Calabi-Yau Varieties and Pencils of K3 Surfaces

S. Hosono$^1$, B.H. Lian$^2$ and S.-T. Yau$^3$

$^1$ Department of Mathematics
Toyama University
Toyama 930, Japan

$^2$ Department of Mathematics
Brandeis University
Waltham, MA 02154

$^3$ Department of Mathematics
Harvard University
Cambridge, MA 02138

Abstract: In this note, we give a list of Calabi-Yau hypersurfaces in weighted projective 4-spaces with the property that a hypersurface contains naturally a pencil of K3 variety. For completeness we also obtain a similar list in the case K3 hypersurfaces in weighted projective 3-spaces. The first list significantly enlarges the list of K3-fibrations of [1] which has been obtained on some assumptions on the weights. Our lists are expected to correspond to examples of the so-called heterotic-type II duality [2].
1. Problems

Let \( w_1, \ldots, w_{n+1} \) be positive integers, and put \( d := \sum w_i \). We call the weight vector \( \hat{w} = (w_1, \ldots, w_{n+1}) \) admissible if the generic weighted degree \( d \) hypersurface in \( \mathbb{C}^{n+1} \) is smooth away from the origin. This means that the weighted projectivized hypersurface in \( \mathbb{P}[w] \) is transversal, i.e., it only acquires singularities from the ambient space \( \mathbb{P}[\hat{w}] \). For \( n = 3 \), there is a list of admissible weights of Reid-Yonemura (see [4]). For \( n = 4 \), there is a list of admissible weights obtained by Klemm-Schimmrigk [5] and Kreuzer-Skarke [6].

Given an admissible weight \( \hat{w} = (w_1, \ldots, w_{n+1}) \) we can consider in \( \mathbb{P}[\hat{w}] \) the generic Calabi-Yau variety given by

\[
\hat{X}_a = \{ z | \sum_{\hat{w} \cdot \nu = d} a_\nu z^\nu = 0 \}.
\] (1.1)

Suppose we intersect this variety with the coordinate hyperplane \( z_{n+1} = 0 \).

**Problem 1.1.** When is \( X_a := \hat{X}_a \cap \{ z_{n+1} = 0 \} \) isomorphic to a transversal Calabi-Yau variety?

Note that by permuting the weights, this includes the cases \( \hat{X}_a \cap \{ z_i = 0 \} \) for any \( i \).

More generally,

**Problem 1.2.** When is there a 1-parameter family of hypersurfaces \( Z_\lambda \) such that \( X_{a,\lambda} := \hat{X}_a \cap Z_\lambda \) is isomorphic to a transversal Calabi-Yau variety?

For \( n = 4 \) and with some assumptions on the weights, a short list of such cases has been tabulated in [4]. We say that \( \nu \) is compatible with the weight \( \hat{w} \) if \( \hat{w} \cdot \nu = d \).

Let \( w = (w_1, \ldots, w_n) \) and \( \bar{w} \) its normalization, i.e. \( \bar{w} := (w_1/\delta_1, \ldots, w_n/\delta_n) \) where \( \delta_i := \text{lcm}(\rho_1, \ldots, \rho_i, \ldots, \rho_n) \) and \( \rho_i := \text{gcd}(w_1, \ldots, \hat{w}_i, \ldots, w_n) \). It is well known that \( \phi : \mathbb{P}[w] \to \mathbb{P}[\bar{w}] \) is an isomorphism under the normalization map \( (z_1, \ldots, z_n) \mapsto (z_1^{\rho_1}, \ldots, z_n^{\rho_n}) \). It is easy to show that \( \delta_1 \rho_1 = \cdots = \delta_n \rho_n \); we call this integer \( k \).

We require that the image \( \bar{X}_a = \phi X_a \) is a transversal Calabi-Yau variety in \( \mathbb{P}[\bar{w}] \). If \( x_1, \ldots, x_n \) are the quasi-homogeneous coordinates of \( \mathbb{P}[\bar{w}] \), then a Calabi-Yau variety can be written as

\[
\bar{X}_b = \{ x | \sum_{\bar{w} \cdot \mu = d} b_\mu x^\mu = 0 \},
\] (1.2)
where \( \bar{d} := \sum_{i=1}^{n} \bar{w}_i = \sum w_i/\delta_i. \) Pulling this back by the normalization map, we see that

\[
\phi^{-1} \bar{X}_b = \{ z \mid \sum_{\bar{w} \cdot \mu = \bar{d}} b_{\mu} \prod \bar{z}^{|\rho_i|} = 0 \} \subset P[w].
\] (1.3)

If we require that \( \phi^{-1} \bar{X}_b = X_a \) for some \( a \), then we conclude that

(a) every \( \mu \in \mathbb{Z}_+^n \) compatible with \( \bar{w} \) has \( \sum w_i \rho_i \mu_i = d \), and \( \nu = (\rho_1 \mu_1, \ldots, \rho_n \mu_n, 0) \) is an exponent compatible with \( \bar{\nu} \).

(b) every exponent \( \nu \in \mathbb{Z}_+^{n+1} \) compatible with \( \bar{\nu} \) having \( \nu_{n+1} = 0 \) is of the form \( \nu = (\rho_1 \mu_1, \ldots, \rho_n \mu_n, 0) \) for some exponent \( \mu \) compatible with \( \bar{\nu} \).

We claim that \( d = k\bar{d} \). This follows from (a) and the fact that \( k = \rho_i \delta_i. \) Since \( \mu = (1, \ldots, 1) \) is compatible with \( \bar{w} \), it follows from (a) that \( w \cdot \rho = d \). Thus our task is to search through the list of normalized admissible weights \( \hat{\nu} \) \((n = 4)\) satisfying

\[
(i) \quad \hat{w} \cdot \nu = d, \quad \nu_5 = 0 \Rightarrow \rho_i | \nu_i \quad \forall i
\]

\[
(ii) \quad w \cdot \rho = d
\]

\[
(iii) \quad \bar{\nu} \text{ admissible}
\]

On the last condition, we will check that \( \bar{w} \) be in the Reid-Yonemura list. It is also clear that (i)–(iii) implies that \( \hat{X}_a \cap \{ z_{n+1} \} \) is isomorphic to \( \bar{X}_b \) in the admissible \( P[\bar{w}] \). Our computer search shows that there are 628 admissible weights \( \hat{\nu} \) of length 5 satisfying (i)–(iii).

**Example:** Take \( \hat{\nu} = (42, 27, 8, 4, 3), \) \( d = 84 \). We consider the intersection \( X := \hat{X} \cap \{ z_3 = 0 \} \). Then \( \rho = (1, 1, 3, 1) \), and so condition (ii) holds. The normalized weight of \( w = (42, 27, 4, 3) \) becomes \( \bar{w} = (14, 9, 4, 1) \), which is an admissible weight of length 4 (see [4]), and so condition (iii) holds. The equations for \( X \) in \( P[\hat{\nu}] \) is \( z_3 = 0 \) plus that of \( \hat{X} \). The latter is given by the generic sum of the monomials with admissible exponent \( \nu \) with \( \nu_5 = 0 \). There are exactly 24 such exponents:

\[
\begin{align*}
&z_5^{28}, z_5^{24} z_4, z_5^{20} z_4^2, z_5^{16} z_4^3, z_5^{12} z_4^4, z_5^8 z_4^5, z_5^4 z_4^6, z_4^7, z_4^3 z_2, \\
z_5^{15} z_4^2 z_2, z_5^{11} z_4^2 z_2, z_5^7 z_4^2 z_2, z_5^3 z_4^2 z_2, z_5^{10} z_2^2, z_5^6 z_2^3 z_2, \\
z_5^6 z_2^2 z_2, z_5^{12} z_2, z_5^8 z_2, z_5^4 z_2, z_5^2 z_2, z_5 z_2, z_2, z_2^3,
\end{align*}
\] (1.5)

Condition (i) holds because the exponent \( \nu_4 \) of \( z_4 \) is always a multiple of \( \rho_3 = 3 \). The equation for the isomorphic image \( \hat{X} \) of \( X \) in \( P[14, 9, 4, 1] \) is the generic sum of the above monomials with the replacement, \( z_1 \mapsto x_1, z_2 \mapsto x_2, z_4 \mapsto x_3^3, z_5 \mapsto x_4 \).

We note that given an admissible weight \( \hat{\nu} \), the Calabi-Yau varieties in \( P[\hat{\nu}] \) can give two distinct transversal Calabi-Yau varieties when intersect with two different coordinate hyperplanes \( z_i = 0 \).
1.1. the second problem

We consider our second problem under the following assumption. We assume that $Z_\lambda$ is of the form

$$\lambda_1 z_{n+1} = \lambda_2 p(z)$$

where $\lambda = [\lambda_1, \lambda_2]$ is regarded as a point in $\mathbb{P}^1$, and $p(z)$ a fixed nonzero quasi-homogeneous polynomial independent of $z_{n+1}$ and has degree $w_{n+1}$. When $\lambda_2 = 0$ this reduces to the case in the first problem. This generalization turns out to require just some minor modification. Specifically, in addition to conditions (i)–(iii), we must require that the weight component (iv) $w_{n+1}$ can be partitioned by the components $w_1, \ldots, w_n$.

This is true iff $p$ exists. Note that as $\lambda$ varies the intersections $\hat{X}_a \cap Z_\lambda$ form a pencil of codimension one subvarieties in $\hat{X}_a$. In the case of $n = 4$ we require that they are transversal K3 varieties when $\lambda_1 \neq 0$. In our list of 628 cases above, we find that all of them admit this description hence enlarging the list of $[\square]$.

The table given in the appendix is the list of the 628 cases. The number denoted $i$ between 1 and 5 in the table indicates $Z_\lambda$ is of the form $\lambda_1 z_i = \lambda_2 p(z)$ as in the case $i = n + 1$ discussed above. Some of the examples in this list have been studied in great details in the context of mirror symmetry (see for example $[7\square8\square9\square]$), and in connection with string duality in $[2\square10\square]$ and others.

We note that the conditions we impose in our method for enumerating K3 pencils are only sufficient but not necessary. There is in fact a criterion given in $[11\square]$ for K3 pencils using the intersection ring of the Calabi-Yau variety. In fact in $[4\square]$ (see the conclusion section there) we have already used this criterion to give a few examples of K3 pencils in which we have computed the intersection ring. For example, the Calabi-Yau hypersurfaces in $\mathbb{P}[8,3,3,1,1]$ was found to have a K3 pencil according to the criterion of $[11\square]$, but this example fails to satisfy conditions (i)-(iii) above. In $[4\square]$, we have also given an algorithm for computing the intersection ring of Calabi-Yau hypersurfaces in weighted projective spaces. This algorithm can in principle be carried out for all of the list $[4\square]$, and be used to check the criterion above. But the actual computation can be enormous.

For completeness, we also do the case of $n = 3$. Thus we search through the list of transversal K3 hypersurfaces in $[4\square]$ which admits a pencil of elliptic curves in one of the following transversal weighted projective spaces $\mathbb{P}[1,1,1], \mathbb{P}[2,1,1], \mathbb{P}[3,2,1]$. The $n = 3$ analogues of conditions (i)-(iii) are satisfied by 18 admissible weights, and all of them satisfy condition (iv).
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| \( \hat{w} = (w_1, \ldots, w_4) \) | \( i \) | \( \hat{w} \) | |---|---|---|
| (4, 3, 3, 2) | 1 | (2, 1, 1) | | (4, 3, 3, 2) | 2 | (3, 2, 1) |
| (2, 2, 1, 1) | 3 | (1, 1, 1) | | (4, 4, 3, 1) | 3 | (1, 1, 1) |
| (4, 2, 1, 1) | 3 | (2, 1, 1) | | (6, 3, 2, 1) | 3 | (2, 1, 1) |
| (10, 5, 4, 1) | 3 | (2, 1, 1) | | (6, 4, 1, 1) | 3 | (3, 2, 1) |
| (9, 6, 2, 1) | 3 | (3, 2, 1) | | (12, 8, 3, 1) | 3 | (3, 2, 1) |
| (21, 14, 6, 1) | 3 | (3, 2, 1) | | (3, 3, 2, 1) | 3 | (1, 1, 1) |
| (9, 4, 3, 2) | 2 | (3, 2, 1) | | (8, 4, 3, 1) | 3 | (2, 1, 1) |
| (12, 7, 3, 2) | 2 | (2, 1, 1) | | (18, 11, 4, 3) | 2 | (3, 2, 1) |
| (15, 10, 4, 1) | 3 | (3, 2, 1) | | (18, 12, 5, 1) | 3 | (3, 2, 1) |
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2. Appendix

| Euler # | $n^1$ | $\psi = (w_1, \ldots, w_5)$ | $i$ | $\phi$ |
|---------|-------|-----------------------------|-----|--------|
| 480     | 287   | (882, 588, 251, 36, 7)     | 3   | (21, 14, 6, 1) |
| 376     | 201   | (280, 140, 109, 16, 15)    | 3   | (14, 7, 4, 3)  |
| 324     | 212   | (630, 420, 179, 24, 7)     | 3   | (15, 10, 4, 1) |
| 256     | 147   | (200, 100, 77, 15, 8)      | 3   | (10, 5, 3, 2)  |
| 240     | 173   | (504, 336, 143, 18, 7)     | 3   | (12, 8, 3, 1)  |
| 216     | 141   | (200, 100, 79, 16, 5)      | 3   | (10, 5, 4, 1)  |
| 192     | 110   | (60, 60, 43, 9, 8)         | 3   | (5, 5, 3, 2)   |
| 180     | 114   | (143, 110, 44, 30, 3)      | 4   | (13, 10, 4, 3) |
| 180     | 114   | (130, 100, 40, 27, 3)      | 4   | (13, 10, 4, 3) |
| 168     | 95    | (144, 67, 48, 20, 9)       | 2   | (12, 5, 4, 3)  |
| 160     | 115   | (160, 80, 61, 15, 4)       | 3   | (8, 4, 3, 1)   |
| 160     | 115   | (160, 80, 63, 12, 5)       | 3   | (8, 4, 3, 1)   |
| 156     | 86    | (77, 56, 42, 30, 5)        | 4   | (11, 8, 6, 5)  |
| 156     | 86    | (66, 48, 36, 25, 5)        | 4   | (11, 8, 6, 5)  |
| 144     | 131   | (378, 252, 107, 12, 7)     | 3   | (9, 6, 2, 1)   |
| 144     | 98    | (162, 99, 32, 27, 4)       | 3   | (18, 11, 4, 3) |
| 144     | 91    | (80, 56, 32, 21, 3)        | 4   | (10, 7, 4, 3)  |
| 120     | 86    | (48, 48, 35, 9, 4)         | 3   | (4, 4, 3, 1)   |
| 120     | 69    | (100, 35, 32, 25, 8)       | 3   | (20, 8, 7, 5)  |
| 120     | 69    | (54, 42, 25, 24, 5)        | 3   | (9, 7, 5, 4)   |
| 120     | 65    | (60, 40, 36, 35, 9)        | 3   | (12, 9, 8, 7)  |
| 120     | 65    | (48, 32, 28, 27, 9)        | 4   | (12, 9, 8, 7)  |
| 112     | 76    | (98, 49, 24, 21, 4)        | 3   | (14, 7, 4, 3)  |
| 112     | 63    | (55, 30, 28, 20, 7)        | 3   | (11, 7, 6, 4)  |
| 112     | 63    | (44, 24, 21, 16, 7)        | 3   | (11, 7, 6, 4)  |
| 108     | 60    | (50, 30, 25, 24, 21)       | 1   | (25, 10, 8, 7) |
| 108     | 60    | (25, 25, 20, 16, 14)       | 1   | (25, 10, 8, 7) |
| 96      | 167   | (225, 200, 150, 24, 1)     | 4   | (9, 8, 6, 1)   |
| 96      | 167   | (216, 192, 144, 23, 1)     | 4   | (9, 8, 6, 1)   |
| 96      | 87    | (120, 60, 47, 8, 5)        | 3   | (6, 3, 2, 1)   |
| Euler # | $h^{1,1}$ | $\hat{\omega} = (w_1, \ldots, w_5)$ | $i$ | $\hat{\omega}$ |
|---------|-----------|---------------------------------|-----|--------------|
| 96      | 79        | $(88, 64, 21, 16, 3)$           | 3   | $(11, 8, 3, 2)$ |
| 96      | 65        | $(63, 42, 35, 24, 4)$           | 4   | $(9, 6, 5, 4)$  |
| 96      | 59        | $(99, 44, 22, 18, 15)$          | 2   | $(33, 22, 6, 5)$ |
| 96      | 59        | $(44, 33, 33, 12, 10)$          | 2   | $(33, 22, 6, 5)$ |
| 96      | 59        | $(44, 39, 22, 15, 12)$          | 1   | $(22, 13, 5, 4)$ |
| 96      | 59        | $(56, 33, 20, 12, 11)$          | 2   | $(14, 11, 5, 3)$ |
| 96      | 59        | $(42, 22, 15, 11, 9)$           | 2   | $(14, 11, 5, 3)$ |
| 96      | 57        | $(44, 32, 24, 15, 5)$           | 4   | $(11, 8, 6, 5)$  |
| 96      | 55        | $(38, 24, 19, 18, 15)$          | 1   | $(19, 8, 6, 5)$  |
| 96      | 55        | $(19, 19, 16, 12, 10)$          | 1   | $(19, 8, 6, 5)$  |
| 84      | 104       | $(294, 196, 56, 39, 3)$         | 4   | $(21, 14, 4, 3)$ |
| 84      | 54        | $(36, 27, 27, 10, 8)$           | 2   | $(27, 18, 5, 4)$ |
| 84      | 54        | $(38, 33, 19, 15, 9)$           | 1   | $(19, 11, 5, 3)$ |
| 84      | 54        | $(22, 19, 19, 10, 6)$           | 2   | $(19, 11, 5, 3)$ |
| 84      | 50        | $(36, 31, 18, 15, 8)$           | 2   | $(6, 5, 4, 3)$   |
| 84      | 50        | $(34, 21, 18, 17, 12)$          | 1   | $(17, 7, 6, 4)$  |
| 84      | 50        | $(17, 17, 14, 12, 8)$           | 1   | $(17, 7, 6, 4)$  |
| 80      | 68        | $(112, 56, 32, 21, 3)$          | 4   | $(14, 7, 4, 3)$  |
| 80      | 51        | $(32, 16, 15, 12, 5)$           | 3   | $(8, 5, 4, 3)$   |
| 72      | 68        | $(108, 49, 36, 20, 3)$          | 2   | $(9, 5, 3, 1)$   |
| 72      | 68        | $(108, 53, 36, 15, 4)$          | 2   | $(9, 5, 3, 1)$   |
| 72      | 65        | $(36, 36, 25, 8, 3)$            | 3   | $(3, 3, 2, 1)$   |
| 72      | 59        | $(56, 35, 18, 14, 3)$           | 3   | $(8, 5, 3, 2)$   |
| 72      | 57        | $(50, 35, 20, 12, 3)$           | 4   | $(10, 7, 4, 3)$  |
| 72      | 50        | $(72, 32, 16, 15, 9)$           | 2   | $(24, 16, 5, 3)$ |
| 72      | 50        | $(44, 27, 20, 9, 8)$            | 2   | $(11, 9, 5, 2)$  |
| 72      | 49        | $(34, 30, 17, 12, 9)$           | 1   | $(17, 10, 4, 3)$ |
| 72      | 49        | $(28, 24, 15, 8, 5)$            | 3   | $(7, 6, 5, 2)$   |
| 72      | 49        | $(28, 11, 11, 10, 6)$           | 2   | $(14, 11, 5, 3)$ |
| 72      | 49        | $(20, 17, 17, 8, 6)$            | 2   | $(17, 10, 4, 3)$ |
| Euler # | $h^{1,1}$ | $\vec{w} = (w_1, \ldots, w_5)$ | $i$ | $\bar{w}$ |
|--------|-----------|--------------------------------|-----|---------|
| 72     | 48        | $(33, 24, 18, 10, 5)$          | 4   | $(11, 8, 6, 5)$ |
| 72     | 47        | $(60, 21, 16, 15, 8)$          | 3   | $(20, 8, 7, 5)$ |
| 72     | 47        | $(27, 21, 12, 10, 5)$          | 4   | $(9, 7, 5, 4)$ |
| 72     | 46        | $(33, 18, 14, 12, 7)$          | 3   | $(11, 7, 6, 4)$ |
| 72     | 44        | $(24, 16, 14, 9, 9)$           | 4   | $(12, 9, 8, 7)$ |
| 64     | 44        | $(24, 21, 20, 12, 7)$          | 2   | $(7, 6, 5, 3)$ |
| 72     | 44        | $(18, 15, 14, 9, 7)$           | 3   | $(7, 6, 5, 3)$ |
| 72     | 44        | $(21, 18, 16, 9, 8)$           | 3   | $(8, 7, 5, 3)$ |
| 64     | 47        | $(36, 16, 15, 8, 5)$           | 3   | $(9, 5, 4, 2)$ |
| 64     | 43        | $(22, 12, 8, 7, 7)$            | 4   | $(11, 7, 6, 4)$ |
| 60     | 194       | $(465, 248, 180, 30, 1)$       | 4   | $(15, 8, 6, 1)$ |
| 60     | 194       | $(450, 240, 180, 29, 1)$       | 4   | $(15, 8, 6, 1)$ |
| 60     | 59        | $(90, 55, 16, 15, 4)$          | 3   | $(18, 11, 4, 3)$ |
| 60     | 49        | $(25, 25, 12, 10, 3)$          | 3   | $(5, 5, 3, 2)$ |
| 60     | 44        | $(22, 16, 12, 5, 5)$           | 4   | $(11, 8, 6, 5)$ |
| 54     | 56        | $(35, 35, 21, 12, 2)$          | 4   | $(5, 5, 3, 2)$ |
| 50     | 44        | $(35, 25, 20, 12, 3)$          | 4   | $(7, 5, 4, 3)$ |
| 48     | 83        | $(156, 91, 39, 24, 2)$         | 4   | $(12, 7, 3, 2)$ |
| 48     | 59        | $(96, 40, 32, 21, 3)$          | 4   | $(12, 5, 4, 3)$ |
| 48     | 53        | $(52, 40, 16, 9, 3)$           | 4   | $(13, 10, 4, 3)$ |
| 48     | 43        | $(28, 21, 21, 10, 4)$          | 2   | $(21, 14, 5, 2)$ |
| 48     | 43        | $(32, 27, 16, 15, 6)$          | 1   | $(16, 9, 5, 2)$ |
| 48     | 41        | $(36, 21, 12, 8, 7)$           | 2   | $(9, 7, 3, 2)$ |
| 48     | 41        | $(27, 14, 9, 7, 6)$            | 2   | $(9, 7, 3, 2)$ |
| 48     | 41        | $(22, 10, 9, 9, 4)$            | 3   | $(11, 9, 5, 2)$ |
| 48     | 39        | $(18, 14, 8, 5, 5)$            | 4   | $(9, 7, 5, 4)$ |
| 48     | 39        | $(26, 18, 15, 13, 6)$          | 1   | $(13, 6, 5, 2)$ |
| 48     | 39        | $(40, 24, 21, 20, 15)$         | 1   | $(20, 8, 7, 5)$ |
| 48     | 39        | $(32, 21, 16, 15, 12)$         | 1   | $(16, 7, 5, 4)$ |
| 48     | 39        | $(15, 15, 14, 12, 4)$          | 1   | $(15, 7, 6, 2)$ |
\begin{tabular}{|c|c|c|c|c|}
\hline
Euler # & $h^{1,1}$ & $\mathbf{w} = (w_1, \ldots, w_5)$ & $\iota$ & $\bar{w}$ \\
\hline
48 & 39 & $(13, 13, 12, 10, 4)$ & 1 & $(13, 6, 5, 2)$ \\
48 & 38 & $(40, 21, 16, 12, 7)$ & 2 & $(10, 7, 4, 3)$ \\
48 & 37 & $(24, 12, 10, 9, 5)$ & 3 & $(8, 5, 4, 3)$ \\
48 & 36 & $(36, 24, 16, 15, 5)$ & 4 & $(9, 6, 5, 4)$ \\
48 & 35 & $(16, 15, 12, 12, 5)$ & 2 & $(5, 4, 3, 3)$ \\
48 & 35 & $(12, 10, 7, 7, 6)$ & 3 & $(7, 6, 5, 3)$ \\
44 & 51 & $(70, 35, 18, 14, 3)$ & 3 & $(10, 5, 3, 2)$ \\
42 & 55 & $(49, 35, 21, 12, 2)$ & 4 & $(7, 5, 3, 2)$ \\
40 & 69 & $(110, 55, 33, 20, 2)$ & 4 & $(10, 5, 3, 2)$ \\
36 & 34 & $(66, 31, 15, 12, 8)$ & 2 & $(11, 5, 4, 2)$ \\
36 & 34 & $(22, 15, 12, 11, 6)$ & 1 & $(11, 5, 4, 2)$ \\
36 & 34 & $(11, 11, 10, 8, 4)$ & 1 & $(11, 5, 4, 2)$ \\
32 & 103 & $(102, 85, 68, 16, 1)$ & 4 & $(6, 5, 4, 1)$ \\
32 & 103 & $(96, 80, 64, 15, 1)$ & 4 & $(6, 5, 4, 1)$ \\
32 & 46 & $(70, 35, 20, 12, 3)$ & 4 & $(14, 7, 4, 3)$ \\
32 & 33 & $(16, 8, 6, 5, 5)$ & 4 & $(8, 5, 4, 3)$ \\
24 & 80 & $(216, 144, 43, 27, 2)$ & 3 & $(12, 8, 3, 1)$ \\
24 & 49 & $(100, 40, 33, 25, 2)$ & 3 & $(10, 5, 4, 1)$ \\
24 & 48 & $(55, 40, 12, 10, 3)$ & 3 & $(11, 8, 3, 2)$ \\
24 & 47 & $(72, 35, 24, 9, 4)$ & 2 & $(6, 3, 2, 1)$ \\
24 & 41 & $(105, 42, 30, 28, 5)$ & 3 & $(15, 6, 5, 4)$ \\
24 & 41 & $(90, 36, 25, 24, 5)$ & 3 & $(15, 6, 5, 4)$ \\
\hline
\end{tabular}
| Euler # | \( h^{1.1} \) | \( \mathbf{w} = (w_1, \ldots, w_5) \) | \( i \) | \( \mathbf{\bar{w}} \) |
|--------|-------------|----------------|-----|----------------|
| 24     | 38          | (42, 21, 9, 8, 4) | 4   | (14, 7, 4, 3)  |
| 24     | 38          | (63, 28, 15, 14, 6) | 2   | (21, 14, 5, 2) |
| 24     | 38          | (32, 20, 9, 8, 3)  | 3   | (8, 5, 3, 2)   |
| 24     | 38          | (27, 18, 16, 8, 3) | 3   | (9, 8, 6, 1)   |
| 24     | 38          | (32, 21, 20, 7, 4) | 2   | (8, 7, 5, 1)   |
| 24     | 38          | (24, 15, 14, 7, 3) | 3   | (8, 7, 5, 1)   |
| 24     | 38          | (33, 16, 9, 8, 6)  | 2   | (11, 8, 3, 2)  |
| 24     | 36          | (35, 20, 12, 10, 3)| 3   | (7, 4, 3, 2)   |
| 24     | 34          | (27, 18, 15, 8, 4) | 4   | (9, 6, 5, 4)   |
| 24     | 33          | (18, 7, 7, 6, 4)  | 2   | (9, 7, 3, 2)   |
| 24     | 33          | (25, 12, 10, 10, 3)| 2   | (5, 3, 2, 2)   |
| 24     | 33          | (27, 12, 10, 6, 5) | 3   | (9, 5, 4, 2)   |
| 24     | 33          | (20, 9, 8, 8, 3)  | 2   | (5, 3, 2, 2)   |
| 24     | 32          | (14, 12, 5, 5, 4) | 3   | (7, 6, 5, 2)   |
| 24     | 32          | (32, 15, 12, 8, 5) | 2   | (8, 5, 3, 2)   |
| 24     | 32          | (30, 17, 12, 9, 4) | 2   | (5, 3, 2, 2)   |
| 24     | 31          | (28, 15, 12, 8, 5) | 2   | (7, 5, 3, 2)   |
| 24     | 30          | (54, 25, 12, 9, 8)| 2   | (9, 4, 3, 2)   |
| 24     | 29          | (15, 15, 12, 10, 8)| 1   | (15, 6, 5, 4)  |
| 24     | 28          | (24, 18, 17, 9, 4)| 3   | (4, 3, 3, 2)   |
| 24     | 27          | (27, 18, 12, 10, 5)| 4   | (9, 6, 5, 4)   |
| 24     | 27          | (30, 14, 12, 9, 7) | 2   | (10, 7, 4, 3)  |
| 24     | 27          | (8, 6, 6, 5, 5)  | 4   | (5, 4, 3, 3)   |
| 20     | 50          | (91, 56, 18, 14, 3)| 3   | (13, 8, 3, 2)  |
| 18     | 53          | (63, 49, 21, 12, 2) | 4   | (9, 7, 3, 2)   |
| 16     | 31          | (18, 8, 5, 5, 4)  | 3   | (9, 5, 4, 2)   |
| 16     | 29          | (20, 16, 9, 8, 3)  | 3   | (5, 4, 3, 2)   |
| 12     | 41          | (54, 33, 9, 8, 4)  | 4   | (18, 11, 4, 3) |
| 12     | 36          | (26, 21, 15, 13, 3)| 1   | (13, 7, 5, 1)  |
| 12     | 36          | (16, 15, 15, 12, 2)| 2   | (15, 8, 6, 1)  |
| Euler # | \( h_{1,1} \) | \( \bar{w} = (w_1, \ldots, w_5) \) | i | \( \bar{i} \) |
|---------|---------------|-----------------|---|-------|
| 12      | 36            | (14, 13, 13, 10, 2) | 2 | (13, 7, 5, 1) |
| 8       | 29            | (28, 16, 9, 8, 3)  | 3 | (7, 4, 3, 2)  |
| 6       | 23            | (21, 10, 9, 6, 5) | 2 | (7, 5, 3, 2)  |
| 0       | 251           | (903, 602, 258, 42, 1) | 4 | (21, 14, 6, 1) |
| 0       | 251           | (882, 588, 252, 41, 1) | 4 | (21, 14, 6, 1) |
| 0       | 131           | (253, 138, 92, 22, 1) | 4 | (11, 6, 4, 1) |
| 0       | 131           | (242, 132, 88, 21, 1) | 4 | (11, 6, 4, 1) |
| 0       | 121           | (153, 136, 102, 16, 1) | 4 | (9, 8, 6, 1) |
| 0       | 119           | (210, 105, 84, 20, 1) | 4 | (10, 5, 4, 1) |
| 0       | 119           | (200, 100, 80, 19, 1) | 4 | (10, 5, 4, 1) |
| 0       | 89            | (225, 150, 45, 28, 2) | 4 | (15, 10, 3, 2) |
| 0       | 89            | (96, 80, 48, 15, 1)  | 4 | (6, 5, 3, 1)  |
| 0       | 89            | (90, 75, 45, 14, 1)  | 4 | (6, 5, 3, 1)  |
| 0       | 83            | (252, 168, 71, 7, 6) | 3 | (6, 4, 1, 1)  |
| 0       | 77            | (70, 56, 42, 13, 1)  | 4 | (5, 4, 3, 1) |
| 0       | 77            | (65, 52, 39, 12, 1)  | 4 | (5, 4, 3, 1) |
| 0       | 71            | (52, 52, 39, 12, 1)  | 4 | (4, 4, 3, 1) |
| 0       | 71            | (48, 48, 36, 11, 1)  | 4 | (4, 4, 3, 1) |
| 0       | 65            | (168, 112, 32, 21, 3) | 4 | (21, 14, 4, 3) |
| 0       | 59            | (165, 110, 30, 22, 3) | 3 | (15, 10, 3, 2) |
| 0       | 59            | (150, 100, 27, 20, 3) | 3 | (15, 10, 3, 2) |
| 0       | 55            | (80, 40, 31, 5, 4)  | 3 | (4, 2, 1, 1) |
| 0       | 55            | (147, 98, 36, 7, 6) | 3 | (21, 14, 6, 1) |
| 0       | 55            | (98, 63, 24, 7, 4)  | 3 | (14, 9, 4, 1) |
| 0       | 55            | (80, 56, 21, 8, 3)  | 3 | (10, 7, 3, 1) |
| 0       | 55            | (70, 49, 18, 7, 3)  | 3 | (10, 7, 3, 1) |
| 0       | 41            | (24, 24, 17, 4, 3) | 3 | (2, 2, 1, 1) |
| 0       | 39            | (50, 25, 16, 5, 4)  | 3 | (10, 5, 4, 1) |
| 0       | 39            | (75, 35, 24, 10, 6) | 3 | (15, 7, 6, 2) |
| 0       | 39            | (35, 20, 12, 5, 3)  | 3 | (7, 4, 3, 1) |
| Euler # | $h^{1,1}$ | $\tilde{w} = (w_1, \ldots, w_5)$ | i  | $\hat{w}$ |
|--------|-----------|---------------------------------|----|----------|
| 0      | 39        | (45, 25, 16, 10, 4)            | 3  | (9, 5, 4, 2) |
| 0      | 38        | (60, 25, 20, 12, 3)           | 4  | (12, 5, 4, 3) |
| 0      | 35        | (63, 28, 18, 14, 3)           | 2  | (21, 14, 6, 1) |
| 0      | 35        | (63, 28, 18, 14, 3)           | 3  | (9, 4, 3, 2) |
| 0      | 35        | (28, 21, 12, 12, 2)           | 2  | (21, 14, 6, 1) |
| 0      | 35        | (28, 21, 12, 12, 2)           | 4  | (4, 3, 3, 2) |
| 0      | 35        | (28, 27, 14, 12, 3)           | 1  | (14, 9, 4, 1) |
| 0      | 35        | (40, 21, 12, 7, 4)           | 2  | (10, 7, 3, 1) |
| 0      | 35        | (30, 14, 9, 7, 3)            | 2  | (10, 7, 3, 1) |
| 0      | 34        | (20, 14, 8, 3, 3)           | 4  | (10, 7, 4, 3) |
| 0      | 31        | (60, 24, 16, 15, 5)          | 4  | (15, 6, 5, 4) |
| 0      | 31        | (20, 20, 12, 5, 3)           | 3  | (4, 4, 3, 1) |
| 0      | 31        | (16, 16, 9, 4, 3)           | 3  | (4, 4, 3, 1) |
| 0      | 31        | (22, 18, 12, 11, 3)          | 1  | (11, 6, 4, 1) |
| 0      | 31        | (24, 19, 15, 12, 2)          | 2  | (5, 4, 2, 1) |
| 0      | 31        | (24, 23, 12, 10, 3)          | 2  | (5, 4, 2, 1) |
| 0      | 31        | (40, 35, 24, 15, 6)          | 3  | (8, 7, 6, 3) |
| 0      | 31        | (35, 16, 15, 10, 4)          | 2  | (7, 4, 3, 2) |
| 0      | 31        | (16, 10, 7, 7, 2)           | 3  | (8, 7, 5, 1) |
| 0      | 31        | (12, 11, 11, 8, 2)          | 2  | (11, 6, 4, 1) |
| 0      | 29        | (45, 20, 10, 9, 6)          | 2  | (15, 10, 3, 2) |
| 0      | 29        | (24, 15, 12, 5, 4)          | 2  | (6, 5, 3, 1) |
| 0      | 29        | (18, 10, 9, 5, 3)          | 2  | (6, 5, 3, 1) |
| 0      | 29        | (13, 12, 12, 9, 2)          | 1  | (3, 2, 2, 1) |
| 0      | 27        | (20, 15, 15, 6, 4)          | 2  | (15, 10, 3, 2) |
| 0      | 23        | (28, 21, 14, 12, 9)        | 1  | (14, 7, 4, 3) |
| 0      | 23        | (14, 9, 7, 6, 6)          | 1  | (7, 3, 2, 2) |
| 0      | 23        | (9, 9, 8, 6, 4)            | 1  | (9, 4, 3, 2) |
| 0      | 23        | (9, 9, 8, 6, 4)            | 3  | (4, 3, 3, 2) |
| 0      | 23        | (7, 7, 6, 4, 4)            | 1  | (7, 3, 2, 2) |
| Euler # | $h^{1,1}$ | $\vec{w} = (w_1, \ldots, w_5)$ | $\iota$ | $\vec{w}$ |
|--------|-----------|-------------------------------|--------|---------|
| 0      | 22        | $(24, 10, 9, 6, 5)$           | 2      | $(8, 5, 3, 2)$ |
| 0      | 18        | $(18, 12, 8, 5, 5)$           | 4      | $(9, 6, 5, 4)$ |
| $-4$   | 26        | $(35, 12, 10, 10, 3)$         | 2      | $(7, 3, 2, 2)$ |
| $-8$   | 29        | $(40, 20, 9, 8, 3)$           | 3      | $(10, 5, 3, 2)$ |
| $-8$   | 25        | $(24, 16, 15, 5, 4)$          | 3      | $(6, 5, 4, 1)$ |
| $-12$  | 38        | $(26, 20, 8, 3, 3)$           | 4      | $(13, 10, 4, 3)$ |
| $-12$  | 30        | $(20, 15, 15, 8, 2)$          | 2      | $(15, 10, 4, 1)$ |
| $-12$  | 30        | $(20, 15, 15, 8, 2)$          | 4      | $(4, 3, 3, 2)$ |
| $-12$  | 30        | $(22, 21, 11, 9, 3)$          | 1      | $(11, 7, 3, 1)$ |
| $-12$  | 30        | $(14, 11, 11, 6, 2)$          | 2      | $(11, 7, 3, 1)$ |
| $-12$  | 25        | $(10, 10, 4, 3, 3)$           | 4      | $(5, 5, 3, 2)$ |
| $-12$  | 24        | $(18, 13, 12, 9, 2)$          | 2      | $(3, 3, 2, 1)$ |
| $-12$  | 16        | $(10, 9, 6, 6, 5)$            | 1      | $(5, 3, 2, 2)$ |
| $-16$  | 23        | $(70, 28, 20, 15, 7)$         | 2      | $(14, 7, 4, 3)$ |
| $-16$  | 23        | $(56, 21, 16, 12, 7)$         | 2      | $(14, 7, 4, 3)$ |
| $-20$  | 15        | $(14, 6, 5, 5, 4)$            | 3      | $(7, 5, 3, 2)$ |
| $-24$  | 110       | $(144, 128, 96, 15, 1)$       | 4      | $(9, 8, 6, 1)$ |
| $-24$  | 77        | $(72, 60, 48, 11, 1)$         | 4      | $(6, 5, 4, 1)$ |
| $-24$  | 60        | $(40, 40, 30, 9, 1)$          | 4      | $(4, 4, 3, 1)$ |
| $-24$  | 51        | $(84, 49, 21, 12, 2)$         | 4      | $(12, 7, 3, 2)$ |
| $-24$  | 29        | $(20, 7, 7, 6, 2)$            | 2      | $(10, 7, 3, 1)$ |
| $-24$  | 27        | $(16, 10, 4, 3, 3)$           | 4      | $(8, 5, 3, 2)$ |
| $-24$  | 26        | $(36, 16, 9, 8, 3)$           | 2      | $(12, 8, 3, 1)$ |
| $-24$  | 26        | $(36, 16, 9, 8, 3)$           | 3      | $(9, 4, 3, 2)$ |
| $-24$  | 26        | $(28, 15, 8, 5, 4)$           | 2      | $(7, 5, 2, 1)$ |
| $-24$  | 26        | $(21, 10, 6, 5, 3)$           | 2      | $(7, 5, 2, 1)$ |
| $-24$  | 25        | $(48, 23, 15, 6, 4)$          | 2      | $(8, 5, 2, 1)$ |
| $-24$  | 23        | $(12, 6, 5, 5, 2)$            | 3      | $(6, 5, 3, 1)$ |
| $-24$  | 22        | $(36, 17, 9, 6, 4)$           | 2      | $(6, 3, 2, 1)$ |
| Euler # | $h^{1,1}$ | $\hat{w} = (w_1, \ldots, w_5)$ | $i$ | $\hat{w}$ |
|--------|---------|---------------------------------|----|---------|
| $-24$  | 22      | $(21, 9, 8, 6, 4)$              | 3  | $(7, 4, 3, 2)$ |
| $-24$  | 21      | $(40, 15, 12, 8, 5)$            | 2  | $(10, 5, 3, 2)$ |
| $-24$  | 21      | $(18, 12, 11, 4, 3)$            | 3  | $(3, 2, 2, 1)$ |
| $-24$  | 20      | $(42, 14, 12, 9, 7)$            | 2  | $(14, 7, 4, 3)$ |
| $-24$  | 20      | $(10, 4, 4, 3, 3)$              | 4  | $(5, 3, 2, 2)$ |
| $-24$  | 18      | $(18, 12, 10, 5, 3)$            | 3  | $(6, 5, 4, 1)$ |
| $-24$  | 17      | $(20, 15, 10, 9, 6)$            | 1  | $(10, 5, 3, 2)$ |
| $-24$  | 15      | $(16, 6, 5, 5, 4)$              | 3  | $(8, 5, 3, 2)$ |
| $-24$  | 12      | $(6, 5, 5, 4, 4)$               | 2  | $(5, 3, 2, 2)$ |
| $-28$  | 17      | $(14, 10, 8, 3, 3)$             | 4  | $(7, 5, 4, 3)$ |
| $-30$  | 24      | $(15, 15, 8, 5, 2)$             | 3  | $(3, 3, 2, 1)$ |
| $-30$  | 23      | $(15, 15, 9, 4, 2)$             | 4  | $(5, 5, 3, 2)$ |
| $-30$  | 17      | $(15, 9, 8, 4, 3)$              | 3  | $(5, 4, 3, 1)$ |
| $-32$  | 87      | $(136, 68, 51, 16, 1)$          | 4  | $(8, 4, 3, 1)$ |
| $-32$  | 87      | $(128, 64, 48, 15, 1)$          | 4  | $(8, 4, 3, 1)$ |
| $-32$  | 29      | $(52, 32, 9, 8, 3)$             | 3  | $(13, 8, 3, 2)$ |
| $-32$  | 19      | $(16, 9, 8, 4, 3)$              | 2  | $(4, 3, 2, 1)$ |
| $-32$  | 17      | $(16, 10, 8, 3, 3)$             | 4  | $(8, 5, 4, 3)$ |
| $-36$  | 148     | $(345, 184, 138, 22, 1)$        | 4  | $(15, 8, 6, 1)$ |
| $-36$  | 102     | $(170, 85, 68, 16, 1)$          | 4  | $(10, 5, 4, 1)$ |
| $-36$  | 98      | $(171, 95, 57, 18, 1)$          | 4  | $(9, 5, 3, 1)$ |
| $-36$  | 98      | $(162, 90, 54, 17, 1)$          | 4  | $(9, 5, 3, 1)$ |
| $-36$  | 44      | $(30, 30, 20, 9, 1)$            | 4  | $(3, 3, 2, 1)$ |
| $-36$  | 44      | $(27, 27, 18, 8, 1)$            | 4  | $(3, 3, 2, 1)$ |
| $-36$  | 26      | $(30, 15, 8, 4, 3)$             | 3  | $(10, 5, 4, 1)$ |
| $-36$  | 26      | $(27, 15, 8, 6, 4)$             | 3  | $(9, 5, 4, 2)$ |
| $-36$  | 20      | $(42, 19, 12, 8, 3)$            | 2  | $(7, 4, 2, 1)$ |
| $-36$  | 20      | $(14, 12, 7, 6, 3)$             | 1  | $(7, 4, 2, 1)$ |
| $-36$  | 20      | $(8, 7, 7, 4, 2)$               | 2  | $(7, 4, 2, 1)$ |
| $-36$  | 20      | $(10, 9, 9, 6, 2)$              | 2  | $(9, 5, 3, 1)$ |
| $\text{Euler} \#$ | $h^{1,1}$ | $\vec{w} = (w_1, \ldots, w_5)$ | $i$ | $\vec{w}$ |
|--------------|---------|---------------------------------|-----|----------|
| −36          | 17      | $(30, 10, 9, 6, 5)$              | 2   | $(10, 5, 3, 2)$ |
| −36          | 14      | $(10, 8, 4, 3, 3)$              | 4   | $(5, 4, 3, 2)$ |
| −40          | 70      | $(66, 55, 44, 10, 1)$           | 4   | $(6, 5, 4, 1)$ |
| −40          | 59      | $(50, 40, 30, 9, 1)$            | 4   | $(5, 4, 3, 1)$ |
| −40          | 49      | $(44, 33, 22, 10, 1)$           | 4   | $(4, 3, 2, 1)$ |
| −40          | 49      | $(40, 30, 20, 9, 1)$            | 4   | $(4, 3, 2, 1)$ |
| −40          | 25      | $(40, 20, 12, 5, 3)$            | 3   | $(8, 4, 3, 1)$ |
| −40          | 19      | $(20, 9, 8, 4, 3)$              | 2   | $(5, 3, 2, 1)$ |
| −42          | 23      | $(21, 15, 9, 4, 2)$             | 4   | $(7, 5, 3, 2)$ |
| −42          | 17      | $(21, 9, 8, 4, 3)$              | 3   | $(7, 4, 3, 1)$ |
| −48          | 77      | $(84, 72, 48, 11, 1)$           | 4   | $(7, 6, 4, 1)$ |
| −48          | 67      | $(165, 110, 33, 20, 2)$         | 4   | $(15, 10, 3, 2)$ |
| −48          | 67      | $(66, 55, 33, 10, 1)$           | 4   | $(6, 5, 3, 1)$ |
| −48          | 59      | $(65, 52, 26, 12, 1)$           | 4   | $(5, 4, 2, 1)$ |
| −48          | 59      | $(60, 48, 24, 11, 1)$           | 4   | $(5, 4, 2, 1)$ |
| −48          | 50      | $(36, 36, 27, 8, 1)$            | 4   | $(4, 4, 3, 1)$ |
| −48          | 43      | $(126, 84, 31, 7, 4)$           | 3   | $(9, 6, 2, 1)$ |
| −48          | 39      | $(24, 24, 16, 7, 1)$            | 4   | $(3, 3, 2, 1)$ |
| −48          | 35      | $(40, 28, 9, 4, 3)$             | 3   | $(10, 7, 3, 1)$ |
| −48          | 35      | $(27, 18, 18, 8, 1)$            | 4   | $(3, 2, 2, 1)$ |
| −48          | 35      | $(24, 16, 16, 7, 1)$            | 4   | $(3, 2, 2, 1)$ |
| −48          | 31      | $(22, 16, 4, 3, 3)$             | 4   | $(11, 8, 3, 2)$ |
| −48          | 22      | $(24, 13, 9, 6, 2)$             | 2   | $(4, 3, 1, 1)$ |
| −48          | 21      | $(14, 5, 5, 4, 2)$              | 2   | $(7, 5, 2, 1)$ |
| −48          | 19      | $(8, 8, 3, 3, 2)$               | 3   | $(4, 4, 3, 1)$ |
| −48          | 19      | $(12, 9, 9, 4, 2)$              | 2   | $(9, 6, 2, 1)$ |
| −48          | 19      | $(12, 9, 9, 4, 2)$              | 4   | $(4, 3, 3, 2)$ |
| −48          | 19      | $(16, 15, 8, 6, 3)$             | 1   | $(8, 5, 2, 1)$ |
| −48          | 17      | $(45, 18, 12, 10, 5)$           | 4   | $(15, 6, 5, 4)$ |
| −48          | 17      | $(15, 8, 6, 4, 3)$              | 2   | $(5, 4, 2, 1)$ |
| Euler # | $h^{1,1}$ | $\psi = (w_1, \ldots, w_5)$ | i   | $\bar{w}$ |
|--------|-----------|-----------------|-----|-----------|
| -48    | 15        | (16, 12, 9, 8, 3) | 1   | (8, 4, 3, 1) |
| -48    | 15        | (16, 12, 9, 8, 3) | 3   | (4, 3, 3, 2) |
| -48    | 15        | (20, 15, 12, 10, 3) | 1   | (10, 5, 4, 1) |
| -48    | 15        | (20, 15, 12, 10, 3) | 3   | (4, 3, 3, 2) |
| -48    | 15        | (12, 11, 6, 4, 3) | 2   | (2, 2, 1, 1) |
| -48    | 15        | (10, 6, 6, 5, 3) | 1   | (5, 2, 2, 1) |
| -48    | 15        | (5, 5, 4, 4, 2) | 1   | (5, 2, 2, 1) |
| -48    | 14        | (14, 8, 4, 3, 3) | 4   | (7, 4, 3, 2) |
| -48    | 12        | (12, 8, 5, 5, 2) | 3   | (6, 5, 4, 1) |
| -48    | 11        | (15, 10, 9, 6, 5) | 2   | (5, 5, 3, 2) |
| -48    | 11        | (20, 15, 12, 8, 5) | 2   | (5, 5, 3, 2) |
| -50    | 24        | (25, 15, 8, 5, 2) | 3   | (5, 3, 2, 1) |
| -54    | 53        | (45, 36, 27, 8, 1) | 4   | (5, 4, 3, 1) |
| -56    | 93        | (160, 80, 64, 15, 1) | 4   | (10, 5, 4, 1) |
| -56    | 76        | (112, 56, 42, 13, 1) | 4   | (8, 4, 3, 1) |
| -60    | 222       | (777, 518, 222, 36, 1) | 4   | (21, 14, 6, 1) |
| -60    | 164       | (465, 310, 124, 30, 1) | 4   | (15, 10, 4, 1) |
| -60    | 164       | (450, 300, 120, 29, 1) | 4   | (15, 10, 4, 1) |
| -60    | 19        | (25, 20, 15, 12, 3) | 4   | (5, 4, 3, 3) |
| -60    | 14        | (9, 9, 4, 3, 2) | 3   | (3, 3, 2, 1) |
| -64    | 39        | (32, 24, 16, 7, 1) | 4   | (4, 3, 2, 1) |
| -64    | 29        | (21, 14, 14, 6, 1) | 4   | (3, 2, 2, 1) |
| -64    | 17        | (28, 14, 8, 3, 3) | 4   | (14, 7, 4, 3) |
| -64    | 15        | (40, 16, 15, 5, 4) | 2   | (8, 4, 3, 1) |
| -64    | 15        | (40, 16, 15, 5, 4) | 3   | (10, 5, 4, 1) |
| -64    | 11        | (28, 8, 7, 7, 6) | 3   | (14, 7, 4, 3) |
| -64    | 11        | (14, 4, 4, 3, 3) | 4   | (7, 3, 2, 2) |
| -64    | 8         | (4, 4, 3, 3, 2) | 3   | (3, 2, 2, 1) |
| -66    | 32        | (21, 21, 14, 6, 1) | 4   | (3, 3, 2, 1) |
| -66    | 23        | (27, 21, 9, 4, 2) | 4   | (9, 7, 3, 2) |
| Euler # | $h^1.1$ | $\tilde{w} = (w_1, \ldots, w_5)$ | $i$ | $\tilde{w}$ |
|---------|----------|-----------------|-----|----------|
| -72     | 88       | (117, 104, 78, 12, 1) | 4   | (9, 8, 6, 1) |
| -72     | 69       | (104, 52, 39, 12, 1)  | 4   | (8, 4, 3, 1) |
| -72     | 57       | (60, 50, 30, 9, 1)    | 4   | (6, 5, 3, 1) |
| -72     | 56       | (54, 45, 36, 8, 1)    | 4   | (6, 5, 4, 1) |
| -72     | 53       | (78, 39, 26, 12, 1)   | 4   | (6, 3, 2, 1) |
| -72     | 53       | (72, 36, 24, 11, 1)   | 4   | (6, 3, 2, 1) |
| -72     | 40       | (28, 28, 21, 6, 1)    | 4   | (4, 4, 3, 1) |
| -72     | 32       | (60, 35, 15, 8, 2)    | 4   | (12, 7, 3, 2) |
| -72     | 29       | (42, 27, 8, 4, 3)     | 3   | (14, 9, 4, 1) |
| -72     | 26       | (40, 20, 13, 5, 2)    | 3   | (4, 2, 1, 1) |
| -72     | 26       | (18, 12, 12, 5, 1)    | 4   | (3, 2, 2, 1) |
| -72     | 23       | (42, 21, 12, 7, 2)    | 3   | (6, 3, 2, 1) |
| -72     | 23       | (35, 21, 14, 12, 2)   | 4   | (5, 3, 2, 2) |
| -72     | 21       | (14, 8, 3, 3, 2)      | 3   | (7, 4, 3, 1) |
| -72     | 20       | (36, 17, 12, 4, 3)    | 2   | (3, 1, 1, 1) |
| -72     | 14       | (20, 10, 4, 3, 3)     | 4   | (10, 5, 3, 2) |
| -72     | 14       | (16, 9, 4, 4, 3)      | 2   | (4, 3, 1, 1) |
| -72     | 13       | (30, 12, 10, 5, 3)    | 2   | (6, 3, 2, 1) |
| -72     | 13       | (30, 12, 10, 5, 3)    | 3   | (10, 5, 4, 1) |
| -72     | 13       | (24, 9, 8, 4, 3)      | 2   | (6, 3, 2, 1) |
| -72     | 13       | (24, 9, 8, 4, 3)      | 3   | (8, 4, 3, 1) |
| -72     | 13       | (24, 11, 6, 4, 3)     | 2   | (4, 2, 1, 1) |
| -72     | 13       | (30, 12, 8, 5, 5)     | 4   | (15, 6, 5, 4) |
| -72     | 13       | (12, 7, 6, 3, 2)      | 2   | (2, 1, 1, 1) |
| -72     | 11       | (7, 6, 6, 3, 2)       | 1   | (1, 1, 1, 1) |
| -72     | 10       | (20, 6, 5, 5, 4)      | 3   | (10, 5, 3, 2) |
| -72     | 9        | (8, 4, 3, 3, 2)       | 3   | (4, 3, 2, 1) |
| -72     | 8        | (12, 9, 8, 4, 3)      | 2   | (3, 3, 2, 1) |
| -72     | 8        | (12, 9, 8, 4, 3)      | 3   | (4, 4, 3, 1) |
| -72     | 7        | (10, 6, 5, 5, 4)      | 3   | (5, 5, 3, 2) |
| $Euler \#$ | $h^{1,1}$ | $\phi = (w_1, \ldots, w_5)$ | $i$ | $w$ |
|---------|---------|----------------------------|-----|-----|
| 39      | 55, 22, 22, 10, 1 | 4 | (5, 2, 2, 1) |
| 39      | 50, 20, 20, 9, 1  | 4 | (5, 2, 2, 1) |
| 33      | 28, 21, 14, 6, 1  | 4 | (4, 3, 2, 1) |
| 11      | 15, 10, 8, 5, 2   | 3 | (3, 2, 2, 1) |
| 76      | 126, 70, 42, 13, 1| 4 | (9, 5, 3, 1) |
| 62      | 105, 60, 30, 14, 1| 4 | (7, 4, 2, 1) |
| 62      | 98, 56, 28, 13, 1 | 4 | (7, 4, 2, 1) |
| 61      | 70, 60, 40, 9, 1  | 4 | (7, 6, 4, 1) |
| 48      | 66, 33, 22, 10, 1 | 4 | (6, 3, 2, 1) |
| 44      | 45, 36, 18, 8, 1  | 4 | (5, 4, 2, 1) |
| 41      | 35, 28, 21, 6, 1  | 4 | (5, 4, 3, 1) |
| 14      | 15, 9, 4, 3, 2    | 3 | (5, 3, 2, 1) |
| 12      | 10, 9, 5, 3, 3    | 1 | (5, 3, 1, 1) |
| 12      | 6, 5, 5, 2, 2     | 2 | (5, 3, 1, 1) |
| 9       | 10, 4, 3, 3, 2    | 3 | (5, 3, 2, 1) |
| 8       | 9, 8, 4, 3, 3     | 2 | (4, 3, 1, 1) |
| 17      | 34, 20, 8, 3, 3   | 4 | (17, 10, 4, 3) |
| 24      | 15, 15, 10, 4, 1  | 4 | (3, 3, 2, 1) |
| 24      | 35, 25, 8, 5, 2   | 3 | (7, 5, 2, 1) |
| 8       | 9, 4, 3, 3, 2     | 2 | (3, 2, 1, 1) |
| 5       | 4, 3, 3, 2, 2     | 2 | (3, 2, 1, 1) |
| 147     | 405, 270, 108, 26, 1| 4 | (15, 10, 4, 1) |
| 119     | 300, 200, 75, 24, 1| 4 | (12, 8, 3, 1) |
| 119     | 288, 192, 72, 23, 1| 4 | (12, 8, 3, 1) |
| 47      | 48, 40, 24, 7, 1  | 4 | (6, 5, 3, 1) |
| 43      | 60, 30, 20, 9, 1  | 4 | (6, 3, 2, 1) |
| 39      | 40, 32, 16, 7, 1  | 4 | (5, 4, 2, 1) |
| 33      | 45, 18, 18, 8, 1  | 4 | (5, 2, 2, 1) |
| 31      | 96, 64, 21, 8, 3  | 3 | (12, 8, 3, 1) |
| 31      | 72, 45, 16, 9, 2  | 3 | (8, 5, 2, 1) |
| Euler # | $h^{1,1}$ | $\mathbf{w} = (w_1, \ldots, w_5)$ | $i$ | $\mathbf{\bar{w}}$ |
|---------|-----------|---------------------------------|-----|-----------------|
| $-96$   | 27        | $(24, 18, 12, 5, 1)$            | 4   | $(4, 3, 2, 1)$  |
| $-96$   | 23        | $(60, 40, 9, 8, 3)$            | 3   | $(15, 10, 3, 2)$|
| $-96$   | 19        | $(30, 15, 9, 4, 2)$            | 4   | $(10, 5, 3, 2)$  |
| $-96$   | 19        | $(15, 10, 10, 4, 1)$           | 4   | $(3, 2, 2, 1)$  |
| $-96$   | 17        | $(12, 12, 6, 5, 1)$            | 4   | $(2, 2, 1, 1)$  |
| $-96$   | 17        | $(14, 14, 7, 6, 1)$            | 4   | $(2, 2, 1, 1)$  |
| $-96$   | 14        | $(24, 10, 8, 3, 3)$            | 4   | $(12, 5, 4, 3)$  |
| $-96$   | 11        | $(18, 8, 4, 3, 3)$             | 2   | $(6, 4, 1, 1)$  |
| $-96$   | 11        | $(18, 8, 4, 3, 3)$             | 4   | $(9, 4, 3, 2)$  |
| $-96$   | 11        | $(8, 3, 3, 2, 2)$              | 2   | $(4, 3, 1, 1)$  |
| $-96$   | 9         | $(20, 9, 4, 4, 3)$             | 2   | $(5, 3, 1, 1)$  |
| $-96$   | 7         | $(8, 6, 4, 3, 3)$              | 1   | $(4, 2, 1, 1)$  |
| $-96$   | 7         | $(8, 6, 4, 3, 3)$              | 4   | $(4, 3, 3, 2)$  |
| $-96$   | 7         | $(9, 6, 4, 3, 2)$              | 3   | $(3, 2, 1, 1)$  |
| $-96$   | 5         | $(6, 4, 3, 3, 2)$              | 2   | $(2, 2, 1, 1)$  |
| $-96$   | 5         | $(6, 4, 3, 3, 2)$              | 3   | $(3, 3, 2, 1)$  |
| $-96$   | 5         | $(4, 3, 3, 3, 2)$              | 1   | $(2, 1, 1, 1)$  |
| $-104$  | 17        | $(12, 8, 8, 3, 1)$             | 4   | $(3, 2, 1, 1)$  |
| $-108$  | 14        | $(26, 16, 4, 3, 3)$            | 4   | $(13, 8, 3, 2)$  |
| $-108$  | 13        | $(10, 10, 5, 4, 1)$            | 4   | $(2, 2, 1, 1)$  |
| $-108$  | 6         | $(3, 3, 2, 2, 2)$              | 1   | $(3, 1, 1, 1)$  |
| $-112$  | 20        | $(49, 21, 14, 12, 2)$          | 4   | $(7, 3, 2, 2)$  |
| $-112$  | 20        | $(21, 14, 7, 6, 1)$            | 4   | $(3, 2, 1, 1)$  |
| $-112$  | 20        | $(24, 16, 8, 7, 1)$            | 4   | $(3, 2, 1, 1)$  |
| $-112$  | 10        | $(16, 8, 3, 3, 2)$             | 3   | $(8, 4, 3, 1)$  |
| $-112$  | 7         | $(20, 8, 5, 5, 2)$             | 2   | $(4, 2, 1, 1)$  |
| $-112$  | 7         | $(20, 8, 5, 5, 2)$             | 3   | $(10, 5, 4, 1)$  |
| $-120$  | 109       | $(240, 128, 96, 15, 1)$        | 4   | $(15, 8, 6, 1)$  |
| $-120$  | 108       | $(264, 176, 66, 21, 1)$        | 4   | $(12, 8, 3, 1)$  |
| $-120$  | 76        | $(132, 72, 48, 11, 1)$         | 4   | $(11, 6, 4, 1)$  |
| Euler # | \( \bar{h}^{1,1} \) | \( \bar{w} = (w_1, \ldots, w_5) \) | \( \bar{t} \) | \( \bar{w} \) |
|-------|-----------------|------------------|---------|---------|
| -120  | 68              | (81, 72, 54, 8, 1)| 4       | (9, 8, 6, 1)|
| -120  | 65              | (110, 55, 44, 10, 1)| 4       | (10, 5, 4, 1)|
| -120  | 49              | (72, 36, 27, 8, 1)| 4       | (8, 4, 3, 1)|
| -120  | 48              | (49, 42, 28, 6, 1)| 4       | (7, 6, 4, 1)|
| -120  | 47              | (77, 44, 22, 10, 1)| 4       | (7, 4, 2, 1)|
| -120  | 39              | (36, 30, 18, 5, 1)| 4       | (6, 5, 3, 1)|
| -120  | 38              | (36, 30, 24, 5, 1)| 4       | (6, 5, 4, 1)|
| -120  | 31              | (35, 28, 14, 6, 1)| 4       | (5, 4, 2, 1)|
| -120  | 29              | (25, 20, 15, 4, 1)| 4       | (5, 4, 3, 1)|
| -120  | 26              | (20, 20, 15, 4, 1)| 4       | (4, 4, 3, 1)|
| -120  | 26              | (16, 16, 12, 3, 1)| 4       | (4, 4, 3, 1)|
| -120  | 25              | (30, 12, 12, 5, 1)| 4       | (5, 2, 2, 1)|
| -120  | 25              | (20, 14, 3, 3, 2)| 3       | (10, 7, 3, 1)|
| -120  | 22              | (60, 40, 12, 5, 3)| 3       | (12, 8, 3, 1)|
| -120  | 22              | (40, 25, 8, 5, 2)| 3       | (8, 5, 2, 1)|
| -120  | 21              | (36, 21, 9, 4, 2)| 4       | (12, 7, 3, 2)|
| -120  | 17              | (12, 12, 8, 3, 1)| 4       | (3, 3, 2, 1)|
| -120  | 17              | (18, 12, 6, 5, 1)| 4       | (3, 2, 1, 1)|
| -120  | 10              | (18, 9, 4, 3, 2)| 3       | (6, 3, 2, 1)|
| -120  | 10              | (8, 8, 4, 3, 1)| 4       | (2, 2, 1, 1)|
| -120  | 10              | (15, 9, 6, 4, 2)| 4       | (5, 3, 2, 2)|
| -120  | 9               | (25, 10, 8, 5, 2)| 3       | (5, 2, 2, 1)|
| -120  | 9               | (10, 5, 5, 4, 1)| 4       | (2, 1, 1, 1)|
| -120  | 9               | (12, 6, 6, 5, 1)| 4       | (2, 1, 1, 1)|
| -120  | 6               | (12, 4, 3, 3, 2)| 2       | (4, 2, 1, 1)|
| -120  | 6               | (12, 4, 3, 3, 2)| 3       | (6, 3, 2, 1)|
| -120  | 5               | (4, 4, 4, 3, 1)| 4       | (1, 1, 1, 1)|
| -120  | 5               | (5, 5, 5, 4, 1)| 4       | (1, 1, 1, 1)|
| -128  | 19              | (16, 12, 8, 3, 1)| 4       | (4, 3, 2, 1)|
| -128  | 7               | (8, 4, 4, 3, 1)| 4       | (2, 1, 1, 1)|
| Euler # | $h_{1,1}$ | $\bar{w} = (w_1, \ldots, w_5)$ | $i$ | $\bar{\omega}$ |
|---------|-----------|-----------------------------|-----|---------|
| -130    | 14        | $(15, 10, 5, 4, 1)$         | 4   | (3, 2, 1, 1) |
| -132    | 56        | $(90, 50, 30, 9, 1)$        | 4   | (9, 5, 3, 1) |
| -132    | 30        | $(42, 21, 14, 6, 1)$        | 4   | (6, 3, 2, 1) |
| -132    | 14        | $(21, 15, 4, 3, 2)$         | 3   | (7, 5, 2, 1) |
| -132    | 7         | $(10, 8, 6, 3, 3)$          | 4   | (5, 4, 3, 3) |
| -132    | 4         | $(10, 3, 3, 2, 2)$          | 2   | (5, 3, 1, 1) |
| -132    | 3         | $(3, 3, 3, 2, 1)$           | 4   | (1, 1, 1, 1) |
| -136    | 34        | $(30, 25, 20, 4, 1)$        | 4   | (6, 5, 4, 1) |
| -138    | 14        | $(9, 9, 6, 2, 1)$           | 4   | (3, 3, 2, 1) |
| -144    | 71        | $(171, 114, 38, 18, 1)$     | 4   | (9, 6, 2, 1) |
| -144    | 71        | $(162, 108, 36, 17, 1)$     | 4   | (9, 6, 2, 1) |
| -144    | 55        | $(90, 45, 36, 8, 1)$        | 4   | (10, 5, 4, 1) |
| -144    | 38        | $(56, 32, 16, 7, 1)$        | 4   | (7, 4, 2, 1) |
| -144    | 26        | $(36, 18, 12, 5, 1)$        | 4   | (6, 3, 2, 1) |
| -144    | 26        | $(81, 54, 16, 9, 2)$        | 3   | (9, 6, 2, 1) |
| -144    | 26        | $(36, 27, 9, 8, 1)$         | 4   | (4, 3, 1, 1) |
| -144    | 26        | $(40, 30, 10, 9, 1)$        | 4   | (4, 3, 1, 1) |
| -144    | 25        | $(25, 20, 10, 4, 1)$        | 4   | (5, 4, 2, 1) |
| -144    | 23        | $(20, 16, 12, 3, 1)$        | 4   | (5, 4, 3, 1) |
| -144    | 19        | $(32, 16, 8, 7, 1)$         | 4   | (4, 2, 1, 1) |
| -144    | 19        | $(36, 18, 9, 8, 1)$         | 4   | (4, 2, 1, 1) |
| -144    | 11        | $(12, 8, 4, 3, 1)$          | 4   | (3, 2, 1, 1) |
| -144    | 11        | $(9, 6, 2, 1)$              | 4   | (3, 2, 1, 1) |
| -144    | 7         | $(6, 6, 3, 2, 1)$           | 4   | (2, 2, 1, 1) |
| -144    | 5         | $(15, 6, 4, 3, 2)$          | 3   | (5, 2, 1, 1) |
| -144    | 5         | $(6, 3, 3, 2, 1)$           | 4   | (2, 1, 1, 1) |
| -150    | 29        | $(30, 25, 15, 4, 1)$        | 4   | (6, 5, 3, 1) |
| -152    | 37        | $(56, 28, 21, 6, 1)$        | 4   | (8, 4, 3, 1) |
| -152    | 17        | $(25, 10, 10, 4, 1)$        | 4   | (5, 2, 2, 1) |
| -152    | 16        | $(28, 14, 7, 6, 1)$         | 4   | (4, 2, 1, 1) |
| Euler # | $h^{1,1}$  | $\psi = (w_1,\ldots,w_5)$ | $i$  | $\tilde{w}$ |
|--------|------------|---------------------------|------|------|
| −156   | 176        | (609, 406, 174, 28, 1)    | 4    | (21, 14, 6, 1) |
| −156   | 66         | (153, 102, 34, 16, 1)     | 4    | (9, 6, 2, 1) |
| −156   | 21         | (63, 42, 12, 7, 2)        | 3    | (9, 6, 2, 1) |
| −156   | 21         | (28, 21, 7, 6, 1)         | 4    | (4, 3, 1, 1) |
| −156   | 8          | (18, 6, 6, 5, 1)          | 4    | (3, 1, 1, 1) |
| −156   | 8          | (21, 7, 7, 6, 1)          | 4    | (3, 1, 1, 1) |
| −160   | 59         | (110, 60, 40, 9, 1)       | 4    | (11, 6, 4, 1) |
| −160   | 15         | (20, 8, 8, 3, 1)          | 4    | (5, 2, 2, 1) |
| −168   | 86         | (204, 136, 51, 16, 1)     | 4    | (12, 8, 3, 1) |
| −168   | 50         | (63, 56, 42, 6, 1)        | 4    | (9, 8, 6, 1) |
| −168   | 32         | (48, 24, 18, 5, 1)        | 4    | (8, 4, 3, 1) |
| −168   | 25         | (24, 20, 16, 3, 1)        | 4    | (6, 5, 4, 1) |
| −168   | 20         | (30, 15, 10, 4, 1)        | 4    | (6, 3, 2, 1) |
| −168   | 18         | (20, 16, 8, 3, 1)         | 4    | (5, 4, 2, 1) |
| −168   | 17         | (24, 18, 6, 5, 1)         | 4    | (4, 3, 1, 1) |
| −168   | 16         | (12, 12, 9, 2, 1)         | 4    | (4, 4, 3, 1) |
| −168   | 13         | (12, 9, 6, 2, 1)          | 4    | (4, 3, 2, 1) |
| −168   | 12         | (20, 10, 5, 4, 1)         | 4    | (4, 2, 1, 1) |
| −168   | 12         | (24, 15, 4, 3, 2)         | 3    | (8, 5, 2, 1) |
| −168   | 8          | (21, 9, 6, 4, 2)          | 4    | (7, 3, 2, 2) |
| −168   | 8          | (9, 6, 3, 2, 1)           | 4    | (3, 2, 1, 1) |
| −168   | 6          | (12, 4, 4, 3, 1)          | 4    | (3, 1, 1, 1) |
| −168   | 2          | (2, 2, 2, 1, 1)           | 4    | (1, 1, 1, 1) |
| −172   | 29         | (49, 28, 14, 6, 1)        | 4    | (7, 4, 2, 1) |
| −172   | 10         | (22, 14, 3, 3, 2)         | 3    | (11, 7, 3, 1) |
| −180   | 26         | (42, 24, 12, 5, 1)        | 4    | (7, 4, 2, 1) |
| −180   | 24         | (50, 30, 10, 9, 1)        | 4    | (5, 3, 1, 1) |
| −180   | 24         | (55, 33, 11, 10, 1)       | 4    | (5, 3, 1, 1) |
| −180   | 17         | (15, 12, 9, 2, 1)         | 4    | (5, 4, 3, 1) |
| −180   | 14         | (45, 30, 8, 5, 2)         | 3    | (9, 6, 2, 1) |
| Euler # | $h^{1,1}$ | $\vec{w} = (w_1, \ldots, w_5)$ | $i$ | $\vec{w}$ |
|---------|----------|-----------------|-----|----------|
| -180 14 | (20, 15, 5, 4, 1) | 4 | (4, 3, 1, 1) |
| -184 9  | (16, 8, 4, 3, 1)  | 4 | (4, 2, 1, 1) |
| -192 77 | (180, 96, 72, 11, 1) | 4 | (15, 8, 6, 1) |
| -192 51 | (117, 78, 26, 12, 1) | 4 | (9, 6, 2, 1) |
| -192 47 | (88, 48, 32, 7, 1)  | 4 | (11, 6, 4, 1) |
| -192 41 | (54, 48, 36, 5, 1)  | 4 | (9, 8, 6, 1) |
| -192 35 | (63, 35, 21, 6, 1)  | 4 | (9, 5, 3, 1) |
| -192 27 | (28, 24, 16, 3, 1)  | 4 | (7, 6, 4, 1) |
| -192 21 | (24, 20, 12, 3, 1)  | 4 | (6, 5, 3, 1) |
| -192 19 | (40, 24, 8, 7, 1)   | 4 | (5, 3, 1, 1) |
| -192 16 | (24, 12, 8, 3, 1)   | 4 | (6, 3, 2, 1) |
| -192 13 | (16, 12, 4, 3, 1)   | 4 | (4, 3, 1, 1) |
| -192 11 | (30, 20, 4, 3, 3)   | 4 | (15, 10, 3, 2) |
| -192 11 | (36, 24, 7, 3, 2)   | 3 | (6, 4, 1, 1) |
| -192 8  | (6, 4, 4, 1, 1)     | 4 | (3, 2, 2, 1) |
| -192 5  | (4, 4, 2, 1, 1)     | 4 | (2, 2, 1, 1) |
| -192 3  | (9, 3, 3, 2, 1)     | 4 | (3, 1, 1, 1) |
| -192 3  | (4, 2, 2, 1, 1)     | 4 | (2, 1, 1, 1) |
| -200 25 | (40, 20, 15, 4, 1)  | 4 | (8, 4, 3, 1) |
| -204 16 | (30, 18, 6, 5, 1)   | 4 | (5, 3, 1, 1) |
| -204 14 | (42, 28, 8, 3, 3)   | 4 | (21, 14, 4, 3) |
| -204 9  | (6, 6, 4, 1, 1)     | 4 | (3, 3, 2, 1) |
| -216 92 | (240, 160, 64, 15, 1) | 4 | (15, 10, 4, 1) |
| -216 68 | (165, 88, 66, 10, 1) | 4 | (15, 8, 6, 1) |
| -216 66 | (156, 104, 39, 12, 1) | 4 | (12, 8, 3, 1) |
| -216 42 | (90, 60, 20, 9, 1)   | 4 | (9, 6, 2, 1) |
| -216 36 | (45, 40, 30, 4, 1)   | 4 | (9, 8, 6, 1) |
| -216 33 | (60, 30, 24, 5, 1)   | 4 | (10, 5, 4, 1) |
| -216 33 | (50, 25, 20, 4, 1)   | 4 | (10, 5, 4, 1) |
| -216 21 | (32, 16, 12, 3, 1)   | 4 | (8, 4, 3, 1) |
| Euler # | $b_{1.1}$ | $\psi = (w_1, \ldots, w_5)$ | $i$ | $\hat{w}$ |
|--------|---------|------------------|----|--------|
| -216   | 18      | (18, 15, 12, 2, 1)| 4  | (6, 5, 4, 1)|
| -216   | 17      | (45, 30, 9, 4, 2)| 4  | (15, 10, 3, 2)|
| -216   | 17      | (18, 15, 9, 2, 1)| 4  | (6, 5, 3, 1)|
| -216   | 13      | (15, 12, 6, 2, 1)| 4  | (5, 4, 2, 1)|
| -216   | 9       | (15, 6, 6, 2, 1)| 4  | (5, 2, 2, 1)|
| -216   | 6       | (12, 6, 3, 2, 1)| 4  | (4, 2, 1, 1)|
| -224   | 17      | (28, 16, 8, 3, 1)| 4  | (7, 4, 2, 1)|
| -228   | 12      | (18, 9, 6, 2, 1)| 4  | (6, 3, 2, 1)|
| -228   | 8       | (27, 18, 4, 3, 2)| 3  | (9, 6, 2, 1)|
| -228   | 8       | (12, 9, 3, 2, 1)| 4  | (4, 3, 1, 1)|
| -232   | 9       | (8, 6, 4, 1, 1)| 4  | (4, 3, 2, 1)|
| -232   | 5       | (6, 4, 2, 1, 1)| 4  | (3, 2, 1, 1)|
| -240   | 137     | (462, 308, 132, 21, 1)| 4  | (21, 14, 6, 1)|
| -240   | 34      | (81, 54, 18, 8, 1)| 4  | (9, 6, 2, 1)|
| -240   | 23      | (72, 48, 12, 11, 1)| 4  | (6, 4, 1, 1)|
| -240   | 23      | (78, 52, 13, 12, 1)| 4  | (6, 4, 1, 1)|
| -240   | 11      | (8, 8, 6, 1, 1)| 4  | (4, 4, 3, 1)|
| -240   | 9       | (20, 12, 4, 3, 1)| 4  | (5, 3, 1, 1)|
| -240   | 7       | (24, 16, 3, 3, 2)| 3  | (12, 8, 3, 1)|
| -252   | 76      | (210, 140, 56, 13, 1)| 4  | (15, 10, 4, 1)|
| -252   | 18      | (54, 36, 9, 8, 1)| 4  | (6, 4, 1, 1)|
| -252   | 18      | (21, 18, 12, 2, 1)| 4  | (7, 6, 4, 1)|
| -252   | 2       | (6, 2, 2, 1, 1)| 4  | (3, 1, 1, 1)|
| -256   | 23      | (40, 20, 16, 3, 1)| 4  | (10, 5, 4, 1)|
| -264   | 48      | (108, 72, 27, 8, 1)| 4  | (12, 8, 3, 1)|
| -264   | 28      | (63, 42, 14, 6, 1)| 4  | (9, 6, 2, 1)|
| -264   | 26      | (36, 32, 24, 3, 1)| 4  | (9, 8, 6, 1)|
| -264   | 20      | (36, 20, 12, 3, 1)| 4  | (9, 5, 3, 1)|
| -264   | 15      | (24, 12, 9, 2, 1)| 4  | (8, 4, 3, 1)|
| -264   | 15      | (42, 28, 7, 6, 1)| 4  | (6, 4, 1, 1)|

24
| Euler # | $h^{1,1}$ | $\psi = (w_1, \ldots, w_5)$ | $i$ | $\bar{w}$ |
|--------|-----------|-----------------------------|-----|----------|
| -264   | 11        | (10, 8, 6, 1, 1)            | 4   | (5, 4, 3, 1) |
| -272   | 7         | (10, 4, 4, 1, 1)            | 4   | (5, 2, 2, 1) |
| -276   | 48        | (105, 56, 6, 1)             | 4   | (15, 8, 6, 1) |
| -276   | 6         | (15, 9, 3, 2, 1)            | 4   | (5, 3, 1, 1) |
| -288   | 115       | (399, 266, 114, 18, 1)      | 4   | (21, 14, 6, 1) |
| -288   | 63        | (165, 110, 44, 10, 1)       | 4   | (15, 10, 4, 1) |
| -288   | 39        | (96, 64, 24, 7, 1)          | 4   | (12, 8, 3, 1) |
| -288   | 23        | (44, 24, 16, 3, 1)          | 4   | (11, 6, 4, 1) |
| -288   | 11        | (30, 20, 5, 4, 1)           | 4   | (6, 4, 1, 1) |
| -288   | 11        | (21, 12, 6, 2, 1)           | 4   | (7, 4, 2, 1) |
| -288   | 9         | (10, 8, 4, 1, 1)            | 4   | (5, 4, 2, 1) |
| -288   | 4         | (8, 4, 2, 1, 1)             | 4   | (4, 2, 1, 1) |
| -300   | 40        | (90, 48, 36, 5, 1)          | 4   | (15, 8, 6, 1) |
| -300   | 15        | (27, 15, 9, 2, 1)           | 4   | (9, 5, 3, 1) |
| -304   | 12        | (12, 10, 8, 1, 1)           | 4   | (6, 5, 4, 1) |
| -312   | 34        | (84, 56, 21, 6, 1)          | 4   | (12, 8, 3, 1) |
| -312   | 20        | (27, 24, 18, 2, 1)          | 4   | (9, 8, 6, 1) |
| -312   | 18        | (45, 30, 10, 4, 1)          | 4   | (9, 6, 2, 1) |
| -312   | 17        | (30, 15, 12, 2, 1)          | 4   | (10, 5, 4, 1) |
| -312   | 11        | (12, 10, 6, 1, 1)           | 4   | (6, 5, 3, 1) |
| -312   | 8         | (12, 6, 4, 1, 1)            | 4   | (6, 3, 2, 1) |
| -312   | 8         | (24, 16, 4, 3, 1)           | 4   | (6, 4, 1, 1) |
| -312   | 5         | (8, 6, 2, 1, 1)             | 4   | (4, 3, 1, 1) |
| -336   | 95        | (315, 210, 90, 14, 1)       | 4   | (21, 14, 6, 1) |
| -336   | 15        | (36, 24, 8, 3, 1)           | 4   | (9, 6, 2, 1) |
| -348   | 12        | (14, 12, 8, 1, 1)           | 4   | (7, 6, 4, 1) |
| -360   | 24        | (60, 40, 15, 4, 1)          | 4   | (12, 8, 3, 1) |
| -360   | 16        | (33, 18, 12, 2, 1)          | 4   | (11, 6, 4, 1) |
| -360   | 5         | (18, 12, 3, 2, 1)           | 4   | (6, 4, 1, 1) |
| -368   | 10        | (16, 8, 6, 1, 1)            | 4   | (8, 4, 3, 1) |
| Euler # | $h^{1,1}$ | $\tilde{w} = (w_1, \ldots, w_5)$ | $i$ | $\tilde{w}$ |
|--------|-----------|---------------------------------|-----|---------|
| −372   | 80        | (273, 182, 78, 12, 1)           | 4   | (21, 14, 6, 1) |
| −372   | 36        | (105, 70, 28, 6, 1)             | 4   | (15, 10, 4, 1) |
| −372   | 8         | (14, 8, 4, 1, 1)                | 4   | (7, 4, 2, 1)   |
| −372   | 4         | (10, 6, 2, 1, 1)                | 4   | (5, 3, 1, 1)   |
| −384   | 25        | (60, 32, 24, 3, 1)              | 4   | (15, 8, 6, 1)  |
| −396   | 32        | (90, 60, 24, 5, 1)              | 4   | (15, 10, 4, 1) |
| −408   | 18        | (48, 32, 12, 3, 1)              | 4   | (12, 8, 3, 1)  |
| −408   | 10        | (27, 18, 6, 2, 1)               | 4   | (9, 6, 2, 1)   |
| −420   | 10        | (18, 10, 6, 1, 1)               | 4   | (9, 5, 3, 1)   |
| −432   | 59        | (210, 140, 60, 9, 1)            | 4   | (21, 14, 6, 1) |
| −432   | 13        | (18, 16, 12, 1, 1)              | 4   | (9, 8, 6, 1)   |
| −432   | 11        | (20, 10, 8, 1, 1)               | 4   | (10, 5, 4, 1)  |
| −456   | 22        | (60, 40, 16, 3, 1)              | 4   | (15, 10, 4, 1) |
| −456   | 18        | (45, 24, 18, 2, 1)              | 4   | (15, 8, 6, 1)  |
| −456   | 14        | (36, 24, 9, 2, 1)               | 4   | (12, 8, 3, 1)  |
| −480   | 47        | (168, 112, 48, 7, 1)            | 4   | (21, 14, 6, 1) |
| −480   | 47        | (147, 98, 42, 6, 1)             | 4   | (21, 14, 6, 1) |
| −480   | 11        | (22, 12, 8, 1, 1)               | 4   | (11, 6, 4, 1)  |
| −480   | 3         | (12, 8, 2, 1, 1)                | 4   | (6, 4, 1, 1)   |
| −528   | 7         | (18, 12, 4, 1, 1)               | 4   | (9, 6, 2, 1)   |
| −552   | 15        | (45, 30, 12, 2, 1)              | 4   | (15, 10, 4, 1) |
| −564   | 29        | (105, 70, 30, 4, 1)             | 4   | (21, 14, 6, 1) |
| −612   | 12        | (30, 16, 12, 1, 1)              | 4   | (15, 8, 6, 1)  |
| −624   | 23        | (84, 56, 24, 3, 1)              | 4   | (21, 14, 6, 1) |
| −624   | 9         | (24, 16, 6, 1, 1)               | 4   | (12, 8, 3, 1)  |
| −720   | 17        | (63, 42, 18, 2, 1)              | 4   | (21, 14, 6, 1) |
| −732   | 10        | (30, 20, 8, 1, 1)               | 4   | (15, 10, 4, 1) |
| −960   | 11        | (42, 28, 12, 1, 1)              | 4   | (21, 14, 6, 1) |