Energy-Efficient Strip Monitoring by Identical Devices Directed to One Side Along the Strip and Having a Coverage Area in the Form of a Sector

Adil Erzin
Sobolev Institute of Mathematics
Novosibirsk State University
Novosibirsk, Russia
e-mail: adilerzin@math.nsc.ru

1. Introduction

Wireless sensor networks (WSN) are often used in security systems to observe (to monitor) the territory \[1\], \[2\], \[3\]. If the object of observation is a road, a pipeline or a corridor, it can be assumed that the strip is monitored \[3\], \[4\], \[5\], \[6\]. The sensor has a coverage area of various shapes within which it collects data. If the sensor is equipped with a video camera, then depending on its location, the coverage area on the surface may be a disk, an ellipse or a sector. In the WSN, however, as well as in wired sensor networks, a scarce resource is the energy of the sensors, which is consumed proportionally to the area covered by them \[1\], \[3\], \[7\], \[8\]. If the sensor coverage areas overlap, this means that extra energy is wasted. Thus, the problem of energy-efficiency systems to observe (to monitor) the territory \[1\], \[2\], \[3\]. In \[2\] the problem of constructing a least density cover, it suffices to consider the covering of one tile.

In the covers of a flat area, various figures can be used. Until recently, circles (disks) were used as figures in the covers \[7\], \[8\], \[9\], \[10\], \[11\], \[12\], \[13\], \[14\]. Recently, coverings using ellipses \[6\], \[15\] and sectors \[10\], \[12\], \[16\], \[17\], \[18\], \[19\], \[20\], \[21\], \[22\], \[23\] have also been considered.

In most publications, the area to be covered is the whole plane \[2\], \[7\], \[8\], \[9\], \[11\], \[15\]. The models of covering bounded regions is more complicated. Effective attempts were made in the construction of strip coverings with circles \[3\], \[4\], \[23\], ellipses \[5\] and sectors \[12\], \[16\], \[17\], \[18\], \[19\], \[20\], \[21\].

It was possible to prove the optimality of the covers only in some classes \[2\], \[11\]. For most covers, accurate estimates of accuracy are found \[3\], \[4\], \[9\], \[13\], \[20\], \[21\], \[23\].

In \[20\] the problem of constructing a least density cover of a strip with identical sectors is studied. Several coverage models are proposed and their comparative analysis is performed which, in particular, allows one to obtain an upper bound for the minimum density of a strip coverage with equal sectors. In \[22\] the problem of constructing a cost-effective cover of a strip with identical sectors is considered, and several efficient coverage models are investigated and their comparative analysis is performed.

In the ordinary cover it is sufficient to cover each point of the strip, regardless of the orientation of the sensors covering the area. Another situation arises when it is necessary to look at the objects from a certain side. For example, in video surveillance systems on the roads, or to identify people walking along the corridor (to passport control or boarding a plane), it is necessary to see their faces. This means that the video cameras must be directed towards the faces of walking people.

Many modern sensors can adjust their coverage area, and the density can be reduced by choosing the optimal values for the parameters of the figures involved in the covering \[1\], \[9\], \[13\], \[24\].

In this paper, we propose several regular covers with identical sectors that observe the strip in the same direction, and we carried out their analysis, which allows choosing the best coverage model and the best parameters of the sector.

The rest of the paper is organized as follows. The mathematical formulation of the problem is given in Section 2. In Section 3, a coverage model S1 is considered. A coverage model S2 is considered in Section 4. A coverage model S3 is presented in Section 5, and the paper is concluded in Section 6.

2. Problem Formulation

We are given a strip whose width, without loss of generality, is set equal to 1, and we need to observe the objects that are being drifted along the strip, for example, from left to right. Let the sensor’s coverage area be a sector \((R, \alpha)\), where \(R\) is the radius and \(\alpha\) is the angle, in the vertex of which the sensor is located. The sensor can be placed anywhere in the strip and can be oriented in any direction.

The point of the strip is covered if it belongs to at least one sector whose vertex is not to the left of the point.
A cover is such placement of sectors, and setting their orientations, that each point of the strip is covered.

The problem is to find the cover of the strip of minimum density.

Let us call the coverage model the mutual arrangement of sectors. If we set the parameters of the sector in the coverage model, we get a specific cover. We consider three coverage models S1, S2 and S3 and find the optimal parameters of the sector for each model, thereby determining the concrete covers S1, S2 and S3.

Only three coverage models are considered in this paper, because they seem to be the most effective. It can be shown that other covers can be reduced to the covers under consideration without increasing the density.

3. Coverage Model S1

In the model S1, the sector angle does not exceed $\pi/3$ and the inequality $R \sin \alpha \geq 1$ holds. One side of the sector lies on the border of the strip, the other side crosses the opposite boundary of the strip, and the sector is oriented to the left. The next sector on the right is symmetric about the center line to the previous sector and shifted to the right to such a distance to cover the maximum part of the strip (Fig. 1a). These two sectors will be called a pair of sectors. Using sector pairs it is possible to construct a regular cover with a tile in the form of a rectangle (in Fig. 1a is a rectangle $BAFE$) whose height coincides with the width of the strip and is equal to 1 and the width is $a$.

Lemma 1. The density of the cover S1 is determined as

$$D_1(R, \alpha) = \frac{R^2 \alpha}{\sqrt{R^2 - (1 - R/2 \tan \alpha)^2 - \frac{1}{\tan \alpha} + R/2}}$$

(1)

and

$$\min_{0 < \alpha \leq \pi/3, R \sin \alpha \geq 1} D_1(R, \alpha) = \frac{2\pi}{3\sqrt{3}} \approx 1.2092,$$

when $\alpha = \pi/3 = 60^\circ$ and $R = 2/\sqrt{3} \approx 1.1547$

Proof. To illustrate the proof, we refer to Fig. 1a. It's obvious that

$$|BC| = |ED|, \ |AO| = \frac{R}{2} \tan \alpha$$

and

$$|OB| = 1 - |AO| = 1 - \frac{R}{2} \tan \alpha.$$

It follows that

$$|BC| = \frac{|OB|}{\tan \alpha} = \frac{1 - \frac{R}{2} \tan \alpha}{\tan \alpha} = \frac{1}{\tan \alpha} - \frac{R}{2}$$

and

$$|BD| = \sqrt{R^2 - |OB|^2} = \sqrt{R^2 - \left(1 - \frac{R}{2} \tan \alpha \right)^2}.$$

Then the width of the tile is

$$a = |BD| - |ED| = \sqrt{R^2 - \left(1 - \frac{R}{2} \tan \alpha \right)^2} - \frac{1}{\tan \alpha} + \frac{R}{2}.$$

Therefore, the density of the cover is

$$D_1(R, \alpha) = \frac{R^2 \alpha}{a}$$

and after substituting $a$, we obtain (1).

Further it is convenient to represent $R$ in the form

$$R = \frac{1}{\sin \alpha + \delta}, \ \delta \geq 0.$$

After substituting $R$ into (1), we obtain the density function $D_1(\alpha, \delta)$, which is growing by $\delta$. Then the optimal value is $\delta = 0$ and, therefore, $R = \frac{1}{\sin \alpha}$, and we get the density function

$$D_1(\alpha) = \frac{\left(\frac{1}{\sin \alpha}\right)^2 \alpha}{\sqrt{\left(\frac{1}{\sin \alpha}\right)^2 - \left(1 - \frac{1}{2} \cos \alpha\right)^2 - \frac{1}{\sin \alpha} + \frac{1}{2 \sin \alpha}}}. $$

The minimum of the density is

$$\min_{\alpha > 0} D_1(\alpha) \approx 1.1767,$$

when

$$\alpha = \alpha_1 \approx 1.1418 > 65^\circ.$$

But if $\alpha > \pi/3$, then the model S1 is not a cover.

However, the function $D_1(\alpha)$ decreases, when $\alpha \in (0, \pi/3]$. Then the optimal value of the angle is $\alpha = \pi/3$, and

$$\min_{0 < \alpha \leq \pi/3; R \sin \alpha \geq 1} D_1(R, \alpha) = D_1(2/\sqrt{3}, \pi/3) = \frac{2\pi}{3\sqrt{3}}.$$

The proof is over.

The optimal cover for the model S1 is shown in Fig. 1b.
Remark. Since function $D_1(\alpha)$ decreases, then if $\alpha \in (0, \alpha_1]$, $\alpha_1 < \pi/3$, then the minimum density is reached when $\alpha = \alpha_1$ and $R = 1/\sin \alpha_1$, and it equals

$$2\alpha_1 \sqrt{\frac{1}{\sin^2 \alpha_1} - \left(1 - \frac{1}{2 \cos \alpha_1}\right)^2} - \sin 2\alpha_1 + \sin \alpha_1$$

4. Coverage Model S2

In model S2, because the angle $\alpha \geq \pi/3$ is greater than in model S1, two successive sectors with a side at one boundary of the strip do not intersect (Fig. 2). A pair of sectors symmetrical with respect to the center line covers a tile ($EACF$ in Fig. 2) that has a height of 1 and a width of $a = |AC|$.

Lemma 2. The density of the cover S2 is determined by function

$$D_2(R, \alpha) = \frac{R^2 \alpha}{2\sqrt{R^2 - 1}}$$

and its minimum

$$\min_{\pi/3 \leq \alpha \leq \pi/2; R \sin \alpha \geq 1} D_2(R, \alpha) = \frac{\pi}{3} \approx 1.0472,$$

when $\alpha = \pi/3$ and $R = \sqrt{2}$.

Proof. Refer to Fig. 2. It’s obvious that

$$|AD| = |CD| = R$$

and

$$|AB| = |BC| = a/2 = \sqrt{R^2 - 1}.$$

Then the density function

$$D_2(R, \alpha) = \frac{R^2 \alpha}{a} = \frac{R^2 \alpha}{2\sqrt{R^2 - 1}}$$

decreases with respect to the angle $\alpha \in [\pi/3, \pi/2]$. Then the optimal value of the angle is $\alpha = \pi/3$, and we get the density function

$$D_2(R) = \frac{R^2 \pi}{6\sqrt{R^2 - 1}}.$$

This is a convex function that reaches the minimum $\pi/3$ when $R = \sqrt{2}$.

The proof is over.

5. Coverage Model S3

The last coverage model, which we consider in this paper, is model S3, which differs from the previous models in that the sensors are located on the same boundary of the strip (Fig. 3a).

Lemma 3. The density of the cover S3 is determined as a function

$$D_3(R, \alpha) = \frac{R^2 \alpha}{2\left(\sqrt{R^2 - 1} - \frac{1}{\tan \alpha}\right)}$$

and

$$\min_{0 < \alpha \leq \pi/2; R \sin \alpha \geq 1} D_3(R, \alpha) = \pi/2 \approx 1.5708,$$

when $\alpha = \pi/2$ and $R = \sqrt{2}$.

Proof. Refer to Fig. 3a. The tile is $AEFC$ with height equals 1 and width $a = |AC|$. We have that

$$|AB| = |BC|, |AD| = \sqrt{R^2 - 1}$$

and

$$|BD| = \frac{1}{\tan \alpha}.$$

Then

$$a = |AB| + |BC| = 2(|AD| - |BD|) = 2\left(\sqrt{R^2 - 1} - \frac{1}{\tan \alpha}\right),$$

and the density function

$$D_3(R, \alpha) = \frac{R^2 \alpha}{a} = \frac{R^2 \alpha}{2\left(\sqrt{R^2 - 1} - \frac{1}{\tan \alpha}\right)}.$$

Minimum of this function

$$\min_{0 < \alpha \leq \pi/2; R \sin \alpha \geq 1} D_3(R, \alpha) = \pi/2,$$

when $R = \sqrt{2}$ and $\alpha = \pi/2$.

The proof is over.

The optimal cover for the model S3 is shown in Fig. 3b.
6. Conclusion

Comparing the proposed coverage models of the strip with identical sectors \((R, \alpha)\), \(\alpha \in (0, \pi/2]\), \(R \geq 1/\sin \alpha\), we can draw the following conclusions.

- If the angle \(\alpha\) can take any values from the domain \((0, \pi/2]\), then the best coverage model is model S2, which gives a coverage of the minimum density \(\min_{\alpha \in (0, \pi/2]} D_2(R, \alpha) = \pi/3 \approx 1.0472\) when \(\alpha = \pi/3 = 60^\circ\) and \(R = \sqrt{2} \approx 1.4142\).
- If the angle \(\alpha \in (0, \alpha_1]\), \(\alpha_1 < \pi/3\) and \(R \geq 1/\sin \alpha\), then model S2 is inadmissible and model S1 should be used with \(\alpha = \alpha_1\) and \(R = 1/\sin \alpha\). In this case, the minimum coverage density \(\min_{\alpha \in (0, \alpha_1]} D_1(R, \alpha)\) cannot be less than \(2\pi/3\sqrt{3} \approx 1.2092\).
- If the sensors can be placed only on one side of the strip, then model S3 should be used. In this case, the minimum coverage density \(\min_{\alpha \in (0, \pi/2]} D_3(R, \alpha) = \pi/2 \approx 1.5708\), when \(\alpha = \pi/2\) and \(R = \sqrt{2}\).

Since it is necessary to observe the strip from a certain side (in our case, from right to left), sectors with angles greater than \(\pi/2\) can be located inside the strip and should be oriented so that the covered points are to the left of the sensor. It can be shown that in this case it is more efficient to locate the sensors on the midline of the strip by orienting the sectors symmetrically relative to the midline (Fig. 4a). But such a cover can be reduced to the cover with sectors with an acute angle, which cover half the strip (Fig. 4b). But this cover is a cover S3 which covers a half of a strip. The density does not change.

In the strip coverings by identical sectors, which have been discussed earlier in [20], [22], and in which it is not important which sector covers this or that point of the strip, it was not possible to find the optimum values of the sensor parameters for any cover model. Consequently, it was not possible to determine the best coverage model. For the models considered in this paper, it is possible to find the optimum values of the sensor parameters. Moreover, in the case when the parameters \(\alpha \in (0, \pi/2], R \geq 1/\sin \alpha\) of the sensor \((R, \alpha)\) can take arbitrary feasible values, the model S2 is preferable.

Let’s compare the effectiveness of the covers. Since the sensing energy consumption is proportional to the sensor’s coverage area, cover S1 consumes \(1.2092 - 1.0472 = 0.162\) extra energy per unit time than the cover S2. Thus, the WSN in which the S2 coverage model is used has a lifetime of 16% more than the sensor network in which the S1 cover is used, and of 52% more than the sensor network in which the S3 cover is used.

In the case when it is required to monitor the strip in opposite directions (for example, when people walk along the corridor in both directions), it is sufficient to add a similar symmetrical cover to the constructed one, in which the devices are directed in the opposite directions. Moreover, to save money, the corresponding devices that are directed...
in different directions need to be mounted on a common site.

Acknowledgments. This research is supported in part by the Russian Foundation for Basic Research (grant No. 16-07-00552), the Ministry of Education and Science of the Republic Kazakhstan (project No. 0115PK00550) and by Russian Ministry of Science and Education under the 5-100 Excellence Programme.

For the numerical calculations we used the Maple 17.02 package licensed to the Novosibirsk State University (serial No. S2AJ447HV7HAJY5V).

References

[1] V. Zalyubovskiy, A. Erzin, S. Astrakov, H. A. Choo, Energy-efficient Area Coverage by Sensors with Adjustable Ranges, Sensors, 2009, 9(4), 2446–2460.
[2] S. N. Astrakov, A. I. Erzin, V. V. Zalyubovskiy, Sensor Networks and Covering the Plane with Disks, Diskretniy Analyz i Issledovanie Operaciy, 2009, 16(3), 3–19. (in Russian)
[3] S. N. Astrakov, A. I. Erzin, Covering an Infinite Strip with Disks of One and Two Radii, Trudi IVMiMG, Informatika, 2009, 9, 143–148. (in Russian)
[4] A. I. Erzin, S. N. Astrakov, Min-Density Stripe Covering and Applications in Sensor Networks, B. Murgante et al. (Eds.): ICCSA 2011, Part V, LNCS 6786, 2011, 9, 152–162.
[5] S. N. Astrakov, A. I. Erzin, Efficient Band Monitoring with Sensors Outer Positioning, Optimization: A J. of Mat. Programming and OR, 2013, 62(10), 1367–1378.
[6] S. N. Astrakov, A. I. Erzin, Sensor Networks and strip covering with Ellipses, Vychislitel’nye Tekhnologii, 2013, 18(2), 3–11. (in Russian)
[7] G. Fan, S. Jin, Coverage Problem in Wireless Sensor Network: Survey, Journal of Networks, 2010, 5(9), 1033–1040.
[8] J. Carle, D. Simplot, Energy-Efficient Area Monitoring by Sensor Networks, IEEE Comput., 2004, 37, 40–46.
[9] L. Fejes Tóth, Covering the Plane with Two Kinds of Circles, Discrete & Computational Geometry, 1995, 13(3), 445–457.
[10] N. Deshpande, A. Shaligram, Energy Saving in WSN with Directed Connectivity, Wireless Sensor Networks, 2013, 5(6), 121–125.
[11] R. Kershner, The Number of Circles Covering a Set, Am. J. Math., 1939, 61, 665–671.
[12] M. A. Guvensan, A. G. Yavuz, On Coverage Issues in Directional Sensor Networks: A Survey, Ad Hoc Networks, 2011, 9(7), 1238–1255.
[13] M. Cardei, J. Wu, M. Lu, Improving Network Lifetime Using Sensors With Adjustable Sensing Ranges, Int. J. Sensor Networks 2006, 1, 41–49.
[14] D. Ismailescu, B. Kim, Packing and Covering with Centrally Symmetric Convex Disks, Discrete Comput. Geom., 2014, 51(2), 495–508.
[15] A. I. Erzin, S. N. Astrakov, Covering a Plane with Ellipses, Optimization: A J. of Mat. Programming and OR, 2013, 62(10), 1357–1366.
[16] J. Ai, A. A. Abouzeid, Coverage by Directional Sensors in Randomly Deployed Wireless Sensor Networks, J. Combin. Optim., 2006, 11(1), 21–41.
[17] H. Ma, Y. Liu, Some Problems of Directional Sensor Networks, Int. J. of Sensor Networks, 2007, 2(1/2), 44–52.
[18] X. Han, X. Cao, E. L. Lloyd, Ch.-Ch. Shen, Deploying Directional Sensor Networks with Guaranteed Connectivity and Coverage. In: 5th Annual IEEE Communication Society Conference on Sensors, Mesh and Ad Hoc Communications and Networks, IEEE eXpress Conf. Publ., Piscataway, 2008, 153–160.
[19] C.-K. Liang, Y.-S. Lo, A Deployment Scheme Based Upon Virtual Force for Directional Sensor Networks, Sensors & Transducers, 2015, 194(11), 35–41.
[20] A. I. Erzin, N. A. Shabelnikova, On the Density of a Strip Covering with Identical Sectors, J. Applied and Industrial Mathematics, 2015, 9(4), 461–468.
[21] A. I. Erzin, L. Oosowa, Comparative Analysis of Regular Covers with One or Two Types of Sectors, NUMTA 2016, AIP AIP Conference Proceedings 1776, 050013, 2016, 5 pages.
[22] A. I. Erzin, Cost-Effective Strip Covering with Identical Directed Sensors, in Proc. DOOR 2016, CEUR-WS, Vladivostok, Russia, 2016, V. 1623, 701–712.
[23] S. N. Astrakov, A. I. Erzin, Construction of Efficient Coverage Models for Monitoring Extended Objects, Vychislitel’nye Tekhnologii, 2012, 17(1), 26–34. (in Russian)
[24] H. Mohamadi, S. Salleh, M. N. Razali, Heuristic methods to maximize network lifetime in directional sensor networks with adjustable sensing ranges, J. of Network and Computer Applications, 2014, 46, 26–35.