Ground test for vibration control demonstrator

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Abstract. In the objective of maximizing comfort in Falcon jets, Dassault Aviation is developing an innovative vibration control technology. Vibrations of the structure are measured at several locations and sent to a dedicated high performance vibration control computer. Control laws are implemented in this computer to analyse the vibrations in real time, and then elaborate orders sent to the existing control surfaces to counteract vibrations. After detailing the technology principles, this paper focuses on the vibration control ground demonstration that was performed by Dassault Aviation in May 2015 on Falcon 7X business jet. The goal of this test was to attenuate vibrations resulting from fixed forced excitation delivered by shakers. The ground test demonstrated the capability to implement an efficient closed-loop vibration control with a significant vibration level reduction and validated the vibration control law design methodology. This successful ground test was a prerequisite before the flight test demonstration that is now being prepared. This study has been partly supported by the JTI CleanSky SFWA-ITD.

1. Introduction

In the objective of maximizing comfort in Falcon jets, Dassault Aviation is developing an innovative vibration control technology based on control surfaces actuation.

The global strategy defined to corroborate the general control law design methodology and demonstrate the efficiency of the designed vibration control laws is first presented in this paper. The main challenges of this demonstration are to design vibration control laws based on numerical model simulations that are robust to aircraft mass variation (to represent fuel consumption during flight). Moreover, the vibrations inside a wide frequency band have to be reduced while the vibration control laws should not have any impact on lower and higher frequencies.

In a second step, the vibration control law design method is detailed. The ground vibration control demonstration that was performed by Dassault Aviation in May 2015 on Falcon 7X business jet is presented with the main results that assess the vibration control law design methodology.

Finally, an innovative frequency-domain interpolatory method providing experimental state-space models of the aircraft is presented and has been tested on several real time data obtained during the ground vibration control test. This method could provide an alternative to design control laws when an accurate numerical model is not available.
2. General vibration control demonstration strategy

The vibration control presented in this paper aims at controlling aircraft response in the [5-15Hz] frequency band (comfort zone).

The objective of the vibration control ground demonstration that was performed by Dassault Aviation in May 2015 on Falcon 7X business jet was to validate the capability to implement vibration control laws using the aircraft control surfaces. This first phase was necessary to validate the control law design methodology. In a second step, a flight test campaign is being prepared in order to demonstrate in flight the efficiency of vibration control. Figure 1 presents the flight control system and the vibration control loop.

![Figure 1: Flight control system and vibration control loop](image)

Theoretical studies have shown two major requirements for the design of an efficient vibration control system:

- Computer sampling rate should be higher than current flight control laws sampling rate
- The analog to digital quantization of sensors should be higher than 12 bits

In the vibration control demonstrator, these requirements are covered by:

- Use of dedicated computer
- Use of dedicated analog sensors specifically chosen for their high sensibility, large bandwidth and low level of noise.

Figure 2 shows the vibration control demonstration of the study workflow.

![Figure 2: Vibration control demonstration of the study workflow](image)
3. Vibration control law design methodology

The vibration control law design methodology developed by ONERA and Dassault Aviation is based on the following three main steps:

- Development of accurate aero-servo-elastic state space models of the aircraft (large-size state-space models)
- Reduction of these models in order to be compatible with control design methods
- Design of vibration control laws

The designed controllers are then analysed on the large models and the most performant ones are selected.

3.1. Business jet aero-servo-elastic modelling

The vibration control laws design methodology presented in this paper is based on an accurate aero-servo-elastic model. This model is used at several steps during the demonstration study logic presented Figure 2:

- Control laws design: laws design and selection
- System architecture: vibration control sensors technology definition
- Ground test analysis and comprehension

The aero-servo-elastic model used is composed of:

- The general Falcon 7X finite element model (Figure 3), adapted to this specific vibration control demonstration (mass and fuel distribution, vibration control laws sensors location, control surfaces deflection, shakers excitation, landing gears extended).
- A rationalized aerodynamic model. Thanks to the enhanced aerodynamic rationalization algorithm developed at Dassault Aviation, models used for frequency-domain simulations and models used for time-domain simulations give very similar results.
- A servo-actuator non-linear model specifically adapted for low amplitude deflections
- Flight control system

![Figure 3: General Falcon 7X finite element model](image)

Several models are built for various flight points and mass cases. State-space model assembly is then performed and leads to models of large size (570 states including aerodynamics in flight, or 250 states on ground). Vibration control study is based on these models. The general linear state-space model equation (taking into account a linearized servo-actuator model) is given below:

\[ \dot{x}(t) = Ax(t) + Bu(t) \]

\[ y(t) = Cx(t) \]

where \(x(t); u(t); y(t)\) are the input output vectors of dimension \(n; n_u; n_y\), respectively (matrices are real of suitable dimension).
3.2. Model reduction

The aero-servo-elastic models considered for vibration control laws design usually result to be of large dimension. In practice, such complex models render the simulation, analysis and optimization tasks very complex or even impossible. Indeed, for the control design step, such a large-scale model set is clearly inappropriate for any numerically-based optimization strategy (or leads to a prohibitive computation time). An approach to bypass this limitation is to perform a model reduction step. The main objective of the dynamical model approximation is to replace the original model, equipped with a large number of internal variables, by another one, with the same number of inputs and outputs but with a drastically lower number of variables. Obviously, a “good” low order model should “well” reproduce the original input-output behaviour. Mathematically, and in the linear framework, given general state-space model equation defined in §3.1, the objective of the model approximation is to find:

\[
\hat{H}: \begin{cases}
\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}u(t) \\
\hat{y}(t) = \hat{C}\hat{x}(t)
\end{cases}
\]

where \(\hat{x}(t)\) is the reduced state vector of dimension \(r \ll n\) and \(\hat{y}(t)\) denotes the approximated output vector (still of same dimension as \(y(t), \text{i.e. } n_y\)). The following optimization problem is traditionally considered:

\[
\hat{H} = \arg \min_G \|H - G\|_{H_2}
\]

where \(G\) belongs to the rational and strictly proper complex valued functions set of order \(r\), with \(n_u\) inputs and \(n_y\) outputs. In this case, the term \(\|H - G\|_{H_2}\) can be viewed as a mismatch mean error over the entire frequency support, which is evaluated through the inner product, called the \(H_2\)-norm. In practice, this problem is non-convex, non-linear and its resolution, especially in a large-scale context, is very complex. However, recent major results in the field of linear algebra and dynamical model approximation theory lead to interpolatory conditions which can be satisfied at a very modest computational cost (Ref. [1] [2]). In addition, since in practice the frequency support of interest for the control design purpose is often limited and bounded, the above presented optimisation problem can be reformulated as the following frequency-limited \(H_2\)-norm approximation problem:

\[
\hat{H} = \arg \min_G \|H - G\|_{H_2,\Omega}
\]

where \(\Omega\) is the frequency support of interest. To solve this problem, similar interpolatory results and algorithms have been developed (Ref. [4]) and implemented in the MORE toolbox, developed at ONERA (Ref. [3]).

Servo-elastic models used for ground control can be reduced from 250 states to less than 40 states. Based on the obtained models, the advanced control synthesis methodology can be applied using enhanced optimization procedures.

3.3. Vibration control law design

As previously mentioned, the main purpose of the vibration controller is (i) to minimise the vibrations inside the aircraft between 5 and 15 Hz (performance objective), while (ii) keeping low frequencies responses unchanged and having no effect on high frequencies responses (constraint).

3.3.1. Forewords and general approach overview

Here, as rooted on the above presented ground-oriented reduced order models, taking into account varying configurations (such as the aircraft mass), this section is dedicated to the description of the methodology to design a dedicated robust vibration controller. More specifically, a more generic framework providing high level tuning parameters to design the vibration control laws is described. This is done by formulating the vibration problem
(performance objective and constraints) into an H-infinity-optimization one, and, based on the recently
developed dedicated numerical tools, this problem can be efficiently solved, allowing synthesizing the
controller in a numerically tractable manner (Ref. [5]). The key point of the proposed solution is the
so-called generalized scheme, illustrated on Figure 4. Secondly, the structured H-infinity controller
design optimization approach is reminded and some controllers are provided to illustrate the proposed
approach effectiveness on the considered models (real validation being presented in the vibration
control ground demonstration section).

3.3.2. Generalized plant (starting point of the controller synthesis). Within the many control design
methods (such as H-infinity or H₂-norm, etc.), the first step consists in writing the so-called
generalized control scheme \( P \), which describes the interconnection of the considered system (here the
ground dynamical model, purple block on Figure 4) and the performance objectives characterized by
the weighting functions (green blocks of Figure 4). This first phase shapes the exogenous input \( w \) to
output \( z \) performance signals, as illustrated on the following Figure 4.

![Figure 4: Generalized scheme - classically used in linear control (left)
and applied to business jet vibration control (right).](image)

Without loss of generality, standard approaches exist to design the performance filters (green
blocks). The choice of these filters is described in Ref. [6]. In the closed-loop H-infinity norm
minimization, these weighting functions determine the bandwidth over which the minimisation of the
closed-loop H-infinity norm will be applied.

3.3.3. Structured H-infinity controller synthesis. Once a generalized plant is available, the following
problem is solved to design the vibration control laws:

\[
K_{opt} = \text{arg min}_{K} \left\| F_{l}(P, K) \right\|_{\infty}
\]

where \( F_{l} \) is the lower fractional operator. To achieve the above objectives, a structured H-infinity
norm-oriented minimization criterion is selected. The main advantage to the structured multi-model
approach is that the frequency response peaks level can be attenuated for all mass configurations, so
that the control laws are robust to aircraft mass. This approach also imposes the controller
mathematical complexity (e.g. its order and frequency shape), which is of great importance when the
control law has to be implemented.

Based on a limited number of sensors, a set of vibrations control laws has been designed and
selected. The structured H-infinity synthesis allowed to shape the vibration control laws in such a way
that they behave like a band-pass filter with gain adaptation in the vibrations frequency range.
A vibration control laws analysis is performed on full size bizjet models to check the vibration reduction level. On the left part of Figure 8, one can see the reduction level of a selected control law. A compromise has to also be found between the slight increase of level around 6.5 Hz and the vibration reduction between 7 and 10 Hz.

A typical vibration control law controller is shown on Figure 5 (order 10 controller) with inputs measured by dedicated accelerometers and the output being the commanded elevator deflection.

4. Vibration control ground demonstration

4.1. Objectives of the ground vibration control test

Once an optimal controller is designed, synthesized and validated on the numerical model, the next step is to perform a validation on Dassault Falcon 7X business jet.

The objectives of the vibration control ground test are:

- Assessment of the overall vibration control law design methodology only based on numerical models
- Validation of the servo-elastic model accuracy with and without control law

These objectives are achieved by being able to understand and model the ground test results.

4.2. Ground test installation

The test was performed on the Falcon 7X S/N 1, property of Dassault Aviation, and based in Istres, France. Prior to the test, the aircraft was equipped with a dedicated vibration control computer and with specific vibration control sensors (7 accelerometers and 1 gyrometer). About 100 external accelerometers were also positioned all over the aircraft for test monitoring and analysis.

Figure 6 shows a picture of the Falcon 7X during the vibration control demonstration.

A test supervision station was installed close to the aircraft, providing shaker excitation, control surfaces excitation and vibration control selection.

Real time measured data was recorded and sent to a test monitoring and analysis station.
4.3. Ground test sequence description

The test sequence is described in Figure 7 and detailed hereafter:

- **Control surfaces excitation without control** (open-loop test). In order to estimate the stability and efficiency of the designed control laws, the vibration control computer calculates the signals that would be sent to the control surfaces in closed loop. These signals are analysed in order to select the most performant laws.

- **Shakers excitation without control** (open-loop test). The shakers provide a fixed force excitation that provides “artificial” vibration in the cockpit and in the passenger cabin. This test brings out a reference aircraft vibration level due to a given force. Both of those open loop tests are also used for model calibration and analyses.

- **Shakers excitation with active control** (closed-loop test). Several control laws are tested with the aircraft excited by the shakers.

In order to correlate pilots and passengers comfort feelings with sensors measurement, some tests were performed with pilots in the cockpit and cabin. Pilots’ point of view on comfort while being seated in the aircraft has been recorded for various control laws.
To validate the robustness of the control laws to the kind of excitation, various excitation levels and signals were performed and two shakers locations were tested. The robustness of the control laws to aircraft mass configuration was tested through three different aircraft fuel tanks filling.

4.4. Vibration control laws experimental results

During the ground vibration control test, the aircraft was first identified in open-loop in order to measure reference transfer functions and validate the numerical servo-elastic model. Then, the selected control laws were activated one by one, and the effect on the aircraft vibrations measured by comparing open-loop with closed-loop transfer functions. These results were finally compared with the numerical model simulations. A typical comparison is presented Figure 8; the simulated transfer functions (open and closed-loop) are compared with the ones measured during the ground test.

![Figure 8: Measured transfer functions of a sensor in the cockpit (shaker excitation), open-loop (blue) and closed-loop (red). Numerical model prediction (left) and test measurements (right)](image)

This figure shows that the vibration control law globally reduces the level of vibration between 5 Hz and 15 Hz. Moreover, the model prediction is quite accurate for both open and closed-loop transfer functions, and the level of attenuation is consistent between the test and the simulation.

Figure 9 compares the transfer functions of the system in open-loop and closed-loop for several laws which reduce some or all of the peaks level during the tests. Several pilots have been seated in the cabin and cockpit so that they could comment on the vibration control laws efficiency compared to their physical experience. The pilots were asked to share their opinion and feelings regarding each law in comparison with the open-loop test reference.

The pilots’ assessment is that vibration control laws clearly reduce the perceived vibration level.
The pilots’ specific comments on each vibration control law confirm the analysis based on the transfer function of pilot sensor with regards to shaker excitation (Figure 9):

- Law #1 is better than open-loop test
- Laws #2 and #3 are both better than law #1

Figure 9: Open-loop and closed loop transfer functions on pilot’s seat (3 different laws are presented)

This ground vibration demonstration test performed on Dassault Aviation Falcon 7X business jet was successful. Three different aircraft mass configurations were tested, and several control laws applied. The results analysis shows that the numerical model is accurate and delivers transfer functions very similar to experimental ones, in both open and closed-loop.

As expected in the simulation step, a substantial level of attenuation is observed when the vibration control is active.

Finally, several laws robust to the aircraft mass were successfully tested.

5. Data interpolatory framework for data-driven interpolation and model generation

In this section, an innovative frequency-domain interpolatory method is presented. It has been applied to several results obtained during the ground vibration control test. With this algorithm, state-space representations are obtained directly from experimental data. This method gives the ability to build experimental state-space models of the tested aircraft, and possibly design control laws even if an accurate numerical model is not available.

In order to derive a model of the open-loop and closed-loop systems based on measured data, the data-driven interpolatory framework is well appropriated. Briefly, let us consider left $(l_j, \mu_j)$ and right $(r_i, \lambda_i)$ interpolation driving frequencies and frequency responses defined as:

\[ l_j^T \tilde{H}_r(\mu_j) = v_j^T \quad \text{and} \quad \tilde{H}_r(\lambda_i)r_i = w_i \]

\[ j = 1, \ldots, q \quad \text{and} \quad i = 1, \ldots, k \]

\( v_j \) and \( w_i \) are the test frequency responses measured during the ground test experiment.

Thanks to the so-called Loewner framework introduced by Ref. [7], for linear systems data-driven interpolation given by:

\[ (\lambda_i, r_i, w_i), i = 1, \ldots, k \quad \text{and} \quad (\mu_j, l_j^T, v_j^T), j = 1, \ldots, q, \]
the left and right interpolation data are gathered as:

\[
\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_k) \in \mathbb{C}^{k \times k}, \quad M = \text{diag}(\mu_1, \ldots, \mu_q) \in \mathbb{C}^{q \times q}
\]

\[
R = [r_1, \ldots, r_k] \quad \text{and} \quad L^T = [r_1, \ldots, r_q]
\]

\[
W = [w_1, \ldots, w_k] \quad \text{and} \quad V^T = [w_1, \ldots, w_q]
\]

Then, by defining the Loewner and shifted Loewner matrices as:

\[
\Sigma = \frac{v_j^T r_i - l_j^T w_i}{\mu_j - \lambda_i} \quad \text{and} \quad \Sigma_{\sigma} = \frac{\mu_j v_j^T r_i - \lambda_i l_j^T w_i}{\mu_j - \lambda_i}
\]

which solve the following Sylvester equations:

\[
\Sigma \Lambda - M \Sigma = LW - VR \quad \text{and} \quad \Sigma_{\sigma} \Lambda - M \Sigma_{\sigma} = LW\Lambda - MVR
\]

it is possible to construct a generalised minimal state space realization:

\[
H_L:\{E_L \dot{x}(t) = A_L x(t) + B_L u(t) \\
y(t) = C_L x(t)
\}
\]

which exactly interpolates the frequency-domain data and where the resulting realization is of minimal McMillian degree. Now, based on this realization, it is possible to re-adjust, if necessary, the original models and re-adjust a control law for future methodology improvements.

Figure 10 provides an illustration of the interpolation results on a single mass case, nicely illustrating the effectiveness of the Loewner framework for interpolating data.

Figure 10 first illustrates the perfect data matching using the Loewner framework (both in complex and real arithmetic – solid blue and dashed red lines). In addition, it is possible to obtain, at the low cost of a singular value decomposition, an approximated model, which is illustrated on Figure 10 with the solid pink line, providing a faithful reproduction of the major frequency phenomena.
Similar results are reported on Figure 11, comparing open and closed-loop results.

![Frequency Response Graphs](image)

Figure 11: Illustration of the interpolatory results obtained by the Loewner approach on both open and closed-loop frequency set obtained in experiments. Here 3 performance outputs have been considered.

6. Conclusion

Dassault Aviation is developing an innovative vibration control technology in order to minimise the vibrations in the cabin and thus increase passengers comfort in business jets.

The vibration control law design methodology is based on accurate aero-servo-elastic models which are used at several steps: for control laws design, for system architecture and for ground test analysis and comprehension. A preliminary size reduction of the models is performed thanks to the MORE toolbox developed at ONERA. The vibration control law design is based on the structured H-infinity approach. This method was selected as it attenuates the frequency response peaks in a restricted frequency range for several mass configurations at a time and it offers the capability to constrain the controller mathematical complexity.

In order to demonstrate the efficiency of this innovative methodology for vibration control design, a ground test was performed in May 2015 on Dassault Aviation Falcon 7X business jet. In a first phase, the vibration control devices and the test instrumentation were installed on Dassault Aviation Falcon 7X S/N 1. Then the vibration control tests were performed. The effect of various vibration control laws have been measured for two shaker locations and three aircraft mass cases. Tests analyses and assessment with the numerical model predictions validates the vibrations control law methodology. The next phases of the demonstration will be prepared in order to reach a good maturity for the vibration control technology with final flight tests.

The research leading to these results has received funding from the European Union’s Seventh Framework Program (FP7/2007-2013) for the Clean Sky Joint Technology Initiative under grant agreement CSJU-GAM-SFWA-2008-001.
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