Time in a Box: Advancing Knowledge Graph Completion with Temporal Scopes

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ABSTRACT
Almost all statements in knowledge bases have a temporal scope during which they are valid. Hence, knowledge base completion (KBC) on temporal knowledge bases (TKB), where each statement may be associated with a temporal scope, has attracted growing attention. Prior works assume that each statement in a TKB must be associated with a temporal scope. This ignores the fact that the uncovering information is commonly missing in a KB. Thus prior work is typically incapable of handling generic use cases where a TKB is composed of temporal statements with/without a known temporal scope. In order to address this issue, we establish a new knowledge base embedding framework, called TIME2BOX, that can deal with atemporal and temporal statements of different types simultaneously. Our main insight is that answers to a temporal query always belong to a subset of answers to a time-agnostic counterpart. Put differently, time is a filter that helps pick out answers to be correct during certain periods. We introduce boxes to represent a set of answer entities to a time-agnostic query. The filtering functionality of time is modeled by intersections over these boxes. In addition, we generalize current evaluation protocols on time interval prediction. We describe experiments on two datasets and show that the proposed method outperforms state-of-the-art (SOTA) methods on both link prediction and time prediction.

CCS CONCEPTS
• Computing methodologies → Knowledge representation and reasoning. Temporal reasoning.

KEYWORDS
Temporal Knowledge Base, TIME2BOX, Link Prediction, Time Prediction

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1 INTRODUCTION
A knowledge base (KB) such as Wikidata and DBpedia stores statements about the world around us. A KB is typically represented as a set of triples in the form of \((s, r, o)\) – short for (subject, relation, object), encoding the association between entities and relations among them. A statement is often temporally scoped, which indicates during which time period it is valid. Two examples are (Albert Einstein, educatedAt, ETH Zurich, 1896 - 1900) and (Albert Einstein, academicDegree, Doctor of Philosophy in Physics, 1906). The former specifies the time period during which Albert Einstein studied at ETH, and the latter points out the specific date when he obtained his degree. Graphs that contain a substantial amount of such time-aware statements are often called temporal knowledge base (TKB) in the machine learning literature. Each statement in a TKB is associated with a validity time as \((s, r, a, t^*)\).

Due to the ever-changing state of the world and missing data, TKBs usually contain inaccurate and incomplete information similar to KBs. The sparsity of TKBs necessitates temporal knowledge base completion (TKBC), namely inferring missing statements from known statements. Temporal link prediction task is proposed to evaluate a TKBC model by testing its performance on answering incomplete temporal queries of the form \((s, r, a, t^*)\) or \((s, r, o, t^*)\).

Despite recent success stories on time-agnostic KBC, research on TKBC is still in its early age and is facing new challenges. The validity time period of a statement is often missing in a KB. As a result, it is difficult to distinguish whether statements in a KB are atemporal (e.g., (Albert Einstein, instanceOf, Human)) or time-dependent (e.g., (United States of America, instanceOf, Historical Unrecognized State\(^2\))). This leads to the question of which statements should be part of a TKB in the first place. Prior works restrict TKBs to a collection of statements where the validity time period for each statement must be available. However, in WIKIDATA114k, a dataset from Wikidata, for instance, 85.1% of all statements are temporal, 56.2% of the temporal statements are missing their validity time information and are excluded in previous studies while only 247,393 out of 1,660,824 statements (i.e., 14.9%) are truly atemporal\(^3\). As the

\(^1\) \(t^*\) could be a time instant or time interval
\(^2\) According to Wikidata that statement holds true between 1776-1784.
\(^3\) For all the statements, we first categorize predicates into two groups – atemporal predicates and temporal predicates. If a predicate has ever been involved in a statement that has temporal scoping, it belongs to temporal predicates; otherwise, it is an atemporal predicate. Atemporal statements are those associated with atemporal predicates.
number of temporal statements with missing validity information is substantial, excluding them from a TKB will significantly reduce the amount of information that could be useful in TKB studies.

Retaining these temporally scoped statements leads to several challenges that need to be addressed. For instance, how to design a TKBC model to handle statements with and without known temporal or pattern of events in the data representation perspective and model design perspective? Clearly, the conventional representation in prior TKBC in the form of (s, r, o, t)\(^4\) falls short. An ideal TKBC model should be more flexible to address cases when the validity information of different types (i.e., point in time, right-open interval (known start time), left-open interval (known end time), closed interval) is presented in a TKB or even no validity information is available for a statement.

The second challenge is how to predict the temporal scope of a statement as it is often missing in TKBs. This task is referred to as time interval prediction, which amounts to answering incomplete queries of the form (s, r, o, ?). How to generate a predicted time interval and evaluate it require further investigation. This problem has only been addressed very recently by Jain et al. [9]. However, at times their evaluation protocols fail to distinguish one predicted interval from another since they do not consider the gap between the predicted and the gold interval in case of no overlap. For instance, the same metric scores are assigned to two predictions [1998, 1999] and [1998, 2010] when a gold interval [2011, 2020] is considered.

In this paper, we present a novel TKBC embedding framework, called TIME2BOX, which relies on the intuition that the answer set in a temporal query (s, r, ?o, t*) is always a subset of answers of its time-agnostic counterpart (s, r, ?o). As illustrated in Fig. 1, there are four correct answer entities to a query (Albert Einstein, employer, 3b). However, when time information is specified (e.g., Albert Einstein, employer, 3b, (1933)) as shown in Fig. 1b, the number of positive answers becomes three. With more temporal information being available (e.g., Albert Einstein, employer, 3b, (1933, 1935)), the answer set shrinks further (see Fig. 1c). Therefore, we propose to model a statement in a TKB by imitating the process of answering its corresponding temporal query (s, r, ?o, t*), which can be achieved in two steps – finding answer entities to its atemporal counterpart (s, r, ?o) by using KBC methods and then picking out entities to be true to the temporal query from preceding answers by including time. We implement this idea by using box embeddings [16, 24], especially inspired by QUERY2BOX [17], which is originally used for answering conjunctive queries [15]. Boxes, as containers, can naturally model a set of answers they enclose. The filtering functionality of time intervals can be naturally modeled as intersections over boxes similarly to Venn diagrams [23]. Meanwhile, performing an intersection operation over boxes would still result in boxes, thus making it possible to design a unified framework to deal with statements/queries of different types.

Our main research contributions are listed as follows:

- We introduce a new evaluation metrics gaeIOU for interval evaluation by taking the gap between a gold interval and a predicted interval into consideration if no overlap between them exists.
- We propose a box-based knowledge graph embedding TKBC framework (TIME2BOX) that can represent and model statements with different types of validity information (i.e., unknown temporal scoping, left/right-open intervals and closed intervals).
- Extensive experiments on two datasets - WIKIDATA12k and WIKIDATA114k - show that TIME2BOX yields the state-of-the-art (SOTA) results in link prediction and outperforms SOTAs in time prediction by significant margins.

TIME2BOX code is available in Github\(^5\).

2 RELATED WORK

Knowledge Base Completion. KBC has been extensively studied in the past [2, 13, 20, 22, 26]. The core insight of these methods is to embed entities and relations in a KB into low-dimensional vectors, which can be utilized in downstream tasks, such as link prediction. These methods can be roughly classified into two groups: transformation-based models and semantic matching energy based models. Transformation-based models treat a relation as a transformation operator. Two well-known assumptions are translation (e.g., TransE [2]) and rotation (e.g., RotatE [20]). For instance, TransE assumes that for a statement (s, r, o), the object embedding can be derived by translating the subject embedding operated by the relation embedding in the embedding space. As such, the presence of a statement in a KG is measured by the distance between the object embedding and the subject embedding after transformation. Semantic matching energy based methods determine the existence of a statement by a score calculated from a function of learned entity and relation embeddings in the latent space [22, 26]. For instance, DistMult [26] uses a 3-way inner product as the scoring function. In addition to these basic triple-based methods, other studies have focused on exploiting higher-order structural information in a KG (e.g., paths, neighbours), such as PTransE [13], R-GCN [19], and TransGCN [4]. All KBC models ignore the temporal scoping of statements, and thus are unable to address temporal statements. However, these models are the foundations for TKBC.

Temporal Knowledge Base Completion. Recently, there has been a surge of interest in taking valid information into consideration as KB statements are usually time-dependent. There are two lines of works on temporal link prediction. The first branch focuses on so-called dynamic knowledge bases (i.e., event KBs (ICEWS) [3]), where each statement is associated with a timestamp. The insight behind this branch is that knowledge in KBs evolves over time and historical statements/events drive the occurrence of new events. Therefore, their focus is more on extrapolation – predicting unseen entity relationships over time by modeling temporal dependencies of statements/events in KBs [6, 10, 21, 25]. The most well-known model is Know-Evolve [21], which assumes that the occurrence of facts/events can be modeled as a multivariate temporal point process.

Unlike the first branch that assumes timestamp-based statements/facts, a TKB in the second branch can associate a statement with time instances or time intervals as its validity information. Moreover, the goal of this line of work is more about interpolation – filling in missing components in TKGs with/without explicitly

\(^4\) denotes a time point

\(^5\)https://github.com/ling-cai/Time2Box
modeling the temporal dependencies between statements. Recent works follow a common paradigm, that is, to encode time as embeddings and then incorporate them into time-agnostic KBC models [7–9, 11, 12, 14]. Leblay and Chekol [12] investigated several extensions of existing KBC models by directly fusing time embeddings with relation embeddings, including TTransE, TRECSCAL, etc. Goel et al. [8] proposed to learn time-varying entity embeddings by replacing a fraction of embedding weights with an activation function of learned frequencies. Unlike previous work, which view time as numeric values, García-Durán et al. [7] concatenated the string representation of the relation and the time, and fed them into an LSTM to obtain time-aware relation representations, which were used in TransE (TA-TransE) and DistMult(TA-DM) afterwards. More recently, Lacroix et al. [11] presented the TKBC problem as a four-way tensor completion problem, and proposed TNTCompEx, which was extended from the time-agnostic CompEx [22]. Jain et al. [9] augmented TNTcomplex with three more time-dependent terms as analogy to the idea of approximating joint distributions by low-order marginals in graph models and incorporated soft ordering and span constraints as temporal inductive biases. Our work belongs to the latter group. However, our proposal is more flexible as we can deal with cases when \( t' \) is a time instant, a (left/right-open) interval, closed interval or even missing, while prior works can only handle one timestamped representation of the form \( (s, r, o, t) \).

3 PRELIMINARIES

3.1 Temporal Knowledge Bases

Prior TKBC methods typically work on TKBs in which each statement has to be associated with validity information. Thereby, for statements that do not have known temporal scopes, they either exclude them from a TKB in the beginning or assume that these statements hold all the time [11]. However, there are limitations in both ways. As discussed in Section 1, excluding them from a TKB will significantly reduce the amount of information that could be beneficial in TKBC studies as the number of such statements is substantial. For the latter, their assumption would be problematic since a lot of them may only hold for a certain time period. For instance, the statement (Warsaw, country, Russian Empire) holds during the time interval [1815-07-09, 1916-11-04]. Following the open-world assumption (OWA), we argue that TKBs are an extension to KBs insofar as the lack of temporal scoping for any given statement does not imply it holding indefinitely.

In the following, we use \( t \) and \( I_{st} = [st, et] \) to denote a time point and a time interval, respectively. The symbol \( \sim \) will stand for unknown temporal validity. There are five types of statements in such a TKB: (1) \( (s, r, o) \) for a statement without a known temporal scope; (2) \( (s, r, o, t) \) for a timestamped statement which holds at a point in time \( t \); (3) \( (s, r, o, I_{st}) \) for a right-open interval-based statement, in which only the time when the statement starts to hold
is known; (4) \((s, r, o, t^*)\) for a left-open interval-based statement, in which only the time when the statement ceases to hold is known; and (5) \((s, r, o, t_{st}^*)\) for a statement which is temporally scoped by a closed interval \([t_{st}, t_{et}]\). Then a TKB is denoted as \(G = \bigcup_{(s, r, o, t^*)}\), namely the union of statements of the five types, where \(s, o \in E\) represent entities, \(r \in R\) denotes a relation and \(t^* \in \{t_{st}, t_{et}, t_{et}^*, \text{None}\}\) denotes different types of valid time or no valid time available.

3.2 The TKBC Problem

Link prediction and time prediction are two main tasks used to evaluate a TKBC model. Statements in TKBs are split into training, validation, and test sets, used for model training, parameter tuning and model evaluation, respectively.

Link prediction. Queries used in this task are of the form \((s, r, ?, o, t^*)\). Performance is evaluated on the rank of a given golden, i.e., ground truth, answer in the list of all the entities sorted by scores in a descending order. Then MRR (mean reciprocal rank), MR (mean rank), HITS@1, HITS@3 and HITS@10 are computed from the ranks over all queries in the test set. However a query may be satisfied by multiple answer entities. Thus another correct answer may be ranked over the given golden answer. In such cases, a KBC/TKBC model should not be penalized. A traditional strategy used in KBC is to filter out those correct answers that are already in the training and validation sets before calculating metrics. This strategy can be directly applied to queries of the form \((s, r, ?, o, t)\) or \((s, r, ?, o, t)\). However, it may not be sufficient for queries of the form \((s, r, ?, o, t)\), as there may exist other answers that are true during a time period within the interval \(I\). For example, suppose two statements – (Albert Einstein, employer, Princeton University, \([1933, 1955]\)) and (Albert Einstein, employer, Leiden University, \([1920, 1946]\)), both Princeton University and Leiden University are correct answers during the period \([1933, 1946]\). One naive way to solve this problem is to discretize the interval \(I\) to a sequence of time points \(t_s\) and then to convert \((s, r, ?, o, t)\) into timestamped queries of the form \((s, r, ?, o, t)\) so that the same filtering process can be performed on each timestamped query. Finally, the ranks over them are averaged to be the rank for a time interval-based query. This idea is well-aligned with the proposal by \([9]\).

Time prediction. Time prediction queries in TKBs are of the form \((s, r, o, \text{?})\). Despite the fact that the validity information could be a point in time or a time interval, a point in time can be viewed as a special time interval, in which start time and end time coincide. Thus, time prediction boils down to time interval prediction. Its performance is evaluated by the overlap between a gold interval and a predicted interval or the closeness between those in case of no overlap. We describe the existing evaluation protocols and propose a generalized evaluation metric in Section 5.

4 METHOD

The key insight of TIME2BOX lies in an intuition that the answer set of a temporal query \((s, r, o, t^*)\) is always a subset of answers of its time-agnostic counterpart \((s, r, o, \text{?})\) and set size decreases by adding more temporal constraints. As illustrated in Fig. 1a, four object entities satisfy the atemporal query (Albert Einstein, employer, \(3\)) while three entities are the correct answers when the query is restricted to the year of 1993 (see Fig. 1b) and only one entity is correct when another time information is further added in the statement, shown in Fig. 1c. Inspired by this observation, we propose to model a temporal statement \((s, r, o, t^*)\) by imitating the process of answering its corresponding temporal query \((s, r, o, \text{?})\), which can be achieved through two steps: 1) finding a set of answer entities that are true for the corresponding atemporal query by using any KBC model and 2) imposing a filtering operation enforced by time to restrict answers afterwards. In following sections, we take a time instant-based statement as example to formalize our idea in a KB space and a vector space, respectively.

4.1 Formalization in a KB Space

For a statement \((s, r, o, t)\) in a TKB, the first step of TIME2BOX, as shown in Fig. 1a, is to project the subject \(s\) to a set of object entities that are true to its corresponding atemporal query in the form of \((s, r, o, \text{?})\) enforced by the relation \(r\). This is a prerequisite for any statement and can theoretically be addressed by any KBC method. Formally, the relation projector is defined as:

Relation Projector - \(\text{OP}_1\): Given the subject entity \(s\) and the relation \(r\), this operator obtains: \(S_\text{r} = \{o' \mid (s, r, o') \in G\} \). \(G\) is the time-agnostic counterpart of \(G\).

Then time information is used to filter out entities that are incorrect during the time of interest from the answer set \(S_\text{r}\). This can be achieved by first projecting the subject \(s\) to a set of object entities that co-occur with \(s\) in statements at a given time point (as shown in blue edges in Fig. 1b) and then finding the intersection over them and \(S_\text{r}\) (see the three entities in red in Fig. 1b). Accordingly, the two involved steps are defined as:

**Time Projector - \(\text{OP}_2\):** Given the subject \(s\) and the timestamp \(t\), this operator obtains: \(S_\text{t} = \{o' \mid o' \in E \text{ and } (s, r, o', t) \in G \text{ and } t \in R\} \).

**Intersection Operator - \(\text{OI}\):** Given \(S_\text{r}\) and \(S_\text{t}\), this operator obtains the intersection \(S_\text{r \cap t} = \{o \mid o \in (S_\text{r} \text{ and } S_\text{t})\} \).

In fact, such a modeling process also fits to left/right-open interval-based statements directly. For a left/right-open interval-based statement, we only consider the known endpoint time in such an interval as we follow the open-world assumption. However, for an atemporal statement, we only need one relation projector to obtain \(S_\text{r}\), which is the final set consisting of correct answer entities to its query form. For a closed interval-based query, one commonly used approach is to randomly pick one timestamp within the interval and to associate it with \((s, r, o)\). Then it can be modeled the same way as an instant-based statement. At training, a timestamp is always randomly picked from the interval to ensure that all the timestamps in the interval are used. In addition to the common strategy, TIME2BOX allows sampling of a sub-time interval within the given interval so that two time constraints (i.e., start time and end time) can be imposed by using two time projectors, as shown in Fig. 1b.\(^6\)

\(^6\)Alternatively, one could also enumerate all the timestamps within the interval and use different \(\text{OP}_1\) to project the subject to multiple sets of entities, each of which is specific for one timestamp. Subsequently, an intersection operator again is performed over all the sets of entities obtained from \(\text{OP}_1\) and \(\text{OP}_2\), in the previous step. However, in spite of its efficiency, this practice is hard to implement in mini-batch training manner since time intervals in different statements usually have varying duration and thus contain different number of timestamps.
4.2 Implementation in a Vector Space

In order to implement this idea in a vector space, two key points are 1) how to model a set of answers returned by a KBC model and 2) how to instantiate two projectors and one intersection operator.

Prior KBC models are incapable of directly representing a set of answer entities in a vector space. Instead, they usually represent entities and relations as single points in the vector space and model point-to-point projections, e.g., TransE. Inspired by QUERY2BOX [17], which is used to deal with complex queries that involve conjunctions, existential quantifiers, and disjunctions, we introduce the idea of boxes in the vector space and thus name the proposed framework TIME2BOX. The reasons for adopting boxes are three-fold. First, boxes are containers that can naturally model a set of answer entities they enclose. Second, finding the intersection set among sets of entities amounts to finding the intersected area over boxes similar to the concept of Venn diagram. Third, the result of performing an intersection operation over boxes is still a box, which makes it possible to deal with statements of different types in a unified framework.

In TIME2BOX, each entity $e \in E$, relation $r \in R$, and timestamp $t \in T$ ($T$ is the set of all discrete timestamps in a TBK) are represented as vector embeddings $e \in \mathbb{R}^d$, $r \in \mathbb{R}^d$, and $t \in \mathbb{R}^d$, respectively, which are illustrated in Fig. 1. Instead, they usually represent boxes in the vector space.

An intersection operation on boxes is still a box, set among sets of entities amounts to finding the intersected area of different boxes. Formally, a box can be represented by its center (i.e., a center point) and its length (i.e., offsets). Formally, a box in a vector space is represented by $b = \{\text{Cen}(b), \text{Off}(b)\}$, where $\text{Cen}(b) \in \mathbb{R}^d$ is its center point and $\text{Off}(b) \in \mathbb{R}^d$ specifies the length/2 of the box in each dimension. If an entity belongs to a set, its entity embedding is modeled as a point inside the box of the set. The interior of a box in a vector space can be specified by points inside it:

$$\text{box}_i = \{e \in \mathbb{R}^d : \text{Cen}(b) - \text{Off}(b) \leq e \leq \text{Cen}(b) + \text{Off}(b)\}$$

where $\leq$ denotes element-wise inequality.

**Projection operators in a vector space.** In previous work, relations are commonly assumed to be projectors that transform a subject embedding to an object embedding in terms of points in a vector space, e.g., TransE [2] and RotatE [20]. Here we adopt a similar idea but take both relations and timestamps as projectors (OP$_r$ and OP$_t$) to project a subject to a set of entities in $S_r$ represented as a time-agnostic box $b_{S_r}$ and to a set of entities in $S_t$ represented as a time-aware box $b_{S_t}$, respectively, which are illustrated in Fig. 1.

The center of a box can be defined as the resulting embedding after applying a projection operator (OP$_r$ or OP$_t$) on the subject embedding. The centers of $b_{S_r}$ and $b_{S_t}$ can be formulated as below:

$$\text{Cen}(b_{S_r}) = e \circ r; \quad \text{Cen}(b_{S_t}) = e \otimes t$$

where $\circ$ and $\otimes$ are projectors OP$_r$ and OP$_t$, respectively. Theoretically, projection operators could be instantiated by any projector in existing KBC models, such as element-wise addition in TransE [2], element-wise product in DistMult [26], and Hadamard product in RotatE [20]. Even though OP$_r$ and OP$_t$ can be different, we choose the same projector for both and implement two TIME2BOX models by taking element-wise addition and element-wise product as operators by following TransE and DistMult, respectively. Accordingly, these two models are named as TIME2BOX-TE and TIME2BOX-DM.

Ideally, the size of the box $b_{S_r}$ should be determined by both the subject entity and the relation, since the box contains all object entities that satisfy a query in the form of $(s, r, o)$. The same applies to $b_{S_t}$. However, as the entity space is usually large in a KB, introducing entity-specific parameters would result in high computational cost. Therefore, in practice, Off($b_{S_r}$) and Off($b_{S_t}$) are only determined by the relation $r \in R$ and the timestamp $t \in T$, respectively. Put differently, the size of $b_{S_r}$ and $b_{S_t}$ is initialized based on $r$ and $t$, which are learned through training.

**Intersection Operators in a vector space.** An intersection operator aims to find the intersection box $b_{\text{inter}} = (\text{Cen}(b_{\text{inter}}), \text{Off}(b_{\text{inter}}))$ of a set of box embeddings $B = \{b_{S_r}, b_{S_t}, ..., b_{S_N}\}$ obtained from the previous step. The intersection operator should be able to deal with $B$ of different sizes, as required in Fig. 1. Thus, both $\text{Cen}(b_{\text{inter}})$ and $\text{Off}(b_{\text{inter}})$ are implemented by using attention mechanisms. Following the idea in Bahdanau et al. [1], the center point $\text{Cen}(b_{\text{inter}})$ is calculated by performing element-wise attention over the centers of boxes in $B$. This can be formulated as follows:

$$\text{Cen}(b_{\text{inter}}) = \sum_i \text{softmax}(\text{NN}([\text{Cen}(b_i)]) \odot \text{Cen}(b_i))$$

where $\text{NN}$ is a one-layer neural network and $b_i \in B$.

Since the intersection box $b_{\text{inter}}$ must be smaller than any of the boxes in $B$, we use element-wise min-pooling to make sure the new box must be shrunk and perform DeepSets [27] over all the $\text{Off}(b_i)$ ($b_i \in B$) to downscale $b_{\text{inter}}$ [17]. This can be written as below:

$$\text{Off}(b_{\text{inter}}) = \text{Min}(\text{Off}) \odot \sigma(\text{DeepSets}(\text{Off}))$$

where DeepSets($[x_1, x_2, ..., x_n]$) = MLP($1/n$) $\cdot \sum_i$ MLP($x_i$), $\sigma$ denotes the sigmoid function, and $\text{Off} = \{\text{Off}(b_i) : b_i \in B\}$.

4.3 Optimization Objective

For a query, TIME2BOX aims to pull correct entity embedding into the final box $b_{\text{inter}}$ while pushing incorrect entity embedding far away from it. The distance-based loss proposed by Sun et al. [20] satisfies this need:

$$\text{Loss} = -\log \sigma(\gamma - D(o, b_{\text{inter}})) - \frac{1}{k} \sum_{i=1}^{k} \log \sigma(D(o_i, b_{\text{inter}}) - \gamma)$$

where $\sigma$ is the sigmoid function, $\gamma$ is a fixed margin, $o$ is the embedding of a positive entity to the given query, and $k$ is the number of negative samples $o_i$. $D(o, b_{\text{inter}})$ measures the distance between entity $o$ and the final box $b_{\text{inter}}$. With the size of a box being considered, the distance is divided into two parts: outside distance $D_{\text{outside}}(o, b_{\text{inter}})$ and inside distance $D_{\text{inside}}(o, b_{\text{inter}})$. For cases when $o$ is outside of $b_{\text{inter}}$, the former refers to the distance of an entity embedding $o$ to the boundary of the box $b_{\text{inter}}$, and the latter
calculates the distance between the box’s center $\text{Cen}(b_{\text{inter}})$ and its boundary. This can be formalized as below:

$$D(o, b_{\text{inter}}) = \alpha \cdot D_{\text{inside}}(o, b_{\text{inter}}) + D_{\text{outside}}(o, b_{\text{inter}})$$ (6)

where $\alpha$ ∈ [0, 1]. When $\alpha = 0$, it means that a positive entity is required to be in a $b_{\text{inter}}$, but its distance to the center is not as important. $D_{\text{inside}}(o, b_{\text{inter}})$ and $D_{\text{outside}}(o, b_{\text{inter}})$ are written as:

$$D_{\text{inside}}(o, b_{\text{inter}}) = ||\text{Cen}(b_{\text{inter}}) - \text{Min}(b_{\text{max}}, \text{Max}(b_{\text{min}}, o))||_1$$

$$D_{\text{outside}}(o, b_{\text{inter}}) = ||\text{Max}(o - b_{\text{max}}, 0) + \text{Max}(b_{\text{min}} - o, 0)||_1$$

where $b_{\text{min}} = \text{Cen}(b_{\text{inter}}) - \text{Off}(b_{\text{inter}})$ and $b_{\text{max}} = \text{Cen}(b_{\text{inter}}) + \text{Off}(b_{\text{inter}})$ are embeddings of the bottom left corner and the top right corner of $b_{\text{inter}}$, respectively.

Compared to answering atemporal queries, finding correct answers to temporal ones is more challenging. Therefore, the loss function should reward more in the optimization direction that is capable of correctly answering temporal queries. For a given query $q_i$, we use $\frac{1}{n_q}$, where $n_q$ is the number of correct answers to $q_i$ that appear in training as a weight to adjust the loss. The core idea here is that time-aware queries often are satisfied with fewer answers, and, thus, are harder to answer compared to atemporal queries.

### 4.4 Time Negative Sampling

Entity negative sampling is widely used in KBC. For a positive sample $(s, r, a)$, negative samples are constructed by replacing $a$ with other entities $a’$, ensuring that $(s, r, a’)$ must not appear in training set. In this paper, we adopt this strategy so that the model is able to learn the association between entities, relations, and time occurring in a positive sample by distinguishing the correct answers from the negative samples. Moreover, for time-aware statements, we perform time negative sampling, which corrupts a statement $(s, r, o, t)$ by replacing $t$ with a number of timestamps $t’$. This is important for statements where only start time or end time is available. As shown in Fig. 1, the proposed architecture cannot distinguish those other entities and, thus, are harder to answer compared to atemporal queries.

### 4.5 Time Smoothness Regularizer

Time is continuous. We may expect that neighboring timestamps would have similar representations in the vector space. Following Lacroix et al. [11], we penalize time difference between embeddings of two consecutive timestamps by using $L_2$:

$$\lambda(T) = \frac{1}{|T| - 1} \sum_{i=1}^{T-1} ||t_{i+1} - t_i||_2^2$$ (8)

During the training step, for batches with temporal statements, we add this regularizer with a weight scalar $\beta$ to the loss function in Eq. 5, where $\beta$ specifies the degree of penalization.

### 5 Evaluation Metrics in Time Prediction

**Time Interval Evaluation.** $\text{gIOU}$ [18] and $\text{aeIOU}$ [9] are two evaluation metrics recently adopted in time interval prediction. Both are built on Intersection Over Union that is commonly used for bounding box evaluation in Computer Vision.

The idea of $\text{gIOU}$ is to compare the intersection between a predicted interval and a gold interval against the maximal extent that the two intervals may expand. It can be formulated as below:

$$\text{gIOU}(\text{gold}, \text{pred}) = \frac{D(\text{gold} \cap \text{pred})}{D(\text{gold} \cup \text{pred})}$$ (9)

where $\text{gold} \cap \text{pred}$ is the overlapping part of two intervals, $\text{gold} \cup \text{pred}$ denotes the shortest contiguous interval (hull) that contains both $\text{gold}$ and $\text{pred}$. As shown in Fig. 2, if $\text{gold} = [2011, 2016]$ and $\text{pred} = [2009, 2013]$, then $\text{gold} \cap \text{pred} = [2011, 2013]$ and $\text{gold} \cup \text{pred} = [2009, 2016]$. $D(I) = l_{\text{max}} - l_{\text{min}} + 1$ is the number of time points at a certain granularity (e.g., year in this paper) during the time interval $I$.

Compared to $\text{gIOU}$, affinity enhanced IOU, denoted as $\text{aeIOU}$, provides a better evaluation in case of non-overlapping intervals and outputs scores in $[0, 1]$. It can be written as follow:

$$\text{aeIOU}(\text{gold}, \text{pred}) = \begin{cases} D(\text{gold} \cap \text{pred}) \quad & D(\text{gold} \cap \text{pred}) > 0 \\ 1 \quad & \text{otherwise} \end{cases}$$ (10)

However, we notice that $\text{aeIOU}$ cannot tell some cases apart. As illustrated in Fig. 2, $\text{aeIOU}$ results in the same scores for \textcircled{5}, \textcircled{6}, and \textcircled{7} when compared to the gold interval[2011, 2016]. Intuitively one would assume that \textcircled{7} is better than the others and \textcircled{5} is the least desirable. The former has a one-year intersection with \textcircled{5} and the gold. For the latter, the gap between \textcircled{5} and \textcircled{6} is smaller than that between \textcircled{6} and the gold, despite the fact that neither \textcircled{5} and \textcircled{6} overlaps with the gold. Its failure lies in that it does not consider the gap between the gold and the predicted interval in case of no overlap.

In the following, we take both the hull and the intersection/gap between a gold interval and a predicted interval into the design of the metric. The intuition is that when the size of the hull remains the same, the metric score of a predicted interval decreases with a larger gap to the gold in case of no overlap and increases with a larger intersection. $\text{aeIOU}$ is therefore generalized to $\text{gaeIOU}$ as below:

$$\text{gaeIOU}(\text{gold}, \text{pred}) = \begin{cases} D(\text{gold} \cap \text{pred}) \quad & D(\text{gold} \cap \text{pred}) > 0 \\ \frac{D(\text{gold} \cap \text{pred})}{D(\text{gold} \cup \text{pred})} \quad & \text{otherwise} \end{cases}$$ (11)
Then we use the greedily coalescing method proposed in Jain et al. [9] to generate time intervals as predictions. Both are rooted in Wikidata. WIKIDATA12k is a widely used benchmark dataset in TKBC where the start time, end time, or both of a statement can remain unspecified. Although this dataset is more appropriate for a TKBC problem, there are two limitations. First, it poses a computational burden as it contains 432k entities and 407 relations, consisting of 7M tuples in the training set. Second, there are several mistakes in the temporal information. For instance, 2014 was written as 2401. We extract a subgraph, named as WIKIDATA114k, and correct temporal information by checking it against Wikidata. More details about data pre-processing and statistics are in Appendix A (All Appendices are available online\(^7\)). Since our focus is on generic knowledge bases, we do not consider event-based datasets, such as ICEWS14 and ICEWS05-15, in which each statement is associated with a timestamp.

6.2 Baselines and Model Variants

In the following experiments, we regard TIME2BOX-TE as our main model, in which both the relation operator and the time projector are instantiated as an element-wise addition. It is denoted as TIME2BOX in resulting tables. We compare it against two SOTAs in TKBC: TNT-Complex and TIMEPLEX base model by using the implementation in Jain et al. [9], both of which are based on the time-agnostic KBC model: ComplEx [22].

In addition to comparison with existing SOTAs, we also conduct an ablation study, in which several variants of the proposed model are compared: (1) TIME2BOX-SI, short for Sample Interval: for a closed interval-based statement, this variant randomly samples a sub time interval from a given interval at each training step and train it as shown in Fig. 1c. (2) TIME2BOX-TR: previous works in TKBC often explicitly fused relations with time information to obtain time-aware relations and empirically demonstrated its effectiveness [7, 9, 11]. We also explicitly model the association between relations and time as a new point \(p_{rt} = r + t\) in the vector space and incorporate it into Eq. 3 to help locate the intersection box. (3)TIME2BOX-DM: this variant implements the relation and time-agnostic KBC model: ComplEx [22].

All these models are trained on statements in training set and evaluated by answering queries where either the object or the time information is missing. Hyper-parameter settings are introduced in Appendix B and comparison of parameters used in different models is summarized in Table 11 in Appendix F. Moreover, we notice there are several limitations in current experimental setups of SOTAs and we detail them in Appendix C.

6.3 Main Results

**Link Prediction Task.** We report main results of link prediction in Table 1. TIME2BOX and all its variants consistently outperform or are on a par with the performance on SOTAs in terms of MRR, statements that do not have known temporal scopes in Wikidata, although they may be time-dependent and useful in TKBC, as discussed in Section 1. The other dataset is a subset of WIKIDATA432k proposed by Lacroix et al. [11], which is the only TKB dataset where the start time, end time, or both of a statement can remain unspecified. Although this dataset is more appropriate for a TKBC problem, there are two limitations. First, it poses a computational burden as it contains 432k entities and 407 relations, consisting of 7M tuples in the training set. Second, there are several mistakes in the temporal information. For instance, 2014 was written as 2401. We extract a subgraph, named as WIKIDATA114k, and correct temporal information by checking it against Wikidata. More details about data pre-processing and statistics are in Appendix A (All Appendices are available online\(^7\)). Since our focus is on generic knowledge bases, we do not consider event-based datasets, such as ICEWS14 and ICEWS05-15, in which each statement is associated with a timestamp.

\[^7\]Link to online Appendices.
Table 1: Link prediction evaluation across two datasets.

| Datasets        | WIKIDATA12k | WIKIDATA114k |
|-----------------|-------------|--------------|
| Metrics         | MRR         | Hits@1       | Hits@10     | MRR         | Hits@1       | Hits@10     |
| TNT-Complex     | 34.99       | 25.69        | 41.73       | 48.99       | 33.99        | 41.73       |
| TIMEPLEX-TR     | 35.99       | 25.69        | 41.73       | 48.99       | 33.99        | 41.73       |
| TIMEPLEX-DM     | 36.79       | 25.69        | 41.73       | 48.99       | 33.99        | 41.73       |
| TIMEPLEX-SI     | 37.25       | 25.69        | 41.73       | 48.99       | 33.99        | 41.73       |
| TIMEPLEX-TNS    | 37.30       | 25.69        | 41.73       | 48.99       | 33.99        | 41.73       |

Table 2: Time prediction evaluation on WIKIDATA12k.

| Metrics         | gIOU@1      | gIOU@10      | aeIOU@1      | aeIOU@10     | gaeIOU@1     | gaeIOU@10    |
|-----------------|-------------|--------------|--------------|--------------|--------------|--------------|
| TNT-Complex     | 31.44       | 35.18        | 18.86        | 40.94        | 11.01        | 29.51        |
| TIMEPLEX-TR     | 35.63       | 60.86        | 18.86        | 37.75        | 62.61        | 32.63        |
| TIMEPLEX-DM     | 36.79       | 62.44        | 21.91        | 41.55        | 14.94        | 37.14        |
| TIMEPLEX-SI     | 38.78       | 70.16        | 25.78        | 50.04        | 17.41        | 47.54        |
| TIMEPLEX-TNS    | 42.30       | 70.16        | 25.78        | 50.04        | 17.41        | 47.54        |

Table 3: Time prediction evaluation on WIKIDATA114k.

| Query Example 1: (Yury Vasilyevich Malyshev, educatedAt, ?o, 1977) |
|---------------------------------------------------------------|
| TIMEPLEX base                                               |
| 1. Rausman Moscow State Technical University,                |
| 2. Gold Star,                                                |
| 3. Communist Party of the Soviet Union,                      |
| 4. Order of Lenin,                                           |
| 5. S.P. Korolev Rocket and Space Corporation Energia,        |
| 6. Hero of the Soviet Union,                                 |
| 7. Gagarin Air Force Academy,                                |
| 8. Kalashnikov Higher Military Aviation School of Pilots,    |
| 9. Ashok Chakra,                                             |
| 10. Heidelberg University,                                   |

| Query Example 2: (Pavel Pablo Kuczynski, countryOfCitizenship, 2013) |
|---------------------------------------------------------------------|
| TIMEPLEX base                                                       |
| 1. France,                                                          |
| 2. Germany,                                                         |
| 3. United States of America,                                        |
| 4. Austria,                                                         |
| 5. Romania,                                                        |
| 6. United Kingdom,                                                  |
| 7. Poland,                                                          |
| 8. Kingdom of Italy,                                                |
| 9. Russian Soviet Federative Socialist Republic,                   |
| 10. Russian Empire,                                                 |

6.4 Qualitative Study

Table 4 showcases examples of timestamp-based link prediction on WIKIDATA12k. The comparison between TIMEPLEX base and TIME2BOX reveals that TIME2BOX is able to learn common characteristics of entities by adopting boxes. For instance, the predicted top 10 returned by TIME2BOX are possible affiliations (e.g., institutes, colleges, universities) in the first query and are countries in the second query. By contrast, TIMEPLEX base returns a mixture of entities with distinct classes for both queries. Furthermore, Table 5 shows an example of time interval-based link prediction, in which TIME2BOX is able to consistently output correct predictions across time and precisely discern the changes of objects over time (i.e., the correct answer shifts from Russian Empire to Ukrainian People’s Republic in 1916), while TIMEPLEX base fails. This can be attributed to the ability of TIME2BOX to capture the order of timestamps and the idea of temporal boxes as a constraint over potential answer entities. Hence, answer entities that are true in two consecutive years can be enclosed in the intersection of temporal boxes.

6.5 Model Variation Study

In this section, we report on observations of results about different model variations, which are shown in Table 1, 2 and 3. Compared to TIME2BOX-DM, which adopts element-wise product as operators, element-wise addition projectors (TIME2BOX) perform better in link prediction and time prediction on both datasets. Moreover, we observe that explicitly modeling association between time...
Table 5: An example of interval-based link prediction on WIKIDATA12k. For time interval-based link prediction, the current strategy is to discretize intervals to timestamps and average ranks for each timestamp-based prediction result as the final evaluation. Only top 1 predictions are shown here.

| Year Range | Answers | TIME2BOX | Russian Empire | Ukrainian People’s Republic |
|------------|---------|----------|-----------------|---------------------------|
| [1905, 1916] | (1) | Russian Empire | | Ukrainian People’s Republic |
| [1917, 1919] | (2) | Ukrainian People’s Republic | | Russian Empire |

REFERENCES

[1] Dmitry Bahdanau, Kyunghyun Cho, and Yoshua Bengio. 2014. Neural machine translation by jointly learning to align and translate. arXiv preprint arXiv:1409.0473 (2014).

[2] Antoine Bordes, Nicolas Usunier, Alberto Garcia-Duran, Jason Weston, and Ok-sana Yakhnenko. 2013. Translating embeddings for modeling multi-relational data. Advances in neural information processing systems 26 (2013), 2787–2795.

[3] Elizabeth Boocher, Jennifer Gutenstalcher, Sean O’Brian, Steve Shellman, James Starz, and Michael Ward. 2015. IECWS Executed Event Data.

[4] Ling Cai, Bo Yan, Gengchen Mai, Krzysztof Janowicz, and Rui Zhu. 2019. Trans- GCN: Coupling Transformation Assumptions with Graph Convolutional Networks for Link Prediction. In K-CAP 2019. 131–138.

[5] Shih Sanker Dasgupta, Swayambhu Nithi Ray, and Parthul Talukdar. 2018. Hyte: Hyperplane-based temporally aware knowledge graph embedding. In EANLP.

[6] Songqian Dong, Huzefa Rangwala, and Yue Nung. 2020. Dynamic Knowledge Graph based Multi-Event Forecasting. In Proceedings of the 26th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining. 1585–1595.

[7] Alberto Garcia-Duran, Sebastian Dumaničić, and Mathias Niepert. 2018. Learning Sequence Encoders for Temporal Knowledge Graph Completion. In EMNLP. Association for Computational Linguistics, Brussels, Belgium. 4816–4821.

[8] Rishab Goel, Seyed Mehran Kazemi, Marcus Brubaker, and Pascal Poupart. 2020. Diachronic embedding for temporal knowledge graph completion. In Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 34. 3988–3995.

[9] Prachi Jain, Sushant Rath, Mousum, and Soumen Chakrabarti. 2020. Temporal Knowledge Base Completion: New Algorithms and Evaluation Protocols. In EMNLP. Association for Computational Linguistics, Online. 3733–3747.

[10] Wooseong Jin, Meng Qiu, Xisen Jin, and Xiang Ren. 2020. Recurrent Event Network: Autoregressive Structure Inference over Temporal Knowledge Graphs. In EMNLP. 6669–6683.

[11] Timothée Lacroix, Guillaume Obozinski, and Nicolas Usunier. 2019. Tensor Decompositions for Temporal Knowledge Base Completion. In IJCAI.

[12] Julien Leblay and Melissa Wudage Chekol. 2018. Deriving Validity Time in Knowledge Graph. In Companion Proceedings of the The Web Conference 2018 (WWW ’18). International World Wide Web Conferences Steering Committee, Republic and Canton of Geneva, CHE. 1771–1776.

[13] Yankai Lin, Zhuyuan Liu, Huan-Bo Luan, Maosong Sun, Siwei Rao, and Song Liu. [n. d.]. Modeling Relation Paths for Representation Learning of Knowledge Bases. In EMNLP.

[14] Yampu Ma, Volker Tresp, and Erik A Daxberger. 2019. Embedding models for episodic knowledge graphs. Journal of Web Semantics 59 (2019), 100490.

[15] Gengchen Mai, Krzysztof Janowicz, Bo Yan, Rui Zhu, Ling Cai, and Ni Lao. 2019. Contextual graph attention for answering logical queries over incomplete knowledge graphs. In K-CAP 2019. 171–178.

[16] Dhruv Patel and Shih Sanker. 2020. Representing joint hierarchies with box embeddings. Automated Knowledge Base Construction (2020).

[17] Hongyu Ren, Weihua Hu, and Jure Leskovec. 2019. Query2box: Reasoning over Knowledge Graphs in Vector Space Using Box Embeddings. In IJCAI.

[18] Hamid Rezaei-gholi, Nathan Tsao, JunYoung Gwak, Amir Sadeghian, Ian Reid, and Silvio Savarese. 2019. Generalized intersection over union: A metric and a loss for bounding box regression. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. 658–666.

[19] Michael Schlichtkrull, Thomas N Kipf, Peter Bloem, Rianne Van Den Berg, Ivan Titov, and Max Welling. 2018. Modeling relational data with graph convolutional networks. In European Semantic Web Conference. Springer, 593–607.

[20] Zhiqing Sun, Zhi-Hong Deng, Jian-Yun Nie, and Jian Tang. 2019. Rotate: Knowledge graph embedding by relational rotation in complex space. arXiv preprint arXiv:1902.10197 (2019).

[21] Rakshit Trivedi, Hanjun Dai, Yichen Wang, and Le Song. 2017. Know-evolve: Temporal episodic knowledge graphs. In Proceedings of the 34th International Conference on Machine Learning-Volume 70: 3462–3471.

[22] Théo Trouillon, Johannes Welbl, Sebastian Riedel, Eric Gaussier, and Guillaume Bouchard. 2016. Complex embeddings for simple link prediction. IJCAI.

[23] John Venn. 1880. I. On the diagrammatic and mechanical representation of propositions and reasonings. The London, Edinburgh, and Dublin philosophical magazine and journal of science 10, 59 (1880), 1–18.

[24] Luke Vilnis, Xiang Li, Shikhar Murty, and Andrew McCallum. 2018. Probabilistic embedding of knowledge graphs with box lattice measures. arXiv preprint arXiv:1805.06627 (2018).

[25] Chengjin Xu, Mojtaba Nayeriy, Fouad Alkhouly, Hamed Sharfizad Yazdi, and Jens Lehmann. 2019. Temporal knowledge graph embedding model based on additive time series decomposition. arXiv preprint arXiv:1911.07893 (2019).

[26] Bishan Yang, Wen-tau Yih, Xiaodong He, Jianfeng Gao, and Li Deng. 2014. Embedding entities and relations for learning and inference in knowledge bases. arXiv preprint arXiv:1412.6575 (2014).

[27] Ismail Zabeer, Satwik Kottulinsky, Sarang Ravanbakhsh, Barnabas Póczos, Ruslan Salakhutdinov, and Alexander J Smola. 2017. Deep Sets. In NIPS. 3394–3404.
A DATA STATISTICS

**Generation of WIKIDATA114k.** We extracted a sport-centric subgraph from WIKIDADA432k. We first picked out statements where the relation `memberOfSportsTeam` appears and obtained an entity set from those statements. Then we find all the statements that entities obtained from the previous step participate in as our initial subgraph. Finally, we ensure that each entity/relation is associated with at least 5 statements and the time period is restricted to [1883, 2023] for temporal statements, which encloses most of the temporal statements in the initial subgraph. This results in 1.7 million statements with 114k entities and 126 relations, and thus named as WIKIDATA114k. See Table 6 for data statistics.

|               | WIKIDATA12k | WIKIDATA114k |
|---------------|-------------|--------------|
| #entities     | 12,544      | 114,351      |
| #relations    | 24          | 126          |
| time period   | [19, 2020]  | [1883, 2023] |
| train         |             |              |
| #all          | 32,497      | 1,670,969    |
| #time instant | 14,099      | 175,637      |
| #start time   | 4,089       | 44,809       |
| #end time     | 1,273       | 2,164        |
| #full time    | 13,035      | 402,135      |
| #no time      | 0           | 1,046,224    |
| valid         |             |              |
| #all          | 4,051       | 11,720       |
| #time instant | 1,857       | 1,177        |
| #start time   | 322         | 342          |
| #end time     | 76          | 11           |
| #full time    | 1,796       | 2,655        |
| #no time      | 0           | 7,535        |
| #test         |             |              |
| #all          | 4,043       | 11,854       |
| #time instant | 1,844       | 1,219        |
| #start time   | 324         | 306          |
| #end time     | 56          | 15           |
| #full time    | 1,819       | 2,790        |
| #no time      | 0           | 7,524        |

**Table 6: Statistics of these datasets used.**

B HYPERPARAMETER SETTINGS

We tune models by the MRR on the validation set. Grid search is performed over negative samples $k = [16, 32, 64, 128]$, learning rate $lr = [0.003, 0.002, 0.001]$, batch size $b = [1500, 2000, 2500, 3000, 3500]$; dimension $d = [200, 300, 400]$; and weight for time smoothness regularizer $\beta = [0.0, 0.1, 0.001, 0.0001]$, as shown in Table 7. We find that effects of different hyperparameters are minimal except for learning rate as the trained model usually converge to similar MRRs as long as they are trained thoroughly. We also observe that time smoothness regularizer is useful in learning time embeddings on WIKIDATA12k while failing to improve the model on WIKIDATA114k. This may be due to data sparsity with regard to time. As the time span of WIKIDATA114k is much smaller, time information is intensive and thus models are capable of learning temporal order between timestamps implicitly.

|               | #negative samples | # learning rate |
|---------------|-------------------|-----------------|
| MRR           | 36.02 36.68 37.06 37.30 | 0.003 0.002 0.001 |
| MR            | 97 100 98 101      | 126 103 101     |
| HITS@1        | 25.96 26.81 27.25 27.38 | 26.83 26.73 27.38 |

**Table 7: Effects of hyper-parameters on WIKIDATA12k**

C EXPERIMENTAL SETUP

Upon inspection on implementations of TKBC models, we find there are two common issues.

First, SOTAs only learn time embeddings for timestamps that appear in training set, which would be problematic at testing. For instance, suppose a sorted (ascending) list of timestamps occurring in training set is [1540, 1569, 1788, 1789, 1790]. SOTAs only learn embeddings for these timestamps, while ignoring intermediate timestamps. As a result, they cannot answer queries when the associated time is not in the list, such as (s, r, o, 1905). This problem would be even worse regarding time interval generation. As when we need to grow a time point to a time interval by extending it to the left or the right, we may jump from one year to a year far away from it. For instance, from 1569 to 1540 (left) or 1788 (right). This is not reasonable and thus may severely affect the evaluation on time prediction. In order to address this issue, we enumerate all the time points in the time span of the training set with a fixed granularity (i.e., year) and use them for all models at training periods.

The other issue is about the evaluation of link prediction task on time interval-based statements (including closed interval-based and left/right-open interval-based statements). In existing works, the evaluation boils down to assessing the correctness of answering a timestamp-based query by randomly picking one timestamp from a set of timestamps within the time interval and then measuring the performance on the newly generated query (i.e., the timestamp-based query). However, this is problematic. For closed interval-based samples, the evaluation results may vary from randomly sampled timestamps and thus may not be stable. For left/right-open interval-based statements, it is more severe. For instance, for a left-open interval-based test sample (Albert Einstein, educatedAt, ?, 1905), Lacroix et al. [11] randomly pick a year before 1905, say 1900, and evaluate whether a model can output the correct answer (University of Zurich) to the new query (Albert Einstein, educatedAt, ?, 1900). Clearly, there is no correct answer at all since he was born in 1879. Therefore, the evaluation on such test samples may not be plausible. In order to address these issues, for a closed interval-based sample, we enumerate all the time points in the interval and do evaluation on each time point separately. Then we use the average performance over them as the overall evaluation. For the latter, we only consider the known endpoint in an interval, namely (s, r, ?, o, st) for right-open cases and (s, r, ?, o, et) for left-open cases.
D  LINK PREDICTION PERFORMANCE BY TYPES OF VALIDITY INFORMATION

Table 8 shows the comparison between different methods in terms of different types of validity information.

| Method         | WIKIDATA12k | WIKIDATA114k |
|----------------|-------------|--------------|
|                | Time Interval (O) | Time Interval (C) | Time Instant | No Time |
| TimePLEX base  | 46.74       | 51.48        | 25.30        | -         |
| Time2BOX       | 30.29       | 38.99        | 18.94        | 18.35     |
|                | 68          | 48          | 94           | 125       |
|                | 32.47       | 18.62        | 11.75        | 16.34     |

Table 8: Link prediction evaluation by types of validity information. Time Interval (O) denotes left/right-open interval-based statements, and Time Interval (C) refers to closed interval-based statements.

E  TIME PREDICTION PERFORMANCE BY DURATION LENGTH

Table 9 and 10 compare the performance of TIMEPLEX and TIME2BOX on the time prediction task across different duration lengths on two datasets. Test samples are first classified into three groups by duration (du) and then evaluate the performance of each group. For an interval $I$, $du = I_{max} - I_{min} + 1$. It shows that our improvements are more pronounced in terms of shorter durations in general.

| Method         | WIKIDATA12k | WIKIDATA114k |
|----------------|-------------|--------------|
|                | du=1        | 1<du<=5      | du=5         |
| TimePLEX base  | 30.29       | 38.99        | 18.94        |
| Time2BOX       | 28.34       | 15.86        | 13.20        |
|                | 11.21       | 22.95        | 22.23        |
|                | 33.30       | 11.07        | 11.16        |

Table 9: Time prediction by duration on WIKIDATA12k

| Method         | WIKIDATA114k |
|----------------|--------------|
|                | du=1        | 1<du<=5      |
| TimePLEX base  | 28.75       | 37.03        |
| Time2BOX       | 25.80       | 34.16        |
|                | 10.50       | 14.50        |

Table 10: Time prediction by duration on WIKIDATA114k

F  MODEL PARAMETER COMPARISON

Table 11 summarizes the number of parameters used in each method.

| Models             | Number of parameters |
|--------------------|----------------------|
| TNTComplex         | $2d(|E| + |T| + 4|R|)$ |
| TIMEPLEX base      | $2d(|E| + |T| + |R|)$ |
| TIME2BOX           | $d(|E| + 2|T| + 2|R|) + 4d^2$ |

Table 11: Number of parameters for each model