The antikaon potential in nuclear matter at finite momentum

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We study the antikaon potential at finite momentum for proton and neutron nuclear matter. Our approach is based on the momentum dependence of the real part of the total $K^-N$ scattering amplitude, which is fixed in free space by experimental data, using the dispersion relation approach. We decompose the contributions to the scattering amplitude from different processes and consider the $\Lambda(1405)$ and $\Sigma(1385)$ to be dissolved in nuclear matter at density $\rho_0$. Within this approach the $K^-$ potential is found to be attractive up to high kaon momenta. Our result is in line with the data on the antikaon potential evaluated from the analysis of kaonic atoms (low momenta), heavy-ion (intermediate momenta) and proton-nucleus (high momenta) collisions.

The antikaon potential in nuclear matter at finite relative momentum at present is a question of vivid interest, which is partly discussed in a controversial manner. The analysis \cite{1} of data on kaonic atoms leads to an antikaon potential of $\approx -180$ MeV at normal nuclear density $\rho_0$, while the studies \cite{2} of $K^-$-meson production from heavy-ion collisions \cite{3} suggest an attractive potential $\approx -100\div120$ MeV. We have proposed in Ref. \cite{4} to attribute this discrepancy to the momentum dependence of the antikaon potential, since the kaonic atoms explore stopped antikaons with $p_K\approx0$, while the heavy-ion experiments have probed the range $300\leq p_K\leq600$ MeV/c. The evaluation of the momentum-dependent potential is a rigorous problem and substantially depends on the model applied. Here we discuss a dispersive approach in which the uncertainties are under theoretical and experimental control.

Within the low density theorem the real part of the antikaon potential at baryon density $\rho_B$ is given in terms of the real forward $K^-$-nucleon scattering amplitude $D$ as

$$
U(\rho_B, E_K) = -\frac{2\pi}{m_K\rho_B} D(E_K),
$$

(1)

where $E_K$ is the kaon energy relative to the nuclear matter rest frame. It is important to note that $D$ here is the total scattering amplitude including all possible processes available at given antikaon energy $E_K$. It is obvious that with increasing $E_K$ the number of open reaction channels becomes very large such that all channels cannot be calculated separately anymore. The contribution from the individual reaction channels then can be controlled only by the relative saturation of the total cross section. In case of the $K^-p$ interaction the $\Lambda(1405)$ resonance dominates at low energy, but apart of its specific role there are contributions from other channels as indicated by the data on the total cross section \cite{5}. Thus models relying exclusively on the $\Lambda(1405)$ properties in the medium have to be taken with great care.

An alternative way is to evaluate the total real forward scattering amplitude $D$ from the total cross section directly, which is controlled by experimental data. The advantages of this method are i) an absolute completeness of the scattering amplitude $D$ with respect to all available channels, and ii) an independence on the hyperon resonance properties in the medium at sufficiently high density where the latter are 'melted'. In free space the real part of the forward total scattering amplitude $D$ can be evaluated from the dispersion relation as \cite{6}.

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\[ D(E_K) = D(E_K=0) + E_K \sum_{j=1}^{2} I_j(E_K) \]
\[ + \sum_{Y=\Lambda}^{g_{KNNY}^2} \frac{g_{KNNY}^2}{16\pi m_N} \frac{E_K [m_K^2 - (m_Y - m_N)^2]}{E_Y(E_K - E_Y)} \]

where \( E_\Lambda = 64 \text{ MeV} \), \( E_\Sigma = 155 \text{ MeV} \) while \( I_1 \) stands for the contribution from the unphysical domain \( E_K < m_K \):
\[ I_1 = \frac{1}{\pi m_N} \int_{E_\Lambda\pi}^{m_K} d\omega A(\omega) \sqrt{m_N^2 + m_K^2 + 2m_N\omega} \frac{\sigma^-(\omega) - \sigma^+(\omega)}{\omega - E_K + \omega + E_K} \]

with \( E_{\Lambda\pi} = 239 \text{ MeV} \). \( A(\omega) \) is the imaginary part of the forward \( K^-N \) scattering amplitude that stems explicitly from the \( \Lambda(1405) \) and \( \Sigma(1385) \) resonances. The contribution from the physical region is given as
\[ I_2 = \frac{1}{4\pi^2} \int_{m_K}^{\infty} d\omega \sqrt{\omega^2 - m_N^2} \frac{\sigma^- (\omega) - \sigma^+ (\omega)}{\omega - \omega + E_K} \]

where \( \sigma^- \) and \( \sigma^+ \) denote the total \( K^-N \) and \( K^+N \) cross sections, respectively, which are fixed by experimental data.

The dispersion relation can be evaluated by fitting \( D(0) \), \( g_{KNN} \) and \( g_{KNNY} \) to the data on the real \( K^-N \) and \( K^+N \) forward scattering amplitudes while taking \( A(\omega) \) from the \( M \)-matrix low energy solution. We note that the relation between the \( K^-N \) and \( K^+N \) scattering amplitudes is given by crossing symmetry.

The solid line in Fig. 1 shows the ratio of the real \( D \) to imaginary part \( A \) of the forward \( K^-p \) scattering amplitude in comparison to the data from [8]. The lines show the dispersion calculations: full (solid) and without the contribution \( I_1 \) from the unphysical domain (dashed), but adjusted to data (dotted) by \( D(E_K=0) \).

Fig. 2 shows the contribution to the total forward \( K^-p \) scattering amplitude from the pole term, the unphysical \( (I_1) \) as well as from the physical region \( (I_2) \). Note that apart from the \( \Lambda(1405) \) contribution and the boundary \( D(0) = -2.73 \text{ fm} \) the other contributions to the real forward scattering amplitude are positive. Furthermore, the \( \Lambda(1405) \) resonance does not couple to the \( K^-n \) system and as shown in Fig. 3 the real part of the \( K^-n \) scattering amplitude remains positive, i.e. the \( K^- \) potential in neutron matter is attractive.

Now, it is by far not obvious that the forward scattering amplitude in nuclear matter is the same as in free space, but it is clear that all terms contributing to \( D \) in the vacuum should be also considered in calculations for nuclear mat-
The experimental result on kaonic atoms [1] indicates that the $K^-N$ forward scattering amplitude is positive at $p_K \approx 0$ already at the rather low nuclear density which the antikaon experiences before decay. This implies that the amplitude $D$ is modified in matter already at rather low densities, which was attributed in Ref. [9] to the 'melting' of the $\Lambda(1405)$ since this resonance gives repulsion. In fact, dynamical calculations [10] on the low energy $K^-p$ interaction in baryonic matter relevant to the in-medium modification of the $\Lambda(1405)$ resonance also indicate an attractive antikaon potential at $p_K=0$.

In order to gain some information on the momentum dependence of the $K^-$ potential we discard the contribution from the unphysical region, but keep the other terms as fixed by the dispersion relation in free space. In principle, it might happen that the $\Lambda(1405)$ is not fully dissolved and might contribute to the total amplitude with a negative component. However, as shown in Fig. 3 and Fig. 4, the amplitude $D$ will be positive above the resonance region $p_K > 1.5$ GeV. Of course, $D$ may change sign going through a resonance. Assuming this hypothesis for a moment, one has to conclude that to saturate both the low energy (kaonic atoms) and high energy limits the amplitude $D$ should change its sign at least twice. However, our analysis as well as the data do not indicate such a strong $K^-$ coupling to resonances (as compared to the non-resonant contributions) which could produce this double oscillation in momentum of the real forward scattering amplitude.

Fig. 3. The real part of the forward $K^-n$ scattering amplitude as a function of the antikaon momentum $p_K$.

Fig. 4 shows the antikaon potential for the proton and neutron nuclear matter calculated at $\rho_B=0.16$ fm$^{-3}$ and averaged over the Fermi distribution with Fermi momentum $p_F \approx 260$ GeV/c. At $p_K=0$ our result is in line with calculations from different dynamical models [11]. With increasing antikaon momentum the potential increases but remains attractive. For neutron mat-
ter it is smaller than for isospin symmetric matter as obtained from averaging over isospin.

In summary, within the experimental uncertainties our result is in agreement with the data on kaonic atoms as well as heavy-ion collisions. Furthermore, the FHS Collaboration recently reported data on $K^-$ production from $p+Be$ collisions where antikaons with $p_K = 1.28 \text{ GeV/c}$ were detected as a function of the beam energy. The analysis of the data indicates an attractive $K^-$ potential of $\simeq -74 \pm 198 \text{ MeV}$ for $p_K = 1.28 \text{ GeV/c}$, which is also compatible with our calculation, however, excludes a repulsive potential.

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REFERENCES

1. E. Friedman, A. Gal and G. Batty, Phys. Lett. B 308 (1993) 6; Nucl. Phys. A 579 (1994) 518; E. Friedman, A. Gal, J. Mareš and A. Cieplý, nucl-th/9804072.
2. G.Q. Li, C. M. Ko and X.S. Fang, Phys. Lett. B 329 (1994) 149; W. Cassing et al., Nucl. Phys. A 614 (1997) 415; G.Q. Li, C.H. Lee and G.E. Brown, Nucl. Phys. A 625 (1997) 372; E. L. Bratkovskaya, W. Cassing and U. Mosel, Nucl. Phys. A 622 (1997) 593; Phys. Lett. B 424 (1998) 244; W. Cassing and E.L. Bratkovskaya, Phys. Rep. 308 (1999) 65.
3. A. Schröter et al., Z. Phys. A 350 (1994) 101; P. Senger et al., Acta Phys. Pol. B 27 (1996) 2993; R. Barth et al., Phys. Rev. Lett. 78 (1997) 4007; F. Laue et al., Phys. Rev. Lett. 82 (1999) 1640; P. Senger and H. Stroebele, J. Phys. G 25 (1999) R59.
4. A. Sibirtsev and W. Cassing, Nucl. Phys. A 641 (1998) 476.
5. Landolt-Börnstein, New Series, ed. Schopper, I/12 (1988); Particle Data Group, Eur. Phys. J. A 3 (1998) 1.
6. N.M. Queen, Nucl. Phys. B 1 (1967) 207; R. Perrin and W.S. Woolcock, Nucl. Phys. B 4 (1968) 671; P. Bailon et al., Phys. Lett. 50 B (1974) 377; O.V. Dumrais, T.Yu. Dumrais

![Figure 4. The antikaon potential at $\rho_B = 0.16 \text{ fm}^{-3}$ as a function of the $K^-$ momentum calculated for proton and neutron nuclear matter.](image-url)
and N.M. Queen, Nucl. Phys. B 26 (1971) 497; R. E. Hendrick and B. Lautrup, Phys. Rev. D 11 (1975) 529; A. D. Martin, Nucl. Phys. B 179 (1981) 33; R. H. Dalitz, Eur. Phys. J. A 3 (1998) 676.

7. J.K. Kim, Phys. Rev. Lett. 14 (1965); 19 (1967) 1079; B. R. Martin and M Sakitt, Phys. Rev. 183 (1969) 1345; A. D. Martin, Nucl. Phys. B 16 (1970) 479.

8. B. Conforto et al., Nucl. Phys. B 8 (1968) 263; R. Armenteros et al., Nucl. Phys. B 21 (1970) 13; O.V. Dumbrais, T.Yu. Dumbrais and N.M. Queen, Fortschr. Phys. 19 (1971) 491.

9. G.E. Brown and M. Rho, Nucl. Phys. A 596 (1996) 503; T. Waas and W. Weise, Nucl. Phys. A 625 (1997) 287.

10. V. Koch, Phys. Lett. B 337 (1994) 7; T. Waas, M. Rho and W. Weise, Nucl. Phys. A 617 (1997) 449; A. Ohnishi, Y. Nara and W. Koch, Phys. Rev. C 56 (1997) 2767; M. Lutz, Phys. Lett. B 426 (1998) 12.

11. G.E. Brown and M. Rho, Phys. Rev. Lett. 66 (1991) 2720; T. Waas, N. Kaiser and W. Weise, Phys. Lett. B 365 (1996) 12; B 379 (1996) 34; K. Tsushima, K. Saito, A.W. Thomas and S. Wright, Phys. Lett B 429 (1998) 239.

12. Yu. T. Kiselev et al., J. Phys. G 25 (1999) 381.

13. T. Kirchner et al., COSY Proposal 21; K. Sistemenich et al., 'Strangeness in nuclei', Cracow (1992) 359; O.W.B. Schult et al., Nucl. Phys. A 583 (1995) 629c; A. Sibirtsev, H. Müller and C. Schneidereit, Z. Phys. A 351 (1995) 333; M. Büscher et al., Acta Phys. Pol. 27 (1996) 3087.