TETRAQUARK CANDIDATES IN THE LHCb’S DI-\(J/\psi\) MASS SPECTRUM

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Abstract

In this article, we study the first radial excited states of the scalar, axialvector, vector and tensor diquark-antidiquark type \(cc\bar{c}\bar{c}\) tetraquark states with the QCD sum rules and obtain the masses and pole residues, then we use the Regge trajectory to obtain the masses of the second radial excited states. The predicted masses support assigning the broad structure from 6.2 to 6.8 GeV in the di-\(J/\psi\) mass spectrum to be the first radial excited state of the scalar, axialvector, vector or tensor \(cc\bar{c}\bar{c}\) tetraquark state, and assigning the narrow structure at about 6.9 GeV in the di-\(J/\psi\) mass spectrum to be the second radial excited state of the scalar or axialvector \(cc\bar{c}\bar{c}\) tetraquark state.

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1 Introduction

The charmonium-like and bottomonium-like states are good subjects to study the exotic states and understand the strong interactions, if they are genuine tetraquark states, there are two heavy valence quarks and two light valence quarks, therefore the dynamics is complex compared to the tetraquark configurations consist of four heavy valence quarks, the attractive interactions between the two heavy quarks (or antiquarks) should dominate at the short distance and favor forming the genuine diquark-antidiquark type tetraquark states rather than the loosely bound tetraquark molecular states, because the light mesons cannot be exchanged between the two heavy quarkonia to provide attractions at the leading order. In recent years, the full-heavy tetraquark states have attracted much attentions and have been studied extensively [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

Recently, the LHCb collaboration reported their preliminary results on the observations of the \(cc\bar{c}\bar{c}\) tetraquark candidates in the di-\(J/\psi\) invariant mass spectrum at \(p_T > 5.2\) GeV [21]. They observed a broad structure above the threshold ranging from 6.2 to 6.8 GeV and a narrow structure at about 6.9 GeV with the significance of more than 5\(\sigma\), furthermore, they also observed some vague structures around 7.2 GeV. The masses of the full-heavy tetraquark states from the phenomenological quark models lie either above or below the di-\(J/\psi\) or di-\(\Upsilon\) threshold, and vary at a large range [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. It is the first time that clear structures in the di-\(J/\psi\) mass spectrum are observed experimentally, which may be evidences for the genuine \(cc\bar{c}\bar{c}\) tetraquark states. The observation of evidences for the \(cc\bar{c}\bar{c}\) tetraquark states provides important experimental constraints on the theoretical models and sheds light on the nature of the exotic states, and plays an important role in establishing the tetraquark states.

In Ref. [7], we study the mass spectrum of the ground states of the scalar, axialvector, vector and tensor full-heavy diquark-antidiquark type tetraquark states with the QCD sum rules, the predicted tetraquark masses lie blow the di-\(J/\psi\) or di-\(\Upsilon\) threshold. In the present work, we extend our previous work to study the mass spectrum of the first radial excited states of the scalar, axialvector, vector and tensor diquark-antidiquark type \(cc\bar{c}\bar{c}\) tetraquark states with the QCD sum rules, then take the masses of the ground states and the first radial excited states as the input parameters, resort to the Regge trajectory to obtain the masses of the second radial excited states, and make possible assignments of the LHCb’s new structures.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the first radial excited states of the \(cc\bar{c}\bar{c}\) tetraquark states in section 2; in section 3, we present

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the numerical results and use the Regge trajectory to obtain the masses of the second radial excited states; section 4 is reserved for our conclusion.

2 QCD sum rules for the first radial excited $cc\bar{c}\bar{c}$ tetraquark states

Let us write down the two-point correlation functions $\Pi(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ in the QCD sum rules firstly,

\[
\Pi(p) = i \int d^4xe^{ip\cdot x} \langle 0 | \{ J(x) J^\dagger(0) \} | 0 \rangle,
\]

\[
\Pi_{\mu\nu\alpha\beta}(p) = i \int d^4xe^{ip\cdot x} \langle 0 | \{ J_{\mu\nu}(x) J^\dagger_{\alpha\beta}(0) \} | 0 \rangle, \tag{1}
\]

where $J_{\mu\nu}(x) = J^1_{\mu\nu}(x), J^2_{\mu\nu}(x), J^3_{\mu\nu}(x)$

\[
J(x) = \varepsilon^{ijk}\varepsilon^{imn} c^{Tj}(x) C^\gamma c^k(x) \bar{c}^m(x) \gamma_\mu C \bar{c}^n(x),
\]

\[
J^1_{\mu\nu}(x) = \varepsilon^{ijk}\varepsilon^{imn} \left\{ c^{Tj}(x) C^\gamma c^k(x) \bar{c}^m(x) \gamma_\mu C \bar{c}^n(x) - c^{Tj}(x) C^\gamma c^k(x) \bar{c}^m(x) \gamma_\nu C \bar{c}^n(x) \right\},
\]

\[
J^2_{\mu\nu}(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} \left\{ c^{Tj}(x) C^\gamma c^k(x) \bar{c}^m(x) \gamma_\mu C \bar{c}^n(x) + c^{Tj}(x) C^\gamma c^k(x) \bar{c}^m(x) \gamma_\nu C \bar{c}^n(x) \right\}, \tag{2}
\]

the $i, j, k, m, n$ are color indexes, the $C$ is the charge conjugation matrix. We choose the currents $J(x), J^1_{\mu\nu}(x)$ and $J^2_{\mu\nu}(x)$ to interpolate the $J^{PC} = 0^{++}$, $1^{+-}$, $1^{--}$ and $2^{++}$ diquark-antidiquark type $cc\bar{c}\bar{c}$ tetraquark states, respectively, as the current $J^1_{\mu\nu}(x)$, where the Lorentz indexes $\mu$ and $\nu$ are antisymmetric, has both the spin-parity $J^P = 1^{+}$ and $1^{-}$ components.

At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J(x), J^1_{\mu\nu}(x)$ and $J^2_{\mu\nu}(x)$ into the correlation functions $\Pi(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ to obtain the hadronic representation [22, 23]. After isolating the ground state contributions of the scalar, axialvector, vector and tensor $cc\bar{c}\bar{c}$ tetraquark states, we obtain the results,

\[
\Pi(p) = \frac{\lambda_X^2}{M_X^2 - p^2 + \cdots},
\]

\[
= \Pi_S(p^2), \tag{3}
\]

\[
\Pi^1_{\mu\nu\alpha\beta}(p) = \frac{\lambda^2_{\mu\nu\alpha\beta}}{M_{Y, +}^2 (M_{Y, +}^2 - p^2)} \left( p^2 g_{\mu\alpha}g_{\nu\beta} - p^2 g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta}P_{\mu\nu}P_{\alpha\beta} + g_{\mu\beta}P_{\alpha\nu}P_{\alpha\beta} + g_{\nu\alpha}P_{\mu\beta}P_{\alpha\beta} \right) + \cdots,
\]

\[
\Pi^2_{\mu\nu\alpha\beta}(p) = \frac{\lambda^2_{\mu\nu\alpha\beta}}{M_{Y, -}^2 (M_{Y, -}^2 - p^2)} \left( -g_{\mu\alpha}P_{\nu\beta} + g_{\mu\beta}P_{\nu\alpha} + g_{\mu\beta}P_{\alpha\nu}P_{\alpha\beta} + g_{\nu\alpha}P_{\mu\beta}P_{\alpha\beta} \right) + \cdots, \tag{4}
\]

\[
\Pi^3_{\mu\nu\alpha\beta}(p) = \frac{\lambda^2_{\mu\nu\alpha\beta}}{M_{Y, +}^2 (M_{Y, +}^2 - p^2)} \left( g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha} + g_{\mu\beta}P_{\alpha\nu}P_{\alpha\beta} - g_{\nu\alpha}P_{\mu\beta}P_{\alpha\beta} + g_{\nu\alpha}P_{\mu\beta}P_{\alpha\beta} \right) + \cdots.
\]

\[
\Pi^4_{\mu\nu\alpha\beta}(p) = \frac{\lambda^2_{\mu\nu\alpha\beta}}{M_{Y, -}^2 (M_{Y, -}^2 - p^2)} \left( -g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha} - g_{\mu\beta}P_{\alpha\nu}P_{\alpha\beta} + g_{\nu\alpha}P_{\mu\beta}P_{\alpha\beta} \right) + \cdots, \tag{5}
\]
where \( \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2} \), the pole residues \( \lambda_X \) and \( \lambda_Y \) are defined by

\[
\langle 0 | J(0) | X(p) \rangle = \lambda_X,
\]

\[
\langle 0 | J_{\mu}^{1}(0) | Y^+(p) \rangle = \frac{\lambda_{Y^+}}{M_{Y^+}} \epsilon_{\mu\alpha\beta} \varepsilon^{\alpha} p^{\beta},
\]

\[
\langle 0 | J_{\mu}^{1}(0) | Y^-(p) \rangle = \frac{\lambda_{Y^-}}{M_{Y^-}} (\varepsilon_{\mu \rho \nu} - \varepsilon_{\nu \rho \mu}) ,
\]

\[
\langle 0 | J_{\mu}^{2}(0) | X(p) \rangle = \lambda_X \varepsilon_{\mu\nu},
\]

the \( \varepsilon_{\mu} \) and \( \varepsilon_{\mu\nu} \) are the polarization vectors of the axialvector, vector and tensor tetraquark states, respectively.

If we take into account (or isolate) the first radial excited states, we obtain

\[
\Pi_{S/T}(p^2) = \frac{\lambda_X^2}{M_X^2 - p^2} + \frac{\lambda_Y^2}{M_Y^2 - p^2} + \cdots,
\]

\[
\Pi_{A/V}(p^2) = \frac{\lambda_{X^+}^2}{M_{X^+}^2 (M_{X^+}^2 - p^2)} + \frac{\lambda_{Y^+}^2}{M_{Y^+}^2 (M_{Y^+}^2 - p^2)} + \cdots. \tag{7}
\]

We project out the axialvector and vector components \( \Pi_A(p^2) \) and \( \Pi_V(p^2) \) by introducing the operators \( P_A^{\mu\nu\alpha\beta} \) and \( P_V^{\mu\nu\alpha\beta} \), respectively,

\[
\tilde{\Pi}_A(p^2) = p^2 \Pi_A(p^2) = P_A^{\mu\nu\alpha\beta} \Pi_{\mu\nu\alpha\beta}(p),
\]

\[
\tilde{\Pi}_V(p^2) = p^2 \Pi_V(p^2) = P_V^{\mu\nu\alpha\beta} \Pi_{\mu\nu\alpha\beta}(p), \tag{8}
\]

where

\[
P_A^{\mu\nu\alpha\beta} = \frac{1}{6} \left( g^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2} \right) \left( g^{\nu\beta} - \frac{p^\nu p^\beta}{p^2} \right),
\]

\[
P_V^{\mu\nu\alpha\beta} = \frac{1}{6} \left( g^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2} \right) \left( g^{\nu\beta} - \frac{p^\nu p^\beta}{p^2} \right) - \frac{1}{6} g^{\mu\alpha} g^{\nu\beta}. \tag{9}
\]

It is straightforward but tedious to carry out the operator product expansion at the deep Euclidean space \( P^2 = -p^2 \to \infty \) or \( \gg \Lambda_{QCD}^2 \), then we obtain the QCD spectral densities through dispersion relation \( \Pi \),

\[
\Pi_{S/T}(p^2) = \int_{16m^2}^{\infty} ds \frac{\rho_{S/T}(s)}{s - p^2},
\]

\[
\tilde{\Pi}_{A/V}(p^2) = \int_{16m^2}^{\infty} ds \frac{\rho_{A/V}(s)}{s - p^2}, \tag{10}
\]

where

\[
\rho_{S/T}(s) = \frac{\text{Im} \Pi_{S/T}(s)}{\pi},
\]

\[
\rho_{A/V}(s) = \frac{\text{Im} \tilde{\Pi}_{A/V}(s)}{\pi}. \tag{11}
\]

We take the quark-hadron duality below the continuum thresholds \( s_0 \) and \( s_0' \), respectively, and perform Borel transform with respect to the variable \( P^2 = -p^2 \) to obtain the QCD sum rules:

\[
\lambda_X^2 \exp \left( \frac{M_X^2}{T^2} \right) = \int_{16m^2}^{s_0} ds \int_{z_i}^{z_f} dz \int_{t_i}^{t_f} dt \int_{r_i}^{r_f} dr \frac{\rho(s, z, t, r)}{s} \exp \left( \frac{r^2}{T^2} \right), \tag{12}
\]
\[ \lambda_{X/Y}^2 \exp \left( -\frac{M_{X/Y}^2}{T^2} \right) + \lambda_{X'/Y'}^2 \exp \left( -\frac{M_{X'/Y'}^2}{T^2} \right) = \int_{16m_c^2}^{s_0} ds \int_{z_i}^{s_f} dz \int_{t_i}^{t_f} dt \rho(s, z, t, r) \exp \left( -\frac{s}{T^2} \right), \] (13)

where the QCD spectral densities \( \rho(s, z, t, r) = \rho_S(s, z, t, r), \rho_A(s, z, t, r), \rho_V(s, z, t, r) \) and \( \rho_T(s, z, t, r), \)

\[ \rho_S(s, z, t, r) = \frac{3m_c^4}{8\pi^6} (s - m_c^2)^2 + \frac{tzm_c^2}{8\pi^6} (s - m_c^2)^2 (5s - 2m_c^2) + \frac{rtz(1 - r - t - z)}{1 - t - z} \frac{1}{32\pi^6} (s - m_c^2)^3 (3s - m_c^2) + \frac{rtz(1 - r - t - z)}{1 - z} \frac{1}{32\pi^6} (s - m_c^2)^3 (3s - m_c^2) \left[ 5 - \frac{t}{1 - t - z} \right] - \frac{rtz^2(1 - r - t - z)}{1 - z} \frac{3}{16\pi^6} (s - m_c^2)^4 + tz \frac{m_c^2}{r^3 12\pi^4} \left[ 2 + s \delta(s - m_c^2) \right] - \frac{tz(1 - r - t - z)}{r^2(1 - z)} \frac{1}{12\pi^4} (3s - 2m_c^2) \left[ 5 - \frac{t}{1 - t - z} \right] + \frac{tz(1 - r - t - z)}{r^2(1 - z)} \frac{1}{\pi} (s - m_c^2) \left[ s + \frac{s^2}{3} \delta(s - m_c^2) - \frac{z}{1 - z} \right] + \frac{1}{r^2 2\pi^4} t \frac{1}{4\pi^4} (3s - 2m_c^2) - \frac{1}{16\pi^4} (3s - 2m_c^2) \right) \]

\[ + \frac{(\alpha_s GG)}{\pi} \left\{ \frac{1}{r^2 6\pi^6} + \frac{t m_c^2}{r 6\pi^6} (3s - 2m_c^2) + \frac{t(1 - r - t - z)}{(1 - t - z)} \frac{1}{12\pi^4} (s - m_c^2) (2s - m_c^2) + \frac{t(1 - r - t - z)}{(1 - z)} \frac{1}{12\pi^4} (s - m_c^2) (2s - m_c^2) \left[ 2 - \frac{t}{1 - t - z} \right] - \frac{tz(1 - r - t - z)}{(1 - z)} \frac{1}{4\pi^4} (s - m_c^2)^2 + t(1 - r - t - z) \frac{1}{12\pi^4} z \left[ 4s - 3m_c^2 - \frac{z}{1 - z} \right] \right\}, \] (14)
\[
\rho_r(s, z, t, r) = \frac{3m_z^4}{16\pi^6} (s - m_e^2)^2 + \frac{t z m_z^2}{8\pi^6} (s - m_e^2)^2 (4s - m_e^2) \\
+ \frac{r t z (1 - r - t - z)}{1 - z} \frac{1}{320\pi^6} (s - m_e^2)^3 (17s - 5m_e^2) \\
+ \frac{r t z (1 - r - t - z)}{1 - z} \frac{1}{320\pi^6} (s - m_e^2)^3 \left[ (21s - 5m_e^2) - \frac{t}{1 - t - z} (17s - 5m_e^2) \right] \\
- \frac{r t z^2 (1 - r - t - z)}{1 - z} \frac{1}{32\pi^6} (s - m_e^2)^4 \\
+ r t z (1 - r - t - z) \frac{s}{80\pi^6} (s - m_e^2) \left[ 28s - 13m_e^2 - \frac{z}{1 - z} 7 (s - m_e^2) \right] \\
+ m_z^2 \frac{\alpha_e \bar{G}_e}{\pi} \left\{ - \frac{1}{r^3} \frac{m_z^4}{12\pi^4} \delta (s - m_e^2) - \frac{1 - r - t - z}{r^2} m_e^2 \frac{12\pi^4 [1 + s \delta (s - m_e^2)]}{12\pi^4 (2s - m_e^2)} \right. \\
- \frac{t z}{r^3} \frac{m_z^2}{12\pi^4} \left[ 1 + s \delta (s - m_e^2) \right] - \frac{t z (1 - r - t - z)}{r^2 (1 - t - z)} \frac{1}{12\pi^4 (2s - m_e^2)} \right. \\
- \frac{t z (1 - r - t - z)}{r^2 (1 - z)} \frac{1}{12\pi^4} \left[ 1 - \frac{t}{1 - t - z} \right] \\
+ \frac{t z^2 (1 - r - t - z)}{r^2 (1 - z)} \frac{1}{6\pi^4} (s - m_e^2) \left\} \\
+ \frac{1}{r^2} \frac{m_z^2}{4\pi^4} + \frac{t z}{r^2} \frac{1}{4\pi^4} (2s - m_e^2) \left\} \\
+ \frac{\alpha_e \bar{G}_e}{\pi} \left\{ - \frac{m_z^2}{48\pi^4} (4s - 3m_e^2) \right. \\
- \frac{r (1 - r - t - z)}{1 - t - z} \frac{1}{32\pi^4} (s - m_e^2) (3s - m_e^2) \right. \\
- \frac{r (1 - r - t - z)}{1 - z} \frac{1}{480\pi^4} (s - m_e^2) \left[ (17s - 5m_e^2) - \frac{t}{1 - t - z} 15 (3s - m_e^2) \right] \\
+ \frac{r z (1 - r - t - z)}{1 - z} \frac{1}{24\pi^4} (s - m_e^2)^2 \left. \\
- r (1 - r - t - z) \frac{1}{240\pi^4} \left[ (14s - 9m_e^2) - \frac{z}{1 - z} 21 (s - m_e^2) \right] \right. \\
- \frac{1}{r z} \frac{m_z^4}{36\pi^4} - \frac{t}{r} \frac{m_z^2}{18\pi^4} (2s - m_e^2) \right. \\
- t (1 - r - t - z) \frac{1}{72\pi^4} (s - m_e^2) (4s - m_e^2) \right. \\
- \frac{t (1 - r - t - z)}{1 - z} \frac{1}{72\pi^4} (s - m_e^2) \left[ 2 (2s - m_e^2) - \frac{t}{1 - t - z} (4s - m_e^2) \right] \\
+ \frac{t z (1 - r - t - z)}{1 - z} \frac{1}{24\pi^4} (s - m_e^2)^2 \\
- t (1 - r - t - z) \frac{1}{72\pi^4} \left[ 7s - 5m_e^2 - \frac{z}{1 - z} 5 (s - m_e^2) \right] \right\}, \tag{15}
\[ \rho_A(s, z, t, r) = \frac{3m_c^4}{16\pi^6} (s - m_c^2)^2 + \frac{tm_c^2}{8\pi^6} (s - m_c^2) (4s - m_c^2) + \frac{rtz(1 - r - t - z)}{16\pi^6} (s - m_c^2) (7s - 4m_c^2) + m_c^2 \frac{\alpha_s GG}{\pi} \left\{ \frac{1}{r^3} \frac{m_c^4}{12\pi^4} \delta (s - m_c^2) - \frac{1 - r - t - z}{r^2} \frac{m_c^2}{12\pi^4} \right\} \left[ 1 + \frac{1}{32\pi^4} \frac{m_c^2}{r^2} \right] \]

\[ (-1 + (s - m_c^2)^2 - \frac{rtz(1 - r - t - z)}{12\pi^4} (s - m_c^2)^2 - \frac{m_c^2}{32\pi^4} \right) \left( \frac{1 - r - t - z}{12\pi^4} \right) \]
and \( s = \frac{\sqrt{s}}{m_c} \). We introduce the notations \( \tau = \frac{1}{\sqrt{2} T} \), \( D^n = (-\frac{d}{d\tau})^n \), and use the subscripts 1 and 2 to represent the ground states \( X, Y \) and the first radially excited states \( X', Y' \) respectively for simplicity. We rewrite the two QCD sum rules in Eqs.(12)-(13) as

\[
\lambda_1^2 \exp \left(-\tau M_1^2\right) = \Pi_{QCD}(\tau), \tag{19}
\]

\[
\lambda_1^2 \exp \left(-\tau M_1^2\right) + \lambda_2^2 \exp \left(-\tau M_2^2\right) = \Pi'_{QCD}(\tau), \tag{20}
\]

here we introduce the subscript \( QCD \) to represent the QCD representation of the correlation functions \( \Pi_{S/A/V/T}\left(p^2\right) \) below the continuum thresholds. We derive the QCD sum rules in Eq.(19) with respect to \( \tau \) to obtain the masses of the ground states,

\[
M_1^2 = \frac{D \Pi_{QCD}(\tau)}{\Pi_{QCD}(\tau)}. \tag{21}
\]

We obtain the masses and pole residues of the ground states of the scalar, axialvector, vector and tensor \( cc\bar{c}\bar{c} \) tetraquark states with the two coupled QCD sum rules shown in Eq.(19) and Eq.(21) [7].

Now we study the masses and pole residues of the first radial excited states. Firstly, let us derive the QCD sum rules in Eq.(20) with respect to \( \tau \) to obtain

\[
\lambda_1^2 M_1^2 \exp \left(-\tau M_1^2\right) + \lambda_2^2 M_2^2 \exp \left(-\tau M_2^2\right) = D \Pi'_{QCD}(\tau). \tag{22}
\]

From Eq.(20) and Eq.(22), we can obtain the QCD sum rules,

\[
\lambda_1^2 \exp \left(-\tau M_1^2\right) = \frac{(D - M_j^2) \Pi'_{QCD}(\tau)}{M_i^2 - M_j^2}, \tag{23}
\]

where the indexes \( i \neq j \). Then let us derive the QCD sum rules in Eq.(23) with respect to \( \tau \) to obtain

\[
M_1^2 = \frac{(D^2 - M_j^2 D) \Pi_{QCD}(\tau)}{(D - M_j^2) \Pi'_{QCD}(\tau)}, \tag{24}
\]

\[
M_1^4 = \frac{(D^3 - M_j^2 D^2) \Pi'_{QCD}(\tau)}{(D - M_j^2) \Pi''_{QCD}(\tau)}. \tag{25}
\]

The squared masses \( M_i^2 \) satisfy the equation,

\[
M_i^4 - b M_i^2 + c = 0, \tag{26}
\]

where

\[
b = \frac{D^3 \otimes D^0 - D^2 \otimes D}{D^2 \otimes D^0 - D \otimes D},
\]

\[
c = \frac{D^3 \otimes D - D^2 \otimes D^2}{D^2 \otimes D^0 - D \otimes D},
\]

\[
D^j \otimes D^k = D^j \Pi'_{QCD}(\tau) D^k \Pi'_{QCD}(\tau), \tag{27}
\]

the indexes \( i = 1, 2 \) and \( j, k = 0, 1, 2, 3 \). Finally we solve the equation analytically to obtain two solutions [26, 27],

\[
M_i^2 = \frac{b - \sqrt{b^2 - 4c}}{2}, \tag{28}
\]
The masses of the ground state, the first radial excited state and the second excited state are well convergent. The dominant contributions come from the perturbative terms, the operator product expansion is well satisfied. In the Borel windows, scales of the QCD spectral densities and Borel windows, which are shown in Table 1. From the QCD sum rules in Eqs.(21)-(28), we can obtain the masses of both the ground states and the first radial excited states. Both the QCD sum rules in Eq.(21) and Eq.(27) have one continuum threshold parameter, both the continuum parameters \( s_0 \) and \( s_0' \) have uncertainties, in this aspect, the ground state masses from the QCD sum rules in Eq.(21) are not superior to the ones from Eq.(27). However, the ground state masses from the QCD sum rules in Eq.(27) suffer from additional uncertainties from the first radial excited states. In calculations, we observe that ground states masses from the QCD sum rules in Eq.(27) underestimate the experimental values [20, 27], so we neglect the QCD sum rules in Eq.(27).

3 Numerical results and discussions

We take the standard value of the gluon condensate [22, 23, 24], and take the \( \overline{\text{MS}} \) mass \( m_c(m_c) = (1.275 \pm 0.025) \text{GeV} \) from the Particle Data Group [25]. We take into account the energy-scale dependence of the \( \overline{\text{MS}} \) mass from the renormalization group equation,

\[
M^2_0 = \frac{b + \sqrt{b^2 - 4c}}{2}.
\]

From the QCD sum rules in Eqs.(27)-(28), we can obtain the masses of both the ground states and the first radial excited states. Both the QCD sum rules in Eq.(21) and Eq.(27) have one continuum threshold parameter, both the continuum parameters \( s_0 \) and \( s_0' \) have uncertainties, in this aspect, the ground state masses from the QCD sum rules in Eq.(21) are not superior to the ones from Eq.(27). However, the ground state masses from the QCD sum rules in Eq.(27) suffer from additional uncertainties from the first radial excited states. In calculations, we observe that ground states masses from the QCD sum rules in Eq.(27) underestimate the experimental values [20, 27], so we neglect the QCD sum rules in Eq.(27).

We should choose suitable continuum threshold parameters \( s_0' \) to avoid contaminations from the second radial excited states and borrow some ideas from the conventional charmonium states. The masses of the ground state, the first radial excited state and the second excited state are \( m_{J/\psi} = 3.0969 \text{GeV} \), \( m_{\psi'} = 3.686097 \text{GeV} \) and \( m_{\psi''} = 4.039 \text{GeV} \) respectively from the Particle Data Group [25]. In this article, we choose the flavor number \( n_f = 4 \) as we study the four-charm-quark states.

We take the standard value of the gluon condensate [22, 23, 24], and take the \( \overline{\text{MS}} \) mass \( m_c(m_c) = (1.275 \pm 0.025) \text{GeV} \) from the Particle Data Group [25]. We take into account the energy-scale dependence of the \( \overline{\text{MS}} \) mass from the renormalization group equation,

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{b_0}{b_0'}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1}{b_0} \log \frac{\mu^2}{m_c^2} + \frac{b_1^2}{b_0^2} \left( \log^2 \frac{\mu^2}{m_c^2} - \log \frac{\mu^2}{m_c^2} - 1 \right) + \frac{b_0 b_2}{b_0^2} \right],
\]

where \( t = \log \frac{\mu^2}{m_c^2} \), \( b_0 = \frac{33-2n_f}{12} \), \( b_1 = \frac{153-19n_f}{48} \), \( b_2 = \frac{2857-633n_f + 225n_f^2}{192} \), \( \Lambda = 213 \text{MeV} \), \( 296 \text{MeV} \) and \( 339 \text{MeV} \) for the flavors \( n_f = 5, 4 \) and \( 3 \), respectively [20]. In this article, we choose the flavor number \( n_f = 4 \) as we study the four-charm-quark states.

In Ref.[7], we obtain the ground state masses of the scalar, axialvector, vector and tensor diquark-antidiquark type full-heavy tetraquark states with the QCD sum rules. In the present work, we take the ground state masses as the benchmark and study the masses of the excited states. After trial and error, we reach the acceptable continuum threshold parameters, energy scales of the QCD spectral densities and Borel windows, which are shown in Table 1. From the Table, we can see that the pole dominance at the hadron side is well satisfied. In the Borel windows, the dominant contributions come from the perturbative terms, the operator product expansion is well convergent.

Now let us take into account all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the first radial excited states, which are also shown explicitly in Table 1 and Fig[1]. The predicted masses and pole residues are rather stable with variations of the Borel parameters, the uncertainties originate from the Borel parameters in the Borel windows are very small, in other words, there appear Borel platforms. Now the three criteria are all satisfied, we expect to make reliable or sensible predictions.
radial excited states, etc satisfy the Regge trajectory, the QCD sum rules \[7\]. If the masses of the ground states, the first radial excited states, the third excited states shown in Table 2 as input parameters to fit the parameters \(\alpha\), where the mass gaps \(\lambda\) can see that the mass gaps \(\Delta M\) can see that the mass gaps \(\lambda\) and obtain the masses of the second radial excited states, which are also shown in Table 2. From the Table, we can see that the predicted masses \(M_{2r}\) of the second radial excited states. The present predictions supports assigning the broad structure from 6.2 to 6.8 GeV in the di-\(J/\psi\) mass spectrum \[21\]. The present predictions supports assigning the broad structure from 6.2 to 6.8 GeV in the di-\(J/\psi\) mass spectrum to be the second radial excited state of the scalar, axialvector, vector or tensor diquark-antidiquark type \(cc\bar{c}\bar{c}\) tetraquark states, and assigning the narrow structure at about 6.9 GeV in the di-\(J/\psi\) mass spectrum to be the second radial excited state of the scalar or axialvector \(cc\bar{c}\bar{c}\) tetraquark state.

### Table 1: The Borel parameters, continuum threshold parameters, energy scales, pole contributions, masses and pole residues of the \(cc\bar{c}\bar{c}\) tetraquark states.

| \(J^{PC}\)   | \(T^z\) (GeV) | \(\sqrt{s_0}\) (GeV) | \(\mu\) (GeV) | pole          | \(M_{X/\psi}\) (GeV) | \(\lambda_{X/\psi}\) (10^{-4} \text{GeV}^3) |
|------------|--------------|-----------------------|-------------|--------------|-----------------------|-----------------------|
| 0^{++} (2S)| 4.4 - 4.8    | 6.80 ± 0.10           | 2.5         | (65 - 79)\%  | 6.48 ± 0.08           | 7.41 ± 1.12           |
| 1^{--} (2S)| 4.4 - 4.8    | 6.85 ± 0.10           | 2.5         | (69 - 82)\%  | 6.52 ± 0.08           | 5.56 ± 0.80           |
| 2^{++} (2S)| 4.9 - 5.3    | 6.90 ± 0.10           | 2.5         | (63 - 76)\%  | 6.56 ± 0.08           | 5.92 ± 0.83           |
| 1^{--} (2P)| 4.5 - 4.9    | 6.90 ± 0.10           | 2.2         | (57 - 73)\%  | 6.58 ± 0.09           | 3.46 ± 0.58           |

### Table 2: The masses of the \(cc\bar{c}\bar{c}\) tetraquark states with the radial quantum numbers \(n = 1, 2\) and 3.

| \(J^{PC}\)   | \(M_1\) (GeV) \[1\] | \(M_2\) (GeV) | \(M_3\) (GeV) |
|------------|-----------------------|--------------|--------------|
| 0^{++}     | 5.99 ± 0.08           | 6.48 ± 0.08  | 6.94 ± 0.08  |
| 1^{--}     | 6.05 ± 0.08           | 6.52 ± 0.08  | 6.96 ± 0.08  |
| 2^{++}     | 6.09 ± 0.08           | 6.56 ± 0.08  | 7.00 ± 0.08  |
| 1^{--}     | 6.11 ± 0.08           | 6.58 ± 0.09  | 7.02 ± 0.09  |

In Table 2 we present the masses of the ground states and the first radial excited states from the QCD sum rules \[7\]. If the masses of the ground states, the first radial excited states, the third radial excited states, etc satisfy the Regge trajectory,

\[ M_n^2 = \alpha(n - 1) + \alpha_0, \]

where the \(\alpha\) and \(\alpha_0\) are constants. We take the masses of the ground states and the first radial excited states shown in Table 2 as input parameters to fit the parameters \(\alpha\) and \(\alpha_0\), and obtain the masses of the second radial excited states, which are also shown in Table 2. From the Table, we can see that the mass gaps \(M_3 - M_1 = 0.91 \sim 0.95\) GeV, which are consistent with the mass gap \(m_{\psi'} - m_{J/\psi} = 0.94\) GeV. Furthermore, from the Table 1 we can see that the continuum threshold parameters \(\sqrt{s_0} \leq M_3\), where the \(M_3\) represents the central values of the masses of the second radial excited states.

From Table 2 we can see that the predicted masses \(M = 6.48 \pm 0.08\) GeV, \(6.52 \pm 0.08\) GeV, \(6.56 \pm 0.08\) GeV and \(6.58 \pm 0.09\) GeV for the first radial excited states of the scalar, axialvector, vector and tensor \(cc\bar{c}\bar{c}\) tetraquark states are consistent with the broad structure above the threshold ranging from 6.2 to 6.8 GeV in the di-\(J/\psi\) mass spectrum \[21\], while the predicted masses \(M = 6.94 \pm 0.08\) GeV and \(6.96 \pm 0.08\) GeV for the second radial excited states of the scalar and axialvector \(cc\bar{c}\bar{c}\) tetraquark states are consistent with the narrow structure at about 6.9 GeV in the di-\(J/\psi\) mass spectrum \[21\]. The present predictions supports assigning the broad structure from 6.2 to 6.8 GeV in the di-\(J/\psi\) mass spectrum to be the first radial excited state of the scalar, axialvector, vector or tensor \(cc\bar{c}\bar{c}\) tetraquark state, and assigning the narrow structure at about 6.9 GeV in the di-\(J/\psi\) mass spectrum to be the second radial excited state of the scalar or axialvector \(cc\bar{c}\bar{c}\) tetraquark state.

### 4 Conclusion

In this article, we construct the scalar and tensor currents to study the first radial excited states of the scalar, axialvector, vector and tensor diquark-antidiquark type \(cc\bar{c}\bar{c}\) tetraquark states with the QCD sum rules and obtain the masses and pole residues. Then we use the Regge trajectory to obtain the masses of the second radial excited states. The present predictions supports assigning
Figure 1: The masses of the first radial excited states of the tetraquark states with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$ and $D$ denote the scalar, axialvector, vector and tensor tetraquark states, respectively.
the broad structure from 6.2 to 6.8 GeV in the di-$J/\psi$ mass spectrum to be the first radial excited state of the scalar, axialvector, vector or tensor $c\bar{c}c\bar{c}$ tetraquark state, and assigning the narrow structure at about 6.9 GeV in the di-$J/\psi$ mass spectrum to be the second radial excited state of the scalar or axialvector $c\bar{c}c\bar{c}$ tetraquark state.

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