Fragmentation of thinking structure: concept construction and problem solving in geometry of junior high school students

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Abstract. Fragmentation is adopted from the term of inefficient data storage in a computer which is used to describe thinking fragmentation. The purpose of this study is to describe the fragmentation of geometrical concepts in junior high school classified into: hole construction, mis-connection, and mis-analogical construction. This study used descriptive qualitative research method with the subjects of 8th grade students in Surakarta. Data collection in this study was conducted by task-based interview method. Qualitative descriptive analysis through the stages of reduction, representation, and interpretation of data. Hole construction occurs when students do not construct concept completely (20%). Mis-connection occurs when students make mistake in making connections between concepts (40%). Mis-analogical construction occurs when students analogizing a concept with another concept in geometry which is a mistake (20%). From the analysis result, defragmentation of thinking structure in geometry can be done with scaffolding to bring up scheme, knit scheme, and improve of analogic thinking.

1. Introduction
Mathematics is considered as an abstract and difficult subject so it has an impact on the low interest in learning mathematics [1]. However, students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding [2]. The concern in learning mathematics is how students can learn mathematics with understanding, building new knowledge actively from experience and previous knowledge [2]. Learning should not only emphasize on memorization and procedures but must be meaningful. When students can build some knowledge on their own accord, it will be meaningful knowledge to them [3]. In developing mathematics learning, teachers can adopt constructivism theories. The application of constructivism in learning mathematics depends on the initial knowledge that students have. Those who have already studied a concept, generally have a misconception about the concept [4].

In the process of constructing concepts and solving mathematical problems, students often experience error but they’re not aware of them [5]. This problem is termed as fragmentation of thinking structure. Fragmentation is adopted from the term of inefficient data storage in a computer which is used to describe thinking fragmentation [6]. Fragmentation in students’ mathematical thinking process and it is classified as follows: (1) pseudo construction occurs when students’ answer seems right but they cannot give the proper reason related concepts underlying the answer, (2) hole construction happens when there is a hole in students’ thinking structure or they do not construct
concept completely, (3) mis-connection occurs when students unable to make or make mistakes in connecting concepts in mathematics (4) mis-analogical construction arises when students consider a concept to be the same as another concept in mathematics which is not related, and (5) mis-logical construction happens when students failed to evaluate whether a mathematical statement is right or whether a concept or rule can apply in a problem [6].

Geometry is a natural area of mathematics for the development of students’ reasoning and justification skills [3]. Mathematical knowledge include geometry is constructed by linking a concepts with another concept. Students’ difficulties in geometrical representation include (1) perceptual organization: Gestalt principles, (2) recognition: bottom-up and top-down processing; and (3) representation of perception-based knowledge: verbal vs. pictorial representation, mental images and hierarchical structure of images [7]. Low spatial ability causes students to make more errors in solving geometry problems, especially the type of transformation error, process skill error, and encoding error [8].

In connection with the explanation above, the purpose of this study is to describe the fragmentation of thinking structures and their impact on the construction of concepts and mathematical problem solving in of junior high school students in Surakarta. Theoretically, the results of this study enrich the theory of learning based on constructivism. Practically, this result is useful for teachers to design learning activities that appropriate for constructing concepts and solving mathematical problems. The novelty of this research is the description of fragmentation of thinking structures and their impact on the construction of concepts and geometrical problem solving.

2. Research Methods

This research uses descriptive qualitative research methods that produces data in the form of written or spoken words from people and observable behavior. This method is believed to solve problems and improve practice through observation, analysis and description [9]. This qualitative research describes fragmentation of students' thinking structures on concept construction and problem solving in geometry by analyzing the results of student work that are confirmed through interview. The subjects in this study consisted of four of 8th grade junior high school students in Surakarta who were selected through purposive sampling. Data were collected using a task-based in-depth interview method. Students are encouraged to examine their own mathematical strategies and thinking so as to broaden their conceptual understanding of the problem. Qualitative descriptive analysis used in this study, including data reduction, data presentation and interpretation. Researcher identifies and stores data based on categorizations of fragmentation, then decides which data can represent each group. The chosen data that interests researcher then discussed and interpreted to create new theories about the fragmentation of thinking structures. This research does not link the thought development process with the fragmentation of existing thinking structures in the cognitive structure of the subject [10].

Here are some problem in the task.

1. A cylinder changes in size, where the radius increases by 30% and the height increases by 20%. Determine the increases percentage of cylinder volume from the original volume!
2. A right triangle with length of the sides: a, b and c. Relating to the length of three sides of triangle, how is the relations of the three sides?
3. Determine the area and circumference of the shades part!

![Figure 1. Circle Problem](image)

3. Results and Analysis

The following data are presented and discussed based on the type of fragmentation of students' thinking structures in geometry.
3.1. Hole Construction

Hole construction happens when there is a hole in students’ thinking structure because of they do not construct concept completely or they do not master initial knowledge that related to the concept. In second problem, students must master the basic concept of a triangle which is shape formed by three sides where the sum of the two sides’ length is always greater than the length of one other side: \( a + b > c \), \( a + c > b \), \( b + c > a \). Considering that the triangle is a right triangle then we can use Pythagorean Theorem to describe length sides relation. However, the problem does not mention whether the hypotenuse side and right angle. Therefore students can make several answers but with the right reasons. For instance we can stated that the square of the hypotenuse length is equal to the sum of the squares of the other two sides’ length. Based on interviews with one of the research subjects known to have the following hole construction. It can be seen that this concept is also related to the operation concept of algebraic form, which is square and square root.

![Figure 2. Student’s written test answer \( c = a^2 + b^2 \)](image)

Interview conducted to dig more information about student answer shown in figure 1.
S: Yes, I only need to draw a right triangle then I will just write letters
R: Why choose the position of the letters like this? It isn’t the one here or the opposite
S: Usually I think the side is \( c \)
R: Okay, what do you know from a right triangle?
S: Triangle that has an angle of 90°
   The sides, here are 3 cm, here are 4 cm, here are 5 cm (while drawing a right triangle and showing the length of each side)
R: What is the concept like?
S: Pythagorean
R: Did you know about Pythagorean, then what was the formula?
S: Oh, the formula, if you want to find \( c \) like this: \( a^2 + b^2 = c \), it means for example \( 3^2 + 4^2 = 9 + 16 = 25 \), it is used as root, right \( \sqrt{25} = 5 \), now that’s \( c \) is
R: If from this formula \( a^2 + b^2 = c \), how come there are no roots?
S: Oh this is the root, \( \sqrt{a^2 + b^2} = c \)

The following illustration shows the students’ fragmentation of thinking structure.

![Figure 3. Students’ fragmentation of thinking structure (hole construction)](image)
Figure 2 indicates that students do not construct concept completely, where students cannot express the concept of sides’ length in common triangles. In formulating a relationship the student also made mistakes because the basic concept of the operation of the square root of the algebraic form was not mastered correctly.

The construction process of students in understanding the triangle is affected by the grouping procedure of right triangle. Students used to check the triangle by checking the size of the three sides meeting the size of the right triangle. The habit of checking the right triangle directly used in this process of assimilation is “checking” with the Pythagorean concept [11].

Fragmentation in thinking process often happens as the result of meaningless learning, especially when it only stresses in memorizing formulas and how a procedure is done [12]. The fragmentation of students’ thinking structures on the geometry concept of 20% is due to the hole construction.

3.2. Mis-connection

Errors in the construction of geometrical concepts caused by mis-connections due to the absence of connections are shown in the following interview transcript.

R: Now that comes in part 2, number 1, what are you doing?
S: Let’s say that the radius is 1 cm, the height is 1 cm, only 1 x 1 = 1 cm is left. If the new volume is increased, the radius is 30%, the height is 20% so the radius is 1 + 30% = 1.3. Then the height is 1 + 20% = 1.2. Staying multiplied, the base tube formula is multiplied high. Only 1.3 x 1.2 = 1.56, the addition is 1.56 - 1 = 0.56. So it's 56%.
R: So the conclusion increased by what percentage?
S: 56%

From the transcript, students have been able to construct the concept of increasing the length of the cylinder correctly: i.e. the radius increases by 30% then the length becomes 1,3 from the original length, the height increases by 20% then the length becomes 1,2 from the original length. The mis-connection arises when a student cannot associate the information he has built with the concept of a cylindrical volume. Students only multiply the quantities so that they lead to the wrong solution.

![Cylinder diagram](image)

**Figure 4.** Students’ fragmentation of thinking structure (mis-connection)

It can be seen that student has been able to construct the concept of increasing length of cylinder elements correctly, but there is a mis-connection, which is when calculating the volume of the tube student only multiplies the radius and height. Student believes that the volume formula used is correct. This can occur because in the previous learning process the concept of the volume of the cylinder is learned only by memorization, not by a construction process so that students easily forget the concept. Most of the fragmentation of students’ thinking structures on the geometry concept of 40% is due to the mis-connection. The importance of mathematical connection ability is it can help the mastery of understanding the meaningful concepts and help solve problem-solving tasks through the interrelationship between mathematical concepts and between mathematical concepts with concepts in other disciplines [13].
3.3. Mis-analogical Construction

Problem solving planning involves a variety of strategies, one of which is by making analogies. Analogy can be a way to solve a problem if it is constructed correctly, otherwise it can lead to a wrong solution if an analogy error occurs.

![Figure 5](image_url). Students’ fragmentation of thinking structure (mis-analogical construction)

Problem solving corresponds to a situation where a person who is facing a problem, they are capable to understand it, since they have knowledge previously learnt, but they do not know how to solve it in that moment, although they try to find a solution by facing such situation [14]. The problem solving corresponds to a task merely perceptive and conceptual. It can be seen that student does not understand the problem well, also seen in the interview "I think the result is 50% i.e. from 30% + 20%". Students analogize that calculating the increase in volume caused by the increase in the length of the radius and height is the same as calculating the number of length percentages of several components, which is obviously wrong analogy. This becomes additional evidence of other research that the thinking structure of false-pseudo of student in solving the problem of inequality is because (1) begins with students' errors in making assumptions when understanding the problem, (2) incompleteness of the students' thinking structure when understanding the problem, and (3) incompleteness of students' thinking substructure in planning ways of completion [15].

R: Okay, lets see next number, number 3, tell me
S: Asked to search circumferences and area of shades part
R: If you go around first, what do you do?
S: K = πd, around this one (pointing to a big circle) 22/7 × 28 = 88, there are 2, all times 2 are 176. The one here (pointing to the middle circle) 22/7 × 21 = 66
R: Where did you get 21?
S: 28, there are sides, this side doesn’t participate, it’s 7, so it’s 28-7 = 21. 22/7 × 21 = 66, times 2 is 132
R: After all that, what do you do?
S: Added
R: Is that the result of everything?
S: Subtracted, this little part is still missing. After meeting the results, added up.

The following illustrations show the students’ fragmentation of thinking structure.

![Figure 6](image_url). Students’ fragmentation of thinking structure (mis-analogy)

The fragmentation of students' thinking structures on the geometry concept of 20% is due to the mis-analogical construction.
4. Conclusion

Based on the results of the analysis of the fragmentation of students' thinking structures in constructing concepts and solving geometry problems, the following conclusions can be concluded: (1) hole construction occurs when students do not construct concept completely and they do not master initial knowledge that related to the concept (20%), in this study this fragmentation occurs there is hole in students’ understanding of concept of sides’ length in common triangles and Pythagorean Theorem; (2) mis-connection occurs when students make mistake in making connections between concepts (40%); in this study it occurs when student failed in making connection between cylinder volume concept and increasing percentages of cylinder component; (3) Mis-analogical construction occurs when students consider a concept to be the same as another concept in geometry which is a mistake (20%); it occurs when student making wrong analogy in circumference and area of shades part. The fragmentation of thinking structures and their impact on the concept construction and problem solving of junior high school students is needed to assist teachers in designing learning activities, especially in geometry, so there are no fragmentation in concept construction and problem solving. The teacher also has to defragment the structure of thinking as soon as possible with scaffolding to to bring up scheme, knit scheme, and improve of analogic thinking. This research should be continued to obtain teaching material based on the description of the fragmentation of thinking structures and their impact on the construction of concepts and mathematical problem solving for junior high school students.

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