Passage of fractal signals via circuit with a ferroelectric capacitor with a negative capacitance

Gadgymurad O. Abdullaev¹, Agalar M.-Z. Agalarov¹, Alexander A. Potapov²,³✉, Alexander A. Rassadin⁴, Anton A. Tronov⁴

¹Department of Theoretical Physics, Amirkanian Institute of Physics of RAS, DSC, Makhachkala, Russia
²Kotel’nikov Institute of Radio Engineering and Electronics of RAS, Moscow, Russia
³JNU-IRE RAS Joint Laboratory of Information Technology and Fractal Processing of Signals, JiNan University, Guangzhou, People’s Republic of China
⁴Nizhny Novgorod Mathematical Society, Nizhny Novgorod, Russia

✉E-mail: potapov@cpire.ru

Abstract: In the report presented, the results of synthesis of two of the newest scientific areas, namely integrated ferroelectrics and fractal radio systems, have been shown on the simplest example. For this goal, the numerically analytical method for an approximate solution of the equation for the electric charge value on a ferroelectric capacitor with negative capacitance connected to a fractal voltage source through a series resistor has been developed. This method combines the regularisation of the input fractal signal (voltage), decomposition of obtained fractal voltage by means of Haar wavelets using the Mallat algorithm, and matching of analytical expressions for charge at the constant input voltage. The accuracy of the method proposed and change of the fractal dimension of voltage before and after schemes mentioned have been discussed.

1 Introduction

At present all over the world great attention is paid to the development of integrated ferroelectrics [1–3]. Having arisen at the end of the last century, this interdisciplinary scientific direction aims at the integration of ferroelectric materials with technologies of microelectronic manufacture.

The discovery [4] of the effect of negative capacitance (NC) in ferroelectric heterostructures, or the so-called NC capacitor, is sure to push powerfully the evolution of this direction. More exactly, under the designation of the NC capacitor we mean a two-layer planar ferroelectric nanoheterostructure from lead zirconate-titanate Pb(Zr₀.₉Tio₀.₁)O₃ and strontium titanate SrTiO₃ with negative differential capacitance at room temperature [4]. The voltage \( U_C \) of the NC capacitor depends on its charge \( q \) as follows [4]:

\[
U_C = -\alpha \cdot q + \beta \cdot q^3, \alpha > 0, \beta > 0, \tag{1}
\]

where according to the experimental data of article [4] \( \alpha \sim 10^{10} \) \( V/C \) и \( \beta \sim 0.5 \times 10^{20} \) \( V/C^3 \).

The application of NC capacitors in different radio devices is certain to give rise to a new branch of radio engineering [5–10]. On the other hand, it is known that since the 80th years of the 20th century Kotel’nikov Institute of Radio Engineering and Electronics of Russian Academy of Sciences has been working intensively on the creation of breakthrough information technologies in the framework of paradigm ‘Fractal Radiophysics and Fractal Radio Electronics’. Corner stones of this interdisciplinary scientific direction are both synergetic principles of the theory of fractals and effects of scaling and deterministic chaos (for more details, see [11–13]). The structural scheme of the paradigm suggested by A.A. Potapov is presented in Fig. 1.

This report demonstrates new opportunities appearing for radio engineering under the joint development of these two above described interdisciplinary scientific directions, namely, in the report, we shall consider the electric circuit shown in Fig. 2, in which the NC capacitor is connected in series with the resistance \( R \). A number of piecewise smooth input voltage \( U(t) \) have been considered in papers [5, 6]. Reference [7] has been devoted to the case of the stochastic input voltage. That is why the rest of the report is organised as follows. In Section 2, we describe in detail the practical scheme of the generator of fractal signals. At first, this scheme has been claimed in brief report [14]. We underline that there are sharp distinctions between our device and devices suggested in [15] and patent [16]. Peculiarities of mathematical modelling of the passage of signals from such generator via the circuit in Fig. 2 are presented in Section 3. In the final section elaborated results are discussed.

2 Generator of fractal signals

Let us consider a van der Pol generator. The electrical scheme of the van der Pol generator on the basis of a field effect transistor is shown in Fig. 3.
Dimensionless current \( y \) in the oscillatory circuit on Fig. 3 is known to obey to the next (see [17] and references therein):

\[
y + y = \mu \left( p - \frac{4}{\tau} \cdot y^2 \right) + \mu \cdot \xi(t),
\]

(2)

where \( y \) and \( \dot{y} \) are the first and the second derivatives of the dimensionless current \( y \) with respect to dimensionless time \( \tau \). This variable is expressed via real time \( t \) as \( \tau = \omega \cdot t \), where \( \omega = (1/L \cdot C) \) is equal to the circular frequency of generation. Furthermore, let us denote the steepness of dependence of the drain current and thermal voltage fluctuations [17].

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Let the output of the electric circuit in Fig. 4 be the input of the generator of fractal signals. Thus, it follows from expression (8) that if \( b \geq 1/a^2 \), then we obtain the physical Weierstrass function again, its fractal dimension being less than the same value for function (5). Otherwise, if \( b < 1/a^2 \), then the fractality of function (8) vanishes.

Fig. 5 shows the behaviour of dimensionless functions (5) and (8) with \( a = 0, 5, b = 5, m = 7, U_0 = 1, \phi_0 = \pi \), and all \( \phi_n = \psi_n = 0 \). From the inequality (7), it follows that in this case the Weierstrass function (6) lies in a strip of width \( 2 \cdot \epsilon \), where \( \epsilon = 2^{-7} \approx 10^{-2} \), around its physical version (5). Under these values of parameters, the fractal dimension of the output voltage from the generator \( D \approx 1.569 \) and the fractal dimension of autocorrelation function of the signal (5),

\[
\frac{U_{\text{in}}(t) \cdot U_{\text{in}}(t + \tau)}{U_0^2} = \frac{\sum_{n=1}^{m} a^n \cdot \cos(b^n \cdot \phi_n \cdot t + \psi_n + \phi_0)}{\sum_{n=1}^{m} a^n}.
\]

(8)

Thus, the function \( \cos \) from expression (8) is nowhere differentiable and has a fractal dimension \( D = 2 + \ln a/\ln b \) [11, 13]. That is why according to [11, 13] we shall call function (5) the physical Weierstrass function.

In order to eliminate the influence of random phase shifts \( \phi_n \) and determine phase shifts \( \psi_n \), one ought to pass the output signal (5) from the summator in Fig. 4 through a correlator. The autocorrelation function of the signal (5),

\[
\frac{U_{\text{in}}(t) \cdot U_{\text{in}}(t + \tau)}{U_0^2} = \frac{\sum_{n=1}^{m} a^n \cdot \cos(b^n \cdot \phi_n \cdot t + \psi_n + \phi_0)}{\sum_{n=1}^{m} a^n}.
\]

(8)

Indeed, because the inequality \( |\cos| \leq 1 \) is true on the entire numerical axis, the inequality

\[
\frac{U(t) - U_0(t)}{U_0} \leq U_0 \cdot \frac{a^{m+1}}{1 - a}
\]

(7)

is also true.

In accordance with (2), one can find that the phase shift \( \phi \) in formula (3) possesses the following conditional probability density [17]:

\[
w(\phi, \tau|\phi_0, 0) = \frac{1}{\pi} + \frac{1}{\pi} \cdot \exp(-\mu \cdot \pi \cdot \phi^2 \cdot \tau \cdot 3 \cdot \pi). \cos[n \cdot (\phi - \phi_0)]
\]

(4)

where \( \phi_0 \) is the initial phase shift. It is obvious that under \( \tau \approx (1/\mu \cdot \epsilon^2) \), expression (4) converges to the uniform distribution \( w(\phi, \tau|\phi_0, 0) = \frac{1}{\pi} \).

Let us now consider a device with a structural scheme presented in Fig. 4. The device contains in the generator of fractal signals that obeys to formula:

\[
U_{\text{in}}(t) = U_0 \cdot \frac{\sum_{n=1}^{m} a^n \cdot \cos(b^n \cdot \phi_n \cdot t + \psi_n + \phi_0)}{\sum_{n=1}^{m} a^n}
\]

(5)

with parameters \( 0 < a < 1, b > 1, \) and \( a \cdot b \geq 1 \), in accordance with series (4), with phase shifts \( \phi_n \) being independent random values uniformly distributed in the interval \([0, 2\pi]\). Phase shifts \( \psi_n \) obey to formula: \( \psi_n = \Phi_n(\omega n) \), where functions \( \Phi_n(\omega) \) are phase-frequency responses of band-pass filters in Fig. 4.

It is easy to observe that function (5) uniformly approximates the well-known Weierstrass function (see [12, 13] and references therein) with the same parameters with any accuracy.
where \( \gamma = \alpha R \) and \( I_0 = U_0 \cdot a^2/R \).

The exact solution of (10) consists of two parts:

\[
g(t) = g_s(t) + g_u(t),
\]

namely, the stable part:

\[
g_s(t) = \sum_{n=1}^{\infty} I_n \cdot \frac{\omega_n \cdot \sin(\omega_n \cdot t)}{\gamma + \omega_n^2}
\]

and the unstable part:

\[
g_u(t) = \left( g_0 + \sum_{n=1}^{\infty} I_n \cdot \frac{\gamma}{\gamma + \omega_n^2} \right) \exp(\gamma \cdot t).
\]

The fractal dimension of the stable part of charge (12) proves to be very sensitive to the value of ratio \( \gamma/\omega_0 \). Temporal dependences of function (12) in the dimensionless form under \( \gamma/\omega_0 = 10^3 \) and \( \gamma/\omega_0 = 1 \) are presented in Fig. 6 (other parameters are: \( a = 0.5, b = 4, m = 7, U_0 = 1, \omega_0 = \pi, R = 1 \)). From Fig. 6b, one can observe that under \( \gamma/\omega_0 = 1 \) the fractality of function (12) disappears. One can explain easily this effect taking into account that amplitude–frequency response of (10), which is

\[
A(\omega) = \left\{ \frac{1}{\gamma^4 + \omega^2} \right\}.
\]

Therefore, if \( \gamma/\omega_0 >> 1 \), then circular frequencies of harmonics of series (5) as a whole are less then the cut-off frequency 1/\( \gamma \). Otherwise, if \( \gamma/\omega_0 \sim 1 \), then series (5) is truncated due to the suppression of its superior harmonics.

At last, from expression (13), we can see that at the \( t > 1/\gamma \) contribution of the second term in formula (11) becomes defining. Also, under \( t > 1/\gamma \), one has to consider the nonlinear term in input (9).

### 3.2 Description of the numerically analytical method

There is no exact solution of (9); nevertheless, we can construct an approximate solution of this equation with the fractal input voltage on the interval of time \( t \in [0, T] \) by other way.

It is well known that one can represent the Hilbert space \( L^2(\mathbb{R}) \) as follows [18]:

\[
L^2(\mathbb{R}) = \bigcup_{j=0}^{+\infty} V_j,
\]

with the orthonormal basis of each subspace \( V_j \) being formed by the following functions:

\[
\phi_{jk}(t) = T_j^{-1/2} \cdot \phi(t/(T_j - k)/T_j) = T \cdot 2^{-j}.
\]

The function \( \phi(\eta) \) is expressed via the Heaviside step function as

\[
\phi(\eta) = \theta(\eta) \cdot \theta(1 - \eta).
\]

Let us project the function \( U_j(t) = U_{\text{in}}(t) \cdot \phi(t/T) \) on the subspace \( V_j \) of space \( L^2(\mathbb{R}) \):

\[
U_j(t) = P_j[U_j(t)] = \sum_{k=-\infty}^{+\infty} a_{jk} \cdot \phi_{jk}(t).
\]

Coefficients of decompositions (15) are defined by the usual way [18]:

**Fig. 5** Output values from the generator of the fractal signal
(a) Dimensionless output voltage from the generator of the fractal signal, (b) Dimensionless autocorrelation function of output voltage from the generator of the fractal signal

**Fig. 6** The stable part of the linear response of NC capacitor on the fractal signal
(a) Graph of stable component of dimensionless charge at \( \gamma/\omega_0 = 10^3 \), (b) Graph of the stable component of the dimensionless charge at \( \gamma/\omega_0 = 1 \)

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\begin{equation}
\alpha_{jk} = \int_{-\infty}^{\infty} U_j(t) \cdot \phi_{jk}(t) \cdot dt.
\end{equation}

Using properties of functions (14), it is easy to establish that there are only 2\(^2\) non-zero coefficients (16), namely
\begin{equation}
a_{jk} = T_j^{-1/2} \int_{\alpha}^{\beta} U_m(t) \cdot dt,
\end{equation}
where \( t_\alpha = k \cdot T_j \) and \( k = 0, 1, \ldots, 2^l - 1 \). Thus, for series (5), integrals (17) are equal to
\begin{equation}
a_{jk} = \frac{2 \cdot U_m}{\sqrt{T_j} \cdot k} \left[ \sum_{n=1}^{m} a_n \cdot \sin \left( \frac{2 \cdot k + 1}{2} \cdot \omega_n \cdot T_j \right) \cdot \cos \left( \frac{2 \cdot k + 1}{2} \cdot \omega_n \cdot T_j \right) \right].
\end{equation}

The example of such an approximation on the basis of formulae (15) and (18) for the dimensionless fractal voltage (5) with the parameters \( a = 0.5, b = 4, m = 7, U_m = 1.0, \alpha_0 = \pi \) is presented in Fig. 7. For the numerical experiment \( j = 6 \) and \( T = 1/3 \) have been chosen. It was also checked that with the increase of \( j \), the precision of approximation grows.

The decomposition (15) on each interval of time \( t \in [t_j, t_{j+k+1}] \) takes a constant value (own for each interval); therefore, substituting it into the right-hand side of (9) instead of the fractal voltage \( U_j(t) \) for each of these intervals, we obtain the following equation:
\begin{equation}
R \cdot \frac{dq}{dt} - a \cdot \dot{q} + \beta \cdot \ddot{q} = a_{jk}, \quad t \in [t_j, t_{j+k+1}].
\end{equation}

Equation (19) is an equation with separable variables. Let us recall the procedure of construction of its exact solution according to [6].

At first, it is necessary to find roots \( \dot{q}_m^j \) (\( l = 1, 2, 3 \)) of the algebraic equation
\begin{equation}
\beta \cdot \ddot{q} - a \cdot \dot{q} - a_{jk} = 0.
\end{equation}

For instance, one can solve this equation by means of the well-known Cardano formula.

After that, it is required to obtain the following partial-fraction decomposition:
\begin{equation}
\frac{R}{a_{jk}} + a \cdot \dot{q} - \beta \cdot \ddot{q} = \sum_{l=1}^{3} A_l^{(j)} \cdot \ln \left| \frac{\dot{q} - \dot{q}_l^j}{\dot{q}_j(t) - \dot{q}_l^j} \right|.
\end{equation}

At last, for \( t \in [t_j, t_{j+k+1}] \), the exact solution of (19) is
\begin{equation}
t - t_j = \sum_{l=1}^{3} \int_{t_j}^{t} A_l^{(j)} \cdot \ln \left| \frac{\dot{q} - \dot{q}_l^j}{\dot{q}_j(t) - \dot{q}_l^j} \right|.
\end{equation}

In expression (22), parameters \( \dot{q}_m^j \) are thought to be real values. However, (20) may possess the pair of complex conjugate roots. Nevertheless, formula (22) is valid in this case too because of the existence of the complex variable \( z \) of the next identity:
\begin{equation}
atan z = \frac{1}{2} \cdot \ln \frac{1 + i \cdot z}{1 - i \cdot z}.
\end{equation}

Thus, starting from \( \ddot{q}(t_j) = 0 \), instead of an exact solution \( \dot{q}(t) \) of (9), we have found its piecewise smooth approximate solution \( \ddot{q}(t) \) in the framework of formulae (20)–(22).

4 Conclusion

All considerations of this report are included into proposed by Potapov in books [11–13] paradigm of fractal radio systems, the scheme of which is shown in Fig. 1. In this report, the method of numerically analytical estimation of the solution of nonlinear (9) arising in the sphere of integrated ferroelectric devices has been suggested. This method is very comfortable for the calculation of the fractal dimension of charge of the NC capacitor. In order to estimate this value in the framework of approaches developed in [11–13], one ought to calculate, according to formulae (20)–(22), two sequences of charges both for projecting the input voltage on subspace \( V_j \) and for projecting it on subspace \( V_{j+1} \). In fact, one should solve \( 2^l (22) \) for \( t = t_{j+k} \) and \( 2^{l+1} (22) \) for \( t = t_{j+k+1} \).

Special attention must be paid to the investigation of accuracy of replacing the charge \( q(t) \) of the NC capacitor by its approximation \( \ddot{q}(t) \). To realise this research program, both wide range of numerical experiments and theoretical considerations are required. For example, one can improve the precision of our method due to the next observation. It is obvious that the orthonormal bases (14) of subspaces \( V_j \) are built from the maternal function \( \psi(t) \) of the Haar wavelet system [13, 18]. It means that one can apply subspaces \( W_j \) with bases \( \psi_{j,k}(t) = \sqrt{2} \cdot \psi(2^j \cdot t \cdot k), \) created from the Haar wavelet \( \psi(t) = \phi(2 \cdot t) - \phi(2 \cdot t - 1) \) [13, 18]. After that, we expand the input subspace \( V_j \) as follows: \( V_j = V_{j-N} \oplus W_{j-N} \oplus \cdots \oplus W_{j-N} \), where \( N \) is the depth of the wavelet decomposition. In this case, formula (15) is reduced to
\begin{equation}
P_j[U_j(t)] = \sum_{k=-\infty}^{\infty} a_{j-N,k} \cdot \phi_{j-N,k}(t) + \sum_{k=1}^{N} d_{j-1,k} \cdot \psi_{j-1,k}(t).
\end{equation}

with the approximating and detailing coefficients of the subsequent levels of resolution being calculated from the coefficients (18) using the Mallat algorithm [18]:
\begin{equation}
a_{j-1,k} = \frac{a_{j,k} + a_{j,k+1}}{\sqrt{2}},
\end{equation}
\begin{equation}
d_{j-1,k} = \frac{a_{j,k} - a_{j,k+1}}{\sqrt{2}}.
\end{equation}

The above-discussed approach is certain to be true because function (23) remains piecewise constant. It is clear that \( N \) strongly depends on the fractal dimension \( D \) of the input voltage.

We emphasise that our choice of the Haar wavelets in expansion (23) and corresponding scaling filters in recurrence formulas (24) is dictated directly by the possibility of finding exact solutions of (19) with a constant input voltage. We consider these wavelets with a scaling parameter equal to 2, but for some fractal functions, for example, for the Knopp function (11, 13), the Haar wavelets with a scaling parameter of 3, 4, 5, etc may be more convenient. The scheme of the generator of the fractal signal of Fig. 4 is compatible with the complementary metal–oxide–semiconductor technology and can be realised on a single chip. Under the modification of band-pass filters, the scheme of the same structure can produce voltage not only in the form of the physical Weierstrass function.
but also in the form of the physical Riemann function, the physical Darboux function, and so on so forth.

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