Anomalies in Universal Intensity Scaling in Ultra-relativistic Laser-Plasma Interactions

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Laser light incident on targets at intensities such that the electron dynamics is ultra-relativistic, gives rise to a harmonic power spectrum extending to high orders and characterized by a relatively slow decay with the harmonic number \(m\) that follows a power law dependence, \(m^{-p}\). Relativistic similarity theory predicts a universal value for \(p = 8/3\) up to some cut-off \(m = m^\ast\). Results presented in this Letter suggest that under conditions in which plasma effects contribute to the emission spectrum, the extent of this contribution may invalidate the concept of universal decay. We report a decay with harmonic number in the ultra-relativistic range characterised by an index \(5/3 \lesssim p \lesssim 7/3\), significantly weaker than that predicted by the similarity model.

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Since the earliest observations of harmonics of laser light reflected from target plasmas by Burnett et al. [1] and Carman, Rhodes and Benjamin [2], the topic of harmonic generation has been one of enduring interest. The range of harmonics excited is governed in large part by the level of harmonic generation has been one of enduring interest. The decay of intensity with harmonic order is found to depend on the details of the interaction physics and that plasma effects contribute to the emission spectrum, the extent of this contribution may invalidate the concept of universal decay. We report a decay with harmonic number in the ultra-relativistic range characterised by an index \(5/3 \lesssim p \lesssim 7/3\), significantly weaker than that predicted by the similarity model.

In this Letter we report results from a set of particle-in-cell (PIC) simulations in the UR range which suggest that the decay of harmonic intensities may be governed by plasma effects. In these circumstances we present evidence of spectral decay more complex than the universal decay proposed by BGP, no longer characterised by a unique index \(p = 8/3\) but by indices spanning a range \(5/3 \lesssim p \lesssim 7/3\).

The effects that appear in the spectrum reflect details of the interaction physics, in particular the plasma oscillations generated by bursts of highly relativistic electrons. We have shown elsewhere that with \(p\)-polarised light, collective effects not only give rise to emission at the plasma frequency and its harmonics [5] but can serve as a source of a modulation at the plasma frequency that appears in the spectrum for suitably chosen parameters [6]. Other work dealing with the effects of plasma oscillations driven by energetic electrons on emission spectra has been reported by Teubner et al. [7] and by Eidmann et al. [8].

In the simulations carried out to generate the harmonic spectra we treat the interaction of a laser pulse of length \(\tau_p\) with a plasma slab of initially uniform density, apart from an interface at the front end. This vacuum-plasma interface is characterised by a density profile with scale length \(\Delta\) that is some prescribed fraction of a wavelength across the ramp. We have used a 1-1/2D fully relativistic and explicit PIC code, that embeds the Bourdier technique [9] to allow for oblique incidence. Two vacuum gaps extend from both the front of the ramp and the planar rear surface of the plasma to the walls of the simulation box to allow for particle and wave propagation. When external fields reach the boundary and propagate outside the system they are not reflected and thus no longer considered in the interaction physics. Plasma particles are reflected at the boundaries by reversing their velocities. The plasma filled a simulation box extending over 4 laser wavelengths with \(2 \times 10^6\) particles distributed across \(10^4\) grids, giving a ratio of grid size to wavelength of \(\sim 10^{-4}\). The number of particles employed and the choice of the simulation param-
eters is sufficient to resolve both the Debye length and the highest frequency mode with acceptable resolution.

The initial electron temperature was chosen to be 100 eV and we used gaussian pulses of variable length and profile. Ions were treated as a neutralizing immobile background, an acceptable approximation given the femtosecond time scales governing these simulations where a 17 fs gaussian pulse is used throughout. The normalized quiver momentum $a_0 \sim 8.5 \left(I_{20} \lambda_L^2\right)^{1/2}$ lay in the range 5-20 with slab densities between 20 $n_c$ and 200 $n_c$, where $n_c$ is the critical density. The reflected emission spectra showing the relative strength of the different oscillation modes are determined from a time-resolved Fourier decomposition of the harmonic contents of the reflected electric field at the vacuum gap, normalized to the fundamental.

In Fig. 1 we contrast PIC spectra from runs at UR intensities with light incident both normally (Fig. 1ab) and obliquely (Fig. 1cd) on the surface of the plasma slab. The plasma density is chosen to be initially uniform with $n_e = 40 n_c$ and the peak intensity of the incident pulse such that $a_0 = 10$. Fig. 1a shows harmonic power levels up to $m = 1000$; harmonics up to $m \sim 200$ are characterised by conversion efficiencies $\geq 10^{-6}$. The decay in peak harmonic power levels with harmonic number $m$ is well represented across the range by the index $p = 8/3$, in agreement with both the similarity analysis and the PIC spectrum due to Baeva, Gordienko and Pukhov [4]. At the same time it appears that across the harmonic range in Fig. 1a there is no marked break in the spectrum predicted by their model at a critical harmonic $m^* = \sqrt{8 \alpha \gamma_s^3}$ where $\gamma_s$ is the maximum value of the relativistic factor associated with the motion of the plasma surface and $\alpha$, the second derivative of the surface velocity, is of order 1. For the parameters in Fig. 1, $m^* \sim 40 - 80$, adopting the BGP estimate for $\gamma_s$.

Fig. 1b presents data from the spectrum in Fig. 1a in which a set of data points is represented over the restricted harmonic range $m = 40$ to $m = 400$ to afford a close-up of the macroscopic intensity scaling applied across the full harmonic range in Fig. 1a. We arbitrarily normalize the harmonic power levels to $P_{100}$ and find from Fig. 1b that across the harmonic range (40 - 400) the scaling with harmonic number falls in a range $2.1 \lesssim p \lesssim 3.0$. In passing we note that this result is not greatly at variance with the scaling reported in [3] where $2.2 \lesssim p \lesssim 2.7$ and $2.5 \lesssim p \lesssim 3.0$ (the latter applied across a much higher harmonic range), depending on intensity. We draw no comparison between our simulated scaling and those from Ref. [3] in view of differences in parameters such as the pulse length (17 fs in the simulations as against 500 fs in the experiments) and without knowing the experimental plasma electron density. That notwithstanding, the results for s-polarised light in Figs. 1a,b are broadly consonant with the BGP scaling $p = 8/3$.

Figs. 1c, 1d highlight changes in the pattern of harmonic emission when $p$-polarised light is incident at 23° to target normal, these spectra showing distinct differences from those at normal incidence or for $s$-polarised light. It is at once apparent that on a macroscopic level, the spectrum is no longer well characterized by a universal decay index $p = 8/3$. In contrast to the spectrum in Fig. 1a there is now a more or less well-defined break in the spectrum in Fig. 1c, centred on $m \sim 100$. We emphasize that this feature is quite distinct from the spectral break deriving from a validity condition in the BGP self-similar model. The break in Fig. 1c occurs with the onset of modulation; below this the spectrum...
is enhanced over that for \( s \)-polarised light (cf. Fig. 1a) by plasma emission with the consequence that the decay coefficient across this region is reduced below the BGP "universal index". We show in Fig. 1d an intensity scaling up to \( m = 400 \) corresponding to that in Fig. 1b, with harmonic power levels normalised to \( P_{100} \), characteristic of the distinctive modulation in the spectrum in Fig. 1c. Below \( m = 100 \) decay is less rapid than the corresponding universal decay, with \( p \sim 5/3 \), while above this the best fit for this choice of parameters is given by \( p \sim 10/3 \).

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\begin{align*}
\text{(a)} & \quad a_0 = 5, n_e/n_c = 20, \quad \text{(b)} & \quad a_0 = 20, n_e/n_c = 40, \\
\text{(c)} & \quad a_0 = 10, n_e/n_c = 50, \quad \text{(d)} & \quad a_0 = 20, n_e/n_c = 100. \\
\end{align*}
\]

The lines indicate a scaling \( P_m = m^{-p} \), where \( p = 7/3 \) in (a), \( p = 5/3 \) in (b),(c) and \( p = 2 \) in (d).

In Fig. 2 we present a range of spectra for additional combinations of \( a_0 \) and \( n_e/n_c \), namely \( a_0, n_e/n_c = (5,20), (20,40), (10,50) \) and \( (20,100) \). All four combinations show spectra with distinctive structure across the range and serve to confirm the deviation from BGP universal scaling already apparent from Fig. 1c. We find some variation in the onset of modulation depending on particular combinations of \( a_0 \) and \( n_e/n_c \), with thresholds ranging from \( m \sim 20 - 30 \) (Fig. 2a,d) to \( m \sim 50 - 70 \) (Fig. 2b,c). Not only is there variation in the onset of modulation, there are also notable differences in modulation frequency across the range of parameters considered. This contrasts with our findings in Ref. [6] where, for a moderately relativistic range, spectra showed fairly regular modulation at the plasma frequency \( \omega_p \), with onset of modulation consistently at \( \sim 1.5 \omega_p \). Despite the variation in detail in the spectra in Fig. 2, the decay index in each case is tolerably well fitted by \( p \) in the range \( 5/3 \lesssim p \lesssim 7/3 \), markedly less than the spectral decay observed in simulations at lower \( a_0 \) [6].

The physics differentiating irradiation by \( p \)-polarized from \( s \)-polarized light derives from bursts of Brunel-excited electrons [10] that drive plasma oscillations to highly nonthermal levels close to the plasma surface. Given the steep gradients in density at the front of the simulation box, the plasma waves excited are no longer uniquely longitudinal but couple strongly to the radiation field, giving rise to emission at the plasma frequency in both back and forward directions, with weaker contributions at higher harmonics of the plasma frequency [5]. Boyd and Ondarza-Rovira further identified a robust feature in the spectrum that appeared between the plasma line and its second harmonic, typically at a frequency \( 1.5 \omega_p \), with width of the order of the plasma frequency. This so-called 'combination line' proved in the event to be the first cycle of a modulation that was evident in the high harmonic spectrum [6]. Across a wide range of combinations of \( (a_0, n_e/n_c) \) Boyd and Ondarza-Rovira found that the frequency at which harmonic intensities were modulated was governed by the plasma frequency. A Fourier decomposition of the plasma electron density showed correspondingly strong modulation at the Plasma frequency so that harmonics of the Laser frequency \( \omega_L \) such that \( m \gtrsim 1.5 (\omega_p/\omega_L) \) reflect this modulation in density, cf. [6].

In the UR-regime the laser-plasma interaction physics is strongly non-linear. Not only is the plasma line itself broadened, but its harmonics may also be strongly driven and likewise broadened. One consequence of the non-linearity is some variability in the order of plasma harmonics excited for different combinations of intensity and electron density. Another effect of emission at harmonics of the plasma frequency \( l \omega_p \) is a widening of the range of \( m \) well beyond the value \( m \sim 1.5 (\omega_p/\omega_L) \) at which the first minimum appeared in the harmonic spectrum at moderately relativistic energies, Ref. 6. Thus one might now expect the threshold for onset of modulation to be shifted to higher frequencies, typically \( m^* \sim (1 - 2)l (\omega_p/\omega_L) \) where \( l \) is the highest plasma harmonic excited, which may contribute to the variation found in Fig. 2.

While Fig. 2 shows departures from the universal \( p = 8/3 \) scaling proposed by BGP, fine detail in the
structure of these spectra is seen more readily over sections of the full harmonic range. In Fig. 3a we show a Fourier representation of the electron density at a point inside the plasma boundary for the parameters used in Fig. 1c. We see from this that the structure in the Fourier decomposition of electron density is reflected in the power spectrum. Above $m \sim 4 \omega_p/\omega_L \sim 25$ is a region extending to $m \sim 100$ over which the Fourier amplitudes show only modest decay, a detail mirrored in the behaviour of $P_m$ found in Fig. 1c, where there is a clear discontinuity in the decay index in this region.

As an illustration of the variability referred to earlier, we show in Fig. 3b a Fourier density representation for the parameter combination in Fig. 2d, i.e. $a_0 = 20, n_e/n_c = 100$, using a log-linear plot to highlight the periodicity of the density modulations. This shows a relatively broad range across which there is comparatively little variation in Fourier amplitudes; between $m \sim 80$ and $m \sim 140$ we find modulation at the plasma frequency. Above this the modulation frequency switches to approximately $2 \omega_p$. The triplet that appears over $170 \lesssim m \lesssim 230$ is the source of broadened emission over this range in Fig. 2d, typically $\sim 6 \omega_p$, wide though this is difficult to discern given the poor resolution in Fig. 2d.

![Figure 3](image)

Figure 3: (a) Fourier decomposition of the electron density corresponding to the parameters used to generate the spectrum shown in Fig. 1c; (b) a log-linear representation of Fourier amplitudes across the range $20 \leq m \leq 350$ for the harmonic spectrum in Fig. 2d, where $a_0 = 20, n_e/n_c = 100$.

The range of values of the decay index in the moderately relativistic range $0.5 \lesssim a_0 \lesssim 2.0$ considered in Ref. [6] varied from $p \sim 4$ to $p \sim 8/3$, for parameter choices such that typically $S = n_e/(a_0 n_c) \gtrsim 20$. In the UR regime $S \lesssim 10$ for the three $a_0$ values used, $a_0 = 5, 10, 20$. Fig. 4 shows the decay index from PIC runs spanning this range of parameters. For $a_0 = 5$ the decay index was found to be $p = 7/3$, irrespective of density. For higher $a_0$ this reduces to $p = 5/3$, or in a few instances, $p = 2$. For each combination considered, the index appears to be distinctly weaker than the universal value proposed by BGP. In contrasting the values of $p$ appearing in Fig. 4 with the BGP universal value $8/3$ we emphasise that nowise do we attribute even quasi-universality to a particular value of $p$, be it $p = 5/3$ or $7/3$.

That aside, results from this set of simulations support our contention that the decay of intensity with harmonic number in the ultra-relativistic regime is more complex than allowed by the “universal” decay index $p = 8/3$. The central tenet of the self-similar approach is that for constant values of the similarity parameter $S = n_e/(a_0 n_c)$, the laser-plasma dynamics remains similar, since $S$-similarity corresponds to a multiplicative transformation group of the Maxwell-Vlasov equations. While this is indeed the case for $s$-polarised or normally incident light, $S$ no longer serves to characterise the dynamical system uniquely when plasma effects contribute to the interaction physics.

![Figure 4](image)

Figure 4: Deviations of $p$ from $8/3$ universality across a range of $S = n_e/(a_0 n_c)$ for $a_0 = 5, 10, 20$ and a range of electron densities. The double solid circles indicate two or more data points at $S=1.5, 2, 4, 5$.

Thus while the combinations of intensity and electron density in Figs. 1c and 2a correspond to $S = 4$, and those in Figs. 2c,d to $S = 5$, the interaction physics, at least as far as its footprints in the emission spectrum is concerned, does not appear to be similar in either case. Note in passing that by similar we intend formal...
S—similarity, as against the conventional usage; in that sense all the spectra are similar. Whereas for the combination in Fig. 1c, \( p = 5/3 \), for Fig. 2a, \( p = 7/3 \); similarly \( p = 5/3 \) for Fig. 2c as opposed to \( p = 2 \) in Fig. 2d. We conclude that plasma-enhanced emission together with plasma-induced modulation can result in decay of the harmonic spectrum significantly weaker than that predicted by similarity theory. At the highest intensity modelled in this work (\( a_0 = 20 \)) we have not found any \( p < 5/3 \); that notwithstanding, we do not rule out yet lower values at higher intensities. Indeed under more extreme UR conditions we have obtained PIC spectra for which \( p \sim 4/3 \).

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