Super-filament formation of a relativistic Gaussian electron beam in a dense collisional plasma

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Abstract. Energy flux transport of a finite high current relativistic electron beam with the Gaussian profile in both the transverse and axial directions in a dense collisional background plasma is studied by means of three-dimensional particle-in-cell simulations. Simulation results reveal the development of a needle-like super-filament formation in both homogeneous as well as inhomogeneous collisional background plasmas. However, in the case of an inhomogeneous background plasma, the beam suffers severe filamentation due to the lower plasma density encountered in the early stage, and only the head of the super-filament survives and travels further inside the plasma. This may not be desirable for the fast ignition fusion scheme.

Energy deposition by a very high current relativistic beam in the core of a highly collisional background plasma is central to the successful realization of the fast ignition (FI) scheme [1]. In a typical FI scenario, an electron beam—generated by the interaction of a short-pulse laser of small focal spot size—travels from the corona to the center of the deuterium–tritium pellet,
where it is supposed to ignite the hot-spot, triggering the fusion reaction. During its propagation from the corona to the center of the pellet, the electron beam faces varying plasma density ranging from merely critical densities at the corona to 1000 times the solid density in the center of the compressed target. Thus, the transport of the beam energy flux gets affected by the local values of the beam to plasma density ratio, the resistivity of the background plasma and the gradient of the background plasma density.

The filamentation of an infinite beam caused by various filamentation instabilities (Weibel and two-stream, oblique mode) has been studied in detail [2–6]. These studies suggest that for a lower background plasma to beam density ratio, the beam filamentation is not dominant. The effect of collisions on the beam filamentation has also been investigated theoretically and numerically suggesting that collisions assist in reviving the beam filamentation [5, 6]. However, the transverse variation in beam density also plays an important role in the development of various instabilities of the electron beam, and affects the propagation of the beam significantly. Such beams with a transverse density variation are expected to be generated during the interaction of a short-pulse laser—which has a small focal spot size—with the fusion pellet. In addition to the known filamentation instabilities [4], such beams are also subject to hosing and modulation instabilities [7]. This renders the dynamics of the beam energy transport very involved, warranting urgent attention to this problem.

In this paper, we investigate the energy transport of a finite beam—with Gaussian profile in both transverse and axial directions—in a strongly collisional background plasma by full three-dimensional particle-in-cell (PIC) simulations. A parametric study of the beam energy flux on various factors such as the plasma to beam density ratio and the collision frequency is performed. We observe that the electron beam propagation in collisional dense background plasmas (higher background plasma to beam density ratio) leads to the strong appearance of filamentation instabilities. These instabilities are even more dominant in the case of an inhomogeneous background plasma due to the lower background plasma to beam density ratio encountered in the coronal region of the target. In the linear stage of the beam filamentation, the electron beam loses significant energy, leading to the generation of a huge quasi-static magnetic field. During the nonlinear stage of the filamentation instability, one notes the formation of a stable super-filament (with a transverse radius of the order of plasma skin-depth), which propagates unhindered for longer distances in a homogeneous collisional background plasma. In the case of an inhomogeneous background plasma, the strong filamentation coupled with hosing/modulation instabilities leads to higher energy loss and only the head of the filament survives. This filament loses energy to the background plasma, mainly due to the excitation of a return plasma current. Thus, the energy loss of the beam is composed of two kinds of processes; the first one is due to filamentation instabilities leading to a huge quasi-static magnetic field generation and the second one is due to excitation of the return plasma current in the background plasma. The energy loss arising due to filamentation instabilities exceeds the loss due to collisional excitation of the return plasma current in the early stage of the interaction. Moreover, one also observes hosing and modulation of the tail of the super-filament. We compare the energy loss of the super-filament with the theoretical estimate of collisional return plasma current excitation and found excellent agreement between the PIC simulation and the analytical results.

For PIC simulations, we set up a virtual box of size $50\lambda_b \times 3\lambda_b \times 3\lambda_b$, where the grid steps are $h_x = h_z = 0.015\lambda_b$ and $h_y = 0.05\lambda_b$, where $\lambda_b := 2\pi c / \omega_b$ is the beam skin depth and $\omega_b = \sqrt{4\pi n_b e^2 / m_e}$ is beam–plasma frequency, $n_b$ is the beam density, $m_e$ is the electron mass and $c$ is...
the speed of the light in vacuum. The electron beam is modeled as an ellipsoidal Gaussian pulse with an initial density defined by
\[ n_e(x, y, z) = n_b \exp(-x^2/(10\lambda_b)^2 - (y^2 + z^2)/\lambda_b^2), \]
whereas the stationary background is set to either a homogeneous density \( n_p \) or a ramp function. The plasma is modeled using 16 PIC particles per cell; the motion of ions is neglected. We do not simulate an actual FI fusion target; however, our simulations are motivated by the characteristics of a compressed FI target and employ simple model plasma distributions. The pulse is initialized with a momentum of \( p_x = 20m_ec \) and we take different background plasma to beam density ratios up to \( n_p/n_b = 200 \). We choose a beam with higher energy as the intensities of the laser being used in FI studies inevitably generate electrons with higher energy in the coronal region of the compressed target. The time evolution of the beam–plasma system is studied for both collisionless and collisional background plasmas. In the latter, different collision frequencies \( \nu_p \) are chosen. Collisions are introduced by a finite conductivity model implemented in the virtual laser plasma laboratory PIC code [8]. Apart from Coulomb collisions in a fully ionized plasma, anomalous resistivity can also arise and plays an important role in beam transportation in overdense plasmas [9]. An accurate but computationally demanding approach for solving the Vlasov–Fokker–Planck equation for FI fusion studies is more appropriate [10]. However, here we omit the details of the collision and damping mechanisms and rather focus on how the effective resistivity impacts the macroscopic beam dynamics. For these simulations, we chose collision frequencies in the range \( \nu_p/\omega_p = [0–0.5] \), where \( \omega_p = \sqrt{4\pi n_p e^2/m_e} \) is the background plasma frequency. We keep track of the total electric and magnetic field energies and the directed energy flux of the beam electrons
\[ F_e(t) = m_ec \int_{\text{Box}} n_e y_e v_{e,\perp} \, dx \, dy \, dz. \]

Additionally, we record an integral projection of the phase space of the beam to its \((x, p_x)\) subspace in order to monitor the deceleration.

Figure 1 shows the filamentation of the beam at different times in a collisional background plasma with the collision frequency \( \nu = 0.1\omega_p \) and plasma to beam density ratio \( n_p/n_b = 200 \). One clearly notes a different dynamics of beam filamentation compared to that studied in [6]. Two distinct differences are easily spotted: the length of the filaments and the hosing/modulation of the tail of the beam. In the case of an infinite beam, the length of the filaments is smaller (a few plasma skin depths), while no hosing/modulation of the beam is observed [6]. This is followed by the formation of a needle-like super-filament at the beam-front which displaces the background plasma electrons. This super-filament also excites a wakefield and the tail of the beam suffers from the hosing/modulation due to it. However, one does not see a resonant excitation of the nonlinear plasma wakefield. Beams with finite transverse radii also experience a strong focusing, which leads to rapid development of the filamentation and subsequent filaments merger. The needle-like super-filament at the beam-front shows a remarkable stability and propagates inside the plasma for longer distances. One may also note the existence of four single filaments at the very front of the filament. These filaments do not merge due to the weaker magnetic pinching at the front (the square of the magnetic field strength is depicted on the inner vertical side of the cubical simulation box, figure 1). This super-filament again leads to the generation of a return plasma current and dissipates energy only due to this collisional current. This excitation of the return plasma current is different from the excitation of return plasma current at the early stages of the beam propagation in the sense that the return plasma current mainly flows outside the super-filament and not inside as in the early stage.
Figure 1. Three-dimensional visualization of the simulation results for the plasma to beam density ratio $n_p/n_b = 200$ and collision frequency $\nu_p = 0.1\omega_p$ at times $16.5(2\pi/\omega_b)$ (frame (a)) and $50(2\pi/\omega_b)$ (frame (b)). Distances are normalized by $(2\pi c/\omega_b)$. The bottom two-dimensional (2D) graphs depict the beam current $J_x/(en_b c)$, while the background shows a cross-section of $B^2(e/m ec\omega_b)^2$. The gray iso-surface shows the beam density for a threshold value of $5n_b$. Panel (a) clearly shows the rapid collapse of the beam along with the generation of a strong magnetic field and the occurrence of hosing instability. Panel (b) shows the single symmetric super-filament, which maintains stability for $\sim 60(2\pi/\omega_b)$.

Figure 2 shows the filamentation of the finite electron beam in an inhomogeneous collisional background plasma at two different instants. The background plasma density has a density ramp profile given by $n_p(x) = n_b \max(x/\lambda_b - 20, 0)$ and the collision frequency is $\nu_p/\omega_p = 0.1$. This case is interesting as it corresponds to laser interaction with a more realistic inhomogeneous plasma target. Due to the density ramp, the electron beam interacts with the low-density plasma in the beginning and suffers severe filamentation as expected from the linear theory of the beam filamentation. The beam filamentation in a low-density collisional homogeneous plasma has been studied earlier \[5, 6\]. This speeds up the filaments merger as is evident from the different time scales compared to figure 1, which has similar parameters except for the background plasma density ramp. The combination of strong filamentation and hosing causes the tail of the filament to dissipate faster. Only the head of the single filament survives and propagates inside the plasma. This is in sharp contrast with the result of figure 1,
Figure 2. Visualization of a simulation similarly to figure 1, except that the background density is a ramp function \( n_p(x) = n_b(x/\lambda_b - 20) \). 2D plots of the beam current (bottom) and squared magnetic field (back wall) are shown, as well as the beam density as an isosurface for a threshold value of 5\( n_b \) at times 8(2\( \pi \)/\( \omega_b \)) and 20(2\( \pi \)/\( \omega_b \)). The lower initial background density causes a quicker collapse, followed by hosing instability and a significant beam spreading. Other parameters are the same as in figure 1.

where one observes the propagation of a single long filament for longer durations. This is particularly significant for FI fusion. According to the linear theory, for higher densities (solid-state densities) encountered in FI fusion, one might expect insignificant filamentation and hence the transport of the beam is not severely affected by the filamentation instabilities. However, PIC simulation results suggest that the lower density encountered by the beam at near-critical density regions can cause significant filamentation and a big part of the beam is lost and does not propagate near the core of the fusion pellet.

Figure 3 compares the energy fluxes of the electron beam for different values of plasma to beam density ratios, and collision frequencies of the background plasma. It is clear that there are two stages of the beam evolution; during the beginning the beam experiences a sudden drop in the energy flux due to filamentation of the beam, which results in the generation of a huge quasi-static magnetic field. At later times when the single-filament formation takes place, a return plasma current is generated at the expense of the beam energy. It may be readily noted from the figure that in a collisionless background plasma, the beam energy loss due to the
Figure 3. The evolution of the beam energy flux with time for different plasma to beam density ratios and background plasma collision frequencies. Curves (1)–(3) depict the energy fluxes of the beam for a collisionless background plasma $\nu_p = 0$ and the density ratios $n_p/n_b = 200$, 100, 10, respectively. Curves (4)–(6) describe the beam energy losses in a collisional plasma for density ratios and collision frequencies $[n_p/n_b = 200, \nu_p/\omega_p = 0.16]$, $[n_p/n_b = 100, \nu_p/\omega_p = 0.16]$, $[n_p/n_b = 200, \nu_p/\omega_p = 0.5]$, respectively. The last line (7) depicts the energy loss in a plasma with density ramp corresponding to the parameters in figure 2.

 filamentation increases with decreasing plasma to beam density ratios (the first three lines in figure 3), in line with the analysis of the linear beam-filamentation theories [4–6]. A similar trend is also observed while accounting for the collisions in the background plasma. In this case, the beam energy loss due to the beam-filamentation instabilities also increases with the decreasing plasma to beam density ratios (solid and dashed red (dark and dotted) lines) for a constant collision frequency. The beam also loses more energy if one further increases the collision frequency (the second lowest line in figure 3). These observations again reaffirm that collisions revive the beam–plasma filamentation instabilities leading to the higher beam energy loss during its propagation [6]. The last line (dashed celeste (dashed light) line) depicts the energy loss in an inhomogeneous plasma, showing a stronger energy loss which is attributed to the severe filamentation in the beginning and loss of beam mass later on as seen in figure 2. One may also observe that in the later stage, when the super-filament formation takes place, the beam also loses its energy due to the return plasma current excitation and wakefield generation by the super-filament. The energy loss during the latter stage increases with higher collision frequency, highlighting the role of the collisional return plasma current excitation. We observe in our simulations that the transverse size of the super-filament is of the order of plasma skin depth. Hence, the return plasma current flows around the super-filament. The super-filament also excites the plasma wakefield and its tail undergoes modulation and hosing. However, the
energy loss due to the self-modulation and hosing is not pronounced due to the absence of the resonant plasma wakefield excitation. Hence, the beam energy loss during this stage may be primarily due to the return plasma current excitation, and we make an estimate of it below and compare with PIC simulation results.

One can estimate the beam-energy loss of the super-filament in homogeneous collisional background plasmas. For \( \nu_p/\omega_p \gg \omega_b/\omega_p \), one can model the background plasma current response by Ohm’s law \( j_p = \sigma E \), \( \sigma = \omega_p^2/4\pi \nu_p \), \( \nu_p \) being the electron–ion collision frequency. After ignoring the displacement current in Maxwell’s equations and for a beam with a smoothly varying transverse density profile, one obtains the following expressions for the field associated with the beam propagation [11]:

\[
B_\phi(r, t) = -\frac{c}{\sigma} \int_0^{(2\pi/\omega_b)} \frac{\partial j_b}{\partial r'} \, dr',
\]

\[
E_r(r, t) = -\frac{c}{4\pi \sigma^2} \frac{\partial j_b}{\partial r},
\]

\[
E_z(r, t) = -\frac{j_b}{\sigma} - \frac{c^2}{4\pi \sigma^2} \int_0^{(2\pi/\omega_b)} \left[ \frac{1}{r} \frac{\partial j_b}{\partial r} + \frac{\partial^2 j_b}{\partial r^2} \right] \, dr'.
\]

**Figure 4.** Comparison of the beam energy losses from PIC simulations with the analytical estimates for beam–plasma parameters \( n_p/n_b = 200, \nu_p/\omega_p = 0.08 \). The red (solid) and blue (dotted) lines correspond to the analytical estimate and the PIC simulations of the beam energy loss, respectively.
where \( j_b \) is the beam current density and \((2\pi/\omega_b)\) is the beam period. For a beam of profile

\[
j_b = j_{b0} \exp\left(-\frac{r^2}{r_b^2}\right) \left[\Theta(t) + \Theta((2\pi/\omega_b) - t)\right], \quad j_{b0} \approx -n_b e c,
\]

where \( n_b \) is the beam density and \( \Theta(t) \) is the Heaviside step function, the normalized beam power loss then reads

\[
P_{\text{norm}} = -\pi \frac{\omega_b \nu_p}{(\gamma_b - 1) \omega_p \nu_p} \left[1 - 4\pi \left(\frac{\omega_b}{\omega_p}\right) \left(\frac{\nu_p}{\omega_p}\right) \left(\frac{c}{r_b \omega_p}\right)^2\right],
\]

where \( P_{\text{norm}} = P_{\text{loss}}/P_{\text{in}} \) and \( P_{\text{in}} = \pi n_b r_b^2 (\gamma_b - 1) m_e c^3 \) is the initial power of the beam and \( \gamma_b \) is the relativistic Lorentz factor. Here, \( \omega_b, \omega_p, r_b \) and \( \gamma_b \) are instantaneous parameters of the beam and the plasma. Figure 4 compares the late time energy loss of the beam from PIC simulations and analytical calculations. In comparing the energy loss we have fitted the profile of the super-filament in PIC simulations to the Gaussian in the transverse and a step function in axial directions, respectively. The values for \( \omega_b, \omega_p \), the radius \( r_b \) as well as the step profile length were automatically computed out of the in-simulation beam density profile using least-squares fitting. One observes excellent agreement between PIC and analytical results confirming the fact that the beam energy loss at late times arises due to excitation of collisional return plasma current.

To summarize, we have studied the filamentation of a finite radius beam in very dense collisional homogeneous as well as inhomogeneous plasmas. We have shown that filamentation instabilities remain a concern even for very-high-density plasmas encountered in inertial confinement fusion as the presence of a plasma density gradient can cause a significant beam-filamentation at the early stage of the beam transport. In the case of a homogeneous background plasma, super-filamentation formation occurs, which shows a remarkable stability and propagates for longer distances. In inhomogeneous background plasmas, the combined effect of filamentation and hosing/modulation of the super-filament leads to a significant loss of the beam energy and a big part of the filament is lost during the early stage of beam propagation. This may not be desirable for fast ignition fusion schemes.

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References

[1] Tabak M et al 1994 Phys. Plasmas 1 1626
Atzeni S et al 2009 Nucl. Fusion 49 055008
Honrubia J J and Meyer-ter-Vehn J 2009 Plasma Phys. Control. Fusion 51 014008
Okabayashi A, Yabuuchi T, Habara H and Tanaka K A 2009 J. Plasma Fusion Res. Ser. 8 1222
Strozzi D J, Tabak M, Larson D J, Divol L, Kemp A J, Bellei C, Marinak M M and Key M H 2012 Phys. Plasmas 19 072711
Scott R H H et al 2012 Phys. Plasmas 19 053104
[2] Weibel E S 1959 Phys. Rev. Lett. 2 83

New Journal of Physics 15 (2013) 035021 (http://www.njp.org/)
[3] Lee R and Lampe M 1973 Phys. Rev. Lett. 31 1390
Pukhov A and Meyer-ter-Vehn J 1996 Phys. Rev. Lett. 76 3975
Califano F, Pegoraro F and Bulanov S V 1997 Phys. Rev. E 56 963
Honda M 2004 Phys. Rev. E 69 016401
Honda M, Meyer-ter-Vehn J and Pukhov A 2000 Phys. Rev. Lett. 85 2128
Bret A, Gremillet L and Dieckmann M 2010 Phys. Plasma 12 120501 and references therein
Meyer-ter-Vehn J 2001 Plasma Phys. Control. Fusion 43 A113
[5] Molvig K 1975 Phys. Rev. Lett. 35 1504
Wallace J M, Brackbill J U, Cranfill C W, Forslund D W and Mason R J 1987 Phys. Fluids 30 1085
Deutsch C, Bret A, Firpo M C and Fromy P 2005 Phys. Rev. E 72 026402
Siemon C, Khudik V and Shevts G 2011 Phys. Plasmas 18 103109
Karmakar A, Kumar N, Pukhov A, Polomarov O and Shvets G 2008 Phys. Rev. E 80 016401
Karmakar A, Kumar N, Shvets G, Polomarov O and Pukhov A 2008 Phys. Rev. Lett. 101 255001
Karmakar A, Kumar N, Pukhov A, Polomarov O and Shvets G 2008 Phys. Plasmas 15 120702
Cottrill L A et al 2008 Phys. Plasmas 15 082108
Kumar N, Karmakar A, Pukhov A and Shvets G 2009 Eur. Phys. J. D 55 415
[7] Whittum D H 1993 Phys. Fluids B 5 4432
Kumar N, Pukhov A and Lotov K 2010 Phys. Rev. Lett. 104 255003
Pukhov A et al 2011 Phys. Rev. Lett. 107 145003
 Schroeder C B, Benedetti C, Esarey E, Grüner F J and Leemans W P 2011 Phys. Rev. Lett. 107 145002
 Schroeder C B, Benedetti C, Esarey E, Grüner F J and Leemans W P 2012 Phys. Rev. E 86 026402
[8] Pukhov A 1999 J. Plasma Phys. 61 425
[9] Sentoku Y, Mima K, Kaw P and Nishikawa K 2003 Phys. Rev. Lett. 90 155001
[10] Thomas A G R, Tzoufras M, Robinson A P L, Kingham R J, Ridgers C P, Sherlock M and Bell A R 2012 J. Comput. Phys. 231 1051 and references therein
[11] Fill E E 2001 Phys. Plasmas 8 1441