Theoretical Analysis of the Variation of Hydraulic Pressure in Cracks Under Uniaxial Compression of Water-saturated Rock Pillar

Mengze YANG  
Liaoning Technical University

Houxu HUANG (✉ wuhanhp14315@163.com)  
Anhui Jianzhu University

Yu YANG  
Liaoning Technical University

Research Article

Keywords: water-saturated rock pillar, uniaxial loading, Maxwell model, tensile stress, clinical hydraulic pressure

DOI: https://doi.org/10.21203/rs.3.rs-687253/v1

License: © This work is licensed under a Creative Commons Attribution 4.0 International License. Read Full License
Abstract
In order to analyse the variation of hydraulic pressure in cracks of water-saturated rock pillar under uniaxial compression, taking the water-saturated rock pillar as the research object, in which the cracks are divided into two types: longitudinal crack and inclined crack, and the elastic-brittle plastic model is used to describe the mechanical behavior of rock. Assuming that the long axial direction of the crack is consistent with the axial direction of the rock pillar, the expression of tensile stress in the direction perpendicular to the long axial direction of the crack under axial compression is derived by using Maxwell model and Inglis formula. Simplifying the crack to flat elliptic, clinical hydraulic pressure in the case of tensile shear failure and compressive shear damage of the cracks are deduced, and the distribution of clinical hydraulic pressure in uniaxial compression cracks with different growth pattern is analysed. The results show that with the propagation of cracks, the clinical hydraulic pressure near the tip is approach to zero, and in case of hydraulic fracturing, the extension should exhibit the characteristic of discontinuity.

Keywords water-saturated rock pillar uniaxial loading Maxwell model tensile stress clinical hydraulic pressure

1 Introduction

The rock is a complexus composed of granular pore microcracks and gas and liquid, which is heterogeneous and contains a large number of fractures of different scales[1-2]. According to the researchs by Malan[3], Hoek and Martin[4], Read[5], and Martino and Chandler[6], rock damage, instability and failure are accompanied by the expansion and growth of cracks. When subjected to uniaxial loading, the brittle rock sample quickly splits into upper and lower parts, forming a new cross-section, which is parallel to the sample axis. However, the newly formed surface is rough and have no shear characteristics, which indicates that the splitting phenomenon is caused by tensile stress.

The rock in the karst tunnel contains in a water-bearing state. When subjected in the natural state, the water inside the rock is not enough to expand the cracks. The pressure on the rock plays an essential role in the expansion of the cracks. Due to the environment of rock in karst tunnel, the water-bearing cracks grow continuously during excavation, and the hydraulic pressure in the cracks also changes with the expansion of the cracks. Karst tunnel construction is often threatened by water inrush, which causes great hidden danger to underground engineering construction safety. Researches on water inrush are mainly carried out from three aspects: water inrush mechanism, water inrush risk analysis and water inrush prevention and control. Wang et al.[7] studied the effects of different factors on splitting
pressure by using a self-developed hydraulic splitting device. Wu et al.\cite{8} carried out hydraulic fracturing tests of fractured rock mass under different injection flow conditions and uniaxial compressive stress, and analysed the hydraulic fracturing characteristics of fractured rock mass under different stress states. Shi et al.\cite{9} conducted a hydraulic fracturing test of rock mass under biaxial compression using a self-made device, and studied the influence of different axial pressures on the hydraulic fracturing process. Existing studies on water-rock interaction are mostly focused on hydraulic tunnels\cite{10,11}. However, different from the small-scale water seepage that may occur in the use stage of hydraulic tunnels, water inrush accidents caused by water seepage may occur in the construction process of karst tunnels. Taking the tunnel-face as an example, the common process of water inrush is as follows: primary and secondary fractures grow under the action of external forces $\rightarrow$ water seeps from the fractures $\rightarrow$ the cracks grow further $\rightarrow$ the water seepage increased further and significantly $\rightarrow$ the fractures develop into water inrush channels $\rightarrow$ water inrush disasters. This process is the result of water-rock interaction. According to the research of Yang\cite{12}, the development rule of hydraulic pressure in fractures is the basis for analysing the hydraulic splitting effect of fractures. As a consequence, we may pay attention to the variation of hydraulic pressure in cracks under uniaxial compression.

Comparing with the dry rock, the karst tunnel contains a large amount of water, and the rock exists in an aqueous environment. Under the action of high hydraulic pressure, the cracks in the rock will expand and connect, resulting in the change of the original structure of the rock. Fracture water inrush in karst tunnel is the result of the fractured rock splitted disturbed by construction and other external forces under the continuous action of karst water and hydraulic pressure\cite{13}. By analysing the variation rule of hydraulic pressure inside the crack under the action of rock compression, grasping the failure mode and failure mechanism of water-bearing crack, thus effectively preventing the occurrence of water inrush disaster in karst tunnel. In this paper, the cracks in cylindrical rock samples are divided into longitudinal cracks and inclined cracks. The elastic brittle plastic model is used to describe the mechanical behavior of the cracks. The tensile stress caused by the cracks in cylindrical rock specimen under uniaxial compression is derived by Maxwell model and Inglis formula. By simplifying the crack into a flat ellipse, the clinical hydraulic pressure of the crack under tension shear failure and compression shear failure is deduced. The expression of maximum tensile stress at crack tip is derived. By analysing the distribution of hydraulic pressure in the fracture under different fracture growth conditions, the variation rule of hydraulic pressure in the fracture under uniaxial compression is obtained.

### 2 Deduction of tensile stress on cracks based on Maxwell model

Before derivating the tensile stress on cracks, we introduce the concept of additional tension stress. Different from the elastic stress, the additional stress belongs to the inelastic stress, which exists around the crack and is the stress component caused by the crack propagation. The process of additional stress concentration and stress relaxation exist during the deformation of rock specimen, and the value of additional stress can be represented by Maxwell model under the combined action of the two processes.

$$\frac{d\Delta s_{ij}^e}{dt} = 2\rho c_s^e \varepsilon_{ij}^e - \nu \frac{\Delta s_{ij}^e}{l}$$  \hspace{1cm} (1)

where $\Delta s_{ij}^e$ represents the additional stress of the crack of size $l$; $\varepsilon_{ij}^e$ represents the deviant strain rate component corresponding to a given loading condition; $\rho$ denotes the density of the rock; $\nu$ denotes the relaxation rate of single or multiple cracks; $c_s^e$ denotes the propagation velocity of shear elastic wave; $l/\nu$ can be understood as
the relaxation time required by the crack of size $l$ in the relaxation process. To simplify the analysis, we assume that the relaxation time for all additional stresses is uniform. The first term on the right of formula (1) represents the elastic load, and the second term describes the relaxation of additional stress in the process of crack propagation.

By giving a constant strain rate, the corresponding additional stress $\Delta s^i_j$ can be expressed as:

$$\Delta s^i_j = 2 \rho c_s^2 \delta \varepsilon^j_l \left(1 - e^{-\nu \tau^i_j} \right)$$  \hfill (2)

The loading time $t$ required by macroscopic fracture of rock specimen is greater than the relaxation time $\tau$, i.e. $t > \tau^{[1]}$. Therefore, formula (2) can be expressed as:

$$\Delta s^i_j \approx 2 \rho c_s^2 \delta \varepsilon^j_l \frac{l}{D}$$  \hfill (3)

Defining the strength of the additional stress as $\Delta \sigma^i_j = \sqrt{3 \Delta s^i_j \Delta s^i_j / 2}$, and then substitute it into formula (3) to get:

$$\Delta \sigma^i_j \approx 3 \rho c_s^2 \delta \varepsilon^j_l \frac{l}{D}$$  \hfill (4)

where $\delta \varepsilon = \sqrt{2 \delta \varepsilon^i_l \delta \varepsilon^j_l}$ represents the intensity of strain rate.

Fig. 1 shows the splitting process of a cylindrical rock sample under uniaxial loading. The OA section represents the elastic deformation process of the specimen, during which the tensile stress increases gradually while the shape of the crack remains unchanged. When the stress increasing to point A, the uniaxial loading reaches the compressive strength $\sigma_c$ and the specimen suddenly splits. The sudden splitting of the specimen at point A is subjected to the joint action of stress concentration and energy concentration in elastic deformation stage OA. According to formula (4), the additional stress intensity perpendicular to the specimen axis can be expressed as:

$$\Delta \sigma^i = 3 \rho c_s^2 \delta \varepsilon \frac{D}{\nu}$$  \hfill (5)

where $D$ is the diameter of specimen. Taking granite as an example, Qi et al. obtained the relationship between the diameter $D$ of granite and the relaxation velocity $\nu(D)$ is:

$$\nu(D) = 112.59D^2 - 1403D + 6056.6$$  \hfill (6)

Substituting $D = 5$ cm into formula (6), we can get $\nu(D) = 1856$ m/s, while the loading process takes a very short time, which is only $\tau \sim 10^{-5}$ s.
Using the elastic-brittle-plastic model to describe the mechanical behavior of rock. It is assumed that elastic deformation occurs before the rock specimen reaches uniaxial compressive strength $\sigma_c$ and during the phase $0 < t < \tau$, the rock suddenly splits at $t = \tau$. In this paper, the compressive and compressive strains are defined as positive, while the tensile and tensile strains are defined as negative. Considering that $E = 2(1 + \nu) \rho c^2_s$, $\varepsilon_s = -\nu \varepsilon_s = -\nu \sigma_s / E$, formula (6) can be rewritten as:

$$\Delta \sigma_s \approx 3 \rho c^2_s \frac{D}{\nu} = 3 \rho c^2_s \varepsilon_s = -\frac{3}{2} \frac{\nu}{1 + \nu} \sigma_s \quad (7)$$

According to formula (7), tensile stress is generated in the direction perpendicular to the long axis of the crack. Ignoring the change of specimen volume in Fig. 1, the induced tensile stress $\Delta \sigma_s$ from point O to point A and its extreme values are respectively:

$$\Delta \sigma_s = -\frac{1}{2} \sigma_s \quad (8)$$
$$\Delta \sigma_{s_{\text{max}}} = -\frac{1}{2} \sigma_s \quad (9)$$

Considering that $\sigma_s \approx 8 \sigma_t$, where $\sigma_c$ and $\sigma_t$ represent the compressive strength and tensile strength respectively. Formula (9) can be further rewritten as:

$$\Delta \sigma_{s_{\text{max}}} \approx 4 \sigma_t \quad (10)$$

Formula (10) indicates that when the rock specimen reaches the uniaxial compressive strength, the induced tensile stress on the crack is almost 4 times of the tensile strength, which is sufficient to cause tensile failure of the rock sample along the direction perpendicular to the axis.

3 Deduction of tensile stress on crack based on Inglis formula

The two widely used failure criteria, Hoek-Brown criterion and Mohr-Coulomb criterion, have been proved to be inapplicable for the interpretation of rock failure\[15,16\]. However, Griffith criterion is different from Hoek-Brown criterion and Mohr-Coulomb criterion, which can research rock fracture by analysing the maximum tensile stress around the cracks\[4\].

The Griffith criterion for rock fracture is described as follows:

$$\begin{cases}
\sigma_s = -\sigma_s & (\sigma_s + 3\sigma_s < 0) \\
\frac{(\sigma_s - \sigma_s)^2}{4(\sigma_s + \sigma_s)} = 2\sigma_t & (\sigma_s + 3\sigma_s \geq 0)
\end{cases} \quad (11)$$

where $\sigma_s$ and $\sigma_s$ represent the maximum and minimum principal stress components, respectively. The description of crack cracking in rock in formula (11) includes two forms of crack cracking. One is the tensile fracture at the crack tip along the crack principal axis caused by the tensile stress. This type of fracture is caused by the stretching or unloading in the direction perpendicular to the crack principal axis (as shown in Fig.2). In this case, the crack belongs to a longitudinal crack, and its long axis is parallel to the axis of rock specimen. The other is the tensile fracture near the crack tip along the tangent direction. In this case, the maximum tensile stress near the crack tip may occur under various compression states such as uniaxial compression and biaxial compression (as shown in Fig.3). In
this case, the crack is inclined crack. According to Griffith criterion, when the cracks are stretched as shown in Fig.2 and Fig. 3, the maximum induced tensile stress acts on the tip of the longitudinal crack and the inclined crack, respectively. Under the given stress condition, the induced tensile stress is the internal reason of the failure of the specimen.

The analysis based on Maxwell model shows that under uniaxial loading, when the long axis of the crack is parallel to the axis of the cylindrical specimen, tensile stress will be generated perpendicular to the direction of the long axis of the crack. However, Griffith criterion does not include this kind of fracture. Therefore, it is necessary to study the possibility of crack propagation along its long axis under uniaxial loading.

Fig.2 The first crack propagation mode of Griffith's criterion (take one longitudinal crack as an example)

Fig.3 The second crack propagation mode of Griffith's criterion (take one slanting crack as an example)

Fig.4 Stress state of Griffith crack

According to Griffith criterion, it is assumed that the rocks are randomly distributed with slender elliptical micro-cracks. When subjected to the compression state, the maximum tensile stress around the crack acts near the crack tip. The crack propagation direction is \( \gamma = -2\beta \) or \( \gamma = \pi - 2\beta \).

As shown in Fig.4, \( \gamma \) is the included angle between the crack propagation direction and the long crack axis, \( \beta \) is the included angle between the crack short axis direction and the maximum principal stress direction \( \sigma_1 \), \( \alpha \) is the central angle of the ellipse, which is determined by the shape of the ellipse, and represents the direction of the maximum
shear stress\[^3\].

According to Griffith criterion, the change of crack inclination is the main factor leading to the change of tangential stress around the crack. In Inglis formula, the tangential stress can be expressed as:

$$
\sigma_p = \frac{\sigma_b \left[ m(m+2) \cos^2 \alpha - \sin^2 \alpha \right] + \sigma_c \left[ (1+2m) \sin^2 \alpha - m^2 \cos^2 \alpha \right] - \tau_{xy} \cdot 2(1+m)^3 \sin \alpha \cos \alpha}{m^2 \cos^2 \alpha + \sin^2 \alpha}
$$

where \( m = b/a \) denotes the ratio of the short axis \( b \) to the long axis \( a \) of an ellipse, \( a \) is infinitesimally tiny in a slender elliptic crack, so \( \sin \alpha \to \alpha \), \( \cos \alpha \to 1 \). Furthermore, formula (12) is simplified as follows:

$$
\sigma_p = \frac{\sigma_b \left[ m(m+2) - \alpha^2 \right] + \sigma_c \left[ (1+2m) \alpha^2 - m^2 \right] - \tau_{xy} \cdot 2(1+m)^2 \alpha}{m^2 + \alpha^2}
$$

Let \( \lambda = \sigma / \sigma, (\sigma \neq 0) \), and combining with formula (14):

$$
\begin{align*}
\sigma_x &= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\beta \\
\sigma_y &= \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \cos 2\beta \\
\tau_{xy} &= -\frac{\sigma_1 - \sigma_3}{2} \sin 2\beta
\end{align*}
$$

Substituting formula (14) into formula (13), and then get:

$$
\begin{align*}
\sigma_y &= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\beta \\
\sigma_x &= \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \cos 2\beta \\
\tau_{xy} &= -\frac{\sigma_1 - \sigma_3}{2} \sin 2\beta
\end{align*}
$$

Under uniaxial loading, the parameter in formula (15) is \( \beta = -\pi/2, \; \alpha = 0, \; \sigma_1 = 0, \; \lambda = 0 \). Therefore, the extreme stress value in formula (15) is:

$$
\sigma_{a} = -\sigma_1
$$

Based on the elastic-brittle-plastic model established above, during the elastic deformation process, even though the increasing of the tangential stress at the crack edge, the shape of the crack remains elongated elliptical, and the tensile stress at the longitudinal crack tip is equivalent to the axial loading value. Stress concentration and energy accumulation process (OA section) exist during the elastic deformation process, which lays down the crack at point A for sudden splitting. In addition, when the extreme value of compressive strength is reached, the extreme value in formula (16) is \( \sigma_{\text{max}} = -\sigma_c \approx -8\sigma_1 \), which is sufficient to cause the fracture of the rock.

4 Determination of clinical hydraulic pressure for water-saturated crack propagation

In order to simplify the analysis, the propagation of water-saturated cracks is considered as a plane problem. In karst tunnels, water inrush accidents are prone to occur at the tunnel face and two sides. As shown in Fig. 5, the stress diagram of crack bearing units on the tunnel face and two sides.
Observe facing straight wall

Observe facing tunnel face

Tunnel face unit containing cracks

Surface of both sides unit containing cracks

Fig. 5 Stress in fractures on the face and sides of an excavated karst tunnel

Fig. 6 Stress in the water-filled fractures on the tunnel

face

Fig. 7 Stress in the water-filled fractures on the surfaces of

both sides of the tunnel

The rock mass element shown in Fig. 6 and Fig. 7 contain an elongated elliptical crack with a long axis $2c$, the included angle between the direction of the long axis of the crack and the longitudinal direction is $\theta$, the hydraulic pressure $p$ in the fracture is uniformly distributed. Based on the stress direction specified in fracture mechanics and rock mechanics, the normal stress and tangential stress on the inner wall of the crack are respectively:

\[
\begin{align*}
\sigma_n &= \left(\frac{\sigma_x + \sigma_y}{2}\right) - \frac{\sigma_y - \sigma_x}{2}\cos2\theta - p \\
\tau &= \frac{\sigma_y - \sigma_x}{2}\sin2\theta
\end{align*}
\]

(17)

Ignoring the stress changes due to tunnel excavation. The force of rock mass element satisfies:

\[
\sigma_x = \lambda p_0, \sigma_y = \sigma_z = p_0
\]

(18)

Substituting formula (18) into formula (17), we can get:

\[
\begin{align*}
\sigma_n &= -\frac{p_0}{2}\left[(1 + \lambda) - (1 - \lambda)\cos2\theta\right] + p \\
\tau &= -\frac{p_0}{2}(1 - \lambda)\sin2\theta
\end{align*}
\]

(19)

where $\sigma_n, \tau$ represent normal stress and tangential stress of crack in palmface element, respectively. Fractures are affected by the joint action of normal and tangential direction, so the propagation of fractures can be regarded as
the I-II complex types fracture problem. According to the tension-shear failure criteria:

\[ K_{Icc} = K_I + K_{II} \]  

where \( K_{Icc} \) denotes Type I fracture toughness, which can be assumed to be an invariant constant. \( K_I \)、\( K_{II} \) represent Type I、II stress intensity factor,and:

\[ K_I = \sigma \sqrt{\pi c} , \quad K_{II} = \tau \sqrt{\pi c} \]  

Substituting formula (19) into formula (21), we get:

\[ K_I = \left\{-\frac{P_0}{2} \left[ (1+\lambda)-(1-\lambda) \cos 2\theta \right]+p \right\} \sqrt{\pi c} \]  

\[ K_{II} = -\frac{P_0}{2} (1-\lambda) \sqrt{\pi c} \sin 2\theta \]  

By substituting formula (22) into formula (20), the clinical water pressure caused by fracture tension-shear failure under the action of hydraulic pressure \( p_{II} \) inside the crack is obtained:

\[ p_{II} = \frac{P_0}{2} \left[ (1+\lambda)+(1-\lambda) \left( \sin 2\theta \cos 2\theta \right) \right] + \frac{K_{Icc}}{\sqrt{\pi c}} \]  

It can be seen from formula (23) that the clinical hydraulic pressure \( p_{II} \) for tensile and shear failure of cracks is affected by the joint effects of crack inclination angle \( \theta \), crack length \( 2c \) and far-field stress.

When subjected to the joint action of the water pressure in the crack and the far-field stress, the compression shear failure will occur along the long axis in the crack. When compression and shear failure occurs, the presence of hydraulic pressure in the crack reduces the normal stress on the two relatively sliding walls, and the friction resistance between the cracks decreases, among which, water plays a lubrication role. The effective shear stress \( \tau_{eff} \) in the joint:

\[ \tau_{eff} = \tau - \sigma_n \tan \varphi_0 \]  

Substituting formula (19) into formula (24), we get:

\[ \tau_{eff} = -\frac{P_0}{2} (1-\lambda) \sin 2\theta + \left\{ \frac{P_0}{2} \left[ (1+\lambda)-(1-\lambda) \cos 2\theta \right] - p \right\} \tan \varphi_0 \]  

At this time, the crack failure form is compression shear failure. Therefore, type II stress intensity factor is used to calculate:

\[ K_{II0} = \tau_{eff} \sqrt{\pi c} \]  

The fracture criterion is:

\[ K_{IIcc} = K_{II0} \]  

Substituting formula (25) into formula (26) , we obtain the fracture toughness \( K_{IIcc} \) of Type II:

\[ K_{IIcc} = \left\{-\frac{P_0}{2} (1-\lambda) \sin 2\theta + \left\{ \frac{P_0}{2} \left[ (1+\lambda)-(1-\lambda) \cos 2\theta \right] - p \right\} \tan \varphi_0 \right\} \sqrt{\pi c} \]  

According to formula (28), the clinical hydraulic pressure of the crack under compression and shear failure is:
Formula (23) and formula obtain the clinical hydraulic pressure of the crack in the case of tension-shear failure and compression-shear failure, from which it can be seen that no matter what kind of failure occurs, its value is affected by crack inclination $\theta$, crack length $2c$, lateral pressure coefficient $\lambda$ and internal friction angle $\varphi_0$ of rock. To analyse the hydraulic pressure of crack growth, here we let $K_{1,ce} = 15$ MN/m$^{\frac{3}{2}}$, $K_{II,ce} = 11$ MN/m$^{\frac{3}{2}}$, $p_0 = 20$ MPa, $\varphi_0 = \frac{\pi}{6}$. At the same time, appropriate parameters are selected respectively to analyse the influence of lateral pressure coefficient fracture length and fracture strike on clinical hydraulic pressure. The influence curves are shown in Fig. 8~10.

\[
P_{r2} = \frac{P_0}{2} (1 + \lambda) - \frac{P_0}{2} (1 - \lambda) (\cos 2\theta + \sin 2\theta \cdot \cot \varphi_0) - \frac{K_{II,ce}}{\sqrt{\pi c}} \cot \varphi_0
\]  

(29)

Fig.8 Influence of lateral pressure coefficient on clinical hydraulic pressure($\theta = \frac{\pi}{4}$, $\sqrt{\pi c} = 1$ m$^2$)

Fig.9 Influence of crack length on clinical hydraulic pressure($\lambda = 1$, $\theta = \frac{\pi}{4}$)
Fig.10 Influence of fracture strike on clinical hydraulic pressure ($\lambda = 1.5, \sqrt{\pi c} = 1 \text{ m}^2$)

The above Figs. (8~10) shows that a certain clinical hydraulic pressure is required regardless of the crack is subjected to tension-shear failure or compression-shear failure. While under the natural state, the hydraulic pressure in the rock is not enough to propagate the crack. So as to say, it is necessary to analyse the variation of hydraulic pressure in the crack under the action of compression.

5 Variation law of hydraulic pressure in crack under uniaxial compression

When subjected to uniaxial compression, the longitudinal crack in the rock produces tensile stress in the direction perpendicular to the long axis, accompanying with the propagation of the crack. The growth of cracks are accompanied by the change of hydraulic pressure in the cracks, and the change of hydraulic pressure also affects the growth of cracks. Before analysing the variation law of hydraulic pressure in the joint, taking the following assumptions:

1. There are three growth modes of elongated elliptical cracks. The first one is the crack grows only along the long axis, the crack length changes and the crack openness stays the same. In the second one, the crack propagates only along the short axis and the openness is changed while the length is unchanged. The third is to expand along the long axis and the short axis at the same time, that is, the length and openness are both changed. (2) Water exists in the rock fissure during the initial state. The hydraulic pressure keeps stable, considering the water viscosity. (3) Water mass conservation in the crack at the moment of crack expansion.

The hydraulic pressure in the seam remains stable. Therefore, the water quality in the crack remains unchanged during a certain period of crack propagation $t$, i.e., $\frac{dm}{dt} = 0$. The following analysis assumes that the long axis of the fracture is distributed horizontally. The elliptic equation is expressed as follows:

$$\frac{x^2}{c^2} + \frac{y^2}{b^2} = 1$$  \hspace{1cm} (30)

where $c$ and $b$ are half of the long and short axes of the crack, respectively.

Considering the visco-resistive effect of water in the crack, according to the principle of fluid mechanics, the flow velocity at any point in the fluid can be written as:
Formula (31) is an axisymmetric parabola. Under the action of boundary viscous resistance, the velocity near the boundary is smaller.

![Diagram](image)

**Fig.11** elongated elliptical crack section model

As shown in Fig. 11, a cross section along the long axis of the elongated elliptic crack is taken at the abscissa coordinate $x_c$ to solve the vibration law of the hydraulic pressure in the crack with the direction. The cross section shape is approximately trapezoidal. The sectional pressure is $u_{ex}$, the section height is $h_x$. The velocity at any point on the section is denoted as $u(x, y)$, where the maximum flow velocity is $u_{ex}$, the average flow velocity is $\bar{u}_{ex}$, section flow is $q_s$. So at the state of $x_s = x_c + dx$, the stress on the section is $p_{ss} = p_s + \frac{\partial p_s}{\partial x} dx$, the section height is denoted as $h_{ss} = h_s + \frac{\partial h_s}{\partial x} dx$, the flow velocity is $u(x_c + dx, y)$, the maximum flow velocity is $u_{ss}$, the average flow velocity is $\bar{u}_{ss}$. Therefore, the sectional flow can be denoted as $q_{ss} = q_s + \frac{\partial q_s}{\partial x} dx$. Formula (31) can be further rewritten as:

$$u(x, y) = u_0 \left[ 1 - \frac{c^2 - y^2}{b^2 (c^2 - x^2)} \right]$$  \hspace{1cm} (31)

The volume of the trapezoidal section can be approximately expressed as:

$$\Delta V = \int dx \approx \frac{1}{2} \left( h_s + h_s + \frac{\partial h_s}{\partial x} dx \right) dx \approx h_s dx$$  \hspace{1cm} (33)

The fluid element is subjected to boundary friction shear force as $f = 2\tau$. According to Newton's shear formula:

$$\tau = \mu \frac{\partial u(x, y)}{\partial y}$$  \hspace{1cm} (34)

Combining with the average velocity defined above, we can get:

$$\bar{u}_{ex} = \frac{q_s}{h_s}$$  \hspace{1cm} (35)
According to fluid mechanics, for laminar flow, the maximum velocity of cross section is 2 times of its average velocity, and it can be obtained as follows:

$$\frac{u_{cx}}{u_{dx}} = 2, \quad \frac{u_{dx}}{u_{cx}} = 2$$  \hspace{1cm} (37)

Substituting formula (31) into formula (34), and using interpolation method to get the maximum value:

$$f = 2\mu \frac{\partial u(x, y)}{\partial y} = \frac{16\mu}{h_x^2} \left[ \frac{1}{2} \left( u_{cx} + u_{dx} \right) \right] = \frac{16\mu q_{cx}}{h_x^2}$$  \hspace{1cm} (38)

For the intercepted section, the difference is almost negligible between $u_{cx}$ and $u_{dx}$, $h_x$ and $h_x$. Substituting formulas (35), (36) and (37) into formula (38), the frictional shear force can be obtained as follows:

$$f = \frac{16\mu q_{cx}}{h_x^2}$$  \hspace{1cm} (39)

The force of the cross-section microunit in the $x$ direction is:

$$\sum F_x = \left[ p_x h_x - \left( p_x + \frac{\partial p_x}{\partial x} \right) \left( h_x + \frac{\partial h_x}{\partial x} \right) - f \right] dx = -\left( p_x \frac{\partial h_x}{\partial x} + h_x \frac{\partial p_x}{\partial x} + \frac{16\mu q_{cx}}{h_x^2} \right) dx$$  \hspace{1cm} (40)

where $\frac{\partial p_x}{\partial x}$ denotes the pressure gradient of the water body inside the crack. According to the law of conservation of mass and the incompressibility of fluid volume, we can get:

$$\frac{dm}{dt} = \int_D \frac{\partial \rho}{\partial t} dV + \iint_D \rho(U \cdot n) dA = 0$$  \hspace{1cm} (41)

Both ends of Eq. (41) are dotted with the velocity vector $U$. The momentum conservation relation of water body is as follows:

$$\frac{dm}{dt} \cdot U = \frac{dP}{dt} = \int_D \frac{\partial \rho U}{\partial t} dV + \iint_D \rho(U \cdot n) dA = \sum F_x$$  \hspace{1cm} (42)

In the analysis of hydraulic pressure in this paper, only one-dimensional laminar flow is considered. Combining with the above assumptions and Eq. (34), (35) and (36), Eq. (42) containing vectors is simplified as:

$$\iint_D (U \cdot n) dA = \int_{x_{cx}}^{x_{dx}} u(x_c, y) dy - \int_{h_{cx}}^{h_{dx}} u(x_c + dx, y) dy = \int_{h_{cx}}^{h_{dx}} u_{cx} \left( 1 - \frac{4y^2}{h_x^2} \right) dy - \int_{h_{cx}}^{h_{dx}} u_{dx} \left( 1 - \frac{4y^2}{h_x^2} \right) dy$$  \hspace{1cm} (43)

Substituting formulas (35), (36) and (37) into formula (43) and simplifying it into the following form:

$$\iint_D (U \cdot n) dA = \frac{2}{3} \left[ u_{cx} h_x - u_{dx} \left( h_x + \frac{\partial h_x}{\partial x} dx \right) \right] = -\frac{4}{3} \frac{\partial q_{cx}}{\partial x} dx$$  \hspace{1cm} (44)

Substituting formulas (33) and (44) into formula (42), the final form of water mass conservation in cracks can be
obtained as follows:

$$\frac{\partial h}{\partial t} = \frac{4}{3}\frac{\partial q_s}{\partial x}$$

(45)

Transforming the momentum conservation relation as follows:

$$\frac{dP}{dt} = \rho \frac{\partial}{\partial t} \int_U u(x, y) dV + \rho \left( \int_{x_c} u^2(x_c, y) dA - \int_{x_c + \Delta x} u^2(x_c, y) dA \right)$$

(46)

Calculating the integral of formula (46) seems to difficult. To simplify the analysis, the velocity at any point in the water body $u(x, y)$ is equivalent to a constant value $u_{const}$. Taking an infinite number of cross-sections between $x_c$ and $x_c + \Delta x$, each section exists an average velocity section $\bar{u}_i (i = 1, 2, 3L)$. Assuming to select $k$ ($k = 1, 2, 3L$) sections, the relationship is as follows:

$$\frac{\partial}{\partial t} \int_U u(x, y) dV = u_{const} \cdot \frac{\partial}{\partial t} \int_U dV = \frac{\partial}{\partial t} \left\{ \frac{1}{k} \left[ \sum_1^k \bar{u}_i \right] \right\} \cdot h_i dx$$

(47)

Substituting formulas (33) and (37) into formula (47), we get:

$$\frac{\partial}{\partial t} \int_U u(x, y) dV = \frac{\partial}{\partial t} \left\{ \frac{1}{k} \left[ \sum_1^k \left( \frac{q_1}{h_1} + \frac{q_2}{h_2} + \cdots + \frac{q_k}{h_k} \right) \right] \right\} \cdot h_i dx$$

(48)

where $q_1 = q_s$, $q_k = q_s$, $h_1 = h_s$, $h_k = h_s$, then other quantities can be expressed as:

$$q_2 = q_s + \frac{1}{k} \frac{\partial q_s}{\partial x} dx$$

$$q_3 = q_s + \frac{2}{k} \frac{\partial q_s}{\partial x} dx$$

$$\cdots$$

$$q_{(k-1)} = q_s + \frac{(k-1)}{k} \frac{\partial q_s}{\partial x} dx$$

(49)

$$h_2 = h_s + \frac{1}{k} \frac{\partial h_s}{\partial x} dx$$

$$h_3 = h_s + \frac{2}{k} \frac{\partial h_s}{\partial x} dx$$

$$\cdots$$

$$h_{(k-1)} = h_s + \frac{(k-1)}{k} \frac{\partial h_s}{\partial x} dx$$

(50)

In this paper, it is assumed that the crack in the cylindrical rock sample is a slender ellipse, so the theoretical correction value of the circular section must be considered. Taking the correction factor as $\beta = \frac{4}{3}$, and substituting formulas (49) and (50) into formula (48), considering that $dx \to 0$, $k \to \infty$, omitting the higher-order subterms, then we get:

$$\frac{\partial}{\partial t} \int_U u(x, y) dV = \beta \frac{\partial q_s}{\partial t} dx = \frac{4}{3}\frac{\partial q_s}{\partial t} dx$$

(51)

Substituting formulas (35), (36), (37) and (49) into formula (46) and carrying out the above processing to get:

$$\int_{x_c} u^2(x_c, y) dA - \int_{x_c + \Delta x} u^2(x_c, y) dA = \beta (u_t^2 s_{x_c} - u_{dx}^2 s_{x_c})$$

(52)

In a similar way, selecting correction factor as $\beta = \frac{4}{3}$, substituting $h_s = h_s + \frac{\partial h_s}{\partial x} dx$, $u_{dx} = u_{dx} + \frac{\partial u_{dx}}{\partial x} dx$ into formula (51), omitting the higher order minor terms to get:

$$\int_{x_c} u^2(x_c, y) dA - \int_{x_c + \Delta x} u^2(x_c, y) dA = \frac{4}{3} \left( -q_s \frac{\partial h_s}{h_s} dx + \frac{2}{h_s^2} \frac{\partial q_s}{\partial x} dx \right)$$

(53)

Substituting formulas (51) and (53) into formula (46), we get:
By combining formulas (42) and (51) and omitting the higher-order small terms, we can get:

$$\frac{\partial p_s}{\partial x} = \frac{4}{3} \rho \left[ \frac{1}{h_s} \frac{\partial q_s}{\partial t} + \left( \frac{-q_s^2}{h_s^2} \frac{\partial h_s}{\partial x} + \frac{2}{h_s^2} \frac{\partial q_s}{\partial x} \right) \right] - \frac{p_s}{h_s} \frac{\partial h_s}{\partial x} - \frac{16\mu q_s}{h_s^3}$$

(55)

According to the cubic law of seepage in rock mass fractures:

$$q = \frac{\sqrt{2}b^3}{12\mu} f$$

(56)

In this paper, only considering cracks along the horizontal direction of the long axis, and the influence of gravity is ignored. Formula (56) is further rewritten as:

$$q_s = \frac{h_s^3}{12\mu} \frac{\partial p_s}{\partial x}$$

(57)

Hydraulic gradient $\frac{\partial p_s}{\partial x}$ is considered as a constant overall. The derivative of the horizontal axis and time in formula (57) is taken separately, and then get:

$$\frac{\partial q_s}{\partial x} = \frac{h_s^2}{4\mu} \frac{\partial h_s}{\partial x} \frac{\partial p_s}{\partial x}$$

(58)

$$\frac{\partial q_s}{\partial t} = \frac{h_s^2}{4\mu} \frac{\partial h_s}{\partial x} \frac{\partial p_s}{\partial x}$$

(59)

Transforming formula (59) as follows:

$$\frac{\partial q_s}{\partial t} = \frac{h_s^2}{3\mu} \frac{\partial q_s}{\partial x} \frac{\partial p_s}{\partial x}$$

(60)

Substituting formulas (57), (58) and (60) into formula (55), we get:

$$\left( \frac{1}{3} + \frac{\rho}{2\mu} \frac{h_s}{\partial t} \right) \frac{\partial p_s}{\partial x} - \frac{p_s}{h_s} \frac{\partial h_s}{\partial x} = 0$$

(61)

Here, we need to calculate the change of fracture openness $h_s$ with the $x$-axis, which can be obtained by elliptic equation:

$$h_s = 2b \sqrt{1 - \frac{x^2}{c^2}} = h_{max} \sqrt{1 - \frac{x^2}{c^2}}$$

(62)

Assuming that the shape of the crack is an elongated ellipse. The length of the long and short axes of the crack increases non-linearly with time, i.e. $c = c(t)$, $b = b(t)$, so we can get:

$$\frac{\partial h_s}{\partial x} = -\frac{h_{max}}{c} \frac{x}{\sqrt{c^2 - x^2}}$$

(63)

$$\frac{\partial h_s}{\partial t} = \frac{h_{max}}{c} \frac{x}{\sqrt{c^2 - x^2}} + \frac{h_{max}^2}{c^2} \frac{x^2}{\sqrt{c^2 - x^2}}$$

(64)
Substituting formulas (63) and (64) into formula (61), we get:

\[
\frac{\partial p_x}{\partial x} + \frac{1}{3} + \frac{1}{2} \frac{\rho h_{max} \kappa^{\max}}{\mu_c} \left( 1 - \frac{x^2}{c^2} \right) + \frac{1}{2} \frac{\rho x^2 h_{max}^2 \kappa^{\max}}{\mu_c c^3} \left( c^2 - x^2 \right) p_x = 0
\]

(65)

The differential equation of formula (65) includes the hydraulic pressure gradient \( \frac{\partial p_x}{\partial x} \) and the growth rate of fractures along the long and short axes. Combining with the classification of crack growth mode above, three cases will be analysed here: ① only consider the rate of change in the long axis, i.e. \( \kappa^{\max} = 0, \kappa = 0 \); ② only consider the rate of change in the short axis, i.e. \( \kappa^{\max} \neq 0, \kappa = 0 \); ③ Consider the rate of change in both the short and long axes, i.e. \( \kappa^{\max} \neq 0, \kappa \neq 0 \). It is assumed that the hydraulic pressure in the \( x \) axial direction of the fracture is constant, i.e. \( p_{x=0} = p_0 \). At this point, under condition ①, formula (65) can be simplified as:

\[
\frac{\partial p_x}{\partial x} + \frac{1}{3} + \frac{1}{2} \frac{\rho h_{max}^2 \kappa^{\max}}{\mu_c c^3} \left( c^2 - x^2 \right) p_x = 0
\]

(66)

To simplify the calculation, let’s take:

\[
\frac{1}{2} \frac{\rho h_{max}^2 \kappa^{\max}}{\mu_c c^3} \kappa = m_0
\]

(67)

Substituting formula (67) into formula (66), and simplifying the equation to obtain:

\[
\frac{1}{p_x} \frac{\partial p_x}{\partial x} = \frac{x}{x^2 - c^2} \frac{3}{1 + 3 m_0 x^2}
\]

(68)

By calculating formula (68), we can get:

\[
p_x = \kappa \left( \frac{x^2 - c^2}{1 + 3 m_0 x^2} \right)^{\frac{1}{6 m_0 c^2 + 2}}
\]

(69)

where \( \kappa \) denotes undetermined coefficient. Substituting the boundary conditions, we get:

\[
p_x = p_0 \left( \frac{c^2 - x^2}{1 + 3 m_0 x^2} \frac{1}{c^2} \right)^{\frac{1}{6 m_0 c^2 + 2}}
\]

(70)

Substituting formula (67) into formula (70), we can get the vibration of hydraulic pressure gradient in the fracture only when the fracture length changes as follows:

\[
p_x = p_0 \left( \frac{c^2 - x^2}{c^2} \frac{1}{1 + 3 m_0 x^2} \right)^{\frac{1}{6 m_0 c^2 + 2}}
\]

(71)

Under the second condition, formula (65) will stays in the following form:
\[
\frac{\partial p_x}{\partial x} + \frac{1}{1 + \frac{1}{2} \frac{\rho}{\mu} h_{\text{max}} \kappa_{\text{max}}} \left(1 - \frac{x^2}{c^2}\right) \frac{x}{c^2 - x^2} p_x = 0
\]  
(72)

By the same taken:

\[
\frac{1}{2} \frac{\rho}{\mu} h_{\text{max}} \kappa_{\text{max}} = n
\]  
(73)

Based on the above equation:

\[
\frac{1}{p_x} \frac{\partial p_x}{\partial x} = \frac{x}{x^2 - c^2} \left(1 + \frac{n}{3} \right) - \frac{n}{c^2} \frac{x}{x^2}
\]  
(74)

After calculating, we get:

\[
p_x = \kappa_x \left[ \frac{x^2 - c^2}{\left(1 + \frac{n}{3} \right) - \frac{n}{c^2} x^2} \right]^{\frac{3}{2}}
\]  
(75)

After substituting the boundary conditions, we get:

\[
p_x = p_0 \left[ \frac{x^2 - c^2}{-c^2 + \frac{3 \rho h_{\text{max}} \kappa_{\text{max}}}{2 \mu + 3 \rho h_{\text{max}} \kappa_{\text{max}}} x^2} \right]^{\frac{3}{2}}
\]  
(76)

When in the third condition, formula (67) and (73) are substituted into formula (65) to obtain:

\[
\frac{1}{p_x} \frac{\partial p_x}{\partial x} = \frac{x}{x^2 - c^2} \left(1 + \frac{n}{c^2} \right) x^2 + \frac{1}{3} + n
\]  
(77)

\[
p_x = \kappa_2 \left[ \frac{x^2 - c^2}{\left(m - \frac{n}{c^2} \right) x^2 + \left(1 + n \right)} \right]^{\frac{3}{2}}
\]  
(78)

After substituting the boundary conditions, we get:

\[
p_x = \kappa_2 \left[ \frac{x^2 - c^2}{\left(m - \frac{n}{c^2} \right) x^2 + \left(1 + n \right)} \right]^{\frac{3}{2}}
\]  
(79)
\[
    p_x = p_0 \left[ \frac{c^2 - x^2}{h_{\text{max}}} \right] \left[ \frac{\rho \left( \frac{h_{\text{max}}}{\mu} \right)}{\frac{h_{\text{max}}}{\mu} \frac{3\rho h_{\text{max}} \mu}{2}} \right] x^2 + c^2
\]

(80)

Formulas (71) (76) (80) represent the analytical solutions for the above three cases. However, the form is complicated and difficult to solve. Therefore, it is necessary to select appropriate parameters to obtain intuitive numerical solutions and conduct more specific analysis on the variation law of hydraulic pressure in the crack under uniaxial compression. Combining with the above three cases of water pressure distribution, let \( c = 0.5 \text{ m} \),

\[
    \frac{\rho h_{\text{max}}^2}{\mu c} = 1, \quad \frac{\rho h_{\text{max}} \mu}{h_{\text{max}}} = 1, \quad \frac{2c}{h_{\text{max}}} = 20.
\]

Assuming that the shape is basically the same during fracture propagation, so we get \( \frac{\rho h_{\text{max}}}{\mu} = 10 \). The above values are substituted and simplified into formulas (78), (79) and (80) to obtain the hydraulic pressure distribution curve in the cracks under three fracture growth modes, as shown in Fig. 12:

![Fig.12 Hydraulic pressure distribution in fractures under different fracture growth modes](image)

Fig. 12 shows the distribution of hydraulic pressure in the cracks along the long axis under three different fracture growth modes. As can be seen from Fig. 12, no matter the crack expands in which way, the greater the distance from the middle of the crack, that is, the closer it is to the crack tip, the smaller the water pressure will be, and finally it approaches zero. The crack exists in the rock, and the actual crack openness is not very large. During the crack propagation process, the vibration of external pressure can be ignored, which indicates that the crack space is difficult to be filled by water under the action of limited crack openness, long length, water flow resistance in the crack and external pressure. In the process of water-bearing crack propagation, the velocity of water flow is less than that of the crack front end, and the closer the crack tip is, the smaller the water pressure in the crack is, which indicates that the continuous propagation of water-bearing crack may not be caused by hydraulic fracture. If hydraulic splitting occurs,
two conditions must be satisfied: (1) The initial hydraulic pressure in the fracture is sufficient. (2) The propagation of the fracture is not continuous, and enough time is essential between the two expansions to fill the fracture with water. However, if the crack length in rock is great, the openness is small, and the hydraulic pressure inside the crack is small, the possibility of hydraulic splitting is almost zero. Therefore, the most likely failure form of water-bearing crack is compression shear failure.

As can be seen from Fig. 12, the distribution of hydraulic pressure $p$ in the crack under three different crack growth modes is significantly different. When the length of the crack changes but the openness of the crack remains unchanged ($\Delta z = 0$, $h_{\text{max}} = 0$), the sharp decrease in the hydraulic pressure inside the cracks is the most obvious. As shown in the black block curve in Fig. 12, the hydraulic pressure in the crack decreases just a little for $0.4 p_0$ in the interval $0 \sim 0.4m$ away from the middle of the crack, while the hydraulic pressure in the crack drops sharply by $0.6 p_0$ in the minimal interval of $0.1m$ away from the tip of the crack. When the length of the crack increases, the hydraulic pressure is transferred inside the crack, but the friction resistance of the inner wall of the cracks and the viscous resistance of the water itself lead to a sharp drop in the hydraulic pressure inside the crack. By comparing the hydraulic pressure distribution curves under the three crack growth modes, only the curve with the change of openness has the most gentle vibration, and the hydraulic pressure in the cracks drops evenly in the whole interval. When the cracks only change along the openness, the curve shows different characteristics from the other two cases. At about $0.42m$ away from the middle part of the cracks, the decreasing trend of hydraulic pressure is weakened, which is due to the increase of crack openness and lead to the relatively larger section of water flow. According to hydromechanics, when other conditions are the same, increasing the ratio between the cross-sectional area of the flow pipe and the length of the flow pipe will reduce the viscous resistance of the fluid, which is conducive to increasing the flow velocity and liquid transfer pressure. This explains the phenomenon that the curve decreases at about $0.42m$ from the middle of the crack. When the length and openness of the crack increase at the same time, the variation trend of the hydraulic pressure in the crack is between the above two cases.

6 Conclusion

In this paper, taking the water-saturated cylindrical rock specimen as the research object. According to the direction of crack propagation, the cracks are divided into two types: longitudinal crack and inclined crack. Deriving the expression of tensile stress that existing in the perpendicular to axial direction under uniaxial compression of rock pillar, the variation of hydraulic pressure in which is analysed, and deriving the clinical hydraulic pressure of the crack in tension shear failure and compression shear failure. Based on the above analysis, the following conclusions are drawn:

(1) The rock is heterogeneous material and contains a large number of cracks with different scales. Under uniaxial compression, the axial direction of the internal longitudinal crack is parallel to the axial direction of rock pillar, and exists the tensile stress caused by the longitudinal crack, which perpendicular to the axial direction of rock pillar.

(2) Maxwell model and Inglis formula are used to derive the expression of the tensile stress caused by the longitudinal crack of rock pillar under uniaxial compression. The results show that the induced tensile stress can be far greater than the uniaxial tensile strength before the uniaxial compressive strength is reached.
(3) When subjected to uniaxial compression, the maximum values of induced tensile stress of either longitudinal crack or inclined crack appear near the crack tip.

(4) Divided the crack failure into tensile shear failure and compression shear failure It is considered that the hydraulic pressure in the rock is not enough to propagate the cracks under natural state.

(5) According to the different ways of crack growth in saturated rock pillar, it can be divided into three situations: only length change, only openness change, and length openness change simultaneously. Analysing and comparing the distribution of hydraulic pressure in the cracks by using fracture mechanics and fluid mechanics. The results show that, no matter how the cracks propagate, the hydraulic pressure in the crack tends to zero near the crack tip. That is to say, it is almost impossible for the cracks with continuous expansion to be fractured by hydraulic power.

(6) Under uniaxial compression, even if there is tensile stress in the longitudinal crack direction perpendicular to its long axis, the hydraulic fracturing of water-bearing crack under this condition requests two conditions: ① The cracks propagation is discontinuous, that is, sufficient residence time is needed to make the cracks filled with water and the hydraulic pressure can be restored; ② The hydraulic pressure in the cracks is sufficient.

Acknowledgements
The authors would like to express their sincere gratitude to the financial support by the National Science Foundation (Grant No. 51774167), Science Foundation of Anhui Province(Grant No. 2008085QE219); Open Foundation of Anhui Province Engineering Technology Research Center of Urban Construction and Underground Space (Grant No. APETRC-2020-3); in addition, their appreciation also goes to the editor and the anonymous reviewers for their comments.

Reference
[1] Qi CZ, Wang MY, Bai JP. and Li KR. (2014), “Mechanism underlying dynamic size effect on rock mass strength”, J. Impact Eng., 68, 1-7. https://doi.org/10.1016/j.ijimpeng.2014.01.005
[2] Qi CZ, Wang MY, Bai JP, Wen XK. and Wang HS. (2016), “Investigation into size and strain rate effects on the strength of rock-like materials”, J. Rock Mech. Min. Sci., 86, 132-146. https://doi.org/10.1016/j.jrmms.2016.04.008
[3] MALAN D F(1999),Time-dependent behavior of deep level tabular excavations in hard rock, Rock Mechanics and Rock Engineering, 32(2),123-155. https://doi.org/10.1007/s006030050028
[4] HOEK E, MARTIN C D(2014),Fracture initiation and propagation in intact rock-A review, Journal of Rock Mechanics and Geotechnical Engineering ,6,287-300. https://doi.org/10.1016/j.jrmge.2014.06.001
[5] READ R S(2004),20 years of excavation response studies at AECL's Underground Research Laboratory, International Journal of Rock Mechanics and Mining Science, 41,1251-1275. https://doi.org/10.1016/j.jrmms.2004.09.012
[6] MARTINO J B, CHANDLER N A(2004),Excavation-induced damage studies at the Underground Research Laboratory, International Journal of Rock Mechanics and Mining Science, 41,1413-1426. DOI: 10.1016/j.jrmms.2004.09.010
[7] Wang T, Yuan DJ, Jin DL, Wang J, Luo WP, Han BY.(2020).Influence factors of splitting pressure in mud water shield [J]. China Civil Engineering Journal, 53(S1):31-36.
[8] Wu Q, Xu LQ, Qiang S, Liu DT, Tao YC, Guo XY. (2020). Experimental study on hydraulic fracturing of fractured rock mass under different stress states [J]. China Rural Water and Hydropower, (01):142-148.

[9] Shi YX, Xu LQ, Tao YC, Liu C, Mao YF. (2020). Experimental study on hydraulic fracturing of rock mass under biaxial compression [J]. Journal of China Three Gorges University (Natural Science Edition), 42(03):23-28.

[10] Gan KR, Y Y, L JS. (2007). Analysis on Karst Water Inflow Mechanisms and determination of thickness of safe rock walls: case study on a tunnel [J]. Tunnel construction, 27(3):13-16. DOI: Oschwald, M. und Hong, L. und Fusetti, A. und De Rosa, M. (2007)

[11] Liu ZW, He MC, Wang SR. (2006). Study on Karst Waterburst mechanism and Prevention countermeasures in Yuanliangshan Tunnel [J]. Rock and Soil Mechanism, 27(2):228-232. DOI: 10.1016/S1872-5805(07)60004-3

[12] Yang KG. (2018). Study on the development law of water pressure in crack under static and dynamic loading [D]. Northwest A & F University.

[13] Li LP, Li SC, Zhang QS. (2010). Study of mechanism of water inrush induced by hydraulic fracturing in karst tunnels [J]. Rock and Soil Mechanics, 31(02):523-528.

[14] Griffith, A.A. (1924). “Theory of rupture”, Proceedings of the 1st International Congress of Applied Mechanics, Delft, The Netherlands.

[15] Bauch, E. and Lemmp, C. (2004), Rock Splitting in the Surrounds of Underground Openings: An Experimental Approach Using Triaxial Extension Test, in Engineering Geology for Infrastructure Planning in Europe, Springer, Berlin, Germany, 244-254.

[16] Lim, S.S., and Martin, C.D. (2010), “Core discing and its relationship with stress magnitude for lac du bonnet granite”, J. Rock Mech. Min. Sci., 47(2), 254-264. https://doi.org/10.1016/j.jrmms.2009.11.007