Properties of non-extremal enhançonns

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Abstract

We study the supergravity solutions describing non-extremal enhançonns. There are two branches of solutions: a ‘shell branch’ connected to the extremal solution, and a ‘horizon branch’ which connects to the Schwarzschild black hole at large mass. We show that the shell branch solutions violate the weak energy condition, and are hence unphysical. We investigate linearized perturbations of the horizon branch and the extremal solution numerically, completing an investigation initiated in a previous paper. We show that these solutions are stable against the perturbations we consider. This provides further evidence that these latter supergravity solutions are capturing some of the true physics of the enhançon.

1 Introduction

The enhançon mechanism [1] provides a novel singularity-resolution mechanism in string theory. As such, it extends the class of physical situations we can understand in terms of weakly-coupled effective theories (this leads to interesting applications in the AdS/CFT context [2, 3]), and it allows us to explore how stringy physics changes our picture of the structure of spacetime. The singularity resolution was originally exhibited in [1] for a BPS geometry, the repulson singularity [4, 5], but both these considerations motivate an interest in exploring the generalisation of this mechanism to non-BPS, finite temperature solutions.

The question of the non-extremal generalisation of the enhançon solution was briefly discussed in [1], and was considered in more detail from a supergravity point of view in [6], where it was found that there are two branches of non-extremal solutions. One of them approaches the usual enhançon in the extremal limit, and always has an enhançon shell, which may or may not have a black hole living inside it; we refer to this as the shell branch. The other branch has a smooth event horizon, and approaches the Schwarzschild geometry far from extremality; we call this the horizon branch.

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branch. The shell branch exists for all masses greater than the BPS value, but for
the standard extremal solution, the horizon branch exists only if the mass difference
from the extremal solution is greater than a certain critical value.

Physically, one expects that adding a small charge to a large black hole should
not drastically change the physics; therefore, it seems that the horizon branch is
the physically relevant solution for masses much greater than the extremal value.
However, there are no horizon branch solutions for small enough mass difference.
Thus, sufficiently close to extremality, we should find a thermally-excited enhançon
shell. If this were to be described by some shell branch solution, we would then
expect that there should be transitions between the two branches as we vary the
parameters. This could provide a useful example of the kind of non-trivial phase
structure we expect to see in such less supersymmetric contexts. Since we have the
explicit solutions, it should be possible to understand the phase structure in detail.
In a previous paper [7], we initiated an investigation of the phase structure for these
solutions, studying the thermodynamics of the two branches and investigating the
linearised perturbations to see if there might be a classical instability which could
provide the mechanism for transitions between them.

In this paper, we will further investigate the physics of these solutions. We discover
that the story is quite different from what we had expected. We will see that the
shell branch of the non–extremal enhançon violates the weak energy condition. We
therefore regard that branch as completely unphysical.\footnote{We would still expect that one can slightly excite the enhançon shell; it is possible that this is
described by some more general ansatz within supergravity. Alternatively, thermal excitation might
smear the branes out so that a thin shell is not a good approximation to the distribution of branes,
implying that no supergravity description is possible in this regime. We will see that features of the
shell branch solutions argue in favor of the latter possibility.}

The shell branch does not represent real physics of the enhançon; what about the
horizon branch? To further investigate this, we have completed our previous study of
the linearised perturbations, finding that there is no instability on the horizon branch
to perturbations of the type we consider. This provides some evidence that the
horizon branch is the physically correct description for the range of parameters where
it exists. We have also analysed the stability of the extremal solution, determining
appropriate boundary conditions at the enhançon shell to supplement the linearised
perturbation equations obtained in [7]. We do not find any instability here either; in
this case, this is very much the expected result. A BPS solution should not have any
instabilities.

We should stress, however, that in neither case have we demonstrated the absence
of an instability; we have merely failed to find one within the class of modes we studied.
In particular, it is worth noting that we have ignored the non-abelian fields coming
from D2-branes, which become light near the enhançon. These might play a central
role in the search for some more general solution in the low mass difference region (a
discussion in the extremal case has appeared in [8]), and it might be interesting to
extend the perturbative analysis to include these fields, as a step towards constructing
such a solution.

In section 2, we review the enhançon solutions, extremal and non–extremal. We
then show that the shell branch of the non–extremal enhançon violates the weak energy condition in section 3. In section 4, we review the linearised perturbation analysis for the extremal and non-extremal enhançon solutions carried out in [7]. In section 5, we study the stability of the horizon branch. We find no instability to the perturbations we are considering. In section 6, we determine the appropriate matching conditions at the shell and carry out the relaxation of the equations in the exterior region for the extreme case. This section corrects flaws in the analysis of this case previously carried out in [9]. Again, no instability is found.

2 The enhançon solutions

The metrics we wish to study are the extremal and non–extremal enhançon solutions obtained in [1, 6]. These geometries describe \( N \) \( D(p + 4) \)-branes wrapped on a K3; as in [7], we focus on the case of two non-compact dimensions, \( p = 2 \). For the extremal case, the Einstein frame metric and fields are

\[
\begin{align*}
    ds^2 &= Z_2^{-5/8} Z_6^{-1/8} \eta_{\mu\nu} dx^\mu dx^\nu + Z_2^{3/8} Z_6^{7/8} (dr^2 + r^2 d\Omega) + V^{1/2} Z_2^{3/8} Z_6^{-1/8} ds_{K3}^2, \\
    e^{2\phi} &= g_s^2 Z_2^{1/2} Z_6^{-3/2}, \\
    C_3 &= (Z_2 g_s)^{-1} dx^0 \wedge dx^1 \wedge dx^2, \\
    C_7 &= (Z_6 g_s)^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge V \varepsilon_{K3},
\end{align*}
\]

where the harmonic functions are

\( Z_6 = 1 + \frac{r_6}{r}, \quad Z_2 = 1 + \frac{r_2}{r} \),

the parameters are related by

\[ r_6 = \frac{g_s N \alpha'^{1/2}}{2}, \quad r_2 = -\frac{V_*}{V} r_6, \]  

\[ r_2 \] is negative with our sign conventions, and \( d\Omega \) denotes the metric on the unit two-sphere. The running K3 volume is

\[ V(r) = V \frac{Z_2(r)}{Z_6(r)}. \]

\[ V = V_* = (2\pi \sqrt{\alpha'})^4 \] at the enhançon radius,

\[ r_e = \frac{2V_*}{V - V_*} r_6. \]

The repulson singularity would occur at \( r = -r_2 < r_e \). In the enhançon mechanism [1], this singularity is resolved by replacing the geometry inside the enhançon radius identified above by a flat space. The geometry inside the shell is

\[ g_s^{1/2} ds^2 = H_2^{-5/8} H_6^{-1/8} \eta_{\mu\nu} dx^\mu dx^\nu + H_2^{3/8} H_6^{7/8} (dr^2 + r^2 d\Omega) + V^{1/2} H_2^{3/8} H_6^{-1/8} ds_{K3}^2. \]

\( ^2\)We are focusing on the simplest case where there are no additional D2-branes.
and the non-trivial fields are
\[ e^{2\phi} = g_s^2 H_2^{1/2} H_6^{-3/2} , \]
\[ C_{(3)} = (g_s H_2)^{-1} dx^0 \wedge dx^1 \wedge dx^2 , \]
\[ C_{(7)} = (g_s H_6)^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge V \varepsilon_{K3} , \]
where
\[ H_2 = 1 + \frac{r_2}{r_c} , \]
\[ H_6 = 1 + \frac{r_6}{r_c} . \]

These constants are introduced to make the continuity of the solution at the shell explicit.

In [6], it was found that this extremal solution could be generalised to obtain two branches of non-extremal solutions, arising from an ambiguity of a choice of sign in the solution of the supergravity equations for the usual ansatz. The non-extremal generalisation of the exterior geometry is
\[ g_s^{1/2} ds^2 = Z_2^{-5/8} Z_6^{-1/8} (-K dt^2 + dx_1^2 + dx_2^2) + Z_2^{3/8} Z_6^{7/8} (K^{-1} dr^2 + r^2 d\Omega_2^2) + V^{1/2} Z_2^{3/8} Z_6^{-1/8} ds_{K3}^2 , \]
the dilaton and R–R fields are
\[ e^{2\phi} = g_s^2 Z_2^{1/2} Z_6^{-3/2} , \]
\[ C_{(3)} = (g_s \alpha_2 Z_2)^{-1} dt \wedge dx^1 \wedge dx^2 , \]
\[ C_{(7)} = (g_s \alpha_6 Z_6)^{-1} dt \wedge dx^1 \wedge dx^2 \wedge V \varepsilon_{K3} , \]
and the various harmonic functions are given by
\[ K = 1 - \frac{r_0}{r} , \]
\[ Z_2 = 1 + \frac{\hat{r}_2}{r} , \quad Z_6 = 1 + \frac{\hat{r}_6}{r} . \]
Here
\[ \hat{r}_6 = -\frac{r_0}{2} + \sqrt{r_6^2 + \left(\frac{r_0}{2}\right)^2} , \]
and \( \alpha_6 = \hat{r}_6 / r_6 \). There are two choices for \( \hat{r}_2 \) consistent with the equations of motion:
\[ \hat{r}_2 = -\frac{r_0}{2} \pm \sqrt{r_2^2 + \left(\frac{r_0}{2}\right)^2} , \]
and \( \alpha_2 = \hat{r}_2 / r_2 \). Here, \( r_2 \) and \( r_0 \) are still given by (3).

There are two branches of solutions. For the upper sign in (14), \( \hat{r}_2 > 0 \), so there is no repulson singularity, and the solution has a regular horizon at \( r = r_0 \). For the
lower choice of sign, however, the repulson singularity always lies outside the would-be horizon, $|\hat{r}_2| > r_0$, and the geometry will be corrected by an enhançon shell. We therefore refer to the former as the horizon branch and the latter as the shell branch.

The shell branch exterior solution is cut off by an enhançon shell at

$$r_e = \frac{V_\ast \hat{r}_6 - V \hat{r}_2}{V - V_\ast}.$$  \hfill (15)

The interior solution can in general contain an uncharged black hole,

$$g_s^{1/2} ds^2 = H_2^{-5/8} H_6^{-1/8} \left( -\frac{K(r_e)}{L(r_e)} Ldt^2 + dx_1^2 + dx_2^2 \right) + H_2^{3/8} H_6^{7/8} (L^{-1} dr^2 + r^2 d\Omega) + V^{1/2} H_2^{3/8} H_6^{-1/8} ds_{K3}^2,$$  \hfill (16)

with accompanying fields

$$e^{2\phi} = g_s^2 H_2^{1/2} H_6^{-3/2},$$

$$C_{(3)} = \left( \frac{K(r_e)}{L(r_e)} \right)^{1/2} (g_s \alpha'_6 H_2)^{-1} dt \wedge dx^1 \wedge dx^2,$$

$$C_{(7)} = \left( \frac{K(r_e)}{L(r_e)} \right)^{1/2} (g_s \alpha'_6 H_6)^{-1} dt \wedge dx^1 \wedge dx^2 \wedge V \varepsilon_{K3},$$  \hfill (17)

where

$$L = 1 - \frac{r'_0}{r},$$

$$H_2 = 1 + \frac{\hat{r}_2}{r_e},$$

$$H_6 = 1 + \frac{\hat{r}_6}{r_e}. $$  \hfill (18)

In this supergravity analysis, there is an independent non-extremality scale $r'_0$ for the interior solution. It was argued in [6] that this is an unphysical degree of freedom, which would be determined uniquely (or possibly up to some discrete ambiguity) in terms of the asymptotic mass and charges if we properly understood the physics of the shell. Unfortunately, this purely supergravity analysis provides too crude a description of the shell in the non-extremal context to do this. All we can say is that $r'_0 < r_e$, in order that the interior black hole actually fits inside the shell.

3 Shell branch violates weak energy condition\textsuperscript{3}

In fact it turns out that it is not just this freedom to specify $r'_0$ which is unphysical: the shell branch solution given above is unphysical for any value of $r'_0$, as the stress

\textsuperscript{3}This observation arose from collaborative discussions with Amanda Peet.
tensor of the shell required to match the exterior and interior solutions violates the weak energy condition.

This is easily seen by considering the \( tt \) component of the stress tensor of the thin shell, resulting from the Israel junction conditions \( \mathbb{6} \):

\[
2\kappa^2 S_{tt} = \frac{1}{\sqrt{G_{rr}}} \left[ \hat{Z}_2' + \hat{Z}_6' \frac{4}{r_i} \left( 1 - \sqrt{\frac{L(r)}{K(r)}} \right) \right] G_{tt}
\]  

(19)

This determines the energy density \( \rho \) of the thin shell, which scales as

\[
\rho \sim -\frac{\hat{Z}_2'}{Z_2} - \frac{\hat{Z}_6'}{Z_6} + \frac{4}{r_i} \left( \sqrt{\frac{L(r)}{K(r)}} - 1 \right)
\]  

(20)

The second term is always positive, as \( \hat{r}_6 > 0 \), implying that \( \hat{Z}_6' < 0 \) (that is, \( \hat{Z}_6 \) is decreasing as \( r \) increases). We generally assume that \( r_0' < r_0 \), so \( L(r) > K(r) \), implying that the third term is positive. However, for solutions on the shell branch, \( \hat{r}_2 < 0 \), so the first term is negative.

If we consider a shell at large radius,

\[
\rho \sim \frac{\hat{r}_2 + \hat{r}_6 + 2(r_0 - r_0')}{r_2^2},
\]  

(21)

and the negative contribution from \( \hat{r}_2 \) is always balanced by the positive contributions from the other terms, to give a positive answer. This is just another way of saying that the exterior solution has a positive ADM mass (in fact, its ADM mass is greater than the mass of the extremal solution).

However, this is not what we want to consider. The radius of the shell is supposed to be close to the enhançon radius. The physical argument for this is that the solution is constructed by bringing branes in from infinity. In the extremal case, the branes see no potential, but the closest radius we can bring them to is the enhançon radius, because they cease to behave like pointlike branes at this radius \( \mathbb{1} \). In the non-extremal case, the branes feel an attractive force in the exterior geometry \( \mathbb{6} \), so they will again not stop until they start to smear out at the enhançon radius.

When we consider a shell at the enhançon radius, the energy density \( \rho \) will be negative. At the enhançon radius, \( \frac{\hat{Z}_2}{Z_2} = \frac{V}{V} \), so \( \rho \) in (20) can be rewritten as

\[
\rho \sim \frac{\hat{r}_2}{r_e} \frac{1}{Z_2} \left( 1 + \frac{V_e \hat{r}_6}{V \hat{r}_2} \right) + \frac{4}{r_e} \left( \sqrt{\frac{L(r_e)}{K(r_e)}} - 1 \right).
\]  

(22)

On the shell branch, \( \hat{r}_2 \) is negative, and \( \frac{\hat{r}_6}{\hat{r}_2} \) is small, so the first term is negative. When \( V_e/V \) is small, \( \hat{Z}_2 \) is small, and the first term will dominate over the second, so that the overall energy density of the shell will be negative. Now we need \( V \gg V_e \) for this supergravity analysis to be relevant; so in the regime where this description is supposed to apply, the energy density of the shell is negative.

One can extend this discussion to consider solutions on the shell branch with additional D2-brane charge, as considered in \( \mathbb{6} \). The expression for the energy density
of the shell becomes more complicated in this case, and we have not been able to find a simple argument that it will always be negative, but numerical investigation shows that the energy density of the shell is negative for all values the parameters we tried also in this more general case.

Thus, the weak energy condition is violated on the shell branch. This implies that the shell branch solution does not correctly describe small perturbations away from the enhançon solution; one possibility is that this signals a breakdown of the thin-shell approximation for small departures from extremality. There is some evidence for this interpretation coming from the study of probe branes. In the BPS case considered previously, we had a moduli space of solutions of the classical equations of motions, and we argued that we could choose all the sources to lie at as small a radius as possible, justifying the thin shell picture. In thermal equilibrium at some non-zero temperature, the constituent branes will carry thermal kinetic energy, and it is not clear that the inter-brane interactions will be sufficient to restrain the branes within a narrow range in \( r \). Near extremality, the average extra energy per brane scales as \( r_0/N \), while the typical scale of the effective potential seen by a probe brane in the exterior region is \( V_{\text{eff}} \sim r_0/r_e \sim r_0/N \), so it is not clear that branes in the enhançon shell will remain confined to a thin layer when we add some thermal energy.\(^4\)

### 4 Linearised perturbation equations

Having shown that the shell branch solutions are unphysical, we would now like to turn our attention to what more we can learn about the other solutions. To further explore the physics of the horizon branch and the extremal metric, it is interesting to continue the analysis of small perturbations of these solutions initiated in [7].

In [7], we considered the linearised perturbations of the exterior geometry, assuming that the perturbation preserves the invariance under spherical symmetry in the \( \theta, \phi \) directions, translational invariance in \( x_1 \) and \( x_2 \), and the discrete symmetries under \( x_1 \to -x_1, x_2 \to -x_2, \phi \to -\phi \). By a suitable choice of coordinates, the most general perturbation consistent with these symmetries can be written as the metric

\[
g_s^{1/2} ds^2 = e^{-\psi_1/2} \left( \bar{Z}_2^{-1/2} \bar{Z}_6^{-1/2} (-\bar{K} e^{\delta \psi_2} dt^2 + e^{-\frac{1}{2} \delta \psi_2 + \delta \psi_3} dx_1^2 + e^{-\frac{1}{2} \delta \psi_2 - \delta \psi_3} dx_2^2) + \bar{Z}_2^{1/2} \bar{Z}_6^{1/2} (\bar{K}^{-1} dr^2 + r^2 d\Omega^2_2) + V^{1/2} \bar{Z}_2 \bar{Z}_6^{-1/2} ds_{K3}^2 \right),
\]

(23)

**dilaton**

\[
\bar{\phi} = \phi + \delta \phi,
\]

(24)

and R–R fields

\[
\bar{C}_{(3)} = C_{(3)} + \delta C_{(3)}, \quad \bar{C}_{(7)} = C_{(7)} + \delta C_{(7)}.
\]

(25)

Here

\[
\psi_1 = \phi + \delta \psi_1, \quad \bar{Z}_2 = Z_2 (1 + \delta Z_2), \quad \bar{Z}_6 = Z_6 (1 + \delta Z_6), \quad \bar{K} = K(1 + \delta K),
\]

(26)

\(^4\)We are grateful to Rob Myers for discussions on this point.
the harmonic functions $Z_2, Z_6, K$ are as in (12), the unperturbed dilaton $\phi$ is as in (11), and the RR potentials are as in (11). The first-order perturbations are all functions of $(r, t)$ only, while the background quantities are functions only of $r$. We look for perturbations of the form $f(r)e^{\sigma t}$.

Writing the metric in the form (23) does not fix the diffeomorphism freedom completely. There are infinitesimal diffeomorphisms which preserve its form. Namely,

$$t \to t' = t + e^{\sigma t} \delta t(r), \quad r \to r' = r + e^{\sigma t} \delta r(r),$$

with

$$\partial_r \delta t = \sigma \frac{Z_2 Z_6}{K^2} \delta r.$$ (28)

This diffeomorphism contains an arbitrary function; since we are not interested in pure gauge perturbations, we should fix this additional gauge symmetry. We can do so by setting one of the perturbations to zero. It is convenient to choose $\delta K = 0$. There remain diffeomorphisms which will preserve $\delta K = 0$: these have

$$\delta r = ar K^{1/2}$$ (29)

and

$$\delta t = \sigma a \left[ -2(r_0 + \hat{r}_2)(r_0 + \hat{r}_6) \frac{1}{\sqrt{K(r)}} + \left( \frac{r}{2} + \frac{7r_0}{4} + \hat{r}_2 + \hat{r}_6 \right) r \sqrt{K(r)} + \right.$$

$$\left. + \left( \frac{15r_0^2}{8} + \frac{3r_0(\hat{r}_2 + \hat{r}_6)}{2} + \hat{r}_2 \hat{r}_6 \right) \ln \frac{1 + \sqrt{K(r)}}{1 - \sqrt{K(r)}} \right] \sigma b.$$ (30)

If we apply this diffeomorphism to the non-extremal enhançon geometry (10), we obtain a metric of the form (23) with

$$\delta \psi_1^d = \left( \phi' - \frac{4}{3r} \right) \delta r - \frac{2}{3} \partial_r \delta r - \frac{2}{3} \sigma \delta t,$$

$$\delta \psi_2^d = -\frac{4}{3r} \delta r + \frac{4}{3} \partial_r \delta r + \frac{4}{3} \sigma \delta t,$$

$$\delta Z_6^2 = \left( \frac{Z_6'}{Z_6} + \frac{2}{r} \right) \delta r,$$

$$\delta Z_2^d = \left( \frac{Z_2'}{Z_2} + \frac{2}{3r} \right) \delta r - \frac{2}{3} \partial_r \delta r - \frac{2}{3} \sigma \delta t,$$

$$\delta \phi^d = \phi' \delta r.$$ (31)

\[\text{Note that our ansatz is slightly more general than the ansatz adopted in the study of perturbations of the extremal enhançon geometry in [9]. We introduced three new perturbation functions, } \delta \psi_1, \delta \psi_2, \text{ and } \delta K. \text{ We can choose to set } \delta K = 0 \text{ by a gauge transformation, and } \delta \psi_3 \text{ decouples—its analysis was completed in [7], so we will set } \delta \psi_3 = 0 \text{ in this paper. However, the additional function } \delta \psi_2 \text{ is needed to solve the full set of linearised field equations, even in the extremal case.}\]
In [7], we showed that if we replace the constants \( a \) and \( b \) by functions \( a(r) \) and \( b(r) \), and rewrite the general perturbation as

\[
\begin{align*}
\delta \psi_1 &= \delta \psi_1^d(a(r), b(r)), \\
\delta \psi_2 &= \delta \psi_2^d(a(r), b(r)) + \Psi_2, \\
\delta Z_6 &= \delta Z_6^d(a(r), b(r)) + Z_6, \\
\delta Z_2 &= \delta Z_2^d(a(r), b(r)), \\
\delta \phi &= \delta \phi^d(a(r), b(r)) + \Phi,
\end{align*}
\]

the full set of linearised equations of motion reduce to two algebraic equations for \( \partial_r a \) and \( \partial_r b \) and the following system of second-order equations for the independent functions \( \Psi_2, Z_6, \Phi \) (where \( ' \) again denotes \( \partial_r \), and we assume that all the perturbations behave as \( e^{\sigma t} \), representing an instability). There is an equation involving \( \Phi'' \),

\[
D(\Phi'' + \frac{2r - r_0}{r^2 K} \Phi' - \frac{Z_2 Z_6}{K^2} \sigma^2 \Phi) + P_2^1 (\Psi_2' + 2Z_6') + Q_1^1 \Phi + Q_2^1 \Psi_2 + Q_3^1 Z_6 = 0, \tag{33}
\]

with the polynomial coefficients

\[
D = r^2 K (8r^2 + 5r \hat{r}_2 + 5r \hat{r}_6 + 2r \hat{r}_2 \hat{r}_6)(4r^2 + 3r \hat{r}_2 + 3r \hat{r}_6 + 2r \hat{r}_2 \hat{r}_6),
\]

\[
P_2^1 = -2r^2 K(-2r^2 \hat{r}_2 + 6r^2 \hat{r}_6 + 8r \hat{r}_2 \hat{r}_6 + 3 \hat{r}_2^2 \hat{r}_6 + \hat{r}_2 \hat{r}_6^2),
\]

\[
Q_1^1 = -r^2(4r_0 \hat{r}_2 + 36r_0 \hat{r}_6 + 3 \hat{r}_2^2 + 6 \hat{r}_2 \hat{r}_6 + 27 \hat{r}_6^2) - r(40r_0 \hat{r}_2 \hat{r}_6 + 6 \hat{r}_2^2 \hat{r}_6 + 30 \hat{r}_2 \hat{r}_6^2) - 12r_0 \hat{r}_2 \hat{r}_6^2 - 8 \hat{r}_2^3 \hat{r}_6,
\]

\[
Q_2^1 = r_0(-2r^2 \hat{r}_2 + 6r^2 \hat{r}_6 + 8r \hat{r}_2 \hat{r}_6 + 3 \hat{r}_2^2 \hat{r}_6 + \hat{r}_2 \hat{r}_6^2),
\]

\[
Q_3^1 = r^2(8r_0 \hat{r}_2 + 24r_0 \hat{r}_6 + 9 \hat{r}_2^2 + 10 \hat{r}_2 \hat{r}_6 + 9 \hat{r}_6^2) + r(24r_0 \hat{r}_2 \hat{r}_6 + 6 \hat{r}_2^2 \hat{r}_6 + 10 \hat{r}_2 \hat{r}_6^2) + 6r_0 \hat{r}_2 \hat{r}_6^2 - 2r_0 \hat{r}_2 \hat{r}_6^2.
\]

The equation involving \( \Psi_2'' \) is

\[
D(\Psi_2'' - \frac{Z_2 Z_6}{K^2} \sigma^2 \Psi_2) + P_2^2 \Psi_2' + P_3^2 Z_6' + Q_1^2 \Phi + Q_2^2 \Psi_2 + Q_3^2 Z_6 = 0, \tag{39}
\]

where \( D \) is as before, and the other polynomial coefficients are

\[
P_2^2 = 64r^5 + r^4(-32r_0 + 120 \hat{r}_2 + 88 \hat{r}_6) + r^3(-76r \hat{r}_2 - 44r \hat{r}_6 + 30 \hat{r}_2^2 + 172 \hat{r}_2 \hat{r}_6 + 30 \hat{r}_6^2) + r^2(-15r \hat{r}_2^2 - 118r_0 \hat{r}_2 \hat{r}_6 - 15r_0 \hat{r}_6^2 + 44 \hat{r}_2 \hat{r}_6^2 + 52 \hat{r}_2 \hat{r}_6^2) + r(-28r \hat{r}_2 \hat{r}_6^2 - 36r_0 \hat{r}_2 \hat{r}_6^2 + 8 \hat{r}_2^2 \hat{r}_6^2 - 4r_0 \hat{r}_2 \hat{r}_6^2, \tag{40}
\]

\[
P_3^2 = -8r^2 \hat{r}_2 K(8r^2 + 16r \hat{r}_6 + 3 \hat{r}_2 \hat{r}_6 + 5 \hat{r}_6^2), \tag{41}
\]

\[
Q_1^2 = 4 \hat{r}_2(r^2(-8r_0 - 6 \hat{r}_2 - 6 \hat{r}_6) + r(4r_0 \hat{r}_6 - 7 \hat{r}_2 \hat{r}_6 + 3 \hat{r}_6^2) + 6r_0 \hat{r}_2 \hat{r}_6 + 2 \hat{r}_2 \hat{r}_6^2), \tag{42}
\]
\[ Q_2^2 = -2r_0 \hat{r}_2 (8r^2 + 16r \hat{r}_6 + 3\hat{r}_2 \hat{r}_6 + 5\hat{r}_6^2), \]  
\[ Q_3^2 = 4\hat{r}_2 (r^2 (16r_0 + 18\hat{r}_2 + 2\hat{r}_6) + r(12r_0 \hat{r}_6 + 21\hat{r}_2 \hat{r}_6 - \hat{r}_6^2) - 3r_0 \hat{r}_2 \hat{r}_6 + 5r_0 \hat{r}_2^2 + 6\hat{r}_2 \hat{r}_6^2). \]  
The equation involving \( Z_6'' \) is
\[
D(Z_6'' - \frac{Z_3 Z_6}{K^2} \sigma^2 Z_6) + P_2^3 \Psi_2 + P_3^3 Z_6 + Q_1^3 \Phi + Q_2^3 \Psi_2 + Q_3^3 Z_6 = 0, 
\]  
where \( D \) is as before, and the other polynomial coefficients are
\[
P_2^3 = -2r^2 K (6r^2 \hat{r}_2 - 2r^2 \hat{r}_6 + 8r \hat{r}_2 \hat{r}_6 + \hat{r}_2^2 \hat{r}_6 + 3r \hat{r}_2 \hat{r}_6^2),
\]
\[
P_3^3 = 64r^5 + r^4 (-32r_0 + 64\hat{r}_2 + 96\hat{r}_6) 
+ r^3 (-20r_0 \hat{r}_2 + 52r_0 \hat{r}_6 + 126r_0 \hat{r}_2 \hat{r}_6 + 30r_0 \hat{r}_2^2 + 76 \hat{r}_2 \hat{r}_6 + 30r_0 \hat{r}_6^2) 
+ r^2 (-15r_0 \hat{r}_2^2 + 172r_0 \hat{r}_2 \hat{r}_6 + 15r_0 \hat{r}_2^3 + 190 \hat{r}_2 \hat{r}_6 + 28 \hat{r}_2 \hat{r}_6^2 + 20 \hat{r}_2 \hat{r}_6^3) 
+ r (-12r_0 \hat{r}_2^3 - 36r_0 \hat{r}_2 \hat{r}_6 + 4r_0 \hat{r}_2^4 + 8r_2 \hat{r}_6 + 6 \hat{r}_2 \hat{r}_6^2 - 4r_0 \hat{r}_2 \hat{r}_6^2),
\]
\[
Q_1^3 = r^2 (12r_0 \hat{r}_2 + 12r_0 \hat{r}_6 + 9r_2^2 + 10r_2 \hat{r}_6 + 9r_6^2) + r (8r_0 \hat{r}_2 \hat{r}_6 + 10r_2^2 \hat{r}_6 + 6 \hat{r}_2 \hat{r}_6^2) - 4r_0 \hat{r}_2 \hat{r}_6^2 \hat{r}_6,
\]
\[
Q_2^3 = r_0 (6r_2 \hat{r}_2 - 2r_2 \hat{r}_6 + 8r \hat{r}_2 \hat{r}_6 + \hat{r}_2^2 \hat{r}_6 + 3r \hat{r}_2 \hat{r}_6^2),
\]
\[
Q_3^3 = -r^2 (24r_0 \hat{r}_2 + 8r_0 \hat{r}_6 + 27 \hat{r}_2^2 + 6 \hat{r}_2 \hat{r}_6 + 3 \hat{r}_6^2) 
- r (24r_0 \hat{r}_2 \hat{r}_6 + 30 \hat{r}_2^2 \hat{r}_6 + 2 \hat{r}_2 \hat{r}_6^2) + 2r_0 \hat{r}_2 \hat{r}_6^2 - 6r_0 \hat{r}_2 \hat{r}_6^2 - 8 \hat{r}_2 \hat{r}_6^2 \hat{r}_6).
\]

The problem of finding unstable perturbations of the enhançon geometries in our ansatz then reduces to finding solutions of this coupled system of equations satisfying some appropriate boundary conditions.

## 5 Horizon-branch stability

Let us first consider the perturbations for the horizon branch solutions. The appropriate boundary conditions are then just that the linearised perturbations should be regular on the horizon \( r = r_0 \) and at infinity. The solutions of the equations (38, 39, 45) behave as
\[
\Phi, \Psi_2, Z_6 \to (r - r_0)^{+\tilde{\sigma}}, \tilde{\sigma} = (r_0 + \hat{r}_2)^{1/2}(r_0 + \hat{r}_6)^{1/2}\sigma
\]
as \( r \to r_0 \), and they behave as
\[
\Phi, \Psi_2, Z_6 \to e^{\pm \sigma r}
\]
as \( r \to \infty \). We wish to know if there is some \( \sigma \) such that we obtain a solution where \( \Phi, \Psi_2, Z_6 \) all have decaying behaviour both at infinity and the horizon.

We have investigated this question numerically, using a simple relaxation method [10] to search for solutions satisfying the falloff conditions at the horizon and infinity, with a smooth
interpolation with no nodes (as we are most interested in the instability with largest \(\sigma\), which we would expect to have no nodes). We then relax \(\Phi, \Psi_2, Z_6, \sigma\) to see if we can reach a solution of the equations of motion. We have explored a wide range of the free parameters \(\hat{r}_6, \hat{r}_2, r_0\) of the background solution, and we never find any instability. The relaxation process fails to converge.

This is the expected result for large non-extremality; in this limit, the horizon branch solutions approach a four-dimensional Schwarzschild metric smeared over the K3 and longitudinal \(x_1, x_2\) directions, and this Schwarzschild metric is known to be stable against the kind of perturbations we are considering [11] (note that our perturbations are assumed independent of the longitudinal coordinates, so the Gregory-Laflamme instability [12] which will appear if the \(x_1, x_2\) are non-compact is absent from this analysis).

The non-trivial result is that this stability persists over the whole of the horizon branch. Thus, the linearised stability analysis has revealed no sign of any transition from this branch of solutions to any other solution as the mass decreases. This is very interesting; although it does not rule out such a transition, the horizon branch has passed the first test we could subject it to, and provides the best available description of the non-extremal enhançon physics in the region where it is available. This leads us to suspect that there is no other supergravity solution describing enhançon physics for the range of parameters where the horizon branch solution exists.

## 6 Extremal stability

To search for instabilities in the extreme case, where there is an with enhançon shell, we need to determine the appropriate matching conditions at the shell for the linearised perturbations \(\Phi, \Psi_2, Z_6\). Since these arise as perturbations of components of the metric which have non-trivial matching conditions relating the discontinuity of their derivatives to the shell stress-energy, obtaining the appropriate matching conditions is a non-trivial problem. We will use the DBI action to obtain these matching conditions. The ensuing numerical study finds no instabilities for the extremal case, as expected from supersymmetry arguments.

Outside the shell, we assume we have the perturbed metric [23], and the associated dilaton and RR fields, with \(r_0 = 0\), so \(K = 1\) (since we have used the diffeomorphism freedom to set \(\delta K = 0\)). Inside the shell, we will have a perturbed flat space,

\[
g_s^{1/2} ds^2 = e^{-\gamma_1/2} \left[ H_2^{-1/2} \tilde{H}_6^{-1/2} (-e^{\delta\gamma_2} dt^2 + e^{-\frac{1}{4}\delta\beta_2} (dx_1^2 + dx_2^2)) + H_2^{1/2} \tilde{H}_6^{1/2} (dr^2 + r^2 d\Omega_2^2) + V^{1/2} \tilde{H}_2^{1/2} \tilde{H}_6^{-1/2} d\sigma_{K3} \right], \tag{53}
\]

dilaton

\[
\bar{\phi} = \phi + \delta\xi, \tag{54}
\]

and R–R fields

\[
\bar{C}_{(3)} = C_{(3)} + \delta C_{(3)}, \quad \bar{C}_{(7)} = C_{(7)} + \delta C_{(7)}. \tag{55}
\]

Here

\[
\gamma_1 = \phi + \delta\gamma_1, \quad H_2 = H_2(1 + \delta H_2), \quad H_6 = H_6(1 + \delta H_6), \tag{56}
\]
the constants $H_2, H_6$ are as in [12] with $r'_0 = 0$, the unperturbed dilaton $\phi$ is as in [17], and the RR potentials are as in [17].

We can analyse the perturbations of the interior geometry following the same route used above for the exterior geometry. The diffeomorphisms preserving the form of our ansatz are now $\delta r = cr$, $\delta t = \sigma H_2 H_6 c r^2 / 2 + \sigma d$. We can write

\[
\begin{align*}
\delta \gamma_1 &= \delta \gamma^d_1(c(r), d(r)), \\
\delta \gamma_2 &= \delta \gamma^d_2(c(r), d(r)) + \Gamma_2, \\
\delta H_6 &= \delta H^d_6(c(r), d(r)) + H_6, \\
\delta H_2 &= \delta H^d_2(c(r), d(r)), \\
\delta \xi &= \delta \xi^d(c(r), d(r)) + \Xi,
\end{align*}
\]

and then we find that the equations for the free perturbation functions are all

\[
\partial_r^2 f + \frac{2}{r} \partial_r f - H_2 H_6 \sigma^2 f = 0,
\]

where $f = \Xi, \Gamma_2, H_6$. The solution regular at $r = 0$ is

\[
f = f_0 \sinh \frac{\bar{\sigma} r}{r},
\]

where $\bar{\sigma} = \sqrt{H_2 H_6} \sigma$. Thus, at the enhançon shell $r = r_e$, these interior perturbations will satisfy the boundary conditions

\[
r_e f'(r_e) + (1 - \bar{\sigma} r_e \coth \bar{\sigma} r_e) f(r_e) = 0.
\]

What we really want is boundary conditions for the exterior perturbations $\Phi, \Psi_2, Z_6$ at the shell. To obtain these from the above conditions on $\Xi, \Gamma_2, H_6$, we need to work out the junction conditions at the shell for the perturbations, and solve for $\Xi, \Gamma_2, H_6$ and their first derivatives in terms of $\Phi, \Psi_2, Z_6$ and their derivatives.

In general, the location of the shell separating the interior and exterior is also perturbed; however, we can use some of the remaining diffeomorphism freedom in the ansatz to fix the coordinate position of the shell to be $r_1 = r_e$, the enhançon radius of the unperturbed metric (by, say, an appropriate choice of the undetermined constant $a$ in [29, 30]). The dynamics of the shell will then find its expression through the variation of the perturbed metric at the shell location. This choice greatly simplifies the problem of matching the perturbations at the shell.

Continuity of the metric and fields at the shell implies

\[
\begin{align*}
\delta \phi(r_e) &= \delta \xi(r_e), \\
\delta \psi_1(r_e) &= \delta \gamma_1(r_e), \\
\delta \psi_2(r_e) &= \delta \gamma_2(r_e), \\
\delta Z_2(r_e) &= \delta H_2(r_e), \\
\delta Z_6(r_e) &= \delta H_6(r_e).
\end{align*}
\]

To relate the first derivatives, we compute the discontinuity in the extrinsic curvature at the surface $r = r_e$ when we patch the two geometries together. This allows us to infer the stress tensor of the perturbed shell from the supergravity point of view, with the result

\[
S_{tt} = \frac{1}{2\kappa^2 \sqrt{g_{rr}}} \left[ \frac{2 \bar{Z}_2'}{Z_2} - 2 \frac{\bar{Z}_6'}{Z_6} - 4 \psi_1' - \delta \psi_2' - 2 \frac{\bar{H}_2'}{H_2} + 2 \frac{\bar{H}_6'}{H_6} + 4 \gamma_1 + \delta \gamma_2 \right] g_{tt}.
\]
\[ S_{\rho\sigma} = \frac{1}{2\kappa^2 \sqrt{g_{rr}}} \left[ 2 \frac{Z_2'}{Z_2} - 2 \frac{Z_6'}{Z_6} - 4\psi'_1 + \frac{1}{2} \delta\psi'_2 - 2 \frac{H'_2}{H_2} + 2 \frac{H'_6}{H_6} + 4\gamma'_1 - \frac{1}{2} \delta\gamma'_2 \right] g_{\rho\sigma}, \quad (63) \]

\[ S_{ij} = \frac{1}{2\kappa^2 \sqrt{g_{rr}}} \left[ \frac{Z_i'}{Z_2} - 3 \frac{Z_6'}{Z_6} - 4\psi_1' - \frac{H'_i}{H_2} + 3 \frac{H'_6}{H_6} + 4\gamma'_1 \right] g_{ij}, \quad (64) \]

\[ S_{ab} = \frac{1}{2\kappa^2 \sqrt{g_{rr}}} \left[ \frac{Z_a'}{Z_2} - 2 \frac{Z_6'}{Z_6} - 4\psi_1' - \frac{H'_a}{H_2} + 2 \frac{H'_6}{H_6} + 4\gamma'_1 \right] g_{ab} \quad (65) \]

(coordinates \( \rho, \sigma \) run over \( t, 1, 2, i, j \) over \( \theta, \phi, \) and \( a, b \) over the K3). We will assume that this shell stress tensor is still sourced by a collection of BPS branes. Using the worldvolume action for a wrapped D6-brane,

\[ S = - \int_{M_2} d^3 \xi e^{-\Phi}(\mu_6 V(r) - \mu_2)(- \det G_{\mu\nu})^{1/2} + \mu_6 \int_{M_2 \times K_3} C(7) - \mu_2 \int_{M_2} C(3), \quad (66) \]

where \( G_{\mu\nu} \) is the induced string-frame metric, and the string-frame volume \( V(r) = V e^{\Phi - \psi_1} Z_2 Z_6^{-1} \), one easily obtains the Einstein-frame stress-energy for a single brane in the exterior geometry,

\[ S^{brane}_{\mu\nu} = -e^{3\phi^4/4} (\mu_6 - \mu_2 V(r)^{-1}) g_{\mu\nu}, \quad (67) \]

\[ S^{brane}_{ab} = -e^{3\phi^4/4} \mu_6 g_{ab} \quad (68) \]

(where \( \mu, \nu \) run over \( t, 1, 2 \)). If we use this to calculate the stress-energy of the shell, we find that the value the stress tensor should take can be written as

\[ S_{\mu\nu}^{shell} = \frac{1}{2\kappa^2 \sqrt{g_{rr}}} e^{3\phi^4/4 + \psi_1^4/4} \left[ \frac{Z_2'}{Z_2} e^{-\phi + \psi_1} + \frac{Z_6'}{Z_6} \right] g_{\mu\nu}, \quad (69) \]

\[ S_{ij}^{shell} = 0, \quad (70) \]

\[ S_{ab}^{shell} = \frac{1}{2\kappa^2 \sqrt{g_{rr}}} e^{3\phi^4/4 + \psi_1^4/4} \frac{Z_6'}{Z_2} g_{ab}. \quad (71) \]

The matching conditions for first derivatives of the metric are then obtained by setting this brane stress tensor equal to the stress tensor calculated from the supergravity point of view above. Similarly, matching the discontinuity in the dilaton to the brane source gives

\[ \phi'_{out} - \phi'_{in} = \frac{e^{3\phi^4/4 + \psi_1^4/4}}{4Z_2^{1/4} Z_6^{-3/4}} \left[ e^{\psi_1 - \phi} \frac{Z_2'}{Z_2} - 3 \frac{Z_6'}{Z_6} \right]. \quad (72) \]

Taking the first-order part of all these equations gives us five equations relating the derivatives of \( \delta\phi, \delta\psi_1, \delta\psi_2, \delta Z_2, \delta Z_6 \) to the corresponding interior quantities.

We then have ten matching equations at the shell. However, we only have nine quantities to specify: we want to solve for the three free interior functions \( \Xi, \Gamma_2, \mathcal{H}_6 \) and their first derivatives at the shell in terms of \( \Phi, \Psi_2, \mathcal{Z}_6 \) and their first derivatives, and we also have three undetermined constants in the diffeomorphisms to fix (since
we already fixed one to satisfy $r_i = r_e$). Remarkably, this over-determined system has a solution, and substituting into the boundary condition \((60)\) gives us, after considerable algebra, the relatively simple expressions

\[
0 = -2\hat{r}_6(v^2 + 1)\Phi'(r_e) - \hat{r}_6(v^2 + 4v - 1)\Psi'_2(r_e) + 4\hat{r}_6(v + 1)Z'_6(r_e) \tag{73}
\]

\[
0 = -8\hat{r}_6(v + 1)\Phi'(r_e) + 4\hat{r}_6(v + 1)\Psi'_2(r_e) \tag{74}
\]

\[
0 = 4\hat{r}_6(v + 1)\Phi'(r_e) + 4\hat{r}_6(v + 1)Z'_6(r_e) \tag{75}
\]

where $v = V/V_*$ and $\hat{\sigma} = \sqrt{H_{12}H_6}/\sigma$.

The problem of finding instabilities of the extremal enhançon solution then reduces to looking for solutions of the equations of motion for $\Phi$, $\Psi_2$, $Z_6$ for some $\sigma$ which satisfy these boundary conditions at the shell and fall off at large distance. We have searched for such solutions using the same relaxation techniques as in the previous section; we do not find any instabilities. This result is, of course, what one would expect on the basis of supersymmetry. A solution which satisfies the BPS bound may have flat directions—it may be marginally stable to some perturbation—but one would not expect that it will have any truly unstable perturbations.

### 7 Conclusions

In this paper, we have investigated the physics of non-extremal enhançon solutions. We have found that the second branch of non-extremal repulson solutions found in \([6]\) appears to be generically unphysical. The singularity in this solution cannot be removed by excising the region inside the enhançon radius and matching to a smooth interior across a physical enhançon shell. If we attempt to impose such a matching, the shell required to achieve it will violate the weak energy condition.

This result provides a resolution of a number of puzzling features of these solutions observed in \([6]\). The first puzzle was that the exterior solution never contains an event horizon, no matter how large the non–extremality parameter $r_0$ became, in contradiction to our expectation that the system would eventually collapse to form a black hole. The second puzzle was that in the limit of large K3 volumes, these solutions do not reproduce the expected non-extremal $D6$–brane. In this limit the

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6We have also checked explicitly that we get the same boundary conditions at the shell for the physical degrees of freedom even if we work in a coordinate system where the coordinate location of the shell is not fixed.
R–R 3-form potential vanishes as expected, but we still have additional dilaton hair. Since the supergravity solutions with these features violate the weak energy condition, these properties do not represent the real physics of wrapped D6-branes.

In [6], it was observed that this solution was presumably just one member of a family of solutions with non-trivial dilaton hair. We would expect that the problem we found here is quite generic in such a family. One can interpret the violation of the weak energy condition at the enhançon radius as saying that the energy in the field configuration outside the enhançon exceeds the total ADM energy. Thus, we need a negative-energy shell to make up the deficit. We would therefore expect any solution with a lot of energy in the dilaton outside the enhançon will similarly need a negative-energy shell to avoid a singularity, and would hence be unphysical. Further investigations of more general non-extremal supergravity solutions are in progress [13].

We remarked earlier that this analysis also appears to extend to the case with additional D2-brane charge. It would be interesting to perform a similar analysis for the system of non-extremal fractional branes considered in [14], where a similar two-branch structure was found.

Investigating numerically the stability of the extremal enhançon, we found that the shell is stable under small radial perturbations. We used a perturbation ansatz, generalising the one adopted in [9], which respected the symmetries of the unperturbed solution. By fixing the diffeomorphism freedom, we were able to reduce the linearized equations of motion to three coupled equations of motion for the perturbed modes. We studied these equations numerically for a wide range of the parameters \( r_2 \) and \( r_6 \), and failed to find a solution which rendered the shell unstable. We concluded that at least for radial perturbations, the extremal enhançon shell is stable. This result is, of course, not a surprise: since this is a supersymmetric solution, we would be very surprised to find an instability.

This result strengthens the argument of [6] for the consistency of the excision procedure [1]. This excision is accomplished by the introduction of a shell of wrapped D6–branes at the enhançon radius. The fact that this solution is stable is another argument in favour of the excision and of the idea that the solution is a sensible construction from the point of view of supergravity.

The horizon branch of the non–extremal solution was also found to be stable, in a numerical investigation for a wide range of the parameters \( \hat{r}_2 \), \( \hat{r}_6 \) and \( r_0 \). Again we use the same generalised ansatz and the diffeomorphism freedom to reduce to a system of three coupled equations. We could not find any unstable solutions to this configuration. This result is also fairly unsurprising, since for large non–extremality the horizon branch solution approaches a four dimensional Schwarzschild metric smeared over a \( K3 \) and two longitudinal directions \( x_1, x_2 \). This geometry is known to be stable against the kind of perturbations that we are investigating.

What is quite interesting is that stability of the horizon branch persists over the whole range of parameters. An instability of the horizon branch could have been interpreted as evidence for the existence of a new family of solutions, which might connect to the extremal enhançon solution. While absence of evidence is not evidence of absence, the stability of the horizon branch makes it seem plausible that it provides the full description of non-extremal enhançon physics for the range of parameters
where it exists.

What of small perturbations away from the extremal enhançon solution, where there is no horizon branch? We do not have a supergravity solution which describes them, but we do not feel this implies some pathology in the physics. It may be that the appropriate solutions lie outside the ansatz we have considered here; further exploration of this possibility is in progress [13]. Alternatively, it may be that the physics of non-extreme enhançons is not captured by a purely supergravity solution. They could involve non-trivial non-abelian gauge fields, or branes distributed in a ‘thick shell’ over some finite range in the radial coordinate. It will be very interesting to investigate this question further. It would also be very interesting to investigate the implications of these results for other applications of the enhançon, such as [15, 16, 17].

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