Research on Input - output Efficiency Evaluation of Regional Power Grid Based on Data Envelopment Analysis

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Abstract: According to the characteristics of the development of regional power grid, the regional power grid evaluation system is studied and explored as viewed from the power grid input-output. Using data envelopment analysis method and taking the power grids with 220 kV~110 kV and 35 kV and below as objective respectively, the input-output evaluation model is established. Through the comprehensive efficiency analysis and projection analysis, the grid efficiency value based on the inputs and outputs, the gap between non-effective unit and the target value and other information elements are obtained, which comprehensively evaluates the input-output efficiency of the regional power grid. It provides a scientific decision support for optimizing the allocation of resources and improving the efficient use of resources, which is conducive to promoting the coordinated development of power grids at all levels. The results of calculation examples demonstrate the effectiveness of the proposed method.

1. Introduction

As the next level of provincial power grid, regional power grid is an important link for the whole power system to contact users, supply and distribute electric energy to users. With the sustained and rapid development of the national economy and the improvement of people's living standards, the construction of regional power grids is gradually increasing and there will be more and more corresponding investment [1]. If the scale of investment is too small or lagging behind, it will inevitably not meet the requirements of economic and social development and load growth, and will also affect the long-term development of enterprises and fail to achieve the expected investment benefits. However, if the investment scale is too large or too advanced, it will inevitably lead to waste of funds and excessive resources, and will also cause pressure on enterprises. Therefore, it is necessary to evaluate the input-output efficiency of the regional power grid, and then judge whether the scale of power grid construction is appropriate and whether the input and output match, which is of great significance to strengthen the guidance of regional power grid construction, realize the accurate investment of power grid, promote the scientific, rational and orderly development of regional power grid, and improve the development capacity of power grid and the development benefit of enterprises[2].
At present, the more common efficiency analysis methods are parameter analysis, stochastic frontier analysis (SFA), data envelopment analysis (DEA) and so on. The parameter method has great limitations and complicated calculation, which is mainly applicable to the calculation of relative efficiency of single output and multiple inputs [3]. DEA is a nonparametric statistical method developed on the basis of the concept of "relative efficiency evaluation" to evaluate the relative efficiency of several decision-making units with the same type of input and output, which is objective and suitable for multi-input-multi-output complex systems [4]. It is a new direction to apply DEA method to the evaluation and analysis of efficiency and investment benefit of power grid enterprises in recent years [5].

Starting from the actual situation of regional power grid enterprises, this paper takes 220 kV and 110 kV power grids and 35 kV and below power grids as research objects, respectively, to construct a power grid investment benefit evaluation model based on data envelopment analysis, to quantitatively analyze the input and output of each level of power grid, and to provide scientific auxiliary decision-making basis for reducing inefficient input of resources.

2. Evaluation model of input-output Efficiency of Regional Power Grid Based on DEA

2.1. Data envelopment analysis

DEA is a new field of cross-research in operations research, management science and mathematical economics. It is a system analysis method developed by Charnes and Cooper et al. in 1978 based on the concept of "relative efficiency evaluation", especially suitable for handling complex systems with multiple inputs and multiple outputs [6]. The C2R model is one of the most widely used DEA models. The mathematical model with Archimedes infinitesimal and relaxation variables is as follows.

\[
\begin{align*}
\min \theta - \varepsilon \left[ \sum_{i=1}^{n} S_{i}^{+} + \sum_{i=1}^{n} S_{i}^{-} \right] \\
\text{s.t.} \sum_{j=1}^{m} \lambda_{j} x_{ij} + S_{i}^{-} - \theta x_{i0} = 0 \\
\sum_{j=1}^{m} \lambda_{j} y_{rj} - S_{r}^{+} = y_{r0} \\
\lambda \geq 0, j = 1, 2, \ldots, n \\
S_{i}^{-} \geq 0, S_{r}^{+} \geq 0
\end{align*}
\]

Where \( n \) is the number of decision making units (DMU); \( \theta \) is the effective utilization degree of input relative to output and the relative benefit value of decision-making unit \((0 \leq \theta \leq 1)\), which reflects the comprehensive allocation efficiency of DMU resources; \( x_{ij} \) is the input of the \( j \)-th decision unit to the \( i \)-th type input; \( y_{rj} \) is the output of the \( j \)-th decision unit for the \( r \)-th type of output; \( \lambda_{j} \) represents a linear combination coefficient of several decision units; \( S_{i}^{-} \) and \( S_{r}^{+} \) are relaxation variables, representing the redundancy of input and the insufficiency of output, respectively; \( \varepsilon \) is a non-Archimedean infinitesimal quantity, in practical applications, \( \varepsilon = 10^{-6} \).

In formula (1), when \( \theta = 1 \) and \( S_{i}^{-} = 0 \), the DMU is DEA effective. Conversely, it is weak effective \((\theta = 1 \text{ and } S_{i}^{-} > 0 \text{ or } S_{r}^{+} > 0)\) or non-DEA effective \((\theta < 1)\).

In order to study the formation factors of technological inefficiency, relevant scholars expand the use scope and ratio concept of C^2R model, add constraint \( \sum_{j=1}^{n} \lambda_{j} = 1 \) in formula (1), and obtain BC^2.

When analyzing DMU which is not DEA effective, we need to get information about the direction of input and output improvement and the goal, so as to provide suggestions for improving production
and management efficiency of enterprises' actual business activities. The method used in DEA model is projection analysis, i.e., \( x_{0}^{\Lambda} = x_{0} - S_{-0}^{\Lambda} \), \( y_{0}^{\Lambda} = y_{0} + S_{+0}^{\Lambda} \), and \( x_{0}^{\Lambda} \) and \( y_{0}^{\Lambda} \) are the final results of optimization, which are the input and output of non-DEA units adjusted to reach DEA efficiency.

2.2. Selection of evaluation indicators

2.2.1. Input indicators

Transformer capacity / line length: The investment in power grid construction depends mainly on the total investment cost of the substation and the line, and is directly related to the transformer capacity and line length [7-8].

Capacity-load ratio: It is the ratio of transformer capacity to maximum load, and an important evaluation index to measure the reliability and economy of power grid. It is easy to waste investment if the capacity-load ratio is too large [9].

Integrated Line Loss Rate: It is the percentage of the loss of power in the circuit and transformer in the process of power transmission in the power supply system. And it is also an important economic and technological index of power supply enterprises, which comprehensively reflects the management level of power grid enterprises [10].

2.2.2. Output indicators

Output indicators need to reflect the ability of power grid input to ultimately translate into direct economic benefits and to maximize the demand for electricity.

Electricity sales: It is an important evaluation index of the output of power grid companies. The most basic requirement of power grid construction at all levels is to meet the local electricity demand to the greatest extent.

"N-1" passing rate: It is an important indicator to characterize the level of power grid security.

Power grid utilization efficiency: It is an important reflection of the operation level of power grid and the operation efficiency of modern power grid enterprises. It is closely related to the economy and is the core issue that enterprises pay close attention to. Power grid utilization efficiency can be measured by maximum load efficiency and equivalent average load efficiency [11].

2.3. Input-output evaluation model of regional power grid based on DEA

As the next level of provincial power grid, the voltage level of regional power grid is mainly composed of 220 kV and below. In order to measure the input-output efficiency of power grid comprehensively and meticulously, combined with the above-mentioned indicators, based on the development characteristics and stages of each level of regional power grid, 220 kV and 110 kV power grids and 35 kV and below power grids are taken as research objects to construct input-output evaluation models. This clearly reflects the problems and shortcomings of input and output of power grids at all levels, and helps to accurately find the corresponding countermeasures and make comprehensive decisions. The input-output benefit evaluation process of regional power grid based on DEA is shown in Figure 1.
3. Simulation

3.1. The Relative Efficiency of DEA

Taking 9 regional power supply enterprises under the jurisdiction of a provincial company as the research decision-making unit, the input-output efficiency of 220 kV and 110 kV power grids and 35 kV power grids of each decision-making unit in 2017 are calculated by using DEAP2.1 software. The C2R model is used to calculate the comprehensive efficiency of each decision-making unit, and then the technical efficiency, scale efficiency and scale reward of each unit are evaluated according to BC2 model of variable scale reward [12]. The calculation results are shown in Tables 1 and 2.

| Decision making unit | Comprehensive efficiency | Technical efficiency | Scale efficiency | Relative validity | Technical efficiency | Scale reward |
|----------------------|-------------------------|----------------------|-----------------|-------------------|----------------------|--------------|
| DMU1                 | 1                       | 1                    | 1               | Effective         | Effective            | Unchanged    |
| DMU2                 | 1                       | 1                    | 1               | Effective         | Effective            | Unchanged    |
| DMU3                 | 0.896                   | 0.914                | 0.981           | Ineffective       | Effective            | Unchanged    |
| DMU4                 | 0.983                   | 1                    | 0.983           | Ineffective       | Effective            | Decrement    |
| DMU5                 | 1                       | 1                    | 1               | Effective         | Effective            | Unchanged    |
| DMU6                 | 0.828                   | 1                    | 0.828           | Ineffective       | Effective            | Decrement    |
| DMU7                 | 1                       | 1                    | 1               | Effective         | Effective            | Unchanged    |
| DMU8                 | 0.764                   | 0.781                | 0.978           | Ineffective       | Invalid              | Decrement    |

Table 1. Efficiency value of 220 kV and 110 kV power grids of power supply enterprises
The table below shows the efficiency values of 35 kV and below power grids of power supply enterprises.

| Decision making unit | Comprehensive efficiency | Technical efficiency | Scale efficiency | Relative validity | Technical efficiency | Scale reward |
|----------------------|--------------------------|----------------------|-----------------|------------------|---------------------|--------------|
| DMU1                 | 0.64                     | 0.742                | 0.862           | Ineffective      | Invalid             | Increment    |
| DMU2                 | 1                        | 1                    | 1               | Effective        | Effective           | Unchanged    |
| DMU3                 | 1                        | 1                    | 1               | Effective        | Effective           | Unchanged    |
| DMU4                 | 1                        | 1                    | 1               | Effective        | Effective           | Unchanged    |
| DMU5                 | 1                        | 1                    | 1               | Effective        | Effective           | Unchanged    |
| DMU6                 | 1                        | 1                    | 1               | Effective        | Effective           | Unchanged    |
| DMU7                 | 1                        | 1                    | 1               | Effective        | Effective           | Unchanged    |
| DMU8                 | 0.611                    | 0.862                | 0.709           | Ineffective      | Invalid             | Increment    |
| DMU9                 | 0.625                    | 0.706                | 0.885           | Ineffective      | Invalid             | Increment    |

### 3.2. Input-output efficiency analysis

From Table 1, it can be seen that in 9 areas, the evaluation results of DMU1, DMU2, DMU5, DMU7, DMU9 are DEA effective, indicating that the input and benefit output of 220 and 110 kV power grid construction in the above 5 areas have reached a good state. This not only guarantees that all input factors are fully utilized, but also meets the local electricity demand to the greatest extent. The remaining 4 decision-making units are DEA-ineffective, which indicates that the combination between input and output is not optimal, and there exists input redundancy or output deficit. Among them, DMU4 and DMU6 are ineffective for DEA, but the technology is effective. It shows that these four areas have good scalability in terms of grid structure and quality, and the reason for their low comprehensive efficiency is scale efficiency. From the perspective of scale reward, all the 4 DEA-ineffective regional power grids have diminished returns to scale, which indicates that the construction scale of 220 and 110 kV power grids in these eight areas has advanced. When input continues to increase, the efficiency of income (output) increase is not high, that is, there is no incentive to continue to increase investment.

From the perspective of 35 kV and below power grids, there are a total of 6 DEA effective units, namely DMU2-7. The remaining 3 are DEA-ineffective, and the value of technical efficiency and scale efficiency are low, indicating that the combination between input and output is optimal, the input and output of power grid are not proportional, and the efficiency value is low. Unlike 220 kV and 110 kV power grids, all ineffective units are increasing returns on scale, indicating that these nine regions have the initiative to continue to increase investment in power grid construction, that is, when input continues to increase, the efficiency of income (output) increase will be greater than that of input increase.

Generally speaking, the comprehensive efficiency, technical efficiency and scale efficiency of 220 and 110 kV power in 9 regions are 0.941, 0.966 and 0.974, respectively, which are superior to those of 35 kV and below power grids of 0.875, 0.923 and 0.940. This shows that the overall efficiency of 220 kV and 110 kV power grids is relatively high. Regionally, the input and output of 220 kV and 110 kV power grids and 35 kV and below power grids are only 5 areas with DEA validity, namely DMU2, DMU5 and DMU7. To some extent, this reflects that the efficiency level of power grids at different levels in most areas is uneven, and the coordination degree of development is not high. Especially in the areas where DEA is ineffective in DMU 8, it is urgent to optimize the distribution of power grids and improve the input and output benefits of power grids [13].
3.3. Projection analysis of inefficient decision making units

In order to transform the ineffective DEA decision-making units into relatively effective ones, the relevant input factors should be reduced or the output should be increased. Using the projection theory, the redundancy of input factors and insufficient output are shown in tables 3 and 4, and the target values are shown in tables 6 and 7. It is noteworthy that the input-output models of 220 kV, 110 kV and 35 kV power grids and below adopt both line loss rate and electricity sales index, so the relaxation variables ($S^-_{5-1}$ and $S^-_{5-2}$, $S^+_{7-1}$ and $S^+_{7-2}$) and objectives ($x^\Lambda_{5-1}$ and $x^\Lambda_{5-2}$, $y^\Lambda_{7-1}$ and $y^\Lambda_{7-2}$) are two groups. The average value of $x^\Lambda_5$ and $y^\Lambda_7$ are obtained as the final target values of line loss rate and electricity sales for each decision-making unit, as shown in Table 5.

### Table 3. Relaxation variables of non-effective decision-making units of 220 kV and 110 kV

| Decision unit | Comprehensive efficiency | $S_1^-$/% | $S_2^-$/km | $S_3^-$/% | $S_4^-$/% | $S_5^-$/% | $S_6^-$/% | $S_7^-$/% | $S_8^-$/% | $S_9^-$/% | $S_10^-$/% |
|---------------|--------------------------|-----------|------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| DMU3          | 0.896                    | -261      | 0          | -0.35     | 0         | 32.3      | 20.9      | 6.7       | 12.9      | 2.3       | 0         |
| DMU4          | 0.983                    | -48       | 0          | 0         | -0.75     | 20.4      | 18.7      | 29.9      | 29.9      | 3.4       | 0         |
| DMU6          | 0.828                    | -260      | -0.822     | -0.88     | 0         | 2.8       | 2.2       | 8.7       | 15        | 3.7       | 0         |
| DMU8          | 0.764                    | -845      | -0.638     | -0.06     | 0         | 63.4      | 56.4      | 49.9      | 18.2      | 33.9      | 0         |

### Table 4. Relaxation variables of non-effective decision-making units of 35 kV

| Decision unit | Efficiency | $S_5^-$/% | $S_6^-$/km | $S_7^-$/% | $S_8^-$/% | $S_9^-$/% | $S_10^-$/% |
|---------------|------------|-----------|------------|-----------|-----------|-----------|------------|
| DMU1          | 0.64       | -0.68     | -53.8      | 0         | 0         | 11.5      | 0          | 4.8        | 7.2       |
| DMU8          | 0.611      | -1.77     | 0          | -3147     | 0         | 23        | 27.4       | 15         |
| DMU9          | 0.625      | -1.25     | 0          | -2824     | 0         | 50.8      | 30.9       | 0          |

### Table 5. Final target recommended values of line loss rate and electricity sales indicators

| Decision making unit | Line loss rate /% | Electricity sales (10^8 kW • h) |
|----------------------|-------------------|-------------------------------|
| DMU1                 | $x^\Lambda_{5-1}$ | $x^\Lambda_{5-2}$ | $x^\Lambda_5$ | $y^\Lambda_{7-1}$ | $y^\Lambda_{7-2}$ | $y^\Lambda_7$ |
| DMU2                 | 4.1               | 4.1                           | 4.1            | 438.4          | 438.4           | 438.4        |
| DMU3                 | 5.63              | 6.28                          | 5.96           | 75.1           | 75.1            | 75.1         |
| DMU4                 | 5.95              | 6.06                          | 6.01           | 100.1          | 100.1           | 100.1        |
| DMU5                 | 6.65              | 6.65                          | 6.65           | 34.8           | 34.8            | 34.8         |
| DMU6                 | 6.91              | 8.35                          | 7.63           | 42.5           | 42.5            | 42.5         |
| DMU7                 | 8.09              | 8.09                          | 8.09           | 119            | 119             | 119          |
| DMU8                 | 7.11              | 3.93                          | 5.52           | 59.4           | 59.4            | 59.4         |
| DMU9                 | 9.85              | 4.9                           | 7.37           | 51.9           | 51.9            | 51.9         |
From Table 3 and Table 4, it can be seen that the problems of excessive line redundancy and relatively high capacity-load ratio exist in areas where the investment benefit value of 220 and 110 kV power grids is poor. This also reflects the continuous increase of 35 kV and below power grid transformation efforts, efforts to solve the weak problem of low and medium voltage distribution network is the top priority of the next regional power grid development. As shown in Table 6 and Table 7, the target value of N-1 throughput index of 110 kV transformer and 110 kV line of DMU4 calculated by DEA model has exceeded 100%, which is inconsistent with reality and should be adjusted to 100%.

4. Conclusion

In view of the regional power grids with different characteristics, considering the construction characteristics and development stages of power grids at different levels, the input-output benefit evaluation models of 220 kV, 110 kV and 35 kV and below power grids based on DEA are established respectively. Management information including comprehensive efficiency, the gap between inefficient decision-making units and their ideal situation, redundancy of input factors or insufficient output, and distribution of technology and scale effectiveness are also obtained. It can provide reference basis for relevant departments to formulate measures to improve the efficiency of power grid, and to formulate investment planning and planning scheme of power grid.
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