.0143034 & 0.0274612 \\
0.0143034 & 1.02782 & 0.0542459 \\
0.0274612 & 0.0542459 & 1.10499
\end{pmatrix}
\]  
(3.14)

We take \( m_b = 4.3 \times 10^9 \text{ eV} \), \( m_\tau = 1.777 \times 10^9 \text{ eV} \) and \( m_{\mu} = 0.105658 \times 10^9 \text{ eV} \) and solve (3.8)-(3.13) after considering a specific \( M_\nu \) in (3.14). For various allowed real values of \( \lambda_{232} \) lower than that mentioned earlier, the solution for other six \( L \) violating couplings do not change by an order. We present below the the values of these couplings considering \( \lambda_{232} \) in the range \((10^{-6} - 10^{-3}) \frac{\tilde{m}_s}{\sqrt{\tilde{m}}} \) (Here and in later discussions \( \tilde{m}_s \) and \( \tilde{m} \) stand for the corresponding magnitude in GeV unit):

\[
\lambda_\ell^q \approx \frac{5.3 \times 10^{-5} \tilde{m}_s}{\sqrt{\tilde{m}}} , \quad \lambda_\mu^q \approx \frac{1.3 \times 10^{-6} \tilde{m}_s}{\sqrt{\tilde{m}}} , \quad \lambda_\tau^q \approx \frac{5.6 \times 10^{-5} \tilde{m}_s}{\sqrt{\tilde{m}}} \]  
(3.15)
\begin{align}
\lambda_\ell^l & \approx \frac{7.3 \times 10^{-6} \tilde{m}_s}{\sqrt{K \tilde{m}}}, & \lambda_\mu^l & \approx \frac{2.3 \times 10^{-4} \tilde{m}_s}{\sqrt{K \tilde{m}}}, & \lambda_{132} & \approx \frac{4.0 \times 10^{-3} \tilde{m}_s}{\sqrt{K \tilde{m}}} \tag{3.16} \\
\end{align}

Another set of real solutions for various $L$ violating couplings for the above case is given below:

\begin{align}
\lambda_\ell^q & \approx \frac{5.3 \times 10^{-5} \tilde{m}_s}{\sqrt{m}}, & \lambda_\mu^q & \approx \frac{1.3 \times 10^{-6} \tilde{m}_s}{\sqrt{m}}, & \lambda_\tau^q & \approx \frac{5.6 \times 10^{-5} \tilde{m}_s}{\sqrt{m}} \tag{3.17} \\
\lambda_\ell^l & \approx \frac{4.2 \times 10^{-6} \tilde{m}_s}{\sqrt{K \tilde{m}}}, & \lambda_\mu^l & \approx \frac{2.3 \times 10^{-4} \tilde{m}_s}{\sqrt{K \tilde{m}}}, & \lambda_{132} & \approx \frac{3.9 \times 10^{-3} \tilde{m}_s}{\sqrt{K \tilde{m}}} \tag{3.18} \\
\end{align}

We have ignored overall + or - sign for the solutions (here and in later cases also) for different $L$ violating couplings. Although there are different set of solutions possible but if we ignore the small changes in the higher decimal places for different solutions then mainly the two set of solutions are found to differ to some extent from each other particularly for the value of $\lambda_\ell^l$ and $\lambda_{132}$ and those two sets of solutions are presented above.

It is important to note here that there is almost no change of the $\lambda_{132}$ value for various real $\lambda_{232}$ value in the range $(0.0 - 10^{-3}) \frac{\tilde{m}_s}{\sqrt{K \tilde{m}}}$ and the two values of $\lambda_{132}$ in (3.16) and (3.18) are very near to the upper bound obtained from the experimental value of $R_\tau = \frac{\Gamma(\tau \rightarrow e\nu\bar{\nu})}{\Gamma(\tau \rightarrow \mu\nu\bar{\nu})}$ \cite{18,20}. This indicates that there is possibility to see the lepton universality violation in future experiments. Same comment is also true for $\lambda_{232}$ coupling as almost same real solutions for various $L$ violating couplings exist for the higher allowed value of $\lambda_{232}$ coupling also.

Of course the statement is based on the present neutrino oscillation data and considering neutrino as Majorana particle , the main contribution in the neutrino mass matrix coming from the earlier mentioned seven $L$ violating couplings and the $L$ violating couplings considered here being real. Although in obtaining the solutions for $L$ violating couplings we have considered here almost degenerate mass for three neutrinos , but such higher values of $\lambda_{132}$ or $\lambda_{232}$ are possible for hierarchical nature of the masses of neutrinos also, as can be seen in the later part of our analysis. If we consider complex or imaginary value of $\lambda_{232}$ coupling considering the experimental upper bound mentioned earlier it is possible to obtain complex solutions for other six $L$ violating couplings from (3.8)-(3.13) for $M_\nu$ in (3.14). For brevity,
we are not presenting those solutions of various $L$ violating couplings for this case of masses and mixing angles and for other cases also. However, at the end of this section we shall make a few general remarks on the complex solutions for these $L$ violating couplings. Existence of the possible solutions for $L$ violating couplings in (3.15) and (3.16) or (3.17) and (3.18) indicates that considering the almost degenerate mass neutrinos (which may be candidate for dark matter also) as mentioned in (2.9) and (2.10), it is possible to reconcile LSND, solar and atmospheric neutrino oscillation data with the $R$ parity violating Supersymmetric Model.

Next, we consider the other possible solutions for the mixing angles as stated in (2.8) and consider again the almost degenerate mass of neutrinos as mentioned in (2.9) and (2.10). In this case, we get the following form of $M_\nu$ after using (3.6):

$$M_\nu = \begin{pmatrix} 1.00704 & 0.0143116 & 0.0274772 \\ 0.0143116 & 1.02788 & 0.054211 \\ 0.0274772 & 0.054211 & 1.105 \end{pmatrix} \quad (3.19)$$

Like earlier case, we again solve equations (3.8)-(3.13) for specific $M_\nu$ in (3.19) and obtain the solutions for six $L$ violating couplings. All the solutions in this case are approximately same as (3.15)-(3.18). So the different set of choices for the mixing angles in (2.8) do not lead to significant change in the values of $L$ violating couplings.

We shall consider next the hierarchical neutrino masses as mentioned in (2.11) and (2.12). For the three mixing angles we consider (2.7). As before using (3.6) we obtain the following form of $M_\nu$:

$$M_\nu = \begin{pmatrix} 0.0534391 & 0.0569347 & 0.10027 \\ 0.0569347 & 0.127333 & 0.202493 \\ 0.10027 & 0.202493 & 0.417771 \end{pmatrix} \quad (3.20)$$

Solving (3.8)-(3.13) for specific $M_\nu$ in (3.20) we obtain the following real solutions for six $L$ violating couplings. We present below the values of these couplings considering $\lambda_{232}$ in the range $10^{-6} - 10^{-3} \frac{\tilde{m}}{\sqrt{m}}$. 

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Another set of real solutions for various $L$ violating couplings for the above case is given below:

$$
\begin{align*}
\lambda_e^q &\approx \frac{1.2 \times 10^{-5}\tilde{m}_s}{\sqrt{m}}, \\
\lambda_\mu^q &\approx \frac{8.3 \times 10^{-6}\tilde{m}_s}{\sqrt{m}}, \\
\lambda_\tau^q &\approx \frac{3.4 \times 10^{-5}\tilde{m}_s}{\sqrt{m}} \\
\lambda_e^l &\approx \frac{5.2 \times 10^{-6}\tilde{m}_s}{\sqrt{Km}}, \\
\lambda_\mu^l &\approx \frac{7.2 \times 10^{-5}\tilde{m}_s}{\sqrt{Km}}, \\
\lambda_{132} &\approx \frac{1.2 \times 10^{-3}\tilde{m}_s}{\sqrt{Km}}
\end{align*}
$$

(3.23)

So it is seen that considering hierarchical nature of the masses of neutrinos also it is possible to reconcile LSND, solar and atmospheric neutrino oscillation experiments with the $R$ violating Supersymmetric Model. In this case only thing to note here is that $\lambda_{132}$ is slightly lower than the earlier cases however not far from the present experimental bound obtained from the lepton universality violation [18–20].

Next we shall consider the hierarchical mass pattern of neutrinos like earlier case but consider the mixing angles as presented in (2.8). In that case, using (3.6) we get the following form of $M_\nu$:

$$
M_\nu =
\begin{pmatrix}
0.0503506 & 0.0572644 & 0.100908 \\
0.0572644 & 0.129869 & 0.201098 \\
0.100908 & 0.201098 & 0.418324
\end{pmatrix}
$$

(3.25)

Like before we solve (3.8)-(3.13) for $M_\nu$ in (3.25) and consider the same range for $\lambda_{232}$ like earlier cases. In this case, the solutions for $L$ violating couplings are almost same as before with hierarchical masses of neutrinos and we are not presenting those solutions seperately.

If the future neutrino oscillation experiments with higher sensitivity and more data support the three flavor mixing scheme as mentioned in section II and the $L$ violating couplings are real, it is expected that experiments on lepton universality violation in future
will find signal for the values of $\lambda_{132}$ or $\lambda_{232}$ couplings at the level required by our analysis. However, if no signals are found for those values of $L$ violating couplings—particularly $\lambda_{132}$ coupling then the explanation for that may be the following. In that case normally it will be expected that $\lambda_{132}$ coupling is very small. However then if one considers again those six equations (3.8)-(3.13) considering $\lambda_{132}$ as effectively zero it is found that for the various cases for masses and mixing angles, real values of $\lambda_{232}$ has to be always of the order of $10^{-3} \bar{m}_s \sqrt{\bar{K}/\bar{m}}$. However if the signal for $\lambda_{232}$ is also not seen through $\tau$-universality violation, it will be necessary to check the role of other couplings for the analysis of neutrino masses and mixing angles. As under this circumstance, $\lambda_{132}$ and $\lambda_{232}$ will be smaller, we may consider the terms next to the leading order in mass in the various elements of the neutrino mass matrix in (3.3). As the mass factor associated with those other non-leading contribution will be less in magnitude, it is expected that the magnitude of some other coupling should be somewhat higher like $\lambda_{132}$ to reproduce the similar forms of $M_\nu$ mentioned earlier and the lepton number violation, in that case, should be observed through that coupling. Depending on the results of future experiments on neutrino oscillation, tau universality violation etc., the analysis with other such couplings may be important.

We like to make a few remarks on the complex solutions for various $L$ violating couplings. Earlier in obtaining all the above-mentioned solutions for different $L$ - violating couplings we have considered value of $\lambda_{232}$ coupling in the range $(10^{-6} - 10^{-3}) \bar{m}_s / \sqrt{\bar{K}/\bar{m}}$. However, if one considers the value of $\lambda_{232}$ in the range $(4.0 - 6.0)10^{-3} \bar{m}_s / \sqrt{\bar{K}/\bar{m}}$ which is very near to the experimental upper bound [18], then from the equations (3.8)-(3.13) considering different form of $M_\nu$ as mentioned earlier, one will obtain the complex solutions for various $L$ violating couplings. Furthermore, if one considers imaginary or complex value of $\lambda_{232}$ in that case also one will obtain the complex solutions for various $L$ violating couplings. Depending on the various choices of the values of $\lambda_{232}$ one may obtain from (3.8)-(3.16) the various solutions for different $L$ violating couplings with various possible complex phases. For brevity, we have not presented various possible complex solutions. For some complex phases the solutions may not be allowed depending on constraint particularly from the value of the electric dipole
moment of electron. We have discussed this in the next section.

The solutions for all \( \lambda \) and \( \lambda' \) couplings are obtained in terms of the parameters \( \tilde{m} \), \( \tilde{m}_s \) and \( K \). Here \( \tilde{m} \) is the Supersymmetry breaking mass parameter which is expected to lie in the range of \( \text{O}(100 \text{ GeV}) \) to \( \text{O}(\text{TeV}) \). To compare our constraint on \( \lambda \) and \( \lambda' \) couplings with the earlier constraints \[18\] we shall consider \( K \approx 1 \) which means slepton mass does not differ much from the squark mass and also shall consider both squark mass and \( \tilde{m} \) of \( \text{O}(100 \text{ GeV}) \). We shall consider those solutions of \( L \) violating couplings for which complex phases are negligible. The earlier constraint from the tau universality violation \[19\] is \( \lambda_{132} \leq 0.06, \lambda_{232} \leq 0.06 \) and \( \lambda_{233} = \lambda_{l_\mu} \leq 0.06 \) which in our analysis from neutrino oscillation, are found to be \( \text{O}(0.01-0.04), \text{O}(0.01) \) and \( \text{O}(0.0007-0.0023) \) respectively. The earlier constraint from \( R_t = \frac{\Gamma_{\text{hadron}}(Z^0)}{\Gamma_l(Z^0)} \[20\] \( \lambda'_{233} = l^q_{\tau} \leq 0.26 \) and \( \lambda'_{233} = l^q_{\mu} \leq 0.39 \) which in our analysis are \( \text{O}(0.0003-0.0005) \) and \( \text{O}((1-8) \times 10^{-5}) \) respectively. The earlier constraint on \( \lambda'_{133} = \lambda_{e} \) obtained from the constraint from neutrino mass \[33\] is \( \lambda_q \approx 0.002 \) and \( \lambda_{133} = \lambda_{e} \approx 0.004 \) which in our analysis are \( \text{O}(0.00008-0.00053) \) and \( \text{O}(0.00004-0.00037) \) respectively. So our analysis indicates somewhat lower values of various \( L \) violating couplings than the upper bound on these couplings obtained from other experiments. Furthermore, if one considers \( \tilde{m} \) to be nearer to TeV region then these values of \( L \) violating couplings will be further lowered. The upper bounds on these couplings obtained from the experimental data on neutral currents, \( \beta \) decay \[18\], muon decay \( (\mu \rightarrow e\gamma, \mu \rightarrow \bar{e}ee) \[34\] \), or tau decay \( (\tau \rightarrow \mu\gamma, \tau \rightarrow e\gamma) \[36\] \) etc. are somewhat higher than the values required in our analysis.

\[3\] When we consider very small value of \( \lambda_{132} \) which can be neglected in the neutrino mass matrix in (3.3), in that case we get this solution for \( \lambda_{232} \) from the equations (3.8)-(3.13). Otherwise various solutions for \( \lambda_{232} \) are possible as mentioned earlier.
IV. CONSTRAINT ON COMPLEX PHASE OF SUPERSYMMETRY BREAKING PARAMETER

In the Standard Model the electric dipole moment of electron is much smaller than their present experimental bound \( d_e < 10^{-26} \text{ecm} \) \[37\]. So the new sources of CP violation which occurs in the supersymmetric model can be studied on the basis of electric dipole moment of electron \[38-40\]. In the minimal supersymmetric standard model apart from the Yukawa couplings there are several complex parameters like three gaugino masses corresponding to SU(3), SU(2) and U(1) groups, the mass parameter \( m_H \) in the bilinear term in the Higgs superfields in the superpotential , dimensionless parameters \( A \) and \( B \) in the trilinear and the bilinear terms of the scalar fields. With suitable redefinition of the fields, some of these parameters can be made real but in that case some others can not be made real like \( A \) parameter \[39\]. The complex \( A \) will contribute to the electric dipole moment (edm) of electron. Furthermore, if we consider the complex \( \lambda' \) couplings, the complex phase associated with those will also contribute to edm of electron. There will be various diagrams in the \( R \)-parity violating Supersymmetry for the edm of electron \[40\]. But the significant contribution to edm comes from the one loop diagram containing top quark in the loop as shown in Figure-2. There will be diagram containing massive neutrinos in the loop . However, the masses of neutrinos are quite small in our discussion and we are ignoring those types of diagrams for our discussion as there will be lesser contribution to the edm of electron. In terms of complex phases we can write \( A_f \) and \( \lambda'_{ijk} \) as

\[
A_{u,d} = |A| \exp(i\alpha_{A_{u,d}}), \quad \lambda'_{ijk} = |\lambda'_{ijk}| \exp(i\beta).
\]

(4.1)

and the mixing angle for the left and the right squark in the familiar way as

\[
\tan 2\theta = 2 |A_u| m_u/(\mu^2_L - \mu^2_R).
\]

(4.2)

Following reference \[40\] and assuming different \( \lambda'_{ijk} \) containing another complex phase as mentioned in (4.1) we can write the edm of electron from figure 2 as

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\[ d_e \approx -\sin \theta \cos \theta \left( (\cos^2 \beta - \sin^2 \beta) \sin \alpha_{A_d} + \cos \beta \sin \beta \cos \alpha_{A_d} \right) \]
\[ \times | \lambda'_{ijk} |^2 \frac{2 | A_d |}{3 \tilde{m}_s^2} m_{u_3} [F_1(x_k) + 2F_2(x_k)] \ 10^{-17} \text{ e cm} \] (4.3)

where \( x_k = (m_{d_k}/\tilde{m}_s)^2 \) and the loop integrals \( F_1 \) and \( F_2 \) are expressed in terms of \( x_k \) as
\[ F_1(x_k) = \frac{1}{2(1-x_k)^2} \left( 1 + x_k + \frac{2x_k \ln x_k}{1-x_k} \right), \]
\[ F_2(x_k) = \frac{1}{2(1-x_k)^2} \left( 3 - x_k + \frac{2 \ln x_k}{1-x_k} \right). \] (4.4)

\( A_d \) and \( \tilde{m}_s \) in (4.3) correspond to the magnitude of those quantities expressed in GeV. We are particularly interested for \( j = 3 \) and \( k = 3 \) case in (4.3). We got a solution for \( \lambda_q^e = \lambda'_{133} \) in section III to explain the neutrino physics data. So we like to constrain here particularly the complex phases associated with \( A \) in (4.3). Considering \( \lambda_q^e \) as real or complex and writing it as \( C\tilde{m}_s/\sqrt{\tilde{m}} \) in the form obtained in section III (where \( C \) is some value depending on the type of solutions), \( A \) and \( \tilde{m}_s \) both from 100 GeV to 1 TeV it is found that the constraint on the complex phase \( \alpha_{A_d} \) and \( \beta \) is
\[ \left( (\cos^2 \beta - \sin^2 \beta) \sin \alpha_{A_d} + \cos \beta \sin \beta \cos \alpha_{A_d} \right) \sin \theta \cos \theta \lesssim \frac{(2 - 16) \times 10^{-14}}{| C |^2} \] (4.5)

In the case for which there will be no contribution to edm in (4.3) is \( \beta = -\alpha_{A}/2 \). For the complex solutions of \( \lambda_q^e \) for which \( | C | \) is not less than about \( 10^{-6} \) we can make the following statements. For \( \beta \approx \pi/4 \) and \( \alpha_{A_d} \approx \pi/2 \) the above inequality can be satisfied for any value of \( \theta \). However such a large phase for \( A \) is not possible as the edm of electron will get contribution from other diagrams involving charginos and neutralinos at the one loop level cancelling this possibility. So \( \beta \approx \pi/4 \) is not possible for any value of \( \theta \). So those set of complex solutions for different \( L \) violating couplings should not be considered when the complex phase associated with \( \lambda_q^e \) is found to be approximately \( \pi/4 \) and \( | C | \) satisfies the above condition. Let us consider that \( \lambda'_{ijk} \) are real and \( \theta = \pi/4 \) in that case it is seen from section III that \( C \approx 10^{-5} \) and the complex phase for the \( A \) parameter \( \alpha_{A_d} < 3.2 \times 10^{-3} \).

Without any specific choice of the mixing angle \( \theta \) one can constrain only the combination of \( \beta, \alpha_A \) and \( \theta \) as shown in (4.5).
V. CONCLUSION

We have shown here that in the minimal supersymmetric model with $R$-parity violating trilinear term in the superpotential in (1.1) it is possible to obtain the appropriate mass square differences and the mixing angles as required to explain the LSND, atmospheric and solar neutrino oscillation experimental data in the three flavor mixing scheme for neutrinos. The validity of the three flavor mixing scheme can be verified in the near future experiments on neutrino oscillation as mentioned in the introduction. The masses for three Majorana neutrinos are generated at the one loop level as shown in Figure-1 and it is possible to satisfy the constraint on the masses and the mixing angles coming from the neutrinoless double beta decay. In each element of the neutrino mass matrix in (3.3) we have considered two leading terms in terms of the magnitude of masses in (3.1) coming from the diagram with slepton and lepton and also coming from the diagram with quark and squark in Figure-1. Under this consideration it is interesting to note that for real values of various $L$ violating couplings at least one of the couplings either $\lambda_{132}$ or $\lambda_{232}$ is expected to be quite high and very near to the experimental upper bound coming from the $\tau$ -universality violation $[18–20]$. Apart from these two particular couplings for some of the $L$ violating couplings the magnitude are such that it might be possible to observe such $L$ violating interaction at the Tevatron or at HERA. At the Tevatron after squark pair production those squarks will decay to LSP (say neutralino) and which will decay via $L^i L^j E^{kc}$ operator giving multilepton signal $[11]$. At HERA one can see $R$ violating Supersymmetry signal for $L^i Q^j D^{kc}$ operator $[12]$ through resonant squark production and its subsequent decay to electron or positron and neutrino giving the signal of high $p_T$ electron or high $P_T$ positron or missing $p_T$ for neutrino. The basic requirement for the observation of such signal is that LSP has to decay inside the detector and this puts bound $[11,12]$

$$\lambda, \lambda' \gtrsim 10^{-5} \left( \frac{m_{\tilde{t}}}{100 \text{ GeV}} \right)^2$$

where $m_{\tilde{t}, \tilde{\ell}}$ stand for the squark and the slepton mass. From the above condition it is seen that if we consider $\tilde{m}$ of the order of squark mass $\tilde{m}_s$ in that case for squark mass or
slepton mass of the order of 300 GeV, it may be possible to observe $L$ violating signal for those couplings discussed in section III for which $\lambda, \lambda' > 5 \times 10^{-6} \frac{\tilde{m}}{\sqrt{m}}$; whereas for squark or slepton mass of the order of TeV the condition is $\lambda, \lambda' > 3 \times 10^{-5} \frac{\tilde{m}}{\sqrt{m}}$. So for various couplings considered in our analysis in section III, it might be possible to observe $L$ violating signal.

If one considers the baryogenesis in the early universe at the GUT scale, after the generation of asymmetry to satisfy the out of equilibrium condition one requires $L$ violating couplings $\sim 10^{-7}$ for squark mass from 100 GeV to 1 TeV range [43] which is significantly smaller than the values of some of the couplings obtained in section III. So if one likes to satisfy the neutrino physics experimental data in the three flavor mixing scheme, it seems in the $R$ violating Supersymmetric scenario the generation of the baryonic asymmetry near the electroweak scale is more favored where the constraint on $L$ violating couplings are not so severe [44]. We have shown that in the $R$ violating Supersymmetric models neutrino can be considered as dark matter candidate also. Our analysis also indicates that to satisfy various experimental data on neutrino oscillation, the lepton number violating couplings are constrained in such a way that some combinations of left and right squark mixing angles and the complex phases of some Supersymmetry parameters - particularly that of $A$ parameter are constrained.

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FIG. 1. One loop diagram involving $L$-violating couplings generating neutrino mass.
FIG. 2. One loop diagram contributing to electric dipole moment of electron.