Research on Underwater Manipulator Control Based on HJI Theory and RBF Neural Network

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Abstract. Aiming at the influence of the underwater complex working environment on the control performance of the manipulator, and the traditional manipulator control method is not effective, a new method based on HJI (Hamilton-Jacobi Inequality) theory and RBF (radial basis function) neural network is proposed. The intelligent controller to realize the stable control of the underwater manipulator. Considering the influence of various additional forces on the manipulator in the underwater environment, the Lagrange method is used to establish the dynamic model of the underwater manipulator. Then the robust conditions of HJI differential inequality are used to ensure the stability of the control system, and the stability of the underwater manipulator system is verified by the Lyapunov stability theory. Finally, MATLAB/Simulink is used to simulate the control of the manipulator. The simulation results verify the effectiveness of the method and can realize the stable control of the underwater manipulator system.

1. Introduction
Underwater manipulators are important equipment for many underwater vehicles, and they play a vital role in applications such as underwater resource acquisition, ocean development, and offshore platform construction. Compared with traditional industrial manipulators, the working environment of underwater manipulators is relatively complicated, and there are many factors that affect its movement. Therefore, how to realize the stable control of the manipulator is a very complicated problem, which requires the control system of the underwater manipulator. It is more robust in compensating for external interference.

Aiming at the influence of uncertain factors on underwater manipulators, Jia [1] proposed a space-time method combining adaptive radial basis function neural network (RBFNN) and parameter adjustable compound nonlinear feedback control (CNF); Yang [2-3] et al. designed the corresponding robust controller for parallel robot and hybrid robot, and achieved good control effect. However, there are obvious differences between this robot and underwater robot in terms of load; Dong [4] et al. established a model-reference neural adaptive control system based on impedance control to adapt to different environments; Huang [5] combined interference observers and proposed model-based adaptive control; Wang [6] implemented a closed-loop system for UVMS Based on the analysis, discrete delay estimation is used to deal with the unknown hydrodynamic disturbance.

Aiming at the complex underwater manipulator system, this paper proposes a robust controller based on HJI theory and RBF neural network. First, the dynamic model of the underwater manipulator is established according to the Lagrange method, and then the HJI differential inequality is used. And
Lyapunov stability theory to verify the stability of the manipulator system. Finally, a simulation analysis of the dual-joint manipulator is carried out to prove that the method proposed in this paper can achieve a good control effect.

2. Mathematical model of underwater manipulator

Based on the Lagrange method, establish the dynamic equation of the controlled object as the n-joint manipulator:

\[
M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) + \Delta(q, \dot{q}) + d = T
\]

(1)

In the formula, \(M(q)\) is a positive definite inertia matrix of order \(n \times n\); \(V(q, \dot{q})\) is the centrifugal force of order \(n \times n\) and the \(G\) formula term \(G(q)\) is the \(n \times 1\) order gravity vector; \(q\) represents the vector of joint variables; \(d\) means external interference.

3. Robust controller design based on HJI and RBF neural network

3.1. HJI Differential Inequality Theorem

Under the assumption of proper regularity and bounded conditions with modulus, it is completely possible to determine whether a nonlinear uncertain system has robust stability through whether an extended HJI differential inequality has a positive solution [7].

For the following models:

\[
\begin{cases}
\dot{x} = f(x) + g(x)d \\
z = h(x)
\end{cases}
\]

(2)

In the formula, \(d\) is external interference, and \(z\) is system evaluation index.

For signal \(d(t)\), its \(L_2\) norm is \(\|d(t)\|_2 = \left\{\int_0^T d^T(t)d(t)dt\right\}^{1/2}\), This norm can measure the energy of \(d(t)\).

In order to evaluate the interference suppression capability of the system, the following performance indicators are set:

\[
J = \sup_{\|W\|_0} \|z\|_2 \|d\|_2
\]

(3)

\(J\) is called the \(L_2\) gain of the system, which represents the robustness of the system; the smaller the \(L_2\) gain of the system, the better the robustness of the system.

Combining the model and defined performance indicators, the HJI (Hamilton-Jacobi Inequality) theorem is described as [8]: For a positive number \(\gamma\), if there exists a positive definite differentiable function \(L(x) > 0\) and

\[
\dot{L} \leq \frac{1}{2}\left\{\gamma^2\|z\|^2_2 - \|z\|_2^2\right\} (\forall d)
\]

(4)

then we have \(J \leq \gamma\).

3.2. Controller design

In order to improve the stability of the system, the HJI inequality is applied to the controller. By designing the appropriate control law, the condition of Equation (4) is satisfied, \(J \leq \gamma\) is obtained, and the robust condition is satisfied. Finally, the stability control of the manipulator is realized.

The ideal tracking trajectory is \(q_d\), the tracking error is defined as \(e = q - q_d\), and the design feed-forward control law is:

\[
T = u + M(q)\ddot{q}_d + V(q, \dot{q})\dot{q}_d + G(q)
\]

(5)
In the formula, \( u \) is the feed-forward control law. Incorporating formula (5) into formula (1), the closed-loop control system can be obtained as:

\[
M(q)\ddot{e} + V(q, \dot{q})\dot{e} + \Delta(q, \dot{q}) + d = u
\]  

(6)

Let \( \Delta f(q, \dot{q}) = \Delta(q, \dot{q}) + d \), put it into equation (5), we can get:

\[
M(q)\ddot{e} + V(q, \dot{q})\dot{e} + \Delta f = u
\]  

(7)

Using RBF neural network to approximate \( F \), its expression is:

\[
\Delta f = W^* \sigma_f + \varepsilon_f
\]  

(8)

In the formula, \( \varepsilon_f \) is the approximation error, \( \sigma_f \) is the RBF Gaussian function, \( W^* \) is the ideal neural network weight.

Comprehensive formulas (7) and (8) can be obtained:

\[
M(q)\ddot{e} + V(q, \dot{q})\dot{e} + W^* \sigma_f + \varepsilon_f = u
\]  

(9)

Let \( x_1 = e \), \( x_2 = \dot{e} + \alpha e \), and \( \alpha > 0 \). Available:

\[
\begin{aligned}
\dot{x}_1 &= x_2 - \alpha x_1 \\
\dot{M}x_2 &= -Vx_2 + \omega - W^* \sigma_f - \varepsilon_f + u
\end{aligned}
\]  

(10)

In the formula, \( \omega = M\dot{e} + Ve \).

Using the HJI inequality, formula (10) is rewritten as follows:

\[
\begin{aligned}
\dot{x} &= f(x) + g(x)d \\
z &= h(x)
\end{aligned}
\]  

(11)

In the formula, \( f(x) = \begin{bmatrix} x_2 - \alpha x_1 \\ \frac{1}{M}(-Vx_2 + \omega - W^* \sigma_f + u) \end{bmatrix} \), \( g(x) = \begin{bmatrix} 0 \\ - \frac{1}{M} \end{bmatrix} \), \( d = \varepsilon_f \).

The evaluation index \( z \) of this system is defined as follows: Since \( d = \varepsilon_f \), the approximation error \( \varepsilon_f \) can be regarded as the external disturbance \( d \), and the evaluation index is defined as \( z = x_2 = \dot{e} + \alpha e \), then its \( L_2 \) gain:

\[
J_R = \sup_{\| \varepsilon_f \|_2 \leq 1} \| z \|_2
\]  

(12)

The adaptive law of the system formula (10) is designed as:

\[
\dot{W}_f = -\eta x_2 \sigma_f^T
\]  

(13)

The feedback control law is designed as:

\[
u = -\omega - \frac{1}{2\gamma^2} x_2 + \dot{W}_f \sigma_f - \frac{1}{2} x_2
\]  

(14)

\( \dot{W}_f \) and \( \sigma_f \) are the neural network weight and Gaussian function output respectively.
3.3. Stability analysis of closed loop system

First, define the Lyapunov function \([9]\) as:
\[
L = \frac{1}{2} x^T \ddot{x} + \frac{1}{2} \eta \text{tr}(\ddot{W}_f \dddot{W}_f)
\]
In the formula
\[
\ddot{W}_f = \dot{W}_f - W'_f.
\]

Combining Equations (10) and (14), and considering the oblique symmetry of the manipulator, the derivative of Lyapunov function can be obtained as follows:
\[
\dot{L} = x^T \dddot{x} + \frac{1}{2} x^T \dddot{M} x + \frac{1}{2} \eta \text{tr}(\dddot{W}_f \dddot{W}_f)
\]
\[
= x^T (V x + \omega - W'_f \sigma_f - \epsilon_f + u) + \frac{1}{2} x^T \dddot{M} x + \frac{1}{2} \eta \text{tr}(\dddot{W}_f \dddot{W}_f)
\]
\[
= x^T (-V x - W'_f \sigma_f - \epsilon_f + \frac{1}{2} x^2 + \dddot{W}_f \sigma_f - \frac{1}{2} x^2 + \frac{1}{2} \dddot{M} x + \frac{1}{2} \eta \text{tr}(\dddot{W}_f \dddot{W}_f)
\]
\[
= x^T (-\epsilon_f - \frac{1}{2} \gamma^2 x + \dot{W}_f \sigma_f - \frac{1}{2} x^2) + \frac{1}{2} x^T (M - 2V) x + \frac{1}{2} \eta \text{tr}(\dddot{W}_f \dddot{W}_f)
\]
\[
= -x^T \epsilon_f - \frac{1}{2} \gamma^2 x^T x + x^T \dot{W}_f \sigma_f - \frac{1}{2} x^T x + \frac{1}{2} \eta \text{tr}(\dddot{W}_f \dddot{W}_f)
\]

Define \(H\) as:
\[
H = \dot{L} - \frac{1}{2} \epsilon^2 \| \epsilon_f \|^2 + \frac{1}{2} \| \| \|^2
\]

Put \(\dot{L}\) into equation (15) to get:
\[
H = -x^T \epsilon_f - \frac{1}{2} \gamma^2 x^T x + x^T \dot{W}_f \sigma_f - \frac{1}{2} x^T x + \frac{1}{2} \eta \text{tr}(\dddot{W}_f \dddot{W}_f) - \frac{1}{2} \gamma^2 \| \epsilon_f \|^2 + \frac{1}{2} \| \| ^2
\]

For the above formula:
\[
1 - x^T \epsilon_f - \frac{1}{2} \gamma^2 x^T x - \frac{1}{2} \gamma^2 \| \epsilon_f \|^2 = -\frac{1}{2} \| \gamma x + \epsilon_f \|^2 \leq 0
\]
\[
2 \dot{W}_f \sigma_f + \frac{1}{2} \text{tr}(\dddot{W}_f \dddot{W}_f) = 0
\]
\[
3 - \frac{1}{2} x^T x + \frac{1}{2} \| \| ^2 = 0
\]

According to the above three conditions, \(H \leq 0\) can be obtained, and then according to the definition of \(H\) in equation (15): \(\dot{L} \leq \frac{1}{2} \gamma^2 \| \epsilon_f \|^2 - \frac{1}{2} \| \| ^2\), and then \(J \leq \gamma\) can be obtained from the HJI theorem, so the closed-loop system of the robotic arm is stable.

4. Simulation analysis of double-joint manipulator

This paper studies the dual-joint manipulator, and its dynamic equation is:
\[
M(q) \ddot{q} + V(q, \dot{q}) \dot{q} + G(q) + D = T
\]
In the formula, \(D = \Delta(q, \dot{q}) + d\).

Set the initial state of the dual-joint manipulator to 0, and the expected values of the two joint angles are: \(q_{1d} = q_{2d} = \sin t\). The mass and joint radius of the manipulator are as follows: \(m_1 = 6\text{ kg}\), \(m_2 = 4\text{ kg}\), \(r_1 = 1\text{ m}\), \(r_2 = 0.8\text{ m}\).
According to the dynamic equation of the underwater manipulator, for the dual-joint manipulator:

1) \[ M(q) = \begin{bmatrix}
(m_1 + m_2) r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos q_2 & m_2 r_2^2 + m_2 r_1 r_2 \cos q_2 \\
2m_2 r_1 r_2 \cos q_2 & m_2 r_2^2
\end{bmatrix} \]

2) \[ V(q) = \begin{bmatrix}
-V_{12} q_2 & -V_{12} (q_1 + q_2) \\
V_{12} q_1 & 0
\end{bmatrix}, \quad V_{12} = m_2 r_1 \sin q_2 \]

3) \[ G(q) = \begin{bmatrix}
(m_1 + m_2) r_1 \cos q_2 + m_2 r_2 \cos(q_1 + q_2) \\
m_2 r_2 \cos(q_1 + q_2)
\end{bmatrix} \]

4) \[ D = \begin{bmatrix}
30 \text{sgn} q_1 \\
30 \text{sgn} q_2
\end{bmatrix} \]

Take the structure of the RBF neural network as 4-7-1 RBF, and take \( \eta = 1500, \alpha = 20 \), and \( \gamma = 15 \). The parameters \( c_i \) and \( b_i \) of the Gaussian function are 
\([-1.5 \quad -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5]\) and 10, respectively. Equation (13) is used for the adaptive law, and the control law is Equation (5) for the feed-forward control law and Equation (14) for the feedback control law. The main simulation program and simulation results are as follows:

**Figure 1.** MATLAB/Simulink simulation main program

Analysis of Figure 2 shows that in the response time of the control system, the actual response speed of joint 1 and joint 2 has reached the expected result compared with the ideal response speed. The control system can be faster under the control of the RBF neural network. Achieve the desired effect.

Analyzing Figure 3 we can see that there is an error between the angular velocities of joint 1 and joint 2 and the ideal angle when the system starts to respond. However, in a short time, the control system of the robotic arm can be quickly adjusted by the controller to make the angular velocities of the two joints reach after the system is stable, the angular velocity of the joint can basically maintain the desired value, that is, the steady-state error of the system is small. It can be obtained that the HJI theory and RBF neural network controller designed in this paper can make the system reach the expected state quickly and stably.
Figure 2. Angle tracking of joint 1 and joint 2 based on RBF compensation

Figure 3. Angular velocity tracking of joint 1 and joint 2 based on RBF compensation
5. Conclusions
For the underwater manipulator, a control method based on HJI theory and RBF neural network is proposed. The high-precision tracking of the underwater manipulator's trajectory is achieved by using the characteristics of the neural network's faster inference speed, and the HJI differential inequality and Lyapunov stability are adopted. The sexual theory verifies the stability of the underwater manipulator system. Through the analysis of the simulation results, both the response time and the steady-state error of the system can reflect the better performance of the control system designed in this article. This article provides an effective and high-precision method for the control of the underwater manipulator.

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