Influence of Proppant Size on the Proppant Embedment Depth

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ABSTRACT: Hydraulic fracturing is a well stimulation technique involving the fracturing of bedrock formations by a pressurized liquid, in which proppants are added to keep the fracture open after the fracturing operation. The scale discrepancy between the rock specimen and the proppant may bring deviations in the analysis of proppant embedment depth if the fluid-deteriorated formation is treated as an isotropic medium. This study tries to uncover the origins of these deviations through numerical and analytical analyses. The fluid-deteriorated formation is first modeled as a layered rock to obtain equivalent elastic parameters under isotropic conditions. Then, the equivalent parameters are used in the numerical modeling of proppant embedment. The numerical simulations indicate that the simplification of the fluid-deteriorated formation into an isotropic rock results in an underestimation of the proppant embedment depth, and this deviation increases with the scale contrast between rock specimens and proppants. Hertz contact theory is utilized to explain this deviation. As a promising technique, the nano/micro-indentation is also proposed to depict the fluid-deterioration effect along the depth. This study provides methods for the calibration of mechanical parameters of fluid-deteriorated rocks in the analysis of proppant embedment.

INTRODUCTION

Hydraulic fracturing is a technique used to stimulate the production of hydrocarbons or other resources, such as geothermal energy, after a wellbore has been drilled.1−6 Proppants are carried by the fracturing fluid into the newly formed fractures to keep them open after the pressure is released and allow hydrocarbons that were trapped in the rock to flow through the fractures more efficiently.7 However, some mechanisms can lead to the reduction of fracture conductivity, such as fine migration,7 proppant diagenesis,8 proppant crushing,9−12 and proppant embedment,12,13 which is defined as proppant particles being embedded into the rock mass under pressure, causing a reduction in the fracture width and conductivity.14 Therefore, it is of vital importance to investigate fracture conductivity after hydraulic fracturing.

Among the above-mentioned reduction mechanisms of fracture conductivity, proppant embedment has been most intensively studied, as shown in Figure 1, through experiments,15−22 numerical simulations,23−25 and analytical modeling.15,26−31 Laboratory experiments showed that proppant embedment in tight gas sandstone reservoirs is related to many factors including shear displacement, fluid type, joint roughness, shear strength, friction angle, and dilation angle (Tang and Ranjith14,21). Among these factors, closure stress was the primary parameter that determined embedment, with proppant size and fluid viscosity also being important (Lacy et al.15,16). Huitt and McGlothlin15 proposed an equation to calculate the proppant embedment, and the impact of the overburden pressure, sizes, and concentrations of proppants was assessed. Volk et al.26 reported the influence of the closure pressure, proppant size and size distribution, proppant concentration, formation hardness, and surface roughness on proppant embedment and proposed empirical equations to describe embedment for non-crushing proppants. In addition, the types and concentrations of proppants and rock types also have a great impact on the proppant embedment in hydraulic fractures (Wen et al.17). Mueller and Amro19 used the indentation hardness of the surface formation to calculate the embedment. However, an experimental investigation is restricted by the test conditions such as high closure stress and the size of rock samples.

Numerical simulation has become a very powerful tool to investigate the proppant embedment of fracture conductivity. Some scholars developed different mechanical models such as analytical solution, discrete element method (DEM) model, and contact mechanics-based models. Alramahi and Sundberg18 presented an analytical model to predict the stress-
dependent conductivity of hydraulic fractures based on simple laboratory measurements of proppant embedment. Based on the fact that the fracture aperture changes with stress, a new mathematical model is built up to calculate the change in fracture aperture, proppant embedment, and deformation. The numerical results concluded that the proppant embedment was the main part that resulted in the change in fracture aperture (Gao et al.27). Guo et al.28,29 developed analytical models to calculate the embedment and conductivity. Features and controlling factors of embedment, residual width, and conductivity were also analyzed. Chen et al.30 modeled the proppant embedment as a function of effective stress by a transformed Hertz contact model and a proposed power law model. However, the analytical model is idealized and cannot represent the complex condition in the subsurface. Deng et al.23 simulated the shale–proppant interaction in hydraulic fracturing with a three-dimensional DEM model and investigated the influence of shale’s property, proppant size, and pressure level on the fracture aperture. Zhang et al.24 experimentally and numerically studied reduced fracture conductivity due to proppant embedment in the shale reservoir. Zhang et al.25 coupled the DEM/computational fluid dynamics to model the proppant embedment and fracture conductivity after hydraulic fracturing. In addition, the Hertz damage mechanics model is adopted to represent the proppant embedment. Ghanizadeh et al.20 used steady-state gas flow tests, high-resolution optical profilometry, microscopic observations, and mechanical (rebound) hardness to characterize unpropped/propped fracture permeability/conductivity and proppant embedment. Xu et al.22 investigated the effect of proppant deformation and embedment on fracture conductivity after fracturing fluid loss. Chen et al.31 proposed a new calculation method for embedment depth considering elastic–plastic deformation based on the mechanism of proppant embedment. However, in almost all these studies on proppant embedment, there may be two limitations. First, the great size contrast between proppants and reservoir rocks was neglected. Second, the reservoir rocks were treated as isotropic.

Before numerical or analytical modeling of proppant embedment is conducted, one essential problem should be addressed—proper calibration of mechanical properties of the reservoir rocks which the proppant particles are indented into. In previous studies, the mechanical properties of the reservoir rocks were either calibrated through core-based compression tests,15,17 sonic velocity and density petrophysical well logs,18 or specified directly. The rock specimens are usually cylinders with dimensions of \( \phi 25 \times 50 \) mm, \( \phi 38 \times 76 \) mm, or \( \phi 50 \times 100 \) mm. However, the proppant particles in petroleum industry are usually generally between 8 and 140 mesh (105 \( \mu \)m to 2.38 mm).29 As shown in Figure 2, the proppant is relatively small when compared with the rock plug. According to Benoit Mandelbrot,34 the magnitudes of physical quantities associate not only with the quantities being measured but also with the “ruler” used. When addressing the specific problem—proppant embedment—the calibration of the mechanical parameters of the core-based sample seems to use a “larger

Figure 1. Factors which affect proppant embedment. Reprinted with permission from Bandara et al.32 Copyright 2019 Elsevier.

Figure 2. Sizes of rock specimens and proppants.
The integrated properties of the entire sample are usually measured for geometrically defined samples and the proppant particles. The rock mechanical properties are often regarded as isotropic when considering the size differences between the rock plug and the rock surface, which results from fluid deterioration, especially inefficient to model the proppant embedded into a layered rock sample, which is idealized as isotropic, may be inadequate for other engineering practices; however, due to the striking scale contrast between proppants and reservoir rocks, whether they provide a resolution fine enough for the analysis of proppant embedment is still in doubt.

The mechanical properties obtained from core-based compression tests are usually regarded as isotropic. When the fracture surfaces are exposed to the fracturing fluid, which breaks down the reservoir rock and transports the proppants down to hold on the fracture, the reservoir rocks near the fracture surfaces may be weakened by this fluid. This deterioration effect decreases with increasing depth from the fracture surfaces. Thus, the mechanical properties calibrated from a rock plug, which is idealized as isotropic, may be inefficient to model the proppant embedded into a layered rock surface, which results from fluid deterioration, especially when considering the size differences between the rock plug and the proppant particles. The rock mechanical properties are usually measured for geometrically defined samples and describe the integrated properties of the entire sample. As a result, the compressive strength is often a poor indicator of the embedment behavior exhibited by proppants in natural formations, particularly in soft materials such as shale. As the disadvantage of modeling the proppant embedment with the mechanical parameters calibrated from traditional triaxial or uniaxial compression tests has been realized, nonstandard-sized rock samples of a small thickness of 7.6 mm were used to minimize the sample deformation contribution to the total deformation in the study of proppant embedment.

In this study, whether a heterogenous layered rock resulting from fluid deterioration could be fully represented by an equivalent isotropic condition in the modeling of proppant embedment is numerically investigated. A numerical experiment of the uniaxial compression test is first conducted on a heterogenous rock to obtain equivalent elastic parameters of an isotropic condition. With these equivalent parameters, the discrepancy of embedment depth between the heterogenous condition and the equivalent isotropic condition is examined via numerical simulations. Later, the problems of equating a heterogenous rock to an isotropic condition are discussed.

The remaining structure of this paper is organized as follows: the methodology section presents the numerical simulation of the embedment of proppants in layered rocks and isotropic rocks; the parameter study section presents the influences of proppant sizes and layers of rocks on the embedment depth; the discussion section uncovers the mechanism of the influence of proppant sizes and layers of rocks on the embedment depth; and finally, we give our findings in the last section.

### METHODOLOGY

**Introduction to ABAQUS.** ABAQUS is a software suite for finite element analysis and computer-aided engineering. ABAQUS offers powerful and complete solutions for both routine and sophisticated engineering problems, covering a vast spectrum of industrial applications. With ABAQUS we can quickly and efficiently create, edit, start, monitor, diagnose, and visualize advanced simulations. With ABAQUS, static stress/displacement analysis, dynamic stress/displacement analysis, steady-state transport analysis, heat transfer and thermal-stress analysis, fluid dynamic analysis, coupled pore fluid flow and stress analysis, electromagnetic analysis, mass diffusion analysis, and acoustic and shock analysis can be easily modeled. In this study, the key procedure used is the contact module, which will be explained in detail later.

**Equivalent Parameter Calibration.** Hou et al. experimentally investigated the effect of fluid damage on proppant embedment under various stresses and temperatures. The authors stated that fluid damage weakened the fracture surface and increased the proppant embedment. According to Mueller and Mohammed’s experimental study, the surface of the formation highly influenced by the treatment fluid can show great differences in the mechanical behavior compared to the untreated area composition of the samples. The greatest decrease was measured for the eagle ford formation, where the fluid contact causes a reduction in surface hardness of almost 50% from the initial value measured under dry conditions. In a more recent study, Song et al. have demonstrated the progressive deterioration effect of the fracturing fluid on rocks. They used the micro-indentation technique to characterize the distance-dependent gradient alteration of the mechanical parameter calibration process.
properties of a rock specimen after THMC treatment. In their study, the elastic modulus $E$ and hardness $H$ were found to vary with the radial distance $d$. They also concluded that the reduction in the Young’s modulus and hardness is caused by the softening induced by the shale-fracturing fluid interactions, and the degree of softening is more severe for longer treatment durations. For the 30- and 60-day treatments, the softening front advances by 5 and 8 mm, respectively. There is something in common with our assumption that the area near the fracture surface is more vulnerable to the fracturing fluid, and this area shows a lower elastic modulus than the area far away from the fracture surface.

As pointed out in the previous section, the deterioration effect of the fracturing fluid on the reservoir rocks will decrease with increasing depth from the fracture surface, as indicated by the gradual change in color in Figure 3a. To check whether the fluid-deteriorated rock plug could be described by homogeneous mechanical parameters, a numerical experiment is conducted. The elastic modulus and Poisson’s ratio of the layered rock are supposed to vary as indicated by Figure 3b, the gradual change in color in Figure 3a. To check whether the effect of the fracturing fluid on the reservoir rocks will decrease fluid, and this area shows a lower elastic modulus than the area far away from the fracture surface.

With the variation in elastic parameters, a numerical simulation of the uniaxial compression test of the layered rock specimen is put forward. In the numerical simulation, nonlinear variations of elastic parameters with a distance from the fracture surface are simplified to linear models as listed in Table 1.

Table 1. Elastic Parameters for Simulations

| layer number $i$ | elastic modulus $E_i$/GPa | Poisson’s ratio $v_i$ |
|------------------|---------------------------|----------------------|
| 1                | 20                        | 0.3                  |
| 2                | 21                        | 0.29                 |
| 3                | 22                        | 0.28                 |
| 4                | 23                        | 0.27                 |
| 5                | 24                        | 0.26                 |
| 6                | 25                        | 0.25                 |
| 7                | 26                        | 0.24                 |
| 8                | 27                        | 0.23                 |
| 9                | 28                        | 0.22                 |
| 10               | 29                        | 0.21                 |

The boundary conditions are indicated as shown in Figure 4a. The bottom of the rock specimen is fixed, and the pressure with a magnitude of 10 MPa is applied at the top. The different colors represent different layers with gradually changing mechanical parameters. The mesh scheme is plotted in Figure 4b. The C3D8R(eight-node brick element with reduced integration) element is adopted. Each layer is divided into 40 segments along the circumferential direction, with five segments in the axial direction. There are 519 elements in each layer. The axial displacement is shown in Figure 4c.

The maximum axial displacement of 0.00207 mm is found to be at the top surface. Thus, the equivalent elastic modulus of $E^*$, the layered rock specimen, is calculated as

$$E^* = \frac{\sigma}{\varepsilon} = \frac{\Delta L}{L} \frac{10 \text{ MPa}}{0.0207 \text{ mm}/50 \text{ mm}} = 24.2 \text{ MPa}$$

where $\sigma$ is the uniaxial stress; $\varepsilon$ is the axial strain; $\Delta L$ is the axial displacement, and $L$ is the height of the rock specimen. The equivalent Poisson’s ratio of the layered rock is

$$\nu^* = \frac{\Delta R/R}{\Delta L/L} = \frac{0.0013051 \text{ mm}/12.5 \text{ mm}}{0.0207 \text{ mm}/50 \text{ mm}} = 0.252$$

where $\Delta R$ is the lateral displacement.

According to the Backus average\(^\text{18}\) of the layered rock, the equivalent Young’s modulus $E^*$ of the layered material is

$$E^* = \left[ \frac{1}{L} \sum_{i=1}^{10} L_i \frac{E_i}{E_i} \right]^{-1} = 24.2 \text{ MPa}$$

where $L_i$ is the height of each layer; and $E_i$ is the elastic modulus of each layer.

The equivalent elastic modulus obtained from the numerical simulation of the uniaxial compression test is consistent with that obtained from the Backus average method. With the equivalent elastic parameters calibrated from the numerical experiment, the proppant embedment into the layered rock and the equivalent isotropic rock will be compared through numerical simulations.

Numerical Simulations of Proppant Embedment. The proppant embedment in the equivalent isotropic rock is first modeled. To lower the computation cost, a 1/4 model is adopted. The deteriorated formation is divided into 10 layers with mechanical parameters listed in Table 1. Each layer is 3 mm $\times$ 3 mm $\times$ 1 mm (length $\times$ width $\times$ height). The proppant is a sphere with a radius of 1 mm. The proppant is modeled as a discrete rigid body. Only 1/8 of the proppant is modeled. The boundary conditions are illustrated in Figure 5a. Due to the symmetry, the axially symmetrical conditions are cast on two vertical surfaces. The bottom of the formation is fixed. The formation is meshed as shown in Figure 5b, and the proppant is meshed as shown in Figure 5c. Each layer is meshed into 20 segments along the length and width and 5 segments along the height. The vertex of the proppant is defined as the reference point, as shown in Figure 5c, and a concentrated force of 94.2478 N along the axial direction is applied via the reference point. The top surface of the formation is specified as the slave surface, while the out surface in contact with the slave surface is defined as the master surface. As can be seen from Figure 5d, the embedment depth at the end of loading is 5.59 $\times$ 10$^{-2}$ mm.

To view proppant embedment in equivalent isotropic formation, such a numerical simulation is repeated, while the only difference is that the layered formation is now represented by the equivalent isotropic mechanical parameters. The boundary conditions are illustrated in Figure 6a and the vertical displacement in the equivalent isotropic case is plotted in Figure 6b. As can be seen from Figure 6b, the proppant embedment depth in the equivalent isotropic formation is 5.11 $\times$ 10$^{-2}$ mm. If the embedment depth in the layered formation is more close to the true value of embedment depth in the underground condition, the relative error of embedment depth of the equivalent isotropic case can be calculated as

$$\delta = \frac{5.11 - 5.59}{5.59} \times 100\% = -8.59\%$$

Here, in eq 4, the negative value indicates that the equivalent isotropic case underestimates the vertical displacement of the proppant normal to the fracture surface.
One question that may arise is whether this relative error of embedment depth would change with the proppant size when a different proppant size is adopted in hydraulic fracturing. Thus, a parameter study is conducted to investigate the influence of proppant size on the embedment depth. Different embedment depths under both layered conditions and equivalent isotropic conditions are simulated and listed in Table 2. In these numerical simulations, only the proppant diameter changes. As a result, the concentrated force applied at the reference point is accordingly changed to ensure that the closure pressure with a magnitude of 30 MPa is kept constant in these numerical simulations.

The relationship between the relative error of proppant embedment depth and the proppant size is also plotted in Figure 7. From Figure 7, equivalent isotropic conditions have smaller proppant embedment depth compared with that in layered conditions, which are regarded more close to the underground conditions. The differences in embedment depth between the two conditions increase with proppant sizes, while the absolute value of relative error decreases with proppant sizes.

To investigate the influence of the number of layers on proppant embedment, different layers are adopted to simulate the variation of mechanical behavior along the depth while keeping the total thickness of the formation. More specifically, the formation is cut into 1, 2, 5, and 10 layers. The case of 1 layer is equivalent isotropic formation, as described in the methodology section. With the same equivalent procedure, the equivalent elastic parameters of 5 layers, 2 layers, and 1 layer are also obtained and listed in Tables 3–5.

For the case of 5 layers, as listed in Table 3, the mechanical parameters of the equivalent layer 1 are obtained by a numerical simulation of a uniaxial compression test of a rock specimen composed of layers 1 and 2 of the original 10 layers. The mechanical parameters of equivalent layers 2 to 5 are obtained by the same procedure. The equivalent mechanical parameters in Tables 4 and 5 are readily available by repeating the method mentioned.

With the equivalent mechanical parameters listed in Tables 3–5, the simulations of proppant embedment under the conditions of different layers are conducted, as shown in Table 6.

The simulations listed in Table 6 differ from each other only in the equivalent mechanical parameters while sharing the same boundary conditions. From the table, the proppant embedment depths of different layers are plotted in Figure 8.

From Figure 8, the case of 1 layer shows the least proppant embedment depth, and it is the way we often adopt in the study of proppant embedment that the nonuniform-deteriorated formation is treated as an isotropic medium. Therefore, in this way, the proppant embedment depth may be underestimated, and then it may lead to a wider aperture than that estimated in the planning stage of hydraulic fracturing. As stated in the previous section, the simulation with 10 layers is a more realistic approximation of the progressively deteriorated formation. The embedment depth increases with the layers at a decreasing rate. In other words, the embedment depth will reach an asymptotic value if the simulation is conducted with more layers. The asymptotic value may be the true embedment depth of the real situation.

**DISCUSSION**

**Significance.** From Figure 7, one may question that the embedment depth is relatively small compared to the proppant size; thus, the study of the proppant may be meaningless. However, proppant embedment may significantly impair fracture conductivity due to a decrease in the fracture width and even a small reduction in fracture width may result in a significant loss of fracture conductivity. Actually,
many reasons, as listed in Figure 1, may affect the proppant embedment, and here, in this study, only the proppant size effect and the cause of the size effect are discussed. In this study, only the instantaneous embedment depth is considered; however, the embedment due to the creep behavior of reservoir rocks may also be significant. 39

**Reason.** In the parameter analysis section, the absolute value of the relative error decreases with proppant sizes. In other words, the relative error of the proppant embedment depth between the layered condition and the equivalent isotropic condition depends on the relative size of the proppant and the rock specimen for mechanical parameter calibration. In the above numerical analysis, the most conservative relative error is made because the largest proppant diameter and the smallest rock specimen size are adopted. In the above analysis, if a smaller proppant diameter and a larger rock specimen size are adopted, the relative error will be greater. The relative size of the proppant diameter and the rock

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**Figure 5.** (a) Boundary conditions; (b) proppant mesh; (c) formation mesh; and (d) vertical displacement.
The specimen used for parameter calibration does affect the accuracy of proppant embedment depth prediction. The mechanism for this size effect may be explained by Saint-Venant’s principle. More specifically, according to Hertz contact theory, at the center of the contact area, the stress $\sigma_z$ normal to the fracture surface follows the relationship with depth as shown in eq 5 and decreases rapidly as shown in Figure 9.

$$\frac{\sigma_z}{p_0} = \frac{1}{1 + (z/a)^2}$$

(5)

where $p_0$ is the maximum value of $\sigma_z$ when depth $z = 0$; $a$ is the radius of the contact area and calculated as

$$a = \sqrt{\frac{3PR}{4E'}}$$

(6)
Table 3. Equivalent Elastic Parameters of 5 Layers

| layer number i | elastic modulus $E_i$/GPa | Poisson’s ratio $v_i$ | equivalent layer | elastic modulus $E_i$/GPa | Poisson’s ratio $v_i$ |
|----------------|----------------------------|----------------------|-------------------|----------------------------|----------------------|
| 1              | 20                         | 0.3                  | 1                 | 20.7                       | 0.299                |
| 2              | 21                         | 0.29                 | 2                 | 22.7                       | 0.278                |
| 3              | 22                         | 0.28                 | 3                 | 24.8                       | 0.258                |
| 4              | 23                         | 0.27                 | 4                 | 26.7                       | 0.237                |
| 5              | 24                         | 0.26                 | 5                 | 28.7                       | 0.217                |

Table 4. Equivalent Elastic Parameters of 2 Layers

| layer number i | elastic modulus $E_i$/GPa | Poisson’s ratio $v_i$ | equivalent layer | elastic modulus $E_i$/GPa | Poisson’s ratio $v_i$ |
|----------------|----------------------------|----------------------|-------------------|----------------------------|----------------------|
| 1              | 20                         | 0.3                  | 1                 | 22.1                       | 0.282                |
| 2              | 21                         | 0.29                 | 2                 | 24.6                       | 0.24                |
| 3              | 22                         | 0.28                 | 3                 | 26.7                       | 0.23               |
| 4              | 23                         | 0.27                 | 4                 | 28.7                       | 0.217               |
| 5              | 24                         | 0.26                 | 5                 | 30.8                       | 0.202               |

Table 5. Equivalent Elastic Parameters of 1 Layer

| layer number i | elastic modulus $E_i$/GPa | Poisson’s ratio $v_i$ | equivalent layer | elastic modulus $E_i$/GPa | Poisson’s ratio $v_i$ |
|----------------|----------------------------|----------------------|-------------------|----------------------------|----------------------|
| 1              | 20                         | 0.3                  | 1                 | 24.2                       | 0.252                |
| 2              | 21                         | 0.29                 | 2                 | 26.7                       | 0.24                |
| 3              | 22                         | 0.28                 | 3                 | 28.7                       | 0.23               |
| 4              | 23                         | 0.27                 | 4                 | 30.8                       | 0.202               |
| 5              | 24                         | 0.26                 | 5                 | 32.9                       | 0.192               |

where $P$ is the total load acting on the proppant, $E^*$ is the equivalent elastic modulus, and $R$ is the equivalent radius expressed as

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

(7)

where $R_1$ and $R_2$ are the radii of the proppant and the fracture surface, respectively. In the analysis of proppant embedment, the fracture surface is regarded as a semi-infinite body and thus $R_2 \to \infty$. Thus, the equivalent radius $R$ in eq 5 can be replaced with the proppant radius $R_1$. The load $P$ is calculated in terms of closure pressure $p$ multiplied by the section area of the proppant

$$P = p \times \pi \times R_1^2$$

(8)

The equivalent elastic modulus $E^*$

$$\frac{1}{E^*} = \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}$$

(9)

As the proppant in the analysis is treated as a rigid body, the elastic modulus of the proppant $E_1 \to \infty$. With eqs 6–8, the radius of contact area in eq 5 is rewritten as

$$a = \sqrt{\frac{3(1-v_1^2)pR_1^3}{4E_2}} = \sqrt{\frac{3(1-v_2^2)pR_1}{4E_2}}$$

(10)

According to eq 5, the normal stress decreases to 0.10$p_a$ at 3a way below the center of the contact area as shown in Figure 9. Scope 3a is regarded as the effective acting depth of contact force, and this effective acting depth is proportional to the radius of the proppant $R_1$. The mechanical parameters of the rock in the range of effective acting depth have the most significant influence on the proppant embedment. In other words, the smaller the proppant radius, the smaller the effective acting depth of proppant embedment. Suppose in eq 10, $v_2 = 0.254$, $p = 20$ MPa, and $E_2 = 25.4$ GPa, the largest effective acting depth $z_{eff} (z_{eff} = 3a = 0.429\, \text{mm})$ is obtained when the proppant radius adopts its peak value of $R_1 = 1.19\, \text{mm}$. That is to say, the characteristics of the rock within the range of 0.429 mm from the fracture surface have the most significant influence on proppant embedment. Even in the most conservative conditions when the largest proppant radius is adopted, the effective acting depth of 0.429 mm is relatively small compared to the size of the rock plug. When smaller proppants are used, the effective acting depth will be even lower. In hydraulic fracturing practice, as shown in Figure 3, from the surface, the deterioration effect of the fracturing fluid decreases with increasing depth. The fracture surface has the lowest strength, and the proppant embedment would be the most severe. The mechanical properties obtained from traditional compression tests of rock plugs should be regarded as averaged parameters which could not provide enough resolution to describe the gradual deterioration effect normal to the fracture surface. It should also be noted that in compression tests, the compressive stress is uniformly distributed throughout the entire rock plug, while in realistic underground conditions, due to the significant contrast between the sizes of proppants and the fracture surface, the severe stress concentration at the contact area makes the application of mechanical properties of rocks calibrated from traditional compression tests conservative in the prediction of proppant embedment depth.

**Solution.** From the above discussion of scale problems involved in the modeling of proppant embedment, it may be questionable to model the fluid-deteriorated rock, which is more similar to a layered rock as an equivalent isotropic rock. The difference in embedment depth between the value predicted by mechanical parameters calibrated from the standard-sized specimen test and the true value increases with a decrease in the proppant size. The proppant particles used in practice with a diameter ranging from 105 $\mu$m to 2.38 mm are rather small compared to rock plugs with dimensions $\phi 25$ $\times$ 50 mm, $\phi 38$ $\times$ 76 mm, or $\phi 50$ $\times$ 100 mm; thus, it would be problematic to obtain mechanical parameters used for proppant embedment modeling by traditional compression tests of rock plugs with dimensions $\phi 25$ $\times$ 50 mm or even larger sizes. Mechanical properties being measured on the core could not provide a fine enough resolution to model the heterogeneities resulting from fracturing fluid deterioration in...
the modeling of proppant embedment. Measurements based on the core scale do not necessarily identify the mechanical mechanisms at work at smaller scales that are responsible for the macroscopic observation. The fluctuation of elastic properties in a rock sample or even a smaller scale could not be identified through traditional compression tests of rock specimens ranging from $\phi 25 \times 50$ mm to $\phi 50 \times 100$ mm. Fortunately, new testing methods, such as micro-indentation and nano-indentation techniques, which can be used to directly measure the elastic properties of a rock sample at a given point, are now available. These indentation techniques can detect fluctuations in the elastic properties of rocks at scales ranging from a few grain diameters up to a few tens of centimeters, which is extremely important in accurate modeling of proppant embedment problems. As measuring the mechanical properties of rocks with a fine resolution of millimeters or less is feasible using micro-indentation tests, it is possible to map the deterioration effect of the fracturing fluid on the mechanical properties of rocks which would be used in proppant embedment modeling. As shown in Figure 10, two methods may be introduced to carry out the conceptual design of the nano/micro-indentation tests of the fluid-deteriorated rock specimens. In method I, the fluid-deteriorated rock specimen, as shown in Figure 10a, is first cut into slices perpendicular to the axle of the specimen, as illustrated in Figure 10b. Then, the nano/micro-indentation tests are conducted on each slice as shown in Figure 10c, and each dot represents one indentation test. The mechanical properties of each slice can be obtained through Mori–Tanaka homogenization techniques.

The differences in the alternative method are the cutting direction and the quantities of slices, as shown in Figure 10d. In method II, the rock specimen is cut into two pieces along the axle center.

### CONCLUSIONS

From the point of view of scales, by numerical simulations, the study answers the question why the elastic parameters of a heterogeneous rock resulting from fracturing fluid deterioration could not be fully equated to a corresponding isotropic condition in the modeling of proppant embedment. The fracturing fluid-deterioration effect makes the rocks layered materials. Traditional compression tests based on core scale
samples could not provide mechanical parameters with enough resolution to be used in the modeling of proppant embedment, as the progressive deterioration effect along the depth could not be identified by these tests. Micro-indentation or nano-indentation seem to be promising techniques in the calibration of mechanical parameters for the modeling of proppant embedment, as they can identify the fluctuations in the elastic properties of rocks at scales ranging from a few grain diameters up to a few tens of centimeters.

**ASSOCIATED CONTENT**

Data Availability Statement
Data sharing not applicable—no new data were generated, as the article is entirely based on analysis and numerical research.

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Notes
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**ABBREVIATIONS**

| Symbol | Description |
|--------|-------------|
| E      | elastic modulus |
| H      | hardness |
| d      | radial distance |
| σ     | uniaxial stress |
| ε     | axial strain |
| ΔL    | axial displacement |
| L     | height of the rock specimen |
| v*    | equivalent Poisson’s ratio of the layered rock |
| ΔR    | lateral displacement |
| E*    | equivalent young’s modulus of the layered material |
| L0    | height of each layer |
| Ed    | elastic modulus of each layer |
| δ     | relative error |
| σc    | contact stress |
| z     | depth |
| P0    | maximum value of contact stress |
| a0    | radius of the contact area |
| P    | total load acting on the proppant |
| Re    | equivalent radius |
| R1    | radius of the proppant |
| R0    | radius of the fracture surface |
| p     | closure pressure |
| v1    | Poisson’s ratio of the proppant |
| v2    | Poisson’s ratio of the reservoir rock |
| zeff  | effective acting depth |

Figure 10. (a) Standard-sized sample; (b) slices perpendicular to the axle of the rock specimen; (c) layout of indentation points on each slice; and (d) layout of indentation points on the axially cut piece.
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