Axisymmetric Bondi–Hoyle accretion on to a Schwarzschild black hole: shock cone vibrations

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ABSTRACT

We study numerically the axisymmetric relativistic Bondi–Hoyle accretion of a supersonic ideal gas on to a fixed Schwarzschild background space–time described with horizon penetrating coordinates. We verify that a nearly stationary shock cone forms and that the properties of the shock cone are consistent with previous results in Newtonian gravity and former relativistic studies. The fact that the evolution of the gas is tracked on a spatial domain that contains a portion of the inner part of the black hole avoids the need to impose boundary conditions on a time-like boundary as done in the past. Thus, our approach contributes to the solution to the Bondi–Hoyle accretion problem at the length-scale of the accretor in the sense that the gas is genuinely entering the black hole. As an astrophysical application, we study for a set of particular physical parameters, the spectrum of the shock cone vibrations and their potential association with Quasi Periodic Oscillations sources.

Key words: Accretion, Accretion discs – black hole physics – hydrodynamics – shock waves.

1 INTRODUCTION

Bondi–Hoyle accretion is the process of evolution of an infinite uniform wind of gas moving near a central object, or conversely a compact object moving on a uniform gas with constant velocity (Bondi & Hoyle 1944). This process is associated with various phenomena related to gas processes near stars or compact objects. It shows interesting properties when considered within the Newtonian and relativistic regimes, which have been explored based on several numerical studies in the classical regime ruled by Newtonian gravity (Shima et al. 1985; Matsuda, Inoue & Sawada 1987; Fryxell & Taam 1988; Sawada et al. 1989; Mastsuda et al. 1991, 1992; Ruffert & Arnett 1994; Ruffert 1994a,b, 1996, 1997; Benensohn 1997; Nagae et al. 2004; Blondin & Raymer 2012), in which the most important subject are the consequences on the morphology of the wind and the supersonic shocks that develop; a summary of results under Newtonian gravity can be found in Foglizzo, Galleti & Ruffert (2005).

Unlike in the Newtonian regime, the relativistic approach allows the study of the Bondi–Hoyle accretion in regions where the gravitational field is strong, for instance, near the event horizon of a black hole. Some studies have been pushed forward in this direction. The first one was performed by Petrich et al. (1989); in this work they carry out axially symmetric numerical simulations in order to study the different accretion patterns developed by the gas during the accretion on to a black hole. Later on, Font & Ibáñez (1998a,b) and Font et al. (1998, 1999) revisited the results using high-resolution shock capturing schemes that have shown to be much more accurate methods than those used in the past consisting in the addition of artificial viscosity. It is worth pointing out that in all these papers the morphology of subsonic and supersonics flows was the most important subject, because when the wind has supersonic velocity, a high-density shock cone forms on the opposite side of the wind source, and such cone has interesting properties like resonant oscillations or flip-flop motion. On the other hand, in the astrophysical context, a recent work by Dönmez, Zanotti & Rezzolla (2011) dedicates important efforts to study a potential relation between QPOs and the behaviour of the gas density in the relativistic Bondi–Hoyle accretion.

Considering full general relativity, Bondi–Hoyle accretion has been studied by Farris, Liu & Shapiro (2010) in the context of the merger of supermassive black hole binaries. Also axisymmetric magnetohydrodynamics Bondi–Hoyle accretion was studied in Penner et al. (2010). On the other hand, Zanotti et al. (2011) studied the case of the relativistic Bondi–Hoyle accretion considering radiative processes and the case of ultrarelativistic axisymmetric Bondi–Hoyle accretion was recently considered in Penner et al. (2012).

In this paper, we present a relativistic study of supersonic axisymmetric Bondi–Hoyle accretion on to a spherically symmetric black hole described with horizon penetrating coordinates. We assume the condition that the gas is a test fluid and does not distort the geometry of the space–time. Previous studies on the subject involve the use

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of coordinates that are singular at the event horizon, and therefore the evolution problem has to be set on a domain that requires an artificial inner boundary outside the black hole’s horizon, which requires the implementation of efficient boundary conditions there, specially when such boundary is close to the event horizon where the state variables tend to diverge, in order to prevent errors to propagate towards the numerical domain and consequently affect the numerical calculations. What the use penetrating coordinates offers is the possibility to define the inner boundary inside the black hole’s horizon and furthermore to avoid the implementation of boundary conditions there, because the light cones using penetrating coordinates remain open and material particles move naturally towards inside the black hole. In other words, the fluid is allowed to fall into the black hole. Some basic models of spherical accretion of dark matter by supermassive black holes already use this condition (Guzmán & Lora 2011a,b).

Even though our approach represents an improvement to the complete study of Bondi–Hoyle accretion, it is only an alternative to solve the problem at the inner boundary related to the length-scale of the accretor. At a different scale, the accretion radius is defined to approximately decide when a particle falls into the compact object or not, and such scale is determined by the velocity of the wind and the equation of state of the gas. Traditionally, the Bondi–Hoyle problem is attended in this second scale, by assuming that the accretor is point-like, whereas in our case we are exactly doing the opposite. Numerically it seems that one has to choose between these two regimes, that is, it is possible to apply wind accretion to astrophysical processes if realistic wind velocities are considered, and consequently, the accretor length-scale is small enough as to be considered a point-like source that cannot be numerically resolved; conversely, if one wants to resolve the accretor (like in our case) the accretion radius length-scale is unresolved unless the velocity of the wind is assumed to be high. This difficulty of studying both scales at the same time is the main reason why the relativistic wind accretion has not been fully resolved at the moment.

The association of numerical results on Bondi–Hoyle accretion to black holes to astrophysical observations involves a series of parameters that in principle should be astrophysically motivated. For instance, the adiabatic index and internal energy of the gas, and the velocity of the wind if one wants to infer the mass and angular momentum of the black hole, or vice versa; furthermore, properties of the gas associated with heat transfer of cooling processes expected to happen would involve an extra set of parameters that should be observationally motivated. Perhaps the most solid bounds among all these parameters is the relative velocity between the accretor and the wind, which is of the order of 100–1000 km s$^{-1}$ in binaries and may achieve from a few thousands of km s$^{-1}$ (González et al. 2007) in superkicked black holes resulting from the collision of two black holes with collimated emission of gravitational radiation up to very recent results indicating 15 000 km s$^{-1}$ (Sperhake et al. 2011) in similar scenarios.

The paper is organized as follows. In Section 2, we define the relativistic Euler equations describing the fluid on to a fixed Schwarzschild black hole in Eddington–Finkelstein coordinates, whereas in Section 3, we explain the numerical methods we use to solve the relativistic Euler equations on a curved background space–time; in Section 4, we show the morphology and mass accretion rates for the accretion of an axisymmetric homogenous wind and in Section 5, we analyse the frequencies associated with the rest-mass density oscillations inside the shock cone. Finally, in Section 6, we present our conclusions.

## 2 Relativistic Hydrodynamic Equations

For a generic space–time, relativistic Euler equations can be derived from the local conservation of the rest mass and the stress–energy tensor $T_{\mu\nu}$ of the fluid model considered:

\[
\nabla_\nu(T^{\mu\nu}) = 0, \\
\nabla_\nu(\rho u^\nu) = 0, \\
\]

where $\rho$ is the rest-mass density, $u^\nu$ is the four-velocity of the fluid and $\nabla_\nu$ is the covariant derivative consistent with the four-metric $g_{\mu\nu}$ of the space–time (Misner, Thorne & Wheeler 1973). Notice that our calculations assume geometric units where $G = c = 1$. We consider that the gas accreted by the black hole is a perfect fluid, which can be described by the following stress-energy tensor

\[
T_{\mu\nu} = \rho u^\mu u^\nu + p g_{\mu\nu},
\]

where $p$ is the pressure and $h$ the relativistic specific enthalpy given by

\[
h = 1 + \epsilon + \frac{p}{\rho},
\]

where $\epsilon$ is the rest-frame specific internal energy density of the fluid.

In order to track the evolution of the fluid, it is convenient to write down the relativistic Euler equations as flux balance laws (Martí, Ibáñez & Miralles 1991; Banyuls et al. 1997; Font et al. 2000), for which we require the space–time metric to be written in the 3+1 splitting approach of general relativity. In this paper, we consider the accretion on to a spherically symmetric black hole for which the most general line element reads

\[
ds^2 = -(\alpha^2 - \gamma_{r\theta} \beta' \beta') \, dr^2 + 2 \gamma_{r\theta} \beta' \, dr \, dt + \gamma_{\theta\theta} \, dr^2 + \gamma_{\phi\phi} (\, d\theta^2 + \sin^2 \theta \, d\phi^2),
\]

where $\alpha$ is the lapse function, the shift vector has only one component $\beta' = (\beta', 0, 0)$, $\gamma_{ij}$ are the components of the spatial metric associated with the space–like hypersurfaces and $x^i = (t, r, \theta, \phi)$ are coordinates of the space–time that we choose to be spherical in the spatial part.

A fingerprint of our paper is that we use penetrating coordinates, because the previous research uses mostly Schwarzschild coordinates, which are singular at the event horizon, and most authors implement an artificial time-like boundary near the event horizon in a region where the gravitational field is extremely strong and numerical boundary conditions in such region might not be as appropriate as in the far region. We instead use horizon penetrating Eddington–Finkelstein slices to describe the black hole, for which $\alpha$, $\beta'$ and $\gamma_{ij}$ read

\[
\alpha = \frac{1}{\sqrt{1 + \frac{2M}{r}}},
\]

\[
\beta' = \left[ \frac{2M}{r} \left( \frac{1}{1 + \frac{2M}{r}} \right), 0, 0 \right],
\]

\[
\gamma_{ij} = \begin{bmatrix}
1 + \frac{2M}{r}, r^2, r^2 \sin^2 \theta
\end{bmatrix}.
\]

Since we consider the test field approximation to be valid in our numerical experiments, there is no need to evolve the geometry of the space–time, instead we keep our background space–time fixed and
describe the evolution of the fluid on top of such space-time. The general equations obtained from (1) for a fluid with axisymmetry in spherical coordinates for metric (4) read

$$\partial_t U + \partial_\alpha (\alpha F^\alpha) + \partial_\mu (\alpha F^\mu) = \alpha S - \frac{\partial \sqrt{\gamma}}{\sqrt{\gamma}} (\alpha F^\mu).$$

(6)

Here, $U$ is a vector of conservative variables, $F^\mu$ are the fluxes in the spatial directions and $S$ is a vector of sources. These quantities are defined in terms of both the primitive variables $(\rho_0, v^\gamma, \theta, \epsilon)$ and the conservative variables themselves as follows:

$$U = \begin{bmatrix} D \\ J_r \\ J_\theta \\ \tau \end{bmatrix} = \begin{bmatrix} \rho_0 W \\ \rho_0 h W^2 v_r \\ \rho_0 h W^2 v_\theta \\ \rho_0 h W^2 - p - \rho_0 W \end{bmatrix},$$

(7)

$$F^\gamma = \begin{bmatrix} (v^\gamma - \frac{\gamma \epsilon}{\gamma - 1}) D \\ (v^\gamma - \frac{\gamma \epsilon}{\gamma - 1}) J_r + p \\ (v^\gamma - \frac{\gamma \epsilon}{\gamma - 1}) J_\theta \end{bmatrix},$$

(8)

$$F^\theta = \begin{bmatrix} \frac{v^\theta D}{\sqrt{\gamma}} \\ \frac{v^\theta J_r}{\sqrt{\gamma}} \\ \frac{v^\theta J_\theta + p}{\sqrt{\gamma}} \\ \frac{v^\theta (\tau + p)}{\sqrt{\gamma}} \end{bmatrix}.$$  

(9)

$$S = \begin{bmatrix} 0 \\ T^{\mu \nu} g_{\alpha \beta} \Gamma^\alpha_{\mu \nu} \\ T^{\mu \nu} g_{\alpha \beta} \Gamma^\beta_{\mu \nu} \\ T^{\alpha \beta} \mu_\alpha - \alpha T^{\mu \nu} \Gamma^\nu_{\mu \nu} \end{bmatrix}.$$  

(10)

In these expressions, $\gamma = \text{det}(\gamma_{ij})$ is the determinant of the spatial three-metric, $\Gamma^\nu_{\mu \sigma}$ are the Christoffel symbols of the space–time and $v^\gamma = (v^r, v^\theta, 0)$ is the three-velocity measured by a Eulerian observer and defined in terms of the spatial part of the four-velocity $u^\gamma$, as $v^\gamma = \frac{\epsilon}{\gamma} v^\gamma + \frac{\gamma - 1}{\gamma} \epsilon$, where $W$ is the Lorentz factor $W = \frac{1}{\sqrt{1 - \gamma v^\gamma v^\gamma}}$.

The system of equations (6)–(10) is a set of four equations for either the primitive variables $\rho_0, v^\gamma, \theta, \epsilon, p$ or the conservative variables $D, J_r, J_\theta, \tau$ and $p$ or $\epsilon$. Therefore, it is necessary to close the system of equations. As usual, we close the system using an equation of state relating the density or pressure, as $\rho = \rho_0 + \rho_0 (\gamma - 1) c^2 (\gamma - 1)^{-\gamma} \epsilon$. In this work, we choose the gas to obey an ideal gas equation of state given by

$$\epsilon = \frac{p}{(\Gamma - 1) \rho_0},$$

(11)

where $\Gamma$ is the ratio of specific heats or adiabatic index. The relativistic speed of sound $c_s$ plays an important role in the definition of the initial conditions of the wind of gas we evolve, and for the equation of state (11) can be written as

$$c_s^2 = \frac{p(\Gamma - 1)}{p + \rho_0 (\Gamma - 1)},$$

(12)

where its asymptotic value or its maximum permitted value is $c_{\text{max}} = \sqrt{\Gamma - 1}$.

3 INITIAL DATA AND NUMERICAL METHODS

3.1 Evolution

We evolve the system of equations (6)–(10) using a high-resolution shock capturing method (LeVeque 1992). We especially use the Harten, Lax, Leer, Einfeldt (HIDDE) Riemann solver (Harten, Lax & van Leer 1983; Einfeldt 1988) in combination with the minmod linear piecewise reconstructor. For the integration in time we used a second-order Runge–Kutta integrator.

We evolve the gas on the domain $[r_{\text{exc}}, r_{\text{max}}] \times [0, \pi] \times [0, 2\pi]$ in spherical coordinates using a two-dimensional code, and a uniformly spaced grid along coordinates $r$ and $\theta$. The radial domain deserves a careful description. On the one hand, $r_{\text{exc}}$ defines a spherical boundary that we can choose inside the black hole’s event horizon called an excision boundary originally implemented to study the evolution of black holes (Seidel & Suen 1992) and recently incorporated to the study of hydrodynamics (Hawke, Löffler & Nerozzi 2005). That is, a chunk of the domain inside the black hole horizon is removed from the numerical domain from $0 < r \leq r_{\text{exc}}$ in order to avoid the black hole singularity and the steep gradients of metric functions near there; since the light cones inside the horizon (located at $r = 2M$) point towards the singularity, there is no need to impose boundary conditions at $r = r_{\text{exc}}$, instead, the fluid simply gets off the domain through such boundary. We choose the excision radius to be $r_{\text{exc}} = 1.5M$, which is both far enough from the singularity at $r = 0$ and provides a buffer zone $1.5M < r < 2M$ for the fluid inside the horizon to flow smoothly from the horizon towards the excision boundary. This is a rather improvement compared to previous results where Schwarzschild coordinates are used and the excision boundary is chosen outside the black hole, in a time-like region that requires the implementation of boundary conditions.

On the other hand, $r_{\text{max}}$ defines an exterior spherical boundary. We distinguish between two hemispheres called downstream and upstream as in Mastusuda et al. (1987) and Font & Ibáñez (1998b). The upstream hemisphere is defined as the part of the sphere where the gas is entering the domain and the downstream is the hemisphere through which the gas leaves the domain. In the downstream hemisphere, we impose outflow boundary conditions and in the upstream boundary, we always consider the initial asymptotic value of all variables as set initially.

Another important issue is that $r_{\text{max}}$ has to be bigger than the accretion radius $r_{\text{acc}}$ (see below) if one wants shocks to form, and experience tells us that using $r_{\text{max}} \sim 10r_{\text{acc}}$ provides adequate time and space scales to see the shock cone until a nearly stationary stage.

At the axis defined by $\theta = 0, \pi$ the sources are extrapolated from the cell next to the boundary and demand $v^\gamma$ to be odd there. We also implemented an atmosphere, that is, we define a minimum value of $\rho_0$ that avoids the divergence of the specific enthalpy and errors that propagate to the other variables. We set the density of the atmosphere to $10^{-11}$, which we found to allow accuracy and consistency of our numerical results. Even though in our numerical experiments we do not measure such a small rest-mass density during the stationary regime, the implementation of the atmosphere works only to prevent the density to be small in rarefaction zones that may form before the stationary stage. Finally, since the fluxes depend both on the primitive and the conservative variables (see equations 8 and 9), we reconstruct the pressure in terms of the conservative and the other primitive variables using a Newton–Raphson algorithm.
We performed tests of our code for different radial domains and resolutions and found optimal number of cells depending on the different models, being particularly interested in maintaining a sufficiently large domain preserving the spatial resolution within a convergence regime. Also in the appendix, we present a set of more numerical standard tests for implementation.

3.2 Initial data

We set the wind to move along the direction of $\zeta$ initially, with constant density and pressure. We characterize the initial velocity field $v_i$ in terms of the asymptotic initial velocity $v_\infty$ as done in Font & Ibáñez (1998b):

\[ v_r = \frac{1}{\sqrt{\gamma}^0} v_\infty \cos \theta, \]
\[ v_\theta = -\frac{1}{\sqrt{\gamma}^0} v_\infty \sin \theta, \]

(13)

(14)

where the relation $v^2 = v_r v_\theta = v_\infty^2$ is satisfied. With these conditions, the gas initially with constant rest-mass density moves along the $\zeta$-direction filling the numerical domain.

The initial density and pressure profiles are chosen in such a way that relation (12) is satisfied. In order to do this, we fix the initial density to a constant $\rho_{ini}$ and give an asymptotic value for the sound speed $c_{ini}$, then using equation (12) the initial pressure can be obtained as follows:

\[ p_{ini} = \frac{c_{ini}^2 \rho_{ini} (\Gamma - 1)}{(\Gamma (\Gamma - 1) - c_{ini}^2 \Gamma)}. \]

(15)

A subtle point here is that we choose a value for $c_{ini}$ to construct the initial pressure, with the only condition that in order to avoid negative or zero pressures, we need the condition $c_{ini} < \sqrt{\Gamma - 1}$ to hold. Finally, using the equation of state (11) we calculate the initial internal energy $\epsilon_{ini}$.

At this point $v_\infty$ and $c_{ini}$ are two important parameters to set up the initial velocity field, and we find useful to define the relativistic Mach number at infinity in order to parametrize our initial data $M_\infty = W v_\infty / W c_{ini} = W M_\infty / W_r$, where $W$ is the Lorentz factor of the gas, $W_r$ is the Lorentz factor calculated with the speed of sound and $M_\infty$ is the asymptotic Newtonian Mach number we use to parametrize our initial configurations. When this number is bigger than 1 it is said that the flow is supersonic otherwise subsonic. In the supersonic case, a high-density shock cone forms and because the density in the cone oscillates the matter transport of such oscillations could be associated with QPO sources.

A very important scale is the accretion radius defined by

\[ r_{acc} = \frac{M}{v_\infty^2 + c_{ini}^2}, \]

(16)

otherwise all the gas in the domain would be accreted and such domain would then be inadequate to observe a shock cone. This radius is the Bondi accretion radius, where $M$ is the accretor mass, in our case a black hole of mass $M = 1$ (Petrich et al. 1989).

The parameter space to study the Bondi–Hoyle accretion is enormous and the free parameters are the initial rest-mass density of the wind $\rho_{ini}$, the value of $\Gamma$, $v_\infty$, $c_{ini}$ or equivalently the internal energy $\epsilon_{ini}$ of the gas or $M$. We decide to fix the initial rest-mass density $\rho_{ini} = 10^{-6}$ in units where the mass of the black hole is $M = 1$, which works in the test fluid approach we are assuming, although there are other precedents where a much higher density is used (Dönmez et al. 2011). We also use this density because for a range $10^{-8} < \rho_{ini} < 10^{-4}$ we find the shock cone and its oscillatory behaviour. In Table 1, we present the parameter space we explore. Another possible parameter would be the mass of the black hole, nevertheless we choose units in which $M = 1$, and automatically spatial and time measures will be in units of $M$; then our results are mass independent, and in order to specialize to a particular astrophysical case where $M$ is given in units of solar masses, we only need to rescale the units of space and time appropriately. We stress that we use parameters that allow the numerical track of the accretion process, which is restricted mainly by the size of the numerical domain; explicitly, if one assumes that either the gas or the black hole moves with speed of the order of $v_\infty \sim \sim c_{ini} \sim 100 \text{ km s}^{-1}$, then $r_{acc} / M \sim 10^2$, and then assuming the outer boundary of the numerical domain is at $r_{max} \sim 10 r_{acc}$ it would be $\sim 10^7$ times the size of the black hole radius. In the most optimistic case, when a black hole is moving with a velocity of the order of $10^3 \text{ km s}^{-1}$ as a result of a superkick as response to the collimated emission of gravitational radiation (Sperhake et al. 2011), the accretion radius would be of the order of $r_{acc} \sim 100 M$ and $r_{max} \sim 1000 M$. This is at the moment a restriction to carry out accurate numerical calculations, because on the one hand one needs a considerably big domain and on the other one needs enough resolution as to resolve the black

\[ \begin{array}{cccc}
\text{Model} & \Gamma & v_\infty & M_\infty & r_{acc} \\
M_{1a} & 4/3 & 0.5 & 5 & 3.84615 \\
M_{1b} & 5/3 & 0.5 & 5 & 3.84615 \\
M_{2a} & 4/3 & 0.4 & 4 & 5.88235 \\
M_{2b} & 5/3 & 0.4 & 4 & 5.88235 \\
M_{3a} & 4/3 & 0.3 & 3 & 10 \\
M_{3b} & 5/3 & 0.3 & 3 & 10 \\
M_{4a} & 4/3 & 0.2 & 2 & 20 \\
M_{4b} & 5/3 & 0.2 & 2 & 20 \\
M_3 & 4/3 & 0.1 & 1 & 50 \\
M_6 & 4/3 & 0.08 & 6.00926 \\
M_7 & 4/3 & 0.32 & 4 & 9.19118 \\
M_8 & 4/3 & 0.24 & 3 & 15.625 \\
M_9 & 4/3 & 0.16 & 2 & 31.25 \\
M_{10} & 4/3 & 0.08 & 1 & 78.125 \\
M_{11} & 4/3 & 0.25 & 5 & 15.3846 \\
M_{12} & 4/3 & 0.2 & 4 & 23.5294 \\
M_{13} & 4/3 & 0.15 & 3 & 40 \\
M_{14} & 4/3 & 0.1 & 2 & 80 \\
M_{15} & 4/3 & 0.05 & 1 & 200 \\
\end{array} \]
hole. What we do here is that in order to cover a numerical domain using the necessary resolution to achieve convergence, we require the domain to be smaller than for the aforementioned astrophysical scenarios. We do this by considering rather high velocities of the gas that imply smaller values of \( r_{\text{acc}} \sim 10M \) (see equation 16).

3.3 Diagnostics

In order to diagnose the physical quantities of the system in our simulations, we implement detectors located at various radii in the numerical domain, that is, we define spheres where we calculate scalar quantities. In particular, we track the mass accretion rate as a function of time. In order to do so, we calculate the mass accretion rate on a sphere specialized to the case of a spherically symmetric space–time in spherical coordinates

\[
M_{\text{acc}} = -2\pi \int Dr^2 \left( v - \frac{\beta v}{\alpha} \right) \sin \theta \, d\theta,
\]

(17)
at various spherical surfaces, including the black hole event horizon. The mass accretion rate helps diagnosing when the accretion has stabilized.

The most important quantity we measure is the rest-mass density of the gas along the axis inside the shock cone, that is, at \( \theta = 0 \) and at the location of the different detectors. This scalar is used to measure how the density oscillates inside the shock cone. At post-step, we perform a Fourier transform of this scalar in order to study the predominant oscillation modes of the density and study the relation between such oscillation and its potential relation to QPO sources.

4 PROPERTIES OF THE SHOCK CONE AND MASS ACCRETION RATES

Depending on the properties of the gas flow, a shock cone appears when the wind is supersonic. This shock cone is a region where the density is significantly higher than the density of the wind itself and it forms behind the black hole on the opposite side of the source of the wind.

In Fig. 1, we show the morphology for the different models we have considered in Table 1. As the parameter space is very large, we only chose two values for the adiabatic index \( \Gamma = 4/3, 5/3 \). Moreover, we only consider four initial wind supersonic velocities of the gas at infinity corresponding to 2, 3, 4, 5 Mach. We then show that the use of penetrating coordinates does not change the well-known morphology found in previous relativistic studies when using time-like internal boundaries, that is, the bigger the Mach number the smaller the shock cone angle, and also when \( \Gamma \) increases the shock cone angle increases.

In Fig. 2, the mass accretion rate for different values of the Mach number and \( \Gamma \) is presented. When the Mach number and the adiabatic index increase, the system reaches the stationary regime faster. This has been already discussed in great detail by Font & Ibáñez (1998b) and to us represents a test. On the other hand, some morphological effects of using horizon penetrating coordinates for non-axial accretion and flip-flop behaviour of the shock have been recently presented (Cruz-Osorio, Lora-Clavijo & Guzmán 2012).

Once we have described some of the properties of the shock cone, we measure the density at various points as a function of time. What happens is that the density vibrates as shown in Fig. 3. In our approach, we define the inner boundary inside the black hole where the light cones are oriented towards the black hole singularity.

Figure 1. Logarithm of the density that illustrates the morphology of the shock cones in various scenarios when the system has achieved a nearly stationary regime. In the top panel, we present the cases for the wind in models (from left to right) \( M_{4a}, M_{3a}, M_{2a} \) and \( M_{1a} \), whereas in the bottom for models \( M_{4b}, M_{3b}, M_{2b} \) and \( M_{1b} \) described in Table 1.
We show the mass accretion rate $\dot{M}_{\text{acc}}$ with respect to the proper time for different values of the Mach number and $\Gamma$. As we expect, the accretion stabilizes faster when the Mach number and $\Gamma$ increase. This quantity is calculated in a detector located at $r = 2.1M$, close to the event horizon. Despite the fact that we are calculating the mass accretion rate near the event horizon, no high oscillations show up and our measurements are accurate.

Figure 3. At the top, we show how the density oscillates in time as measured by the detector at 12.1M. In the second plot, we show a zoomed time window showing the signals as measured by various detectors equally spaced being the first one located at 2.1 and the farthest one at 16.1; the correlation among the signals measured by successive detectors can be seen; it is also shown that high-frequency modes that are measured by detectors close to the hole are not seen by farther detectors. This result is complemented with power spectra below.

5 A POSSIBLE MODEL OF QPO SOURCES

A rather appealing potential application of the vibrations of the density of the gas within the shock cone is related to the QPO type of emission. In Dönmez et al. (2011), it is found that the first work studying the possibility that the vibrations of the density within the shock cone could be associated with QPO frequencies. Some differences in our approach are that our analysis uses penetrating coordinates in the description of the black hole, whereas in Dönmez et al. (2011) singular coordinates are used, and our study is axisymmetric whereas in Dönmez et al. (2011) slab symmetry is used. With these tools, we ask whether the most powerful peaks are approximately within the range of frequencies observed for QPOs in terms of the black hole mass.

It is worth to mention that in our relativistic approach, aside of the restrictions of the realistic velocity of the wind, we do not include non-adiabatic cooling and heating processes since this is a first step towards an exhaustive and realistic analysis in the general relativistic regime truly allowing the gas entering the black hole. The first steps on that direction involving radiation terms have been recently published (Zanotti et al. 2011; Roedig, Zanotti & Alic 2012).

We now analyse the oscillations as measured at various distances from the black hole. In Fig. 3, we show various aspects of the oscillations. First, we show the oscillations as measured by a detector located at distance 12.1M in proper time; it can be seen that after an initial transient the rest-mass density approaches a nearly stationary regime. Secondly, we show a time window showing the signal as measured by detectors located at various distances, ranging from 2.1M to 16.1M. From this plot, we learn that detectors closer to the black hole measure oscillations dominated by high-frequency modes, whereas detectors far from the black hole measure signals dominated by lower frequency oscillations. Moreover, this plot shows that signals among detectors are well correlated and that we are not measuring only numerical noise. We support our calculations with the a self-convergence test of the density in Fig. 4.

In order to interpret the oscillations of the density, we perform a Fourier transform of the signals measured by different detectors like those in Fig. 3 using the proper time at the location of each detector. The results appear in Fig. 5, where the high-frequency modes are shown to dominate near the black hole and do not appear far from the black hole. We concentrate in modes that may be global and detected in a big part of the domain as suggested by the analysis in Dönmez et al. (2011). As a particular case, we point out with an arrow in Fig. 5 a frequency peak that appears in all the detectors. That it appears in all our detectors is an indication that the corresponding mode is global and this is the reason why we choose it to correspond to the frequency in our analysis. In all the cases, in our parameter space the spectra show the presence of such mode. This is the reason why in all the cases studied we consider the spectrum as measured with a detector located at $r = 12.1M$ where the spectrum is very clean and the most powerful peak corresponds precisely to this mode.
Self-convergence of the $L_2$ norm of the density along the positive $c$-axis, where the shock cone forms. The ratio among successive resolutions is 1.5, and the convergence factor is defined by 1.5$^5$. The accuracy of our algorithms is expected to be first order due to the development of shocks.

In Fig. 4, we show the frequencies of oscillation of the rest-mass density in the shock cone in terms of the wind velocity for different values of the speed of sound.

In Fig. 5, we show the frequency window we find in all our simulations with observational results. We ask about the value of the speed of sound $c_\infty$ that a black hole exists, e.g Cyg X-1 (Liu, Bregman & Seitzer 2004). As we can see, this figure predicts a variety of possible configurations for the observed frequencies. For example, the allowed mass of the black hole for various values of $c_\infty$. For each value of the speed of sound (each plot in Fig. 7), we see that the bigger the velocity of the wind the bigger the frequency allowed.

In order to compare with observations, we include in Fig. 7 the masses and frequencies corresponding to the case of Sgr A$^*$, where the mass of the supermassive black hole is $M \approx 4.1 \pm 0.6 \times 10^6 M_\odot$ (Ghez et al. 2008) and the frequency range was the one reported in Aschenbach et al. (2004) and Abramowicz et al. (2004). We also show a shadowed region corresponding to the range of masses and frequencies associated with high-mass X-ray binaries (HMXBs).

Based on our numerical results in Fig. 7, we notice that for the range of velocities analysed, for the model with $\Gamma = 4/3$ and various wind velocities, the frequencies of the density oscillations and mass of the black hole for Sgr A$^*$ lie partially within that of observations. The trend observed between panels in Fig. 7 indicates that when $c_\infty$ increases, a bigger range of our parameters contain Sgr A$^*$.

Fig. 7 also shows a shadowed box corresponding to QPOs between 1 and 400 mHz, observed in the spectra of HMXBs. Some of these measurements are associated with regions where it is believed that a black hole exists, e.g Cyg X-1 (Liu, Bregman & Seitzer 2004). As we can see, this figure predicts a variety of possible configurations for the observed frequencies. For example, the allowed mass of the black hole that is hosted in the observed regions can run from hundreds to millions of solar masses for all the models presented here.

The vertical dashed line in Fig. 7 corresponds to another QPO source of 1.27 Hz (Kaur et al. 2007). Consistent black hole masses would be of about $10^5$ to $10^6 M_\odot$.

An exhaustive further study of the parameter space, would include information on the equation of state, additional conditions involving non-adiabatic and radiative processes like in Zanotti et al. (2011) and Roedig et al. (2012) in more realistic models. However, it would be interesting as a first step the use of realistic wind
Figure 7. We summarize our study of the parameter space. From top to bottom, the speed of sound used is $c_s = 0.1$, 0.08, 0.05. We also show a vertical line indicating a special frequency of 1.27 Hz associated with HMXBs. We also indicate a shadowed region corresponding to HMXBs in the range 1–400 mHz. The black box corresponds to the black hole mass and frequency range observed in Sgr A*.

**6 CONCLUSIONS AND DISCUSSION**

We have studied numerically the axisymmetric relativistic Bondi–Hoyle accretion of a supersonic ideal gas on to a fixed Schwarzschild background space–time described using horizon penetrating coordinates. Our results reproduce the morphology of the distribution of the rest-mass density of the gas obtained in the Newtonian regime and in previous relativistic studies, and verify that a stable shock cone forms.

With our approach, we measured the consistency in the rest-mass density within the shock cone, which is an indication that the excision implemented inside the black hole works fine with accuracy and convergence. Our treatment contributes to the solution of the Bondi–Hoyle accretion problem at the scale of the accretor scalelength. This will provide a better understanding of the gas dynamics near the black hole. The restriction of this local type of study in astrophysical scenarios at the moment is that one needs the use of a small accretion radius in order to cover a sufficiently large numerical domain, which in turn implies that the velocities that can be studied are higher than those observed or estimated by theoretical models.

One of the properties of the accretion of a relativistic supersonic wind is that the density within the shock cone vibrates. We explore the possibility that such vibrations show frequencies within the range of those observed in QPO sources. Our approach is the first one for axisymmetric flows, in the relativistic regime, on a space–time corresponding to a black hole, which additionally is described in coordinates that truly allow the gas flow inside the black hole horizon.

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**REFERENCES**

Abramowicz M. A., Kluźniak W., McClintock J. E., Remillard R. A., 2004, ApJ, 609, L63
Aschenbach B., Grosso N., Porquet D., Predehl P., 2004, A&A, 417, 17
Benensohn J. S., Lamb D. Q., Taam R. E., 1997, ApJ, 478, 723
Blondin J. M., Rayner E., 2012, ApJ, 752, 30
Bondi H., Hoyle F., 1944, MNRAS, 104, 273
Cruz-Osorio A., Guzmán F. S., Lora-Clavijo F. D., 2011, J. Cosmol. Astropart. Phys., 06, 029
Cruz-Osorio A., Lora-Clavijo F. D., Guzmán F. S., 2012, MNRAS, 426, 732
Del Zanna L., Bucciantini N., 2002, A&A, 390, 1177
Del Zanna L., Zanotti O., Bucciantini N., Londrillo P., 2007, A&A, 473, 11
Donat R., Font J. A., Ibáñez J. M., Marquina A., 1998, J. Comp. Phys., 146, 58
Domínguez O., Zanotti O., Rezzolla L., 2011, MNRAS, 412, 1659
Einfeldt B., 1988, SIAM J. Numer. Anal., 25, 294
Farris B. D., Liu T. Y., Shapiro S. L., 2010, Phys. Rev. D, 81, 084008
Foglizzo T., Galletti P., Ruffert M., 2005, A&A, 435, 397
Font J. A., Ibáñez J. M., 1998a, MNRAS, 298, 835
Font J. A., Ibáñez J. M., 1998b, ApJ, 494, 297
Font J. A., Ibáñez J. M., Papadopoulos P., 1998, ApJ, 507, L67
Font J. A., Ibáñez J. M., Papadopoulos P., 1999, MNRAS, 305, 920
Font J. A., Miller M., Suen W-M., Tobias M., 2000, Phys. Rev. D, 61, 044011
Fryxell B. A., Taam R. E., 1988, ApJ, 335, 862
Ghez A. M. et al., 2008, ApJ, 689, 1044
Gonzalez J. A., Hannam M. D., Sperhake U., Bruegmann B., Husa S., 2007, Phys. Rev. Lett., 98, 231101
We show the gas variables at time \( t = 0.6 \) and the exact solution. As a global convergence test involves regions where constant states are involved, the velocity of the shock and the shell of density at the front shock are important quantities to monitor. We resolve the shock with only a few points and the velocity at the time of the snapshot is very accurate. We use for this test 1000 cells and \( \Gamma = 5/3 \).

**Figure A1.** We show the gas variables at time \( t = 0.6 \) and the exact solution. As a global convergence test involves regions where constant states are involved, the velocity of the shock and the shell of density at the front shock are important quantities to monitor. We resolve the shock with only a few points and the velocity at the time of the snapshot is very accurate. We use for this test 1000 cells and \( \Gamma = 5/3 \).
Figure A2. This test shows that our implementation is capable to track the evolution of velocities that correspond to Lorentz factors of the order of $\sim 1000$. The snapshot is taken at $t = 0.7$. In this test, we use $\Gamma = 4/3$. Again, we use 1000 cells.

Table A1. Initial data for the 2D shock.

| Type            | Top-left | Top-right |
|-----------------|----------|-----------|
| $\rho$          | 0.1      | 0.1       |
| $p$             | 1.0      | 0.01      |
| $v^x$           | 0.99     | 0         |
| $v^y$           | 0        | 0         |

| Type            | Bottom-left | Bottom-right |
|-----------------|-------------|--------------|
| $\rho$          | 0.5         | 0.1          |
| $p$             | 1.0         | 1.0          |
| $v^x$           | 0           | 0            |
| $v^y$           | 0           | 0.99         |

Figure A3. Logarithm of the rest-mass density for the 2D relativistic Riemann problem at $t = 0.35$. We illustrate the morphology developed by the process. We use $1200 \times 1200$ cells to cover the domain.

Figure A4. Snapshots of the logarithm of the rest-mass density for the Emery wind tunnel at times $t = 1.56, 3.125, 4.69$. The step size is restricted to the domain $[-0.9, 1.5] \times [-0.5, 0.3]$. The number of cells used is $1200 \times 400$. The bow shock can be appreciated when it forms and grows (first panel), when it bounces from the upper boundary (second panel) and when it bounces again from the step (third panel).
Figure A5. Numerical and exact solution of the Michel accretion at time $t = 0.990M$. The solid line is the exact solution and the dots are the numerical solution. The last plot shows the convergence order calculated using 100, 200 and 400 cells; this shows the nearly second-order convergence of our implementation of this test. The time is in units of the black hole mass $M$.

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