Conformally invariant teleparallel theories of gravity

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Abstract

We analyze the construction of conformal theories of gravity in the realm of teleparallel theories. We first present a family of conformal theories which are quadratic in the torsion tensor and are constructed out of the tetrad field and of a scalar field. For a particular value of a coupling constant, and in the gauge where the scalar field is restricted to assume a constant value, the theory reduces to the teleparallel equivalent of general relativity, and the tetrad field satisfies Einstein’s equations. A second family of theories is formulated out of the tetrad field only, and the theories are not equivalent to the usual Weyl Lagrangian. Therefore the latter is not the unique genuinely geometrical construction that yields a conformally invariant action. The teleparallel framework allows more possibilities for conformal theories of gravity.

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1 Introduction

Conformal invariance is a natural extension of the symmetries of Einstein’s general relativity, namely, the covariance of the metric tensor and of Einstein’s equations under space-time coordinate transformations. The invariance of a possible theory of gravity under conformal transformations requires either that one adds a scalar field to the theory, or adopt the Lagrangian based on the square of Weyl’s tensor. The addition of a scalar field to the Hilbert-Einstein action was already considered by Dirac [1] and Utyiama [2]. They established conformal theories of gravity in terms of the metric tensor, and of a vector and scalar fields. Dirac [1] noted that the addition of a scalar field to the Lagrangian density would avoid the complications that arise with Weyl’s action principle, that yields field equations much more complicated than Einstein’s equations.

A conformal theory of gravity is a viable candidate for the quantum theory of gravity [3, 4], since it is expected to be renormalizable. In the latter references a conformal transformation is performed in the non-conformally invariant Hilbert-Einstein action such that the conformal factor \( \omega(x) \) is extracted from the metric tensor \( g_{\mu\nu} \) according to \( g_{\mu\nu} = \omega^2 \tilde{g}_{\mu\nu} \), and considered as an independent degree of freedom. In ref. [3] it is suggested the condition \( \det \tilde{g} = -1 \) on \( \tilde{g}_{\mu\nu} \). Because of this condition, the quantity \( \tilde{g}_{\mu\nu} \) is not exactly a tensor, since it does not transform as an ordinary tensor. But it is argued that the effective theory constructed out of \( \tilde{g}_{\mu\nu} \) is conformally (scale) invariant.

A theory of gravity with conformal symmetry has been addressed recently in connection with the dark matter and dark energy problems [5, 6], and in the investigation of anisotropic cosmological solutions [7]. In this approach the Lagrangian density is given by the square of Weyl’s tensor, and is ultimately written in terms of the square of the Ricci tensor and of the scalar curvature. It is usually argued in the literature that Weyl’s Lagrangian density is the unique combination constructed out of the curvature tensor that leads to a conformally invariant theory. While this statement might be correct, Weyl’s Lagrangian is not the unique genuinely geometrical quantity that displays conformal invariance.

In this article, we show that it is possible to establish a conformally (scale) invariant family of theories in the framework of teleparallel theories of gravity constructed out of the tetrad field. We will show that a quadratic combination of the torsion tensor that slightly deviates from the Lagrangian
density of the teleparallel equivalent of general relativity (TEGR) can be made conformally invariant, with the help of a scalar field. The addition of a suitable kinetic term for the scalar field yields a theory that reduces to the TEGR when the scalar field is gauge fixed to a constant value. We remark that the role of conformal transformations in $f(T)$ type theories has already been analyzed in ref. [8].

We will also show that a family of Lagrangian densities that depend on the fourth power of the torsion tensor, with no addition of a scalar field, is invariant under conformal transformations and yields conformal theories of gravity that so far have not been presented and investigated in the literature. These theories are not equivalent to the usual Weyl’s theory. Since the theories presented here are invariant under conformal transformations, a possibility exists that they could contribute to the formulation of the quantum theory of gravity.

**Notation:** space-time indices $\mu, \nu, ...$ and SO(3,1) indices $a, b, ...$ run from 0 to 3. Time and space indices are indicated according to $\mu = 0, i, a = (0), (i)$. The tetrad field is denoted $e^a_{\mu}$, and the torsion tensor reads $T_{a\mu\nu} = \partial_\mu e_{a\nu} - \partial_\nu e_{a\mu}$. The flat, Minkowski spacetime metric tensor raises and lowers tetrad indices and is fixed by $\eta_{ab} = e_{a\mu} e_{b\nu} g^{\mu\nu} = (-1, +1, +1, +1)$. The determinant of the tetrad field is represented by $e = \det(e^a_{\mu})$.

The torsion tensor defined above is often related to the object of anholonomity $\Omega^a_{\mu\nu}$ via $\Omega^a_{\mu\nu} = e_{a\lambda} T^a_{\mu\nu}$. However, we assume that the (non-scale invariant) spacetime geometry is defined by the tetrad field only, and in this case the only possible nontrivial definition for the torsion tensor is given by $T^a_{\mu\nu}$. This torsion tensor is related to the antisymmetric part of the Weitzenböck connection $\Gamma^\lambda_{\mu\nu} = e_{a\lambda} \partial_\nu e_{a\mu}$, which establishes the Weitzenböck spacetime. The curvature of the Weitzenböck connection vanishes. However, the tetrad field also yields the metric tensor, which determines the Riemannian geometry. Therefore in the framework of a geometrical theory based on the tetrad field, one may use the concepts of both Riemannian and Weitzenböck geometries.

## 2 TEGR with conformal invariance

A conformal transformation on the space-time metric tensor $g_{\mu\nu}$ affects the length scales. It transforms $g_{\mu\nu}$ into $\bar{g}_{\mu\nu} = e^{2\theta(x)} g_{\mu\nu}$, where $\theta(x)$ is an arbi-
trary function of the space-time coordinates. The same transformation on both the tetrad field and its inverse is defined by
\[
\tilde{e}_{a\mu} = e^{\theta(x)} e_{a\mu}, \quad \tilde{e}^{a\mu} = e^{-\theta(x)} e^{a\mu}.
\] (1)

The expressions above determine the transformation properties of all geometrical quantities of interest in this analysis. The transformation of the projected components of the torsion tensor \( T_{abc} = e_b^\mu e_c^\nu (\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu}) \) is given by
\[
\tilde{T}_{abc} = e^{-\theta} (T_{abc} + \eta_{ac} e_b^\mu \partial_\mu \theta - \eta_{ab} e_c^\mu \partial_\mu \theta),
\]
\[
\tilde{T}^{abc} = e^{-\theta} (T^{abc} + \eta^{ac} e_b^\mu \partial_\mu \theta - \eta^{ab} e_c^\mu \partial_\mu \theta).
\] (2)

It follows from the equation above that for the trace of the torsion tensor \( T_a = T^b_{\, ba} \), we have
\[
\tilde{T}_a = e^{-\theta} (T_a - 3 e_a^\mu \partial_\mu \theta),
\]
\[
\tilde{T}^a = e^{-\theta} (T^a - 3 e^{a\mu} \partial_\mu \theta).
\] (3)

With the help of equations (2) and (3), it is possible to verify that the behaviour of the three torsion squared terms that determine the Lagrangian density of teleparallel theories of gravity is given by
\[
\tilde{T}^{abc} \tilde{T}_{abc} = e^{-2\theta} (T^{abc} T_{abc} - 4T^{\mu} \partial_\mu \theta + 6g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta),
\]
\[
\tilde{T}^{abc} \tilde{T}_{bac} = e^{-2\theta} (T^{abc} T_{bac} - 2T^{\mu} \partial_\mu \theta + 3g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta),
\]
\[
\tilde{T}^a \tilde{T}_a = e^{-2\theta} (T^a T_a - 6T^{\mu} \partial_\mu \theta + 9g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta).
\] (4)

In view of the equations above it is straightforward to check that the quantity
\[
L = \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - \frac{1}{3} T^a T_a
\] (5)
transforms under a conformal transformation according to \( \tilde{L} = e^{-2\theta} L \). Equation (5) is the point of departure for the construction of conformally invariant theories.
As a consequence of eq. (1) we find that for the determinant \( e \) of the tetrad field we have \( \tilde{e} = e^{4\theta} e \). Therefore, we introduce a scalar field \( \phi \) that is assumed to transform as

\[
\tilde{\phi} = e^{-\theta} \phi. \tag{6}
\]

With the help of eq. (6) it is easy to verify that

\[
e\phi^2 \left( \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - \frac{1}{3} T^a T_a \right) \tag{7}
\]

is invariant under coordinate transformations and conformal transformations. We note that in the TEGR [9, 10] the coefficient of the term \( T^a T_a \) is \(-1\), and not \(-1/3\) as it appears in the expression above.

The Lagrangian density for the scalar field is established by noting that a covariant derivative may be defined as

\[
D_\mu \phi = \left( \partial_\mu - \frac{1}{3} T_\mu \right) \phi, \tag{8}
\]

where \( T_\mu = T^\lambda \lambda_\mu = T^a a_\mu \). Under the transformation (1) we have \( \tilde{T}_\mu = e^{-\theta}(T_\mu - 3\partial_\mu \theta) \), and thus \( \tilde{D}_\mu \tilde{\phi} = e^{-\theta} D_\mu \phi \).

Therefore the Lagrangian density

\[
\mathcal{L} = k e \left[ -\phi^2 \left( \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - \frac{1}{3} T^a T_a \right) + k' g^{\mu \nu} D_\mu \phi D_\nu \phi \right], \tag{9}
\]

where \( k = 1/(16\pi G) \) is invariant under conformal transformations. The constant coefficient \( k' \) is fixed by requiring that the condition \( \phi = 1 \) (or \( \phi = \phi_0 = \text{constant} \)) in \( \mathcal{L} \) leads to the usual teleparallel Lagrangian that establishes the TEGR. It is easy to verify that for this purpose we must have \( k' = 6 \).

Thus the Lagrangian density that is invariant under space-time coordinate transformations and under conformal transformations, and that yields the Lagrangian density of the TEGR in the limiting case \( \phi \to \phi_0 = \text{constant} \), is given by

\[
\mathcal{L} = k e \left[ -\phi^2 \left( \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - \frac{1}{3} T^a T_a \right) + 6 g^{\mu \nu} D_\mu \phi D_\nu \phi \right]. \tag{10}
\]
The expression above may be simplified in two steps. First we rewrite
the right hand side of (5) as
\[
\frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - \frac{1}{3} T^a T_a = \Sigma^{abc} T_{abc} + \frac{2}{3} T^a T_a ,
\]
where
\[
\Sigma^{abc} = \frac{1}{4} (T^{abc} + T^{bac} - T^{cab}) + \frac{1}{2} (\eta^{ac} T^b - \eta^{ab} T^c) ,
\]
and
\[
\Sigma^{abc} T_{abc} = \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - T^a T_a
\]
is the scalar that establishes the Lagrangian of the TEGR \[9, 10\], and ultimately yields Einstein’s equations. Second, we observe that
\[
6 g^{\mu\nu} D_\mu \phi D_\nu \phi = 6 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 4 g^{\mu\nu} \phi (\partial_\mu \phi) T_\nu + \frac{2}{3} \phi^2 T^a T_a .
\]
By substituting eqs. (11) and (14) into eq. (10), we find
\[
\mathcal{L}(e_{a\mu}, \phi) = k e \left[ -\phi^2 \Sigma^{abc} T_{abc} + 6 g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - 4 g^{\mu\nu} T_\nu \phi (\partial_\mu \phi) \right] .
\]
It is clear that by requiring \( \tilde{\phi} = 1 \), for instance, by means of a gauge conformal transformation, eq. (15) reduces to the Lagrangian density of the TEGR.

The Euler-Lagrange field equations obtained by varying \( \mathcal{L}(e_{a\mu}, \phi) \) with respect to \( e^{a\mu} \) and \( \phi \) are, respectively,
\[
e_{av} e_{b\mu} \partial_\lambda (e \phi^2 \Sigma^{b\nu\lambda}) - e \phi^2 (\Sigma^{b\nu\lambda} T_{bc\mu} - \frac{1}{4} e_{a\mu} \Sigma^{bcd} T_{bcd}) \\
- \frac{3}{2} e e_{a\mu} g^{\lambda\nu} (\partial_\lambda \phi)(\partial_\nu \phi) + 3 e e_{a\nu} (\partial_\mu \phi)(\partial_\nu \phi) \\
+ e e_{a\mu} g^{\lambda\nu} T_\lambda \phi (\partial_\nu \phi) - e \phi e_{a\nu} (T_\nu \partial_\mu \phi + T_\mu \partial_\nu \phi) \\
- e g^{\lambda\nu} \phi (\partial_\lambda \phi) T_{a\mu\nu} \\
- e_{av} e_{b\mu} \partial_\sigma [e g^{\lambda\nu} \phi (\partial_\lambda \phi) e^{br}] + e_{as} e_{b\mu} \partial_\nu [e g^{\lambda\nu} \phi (\partial_\lambda \phi) e^{br}] = 0 ,
\]
and
\[ \partial_\nu (e g^{\mu\nu} \partial_\mu \phi) + \frac{1}{6} \phi [e \Sigma^{abc} T_{abc} - 2 \partial_\mu (e T^\mu)] = 0. \] (17)

The field equation for \( \phi \) is obtained exactly in the form presented above. However, we note that the last two terms of the equation (the terms between brackets) define a quantity that is identically equal to the scalar curvature constructed out of the tetrad field only [10].

\[ e \Sigma^{abc} T_{abc} - 2 \partial_\mu (e T^\mu) \equiv -e R(e). \] (18)

It is precisely this identity that establishes the equivalence between the field equations of the TEGR and Einstein’s equations. Therefore the final form of the field equation for \( \phi \) is

\[ \partial_\nu (e g^{\mu\nu} \partial_\mu \phi) - \frac{1}{6} e \phi R(e) = 0. \] (19)

The solutions of the field equations (16) and (19) are related to the solutions of Einstein’s equations in vacuum. In order to prove this statement, we need first to verify whether \( \phi = \phi_0 = \text{constant} \) is a solution of the field equation (19).

By making \( \phi = 1 \), for instance, in eq. (16), the latter is reduced to

\[ e_{\alpha\nu} e_{\beta\mu} \partial_\lambda (e \Sigma^{b\nu\lambda}) - e (\Sigma^{bc} a T_{bc\mu} - \frac{1}{4} e_{\alpha\mu} \Sigma^{bcd} T_{bcd}) = 0. \] (20)

This equation is equivalent to Einstein’s equations in vacuum. The left hand side of the equation is identically equal to \( 1/2 [(eR_{\alpha\mu} - \frac{1}{2} e_{\alpha\mu} R)] \) [10]. A solution of the field equation (19) of the type \( \phi = \phi_0 = \text{constant} \) is possible provided \( R = 0 \). However, it follows from the Einstein’s equations in vacuum that the scalar curvature \( R \) vanishes. Therefore the set \( (\phi_0, e_{\alpha\mu}) \), where \( \phi_0 = \text{constant} \) and \( e_{\alpha\mu} \) is solution of Einstein’s equations in vacuum, is solution of the field equations (16) and (19).

Since \( \phi = 1 \) is a possible solution of the field equations, integration by parts in the Lagrangian density (15) is not a trivial and straightforward procedure. By performing integration by parts in the last term of (15), namely, in \(-4e g^{\mu\nu} T_{\nu} \phi (\partial_\mu \phi)\), the surface term that arises in the procedure does not vanish in the context of asymptotically flat space-times. Yet, neglecting the nonvanishing of this term, integration by parts in the term above yields \( 2\partial_\mu (e T^\mu) \phi^2 \). By adding to the latter the first term of (15), i.e., \(-e \phi^2 \Sigma^{abc} T_{abc}\),
and making use of the identity (18), we conclude that the Lagrangian density (15) is precisely given by $e [\phi^2 R + 6 g^{\mu \nu} (\partial_\mu \phi)(\partial_\nu \phi)]$. Therefore the theory defined by (10) or (15) is equivalent to the usual formulation of the Hilbert-Einstein Lagrangian endowed with conformal symmetry, but not the theory defined by (9) with $k' \neq 6$. Therefore theories for which $k' \neq 6$ in eq. (9) are conformal theories of gravity that have not been considered so far.

3 A purely geometrical conformal theory

A more complicated geometrical construction of a theory of gravity with conformal invariance is based on the square of the quantity given by eq. (5). The theory defined by

$$\mathcal{L}(e_{a\mu}) = \alpha e L^2, \quad (21)$$

where $\alpha$ is a dimensionless constant, is invariant under conformal transformations. No scalar field is needed for the conformal symmetry. Moreover, it is not possible to envisage any possible relation between $L^2$ and $C^a_{\alpha \beta \mu \nu} C_{\alpha \beta \mu \nu}$, where $C_{\alpha \beta \mu \nu}$ is the Weyl tensor, since it is not possible to write $L$ given by (5) in terms of the metric tensor. The field equation obtained by varying eq. (21) is given by

$$e_{a\nu} e_{b\mu} \partial_\lambda (e L \Lambda^{b\nu \lambda}) - \frac{1}{8} e_{a\mu} L = 0, \quad (22)$$

where

$$\Lambda^{abc} = \Sigma^{abc} + \frac{1}{3} (\eta^{ab} T^c - \eta^{ac} T^b)$$

$$= \frac{1}{4} (T^{abc} + T^{bac} - T^{cab}) + \frac{1}{6} (\eta^{ac} T^b - \eta^{ab} T^c). \quad (23)$$

By contracting eq. (22) with $e^{a\mu},$ it is not difficult to see that the equation is trace free, as it is expected to be. Evidently the field equation (22) is quite intricate, and it is not easy to find an analytic solution of the equation.

The Lagrangian density given by eq. (21) is not the only combination of the torsion tensor that yields a conformally invariant action integral. By inspecting eq. (4) it is easy to verify that
\[ L_1 = A T^{abc} T_{abc} + B T^{abc} T_{bac} + C T^{a} T_{a}, \]  
where \( A, B, \) and \( C \) are constants that satisfy
\[ 2A + B + 3C = 0, \]  
transforms covariantly under conformal transformations: \( \tilde{L}_1 = e^{2\theta} L_1. \) Therefore the family of theories defined by the Lagrangian density
\[ \mathcal{L}(e_{\alpha\mu}) = e L_1 L_2, \]  
where \( L_2 = A' T^{abc} T_{abc} + B' T^{abc} T_{bac} + C' T^{a} T_{a}, \) and \( A', B' \) and are \( C' \) constants that also satisfy \( 2A' + B' + 3C' = 0, \) is invariant under conformal transformations.

4 Final remarks

In this article we have presented families of conformal theories of gravity constructed out of the torsion tensor. The theory defined by the Lagrangian densities (10) or (15) is equivalent to the Hilbert-Einstein action endowed with conformal symmetry. The theory defined by eq. (9), with \( k' \neq 6, \) deviates from the standard formulation of general relativity. In the Lagrangians (9), (10) and (15) a scalar field is necessary in order to ensure the conformal symmetry.

On the other hand, the theory defined by eq. (21) is genuinely geometrical. Moreover it is not equivalent to the usual Weyl Lagrangian density. There is no geometrical relation between \( L^2, \) which is a functional of \( e_{\alpha\mu}, \) and the square of the Weyl tensor \( C_{\alpha\beta\mu\nu}, \) constructed out of the metric tensor. By combining eqs. (11) and (18) we see that
\[ L = -R + \frac{2}{e} \partial_\mu (e T^\mu) + \frac{2}{3} T_\mu T^\mu, \]  
where \( R \) is the scalar curvature. It is impossible to write a vector quantity like \( T^\mu \) as a covariant functional constructed out of the second rank tensor \( g_{\mu\nu}. \)

For the sake of completeness, we will indicate the construction of the teleparallel version of the Weyl Lagrangian density. Let us consider the
Weitzenböck connection $\Gamma^\lambda_{\mu\nu} = e^{a\lambda} \partial_\mu e_{a\nu}$, the Christoffel symbols $^0\Gamma^\lambda_{\mu\nu}$ and the contorsion tensor $K^\lambda_{\mu\nu}$ defined by

$$K^\lambda_{\mu\nu} = \frac{1}{2}(T^\lambda_{\mu\nu} + T^\lambda_{\nu\mu} + T^\lambda_{\nu\mu}),$$

where $T^\lambda_{\mu\nu} = e^{a\lambda}(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu})$. These quantities are identically related by

$$\Gamma^\lambda_{\mu\nu} = ^0\Gamma^\lambda_{\mu\nu} + K^\lambda_{\mu\nu}. \quad (28)$$

The curvature tensor constructed out of the Weitzenböck connection, on the left hand side of eq. (28), vanishes identically. Thus by substituting this connection into the standard expression of the Riemann tensor,

$$R^\lambda_{\sigma\mu\nu} = \partial_\mu \Gamma^\lambda_{\sigma\nu} - \partial_\nu \Gamma^\lambda_{\sigma\mu} + \Gamma^\lambda_{\beta\mu} \Gamma^\beta_{\sigma\nu} - \Gamma^\lambda_{\beta\nu} \Gamma^\beta_{\sigma\mu},$$

we arrive at the identity

$$R^\lambda_{\sigma\mu\nu}(^0\Gamma) = -\nabla_\mu K^\lambda_{\sigma\nu} + \nabla_\nu K^\lambda_{\sigma\mu} - K^\beta_{\lambda\mu} K^\lambda_{\beta\nu} + K^\lambda_{\beta\nu} K^\lambda_{\nu\beta}, \quad (29)$$

where the left hand side is the Riemann-Christoffel tensor. The covariant derivative $\nabla_\mu$ is constructed out of the Christoffel symbols $^0\Gamma^\lambda_{\mu\nu}$. It is clear that substitution of the left hand side of the expression above into the Weyl Lagrangian density,

$$\sqrt{-g} C^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu} = \sqrt{-g} \left(R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} - 2 R^{\mu\nu} R_{\mu\nu} + \frac{1}{3} R^2 \right), \quad (30)$$

or even into the simplified, reduced form

$$\sqrt{-g} C^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu} = -2 \sqrt{-g} (R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2), \quad (31)$$

obtained with the help of the Lanczos identity, yields a very complicated structure of a conformally invariant teleparallel theory, which contains derivatives of the torsion tensor, and is in no way related to expression (21) or to the square of eq. (27).

Therefore the Weyl Lagrangian density is not the unique purely geometrical entity that yields a conformal theory of gravity. The Lagrangian density given by eq. (26) seems to be the most general family of theories constructed out of the torsion tensor $T_{a\mu\nu}$ only (i.e., with no derivatives of $T_{a\mu\nu}$) that are
invariant under conformal transformations. The theories established here may play a role in the formulation of the quantum theory of gravity.

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