The global existence and large time behavior of smooth compressible fluid in an infinitely expanding ball, III: The 3-D Boltzmann equation

Huicheng Yin a,⁎, 1, Wenbin Zhao b, 1

a School of Mathematical Sciences, Jiangsu Provincial Key Laboratory for Numerical Simulation of Large Scale Complex Systems, Nanjing Normal University, Nanjing 210023, China
b Department of Mathematics and IMS, Nanjing University, Nanjing 210093, China

Received 13 April 2017; revised 16 August 2017

Abstract

This paper is a continuation of the works in [35] and [37], where the authors have established the global existence of smooth compressible flows in infinitely expanding balls for inviscid gases and viscid gases, respectively. In this paper, we are concerned with the global existence and large time behavior of compressible Boltzmann gases in an infinitely expanding ball. Such a problem is one of the interesting models in studying the theory of global smooth solutions to multidimensional compressible gases with time dependent boundaries and vacuum states at infinite time. Due to the conservation of mass, the fluid in the expanding ball becomes rarefied and eventually tends to a vacuum state meanwhile there are no appearances of vacuum domains in any part of the expansive ball, which is easily observed in finite time. In the present paper, we will confirm this physical phenomenon for the Boltzmann equation by obtaining the exact lower and upper bound on the macroscopic density function.

© 2017 Elsevier Inc. All rights reserved.

MSC: 35L70; 35L65; 35L67; 76N15

Keywords: Boltzmann equation; Expanding ball; Weighted energy estimate; Global existence; Vacuum state

⁎ Corresponding author.
E-mail addresses: huicheng@nju.edu.cn, 05407@njnu.edu.cn (H. Yin), zhaowb1989@gmail.com (W. Zhao).
1 Huicheng Yin and Wenbin Zhao are supported by the NSFC (No. 11571177, No. 11731007) and A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

http://dx.doi.org/10.1016/j.jde.2017.08.064
0022-0396/© 2017 Elsevier Inc. All rights reserved.
1. Introduction

The compressibility of gases plays a basic role in gas dynamics. When one squeezes a soft container filling with gases, the gases will become denser and the corresponding temperature will get higher in the adiabatic process. In this paper, as in [35] and [37], we consider an opposite situation for the compressible gases filling a 3-D expansive ball. It is assumed that the expansive ball is described by

\[ \Omega_t = \{ x = (x_1, x_2, x_3) \in \mathbb{R}^3 : |x| = \sqrt{x_1^2 + x_2^2 + x_3^2} < R(t), t \geq 0 \} \]

at the time \( t \), where \( R(t) = (1 + h^2 t^2)^{1/2} \) for some positive constant \( h \). From the expression of \( \Omega_t \), we know that the expansive ball at time \( t \) is formed by pulling out the initial unit ball \( \Omega_0 = \{ x : |x| < 1 \} \) with smooth speed and acceleration (see Fig. 1). The pulling speed on the boundary is \( R'(t) = h^2 t (1 + h^2 t^2)^{-1/2} \), which increases smoothly from 0 to \( h \). We denote the time–space domain by \( S = \{(t, x) : t > 0, |x| < R(t)\} \). Suppose that the movement of the gases in \( \Omega_t \) is described by the 3-D Boltzmann equation:

\[
\partial_t f + \xi \cdot \nabla_x f = Q(f, f),
\]

where \( f = f(t, x, \xi) \) stands for the distribution function of gas particles at time \( t \), position \( x \in \Omega_t \) and velocity \( \xi \in \mathbb{R}^3 \), the collision operator \( Q(f, g) \) with hard-sphere interaction is given by

\[
Q(f, g) = Q(f, g)(t, x, \xi) = \frac{1}{2} \int_{\mathbb{R}^3 \times S^2} |(\xi - \xi_s) \cdot \omega| (f' \, g' + f \, g - f \, g_s - f_s \, g) \, d\xi_s d\omega \quad (1.2)
\]

with \( \omega \in S^2 \) being the unit sphere in \( \mathbb{R}^3 \), and

\[
\begin{align*}
f_s &= f(t, x, \xi_s), & f' &= f(t, x, \xi'), & f_s' &= f(t, x, \xi'_s), \\
\xi' &= \xi - [(\xi - \xi_s) \cdot \omega] \omega, & \xi'_s &= \xi_s + [(\xi - \xi_s) \cdot \omega] \omega.
\end{align*}
\]
