Delay-Aware Data Transmission of Multi-carrier Communications in the Presence of Renewable Energy

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Abstract In the paper, we investigate the data transmission delay in renewable energy aided multi-carrier system. Besides utilizing the local renewables, the transmitter can also purchase grid power. By scheduling the amount of transmitted data (the data are stored in a buffer before transmission), the sub-carrier allocation, and the renewable allocation in each transmission period, the transmitter aims to minimize the purchasing cost under a buffer delay constraint. By theoretical analysis of the formulated stochastic optimization problem, we find that transmitting the scheduled data through the subcarrier with best condition is optimal and greedy renewable energy is approximately optimal. Furthermore, based on the theoretical derives and Lyapunov optimization, an on-line algorithm, which does NOT require future information, is proposed. Numerical results illustrate the delay and cost performance of the proposed algorithm. In addition, the comparisons with the delay-optimal policy and cost-optimal policy are carried out.

Keywords Multi-carrier communications · Delay · Renewable energy

1 Introduction

With the rapid growth of traffic (e.g., speech, data, video, etc.) in wireless communications, the power consumption becomes huge, which has incurred severe environmental problems. Meanwhile, high-rate based new business, such as mobile internet, cloud computing, and big data service, make the increasing trend for power consumption more apparent. Improving energy efficiency has been an important aim in communication system design [1]. On the other hand, QoS guarantee such as delay is also important especially for delay-
sensitive traffic. For example, in real-time multimedia services (e.g., video transmission), if a received packet violates its delay limit then it is considered useless and must be discarded. Generally speaking, power saving and QoS (i.e., delay) improvement are two conflicting aspects in wireless communications: Reducing power consumption will degrade delay performance, and the delay performance improves at the expense of increasing power consumption. Power and delay tradeoff is a fundamental problem in wireless communications [2, 3].

Renewable energy has attracted much attention due to its naturality, renewable and pollution-free characteristics. Energy harvesting technique is capable of converting the renewable energy from the environment into electrical energy [4]. Recently, incorporating renewable energy in wireless communications system (or energy harvesting aided wireless communications) becomes a hot topic in the literature [5, 6]. Meanwhile, renewable energy aided base station begins to leak in practical cellular mobile communications systems.

High data rates communications is significantly limited by inter-symbol interference (ISI) due to the existence of the multiple paths. Multi-carrier modulation techniques, including orthogonal frequency division multiplexing (OFDM) modulation are considered as the most promising technique to combat this problem. Multi-carrier modulation techniques (e.g., OFDM) have been widely selected as the physical layer technique in broadband wireless system (e.g., LTE). Since the delay is generally more sensitive in high rate multi-carrier communications, the study on power and delay is especially vital therein.

The joint investigation of power and delay in multi-carrier communications system has not gained much attention yet, particularly when local renewable energy is available. In the paper, we study the delay constrained power allocation in renewable energy aided point-to-point multi-carrier communications. The transmitter is equipped with renewable energy generation device. Meanwhile, the transmitter can purchase power from the grid so as to alleviate the renewable energy’s intermittence for a stationary minimum QoS guarantee. The upper layer of the transmitter generates data stochastically, and the data wait in an FIFO (First-In-First-Out) buffer before transmission. In each period, the transmitter decides the number of data for each sub-carrier in the period, and sends to the receiver (Meanwhile, the transmitted data are removed from the buffer). In addition, the transmitter settles how much renewable energy being allocated from the storage battery (and the rest required energy is purchased from the grid). The objective is to minimize the cost under a buffer delay constraint. A stochastic optimization problem is formulated accordingly. By theoretical analysis, we prove that transmit all scheduled data through the best subcarrier in each period is optimal. We give the approximate optimal renewable allocation. Furthermore, an on-line algorithm (referred to as BGL algorithm) is proposed based on the theoretical results.

Specifically, the advances of this paper lie in two aspects: 1) Multi-carrier communication is considered, which is vital in ISI alleviation and necessary in frequency selective fading channels. 2) An on-line algorithm combining our theoretical derives and the Lyapunov optimization is proposed.

The rest of the paper is structured as follows: In Sect. 2, the system model is described, and the stochastic problem is formulated accordingly. The analysis of the formulated problem is performed in Sect. 3. Next, an on-line algorithm, i.e., the BGL algorithm, is proposed in Sect. 4. Numerical results are given in Sect. 5. Finally, Sect. 6 concludes the paper.
We consider a discrete-time model of point-to-point multi-carrier wireless communications. Time is divided into periods with length $\tau$ each. The $n$-th period is the time interval $[n\tau, (n+1)\tau)$ as illustrated in Fig. 1. There are $M$ sub-carriers, each suffers block flat fading. For a sub-carrier, the channel state remains constant during a period and is variable over periods. The channel power gain of the $i$-th sub-carrier during the $n$-th period is denoted as $H_i[n]$. 

$$H_n = (H_1[n], \ldots, H_M[n])$$

is the channel state vector of the $M$ sub-carriers during the $n$-th period. The transmitter is equipped with renewable energy generation device(s), e.g., photovoltaic cells. The renewables are stored in a storage battery with capacity $B$ before usage. The arrived renewables at the end of the $n$-th period is denoted as $E[n]$ ($E[n]$ can be understood as the accumulated renewables during the $n$-th period). Meanwhile, the transmitter is connected to the power grid, and it can purchase power from the grid. The upper layer of the transmitter generates $A[n]$ packages at the end of the $n$-th period ($A[n]$ can be viewed as the total generated data during the $n$-th period). It is assumed that each package contains $b$ bits. The generated data are stored in an FIFO data buffer. During the $n$-th period, the transmitter removes $R[n]$ packages from the data buffer, and sends to the receiver through the considered $M$ sub-carriers. For the $i$-th sub-carrier, the transmitted data number is $R_i[n]$ during the $n$-th period, and $R[n] = \sum_{i=1}^{M} R_i[n]$ is the total scheduled packages for transmission.

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1 The capacity denotes the maximal energy that can be stored in the battery. Unless specified, the energy unit is J.

2 The rate is $R_i[n]$ package/period/Hz.
\( \mathbf{R}_n = (\mathbf{R}_1[n], \ldots, \mathbf{R}_M[n]) \)

is the rate vector of the \( M \) sub-carriers in the \( n \)-th period. Denote the data buffer length at instance \( n \tau \) as \( Q[n] \). The evolution of the buffer length can be given as

\[
Q[n + 1] = (Q[n] - R[n])^+ + A[n],
\]

where \((\cdot)^+ = \max\{\cdot, 0\} \). Assume that the additive white Gaussian noise at the \( i \)-th sub-carrier is with zero mean and variance \( \sigma_i^2 \). The consumed power for reliable transmission (i.e., error-free according to capacity) of the \( i \)-th sub-carrier is

\[
P_i[n] = \frac{\sigma_i^2}{H_i[n]} \left( e^{\theta R_i[n]} - 1 \right),
\]

where \( \theta = 2 \ln(2)b/L \) with \( L \) being the channel uses in each period. The total consumed power during the \( n \)-th period is

\[
P[n] = \sum_{i=1}^{M} P_i[n].
\]

In the \( n \)-th transmission, the transmitter allocates \( W[n] \) power from the storage battery, and other power, i.e., \( \max\{P[n] - W[n], 0\} \), is purchased from the power grid. That is to say, the purchased grid power in the \( n \)-th period is

\[
P_{grid}[n] = (P[n] - W[n])^+.
\]

Denote the stored renewables in the battery at instance \( n \tau \) as \( E_b[n] \). The evolution of the battery energy is

\[
E_b[n + 1] = \min\{(E_b[n] - W[n] \tau)^+ + E[n], B\}.
\]

Denote the grid power price in the \( n \)-th period as \( \zeta[n] \), which is constant during the \( n \)-th period but may change across different periods, then the cost of purchasing the grid power in the \( n \)-th period is

\[
C[n] = P_{grid}[n] \zeta[n]
\]

The time-average cost over a sufficient large but finite time horizon with \( n_{end} \) periods is expressed as

\[
\bar{C} = \frac{1}{n_{end}} \sum_{n=0}^{n_{end}-1} C[n]
\]

For notational simplicity, let \( Z[n] = (\mathbf{R}_n, W[n]) \) be the control action made by the transmitter at the beginning of the \( n \)-th period. Meanwhile, define

\[
\mathcal{Z}[n] = \left\{ \left( \mathbf{R}_n, W[n] \right) \left| R[n] \leq Q[n], W[n] \leq \frac{E_b[n]}{\tau} \right. \right\}
\]

as the corresponding action space. Let \( \mu \) be the maximal data buffer length (i.e., the buffer delay constraint). We have the following constrained stochastic optimization problem.
\[
\min_{(Z[n])_{n=0}^{n_{\text{end}}-1}} \bar{C} = \frac{1}{n_{\text{end}}} \sum_{n=0}^{n_{\text{end}}-1} C[n]
\]

s.t. \[
\begin{align*}
\bar{Q} &= \frac{1}{n_{\text{end}}} \sum_{n=0}^{n_{\text{end}}-1} Q[n] < \mu, \\
Z[n] &\in \mathcal{Z}[n]
\end{align*}
\]

3 Problem Analysis

Define a policy as \( p = (p_0, p_1, \ldots) \) that \( p_n \) generates action \( Z[n] = (R_n, W[n]) \) in the \( n \)-th period. In the problem, a policy includes the rate vector allocation policy, \( R(\cdot) \) (which generates \( R_n \) in the \( n \)-th period), and the renewable allocation policy, \( W(\cdot) \) (which generates \( W[n] \) in the \( n \)-th period). In addition, the rate vector policy can be decomposed into two aspects: How many packages will be transmitted in each period (i.e., the one period rate) and how to allocate these packages over \( M \) sub-carriers. They are referred to as the rate allocation policy \( R(\cdot) \) (which generates \( R[n] \) in the \( n \)-th period) and the sub-carrier allocation policy, respectively.

Intuitively, to reduce the cost given the total data for the \( n \)-th period transmission, \( R[n] \), all the data should be transmitted through the “best” subcarrier for power savings. In addition, power should be allocated from the battery as much as possible since the renewable energy is free. Formally, the one period cost in the \( n \)-th period is

\[
P_{\text{M}} = \sum_{i=1}^{M} \sigma_i \left( e^{\beta R_i[n]} - 1 \right) - W[n] + \frac{1}{n_{\text{end}}} \sum_{n=0}^{n_{\text{end}}-1} \xi[n].
\]

As \( R[n] = \sum_{i=1}^{M} R_i[n] \) is fixed, the optimal \( R_n \) for minimizing \( \sum_{i=1}^{M} \sigma_i \left( e^{\beta R_i[n]} - 1 \right) - W[n] \) is \( R_n^* = (R_1[n], \ldots, R_M[n]) \) with

\[
R_j^*[n] = \begin{cases} 
R[j], & \text{arg max } \frac{H_j[n]}{\sigma_i} \\
0, & \text{otherwise}
\end{cases}
\]

Since all scheduled data are transmitted through the subcarrier with best channel condition, we call the strategy as “best-subcarrier” policy. Apparently, the optimal \( W[n] \) for minimizing \( \sum_{i=1}^{M} \sigma_i \left( e^{\beta R_i[n]} - 1 \right) - W[n] \) is allocating the renewable energy as much as possible. And we refer this strategy as greedy renewable energy allocation policy. Then, we have the following lemma.

**Lemma 1** Given the transmitting package number in a period, the “best-subcarrier” policy and greedy renewable energy allocation policy will be utilized to minimize the one period cost.

We have derived that the “best-subcarrier” policy and the greedy policy are the optimal sub-carrier allocation policy and optimal renewable energy allocation policy, respectively, for one period given transmitting data number. Next, there is a natural question “Whether the optimality of the ‘best-subcarrier’ & greedy renewable energy allocation policy holds

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3 State refers to \( H_n, Q[n], E_p[n] \) and \( \xi[n] \); control action is generated according to state under policy.
for the average cost over several periods given the rate allocation policy?" The following lemma reveals the answer.

**Lemma 2** Given the rate allocation policy $R(\cdot)$, the "best-subcarrier" policy is optimal. In contrast, the greedy renewable policy is NOT always optimal.

**Proof** See the "Appendix".

Remark For one period, the "best-subcarrier" policy and the greedy policy are optimal sub-policies. Why the optimality of "best-subcarrier" policy can be extended from one to several periods but the greedy renewable policy can not? The reasons are as follows: The variation of the subcarrier allocation policy in one period can be thoroughly reflected by the one-period cost, and accordingly does not influence the state of the following period. Then, the optimality for one period can be extended to several periods because of the independence between two consecutive periods. By contrast, the renewable allocation policy in one period will definitely affect the state (e.g., $E_b$) of the following period, furthermore the corresponding action. That is to say, the variation of the renewable allocation policy in one period can NOT be thoroughly reflected by the one-period cost, the effect can be "propagated" to the following periods. The independence can not hold for the renewable allocation policy. Thus, for greedy renewable policy, optimality in each period does not mean optimality for several periods.

Although the greedy renewable policy is NOT strictly optimal given the rate policy, it is nearly optimal [6].

Based on the above analysis, we have the following theorem.

**Theorem 1** The (approximately) optimal solution of (10), $\{R_n^*, W_n^*[n]\}_{n=0}^{n_{\text{end}}-1}$, can be given as follows: Denote $v_n := \arg \min_i \{\frac{\sigma^2}{H_n[i]}\}$, $\eta_n := \frac{\sigma^2}{H_n[n]}$ and $P(R[n]) := \eta_n(e^{\eta_n} - 1)$. The optimal rate vector allocation $R_n^* = (R_1^*[n], \ldots, R_M^*[n])$ is given by

$$R_j^*[n] = \begin{cases} R_j^*[n], & j = v_n \\ 0, & \text{otherwise} \end{cases}$$

where $R_j^*[n]$ is the solution of

$$\min_{\{R[n]\}} \bar{C} = n_{\text{end}}^{-1} \sum_{n=0}^{n_{\text{end}}-1} \left[ P(R[n]) - \min\{E_b[n], P(R[n])\} \right]$$

s.t.

$$Q < \mu, \quad R[n] \leq Q[n].$$

The approximately optimal renewable allocation

$$W^*[n] = P(R^*[n]) - \min\{E_b[n], P(R^*[n])\}.$$

**Proof** The theorem can be verified by the first half of Lemma 2 and the approximate optimality of greedy renewable allocation [6]. The detailed proof is omitted for brevity.
4 On-Line Algorithm

In many scenarios, the future information is unavailable. We need to make decisions according to current (and past) information (on-line decision). In this section, we propose an on-line algorithm “Best-Greedy-Lyapunov” (BGL algorithm) (in Table 1) based on Theorem 1 and recently developed Lyapunov optimization [7].

The proposed BGL algorithm is purely an on-line algorithm, which requires only the current system state, e.g., current channel information $H_n$, current grid power price $\xi[n]$, current data queue length $Q[n]$, and current battery energy queue length $E_b[n]$. In the algorithm, we only need to solve a simplified deterministic optimization problem, (12).

In the following, we investigate the problem quantitatively. That is to say, we concentrate on finding the optimal solution of (12) mathematically to give the answer for “How much data will be transmitted in each period?”

If

$$R_{th} := \frac{1}{\theta} \ln \left( \frac{E_b[n]}{\eta_n} + 1 \right) \geq Q[n],$$

(12) is reduced to

$$\min_{0 \leq R[n] \leq Q[n]} -Q[n]R[n].$$

Thus, the optimal solution is $R^*[n] = Q[n]$. Otherwise, if $R_{th} < Q[n]$, (12) becomes

$$\min_{R_{th} \leq R[n] \leq Q[n]} V \cdot \left[ \eta_n \left( e^{R[n]} - 1 \right) - E_b[n] \right] \xi[n] - Q[n]R[n]$$

(14)

combined with

$$\min_{0 \leq R[n] \leq R_{th}} -Q[n]R[n].$$

(15)

The optimal solution of (15) is $R^*[n] = R_{th}$, and corresponding object value is $(-Q[n]R_{th})$; For (14), if

Table 1.

Algorithm: BGL algorithm

| Step 1: At the beginning of each period $n$, observe the current state including |
| current environment information (i.e., $H_n$ and $\xi[n]$), current data queue length $Q[n]$, and current battery energy queue length $E_b[n]$. |
| Step 2: Choose $R[n] \in [0, Q[n]]$ to minimize |
| $V \cdot \left[ \eta_n \left( e^{R[n]} - 1 \right) - \tilde{W}[n] \right] \xi[n] - Q[n]R[n],$ |
| where $\tilde{W}[n] = \min \{ E_b[n], \eta_n \left( e^{R[n]} - 1 \right) \}$, and $V > 0$ is a constant. |
| Step 3: Denote the selected $R[n]$ in step 2 as $R^*[n]$. Choose the optimal $W[n]$ as $W^*[n] = \min \{ E_b[n], \eta_n \left( e^{R^*[n]} - 1 \right) \}$ |
| Step 4: Update $Q[n]$ and $E_b[n]$ according to (1) and (5), respectively. |
the optimal solution is \( R^*[n] = R_s^4 \) and the corresponding object value is 
\[
\frac{Q[n]}{\theta V \eta_s \xi[n]} - V \eta_s \xi[n] - VE_b[n] \xi[n] - \frac{Q[n]}{\theta} \ln \frac{Q[n]}{\theta} \eta_s \xi[n] < - Q[n]R_{th}. 
\]
If \( R_s < R_{th} \), \( R^*[n] = R_{th} \) and the corresponding object value is 
\[
(-Q[n]R_{th}). 
\]
If \( R_s > Q[n] \), \( R^*[n] = Q[n] \) and the corresponding object value is 
\[
V \cdot \left[ \eta_s (e^{\theta Q[n]} - 1) - E_b[n] \xi[n] - Q[n]Q[n] < - Q[n]R_{th}. \right. 
\]

In conclusion,
\[
R^*[n] = \begin{cases} 
R_s, & R_s \in [R_{th}, Q[n]] \\
R_{th}, & R_s < R_{th} \\
Q[n], & R_s > Q[n] 
\end{cases} 
\tag{16} 
\]

**Remark**  
\( R_{th} \) is the maximal packet number that can be transmitted when using the stored renewable energy only. \( R_{th} > Q[n] \) means that the stored renewables can support transmitting all the buffer data. Based on Theorem 1, transmitting all the buffer data is optimal in this scenario. That is to say, \( R^*[n] = Q[n] \). On the other hand, when \( R_{th} < Q[n] \), the renewables can NOT support emptying the data buffer, i.e., we may need purchasing power from grid in this case: If \( R_s < R_{th} \) (e.g., \( V \) is large), cost is the key factor. We transmit \( R_{th} \) packages only using the renewables and no grid power will be purchased. We trade the delay for cost decrease. If \( R_s > Q[n] \) (e.g., \( V \) is small), the delay requirement is sharp. Together with the renewables, we purchase the deficient power from power grid to empty the data buffer. We trade cost for delay performance. The third scenario, \( R_s \in [R_{th}, Q[n]] \), both the cost and delay are important, we can NOT trade one totally for the other. Besides the renewables, we buy some grid power for transmitting \( R_s \) packages (NOT all buffer data).

Finally, we analyze the effect of parameter \( V \) on the data transmission qualitatively (or semi-quantitatively). In step 2, \( V > 0 \) is some constant to tradeoff the cost and queue length (i.e., delay). Specifically, given a small value of \( V \), the data buffer queue length is the focus. Then we transmit as much data as possible, \( R^*[n] = Q[n] \). That is to say, we trade cost for delay. For large \( V \), the reducing the cost is dominant, then we do NOT purchase power from the grid and only part (NOT always all) of the buffer data will be transmitted. That is to say, we trade delay performance for cost.

### 5 Numerical Results

In this section, simulations are carried out to illustrate the performance of the BGL algorithm. First, the simulation settings are given. Next, we show the delay and cost performance of BGL algorithm. Third, we compare the BGL algorithm against two other algorithms (i.e., the DOP and the COP) explained later.

#### 5.1 Simulation Setup

In the simulations, 3 sub-carriers are considered, i.e., \( M = 3 \), and we set \( \tau = 1, b = 1, N = 5, \sigma^2_t = 1 \). We consider Rayleigh fading in each sub-channel. That is to say, the power

\[ R_s := \frac{1}{\theta} \ln \frac{Q[n]}{\theta V \eta_s \xi[n]} \in [R_{th}, Q[n]], \]

**Remark**  
\( R_{th} \) is the stationary point of the object function in (14).
gain of each sub-channel is exponentially distributed. It is assumed that the 3 sub-channels have same mean power gain. The data arrival is i.i.d., and the arrived package number in each period chooses 0, 10, 20, 30 with probability 0.1, 0.5, 0.3, 0.1, respectively. The renewable energy arrives i.i.d. [8], and the arrived renewable energy in each period is 100, 300, 500, 800 with probability 0.1, 0.6, 0.2, 0.1, respectively. The grid power price is 0.02 and 0.05 with probability 0.3 and 0.7, respectively. The performance is averaged over \( n_{\text{end}} = 10^6 \) periods.

5.2 Delay and Cost Performance of the BGL Algorithm

Figure 2 plots the average delay and corresponding cost performance of the BGL algorithm with respect to \( V \) under different battery capacities. The mean power gain is set as 0.3. We consider two battery capacities \( B = 2500 \) and \( B = 1000 \). We can see that with the increase of \( V \), the data buffer length increases at first, and then remains static. Meanwhile, the cost decreases to zero and remains as zero then. It can be explained as follows: \( V \) is the trade-off factor. When increase \( V \), the “importance” of cost increases and the “importance” of delay decreases [see (12)]. The BGL algorithm trades the delay performance degradation for the cost decrease. Mathematically, when \( V \) increases, according to (16), the

![Figure 2](image-url)
average transmitted package number decreases at first. Given the mean data arrival and renewable energy arrival, less transmitted data results in longer average data buffer length and less average cost. Hence the data buffer length increases and the cost decreases. When $V$ is very large, $R_t < R_{th}$ satisfies almost always, then according to (16), the transmitted package number remains as $R_{th}$. The average buffer data length remains static given mean data arrival. Furthermore, the renewable energy can support $R_{th}$ and no grid power needs purchasing. Thus, the cost remains as zero. In addition, by comparing the two lines of $B = 2500$ and $B = 1000$ in each sub-figure, we can find that larger battery capacity improves the delay and cost performance. This is evident since larger capacity stores more (at least no less) renewables and leads to more data transmission and/or less cost.

Figure 3 illustrates the delay and corresponding cost performance of BGL algorithm versus the mean power gain under different $V$. The battery capacity $B = 2500$. It is observed that the buffer length and cost decrease when the channel condition improves (i.e., increase of mean power gain) at first, and remains static then. The reason is as follows: When the channel condition improves, the same amount energy supports more data transmission [see (3)]. Given the mean data arrival, the average data buffer length decreases. Given the mean renewable arrival, the cost decreases. By comparing the lines of
different $V$, we can also find that larger $V$ leads to longer buffer length and less cost. The explanation is same as in Fig. 2.

5.3 Algorithm Comparison

In this subsection, we compare the BGL algorithm against two other algorithms described as follows:

- Delay-optimal policy (DOP): In each period, the transmitter sends all the buffer data through the best sub-carrier, and utilizes the renewable energy as much as possible. Formally,

  $$R_j[n] = \begin{cases} Q[n], & j = v_n \\ 0, & \text{otherwise} \end{cases} \quad W[n] = \min \left\{ E_h[n], \eta_n \left( e^{\theta Q[n]} - 1 \right) \right\}. $$

- Cost-optimal policy (COP): In each period, utilize the renewables only to transmit data as much as possible through the best sub-carrier, and no grid power is purchased. That is to say, the cost is zero. Formally,
\[ R_j[n] = \begin{cases} \min\{Q[n], R_{ih}\}, & j = v_n \\ 0, & \text{otherwise}; \end{cases} \]
\[ W[n] = \min\left\{ E_b[n], \eta_e \left( e^{\frac{Q[n]}{C_1}} - 1 \right) \right\}. \]

Figure 4 shows the comparisons of delay and corresponding cost, respectively, for the BGL algorithm against the DOP as well as the COP. The mean power gain is 0.3, and the battery capacity is \( B = 2500 \). With the increase of \( V \), the BGL algorithm reaches the COP both in delay and cost. On the contrary, when decrease \( V \), the BGL algorithm approaches the the DOP. In terms of performance, the BGL policy can be viewed as a “mixed” policy of the DOP and the COP. By adjusting \( V \), the delay and cost can be traded off. The BGL algorithm with varying \( V \) can be applied as follows: Considering a constraint on delay (i.e., a value of \( \mu \)), we can get the maximal \( V \) from Fig. 4a. Then we get the optimal cost from Fig. 4b.

6 Conclusion

Delay-constrained data transmission in multi-carrier communications is studied in the presence of local renewable energy. We formulate a stochastic constrained optimization problem. By theoretical analysis, we derive that transmit data thorough the best sub-carrier is optimal, and greedy renewable allocation is nearly optimal. Additionally utilizing the Lyapunov optimization, the BGL algorithm is proposed. Using a tradeoff factor, the cost and delay performance can be adjusted in the BGL algorithm. In the end, simulations show the effectiveness and advance of the BGL algorithm.

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Appendix: Proof of Lemma 2

First, the optimality of the “best-subcarrier” policy can be verified by contradiction. Suppose that an optimal policy of the problem (10) does not utilize “best-subcarrier” policy as its subcarrier allocation policy. Consider a period, e.g., the \( i \)-th period, since \( R(\cdot) \) is given, \( R[i] \) is fixed, and is not transmitted totally through the best subcarrier. Now let a “novel” policy that transmit all the \( R[i] \) data through the best subcarrier and keep others same as the supposed optimal policy. Then there will be some power savings, and the cost in the \( i \)-th period will be not more. Meanwhile the delay is the same. As the “novel” policy only changes the sub-carrier allocation, it will not change the beginning state of the following periods. In this sense, the periods are independent, i.e., for the following periods, the cost under the “novel” policy for each period is not more and delay is the same (Note that the novel policy is feasible) compared to the supposed optimal policy. As \( n_{end} \) is sufficiently large, there is at least one period, the cost is less. Hence the average cost is less (Observe that \( n_{end} \) is finite). This contradicts with the assumption. The proof of the optimality of the “best-subcarrier” policy completes.

\(^5\) If the transmitter purchases from the power grid in a period under the supposed optimal policy, then in this period, the “novel” policy produces less cost.
Second, the Lagrange relaxed problem of (10) (with multiplier \( \lambda > 0 \)) can be expressed as

\[
\min_{\{Z[n]\}_{n=0}^{n_{end}-1}, Z[n] \in \mathbb{Z}[n]} \bar{C} = \frac{1}{n_{end}} \sum_{n=0}^{n_{end}-1} C[n] + \lambda Q[n]
\] (17)

The optimal renewable energy allocation policy of (10) must be optimal for (17). In other word, if we can prove that the greedy renewable policy is not optimal for (17), then we prove that the greedy renewable policy is not optimal for (10). The non-optimality of the greedy renewable policy for (17) can be verified similarly as the proof of Lemma 10 in [6]. Then the proof of the greedy renewable policy’s non-optimality completes.

References

1. Feng, D., Jiang, C., Lim, G. Jr., Cimini, L. J., Feng, G., & Li, G. Y. (2013). A survey of energy-efficient wireless communications. *IEEE Communications Surveys & Tutorials, 15*(1), 167–178.
2. Berry, R., & Gallager, R. (2002). Communication over fading channels with delay constraints. *IEEE Transactions on Information Theory, 48*(5), 1135–1149.
3. Zhang, X., & Tang, J. (2013). Power-delay tradeoff over wireless networks. *IEEE Transactions on Communications, 62*(9), 3673–3684.
4. Prasad, R. V., Devasenapathy, S., Rao, V. S., & Vazifehdan, J. (2014). Reincarnation in the ambiance: Devices and networks with energy harvesting. *IEEE Communications Surveys & Tutorials, 16*(1), 195–213.
5. Li, H., Huang, C., Cui, S., & Zhang, J. (2014). Distributed opportunistic scheduling for wireless networks powered by renewable energy sources. In *Proceedings of IEEE INFOCOM’14*, Toronto, Canada, April 27–May 2, 2014.
6. Zhang, T., Chen, W., Han, Z., & Cao, Z. (2015). A cross-layer perspective on energy harvesting aided green communications over fading channels. *IEEE Transactions on Vehicular Technology, 64*(4), 1519–1534.
7. Neely, M. J. (2010). Universal scheduling for networks with arbitrary traffic, channels, and mobility. In *Proceedings of IEEE CDC’10*, Atlanta, GA, USA.
8. Ozel, O., & Ulukus, S. (2012). Achieving AWGN capacity under stochastic energy harvesting. *IEEE Transactions on Information Theory, 58*(10), 6471–6483.

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