Excitation of Spin Waves in Superconducting Ferromagnets

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This Letter presents a theoretical analysis of propagation of spin waves in a superconducting ferromagnet. The surface impedance was calculated for the case when the magnetization is normal to the sample surface. We found the frequencies at which the impedance and the power absorption have singularities related to the spin wave propagation, and determined the form of these singularities. With a suitable choice of parameters, there is a frequency interval in which two propagating spin waves of the same circular polarization are generated, one of them having a negative group velocity.

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Coexistence of superconductivity (SC) and ferromagnetism (FM) has been a long-standing problem since the beginning of the modern theory of SC [1, 2, 3, 4]. There has been a renewed interest to this problem, and SC-FM coexistence was discovered recently in various materials such as ruthenocuprates [4, 5], ZrZn$_2$ [6], UGe$_2$ [7], and URhGe [8]. It was also suggested theoretically that in $p$-wave superconductors, time reversal symmetry is broken, and a non-zero magnetic moment arises [9].

The existence of the FM order in a superconductor is hidden by Meissner currents, which create a magnetic moment opposite to the spontaneous magnetic moment. Moreover, in the Meissner state SC eliminates another consequence of the FM order: the equilibrium domain structure [10]. However, the Meissner currents cannot shade such an important manifestation of the FM order as spin waves. A conventional technique to probe spin modes in ferromagnets, both insulators and metals, has been observation of Ferromagnetic Resonance (FMR) [11]. There are also reports on FMR observations in materials with SC–FM coexistence [12].

In this work we theoretically investigated propagation of spin waves in a material, which possesses both SC and FM order and is irradiated by an electromagnetic (EM) wave. The first step in this direction was done by Ng and Varma [13], who studied the spectrum of spin waves interacting with vortex modes in the spontaneous vortex phase (mixed state in zero external magnetic field). Here we analyze the boundary problem for the Meissner state: how an incident EM wave can generate spin waves inside a sample. We solved the Landau-Lifshitz equation for the magnetization and the equations of London electrodynamics, assuming that the equilibrium magnetization is normal to the surface and there is no external static magnetic field and, as a result of it, no Meissner currents at equilibrium. We found the spectrum of spin waves for two cases, depending on the stiffness of the spin system. If the stiffness is large enough, the spectrum looks similar to that of a FM insulator, with the minimum wave frequency (threshold for spin wave propagation) equal to the frequency of the uniform FMR, i.e. corresponding to zero wave vector [11]. But the opposite case of small spin stiffness is more interesting. Then propagation of spin waves becomes possible at frequencies lower than that of the uniform FMR, and waves near the threshold have finite wave vectors. Moreover, a monochromatic incident EM wave generates two propagating spin waves with slightly different wave vectors. One of them has a negative group velocity, i.e., its direction is opposite to that of the phase velocity. We found the surface impedance, which is intimately connected with the spin wave spectrum. The analysis was focused on the case without explicit dissipation terms in the dynamical equations, when all energy absorbed from the incident radiation is carried away by spin waves propagating deep into the sample bulk. But we discuss also the role of various dissipation sources. The analysis demonstrates that the microwave probing of spin modes, which has been so fruitful for normal ferromagnets, should be also an effective technique for investigation of unusual properties of spin modes in superconducting ferromagnets (SCFM’s).

We consider a SCFM occupying the semispace $z > 0$ as shown in Fig. 1. This is a good approximation for a slab so thick that one can neglect a possible reflection of waves from the second boundary. Uniting the magnetism approach for ferromagnets and the London approximation for superconductors leads to the following free energy functional for our material:

$$F = \int d^3x \left\{ 2\pi \alpha [M_z^2 + \ell_2^2 (\partial_i M_j) (\partial_i M_j)] \right\}$$
\[ \frac{1}{8\pi \lambda^2} \left( \frac{\Phi_0}{2\pi} \nabla \phi - \mathbf{A} \right)^2 + \frac{B^2}{8\pi} - \mathbf{B} \cdot \mathbf{M} \right), \tag{1} \]

where \( \phi \) is the phase of the SC order parameter, \( \Phi_0 = \hbar c/2e \) is the magnetic flux quantum, \( \mathbf{M} \) is the FM magnetization, and \( M_L \) is the magnetization component perpendicular to the easy axis \( \hat{z} \), which is normal to the sample surface. At equilibrium the magnetization is \( \mathbf{M}_0 = M_0 \hat{z} \) \((M_L = 0)\), and the large anisotropy parameter \( \alpha > 1 \) (the ratio of the anisotropy energy to the magnetostatic energy) ensures that our configuration is stable against the flip of \( \mathbf{M}_0 \) into the plane parallel to the surface. In our geometry the static magnetic induction \( \mathbf{B}_0 \) vanishes both inside and outside the sample, but the magnetization creates a magnetic field inside the sample: \( \mathbf{H}_0 = -4\pi \mathbf{M}_0 \). In accordance with the micromagnetism approach, the absolute value of \( \mathbf{M} \) is assumed to be constant \( M = M_0 \), and hence terms dependent on it were omitted. The length \( l_d \) which is of the order of the domain wall thickness in normal ferromagnets, characterizes the stiffness of the spin system. We consider a single-domain sample, since in SCFM’s in the Meissner state there are no domains at equilibrium \([10]\).

The spin dynamics is governed by the Landau-Lifshitz equation \([11]\)

\[ \frac{d\mathbf{M}}{dt} = -g \left( \mathbf{M} \times \frac{\delta F}{\delta \mathbf{M}} \right), \tag{2} \]

where \( g \) is the gyromagnetic factor. Since we are concerned here with motion near the equilibrium, we can decompose

\[ \mathbf{M} = \mathbf{M}_0 + \mathbf{m}, \tag{3} \]

where \( \mathbf{m} \perp \mathbf{M}_0 \) is the dynamic part. For plane waves \( \propto e^{i(kz - \omega t)} \), Eq. \((2)\) yields:

\[ -i\omega \mathbf{m} = -g \mathbf{M}_0 \times \left\{ 4\pi \alpha (1 + i\frac{\lambda^2}{k^2}) \mathbf{m} - \mathbf{b} \right\}, \tag{4} \]

where, since the equilibrium magnetic induction vanishes, there is only a dynamic part of the induction: \( \mathbf{B} = \mathbf{b} \). To find \( \mathbf{b} \), we minimize the free energy with respect to the vector potential \( \mathbf{A} \). This yields the London equation:

\[ \left( 1 + k^2 \lambda^2 \right) \mathbf{b} = 4\pi k^2 \lambda^2 \mathbf{m}. \tag{5} \]

Substituting this into Eq. \((4)\), we obtain the equation of motion for the magnetization:

\[ -i\omega \mathbf{m} = -4\pi g \mathbf{M}_0 \times \mathbf{m} \left[ \alpha (1 + i\frac{\lambda^2}{k^2}) - \frac{k^2 \lambda^2}{1 + k^2 \lambda^2} \right], \tag{6} \]

with the dispersion relation

\[ \omega = \pm \omega_{fm} \left( 1 + i\frac{\lambda^2}{k^2} - \frac{k^2 \lambda^2}{\alpha (1 + k^2 \lambda^2)} \right), \tag{7} \]

where \( \omega_{fm} = 4\pi \alpha g M_0 \) is the frequency of the uniform FMR at zero internal magnetic field for normal FM’s. In the limit \( \lambda \to \infty \) the spin wave spectrum should transform to that of an insulator. Neglecting the spin stiffness \((l_d \sim 0)\) the latter can be presented as \( \omega = g(H_a + H) \), where \( H_a = 4\pi M_0 \alpha \) is the anisotropy field \([11]\). This agrees with Eq. \((7)\) (at finite \( k)\) bearing in mind that \( H = -4\pi M_0 \) in our geometry (see Fig. 1). However, whatever large \( \lambda \) could be, at very small \( k \ll 1/\lambda \) the dynamic induction is screened out, as evident from Eq. \((3)\), so at these scales the spin-wave spectra in superconducting and in insulating ferromagnets are different.

The Maxwell equation \( \partial \mathbf{b}/\partial t = -e \nabla \times \mathbf{e} \) and the London equation, Eq. \((4)\), relate the EM field inside the sample to the magnetization:

\[ \mathbf{b} = \mathbf{b} - 4\pi \mathbf{m} = -\frac{4\pi}{1 + k^2 \lambda^2} \mathbf{m}, \]

\[ \mathbf{e} = \frac{\omega}{kc} \mathbf{b} \times \hat{z} = \frac{\omega \lambda}{c} \frac{4\pi k \lambda}{1 + k^2 \lambda^2} \mathbf{m} \times \hat{z}. \tag{8} \]

The two signs in Eq. \((7)\) correspond to two senses of circular polarization: \( \mathbf{m}^{(e)} = \mathbf{m}^{(\lambda)} (\hat{x} \pm i \hat{y}) \). Only positively polarized waves \( \mathbf{m}^{(+)\lambda} \), which correspond to the upper sign, can propagate inside the sample, and in the following we shall focus on this polarization. The form of the spectrum depends on the ratio of the domain wall thickness \( l_d \) to the London penetration depth \( \lambda \), as shown in Fig. 2. If \( k^2 \lambda^2 \gg 1 \), the minimal frequency (threshold for spin-wave propagation) is \( \omega = \omega_{fm} \) at \( k = 0 \). On the other hand, if \( l_d \gg \lambda \), then the minimal frequency is at a finite wave vector, \( k_m = (1/\sqrt{\alpha}, l - 1/\lambda^2)^{1/2} \), and has a lower value, \( \omega_m = \omega_{fm} \left[ 1 - (1/\sqrt{\alpha} - l_d/\lambda^2) \right] \). This dip in the spin-wave spectrum was revealed by Ng and Varma \([13]\) for the spontaneous vortex phase. Waves with wave vectors satisfying \(|k| < k_m\) have a negative group velocity \( d\omega(k)/dk \), i.e., its sign is opposite to that of the phase velocity \( \omega(k)/k \). Thus in this regime a positively polarized incident EM wave with the frequency between \( \omega_m \) and \( \omega_{fm} \) excites in the material two propagating waves, one with a positive and another with a negative group velocity. This unusual property is unique to SCFM’s.

Now we turn to calculation of the surface impedance, which specifies the boundary conditions for the EM field outside the sample. The dispersion relation is a quadratic equation for \( k^2 \), so for a given frequency there should be two solutions for \( k^2 \): \( k_1^2 \) and \( k_2^2 \). These two solutions correspond to two modes generated by the incident EM wave inside the sample. Thus the situation is analogous to that in two mode electrodynamics, the theory suggested for calculation of the surface impedance of a superconductor in the mixed state taking into account the elastic degree of freedom of the vortex array \([14]\). In the present case the second mode (the second solution for \( k^2 \)) originates from the spin degree of freedom. Moreover, in contrast to Ref. \([14]\), where both modes inside the sample were evanescent \((k_{1,2}^2 < 0)\), now at \( \omega > \omega_{fm} \) one mode, and at
\( \omega_m < \omega < \omega_{fm} \) both modes are propagating.

In order to find a proper superposition of the two modes inside the sample, \( \mathbf{m}^{(\pm)}(z) = \mathbf{m}_1^{(\pm)} e^{ik_1 z} + \mathbf{m}_2^{(\pm)} e^{ik_2 z} \), we need two boundary conditions. One of them is the usual continuity of the EM field across the sample boundary. Polarization of the excited spin wave is defined by the polarization of the incident EM wave outside the sample, and the surface impedance tensor is diagonal in the circular wave basis. In this basis \( \mathbf{h} = h^{(\pm)}(z) (\hat{x} \mp i\hat{y}) \) and \( \mathbf{e} = e^{(\pm)}(z) (\hat{x} \mp i\hat{y}) \), where \( h^{(\pm)}(z) = h_1^{(\pm)} e^{ik_1 z} + h_2^{(\pm)} e^{ik_2 z} \) (and similarly for \( e^{(\pm)} \)). Then the surface impedance is

\[
\zeta^{(\pm)} = \pm \frac{e^{(\pm)}(0)}{h^{(\mp)}(0)} .
\]  

The second boundary condition should be imposed on the magnetization. The simplest possible boundary condition is \( \partial \mathbf{m} / \partial z = 0 \) at \( z = 0 \), which means absence of spin currents through the sample surface. This gives

\[
k_1 m_1^{(\pm)} + k_2 m_2^{(\pm)} = 0 .
\]  

Together with Eq. (8) this yields the field amplitudes at the surface:

\[
e^{(\pm)}(0) = \mp i \frac{4\pi \alpha \lambda}{c} \left( \frac{k_1 \lambda}{1 + k_1^2 \lambda^2} \right) \left( \frac{k_2 \lambda}{1 + k_2^2 \lambda^2} \right) m_1^{(\pm)},
\]

\[
h^{(\pm)}(0) = -4\pi \left( \frac{1}{1 + k_1^2 \lambda^2} - \frac{1}{1 + k_2^2 \lambda^2} \right) m_1^{(\pm)} .
\]  

Substituting this into Eq. (9), we obtain after simplifications:

\[
\zeta^{(\pm)} = -\frac{\omega \lambda^4}{c} \frac{k_1 k_2 (k_1 + k_2)}{1 + (k_1^2 + k_2^2 + k_1 k_2) \lambda^2} .
\]  

After finding the two roots \( k_1^2 \) and \( k_2^2 \) of Eq. (7) we need to choose the signs of \( k_1 = \pm \sqrt{k_1^2} \) and \( k_2 = \pm \sqrt{k_2^2} \). If the wave vectors have imaginary parts, they should be positive in order to ensure attenuation of the wave \( e^{ik_2 z - \omega t} \) inside the sample. But if \( k \) is purely real (propagating wave), the the sign is fixed by the condition that the energy must be carried away from the surface. One can check that the energy flux for each mode normal to the sample surface is proportional to the group velocity:

\[
P_z = -4\pi \alpha^2 \lambda^2 \frac{\partial \mathbf{m}_z}{\partial z} + \frac{c}{4\pi} \mathbf{E} \times \mathbf{H} = 2\pi \omega \alpha \lambda \mathbf{m}_z \frac{\partial^2 \omega}{\partial k^2} .
\]  

Hence in order to carry energy away from the boundary, waves with a negative group velocity should have wave vectors directed toward the boundary. Bearing this in mind, we obtain from Eq. (12) the general expression for the surface impedance:

\[
\zeta^{(\pm)} = \frac{\omega \lambda}{c} \sqrt{Q (\alpha^{-1} - \lambda^2 - 2\lambda^2 Q^2 - Q)}
\]

\[
\alpha^{-1} + \lambda Q^2 \lambda^2 - Q.
\]  

where \( Q = 1 \mp \omega / \omega_{fm} \), and the signs of the square roots are determined by the above requirements on the wave vectors (this is consistent with the condition \( \text{Re} \zeta > 0 \)).

Now we shall analyze the most interesting particular cases, when the frequency is close to thresholds at which spin-wave propagation becomes possible. Two regimes should be considered, depending on the value of \( \lambda \).

1. Stiff spin system: \( \sqrt{\alpha l_d} > \lambda \). Then the threshold frequency for spin-wave propagation is \( \omega_{fm} \), and near the threshold \( k_1 \ll |k_2| \):

\[
k_1^2 \simeq \frac{\omega - \omega_{fm}}{\omega_{fm}} \frac{\alpha}{\alpha l_d^2 - \lambda^2} , \quad k_2^2 \simeq \frac{1}{\alpha l_d^2} - \frac{1}{\lambda^2} .
\]  

Since \( \sqrt{\alpha l_d} > \lambda \), the short-wavelength mode is evanescent \( (k_2^2 < 0) \), while the long-wavelength mode is evanescent \( (k_1^2 < 0) \) at \( \omega < \omega_{fm} \) and propagating \( (k_1^2 > 0) \) at \( \omega \gtrsim \omega_{fm} \). Finally the surface impedance is

\[
\zeta^{(\pm)} \simeq -\frac{\omega \lambda^4}{c} \frac{k_1 k_2}{1 + k_2^2 \lambda^2} = \frac{\sqrt{(\omega - \omega_{fm}) \omega_{fm}}}{c} \sqrt{\alpha (\alpha l_d^2 - \lambda^2)} .
\]  

Thus, \( \zeta^{(\pm)} \) is purely imaginary below the threshold and purely real above it, where it grows as \( \sqrt{\omega - \omega_{fm}} \) (actually, there is also nonzero \( \text{Im} \zeta^{(\pm)} \) above the threshold, but it is of higher order in small \( \sqrt{\omega - \omega_{fm}} \)).

2. Soft spin system: \( \sqrt{\alpha l_d} < \lambda \). In this regime the threshold frequency has a lower value \( \omega_m < \omega_{fm} \). Near the threshold \( |k_1| < k_m \) and \( |k_2| > k_m \) are close to each other:

\[
k_{1,2}^2 \simeq k_m^2 \mp \frac{\sqrt{\omega - \omega_m}}{\omega_{fm}} \frac{1}{\alpha^{1/4} \lambda^2 / 2 l_d^2} .
\]  

Also, one should take into account that the mode with \( k_1 < k_m \) has a negative group velocity above the threshold, so the negative sign of \( k_1 \) should be chosen. Then \( \zeta^{(\pm)} \) near the threshold is given by

\[
\zeta^{(\pm)} \simeq -\frac{\omega \lambda^4}{c} \frac{k_2^2}{1 + \lambda^2 k_2^2} \frac{[k_2 - |k_1|]}{\omega_{fm}} \sqrt{\frac{\omega - \omega_m}{\omega_{fm}} \frac{\omega_{m}}{c l_d} \sqrt{1 - \sqrt{\alpha l_d} \lambda} .
\]  


Again the surface impedance is real above the threshold $(\omega > \omega_m)$ and imaginary below it $(\omega < \omega_m)$.

In addition, there is still a singularity at $\omega = \omega_{fm}$. We can use Eqs. 16 and 19, but now, since $\sqrt{\alpha l_d} < \lambda$, the short-wavelength mode became propagating ($k_2^0 > 0$).

The long-wavelength mode is now propagating ($k_1^0 > 0$) at $\omega < \omega_{fm}$ and evanescent ($k_1^0 < 0$) at $\omega > \omega_{fm}$. Thus at $\omega = \omega_{fm}$ there is a transition from two propagating modes to one. On the other hand, at $\omega \gg \omega_{fm}$ Eq. 16 yields a purely imaginary impedance despite the presence of a propagating mode with real $k_2$. In order to obtain the real part of the surface impedance we need to go to next order of the expansion in the small parameter $k_1/k_2$:

$$\text{Re} \zeta^{(+)}(\omega \gg \omega_{fm}) \approx -\frac{\omega^4 k_1^2 k_2^2}{c (1 + k_3^0 \lambda^2)^2} \frac{\lambda c}{\sqrt{\lambda^2 - \alpha l_d^2}}.$$  

In the opposite limit $\lambda \to \infty$, $\lambda^2 = i \delta^2/2$, and we obtain the surface impedance for normal FM metals [13]. However, unlike in SCFM’s, in FM metals the dissipation is an essential part of spin dynamics, which can not be assumed small.

Note that at $\omega = \omega_{fm}$ [see Eq. 16], $\zeta^{(+)}$ vanishes even after renormalization of the penetration depth, since at uniform FMR no currents are generated, so the dissipation due to normal currents is ineffective. Then, other sources of dissipation become important, in particular, the transverse spin relaxation, which is well known from studies of normal ferromagnets [11].

Our phenomenological approach is valid for any material with coexisting SC and FM order parameters, independent of their microscopic origins. Moreover, we even believe that it is not essential whether FM originates from electron spins or from the orbital moment of Cooper pairs, which characterizes the p-wave pairing in some materials [4]. In the latter case one cannot call the modes which were analyzed here, “spin waves”: they should be called “orbital waves” as in $^3$He physics. It is known from $^3$He studies that the dynamics of the orbital moment is similar to spin dynamics [11]. In the case of electron pair-

In conclusion, we have demonstrated that propagating spin waves are possible in superconducting ferromagnets in the Meissner state, similar to those in insulating ferromagnets. In contrast to the latter, in SCFM’s with low spin stiffness there is a frequency interval, in which there are two propagating spin modes of the same circular polarization, one of them having a negative group velocity, i.e., with the direction opposite to that of the phase velocity. We solved the boundary problem and calculated the surface impedance for the case of the easy magnetization axis normal to the surface of the sample. Revealed singularities of the dependence of the surface impedance on the frequency provide experimentalists with a tool for probing spin-wave dynamics by microwave measurements of materials with coexisting FM and SC.

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