Decomposition Without Regret

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Programming languages are embracing both functional and object-oriented paradigms. A key difference between the two paradigms is the way of achieving data abstraction. That is, how to organize data with associated operations. There are important tradeoffs between functional and object-oriented decomposition in terms of extensibility and expressiveness. Unfortunately, programmers are usually forced to select a particular decomposition style in the early stage of programming. Once the wrong design decision has been made, the price for switching to the other decomposition style could be rather high since pervasive manual refactoring is often needed. To address this issue, this paper presents a bidirectional transformation system between functional and object-oriented decomposition. We formalize the core of the system in the FOOD calculus, which captures the essence of functional and object-oriented decomposition. We prove that the transformation preserves the type and semantics of the original program. We further implement FOOD in Scala as a translation tool called COOK and conduct several case studies to demonstrate the applicability and effectiveness of COOK.

Additional Key Words and Phrases: Bidirectional program transformation, Functional decomposition, Object-oriented decomposition

1 INTRODUCTION

Programming languages are embracing multiple paradigms, in particular functional and object-oriented paradigms. Modern languages are designed to support multi-paradigms. Well-known examples are OCaml, Swift, Rust, TypeScript, Scala, F#, Kotlin, to name a few. Meanwhile, mainstream object-oriented languages such as C++ and Java are gradually extended to support functional paradigms. When multiple paradigms are available within one programming language, a natural question to ask is: which paradigm should a programmer choose when designing programs?

A fundamental difference between functional and object-oriented paradigms is the way of achieving data abstraction [Cook 2009; Reynolds 1978]. That is, how to organize data with associated operations. Typically, object-oriented decomposition is operation first: we first declare an interface that describes the operations supported by the data and then implement that interface with some classes. Conversely, functional decomposition is data first: we first represent the data using an algebraic datatype and then define operations by pattern matching on that algebraic datatype.

Figure 1 manifests the difference between object-oriented and functional decomposition by implementing an evaluation operation on literals and subtractions with both styles in Scala. The object-oriented decomposition version shown on the left hand side models expressions as a class hierarchy, where the interface Exp declares an abstract eval method and two classes Lit and Sub concretely implement eval. An Exp object is created via calling new on the classes and then evaluated by invoking the eval method. In contrast, the functional decomposition version shown on the right-hand side defines Exp as an algebraic datatype with Lit and Sub being the constructors.

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// class hierarchy
trait Exp {  
    def eval: Int
}

class Lit(n: Int) extends Exp {  
    def eval: Int = n
}

class Sub(l: Exp, r: Exp) extends Exp {  
    def eval: Int = l.eval - r.eval
}

// client code
new Sub(new Lit(1), new Lit(2)).eval

// algebraic datatype
sealed trait Exp

case class Lit(n: Int) extends Exp

case class Sub(l: Exp, r: Exp) extends Exp

// pattern matching function
def eval(exp: Exp): Int =  
    exp match {  
      case Lit(n) => n  
      case Sub(l,r) => eval(l) - eval(r)
    }

// client code
val exp = Sub(Lit(1), Lit(2))
val result = eval(exp)

Fig. 1. Object-oriented decomposition (left) vs. functional decomposition (right) in Scala.

The eval operation is separately defined as a pattern matching function on Exp. An Exp instance is created by calling the constructors and then evaluated by applying the eval function.

There are important tradeoffs between functional and object-oriented decompositions in terms of extensibility and expressiveness. As acknowledged by the notorious Expression Problem [Cook 1991; Reynolds 1978; Wadler 1998], these two decomposition styles are complementary in terms of extensibility. Object-oriented decomposition makes it easy to extend data variants through defining new classes. For example, negations can be added to the Exp hierarchy modularly:

class Neg(e: Exp) extends Sub(new Lit(0), e)

On the other hand, functional decomposition makes it easy to add new operations such as simplification on expressions:

def simplify(exp: Exp): Exp =  
    exp match {  
      case Sub(l,Lit(0)) => l  
      case _ => exp
    }

Besides extensibility, object-oriented and functional decomposition have different expressive power. Object-oriented decomposition facilitates code reuse through inheritance (e.g., Neg) and enables interoperability [Aldrich 2013] between different implementations of the same interface whereas functional decomposition allows inspection on the internal representation of data through (nested) pattern matching, simplifying abstract syntax tree transformations (e.g., simplify).

Unfortunately, programmers are forced to decide a particular decomposition style in the early stage of programming. A proper choice, however, requires predictions on the extensibility dimension and kinds of operations to model, which may not be feasible in practice. Once the wrong design decision has been made, the price for switching to the other decomposition style could be rather high since pervasive manual refactoring is often needed. For example, to convert the left-hand side of Figure 1 to its right hand side, one has to: 1) change classes to case classes; 2) extract eval out of the class hierarchy, add an argument of type Exp, and merge the implementations using case clauses; 3) switch method selections to function applications, e.g. l.eval to eval(l); 4) remove newss on Lit and Sub. Such manual transformation from object-oriented decomposition to functional decomposition is rather tedious and error-prone and so as for the other way round.

A better way, however, allows programmers to choose a decomposition style for prototyping without regret. When the design choice becomes inappropriate, a tool automatically transforms...
their code into the other style without affecting the semantics. The need for switching decomposition styles actually happens in the real world. For example, to grow their simple SQL processor into a realistic SQL engine, Rompf and Amin [2019] switch the decomposition style from functional to object-oriented for supporting a large number of operators. Even at later stages, such an automatic translation tool could be used to make extensions of data variants or operations easier by momentarily switching the decomposition, adding the extension, and then transforming the program back to the original decomposition. Furthermore, studying the transformation between the two styles can provide a theoretical foundation for compiling multi-paradigm languages into single-paradigm ones. From an educational perspective, the tool can help novice programmers to understand both decomposition styles better.

To address this issue, we propose a bidirectional transformation between functional and object-oriented decomposition based on the observation that restricted forms of functional and object-oriented decomposition are symmetric. Our bidirectional transformation is greatly inspired by Cook [2009]’s pure object-oriented programming and the line of work on the duality of data and codata [Binder et al. 2019; Downen et al. 2019; Ostermann and Jabs 2018; Rendel et al. 2015]. The main novelty of our work is an automatic, type-directed transformation formalized in the core calculus FOOD, which captures the essence of Functional and Object-Oriented Decomposition. Unlike existing work, FOOD does not require new language design and can be directly applied to existing multi-paradigm languages. We implement FOOD in Scala as a translation tool called Cook and conduct several case studies to demonstrate the applicability of Cook.

In summary, the contributions of our work are:

- We introduce functional and object-oriented decomposition and show their correspondence (Section 2).
- We formalize the type-directed bidirectional transformation between functional and object-oriented decomposition in the FOOD calculus and show how to concretely transform programs using FOOD (Section 3).
- We give the semantics of FOOD and prove that the transformation preserves the type and semantics of the original program (Section 4).
- We implement the approach in Scala as a translation tool called Cook, and conduct several case studies to demonstrate its applicability (Section 5).
- We discuss features beyond FOOD and sketch how FOOD can be ported to other multi-paradigm languages (Section 6).

Examples, case studies, complete rules, proofs, and the implementation of Cook can be found in the supplementary material.

2 BACKGROUND

In this section, we introduce functional and object-oriented decomposition and show their correspondence. To facilitate the discussion, we reuse the integer sets presented by Cook [2009] as a running example.

2.1 Object-oriented decomposition

The object-oriented decomposition style used throughout this paper follows Cook [2009]’s definition of pure object-oriented programming (OOP), which is arguably the essence of OOP. Here are the key principles of pure OOP:

(1) Objects are first-class values;
(2) Interfaces are used as types not classes;
(3) Objects are accessed through their public interfaces...
trait Set {
    def isEmpty: Boolean
    def contains(i: Int): Boolean
    def insert(i: Int): Set =
        if (this.contains(i)) this
        else new Insert(this, i)
    def union(s: Set): Set = new Union(this, s)
}

object Empty extends Set {
    def isEmpty = true
    def contains(i: Int) = false
    override def union(s: Set) = s
}

class Insert(s: Set, n: Int) extends Set {
    def isEmpty = false
    def contains(i: Int) = i == n || s.contains(i)
}

class Union(s1: Set, s2: Set) extends Set {
    def isEmpty = s1.isEmpty && s2.isEmpty
    def contains(i: Int) = s1.contains(i) || s2.contains(i)
}

Empty.insert(1).union(Empty.insert(3)).contains(3)

Fig. 2. Object-oriented decomposition for integer sets

In pure OOP, a class is an object generator, which is a procedure that returns a value satisfying an interface. The internal representation of an object is hidden to other objects. Although most OOP languages do not strictly follow the pure OOP style, programming in pure OOP style brings several advantages, resulting in more flexible software systems. More importantly, as we shall see later, it allows the code to be transformed to functional decomposition style.

**Interfaces.** A pure OOP implementation of integer sets is given in Figure 2. The interface Set describes four operations supported by an integer set: isEmpty checks whether a set is empty; contains tells whether an integer is in the set; insert puts an integer to a set if it is currently not in that set; union merges two sets. These methods are often referred as destructors, which tear down an object to some other type. Special destructors whose return types are Set (e.g. insert and union) are also known as producers or mutators.

**Classes.** The Set interface is implemented by three classes Empty, Insert, and Union. The singleton class Empty (object in Scala) represents an empty set; The Insert class models the insertion of an integer to a set; The Union class models the union of two sets. As shown by insert and union, the sole use of classes is in new expressions for creating objects, following the principles of pure OOP.
sealed trait Set

case object Empty extends Set

case class Insert(s: Set, n: Int) extends Set

case class Union(s1: Set, s2: Set) extends Set

def isEmpty(self: Set): Boolean = self match {
  case Empty => true
  case Insert(_, _) => false
  case Union(_, _) => isEmpty(s1) && isEmpty(s2)
}

def contains(self: Set)(i: Int): Boolean = self match {
  case Empty => false
  case Insert(_, _) => n == i || contains(s)(i)
  case Union(_, _) => contains(s1)(i) || contains(s2)(i)
}

def insert(self: Set)(i: Int): Set = 
  if (contains(self)(i)) self 
  else Insert(self, i)

def union(self: Set)(that: Set): Set = self match {
  case Empty => that
  case _ => Union(self, that)
}

contains(union(insert(Empty)(1))(insert(Empty)(3)))(3)

Fig. 3. Functional decomposition for integer sets

Inheritance and method overriding. Although inheritance is not an OOP-specific feature [Cook and Palsberg 1989], it is commonly available in OOP languages for code reuse. To avoid duplicating the same implementation in different classes, both insert and union come with a default implementation in the Set interface. These default implementations are inherited by Set’s classes so that explicit definitions for insert and union can be omitted. An exception is Empty, which overrides the union destructor to return the other set immediately instead of creating a Union object. Through dynamic dispatching, the overridden definitions will be selected instead of the inherited ones.

This. A special variable this (self or me in other OOP languages) is in the scope of an interface or a class for referring to the object itself. As illustrated by the default implementation of insert, this is used for constructing a (new Insert(this, i)) or invoking other methods (can often be omitted contains(i)).

There are other features that are often (wrongly) recognized as OOP features, such as mutable state and subtyping. As Cook [2009] argues, these features are not essentials of OOP. We will discuss these features in Section 6.1.
2.2 Functional decomposition

The essence of functional decomposition is arguably algebraic datatypes plus pattern matching functions. A functional decomposition implementation of integer sets is given in Figure 3.

Algebraic datatypes. Now, Set is modeled as an algebraic datatype (sealed trait in Scala) with Empty, Insert and Union captured as constructors (case class in Scala) of Set. Unlike classes, a Set object is created by calling the constructors without new, e.g. Insert(Empty, 3).

Pattern matching functions. Four functions, isEmpty, contains, insert, and union are defined on the Set datatype. These functions are called consumers on Set since their first argument self is a datatype. A consumer is typically defined through pattern matching (match clause in Scala) on the datatype and giving a case clause for each constructor of that datatype. For example, isEmpty and contains are implemented in this manner.

Wildcard pattern. Sometimes it is tedious to repeat case clauses for constructors that have a common behavior. For such cases, the wildcard pattern is often used for giving a default behavior to all boring cases at once. For instance, the union consumer uses a wildcard pattern, case _ => Union(self, that), to handle non-Empty cases altogether.

2.3 The correspondence between functional and object-oriented decomposition

Having a closer look at Figure 2 and Figure 3, we can see that they are essentially symmetric. Their correspondence is revealed by Figure 4, where features in one style has a one-to-one mapping in the other style. In other words, it is possible to transform from one decomposition style to the other back and forth without losing information. Section 3 will formalize the bidirectional transformation and concretely show how to transform the two versions of integer sets discussed here.

3 FORMALIZATION

In this section, we first formalize the type-directed bidirectional transformation between functional and object-oriented decomposition in the FOOD calculus. Our formalization combines ideas from Featherweight Java [Igarashi et al. 2001] and Binder et al. [2019]’s work. We then concretely show how to transform Figure 2 and Figure 3 back and forth using FOOD.

3.1 Syntax

Figure 5 gives the syntax of FOOD, which is a minimal calculus capturing essential features for functional and object-oriented decomposition with a Scala-like syntax. To clarify the formalization, we use keywords different from Scala, where Scala’s trait maps to interface; sealed trait maps to data; and case class maps to case.

A FOOD program (L) consists of some definitions (Def) followed by an expression (e).
Program  \( L ::= Def; L \mid e \)
Definition  \( Def ::= Dt \mid It \mid Ctr \mid Gen \mid Csm \)
Datatype  \(Dt ::= data \ D\)
Interface  \(It ::= interface \ D \{Dtr\}\)
Destructor  \(Dtr ::= Dec \mid Fun\)
Constructor  \(Ctr ::= case \ C(x:T) \ extends \ D\)
Generator  \(Gen ::= class \ C(x:T) \ implements \ D \{Fun\}\)
Consumer  \(Csm ::= def \ f(self:D)(x:T) : T = case \ P \Rightarrow e\)
Declaration  \(Dec ::= def \ f(x:T) : T\)
Function  \(Fun ::= Dec = e\)
Expression  \(e ::= x \mid e_1.f(\overline{e_2}) \mid f(e_1)(\overline{e_2}) \mid C(\overline{e}) \mid new \ C(\overline{e})\)
Pattern  \(P ::= C(\overline{e}) \mid _\)
Types  \(T ::= D \mid T \rightarrow T\)
\(D ::= \) datatype or interface names
\(C ::= \) constructor or generator names
\(f ::= \) consumer or destructor names

Fig. 5. Syntax of FOOD

Definitions. A definition can be a datatype (Dt), an interface (It), a constructor (Ctr), a generator (Gen), or a consumer (Csm). An interface \( D \{Dtr\} \) declares some destructors (Dtr), which can have an optional default implementation. A class \( C(x:T) \ implements \ D \{Fun\}\) implements the destructors declared by the interface \( D\), no more and no less. An instance of data \( D\) can be constructed via its constructors case \( C(x:T) \ extends \ D\). A consumer \( def \ f(self:D)(x:T) : T = case \ P \Rightarrow e\) has two parameter lists, where the first parameter has a reserved variable name self and must be of a datatype \( D\). The body of a consumer is some named patterns with an optional wildcard pattern in the end.

Expressions. Metavariable \( e\) ranges over expressions. Expressions include variables \( x\), method selections \( e_1.f(\overline{e_2})\), consumer applications \( f(e_1)(\overline{e_2})\), constructor calls \( C(\overline{e})\), or new expressions \( new \ C(\overline{e})\). We assume that the special variable \( this\) is used only in destructors and the reserved variable self is only used for the datatype argument in consumers.

Notations. We write \( \overline{e}\) as a shorthand for a possibly empty sequence. Semicolon denotes the concatenation of sequences. Also, in the rest of the paper, we consider that variables with and without an overline are distinct, e.g. \( x\) is distinct from \( \overline{x}\).

3.2 Type-directed transformation

There is a preprocessing pass for collecting a global context \( \Delta\) before the transformation actually happens. The information contained in \( \Delta\) is given in Figure 6. These definitions are trivial and, due to lack of space, are explained in plain text. Intuitively, one can think of this preprocessing step as turning some of the program’s syntax into a logical encoding as sets and maps. As we shall see later, having \( \Delta\) precomputed will simplify the subsequent translation rules.

\( \Delta; \Gamma \vdash L \Rightarrow T \leadsto L’\) denotes the type-directed translation, which states that under the context of \( \Delta\) and \( \Gamma\), a program \( L\) of type \( T\) is translated to \( L’\). Core translation rules are split into two sets respectively for object-oriented decomposition (Figure 7) and functional decomposition (Figure 8).
DT      Datatypes
It       Interfaces
Ctr(D)   Constructors of datatype D
Dtr(D)   Deststructors of interface D
Gen(D)   Generators of interface D
Csm(D)   Consumers of datatype D
Sig(N)   Signature of constructor/generator/consumer indexed by name N
dtrSig(f, D) Signature of destructor f for interface D
DEF(N)   Definitions indexed by name N

Fig. 6. Preprocessed global context Δ for transformation

\[
\begin{array}{c}
\text{IT2Dt} \\
D \in \text{It} \\
\Delta; \Gamma \vdash \text{Dtr} \sim Csm \\
\Delta; \Gamma \vdash L \Rightarrow T \rightsquigarrow L'
\end{array}
\]

\[
\begin{array}{c}
\text{GEN2Ctr} \\
D \in \text{It} \\
\Delta; \Gamma \vdash L \Rightarrow T \rightsquigarrow L'
\end{array}
\]

\[
\begin{array}{c}
\text{DEC2Csm} \\
C = \text{GEN(D)} \\
\Delta; \Gamma \vdash \text{Def(C)} \sim \text{case P} \Rightarrow e
\end{array}
\]

\[
\begin{array}{c}
\text{FUN2Csm} \\
\Delta; \Gamma \vdash \text{D} \{\text{Fun}\} \Rightarrow \text{case C(y:S)} \Rightarrow \text{[this \mapsto \text{self}]e'}
\end{array}
\]

\[
\begin{array}{c}
\text{FUN2Case} \\
\Delta; \Gamma \vdash \text{Def} \{\text{Fun}\} \sim \text{case C(y:S)} \Rightarrow \text{[this \mapsto \text{self}]e'}
\end{array}
\]

\[
\begin{array}{c}
\text{Sel2App} \\
\Delta; \Gamma \vdash e_1 \Rightarrow D \rightsquigarrow e_1' \\
f \in \text{DTR(D)} \\
\text{dtrSig}(f, D) = \overline{T} \rightarrow T \\
\Delta; \Gamma \vdash e_2 \Rightarrow T \rightsquigarrow e_2'
\end{array}
\]

\[
\begin{array}{c}
\text{NEW2Obj} \\
\text{Sig}(C) = \overline{T} \rightarrow D \\
C \in \text{GEN(D)} \\
\Delta; \Gamma \vdash \text{new } C(\overline{e}) \Rightarrow D \rightsquigarrow C(\overline{e'})
\end{array}
\]

Fig. 7. Core translation rules for object-oriented decomposition

Auxiliary translation rules may require additional information collected along the way. For instance, r_D in IT2Dt passes the interface name D to the translation rules on destructors. Similarly,
we use \( \vdash_f \) in Fun2Case to specialize the generator to case clauses translation for a given destructor \( f \). Lastly, \( \vdash_C \) translates a specific case clause for constructor \( C \) to a destructor in Case2Fun.

Apart from the core rules, Figure 9 gives some of the translation rules for skipping over code that is not meant to be translated (Complete rules can be found in Appendix A). All the rules in the figure work by only transforming the inner expressions. This is explained in more detail in Section 3.5. Our main translation rules are explained by example in Section 3.3 and Section 3.4. Notably, our translation rules make use of type information. Section 3.6 provides concrete examples of why type information is essential.

### 3.3 From object-oriented decomposition to functional decomposition

Recall the code snippet shown in Figure 2. After preprocessing, the global context contains:
\[
\begin{align*}
\text{It2It} & \quad \Delta; \Gamma, \text{this} : D \vdash \text{Dtr} \Rightarrow T \leadsto \text{Dtr}' \quad \Delta; \Gamma \vdash L \Rightarrow T \leadsto L' \\
\text{GEN2Gen} & \quad \Delta; \Gamma, \text{this} : D, x : T \vdash \text{Fun} \Rightarrow T \leadsto \text{Fun}' \quad \Delta; \Gamma \vdash L \Rightarrow T \leadsto L' \\
\text{Sel2Sel} & \quad \Delta; \Gamma \vdash e_1 \Rightarrow D \leadsto e'_1 \quad \text{DTRSig}(f, D) = \overline{T} \rightarrow T \\
& \quad \Delta; \Gamma \vdash e_2 \Rightarrow T \leadsto e'_2 \\
\text{New2New} & \quad \text{Sig}(C) = \overline{T} \rightarrow D \\
& \quad \Delta; \Gamma \vdash e \Rightarrow T \leadsto e'
\end{align*}
\]

Fig. 9. Some of the rules for types not selected for transformation

\[
\begin{align*}
\text{It} & = \{\text{Set}\} \\
\text{DTR}(& \text{Set}) & = \{\text{isEmpty, contains, insert, union}\} \\
\text{GEN}(& \text{Set}) & = \{\text{Empty, Insert, Union}\}
\end{align*}
\]

The translation starts from the Set interface. Since Set is in It, it is translated to a datatype together with some consumers by the rule \text{It2Dtr}. As indicated by \text{DTR(Set)}, isEmpty, contains, insert and union are destructors of Set, which are translated to consumers under the term context that \text{this} is of type Set. These destructors are treated differently depending on whether they have a default implementation. isEmpty and contains are declarations and hence are dealt by DEC2CSM. For instance, isEmpty is augmented with the variable self of type Set and its body is represented by case clauses collected from generators of Set. FUN2CASE is applied to every generator of GEN(Set) (i.e. Empty, Insert, and Union). FUN2CASE finds out the definition of isEmpty, translates the body and wraps it into a case clause. Particularly interesting is the Union class, where isEmpty is recursively called on its components. With isEmpty being a consumer afterwards, destructor selections on isEmpty (e.g. s1.isEmpty) should be translated into consumer calls (e.g. isEmpty(s1)). This is done by the rule \text{Sel2App} in a type-directed manner, which makes sure that s1 is of type Set and isEmpty is in DTR(Set). Therefore, the case clause for Union is \text{case Union(s1, s2) \Rightarrow isEmpty(s1) \\ \\ \& \& isEmpty(s2)}.

On the other hand, destructors with a default implementation like insert and union are dealt by FUN2CSM. Building upon DEC2CSM, FUN2CSM additionally appends a wildcard clause constructed from the default implementation to the case clauses collected from generators. Let us take a closer look at the insert defined inside Set. It calls new on Insert, which is in GEN(Set) and will be a constructor after translation. Thus, new Insert(this, i) is translated to Insert(this, i) by the rule NEW2CTR. Also, the this variable is substituted by the variable self.

The translation then goes to generator definitions, Empty, Insert, and Union. They are simply translated to constructors with their body dropped by the rule GEN2CTR.

Finally, the expression is translated, where SELDTR is repeatedly applied to convert destructor selections to consumer applications.
3.4 From functional decomposition to object-oriented decomposition

Preprocessing Figure 3 gives us the following global context:

\[
\begin{align*}
\text{DT} & = \{ \text{Set} \} \\
\text{CTR}(\text{Set}) & = \{ \text{Empty}, \text{Insert}, \text{Union} \} \\
\text{CSM}(\text{Set}) & = \{ \text{isEmpty}, \text{contains}, \text{insert}, \text{union} \}
\end{align*}
\]

Now that Set is a datatype in DT, the rule DT2IT is applied, which translates a datatype into an interface. Consumers of Set, revealed by CSM(Set), are translated to destructors using either the rule CSM2DEC or CSM2FUN depending on whether they contain a wildcard pattern. If a wildcard pattern is not contained (e.g. isEmpty and contains), the rule CSM2DEC is applied, which returns the signature of the consumer with first parameter list (self: Set) dropped. If a wildcard pattern exists\(^1\) (e.g. insert and union), a default implementation is constructed from the right hand side of the wildcard pattern. The translation on insert deserves some explanation. Since Set will become an interface after translation, consumer applications on Set (e.g. contains(self)(i)) are rewritten as destructor selections set.contains(i) by the rule App2SEL. Also, constructor applications on generators of Set (e.g. Insert(self,i)) are replaced by new expressions (e.g. new Insert(self,i)) using the rule Obj2New. Moreover, references to the self variable are substituted as this.

The CTR2GEN rule is then applied to every constructor of Set, namely Empty, Insert, and Union, for producing a generator. For the case of Union, the translation walks through the consumers of Set. If there exists a case clause for Union, a destructor will be produced by the rule CASE2FUN.

Next goes to the consumer definitions. Since insert, isEmpty, contains, and union are all in CSM(Set), they are eliminated by the rule CSMELIM.

Finally, App2SEL repeatedly rewrites consumer applications to destructor selections for the last expression.

3.5 Transforming selected types

The programs discussed so far contain only one datatype or one interface. However, multiple datatypes and interfaces may coexist in a complicated program. For such programs, a user may want to select certain algebraic datatypes or interfaces rather than all of them for transformation. Our transformation is also applicable in this scenario. The only additional step required is modifying the global context according to the selected types.

Let \( \overline{D} \) be the types selected for transformation, then for every type \( D \) in the program we do:

\[
D \not\in \overline{D} \land D \in \text{DT} \Rightarrow \begin{cases}
\text{DT} = \text{DT}/D \\
\text{CTR}(D) = \emptyset \\
\text{CSM}(D) = \emptyset
\end{cases}
\quad D \not\in \overline{D} \land D \in \text{IT} \Rightarrow \begin{cases}
\text{IT} = \text{IT}/D \\
\text{GEN}(D) = \emptyset \\
\text{DTR}(D) = \emptyset
\end{cases}
\]

That is, if a datatype \( D \in \text{DT} \) (interface \( D \in \text{IT} \)) is not in the set of types selected for transformation \( \overline{D} \), then we remove it from DT (IT). We also set the corresponding set of constructors and consumers (generators and destructors) to \( \emptyset \). Basically, DT or IT will only contain the types selected for transformation. Some of the rules for skipping over code that is not meant to be translated (by only translating the inner expressions) are given in Figure 9.

Transforming selected interfaces. To see concretely how the transformation works only on selected types, suppose the following class hierarchy for integer lists is defined together with Figure 2:

\[
\text{trait List} \{ \text{def contains}(i: \text{Int}): \text{Boolean} \}
\]

\[
\text{object Nil extends List} \\
\quad \text{def contains}(i: \text{Int}) = \text{false}
\]

\(^1\)If the body of a consumer is an ordinary expression \( e \), it is viewed as a syntactic sugar of case \( _{} \Rightarrow e \)
class Cons(n: Int, xs: List) extends List {
  def contains(i: Int) = i == n || xs.contains(i)
}

We would only like to transform Set. Since List is not one of the selected types, it is removed from \text{It} and \text{Gen(List)}/\text{Dtr(List)} are deleted, resulting in a $\Delta$ similar to that in \text{Section 3.3}. Consequently, different sets of rules will be applied to the Set and List hierarchies. The rule \text{It2It} is applied for List and \text{Gen2Gen} is applied to Nil and Cons, which translate their inner expressions only. Moreover, destructor selections on List objects and new on Cons are preserved by the rule \text{Sel2Sel} and \text{New2New}. Note that the translation rules \text{It2It}, \text{Gen2Gen}, \text{Sel2Sel} and \text{New2New} can be found in \text{Figure 9}.

\textit{Transforming selected datatypes.} Oppositely, suppose List is implemented in a functional way together with sets shown in \text{Figure 3}:

\begin{verbatim}
sealed trait List
case object Nil extends List
case class Cons(n: Int, xs: List) extends List
\end{verbatim}

And Set is the only candidate for transformation. Then we get a $\Delta$ similar to that in \text{Section 3.4}. Unlike Set, List is processed by the rule \text{Dtr2Dtr} since it is not in \text{Dt}. Therefore, List is still a datatype after translation. Nil and Cons are also unchanged by the rule \text{Ctr2Ctr} and contains remains a consumer on List by the rule \text{Csm2Csm}. Moreover, constructor calls on Cons and application of contains on Lists are retained by the rule \text{Obj2Obj} and \text{App2App}.

3.6 Why the transformation should be type-directed

Importantly, the List example illustrates why the transformation should be type-directed. Notice that contains is defined on both Set and List. For the case when contains is a destructor, expressions of the form $x$.contains(i) should be transformed differently according to the type of $x$. If the type of $x$ is Set which is in \text{It}, the rule \text{Sel2App} is applied, which transforms the expression to contains($x$)(i). Otherwise, the rule \text{Sel2Sel} is applied and $x$.contains(i) will be the same after transformation. Likewise, for the case when contains is an overloaded consumer on both Set and List, expressions of the form contains($x$)(i) should be distinguished by the type of $x$. When $x$ is of type Set, the rule \text{App2Sel} transforms the expression to $x$.contains(i) due to the fact that Set is in \text{Dt}. In contrast, the rule \text{App2App} gives back contains($x$)(i) when $x$ is a List object.

However, without being guided by types, a syntax-directed approach such as the one used by Binder et al. [2019] would transform expressions of the same form uniformly, resulting in an erroneous program after transformation.

4 SOUNDNESS RESULTS

In this section, we discuss our main soundness results. We start by giving the operational semantics of \text{FOOD} in \text{Section 4.1}. Then, we proceed to provide the soundness theorems in \text{Section 4.2}.

4.1 Small-step semantics

We introduce a call-by-value small-step semantics for \text{FOOD}, which will be used when proving the soundness of the translation rules. The rules for our evaluation relation $\rightarrow$ are given in \text{Figure 10}.

An evaluation context is an expression with a hole $\square$, where the hole denotes the next expression to be evaluated. $E[e]$ replaces the hole with the expression. We assume that $\text{obj}(C, \overline{\sigma})$ denotes an object created from a generator (E-\text{New}) or a constructor (E-\text{Ctr}) $C$, where $\overline{\sigma}$ are the values of the corresponding fields. Rule E-\text{CONGR} applies one evaluation step to expression $e_1$ in the evaluation
\[ v := \text{obj}(C, \overline{v}) \]

Evaluation contexts

\[ E := \Box | C(\overline{v}, \Box, \overline{v}) | \text{new } C(\overline{v}, \Box, \overline{v}) | f(\Box)(\overline{v}) | f(v)(\overline{v}, \Box, \overline{v}) | \Box.f(\overline{v}) | v.f(\overline{v}, \Box, \overline{v}) \]

\[ e \rightarrow e' \]

E-Congr

\[ e_1 \rightarrow e_2 \]

\[ E[e_1] \rightarrow E[e_2] \]

E-Ctrl

\[ C(\overline{v}) \rightarrow \text{obj}(C, \overline{v}) \]

E-New

\[ \text{new } C(\overline{v}) \rightarrow \text{obj}(C, \overline{v}) \]

E-CSM

\[ \text{CSMBody}(f, C) = (\overline{y}, \overline{x}, e) \]

\[ f(\text{obj}(C, \overline{v_1}))(\overline{v_2}) \rightarrow [\text{self} \mapsto \text{obj}(C, \overline{v_1}), \overline{y} \mapsto \overline{v_1}, \overline{x} \mapsto \overline{v_2}]e \]

E-DTR

\[ \text{DTRBody}(f, C) = (\overline{y}, \overline{x}, e) \]

\[ \text{obj}(C, \overline{v_1}).f(\overline{v_2}) \rightarrow [\text{this} \mapsto \text{obj}(C, \overline{v_1}), \overline{y} \mapsto \overline{v_1}, \overline{x} \mapsto \overline{v_2}]e \]

Fig. 10. Small-step semantics of \textsc{Food}

DTRGen

\[ \text{Def}(C) = \text{class } C(y : T) \text{ implements } D \{ \text{Fun} \} \quad \text{def } f(x : T) : T = e \in \text{Fun} \]

\[ \text{DTRBody}(f, C) = (\overline{y}, \overline{x}, e) \]

DTRIt

\[ \text{Def}(C) = \text{class } C(y : T) \text{ implements } D \{ \text{Fun} \} \]

\[ \text{Def}(D) = \text{interface } D \{ \text{Dtr} \} \quad \text{def } f(x : T) : T = e \in \text{Dtr} \]

\[ \text{DTRBody}(f, C) = (\Box, \overline{x}, e) \]

\[ \text{CSMBody}(f, C) = (\overline{y}, \overline{x}, e) \]

CSMCTR

\[ \text{Def}(f) = \text{def } f(\text{self} : D)(x : T) : T = \text{case } P \Rightarrow e \quad \text{case } C(\overline{y}) \Rightarrow e \in \text{case } P \Rightarrow e \]

\[ \text{CSMBody}(f, C) = (\overline{y}, \overline{x}, e) \]

CSMDT

\[ \text{Def}(f) = \text{def } f(\text{self} : D)(x : T) : T = \text{case } P \Rightarrow e \quad \text{case } _{} \Rightarrow e \in \text{case } P \Rightarrow e \]

\[ \text{CSMBody}(f, C) = (\Box, \overline{x}, e) \]

Fig. 11. Destructor/consumer lookup

context \(E[e_1]\), thus evaluating \(E[e_1]\) to \(E[e_2]\). For the E-DTR rule, we bound the receiver object to this, and all the fields of \(f\) to their corresponding values, in order to evaluate its body \(e\). The rule
E-Csm is very similar to E-Dtr with the difference that, instead of binding the object \( \text{obj}(C, \bar{x}) \) to this, we bind it to the datatype argument self. Note that before E-Dtr and E-Csm apply, the repeated application of E-CONGR evaluated all the sub-expressions to values. E-Dtr and E-Csm rely on two auxiliary definitions given in Figure 11. For dtrbody, we compute a tuple containing the class fields, destructor arguments and destructor body by looking up destructor \( f \) defined on generator \( C \). Similarly, for csmbody the elements of the resulting tuple are: constructor field names, parameter names in the second list, and the right-hand side of the case clause \( C \) on consumer \( f \). We assume the usual rules for substitution.

4.2 Soundness theorems

Theorems 4.1 and 4.2 state the type safety of FOOD. Then, Theorem 4.3 and Theorem 4.4 state the soundness of our translation. In particular, Theorem 4.3 captures the type and syntax preservation of the translation, and Theorem 4.4 states the preservation of semantics. Additional lemmas and proofs are left in Appendix B. We also have a discussion on well-formedness.

**Type safety of FOOD.** Given that our typing rules are integrated with the type-directed translation rules in Figure 7 and Figure 8, the type safety theorems ignore the result of the actual translation (by using _), and only focus on type derivations.

**Theorem 4.1 (Preservation).** Suppose \( e \) is a well-typed expression. If \( \Delta; \cdot \vdash e : T \leadsto \_ \) and \( e \rightarrow e' \), then \( \Delta; \cdot \vdash e' : T \leadsto \_ \).

**Theorem 4.2 (Progress).** If \( \Delta; \cdot \vdash e : T \leadsto \_ \), then either \( e \rightarrow e' \) or \( e \) is a value.

**Soundness of the translation.** Theorem 4.3 states that, if a well-typed program \( P \) is translated to \( P' \) under the typing environment \( \Gamma \) and global context \( \Delta \), then \( P' \) can be translated back to \( P \) under the same typing environment and the translated global context \( \Delta' \). Intuitively, transforming a FOOD program twice will give us back the original program, thus being syntax preserving. Also, \( P' \) has the same type \( T \) as \( P \) under the typing environment \( \Gamma \) and global context \( \Delta' \), thus being type preserving too.

**Theorem 4.3 (Transformation Preservation).** If \( \Delta; \Gamma \vdash P \Rightarrow T \leadsto P' \) and \( \Delta \leadsto \Delta' \), then \( \Delta'; \Gamma \vdash P' \Rightarrow T \leadsto P \).

Theorem 4.4 states that if the evaluation of \( P \) terminates with result \( v \), then the evaluation of \( P' \) obtained through the translation also terminates with result \( v \). Alternatively, if the execution of \( P \) doesn’t terminate, neither does the execution of \( P' \).

**Theorem 4.4 (Semantics preservation).** Given \( \Delta; \Gamma \vdash P \Rightarrow T \leadsto P' \), the following two conditions must hold: (1) if \( P \rightarrow^* v \), then \( P' \rightarrow^* v \), and (2) if \( P \) diverges, then so does \( P' \).

**Well-formedness.** Another property of interest for our transformation besides type and semantics preservation, is the preservation of well-formedness. Informally, a FOOD program is well-formed if variables are in scope, a class implements the exact set of methods declared by the interface, and patterns are exhaustive. Intuitively, it can be seen that proving preservation of well-formedness is straightforward. We leave the formal proof as future work.

5 Implementation and Case Studies

Based on FOOD, we develop a bidirectional transformation tool called Cook in Scala and conduct several case studies to demonstrate its applicability and effectiveness. In this section, we discuss the implementation and some of the most interesting case studies.
5.1 Implementation and limitations of C/o.sc/o.sc/k.sc

C/o.sc/o.sc/k.sc employs the Scalameta library [sca [n.d.]] for analyzing, transforming, and pretty-printing Scala programs. Given a Scala program, C/o.sc/o.sc/k.sc gets its AST from the parser provided by Scalameta. Following the formalization in Section 3, the AST is processed by two major passes. The first pass collects the global context $\Delta$ and the second pass does the type-directed transformation. To accept more Scala programs, C/o.sc/o.sc/k.sc has a few more passes in-between such as making the use of this explicit for classes or traits. After the transformation, we obtain a new AST. Finally, we pretty-print the new AST to produce the transformed program.

Compared to FOOD, Scala is so rich in terms of language constructs that C/o.sc/o.sc/k.sc cannot handle all of them. In order to transform programs used in our case studies, C/o.sc/o.sc/k.sc does extend FOOD to handle if-expressions, ordinary def functions, val declarations, etc. Some constructs like if-expressions have straightforward translation rules that just apply the translation to subexpressions like what Figure 9 shows. Some constructs may affect contexts such as def and val declarations, where contexts are changed accordingly before the translation goes on to the remaining program. Still there are many features that C/o.sc/o.sc/k.sc cannot handle, which will be discussed in Section 6.1.

As a prototype implementation, C/o.sc/o.sc/k.sc has certain limitations. One limitation is that the soundness of translation on programs that use features beyond FOOD is not guaranteed, although the Scala compiler and test suites can be used to check the correctness of the translated programs. Another limitation is that when applying C/o.sc/o.sc/k.sc to existing code that is not written in FOOD style, one has to adjust the code manually. Typical adaptions are moving the datatype parameter to first, fixing name inconsistencies between case class and patterns, rewriting nested patterns into top-level patterns or adding explicit type annotations for C/o.sc/o.sc/k.sc that can essentially be inferred by the Scala compiler. These manual adaptions could potentially be automated by C/o.sc/o.sc/k.sc.

5.2 Case studies overview

Table 1 summarizes the case studies and examples (separated by the line) with the associated file name, the description, the style, SLOC and translation time displayed for each case study. The case studies are interesting because

- They are written in different styles, covering object-oriented, functional, or mix styles;
- They are extended in various ways, including data variant and operation extensions;
- They are non-trivial, e.g. the SQL query processor [Rompf and Odersky 2010] and the prettier printer [Wadler 2003] are used as core of industrial-strength databases or compilers.

Besides larger case studies, there are also smaller examples ported from different textbooks for introducing functional or object-oriented programming [Felleisen et al. 2010; Odersky et al. 2008] or literatures for data structure implementations [Fredman et al. 1986; Okasaki 1999]. Overall, these case studies are rather comprehensive in examining different aspects of C/o.sc/o.sc/k.sc. The remaining of this section will focus on the most interesting ones.

5.3 SQL Processor

Starting from an interpreter, Rompf and Amin [2019] show how to turn the simple interpreter into an efficient compiler through staging à la LMS [Rompf and Odersky 2010]. Their SQL processor implementation uses functional decomposition in Scala.

A SQL query is parsed into a relational algebra operator. Initially, the Operator datatype has 5 constructors: Scan, Project, Filter, Join and Print. The core consumer on Operator is execOp, which accumulates the action to the record in the parameter yld according to the Operator. The following excerpt is copied from Rompf and Amin [2019]’s paper:

```scala
def execOp(o: Operator)(yld: Record => Unit): Unit = o match {
```
Table 1. Summary of case studies.

| File name      | Description                                             | Style | SLOC | Time (ms) |
|----------------|---------------------------------------------------------|-------|------|-----------|
| SqlFP          | SQL query processor [Rompf and Amin 2019]               | FP    | 176  | 54.1      |
| PrettierPrinter| Pretty printer with alternative layouts [Wadler 2003]    | FP    | 195  | 33.2      |
| BoolNormalizer | Boolean formula normalizer [Binder et al. 2019]         | Mixed | 92   | 14.8      |
| ProgInScala    | Rendering library & expression formatter [Odersky et al. 2008] | Mixed | 106  | 26.9      |
| Json           | JSON formatter and validator                           | Mixed | 90   | 19.7      |
| Scans          | Circuit DSL [Zhang and Oliveira 2019]                   | FP    | 63   | 10.7      |
| RiverSystem    | River location [Felleisen et al. 2010]                  | OOP   | 23   | 3.9       |
| Shape          | Shape representation [Felleisen et al. 2010]            | OOP   | 31   | 2.5       |
| Stack          | Stack implementation [Felleisen et al. 2010]            | OOP   | 18   | 4.4       |
| PairingHeap    | Efficient heap implementation [Fredman et al. 1986]     | FP    | 29   | 5.0       |
| ListFP         | List implementation [Okasaki 1999]                      | FP    | 38   | 3.9       |
| Bst            | Binary search tree implementation [Okasaki 1999]         | FP    | 40   | 4.9       |

```scala
case Scan(filename) => processCSV(filename)(yld)
case Print(parent) => execOp(parent) { rec => printFields(rec.fields) }
case Filter(pred, parent) =>
  execOp(parent) { rec => if (evalPred(pred)(rec)) yld(rec) }
...
```

which is already written in a style close to **Cook**. In fact, their entire implementation is written in this manner. Consequently, a few renaming on variables is sufficient to fit their implementation into **Cook**.

Later on, Rompf and Amin [2019] introduce two new operators, Group and HashJoin, for supporting group by syntax and a more efficient hash-join operator. With the functional decomposition version, these extensions have to be added by defining new case classes of `Operator` and modifying existing consumers on `Operator` scattered around the file. In contrast, it can be simplified by switching to object-oriented style and adding the extensions modularly as **classes** with **Cook**:

```scala
class Group(keys: Schema, agg: Schema, op: Operator) extends Operator {
  def resultSchema = keys ++ agg
  def execOp(yld: Record => Unit) = ...
}
class HashJoin(left: Operator, right: Operator) extends Operator {
  def resultSchema = resultSchema(left) ++ resultSchema(right)
  def execOp(yld: Record => Unit): Unit = ...
}
```

The transformed object-oriented code looks very much like the core part of Zhang and Oliveira [2019]'s hand-written implementation. Similarly, when the need of new operations such as compiling SQL queries to C backend arises, we can transform the implementation back to functional style to facilitate operation extensions. Again, the **Cook**-transformed functional version with extensions is almost identical to the manually extended version done by Rompf and Amin [2019].

In fact, as acknowledged by Rompf and Amin [2019], the decomposition style switch happens when they grow this simple SQL processor into a realistic SQL engine. The decomposition style is switched from functional to object-oriented to support a large number of operators such as various join operators (semi joins, anti joins, outer joins, etc.). With **Cook**, however, this switch can be done seamlessly without the pain of pervasive refactoring on the original code.
5.4 Prettier printer

Wadler [2003] presents a pretty printer library that allows a document to be printed with different layouts efficiently. The implementation was originally written in Haskell using functional decomposition. Wadler [2003] shows a simpler and less efficient version first. A document is represented by a datatype \( \text{TDoc} \) with 3 constructors for creating empty documents (\( \text{TNil} \)), lifting strings into documents (\( \text{TText} \)) and inserting line breaks (\( \text{TLine} \)). Other combinators on \( \text{TDoc} \) for adding indentations (\( \text{nest} \)) and concatenating documents (\( <> \)) are defined as consumers on \( \text{TDoc} \), along with a layout consumer for printing a \( \text{TDoc} \) as a string.

The simple version is then extended to support alternative layouts. \( \text{TDoc} \) is extended with both new consumers (e.g. \( \text{group} \), \( \text{flatten} \) and \( \text{pretty} \)) as well as a new constructor \( \text{Union} \). To further improve efficiency, another document datatype \( \text{DOC} \) is defined and several consumers are introduced for compiling \( \text{DOC} \) to \( \text{TDoc} \) efficiently.

We ported the Haskell implementation into Scala and then adapted the ported implementation into \text{Coq} \text{Style}. Most of the adaptions are trivial. Only two consumers deserve some attention since they use advanced forms of pattern matching.

The first is the \( \text{fits} \) consumer which uses a guarded wildcard pattern in the beginning:

```scala
def fits(w: Int, x: TDoc): Boolean = x match {
  case _ if w < 0 => false
  case TText(s, x) => fits(w - s.length, x)
  case TNil => true
  case TLine(i, x) => true
}
```

We split \( \text{fits} \) into two consumers and replace the guarded wildcard pattern as an if-expression:

```scala
def fits(self: TDoc)(w: Int): Boolean = if (w < 0) false else fitsAux(self)(w)
def fitsAux(self: TDoc)(w: Int): Boolean = self match {
  case TText(s, x) => fits(x)(w - s.length)
  case TNil => true
  case TLine(i, x) => true
}
```

The other one is \( \text{be} \) function, which deeply pattern matches on a list of pairs.

```scala
def be(w: Int, k: Int, xs: List[(Int, DOC)]): TDoc = xs match {
  case List() => TNil
  case (i, NIL) :: z => be(w, k, z)
  case (i, :<>(x, y)) :: z => be(w, k, (i, x) :: (i, y) :: z)
  ...
}
```

Similar to \( \text{fits} \), we break \( \text{be} \) into two mutually recursive functions with nested patterns rewritten as top-level patterns:

```scala
def be(w: Int, k: Int, xs: List[(Int, DOC)]): TDoc = xs match {
  case List() => TNil
  case (n, x) :: z => beAux(x: DOC)(w, k, n, z)
}
def beAux(self: DOC)(w: Int, k: Int, n: Int, z: List[(Int, DOC)]): TDoc = self match {
  case NIL => be(w, k, z)
  case :<>(x, y) => be(w, k, (n, x) :: (n, y) :: z)
  ...
}
```
With these adaptations done, we obtained an object-oriented implementation of Wadler’s printer transformed by Cook. Here is an excerpt of the DOC interface produced by Cook:

```scala
trait DOC {
  def <>:(y: DOC): DOC = new <>::<(this, y)
  def nest(i: Int): DOC = new NEST(i, this)
  def group: DOC = new <|>(this.flatten, this)
  def flatten: DOC
  def pretty(w: Int): String = this.best(w, 0).layout
}
```

A simple example to demonstrate the transformed library could be:

```scala
val d: DOC =
  text("[" <> //
    line <> text("Hello") <> // Hello
    line <> text("world").nest(2) <> // world
    line <> text("]") //
where the result of calling d.pretty(10) is shown in comments. Larger examples contained in the original implementation like trees and XMLs all work well in the transformed library.

Similar to the extensions discussed in Section 5.3, adding extensions mentioned above can also be simplified by switching decomposition style back and forth with the help of Cook.

### 5.5 Boolean formula normalizer

The boolean formula normalizer case study is reproduced from Binder et al. [2019]’s work. The goal is to develop a normalizer that evaluates a boolean formula to its negation normal form by repeatedly applying De Morgan’s laws or double negation elimination rules. The development is done through several iterations by switching the decomposition style on a particular type back and forth for extensions. This case study further examines Cook’s ability to transform selected types.

The implementation contains several datatypes for modeling boolean formulas (Expr), redexes (Redex), values (Value) and found value/redex (Found). Core consumers are search, searchPos and searchNeg, which search for a redex in an Expr using the continuation-passing style.

```scala
trait Value2Found {
  def apply(value: Value): Found }

def search(self: Expr): Found = searchPos(self)(EmptyCnt)
def searchPos(self: Expr)(cnt: Value2Found): Found = self match {
  case EVar(n) => cnt.apply(ValPosVar(n))
  case ENot(e) => searchNeg(e)(cnt)
  case EAnd(l,r) => searchPos(l)(new AndCnt1(r,cnt))
  case EOr(l,r) => searchPos(l)(new OrCnt1(r,cnt))
}
```

Initially, Value2Found was modeled as an interface with only destructor apply. Some instances of Value2Found used in the definition above are given below:

```scala
object EmptyCnt extends Value2Found {
  def apply(value: Value): Found = FoundValue(value)
}

class AndCnt1(e: Expr, cnt: Value2Found) extends Value2Found {
  def apply(value: Value): Found = searchPos(e)(new AndCnt2(value,cnt))
}
```
We adapt local comatches (similar to anonymous objects) used by Binder et al. [2019] to ordinary generators in Cook.

In the second iteration, Value2Found was switched to a datatype, renamed as Context and extended with a new consumer findNext. In the third iteration, Context datatype was extended with another consumer substitute and the Expr datatype was extended with an evaluate consumer. By switching Context back to an interface, we derived the final implementation.

Unsurprisingly, Cook makes the iterative development of the boolean formula normalizer smoothly by allowing style switches on selected types.

5.6 Findings

We have already established the semantic preserving nature of the translation in Section 4.2. The case studies above additionally allow us to conclude a number of findings below on the effectiveness and applicability of Cook.

Cook facilitates extensions. As we can see in Section 5.3, the translated object-oriented code behaves exactly as how well-written object-oriented code should be in terms of extensibility. We can add new classes such as Group and HashJoin without touching any of the existing code. This very desirable behavior demonstrates the effectiveness of the translation and can be observed throughout the case studies. Moreover, the extended code could be readily translated back to the original style, resulting in code similar to the hand-written one.

Cook is applicable to a lot of existing code with moderate adaptions. As demonstrated in the case studies, most adaptions are very straightforward and only involve moving the datatype parameter to the first parameter lists, fixing the name inconsistency of variables between case class and case clauses, or providing type information that Cook currently cannot infer. The only slightly complicated case is when advanced forms of pattern matching are used. They need to be rewritten into the normal forms as illustrated.

Cook transforms programs fast. The case study programs are compiled using Scala 2.12.13, JDK 1.8.0_292 and executed on a MacBook Pro (M1) with 8 cores and 16 GB memory. We use ScalaMeter 0.8.2 [scala [n.d.]b] microbenchmark framework to measure the average time for 100 runs of transformation. As shown in the last column of Table 1, the translation time is negligible and is positively correlated with the SLOC of the program being transformed.

6 DISCUSSION

In this section, we discuss features not covered by FOOD and how FOOD can be applied to other multi-paradigm languages.

6.1 Features beyond FOOD

As a core calculus, there are a lot of language features not covered by FOOD. As discussed in Section 5.1, FOOD can be extended to handle more language features. Here, we discuss additional features that are not yet supported or cannot be supported in principle by FOOD.

Type tests/type casts. Violating pure OOP principles, classes are sometimes used as types for type tests and type casts. Type tests/type casts are frequently seen in binary methods [Bruce et al. 1996] for inspecting how the other object is constructed. For example, to compare structural equality of two sets, one may change Insert to:

```scala
class Insert(val s: Set, val n: Int) extends Set { // val added for getters
    
    def equals(that: Set) =
```


if (that.isInstanceOf[Insert]) {
    val thatInsert = that.asInstanceOf[Insert]
    this.n == thatInsert.n && (this.s equals thatInsert.s)
} else false

Besides being used as types, fields of Insert are made public, exposing its internal representation. Nonetheless, equals can be rewritten without casts by adding a pair of isInsert and fromInsert destructors [Emir et al. 2007] for checking whether an object is created via Insert and then extracting fields from an Insert object:

```
trait Set {
    ...
    def isInsert: Boolean = false
    def fromInsert: (Set,Int) = throw new RuntimeException
}
```

In Insert, isInsert and fromInsert are overridden accordingly:

```
class Insert(s: Set, n: Int) extends Set {
    ...
    override def isInsert = true
    override def fromInsert = (s,n)
    def equals(that: Set) =
        that.isInsert && s.equals(that.fromInsert._1) && (n == that.fromInsert._2)
}
```

This implementation still has issues like polluting the interface with implementation details due to the fact that object-oriented decomposition discourages inspections on internal representation of other objects. If such an ability is a must, functional decomposition might be a better option.

**Complex patterns.** Functional decomposition allows inspection internal representation of data through pattern matching. Moreover, patterns can be nested or guarded by a condition. Such complex patterns correspond to multi-methods [Chambers 1992], which are neither available in pure OOP nor in many multi-paradigm languages like Scala. Nevertheless, they can be rewritten into standard forms. As illustrated in Section 5.4, guarded patterns can be rewritten as if expressions and nested patterns can be replaced as multiple top-level patterns. If that becomes a burden, then it is time to fix the decomposition style to be functional.

**Inheritance.** The original pure OOP [Cook 2009] excludes inheritance since it is not a feature specific to OOP. In Section 2, we have shown a lightweight use of inheritance based on interfaces corresponding to wildcard pattern in functional decomposition. There are other forms of inheritance such as class-based inheritance or even multiple inheritance. The use of interface-based inheritance is considered less harmful than other forms as it neither introduces additional dependencies nor causes the diamond problem. For other forms of inheritance that do not have a counterpart in functional decomposition, we rewrite them as explicit delegations. For example Neg shown in Section 1 can be rewritten as:

```
class Neg(e: Exp) extends Exp { def eval = new Sub(new Lit(0), e).eval }
```

**Mutable state.** Mutable state is typically viewed as a object-oriented programming feature, which is indeed a feature from imperative programming. We can both write functional and object-oriented
programs that manipulate mutable state. Supporting mutable state in FOOD is possible by allowing mutable fields in both generators and constructors. We leave a formalization of FOOD with mutable state as future work.

**Subtyping.** Subtyping is another feature that is often (mistakenly) considered as an object-oriented feature. Nevertheless, object-oriented languages do have a better support for subtyping by allowing user-defined subtyping relations. In OOP, interfaces can be extended with new destructors:

```trait ExtSet extends Set { def intersect(other: ExtSet): ExtSet }
```

The extended interface ExtSet is a subtype of Set and objects of ExtSet can be passed as arguments to functions that accept Sets. However, a datatype is typically not allowed to be extended by another datatype. Even if it is allowed, e.g. in Scala:

```sealed trait ExtSet extends Set
case class Intersect(s1: ExtSet, s2: ExtSet) extends ExtSet
```

the extended datatype ExtSet is treated as a subtype of Set and existing consumers on Set would be warned with a missing case for Intersect. However, ExtSet should be indeed a supertype of Set. Therefore, FOOD currently does not handle extended interfaces/datatypes as the goal of FOOD is to be applicable to existing multi-paradigm languages.

**Parametric polymorphism.** Parametric polymorphism is a useful feature that is not yet supported by FOOD. Different forms of parametric polymorphism pose different level of challenges. Supporting parametric data types and generic functions is relatively simple. For instance, Set can be defined as a parametric algebraic datatype with the element type captured as a type parameter:

```sealed trait Set[A]
case class Insert[A](s: Set[A], n: A) extends Set[A]
```

Accordingly, consumers on Set[A] are also parameterized by A. In principle, this generic version of Set can be transformed to parameterized class hierarchies and vice versa. However, supporting generalized algebraic datatypes (GADTs) [Xi et al. 2003] would be problematic as existing multi-paradigm languages may not support GADTs and their counterpart GACoDTs [Ostermann and Jabs 2018]. For example, GADTs are known to have soundness issues in Scala [Giarrusso 2013]. What makes it even trickier is that type parameters may interact with subtyping. For example, type parameters be constrained with variance or bounds in Scala. Therefore, how to fully support parametric polymorphism remains an open problem.

### 6.2 Applying FOOD to Scala 3

The latest version of Scala, Scala 3, opens up many possibilities to further simplify FOOD. Here, we summarize the key changes that are particularly relevant to our work.

**Enumerations.** In Scala 3, closed algebraic datatypes can be defined using `enum`, e.g.

```enum Set:
  case Empty
  case Insert(s: Set, n: Int)
  case Union(s1: Set, s2: Set)
```

where constructors of Set are defined in one place and explicit `extends` clauses can be omitted.

**Extension methods.** Scala 3 introduces `extension methods`, which allows adding methods to existing types. Moreover, extension methods are invoked using the dot notation just like instance methods. Altogether, we can define consumers on datatype as extension methods, e.g. contains:

```extension (self: Set) def contains(i: Int): Boolean = self match
  case Empty => false
```

, Vol. 1, No. 1, Article . Publication date: April 2022.
interface Exp { fun eval(): Int }
sealed class Exp
class Lit(val n: Int): Exp {
    override fun eval(): Int { return n } }
data class Lit(val n: Int): Exp()
class Sub(val l: Exp, val r: Exp): Exp {
    override fun eval(): Int {
        return l.eval() - r.eval()
    }
}
data class Sub(val l: Exp, val r: Exp): Exp()

fun main() {
    print(Sub(Lit(1),Lit(2)).eval())
}

fun eval(self: Exp): Int {
    return when (self) {
        is Lit -> self.n
        is Sub -> eval(self.l) - eval(self.r)
    }
}

fun main() {
    print(eval(Sub(Lit(1),(Lit(2)))))
}

Fig. 12. Object-oriented decomposition vs functional decomposition in Kotlin

case Insert(s,n) => n == i || s.contains(i)
case Union(s1,s2) => s1.contains(i) || s2.contains(i)

This change brings two main advantages. First, extension methods distinguish consumers from ordinary functions through the surface syntax. As shown by the definition above, it clearly indicates that eval is a consumer on Set. Second, recursive calls on contains are written as just like it is a destructor.

Optional new. Scala 3 allows classes to be instantiated without new, making it consistent with ordinary classes. Together with consumers defined as extension methods, client code for functional and object-oriented decomposition is made identical. As a consequence, the bidirectional transformation can be simplified with only rules for definitions and the switch of decomposition style will not affect client code.

6.3 Applying FOOD to other multi-paradigm languages

Although Scala is used throughout this paper for demonstration, our approach is indeed not Scala-specific. The core features we are focusing on are commonly available in other multi-paradigm languages. Modulo syntax differences, languages such as OCaml, F#, Swift, Rust, and Kotlin are possible candidates for FOOD. As an example, Figure 12 ports the simple arithmetic expression language shown in Figure 1 to Kotlin. We can see that the Kotlin version looks very much like the Scala version. Therefore, most of the rules shown in Section 3 can be directly applied to Kotlin. Of course, adjustments may be necessary for dealing with syntax differences. For example, Swift separates the field declarations and the initializer for generators, then the abstract syntax and rules named with GEN need to be changed accordingly. There is an ongoing work that implements FOOD in Rust. Preliminary results show that FOOD is also applicable to Rust, although Rust’s type system, in particular smart pointers, poses new challenges that have not yet solved.

7 RELATED WORK

Expression Problem. The Expression Problem dates back to Reynolds [1978], who first pointed out that user-defined types (abstract datatype) and procedural data structures are two complementary approaches to data abstraction in terms of extensibility. Cook [1991] further distilled the tradeoffs between the two data abstraction approaches and argued that objects are essentially procedural data structures. Wadler [1998] popularized the problem by coining the term “Expression
Problem” and describing the requirements that a proper solution should meet. Thereafter, many solutions to the Expression Problem have been proposed [Carette et al. 2009; Hofer et al. 2008; Oliveira and Cook 2012; Swierstra 2008; Wang and Oliveira 2016; Zenger and Odersky 2001, 2005; Zhang et al. 2021], to list a few. However, solutions to the Expression Problem typically introduce extra complexity, parameterization, and indirections that may cause performance penalty or require boilerplate code [Zhang and Oliveira 2020, 2017]. In contrast, FOOD allows programmers to write the code in their familiar style without performance penalty incurred by indirections.

The duality of data and codata. There is a line of work studying the duality of data and codata [Binder et al. 2019; Downen et al. 2019; Laforgue and Régis-Gianas 2017; Ostermann and Jabs 2018; Rendel et al. 2015]. In particular, Binder et al. [2019]’s work is most closely related and our formalization is greatly inspired by their work. The major differences are that our transformation is type-directed whereas theirs is syntax-directed and our transformation can be applied to existing languages whereas theirs requires a new language design. In their language design, destructors and consumers are in the same namespace with distinct names, which is quite different from the setting of existing languages where classes have their own namespace and names can be overloaded. As discussed in Section 3.6, type information is critical for transforming overloaded names correctly. Their work additionally supports local (co)pattern matching. However, specialized syntax is invented for explicitly capturing the environment and providing a name for transformation. This can hardly be enforced for existing languages and some convenience of local (co)pattern matching is lost. Such local pattern matching and copattern matching are rewritten as top-level definitions in FOOD. Ostermann and Jabs [2018] investigate the duality of generalized algebraic datatype and codatatype to their generalized versions. How to port their formalization to existing languages remains a problem as there is inadequate support for these features in existing languages. There are other transformation schemes between data to codata with a focus on compositionality [Downen et al. 2019; Laforgue and Régis-Gianas 2017]. Unlike FOOD, Downen et al. [2019] compile data to codata using the VISITOR pattern, which does not switch the dimension of extensibility.

Multiple views of a program. As a program can be written in multiple ways, intentional programming [Simonyi 1995] aims at capturing the intents of a programmer underneath the surface syntax. The intents are a high-level representation of the program maintained in some database and the view, code, is generated on the fly. Following the idea of intentional programming, Decal [Janzen and De Volder 2004] is a tool on an OOP language with open classes for allowing crosscutting concerns to be added in one place. It maintains an internal representation of the program using a SQL engine and generates views from the internal representation. Decal users can either choose modules view (similar to functional decomposition in FOOD) or classes view (similar to object-oriented decomposition in FOOD) for editing and the changes to the view will be reflected to the internal representation. Compared to Decal, FOOD does not need to maintain such an internal representation, simplifying the implementation. Moreover, unlike FOOD, Decal does not have a formalized transformation.

8 CONCLUSION AND FUTURE WORK
In this paper, we have shown that restricted forms of functional and object-oriented decomposition are symmetric. We propose a type-directed bidirectional transformation between functional and object-oriented decomposition in the FOOD calculus and proved the soundness of FOOD. Moreover, we have implemented FOOD in Scala called Cook and conducted several case studies to demonstrate the applicability and effectiveness of Cook.
In future work, we would like to explore more features such as mutable state and parametric polymorphism in FOOD. How these additional features interact with existing features and affect the duality needs further investigation. We would also like to improve the usability of Cook by reducing manual adaptations through automatic rewriting nested patterns, better type inference, etc. and develop similar tools on other multi-paradigm languages such as Rust. Another direction of future work is to mechanize our manual proofs in a theorem prover like Coq, where Binder et al. [2019]’s work is a good start point.

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A ADDITIONAL FORMALIZATION

In this section, we provide some additional formalization that, for brevity reasons, was left out of Section 3. Signatures for generators, constructors, consumers, and destructors are collected using
Figure 13 gives the translation rules for skipping over code that were not captured in Figure 9. Similarly to Figure 9, all the rules in Figure 13 work by only transforming the inner expressions.
B Lemmas and Proofs

In this section, we provide the proofs for the soundness theorems stated in Section 4, together with some helper lemmas. To begin with, Lemmas B.1-B.5 describe the relationship between the global context before and after the translation. Lemmas B.6 and B.7 describe the relation between the consumer and destructor bodies before and after the translation.

**Lemma B.1 (Constructor to generator).** If \( \text{Sig}(C) = \overline{T} \rightarrow D \) and \( C \in \text{Ctr}(D) \) then, after the translation, \( \text{Sig}(C)' = \overline{T} \rightarrow D \) and \( C \in \text{Gen}(D)' \).

**Proof.** Trivial by \( \text{Ctr2Gen} \). □

**Lemma B.2 (Generator to constructor).** If \( \text{Sig}(C) = \overline{T} \rightarrow D \) and \( C \in \text{Gen}(D) \) then, after the translation, \( \text{Sig}(C)' = \overline{T} \rightarrow D \) and \( C \in \text{Ctr}(D)' \).

**Proof.** Trivial by \( \text{Gen2Ctr} \). □

**Lemma B.3 (Destructor to consumer).** If \( f \in \text{Dtr}(D) \) and \( \text{dtrSig}(f, D) = \overline{T} \rightarrow T \) then, after the translation, \( f \in \text{Csm}(D)' \) and \( \text{Sig}(f)' = D \rightarrow \overline{T} \rightarrow T \).

**Proof.** Trivial by \( \text{It2Dtr} \) and \( \text{Dec2Csm} \). □

**Lemma B.4 (Consumer to destructor).** If \( f \in \text{Csm}(D) \) and \( \text{Sig}(f) = D \rightarrow \overline{T} \rightarrow T \) then, after the translation, \( f \in \text{Dtr}(D)' \) and \( \text{dtrSig}(f, D)' = \overline{T} \rightarrow T \).

**Proof.** Trivial by \( \text{Dt2It} \) and \( \text{Csm2Dec} \). □

**Lemma B.5 (Global context translation).** If \( \Delta \) is the global context collected through the pre-processing of \( P \), and \( \Delta' \) is the context collected through the preprocessing of \( P' \) such that \( \Delta; \Gamma \vdash P \Rightarrow T \leadsto P' \), then: \( \text{It} = \text{Dtr}' \) and \( \text{Dt} = \text{It}' \) and \( \forall D \in \text{It}. \text{Dtr}(D) = \text{Csm}(D)' \land \text{Gen}(D) = \text{Ctr}(D)' \) and \( \forall D \in \text{Dt}. \text{Csm}(D) = \text{Dtr}(D)' \land \text{Ctr}(D) = \text{Gen}(D)' \). We write \( \Delta \leadsto \Delta' \).

**Proof.** Trivial by \( \text{It2Dtr, Dt2It and Lemmas B.1-B.4} \). □

**Lemma B.6 (Destructor to consumer case).** If \( C \in \text{Gen}(D), f \in \text{Dtr}(D) \), then \( \text{dtrBody}(f, C) = (\overline{y}, \overline{x}, e) \) and \( \Delta; \Gamma \vdash e \Rightarrow T \leadsto e' \) then, after the translation, \( C \in \text{Ctr}(D)' \), \( f \in \text{Csm}(D)' \) and \( \text{csBody}(f, C) = (\overline{y}, \overline{x}, [\text{this} \mapsto \text{self}]e') \).

**Proof.** By induction.

\[ \text{DtrGen} \]

\[
\begin{align*}
\text{Case} & \quad \text{Def}(C) = \text{class}\ C(y : T)\ \text{implements}\ D\ \{\text{Fun}\} \quad \text{Def}(x : T) : T = e \in \text{Fun} \\
\text{dtrBody}(f, C) = (\overline{y}, \overline{x}, e)
\end{align*}
\]

\[ \Delta; \Gamma \vdash_f\ \text{class}\ C(y : T)\ \text{implements}\ D\ \{\text{Fun}\} \quad \text{case}\ C(\overline{y}) \Rightarrow [\text{this} \mapsto \text{self}]e' \quad \text{By Fun2Case} \\
\text{csBody}(f, C) = (\overline{y}, \overline{x}, [\text{this} \mapsto \text{self}]e') \quad \text{By CsmCtr} \\
\]

**DtrIt**

\[
\begin{align*}
\text{Def}(C) & = \text{class}\ C(y : T)\ \text{implements}\ D\ \{\text{Dtr}\} \\
\text{Def}(x : T) : T = e \in \text{Dtr} \\
\text{dtrBody}(f, C) = (\varnothing, \overline{x}, e)
\end{align*}
\]

\[ \Delta; \Gamma \vdash_D\ \text{Def}(x : T) : T = e \leadsto \text{def}\ f(\text{self} : D)(x : T) : T = \text{case}\ P \Rightarrow e;\ \text{case} \_ \Rightarrow [\text{this} \mapsto \text{self}]e' \quad \text{By Fun2Csm} \\
\text{csBody}(f, C) = (\varnothing, \overline{x}, [\text{this} \mapsto \text{self}]e') \quad \text{By CsmDt}
\]
LEMMA B.7 (Consumer to destructor case). If $C \in \text{Ctr}(D)$, $f \in \text{Csm}(D)$, $\text{csmbody}(f, C) = (\overline{\gamma}, \overline{x}, e)$ and $\Delta; \Gamma \vdash e \Rightarrow T \sim e'$ then, after the translation, $C \in \text{Gen}(D)'$, $f \in \text{dtr}(D)'$, $\text{dtrbody}(f, C) = (\overline{\gamma}, \overline{x}, [\text{self} \mapsto \text{this}] e')$

\begin{proof}
By induction.

\begin{align*}
\text{csmbody}(f, C) & = (\overline{\gamma}, \overline{x}, e) \\
\text{dtrbody}(f, C) & = (\overline{\gamma}, \overline{x}, [\text{self} \mapsto \text{this}] e')
\end{align*}

\begin{itemize}
\item Case $\text{self} \mapsto \text{this}$
\item Case $\text{self} \mapsto \text{this}$
\item Case $\text{self} \mapsto \text{this}$
\item Case $\text{self} \mapsto \text{this}$
\item Case $\text{self} \mapsto \text{this}$
\item Case $\text{self} \mapsto \text{this}$
\end{itemize}
\end{proof}

B.1 Proof for Theorem 4.1

\begin{proof}
By rule induction on the given typing derivation.

\begin{itemize}
\item Case Sel2App. Given $e_1, f(\overline{e_2})$, either E-Congr or E-Dtr apply.
\begin{itemize}
\item If E-Congr applies, then either $e_1 \rightarrow e_1'$, or, there exists some $e_2$ in $\overline{e_2}$ such that $e_2 \rightarrow e_2'$. For the first case, by the induction hypothesis, if $\Delta; \cdot \vdash e_1 : T_1 \sim \_\_\_$, then $\Delta; \cdot \vdash e_1' : T_1 \sim \_\_\_$. Consequently, it must be the case that if $\Delta; \cdot \vdash e_1, f(\overline{e_2}) : T \sim \_\_\_$, then $\Delta; \cdot \vdash e_1', f(\overline{e_2}) : T \sim \_\_\_$. Similarly, for the second case, if $\Delta; \cdot \vdash e_2 : T_2 \sim \_\_\_$, then $\Delta; \cdot \vdash e_2' : T_2 \sim \_\_\_$. by the induction hypothesis. Consequently, if $\Delta; \cdot \vdash e_1, f(\overline{e_2}) : T \sim \_\_\_$, then $\Delta; \cdot \vdash e_1, f(\overline{e_2}) : T \sim \_\_\_$. (2) If E-Dtr applies, then the conclusion results from Fun2Csm, destructor lookup and standard substitution rules.
\item Case App2Sel. Given $f(e_1)(\overline{e_2})$, either E-Congr or E-Csm apply.
\begin{itemize}
\item If E-Congr applies, then either $e_1 \rightarrow e_1'$, or, there exists some $e_2$ in $\overline{e_2}$ such that $e_2 \rightarrow e_2'$. For the first case, by the induction hypothesis, if $\Delta; \cdot \vdash e_1 : T_1 \sim \_\_\_$, then $\Delta; \cdot \vdash e_1' : T_1 \sim \_\_\_$. Consequently, it must be the case that if $\Delta; \cdot \vdash f(e_1)(\overline{e_2}) : T \sim \_\_\_$, then $\Delta; \cdot \vdash f(e_1')(\overline{e_2}) : T \sim \_\_\_$. Similarly, for the second case, if $\Delta; \cdot \vdash e_2 : T_2 \sim \_\_\_$, then $\Delta; \cdot \vdash e_2' : T_2 \sim \_\_\_$. by the induction hypothesis. Consequently, if $\Delta; \cdot \vdash f(e_1)(\overline{e_2}) : T \sim \_\_\_$, then $\Delta; \cdot \vdash f(e_1)(\overline{e_2}) : T \sim \_\_\_$. (2) If E-Csm applies, then the conclusion results from Csm2Fun, consumer lookup and standard substitution rules.
\item Case Obj2New. Given $C(\overline{e})$, either E-Congr or E-Ctr apply.
\end{itemize}
\end{itemize}
\end{proof}
(1) If E-CONGR applies, then there exists some e in Π such that e → e'. By the induction hypothesis, if Δ;· ⊢ e : T₁ ⊑ _, then Δ;· ⊢ e' : T₁ ⊑ _. Consequently, it must be the case that if Δ;· ⊢ C(Π) : T ⊑ _, then Δ;· ⊢ C(Π') : T ⊑ _.

(2) If E-CTR, then the conclusion follows from the typing rule OBJ, where we ignore the translation part (as no translation is needed for obj):

\[ \text{OBJ} \]

\[
\Delta; \Gamma \vdash \text{obj}(C, \Pi) \Rightarrow D \rightsquigarrow _
\]

- Case NEW2OBJ. Given \( \text{new } C(\Pi) \), either E-CONGR or E-NEW apply.

  (1) If E-CONGR applies, then there exists some e in Π such that e → e'. By the induction hypothesis, if Δ;· ⊢ e : T₁ ⊑ _, then Δ;· ⊢ e' : T₁ ⊑ _. Consequently, it must be the case that if Δ;· ⊢ \text{new } C(\Pi) : T ⊑ _, then Δ;· ⊢ \text{new } C(\Pi') : T ⊑ _.

  (2) If E-NEW, then the conclusion follows from the typing rule OBJ above.

\[ \square \]

### B.2 Proof for Theorem 4.2

**PROOF.** By rule induction on the given typing derivation.

- Case SEL2APP. Given \( e₁.f(Π₁) \), either E-CONGR applies, or \( e₁ = \text{obj}(C, Π₁) \) and \( Π₁ = Π₂ \). By E-DTR, the latter scenario evaluates to [this \( \mapsto \text{obj}(C, Π₁), \bar{\gamma} \mapsto \bar{\eta}, \bar{x} \mapsto \bar{y} \)] \( e₃ \), where

  \[ \text{dtrBody}(f, C) = (Π₁, Π₂, e₃) \].

  By the induction hypothesis and standard substitution rules

  \[ \text{this \( \mapsto \text{obj}(C, Π₁), \bar{\gamma} \mapsto \bar{\eta}, \bar{x} \mapsto \bar{y} \)} \text{either evaluates to } e₃', \text{or it’s a value}. \]

- Case APP2SEL. Given \( f(e₁)(Π₂) \), either E-CONGR applies, or \( e₁ = \text{obj}(C, Π₁) \) and \( Π₁ = Π₂ \). By E-CSM, the latter scenario evaluates to [sel₁f \( \mapsto \text{obj}(C, Π₁), \bar{\gamma} \mapsto \bar{\eta}, \bar{x} \mapsto \bar{y} \)] \( e₃ \), where

  \[ \text{csmBody}(f, C) = (Π₁, Π₂, e₃) \].

  By the induction hypothesis and standard substitution rules

  \[ \text{sel₁f \( \mapsto \text{obj}(C, Π₁), \bar{\gamma} \mapsto \bar{\eta}, \bar{x} \mapsto \bar{y} \)} \text{either evaluates to } e₃', \text{or it’s a value}. \]

- Case OBJ2NEW. Given \( C(Π) \), either E-CONGR applies, or \( Π \) is Π. By E-CTR, the latter evaluates to \( \text{obj}(C, Π) \), which is a value.

- Case NEW2OBJ. Given \( \text{new } C(Π) \), either E-CONGR applies, or \( Π \) is Π. By E-NEW, the latter evaluates to \( \text{obj}(C, Π) \), which is a value.

\[ \square \]

### B.3 Proof for Theorem 4.3

**PROOF.** By induction.

\[
\text{Obj2NEW}
\]

\[
\text{Sig}(C) = \text{T} \rightarrow D \quad C \in \text{CTR}(D) \quad \Delta; \Gamma \vdash e \Rightarrow \text{T} \rightsquigarrow e' \\
\Delta; \Gamma \vdash C(\Pi) \Rightarrow D \rightsquigarrow \text{new } C(e')
\]

\[
\text{Sig}(C') = \text{T} \rightarrow D \text{ and } C \in \text{GEN}(D) \quad \text{By Lemma B.1}
\]

\[
\Delta; \Gamma \vdash e' \Rightarrow \text{T} \rightsquigarrow e \quad \text{By i.h.}
\]

\[
\Delta; \Gamma \vdash \text{new } C(e') \Rightarrow D \rightsquigarrow C(\Pi) \quad \text{By NEW2OBJ}
\]
Theorem 4.4

Regarding the proof for condition (2), if program $P$ does not diverge, we must have $e_1 \rightarrow^* v_1$ and $\overline{v}_2 \rightarrow^* \overline{v}_2$ (for the latter, we abuse the notation to mean that each individual $v_2$ evaluates to a corresponding $\overline{v}_2$). Then, by induction hypothesis $e'_1 \rightarrow^* v_1$ and $\overline{e}_2' \rightarrow^* \overline{v}_2$. The conclusion follows from Lemma B.7 and rules E-Csm and E-Dtr in Figure 10.

Case OBJ2NEW. Given that $P$ does not diverge, we must have $\overline{e} \rightarrow^* \overline{v}_1$. Then, by induction hypothesis $\overline{e}' \rightarrow^* \overline{v}_1$. From rules E-CTR and E-NEW in Figure 10, it follows that both $C(\overline{v}_1)$ and new $C(\overline{v}_1)$ evaluate to the same value.

Case NEW2OBJ. Follows exactly the same pattern as the case for OBJ2NEW.

Regarding the proof for condition (2), if program $P$ can diverge, then let $T$ be an infinite trace of $P$. Given the rules in Figure 10 and the fact $P$ contains a finite number of instructions, one of the following situations must happen:
(i) There exists a method invocation $e_1.f(\overline{e_2})$ that appears an infinite number of times in $T$. According to $\text{Sel2App}$, each method invocation $e_1.f(\overline{e_2})$ is translated to a corresponding function application $f(e_1')(\overline{e_2'})$ in $P'$. By the induction hypothesis, we have that if $e_1 \rightarrow^* v_1$ and $\overline{e_2} \rightarrow^* \overline{v_2}$, then $e_1' \rightarrow^* v_1$ and $\overline{e_2'} \rightarrow^* \overline{v_2}$. Then, the execution of $P'$ must contain an infinite number of function applications $f(e_1')(\overline{e_2'})$.

(ii) There exists a function application $f(e_1)(\overline{e_2})$ that appears an infinite number of times in $T$. According to $\text{App2Sel}$, each function application $f(e_1)(\overline{e_2})$ is translated to a corresponding method invocation $e_1'.f(\overline{e_2'})$ in $P'$. By the induction hypothesis, we have that if $e_1 \rightarrow^* v_1$ and $\overline{e_2} \rightarrow^* \overline{v_2}$, then $e_1' \rightarrow^* v_1$ and $\overline{e_2'} \rightarrow^* \overline{v_2}$. Then, the execution of $P'$ must contain an infinite number of method invocations $e_1'.f(\overline{e_2'})$.

$\square$