The fabrication and study of devices that combine triplet superconductivity and magnetism in a controlled manner will undoubtedly lead to new insights about the unique interplay of these two phases, and possibly also novel spintronic devices. Much attention has therefore been directed at developing the theoretical understanding of such systems [1, 2, 3, 4, 5, 6]. These efforts have already revealed many unconventional effects in the transport properties of such devices: for example, the Josephson charge current through a triplet-superconductor–ferromagnet–triplet-superconductor (TFT) junction depends not only upon the magnitude (as in a singlet junction, see Ref. 7) but also upon the orientation of the ferromagnetic moment at the barrier [2, 6]. The origin of this behaviour is the interaction of the barrier moment with the spin-structure of the triplet state. Despite the clear importance of the spin, only a few works have considered the appearance of a spontaneous spin current in materials where the pairing has a triplet component [4, 6, 8, 9, 10].

In this letter we study the appearance of a Josephson spin current in a minimal model of a TFT junction [2, 6]. We find three general mechanisms for producing a spin current: spin-filtering, misalignment of the triplet vector order parameters (d-vectors), and spin-flipping off a transverse barrier moment. The symmetry properties of the spin currents due to each mechanism are analyzed in detail, and the physical origins are understood by examining the contributions to the charge and spin currents from each spin sector. Unlike the Josephson charge current, we show that the Josephson spin currents on each side of the barrier can be different. By judicious choice of the tunnelling barrier and the d-vector alignment, we can exercise fine control over the magnitude, direction and orientation in spin space of the spin currents on either side of the barrier.

The Hamiltonian describing the TFT junction is given by $H = \int \mathcal{H}(z, z') dzdz'$ with Hamiltonian density

$$\mathcal{H}(z, z')$$

$\hat{\mathcal{H}}(z, z')$
\[ \mathbf{M}(z, z') = (M_L \cos \alpha, M_L \sin \alpha, M_\parallel) \delta(z) \delta(z'); \] 
\[ \hat{\sigma}_{\alpha \beta} \] denotes the vector of Pauli matrices. In what follows, we set \( \hbar = 1 \) and quote values of \( M_L, M_\parallel \) and \( U_P \) in units of \( m/k_F \) where \( k_F \) is the Fermi momentum. A schematic diagram of the junction is shown in Fig. (1).

We construct the \( 4 \times 4 \) Nambu-spin space Green’s function \( \hat{G}(z, z', \Omega; i\omega_n) \) of the junction in terms of quasi-classical scattering wavefunctions. A full account of this method for singlet superconductors is given in Ref. [11]; the generalization to the triplet case is straightforward.

The spin quantization plane of each superconductor. The class of the Josephson currents are best appreciated by isolating the potentials \( \mu_L \) and \( \mu_R \) produced by the spin-filter, we find that the spin current can flow in the absence of a charge current \[ \text{Fig. (3)(d)} \]. Unlike the spin-filter, the spin current is antisymmetric in both \( x \)-axis. 

The most direct way to produce a spin current is by the spin-filtering effect [6]. For a spin-filter between equal-spin-pairing superconductors, we require both a finite \( U_P \) and a component of the magnetic moment \( \mathbf{M} \) in the spin quantization plane of each superconductor. The component of \( \mathbf{M} \) in the spin quantization planes then defines a preferred direction, with the spin projections along this axis not mixed by the scattering at the barrier. Rather, we find spin-dependent effective scattering potentials \( U_P \pm |\mathbf{M}| \). The transparency of the barrier is therefore different in each spin sector, favouring the transmission of one spin species of Cooper pair over the other, i.e., the barrier selectively filters out one spin state, see Fig. (2)(a).

By time-reversal symmetry and mirror reflection of the junction in the \( x-z \) plane, we find that the spin current is antisymmetric in both \( \phi \) and \( \mathbf{M} \). We now specialize to the case when \( \mathbf{M} = M_\parallel \hat{z} \), as this remains a spin-filter for arbitrary \( \theta \). The spin structure of the Josephson currents are best appreciated by isolating the contributions to the total charge and spin currents from each spin sector: we define the spin-\( \sigma \) charge current as \( I_{\nu,\sigma} = (I_{\nu} - \sigma 2e I_{\nu,\parallel})/2 \), and the spin-\( \sigma \) spin current \( I_{\nu, z, \sigma} = -\sigma I_{\nu,z,\parallel}/2e \). For the spin-filter, we find that for \( U_P, M_\parallel > 0 \) the magnitude of \( I_{\nu,\parallel} \) is greater than \( I_{\nu,\parallel} \) [Fig. (3)(a)]. As such, \( I_{\nu, z, \parallel} \) and \( I_{\nu, z, \parallel} \) do not cancel, giving us the finite spin current shown in Fig. (3)(b).

Asano has shown that a spin current is also generated in a non-magnetic junction by misaligning the \( d \)-vectors of the superconductors [8]. This produces a gradient of the order parameter in spin space, driving a spin current in analogy to the situation in superfluid \( ^3 \)He [12]. For the TFT junction studied here, this results in distinct phase differences in each of the spin channels, \( \phi_\nu = \phi - \sigma \theta \) [6, 13], as illustrated schematically in Fig. (2)(b). We show the charge currents through each spin sector in Fig. (3)(c): they are 2\( \theta \) out of phase with respect to one another but otherwise identical. Although each spin sector has a finite current flowing through it at \( \phi = 0 \), these cancel and so the total charge current is vanishing: this is not true of the spin current, demonstrating that the spin current can flow in the absence of a charge current [Fig. (3)(d)]. Unlike the spin-filter, the spin current produced by the \( d \)-vector misalignment is symmetric with respect to \( \phi \), which follows from time-reversal symmetry. When \( d \)-vector misalignment is combined with a spin-filter, the charge currents do not cancel at \( \phi = 0, \pi \) and the junction is then in a fractional state [13, 14].

Both the spin-filter and the \( d \)-vector misalignment mechanisms preserve the spin of the tunnelling Cooper pairs. We hence find that, like the charge current, the spin currents on either side of the junction are the same. This is not the case, however, for the third mechanism,
FIG. 3: (color online) Current vs phase relations for the three basic mechanisms. (a) Charge current and (b) z-component of spin current for a spin-filter barrier \( (M_\|, U_P = 1) \), decomposed into contributions from each spin sector. (c) Charge current and (d) spin current for a non-magnetic barrier and with misaligned \( d \)-vectors of the two superconductors \( (U_P = 1, \theta = 0.2\pi) \), other parameters vanishing. (e) Charge current and (f) spin current on the LHS of a barrier with a transverse moment \( (M_\perp = 0.5, \alpha = 0.3\pi) \), other parameters vanishing. The spin current on the RHS is identical in magnitude but has opposite sign. In all figures we set \( T = 0.4T_c \).

where the barrier magnetic moment has a component outside the quantization plane of the two superconductors. We consider the least complicated such scenario: we have aligned \( d \)-vectors, and there is only a transverse moment at the junction (i.e. \( M_\perp \neq 0, M_\parallel = U_P = \theta = 0 \)). We find that only the z-component of the spin current is non-vanishing for \( 0 < \alpha < \pi/2 \). Although the magnitude of the spin current is the same on each side of the junction, the *sign* is reversed, i.e. the \( \nu = L \) and \( \nu = R \) spin currents flow in opposite directions. This remarkable result is confirmed by a symmetry analysis: treating the spin current as a function only of the angle \( \alpha \) and the phase \( \phi \), we find by time-reversal and inversion symmetry that \( I_{\nu z}^z(\alpha, \phi) = -I_{R z}^z(\alpha, \phi) \) [15]. For \( M_\perp = 0.5 \) and \( \alpha = 0.3\pi \), we show the charge current and its decomposition into spin-\( \uparrow \) and spin-\( \downarrow \) components on each side of the barrier in Fig. (3)(e), while the z-component of the spin currents on the LHS are shown in Fig. (3)(f).

The physical interpretation of this behaviour follows by examining the charge currents through each spin sector: from Fig. (3)(e), we observe that the current through the spin-\( \uparrow \) \( (\downarrow) \) sector on the LHS of the junction is equal to the current through the spin-\( \downarrow \) \( (\uparrow) \) sector on the RHS. That is, the spin of some of the tunnelling Cooper pairs is flipped by the transverse moment; this spin-relaxation process prevents a spin-accumulation occurring at the barrier. When a spin-\( \sigma \) electron-like (hole-like) quasiparticle tunnelling between two normal regions undergoes a spin-flip at the tunnelling barrier, the transmitted and reflected waves acquire a phase shift of \( +(-)\sigma \). Although the spin components of the charge current indicate that the situation is somewhat more complicated for tunnelling between two superconductors, this spin-dependent phase shift is ultimately the origin of the spin current. This conclusion is immediately apparent from the presence of the normal-normal scattering matrix in the boundary conditions for the quasiclassical Green’s function at a spin-active interface [14]. We illustrate the spin-flipping mechanism schematically in Fig. (2)(c).

We have now discussed in detail the three basic mechanisms for the production of a spin current in the TFT junction. We have already seen that the sign of the spin current need not be the same on either side of the junction. By combining the three mechanisms, however, we demonstrate that we can also vary the magnitude and spin-orientation of the current.

Upon combining the transverse moment with a spin-filter \( (M_\perp, M_\parallel, U_P \neq 0, \theta = 0) \), we find two significant changes: a \( y \)-component of the spin current appears in addition to the \( z \)-component, and the magnitude of the spin currents on either side of the barrier are no longer the same for \( 0 < \alpha < \pi/2 \). We show the spin currents on the LHS and RHS of such a barrier in Fig. (3)(a) and (b) respectively. We consider first the origins of the \( y \)-component. For \( \alpha \neq 0, \pi \), this follows immediately from the spin-filter effect: for aligned \( d \)-vectors, the \( y \)-axis lies within the quantization plane of both superconductors. As the magnetization lying in the \( y-z \) plane has a finite component along the \( y \)-axis, the resulting spin-filter selects a direction with a finite \( y \)-component, and so \( I_{s,y}^z \neq 0 \). This is the only magnetic interaction at the barrier when \( \alpha = \pi/2 \), and the spin current is then the same on either side of the junction. For \( 0 < \alpha < \pi/2 \), however, there is a finite component of the magnetization along each axis: the scattering off the components transverse to the \( y \)-axis and off those transverse to the \( z \)-axis also produces a \( y \)- and a \( z \)-component of the spin current respectively. As above, the spin currents produced by the spin-flipping flow in opposite directions on either side of the junction. The total spin current on each side of the junction is then the sum of a contribution which is symmetric and a contribution which is antisymmetric with respect to \( \nu \). As the former is antisymmetric in \( \phi \) while the latter is symmetric, and both
We finally discuss the combination of a spin non-conserving moment with a misalignment of the $d$-vectors. For a finite $\theta \neq 0, \pi$, the quantization planes of the two superconductors only intersect along the $z$-axis: any barrier moment which has a component transverse to the $z$-axis is not spin-conserving for the tunnelling Cooper pairs. We therefore consider the case where, in the absence of the $d$-vector misalignment, the magnetic scattering produces a $y$-component of the spin current. When the $d$-vectors are misaligned, a $y$-component of the spin current still flows on the LHS; on the RHS, however, we have an additional $x$-component of current, which is related to the $y$-component by $I_{s,x}^R = \tan(\theta) I_{s,y}^R$. By using a magnetic moment at the barrier, we are therefore able to produce a spin current on the RHS with a component along the axis perpendicular to the quantization plane in the LHS superconductor. This situation is displayed in Fig. 4(c) and (d), which respectively show the spin currents on the LHS and RHS of a spin-filter for the $y$-component of spin ($M_\perp U_P \neq 0, \alpha = \pi/2$). Note that the finite $z$-component of the spin current is composed of contributions due to the misaligned $d$-vectors and also from the spin-flip scattering. Unlike the previous cases, there is no simple relationship between the spin currents on either side of the barrier, which is reflected in Fig. 4(c) and (d) by the considerable differences between the $\nu = L$ and $\nu = R$ $z$-component of the spin current: this is strongly enhanced at $\phi = \pi$ on the RHS over that on the LHS, while its value at $\phi = 0$ is much the same in the two cases.

In conclusion, we have used the quasiclassical method to study the interaction of triplet superconductivity and magnetism in a model Josephson junction. Our main result is the classification of the three distinct mechanisms for the production of a spin current. The signature properties of each of these mechanisms are given intuitive physical explanations by decomposing the total charge and spin currents into contributions from each spin sector [Fig. 2]. We demonstrate that in the presence of a spin-relaxing moment at the barrier, neither the magnitude, direction or spin orientation of the spin currents on each side of the junction need be identical. Our work theoretically opens the route for the inclusion of triplet Josephson junctions in spintronic applications.

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