Cosmological consequences of a scalar field with oscillating equation of state: 
A possible solution to the fine-tuning and coincidence problems

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We propose a new dark energy model for solving the cosmological fine-tuning and coincidence problems. The key idea is that the Universe may have several acceleration phases across the history. The specific example we study is a quintessence model with repeated approximately double exponential potential, which only introduces one Planck scale parameter and three dimensionless parameters of order unity. The cosmological background evolution equations can be recast into a four-dimensional dynamical system. The main properties of this system are discussed in details. As a bonus, our model provides a natural unification of early inflation, late-time dark energy and early dark energy recently proposed to ease the Hubble tension.

I. INTRODUCTION

The cosmic late-time acceleration has been confirmed for two decades [1, 2]. The simplest model to explain this phenomenon is the ΛCDM model with the cosmological constant $\Lambda = O(H_0^2/c^2)$, where $H_0$ is the Hubble constant. However, this model suffers from serious theoretical problems. One is the fine-tuning problem, which can be learned in the following way. The energy density of vacuum given by quantum field theory should be of the order of Planck scale density $\rho = 5.1 \times 10^{96}$ kg/m³ while the effective energy density of $\Lambda$ is $\rho_\Lambda = 1.4 \times 10^{-26}$ kg/m³. The ratio of $\rho_\Lambda$ and $\rho_P$ is approximately $O(10^{-120})$. If one interprets the origin of $\Lambda$ as the vacuum energy, then how to obtain $\rho_\Lambda$ from $\rho_P$ is a fine-tuning problem. There are many theories to explain the origin of this extremely small ratio, e.g., spacetime foam [3–5] and quantum gravity discreteness [6, 7]. However, a mature theory in this way seems far away from us. If the late-time acceleration is not driven by $\Lambda$ but a dynamical field, then it is possible to hide the vacuum energy at macroscopic scales (i.e., to solve the old cosmological constant problem) with reasonable theories [8, 9].

Another problem exist in the ΛCDM model is the coincidence problem, which states why the dark energy density is comparable to the normal matter density at today (see Fig. 3 in Ref. [10] for an intuition). It is believed that some dynamical dark energy models can alleviate, but not solve, this problem with the tracker property [11–15]. However, we may rephrase the coincidence problem as why the transition from matter-dominated Universe to dark energy-dominated Universe occurs today. In this sense, the tracker property does nothing to alleviate the coincidence problem. We think the coincidence problem is related to the fact that the wide-used dark energy models generally need a parameter related to $H_0$. Note that $H_0$ is the value of the Hubble parameter at today (the time human exist). Introducing such a human-related parameter into the fundamental theory will inevitably make the Universe special at today. Thus, abandoning $H_0$-related parameters may be the key to solving the coincidence problem.

In this paper, we start from the specific question: Can we explain the cosmic late-time acceleration without $H_0$-related parameters? Furthermore, we require the theory only introduces Planck scale parameters and dimensionless parameters of order unity. Technically, in the framework of quintessence, we might be able to realize this with a repeated double exponential potential (see Fig. 1 for an intuition). Note that the single steep exponential potential could realize the dark energy density tracks the energy density of normal matters [16]. The model with double exponential potential, in which one is steep and one is flat, presents both early scaling and late-time accelerating solutions [17]. If we repeat the double exponential potential periodically with the scalar field increasing, we may obtain a $H_0$-scale acceleration under conditions that meet our requirements. The equation of state (EoS) of the scalar field with such potential should be oscillating. The desired theory may be able to solve the fine-tuning and coincidence problems.

II. THE MODEL

We consider cosmic expansion driven by a single scalar field with normal matters including radiation and pressureless fluid. The action for this physical system is of the form

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa} - \frac{\mathcal{L}_\phi}{\kappa} \right] + S_m,$$

where $\kappa = 8\pi G/c^4$. For the normal matters, we know the variation $\delta S_m = -\frac{1}{2} \int d^4x \sqrt{-g} T_{\mu\nu} \delta g^{\mu\nu}$ and $T_{\mu\nu} = (\rho_m + p_m/c^2) u_\mu u_\nu + p_m g_{\mu\nu}$. The EoS of normal matters is defined as $w_m = p_m/(\rho_m c^2)$. We know $w_m = 0$ for the pressureless fluid and $w_m = 1/3$ for the radiation. For the scalar field, we adopt $\mathcal{L}_\phi = X + V(\phi)$, where

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The double exponential potential exactly, but assume $X = \frac{1}{2} \rho^{\mu \nu} \partial_\mu \phi \partial_\nu \phi$. For the potential, we do not repeat the double exponential potential exactly, but assume

$$V(\phi) = V_0 \exp \left[-\frac{\lambda_1 + \lambda_2}{2} \phi - \frac{\alpha(\lambda_1 - \lambda_2)}{2} \sin \frac{\phi}{\alpha}\right], \quad (2)$$

where $V_0$, $\lambda_1$, $\lambda_2$, $\alpha$ are parameters. This can be regarded as an approximate but simple realization of the repeated double exponential potential. In our conventions, $\lambda_i$ and $\alpha$ are dimensionless and $[V_0] = \text{length}^{-2}$. Note that $V_0 > 0$ as required by Eq. (5a). For the first step, we can assume $\lambda_1 > 0$, $|\lambda_2| < \lambda_1$ and $\alpha > 0$. Figure 1 plots $V(\phi)$ for four cases. The $\alpha$ controls the period of oscillation. This potential satisfies our requirement that $\lambda$ varies as $\lambda_1 \rightarrow \lambda_2 \rightarrow \lambda_1 \rightarrow \lambda_2 \rightarrow \cdots$ with $\phi$ increasing [see Eq. (7b) for the definition of $\lambda$]. For suitable parameter settings, we hope the Universe is decelerating when $\lambda \approx \lambda_1$ and accelerating when $\lambda \approx \lambda_2$. However, this is just our initial idea. The system behaves much more complex as we will see in Sec. III. Variation of the action with respect to the metric gives the gravitational field equations

$$G_{\mu \nu} = \kappa T_{\mu \nu} + \Phi_{\mu \nu}, \quad (3)$$

where $\Phi_{\mu \nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu \nu} \Theta$. Variation of the action with respect to $\phi$ gives the scalar field equation $\Box \phi = V'$, where $V' = dV/d\phi$. Hereafter we call the model described by Eq. (2) as the sine oEoS model, where the first letter o means oscillating and EoS means the equation of state. Replacing the sine with the cosine in Eq. (2) does not change the essence of the model as $\cos(x) = \sin(x + \pi/2)$.

To be consistent with current observations [18], we assume the Universe is described by the flat Friedmann-Lemaître- Robertson-Walker (FLRW) metric

$$ds^2 = -c^2 dt^2 + a^2(dx^2 + dy^2 + dz^2), \quad (4)$$

where $a = a(t)$. For the normal matter, the energy-momentum tensor is $T_{\mu \nu} = \text{diag}\{-\rho_m c^2, p_m, p_m, p_m\}$ and energy conservation is described by $\dot{\rho}_m + 3(1 + w_m) H \rho_m = 0$. For the scalar field, we can assume $\phi = \phi(t)$, which gives $X = -\frac{\phi^2}{2c^2}$ and $\Phi_{\mu \nu} = \text{diag}\{X - V, -X - V, -X - V, -X - V\}$. Substituting the above results into the gravitational and scalar field equations, we obtain

$$H^2 = \frac{8\pi G}{3} \rho_{\text{tot}}, \quad (5a)$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho_{\text{tot}} + \frac{3\rho_{\text{tot}}}{c^2}\right), \quad (5b)$$
$$\dot{\phi} + 3H \phi + c^2V' = 0, \quad (5c)$$

where $\rho_{\text{tot}} = \rho_m + \rho_\phi$, $\rho_{\text{tot}} = \rho_m + p_\phi$, $\rho_\phi \equiv \left(-X + V)/(\kappa c^2\right)$ and $p_\phi \equiv \left(-X - V)/\kappa\right)$. Equivalently, we can define the EoS of the scalar field as $w_\phi = p_\phi/(\rho_\phi c^2)$ and Eq. (5c) can be written as $\dot{\rho}_\phi + 3(1 + w_\phi) H \rho_\phi = 0$. Equation (5c) can be derived from Eqs. (5a) and (5b) as we expected. In order to compare theoretical results with observations, we define

$$\Omega_\phi \equiv \frac{8\pi G}{3H^2} \rho_\phi, \quad \Omega_m \equiv \frac{8\pi G}{3H^2} \rho_m = 1 - \Omega_\phi, \quad (6)$$

The $\omega_{\text{tot}}$ and $H_0$ are enough for the observational constraints about cosmic background evolution. For example, the luminosity distance $D_L(z) = \left(\frac{1 + z}{H_0}\right) \int_0^z \frac{dz}{E(z)}$, where $E^2(z) = \exp\left(\int_0^z \frac{3H \rho_{\text{tot}}^{1/3}}{c}\mathrm{d}z\right)$. In this paper, we do not fit real data because the chaos phenomenon in the model makes the classical statistical methods invalid here (see Sec. III for detailed discussions). Instead, in the next section, we discuss how well the sine oEoS model with certain parameter settings can recover the $\Lambda$CDM model in the late-time era.

### III. MAIN PROPERTIES

Phase space analysis is a powerful tool for quantitatively understanding the cosmological dynamics [10]. As in the case of exponential potential [16], we define the dimensionless variables

$$x_1 \equiv \frac{\dot{\phi}}{\sqrt{6}H}, \quad x_2 \equiv \frac{c \sqrt{\sqrt{V}}}{\sqrt{3}H}. \quad (7a)$$

As declared in the review paper [10], these two variables were first introduced in Ref. [16]. Based on this definition, we have $\Omega_\phi = x_1^2 + x_2^2$ and $w_\phi = (x_1^2 - x_2^2)/\Omega_\phi$. For the sine oEoS model, we also need

$$\lambda \equiv \frac{\lambda_1 + \lambda_2}{2} \equiv \frac{\lambda_1 - \lambda_2}{2} \equiv \frac{\cos \phi}{\alpha}, \quad \lambda \equiv \frac{\lambda_1 + \lambda_2}{2} \equiv \frac{\lambda_1 - \lambda_2}{2} \equiv \frac{\cos \phi}{\alpha}, \quad (7b)$$
$$\nu \equiv \sqrt{6}(\lambda_2 - \mu) \equiv -\frac{\sqrt{6}(\lambda_1 - \lambda_2)}{2} \sin \frac{\phi}{\alpha}, \quad (7c)$$

where $\mu \equiv V'/V$. The cosmic evolution equations can then be written as

$$\frac{dx_1}{dN} = -3x_1 + \sqrt{6} \lambda x_2^2 + \frac{3}{2} \lambda_1 L, \quad (8a)$$

[FIG. 1. The potential of the sine oEoS model.]

\[ X = \frac{1}{2} \rho^{\mu \nu} \partial_\mu \phi \partial_\nu \phi. \]
\[
\frac{dx_2}{dN} = -\frac{\sqrt{6}}{2} \lambda x_1 x_2 + \frac{3}{2} x_2 L,
\]
\[
\frac{d\lambda}{dN} = \nu x_1,
\]
\[
\frac{d\nu}{dN} = \frac{3x_1}{\alpha^2} (\lambda_1 + \lambda_2 - 2\lambda),
\]
where \( L = (1 - w_m)x_1^2 + (1 + w_m)(1 - x_2^2) \), \( N = \ln(a/a_i) \) and \( a_i \) is the value of the scale factor at any fixed time point. Similar to the exponential and power-law potentials [16, 19], the parameter \( V_0 \) disappears in the dynamical system equations in our model. The evolution of this physical system is completely described by trajectories within the region \( x_1^2 + x_2^2 \leq 1, x_2 \geq 0, \lambda_2 \leq \lambda \leq \lambda_1 \) and \( |\nu| \leq \sqrt{6} (\lambda_1 - \lambda_2)/(2\alpha) \). One constraint equation given by Eq. (7) is
\[
\nu(\lambda) = \nu(\lambda) = \pm \frac{\sqrt{6}}{\alpha} \sqrt{\lambda(\lambda_1 + \lambda_2) - \lambda^2 - \lambda_1 \lambda_2}. \tag{9}
\]
The sign of the above equation is changeable with the system evolution. This is why we do not substitute Eq. (9) into Eq. (8c) to eliminate \( \nu \) to obtain a three-dimensional dynamical system. Using the four-dimensional dynamical system Eq. (8) to characterise the evolution of the Universe can avoid the sign selection problem, which is useful to the following analyses.

### A. Critical points and stability

We are now ready to find the critical points of the four-dimensional dynamical system Eq. (8) and to perform the stability analysis. We only consider the case where \( w_m = 0 \) or \( 1/3 \). Depending on the value of \( \lambda_2 \), we have up to four critical points which are listed in Table I. In principle, one can directly obtain the stability of points A, B and C from Fig. 1. But we also perform the standard stability analysis (see Ref. [10] for a lecture).

Point O means the Universe is dominated by normal matter, which is unstable as we expected. Point C is the only stable attractor and stands for the cosmological solution where the Universe is dominated by scalar field with \( w_\phi = -1 \). This is inconsistent with the physical scenario we expected. Thus we would require \( \lambda_2 \geq 0 \) to avoid point C for the viable model. This requirement also makes the saddle point B disappear. Point A is saddle point and we will discuss more about it later. The terrible thing is, unlike the results of most dark energy models (see Ref. [10] for a review), here the critical points do not provide any quantitative information about the evolution of the system for \( \lambda_2 > 0 \).

### B. The viable parameter space

We want to find the allowed region in the parameter space in which the sine oEoS model could present a reasonable cosmological background evolution. Our discussion strongly depends on the results of the exponential potential. Here we summarize the main results obtained in Ref. [16]. For the potential \( V(\phi) = V_0 \exp(-\lambda \phi) \), if \( \lambda^2 < 3(1 + w_m) \), then point \((x_1, x_2) = (\lambda/\sqrt{6}, \sqrt{1 - \lambda^2/6})\) is stable and represents a Universe dominated by the scalar field with \( w_\phi = -1 + \lambda^2/3 \). If \( \lambda^2 > 3(1 + w_m) \), then point \((x_1, x_2) = (\sqrt{3/2(1 + w_m)}/\lambda, \sqrt{3(1 - w_m^2)/(2\lambda^2)})\) is stable and represents a scaling solution with \( \Omega_\phi = 3(1 + w_m)/\lambda^2 \) and \( w_\phi = w_m \). Intuitively, in the sine oEoS model, we may use a large \( \lambda \) to achieve \( \rho_\phi \) tracks \( \rho_m \) and use a small \( \lambda \) to accelerate the Universe.

To solve the coincidence problem, we expect \( \rho_\phi \) and \( \rho_m \) in the same order of magnitude many times across the history of the Universe. In the limit of \( \alpha \ll 1 \), Eq. (2) can be approximately regarded as an exponential potential with \( \lambda = (\lambda_1 + \lambda_2)/2 \). In this case we expect \( \rho_\phi \) tracks \( \rho_m \) in both radiation and matter era, which require \( (\lambda_1 + \lambda_2)^2/4 \geq \max(3[1 + w_m]) = 4 \), i.e., \( \lambda_1 + \lambda_2 > 4 \). Increasing \( \lambda \) makes the scaling solution disappear and \( \rho_\phi/\rho_m \) time-dependent. However, numerical results show that generally \( \lambda_1 + \lambda_2 > 4 \) is sufficient to satisfy our requirement that \( \rho_\phi \) was in coincidence with \( \rho_m \) many times even for \( \alpha = \mathcal{O}(1) \). In addition, very large \( \alpha \) is not allowed because increasing \( \alpha \) reduces the frequency of coincidence as shown in Fig. 2. Comparing the left and right sides of Fig. 2, we find the relation between \( \alpha \) and the coincidence frequency is independent of the initial conditions. The exact upper limit on \( \alpha \) may be subjective and a reasonable one can be \( \alpha \leq \mathcal{O}(1) \).

To explain the cosmic late-time acceleration, we need \( w_\phi \) can be very close to \(-1 \) in some time period (see Eq. (51) in Ref. [18] for observational constraints). In the limit of \( \alpha \gg 1 \), locally we may can regard Eq. (2) as a single exponential potential. The minimum value of \( w_\phi \) should be reached at \( \lambda \approx \lambda_2 \) and \( w_{\phi_{\text{min}}} \approx -1 + \lambda_2^2/3 \). This result is also numerically verified for \( \alpha = \mathcal{O}(1) \). If we require \( w_{\phi_{\text{min}}} < -0.95 \) as given in Ref. [18], then \( \lambda_2 < 0.39 \). In addition, very small \( \alpha \) is not allowed because decreasing \( \alpha \) increases the value of \( w_{\phi_{\text{min}}} \) (and also \( w_{\text{hot_{min}}} \) as shown in Fig. 2). Unfortunately, the exact lower limit on \( \alpha \) is not obtained here. One important thing is worth mentioning for \( \lambda_2 = 0 \). The stability analysis summarized in Table I shows point A is a saddle point. Interestingly, it can however attract many non-trivial solutions (see Fig. 3 for an example, which shows the scalar field with sufficient low kinetic energy will be trapped into point A). In order to improve the robustness of the sine oEoS model, it is reasonable to require \( \lambda_2 > 0 \). In summary, the viable parameter space should be \( \lambda_1 + \lambda_2 > 4, 0 < \lambda_2 < 0.39, \alpha = \mathcal{O}(1) \) and \( V_0 \) is arbitrary. If we assume \( \phi = \mathcal{O}(1) \) at the onset of the cosmic Big Bang, it is reasonable to assume \( V_0 = \mathcal{O}(l_p^2) \), where \( l_p \) is the Planck length. The sine oEoS model can explain the late-time acceleration with only one Planck scale parameter and several dimensionless parameters of order unity. In this sense no parameters need fine-tuning in the sine oEoS model.
TABLE I. Critical points of the dynamical system Eq. (8) with existence and physical properties. The label column is consistent with the labels in Fig. 1. The methods to analyze the stability are listed in the last column, in which linear stability theory is performed to Eq. (8) while center manifold theory is performed to Eq. (5). Here $b_{\pm} = (-3 \pm \sqrt{9 - 12\sqrt{-\lambda_1 \lambda_2}/\alpha})/2$ and $c_{\pm} = (-3 \pm \sqrt{9 - 12\sqrt{-\lambda_1 \lambda_2}/\alpha})/2$, which give $b_+ > 0$, $b_- < 0$ and $\text{Re}(c_{\pm}) < 0$.

| Label | $(x_1, x_2, \lambda, \nu)$ | Existence | $\Omega_\phi$ | Eigenvalues | Stability | Method |
|-------|-----------------------------|-----------|--------------|-------------|-----------|--------|
| O     | $(0, 0, \lambda, \nu)$     | All $\alpha_2$ | 0            | $[0, 0, -\frac{3\nu m - \frac{11}{2}, \frac{(1 + w_m)}{2}]$ | saddle | linear stability theory |
| A     | $(0, 1, 0, 0)$              | $\lambda_2 = 0$ | 1            | $[0, 0, -3, -3(1 + w_m)]$ | saddle | center manifold theory |
| B     | $(0, 1, 0, \sqrt{-6\lambda_1 \lambda_2}/\alpha)$ | $\lambda_2 < 0$ | 1            | $[0, -3(1 + w_m), b_+, b_-]$ | saddle | linear stability theory |
| C     | $(0, 1, 0, -\sqrt{-6\lambda_1 \lambda_2}/\alpha)$ | $\lambda_2 < 0$ | 1            | $[0, -3(1 + w_m), c_+, c_-]$ | stable | center manifold theory |

FIG. 2. Evolution of the dark energy relative energy density $\Omega_\phi$ and the total effective EoS parameter $w_{tot}$ for the sine oEoS model. The parameters are $w_m = 0$, $\lambda_1 = 4.5$, $\lambda_2 = 0.2$, $\alpha = 0.5, 0.75$ and 1.0 for the first, second and third row, respectively. The initial conditions are $x_1 = 0.75$, $x_2 = 0.5$, $\lambda_0 = 0.3$, $\nu_0 = \nu_+ (\lambda_0)$ and $\nu_0 = \nu_- (\lambda_0)$ for the first and second column, respectively. The plots start at $N = 0$ and end at $N = 60$. Here $w_{\lambda_2} = -1 + \lambda_2^2/3$ and $\lambda_{\text{normalized}} = (2\lambda - \lambda_1 - \lambda_2)/(\lambda_1 - \lambda_2) \equiv \cos(\phi/\alpha)$, which is plotted in the subplots (d) and (f) and can be used to track the position of $\phi$ in $V(\phi)$. Note that $\lambda_{\text{normalized}} \approx 1$ corresponds to $\lambda \approx \lambda_1$ and $\lambda_{\text{normalized}} \approx -1$ corresponds to $\lambda \approx \lambda_2$.

In this paper, we do not perform complete parameter constraints with real data (see the next subsection for reasons), but we do find a set of parameters that make the sine oEoS model very close to the standard ΛCDM model in the late-time Universe. For example, one can easily verify $\Omega_m = 0.29$, $\Omega_\phi = 0.71$, $w_{tot} = -0.70$, and $dw_{tot}/dN = -0.61$ at $N = 59.28$ in Fig. 2 (d), where $dw_{tot}/dN$ can be calculated based on Eqs. (6) and (8). In principle, we can set $N = 59.28$ as today and set $N$ equals to a number smaller than zero as the beginning of the Big Bang if necessary. For the ΛCDM model, we know $\Omega_m \approx 0.3$, $\Omega_\lambda \approx 0.7$, $w_{tot} \approx -0.7$ and $dw_{tot}/dN \approx -0.63$ at today. Therefore, it is reasonable to believe that the sine oEoS model can well fit the observations about the late-time Universe.

C. Chaos

Chaos appears in Fig. 2 and Fig. 3. We think two phenomena are related to the emergence of chaos. One is that no critical point is stable for $\lambda_2 \geq 0$. The other is that the evolution of the scalar field is not attracted to the scaling solution when $\lambda \approx \lambda_1$ as shown in the shaded region in Fig. 2 (f). This is understandable since the only stable critical point for $V(\phi) \propto \exp(-\lambda_1 \phi)$ is a spiral [16], not a node, and the $\lambda \approx \lambda_1$ part in the sine oEoS model is too short to successfully attract the scalar field. In contrast, the only stable critical point for $V(\phi) \propto \exp(-\lambda_2 \phi)$ is a node [16] and can attract the scalar field faster. This result violates our initial idea that $\rho_\phi$ tracks $\rho_m$ when $\lambda \approx \lambda_1$ and the Universe is accelerating when $\lambda \approx \lambda_2$. However, our model can still be used to solve the fine-tuning and coincidence problems (see discussions before).

The worse thing is that chaos make cosmological constraints tricky. In some classical dark energy models, the tracker property makes the late-time cosmic evolution independent of the dark energy initial conditions [13–15]. In this case, we do not need to consider these initial conditions in the cosmological constraints. However, in the sine oEoS model, the late-time evolution depends on the
initial conditions of the scalar field. Furthermore, if we set \( N = 0 \) as the beginning of the Big Bang and set \( N = 60 \) as today, then the dependence should be quite strong. The consequence is that we have to consider the initial conditions as fitting parameters in the cosmological constraints and the posterior distribution used in the classical statistical analysis changes dramatically with parameter changes. There should be many peaks in the posterior distribution, which makes the contour plots not reflect the parameter distributions correctly.

IV. DISCUSSION

In conclusion, qualitatively, the oscillating EoS scenario seems like a natural and simple way to eliminate both the fine-tuning and coincidence problems. Quantitatively, we have demonstrated the availability of the sine oEoS model proposed in this paper.

Another promising feature is that our model provides a natural unification of early inflation [20], late-time dark energy [1, 2], and early (redshift \( z \gtrsim 3000 \)) dark energy recently proposed to ease the Hubble tension [21–23]. The possible unification of early inflation and late-time dark energy has been widely discussed in the literatures (see Refs. [24–27] for examples). Compared with these models, we believe that our model is more natural because we only need one Planck scale parameter and several dimensionless parameters of order unity. So far, the early dark energy only attracted limited attention. One important follow-up is Ref. [28], which proposed an explanation for the coincidence between matter-radiation equality and early dark energy with neutrino physics. In our model, it is not a coincidence that an accelerating phase appears at \( z \gtrsim 3000 \), because there are many similar periods in the history.

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