We study toroidal compactifications of Type II string theory with D-branes and nontrivial antisymmetric tensor moduli and show that turning on these fields modifies the supersymmetry projections imposed by D-branes. These modifications are seen to be necessary for the consistency of T-duality. We also show the existence of unusual BPS configurations of branes at angles that are supersymmetric because of conspiracies between moduli fields. Analysis of the problem from the point of view of the effective field theory of massless modes shows that the presence of a 2-form background must modify the realization of supersymmetry on the brane. In particular, the appropriate supersymmetry variation of the physical gaugino vanishes in any constant field strength background. These considerations are relevant for the $E_7(7)$-symmetric counting of states of 4-dimensional black holes in Type II string theory compactified on $T^6$. 

I. INTRODUCTION

D-branes have played an important role in recent advances in string theory. The realization by Polchinski [1] that D-branes are the carriers of Ramond-Ramond (RR) charges, coupled with detailed analyses of the interactions of D-branes with string modes [2,3], has contributed greatly to the current understanding of non-perturbative aspects of string theory. D-branes are BPS states, and so break half of the available supersymmetry and configurations of more than one D-brane can break supersymmetry further. Configurations consisting of various types of D-branes intersecting at right angles have been studied by many authors. Indeed, this has been important in the analysis of black hole entropy using D-brane states [4,5]. The corresponding brane configurations have also been studied in M-theory [6]. Recently it has been realized [7] that more general brane configurations can preserve some supersymmetry - branes may intersect at angles, given some simple restrictions on those angles.

In this note we extend the work of Ref. [7] by discussing toroidal compactifications of D-branes in Type II string theory with moduli. In Sec. II we present the general formulation of D-brane compactifications on bent tori with NS 2-form backgrounds, and show that turning on $B_{\mu\nu}$ modifies the supersymmetry projections imposed by the presence of the D-branes. A non-vanishing $B_{\mu\nu}$ or world-volume gauge field on a $p$-brane also induces the charges of smaller branes via world-volume Chern-Simons couplings. In Sec. III we review the general solution of branes at angles given in [7] and develop techniques that will be used in subsequent sections. In Sec. IV we apply T-duality to BPS configurations of branes at angles and show that the general formulation of Sec. III is needed to correctly account for BPS saturation after the duality transformation. Including the effects of the 2-form moduli leads to some surprises. For example, as discussed in Sec. IV, a 1-brane orthogonal to a 5-brane can be supersymmetric if a suitable background B field (or gauge field on the 5-brane) is turned on. In fact, after compactifying on a 6-torus there are spaces of $N=1, d=4$ supersymmetric configurations of 1-branes at angles to 5-branes. In the limit that the 1-brane is embedded in the 5-brane, these become $N=2, d=4$ configurations.

In Sec. V we examine these BPS configurations from the point of view of the effective field theory on the brane and conclude that the non-vanishing 2-form background must modify the realization of world-volume supersymmetry. In particular, the supersymmetry variation of the physical gaugino field must be modified in the presence of a background field strength. Working in light cone gauge, we derive the modified variation and show that any constant 2-form background (not necessarily self-dual or anti-self-dual) preserves world-brane supersymmetry. This cannot be understood by keeping the leading (Yang-Mills) terms in the world-volume effective theory and suggests that the non-linear structure of the Dirac-Born-Infeld D-brane effective action must come into play in an interesting way.

Finally, in Sec. VI we discuss briefly the relation of the D-brane configurations uncovered here to classical p-brane solutions of the Type II supergravity theories. In particular, the general black hole solutions of these theories

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* vijayb@puhep1.princeton.edu
† rgleigh@uiuc.edu
compactified on a six-torus contain branes compactified at angles. Therefore, the considerations of this paper are relevant for the $E_7(7)$-symmetric counting of states of 4-dimensional black holes in Type II string theory.

II. GENERAL FORMULATION

In this section we will derive the supersymmetry projections imposed by the presence of D-branes compactified on tori with non-vanishing NS 2-form moduli. Since there is a local symmetry connecting the world-volume field-strength $F$ with the (pullback of the) NS-NS 2-form $B$, it is $\hat{F} = F + B$ that will appear. We begin with a single 9-brane in the absence of any background fields. The supersymmetry condition is (in the boundary state formalism):

$$(Q^A + \Omega_9(\Gamma) \hat{Q}^A)|B\rangle = 0 \quad (1)$$

where $Q^A$ is a spacetime supersymmetry generator and is just the zero-momentum fermion vertex operator taken in some fixed ghost picture. The factor $\Omega_9(\Gamma) \equiv \pm \Gamma_{11}$ in (1) refers to the orientation of the brane. In a constant background gauge field, Ref. [10] gives the corresponding relation:

$$(Q + \Omega_9(\Gamma) M(\hat{F}) \hat{Q})^A|B\rangle = 0 \quad (2)$$

(Also see [11] in a D-brane context.) Turning on $\hat{F}$ produces a relative rotation of left and right moving vectors by the matrix $R = \left( \begin{array}{cc} 1 & a \\ -a & 0 \end{array} \right)$, and $M(\hat{F})$ is just the spinorial representation of this rotation. For example, if $\hat{F}$ is $2 \times 2$:

$$\hat{F} = \left( \begin{array}{cc} 0 & a \\ -a & 0 \end{array} \right) \quad a = \tan \beta/2 \quad (3)$$

then $R$ is just a rotation by the angle $\beta$. $M(F)$ is given by:

$$M(\hat{F}) = \det(1 + \hat{F})^{-1/2} \mathcal{E}(-1/2 \hat{F}_{\mu\nu}\gamma^\mu\gamma^\nu) \quad (4)$$

where $\mathcal{E}$ is the exponential function defined through its power series expansion, with the product of $\gamma$-matrices in each term antisymmetrized.

Now consider the case of a general $p$D-brane with both metric and antisymmetric tensor backgrounds. Let $\{e^i_\mu\}$ and $\{f^i_\mu\}$ be vielbeins spanning the spaces tangent and normal to the D-brane and let $\Gamma^i = e^i_\mu \gamma^\mu$ and $\tilde{\Gamma}^i = f^i_\mu \gamma^\mu$. We start with eq. (1) for a 9-brane and construct $p$D-branes by T-dualizing $9-p$ of the dimensions. T-duality changes the sign of the right-moving coordinates and worldsheet supersymmetry requires a similar change in the worldsheet fermions. The spacetime supersymmetry generators are built out of zero-momentum Ramond vertex operators and so we expect that the right-moving supersymmetry generator picks up a factor of $\tilde{\Gamma}^i \Gamma^{11}$ for each T-dualized dimension [12]. This gives the supersymmetry relation $\left[ Q \pm \prod_j (\tilde{\Gamma}^j \Gamma^{11}) \hat{Q} \right]^A|B\rangle = 0$. When an $\hat{F}$ background on the D-brane is turned on, open strings sense the background through their endpoints, causing a Lorentz-rotation of right-moving fields relative to left-moving fields. This further modifies the relation to $\left[ Q \pm \prod_j (\tilde{\Gamma}^j \Gamma^{11}) M(\hat{F}) \hat{Q} \right]^A|B\rangle = 0$. Commuting the factors of $\Gamma^{11}$ through the other gamma matrices we arrive at the supersymmetry condition:

$$(Q + \Omega_p(\Gamma) M(\hat{F}) \hat{Q})^A|B\rangle = 0 \quad (5)$$

where $\Omega$ is the volume form of the brane (normalized to unity) and

$$\Omega_p(\Gamma) = \Omega_{i_1 \ldots i_{p+1}} \Gamma^{i_1} \ldots \Gamma^{i_{p+1}}. \quad (6)$$

The sign is fixed by comparing to the 9-brane equation (1).

We may also check the form of eq. (5), at least for branes of Type IIB, by taking a limit of eq. (2). The boundary condition for open strings in a background field takes the form:

$$\partial_n X_i + \left( g^{-1} \hat{F} \right)_{ij} \partial_\tau X^j = 0 \quad (7)$$

1We take $\Gamma_{11} \hat{Q} = +\hat{Q}$.
Thus, if we take \( \hat{F} \) from zero to infinity, we will convert two boundary conditions from Neumann to Dirichlet. In fact, there are many directions in which to go to infinity, and these correspond via T-duality to D-branes at angles. We will explore this in detail in a later section, but for now we simply note (for simplicity, we take \( 2 \times 2 \hat{F} \)):

\[
M(\hat{F} \to \infty) \sim \lim_{\hat{F} \to \infty} \left( \frac{1}{F} + \ldots \right) \left( 1 + F\gamma\gamma + \ldots \right) = \gamma\gamma
\]

(8)

This is in agreement with eq. (8): the \( \gamma\gamma \) factor here removes the corresponding \( \gamma \)-matrices in \( \Gamma_{11} \) of (2), reducing it to eq. (5).

Eq. (5) gives the supersymmetry projection imposed by D-branes in a general constant background of NS fields. (See [14] for a related discussion in light cone frame.) We now proceed to construct BPS configurations of branes in such backgrounds.

III. BRANES AT ANGLES

In this section we review the general solution of branes at angles with \( \hat{F} = 0 \), which was found in Ref. [7]. Later we will use T-duality on such configurations to produce other \( \hat{F} \neq 0 \). These situations will illustrate how conspiracies of the moduli in Eq. (5) can restore BPS saturation in configurations which naively break supersymmetry.

To begin we establish some notation and conventions that are useful in analyzing supersymmetry in the presence of D-branes. Each D-brane will induce a condition (5). It is convenient to introduce a Fock basis in which

\[
a_j = \frac{1}{2}(\Gamma^{2j} + i\Gamma^{2j+1})
\]

(9)

\[
a_j^\dagger = \frac{1}{2}(\Gamma^{2j} - i\Gamma^{2j+1}).
\]

(10)

These operators satisfy the algebra \( \{a_j, a_k^\dagger\} = \delta_j^k \) and so are lowering and raising operators. We denote the vacuum state \( |0\rangle \), which is annihilated by all \( a_j \). Note for later use that \( \Gamma^{2j}\Gamma^{2j+1} = -i(2n_j - 1) \) where the \( n_j \) are occupation numbers. We will analyze the supersymmetry projections induced by D-branes using the raising and lowering operators introduced here.

A. General Solution of Branes at Angles

Consider two \( n \)-branes\(^2\) embedded in \( 2n \) compact dimensions where the two branes are related by an \( O(2n) \) rotation. We introduce complex coordinates \( \{z_i = x^a \pm ix^b\} \) where the \( x \) are orthonormal real coordinates of the target space. Without loss of generality, assume the first brane lies along \( \text{Re} z_i \). Supersymmetry is preserved when the \( O(2n) \) rotation acts as an \( SU(n) \) or \( ASU(n) \) rotation, for some choice of complex structure:

\[
z_i \to R^k_i z^l
\]

(11)

In this case the two supersymmetry conditions reduce to:

\[
\prod_{k=1}^{n} (a^k + a_k) \hat{Q} = \pm \prod_k \left( R^k_i a^l_i + R^k_i a^l_i \right) \hat{Q}
\]

(12)

Solutions for the plus (minus) sign are \( Q = \{0\}, \prod_k a_k^\dagger \{0\} \) provided that \( R \) is an \( SU(n) \) (\( ASU(n) \)) rotation. Quantum numbers of the \( a_k \) associated with the common dimensions fill out the spinorial representations.

In particular, consider 10-dimensional models compactified on tori \((T^2)^n \) (although this is a simplification, and we could easily turn on moduli to get to \( T^{2n} \)). We will take an orthonormal frame with \( e_{2j} \) along the A-cycle of the \( j^{th} \) \( T^2 \). We take the \( a^{th} \) D-brane to lie along the directions \( \cos \theta_{j}^{(a)} e_{2j} + \sin \theta_{j}^{(a)} e_{2j+1} \). Note that on tori of finite size,

\(^2\)Following [7], \( n \) refers to the dimensions of the branes which do not overlap. Each brane may or may not share additional dimensions.
there is a kind of quantization condition (or rather a rationality condition) on the angles \( \theta^{(a)}_j \): in order not to be space-filling, the D-brane should lie along some \((m_j, n_j)\)-cycle of the tori. Thus it is simple to see that:

\[
\tan \theta^{(a)}_j = \frac{m^{(a)}_j \text{Im} \tau_j}{n^{(a)}_j + m^{(a)}_j \text{Re} \tau_j}
\]

(13)

where \( \tau_j \) is the modular parameter of the \( j \)-th torus. The supersymmetry conditions for each D-brane are then

\[
Q \pm \prod_{j=1}^{n} \left( \cos \theta^{(a)}_j \Gamma^{2j} + \sin \theta^{(a)}_j \Gamma^{2j+1} \right) \tilde{Q} = 0
\]

(14)

or

\[
Q \pm \prod_{j=1}^{n} \left( e^{-i\theta^{(a)}_j} a_j + e^{+i\theta^{(a)}_j} a_j^\dagger \right) \tilde{Q} = 0
\]

(15)

We have assumed so far that each D-brane has undergone a \( U(n) \) rotation from a reference configuration lined up along the A-cycles of all tori. Ref. [7] found the condition for supersymmetry: the rotation required to bring one D-brane in line with the other should be in \( SU(n) \) (or \( ASU(n) \) in the case of opposite orientation). With the present conventions, for branes on \( (T^2)^n \), the \( SU(n) \) case corresponds to the statement that

\[
\sum_{j=1}^{4} (\theta^{(1)}_j - \theta^{(2)}_j) = 0 \pmod{2\pi}
\]

(16)

for any pair of branes. This is true up to changes in complex structure: for example, if we complex conjugate one of the coordinates (so that the corresponding angle changes sign) we still find solutions, if the angles satisfy the corresponding condition. There is also a suitable change in the Fock states.

### IV. T-DUALITY

In this section we discuss the action of T-duality on the BPS configurations of branes at angles from Sec. III A. We will see that the more general formulation of eq. [3] is necessary to consistently account for BPS saturation after T-duality.

#### A. 2-2 versus 0-4

Our first example concerns a BPS saturated configuration of 2-branes at angles. T-duality performed along directions spanned by one of the branes turns the 2-2 configuration into a 0-brane bound to a 4-brane with gauge fields on the 4-brane. The strength of the gauge fields is directly related to the angles in the 2-2 configuration. The 0-4 system is BPS by itself [12] and so we must show that gauge fields produced by T-duality do not spoil the supersymmetry due to their appearance in the supersymmetry relation [3].

1. 2-2

We consider two 2-branes on a \( (T^2)^2 \) in the 6789-directions, in the setup described in Sec. III A. The supersymmetry conditions reduce to:

\[
\tilde{Q} = \pm e^{-i} \sum_{j=1}^{4} (\theta^{(2)}_j - \theta^{(1)}_j)(2n_j - 1) \tilde{Q}
\]

(17)

Thus in the case that the branes have the same orientation, we have solutions when:

\[
\sum_{j=0}^{4} (\theta^{(1)}_j - \theta^{(2)}_j)(2n_j - 1) = 0 \pmod{2\pi}
\]

(18)
For $n_3 = n_4$ this yields the same solutions as eq. (16) and the surviving spinors are $\tilde{Q} = |00\rangle, |11\rangle = a_3^{\dagger}a_4^{\dagger}|00\rangle$. For $n_3 \neq n_4$ the surviving spinors are $|10\rangle$ and $|01\rangle$. The latter solution corresponds to a change of complex structure, in which the complex coordinate of one of the $T^2$ factors is conjugated. In the case of opposite orientation, we have solutions when

$$
\sum_{j=3}^{4} (\theta_j^{(1)} - \theta_j^{(2)}) (2n_j - 1) = \pi \mod 2\pi
$$

(19)

which is the modification of eq. (16) and eq. (18) for an ASU$(n)$ rotation. This result has a simple interpretation: a brane-antibrane pair at angles is identical to a brane-brane configuration at different angles: the solution in eq. (19) is obtained by turning a brane into an antibrane by a $\pi$-rotation in one 2-plane.

2. 0-4

The 2-2 configuration of the previous section can be turned by T-duality into a 0-brane bound to a 4-brane with gauge fields on the 4-brane. In this section we study possible solutions to the supersymmetry condition (5) for the 0-4 case. For simplicity, consider a 4-brane wrapped on a $T^4$ in the 6789 directions with a gauge field of the form:

$$
\hat{F} = \begin{pmatrix} f_3 \sigma & 0 \\ 0 & f_4 \sigma \end{pmatrix} \quad \sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
$$

(20)

$\hat{F}$ modifies the supersymmetry projections imposed by the 4-brane and also induces 2-brane and 0-brane charge via the Chern-Simons couplings on the 4-brane. Given the general formulation of Section 2, after a little algebra the supersymmetry conditions read:

$$
Q \pm \hat{Q} = 0
$$

(21)

$$
Q \pm \prod_{j=3}^{4} (-i)(2n_j - 1) \sqrt{1 + if_j(2n_j - 1)} \hat{Q} = 0
$$

(22)

If we define $f_j = -\cot \alpha_j$, the second equation becomes:

$$
Q \pm e^{i\sum_{j=3}^{4} \alpha_j (2n_j - 1)} \hat{Q} = 0
$$

(23)

(Note here that $0 < \alpha_j < \pi$ corresponds to finite $f_j$.). Thus, in the brane-brane (brane-antibrane) case, the two conditions are compatible if:

$$
\sum_{j=3}^{4} \alpha_j (2n_j - 1) = 0 \ \text{(mod 2\pi)}
$$

(24)

This should be compared to Eqs. (18) and (19). Because of the restriction in the range of $\alpha_j$, we conclude that the brane-antibrane (BA) case is solved by the spinors $|00\rangle$ and $|11\rangle$ for which $\alpha_3 + \alpha_4 = \pi \mod 2\pi$. In contrast, the brane-brane (BB) case has solutions $|01\rangle$ and $|10\rangle$ for which $\alpha_3 - \alpha_4 = 0 \mod 2\pi$. In terms of the physical gauge field $F$, the BA and BB cases correspond to anti-self-dual ($f_3 = -f_4$) and self-dual ($f_3 = f_4$) fields. This is related to the fact that 0-branes and anti-0-branes marginally bound to 4-branes can be understood as small instantons and anti-instantons of the 4-brane gauge theory. We have seen in Sec. 1 that in the presence of a constant gauge field background the supersymmetries that are present on the brane are different from the supersymmetries that survive in the absence of the background. The BPS configurations derived in this section are self-dual in order that the supersymmetries that survive the introduction of the 0-brane (instanton) on the 4-brane are a subset of the supersymmetries that are present on the 4-brane with constant gauge fields. We can also understand this from T-duality. Whether T-duality of a 2-2 configuration produces a 0-brane or an anti-0-brane bound to a 4-brane depends on the relative orientations of the original 2-branes after the $SU(2)$ (ASU$(2)$) rotations. This is also reflected in the gauge fields produced by T-duality.
3. T-duality

Now we show that T-duality maps the 2-2 configurations of Sec. [IV A 1] into precisely the BPS 0-4 configurations with gauge fields described above. Indeed, the angles between branes in the 2-2 case are mapped directly into gauge fields on the 4-brane. It is easiest to study T-duality by examining its effects on the boundary conditions for open strings propagating on the D-branes. (With a little more effort it is possible to examine the effects of T-duality directly on the background metric of the bent torus and the antisymmetric tensor. That analysis is consistent with the vertex operator manipulations below, and will not be presented here.)

Let us perform T-duality along the dimensions spanned by one of the two branes in Sec. [IV A 1]. Without loss of generality we can pick orthonormal coordinates \{X^6, \ldots, X^9\} where the first brane lies along \(X^6\) and \(X^8\). Then open strings propagating on the two branes have boundary conditions:

1. \(\partial_n X^{6,8} = \partial_\tau X^{7,9} = 0\)
2. \(\partial_n [\cos \alpha_{3,4} X^{6,8} + \sin \alpha_{3,4} X^{7,9}] = 0\)
   \(\partial_\tau [-\sin \alpha_{3,4} X^{6,8} + \cos \alpha_{3,4} X^{7,9}] = 0\) (25)

where we have defined \(\alpha_{3,4} = \theta_{3,4}^{(2)} - \theta_{3,4}^{(1)}\). T-duality exchanges Neumann and Dirichlet boundary conditions. So, T-dualizing along \(X^6\) and \(X^8\) gives:

1. \(\partial_\tau X^{6,7,8,9} = 0\)
2. \(\partial_n X^{7,9} + \cot \alpha_{3,4} \partial_\tau X^{6,8} = 0\)
   \(\partial_n X^{6,8} - \cot \alpha_{3,4} \partial_\tau X^{7,9} = 0\) (26)

These boundary conditions can be interpreted as a 0-brane or an anti 0-brane bound to a 4-brane with a gauge field \(F_{78} = \cot \alpha_3\) and \(F_{98} = \cot \alpha_4\) [3]. The two possible BPS conditions for 2-branes at angles \(\alpha_3 \pm \alpha_4 = 0\) translate into precisely the self-dual and anti-self-dual solutions in the 4-4 case.

B. 3-3 versus 1-5

In the example in the previous section T-duality of the 2-branes produced a 0-brane bound to a 4-brane. This configuration is already BPS saturated and our task was to show that the resulting gauge fields did not interfere with BPS saturation via their appearance in the supersymmetry projection Eq. (5). An even more instructive example arises from T-duality of 3-branes at angles. These can be converted to a 1-brane at an angle relative to a 5-brane. Such a configuration would normally break all the supersymmetries. We show here that the gauge fields that are simultaneously produced by T-duality restore BPS saturation via action of the matrix \(M(F)\) appearing in the supersymmetry relation (5).

1. 3-3

We consider two 3-branes\(^3\) on a \((T^2)^3\) in the 456789-directions, in the setup described in Sec. [III A]. The supersymmetry conditions reduce to:

\[ \tilde{Q} = \pm e^{-i} \sum_{j=2}^{4} (\theta_j^{(2)} - \theta_j^{(1)}) (2n_j - 1) \tilde{Q} \] (27)

When the branes have the same orientation, we have solutions when:

\[ \sum_{j=2}^{4} (\theta_j^{(1)} - \theta_j^{(2)}) (2n_j - 1) = 0 \text{ mod } 2\pi \] (28)

\(^3\)Again, these branes may intersect in additional non-compact dimensions.
For \( n_2 = n_3 = n_4 \) we find the same solutions as in Eq. (16) and the surviving spinors are \( \tilde{Q} = |000\rangle, |111\rangle = a_1^0 a_2^0 a_3^0 |000\rangle \). Filling out the spinorial representations in the four non-compact dimensions yields \( N = 1 \) supersymmetry in \( d = 4 \) for the generic case where the 3-branes intersect at a point. The solutions for unequal \( n_i \) preserve the same amount of supersymmetry albeit with different surviving spinors and correspond, as in the 2-2 case, to \( SU(3) \) rotations in complex structures where the complex coordinates of some of the \( T^2 \) factors have been conjugated. Again as in the 2-2 case, the solutions in the case of opposite orientations replace the right side of Eq. (28) with \( \pi \) mod \( 2\pi \) and reflect the fact that a brane is turned into an anti-brane by a rotation by \( \pi \) in one 2-plane.

An interesting phenomenon in the 3-3 case that differs from the 2-2 situation is supersymmetry enhancement in some parts of the moduli space of BPS configurations. For example, if \( \theta_2^{(2)} - \theta_1^{(1)} = 0 \), so that the 3-branes intersect along a line rather than on a point, the supersymmetry in enhanced to \( N = 2 \) in \( d = 4 \) because the spinors with \( n_3 = n_4 \) or \( n_3 \neq n_4 \) in the above analysis will have the solutions Eq. (28) regardless of the value of \( n_2 \).

2. 1-5

T-duality of 3-3 configurations of the previous section along two of the dimensions spanned by one of the D-branes yields a 1-brane at an angle to a 5-brane with a gauge field background. Consider a 5-brane on a \( T^5 \) in the 56789 directions with gauge fields of the form:

\[
\tilde{F} = \begin{pmatrix}
0 & 0 & 0 \\
0 & f_3\sigma & 0 \\
0 & 0 & f_4\sigma
\end{pmatrix}
\]

\[\sigma = \begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}\] (29)

Besides modifying the supersymmetry projections imposed by the 5-brane, \( \tilde{F} \) induces 3-brane and 1-brane charge. Now add a 1-brane at an angle \( \alpha_2 \) relative to the 5-brane on the \( T^2 \) in the 45-directions. Then the general formulation of Sec. [4] and an analysis parallel to Sec. [IV A 2] yields the supersymmetry condition:

\[
\tilde{Q} = \pm e^{i \sum_{j=2}^4 \alpha_j (2n_j - 1)} \tilde{Q}
\]

where we have defined \( f_j = -\cot \alpha_j \) for \( j = 3, 4 \) so that \( 0 < \alpha_j < \pi \) as in Sec. [IV A 2]. Therefore solutions exist in the brane-brane (brane-antibrane) case when

\[
\sum_{j=2}^4 \alpha_j (2n_j - 1) = 0(\pi) \text{ mod } 2\pi
\]

(31)

which should be compared to eq. (28). If we take \( \alpha_2 \geq 0 \), then given the restriction of the range of \( \alpha_3 \) and \( \alpha_4 \), the brane-antibrane (BA) case is solved by the spinors \( |000\rangle \) and \( |111\rangle \) when \( \alpha_2 + \alpha_3 + \alpha_4 = \pi \) mod \( 2\pi \). The BB case is solved by the spinors \( |110\rangle \) and \( |001\rangle \) when \( \alpha_2 + \alpha_3 - \alpha_4 = 0 \). These are solutions with \( N = 1 \) supersymmetry in the four non-compact dimensions.

It is instructive to examine the limit \( \alpha_2 \to 0 \) in which the 1-brane lies within the 5-brane. In this case, the restrictions on the range of \( \alpha_3 \) and \( \alpha_4 \) (which are equivalent to requiring finite field strengths on the 5-brane) gives solutions in the BB case when \( \alpha_3 - \alpha_4 = 0 \) and in the BA case when \( \alpha_3 + \alpha_4 = \pi \). In terms of the physical gauge fields these correspond to self-dual \( f_3 = f_4 \) and anti-self-dual \( f_3 = -f_4 \) gauge fields in the four dimensions on the 5-brane that are transverse to the 1-brane. This self-duality arises for the same reason as the self-duality of the gauge fields in the 0-4 configurations in Sec. [IV A 2]. Furthermore, the self-dual and anti-self-dual fields are consistent with four choices of spinors each: in the BB case solutions are \( |110\rangle, |010\rangle, |101\rangle \) and \( |001\rangle \). After filling out the spinorial representations in the non-compact dimensions this gives \( N = 2 \) supersymmetry in \( d = 4 \). In other words, as the relative angle between the 1-brane and 5-brane is decreased to zero we are seeing an enhancement of supersymmetry \( N = 2 \).

We know from Ref. [3] and from the Chern-Simons terms in the D-brane effective action that a 1-brane within a 5-brane can be understood as self-dual gauge field in the four dimensions transverse to the 1-brane on the 5-brane worldvolume. The conditions \( \alpha_2 - \cot^{-1} f_3 + \cot^{-1} f_4 = 0 \) and \( \alpha_2 - \cot^{-1} f_3 - \cot^{-1} f_4 = \pi \) that solve the BB and BA cases can be understood as a generalization of self-duality. Branes at angles arise in worldvolume terms as gauge fields shifted away from self-duality in a specific way.

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4 Additional solutions exist which reverse the signs of the \( \alpha_j \) with corresponding changes in the spinors.

5 We thank M. Cvetič for discussions regarding this point and its analogue in the language of classical p-brane solutions.
3. T-duality

To show the T-dual relation between the 3-3 and 1-5 configurations displayed above we repeat the analysis of Sec. IV A 3 that related the 2-2 and 0-4 configurations. We only want to T-dualize along two of the dimensions of one of the 3-branes in order to turn it into a 1-brane. So the treatment is identical to Sec. IV A 3 and the 3-3 system T-dualizes to a 1-5 system with $F_{79} = \cot \alpha_3$ and $F_{98} = \cot \alpha_4$. Here $\alpha_{3,4} = \theta_{3,4}^{(1)} - \theta_{3,4}^{(2)}$. This exactly matches the 1-5 configurations of the previous section.

V. THE FIELD THEORY PERSPECTIVE

In this section we re-examine the question of BPS saturation from the point of view of the world-brane effective theory. The world-brane theory has the field content of the dimensional reduction of $N = 1$ super Yang-Mills from 10 dimensions down to the brane [13]. To check if supersymmetry is preserved in the presence of a nonvanishing $F$ background we must study the variation of the gaugino. The super Yang-Mills variation of the gaugino is given by

$$\delta \chi = (-1/4) \Gamma^{\mu\nu} F_{\mu
u} \epsilon. \quad (32)$$

This would seem to be in direct contradiction of the assertion in Eq. (5) that any constant gauge field (or NS 2-form) background on a single D-brane is a BPS state that merely redefines the supersymmetries surviving the presence of the brane. In this section we resolve this conundrum by showing the physical gaugino vertex operator is modified in the presence of background gauge fields and that the variation of this field under the surviving supersymmetries vanishes identically. (Related work appears in Ref. [14].)

A. Light Cone Gauge

It is easiest to carry out the analysis in light-cone gauge with 10 dimensional fields. The analysis for any D-brane can be carried out by a straightforward dimensional reduction of our fields to the brane world-volume. To begin we recall the light-cone decomposition of fields and supersymmetries that appears in Sec. 5.3 of [13]. The left and right moving massless boson vertex operators transform in the $8_s$ representation of the transverse SO(8) in light-cone frame and are denoted $V^a_{BL}$ and $V^a_{BR}$, where $i = 1 \cdots 8$. States are constructed by the action of $V^a_{\tilde{B}} \xi^i$ where $\xi^i$ is the boson wavefunction. Ten dimensional 16-component Majorana-Weyl spinors decompose under the transverse SO(8) into 8-component dotted and undotted spinors that transform under the $8_s$ and $8_s$ representations. So we can split the left and right moving supersymmetries into $Q^a$, $Q^i$, $\tilde{Q}^a$ and $\tilde{Q}^i$. The massless spinor (gaugino) vertex operator can be split into dotted and undotted pieces, but the Dirac equation implies that the undotted fermion wavefunction is fully determined in terms of the dotted fermion components. So, on shell, fermion states are created by left and right moving operators $V^a_{FL} u^a$ and $V^a_{FR} u^a$ where $u^a$ is the gaugino wavefunction.

The action of the undotted supersymmetry on the bosons is summarized by [14]:

$$\begin{align*}
[\eta^a Q^a, V^a_{\tilde{B}L} \xi^i] &= V^a_{\tilde{B}L} \tilde{u}^a \\
[\eta^a \tilde{Q}^a, V^a_{\tilde{B}R} \xi^i] &= V^a_{\tilde{B}R} \tilde{u}^a
\end{align*} \quad (33)$$

where $\tilde{u}^a(\eta^a, \xi^i) = k^+ \eta^a \gamma^a_{\tilde{B}L} \xi^i$. Here $k^+$ and $k^-$ are transverse and light-cone components of the momentum in the boson and the gamma matrices are in the representation given in Appendix 5.B of [13]. (The dotted supersymmetries $\tilde{Q}^a$ act in a similar fashion except that $\tilde{u}$ is different in this case.) The transformed fermion wavefunction $\tilde{u}$ gives precisely the light cone decomposition of the ten dimensional super Yang-Mills gaugino variation in Eq. (32). In what follows we will construct the physical gaugino operator on a D-brane with background fields and show that its commutator with the surviving supersymmetry in Eq. (3) vanishes.

B. Construction of Vertex Operators

In the presence of a gauge field background, the bosonic coordinate $X^i = X^i_L(\tau - \sigma) + X^i_R(\tau + \sigma)$ of open strings propagating on the brane satisfying a boundary condition of the form $X^i_L = M_{ij} X^j_R$ where $M$ is the light-cone vector representation of the rotation $-\Omega(\Gamma)(1-F)/(1+F)$ that appears in eq. (3) [13]. Define a new coordinate $\tilde{X}^i_L = M_{ij} X^j_L$.\[\text{9}\]
Then the left and right moving parts of the dual coordinate $\tilde{X}^i = \tilde{X}^i_L + \tilde{X}^i_R$ satisfy Neumann boundary conditions. The bosonic part of the vertex operator for the massless boson is therefore proportional to $\partial_\tau \tilde{X}^i = \partial_\tau \tilde{X}^i_L + \partial_\tau \tilde{X}^i_R$. In terms of the original coordinate $X$, this means that the operator in proportional to $[(\partial_\tau + \partial_\sigma)X^i + M_{ij}^{-1}(\partial_\tau - \partial_\sigma)X^j]$. Multiplying by an overall factor of $M$, and supersymmetrizing the vertex operator tells us that the massless boson vertex operator that satisfies the boundary conditions imposed by the gauge field is proportional to the the following combination left and right moving operators:

$$V^i_B = V^i_{BL} + M_{ij}V^j_{BR}$$  \hspace{1cm} (34)

Supersymmetry then gives us a similar expression for the physical gaugino vertex operator:

$$V^\dot{a}_F = V^\dot{a}_{FL} + M_{\dot{a}b}V^b_{FR}$$  \hspace{1cm} (35)

where $M_{\dot{a}b}$ is an SO(8) dotted spinor representation of the rotation $M$, the lightcone decomposition of eq. (5). Note that we can construct another combination of $V_{FL}$ and $V_{FR}$:

$$\phi^\dot{a}_F = V^\dot{a}_{FL} - M_{\dot{a}b}V^b_{FR}$$  \hspace{1cm} (36)

but $\phi$ vanishes when acting on a physical state because of boundary conditions imposed by the presence of the gauge field. Finally, the light cone decomposition of the supersymmetry condition in eq. (11) is:

$$Q^a_i |B\rangle = (Q^a - M_{ab}\tilde{Q}^b)|B\rangle = 0$$  \hspace{1cm} (37)

with a similar equation for the dotted supersymmetries[14].

C. Variation of the Gaugino

As discussed in Sec. V A, in order to study the variation of the gaugino, we commute the supercharge with the massless boson vertex operator, and read off the wavefunction of the resulting fermion. Using Eq. (33) for the action of supersymmetries we find that:

$$[\eta^a Q^a_+, V^i_B \zeta^i] = V^\dot{a}_{FL} \tilde{u}^\dot{a}(\eta^a, \zeta^i) - V^\dot{a}_{FR} \tilde{s}^\dot{a}(\eta^a, \zeta^i)$$  \hspace{1cm} (38)

$$\tilde{s}^\dot{a}(\eta^a, \zeta^i) = \tilde{u}^\dot{a}(\eta^a M_{ca}, \zeta^i M_{ki})$$  \hspace{1cm} (39)

where $\tilde{u}^\dot{a}(\eta^a, \zeta^i)$ is defined below eq. (33). Using the definition of $V_F$ and $\phi_F$ in terms of $V_{FL}$ and $V_{FR}$ we can rewrite this as:

$$[\eta^a Q^a_+, V^i_B \zeta^i] = (1/2)V^\dot{a}_F \left[ \tilde{u}^\dot{a} - M_{\dot{a}b} \tilde{s}^b \right] \equiv (1/2)V^\dot{a}_F t^\dot{a}$$  \hspace{1cm} (40)

where we have dropped terms proportional to $\phi_F$ since this operator evaluates to zero on physical states. The wavefunction $t^\dot{a}$ can be decomposed as:

$$t^\dot{a} = k^+ \eta^a \left[ \gamma^i_{ab} - M_{ij}M_{\dot{a}b}M_{ab}\gamma^j_{lb} \right] \zeta^i$$  \hspace{1cm} (41)

The quantity in parentheses relates[14] the vector, dotted spinor and undotted spinor representations of SO(8) rotations and vanishes identically. As discussed in Sec. V A, the wavefunction $t^\dot{a}$ is the lightcone decomposition of $\delta \chi$, the variation of the physical gaugino field. We have therefore shown that the variation of the physical gaugino under the unbroken undotted supersymmetries vanishes identically. The dotted supersymmetries can be treated similarly.

The physical significance of this is clear: it is not enough, in seeking to identify all supersymmetric configurations, to simply look at the supersymmetry variations of super-Yang-Mills. The background field induces important mixing effects, both in the generator of supersymmetry, as well as the physical mass eigenstates. Presumably, in the presence of a constant background field, it is necessary to consider a supersymmetrized version of the full Dirac-Born-Infeld action which is correct to all orders in $\alpha'$. 

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[14] The symbol $\tilde{\cdot}$ denotes the action of a supersymmetry variation on a field, and $\cdot$ denotes the action of a supersymmetry variation on a superspinor.
VI. CLASSICAL SOLUTIONS AND BLACK HOLE ENTROPY

In this paper we have demonstrated that there are compactifications of D-branes at angles that are supersymmetric only when suitable antisymmetric tensor moduli fields are turned on at the position of the brane. It would be very attractive to demonstrate this from the point of view of the classical p-brane solutions of supergravity. To date, the general procedure for constructing intersecting brane solutions is only understood for orthogonal branes \cite{6} and for branes embedded in larger branes \cite{17}. Furthermore, there are no available intersecting p-brane solutions where the branes are localized on each other. Both of these difficulties impede an analysis of the issues of this paper from the viewpoint of classical solutions. The p-brane solutions corresponding to the D-brane configurations presented here will have the unusual property that the antisymmetric tensor field moduli will have to converge to some specific value as the core of the solution is approached regardless of the asymptotic value. When the intersecting p-branes at angles are compactified, this should yield a class of “fixed scalar” fields in the non-compact space. (See \cite{18} for discussions of such “fixed scalars” and their appearance in the physics of black holes.)

The results of this paper also have interesting applications to computations of extremal black hole entropy in Type II string theory. The generating solution for NS-NS charged black hole solutions of Type II on $T^6$ has been discussed in \cite{19,8}, and contains five quantized charges as opposed to the four that have appeared in the extant computations of four dimensional black hole entropy \cite{5}. The fifth charge, which is necessary in order to account for the $E_7(7)$ symmetry of the complete entropy formula, is associated with branes at angles. Indeed, the solutions of \cite{19,8} contain fundamental strings at angles relative to NS 5-branes and various dualities can be applied to produce black holes containing D-branes at angles. Our work has shown how configurations of branes at angles can be made supersymmetric by turning on NS 2-form backgrounds. These backgrounds also induce RR-charges via Chern-Simons couplings on the brane world-volume. These charges must be accounted for in the $E_7(7)$ counting of states. The dualities that produce the D-branes also produce Ramond-Ramond background fields and so it is necessary to understand these also in order to account for the black hole entropy. Work is in progress to understand these issues as well as to construct the classical solutions discussed in the previous paragraph \cite{9}.

VII. CONCLUSION

In this paper we have shown that non-vanishing antisymmetric tensor backgrounds modify the supersymmetry projections imposed by D-branes. These modifications are necessary for the consistency of T-duality since BPS configurations of branes at angles are dual to branes with antisymmetric tensor backgrounds. We argued that the realization of supersymmetry on the brane must also be modified in the presence of an NS 2-form and showed, in particular, that the variation of the physical gaugino vanishes in any constant background. One consequence of these results is the existence of unusual BPS configurations of branes at angles - for example, a 1-brane at an angle to a 5-brane is supersymmetric if a suitable background is turned on. The 1-5 system is also interesting in that the generic configuration has N=1 supersymmetry in the four non-compact dimensions, and these configurations are related to N=2 systems where the 1-brane lies within the 5-brane. We also sketched a forthcoming application of this work to the study of black hole entropy in string theory \cite{9}.

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