1. Introduction

There are several types of mechanisms which are suitable for use in a Stirling heat engine. Main advantage of crank mechanism is relative design simplicity. Amount of experiences with mathematical modelling and dynamic calculations of crank mechanism allows its optimisation on the operating conditions [1 - 4]. The Stirling engines with non-conventional mechanisms have several advantages in practical applications, but the design of calculation model is often difficult [5 - 6].

FIK mechanism (Fig. 1) is a swinging system. Centre (centre of gravity) of the swinging plate makes circular motion during the rotation of the shaft. Bevel gear is used against parallel rotation of swinging plate with shaft. Reciprocating movement of the pistons is transformed to the rotational movement of shaft through ball joint segments [7]. This schematic and virtual representation was designed for application in type of a Stirling engine, specifically for alpha configuration [8].

1.1 Basic dimensions for calculation of kinematic and dynamic parameters

Basic dimensions of schematic model are the most important parameters for creating a kinematic model which is described by mathematical equations [9]. Figure 2 (left) shows some of basic dimensions and coordinate system. Main dimensions, their individual shortcuts and their values are:

- \( D_v = 0.275\,\text{m} \) - distance between opposite cylinders (distance of cylinder axis),
- \( D = 0.075\,\text{m} \) - bore,
- \( Z = 0.069\,\text{m} \) - stroke,

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• $R = 0.294$m - radius of base rolling cone,
• $R_{ko} = 0.1346$m - distance between angle cranked axis of shaft and centre of lower ball joint centre,
• $\varphi = 15^\circ$ - angle between main axis and axis of cranked part of the shaft.

Schematic view of engine (left part) and dynamic model with the specified points of the reduced mass (right part) are in Fig. 2. Masses of these specified points are:

• $m_P = 0.6691$kg - reduced mass of the piston and connecting rods,
• $m_{Av} = 0.572$kg - reduced mass of the connecting rods,
• $m_{sw. p a cs} = 5.73$kg - reduced mass of the swinging plate and cranked part of the shaft.

The formulas for computing trajectories of the point $Av_1$ (first cylinder) in $p$-$q$-$z$ directions in a polar coordinate system are:

\[
p_{Av_1} = R \cdot \cos \alpha_t - \left[ (R - r_A) \cdot \cos \left( \alpha_t - \beta_A \right) \right] \cdot \cos \varphi \cdot \cos \alpha_t - \left[ (R - r_A) \cdot \sin \left( \alpha_t - \beta_A \right) \right] \cdot \sin \alpha_t.
\]

(1)

\[
q_{Av_1} = R \cdot \sin \alpha_t - \left[ (R - r_A) \cdot \cos \left( \alpha_t - \beta_A \right) \right] \cdot \cos \varphi \cdot \sin \alpha_t + \left[ (R - r_A) \cdot \sin \left( \alpha_t - \beta_A \right) \right] \cdot \sin \alpha_t.
\]

(2)

\[
z_{Av_1} = \left[ (R - r_A) \cdot \cos \left( \alpha_t - \beta_A \right) \right] \cdot \sin \varphi.
\]

(3)

where: $\alpha_t$ - rotation angle of shaft, $r_A$ - means $R_{ko}$, $\beta_A$ - fixed angular position of point $Av_1$ to wobble plate.

Then formulas for calculating acceleration in $p$-$q$-$z$ (general entry) are:

\[
a_{p_{Av_1}} = \frac{d^2}{dt^2} p_{Av_1}, a_{q_{Av_1}} = \frac{d^2}{dt^2} q_{Av_1}, a_{z_{Av_1}} = \frac{d^2}{dt^2} z_{Av_1}.
\]

(4)

The resulting equation for calculating the stroke in point P1 is:

\[
z_P = \left[ \left[ (R - r_A) \cdot \cos \left( \alpha_t - \beta_A \right) \right] \cdot \sin \varphi \right] + I_0 \cdot \cos \left[ \arcsin \left( \frac{\sqrt{(q_{Av_1} - q_{P0})^2 + (p_{Av_1} - p_{P0})^2}}{I_0} \right) \right],
\]

(5)

where: $l_0 = 0.193$m - length of ball joint segment (length of the connecting rod), $q_{Av_1}, p_{Av_1}$ - position in coordinates of the cylinder axis in the $p$-$q$ plane.

Then formula for calculating of acceleration in $z$ direction (general entry) is:

\[
a_P = \frac{d^2}{dt^2} z_P.
\]

(6)

From the dynamic model were determined equations for the total inertial forces and its moments in $p$-$q$-$z$ directions [10 - 11]. The courses of inertial forces and moments resulting from the final calculation of the dynamical model are shown in Fig. 3.

Following projections (Fig. 4) of the trajectory of the point $Av_1$ in $p$-$q$-$z$ planes were obtained from formulas (1), (2) and (3).

2. Design of balancing mass

Figure 5 shows a virtual model, real model and sensor placement (attaching of sensor in $p$ axis).

Balancing system requires the following proposal of balancing mass:

• $r_v = 0.15$m - the distance between the centre of gravity of balancing mass and shaft axis ($z$ - axis),
• $m_v = 0.2$kg - the weight of balancing mass,
• $v_p = 0.5$m - the distance between the centre of gravity of balancer mass and the plane $p$-$q$.

It is impossible to keep the distance $v_p = 0.5$m for structural reasons (Fig. 5, virtual model, the distance between the upper and lower plates is 0.247$m). Based on the above parameters the following alternative solutions were proposed.
Fig. 4 Trajectories of Av1 point in p-q, p-z and q-z planes and the sequence of movements of the point Av1 according to the angle of rotation of the shaft.

Fig. 5 Construction of a machine designed for the measurement of basic dynamic parameters and sensor placement (1- acceleration sensor p direction)
3. First type of balancing mass

This distance \( v \) was modified to 0.07m due to real possibilities of the machine structure and mounting of balancing mass. The balancing system is designed to eliminate the inertial forces in directions p, and q, but the total torque of the system is able to balance only partially. Inertial forces in the z direction and the total moment around z axis was ignored in the design of the balancing system (Fig. 6).

![Fig. 6 Resulting courses with and without balancing mass (first type, left); Balancing system applied on real model (right), 2 - balancing mass](image)

4. Second type of balancing mass

This balancing system was designed for better elimination of moments from inertial forces than the first type (this improvement was achieved by increasing the distance \( v \)), but it was impossible to keep the distance between the centre of gravity of the balancing mass and shaft axis (z-axis) as the first type, so the nearest possible distance \( r \) was chosen (Fig. 7).

![Fig. 7 Resulting courses with and without balancing mass (second type, left); Balancing system applied on real model (right)](image)

The parameters for positioning and adjustment of the second type of balancing mass are:
- \( r_v = 0.155m \) - the distance between the centre of gravity of the balancing mass and shaft axis (z-axis),
- \( m_v = 0.22kg \) - weight of the balancing mass,
- \( v_v = 0.14m \) - the distance between the centre of gravity of the balancing mass and the plane p-q.

5. Measurement results

Spectral analysis:
The measuring device VIBROTEST 60 (Fig. 8) was used for the measurement of spectral analysis. Basic settings of the device:
- spectral range 0-200Hz,
- averaging RMS.
Measurement of vibration mightiness:

The following comparison shows the values of the vibration mightiness for a given speed range in p-axis and the axis of

- 1600 numbers of lines (device settings for integer values of frequencies by spectral range) [12 - 13].
the first cylinder. The vibration mightiness is the most effective vibration velocity (in a given time interval) at selected locations [11]. Measurements of vibration mightiness were performed for purpose of comparing systems without and the system with balancing mass designed by the calculation [12]. The selected measurement time interval was 60 seconds (Fig 11).

6. Conclusion

It is clearly visible from the spectral analysis shown in Figs. 9 and 10 how big is the difference of oscillation courses for the measurement with and without balancing mass at the excitation frequency 7Hz. The course of the first type of balancing mass has worse elimination of the moment from inertial forces of piston group P1in the z direction. It can be seen that balancing mass was able to eliminate the moments from inertial forces in the z direction at the excitation frequency of 7Hz only partially. The courses of the second type of balancing mass in the p direction has better elimination of inertia and oscillations in the z direction, but it was difficult to set the minimal position of the arm of balancing mass (r V). The measurements were performed also in other possible adjustments of the balancing system and it shows that with these two types of balancing masses was FIK machine able to achieve the highest RPM without endangering the stability of the model. There are visible significant differences in higher revolutions in the case of vibration mightiness measurement but interpretability of the results corresponds to the computational model as the results of spectral analysis show.

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