An optimized model of electricity price forecasting in the electricity market based on fuzzy time series

Behrooz Safarinejad*, Masihollah Gharibzadeh and Mohsen Rakhshan

Control Engineering Department, Shiraz University of Technology, Modarres Blvd., PO Box 71555-313, Shiraz, Iran

(Received 2 June 2014; final version received 25 September 2014)

Electricity price forecasting in the electricity market is one of the important purposes for improving the performance of market players and increasing their profits in a competitive electricity market. Since the system load is one of the important factors affecting electricity price changes, a two-factorial model based on fuzzy time series is presented in this paper for electricity price forecasting using the electricity prices of the previous days and the system load. In the proposed method, price and system load time series are fuzzified by fuzzy sets created based on the fuzzy C-means clustering algorithm. After determining proposed model coefficients by the Teaching–Learning-Based Optimization algorithm, this model is used for forecasting the next day electricity price. The promising performance of the proposed model is examined using Australia and Singapore electricity markets data.

Keywords: electricity market; price forecasting; fuzzy time series; teaching–learning-based optimization algorithm

1. Introduction

One of the major goals of restructuring and the establishment of the electricity market in many countries is to create a competitive environment in order to achieve goals such as increase in efficiency and investment encouragement. Within the new structure, electrical energy price is determined based on the proposed price of the producers and the consumers in the market. In many countries, the electricity market is usually held by the combination of the pool market and the mutual contracts market (Amjadi & Heemati, 2006).

In the pool market, the suggestions of the producers are sorted from the lowest price to the highest price and the requests of the consumers are sorted from the highest price to the lowest price. Then, the intersection of these two curves indicates the market clearing price. Forecasting the prices can be used for various purposes. Based on the applications, one can classify the price forecasting in three categories, short-term forecasting (for a few days), mean-term forecasting (for several months) and long-term forecasting (for several years) (Kirschen & Strbac, 2004; Shahidehpour, Yamin, & Li, 2002). Long-term forecasting is usually used for reducing the risk of investment, production development planning and other similar purposes. Mean-term forecasting is usually employed for repair planning of the production units and also water and heating units. One can use short-term forecasting for bidding strategy in the market. In comparison with the price of other goods, the electricity energy price curve has some properties such as non-stationary mean and variance and high frequency. Moreover, seasonal changes and the calendric time cause severe abnormal changes in the price that makes the forecasting harder. These properties of electricity energy price forecasting could be the result of non-storable property of electrical energy at the macro level, necessity of equilibrium between the offer and demand for stability of the system at each moment, the effect of transmission lines on the exchanges, the low elasticity of the loads over the short term, the sudden errors in the network and the production system and the effect of the atmospheric conditions on changes in the loads and the market power. In general, one can divide the electricity price forecasting into three general categories (Aggarwal, Saini, & Kumar, 2009):

- the game theory,
- the simulation approach and
- the time series approach.

In the game theory, the competition between the producers for selling the electricity and maximizing their own profits can be modelled as a game, in which electricity price will be estimated from the result of the game (Ventosa, Baillo, Ramos, & Rivier, 2005). Bertrand models, supply function equilibrium and Cournot are the three main models used for the electricity market based on the level of competition in the market (Day, Hobbs, & Pang, 2002).

*Corresponding author. Email: safarinejad@sutech.ac.ir

© 2014 The Author(s). Published by Taylor & Francis. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. The moral rights of the named author(s) have been asserted.
In the simulation approach, one creates a model of the system to calculate the price of the electricity based on the production cost using an optimal distribution of the load (Bastian, Zhu, Banunaryan, & Mukherji, 1999; Kavousi-Fard, Niknam, & Golmaryami, 2014). This method has high accuracy due to considering various factors and imitates the actual power flows in the system. The main disadvantage of this method is that the algorithm requires extensive data entry. Moreover, the calculations are highly complex. In the time series approach, the price curves of the past are used to model and forecast the price (Nogales, Contreras, Conejo, & Espinola, 2002).

Recently, intelligent approaches, such as artificial neural networks (ANN), have been used to forecast the energy price. This application comes from their abilities in data forecasting and pattern recognition. These two abilities are used to predict the load and find the load pattern of the system (Ramos, Expósito, Santos, Lora, & Guerra, 2002; Troncoso, Riquelme, & Riquelme, 2002). ANN process the input data and the output data. In our case study input data are the past prices of the electricity and the other factors like load of the system which effect on the price of electricity. Moreover, the output in our case study is the pattern of the electricity price. Rodriguez and Anders (2004) proposed a new approach by combining the ANN and fuzzy logic is proposed.

Weighted nearest neighbours (WNN) algorithms (Dasarathy, 1991; Dudani, 1975) are methods which are mainly using for pattern classification. These methods use the similarity of the properties of a population. The members of the cluster have similar properties. The proposed idea is used to train the WNN classifier. Most statistical methods intricate a model from the information of the database, but, in the WNN approach the training set is considered as a model itself.

Lately, many researchers are using the WNN method not only for pattern recognition, but also in new environments such as game theory, expert systems or time series forecasting. Several researchers have used these approaches in their papers to predict the next-day hourly energy consumption (Troncoso, Riquelme, Martínez, Riquelme, & Gómez, 2003; Troncoso, Riquelme, Riquelme, Gómez, & Martínez, 2004), and they had promising results.

WNN algorithm and time series method have been used together to predict the electricity price with a competitive result in Lora, Santos, Expósito, Ramos, and Santos (2007).

In cases where sufficient data from the production and system are not accessible, the past information is useful for price forecasting. Autoregressive integrated moving average linear model (Conejo, Plazas, Espinola, & Molina, 2005) and the nonlinear neural network models (Singhal & Swarup, 2011) and fuzzy-neural networks (Wei, 2011) are the main approaches for forecasting the time series. One of the disadvantages of these approaches is that, when the past data contain linguistic variables, they lose their efficiency. For fixing this issue, Song and Chissom (1993) proposed the fuzzy time series based on which, Chen (1996) presented a model for forecasting the number of registrations in Alabama University. Later, many researchers modified this method and presented many papers to study the application of the proposed method. For example, the reader can study (Chen & Hwang, 2000) for temperature forecasting. (Chu, Chen, Cheng, & Huang, 2009) for forecasting the stock index and (Liu, Bai, Fang, & Luo, 2010) for load forecasting.

In this paper, we present a model based on fuzzy time series for forecasting the electricity price in the electricity market. Since the load of the system effects the price of electricity, in the proposed model, in addition to the past day’s electricity price, we consider the load of the system as an important factor for forecasting the price.

In the proposed model, forecasting the next day’s price is based on fuzzifying the time series of the electricity price of the previous days and also the system load. Fuzzy C-means clustering algorithm is used to determine the fuzzy sets. The coefficients of the proposed model are also optimized using Teaching–Learning-Based Optimization (TLBO) algorithm.

The main contributions of this work include: (1) presenting a model based on fuzzy time series for forecasting electricity price in the electricity market. (2) Considering the load of the system as an important factor for forecasting the price. (3) Using fuzzy C-means clustering algorithm to determine the fuzzy sets. (4) Determining the coefficients of the proposed model using TLBO algorithm.

The rest of this paper is organized as follows. In Section 2, the primary concepts of the fuzzy time series are presented. In Section 3, fuzzy C-means clustering algorithm is investigated. In Section 4, TLBO algorithm is presented. Section 5 investigates the proposed approach of the paper. In Section 6, simulation examples are given to illustrate the merits of the proposed approach for the Australia and Singapore electricity markets. In Section 7, a conclusion is drawn.

### 2. Fuzzy time series

In this section, the concept of fuzzy time series is presented in brief (Qiu, Liu, & Li, 2011). Suppose that $U$ is the space of our study, where $U = \{u_1, u_2, \ldots, u_n\}$. The fuzzy set $A_i$ in the space $U$ is defined as follows:

$$
A_i = \frac{f_{Ai}(u_1)}{u_1} + \frac{f_{Ai}(u_2)}{u_2} + \ldots + \frac{f_{Ai}(u_n)}{u_n},
$$

(1)

In the above equation, $f_{Ai}$ is the membership function of the fuzzy set $A_i$, $f_{Ai} : U \rightarrow [0, 1]$ and $f_{Ai}(u_k)$ is the membership grade of the linguistic variable $u_k$ to the fuzzy set $A_i$ where $f_{Ai}(u_k) \in [0, 1]; 1 \leq k \leq n$.

**DEFINITION 1** Suppose that $Y(t)(t = \ldots, 0, 1, 2, \ldots)$ is a subset of the real numbers and also the space of the
study with the fuzzy sets \( f_1(t) \). When \( F(t) \) is a set of \( f_1(t), f_2(t), \ldots \) then \( F(t) \) is a fuzzy time series on \( Y(t) \).

**Definition 2** Suppose that \( F(t) \) is made by \( F(t) = F(t - 1) \). This dependency is shown as \( F(t) = F(t - 1) \circ R(t, t - 1) \), where \( R(t, t - 1) \) is a fuzzy relation between \( F(t) \) and \( F(t - 1) \) and “\( \circ \)” is the fuzzy operator. This model is the first-order model of \( F(t) \).

**Definition 3** If \( F(t) \) is made by \( F(t - 1) \) and \( R(t, t - 1) \) is constant for each \( t \), \( F(t) \) is a time invariant fuzzy time series. Otherwise, it is called time-varying fuzzy time series.

**Definition 4** The fuzzy relations of data points are classified according to the same time index. For example, the fuzzy relations \( A_1 \rightarrow A_2, A_1 \rightarrow A_3 \), and \( A_1 \rightarrow A_4 \) are classified in the \( A_1 \rightarrow A_2, A_1 \rightarrow A_3, A_1 \rightarrow A_4 \) category.

### 3. Fuzzy C-means clustering algorithm

Fuzzy C-means clustering algorithm is an iterative algorithm for clustering the data (Niknam, Taherian Fard, Ehrampoosh, & Rousta, 2011; Wang, 1997). The objective of the algorithm is to determine the centres of the clusters and the membership degree of each data to each cluster, to minimize the following function:

\[
J_m(U, V) = \sum_{k=1}^{n} \sum_{i=1}^{c} (u_{ik})^m \| x_k - v_i \|^2, \tag{2}
\]

In the above equation, \( c \) is the number of clusters, \( n \) is the number of data points, \( U = [u_{ik}] \) is the degree of membership matrix and \( V = [v_1, \ldots, v_c] \) is the cluster centres vector and \( m \) is a constant scalar. Fuzzy C-means clustering algorithm has the following steps:

**Step 1** Determine the initial values for membership degree matrix \( U^{[0]} \).

**Step 2** Determine the cluster centres using the following equation:

\[
v_i = \frac{\sum_{k=1}^{n} (u_{ik})^m \cdot x_k}{\sum_{k=1}^{n} (u_{ik})^m}, 1 \leq i \leq c \tag{3}
\]

**Step 3** Update the membership degree matrix using the following equation:

\[
u_{ik} = \frac{1}{\sum_{j=1}^{c} \left(\| x_k - v_i \| / \| x_k - v_j \| \right)^{2/(m-1)}},
\]

\[1 \leq i \leq c, \quad 1 \leq k \leq n \tag{4}
\]

**Step 4** If the changes in the membership degree matrix in each iteration become smaller than a threshold, the algorithm stops, otherwise, we return to **Step 2**.

### 4. TLBO algorithm

These algorithms are derived from the process of teaching and learning in the classroom (Azizipanah-Abarghooei, Niknam, Bavafa, & Zare, 2014; Rao, Savsani, & Vakharia, 2012; Shabanpour-Haghighi, Seifi, & Niknam, 2014). In the class, the teacher is trying to increase the mean knowledge of the students through education. Moreover, the students are trying to increase their knowledge through the exchange of information with each other. TLBO algorithm models these two processes. This algorithm is described as follows:

First, we produce some initial population in the permissible region and the point with the best answer will be selected as a teacher and the other points are considered as students. Now the process of learning begins and repeats until convergence. These two phases are modelled as follows.

#### 4.1. Teacher phase

As we mentioned before, the learning process of the teacher is done to increase the mean knowledge of the class. Therefore, the teaching process can be modelled as follows:

\[
X_{\text{new}} = X_{\text{old}} + r(X_{\text{teacher}} - T_F \times X_{\text{mean}}), \tag{5}
\]

where \( X_{\text{old}} \) and \( X_{\text{new}} \) are the student status before the teaching process and the student status after the teaching process, respectively. \( X_{\text{teacher}} \) is the teacher status, \( X_{\text{mean}} \) is the mean status of the class and \( r \) is a random number between 0 and 1. Moreover, \( T_F \) is a random number which has a value of 1 or 2 that indicates the learning rate of a student. This parameter can be calculated using the following equation:

\[
T_F = \text{round}[1 + \text{rand}(0, 1)]. \tag{6}
\]

Equation (5) expresses that each student will learn something from the teacher and the knowledge of the student will increase. Therefore, in each iteration the students find a better place and improve their former positions.

#### 4.2. Student phase

In this phase, each student exchanges the knowledge with a random student. The student with a higher knowledge teaches the other. This process is modelled as follows.

\[
\begin{align*}
X_{\text{new}} &= X_{\text{old}} + r(X_i - X_j) & \text{if } f_i > f_j \\
X_{\text{new}} &= X_{\text{old}} + r(X_j - X_i) & \text{if } f_i < f_j
\end{align*}
\tag{7}
\]

where \( i \) and \( j \) are the indices of the students, \( r \) is a random number between 0 and 1 and \( f_i \) indicates the level of knowledge of the \( i \)th student. The smaller objective function in the minimization problem denotes the higher knowledge of the student.
In both teacher and student phases, the new point will be accepted if it improves the answer. The advantages of the TLBO algorithm are simplicity, low computational complexity, high searching power to find the global optimum and lack of tuning parameters, except for the initial population.

5. The proposed method

In the proposed method, electricity price in each hour depends on two factors: the predicted load in that hour and the price of the previous day at the same hour. If for a specific time of day, the forecasting load for the day \((t + 1)\) fuzzifies by \(m\) fuzzy sets, electricity price in day \(t\) fuzzifies by the \(n\) fuzzy sets, \(A_{t+1} = [a_1 a_2 \ldots a_m]\) represents the membership degree vector of system load for each of the \(n\) fuzzy sets and \(B_t = [b_1 b_2 \ldots b_n]\) represents the membership degree of price for each of \(m\) fuzzy sets, we can estimate the price in the day \((t + 1)\) using the following relation:

\[
P_{t+1} = \sum_{i=1}^{m} \sum_{j=1}^{n} K_{ij} \times G_{ij} + M,
\]

where \(M\) is the mean of the previous days price, \(P_{t+1}\) and \(K\) are the price in the day \((t + 1)\) and model coefficient matrix, respectively, which are determined using the TLBO algorithm.

Thus, with the availability of price data and load for \(r\) day before and the predicted load for the next day, the proposed method with the following steps are applied to forecast the price.

\textit{Step 1} Form the time series of price and the load based on the data of the previous days at the same time.

\textit{Step 2} Form the fuzzy sets for each time series.

In this paper, we have used Gaussian membership functions. In order to determine the centres of the Gaussian functions, the data of time series are clustered first using fuzzy \(C\)-means algorithm and then the centres of the clusters are set as the centres of the Gaussian membership functions.
Figure 3. Price forecasting of the electricity market in Australia on 11 May 2011 with the training process.

Figure 4. Price forecasting of the electricity market in Singapore for 11 September 2003.

Step 3 Fuzzify the time series of price and load.

Step 4 Use the TLBO algorithm to determine the model coefficients which we discussed before. The coefficients should be determined so as to minimize the following cost function:

\[ F(K) = \sum_{t=2}^{T} (y_t - p_t(K))^2, \quad (11) \]

where \( p_t \) is the series predicted value at time \( t \), \( y_t \) is the series true value at time \( t \), \( K \) represents the model coefficients matrix and \( T \) is the length of the time series.

Step 5 Use the model for forecasting the price.

Figure 1 denotes the flowchart of the proposed model with TLBO.

6. Case study

To examine the proposed method, we use the data of the Australia and Singapore electricity markets (AEMO – see https://www.aemo.com.au/; and EMCSG – see https://www.emcsg.com/). The algorithm is used to forecast the electricity price of Australia on 11 May 2011 which is shown in Figure 2. Figure 3 denotes the training data and the forecasted price in comparison with the real price. Furthermore, Figure 4 shows the forecasting of the electricity price in Singapore on 11 September 2003. The training data and the comparison of the forecasting data with the real data are also shown in Figure 5.

In this paper, mean absolute percentage error (MAPE) and mean squared error (MSE) are used to evaluate the results of the forecasting. These two criteria are defined as follows:

\[ \text{MAPE} = \frac{1}{N} \sum_{i=1}^{N} \frac{|P_a(i) - P_f(i)|}{P_{\text{AVE}}}, \quad (12) \]

\[ P_{\text{AVE}} = \frac{1}{N} \sum_{i=1}^{N} P_a(i), \quad (13) \]

\[ \text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (P_a(i) - P_f(i))^2, \quad (14) \]

where \( P_a \) is the real value and \( P_f \) is the predicted value.

The above two criteria are applied for forecasting the prices in the Australia electricity market for 11–20 May 2011 and the Singapore electricity market for 11–20 September 2003. The results are presented in Tables 1 and 2, respectively.

Figure 5. Price forecasting of the electricity market in Singapore for 11 September 2003 with training process.
In the simulations of the proposed model, we used five fuzzy sets for fuzzifying the price and load. Moreover, as we mentioned before, the sets are formed using clustering.

As we can see from the two tables, the error of forecasting is acceptable. Thus, the proposed method has enough accuracy to forecast the electricity price in the market.

7. Conclusion

In this paper, a method with two factors was proposed for forecasting the price of electricity in the electricity market. In this method, the price during each hour was influenced by the predicted load in that hour and the price of electricity in the previous days in the same hour. After fuzzifying the data with the fuzzy sets which were formed using fuzzy C-means clustering, the model coefficients were determined using TLBO. This model was used to forecast the electricity price for the next day. This method was applied to the real data of the electricity markets in Australia and Singapore to show the merit of the proposed method.

Other optimization methods can also be used instead of the TLBO algorithm in other researches. Furthermore, using other time series prediction methods for forecasting the price of the electricity in the electricity market can be considered as a further research direction.

References

Aggarwal, S. K., Saini, L. M., & Kumar, A. (2009). Electricity price forecasting in deregulated markets: A review an evaluation. International Journal of Electrical Power & Energy Systems, 31, 13–22.

Amjadi, N., & Hemmati, M. (2006). Energy price forecasting – problems and proposals for such predictions. IEEE Power and Energy Magazine, 4(2), 20–29.

Azizipanah-Abarghoee, R., Niknam, T., Bavafa, F., & Zare, M. (2014). Short-term scheduling of thermal power systems using hybrid gradient based modified teaching–learning optimizer with black hole algorithm. Electric Power Systems Research, 108, 16–34.

Bastian, J., Zhu, J., Banunaryanan, V., & Mukherji, R. (1999). Forecasting energy prices in a competitive market. IEEE Computer Applications in Power, 12(3), 40–45.

Chen, S. M. (1996). Forecasting enrollments based on fuzzy time series. Fuzzy Sets and Systems, 81, 311–319.

Chen, S. M., & Hwang, J. (2000). Temperature prediction using fuzzy time series. IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, 30(2), 45–60.

Chu, H. H., Chen, T. L., Cheng, C. H., & Huang, C. C. (2009). Fuzzy dual-factor time-series for stock index forecasting, expert systems with applications. Expert Systems with Applications, 36, 165–171.

Conejo, A. J., Plazas, M. A., Espinola, R., & Molina, A. B. (2005). Day-ahead electricity price forecasting using the wavelet transform and ARIMA models. IEEE Transactions on Power Systems, 20(2), 1035–1042.

Dasarathy, B. V. (1991). NN pattern classification techniques. Washington, DC: IEEE Computer Society Press.

Day, C. J., Hobbs, B. F., & Pang, J. S. (2002). Oligopolistic competition in power networks: A conjectured supply function approach. IEEE Transactions on Power Systems, 17, 597–607.

Dudani, S. (1975). The distance-weighted K-nearest-neighbor rule. IEEE Transactions on Systems, Man, and Cybernetics, SMC-6(4), 325–327.

Kavousi-Fard, A., Niknam, T., & Golmaryami, M. (2014). Short term load forecasting of distribution systems by a new hybrid modified FA-backpropagation method. Journal of Intelligent and Fuzzy Systems, 26(1), 517–522.

Kirschchen, D., & Sircac, G. (2004). Fundamentals of power system economics. New York: John Wiley & Sons.

Liu, X., Bai, E., Fang, J., & Luo, L. (2010). Time-variant slide fuzzy time-series method for short-term load forecasting. IEEE International Conference on Intelligent Computing and Intelligent Systems (ICIS), Xiamen, pp. 65–68. doi:10.1109/ICICISYS.2010.5658722

Lora, A. T., Santos, J. M. R., Expósito, A. G., Ramos, J. L. M., & Santos, J. C. R. (2007). Electricity market price forecasting based on weighted nearest neighbors technique. IEEE Transactions on Power Systems, 22(3), 1294–1301.

Niknam, T., Taherian Fard, E., Ebrahimpooosh, S., & Rousta, A. (2011). A new hybrid imperialist competitive algorithm on data clustering. Sadhana, 36(3), 293–315.

Nogales, F. J., Contreras, J., Conejo, A. J., & Espinola, R. (2002). Forecasting next-day electricity prices by time series models. IEEE Transactions on Power Systems, 17(2), 342–348.

Qiu, W., Liu, X., & Li, H. (2011). A generalized method for forecasting based on fuzzy time series. Expert Systems with Applications, 38, 10446–10453.

Ramos, J. L. M., Expósito, A. G., Santos, J. M. R., Lora, A. T., & Guerra, A. R. M. (2002). Influence of ANN-based market price forecasting uncertainty on optimal bidding.
Proceedings of PSCC Power System Computation Conference, Seville, Spain, pp. 45–56.
Rao, R., Savsani, V., & Vakharia, D. (2012). Teaching-learning-based optimization: An optimization method for continuous non-linear large scale problems. *Information Sciences, 183*, 1–15.
Rodriguez, C. P., & Anders, G. J. (2004). Energy price forecasting in the Ontario competitive power system market. *IEEE Transactions on Power Systems, 19*(1), 366–374.
Shabanpour-Haghighi, A., Seifi, A., & Niknam, T. (2014). A modified teaching-learning based optimization for multi-objective optimal power flow problem. *Energy Conversion and Management, 77*, 597–607.
Shahidehpour, M., Yamin, H., & Li, Z. (2002). *Market operations in electric power systems*. Hoboken, NJ: John Wiley & Sons.
Singhal, D., & K. S. (2011). Electricity price forecasting using artificial neural networks. *International Journal of Electrical Power & Energy Systems, 33*, 550–555.
Song, Q., & Chissom, B. S. (1993). Fuzzy time series and its models. *Fuzzy Sets and Systems, 54*, 269–277.
Troncoso, A., Riquelme, J., & Riquelme, J. (2002). Electricity market price forecasting: Neural networks versus weighted-distance k nearest neighbours. *Lecture Notes in Computer ScienceLect. Notes Comput. Sci., 245*, 321–330.
Troncoso, A., Riquelme, J. C., Martínez, J. L., Riquelme, J. M., & Gómez, A. (2003). Influence of kNN-based load forecasting errors on optimal energy production. *Lecture Notes in Artificial Intelligence, 2902*, 189–203.
Troncoso, A., Riquelme, J. M., Riquelme, J. C., Gómez, A., & Martínez, J. L. (2004). Time-series prediction: Application to the short-term electric energy demand. *Lecture Notes in Artificial Intelligence, 3040*, 577–586.
Ventosa, M., Baïllo, A., Ramos, A., & Rivier, M. (2005). Electricity market modeling trends. *Energy Policy, 33*(7), 897–913.
Wang, L. X. (1997). *A course in fuzzy system and control*. Upper Saddle River, NJ: Prentice Hall.
Wei, X. M. (2011). Fuzzy neural network market clearing power price forecasting based on K-means clustering. *Applied Mechanics and Materials, 121–126*, 2035–2039.