Improving the Lattice QED Action

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Strongly coupled QED is a model whose physics is dominated by short-ranged effects. In order to assess which features of numerical simulations of the chiral phase transition are universal and which are not, we have formulated a quenched version of the model in which photon degrees of freedom are defined on a lattice of spacing $a$, but fermions only on a lattice of spacing $2a$. The fermi-photon interaction is then obtained via a blocking procedure, whose parameters allow a degree of control over the relative importance of short wavelength modes. Results from a variety of models are presented; the critical exponents $\delta$ and $\beta_m$ governing the transition appear to be independent of the blocking, or even of whether a gauge-invariant action is used for the photons.

In this talk I will describe work done in collaboration with John Kogut. We have been experimenting with different variants of the non-compact lattice QED action. The motivation is twofold: firstly we wish to get a better understanding of chiral symmetry breaking, which occurs at strong coupling, and in particular to disentangle the role of various features, eg. whether the physics is dominated by short or long ranged effects; the relative importance of photon exchange and four-fermi contact interactions; and the role, if any, of magnetic monopole excitations. Secondly, if there were a UV fixed point in strongly coupled QED, then there should be the possibility of some renormalization group “improvement” which would widen the scaling region seen in simulations and give cleaner signals.

A specific example is the hypothesis that the fixed point can be described by the gauged Nambu – Jona-Lasinio model:

$$\mathcal{L} = \bar{\psi}D\psi + \frac{1}{4}F_{\mu\nu}^2 + G[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2].$$

(1)

Studies using the quenched ladder approximation predict a line of fixed points running from the NJL mean field transition at $(\alpha, G) = (0, 4)$ to the essential singularity discussed by Miransky and Bardeen et al at $(\alpha_c, 1)$ with $\alpha_c = \pi/3$. Critical exponents of the chiral transition are analytic functions of $\alpha/\alpha_c$ along the fixed line. Numerical simulations of quenched non-compact QED (NC QED) yield estimates of exponents consistent with crossing the fixed line at $(0.44 \alpha_c, 3.06)$. Why might the gauged NJL model be a good effective theory for lattice QED? One clue is the lattice photon propagator. In Feynman gauge,

$$D_{\mu\nu}^{-1}(k) = \delta_{\mu\nu} \sum_{\rho} 4 \sin^2 \frac{k_{\rho}}{2}.$$  

(2)

This falls below the continuum form $k^2$ at the edge of the Brillouin zone, and generates an effective contact interaction between vector currents, as well as the term due to photon exchange.

Our proposal is to modify the relative weights of large and small $k$ modes in the propagator by modifying the lattice photon action. The approach is simple: from photon fields $\theta_{\mu}(x)$ generated on a $(2N)^4$ lattice using the standard non-compact action we form blocked links $\Theta_{\mu}(x)$ on a $N^4$ lattice via:

The action remains Gaussian under blocking and it is straightforward to examine the new dispersion curves (fig. 1). From the large family of possible blocked actions we chose two forms to examine in simulations: (1) $a = 1, b = c = d = 0$. We found this to enhance short wavelength modes.
over long, effectively increasing the “screening” in the model; and (II) $a = 1, b = \frac{1}{4}, c = d = \frac{1}{16}$. This choice gave a relative suppression of short wavelength modes. For comparison we also tried a “totally screened” random link (RL) model in which the $\{\Theta\}$ are generated independently on each link with Gaussian weight $\exp(-16\Theta^2)$.

To examine the critical properties of the chiral transition, we estimated the chiral condensate $\langle \bar{\psi}\psi \rangle$ in the chiral limit bare mass $m \rightarrow 0$ on (blocked) lattice sizes $8^4, 12^4, \text{ and } 16^4$ in two complementary ways \cite{1}. First we measure $\langle \bar{\psi}\psi \rangle$ as a function of $m$ by inverting $(D/\Theta + m)$, where $D$ is the staggered fermion covariant derivative, using a conjugate gradient algorithm. We used a sequence of $m$ values $0.005, 0.004, \ldots, 0.001$. Secondly, we estimate the spectral density function $\rho(\lambda)$ by using the Lanczos algorithm to find the eigenvalues of $D/\Theta$. The spectral density is related to a branch cut in $\langle \bar{\psi}\psi(m) \rangle$ along the imaginary $m$ axis:

$$2\pi \rho(\lambda) = \lim_{\varepsilon \rightarrow 0} \langle \bar{\psi}\psi(-i\lambda + \varepsilon) \rangle - \langle \bar{\psi}\psi(-i\lambda - \varepsilon) \rangle.$$  \hspace{1cm} (3)

On the assumption of power-law scaling at criticality, we have a relation between the critical coupling $\beta_c \equiv 1/\epsilon_c^2$ and the critical exponent $\delta$:

$$\langle \bar{\psi}\psi(m) \rangle |_{\beta = \beta_c} = Am^{1/\delta}.$$  \hspace{1cm} (4)

The two parameters can be estimated by looking for the best fit of eqn. (4) as a function of $\beta$ to the conjugate gradient data. Alternatively, the Lanczos data can be fitted using the ansatz

$$\rho(\lambda)|_{\beta = \beta_c} = B\lambda^{1/\delta'}.$$  \hspace{1cm} (5)

As a cross check, using (3) we have the relations

$$\delta = \delta'; \quad A = \frac{\pi B}{\cos(\pi/2\delta')}.$$  \hspace{1cm} (6)

By demanding consistency between the two sets of measurements, we can achieve accuracy of better than a percent in estimating $\beta_c$, and 5% in $\delta$. It is also possible to estimate the magnetic exponent $\beta_m$ from the $\rho(\lambda)$ data, though with less precision:

$$\rho(\lambda = 0) = C(\beta - \beta_c)^{\beta_m}.$$  \hspace{1cm} (7)

Another observable we kept track of was the size of the largest connected monopole cluster, along with the associated cluster susceptibility \cite{3}. By monitoring this on a series of lattice sizes $8^4, 10^4, \ldots, 20^4$ we were able to estimate the critical coupling for the percolation transition. This is worth studying, because the percolation threshold is close to the chiral transition in NC QED, and actually appears to coincide once dynamical fermions are introduced. Although the relevance of monopoles to the chiral transition is controversial \cite{4}, the percolation threshold is at least a characteristic property of the “QED vacuum”, and it will prove interesting in this study.

Our results for the chiral transition are presented for each of the four quenched models we have studied in the following table. The suscepti-

|            | $\beta_c$ | $\delta$ | $\beta_m$ | $\gamma$ |
|------------|-----------|----------|-----------|----------|
| NC QED     | 0.257(1)  | 2.1(1)   | 0.86(3)   | 1.03     |
| Block I    | 0.664(4)  | 2.3(1)   | 0.82(6)   | 1.07     |
| Block II   | 0.112(4)  | 2.3(1)   | 0.96(7)   | 1.25     |
| RL         | 0.056(1)  | 2.3(2)   | 0.73(3)   | 0.95     |

bility exponent $\gamma$ was calculated using the scaling relation $\gamma = \beta_m(\delta - 1)$: it is included because the quenched ladder approximation predicts it to be
unity \[1\][2]. We see that the main trend is for the models which are screened relative to the standard formulation (ie. short wavelength modes less suppressed) to break chiral symmetry at weaker bare coupling, whereas the less screened model (Block II) has the transition at stronger coupling. The critical inverse photon propagators \[D^{-1}_{\mu\nu}(k)\] are shown for each model in fig. 2, averaged over the corners of the Brillouin zone, since the blocking violates hypercubic symmetry to an extent. Also shown for comparison is the critical inverse propagator for 4 flavor QED, assuming a critical coupling \[\beta_c \simeq 0.205\] and logarithmic screening using the perturbative formula. The crucial observation is that all the curves differ strongly in the \(k \to 0\) limit, and approach each other near the zone edge. This is conclusive evidence that chiral symmetry breaking in QED is due to short-ranged interactions. It is also striking how similar the estimates for the exponent \(\delta\) are in the four models, even for the RL model which violates gauge invariance. The universality of \(\delta\) and (less convincingly) \(\beta_m\) remains to be understood – perhaps it is a feature of the staggered fermion formulation. It will be extremely valuable to get quantitative results using other formulations.

In the next table we compare the critical couplings for the chiral and monopole percolation thresholds. From this we learn that the two transitions can be made distinct if screening is lessened: however for “screened” models (Block I, \(N_f = 2, 4 \text{ QED}\) the two transitions appear to coincide \[3\][4]. Figure 2 also supports a crude division of models into “screened” and “unscreened”.

In the quenched theory, we also see that

\[
\beta_c(\chi_{SB}) \geq \beta_c(\text{percolation}) \tag{8}
\]

for all models in which the photon fields are generated using a gauge-invariant action.

The results of this study have raised more questions than have been answered. Little support has been found for the scenario predicted by the ladder approach to the gauged NJL model: we have been unable to affect the critical exponents by tuning the short-ranged properties of the model. Perhaps a more radical approach, such as a formulation in momentum space \[3\], is needed. We have also shown that monopole percolation can be made separate from the chiral transition, at least in the quenched case, but have found little evidence for the percolation threshold occurring at a weaker coupling than the chiral transition, except in the RL model. It would appear desirable to continue to explore this problem from many different directions.

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