Three Applications of a Bonus Relation for Gravity Amplitudes

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Abstract

Arkani-Hamed et. al. have recently shown that all tree-level scattering amplitudes in maximal supergravity exhibit exceptionally soft behavior when two supermomenta are taken to infinity in a particular complex direction, and that this behavior implies new non-trivial relations amongst amplitudes in addition to the well-known on-shell recursion relations. We consider the application of these new ‘bonus relations’ to MHV amplitudes, showing that they can be used quite generally to relate $(n-2)!$-term formulas typically obtained from recursion to $(n-3)!$-term formulas related to the original BGK conjecture. Specifically we provide (1) a direct proof of a formula presented by Elvang and Freedman, (2) a new formula based on one due to Bedford et. al., and (3) an alternate proof of a formula recently obtained by Mason and Skinner. Our results also provide the first direct proof that the conjectured BGK formula, only very recently proven via completely different methods, satisfies the on-shell recursion.

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I. INTRODUCTION

The enormous progress we have witnessed over the past few years in our understanding of the structure and calculability of gluon scattering amplitudes amply demonstrates that simple amplitudes do not require simple Lagrangians. Perhaps counterintuitively, it has recently been suggested [1] that the quantum field theory with the simplest amplitudes may in fact be \( \mathcal{N} = 8 \) supergravity (SUGRA).

However it is clear that we are still very far from completely exposing the alleged simplicity of SUGRA amplitudes, even at tree level. To see this one need look no further than the maximally helicity violating (MHV) graviton amplitudes. The formula originally conjectured by Berends, Giele and Kuijf [2], not proven until 20 years later by Mason and Skinner [3], has for \( n > 4 \) particles the form

\[
\mathcal{M}_n = \sum_{\mathcal{P}(2,\ldots,n-2)} \frac{[1,2][n-2,n-1]}{(1-n-1)} \left( \prod_{i=1}^{n-3} \prod_{j=i+2}^{n-1} \langle i\ j \rangle \right) \prod_{k=3}^{n-3} \langle k|p_{k+1} + p_{k+2} + \cdots + p_{n-1}|n \rangle , \tag{1.1}
\]

where the sum runs over all permutations of the labels \( \{2,\ldots,n-2\} \). This expression, obtained from the KLT relations [4] between open and closed string amplitudes (reviewed in [5]) does not seem particularly simple, especially when contrasted with the remarkable Parke-Taylor formula for color-ordered MHV gluon amplitudes [6, 7]

\[
A(1,2,\ldots,n) = \frac{1}{\langle 1\ 2 \rangle \langle 2\ 3 \rangle \cdots \langle n\ 1 \rangle} . \tag{1.2}
\]

A number of authors have observed that some gravity amplitudes have exceptionally soft behavior as the momenta of two particles are taken to infinity in a certain complex direction\(^2\) (see for example [8, 10, 11, 12, 13]). One result of [1] is the finding that in fact all SUGRA amplitudes fall off like \( 1/z^2 \) for large \( z \) as the supermomenta of two particles are taken to infinity in a particular complex superdirection. This stands in contrast to SYM, where amplitudes only fall off like \( 1/z \) (see [14, 15] for treatments of other theories). The \( 1/z \)

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1 For simplicity we avoid committing to a choice of which two particles \( i \) and \( j \) have negative helicity. This means that (1.1) and (1.2) are to be understood as superspace expressions with the overall delta-function \( \delta^{2\mathcal{N}}(q) = \delta^{2\mathcal{N}}(\sum \tilde{\lambda}^a \eta_A) \) of supermomentum conservation suppressed. To restore the helicity information one would multiply (1.1) by \( \langle i\ j \rangle^8 \) for negative helicity gravitons, and (1.2) by \( \langle i\ j \rangle^4 \) for negative helicity gluons.

2 Moreover, it has been argued [8] that the soft behavior of tree amplitudes is of direct importance for UV cancellations in SUGRA loop amplitudes [9].
falloff common to SYM and SUGRA allows one to rewrite the contour integral $\oint \frac{dz}{z} \mathcal{M}(z)$ as a sum over residues with no contribution at infinity $[16]$, leading to the well-known BCF recursion relations $[17]$ for the physical amplitude $\mathcal{M}(0)$ (studied for gravity in $[10, 11]$ and fully proven first in $[12]$). The $1/z^2$ falloff special to gravity allows the same to be done for $\oint dz \mathcal{M}(z)$, implying new non-trivial relations between tree amplitudes which we call the ‘bonus relations’.

All known $n$-graviton MHV formulas in the literature $[2, 3, 8, 10, 18, 19]$ fall into two categories: those which come from solving the on-shell recursion have $(n-2)!$ terms (since the recursion treats two lines as special), while those obtained from manipulating the BGK formula preserve its $(n-3)!$ terms.

In this paper we show that the MHV bonus relation may be used quite generally to relate an $(n-2)!$-term formula to one with $(n-3)!$-terms that obey a slightly simplified recursion. Three specific applications are presented: a proof of a formula proposed by Elvang and Freedman $[19]$, a new formula based on one due to Bedford et. al. $[10]$, and an alternate proof of a formula recently obtained by Mason and Skinner $[3]$.

Since two of the formulas we prove ($3.3$ and $3.14$) are known $[3, 8, 19]$ to be equivalent to the original BGK conjecture $1.1$, our work provides as a byproduct the first direct proof that the BGK formula (which Mason and Skinner derived using completely different methods) satisfies the on-shell recursion.

Our work is only a small step in the decades-long march towards a better understanding of the structure of tree-level graviton amplitudes. Over the years a variety of different approaches have shed light on this problem in addition to those mentioned above, including string-based methods $[20]$, Lagrangian-level manipulations $[21]$, twistor string inspired ideas $[22, 23]$ such as the MHV vertex approach $[13, 24]$, and exploitation of $E_{7(7)}$ symmetry $[1, 25, 26, 27]$. Despite all of this progress it seems clear that much of the structure is still elusive (see for example $[28]$ for some specific open questions). In particular, in this paper we only consider MHV amplitudes although the bonus relations have implications for all amplitudes. Moreover, in SYM it has been found that NMHV amplitudes, for example, satisfy certain sum rules $[29]$ which are not a consequence of large $z$ behavior. If the promise of $[1]$ is realized then we can expect an even richer story to emerge for SUGRA.
II. CASHING IN THE BONUS FOR MHV AMPLITUDES

We follow the conventions of [1] in choosing the supersymmetry-preserving shift [1, 30]

\[
\begin{align*}
\lambda_1(z) &= \lambda_1 + z\lambda_2, \\
\tilde{\lambda}_2(z) &= \tilde{\lambda}_2 - z\tilde{\lambda}_1, \\
\eta_1(z) &= \eta_1 + z\eta_2.
\end{align*}
\]

(2.1)

Although bonus relations hold for all amplitudes, their application is simplest for MHV amplitudes to which we now restrict our attention. In this case there is only a single relevant type of factorization, shown in Fig. 1. Let us define the subamplitude

\[
M_k = \int d^8 \eta \, \mathcal{M}_L(\hat{1}, k, -\hat{P}(z_k)) \frac{1}{(p_1 + p_k)^2} \mathcal{M}_R(\hat{2}, 3, \ldots, k, \ldots, n, \hat{P}(z_k))
\]

(2.2)
as the expression corresponding to the diagram in Fig. 1 with particle \( k \in \{3, \ldots, n\} \) joining particle 1 on the left side of the factorization. Here

\[
\hat{P}(z) = p_1 + p_k + z\lambda_k \tilde{\lambda}_1
\]

(2.3)
is the shifted intermediate momentum crossing the diagram and

\[
z_k = -\left\langle 1 \, k \right\rangle \left\langle 2 \, k \right\rangle
\]

(2.4)
is the value of \( z \) at which \( \hat{P}(z) \) goes on-shell. In terms of the subamplitudes defined in (2.2) the BCF recursion is simply

\[
\mathcal{M}_n = M_3 + M_4 + \cdots + M_n,
\]

(2.5)
while the bonus relation takes the form

\[
0 = z_3 M_3 + z_4 M_4 + \cdots + z_n M_n.
\]

(2.6)
We can use this equation to delete any one $M_k$, say $M_3$, in $(2.5)$, arriving at

$$\mathcal{M}_n = \sum_{k=4}^{n} \left( 1 - \frac{z_k}{z_3} \right) M_k = \sum_{k=4}^{n} \frac{\langle 1 2 \rangle \langle 3 k \rangle}{\langle 1 3 \rangle \langle 2 k \rangle} M_k$$

(2.7)

with the help of the Schouten identity. This is almost identical to the BCF recursion $(2.5)$ except that one buys a reduction in the number of terms from $n-2$ to $n-3$ at the price of inserting a relatively simple factor into the sum. When applied recursively this method reduces the number of terms in an $n$-graviton amplitude from $(n-2)!$ to $(n-3)!$.

III. APPLICATIONS

A. Direct proof of a formula due to Elvang and Freedman

In [19] Elvang and Freedman presented two new formulas for MHV gravity amplitudes in terms of squared MHV SYM amplitudes. First, the BCF recursion was used to prove the formula

$$\mathcal{M}_n = \sum_{\mathcal{P}(3,\ldots,n)} F(1,2,\ldots,n)$$

(3.1)

with

$$F(1,2,\ldots,n) = \langle 1 n \rangle \langle n 1 \rangle \left( \prod_{s=4}^{n-1} \beta_s \right) A(1,2,\ldots,n)^2,$$

$$\beta_s = -\frac{s s + 1}{2 s + 1} \langle 2 | p_3 + p_4 + \cdots + p_{s-1} | s \rangle.$$  (3.2)

The second formula is

$$\mathcal{M}_n = \sum_{\mathcal{P}(4,\ldots,n)} \frac{\langle 1 2 \rangle \langle 3 4 \rangle}{\langle 1 3 \rangle \langle 2 4 \rangle} F(1,2,\ldots,n),$$

(3.3)

where the distinguished particle 3 can be chosen freely from the set \{3,\ldots,n\} without changing the result as long as the sum includes all permutations of the remaining $n-3$ elements. This formula was obtained by manipulating a slightly reprocessed version of the BGK formula from [8]. We will now show that $(3.3)$ follows directly from $(3.1)$ and $(2.7)$.

The proof proceeds by induction, beginning with the cases $n = 4,5$ already shown to be correct in [19]. We now have to show that $(3.3)$ continues to hold for $n+1$ gravitons if we allow ourselves to assume that it holds up to and including $n$ gravitons. Now the factor
$F(1,2,\ldots,n)$ was shown in [19] to satisfy the BCF recursion already, which means that we know it satisfies
\[
\int d^8 \eta \, \mathcal{M}_L(\hat{1}, n + 1, -\hat{P}_{n+1}) \frac{1}{(p_1 + p_{n+1})^2} F(\hat{2}, 3, \ldots, n, \hat{P}_{n+1}) = F(1,2,\ldots,n+1), \quad (3.4)
\]
where
\[
\hat{P}_{n+1} = p_1 + p_{n+1} + z_{n+1} \lambda_2 \tilde{\lambda}_1, \quad z_{n+1} = -\frac{\langle 1n+1 \rangle}{\langle 2n+1 \rangle}.
\]
Then we use (2.7) to write the $n + 1$ graviton amplitude as
\[
\mathcal{M}_{n+1} = \sum_{k=4}^{n+1} \frac{\langle 12 \rangle\langle 3k \rangle}{\langle 13 \rangle\langle 2k \rangle} M_k
\]
\[
= \frac{1}{(n-2)!} \sum_{\mathcal{P}(4, \ldots, n+1)} \sum_{k=4}^{n+1} \frac{\langle 12 \rangle\langle 3k \rangle}{\langle 13 \rangle\langle 2k \rangle} M_k
\]
\[
= \frac{1}{(n-3)!} \sum_{\mathcal{P}(4, \ldots, n+1)} \frac{\langle 12 \rangle\langle 3n+1 \rangle}{\langle 13 \rangle\langle 2n+1 \rangle} M_{n+1}. \quad (3.6)
\]
On the second line we have used the fact that $\mathcal{M}_{n+1}$ is fully symmetric under the exchange of any labels to introduce a sum over permutations together with $1/(n-2)!$ to compensate for overcounting. Inside the sum over permutations we can then without loss of generality set $k = n+1$ while including a factor of $n-2$ counting the number of terms in the sum. Now according to the definition (2.2) we have
\[
M_{n+1} = \int d^8 \eta \, \mathcal{M}_L(\hat{1}, n + 1, -\hat{P}_{n+1}) \frac{1}{(p_1 + p_{n+1})^2} \mathcal{M}_R(\hat{2}, 3, \ldots, n, \hat{P}_{n+1}). \quad (3.7)
\]
Plugging in (3.3) for $\mathcal{M}_R$ we find that the extra factor in front goes along for the ride as we apply the recursion (3.4), leading to
\[
M_{n+1} = \sum_{\mathcal{P}(4, \ldots, n)} \frac{\langle \hat{P}_{n+1} 2 \rangle\langle 34 \rangle}{\langle \hat{P}_{n+1} 3 \rangle\langle 24 \rangle} F(1,2,\ldots,n+1). \quad (3.8)
\]
Substituting this into (3.6) we find that the redundant inner sum over $\mathcal{P}(4, \ldots, n)$ cancels the $1/(n-3)!$ factor leading to
\[
\mathcal{M}_{n+1} = \sum_{\mathcal{P}(4, \ldots, n+1)} \frac{\langle 12 \rangle\langle 3n+1 \rangle\langle \hat{P}_{n+1} 2 \rangle\langle 34 \rangle}{\langle 13 \rangle\langle 2n+1 \rangle\langle \hat{P}_{n+1} 3 \rangle\langle 24 \rangle} F(1,2,\ldots,n+1). \quad (3.9)
\]
Finally an elementary manipulation reveals that
\[
\frac{\langle 12 \rangle\langle 3n+1 \rangle\langle \hat{P}_{n+1} 2 \rangle\langle 34 \rangle}{\langle 13 \rangle\langle 2n+1 \rangle\langle \hat{P}_{n+1} 3 \rangle\langle 24 \rangle} = \frac{\langle 12 \rangle\langle 34 \rangle}{\langle 13 \rangle\langle 24 \rangle}, \quad (3.10)
\]
thereby completing the inductive proof of (3.3). Note that we could have used a version of (2.7) to delete any one subamplitude of our choice, not necessarily $M_3$, so a byproduct of our analysis is a demonstration that (3.3) is not dependent on the choice of the distinguished particle 3. This feature was only checked numerically in [19].

B. A new formula based on one by Bedford, Brandhuber, Spence and Travaglini

Here we apply the same idea to another $(n-2)!$-type formula, given in (1.5) of [10] and reproduced here in a slightly relabeled form,

$$M_n = \frac{1}{2} \sum_{P(3,\ldots,n)} \frac{[1\,n]}{\langle 1\,n\rangle \langle 1\,2\rangle^2 \langle 2\,3\rangle \langle 2\,4\rangle \langle 3\,4\rangle \langle 3\,5\rangle \langle 4\,5\rangle} \prod_{s=5}^{n-1} \frac{\langle 2|p_3 + p_4 + \cdots + p_{s-1}|s\rangle}{\langle s\,s+1\rangle \langle 2\,s+1\rangle}.$$  \hspace{1cm} (3.11)

Note that this formula is only valid for $n \geq 5$ (and for $n=5$ one simply omits the product). It was proven in [19] that this formula is equivalent to (3.1).

Following the example set in the previous subsection it is clear that we can simplify this formula by omitting one particle, say 3, from the sum over permutations at the expense of introducing an appropriate factor like in (3.3). Here we should be careful though because the formula (3.11) starts only from $n=5$ points so we should not choose the factor to involve particle 4, but rather the factor must be

$$1 - \frac{z_5}{z_3} = \frac{\langle 1\,2\rangle \langle 3\,5\rangle}{\langle 1\,3\rangle \langle 2\,5\rangle}.$$  \hspace{1cm} (3.12)

Inserting this factor into (3.11) leads to the simplified formula

$$M_n = \frac{1}{2} \sum_{P(4,\ldots,n)} \frac{[1\,n]}{\langle 1\,n\rangle \langle 1\,2\rangle \langle 1\,3\rangle \langle 2\,3\rangle \langle 2\,4\rangle \langle 2\,5\rangle \langle 3\,4\rangle \langle 4\,5\rangle} \prod_{s=5}^{n-1} \frac{\langle 2|p_3 + p_4 + \cdots + p_{s-1}|s\rangle}{\langle s\,s+1\rangle \langle 2\,s+1\rangle}.$$  \hspace{1cm} (3.13)

We have checked numerically that this modified version of (3.11) agrees with all of the other MHV formulas ((1.1), (3.1), (3.3) and (3.14)) through $n=12$ gravitons. Of course it is also simple to prove analytically that it is correct, following exactly the same steps as in the previous subsection. One ends up with the same factors as in (3.10) except with $4 \rightarrow 5$, thereby establishing that (3.13) is correct.
C. Alternate proof of a formula due to Mason and Skinner

Our final application concerns a formula established recently by Mason and Skinner [3]

\[ M_n = \sum_{P(2, \ldots, n-2)} \frac{A(1, 2, \ldots, n)}{(1-n-1)(n-1)n1} \prod_{m=2}^{n-1} \frac{[m|p_{k+1} + p_{k+2} + \cdots + p_{n-1}|n]}{\langle m n \rangle}. \quad (3.14) \]

via a background field calculation of a single graviton scattering off a self-dual gravitational background. It was also shown to be equivalent to the original BGK formula (1.1), thereby providing the first analytic proof of the BGK conjecture.

Here we provide an alternate proof of this formula by showing directly that it satisfies the simplified on-shell recursion (2.7). As usual the proof proceeds by induction, beginning with the case \( n = 4 \) which is simple to verify explicitly. We then assume that (3.14) holds for \( n \) gravitons and apply the recursion (2.7) to calculate the \( n+1 \)-graviton amplitude. Choosing now for convenience to shift \( \lambda_1 \) and \( \tilde{\lambda}_{n+1} \) instead of (2.1) leads to the \( n-1 \) factorizations shown in Fig. 2. Furthermore we use the bonus relation to delete the last subamplitude, so that the \( k \)-th subamplitude picks up the factor

\[ 1 - \frac{z_k}{z_n} = \frac{\langle 1 n + 1 \rangle \langle n k \rangle}{\langle 1 n \rangle \langle n + 1 k \rangle}. \quad (3.15) \]

The combinatorics work out just as in section III.C, so that inside the appropriate sum over permutations we can focus without loss of generality on just the first subamplitude \( k = 2 \) in Fig. 2. Therefore let us now take a look at the factors which appear when (3.14) is inserted into this diagram,

\[ \frac{\langle 1 n + 1 \rangle \langle n 2 \rangle}{\langle 1 n \rangle \langle n + 1 2 \rangle} \times \left[ \frac{1}{[2 P_2][P_2 1][1 2]} \right]^2 \times \frac{1}{(p_1 + p_2)^2} \times \frac{A(\hat{P}_2, 3, \ldots, n + 1)}{\langle \hat{P}_2 n \rangle \langle n n + 1 \rangle \langle n + 1 \hat{P}_2 \rangle} \prod_{m=3}^{n} \frac{[m|p_{m+1} + p_{m+2} + \cdots + p_n|n + 1]}{\langle m n + 1 \rangle}. \quad (3.16) \]
The first term comes from (3.15), the second term is the 3-particle MHV amplitude on the left side of the factorization, the third term is the propagator, and the second line comes from inserting the $n$-graviton amplitude (3.14) on the right side of the factorization. Now the factors
\[
\frac{1}{[2\hat{P}_2][\hat{P}_21][12]} \times \frac{1}{(p_1 + p_2)^2} \times A(\hat{P}_2, 3, \ldots, n + 1)
\] (3.17)
are precisely those that would appear in the recursion for the MHV SYM amplitude; these therefore combine to give $A(1, 2, \ldots, n + 1)$. Next we take a look at the factors
\[
\frac{\langle 1 + n + 1 \rangle \langle n 2 \rangle}{\langle 1 n \rangle \langle n + 1 2 \rangle} \times \frac{1}{[2\hat{P}_2][\hat{P}_21][12]} \times \frac{1}{\langle \hat{P}_2 n \rangle \langle n n + 1 \rangle \langle n + 1 \hat{P}_2 \rangle}
= \frac{1}{\langle 1 n \rangle \langle n + 1 n \rangle \langle n + 1 1 \rangle} \times \frac{[2|p_1|n + 1]}{\langle n + 1 2 \rangle}.
\] (3.18)

The second term here exactly supplies the missing $m = 2$ term in the product on the second line of (3.16), so that when everything is finally combined we end up with
\[
\frac{A(1, 2, \ldots, n + 1)}{\langle 1 n \rangle \langle n n + 1 \rangle \langle n + 1 1 \rangle} \prod_{m=2}^{n} \frac{|m|p_{m+1} + p_{m+2} + \cdots + p_n|n + 1)}{\langle m n + 1 \rangle},
\] (3.19)
in precise agreement with (3.14), thereby completing the inductive proof.

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