Influence of Anisotropy and Nonhomogeneity on Stability Analysis of Fissured Slopes Subjected to Seismic Action

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1.Introduction

Cracks or fissures are often found at the crest of cohesive soil slopes because of the development of tension. The presence of cracks poses significant challenges to the assessment of slope stability because cracks introduce one discontinuity in both the static and kinematic fields [1, 2]. Taylor [3] pointed out that “...the action within the tension zone is a subject that is worthy of much study.” Therefore, stability analysis of the fissured slopes is a very important practical engineering problem. The used traditional methods to analyze the effects of cracks on the stability of slopes have been categorized as follows: finite element limit analysis (FELA) methods [4, 5] and limit equilibrium (LE) methods [6–10]. Baker [11] described the procedure of the limit equilibrium method with variational extremization for the evaluation of the effect of tensile cracks of the cohesive-frictional soil slope. After this, other researchers also developed the variational limit equilibrium methods to obtain the factor of safety for giving fissured slopes [12, 13]. Another approach to analyzing the stability of fissured slopes is to use the upper bound theorem of limit analysis (LA). Limit analysis provides a powerful method for analyzing stability problems [14, 15]. Based on the LA approach, Utili [1] and Utili and Abd [2] investigated the stability of homogeneous slopes manifesting vertical cracks, and the depth and locations of cracks. Michalowski [16, 17] introduced a crack-opening process within the strict framework of LA. Because the LA approach is comparatively more convenient to use than the static approach [18], adopting it to solve the stability problem has almost exclusively concentrated on the kinematic theorem [19]. Moreover, the results from the upper bound theorem are nearly identical to those from the variational limit equilibrium closed-form solutions [20]. Therefore, the LA approach is appropriate for assessing the stability of the fissured slopes.

Seismic action can trigger slope instability, especially in the case of slopes with cracks [2, 21, 22]. The conventional
method for estimating the effect of a seismic force on the stability of a slope is the so-called "pseudostatic analysis method," which was employed by Terzaghi [23]. According to this method, the inertia force is treated as an equivalent concentrated horizontal force at the center of gravity of the critical sliding mass [24]. An analytical method based on the upper bound theorem in combination with the pseudostatic method is widely presented for the stability of the fissured slopes and subjected to seismic action [2, 22, 24–26]. In the previous study of fissured slope stability, soil was treated as an isotropic and homogeneous geotechnical material. However, due to the impact of inherent soil structures [27], cutting action [28], consolidation [29, 30], cementation [31], and soil strength linear increase with depth [18–20, 24, 32–34], soil deposits can exhibit remarkably anisotropy and nonhomogeneity behavior. Thus, ignoring the anisotropic and nonhomogeneous behavior can cause the geotechnical materials to be unrealistic in fissured slope stability analysis.

In this paper, the upper bound theorem of limit analysis together with the pseudostatic method is used to assess the stability of fissured slopes subjected to seismic action where the anisotropy and nonhomogeneity of the soil strength is considered. A rotational two-dimensional (2D) failure mechanism proposed by Chen et al. [19], Chen and Sawada [24], and Ukritchon and Keawawasvong [35] is adopted. The explicit functions about the stability factors of the fissured slope and the location of the cracks are derived, which are optimized to find out the minimum stability factor and the critical crack location, respectively. Next, the procedures for getting the safety factor of fissured slopes are presented where the effects of soil anisotropy and nonhomogeneity and seismic action are both taken into account. Furthermore, the variation in the safety factor of fissured slopes under the impact of the crack depth and the horizontal and vertical seismic acceleration is also given in this article.

2. Upper Bound Theorem of Fissured Slope Stability

2.1. Basic Assumption. (1) The research problem can be studied in a plane-strain problem; (2) the Mohr–Coulomb criterion is adopted in this study; (3) when fissured slope soil is in the critical state, a failure surface passes through the toe of the slope; (4) the anisotropic and nonhomogeneous qualities of slope soil lie only in its cohesive strength; (5) the seismic action of the sliding wedge is simplified to the static load.

2.2. Anisotropy and Nonhomogeneity of Soil Strength. Figure 1 illustrates a fissured slope with soil anisotropy and nonhomogeneity and its corresponding potential failure surface. In previous studies, the slope soil was regarded as an isotropic and homogeneous material. However, plenty of experimental tests showed that the actual soil exhibits remarkable anisotropy and nonhomogeneity naturally, which should not be ignored in slope stability analysis.

Soil anisotropy indicates that the behavior of soil shear strength changes in different directions. As shown in Figures 1(a) and 1(b), the cohesion strength $c_i$ is in the direction of the maximum principal stress, which is inclined at an angle of $i$ with the vertical direction. The cohesion strength $c_i$ formulation follows the notations presented by Lo [36] and Casagrande and Carillo [37]. A brief description of the relevant formula can be expressed as

$$c_i = c_h + (c_v - c_h)\cos^2 i = c_h \left(1 + \frac{1 - k}{k} \cos^2 i\right), \quad (1)$$

where $c_h$ and $c_v$ represent the horizontal and vertical cohesive strength, respectively. The ratio of $c_h$ to $c_v$ can be defined as $k$ ($k = c_h/c_v$), which represents the anisotropy of the slope soil. As Lee and Rowe [27] discussed, $k = 1$ for isotropic soils, $k < 1$ referred to normally consolidated or slightly overconsolidated clays, and $k > 1$ referred to heavily overconsolidated clays.

Based on the geometry illustrated in the inset of Figure 1(a), the angle, $i$, can be calculated as

$$i = \theta - \frac{\pi}{2} - \varphi + \delta, \quad (2)$$

where $\theta$ is any counterclockwise positive angle from the initial radius and $\varphi$ is the friction angle of slope soil. The parameter $\delta$ represents the angle between the failure surface and the surface perpendicular to the direction of the principal cohesion strength. As presented by Chen et al. [19], Su et al. [38], and Pan and Dias [39], $\delta$ is taken as $\delta = \pi/4 + \varphi/2$. In addition, $\delta$ is a constant parameter regardless of the direction of the major principal stress $\sigma_1$.

For nonhomogeneous soils, nonhomogeneity is the term of soil strength and is used to describe the behavior that soil cohesion varies linearly with depth from the ground surface (Figure 1(c)). As defined by Chen et al. [19], nonhomogeneity only refers to the cohesive strength. In this study, the cohesive strength nonhomogeneous coefficient is called $n_0$. Note that herein the coefficient parameter $n_0$ is $n_0 = 1$, when the soil mass shows homogeneous cohesion.

2.3. Definition and Formulation of Problem. To establish the 2D limit analysis method, Utili [1] and Michalowski [16] conducted some research on the 2D rotational mechanisms for fissured slopes considering the log-spiral failure surface. Afterwards, Utili and Abd [2] adopted the 2D failure mechanisms for toe failure to investigate the effect of seismic action on fissured slopes employing the upper bound method together with the pseudostatic approach. Hence, this study utilized the 2D upper bound method of Utili and Abd [2] to derive the crack location, stability factors, and safety factors for soil fissured slopes with seismic action.

The failure mode is shown in Figure 1(a) with the slope angle $\beta$, self-weight $y$, and height $H$. Therein, $BC$ represents a vertical crack, which is through the upper surface of a slope. The depth of the crack BC gives $H_1$. The length of EF is $L_1$, and of BF is $L_2$. The slope is represented as a rigid body that slides along the log-spiral surface with center $O$ and angular
velocity \omega. The vertical crack BC forms the sliding surface combined with the shear slip surface, namely, surface BCD.

The logarithm-spiral formula can be written as

\[ r_h = r_0 \exp[(\theta_h - \theta_0)\psi], \]

where the parameters \( r_0, \theta_0 \), and \( \theta_h \) refer to the geometrical relationship in Figure 1(a) and \( \psi = \tan \phi_m \).

For presentation of the location of the crack, the normalized parameters \( L_{1r} \) and \( L_{2r} \) are adopted in this study. The functions \( L_{1r} \) and \( L_{2r} \) are determined by

\[ L_{1r} = \frac{\sin(\theta_0 + \beta)}{\sin \beta} - \frac{\sin(\theta_h + \beta)}{\sin \beta} \exp[(\theta_h - \theta_0)\psi], \]

\[ L_{2r} = \sin \theta_e \exp[(\theta_e - \theta_0)\psi] - \sin \theta_0, \]

where the parameter \( \theta_e \) refers to the geometry in Figure 1(a).

The rate of external work for sliding EBCD, \( W_r \), is calculated as the work of block EFD minus the work of block BFC. Moreover, the work of blocks EFD and BFC is calculated with the same method. Thus, the external work rate of the soil weight can be written as

\[ W_r = \gamma r_0^3 \omega (f_1 - f_2 - f_3 - f_4 + f_5 + f_6), \]

where

\[
\begin{align*}
f_1 &= \frac{(3\psi \cos \theta_h + \sin \theta_h)\exp[3(\theta_h - \theta_0)\psi] - (3\psi \cos \theta_0 + \sin \theta_0)}{3(1 + 9\psi^2)}, \\
f_2 &= \frac{L_{1r} (2\cos \beta_0 - L_{1r})\sin \theta_0}{6}, \\
f_3 &= \frac{[\sin(\theta_h - \theta_0) - L_{1r}\sin \theta_0]\exp[(\theta_h - \theta_0)\psi]}{6} \cos \theta_0 - L_{1r} + \cos \theta_h \exp[(\theta_h - \theta_0)\psi], \\
f_4 &= \frac{(3\psi \cos \theta_e + \sin \theta_e)\exp[3(\theta_e - \theta_0)\psi] - (3\psi \cos \theta_0 + \sin \theta_0)}{3(1 + 9\psi^2)}, \\
f_5 &= \frac{L_{2r} (2\cos \beta_0 - L_{2r})\sin \theta_0}{6}, \\
f_6 &= \frac{\cos^2 \theta_e \exp[2(\theta_h - \theta_0)\psi] \sin \theta_e \exp[(\theta_e - \theta_0)\psi] - \sin \theta_0}{3}.
\end{align*}
\]
In addition to the weight force, a horizontal pseudostatic force, \( K_h W \), and a vertical one, \( K_v W \), are added to account for the seismic action. The method for calculating the rate of external work due to the seismic forces is similar to that for determining the rate of external work due to the soil weight; the only difference is in the velocity direction when calculating the rate of external work due to the seismic force which is horizontal [22].

The rate of external work attributing to vertical pseudostatic force can be written as

\[
W_{ev} = \lambda K_h r_0^3 \omega (f_1 - f_2 - f_3 - f_4 + f_5 + f_6),
\]

where \( \lambda = K_v / K_h \). From Figure 1(a), it can be seen that the + sign indicates vertical downward acceleration, whereas the – sign indicates vertical upward acceleration.

Moreover, the rate of external work attributing to horizontal pseudostatic force can be written as

\[
W_{eh} = K_h r_0^3 \omega (f_7 - f_8 - f_9 - f_{10} + f_{11} + f_{12}),
\]

where

\[
\begin{align*}
f_7 &= \frac{3 (3 \sin \theta_h + \cos \theta_h) \exp \left[ \frac{3 (\theta_h - \theta_0) \psi}{3(1 + 9 \psi^2)} \right]}{3(1 + 9 \psi^2)}, \\
f_8 &= \frac{L_1 \sin^2 \theta_0}{3}, \\
f_9 &= \frac{\left[ \sin (\theta_h - \theta_0) - L_1 \sin \theta_h \exp \left[ (\theta_h - \theta_0) \psi \right] \sin \theta_0 + \sin \theta_h \exp \left[ (\theta_h - \theta_0) \psi \right] \right]}{6}, \\
f_{10} &= \frac{3 (3 \sin \theta_e + \cos \theta_e) \exp \left[ \frac{3 (\theta_e - \theta_0) \psi}{3(1 + 9 \psi^2)} \right]}{3(1 + 9 \psi^2)}, \\
f_{11} &= \frac{L_2 \sin^2 \theta_0}{3}, \\
f_{12} &= \frac{\cos \theta_e \exp \left[ (\theta_e - \theta_0) \psi \right] \sin^2 \theta_e \exp \left[ (\theta_h - \theta_0) \psi \right] - \sin^3 \theta_0}{6}.
\end{align*}
\]

Knowledge on the known depth of the crack, \( H_1 \), brings in an additional equation about \( \theta_e \), which can be obtained by combining equation (13) and the geometry in Figure 1(a) so that the following constraint is found:

\[
\sin \theta_e \exp (\theta_e \psi) = \sin \theta_0 \left( 1 - \frac{H_1}{H} \right) \exp (\theta_h \psi) + \sin \theta_0 \left( \frac{H_1}{H} \right) \exp (\theta_h \psi).
\]

The parameter \( F_S \) is the strength reduction factor, which is selected as the factor for evaluation of slope stability. To obtain the true factor of safety, the strength reduction factor is gradually increased until the failure of the slope [20]. When this critical value has been found, the factor of safety of the fissured slope is equal to the strength reduction factor. The shear strength parameters \( c \) and \( \varphi \) are divided by the safety factor \( F_S \), which are analytically defined as

\[
\begin{align*}
c_m &= \frac{c}{F_S}, \\
\psi &= \tan \varphi_m = \frac{\tan \varphi}{F_S}.
\end{align*}
\]

Expressions of the rate of external work done by the soil weight \( W_s \), the vertical pseudostatic force \( W_{ev} \), the horizontal pseudostatic force \( W_{eh} \), and the rate of internal energy dissipation are given by

\[
\begin{align*}
&f_7 = \frac{3 (3 \sin \theta_h + \cos \theta_h) \exp \left[ \frac{3 (\theta_h - \theta_0) \psi}{3(1 + 9 \psi^2)} \right]}{3(1 + 9 \psi^2)}, \\
&f_8 = \frac{L_1 \sin^2 \theta_0}{3}, \\
&f_9 = \frac{\left[ \sin (\theta_h - \theta_0) - L_1 \sin \theta_h \exp \left[ (\theta_h - \theta_0) \psi \right] \sin \theta_0 + \sin \theta_h \exp \left[ (\theta_h - \theta_0) \psi \right] \right]}{6}, \\
&f_{10} = \frac{3 (3 \sin \theta_e + \cos \theta_e) \exp \left[ \frac{3 (\theta_e - \theta_0) \psi}{3(1 + 9 \psi^2)} \right]}{3(1 + 9 \psi^2)}, \\
&f_{11} = \frac{L_2 \sin^2 \theta_0}{3}, \\
&f_{12} = \frac{\cos \theta_e \exp \left[ (\theta_e - \theta_0) \psi \right] \sin^2 \theta_e \exp \left[ (\theta_h - \theta_0) \psi \right] - \sin^3 \theta_0}{6}.
\end{align*}
\]

Note that there is no internal dissipation of energy along the crack since the crack is prior to the slope failure. Thus, the energy is just dissipated internally along the log-spiral failure surface CD. The rate of internal energy dissipation is found by multiplying the differential area \( r \, d\theta / \cos \varphi \) by \( c_i \), times the discontinuity in \( v \cos \varphi \), and the total internal dissipation rate of energy is calculated by

\[
D = \int_{\theta_i}^{\theta_h} c_i (v \cos \varphi) \frac{r \, d\theta}{\cos \varphi}
\]

The influence of anisotropy and nonhomogeneity on the cohesive strength \( c_i \) can then be obtained as follows:

\[
c_i = c_m K = \frac{cK}{F_S},
\]

where

\[
K = \left[ 1 + \frac{1 - k}{k} \cos^2 \varphi \right] \\
\left. \cdot \left( n_0 + \frac{1 - n_0}{H_r} \left( \sin \theta_e (\theta_h - \theta_0) - \sin \theta_0 \right) \right) \right] (12)
\]

\( H_r \) is calculated by

\[
H_r = \sin \theta_h \exp \left[ (\theta_h - \theta_0) \psi \right] - \sin \theta_0.
\]
dissipation $D$ for the rotational mechanism can be obtained. The work-energy balance equation, which defines that the rate of external work equals the rate of energy dissipation, is as follows:

$$ W_r + W_{ev} + W_{eh} = D. \quad (16) $$

$$ N_S = \frac{c}{\gamma HF_S} = \frac{(1 \pm \lambda K_h)(f_1 - f_2 - f_3 + f_4 + f_5 + f_6) + K_h(f_7 - f_8 - f_9 + f_{10} + f_{11} + f_{12})}{H_f d}, \quad (17) $$

where

$$ f_d = f_{d1} + f_{d2}. \quad (18) $$

The functions $f_{d1}$ and $f_{d2}$ appearing in the above equation are calculated as

$$ f_{d1} = \frac{n_0}{\exp(2\theta_0\psi)} \left[ \frac{\exp(2\theta_0\psi)}{2} + \frac{1-k}{k} f_{d11} \right] \frac{\theta_0}{\theta_0}, $$

$$ f_{d11} = \frac{\exp(2\theta_0\psi)}{2} \left[ \cos\left( \frac{\pi}{2} - \varphi_m \right) \left( \psi \cos 2\theta + \sin 2\theta \right) \frac{2(1 + \psi^2)}{2} - \sin\left( \frac{\pi}{2} - \varphi_m \right) \left( \psi \sin 2\theta - \cos 2\theta \right) \right], $$

$$ f_{d12} = \frac{3\sin \theta - \cos \theta \exp(3\theta_0\psi)}{(1 + 9\psi^2)}, $$

$$ f_{d13} = \frac{\exp(3\theta_0\psi)}{2} \left[ \cos\left( \frac{\pi}{2} - \varphi_m \right) \left( \cos \theta - 3\psi \sin \theta \right) \frac{2(1 + 9\psi^2)}{6(1 + \psi^2)} + \left( \psi \sin 3\theta - \cos 3\theta \right) \right], $$

$$ - \sin\left( \frac{\pi}{2} - \varphi_m \right) \left[ \psi \sin 3\theta - \cos 3\theta \right] + \left( 3\psi \sin \theta - \cos \theta \right) \right] $$

A stability number, $N_S = c/\gamma HF_S$, is defined here as presented by Taylor [40]. From equation (16), $N_S$ is obtained by the following expression:

Based on the mathematical optimization method, the minimum of $N_S = f_1(\theta_0, \theta_0, \theta_0, \varphi, \psi, \lambda, n_0, k, K_h)$ and $\lambda_{cp} = f_2(\theta_0, \theta_0, \theta_0, \psi, \varphi, \lambda, n_0, k, K_h)$ over the two geometrical variables $\theta_0$ and $\theta_0$ provides the best upper bound on $N_S$ and $\lambda_{cp}$, respectively. In this study, a computer program is developed by using the Matlab software to implement the optimum algorithm.

### 3. Results and Discussion

#### 3.1. Comparisons

In the isotropy and homogeneity condition (i.e., $k = 1.0, n_0 = 1.0$), and in the static conditions (i.e., $K_h = K_v = 0$), the calculated solutions can be compared with upper bound results given by Utili [1], as shown in Table 1. The upper bound on the stability factor, $N = \gamma HF/c$, is obtained. When different $\varphi$ values (i.e., $\varphi = 10^\circ, 20^\circ$) and different crack depth (i.e., $H_i/H = 0.2 \sim 0.8$) were considered in fissured slope analysis, it should be noted that the upper bound solutions presented in this study are consistent with the upper bound results of Utili [1].

In the pseudostatic condition (i.e., $K_h \neq 0, K_v = 0$), Utili and Abd [2] introduced the 2D upper bound theorem to conduct fissured slope stability, and the stability factor $N$ was also chosen as the evaluation index for parameter analysis. The stability factor is plotted in Figure 2 for comparison with the parameters corresponding to $\beta = 40^\circ \sim 90^\circ$ and
### Table 1: Comparison of the stability factor $yH/c$ between upper bound results of Utili [1] and the results of this study.

| $\beta$ (°) | $H_1/H$ | Utili [1] | This study |
|-------------|---------|-----------|------------|
| 10°         | 0.2     | 3.92      | 3.91       |
|             | 0.4     | 3.39      | 3.38       |
|             | 0.6     | 2.96      | 2.97       |
|             | 0.8     | 2.63      | 2.65       |
| 20°         | 0.2     | 4.70      | 4.69       |
|             | 0.4     | 4.06      | 4.06       |
|             | 0.6     | 3.55      | 3.56       |
|             | 0.8     | 3.15      | 3.17       |

$K_h = 0.1 \sim 0.4$. In this case, the parameter $\lambda$ is taken as 0.5, and $\varphi = 30^\circ$. As can be seen in Figure 2, the trends of the two results are the same. In addition, the dotted lines, which represent the results of this study, are in good agreement with the solid lines, representing the results of Utili and Abd [2].

The two comparisons indicated that the solution derived from this study is reasonable for evaluating the stability of the 2D fissured slopes with soil anisotropy and nonhomogeneity subjected to seismic action.

#### 3.2. Stability Charts

Based on comparative studies in different combinations of the related parameters, the calculation results of the stability factor $N$ are given in Figure 3 for different coefficients of anisotropy and nonhomogeneity. We select four slopes with $H_1/H = 0.1$, 0.2, 0.3, and 0.4 as examples. Figure 3 presents that the stability factor $N$ as the $x$-coordinate is the nonhomogeneity coefficient $n_0$.

Considering different nonhomogeneity coefficients $n_0$ ($n_0 = 0 \sim 1.0$) and different anisotropy coefficients $k$ ($k = 0.6 \sim 1.4$), five variation lines are presented in each figure. Furthermore, a fissured slope with the following properties was assumed: slope angle $\beta = 60^\circ$, internal friction angle $\varphi = 30^\circ$, seismic coefficient $K_s = 0.3$, and $\lambda = 0.5$. In Figure 3, the slope of the calculated line decreases with an increase in the crack depth $H_1/H$. It is noted that the stability factor $N$ increases linearly with an increase in the cohesion nonhomogeneity coefficient $n_0$ and decreases as the cohesion anisotropy coefficient $k$ increases. It implies that the impact of soil anisotropy and nonhomogeneity should be considered in the excavations of cuttings.

#### 3.3. Determination of Crack Location

With a crack of known depth $H_1$, through the computations, the most adverse location of the crack can be obtained, as shown in Figures 4 and 5. The dimensionless crack location $L/H$ ($L = L_1 − L_2$) is plotted against the dimensionless crack depth $\delta/H$ for $K_h = 0.1, 0.3$, and $\lambda = 0.5, 0.5$. The nonhomogeneity coefficient $n_0 = 1.0$ for Figure 4, and the anisotropy coefficient $k$ ranges from 0.6 to 1.4 with increments of 0.2. In Figure 5, the anisotropy coefficient $k = 1.0$, and the nonhomogeneity coefficient $n_0$ ranges from 0 to 1.0 with increments of 0.5. It can be seen in Figures 4 and 5 that the location of the crack $L/H$ increases with increasing $K_h$. However, the location of the crack $L/H$ decreases with increasing $\lambda$. It implies that, with an increase in the horizontal seismic acceleration coefficient $K_v$ ($K_v = \lambda K_h$), the critical location of the crack became deeper (a smaller $L_2$). It can also be seen that as the anisotropy coefficient $k$ and nonhomogeneity coefficient $n_0$ increased, the location of the crack $L/H$ decreased. Meanwhile, it was found that the bigger the values of the known depth of the crack $H_1/H$ were, the lower the difference of the location of the crack $L/H$ was. Especially when $H_1/H$ increased to a certain value, the location of the crack $L/H$ with various anisotropy and nonhomogeneity coefficients reached the same value.

#### 3.4. Solution of Safety Factors

To further study the impact of anisotropy and nonhomogeneity on the stability of given fissured slopes, the factor of safety $F_S$ can be obtained from a series of stability charts. To get the factor of safety $F_S$, some conditions are considered in this section: A fissured slope has a height of 15 m, $H_1/H = 0.2$, $\beta = 60^\circ$, and $\gamma = 18 \text{kN/m}^3$. The frictional cohesive soil properties $\varphi = 30^\circ$ and $c = 40 \text{kPa}$. The seismic coefficient $K_s = 0.3$ and $\lambda = 0.5$. The anisotropy coefficient $k = 1.0$ for case 1, and the cohesion nonhomogeneity coefficient $n_0$ ranges from 0.2 to 1.0 with the increments of 0.2. In case 2, the cohesion nonhomogeneity coefficient $n_0 = 1.0$, and the anisotropy coefficient $k$ ranges from 0.6 to 1.4 with the increments of 0.2.

To explore the stability charts with the change of the anisotropy and nonhomogeneity coefficient for a fissured slope, Figures 6(a) and 6(b) present the different values of $\lambda_{\varphi} = c/\gamma H \tan \varphi$ by taking the parameter $\psi = \tan \varphi/F_S$ as the
As expected, the values of $\lambda_{cp}$ were not linearly distributed with the parameter $\psi$. In Figure 6(a) for case 1, the change of the nonhomogeneity coefficient $n_0$ significantly affected the values of $\lambda_{cp}$ and $\psi$. It can be concluded that the value of $\psi$ increases as the nonhomogeneity coefficient $n_0$ decreases, and also note that as $\psi$ increases, the value of $\lambda_{cp}$ decreases. However, in Figure 6(a) for case 2, the anisotropy coefficient $k$ has minor effects on the values of $\lambda_{cp}$ and $\psi$.

Using the presented stability charts (Figures 6(a) and 6(b)), one can easily get the factor of safety for given fissured slope and soil properties. A simple example is given here to illustrate it. Firstly, take case 1 as an example and calculate $\lambda_{cp} = c/\gamma H \tan \phi = 0.257$. Secondly, with the anisotropy coefficient $k = 1.0$ and the given nonhomogeneity coefficient $n_0$ being 0.2, 0.6, and 1.0, it is determined from Figure 6(a) that the values of $\psi$ are 0.688, 0.630, and 0.570, respectively. Finally, because the parameter $\phi$ is given, the factors of safety can be calculated as $F_s = \tan \phi/\lambda_{cp} = 0.84, 0.91$, and 1.01, respectively.

To completely study the influence of anisotropy and nonhomogeneity, the change of the factor of safety for a fissured slope is obtained from Figure 7, which covers the overall range of anisotropy and nonhomogeneity coefficients. Note that as the nonhomogeneity coefficient $n_0$ increases, the factor of safety $F_s$ increases, which leads to the fissured slope becoming safe. However, the factor of safety $F_s$ decreases as the anisotropy coefficient $k$ increases, which leads to the fissured slope becoming unsafe.

Considering different anisotropy and nonhomogeneity conditions, Figure 8 gives the values of the factor of safety $F_s$ for three different factors ($K_h$, $H_i/H$, and $\lambda$). It can be seen from Figure 8(a) that as the seismic acceleration increases, the factor of safety $F_s$ decreases. Taking Figure 8(b) into consideration, an interesting phenomenon is that the factor of safety $F_s$ increases as the known depth of the crack $H_i/H$ increases. One possible reason is that the variation crack depth $H_i/H$ will correspond to different crack locations ($L/H$) when searching for the most dangerous sliding surface in upper bound analysis, resulting in the safety factors being

\(\begin{align*}
N &= yH/c \\
\text{Figure 3: Influence of soil anisotropy and nonhomogeneity on stability charts with slope cracks of known depth ($\beta = 60^\circ$, $\phi = 30^\circ$, $K_h = 0.3$, $\lambda = 0.5$): (a) $H_i/H = 0.1$; (b) $H_i/H = 0.2$; (c) $H_i/H = 0.3$; (d) $H_i/H = 0.4$.}
\end{align*}\)
Figure 4: Influence of soil anisotropy on the location of the cracks for fissured slopes ($\beta = 60^\circ$, $\varphi = 30^\circ$): (a) $K_h = 0.1, 0.3$; (b) $\lambda = -0.5, 0.5$.

Figure 5: Influence of soil nonhomogeneity on the location of the cracks for fissured slopes ($\beta = 60^\circ$, $\varphi = 30^\circ$): (a) $K_h = 0.1, 0.3$; (b) $\lambda = -0.5, 0.5$. 
Meanwhile, a higher nonhomogeneity coefficient can lead to the factor of safety $F_S$ decrease. Figure 8(c) shows the effects of the parameter $\lambda$ on the factor of safety $F_S$. When the parameter $\lambda$ is negative, the factor of safety $F_S$ obtained is larger than that when the parameter $\lambda$ is positive. So it can be concluded that a fissured slope under vertical downward seismic acceleration is more dangerous than that under vertical upward seismic acceleration.
Figure 8: Continued.
4. Conclusions

Based on the upper bound theorem of limit analysis, solutions are given to evaluate the stability of the fissured slopes subjected to seismic action with soil anisotropy and nonhomogeneity properties. A numerical optimization procedure is applied to predict the adverse location of a crack and the safety factor of a fissured slope. To better estimate the stability of the fissured slope, a series of stability charts are presented, and the procedures for getting the factor of safety are put forward. Besides, from this study, the following conclusions can be drawn:

(1) The stability factor $N$ increases as the nonhomogeneity coefficient $n_0$ increases, but it decreases with the increasing anisotropy coefficient $k$. It is found from a series of stability charts that the influence of soil anisotropy and nonhomogeneity should not be ignored in the theoretical analysis of fissured slope stability.

(2) The location of the crack $L/H$ decreases as the anisotropy coefficient $k$ and nonhomogeneity coefficient $n_0$ increase. The seismicity has dramatic influence on the adverse location of the crack. As the horizontal seismic acceleration coefficient $K_h$ increases or the vertical seismic acceleration coefficient $K_v$ decreases, the critical location of the crack becomes deeper. If the values of the known depth of the crack $H_i/H$ are bigger, the difference of the location of the crack $L/H$ is lower.

(3) The change of the nonhomogeneity coefficient $n_0$ dramatically affected the values of $\lambda_{cp}$ and $\psi$, but the anisotropy coefficient $k$ had minor effects on them. The factor of safety $F_S$ of the fissured slope obtained from the relevant stability charts is appropriate. An increase in the nonhomogeneity coefficient $n_0$ and a decrease in the anisotropy coefficient $k$ could lead to an increase in the factor of safety $F_S$. As the seismic acceleration increases, the factor of safety $F_S$ decreases. Moreover, a fissured slope under vertical downward seismic acceleration is more dangerous than that under vertical upward seismic acceleration. It is valid under the conditions studied in this article.

Data Availability

The generated or analyzed data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest related to this paper.
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