$b \rightarrow s\gamma$ and $\epsilon_1$ Constraints on Supergravity Models

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Abstract

In the light of the top quark discovered very recently by CDF, we investigate the possibility of narrowing down the allowed top quark masses by combining for the first time only two strongest constraints present in the no-scale $SU(5) \times U(1)$ supergravity model, namely, the ones from the flavor-changing radiative decay $b \rightarrow s\gamma$ and the precision measurements at LEP in the form of $\epsilon_1$. It turns out that even without including the most devastating constraint from $Z \rightarrow b\bar{b}$ measurement at LEP in the form of $R_b$ directly or $\epsilon_b$ indirectly, the combined constraint from $b \rightarrow s\gamma$ and $\epsilon_1$ alone in fact excludes $m_t(m_t) \gtrsim 180 \text{GeV}$ altogether in the no-scale model, providing a constraint on $m_t$ near the upper end of the CDF values. The resulting upper bound on $m_t$ is stronger and 5 GeV lower than the one from combining $\epsilon_1$ and $\epsilon_b$ constraints and also combining $b \rightarrow s\gamma$ and $\epsilon_b$ constraints in the previous analysis.
With the top quark discovered very recently by the CDF Collaboration from Fermi Laboratory in \( \bar{p}p \) collisions with the measured top mass \( m_t = 176 \pm 8 \pm 10 \) GeV, the Standard Higgs mass \( m_H \) is now the only unknown parameter in the Standard Model (SM). Despite the remarkable successes of the SM in its complete agreement with current experimental data, there is still no experimental information on the nature of its Higgs sector. The unknown \( m_t \) has long been one of the biggest obstacles in studying the phenomenology of the SM and its extensions of interest. Now that \( m_t \) is measured, one should be able to narrow down the values of \( m_t \) to the vicinity of the above central value considering the large experimental uncertainties in the measured top mass. In the context of supersymmetry, such a task can be performed within the Minimal Supersymmetric Standard Model (MSSM) \[2, 3, 4\]. The problem with such calculations is that there are too many parameters in the MSSM and therefore it is not possible to obtain precise predictions for the observables of interest. In the context of supergravity models (SUGRA), on the other hand, any observable can be computed in terms of at most five parameters: the top-quark mass, the ratio of Higgs vacuum expectation values (\( \tan \beta \)), and three universal soft-supersymmetry-breaking parameters (\( m_{1/2}, m_0, A \)). This implies much sharper predictions for the various quantities of interest, as well as numerous correlations among them. Of even more experimental interest is \( SU(5) \times U(1) \) SUGRA where string-inspired ansätze for the soft-supersymmetry-breaking parameters allow the theory to be described in terms of only three parameters: \( m_t, \tan \beta, \) and \( m_\tilde{g} \).

In this letter, we would like to present the possibility of narrowing down the allowed top quark masses by combining for the first time two strongest constraints present in the no-scale \( SU(5) \times U(1) \) SUGRA, namely, the ones from the flavor-changing radiative decay \( b \to s\gamma \) and the precision measurements at LEP in the form of \( \epsilon_1 \). After the first observation by CLEO on the exclusive decay \( B \to K^*\gamma \) \[5\] which provides only the upper bound on
the inclusive branching ratio of \( b \to s\gamma \) decay, CLEO has recently measured the inclusive branching ratio of \( b \to s\gamma \) decay \([3]\), which can provide much more precise constraint. Among four variables \( \epsilon_{1,2,3,b} \) introduced in Ref. \([3, 4]\), only \( \epsilon_1 \) and \( \epsilon_b \) lead to significant constraints in supersymmetric models \([7, 8]\). Although \( \epsilon_b \), which encodes the vertex corrections to \( Z \to bb \), provides even stronger constraint than \( \epsilon_1 \) for \( m_t < 170 \text{ GeV} \) \([8, 9]\), we do not include in our analysis here the \( \epsilon_b \) constraint in an attempt to isolate and exclude the most devastating impact on the precision test from the recent LEP measurement on \( R_b \) (\( \equiv \frac{\Gamma(Z \to bb)}{\Gamma(Z \to \text{hadrons})} \)) whose latest experimental values \([10]\) lie more than three standard deviations above the SM predictions for the values of \( m_t \) from the CDF. In fact, the possibility of improving the situation to a certain extent in this so-called “\( R_b \)-crisis” has been studied in the context of SUGRA in Ref. \([8]\) \(*\). An analysis similar to the one here has been done previously in Ref. \([9]\) where the combined constraints from \( \epsilon_1 \) and \( \epsilon_b \) and also from \( b \to s\gamma \) and \( \epsilon_b \) were studied. However, the correlated constraint from \( b \to s\gamma \) and \( \epsilon_1 \) has never been studied.

The \( SU(5) \times U(1) \) SUGRA contains, at low energy, the SM gauge symmetry and the particle content of the MSSM. The procedure to restrict 5-dimensional parameter spaces is as follows \([11]\). First, upon sampling a specific choice of \((m_{1/2}, m_0, A)\) at the unification scale and \((m_t, \tan \beta)\) at the electroweak scale, the renormalization group equations (RGE) are run from the unification scale to the electroweak scale, where the radiative electroweak breaking condition is imposed by minimizing the effective 1-loop Higgs potential, which determines the Higgs mixing term \( \mu \) up to its sign. Here the sign of \( \mu \) is given as usual \([12]\), and differs from that of Ref. \([8, 16]\); i.e. , we define \( \mu \) by \( W_\mu = \mu H_1 H_2 \).

We also impose consistency constraints such as perturbative unification and the naturalness bound of \( m_\tilde{g} \lesssim 1 \text{ TeV} \). Finally, all the known experimental bounds on the sparticle

\* As can be seen in Ref. \([8]\) by imposing the latest experimental data on \( R_b \) alone, \( m_t(m_t) \gtrsim 170 \text{ GeV} \) may be excluded at 99% C. L. in the no-scale \( SU(5) \times U(1) \) SUGRA.
masses are imposed. This procedure yields the restricted parameter spaces for the model. Further reduction in the number of input parameters in $SU(5) \times U(1)$ SUGRA is made possible because in specific string-inspired scenarios for $(m_{1/2}, m_0, A)$ at the unification scale these three parameters are computed in terms of just one of them \cite{13}. One obtains $m_0 = A = 0$ in the no-scale scenario.

The inclusive branching ratio of $b \rightarrow s\gamma$ decay has been recently measured for the first time by CLEO to be at 95% C. L. \cite{6},

$$1 \times 10^{-4} < Br(b \rightarrow s\gamma) < 4 \times 10^{-4}.$$  

This follows the renewed surge of interests on the $b \rightarrow s\gamma$ decay, spurred by the CLEO bound $Br(b \rightarrow s\gamma) < 8.4 \times 10^{-4}$ at 90% C.L. \cite{4}, with which it was pointed out in Ref. \cite{15} that the CLEO bound can be violated due to the charged Higgs contribution in the 2 Higgs doublet model (2HDM) and the MSSM basically if $m_{H^\pm}$ is too light, excluding large portion of the charged Higgs parameter space. It has certainly proven that this particular decay mode can provide more stringent constraint on new physics beyond SM than any other experiments\cite{1, 10, 17}. As we know, with the increasing accuracy of the LEP measurements, it has become extremely important performing the precision test of the SM and its extensions\cite{4}. Among several different schemes to analyze precision electroweak tests, we choose a scheme introduced by Altarelli et al.\cite{4} \cite{19} where four variables, $\epsilon_{1, 2, 3}$ and $\epsilon_b$ are defined in a model independent way. These four variables correspond to a set of observables $\Gamma_l, \Gamma_b, A'_{FB}$ and $M_W/M_Z$. Among these variables, $\epsilon_b$ encodes the vertex corrections to $Z \rightarrow b\bar{b}$.

In the 2HDM and the MSSM, $b \rightarrow s\gamma$ decay receives significant contributions from penguin diagrams with $W^\pm - t$ loop, $H^\pm - t$ loop \cite{20} and the $\chi_{1, 2}^\pm - \tilde{t}_{1, 2}$ loop \cite{21} only in the  

\footnotetext[1]{We use the following experimental lower bounds on the sparticle masses in GeV in the order of gluino, squarks, lighter stop, sleptons, and lighter chargino: $m_{\tilde{g}} \gtrsim 150$, $m_{\tilde{q}} \gtrsim 100$, $m_{\tilde{t}_1} \gtrsim 45$, $m_{\tilde{l}} \gtrsim 43$, $m_{\tilde{\chi}^\pm} \gtrsim 45$.}

\footnotetext[2]{A standard model fit to the latest LEP data yields the top mass, $m_t = 178 \pm 11^{+18}_{-19}$ GeV \cite{18}, which is in perfect agreement with the measured top mass from CDF.}
MSSM. The expression used for $Br(b \to s\gamma)$ in the leading logarithmic (LL) calculations is given by $^{22}$

$$
\frac{Br(b \to s\gamma)}{Br(b \to ce\bar{\nu})} = \frac{6\alpha}{\pi} \left[ \eta^{16/23} A_\gamma + \frac{8}{3}(\eta^{14/23} - \eta^{16/23}) A_g + C \right]^2 I(m_c/m_b) \left[ 1 - \frac{2}{3\pi}\alpha_s(m_b) f(m_c/m_b) \right],
$$

(1)

where $\eta = \alpha_s(M_W)/\alpha_s(m_b)$, $I$ is the phase-space factor $I(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$, and $f(m_c/m_b) = 2.41$ the QCD correction factor for the semileptonic decay. $C$ represents the leading-order QCD corrections to the $b \to s\gamma$ amplitude when evaluated at the $\mu = m_b$ scale $^{22}$. We use the 3-loop expressions for $\alpha_s$ and choose $\Lambda_{QCD}$ to obtain $\alpha_s(M_Z)$ consistent with the recent measurements at LEP. In our computations we have used: $\alpha_s(M_Z) = 0.118$, $Br(b \to ce\bar{\nu}) = 10.7\%$, $m_b = 4.8$ GeV, and $m_c/m_b = 0.3$. The $A_\gamma, A_g$ are the coefficients of the effective $bs\gamma$ and $bsg$ penguin operators evaluated at the scale $M_W$. Their simplified expressions are given in Ref. $^{23}$ in the justifiable limit of negligible gluino and neutralino contributions $^{21}$ and degenerate squarks, except for the $\tilde{t}_{1,2}$ which are significantly split by $m_t$. Regarding large uncertainties in the LL QCD corrections, which is mainly due to the choice of renormalization scale $\mu$ and is estimated to be $\approx 25\%$, it has been recently demonstrated by Buras et al. in Ref. $^{24}$ that the significant $\mu$ dependence in the LL result can in fact be reduced considerably by including next-to-leading logarithmic (NLL) corrections, which however, involves very complicated calculations of three-loop mixings between certain effective operators and therefore have not been completed yet.

The expression for $\epsilon_1$ is given as $^{3}$

$$
\epsilon_1 = e_1 - e_5 - \frac{\delta G_{V,B}}{G} - 4\delta g_A,
$$

(2)
where $e_{1,5}$ are the following combinations of vacuum polarization amplitudes

$$e_1 = \frac{\alpha}{4\pi \sin^2 \theta_W M_W^2} [\Pi_{T}^{33}(0) - \Pi_{T}^{11}(0)],$$  \hfill (3)

$$e_5 = M_Z^2 F'_{ZZ}(M_Z^2),$$  \hfill (4)

and the $q^2 \neq 0$ contributions $F_{ij}(q^2)$ are defined by

$$\Pi_{T}^{ij}(q^2) = \Pi_{T}^{ij}(0) + q^2 F_{ij}(q^2).$$  \hfill (5)

The $\delta g_A$ in Eq. (2) is the contribution to the axial-vector form factor at $q^2 = M_Z^2$ in the $Z \to l^+l^-$ vertex from proper vertex diagrams and fermion self-energies, and $\delta G_{V,B}$ comes from the one-loop box, vertex and fermion self-energy corrections to the $\mu$-decay amplitude at zero external momentum. These non-oblique SM corrections are non-negligible, and must be included in order to obtain an accurate SM prediction. As is well known, the SM contribution to $\epsilon_1$ depends quadratically on $m_t$ but only logarithmically on the SM Higgs boson mass ($m_H$). In this fashion upper bounds on $m_t$ can be obtained which have a non-negligible $m_H$ dependence: up to 20 GeV stronger when going from a heavy ($\approx 1$ TeV) to a light ($\approx 100$ GeV) Higgs boson. It is also known (in the MSSM) that the largest supersymmetric contributions to $\epsilon_1$ are expected to arise from the $\tilde{t}-\tilde{b}$ sector, and in the limiting case of a very light stop, the contribution is comparable to that of the $t-b$ sector. The remaining squark, slepton, chargino, neutralino, and Higgs sectors all typically contribute considerably less. For increasing sparticle masses, the heavy sector of the theory decouples, and only SM effects with a light Higgs boson survive. (This entails stricter upper bounds on $m_t$ than in the SM, since there the Higgs boson does not need to be light.) However, for a light chargino ($m_{\chi_1^\pm} \to \frac{1}{2}M_Z$), a $Z$-wavefunction renormalization threshold effect can introduce a substantial $q^2$-dependence in the calculation, i.e., the presence of $e_5$ in Eq. (2) [3].

The complete vacuum polarization contributions from the Higgs sector, the supersymmetric
chargino-neutralino and sfermion sectors, and also the corresponding contributions in the SM have been included in our calculations.

In Fig. 1 we show the results of the calculation of $\epsilon_1$ for all allowed points in the no-scale $SU(5) \times U(1)$ SUGRA for the running top mass $m_t(m_t) = 170, 180 \text{ GeV}$. We use in the figure the following experimental value for $\epsilon_1$,

$$\epsilon_1^{\exp} = (3.5 \pm 1.8) \times 10^{-3},$$

determined from the latest $\epsilon$- analysis using the LEP and SLC data in Ref. [25]. In the figure points between the two horizontal lines are allowed by the $\epsilon_1$ constraint at the 90% C. L. Since all sparticle masses nearly scale with the gluino mass (or the chargino mass), it suffices to show the dependences of the parameter on, for example, the chargino mass. Therefore, we show the explicit dependence only on the chargino mass in Fig. 1. The significant drop in $\epsilon_1$ comes from the threshold effect of Z-wavefunction renormalization as discussed earlier. As can be seen in Fig. 1, the current experimental values for $\epsilon_1$ prefer light but not too light chargino in the no-scale model:

$$50 \text{ GeV} \lesssim m_{\tilde{\chi}^\pm_1} \lesssim 70 \text{ GeV}, \quad m_t(m_t) = 180 \text{ GeV},$$

$$50 \text{ GeV} \lesssim m_{\tilde{\chi}^{\pm}_1}, \quad m_t(m_t) = 170 \text{ GeV},$$

In order to deduce the bounds on any of the other masses from the above bounds, one can use the scaling relations in the model, $m_{\tilde{q}} \approx 0.97m_{\tilde{g}}$ and $2m_{\chi_1^0} \approx m_{\chi_2^0} \approx m_{\tilde{\chi}^\pm_1} \approx 0.28m_{\tilde{g}}$. For $m_t(m_t) < 170 \text{ GeV}$, it is fairly clear that there will be only lower bound on the chargino mass. In addition to the $\epsilon_1$ constraint there is another very strong constraint coming from the $b \rightarrow s\gamma$ decay as mentioned earlier. In order to see if it provides additional constraint, in Fig. 2 we show the results of the calculation of $Br(b \rightarrow s\gamma)$ versus $\epsilon_1$. It is very interesting for one to see that the combined constraint is much stronger than each individual constraint.
and it nearly excludes $m_t(m_t) = 180$ GeV in the model, leaving only one point (out of a few thousand points) still allowed for $\mu < 0$. However, this one point is also excluded by imposing the preliminary but conservative bound from all 4 LEP collaborations on the lightest chargino mass, $m_{\chi^\pm_1} \gtrsim 65$ GeV \cite{26}. Therefore, $m_t(m_t) \gtrsim 180$ GeV is excluded altogether in the no-scale model, and this provides a constraint on $m_t$ near the upper end of the CDF values. Without imposing the new bound $m_{\chi^\pm_1} \gtrsim 65$ GeV, the resulting constraint is in fact even stronger than the one from the combined constraints from $\epsilon_1$ and $\epsilon_b$ and also from $b \to s\gamma$ and $\epsilon_b$ in Ref. \cite{9}. On the other hand, the combined constraint for $m_t(m_t) = 170$ GeV becomes less severe although the $b \to s\gamma$ constraint excludes additionally a large fraction of the parameter space. The large suppression in $Br(b \to s\gamma)$ for $\mu < 0$ in the model is worth further explanation. As first noticed in Ref. \cite{16}, what happens is that in Eq. (1), the $A_\gamma$ term nearly cancels against the QCD correction factor $C$; the $A_g$ contribution is small. The $A_\gamma$ amplitude receives three contributions: from the $W^{\pm}-t$ loop, from the $H^{\pm}-t$ loop, and from the $\chi^{\pm_1,2}_{1,2}-\tilde{t}_{1,2}$ loop. The first two contributions are always negative \cite{22}, whereas the last one can have either sign, making it possible having cancellations among three contributions.

In conclusion, in the light of the top quark discovered very recently by CDF, we investigate the possibility of narrowing down the allowed top quark masses by combining for the first time only two strongest constraints present in the no-scale $SU(5) \times U(1)$ supergravity model, namely, the ones from the flavor-changing radiative decay $b \to s\gamma$ and the precision measurements at LEP in the form of $\epsilon_1$. It turns out that even without including the most devastating constraint from $Z \to b\bar{b}$ measurement at LEP in the form of $R_b$ directly or $\epsilon_b$ indirectly, the combined constraint from $b \to s\gamma$ and $\epsilon_1$ alone in fact excludes $m_t(m_t) \gtrsim 180$ GeV altogether in the no-scale model, providing a constraint on $m_t$ near the upper end of the CDF values. The resulting upper bound on $m_t$ is stronger and 5 GeV lower than the one from combining $\epsilon_1$ and $\epsilon_b$ constraints and also combining $b \to s\gamma$ and $\epsilon_b$ constraints in
the previous analysis.

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References

[1] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 74 (1995) 2626.

[2] E. Eliasson, Phys. Lett. B 147 (1984) 67; S. Lim, et al., Phys. Rev. D 29 (1984) 1488; J. Grifols and J. Sola, Nucl. Phys. B 253 (1985) 47; B. Lynn, et al., in Physics at LEP, eds. J. Ellis and R. Peccei, CERN Yellow Report CERN86-02, Vol. 1; R. Barbieri et al., Nucl. Phys. B 341 (1990) 309; A. Bilal, J. Ellis, and G. Fogli, Phys. Lett. B 246 (1990) 459; M. Drees and K. Hagiwara, Phys. Rev. D 42 (1990) 1709; M. Drees, K. Hagiwara, and A. Yamada, Phys. Rev. D 45 (1992) 1725.

[3] R. Barbieri, M. Frigeni, and F. Caravaglios, Phys. Lett. B 279 (1992) 169.

[4] G. Altarelli, R. Barbieri, and F. Caravaglios, Nucl. Phys. B 405 (1993) 3.

[5] CLEO Collaboration, R. Ammar et al. Phys. Rev. Lett. 71 (1993) 674.

[6] CLEO Collaboration, M. S. Alam et al., Phys. Rev. Lett. 74 (1995) 2885.

[7] J. L. Lopez, D. V. Nanopoulos, G. T. Park, H. Pois, and K. Yuan, Phys. Rev. D 48 (1993) 3297; J. L. Lopez, D. V. Nanopoulos, G. T. Park, and A. Zichichi, Phys. Rev. D 49 (1994) 4835.
[8] J. E. Kim and G. T. Park, Phys. Rev. D 50 (1995) R6686.

[9] J. E. Kim and G. T. Park, Phys. Rev. D 51 (1995) 2444.

[10] P. B. Renton, talk given at 17th International Symposium on Lepton Photon Interactions, August 1995, Beijing, China.

[11] See e.g., S. Kelley, J. L. Lopez, D. V. Nanopoulos, H. Pois, and K. Yuan, Nucl. Phys. B 398 (1993) 3.

[12] See e.g., V. Barger, M. Berger, and P. Ohmann, Phys. Rev. D 49 (1994) 4908; J. E. Kim and G. T. Park, Phys. Rev. D 51 (1995) 2444.

[13] See e.g., L. Ibáñez and D. Lüst, Nucl. Phys. B 382 (1992) 305; V. Kaplunovsky and J. Louis, Phys. Lett. B 306 (1993) 269; A. Brignole, L. Ibáñez, and C. Muñoz, Nucl. Phys. B 422 (1994) 125.

[14] M. Battle et al. (CLEO Collab.) in Proceedings of the joint Lepton-Photon and Europhysics Conference on High-Energy Physics, Geneva 1991.

[15] J. Hewett, Phys. Rev. Lett. 70 (1993) 1045; V. Barger, M. Berger, and R. J. N. Phillips, Phys. Rev. Lett. 70 (1993) 1368.

[16] J. L. Lopez, D. V. Nanopoulos, and G. T. Park, Phys. Rev. D 48 (1993) R974; G. T. Park, P. 70, the Proceedings of the HARC workshop “Recent Advances in the Superworld”, The Woodlands, Texas, April 1993; J. L. Lopez, D. V. Nanopoulos, G. T. Park, and A. Zichichi, Phys. Rev. D 49 (1994) 355.

[17] G. T. Park, Phys. Rev. D 50 (1994) 599; G. T. Park, Mod. Phys. Lett. A 9 (1994) 321; G. T. Park, Mod. Phys. Lett. A 10 (1995) 967.
[18] D. Schaile, the Proceedings of the 27th International Conference on High Energy Physics, Glasgow, July 1994.

[19] G. Altarelli, CERN-TH.6867/93 (April 1993).

[20] B. Grinstein and M. Wise, Phys. Lett. B 201, 274 (1988); B. Grinstein et al., Nucl. Phys. B 399, 269 (1990); T. Rizzo, Phys. Rev. D 38, 820 (1988); C. Geng and J. Ng, Phys. Rev. D 38, 2858 (1988); W. Hou and R. Willey, Phys. Lett. B 202, 591 (1988).

[21] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. B 353 (1991) 591.

[22] B. Grinstein, R. Springer, and M. Wise, Phys. Lett. B 202, 138 (1988); R. Grigjanis et al., Phys. Lett. B 213, 355 (1988); Phys. Lett. B 224, 209 (1989); M. Misiak, Phys. Lett. B 269, 161 (1991); M. Misiak, Nucl. Phys. B 393, 23 (1993).

[23] R. Barbieri and G. Giudice, Phys. Lett. B 309 (1993) 86.

[24] A. Buras, M. Misiak, M. Münnz and S. Pokorski, Nucl. Phys. B 424, 374 (1994).

[25] G. Altarelli, the Proceedings of the 1st International Conference on Phenomenology of Unification from Present to Future, Rome, CENR-TH.7319/94, June 1994.

[26] L. Rolandi, H. Dijkstra, D. Strickland and G. Wilson, representing the ALEPH, DELPHI, L3 and OPAL Collaborations, Joint Seminar on the First Results from LEP 1.5, CERN, Dec. 1995.
Figure Captions

• Figure 1: The predictions for $\epsilon_1$ versus the chargino mass in the no-scale $SU(5) \times U(1)$ SUGRA for the running top mass $m_t = 180$ GeV (top row) and $m_t = 170$ GeV (bottom row). In the figure, points between the horizontal lines are allowed at the 90% C. L.

• Figure 2: The correlated predictions for $BR(\equiv Br(b \rightarrow s\gamma))$ and $\epsilon_1$ in the no-scale $SU(5) \times U(1)$ SUGRA for the running top mass $m_t = 180$ GeV (top row) and $m_t = 170$ GeV (bottom row). Points between the two horizontal lines or below the horizontal line as pointed by the arrow are allowed by the $b \rightarrow s\gamma$ constraint at the 95% C. L. while points between the two vertical lines are allowed by the $\epsilon_1$ constraint at the 90% C. L.