Quark distributions in polarized and nonpolarized hadrons

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Valence quark distributions in pion, polarized $\rho$ mesons and nucleon in the region of intermediate $x$ are obtained by generalized QCD sum rules. It is shown, that polarization effects are very significant. The strong suppression of quark and gluon sea distributions in longitudinally polarized $\rho$ mesons is found.

Determination of quark distribution functions in a model-independent way in QCD sum rules based only on QCD and the operator product expansion (OPE) seems to be very important task especially for polarized hadron. A method to determinate valence quark distribution in QCD sum rules at intermediate $x$ was suggested in [1] and generalized in [2],[3]. Let me briefly present the method. We start from consideration of the 4-point correlator with 2 hadron and 2 electromagnetic currents.

$$\Pi = -i \int d^4x d^4y d^4z e^{ip_1 x + i q y - i p_2 z} \langle 0 | T \{ j^h(x), j^{el}(y), j^{el}(0), j^h(z) \} | 0 \rangle$$

Here $p_1$ and $p_2$ are the initial and final momenta carried by hadronic the current $j^h$, $q$ and $q' = q + p_1 - p_2$ are the initial and final momenta carried by virtual photons (Lorentz indices are omitted) and $t = (p_1 - p_2)^2 = 0$. To find structure function one should compare dispersion representation of the forward scattering amplitude in terms of physical states with those in OPE and use Borel transformation. Though we should consider the case of forward scattering, i.e. $p_1 = p_2$, but, and it is significant to method, we should keep $p_1^2$ not equal to $p_2^2$ in all intermediate stages and only in final result take the forward scattering limit. Only in such way, and it was the main idea of generalization of sum rules (see [2]) it was found to effectively (exponentially) suppress all terms in sum rules except those which are proportional to structure function.

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so we can found structure functions of hadron with good accuracy. Finally equating the physical and QCD representations (for detail see [2]) after double borelization (on $p_1^2$ and $p_2^2$) we found

$$Im \Pi_{QCD}^0 + \text{Power correction} = 2\pi F_2(x, Q^2) g_h^2 e^{m_h^2/(M_1^2 + M_2^2)}$$

Here $Im \Pi_{QCD}^0$ is the bare loop contribution, (where continuum contribution is eliminated) and $g_h$ is defined as $\langle 0| j_h | h \rangle = g_h$. In what follows, we put $M_1^2 = M_2^2 \equiv 2M^2$.

Let me briefly illustrate the main points of the calculation for the case of pion. It can be treated as a check of the accuracy of the method due to fact that for pion the experimental results are available. To find the pion structure function one should choose the imaginary part of 4-point correlator with two axial (hadron) and two electromagnetic currents and consider the invariant amplitude at tensor structure, $P_\mu P_\nu P_\lambda P_\sigma / \nu$, where $P = (p_1 + p_2)/2$, $\mu, \nu$ are vector current indexes and $\lambda, \sigma$ - hadron current indexes. I shall briefly note the main points of calculations, for detail see [2]. In QCD part of sum rules we take into account the following terms:

1. Unit operator contribution (bare loop) and leading order (LO) perturbative corrections proportional to $ln(Q^2/\mu^2)$, where $\mu^2$ is the normalization point. In what follows, the normalization point will be chosen to be equal to the Borel parameter $\mu^2 = M^2$.

2. Power corrections – higher order terms of OPE. Among them, the dimension-4 correction, proportional to the gluon condensate $\langle 0| G_{\mu \nu} G_{\alpha \beta} | 0 \rangle$ was first taken into account, but it was found that the gluon condensate contribution to the sum rule vanishes after double borelization. There are two types of vacuum expectation values of dimension 6. One, involving only gluonic fields: $\frac{2\pi}{\alpha_s} f^{abc} (G^a_{\mu \nu} G^b_{\nu \lambda} G^c_{\lambda \mu} | 0 \rangle$ and the other, proportional to the four-quark operators (after factorization) $a^2 = \alpha_s (2\pi)^4 \langle 0| \bar{\psi} \psi | 0 \rangle^2$. Terms of the first type cancel in the sum rule for pion and only terms of the second type survive. Finally quark distribution function has the following form:

$$x u_\pi(x) = \frac{3}{2\pi} \frac{M^2}{f_\pi^2} x^2 (1-x) \left[ 1 + \left( \frac{a_s(M^2) \cdot ln(Q_0^2/M^2)}{3\pi} \right) \right]$$

$$\times N(x) \left( 1 - e^{-s_0/M^2} \right) \left[ \frac{4\pi a_s(M^2) \cdot 4\pi a^2}{(2\pi)^4 \cdot 3^7 \cdot 2^6 \cdot M^6} \cdot \frac{\omega(x)}{x^3(1-x)^3} \right], \quad (1)$$
where \( N(x) = (1 - x)(1 + 4x\ln(1 - x)) - 2x(1 - 2x)\ln x \) and \( \omega(x) \) is the fourth degree polynomial in \( x \). This function \( u_x(x) \) may be used as an initial condition at \( Q^2 = Q_0^2 \) for solution of QCD evolution equations.

Figure 1: Quark distribution function for pion, transversally and longitudinally polarized \( \rho \) - dash, solid and line with squares correspondingly

In the numerical calculations we choose: the effective \( \Lambda_{QCD}^{LO} = 200 \ MeV \), \( Q_0^2 = 2 \ GeV^2 \). \( a^2 \) is varied \( a^2 = 0.23\pm0.1 GeV^6 \), the upper limit is close to those found from recently from \( \tau \) decays analysis [4], the middle is close to standard choose. The continuum threshold was varied in the interval \( 0.8 < s_0 < 1.2 GeV^2 \) and it was found, that the results depend only slightly on it’s variation. The analysis of the sum rule (1) shows, that it is fulfilled in the region \( 0.2 < x < 0.7 \), where both power correction and continuum contribution are less than 30% and the stability in the Borel mass parameter \( M^2 \) dependence in the region \( 0.4 GeV^2 < M^2 < 0.7 GeV^2 \) is good. The accuracy of our results is about 20–30%, the main source of uncertainty is quark condensate value and possible non-logarithmic perturbative corrections. The result of our calculation of the valence quark distribution function (QDF) \( xu_\pi(x,Q_0^2) \) in the pion is shown in Fig.1. Comparison with various experimental result lead us to conclusion, that
agreement with experiment is good. I want to note, that this result of QDF is based only on the QCD sum rules.

If we use some natural additional assumptions, we also can calculate the second moments of QDF. Assume, that $u_\pi(x) \sim 1/\sqrt{x}$ at small $x \lesssim 0.15$ according to the Regge behaviour and $u_\pi(x) \sim (1-x)^2$ at large $x \gtrsim 0.7$ according to quark counting rules. Then, matching these functions with our result, one may find the numerical values of the first and the second moments of the $u$-quark distribution: $M_1 = \int_0^1 u_\pi(x) dx \approx 0.84$, $M_2 = \int_0^1 x u_\pi(x) dx \approx 0.21$

where the values various slightly at reasonable changes of QDF behaviour at large or small $x$. The moment $M_1$ has the meaning of the number of $u$ quarks in $\pi^+$ and it should be $M_1 = 1$. The deviation of $M_1$ from 1 characterizes the accuracy of our calculation. The moment $M_2$ has the meaning of the pion momentum fraction carried by the valence $u$ quark. Therefore, the valence $u$ and $\bar{d}$ quarks carry about 40% of the total momentum what is close to experimental results.

In the same way one can calculate valence $u$-quark distribution in the $\rho^+$ meson. The choice of hadronic current is evident. $j_\rho^\mu = \bar{d}\gamma_\mu u$ and the matrix element is $\langle \rho^+ | J_\mu^\rho | 0 \rangle = m_\rho^2 e_\mu$ where $m_\rho$ is the $\rho$-meson mass, $g_\rho$ is the $\rho - \gamma$ coupling constant, $g_\rho^2/4\pi = 1.27$ and $e_\mu$ is the $\rho$ meson polarization vector.

In the non-forward amplitude the tensor structure for determination of $u$-quark distribution in the longitudinal $\rho$ meson is $P_\mu P_\nu \delta_{\lambda\sigma}$, while for transverse $\rho$ it is $-P_\mu P_\nu \delta_{\lambda\sigma}$ (see [3]).

In the case of longitudinal $\rho$ meson the tensor structure, that is separated is the same as in the case of the pion. It was shown, that $u$-quark distribution in the longitudinal $\rho$ meson can be found from Eq.(1) by substituting $m_\pi \rightarrow m_\rho$, $f_\pi \rightarrow m_\rho/q_\rho$. Sum rules for $u_\rho^\nu(x)$ are satisfied in the wide $x$ region: $0.1 < x < 0.85$ with high accuracy (about 10%). Figure. 1 (curve with squares) presents $x u_\rho^\nu(x)$ as a function of $x$. The values $M^2 = 1$ GeV$^2$ and $s_0 = 1.5$ GeV$^2$, $Q_0^2 = 4$ GeV$^2$ were chosen.

Let us now consider the case of transverse $\rho$-meson, i.e., the term proportional to the structure $P_\mu P_\nu \delta_{\lambda\sigma}$. The procedure of calculations are the same except two points.

1. In contrast to the pion case, the $\langle G^{a\mu}_\rho G^{a\nu}_\rho \rangle$ correction for transversally polarized $\rho(\rho_T)$, does not vanish.
2. In contrast to \( \pi \) and \( \rho_L \)-meson cases, the terms proportional to 
\[
\langle 0 \mid g^3 f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \mid 0 \rangle
\]
are not cancelled for \( \rho_T \) and one should estimate it. But 
\[
\langle 0 \mid g^3 G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} f^{abc} \mid 0 \rangle
\]
is not well known; so we need to use estimations based there are only some estimates based on the instanton model. Result for QDF in transversally polarized \( \rho \) meson is shown on Fig.1 for the region \( 0.2 < x < 0.65 \) (dash line). One can find complete analytical form and its detail analysis in our paper [3], I do not write it down since it is very large. The choose of parameters are the same as in previous cases. The main sources of uncertainties are \( d = 6 \) gluon condensate (gives about 20\% and \( d = 4 \) gluon condensate (due the uncertainty of it own value about factor 1.5, which lead to uncertainty in QDF about 20\%). We estimate accuracy about 30–50\% (better in the middle of region and worser at the end). From fig.1 one can see, that difference between QDF for longitudally \( \rho_L \) and \( \rho_T \) transversally polarized \( \rho \) meson is very large, many times larger that uncertainties.

So we can conclude that QDF significantly depend of polarization.

Moreover, if we construct the moment of QDF for \( \rho_L \) in the same way as for pion then we found \( M_1 = 1.06 \) and \( M_2 = 0.39 \) (for one valence quark). So we see that valence quark carry about 80\% of the total momentum of longitudinally polarized \( \rho \) meson, so gluon sea there should be strongly suppressed. One should note, that accuracy of this prediction of moment of \( \rho_L \) is very high, because in this case sum rules for QDF are covered almost all \( x \) region, as I noted before, so extrapolation procedure contribution numerically is negligible.

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