Inelastic clump collision model for non-Gaussian velocity distribution in molecular clouds

Shigeru Ida

Department of Earth and Planetary Science, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152, Japan

Y-h. Taguchi

Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152, Japan

ABSTRACT

Non-Gaussian velocity distribution in star forming region is reproduced by inelastic clump collision model. We numerically calculated the evolution of inelastic hard spheres in sheared flow, which corresponds to cloud clumps in differential galactic rotation. This system fluctuates largely around equilibrium state, creating clusters with inelastic collisions and destroying them with shear motion. The fluctuation makes spheres have non-Gaussian velocity distribution with nearly exponential tail. How far from Gaussian distribution depends upon coefficient of restitution, which can produce the variety of degree of deviation from Gaussian among regions.

Subject headings: Interstellar: Clouds – line profiles
1. Introduction

In general, the velocity distribution in molecular clouds has an excess high velocity tail compared with Gaussian (e.g., Falgarone and Phillips 1990, Miesch and Scalo 1995). In particular, Miesch and Scalo pointed out that the most important results are variety between subregions of molecular cloud (in some subregions the distribution is nearly Gaussian while it deviates significantly from Gaussian in other subregions) and nearly exponential tail in the non-Gaussian cases (Miesch and Scalo 1995).

Several explanations for the non-Gaussian distribution have been proposed: the high-velocity component came from unbound interclump medium (Blitz and Stark 1988), high-speed clump collisions (Keto and Lattanzio 1989), nonlinear Alfvén waves (Elmegreen 1990), or the intermittency of interstellar turbulence (Falgarone and Phillips 1990). However, it is difficult for these mechanisms to explain the variety and the exponential tail (Miesch and Scalo 1995).

We propose an alternative mechanism, clustering of clumps through mutual inelastic collisions. This mechanism causes the non-Gaussian distribution with the variety and the nearly exponential tail with simple physics. Molecular clouds have hierarchical structure. Here we generically call density-contrasted areas with any sizes “clumps”. Clumps interact with one another through direct collisions and gravitational scatterings. In general, interparticle interactions in the external gravitational potential converts orbital energy to random motions (e.g., Goldreich and Tremaine 1982, Ida 1990, Ohtsuki 1993). Sheared motion between particles, which is due to differential rotation of galactic disk, causes encounters between them, even if they have no random motion. Thus random motions are produced by interclump collisions and/or gravitational scatterings. On the other hand, interclump collisions also have effect to dissipate random motions, since they are supersonic, i.e., inelastic. Hence, some equilibrium state of random motions of clumps are maintained.

To make physics clear, we consider a simple model, inelastic hard spheres in sheared motion and numerically calculated the dynamical evolution of the system. We neglected gravitational scattering, because what makes non-Gaussian distribution is inelasticity of collisions (see below) and the effects of gravitational scatterings are similar to those of elastic collisions (e.g., Cuzzi et al. 1979, Ohtsuki 1993).

As we will show, the fluctuation around the equilibrium state is large in the case with relatively large inelasticity and densely packed spheres. That is, the mean value of the velocity dispersion fluctuates considerably and nonsteady spatial inhomogeneity of the distribution of the spheres (“cluster”) is formed. This behavior is characteristic of a system with both energy input and dissipation and does not depend on details of dissipation mechanism. Hence, the essential features of realistic clump collisions can be described by our simple model, though dissipation is much more complicated in realistic clump collisions.

The clustering of clumps produces non-Gaussian velocity distribution with nearly exponential tail. The deviation from Gaussian is closely related to the degree of clustering, which is regulated by degree of inelasticity (restitution coefficients). In this model, the variety is naturally explained as different “effective” restitution coefficients among subre-
regions.

2. Method of numerical calculation

Numerical setup of our model is as follows. We adopt rotating local Cartesian coordinates in a galactic disk. Hard spheres with normal restitution coefficient \( e \) are packed into two dimensional box having width \( L_y \) and height \( L_x \). The \( x \) and \( y \) directions correspond to radial and azimuthal directions of the galactic disk, respectively. We neglected “softness” of clumps, tangential restitution coefficient, and the thickness of the disk for simplicity, which hardly affects the properties of the results here.

For the limitation of calculation, number of spheres, i.e., \( L_x \) and \( L_y \), cannot be taken so large. Hence the box corresponds to the area which is much smaller than the area identified to one subregion by observation. We describe dynamics of a whole subregion by time average of this box instead of by summation of such boxes which cover a whole subregion.

This box has periodic boundary condition in the \( x \) direction and sheared boundary condition in the \( y \) direction, corresponding to sheared motion in a galactic potential (Fig.1). When a sphere passes through lower horizontal side of the box downward, it re-enters into the box from upper horizontal boundary. However, horizontal position differs between exiting position and entering position. Suppose that \( y \) coordinate of the position where the sphere goes out of box is \( y_{out} \) and that of entering is \( y_{in} \). Then \( y_{in} \) is equal to \( y_{out} + Ut \), where and \( t \) is time and \( U \) is sheared velocity between positions at upper and lower boundaries. If \( y_{out} + Ut \) is outside the box, \( y_{in} \) is taken equal to be \( y_{out} + Ut - L_y \). The \( y \) component of velocity after re-entering box also increase by \( U \); \( v_{in} = v_{out} + U \) where \( v_{in} \) and \( v_{out} \) are the \( y \) component of velocity when the sphere exits from and re-enters the box respectively. When a sphere passes through horizontal boundary upward, \( y_{out} \) should be \( y_{in} - Ut \) and \( v_{in} = v_{out} - U \). On the other hand, if a particle passes horizontally the vertical boundaries, \( x_{in} = x_{out} \) and \( u_{in} = u_{out} \), where \( u \) is the \( x \)-component of velocity.

In addition to self-gravity of clumps, we neglected Coriolis force and tidal force (the difference between galactic potential force and centrifugal force) in the equations of motion, because mean collision time is much shorter than galactic rotation time, i.e., characteristic time of Coriolis and tidal forces. (Preliminary results by 3D \( N \)-body calculation with self-gravity, Coriolis and tidal forces show essentially the same results.)

3. Results and their physical interpretation

By using this model, we perform numerical simulations. 256 spheres with diameter 1.0 are considered. The box size \( L_x \times L_y \) is taken to be \( 18.2 \times 15.2 \) and sheared velocity \( U = 0.5 \). The box size is taken to realize dense packing (in this case \( \sim 70\% \)). Clusters of clumps are created by inelastic collisions and are smeared out by sheared motion. If packing is so sparse that mean collision time is longer than smearing timescale, clusters are not maintained. Hence dense packing is required to produce non-Gaussian distribution due to clustering. The choices of values of \( L_x \), \( L_y \), and \( U \) do not change results, as long as dense packing is realized.

Restitution coefficient \( e \) is taken to be 0.6 and 0.9. Result is sensitive to value of \( e \). After discarding transient period starting from initial condition with equally spacing and random velocity \( (\in [-0.05, 0.05]) \), we measure
distribution of velocity component $u^1$ over 120 (180) snapshots for $e = 0.9$ ($e = 0.6$) every 256 (= total number of spheres) collisions (Fig. 2). The distribution in Figure 2(a) clearly deviates from Gaussian distribution. It has excess high-velocity tail that is nearly exponential. This property is independent of initial conditions when $e$ is relatively small. This universal property is consistent with observations. The degree of the deviation increases with deviation of the restitution coefficient $e$ from unity. The result with $e = 0.9$ shows almost Gaussian distribution (See Fig. 2(b)).

We found that non-Gaussian distribution is made by unsteady clustering due to inelastic collision. In Fig. 3, time evolutions of mean velocity dispersion are shown. The vertical and horizontal axes are mean velocity dispersion and cumulative collision numbers. The velocity dispersion appreciably varies with time in the $e = 0.6$ case. In the $e = 0.9$ case, the variation is smaller. In the limit of $e \to 1$, the velocity dispersion fluctuates only within statistical one. We found that this variance is induced by clustering of clumps. Generally speaking, introduction of inelastic collision causes spatial inhomogeneity. Goldhirsch and co-workers (Goldhirsch and Zanetti 1993, Goldhirsch et al. 1993) have already pointed out that spatially homogeneous distribution of hard spheres with inelastic collisions are unstable and collapse into several clusters. In our case, shear motion destroys the clusters. Hence creation and destruction of clusters are repeated. Through these processes, energy input from shear motion is intermittently dissipated by inelastic collisions, which makes large variation of velocity dispersion.

This time variance of dispersion does make the deviation from Gaussian distribution, since a composition of many Gaussian-like distributions with different dispersions is non-Gaussian distribution with excess high-velocity tail. Thus non-Gaussian velocity dispersion can be explained by nonsteady clustering due to inelastic collisions.

As noted before, the time variance of the present calculation corresponds to spatial variance in subregion of molecular cloud. Hence, the observed non-Gaussian distribution with excess high-velocity tail can be explained by spatial variance of clustering within a subregion caused by inelastic interclump collisions.

This model explains the observed variety of the distribution as the variety of “effective” restitution coefficient. As shown in Fig. 2, the deviation from Gaussian depends upon restitution coefficient. Effective restitution coefficient would be different among subregions in clouds. For example, effective $e$ can change with $\rho$ as follows. The relative importance between inelastic collisions and gravitational scatterings is indicated by the ratio of sizes $r$ of clumps to their tidal (or Roche-lobe) radii $r_t$ defined by $(GM/\Omega^2)^{1/3}$ where $M$ is clump mass and $\Omega$ is galactic rotational frequency; if $r/r_t \ll 1$, gravitational scatterings are more important, and vice versa (e.g., Ohtsuki 1993). For clumps of molecular cloud in the neighborhood of the sun, we obtain

$$\frac{r}{r_t} \approx 0.1 \left( \frac{\rho}{10M_\odot pc^{-2}} \right)^{-1/3},$$

where $\rho$ is mass density of clump. This means that direct collisions are as effective as gravitational scatterings in evolution of ran-
dom motion of clumps, analogous to planetary rings. Since gravitational scattering is
equivalent to $e = 1$ collisions, effective $e$ can change with $\rho$ through Eq. (1). Furthermore,
different configurations and random velocities of clumps may change effective $e$.

Thus inelastic interclump collisions explain the nearly exponential high-velocity tail and
the variety of distribution, and hence it can be a basic mechanism why velocity distribution
in molecular cloud deviates from Gaussian.

We would like to thank Hideki Takayasu for fruitful discussion.

REFERENCES
Blitz, L., and Stark, A. A. 1988, ApJ, 333, 353.

Cuzzi, J. N., Drisen, and R. H., Burns, J. A.,
Hamill, P. 1979, Icarus, 38, 54.

Elmegreen, B. G. 1990, ApJ, bf 361, L77.

Falgarone, E., and Phillips, T. G. 1990, ApJ,
359, 344.

G. Goldhirsch, and I., Zanetti, G. 1993,
Phys. Rev. Lett., 70 1619.

Goldhirsch, I., Tan, M.-L., and Zanetti G.
1993, J. Sci. Comp., 8 1.

Goldreich, P., and Tremaine, S. 1982,
ARA&A, 20, 249.

Ida, S. 1990, Icarus, 88, 129.

Kato, E. R., and Lattanzio, J. C. 1989, ApJ,
355, 190.

Miesch, M. S., and Scalo, J. M. 1995, ApJ, in
press.

Ohtsuki, K. 1993, Icarus, 106, 228.
Fig. 1.— Local Cartesian coordinates with sheared boundary condition.

Fig. 2.— Velocity distribution (a) $e = 0.6$ (b) $e = 0.9$ $u$ is renormalized so as to have variance of unity.

Fig. 3.— Time evolutions of mean velocity $u$ dispersion are shown. Upper: $e = 0.9$, Lower: $e = 0.6$. Data before 5120th collision are discarded.
Fig. 1
Fig 2a

Distribution of $u$

velocity $u$
Distribution of $u$

Fig 2b
velocity (u) dispersion vs. Number of collisions