ISOTHERMAL DISTRIBUTIONS IN MONDian GRAVITY AS A SIMPLE UNIFYING EXPLANATION FOR THE UBQUITOUS $\rho \propto r^{-3}$ DENSITY PROFILES IN TENUOUS STELLAR HALOS

X. HERNÁNDEZ, M. A. JIMÉNEZ, AND C. ALLEN
Instituto de Astronomía, Universidad Nacional Autónoma de México, Apartado Postal 70-264, C.P. 04510 México D.F., Mexico; xavier@astro.unam.mx

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ABSTRACT
That the stellar halo of the Milky Way has a density profile which, to first approximation, satisfies $\rho \propto r^{-3}$ and has been known for a long time. More recently, it has become clear that M31 also has such an extended stellar halo, which approximately follows the same radial scaling. Studies of distant galaxies have revealed the same phenomenology. Also, we now know that the density profiles of the globular cluster systems of our Galaxy and Andromeda to first approximation follow $\rho \propto r^{-3}, \Sigma \propto R^{-2}$ in projection. Recently, diffuse populations of stars have been detected spherically surrounding a number of Galactic globular clusters, extending much beyond the Newtonian tidal radii, often without showing any evidence of tidal features. Within the standard Newtonian and general relativity scenario, numerous and diverse particular explanations have been suggested, individually tailored to each of the different classes of systems described above. Here we show that in a MONDian gravity scenario any isothermal tenuous halo of tracer particles forming a small perturbation surrounding a spherically symmetric mass distribution will have an equilibrium configuration which to first approximation satisfies a $\rho \propto r^{-3}$ scaling.

Key words: galaxies: kinematics and dynamics – galaxies: star clusters: general – gravitation – stars: kinematics and dynamics

1. INTRODUCTION

Progress in the debate between the hypothetical physical reality of astrophysical dark matter and the option of modifying gravity at the low acceleration regime will hinge upon the exploration of as many independent lines of enquiry as possible. Whereas the rotation curves of galaxies can be adequately reproduced from either point of view (Milgrom 1983, 1994; Sanders & McGaugh 2002; Swaters et al. 2010), a variety of recent studies have shown results in tension with the standard scale-invariant gravity plus dark matter paradigm, and being more in line with generic MONDian gravity approaches.

We shall use the term MONDian to refer to modified gravity theories in which, in the low velocity limit, the force per unit mass between a test particle and a spherical mass distribution will shift from the Newtonian expression of $GM/r^2$ to an $(GMa_0)^{1/2}/r$ behavior for acceleration scales below Milgrom’s $a_0$, independently of the fundamental underlying theory of gravity which might lead to such a behavior, e.g., Bekenstein (2004), Moffat & Toth (2008), Zhao & Famaey (2010), Bernal et al. (2011), Mendoza et al. (2011), Capozziello & De Laurentis (2011), and Famaey & McGaugh (2012). The most salient features of such schemes are equilibrium velocities which become independent of distances at a value $(GMa_0)^{1/4}$.

In Lee & Komatsu (2010) and Thompson & Nagamine (2012) it has been shown that the infall velocity of the two components of the Bullet cluster is incompatible with expectations of full $\Lambda$CDM predictions, and is actually a challenge to the standard gravity theory, as it surpasses the escape velocity of the combined system. Recently, Kroupa (2012) has shown that tidal dwarf galaxies, which, under the standard scenario, are transient tidal clumps of galactic material having out of equilibrium dynamics, actually fall on the same Tully–Fisher relation as all dwarf galaxies. This appears as an uncanny coincidence from the standard gravity perspective, where the dynamics of normal dwarfs are thought to be determined by their dominant dark matter halos. The result is expected under MONDian gravity schemes, where, once the baryonic mass of the system is fixed, the dynamics will uniquely follow, as it is indeed observed.

Along the same lines, in Haghi et al. (2011), Scarpa et al. (2011), Hernandez & Jiménez (2012), and Hernandez et al. (2013), it has been shown that the velocity dispersion profiles of a number of Galactic globular clusters stop falling radially along Keplerian expectations and settle to finite asymptotic values on crossing the $a < a_0$ threshold. The standard gravity explanation for these profiles, that it is the tides of the Milky Way (MW) that dynamically heat the outskirts of the clusters observed, e.g., Lane et al. (2010), appears suspect, as the Newtonian tidal radii can be shown to exceed the points where the velocity dispersion profiles flatten by large factors, and because the total globular cluster masses and asymptotic velocity dispersion values follow the galactic Tully–Fisher relation, as expected under MONDian gravity schemes. Finally, in Hernandez et al. (2012) we showed that the relative velocities of extremely wide binaries do not follow the expectations of full galactic dynamical Newtonian simulations, but diverge from the Keplerian decline with radius to settle also at finite relative velocities on crossing the $a < a_0$ threshold of MONDian gravity proposals.

In this paper we show that the density profile of an isothermal population of trace particles surrounding a spherical mass distribution in MONDian gravity will naturally follow an approximately $\rho \propto r^{-3}$ profile. Under standard gravity approaches, the ubiquitous nature of $\rho \propto r^{-3}$ profiles has to be addressed through a variety of highly specific explanations, individually tailored to each of the diverse classes of systems where these profiles have been observed. Examples of the above are mergers and tidal dissolution of accreted substructure for the tenuous stellar halos surrounding our Galaxy, M31, and also the recently detected ones around external galaxies (e.g., Bullock et al. 2001; Abadi et al. 2006), and the compression of tidal tails at apocenter or disk shock heating for the “extra tidal” features surrounding Galactic globular clusters, e.g., Da Costa (2012).

The above situation contrasts with the appearance of a direct equilibrium solution for tracer populations having the
simplest isotropic and isothermal Maxwellian distribution function, which, to first order, yields $\rho \propto r^{-3}$ profiles under MONDian gravity schemes, which we show here. We suggest that this is one further piece of evidence pointing in the direction of the necessity of a modified gravity regime at low acceleration scales.

In Section 2 we derive the first order predictions for equilibrium tenuous stellar halos under a MONDian gravity scheme, and also under Newtonian gravity within dark matter halos, and under Newtonian gravity in the absence of dark matter halos. In Section 3 we compare the theoretical estimates with the observed profiles for a variety of systems, finding the MONDian prediction a good fit to the observed situation, over a wide variety of scales and classes of astronomical systems. Section 4 presents a final discussion of our results.

2. FIRST ORDER DYNAMICAL EXPECTATIONS

We shall model the physical situation which applies generically for a MONDian scenario, where in the $a \ll a_0$ limit, the force between a test particle and a spherically symmetric mass distribution becomes $-(GM(r)a_0)^{1/2}/r$, when one introduces no modifications to Newton’s second law, e.g., Hernandez et al. (2010) and Mendoza et al. (2011).

Assuming spherical symmetry, and taking the derivative of the kinematic pressure, the equation of hydrostatic equilibrium for a polytropic equation of state $P = K\rho^\gamma$ is

$$\frac{d(K\rho^\gamma)}{dr} = -\rho \nabla \phi. \quad (1)$$

In going to isothermal conditions, $\gamma = 1$ and $K = \sigma^2$, e.g., Binney & Tremaine (1987), and we get

$$\sigma^2 \frac{d\rho}{\rho \, dr} = -\nabla \phi. \quad (2)$$

Writing $\rho = (4\pi r^2)^{-1}dM(r)/dr$, the above equation can be written as

$$\sigma^2 \left[ \frac{dM(r)}{dr} \right]^{-1} d^2M(r) - \frac{2}{r} = -\nabla \phi(r) \quad (3)$$

where $\sigma$ is the isotropic Maxwellian velocity dispersion for the population of stars. The above treatment is common, and can be found in, e.g., Hernandez et al. (2010), where we used it in the modeling of dSph galaxies, systems characterized by flat velocity dispersion profiles, obtaining mass models consistent with observed velocity dispersion, half mass radii and total masses, in the absence of dark matter. Other recent examples of similar treatments can be found in, e.g., Drukier et al. (2007), Sollima & Nipoti (2010), and Hernandez & Jiménez (2012), all modeling stellar populations using isotropic Maxwellian distribution functions.

As an illustrative example we can take $\nabla \phi(r) = GM(r)/r^2$ for the right-hand side of Equation (3), the Newtonian expression appearing for $a \gg a_0$. Looking for a power law solution for $M(r) = M_0 (r/r_0)^m$, we get

$$\sigma^2 \left[ \frac{m-3}{r} \right] = -\frac{GM_0}{r^2} \left( \frac{r}{r_0} \right)^m, \quad (4)$$

and hence $m = 1$, the standard isothermal halo, $M(r) = 2\sigma^2 r/G$, having a constant centrifugal equilibrium velocity $v^2 = 2\sigma^2$ and infinite extent. In going to the MONDian limit of $a \ll a_0$, $\nabla \phi(r) = (GM(r)a_0)^{1/2}/r$, Equation (3) yields

$$\sigma^2 \left[ \frac{m-3}{r} \right] = -\frac{[GM_0a_0]^{1/2}}{r} \left( \frac{r}{r_0} \right)^m. \quad (5)$$

In this limit $m = 0$, we obtain $M(r) = M_0$ and $\sigma^2 = [GM_0a_0]^{1/2}$, the expected Tully–Fisher scaling of the circular equilibrium velocity with the fourth root of the mass, with rotation velocities that remain flat even after the mass distribution has converged, thus, rigorously isothermal halos are naturally limited in extent, as already shown by Milgrom (1984). It is interesting that in this limit the scaling between the circular rotation velocity and the velocity dispersion is only slightly modified as compared to the Newtonian case, with the proportionality constant changing from 2 to 3, for the squares of the velocities. Note also that a fuller asymptotic analysis (Milgrom 1984) not imposing a power law solution shows that the factor of three obtained above will in general lie in the range 3–4.5.

We can now look for the behavior of a tenuous stellar halo in the MONDian regime, and hence at large distances, around a mass distribution which has essentially converged, by looking at Equation (2) and writing the right-hand side as $-(GM_0a_0)^{1/2}/r$, where $M_0$ is the total mass of the galactic or stellar system. The $\rho$ in the left-hand side of this same equation now refers to the density distribution of essentially test particles making up a trace population, e.g., a stellar galactic halo, the globular cluster distribution around a large galaxy, or the faint halos of “extra-tidal” stars surrounding Galactic globular clusters. Using also the result of Equation (5) of $3\sigma^2 = (GM_0a_0)^{1/2}$, Equation (2) yields

$$\frac{d\rho}{dr} = -3\frac{\rho}{r^2}, \quad (6)$$

which can then be integrated directly to yield

$$\rho(r) = \rho_0 (r_0/r)^3. \quad (7)$$

In deriving Equation (7) we have introduced the assumptions of a tracer population and the results of having forced a power law solution in Equation (5), which significantly simplify the calculations with respect to a full numerical solution (e.g., Milgrom 1984 or Hernandez & Jiménez 2012), or even with respect to the asymptotic analysis of Milgrom (1984). The above assumptions will certainly never be strictly valid in a real astrophysical system, still, provided they are approximately valid, the solution of Equation (7) will represent a first order description. Our simplified approach, however, allows a transparent handling of the physics, and permits a clear understanding for the generic appearance of a $\rho(r) \propto r^{-3}$ region, a feature already noted in the numerical solutions presented in Milgrom (1984), and apparent in many astrophysical systems, as discussed in the following section. The volumetric distribution of Equation (7) can be projected analytically along one direction to yield the projected surface density profile

$$\Sigma(R) = \frac{\pi \rho_0 r_0^3}{2R^2}. \quad (8)$$

Of course, $\rho \propto r^{-3}$ is an approximation which will only be valid over a limited radial range. In fact, mass profiles for isothermal solutions converge to finite total masses and radii, as
shown in, e.g., Milgrom (1984) and Hernandez et al. (2010). In closer detail, the density profiles will steepen beyond $r^{-3}$ as one moves farther out as the total mass converges, e.g., as seen in the broken power law fits for the Galactic stellar halo reported by Sesar et al. (2011).

Under the Newtonian expression of $\nabla \phi = G M(r)/r^2$, the equivalent development for a trace population in the halo of a galaxy having the same rotation curve as the one leading to Equation (6), $M(r) = 2\sigma^2r/G$, where this time $M(r)$ refers mostly to the hypothetical dark matter component, yields

$$\rho(r) = \rho_0(r_0/r)^2$$

(9)

as the expression corresponding to Equation (7). The case of a trace population around an essentially converged total mass in Newtonian dynamics, e.g., the tenuous stellar halos surrounding the Galactic globular clusters, $\nabla \phi = G M_{\text{tot}}/r^2$ yields

$$\rho(r) = \rho_0 e^{(GM_{\text{tot}}/\sigma^2r)}$$

(10)

a density distribution which tends to a constant at large radii. Thus, we see that equilibrium configurations of isothermal tracer populations in the MONDian regime will approximately follow $\rho(r) \propto r^{-3}$ density profiles, while under Newtonian gravity the same populations within the corresponding dark matter halos will show much shallower $\rho(r) \propto r^{-2}$ profiles, which, in the absence of dark matter halos, e.g., tenuous stellar halos about globular clusters, will have density profiles as given by Equation (10).

3. OBSERVATIONAL COMPARISONS

In this section we review the observational situation of tenuous tracer population halos, which now spans a very wide range of systems and astrophysical scales. We begin with a number of recent determinations of the density structure of the stellar halo of our Galaxy. Morrison et al. (2000) implement a careful disk/halo star separation criteria, and obtain $\rho(r) \propto r^{-3}$ for the stellar halo of the MW. Jurić et al. (2008) report a single power law fit $\rho(r) \propto r^{-2.8\pm0.3}$, while Bell et al. (2008) find halo profiles having more structure than simple power laws to yield better fits, but still, $\rho(r) \propto r^{-3}$ for the preferred single power law model. Finally, Sesar et al. (2011) obtain a broken power law as the most accurate description, but again, a best fit single power law of $\rho(r) \propto r^{-2.9}$. There is clearly a broad radial range over which the best fit single power law model yields a slope as expected from Equation (7).

Recent detailed studies of the stellar halo of Andromeda using a variety of techniques and data from the largest modern facilities have reached a consensus for a $\rho \propto r^{-3}$ structure. Ibata et al. (2007) obtain $\Sigma(R) \propto R^{-1.91\pm0.12}$ with data from the Canada–France–Hawaii Telescope, Tanaka et al. (2010) using the Suprime-Cam instrument on the Subaru telescope found $\Sigma(R) \propto R^{-2.17\pm0.15}$, and lastly Gilbert et al. (2012) measure $\Sigma(R) \propto R^{-2.2\pm0.2}$ out to very large distances, 175 kpc, coming to $\Sigma(R) \propto R^{-2.0\pm0.5}$ for 20 kpc $< R < 90$ kpc once the kinematical substructure is removed. It appears that the stellar halo of M31 is a classic example of the tenuous populations described by Equation (7).

Regarding more external galaxies, tenuous extended stellar halos have been detected over the past few years surrounding numerous systems. Recent detections include Cockcroft et al. (2013), who find evidence for an extended stellar halo about M33 and Bakos & Trujillo (2012), who report finding such structures about a sample of seven late-type spirals from the Sloan Digital Sky Survey (SDSS), in all cases with total masses amounting to only a few percent of the total baryonic mass of the host galaxies. Although the above observations cannot yet yield secure projected density profiles, Jablonka et al. (2010) report a tenuous stellar halo about NGC 3957 with a projected scaling $\Sigma(R) \propto R^{-2.76\pm0.43}$, while Barker et al. (2009) find a faint stellar halo about M81 with a projected scaling $\Sigma(R) \propto R^{-2.0\pm0.2}$, and Bailin et al. (2011) observe a stellar halo about NGC 253 with $\Sigma(R) \propto R^{-2.8\pm0.6}$. In going to larger samples, Zibetti et al. (2004) showed through the stacking of images from 1047 edge-on spiral galaxies from the SDSS that these very generally present extended tenuous stellar halos with volumetric radial density profiles well described by $\rho(r) \propto r^{-3}$.

In going to the spatial distribution of a different tracer population, this time the globular cluster systems of galaxies, it has been well known for many years that the density profile of the GC system of the MW very accurately follows a $\rho(r) \propto r^{-3}$ profile, e.g., Surdin (1994), Racine & Harris (1989). Looking in more detail, more recently Bica et al. (2006) find a $\rho(r) \propto r^{-3}$ profile with $3.2 < n < 3.9$ for all Galactic globular clusters, while the metal-rich population also follows a power law, this time with $\rho(r) \propto r^{-3.2\pm0.2}$ for large radii, and $\rho(r) \propto r^{-3.2\pm0.9}$ if one includes the effects of oblateness in the distribution. The globular cluster system of M31 has a projected power law density profile also in agreement with the expectations of Equation (7), e.g., Racine (1991) determined $\Sigma(R) \propto R^{-2}$. More recently and in more detail, Huxor et al. (2011) obtained a best fit profile composed of three distinct power laws, which, however, if modeled as a single power law for $r > 1$ kpc, can be approximated by the same $\Sigma(R) \propto R^{-2}$ law found earlier by Racine (1991).

Going to more external galaxies, Perelmuter & Racine (1995) found a best fit $\Sigma(R) \propto R^{-2}$ scaling for the globular cluster system of M81. Harris et al. (1984) found the outer projected radial distribution of globular clusters in NGC 4594, the Sombrero galaxy, to be well described by a $\Sigma(R) \propto R^{-2}$ profile. Harris & van den Bergh (1981) also found $\rho(r) \propto r^{-3}$ scalings for the globular cluster systems around seven elliptical galaxies. More recent studies have found a spread in the power law slopes of projected density profiles for globular cluster systems, but taken as a whole, “typical projected power-law indices range from 2 to 2.5 for some low-luminosity Es to 1.5 or a bit lower for the most massive giant ellipticals” Brodie & Strader (2006, p. 211).

Since the studies of Grillmair et al. (1995) and Leon et al. (2000), a number of tenuous stellar halos associated with Galactic globular clusters have been detected. The problem of determining structural parameters is harder than in the cases of the stellar halos surrounding galaxies as the overall numbers of stars are much lower, and the problem of contamination by foreground and background sources, as well as by obscuration, is significant. More modern studies have found a large range of power law slopes in the outskirts of globular clusters, for example, McLaughlin & van der Marel (2005) find projected indexes going from $-2$ to $-6$, but report that a population of the most massive clusters shows indexes close to $\Sigma(R) \propto R^{-2}$ and conclude that the extended halos enveloping the clusters they study are suggestive of a generic equilibrium feature, rather than being transient structures. Jordi & Grebel (2010) report projected power law indexes for tenuous stellar halos surrounding 17 Galactic globular clusters; their most reliable results span values from $-1$ to $-4$. These authors also note features which are problematic for a standard gravity
interpretation, in the cases of, e.g., NGC 7089 and Pal 1, whose stellar halos are clearly spherical with no sign of any tidal features, in spite of extending in both cases much beyond their Newtonian Jacobi radii. Carballo-Bello et al. (2012) perform a similar study (also including careful CMD modeling to limit contamination) for the extra-tidal halos of 19 Galactic globular clusters. Again, the reported projected power law indexes span a broad range from close to −2 to about −4, excluding clusters showing clear tidal features. Note that for two of the three clusters which overlap with the Jordi & Grebel (2010) sample, NGC 4147 and NGC 5272, Carballo-Bello et al. (2012) report power law indexes of −2.8±0.07 and −3.18±0.08, while Jordi & Grebel (2010) assign to these same clusters values of −1.48±0.24, and of −0.94±0.38, respectively, for a comparable radial range. This simply illustrates that the observational situation is far from converging to definitive answers regarding these systems.

Further, in the case of globular clusters the problem is intrinsically less clear than for the galactic stellar halos, as internal dynamical evolutionary effects might play a part, as well as the gravitational perturbations due to the crossing of the Galactic disk. It is, however, clear that Galactic globular clusters often, if not always, are surrounded by tenuous stellar halos, which, from a Newtonian point of view, must be regarded as “extra tidal” structures. The smooth and round appearances often observed, with all absence of tidal tails, hence become a problem. This problem does not appear under MONDian gravity, where satellite systems are generally expected to be much more robust to tides, e.g., Hernandez & Jiménez (2012).

Interestingly, Mackey et al. (2010) find a Σ(R) ∝ R−n power law structure for the tenuous stellar halo surrounding an extremely isolated globular cluster in M31, which begins as n ≈ 2.5, and then breaks further outward to n ≈ 3.5. Also, note that most of the Σ(R) ∝ R−n indexes in the two recent Jordi & Grebel (2010) and Carballo-Bello et al. (2012) studies cluster about n = 3. The presence of as yet undiscovered tidal features among these clusters would tend to artificially steepen their profiles. Note also that Grillmair et al. (1995) cautioned that the difficulties of background subtraction and obscuration corrections will lead to systematics which tend to yield overestimates in n. On the other hand, the large indexes sometimes reported for globular clusters could also be detections of the steepening in the profile expected under MONDian gravity models on approaching the final radius, already mentioned in the discussion following Equation (8).

It thus appears clear that extended tracer population halos, stars, or globular clusters having a small fraction of the light of their host systems, galaxies, or globular clusters are a common feature. Also, such halos are generally never far from the predictions of Equation (7) for equilibrium configurations of isothermal tracer populations in the MONDian regime, to first approximation ρ(r) ∝ r−3 or Σ(R) ∝ R−2. Within the standard gravity interpretation, explanations of the power law density profiles of galactic stellar halos have been proposed in terms of the accretion and tidal dissolution of substructure falling into the main galaxy, e.g., Bullock & Johnston (2005), and Abadi et al. (2006). However, the overall smoothness and uniformity in stellar properties of these systems has been pointed out as problematic for the standard explanation, which naturally implies a degree of randomness in the accreted material, e.g., Ibata et al. (2007), and Bell et al. (2008), who also find from simulations stellar halos not matching observations in terms of the substructure details. Ibata et al. (2007) also show that standard simulations sometimes yield exponents of ρ(r) ∝ r−n inconsistent with observations, with n = 4 or even n = 5. Also, a further explanation must be sought for the observed profiles of the globular cluster systems surrounding galaxies, and yet another for the remarkably smooth “extra-tidal” stellar halos surrounding many of the globular clusters of the MW.

4. DISCUSSION

The relevance of the ρ(r) ∝ r−3 solution presented here to the observations listed above depends crucially on the validity of three assumptions regarding the tracer population in question, and which enter into the derivation of Equation (7): (1) that it lies within the a < a0 region over which the modified gravity regime is thought to apply; (2) that its velocity dispersion does not depend on radius, i.e., that it is isothermal; and lastly, (3) that there is no orbital anisotropy present in its velocity dispersion, i.e., that it is isotropic. The validity of the first assumption is easy to verify; the radial ranges over which galactic stellar halos and globular cluster populations are observed to comply with the ρ(r) ∝ r−3 profiles are within the radial ranges where flat rotation curves are seen, and hence, from the accurate rotation curve modeling which MOND affords (e.g., Swaters et al. 2010), also within the a < a0 region. In the case of the tenuous stellar halos surrounding Galactic globular clusters, these appear at radial distances comparable to, but mostly larger than, the regions where the a < a0 threshold is crossed and the velocity dispersion profiles flatten, in the cases where this last have been measured, e.g., Scarpa et al. (2011) and Hernandez et al. (2013).

Regarding the second assumption, in all of the cases listed in the previous section, wherever a radial profile has been measured for the velocity dispersion of the tracer populations in question, these have been shown to be consistent with a constant isothermal solution. Examples of this last point are Battaglia et al. (2006) who, from a sample of 240 halo objects, obtain a velocity dispersion profile for the halo stars in the MW consistent, within errors, with a constant value from 15 kpc to about 70 kpc, and with an inferred anisotropy consistent with an isotropic distribution. Brown et al. (2010) do find a falling trend for the velocity dispersion profile of stars in the MW halo, but only a very mild radial drop, while more recently Samurovic & Lalovic (2011) obtain a velocity dispersion profile for a large sample of 2557 blue horizontal branch stars in the MW from Xue et al. (2008) which is consistent with a constant value out to 70 kpc. Finally, Kafle et al. (2012), using 4664 blue horizontal branch stars from Xue et al. (2011) in the MW halo, observe a radial velocity dispersion which is indistinguishable from flat outward of about 15 kpc, out to their last measured point at close to 60 kpc. Also, in all cases where the velocity dispersion profiles of stars in Galactic globular clusters have been measured out to large radii, these can be seen to be consistent with constant σ values, e.g., Scarpa et al. (2011) and Hernandez et al. (2013). An interesting feature of the two most recent references measuring the velocity dispersion profile of the stellar halo of our Galaxy listed above is that the level for the constant one-dimensional velocity dispersion found is of between 100 and 110 km s−1, which would bring it in accordance with the expectations of the 3σ2 = v2 condition we derive in Section 2, since \( \sqrt{3} \times 110 = 190 \), the observed asymptotic rotation velocity of the MW.

The third assumption is much harder to test empirically, as no reliable measurements of orbital anisotropy exist for any of the astronomical systems treated here. Orbital isotropy is, however,
a natural first order approximation commonly used in the modeling of self-gravitating systems, e.g., Binney & Tremaine (1987), or Drukier et al. (2007), Sollima & Nipoti (2010), and Hernandez & Jiménez (2012), in the modeling of globular clusters under either Newtonian or MONDian approaches. Although an idealization, it provides a convenient reference solution for a variety of dynamical studies of self-gravitating systems; for instance, studies of dynamical friction due to both the hypothetical dark matter and stellar components of dSph galaxies routinely assume isotropic distribution functions for the stars in question, even though this assumption is understood as only a first order approximation (e.g., Sanchez-Salcedo et al. 2006; Goerdt et al. 2006; Cole et al. 2012 to cite a few recent examples). In the absence of any evidence suggesting orbital anisotropy for the systems and radial ranges treated here, e.g., any observed dominant flattening in the light distribution, we chose not to introduce any at this initial point. Further, in the particular case of MOND gravity, it was already shown in Milgrom (1984) that the introduction of a slight degree of anisotropy, which could in principle be present, modifies only slightly the resulting density profiles of self-gravitating systems.

It is of course true that under Newtonian gravity a $\rho(r) \propto r^{-3}$ profile for a tracer population can also be found, but only if one allows for more complex $\sigma(r)$ and radially varying anisotropy parameters. Since the observed density profiles for tenuous halos match the simplest isothermal (as observed in all cases where this function has been measured) and isotropic distribution functions under MONDian gravity, this solution is to be preferred to the fitting of contrived, ad hoc, radial variations in the velocity dispersion and anisotropy parameters of the tracer populations in question under Newtonian gravity, especially as none such variations have been detected.

Clearly, for any astrophysical system where the halo population ceases to be a small perturbation on the total mass, or where the velocity dispersion profile is seen to deviate significantly from the isothermal condition assumed here, our solution will not be relevant. In spite of the approximate nature of the solution (due to the tracer population assumption, the strict isothermal assumption and the forcing of a power law solution), the approximately isothermal profile of various systems where this has been observed, the very low mass contribution of the tracer populations treated, and the good match to a $r^{-3}$ density profile which a wide variety of systems present, give us confidence in that some of the physics has been captured by the modeling.

Finally, it is interesting that some of the systems mentioned in the previous section are not in the deep MOND regime, therefore, from the point of view strictly of MOND as such, no significant modifications to gravity should be apparent. We note that the external field effect of MOND will be substantially modified for different modified gravity theories, of the various types listed in the introduction. Indeed, MOND variants have been discussed where the external field effect is substantially reduced, or even practically disappears, e.g., Milgrom (2011). We note also that our previous results of Hernandez et al. (2012) looking at the observed relative velocities of wide binaries in the solar neighborhood, or of Hernandez et al. (2013) finding MONDian phenomenology in the observed outer dynamics of Galactic globular clusters, both classes of systems not in the deep MOND regime, strongly suggest a modified gravity theory where no external field effect appears.

To summarize, we have shown that under a MONDian gravity force law, the density profiles of isothermal tenuous tracer population halos with isotropic Maxwellian velocities surround-ing spherical mass distributions will be well approximated by $\rho \propto r^{-3}$ scalings. We suggest that such equilibrium configurations provide a natural, and certainly general, explanation for the observed close to $\rho \propto r^{-3}$ behavior of: the stellar halos surrounding the MW, M31, and a variety of external galaxies, the density profiles of the globular cluster systems in our Galaxy and Andromeda, and the radial structure of the ”extra tidal” stellar halos recently observed surrounding a number of Galactic globular clusters.

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