Pion transverse momentum spectrum, elliptic flow and interferometry in the granular source model for RHIC and LHC heavy ion collisions

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We systematically investigate the pion transverse momentum spectrum, elliptic flow, and Hanbury-Brown-Twiss (HBT) interferometry in the granular source model for the heavy ion collisions of Au-Au at $\sqrt{s_{NN}} = 200$ GeV and Pb-Pb at $\sqrt{s_{NN}} = 2.76$ TeV with different centralities. The granular source model can well reproduce the experimental results of the heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). We examine the parameters involved in the granular source model. The experimental data of the momentum spectrum, elliptic flow, and HBT radii for the two collision energies and different centralities impose very strict constraints on the model parameters. They exhibit certain regularities for collision centrality and energy. The space-time structure and expansion velocities of the granular sources for the heavy ion collisions at the RHIC and LHC energies with different centralities are investigated.

PACS numbers: 25.75.-q, 25.75.Gz, 25.75.Ld

I. INTRODUCTION

The main purpose of relativistic heavy ion collisions is to probe the new matter, quark-gluon plasma (QGP), and study its properties. Because of the complexity of the nucleus-nucleus collisions, model investigation plays important roles in determining and characterizing the QGP. Single particle spectrum, elliptic flow, and two-particle Hanbury-Brown-Twiss (HBT) correlations are crucial final-state observables in relativistic heavy ion collisions \cite{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15}. They reflect the characteristics of the particle-emitting sources in different aspects at different stages. A combined investigation of these observables can provide very strong constraints for the source models. So far, much progress has been made in understanding the experimental data of the heavy ion collisions at the top energies of the Relativistic Heavy Ion Collider (RHIC) \cite{13,14}. However, more detailed investigations of the physics beneath the data through multi-observable analyses are still needed. On the other hand, the experimental data of several TeV heavy ion collisions at the Large Hadron Collider (LHC) have been recently published \cite{15,16,17,18,19,20,21,22,23,24,25,26,27,28,29}. It is an ambitious goal for the models to explain consistently the data of the heavy ion collisions at the RHIC and LHC energies.

In Refs. \cite{42,43}, W. N. Zhang et al. proposed and developed a granular source model of QGP droplets to explain the HBT data of the RHIC experiments \cite{3,4}. Their investigations indicate that the short evolution lifetime and wide initial distribution of the QGP droplets in the granular source can lead to the result of the HBT radii, $R_{\text{out}} \sim R_{\text{side}}$. Here the labels “out” and “side” denote the transverse directions parallel and perpendicular to the transverse momentum of the pion pair \cite{46,47}.

And, the granular source results of the pion transverse momentum spectrum \cite{43,44}, elliptic flow \cite{43}, and HBT radii \cite{43,44} are well in agreement with the experimental measurements for the Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV at the RHIC \cite{2,6,11,12}. They also find \cite{43,44} that the granular source model may reproduce the main characteristics of the two-pion source functions, extracted by the imaging techniques \cite{18,52}, in the RHIC experiments \cite{13,14}. In Refs. \cite{53,54}, the fluctuating signatures of the single-event HBT correlation functions of granular sources are investigated. The detection of source inhomogeneity through the fluctuating single-event HBT correlation functions is discussed in Ref. \cite{55}, with the smoothed particle hydrodynamics (SPH) \cite{56,57}. Recently, the HBT analyses in the granular source model for the experimental data of the most central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the LHC \cite{24} are performed \cite{58}. The model parameters of the granular sources for the most central collisions at the RHIC and LHC energies are compared and discussed \cite{58}.

Although the granular source model explained the pion HBT radii in the most central Au-Au and Pb-Pb collisions at the RHIC and the LHC respectively \cite{58}, it is still a challenge to models to explain consistently the experimental HBT measurements in the different centrality regions of the collisions at the RHIC and the LHC energies. On the other hand, pion momentum spectrum and elliptic flow are very sensitive to collision centrality. A combined investigation of pion momentum spectrum, elliptic flow, and HBT interferometry in the different centrality regions of the collisions at the RHIC and the LHC energies, in the granular source model, is of great interest. In this work, we systematically investigate the pion transverse momentum spectrum, elliptic flow and HBT interferometry in the granular source model for the heavy ion collisions at the RHIC and LHC energies with different centralities. By comparing the granular source results of pion transverse momentum spectrum, elliptic flow, and HBT radii with the experimental data of the
Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV with 0–5%, 10–20%, and 30–50% centralities, and the experimental data of the Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with 10–20% and 40–50% centralities, we obtain the model parameters as a function of collision centrality and energy. We investigate the space-time structure and expansion velocities of the granular sources at the RHIC and LHC energies with the different centralities. Our investigations indicate that the granular source model can reproduce the experimental data of the pion transverse momentum spectra, elliptic flow, and HBT radii of the Au-Au collisions at RHIC \[1, 2, 7, 12\] and the Pb-Pb collisions at LHC \[17, 22, 23\]. The parameters in the granular source model exhibit certain regularities for collision centrality and energy. The space-time structure and expansion velocities of the granular sources are consistent with that reflected by the observables.

The rest of this paper is arranged as follows. In Sec. II, we describe the basic ingredients of the granular source model used in this work. In Sec. III, we present the pion transverse momentum spectrum, elliptic flow and HBT results of the granular sources for the heavy ion collisions at the RHIC and LHC energies with different centralities. The regularities of the model parameters are also discussed in this section. In Sec. IV, we investigate the space-time structure and expansion velocities of the granular sources. Finally, the summary and discussions are given in Sec. V.

II. GRANULAR SOURCE MODEL

In the heavy ion collisions at the RHIC top energies and the LHC energy, the created strong-coupled QGP (sQGP) systems in the central rapidity region may reach local equilibrium at a very short time, and then expand rapidly along the beam direction (z-axis). Because of the random variations in the distribution of collision nucleons due to quantum fluctuations, the local equilibrium system is not uniform in the transverse plane (x-y plane) \[59\]. It may form many tubes along the beam direction during the subsequent fast longitudinal expansion and finally fragment into many QGP droplets with the effects of "sausage" instability, surface tension, and bulk viscosity \[13, 60, 62\]. As a first-step idealized approximation, granular source model regards the whole source evolution as the superposition of the individual evolutions of the QGP droplets. Each droplet has a position-dependent initial velocity and evolves hydrodynamically.

As in Ref. \[13\], we suppose the QGP droplets in the granular source initially distribute within a cylinder along z-axis by

$$\frac{dN_d}{dx_0dy_0dz_0} \propto \left[1 - e^{-\left(\frac{x_0^2+y_0^2}{\Delta R_T^2}\right)}\right] \theta(R_T - \rho_0) \times \theta(R_z - |z_0|),$$

where $\rho_0 = \sqrt{x_0^2 + y_0^2}$ and $z_0$ are the initial transverse and longitudinal coordinates of the droplet centers. The parameters $R_T$ and $R_z$ describe the initial sizes of the source, and $\Delta R_T$ is a shell parameter in the droplet frame \[43\].

In Ref. \[58\], the Bjorken hypothesis \[63\] is used to describe the longitudinal velocity of droplet for the most central collisions, and the transverse velocity of droplet has a form of exponential power. Considering the longitudinal velocity of droplet varying with collision centrality, we also introduce a longitudinal power parameter, which will be determined by experimental data, to describe the longitudinal velocity phenomenologically. The initial velocities of the droplets in granular source frame are assumed as \[13\]

$$v_{di} = \text{sign}(r_{0i}) \cdot a_i \left(\frac{|r_{0i}|}{R_i}\right)^b_i, \quad i = 1, 2, 3, \quad (2)$$

where $r_{0i}$ is $x_0$, $y_0$, or $z_0$ for $i = 1$, 2, or 3, and $\text{sign}(r_{0i})$ denotes the signal of $r_{0i}$, which ensures a outward droplet velocity. In Eq. (2), $R_i = (R_T, R_T, R_z)$, $a_i = (a_x, a_y, a_z)$ and $b_i = (b_x, b_y, b_z)$ are the magnitude and exponent parameters in $x$, $y$, and $z$ directions, which are associated with the early thermalization and pressure gradients of the system at the fragmentation. It is convenient to use the equivalent parameters $\pi_T = (\pi_T + a_y)/2$ and $\Delta a_T = a_x - a_y$ instead of $a_x$ and $a_y$. The parameters $\pi_T$ and $\Delta a_T$ describe the transverse expansion and asymmetric dynamical behavior of the system at the fragmentation, respectively. For simplicity, we take $b_x = b_y = b_T$ in calculations. The parameters $b_T$ and $b_z$ describe the coordinate dependence of exponential power in transverse and longitudinal directions.

In the calculations of the hydrodynamical evolution of the droplet, we use the equation of state (EOS) of the S95p-PEC165-v0 \[64\], which combines the lattice QCD data at high temperature with the hadron resonance gas at low temperature. We assume systems fragment when reaching a certain local energy density, and take the initial energy density of the droplets to be 2.2 GeV/fm$^3$ for all considered collisions for simplicity \[58\]. The initial droplet radius is supposed satisfying a Gaussian distribution with the standard deviation $\sigma_d = 2.5$ fm in the droplet local frame \[58\].

The final identical pions are considered to be emitted out of the surfaces of droplets with momenta obeying the Bose-Einstein distribution in the local frame at freeze-out temperature $T_f$. To include the resonance decayed pions later as well as the directly produced pions at chemical freeze out early, a wide region of $T_f$ is considered with the probability \[58\]

$$\frac{dP}{dT_f} \propto f_{\text{dir}} e^{-\frac{T_{\text{chem}} - T_f}{\Delta T_{\text{dir}}}} + (1 - f_{\text{dir}}) \times e^{-\frac{T_{\text{chem}} - T_f}{\Delta T_{\text{dec}}}}, \quad (T_{\text{chem}} > T_f > 80 \text{ MeV}), \quad (3)$$

where $f_{\text{dir}}$ is the fraction of the direct emission around the chemical freeze out temperature $T_{\text{chem}}$. $\Delta T_{\text{dir}}$ and $\Delta T_{\text{dec}}$ are the temperature widths for the direct and decay emissions, respectively. In the calculations, we take
f_{\text{dir}} = 0.75$, $\Delta T_{\text{dir}} = 10$ MeV, and $\Delta T_{\text{rec}} = 90$ MeV as in Ref. [58]. The value of $T_{\text{chem}}$ is taken to be 165 MeV as it be taken in the S95p-PCE165-v0 EOS [64].

After fixing the parameters used in the calculations of hydrodynamical evolution and freeze-out temperature, the free model parameters are the three source geometry parameters ($R_T$, $\Delta R_T$, $R_z$) and the five droplet velocity parameters ($\vec{v}$, $\Delta r$, $a_z$, $b_r$, $b_z$). They are associated with the initial size, expansion, and directional asymmetry of system, and have significant influence on the observables of pion momentum spectra, elliptic flow, and HBT radii. We will combine the experimental data of these observables to investigate the parameters of the granular source as a function of the collision centrality and energy in the heavy ion collisions at the RHIC and LHC energies in next sections.

III. PION MOMENTUM SPECTRUM, ELLIPTIC FLOW, AND HBT RESULTS

In high energy heavy ion collisions, the invariant momentum distribution of final particles can be written in the form of a Fourier series [62, 66],

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left[ 1 + \sum_n 2v_n \cos(n\phi) \right], \quad (4)$$

where $E$ is the energy of the particle, $p_T$ is the transverse momentum, $y$ is the rapidity (it should not bring a mistake with coordinate from the context), and $\phi$ is the azimuthal angle with respect to the reaction plane. In Eq. (4), the first term on right is the transverse momentum spectrum in the rapidity region $dy$, and the second harmonic coefficient $v_2$ in the summation is called elliptic flow.

At the RHIC and LHC energies, the spectators in nucleus-nucleus collisions depart from the reaction region rapidly after collision, and a very hot and dense fireball is formed in an almden shape perpendicular to the reaction plane. The particle spectra of transverse momentum at low $p_T$ ($p_T \leq 3$ GeV/$c$) contain the information about the transverse expansion and thermal properties of the particle-emitting sources at freeze-out temperature [13–19]. By comparing the pion transverse momentum spectra of the granular sources with experimental data, we can constrain the velocity parameters $\vec{v}$ and $a_z$ of the granular sources.

Choosing $x$ axis on the reaction plane, elliptic flow $v_2$ can be expressed as

$$v_2(p_T) = \langle \cos(2\phi) \rangle = \frac{\left\langle \frac{p_T^2 - \bar{p}_T^2}{p_T^2} \right\rangle}{\bar{p}_T^2}. \quad (5)$$

Since the reaction plane orientation is hardly to estimate exactly in experiment, an alternative technique for elliptic flow analysis is the measurement of the two-particle cumulant of azimuthal correlations, $v_2\{2\}$ [4, 6, 7, 20–22, 67], which gives essentially the same results as the reaction-plane method [4, 6, 7, 68].

In non-central nucleus-nucleus collisions, the space-asymmetry of the system can bring anisotropic pressure gradients, which lead to the anisotropic transverse-momentum distributions of final particles and nonzero $v_2$. For the granular source, the results of elliptic flow are very sensitive to the parameters $\Delta r$, $b_r$, and $b_z$ [43]. The experimental data of the transverse momentum spectrum and elliptic flow impose strict constraints on the velocity parameters in the granular source model.

Two-particle Hanbury-Brown-Twiss (HBT) correlation function is defined as the ratio of the two-particle momentum spectrum $P(p_1, p_2)$ to the product of two single-particle momentum spectra $P(p_1)P(p_2)$. It has been widely used to extract the space-time geometry, dynamic and coherence information of the particle-emitting source in high energy heavy ion collisions [65, 73]. In the usual HBT analysis in high energy heavy ion collisions, the two-pion correlation functions are fitted by the Gaussian parameterized formula

$$C(q_{\text{out}}, q_{\text{side}}, q_{\text{long}}) = 1 + \lambda e^{-R_{\text{out}}^2 q_{\text{out}}^2 - R_{\text{side}}^2 q_{\text{side}}^2 - R_{\text{long}}^2 q_{\text{long}}^2}. \quad (6)$$

where $q_{\text{out}}$, $q_{\text{side}}$, and $q_{\text{long}}$ are the Bertsch-Pratt variables [16, 17], which denote the components of the relative momentum $q = p_1 - p_2$ in transverse and side directions and in longitudinal direction, respectively. In Eq. (6) $\lambda$ is chaotic parameter of source, $R_{\text{out}}$, $R_{\text{side}}$, and $R_{\text{long}}$ are the HBT radii in out, side, and long directions. The results of HBT radii are related to the source geometry as well as expansion. We can finally determine the geometry parameters ($R_T$, $\Delta R_T$, $R_z$) and the velocity parameters ($\vec{v}$, $\Delta r$, $a_z$, $b_r$, $b_z$) in the granular source model by the multi-observable analyses of the pion spectrum, elliptic flow, and HBT interferometry.

In the multi-observable analyses, we choose the experimental data of the Au-Au collisions at $\sqrt{S_{\text{NN}}} = 200$ GeV at the RHIC [12] and the Pb-Pb collisions at $\sqrt{S_{\text{NN}}} = 2.76$ TeV at the LHC [17, 22, 23] for determining the geometry and velocity parameters of the granular sources. These experimental data provide the identical pion $p_T$ spectrum, elliptic flow, and HBT radii simultaneously in the same centrality regions (RHIC: 0–5%, 10–20% and 30–50%; LHC: 10–20% and 40–50%).

In Fig. 11 we plot the pion transverse momentum spectra (left panel) and elliptic flow (right panel) of the granular sources for the heavy ion collisions at the RHIC and LHC energies with different centralities. The experimental data of the pion transverse momentum spectra and elliptic flow of the Au-Au collisions with 0–5%, 10–20% and 30–50% centralities at $\sqrt{S_{\text{NN}}} = 200$ GeV at the RHIC [1, 2, 4], and the Pb-Pb collisions with 10–20% and 40–50% centralities at $\sqrt{S_{\text{NN}}} = 2.76$ TeV at the LHC [17, 22, 23] are shown simultaneously. Comply with the experimental measurements, we use the rapidity cuts $|y| < 0.1$ [1] and $|y| < 0.5$ [16, 17] in the calculations of the $p_T$
spectra of the granular sources at the RHIC energy and the LHC energy, respectively. In the calculations of the elliptic flow of the granular sources, the pseudorapidity cuts $|\eta| < 1.0$ and $|\eta| < 0.8$ are adopted as the same as in the experimental analyses at the RHIC [7] and LHC [22], respectively.

As shown in Fig. 1, the pion transverse momentum spectra of the granular sources agree with the experimental data with different centralities at the RHIC and LHC energies simultaneously. The spectra at the LHC energy exhibit clear up-warp at $p_T > 1.5$ GeV/c as compared to those at the RHIC energy. However, the results of elliptic flow at the LHC energy almost match the elliptic flow results at the RHIC energy with the same and near centrality regions. The results of elliptic flow of granular sources exhibit clear centrality dependence as the experimental data with. The $v_2$ results decrease with increasing collision centrality. At $p_T > 2.5$ GeV/c, the granular source results of elliptic flow are a little higher than those of experimental data. It reflects the limitations at high $p_T$ of the model based on hydrodynamical evolution.

In Fig. 2 we show the granular source results of the pion HBT radii and chaotic parameter as a function of the transverse momentum of pion pair, $k_T = |p_1 + p_2|/2$, obtained by Gaussian parameterized formula fit in the longitudinally comoving system (LCMS) [71–73]. The experimental data of STAR Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV [12] and ALICE Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [25], which have the same centralities as the experimental data of pion $p_T$ spectra and $v_2$ shown in Fig. 1 are also plotted for comparing. In the HBT analyses of the granular sources for the RHIC and LHC energies, we applied the rapidity cut $|y| < 0.5$ and the pseudorapidity cut $|\eta| < 0.8$ as the same in the experimental analyses of STAR [12] and ALICE [25], respectively.

From Fig. 2 it can be seen that the HBT radii $R_{out}$, $R_{side}$, and $R_{long}$ of the granular sources simultaneously agree with the experimental data at the RHIC and LHC energies. Both the transverse and longitudinal HBT radii increase with increasing collision centrality. At the LHC energy the results of the HBT radii $R_{out}$, $R_{side}$, and $R_{long}$ are larger than those at the RHIC energy, respectively. However, the results of the ratio of $R_{out}/R_{side}$ are always about 1 and independent of the collision centrality and energy. In experimental HBT measurements, many effects, such as the Coulomb interaction between the final particles, particle missing-identification, source coherence, etc., can influence the results of the chaotic parameter $\lambda$ [69–73]. Because these effects do not be considered in the granular source model, the $\lambda$ results of the granular sources are larger than the experimental data.

In Table I we present the values of the geometry and velocity parameters of the granular sources used in the multi-observable analyses. One can see that the value of $R_T$ for a certain collision centrality and energy is larger than that of $R_z$. So the initial geometry of the granular source is a short cylinder. The initial transverse and longitudinal sizes of the granular sources $R_T$ and $R_z$ increase with increasing collision centrality. The transverse shell parameter $\Delta R_T$ also increases with increasing collision centrality. Because $\Delta R_T \ll R_T$, the shell effect is small and the initial distributions of droplets are almost volume distribution. For 10–20% centrality, the values of the geometry parameters $R_T$ and $R_z$ of the granular source for the RHIC energy are smaller than that for the LHC energy, respectively. The QGP droplets in the granular sources initially distribute in larger transverse...
TABLE I: The geometry and velocity parameters of the granular sources.

| Centrality     | $R_{\text{out}}$ (fm) | $\Delta R_{\text{out}}$ (fm) | $R_{\text{side}}$ (fm) | $\Delta R_{\text{side}}$ (fm) | $\pi_T$ | $\Delta \pi_T$ | $a_z$ | $b_T$ | $b_z$ |
|----------------|------------------------|------------------------------|------------------------|------------------------------|--------|---------------|------|------|------|
| RHIC, 0-5%     | 5.8                    | 0.6                          | 3.9                    | 0.49                        | 0.066  | 0.593         | 0.76 | 0.13 |
| RHIC, 10-20%   | 4.5                    | 0.5                          | 2.9                    | 0.454                       | 0.115  | 0.593         | 0.56 | 0.11 |
| RHIC, 30-50%   | 2.5                    | 0.3                          | 0.5                    | 0.437                       | 0.156  | 0.593         | 0.37 | 0.06 |
| LHC, 10-20%    | 6.0                    | 0.9                          | 5.5                    | 0.431                       | 0.092  | 0.592         | 0.35 | 0.13 |
| LHC, 40-50%    | 2.5                    | 0.4                          | 1.8                    | 0.407                       | 0.131  | 0.590         | 0.23 | 0.03 |

and longitudinal regions for the more central and higher energy collisions.

In transverse direction, the velocity parameters $\pi_T$ and $b_T$ increases with increasing collision centrality. For a fixed $b_T$, the larger the parameter $\pi_T$, the larger the average transverse velocity of droplet is. Because the values of $b_T$ are less than one, the larger $b_T$, the smaller the average transverse velocity of droplet is, if $\pi_T$ is fixed. In longitudinal direction, the parameter $a_z$ is almost independent of collision centrality and energy. The values of the parameter $b_z$ are much smaller than that of $b_T$, while $b_z$ increases with increasing collision centrality as $b_T$. The large difference between the values of the transverse and longitudinal exponent parameters $b_T$ and $b_z$, and the different centrality dependence of $\pi_T$ and $a_z$ reflect the different dynamical behaviors in transverse and longitudinal directions in the heavy ion collisions at the RHIC and LHC energies.

In Fig. 2, we plot the droplet velocities $v_{d_T} = \pi_T (\rho/R_T)^b_T$ and $v_{d_z} = a_z (|z|/R_z)^b_z$ of the granular sources. One can see that the droplet transverse velocity of central collision is smaller than that of peripheral collision in the center region of the source, although the droplet transverse velocity of central collision is larger at the edge of the source. The average longitudinal velocity of droplet is larger than the average transverse velocity. In peripheral collisions, the droplet longitudinal velocity is almost a constant in the source. From Table I, it can be seen that the velocity parameter $\Delta \pi_T$ decreases with collision centrality. This leads to the increase of $v_2$ with decreasing collision centrality.

In Fig. 3, we plot the droplet transverse and longitudinal velocities $v_{d_T} = \pi_T (\rho/R_T)^b_T$ and $v_{d_z} = a_z (|z|/R_z)^b_z$. 

FIG. 2: (Color online) The results of pion HBT radii and chaotic parameter of the granular sources (open circle) with different centralities for the highest RHIC energy (left column) and LHC energy (right column). The solid circle, star, and solid diamond symbols in the left column are the STAR data of the Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV with 0–5%, 10–20% and 30–50% centralities respectively. The star and solid triangle-down symbols in the right column are the ALICE data of the Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with 10–20% and 40–50% centralities respectively.

FIG. 3: (Color online) The droplet transverse and longitudinal velocities $v_{d_T} = \pi_T (\rho/R_T)^b_T$ and $v_{d_z} = a_z (|z|/R_z)^b_z$. 

IV. GRANULAR SOURCE SPACE-TIME AND EXPANSION

In the granular source model, the results of single particle $p_T$ spectrum, elliptic flow, and HBT radii are strongly related to the source space-time and expansion properties. The successes of the granular source model in explaining the experimental data of pion $p_T$ spectrum, elliptic flow, and HBT radii in the heavy ion collisions at the RHIC and LHC inspire us to further study the granular source space-time and expansion features.

In Fig. 4, we show the pictures of the distributions of pion source points in one granular source event for the RHIC central collision. The $z$ region for the panels (a), (b), and (c) is $|z| < 1$ fm. The $y$ region for the panels (d), (e), and (f) is $|y| < 1$ fm. The time for the panels (a) and (d), (b) and (e), and (c) and (f) is $t = 4$, 8, and 12 fm/c, respectively. The exposure time for these pictures is 0.5 fm/c.

In Fig. 5, we plot the space-time distributions of the pion source points of the granular sources projected on $t$-$r_{out}$, $t$-$r_{side}$, and $t$-$r_{long}$ planes, for five hundred events for the heavy ion collisions at the RHIC energy with 0–5% and 30–50% centralities. Here, the dashed lines are the average values of $r_i$ for all of the source points, and the bullets are the average values of $r_i$ obtained from the same $t$ bins.

In the calculations the same rapidity cut $|y| < 0.5$ as in the experimental HBT analyses is used. We take the relative momentum cut $|q_i| < 100$ MeV/c for pion pairs because most of the contributions in HBT correlation functions come from the particle pairs with small relative momenta. One can see that the distributions for many events are smoothed. The widths of the $r_i$-distributions for the central collision are wider than those for the peripheral collision. In side and long di-
rections, the distributions are symmetric with respect to $r_i = 0$. However, one can observe a time-increased asymmetry for the distributions in out direction. It is because the coordinate-dependent source transverse expansion boosts particle momenta along the direction out of the source, and this coordinate-momentum correlation leads the result that the average angle between particle momentum and emitting coordinate trends to be smaller than isotropic emission. For the central and peripheral collisions, the average values of $r_{\text{out}}$ are 2.52 and 2.78 fm respectively. The effect of the coordinate-momentum correlation is larger for the peripheral collision because of the larger source transverse velocity that the peripheral collision with (will be seen in Fig. 7(a)).

In Fig. 6 we compare the space-time distributions of the pion source points of the granular sources for the RHIC and LHC collisions with 10–20% centrality. The dashed lines are the average values of $r_i$ and $t$ for all of the source points. The bullets are the average values of $r_i$ obtained from the same $t$ bins.

![Figure 6: (Color online) The source-point distributions of final identical pions in $r_i = t$ plane (i = out, side, long), for the granular sources of the RHIC and LHC collisions with 10–20% centrality. The dashed lines are the average values of $r_i$ and $t$ for all of the source points. The bullets are the average values of $r_i$ obtained from the same $t$ bins.](image)

Figure 6 (a) and (b) display the average transverse and longitudinal droplet velocities of the granular sources as a function of time at the RHIC energy. Figure 7 (c) and (d) display the average transverse and longitudinal droplet velocities of the granular sources as a function of time at the LHC energy. It can be seen that the average velocities increase with decreasing collision centrality at both the energies. The larger average transverse velocities of sources at the LHC energy are consistent with the results that the $p_T$ spectra at the LHC energy exhibit up-warp at larger $p_T$ as compared to those at the RHIC energy (see the left panel of Fig. 1). The differences of the transverse velocities for different centralities become small at the higher energy, and the differences of the longitudinal velocities for different centralities are larger at the LHC energy. In Fig. 7 (e) and (f) we present the average transverse and longitudinal droplet velocities for the granular sources at the RHIC and LHC energies. The average transverse and longitudinal droplet velocities increase with the decreasing collision centrality for both the energies. In our granular source model, the droplet evolution in the local frame is independent of the collision energy and centrality. However, the different droplet

![Figure 7: (Color online) (a)–(d) The average transverse and longitudinal velocities of the pion source points, $v_{\text{ST}}$ and $v_{\text{SL}}$, versus the emission time of the granular sources. (e) and (f) The average transverse and longitudinal droplet velocities $v_{\text{dT}}$ and $v_{\text{dL}}$ of the granular sources for the RHIC and LHC collisions.](image)
velocities lead to the difference between the average emission time in the source center-of-mass frame because of the different Lorentz time delays. The larger the droplet velocity the larger the emission time is. For example, the average emission time of the granular sources for the RHIC central and peripheral collisions are 8.39 and 9.11 fm/$c$, respectively (see Fig. 5). The average emission time for the granular source for LHC peripheral collision is the largest because the average droplet velocity is the largest in this case (see Fig. 7(e)).

In the two-pion interferometry in high energy heavy ion collisions, the difference between the transverse HBT radii $R_{out}$ and $R_{side}$ satisfies [71, 72, 73],

$$R_{out}^2(k_T) - R_{side}^2(k_T) \approx \left[\langle \beta_T \tilde{r}^2 \rangle - 2\langle \beta_T \tilde{r}_{out} \tilde{t} \rangle \right](k_T),$$

where $\beta_T = |p_{1T} + p_{2T}|/(E_1 + E_2)$ is the transverse velocity of the pair, $\tilde{t} = t - \langle t \rangle$ and $\tilde{r}_{out} = t_{out} - \langle t_{out} \rangle$ are the deviations of source time and coordinate $r_{out}$ from their averages, respectively.

![FIG. 8: (Color online) The granular source $\langle \beta_T^2 \tilde{r}^2 \rangle$, $2\langle \beta_T \tilde{r}_{out} \tilde{t} \rangle$, $\beta_T$, and $\beta_L$ versus $k_T$ for the RHIC and LHC collisions.](image)

In Fig. 8 we plot $\langle \beta_T^2 \tilde{r}^2 \rangle$, $2\langle \beta_T \tilde{r}_{out} \tilde{t} \rangle$, and the average transverse and longitudinal velocities of the pion pair as a function of $k_T$. It can be seen that $\langle \beta_T^2 \tilde{r}^2 \rangle$ and $2\langle \beta_T \tilde{r}_{out} \tilde{t} \rangle$ increase as $k_T$ increases. At small $k_T$, the values of $\langle \beta_T^2 \tilde{r}^2 \rangle$ are larger than those of $2\langle \beta_T \tilde{r}_{out} \tilde{t} \rangle$. But at large $k_T$, the values of $2\langle \beta_T \tilde{r}_{out} \tilde{t} \rangle$ are close even larger than the corresponding results of $\langle \beta_T^2 \tilde{r}^2 \rangle$. Both $\langle \beta_T^2 \tilde{r}^2 \rangle$ and $2\langle \beta_T \tilde{r}_{out} \tilde{t} \rangle$ decrease with collision centrality. The near values of $\langle \beta_T^2 \tilde{r}^2 \rangle$ and $2\langle \beta_T \tilde{r}_{out} \tilde{t} \rangle$ at a fixed $k_T$ lead to the HBT results $R_{out}/R_{side} \approx 1$ for the granular sources. From Fig. 8(e) and (f) one can see that the average transverse velocities of the pair at the RHIC energy are higher than those at the LHC energy at large $k_T$. However, the results of the average longitudinal velocities are opposite. For a fixed $k_T$, a larger $\beta_T$ means a larger $k_L$, and therefore a larger $E_k (E_k^2 = k_T^2 + k_L^2 + m_0^2)$ and smaller $\beta_T$. It will be seen that the reason for the larger $\beta_L$ at large $k_T$ at the LHC energy is the larger longitudinal expansion of the granular source at the LHC energy as compared to that at the RHIC energy (see Fig. 9(c) and (d)).

![FIG. 9: (Color online) The $k_T$ dependence of the average emission time $\tilde{t} = \langle t \rangle(k_T)$, the time variance root $\Delta t = [(\tilde{t}^2)^{1/2}]$, and the average source transverse and longitudinal velocities $\langle \bar{v}_{ST} \rangle$ and $\langle \bar{v}_{SL} \rangle$ for the granular sources at the RHIC and LHC energies.](image)

In Fig. 9 we show the average emission time $\tilde{t} = \langle t \rangle(k_T)$, the time variance root $\Delta t = [(\tilde{t}^2)^{1/2}(k_T)]$, and the average source transverse and longitudinal velocities $\langle \bar{v}_{ST} \rangle$ and $\langle \bar{v}_{SL} \rangle$ for the granular sources at the RHIC and LHC energies. The error bars for the results are statistical errors. One can see that the average emission time decreases with increasing collision centrality and increases with increasing collision energy. It is because that the average droplet velocity increases with decreasing collision centrality and increasing collision energy. The larger the droplet velocity, the larger time delay is. The values of $\Delta t$, which is also referred as the source lifetime, are almost independent of $k_T$ and smaller as compared to the values of $\tilde{t}$. For a fixed $k_T$, the average source velocities increase with decreasing collision centrality. At large $k_T$, the average longitudinal velocities of the granular sources at the LHC energy are larger than the corresponding values at the RHIC energy. It is because that the pion pairs with larger $k_T$ at the LHC energy correspond to a larger average emission time in the source center-of-mass frame (see Fig. 9(b)), and
therefore have larger average longitudinal source velocities as compared to those at the RHIC energy (see Fig. 7(b) and (d)). This result is consistent with the results of $\beta_L$ shown in Fig. 5(e) and (f). The large longitudinal velocity of the granular source at the LHC energy boosts strongly the pair momentum of the particles with almost the same emission direction, and leads to the large $\beta_L$ results.

Finally, it should be noted that the emission time mentioned in the paper is the time recorded from the initial state of the granular source. The real emission time from the beginning of collision should also plus the system pre-equilibrium time $\tau_0$ and the breakup time $t_0$, which should be different for different collision energies [58] and centralities. However, the lifetime of the granular source is independent of the time original point. The small source lifetime is a character of the granular source [42, 43, 58, 74, 75].

V. SUMMARY AND DISCUSSION

We systematically investigate the pion transverse momentum spectrum, elliptic flow, and HBT interferometry in the granular source model for the heavy ion collisions at the RHIC highest energy and the LHC energy. The centrality and energy dependence of the observables at the two energies are examined. By comparing the granular source results with the experimental data of the Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV at the RHIC and the Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the LHC with different collision centralities, we investigate the geometry and velocity parameters in the granular source model as a function of collision centrality and energy. The space-time structure and expansion velocities of the granular sources at the RHIC and LHC energies with different centralities are examined. Our investigations indicate that the granular source model can well reproduce the experimental data of pion transverse momentum spectra, elliptic flow, and HBT radii in the Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV with 0–5%, 10–20%, and 30–50% centralities [1, 2, 7, 12], and in the Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with 10–20% and 40–50% centralities [17, 22, 27]. The experimental data of pion momentum spectra, elliptic flow, and HBT radii impose very strict constraints on the parameters in the granular source model. They exhibit certain regularities for collision centrality and energy. The space-time structure and expansion velocities of the granular source are consistent with that reflected by the observables.

In the granular source model we assume that the system created in the ultrarelativistic heavy ion collisions occurs fragmentation and forms the granular source of QGP droplets due to the dynamical instability in the fast expansion at the early stage and the surface tension of the strongly coupled QGP. We use ideal hydrodynamics to describe the droplet evolution and assume a Gaussian distribution for the droplet radius for simplicity. Because many droplets evolve simultaneously, the source lifetime is smaller as compared to that of a continued big source, which evolves in whole and freeze out from the source surface. For the granular source, the distribution of particle-emitting points of single event presents a clump-structure during the source evolution. However, the distribution of source points for many events is continued and presents a volume distribution because of the contribution from the droplets in the central region of the source. The short source lifetime, clump-structure of source points distribution of single event, and volume distribution of source points for many events are the characters of the granular source.

The investigations for the granular source parameters indicate that the QGP droplets initially distribute in larger transverse and longitudinal regions for the more central and higher energy collisions. So, the distribution width of source points increases with increasing collision centrality and energy. In transverse direction, the droplet velocity of central collision is smaller than that of peripheral collision in the center region of the source, although the droplet transverse velocity of central collision is larger at the edge of the source. The average longitudinal velocity of droplet is larger than the average transverse velocity of droplet. The droplet longitudinal velocity increases with decreasing collision centrality. The larger droplet velocities in the peripheral collisions lead to larger average velocities of source. Because the difference of the source transverse velocities in and out of reaction plane decreases with collision centrality, the elliptic flow decreasing with collision centrality. In HBT interferometry, the difference between the transverse HBT radii $R_{\text{out}}$ and $R_{\text{side}}$ is related to the transverse velocity of particle pair, the source lifetime, and the space-time correlation of source points. Both the quantities $\langle \beta_T T \rangle$ and $2R_{\text{out}}^2$ increase with the decreasing collision centrality and increasing transverse momentum of the pair, $k_T$. However, the difference of the two quantities for a fixed $k_T$ is approximately equal to zero. This leads to the results of $R_{\text{out}}(k_T) \approx R_{\text{side}}(k_T)$.

While one may argue on the details and the number of parameters used in the granular source model, the consistent explanation of a large number of measured one-particle and two-particle correlation quantities suggests that this model description captures some fundamental features of the space-time dynamics. After all, it is a challenge to a model to explain the experimental data of the momentum spectra, elliptic flow, and HBT radii simultaneously at the RHIC and LHC energies, and the final criterion for a model is experiments. It will be of interest to improve the granular source model and investigate the effects of QGP viscosity and droplet interaction on the granular source parameters. On the other hand, the studies of the forming mechanism and signals of the granular source will be of interest.
Acknowledgments

We thank Dr. L. Cheng, Dr. U. Heinz, Dr. S. Jeon, and Dr. H. C. Song for helpful discussions. This research was supported by the National Natural Science Foundation of China under Grant No. 11275037.

[1] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 92, 112301 (2004).
[2] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. C 69, 034909 (2004).
[3] B. B. Back et al. (PHOBOS Collaboration), Phys. Rev. C 75, 024910 (2007).
[4] C. Adler et al. (STAR Collaboration), Phys. Rev. C 66, 034904 (2002).
[5] K. Adcox et al. (PHENIX Collaboration), Phys. Rev. Lett. 89, 212301 (2002).
[6] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 91, 182301 (2003).
[7] J. Adams et al. (STAR Collaboration), Phys. Rev. C 72, 014904 (2005).
[8] S. Afanasiev et al. (PHENIX Collaboration), Phys. Rev. C 80, 024909 (2009).
[9] C. Adler et al. (STAR Collaboration), Phys. Rev. Lett. 87, 082301 (2001).
[10] K. Adcox et al. (PHENIX Collaboration), Phys. Rev. Lett. 98, 192302 (2002).
[11] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 93, 152302 (2004).
[12] J. Adams et al. (STAR Collaboration), Phys. Rev. C 71, 044906 (2005).
[13] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 98, 132301 (2007).
[14] S. Afanasiev et al. (PHENIX Collaboration), Phys. Rev. Lett. 100, 232301 (2008).
[15] M. Floris (ALICE Collaboration), J. Phys. G 38, 124025 (2011).
[16] R. Pregenella (ALICE Collaboration), Acta Phys. Polon. B 43, 555 (2012); [arXiv:1111.7080].
[17] B. Abelev et al. (ALICE Collaboration), Phys. Rev. C 88, 044910 (2013).
[18] B. Abelev et al. (ALICE Collaboration), Phys. Rev. Lett. 109, 252301 (2012).
[19] B. Abelev et al. (ALICE Collaboration), Phys. Lett. B 720, 52 (2013).
[20] K. Aamodt et al. (ALICE Collaboration), Phys. Rev. Lett. 105, 252302 (2010).
[21] K. Aamodt et al. (ALICE Collaboration), Phys. Rev. Lett. 107, 032301 (2011).
[22] R. Snellings (ALICE Collaboration), J Phys. G 38, 124013 (2011).
[23] M. Krzewicki (ALICE Collaboration), J Phys. G 38, 124047 (2011).
[24] K. Aamodt et al. (ALICE Collaboration), Phys. Lett. B 696, 328 (2011).
[25] A. Kisiel on behalf of ALICE Collaboration, talk at The Seventh Workshop on Particle Correlations and Femtoscopy, September 20-24, 2011, Tokyo, Japan; PoS (WPCF2011) 003.
[26] Bao-Chun Li, Yuan-Yuan Fu, Li-Li Wang, Fu-Hu Liu, Advances in High Energy Physics, 2013, 908046 (2013).
[27] Yu. M. Sinyukov, S. V. Akkelin, Iu. A. Karpenko, V. M. Shapoval, Advances in High Energy Physics, 2013, 198928 (2013).
[28] Fu-Hu Liu, Ya-Hui Chen, Hua-Rong Wei, and Bao-Chun Li, Advances in High Energy Physics, 2013, 965735 (2013).
[29] I. Arsene et al. (BRAHMS Collaboration), Nucl. Phys. A 757, 1 (2005); B. B. Back et al. (PHOBOS Collaboration), ibid., 28; J. Adams et al. (STAR Collaboration), ibid., 102; K. Adcox et al. (PHENIX Collaboration), ibid., 184.
[30] J Lajoie (PHENIX Collaboration), J. Phys. G 34, S191 (2007); L. Ruan (STAR Collaboration), ibid., S199; I. G. Bearden (BRAHMS Collaboration), ibid., S207; D. J. Hofman (PHOBOS Collaboration), ibid., S217; M. C. de la Barca (STAR Collaboration), ibid., S225.
[31] J. P. Blaizot, J. Phys. G 34, S243 (2007); L. McLerran, ibid., S583.
[32] A. Franz (PHENIX Collaboration), J. Phys. G 35, 104002 (2008); R. Debbe (BRAHMS Collaboration), ibid., 104004; B. Wosiek (PHOBOS Collaboration), ibid., 104005; B. Mohanty (STAR Collaboration), ibid., 104006; T. C. Awes (PHENIX Collaboration), ibid., 104007.
[33] L. McLerran, J. Phys. G 35, 104001 (2008); R. Venugopalan, ibid., 104003; R. K. Seto, ibid., 104043; E. Shuryak, ibid., 104044.
[34] W. A. Zajc, Nucl. Phys A 830, 3c (2009).
[35] S. Bathe (PHENIX Collaboration), J. Phys. G 38, 124001 (2011). H. Masui (STAR Collaboration), ibid., 124002; S. Esumi (PHENIX Collaboration), ibid., 124006; P. Sorenson (STAR Collaboration), ibid., 124029.
[36] B. Schenke, J. Phys. G 38, 124009 (2011); F. Antinori, ibid., 124038.
[37] U. A. Wiedemann, Nucl. Phys A 904-905, 3c (2013); B. Hippolyte, D. H. Rischke, ibid., 318c.
[38] K. Aamodt et al. (ALICE Collaboration), Phys. Rev. Lett. 106, 032301 (2011).
[39] J. Velkovska (CMS collaboration), J. Phys. G 38, 124011 (2011).
[40] K. Aamodt et al. (ALICE Collaboration), Phys. Lett. B 708, 249 (2012).
[41] K. Šafrářík (ALICE Collaboration), Nucl. Phys. A 904-905, 27c (2013); B. Wosiek (ATLAS Collaboration), ibid., 35c (2013); S. J. Sanders (CMS Collaboration), ibid., 98c (2013).
[42] W. N. Zhang, M. J. Efaaf, and C. Y. Wong, Phys. Rev. C 70, 024903 (2004).
[43] W. N. Zhang, Y. Y. Ren, and C. Y. Wong, Phys. Rev. C 74, 024908 (2006).
[44] W. N. Zhang, Z. T. Yang, and Y. Y. Ren, Phys. Rev. C 80, 044908 (2009).
[45] Z. T. Yang, W. N. Zhang, L. Huo, and J. B. Zhang, J. Phys. G 36, 015113 (2009).
[46] G. Bertsch, M. Gong, and M. Tohyama, Phys. Rev. C 37, 1896 (1988); G. Bertsch, Nucl. Phys. A 498, 173c
(1989).

[47] S. Pratt, T. Csörgo, and J. Zimányi, Phys. Rev. C 42, 2646 (1990).

[48] D. A. Brown and P. Danielewicz, Phys. Lett. B 398, 252 (1997).

[49] D. A. Brown and P. Danielewicz, Phys. Rev. C 57, 2474 (1998).

[50] D. A. Brown and P. Danielewicz, Phys. Rev. C 64, 014902 (2001).

[51] P. Danielewicz and S. Pratt, Phys. Lett. B 618, 60 (2005).

[52] P. Danielewicz and S. Pratt, Phys. Rev. C 75, 034907 (2007).

[53] C. Y. Wong and W. N. Zhang, Phys. Rev. C 70, 064904 (2004).

[54] W. N. Zhang, S. X. Li, C. Y. Wong, and M. J. Efaaf, Phys. Rev. C 71, 064908 (2005).

[55] Y. Y. Ren, W. N. Zhang, and J. L. Liu, Phys. Lett. B 669, 317 (2008).

[56] C. E. Aguiar, T. Kodama, T. Osada, Y. Hama, J. Phys. G 27, 75 (2001).

[57] Y. Hama, T. Kodama, O. Socolowski Jr, Braz. J. Phys. 35, 24 (2005); arXiv:hep-ph/0407264.

[58] W. N. Zhang, H. J. Yin, and Y. Y. Ren, Chin. Phys. Lett. 28, 122501 (2011).

[59] A. Adare, M. Luzum, H. Petersen, Phys. Scripta 87, 048001 (2013); arXiv:1212.5888 [nucl-th]

[60] J. Takahashi et al., Phys. Rev. Lett. 103, 242301 (2009).

[61] C. Y. Wong, Ann. Phys. 77, 279 (1973).

[62] G. Torrieri, B. Tomášik, I. Mishustin, Phys. Rev. C 77, 034903 (2008).

[63] D. J. Bjorken, Phys. Rev. D 27, 140 (1983).

[64] C. Shen, U. Heinz, P. Huovinen, and H. C. Song, Phys. Rev. C 82, 054904 (2010).

[65] S. Voloshin and Y. Zhang, Z. Phys. C 70, 665 (1996).

[66] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C 58, 1671 (1998).

[67] N. Borghini, P. M. Dinh, J. Y. Ollitrault, Phys. Rev. C 64, 054901 (2001).

[68] R. S. Bhalerao, J. Y. Ollitrault, Phys. Lett. B 641, 260 (2006).

[69] M. Gyulassy, S. K. Kauffmann, and Lance W. Wilson, Phys. Rev. C 20, 2267 (1979).

[70] C. Y. Wong, Introduction to High-Energy Heavy-Ion Collisions (World Scientific, Singapore, 1994), Chap. 17.

[71] U. A. Wienemann and U. Heinz, Phys. Rep 319, 145 (1999).

[72] R. M. Weiner, Phys. Rep 327, 249 (2000).

[73] M. A. Lisa, S. Pratt, R. Soltz, U. Wiedemann, Annu. Rev. Nucl. Part. Sci 55, 357 (2005).

[74] W. N. Zhang and C. Y. Wong, Int. J. Mod. Phys. E 16, 3262 (2007).

[75] Wei-Ning Zhang, talk at The Seventh Workshop on Particle Correlations and Femtoscopy, September 20-24, 2011, Tokyo, Japan; PoS (WPCF2011) 051.

[76] M. Herrmann, G. F. Bertsch, Phys. Rev. C 51, 328 (1995).

[77] S. Chapman, P. Scotto, U. Heinz, Phys. Rev. Lett. 74, 4400 (1995).