Cross-stream migration of asymmetric particles driven by oscillating shear

M. Laumann¹, P. Bauknecht², S. Gekle², D. Kienle¹(α) and W. Zimmermann¹

¹Theoretische Physik I, Universität Bayreuth - 95440 Bayreuth, Germany
²Biofluid Simulation and Modeling, Universität Bayreuth - 95440 Bayreuth, Germany

received 23 December 2016; accepted in final form 13 March 2017
published online 6 April 2017

PACS 47.15.G – Low-Reynolds-number (creeping) flows
PACS 47.57.ef – Sedimentation and migration
PACS 83.50.-v – Deformation and flow

Abstract – We study the dynamics of asymmetric, deformable particles in oscillatory, linear shear flow. By simulating the motion of a dumbbell, a ring-polymer, and a capsule we show that cross-stream migration occurs for asymmetric elastic particles even in linear shear flow if the shear rate varies in time. The migration is generic as it does not depend on the particle dimension. Importantly, the migration velocity and migration direction are robust to variations of the initial particle orientation, making our proposed scheme suitable for sorting particles with asymmetric material properties.

Introduction. – During the recent years, microfluidics has evolved to a cross-disciplinary field, ranging from basic physics to a plethora of biological and technical applications [1–7], including the control of small amounts of fluids, chemical synthesis [8,9], biological analysis [10,11], and the study of the deformation dynamics of droplets, vesicles, capsules, or blood cells [12–40]. An important transport mechanism in microfluidic flows is the cross-stream migration (CSM), where particles move across streamlines and can be sorted due to their particle-specific properties [41,42].

The CSM effect has been first reported in 1961 by Segre and Silberberg for rigid particles at finite Reynolds number in pipes with diameters of several millimeters [43]. When channels approach the micrometer scale, the Reynolds number vanishes (Stokes regime) and fluid inertia does not matter; likewise, for μm-sized particles thermal effects can be discarded. In the Stokes regime, CSM arises in curvilinear [12–15] and rectilinear flow [16–20], if the particle is elastic and, in case of rectilinear flow, the flow’s fore-aft symmetry is broken, requiring intra-particle hydrodynamic interaction [16–19]. Such symmetry breaking occurs near boundaries via wall-induced lift forces [19,21–23] or by space-dependent shear rates, so that dumbbells [16–18], droplets [24–26], vesicles and capsules [27–29] exhibit CSM even in unbounded flow. These parity breaking mechanisms may be accompanied by other effects due to viscosity contrast [24,44] or particle chirality [45], which further impact the CSM.

Here we show that a controlled cross-stream migration is possible even in unbounded linear shear flow, provided that 1) the particle holds an intrinsic asymmetry (parity breaking), and 2) the shear rate varies in time, causing time-dependent particle deformations. Importantly, the cross-stream migration occurs irrespective of the dimensionality of the particle, accentuating its generic nature, as we show by studying particles extending in one (1D), two (2D), and three (3D) dimensions. We demonstrate that the CSM depends on external flow parameters such as switching period, which can be controlled conveniently to achieve an optimized migration.

Model and approach. – To reveal the generic behavior of the CSM in oscillatory shear flow, we use three kinds of particles, which share the common features that they are deformable, asymmetric, and their constituent parts interact hydrodynamically. The first two particle types are a dumbbell (1D) and a ring-polymer (2D), modeled by a sequence of bead-spring units with the i-th bead located at \( \mathbf{r}_i \) and connected to its nearest neighbors by linear springs.

(α)E-mail: diego.kienle@uni-bayreuth.de (lead author and project coordination)
with an equilibrium bond length \( b \) and force constant \( k \). The dumbbell asymmetry is modeled by assigning different friction coefficients \( \zeta_1 \) and \( \zeta_2 \) to unequal sized beads 1 and 2 with \( r_2 = \zeta_2/\zeta_1 = 3 \) (fig. 1(a), inset). The asymmetry of the \( N \)-bead ring-polymer is realized by a space-dependent bending stiffness \( \kappa(\{r\}) \) along the ring contour \( \{r\} \). The third particle is an elastic capsule (3D), the stiff and bendy portion in either case has a bending stiffness of \( \kappa_2 \) and \( \kappa_1 \) with a ratio \( r_\kappa = \kappa_2/\kappa_1 = 1.5 \) (fig. 1(b) and (c), inset).

The migration behavior of all three particle kinds is obtained from their non-Brownian trajectories. The trajectories for the dumbbell and ring-polymer, exposed to an unperturbed flow field \( \mathbf{u}(\mathbf{r}) \), are determined by solving the standard Stokesian dynamics for bead-spring models [46],

\[
\mathbf{r}_i = \mathbf{u}(\mathbf{r}_i) + \sum_{j=1}^{N} \mathbf{H}_{ij} \cdot \left[ \mathbf{F}_{ij}^{bo} + \mathbf{F}_{ij}^{be} \right].
\]

\( \mathbf{F}_{ij}^{bo} \) and \( \mathbf{F}_{ij}^{be} \) refer to harmonic bonding and bending forces, obtained from the potentials \( U^{bo} = \sum_{i=1}^{N} \frac{k}{2}(R_i - b)^2 \) and \( U^{be} = -\sum_{i=1}^{N} \kappa(r_i) \ln[1 + \cos \alpha_i] \). \( R_i = |\mathbf{r}_i - \mathbf{r}_{i+1}| \) denotes the absolute value of the bond vector and \( \cos \alpha_i = \mathbf{e}_{R_{i-1}} \cdot \mathbf{e}_{R_i} \) is the angle between the bond vectors \( \mathbf{R}_{i-1} \) and \( \mathbf{R}_i \) with \( \mathbf{e}_{R_i} = \mathbf{R}_i/|\mathbf{R}_i| \) the bond unit vector. The hydrodynamic interaction (HI) between bead \( i \) and \( j \), inducing a hydrodynamic backflow (HB), is included in eq. (1) via the mobility matrix \( \mathbf{H}_{ij} \) within the Oseen tensor [46],

\[
\mathbf{H}_{ij} = \begin{cases} \frac{1}{8\pi N ij} \left[ I + \mathbf{e}_{R_{ij}} \otimes \mathbf{e}_{R_{ij}} \right] & i \neq j, \\ \frac{1}{6} & i = j \end{cases}
\]

with \( \mathbf{e}_{R_{ij}} = \mathbf{R}_{ij}/|\mathbf{R}_{ij}| \) and \( \mathbf{R}_{ij} = \mathbf{r}_i - \mathbf{r}_j \). The capsule path is calculated using the immersed boundary method in conjunction with the lattice Boltzmann method for the flow [47–49], employing an adapted version of the ESPResSo package [50]. Throughout we assume a time-dependent (td), linear shear flow \( \mathbf{u}(x, y) = S(t) y \mathbf{e}_x \) along the \( \mathbf{e}_x \)-axis; the shear rate \( S(t) \) has a period \( T \) with \( S(t) = +\gamma \) during the first half-period \( T_1 \) and \( S(t) = -\gamma \) during the second half-period \( T_2 \) with \( T_1 = T_2 = T/2 \). The initial orientation of all three particles is \( \phi_0 = 2.0 \pi \) with the small \( \zeta_1 \)-bead, respectively, the stiff \( \kappa_2 \)-contour/surface being located to the left.

**Generic behavior.** – Figure 1 shows the transverse component of the center of drag \( y_c(t) \) (\( \zeta \)-weighted), scaled with respect to the bond length \( b \) of the bead-spring unit or the capsule radius \( a \), as a function of the scaled time \( t_{\gamma} \) with fixed \( \gamma \) for all three particles. For symmetric particles \( (r_\kappa = 1.0) \), the cross-stream migration is zero at any time (dashed line) [14,17] as parity breaking does not occur irrespective of whether the shear flow is stationary (ss) or time dependent (td). For asymmetric particles \( (r_\kappa > 1.0) \) in stationary shear flow parity breaking exists, resulting indeed in a temporary CSM, as reflected in the oscillatory behavior of \( y_c(t) \), whereas the net migration over one shear-cycle is still zero (red solid line). This interim migration of asymmetric particles can be exploited to attain a net cross-stream migration, if the shear rate \( S(t) \) is made time dependent by switching \( S(t) \) at a frequency \( 1/T \), as shown in fig. 1(a)–(c) by the blue solid.

![Fig. 1: (Color online) Lateral position \( y_c(t) \) vs. scaled time \( t_{\gamma} \) for the asymmetric (a) 1D dumbbell, (b) 2D ring, and (c) 3D capsule, sketches of which are shown in the inset. Irrespective of the model details, all particle types perform a net cross-stream migration in linear shear (blue solid), if the shear rate is time-dependent (td). At steady shear (ss), \( y_c(t) \) oscillates around a constant mean (red solid), so that the net migration vanishes [14,17]. For symmetric particles \( (r_\kappa = 1.0) \), the migration is zero. The initial orientation is \( \phi_0 = 2.0 \pi \).](image-url)
line. The fact that all three particles display cross-stream behavior irrespective of their dimensionality and model details is an indication of a **generic** property\(^1\), which can be attributed to the different **mean** shapes the particle acquires during each half-period, as discussed next.

**Migration mechanism.** – To understand the CSM mechanism, we take a closer look at the cross-stream dynamics of an asymmetric dumbbell \((r_\zeta > 1)\) and an asymmetric ring \((r_\kappa > 1)\), consisting of \(N\)-bead-spring units; we note that the discussion provided for the ring is general insofar as it applies for the 3D capsule as well where the ring is viewed as a 2D cut through the capsule plane of symmetry. To keep the explanation of the CSM mechanism transparent, we focus in either case on the **steady-state** regime (approached by all three particles after a transient), where the dumbbell and the ring have adopted a stable **mean** orientation \(\langle \phi \rangle_{T/2} \) or \(\langle \phi \rangle_T\), as determined by averaging their orientation angle \(\phi(t)\) over a half or full shear-cycle, respectively.

Starting with the dumbbell, one can derive from eq. (1) a closed-analytical expression for the instantaneous cross-stream velocity \(v_m(t)\) of its \(\zeta\)-weighted center \(y_c(t)\), as detailed in footnote \(^1\) and given by

\[
v_m(t) = \frac{k}{4\eta} \frac{r_\zeta - 1}{r_\zeta + 1} \frac{R(t) - b}{R(t)} \sin(\phi(t)),
\]

with \(\eta\) the viscosity, \(\phi(t)\) the orientation angle, and \(R(t) = |r_1 - r_2|\) the distance between bead 1 and 2, as introduced in fig. 1(a). Equation (3) facilitates reading off various, well-known limiting cases: in linear shear flow CSM does not occur at any time \((v_m(t) = 0)\), irrespective of whether the flow is stationary or time-dependent, if the dumbbell is i) symmetric \((r_\zeta = 1)\), ii) very soft (small \(k\)), or iii) if \(\text{HI is absent (free-draining)}\) or weak, as realized for large bond lengths \(b\) [17].

Once the dumbbell is asymmetric \((r_\zeta > 1)\) and simultaneously deformed (finite \(k\)), \(v_m(t) \neq 0\), a **net** migration may be possible. Even though the precise conditions for a net migration step \(\Delta y_c\) during one half-cycle can be obtained only by integrating eq. (3), one can still gain important insights on the CSM mechanism by a qualitative inspection of eq. (3) and how the various terms interplay. First, as long as the switching period \(T\) and the shear rate \(\dot{\gamma}\) are not too large to avoid full turnovers, the dumbbell orientation \(\phi(t)\) oscillates (after a transient regime) around a mean angle \(\langle \phi \rangle_{T/2} = 3\pi/2\) or \(\pi/2\), depending on the initial orientation \(\phi_0\). When \(\langle \phi \rangle_{T/2} = 3\pi/2\) \((\pi/2)\), we observe that \(\sin(\phi(t))\) remains negative (positive) over the entire half-cycle and becomes largest once \(\phi(t) \approx 3\pi/2\) \((\pi/2)\), i.e., the dumbbell is perpendicular to the flow direction, as shown in fig. 2(a). The bond length \(R(t)\) oscillates also around the equilibrium bond length \(b\), so that the term \(R(t) - b\) in eq. (3) alters its sign (fig. 2(a)), causing the instantaneous CSM velocity \(v_m(t) = \dot{y}_c(t)\) to oscillate (fig. 1(a)). Therefore, the sign of the net CSM depends on whether the positive or negative migration increments to \(\Delta y_c\) contribute most during the half-cycle.

Based on eq. (3), one may expect that \(\Delta y_c < 0\) because \(\sin(\phi(t) < 0\) over the whole half-cycle while the deformation \(R(t) - b\) is asymmetric such that the dumbbell is stretched more strongly \((R(t) - b > 0)\) than being compressed over \(T/2\), as demonstrated in fig. 2(a) by plotting the respective terms of eq. (3); the asymmetry of \(R(t) - b\) can be ascribed to the larger difference of the flow velocity between and hence larger drag on the beads when the dumbbell is stretched. However, a mean CSM with \(\Delta y_c < 0\) is in clear contradiction to our numerical results, shown in fig. 1(a). The origin for the net migration step \(\Delta y_c\) being **positive** can be attributed to the non-linear behavior of the hydrodynamic interaction, appearing in eq. (3) via the \(1/R(t)\)-term, so that deformations of a compressed dumbbell receive a larger negative weight; fig. 2(a) displays the respective behavior of \((R(t) - b)/R(t)\), which is amplified furthermore by the peaking of \(\sin(\phi(t)\) when \(R(t) - b < 0\). Hence, the positive contributions during the dumbbell compression outbalance the negative ones when the dumbbell is stretched. Our qualitative analysis, based on eq. (3), indicates that \(\Delta y_c\) is positive (negative) when the dumbbell swings around \(\langle \phi \rangle_{T/2} = 3\pi/2\) \((\pi/2)\). This qualitative picture is consistent with fig. 2(b), showing the evolution of \(y_c(t)\) by integrating eq. (3) with the migration step \(\Delta y_c > 0\) at the end of one half-cycle \(T/2\), and is in full agreement with our numerical result shown

---

\(^1\)The Supporting Information (SI) contains further details on the models, a derivation of eq. (4), plots and movies on the generic behavior (including an extension of the orientation robustness) of the dumbbell and capsule (Movie1.mp4, Movie2.mp4, Movie3.mp4, Movie4.mp4, Movie5.mp4, Movie6.mp4, Movie7.mp4, Movie8.mp4, Movie9.mp4, and SupplementaryMaterial.pdf). A link of the abrupt \(v_m\)-drop to the \(\langle \phi \rangle_T\)-attractors is provided, too.
in fig. 1(a). Finally, we note that for a rigid dumbbell \( R(t) = 0 \) the migration step \( \Delta y_c = 0 \) when swinging around \( \langle \phi \rangle_{T/2} = 3\pi/2 \) (\( \pi/2 \)), as one can show by solving eq. (3) analytically (see footnote \(^1\)).

We now inspect the CSM behavior of the 2D ring and assume again that both the period \( T \) and the shear rate \( \dot{\gamma} \) are not too large as to prevent the ring dynamics being dominated by tank-treading, causing a net zero migration, as discussed in the following paragraphs. Under this condition and for an initial orientation \( \phi_0 \), the ring adopts (after a transient) one stable mean orientation \( \langle \phi \rangle_T \) over one shear-cycle \( T \). Specifically, for \( \phi_0 = 2.0 \pi \) the mean orientation of the ring is \( \langle \phi \rangle_T \approx 1.75 \pi \) with the stiff contour located in the upper (left) half-space and referred to below. We note that other initial orientations \( \phi_0 \) may lead to one of the other possible mean orientations with \( \langle \phi \rangle_T \approx 1.25 \pi, \approx 0.75 \pi, \) or \( \approx 0.25 \pi \), where the ring (and capsule) displays CSM. Importantly, the explanation of the CSM mechanism provided for the mean orientation \( \langle \phi \rangle_T \approx 1.75 \pi \) and shown in fig. 1(b) applies irrespective of the specific value of \( \langle \phi \rangle_T \).

Since a closed semi-analytic expression of the CSM velocity similar to eq. (3) is not possible beyond dumbbell models, we analyze the migration of the ring in terms of the mean steady-state CSM velocity \( v_m^s = \langle v_m(\infty) \rangle_T \) (along the \( y \)-axis) for each half-period \( T_i \), obtained by averaging the velocity \( v_m(t) \) over \( T_i \) (see footnote \(^1\)),

\[
v_m^s = \langle e_y \cdot \dot{r}_e(\infty) \rangle_T = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq i} \langle e_y \cdot H_{i,j} \cdot F_j \rangle_T. \tag{4}
\]

Equation (4) implies that for rectilinear flows with \( e_y \cdot u = 0 \), the cross-stream transport is entirely driven by the particle drag due to the hydrodynamic backflow, induced by the potential forces \( F \). Further, the magnitude and direction of each HB (and hence of \( v_m^s \)) depend on the particle shape via the force profile \( F \) and the dyadic mobility matrix \( H_{i,j} \). An expression similar to eq. (4) can be used to determine the mean HI-induced flow field \( v(r) \) for each half-period (see footnote \(^1\)). The respective 2D backflow \( v(r) \), shown in fig. 3(a) and (b) when \( \langle \phi \rangle_T \approx 1.75 \pi \), corresponds to an elongational flow, whose flow lines are reversed (sign change) as a result of the altering ring deformation during the \( S(t) \)-switching (+\( \gamma \rightarrow -\gamma \)), and displayed more clearly in fig. 3(c) and (d). The ring asymmetry causes generally a break of the parity symmetry (PS) of the elongational HB, but the extent of the PS violation depends on the strength of the mean deformation during each half-cycle \( T_i \) (fig. 3(c) and (d)).

Comparing the mean deformation for each half-cycle, one observes that the ring asymmetry is enhanced during the first \( T_1 \) shear-cycle, causing an increased parity break of the elongational HB (fig. 3(c)). But this implies that the difference between the mean opposing partial HB-drags at the stiff and bendy side, \( v_m^s \) and \( v_m^b \) (see footnote \(^1\); eq. (9) of the SI), becomes larger with \( |v_m^s| > |v_m^b| \) since \( \kappa_2 > \kappa_1 \).

Hence, the mean migration velocity during the \( T_1 \)-cycle, \( v_m^s = v_m^s + v_m^b \), is large and positive. In turn, during the \( T_2 \) shear-cycle the situation is reversed as the ring shape is roughly circular, i.e., the ring asymmetry is reduced with the result that the parity of the elongational backflow is partly recovered (fig. 3(b) and (d)), and the opposing partial HBs almost cancel. The reason for the residual backflow is because the HB-drag at the stiff contour part is slightly larger than at the bendy side (\( |v_m^s| > |v_m^b| \)), as a result of the larger stiffness. During the \( T_2 \)-cycle the mean migration step \( v_m^s = v_m^s + v_m^b \) is thus small and negative. Over the course of one shear-cycle \( T = T_1 + T_2 \), the net migration \( v_m = v_m^s + v_m^b \) is therefore positive, as displayed by all three kinds of particles (fig. 1(a)–(c)).

**Orientation robustness.** We now demonstrate that the CSM effect is quite robust against a dispersal of initial orientations by varying the angle \( \phi_0 \), while keeping the orientation axis within the \( y \)-x shear plane (tilt angle \( \theta_0 = 0 \)). Figure 4 shows the instantaneous migration velocity of the ring \( v_m(t) \tau_{T}/b \), averaged over one cycle \( T \), vs. time \( t \) (\( \phi = 0.1 \) fixed, \( \tau_{T} = \eta \beta \gamma^{-1/2} \)) for various orientations \( \phi_0 \) within the intervals \( I^+ = [1.1; 2.0] \pi \) and \( I^− = [0.1; 1.0] \pi \). The interval \( I^+ \) (\( I^− \)) corresponds to ring
orientations, where during migration the stiff part lies in the mean within the upper (lower) half-space with the orientation angle $\phi(t)$ oscillating either around the mean $\langle \phi \rangle_T \approx 1.75 \pi$ or $1.25 \pi$ ($\langle \phi \rangle_T \approx 0.75 \pi$ or $0.25 \pi$). As discussed before, this implies that the final, steady-state migration velocity $v_m \equiv \langle v_m(\infty) \rangle_T$, is positive for $\phi_0 \in I^+$ and negative for $\phi_0 \in I^-$, as disclosed in fig. 4 for $t\dot{\gamma} > 9.0 \cdot 10^3$. Remarkably, the ring migrates always at the same steady-state speed $v_m^{\infty}$ even though the $\phi_0$-orientation varies by almost $\Delta \phi_0 \approx \pi$, meaning $v_m^{\infty}$ is independent of $\phi_0$. In turn, the choice of $\phi_0$ determines strongly the short-time dynamics of $\langle v_m(t) \rangle_T$, as shown in fig. 4 for $t\dot{\gamma} < 9.0 \cdot 10^3$. This imbalance of the magnitude and in part the sign of $\langle v_m(t) \rangle_T$ is a transient signature and exists as the orientation angle $\phi(t)$ is not yet in-phase with the shear signal $S(t)$; the phase synchronization of the angle takes place gradually within the transient regime over many shear-cycles $T$ before a phase-locking is established.

The behavior of $\langle v_m(t) \rangle_T$, shown for the ring in fig. 4, is generic and displayed by the other particle types (see footnote 1).

We note that while the migration persists ($v_m \neq 0$) in most cases when the tilt angle $\theta_0$ of the particle axis and the $y$-$x$ shear plane is non-zero (accentuating the robustness of the CSM effect), some signatures of the migration alter when $\theta_0 \neq 0$ and depend on the particle type, which we briefly summarize below with more details provided in the SI (see footnote 1). In case of the dumbbell, the tilt angle $\theta(t)$ always relaxes back towards the $y$-$x$ shear plane ($\theta(\infty) = 0$) for any value of $\theta_0 \in [0.0; \pi/2]$, so that $\langle \theta \rangle_T^F = 0$ is an asymptotically stable fixed point; only for one tilt angle $\theta_0 = \pi/2$, the dumbbell retains its initial orientation within the $z$-$x$ plane, in which case $\langle \theta \rangle_T^F = \frac{\pi}{2}$ is a neutral stable fixed point and corresponds to a non-migrating state ($v_m = 0$). The dumbbell migration is, therefore, robust against $\theta_0$-variations over the entire interval $\pi/2$. The capsule behaves likewise and exhibits orientational relaxation as well, except that the previous robustness interval for $\theta_0$ is reduced to $[0; \frac{\pi}{4}]$ with $v_m \neq 0$, while the residual interval $[\frac{\pi}{4}; 1.0] \frac{\pi}{2}$ leads to zero migration, as the capsule axis relaxes to the other stable fixed point $\langle \theta \rangle_T^F = \frac{\pi}{2}$. The ring migration differs from the dumbbell and capsule insofar as all tilt angles $\theta_0$ are neutral stable fixed points, i.e., $\langle \theta \rangle_T^F = \theta_0 \in [0; \frac{\pi}{2}]$, implying that the ring keeps its initial $\theta_0$-orientation. The non-relaxation of the tilt angle has the consequence that the ring moves at a different (but constant) speed for each value $\theta_0 \in [0; \frac{\pi}{2}]$. Again, the CSM of the ring is robust over the entire $\theta_0$-interval of $\pi/2$.

**Frequency dependence.** — The migration process is not entirely determined by the material properties of the particle (e.g., stiffness), but can be controlled also by external parameters such as the shear rate $\dot{\gamma}$ or the switching period $T$, the latter being discussed next. Figure 5 shows the steady-state migration velocity $v_mT/\theta_0$ for a fixed $\dot{\gamma} = 0.1$ vs. the period $T\dot{\gamma}$, which sets the time scale for the sign change of the shear rate $S(t)$. When $T\dot{\gamma}$ is small, the migration speed $v_m$ is rather low (regime (1)) since the quickly alternating shear rate $S(t)$ induces only a small shear deformation of the ring shape, so that the ring has not sufficient time to reorient and to fully develop its mean conformation within each half-period $T_1$ and $T_2$, respectively; at these short times tank-treading is still marginal, as sketched in fig. 5. For larger periods $T$, the ring has now more time within each half-cycle to deform and fully adopt the migration state, so that $v_m$ monotonously grows first (regime (2)), approaching a maximum at $T\dot{\gamma} \approx 7$. At this stage, a weak partial tank-treading (TT) of the contour is initiated, but the ring dynamics is still dominated by oscillatory shear deformations, driving the CSM. Beyond a value of $T\dot{\gamma} > 7$ the migration gradually decays since tank-treading becomes increasingly important insofar as a larger fraction of time of each half-cycle $T_1$ is spent on tank-treading.

---

Cross-stream migration in oscillating shear

Fig. 4: (Color online) Instantaneous migration velocity $\langle v_m(t) \rangle_T T/\theta_0$ of the ring, averaged over one period $T$, vs. time $t\dot{\gamma}$ for different initial angles $\phi_0$ taken out of the interval $I^+ = [1.1; 2.0] \pi$ or $I^- = [0.1; 1.0] \pi$. The $\langle v_m(t) \rangle_T$-transient depends on $\phi_0$, while the steady-state value $v_m = \langle v_m(\infty) \rangle_T$ is $\phi_0$-independent. Parameters: $\dot{\gamma} = 0.1, T = 20, r_n = 1.5$.

Fig. 5: (Color online) Steady-state migration velocity $v_mT/\theta_0$ of the ring vs. switching period $T\dot{\gamma}$. Four dynamic regimes are identified: (1) oscillatory shear deformation, indicated by the horizontal arrow, at small $T$; (2) weak tank-treading (TT) superposed with (1), marked by the half-circle arrow; (3) enhanced tank-treading at large $T$; (4) TT-dominated with zero net migration for $T\dot{\gamma} > 11$. Inset: $T$-averaged mean orientation $\langle \phi \rangle_T$ vs. $T\dot{\gamma}$. Parameters: $\phi_0 = 2.0 \pi, \dot{\gamma} = 0.1, r_n = 1.5$. 44001-p5
This implies that a portion of the stiff/bendy contour is now partly shuffled from the upper/lower half-space to the lower/upper one (regime (3)), i.e., the dynamics of the entire shear-cycle takes now place within two half-spaces (with an unequal amount) and each contributes to the CSM with opposite sign. The net velocity \( v_m \) is still positive, since the mean orientation of the ring \( \langle \phi \rangle_T \approx 1.75\pi \) (fig. 5, inset) with the stiff/bendy contour part residing on average within the upper/lower half-space. Beyond \( T\dot{\gamma} > 11 \) the CSM comes to a halt since tank-treading dominates now the dynamics within each half-period, so that even a larger fraction of the stiff/bendy contour is re-shuffled between the upper-lower half-space. Within this TT-dominated regime, the mean orientation flips from \( \langle \phi \rangle_T \approx 1.75\pi \) \((v_m > 0)\) to \( \langle \phi \rangle_T \approx 2.0\pi \) (fig. 5, inset), which corresponds to a symmetric state where equal amounts of the stiff/bendy contour lie in the mean within both half-spaces, so that \( v_m \) is zero (see footnote 1). The abrupt \( v_m \) drop is hence inherently connected with the abrupt change of the mean orientation \( \langle \phi \rangle_T \), which can be understood by inspecting the phase-space \( \langle \phi(t) \rangle_T - \langle \phi(t) \rangle_T \) (see SI in footnote 1 for details). Here we just note that the phase-space features a pattern of discrete, asymptotically stable orientations \( \langle \phi \rangle_{FP} ^T \) (fixed points), which the ring can access. Importantly, the number and value of available \( \langle \phi \rangle_{FP} ^T \) depend sensitively on the switching period \( T \) (see footnote 1). In our case with \( \phi_0 = 2.0\pi \), the only stable orientation the ring can adopt is \( \langle \phi \rangle_{FP} ^T \approx 1.75\pi \) as long as \( T\dot{\gamma} < 11 \) while \( \langle \phi \rangle_T \approx 2.0\pi \) is unstable2. When \( T\dot{\gamma} > 11 \), the previous fixed point at \( 1.75\pi \) disappears, so that the orientation \( \langle \phi(t) \rangle_T \approx 1.75\pi \) is acquired only temporarily while a new orientational attractor appears at \( \langle \phi \rangle_{FP} ^T = 2.0\pi \). Since the value \( 1.75\pi \) lies within the (extended) range of the \( 2\pi \)-attractor, the ring locks in to the mean orientation of \( \langle \phi \rangle_T = 2.0\pi \) (fig. 5, inset), corresponding to a non-migrating state \( (v_m = 0) \) (see footnote 1).

**Conclusions.** — We have shown that deformable particles, which hold an intrinsic asymmetry (parity breaking), display cross-stream migration (CSM) in time-periodic, linear shear flow for medium switching frequencies. The net migration can be attributed uniquely to the particle asymmetry as it leads to an asymmetric force distribution within the periodically deformed particle, inducing asymmetric, non-compensating hydrodynamic backflows (HBs). Since the magnitude and direction of the HBs depend on the actual particle deformation, which is different within the first and second half-period, the HBs averaged over one shear-cycle \( T \) are non-zero, thus leading to a finite CSM (fig. 3). The CSM is generic inasmuch as it does not depend on the particle dimension nor on the specific details of its asymmetry (fig. 1(a)–(c)). While the migration direction is sensitive to whether the stiff/bendy part of the particle resides during one shear cycle in the mean within the upper or lower half-space, the CSM speed approaches after a transient phase a constant value and is independent of the initial particle orientation (fig. 4).

Given that even a small asymmetry in the bending modulus (factor 1.5 or less) of micron-sized particles can trigger for medium channel lengths a sizable migration velocity of \( 20 \text{ mm s}^{-1} \) under realistic flow conditions with a shear rate of \( \dot{\gamma} = 22 \text{ s}^{-1} \) and a period of \( T = 1.75\text{ Hz} \) (see footnote 1), our proposed scheme facilitates appreciable migration distances in compact microfluidic setups just by independently tuning the amplitude and frequency of the shear rate. Investigating effects due to random material inhomogeneities will be an interesting subject for future studies.

**REFERENCES**

[1] Squires T. M. and Quake S. R., Rev. Mod. Phys., **77** (2005) 978.

[2] Whitesides G. M., Nature, **442** (2006) 368.

[3] Popel A. S. and Johnson P. C., Annu. Rev. Fluid Mech., **37** (2005) 43.

[4] Graham M. D., Annu. Rev. Fluid Mech., **43** (2011) 273.

[5] Dahl J. B., Lin J.-M. G., Muller S. J. and Kumar S., Annu. Rev. Chem. Biomol. Eng., **6** (2015) 293.

[6] Sackmann E. K., Fulton A. L. and Beebe D. L., Nature, **507** (2014) 181.

[7] Amini H., Lee W. and Di Carlo D., Lab Chip, **14** (2014) 2739.

[8] Jähnisch K., Hessel V., Löwe H. and Baerns M., Angew. Chem., Int. Ed., **43** (2004) 406.

[9] Elvira K. S., Casadevall I Solvas X., Wootton R. C. R. and deMello A. J., Nat. Chem., **5** (2013) 905.

[10] Yi C., Li C.-W., Ji S. and Yang M., Anal. Chim. Acta, **560** (2006) 1.

[11] Chen J., Li J. and Sun Y., Lab Chip, **12** (2012) 1753.

[12] Shaffer R. H., Laiken N. and Zimm B. H., Biophys. Chem., **2** (1974) 180; Shaffer R. H., Biophys. Chem., **2** (1974) 185.

[13] Aubert J. H. and Tirrell M., J. Chem. Phys., **72** (1980) 2694; Aubert J. H., Prager S. and Tirrell M., J. Chem. Phys., **73** (1980) 4103.

[14] Nitsche L. C., AIChE J., **42** (1996) 613.

[15] Ghigliotti G., Rahimian A., Diros G. and Misset B., Phys. Rev. Lett., **106** (2011) 028101.

[16] Sekhon G., Armstrong R. C. and Jhon M. S., J. Polym. Sci., **20** (1982) 947.

[17] Brunn P. O., Int. J. Multiphase Flow, **9** (1983) 187.

PB and SG thank the Volkswagen foundation for support and gratefully acknowledge the Leibniz Supercomputing Center Munich for the provision of computing time. ML and WZ acknowledge support by the DFG priority program on Micro- and Nanofluidics.
Cross-stream migration in oscillating shear

[18] Brunn P. O. and Chi S., Rheol. Acta, 23 (1984) 163.
[19] Jhon M. S. and Freed K. F., J. Polym. Sci., 23 (1985) 955.
[20] Agarwal U. S., Datta A. and Mashelkar R. A., Chem. Eng. Sci., 49 (1994) 1693.
[21] Cantat I. and Misbah C., Phys. Rev. Lett., 83 (1999) 880.
[22] Seifert U., Phys. Rev. Lett., 83 (1999) 876.
[23] Ma H. B. and Graham M. D., Phys. Fluids, 17 (2005) 083103.
[24] Haber S. and Hetsroni G., J. Fluid Mech., 49 (1971) 257.
[25] Leal L. G., Annu. Rev. Fluid Mech., 12 (1980) 435.
[26] Mandal S., Bandopadhyay A. and Chakraborty S., Phys. Rev. E, 92 (2015) 023002.
[27] Kaoui B., Ristow G. H., Cantat I., Misbah C. and Zimmermann W., Phys. Rev. E, 77 (2008) 021903.
[28] Danker G., Vlahovska P. M. and Misbah C., Phys. Rev. Lett., 102 (2009) 148102.
[29] Dodd S. K. and Bagchi P., Int. J. Multiphase Flow, 34 (2008) 966.
[30] Sibillo V., Pasquariello G., Simeone M., Cristini V. and Guido S., Phys. Rev. Lett., 97 (2006) 054502.
[31] Abkarian M. and Viallat A., Soft Matter, 4 (2008) 653.
[32] Dupire J., Abkarian M. and Viallat A., Phys. Rev. Lett., 104 (2010) 168101.
[33] Baroud C. N., Gallaire F. and Dangra R., Lab Chip, 10 (2010) 2032.
[34] Deschamps J., Kantsler V., Segre E. and Steinberg V., Proc. Natl. Acad. Sci. U.S.A., 106 (2009) 11444.