TeV scale mirage mediation
in NMSSM

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Abstract

We study the next-to-minimal supersymmetric standard model. We consider soft
supersymmetry breaking parameters, which are induced by the mirage mediation
mechanism of supersymmetry breaking. We concentrate on the mirage mediation,
where the so-called mirage scale is the TeV scale. In this scenario, we can realize
the up-type Higgs soft mass of $\mathcal{O}(200)$ GeV, while other masses such as gaugino
masses and stop masses are heavy such as 1 TeV or more. Cancellation between the
effective $\mu$-term and the down-type Higgs soft mass ameliorates the fine-tuning in
the electroweak symmetry breaking even for $\mu = \mathcal{O}(500)$ GeV. The mixing between
the doublet and singlet Higgs bosons is suppressed by $(\lambda/\kappa) \tan^{-1} \beta$. Then the
lightest doublet Higgs mass naturally reaches 125 GeV lifted by the new quartic
coupling. The higgsino and singlino are light and their linear combination is the
lightest superparticle.
1 Introduction

Supersymmetric extension is a good candidate for physics beyond the standard model (SM). The minimal supersymmetric standard model (MSSM) is the simplest extension. The MSSM is quite interesting because of its minimality and its detailed studies have been done for several aspects.

However, the MSSM has the fine-tuning problem. Within the framework of the MSSM, the $Z$-boson mass, $m_Z$, is obtained as

$$\frac{m_Z^2}{2} \approx -m_{H_u}^2 - |\mu|^2,$$

(1.1)

where $m_{H_u}^2$ is the soft supersymmetry (SUSY) breaking scalar mass squared of the up-sector Higgs field and $\mu$ is the supersymmetric mass. The radiative corrections on $m_{H_u}^2$ are obtained as $m_{H_u}^2 \sim -m_t^2 \sim -M_3^2$, where $m_t$ and $M_3$ denote the stop and gluino masses, respectively. In most of cases, the stop and the gluino masses are much larger than $m_Z$. Thus, we need fine-tuning between $m_{H_u}^2$ and $|\mu|^2$ to realize the correct value of $m_Z$. Furthermore, it is required that $m_t = \mathcal{O}(1)$ TeV or larger in order to obtain the Higgs mass such as $m_h \approx 125$ GeV which is recently reported by ATLAS and CMS collaborations[2, 3].

The mirage mediation is one of the interesting mediation mechanisms of SUSY breaking [4, 5, 6]. The mirage mediation is a mixture of the modulus mediation [7] and the anomaly mediation [8] with a certain ratio. In particular, it was pointed out that the TeV-scale mirage mediation can ameliorate the above fine-tuning problem of the MSSM [9, 10, 11]. In the TeV-scale mirage mediation, the above radiative corrections on $m_{H_u}^2$ and the anomaly mediation contributions are canceled each other. Then, the value of $|m_{H_u}^2|$ at the electroweak scale can be smaller than stop and gluino masses. The TeV scale mirage mediation also leads several phenomenologically interesting aspects [12] because its SUSY particle spectrum is quite compressed.

The next-to-minimal supersymmetric standard model (NMSSM) is the extension of the MSSM by adding a singlet $S$ [13] (see for review e.g. [14]). Here, we also impose the $Z_3$ symmetry. The NMSSM does not have the $\mu$-term, $\mu H_u H_d$, in the superpotential, where $H_u$ and $H_d$ denote the up and the down-sector Higgs superfields, respectively. On the other hand, the term $\lambda S H_u H_d$ is allowed in the NMSSM superpotential. After $S$ develops its vacuum expectation value (VEV), the effective $\mu$-term is generated. That gives us a solution for the so-called $\mu$-problem [15]. The NMSSM is also interesting in the light of the recent indication of the relatively heavy Higgs boson reported by ATLAS and CMS, endowed with an additional Higgs self-coupling. In the Higgs sector, the doublet Higgs and singlet fields mix each other. Then, the Higgs sector in the NMSSM has a quite rich structure.

The NMSSM also leads the same relation as (1.1), and the NMSSM has the fine-tuning problem similar to the one in the MSSM. Thus, it is interesting to apply the TeV-scale mirage mediation to the NMSSM. In this paper, we study the NMSSM with the soft SUSY breaking terms induced through the TeV-scale mirage mediation [1].
This paper is organized as follows. In section 2, we give a brief review on the mirage mediation, and the TeV scale mirage mediation. In section 3, we apply the TeV scale mirage mediation to the NMSSM, and study its spectrum. Section 4 is devoted to conclusion and discussion. In Appendix A, we show explicitly initial conditions of soft parameters, which are induced through the mirage mediation in the NMSSM.

2 TeV-scale mirage mediation

Here, we review briefly the mirage mediation [4]. The mirage mediation is the mixture between the modulus mediation and the anomaly mediation. Then, the gaugino masses are obtained as

\[ M_a = M_0 + \frac{m_{3/2}}{8\pi^2} b_a g_a^2, \]

(2.1)

where \( g_a \) and \( b_a \) are the gauge couplings and their \( \beta \) function coefficients, and \( m_{3/2} \) denotes the gravitino mass. The first and the second terms in the right-hand side of Eq. (2.1) correspond to the contributions due to the modulus mediation and the anomaly mediation, respectively.

Similarly, we obtain the so-called \( A \)-terms corresponding to the Yukawa couplings, \( y_{ijk} \), and the soft scalar masses \( m_i \) as

\[ A_{ijk}(M_{\text{GUT}}) = a_{ijk}M_0 - (\gamma_i + \gamma_j + \gamma_k)\frac{m_{3/2}}{8\pi^2}, \]

\[ m_i^2(M_{\text{GUT}}) = c_iM_0^2 - \gamma_i(\frac{m_{3/2}}{8\pi^2})^2 - \frac{m_{3/2}}{8\pi^2}M_0\theta_i, \]

(2.2)

where

\[ \gamma_i = 2 \sum_a g_a^2 C_2^a(\phi^i) - \frac{1}{2} \sum_{jk} |y_{ijk}|^2, \]

\[ \theta_i = 4 \sum_a g_a^2 C_2^a(\phi^i) - \sum_{jk} a_{ijk}|y_{ijk}|^2, \]

\[ \dot{\gamma}_i = 8\pi^2 \frac{d\gamma_i}{d\ln\mu_R}. \]

(2.3)

Here, \( C_2^a(\phi^i) \) denotes the quadratic Casimir corresponding to the representation of the matter field \( \phi^i \). In addition, \( a_{ijk} \) and \( c_i \) parametrize the A-term and the scalar mass squared generated through the modulus mediation in the unit of the universal gaugino mass, \( M_0 \). These coefficients are determined by modulus-dependence of the Kähler metric as well as the Yukawa coupling. One can write \( c_i \) as

\[ c_i = c_i^{\text{(tree)}} + \delta c_i^{\text{(loop)}}. \]

(2.4)

Here, \( c_i^{\text{(tree)}} \) is calculated from the tree-level Kähler metric of the matter field \( \phi^i \) and they are ratios of small integers including 0 and 1 [7, 10, 13]. In addition, \( \delta c_i^{\text{(loop)}} \) is obtained
with the one-loop Kähler metric of the matter field, but such a loop correction to the Kähler metric depends on the detail of the ultraviolet-model and is hard to calculate (see e.g. Ref. [17]). That is, $c_i$ is ambiguous at the one-loop level, although such ambiguity is subdominant and less important in most of cases. Here, we consider the case with

$$a_{ijk} = c_i + c_j + c_k.$$  \hspace{1cm} (2.5) 

We input the values of $c_i$ at $M_{\text{GUT}} = 2 \times 10^{16}$ GeV.

It is convenient to use the following parameter \cite{5},

$$\alpha \equiv \frac{m_{3/2}}{M_0 \ln(M_{pl}/m_{3/2})},$$  \hspace{1cm} (2.6) 

to represent the ratio of the anomaly mediation to the modulus mediation. Here $M_{pl}$ is the reduced Planck scale.

One of the interesting aspects in the mirage mediation is that the above spectrum (2.1) and (2.2) has a special energy scale, that is, the mirage scale,

$$M_{\text{mir}} = \frac{M_{\text{GUT}}}{(M_{pl}/m_{3/2})^{\alpha/2}}.$$  \hspace{1cm} (2.7) 

At this scale, the gaugino masses are obtained as \cite{5},

$$M_a(M_{\text{mir}}) = M_0.$$  \hspace{1cm} (2.8) 

That is, the anomaly mediation contribution and the radiative corrections cancel each other, and the pure modulus mediation appears at the mirage scale. Furthermore, the $A$-terms and the scalar masses squared also satisfy\footnote{The scaler masses at the Mirage scale can be modified due to the U(1) tadpole contribution in the renormalization group running when the different values of $c_{H_u}$ and $c_{H_d}$ are chosen. However, such a modification is small and can be included in ambiguities of $c_i$ if couplings are small. See \cite{5} for detailed discussions. We include this contribution in our numerical analysis.}

$$A_{ijk}(M_{\text{mir}}) = (c_i + c_j + c_k)M_0, \quad m_i^2(M_{\text{mir}}) = c_iM_0^2,$$  \hspace{1cm} (2.9) 

if the corresponding Yukawa couplings are small enough or if the following conditions are satisfied,

$$a_{ijk} = c_i + c_j + c_k = 1,$$  \hspace{1cm} (2.10) 

for non-vanishing Yukawa couplings, $y_{ijk}$ \cite{5}.

When $\alpha = 2$, the mirage scale $M_{\text{mir}}$ is around 1 TeV. Then, the above spectrum (2.8) and (2.9) is obtained at the TeV scale. That is the TeV scale mirage mediation scenario. In particular, there would appear a large gap between $M_0$ and the scalar mass $m_i$ with $c_i \approx 0$. We will apply the TeV scale mirage scenario to the NMSSM in the next section.

In the TeV scale mirage scenario, the stop mass squared becomes negative at high energy \cite{18}, while it is positive at low energy below $10^6$ GeV. Thus, the vacuum which
breaks the electroweak symmetry at the electroweak scale might be a local minimum, but instead there would be a color and/or charge breaking vacuum with field values larger than $10^6$ GeV. Here, we assume the thermal history of the Universe such that field values remain around the origin until the temperature reaches the electroweak scale. In addition, we need to confirm that the tunnelling rate is small enough, i.e. less than the Hubble expansion rate. In Refs. [19], it has been shown that such a rate is small enough, as long as the squark/slepton masses squared are vanishing or positive around $10^4$ GeV. This condition is satisfied in our TeV scale mirage mediation scenario.

3 TeV scale mirage in NMSSM

In this section, we apply the TeV scale mirage mediation scenario to the NMSSM.

3.1 NMSSM

Here, we briefly review on the NMSSM, in particular its Higgs sector before we apply the TeV scale mirage mediation scenario to the NMSSM. In the NMSSM, we extend the MSSM by adding a singlet chiral multiplet $S$ and imposing a $Z_3$ symmetry. Then, the superpotential of the Higgs sector is written as

$$W_{\text{Higgs}} = -\lambda SH_uH_d + \frac{\kappa}{3}S^3. \quad (3.1)$$

Here and hereafter, for $S$, $H_u$ and $H_d$ we use the convention that the superfield and its lowest component are denoted by the same letter. The full superpotential also includes the Yukawa coupling terms between the matter fields and the Higgs fields, which are the same as those in the MSSM.

The following soft SUSY breaking terms are induced in the Higgs sector,

$$V_{\text{soft}} = m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 - \lambda A\lambda SH_uH_d + \frac{\kappa}{3}A\kappa S^3 + \text{h.c.} \quad (3.2)$$

Then, the scalar potential of the neutral Higgs fields is given as

$$V = \lambda^2|S|^2(|H_d^0|^2 + |H_u^0|^2) + |\kappa S|^2 - \lambda H_d^0H_u^0|^2 + V_D$$
$$+ m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 - \lambda A\lambda SH_uH_d + \frac{\kappa}{3}A\kappa S^3 + \text{h.c.}, \quad (3.3)$$

with

$$V_D = \frac{1}{8}(g_1^2 + g_2^2)(|H_d^0|^2 - |H_u^0|^2)^2, \quad (3.4)$$

where $g_1$ and $g_2$ denote the gauge couplings of U(1)$_Y$ and SU(2).
The minimum of the potential is obtained by analyzing the stationary conditions of the Higgs potential,

\[
\frac{\partial V}{\partial H_d^0} = \lambda v^2 \cos(\beta + v^2 \sin^2 \beta) - \lambda \kappa v^2 \sin \beta + \frac{1}{4} g^2 v^3 \cos \beta \cos 2\beta \\
+ m_{H_d^0}^2 v \cos \beta - \lambda A v \sin \beta = 0, \quad (3.5a)
\]

\[
\frac{\partial V}{\partial H_u^0} = \lambda v^2 \sin(\beta + v^2 \cos^2 \beta) - \lambda \kappa v^2 \cos \beta - \frac{1}{4} g^2 v^3 \sin \beta \cos 2\beta \\
+ m_{H_u^0}^2 v \sin \beta - \lambda A v \cos \beta = 0, \quad (3.5b)
\]

\[
\frac{\partial V}{\partial S} = \lambda v^2 s^2 + 2\kappa^2 v^2 \sin 2\beta + m_S^2 - \frac{1}{2} \lambda A v^2 \sin 2\beta + \kappa A v^2 s^2 = 0, \quad (3.5c)
\]

where \( g^2 = g_1^2 + g_2^2 \). Here, we denote VEVs as

\[
v^2 = \langle|H_d^0|^2 + |H_u^0|^2\rangle, \quad \tan \beta = \frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle}, \quad s = \langle S \rangle. \quad (3.6)
\]

Using the above stationary conditions, we obtain the Z boson mass \( m_Z^2 = \frac{1}{2} g^2 v^2 \) as

\[
m_Z^2 = \frac{1 - \cos 2\beta}{\cos 2\beta} m_{H_u^0}^2 - \frac{1 + \cos 2\beta}{\cos 2\beta} m_{H_d^0}^2 - 2\mu^2, \quad (3.7)
\]

where \( \mu = \lambda s \). For \( \tan \beta \gg 1 \), this equation becomes

\[
m_Z^2 \simeq -2m_{H_u^0}^2 + \frac{2}{\tan^2 \beta} m_{H_d^0}^2 - 2\mu^2. \quad (3.8)
\]

This relation is the same as the one in the MSSM. Indeed, when we neglect the second term in the right-hand side, the above relation is nothing but Eq. (1.1). Thus, the natural values of \( |m_{H_u^0}| \) and \( |\mu| \) would be of \( \mathcal{O}(100) \) GeV. Furthermore, the natural value of \( |m_{H_d^0}|/\tan \beta \) would be of \( \mathcal{O}(100) \) GeV or smaller. Alternatively, \( |\mu| \) and \( |m_{H_u^0}|/\tan \beta \) could be larger than \( \mathcal{O}(100) \) GeV when \( \mu^2 \) and \( m_{H_d^0}^2/\tan^2 \beta \) are canceled each other in the above relation at a certain level. Even in such a case, \( |m_{H_u^0}| \) would be naturally of \( \mathcal{O}(100) \) GeV. On the other hand, other sfermion masses as well as gaugino masses must be heavy as the recent LHC results suggested. To realize such a spectrum, we apply the TeV scale mirage mediation in the next section, where we take \( c_{H_u} = 0 \) to realize a suppressed value of \( |m_{H_u^0}| \) compared with \( M_0 \).

### 3.2 TeV scale mirage mediation in NMSSM

Here, we study the TeV scale mirage mediation scenario in the NMSSM. Soft SUSY breaking terms are obtained through the generic formulas (2.1) and (2.2) with taking \( \alpha = 2 \). For concreteness, we give explicit results of all the soft SUSY breaking terms for the NMSSM in Appendix A. We concentrate on the Higgs sector as well as gauginos and stops.
We consider the following values of \( c_i \),
\[
    c_{H_d}^{(\text{tree})} = 1, \quad c_{H_u}^{(\text{tree})} = 0, \quad c_S^{(\text{tree})} = 0, \quad c_{t_L}^{(\text{tree})} = c_{t_R}^{(\text{tree})} = \frac{1}{2},
\]
for \( H_d, H_u, S \), and left and right-handed \((s)\)top fields, respectively. This is the same assignment as the pattern II in Ref. [11] for the MSSM except for \( c_S \). Then, the soft parameters due to only modulus mediation contribution are given by
\[
    \begin{align*}
        (A_t)^{\text{modulus}} &= (A_\lambda)^{\text{modulus}} = M_0, \quad (A_\kappa)^{\text{modulus}} = 0, \\
        (m_{H_d}^2)^{\text{modulus}} &= M_0^2, \quad (m_{t_L}^2)^{\text{modulus}} = (m_{t_R}^2)^{\text{modulus}} = \frac{1}{2}M_0^2, \\
        (m_{H_u}^2)^{\text{modulus}} &= (m_S^2)^{\text{modulus}} = 0,
    \end{align*}
\]
when we neglect \( \delta c_i^{(\text{loop})} \). The above assignment of \( c_i \) satisfies the condition, (2.10) for the top Yukawa coupling and the coupling \( \lambda \), but not for the coupling \( \kappa \). However, we do not consider a large value of \( \kappa \) to avoid the blow-up of \( \kappa \) and \( \lambda \) as will be shown later. Thus, we obtain the following values,
\[
    A_t \approx A_\lambda \approx M_0, \quad m_{H_d}^2 \approx M_0^2, \quad m_{t_L}^2 \approx m_{t_R}^2 \approx \frac{1}{2}M_0^2, \quad (3.11)
\]
up to \( \mathcal{O}(\kappa^2/8\pi^2) \) at the TeV scale. Note that \( \delta c_i^{(\text{loop})} \) has negligible effects for these values.

Similarly, we can obtain the values of \( A_\kappa, |m_{H_u}| \) and \( |m_s| \) at the TeV scale, however those are suppressed compared with \( M_0 \). For such suppressed values, sub-leading corrections e.g. the one-loop correction on the Kähler metric are not negligible anymore. That introduces one-loop ambiguity into the model. Including such corrections, at the TeV scale we obtain
\[
    m_{H_u}^2 \approx \delta c_{H_u}^{(\text{loop})}M_0^2, \quad m_S^2 \approx \delta c_S^{(\text{loop})}M_0^2, \quad (3.12)
\]
with \( \delta c_{H_u}^{(\text{loop})}, \delta c_S^{(\text{loop})} = \mathcal{O}(1/8\pi^2) \). Note that similar to Eq.(3.11), Eq.(3.12) also includes corrections of \( \mathcal{O}(\kappa^2M_0^2/8\pi^2) \) due to the violation of the mirage unification by \( \kappa \). That is, we obtain \( m_{H_u}^2 = 0, m_S^2 = 0 \) up to \( \mathcal{O}(M_0^2/8\pi^2) \) at the TeV scale. Similarly, at the TeV scale we can obtain,
\[
    A_\kappa = 0, \quad (3.13)
\]
up to \( \mathcal{O}(M_0/8\pi^2) \). Because of such ambiguity, we use \( A_\kappa \) as a free parameter, which must be small compared with \( M_0 \). In addition, we determine the values of \( m_{H_u}^2, m_S^2 \) and \( \mu (=\lambda s) \) at the electroweak scale from the stationary conditions, (3.5), where we use the experimental value \( m_Z = \frac{1}{\sqrt{2}}gv = 91.19 \text{ GeV} \) and \( \tan \beta \) as a free parameter.

Through the above procedure, the parameters, \( m_{H_u}^2, m_S^2 \) and \( \mu \), at the electroweak
scale are expressed by $\tan \beta$, $m_{H_d}^2$, $A_\lambda$ as follows,

$$
\begin{align}
\mu &= \frac{\lambda \tan \beta}{A_\lambda \tan \beta} \left\{ 1 - \sqrt{1 - 4X} \right\}, \\
m_S^2 &= -2 \left( \frac{\kappa}{\lambda} \right)^2 \mu^2 - \left( \frac{\kappa}{\lambda} \right) A_\kappa \mu + \frac{\lambda^2}{g^2} m_Z^2 \left\{ \left( \frac{A_\lambda}{\mu} + 2 \frac{\kappa}{\lambda} \right) \sin 2\beta - 2 \right\}, \\
m_{H_u}^2 &= \frac{\tan^2 \beta - 1}{\tan^2 \beta - \mu^2 - \frac{m_Z^2}{2}} \left( \frac{m_{H_d}^2}{\tan^2 \beta - \mu^2 - \frac{m_Z^2}{2}} \right),
\end{align}
$$

(3.14)

where,

$$
X = \frac{m_{H_d}^2 (1 - \frac{\kappa}{\lambda} \tan \beta)}{A_\lambda \tan^2 \beta} \left( 1 + \frac{\tan^2 \beta}{\tan^2 \beta - 1} \left( \frac{2 \lambda^2}{g^2} - \frac{\tan^2 \beta - 1}{2 \tan^2 \beta} \right) \frac{m_Z^2}{m_{H_d}^2} \right).
$$

(3.15)

For $\tan \beta \gg \max(1, \kappa/\lambda)$, these parameters are approximated as,

$$
\begin{align}
\mu &= \frac{m_{H_d}^2}{A_\lambda \tan \beta}, \\
m_S^2 &\sim -2 \left( \frac{\kappa}{\lambda} \right)^2 \left( \frac{m_{H_d}^2}{A_\lambda \tan \beta} \right)^2 - \left( \frac{\kappa}{\lambda} \right) A_\kappa \left( \frac{m_{H_d}^2}{A_\lambda \tan \beta} \right) + 2 \frac{\lambda^2}{g^2} A_\lambda^2 m_Z^2, \\
m_{H_u}^2 &\sim \frac{m_{H_d}^2}{A_\lambda \tan^2 \beta} - \frac{m_Z^2}{2}.
\end{align}
$$

(3.16a)\text{-(3.16c)}

When $\tan \beta = \mathcal{O}(10)$, the values of $\mu$, $|m_{H_u}|$ and $|m_S|$ are smaller than $M_0$ by the factor $\tan \beta$ because $m_{H_d} \simeq A_\lambda \simeq M_0$. Thus, the values of $\mu$ and $|m_{H_u}|$ could be of $\mathcal{O}(100)$ GeV while the other masses of the superpartners are of $\mathcal{O}(M_0) = \mathcal{O}(1)$ TeV. Then, the fine-tuning problem can be ameliorated. Furthermore, one can see that the first and the second terms in the last equation cancel each other for our choice of $c_i$. The next leading contributions are of $\mathcal{O}(m_{H_d}^2/\tan^4 \beta)$ or $\mathcal{O}(m_{H_d}^2 A_\lambda/\tan^2 \beta)$. Thus, $m_Z^2$ is almost determined by $m_{H_u}^2$ alone and insensitive to the value of $\mu$. This means that actually $\tan \beta \approx 3$ is enough to obtain the fine-tuning of $|\partial \ln m_{H_u}^2/\partial \ln m_{H_d}^2|^{-1} = m_Z^2/(2m_{H_u}^2) = \mathcal{O}(100)$% for $M_0 \approx 1$ TeV. In this case, $\mu$ can be as heavy as $\mathcal{O}(400)$ GeV without deteriorating the fine-tuning. The origin of this cancellation is easily understood by examining the doublet mass matrix,

$$
\mathcal{L}_M = -(H_d, H_u^*) \mathcal{M}_H^2 \begin{pmatrix} H_d^* \\ H_u \end{pmatrix},
$$

(3.17)

where

$$
\mathcal{M}_H^2 = \begin{pmatrix} m_{H_d}^2 + \mu^2 & -A_\lambda \mu \\ -A_\lambda \mu & m_{H_u}^2 + \mu^2 \end{pmatrix} \approx \begin{pmatrix} M_0^2 + \mu^2 & -M_0 \mu \\ -M_0 \mu & \mu^2 \end{pmatrix}.
$$

(3.18)

The modulus mediated contribution $M_0$ cancels in the determinant of the mass matrix, $\det(\mathcal{M}_H^2) \approx \mu^4$. The heavy mode has mass of $\mathcal{O}(M_0)$, then the mass of the light mode
is suppressed as $\mu^2/M_0 \approx \mu/\tan \beta$ and a flat direction appears along $H_u/H_d \approx M_0/\mu \approx \tan \beta$. This mechanism was previously observed in $\mu$ in the context of the MSSM. In the NMSSM, the relation $m_{H_d} \approx A_\lambda$ is well controlled up to the leading contribution of the modulus mediation, in contrast to the $B$-term (in place of $A_\lambda$) in the MSSM, which is a remnant of the fine-tuned cancellation between the terms of $O(m_{3/2})$ and subject to uncontrolled corrections.

In the following section, we show numerically the spectrum of our model.

### 3.3 Spectrum

Here, we study numerically the spectrum of our model. Before showing numerical results, we recall our parameters. In our analysis, the free parameters are $\lambda, \kappa, \tan \beta$ and $A_\kappa$ given at the SUSY scale and $M_0$ given at $M_{\text{GUT}}$. As the SUSY scale, we choose $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}} \approx M_0/\sqrt{2}$. Using them, we determine all of the soft SUSY breaking parameters except for $m_{H_u}^2, m_S^2$ and $\mu$, which are determined by using the stationary condition of the Higgs potential. Note that in the following numerical analysis we include corrections from the 1-loop effective potential,

$$V_{\text{1-loop}} = \frac{1}{64\pi^2} \text{Str} \left[ M^4 \left\{ \ln \left( \frac{M^2}{M_{\text{SUSY}}^2} \right) - \frac{3}{2} \right\} \right],$$

(3.19)

in the stationary conditions (3.5), where $M$ represents the mass matrix of our model and $\text{Str}$ denotes the supertrace. In all of the following numerical analysis, we use $A_\kappa = -100$ GeV as a typical value of $A_\kappa$. When $\lambda$ and/or $\kappa$ are large at the electroweak scale, they blow up below the GUT scale. Thus, we have constrains on large values of $\lambda$ and $\kappa$ by requiring that those do not blow up below the GUT scale.

Figure 1 shows the lightest CP-even Higgs mass $m_{h_1}$, soft scalar masses of $H_u$ and $S$, $m_{H_u}$ and $m_S$, and $\mu$ for $M_0 = 1200$ GeV and $\tan \beta = 3$, in panels (a), (c), (d) and (e), respectively. The panel (b) in the figure shows the coupling squared between the lightest CP-even Higgs and the Z bosons, $g^2_{ZZh_1}$, as the ratio to the one in the SM, i.e. $g^2_{ZZh_1}/g^2_{SM}$. The second lightest CP-even Higgs mass $m_{h_2}$ is also plotted in panel (f).

In the figure, the red curve corresponds to the values of $\lambda$ and $\kappa$ at the electroweak scale, which blow up at the GUT scale. Thus, we exclude the outside of this curve. The gray region around $\kappa = 0$ corresponds to the region where the tachyonic mode appears in the Higgs sector. The yellow region is excluded because the Higgs potential has the false vacua studied in Ref. [20], deeper than the realistic vacuum. From these constraints, the region with small $\kappa/\lambda$ is disfavored. The gray region around the red curve indicates the region where the tree level Higgs mass becomes tachyonic and the iterative procedure we employed does not work to estimate the stationary conditions. The quantum corrections (3.19) could lift the tachyonic mass, however, we do not calculate it because the region has already been excluded by the LEPII bound ($m_{h_1} > 114.4$ GeV).

The value of $\mu$ is around 200–400 GeV, which is consistent with the rough estimation in Eq. (3.16a), i.e. $\mu \sim m_{H_d}^2/(A_\lambda \tan \beta) \sim M_0/\tan \beta$. Obviously the expansion in Eq. (3.16a) becomes worse for $\kappa/\lambda \gtrsim \tan \beta$, while the expansion holds well for $\kappa/\lambda \lesssim 0.3$ where the

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value of $m_{H_u}$ is around 100–200 GeV. In this region the fine-tuning of the parameters for the electroweak symmetry breaking is of $O(10)$%, even though most of the superpartners have heavy masses of $O(M_0)$. A large value of $\mu \simeq 400$ GeV potentially degrades the fine-tuning, however the cancellation renders $\mu$ irrelevant to $m_Z$ at the leading order of the expansion by $1/\tan \beta$.

The value of $|m_S|$ is roughly estimated as $|m_S| \sim (\sqrt{2}\kappa/\lambda)\mu$ in Eq. (3.16b). Thus, it is found that $|m_S| \approx 400$ GeV for $\kappa \approx \lambda$ and $|m_S|$ increases (decreases) as $\kappa/\lambda$ increases (decreases). The value of $|m_S|$ is expected to be suppressed in the TeV scale mirage mediation scenario. However, a large value of $\kappa/\lambda$ leads large $|m_S|$ through the stationary condition like $|m_S| \sim 500$ GeV. Such a large value would not be realized in our TeV scale mirage mediation exactly corresponds to the region where the fine-tuning is ameliorated. Note that the condition (2.10) also holds well for this region.

In Fig. 1.(a), the lightest CP-even Higgs boson, which dominantly consists of the doublet scalar here, has a mass $m_{h_1}$ around 80–125 GeV. We estimated $m_{h_1}$ using NMHDECAY in the NMSSMTools package [21]. We calculated the minimum of the effective potential renormalized at $M_{SUSY}$ by the iteration starting from the tree-level minimum. Then we used the resultant $\mu$ as an input of the NMSSMTools. The pole mass $m_t = 172.9$ GeV was used in our calculation. The qualitative behavior of the mass of the SM-like Higgs boson is given by

$$m_{2SM} \simeq m_Z^2 \cos^2 2\beta + \lambda^2 v^2 2\beta - \frac{\lambda^2}{\kappa^2} v^2 (\lambda - \kappa \sin 2\beta)^2 + \frac{3m_t^4}{4\pi^2 v^2} \left( \ln \left( \frac{m_t^2}{m_t^2} \right) + \frac{A_t^2}{m_t^2} \left( 1 - \frac{A_t^2}{12m_t^2} \right) \right),$$

(3.20)

for $\kappa s \gg |A_{\kappa}|, |A_{\lambda}|$ in [14]. The first term and the fourth term are the tree-level contribution and the radiative correction in the MSSM, respectively. The second term comes from the new quartic Higgs couplings in the NMSSM, and the third term comes from the mixing of the doublet scalars with the singlet. The third term is always negative because the mixing reduces the lightest eigenvalue of the mass matrix. Although the above approximation does not apply in the figure, the behavior of the higgs mass can be understood in terms of the mixing. The effect of the mixing undermines that of the additional quartic coupling $\lambda^2 \sin^2 2\beta$ and suppresses the Higgs mass except for the narrow region $4 \lesssim \lambda/\kappa \lesssim 8$. The LHC observation, $m_{h_1} \approx 125$ GeV, is satisfied only around $(\lambda, \kappa) = (0.7, 0.11)$ where the mixing vanishes. Such a region may be realized as a quasi-infrared fixed point if $\lambda$ has a strong dynamics origin at the GUT scale, while $\kappa$ is suppressed due to the approximate Peccei-Quinn symmetry. It is important to stress again that this region is favored by the TeV scale mirage mediation and the fine-tuning. The mixing of the doublet and the singlet Higgs scalars in the Lagrangian is obtained by rotating the doublet Higgs by
\[ \Delta L = -2v \left[ \mu - (A_\lambda + 2\kappa s) \cos \beta \sin \beta \right] h \Delta S_r, \]  
\[ \Delta L = 4v \frac{\mu}{\tan \beta} \left[ \frac{\kappa}{\lambda} + O \left( \frac{1}{\tan^2 \beta} \right) \right] h \Delta S_r. \]  

Thus the mixing is automatically suppressed by \((\kappa/\lambda)/\tan \beta\) in our scenario.

The coupling of the lightest CP-even Higgs boson to the \(Z\) boson, \(g_{ZZh_1}\), is almost the same as the one in the standard model in most of the parameter space as expected. We show the ratio \(g_{ZZh_1}^2/g_{SM}^2\) in the figure, where \(g_{SM}\) denotes the Higgs coupling to the \(Z\) boson in the standard model. The mixing between the doublet and the singlet is minimized around the \(Z\) boson in the standard model. The mixing between the doublet and the singlet is

\[ \frac{\kappa}{\lambda} \approx \frac{\beta}{\tan \beta} \approx 0.10, \]  

\[ \frac{\kappa}{\lambda} \approx \frac{\beta}{\tan \beta} \approx 0.04. \]

Table 1 shows examples of spectra and \(g_{ZZh_1}^2/g_{SM}^2\) for \((\lambda, \kappa) = (0.10, 0.40), (0.40, 0.10)\) and \((0.70, 0.11)\). In the table, \(m_{h_i}\) for \(i = 1, 2, 3\) and \(m_{\tilde{a}_i}\) for \(i = 1, 2\) denote three CP-even Higgs masses and two CP-odd Higgs masses, respectively. Also, \(m_{\tilde{t}_{i,2}}\) denote two eigenvalues of stop masses. Other squark and slepton masses depend on their values of \(c_i\), and they are of \(O(M_0)\) unless \(c_i = 0\). The second lightest CP-even Higgs boson \(h_2\) is lighter for \((\lambda, \kappa) = (0.40, 0.10)\) and \((0.70, 0.11)\) than the one for \((\lambda, \kappa) = (0.10, 0.40)\). Its dominant component is the singlet Higgs boson \(S\). Its mass decreases as \(\kappa/\lambda\) decreases. Then, the above behavior of \(m_{h_2}\) occurs. On the other hand, the dominant component of the heaviest CP-even Higgs boson \(h_3\) is the down-type Higgs boson \(H_d\). Its mass is heavy and almost equal to \(M_0\), independent of \(\lambda\) and \(\kappa\). By the same reason, the mass of the heavier CP-odd Higgs boson \(a_2\) is almost the same as \(m_{h_3}\) as well as \(M_0\). The lightest CP-odd Higgs boson \(a_1\) is lighter for small \(\kappa\) due to the approximate Peccei-Quinn symmetry. Also, three gaugino masses are almost the same as \(M_0\). It might be challenging but interesting subject to observe these light extra-Higgs bosons through the small mixing with the doublets in LHC and ILC.

Figure 2 shows the same as Fig. 1 except for \(M_0 = 1500\) GeV. Most of mass parameters become larger than those for \(M_0 = 1200\) GeV. The lightest CP-even Higgs mass becomes heavy due to the heavier stop and the portion of the region with \(m_{h_1} \approx 125\) GeV increases. Table 2 is the same as Table 1 except \(M_0 = 1500\) GeV. Obviously, all of masses become heavier than those in Table 1. The behavior of \(m_{h_2}, m_{h_3}\) and \(m_{a_2}\) is the same as the one in Table 1. For completeness, we also plot the figure for \(M_0 = 1700\) GeV in Fig. 3 and list the spectra and the coupling for \(M_0 = 1700\) GeV in Table. 3.

Figure 4 shows the same as Fig. 1 except for \(\tan \beta = 5\). The approximation in Eq. (3.16) and the cancellation work well for \(\kappa/\lambda \lesssim 5\). The up-type Higgs mass \(|m_{H_u}|\) is around \(100 - 250\) GeV in most of the parameter space, while the region, \(\kappa/\lambda \lesssim 0.6\), satisfies \(|m_S| \lesssim 100\) GeV and is favored by the TeV scale mirage mediation. In this region, the singlet becomes lighter than the doublet and \(g_{ZZh_1}^2/g_{SM}^2\) can decrease to \(O(10)\) % near
Table 1: Spectra for \((\lambda, \kappa) = (0.10, 0.40), (0.40, 0.10)\) and \((0.70, 0.11)\) with \(\tan \beta = 3\) and \(M_0 = 1200\text{GeV}\).

| \((\lambda, \kappa)\)       | (0.10, 0.40) | (0.40, 0.10) | (0.70, 0.11) |
|----------------------------|-------------|--------------|--------------|
| \(m_{h_1}\)                | 105 GeV     | 107 GeV      | 126 GeV      |
| \(m_{h_2}\)                | 1261 GeV    | 182 GeV      | 138 GeV      |
| \(m_{h_3}\)                | 1700 GeV    | 1312 GeV     | 1320 GeV     |
| \(m_{a_1}\)                | 512 GeV     | 181 GeV      | 158 GeV      |
| \(m_{a_2}\)                | 1260 GeV    | 1311 GeV     | 1321 GeV     |
| \(g_{ZZh_1}^2/g_{SM}^2\)   | 1.00        | 0.95         | 0.94         |
| \(m_{\tilde{t}_1}\)        | 823 GeV     | 823 GeV      | 823 GeV      |
| \(m_{\tilde{t}_2}\)        | 849 GeV     | 849 GeV      | 849 GeV      |
| \(\mu\)                    | 217 GeV     | 396 GeV      | 407 GeV      |

Table 2: Spectra for \((\lambda, \kappa) = (0.10, 0.40), (0.40, 0.10)\) and \((0.70, 0.11)\) with \(\tan \beta = 3\) and \(M_0 = 1500\text{GeV}\).

| \((\lambda, \kappa)\)       | (0.10, 0.40) | (0.40, 0.10) | (0.70, 0.11) |
|----------------------------|-------------|--------------|--------------|
| \(m_{h_1}\)                | 107 GeV     | 110 GeV      | 128 GeV      |
| \(m_{h_2}\)                | 1574 GeV    | 231 GeV      | 172 GeV      |
| \(m_{h_3}\)                | 2134 GeV    | 1637 GeV     | 1641 GeV     |
| \(m_{a_1}\)                | 572 GeV     | 200 GeV      | 172 GeV      |
| \(m_{a_2}\)                | 1573 GeV    | 1636 GeV     | 1645 GeV     |
| \(g_{ZZh_1}^2/g_{SM}^2\)   | 1.00        | 0.98         | 0.99         |
| \(m_{\tilde{t}_1}\)        | 2.25 \times 10^6\text{GeV}^2 | 2.18 \times 10^6\text{GeV}^2 | 2.18 \times 10^6\text{GeV}^2 |
| \(m_{\tilde{t}_2}\)        | 2.03 \times 10^6\text{GeV}^2 | 5.02 \times 10^4\text{GeV}^2 | 3.84 \times 10^4\text{GeV}^2 |
| \(m_{\tilde{t}_3}\)        | -2.25 \times 10^6\text{GeV}^2 | -1.92 \times 10^4\text{GeV}^2 | -9.38 \times 10^4\text{GeV}^2 |
| \(m_{\tilde{t}_4}\)        | 1023 GeV    | 1023 GeV     | 1023 GeV     |
| \(\mu\)                    | 1055 GeV    | 1055 GeV     | 1055         |
|                            | 272 GeV     | 495 GeV      | 508 GeV      |
the border of the excluded region by the false vacuum. In such a region the lightest CP-even Higgs boson \((m_{h_1} \simeq 80 - 90 \text{ GeV})\) could escape the LEPII bound and the second lightest CP-even Higgs boson \(m_{h_2} \approx 125 \text{ GeV}\) gives the signal observed in LHC [22]. Note that the small mixing with the singlet scalar enhances the second lightest Higgs mass in contrast to the lightest Higgs case and makes it easier to obtain the LHC value. The production cross section of \(h_2\) (mainly composed of \(H_u\)) through the gluon fusion or the vector boson fusion is reduced by \(O(10)\%\) relative to the SM due to the mixing with the singlet. The branching ratio \(BR(h_2 \rightarrow ZZ, WW, \gamma\gamma)\) is also reduced. This is because the width \(\Gamma(h_2 \rightarrow bb)\) does not change due to the suppressed mixing between the heavy \(H_d\) and the singlet, while the widths \((h_2 \rightarrow ZZ, WW, \gamma\gamma)\) are reduced due to the mixing between \(H_u\) and the singlet. Such a reduction will be confirmed or excluded by the future measurement of the Higgs coupling in LHC and ILC. Table 4 shows the examples of the mass spectra and \(g^2_{ZZh_1}/g^2_{SM}\) for \((\lambda, \kappa) = (0.10, 0.40)\), \((0.40, 0.10)\) and \((0.70, 0.11)\).

When we increase \(\tan \beta\), \(|\mu|\) decreases as expected from the rough estimation, \(|\mu| \sim M_0/\tan \beta\) in Eq. (3.16a). For example, the value \(\tan \beta = 10\) leads \(\mu = 100 \text{ GeV}\) or less for \(M_0 \simeq 1 \text{ TeV}\), which is excluded by the chargino mass bound by LEPII. The value of \(|m_S|\) also decreases as \(\tan \beta\) increases. In opposite view, this means that tuning of \(|m_S|\) (or \(\delta c_{i}^{\text{(loop)}}\)) increases to obtain large \(\tan \beta\) for fixed \((\lambda, \kappa)\) and small \(\tan \beta\) is favored in our scenario.

Finally we comment on the fermionic sector. The singlino mass is estimated as \(2\kappa\mu/\lambda\). Since we have \(\mu\) around \(200 - 500 \text{ GeV}\), the singlino can also be light. Note that the region with small \(\kappa/\lambda\) is excluded by the appearance of the tachyonic modes and/or the false vacua. Thus, we can not lead the singlino much lighter than the higgsino. Also, large \(\kappa/\lambda\) leads large \(|m_S|\), which could not be derived in our TeV scale mirage scenario. Thus,

| \((\lambda, \kappa)\) | \((0.10, 0.40)\) | \((0.40, 0.10)\) | \((0.70, 0.11)\) |
|-----------------|----------------|----------------|----------------|
| \(m_{h_1}\) | 108 GeV | 112 GeV | 128 GeV |
| \(m_{h_2}\) | 1782 GeV | 264 GeV | 196 GeV |
| \(m_{h_3}\) | 2422 GeV | 1853 GeV | 1861 GeV |
| \(m_{a_1}\) | 609 GeV | 202 GeV | 181 GeV |
| \(m_{a_2}\) | 1782 GeV | 1854 GeV | 1861 GeV |
| \(g^2_{ZZh_1}/g^2_{SM}\) | 1.00 | 0.99 | 1.00 |
| \(m^2_{H_d}\) | \(2.79 \times 10^6\text{GeV}^2\) | \(2.80 \times 10^6\text{GeV}^2\) | \(2.80 \times 10^6\text{GeV}^2\) |
| \(m^2_{H_u}\) | \(2.62 \times 10^6\text{GeV}^2\) | \(6.56 \times 10^4\text{GeV}^2\) | \(5.09 \times 10^4\text{GeV}^2\) |
| \(m^2_S\) | \(-2.91 \times 10^6\text{GeV}^2\) | \(-2.66 \times 10^4\text{GeV}^2\) | \(-1.31 \times 10^4\text{GeV}^2\) |
| \(m_{\tilde{t}_1}\) | 1157 GeV | 1157 GeV | 1157 GeV |
| \(m_{\tilde{t}_2}\) | 1193 GeV | 1193 GeV | 1193 GeV |
| \(\mu\) | 308 GeV | 560 GeV | 575 GeV |

Table 3: Spectra for \((\lambda, \kappa) = (0.10, 0.40), (0.40, 0.10)\) and \((0.70, 0.11)\) with \(\tan \beta = 3\) and \(M_0 = 1700\text{GeV}\).
singlino much heavier than the higgsino is disfavored. Thus, both masses of the higgsino and singlino would be of the same order. Since all of gaugino masses as well as squark and slepton masses are much heavier, the lightest superparticle would be a linear combination between the higgsino and singlino, depending on $\kappa/\lambda$. The gravitino is quite heavy such as $m_3/2 \sim 4\pi^2 M_0$, as already known in the mirage mediation mechanism [4, 5, 6].

The lightest neutralino in this model behaves like the linear combination between the higgsino and bino in the MSSM and could reproduce the observed abundance of the cold dark matter assuming the thermal relic saturates it [23]. While the extra Higgs bosons play an important role in the direct detection of them, which could be significantly different from the results in the MSSM [24]. The detailed study of the phenomenology of the TeV scale mirage mediation in the NMSSM is interesting for its distinct mass spectrum, however, beyond the scope of this work [25].

4 Conclusion

We have studied the NMSSM with the TeV scale mirage mediation. The region with large $\kappa/\lambda$ requires a large value of $|m_S|$ to satisfy the stationary conditions of the Higgs potential. Such a large value could not be realized in our TeV scale mirage scenario and therefore such parameter region is disfavored. In the favored region, it is found that we can realize $|m_{H_u}| \sim 200$ GeV, while other masses are heavy such as 1 TeV. Then, the fine-tuning problem is ameliorated. The cancellation between the effective $\mu$-term and the down-type Higgs soft mass reduces the sensitivity of $\mu$ to $m_Z$ and $\mu = \mathcal{O}(500)$ GeV is possible without significantly deteriorating the fine-tuning. The mixing between the light doublet and the singlet is suppressed by $(\kappa/\lambda)\tan^{-1}\beta$. For small $\tan\beta$ the lightest

| $(\lambda, \kappa)$ | (0.10, 0.40) | (0.40, 0.10) | (0.70, 0.11) |
|-----------------|-------------|-------------|-------------|
| $m_{h_1}$       | 113 GeV     | 79 GeV      | 65 GeV      |
| $m_{h_2}$       | 1136 GeV    | 126 GeV     | 130 GeV     |
| $m_{h_3}$       | 1209 GeV    | 1222 GeV    | 1228 GeV    |
| $m_{a_1}$       | 420 GeV     | 138 GeV     | 121 GeV     |
| $m_{a_2}$       | 1205 GeV    | 1221 GeV    | 1227 GeV    |
| $g_{Z\bar{Z}h_1}/g_{SM}^2$ | 1.00 | 0.24 | 0.13 |
| $m_{H_d}^2$     | $1.39 \times 10^6$ GeV$^2$ | $1.39 \times 10^6$ GeV$^2$ | $1.39 \times 10^6$ GeV$^2$ |
| $m_{H_u}^2$     | $4.91 \times 10^4$ GeV$^2$ | $2.00 \times 10^4$ GeV$^2$ | $1.97 \times 10^4$ GeV$^2$ |
| $m_S^2$         | $-6.23 \times 10^6$ GeV$^2$ | $8.11 \times 10^5$ GeV$^2$ | $4.85 \times 10^5$ GeV$^2$ |
| $m_{\tilde{t}_1}$ | 822 GeV | 822 GeV | 822 GeV |
| $m_{\tilde{t}_2}$ | 847 GeV | 847 GeV | 847 GeV |
| $\mu$           | 146 GeV     | 228 GeV     | 232 GeV     |

Table 4: Spectra for $(\lambda, \kappa) = (0.10, 0.40), (0.40, 0.10)$ and $(0.70, 0.11)$ with $\tan\beta = 5$ and $M_0 = 1200 GeV$. 
CP-even Higgs is mainly the doublet and its mass reaches 125 GeV without suppression by the mixing. When we increase $M_0$, the Higgs mass $m_{h_1}$ slowly increases, and also the value of $|\mu|$ increases. However, the required fine-tuning is still mild e.g. for $M_0 = 1700$ GeV.

The coupling between the lightest CP-even Higgs boson and the $Z$ boson is almost the same as one in the standard model for small $\tan \beta$, however, can decrease to $O(10)$ % for moderate $\tan \beta$ where the lightest CP-even Higgs is mainly singlet. If the lightest CP-even Higgs escapes the LEPII bound, the second lightest CP-even Higgs boson could be the boson observed in LHC. The mass of the second lightest CP-even Higgs boson depends on $\kappa$ and $\lambda$, and it can be light in the parameter region favored in our scenario. The heaviest CP-even Higgs mass as well as the heaviest CP-odd and charged ones is of $O(M_0)$. The lightest CP-odd Higgs can also be light. Thus, the Higgs sector has a rich structure.

In our scenario, the higgsino is light compared with three gauginos. In addition, the singlino is also light. Both masses of the higgsino and the singlino are of the same order. Then, the lightest superparticle is a linear combination between the higgsino and singlino. Such a neutralino sector and Higgs sector would lead to several phenomenologically interesting aspects.

**Note added**

While this work was being completed, we received Ref. [26], which also considered relevant aspects.

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**A Soft SUSY breaking terms**

Here we give explicitly soft SUSY breaking terms induced by the mirage mediation mechanism in the NMSSM.

In the mirage mediation, the soft parameters at the scale just below $M_{GUT}$ are given
by

\[ M_a(M_{\text{GUT}}) = M_0 + \frac{m_3/2}{8\pi^2} b_a g_a^2, \]

\[ A_{ijk}(M_{\text{GUT}}) = (c_i + c_j + c_k)M_0 - (\gamma_i + \gamma_j + \gamma_k) \frac{m_3/2}{8\pi^2}, \]

\[ m_i^2(M_{\text{GUT}}) = c_i M_0^2 - \gamma_i \left( \frac{m_3/2}{8\pi^2} \right)^2 - \frac{m_3/2}{8\pi^2} M_0 \theta_i, \quad (A.1) \]

where

\[ b_a = -3 \text{tr}(T_a^2(\text{Adj})) + \sum_i \text{tr}(T_a^2(\phi^i)), \]

\[ \gamma_i = 2 \sum_a g_a^2 C_2^a(\phi^i) - \frac{1}{2} \sum_{jk} |y_{ijk}|^2, \]

\[ \theta_i = 4 \sum_a g_a^2 C_2^a(\phi^i) - \sum_{jk} a_{ijk} |y_{ijk}|^2, \]

\[ \dot{\gamma}_i = 8 \pi^2 \frac{d\gamma_i}{d\ln \mu_R}. \quad (A.2) \]

Here, \( T_a^2(\text{Adj}) \) and \( T_a^2(\phi^i) \) denote Dynkin indices of the adjoint representation and the representation of matter fields \( \phi^i \). We have assumed \( \omega_{ij} = \sum_{kl} y_{ijkl} y_{ijkl}^* \) to be diagonal.

Within the framework of the NMSSM, the \( \beta \)-function coefficients, anomalous dimensions and other coefficients in the above equations are obtained as

\[ b_3 = -3, \ b_2 = 1, \ b_1 = 11, \]

\[ \gamma_{H_u} = \frac{3}{2} g_2^2 + \frac{1}{2} g_1^2 - 3y_t^2 - \lambda^2, \]

\[ \gamma_{H_d} = \frac{3}{2} g_2^2 + \frac{1}{2} g_1^2 - \lambda^2, \]

\[ \gamma_S = -2\kappa^2 - 2\lambda^2, \]

\[ \gamma_{Q_a} = \frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{1}{18} g_1^2 - (y_t^2 + y_b^2) \delta_{3a}, \]

\[ \gamma_{U_a} = \frac{8}{3} g_3^2 + \frac{8}{9} g_1^2 - 2y_t^2 \delta_{3a}, \]

\[ \gamma_{D_a} = \frac{8}{3} g_3^2 + \frac{2}{9} g_1^2 - 2y_b^2 \delta_{3a}, \]

\[ \gamma_{L_a} = \frac{3}{2} g_2^2 + \frac{1}{2} g_1^2 - y_t^2 \delta_{3a}, \]

\[ \gamma_{E_a} = 2g_1^2 - 2y_t^2 \delta_{3a}, \quad (A.3) \]
\[ \begin{align*}
\theta_{H_u} &= 3g_2^2 + g_1^2 - 6g_2^2 a_{H_u Q_3 U_3^c} - 2\lambda^2 a_{H_u H_4 S}, \\
\theta_{H_d} &= 3g_2^2 + g_1^2 - 6g_2^2 a_{H_d Q_3 D_3^c} - 2y_f^2 a_{H_d L_3 E_3^c} - 2\lambda^2 a_{H_u H_4 S}, \\
\theta_S &= -2\lambda^2 a_{H_u H_4 S} - \kappa^2 a_{SSS}, \\
\theta_{Q_a} &= \frac{16}{3} g_3^2 + 3g_2^2 + \frac{1}{9} g_1^2 - 2(y_1^2 a_{H_u Q_3 U_3^c} + y_b^2 a_{H_d Q_3 D_3^c})\delta_{3a}, \\
\theta_{U_a} &= \frac{16}{3} g_3^2 + \frac{16}{9} g_1^2 - 4y_f^2 a_{H_u Q_3 U_3^c} \delta_{3a}, \\
\theta_{D_a} &= \frac{16}{3} g_3^2 + \frac{4}{9} g_1^2 - 4y_b^2 a_{H_d Q_3 D_3^c} \delta_{3a}, \\
\theta_{L_a} &= 3g_2^2 + g_1^2 - 2y_f^2 a_{H_u L_3 E_3^c} \delta_{3a}, \\
\theta_{E_a} &= 4g_1^2 - 4y_f^2 a_{H_d L_3 E_3^c} \delta_{3a},
\end{align*} \]

(A.4)

\[ \begin{align*}
\dot{\gamma}_{H_u} &= \frac{3}{2} g_2^4 + \frac{11}{2} g_1^4 - 3y_f^2 b_{yt} - \lambda^2 b_{\lambda}, \\
\dot{\gamma}_{H_d} &= \frac{3}{2} g_2^4 + \frac{11}{2} g_1^4 - 3y_b^2 b_{yb} - y_f^2 b_{yr} - \lambda^2 b_{\lambda}, \\
\dot{\gamma}_{S} &= -2\kappa^2 b_{\kappa} - 2\lambda^2 b_{\lambda}, \\
\dot{\gamma}_{Q_a} &= -8g_3^4 + \frac{3}{2} g_2^4 + \frac{11}{18} g_1^4 - (y_1^2 b_{yt} + y_b^2 b_{yb}) \delta_{3a}, \\
\dot{\gamma}_{U_a} &= -8g_3^4 + \frac{88}{9} g_1^4 - 2y_f^2 b_{yt} \delta_{3a}, \\
\dot{\gamma}_{D_a} &= -8g_3^4 + \frac{22}{9} g_1^4 - 2y_b^2 b_{yb} \delta_{3a}, \\
\dot{\gamma}_{L_a} &= \frac{3}{2} g_2^4 + \frac{11}{2} g_1^4 - y_f^2 b_{yr} \delta_{3a}, \\
\dot{\gamma}_{E_a} &= 22g_1^4 - 2y_f^2 b_{yr} \delta_{3a},
\end{align*} \]

(A.5)

where

\[ \begin{align*}
b_{yt} &= -\frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{9} g_1^2 + 6y_f^2 + y_b^2 + \lambda^2, \\
b_{yb} &= -\frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{9} g_1^2 + y_f^2 + 6y_b^2 + y_r^2 + \lambda^2, \\
b_{yr} &= -3g_2^2 - 3g_1^2 + 3y_b^2 + 4y_r^2 + \lambda^2, \\
b_{\kappa} &= 6\lambda^2 + 6\kappa^2, \\
b_{\lambda} &= -3g_2^2 - 9g_1^2 + 3y_f^2 + 3y_b^2 + y_r^2 + 4\lambda^2 + 2\kappa^2.
\end{align*} \]

(A.6)

Here, \(Q_a, U_a, D_a, L_a\), and \(E_a\) denote left-handed quark, right-handed up-sector quark, right-handed down-sector quark, left-handed lepton, and right-handed lepton fields, respectively, and the index \(a\) denotes the generation index. We have included effects due to Yukawa couplings, \(y_t, y_b, \) and \(y_r\), only for the third generations.
Figure 1: Constraints and SUSY breaking parameters. We use $M_0 = 1200 GeV, \alpha = 2, c_{H_d} = 1, c_{H_u} = 0, c_S = 0, c_{\tilde{Q}} = 0.5, c_{\tilde{t}} = 0.5$ and $\tan \beta = 3, A_\kappa = -100 GeV$. For detail of the shaded regions, see the text. The TeV scale mirage mediation disfavors the region $|m_S| \gtrsim M_0/\sqrt{8\pi^2} \sim 100$ GeV (pink shaded).
Figure 2: Constraints and SUSY breaking parameters. We use $M_0 = 1500\text{GeV}$, $\alpha = 2$, $c_{Hd} = 1$, $c_{Hu} = 0$, $c_S = 0$, $c_\tilde{Q} = 0.5$, $c_\tilde{t} = 0.5$ and $\tan \beta = 3$, $A_\kappa = -100\text{GeV}$. 
Figure 3: Constraints and SUSY breaking parameters. We use $M_0 = 1700\text{GeV}, \alpha = 2, c_{H_d} = 1, c_{H_u} = 0, c_S = 0, c_{\tilde{Q}} = 0.5, c_{\tilde{t}} = 0.5$ and $\tan \beta = 3, A_\kappa = -100\text{GeV}$. 
Figure 4: Constraints and SUSY breaking parameters. We use $M_0 = 1200 \text{GeV}, \alpha = 2, c_{H_d} = 1, c_{H_u} = 0, c_S = 0, c_{\tilde{Q}} = 0.5, c_{\tilde{t}} = 0.5$ and $\tan \beta = 5, A_\kappa = -100 \text{GeV}$. 
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