Three-dimensional inverse method for aerodynamic optimization in compressor

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Abstract. Design experience plays an important role in compressor design. Accumulated design experience is used to reduce the number of simulations and to make time for the whole optimization process to be compatible with industrial standards. However, the major drawbacks of this design strategy are that the design result depends on talented designers with rich design experience and this method does not easily produce better configurations than existing designs. These drawbacks are related to the parametric description of the blade, which is conventionally performed using only geometric parameters. A good solution to this problem is to use a blade parametrization based on an inverse design method. Inverse design methods have been widely used for the design of various kinds of turbomachines, proving that it is a valuable alternative to the iterative use of direct methods. Inverse design parameter in the inverse design approach is the blade loading on both the hub and the shroud along the meridional direction. The blade loading distributions have a more direct relationship to the aerodynamic performance because they influence the flow field in a more straight-forward way. Fewer design parameters are then required to describe the blade shape than a purely geometric expression of the blade. Therefore, an optimization design method using the inverse method to parameterize the blade geometry can reduce the overall optimization time. The optimization design process then gives the optimal blade loading distributions, instead of the optimal combination of the geometric parameters. This is a more general result which can be applied to similar design problems without repeating the optimization process.

1. Introduction

There are two kinds of problem in fluid dynamics for compressors. The first is called direct calculation. The other is called inverse design. For inverse design problem the flow field information such as velocity and pressure are prescribed then the blade geometry is determined according to the specified flow field. It is a tough job to design a high performance compressor because of rotation effect and complicated geometry. It is difficult to make a breakthrough for compressor based on conventional design method. Since the geometric parameters are mainly dealt with in conventional design. For compressor there are so many geometric parameters to express the blade and it is hard to know the relationship between them and the compressor performances. Compared with the conventional design method inverse design method is more direct. In inverse design the blade geometry is calculated based on the prescribed velocity or pressure. The main design parameters are flow field variables and they have direct influence on aerodynamic performance of the compressor.

Inverse design developed from two dimensions\cite{1} to three dimensions\cite{2-4}, from ideal fluid to viscous fluid\cite{5}, from incompressible fluid to compressible fluid\cite{6,7}. Inverse design method has already
become a mature methodology and been used in many kinds of turbomachines\cite{8-21}. In the present inverse design method the main input parameter is the blade loading distribution which has direct influence on aerodynamic performance of the compressor. The performance can be controlled by adjusting the blade loading distributions. For example the secondary flow in the impeller can be suppressed by giving a reasonable blade loading distributions. On the other hand the blade geometry can be parameterized through the blade loading which reduces the number of the parameters dramatically compared with the geometrical parameterization. As a result optimization based on inverse design can be done efficiently and have a better result than conventional design. An optimized blade loading distribution instead of blade geometry is obtained after optimization which is useful to similar compressor design.

2. Description of three-dimensional inverse design method
In the method which will be presented in this paper the following assumptions will be made\cite{6}.
- The flow is steady, inviscid and uniform at the inlet, so that the only vorticity is the bound vorticity on the blades.
- There is no trailing shed vorticity.
- The blades have zero thickness, so that they can be represented by a single sheet of vorticity. However, the blade blockage effects are accounted for by using a mean stream surface thickness parameter in the continuity equation of the mean flow.
- The working fluid is a perfect gas and the flow is subsonic.

Based on the above assumptions according to Kelvin’s theorem the vorticity may be expressed in terms of a periodic delta function as

\[ \vec{\Omega} = \nabla \times \vec{V} = (\nabla \times \vec{V}) \delta_p(\alpha) \]  

where

\[ \alpha = \theta - f(r, z) = m \frac{2\pi}{B}, \quad m = 0, 1, 2, 3, \ldots \]  

represents the blade faces, \( \theta \) is the tangential co-ordinate of a cylindrical-polar co-ordinate system and \( f(r, z) \) is the angular co-ordinate of the point on the thin blade surface, or the so called wrap angle. \( \delta_p(\alpha) \) is the periodic delta function given by

\[ \delta_p(\alpha) = \frac{2\pi}{B} \sum_{m=-\infty}^{\infty} \delta \left( \alpha - m \frac{2\pi}{B} \right) = \Re \sum_{m=-\infty}^{\infty} e^{i m \alpha} \]  

The tangential mean of the delta function, \( \delta_p(\alpha) \), is unity and hence the mean vorticity is given by

\[ \overline{\Omega} = \nabla \times \overline{\vec{V}} = \nabla r \vec{V}_\theta \times \nabla \alpha \]  

2.1. Calculation of flow field
Here the pitchwise variation in density is negligible. In the relative frame of reference the continuity equation in steady flow is given by

\[ \nabla \cdot (\rho \vec{W}) = 0 \]  

where \( \vec{W} \) is the relative velocity vector, \( \rho \) is the density. Since the flow field is to be solved by decomposing it into circumferentially averaged and periodic components, the relative velocity vector \( \vec{W} \) can be written as

\[ \vec{W} = \vec{V} - \omega \times r + \vec{v} = \overline{\vec{W}}(r, z) + \vec{v}(r, \theta, z) \]  

where \( \vec{v} \) is the periodic component of velocity, \( \overline{\vec{W}} \) is the circumferentially averaged component of relative velocity, \( \omega \) is the rotational velocity and \( r \) is the position vector.

To account for blade blockage effects, a stream sheet thickness distribution can be included in the continuity equation, namely

\[ \nabla \cdot (\rho B \overline{\vec{W}}) = 0 \]
where $\bar{\rho}$ is circumferential averaged density and
\begin{equation}
B_f = 1 - \frac{t_\theta}{2\pi} \frac{B}{r}
\end{equation}
where $B_f$ is the blockage factor, $t_\theta$ is the tangential thickness and $r$ is the radius. The normal thickness distribution is used in conjunction with the estimated blade wrap angles to compute the tangential thickness from
\begin{equation}
t_\theta^2 = t_\theta^2 \left[ 1 + r^2 \left( \frac{\partial f}{\partial r} \right)^2 + r^2 \left( \frac{\partial f}{\partial z} \right)^2 \right]
\end{equation}

To satisfy the continuity equation (7) a stream function $\Psi(r,z)$ is defined to make $\bar{\rho} \vec{V} = \nabla \Psi \times \nabla \theta$, namely
\begin{align}
\vec{V}_r &= -\frac{1}{r B_f} \left( \frac{\rho_i}{\rho} \right) \frac{\partial \Psi}{\partial z}, \\
\vec{V}_z &= \frac{1}{r B_f} \left( \frac{\rho_i}{\rho} \right) \frac{\partial \Psi}{\partial r}
\end{align}
where $\rho_i$ is a reference density and $\Psi$ is the so-called Stoke stream function for three-dimensional axisymmetric flow. To obtain an equation for the unknown stream function, let us consider the tangential component of the mean velocity,
\begin{equation}
\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{\partial}{\partial z} \frac{\partial \Psi}{\partial z} = \bar{\rho} \frac{\partial \vec{V}}{\partial r} \cdot \vec{\Omega}
\end{equation}
where the right side is zero outside the blade region. This elliptic equation can be solved subject to boundary conditions at the endwalls and upstream and downstream boundaries. At endwalls stream function $\Psi$ equals constant. The far upstream (or downstream) boundary condition is given by
\begin{equation}
-\frac{1}{r} \frac{\partial \rho}{\partial s} \frac{\partial \Psi}{\partial z} = \vec{V}_z \cdot \vec{n}
\end{equation}
where $s$ is the distance along the far upstream (or downstream) boundary, $\vec{n}$ is the unit vector in the meridional plane normal to the far upstream (or downstream) boundary and $\vec{V}_z$ is circumferentially averaged velocity vector at the far upstream (or downstream) boundary. By solving equation (12) subject to the above boundary conditions, the mean flow velocity field is determined.

For the calculation of the periodic flow let us consider a Clebsch formulation for the periodic velocity
\begin{equation}
\vec{v} = \nabla \Phi(r,\theta,z) - S(\alpha) \nabla r \vec{V}_\phi
\end{equation}
where $\Phi$ is the potential function of the periodic flow. In the absence of circumferential variations in density the periodic component of the continuity equation can be written as
\begin{equation}
\nabla \cdot \vec{v} = -\nabla \cdot \vec{V}_\phi \ln \bar{\rho}
\end{equation}

Substitute (14) into (15) we get
\begin{equation}
\nabla^2 \Phi = S(\alpha) \nabla^2 r \vec{V}_\phi + (\nabla \alpha \cdot \nabla r \vec{V}_\phi) S'(\alpha) - \nabla \cdot \nabla \ln \bar{\rho}
\end{equation}
where the first two terms on the right side of the will be zero outside the blade row. Since the flow is periodic in the pitchwise direction, the potential function can be expressed in terms of a Fourier series of the form
\begin{equation}
\Phi(r,\theta,z) = \sum_{m=-\infty}^{\infty} \Phi_m(r,z) e^{im\theta}
\end{equation}
Substitute (14), (15) and (17) into (16) we can get
\[
\frac{\partial^2 \Phi_m}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi_m}{\partial r} + \frac{\partial^2 \Phi_m}{\partial z^2} + \frac{\partial \Phi_m}{\partial z} \frac{\ln(\rho / \rho_i)}{\partial r} + \frac{\partial \Phi_m}{\partial r} \frac{\ln(\rho / \rho_i)}{\partial z} - \frac{m^2 B^2}{r^2} \Phi_m = e^{-i m B f(r, z)} \]

(18)

In order to solve (18) boundary conditions should be applied on the four boundaries of the physical domain. At the endwalls the periodic velocity normal to the hub and shroud must be zero. This condition can be expressed by

\[
\frac{\partial \Phi_m}{\partial n} = \frac{\partial r \tilde{V}_\theta}{\partial n} e^{-i m B f(r, z)}
\]

(19a)

Far upstream and downstream the flow is uniform. This condition can be implemented by imposing \( \Phi_m = 0 \)

(19b)

The equations modeling the flow field have to be solved subject to the Kutta-Joukowski condition, namely

\[ W_{bl} \cdot \nabla r \tilde{V}_\theta = 0 \]

where \( W_{bl} \) is the relative velocity at the blade, \( \frac{W_{bl}}{2} = (W^+ + W^-) / 2 \) where \( W^+ \) and \( W^- \) are the velocities on the upper and lower surfaces of the blades.

2.2. Calculation of density

For a steady flow process in the absence of viscous and body forces in the case of uniform flow at the inlet the first law of thermodynamics equation can be reduced to

\[ I = h + \frac{1}{2} W^2 - \frac{1}{2} u^2 = \text{const} \]

(21)

where \( I \) is the rotary stagnation enthalpy and \( h \) is the rothalpy, \( u \) is the peripheral speed. By using the perfect gas equation and isentropic relations, the circumferential averaged density \( \overline{\rho} \) can be calculated through by neglecting the pitchwise variation of the density

\[ \overline{\rho} = \left( \frac{1}{1 + \frac{\omega^2 r^2 - (W \cdot W)}{2 C_p T_i}} \right)^{1/(y - 1)} \]

(22)

where \( \omega \) is the rotational speed, \( C_p \) is the specific heat, \( T \) is the temperature, subscript \( i \) represents the reference value.

2.3. Calculation of blade shape

Once the flow field has been determined, it is then possible to compute the blade shape by using the blade boundary condition that the blade must be aligned to the velocity vector there. This condition can be expressed as

\[ W_{bl} \cdot \nabla \alpha = 0 \]

where \( \nabla \alpha \) is a vector normal to the blade surface and \( W_{bl} \) is the relative velocity at the blade. Expanding (23),

\[ (\tilde{V}_z + v_{zh}) \frac{\partial f}{\partial z} + (\tilde{V}_r + v_{rh}) \frac{\partial f}{\partial r} = \frac{r \tilde{V}_\theta}{r^2} + \frac{v_{zh}}{r} - \omega \]

(24)

where \( f \) is the wrap angle. The initial value of \( f \) along a quasi-orthogonal must be given as input values for integrating (24). This initial value is called the stacking condition of the blade.

Equation (12), (18), (22) and (24) constitute the equation set of the inverse method subject to the boundary conditions (13), (19) and (20). The equation set can be solved in an iterative way on the
meridional channel. After the calculation both blade shape and the corresponding flow field are obtained.

3. Suppression of secondary flows
Meridional channel shape, blade loading distribution, thickness distribution and stacking condition are main input parameters for inverse design method. The meridional derivative of $rV_\theta$ which is called blade loading here has direct relationship with the rothalpy difference between the two sides of the blades through the following expression for compressible flow:

$$h^+ - h^- = \frac{2\pi}{B}W_{mb} \frac{\partial rV_\theta}{\partial m}$$

where $h$ is the rothalpy, superscripts $+$ and $-$ correspond to values on either side of the blades, $B$ is the blade number, $W_{mb}$ is the meridional velocity on the blades, and $m$ is in the direction of streamlines in the meridional plane. According to (25) by making changes to the meridional derivative of $rV_\theta$ it should then be possible to control the blade pressure loading. Based on that turbomachinery performance can be improved more conveniently than conventional design process [22-24]. A general distribution type of the blade loading is called three-segment style as shown in Figure 1.

![Figure 1. Three-segment type blade loading distribution.](image)

Secondary flow in impeller can be suppressed by adjusting the blade loading based on inverse design method [25-28]. For a qualitative discussion, it is useful to consider the following simple kinematic equation derived by Zangeneh et al. [25]:

$$\mathbf{W} \times (\mathbf{W} \cdot \mathbf{\Omega}) = 2\mathbf{\Omega} \cdot (\mathbf{W} \times \mathbf{W}) + \mathbf{\Omega} \times (2\mathbf{\omega} \times \mathbf{W})$$

where $(\mathbf{W} \cdot \mathbf{\Omega})$ represents the streamwise component of absolute vorticity. According to (26) streamwise components of vorticity (and therefore secondary flows) are generated when there is a component of acceleration (streamline curvature or Coriolis) in the direction of absolute vorticity.

Although the magnitude of streamline curvature or Coriolis acceleration can be found easily in different parts of the impeller, it is perhaps more useful from a designer’s point of view to relate the secondary flow generation to an easily calculated variable such as pressure, Mach number, etc. This can be done quite easily by considering the momentum equation in inviscid flow, which can be written as:
where the term in brackets on the right side is the well-known “reduced static pressure” usually denoted by $Pr$. Based on \((27)\) secondary flow is generated whenever there is a gradient of reduced static pressure in the direction of vorticity in the flow field.

They analyzed the $C_p$ distribution as shown in Figure 2 for a conventional pump impeller, where $C_p = (\text{Rotary stagnation pressure} - Pr)/(1/2 \rho U_m^2)$.

![Figure 2. Predicted $C_p$ distribution for conventional pump impeller\[25\].](image)

By analyzing the $C_p$ distribution and the internal flow field they find three different ways to suppress secondary flows by minimizing $\Delta C_p$ in specific region of the impeller.

1. First way is to decrease the value of $C_p$ on the shroud suction surface, which can be achieved by decreasing the blade pressure loading (or $\Delta C_{p_{sh}}$ as shown in Figure 2) on the shroud in the aft part of the impeller.

2. Increasing the value of $C_p$ on the hub suction surface also helps to reduce $\Delta C_p$. This can be achieved by increasing the blade pressure loading (or $\Delta C_{p_{ss}}$) near the trailing edge region.

3. In many circumstances, the degree that the shroud suction surface $C_p$ can be reduced is limited by the diffusion ratio or other design criteria such as shockwave control. So in practice it is important to use a combination of (1) and (2) in which the shroud suction surface $C_p$ is reduced in the aft part of the impeller, while at the same time the hub suction surface $C_p$ is increased there.

Since the blade loading distribution is direct related to the pressure loading in the impeller according to \((25)\). One can control the $C_p$ distribution by changing the blade loading distribution based on the above three rules. See Zangeneh at al.\[25\] for more details.

The other important input specification for the inverse design method, which affects the $C_p$ (or $M$) distribution on the blades, is the stacking condition. As discussed above, this condition is required in order to compute the wrap angle throughout the blade region by integrating the first-order hyperbolic equation \((24)\). For this condition, the distribution of wrap angle $f$ should be specified along a quasi-orthogonal (called the stacking quasi-orthogonal) from hub to shroud. By specifying different value of $f$ at the shroud and the hub one can control the blade lean angle which can introduce a spanwise blade force as shown in Figure 3. The blade force can influence the $C_p$ distribution. Based on the above $C_p$ control rules the ideal type of stacking condition for suppression of secondary flows in radial and
mixed flow turbomachines is to lean the blades linearly against the direction of rotation, as shown in Figure 3, in the aft part of the impeller.

![Figure 3. Effect of blade lean](image)

4. Optimization strategy based on inverse method

Automatic optimization strategies based on automating the conventional design process by coupling an optimization method, CAD based blade generators and a computational fluid dynamics code present some drawbacks. The main one is the high computational cost, which is due to the large number of simulations required. This is related to the large number of geometrical parameters necessary to accurately represent the blade geometry and to the shape of the objective function to be optimized, which is usually quite complex since there is not a direct relationship between the geometrical design parameters and the aerodynamic performance. For multiobjective turbomachinery designs more CFD simulations for each configuration are necessary. Another major drawback of optimization strategies is that at the end of each optimization, despite the large number of CFD simulations, it is difficult to derive a general know-how that can be exploited for similar design problems.

The main advantage of the inverse design approach is the closer relationship between the design parameters and the aerodynamic flow field, and then a more direct control of the aerodynamic performance. Therefore, the use of an inverse method to parametrize the blade geometry in an optimization system reduces the complexity of the objective function with a potential reduction of the overall optimization time.

Design specifications such as the design mass flow rate and the work coefficient or flow turning are directly imposed in the inverse design parameterization, while in the conventional approach such parameters have to be considered as constraints, which reduces the optimization convergence rate.

At the end of the design process, instead of the optimal combination of geometrical control points, the optimal blade loading distribution is obtained. This is a general result and can be applied to similar design problems without repeating the optimization process.

Optimization strategies based on inverse design method have been applied to different turbomachinery. In all these optimizations the blade was parameterized by blade loading distributions. The meridional channel can also be parameterized coupled with the blade parameterization for the impeller optimization design. Compared with optimization approach based on conventional design these optimization strategies based on inverse design method have reasonable time cost and can give a general know-how that can be exploited for similar design problems.

5. Conclusions

In three-dimensional inverse design method the compressor blade can be calculated subject to a specified flow field such as velocity or pressure distributions. In the present method blade loading distributions are given to calculate the blade geometry. Since the blade loading distributions have direct relationship with the flow field in the compressor the aerodynamic performance can be controlled accordingly by changing the blade loading. For example secondary flow in the impeller can
be suppressed by giving an appropriate blade loading distribution and blade stacking condition (blade lean angle).

Based on the above advantages of the inverse design method optimization strategies by coupling the inverse method, CFD and optimization method can be set up for compressor designs. By using these strategies the computational cost decrease dramatically and after optimization more general design know-how can be achieved which is useful for similar design problems. The strategy is suitable for multiobjective turbomachinery design. And it is also suitable for multidisciplinary optimizations, where stress and vibration analyses can be coupled with the aerodynamic ones.

References
[1] Hawthorne W R, Wang C, Tan C S, et al 1984 ASME Journal of Engineering for Gas Turbines and Power 106(2) 346-353
[2] Tan C S, Hawthorne W R, McCune J E, et al 1984 ASME Journal of Engineering for Gas Turbines and Power 106(2) 354-365
[3] Borges J E 1990 ASME Journal of Turbomachinery 112 346-354
[4] Zangeneh M 1994 ASME Journal of Turbomachinery 116 280-290
[5] Tiow W T and Zangeneh M 1998 A viscous transonic inverse design method for turbomachinery blades part I: 2D cascades Int. Gas Turbine & Aeroengine Congress & Exhibition (Stockholm, Sweden, 2-5 June 1998)
[6] Zangeneh M 1991 International Journal of Numerical Methods in Fluids 13 599-624
[7] Zangeneh M and Hawthorne W R 1990 A fully compressible three dimensional inverse design method applicable to radial and mixed flow turbomachines Gas Turbine and Aeroengine Congress and Expositon (Brussel, Belgium, 11-14 June 1990)
[8] Cao S L, Peng G Y and Yu Z Y 2005 Journal of Fluids Engineering 127 330-338
[9] Peng G Y, Cao S L, Ishizuka M, et al 2002 International Journal for Numerical Methods in Fluids 39 517-531
[10] Zangeneh M 1997 ASME paper No.97-GT-208
[11] Ashihara K, Goto A and Kamijo K 2001 Study on turbopump inducers design by 3-D inverse design method Proc. of the 1st Int. Symp. on Advanced Fluid Information (Miyagi Zao, Sendai, Japan, 4-5 October 2001)
[12] Goto A, Nohmi M, Sakurai T, etc 2002 Journal of Fluids Engineering 124 329-335
[13] Ashihara K and Goto A 2000 Study on pump impeller with splitter blades designed by 3-D inverse design method ASME 2000 Fluids Engineering Division Summer Meeting (Boston, Massachusetts, USA, 11-15 June 2000)
[14] Goto A and Ashihara K 1999 Compact design of diffuser pumps using three-dimensional inverse method Proc. of the 3rd ASME/JSME Joint Fluids Engineering Conf. (San Francisco, California, USA, 18-23 July 1999)
[15] Sakurai T, Saito S, Goto A, etc. Pump design system based on inverse design method and its application to development of diffuser pump series Proc. of the 3rd ASME/JSME Joint Fluids Engineering Conf. (San Francisco, California, USA, 18-23 July 1999)
[16] Ashihara K and Goto A 2002 Effects of blade loading on pump inducer performance and flow fields Proc. of ASME 2002 Fluid Engineering Division Summer Meeting (Montreal, Quebec, Canada, 14-18 July 2002)
[17] Goto A and Zangeneh M 1998 Hydrodynamic design of pump diffuser using inverse design method and CFD ASME Fluid Engineering Division Summer Meeting (Washington, USA, 21-25 June 1998)
[18] Zangeneh M 1996 Journal of Turbomachinery 118 385-393
[19] Okamoto H and Goto A 2002 Proc. of the 2002 Joint US ASME-European Fluids Engineering Summer Conf. (Montreal, Quebec, Canada, 14-18 July 2002)
[20] Roddis M E and Zangeneh M 1993 The Royal Institution of Naval Architects 135 175-189
[21] Goto A and Zangeneh M 124 Journal of Fluids Engineering 124 319-328
[22] Ashihara K and Goto A 1999 Improvements of pump suction performance using 3D inverse design method Proc. of the 3rd ASME/JSME Joint Fluids Engineering Conf. (San Francisco, California, USA, 18-23 July 1999).

[23] Ashihara K, Goto A, Kamijo K, Yamada H and Uchiumi M 2002 Improvements of inducer inlet backflow characteristics using 3-D inverse design method Proc. of 38th AIAA/ASME/SAE/ASEE Joint Propulsion Conf. (Indianapolis, Indiana, USA, 7-10 July 2002).

[24] Zangeneh M, Vogt D and Roduner C 2002 Improving a vaned diffuser for a given centrifugal impeller by 3D inverse design ASME Turbo Expo: Land, Sea & Air (Amsterdam, The Netherlands, 3-6 June 2002).

[25] Zangeneh M and Goto A 1998 Journal of Turbomachinery 120 723-733.

[26] Watanabe H and Harada H 1999 Suppression of secondary flows in a turbine nozzle with controlled stacking shape and exit circulation by 3D inverse design method Int. Gas Turbine & Aeroengine Congress & Exhibition (Indianapolis, Indiana, USA, 7-10 June 1999).

[27] Zangeneh M, Goto A and Takemura T 1996 Journal of Turbomachinery 118 536-543.

[28] Zangeneh M, Goto A and Takemura T 1996 Journal of Turbomachinery 118 536-543.

[29] Daneshkah K and Zangeneh M 2010 Parametric Design of A Francis Turbine Runner by Means of A Three-Dimensional Inverse Design Method IOP Conference Series: Earth and Environmental Science (Timisoara, Romania, 20-24 September 2010) p 012058.

[30] Bonaiuti D, Zangeneh M, Aartojarvi R and Eriksson J 2010 ASME J. Fluids Eng. 132(3) 031104.

[31] Yiu K F C and Zangeneh M 2000 J. Propul. Power 16(6) 1174–1181.

[32] Bonaiuti D and Zangeneh M 2009 ASME J. Turbomach. 131(2) 021014.