KINEMATIC ORIGIN OF CORRELATIONS BETWEEN GAMMA-RAY BURST OBSERVABLES

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ABSTRACT

Recently, several new correlations between gamma-ray burst (GRB) observables have been discovered. Like previously well-established correlations, they challenge GRB models. Here, we show that in the cannonball (CB) model of GRBs, the newly discovered correlations have the same simple kinematic origin as those discovered earlier. They all result from the strong dependence of the observed radiations on the Lorentz and Doppler factors of the jet of highly relativistic plasmoids (CBs) that produces the observed radiations by interaction with the medium through which it propagates.

Key words: gamma-ray burst: general – gamma-ray burst: individual (090529A)

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1. INTRODUCTION

Despite the enormous complexity and diversity of gamma-ray bursts (GRBs) and their afterglows, various well-established correlations between GRB observables were found during the past years. Such correlations challenge the theoretical models of GRBs. Most of these correlations have been neither predicted nor explained by the standard fireball model of GRBs, which has been extensively employed to explain the GRB phenomenon (for reviews see, e.g., Meszaros 2002; Zhang & Meszaros 2004; Piran 2004; Zhang 2007). In the cannonball (CB) model of GRBs, these correlations were shown to be a simple consequence of the strong dependence of GRB observables on the Lorentz factor $\gamma$ and Doppler factor $\delta$ of the highly relativistic jet of plasmoids (CBs) whose interaction with the medium along its path produces the observed radiations (Dar & De Rújula 2000, 2004; Dado et al. 2007, 2009).

Recently, many new correlations were discovered between pairs of observables characterizing the prompt gamma ray and early optical emissions in a sample of GRBs rich in X-ray flashes (XRFs; Liang et al. 2010). Here, we show that in the CB model all these new correlations also follow from the strong dependence of the GRB observables on the Lorentz factor $\gamma$ and the Doppler factor $\delta$ of the highly relativistic jet of CBs.

In particular, from their selected GRB sample, Liang et al. (2010) inferred tight pair correlations between the initial Lorentz factor $\Gamma_0$ of the relativistic ejecta in GRBs and other GRB observables, such as the isotropic equivalent gamma-ray energy $E_{\text{iso}, \gamma}$ of the prompt emission, the peak luminosity $L_{p, O}$ of the early optical emission, and the peak time $t_p^\prime$ of the optical emission in the GRB rest frame. The standard fireball model with internal and external shocks does not explain the physical origin of these newly discovered correlations, nor the origin of other well-established correlations between GRB observables.

Moreover, in the internal shock fireball model, the tight $\Gamma_0 - E_{\text{iso}, \gamma}$ correlation yields a peak energy $E_{\text{iso}, \gamma}^\prime$, that is practically independent of $E_{\text{iso}, \gamma}$ (Liang et al. 2010), in contradiction with the well-established Amati relation (Amati et al. 2002; Amati 2006). However, the initial value of the Lorentz factor of the relativistic ejecta that produced the GRBs was not truly measured. Instead, it was inferred (Liang et al. 2010) from an assumed relation, $\Gamma_0 \propto \left[ E_{\text{iso}, \gamma} / (t_p^\prime)^3 \right]^{1/8}$. This relation follows from the standard fireball model expression for $\Gamma_0$ (Sari & Piran 1999), neglecting its weak dependence on the circumburst density and on the conversion efficiency of the relativistic kinetic energy of the ejecta to radiation. The validity of this fireball model relation has never been proven. Thus, hereafter, we shall distinguish between the true initial value of the bulk motion Lorentz factor $\gamma_0$ of the relativistic ejecta and $\Gamma_0$, its alleged value inferred from fireball modeling of GRB data. We shall show that, in the CB model, the strong dependence of $E_{\text{iso}, \gamma}$ and $t_p^\prime$ and consequently of $\Gamma_0$ on the true initial values of the Lorentz factor $\gamma$ and the Doppler factor $\delta$ also yields the tight pair correlations between the fireball model parameter $\Gamma_0$ and the GRB observables which were discovered by Liang et al. (2010) and by Lu et al. (2011), and could not be explained by the standard fireball model.

2. KINEMATIC ORIGIN OF CORRELATIONS IN THE CB MODEL

2.1. Origin of the Observed Radiations in the CB Model

In the CB model of GRBs (Dado et al. 2002; Dar & De Rújula 2004; Dado et al. 2009), GRBs and their afterglows are produced by the interaction of bipolar jets of highly relativistic plasmoids (CBs) of ordinary matter with the radiation and matter along their trajectory (Shaviv & Dar 1995; Dar 1998). Such jetted CBs are presumably ejected in accretion episodes on the newly formed compact stellar object in core-collapse supernova (SN) explosions (Dar et al. 1992; Dar & Plaga 1999; Dar & De Rújula 2000) in mergers of compact objects in close binary systems (Goodman et al. 1987; Shaviv & Dar 1995) and in phase transitions in compact stars (Dar 1998; Dar & De Rújula 2000; Dado et al. 2009). For instance, in GRBs associated with SNe, it is hypothesized that an accretion disk or a torus is produced around the newly formed compact object, either by stellar material originally close to the surface of the imploding core and left behind by the explosion-generating outgoing shock, or by more distant stellar matter falling back after its passage (Dar & De Rújula 2004). As observed in microquasars, each time part of the accretion disk falls abruptly

1 See, however, Thompson et al. (2007) and Zhang & Yan (2011).

2 See, however, Lu et al. (2011).
onto the compact object, two CBs made of ordinary-matter plasma are emitted with large bulk-motion Lorentz factors \(\gamma \gg 1\) in opposite directions along the rotation axis from where matter has already fallen back onto the compact object due to lack of rotational support. The prompt \(\gamma\)-ray pulses and early-time and X-ray flares are dominated by inverse Compton scattering (ICS) of glory photons—a light halo surrounding the progenitor star that was formed by scattered stellar light from the pre-SN wind blown from the progenitor star. The ICS is overtaken by synchrotron radiation (SR) when the CBs enter the pre-SN wind ejecta of the progenitor star (see, e.g., Dado et al. 2009). The SR dominates the early-time optical/NIR emission and the broadband afterglows produced by the CBs when they continue to propagate in the interstellar medium (ISM). ICS of the SR produces the emission of very high energy photons during the early-time optical/NIR emission and the broadband afterglow (Dado & Dar 2009a).

### 2.2. Kinematic Correlations

GRBs are not standard candles because of the diversity of their central engines and environments. But, because of the large bulk motion Lorentz factor \(\gamma\) of the jet of CBs, their emitted radiation at redshift \(z\), which is observed at a small angle \(\theta\) relative to the direction of the jet, is boosted by a large Doppler factor \(\delta = 1/\gamma (1 - \beta \cos \theta)\) and collimated through relativistic beaming by a factor \(\delta^2\). Moreover, the time difference \(dt\) in the observer frame between the arrival of photons emitted by the point-like CBs at two different points along their path, which are separated by a distance \(dr\) and time \(dt' = dr/c\) in the progenitor’s rest frame (hereafter a prime indicates an observable in the progenitor’s rest frame), is shortened (aberrated) according to \(dt = dt'/1/(1 + z)/\gamma \delta\). The large Doppler boosting, relativistic collimation, and time aberration produce correlations between GRB observables, despite their dependence on the CBs’ intrinsic (rest-frame) properties and on the environment along their trajectories (which produce a significant spread around these simple kinematic correlations).

The redshift \(z\) of the GRB location is measurable, and the dependence of the GRB observables on redshift can be taken into account explicitly, unlike their dependence on the values of the Lorentz factor and the viewing angle of the jet, which can only be inferred with model-dependent assumptions. However, the strong dependence on \(\gamma\) and \(\delta\) can be used to correlate triplets of independent observables without knowing the values of \(\gamma\) and \(\delta\). Moreover, several observables depend on the same combination of \(\gamma\) and \(\delta\) that results in pair correlations. Finally, due to selection effects in the observations, various observables depend strongly on only \(\gamma\) or \(\delta\), which also yields pair correlations. In particular, the dependence on the viewing angle of the jet can be eliminated in two general cases. For \(\gamma^2 \gg 1\) and small viewing angles \(\theta^2 \ll 1\), the Doppler factor satisfies \(\delta \approx 2\gamma/(1 + \gamma^2 \theta^2)\) to an excellent approximation. For \(\theta^2 \gg 1\), the Doppler factor decreases rather slowly with increasing \(\theta\). But for \(\theta^2 \gg 1\), the Doppler factor decreases with increasing viewing angle like \(\theta^{-2}\), and the observed fluence of gamma rays, which in the CB model is amplified by a factor \(\delta^2\) due to relativistic beaming, decreases like \((1 + \gamma^2 \theta^2)^{-2}\). The geometrical probability to view a bipolar GRB from a small angle \(\theta\) increases like \((1 - \cos \theta) \approx \theta^2/2\) and the product \(\theta^2(1 + \gamma^2 \theta^2)^{-2}\) has a maximum when \(\gamma^2 \theta^2 = 1\). Consequently, \(\delta = \gamma\) for the most probable viewing angle \(\theta = 1/\gamma\) of GRBs (Shaviv & Dar 1995).

In “soft” GRBs with \(\gamma^2 \theta^2 \gg 1\), such as XRFs with \(\gamma^2 \theta^2 \gtrsim 10\), the dependence on the exact value of the Lorentz factor can be ignored compared to the strong dependence on the Doppler factor. This yields a slightly different correlation. Thus, we shall derive the CB model correlations for the above two different situations, for \(\theta \ll \gamma\) and for \(\gamma^2 \theta^2 \gg 1\), i.e., \(\delta \ll \gamma\) where the dependence on \(\gamma\) can be neglected compared to that on \(\delta\). For a mixed population of ordinary GRBs, soft GRBs, and XRFs one may expect an approximate correlation to be obtained by using the average index of the two power-law correlations. (Alternatively, one can derive triple correlations, i.e., correlations that involve three independent observables, each of which depends both on \(\gamma\) and \(\theta\). Below, we derive the CB model pair correlations that correspond to those discovered empirically by Liang et al. (2010) and Lu et al. (2011), and we compare them in the text and in Table 1.

### 3. PAIR CORRELATIONS BETWEEN GRB OBSERVABLES

#### 3.1. Correlations between Pulse-shape Parameters

In the CB model, \(\gamma\) and \(\delta\) stay put at their initial values until the CB sweeps in a relativistic mass/energy comparable to its initial rest mass, e.g., during the prompt emission and the early afterglow until the “jet break” (Dado et al. 2009 and references therein) or during flares due to crossing of density bumps with a wind-like density profile. Because of time aberration, all time measures of the prompt emission pulses and early-time flares, such as their rise time \(t_r\), from half-maximum to peak value, the peak time \(t_p\) after the beginning of the flare/pulse, the decay time \(t_d\) from peak value to half-maximum, and its full width at half-maximum (FWHM) \(t_w\), are proportional to \((1 + z)/\gamma \delta\). Hence,

\[
\log t_i = a_{ij} + \log t_j, \quad \text{where } i, j = r, d, p, w. \tag{1}
\]

This explains the origin of the “universal” power-law index \(\sim 1\) of the “power-law correlations” (proportionality) between the temporal parameters of the prompt \(\gamma\)-ray pulses.

Moreover, the prompt ICS emission pulses and early-time flares have an approximate light curve (see, e.g., Dado et al.
where $\Delta$ is the characteristic time before the fast expansion of a CB, roughly like $R^2 \propto t^2/(t^2 + \Delta^2)$, stops by cooling or by collision with the wind ejecta, which, like the glory, has roughly $1/(r^2 + r_h^2) \propto 1/(t^2 + \Delta^2)$ decline. As long as the spectral evolution is slow during a pulse/flare, the pulse shape can be well approximated by $dN_\gamma/dt \propto t^2/(t^2 + \Delta^2)$, which peaks at $t_p = \Delta$, has an FWHM $t_w = 2\Delta$, a rise time from half-maximum to peak value, $t_r \approx 0.59\Delta$, and a decay time from peak value to half-maximum $t_d \approx 2.84\Delta$. Consequently, the above “universal” pulse/flare shape yields the simple redshift independent relations (Dar & De Rújula 2004; Dado et al. 2009):

\[
\begin{align*}
  t_d &\approx 2.39 t_r, \quad \text{i.e., } \log t_d \approx 0.38 + \log t_r, \\
  t_d &\approx 1.41 t_p, \quad \text{i.e., } \log t_d \approx 0.15 + \log t_p, \\
  t_r &\approx 0.59 t_p, \quad \text{i.e., } \log t_r \approx -0.23 + \log t_p, \\
  t_w &\approx 2.00 t_p, \quad \text{i.e., } \log t_w \approx 0.30 + \log t_p, \\
  t_w &\approx 3.39 t_r, \quad \text{i.e., } \log t_w \approx 0.53 + \log t_r, \\
  t_w &\approx 1.42 t_d, \quad \text{i.e., } \log t_w \approx 0.15 + \log t_d,
\end{align*}
\]

where time is measured in seconds. These relations are well satisfied within observational errors by well-resolved GRB peaks/flares as was found, e.g., in Kocevski et al. (2003) through empirical parametrization of these peaks/flares. In the CB model, these correlations are also approximately valid for the prompt/early-time optical flares because both the prompt/early-time ICS pulses and the prompt/early-time ICS and SR flares rise and a decay like $(\Delta^2 + t^2)^{-1}$. Indeed, the above predicted correlations are in good agreement with those found by Liang et al. (2010) for the early-time optical peak:

\[
\begin{align*}
  \log t_d &= (0.48 \pm 0.13) + (1.06 \pm 0.06) \log t_r, \\
  \log t_d &= (-0.09 \pm 0.29) + (1.17 \pm 0.11) \log t_p, \\
  \log t_r &= (-0.54 \pm 0.22) + (1.11 \pm 0.08) \log t_p, \\
  \log t_w &= (0.05 \pm 0.27) + (1.16 \pm 0.10) \log t_p, \\
  \log t_w &= (0.61 \pm 0.11) + (1.05 \pm 0.05) \log t_r, \\
  \log t_w &= (0.15 \pm 0.02) + (0.98 \pm 0.01) \log t_d.
\end{align*}
\]

In the CB model, the optical emission is dominated by SR and its light curve is given by (see, e.g., Dado et al. 2009 and references therein)

\[
F_\nu(t) \propto n^{(1+\beta)/2} R^2 \gamma^3 \delta^{-1} \delta^{3+\beta},
\]

where $n[t]$ is the density along the CB trajectory, $R$ is the CB radius, and $\beta$ is the spectral index of the SR. Typically, the spectral index in the optical band is $\beta_O \approx 0.5$ at early time and gradually approaches $\beta_X \approx 1$ at late time. The peak luminosity of the prompt optical emission flare is obtained when the CB reaches the peak density of the progenitor’s wind ejecta (Dado & Dar 2010) while $\gamma$ and $\delta$ stay put at their initial values. Thus, for the most probable viewing angle of GRBs $\theta \approx 1/\gamma$, i.e., $\delta \approx \gamma$, the CB model predicts for the peak optical luminosity $L_{p,O} \propto \gamma^4$, and $t'_w \propto 1/\gamma \delta \approx 1/\gamma^2$ for the FWHM of the GRB rest frame. These dependences yield the correlation $L_{p,O} \propto (t'_w)^{-2}$, in agreement with that found in Liang et al. (2010) and reported in their Equation (11) and in Table 1. Similar correlations are expected between $L_{p,O}$ and $t_r$, $t_d$, and $t_p$, respectively.

Note, however, that in the CB model, XRFs that are far off-axis GRBs initially have $\gamma^2 \delta^2 \gg 1$. The deceleration of the CBs in XRFs yields $\delta(t)$, which first rises slowly as $\gamma(t)$ decreases with increasing time until it reaches a maximum when $\gamma(t)\delta(t) = 1$, i.e., when $\delta(t) \approx \gamma(t)$. Hence, for a constant density ISM, a typical early-time optical spectral index of $\beta_O \approx 0.5$, and $\delta(t) \approx \gamma(t)$, Equation (3) yields $L_{p,O} \propto [\gamma(t)]^4$. Hence, for $t'_w \propto 1/\gamma(t)\delta(t) \approx [\gamma(t)]^{-1}$, the CB model predicts the correlation $L_{p,O} \propto (t'_w)^{-2}$.
3.3. Correlations Involving Total Energy, Peak Energy, and Peak Optical Luminosity

In the CB model, the peak energy $E_{p,\gamma}$ and the isotropic equivalent gamma-ray energy $E_{\text{iso,}\gamma}$ of a single gamma-ray pulse or an early-time flare satisfy (Dar & De Rújula 2000)

$$(1 + z)E_{p,\gamma} \propto \gamma \delta, \quad E_{\text{iso,}\gamma} \propto \delta^3,$$  \hspace{1cm} (4)

where $E_{\text{iso,}\gamma}$ and $E_{p,\gamma}$ refer to that single pulse/flare. For $\delta \approx \gamma$, Equation (4) yields $(1 + z)E_{p,\gamma} \propto \gamma^2$, $E_{\text{iso,}\gamma} \propto \gamma^3$, and the correlations $(1 + z)E_{p,\gamma} \propto E_{\text{iso,}\gamma}^{2/3}$. In soft GRBs and XRFs, i.e., GRBs with a large viewing angle, the CB model predicts $(1 + z)E_{p,\gamma} \propto \delta \propto E_{\text{iso,}\gamma}^{5/3}$ (Dar & De Rújula 2000). For a mixed population of soft and hard GRBs, it yields the mean correlation

$$(1 + z)E_{p,\gamma} \propto E_{\text{iso,}\gamma}^{1/2 \pm 1/6}.$$  \hspace{1cm} (5)

Since $E_{p,\gamma}$ at peak luminosity is observed to be proportional to the $E_{p,\gamma}$ of the time-integrated spectrum over the entire GRB (Goldstein et al. 2012), the above follows is also valid for the entire GRB (Dar & De Rújula 2000), as was discovered empirically (Amati et al. 2002; Amati 2006).

In the CB model, where the early optical emission is dominated by SR with a canonical spectral index $\beta_0 \approx 0.5$, Equation (3) predicts that ordinary GRBs with $\delta \approx \gamma$ have a peak optical luminosity of $L_{p,\gamma} \propto \gamma^4$, and consequently

$L_{p,\gamma} \propto E_{\text{iso,}\gamma}^{5/3}$. In soft GRBs and XRFs where the dependence on $\gamma$ at early times can be neglected compared to the dependence on $\delta$, the resulting correlation is $L_{p,\gamma} \propto \delta^{7/2} \propto E_{\text{iso,}\gamma}$. Thus, the mean correlation in a mixed population of soft and hard GRBs that is predicted by the CB model is

$L_{p,\gamma} \propto E_{\text{iso,}\gamma}^{5/4 \pm 1/12},$  \hspace{1cm} (6)

which is in agreement with the power-law correlation with an index $1.17 \pm 0.13$ found by Liang et al. (2010) and reported in their Equation (12) and in Table 1.

The equivalent isotropic optical energy of the prompt optical flare that dominates $E_{\text{iso,}\gamma}$ is roughly given by $L_{p,\gamma} t'_w \propto \gamma^{-1/2} \delta^{5/2}$. Thus, in ordinary GRBs where $\delta \approx \gamma$, the CB model predicts $E_{\text{iso,}\gamma} \propto \gamma^3$, i.e., $E_{\text{iso,}\gamma} \propto E_{\text{iso,}\gamma}^{2/3}$. In soft GRBs, $E_{\text{iso,}\gamma} \propto \delta^{5/2} \propto (E_{\text{iso,}\gamma})^{5/6}$, and the mean effective correlation that follows is

$E_{\text{iso,}\gamma} \propto E_{\text{iso,}\gamma}^{3/4 \pm 1/12},$  \hspace{1cm} (7)

which is in agreement with the correlation found by Liang et al. (2010) and reported in their Equation (14) and in Table 1.

3.4. Triple Correlations

Many correlations involving triplets of GRB observables can be derived using their strong dependence on $\gamma$ and $\delta$. For instance, the peak of the equivalent isotropic luminosity of a single $\gamma$-ray pulse or an early-time X-ray flare satisfies

$L_{p,\gamma} \propto E_{\text{iso,}\gamma} t'_w \propto E_{\text{iso,}\gamma} E_{p,\gamma}^{t'_w}$,$  \hspace{1cm} (8)

where $L_{p,\gamma}$, $E_{\text{iso,}\gamma}$, and $E_{p,\gamma}$ are of the same single pulse/flare.

Since the $E_{p,\gamma}$ at peak luminosity is observed to be proportional to the $E_{p,\gamma}$ of the time-integrated spectrum over the entire GRB (Goldstein et al. 2012), the CB model yields the approximate binary correlation

$E_{p,\gamma} \propto L_{p,\gamma}^{3/4 \pm 1/6}$. This predicted correlation is compared in Figure 1 with Fermi Gamma-ray Burst Monitor (GBM) observations of 26 GRBs with known redshifts (Gruber et al. 2011). Note, however, that the time-averaged luminosity of multipeak GRBs satisfies

$$\langle L_{\text{iso,}\gamma} \rangle \approx 0.9 E_{\text{iso,}\gamma}/[T_{90}/(1 + z)],$$  \hspace{1cm} (9)

where $T_{90}/(1 + z)$ is the GRB duration during which 90% of the observed prompt GRB energy is emitted. In the CB model, $T_{90}/(1 + z)$ of multipeak long GRBs is intrinsic and does not depend on $\gamma$ and/or $\delta$. Hence, the last equation yields the binary correlations

$$\langle L_{\text{iso,}\gamma} \rangle \propto E_{\text{iso,}\gamma}; \quad E_{p,\gamma} \propto \langle L_{\text{iso,}\gamma} \rangle^{1/2 \pm 1/6}.\hspace{1cm} (10)$$

In the CB model, the break time of a canonical X-ray afterglow satisfies (Dado et al. 2009), $t_{b,X} \propto (1 + z)/(\delta^2 \gamma)$. Then, using the relations in Equation (4), the following correlation is obtained:

$t_{b,X} \propto 1/[E_{p,\gamma}^{1/3}] \propto 1/[E_{\text{iso,}\gamma}^{5/6 \pm 1/6}].$  \hspace{1cm} (11)

4. ORIGIN OF THE TIGHT $\Gamma_0 - E_{\text{iso,}\gamma}$ AND $\Gamma_0 - L_{\text{iso,}\gamma}$ CORRELATIONS

Liang et al. (2010) and Lu et al. (2011) reported the “tight correlations,” $\Gamma_0 \approx 118[E_{\text{iso,}\gamma}/10^{52} \text{erg}]^{0.26 \pm 0.04}$ and $\Gamma_0 \approx 264[L_{\text{iso,}\gamma}/10^{52} \text{erg}]^{0.27 \pm 0.03}$ between the initial bulk motion Lorentz factor $\Gamma_0$ of the ejecta and $E_{\text{iso,}\gamma}$ or the effective luminosity $L_{\text{iso,}\gamma} = E_{\text{iso,}\gamma}/(1 + z) T_{90}$, where $T_{90}/(1 + z)$ is the intrinsic duration during which 90% of the observed prompt GRB energy is emitted. However, $\Gamma_0$ was not a measured value
of the initial Lorentz factor of the jetted ejecta. For instance, Liang et al. (2010) assumed that
\[ \Gamma_0 \simeq 192 \left[ E_{\text{iso},52,y}/(t'_p)^3 \right]^{1/8}, \] (13)
where \( E_{\text{iso},52,y} = E_{\text{iso},y}/(10^{52} \text{ erg}) \) and \( t'_p \) is in seconds. This relation follows from a fireball model expression for \( \Gamma_0 \) (Sari & Piran 1999) after neglecting its weak dependence on the unknown circumburst density and on the fraction of the relativistic kinetic energy of the ejecta that is converted to radiation.

Substitution of the CB model relations \( E_{\text{iso},y} \propto \delta^0 \) and \( t'_p \propto 1/(\gamma \delta) \) into Equation (13) yields \( \Gamma_0 \propto \gamma^{3/8} \delta^{-6/8} \). Consequently, for a mixed population of ordinary GRBs and XRFs, the CB model predicts the following correlations compared to the observed (obs) ones:

- \( \Gamma_0 \propto \left[ E_{\text{iso},y} \right]^{5/11+1/16} \simeq \left[ E_{\text{iso},y} \right]^{0.31 \pm 0.06}, \) \[ \text{obs: } \Gamma_0 \propto \left[ E_{\text{iso},y} \right]^{0.26 \pm 0.04}, \]
- \( \Gamma_0 \propto \left[ L_{\text{iso},y} \right]^{5/11+1/16} \simeq \left[ E_{\text{iso},y} \right]^{0.31 \pm 0.06}, \) \[ \text{obs: } \Gamma_0 \propto \left[ L_{\text{iso},y} \right]^{0.27 \pm 0.03}, \]
- \( \Gamma_0 \propto \left[ L_{p,0} \right]^{(141\pm15)/448} \simeq \left[ L_{p,0} \right]^{0.25 \pm 0.03}, \) \[ \text{obs: } \Gamma_0 \propto \left[ L_{p,0} \right]^{0.20 \pm 0.03}, \]
- \( \Gamma_0 \propto \left[ t'_p \right]^{(-21\pm3)/32} \simeq \left[ t'_p \right]^{0.66 \pm 0.09}, \) \[ \text{obs: } \Gamma_0 \propto \left[ t'_p \right]^{0.59 \pm 0.03}. \]

Note that for a GRB population dominated by XRFs and soft GRBs, where the dependence on \( \gamma \) can be neglected compared to that on \( \delta \), the CB model predicts; \( \Gamma_0 \propto \left[ E_{\text{iso},y} \right]^{0.25}, \) \( \Gamma_0 \propto \left[ L_{\text{iso},y} \right]^{0.25}, \) \( \Gamma_0 \propto \left[ L_{p,0} \right]^{0.21}, \) and \( \Gamma_0 \propto \left[ t'_p \right]^{-0.75}. \)

5. SUMMARY AND CONCLUSIONS

In a long series of publications, we have demonstrated that the CB model of GRBs correctly predicted the main observed properties of GRBs, including the well-established correlations between GRB observables, and can reproduce successfully the broadband light curves of GRBs and their afterglows from onset until very late times, despite their enormous complexity and diversity (see, e.g., Dar & De Rújula 2004; Dado et al. 2009; Dado & Dar 2009a, 2009b, 2010, and references therein). In this paper, we have shown that in the CB model, all the newly discovered pair correlations between gamma-ray burst observables that were reported by Liang et al. (2010) and by Lu et al. (2011), like the previously well-established correlations, are a simple consequence of the strong dependence of the GRB observables on the Lorentz factor and the viewing angle of the highly relativistic and narrowly collimated jets whose interaction with the medium along their path produces the observed radiations.

One of the newly discovered correlations by Liang et al. (2010) was the tight relationships, \( \Gamma_0 \simeq 182 \left[ E_{\text{iso},52,y}/10^{52} \text{ erg} \right]^{0.25 \pm 0.04}. \) The authors pointed out in their paper that “there is no straightforward theory that predicts this relationship between \( \Gamma_0 \) and \( E_{\text{iso},y} \),” and that this tight correlation is inconsistent with the well-established Amati relation (Amati 2006) if the prompt gamma-ray emission in GRBs is produced by the internal shock mechanism of the fireball model. However, the tight \( \Gamma_0 - E_{\text{iso},y} \) correlation (and the \( \Gamma_0 - E_{\text{iso},y} \) correlation discovered by Lu et al. 2011) was based on values of \( \Gamma_0 \) inferred from a fireball model relation \( \Gamma_0 \propto \left[ E_{\text{iso},y}/(t'_p)^3 \right]^{1/8} \) and from measured values of \( E_{\text{iso},y} \) and \( t'_p \), not on reliable measurements of the initial Lorentz factor \( \gamma \). Using the CB model dependences of \( E_{\text{iso},y} \) and \( t'_p \) on \( \gamma \) and \( \delta \), one reproduces the power-law correlation between \( \Gamma_0 \) and \( E_{\text{iso},y} \) or \( \Gamma_0 \) with the observed index 0.25 for a population rich in soft GRBs and XRFs. Indeed, the GRB sample that was used in Liang et al. (2010) to infer the tight correlation contains a large fraction of XRFs (e.g., 060904B, 070318, 070419A, 071010A, 080330, 080710, 070208) and soft GRBs. We conclude that the tight correlation between \( \Gamma_0 \) and \( E_{\text{iso},y} \) is that expected in the CB model, as well as the Amati relation (Equation (5)), which actually was predicted by the CB model (see Dar & De Rújula 2000, Equation (40)) long before it was discovered empirically (Amati et al. 2002; Amati 2006). In the case of the standard fireball model, where the prompt emission pulses are produced by synchrotron emission from internal shocks, the tight \( \Gamma_0 - E_{\text{iso},y} \) correlation yields essentially \( E_{\text{iso},y} \), which is constant for different \( E_{\text{iso},y} \) values, in contradiction with the Amati relation (Liang et al. 2010). This, perhaps, is not surprising in view of the fact that the standard fireball model was never shown to correctly predict the shape of the prompt emission gamma-ray pulses and their spectral evolution, nor their typical photon energy and total GRB energy.

Finally, we would like to caution that the new correlations discovered by Liang et al. (2010) were inferred from a sample of selected GRBs. Only GRBs with a single optical peak that could be modeled with the empirical formula of Kocevski et al. (2003) seem to be included in the sample. Bright GRBs with clear early-time multipeak optical emission were not included in the GRB sample. In fact, a single late-time peak can also be a late-time flare due to a density bump in the ISM (Dado et al. 2002), such as observed in GRB 970508 (Piro 1998) or the blended sum of unresolved peaks like that observed in XRF 071031 (Kruhler et al. 2009; Dado & Dar 2010). Moreover, soft GRBs and XRFs usually have a slow rebrightening of their optical and X-ray afterglows that probably has a completely different origin—an initial rise in \( \delta(t) \) as \( \gamma(t) \) decreases due to deceleration. Thus, most XRFs and soft GRBs show, after a prompt emission flare(s), a slowly rising afterglow followed by a power-law decay like that of ordinary GRBs (Dado & Dar 2009b).

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\(^{5}\) An interpretation of the \( \Gamma_0 - E_{\text{iso},y} \) correlation discovered by Liang et al. (2010) was recently proposed by Liu et al. (2011) within the framework of a neutrino cooling dominated central engine model.
