Soft Dipole Pomeron in hadronic elastic and in deep inelastic scattering

E. Martynov
Bogoliubov Institute for Theoretical Physics, 03143, Kiev-143, Ukraine
E-mail: martynov@bitp.kiev.ua

Abstracts. A brief review on the Dipole Pomeron model is given. The model not only describes data on hadron-hadron interactions, but also allows to describe data on the proton structure function with a $Q^2$ independent intercept. Moreover the chosen Dipole Pomeron has an intercept equal to one and does not violate unitarity limit on the total elastic cross-section.

The Regge approach [1] is one of the most powerful method to investigate hadronic interactions at high energies. In spite of many unsolved problems, its actual application is slowed down because of lacking new experimental data. Now, the main efforts of the experimentalists and theoreticians are concentrated on the deep inelastic scattering (DIS) and related processes. Again the Regge method shows its effectiveness in a wide kinematical domain.

It follows quite evidently from the observed growth of the hadronic total cross-sections and of the structure functions that the true Pomeron singularity is more complicated than that a simple $j$-pole. Even if a simple pole with an intercept larger than unity is used as an input, it should be unitarized in order to restore the Froissart-Martin bound [2], $\sigma_\text{tot}(s) < C \ln^2 s$. Unfortunately a well defined strict procedure of unitarization is still absent, while approximate methods are used; as consequence, we do not know what is really the true Pomeron.

The well known and quite popular example of a model with an input Pomeron violating the unitarity bound (because it has $\alpha_P(0) \approx 1.08 > 1$) is the Domnachie-Landshoff Pomeron (D-L) [3]. Moreover, and regrettably, in some papers this model is identified with the Regge theory and its inability to describe some experimental data is declared as a problem of Regge theory. However, and happily, the D-L Pomeron is not the only phenomenological possibility to describe the above mentioned growth of observed quantities. Another way to satisfy this property is to suppose that the Pomeron is harder than a simple pole at $j = 1$. The simplest realization of such an hypothesis is the Dipole Pomeron model [4].

The Dipole Pomeron (DP) is a combination of the double and simple poles in the partial $j$-plane amplitude

$$\Phi(j, t) \propto \frac{\phi_1(t)}{|j - \alpha_P(t)|^2} \approx \frac{\phi_1(t)}{|j - \alpha_P(t)|^2} + \frac{\phi_2(t)}{|j - \alpha_P(t)|^2}$$

with the trajectory $\alpha_P(t) \approx 1 + \alpha_P^\prime t$, linear at small $t$. The Dipole Pomeron is the unitarity limit for an isolated $j$-singularity with a unit intercept and with a linear trajectory at small $t$. If

$$\Phi(j, t) \propto (j - 1 - \alpha^' t)^{-\nu - 1}$$

then

$$\sigma_\text{tot} \propto \ln^\nu s/s_0, \quad \sigma_\text{el} \propto \ln^{2\nu - 1} s/s_0,$$

$$\sigma_\text{el}/\sigma_\text{tot} \leq 1 \quad \Rightarrow \quad \nu \leq 1.$$

Thus in the present Dipole Pomeron model at $s \to \infty$:

$$\sigma_\text{el}^{(P)}(s) \propto \sigma_\text{el}^{(P)}(s) \propto \sigma_\text{el}^{(P)}(s) \propto \ln(s/s_0).$$

The Dipole Pomeron in hadronic processes. At preasymptotic energies the contribution of other reggeons ($f, \omega \text{ etc...}$) should be added. If only $pp$ and $\bar{p}p$ amplitudes are considered one can write

$$A_{\text{(pp)}}^{(p)}(s, t) = \mathcal{P}(s, t) + f(s, t) \pm \omega(s, t)$$

with

$$\mathcal{P}(s, t) = i [g_1(t) \ln(-is/s_0) + g_2(t)(-is/s_0)^{\alpha_P(t) - 1}, \quad s_0 = \text{const},$$

$$r(s, t) = \eta f g_r(t)(-is/s_0)^{\alpha_r(t) - 1}, \quad r = f, \omega; \quad \eta_f = i, \quad \eta_\omega = 1.$$
The DP model was applied \(^{3}\) to the analysis of available data on the total cross-sections and on the real to imaginary part ratios (at \(t = 0\) and \(\sqrt{s} \geq 5\) GeV) for meson-nucleon and nucleon-nucleon interactions. It leads to the best description \((\chi^2/d.o.f. \approx 1.12)\) comparing with other models. A new very detailed and circumstantial analysis of these data confirming our conclusions is presented at this conference in the talk of V. Ezhela \(^{3}\).

Furthermore, in the framework of a modified additive quark model (where corrections for the coupling of Pomeron with two quark lines as well as new counting rules for the secondary reggeons were taken into account) \(^{3}\) the Dipole Pomeron was used to describe simultaneously cross-sections of \(p^\pm p, \pi^\pm p, \gamma p, \gamma \gamma\) interactions, related by factorization conditions.

An important property of the Dipole Pomeron model is that all fits give a high value of the \(f\)-reggeon intercept, \(\alpha_f(0) \approx 0.8 \pm 0.82\). Does such an intercept contradict to the data on the \(f\)-trajectory known from a resonance region? The answer is "yes" if trajectory is assumed to be linear. However besides of general theoretical arguments in favor of nonlinear trajectories, the experimental data on resonances lying on \(f\)-trajectory indicate its nonlinearity (see Fig.1). A more realistic trajectory than those shown in the Fig.1, \(\alpha_f(t) = \alpha_f(0) + \gamma_1 (\sqrt{4m_p^2 - \sqrt{4m_p^2 - t}}) + \gamma_2 (\sqrt{t_1} - \sqrt{t_1 - t})\), gives \(0.77 < \alpha_f(0) < 0.87\).

In the Fig.2, we show how \(\alpha_f(0)\) is correlated with a power of \(\ln s\) in behaviour of the total cross-section if the amplitudes are parameterized in the form (1-3), but with the replacement \(\ln(-is/s_0) \rightarrow \ln^2(-is/s_0)\) (see \(^{3}\) for details).

![Figure 1: Real part of f-trajectory](image1.png)

![Figure 2: Intercept of f-trajectory vs. γ.](image2.png)

**The Dipole Pomeron in deep inelastic scattering.** We support and investigate the point of view that there is only one Pomeron (two Pomerons, "soft" and "hard", are considered in \(^{3,4}\)). This unique Pomeron is universal and factorizable (at least the main term should satisfy factorization at c.m.s. energy \(W \gg m_p\)), i.e. it is the same in all processes, only vertex functions depend on interacting particles. Thus we think that Pomeron has a \(Q^2\) independent trajectory. As in the pure hadron case we use a Pomeron trajectory with a unit intercept \((\alpha_p(0) = 1)\).

Defining the Dipole Pomeron model for DIS, we start from the expression connecting the transverse cross-section of \(\gamma^p p\) interaction to the proton structure function \(F_2^p\) and the optical theorem for forward scattering amplitude

\[
\sigma_T^{\gamma^p p} = 3mA(W^2, Q^2; t = 0) = \frac{4\pi^2\alpha}{Q^2(1-x)}(1 + 4m_p^2x^2/Q^2)F_2^p(x, Q^2),
\]

where \(\sigma_T^{\gamma^p p} = 0\) is assumed. The forward scattering at \(W^2 = Q^2(1/x - 1) + m_p^2\) far from the threshold \(W_{th} = m_p\) is dominated by the Pomeron and the \(f\)-reggeon \(^{3}\)

\[
A(W^2, Q^2; t = 0) = P(W, Q^2) + f(W, Q^2), \quad P(W^2, Q^2) = P_1 + P_2, \quad P_1 = iG_1(Q^2)\ln(-iW^2/m_p^2)(1-x)B_1, \quad P_2 = iG_2(Q^2)(1-x)^{B_2}, \quad f(W^2, Q^2) = iG_f(Q^2)(-iW^2/m_p^2)^{\alpha_f(0)-1}(1-x)^{B_f}.
\]

\(^{2}\)We ignored an \(a_2\)-reggeon considering the \(f\)-term as an effective one at \(W > 3\) GeV.
It evidently follows from the experimental data (Fig.3) that $Q^2\sigma^\gamma p$ decreases with $Q^2$ at least at high $Q^2$. Therefore we choose

$$G_i(Q^2) = g_i/(1 + Q^2/Q_i^2)^{D_i},$$

expecting $D_i > 1$ at high $Q^2$. Here we only note that as it follows from the fit, $D_i$ and $B_i$ should be functions of $Q^2$. By hypothesis each of them varies between two constants when $Q^2$ goes from 0 to $\infty$. The details of the parameterization of the real functions $D_i(Q^2), B_i(Q^2)$ (as well as the references for the data) are given in the paper [10].

A detailed analysis [10] of the fit leads to the following conclusions:

- The possibility to describe available data with the Pomeron that does not violate the Froissart-Martin limit is shown.
- The preasymptotic contributions, $P_2$ and $f$, play an important role in the considered kinematical domain.
- The Dipole Pomeron has a $Q^2$ independent intercept. There is no contradictions with the "experimental data" on $\Delta(Q^2) = \alpha_P - 1$.

Indeed, the effective Pomeron intercept is extracted from the data on $F_2^p(x, Q^2)$ making use of the simplified parameterization of the structure function $F_2^p = a + bx^{-\Delta}$ at fixed $Q^2$. On the second hand, one can write

$$F_2^p(x, Q^2) = G(Q^2)(1/x)^{\Delta_{eff}(x,Q^2)}$$

and consequently

$$\frac{\partial \ln F_2^p}{\partial \ln (1/x)} = \Delta_{eff} + x \frac{\partial \Delta_{eff}}{\partial x} \ln(1/x)$$

If $\Delta_{eff}(x,Q^2) \approx \text{const}$ at $x \ll 1$ then

$$\frac{\partial \ln F_2^p}{\partial \ln (1/x)} \approx \Delta_{eff}.$$ 

Comparison of this derivative in our model with the experimental data on $\Delta_{eff}$ (or $\lambda_{eff}$ in the notation of the experimentalists) is shown in Fig.4. The open triangles and the curve correspond to the $\frac{\partial \ln F_2^p}{\partial \ln (1/x)}$ calculated in the given model.

There is an important prediction of the model concerning $\frac{\partial \ln F_2^p}{\partial \ln (1/x)}$ at fixed $x$. This $x$-slope begins to decrease when $Q^2 > \sim 100 \text{GeV}^2$. It would be interesting to compare the model with new high $Q^2$ data.
when they are published. A comparison of this quantity in our model [10], in a QCD motivated ALLM model [11] and in the interpolating model [12] (between a Regge-type behaviour and a solution of the evolution equation) is shown in Fig. 5.

The Soft Dipole Pomeron model well describes also the $Q$-slope or $\frac{\partial F_2^p(x,Q^2)}{\partial \ln Q^2}$. A more detailed discussion of the properties of the Dipole Pomeron Model (comparison with the data on $\sigma^{\gamma p}$, $F_2^p$ and so on) as well as some predictions of the model are given in the paper [10].

**Conclusion.** The available data on hadron-hadron and photon-hadron cross-sections can be described in a traditional Regge approach with a Soft Pomeron. It is not necessary to use an intercept depending on $Q^2$ and higher than one. A scale in $Q^2$, where a nonperturbative Regge behaviour is transformed to a QCD perturbative one, is still unknown in the presented approach and requires an additional consideration.

I thank very much my coworkers P. Desgrolard, M. Giffon, A. Lengyel and E. Predazzi for many fruitful discussions, for interesting and useful collaboration.

**References**

[1] P. D. B. Collins, *An introduction to Regge theory & high energy physics*, Cambridge University press (Cambridge) 1977.

[2] M. Froissart, *Phys. Rev.* 123, 1053 (1961); A. Martin, *Phys. Rev.* 129, 432 (1963) and *Nuovo Cimento* 29, 993 (1963).

[3] A. Donnachie and P. Landshoff, *Phys. Lett.* B 296, 227 (1992);

[4] T. Sawada, *Nuovo Cimento A* 48, 534 (1967); R. J. N. Phillips, Preprint of Rutheford Lab. RL-73-04, 1973; A. I. Bugrij, L. L. Jenkovszky, N. A. Kobylinsky and V. P. Shelest, *Lett. Nuovo Cimento* 6, 577 (1973); L. L. Jenkovszky, E. S. Martynov and B. V. Struminsky, *Phys. Lett.* B 249, 535 (1990).

[5] P. Desgrolard, M. Giffon, A. Lengyel and E. Martynov, *Nuovo Cimento A* 107, 637 (1994); P. Desgrolard, M. Giffon and E. Martynov, *Nuovo Cimento A* 110, 537 (1997).

[6] V. Ezhela et al., talk at this Conference, hep-ph/9908215.

[7] P. Desgrolard, M. Giffon, E. Martynov and E. Predazzi, Eur. Phys. Journ. C 9, 623 (1999).

[8] M. Bertini, *et al.*, *Rivista Nuovo Cimento* 19, 1 (1996).

[9] A. Donnachie and P. Landshoff, *Phys. Lett.* B 437, 408 (1998).

[10] P. Desgrolard, A. Lengyel and E. Martynov, Eur. Phys. Journ. C 7, 655 (1999); hep-ph/9811380.

[11] H. Abramovicz, A. Levy, hep-ph/9712415.

[12] P. Desgrolard, L. Jenkovszky, F. Paccanoni, Eur. Phys. Journ. C 7, 263 (1999).