Fast high fidelity quantum non-demolition qubit readout via a non-perturbative cross-Kerr coupling

R. Dassonneville, T. Ramos, V. Milchakov, L. Planat, É. Dumur, F. Foroughi, J. Puertas, S. Leger, K. Bharadwaj, J. Delaforce, C. Naud, W. Hasch-Guichard, J. J. Garcia-Ripoll, N. Roch, and O. Buisson

1 Univ. Grenoble-Alpes, CNRS, Grenoble INP, Institut Néel, 38000 Grenoble, France
2 Institute of Fundamental Physics, IFF-CSIC, Calle Serrano 113b, 28006 Madrid, Spain
3 Centro de Óptica e Información Cuántica, Facultad de Ciencias, Universidad Mayor, Chile

(Dated: November 21, 2019)

Qubit readout is an indispensable element of any quantum information processor. In this work we propose an original coupling scheme between qubit and cavity mode based on a non-perturbative cross-Kerr interaction. It leads to an alternative readout mechanism for superconducting qubits. This scheme, using the same experimental techniques as the perturbative cross-Kerr coupling (dispersive interaction), leads to an alternative readout mechanism for superconducting qubits. This new process, being non-perturbative, maximizes speed of qubit readout, single-shot fidelity and its quantum non-demolition (QND) behavior at the same time, while minimizing the effect of unwanted decay channels such as, for example, the Purcell effect. We observed 97.4 % single-shot readout fidelity for short 50 ns pulses. Using long measurement, we quantified the QND-ness to 99 %.

I. INTRODUCTION

In Noisy Intermediate Scale Quantum (NISQ) devices [1], measurements are usually the last step of the algorithm. Here, a high-fidelity readout is an interesting asset that reduces the overhead in error mitigation [2] and in the characterization of gate fidelities [3]. However, high-fidelity quantum non-demolition (QND) single-shot measurements become a requirement once we consider scaling up quantum technologies [4] to large devices, using quantum error correction [5, 6] and fault-tolerant quantum computation [7, 8]. In this context, lowering the readout and QND-errors is as important as decreasing the single- and two-qubit gate errors below the scaling thresholds.

A fast and high-fidelity QND measurement demands a strong coupling to the measurement device combined with a good preservation of the qubit state. In trapped ion qubits, this dilemma is solved by encoding information in two long-lived states, only one of which couples to incoming radiation [9]. Fluorescence counting gives a measurement in two long-lived states, only one of which couples to incoming radiation [9]. Fluorescence counting gives a good QND projective measurement [14, 16]. This transverse coupling has been extensively used in most circuit-QED experiments. State-of-the-art measurement fidelities and speeds using this standard dispersive technique are summarized in Table I, will be discussed. Qubits and resonators are usually coupled together via the electrical interaction between the dipole moment field \( \hat{q} \) of the qubit and the field amplitude \( \hat{c} \) of the resonator. This field-field interaction is known as the transverse coupling resulting in a Jaynes-Cummings model with the coupling term \( g_x (\hat{q}^\dagger \hat{c} + \hat{q} \hat{c}^\dagger) \) in the Hamiltonian [11, 13]. In the dispersive limit [15], the qubit-cavity detuning \( \Delta = \omega_c - \omega_q \) exceeds the coupling strength \( |g_x| \ll |\Delta| \), and the cavity experiences an effective energy-energy interaction \( \frac{g_x^2}{2i(\Delta - \omega_q)} \hat{\sigma}_z \hat{c}^\dagger \hat{c} \), known as the dispersive or cross-Kerr interaction. It gives rise to a qubit-dependent frequency shift, mapping the state of the qubit to the signal phase probing the resonator and thus providing a good QND projective measurement [14, 16]. This transverse coupling has been extensively used in most circuit-QED experiments. State-of-the-art measurement fidelities and speeds using this standard dispersive technique are summarized in the first row of Table I. However, the dispersive readout is fundamentally limited by unavoidable higher order corrections to perturbation theory, which distort the qubit dynamics [17–19], and induce additional decay channels [20].

Several works have investigated how to overcome these limitations, designing new quantum circuits [21–28]. Implementing a coupling scheme that involves natively the energy of the qubit – as opposed to an effective energy interaction – resolves these limitations. Along this line, the longitudinal coupling \( g_x \hat{q} \hat{q}^\dagger (\hat{c}^\dagger + \hat{c}) \) [cf. second row of Table I] is remarkable. It induces a qubit-dependent displacement of the cavity field \( \hat{c} \) [27]. When combined with a parametric modulation \( g_x(t) \) at the cavity frequency \( \omega_c \), this interaction results in a faster separation of the pointer states with a QND-ness as high as \( Q = 98.4 \% \) [29, 30].

In this work we propose a new qubit-cavity coupling

\[
\hat{H}_q \simeq \hbar \omega_q \hat{q}^\dagger \hat{q} - \frac{\hbar}{2} \omega_c \sigma_z,
\]

a slightly anharmonic oscillator with frequency \( \omega_q \) and anharmonicity strength \( \alpha_q \). Three types of couplings, summarized in Table I, will be discussed. Qubits and resonators are usually coupled together via the electrical interaction between the dipole moment field \( \hat{q} \) of the qubit and the field amplitude \( \hat{c} \) of the resonator. This field-field interaction is known as the transverse coupling resulting in the Jaynes-Cummings model with the coupling term \( g_x (\hat{q}^\dagger \hat{c} + \hat{q} \hat{c}^\dagger) \) in the Hamiltonian [11, 13]. In the dispersive limit [15], the qubit-cavity detuning \( \Delta = \omega_c - \omega_q \) exceeds the coupling strength \( |g_x| \ll |\Delta| \), and the cavity experiences an effective energy-energy interaction \( \frac{g_x^2}{2i(\Delta - \omega_q)} \hat{\sigma}_z \hat{c}^\dagger \hat{c} \), known as the dispersive or cross-Kerr interaction. It gives rise to a qubit-dependent frequency shift, mapping the state of the qubit to the signal phase probing the resonator and thus providing a good QND projective measurement [14, 16]. This transverse coupling has been extensively used in most circuit-QED experiments. State-of-the-art measurement fidelities and speeds using this standard dispersive technique are summarized in the first row of Table I. However, the dispersive readout is fundamentally limited by unavoidable higher order corrections to perturbation theory, which distort the qubit dynamics [17–19], and induce additional decay channels [20].

Several works have investigated how to overcome these limitations, designing new quantum circuits [21–28]. Implementing a coupling scheme that involves natively the energy of the qubit – as opposed to an effective energy interaction – resolves these limitations. Along this line, the longitudinal coupling \( g_x \hat{q} \hat{q}^\dagger (\hat{c}^\dagger + \hat{c}) \) [cf. second row of Table I] is remarkable. It induces a qubit-dependent displacement of the cavity field \( \hat{c} \) [27]. When combined with a parametric modulation \( g_x(t) \) at the cavity frequency \( \omega_c \), this interaction results in a faster separation of the pointer states with a QND-ness as high as \( Q = 98.4 \% \) [29, 30].
### Table I. State-of-the-art parameters for three different coupling types between an harmonic readout mode and a superconducting qubit.

| Type            | Elementary readout coupling | Effective QND readout coupling | QND fidelity | Single-shot readout fidelity | Detection time | State-of-the-art references |
|-----------------|-----------------------------|--------------------------------|--------------|------------------------------|----------------|-----------------------------|
| Transverse      | \( \sim g_s(\hat{q} + \hat{q}^\dagger)(\hat{c} + \hat{c}^\dagger) \) | \( \sim \frac{(g_s)^2\Delta}{\Delta_{\text{pump}}\Delta_{\text{det}}}(\hat{c}^\dagger \hat{c}) \) | Not given | 99.1%–99.6% | 48 ns–88 ns | [14] |
| Longitudinal    | \( \sim g_s(t\hat{q} \hat{q}^\dagger + \hat{c} \hat{c}^\dagger) \) | \( \sim g_s(t)(\hat{c}^\dagger + \hat{c}) \) | 98.4% | 98.9% | 750 ns | [29] |
| Cross-Kerr      | \( \sim g_u(\hat{q} + \hat{q}^\dagger)^2(\hat{c} + \hat{c}^\dagger)^2 \) | \( \sim g_u \hat{\sigma} \hat{c} \hat{c}^\dagger \) | 99% \pm 0.6% | 97.4% \pm 0.7% | 30 ns–50 ns | Present work |

This table compares the performance of different coupling schemes for qubit readout. The first column lists the type of coupling, followed by the elementary readout coupling and the effective QND readout coupling. The second column shows the QND fidelity, with references to the corresponding detection times. The state-of-the-art is indicated in the last column.

---

### Scheme based on a non-perturbative cross-Kerr interaction

The scheme presented here is based on a non-perturbative cross-Kerr interaction [cf. third row of Table I]. It leads to an alternative readout mechanism for superconducting qubits. This new process is fast, has a large single-shot fidelity, maximizes the QND nature of the process, and does not require any parametric modulation. Similar non-perturbative cross-Kerr couplings have been recently proposed for the readout of a flux qubit [31] and of a spin qubit [32]. However, our experimental setup builds on ideas previously proposed in Ref.[22] and it is realized with an artificial transmon molecule with one emergent qubit-like transmon degree of freedom and a bosonic ancilla that couples to the readout cavity [cf. Fig. 1a]. The qubit develops a Kerr-type interaction with the ancilla-cavity polariton branches [cf. Fig. 1b]. This interaction enables a detection scheme analogous to the standard dispersive measurement. Nevertheless, unlike transverse dispersive, since the coupling is not perturbative, it does not imply any cavity-mediated excitations or decay. Moreover, the strength of the readout shift can be made large, and is independent of the detuning, allowing to neglect any spurious qubit-resonator coupling via an increased detuning. This results in a very efficient single-shot QND readout of the qubit even in its first demonstration: it has a record QND-ness of 99\%, a fidelity of 97.4\%, while only requires a short measurement time of 50 ns. This readout mechanism can be combined with other paradigms of direct qubit-qubit interactions [33], as an upgrade to existing quantum computing and simulation architectures.

---

### II. TRANSMON MOLECULE INSIDE A CAVITY

In this section we give details on the physical mechanisms for the qubit readout using a non-perturbative cross-Kerr coupling. The setup demonstrating this new readout mechanism uses a transmon molecule (two coupled transmons) circuit [cf. Fig. 2c] inserted inside a cavity. We start by introducing the specific experimental system in Sec. II A, and then, in Sec. II B, we write down the theoretical model describing the open quantum dynamics of the system. We consider the strong coupling regime between cavity and ancilla, getting two strongly hybridized polariton modes. The qubit then couples strongly to these two polaritons via non-perturbative cross-Kerr couplings \( \chi_J \). This allows for an efficient readout of the qubit state via the transmission output of the cavity as shown below.

#### A. Physical implementation

The device consists of an aluminium Josephson circuit, which is deposited on an intrinsic silicon wafer and inserted in a 3D copper cavity [cf. Fig. 2a]. An optical...
The phase average and phase difference between the two external pads and the center plane is sketched in red. The cavity directions (ac, bc, cc) and sample directions (as, bs, cs) are represented. (c) Lumped element of the transmon molecule circuit. (d) Optical microscope and SEM pictures of the transmon molecule sample. The Josephson junctions are highlighted in red. The SQUID Josephson junctions implementing the coupling inductance $L_a$ are highlighted in green.

The image of the molecule circuit is shown Fig. 2d, which implements the lumped element circuit of Fig. 2c. The molecule is realized by coupling two identical transmon qubits through a large inductance. The two small Josephson junctions of the transmon are shunted by capacitance $C_s$ between the two external pads and the central one. The two transmon qubits are coupled by a large inductance $L_a$, obtained by a chain of 10 small SQUIDs of surface $S_{SQUID}$. Therefore this coupling inductance is tunable by an external flux $\Phi_s$. The circuit contains a second loop of surface $S$ linking the two transmon Josephson junction and the inductance $L_a$. It defines a second external flux $\Phi$ with $\Phi = r\Phi_s$, and $r = S/S_{SQUID} \simeq 26$. An additional capacitance $C_t$ is coupled the two external pads. As already discussed in previous work [21], the quantum dynamics of the transmon molecule in the case $\Phi = n\Phi_0$ (with $n$ an integer) can be described by

$$
\hat{H}_{mol} = 2E_{C_q}\hat{n}_q^2 - 2E_J\cos(\hat{x}_a) + 2E_{C_s}\hat{n}_s^2 - 2E_J\left(\cos(\hat{x}_a) - 2\frac{L_J}{L_a}(\hat{n}_s^{\dagger}\hat{n}_s)\right) - \frac{E_J}{2}\hat{x}_a^2\hat{x}_a^2,
$$

where the first line describes the qubit, the second line the anharmonic oscillator called hereafter ancilla mode and the last one the coupling between them. Here, the phase average and phase difference between the two transmon Josephson junctions are denoted by $\hat{x}_a$ and $\hat{x}_s$, respectively, whereas their conjugate charge operators, normalized by a Cooper pair charge $2e$, are denoted by $\hat{n}_a$ and $\hat{n}_a$. The quantity $E_{C_q} = e^2/2C_q$ corresponds to the charging energy of a single transmon, whereas, $E_{C_s} = e^2/2(2C_t + C_s)$ is also related to the capacitance $C_t$ between the two external pads. $E_J$ is the Josephson energy of the transmons, and $L_J = (\Phi_0/2\pi)^2/\pi^2$ the Josephson inductance. Except for the first two lines, we derived Eq. 1 by expanding to fourth order in $\hat{x}_a$ and $\hat{x}_s$.

To measure the transmon molecule, we insert the silicon chip inside a 3D copper cavity with a volume $24.5\, \text{mm} \times 5\, \text{mm} \times 35\, \text{mm}$ (length $\times$ height $\times$ width) along the $\text{ac}$, $\text{bc}$, and $\text{cs}$ directions, respectively (Fig. 2b). The cavity mode considered hereafter is the fundamental TE$_{001}$ mode with the microwave electric field aligned along the $\text{bs}$ direction. It is modeled as a harmonic oscillator with frequency $\omega_c$ and annihilation operator $\hat{c}$.

### B. Qubit-polaritons model

The first line in Eq. (1) corresponds to the Hamiltonian of a transmon qubit rewritten as $\hat{H}_q = \hbar \omega_q \hat{q}^\dagger \hat{q} - \alpha_q \hat{q}^{\dagger 2} \hat{q}^2$, with frequency $\omega_q$ and anharmonicity $\alpha_q$. The second line in Eq. (1) describes the ancilla mode $\hat{a}$, with frequency $\omega_a$ and anharmonicity $U_a$. Due to the presence of the coupling inductance $L_a$ and capacitance $C_t$, the ancilla anharmonicity $U_a$ is much weaker than the qubit anharmonicity $\alpha_q$. In our experiments, the ancilla will be weakly populated ($\langle \hat{a}^\dagger \hat{a} \rangle \lesssim 2$), allowing us to safely neglect the anharmonicity $U_a$, and regard the ancilla as an harmonic oscillator $\hat{H}_a = \hbar \omega_a \hat{a}^\dagger \hat{a}$ [cf. appendix B]. Interesting nonlinear and bistability effects arise when the ancilla is strongly populated ($\langle \hat{a}^\dagger \hat{a} \rangle \gg 1$), but these effects will be discussed elsewhere. Because of the circuit symmetry, there is neither a field-field (transverse) nor a field-energy (longitudinal) coupling between qubit and ancilla. The lowest order coupling corresponds to the last term in Eq. (1) and it is a direct consequence of the non-linearity of the Josephson junctions [34]. It is this term which will produce the cross-Kerr coupling between the qubit and the polariton modes.

To obtain such effect, we hybridize strongly the ancilla and the cavity mode by aligning the sample direction $\text{bs}$ to the cavity direction $\text{bc}$. In this way, we maximize the
coupling $g_a$ between the ancilla and the cavity. When neglecting residual asymmetry between the two transmons and misalignment between the sample and the cavity, the qubit-cavity transverse coupling is zero. This is guaranteed by the symmetry of the transmon molecule and of the TE$_{101}$ mode of the cavity. Consequently the cavity Hamiltonian and its interaction with the molecule circuit takes the simple form: $\hat{H}_{\text{cav}} = \hbar \omega_c \hat{c}^\dagger \hat{c} + \hbar g_a (\hat{a}^\dagger + \hat{a}) (\hat{c}^\dagger + \hat{c})$.

The total Hamiltonian of the system which includes the transmon molecule and the properly oriented cavity is then given by (cf. appendix B for details):

$$\frac{\hat{H}_{\text{tot}}}{\hbar} = \omega_q \hat{q}^\dagger \hat{q} - \alpha_q \hat{q}^\dagger \hat{q}^2 + \omega_a \hat{a}^\dagger \hat{a} + \omega_c \hat{c}^\dagger \hat{c} - \frac{g_{zz}}{2} (\hat{q}^\dagger + \hat{q})^2 (\hat{a}^\dagger + \hat{a})^2 + g_a (\hat{a}^\dagger + \hat{a}) (\hat{c}^\dagger + \hat{c}). \quad (2)$$

In our experiment cavity and ancilla modes are close to resonance and thus strongly hybridized. This leads to two new normal modes called upper and lower polariton modes, $\hat{c}_u$ and $\hat{c}_l$, which are a linear combination of ancilla and cavity fields, $\hat{a}^\dagger + \hat{a}$ and $\hat{c}^\dagger + \hat{c}$. In the rotating-wave approximation (RWA), they are given by a rotation $\hat{c}_u = \cos(\theta) \hat{a} + \sin(\theta) \hat{c}$, and $\hat{c}_l = \cos(\theta) \hat{c} - \sin(\theta) \hat{a}$ where the cavity-ancilla hybridization angle reads $\tan(2\theta) = 2g_a/(\omega_a - \omega_c)$. In terms of these polariton modes, the total Hamiltonian takes the form (cf. appendix B)

$$\frac{\hat{H}_{\text{tot}}}{\hbar} = \omega_u \hat{\sigma}_z + \sum_{j=u,l} \omega_j \hat{c}_j^\dagger \hat{c}_j - \sigma_z \sum_{j=u,l} \chi_j \hat{c}_j^\dagger \hat{c}_j \chi_j, \quad (3)$$

where $\omega_u \simeq \sin^2(\theta) \omega_c + \cos^2(\theta) \omega_a + \sin(2\theta) g_a$ and $\omega_l \simeq \cos^2(\theta) \omega_c + \sin^2(\theta) \omega_a - \sin(2\theta) g_a$. These are the frequencies of the upper and lower polariton modes, respectively. In addition, $\sigma_z = 2\hat{q}^\dagger \hat{q} - 1$ is the Pauli matrix of the transmon in the two-level system approximation, which interacts with the upper and lower polariton via non-perturbative cross-Kerr couplings $\chi_u = g_{zz} \cos^2(\theta)$ and $\chi_l = g_{zz} \sin^2(\theta)$, respectively. Each polariton is in some proportion cavity-like and therefore can be used as readout mode. Similarly, each polariton is also ancilla-like and therefore develop a cross-Kerr coupling with the qubit. We retrieve here the coupling between a qubit and a readout mode presented in the third row of Table I. It is important to note here that these cross-Kerr coupling strengths $\chi_j$ are non-perturbative, meaning they do not depend on the qubit-resonator detuning but only on the hybridization angle $\theta$ and the initial ancilla-qubit cross-Kerr coupling $g_{zz}$.

C. Conditional polaritons spectroscopy

Inspecting Eq. (3) we see that, except for dissipation and dephasing effects treated in appendix B, the population of the qubit $|\langle \hat{\sigma}_z \rangle_t_0 \rangle$ remains constant during the dynamics, with $t_0$ the initial time. The qubit’s main effect is thus simply to shift the transition frequencies of each polariton mode $\hat{c}_j$ as $\omega_j \rightarrow \omega_j - \chi_j \langle \hat{\sigma}_z \rangle_t_0$. The shift of the polariton resonances can be measured by shining a weak continuous coherent drive on the cavity and recording the amplitude of the field at the transmission output $|\langle \hat{a}_{\text{out}} \rangle_{ss} \rangle$ [cf. Fig. 1]. Fig. 3 shows typical spectroscopic measurements as a function of the driving frequency $\omega_d$, with the blue and red curves corresponding to the case the qubit is prepared in states $|g \rangle$ ($\langle \hat{\sigma}_z \rangle_{t_0} \approx -1$) and in $|e \rangle$ ($\langle \hat{\sigma}_z \rangle_{t_0} \approx +1$), respectively.

A complete description of the system, including losses and dephasing of the qubit, ancilla, and cavity, can be obtained by a master equation formalism as shown in appendix B. Using this formalism we derive a compact expression describing the transmission amplitude of output field, which reads

$$|\langle \hat{c}_{\text{out}} \rangle_{ss} \rangle = \sqrt{\kappa_{\text{out}}} \sin(\theta) \langle \hat{c}_u \rangle_{ss} \cos(\theta) \langle \hat{c}_l \rangle_{ss} \right)$$

$$= -i\Omega \sqrt{\kappa_{\text{out}}} \kappa / 2 + \Gamma_{\text{eff}} - i \delta_{\text{eff}}.$$  \quad (5)

Here, $\Omega = \sqrt{\kappa_P \kappa_a}$ is the strength of the microwave drive, with $\kappa_a$ the coupling to the input port and $P_a$ the input power. In addition, $\kappa_{\text{out}}$ describes the coupling to the output of the cavity and $\kappa = \kappa_{\text{out}} + \kappa_{\text{in}}$ is the total cavity decay. The resonances of the lower and upper modes appear at the two driving frequencies $\omega_d$ that satisfy $\delta_{\text{eff}} = 0$, where $\delta_{\text{eff}}$ is the effective detuning of the cavity given by

$$\delta_{\text{eff}} = \omega_d - \omega_c = -g_a^2 (\omega_d - \omega_c + g_{zz} \langle \sigma_z \rangle_{t_0}) \left(\frac{2 \omega_d - \omega_a + g_{zz} (\sigma_z)_{t_0}}{(\omega_d - \omega_a + g_{zz} (\sigma_z)_{t_0})^2 + (\Gamma_a)^2} \right)^2. \quad (6)$$

Here, $\Gamma_a$ is the ancilla decoherence, including dissipation and pure dephasing as shown in appendix B. On the
other hand, the widths and heights of the resonances are determined not only by the cavity decay \( \kappa \), but also by the ancilla effective decoherence \( \Gamma_a^{\text{eff}} \) given explicitly by

\[
\Gamma_a^{\text{eff}} = \frac{g_2^2 \Gamma_a}{(\omega_d - \omega_a + g_{zz}(\sigma_z)_B)^2 + (\Gamma_a)^2}.
\]  

In Fig. 3 the transmitted signal is measured using a 500 ns square microwave pulse applied immediately after preparing the qubit in \( |g\rangle \) or \( |e\rangle \) states. Two resonance peaks are observed corresponding to the two polariton modes and qubit state dependent frequency shifts are clearly visible. The lineshapes are fitted using Eq. (5). The peaks of the lower and upper polariton branches are indeed shifted by \( \sim 2 \chi_j \), up to small errors in the calibration and initial state preparation of the qubit states \( |g\rangle \) and \( |e\rangle \).

### III. SINGLE-SHOT QUANTUM NON-DEMOLITION MEASUREMENTS

#### A. Individual measurement records and quantum trajectories

Readout is performed using a standard microwave set-up including a high saturation-power Josephson parametric amplifier made from a SQUID chain [35]. Next we consider the readout performance at zero flux measuring the signal transmitted through the lower polariton \( j = l \).

To readout the qubit state a coherent microwave tone is applied at frequency \( \omega_a \) and initial state preparation of the qubit states \( |g\rangle \) and \( |e\rangle \). We define \( I(t) \) and \( Q(t) \) the in-phase and the quadrature microwave signal. Its phase has been adjusted so the information about the qubit state is only contained in \( I(t) \).

One thousand individual trajectories have been measured when the qubit is prepared either in \( |g\rangle \) or \( |e\rangle \) state. Four typical individual records are plotted in Fig. 4. The duration pulse is 1000 ns acquired over a larger time window (around 1300 ns). These measurement records give an insight on the real time dynamics of the qubit from single-shot trajectories. Notice that after a time of few \( \kappa_l^{-1} \sim 15 \text{ ns} \), the qubit state can already be inferred from a single trajectory, and that in Fig. 4(b) a quantum jump [38] of the qubit appears clearly. In addition to the individual trajectories, the mean value averaged over the one thousand trials, as well as the related standard deviation, is plotted as function of time. Due to qubit relaxation, the averaged excited state response (red solid line) decays towards the ground state response, while its corresponding standard deviation (red shaded area) grows in time. This finite qubit lifetime, limits the distinguishability for long measurement and highlights the need for a fast readout. The qubit decay under drive \( T_{1,\text{drive}} \) is similar to the one measured without drive \( T_1 \sim 3.3 \mu\text{s} \). Therefore, the measurement does not disturb the qubit relaxation, indicating a QND measurement.

#### B. Quantum non-demolition fidelity

To check the QND-ness of the measurement, we quantify the repeatability of successive measurements. We
now consider only the measurement records between time $100 \text{ns}^{-1} \sim 150 \text{ns}$ and $1000 \text{ns}$ to be in the steady state regime of the applied squared pulse. It corresponds to the ground state if $I(t) < I_{th}$ or to the excited state if $I(t) > I_{th}$ with $I_{th} = 15.5 \text{mV}$. We define four conditional probabilities, $P_{\alpha,\beta}$, the probability to measure $\alpha$ in the first measurement and $\beta$ in the second measurement, where $\alpha, \beta = g, e$ can correspond to ground or excited states. From these probabilities, the QND fidelity [29] is obtained to be $Q = \frac{P_{a,e} + P_{e,a}}{2} = 99\%$. Here, the imperfect value of $P_{a,e} = (98.3 \pm 0.7)\%$ is explained by the relaxation during measurement, and the value $P_{g,g} = (99.6 \pm 0.02)\%$ is justified by the thermal excitations during measurement. Moreover, each probability has an extra uncertainty due to finite counting of $\pm 0.6\%$. These results are comparable to the QND fidelity obtained in Touzard et al [29] using a parametric modulation scheme and corresponds, to the best of our knowledge, to the state-of-the-art values.

C. Single-shot readout fidelity

In the early days of circuit-QED, averaging was necessary to infer the qubit state with high fidelity. However, thanks to the advent of Josephson-based amplifier [39–41], high fidelity, single shot discrimination of the qubit state is now possible [42]. Since then, works have been performed on Purcell filters and amplifiers in an attempt to increase further the readout fidelity [16, 43–45], which is now culminating at $99.6\%$ in 88 ns [14]. Readout fidelity is currently limited by the balance between the time needed to discriminate the qubit state and the qubit $T_1$.

To quantify the readout fidelity, we perform heralding [46] by applying first a 50 ns square readout pulse. In the analysis, we keep only the sequences where the qubit is found in the ground state for this first measurement. After this pulse, we wait $300 \text{ns} \sim 200 \text{ns}^{-1}$ for the resonator to decay back into its vacuum state before preparing the qubit in the ground or in the excited state. Then another 50 ns square readout pulse is applied. The two measurement pulses correspond to a steady state amplitude of $\pi_t \approx 2$. In Fig. 5, histograms of $24 \cdot 10^3$ single shot readouts are plotted as the function of the in-phase amplitude when the qubit is prepared in $|g\rangle$ and $|e\rangle$ states. A weight function is used to maximize the distinguishability between the two qubit states [14]. The histograms are fitted by the sum of two Gaussians (colored solid lines). The intersection of these two fitted histograms defines a threshold $I_{th}$ (vertical dash line) distinguishing the two qubit state. The readout fidelity is defined as $F = 1 - (P(e|g) + P(g|e))/2 \approx 1 - (\epsilon_g + \epsilon_e)/2$, where $P(x|y)$ is the probability of reading out $x$ while having prepared the state $y$. In addition, $\epsilon_g$ and $\epsilon_e$ are the fraction of measured events of detecting $I \geq I_{th}$ when the qubit was prepared in $g$ and $I \leq I_{th}$ when the qubit was prepared in $e$, respectively. We obtained a readout fidelity of $F = 97.4\%$ affected by the imperfections $\epsilon_g = 1.0\%$, and $\epsilon_e = 4.3\%$. Here, we distinguish different sources of error as $\epsilon = \epsilon_a + \epsilon_o = \epsilon_o + \epsilon_r + \epsilon_l$ where $\epsilon_a$ is the overlap error (green shaded area), $\epsilon_o$ is the assignment error (red and blue shaded areas) which can be decomposed into $\epsilon_r$, the preparation error and $\epsilon_l$, the transition during measurement error. We also have overlap errors $\epsilon_o,g = \epsilon_o,e = 0.4\%$. In total, we have an assignment error of $\epsilon_o,e = 3.9\%$ in which we expect $\sim 1.5\%$ due relaxation during measurement and $\sim 1.9\%$ error due to imperfections in the $\pi$-pulse. The leftover errors are within the uncertainty due to finite counting of $\pm 0.7\%$, but may be attributed to a not perfect heralding procedure or possibly to measurement-induced transitions [18]. We believe that the readout fidelity can be further increased by implementing pulse envelop optimization such as DRAG pulse [47] to have less excited state preparation error, or CLEAR pulse [48] to achieve better discrimination in a shorter integration time and therefore reduce error due to relaxation during measurement.

D. Coherence and readout quality factor

Both QND-ness and single-shot readout fidelity are limited by the finite $T_1$ of the qubit. To understand qubit lifetime limitations, we have measured its relax-
SNR can be further improved.

IV. CONCLUSION AND OUTLOOKS

In conclusion, we have developed an original qubit readout scheme relying on a non-perturbative cross-Kerr coupling, in contrast to the usual cross-Kerr coupling that is perturbatively obtained from the transverse coupling in the dispersive regime. Therefore, our new experimental measurement design does not suffer from cavity-mediated excitations or decay, and the strength of the readout shifts can be made large and independent of the detuning. This allows for a fast readout of the qubit, with a large single-shot fidelity, and a maximization of the QND-nature of the measurement. The qubit and readout performances are currently limited because of residual qubit–cavity transverse couplings. However, no fundamental reason prevents further suppression of this transverse coupling. In fact, we can obtain the same readout shifts 2χg from the dispersive readout, but having a much larger qubit-polaritons detuning, which may allow us to significantly reduce the unwanted consequences of the residual transverse coupling in the future.

According to our readout error budget and to our QND-ness analysis, the measurement-induced qubit state mixing is particularly low compared to the standard literature. This could be explained by the non-perturbative nature of our cross-Kerr coupling and will be the topic of future investigations.

ACKNOWLEDGMENTS

The authors thank D. Basko, D. Divincenzo, B. Huard and K. Rafsanjani for fruitful discussions. R.D. acknowledges support from Fondation CFM pour la recherche. R.D., V.M. and O.B. acknowledge support from ANR REQUIEM (ANR-17-CE24-0012-01), J.J.G.-R. and T.R. acknowledge support from MINECO/FEDER Project No. FIS2015-70856-P and CAM/FEDER Project No. S2018/TCS-4342 (QUITEMAD-CM). T.R. further acknowledges funding from the EU Horizon 2020 program under the Marie Sklodowska-Curie grant agreement No. 709837. S. L acknowledges the Agence Nationale de la Recherche under the programme Investissements d’avenir (ANR-15-IDEX-02). J. D. and K. B. acknowledges the European Union’s Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 754303.

Appendix A: Experimental setup

In this section we describe the measurement setup shown in Fig. 7. Qubit and readout pulses are sent through the same input line. The transmitted signal

FIG. 6. T1 versus flux. Black points and errorbars are the extracted Gaussian means and standard deviations respectively. Red shadows are computed Purcell-limited T1 with a one-mode cavity and parameters described in Appendix C and F with a 10% uncertainty margin. For the dashed line contour, only the asymmetry in critical current is considered, for the dotted line contour, only the misalignment is considered and for the solid line contour, both imperfections are considered.

\[ T_1 = \frac{\text{signal-to-noise ratio (SNR) per photon number and a magnitude lower than the measured } T_1}{\text{fundamental reason prevents further suppression of this transverse coupling. In fact, we can obtain the same readout shifts } 2\chi g \text{ of the dispersive readout, but having a much larger qubit-polaritons detuning, which may allow us to significantly reduce the unwanted consequences of the residual transverse coupling in the future.}} \]

\[ \text{Assuming our readout shift was obtained by the usual dispersive transverse coupling as } \chi = \frac{(\alpha_j)^2}{\Delta(\alpha_j^2 - \sigma_j^2)} = 2\pi \text{ MHz, with other parameters constant, we found a transverse coupling } g_x = 2\pi \times 169 \text{ MHz, resulting in a Purcell-limited } T_1 \approx \frac{\kappa^{-1} (\Delta/g_x)^2}{\kappa^2} \approx 0.3 \mu s, \text{ one order of magnitude lower than the measured } T_1. \]

\[ \text{Despite this limited } T_1, \text{ we achieve a good steady state signal-to-noise ratio (SNR) per photon number and a good readout quality factor defined as } Q_r = 4\chi^2/(\kappa^2/4 + \chi^2)\kappa T_1 \approx 427. \text{ Indeed, the optimal steady state SNR is given by } [49] \text{ SNR} = \eta n Q_r \text{ with } n \text{ the photon number and } \eta \text{ the quantum efficiency. As a comparison, Ref. [16] reports } Q_r = 540 \text{ and Ref. [14] } Q_r = 1075. \text{ In the future, our currently Purcell-limited } T_1 \text{ can be largely improved, without sacrificing on } \kappa \text{ and } \chi, \text{ by a better optimization of parameters. In this way, we believe that an increase of one order of magnitude in } Q_r \text{ is within reach. Moreover, we expect the photon number limitation to be less restrictive for the non-perturbative cross-Kerr coupling than for the perturbative one and thus the steady state SNR can be further improved.} \]
for the transmon molecule reads,

$$\frac{\hat{H}_{\text{mol}}}{\hbar} = \tilde{\omega}_q \hat{q}^\dagger \hat{q} - \frac{K_q}{4} (\hat{q} + \hat{q}^\dagger)^4 + \tilde{\omega}_a \hat{a}^\dagger \hat{a}$$  \hspace{1cm} (B1)

$$- \frac{K_a}{4} (\hat{a} + \hat{a}^\dagger)^4 - \frac{g_{zz}}{2} (\hat{q} + \hat{q}^\dagger)^2 (\hat{a} + \hat{a}^\dagger)^2,$$

where $\hat{q}$ and $\hat{a}$ are the annihilation operators for qubit and ancilla modes, satisfying $\hat{x}_q = (\hat{q} + \hat{q}^\dagger)/\sqrt{2}$ and $\hat{x}_a = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$. In case of no asymmetry, the Hamiltonian parameters are given in terms of microscopic circuit parameters as $\tilde{\omega}_q = \frac{\hbar}{2} \sqrt{2E_C E_J}$, $\tilde{\omega}_a = \frac{1}{\hbar} \sqrt{2E_J E_C a} (1 + \frac{2L_L}{L_0})$, $K_q = \frac{E_C a}{24 \hbar}$, $K_a = \frac{E_C a}{24 \hbar} \frac{1}{1 + 2 \frac{L_L}{L_0}}$, and $g_{zz} = \frac{\sqrt{E_C E_C a}}{8 \hbar} \sqrt{\frac{1}{1 + 2 \frac{L_L}{L_0}}}$. We can write the above Hamiltonian in normal ordering and perform the RWA, provided the couplings are much smaller than the free frequencies, i.e. $K_q, K_a, g_{zz} \ll \tilde{\omega}_q, \tilde{\omega}_a$, and obtain

$$\frac{\hat{H}_{\text{mol}}}{\hbar} = \omega_q \hat{q}^\dagger \hat{q} - \alpha_q \hat{q}^\dagger \hat{q} \hat{q} ^\dagger \hat{q} + \omega_a \hat{a}^\dagger \hat{a} - U_a \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} + g_{zz} \hat{a}^\dagger \hat{a} (2 \hat{q}^\dagger \hat{q} - 1) - \frac{g_{zz}}{2} (\hat{a}^\dagger \hat{a} \hat{q} \hat{q} + \text{h.c.})$$ \hspace{1cm} (B2)

Here, we have done the identifications $\omega_q = \tilde{\omega}_q - g_{zz} - 3K_q$, $\omega_a = \tilde{\omega}_a - 2g_{zz} - 3K_a$, $U_a = 3K_a/2$, and $\alpha_q = 3K_q/2$. Since in our experiments qubit and ancilla are largely detuned, $|\omega_q - \omega_a| \gg g_{zz}$, we can neglect the last term in Eq. (B2) applying an additional RWA. In addition, since the population and anharmonicity of the ancilla are small for the parameter regime considered throughout this work, i.e. $(\hat{a}^\dagger \hat{a}) \lesssim 1$ and $U_a \ll g_{zz}, \alpha_q$, respectively, we can also neglect the ancilla anharmonic term in the first line of Eq. (B2). Regarding the coupling between ancilla and cavity mode, $H_{\text{cav}} = \hbar \omega_c \tilde{c}^\dagger \tilde{c} + \hbar g_a (\tilde{c}^\dagger + \tilde{c}) (\hat{a}^\dagger \hat{a})$, we also apply the RWA provided $g_a \ll \omega_c \sim \omega_q$. After all the above approximations, the total Hamiltonian $\hat{H}_{\text{tot}} = \hat{H}_{\text{mol}} + \hat{H}_{\text{cav}}$ can be written as

$$\frac{\hat{H}_{\text{tot}}}{\hbar} = \omega_q \hat{q}^\dagger \hat{q} - \alpha_q \hat{q}^\dagger \hat{q} \hat{q} ^\dagger \hat{q} + \omega_a \hat{a}^\dagger \hat{a} + \omega_c \tilde{c}^\dagger \tilde{c}$$ \hspace{1cm} (B3)

$$- g_{zz} \hat{a}^\dagger \hat{a} (2 \hat{q}^\dagger \hat{q} - 1) + g_a (\tilde{c}^\dagger \tilde{c} + \hat{a}^\dagger \hat{a} + \hat{a}^\dagger \hat{a}^\dagger \hat{c}).$$

Finally, we notice that in our experiments the cavity and ancilla are close to resonance, so that these two modes become strongly hybridized into upper and lower polariton modes defined as $\hat{c}_u = \cos(\theta) \hat{a} + \sin(\theta) \tilde{c}$, and $\hat{c}_l = \cos(\theta) \tilde{c} - \sin(\theta) \hat{a}$, respectively, with $\tan(2\theta) = 2g_a/(\omega_a - \omega_c)$. Re-expressing the total Hamiltonian (B3) in terms of terms of these porlation modes, and doing the two-level approximation in the qubit subspace $\sigma_0 = 2 \hat{q}^\dagger \hat{q} - 1$ due to its large anharmonicity $\alpha_q$, we obtain the final Hamiltonian in Eq. (3) of the main text.

On the other hand, a realistic circuit setup will also present dissipation and dephasing of the different components, so that the full dynamics of the open system is...
obtains a linear system of equations for (B4). Indeed, in the case of weak driving $\Omega \ll \kappa$, the asymmetry of the cavity field at the transmission output port.

- suppress fully this asymmetry. Considering this asymmetry, the molecule circuit. It is experimentally challenging to describe the pulses for the transmission and QND measurements performed throughout this work.

- Using input-output relation, $\hat{c}_{\text{out}}(t) = \hat{c}_{\text{in}}(t) - \sqrt{\kappa_{\text{out}}}(\hat{c}_{\text{ss}})$, we can find a simple expression for the amplitude of the cavity field at the transmission output port. Taking expectation values on steady state, we obtain $\langle \hat{c}_{\text{out}} \rangle_{\text{ss}} = \sqrt{\kappa_{\text{out}}} \langle \hat{c} \rangle_{\text{ss}}$, where $\langle \hat{c} \rangle_{\text{ss}}$ can be easily obtained from the steady state solution of the master equation (B4). Indeed, in the case of weak driving $\Omega \ll \kappa$, one obtains a linear system of equations for $\langle \hat{c} \rangle_{\text{ss}}$, and $\langle \hat{a} \rangle_{\text{ss}}$, and the solution of the former is shown in Eq. (5) of the main text.

**Appendix C: Imperfections**

We have identified two main sources of imperfections that can lead to a non-zero qubit-cavity transverse coupling, therefore limiting the readout performances.

- The first one is the Josephson junction asymmetry $d_J = |E_{J_1} - E_{J_2}|/(E_{J_1} + E_{J_2})$ in the transmon molecule circuit. It is experimentally challenging to suppress fully this asymmetry. Considering this asymmetry, we need to add in the Hamiltonian (1) a new term $-2E_Jd_J \sin(\hat{x}_q)\sin(\hat{x}_a)$, where now $E_J$ is the mean Josephson energy of the two Josephson junctions. At first order, this new term corresponds to a transverse coupling between the qubit and the ancilla. It is therefore important to be able to characterize it. We measured the room temperature resistances between each pad of the sample. These resistances have contributions from the Josephson junction resistances $R_{J_1}, R_{J_2}$, the resistance of the array of SQUID and resistances from the connecting wires. The wire resistances are estimated via measurement of wires-only test structures on a dedicated test-chip fabricated during the same process. In the end, we solve a set of 3 equations with 3 unknowns and found an asymmetry $d_J = |R_{J_1} - R_{J_2}|/(R_{J_1} + R_{J_2}) = 1.3\%$.

- The second imperfection is a misalignment of the sample inside the 3D cavity. Considering the size of the cavity groove and of the sample, we estimate a misalignment angle up to $\theta_m = \pm 5$ deg. Assuming roughly that the ratio of transverse coupling $g_t/g_a$ is given by $\tan(\theta_m)$, we estimate a qubit-cavity transverse coupling of $|g_t|/2\pi \lesssim 25$ MHz $\ll g_a$.

**Appendix D: Hamiltonian valid at all flux**

When a non-integer quantum flux is applied, the symmetry is broken and other terms arise in the molecule circuit Hamiltonian, namely

$$
\hat{H}_{\text{mol,tot}} = 2E_C\hat{n}_q^2 + 2E_C\hat{n}_a^2 - 2E_J\cos(\hat{x}_q)\cos(\hat{x}_a) + d_J \sin(\hat{x}_q)\sin(\hat{x}_a) - 2\frac{L_J}{L_a(\Phi)}(\hat{x}_a - \frac{\pi \Phi}{\Phi_0})^2.
$$

To fit the energy spectrum versus flux, we numerically diagonalize, in a $(8 \times 8 \times 8)$ basis, the total Hamiltonian $H_{\text{tot}} = H_{\exp}^{\text{mol,tot}} + \hat{H}_{\text{cav}}$, where $H_{\text{mol,tot}}$ is the Taylor expansion to fourth order of Eq. (D1) and $H_{\text{cav}} = \hbar \omega_c \hat{c}^\dagger \hat{c} + \hbar g_a(\hat{a}^\dagger + \hat{a})(\hat{c}^\dagger + \hat{c})$. This Taylor expansion is valid at integer quantum flux but becomes less valid around frustration points [50]. Nonetheless, the eigenenergies are fitted within 2 % errors.

**Appendix E: System characterization**

1. Qubit-polaritons spectroscopy

Fig. 8(a) presents the single tone spectroscopy performed by measuring the cavity transmission versus magnetic flux $\Phi$ and frequency. The two resonant polariton modes are observed as two maximal transmission peak which strongly vary with $\Phi$. It demonstrates a direct coupling to the traveling microwave signal. The bare cavity resonant frequency $\omega_c^{\text{bare}}/2\pi = 7.169$ GHz of the fundamental mode has been measured at 4 K but it is no longer visible at this frequency. Indeed because of its strong hybridization with the ancilla mode, the cavity is now split in the two polariton modes. From the cavity, they inherit their direct coupling to traveling microwave signal. From the ancilla, they get a flux dependence. The two polariton frequencies vary rapidly in flux with a period given by flux quantization in the large circuit loop. In addition a slow variation is superimposed and this affects differently to the two modes. The two polariton modes present a non linear response inherited from the ancilla anharmonicity. When the input microwave power is large, the polariton dynamics shows a bi-stability behaviour. This regime is beyond the scope of this article and we consider here only the low input power in the linear regime.

No qubit resonance is directly detected via single tone spectroscopy. Therefore two-tone spectroscopy is needed to reveal it. One tone is swept between 5.5 GHz and 6.4 GHz in the vicinity of the qubit resonance. The second tone measures the transmission signal at the resonant frequency of one of the polariton modes. This two tone spectroscopy reveals the qubit flux dependence (cf. Fig. 8(b)). We observed a flux dependence periodic in $\Phi_o$ but without any superimposed slow variation.
FIG. 8. (a) Single tone transmission $S_{21}$ measurements as function of frequency and flux (coil current). (b) Two-tone measurement, where the corresponding transmission $S_{21}$ is normalized by its value without second tone. (c) The extracted resonant frequencies (color lines: qubit in blue, lower polariton in orange and upper polariton in purple) are fit via the numerical model (black dash lines) discussed in Appendix D.

The resonant frequencies of the qubit and the two polariton modes are extracted from the single and two tone spectroscopy and plotted in Fig. 8(c) as function of flux $\Phi$. They are perfectly fitted by the numerical model presented in appendix D. The model precisely described the flux variation of the qubit as well as the two polariton modes. From the fit shown in Fig. 8(b), the circuit parameters are determined and are listed in appendix F. Their values are consistent with estimation based on HFSS simulation and room temperature resistance measurements of the Transmon Josephson junction and SQUID chains. At non-integer reduced flux ($\Phi/\Phi_0 \neq n$), the symmetry is broken and the molecular Hamiltonian in Eq. (1) must be considered, which takes into account additional coupling terms such as $\hat{x}_a^2 \hat{x}_a$ [21]. These terms complexify the quantum dynamics of the system and understanding their effect is beyond the scope of this paper. Hereafter we will only consider the working points at integer reduced flux ($\Phi/\Phi_0 = n$). At these fluxes, the qubit frequency is set to $\omega_q/2\pi = 6.280\text{GHz} \pm 4\text{MHz}$ (with small variations due to frequency renormalization) and only the ancilla frequency, and thus the polariton frequencies, can vary.

2. Polaritons tunability

Interestingly, the different flux working points allow to tune the ancilla-cavity hybridization angle without affecting the qubit frequency [cf. Fig. 8]. Therefore, we can in-situ tune the parameters $\omega_i$ and $\chi_j$ in Eq. (3).

In Fig. 9(a), the two polaritons resonant frequencies are plotted at integer flux quantum $n$. They are quantitatively described by the lower and upper polariton modes $\hat{c}_l$ and $\hat{c}_u$ previously discussed. Here we set the bare cavity frequency to the value measured at 4 K and the bare ancilla is extracted from the expression $\omega_a = \omega_i + \omega_u - \omega_c$. On resonance ($\omega_i = \omega_c$), the two polaritons are maximally hybridized. We measure $g_a/2\pi = 295\text{MHz}$ from

FIG. 9. (a) The lower (orange) and upper (purple) polariton resonant frequencies as function of integer quantum flux. They are fitted (black lines) using the numerical model discussed in Appendix D. The grey dashed lines correspond to the bare cavity and bare ancilla frequencies. An avoided crossing between ancilla and cavity can thus be seen. (b) Cross-Kerr strengths between qubit and lower (orange) and upper (purple) polaritons. Black lines are the expected cross-Kerr coupling using $\chi_l = g_{zz} \sin^2(\theta)$ and $\chi_u = g_{zz} \cos^2(\theta)$ with $g_{zz}/\pi = 69\text{MHz}$. The grey diamonds are simulated points computed using Black Box Quantization [51] with EM simulation.
the anti-level crossing. The hybridization weights \(\sin^2(\theta)\) and \(\cos^2(\theta)\) between cavity and ancilla are then fitted. At zero flux the upper polariton mode is mainly ancilla-like while the lower polariton is mainly cavity-like. When the cavity and ancilla are resonant, the hybridization weight is 50%.

Each polariton resonance is shifted by the cross-Kerr coupling strength \(2\chi_j\) conditioned on the qubit state. The cross-Kerr coupling between the qubit and the two polariton modes are plotted in Fig. 9(b) as a function of integer flux quantum. A single tone spectroscopy is performed around the polariton resonance when a \(\pi\)-pulse is applied or not. Because of relaxation, these experiments are performed in the time domain with a 30 ns \(\pi\)-pulse immediately followed by a 500 ns readout pulse. The cross-Kerr coupling is quantitatively described by \(2\chi_j\) as predicted by the effective polariton model. We measured large readout shifts \(\chi_j/\pi\) from 9 to 58 MHz thanks to the non-perturbative cross-Kerr coupling. These readout shifts are neither limited by the validity of the dispersive approximation nor by the multi-level aspects of the transmon. For instance, in Ref. [14] the effective coupling for readout has been optimized and is reported to be \(\chi = \frac{(g_{\pi})^2\alpha_j}{\Delta(\Delta - \omega_j)} = 2\pi \cdot 7.9\) MHz. This is on the order or below of what we can achieve with the present setup without doing an intense optimization of our parameters. Interestingly, at zero flux, the upper polariton, which is further detuned from the qubit than the lower polariton, has a stronger readout shift than the lower polariton.

### Appendix F: Circuit parameters

| \(\omega_q/2\pi\) | \(\omega_{q}/2\pi\) | \(\omega_{l}/2\pi\) | \(\omega_{u}/2\pi\) | \(\omega_{a}/2\pi\) |
|------------------|------------------|------------------|------------------|------------------|
| 6.284            | 7.780            | 7.169            | 7.038            | 7.911            |

| \(g_{zz}/\pi\) | \(g_{uu}/\pi\) | \(g_{zz}/\pi\) | \(g_{uu}/\pi\) |
|----------------|----------------|----------------|----------------|
| 69             | 295            | 9              | 57             |

**TABLE II. Frequencies and interaction strengths at zero flux**

\[
T_{1} \quad T_{2} \quad C_{a}/2\pi \quad \kappa_{a}/2\pi \quad \kappa_{l}/2\pi \quad \kappa_{u}/2\pi \\
3.3 \mu s \quad 3.2 \mu s \quad 4.9 MHz \quad 18 MHz \quad 10 MHz \quad 3.5 MHz
\]

**TABLE III. Coherence and decay of the different modes at zero flux.**

| \(I_{C}\) (nA) | \(|L_{a}|\) (nH) | \(C_{q}\) (fF) | \(C_{l}\) (fF) | \(d_{f}\) (%) |
|----------------|-----------------|----------------|----------------|-----------|
| 49.6           | 8.6             | 92             | 32             | 1.3       |

**TABLE IV. Circuit parameters**

We summarize in Table II the different frequencies of the modes and the different coupling strengths at zero flux. We also detail in Table III the decay and coherence rates of the modes at zero flux. The microscopic circuit parameters are displayed in Table IV.

[1] J. Preskill, *Quantum* 2, 79 (2018).
[2] Y. Li and S. C. Benjamin, *Phys. Rev. X* 7, 021050 (2017).
[3] E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland, *Phys. Rev. A* 77, 012307 (2008).
[4] D. P. Divincenzo, *Fortschritte der Physik* 48, 771 (2000), arXiv:quant-ph/0002077 [quant-ph].
[5] J. Kelly, R. Barends, A. G. Fowler, A. Megrant, E.Jeffrey, T. C. White, D. Sank, J. Y. Mutus, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, I.-C. Hoi, C. Neill, P. J. O’Malley, C. Quintana, P. Roushan, A. Vainsencher, J. Wenner, A. N. Cleland, and J. M. Martinis, *Nature* 519, 66 (2015).
[6] P. Schindler, J. T. Barreiro, T. Monz, V. Nebendahl, D. Nigg, M. Chwalla, M. Hennrich, and R. Blatt, *Science* 332, 1059 (2011).
[7] A. Bermudez, X. Xu, R. Nigmatullin, J. O’Gorman, V. Negnevitsky, P. Schindler, T. Monz, U. G. Poschinger, C. Hempel, J. Home, F. Schmidt-Kaler, M. Biercuk, R. Blatt, S. Benjamin, and M. M"uller, *Phys. Rev. X* 7, 041061 (2017).
[8] J. M. Gambetta, J. M. Chow, and M. Steffen, *npj Quantum Information* 3, 2 (2017).
[9] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, *Rev. Mod. Phys.* 75, 281 (2003).
[10] C. J. Ballance, T. P. Harty, N. M. Linke, M. A. Sepiol, and D. M. Lucas, *Phys. Rev. Lett.* 117, 060504 (2016).
Korotkov, and J. M. Martinis, Phys. Rev. Lett. 117, 190503 (2016).

[19] R. Lescanne, L. Verney, Q. Ficheux, M. H. Devoret, B. Huard, M. Mirrahimi, and Z. Leghtas, Physical Review Applied 11 (2019).

[20] A. A. Houck, J. A. Schreier, B. R. Johnson, J. M. Chow, J. Koch, J. M. Gambetta, D. I. Schuster, L. Frunzio, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Physical Review Letters 101 (2008).

[21] F. Lecocq, J. Claudon, O. Buisson, and P. Milman, Phys. Rev. Lett. 107, 197002 (2011).

[22] I. Diniz, E. Dumur, O. Buisson, and A. Auff´eves, Phys. Rev. A 87, 033837 (2013).

[23] A. J. Kernen, New J. Phys. 15, 123011 (2013).

[24] E. Dumur, B. Küng, A. K. Feofanov, T. Weissl, N. Roch, C. Naud, W. Guichard, and O. Buisson, Physical Review B 92 (2015).

[25] P.-M. Billangeon, J. S. Tsai, and Y. Nakamura, Phys. Rev. B 91, 094517 (2015).

[26] S. Richer and D. DiVincenzo, Phys. Rev. B 93, 134501 (2016).

[27] ´E. Dumur, B. K¨ung, A. K. Feofanov, T. Weissl, N. Roch, C. Naud, W. Guichard, and O. Buisson, Physical Review B 92 (2015).

[28] J. Ikonen, J. Goetz, J. Ilves, A. Kernen, A. M. Gunyho, M. Partanen, K. Y. Tan, D. Hazra, L. Grnberg, V. Vesterinen, S. Simbierowicz, J. Hassel, V. Weinfang, J. de Sterke, A. Bruno, R. Schouten, and L. DiCarlo, Phys. Rev. Applied 6, 034008 (2016).

[29] J. Gambetta, A. A. Houck, D. I. Schuster, L. Frunzio, J. M. Devoret, S. M. Girvin, and R. J. Schoelkopf, Physical Review Letters 94 (2005).

[30] J. Gambetta, A. Blais, D. I. Schuster, A. Wallraff, L. Frunzio, J. M. Devoret, S. M. Girvin, and R. J. Schoelkopf, Physical Review A 74 (2006).

[31] J. Ikonen, J. Goetz, J. Ilves, A. Kernen, A. M. Gunyho, M. Partanen, K. Y. Tan, D. Hazra, L. Grnberg, V. Vesterinen, S. Simbierowicz, J. Hassel, V. Weinfang, J. de Sterke, A. Bruno, R. Schouten, and L. DiCarlo, Phys. Rev. Applied 6, 034008 (2016).

[32] J. Ikonen, J. Goetz, J. Ilves, A. Kernen, A. M. Gunyho, M. Partanen, K. Y. Tan, D. Hazra, L. Grnberg, V. Vesterinen, S. Simbierowicz, J. Hassel, V. Weinfang, J. de Sterke, A. Bruno, R. Schouten, and L. DiCarlo, Phys. Rev. Applied 6, 034008 (2016).

[33] J. Ikonen, J. Goetz, J. Ilves, A. Kernen, A. M. Gunyho, M. Partanen, K. Y. Tan, D. Hazra, L. Grnberg, V. Vesterinen, S. Simbierowicz, J. Hassel, V. Weinfang, J. de Sterke, A. Bruno, R. Schouten, and L. DiCarlo, Phys. Rev. Applied 6, 034008 (2016).

[34] J. Ikonen, J. Goetz, J. Ilves, A. Kernen, A. M. Gunyho, M. Partanen, K. Y. Tan, D. Hazra, L. Grnberg, V. Vesterinen, S. Simbierowicz, J. Hassel, V. Weinfang, J. de Sterke, A. Bruno, R. Schouten, and L. DiCarlo, Phys. Rev. Applied 6, 034008 (2016).