Effect of space-momentum correlations on the constituent quark number scaling of hadron elliptic flows

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Abstract

Using models ranging from schematic one with a simple quark distribution to more realistic blast wave, we study the elliptic flow of hadrons produced from coalescence of quarks and antiquarks in the quark-gluon plasma that is formed in ultrarelativistic heavy ion collisions. In particular, we study effects due to azimuthal anisotropy in the local transverse momentum distribution of quarks, as generated by their position-momentum correlations as a result of radial flow and/or jet quenching. We find that even if quarks have large local non-elliptic anisotropic flow, the elliptic flow of produced hadrons can still scale with their constituent quark numbers. This scaling is, however, broken if the radial flow of coalescing quarks is anisotropic and/or if the momentum dispersion of quarks inside hadrons is included.

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I. INTRODUCTION

In searching for the signatures of the Quark-Gluon Plasma (QGP) produced at the Relativistic Heavy Ion Collider (RHIC) and to study its properties, the elliptic flow, i.e., the second harmonic in the azimuthal distribution of the transverse momenta of produced particles, has played an important role. It was shown in Ref. [1] using the parton cascade model and in Refs. [2, 3] using the hydrodynamic model that the elliptic flow in heavy ion collisions at RHIC is sensitive to the properties of formed partonic matter. The observed large elliptic flow of charged hadrons at RHIC [4, 5] is indeed consistent with an initial partonic matter that is not only dense but also strongly interacting [6]. For identified midrapidity hadrons except pions, it was further observed that the dependence of the elliptic flow on transverse momentum becomes similar if both are divided by the number of constituent quarks in the hadrons, i.e., two for mesons and three for baryons. Such scaling of the elliptic flow of hadrons according to their constituent quark numbers [7, 8] is consistent with the picture of constituent or massive quarks coalescing into hadrons [9, 10, 11]. In particular, the quark number scaling of hadron elliptic flow is exact in the so-called naive coalescence model, which only allows quarks and antiquarks (generally called quarks in the rest of the paper) with same transverse momentum to coalesce into hadrons. This scaling is, however, violated in more realistic quark coalescence models that takes into account the momentum distribution of quarks inside hadrons [9, 12, 14, 15] or higher Fock states in hadron wave functions [16]. According to Ref. [15], the former leads to a violation of the quark number scaling of hadron elliptic flow by about 10% for mesons and 15% for baryons, resulting in a relative baryon/meson violation of about 5% which is similar to that observed in the most recent experimental results [17]. The hadron elliptic flow is also affected by resonance decays [15]. Although this effect has been estimated to be small, including the resonance decay effect can largely account for the observed quark number scaling violation in the pion elliptic flow [15, 18].

Other sources for the violation of the quark number scaling in hadron elliptic flow have also been studied recently. For example, it was shown that anisotropies in the quark phase-space density and effective emission volume at hadronization can lead to the breaking of hadron elliptic flow according to their constituent quark numbers [19]. Furthermore, it was pointed out in Ref. [20] that in usual derivation of the quark number scaling of the elliptic
flow based on the naive quark coalescence model \cite{9, 10, 11}, correlations between positions and momenta of quarks, as expected from the hydrodynamic or the parton cascade model, were ignored as only the spatially averaged quark momentum distributions were used. Since the position-momentum correlations can generate large non-elliptic anisotropy in the quark local momentum distribution, which may not be averaged out after integrating over the quark spatial distribution, the quark number scaling of hadron elliptic flows may thus be strongly violated.

To understand more clearly what role is played by quark position-momentum correlations and how the quark number scaling of hadron elliptic flow appears in the quark coalescence model, we examine in the present paper the relation between the elliptic flows of quarks and mesons in models that range from a simple schematic model to the more realistic blast wave model. Our analyses indicate that although large azimuthal anisotropy generally exists in the quark local transverse momentum distributions in these diverse models as well as in the model used in Ref.\cite{12}, it does not necessarily lead to the violation of the quark number scaling of hadron elliptic flows if the quark global transverse momentum distribution does not have odd-order anisotropic flows.

II. ANISOTROPIC FLOW

Following the notations of Ref.\cite{20}, the spatial and transverse momentum distributions \(S(x, p_T, y)\) of particles produced in relativistic heavy ion collisions can be expanded in Fourier series as

\[
S(x, p_T, y) = S_0(x, p_T, y) \left\{ 1 + 2 \sum_{n=1}^{\infty} \text{Re} \left( c_n(x, p_T, y) e^{-in\phi_p} \right) \right\},
\]

where \(p_T \equiv p_T(\cos \phi_p, \sin \phi_p)\) is the transverse momentum of a particle with \(\phi_p\) denoting its azimuthal angle with respect to the reaction plane, and \(y\) is its rapidity. The Fourier coefficient \(c_n(x, p_T, y)\) denotes the \(n\)th-order local anisotropic flow of particles with momentum \(p_T\) in an infinitesimal space-time volume around \(x\). In general, \(c_n\) is complex and is written as \(c_n = v_n + iu_n\), with the real and imaginary parts given by averaging over the azimuthal angle \(\phi_p\) of particle momentum, i.e., \(v_n = \langle \cos(n\phi_p) \rangle\) and \(u_n = \langle \sin(n\phi_p) \rangle\).

Experimentally, only the spatially averaged global anisotropic flows are measured, i.e.,

\[
\tau_n(p_T, y) \equiv \langle c_n(x, p_T, y) \rangle = \frac{\int d^4x c_n(x, p_T, y) S(x, p_T, y)}{\int d^4x S(x, p_T, y)}. \tag{2}
\]
The real part of the two lowest orders $\mathbf{v}_1$ and $\mathbf{v}_2$ are the directed and elliptic flows, respectively. The imaginary part $\mathbf{u}_n$ always vanish in collisions between identical spherical nuclei due to reflection symmetry with respect to the reaction plane, i.e., invariant under the transformation $\phi \rightarrow -\phi$. Furthermore, for particles at midrapidity in collisions with equal mass nuclei, anisotropic flows of odd orders vanish as a result of the additional symmetry $\phi \rightarrow \phi + \pi$. In the present study, we consider only particles at midrapidity in collisions of identical spherical nuclei.

In the naive coalescence model [9, 10, 11], which only considers the momentum distribution of quarks, thus neglecting correlations between quark positions and momenta, and allows quarks with same transverse momentum to coalesce into hadrons, one obtains a simple relation between meson and quark elliptic flow if all $\mathbf{v}_n$ of quarks vanish except for $n = 2$; i.e.,

$$\mathbf{v}_{2,M} = \frac{2\mathbf{v}_{2,q}}{1 + 2\mathbf{u}_{2,q}}.$$  \hfill (3)

Neglecting $\mathbf{u}_{2,q}$ in the denominators due to its small value, a scaling of the elliptic flow of mesons according to their constituent quark number then follows.

Including quark position-momentum correlations, the meson local elliptic flow $v_{2,M}$ is then related to quark local $v_n$ and $u_n$ via [20]

$$v_{2,M} = \frac{2v_2 + v_1^2 - u_1^2 + 2 \sum_{k=1}^{\infty} (v_k v_{2+k} + u_k u_{2+k})}{1 + 2 \sum_{k=1}^{\infty} (v_k^2 + u_k^2)}.$$  \hfill (4)

Although higher-order quark local anisotropic flows $v_n$ and $u_n$ with $n > 3$ are likely small, quark local $v_1$ and $u_1$ can be large as first pointed out in Ref. [20]. It is not obvious that in the spatial average the contribution of quark local directed flow $v_1$, $u_1$ vanishes. The observed approximate scaling of hadron elliptic flow according to the constituent quark number may simply due to accidental cancellations of quark local directed flow as suggested in Ref. [20].

In the present paper, we study the effect of nonvanishing quark local directed flow on the meson elliptic flow in various models. Our analysis shows that after spatial average its effect either disappears or is small, thus having negligible or only small effect on the elliptic flow of hadrons.
III. A SCHEMATIC MODEL

We first consider a schematic model in which quarks are distributed uniformly in space with their momenta pointing radially outward, i.e., same azimuthal angles $\phi_p$ and $\phi_r$ for the momentum and position vectors of a quark. This quark distribution is qualitatively similar to that of quenched jets produced in heavy ion collisions at RHIC [21, 22]. Including also an azimuthal anisotropy of the form $\cos(2\phi_p)$ in the quark transverse momentum distribution, the quark distribution becomes

$$\frac{dN}{d^2r d^2p} \propto (1 + 2a_2 \cos(2\phi_p)) \delta(\phi_r - \phi_p),$$

where the coefficient $a_2$ is the quark elliptic flow $v_{2,q}$.

Considering only quarks with spatial azimuthal angles less than $\phi'_r$, the Fourier coefficient $c_{n,q}$ of resulting momentum distribution can be evaluated by integrating $\phi_r$ from 0 to $\phi'_r$, i.e.,

$$c_{n,q} = \frac{1}{N} \int_0^{\phi'_r} d\phi_r \int_0^{2\pi} d\phi_p \left[ 1 + 2a_2 \cos(2\phi_p) \right] \delta(\phi_r - \phi_p) e^{i n \phi_p}$$

$$= \frac{i}{N} \left\{ \frac{1}{n} + a_2 \frac{2n}{n^2 - 4} - \frac{e^{i n \phi'_r}}{n} - 2a_2 \left[ \frac{e^{i(n-2)\phi'_r}}{n-2} + \frac{e^{i(n+2)\phi'_r}}{n+2} \right] \right\},$$

where $N = \phi'_r + a_2 \sin(2\phi'_r)$ is the normalization factor shown in the denominator of Eq.(2).

For $n = 2$, the above equation gives

$$c_{2,q} = \frac{1}{N} \left\{ a_2 \phi'_r + i \left[ \frac{1}{2} + \frac{a_2}{4} - \left( \frac{1}{2} + \frac{a_2}{4} e^{2i\phi'_r} \right) e^{2i \phi'_r} \right] \right\}. \tag{7}$$

The global quark elliptic flow is obtained from Eq.(7) by letting $\phi'_r = 2\pi$, and this gives $v_{2,q} = a_2$ as expected. It can be checked that all higher-order global quark anisotropic flows vanish as the first two terms in Eq.(6) are exactly canceled by the last two terms when $\phi'_r = 2\pi$. On the other hand, for quarks in a localized space the distribution contains an infinite number of Fourier coefficients. For example, for quarks in the first quadrant, which corresponds to integrating $\phi_r$ from 0 to $\phi'_r = \pi/2$, all $c_n$’s are nonzero. Assuming an anisotropy parameter $a_2 = 0.1$, we find that for quarks in the first quadrant, the local directed flow is $v_1 = 0.68$ and its imaginary part is $u_1 = 0.59$. Both are close to those from the MPC code for Au+Au collisions at $\sqrt{s} = 200$ GeV with $b = 8$ fm [20]. Except for the local elliptic flow $v_{2,q} = \overline{v}_{2,q} = 0.1$, other real even Fourier coefficients all vanish, and this is similar to that in the midrapidity region of heavy ion collisions at RHIC. The imaginary even Fourier
coefficients are in general nonzero, and we have for the second Fourier coefficient \( u_{2,q} = 0.64 \).

We note that the large azimuthal anisotropy in the quark local momentum distribution in the present model is caused by the strong position and momentum correlations in the quark distribution, which is also present in the blast wave model \([23]\) as well as in the transport model \([24]\). These models further include, however, effects due to thermal motions of quarks.

In the naive coalescence model, the transverse momentum spectrum of produced mesons is given by \([12]\)

$$
\frac{dN_M}{d^2p} = g_M \int \prod_{i=1}^{2} d^2 r_i d^2 p_i \frac{dN_q}{d^2 r_i d^2 p_i} (2\pi)^2 \delta^{(2)}(r_1 - r_2) \delta^{(2)}(p_1 - p_2) \delta^{(2)}(p - p_1 - p_2),
$$

where \( g_M \) is the statistical factor for a quark and an antiquark to form a colorless meson with certain spin and isospin.

With the quark distribution of Eq.(5), carrying out the integral in Eq.(8) over the azimuthal angle from 0 to \( \phi'_r \) gives following meson anisotropic flow in a restricted space:

$$
c_n,M = \frac{i}{N} \left\{ \frac{1 + 2a_2^2}{n} + \frac{4na_2}{n^2 - 4} + \frac{2na_2^2}{n^2 - 16} - \frac{1 + 2a_2^2}{n} e^{in\phi'_r} \right\}
- 2a_2 \left[ \frac{e^{i(n-2)\phi'_r}}{n - 2} + \frac{e^{i(n+2)\phi'_r}}{n + 2} \right] - a_2^2 \left[ \frac{e^{i(n-4)\phi'_r}}{n - 4} + \frac{e^{i(n+4)\phi'_r}}{n + 4} \right],
$$

where \( N = (1 + 2a_2^2)\phi'_r + 2a_2 \sin(2\phi'_r) + a_2^2/2 \sin(4\phi'_r) \). The local elliptic flow in this restricted space is then

$$
c_{2,M} = \frac{1}{N} \left\{ 2a_2 \phi'_r + i \left[ \frac{1 + 2a_2^2}{2} + \frac{a_2}{2} - \frac{2a_2^2}{3} - \frac{1 + 2a_2^2}{2} e^{i\phi'_r} \right] e^{4i\phi'_r} \right\}
- a_2 \left[ \frac{e^{6i\phi'_r}}{6} - \frac{e^{-2i\phi'_r}}{2} \right].
$$

Including all quarks, i.e., letting \( \phi'_r = 2\pi \) in Eq.(9), cancellations similar to those in the evaluation of the quark global anisotropy flows lead to the following meson transverse momentum distribution:

$$
\frac{dN_M}{d^2p} \propto 1 + 2a_2^2 + 4a_2 \cos(2\phi_p) + 2a_2^2 \cos(4\phi_p).
$$

The above equation shows that the meson elliptic flow is

$$
\overline{v}_{2,M} = \frac{2\overline{v}_{2,q}}{1 + 2\overline{v}_{2,q}^2},
$$

and there is also a 4th-order anisotropic flow given by

$$
\overline{v}_{4,M} = \frac{\overline{v}_{2,q}^2}{1 + 2\overline{v}_{2,q}^2}.
$$
These results are exactly what one expects from the naive coalescence model when all quark anisotropic flows except $v_2$ are zero [11].

The above result holds for any value of the anisotropy parameter $a_2 = \bar{v}_{2,q}$, and thus for any value of local anisotropic flow $c_{n,q}$. In other words, despite the fact that locally there can be very large anisotropy in the quark momentum distribution, they are not relevant for the global elliptic flow of mesons that are produced from the coalescence process. The quark number scaling of hadron elliptic flow is exact as expected when the quark distribution has only non-zero global elliptic flow.

IV. THE BLAST WAVE MODEL

The schematic model introduced above can be made more realistic by including thermal motions of quarks. Adding a random component to the quark momentum weakens the extreme angular correlation between the positions and momenta of quarks in the schematic model. As shown in the following, mesons produced from the naive coalescence remain to have an elliptic flow that follows the quark number scaling if the radial flow is isotropic, i.e. independent of azimuthal angle. The large anisotropy in the quark local momentum distribution, particularly the $v_{1,q}$ and $u_{1,q}$, does not affect the scaling relation between quark and meson elliptic flows. On the other hand, the quark number scaling of hadron elliptic flow can be violated when the quark radial flow is anisotropic, i.e., its velocity varies with the spatial azimuthal angle $\phi_r$.

A. Blast wave with isotropic radial flow

We first consider the case that quarks have a relativistic Boltzmann distribution with isotropic radial flow velocity [25], i.e.,

$$\frac{dN_{q,q}}{d^2p_T d^2r} \propto \exp \left[ -\frac{m_T}{T} \cosh \rho_0 + \frac{p_T}{T} \sinh \rho_0 \cos(\phi_r - \phi_p) \right] \left[ 1 + 2a_2(p_T) \cos(2\phi_p) \right], \quad (14)$$

where $m_T = (p_T^2 + m_q^2)^{1/2}$. In the above, dependence on the spatial and momentum rapidities $y$ and $\eta$ has been dropped as we consider a Bjorken boost invariant distribution at $y = \eta = 0$. For simplicity, both the quark density distribution and their flow profile $\beta_T = \tanh \rho_0$ are taken to be independent of the radial position. Also, the spatial shape of the quark
distribution is assumed to be cylindrical. Although spatial ellipticity can cause the breaking of quark number scaling of hadron elliptic flow as discussed in Ref. [19], it is not essential for the purpose of present study.

For quarks with spatial azimuthal angle less than $\phi_r$, the different harmonics in the Fourier expansion of the quark distribution in Eq. (14) are given by

$$c_{n,q}(p_T) = \frac{1}{N} \int_0^{2\pi} d\phi_p \int_0^{\phi_r} d\phi_r \exp \left[ \alpha(p_T) \cos(\phi_r - \phi_p) + i n \phi_p \right] \times \left[ 1 + 2 a_2(p_T) \cos(2\phi_p) \right],$$

(15)

with $N = 2\pi \{ I_0[\alpha(p_T)]\phi_r' + a_2(p_T)I_2[\alpha(p_T)]\sin(2\phi_r') \}$ and $\alpha(p_T) = (p_T/T) \sinh \rho_0$. If all quarks are included, i.e., $\phi_r' = 2\pi$, then all orders of anisotropic flow except $v_2$ vanish. This can be easily seen as the spatial integral can now be expressed in terms of the Bessel function $I_0$ and is thus canceled by the same factor in the normalization factor $N$ in the denominator. What is left is then the integral over $\phi_p$ which is non-zero only for $n = 2$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Quark local anisotropic flow as a function of transverse momentum from averaging over quarks in the first quadrant of the transverse plane.}
\end{figure}

On the other hand, because of quark radial flow, none of the harmonics in the Fourier expansion vanishes if only quarks in a limited space are considered. To estimate the values of the different harmonics in the present model for heavy ion collisions at RHIC, we take a slope parameter $T = 170$ MeV for the quark transverse momentum distribution, which is consistent with the phase transition temperature from lattice QCD calculations. Masses of quarks are taken to be $m_q = m_{\bar{q}} = 300$ MeV as in Ref. [12]. For the strength of quark
anisotropic flow, we take $\rho_0 = 0.4$ and $a_2(p_T) = 0.1 \tanh(1.2p_T)$ in order to have radial and elliptic flows with magnitude similar to those observed in experiments. Results for the first two local harmonics $v_{1,q}$, $u_{1,q}$, $v_{2,q}$ and $u_{2,q}$ in the first quadrant are shown in Fig. 1 and they are seen to have values similar to those from the MPC quark cascade model [24].

The meson spectrum obtained from the naive coalescence of quarks, i.e., only quarks in the same phase space point can coalesce, can be easily derived from the quark distribution of Eq.(14) by integrating over both spatial azimuthal angle $\phi_r$ and momentum azimuthal angle $\phi_p$, and the result is given by

$$\frac{dN_M}{d^2p_T} \propto e^{-\tilde{\alpha}(p_T/2)} I_0[2\alpha(p_T/2)] \times \left[ 1 + 2a_2^2(p_T) + 4a_2(p_T) \cos(2\phi_p) + 2a_2^2(p_T) \cos(4\phi_p) \right],$$

where $\tilde{\alpha}(p_T) = (m_T/T) \cosh \rho_0$. Comparing to Eq.(11), we see that the quark number scaling of meson elliptic flow appears as in the schematic model of Section III. This example thus demonstrates again that the naive coalescence model leads to the quark number scaling of hadron elliptic flow, independent of the presence of large anisotropy in the quark local momentum distribution.

If we had considered quarks in a finite rapidity range, integrating over the rapidity leads to the Bessel function $K_1(\tilde{\alpha})$ instead of the $e^{-\tilde{\alpha}}$ in Eq.(16). This does not, however, affect the conclusion from the simpler picture considered in the above.

**B. Blast wave with anisotropic radial flow**

The above blast wave model can be made more general by allowing the radial flow velocity to depend on both the azimuthal angle in space and the transverse momentum. Specifically, we introduce a radial flow profile $\beta_T(p_T, \phi_r) = \tanh(\rho(p_T, \phi_r))$, with $\rho(p_T, \phi_r) = \rho_0 + \rho_2(p_T) \cos(2\phi_r)$. This is similar to that in the blast wave model of Ref. [26], except for the $p_T$ dependence of the azimuthal anisotropy parameter $\rho_2$. The latter makes it possible to parameterize the typical increase and then saturation (or slight decrease) of the elliptic flow with transverse momentum, that is observed in experiments [4, 5] and predicted by transport models [27, 28, 29]. With anisotropic radial flow, quarks can acquire an elliptic flow without introducing explicitly anisotropic azimuthal distributions in space and momentum as in the models discussed so far. The quark distribution in the present model
is then given by
\[ \frac{dN_{q,q}}{d^2p_Td^2r} \propto \exp \left[ -\frac{m_T}{T} \cosh \rho(p_T, \phi_r) + \frac{p_T}{T} \sinh \rho(p_T, \phi_r) \cos(\phi_r - \phi_p) \right]. \] (17)

Defining \( \tilde{\alpha}(p_T, \phi_r) = \left( \frac{m_T}{T} \right) \cosh \rho(p_T, \phi_r) \) and \( \alpha(p_T, \phi_r) = \left( \frac{p_T}{T} \right) \sinh \rho(p_T, \phi_r) \), the transverse momentum dependence of the quark elliptic flow can be easily calculated as in Ref. [26], and the result is given by
\[ v_{2,q}(p_T) = \int_0^{2\pi} d\phi_r \exp \left[ -2\tilde{\alpha}(p_T/2, \phi_r) \right] \frac{I_2[\alpha(p_T/2, \phi_r)] \cos(2\phi_r)}{I_0[\alpha(p_T, \phi_r)]}. \] (18)

In the naive quark coalescence model, which only allows quarks with the same momentum to coalesce into hadrons, the meson distribution is simply given by the following one-dimensional integral:
\[ \frac{dN_M}{d^2p} \propto \int_0^{2\pi} d\phi_r \exp \left[ -2\tilde{\alpha}(p_T/2, \phi_r) \right] \exp \left[ 2\alpha(p_T/2, \phi_r) \cos(\phi_r - \phi_p) \right]. \] (19)

The elliptic flow of mesons is then
\[ v_{2,M}(p_T) = \int_0^{2\pi} d\phi_r \exp \left[ -2\tilde{\alpha}(p_T/2, \phi_r) \right] \frac{I_2[\alpha(p_T/2, \phi_r)] \cos(2\phi_r)}{I_0[\alpha(p_T/2, \phi_r)]}. \] (20)

Contrary to the case of isotropic radial flow, the quark number scaling of hadron elliptic flow is not obvious when the radial flow is anisotropic, and a violation is generally found as shown below.

Using the anisotropic function \( \rho_2 = 0.25 / \{ 1 + \exp(p_T - 0.5) / 0.75 \} \) + 0.014 in the quark radial flow, we have evaluated numerically both the quark and scaled meson elliptic flows, and they are shown in Fig.2 by the solid and dashed thick lines, respectively. It is seen that the quark number scaling of hadron elliptic flow is appreciably broken (about 20%) at \( p_T/n \sim 1.5 \) \( \text{GeV} \). Furthermore, the meson elliptic flow in the present model is larger than the one expected from the quark number scaling, in contrast with the smaller meson elliptic flow from most other sources of quark number scaling violation [14, 15, 16, 19]. Since the local harmonics in the quark momentum distribution in the present case have values very similar to those in the previous blast wave model with isotropic radial flow, they are not the reason that the quark number scaling of meson flow is violated. The violation is caused instead by the larger coalescence probability for quarks in the reaction plane than out of the reaction plane as a result of anisotropy in the radial flow. This can be seen by examining the second harmonic in the spatial distribution \( s_2 = \langle \cos(2\phi_r) \rangle \) of quarks and mesons. As
FIG. 2: Scaled momentum elliptic flow $v_2$ and spatial elliptic deformation $s_2$ as a function of scaled transverse momentum. Solid lines are the quark $v_{2,q}$ (thick) and $s_{2,q}$ (thin), while dashed lines are $v_{2,M}$ (thick) and $s_{2,M}$ (thin) of mesons.

shown in Fig. 2, the scaled spatial elliptic deformation $s_2$ of mesons (thin dashed line) is indeed larger than that of the quarks (thin solid line), except at low transverse momenta.

Our results at low transverse momenta needs, however, to be interpreted with caution as the coalescence model violates unitary if applied to the entire phase space. Although the coalescence model is applicable for describing the production of rare particles such as those with intermediate transverse momenta, a more dynamical approach that takes into account properly energy and entropy conservation is needed for more abundant low transverse momenta particles.

We would like to point out that for quarks with transverse momentum $p_T/n \geq 1.5$ GeV, their elliptic flow is not likely affected by anisotropy in their radial flow. The momentum distribution of these quarks is thus more similar to that in the schematic model of Sect. III for which the quark number scaling of hadron elliptic flow holds exactly.

V. HADRON WAVE FUNCTION EFFECT ON ELLIPTIC FLOW

Besides anisotropic radial flow, the quark number scaling of hadron elliptic flow can also be violated by the coalescence of quarks with different momenta and at different positions when the hadron wave function is taken into consideration as in Refs. 12, 15. To see this
explicitly, we generalize Eq. (8) to three dimensional space and replace the resulting two delta functions \((2\pi)^3\delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2)\delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_2)\) by the meson Wigner function from Ref. [12], i.e.,

\[
f_M(x_1, x_2; p_1, p_2) = \frac{9\pi}{2} \Theta \left( \Delta_x^2 - (x_1 - x_2)^2 \right) \Theta \left( \Delta_p^2 - (p_1 - p_2)^2 \right),
\]

where \(\Delta_x = \Delta_p^{-1}\) is the width of the quark momentum distribution in the meson.

\[\text{FIG. 3: Scaled elliptic flow as a function of scaled transverse momentum. The solid line is the quark elliptic flow } v_{2,q}. \text{ Meson elliptic flow from the coalescence model is shown by open circles for a width of hadron Wigner function } \Delta_p = 0.48 \text{ GeV, by filled squares for } \Delta_p = 0.24 \text{ GeV, and by filled circles for } \Delta_p = 0.1 \text{ GeV. Dashed lines are drawn to guide the eye.} \]

Using the same quark distribution function of Ref. [12], which is essentially the blast wave model with isotropic radial flow studied in Sect. IV A, we show in Fig. 3 the result for mesons from the coalescence model based on a Monte Carlo evaluation of the coalescence integral. Three different values of \(\Delta_p\) are used. The filled squares are the result with \(\Delta_p = 0.24 \text{ GeV}\) that was used in Ref. [12, 15] to fit the experimental meson spectra at RHIC. In this case, the quark number scaling is violated by about 10% in the scaled momentum region \(p_T/n \simeq 1 - 2\) GeV, as already found in a previous work [12, 15] and mentioned in the Introduction. In Ref. [24], a larger violation of about 20% has been found in the quark number scaling of hadron elliptic flow. This may be largely due to the fact that the corresponding value for
\[ \Delta p \] used in Ref. \[24\] is about a factor of two larger than our value due to the absence of the factor \(9\pi/2\) in the hadron Wigner function. Indeed, using \(\Delta p = 0.48\) GeV in the present calculation leads to a much smaller scaled meson elliptic flow as shown by open circles, thus a much stronger violation of the quark number scaling of the meson elliptic flow. In the scaled momentum region \(p_T/n \simeq 1 - 2\) GeV, the violation of the quark number scaling is consistent with the 20\% effect seen in Ref. \[24\]. Also shown in Fig. \[3\] by filled circles are results obtained with a smaller \(\Delta p = 0.1\) MeV. It is seen that they are now very close to the quark elliptic flow given by the solid line, as expected in the limit of \(\Delta p \to 0\).

![Figure 4](image_url)

**FIG. 4:** Local harmonics of quark momentum distributions as a function of transverse momentum from averaging over the first quadrant of the transverse plane.

We have also evaluated the local harmonics of the momentum distribution of quarks in the first quadrant. They are shown in Fig. \[4\] for \(v_{1,q}\) (filled squares), \(u_{1,q}\) (open squares), \(v_{2,q}\) (filled circles), and \(u_{2,q}\) (open circles). Both the real and imaginary local directed flows \(v_{1,q}\) and \(u_{1,q}\) are large, as in the anisotropic blast wave model of Sect. \[1B\] shown in Fig. \[2\].

Our results thus demonstrate that violation of the quark number scaling of hadron elliptic flow in the realistic model of Ref. \[12, 15\] is not due to the large local anisotropic flow but is rather a result of the finite dispersion of quark momentum inside hadrons.
VI. SUMMARY

To understand the observed quark number scaling in the elliptic flow of identified hadrons at RHIC, we have studied a number of models that range from the schematic one with a simple quark distribution to the realistic blast wave model that is consistent with the underlying hydrodynamic and transport models. In the naive coalescence model, in which only quarks with same momentum can coalescence, all models, except the blast wave model with an anisotropic radial flow, lead to scaling of hadron elliptic flow according to the constituent quark number. In the anisotropic blast wave model, the violation of this scaling is due to the azimuthal asymmetry in the quark coalescence probability as a result of the azimuthal angle dependence of the radial flow. In all cases, large local anisotropic flows are present, but they do not play any role in the final hadron elliptic flow. We have further demonstrated in a blast wave model that the quark number scaling can also be violated in a more realistic coalescence model which takes into account the momentum dispersion of quarks in hadron wave functions. Since effects due to anisotropic radial flow and hadron wave functions are small if realistic values are used for these quantities, observation of the quark number scaling of hadron elliptic flow in experiments thus provides a unique signature for hadronization via the quark coalescence as well as for the existence of the quark-gluon plasma prior the production of hadrons.

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[1] B. Zhang, M. Gyulassy and C.M. Ko, Phys. Lett. B 455, 45 (1999).
[2] D. Teaney, J. Laureant, and E.V. Shuryak, Phys. Rev. Lett. 86, 4783 (2001).
[3] P.F. Kolb, P. Huovinen, U. Heinz, and H. Heiselberg, Phys. Lett. B 500, 232 (2001).
[4] S.S. Adler et al., PHENIX Collaboration, Phys. Rev. Lett. 91, 182301 (2002).
[5] J. Adams et al., STAR Collaboration, Phys. Rev. Lett. 92, 052302 (2003).
[6] E. Shuryak, J. Phys. G 30, S1221 (2004).
[7] P. Sorensen, J. Phys. G. 30, S217 (2004).
[8] S.A. Voloshin, Nucl. Phys. A715, 379 (2003).
[9] R.J. Fries, B. M"uller, C. Nonaka, and S.A. Bass, Phys. Rev. C 68, 044902 (2003); Phys. Rev. Lett. 90, 202303 (2003).
[10] D. Molnar and S.A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003).
[11] P. Kolb, L.W. Chen, V. Greco, and C.M. Ko, Phys. Rev. C 69, 051901(R) (2004).
[12] V. Greco, C.M. Ko, and P. Lévai, Phys. Rev. C 68, 034904 (2003).
[13] R.C. Hwa and C.B. Yang, Phys. Rev. C 67, 034902 (2003); 064902 (2003).
[14] Z.W. Lin and D. Molnar, Phys.Rev. C 68 044901 (2003).
[15] V. Greco and C.M. Ko, Phys. Rev. C 70, 024901 (2004)
[16] B.Muller, R.J. Fries, and S.A. Bass, Phys. Lett. B 618, 77 (2005).
[17] P. Sorensen, nucl-ex/0510052.
[18] X. Dong, S. Esumi, P. Sorensen, and N. Xu, Phys. Lett. B 597, 328 (2004).
[19] S. Pratt and S. Pal, Phys. Rev.C 71, 014905 (2005).
[20] D. Molnar, nucl-th/0408044.
[21] M. Gyulassy, I. Vitev, X.N. Wang, Phys. Rev. Lett. 86, 2537 (2001).
[22] A. Dainese, C. Loizides, G. Paic, Eur. Phys. J. C 38, 461 (2005).
[23] V. Greco, C.M. Ko, and P. Lévai, Phys. Rev. Lett. 90, 202302 (2003); V. Greco, C.M. Ko, and R. Rapp, Phys. Lett. B 595, 202 (2004); V. Greco and C.M. Ko, J. Phys. G 31, S407 (2005).
[24] D. Molnar, J. Phys. G 30, S1239 (2004)
[25] U. Heinz, K.S. Lee, and E. Schnedermann, in Advanced Series on High Energy Physics 6, 471-517, Ed. by R. Hwa (World Scientific, Singapore, 1990).
[26] P. Huovinen, P.F. Kolb, U. Heinz, P.V. Ruuskanen, and S.A. Voloshin, Phys. Lett. B 503, 58 (2001).
[27] Z.W. Lin and C.M. Ko, Phys. Rev. C 65, 034904 (2002).
[28] L.W. Chen, C.M. Ko, and Z.W. Lin, Phys. Rev. C 69, 031901(R) (2004); L.W. Chen and C.M. Ko, J. Phys. G 31, S49 (2005).
[29] D. Molnar and M. Gyulassy, Nucl. Phys. A698, 379 (2002).