Universality in SNIae and the Phillips Relation

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The use of supernovae of Type Ia for the determination of accurate distances rests upon the empirical Phillips relation, in which the brightest events are the broadest in time. Implications of new data upon the homogeneity of light curves under the operation of a stretch in time, of the parabolic luminosity increase at the earliest times, and of the time from explosion to maximum light are discussed. The early luminosity is in excellent agreement with the predictions of [Arnett (1982)] and the lack of prominent higher modes of diffusion constrain progenitor and explosion models. Difficulties with reproducing the observed rise time are restricted to radiative transfer models (e.g., Höflich & Khokhlov (1996)), and probably due to an overestimate of thermal photon escape due to inadequate line lists. Because of the strong dependence of luminosity on \[56\]Ni mass, some simple models can give a Phillips relation of the correct sense.

1. Introduction

Supernovae of Type Ia (SNIae) have been found at high redshift (up to \(z \approx 1\)) [Perlmutter, et al. (1997b), Schmidt, et al. (1999), Perlmutter, et al. (1999a)]. Being about \(10^6\) times brighter than Cepheids, SNIae can be seen about \(10^3\) times further, and consequently provide a more interesting possibility for determining truly cosmic distances. Although the SNIae are not identical, so that each event is not strictly like the standard one, [Phillips (1993)] found that the brighter events are broader in time (the Phillips relation). Calibration of this variation, with a set of relatively nearby supernovae, the Calan-Tololo sample of [Hamuy, et al. (1995)], allows individual events to be placed upon a standard scale [Riess, Press & Kirshner (1995)] and so doing reduces the scatter in the Hubble diagram (a plot of redshift versus calibrated brightness).

To properly use this breakthrough, we must understand the underlying physics of Phillips’ empirical relation. Several recent observational developments,

(a) The explicit demonstration, by [Perlmutter et al. (1997a), Perlmutter (1999b)], that the application of a time stretch operation to SNIae and a luminosity normalization, give a universal light curve shape, and

(b) Measurement of the rise to maximum from low luminosities [Riess, et al. (1999a)], gives the detailed structure of the early light curve, provide interesting information on the nature of the supernova event. The fact that SNIa light curves, except for a few odd events that can be easily recognized by their spectra, represent a single parameter family of shapes which can be “stretched” to a single universal shape, needs to be explained. It is this universality that allows the nature of the individual event to be identified, independent of distance, and their use as distance indicators to be precise.

2. Perspective on Analytic Solutions

The luminosity of SNIae is provided almost entirely by the decay of \(^{56}\)Ni and \(^{56}\)Co; other radioactive sources become important well after maximum light, and thermal emission from shocks is small. Determining the shape of the light curve, that is, the luminosity as a function of time, involves three separate issues:
\( (a) \) The transfer of radioactive energy by gamma-rays and positrons to the thermal energy of the plasma, properly including energy which escapes in the form of positrons, gamma and x-rays,

\( (b) \) The work done on the expanding plasma by the thermal radiation (or alternatively, the reduction in radiation energy by the accumulated redshift of the trapped photons), and,

\( (c) \) The energy escaping as thermalized radiation, which forms the supernova spectrum observed from the infra-red through the visible to ultra-violet (UVOIR).

The analytic solutions used approximate the last item, the escape probability, by a grey diffusion operator, parameterized by an effective opacity. The actual physics of the escape of thermal radiation is complex, and this complexity has obscured some simple and general features of the light curves. In particular, a reasonable reproduction of the observed spectrum cannot guarantee a reasonable representation of the thermal photon escape probability.

Here we will use the simple model to illustrate some general points (a more accurate discussion is in preparation with P. Nugent and P. Pinto). Note that the escape operator for thermal photons might be better thought of as diffusion down in energy space as well as out in radius; see [Pinto & Eastman (1999)]. It will be an interesting challenge to develop a better approximation to the thermal escape operator, which is simple enough to allow analytic solutions.

In this analysis we will use only the solutions presented in [Arnett (1982)] and earlier to emphasize the phenomena already contained in these early efforts, which have only now had observational confirmation.

### 3. The Shape of the Light Curve

The bolometric luminosity may be written as

\[
L = \epsilon_{\text{Ni}} M_{\text{Ni}} \Lambda(x, y).
\]  \hspace{1cm} (3.1)

Here \( M_{\text{Ni}} \) is the mass of \( ^{56}\text{Ni} \); \( \epsilon_{\text{Ni}} \) is the energy of radioactive decay of \( ^{56}\text{Ni} \) per unit mass, divided by the mean lifetime \( \tau_{\text{Ni}} \). Actually \( ^{56}\text{Co} \) contributes as well. This may be included in the \( \Lambda \) function, with the effect that the peak shifts higher in luminosity and later in time. In either case, \( \epsilon_{\text{Ni}} M_{\text{Ni}} \) is a convenient scale factor for the luminosity.

The shape of the light curve, which is independent of distance, is contained in \( \Lambda(x, y) \), where

\[
x = t/\tau_m,
\]  \hspace{1cm} (3.2)

and

\[
y = \tau_m/2\tau_{\text{Ni}}.
\]  \hspace{1cm} (3.3)

The effective escape time in a diffusing and expanding medium is a logarithmic average of the expansion time \( \tau_h \) and the diffusion time \( \tau_0 \),

\[
\tau_m = \sqrt{2\tau_h \tau_0},
\]  \hspace{1cm} (3.4)

see [Arnett (1982)]. Note that \( xy \) is simply time in units of two mean lives of \( ^{56}\text{Ni} \), or about 17.6 days. Although \( y \) was defined within the particular analytic context of simple diffusion in an expanding medium, it may be more generally interpreted as a measure of the probability of escape of energy by photon transport.

The general character of these light curves is shown in figure 1, which represent the solutions for \( ^{56}\text{Ni} \) decay alone. For the observationally interesting case of maxima occurring
around $xy \approx 1$ as shown, we have

$$\Lambda_{\text{max}} \approx 0.165/y,$$

(3.5)

and

$$t_{\text{max}}/2\tau_{\text{Ni}} \approx 0.42 + 0.48y.$$  

(3.6)

These approximations apply to the simple case in which only the $^{56}\text{Ni}$ heating is included, not that of $^{56}\text{Co}$ decay (the more general case was calculated but not tabulated in Arnett (1982)). Near maximum light ($t \approx 2\tau_{\text{Ni}}$), the heating from $^{56}\text{Co}$ is equal to that from $^{56}\text{Ni}$, and dominates at later times. This additional heating will increase both $t_{\text{max}}$ and $\Lambda_{\text{max}}$ relative to the values given by these approximations, but make no qualitative change. The lower panel shows the effect of (1) renormalizing the luminosity ($\Lambda \to \Lambda/\Lambda_{\text{max}}$), and (2) stretching the time scale to line up the maxima ($t \to t/t_{\text{max}}$ was used here). The curves lie almost on top of each other, so that in this sense, the shape is “universal.”

For theoretical light curves the time of explosion is easily defined, which is not true observationally. The observational stretch includes a shift in time as well, $t \to (t - t(0))/t_{\text{max}}$.

The light curves shown are those first presented by Arnett (1982). However, it was only after the discovery by Perlmutter et al. (1997a) that the observational data could be mapped into a universal curve by a normalization of luminosity and a stretch of time scale relative to the time of peak luminosity, that the analytic solutions were plotted in this form. For this simple case, the analytic solutions have this same property of (approximate) universality as the data.

4. The Early Light Curve

Figure 1 also shows that all the light curves have a parabolic dependence upon time during the earliest times after the explosion. Riess, et al. (1999a) present measurements of the earliest detections of nearby SNIae, which delineate the rise behavior for 18 to 10 days before maximum.

According to Riess, et al. (1999a), Goldhaber has proposed a method of determining the
rise time of SNIae which is based upon the “stretch” method of Perlmutter et al. (1997a), Riess, et al. (1999a) and Riess, et al. (1999b) have applied a similar approach to the B-band light curves of a number of SNIae. Goldhaber proposed that we describe the young SNIa as a homologously expanding fireball whose luminosity is most sensitive to its increasing radius, rather than effective temperature. The luminosity is then

\[ L = \alpha(t_{\text{max}} + t_r)^2, \]

where \( t_{\text{max}} \) is the time elapsed relative to maximum, \( t_r \) is the rise time, and \( \alpha \) is the “speed” of the rise.

Figure 2 shows the data from Riess, et al. (1999a). The squares represent their ten SNIae; upper limits have not been plotted. The time coordinate has been stretched by their stretch factors, and shifted so that \( t = 0 \) corresponds to the onset of the explosion. Thus, after stretching, the new time coordinate is \( t = t_{\text{max}} + t_r \). Their \( B \) magnitude has been rudely converted to solar luminosities by simply ignoring bolometric corrections (this is roughly correct, see Riess, et al. (1999a)). In this linear plot, the quadratic nature of the time dependence is obvious.

This behavior was predicted in Arnett (1982). Ten days before maximum is roughly ten days after explosion, at which time the Co luminosity is about 0.3 of that of Ni, and should not yet make a qualitative difference. Note that \( xy = t/2\tau_{Ni} \), so for early times \( t << \tau_{Ni} \), \( \Lambda \approx x^2 = (t/2\tau_{Ni})^2/y^2 \), so that

\[ L = \epsilon_{Ni}M_{Ni}(t/2\tau_{Ni}y)^2. \]  

(4.8)

The luminosity scale is set by the mass of \( ^{56}\text{Ni} \) and the shape parameter \( y \), the time dependence is quadratic in \( t \), and

\[ \alpha = \epsilon_{Ni}M_{Ni}/(2\tau_{Ni}y)^2. \]  

(4.9)

This identifies Goldhaber’s \( \alpha \) with the solution parameters, \( 2\tau_{Ni}^2y^2 = \kappa M/2\beta cv_{sc} \), where \( \kappa \) is the effective opacity and \( \beta \approx 13.7 \) (see Arnett (1980) Table 2), and \( M_{Ni} \) which is the mass of \( ^{56}\text{Ni} \).

The solid lines in figure 2 represent this solution for \( (M_{Ni}/M_\odot)/y^2 = 0.25, 0.5 \) and 1.0. At early times \( (M_{Ni}/M_\odot)/y^2 \approx 0.4 \). For a popular estimate of \( (M_{Ni}/M_\odot) = 0.6 \), we have \( y = 1.2 \). As we shall see, this is a plausible value.
5. Higher Modes

The behavior shown in figure 1 and figure 2 is based on a theoretical model which assumes that the higher modes in the spatial solution of the diffusion equation are small \( \text{(Arnett (1980))} \). These higher modes can be driven by

- \( (a) \) a distribution of \(^{56}\text{Ni} \) which is different from the distribution of energy in the fundamental mode for diffusion, \( \text{Pinto & Eastman (1999)} \).
- \( (b) \) a time dependence in the opacity (effective escape parameter \( y \)), or
- \( (c) \) the interaction of the exploding star with surrounding matter or a companion.

Such overtones modify the shape of the light curve, and in principle can be detected as a distance independent characteristic of SNIae. The \( \text{Riess, et al. (1999a)} \) data in figure 2 place limits on these effects.

6. The Risetime

Most theoretical models of SNIae predict significantly shorter risetimes than are found \( \text{(Vacca & Leibundgut (1996), Riess, et al. (1999a))} \). For example, \( \text{H"offlich & Khokhlov (1996)} \) give risetimes to visual maximum of 9 to 16 days, with an average value of 14 days for single white dwarf explosions. The same difference is seen by \( \text{Pinto & Eastman (1999)} \).

\( \text{Riess, et al. (1999a)} \) state:

“If these models are otherwise accurate, we concur with the conclusion of Vacca & Leibundgut (1996) that the model atmospheric opacity has been significantly underestimated. Past work suggests that deficient resonance line lists may be the culprit. By increasing the number of resonance lines from 500 to 100,000, the risetime for models by Harkness (1991) increased by 8 days.”

By including the additional lines, Harkness decreased the escape probability for thermal photons, which in our language means increasing \( y \). This means not only atmospheric opacity, but opacity at all depths. Because the opacity is strongly frequency dependent \( \text{(Pinto & Eastman (1999))} \), the atmosphere is not a sharply defined radius. This makes the escape probability view a clearer one. If maximum light occurs at 19.5 days, about 3.2 half-lives, the \(^{56}\text{Ni} \) has dropped to about 0.10 of its initial value, and most of the decayed Ni is in the form of Co. The line lists for Co and Ni are less extensive than for Fe, although these elements have comparably complex atomic and ionic states. While this may not affect spectral synthesis, which is often more sensitive to strong lines (which are likely to be in even poor line lists), it would affect the thermal escape probability.

If the spectral synthesis modelling of SNIae is deficient in this way, then attempts to use these models to infer global properties of SNIae will inevitably be biased, and their use for detailed inferences concerning distances and evolution of SNIae suspect. This argues for the relevance of the simpler approach pioneered by Dave Branch (see \( \text{Nugent, et al. (1995)} \)), which focuses on the atmosphere, and for the systematic effort to understand the physics of the complex models in order to make them adequately robust \( \text{(Baron, et al. (1999), Pinto & Eastman (1999))} \).

If we use a value of \( y = 1.2 \) (see above), then we expect \( \Lambda_{\text{max}} \approx 0.138 \) and \( t_{\text{max}} \approx 17.5 \) days. This is less that the value of 19.5 of \( \text{Riess, et al. (1999a)} \) but it is an underestimate because heating from \(^{56}\text{Co} \) decay will shift the maximum luminosity to later times. The luminosity at maximum is then \( L/L_\odot = 32.0 \times 10^9 \), or a B magnitude of -18.35. The addition Co luminosity brightens this by about -0.75 to -19.1. This is to be compared to \( M_B = -19.45 \) of \( \text{Riess, et al. (1999a)} \), which is encouragingly consistent, given the crudeness of our analysis.
7. The Phillips Relation

As is clear from the top panel in figure 1, the Λ curves which peak earlier (the ones with smaller $y$), have larger values at peak. Thus, to the extent that the nickel mass $M_{Ni}$ does not change with $y$ (from event to event), we have an anti-Phillips relation (Phillips (1993)). How should $M(Ni)$ change with $y$?

Let us begin by examining a carbon-oxygen white dwarf of near Chandrasekhar mass, which makes a transition to detonation after expanding to some lower central density $\rho_{tran}$. The material will be heated to a temperature which depends primarily upon its current density, and the energy available from burning. We may divide the star into layers, depending upon whether its peak temperature allows explosive carbon burning, oxygen burning, or silicon burning (see Woosley, Arnett, & Clayton 1973, Arnett (1996)). We will ignore deviations of the mass-density structure from that of an $n = 3$ polytrope. The transition to detonation may occur in a violent deflagration or in the compressional phase following a mild deflagration. Details may vary from the simple model we use, depending upon the explosion mechanism and the progenitor characteristics.

The top panel of figure 3 shows the final composition expected for a white dwarf, as a function of the central density it has when it detonates. At low density there is no burning, and the initial CO abundances are preserved. Carbon burning produces mostly O and Mg at these explosive temperatures. At still higher density, oxygen burning makes Si, S, Ar, and Ca (SiCa). At the highest density, nickel is the dominant ash.

If all these changes in composition, throughout the white dwarf, are converted into an implied explosion energy, we may relate the explosion energy to the mass of nickel produced. This is shown in the lower panel. At the lowest densities for detonation, the burning does not proceed through silicon burning to make Ni, but energy is released from burning up to SiCa. There is an abrupt rise to about 0.7 foe ($10^{51}$ ergs), and then a gradual increase to over 1.3 foe, so

$$E_{SN}/(10^{51}\text{erg} \approx 0.7 + (6/7)(M_{Ni}/M_\odot).$$

(7.10)

This is related to the velocity scale through the distribution of post-explosion velocity.
with density:

$$E_{SN} = \frac{1}{2} M < v^2 > .$$  \hspace{1cm} (7.11)

If the composition has no effect on the opacity, or more precisely, the thermal photon escape time, then the \( y \) parameter depends upon \( M(Ni) \) through the velocity scale, or equivalently, the explosion energy. Thus,

$$y \propto \sqrt{M/v_{sc}} \rightarrow M^{3/4}/E_{SN}^{1/4},$$  \hspace{1cm} (7.12)

which is a relatively weak dependence on \( E_{SN} \).

Collecting results,

$$L = \epsilon_{Ni} M_{Ni} \Lambda_{max}$$

$$\propto M_{Ni}/y$$

$$\propto y^3$$  \hspace{1cm} (7.13)

where the last result assumes \( E >> 0.7 \) foe. This gives the observed sense of the Phillips relation: brighter SN have broader light curves. However, there is a potential difficulty here (Pinto & Eastman (1999)): if the \( ^{56}Ni \) is distributed as a central sphere of pure Ni, then the average distance to the surface increases with \( M_{Ni} \), and so does the probability of leakage from gamma and x-ray escape. This changes the light curve shape, and may destroy the sense of the Phillips relation. Alternatively, if the Ni distribution is not a strong function of \( M_{Ni} \) at a time several days after the explosion, the previous result holds. The production of a Phillips relation that agrees with observation depends upon the nature of the explosion assumed, but may not be difficult to get for some simple and attractive models.

8. Conclusions

The new data allow us to independently determine the parameters in the analytic models of SNIae. The predictions for the premaximum behavior of the light curves are confirmed, and it will be possible to place new contraints on the nature of the progenitors and explosions. Even at the crude level sketched here, it is possible to get self consistent values, and a more accurate computation following this logic is warranted. In contrast, the radiative transfer models of Höflich & Khokhlov (1996) fail to give the correct rise time, probably due to incomplete line lists, and are likely to give biased results when applied to the description of such global properties of SNIae, at least until this weakness is corrected.

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Universality in SNIae
$M_{Ni}/y^2 = 1$

$L_{solar}$ vs. stretched days

- $2.5 \times 10^9$
- $2 \times 10^9$
- $1.5 \times 10^9$
- $1 \times 10^9$
- $0.5 \times 10^9$
- $0 \times 10^9$
- $0.5 \times 10^8$
- $0 \times 10^8$

- $0.25$
- $0$
- $2$
- $4$
- $6$
- $8$
- $10$
- $12$
- $14$
- $16$
- $18$
- $20$

stretched days
