Influence of spatial position of gas bubbles in liquid on their joint dynamics

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Abstract. The influence of the mutual arrangement of initially identical spherical equidistant gas bubbles in liquid on their dynamics at the antinode pressure of an ultrasonic standing wave is studied. A mathematical model is used, in which small deformations of bubbles are taken into account. Two configurations of their relative position are considered: linear and planar. The first configuration is composed from three bubbles with their centers on one straight line, the second configuration is composed from five bubbles with their centers on two mutually orthogonal straight lines. It is shown that the magnitude of the deformations of the central bubbles in the linear and planar configurations is approximately the same.

1. Introduction
In the case of a relatively close arrangement of gas-vapor bubbles in liquid to each other, the hydrodynamic interaction between them can have a significant effect on their dynamics [1-4]. For example, the pressure acting on the bubbles in the center of the cluster can be much larger than at the periphery. On the other hand, the interaction between the bubbles can lead to their large deformations and destruction.

The interaction of bubbles may depend on various factors. In the present work, the influence of their mutual arrangement in the pressure antinode of an ultrasonic standing wave is studied. Two configurations of the mutual arrangement of bubbles are considered. In the first configuration, the bubble centers are on one straight line, and in the second configuration they are located on two mutually orthogonal straight lines. The dynamics of bubbles is described by a system of ordinary differential equations obtained by the method of spherical functions [3, 4] under the assumption that the liquid is weakly compressible, weakly viscous, and the bubbles are homobaric. In the equations, the possibility of strong radial oscillations of bubbles, their displacements in liquid, as well as their deformations, small in magnitude but arbitrary in shape, is taken into account.

2. Statement of the problem
We study the dependence of the dynamics of weakly-nonspherical gas bubbles on their mutual arrangement in liquid at the pressure antinode of a standing wave, where the pressure \( p_e \) varies according to the harmonic law

\[ p_e = p_0 - p_n \sin \omega t. \]
Here $p_0$ is the static pressure of the liquid; $p_a$, $\omega$ are the amplitude and frequency of oscillations; $t$ is time.

The considered configurations of the mutual arrangement of bubbles are illustrated in figure 1. In the first (linear) configuration, the bubble centers are on one straight line, whereas in the second (planar) configuration they are located on two mutually orthogonal straight lines. In the linear configuration, three equidistant bubbles are taken. The planar configuration is composed of five bubbles.

![Figure 1. Linear (left) and planar (right) configurations of the mutual arrangement of bubbles at the beginning of their interaction.](image)

Initially, all the bubbles are spherical, equal in radius, filled with air, and the peripheral equidistant from the central ones. The liquid is water at room conditions. The input parameters of the problem are the following: the sound velocity in liquid is $c_0 = 1500 \text{ m/s}$, the liquid density is $\rho_0 = 998 \text{ kg/m}^3$, the static liquid pressure is $p_0 = 1 \text{ bar}$, the surface tension coefficient is $\sigma = 0.0725 \text{ N/m}$, the dynamic viscosity of liquid is $\mu = 10^{-3} \text{ kg/(m s)}$, the adiabatic index is $\kappa = 1.4$, the amplitude of pressure oscillations in the wave is $p_a = 1.2 \text{ bar}$, the frequency of their oscillations is $\omega / 2\pi = 20 \text{ kHz}$. At the initial time $t = 0$, the bubble radius is $3 \mu \text{m}$, the distance between the centers of adjacent bubbles is $d_0 = 200 \mu \text{m}$.

### 3. Mathematical model

The equation of the surface of the $k$-th bubble ($k = 1, 2, \ldots, K, K$ is the number of the bubbles) is

$$F_k(r_k, \theta_k, \varphi_k, t) = r_k^2 - R_k(t) - \sum_{n=2}^{N} a_{nk}(t) \cdot Y_n(\theta_k, \varphi_k) = 0.$$  

Here $r_k$, $\theta_k$, $\varphi_k$ is the spherical coordinate system with the origin at the center of the $k$-th bubble; $Y_n(\theta_k, \varphi_k) = (Y_{nk} \cdot Y_{nk-1} \cdots Y_{nk} \cdots Y_{nk} \cdots Y_{nk})$. $Y_m = p_n^m(\cos \theta_k) e^{in\varphi_k}$ is a spherical function; $p_n^m$ is the associated Legendre polynomial of degree $n$ of order $|m|$; $i$ is the imaginary unit; $a_{nk} = (a_{nk} + ia_{nk}) / 2, (a_{nk} + ia_{nk}) / 2, \ldots a_{nk} \cdots, (a_{nk} - ia_{nk}) / 2, (a_{nk} - ia_{nk}) / 2 \}$; $a_{nk}$ is the amplitude of the deviation of the surface of the $k$-th bubble from the spherical shape $r_k = R_k$ in the form of the surface harmonics with the number $n$ and the order $m$; $N$ is the maximum of the number of spherical harmonics composing the bubbles surface.

To describe the joint dynamics of the bubbles we use the following system of ordinary differential equations of second order in the bubble radii $R_{ik, k}$, the radius vectors of their centers $p_{ik}$ and the vectors $a_{n,k}$, characterizing the deviation of the bubble surface from spherical,

$$R_{ik} = \frac{3R_{ik}^2}{2} - \frac{3p_{ik} \cdot B_{100} \cdot p_{ik}^T}{8} + \frac{2\sigma}{\rho_oR_{ik}} - \frac{p_k - p_{0k}}{\rho_0} + \psi_{ik} + \Delta_k = \sum_{j=1, j \neq k}^{K} \left[ \frac{B_{0j}}{d_{kj}^2} - \frac{B_{0j}}{d_{kj}^2} \left( R_{jk} \cdot C_{014} \right) \right] - \frac{9B_{0j}}{4d_{kj}^2} + \sum_{s=1, s \neq k}^{K} \left( R_{jk} \cdot B_{0s} \cdot C_{014} \cdot C_{014} \right) + \frac{6B_{0j}}{8d_{kjs}^2} \left( R_{jk} \cdot C_{014} \cdot C_{014} \right),$$  

(1)
Here the dashes and the dots denote differentiation with respect to time, $m = 2, 3, \ldots, N$, $\mathbf{p}_k = (x_k + iy_k)/2, z_k, (x_k - iy_k)/2)$, $x_k, y_k, z_k$ are the coordinates of the center of the $k$-th bubble, $\varepsilon_{ak} = a_{ak}/R_k, d_{kj}$ is the distance between the centers of the $k$-th and $j$-th bubbles, $p_i$ is the gas pressure in the $k$-th bubble, $\rho_0$ is the density of the liquid, $\sigma$ is the surface tension coefficient, $\psi_{ok}, \psi_{lk}, \psi_{mk}, \Lambda_k$ are corrections for of the effects of the liquid viscosity and compressibility [4], $\delta_{mn}$ is the Kronecker symbol; $B_{ok} = -R_k^2 \dot{R}_k^2, C^\gamma_{\xi k j} = (\gamma^\xi_{\gamma k j})$, $a_{\gamma \varphi} = (a_{\gamma \varphi}^\gamma_{\xi \varphi})$ are the matrices of numbers, $\zeta = -\gamma, -\gamma + 1, \ldots, \gamma$, $\xi = -\zeta, -\zeta + 1, \ldots, \zeta$, $C^\xi_{\xi k j} = Y_{\zeta k j}^\xi(\theta_k, \varphi_k) (-1)^{[\xi][\zeta][\varphi]}(1 + \zeta - \xi)|\xi - \zeta|!|\gamma - \xi|!|\gamma - \zeta|! \cdot \theta_k, \varphi_k$ are the angular coordinates of the center of the $j$-th bubble in a spherical system with the origin at the center of the $k$-th bubble, $\alpha_{\gamma \varphi}^\gamma_{\xi \varphi} = 2 \varphi + 1 \left(0 - 1\right)^{[\xi][\varphi]} 2\pi \int_0^\pi \int_0^{2\pi} \sin\theta_\xi \theta_\varphi Y_{\xi \varphi}(\theta_\xi, \varphi_\xi) Y_{\gamma \varphi}(\theta_\varphi, \varphi_\varphi) \rho_0 d\theta_\xi d\varphi_\varphi$, $\Theta^\gamma_\varphi = p a_{\gamma \varphi} + b_{\gamma \varphi}$, \( Y_{\gamma \varphi} = \gamma(\gamma + 1) + \zeta(\xi + 1) - \rho(\xi + 1) \mathbf{a}_{\gamma \varphi}^\gamma_{\xi \varphi}/2 \). Equations (1)-(3) are obtained by the method of spherical functions [3, 4] under assumption that the bubbles are not very close to each other, and their nonsphericity and spatial displacement velocities are small. Moreover, equations (1)-(3) take into account the deformations of the central bubbles only in the form of the second harmonic.

4. Results of calculations
Figure 2 shows the change in the radius $R_c$ of the central bubble in the linear configuration of three bubbles during one oscillation period of the liquid pressure $p_c$. It can be seen that the bubble experiences strong radial expansion and contraction. Thus, at the stage of lowering the pressure of the
liquid, it expands almost fourfold. The radii of other bubbles in this configuration, as well as the radii of all the bubbles in the planar configuration, vary approximately by the same law.

As a result of the interaction between the bubbles the surfaces of the central and other bubbles are deformed. Moreover, in all the configurations the deformations of the motionless central bubble appear lower than the deformations of the neighboring bubbles displacing to the central one. Figure 3 shows the changes of the mean-square dimensionless amplitude \[ |e_{2c}| = \sqrt{\sum_{m=-2}^{1} (\varepsilon_{2c})^2} \] of the deformations of the surface of the central bubbles in the form of the second harmonic (\( \varepsilon_{2c}^m = a_{2c}^m / R_c \)) during one period of the liquid pressure oscillations for the both configurations under consideration. It can be seen that the maximum deformations of the central bubbles arise in the vicinity of the intense radial pulsations, while at the beginning and at the end of the period the bubbles are practically spherical. At that, the mean-square amplitude of deformations of the central bubbles in the linear and flat configurations change approximately identically.

**Figure 2.** Change in the radius \( R_c \) of central bubbles (thick curve) during one period of the liquid pressure oscillations \( p_c \) (thin curve).

**Figure 3.** Change in the mean-square amplitude of the second harmonic, which determines the nonsphericity of the central bubbles in the configurations in question (figure 1).

**References**

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