Features of localized plasmons formation in four-particle spaser systems

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Abstract. The article is focused on the management problem of quantum dynamics of localized plasmons in model of a four-particle spaser which consists of metal nanoparticles and semiconductor quantum dots. The conditions for observation stable stationary regime of plasmon formation in the presented model have been determined by the use of mean field approximation.

1. Introduction
The recent rapid progress in the field of nanotechnology has led to practical possibility for generation and control of N-photon states using single quantum emitters [1] and high-quality micro- [2] and nanoresonators [3]. The key position in the development of such devices is creation of a unique device – nanolaser [4]. Functional principles of the nanolaser require a reformulation of the known rules of laser generation for subwavelength scales [5]. This can be done for systems containing a localized spaser [6]. In the simple case this device consists of a semiconductor quantum dot (QD) and a metal nanoparticle (NP) coupled by nearfield interaction. QD is used here as an effective nearfield pump source, because the excitonic decay in the QD leads to perturbations of NP near field. Under the conditions of a plasmon-exciton resonance this perturbation leads to generation of the plasmons localized on the NP surface [7, 8].

Realistic model of the spaser can be based on a compound nanoobject consisting of a metal core and a semiconductor shell [9] or on a distributed system of nanoparticles with the complex geometry [10, 11, 12, 13]. Currently, both localized subwavelength [4] and waveguide [14] spaser-like systems realized in practice.

In this article we particularly consider the possibility dynamics of plasmons generated in a multiparticle spaser system. For this, the model suggested in [15] has been extended for a $2 \times 2$ spaser system, which consists of two closely located NPs and two QDs coupled by strong local fields. Importantly the considered spaser systems can be integrated in the individual plasmonic waveguide [16] and information plasmonic circuit [17, 18, 19].

2. The formation of localized plasmons in the system of two spasers coupled by strong dipole-dipole interactions
Let us consider the system of two spasers, consisting of 2 QDs and 2 NPs (so-called Spaser $2 \times 2$), see Fig. 1. First of all, the efficiency of interaction in the system will depend on the
in particular, interaction Hamiltonian between NP and QD can be written in the form of dipole-dipole energy exchange, corresponding interband transitions in QD, where

\[
\sum_{i} (2n+1) P_{nm}^{m} (\cos \theta) e^{i m \varphi} \text{ are spherical functions expressed via Legendre polynomials, } c_{NP} \text{ defines the orientation of NP dipole moment, } E_{nm} = \sqrt{\frac{\hbar \omega_{nm}}{2a_{NP}(2n+1)\varepsilon_{0}}} \text{ is the dimensional factor, where } n \text{ is the general quantum and } m \text{ is the magnetic quantum numbers [8], } a_{NP} \text{ is the radius of } i\text{-th NP.}
\]

Then, near field of single QD is written as

\[
\hat{E}_{QDi} = \frac{1}{4\pi\varepsilon_{0}} \frac{3\hat{n} \cdot \hat{c}_{QDi}}{r^{3}} \hat{c}_{QDi} \hat{d}_{QDi},
\]

where dipole moment operator \(\hat{d}_{QDi} = \mu_{QDi} (\hat{S}_{i} + \hat{S}_{i}^{+}) \hat{c}_{QD} \) expressed via creation operator \(\hat{S}_{i}^{+} = |e\rangle_{i} \langle g|_{i}\) and annihilation operator \(\hat{S}_{i} = |g\rangle_{i} \langle e|_{i}\) of excitons and matrix element \(\mu_{QDi}\) corresponding interband transitions in QD, where \(|e\rangle_{i}\) corresponds to exited and \(|g\rangle_{i}\) ground states of the system. Presented operators satisfy commutation relationships \([\hat{S}_{i}^{+}, \hat{S}_{i}] = D_{i}\) and \([\hat{S}_{i}, D_{i}] = 2\hat{S}_{i}\), where \(D_{i} = \hat{S}_{i}^{+} \hat{S}_{i} - \hat{S}_{i} \hat{S}_{i}^{+}\) is the population imbalance operator; \(\hat{c}_{QDi}\) determines the QD dipole momentum orientation.

In conditions \(\lambda_{1,2} \gg r > a_{NPi}, a_{QDi}\) where \(a_{QDi}\) is the QD radius, and \(\lambda_{1,2}\) is the transition wavelength in QD, all pair interactions correspond to dipole-dipole energy exchange, in particular, interaction Hamiltonian between NP and QD can be written in the form

\[
V_{i}^{QN} = -\hat{E}_{NP}^{\parallel} \hat{d}_{QDi}. \text{ Presented in Fig. 1 geometry corresponds to the case } \theta = 0 \text{ and therefore the Legendre polynomials take the form } P_{1}^{\parallel} (\cos \theta) = 1, P_{1}^{\parallel \neq 0} (\cos \theta) = 0. \text{ Thus, the near-field of NP in the position of QD location has the form}
\]

\[
\hat{E}_{NPi} = \sqrt{\frac{\hbar \omega_{c} a_{NPi}^{3}}{2\pi\varepsilon_{0}}} \frac{1}{r^{3}} (\hat{c}_{i} + \hat{c}_{i}^{+}) \hat{c}_{NPi},
\]
where \( \omega_{pl} \) determine plasmon frequencies of NP\(_1\). Interaction Hamiltonian for adjacent NPs \( V^{NN} = -\hat{E}_{NP1} \hat{d}_{NP2} \) is determined by orientation \( \theta = \pi/2 \), for which \( P^1_{\theta} (\cos \theta) = 1 \) and \( P^{m\neq1}_{\theta} (\cos \theta) = 0 \). In this case, the expression for the field takes the form

\[
\hat{E}_{NP1} = \sqrt{\frac{\hbar \omega_{pl} a_{NP1}^3}{4\pi \varepsilon_0}} \frac{1}{r^3} (\hat{c}_i + \hat{c}_i^+) \hat{e}_{NP1},
\]

and corresponding induced dipole moment of NP\(_1\). This dipole moment can be obtained from comparison (3) and the expression for near field of NP\(_1\) in analogy with (1). This expression has the form \( \hat{d}_{NP1} = \mu_{NP1} (\hat{c}_i + \hat{c}_i^+) \hat{e}_{NP} \), where \( \mu_{NP1} = \sqrt{4\pi \varepsilon_0 \hbar \omega_{pl} a_{NP1}^3} \). Finally, interaction Hamiltonian \( V^{QQ} = -\hat{E}_{QD1} \hat{d}_{QD2} \) between single QD in spaser structure is determined by the given geometry, for which \((\vec{n} \cdot \hat{e}_{QD1}) = 0\).

Based on the necessity of internal symmetry of layer arrangement for QD and NP in case of scaling spaser system to 2D massive of spasers we assume \( r_{NP} = r_{QD} = r_1, r_{QN} = r \). The working regime of coupled spasers significantly depends on the ratio between frequencies \( \omega_{1,2} \) of transitions in QDs and plasmon frequencies \( \omega_{pl1,2} \). They are usually [6, 20] considered almost equal to each other so that there are mainly implemented linear plasmon interactions in the system. Then, in the case \( \omega_i \approx \omega_{pi} \), corresponding Hamiltonian of interaction takes the form:

\[
H = \hbar \omega_{pl1} \hat{c}_1^+ \hat{c}_1 + \hbar \omega_{pl2} \hat{c}_2^+ \hat{c}_2 + \frac{\hbar \omega_1}{2} D_1 + \frac{\hbar \omega_2}{2} D_2
\]

\[
+ \hbar \Omega_1 \left( \hat{c}_1 \hat{S}_1^+ + \hat{c}_1^+ \hat{S}_1 \right) + \hbar \Omega_2 \left( \hat{c}_2 \hat{S}_2^+ + \hat{c}_2^+ \hat{S}_2 \right)
\]

\[
+ \hbar \Omega_{QQ} \left( \hat{S}_1 \hat{S}_2^+ + \hat{S}_1^+ \hat{S}_2 \right) + \hbar \Omega_{pp} \left( \hat{c}_1 \hat{c}_2^+ + \hat{c}_1^+ \hat{c}_2 \right),
\]

where 5-th and 6-th terms with \( \Omega_i = \sqrt{\frac{\omega_{pl1} a_{NP1}^3}{2\pi \varepsilon_0 \hbar \omega_{pl1} \omega_{pl2}} \mu_{PP}} \) correspond to \( V^{QN} \), 7-th with \( \Omega_{QQ} = \frac{\mu_{QQ}}{4\pi \varepsilon_0 \hbar \omega_{pl1}} \) appears from \( V^{QQ} \) and term with \( \Omega_{pp} = \frac{\mu_{pp}^2 \pi}{4\pi \varepsilon_0 \hbar \omega_{pl1}} \) is defined by Hamiltonian \( V^{NN} \). The cross-interaction between NP and QD from neighboring spasers we neglect. In (4) we don’t take into account the Ferster transfer of energy between QDs, although in a real situation it must be done.

As a model we choose a spaser consisting of metal NP and QD based on semiconductor CdS, for which the chosen states \( |g\rangle_i \) and \( |e\rangle_i \) correspond to hole level 1S (\( h \)) in valence band and electron level 1S (\( e \)) in conduction band. Estimates of QD sizes can be performed proceeding from the plasmon mode frequency \( \omega_p = \omega_{pl} = \omega_{pl2} \), which for spherical metal NP is determined by expression \( \omega_p = \frac{\omega_{pl}}{\sqrt{3}} \). For satisfying exact resonance condition \( \omega_p = \omega \) between NP and QD, the size of QD is determined by known dependence [21] of the interband transition energy 1S (\( e \)) → 1S (\( h \)) on its diameter \( D_{QD} = 2a_{QD} \):

\[
E_{hh} = \hbar \omega_p = E_g + 2 \frac{\hbar^2 \pi^2}{D_{QD}^2} \left( \frac{1}{m_e} + \frac{1}{m_h} \right) - \frac{3.56 \cdot e^2}{\varepsilon \cdot D_{QD}},
\]

where \( e \) is the electron charge, \( h \) is the Planck constant, \( m_e \) and \( m_h \) are the effective masses of electron and hole in bulk of QD material, respectively, with dielectric permittivity \( \varepsilon \) and bandgap \( E_g \). For CdS corresponding parameters are \( E_g/e = 2.42 \) eV, \( m_e = 0.19m_0 \), \( m_h = 0.8m_0 \), \( \varepsilon = 9 \) and \( D_{QD} = 1.88 \) nm. Bohr radius of exciton \( R_{ex} \) for CdS is 2.5 nm [22] therefore strong confinement regime [23] will be observed for the considered QDs, and energy sublevels of
conductivity zone will be essentially separated. Therefore, it can be assumed that the two-level model will be valid for QDs. However, the size of QDs is sufficient for multiple generation of electron-hole pairs under the influence of an external pump. The subsequent decay of excitons leads to the formation of multi-particle plasmon states, which are localized on the NPs. However, assuming that the electron-hole pairs generated in the QDs have similar values of the energy, the summation symbol in (4) can be eliminated by use of a single-mode Hamiltonian.

The value of dipole moment for corresponding interband transition [24] can be approximately determined as $|\mu_{QD}|^2 = \frac{e^2}{6\pi\varepsilon_0} \left( \frac{m_0}{m_e} - 1 \right) E_{bb}$ and in selected conditions equals to $\mu_{QD} = 0.16 \cdot 10^{-28}$ C · m. The dipole moment of NP with radius, which exactly coincide with radius of QD, equals to $S^{2/3} \cdot \mu_m$. We suppose that spaser 2 × 2 is a square with characteristic sizes $r_1 = r = 3$ nm and $a_{NP} = a_{QD}$, corresponding Rabi frequencies in (4) take a value $\Omega_1 = \Omega_2 = \Omega = 2 \cdot 10^{13}$ s$^{-1}$, $\Omega_{QQ} = 8.46 \cdot 10^{11}$ s$^{-1}$, $\Omega_{pp} = 1.54 \cdot 10^{14}$ s$^{-1}$. It is shown that the efficiency of dipole-dipole interaction between QDs in the presented geometry is significantly lower for the similar efficiency both between the adjacent NPs, and in a pair of NP-QD. Thus, for further consideration of the limit of a single-particle states, the term with $\Omega_{QQ}$ can be neglected, and then we can proceed to the consideration of the following Heisenberg-Langevin system of equations obtained from (4):

$$
\dot{\hat{c}}_1 = i \left( \Delta_1 + \frac{i}{\tau_{c1}} \right) \hat{c}_1 - i\bar{\Omega}_1 \hat{S}_1 - i\Omega_{pp} \hat{c}_2 + \hat{F}_{c1}, \quad (6a)
$$

$$
\dot{\hat{c}}_2 = i \left( \Delta_2 + \frac{i}{\tau_{c2}} \right) \hat{c}_2 - i\bar{\Omega}_2 \hat{S}_2 - i\Omega_{pp} \hat{c}_1 + \hat{F}_{c2}, \quad (6b)
$$

$$
\dot{\hat{S}}_1 = i \left( \delta_1 + \frac{i}{\tau_{S1}} \right) \hat{S}_1 + i\bar{\Omega}_1 \hat{D}_1 \hat{c}_1 + \hat{F}_{S1}, \quad (6c)
$$

$$
\dot{\hat{S}}_2 = i \left( \delta_2 + \frac{i}{\tau_{S2}} \right) \hat{S}_2 + i\bar{\Omega}_2 \hat{D}_2 \hat{c}_2 + \hat{F}_{S2}, \quad (6d)
$$

$$
\dot{\hat{D}}_1 = -2i\Omega_1 \left( \hat{S}^\dagger \hat{c}_1 - \hat{S} \hat{c}_1^\dagger \right) - \frac{D_1 - D_{01}}{\tau_{D1}} + \hat{F}_{D1}, \quad (6e)
$$

$$
\dot{\hat{D}}_2 = -2i\Omega_2 \left( \hat{S}^\dagger \hat{c}_2 - \hat{S} \hat{c}_2^\dagger \right) - \frac{D_2 - D_{02}}{\tau_{D2}} + \hat{F}_{D2}, \quad (6f)
$$

where $\Delta_1 = \bar{\omega} - \omega_{p1}$, $\Delta_2 = \bar{\omega} - \omega_{p2}$, $\delta_1 = \bar{\omega} - \omega_1$, $\delta_2 = \bar{\omega} - \omega_2$, and parameters $\bar{\omega}$ and $D_{01(02)}$ correspond to the frequency and value of spaser pump, respectively. In the deriving system (6) we used rotating-wave approximation $\hat{c} = \hat{c} \cdot \exp(-i\bar{\omega}t)$ and $\hat{S} = \hat{S} \cdot \exp(-i\vartheta t)$ upon passage to the new slow-varying operators $\hat{c}(\hat{c}^\dagger)$ and $\hat{S}(\hat{S}^\dagger)$. In equations (6) the characteristic parameters of decay rate for plasmons $\frac{1}{\tau_{c1(c2)}}$ in NP, the decay rate of excitons $\frac{1}{\tau_{S1(S2)}}$ in excited QDs, and also the operators of Langevin noises $\hat{F}_{c1(c2)}$ ($\hat{F}_{S1(S2)}$, $\hat{F}_{D1(D2)}$) are introduced phenomenologically [25], proceeding from a condition of system interaction with the reservoir. The pump operator and appropriate time, generally, have the forms $D_{01(02)} = \frac{2\tau_{S1(S2)} - \tau_{p1(p2)}}{2\tau_{S1(S2)} + \tau_{p1(p2)}} H$ and $\tau_{D1(D2)} = \left( \frac{1}{\tau_{S1(S2)}} + \frac{1}{\tau_{p1(p2)}} \right)^{-1}$ [26], where parameters $\tau_{p1(p2)}$ are presented by characteristic pumping times. Further, we assume that both NPs (and also both QDs) in spaser 2 × 2 are identical to each other, consequently, it is possible to suppose $\Omega_1 = \Omega_2$, $\tau_{c1} = \tau_{c2} = \tau_c$, $\tau_{S1} = \tau_{S2} = \tau_S$, $\Delta_1 = \Delta_2 = \Delta (\omega_{p1} = \omega_{p2} = \omega_p)$, $\delta_1 = \delta_2 = \delta (\omega_1 = \omega_2 = \omega)$. Besides, we assume that populations in both QDs change synchronously, i.e. $D_1 = D_2 = D$.

Going over to the estimates of relaxation parameters of the problem, it should be noted, that decay rate of plasmon mode $\gamma_p = \frac{1}{\tau_c} = \frac{1}{\tau_f} + \frac{1}{\tau_m}$ is determined by characteristic time of radiation...
\( \tau_R \) and Joule \( \tau_I \) losses. However, under the condition \( \frac{1}{\tau} \approx 30 \frac{1}{\tau_R} \) \[27\] we do not have to take radiation losses into account and Joule losses are normally determined by electron collisional frequency in metal \( \gamma_s \), i.e. \( \gamma_p \approx \gamma_s \), where \( \gamma_s = 4 \cdot 10^{13} \text{ s}^{-1} \). A more correct calculation of the decay rate of plasmons localized on the NPs requires consideration of its geometrical features, as well as a more precise chemical composition \[28\].

The parameter \( \frac{1}{\tau} = \frac{1}{\tau_R} + \frac{1}{\tau_F} \) presents the total rate of radiative (with time \( \tau_R \)) and nonradiative (with time \( \tau_F \)) losses in QD. At the same time, parameter \( \frac{1}{\tau_F} \) gives the main contribution, since the processes of nonradiative recombination of excitons (with creating phonon modes) occur on short times \( \tau_F \) when annealing technology \[29\] is used, this parameter can be essentially increased \[30\]. Therefore, following the work \[6\] we accept the estimate \( \frac{1}{\tau_F} = 4 \cdot 10^{10} \text{ s}^{-1} \). A more correct approach requires taking into account the binding energy of excitons and the accurate derivation of radiative damping rate dependencies by analogy with quantum well model \[31\].

3. The stationary solutions for four-particle spaser

We determine characteristic spaser generation frequency (spasing frequency) \( \dot{\omega} \), spasing threshold \( D_{th} \), and also find the possible stationary regimes of its time evolution. For this purpose, at the initial stage we write the system of equations for average values, analogical to operator system (6), assuming that now \( c = \langle \hat{c} \rangle \), \( c^* = \langle \hat{c}^+ \rangle \) and consider that Langevin noise operators are equal to zero. Then, assuming \( \dot{c}_1(2) = \hat{D} = \dot{S}_1(2) = 0 \), we express \( c_2 \) from equation on average values (6a) and substitute obtained expression in equations on average values (6b) and (6d). After it, we express parameter \( S_2 \) from (6d) and substitute it in (6b) and (6d). Obtained new system of algebraic equations on the average values will be written in the form:

\[
\begin{align*}
\Omega D c_1 + B S_1 &= 0, \\
\left( \frac{A^2}{\Omega_{pp}} + \frac{A \Omega^2 D}{B \Omega_{pp}} - \Omega_{pp} \right) c_1 + \left( -\frac{\Omega A}{\Omega_{pp}} - \frac{\Omega^3 D}{\Omega_{pp} B} \right) S_1 &= 0,
\end{align*}
\]

where new definitions \( A = A_R + i A_I = \Delta + \frac{i}{\tau} \) and \( B = B_R + i B_I = \delta + \frac{i}{\tau} \) are introduced. The system (7) can have non-trivial solutions in the case, when matrix determinant for the left side of system of equations is equal to zero. Thus, we obtain the system of two self-consistent equations for the real and imaginary parts of such determinant

\[
\begin{align*}
\Omega^4 D^2 + 2 (A_R B_R - A_I B_I) \Omega^2 D \\
+ (B_I^2 - B_R^2) (A_I^2 - A_R^2 + \Omega_{pp}^2) \\
- 4 A_R B_R B_I B_R &= 0, \quad \text{(8a)} \\
2 (A_R B_I + A_I B_R) (A_R B_R - A_I B_I + \Omega^2 D) \\
- 2 \Omega_{pp}^2 B_I B_R &= 0. \quad \text{(8b)}
\end{align*}
\]

Solving these equations (8a) and (8b) together, it is possible to obtain expressions for spasing frequency \( \dot{\omega} \) and for threshold value \( D_{th} \). In this case, the system of equations has two roots in the form

\[
\dot{\omega}_\pm = \frac{\tau_S \omega + \tau_c (\omega_p \pm \Omega_{pp})}{\tau_c + \tau_S}, \quad \text{(9a)}
\]

\[
D_{th, \pm} = \frac{1 + \left( \frac{\tau_c \tau_S}{\tau_c + \tau_S} \right)^2 (\omega - \omega_p \pm \Omega_{pp})^2}{\tau_c \tau_S \Omega^2}. \quad \text{(9b)}
\]
one of which \((\bar{\omega}_-, D_{th,-})\) will not give stable solution for (6), and another we will assume that \(D_{th} = D_{th,+}\).

It is should be noted, that in the limit case \(\Omega_{pp} = 0\) solutions (9) coincide with a known model of spaser \(1 \times 1\) which consists of one QD and one NP [7]. However, the presence of near-field interaction between two NPs significantly increases the spasing threshold. The obtained in (9) solutions correspond only to the situation for the regime of linear energy exchange between NP and QD determined by Hamiltonian (4), which can be implemented under conditions of small frequency detunings \(\Delta\) and \(\delta\). Under selected parameters of interaction, the values of thresholds are \(D_{th}^{1\times1} = D_{th}(\Omega_{pp} = 0) = 0.004\) and \(D_{th}^{2\times2} = D_{th}(\Omega_{pp} = 1.54 \cdot 10^{14} \text{s}^{-1}) = 0.0632\), and in further simulation pumps are supposed to equal each other for both types of spaser and are chosen according to the conditions \(D_0 > \max(D_{th}^{1\times1}, D_{th}^{2\times2})\) and \(D_0 = 0.1\).

Assuming \(c_1 = c_2 = c\) \((S_1 = S_2 = S)\), we express \(c\) from equation (7b) and substitute obtained expression in the result of summing equations on average values (6e) and (6f), which are obtained from condition \(D = 0\). As a result, steady-state solutions for the amplitudes of generating on NPs plasmons and forming in QD excitons take forms which are determined up to the phase \(\phi\):

\[
\bar{c} = e^{i\bar{\phi}} \sqrt{\frac{\tau_c}{4\tau_p}} (D_0 - D_{th}), \tag{10a}
\]

\[
\bar{S} = \frac{i + \frac{\tau_c\tau_p}{\tau_c + \tau_p} (\omega - \omega_p - \Omega_{pp})}{\tau_c\Omega} \bar{c}. \tag{10b}
\]

The test on stability of obtained solutions was carried out by analysis of eigenvalues \(\lambda_i\) of the linearized system of equations on average values (6) near fixed points (10), as well as by using a direct numerical simulation of this system. At the same time, for obtained solutions (10) one of the roots of characteristic equation for linearized system (6) on average values is always equal to zero, while other eigenvalues are satisfied to inequality \(\text{Re}(\lambda_i) < 0\). Thus, the system moves to the boundary of aperiodic stability in the absence of the external synchronization. From the mathematical point of view, in such case further analysis full nonlinear system (6) should be carried out. Nevertheless, numerical analysis of the system (6) demonstrates the stability of obtained solutions (10).

The parametric plane formed by the combination of the parameters pump value \(D_0\) and the characteristic time of excitons decay \(\tau_S\) in the system with the depicted stability area for the solutions (10) verified by numerical simulation of system (6) is presented in Fig. 2. Obviously, in the case of using chemical inhomogeneous (not pure in composition) QDs jointly leading to decreasing value \(\tau_S\), it is necessary to increase pump value \(D_0\) for supporting stationary conditions for spasing. At the same time, increasing \(D_0\) leads to the linear growth of the number of plasmons \(|c|^2\) which are generated by spaser provided that the chosen characteristic pumping time is \(\tau_D = 2.7 \cdot 10^{-15}\text{s}\), the inset in Fig. 2.

The validity test of the obtained solutions (10) with taking into account correction \(\Omega_{pp}\) has been carried out by their comparing with direct numerical simulation of the system (6) for average values. In particular, in Fig. 3, performed for parameter \(D\), the consent of analytical and numerical results is clearly observed, when choosing the parameters of interaction for point A from presented area of stability in Fig. 2. At the same time, the dynamics of the transition process to the stationary values for \(D\) and \(|c|^2\) also has a significant dependence on \(\Omega_{pp}\). In particular, fast synchronization regime of energy exchange processes between QD and NP under condition \(\Omega_{pp} = 0\) is replaced by more "protracted" in time stabilization process taking into account the contribution of \(\Omega_{pp}\), from Fig. 3, 4. Besides, when accounting \(\Omega_{pp}\) more slowly and intensive energy exchange is established in the system, which can be observed by using temporal dynamics of changing amount \(\sin(\Delta \phi(t))\), where parameter \(\Delta \phi(t) = \text{Arg}\left(\frac{c(t)}{S(t)}\right)\) determines a
Figure 2. The parametric plane (pump value $D_0$, decay time of excitons in QD $\tau_S$) with depicted stability area of $2 \times 2$ spaser and a point A ($0.5, 0.25 \cdot 10^{-10} \text{ s}$). In the inset: the dependences of average numbers of plasmons $|\bar{c}|^2$ (blue thick lines) and excitons $|\bar{S}|^2$ (red thin lines) on the value of pump for spaser $2 \times 2$ with accounting $\Omega_{pp}$ (solid lines) and for spaser $1 \times 1$ without accounting $\Omega_{pp}$ (dashed lines). The parameters of interaction: $\omega_p = \omega = 7.9 \cdot 10^{15} \text{ s}^{-1}$, $\Omega = 2 \cdot 10^{13} \text{ s}^{-1}$, $\Omega_{pp} = 1.54 \cdot 10^{14} \text{ s}^{-1}$, $\tau_c = 0.25 \cdot 10^{-13} \text{ s}$, $\tau_D = 2.7 \cdot 10^{-15} \text{ s}$.

Figure 3. The time dependences of population imbalance parameter $D$ for spasers $1 \times 1$ (red thin lines) and $2 \times 2$ (blue thick lines), calculated by using formulas (9) (dashed lines) and by using direct numerical simulation (solid lines) of system (6). The initial values: $c(0) = \bar{c} \cdot 1.05$, $S(0) = \bar{S} \cdot 0.95$, $D(0) = 0.1$, where steady-state solutions $\bar{c} = 0.7110 + 0.7110i$, $\bar{S} = -6.8917 - 4.0475i$ for spaser $1 \times 1$ and $\bar{c} = 0.7577 + 0.7577i$, $\bar{S} = -1.5154 + 1.5154i$ for spaser $2 \times 2$ when choosing phase $\phi(0) = \pi/4$. The simulation parameters correspond to point A from Fig. 2.
Figure 4. The time dependences of average number of plasmons $|c|^2$ for spaser $1 \times 1$ (red thin lines) and for spaser $2 \times 2$ (blue thick lines), calculated by using formulas (10a) (dashed lines) and by using direct numerical simulation (solid line) of system (6). In the insets: dependences of the relative phase between plasmon and exciton modes for spaser $2 \times 2$ (bottom inset) and for spaser $1 \times 1$ (top inset). The initial values and parameters of simulation correspond to Fig. 3.

4. Conclusions
In this paper we describe the scheme of double spaser and observe the stationary regime formation of localized plasmons in this model. The scheme consists of two NPs and two QDs located in the vertices of a square and coupled with each other by means of a nearfield. It is shown that the plasmon-exciton kinematics in the scheme significantly depends on contribution of the energy of dipole-dipole interactions between the pair of NPs, while the contribution of dipole-dipole interactions between the pair of QDs can be neglected. Based on the realistic parameters of metal NPs and semiconductor QDs of $CdS$, the optimal conditions for observation of steady-states solutions for average numbers of plasmons and excitons in proposed scheme of spaser have been determined. Further development of this work is aimed at a complex simulation of quantum dynamics and a nonclassical states control of localized plasmons by analogy with optics [32].

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