Semi-Analytical Model Based on the Volumetric Source Method to Production Simulation from Nonplanar Fracture Geometry in Tight Oil Reservoirs

Ting Li, Yongsheng Tan,* Shuhui Dai, Xun Zhou, Jinzhu Yu, and Qi Yang

ABSTRACT: Fracturing measures are common practice for horizontal wells of tight oil reservoirs. Thus, production estimation is a significant problem that should be solved. However, previous models for the production of fractured horizontal wells of tight oil reservoirs have some problems. In this paper, we present a semi-analytical model based on the volumetric source method to simulate production from nonplanar fracture geometry in a tight oil reservoir. First, we developed an analytical model based on the volumetric source method in nonplanar fracture geometry with varying widths. Second, the model was coupled with fracture flow and solved by the Gauss–Seidel iteration. Third, the semi-analytical model was verified by a numerical reservoir simulator. Finally, sensitivity analysis was conducted for several critical parameters. Results of validations showed good agreement between this paper’s model and the numerical reservoir simulator. The results from the sensitivity analysis showed that (1) production increases with an increased number of fracture segments; (2) production drops more quickly with a smaller fracture half-length in the first stage, and it drops slowly with a smaller fracture half-length in the second stage; (3) cumulative production increases more quickly with a bigger fracture conductivity; and (4) cumulative oil production from a fracture with a constant width and without stress sensitivity coefficient is smaller than that from a fracture with varying widths and with stress sensitivity coefficient. This research provides a basis and reference for production estimation in tight oil reservoirs.

1. INTRODUCTION

In recent years, hydraulic fracturing technology has been used widely in unconventional resources around the world.1 Undoubtedly, production evaluation of these wells holds important issues that should be solved by petroleum engineers.2−4 At present, there are three methods to predict production from fractured wells: (1) analytical models, (2) numerical models, and (3) semi-analytical models. Analytical modeling is a simple and feasible method to predict production, but it does not provide an accurate prediction of production because it ignores the pressure drop in the fracture. Semi-analytical models can predict production accurately, but numerical modeling requires a lot of basic data.5−7 Semi-analytical models take advantage of the benefits of analytical and numerical models, and do not have some of their major drawbacks. Semi-analytical modeling considers more factors and makes less assumptions than analytical models, and its computational time is less than that of numerical models. Therefore, the semi-analytical model is a simple and practical method for predicting production.

In previous studies, researchers have built semi-analytical models based on the point source function for predicting the production of fractured wells.8−18 However, point source functionality has a disadvantage: its solution has a singular point because the source of the model has no volume.19−24 To deal with this issue, the volumetric source function was proposed by Valko and Amini;19 this function has higher computational efficiency and accuracy than the point source function. Furthermore, a series of production prediction models for fractured wells were built by the volumetric source function.20−22

To simplify numerical calculations, fracture geometry is usually assumed as the constant fracture width (fracture geometry constant width [FGCW]). Based on this assumption, a series of production prediction models for fractured wells have been built by analytical solutions, semi-analytical solutions, and numerical solutions.9−15 For example, local grid refinement (LGR), which represents greater permeability with a constant and small width, was used to model hydraulic fracturing with...
advanced reservoir simulation software. However, the fracture geometry proved more complex than in previous studies. The production model of the fractured well was developed under a complex fracture geometry with varying widths along the fracture (fracture geometry varying widths [FGVW]). An elliptical fracture network was described by a wire-mesh model in naturally fractured formation. A nonplanar fracture was used to model production in unconventional resources, a method that can generate an unstructured grid to simulate production with a numerical simulator. A semi-analytical model was presented to model production for a complex fracture geometry in the Barnett shale. The model did not consider, however, the fracture width change. The model then was extended to simulate production in tight oil reservoirs with nonplanar fracture geometry, but it was built based on point source functionality.

After reviewing and summarizing the literature, we found that much effort is needed to develop a model for FGVW based on the volumetric source function to model production in unconventional resources. In this paper, we propose a semi-analytical model based on the volumetric source function to predict production from nonplanar fracture geometries in tight oil reservoirs. The semi-analytical model was verified by numerical reservoir simulation software (ECLIPSE 2011). After that, sensitivity analysis was conducted for several critical parameters. This research provides an important basis and reference for estimating the production of FGVW in tight oil reservoirs.

2. SEMI-ANALYTICAL MODEL AND SOLUTION

2.1. Assumptions. The physical model of a fractured well (Figure 1) with vertical fractures (FVW) is shown in Figure 2. The fractured well is located in a tight oil reservoir with box-type closed boundaries. Other important assumptions are made as follows:

- The reservoir is homogeneous and isotropic.
- The oil reservoir is fully penetrated by the fracture, and the fracture is finite conductivity.
- The temperature and initial reservoir pressure are constant.
- A single-phase fluid (oil) exists in the reservoir and fracture.
- The fluid flow in the reservoir and fractures obeys Darcy’s law.

2.2. Oil Flow from Reservoir to Fracture. Based on the assumptions, only a single-phase oil exists in the tight oil reservoir. Because point source functions also have the disadvantage that their solution has a singular point, we applied the volume source function proposed by Valkó and Amini. As shown in Figure 3, the single-phase fluid flow from...
the width of the reservoir in the $x$ direction, m; $m$ is the permeability in the $x$ direction, mD; $k_y$ is the permeability in the $y$ direction, mD; $k_z$ is the permeability in the $z$ direction, mD; $\phi$ is the porosity, fraction; $\mu$ is the viscosity, $10^{-3}$ $\mu$m$^3$; $B$ is the oil volume factor, m$^3$/m$^3$; $t$ is the production time, h; $q$ is the source strength, source strength vector; $\varepsilon_c$ is the total compressibility, MPa$^{-1}$.

As is shown in Figure 4, the dimensionless pressure ($p_{Dn}$) from all $N_f$ segments can be expressed as follows

$$p_{Dn} = \sum_{1}^{N_f} q_{ij} p_{Dj}$$

(13)

Figure 4. Schematic of a fracture with varying widths and fluid flow through the $j$th segment of the fracture with finite conductivity.

Based on the dimensionless definition of pressure, the pressure here can be expressed as follows

$$p = R = \frac{Bq_{Dn}}{86.4kL}$$

(14)

2.3. Oil Flow from Fracture to Wellbore. Assuming that the fracture has finite conductivity, the pressure drop in the $j$th fracture segment caused by fluid flow can be simplified to a one-dimensional model, as shown in Figure 3. Assuming that the fluid flow of the fracture conforms to Darcy’s law, the pressure drop in the $j$th fracture segment can be expressed as follows

$$p_j - p_{j+1} = \int_{y_j}^{y_{j+1}} \left( \frac{\mu}{\rho k y_j d} \right) [q_j (y - y_j)] dy, j
= 1, ..., N_f$$

(15)
For fracture width and permeability, an analytical method to present fracture width in planar fracture geometry was proposed by Sneddon in 1951,26 the fracture width can be expressed as follows

\[ w_f = w_{\text{max}} \left( 1 - \frac{x}{x_N} \right)^2 \]  

(16)

2.4. Permeability-Pressure Equation. If the influence of proppant is not considered, fracture permeability can be expressed as follows26

\[ k_f = \frac{w_f^2}{12} \]  

(17)

The stress sensitivity coefficient of permeability can be expressed as follows27

\[ k_f = k_{\text{fracture}} e^{-a(q_i - p)} \]  

(18)

2.5. Solution Method. The well was assumed to produce under constant bottomhole pressure, with the specific variables as follows6

- \( N_f \) is the flow rate of each segment center, but the flow rate at the point of the tight oil reservoir is unknown; so the number of unknown points is \( N_f \).
- \( q_{\text{in}} \) is the inflow rate of fracture segments \( q_{\text{in}} = 1, 2, ..., N_f \).
- \( q_f \) is the pressure at each fracture segment, but we know the pressure of the wellbore at the fracture–wellbore intersection; so the number of unknown points is \( N_p, \) \( p_{\text{f}} = 1, 2, ..., N_f \).

These unknown variables can be expressed in the following vector form

\[ x = [q_{\text{in}}, q_{\text{out}}, q_f, q_{\text{in}}, q_{\text{out}}^\prime, q_f^\prime, q_{\text{in}}^\prime, q_{\text{out}}^\prime, p_1, p_2, ..., p_N] \]  

(19)

For the whole equations, there are \( 2N_f + N_p \) unknowns. In other words, there are \( 2N_f + N_p \) equations. The specific descriptions are as follows:

- Each point has \( 2N_f + N_p \) mass-balance equations. In fact, every point in Figure 3 conforms to the mass-balance equations. The inflow was assumed to be equal to the outflow for fluid flow in fracture segments.8 The specific equation expression is shown as follows

\[ (q_{\text{in}})_i = (q_{\text{out}})_i, \quad i = 1, 2, ..., N_f \]  

(20)

- The wellbore pressure drop along the fracture includes \( N_f \) equations (eq 14).
- Pressure at the center of each segment includes \( N_f \) equations (eq 13). Meanwhile, this section can be linked to fractures and reservoirs.

In this study, the equations are solved by the Newton–Raphson method, and the solution process is shown in Figure 5. The specific expression of this method is as follows

\[ J dx = -F(x) \]  

(21)

\[ x_{k+1} = x_k + dx \]  

(22)

In which

\[ J = \frac{\partial F(x)}{\partial x} \]  

(23)

where

\[ J \] is the Jacobian matrix;
\[ dx \] is the increment of variable;
\[ F(x) \] is the residual term consisting of eqs 13 and 14;
\( x_{k+1} \) is the solution of the \( k+1 \) iterative number;
\( x_k \) is the solution of the \( k \) iterative number.

3. MODEL VERIFICATION

The fracture is wedge-shaped with a width of 9.2 mm and a tip width of 1.2 mm, the fracture is divided into 10 segments, and the length of each section is 0.8 mm. Then, the fracture widths calculated by eq 16 are as follows: 9.2, 8.4, 7.6, 6.8, 6.0, 4.2, 3.4, 2.6, and 2 mm.

To verify the approach, a commercial numerical simulator (ECLIPSE 2011) was introduced. The basic data from the Junggar basin in the tight oil field is provided in Table 1. The three-dimensional size for \( x, y, \) and \( z \) directions is \( 10 \times 10 \times 1 \) m, the grid dimension is \( 80 \times 30 \times 40 \), and the time step is 10 days. As shown in Figure 6, the results show that the production from this paper’s model agrees with that of the commercial simulator (ECLIPSE 2011).

4. RESULTS AND DISCUSSION

4.1. Effect of Number of Fracture Segments. As shown in Figure 7, we can see the effect of the number of fracture segments on production. In this case, the numbers of fracture segments were 20, 25, 30, and 35. From the figure, it can be seen that the production from 25 fracture segments reached that from 30 or 35 fracture segments. Therefore, 25 fracture segments are recommended as the best scheme in this case. The length of each segment corresponding to 25 segments is 9.6 m, suggesting that the fracture length of 9.6 m achieves accurate simulation results.

4.2. Effect of Fracture Half-LENGTHS. Figure 8 shows the effect of fracture half-length on production. As shown, the production drops more quickly with the smaller fracture half-length in the first stage, and it drops slowly with the smaller
fracture half-length in the second stage. In this case, the production fell sharply at 0−50 days, and the production stabilized after 50 days. The first stage of reservoir production represents an unsteady state; the production dropped quickly. However, the second stage of reservoir production represents a quasi-steady state; the production reached stability. Production increased with increasing fracture half-length because the longer fracture half-length made a bigger contact area in the tight reservoir and fracture system.

4.3. Effect of Fracture Conductivities. Figure 9 shows cumulative production over a 300-day period with different fracture conductivities. In Figure 8, it can be seen that the cumulative production increased more quickly with the bigger fracture conductivity. The increments of cumulative production in different fracture conductivities gradually increased during the initial production stage of 0−50 days, but the increments tended to stabilize at 51−300 days. This result is because the fracture system was in an unsteady state at 0−50 days, and it reached a quasi-steady state at 51−300 days. Hence, fracture conductivity should be increased as much as possible during fracturing operations under the economic and technical limit.

4.4. Effect of Variable Fracture Widths. Figure 10 compares cumulative oil production between varying-width and constant-width fractures. As shown in Figure 10, the cumulative oil production of the fracture with constant width is

Table 1. Basic Physical Parameters of Tight Reservoir

| Parameter                  | Value | Unit  |
|----------------------------|-------|-------|
| Reservoir length           | 800   | m     |
| Reservoir width            | 300   | m     |
| Length of horizontal well  | 600   | m     |
| Initial oil reservoir pressure | 46.2 | MPa  |
| Permeability               | 0.012 | mD    |
| Porosity                   | 10.6  | %     |
| Reservoir thickness        | 40    | m     |
| Oil viscosity              | 6     | mPa·s |
| Fracture half-length       | 120   | m     |
| Fracture conductivity      | 9.2   | D·cm  |
| Comprehensive compressibility factor | $3 \times 10^{-4}$ | MPa$^{-1}$ |
| Formation volume factor    | 9.2   | m$^3$/m$^3$ |
| Reservoir temperature      | 102.95| °C    |
| Production time            | 300   | days  |
| Wellbore radius            | 0.12  | m     |
| Bottomhole pressure        | 35    | MPa   |
| The number of fractures    | 12    |       |

Figure 6. Comparison of production rate obtained from this paper's model, field production data, and numerical simulation (ECLIPSE 2011).

Figure 7. Effect of fracture segments on production.
smaller than that of the fracture with varying widths. The relative difference between the cumulative oil production of the two fractures at 300 days is about 9.08%. Therefore, to predict production in fractured wells in tight reservoirs, fractures with varying widths should be considered.

4.5. Effect of Stress Sensitivity Coefficients. Figure 11 compares cumulative oil production at 300 days with and without the stress sensitivity coefficient. As shown in Figure 11, the cumulative oil production with stress sensitivity coefficient is larger than that without the stress sensitivity coefficient. The relative difference between the cumulative oil production with and without stress coefficient sensitivity at 300 days is about 3.21%. Therefore, to predict the production of fractured wells in tight oil reservoirs, stress sensitivity should be considered.

5. FIELD CASE STUDY

We performed an analysis of horizontal well performance in a tight oil reservoir. As shown in Figure 12, the bottomhole flow pressure was used to calculate oil production from a horizontal well in the tight oil reservoir. The horizontal well was completed with 12 fractures with a half-length of approximately 100 m. The other parameters were set the same as the previous case (Table 2). The varying fracture width was from 4.2 to 0.8 mm. Based on this, the fracture permeability in eq 17 is from $1.47 \times 10^6$ to $5.3 \times 10^4 \mu m^2$. After detailed analysis, Figure 13 shows that the results of this paper are in good agreement with the oil field data. Hence, this model can be used to predict oil production in tight oil reservoirs.

6. CONCLUSIONS

This paper proposed a semi-analytical model based on the volumetric source method to simulate production from nonplanar fracture geometry in tight oil reservoirs. To verify the proposed semi-analytical model, validation was conducted by a numerical reservoir simulator (ECLIPSE 2011). Finally, the sensitivity of several critical parameters was analyzed. Conclusions were drawn as follows:

1. The results of the semi-analytical model based on the volumetric source method match well with those of a numerical reservoir simulator (ECLIPSE 2011), indicating the reliability of this paper’s model.

2. For this case, the production increased with an increasing number of fracture segments. But the production reached its highest point with a fracture length of 24 m and a

Table 2. Basic Physical Parameters of Tight Reservoir for the Field Case Study

| Parameter                      | Value  | Unit |
|--------------------------------|--------|------|
| Reservoir length               | 600    | m    |
| Reservoir width                | 400    | m    |
| Length of horizontal well      | 500    | m    |
| Initial oil reservoir pressure | 39.4   | MPa  |
| Permeability                   | 0.018  | mD   |
| Porosity                       | 13.4   | %    |
| Reservoir thickness            | 34     | m    |
| Oil viscosity                  | 8.2    | mPa·s|
| Fracture half-length           | 116    | m    |
| Fracture conductivity          | 10.2   | D·cm |
| Comprehensive compressibility factor | $4.2 \times 10^{-4}$ | MPa$^{-1}$ |
| Formation volume factor        | 8.9    | m$^3$/m$^3$ |
| Reservoir temperature          | 92.5   | ºC   |
| Production time                | 300    | days |
| Wellbore radius                | 0.12   | m    |
| Bottomhole pressure            | 32     | MPa  |
| The number of fractures        | 12     |      |
fracture segment number of 25. Therefore, the number of fracture segments should be optimized in production calculation.

(3) Production drops more quickly with a smaller fracture half-length in the first stage, and it drops slowly with a smaller fracture half-length in the second stage. Cumulative production increases more quickly with a bigger fracture conductivity.

(4) Cumulative oil production of a fracture with a constant width and without stress sensitivity coefficient is smaller than that of a fracture with varying widths and with stress sensitivity coefficient.

■ AUTHOR INFORMATION

Corresponding Author
Yongsheng Tan — State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Hubei 430071, China; University of Chinese Academy of Sciences, Beijing 100049, China; orcid.org/0000-0003-1251-6066; Phone: +8618971369196; Email: tanyongsheng2012@163.com

Authors
Ting Li — State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum, Beijing 102249, China; State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Chengdu University of Technology, Chengdu 610059, China; Petroleum Engineering College of Yangtze University, Wuhu 430100, China
Shuhui Dai — Exploration and Development Management Department, China ZhenHua Oil Co., Ltd., Beijing 100031, China
Xun Zhou — Engineering and Technology Research Institute, CNPC Bohai Drilling Company, Tianjin 300457, China
Jinzhu Yu — Oil and Gas Technology Research Institute, PetroChina Changqing Oilfield, Xi’an 710018, China
Qi Yang — China United Coalbed Methane Company, Beijing 100011, China

Complete contact information is available at:
https://pubs.acs.org/10.1021/acsomega.0c05119

Notes
The authors declare no competing financial interest.

■ ACKNOWLEDGMENTS

This work is supported by the Foundation of State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum, Beijing (No. PRP/open-1901) and fund by Open Fund of State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Chengdu University of Technology (No. PLC20190703).

■ REFERENCES

(1) Zhou, X.; Yuan, Q.; Rui, Z.; Wang, H.; Feng, J.; Zhang, L.; Zeng, F. Feasibility study of CO2 huff ‘n’ puff process to enhance heavy oil recovery via long core experiments. Appl. Energy 2019, 236, S26–S39.
(2) Chen, Z.; Liao, X.; Zhao, X.; Lyu, S.; Zhu, L. A comprehensive productivity equation for multiple fractured vertical wells with non-linear effects under steady-state flow. J. Pet. Sci. Eng. 2017, 149, 9–24.
(3) Kou, Z.; Dejam, M. Dispersion due to combined pressure-driven and electro-osmotic flows in a channel surrounded by a permeable porous medium. Phys. Fluids 2019, 31, No. 056603.
(4) Kou, Z.; Dejam, M. Control of shear dispersion by the permeable porous wall of a capillary tube. Chem. Eng. Technol. 2020, 43, 2208–2214.
(5) Wan, J.; Dale, B. A.; Ellison, T. K.; Benish, T. G.; Grubert, M. A. In Coupled Well and Reservoir Simulation Models to Optimize Completion Design and Operations for Subsurface Control, SPE Europe/EAGE Conference and Exhibition, Rome, Italy; Society of Petroleum Engineers, 2008; p 6.
(6) Neylon, K. J.; Reiso, E.; Holmes, J. A.; Nesse, O. B. In Modeling Well Inflow Control with Flow in Both Annulus and Tubing, SPE Reservoir Simulation Symposium; Society of Petroleum Engineers, 2009; p 2.
(7) Marzouqi, A.; Al Helmy, H.; Kesha, A.; Elasmar, M.; Shafia, S. In Wellbore Segmentation Using Inflow Control Devices: Design and Optimization Process, Abu Dhabi International Petroleum Exhibition and Conference; Society of Petroleum Engineers, 2010; p 11.
(8) Zhou, W.; Banerjee, R.; Poe, B.; Spath, J.; Thambynayagam, M. Semi-analytical production simulation of complex hydraulic-fracture networks. SPE J. 2013, 19, 6–18.
(9) Zongxiao, R.; Wu, X.; Liu, D.; et al. Semi-analytical model of the transient pressure behavior of complex fracture networks in tight oil reservoirs. J. Nat. Gas Sci. Eng. 2016, 35, 497–508.
(10) Wang, J.; Xu, J.; Wang, Y.; Li, H.; Liu, T.; Wen, X. Productivity of hydraulically-fractured horizontal wells in tight oil reservoirs using a linear composite method. J. Pet. Sci. Eng. 2018, 164, 450–458.
(11) Yao, S.; Zeng, F.; Liu, H.; Zhao, G. A semi-analytical model for multi-stage fractured horizontal wells. J. Hydrod. 2013, 507, 201–212.
(12) Lin, J.; Zhu, D. Modeling well performance for fractured horizontal gas wells. J. Nat. Gas Sci. Eng. 2012, 18, 180–193.
(13) He, Y.; Cheng, S.; Rui, Z.; Qin, J.; Fu, L.; Shi, J.; Wang, Y.; Li, D.; Patil, S.; Yu, H.; et al. An improved rate-transient analysis model of multi-fractured horizontal wells with non-uniform hydraulic fracture properties. Energies 2018, 11, No. 393.
(14) Chen, Z.; Zhao, X.; Dou, X.; Zhu, L.; Lyu, S.; et al. A finite-conductivity horizontal-well model for pressure-transient analysis in multiple-fractured horizontal wells. SPE J. 2017, 22, 1112–1122.
(15) Yu, W.; Wu, K.; Sepehrnoori, K. A semi-analytical model for production simulation from nonplanar hydraulic-fracture geometry in tight oil reservoirs. SPE J. 2016, 21, 1028–1040.
(16) Xu, W.; Thiercelin, M. J.; Ganguly, U.; Weng, X.; Gu, H.; Onda, H.; Sun, J.; Le, C. In Wir meshes: A Novel Shale Fracturing Simulator, Proceedings of the International Oil and Gas Conference and Exhibition; Society of Petroleum Engineers, 2010; p 6.
(17) Mirzaei, M.; Cipolla, C. L. In A Workflow for Modeling and Simulation of Hydraulic Fractures in Unconventional Gas Reservoirs, Proceedings of SPE Middle East Unconventional Gas Conference and Exhibition; Society of Petroleum Engineers, 2012; p 2.
(18) Wei, Y.; Hu, X.; Wu, K.; Wu, K.; Sepehrnoori, K.; Olson, J. E. Coupled fracture-propagation and semianalytical models to optimize shale gas production. SPE Reservoir Eval. Eng. 2017, 20, 1028–1040.

(19) Valko, P. P.; Amini, S. In The Method of Distributed Volumetric Sources for Calculating the Transient and Pseudosteady-State Productivity of Complex Well-Fracture Configurations, SPE Hydraulic Fracturing Technology Conference; Society of Petroleum Engineers, 2007; p 1.

(20) Li, H.; Tan, Y.; Jiang, B.; Wang, Y.; Zhang, N. A semi-analytical model for predicting inflow profile of horizontal wells in bottom-water gas reservoir. J. Pet. Sci. Eng. 2018, 160, 351–362.

(21) Tan, Y.; Li, H.; Zhou, X.; Jiang, B.; Wang, Y.; Zhang, N. A semi-analytical model for predicting horizontal well performances in fractured gas reservoirs with bottom-water and different fracture intensity. J. Energy Resour. Technol. 2018, 140, No. 102905.

(22) Tan, Y.; Li, Q.; Li, H.; Zhou, X.; Jiang, B. A Semi-analytical model based on the volumetric source method to predict acid injection profiles of horizontal wells in carbonate reservoirs. J. Energy Resour. Technol. 2020, 143, No. 053001.

(23) Zhang, L.; Kou, Z.; Wang, H.; Zhao, Y.; Dejam, M.; Guo, J.; Du, J. Performance analysis for a model of a multi-wing hydraulically fractured vertical well in a coalbed methane gas reservoir. J. Pet. Sci. Eng. 2018, 166, 104–120.

(24) Kou, Z.; Wang, H. Transient pressure analysis of a multiple fractured well in a stress-sensitive coal seam gas reservoir. Energies 2020, 13, No. 3849.

(25) Sneddon, I. N. Fourier Transforms; McGraw-Hill: New York, 1951.

(26) Witherspoon, P. A.; Wang, J.; Iwai, K.; Gale, J. E. Validity of cubic law for fluid flow in a deformable rock fracture. Water Resour. Res. 1980, 16, 1016–1024.

(27) Tong, D.; Chen, Q.; Liao, X. Nonlinear Flow Dynamics in Porous Media; Petroleum Industry Press: Beijing, 2003; pp 97–104.