Relations between Electromagnetic Form Factors of Baryons

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The inclusion of two-body exchange currents in the constituent quark model leads to new relations between the electromagnetic properties of octet and decuplet baryons. In particular, the $N \rightarrow \Delta$ quadrupole transition form factor can be expressed in terms of the neutron charge form factor.

1. Neutron and $\Delta$ charge form factors

In Ref. \textsuperscript{[1]} we have shown that the Sachs charge form factor $G_E^n(q^2)$ and charge radius $r_n^2 = -6(d/dq^2)G_E^n(q^2) \big|_{q^2=0}$ of the neutron are dominated by quark-antiquark pair exchange currents shown in Fig.\textsuperscript{1}(b-c). The latter describe the gluon and pion degrees of freedom, while the one-body currents in Fig.\textsuperscript{1}(a) describe the valence quark degrees of freedom in the nucleon. The two-body exchange charge operator contains a spin-dependence of the schematic form \[ \rho_{[2]} \propto \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j Y^0(q) - \frac{\sqrt{6}}{2} \left[ [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]^2 \times Y^2(q) \right]^0. \] This gives different matrix elements for quark pairs in spin 0 and spin 1 states. Because a down-down pair in the neutron is always in a spin 1 state, while an up-down pair can be in spin 0 or spin 1 states, an asymmetry in the charge distribution of up and down quarks arises. Consequently, a nonvanishing neutron charge form factor (see Fig.2) and radius is obtained \textsuperscript{[1]}:

\[ r_n^2 = -\frac{b^2}{3m_q} \left( \delta_g(b) + \delta_\pi(b) \right) = -b^2 \frac{M_\Delta - M_N}{M_N}. \]  

Here, $b$ is the quark core (matter) radius of the nucleon, $m_q$ is the constituent quark mass, and the functions $\delta_g(b)$ and $\delta_\pi(b)$ are the gluon and pion contributions to the $N-\Delta$ mass splitting, which satisfy $M_\Delta - M_N = \delta_g(b) + \delta_\pi(b)$.

The spin-spin term in the charge operator is also responsible for the following relation between the charge form factors of the nucleon and $\Delta$:

\[ G_E^n(q^2) - G_E^\Delta^+(q^2) = G_E^n(q^2), \quad r_p^2 - r_\Delta^+ = r_n^2, \]  

where $r_\Delta^+$ is the charge radius of the $\Delta^+$ and $q$ is the three-momentum transfer of the photon. An analogous result holds for the difference of neutron and $\Delta^0$ form factors. The charge form factor of the $\Delta^0$ and the corresponding charge radius are zero in the present

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model as it should be on general grounds \[3\]. From its derivation \[4\] it is evident that the charge radius relation is independent of whether gluon or pion exchange is the dominating residual interaction between constituent quarks. Using a general parametrization method, Dillon and Morpurgo \[5\] have recently shown that, if three-quark currents and strange quark loops are neglected, \( r^2_p - r^2_{\Delta^+} = r^2_n \) is a consequence of the symmetries and dynamics of QCD that is model-independently valid. They have also shown that three-body currents slightly modify, but do not invalidate the general relation between the proton, neutron, and \( \Delta \) charge radii.

We repeat that Eq. (2) is a consequence of the spin-spin \( \sigma_i \cdot \sigma_j / (m_i m_j) \) term in \( \rho^{[2]} \), which leads to a \( \Delta \) charge radius that is larger than the proton charge radius. This effect is of the same generality as, and closely connected with the \( N - \Delta \) mass splitting due to the spin-spin interaction in the Hamiltonian. The latter is repulsive in quark pairs with spin 1 and makes the \( \Delta \) heavier than the nucleon.\[6\]

2. Electromagnetic \( N \rightarrow \Delta \) transition form factors

In the constituent quark model with exchange currents a connection between the neutron charge form factor \( G^m_N(q^2) \) \[1\] and the \( N \rightarrow \Delta \) quadrupole transition form factor \( F^{p\rightarrow\Delta^+}_Q(q^2) \) \[2,4\] emerges:

\[
F^{p\rightarrow\Delta^+}_Q(q^2) = \frac{3\sqrt{2}}{q^2} G^m_N(q^2), \quad Q_{p\rightarrow\Delta} = \frac{r^2_n}{\sqrt{2}}, \quad r^2_{Q,p\rightarrow\Delta^+} = \frac{11}{20} b^2 + r^2_{\gamma q}. \tag{3}
\]

The \( N \rightarrow \Delta \) quadrupole transition form factor is a measure of the intrinsic deformation of the nucleon and the \( \Delta \). The above results for the \( N \rightarrow \Delta \) transition quadrupole moment, \( Q_{p\rightarrow\Delta^+} \), and the transition quadrupole radius\[4\], \( r^2_{Q,p\rightarrow\Delta^+} \), were derived before\[2,4\]. They are seen here to be the 0th and 1st moment of the more general relation between \( G^m_N \) and

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\[3\] Combining Eq. (1) and Eq. (2) yields \( r^2_{\Delta^+} - r^2_p = [b^2/(3m_q)](M_{\Delta^+} - M_p) \), i.e., a relation between the mass difference between \( \Delta^+ \) and proton and a corresponding charge radius difference.

\[4\] The term \( 1/10 b^2 \) in Eq. (53) of Ref. [2] should be replaced by \( 11/20 b^2 \).
Figure 2. Neutron charge form factor $G_E^n(Q^2)$ and the quadrupole transition form factor $F_Q^{p\rightarrow\Delta^+}(Q^2)$ as a function of four-momentum transfer $Q^2 = -q^2$. The crosses, triangles, circles are the recent data \[6\]. The upper curve is a quark model calculation with exchange currents \[1,2\]. The gluon and pion contributions to $G_E^n(Q^2)$ are shown separately. The lower curve is $F_Q^{p\rightarrow\Delta^+}(Q^2) = -3\sqrt{2} G_E^n(Q^2)/Q^2$.

$F_Q^{p\rightarrow\Delta^+}$, plotted as the lower curve in Fig.\[4\]. The quark model with exchange currents explains $Q_{p\rightarrow\Delta^+}$ as a double spin flip of two quarks, with all valence quarks remaining in the dominant, spherically symmetric $L = 0$ state. The double spin-flip comes from the tensor term in $\rho_2$. The latter is closely related to the tensor term in the Hamiltonian. The quark core ($D$ waves in the nucleon) also contributes to $Q_{p\rightarrow\Delta^+}$. This valence quark contribution amounts to about 20% (due to the smallness of the $D$ wave amplitudes) of the double spin flip amplitude \[4\]. We conclude that the collective gluon and pion degrees of freedom are mainly responsible for the deformation of the $N$ and $\Delta$.

Due to the first relation in Eq.(\[3\]), the quadrupole transition radius can also be expressed as the 2nd moment of $G_E^n(q^2)$, namely, $r_{Q,p\rightarrow\Delta^+}^2 = (18/r_n^2) (d/dq^2)G_E^n(q^2)|_{q^2=0}$. Because the quark core radius $b$ is fixed by Eq.(\[1\]), one could extract the charge radius of the light constituent quarks, $r_{\gamma q}^2$, from both the $G_E^n(q^2)$ data, and from the slope of $F_Q^{p\rightarrow\Delta^+}(q^2)$ at $q^2 = 0$. Both determinations of $r_{\gamma q}^2$ should agree.

It is interesting that the additive quark model relation between the magnetic $N \rightarrow \Delta$ transition and the neutron magnetic moments $\mu_{p\rightarrow\Delta^+} = -\sqrt{2}\mu_n$ remains unchanged after including the gauge-invariant two-body exchange currents of Fig.(\[b-d\]); and that it continues to hold even at finite momentum transfers

$$F_{M}^{p\rightarrow\Delta^+}(q^2) = -\sqrt{2} G_M^n(q^2), \quad \mu_{p\rightarrow\Delta^+} = -\sqrt{2} \mu_n, \quad r_{M,p\rightarrow\Delta^+}^2 = -\sqrt{2} \frac{\mu_n}{\mu_{p\rightarrow\Delta^+}} r_{M,n}^2, (4)$$
where \( r_{M,p\rightarrow\Delta^+}^2 \) is the magnetic \( N \rightarrow \Delta \) transition radius, and \( r_{M,n}^2 \) the magnetic radius of the neutron. The transition magnetic moment predicted by Eq. (4) underestimates the empirical value by 30%. This discrepancy between theory and experiment can presumably be explained by including spatial three-body currents in the theoretical description [4].

Combining Eq. (3) and Eq. (4) we find that the ratio of the charge quadrupole and magnetic dipole \( N \rightarrow \Delta \) transition form factors can be expressed in terms of the experimentally better known elastic neutron form factors

\[
\frac{F_Q^{p\rightarrow\Delta^+}(q^2)}{F_M^{p\rightarrow\Delta^+}(q^2)} = \frac{3}{q^2} \frac{G_E^n(q^2)}{G_M^n(q^2)}, \quad \frac{C2}{M1} = \frac{M_N\omega_{cm} G_E^n(q^2)}{2q^2 G_M^n(q^2)}. \tag{5}
\]

where \( \omega_{cm} = 258 \text{ MeV} \) is the center of mass energy of the photon-nucleon system at the \( \Delta \) resonance. For example, this yields \( F_Q^{p\rightarrow\Delta^+}(q^2=0) = 1.04 \frac{r_n^2}{(2\mu_n)} = -0.030 \) and \( F_M^{p\rightarrow\Delta^+}(q^2=4.2 \text{ fm}^{-2}) = -0.042 \). Sign and magnitude of these theoretical predictions are in agreement with recent experimental data \( C2/M1(q^2=4.2 \text{ fm}^{-2})_{exp} = -0.046(8) \) [8].

3. Relations between octet and decuplet hyperon charge radii

Using the general parametrization method of Refs. [6,7], we find the following relations between octet and decuplet charge radii for strange hyperons:

\[
r_{\Sigma^-}^2 - r_{\Sigma^+}^2 = r_{\Xi^-}^2 - r_{\Xi^+}^2 = r_n^2 \left( x + x^2 \right), \quad r_{\Sigma^+}^2 - r_{\Sigma^0}^2 = r_{\Xi^0}^2 - r_{\Xi^+}^2 = r_n^2 \left( 2x - x^2 \right), \tag{6}
\]

where \( x = m_u/m_s \) is the ratio of nonstrange to strange quark masses. Again, it is the \( \sigma_i \cdot \sigma_j/(m_i m_j) \) term in the charge operator that leads to Eq. (6).

In summary, by including two-body currents in the constituent quark model we have found hitherto unknown relations between the electromagnetic form factors of octet and decuplet baryons. In particular, the \( C2/M1 \) ratio in the electromagnetic \( N \rightarrow \Delta \) transition can be expressed in terms of the elastic form factors of the neutron.

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