Scaffolding trajectory: depicting teacher's thinking

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Abstract. Contingency is the main condition of scaffolding, where scaffolding is provided in accordance with the students' understanding so that the relationship between teacher and student in the problem-solving process is very close. In that condition, it is worth to learn about how the teacher thought is in diagnosing student' obstacle in dealing with the problem in mathematics learning. This qualitative research aims to investigate the scaffolding trajectory in solving mathematics problems that is limit of trigonometric. Scaffolding trajectory is used to analyze how the teacher thinks in providing assistance. Participants in this case study consisted of two teachers and 41 students of XII grader from two schools in Malang, Indonesia. The results showed that there were differences in the teachers’ scaffolding trajectories. There were two categories of teacher's thinking in guiding students; diagnosis and lack of diagnosis. The category of diagnosis is identified from the assistance provided by the teacher that is appropriate to the needs of the students, while lack of diagnosis is identified from the inappropriate assistance provided by the teacher for the student's needs.

1. Introduction
Students’ involvement in the learning process cannot be separated from any assignment proposed by the teachers. The assignment, which teachers give to the students, is aimed to achieve the learning objectives. The description of the assignment sequences in the learning and of student’s thinking is conceptualized in learning trajectory. Learning trajectory, in brief, as a description of student’s thinking and learning designed to support the student in understanding specific mathematical domain [1]. One aspect in learning trajectory is an instructional sequence involving assignment. The assignment is composed by a teacher to improve effectively student's understanding in constructing concepts and skills in accordance with the initial knowledge of the student. Therefore, scaffolding is an important element that cannot be avoided from the learning trajectory discourse. Based on the term of learning trajectory, in this study, scaffolding trajectory is conceptualized as a depiction of the teacher's thinking in providing sequences of student’s assignments in guiding that aims to improve student's comprehension and skills in solving mathematical problems. The problem creates an opportunity for the teacher to assess what his
or her students are learning and where they are experiencing difficulty and to teach mathematics concepts and procedures [2,3]. Students need scaffolding in the problem-solving process to become successful [4].

Scaffolding term was initially introduced by David et al. to describe a support by a person who has the capacity in the instructional process for students, and this support is given during the process in order that the students are able to solve the problem independently [5]. Scaffolding has been developed by researchers, one of them is Derek and David, defining that scaffolding is an action in instructional process in supporting students’ knowledge construction and giving students a foundation so that the students are able to work independently [6]. From scaffolding concepts developed by former researchers, scaffolding can be viewed as a help provided by teachers to improve students' understanding where students cannot achieve without guidance.

In the process of mathematics instruction, scaffolding is the main component that should be provided by teachers, especially in problem-solving activities. In this activity, students may have difficulties so they need help to solve the problem posed by the teacher. One of the mathematics material, which is considered difficult by students, is limit [7, 8, 9]. In fact, limit comprehension is very important as a foundation for students to be able to learn the next calculus material.

One of the scaffolding characteristics is contingent [10]. Contingent is a condition where scaffolding is provided according to student understanding. Contingency is a support to students adapted to existing levels of understanding [11]. Implementation of contingent characteristics in the learning process requires the ability of teachers to diagnose students' understanding. Teachers should understand how the existing knowledge domain of students and what knowledge is needed in solving problems.

Based on the background of the importance of diagnosing in contingency, the focus of this study is to investigate the implementation of teacher diagnosis to students in providing scaffolding in mathematics problem-solving. The results will complement the previous studies, where studies that provide attention to teacher’s diagnosis [12] and evidence of contingency in the learning process are limited [13], especially in mathematics instructions. Teacher's scaffolding is tracked in the form of a trajectory to find out how teacher thinking when responding to the student by providing scaffolding in the process of solving a mathematics problem.

2. Methods
This qualitative research uses a case study approach aimed at analyzing a particular case [14], that is how the implementation of diagnostic strategies in contingency scaffolding during solving mathematics problem process. Case study approach was chosen in aiming at providing facts on the ground as the basis for further related studies. Participants of the study included two teachers and forty-one students with the moderate ability of grade XII from two schools in Malang, Indonesia. The collected qualitative data consisted of instructional videos, teachers’ interviews, and students' works. This article presents illustrations of examples of teachers scaffolding moments to students.

The data from the video was a recording of the teacher scaffolding action to the student in solving the trigonometric limit problem that was determining the result of $\lim_{x \to 0} \left( \frac{1}{x \tan x} - \frac{\cos x}{x \sin x} \right)$. The focus of the recording was the teacher's action in providing scaffolding [10] when the teacher gave scaffolding to a student [15]. The collected data were then analysed for exploring the data to obtain a general understanding of the data. Coding data was done to categorize scaffolding teacher’ action (Sc), student’ obstacle (Ob), student’ response (Res), diagnosis (Di) and lack of diagnosis (LcDi). Student’ obstacles showed student’ difficulty, which was identified as a wrong way in problem solving. Coding was the process of segmenting in labelling text to describe and interpret data [16]. The description was to make a detail of the data as information that can help researchers in tracking the teachers’ scaffolding in the process of assisting students solving in the trigonometric limit problem.

3. Results and discussion
The framework of this study is the scaffolding characteristics of contingency [10] and the implementation of diagnostic strategies to determine the level of students’ understanding and guidance
that will be given to the students by the teachers using asking questions, giving hints or prompt after paying attention to the students’ work [17]. The results of the observations indicated that there were differences in scaffolding trajectories depicting teachers’ thinking. There were two categories of teachers’ thinking; diagnosis and lack of diagnosis in the implementation of scaffolding contingency in the process of solving the problem that will be described below.

3.1. Diagnosis
Teacher (T1) observed the work process of the student (S1) in solving the problem. S1’s work is presented in Figure 1. T1 identified student' difficulties in constructing fractional forms according to trigonometric limit theorem [7]. S1’s error caused S1 to fail to continue the fractional form on the first line, which was seen from the student’s writing (Ob1) that changes $\frac{1}{x\tan x}$ becomes $\frac{1}{x\tan x}$.

The result of S1’s work (Figure 1) showed that S1 multiplied $\frac{1}{x\tan x}$ with $\frac{1}{x\tan x}$ in the numerator and denominator. Then, S1 tried to build another form of the fraction that was changing the problem into $\frac{\sin x - (1 - \sin^2 x)\tan x}{x\tan x \sin x}$ by equating the first and second denominators and changing the form $\cos^2 x$ into $1 - \sin^2 x$. The following obstacle was identified from S1 mistake when he tried to transform

Figure 1. The result of S1’s works.
fractions to the form \( \frac{\sin x}{x} \), which is one of the trigonometric function limits theorem (Ob2). The result of the student work, \( 0 + 1 \), is a mistake, due to wrong process in problem solving.

From the S1’s work on Figure 1, T1 made a diagnosis. The diagnosis (Di) was based on the initial error (Ob1) that is S1 was not able to build fractional form according to limit theorem. The teacher tried to figure out the students’ thinking sequences to start giving scaffolding actions. The teacher began to give scaffolding illustrated briefly in Table 1, which contains the excerpt of the dialogue between T1 and S1.

Table 1. Transcript excerpt between T1 and S1.

| Subject | Dialogue | Coding |
|---------|----------|--------|
| T1      | Why is it 1? [T1 points to S1’s work] | Sc1; Di |
| S1      | So, Sir [S1 is writing how to obtain \( I \) (Ob3)] | Res1 |
| T1      | How to get this "\( I \)"? | Sc2; Di |
| S1      | \( \tan x \) per \( x \) ... separated | Res2 |
| T1      | You separated fraction, but another is not using limit. Notice, you have already applied the limit law. One part used the limit law, but another did not. Please reread the problem. Pay attention to the peculiarities of the \( \tan x \) form | Sc3; Di |

The dialogue between T1 and S1 (Table 1) shows the teacher’s diagnosis upon the student’s understanding. From S1 writing, the teacher was interested in how S1 thought to get the number 1 by using the question (Sc1). Response S1 was to explain the origin of the number 1 (Res1) by writing \( \frac{\sin x}{x \tan x \sin x} - \frac{\tan x}{x \tan x \sin x} + \frac{\sin^2 x \tan x}{x \tan x \sin x} \). To clarify S1’s answer, T1 used the question (Sc2). S1 explained the origin of number 1 by answering the T1’s question (Res2). Error S1 obtaining number 1 (Ob3) was used T1 as the base in giving the next scaffolding (Sc3) using explanation and hint.

Before providing scaffolding, the teacher focused on student’s error and diagnosed student’s thinking to know his understanding. The diagnosis was done through reading student's writing and providing questions (Sc1, Sc2) and hint (Sc3) as scaffolding means [8]. In this case, the teacher gave 15 scaffolding (Sc15) until the student was able to solve the problem by himself. T1’s scaffolding trajectory is presented in Figure 2.

Figure 2. T1’s scaffolding trajectory.

T1’s scaffolding trajectory (Figure 2) shows the sequence of the teacher thinking in providing scaffolding using appropriate diagnosis. Providing scaffolding that fits the students' needs, (red lines) will influence effective scaffolding. This strategy refers to the adaptive characteristic [18] that is the given scaffolding should be in accordance with the students' understanding by involving scaffolding means in the form of questions, clues, or explanations. The diagnosis strategy is essential in implementing contingent in scaffolding [19]. Teacher’s diagnosis is used to plan and implement scaffolding in the instructional process.
3.2. Lack of diagnosis

The teacher (T2) scaffolding moment to the student (S2) was done after the teacher identified the student’s error presented in Figure 3. The student's obstacles were identified from the S2 writing, they were

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\lim_{x \to 0} \frac{1 - 2\sin^2 x}{x \tan x} (Ob1), \quad \lim_{x \to 0} \frac{1 - (1 - 2\sin^2 x)}{x \tan x \sin x} (Ob2), \quad \text{and} \quad \lim_{x \to 0} \frac{\sin x \sin x}{x} (Ob3). \]

T2 is less sensitive to S2’s errors in failing building fractional forms. From the results of S2’s works, it could be identified that S2 had difficulties in constructing fractions to determine the trigonometry functions limit.

\[\text{Figure 3. The result of S2's works.}\]

S2 obstacles (Figure 3) are to transform a form of \(\cos^2 x\). S2 wrote \(\cos^2 x\) as \(1 - 2\sin^2 x\) (Ob1), but the proper form was \(1 - \sin^2 x\). The following step showed that S2 had an inability in transforming limit fraction, i.e. \(\lim_{x \to 0} \frac{1 - (1 - 2\sin^2 x)}{x \tan x \sin x}\) (Ob2) and \(\lim_{x \to 0} \frac{\sin x \sin x}{x}\) (Ob3). S2 had an obstacle in equating the denominator of the two fractions. S2 did not change the form of the numerator after matching its denominator. Moment teacher scaffolding is presented in table 2.

\[\text{Table 2. Transcript excerpt between T2 and S2.}\]

| Subject | Dialogue | Coding |
|---------|----------|--------|
| T2      | Please reread and examine. You already applied two ways (Ob1 and Ob2), but it does not work. Please do again by applying another way, the third way | Sc1; LcDi |
| S2      | Yes, Mom [S2 applying the third way (Ob3)] | Res1 |
| T2      | Please try to redo from the beginning, reread the problem. So, what is \(\tan\)? | Sc2; LcDi |
| S2      | \(\sin\) per \(\cos\) | Res2 |
| T2      | Yes... Now, please do | Sc3; LcDi |

Excerpt between T2 and S2 (Table 2) shows T2 was less in diagnosing student’ difficulties. This is clearly observed from Sc1 in the form of prompt where T2 asked S2 to try to continue her work (LcDi). While in the S2 previous work it had been identified errors (Ob1 and Ob2). The S2 response (Res1) when continuing to do the work of the previous error had resulted in another error (Ob3). Scaffolding by T2 in the form of a given question (Sc2) was not enough to aim student realize the previous error (Res2). The T2 insensitivity in diagnosing student’ difficulties (LcDi) has resulted in the next scaffolding in the form of prompt (Sc3) being less effective, which is identified from the appearance of student errors (Ob3) due to continuing the previous work indicated an error. In this case, the T2 gave 26
scaffolding (Sc26) until the S2 was able to solve the problem by herself. Teacher’s scaffolding trajectory is presented in Figure 4.

T2’s scaffolding trajectory (Figure 4) showed that teacher lack of sensitivity in diagnosis, caused the scaffolding path tends to be longer (red line) than if the teacher was able to diagnose student’ difficulties appropriately (blue line). Due to lack diagnosis, the student’s work indicates she confused with the help provided by the teacher, which is indicated by the student’s work that does not focus on fixing errors that have been made before. Awareness of teachers is needed in diagnosis [20] to determine the level of students’ understanding of their zone of proximal development (ZPD) [17]. ZPD is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with the more capable peers [21]. So, learning will only happen if the teacher provides assistance, according to the student’s ZPD. Inappropriate scaffolding with ZPD has implications for students’ thinking.

In the process of problem-solving involving the construction of an action [22], it needs a good ability of the teacher in diagnosing student’ understanding to provide appropriate scaffolding for the student. Paying attention to student responses after being provided scaffolding have implications for teacher decisions, not only in determining scaffolding means but also in shifting degree of assistance. If a student needs high assistance, then the degree of one is raised. Instead, if student show increased understanding, then the assistance will be reduced [10].

Teacher scaffolding in solving problems is a bridge that connects what is already known to a student with the new knowledge to be mastered. Scaffolding is provided not to change the nature or degree of difficulty of the problem, but to help the student successfully solve the given problem. Thus, a careful diagnosis will help the student on what the student needs so that teacher scaffolding is effective. This finding is in line with [23, 24] which stated that understanding students thinking affects teaching effectiveness. Information about student thinking is used by teachers to improve instructional practices [25]. The results of this study provide evidence of the importance of understanding teacher's thinking in the instructional process [26], which can be used as a basis for following studies, especially in mathematics instruction.

4. Conclusion
Scaffolding trajectory as a teacher’s strategy track in providing scaffolding given to students in solving mathematics problems process. Scaffolding trajectory describes how the teacher thinks in assisting the student. The results showed that there were two categories of teachers’ thinking; diagnosis and lack of diagnosis. The ability of teachers in diagnosis determines the effectiveness of scaffolding. The right diagnosis will determine the right scaffolding mean so that it supports the success of student learning in solving mathematics problems. How the implementation of more complete diagnosis strategy for whole class scaffolding in mathematics instructions needs to be further investigated.

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