A Study on the Structure Functions and the Radius of the Nucleons

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Abstract
We have investigated the properties of the structure functions of the nucleons in the context of the statistical model using only the radius parameter of the respective nucleons. The radius of the neutron has been estimated and is found to be 0.8fm. It is interesting to observe that the proton radius 0.865fm which exactly equals to the most accepted charge radius of the proton and which in turn characterizes proton as a meso object, yields reasonable results for the structure function and its properties. GT sum rule has been investigated with some interesting conclusions.

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There have been several attempts towards the understanding and estimating of the structure function of the nucleons [1]. It is well known that the measurement of nucleon structure function is important for the understanding of the internal structure of nucleons. The most current interest in structure function is due to the violation of GT Sum rule which indicates the sea quark asymmetry inside the nucleons. The difference between the neutron and the proton structure functions is observed to be sensitive to the u and d quark distribution. NMC [2] result describes the deviation of the GT Sum rule [3] in nucleons. It has been suggested that the pion cloud contribution plays a crucial role in the long range structure of the nucleons. Zamani [4] has investigated the $F_2$ structure function of nucleons using meson cloud in a light cone frame. They have investigated GT Sum rule violation and argued that the contribution from the meson cloud is small which indicates the other sources of violation. Farrar et al [5] have investigated pion nucleon structure function near $x \rightarrow 1$ in coloured quark and vector gluon model. Osipenko et al [6] have investigated the contribution of leading twist moment of neutron structure function. In a subsequent analysis [7] they have investigated proton $F_2$ structure function in the resonance region and investigated the evolution of its moments. Broadhurst et al [8] have shown that at the two loop level the GT sum rule is suppressed by a factor $\frac{1}{N_c}$. Cheng et al [9] have investigated the contribution of the strange quark content of the nucleon to the deviation of the GT sum rule in the context of the chiral quark model. In the present work we have investigated the $F_2$ structure function of the proton and neutron in the context of the statistical model [10]. In the framework of the model the structure functions can be investigated via the radius parameter of the respective nucleons. It has been observed that the neutron radius 0.80fm and proton radius 0.865fm reproduces the structure functions and its properties reasonably well within the other theoretical and experimental estimates. The violation of the GT sum rule has been investigated and the violation has been suggested to be the manifestation of the difference of the spacial extension of the nucleons which in turn may be related to the fractal behaviour of the hadrons [11].
The Model

In statistical model [10] the probability density for a nucleon is obtained as:

$$|\Psi(r)|^2 = \frac{315}{64\pi r_0^{9/2}} (r_0 - r)^{3/2} \theta(r_0 - r)$$  

(1)

where \( r_0 \) is the radius parameter of the nucleon and \( \theta \) is usual step function. The momentum space wave function \( \psi(k) \) is derived from the Fourier transform of the wave function \( \psi(r) \) as:

$$\psi(k) = \frac{c}{k} \int_{0}^{\infty} r \psi(r) dr \cdot \sin kr$$  

(2)

where \( c \) is appropriate normalisation constant. The normalised momentum space wave function with (1) is obtained as:

$$\psi(k) = 2\sqrt{3\pi r_0} k^{-1} j_1(kr_0)$$  

(3)

It is to be noted that \( \psi(k) \) depends only on the corresponding size parameter of the nucleon. In our subsequent analysis we would investigate the nucleon structure function with the above \( \psi(k) \) as an input.

Radius of Neutron and Proton:

The radius of the neutron has been derived by adjusting against the experimentally observed value of the mean life of neutron decay to proton. As the confined radius of the neutron is not precisely known, we use the well-known weak decay of neutron i.e, \( n \rightarrow p e^{-} \nu \). The beta decay coupling constant obtained from Fermi theory using the decay rate \( \Gamma_{fi} \approx 4[2\pi]^{-3} G^2 m^2_e \). With the observed neutron life time = 0.93*10^3 sec, we have obtained the \([Gm_e]^2\) and \([Gm_p]^2\) as 0.29x10^{-11} and 1.10x10^{-5} respectively. Now the neutron and the proton are subject to strong interaction. We allow the possible strong interaction corrections i.e, pion correction introducing a form factor \( F \), an invariant function of the proton and the neutron momenta so that the matrix element can be recast as \( F[\bar{u}_p\gamma_\mu u_n] \) where symbols have their usual meanings. Hence the decay amplitude \( \Gamma_{fi} = [(-16Gm_p m_n m_e m_\nu)^{1/2} (\bar{u}_p\gamma_\mu u_n)(\bar{u}_e\gamma_\mu u_\nu)] \) gets modified with this assumption. Now since the change in momentum involved is very small compared to the hadron masses, we do not expect much variation.
in $F(q^2)$. We assume $|q^2| = (m_n - m_p)^2 = \Delta^2$ which is almost constant. So when strong interaction is taken into account we may assume,

$$F[Gm_p]^2 = 1.10 \times 10^{-5} \tag{4}$$

Now the form factor can be expressed as:

$$F(q^2) = \int e^{iqr} |\psi(r)|^2 \tag{5}$$

In order to make some allowance of the strong interaction effect like pionic correction we assume that when two quarks come closer they interact strongly to yield pionic correction to the decay rate. Poggio et al [12] have pointed out that it would be good approximation that the maximum contribution would result from the ud di-quark region i.e, in the region $r = \frac{1}{2\mu}$, where $2\mu$ is the reduced mass of the ud diquark. For the small values of the $q^2$ we may recast the expression (5) as:

$$F(q^2) = \frac{4}{3} \pi r_0^3 |\psi(r = \frac{1}{2\mu})|^2 \tag{6}$$

With $|\psi(r)|^2$ as input from (1) we obtain,

$$F = \frac{105}{16} (1 - \frac{1}{m_qr_0})^2 \tag{7}$$

From (4) and (7) we estimated $r_n$, the radius of the neutron as 0.80 fm using $m_u = m_d = 360$ MeV. We would use this value of neutron radius in our subsequent analysis.

For proton we use the radius as 0.856 fm. It may be mentioned here that Hefter [13] has applied the Non-Linear Schrodinger Equation (NLSE) [14] with an inhomogeneous term to the atomic physics and nuclear physics. The typical length of the system as described by NLSE is found to be related to the non-linear term of the equation. The objects are classified to be micro, meso and macro objects according to the characteristics the gausson solution. The solution involves a typical length which in turn related to the centre of mass gausson solution of a spherical object with a typical radius $R$ (say). It has been pointed out that the properties of meso object are different from the other two types of objects. They have
suggested that the nucleons and the $\alpha$ particles are subjected to the meso object. Hefter [13] has argued that the non-linearities are related to the compressibility of the object. From the compressibility $\kappa$ of the proton it has been suggested that the proton belongs to a meso object with the one of the feature that its charge radius is equal to the radius parameter of the proton. They have indicated that the proton radius is $r_p = 0.865$ fm which equals to the most accepted value of the charge radius of proton. We have used this value of proton radius parameter in our subsequent analysis. Hefter [13] has mentioned that the maximum value of the nucleon matter radius should be 1.2 fm from the study of the compressibility. We have also used $r_p = 1.2$ fm in some of the calculations.

The Structure function:

The free nucleon $F_2$ structure function in the non-linear limit runs as:

$$F(x) = \frac{M}{8\pi^2} \int_{k_{\text{min}}}^{\infty} |\psi(k)|^2 dk^2$$

where $M$ is the mass of the nucleon and $k_{\text{min}} = M \left| x - \frac{1}{3} \right|$. With $\psi(k)$ as an input in (8) the structure function $F(x)$ for the proton and neutron has been estimated using the relevant radius parameter as stated before. The results have been displayed in Fig-1 and in Fig-2. The difference between the structure functions has been estimated and results are displayed in the Fig-3. The ratio of the neutron proton structure functions has been displayed in the Fig-4. The ratios are estimated radius parameter of the proton as 0.856 fm and 1.2 fm.

The Gottfried Sum rule runs as:

$$\int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x} dx = \frac{1}{3}$$

The deviation of GT sum rule is a problem worth pursuing. It has been suggested that the deviation from the GT sum rule manifests the asymmetry between the $\bar{u}$ and $\bar{d}$ sea. We have estimated the GT sum rule in the context of our formalism with $r_p = 0.865$ fm and $r_n = 0.8$ fm and obtained the value as 0.240 which agrees closely with corresponding result by NMC [2] which runs as $\pm 0.024 \pm 0.034 \pm 0.021$. The ratio of the structure functions $\frac{F_2^n}{F_2^p}$ as $x \to 1$ is estimated as 0.73 for aforesaid values of the neutron and proton radius. It may be mentioned
that for \( r_p = 1.2 \text{fm} \) and \( r_n = 0.8 \text{fm} \) we have obtained the ratio as 0.224. The ratio of the second moments \( \frac{M_4^A}{M_4^p} \) has been obtained as 0.532 with the input of neutron and proton radius as 0.80fm and 0.865fm respectively which agrees well with the estimation of Osipenko et al [6].

**Results and Discussions**

In the present work we have investigated the structure function properties of the neutron and the proton. The model we have used in the current investigation enable us to investigate the structure functions using only the size parameter of the respective nucleons. The results are presented in the range 0.1 \( \langle x \rangle \langle 0.5 \). Signal et al [15] have investigated the structure function for nucleon in the two dimensional MIT bag model with the cavity radius 1fm. \( F_p - F_n \) estimated in the present work yields good agreement with the NMC results in the aforesaid region of \( x \). Deviation from the GT sum rule estimated is found to be in good agreement with the NMC [2] results. Cheng et al [9] has indicated that a significant contribution to the deviation may come from the strange quark sea contribution of the nucleon. Broadhurst et al [8] have estimated GT sum rule \( I_g \) as 0.219 to 0.178. They have shown that the deviation persists even at the large \( N_c \) limit. The ratio of the neutron proton structure functions estimated in the current work shows reasonable agreement with the work of Zamani et al [4] in which they have investigated the ratio in the light cone frame where a dressed nucleon is supposed to be superposition of the bare nucleon and a virtual light cone Fock state of baryon meson pairs. It may be mentioned that the ratio of the structure function is sensitive to the \( u \) and \( d \) quark distribution, particularly in the large \( x \) limit the ratio is sensitive to the valance distribution of \( u \) and \( d \) quark. With the spin-flavour symmetry, the SU(6) predicts the ratio as \( \frac{2}{3} \) [16]. The \( \frac{F_n^2}{F_p^2} \) in the large \( x \) limit \( (x \to 1) \) in shell formalism has been estimated to be \( \frac{1}{4} \) [17] whereas the helicity conservation in perturbative QCD with unperturbed spin flavour symmetric wave function yields the asymptotic value of the ratio as \( \frac{2}{7} \) [18]. Osipenko et al [7] have investigated the ratio in resonance region and have extracted the ratio as \( \frac{F_n^2}{F_p^2} = 0.34 \pm 0.12 \pm 0.13 \) for \( x = 0.7 \). Melnitchouk et al [19] have extracted \( \frac{F_n^2}{F_p^2} \) from the deuteron data and found that the result is consistent with the QCD prediction. Farrar et al
[5] have investigated the problem in the coloured quark and vector gluon model of hadron and observed that the result is in agreement with the QCD prediction. The result we obtain for the ratio is 0.73 for proton radius 0.865fm whereas with \( r_p = 1.2 \text{fm} \) we get the ratio as 0.22 close to the QCD prediction.

The radius parameter is a very important quantity which is yet to be determined precisely. In the present work we have tried to investigate the properties of the nucleon structure function using only the radius parameter or in other words we have tried to investigate the value of the radius parameter of the proton and neutron through the investigation of the structure functions of the respective nucleons. The \( r_p = .865 \text{fm} \) and \( r_n = .80 \text{fm} \) are observed to be doing well in reproducing the structure function properties. The proton thus may has the possibility of behaving like a meso object. We have also used the radius parameter of the proton as 1.2 fm which Hefter [13] has pointed out as the maximum value of the nucleon matter radius and obtained the good agreement with the experimental results. It is interesting to note that deviation of GT sum rule which arises due to the light quark sea asymmetry has been found to be manifested by the spacial extension of the proton and neutron. The values of the structure functions obtained in the current work are found to be reasonably well whereas the GT sum rule and higher moments shows very good agreement. It may be mentioned that at large x symmetric sea vanishes more rapidly which may affect the experimental data. The results which are reproduced fitting with only one parameter is found to be very encouraging. It can be said that geometry plays an important role in exhibiting the structure function properties. It is pertinent to point out here that the statistical model predicted the hadron as a fractal object with fractal dimension \( \frac{9}{2} \) [11] and in the context of the model the scaling violation has been attributed to the fractal behaviour of the hadron. From the present study of the structure function using the model, it may not be irrelevant to emphasize that the violation of GT sum rule may be attributed to the fractal behavior of the nucleons. However further investigation would be done in our future works particularly considering the strange quark sea contribution to the GT sum rule.
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Figure captions:

1. Fig 1. The proton structure function with $r_p = 0.865$fm.

2. Fig 2. The Neutron structure function with $r_n = 0.80$fm.

3. Fig 3. The difference in proton neutron structure function $F^p_2(x) - F^n_2(x)$ with $r_p = 0.865$fm and $r_n = 0.80$fm.

4. Fig 4. The ratio of the neutron proton structure function $\frac{F^n}{F^p_2}$ for $r_p = 0.865$fm. and 1.2fm whereas $r_n = 0.80$fm.