Dynamics of Crime In and Out of Prisons

Jongo Park* and Pilwon Kim†
Department of Mathematical Sciences
Ulsan National Institute of Science and Technology (UNIST)
Ulsan Metropolitan City
44919, Republic of Korea

March 4, 2019

Abstract

The accumulated criminal records shows that serious and minor crimes differ in many measures and are related in a complex way. While some of those who have committed minor crime spontaneously evolve into serious criminals, the transition from minor crime to major crime involves many social factors and have not been fully understood yet. In this work, we present a mathematical model to describe how minor criminals turns in to major criminals inside and outside of prisons. The model is design to implement two social effects which respectively have been conceptualized in popular terms “broken windows effect” and “prison as a crime school.” Analysis of the system shows how the crime-related parameters such as the arrest rate, the period of imprisonment and the in-prison contact rate affect the criminal distribution at equilibrium. Without proper control of contact between prisoners, the longer imprisonment rather increases occurrence of serious crimes in society. An optimal allocation of the police resources to suppress crimes is also discussed.

1 Introduction

Understanding what factors cause a high crime rate is essential to developing effective measures to prevent crime in a community. The accumulated criminal records[8,15] shows that serious and minor crimes differ in many measures such as occurrence rate, arrest rate and rehabilitation rate. There is also difference between control activity of the police devoted to serious crimes and that devoted to minor crimes[2].

In recent years, there have been substantial progresses in developing mathematical tools to investigate criminal activity. Mathematical models based on the reaction-diffusion equations have been proposed[29,27,26] to study dynamics of localized patterns of criminal activity, especially focusing on the re-victimisation phenomena and the hot-spot formation. Some other models have adapted their basis from population biology, such as infectious disease models[4,22,18] and predator-prey models[32,6,20]. A similar approach was used for modeling organized crimes[7,31], where gang membership is treated as an infection that multiplies through peer contagion.

*carlorton625@unist.ac.kr
†pwkim@unist.ac.kr
In this study, we present a mathematical model to describe how minor criminals turn into major criminals. While some of those who have committed minor crime spontaneously evolve into serious criminals [21], the transition from minor crime to major crime involves many social factors and have not been fully understood yet. Besides the basic progressive nature of crime, we are interested in finding extra factors that accelerate the transition from minor crimes to major crimes. This paper focuses on criminal transitions occurring inside and outside of prison, which respectively have been conceptualized in popular terms “broken windows effect” and “prison as a crime school”: the broken windows theory states that accumulation of low level offenses in a community, if not adequately controlled, acts as a social pressure that leads to more serious crimes [33, 12, 5]. On the contrary, in prisons, staying with many criminal peers in a limited facility makes minor criminals frequently contact with hard-core and skilled criminals and possibly deepens their illegal involvement [11, 9, 13]. The peer-effect on crime recidivism is strongly supported by recent empirical research in many countries [1, 10, 24].

We are especially interested in the influence of over-crowded prison facilities on criminal transition. A mathematical model reflecting the effect of incarceration on recidivism has been proposed in [18]. However, to the authors’ knowledge, the in-prison dynamics between minor and major criminals and the effect of the prison capacity on it have never been studied before in a mathematical framework. The behavior of the model is investigated through stability analysis, bifurcation analysis, and numerical simulations. By analyzing the corresponding system of equations, we demonstrate how the crime-related parameters such as the arrest rate, the rehabilitation rate and the capacity of the prison affect the criminal distribution at equilibrium. The results also suggest an optimal allocation of the police resources to minimize occurrence of serious crimes. This may be used to assist policy-makers in the development of effective crime control strategies.

2 Model

Basic assumptions

We first formulate our proposal as a compartmental model, with the population \(T > 0\) being divided into five disjoint groups \(N, M, F, P_M\) and \(P_F\). The group \(N\) represents non-criminals who have never involved any criminals or have finished serving their prison sentence. \(M\) and \(F\) denote individuals who have committed misdemeanor and felony respectively, but have not get arrested yet. Once they are arrested and are sentenced to be imprisoned, they become inmates, \(P_M\) and \(P_F\), respectively. They may return to normal civilian \(N\) after serving their sentence in prison. However, part of them commit a crime again, reverting back to \(M\) or \(F\). Figure 1 shows the structure of transitions occurring between groups in the basic model.

In the absence of good evidence to the contrary, we follow the common rules in population dynamics: 1) the transfer out of any particular group is proportional to its size. 2) if the transfer is caused by contact between two group members, it is proportional to the size of both groups. Based on the rule 1, we set \(c \geq 0\) and \(d \geq 0\) to express the transition rates from \(N\) to \(M\), and from \(M\) to \(F\), respectively.

\[
N \xrightarrow{c} M \quad \text{and} \quad M \xrightarrow{d} F
\]  

We assume the direct transition from \(N\) to \(F\) is negligible compared to that from \(M\) to \(F\). The sequential transition from \(N\) through \(M\) to \(F\) is justified from the report that most of those people who commit major crime have committed minor crimes before [21].
The second rule (transition by contact) is based on the assumption that the population is homogeneously mixed, and should be dealt with in our model with care. While the models for organized gang crimes [7, 31] treat the transition between gang members as contagion by frequent contact, such analogy with epidemic process may be not adequate for daily contacts occurring in a general society. Considering the nature of criminals that hide their true intention from others, we can presume that assimilation with criminals occurring by random contact is rare and negligible compared to other factors that we will consider in the following sections.

Once criminals are caught and convicted, they are imprisoned to serve their sentence. If the parameters $a_M \geq 0$ and $a_F \geq 0$ are respectively the arrest-and-conviction rates for misdemeanor and felony, the corresponding transitions are

$$M \xrightarrow{a_M} P_M \quad \text{and} \quad F \xrightarrow{a_F} P_F \quad (2)$$

Let $i_M > 0$ and $i_F > 0$ be the period of imprisonment for minor and major criminals, respectively. In addition, let $0 \leq r_M \leq 1$ and $0 \leq r_F \leq 1$ denote the rehabilitation rates for minor and major criminals, respectively, which determines the proportion of prisoners moving back to after release. Then we have the transitions

$$P_M \xrightarrow{r_M/i_M} N \quad \text{and} \quad P_F \xrightarrow{r_F/i_F} N \quad (3)$$

This implies that the other portions, $1 - r_M$ and $1 - r_F$, of inmates commit a crime again, as

$$P_M \xrightarrow{(1-r_M)/i_M} M \quad \text{and} \quad P_F \xrightarrow{(1-r_F)/i_F} F \quad (4)$$

**Broken windows effect out of prisons**

Although the above transitions describe the basic structure of dynamics between criminals and non-criminals, it does not properly reflect important interactions between them. We extend the
basic model to incorporate the broken windows theory. The theory states that seemingly petty signals of mischief, if not adequately controlled, elicit more serious crime. In order to describe the atmospheric pressure that accelerates transition from minor criminals to major criminals, we add to (1) the quadratic size effect as

\[ M + M \xrightarrow{b} F + F \]  

(5)

where \( b \geq 0 \) is the coefficient that represents the broken-windows effect.

### Crime-school effect in prisons

We further extend the model to investigate how interactions in prison influence post-release behavior of criminals. The beneficial deterrent effect of prison may be weakened by the negative side-effects of incarceration: Through routine contacts in a limited facility area, criminals get to learn from each other, build new networks and find new opportunities for crime[13].

We assume that, in prison, criminal motivations and skills are spread between inmates and the “contagion” occurs by frequent contacts between them. To describe such in-prison transition from the minor criminals to the major criminals, we define \( P'_M \) as those who used to be minor criminals and turn to be major criminals by assimilation with them. They emerged from the transitions as

\[ P_M + P_F \xrightarrow{\beta} P'_M + P_F \quad \text{and} \quad P_M + P'_M \xrightarrow{\beta} P'_M + P'_M \]  

(6)

where \( \beta \geq 0 \) is the transmission contact rate of prisoners. It is desirable to maintain the total number of the prisoners under the capacity of the prison facilities. Overcrowding in prison increases intensity of interactions between prisoners and raises risk of recidivism[19]. The existence of \( P'_M \) is problematic, as they are virtually major criminals, while they are treated as minor ones: they are “disguised major criminals” and are released after short detention with a low rehabilitation rate.

\[ P'_M \xrightarrow{r_F / i_M} N \quad \text{and} \quad P'_M \xrightarrow{(1 - r_F) / i_M} F. \]  

(7)

Now, based on the transitions (1) to (7), we derive the corresponding model as

\[
\begin{align*}
\frac{dN}{dt} &= -cN + \frac{r_M}{i_M} P_M + \frac{r_F}{i_F} P_F + \frac{r_{FM}}{i_M} P'_M \\
\frac{dM}{dt} &= cN - (a_M + d)M - bM^2 + \frac{1 - r_M}{i_M} P_M \\
\frac{dF}{dt} &= dM + bM^2 - a_F F + \frac{1 - r_F}{i_F} P_F + \frac{1 - r_{FM}}{i_M} P'_M \\
\frac{dP_M}{dt} &= a_M M - \frac{1}{i_M} P_M + \beta P_M (P_F + P'_M) \\
\frac{dP_F}{dt} &= a_F F - \frac{1}{i_F} P_F \\
\frac{dP'_M}{dt} &= -\frac{1}{i_M} P'_M + \beta P_M (P_F + P'_M)
\end{align*}
\]  

(8)

where \( N(0) \geq 0, M(0) \geq 0, F(0) \geq 0, P_M(0) \geq 0, P_F(0) \geq 0 \) and \( P'_M(0) \geq 0 \). The model is completed by the identity \( N + M + F + P_M + P_F + P'_M = T \) which states that the population is closed.
3 Results

The model (8) is a 6-dimensional nonlinear systems and is hard to analytically investigate. However, if the transitions in prison are completely suppressed with $\beta = 0$, then asymptotic behavior of the model (8) is determined by a four dimensional dynamics of which equilibrium solution can be obtained.

**Theorem 1.** Suppose all the parameters in the system (8) are positive except $\beta = 0$. There is a unique equilibrium $(N^*, M^*, F^*, P^*_M, P^*_F, P'_M)$ of the system (8) such that

$$
M^* = \frac{-p + \sqrt{w}}{2q},
$$

$$
F^* = \frac{cT - s_M M^*}{s_F},
$$

$$
P^*_M = i_M a_M M^*,
$$

$$
P^*_F = i_F a_F F^*,
$$

$$
P'_M = 0,
$$

$$
N^* = T - M^* - F^* - P^*_M - P^*_F,
$$

where

$$
p = cd + cda_F i_F + ca_F r_F + da_F r_F + ca_M a_F i_M r_F + a_M a_F r_M r_F,
$$

$$
q = bc + bca_F i_F + ba_F r_F,
$$

$$
w = p^2 + 4ca_F r_F q T,
$$

$$
s_M = c + a_M i_M + a_M r_M,
$$

$$
s_F = c + a_F i_F + a_F r_F.
$$

The equilibrium is locally asymptotically stable either for sufficiently small values of $a_M$ and $a_F$, or for sufficiently large values of $i_M$ and $i_F$.

In this section, we perform the bifurcation analysis of the proposed model (8). For a typical simulation, we set the parameters as

$$
b = 0.00001, c = 0.00012, d = 0.0004, a_M = 0.1, a_M = 0.1, r_M = 0.4, r_F = 0.2
$$

$$
\beta = 0.001, i_M = 0.5, i_F = 5.
$$

These parameters are calibrated such that the distribution of major/minor criminals and their arrest rates largely agree with crime statistics in several countries[8, 15]. We set the total population $T = 1,000,000$ through out the analysis. In Figure 2, the bifurcation diagram for the broken-windows effect $b$ is illustrated. The minor crime tends to decrease and the major crime increases as $b$ grows. This implies that elimination of environmental factors that reveal misdemeanors is important to prevent occurrence of more serious crimes. Note that keeping $b$ near zero cuts down major criminals to a negligible level. This implies that the broken widows effect becomes even more important in a safe society where the ratio of major criminals is relatively low.
While suppressing $b$ promotes the preventive effect on serious crimes out of prison, controls inside prison can work as a more practical measure against crimes. The bifurcation diagram in Figure 3 shows how the distribution of criminals changes with the contact rate $\beta$ in prison. The
rise of $\beta$ increases the number of major criminals in society.

Another in-prison measure to control crimes is the period of imprisonment for criminals. There are mixed evidence regarding the question of whether spending more time in prison increases the rehabilitation rate \cite{14, 16}. However, here we assume that the period of imprisonment and the rehabilitation rate are at least weakly positively correlated, as long as criminals are held in custody in a effectively managed facility. Let us denote $w \geq 1$ as a general weight of offense and set the period of imprisonment as

\[ i_M = w i_M^{\text{min}} \quad \text{and} \quad i_F = w i_F^{\text{min}} \]  \hspace{1cm} (10)

where $i_M^{\text{min}} = 0.5(\text{year})$ and $i_F^{\text{min}} = 0.5(\text{year})$ are the minimum period for minor and major criminals, respectively. The weight of offense is also related with the rehabilitation rate. We set as

\[ r_M = 0.06w + 0.34 \quad \text{and} \quad r_F = 0.03w + 0.17 \]  \hspace{1cm} (11)

so that $r_M$ and $r_F$ slightly increases with $w$. Note that this agrees with (9) when $w = 1$.

![Figure 4: Equilibrium distribution according to the weight of offense $w$](image)

Figure 4 shows how the weight of offense contributes to the criminal distribution in two cases: (a) with $\beta = 0.0002$ and (b) $\beta = 0.001$. It is no surprising that the number of inmates increases with $w$ in both cases, since a higher weight of offense means a longer detention. More noteworthy differentiation between (a) and (b) is the change in $F$, the number of major criminals in society, according to $w$. When the transmission contact rate is as low as $\beta = 0.0002$, a higher weight of offense reduces the number of criminals. On the contrary, when $\beta = 0.001$, assigning more weight of offense leads to increase of major crimes in society. Hence a higher weight of offense has a positive reform effect only when the frequent contact between prisoners is effectively prohibited.

Changes in the security measures are likely to have a greater impact on crime \cite{17}. Let us investigate how the allocation of the police resource to control activity of major/minor crime affects the distribution of criminals. Let $c_T$ be the total budget for security. Also let $c_M$ and $c_F$ be the budget for control of minor crime and major crime, respectively. Note $c_T = c_M + c_F$. We assume that the arrest rate is proportional to the budget used to control the crime. Then we can set $a_M = e_M c_M$ and $a_F = e_F c_F$ where $p_M$ and $p_F$ are the police efficiency for minor and major
crime, respectively. In the example, $e_M = 0.2$ and $e_F = 0.04$ are used. Figure 5 shows how the budget ratio $c_F/c_T$ affects the number of major criminals. The minimum of $F$ is achieved at around $c_F/c_T \approx 0.4$. Spending more portion of the budget for the major crime control brings negligence on minor crimes, which eventually leads to excessive occurrence of major crimes due to the broken windows effect and the crime school effect.

![Figure 5: Equilibrium distribution according to the allocation of police resource. $e_M = 0.2, e_F = 0.04$](image)

4 Discussion

We here present the mathematical models for crime dynamics that mainly focus on transition from minor to major criminals occurring in and out of prisons. It is confirmed that both the broken windows effect and the crime-school effect greatly change the criminal distribution. While utilizing the broken windows effect is a preventive measure, improving conditions in correctional facilities can provide a more direct and efficient measure against crimes. The presented work showed that suppressing interactions between overcrowded inmates in prisons is crucial in controlling crimes in society. If not keeping the in-prison contact rate at a low level, extension of the period of imprisonment only results in rapid increase in major crimes.

The model also shows the importance of an balanced resource allocation between control activity devoted to serious crimes and that devoted to minor crimes. The analysis confirms that, due to the broken windows effect and the crime-school effect, targeting only major crimes can be very inefficient and even bring an opposite result that increases major criminals. While the results in this work are not predictions, we hope that they can provide useful insights into crime dynamics and possibly suggest effective policies towards crime abatement.
Acknowledgements
This work was supported by the Ministry of Education of the Republic of Korea and the National Research Foundation of Korea (NRF-2017R1D1A1B04032921). The funder had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

References

[1] Patrick Bayer, Randi Hjalmarsson, and David Pozen, Building criminal capital behind bars: Peer effects in juvenile corrections, The Quarterly Journal of Economics 124 (2009), no. 1, 105–147.

[2] David W Britt and Charles R Tittle, Crime rates and police behavior: A test of two hypotheses, Social Forces 54 (1975), no. 2, 441–451.

[3] Courtney Brown, Serpents in the sand: Essays on the nonlinear nature of politics and human destiny, University of Michigan Press, 1995.

[4] Michael Campbell and Paul Ormerod, Social interaction and the dynamics of crime, Volterra Consulting Ltd (1997).

[5] Magdalena Cerdá, Melissa Tracy, Steven F Messner, David Vlahov, Kenneth Tardiff, and Sandro Galea, Misdemeanor policing, physical disorder, and gun-related homicide: a spatial analytic test of "broken-windows" theory, Epidemiology (2009), 533–541.

[6] Donna Marie Giselle Comissiong, Joanna Sooknanan, and Balswaroop Bhatt, Criminals treated as predators to be harvested: a two prey one predator model with group defense, prey migration and switching, Journal of Mathematics Research 4 (2012), no. 4, 92.

[7] ______., Life and death in a gang-a mathematical model of gang membership, Journal of Mathematics Research 4 (2012), no. 4, 10.

[8] National Research Council et al., Understanding crime trends: Workshop report, National Academies Press, 2009.

[9] Francis T Cullen, Cheryl Lero Jonson, and Daniel S Nagin, Prisons do not reduce recidivism: The high cost of ignoring science, The Prison Journal 91 (2011), no. 3 _suppl, 48S–65S.

[10] Anna Piil Damm and Cedric Gorinas, Deal drugs once, deal drugs twice: peer effects on recidivism from prisons, Essays on Marginalization and Integration of Immigrants and Young Criminals—A Labor Economics Perspective. Aarhus University (2013).

[11] Anna Piil Damm and Cédric Gorinas, Prison as a criminal school: Peer effects and criminal learning behind bars, The Rockwool Foundation Research Unit Study Paper (2016), no. 105.

[12] Bernard E Harcourt and Jens Ludwig, Broken windows: New evidence from new york city and a five-city social experiment, The University of Chicago Law Review (2006), 271–320.

[13] Anaïs Henneguelle, Benjamin Monnery, and Annie Kensey, Better at home than in prison? the effects of electronic monitoring on recidivism in france, The Journal of Law and Economics 59 (2016), no. 3, 629–667.
[14] Young-Oh Hong, *A study on recidivism rates and recidivism prediction for violent crimes*, Korean Institute of Criminology, 2000.

[15] J van Kesteren, Patricia Mayhew, and Paul Nieuwbeerta, *Criminal victimization in seventeen industrialized countries*, WODC, 2000.

[16] Patrick A Langan and David J Levin, *Recidivism of prisoners released in 1994*, Fed. Sent. R. 15 (2002), 58.

[17] YongJei Lee, John E Eck, and Nicholas Corsaro, *Conclusions from the history of research into the effects of police force size on crime—1968 through 2013: a historical systematic review*, Journal of Experimental Criminology 12 (2016), no. 3, 431–451.

[18] David McMillon, Carl P Simon, and Jeffrey Morenoff, *Modeling the underlying dynamics of the spread of crime*, PloS one 9 (2014), no. 4, e88923.

[19] Benjamin Monnery, *Incarceration length and recidivism: qualitative results from a collective pardon in france*, Post-print, HAL, 2015.

[20] Juan C Nuno, Miguel A Herrero, and Mario Primicerio, *A triangle model of criminality*, Physica A: Statistical Mechanics and its Applications 387 (2008), no. 12, 2926–2936.

[21] Australia. Parliament. House of Representatives. Standing Committee on Legal, Constitutional Affairs, and 1942 Bishop, Bronwyn, *Crime in the community : victims, offenders and fear of crime*, [Canberra, A.C.T. : House of Representatives, Standing Committee on Legal and Constitutional Affairs], 2004 (English), Title from title frame of PDF file ; viewed 5 Dec. 2004.

[22] Paul Ormerod, Craig Mounfield, and Laurence Smith, *Non-linear modelling of burglary and violent crime in the uk*, Volterra Consulting Ltd (2001).

[23] D Wayne Osgood, *Statistical models of life events and criminal behavior*, Handbook of quantitative criminology, Springer, 2010, pp. 375–396.

[24] Aurelie Ouss, *Prison as a school of crime: Evidence from cell-level interactions*, (2011).

[25] Jovan Rajs, Teet Härm, and Ulf Brodin, *A statistical model examining repetitive criminal behavior in acts of violence*, The American journal of forensic medicine and pathology 8 (1987), no. 2, 103–106.

[26] Nancy Rodriguez and Andrea Bertozzi, *Local existence and uniqueness of solutions to a pde model for criminal behavior*, Mathematical Models and Methods in Applied Sciences 20 (2010), no. supp01, 1425–1457.

[27] Martin B Short, Andrea L Bertozzi, and P Jeffrey Brantingham, *Nonlinear patterns in urban crime: Hotspots, bifurcations, and suppression*, SIAM Journal on Applied Dynamical Systems 9 (2010), no. 2, 462–483.

[28] Martin B Short, P Jeffrey Brantingham, Andrea L Bertozzi, and George E Tita, *Dissipation and displacement of hotspots in reaction-diffusion models of crime*, Proceedings of the National Academy of Sciences (2010).
[29] Martin B Short, Maria R D’orsogna, Virginia B Pasour, George E Tita, Paul J Brantingham, Andrea L Bertozzi, and Lincoln B Chayes, *A statistical model of criminal behavior*, Mathematical Models and Methods in Applied Sciences **18** (2008), no. supp01, 1249–1267.

[30] Martin B Short, Maria R D’orsogna, Patricia J Brantingham, and George E Tita, *Measuring and modeling repeat and near-repeat burglary effects*, Journal of Quantitative Criminology **25** (2009), no. 3, 325–339.

[31] J Sooknanan, B Bhatt, and DMG Comissiong, *Catching a gang—a mathematical model of the spread of gangs in a population treated as an infectious disease*, International Journal of Pure and Applied Mathematics **83** (2013), no. 1, 25–43.

[32] Louis G Vargo, *A note on crime control*, The bulletin of mathematical biophysics **28** (1966), no. 3, 375–378.

[33] James Q Wilson and George L Kelling, *The police and neighborhood safety: Broken windows*, Atlantic monthly **127** (1982), no. 2, 29–38.