The kaonic atoms research program at DAΦNE: from SIDDHARTA to SIDDHARTA-2

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Abstract. The interaction of antikaons with nucleons and nuclei in the low-energy regime represents an active research field in hadron physics with still many important open questions. The investigation of light kaonic atoms, in which one electron is replaced by a negatively charged kaon, is a unique tool to provide precise information on this interaction; the energy shift and the broadening of the low-lying states of such atoms, induced by the kaon-nucleus hadronic interaction, can be determined with high precision from the atomic X-ray spectroscopy, and this experimental method provides unique information to understand the low energy kaon-nucleus interaction at the production threshold. The lightest atomic systems, like the kaonic hydrogen and the kaonic deuterium deliver, in a model-independent way, the isospin-dependent kaon-nucleon scattering lengths. The most precise kaonic hydrogen measurement to-date, together with an exploratory measurement of kaonic deuterium, were carried out in 2009 by the SIDDHARTA collaboration at the DAΦNE electron-positron collider of LNF-INFN, combining the excellent quality kaon beam delivered by the collider with new experimental techniques, as fast and very precise X-ray detectors, like the Silicon Drift Detectors. The SIDDHARTA results triggered new theoretical work, which achieved major progress in the understanding of the low-energy strong interaction with strangeness reflected by the antikaon-nucleon scattering lengths calculated with the antikaon-proton amplitudes constrained by the SIDDHARTA data. The most important open question is the experimental determination of the hadronic energy shift and width of kaonic deuterium; presently, a major upgrade of

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the setup, SIDDHARTA-2, is being realized to reach this goal. In this paper, the results obtained in 2009 and the proposed SIDDHARTA-2 upgrades are presented.

1 Introduction

The strong interaction, described by the QCD in the framework of the Standard Model, is still hiding many mysteries, especially in the low-energy limit, the so called non-perturbative regime. Particularly interesting is the strong interaction involving the strange quark which, belonging to the light quarks but having a mass of about 100 $MeV/c^2$, much heavier than the few $MeV/c^2$ up and down quark masses, plays a peculiar role. Since kaons and antikaons are the lowest mass particles containing strange quarks, for decades their interaction with nucleons and nuclei in the low-energy regime has been subject of intensive studies in experiment and theory (for reviews see [1,2]). Effective field theories contain appropriate degrees of freedom to describe physical phenomena occurring at the nucleon-meson scale and chiral perturbation theory was extremely successful in describing systems like pionic atoms; however it is not directly applicable for the kaonic systems, where non-perturbative coupled-channel techniques can be used ([3]). These theories are still waiting to be verified by experimental data. There are two experimental approaches to probe the kaon-nucleus strong interaction, both exploited at LNF-INFN. One is by studying the scattering and the reaction channels between the kaon and the nucleus, to directly search for bound states and extract the potential of the strong interaction; this is the experimental method followed by the AMADEUS collaboration ([4–6]). The other method consists in the precision X-ray measurement of the shift and the broadening of the energy levels of kaonic atoms caused by the kaon-nucleus strong interaction. This latter one, exploited by the SIDDHARTA and SIDDHARTA-2 collaborations ([7–13]), is of significant importance, since it is the only method able to provide direct information on the kaon-nucleus system at threshold.

1.1 Kaonic atoms

Kaonic atoms are formed when a negatively charged kaon is stopped in a target and replaces one of the electrons in an atom; due to the much higher $K^-$ mass with respect to the $e^-$ one, this newly formed exotic atom radius is smaller than the one of the usual atom and the $K^-$ is stopped at a distance from the nucleus corresponding to the $n \approx 25$ excited state. The subsequent cascade of the antikaon to the ground level occurs via several different processes; in particular, the last transitions on the 1s level are radiative and photons are emitted in the X-ray region. For light atoms, especially for the hydrogen and the deuterium, a detectable energy shift from the electromagnetic value of the ground state is expected, as well as a broadening of the ground state level, caused by nuclear absorption (see fig.1).

By measuring these observables, kaonic atoms offer the unique possibility to determine the s-wave antikaon-nucleon scattering lengths at vanishing energy, alternatively to scattering experiments were an extrapolation to zero energy is necessary. With the advance of the experimental techniques, both in the accelerator and detector sectors, we are presently able to perform very high precision measurements, resulting in a deeper and more complete understanding of the many open questions in the QCD. In particular, with the advent of clean kaon beams, as the one provided by the DAΦNE collider, and of performant, fast, X-ray detectors as the Silicon Drift Detectors, the kaonic atoms studies entered the precision era.
After the atomic capture of the kaon a kaonic atom is formed in a highly excited state and a few kaons will cascade to ground state. $1s$ level is shifted and broadened by the strong interaction \((14)\).

### 2 SIDDHARTA experiment

In 2009, with the SIDDHARTA experiment at DAΦNE, the strong interaction induced shift of the ground state of kaonic hydrogen atoms and the absorption width were measured with the highest accuracy up to now \([8, 9]\). Measurements of the $2p$ shift and width of kaonic helium ($^4He$ as well as $^3He$) \([10–12]\) completed the SIDDHARTA program. Conscient about the importance of the $Kd$ measurement an attempt was also done in this direction but, due to the very low yield of the transition, only an upper limit could be extracted and no values for the $1s$ level shift and width were delivered \([8]\).

#### 2.1 Experimental setup

The SIDDHARTA setup consisted of two main components, the light-weight cryogenic target cell and a specially developed large area, high resolution X-ray detector system made of Silicon Drift Detectors (SDDs). The experiment made use of the $K^+K^-$ pairs coming from the $\Phi$ decays with a 49% branching ratio. The kaons leaving the interaction point through the SIDDHARTA beam pipe were degraded in energy and entered the cryogenic gaseous hydrogen (helium) target placed above the beam pipe, forming a kaonic atom and emitting X-rays during the $2p \rightarrow 1s$ (hydrogen) and $3d \rightarrow 2p$ (helium) transitions (see fig. 2).

The $K^+K^-$ pairs were emitted in back-to-back configuration and identified by the kaon detectors, made of two plastic scintillators placed above and below the interaction point as illustrated in fig. 2. The kaons were distinguished from the minimum ionizing particles using the time of flight information at the kaon detectors, and the coincidence of the two scintillators defined the kaon trigger, which marked the timing of the incident kaons. A fraction of the negatively charged kaons activating the kaon trigger were successfully stopped inside the volume of the gaseous target placed about 20 cm above the interaction point to form kaonic atoms. A working pressure of 0.3 MPa was achieved, which led to a hydrogen gas density of 2% of liquid hydrogen density, at a working temperature of 25 K. Special Silicon Drift Detectors were developed with excellent energy resolution ($FWHM \approx 150\, eV @ 6\, keV$) and timing capability in the order of $\mu s$ \([15]\) when operated at 140 K temperature. Using the X-ray signal from the SDDs in coincidence with the $K^+K^-$ pair, the continuous machine background, as well as unwanted fluorescence X-rays, could be efficiently suppressed. The SDDs had a total active
area of 144 cm\(^2\), covering about 10% of the solid angle around the target cell. The energy calibrations of the SDDs were done every few hours, using the X-ray tube activated K\(\alpha\) lines of Ti (4.5 keV), Mn (5.5 keV) and Cu (8.0 keV) to determine the scale of the energy spectra. The energy resolution at 6 keV was stable at about 150 eV (FWHM) throughout the measurement. More details about the configuration and the performance of the detectors can be found in \cite{8–12}. Data were accumulated with gaseous targets of hydrogen (1.18 g/l), deuterium (2.250 g/l), helium-3 (0.355 g/l), and helium-4 (1.65 g/l and 2.15 g/l).

2.2 Kaonic Helium measurements

For what concerns the kaonic helium, its transition to 1s level couldn’t be observed due to the very low yield and to a transition energy out of the SDD dynamic range. Before the SIDDHARTA experiment, there existed only four measurements and the situation was rather ambiguous: three measurements \cite{16–18}, performed more than 20 years ago, giving, within few \(\sigma\)s, results which are more than an order of magnitude higher with respect to the theoretical predictions and a more recent one, performed at KEK \cite{19}, which was, instead, compatible with the theoretical predictions (\cite{20–21}), but incompatible with the previous experiments. A conclusive precise measurement on \(K^4\text{He}\) was then needed in order to solve the "puzzle", together with the first measurement of \(K^3\text{He}\), fundamental to obtain valuable information on the \(K^- p\) and the \(K^- n\) interactions. The kaonic helium spectra obtained by the SIDDHARTA experiment are shown in fig. 3\cite{8–12}.

In the left picture, the peak seen at 6.2 keV is identified as the \(K^3\text{He} L_\alpha\) line (the 3\(d \rightarrow 2p\) transition). In the right one, the peak seen at 6.4 keV is identified as the \(K^3\text{He} L_\alpha\) line. In addition to these lines, other smaller peaks are clearly visible which are the kaonic atom X-ray lines produced by kaons stopping in the target window made of Kapton, and the Ti and Mn lines. The strong-interaction
shifts of the kaonic helium 2p states were obtained from the difference between the experimentally determined values and the QED calculated ones [22, 23]. The results are:

\[ \varepsilon_{2p}(K^3He) = -2 \pm 2 \text{ (stat)} \pm 4 \text{ (syst)} \text{ eV} \]
\[ \varepsilon_{2p}(K^4He) = 5 \pm 5 \text{ (stat)} \pm 4 \text{ (syst)} \text{ eV} \]

Thus a shift compatible with 0 eV of the 2p level from experiment is established, which is in agreement with the theoretical estimations and, within the errors, with the results reported by the E570 [19] collaboration, while is not in agreement with the previous measurements ([16–18]). This is probably due to the fact that, in those experiments, the helium target was a liquid one in which, as indeed happened also in the KEK experiment ([19]), a big Compton effect is present and has to be taken into account in the analysis procedure.

### 2.3 Kaonic Hydrogen and Kaonic Deuterium measurement

The lightest kaonic atom is the \( K^- p \) atom in which the principal electromagnetic interaction is accompanied by the strong interaction of the kaon with the proton which is measurable by X-ray spectroscopy of the radiative transitions from the np states (2p, 3p, ...) to the 1s ground state (K transitions). The \( K^- p \) scattering length \( a_{K^- p} \) can be obtain from the equation

\[ \varepsilon_{1s} + \frac{i}{2} \Gamma_{1s} = 2\alpha^3 \mu_e^2 a_{K^- p}(1 - 2\alpha\mu_e(\ln \alpha - 1))a_{K^- p} \]

being \( \varepsilon_{1s} \) and \( \Gamma_{1s} \) the shift and width of the transition to the 1s level, (for more details on the various terms see [24]) and it is related to the isospin dependent scattering lengths by

\[ a_{K^- p} = \frac{1}{2}(a_0 + a_1) \]

Historically there were several measurements of the strong-interaction shift \( \varepsilon_{1s} \) and width \( \Gamma_{1s} \) of kaonic hydrogen ([25–29]). In the 1970s and the 1980s three groups ([25–27]) reported a measured
attractive shift (positive $\epsilon_{1s}$), while the information extracted from the analyses of the low energy KN data ([20–32]) shows a repulsive shift (negative $\epsilon_{1s}$). This contradiction has been known as the "kaonic hydrogen puzzle". In 1997, the first distinct peaks of the kaonic-hydrogen X-rays were observed by the KEK-PS E228 group [28] with a significant improvement in the signal-to-background ratio by the use of a gaseous hydrogen target, where previous experiments had employed liquid hydrogen. It was crucial to use a low-density target, namely a gaseous target, because the X-ray yields quickly decrease towards higher density due to the Stark mixing effect. The observed repulsive shift was consistent in sign with the analysis of the low energy KN scattering data, resolving the long-standing discrepancy. More recent values reported by the DEAR group in 2005 [29], with substantially reduced errors, firmly established the repulsive shift obtained in the previous E228 experiment. The latest kaonic hydrogen and deuterium X-ray energy spectra, obtained by the SIDDHARTA experiment, are shown in fig. 4 [8].

The K-series X-rays of kaonic hydrogen were clearly observed while those for kaonic deuterium were not visible. This appears to be consistent with the theoretical expectation of lower X-ray yield and greater transition width for deuterium ([33]) than for kaonic hydrogen. However, the kaonic deuterium spectrum can be used to characterize the background. The vertical dot-dashed line in fig. 4 indicates the X-ray energy of kaonic hydrogen $K_\alpha$ calculated using only the electromagnetic interaction (EM). Comparing the kaonic hydrogen $K_\alpha$ measured peak and the EM value, a repulsive shift of the kaonic hydrogen 1s energy level is easily seen. Many other lines from kaonic atom X-rays were detected in both spectra as indicated with arrows in the figure. These kaonic atom lines result from high n X-ray transitions of kaons stopped in the target cell wall made of kapton ($C_{22}H_{10}O_5N_2$) and its support frames made of aluminium. There are also characteristic X-rays from titanium and copper foils installed for X-ray energy calibration. A global simultaneous fit of the hydrogen and deuterium spectra has been performed, whose results are shown in fig. 4 (b) and (c). The kaonic hydrogen lines were represented by Lorentz functions convoluted with the detector response function, where the Lorentz width corresponds to the strong interaction broadening of the 1s state. The region of interest of Kd X-rays is illustrated in fig. 4 (c). The 1s level shift $\epsilon_{1s}$ and width $\Gamma_{1s}$ of kaonic hydrogen were determined to be [8]:

$$\epsilon_{1s} = -283 \pm 36 \text{ (stat)} \pm 6 \text{ (syst)} \text{ eV}$$

$$\Gamma_{1s} = 541 \pm 89 \text{ (stat)} \pm 22 \text{ (syst)} \text{ eV}$$

The quoted systematic error is a quadratic summation of the contributions from the systematic errors on the SDD gain shift, the SDD response function, the ADC linearity, the low energy tail of the kaonic hydrogen higher transitions, the energy resolution, and the procedural dependence shown by independent analysis. This measurement represents the best precision available until today, and it has been used to set constraints on the calculated real and imaginary part of the $K^- p$ amplitude ([34]).

3 SIDDHARTA-2 experiment

The case of kaonic deuterium is more challenging than kaonic hydrogen mainly due to the larger widths of the K lines and the lower X-ray yield expected. Experimentally the case of kaonic deuterium is still open; the SIDDHARTA experiment measured the X-ray spectrum with a pure deuterium target but, due to the limited statistics and high background, the determination of $\epsilon_{1s}$ and $\Gamma_{1s}$ was impossible. An upper limit for the X-ray yield of the K lines could be extracted from the data: total yield < 0.0143, $K_\alpha$ yield < 0.0039 [13].
Figure 4. A global simultaneous fit result of the X-ray energy spectra of hydrogen and deuterium data. (a) Residuals of the measured kaonic-hydrogen X-ray spectrum after subtraction of the fitted background, clearly displaying the kaonic-hydrogen K-series transitions. The fit components of the K-$$\beta$$ transitions are also shown, where the sum of the function is drawn for the higher transitions (greater than K$$\beta$$). (b), (c) Measured energy spectra with the fit lines. Fit components of the background X-ray lines and a continuous background are also shown. The dot-dashed vertical line indicates the EM value of the kaonic-hydrogen K$$\alpha$$ energy. (Note that the fluorescence K$$\alpha$$ line consists of K$$\alpha$$1 and K$$\alpha$$2 lines, both of which are shown.)

Since the kaonic deuterium X-ray measurement represents the most important experimental information missing in the field of the low-energy antikaon-nucleon interactions today, a new experiment (SIDDHARTA-2) is planned, based on a much improved apparatus.

3.1 Experimental setup upgrade

Thanks to the knowledge acquired in 2009 with the SIDDHARTA experiment, a new version of the experimental apparatus, aiming to increase the signal over background ratio (S/B) by a factor about 20 allowing the kaonic deuterium measurement, has been developed by the SIDDHARTA-2 collabo-
ration. The major upgrades introduced to fulfill the kaonic deuterium measurement goal, are shown in fig. 5 and listed below: 

- Larger area and faster SDD detector array; the solution to improve the SDD time resolution consists in the reduction of the single element size (from 10x10 to 8x8 mm$^2$) and the replacement of the integrated J-FET (thermally limited), by a newly-developed amplifier on the ceramics, able to operate at very low temperatures (below 50 K). The shorter path and the higher carrier mobility consent a faster charge packet drift to the anode and therefore, a reduced time window (350 ns instead of 900 ns) for each trigger, with consequent a suppression of the asynchronous background. The new SDD detectors are produced by Fondazione Bruno Kessler (FBK) in Trento, Italy.
- A veto detector to measure the prompt time of the secondaries from $K^-$ absorption on nuclei. The system consists in scintillators surrounding the vacuum chamber, read at both ends by PMs coupled to mirrors and light-guides (to cope with the narrow space between the setup and the shielding against beam background).
- A new cryogenic target in reinforced kapton (13 cm diameter, 7 cm height), operating few hundred mK above the liquid point (25 K) at a pressure of 4 bar (5% LHD), for more efficient kaon stopping.
- A veto system, consisting in scintillators read by SiPMs, placed behind each SDD array, to reject the hadronic background coming from border hits of Minimum Ionizing Particles (MIPs), depositing energy in the X-ray range.
- A kaon trigger with geometric acceptance optimized to match the kaon gas stopping distribution.
- An improved X-ray calibration system, providing low-rate in-situ calibration, as well as high rate calibration between physics runs, to compensate very small fluctuations in each single SDD response.
- Mechanical and cryogenic improvements of the vacuum chamber, necessary to add more cooling power to the SDDs and to the cryogenic target.

3.2 Kaonic deuterium expected measurement

All the above described items were optimized by GEANT4 simulation, considered reliable after having reproduced, in the same framework, the SIDDHARTA results, both in terms of signal and background.
ground, to 7% accuracy. Using theoretical values as inputs to the Monte Carlo simulation, the expected spectrum of the $K^-d$ transitions, for an acquired luminosity of $800 \text{ pb}^{-1}$ and assuming a yield of 0.1%, is shown in fig. 6.

![Simulated $K^-d$ spectrum for SIDDHARTA-2 for 800 pb$^{-1}$ integrated luminosity](image)

**Figure 6.** The simulated $K^-d$ spectrum for SIDDHARTA-2 for 800 pb$^{-1}$ integrated luminosity (the $K_\alpha$ line is at 7 keV, while from 8 to 10 keV there is the K-complex.

The fit of the simulated spectrum shows how the $K^-d$ $2p \rightarrow 1s$ transition, influenced by the strong interaction, can be determined with a precision of 30 eV for the shift and 70 eV for the width.

4 Conclusions

The SIDDHARTA experiment at the DAΦNE electron-positron collider measured the strong interaction induced shift and width of kaonic hydrogen and helium transitions with unprecedented accuracy. Important implications for the theory of the strong interaction with strangeness in the low energy regime were provided by the SIDDHARTA constraints. In addition, SIDDHARTA measured for the first time ever the kaonic helium 3 transitions to the $2p$ level and the first, similar, measurement, of kaonic helium 4 with a gaseous target. Still open is the challenging case of kaonic deuterium which will be attacked by the follow-up experiment SIDDHARTA2, aiming at the determination of the hadronic shift and width to enable the extraction of the antikaon-nucleon isospin dependent scattering lengths $a_0$ and $a_1$. The SIDDHARTA-2 collaboration is preparing a series of modifications and upgrades of the apparatus and experimental configurations, aiming to measure the kaonic deuterium x-rays with a precision compatible with that achieved in the hydrogen target measurement starting with 2018, when the apparatus will be installed on the DAΦNE collider.

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References

[1] E. Friedman, A. Gal, Phys. Rep. 452, 89 (2007)
[2] C.J. Batty, E. Friedman, A. Gal, Phys. Rep. 287, 385 (1997).
[3] J. Caro-Ramon, et al., Nucl. Phys. A 672, 249-269 (2000).
[4] The AMADEUS collaboration, LNF preprint, LNF/9607/24(IR).

http://www.lnf.infn.it/esperimenti/siddharta/LOI_AMADEUS_March2006.pdf.

[5] K. Piscicchia et al., PoS Bormio2013 034 (2013).
[6] O. Vazquez Doce et al. Phys. Lett. B 758 134-139 (2016).
[7] The SIDDHARTA-2 collaboration, The SIDDHARTA-2 proposal.

https://www.lnf.infn.it/committee/private/documenti/SIDDHARTA2-proposal_FINAL.pdf.

[8] M. Bazzi et al., Phys. Lett. B 704, 113 (2011).
[9] M. Bazzi et al., Nucl. Phys. A 881, 88 (2012).
[10] M. Bazzi et al., Phys. Lett. B 681, 310 (2009).
[11] M. Bazzi et al., Phys. Lett. B 714, 40 (2012).
[12] M. Bazzi et al., Phys. Lett. B 697, 199 (2011).
[13] M. Bazzi et al., Nucl. Phys. A 907, 69 (2013).
[14] K. Phelan et al., Nucl. Inst. and Meth. in Phys. Res. A, 845, 533-536 (2017).
[15] M. Bazzi et al., Nucl. Inst. Meth. in Phys. Res. A 628, 264 (2011).
[16] C.E. Wiegand and R.H. Pehl, Phys. Rev. Lett 27, 1410 (1971).
[17] C.J. Batty et al., Nucl. Phys. A 326, 455 (1979).
[18] S. Baird et al., Nucl. Phys. A 392, 297 (1983).
[19] S. Okada et al., Phys. Lett. B 653, 387 (2007).
[20] C.J. Batty, Nucl. Phys. A 508, 89c (1990).
[21] R. Seki, Phys. Rev. C 5, 1196 (1972).
[22] V. Lucherini, et al., Nucl. Instr. Meth. A 496 (2003) 315.
[23] S.G. Karshenboim, et al., Can. J. Phys. 84 (2006) 107.
[24] T. Hyodo and D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012).
[25] J. D. Davies, et al., Phys. Lett. B 83, 55 (1979).
[26] M. Izycki, et al., Z. Phys. A 297, 11 (1980).
[27] P. M. Bird, et al., Nucl. Phys. A 404, 482 (1983).
[28] M. Iwasaki, et al. Phys. Rev. Lett. 78, 3067 (1997).
[29] G. Beer, et al. (DEAR Collaboration), Phys. Rev. Lett. 94, 212302 (2005).
[30] W.E. Humphrey and R.R. Rose, Phys. Rev. 127, 1305 (1962).
[31] J.K. Kim, Phys. Rev. Lett. 19, 1074 (1967).
[32] A.D. Martin, Nucl. Phys. B 179, 33 (1981).
[33] T. Koike, T. Harada, Y. Akaishi, Phys. Rev. C 53, 79 (1996).
[34] Y. Ikeda, T. Hyodo, and W. Weise, Phys. Lett. B 706, 63 (2011).
[35] M Bazzi et al. Journ. of. Instr. 8, T11003 (2013).
[36] M. Iliescu et al., Journal of Physics: Conference Series 770, 012034 (2016).