Freeform surface characterisation: theory and practice

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Abstract. The specification and characterisation of freeform surfaces is immature: many CAD packages only allow the nominal freeform geometry to be specified. To control manufacturing and function, the allowable geometric variability also needs to be specified. Currently International standards specify allowable geometry through geometrical tolerancing. Geometrical tolerancing has proved to be a very blunt instrument with many cases, particularly in the aerospace and biomedical industries, of failure of the function of freeform surfaces due to inadequate specification. The paper begins by discussing the importance of the decomposition of the surface geometry into different scales for both specification and characterisation. The three main types of geometrical decomposition: linear, morphological and segmentation are briefly discussed. The Laplace-Beltrami operator (LBO) is the generalization of the Laplace operator to manifolds (i.e. freeform surfaces). By taking Eigenfunctions/Eigenvalues of the LBO a spectral decomposition of the freeform geometry can be realized. The Eigenfunctions are called manifold harmonics and are a direct generalization of Fourier harmonics for freeform surfaces. Tolerancing the Eigenfunctions provides a very powerful and flexible system to control geometrical variability of freeform surfaces; extending the geometrical tolerancing system by allowing decomposition of the geometrical shapes within tolerance zones. Further a generalized Gaussian filter can also be realized by Gaussian weighting the LBO spectra and reconstructing the weighted Eigenfunctions.

1. Introduction
Geometrical tolerancing consists of a system of specifying tolerance zones in engineering drawings in order to control workpiece variability. It was developed to improve the weakness of previous tolerance systems to handle imperfect form and ambiguous references and was primarily developed for assembly of components. Geometrical tolerancing is still at the core of today’s tolerancing systems: Geometrical Product Specification and Verification (GPS) defined by the International Organisation for Standardization (ISO). The use of tolerance zones is causing real industrial problems in the specification of high valued precision products; two illustrative examples are:

1. In the aerospace industry, the geometry of the tips of turbine blades cannot be controlled by specifying a tolerance zone alone. To distinguish between good and failing functional geometries,
that both satisfy the tolerance zone, requires further mathematical specification to identify allowable geometries from those that are not allowed [1].

2. In healthcare, the specification of the geometrical shape of the cup in total hip-joint replacements by a simple tolerance zone is allowing some cups to fail, by dislocating out of position, prematurely. A new design of non-spherical head is beginning to appear and the market requires improved specification. Again further mathematical specification is required to distinguish between good and failing functional geometries [2,3].

**Traditional metrology** is essentially analogue, using hard and hand gauges to verify an artifact is to specification. The mathematics is encapsulated within the mechanical set-up (jigs and fixtures) and the relationships of the gauges (see figure 1). Post-measurement calculation is relatively simple and, traditionally, carried out by hand. Traditional metrology is restricted mainly to simple geometrical shapes (planes, spheres, cylinders, cones, etc.) that described the idealised geometry of artifacts of the day [4].

**Digital metrology**, through sampling in a co-ordinate system, provides a digital representation of an artifact, enabling computational techniques to be applied (see figure 2). The mathematics is encapsulated in the post-measurement calculations through software implementing algorithms that are becoming increasingly complex. Digital techniques thus enable GPS specification (and artifact manufacture) to move beyond the traditional metrology domain of simple geometrical shapes: to manufacture, measure and analyse almost any desired geometry (including freeform geometries).

![Figure 1 Measuring taper angle with traditional metrology](image1)

![Figure 2 Measuring taper angle with the digital paradigm](image2)

Although there have been recent modifications to increase GPS’s utility, specification is still basically a go/no-go system with components being in or out of tolerance. It still reflects the language, methodologies, and concepts of traditional metrology and has not truly entered the modern digital era.

The Grand Challenge is to make a step change to the geometrical tools in the specification toolbox, beyond tolerance zones, to break the formidable barriers facing a designer in specifying geometries (particularly freeform geometries) and their allowable variability optimised for final functionality and manufacturing processes. In this paper a possible solution to the challenge is discussed and mathematical tools developed to realize a geometrical specification toolbox that has potential utility for current and future manufacturing requirements.

2. **The structure of GPS specification**

The GPS specification process starts by identifying the geometrical features of interest before putting limits on both intrinsic characteristics of the identified features and/or relationships between these features. Geometrical features are defined by a series of four basic operations carried out on a geometrical surface: **sampling**, moving from a continuous object to a discrete object, its dual **reconstruction**, moving from a discrete object to a continuous object, **decomposition** of a surface into different features, and its dual **composition**, building a surface up from different features.
Decomposition is fundamental for specification: it is the operator that creates the different geometrical features of interest. There are currently many different sub-types of decomposition within ISO standards [5] including: partitioning an artefact into different functional surfaces; filtering a surface into features of different sizes; association, i.e., determining the best-fit specified nominal geometrical surface to the sampled points, etc. It is the structure of decomposition that is focused on in this paper. The basic idea is to decompose a tolerance zone into features of different scales.

2.1. Geometrical scales
It is the concept of geometrical scales and their association with decomposition that is introduced in this paper. So what is a geometrical scale and why are they so useful for decomposition? An analogy between GPS and music is used here to illustrate the basic idea.

Beethoven specifies some music (composes). This is manufactured (by playing), see figure 3. How do we know it is manufactured to specification?

Figure 3 Beethoven composes some music which is played

The played music is measured (by listening) and the notes written down on a MUSICAL SCALE (decomposition) to produce a musical score, see figure 4. This score can be then compared to the original specification to check for conformity. The important concept here is that the decomposition requires a scale in order to carry out the decomposition.

Figure 4 Identifying the individual notes on a musical scale to produce a musical score

One very well-known scale is the Fourier scale based on the harmonics of sine and cosine functions. Unfortunately the Fourier transform can only be applied, without distortion, to surfaces with zero Gaussian curvature (i.e. planes, cylinders etc.). Fortunately there is an approach to extend Fourier like scales to surfaces with non-zero Gaussian curvature (i.e. freeform surfaces) without distortion. The missing connection is that sine and cosine functions are Eigenfunctions of the Laplace operator. This extension will be used as an illustrative example of the use of geometrical scales for decomposition in the next section.

2.2. The Laplace-Beltrami operator
The Laplace-Beltrami operator (LBO) is the generalization of the Laplace operator (heat equation) to manifolds (i.e. freeform surfaces) [6]. Since the discrete version of the LBO, for triangular meshes, is very well known it is natural to choose the Eigenfunctions of the LBO as a functional basis for decomposition [7,8]. The Eigenfunctions are called manifold harmonics and are a direct generalization of Fourier harmonics for freeform surfaces. The Eigenvalues correspond to the square of the angular frequency of the surface and form a natural geometrical scale. Figure 5 shows a rendered version of triangular mesh data of a measured artificial knee joint (47,638 vertices and
94,243 faces). Figure 6 shows the corresponding first 16 Eigenfunctions from the LBO of this triangular mesh data (2000 Eigenfunctions and Eigenvalues were calculated in total). As can be clearly seen, starting from the top left and preceding row by row, the Eigenfunctions become progressively more detailed and form a basis for describing the form errors, shape and texture of this particular freeform surface. The one disadvantage of using this approach with the LBO is that the form needs to be removed first before calculating the LBO Eigenfunctions and Eigenvalues otherwise the size changes.

![Figure 5 Measured knee joint](image)

![Figure 6 First 16 Eigenfunctions from the LBO on the knee joint data](image)

Tolerancing the Eigenfunctions provides a very powerful and flexible system to control geometrical variability of freeform surfaces; extending the geometrical tolerancing system by allowing decomposition of the geometrical shapes within tolerance zones. Further, the Eigenfunctions can be used to filter noisy data through a weighted reconstruction (such as Gaussian weights) of the Eigenfunctions using the Eigenvalues as a nesting index [7,8]. When Gaussian weights are used the resulting filter generalizes the Gaussian filter to freeform surfaces.

2.3. Other Geometrical Scales
There are many different types of geometrical scales that have potential utility in GPS from scales based on morphological operations (scale space), segmentation (Wolf pruning), to linear operators (Partial Differential Equations (PDE), Wavelets, Spectral polynomials etc.), see figure 7. Different scales will correlate with different surface functions for example it is anticipated that morphological scales will correspond to contact type phenomenon. With all the above scales once the required surface features have been identified it is fairly straightforward to define both intrinsic characteristics of the identified features and/or relationships between these features and to calculate statistics of these characteristics to define measurands for specification and verification.

3. Summary and further work
This paper has elucidated that whereas metrology and manufacturing have both moved beyond the traditional metrology domain of simple geometrical shapes, specification still reflects the language, methodologies, and concepts of traditional metrology and has not truly entered the modern digital era. This is a severe limitation on the specification of complex geometries such as freeform surfaces. The structure of specification is then discussed showing the importance of decomposition to identify features of interest. The connection between decomposition and geometrical scales is introduced before an example of a geometrical scale, based on the Eigenfunctions and Eigenvalues of the Laplace-Beltrami operator, is illustrated. Finally the characterisation of identified features together with suitable statistics is outlined to define measurands for specification and verification is briefly mentioned.
This paper is a manifesto for an innovative approach to allow specification of complex geometries, particularly specification of the allowable geometrical variation in shape and texture beyond tolerance zones. There are still details to be explored, particularly the correlation between the different scales and function. Another unanswered question is: do PDE scales exist that do not require the form to be removed first before being applied? The future is going to be very exciting developing the concepts and addressing the challenges presented by this manifesto for specification and verification of complex geometries.

4. References

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