Unravelling Tutors’ Conceptions of Teaching Mathematics in Virtual School Using a Goal-Action Model

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Abstract
This study explored the unique integration of mathematics tutors in teaching high school mathematics in a Virtual mathematics school (VMS). The tutors were three excelling STEM students who did not have any formal preparation for teaching before their work in the VMS. The goal of the study presented in this paper was to design a model of proficiency for these tutors. The model was designed using multiple case study methodology by tracking the learning through teaching of the tutors. Activity theory (Leontiev, 1978) framed the design of the study and led to a “goal-action” model of the tutors’ proficiency in the VMS. The goals part of the model is rooted in the construct of students’ mathematical potential (Leikin, 2021), and the action part was unraveled using the construct of a Teaching Triad (Jaworski in For the Learning of Mathematics, 12(1), 8–14, 1992). In this paper, we illustrate how our model can be used to analyze changes in the tutors’ proficiency, and how it can be applied to descriptive, explanatory, and analytical power.

Keywords Actions · Conceptions · Goals · Tutors · Virtual mathematics school

Introduction
The main rationale for establishing virtual schools in general (Cleveland-Innes & Garrison, 2021) and virtual mathematics schools (VMSs) in particular (Engelbrecht et al., 2020) was the necessity of providing access to high quality teaching for groups living in outlying areas. “Pedagogic and not geographic theories of distance education began to emerge as we approached the twenty-first century” (Garrison, 2000). Presently, under the pandemic conditions of COVID-19, distance, online learning is essential, and a broad range of distance learning programs have been implemented. The study presented in this paper provides important evidence about proficiency of teaching in online settings.
In 2012, a VMS was established in Israel. The school was established for pupils with high mathematical potential from rural areas whose schools did not provide the option of studying high level mathematics. The staff included three main groups of employees: instructional designers, teachers, and tutors who trained students in small groups of three or four. The tutors were excellent university STEM students with high GPAs with no formal training as teachers and no prior pedagogical knowledge or experience.

This article is part of a larger research study that explored the professional development of designers, teachers, and tutors in the course. The part of the study presented here focuses exclusively on the mathematics tutors since, to the best of our knowledge, no studies have been conducted on tutors in a virtual high school. Through exploring the development of tutors’ proficiency, we devised a model described in this paper. The role of the model is threefold: to provide theoretical characterization of the proficiency of the tutors in teaching in a VMS, to analyze the development of the proficiency of the tutors, and to distinguish between the profiles of different tutors.

**Theoretical Background**

**Virtual Schools as an Instance of Distance Learning**

Distance education is increasing exponentially since the onset of the COVID-19 epidemic, and the goal is to increase access, affordability, and quality in educational settings. Consequently, there is a growing need for research on the development, design, delivery, and navigation of distance education as an instructional environment (Cleveland-Innes & Garrison, 2021). Distance education is defined as internet-based learning that delivers content and enables communication between the instructor and the students with computer and communication technology (Cleveland-Innes, 2021).

The focus of this study is on a virtual school, which is one of the forms of distance education. There are different types of virtual schools with a variety of online and off-line formats, content, and instructional settings. Virtual schools differ one from another also in the number and goals of the courses offered to students. In some virtual schools, students study one or two enrichment courses (Smith et al., 2005), whereas in other schools, students participate in online learning programs to earn academic credit for graduation or promotion to the next educational level (Beck & Beasley, 2021). There is evidence that distance education facilitates quality learning, improves student achievements, self-regulation, higher order thinking, collaboration, and technological skills (Barbour & Reeves, 2009; Cleveland-Innes & Garrison, 2021; Miron & Gulosino, 2016; Posey et al., 2010). At the same time, there are possible disadvantages of virtual learning environment related to technological disturbances (like internet disconnection, inadequate voice, and image quality) or quality of instructional interactions including a lack of student initiative for independent work, feelings of social isolation, and the loss of visual cues (Kerka, 1996; Posey et al., 2010).
Note, this research was conducted before the outbreak of COVID-19. The classes were held synchronously. The platform used in VMS allowed a tutor or student to share a screen to show, for example, a presentation, a solution, or a website. However, the platform did not allow the tutor and students to see each other. In this VMS, expert teachers conducted the introductory lessons, which were accompanied by problem-solving tutoring conducted by tutors. Mathematics Tutors.

Most studies on tutors have been carried out in the context of university education where tutors play a vital role in the success of the mathematics courses. In contrast, this study focused on tutors in a VMS. The tutors usually supplement the roles of lecturers in providing students with an opportunity to use their problem-solving skills. They have two main roles: supporting development of cognitive field-dependent skills related to the content of courses and providing emotional support during learning. In the context of mathematics courses, the tutors are expected to have a solid background in mathematics, and should be able to create a comfortable, open, and accessible atmosphere directed at students’ learning (Luo et al., 2001; Tawfeeq, 2010). Studies conducted among tutors in the universities focus on two main issues: exploring tutors’ experiences and examining professional development programs for tutors (Houssart, 2012; Luo et al., 2000; Speer et al., 2005). Studies have shown that tutors themselves often need training in creating an interactive classroom, developing communication skills, and developing diverse skills to meet the challenge of a variety of students, and thus researchers suggest guidelines for training programs (Luo et al., 2001; Speer et al., 2005).

The tutors’ task is not simple, especially because they have neither formal knowledge nor informal awareness about the teaching and learning of mathematics: “To know mathematics “for self” is entirely different from mathematics for the sake of others.” (Tawfeeq, 2010, p. 6). Most tutors have the required mathematical knowledge but have little or no pedagogical knowledge or skills (in the sense of Shulman, 1986). Consistent with Thompson (1992), who described teachers’ conceptions of the nature of mathematics as their knowledge and beliefs concerning the discipline of mathematics and the nature of teaching, the present study focuses on tutors’ conceptions as a combination of their knowledge and beliefs. Considering the complex relationship between tutors’ conceptions and their practices, we analyze their development in craft mode (i.e., through teaching) (Kennedy, 2002; Lloyd, 2002).

In our research, the role of the tutors in the VMS was to assist the teacher in developing students’ problem-solving skills, to encourage student learning, to review homework, and to provide feedback to pupils. Our focus in this study was to analyze development of tutors’ conceptions about their roles and their teaching skills. Researchers and educators emphasize the fundamental change that has taken place in the role of teachers and in their conceptions about teaching in a virtual environment (Hawkins et al., 2012; Martin et al., 2019; Palloff & Pratt, 2001; Thach & Murphy, 1995; Wiesenberg & Stacey, 2008).

The tutors were excellent university STEM students who took a 2-h introductory workshop on VMS technology and the role of tutors in the learning process. Thus, our study tracked the development of tutors’ proficiency as well as changes in their conceptions using a learning-through-teaching perspective (Leikin, 2006; Leikin & Zazkis, 2010). To follow these changes, we first developed a model of tutors’
conceptions related to conducting tutorial lessons in VMS. The model was framed by Leontiev’s (1978) activity theory through zooming in on their goals linked to the developing students’ mathematical potential (Leikin, 2021), and their actions expressed in terms of a teaching triad (Jaworski, 1992).

**Activity Theory**

Activity theory makes connections between motives and activity, goals and actions, and conditions and tools (Leontiev, 1978). The model is hierarchical, in which motive and activity are above goal-oriented actions and underlie tasks or conditions that determine operations and tools (see Fig. 1). Central for activity theory is goal-oriented activity, which is a crucial part of human behavior. Goal orientation directs actions towards anticipated results. Activity is performed by groups of actions, which again may be decomposed into operations in according with the conditions in which activity undergo. The connections between goals and actions that characterize any human activity is applied to the theoretical framing of this study. The model that combines goals and actions of tutors in terms of student mathematical potential and the teaching triad was produced through a content analysis of the data.

**The “Student’s Mathematical Potential” Model**

The concept of students’ mathematical potential is one of the bases of the education and professional development of teachers. Successful realization of mathematical potential can serve as an indicator of teachers’ proficiency (Leikin, 2021). The mathematical potential of a student, as suggested by Leikin (2021), includes the following variables: analytical and creative abilities, emotional factors such as belief and motivation, student commitments to learning mathematics, and learning opportunities according to differences between students (see Fig. 2). The mathematical potential of students should be an integral part of teachers’ knowledge. Teachers need to be aware that student learning includes mathematical knowledge and skills, but also depends on general cognition, motivation, beliefs, and students’ learning history (Leikin, 2021). In our research we used the “student’s mathematical potential” model to sort out the stated tutors’ goals in tutoring in the VMS. The model was chosen as suitable for analyzing the goals since the analysis of the data in the study revealed that the tutors’ goals, as identified in their interviews, corresponded to the

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**Fig. 1** Activity theory model (Leontiev, 1978)

![Activity theory model](image-url)
components of the “student’s mathematical potential” model. Later in the findings, we will present the model developed in the study in greater detail.

**Teaching Triad**

The “teaching triad” model (Jaworski, 1992; Potari & Jaworski, 2002) consists of three aspects representing core elements of any teaching situation: management of learning, sensitivity to students, and mathematical challenges. The model links these three aspects of the teaching activity and enables exploring the instruction according to them.

Management of learning (ML) describes the role of teachers in creating a supportive learning environment, their ways of developing social norms within the classroom setting, and their creation of opportunities to engage in mathematics. It includes classroom groupings, planning of tasks and activities, and setting of norms. Sensitivity to student (SS) describes the teachers’ knowledge and attention to their students’ cognitive and affective needs. This is reflected, for example, in the ways in which tasks are presented, how teachers respond to the students’ answers, and the attention they devote to their way of thinking. It is also reflected in the ways in which the teacher fosters students’ personal beliefs in, and valuing of, their ability to do mathematics and think mathematically and students’ wellbeing and positive attitude within the classroom setting. Mathematical challenge (MC) represents the challenges offered to students to stimulate their mathematical thinking. The aspect of mathematical challenge refers to the way the student is motivated, the methods used by the teacher to encourage mathematical thinking, and the degree of interest and challenge embedded in the mathematical content. This is reflected in the types of questions, the degree of mathematical connectivity, and the consideration of the individual needs of each student. These three components of the teaching triad are intertwined and mutually supportive as shown in Fig. 3.

In our research, we used the “teaching triad” model (Jaworski, 1992; Potari & Jaworski, 2002) to characterize the actions reflectively reported by the tutors during
the interviews. The results section describes the model devised in this study through integration of the teaching triad.

**Research Goals and Context**

The proposed study was conducted in a virtual school where online synchronized learning took place. In the virtual school, students from around the country learned mathematics and physics virtually and synchronously. The staff includes educators who develop the curriculum, teachers, and tutors who teach small groups. The tutors are university students with high GPAs, not formally trained as teachers, and with little knowledge or experience in teaching or pedagogy. The mathematics classes are conducted in groups of 20 students with experienced mathematics teachers, but who were not experienced at teaching in a virtual environment. Teachers teach five hours weekly in classes. They usually introduce the new mathematical concepts, theorems, and proofs and demonstrate solutions of exercises. The tutoring groups include three or four students. They teach 2 h per week, and, in class, they solve exercises and answer problems students have encountered. Once in 2 weeks, the tutors meet with the teachers for one hour and are inform about the topics and lesson themes that are taught and planned for the whole class. The tutor has autonomy in choosing the exercises. The study focused on the tutorial groups. The goal of the large study was to analyze the development of the expertise of mathematics tutors in a VMS. This paper focuses on presenting the model developed in the study and demonstrates how it can be used to analyze changes in tutor’s conceptions and the differences between the conceptions of different tutors.

**Participants**

Three mathematics tutors participated in the study, all of whom are undergraduate students who were hired to teach high-level math to tenth or eleventh grade students in a virtual environment. They all study STEM subjects and excel in their studies. They were carefully selected by the virtual school staff and were given a brief course on working in a VMS. They had no formal training as teachers. In this paper, our aim was to demonstrate the use of the model through two tutors who represent the conceptions of the tutors in the large study. They were chosen as representatives of the cohort of all tutors as they all had similar characteristics.

![The teaching triad (Jaworski, 1992)](image-url)
of age, academic excellence, and no formal training for teaching. Shira was a 28-year-old university student studying for a bachelors’ degree (B.Sc.) in mathematics and computer science. She was an excellent third-year student without teaching experience preceding her work in VMS. At the time of the study, she was in her first year of work as a tutor at the virtual school. Harry was a 35-year-old outstanding medical student who had competed at the highest level of mathematics in high school. During the study he was in his fifth year of medical school. This was his first experience teaching mathematics at the VMS.

Tools and Data Collection

Inspired by Leontiev’s activity theory, this paper introduces the Goal-Action Model of tutors’ conceptions. The data collection included interviews with the tutors to track changes in tutors’ conceptions about the goals of tutoring in VMS. The interviews were semi-structured and were conducted twice: once at the beginning of the tutors’ instruction, and again after 1 year of instruction. The tutors were asked a general question and, if necessary, the interviewer (the first author of this paper) asked for clarification questions to clarify the answer. Overall, the interviews included 12 general questions accompanied by two or three clarification questions. Both pre and post interviews lasted about an hour and included similar questions. (The post interview asked the tutors to provide examples from their experience during the school year.) All the interviews were audio-recorded and transcribed. The following are examples of general questions: What do you think are the goals of teaching mathematics at a high level? What do you believe is the goal of tutoring? What is your opinion regarding the role of a virtual environment in teaching mathematics? What is the difference between teaching in the VMS and regular teaching? How do you plan and prepare a lesson? On what do you base your lesson planning? What elements of tutor’s work do you believe are essential?

Additionally, we performed ongoing observations of the videotaped lessons along the school year to analyze tutors’ actions during the lessons and changes in these actions. Meetings with the VMS staff allowed triangulation and validation of data analysis. The large study relied on three sources: interviews, observations, and meetings with the VMS team. The model was built based on the interviews. In what follows, we describe in detail the data analysis with examples, present the model, and illustrate its uses to demonstrate differences between the two tutors and changes in their conceptions and actions over the year. In this paper, we are unable to include all the data from the interviews.

Data Analysis

Data analysis was performed in three main steps and combined directed and inductive content analysis (see Fig. 4). The first step was the analysis of the interviews we
 conducted using Leontiev’s (1978) distinction between the tutors’ goals and their actions to achieve their goals (see Fig. 5).

In our study, any statement by the tutors hinting at an expected outcome was interpreted as a goal they want to achieve. Any statement hinting at a way to achieve a goal was interpreted as an action. The second step, the directed analysis of the tutor’s goals was performed using students’ mathematical potential (Leikin, 2021) and the directed analysis of the tutor’s actions was performed using Jaworski’s teaching triad (Jaworski, 1992). The directed analysis devised the main categories of the tutors’ goals and actions. It was followed by an inductive analysis of the interviews (Patton, 2002) to devise subcategories for the categories of goals and actions at the level of operations and tools. Figure 5 describes the theoretical framework related to this study in accordance with the theoretical constructs used in the data analysis. Note that the magnifying glass signs the focus of the current study.

![Fig. 4 The process of building the “goals-actions” model](image)

![Fig. 5 Research foci framed by activity theory](image)
Findings

The “goals-actions” model of tutors’ conceptions related to teaching in VMS is one of the main findings of the larger study. The model is of analytical and explanatory power: it can be used to analyze and characterize tutors’ conceptions about teaching in VMS and changes in conceptions over time while teaching. In this paper, we present the development of the model in the inductive mode through exemplification of the two tutor’s (Harry and Shira) conceptions at the initial state of their work in VMS. Afterwards, we describe the model and illustrate its uses for (a) tracking the changes in tutors’ conceptions and (b) comparing conceptions of the two tutors.

Developing the Model

As mentioned, the model focuses on two aspects of the conceptions: the goals of teaching mathematics, and the actions taken to reach these goals. The three major categories of goals — MAG (mathematical ability goals), LOG (learning opportunity goals), and AG (attitudinal goals)—are used for the directed analysis included three components of students’ mathematical potential, assuming that VMS is providing opportunities for the realization the students’ potential. Three major categories of actions were devised based on the teaching triad model: MA (mathematical actions) includes consideration of teachers’ work on mathematical tasks and their choices and analysis of mathematical tasks devised for the students. DA (didactical Actions) are directed at regulation of learning process and learning interactions using didactical tools, settings and learning methods (e.g., individual vs. cooperative). SA (sensitivity-to-students actions) included tutors’ activity that was based on students’ responses, and the adjustment of planned learning trajectories to students’ needs while teaching. In addition, a category TA (technological actions) made use of the technological tools such as PowerPoint or dynamic mathematics software or other tools available in VMS. The internet communication was also supported by the VMS tools that supported teacher-student communication during the lesson. Distinguishing between the different categories of actions was not always easy, and we were supported by the context to decide about categories. We start from illustrative diagrams that emerged from the interviews with Shira and Harry, which are accompanied by excerpts from the interviews to explain how these diagrams were designed. Then we turn to the model, and then we demonstrate the uses of the model.

An Example of Excerpt Analysis — Goal & Actions — Harry

We chose to exemplify Harry’s view on the goals of tutoring in VMS using an excerpt in which he clearly determined the goal “Adapt teaching to students’ level” (LOG3) (see Fig. 8 at the top) since this goal was identified in interviews with each tutor. This goal is unique to the tutorial classes and includes adjusting the material, the pace, and the level of study to each student’s abilities. This is
one of four goals that have been assigned to the main category LOG—learning opportunity goals. This category was designed to provide special opportunities for students in their tutorial classes (the number of students is small and there is an opportunity for personal attention) where tutors can achieve goals that are difficult to achieve during teachers’ lessons. As mentioned, two semi-structured interviews were conducted. Attached are excerpts from an interview with Harry at the beginning of his teaching.

**Excerpts from an interview with Harry**

**Interviewer:** What do you believe is the goal of tutoring?

**Harry:** In my lessons, my role is to give the pupils a chance to practice at a high level according to their ability [H-1]. In classes with their teachers, on the other hand, teachers teach the material, and show basic exercises to demonstrate the subject, but there is no sequence of exercises graded by difficulty. In the tutorial classes there is a sequence of exercises, there are almost no study sections [H-2]

**Interviewer:** How do you organize the work with the students in class?

**Harry:** Usually, I deal with topics that pupils did not understand in their lessons with their teachers. [H-3]. In most cases I present an exercise and ask them to start solving [H-4], I see where everyone is stuck and from there, I help [H-5]. I choose exercises for them and tell them what problems to solve for homework [H-6]

**Interviewer:** How do you help students deal with the difficulties?

**Harry:** Based on their answers I identify the difficulties and try to address any difficulty [H-7]. As they solve the problems, I identify the difficulties of each pupil [H-8], which allows me to strengthen the skills of that specific pupil [H-9]

**Interviewer:** How do you choose the tasks?

**Harry:** I choose the tasks at the level of the pupils [H-10]. This way, I can tell what they know or what they must learn [H-11]. I present the mathematical problem and give them time to work on it; they offer solutions [H-12]. I immediately identify the difficulties; I ask them to explain how they arrived at an answer [H-13]. In the lessons with their teacher, they are more passive, and in the tutorials, they are the ones responsible for their learning

**Interviewer:** How do you identify the difficulties?

**Harry:** They can ask in class, and if I see that they have not been able to solve the homework problems, they ask me during the tutorial classes [H-14]

**Interviewer:** What helps you answer students’ questions?

**Harry:** There is continuous communication on WhatsApp, which is another channel for asking questions [H-15]. To answer their questions, I use presentations [H-16]. Sometimes I must go back and explain again [H-17] …. I sometimes offer a more detailed explanation [H-18]

Harry stated in the first interview that it was important for him to identify the difficulties of his students and to adapt the teaching to their needs. He noted that his primary role in tutorial classes is to work with students according to their ability [H-1]. Initially, he maps students’ knowledge and locates their difficulties by giving them exercises [H-3]. After identifying their difficulties, he helps according to their needs and adjusts his teaching [H-5, H-7]. This enables him to obtain accurate information about each student’s understanding of the material being studied [H-8], and subsequently he can gradually assign tasks to the students and identify their level [H-9]. Homework exercises are also adjusted to students’ needs [H-2, H-6]. We see these statements as an indicator of his view of his role as a teacher in achieving the goal of adapting teaching to students’ level. Referring to the actions (see Fig. 8 at the bottom), from Harry’s statements,
we identified six actions he used to achieve this goal: (a) He offers each of the students exercises that match their level [H-10], and thus he identifies their difficulty [H-11]- here we identify mathematical action (MA7 — *provide tasks tailored to the level of knowledge*); (b) He encourages his students to solve tasks independently. The students are the ones who offer the solutions, and through their answers, he identifies the difficulties they encounter [H-4, H-12, H-13]. We adapted it to didactic action (DA1 — *encourage self-solving tasks*); (c) Harry encourages them to ask questions during class [H-14]. This is a mathematical action (MA5 — *allow questions*), which is not always achieved during the lessons with the teacher. (d) He also allows them to ask questions after the lesson via WhatsApp [H-15], which means he uses technological action (TA3 —*use technological communication*). (e) To answer their questions, he uses presentations built by the development team [H-16] also technological action (TA1— *use presentation*). (f) Finally, he gives detailed explanations [H-17, H-18], which is another didactic action (DA4 — *provide explanations by the tutor*).

For this example, we built the following diagram (see Fig. 6) that describes the goal and actions that Harry declared that he uses to achieve it. In this way, diagrams were constructed for all the goals and actions identified in the interviews with all three tutors.

**An Example of Excerpt Analysis — Goal & Actions—Shira**

We will demonstrate another category for a goal from the main category — develop mathematical abilities — “Develop Mathematical Understanding” (MAG1). Attached are citations from an interview with Shira at the beginning of her teaching.

![Fig. 6 A diagram of goals-action — Harry’s first Interview](image-url)
Excerpts from an interview with Shira

**Interviewer:** What do you think are the goals of teaching mathematics at a high level?

**Shira:** My goal is to give students the ability to learn mathematics from comprehension [S-1]. For example, not only finding the extreme points of a function, but also asking how many intersection points a function will have with the line $y = k$

**Interviewer:** What, in your opinion, is the goal of tutoring?

**Shira:** I direct students to understanding beyond the technique, so they will truly understand what they are doing [S-2]. This is something that opens doors later in school and life [S-3]. I tell my pupils that I think learning mathematics at a high level has more depth; it touches a bit on academic material, so it is important to understand what is going to happen next if they want to continue to learn science [S-4]

**Interviewer:** What elements of a tutor’s work with students seem to be essential for you?

**Shira:** In terms of practice, I think the goal is to understand things in depth [S-5]. If there is an unclear point, then it is possible to give an individual lesson where I can help the pupil arrive at an understanding of the subject. This can help them advance [S-6]. In lessons with the teacher, this does not happen in the same way because there are 25 students. In the tutorial classes, there is an opportunity for reinforcement and for reaching every student

**Interviewer:** How do you plan and prepare a lesson? On what do you base your lesson planning?

**Shira:** I am very pedantic and strict about having an in-depth understanding of the material, trying to teach them to understand [S-7]. For example: if you give a lot of data in geometry, then you first need to organize it, and sort what data needs to be found and see if the $x$ we found is appropriate, and of course review the answer [S-8]. I ask them to test themselves in other ways and solve by other methods [S-9]

One of the most important goals of the tutor, Shira notes in the interview, is to get students to learn based on understanding. In her view, students should not be satisfied with learning procedures and technique, but they should strive to understand everything in depth. She sees this as a unique goal for tutorial classes. Understanding the material is a principle that guides Shira in planning her lessons [S-1, S-2, S-4, S-5, S-7].

Referring to the actions (see Fig. 8 at the bottom) to achieve this goal, Shira emphasizes four actions: (a) She believes that when one learns based on understanding, it “opens doors for the future” especially if one plans to continue in scientific academic studies [S-3]. We attributed this as sensitivity-to-student action (SA3 — link between mathematics and future events); we identified three more mathematical actions: (b) To develop mathematical understanding, she encourages her pupils to look for solutions in other ways [S-9]—(MA1 — encourage solutions in a variety of ways); (c) she encourages them to test and evaluate their solutions [S-8] — (MA2 — encourage criticism of the solution); (d) she conducts individual lessons in which she assigns pupils tasks tailored to their level of knowledge and focuses on the point where the pupil has difficulty understanding. She says this move “gives him a jump” [S-6] — (MA7 — provide tasks tailored to the level of knowledge). The following diagram (see Fig. 7) describes Shira’s goal and the actions she took to implement it.

The other categories were similarly constructed based on quotes from tutors’ interviews. Below, we present the model that emerged throughout this process of analysis.
The Model

Activity theory (Leontiev, 1978) framed the study design and led to “Goal-Action” model of tutor’s proficiency in VMS. The goals part of the model is rooted in the construct of students’ mathematical potential (Leikin, 2021) and the Action part was unraveled using the construct of the Teaching Triad (Jaworski, 1992). The model is used for comparative analysis of conceptions and changes in conceptions of tutors. The main categories emerged from the directed analysis (research literature) and the sub-categories emerged from the inductive analysis (the interviews). Figure 8 describes the “goals-actions” model of tutors’ conceptions related to teaching in VMS. According to the “mathematical potential” model, three main categories were defined for goal analysis: (a) MAG (mathematical ability goals) — development of students’ mathematical abilities, which include understanding, thinking, and advancing work proficiency; (b) LOG (learning opportunity goals) — providing special opportunities for students in the practice classes, which include completing material, solving high-level problems, adapting teaching to difficulties and transferring responsibility for learning to students; and (c) AG (attitudinal goals) — develop a positive attitude towards mathematics for example, believing in their ability and motivation. Nine sub-categories for goals were identified (see Fig. 8 at the top). The framework chosen as appropriate for classifying and characterizing the actions in the “goals-actions” model was built on the “teaching triad” model (Jaworski, 1992; Potari & Jaworski, 2002). The model links teaching characteristics to three areas in which the teacher is involved: learning management, students’ sensitivity, and a mathematical challenge. Based on this model, the tutors’ statements about the actions they performed with the students were classified as corresponding to one of the four main categories. They include the types of actions associated with MA (mathematical actions) aimed at promoting mathematics learning, deepening reasoning, the ability to ask questions and critical thinking. Actions related to sensitivity towards students (SA) reflected tutors’ consideration of students’ needs both
in terms of the cognitive aspects and in cultivating a personal attitude. DA (didactic actions) described the actions of the tutor to improve learning management and TA (technological actions) were those that indicated the use of technology in the virtual learning environment. Twenty-three subcategories were identified for the actions to achieve the goals. (See Fig. 8 at the bottom).
**Implementing the Model for Comparative Analysis of Conceptions**

Below, we present examples that illustrate the applicability of “Goals-Action” model for analyzing tutors’ proficiency. In Example 1, we used the model to show an analysis of a goal that we found in the two interviews with Harry and Shira, one at the beginning of the year and the other after 1 year of instruction, and the actions they used to achieve that goal. With the help of the model, we can reveal the changes in their conceptions regarding the fulfilment of the goal. In the larger study, we analyzed the goals of all the tutors and their actions to get a full picture of the changes that took place in their professional development during a year of teaching. In example 2, we used the model to analyze a goal we found at the same time among two different tutors. Using the model, we can reveal the differences in different tutors’ conceptions regarding the achievement of the goal, and hence, we can draw conclusions about the differences between the conceptions of the different tutors.

**Example 1: Change in diagrams—same tutor—different times—same goal**

The following diagrams (see Figs. 9 and 10) describe the same goal that Harry and Shira stated they wanted to achieve in both interviews and the actions they took to achieve it. The goal is: “Adapt teaching to students’ level.” From looking at the diagrams, we learn that this goal is important to them throughout their teaching. By comparing the actions Harry used to achieve the goal (see Fig. 9), we can see that the didactic action he consistently used in both sessions is encourage self-resolving tasks, which indicates the importance of this action to Harry’s conception. The change in didactic actions was that at the beginning of the year he provided explanations to the pupils, but at the end of the year he discussed topic chosen by the students, and as a result, adjusted the instruction to their level.

From the aspect of mathematical actions both at the beginning of the year and at the end of the year, he saw the importance of provide tasks tailored to the level of knowledge of the pupils and allow questions. At the end of the year, another mathematical action was added – encourage criticism of the solution, which also provided him with information about the student’s level and helped him plan the continuation of the teaching accordingly. The technological actions he used (presentations and technological communication) were mentioned in the first interview, while at the end of the year he did not mention these actions, perhaps because they became routine in the virtual environment, and he did not see the need to describe them in detail. At the end of the year, however, Harry notes sensitivity-to-students actions (refer errors, personal contact with students) that he used to achieve this goal, which indicates a change in his conception of adapting teaching to the level of students not only cognitively, but also emotionally.

In conclusion, from the example, we can see that Harry’s conceptions changed. The change was reflected in the addition of sensitivity-to-students actions not mentioned earlier this year and the disappearance of the technological actions he mentioned earlier this year. Another change is reflected in the transition from the tutor’s conception as explanatory (teacher-focused) to the conception that the students are...
the ones navigating, choosing the topic of discussion, and criticizing the solution (student-focused). This finding is significant and implies his learning while teaching as he did not receive teaching training.

By comparing actions that Shira used to achieve the goal (see Fig. 10), we can see that the didactic actions used in both sessions were encourage self-solving tasks and discuss topic chosen by students. Mathematical actions performed by Shira both at the beginning and at the end of the year provide tasks tailored to the level of knowledge. At the end of the year, Shira integrated encourage solutions in a variety of ways among actions that were important to her. Whereas at the beginning of the year she used technological action as use of presentations, at the end of the year, she broadened technological actions to use technological communication, use of websites, and complete lesson by watching the recording.
To summarize, from the example, we can see that the change in Shira’s conceptions was mainly the addition of technological actions, addition of mathematics actions, and maintaining the same didactic actions. Shira did not mention any sensitivity-to-student actions at all (either at the beginning of the year or at the end of the year).

**Example 2: Comparison Between Two Tutors**

The following diagrams (see Fig. 11) describe the same goal that both Harry and Shira stated they want to achieve after a year of teaching: Adapt teaching to the students’ level. The diagrams also describe the actions each of them used to achieve the goal and differences at the same time-same goal. From the example we can see that in the interview after a year of teaching, Harry and Shira emphasize the importance of the goal of adapt their teaching to the level of the students. This goal is from the field of “provide learning opportunities for students” and is expected from tutors’
classes, where the number of students is small, and there is an opportunity to pin-
point students’ difficulties and adapt their teaching to these difficulties. Regarding
the actions they declare to use to achieve the goal of adapt teaching to students’
level, they both seem to use identical didactic actions, allowing students to select a
topic for discussion and encouraging them to solve the tasks independently. In terms
of mathematical actions, they both indicate that in the tutorial classes, they try to
give students assignments consistent with their level. Shira also encourages them
to solve problems in different ways, while Harry allows them to ask questions and
e ncourages them to review the solution. The main difference between them is that

Fig. 11  Diagrams of goals-action from Harry’s second interview and Shira’s second interview
Shira indicates more use of technological actions that are appropriate for the teaching environment and does not mention sensitivity-to-students actions at all, while Harry does not mention technological actions. In addition, Harry emphasizes sensitivity-to-students actions he uses from both the cognitive domain by referring to students’ mistakes and from the emotional domain by indicating personal contact with them, while Shira does not.

Discussion

The purpose of this paper is to introduce the “goals-actions” model of instructional skills linked to tutors’ practices in a VMS. We demonstrated how the model can be used to analyze changes in tutors’ conceptions about teaching in a VMS. To achieve this goal, we used a multiple case study methodology through tracing learning through the teaching of three tutors. Activity theory (Leontiev, 1978) framed the analysis of individual interviews with the tutors. Data analysis combined directed and inductive content analysis. The directed analysis regarding tutors’ conceptions of instructional goals relied on the model of students’ mathematical potential (Leikin, 2021). The directed analysis of the tutor’s actions was performed using Jaworski’s teaching triad (Jaworski, 1992). These two theoretical models framed the two major components of the “goal-action” model devised by the research described here. In this way, the goal component of the model comprised three main categories and the action component integrated four main categories. The inductive analysis of the interviews elaborated conceptualization of the goals and actions through identification of the sub-categories of tutors’ conceptions of instructional goals and the ways the actions they took to implement these goals in their teaching (see Fig. 8).

As recommended by Schoenfeld (2000), models in mathematics education should be descriptive and explanatory and have a wide scope. The descriptive power of the “goal-action” model presented is reflected in several domains. First, the model allows for a better understanding of the conceptions of tutors/teachers regarding goals and actions. The diagrams describe the goals and actions. Moreover, in the larger study, we used the model to explain the tutors’ practice in the lessons in terms of goals and actions and the connection between the declarative conceptions and implementation of goals in the lessons. Therefore, it will be possible to follow the professional development of tutors. With the help of the model, we can explain the changes in the conceptions of the tutors during their teaching. In Example 1, Harry’s change in proficiency during the school year was expressed in the use of a wider variety of actions to achieve the goal of \textit{adapting teaching to students’ level}. In particular, the change during the school year was reflected in the addition of sensitivity-to-students actions not identified at the beginning of the year. These actions, such as \textit{refer to errors} and \textit{personal contact with students}, reflect sensitivity to students and alertness to the psychological and social aspects of mathematics learning considered in the research literature to be crucial for supporting student learning (Luo et al., 2001; Tawfeeq, 2010). Especially in an online environment, one of the most important roles ascribed to the online instructor is to create “personal connections” (Martin et al., 2019).
Another notable finding was related to the change in the didactic actions taken by Harry reflected in the transition from *provide explanations* to *discuss a topic chosen by students*. This shift is not trivial since Harry has no training as a teacher. According to research, one major difficulty for virtual classroom teachers relates changing the role of the teacher to enable students to be at the center and the teacher as the facilitator (Chang et al., 2011; Dhawan, 2020; Martin et al., 2019).

The explanatory power of the model is expressed in the possibility to explain the differences between different instructional practices by comparing instructor’s goals. For example, in example 2, we saw that after a year of teaching Harry and Shira both noted the importance of the goal of *adapting teaching to students’ level*. They used common declared actions: didactic actions, such as ‘*encourage self-solving tasks*’ and mathematical actions, such as *provide tasks tailored to the level of knowledge*. This indicates their pedagogical conception of the role of the tutor in this environment where the student is at the center and the tutor serves as a guide (Beck & Ferdig, 2008; Bennett & Lockyer, 2004; Martin et al., 2019; Wiesenberg & Stacey, 2008). However, noteworthy differences were found as well. To adapt the teaching to the level of the students, Harry used sensitivity-to-students actions both cognitive and emotional. For example, ‘*personal contact with students*’ is considered in the research literature as showing sensitivity to students, which is expressed in making personal contact with students (Potari & Jaworski, 2002). Shira, on the other hand, emphasized technological actions that the environment enables, for example, *complete lesson by watching the recording*. She used technological actions which suggests her belief that technology can transfer responsibility for learning to students as recommended in studies, which hold that in distance learning students must be independent and take responsibility for their learning (Dhawan, 2020). Overall, these examples illustrate the applicability of the model by capturing the change that Harry declared in using actions to achieve the same goal after a year of teaching, as well as the differences stated by the two tutors in terms of the actions they used to achieve the same goal.

The model has a wider scope than that presented here. The main categories in the model were constructed based on models from the literature and provide a framework for looking at different aspects of teaching and learning. In this study it was found that these frameworks match the categories that emerged from the field in the inductive analysis. Although the study presents a case of tutors using a model built specifically for them, we can assume that the main categories in the model can also be used with other populations, such as analyzing preservice teachers’ classes in the teachers’ internship year or in teacher development evaluation frameworks. In addition, the model can be used in teaching mathematics in different environments (not just in virtual school), in research about using a particular approach to teaching, and even in research in other fields (not just in mathematics) with matching of certain categories. The main categories of the model regarding goals and actions can serve as cornerstones for the adapted models, while the sub-categories can serve as a basis for change depending on the analysis of the interviews with the studied population.

To the best of our knowledge, there have been no studies that examine the work of tutors in a virtual high school environment. Our research is based on an in-depth observation of work done by tutors not formally trained in teaching mathematics,
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who played a novel role in high school math classes. This study contributes to previous research and makes a theoretical as well as a methodological contribution by expanding upon and integrating previous models. The model provides tools for characterizing tutors’ instruction and a language in which goals and actions can be discussed. The results of the study allow teachers, researchers, and developers to understand in depth the conceptions of tutors in a virtual classroom. The model also serves as a tool for analyzing and characterizing tutors’ conceptions in mathematics tutorial groups in a virtual environment. The categories in the model suggest a conceptualization of the goals and actions. Studies conducted among university teachers found that knowledge of tutors’ conceptions and the problems that awaited them helped identify ways to improve their teaching (Luo et al., 2001). Based on these studies, it seems reasonable that to build teacher training programs in virtual schools, one must first learn about the tutors’ conceptions of teaching in a VMS. The model proposed in this paper serves as a first step toward achieving this goal. The findings and examples presented may help teacher educators create workshops to increase tutors’ awareness of the essential characteristics of tutoring in a virtual environment such as highlighting changes in the role of the teacher and how to use the technology in this environment (Chang et al., 2011; Martin et al., 2019; Prensky, 2008).

There are several limitations in this qualitative study. The sample size of three online tutors was very small. Conducting the study on a larger scale with more tutors and across other disciplines might strengthen our findings. In addition, due to time constraints, we were unable to perform a responsive verification of the interviews (i.e., a peer review), which could have improved the accuracy, reliability of our results, and the implications of our findings for other populations. Another limitation of this study is the unique student populations which may impact the categories obtained. Many research questions remain unanswered in the field of online learning in mathematics tutorial groups. This article contributes to our understanding of the conceptions of online tutors and the changes that occurred in their teaching during the year. In the large study, we continued to use the categories both for examining the tutors’ lessons and for exploring the connections between their declared conceptions and their practices in their lessons. For example, we can examine whether the change in Harry’s conceptions in the context of the goal of adapting teaching to the students’ level is also reflected in his lessons, and whether there has been a change in the structure of the lessons and the type of questions asked. Preliminary findings show that there are conceptions that are both declared and expressed in the lessons, such as the goal of adapting teaching to students’ level and the didactic action encourage self-solving tasks that were implemented in the tutors’ lessons. On the other hand, there were declared conceptions that were not found at all in the lessons, for example, the goal of encourage thinking. These preliminary findings indicate the power of the model.

This study opened new doors for future research that may examine conceptions of other populations, give a perspective on teacher development, and compare the development of different teachers. This type of information is important not only for researchers, but for online tutors as well as for schools planning an online teacher training program.
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Declarations

Ethical Approval and Consent to Participate The study reported in this article has informed consent from all the tutors included in the study.

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