Sequential Model Adaptation Using Domain Agnostic Internal Distributions

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Abstract

We develop an algorithm for sequential adaptation of a classifier that is trained for a source domain to generalize in an unannotated target domain. We consider that the model has been trained on the source domain annotated data and then it needs to be adapted using the target domain unannotated data when the source domain data is not accessible. We align the distributions of the source and the target domains in a discriminative embedding space via an intermediate internal distribution. This distribution is estimated using the source data representations in the embedding. We conduct experiments on four benchmarks to demonstrate the method is effective and compares favorably against existing methods.

1 Introduction

Advances in deep learning have led to significant performance improvement in a wide range of supervised learning tasks. Similar to other machine learning (ML) techniques, deep learning is vulnerable with respect to distributional shifts during the model execution time when the model is tested. Distributional shifts would lead to model performance degradation which makes model adaptation, inevitable to make the model generalizable again in new domains. Unfortunately, adapting deep neural networks is conditioned on availability of massive labeled datasets. This may not, however, be always feasible due to prohibitive costs of manual data annotation [33]. A naive solution to improve model generalization is to finetune the previously trained network in the target domain. Despite being effective in reducing the effect of domain shift, finetuning still requires annotated data on the target domain.

Domain adaptation is a similar problem in which only unlabeled data is accessible in the target domain [11], while the source domain labeled data is accessible at the same time. The goal in UDA is to leverage from the source domain data to acclimate the deep neural network to generalize well in the target domain. An effective approach for domain adaptation is to align distributions of both domains by mapping data into a latent domain-invariant space [6, 32]. As a result, a classifier that is trained using the source labeled data features in this space will generalize well
Recent domain adaptation methods model this latent space as the output of a shared deep encoder. The encoder network is trained such that the source and the target domains share a similar distribution in its output. This training procedure has been implemented using either adversarial learning [13, 26, 30, 38, 45] or by directly minimizing the distance between the two distributions in the embedding [9, 17, 24, 25, 34, 35]. Adversarial learning [13] has been used to extract features that are maximally discriminative in the source domain and at the same are as indistinguishable as possible in the target domain. As a result, the distributions are aligned indirectly. Despite being effective, adversarial learning often requires delicate optimization initialization and architecture engineering to be stable [36]. In contrast, choosing the metric is the challenge for direct matching.

Most existing unsupervised domain adaptation (UDA) algorithms consider a joint learning setting, where the model is trained jointly on both the target domain unlabeled data and the source domain labeled data. As a result, these algorithms cannot be used for sequential model adaptation. Note that although a few source-free domain adaptation algorithms have been developed recently [18, 19, 22], these methods use adversarial learning to memorize the source domain to generate source domain pseudo-data points for model retraining which necessitates using additional networks, rather directly adapting the base classifier network. Our goal is to adapt the classifier model to generalize well in the target domain using solely the target domain unlabeled data. This setting can be considered as an improvement over using an off-the-shelf pre-trained model when unlabeled target domain data is available. Our approach also relaxes the necessity of sharing training data between the domains.

Contributions: our main contribution is to develop a sequential model adaptation algorithm which is based on learning a parametric internal distribution for the source domain data distribution in a shared embedding space. This internally learned distribution is used to align the source and the target distributions. We approximate this multi-modal distribution using a Gaussian mixture modal (GMM). In order to adapt the model to work well the target domain, we draw samples from this internal distribution and enforce the target domain to share the same distribution in the embedding space by minimizing the distance between the two distributions. We conduct experiments on four standard benchmark datasets and observe that our algorithm compares favorably even against state-of-the-art UDA methods.

2 Background and Related Work

UDA is closely related to sequential model adaptation. Several discrepancy measures have been used in the literature to align two distributions to address UDA. A group of methods match the first-order and the second order statistics of the source and the target domains. This includes methods that use the Maximum Mean Discrepancy (MMD) [24, 25] and correlation alignment [41]. A more effective approach is to use a probability distance metric that captures distributional differences in higher order statistics. The Wasserstein distance (WD) [1, 5] is such an example that is also a suitable metric for deep learning due to having non-vanishing gradients. This property is helpful because deep learning optimization problems are usually solved using the first-order optimization methods that rely on the objective function gradient. Damodaran et al. [1] used the WD for domain alignment in a UDA setting which led to considerable performance improvement compared to the methods that rely
on matching only lower-order statistics \cite{24, 41}. In this work, we rely on the sliced Wasserstein distance (SWD) variant of WD \cite{20} for domain alignment which possess similar properties but can be computed efficiently due to its closed-form formulation.

Most existing UDA methods use a strong assumption. It is assumed that the source and the target domain data are accessible simultaneously and the model is trained jointly on both datasets. Sequential model adaptation can be considered as a more challenging setting when the source domain data is unavailable. By addressing this learning setting, we can also address UDA when the source domain data cannot be shared due to privacy or security concerns. This learning setting for domain adaptation has been explored for non-deep models \cite{7, 16, 44}. However, these works address sequential model adaptation when the input distribution can be estimated with a parametric distribution and the base models have a small number of parameters. Hence, it is not trivial to extend the above works for the end-to-end training procedure of deep networks when deep learning is necessary for decent performance. Recently, adversarial learning has been used to address source-free DA \cite{18, 22}, but these methods memorize the source domain using additional networks to generate a source pseudo-dataset that can be used as a surrogate for the real source dataset. Quite differently, we rely an internally learned distribution by a base classifier model for the source domain to align the two source and target domain distributions indirectly in an embedding to adapt the base classifier sequentially. This approach eliminates the need for any additional network when the source data is inaccessible.

When a deep network is trained for solving a classification problem, the model would have decent performance only if the data points that belong to each class form a single cluster in an embedding space which is modeled by the network responses at higher layers. In other words, the source domain input distribution is transformed into a multi-modal internal distribution, where each mode of the distribution encodes one of the classes. The internal distribution encodes the knowledge gained from supervised learning in the source domain. Domain shift occurs when changes in the input distribution lead to discrepancies between the transformed input distribution and the internally learned distribution. UDA can be addressed by aligning the target domain distribution internally with the source domain internal distribution. We estimate the internal distribution in the embedding space via a parametric Gaussian mixture model. We develop an algorithm for sequential model adaptation by enforcing the target domain to share the same internal distribution in the embedding.

3 Problem Statement

Consider a source domain $S$ which consists of the distribution $p_S(x)$ and the labeling function $f(\cdot): \mathbb{R}^d \to \mathcal{Y} \subset \mathbb{R}^k$. Given a family of parametric functions $f_\theta: \mathbb{R}^d \to \mathcal{Y}$, e.g., a deep neural network with learnable parameter $\theta$, our goal is to solve for an optimal model with minimum expected risk, i.e., $\hat{\theta} = \arg\min_{\theta} \hat{e}_{\theta}(\mathbf{X}_S, \mathbf{Y}_S, \mathcal{L}) = \arg\min_{\theta} \sum_i \mathcal{L}(f_\theta(x_i^s), y_i^s)$, as a surrogate for the true risk. Consider that after training on the source domain, we also encounter a target domain sequentially with an
unlabeled dataset $D_T = (X_T)$, where $X_T = [x_1^T, \ldots, x_M^T] \in \mathbb{R}^{d \times M}$ and $x_i^T \sim p_T(x)$, e.g., due to occurrence of domain shift. As a result, using ERM is not feasible in the target domain. We know a priori that the two domains share the same classes but despite this relation, distributional discrepancy exists between the domains, i.e., $p_S \neq p_T$. Domain discrepancy leads to poor generalization of $f_\theta$ in the target domain. Our goal is to adapt the classifier $f_\theta$ using the dataset $D_T$ such that it generalizes well in the target domain when the source dataset is inaccessible (see Figure 1).

In order to circumvent the challenge of distributional gap between the two domains, we can decompose the mapping $f_\theta(\cdot)$ into a deep encoder $\phi_{\theta}(\cdot): \mathcal{X} \rightarrow \mathcal{Z} \subset \mathbb{R}^p$ and a classifier subnetwork $h_w(\cdot): \mathcal{Z} \rightarrow \mathcal{Y}$ such that $f_\theta = h_w \circ \phi_{\theta}$, where $\theta = (w, v)$. Here, $\mathcal{Z}$ denotes an intermediate embedding space between the input space and the label space in which we assume that the classes have become geometrically separable. Given $\hat{\theta}$ and $D_T$, if we adapt $\phi_{\theta}$ such that the discrepancy between the source and target distributions, i.e., the distance between $\phi(p_s(x^s))$ and $\phi(p_T(x^t))$, is minimized in the embedding space (making the embedding domain agnostic), then the classifier $h_w$ will generalize well on the target domain, despite being trained using only the source labeled data points. Many prior classic UDA algorithms use this strategy but implement it by assuming that $D_S$ is always accessible. This makes computing the distance between the distributions $\phi(p_s(x^s))$ and $\phi(p_T(x^t))$ feasible and hence UDA reduces to selecting a proper probability metric and then solving a typical deep learning minimization problem [1, 1, 5, 24, 25]. The major challenge in the sequential model adaptation setting is that the term $\phi(p_s(x^s))$ cannot be computed directly.

**Figure 1:** Architecture of the proposed model adaptation framework.

4 Proposed Algorithmic Solution

We propose to solve the challenge of sequential model adaptation through aligning the source and the target distribution indirectly via an intermediate internally learned distribution in the embedding space. We set a softmax function at the output of the encoder just before passing data representations into the classifier subnetwork. As a result, the classifier can be assumed as a maximum a posteriori (MAP) estimator which assigns a membership probability distribution to any given data point. As a result, when an optimal model is trained for the source domain, the encoder would transform the source distribution into a multi-modal distribution $p_f(\mathbf{z})$ with $k$ components in the embedding space (see Figure 1, right). This transformation occurs because the classes should become separable in the embedding space as the result of learning a generalizable model. The internal distribution is a multi-modal distribution in which each mode would capture one of the classes. If we update the model such that the internal distribution remains stable in the target domain after model adaptation, i.e., the source and the target domain would share similar
internal distributions in the embedding space, then the classifier subnetwork would
generalize well in the target domain due to negligible domain gap in the embedding.

The empirical version of the internal distribution is encoded by the source
data samples \( \{(\phi_\theta(x_i^s), y_i^s)\}_{i=1}^N \). We consider that \( p_J(z) \) is a GMM with \( k \) com-
ponents:

\[
p_J(z) = \sum_{j=1}^k \alpha_j N(z|\mu_j, \Sigma_j),
\]

where \( \alpha_j \) denote mixture weights, i.e.,

prior probability for each class, \( \mu_j \) and \( \Sigma_j \) denote the mean and co-variance for
each component. Since we have labeled data points, we can compute the GMM
parameters using MAP estimates. Let \( \mathcal{S}_j \) denote the support set for class \( j \)
in the training dataset, i.e., \( \mathcal{S}_j = \{(x_i^s, y_i^s) \in \mathcal{D}_s | \arg \max y_i^s = j\} \). Then, the MAP esti-
mate for the GMM parameters is given as the following:

\[
\hat{\alpha}_j = \frac{|\mathcal{S}_j|}{N}, \quad \hat{\mu}_j = \frac{1}{|\mathcal{S}_j|} \sum_{(x_i^s, y_i^s) \in \mathcal{S}_j} \phi_\theta(x_i^s), \quad \hat{\Sigma}_j = \frac{1}{|\mathcal{S}_j|} \sum_{(x_i^s, y_i^s) \in \mathcal{S}_j} (\phi_\theta(x_i^s) - \hat{\mu}_j)^\top (\phi_\theta(x_i^s) - \hat{\mu}_j).
\]

Our major idea is to use this internal distributional estimate to circumvent the major
challenge of sequential UDA. In order to adapt the model to work well for the target
domain, we update the model such that the encoder matches the target distribution
into the internal distribution in the embedding space. To this end, we can draw
random samples from the internal distributional estimate and generate a labeled
pseudo-dataset: \( \hat{\mathcal{D}} = (\mathcal{Z}_\mathcal{P}, \mathcal{Y}_\mathcal{P}) \), where \( \mathcal{Z}_\mathcal{P} = [z_1^p, \ldots, z_{N_p}^p] \in \mathbb{R}^{p \times N_p}, \mathcal{Y}_\mathcal{P} = [y_1^p, \ldots, y_{N_p}^p] \in \mathbb{R}^{k \times N_p}, z_i^p \sim \hat{p}_J(z) \), and the labels are ascribed according to the classifier subnetwork
prediction. To generate a clean pseudo-dataset, we also set a threshold \( \tau \) and include
only those generated samples for which the classifier prediction confidence is greater
than \( \tau \). The sequential UDA problem then reduces to solving the alignment problem:

\[
\min_{\theta, \theta_0} \sum_{i=1}^N \mathcal{L}(h_{\theta}(z_i^p), y_i^p) + \lambda D(\phi_\theta(p_T(\mathcal{X}_T)), \hat{p}_J(\mathcal{Z}_\mathcal{P})),
\]

where \( D(\cdot, \cdot) \) denotes a probability metric to measure the distributional discrepancy,
and \( \lambda \) is a trade-off parameter between the two terms (see Figure 1, left).

The first term in Eq. (3) is used to ensure that the classifier continues to perform
well on the internal distribution (note that the pseudo-dataset approximates this
distribution). The second term is the domain alignment matching loss which enforces
the target domain to share a similar internal distribution in the embedding space.

### Algorithm 1 SMAUI (\( \lambda, ITR \))

1. **Initial Training:**
   1. Input: source dataset \( \mathcal{D}_S = (\mathcal{X}_S, \mathcal{Y}_S) \),
   2. Training on Domain Source:
   3. \( \theta_0 = (\hat{\theta}_0, \hat{\theta}_0) = \arg \min_{\theta} \sum_i \mathcal{L}(f_\theta(x_i^s), y_i^s) \)
   4. Internal Distribution Estimation:
   5. Use Eq. (2) and estimate \( \alpha_j, \mu_j, \) and \( \Sigma_j \)
   6. Model Adaptation:
   7. Input: target dataset \( \mathcal{D}_T = (\mathcal{X}_S) \)
   8. Pseudo-Dataset Generation:
   9. \( \hat{\mathcal{D}} = (\mathcal{Z}_\mathcal{P}, \mathcal{Y}_\mathcal{P}) = (\mathcal{Z}_T, \mathcal{Y}_T) = (\{z_1^p, \ldots, z_{N_p}^p\}, \{y_1^p, \ldots, y_{N_p}^p\}), \) where:
   10. \( z_i^p \sim \hat{p}_J(z), 1 \leq i \leq N_p \)
   11. \( y_i^p = \arg \max_{j} \{h_{\theta_0}(z_i^p)\} \)
   12. for \( itr = 1, \ldots, ITR \) do
   13. draw data batches from \( \hat{\mathcal{D}} \) and \( \hat{\mathcal{D}}_\mathcal{P} \)
   14. Update the model by solving Eq. (3)
   15. end for

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5
The major remaining question is selection of the distribution metric. We choose SWD metric \([5]\) to compute \(D(\cdot, \cdot)\) due to its suitability for deep learning due to possessing non-vanishing gradients when two distributions have non-overlapping supports \([5, 20]\). SWD inherits this property from WD, yet the advantage of using SWD over WD is that SWD can be computed efficiently using a closed form solution. Additionally, empirical version of SWD can be computed using the samples that are drawn from the corresponding two distributions, as it is the case in Eq. (3). Hence, Eq. (3) can be solved using first-order optimization techniques (see appendices for more details on properties of SWD). Our proposed solution, named Sequential Model Adaptation Using Internal distribution (SMAUI), is presented and visualized in Algorithm 1. Figure 1 also presents the high-level description of our algorithm.

5 Experimental Validation

Sequential model adaptation is quite an unexplored learning setting. For this reason, we compare SMAUI against several recently developed UDA algorithms using standard UDA tasks due to closeness of the UDA learning setting to the sequential model adaptation setting. We consider different UDA tasks in terms of difficulty and the base network structure. Our implemented code is available at: https://github.com/mrostami1366/SequentialUDA.

5.1 Datasets

We validate our method on four standard benchmark set of UDA tasks.

Digit recognition tasks: MNIST (\(\mathcal{M}\)), USPS (\(\mathcal{U}\)), and SVHN (\(\mathcal{S}\)), datasets are used as the three digit recognition domains. Following the the UDA literature, we report performance on the three UDA tasks: \(\mathcal{M} \rightarrow \mathcal{U}, \mathcal{U} \rightarrow \mathcal{M}, \text{and } \mathcal{S} \rightarrow \mathcal{M}\).

Office-31 Detest: this dataset is a visual recognition dataset with 4,652 images which are categorized into ten classes in three distinct domains: Amazon (\(\mathcal{A}\)), Webcam (\(\mathcal{W}\)) and DSLR (\(\mathcal{D}\)) with six pair-wise definable UDA tasks among the domains.

ImageCLEF-DA Dataset: this image classification dataset is a generated using the 12 shared classes between the Caltech-256 (\(\mathcal{C}\)), the ILSVRC 2012 (\(\mathcal{I}\)), and the Pascal VOC 2012 (\(\mathcal{P}\)) datasets. Each class has 50 images which results in 600 images for each domain. We perform experiments on the six possible UDA tasks.

VisDA-2017: the goal is to train a model on a synthetic domain and adapt it to work on real images. The synthetic images are generated by renderings of 3D models from different angles and lightning conditions across 12 classes with 280K images.

5.2 Empirical Evaluation:

We follow the UDA literature for evaluation due to the topic proximity. We use the metrics and the features used in the UDA literature for fair comparison. We use the VGG16 network as the base model for the digit recognition tasks. The network is initialized with random weights. For the other tasks, we use the ResNet-50 network which is pre-trained on the ImageNet as the backbone network. We set \(\tau = 0.99\) and \(\lambda = 10^{-3}\).

We report the average classification rate on the target domain and the standard deviation based on ten runs for each UDA task. We train the base model using
the source labeled data. We report the performance of the model before adaptation as a baseline which also demonstrates the effect of domain shift. Then we adapt the model using the target unlabeled data using SMAUI algorithm and report the target domain performance. In our Tables, bold font denotes the best performance. The baseline performance before model adaptation is reported in the first row, then the UDA methods based on adversarial learning, then the methods based on direct matching, separated by a line in the middle, followed by our result in the last row.

In our comparison, we include both pioneer and recent works to be representative of the literature. We include methods based on adversarial learning: GtA [38], DANN [8], ADDA [4], MADA [43], SymNets [45], CDAN [26], and MMAN [27]. We also include methods based on direct matching: DAN [24], DRCN [10], CORAL [42], RevGrad [9], CAN [17], JAN [25], WDGRL [40], JDDA [4], and ETD [21]. We also include source-free UDA methods UDAwSD [22] and SHOT [23]. For each dataset, we include results of the works for which the original paper has used that dataset. For more details on the experimental setup, please refer to the appendices.

5.3 Results

Results for the digit recognition tasks are reported in Table 1. Despite the sequential training regime, we observe SMAUI outperforms the other methods in one of these tasks and its performance is quite competitive in the other two tasks. SMAUI leads to strong results compared to the joint UDA methods because stabilizing the internal distribution for both domain would mitigate domain shift. We also observe that performance of the methods based on direct probability match has improved recently which might because of using secondary mechanisms to improve generalization.

Table 2 summarizes the results for Office-31 dataset. We see in two of the tasks SMAUI leads to the best results and for the rest of the tasks is still competitive. Note, however, it seems there is no clear winner algorithm across the tasks of this dataset. This maybe because some labels in this dataset are noisy and some images contain objects that belong to other classes [3]. The approach we use to align the two distributions is sensitive with respect to existence label pollution which makes aligning the distributions class-conditionally more challenging.

Results for UDA tasks of the ImageCLEF-DA dataset are reported in Table 3. We see that although SMAUI does not use the source samples during model adaptation, it leads to a significant performance boost over the prior methods. This may be because ImageCLEF-DA is balanced across all the domains and classes. Hence, matching the source and the target distributions to the same internal multi-modal distribution is more straightforward. This is because we rely on empirical distributions for alignment and balanced datasets represent true distributions better.

Results for VisDA task is presented in Table 4. We observe a significant boost in performance for VisDA task. From inspecting Tables 1–5, we conclude that there is no single method with the best performance on all the tasks. This is natural because these datasets are diverse in terms of difficulty, cross-domain gap, dataset size, label-pollution, etc, and any particular method may be more vulnerable in special cases. However, we note that although these UDA methods should serve an upperbound for SMAUI- as a sequential model adaptation method- SMAUI works reasonably well on all these tasks with state-of-the-art performance on many of the tasks. These results demonstrate that although our motivation was to address sequential model
adaptation, SMAUI can also be used as a standard joint training UDA algorithm with competitive results. Note that SMAUI mostly outperforms the recently developed source-free UDA methods. Hence, it can also be used as a source-free UDA algorithm to preserve privacy. We hope that emergence of sequential model adaptation methods would allow comparing our results against more similar methods.

5.4 Analysis and Ablative Studies

To provide a better intuition about effectiveness of SMAUI, we have used the UMAP visualization tool to reduce the dimension of the data representations in the embedding space for 2D visualization. Figure 2 represents the testing splits of the source and the target domains data and samples of the internal distribution for the $S \rightarrow M$ digit recognition task. In this figure, each point represents one data point and each color represents one of the ten digit classes. Comparing Figures 2a

| Method  | $M \rightarrow U$ | $U \rightarrow M$ | $S \rightarrow M$ |
|---------|------------------|------------------|------------------|
| SMAUI   |                  |                  |                  |

Table 1: Classification accuracy for UDA tasks between the digit recognition datasets.

| Method | $A \rightarrow W$ | $D \rightarrow W$ | $W \rightarrow D$ | $A \rightarrow D$ | $D \rightarrow A$ | $W \rightarrow A$ | Average |
|--------|------------------|------------------|------------------|------------------|------------------|------------------|---------|
| Source Only | 68.4±0.2 | 96.7±0.1 | 99.3±0.1 | 68.9±0.2 | 62.5±0.3 | 60.7±0.3 | 75.6 |
| GA [6] | 89.5±0.5 | 97.9±0.3 | 99.8±0.4 | 87.7±0.5 | 72.8±0.3 | 71.4±0.4 | 86.5 |
| DANN [8] | 82.0±0.4 | 96.9±0.2 | 99.1±0.1 | 79.7±0.4 | 68.2±0.4 | 67.4±0.5 | 82.2 |
| ADDA [19] | 86.2±0.5 | 96.2±0.3 | 98.4±0.3 | 77.8±0.3 | 69.5±0.4 | 68.9±0.5 | 82.8 |
| SymNets [20] | 90.8±0.1 | 98.8±0.3 | 100.0±0.0 | 93.9±0.5 | 74.6±0.6 | 72.5±0.5 | 88.4 |
| MADA [21] | 82.0±0.4 | 96.9±0.2 | 99.1±0.1 | 79.7±0.4 | 68.2±0.4 | 67.4±0.5 | 82.2 |
| CDAN [22] | 93.1±0.2 | 98.2±0.2 | 100.0±0.0 | 89.8±0.3 | 70.1±0.4 | 68.0±0.4 | 86.6 |
| UDAwSD [24] | 93.7±0.2 | 98.5±0.1 | 99.8±0.2 | 92.7±0.4 | 75.3±0.5 | 77.8±0.1 | 89.6 |

Table 2: Classification accuracy for UDA tasks for Office-31 dataset.

| Method | $I \rightarrow P$ | $P \rightarrow I$ | $I \rightarrow C$ | $C \rightarrow I$ | $C \rightarrow P$ | $P \rightarrow C$ | Average |
|--------|------------------|------------------|------------------|------------------|------------------|------------------|---------|
| Source Only | 74.8±0.3 | 83.9±0.1 | 91.5±0.3 | 78.0±0.2 | 65.5±0.3 | 91.2±0.3 | 80.8 |
| DANN [8] | 82.0±0.4 | 96.9±0.2 | 99.1±0.1 | 79.7±0.4 | 68.2±0.4 | 67.4±0.5 | 82.2 |
| SymNets [20] | 80.2±0.3 | 93.6±0.2 | 97.0±0.3 | 93.4±0.3 | 78.7±0.3 | 96.4±0.1 | 89.9 |
| MADA [21] | 75.0±0.3 | 87.9±0.2 | 96.0±0.3 | 88.8±0.3 | 75.2±0.2 | 92.2±0.3 | 85.9 |
| CDAN [22] | 76.7±0.3 | 90.6±0.3 | 97.0±0.4 | 90.5±0.4 | 74.5±0.3 | 93.5±0.4 | 87.1 |
| SMAUI | 74.3±0.4 | 82.2±0.2 | 92.8±0.2 | 86.3±0.4 | 69.2±0.4 | 89.8±0.4 | 82.4 |
| Jan [25] | 75.0±0.6 | 86.0±0.3 | 96.2±0.4 | 87.0±0.5 | 74.3±0.5 | 91.5±0.6 | 85.0 |
| ETD [26] | 76.8±0.4 | 88.0±0.2 | 94.7±0.2 | 89.5±0.3 | 74.2±0.3 | 91.7±0.3 | 85.7 |
| SMAUI | 88.7±1.2 | 99.5±0.2 | 100±0.0 | 94.9±0.3 | 88.8±0.9 | 99.8±0.0 | 95.3 |

Table 3: Classification accuracy for UDA tasks for ImageCLEF-DA dataset.

| Task | JAN [6] | DJT [7] | GTA [8] | SimNet [24] | CDAN [26] | MCD [30] | SMAUI |
|------|---------|---------|---------|------------|-----------|----------|--------|
| Syn.→Real.| 61.6 | 66.9 | 69.5 | 69.6 | 70.0 | 71.9 | 76.9±0.7 |

Table 4: Classification accuracy for the VisDA UDA task.
and 2b, we can see that the high-confidence internal GMM distribution samples approximate the source domain distribution reasonably well. Figure 2c denotes that the target domain samples are separable prior to adaptation to some extent due to domain similarity, but we can observe more regions with overlapped classes, i.e., less separability, that lead to performance degradation. This is due to distributional gap between the two domains. Figure 2d denotes that SMAUI algorithm has aligned the source and the target distributions using the intermediate internal distribution.

For a class-level analysis of effect of model adaptation, Figures 3a–3d visualize the confusion matrices for the classifier with explanations in the caption. We can see in Figure 3b that domain shift causes confusion between digit classes that are in visually similar classes, e.g., digits “3” and “8” or digits “4”, “7”, and “9”. As seen in Figure 3c, the confusion is reduced for all classes using SMAUI algorithm. Comparing Figure 3c with Figures 3a and 3d, we see that the initial confusions between the classes in the source domain translate into the target domain, despite the fact that the target domain (M) is an easier problem (Figures 3d).

We have plotted the test error versus optimization epochs for SMAUI during training in Figure 3e. We observe that as more training epochs are performed and the distributions are aligned, i.e., domain discrepancy is minimized, the testing accuracy on the target domain accuracy constantly increases, as anticipated by the algorithm.

6 Conclusions

We addressed the problem of model adaptation in a sequential task learning setting. We minimized the cross-domain distributional discrepancy in a shared embedding space using an intermediate multi-modal internal distribution which is estimated
using a GMM distribution. As a result indirect alignment, we can adapt the source-trained classifier to generalize better on the target domain. A future research direction is to improve the algorithm performance via class-conditional alignment techniques.

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A Sliced Wasserstein distance

In our work, we have used the Sliced Wasserstein (SWD) distance for measuring distribution discrepancy. We provide a short background for the interested reader here. SWD is defined based on the Wasserstein distance (WD). The Wasserstein distance between two probability distributions \( p_S \) and \( p_T \), can be defined as:

\[
W_c(p_S, p_T) = \inf_{\gamma \in \Gamma(p_S, p_T)} \int_{X \times Y} c(x, y) d\gamma(x, y)
\]  

(4)

where \( \Gamma(p_S, p_T) \) is the set of all joint distributions \( p_{ST} \) with marginal single variable distributions \( p_S \) and \( p_T \), and \( c : X \times Y \to \mathbb{R}^+ \) is the transportation cost which normally is assumed to be \( \ell_2 \)-norm Euclidean distance. As can be seen, computing the Wasserstein distance is not trivial and requires solving an optimization problem which is a special case of linear programming problems as the subjective function in Eq. (4) and the constraint on \( \gamma \) are both linear. However, when the distributions are \( 1 \)-dimensional, computing the Wasserstein distance reduces to a closed-form solution as follows:

\[
W_c(p_S, p_T) = \int_{0}^{1} c(P_{S}^{-1}(\tau), P_{T}^{-1}(\tau))d\tau,
\]  

(5)

where \( P_S \) and \( P_T \) are the cumulative distributions of the \( 1 \)-dimensional distributions \( p_S \) and \( p_T \). This closed-form solution that has a much lower computational complexity compared to Eq. (4), motivates the definition of SWD in order to extend application of Eq.(5) on higher dimensional distributions.

The idea behind the SWD is based on the slice sampling [29]. The idea is to project two \( d \)-dimensional probability distributions into their marginal one-dimensional distributions, i.e., slicing the high-dimensional distributions, and to approximate the Wasserstein distance by integrating the Wasserstein distances between the resulting \( 1 \)-dimensional marginal probability distributions over all possible one-dimensional subspaces, which have closed form solution. This can be a good estimate for the optimal transport as any probability distribution can be represented uniquely via the set of \( 1 \)-dimensional marginal projection distributions [14]. For the distribution \( p_S \), a one-dimensional slice of the distribution is defined:

\[
\mathcal{R}p_S(t; \gamma) = \int_{S^{d-1}} p_S(x) \delta(t - \langle \gamma, x \rangle) dx,
\]  

(6)

where \( \delta(\cdot) \) denotes the Kronecker delta function, \( \langle \cdot, \cdot \rangle \) denotes the vector inner dot product, \( S^{d-1} \) is the \( d \)-dimensional unit sphere, and \( \gamma \) is the projection direction. In other words, \( \mathcal{R}p_S(\cdot; \gamma) \) is a marginal distribution of \( p_S \) obtained from integrating \( p_S \) over the hyperplanes orthogonal to \( \gamma \). The SWD then is defined as integral of the Wasserstein distance between the sliced distributions over all \( 1 \)-dimensional subspaces \( \gamma \) on the unit sphere:

\[
SW(p_S, p_T) = \int_{S^{d-1}} W(\mathcal{R}p_S(\cdot; \gamma), \mathcal{R}p_T(\cdot; \gamma)) d\gamma
\]  

(7)

where \( W(\cdot) \) denotes the Wasserstein distance. The main advantage of using the SWD is that as evident from Eq (7), unlike the Wasserstein distance, calculation of the
SWD does not require a numerically expensive optimization. This is due to the fact that the Wasserstein distance between two one-dimensional probability distributions has a closed form solution. Since only samples from distributions are available, the one-dimensional Wasserstein distance can be approximated as the $\ell_p$-distance between the sorted samples. Note however, this way we can compute merely the integrand function in Eq. (7) for a known $\gamma$. To approximate the integral in Eq. (7), we can use a Monte Carlo style integration. First, we sample the projection subspace $\gamma$ from a uniform distribution that is defined over the unit sphere and then compute one-dimensional Wasserstein distance on the sample. We can then approximate the integral in Eq. (7) by computing the arithmetic average over a suitably large enough number of drawn samples. Formally, the SWD between $f$-dimensional samples $\{\phi(x_i^S) \in \mathbb{R}^f \sim p_S\}_{i=1}^M$ and $\{\phi(x_i^T) \in \mathbb{R}^f \sim p_T\}_{j=1}^M$ in our problem of interest can be approximated as the following sum:

$$SW^2(p_S, p_T) \approx \frac{1}{L} \sum_{l=1}^L \sum_{i=1}^M |\langle \gamma_l, \phi(x^S_{s_l[i]}) \rangle - \langle \gamma_l, \phi(x^T_{t_l[i]}) \rangle|^2$$  \hspace{1cm} (8)

where $\gamma_l \in S^{f-1}$ is uniformly drawn random sample from the unit $f$-dimensional ball $S^{f-1}$, and $s_l[i]$ and $t_l[i]$ are the sorted indices of $\{\gamma_l \cdot \phi(x_i)\}_{i=1}^M$ for source and target domains, respectively. We utilize the empirical version of SWD in Eq. (8) as the discrepancy measure between the probability distributions to match them in the embedding space. Note that the function in Eq. (8) is differentiable with respect to the encoder parameters and hence we can use gradient-based optimization techniques that are commonly used in deep learning to minimize it with respect to the model parameters.

**B Details of Experimental Implementation**

In the digit recognition experiments, we resized the images of SVHN dataset to $28 \times 28$ images to have the same size of the MNIST and the USPS datasets. This is necessary because we use the same encoder across all domains.

In our experiments, we used cross entropy loss as the discrimination loss. At each training epoch, we computed the combined loss function on the training split of data and stopped training when the loss function became constant. We used Keras for implementation and ADAM optimizer with learning rate $lr = 10^{-4}$. We have run our code on a cluster node equipped with 2 Nvidia Tesla P100-SXM2 GPU’s. The implemented code is provided as a supplement.

All the datasets have their own standard training/testing splits in all domains. For each experiment, we used these testing splits to measure performance of the methods that we report in terms of classification accuracy. We used the classification rate on the testing set to measure performance of the algorithms. We performed 10 training trials and reported the average performance and the standard deviation on the testing sets for these trials.