Interaction induced Fermi-surface renormalization in the $t_1-t_2$ Hubbard model close to the Mott-Hubbard transition

Luca F. Tocchio, Federico Becca, and Claudius Gros

1 Institute for Theoretical Physics, Frankfurt University, Max-von-Laue-Straße 1, D-60438 Frankfurt a.M., Germany
2 CNR-IOM-Democritos National Simulation Centre and International School for Advanced Studies (SISSA), Via Beirut 2, I-34151, Trieste, Italy

(Dated: May 12, 2010)

We investigate the nature of the interaction-driven Mott-Hubbard transition of the half-filled $t_1-t_2$ Hubbard model in one dimension, using a full-fledged variational Monte Carlo approach including a distance-dependent Jastrow factor and backflow correlations. We present data for the evolution of the magnetic properties across the Mott-Hubbard transition and on the commensurate to incommensurate transition in the insulating state. Analyzing renormalized excitation spectra, we find that the Fermi surface renormalizes to perfect nesting right at the Mott-Hubbard transition in the insulating state, with a first-order reorganization when crossing into the conducting state.

PACS numbers: 71.27.+a, 71.10.Fd, 71.30.+h, 75.10.Jm

I. INTRODUCTION

Low-dimensional Fermi gases and insulators with intermediate and strong couplings show a plethora of interesting phenomena, both in the domains of synthesizable materials and of ultra-cold atom gases with the proximity of metallic, magnetic, superconducting, and insulating phases being a key target for experimental and theoretical studies. One-dimensional correlated electron systems are hence good targets, to give an example, for the exploration of photo-induced phase transitions having in part extremely large third-order non-linear optical susceptibilities, with possible applications to all-optical switching devices.

Here, we are interested in the nature of the interaction-driven Mott-Hubbard transition which occurs in the one-dimensional $t_1-t_2$ Hubbard model at half filling. In particular, we will assess the evolution of the Fermi surface by varying the Coulomb interaction. In the insulating state, the underlying Fermi-surface is given by the boundary of the occupied states of the renormalized dispersion relation, when the residual interactions giving rise to the charge gap are turned off in a Gedanken experiment. Mathematically, the underlying Fermi surface is defined in a non Fermi-liquid state as the locus in $k$-space where the real part of the one-particle Green’s function changes its sign. By investigating magnetic and charge properties, we find that the Fermi surface reconstructs in a first-order manner right at the Mott transition. In particular, the Fermi surface is generic, namely non-nesting, in the metallic side, whereas it has perfect nesting properties in the insulating state, at the transition point.

The paper is organized as follow: in section II, we introduce the Hamiltonian; in section III we describe our variational wave function; in section IV we present our numerical results and, finally, in section V we draw the conclusions.

II. MODEL

We consider the one-dimensional $t_1-t_2$ Hubbard model

$$\mathcal{H} = - \sum_{i,\sigma, n=1,2} t_{1,2} c_{i,\sigma}^\dagger c_{i+n,\sigma} + \text{H.c.} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}, \tag{1}$$

where $c_{i,\sigma}^\dagger$ is the electron creation operator, $\sigma = \uparrow, \downarrow$ the electron spin, $i = 1, \ldots, L$ the site index, $n_{i,\sigma} = c_{i,\sigma}^\dagger c_{i,\sigma}$ the electron density, $t_1$ and $t_2$ the nearest and next-nearest neighbor hopping amplitudes, and $U$ the on-site Coulomb repulsion. In this work, we focus our attention on the half-filled case with $L$ electrons on $L$ sites.

The ground state of the $t_1-t_2$ Hubbard model at half filling is predicted to be an insulator with gapless spin excitations (conventionally labeled as C0S1) for $t_2/t_1 < 1/2$ and every finite $U/t_1$ and small $U/t_1$ for a spin-gapped metal (C1S0) with strong superconducting fluctuations for $t_2/t_1 > 1/2$ and $U/t_1$, a fully-gapped spontaneously dimerized insulator (C0S0) for $t_2/t_1 > 1/2$ and large $U/t_1$. Our findings, which are summarized in Fig. I, are in very good agreement with these results. The locus of the metal-insulator transition has been investigated by several groups with slightly varying outcomes. Remarkably, a transition between incommensurate and commensurate spin excitations is expected to take place inside the C0S0 phase. Finally, we would like to mention that a tiny C2S2 phase could be stable for $U/t_1 \to 0$, as suggested by a weak-coupling renormalization group approach. Recent calculations showed that this phase can be further stabilized in presence of long-range interactions.

III. VARIATIONAL APPROACH

In this paper, we present a variational Monte Carlo study of the Hubbard model for $t_2/t_1 > 1/2$, which allows us to determine accurately the locus of the metal-
insulator transition, to study the transition between commensurate and incommensurate spin-spin correlations in the large-$U$ (dimerized) phase and to investigate its underlying Fermi surface. In particular, we will show that the magnetic correlations are related to the single-particle spectrum in the optimized variational wave function. Moreover, we will propose that the metal-insulator transition is driven in the Mott state by a renormalization of the underlying Fermi surface to perfect nesting.

Both the metallic and the insulating phases can be constructed, in a variational approach. In a first step, one constructs uncorrelated wave functions given by the ground state (BCS) of a superconducting Bardeen-Cooper-Schrieffer (BCS) Hamiltonian.23,24

\[ \mathcal{H}_{\text{BCS}} = \sum_{\mathbf{q},\sigma} \epsilon_{\mathbf{q}} c_{\mathbf{q},\sigma}^{\dagger} c_{\mathbf{q},\sigma} + \sum_{\mathbf{q}} \Delta_{\mathbf{q}} c_{\mathbf{q}+\mathbf{Q},\uparrow}^{\dagger} c_{\mathbf{q},\downarrow} + \text{H.c.}, \]

where both the free-band dispersion $\epsilon_{\mathbf{q}}$ and the pairing amplitudes $\Delta_{\mathbf{q}}$ are variational functions. We use the parametrization

\[ \begin{align*}
\epsilon_{\mathbf{q}} &= -2\tilde{t}_1 \cos q - 2\tilde{t}_2 \cos(2q) - \mu,
\Delta_{\mathbf{q}} &= \Delta_1 \cos q + \Delta_2 \cos(2q) + \Delta_3 \cos(3q),
\end{align*} \]

where the effective hopping amplitudes $\tilde{t}_1$ and $\tilde{t}_2$, as well as the effective chemical potential $\mu$ and the local pairing fields $\Delta_1$, $\Delta_2$, and $\Delta_3$ are variational parameters to be optimized. The excitation spectrum for Bogoliubov excitations is given by

\[ E_q = \sqrt{\epsilon_q^2 + \Delta_q^2}. \]

The correlated state $|\Psi_{\text{BCS}}\rangle$ is then given by

\[ |\Psi_{\text{BCS}}\rangle = \mathcal{J}|\Psi_{\text{BCS}}\rangle, \]

where $\mathcal{J} = \exp(-1/2 \sum_{i,j} v_{i,j} n_i n_j)$ is a density-density Jastrow factor (including the on-site Gutzwiller term), with the $v_{i,j}$ being optimized independently for ever distance $|i-j|$. Notably, within this kind of wave function, it is possible to obtain a pure (i.e., non-magnetic) Mott insulator by considering a sufficiently strong Jastrow factor,25 i.e., $v_q \sim 1/q^2$ ($v_q$ being the Fourier transform of $v_{i,j}$) and a Luttinger-liquid wave function with arbitrary critical exponents26 whenever $v_{i,j} \sim \log |i-j|$. In addition, a dimerized phase can be obtained just by considering a gapped BCS spectrum $E_q$ together with $v_q \sim 1/q^2$ (that is the case whenever $t_2/t_1 > 1/2$ and $U/t_1$ is large enough).27 Remarkably, in this case, finite dimer-dimer correlations are found at large distance, even though the wave function does not break the translational symmetry. Here, we do not report results on dimer-dimer correlations, that are found in the CO50 phase (see Ref. 23), but we concentrate on spin and charge properties, with a particular emphasis on the evolution of the Fermi surface by changing $t_2/t_1$ and $U/t_1$.

As we demonstrated recently,27 the projected BCS state $|\Psi_{\text{BCS}}\rangle$ can be improved further by considering backflow correlations, which modify the single-particle orbitals, in the same spirit as proposed by Feynman and Cohen.28 In this way, already the determinant part of the wave function includes now correlation effects. All results presented here are obtained by fully incorporating the backflow corrections and optimizing individually every variational parameter in $\epsilon_q$ and $\Delta_q$ of Eq. (3), in the Jastrow factor $\mathcal{J}$ of Eq. (5), as well as backflow corrections.
IV. RESULTS

A. Mott-Hubbard transition

The ground-state properties can be easily assessed by computing density and magnetic structure factors:

\[
N(q) = \frac{1}{L} \sum_{k,l} e^{i q (k-l)} \langle n_k n_l \rangle, \tag{6}
\]

\[
S(q) = \frac{1}{L} \sum_{k,l} e^{i q (k-l)} \langle S_k^z S_l^z \rangle, \tag{7}
\]

where \(n_k\) and \(S_k^z\) are the total density and the \(z\)-component of the spin operator on site \(k\), respectively.

The static density-density correlations behave qualitatively different in a metallic and a Mott-insulating state for small momenta \(q\), with the metallic state being characterized by a linear dependence of \(N(q) \sim q\), while in the insulating phase \(N(q) \sim q^2\). In Fig. 2 we present the behavior of \(N(q)/q\) across the transition for three values of the ratio \(t_2/t_1\). The locus of the Mott-Hubbard transition can be determined easily, allowing us to draw the phase diagram in Fig. 1. Our determination of the line separating the metallic and the insulating phase is in good agreement with Refs. 16,18.

The metallic region in the phase diagram can be described as a Luther-Emery liquid, with a finite gap in the spin excitation spectrum and gapless charge excitations.\textsuperscript{29} The charge stiffness \(K_p\) can be extracted, for example, from the long-distance behavior of the density-density correlations. In any conducting phase, we expect that \(K_p\) is also related to the slope of \(N(q)\) at small \(q\), i.e., \(N(q) \sim K_p q^2/\pi\). In fact, the latter equation, which is definitely valid in Luttinger liquids, should hold whenever the charge degrees of freedom are gapless.\textsuperscript{18} This procedure to obtain \(K_p\) works very well for the doped single-bond Hubbard model, namely for the model of Eq. \(1\) with \(t_2 = 0.2\) in comparison with the exact results, as obtained by Bethe ansatz.\textsuperscript{16} Unfortunately, in the metallic phase of the \(t_1-t_2\) model, the existence of very small gaps prevents us to obtain accurate results for \(K_p\).

B. Magnetic properties

The presence of short-range magnetic order is signaled by the appearance of a peak in \(S(q)\), for a certain momentum \(Q\). In the following, we will compare the magnetic properties with the renormalized single-particle spectrum \(E_q\) of the optimized variational wave function. The energy scale for \(E_q\) will be taken as the bandwidth \(W\) of the original free dispersion \(E_q^0 = \pm 2t_1 \cos(q) - 2t_2 \cos(2q)\). Note that, in the non-interacting case, there is only a single, perfectly nested Fermi surface for \(t_2/t_1 < 0.5\), with two Fermi points separated by \(\pi\). Instead, for \(t_2/t_1 > 0.5\) there are two Fermi seas and four Fermi points.

In the metallic phase, the spin properties are only slightly modified by the presence of a small but finite interaction \(U\), with respect to the \(U = 0\) behavior; for \(t_2/t_1 > 0.5\) the single-particle spectrum \(E_q\) exhibits four minima, at \(\pm k_1\) and \(\pm k_2\), and the peak of \(S(q)\) is located at \(Q^{\text{met}} = k_2 - k_1 = \pi/2\). The condition \(Q^{\text{met}} = \pi/2\) is determined by the Luttinger sum-rule for the metal, which states that the total volume of the Fermi sea equals the number of electrons. In the insulating phase, the situation changes qualitatively and the magnetic properties of the system become strongly affected by the value of \(t_2/t_1\).

In Fig. 3 we show the behavior of \(S(q)\) across the metal-insulator transition for \(t_2/t_1 = 0.75\), in comparison with the variationally determined renormalized single-particle spectrum \(E_q\). It can be observed that, when entering the insulating phase, the single-particle spectrum becomes strongly gapped and the two central minima collapse into a unique relative minimum at \(q = 0\), that subsequently disappears, as \(U/t_1\) increases. At the same time, the peak in \(S(q)\) shifts from \(Q^{\text{met}} = \pi/2\) to \(Q^{\text{ins}} = \pi\). Remarkably, just above the Mott transition, namely for \(5 < U/t_1 < 10\), the quantity \(2k_1\) (i.e., the distance between the two absolute minima of \(E_q\)) is slightly different from \(\pi\) and becomes commensurate.
only after a second transition (e.g., \( U/t_1 \simeq 9 \)), inside the insulating phase.\(^\text{18}\) However, the degree of incommensurability is very small and does not show up in corresponding shift of the peak in \( S(q) \) from \( Q^{\text{ins}} = \pi \). Indeed, the magnetic correlations are short-ranged and the peak in \( S(q) \) is consequently broad. A shift of the momentum away from \( \pi \) will therefore result in a shift of the maximum in \( S(q) \) only for a substantial degree of incommensurability.

In Fig. 4 we plot the spin-spin correlations \( S(q) \) and the single-particle spectrum \( E_q \) for the ratio \( t_2/t_1 = 0.9 \). In the metallic phase, the spin-spin correlations are always peaked at \( Q^{\text{met}} = \pi/2 \), while in the insulating phase the peak slowly shifts to \( Q \simeq 0.6\pi \). For \( U = 6 \) and \( 7 \) the single-particle spectrum \( E_q \) is qualitatively different from the one for larger \( U \)’s. As shown later, this is related to the different behavior of the variational hopping ratio \( t_2/t_1 \) close to the metal-insulator transition with respect to the strong-coupling regime. For larger values of the ratio \( U/t_1 \), \( E_q \) shows four local minima, with the peak in \( S(q) \) located at \( Q^{\text{ins}} = 2k_1 \), where \( 2k_1 \) is the distance between the two absolute minima.

Finally, in Fig. 5 we summarize the spin-spin correlations for different \( t_2/t_1 \), at a given value of \( U/t_1 \), chosen to be far enough from the metal-insulator transition in order to describe the large-\( U \) behavior of \( S(q) \). The peak in the spin-spin correlations exhibits the commensurate-incommensurate transition moving far from \( Q = \pi \), as the ratio \( t_2/t_1 \) is increased. When \( t_2/t_1 = 1.5 \) the system behaves already like in the \( t_2/t_1 \to \infty \) limit, with the peak commensurate to a lattice with a doubled unit cell (\( Q = \pi/2 \)). These results are in agreement with previous studies for the Heisenberg\(^\text{32}\) and the Hubbard model.\(^\text{15}\)

![FIG. 4: (Color online) Evolution of \( 2k_1 \) (distance between the minima of the single-particle spectrum \( E_q \)) in the insulating phase for \( t_2/t_1 = 0.75 \). Note the transition from an incommensurate to a commensurate value for \( U/t_1 \simeq 9 \).](image)

![FIG. 5: (Color online) Upper panel: Spin-spin correlations \( S(q) \) at \( t_2/t_1 = 0.9 \), for a \( L = 120 \) chain. Data are shown for \( U/t_1 = 5 \) (metal) and \( U/t_1 = 6, 7, 8, 12 \) (insulator). Lower panel: Single-particle spectrum \( E_q/W \) at \( t_2/t_1 = 0.9 \) for the same values of the electron-electron repulsion \( U \) and the same chain length.](image)

### C. Fermi surface renormalization

Finally, we present our central result, namely the fact that the metal-insulator transition is driven, in the Mott-insulating state, by a renormalization of the underlying Fermi surface to perfect nesting. With underlying Fermi surface, we mean the locus of the highest occupied momenta in the non-interacting spectrum \( \epsilon_q = -2t_1 \cos(q) - 2t_2 \cos(2q) \), obtained from the optimized variational hopping parameters. We would like to stress that the concept of an underlying Fermi surface is of central importance for the angular resolved photoemission spectroscopy (ARPES) studies of strongly correlated systems, like the high-temperature superconductors.\(^\text{5, 2} \)

Note, that \( E_q = \sqrt{\epsilon_q^2 + \Delta_q^2} \) corresponds within renormalized mean-field theory\(^\text{22}\) to the excitation spectrum of projected Bogoliubov quasiparticles and \( \epsilon_q \) hence to the dispersion of the renormalized quasiparticles. Moreover, recent calculations on the \( t–J \) and the periodic Anderson models highlighted the possibility to assess the Fermi surface from the parametrization of a variational wave function.\(^\text{22, 23}\) Here, the renormalization of the hopping...
parameters made it possible to show non-trivial deformations of the non-interacting Fermi surface, due to the Gutzwiller projection.

We show in Fig. 6 that the ratio $\tilde{t}_2/\tilde{t}_1$ in the metallic phase is almost equal to the bare value $t_2/t_1$, regardless of the degree of interaction. This weak renormalization of the band-structure in the metallic state is in agreement with a renormalization-group study, which predicts that the renormalization of the Fermi surface is proportional to $U^2$. Then, after the metal-insulator transition, the ratio jumps to a smaller value, very close to $1/2$. According to our data, we propose that the optimized variational ratio of $\tilde{t}_2/\tilde{t}_1$ is renormalized to $1/2$, exactly at the metal-insulator transition. This discontinuous behavior of the renormalized band structure is also evident in Fig. 6 e.g., for $t_2/t_1 = 0.9$, with the number of minima of the single-particle spectrum $E_q$ jumping four to two when entering the Mott-insulating state. We note that an analogous tendency towards a Fermi surface symmetrization in the insulating state has been observed in a study of a two-dimensional frustrated lattice.

A renormalization of the variational hopping ratio to $\tilde{t}_2/\tilde{t}_1 = 1/2$ implies that the Fermi surface is nested, with two Fermi points separated by a vector $\pi$. This perfect nesting condition drives the system to be an insulator, generating a charge gap as soon as electron-electron interaction is switched on. Remarkably, while the metal-insulator transition is driven by the renormalized dispersion $\epsilon_q$, the pairing terms $\Delta_q$ are crucial in determining the spin properties of the model, via the renormalized excitation spectra $E_q = \sqrt{\epsilon_q^2 + \Delta_q^2}$. Indeed, as shown for example in Fig. 7, the minima of the single-particle spectrum at $t_2/t_1 = 0.9$ are connected by an incommensurate vector, leading to an incommensurate peak in $S(q)$.

We have presented an extensive study of the phase diagram of the one-dimensional $t_1-t_2$ Hubbard model at half filling, with emphasis on the evolution of the magnetic properties and of the underlying Fermi surface across the interaction-driven Mott-Hubbard transition. We have shown that the magnetic correlations are related to the single-particle spectrum in the optimized variational wave function and we have described how they are affected by the metal-insulator transition. In the insulating phase, the peak in the spin-spin correlations exhibit the commensurate-incommensurate transition moving far from $Q = \pi$, as the ratio $t_2/t_1$ is increased, and then becomes commensurate to a doubled unit cell when $t_2/t_1 \geq 1.3$.

Our main findings culminate in the hypothesis that the underlying Fermi surface renormalizes to perfect nesting right at the transition in the insulating phase, with a first-order reorganization when crossing the transition into the metallic state. Similar renormalizations of the Fermi surface have been observed in two-dimensional models. Therefore, we believe that our results are important for an improved understanding of Mott-Hubbard transitions quite in general, transcending the specific one-dimensional physics.

L.F.T. and C.G. acknowledge the support of the German Science Foundation through the Transregio 49.

V. CONCLUSIONS

We have presented an extensive study of the phase diagram of the one-dimensional $t_1-t_2$ Hubbard model at half filling, with emphasis on the evolution of the magnetic properties and of the underlying Fermi surface across the interaction-driven Mott-Hubbard transition. We have shown that the magnetic correlations are related to the single-particle spectrum in the optimized variational wave function and we have described how they are affected by the metal-insulator transition. In the insulating phase, the peak in the spin-spin correlations exhibit the commensurate-incommensurate transition moving far from $Q = \pi$, as the ratio $t_2/t_1$ is increased, and then becomes commensurate to a doubled unit cell when $t_2/t_1 \geq 1.3$.

Our main findings culminate in the hypothesis that the underlying Fermi surface renormalizes to perfect nesting right at the transition in the insulating phase, with a first-order reorganization when crossing the transition into the metallic state. Similar renormalizations of the Fermi surface have been observed in two-dimensional models. Therefore, we believe that our results are important for an improved understanding of Mott-Hubbard transitions quite in general, transcending the specific one-dimensional physics.

L.F.T. and C.G. acknowledge the support of the German Science Foundation through the Transregio 49.

1. N. Toyota, M. Lang, and J. Müller, *Low-Dimensional Molecular Metals* (Springer, New York, 2007).

2. S. Giorgini, L.P. Pitaevskii, and S. Stringari, Rev. Mod.
Phys. 80, 1215 (2008).
3 I. Shinichiro and H. Okamoto, J. Phys. Soc. Jpn. 75, 011007 (2006).
4 H. Kishida, H. Matsuzaki, H. Okamoto, T. Manabe, M. Yamashita, Y. Taguchi, and Y. Tokura Nature (London) 405, 929 (2000).
5 C. Gros, B. Edegger, V.N. Muthukumar, and P.W. Anderson, PNAS 103, 14298 (2006).
6 T. Yoshida et al., Phys. Rev. B 74, 224510 (2006).
7 B. Edegger, V.N. Muthukumar, and C. Gros, Adv. Phys. 56, 927 (2007).
8 R. Sensarma, M. Randeria, and N. Trivedi, Phys. Rev. Lett. 98, 027004 (2007).
9 J. Kokalj and P. Prelovsek, Phys. Rev. B 75, 045111 (2007); Phys. Rev. B 78, 153103 (2008).
10 I. Dzyaloshinskii, Phys. Rev. B 68, 085113 (2003).
11 At half-filling the relative sign between $t_1$ and $t_2$ is irrelevant.
12 E.H. Lieb and F.Y. Wu, Phys. Rev. Lett. 20, 1445 (1968).
13 M. Fabrizio, Phys. Rev. B 54, 10054 (1996).
14 R. Arita, K. Kuroki, H. Aoki, and M. Fabrizio, Phys. Rev. B 57, 10324 (1998).
15 S. Daul and R.M. Noack, Phys. Rev. B 61, 1646 (2000).
16 M.E. Torio, A.A. Aligia, and H.A. Ceccatto, Phys. Rev. B 67, 165102 (2003).
17 C. Aebischer, D. Baeriswyl, and R.M. Noack, Phys. Rev. Lett. 86, 468 (2001).
18 C. Gros, K. Hamacher, and W. Wenzel, Europhys. Lett. 69, 616 (2005).
19 G.I. Japaridze, R.M. Noack, D. Baeriswyl, and L. Tincani, Phys. Rev. B 76, 115118 (2007).
20 C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, Phys. Rev. Lett. 97, 136401 (2006).
21 K. Louis, J.V. Alvarez, and C. Gros, Phys. Rev. B 64, 113106 (2001).
22 H.-H. Lai and O.I. Motrunich, Phys. Rev. B 81, 045105 (2010).
23 C. Gros, Phys. Rev. B 38, 931(R) (1988).
24 F.C. Zhang, C. Gros, T.M. Rice, and H. Shiba, Supercond. Sci. Technol. 1, 36 (1988).
25 M. Capello, F. Becca, M. Fabrizio, S. Sorella, and E. Tosatti, Phys. Rev. Lett. 94, 026406 (2005).
26 C.S. Hellberg and E.J. Mele, Phys. Rev. Lett. 67, 2080 (1991); R. Valentí and C. Gros, Phys. Rev. Lett. 68, 2402 (1992).
27 L.F. Tocchio, F. Becca, A. Parola, and S. Sorella, Phys. Rev. B 78, 041101(R) (2008).
28 R.P. Feynman and M. Cohen, Phys. Rev. 102, 1189 (1956).
29 J. Solyom, Adv. Phys. 28, 201 (1979).
30 M. Capello, F. Becca, S. Yunoki, M. Fabrizio, and S. Sorella, Phys. Rev. B 72, 085121 (2005).
31 H.J. Schulz, Int. J. Mod. Phys. B 5, 57 (1991).
32 R. Bursill, G.A. Gehring, D.J.J. Farnell, J.B. Parkinson, T. Xiang, and C. Zeng, J. Phys. C: Condens. Matter 7, 8605 (1995).
33 A. Himeda and M. Ogata, Phys. Rev. Lett. 85, 4345 (2000).
34 H. Watanabe and M. Ogata, Phys. Rev. Lett. 99, 136401 (2007); J. Phys. Soc. Jpn. 78, 024715 (2009).
35 J. Liu, J. Schmalian, and N. Trivedi, Phys. Rev. Lett. 94, 127003 (2005).
36 L.F. Tocchio, A. Parola, C. Gros, and F. Becca, Phys. Rev. B 80, 064419 (2009).