Physics-Informed Deep Reversible Regression Model for Temperature Field Reconstruction of Heat-Source Systems

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Abstract

Temperature monitoring during the life time of heat source components in engineering systems becomes essential to guarantee the normal work and the working life of these components. However, prior methods, which mainly use the interpolate estimation to reconstruct the temperature field from limited monitoring points, require large amounts of temperature tensors for an accurate estimation. This may decrease the availability and reliability of the system and sharply increase the monitoring cost. To solve this problem, this work develops a novel physics-informed deep reversible regression models for temperature field reconstruction of heat-source systems (TFR-HSS), which can better reconstruct the temperature field with limited monitoring points unsupervisedly. First, we define the TFR-HSS task mathematically, and numerically model the task, and hence transform the task as an image-to-image regression problem. Then this work develops the deep reversible regression model which can better learn the physical information, especially over the boundary. Finally, considering the physical characteristics of heat conduction as well as the boundary conditions, this work proposes the physics-informed reconstruction loss including four training losses and jointly learns the deep surrogate model with these losses unsupervisedly. Experimental studies have conducted over typical two-dimensional heat-source systems to demonstrate the effectiveness of the proposed method.

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1. Introduction

Heat management plays an important role in heat-source systems where heat may be generated internally, especially over systems consisting of components (electronic devices) with smaller size and higher power density [1, 2, 3, 4]. It can provide the working status and even improve the life time and reliability of the system. Temperature monitoring, which can monitor and provide the real-time operating temperature, tends to be an irreplaceable process in heat management system. Generally, temperature transducer [5, 6] is used as the engineering device to convert the temperature information to available signal as output. However, too many tranducers would negatively affect the availability and reliability of the system and further sharply increase the monitoring cost, and only limited number of transducers can be assigned for real-time temperature monitoring in real-world system. This would increase the difficulty to obtain the whole temperature field of the heat-source systems. Therefore, research on Temperature field reconstruction task of heat-source systems (TFR-HSS) [7] has become an urgent but challenging task in engineering systems, such as printed circuit board (PCB) in electronic equipments and honeycomb board in satellite, and others.

Traditional methods utilize the interpolation methods for temperature field reconstruction [8]. Interpolation is a type of estimation to reconstruct the temperature of interest area within the range of a discrete set of known monitoring temperature points, such as the linear interpolation, polynomial interpolation, spline interpolation [9]. These interpolation methods mainly take advantage of the fixed local and global relative correlation and are generally fast to calculate and easy to operate. However, they cannot be adaptive to learn the correlation between the interest points and the monitoring points and their performance.
is guaranteed by the prior knowledge. Besides, they ignore the physical characteristics of the temperature field and thus usually provide the constructed temperature field with high prediction error.

Faced with these circumstances, most of the research take advantage of surrogate modeling based on traditional regression models [10], such as Gaussian process regression (Kriging method) [11], support vector regression (SVR) [12], polynomial regression [13], GRBF-based kernel regression [14], gappy proper orthogonal decomposition (GOP) [15], in the literature of field reconstruction. Benefit from machine learning, these surrogate models can learn the nonlinear physical correlation adaptively and present stronger generalization and more universal. Nevertheless, these methods which consist of limited number of parameters cannot be well applied to the ultra-dimensional TFR-HSS task. Therefore, exploring surrogate models with higher representational ability and better performance in describing linear and nonlinear physical information is essential for current ultra-dimensional task.

In recent years, deep neural networks (DNNs), as a non-linear ultra-dimensional fitting [16], have achieved good performance in extracting high-level information in many fields, such as image classification [17] [18]. As a representative, convolutional neural networks (CNNs), which can extract both the local and global information and describe the complex physical correlation between different input pixels, have already been applied in thermal analysis of electronic systems [19]. Considering the ultra-dimension and nonlinear physical characteristics of the mapping from observation data to temperature field, deep models with potential ability to learn the latent complex physical correlation is an effective method for the specific task. In this work, due to the good performance, the CNNs will be used as the deep surrogate model to extract the physical correlation [20].

However, there still exist two difficulties faced in applying deep surrogate models in TFR-HSS task.

- Appropriate numerical modelling which can fit for deep surrogate models is 

required for TFR-HSS task. Besides, a proper deep surrogate model which can capture physical characteristics of temperature field is also needed to reconstruct the field given limited monitoring points.

- General CNN is data-driven where large amounts of labeled samples are required for the training of the deep model. However, for TFR-HSS task, the temperature field corresponding to the given monitoring value is expensive to obtain, sometimes even unavailable.

These problems would make it more challenging to apply deep surrogate models for TFR-HSS task.

To solve the first problem, this work models the heat-source system as a two-dimensional domain and discretize the domain with $N \times N$ grid. Then, the information from monitoring points and the temperature field of the system can be numerically modelled as $N \times N$ matrix, respectively. The TFR-HSS task is transformed as the optimization problem to find the mapping from the matrix of monitoring points to the whole temperature field. The problem can be seen as the image-to-image regression problem and can be solved by general deep regression models. However, due to the law of forward and backward propagation of convolutional layers in general models \cite{21} and the law of heat conduction \cite{22}, general models, such as the fully convolutional networks (FCN) \cite{23}, cannot provide expected reconstruction performance, especially in the area near the upside and rightside boundaries. Considering both the laws and fitting the deep models for the task, this work develops the reversible regression models for this specific TFR-HSS task which can solve the field reconstruction near boundaries through twice encoder-decoder process and the diagonal flip operation.

To solve the latter one, this work utilizes the physical characteristics of temperature field and develops the physics-informed deep learning methods to train the deep model unsupervisedly for the TFR-HSS task. Generally, model-based deep learning methods, which can use the model prior of the data for the training of the deep model, is an effective way for the deep learning with limited samples \cite{24,25}. These model priors, such as the statistical property, topological
structure correlation, can assist the training of deep models. As a specific form of model priors, physics properties can also be used as the model prior for the training of the deep surrogate model. Many prior works have focused on such physics prior for the learning of neural networks to solve the engineering problem, such as the elastodynamics [26], fluid dynamics [27], thermochemical curing process [28]. Even though the applied field varies from each other, the main idea of such problem is to formulate the training loss based on the partial differential equation (PDE) from the physical process [29, 30]. Faced with the TFR-HSS task, this work will develop the physical informed training loss for the training of proposed surrogate model to provide an real-time prediction from the monitoring value to the whole temperature field.

For current task, the physical properties, including the heat conduction, boundary conditions, spatial smoothness, can be used for the training of the surrogate model. First, considering the heat conduction correlation between different points in the system, this work develops the Laplace loss which constraints the predicted temperature field with the steady-state conduction equation with heat sources. Then, the boundary conditions are constrainted by the formulated BC loss. Besides, the point loss is constructed to ensure the consistency of the temperature value of monitoring points in the reconstructed temperature field. More importantly, the total variation loss is used to guarantee the spatial smoothness of the reconstructed temperature field. These four constructed losses encourage the deep surrogate model to learn the physical correlation and hence joint learning with these four losses can make the surrogate model provide impressive reconstruction performance for the TFR-HSS task.

Considering the merits of deep reversible regression model and physics-informed deep learning, this work develops a novel physics-informed deep reversible regression model as surrogate model for temperature field reconstruction of heat-source systems. Deep reversible regression model aims to increase the representational ability to extract the high-level and subtle physical information. Besides, the physics-informed reconstruction loss, which is jointed by the Laplace loss, the BC loss, the point loss, and the TV loss, is developed to
capture the complex physical heat transfer correlation, which makes it possible to train the deep surrogate models unsupervisedly. To sum up, this work makes the following contributions.

- This work defines the temperature field reconstruction of heat-source systems (TFR-HSS) task from real-world engineering applications and provides mathematical formulation as well as the numerically modeling of the task.

- Based on the numerically modeling of the task as well as the characteristics of vanilla deep regression models, this work further proposes the reversible regression model which can better fit for the TFR-HSS task.

- This work develops the physics-informed training losses for TFR-HSS task, including the Laplace loss for preserving physical heat conduction property, the point loss for maintaining the temperature value by transducers, the BC loss for satisfying the boundary conditions, and the TV loss for realising spatial smoothness.

Moreover, the experiments over typical two-dimensional heat-source systems have been conducted and the comparison results with the most recent methods have demonstrated the superiority of the proposed method for TFR-HSS task.

The remainder of this paper is arranged as follows. In Section 2 the mathematical form of the TFR-HSS task is defined. Section 3 numerically models the TFR-HSS task, and further develops the physics-informed deep reversible regression models for the reconstruction task, including the proposed reversible regression models and the developed physics-informed training losses. The experimental studies over typical two-dimensional heat-source systems are presented to validate the effectiveness of the proposed method in Section 4. Finally, we conclude this paper with some discussions in Section 5.
2. Temperature Field Reconstruction of Heat-Source Systems (TFR-HSS)

This work focuses on the heat-source systems where heat may be generated internally and the principles of heat transfer are mainly concerned with. The heat-source systems can be modeled as a two-dimensional domain where each electronic component is defined as the heat source with different shapes and thermal conduction occurs over this specific domain [31, 32]. Given the system, the TFR-HSS task aims to reconstruct the overall temperature field with specified temperature values from monitoring tensors and is an important part of real-time health detection system of electronic equipment in engineering.

Given a heat-source system with $\Lambda$ heat sources and $m$ transducers are placed on the layout domain for temperature monitoring. Generally, the mathematical formulation of the TFR-HSS task can be formulated as

$$T^* = \arg \min_T \left( \sum_{i\in[1,m]} |T(x_{s_i}, y_{s_i}|\phi_1, \cdots, \phi_\Lambda) - f(x_{s_i}, y_{s_i})| \right)$$ (1)

where $T(\cdot)$ describes the reconstructed temperature field of the system and $f(\cdot)$ is the monitoring temperature value. $\phi_i (i = 1, 2, \cdots, \Lambda)$ represents the intensity distribution of the $i$-th heat source. For simplicity, the intensity of each heat source is uniformly distributed and set to a constant value. $O_1, O_2, \cdots, O_m$ denote the monitoring points where $(x_{s_i}, y_{s_i})$ describes the positions of the $O_i$ monitoring point and $m$ is the number of the points.

Therefore, the key process of this specific task is to obtain the optimal temperature field using Eq. (1) while satisfying the physical equation of thermal conduction. As for the two-dimensional heat conduction, the steady-state satisfies the Laplace equation, which can be formulated as

$$\frac{\partial}{\partial x} (\lambda \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (\lambda \frac{\partial T}{\partial y}) + \sum_{i=1}^{\Lambda} \phi_i(x, y) = 0$$ (2)

where $\lambda$ represents the thermal conductivity of the domain. In addition to the thermal conduction equation, physical properties over boundary conditions
should also be constrained and can be generally written as

\[ T = T_0 \text{ or } \lambda \frac{\partial T}{\partial n} = 0 \text{ or } \lambda \frac{\partial T}{\partial n} = h(T - T_0) \quad (3) \]

where \( T_0 \) is a constant temperature value, \( n \) denotes the (typically exterior) normal to the boundary, and \( h \) represents the convective heat transfer coefficient. The three boundaries are known as the Dirichlet boundary conditions (Dirichlet BCs) where \( T_0 \) is the isothermal boundary temperature, the Neumann boundary conditions (Neumann BCs) where zero heat flux is exchanged, and the Robin boundary conditions (Robin BCs) where \( T_0 \) represents the surrounded fluid temperature value. Overall, the TFR-HSS task can be transformed as the following optimization problem:

\[
\min_T \left( \sum_{i \in [1, m]} \left| T(x_{s_i}, y_{s_i}|\phi_1, \cdots, \phi_\Lambda) - f(x_{s_i}, y_{s_i}) \right| \right)
\]

s.t. \( \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \sum_{i=1}^{\Lambda} \phi_i(x, y) = 0 \quad (4) \)

\[
T = T_0 \text{ or } \lambda \frac{\partial T}{\partial n} = 0 \text{ or } \lambda \frac{\partial T}{\partial n} = h(T - T_0)
\]

As a representative but without loss of generality, the volume-to-point (VP) heat conduction problem in a two-dimensional rectangular domain as Fig. 1 shows is taken for the validation of the proposed method (just as [19] and [31]). In this problem, all the boundaries are adiabatic except the small patch of heat sink which is represented as \( \delta \) in the figure. The objective of the reconstruction problem is to reconstruct the temperature field of the domain from the \( m \) monitoring points.

Through mathematical modeling, the TFR-HSS task can be transformed to the continuous optimistic problem as Eq. 4 shows. To solve this optimization, this work numerically models the problem and develops the physics-informed deep reversible regression model as deep surrogate model for the task. Following we will introduce the numerical modelling process, the construction of the reversible regression model and the physics-informed deep learning methods for temperature field reconstruction in detail.
3. Physics-informed Deep Reversible Regression Model

For convenience, $\Omega$ is used to represent the square domain, $\Omega_e$ represents the layout area without heat sources laid on, $\Omega_l$ describes the area with heat sources laid on, and $\Omega_b$ stands for the boundary area. The size of the domain is set to $L = 0.1m$ and the length of the heat sink is set to $\delta = 0.01m$.

3.1. Numerical Modelling for TFR-HSS Task

In order to facilitate the computing process, numerical modelling of the TFR-HSS task is necessary. At first, just as Fig. 2(a) shows, the layout domain is meshed by $N \times N$ grid. The area within a certain grid is supposed to share a constant temperature value. The monitoring points are arranged in the grids to obtain the temperature of the grid. Then, two-dimensional matrix $f$ of $N \times N$ with monitoring temperature can be obtained and used as the input.
Figure 2: Numerical modelling for TFR-HSS task. (a) Layout board; (b) Discreted monitoring matrix; (c) Discreted temperature field.

to reconstruct the overall temperature field in the layout domain. As Fig. 2(b) shows, the discreted monitoring matrix $f$ is filled with the monitoring temperature value at the monitoring point and the remainder points will be filled with the constant value $T_0$.

Through numerical modelling, the objective of TFR-HSS task is to obtain the temperature value of other grid in the matrix. Then, as Fig. 2(c) shows, the output of the task is the reconstructed temperature field which has been numerically modelled as a $N \times N$ matrix $T$. Therefore, the task can be seen as a discrete optimization problem which tries to find the surrogate mapping $\Phi$ from monitoring matrix $f$ to reconstructed temperature field $T$, and it can be written as

$$f \xrightarrow{\Phi} T$$

Since $f$ and $T$ are both $N \times N$ matrix, the problem can be seen as the regression problem and general image-to-image regression methods can be applied as the deep surrogate model for the TFR-HSS task.

However, due to special characteristics of the HFR-HSS task, general deep regression methods usually cannot well work for the task. Through fully considering these characteristics, this work develops a novel reversible regression model for TFR-HSS task. Besides, since limited labelled training samples are available for TFR-HSS task, this work develops the physics-informed training...
loss based on the physical properties of TFR-HSS task and learns the deep model unsupervisedly. Following we will introduce the proposed method in detail.

### 3.2. Reversible Regression Model

Due to the calculation order of convolution operation in general deep regression models, the reconstructed temperature field usually has jagged boundary temperature, especially the reconstructed temperature field near the boundary of upside and rightside. The reason is that the forward and backward of convolutional layer in deep model is conducted orderly, and the final calculated area cannot capture enough physical information to update the temperature under only one encoder-decoder process.

Taking these characteristics of TFR-HSS task into consideration, this work develops the reversible regression model which consists of two encoder-decoder operations and one flip operation to guarantee the reconstruction performance of the surrogate model. Fig. 3 presents the architecture of the proposed model. As the figure shows, the proposed model, which is written as $Net_1 - Net_2$, is divided into two parts, e.g. $Net_1$ and $Net_2$. Both $Net_1$ and $Net_2$ are independent encoder-decoder models, which can have the same or different structures. Between $Net_1$ and $Net_2$, the flip operation with diagonal flipping is conducted to make the computing process reversible. Such operation makes the reversible regression model better fit for the TFR-HSS task.

The proposed reversible regression model is easy to implement and can provide remarkable reconstruction performance for TFR-HSS task. Benefit from the flip operation and two successive encoder-decoder processes, the temperature field can be reconstructed well with limited monitoring points over the boundaries.

### 3.3. Physics-Informed Reconstruction Loss

Considering the characteristics of the TFR-HSS task and the strong dependency of large amounts of training samples for deep learning, this work develops the novel physics-informed reconstruction loss for the task and train
Figure 3: The architecture of proposed reversible regression model for the TFR-HSS task. Here, $Net_1$ and $Net_2$ represent the base models in the proposed model which can have the same or different structures.

the reversible regression model unsupervisedly. The loss consists of four significantly different loss terms which describe different physical characteristics from different perspectives.

First, it is generally acknowledged that MSE loss can be well work for reconstruction. In this work, we formulate the point loss $L_{\text{point}}$ to make the predicted temperature satisfy the temperature over monitoring points. Therefore, the point loss can be formulated as

$$L_{\text{point}} = \sum_{i=1}^{m} |T(x_{s_i}, y_{s_i}) - f(x_{s_i}, y_{s_i})|^2_2$$  \hspace{1cm} (6)$$

where $| \cdot |^2_2$ represents the $L_2$ norm.
In addition to point loss for reconstruction of monitoring points, the boundary conditions in Eq. 4 also have significant effects on the temperature field. To use the boundary conditions for reconstruction, this work designs the BC loss $L_{bc}$ to ensure the physical properties of the boundaries of the temperature field. Since this work mainly considers the Romann B.C. and Dirichlet B.C., the $L_{bc}$ is formulated over this two kinds of boundary conditions separately. As introduces in Section 2, the Romann B.C. satisfies $|\frac{\partial T(x,y)}{\partial n}|(x,y)\in\Omega_{romann} = 0$, and the Dirichlet B.C. satisfies $|T(x,y) - T_0|(x,y)\in\Omega_{dirichlet} = 0$. Therefore, the continuous form of the $L_{bc}$ loss can be formulated as

$$L_{bc} = |T(x,y) - T_0|(x,y)\in\Omega_{dirichlet}^\text{b} + |\frac{\partial T(x,y)}{\partial n}|(x,y)\in\Omega_{romann}^\text{b} \quad (7)$$

where $\Omega_{dirichlet}^\text{b}$, $\Omega_{romann}^\text{b}$ describes the boundary regions satisfying the Dirichlet B.C. and the Romann B.C., respectively. Here, the BC loss is also processed discretely. For Dirichlet B.C., the $bc$ loss can be written as

$$L_{bc}^\text{Dirichlet} = \sum_{(x_i,y_j)\in\Omega_{dirichlet}^\text{b}} |T(x_i,y_j) - T_0| \quad (8)$$

For Romann B.C., the temperature over the boundary should satisfy $T(x_i,y_N) - T(x_i,y_{N-1}) = 0$ or $T(x_i,y_2) - T(x_i,y_1) = 0$ or $T(x_N,y_j) - T(x_{N-1},y_j) = 0$ or $T(x_2,y_j) - T(x_1,y_j) = 0(i,j = 1, 2, \cdots, N)$. In this work, the BC loss over Romann B.C. is implemented through replicate padding of the temperature field with one dimension over the surrounded boundaries. Since the physical property with Romann B.C. can be satisfied through replicate padding operation, the $L_{bc}$ loss could mainly concern about the physical property over the Dirichlet B.C., and it can be written as

$$L_{bc} = \sum_{(x_i,y_j)\in\Omega_{dirichlet}^\text{b}} |T(x_i,y_j) - T_0| \quad (9)$$

For the heat-source systems, the thermal conduction satisfies the two-dimensional laplace equation as Eq. 2 shows. It builds the relationship between different points inside the heat-source systems and can guide the reconstruction of the temperature field with the help of the monitoring points. In order to take advantage of such kind of physical information, reconstruction-based Laplace loss
$L_{\text{laplace}}$ is built to preserve the physical characteristics of the predicted temperature on the domain. Based on Eq. 2, the reconstruction-based laplace loss can be formulated as

$$L_{\text{laplace}} = \sum_{(x,y)\in \Omega} \left| \frac{\partial}{\partial x} (\lambda \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (\lambda \frac{\partial T}{\partial y}) + \sum_{i=1}^{\Lambda} \phi_i(x,y) \right|$$

(10)

where $\lambda$ is set to 1 in this work, namely constant thermal conductivity is assumed. Then Eq. 10 can be reformulated as

$$L_{\text{laplace}} = \sum_{(x,y)\in \Omega} \left| \frac{\partial}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} \frac{\partial T}{\partial y} + \sum_{i=1}^{\Lambda} \phi_i(x,y) \right|$$

(11)

For the current task, the real-time intensity of heat sources in the system remains unknown, and therefore in this work, the laplace loss is constructed by the thermal conduction characteristics over the domain without the heat sources laid where the intensity can be seen as zero for simplicity. Then, the continuous form of the final laplace loss $L_{\text{laplace}}$ can be written as

$$L_{\text{laplace}} = \sum_{(x,y)\in \Omega_e} \left| \frac{\partial}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} \frac{\partial T}{\partial y} \right|$$

(12)

where $\Omega_e$ describes the domain without heat source placed. Without exception, the laplace loss is implemented discretely. Based on the difference equation, $\frac{\partial}{\partial x} \frac{\partial T}{\partial x}$ at $(x_i, y_j)$ can be calculated as

$$\frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) \bigg|_{x=x_i} = \frac{\partial}{\partial x} \left( \frac{T(x_{i+1}, y_j) - T(x_i, y_j)}{x_{i+1} - x_i} \right) = \frac{T(x_{i+1}, y_j) - T(x_i, y_j)}{x_{i+1} - x_i} - \frac{T(x_i, y_j) - T(x_{i-1}, y_j)}{x_i - x_{i-1}}$$

(13)

Since the temperature field is calculated by the uniform square mesh, the equation can be reformulated as

$$\frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) \bigg|_{x=x_i} = \frac{T(x_{i+1}, y_j) + T(x_{i-1}, y_j) - 2T(x_i, y_j)}{\Delta x^2}$$

(14)

where $\Delta x = x_{i+1} - x_i$. Similarly, $\frac{\partial}{\partial y} \frac{\partial T}{\partial y} \big|_{y=y_j}$ can be reformulated as

$$\frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} \right) \bigg|_{y=y_j} = \frac{T(x_i, y_{j+1}) + T(x_i, y_{j-1}) - 2T(x_i, y_j)}{\Delta y^2}$$

(15)
where $\Delta y = y_{j+1} - y_j$. Therefore, the discrete form of $L_{\text{laplace}}$ can be written as

$$L_{\text{laplace}} = \sum_{(x_i, y_j) \in \Omega_e} |\frac{\partial}{\partial x} \frac{\partial T}{\partial x}(x_{i+1}, y_j) - \frac{\partial}{\partial y} \frac{\partial T}{\partial y}(x_i, y_{j-1})|$$

$$= \sum_{(x_i, y_j) \in \Omega_e} \frac{1}{\Delta x^2} \left( T(x_{i+1}, y_j) + T(x_{i-1}, y_j) - 2T(x_i, y_j) \right)$$

$$+ \frac{1}{\Delta y^2} \left( T(x_i, y_{j+1}) + T(x_i, y_{j-1}) - 2T(x_i, y_j) \right)$$

In the numerical modelling process, we set $\Delta = \Delta x = \Delta y$. Denote $D_{x, y} = T(x_{i+1}, y_j) + T(x_{i-1}, y_j) + T(x_i, y_{j+1}) + T(x_i, y_{j-1}) - 4T(x_i, y_j)$. Then, $L_{\text{laplace}}$ can be reformulated as

$$L_{\text{laplace}} = \sum_{(x_i, y_j) \in \Omega_e} \left| \frac{D_{x, y}}{\Delta x^2} \right|$$

Interestingly, $D_{x, y}$ is the typical two-dimensional difference format and can be seen as a special form of convolutional operation. Therefore, in this work, $L_{\text{laplace}}$ can be calculated using convolutional operation and $L_1$ norm.

Generally speaking, the temperature field changes gently and there is no drastic changes in steady-state temperature field. Considering such property, this work further uses the total variation (TV) regularization \[33\] for the temperature field reconstruction task. The TV regularization $L_{tv}$ encourages the spatial smoothness of the reconstructed temperature field, especially over the region near boundaries and with the heat sources laid on, and further helps the reconstruction process. It mainly considers the one-order gradient information of the temperature field and can be formulated as

$$L_{tv} = \int_{\Omega} \left( \frac{\partial T}{\partial x}(x, y)^2 + \frac{\partial T}{\partial y}(x, y)^2 \right) dx$$

where $\rho$ describes the order of the TV regularization. The gradient information describes the local changes of the temperature field and can be re-written as the relationship of neighboring points of the field discretely. Therefore, Eq. 18 can
be calculated by

\[ L_{tv} = \sum_{i=1}^{N} \sum_{j=1}^{N} ((T(x_i, y_j + 1) - T(x_i, y_j))^2 + (T(x_{i+1}, y_j) - T(x_i, y_j))^2)^{\frac{2}{\rho}} \]  \hspace{1cm} (19)

In this work, \( \rho \) is set to 2 and the used \( L_{tv} \) can be written as

\[ L_{tv} = \sum_{i=1}^{N} \sum_{j=1}^{N} ((T(x_i, y_j + 1) - T(x_i, y_j))^2 + (T(x_{i+1}, y_j) - T(x_i, y_j))^2) \]  \hspace{1cm} (20)

Based on Eqs. 6, 9, 16, and 20, the final physics-informed reconstruction loss (PIRL) can be formulated as

\[ L = L_{point} + \alpha L_{bc} + \beta L_{laplace} + \gamma L_{tv} \]  \hspace{1cm} (21)

where \( \alpha, \beta, \gamma \) stands for the tradeoff parameters.

### 3.4. Evaluation Metrics for the Reconstruction Performance

To evaluate the reconstruction performance of the proposed method quantitatively, this work designs the following four metrics based on the temperature field information we concern about, namely the mean absolute error (MAE), the maximum of component-constrained absolute error (M-CAE), the component-constrained mean absolute error (CMAE), and the boundary-constrained mean absolute error (BMAE).

**Mean absolute error (MAE)** calculates the mean absolute error of the whole reconstructed temperature field, which can be formulated as

\[ E_{MAE} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} |T(x_i, y_j) - \hat{T}(x_i, y_j)| \]  \hspace{1cm} (22)

where \( \hat{T} \) represents the real temperature field obtained by numerical simulation.

**Component-constrained mean absolute error (CMAE)** calculates the mean absolute error of area with heat sources laid on and it can be written as

\[ E_{CMAE} = \frac{1}{|\Omega_t|} \sum_{(x_i, y_j) \in \Omega_t} |T(x_i, y_j) - \hat{T}(x_i, y_j)| \]  \hspace{1cm} (23)
where $\Omega_l$ represents the area with heat sources laid on.

**Maximum of Component-constrained absolute error (M-CAE)** calculates the maximum absolute error of the whole reconstructed temperature field, which can be formulated as

$$E_{M-CAE} = \max_{(x_i, y_j) \in \Omega_l} |T(x_i, y_j) - \hat{T}(x_i, y_j)|$$  \hspace{1cm} (24)

**Boundary-constrained mean absolute error (BMAE)** calculates the mean absolute error over the boundary area and it can be written as

$$E_{BMAE} = \frac{1}{|\Omega_b|} \sum_{(x_i, y_j) \in \Omega_b} |T(x_i, y_j) - \hat{T}(x_i, y_j)|$$  \hspace{1cm} (25)

where $\Omega_b$ denotes the boundary regions.

In conclusion, the MAE measures the overall reconstruction performance of the proposed method, the M-CAE and CMAE indicate the reconstruction performance of local area of interest, and the BMAE concerns the reconstruction performance over the boundaries. Such four metrics can thoroughly evaluate the performance of the proposed method.

### 3.5. Implementation of the Proposed Method for TFR-HSS task

The pseudocode of the training process of the proposed method is given in Algorithm 1. The implementation can be divided into three parts: training process, prediction process, and evaluation process.

**Step 1:** Prepare the training samples, construct the reversible regression model as surrogate mapping to extract the physical information and define the hyper-parameters.

**Step 2:** Train the reversible regression model unsupervisedly following step 2-11 in Algorithm 1 and provide the optimized surrogate mapping $\Phi^*$.

**Step 3:** Predict the temperature field with obtained surrogate mapping $\Phi^*$.

**Step 4:** Evaluate the surrogate mapping $\Phi^*$ under the given metrics.
4. Experimental Studies

4.1. Experimental Setups

Datasets: To validate the effectiveness of the proposed method for temperature field reconstruction, this work constructs four typical thermal analysis datasets with different heat sources and boundary conditions, which are described as Data A, Data B, Data C, Data D, respectively. The domain of the systems is denoted as $0.1m \times 0.1m$ board. For convenience but without loss of generality, the commonly used heat sinks with 0.01m are used as the boundary conditions for heat dissipation.

As Fig. 4 shows, Data A and Data B are designed with the same heat sources layout but with heat sinks on different boundaries. Compared with Data A and Data B, Data C and Data D are designed with more complex heat sources. Table 4.1 lists the number of monitoring points for these datasets. In this work, for temperature field reconstruction, the monitoring points are placed near the boundary, between the heat-sources and on the heat-sources, respectively. As the table shows, 16 monitoring points are placed near the boundary. 18 points are put between components for Data A, B, D and 16 points are put for Data C.

\footnote{These datasets are generated by our developed data generator which is released at \url{https://github.com/shendu-sw/recon-data-generator}}
Table 1: Number of monitoring points used in this work for different datasets. NB, BC and OC represent near the boundary, between components and on the components, respectively.

| Positions | NB  | BC  | OC              | Total |
|-----------|-----|-----|-----------------|-------|
| Data A & B| 16  | 18  | 9 × 10          | 124   |
| Data C    | 16  | 16  | 9 × 5 + 12 × 3 + 10 × 2 | 133   |
| Data D    | 16  | 18  | 9 × 12          | 142   |

For Data A, B, and D, nine monitoring points are placed on each component. While for Data C, nine monitoring points are placed for general heat sources, twelve are placed for complex sources with three parts and ten are placed for complex sources with two parts. Such complex heat sources in Data C describe the complex components made of several different materials in engineering, such as central processing units (CPUs).

Furthermore, the power of the heat sources in Data A and B is distributed uniformly. While in order to better evaluate the performance of the proposed method, several heat sources in Data C and D are supposed as complex power distribution. For simplicity, the complex heat source is divided into several parts (two or three in this work) and each part owes an independent uniform distributed power. For each dataset, the power of each heat source ranges from 0-30000 W/m² (includes different parts of the complex heat source).

For all the data, FEniCS, one of the finite element simulation software, is used to generate the groundtruth temperature field given a specific heat source system[^1].

**Optimization and hyperparameters:** Considering the orders of magnitude of the four losses, α, β, γ are set to 1e⁻³, 1e⁻², 1e⁻², respectively. For each simulation data, we choose 40000 samples for training process where 80% is for training and 20% for validation, and 10000 samples for testing. The training epoch is set to 50.

[^1]: https://fenicsproject.org/
Compute Infrastructure: A very common machine with a 2.8-GHz Intel(R) Xeon(R) Gold 6242 CPU, 256-GB memory, and NVIDIA GeForce RTX 3090 GPU was used to test the performance of the proposed method.

4.2. General Reconstruction Performance

At first, we present a brief overview of the merits of the proposed physics-informed deep surrogate learning method for TFR-HSS task. In this set of experiments, the SegNet-AlexNet [34] (SegNet with AlexNet backbone) is used as the base regression model of proposed reversible regression model. Both Net1 and Net2 in the reversible regression model uses the SegNet-AlexNet.

Table 2 lists the performance over Data A, B, C, D under different metrics and Fig. 5 presents several reconstruction examples of the proposed method over these datasets. As the results show, the MAE of proposed method over all the datasets is less than 0.5K. Especially, over the components, the MAE is about 0.1K over all the datasets. Besides, the BMAE is about or less than 1K over all the datasets. This means that the proposed method can better reconstruct the temperature both over the components and near the boundaries. As Fig. 5 shows, the reconstruction error of only few points near the boundaries is larger than 1K. This also indicates that the proposed reversible regression model can well work for the current task.

Obviously, the base models can significantly affect the performance of the proposed method. Therefore, following we will present the general reconstruction performance of the proposed method with different surrogate models.

4.2.1. Performance with Different Surrogate Models

The base model of Net1 and Net2 in the proposed reversible regression model can significantly affect the reconstruction performance of the proposed method. In addition to the SegNet as the base model, based on the architecture

\[\text{https://github.com/shendu-sw/PIRL}\]

\[\text{https://github.com/shendu-sw/PIRL}\]

More details about the models are provided in attachment.
Figure 5: Examples of proposed method over Data A, B, C and D for TFR-HSS task.

Table 2: General reconstruction performance (K) of proposed method over datasets for TFR-HSS task (32000, 8000 samples for training, and validation, respectively).

| Data   | MAE  | M-CAE | CMAE  | BMAE  |
|--------|------|-------|-------|-------|
| Data A | 0.3436 | 1.4340 | 0.0883 | 1.0483 |
| Data B | 0.2655 | 3.6059 | 0.1006 | 0.9003 |
| Data C | 0.1727 | 1.7394 | 0.1157 | 0.4560 |
| Data D | 0.2456 | 1.7043 | 0.1902 | 0.5751 |
of Feature Pyramid Networks (FPN) \cite{Lin_2017_CVPR}, Fully Convolutional Networks (FCN) \cite{Long_2015_ICCV}, and UNet \cite{Ronneberger_2015_CVPR} which are proposed for general computer vision task, such as object detection \cite{Redmon_2016_ICCV} and image segmentation \cite{Ronneberger_2015_CVPR}, this work designs several base model architectures for current task. Based on AlexNet \cite{Krizhevsky_2012_NIPS}, this work designs the specific FCN for current TFR-HSS task. Based on VGG \cite{Simonyan_2015_ICCV}, this work designs the specific UNet. Based on ResNet \cite{He_2016_CVPR}, this work designs the specific FPN for this task.

In this work, we construct four forms of the proposed reversible regression models, namely Segnet-Segnet, FCN-FCN, UNet-UNet, FPN-SegNet, respectively. It should be noted that instead of FPN-FPN, we use the FPN-SegNet since the FPN-FPN does not converge for current TFR-HSS task. Table 3 lists the performance of the four configurations over the four datasets and Fig. 6 shows the examples of the reconstruction results over Data A with different configurations.

From the table, it can be find that FPN-SegNet performs better than other configurations over Data B and Data C under MAE while over Data A, UNet-UNet performs the best and over Data D, FCN-FCN performs performs the best. From the view of M-CAE, FPN-SegNet performs the best than other configurations while under CMAE and BMAE, UNet-UNet is better than other three configurations. The SegNet-SegNet performs the worst performance among all the four configurations.

For simplicity and better presenting the performance of the proposed method affected by other variables, the SegNet-SegNet is used as the base model in the proposed method unless otherwise specified.

4.3. Comparisons with vanilla Deep Regression Models

In this set of experiments, we compare the proposed reversible regression model with vanilla deep regression models which are both trained by the proposed physics-informed reconstruction methods. Here, the proposed reversible regression model is divided into two forms: the reversible surrogate model with
(a) Results by SegNet-SegNet

(b) Results by FCN-FCN

(c) Results by FPN-SegNet

(d) Results by Unet-Unet

Figure 6: Examples of the reconstruction results over Data A by proposed reversible regression models with different base models.

Table 3: Reconstruction performance (K) of proposed physics-informed deep learning on reversible regression model with different base models for TFR-HSS task.

| Models         | Metrics | Data A | Data B | Data C | Data D |
|---------------|---------|--------|--------|--------|--------|
| SegNet-SegNet  | MAE     | 0.3436 | 0.2655 | 0.1727 | 0.2456 |
|               | M-CAE   | 1.4340 | 3.6059 | 1.7394 | 1.7043 |
|               | CMAE    | 0.0883 | 0.1006 | 0.1157 | 0.1902 |
|               | BMAE    | 1.0483 | 0.9003 | 0.4560 | 0.5751 |
| FCN-FCN       | MAE     | 0.1309 | 0.1605 | 0.1051 | 0.1487 |
|               | M-CAE   | 0.6310 | 0.6672 | 0.4913 | 0.6505 |
|               | CMAE    | 0.0865 | 0.1065 | 0.1078 | 0.1674 |
|               | BMAE    | 0.1614 | 0.1847 | 0.1589 | 0.1941 |
| FPN-SegNet     | MAE     | 0.1503 | 0.1510 | 0.0931 | 1.1305 |
|               | M-CAE   | 0.4059 | 0.4631 | 0.2941 | 4.5548 |
|               | CMAE    | 0.1429 | 0.1393 | 0.0820 | 0.8288 |
|               | BMAE    | 0.1654 | 0.1656 | 0.1502 | 2.7322 |
| UNet-Unet     | MAE     | 0.1265 | 0.1540 | 0.1146 | 0.1979 |
|               | M-CAE   | 0.3283 | 0.5230 | 0.3035 | 0.6429 |
|               | CMAE    | 0.0703 | 0.0983 | 0.0700 | 0.2277 |
|               | BMAE    | 0.1582 | 0.1587 | 0.1250 | 0.2297 |
flip operation (RSM$_{w}$), and the reversible surrogate model without flip operation (RSM$_{wo}$).

Table 4 lists the comparison results. In the table the vanilla deep regression models is general SegNet model. For the proposed method, both Net$_1$ and Net$_2$ select the SegNet model. The proposed method including the RSM$_{w}$ and RSM$_{wo}$ is better than vanilla deep regression model. Especially, the proposed method can better reconstruct the temperature field near the boundaries. From the view of BMAE, the proposed method with flip operation presents a BMAE of 1.0483K, 0.9003K, 0.4560K, 0.5751K over Data A, B, C, D, respectively, is better than the performance of vanilla regression models where the BMAE is 2.3134K, 8.2632K, 2.9737K, 14.0953K, respectively. From Fig. 7 it is also obvious that the proposed method can better reconstruct the temperature field, especially the field near the boundaries. Furthermore, compare the experimental results between the RSM$_{w}$ and RSM$_{wo}$, and we can find that the flip operation can improve the reconstruction performance. This is because that the forward and backward of convolutional layer in deep model is conducted orderly, which makes the final calculated area cannot capture the physical information well. Just as Fig. 7(b), 7(d), 7(f), and 7(h) shows, the temperature field near the upside and rightside boundary cannot be well reconstructed.

4.4. Ablation Studies
4.4.1. Performance with Different Training Samples

In this subsection, we conduct experiments of the proposed method with different number of training samples. The number of training samples is chosen from \{1000, 2000, 5000, 10000, 20000, 40000\}. Fig. 8 shows the reconstruction performance under different metrics over the four datasets.

Just the figure shows, the reconstruction performance tends to be better with the increase of the training samples. More training samples can help the proposed surrogate model to better learn the physical correlation between different points in the system. It should also be noted that when the number of training samples increases to a certain level, the improvement of performance
Figure 7: Examples of the reconstruction results over different datasets by proposed reversible regression model with flip operation and vanilla deep regression model. (a), (c), (e), (g) describes example by proposed method over Data A, B, C, D, respectively; (b), (d), (f), (h) describes example by vanilla model over Data A, B, C, D, respectively.
Table 4: Reconstruction performance (K) of proposed reversible regression model and the vanilla deep regression model for TFR-HSS task. In the table, ‘Vanilla’ represents the vanilla deep regression model, ‘RSM\_wo’ means the reversible surrogate model without flip operation and ‘RSM\_w’ means the reversible surrogate model with flip operation.

| Models | Metrics | Data A  | Data B  | Data C  | Data D  |
|--------|---------|---------|---------|---------|---------|
| Vanilla| MAE     | 0.7640  | 2.4992  | 0.9532  | 3.5093  |
|        | M-CAE   | 4.4597  | 14.1941 | 6.5920  | 15.7004 |
|        | CMAE    | 0.2275  | 0.8505  | 0.3544  | 1.4107  |
|        | BMAE    | 2.3134  | 8.2632  | 2.9737  | 14.0953 |
| RSM\_wo| MAE     | 0.5729  | 0.3028  | 0.6559  | 0.3325  |
|        | M-CAE   | 3.3061  | 1.8298  | 5.6617  | 3.1053  |
|        | CMAE    | 0.1949  | 0.1220  | 0.2570  | 0.3202  |
|        | BMAE    | 1.7699  | **0.8976** | 2.1814 | **0.3654** |
| RSM\_w | MAE     | **0.3436** | **0.2655** | **0.1727** | **0.2456** |
|        | M-CAE   | **1.4340** | 3.6059  | **1.7394** | **1.7043** |
|        | CMAE    | **0.0883** | **0.1006** | **0.1157** | **0.1902** |
|        | BMAE    | **1.0483** | 0.9003  | **0.4560** | 0.5751  |
Figure 8: Performance of proposed method with different number of unlabelled training samples for TFR-HSS task.

tends to be slight due to the limited of the training loss as well as the deep model. This indicates that other effective training methods and representative deep models are required to extract more useful physical information for better reconstruction performance.

4.4.2. Performance with Different Hyperparameters

In order to reconstruct the temperature field unsupervisedly with only monitoring information, the proposed method construct four different losses in order to use the physical information of temperature field, namely the Point loss, the BC loss, the Laplace loss, and the TV loss (see subsection 3.3 for details). These four losses play a different role in the training of the deep model. To show the effect of different losses have on the reconstruction performance, this subsection compares the performance of the proposed method without the supervision
of one of these losses.

Table 5 shows the comparison results where \( \alpha = 0 \) denotes the proposed training without BC loss, \( \beta = 0 \) denotes the proposed training without Laplace loss, and \( \gamma = 0 \) describes the proposed training without the TV loss. First of all, from the table we can find that even though some performance obtains a slight drop with the proposed method, the proposed method can help to improve the most of the performance under all these four metrics. For Data A, when \( \beta = 0 \) the MAE is better than the original one. We can find that when \( \beta = 0 \), the BMAE tends to be 0.6108K which is better than 1.0483K by original one. This means that for Data A, the slight drop of performance of original training using laplace loss is mainly because of the BC loss instead of the laplace loss.

Inspect the reconstruction performance with \( \gamma = 0 \), and we can find that the TV loss which takes advantage of the neighbor correlations plays an important role in the reconstruction of temperature field near the boundary as well as the field over the component. From the table, it can be also noted that the performance drops the most without the TV loss when compared with all the other losses.

In addition, compare the performance when \( \alpha = 0 \) with the original one, we can find that the BMAE drops the most. This indicates that the BC loss plays an important role in the temperature field reconstruction near the boundaries.

Overall, all the training losses in the proposed method play an important role in temperature field reconstruction. Under jointly learning of all these losses, the proposed model can better learn the physical information of temperature field and reconstruct the temperature field with expected small error.

4.5. Comparisons with Other Methods

This work uses the global gaussian interpolation (GGI), Gaussian Process Regression (GPR) [42], support vector regression (SVR) [12], polynominal regression(PR) [43], neural networks (NN) [21] as baselines.

The global gaussian interpolation is proposed by us which can utilize the global information instead of the local information. The reconstructed temper-
Table 5: Reconstruction performance (K) of proposed method with different hyperparameters for TFR-HSS task.

| Setting | Metric | Data A | Data B | Data C | Data D |
|---------|--------|--------|--------|--------|--------|
| α = 0   | MAE    | 0.5582 | 0.6050 | 0.6372 | 0.6942 |
| β = 0   |        | 0.2144 | 0.5583 | 0.6432 | 0.3930 |
| γ = 0   |        | 1.3561 | 1.7778 | 1.5841 | 2.6735 |
| original|        | 0.3436 | 0.2655 | 0.1727 | 0.2456 |
| α = 0   | M-CAE  | 2.4146 | 1.4672 | 2.2615 | 5.2712 |
| β = 0   |        | 1.3816 | 6.6986 | 4.1077 | 2.2869 |
| γ = 0   |        | 9.3194 | 14.1046| 6.9657 | 25.2049|
| original|        | 1.4340 | 3.6059 | 1.7394 | 1.7043 |
| α = 0   | CMAE   | 0.1471 | 0.1310 | 0.2799 | 0.3935 |
| β = 0   |        | 0.1180 | 0.1787 | 0.2309 | 0.3156 |
| γ = 0   |        | 0.7397 | 0.6592 | 0.6852 | 2.9438 |
| original|        | 0.0883 | 0.1006 | 0.1157 | 0.1902 |
| α = 0   | BMAE   | 1.6682 | 1.5572 | 1.6145 | 1.9514 |
| β = 0   |        | 0.6108 | 1.9804 | 1.9854 | 0.8724 |
| γ = 0   |        | 3.8190 | 9.5846 | 5.5920 | 4.8308 |
| original|        | 1.0483 | 0.9003 | 0.4560 | 0.5751 |
ature at \((x_0, y_0)\) is related to all the monitoring points, and it can be formulated as

\[
T(x_0, y_0) = \sum_{i=1}^{m} e^{-| (x_0 - x_{s_i})^2 + (y_0 - y_{s_i})^2 |} f(x_{s_i}, y_{s_i}).
\] (26)

The Gaussian process regression is implemented with the dace toolbox. For polynomial regression, the degree of the polynomial fit is set to 5. For neural network, the structure of the network is set to ‘2-10-10-1’. The support vector regression is realised with the ‘sklearn.svm’ package.

Table 6 lists the comparison results with these former methods. Inspect the table and we can obtain the following conclusions. First, the proposed method can obtain a better performance when compared with other methods. The proposed method can obtain a MAE of 0.1503K, 0.1510K, 0.0931K, and 0.1487K over the four datasets, respectively, outperforms all the other methods. Besides, the proposed method costs less time for prediction. Table 6 shows the prediction time of 10000 samples. We can find that for all the four datasets, the proposed method costs about 50s for prediction of 10000 testing samples benefiting from the use of GPUs while the fastest of other methods cost 4391.42s (NN over Data B). Especially, the SVR costs 88474.17s for prediction of Data C.

In conclusion, the proposed physics-informed deep learning over reversible regression model can provide a fast prediction, as well as a well reconstruction performance and be better fit for TFR-HSS task.

5. Conclusions

In this work, we give the definition of TFR-HSS task systematically and further develop the physics-informed deep reversible regression model for the task. First, a novel deep reversible regression model with two successive encoder-decoder processes and one flip operation is proposed as the deep surrogate model for the TFR-HSS task. Experiments have shown that the proposed deep surrogate model can better reconstruct the temperature field than general deep regression models, especially over the boundaries. Then, we develop a novel
Table 6: Reconstruction performance (K) of different methods for TFR-HSS task. The time, namely the prediction time is calculated over 10000 test samples.

| Data | Method | Time (s)   | MAE    | M-CAE  | CMAE   | BMAE  |
|------|--------|------------|--------|--------|--------|-------|
|      | GGI    | 13884.73s  | 0.6148 | 3.3857 | 1.4215 | 2.6057|
|      | GPR    | 8596.87s   | 0.2799 | 1.2582 | 0.5000 | 2.4980|
|      | SVR    | 36164.35s  | 0.2703 | 0.5196 | 0.0707 | 2.9585|
|      | PR     | 16421.41s  | 0.5852 | 1.6478 | 1.3492 | 6.2793|
|      | NN     | 4411.96s   | 0.4844 | 3.5185 | 1.1381 | 3.6544|
|      | PIRL   | 52s        | 0.1503 | 0.4059 | 0.1429 | 0.1654|
| Data A| GGI    | 14143.03s  | 0.7128 | 13.5829| 2.6513 | 3.2147|
|      | GPR    | 8783.92s   | 0.3026 | 5.3661 | 1.0045 | 2.1256|
|      | SVR    | 30420.54s  | 0.3190 | 0.4985 | 0.0698 | 3.6980|
|      | PR     | 16766.82s  | 0.6410 | 6.2734 | 2.0138 | 5.9980|
|      | NN     | 4391.42s   | 0.5188 | 9.2131 | 1.7123 | 3.4599|
|      | PIRL   | 47s        | 0.1510 | 0.4631 | 0.1393 | 0.1656|
| Data B| GGI    | 16517.52s  | 0.7693 | 19.3481| 2.4615 | 3.6707|
|      | GPR    | 11754.00s  | 0.2998 | 7.9193 | 0.8648 | 2.0172|
|      | SVR    | 88474.17s  | 0.2563 | 0.3905 | 0.0711 | 3.5960|
|      | PR     | 18569.39s  | 0.7478 | 9.0079 | 1.7457 | 8.8377|
|      | NN     | 6796.88s   | 0.6091 | 13.6171| 1.6593 | 5.1221|
|      | PIRL   | 47s        | 0.0931 | 0.2941 | 0.0820 | 0.1502|
| Data C| GGI    | 16812.54s  | 0.541 | 3.9    | 0.331  | 0.709 |
|      | GPR    | 11673.16s  | 0.458 | 1.7    | 0.303  | 1.26  |
|      | SVR    | 73571.12s  | 0.921 | 15.8   | 0.652  | 2.12  |
|      | PR     | 18654.51s  | 0.813 | 3.48   | 0.585  | 2.43  |
|      | NN     | 6857.72s   | 0.693 | 3.3    | 0.539  | 1.64  |
|      | PIRL   | 47s        | 0.1487 | 0.6505 | 0.1674 | 0.1941|
| Data D| GGI    | 16812.54s  | 0.541 | 3.9    | 0.331  | 0.709 |
|      | GPR    | 11673.16s  | 0.458 | 1.7    | 0.303  | 1.26  |
|      | SVR    | 73571.12s  | 0.921 | 15.8   | 0.652  | 2.12  |
|      | PR     | 18654.51s  | 0.813 | 3.48   | 0.585  | 2.43  |
|      | NN     | 6857.72s   | 0.693 | 3.3    | 0.539  | 1.64  |
|      | PIRL   | 47s        | 0.1487 | 0.6505 | 0.1674 | 0.1941|
physics-informed reconstruction loss for TFR-HSS task, including the laplace loss, the point loss, the bc loss and the tv loss. With the proposed method, deep surrogate model can be trained unsupervisedly. The experimental results also demonstrate that the temperature field can be well reconstructed. Besides, compared with commonly used gaussian regression method and other machine learning methods, the proposed method can provide better reconstruction performance.

As future work, it would be interesting to design the special layer which can better extract the intrinsic physical properties for TFR-HSS task. Besides, investigating other physics-informed training loss which can better train the deep surrogate model is another interesting topic. Reducing redundant monitoring points while maintaining the reconstruction performance is also worthy of deep study.

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Algorithm 1 The framework of the proposed method for TFR-HSS task

**Input:** Training samples \( \{f_1, f_2, \cdots, f_n\} \), Testing samples \( \{f_{t1}, f_{t2}, \cdots, f_{tk}\} \), surrogate mapping \( \Phi \), hyperparameter \( \alpha, \beta, \gamma \).

**Output:** \( \Phi^* \)

1: // Training process: Train the model
2: while not converge do
3:   Reconstruct the temperature field with surrogate mapping by \( T_i = \Phi(f_i)(i = 1, 2, \cdots, n) \).
4:   Compute the point loss \( L_{point} \) using Eq. 6.
5:   Compute the laplace loss \( L_{laplace} \) using Eq. 16.
6:   Compute the TV loss \( L_{tv} \) using Eq. 20.
7:   Compute the bc loss \( L_{bc} \) using Eq. 9.
8:   Compute the training loss \( L \) using Eq. 21.
9:   Update \( \Phi \) using training loss \( L \) by auto-grad.
10: end while
11: Provide the optimized surrogate mapping \( \Phi^* \).

12: // Prediction process: Predict temperature field with trained model
13: Predict the temperature field of testing samples using \( T_{ti} = \Phi^*(f_{ti})(i = 1, 2, \cdots, k) \).

14: // Evaluation process: Evaluate the performance of trained model
15: Evaluate surrogate mapping \( \Phi^* \) under MAE, CMAE, M-CAE, and BMAE using Eq. 22-25.
16: return \( \Phi^* \).