Black hole and baby universe in a thin film of $^3\text{He}$-A

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Abstract: Condensed matter black hole analogues may provide guidance in grappling with difficult questions about the role of short distance physics in the Hawking effect. These questions bear on the very existence of Hawking radiation, the correlations it may or may not carry, the nature of black hole entropy, and the possible loss of information when a black hole evaporates. We describe a model of black hole formation and evaporation and the loss of information to a disconnected universe in a thin film of $^3\text{He}$-A, and we explain why the existence of Hawking radiation has not yet been demonstrated in this model.

[Chapter for book, Artificial Black Holes, eds. M. Novello, M. Visser, and G. Volovik (World Scientific, 2002), based on a talk by TJ at the Workshop on Analog Models of General Relativity, held at CBPF in Rio de Janeiro, October 16-20, 2000.]
1 Introduction and motivation

From the condensed matter point of view, black hole analogues are a curiosity. They certainly generate new questions, and they may lead to new insights into the interplay between bulk and microscopic physics. From the quantum gravity point of view, there is much more at stake. We are looking to condensed matter systems for the guidance they may provide in grappling with difficult questions about the role of short distance physics in the Hawking effect. These questions bear on the very existence of Hawking radiation, the correlations it may or may not carry, the nature of black hole entropy, and the possible loss of information when a black hole evaporates.

Although the Hawking effect is a low-energy phenomenon for black holes much larger than the Planck mass, it cannot be deduced strictly within a low energy effective theory. The gravitational redshift from the event horizon is infinite, so the outgoing modes that carry the Hawking radiation emerge from the Planck regime. To derive the Hawking effect one needs only the assumption that near the horizon these modes emerge in their local ground state at scales much longer than the Planck length but still much shorter than the black hole radius. This assumption is plausible, since the background is slowly varying in time and space compared to these scales. However, it is not derived.

A condensed matter analogy has already provided some support for this picture. Unruh introduced a sonic analogue, in which the black hole is modeled by a fluid flow with a supersonic “horizon”, and the quantum field is replaced by the quantized perturbations of the flow. In a continuum treatment, Unruh argued that the horizon would radiate thermal phonons at the Hawking temperature $\frac{\hbar \kappa}{2\pi}$, where $\kappa$, which would be the surface gravity for a black hole, is here the gradient of the velocity field evaluated at the sonic horizon. The short-distance (atomic) physics of this analogue is fully understood in principle, hence it should be possible to understand the origin and state of the Hawking modes. As a first step inspired by this model, a number of studies have been carried out where the linearity of the quantum field equation is preserved but the short distance behavior is modified either by introducing nonlinear dispersion or a lattice cutoff, designed to mimic some aspects of the real atomic fluid. The consequences have been discussed in detail elsewhere (see e.g. [3] for a review). The main point is that, despite the exotic origin of the outgoing modes via “mode conversion” near the horizon, the short-distance physics does indeed deliver
these modes in their local ground state near the horizon if they originate far from the horizon in their ground state. These models thus lend some (linear but nontrivial) support to the contention that Planck scale effects deliver the local vacuum at a black hole horizon.

A controversial consequence of this simple picture, however, is that the Hawking effect produces a loss of information from the world outside the horizon. The reason is that the local vacuum condition at the horizon entails correlations between the field fluctuations inside and outside the horizon. The radiated Hawking quanta are thus correlated to “partners” that fall into the black hole. The origin of these correlations is precisely the same as in the vacuum of flat spacetime, so it is difficult to see any reason to doubt this account. It is often doubted, however, since it implies that the process of formation and complete evaporation of a black hole entails non-unitary evolution in the Hilbert space restricted to the outside world, which is considered by many (not including the authors) to be a violation of quantum mechanics. It would be useful to have a down-to-earth condensed matter analogue in which the information loss question arises but the fundamental physics is understood.

Related to the issue of information loss is the nature of black hole entropy. A spherical black hole of mass $M$ emitting an energy $dE = dMc^2$ in thermal radiation at the Hawking temperature $T_H = \frac{hc^3}{8\pi GM}$ loses an entropy $dS = dE/T_H = d(A/4l_P^2)$, where $A = 4\pi R_s^2$ is the area of the event horizon of radius $R_s = \frac{2GM}{c^2}$, and $l_P = (\hbar G/c^3)^{1/2} \approx 10^{-33}$ cm is the Planck length. A black hole thus has one unit of entropy for every four units of Planck area. To understand the nature of the microscopic degrees of freedom counted by this entropy remains one of the outstanding problems of quantum black hole physics. To solve this problem will presumably require understanding the nature of the short-distance cutoff. Without a cutoff the entropy would seem to be infinite due to the quantum entanglement between field degrees of freedom on either side of the event horizon discussed in the previous paragraph. If a condensed matter horizon analogue produces Hawking radiation, then it would seem to also carry an entanglement entropy, so it may provide some guidance on the nature of black hole entropy. (There are important differences however, since in the condensed matter setting there is

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1In addition to the information of correlations between Hawking quanta and their partners, any information that simply falls into the black hole from the outside appears to be lost.
no relation between the energy of the system and the area of the horizon, the Einstein equation of course does not pertain to the evolution of the horizon area, and the area need not even change during the evolution of the system.)

The basic question of the existence of the Hawking effect, as well as the issues of information loss and black hole entropy should be approachable in condensed matter analogues. The system we focus on here is particularly interesting in that it provides a model of the formation and evaporation of a black hole, with a disconnected part of the “universe”, analogous to a so-called “baby universe”, into which information can potentially be lost.

It should be stated at the outset that at this stage we have only a model of the background spacetime geometry. A detailed analysis of issues pertaining to the Hawking effect remains to be carried out, and it is not yet clear that this system would produce analogue Hawking radiation. Another point worth stressing is that, for the purposes of obtaining guidance in quantum gravity issues, the experimental feasibility of actually setting up and observing a condensed matter analogue is not essential. Nevertheless, experimental observations are certainly one of the goals of the whole program, both to confirm the basic properties of the Hawking effect, and to gain insight into the short-distance physics at play.

In the next section we introduce $^3$He-A and discuss how this medium can be used to construct various black hole analogues. This is followed by a section focusing on the effective geometry and Hawking radiation in the moving thin-film domain-wall model. The last section describes the analogue of black hole formation and evaporation and the loss of information to a disconnected universe in the that model. We would like this article to be accessible to researchers in both condensed matter and gravitational physics, hence we include more than the usual amount of introductory material. We use units with $k_B = 1$.

## 2 Black hole analogues using $^3$He

The Hawking effect is a quantum tunneling process that produces a gentle instability of the ground state due to the presence of unoccupied negative energy states in the ergoregion behind the horizon. The instability creates a flux of particles in a thermal state at the Hawking temperature. If a condensed matter system is to produce identifiable analogue Hawking radiation, therefore, the system should presumably be at least be as cold as the Hawk-
ing temperature,\footnote{This condition may be avoided by observing instead a runaway quantum instability related to the Hawking effect that is expected to occur for bosonic fields when there is an inner horizon in addition to an outer horizon \cite{4,5}.} which is very low temperature for reasonable laboratory parameters. Moreover, there should be no other dissipation mechanisms that could swamp the Hawking effect.

A natural place to start looking is therefore at superfluid systems at zero temperature. The case of superfluid $^4$He was initially examined in \cite{6}, and further discussed in \cite{7}. It was concluded that a sonic horizon cannot be established in superflow, because the flow is unstable to roton creation at the Landau velocity which is some four times smaller than the sound velocity.

## 2.1 $^3$He-A

Potentially more promising \cite{8,9,10,11} is the (anisotropic) A-phase of superfluid $^3$He, which has a rich spectrum of massless quasiparticle excitations. In particular, there are fermionic quasiparticles — the “dressed” helium atoms — which have gapless excitations near the gap nodes at $\vec{p} = \pm p_F \hat{l}$ on the anisotropic Fermi surface, and therefore can play the role of a massless relativistic field in a black hole analogue. The unit vector $\hat{l}$ is the direction of orbital angular momentum of the $p$-wave Cooper pairs and $p_F$ is the Fermi momentum.

For the benefit of readers not familiar with $^3$He-A we inject here a lightning sketch of the basics. (For complete introductions see \cite{12,13,14}.) $^3$He is a spin-1/2 fermion, and is described in a many-body fluid by a second quantized field operator $\psi^A(x)$, where $A$ is a two-component spinor index. The fluid has a phase transition at 2.7 mK to a superfluid state in which the order parameter $\langle \psi^A(x) \psi^B(y) \rangle = f^{AB}(x-y)$ is non-zero. Below a pressure of about 33 bars and a temperature of order 1 mK the fluid is in the so-called B-phase. The A-phase is a very long-lived metastable phase that is coexistent with the B-phase and is stable in the region between 20 and 33 bars from 2.7 mK down to around 2 mK at the higher pressure. In the A-phase the order parameter, which can be thought of as the wave function of a Cooper pair, is a spatial p-wave and a spin triplet, and has the structure $|L = 1, m_L = 1\rangle \otimes |S = 1, m_S = 0\rangle$. The coherence length $\xi$, which corresponds to the size of the Cooper pair wavefunction, is of order 500 Å. The Fourier transform of $f^{AB}(x-y)$ evaluated near the Fermi momentum

\textbf{Footnotes:}

\footnote{This condition may be avoided by observing instead a runaway quantum instability related to the Hawking effect that is expected to occur for bosonic fields when there is an inner horizon in addition to an outer horizon \cite{4,5}.}
\[ |\mathbf{p}| = p_F \] is proportional to the energy gap

\[
\Delta(p) = \frac{\Delta}{p_F} \left( |\mathbf{e}_1 + i\mathbf{e}_2|^i p_i \left( |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) \right), \tag{1}
\]

where the three unit vectors \( \mathbf{l}, \mathbf{e}_1 \) and \( \mathbf{e}_2 \) form a right-handed orthonormal triad. The gap function (1) has nodes at \( \vec{p} = \pm p_F \hat{l} \). Near these nodes the fermion quasiparticles can have arbitrarily low energies (above the Fermi energy), and they behave like massless relativistic particles. The velocity of these quasiparticles parallel to \( \hat{l} \) in \(^3\text{He}-\Lambda\) is the Fermi velocity \( v_F \sim 55 \text{ m/s} \), while their velocity perpendicular to \( \hat{l} \) is only \( c_\perp = \Delta/p_F \sim 3 \text{ cm/s} \), where \( \Delta \sim T_c \sim 1 \text{ mK} \) is the energy gap.

### 2.2 Black hole candidates

#### 2.2.1 Superflow

It should be possible to set up an inhomogeneous superflow exceeding the slow speed \( c_\perp \) in a direction normal to \( \hat{l} \), thus creating a horizon for the fermion quasiparticles. There is a catch, however, since the superflow is unstable when the speed relative to a container exceeds \( c_\perp \) \([9]\). A possible way around this was suggested by Volovik \([11]\), who considered a thin film of \(^3\text{He}-\Lambda\) flowing on a substrate of superfluid \(^4\text{He}\), which insulates the \(^3\text{He}\) from contact with the container. In such a film, the vector \( \mathbf{l} \) is constrained to be perpendicular to the film. A radial flow on a torus, such that the flow velocity near the inner radius exceeds \( c_\perp \), would produce a horizon. Theoretically this looks promising, however the Hawking temperature for a torus of size \( R \) is

\[
T = \left( \frac{\hbar}{2\pi} \right) \left( \frac{dv}{dr} \right) \sim \hbar c_\perp / R \sim (\lambda_F / R) \text{ mK},
\]

where \( \lambda_F \) is the Fermi wavelength, which is of the order of Angstroms. Thus, even for a micron sized torus, the Hawking temperature would be only \( \sim 10^{-7} \text{ K} \).

#### 2.2.2 Moving solitonic texture

An alternative is to keep the superfluid at rest with respect to the container, but arrange for a texture in the order parameter to propagate in such a way as to create a horizon. For example, in \([8]\) a moving “splay soliton” is considered. This is a planar texture in which the \( \mathbf{l} \) vector rotates from \( +\hat{x} \) to \( -\hat{x} \) along the \( x \)-direction perpendicular to the soliton plane. A quasiparticle moving in the \( x \)-direction thus goes at speed \( v_F \) far from the soliton and...
at speed $c_\perp$ in the core of the soliton. If the soliton is moving at a speed greater than $c_\perp$, the quasiparticles will not be able to keep up with it, so an effective horizon will appear. This example turns out to be rather interesting and complicated in the effective relativistic description. The null rays on the horizon have a transverse velocity, making it like that of a rotating black hole rather than a static black hole. In addition, since the $\hat{l}$ vector couples to the quasiparticles like an electromagnetic vector potential, its time and space dependence generates a strong “pseudo-electromagnetic” field outside the “black hole” which would produce quasiparticle pairs by analogy with Schwinger pair production \cite{13}. (This latter process may be the same as what produces the so-called “orbital viscosity” \cite{12} of a time-dependent texture.) The Hawking temperature also tends to be very low, and it seems likely that the Hawking effect would be masked by the pseudo-Schwinger pair production, though this has not been definitively analyzed.

2.2.3 Thin film with a moving domain wall

In \cite{10} a simpler system was studied, that of a thin film of $^3$He-A, perhaps on a $^4$He substrate, with a domain wall. The vector $\hat{l}$, which is perpendicular to the film, has opposite sign on either side of the wall, and in the wall region the condensate is in a different superfluid phase. At the core of the wall the group velocity of the quasiparticles goes to zero, so if the wall itself is moving, a horizon will appear. For the rest of this article we focus on this example. The effective spacetime geometry of this system was first studied in \cite{10}. Here we extend that analysis to the case of a wall that accelerates and then comes to rest again.

3 Effective spacetime and Hawking effect from a moving domain wall texture

3.1 Texture and spectrum

The order parameter for a domain wall texture in a thin film is described by a gap function of the same form as \cite{14}, with the unit vector $e_1$ replaced by something like $\tilde{e}_1 = \tanh(x/d) e_1$, where $d \sim \xi$ is the width of the wall. Here it is assumed that the film lies in the x-y plane with the wall along the y-axis.
The thickness of the film should be not much more than the coherence length $\xi$ in order for the domain wall to be stable.

\[ E^2(p) = v_F^2 (p - p_F)^2 + c_\perp^2 (e_1 \cdot p)^2 + c_\perp^2 (e_2 \cdot p)^2. \]  

(2)

There is no excitation perpendicular to the thin film, hence we have $p_z = p_F$, so

\[ p = \sqrt{p_F^2 + p_x^2 + p_y^2} = p_F + \frac{1}{2p_F}(p_x^2 + p_y^2) + \cdots. \]  

(3)

Using this expansion in (2) together with the replacement $e_1 \to \tilde{e}_1$ we obtain the low energy spectrum for motion in the $x$–$y$ plane in the domain wall texture:

\[ E^2 = c(x)^2 p_x^2 + c_\perp^2 p_y^2 + \frac{c_\perp^2}{\hbar^2} \xi^2 (p_x^2 + p_y^2)^2 + \cdots. \]  

(4)

In the quartic term we have replaced $v_F/2\Delta$ by $\xi/\hbar$, to which it is roughly equal. The speed $c(x)$, defined by

\[ c(x) = c_\perp \tanh(x/d), \]  

(5)

goes to zero at the core of the domain wall.

In the low energy limit the quartic term is negligible and the spectrum becomes that of a massless relativistic particle in two dimensions. The non-relativistic quartic corrections become important at higher energy, when the wavelength is of order the coherence length $\xi$. Note that this is of order 500 Å.
much longer than the interatomic spacing. The corrections produce “superluminal” group velocities at high momentum. If a quasiparticle is localized near the domain wall then these nonrelativistic corrections are important, since the width of the wall is of order $\xi$.

3.2 Spacetime of the stationary wall

The relativistic limit of (4) is that of a massless particle in a 2+1 dimensional spacetime with the line element

$$ds^2 = -dt^2 + c(x)^{-2}dx^2 + c_{\perp}^{-2}dy^2.$$  \hfill (6)

The metric has translation invariance in the y-direction, so we will make a “dimensional reduction” to the 1+1 dimensional spacetime

$$ds^2 = -dt^2 + c(x)^{-2}dx^2.$$  \hfill (7)

Clearly this spacetime is flat, since one can introduce a new spatial coordinate by $dx_* = dx/c(x)$ in terms of which the line element takes the manifestly flat form $ds^2 = -dt^2 + dx_*^2$. Note however that since $c(x)$ goes to zero linearly as $x \to 0^+$, the coordinate $x_*$ goes to $-\infty$ logarithmically as the domain wall is approached from the side of positive $x$. Therefore the film really corresponds to two complete copies of flat spacetime, joined “at infinity” at the wall.

3.3 Spacetime of the moving wall

Now suppose the domain wall texture is moving to the right at speed $v < c_{\perp}$ relative to the superfluid condensate. Then right moving quasiparticles sufficiently far from the wall will stay ahead of the wall, but those inside the point where $c(x) = v$ will fail to stay ahead. There will be a black hole horizon where $c(x) = v$ and a white hole horizon where $c(x) = -v$ on the left hand side of the wall. In between the two horizons all low energy quasiparticles are purely left-moving relative to the wall texture (see figure 2). The closer $v$ is to $c_{\perp}$ the farther apart the horizons lie.

In the analogy with quantum gravity, it would appear that the Planck scale should be identified with $\xi$ since this measures the “elasticity” of the background, and there is at present no analogue of the underlying atomic scale in fundamental theory except perhaps the string scale, which is usually taken to be longer than the Planck length, rather than shorter.
To determine the spacetime metric for the moving wall, we introduce coordinates $x_s$ and $x_w$ at rest with respect to the superfluid and the wall, respectively. These are related by the Galilean transformation $x_s = x_w + vt$.

The dispersion relation is determined in the superfluid frame, so the line element (7) applies in the superfluid frame, however the argument of the function $c(x)$ should be $x_w$ since this function describes the texture which is at rest in the wall frame. For the moving wall we thus have

$$ds^2 = -dt^2 + c(x_w)^{-2}dx_s^2$$  
$$= -dt^2 + c(x_w)^{-2}(dx_w + vd\tau)^2$$  
$$= -(1 - v^2c(x_w)^{-2})dt^2 + 2vc(x_w)^{-2}dt\,dx_w + c(x_w)^{-2}dx_w^2. \tag{9}$$

Perhaps surprisingly, this is no longer a flat spacetime. It has black and white hole horizons at $c(x_h) = \pm v$. The wall core at $x_w = 0$ is now a spacelike line (since the coefficient of $dt^2$ is positive there), and it lies at finite proper time along geodesics. The Ricci curvature scalar is given by

$$R = \frac{-4v^2}{d^2}\left(\frac{c_\perp^2}{c(x_x)^2} - \frac{c(x_x)^2}{c_\perp^2}\right). \tag{11}$$

This diverges like $-(2v/x_w)^2$ as $x_w = 0$ is approached, so there is a curvature singularity at the core. The spacetime therefore looks rather like that of an eternal Schwarzschild black hole. The curvature at the horizon is given by

$$R_{\text{horizon}} = -(2c_\perp/d)^2 \left[1 - (v/c_\perp)^4\right]. \tag{12}$$

Unlike the maximally extended Schwarzschild black hole, however, the spacetime of the moving wall is incomplete, in that geodesics can run off the edge of the coordinate system $(t, x_w)$ in a finite proper time or affine parameter. The location of the incomplete boundaries will be indicated below.
course physical quasiparticles cannot escape, because this really is the entire physical spacetime. What happens is that as a quasiparticle heads in the direction of an incomplete boundary (either forward or backward in time), it is blueshifted into the part of the spectrum where the nonrelativistic corrections become important, at which point it propagates superluminally and the geodesics of the effective metric no longer determine its trajectory.

3.4 Hawking effect

Since the effective spacetime is that of a black hole, it is natural to suppose the horizon would radiate fermion quasiparticles at the Hawking temperature \( T_H = (h/2\pi)\kappa \), where \( \kappa = dc/dx(x_h) \) is the surface gravity of the horizon. For the metric (10) with (5) we have explicitly

\[
T_H(v) = T_H(0) \left(1 - \frac{v^2}{c_\perp^2}\right), \quad T_H(0) = \frac{hc_\perp}{2\pi d}.
\]

Putting in the numbers we have \( T_H(0) \sim 1 \, \mu\text{K} \). Equation (13) gives the Hawking temperature in the wall reference frame (the “Killing temperature” in the language of general relativity), which is related to the temperature in the asymptotic frame of the superfluid by a Doppler shift factor:

\[
T_{H,sf} = T_H(0) \left(1 + \frac{v}{c_\perp}\right).
\]

The Hawking temperature in the frame of the superfluid is thus never less than \( T_H(0) \). Although this is three orders of magnitude below the critical temperature, and extremely low in practical terms, it is nevertheless close to where the non-relativistic corrections become important (assuming \( d \sim \xi \)).

While the black hole analogy looks compelling, it should be emphasized that the Hawking effect depends on behavior of the quantum field that may not be valid in this context. As discussed in the introduction, the required condition is that the high frequency outgoing modes near the horizon be in their quantum ground state. In this case these modes come from the singularity, since they propagate “superluminally”. The propagation of these modes though the singularity may excite them. This has not yet been analyzed\(^4\).

\(^4\)However the related problem of quasiparticle tunneling across the stationary domain wall has been studied by Volovik in section 11.1 of [15].
produced extra, induced emission.) Another difference from the black hole case is that once a Hawking pair is produced, the negative energy partner rattles around trapped in the ergoregion between the two horizons. Once these available states fill up further Hawking radiation would be suppressed. (In the case of a black hole, by contrast, the negative energy partners fall into the singularity never to return.) For a discussion of aspects of the behavior of superfluids in the presence of a quasiparticle horizon see reference [13].

For the remainder of this article we assume that, in spite of Pauli-blocking effects, the moving domain wall produces at least some Hawking radiation, and we go on to study the analogue of the process of formation and evaporation of a black hole. According to (13) the Hawking temperature approaches a nonzero constant as \( v \to 0 \). Nevertheless it is clear that at \( v = 0 \) there can be no radiation since the wall is static and there is no horizon, hence there is no ergoregion with negative energy states to be filled. The radiation rate must therefore go to zero as \( v \) goes to zero. If it goes as a power of \( v \) then we have \( \frac{dE}{dt} = -b v^n \). The kinetic energy of the moving domain wall is proportional to \( v^2 \) if, as seems plausible, the action is dominated by quadratic terms in the time and space derivatives. In this case \( E = \mu v^2/2 \) for some constant \( \mu \). Integrating the energy loss, we find that it takes a finite time for the wall to come to rest if \( n < 2 \), but an infinite time if \( n \geq 2 \).

Finally, a comment about entropy. It is tempting to try and define a thermodynamic entropy \( S \) for the moving domain wall, however it is by no means clear that this should be meaningful. In analogy to the black hole entropy, one might define \( S \) via \( dS = dE/T_{H,sf} \), where \( E = \mu v^2/2 \) as above and \( T_{H,sf} \) is the Hawking temperature in the superfluid frame, (14). This yields the formula \( S = \left( 2\pi \mu c_\perp d/h \right) \left( v/c_\perp - \ln(1 + v/c_\perp) \right) \). This analogy seems flawed however, as the domain wall is not stationary in the superfluid frame so does not represent an “equilibrium” system. If we try to correct this by passing to the frame of the moving wall, we run into the problem that, as the wall slows down due to Hawking radiation, this comoving frame changes, unlike in the black hole case where the asymptotic rest frame of the black hole is fixed even as the black hole evaporates (or absorbs radiation).

4 Black hole formation and evaporation in the thin-film domain-wall model
4.1 Carter–Penrose causal diagrams

To best exhibit the incompleteness of the black hole spacetime discussed in the previous section, as well as features of the case where the hole forms and then evaporates, it is helpful to use Carter–Penrose diagrams. We therefore pause here to explain what such a diagram is for the benefit of readers from the condensed matter side.

The basic idea is to draw a picture representing the causal structure of a spacetime by showing light rays at 45°, with regions at infinite time or space brought into a finite location by a spacetime dependent conformal rescaling of the line element, $d\tilde{s}^2 = \Omega^2 d s^2$, where $\Omega \to 0$ at infinity. Since the causal structure is determined by the light cones, which are the solutions of $d s^2 = 0$, the causal structure of $d\tilde{s}^2$ is identical to that of $d s^2$. (See for example [16].)

As an example, consider 1+1 dimensional flat spacetime given by the line element $d s^2 = -d t^2 + dx^2 = -du dv$, where $u = t - x$ and $v = t + x$. The conformal factor $\Omega(u, v) = (\cosh u \cosh v)^{-1}$ brings infinity into a finite location in the sense that the points at infinity for $d s^2$ lie at a finite proper distance for $d\tilde{s}^2$. A diagram of the tilde spacetime is then just a diamond, figure 3 (a). The boundaries of the spacetime are at infinite time and/or space. Timelike geodesics (straight lines in this case) emerge from past timelike infinity $i^- \text{ and terminate at future timelike infinity } i^+$. Spacelike geodesics go from left spacelike infinity $i_0^L \text{ to right spacelike infinity } i_0^R$, and null geodesics or light rays go from left or right past null infinity $I^- \text{ ("scri-minus") to right or left future null infinity } I^+ \text{ ("scri-plus").}$

In four-dimensional flat spacetime the spherical symmetry can be used to reduce to a two dimensional diagram. In spherical coordinates the metric is $d s^2 = -d t^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$. If we now define $u = t - r$ and $v = t + r$, the geometry of the $t-r$ subspace for each set of polar angles is identical to the 1+1 dimensional case except that now only $v > u$ is physical. The spacetime is thus represented by figure 3 (b), a diagram that is half of a diamond, with each point representing a 2-sphere except those on the vertical line on the left, which represents $r = 0$. This is shown as a dotted line.

The spacetime of spherical matter that collapses to form a black hole looks like figure 3 (c). The shaded region represents the collapsing matter. The dashed line represents the event horizon, and the thick-dashed line represents the curvature singularity inside the black hole. Note that the singularity is spacelike, and is the future terminus of any causal curve that goes beyond the horizon. It is unknown how or even if spacetime develops in any form to the
future of the singularity, so a question mark is placed there. One common hypothesis is that a “baby universe” is born there, which is disconnected from the outside world. A controversial question is whether such a baby universe can harbor information unavailable to the outside world.

The diagram for a black hole that forms by collapse and then evaporates away is shown in figure 3(d). After the black hole is gone, the origin of spherical coordinates appears shifted over in the diagram. No spacelike slice can enter the upper diamond (region $F$) and still be a Cauchy surface, since causal curves that cross the horizon into the black hole will never reach such a surface. This is the basis of the claim that only incomplete information is available to observers outside the horizon after the black hole is gone.

Outside the matter the spacetime is the static Schwarzschild line element, which can be analytically extended to a complete spacetime, the diagram of which is given in figure 3(e). This so-called “eternal” black hole spacetime is time-symmetric, with a white hole singularity in the past to match the
black hole singularity in the future. It has two asymptotic spatial regions, connected through a “throat” at the center of the diagram.

4.2 Diagrams of static and uniformly moving walls

4.2.1 Static wall

As explained in section 3.2 the spacetime of the static wall is just two complete copies of Minkowski spacetime. The causal diagram is figure 4. In strictly relativistic terms there is no connection at all between the two spacetimes, however the quasiparticles can travel superluminally and thus pass from one side of the wall to the other at a finite value of the time coordinate $t$. We indicate this physical connection by depicting the spacetimes as joined at the wall at spatial infinity.

![Static wall diagram](image)

**Figure 4: Static wall.**

4.2.2 Moving wall

The causal structure of the spacetime of the moving wall (10) is shown in figure 5 (a). It looks like what one would obtain by cutting the diagram for the eternal black hole (figure 3 (e)) along the white hole horizon, and sliding the lower half up so that the white hole singularity coincides with the black hole singularity. The result of this cut is to leave the spacetime incomplete along the cut, which corresponds to the pair of long-dashed lines in figure 5 (a). Geometrically it is not well-defined to extend the spacetime across the singularity, but physically in the thin film there is continuity in passing through the core of the domain wall. The other side of the wall thus plays the role of a baby universe.

To clarify the relation between the conformal diagram and the physical spacetime we include in figure 5 (a) lines of constant $t$, $x_s$, and $x_w$. Note
that the incomplete boundaries are at $t \to \pm \infty$. They are the terminus of a null ray that runs parallel to the black hole horizon backward in time, or the white hole horizon forward in time. Such null rays asymptotically approach the horizon, blueshifting until superluminal terms in the dispersion relation become important, at which stage a quasiparticle would cross the horizon. Note also that the Newtonian time slices cross the singularity progressively from left to right.

4.3 Black hole creation and evaporation

4.3.1 Creation

To construct an analogue of a black hole that forms by collapse we imagine that the wall velocity $v$ is now a function of Newtonian time $v(t)$ which is equal to zero before $t_1$ and thereafter smoothly approaches $v$. (We do not attempt at this stage to devise a mechanism for actually accelerating the wall in this manner.) The resulting spacetime should look like a portion of the static wall figure 4 below $t_1$ and a portion of the moving wall figure 5 (a) above $t_1$. This yields figure 5 (b). Note that the past incomplete boundary is now absent because the black hole forms at a finite time.

4.3.2 Evaporation

In the case of a real black hole the energy for the Hawking radiation comes from the mass of the hole. As the hole radiates it loses mass until it evaporates away completely, unless stabilized by conserved charges it cannot shed. In the case of the domain wall, the radiation occurs only when the wall is moving, and it is possible that the back-reaction to the radiation causes the wall to slow down. On the other hand, as discussed in subsection 3.4, Pauli blocking may well terminate the Hawking process before the wall comes to rest. There is presumably another dissipation mechanism, such as pair-breaking due to the time-dependence of the moving texture, that eventually would stop the wall. In any case, for the purposes of creating a model of black hole evaporation, we can imagine simply that somehow or another the wall comes to rest at a time $t_2$. The resulting spacetime should then again look like a portion of the static wall above $t_2$, as shown in figure 5 (c).

The causal structure for the wall that accelerates and then stops is similar but not entirely analogous to the spacetime of a black hole that evaporates.
Figure 5: Causal diagrams of moving domain-wall textures.
There is no region analogous to region $F$ of figure 3(d), and in fact the spacetime is globally hyperbolic. The analogy is improved if we lean on the role of the Newtonian simultaneity to define what is accessible “at a given time”. Thus, subsequent to $t_2$, the spacetime consists again of two causally disconnected pieces analogous to $F$ and the black hole interior of figure 3(d).

To understand better what is going on it is helpful to use also a non-conformal diagram, in which the domain wall worldline is drawn vertically, figure 6. The singularity appears when the wall starts moving, and disappears when it comes to rest. During the motion the singularity is a window to the other side of the wall. Figure 6 shows the black and white hole horizons as dashed lines, as well as a quasiparticle worldline that crosses from right to left, and a Hawking pair that is created at the temporary horizon. Note that

Figure 6: *Temporary one-way window.*

as it approaches the white hole horizon, the partner of the Hawking quantum is turned back toward the singularity since it is rightmoving with respect to the superfluid condensate. It “rattles” back and forth between the horizons until the wall stops moving.
4.4 Information loss

It seems clear from the previous discussion that quasiparticle information can fall across the horizon and be lost to the outside world. The information could come back carried by superluminal high energy quasiparticles, but it need not and there is no reason to suppose it does. The question of information loss by Hawking radiation is thornier. The partners remain in the ergoregion and fill the negative energy states. It seems that roughly half their information would wind up on the right side of the singularity to be available after the black hole “evaporates”. Still, that leaves the other half that is lost.

5 Conclusion

This is just the beginning of the story. Clearly a lot remains to be done to understand the nature of the Hawking effect in the setting of the thin-film domain wall. Nevertheless, we hope that this analogue model will prove stimulating to researchers pondering the nature of Hawking radiation and information loss in quantum gravity on the one hand, and the physics of moving superfluid textures on the other.

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