Pressure Equalization Simulation Method for Docking Process

Guoxiang Li, Liping Pang*, Yufeng Fang, Tong Zheng and Hongquan Qu

ABSTRACT

Pressure equalization process will occur when a space ship docking to a space station through a nonlinear Pressure Equalization Assembly (PEA). The real-time simulation technology has great significance to understand the pressure equalization process and to test the working performance of PEA rapidly. However, the steady-oscillation will emerge when the Euler method is used. This paper firstly established a one-order nonlinear material flow model to study the docking process and then proposed a double-model method to simulate the pressure equalization process correctly. The double-model includes a Nonlinear Equalization (NR) model and an Approximate Linear Equalization (ALR) model. Based on the stability analysis theory, the mass flow can be divided into two intervals by the minimum stable mass flow, \( w_{\min} \) and the NR model and ALR model will be applied in \([w_{\min}, w_{\max}]\) and \([0, w_{\min}]\), separately. The simulation results show that the steady-state performance of docking process can be improved greatly by using the presented method.

KEYWORD: Docking process; pressure equalization; real-time simulation; Euler method; steady-oscillation

INSTRUCTION

Air material will be transferred through the PEA in a pressure equalization process until its docking pressure difference is close to zero (Zhang, X. et al. 2014, Zhang, Y.W. et al. 2012). The pressure and mass flow models for two docking process can be treated as a one-order nonlinear system due to the nonlinear feature of the PEA.

Guoxiang Li, Liping Pang*, Yufeng Fang. School of Aviation Science and Engineering, Beihang University, Beijing 100191, China
Tong Zheng, Hongquan Qu. College of Information Engineering, North China University of Technology, Beijing 100041, China
Simulation technology can help designers understand the real system and control its working performance (Ma et al. 2012). In order to improve the computational efficiency, the Euler method with fixed time step is usually used. However, the steady-state oscillation will emerge for a nonlinear system. Some numerical methods should be studied further in order to overcome the distortion of steady-state performance and guarantee stable numerical outcomes for some nonlinear systems (Pavlov et al. 2013, Hilscher 2012).

Due to a specific application, this paper uses the explicit Euler solver to simulate the nonlinear pressure equalization process. First, the analytic method of stability for linear system is borrowed to reveal the fundamental reason for the steady-state oscillation. Second, the pressure equalization process of docking spacecraft is proved to have a characteristic of $2^n$ periodic.

In order to overcome the distortion of steady-state performance and guarantee stable numerical results, a simulation method based on double models is presented in this paper. This research may have a meaning to understand steady-state oscillation in a one-order nonlinear system, and provide a reference to solve this problem by only using a fixed time-step Euler solver for some nonlinear systems.

**MATERIAL FLOW MODEL FOR DOCKING PROCESS**

The pressure equalization process between space station and spaceship is shown in Figure 1, where $P_1$ and $P_2$ represent the air pressures in Cabin 1 and Cabin 2, which represent the space station and the spaceship, separately.

![Figure 1. Pressure equalization for a space station and a spaceship.](image)

A nonlinear PEA is used in the docking process. Its material flow can be expressed in Equation 1:

$$P_2 - P_1 = R_{nl} w^2$$

where the subscripts, 1 and 2, represent Cabin 1 and Cabin 2, separately; $P$ is the air pressure, Pa; $w$ is the mass flow, kg/s; $R_{nl}$ is the friction coefficient of the nonlinear flow component, Pa/(kg/s)$^2$.

The air is assumed as the ideal gas which obeys the ideal-gas equation:

$$PV = mR_g T$$

where $m$ is the cabin air mass, kg; $T$ is the cabin temperature, K; $V$ is the cabin volume, m$^3$; $R_g$ is the air constant, $R_g=296.8$ J/(kg $\cdot$ K).

Assume the cabin temperature changes close to zero during the pressure equalization process, and the changes of cabin air pressure can be obtained:
where \( c \) is the volume coefficient, \( c = V/(R_g T) \), kg/Pa; \( t \) is time, s.

One-order nonlinear equation can be obtained:

\[
\frac{dP_1}{dt} = -\frac{1}{c_1 \sqrt{R_g}} \left[ \frac{P_2 - P_1}{R_g} + \frac{c_1}{c_2} P_1 - \frac{c_1}{c_2} P_1 \right]
\]

(4a)

\[
P_2 = P_{20} + \frac{c_1}{c_2} P_{10} - \frac{c_1}{c_2} P_1
\]

(4b)

where \( P_{10} \) and \( P_{20} \) represent the original cabin pressures of Cabin 1 and 2, separately.

From Equation 4, it is clear that the relationship of the pressure change models during the docking process is nonlinear. Due to the linear correlation of \( P_1 \) and \( P_2 \), the characteristic of docking system can degenerate from a two-order nonlinear system to a one-order nonlinear one.

SIMULATION FOR DOCKING PROCESS

It is easy to analyze the system stability for a linear system. However, for a nonlinear system, its stability and steady-state performance should be analyzed together, such as fixed-point, cycle-orbit and chaos. In the following section, the stability analysis of the numerical solution method is discussed in detail firstly. Then the steady-state performance of nonlinear pressure equalization process is studied.

Stability Analysis Of Numerical Solution Method

Unlike a continuous system, the convergent region of any discrete system is finite by using the numerical solution method (André, 2012). For example, when the Euler method is used to solve a linear continuous system, \( \frac{dy}{dx} = \lambda f(x, y) \). In order to ensure the system stability, the eigenvalues and time step of the system need to meet Equation 5.

\[-2 < |\lambda h| < 0 \]

(5)

For a nonlinear system, the implicit numerical solving methods are often used to ensure the system stability. However, implicit methods consume large computational time, especially for a huge system. As the convergent region of the Euler method is the subset of the other numerical methods, the explicit Euler method with fixed time step is adopted in the following numerical study.
Steady-state Oscillation Analysis

In order to control the eigenvalue within the convergent region by using Euler method, the characteristic of the eigenvalue is studied first. Then a stable simulation method is established.

The eigenvalue of Equation 4 can be expressed in Equation 6:

\[ \lambda = -\frac{c_1 + c_2}{2R_{nl}c_1c_2w} \]  

(6)

As the parameters are fixed, such as \( R_{nl}, c_1 \) and \( c_2 \), the changes of mass flow in system is studied in detail. The minimum value of mass flow can be obtained with Equation 5:

\[ w_{\text{min}} = \frac{h(c_1 + c_2)}{4R_{nl}c_1c_2} \]  

(7)

Hence, the docking mass flow can be divided into two intervals, \([w_{\text{min}}, w_{\text{max}}]\) and \([0, w_{\text{min}}]\). When the mass flow is in the interval \([w_{\text{min}}, w_{\text{max}}]\), the discrete system is stable. Otherwise, it is steady-state oscillation. The fundamental reason of oscillation is studied carefully in the following part.

The pressure equalization model is shown as Equation 8:

\[ f(t_n,y_n) = A \left[ \frac{1}{D}(B - Cy_n) \right]^{1/2} \]  

(8)

where \( A = 1/c_1; B = P_{20} + c_1/c_2P_{10}; C = c_1/c_2 + 1; D = R_{nl} \).

We set up the following two definitions to study the above question.

**Definition 1.** For arbitrary nonlinear discrete system, let \( x_0 \in M \), there is an equation of iterated, \( g(x_0) = x_0, g^k(x_0) \neq x_0 \), where \( k \) which is a natural number is smaller than \( n \), we say that \( x_0 \) is a \( n \)-cycle-point of \( g(x_n) \).

**Definition 2.** Suppose \( x_0 \) is a \( n \)-cycle point of \( g(x) \), if \( |dg^n(x_0)/dx|<1 \), we will say that \( x_0 \) is an attractive cycle-point; if \( |dg^n(x_0)/dx|>1 \), we will say that \( x_0 \) is an excluded cycle-point.

According the Def.1, 2-cycle-points of the system can be obtained, that is, \( y_1 = a_1 \) and \( y_2 = a_2 \).

\[ a_1 = \frac{B}{C} \frac{A^2h^2}{8D} - \frac{1}{8C} \sqrt{\left( \frac{8B}{D} - \frac{A^2Ch^2}{D} \right)^2 - 16 \left( \frac{4B^2 - A^2BC^2h}{D} \right)} \]  

\[ a_2 = \frac{B}{C} \frac{A^2h^2}{8D} - \frac{1}{8C} \sqrt{\left( \frac{8B}{D} - \frac{A^2Ch^2}{D} \right)^2 + 16 \left( \frac{4B^2 - A^2BC^2h}{D} \right)} \]

As \( |df^2(a_1)/dy_n|<1, |df^2(a_2)/dy_n|<1 \). According to Def.2, \( a_1 \) and \( a_2 \) are attractive cycle-points. So the simulation result may have three cases with the different initial values, \( y_i \), as shown in Figure 3.

\[ a_1 \quad a_{\text{true}} \quad a_2 \]

Figure 3. Three regions of original value.
(1) When $y_i$ is set in the region I, $y_i < a_1$, the system value, $y_n$, will firstly arrive at $a_1$, and then oscillate between $a_1$ and $a_2$ periodically.

(2) When $y_i$ is set in the region II, $y_i > a_2$, $y_n$ will firstly arrive at $a_2$, and then oscillate between $a_1$ and $a_2$ periodically.

(3) When $y_i$ is set in the region III, $a_1 < y_i < a_2$, and $y_n \neq a_{true}$, $y_n$ will finally oscillate between $a_1$ and $a_2$ periodically.

**Pressure Equalization Simulation With Euler Solver**

The simulation conditions of pressure equalization are given in Table 1. The initial temperatures of two docking spacecraft are both equal to 20°C and the time step is 0.1s, so $w_{min}=0.00145$kg/s.

| Setting          | Cabin 1 | Cabin 2 | Unit   |
|------------------|---------|---------|--------|
| Volume           | 119     | 40      | m³     |
| Volume Coefficient | 0.0014 | 4.5953×10^{-4} | kg/Pa  |
| Friction Coefficient | 10^4  | 10^4   | Pa/(kg/s)^2 |

Solve Equation 4a by fixed time-step Euler method, Equation 12 is obtained.

$$y_{n+1} = g(y_n) = y_n + 703 \times \left( \frac{4.0566e5 - 4.0466y_n}{1 \times 10^4} \right)^{1/2}$$

(9)

According to Def.1, we can get two 2-cycle-points of this system, $a_1=100246.62$ and $a_2=100247.62$.

![Figure 4. Change of space station pressure.](image)

The change of space station pressure is shown in Figure 4. Analyzing Figure 4, we can find that the pressure emerge oscillation between $a_1$ and $a_2$ when the system is near its steady state. According to Def.2, as $|g^{(2)}(a_1)|=0.0249<1$, $|g^{(2)}(a_2)|=0.0250<1$, $a_1$ and $a_2$ are attractive cycle-points of this system. Hence, simulation results of the pressure change with the different initial values are shown in Figure 5.

![Figure 5. Simulation results of pressure change with different original values.](image)
DOUBLE MODEL METHOD

Method Principle

In order to overcome the steady-state oscillation problem in the nonlinear system, a double-model method is presented in this paper.

1. The minimum mass flow, \( w_{\text{min}} \), can be obtained by Equation 7. Then the mass flow is divided into two intervals, \([w_{\text{min}}, w_{\text{max}}]\) and \([0, w_{\text{min}}]\).

2. Double models which compose of NR model and ALR model is set up and applied in \([w_{\text{min}}, w_{\text{max}}]\) and \([0, w_{\text{min}}]\), separately.

ALR Model

The ALR model of the system follows:

\[
\frac{dP}{dt} = \frac{1}{c_1 R_l} \left[ P_{20} + \frac{c_1}{c_2} P_{10} - \left( \frac{c_1}{c_2} + 1 \right) P_1 \right]
\]

where \( R_l \) is the friction coefficient of linear flow component, Pa/(kg/s)^2.

The friction coefficient of ALR can be expressed in Equation 11.

\[
R_l = R_w \frac{w}{c}
\]

The eigenvalue of the linear system is \( \lambda = -\frac{(c_1 + c_2)}{2 c_1 c_2 R_l} \). In order to keep the system convergent, the friction coefficient of ALR should satisfy:

\[
R_l > \frac{(c_1 + c_2) h}{2 c_1 c_2} = 289.04
\]

Hence, the friction coefficient of the linear component is assumed as 290Pa/(kg/s)^2 when ALR model is used.

Comparison Of Double Model And Single Model

The simulation results of the ode45 solver in Simulink can represent the approximately real solutions (Zhu et al. 2012). The following section gives some simulation comparisons between the double-model and the original model. The simulation of the double-model and the original model are obtained by using the fixed time-step Euler solver and the variable time-step ode45 solver, as shown in Figure 7.

From Figure 7, we can find that the simulation result of double model method is stable. Compared with Figure 4, steady-state performance of this nonlinear system is improved very well, and the simulation result of the double model method is close to the approximately real solution.
CONCLUSIONS

A pressure equalization process of the docking spacecraft has one-order nonlinear feature. In order to simulate this nonlinear system in real-time simulator, the fixed time-step method is forced to use. However, steady-state oscillation occurs when the fixed time-step solver is used in this nonlinear system. In order to overcome the drawback, we conduct the following study:

1. The steady-state oscillation of docking process is analyzed by using the methods including the linear stability analysis and the steady-state oscillation analysis for one-order discrete nonlinear system. Nonlinear characteristic leads to its eigenvalue and mass flow having an opposite relation. The eigenvalue will approach to infinity and won’t satisfy the steady condition when the mass flow tends to be zero. Hence, we can get a minimum mass flow, $w_{min}$, which ensures the system stability.

2. In order to ensure the simulation performance when the mass flow is lower than $w_{min}$, we put forward a pressure equalization simulation method based double models, which includes a NR model and an ALR model. If the mass flow is larger than $w_{min}$, the NR model is used; otherwise, the ALR model is used.

3. In order to compare the simulation performance, we compare the simulation results by using two kind of numerical solvers, ode1 with fixed time step and ode45 with variable time step. The latter result can represent the approximately real solutions.

Above all, the study in this paper can provide a way to understand the steady-state oscillation of one-order system and put a reference to solve the problem of steady-state oscillation for a nonlinear system by using a fixed time-step Euler numerical method.

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