Double D-term inflation

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Abstract

Comparisons of cosmological models to current data show that the presence of a non-trivial feature in the primordial power spectrum of fluctuations, around the scale $k \sim 0.05h\text{Mpc}^{-1}$, is an open and exciting possibility, testable in a near future. This could set new constraints on inflationary models. In particular, current data favour a $\Lambda$CDM model with a steplike spectrum, and more power on small scales. So far, this possibility has been implemented only in toy models of inflation. In this work, we propose a supersymmetric model with two $U(1)$ gauge symmetries, associated with two Fayet-Iliopoulos terms. Partial cancellation of the Fayet-Iliopoulos by one of the charged fields generates a step in the primordial power spectrum of adiabatic perturbations. We show that when this field is charged under both symmetries, the spectrum may have more power on small scales.

1 Introduction

A most exciting aspect of inflationary models is the possibility of constraining them through observations of cosmological perturbations. Generally, the power spectrum of primordial fluctuations can be easily related to the inflation potential [1]. In more complicated models, like multiple-stage inflation [2–4], slow-roll conditions may be violated; then, the primordial spectrum has to be derived from first principles, and exhibits a non-trivial feature at some scale. These scenarios shouldn’t be regarded as unrealistic: inflation has to take place within the framework of a high-energy theory, with many scalar fields. So, the real question is not to know whether multiple-stage inflation can occur, but whether it is predictive, or just speculative. Multiple-stage inflation has already been invoked for many purposes: generation of features in the power spectrum at observable scales, setting of initial conditions, description of the end of inflation. Here we are interested in the first possibility. The range of cosmological perturbations observable today in the linear or quasi-linear regime has been generated during approximately 8 $e$-folds, 50 $e$-folds
before the end of inflation. So, a feature in the primordial power spectrum
could very well be observable, without any fine-tuning; only observations can
rule out this possibility.

In the radiation and matter dominated Universe, primordial fluctuations of the
metric perturbations couple with all components, leading to the formation of
Cosmic Microwave Background (CMB) anisotropies and Large Scale Structure
(LSS). It is well established that a pure Cold Dark Matter (CDM) scenario,
in a flat universe, and with a scale-invariant or even a tilted primordial spec-
trum, is in conflict with LSS data. So, many variants of this scenario have
been considered, including a Broken Scale Invariant (BSI) power spectrum.
Indeed, standard CDM predictions are improved when less power is predicted
on small scales. Specific cases have been compared accurately with both LSS
data and recent CMB experiments, including the double-inflationary spectrum
of \[5\], and the steplike spectrum of \[6\]. However, even with such spectra, it has
been shown \[7–9\] that standard CDM cannot be made compatible with all ob-
servations. Independently, recent constraints from Ia supernovae \[10\] strongly
favour the ΛCDM scenario, with a cosmological constant \(\Lambda \simeq 0.6 - 0.7\) and
a Hubble parameter \(h = 0.6 - 0.7\). In this framework, a reasonable fit to all
CMB and LSS data can be obtained with a flat or a slightly tilted spectrum.
So, ΛCDM is very promising and should be probably considered as a current
standard picture, from which possible deviations can be tested. In this respect,
in spite of large error bars, some data indicate a possible sharp feature in the
primordial power spectrum around the scale \(k \simeq 0.05h\text{Mpc}^{-1}\). First, APM
redshift survey data seem to point out the existence of a step in the present
matter power spectrum \[11,3\]. Second, Abell galaxy cluster data exhibit a sim-
ilar feature \[12\] (at first sight, it looks more like a spike than like a step, but in
fact a steplike primordial spectrum multiplied with the ΛCDM transfer func-
tion reproduces this shape \[8\]). Third, this scale corresponds to \(l \sim 300\), i.e. to
the first acoustic peak in the CMB anisotropies, and increasing power at this
scale, through a bump or a step, would lead to a better agreement with Sask-
toon. In the next years, future observations will either rule out this possibility
(which will then be attributed to underestimated errorbars), or confirm the
existence of a feature, and precisely constraint its shape. In \[8,9\], we compared
a specific BSI ΛCDM model with CMB and LSS data. The primordial spec-
trum was taken from a toy model proposed by Starobinsky \[6\], in which the
derivative of the inflaton potential changes very rapidly in a narrow region. In
this case, the primordial spectrum consists in two approximately flat plateaus,
separated by a step. This model improves the fits to CMB and LSS data (even
when the cluster data are not taken into account). It also reproduces fairly
well the feature of Einasto et al. \[12\] when the step is inverted with respect
to the usual picture, i.e., with more power on small scales. Independently, in
a preliminary work \[13\] (motivated by the inflationary model of \[14\]), a pri-
modial spectrum with a bump centered at \(k \simeq 0.06h\text{Mpc}^{-1}\) was compared
with CMB and LSS data, including more redshift surveys than in \[8\], and not
taking Einasto et al. [12] data into account. It is remarkable that among many cosmological scenarios, this BSI spectrum combined with ΛCDM yields one of the best fits.

So, we have good motivations for searching inflationary models based on realistic high-energy physics, that predict a bump or an inverted step at some scale, and approximately scale-invariant plateaus far from it. Successful comparison with the data requires the deviation from scale invariance to be concentrated in a narrow region $k_1 \leq k \leq k_2$; roughly, $k_2/k_1$ should be in the range 2-10. This is quite challenging, because double inflationary models studied so far predict systematically less power on small scale, with a logarithmic decrease on large scale rather than a plateau. However, these models were based on the general framework of chaotic inflation. Today, the best theoretically motivated framework is hybrid inflation [15]. Indeed, hybrid inflation has many attractive properties, and appears naturally in supersymmetric models: the inflaton field(s) follow(s) one of the flat directions of the potential, and the approximately constant potential energy density is provided by the susy-breaking F or D-term. When the F-term does not vanish, conditions for inflation are generally spoiled by supergravity corrections, unless interesting but rather complicated variants are considered (for a very good review, see [16]). On the contrary, the simple D-term inflation mechanism proposed by Binétruy & Dvali [17] and Haylo [18] is naturally protected against supergravity corrections, and can be easily implemented in realistic particle physics models, like the GUT model of reference [19], without invoking ad hoc additional sectors. If supergravity is considered as an effective theory from superstrings, D-term inflation is hardly compatible with heterotic strings theory [16,20], but consistent with type I string theory [21].

Our goal is to show that in this framework, a simple mechanism can generate a steplike power spectrum with more power on small scales. This mechanism is based on a variant of D-term inflation, with two Fayet-Iliopoulos terms. However, the fact that it is D-term inflation is not crucial for our purpose: a similar lagrangian could be obtained with two non-vanishing F-terms, or one D-term plus one F-term, like in [4]. We will not consider the link between the model of this paper and string theory, putting by hand the Fayet-Iliopoulos terms from the beginning. In a very interesting paper, Tetradis & Sakellariadou [4] studied a supersymmetric double inflationary model with a quite similar lagrangian. However, the motivation of these authors is to save standard CDM. So, they are pushed to regions in parameter space quite different from us, and do not consider a steplike spectrum with flat plateaus, but rather a power-law spectrum with $n = 1$ on large scales and $n < 1$ on small scales.
2 The model

We consider a supersymmetric model with two gauge symmetries $U(1)_A \times U(1)_B$, and two associated Fayet-Iliopoulos positive terms $\xi_A$ and $\xi_B$ (there is no motivation from string theory to do so, at least at the moment, but SUSY and SUGRA allow an arbitrary number of Fayet-Iliopoulos terms to be put by hand in the lagrangian). In this framework, the most simple workable model involves six scalar fields: two singlets $A$ and $B$, and four charged fields $A_{\pm}, B_{\pm}$, with charges $(\pm 1, 0), (\pm 1, \pm 1)$. Let us comment this particular choice. First, the presence of two singlets is crucial. With only one singlet coupling to both $A_{\pm}$ and $B_{\pm}$, we would still have double-inflation, but the second stage would be driven by both F and D-terms, and no sharp feature would be predicted in the primordial spectrum. Second, each charged field could be charged under one symmetry only; then, a steplike spectrum would be generated, but with necessarily less power on small scales. Here, the fact that $B_-$ has a charge $-1$ under both symmetries is directly responsible for the inverted step, as will become clear later. Finally, both global susy and supergravity versions of this model can be studied: supergravity corrections would change the details of the scenario described thereafter, but not its main features. We consider the superpotential:

$$W = \alpha A A_+ A_- + \beta B B_+ B_-$$

(with a redefinition of $A$ and $B$, we have suppressed terms in $B A_+ A_-, A B_+ B_-$, and made $(\alpha, \beta)$ real and positive). In global susy, the corresponding tree-level scalar potential is:

$$V = \alpha^2 |A|^2(|A_+|^2|A_-|^2) + \alpha^2 |A_+ A_-|^2 + \frac{g_A^2}{2}(|A_+|^2 - |A_-|^2 + |B_+|^2 - |B_-|^2 + \xi_A)^2$$

$$+ \beta^2 |B|^2(|B_+|^2 - |B_-|^2) + \beta^2 |B_+ B_-|^2 + \frac{g_B^2}{2}(|B_+|^2 - |B_-|^2 + \xi_B)^2, \quad (1)$$

with a global supersymmetric vacuum in which all fields are at the origin, excepted $|B_-| = \sqrt{\xi_B}$, and, depending on the sign of $(\xi_A - \xi_B)$, $|A_-|$ or $|A_+| = \sqrt{\xi_A - \xi_B}$.

3 The two slow-roll inflationary stages

3.1 Inflationary effective potential

There will be generically two stages of inflation, provided that initial conditions for $(A, B)$ obey to :

$$|A|^2 \geq \frac{g_A^2 \xi_A}{\alpha^2}, \quad |B|^2 \geq \frac{g_A^2 \xi_A + g_B^2 \xi_B}{\beta^2}. \quad (2)$$
Then, charged fields have positive squared masses and stand in their (local) minimum \( A_\pm = B_\pm = 0 \) (for a discussion of the charged fields initial conditions, see for instance [22,23]). \( A \) and \( B \) have a constant phase, while their moduli \( \tilde{A} \equiv |A|/\sqrt{2} \) and \( \tilde{B} \equiv |B|/\sqrt{2} \) behave like two real inflaton fields and roll to the origin, until one inequality in (2) breaks down. We assume that the condition on \( B \) breaks first.

During this first stage, the field evolution is driven by the effective potential: 
\[
V_1 = \frac{g_A^2 \xi_A^2 + g_B^2 \xi_B^2}{2} + \Delta V_1.
\]
The one-loop correction \( \Delta V_1 \) is small \( (\Delta V_1 \ll V_1) \) [21], but crucial for the field evolution. It consists in two terms with a logarithmic dependence on \( A \) and \( B \). The former takes a simple form following from [24], because we can assume \( g_A^2 \xi_A A \ll \alpha^2 |A|^2 \). The latter is more complicated because the dimensionless quantity
\[
b \equiv \frac{g_A^2 \xi_A A + g_B^2 \xi_B}{g_B^2 + g_B^2} \to 1
\]
when \( B \) reaches its critical value. The exact expressions are:
\[
\Delta V_1 = \frac{g_A^2 \xi_A^2}{32\pi^2} \left( \ln \frac{\alpha^2 |A|^2}{\Lambda^2} + \frac{3}{2} \right) \\
+ \frac{(g_A^2 \xi_A + g_B^2 \xi_B)^2}{32\pi^2} \left( \ln \frac{\beta^2 |B|^2}{\Lambda^2} + \frac{(1 + b)^2 \ln(1 + b) + (1 - b)^2 \ln(1 - b)}{2b^2} \right)
\]
(\( \Lambda \) is the renormalization energy scale at which \( g_A \) and \( g_B \) must be evaluated).

When \( b \ll 1 \), the term involving \( b \) tends to \( \frac{3}{2} \). Even at \( b = 1 \), this term only increases the derivative \( (\partial V_1/\partial \tilde{B}) \) by a factor \( 2 \ln 2 \) : so, it can be neglected in qualitative studies. In this approximation, it is easy to calculate the trajectories of \( A \) and \( B \), and to note that \( B \) reaches its critical value before \( A \) only if the initial field values obey to:
\[
\frac{|B|_0}{|A|_0} < 1 + \frac{g_B^2 \xi_B}{g_A^2 \xi_A}.
\]

This condition is natural, in the sense that it allows \( |A|_0 \) and \( |B|_0 \) to be of the same order of magnitude, whatever the values of \( g_{A,B} \) and \( \xi_{A,B} \).

At the end of the first stage, \( (B, B_-) \) evolve to another false vacuum: \( B = 0 \), \( |B_-|^2 = (g_A^2 \xi_A + g_B^2 \xi_B)/(g_A^2 + g_B^2) \). During this transition, the charged fields \( B_+, A_\pm \) remain automatically stable if we impose \( \xi_B \leq 2 \xi_A \). Afterwards, a second stage occurs: \( A \) rolls to the origin, driven by the potential:
\[
V_2 = \frac{g_A^2 g_B^2 (\xi_A - \xi_B)^2}{2(g_A^2 + g_B^2)} \left( 1 + \frac{g_A^2 g_B^2}{16\pi^2(g_A^2 + g_B^2)} \left( \ln \frac{\alpha^2 |A|^2}{\Lambda^2} + \frac{3}{2} \right) \right),
\]
until \( |A_+| \) or \( |A_-| \) becomes unstable, and quickly drives the fields to the supersymmetric minimum.
3.2 Second stage of single-field inflation

Let us focus first on the second stage of inflation, in order to find the small scale primordial power spectrum \(i.e.,\) if the second stage starts at \(t = t_2,\) and \(k_2 \equiv a(t_2)H(t_2),\) the power spectrum at scales \(k > k_2).\) This stage should last approximately \(N \simeq 50\) e-folds, so that the transition takes place when scales observable today cross the Hubble radius.

A standard calculation shows that for \(\alpha\) of order one, the second slow-roll condition breaks before \(A\) reaches its critical value (which is given by \(\alpha^2|A_C|^2 = \frac{g_A^2g_B}{g_A^2 + g_B^2}|\xi_A - \xi_B|\)). Integrating back in time, we find that \(N\) e-folds before the end of inflation,

\[
|A| = \sqrt{\frac{N}{2\pi^2}} \frac{g_A g_B}{\sqrt{g_A^2 + g_B^2}} M_P
\]  

(we are using the reduced Planck mass \(M_P \equiv (8\pi G)^{-1/2} = 2.4 \times 10^{18}\) GeV).

Then, the primordial spectrum can be easily calculated, using the single-field slow-roll expression [1]:

\[
\delta_H^2 = \frac{1}{75\pi^2 M_P^6 \left(dV_2/dA\right)^2} = \frac{16\pi^2 g_A^2 + g_B^2}{75 M_P^6} (\xi_A - \xi_B)^2 |A|^2. \tag{7}
\]

To normalize precisely this spectrum (7) to COBE, it would be necessary to calculate the contribution of cosmic strings generated by symmetry breaking [25], to make assumptions about the geometry and matter content of the universe, and to fix the amplitude of the step in the primordial power spectrum (between COBE scale and \(k_2\)). However, if perturbations generated by cosmic strings are not dominant, and if the primordial spectrum is approximately scale-invariant as required by observation, we can estimate the order of magnitude of the primordial power spectrum at all observable scales: \(\delta_H^2 \sim 5 \times 10^{-10}.\)

So, at \(k = k_2,\) inserting (6) into (7), we find the constraint:

\[
\sqrt{|\xi_A - \xi_B|} \sim 3 \times 10^{-3} M_P \sim 10^{15-16}\) GeV. \tag{8}
\]

At first sight, this constraint could be satisfied when \((\sqrt{\xi_A}, \sqrt{\xi_B})\) are both much greater than \(10^{-3} M_P,\) and very close to each other. Then, however, the amplitude of the large-scale plateau would violate the COBE normalization, as can be seen from the following. Also, there is no reason to believe that one term is much smaller than the other: this would raise a fine-tuning problem.

So, we will go on assuming that both Fayet-Iliopoulos terms have an order of magnitude \(\sqrt{\xi_A} \sim \sqrt{\xi_B} \sim 10^{-3} M_P,\) just as in single D-term inflation.
The spectrum tilt $n_S$ at $k = k_2$ can be deduced from the slow-roll parameters $(\epsilon, \eta)$ [1]. Like in any model of single-field D-term inflation, $\epsilon$ can be neglected, and $n_S(k_2) = 1 + 2\eta(k_2) = 1 - 1/N \simeq 0.98$. So, the spectrum on small scales is approximately scale-invariant.

### 3.3 First stage of two-field inflation

During the first inflation, the primordial spectrum calculation must be done carefully. If slow-roll conditions were to hold during the transition between both inflationary stages, the evolution of metric perturbations (for modes outside the Hubble radius) would be described at first order by the well-known slow-roll solution (see for instance [26]):

$$\Phi = -C_1 \frac{\dot{H}}{H^2} - H \frac{d}{dt} \left( \frac{d_A V_A + d_B V_B}{V_A + V_B} \right), \quad (9)$$

where $\Phi$ is the gravitational potential in the longitudinal gauge. Here, $C_1$ is the time-independent coefficient of the growing adiabatic mode, while $d_A$ and $d_B$ are coefficients related to the non-decaying isocurvature mode (in fact, only $d_A - d_B$ is meaningful [26]). The expression of $V_A$ and $V_B$ at a given time can be calculated only if the whole field evolution is known, between the first Hubble crossing and the end of inflation. Formally, in the general case of multiple fields $\phi_i$, the $V_i$’s can be found by integrating $dV_i = (\partial V_i / \partial \phi_i) d\phi_i$ back in time, starting from the end of inflation. This just means here that during the second slow-roll, $V_A = V_2$, $V_B = 0$, and during the first slow-roll:

$$V_A = \frac{g_A^2 g_B^2 (\xi_A - \xi_B)^2}{2(g_A^2 + g_B^2)} + \frac{g_A^4 \xi_A^2}{32\pi^2} \left( \ln \frac{\alpha^2 |A|^2}{\Lambda^2} + \frac{3}{2} \right),$$

$$V_B = \frac{(g_A^2 \xi_A + g_B^2 \xi_B)^2}{2(g_A^2 + g_B^2)} + \frac{(g_A^4 \xi_A + g_B^4 \xi_B)^2}{32\pi^2} \left( \ln \frac{\beta^2 |B|^2}{\Lambda^2} + \frac{3}{2} \right). \quad (10)$$

We see immediately from (9) with $V_B = 0$ that the isocurvature mode is suppressed during the second inflationary stage. On the other hand, it must be taken into account during the first stage. This leads to the well-known expression [26]:

$$\delta_H^2(k) = \frac{V}{75\pi^2 M_P^3} \left( \frac{V_A^2}{(dV_A/dA)^2} + \frac{V_B^2}{(dV_B/dB)^2} \right)_k, \quad (11)$$

where the subscript $k$ indicates that quantities are evaluated at the time of Hubble radius crossing during the first stage.
If slow-roll is briefly disrupted during the transition (this is the interesting case if we want to generate a narrow step or bump), the solution (9) doesn’t hold at any time, but we still have a more general exact solution, describing the adiabatic mode (in the long-wavelength regime):

\[ \Phi = C_1 \left( 1 - \frac{H}{a} \int_0^t a dt \right) + \text{(other modes)}. \]

If, during the transition, the other modes are not dominant (as usually expected), the matching of the three solutions: (9) before the transition, (12) during the transition, and (9) afterwards, shows that \( C_1 \) is really the same number during all stages: the slow-roll disruption doesn’t leave a signature on the large-scale power spectrum (11). On the other hand, if a specific phenomenon amplifies significantly isocurvature modes during the transition, the same matching shows that during the second stage, there will be an additional term contributing to the adiabatic mode. This role may be played by parametric amplification of metric perturbations, caused by oscillations of \( B_- \). Generally speaking, the possibility that parametric resonance could affect modes well outside the Hubble radius is still unclear, and might be important in multi-field inflation [27]. In our case, this problem would require a careful numerical integration, and would crucially depend on the details of the one-loop effective potential during the transition. So, we leave this issue for another publication, and go on under the standard assumption that expression (11) can be applied to any mode that is well outside the Hubble radius during the transition.

Let us apply this assumption to our model, and find the large scale primordial power spectrum (i.e., if the first stage ends at \( t = t_1 \), and \( k_1 \equiv a(t_1)H(t_1) \), the power spectrum at scales \( k < k_1 \)). The contribution to \( \delta_H \) arising from perturbations in \( A \) reads:

\[ \frac{\delta_H^2}{|A|} \equiv \frac{V}{75\pi^2 M_p^6 (dV_A/dA)^2} \frac{V_A^2}{75 M_p^6} \frac{16\pi^2 g_B^4 (g_A^2 \xi_A^2 + g_B^2 \xi_B^2)(\xi_A - \xi_B)^4}{g_A^4 (g_A^2 + g_B^2)^2 \xi_A^4}|A|^2. \]

The transition lasts approximately 1 e-fold (indeed, during this stage, the evolution is governed by second-order differential equations with damping terms \( +3H \dot{\phi}_i \), and \((B, B_-)\) stabilize within a time-scale \( \Delta t \sim H^{-1} \)). During that time, \( A \) is still in slow-roll and remains approximately constant. So, using eqs. (7) and (13), it is straightforward to estimate the amplitude of the step in the primordial spectrum, under the assumption that \( \delta_H(k_2)|_B \) is negligible:

\[ p^2 \equiv \frac{\delta_H^2(k_1)|_A}{\delta_H^2(k_2)} = \frac{(1 - \xi_B/\xi_A)^2 (1 + g_B^2 \xi_B^2/g_A^2 \xi_A^2)}{(1 + g_A^2/g_B^2)^3}. \]
Since we already imposed $\xi_B \leq 2\xi_A$, $p$ can easily be smaller than one, so that we obtain, as desired, more power on small scales. The simple explanation is that with $B_-$ charged under both symmetries, the transition affects not only the dynamics of $(B, B_-)$, but also the one-loop correction proportional to $(\ln |A|^2)$, in such way that the slope $(\partial V/\partial \tilde{A})$ can decrease by the above factor $p$.

However, perturbations in $B$ must also be taken into account. Their contribution to the large scale primordial spectrum reads:

$$\delta^2_H|_B \equiv \frac{V}{75\pi^2 M_P^4} \frac{V_B^2}{(dV_B/dB)^2} = \frac{16\pi^2}{75 M_P^6} \frac{(g_A^2 \xi_A^2 + g_B^2 \xi_B^2)}{(g_A^2 + g_B^2)^2} |B|^2. \quad (15)$$

At the end of the first stage, the value of $|B|$ is roughly given by $M_P^2/(\partial^2 V_1/\partial \tilde{B}^2) = V_1$, since after the breaking of this slow-roll condition, eq. (11) is not valid anymore, and $B$ quickly rolls to its critical value. This yields:

$$|B| = \frac{g_A^2 \xi_A + g_B^2 \xi_B}{2\pi \sqrt{g_A^2 \xi_A^2 + g_B^2 \xi_B^2}} M_P. \quad (16)$$

We can now compare $\delta^2_H|_A$ and $\delta^2_H|_B$ at the end of the first stage. It appears that for a natural choice of free parameters ($g_A \sim g_B, \xi_A \sim \xi_B \sim |\xi_A - \xi_B|$), both terms can give a dominant contribution to the global primordial spectrum. If $\delta^2_H|_B$ dominates, we expect $p > 1$, and a tilted large-scale power spectrum (due to the decrease of $|B|$, which is at the end of its slow-roll stage). On the other hand, if $\delta^2_H|_B$ is negligible, the step amplitude is directly given by (14), and the large scale plateau is approximately flat, with $n_S \simeq 0.98$ like in single-field D-term inflation. As we said in the introduction, this latter case is the most interesting one in the framework of ΛCDM models. A numerical study shows that $\delta^2_H|_A \gg \delta^2_H|_B$ holds in a wide region in parameter space. Indeed, we explored systematically the region in which $0.1 \leq g_B/g_A \leq 10$, and $(\sqrt{\xi_A}, \sqrt{\xi_B})$ are in the range $(0.1-10) \times 10^{-3} M_P$. We find that $\delta^2_H|_A \geq 10 \delta^2_H|_B$ whenever:

$$(g_B \geq 0.8g_A, \sqrt{\xi_A} \geq 1.1 \sqrt{\xi_B}) \quad \text{or} \quad (\forall g_A, \forall g_B, \sqrt{\xi_B} \geq 1.1 \sqrt{\xi_A}). \quad (17)$$

So, inside these two regions, the primordial spectrum has two approximately scale-invariant plateaus ($n_S \simeq 0.98$), and the step amplitude is given by (14). Further, a good agreement with observations requires a small inverted step, $0.75 \leq p \leq 0.85$ [8], and of course, a correct order of magnitude for the amplitude, $\delta^2_H(k_1) \sim \mathcal{O}(10^{-10})$. These additional constraints single out two regions in parameter space:
\[
2.2 g_A \leq g_B \leq 4g_A, \ 13(\sqrt{\xi_A} - 4.5 \times 10^{-3}) \leq 10^3 \xi_B \leq 8(\sqrt{\xi_A} - 2.5 \times 10^{-3})
\]

or \[
2.2g_A \leq g_B \leq 4g_A, \ 13(\sqrt{\xi_A} - 4.5 \times 10^{-3}) \leq 10^3 \xi_B \leq 8(\sqrt{\xi_A} - 2.5 \times 10^{-3})
\]

In this section, we only studied the primordial spectrum of adiabatic perturbations. Indeed, it is easy to show that tensor contributions to large-scale CMB anisotropies are negligible in this model, like in usual single-field D-term inflation (a significant tensor contribution would require \(g_{A,B} \sim \mathcal{O}(10)\) or greater, while the consistency of the underlying SUSY or SUGRA theory requires \(g_{A,B} \sim \mathcal{O}(10^{-1})\) or smaller \([21]\)).

### 3.4 Transition between slow-roll inflationary stages

We will briefly discuss the issue of primordial spectrum calculation for \(k_1 < k < k_2\). During this stage, \(A\) is still slow-rolling, but \(B\) and \(B_-\) obey to the second-order differential equations in global susy:

\[
\ddot{B}_{(-)} + 3H \dot{B}_{(-)} + \frac{\partial V}{\partial B_{(-)}} = 0.
\]

The derivatives of the potential are given by the tree-level expression (1), plus complicated one-loop corrections. In supergravity, the tree-level potential will be different from (1), even with a minimal gauge kinetic function and Kähler potential. With a non-minimal Kähler potential, an additional factor will also multiply both inertial and damping terms \(\ddot{B}_{(-)} + 3H \dot{B}_{(-)}\). The evolution of background quantities has been already studied in a similar situation in \([22]\).

As far as perturbations are concerned, the simplest possibility would be to recover the generic primordial power spectrum introduced by Starobinsky \([6]\), also in the case of a transition between two slow-roll inflationary stages, with a jump in the potential derivative. This would be possible if: (i) \(H\) was approximately constant in the vicinity of the transition; (ii) the number of \(e\)-folds separating both slow-roll regimes was much smaller than one (in other terms, the time-scale of the transition must be much smaller than \(H^{-1}\)). However, in the model under consideration, (i) is not a good approximation, because during the transition there is a jump in the potential itself. Moreover, (ii) is in contradiction with the equations of motion (19): \(B\) and \(B_-\) only stabilize after a time-scale \(\delta t \sim H^{-1}\), due to the damping factor \(+3H\). This statement is very general, and holds even in supergravity with a non-minimal Kähler potential. So, it seems that only a first-order phase transition can reproduce the exact power spectrum of \([6]\), as previously noticed by Starobinsky himself.
A model of inflation with a first-order phase transition has been proposed in [14]. Further, we don’t believe that the power spectrum in our model can be approximated by the numerical solution of double chaotic inflation [5]. Indeed, in chaotic double inflation, fields masses are typically of the same order as the Hubble parameter: so, background fields have no strong oscillatory regime. In our case, estimating the effective mass for $B_-$ from the tree-level potential, and comparing it with $V$ (which gives a lower bound on $H$), we see that before stabilization, $B_-$ will undergo approximately $\sim \frac{M_P}{\sqrt{\xi_B}} \sim 10^3$ oscillations. These oscillations are relevant for the primordial spectrum calculation, because $B_-$ perturbations and metric perturbations will strongly couple.

So, for each particular model, a numerical integration of the background and perturbation equations should be performed in order to find the shape, and even the width of the transition in the primordial power spectrum. The result could be either a step or a bump, and since the fields stabilize in $\delta t \sim H^{-1}$, it is reasonable to think that the width of the feature will not be too large, $(k_2/k_1) \leq 10$. So, in any case, the primordial spectrum should be in good agreement with current observations, but precise comparison with future data requires numerical work.

4 Conclusion

We introduced a model of double D-term inflation, with two $U(1)$ gauge symmetries, and two associated Fayet-Iliopoulos terms. A phase of instability for two fields $(B, B_-)$ separates two slow-roll inflationary stages. During this transition, $B_-$ partially cancels the Fayet-Iliopoulos terms, causing a jump in the potential; also, since it is charged under both symmetries, it affects the one-loop corrections in such way that the potential can become less steep in the direction of one inflaton field, $\tilde{A}$. As a result, for a wide window in the space of free parameters (the Fayet-Iliopoulos terms $\xi_{A,B}$ and the gauge coupling constants $g_{A,B}$), the primordial spectrum of adiabatic perturbations consists in two approximately scale-invariant plateaus, separated by an unknown feature, presumably of small width, and with more power on small scales. The amplitude of the step, $p$, is given by a simple function of parameters (14). In the framework of $\Lambda$CDM, spectra with $0.75 \leq p \leq 0.85$ are likely to fit very well current LSS and CMB data, as argued in [8,9]. However, for a detailed comparison of our model with observations, we need the shape of the primordial power spectrum between the plateaus. This issue requires a numerical integration, and the result will be model-dependent, in contrast with predictions for the slow-roll plateaus. Finally, before any precise comparison with observations, one should also consider the production of local cosmic strings, which is
a typical feature of D-term inflation [25]. Indeed, CMB anisotropies and LSS may result from both local cosmic strings and inflationary perturbations [29].

Acknowledgements

I would like to thank D. Polarski, A. Starobinsky and N. Tetradis for illuminating discussions. G. Dvali, E. Gawiser and R. Jeannerot also provided very useful comments on this work. I am supported by the European Community under TMR network contract No. FMRX-CT96-0090.

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