CosmoBolognaLib: C++ libraries for cosmological calculations

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Abstract

We present the CosmoBolognaLib, a large set of Open Source C++ numerical libraries for cosmological calculations. CosmoBolognaLib is a living project aimed at defining a common numerical environment for cosmological investigations of the large-scale structure of the Universe. Several public libraries for astronomical calculations are nowadays available, in different languages, such as e.g. CiunBASE (Taghizadeh-Popp, 2010), CosmoPMC (Kilbinger et al., 2011), AstrolML (VanderPlas et al., 2012), CUTE (Alonso, 2012), Astropy (Astropy Collaboration et al., 2013), Cosmo++ (Aslanyan, 2014), CosmoloPy\textsuperscript{1}, NumCosmo (Dias Pinto Vitenti and Penna-Lima, 2014), Tree_corr (Jarvis, 2015).

Aiming at defining a common environment for handling extragalactic source catalogues, performing statistical analyses and extracting cosmological constraints, we implemented a large set of C++ libraries, called CosmoBolognaLib (hereafter CBL), specifically focused on numerical computations for cosmology, thus complementing the available software. In particular, the CBL provide highly optimised algorithms to measure two-point (2PCF) and three-point correlation functions (3PCF), exploiting a specifically designed parallel chain-mesh algorithm to count pairs and triplets. Several types of correlation functions can be computed, such as the angle-averaged 2PCF, the 2D 2PCF in both Cartesian and polar coordinates and its multipole moments, the angular, projected and deprojected 2PCF, the clustering wedges, the filtered 2PCF, and the connected and reduced 3PCF (see §4.1). Moreover, a large set of methods are provided to construct random catalogues, to estimate errors and to extract cosmological constraints from clustering analyses (see §4.3). These features represent the main novelty of the presented libraries.

The CBL are fully written in C++. They can be included either in C++ codes or, alternatively, in high-level scripting languages through wrapping. We provide an example code that shows how to include the CBL in Python scripts in Appendix B.3.

This effort can be considered as a living project, started a few years ago and intended to be continued in the forthcoming years. The following is the list of scientific publications that have been fully or partially performed using the presented libraries: Marulli et al. (2011, 2012a,b, 2013, 2015); Giocoli et al. (2013); Villaescusa-Navarro et al. (2014); Moresco et al. (2014); Veropalumbo et al. (2014, 2015); Sereno et al. (2015); Moresco et al. (in preparation); Petracco et al. (2015). Thanks mainly to the adopted object-oriented programming technique, the CBL are flexible enough to be significantly extended.

In this paper, we present the main features of the current version of the CBL, that is fully publicly available\textsuperscript{2}, together with the documentation obtained with doxygen\textsuperscript{3}. A set of sample codes, that explain how to use these libraries in either C++ or Python software, is provided at the same webpage.

The paper is organised as follows. In §2 we describe the CBL class for cosmological computations. In §3 we present

\textsuperscript{1}http://apps.difa.unibo.it/files/people/federico.marulli3
\textsuperscript{2}https://github.com/federicomarulli/CosmoBolognaLib
\textsuperscript{3}http://apps.difa.unibo.it/files/people/federico.marulli3
\textsuperscript{4}http://roban.github.com/CosmoloPy/
the classes implemented for handling catalogues of extragalactic sources. 2PCF and 3PCF can be measured and modelled with specific classes that are described in §4. §5 presents the CBL methods for statistical analyses. In §6 we provide a brief description of the other CBL functions used for several generic calculations. Finally, in §7 we draw our conclusions. Compiling instructions and a few sample codes are reported in Appendix A and Appendix B, respectively.

2. Cosmology

All the cosmological functions defined in the CBL are implemented as public members of the class cosmobl::Cosmology. The private parameters of this class are the following: the matter density, that is the sum of the density of baryons, cold dark matter and massive neutrinos (in units of the critical density) at $z=0$, $\Omega_{\text{matter}}$; the density of baryons at $z=0$, $\Omega_{\text{baryons}}$; the density of massive neutrinos at $z=0$, $\Omega_{\nu}$; the effective number of relativistic degrees of freedom, $N_{\text{eff}}$; the number of massive neutrino species; the density of dark energy at $z=0$, $\Omega_{\text{DE}}$; the density of radiation at $z=0$, $\Omega_{\text{radiation}}$; the Hubble parameter, $\hat{h}=H_0/100$; the initial scalar amplitude of the power spectrum, $A_s$; the primordial spectral index, $n_{\text{spec}}$; the two parameters of the dark energy equation of state in the Chevalier-Polarski-Linder parameterisation (Chevallier and Polarski, 2001; Linder, 2003), $w_0$ and $w_\gamma$; the non-Gaussian amplitude, $f_{NL}$; the non-Gaussian shape – local, equilateral, unfolded, orthogonal (Fedeli et al., 2011); the model used to compute distances (used only for some specific interacting dark energy models, see Marulli et al. (2012a); a variable called unit, used to choose between physical units or cosmological units (that is in unit of $h$). If the above parameters are not specified when creating an object of this class, default values from Planck cosmology will be used (Planck Collaboration et al., 2014). In any case, each cosmological parameter can be set individually, when required.

Once the cosmological model has been chosen by setting the parameters described above, a large set of cosmological functions can then be used. We provide here a brief overview of the main functions of the class. The full explanation of the whole set of class members can be found in the documentation at the CBL webpage.

Several functions are available to estimate the redshift evolution of all the relevant cosmological parameters, to compute the lookback and cosmic times, to estimate cosmological distances and volumes, and to convert redshifts into comoving distances and viceversa.

There are methods to estimate the number density and mass function of dark matter haloes. Specifically, the code implements the following equation (see e.g. Marulli et al., 2011):

$$\frac{MdM}{\bar{\rho}} \frac{dn(M,z)}{dM} = \xi f(\xi) \frac{dc}{\zeta},$$

with $\zeta \equiv [\delta_m(z)/\sigma(M)]^2$, where $\delta_m(z)$ is the overdensity required for spherical collapse at $z$, $\bar{\rho} = \Omega_{\text{matter}}\rho_c$, $\rho_c$ is the critical density of the Universe, and $dn(M,z)$ is the halo number density in the mass interval $M$ to $M + dM$. The variance of the linear density field is given by

$$\sigma^2(M) = \int dk \frac{k^2 P_{\text{lin}}(k)}{2\pi^2} [W(kR)]^2,$$

where the top-hat window function is $W(R) = (3/4\pi^2)(3M/4\pi \bar{\rho})^{1/3}$. At the moment, the implemented mass function models are the following: Press and Schechter (1974); Sheth and Tormen (1999); Jenkins et al. (2001); Warren et al. (2006); Shen et al. (2006); Reed et al. (2007); Pan (2007); Tinker et al. (2008); Angulo et al. (2012).

Methods to estimate the effective linear bias of dark matter haloes are provided as well. The effective bias is computed through the following integral:

$$b(z) = \frac{\int_{M_{\text{min}}}^{M_{\text{max}}} n(M,z)b(M,z)dM}{\int_{M_{\text{min}}}^{M_{\text{max}}} n(M,z)dM},$$

where $b(M,z)$ is the linear bias and $n(M,z)$ is the halo number density. The available parameterisations are: Sheth and Tormen (1999); Sheth et al. (2001); Tinker et al. (2010).

A large set of functions is provided to estimate the real-space and redshift-space power spectra and 2PCF (see §Appendix B.1), and to assess the cosmic mass accretion history (Giocoli et al., 2013). To estimate the dark matter power spectrum and all the derived quantities, such as the mass variance used to compute the mass function and bias, the user can choose between one of the following external codes: CAMB (Lewis et al., 2000), MPTbreeze (Crocce et al., 2012), CLASS (Lesgourgues, 2011; Blas et al., 2011), Eisenstein&Hu code (Eisenstein and Hu, 1998, 1999). The latter will be exploited a posteriori by the CBL via specific functions used to set the parameter files conveniently.

3. Catalogues

The class cosmobl::Catalogue is used to handle samples of astronomical objects. The present version of the CBL provides specific classes for galaxies, clusters of galaxies, dark matter haloes and generic mock objects. However, the code structure is sufficiently versatile to easily include new objects or to extend the present ones, e.g. by adding new properties. Once the catalogue is created, several operations can be performed, such as estimating the distribution of any property of the object members, dividing the catalogues in sub-samples, or creating a smoothed version of the original catalogue. Moreover, a catalogue can be passed to other objects as an input, e.g. to estimate 2PCF and 3PCF (see §4.1), or to assess errors through the jackknife or bootstrap techniques (see §4.2). Catalogues can also be added together, or they can be enlarged by adding new single objects.

For a fast spatial search of objects in the catalogues, we implemented a highly optimised chain-mesh method, specifically designed for counting object pairs and triplets in a specified range of scales. The algorithm implements a pixelization...
scheme, similar to the one described in Alonso (2012). First, the catalogue is divided into cubic cells, and the indexes of all the objects in each cell are stored in vectors. Then, to find all the objects close to a given one, the search is performed only on the cells in the chosen scale range, thus minimizing the amount of useless counts of objects at too large separations. In this way, the efficiency of the method depends primarily on the ratio between the scale range of the searching region and the maximum separation between the objects in the catalogue. This is particularly useful when measuring 2PCF and 3PCF (see §4). For alternative searching algorithms, such as kd-tree and ball-tree methods, see e.g. Jarvis (2015).

The chain-mesh method is implemented in the four classes: cosmobl::ChainMesh, cosmobl::ChainMesh1D, cosmobl::ChainMesh2D and cosmobl::ChainMesh3D, designed to handle chains in 1, 2 and 3 dimensions. An example that shows how to create and use objects of these classes is provided in Appendix B.2 and at the CBL webpage.

4. Clustering

One of the main focuses of the CBL is to provide functions to measure and model the clustering properties of astronomical sources. In this section, we present a general description of the main features of the current version of CBL methods for clustering analyses.

4.1. Measurements

Two CBL classes, cosmobl::TwoPointCorrelation and cosmobl::ThreePointCorrelation, can be used to measure 2PCF and 3PCF, respectively. The 2PCF, $\xi(r)$, is implicitly defined as $dP_{12} = n^2[1 + \xi(r)]dV_1dV_2$, where $n$ is the average number density, and $dP_{12}$ is the probability of finding a pair with one object in the volume $dV_1$ and the other one in the volume $dV_2$, separated by a comoving distance $r$. To estimate this function, the CBL provide an implementation of the Landy and Szalay (1993) estimator:

$$\xi(r) = \frac{DD(r) + RR(r) - 2DR(r)}{RR(r)},$$

(4)

where $DD$, $RR$ and $DR$ are the data-data, random-random and data-random normalised pair counts, respectively, for a separation bin $r \pm dr/2$. The following types of clustering functions can be computed:

- the angle-averaged 2PCF, $\xi(r)$;
- the 2D 2PCF in both Cartesian and polar coordinates, $\xi(r_p, \pi)$ and $\xi(r, \mu)$, that is as a function of perpendicular, $r_p$, and parallel, $\pi$, line-of-sight separations, and as a function of absolute separation, $r = \sqrt{r_p^2 + \pi^2}$; and the cosine of the angle between the separation vector and the line of sight, $\mu \equiv \cos \theta \equiv r_1/r$, respectively;
- the projected 2PCF:

$$w(r_p) = 2 \int_0^{r_{\text{max}}} dr' \xi(r_p, r'),$$

(5)

- the deprojected 2PCF:

$$\xi(r) = \frac{1}{\pi} \int_0^{r_{\text{max}}} dr' \frac{dP_{12}(r')/dr_p}{\sqrt{r_p^2 - r^2}};$$

(6)

- the angular 2PCF, $w(\theta)$, where $\theta$ is the angular separation;
- the multipole moments of the 2PCF:

$$\xi(r) = \frac{2l + 1}{2} \int_0^{\pi} \xi(x, \mu) L_l(\mu) d\mu ,$$

(7)

where $L_l(\mu)$ is the Legendre polynomial of order $l$;
- the clustering wedges (Kazin et al., 2012):

$$\xi(r, \Delta \mu) \equiv \frac{\int_{r_{\text{max}}}^{r_{\text{max}}} dr' \xi(r, \mu')}{\int_{r_{\text{max}}}^{r_{\text{max}}} dr'};$$

(8)

- the filtered correlation function (Xu et al., 2010; Cervantes et al., 2012):

$$\omega_0(r_x) = 4\pi \int_0^r dr \left( \frac{r}{r_x} \right)^2 \xi(r) W(r, r_x),$$

(9)

where the filter is:

$$W(x) = \begin{cases} 4x^2(1-x)(\frac{1}{2} - x) & 0 < x < 1, \\ 0 & \text{otherwise}; \end{cases}$$

(10)

and $x \equiv (r/r_x)^3$.

The CBL provide also methods both to construct random catalogues with different geometries and to read them from files, in case they have been already computed. Specifically, there are functions for both cubic and conic geometries, in order to construct random catalogues both for cubic simulation snapshots, and for mock or real catalogues in light-cones.

Analogously to the 2PCF, the 3PCF, $\zeta(r_{12}, r_{23}, r_{31})$, is defined as $dP_{123} = n^3[1 + \zeta(r_{12}) + \xi(r_{23}) + \xi(r_{31}) + \zeta(r_{12}, r_{23}, r_{31})]dV_1dV_2dV_3$, where $n$ is the average density of objects, and $V_i$ are comoving volumes. It is calculated using the Szapudi and Szalay (1998) estimator:

$$\zeta(r_{12}, r_{23}, r_{31}) = \frac{DDD - 3DDR + 3DRR - RRR}{RRR},$$

(11)

where $DDD$, $RRR$, $DDR$, and $DRR$ are the normalised numbers of data triplets, random triplets, data-data-random triplets, and data-random-random triplets, respectively. The algorithm fixes two sides of the triangles and varies the angle, $\theta$, between them. The 3PCF can be measured both in Cartesian coordinates, $\zeta(r_{12}, r_{23}, r_{31})$, and as a function of the angle between the triangle sides, $\xi(\theta)$. Also the reduced 3PCF, $Q(r_{12}, r_{13}, \theta)$, can be computed. The latter is defined as follows:

$$Q = \frac{\zeta(r_{12}, r_{23}, \theta)}{\zeta(r_{12})\xi(r_{23}) + \xi(r_{23})\xi(r_{31}) + \xi(r_{31})\xi(r_{13})}.$$  

(12)
The minimum and maximum separations used to count pairs and triplets and the binning size are free parameters that can be set by the user.

The algorithms to measure the above clustering functions use the chain-mesh method described in §3. The code exploits also multithreaded parallelism. Specifically, all the loops to count the number of object pairs and triplets are parallelized via OpenMP\(^3\). The code performances scale almost linearly with the number of threads.

All the operations related to pair and triplet counts are implemented in the classes cosmobl::Pairs2D, cosmobl::Pairs3D, cosmobl::Triplets, cosmobl::Triplets2D and cosmobl::Triplets3D. These functions have been deeply tested with both simulated catalogues (Marulli et al., 2011, 2012a,b, 2015; Villaescusa-Navarro et al., 2014; Moresco et al., 2014; Petracca et al., 2015), and real catalogues of galaxies (Marulli et al., 2013; Moresco et al., in preparation) and galaxy clusters (Veropalumbo et al., 2014, 2015; Sereno et al., 2015). Appendix B.2 provides an example that shows how to create a random catalogue and measure the 2PCF. Further examples can be found at the CBL webpage.

4.2. Errors

To estimate the errors in 2PCF measurements, the CBL provide specific functions to estimate the covariance matrix defined as follows:

\[
C_{i,j} = \mathcal{F} \sum_{k=1}^{N} (\hat{\xi}_i^k - \bar{\xi}_i)(\hat{\xi}_j^k - \bar{\xi}_j),
\]

where the indexes \(i\) and \(j\) run over the spatial bins of the 2PCF, the index \(k\) refers to the 2PCF of the \(k\)th realisation, \(\bar{\xi}\) is the mean 2PCF over all the \(N\) realisations. The factor \(\mathcal{F}\) is equal to either \((N-1)/N\) or \(1/N\), in case of jackknife or bootstrap errors, respectively. By definition, the diagonal elements of this matrix are the variance of the \(i\)-th spatial bin: \(\sigma_i^2\). The covariance matrix can be estimated with three alternative methods (see e.g. Norberg et al., 2009):

- **analytic errors**: 2PCF errors can be estimated analytically, assuming Poisson statistics. The CBL contain functions to compute analytic errors used to set the diagonal elements of \(C_{i,j}\);
- **internal errors**: the CBL provide functions to estimate errors by sub-sampling the data catalogue and measuring the 2PCF for all but one region – jackknife, or for a random extraction of regions – bootstrap. The volume can be partitioned either in cubic sub-regions, useful e.g. when analysing simulation snapshots, or in sub-regions of generic geometry using the external software MANGLE to reconstruct the angular mask (Swanson et al., 2008);
- **external errors**: the CBL class cosmobl::LogNormal can be used to generate lognormal mock catalogues (Coles and Jones, 1991), with a specified power spectrum, from which the covariance matrix can be estimated.

\(^3\)http://openmp.org/wp/

Analogous methods for the 3PCF will be included in a forthcoming version of the CBL.

4.3. Models

The modelling of the 2PCF is managed by the class cosmobl::ModelTwoPointCorrelation, which provides methods to model all the two-point statistics described in §4.1. The following sections present an overview of the available facilities.

4.3.1. The angle-averaged 2PCF

The angle-averaged 2PCF of cosmic tracers, \(\xi(r)\), can be modelled both in real space and in redshift space. In real space, we implement the simple model:

\[
\xi(r) = b^2 \xi_{DM}(ar),
\]

where \(\xi(r)\) is the model 2PCF of the tracers, \(\xi_{DM}(r)\) is the dark matter 2PCF (see §2), \(b\) is the linear bias of dark matter haloes or galaxies, and \(a\) is the shift parameter defined as:

\[
a \equiv \frac{D_V(r)}{r_s(D_V)},
\]

where \(D_V\) is the isotropic volume distance and \(r_s\) is the position of the sound horizon at decoupling. This quantity is used when exploiting the standard ruler technique in case of baryon acoustic oscillations (BAO) fitting (Veropalumbo et al., 2014).

To model the angle-averaged 2PCF in redshift space, we compute the Fourier anti-transform of the redshift-space power spectrum, that can be written as follows:

\[
P(k,\mu) = P_{DM}(k) \left( b^2 + f \mu^2 \right)^2 \exp \left( -k^2 \mu^2 \sigma^2 \right),
\]

where \(P_{DM}(k)\) is the dark matter power spectrum (see §2), \(f\) is the linear growth rate of cosmic structures, and \(\sigma\) is a damping scale term introduced to describe the effect of Gaussian redshift errors. Specifically, the relation between \(\sigma\) and the redshift error \(\sigma_z\) is:

\[
\sigma = \frac{c \sigma_z}{H(z)},
\]

where \(c\) is the speed of light and \(H(z)\) is the Hubble function at redshift \(z\). Ignoring the non-linear damping term, the angle-averaged 2PCF model reads:

\[
\xi(s) = \left[ b^2 + \frac{2f}{3b} + \frac{1}{5} \left( \frac{f}{b} \right)^2 \right] \cdot \xi_{DM}(as),
\]

while in the most general case it becomes:

\[
\xi(s) = b^2 \xi'(s) + b \xi''(s) + \xi'''(s),
\]

where \(\xi'(s), \xi''(s)\) and \(\xi'''(s)\) are, respectively, the Fourier anti-
transforms of:
\[ P'(k) = P_{\text{DM}}(k) \frac{\sqrt{\pi}}{2k\sigma} \text{erf}(k\sigma); \]
\[ P''(k) = \frac{f}{(k\sigma)^3} P_{\text{DM}}(k) \left[ \frac{\sqrt{\pi}}{2} \text{erf}(k\sigma) - k\sigma \exp(-k^2 \sigma^2) \right]; \]
\[ P'''(k) = \frac{f^2}{(k\sigma)^5} P_{\text{DM}}(k) \left\{ \frac{3\sqrt{\pi}}{8} \text{erf}(k\sigma) \right. \]
\[ - \frac{k\sigma}{4} \left[ 2(k\sigma)^2 + 3 \right] \exp(-k^2 \sigma^2) \}. \]

where \( P'(k), P''(k) \) and \( P'''(k) \) are the terms obtained by integrating Eq. (16) along \( \mu \).

The cosmobl::ModelTwoPointCorrelation class contains also a model for the de-wiggled power spectrum, used to describe non-linear damping effects on BAO. In this case, the dependence on non-linear effects is explicit, via the \( \Sigma_{\text{NL}} \) parameter (Eisenstein et al., 2007). The de-wiggled power spectrum for the dark matter is:

\[ P_{\text{DM}}(k) = [P_{\text{lin}}(k) - P_{\text{nw}}(k)] e^{k^2 \Sigma_{\text{NL}}^2} + P_{\text{nw}}(k), \]

where \( P_{\text{lin}} \) is the linear dark matter power spectrum, \( P_{\text{nw}} \) is the linear dark matter power spectrum without BAO, and \( \Sigma_{\text{NL}} \) describes the non-linear damping effect. Then the 2PCF is computed as the Fourier anti-transform of the power spectrum.

Finally, the CBL provide a simple empirical model used to recover the BAO peak position. It approximates the 2PCF as a combination of a power law and a Gaussian function to reproduce the BAO peak shape (Smith et al., 2008):

\[ \xi(s) = \left( s/s_0 \right)^{-\gamma} + \frac{A}{2\sqrt{2\pi} \sigma} \exp \left( \frac{(s - s_m)^2}{2\sigma^2} \right), \]

where \( s_0 \) and \( \gamma \) are the power-law normalisation and slope, while \( A \), \( \sigma \) and \( r_m \) are the Gaussian normalisation factor, the standard deviation and the mean, respectively.

4.3.2. Projected correlation function

The model for the projected correlation function relies on the assumption that the integration of the 2PCF along the line of sight cancels out redshift-space distortion effects. The model is derived by changing the variable of the integration in Eq. (5) from \( \pi \) to \( r \equiv \sqrt{\pi^2 + r_p^2}; \)

\[ w(r_p) = b^2 \int_0^{r_{\text{max}}} dr' \frac{\xi_{\text{DM}}(r)}{\sqrt{r^2 - r_p^2}}. \]

4.3.3. 2D 2PCF

The CBL provide methods to model the redshift-space 2D 2PCF and its multipole moments. The so-called dispersion model is currently implemented. In the linear regime, the redshift-space 2PCF can be written as follows:

\[ \xi^{\text{lin}}(s, \mu) = \xi_0(s) P_0(\mu) + \xi_2(s) P_2(\mu) + \xi_4(s) P_4(\mu) \],

where \( P_l \) are the Legendre polynomials (Kaiser, 1987). The multipole moments are:

\[ \xi_0(s) = \left[ 1 + \frac{2}{3} \beta + \frac{1}{3} \beta^2 \right] \cdot \xi(r); \]
\[ \xi_2(s) = \left( b_0 \sigma_s \right)^2 + \frac{2}{3} f \sigma_s \cdot b_0 \sigma_s + \frac{1}{5} (f \sigma_s)^2 \cdot \xi_{\text{DM}}(r) \sigma_s^2; \]
\[ \xi_4(s) = \frac{4}{3} f \sigma_s \cdot b_0 \sigma_s + \frac{4}{7} (f \sigma_s)^2 \left[ \frac{\xi_{\text{DM}}(r) \sigma_s}{\sigma_8^2} - \overline{\xi_{\text{DM}}(r)} \right]. \]

where the barred functions are:
\[ \overline{\xi_{\text{DM}}(r)} \equiv \frac{3}{r^2} \int_{0}^{r} dr' \xi_{\text{DM}}(r') r'^2; \]
\[ \overline{\xi_{\text{DM}}(r)} \equiv \frac{5}{r^2} \int_{0}^{r} dr' \xi_{\text{DM}}(r') r'^4. \]

To account for non-linear dynamics, the linearly-distorted correlation function is then convolved with the distribution function of pairwise velocities, \( f(v), \) (Peacock and Dodds, 1996; Peebles, 1980):

\[ \xi(s, \mu) = \frac{1}{H(z)} \left( 1 + z \right) \]

where the pairwise velocity \( v \) is expressed in physical coordinates. Both exponential and Gaussian functions can be used to model the distribution function \( f(v) \) (see e.g. Marulli et al., 2012b).

More accurate models for redshift-space distortions will be included in a forthcoming version of the CBL.

4.3.4. Other statistics

The CBL provide methods to model both the wedges of 2D 2PCF and the filtered 2PCF described in §4. This can be done by assuming a model for the redshift-space distorted \( \xi(s, \mu) \) (see §4.3.3), and then using Eqs.(8)-(9). Methods to model the angular 2PCF and the 3PCF are not yet available and will be added in a future version of the CBL.

5. Tools for statistical analyses

We implemented generic classes to model measured quantities and derive cosmological constraints. Input data and models are fully customizable. The implementation is done in the classes: cosmobl::Data, cosmobl::Model and
cosmobl::Parameter. The class cosmobl::Chi2 implements standard minimum $\chi^2$ fitting techniques. The CBL provide also Bayesian inference methods based on the Bayes’ theorem:

$$p(\hat{\theta}|\hat{X}) = \frac{p(\hat{X}|\hat{\theta}) p(\hat{\theta})}{p(\hat{X})},$$

(33)

where $\hat{X}$ are the data, $\hat{\theta}$ are the model parameters, $p(\hat{\theta})$ is the prior probability distribution of the parameters, and $p(\hat{X}|\hat{\theta})$ and $p(\hat{\theta}|\hat{X})$ are the likelihood function and the parameter posterior probability distribution, respectively. The CBL provide methods to perform the Markov Chain Monte Carlo (MCMC) likelihood sampling technique. The latter consists in sampling a target distribution using a correlated random walk: every step is extracted after a trial that depends only on the previous one (Markov process). The steps are collected in chains, that define marginalised posterior probability of the model parameters $p(\hat{\theta}|\hat{X})$. We implemented two MCMC algorithms:

- the Metropolis-Hastings algorithm (Hastings, 1970). It consists in a single-particle sampling of the parameter space. At each step, $t$, the proposed parameter vector, $\hat{\theta}_{\text{prop}}$, is extracted from the distribution $q(\hat{\theta}|\hat{\theta}(t))$, centered on $\hat{\theta}(t)$;
- the stretch-move algorithm (Goodman and Weare, 2010). It represents a multi-particle approach. At each step, $t$, the proposed position $\hat{\theta}_{\text{prop}}^i$ for the $i$-th particle is located on the line connecting $\hat{\theta}(t)$ and $\hat{\theta}_{\text{old}}(t)$, where the latter is randomly extracted from the particle ensemble. This allows an exchange of information between particles in the cloud.

The current version of the CBL implements Gaussian priors, though minor modifications are required to include different parameterisations. Examples of scientific results obtained using the implemented Bayes methods are provided e.g. in Veropalumbo et al. (2014, 2015).

The functions used to handle the likelihood sampling and the parameter posteriors are implemented in the classes cosmobl::Prior, cosmobl::Chain and cosmobl::MCMC.

6. Other functions

In addition to the classes described above, a large set of generic functions are included in the cosmobl namespace. Among them, the set includes: i) functions of generic use, such as to handle errors and warning messages or endian conversions; ii) functions to manipulate vectors and matrices; iii) functions for statistical analyses; iv) functions to calculate distances; v) special functions (e.g. Legendre polynomials). A full documentation can be found at the CBL webpage.

7. Conclusions

We presented the CosmoBolognaLib, a large set of publicly available C++ libraries for cosmological calculations. This represents a living project with the primary goal of defining a common numerical environment for cosmological investigations of the large-scale structure of the Universe. These libraries provide several classes and methods that can be used to handle catalogues of extragalactic sources, measure statistical quantities, such as 2PCF and 3PCF, and derive cosmological constraints. The doxygen documentation is provided at the same webpage where the libraries can be downloaded, together with a set of sample codes that show how to use this software in either C++ or Python codes.

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Appendix A. How to compile

To compile the CBL, the following steps have to be followed:

1. if not already present in the system, install the following external libraries: GSL (GNU Scientific Library)\(^6\), FFTW\(^7\), OpenMP\(^8\) and Numerical Recipes (Third Edition)\(^9\);
2. download the CosmoBolognaLib.tar archive\(^10\) and unpack all the files in a folder called CosmoBolognaLib/;
3. enter the CosmoBolognaLib/ folder and type: make

In this way, the full set of libraries will be compiled, using the GNU project g++ compiler. Other Makefile options are the following:

- make lib* → compile the * library (e.g. make libFUNC will compile the library libFUNC.so)
- make python → compile the Python wrappers
- make clean → remove all the object files that have been already compiled
- make purge → make clean + remove all the library files (i.e. *.so)
- make purgeALL → make purge + remove all the files stored for internal calculations
- make CAMB → compile the external software CAMB
- make CLASS → compile the external software CLASS
- make CLASSpy → compile the Python wrapper for the external software CLASS

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\(^6\) http://www.gnu.org/software/gsl
\(^7\) http://www.fftw.org
\(^8\) http://openmp.org/
\(^9\) http://www.nr.com
\(^10\) http://apps.difa.unibo.it/files/people/federico.marulli3/


**make MPTbreeze → compile the external software MPTbreeze**

To compile on MAC OS, the command to be used is: **make SYSTM='MAC'**. If needed, both the compiler and the compilation flags can be specified by modifying the variables C and FLAGS defined in the Makefile, e.g. typing: **make C=icc Flags=-O0 -g**.

**Appendix B. Examples**

In this section, we provide some key examples that can help the user to understand the main functionalities of the CBL. Many other example codes are provided at the CBL webpage.

**Appendix B.1. Cosmology**

The following code shows how to use the class cosmobl::Cosmology to estimate the value of $f(r)$ at $z = 1$. When the object is created, at line 13, default cosmological parameters are used. Alternatively, the cosmological parameters can be set directly by using a different constructor, as shown in the example code provided at the CBL webpage.

```
int main () {
    // create an object of class Cosmology
    cosmobl::Cosmology cosm;
    // the current directory
    const string pdir = "C:\\Users\\user\\Documents\\CosmoBolognaLib\\cosmobl\\Cosmology.h"
    // include the header file of libCOSM.so
    #include "Cosmology.h"
    // the CosmoBolognaLib directory
    const string cosmobil::par::DirCosmo = DIRCOSMO;
    // create the object of class RandomCatalogue
    random_catalogue_box(cat, ran);
    // the current directory
    const string cosmobil::par::DirLoc = DIRL;
    // read the coordinates
    ifstream fin(cosmobl::par::DirLoc+"cat.dat");
    // create an object of class TwoPointCorrelation
    TwoP (cat, ran);
    // create the object of class Catalogue
    shared_ptr<cosmobl::Catalogue> cat(cosmobl::par::DirLoc+"cat.dat");
    // create the object of class TwoPointCorrelation
    shared_ptr<cosmobl::TwoPointCorrelation> TwoP (cat, ran);
    // set the redshift
    double z = 1.0;
    // set the external code used to estimate $f(r)$
    string method = "CAMB";
    // print the value of $f(z)\cdot\sigma_8(z)$
    cout << cosm.fsigma8(z, method) << endl;
    return 0;
}
```

Listing 1: Example of how to use the class cosmobl::Cosmology.

**Appendix B.2. The 2PCF**

The following code illustrates how to handle a catalogue of galaxies and measure the 2PCF. Firstly, a vector of objects of class cosmobl::Galaxy is created (lines 21-27). Then, two objects of class cosmobl::Catalogue are created, one containing the galaxies (line 30), and the other containing objects randomly distributed in the same box (lines 32-35). Finally, the 2PCF is computed by exploiting the methods of the class cosmobl::TwoPointCorrelation (lines 52-64).

```
int main () {
    // create an object of class Cosmology
    cosmobl::Cosmology cosm;
    // the current directory
    const string cosmobil::par::DirCosmo = DIRCOSMO;
    // create the object of class Cosmology
    cosmobl::Cosmology cosm;
    // the current directory
    const string cosmobil::par::DirLoc = DIRL;
    // read the coordinates
    ifstream fin(cosmobl::par::DirLoc+"cat.dat");
    // create an object of class TwoPointCorrelation
    TwoP (cat, ran);
    // create the object of class Catalogue
    shared_ptr<cosmobl::Catalogue> cat(cosmobl::par::DirLoc+"cat.dat");
    // create the object of class TwoPointCorrelation
    shared_ptr<cosmobl::TwoPointCorrelation> TwoP (cat, ran);
    // set the redshift
    double z = 1.0;
    // set the external code used to estimate $f(r)$
    string method = "CAMB";
    // print the value of $f(z)\cdot\sigma_8(z)$
    cout << cosm.fsigma8(z, method) << endl;
    return 0;
}
```

Listing 2: Example of how to measure the two-point correlation function.

**Appendix B.3. Python**

This last example shows how to import the CBL as a module in a Python code. Specifically, the following code computes the comoving distance at $z = 1$. Only a subset of the CBL has currently a Python wrapper. We plan to extend this to all the CBL functions in a future version of the libraries. For now, this example is just meant to illustrate the potentiality of this approach.

```
# import the module Cosmology of CosmoBolognaLib
from CosmoBolognaLib import Cosmology
# set the cosmology, using default parameters
```
cosmology

set the redshift

z = 1

calculate the comoving distance
dc = cosmo.D_C(z)

Listing 3: Example of how to use the CosmoBolognaLib in Python.

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Cosmology

d = cosmo.D_C(z)

Listing 3: Example of how to use the CosmoBolognaLib in Python.

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Cosmology

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Listing 3: Example of how to use the CosmoBolognaLib in Python.

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