Determining the gluon condensate with DIS experiments at small Bjorken parameter regime

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Abstract

Using gauge/gravity duality we study deep inelastic scattering (DIS) of a lepton with a proton target in presence of gluon condensation. We adopt a modified AdS5 background in which the modification parameter $c$ corresponds to the gluon condensation in the boundary theory. Firstly we study electromagnetic field in such a scattering. Our results show that presence of $c$ strengthen magnitude of electromagnetic field. In the next step, we find baryonic states wave function equations in which mass of the proton target demands contribution of only small values of $c$. As our main aim is determining value of this parameter, we find $c = 0.0120 \pm 0.0005$ GeV$^4$ from experimental data. Proceeding by electromagnetic field and baryonic states, we derive the holographic interaction action that is related to amplitude of the scattering. Eventually we compute the corresponding structure functions as functions of Bjorken parameter $x$ and the momentum lepton transfers $q$ numerically. Comparing Jlab Hall C data with our theoretical results at small values of $x$, we find that at regime of small Bjorken parameter there is a good agreement between theory and phenomenology.

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1 Introduction

Deep inelastic scattering (DIS) process has the usage of studying proton internal structure which is a puzzle in quantum chromodynamic (QCD). The main difficulty comes from the fact that massless gluons and nearly massless quarks give rise to the mass of the proton $M \sim 1 \text{ GeV}$. One may say that from special relativity point of view the missing mass could be considered initiating from kinetic energy of quarks and gluons inside the proton, but it turns out that the mentioned kinetic energy is not sufficient to describe the total vanishing mass. To explore more on this issue, breaking of conformal symmetry in QCD is another reason which should be considered. It leads to an anomalous dimension contributes in the DIS. Let us recall that QCD coupling is large at low energies, hence, perturbative calculations can not be used for studying many properties of hadrons. The alternative way for this purpose, is using holographic approach to study DIS.

In AdS/CFT description a strongly coupled field theory on the boundary of the AdS space could be described by a weakly coupled gravity theory in the bulk of the AdS. In other words a ten dimensional geometry on the corresponding boundary is an exact dual of a supersymmetric SU($N$) gauge theory with large $N$ in the bulk. Specifically, string theory in $AdS_5 \times S_5$ space is a dual of a four dimensional gauge theory. AdS/CFT duality is for conformal theories originally, but fortunately it has been generalized to non-conformal theories like QCD, so in any case of interest by adopting an appropriate AdS background one can study a QCD problem. In current study the proton internal structure will be considered in a DIS process by using a modified $AdS_5$ background. In a phenomenological holographic approach, AdS/QCD tries to fit a five-dimensional effective field theory to QCD as much as possible. Doing so, mass gap, confinement and supersymmetry breaking are included by considering some modifications in gravity duals. To break conformal symmetry one may modify radial coordinate, the 5th dimension of space-time. This method was represented in the reference [4].
In current study we should adopt a modified AdS which introduces the gluon condensation in QCD side of duality. Originally gluon condensation was a measure for nonperturbative physics in QCD at zero temperature \[5\]. Later it was identified as an order parameter for confinement to study some nonperturbative phenomena \[6–8\].

Well-known modified holographic model which introduces gluon condensation in the boundary theory is given with the following action in Minkowski spacetime \[9\],

\[
S = -\frac{1}{2k^2} \int d^5x \sqrt{g} \left( R + \frac{12}{L^2} - \frac{1}{2} \partial_\lambda \varphi \partial^\lambda \varphi \right),
\]

where \(k\) is gravitational coupling in 5-dimensions, \(R\) is Ricci scalar, \(L\) is the radius of the asymptotic AdS$_5$ spacetime, and \(\varphi\) is a massless scalar which is coupled with the gluon operator on the boundary. Considering a suitable ansatz as follows,

\[
ds^2 = \frac{L^2}{z^2}(\sqrt{1 - c^2z^8}(-dt^2 + \sum_i dx_i^2) + dz^2),
\]

\[
\varphi(z) = \frac{3}{2} \ln \frac{1 + cz^4}{1 - cz^4} + \varphi_0.
\]

helps to solve the Einstein equation and the dilaton equation of motion. In the above dilaton-wall solution, \(\varphi_0\) is a constant, \(c = \frac{1}{z_c^4}\) and \(z_c\) denotes the IR cutoff. This is the background solution on which we would like to work. Also we compute in the unit where \(L = 1\). Recall that \(i = 1, 2, 3\) are orthogonal spatial boundary coordinates, \(z\) denotes the 5th dimension, radial coordinate and \(z = 0\) sets the boundary. To investigate how the forth correction of radial coordinate appears in the metric (which it’s coefficient denoted by \(c\)), let us mention that the dilaton field is dual to a scalar operator and the metric is dual to the energy-momentum tensor of the dual field theory \[10\] (for more discussion see \[11–13\]). Expanding the dilaton profile near \(z = 0\) will give,

\[
\varphi(z) = \varphi_0 + \sqrt{6}cz^4 + ....
\]

According to the holographic dictionary \(\varphi\) and \(c\) are the source and the parameter associated with the confinement respectively. One clearly notices that \(c\) in the background metric breaks the conformal symmetry and the gluon condensation in the boundary theory appears. The relevant phenomenological information show its value generally lie in the range \(0 < c \leq 0.9\) GeV$^4$ \[14 \[16\] however we are interested to find the exact value of \(c\) during a DIS process with proton target.

Holographic description of gluon condensation allows one to explore many physical quantities in presence of such a parameter. Firstly in order to get familiar with its phenomenological aspects, let us give a comment. The dilaton wall Solution \[2\], \[3\] is related to the zero temperature case. Thus using this, is appropriate to study DIS and its physics. On the other hand, the advantage of

\[\text{It shows that } z \text{ is defined from zero to IR limit as usual, so one should not misunderstand that parameter } c \text{ bounds upper limit of } z \text{ to values less than cutoff! in fact it should be interpreted as always } c < \frac{1}{z_c}.\]
working with this solution is the presence of parameter $c$ with a specific phenomenological domain of values. As it must be, one can readily check that in the limit $c \rightarrow 0$ the solution reduces to $AdS_5$ background which although has nothing to do with mass gap, but as one could modify it on radial coordinate, more phenomenological results would turn out. In fact it has became an approach to discuss more phenomenological aspects by using modified $AdS$ \[17\] \[19\].

Our system of interest is described as a deep inelastic scattering with a proton target. In the most related work \[20\], by using the deformed AdS such a scattering has been studied. In our present case, the modified $AdS_5$ denotes the gluon condensation as it turns out from exact solution of \[11\]. Such a solution demands one to consider the coupled dilaton field \[3\] with the background \[2\]. With regard to all the mentioned motivations, in the current work we will use holographic model of gluon condensation to study DIS with proton target. Since models with anomalous dimension in AdS/QCD lead to generation of a mass scale of fermionic fields many works have used them to deal with DIS \[20\] \[53\].

This paper is organized as follows, after brief review of DIS properties via holography in section 2 we will study electromagnetic interactions and baryonic states in deep inelastic scattering in sections 3 and 4 respectively. Proceeding by these results, interaction action will be given in section 5 then according to the relation between such action and scattering amplitude we will study structure functions. In section 6 we briefly review and discuss our results.

2 DIS parametrization and holography

We start this section with a brief review of deep inelastic scattering, to explicit our motivation and aims. The main usage of DIS in particle physics is exploring the inner hadronic structure and strong interactions. In a DIS process in which a lepton scattered off a proton target through the exchange of a virtual photon, proton fragmentation produces a lepton and some final hadronic states. It should be noted that production of final hadronic states depends on four momentum the initial lepton transfers. Therefore, the mentioned four momentum causes the inner quark and gluon of proton expelled out, eventually in the next step quark anti-quark pairs hadronize. To detect the final lepton, one needs to sum over all possible final hadronic states labeled by $X$. According to \[54\] DIS is parametrized by Bjorken dynamical variable which is defined as,

\[ x = -\frac{q^2}{2P \cdot q}, \]  

(5)

needless to say that lepton transfers momentum $q$ to the proton target via a virtual photon and $P$ is the initial proton momentum. We adopt the method have been explained specifically in the above mentioned reference and rederived in \[55\] \[56\]. Doing so, hadronic transition amplitude is given as,

\[ W^{\mu \nu} = F_1(q^{\mu \nu} - \frac{q^\mu q^\nu}{q^2}) + 2x \frac{q^\mu}{q^2} F_2(P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x}), \]  

(6)
where \( F_{1,2} = F_{1,2}(x, q^2) \) are some structure functions.

Now, let us relate the above matrix to holography. From AdS/QCD dictionary, elements of \( \bar{G} \) in QCD side is connected to the interaction action in AdS side as \([21]\),

\[
\eta_\mu < P + q, s_X | J^\mu (0) | P, s_i >= K_{eff} S_{int},
\]

where \( \eta_\mu \) is polarization of virtual photon, \( | P, s_i > \) represents a normalizable proton state with spin \( s_i \), \( J^\mu \) is the electromagnetic quark current and \( s_X \) denotes the final state. It is worth to mention that \( K_{eff} \) is an effective factor that phenomenologically adjust the bulk supergravity quantities to the boundary, based on a different point of view that bulk/boundary duality implies that these quantities are proportional, rather than necessarily equal \([10]\). The interaction action is written as,

\[
S_{int} = g_V \int dz d^4x e^{-\varphi} \sqrt{-g} \phi^\mu \bar{\Psi}_X \Gamma_\mu \Psi_i,
\]

and \( g_V \) is a coupling constant related to the electric charge of the baryon, \( \varphi \) is the dilaton field and \( \sqrt{-g} \) is given by the metric, \( \phi^\mu \) is the electromagnetic gauge field, \( \Psi_i \) and \( \Psi_X \) are the initial and final state spinors for the baryon respectively and \( \Gamma_\mu \) are Dirac gamma matrices in the curved space. By computing all above quantities in accordance with \([2]\) and \([3]\), we study DIS with a proton target.

### 3 Electromagnetic interactions in deep inelastic scattering

There are electromagnetic interactions based on photon involves in the scattering. It can be described as the presence of photon in modified AdS. The action for a five dimensional massless gauge field \( \phi^n \) is given by,

\[
S = -\frac{1}{4} \int d^5x e^{-\varphi} \sqrt{-g} F^{mn} F_{mn},
\]

where \( F^{mn} = \partial^m \phi^n - \partial^n \phi^m \), and \( m,n \) refer to the 5-dimensional space includes Minkowski spacetime coordinates, \( \mu, \nu \) and \( z \), and \( \varphi \) is the dilaton field given by \([3]\). Notice that \( \varphi \) and \( \phi \) should be differentiated by reader. In fact \([2]\) is an action showing the gauge field \( \phi \) on a background coupled to a dilaton field \( \varphi \). From \([3]\) the equation of motion of such an electromagnetic field is derived as,

\[
\partial_m [e^{-\varphi} \sqrt{-g} F^{mn}] = 0.
\]

Considering \( m, n \equiv \mu, \nu, z \) the relation \([10]\) leads to,

\[
\partial_\mu \left[ \frac{1}{z} (1 + cz^4)^{1-\sqrt{\frac{4}{3}}} (1 - cz^4)^{1+\sqrt{\frac{4}{3}}} F_{\mu z} \right] = 0,
\]

\[
\partial_z \left[ \frac{1}{z} (1 + cz^4)^{1-\sqrt{\frac{4}{3}}} (1 - cz^4)^{1+\sqrt{\frac{4}{3}}} F^{\mu z} \right] = 0.
\]

5
To solve the equations of motion of the gauge field in (11), first we should fix the gauge. Let us consider there is an electromagnetic field in the bulk with the geometry metric (2) defines. This obeys the 5–dimensional Maxwell equation supplemented by a gauge condition which we take to be,

\[ e^{-\varphi} \sqrt{-g} \partial_{\mu} \phi^{\mu} + \partial_{z} (e^{-\varphi} \sqrt{-g} \phi_{z}) = 0, \tag{12} \]

From (12) one can write,

\[ \partial_{\mu} \phi^{\mu} + \frac{z}{(1 + c z^4)^{1 - \sqrt{\frac{7}{2}}}} \partial_{z} \left( \frac{(1 + c z^4)^{1 - \sqrt{\frac{7}{2}}}}{z} (1 - c z^4)^{1 + \sqrt{\frac{7}{2}}} \phi_{z} \right) = 0, \tag{13} \]

so,

\[ \Box \phi_{\mu} + \partial_{\mu} \partial_{z} \phi_{z} - \frac{1 + 4 \sqrt{6} c z^4 + 7 c^2 z^8}{z (1 - c^2 z^8)} \partial_{\mu} \phi_{z} = 0. \tag{14} \]

Using the gauge (14) together with (11) leads to the following equations,

\[ \Box \phi_{z} - \partial_{\mu} \partial_{z} \phi^{\mu} = 0, \tag{15} \]

\[ \Box \phi_{\mu} + \partial_{z}^{2} \phi_{\mu} - \frac{1 + 4 \sqrt{6} c z^4 + 7 c^2 z^8}{z (1 - c^2 z^8)} \partial_{z} \phi_{\mu} = 0. \tag{16} \]

At this point one could continue by considering a photon with a particular polarization as \( \eta_{\mu} q^{\mu} = 0 \) for simplicity, hence only the \( \phi^{\mu} \) component contributes in the scattering \([21, 24, 55]\). In the latter case we need to solve only (16). The above electromagnetic field equation can not be solved analytically and we need to use numerical methods. Let us consider \( \phi_{\mu}(z, q, y) = \eta_{\mu} e^{i q \cdot y} \phi_{1}(z, q) \), and apply it in (16). With the boundary condition \( \phi_{\mu}(z, q, y)\big|_{z=0} = \eta_{\mu} e^{i q \cdot y} \) one may study the electromagnetic field numerically.

In figure, plots of component \( \phi_{1}(z, q) \) of electromagnetic filed have been shown for different values of \( q^{2} \) and \( c \). Remind that \( q \) is the momentum that lepton transfers and \( c \) is a modification parameter in the background metric which introduces the gluon condensation phenomena in the boundary theory. Thus, studying the behaviour of \( \phi_{1} \) along \( z \) coordinate with respect to both \( q^{2} \) and \( c \), can give us some understanding of electromagnetic field behaviour. Plot a) shows that at large values of \( q^{2} \) increasing of parameter \( c \) does not affect neither the magnitude nor the behaviour of field significantly. Going from boundary \( (z = 0) \), \( \phi_{1} \) descends along \( z \). In the plot b) at small values of \( q^{2} \), increasing of parameter \( c \), increases magnitude of \( \phi_{1} \), as at larger \( z \), the effect of \( c \) on \( \phi_{1} \) is more visible. In c) one can compare \( \phi_{1} \) for different values of \( q^{2} \) but at a fixed \( c \). Obviously the smaller \( q^{2} \) is, the stronger \( \phi_{1} \) is. In other words the magnitude of electromagnetic field is larger at small \( q^{2} \) rather than large \( q^{2} \).
4 Baryonic state equations in deep inelastic scattering

In this section, we study the baryonic initial and final states for further requirements of the interaction action (8). The equation of motion of fermionic states are,

\[(\mathcal{D} - m_5)\Psi = 0,\]

where \(m_5\) is the baryon bulk mass and the operator \(\mathcal{D}\) is defined as,

\[\mathcal{D} = g^{mn}e^a_n\gamma_a (\partial_m + \frac{1}{2}\omega_{m}^{bc}\Sigma_{bc}),\]

in which \(\gamma_a = (\gamma_{\mu}, \gamma_5)\), \(\{\gamma_a, \gamma_b\} = 2\eta_{ab}\) and \(\Sigma_{\mu 5} = \frac{1}{4}[\gamma_{\mu}, \gamma_5]\) \([57] [60]\). \(\gamma_\mu\) are Dirac’s gamma matrices. a, b, c are flat space and, m, n, p, q are AdS space indices respectively. As before \(\mu, \nu\) represent...
the Minkowski space. With the metric (2), Vielbein are computed as,
\[ e^a_n = \frac{(1 - c^2 z^8)^{1/4}}{z} \delta^a_n, \]
\[ a = t, x_1, x_2, x_3, x_4, \]
\[ e^b_n = \frac{1}{z} \delta^b_n, \]
\[ b = z. \]

(19)

The above terms will give us first term of (18). Now we should compute the second term of that. Spin connection is given by,
\[ \omega^{ab}_m = e^a_n \partial_m e^{nb} + e^a_n e^{pb} \Gamma^m_{pn}, \]
where the Christoffel symbols are,
\[ \Gamma^p_{mn} = \frac{1}{2} g^{pq} \left( \partial_n g_{mq} + \partial_m g_{nq} - \partial_q g_{mn} \right). \]

(21)

From the metric (2), one may write,
\[ g^{\mu \nu} = \sqrt{1 - c^2 z^8} z^2 \eta^{\mu \nu} \]
and
\[ g^{zz} = \frac{1}{z^2}. \]
So the only non vanishing terms are, \( \Gamma^z_{\mu \nu}, \Gamma^z_{zz}, \Gamma^\mu_{\nu z} \). After computation they are written as,
\[ \Gamma^z_{\mu \nu} = \frac{1}{z} \left( 1 - c^2 z^8 \right)^{1/4} \delta^\nu_\mu, \]
\[ \Gamma^z_{zz} = \frac{1}{z^2}, \]
\[ \Gamma^\mu_{\nu z} = \frac{(1 + c^2 z^8)^4}{z(1 - c^2 z^8)} \delta^\mu_\nu. \]

(22)

Also from (2) together with (19) and (21) the relation (20) turns to,
\[ \omega^z_{\mu} = -\omega^\mu_z = -\frac{(1 + c^2 z^8)}{z(1 - c^2 z^8)^{1/4}} \delta_\mu^\nu, \]

(23)

hence other components of \( \omega^{ab}_m \) are zero. Using these solutions, (18) is given by,
\[ D = z\gamma^5 \partial_z + \frac{z}{(1 - c^2 z^8)^{1/4}} \gamma^\mu \partial_\mu - 2 \frac{(1 + c^2 z^8)}{z(1 - c^2 z^8)^{3/4}} \gamma^5 - m_5 \Psi = 0. \]

(24)

and the EOM (17) is written as,
\[ [z\gamma^5 \partial_z + \frac{z}{(1 - c^2 z^8)^{1/4}} \gamma^\mu \partial_\mu - 2 \frac{(1 + c^2 z^8)}{z(1 - c^2 z^8)^{3/4}} \gamma^5 - m_5] \Psi = 0. \]

(25)

According to the fact that spinor is either left handed or right handed, and since Kaluza-Klein modes are dual to the chirality spinors we decompose these components and expand as,

\[ \Psi_{L/R}(x^\mu, z) = \sum_n f_{L/R}^n(x^\mu) \chi_{L/R}^n(z), \]

(26)
applying (26) in the equation of motion (25) we find the coupled equations as,

\[ (\partial_z - 2 \frac{(1 + c^2 z^8)}{z^2 (1 - c^2 z^8)^2}) \chi_L(z) = \frac{M_n}{(1 - c^2 z^8)^\frac{3}{4}} \chi_R(z), \]  

(27)

\[ (\partial_z - 2 \frac{(1 + c^2 z^8)}{z^2 (1 - c^2 z^8)^2}) - \frac{m_5}{z} \chi_R(z) = \frac{-M_n}{(1 - c^2 z^8)^\frac{3}{4}} \chi_L(z), \]  

(28)

Decoupling these two equations we lead to the following equation which describes both left handed and right handed sectors as,

\[ -(1 - c^2 z^8)^\frac{3}{4} \left( \partial_z - 2 \frac{(1 + c^2 z^8)}{z^2 (1 - c^2 z^8)^2} \right) \pm \frac{m_5}{z} \chi_L(z) = M_n \chi_R(z). \]  

(29)

In continue we try to make a Schrödinger-like equation by applying a transformation as follows,

\[ \chi_{R/L}(z) = e^{-2(1 - c^2 z^8)^\frac{1}{4} \frac{m_5}{z}} \psi_{R/L}(z), \]  

(30)

so the equation (29) is written as,

\[ \sqrt{1 - c^2 z^8} \left( -\psi''_{R/L}(z) + \frac{m_5(m_5 + 1) - c^2 z^8(2m_5^2 + 7) + c^4 z^{16} m_5(m_5 + 1)}{z^2 (1 - c^2 z^8)^2} \psi_{R/L}(z) \right) = M_n^2 \psi_{R/L}(z). \]  

(31)

In [31], \( m_5 \) is a parameter in AdS side of gauge/ gravity duality and related to baryon mass in the gauge side, so the normalizable solutions of the above equations are dual to the states in the boundary theory. In pure AdS space, the bulk mass is related to the canonical conformal dimension \( \Delta_{can} \) of a boundary operator.

\[ |m_5^{AdS}| = \Delta_{can} - 2. \]  

(32)

Remind that QCD is not a conformal field theory since it has mass gap. Therefore the gravity side should be modified somehow and it is not pure AdS any more. Modifying AdS, the canonical dimension \( \Delta_{can} \) of an operator has an anomalous contribution of \( \gamma \) which implies an effective scaling dimension.

\[ |m_5| = \Delta_{can} + \gamma - 2. \]  

(33)

Contribution of anomaly is related to how one modifies the theory. For example in [35] modification of the scale introduces the mass gap in the theory. Therefore the anomalous contribution is related to any factor which represents the energy scale in the theory and leads to mass spectra. Hence, the main task is finding bulk mass after anomalous contribution. In AdS/CFT dictionary, the bulk mass is related to the dimension, means the energy scale of the boundary theory is holographically
related to the localization in the $z$ coordinate, therefore we have $z$ dependent mass in the bulk. Let us focus on $m_5$. Numerically, one may fit $m_5$ as the equations \textsuperscript{[31]} have normalizable solutions. Fixing $M$ as proton mass, we should find suitable values for $c$ and $m_5$ which give us well defined answers.

Figures 2 and 3 show initial (n=1) and final (for two excited states as n=2,3) chiral components of the wave function respectively. Solving the equations \textsuperscript{[31]} numerically, we fix $M$ as proton mass, therefore $m_5$ and $c$ should be found as the equations have answers. The values $c = 0.0120 \pm 0.0005 \text{GeV}^4$ and $m_5 = 0.081 \text{GeV}$ are demanded by the proton mass as eigenvalue of equation. Interestingly, the value of parameter $c$ in AdS side is very close to the phenomenological GC value of QCD as it has been found $G_2 = 0.010 \pm 0.0023 \text{GeV}^4$ in the reference \textsuperscript{[17]}. Another consequence of presence of $c$ is that the anomaly $\gamma$ in \textsuperscript{[33]} is so intense that affects the bulk mass significantly.

![Figure 2](image1.png)

**Figure 2:** Left handed (dashed) and right handed (solid) sectors of wave function from \textsuperscript{[31]} for the initial state (target proton), by considering $c = 0.0120 \pm 0.0005 \text{GeV}^4$ and $m_5 = 0.081 \text{GeV}$.

![Figure 3](image2.png)

**Figure 3:** Left handed (dashed) and right handed (solid) sectors of wave function from \textsuperscript{[31]} for the final state a) n=2 and b) n=3, by considering $c = 0.0120 \pm 0.0005 \text{GeV}^4$ and $m_5 = 0.081 \text{GeV}$.
After finding both left handed and right handed modes from (31) we may consider wave functions,

\[ \Psi_i = \frac{e^{-\frac{2(1-c^2 z^2)^{\frac{1}{4}}}{z}}}{(1 - c^2 z^8)^{\frac{1}{8}}} e^{iP_y[(\frac{1 + \gamma_S}{2})\psi^X_L + (\frac{1 - \gamma_S}{2})\psi^X_R]}u_{s_i}(p), \]  

(34)
as initial wave function for the target proton and,

\[ \Psi_X = \frac{e^{-\frac{2(1-c^2 z^2)^{\frac{1}{4}}}{z}}}{(1 - c^2 z^8)^{\frac{1}{8}}} e^{iP_x \cdot y[(\frac{1 + \gamma_S}{2})\psi^X_L + (\frac{1 - \gamma_S}{2})\psi^X_R]}u_{s_X}(p), \]  

(35)
as final wave function for hadronic state.

5 Modified geometry and action of deep inelastic scattering

In accordance with the relation (7) and (8) we may find interaction action from electromagnetic field obtained from (36) and baryonic states from (34) - (35). Then the interaction action is given by,

\[ S_{int} = g_V \int dz d^4y e^{-\frac{z}{c} \sqrt{-g} \phi^\mu \Psi_X e^{\frac{1}{2} \gamma_\mu \gamma_\alpha \Psi_i} } \]

\[ = g_V \int dz^4 y \frac{1}{z} (1 - c^2 z^4)^{1+\sqrt{2}} (1 + c^4)^{1-\sqrt{2}} \phi^\mu \Psi_X \frac{1}{z} (1 - c^2 z^8)^{\frac{1}{8}} \delta^\alpha_{\gamma \gamma} \psi_i \]

\[ = g_V \int dz d^4 y \frac{1}{z^2} (1 - c^2 z^4)^{\frac{5}{8} + \sqrt{2}} (1 + c^4)^{\frac{5}{8} - \sqrt{2}} \phi^\mu \Psi_X \gamma_\mu \psi_i, \]  

(36)
and from (35) one may write,

\[ \Psi_X = \frac{e^{-\frac{2(1-c^2 z^2)^{\frac{1}{4}}}{z}}}{(1 - c^2 z^8)^{\frac{1}{8}}} e^{-iP_x \cdot y \bar{u}_{s_X}(p) \{[(\frac{1 + \gamma_S}{2})\psi^X_L + (\frac{1 - \gamma_S}{2})\psi^X_R]}. \]  

(37)
Therefore the interaction action is as follows,

\[ S_{int} = \frac{g_V}{2} \int dz^4 y e^{-i(P_x - P_y) \eta \mu \phi_1} \left[ \bar{u}_{s_X} \left( \hat{P}_L \psi^X_L + \hat{P}_R \psi^X_R \right) \gamma_\mu (\hat{P}_L \psi^X_L + \hat{P}_R \psi^X_R) u_{s_i} \right] \]

\[ = \frac{g_V}{2} (2\pi)^4 \delta^4(P_x - P_q) \eta \mu \int dz \frac{1}{z^2} (1 - c^2 z^4)^{\frac{5}{8} + \sqrt{2}} (1 + c^2 z^4)^{\frac{5}{8} - \sqrt{2}} \]

\[ \left[ \bar{u}_{s_X} \gamma_\mu \hat{P}_R u_{s_i} \bar{\psi}^X_L \psi^X_L + \bar{u}_{s_X} \gamma_\mu \hat{P}_L u_{s_i} \bar{\psi}^X_R \psi^X_R \right]. \]  

(38)
Let us define the following integral as,

\[ B_{R,L} = \int dz \frac{\sqrt{1 - c z^4} \sqrt{(1 + c z^4)^{\frac{3}{2}}}}{1 - c^2 z^8} \left( \frac{1}{2} \right) \phi \psi^{\frac{1}{2}} X^{\frac{1}{2}} \phi R L \psi R L, \]  

(39)

which leads to the interaction action as,

\[ S_{int} = \frac{g V}{2} (2\pi)^4 \delta^4(P_X - P - q) \eta \mu [\bar{u}_{s_X} \gamma_{\mu} \hat{P} R u_{s_R} B L + \bar{u}_{s_X} \gamma_{\mu} \hat{P} L u_{s_L} B R], \]  

(40)

and (7) is written as,

\[ \eta \mu < P_X | J_\mu (q) | P_i > = \frac{g_{eff}}{2} (P_X - P - q) \eta \mu [\bar{u}_{s_X} \gamma_{\mu} \hat{P} R u_{s_R} B L + \bar{u}_{s_X} \gamma_{\mu} \hat{P} L u_{s_L} B R] \]  

(41)

where \( g_{eff}^2 = K_{eff}^2 g_V^2 (2\pi)^6 \), and \( g_V^2 = \frac{1}{137} \). \( K_{eff}^2 \) should be fitted numerically as it has been shown in table 1. Considering above equations and after some calculation (for details see [21, 24, 56]) the relation (6) could be written as,

\[ \eta \mu \eta \nu W_{\mu \nu} = \eta^2 F_1(q^2, x) + \frac{2x}{q^2} (\eta.P)^2 F_2(q^2, x), \]  

(42)

where \( F_1 \) and \( F_2 \) are,

\[ F_1(q^2, x) = \frac{g_{eff}^2}{4} \left[ M_0 M_X B_L B_R + (B^2_L + B^2_R) \right] \frac{1}{M_X^2}, \]  

(43)

and

\[ F_2(q^2, x) = \frac{g_{eff}^2 q^2}{8} \frac{q^2}{x} (B^2_L + B^2_R) \frac{1}{M_X^2}, \]  

(44)

respectively. Also \( M_0 \) is the mass of the initial hadron and \( M_X \) is the mass of the final hadron as,

\[ M_X = \sqrt{M_0^2 + q^2 (1 - \frac{x}{x})}. \]  

(45)

Let us consider the limit of small values of \( x \), and \( M_X \gg M_0 \) then from (43) and (44) we find,

\[ F_1(q^2, x) \approx \frac{1}{2} \left( 1 + 2x \frac{M_0^2}{q^2} \right) F_2(q^2, x), \]  

(46)

which gives us some approximation of ratio of structure functions at small Bjorken parameter.
Numerical strategy

As we mentioned about hadron wave functions, the eigenvalue of the ground state should be close to the square of the mass of the proton. So, we consider ranges $0 < m_5 < 1\,\text{GeV}$ and $0.001 < c < 1\,\text{GeV}^4$, while the eigenvalue of the equation of ground state, is in the range from $0.876\,\text{GeV}$ to $1\,\text{GeV}$ ($M_{\text{proton}} = 0.938\,\text{GeV}$). Accordingly we obtain a series of suitable values of parameters $m_5$ and $c$. Their suitable approximate ranges are $0.001 < m_5 < 0.2\,\text{GeV}$ and $0.006 < c < 0.02\,\text{GeV}^4$. Looking for the appropriate values of $K_{\text{eff}}^2$ as $m_5$ and $c$ satisfy their ranges and our theoretical calculations can be fitted with the experimental data for $F_2$, we continue by focusing on only small $x$ and small $q^2$. In the form of sweep spectrum, we determine a set of $m_5$ and $c$ and then for each set, we fit the experimental data for $F_2$ to obtain $K_{\text{eff}}^2$ and the error bar. What needs to be mentioned here is that for $m_5$, the search step is 0.01, and for $c$, the search step is 0.001. The optimal values of parameter with smallest error are $m_5 = 0.081\,\text{GeV}$, $c = 0.0120 \pm 0.0005\,\text{GeV}^4$, $K_{\text{eff}}^2 = 37.3259$. Using this set of parameters, we will get the proton structure function $F_2$ as function of $q^2$, to compare them with the experimental data.

| $x$    | $m_5 / \text{GeV}$ | $c / \text{GeV}^4$ | $k_{\text{eff}}^2$ |
|--------|--------------------|--------------------|-------------------|
| 0.015  | 0.081              | 0.012$\pm 5 \times 10^{-4}$ | 37.3259         |
| 0.025  |                    |                    |                  |
| 0.04   |                    |                    |                  |

Table 1: Adjustment of parameter $k_{\text{eff}}^2$ at different $x$ with $c = 0.0120 \pm 0.0005\,\text{GeV}^4$. Remind that the value of parameter $c$ is demanded by phenomenological value of proton mass.

Figure 4 is a comparison between Jlab Hall C data [61] and our theoretical results at small $x$. In plots a), b), c) our results have good agreements with experimental data at $x = 0.015$, $x = 0.025$ and $x = 0.04$.

Plot d) is the ratio of structure functions from [10] as it should be $F_2(1+2x \frac{m_0^2}{q^2})$, is near one, especially at small $q^2$ and for the smallest value of Bjorken parameter.
Conclusions

In a holographic description of DIS we found effects of parameter $c$ which represents gluon condensation in the boundary theory. Since there is proton target in the scattering, mass of proton and value of parameter $c$ both play important role in this study. One of our main aim in this study was determining the value of $c$ by experimental data. Solving the equation of baryonic wave function numerically, one sets the proton mass as eigenvalue to find best values of bulk mass and parameter $c$. In accordance with it, only small values of $c$ lead to renormalizable answer of the equation. It could be suggested that since parameter $c$ breaks the conformal symmetry, it’s value represents the confinement. So in our case of study, confinement is not strong. Proceeding by this result we discussed structure functions in the scattering at small values of Bjorken parameter in comparison with Jlab Hall C data. Our theoretical results are in a good agreement with data, which shows our model works better for small $x$, weak confinement and small momentum.
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