Supersymmetry and Gauge Invariance Constraints in a U(1)×U(1)′-Higgs Superconducting Cosmic String Model

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A supersymmetric extension of the $U(1) \times U(1)'$-Higgs bosonic superconducting cosmic string model is considered, where the action is constructed from scalar chiral and vector superfields. The chiral superfields are assumed to transform separately under the Abelian gauge groups, and the constraints imposed upon such a model due to renormalizability, supersymmetry, and gauge invariance are examined. For a simple model with a single $U(1)$ chiral superfield and a single $U(1)'$ chiral superfield, the Witten mechanism for bosonic superconductivity (giving rise to long range gauge fields outside of the string) does not exist without the introduction of extra chiral supermultiplets. The simplest model that can accommodate the requisite interactions for a superconducting cosmic string solution with long range $U(1)$ gauge fields requires five chiral supermultiplets. An investigation of the bosonic sector of this model indicates that a solution is admitted describing a gauge string surrounded by supersymmetric vacuum. By considering the effects of the gauge string background upon the remaining scalar fields of the theory, it is concluded that a parameter range exists for which the $U(1)$ charged scalar fields condense within the core of the string, making it a superconducting string that can carry charge and/or current. It is also found that the neutral scalar field can be associated with particle bound states that are localized within or near the string core.

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I. INTRODUCTION

The intriguing possibility exists that the early universe may have undergone a series of symmetry breaking phase transitions resulting in the production of topological defects, such as cosmic strings [1]- [4]. As pointed out by Witten [5], it is possible that cosmic strings may be superconducting, owing to the presence of a charged field condensate in the core of the string. Superconducting strings can therefore be endowed with a charge and/or current generating long range gauge fields outside of the string. It has also been argued that Grand Unified Theory (GUT) scale cosmic strings may quite generically exhibit superconducting properties [6], [7]. The additional possibility exists that supersymmetry was physically realized in the early universe, and that the supersymmetry was broken at the same time as, or subsequent to, the formation of cosmic strings. The investigation of basic models and scenarios involving superconducting cosmic strings within a supersymmetric context therefore seems relevant.

The basic prototype bosonic superconducting string model [5] is based upon a $U(1) \times U(1)'$ gauge symmetry. By the Higgs mechanism, the $U(1)'$ gauge symmetry is spontaneously broken by the vacuum state of the theory, giving rise to the existence of a topologically stable cosmic string solution. While the $U(1)$ symmetry is respected by the vacuum, it can get broken within the string core to give rise to a bosonic condensate. Here, attention is focused upon the supersymmetric extension of the $U(1) \times U(1)'$-Higgs bosonic superconducting string model. In the nonsupersymmetric version of the model containing one $U(1)$ complex scalar field and one $U(1)'$ complex scalar field, along with $U(1)$ and $U(1)'$ vector gauge fields, a scalar potential can readily be constructed which allows the Witten mechanism to operate, giving rise to a bosonic superconducting cosmic string solution. The scalar potential of this model is subject to the constraints imposed by renormalizability and gauge invariance. In the supersymmetric version of the $U(1) \times U(1)'$-Higgs model, however, the scalar potential which describes interactions between various fields is derived from the superpotential and $D$-type auxiliary terms, and there is an additional constraint imposed by supersymmetry. The supersymmetric version of the model can therefore have a different appearance from that of the nonsupersymmetric version. In fact, what is found is that the supersymmetric version of the model, with one $U(1)$ chiral superfield and one $U(1)'$ chiral superfield, along with the Abelian gauge superfields, does not allow the Witten mechanism to exist, so that no superconducting string solutions with long range gauge fields are supported by this simple model. The simplest version of the supersymmetric model that can accommodate the Witten mechanism apparently requires five chiral superfields, i.e. two $U(1)$ chiral superfields, two $U(1)'$ chiral superfields, and a neutral chiral superfield, along with the gauge vector superfields. The superconducting string solution that is admitted by this model is examined. By
focusing upon the bosonic sector of this model, it is found that the two \( U(1)' \) scalar fields conspire to give rise to a gauge string, while the two \( U(1) \) scalar fields form a condensate within the string core, and the neutral scalar field can be associated with bound states localized within or near the string core.

The nonsupersymmetric \( U(1) \times U(1)' \)-Higgs model is briefly reviewed in section II. In section III a supersymmetric Abelian-Higgs model with one chiral superfield and a vector gauge field admitting a Nielsen-Olesen cosmic string solution \([1]\) is presented, where notation and orientation can be established. A supersymmetric extension of the \( U(1) \times U(1)' \)-Higgs model forms section IV, where the relevance of supersymmetry and gauge invariance constraints becomes evident. Section V involves an examination of the superconducting string solution in this model, and a summary forms section VI.

II. THE \( U(1) \times U(1)' \)-HIGGS BOSONIC SUPERCONDUCTING STRING MODEL

Cosmic gauge strings \([1]–[4]\) can emerge as solutions in an Abelian-Higgs model where the Abelian gauge group undergoes a spontaneous symmetry breaking. Outside the string core, where the scalar field becomes nonvanishing, the vector gauge field acquires mass so that there is no long range gauge field generated by the gauge string configuration. However, a long range gauge field can be generated if an additional Abelian gauge group is included \([3]\). In a \( U(1) \times U(1)' \)-Higgs model \([3, 4]\) containing two complex scalar fields, with each scalar field transforming separately under the gauge groups, one Abelian gauge group can be spontaneously broken, giving rise to cosmic strings, while the other gauge group remains unbroken. With a suitable potential (and within a particular parameter range), the charged scalar field associated with the unbroken Abelian symmetry can stabilize the system through the formation of a scalar condensate within the core of the string, giving rise to a current density which generates a nonvanishing long range gauge vector field outside of the string. The Lagrangian for such a model can be displayed as

\[
L = (D^\mu \phi)^* (D_\mu \phi) + (D^\mu \sigma)^* (D'_\mu \sigma) - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} - \frac{1}{4} B^{\mu \nu} B_{\mu \nu} - V, \tag{1}
\]

where \( D'_\mu \sigma = (\nabla_\mu + igB_\mu) \sigma \) is the gauge covariant derivative\([4]\) associated with the spontaneously broken \( U(1)' \) gauge group, \( D_\mu \phi = (\nabla_\mu + ieA_\mu) \phi \) is the gauge covariant derivative associated with the \( U(1) \) gauge group, which is unbroken outside of the string, \( \sigma \) is the string scalar field which carries a \( U(1)' \) charge \( Q \) and a \( U(1) \) charge

\[^1\nabla_\mu \) is the ordinary spacetime covariant derivative, for use in curvilinear coordinate systems.
q = 0, and φ is the scalar field associated with U(1) which carries U(1)′ and U(1) charges of \( Q = 0 \) and \( q \), respectively. The field strength tensors are \( B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \) and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \).

The potential can be written in the form

\[
V = \frac{1}{2} \lambda_\sigma (\bar{\sigma} \sigma - \eta^2)^2 + f (\bar{\sigma} \sigma - \eta^2) \bar{\phi} \phi + m^2 \bar{\phi} \phi + \frac{1}{2} \lambda_\phi (\bar{\phi} \phi)^2. \tag{2}
\]

The vacuum state is described by \( |\sigma| = \eta, \phi = 0 \), and it is assumed that the parameters \( \lambda_\sigma, \lambda_\phi, f, \) and \( m \) are real-valued positive quantities. In the string core \( \sigma \to 0 \), and by the Witten mechanism \(^5\) there exists a range of parameters for which the condition \( |\phi| \to |\phi_0| \), where

\[
|\phi_0|^2 = \frac{(f \eta^2 - m^2)}{\lambda_\phi}, \tag{3}
\]

is required for stability of the system, provided that \( (f \eta^2 - m^2) > 0 \). The electromagnetic current density \( e_j = ie[\bar{\phi}(D_\mu \phi) - \phi(D_\mu \phi)^*] \) can be nonvanishing in the string core and can therefore generate long range electromagnetic fields outside the string. These fields allow long range interactions to exist between strings and between strings and particles carrying a U(1) charge.

It should also be noted that fermionic superconducting strings can exist \(^6\), where fermionic zero modes, rather than a bosonic condensate, exist within the string core \(^8\), \(^9\), with the fermions carrying a nonzero U(1) charge. These fermion modes can arise from Yukawa couplings to scalar fields.

III. SUPERSYMMETRIC EXTENSION OF THE ABELIAN-HIGGS MODEL

Let us consider a supersymmetric extension of the Abelian-Higgs model \(^8\), \(^9\) which can give rise to gauge string solutions when the Abelian gauge group is spontaneously broken. The model consists of a chiral superfield \( \Sigma \) coupled to a vector superfield \( B \). (We employ the Wess-Zumino gauge throughout.)

The chiral supermultiplet contains a complex scalar field \( \sigma \), a Weyl 2-spinor \( \psi_\alpha \), along with an auxiliary complex scalar field \( F \). The chiral superfield has a superspace representation \(^9\), \(^10\)

\[
\Sigma(x, \theta, \bar{\theta}) = \sigma(y) + \sqrt{2} \theta \psi(y) + \theta^2 F(y), \tag{4}
\]

where \( y^\mu = x^\mu + i \theta \sigma^\mu \bar{\theta} \). In the Wess-Zumino gauge, the vector supermultiplet contains a “photon” \( B_\mu \), the Weyl spinor “photino” fields \( \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}} \), and a \( D \)-term auxiliary field \( D(x) \). The vector superfield \( B \) has a superspace representation \(^9\), \(^10\)

\(^2\)Note that the “bar” and “star” symbols are used interchangeably to denote complex conjugation, so that \( \bar{\phi} = \phi^* \), for example.
where \( \theta^2 = \theta \theta \equiv \theta^\alpha \theta^\alpha \) with \( \alpha = 1, 2 \), \( \bar{\theta}^2 = \bar{\theta} \bar{\theta} \equiv \bar{\theta}_\alpha \bar{\theta}^\alpha \) with \( \bar{\alpha} = 1, 2 \), and \( \theta \lambda = \theta^\alpha \lambda_\alpha \), \( \bar{\theta} \bar{\lambda} = \bar{\theta}^\alpha \bar{\lambda}_\alpha \). [Aside from a metric \( g_{\mu \nu} \) with signature \((+,-,-,-)\), we adopt the conventions (see Appendix) and gamma matrices of ref. [9].] The Lagrangian, aside from surface terms, is given by [9], \( \text{\( [\text{12}] \)} \)

\[
L = \frac{1}{4} \left[ W^\alpha W_\alpha |_{\theta^2} + \bar{W}_\bar{\alpha} \bar{W}^{\bar{\alpha}} |_{\bar{\theta}^2} \right] + \left( \Sigma \varepsilon^{\rho\xi\eta\nu} \Sigma \right) |_{\theta^2 \bar{\theta}^2},
\]

where \( [\text{10}] \) \( W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha B \) is the field strength chiral superfield \((D_\alpha \) is the supersymmetric covariant derivative), \( W \) is the superpotential, \( W |_{\theta^2} \) represents the \( \theta^2 \) part of \( W \), etc., \( \kappa D(x) \) is the Fayet-Illiopoulos \( [\text{11}] \) \( D \)-term, with \( \kappa \) =constant, and \( \Sigma \) is the complex conjugate of \( \Sigma \), etc.

Under an Abelian gauge transformation,

\[
\Sigma \rightarrow e^{-i\Lambda Q} \Sigma, \quad \bar{\Sigma} \rightarrow \bar{\Sigma} e^{i\bar{\Lambda} Q}, \quad B \rightarrow B + \frac{i}{2} (\Lambda - \bar{\Lambda}), \quad W_\alpha \rightarrow W_\alpha,
\]

where \( \Lambda(x, \theta, \bar{\theta}) \) is a gauge parameter chiral superfield, and \( Q \) is the Abelian charge. The action associated with the Lagrangian \( L \) is invariant under supersymmetry transformations and the Abelian gauge transformations in (7).

The model can be extended to accommodate \( N \) chiral superfields \( \Sigma_i = (\sigma_i, \psi_i, F_i) \), \( i = 1, \ldots, N \), with associated Abelian charges \( Q_i \), with the replacement \( \Sigma \rightarrow \Sigma_i \), \( Q \rightarrow Q_i \) and summing over the index \( i \) in the expression for the Lagrangian in (7).

Solving for the auxiliary fields then yields \( [\text{9}] \)

\[
\bar{F}_i = -\frac{\partial W}{\partial \sigma_i}, \quad F_i = -\frac{\partial W}{\partial \bar{\sigma}_i}, \quad D = -\left[ \kappa + \frac{1}{2} g \sum_i Q_i \bar{\sigma}_i \sigma_i \right].
\]

The Lagrangian can be written in terms of the component fields as

\[
L = L_B + L_F + L_Y,
\]

where

\[
L_B = (\bar{D}^\mu \bar{\sigma}_i)(D_\mu \sigma_i) - \frac{1}{4} B^{\mu \nu} B_{\mu \nu} - V,
\]

\[
L_F = -i \psi_i \sigma^\mu \bar{D}_\mu \psi_i - i \lambda \sigma^\mu \bar{\partial}_\mu \bar{\lambda},
\]

\[
L_Y = \frac{ig}{\sqrt{2}} Q_i \left[ \bar{\sigma}_i \psi_i \lambda - \sigma_i \bar{\psi}_i \bar{\lambda} \right] - \frac{1}{2} \sum_{k,l=1}^N \left( \frac{\partial^2 W}{\partial \sigma_k \partial \sigma_l} \right) \psi_k \psi_l + CC,
\]

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where a sum over \( i \) is implied, and where \( D^i_\mu = (\nabla_\mu + igQ_iB_\mu) \) and \( \bar{D}^i_\mu = (\nabla_\mu - igQ_iB_\mu) \) are the ordinary gauge covariant derivatives. The scalar potential in (10) is given by

\[
V = \sum_i \bar{F}_i F_i + \frac{1}{2} D^2 = \sum_i \left| \frac{\partial W}{\partial \sigma_i} \right|^2 + \frac{1}{2} \left[ \kappa + \frac{1}{2} g \sum_i Q_i \bar{\sigma}_i \sigma_i \right]^2.
\]  

(13)

For a renormalizable theory (in four spacetime dimensions) the most general form of the superpotential is described by

\[
W = a_i \Sigma_i + b_{ij} \Sigma_i \Sigma_j + c_{ijk} \Sigma_i \Sigma_j \Sigma_k ,
\]  

(14)

where \( a_i, b_{ij}, \) and \( c_{ijk} \) are constants and a sum over the contracted indices is implied. The superpotential \( W \) is also constrained by gauge invariance, i.e. each term in \( W \) must have a net charge of zero, so that \( a_i = 0 \) for \( Q_i \neq 0, b_{ij} = 0 \) for \( Q_i + Q_j \neq 0, \) and \( c_{ijk} = 0 \) for \( Q_i + Q_j + Q_k \neq 0. \)

In the Fayet model [8, 9] with a single chiral superfield \( \Sigma_i, (i = 1), \) gauge invariance forces the superpotential \( W \) to vanish identically, so that the scalar potential reduces to

\[
V = \frac{1}{2} D^2 = \frac{1}{2}(\kappa + \frac{1}{2} gQ\bar{\sigma}\sigma)^2. 
\]  

(15)

and the supersymmetric vacuum state, located by \( |\sigma| = \eta, \) spontaneously breaks the Abelian gauge symmetry. In the limit of vanishing fermion fields, we have \( L = L_B, \) with the scalar potential given by (13), which coincides with a broken symmetric Abelian-Higgs model, which admits as a solution, an ordinary Nielsen-Olesen cosmic Abelian gauge string. With the inclusion of several chiral superfields \( \Sigma_i, \) the theory, in general, supports cosmic strings formed through the breaking of the Abelian gauge symmetry, along with interactions involving charged particles which transform under the same Abelian gauge group as gives rise to the strings. However, no long range forces exist, since the vector field \( B_\mu \) acquires mass. Therefore, even if bosonic condensates or fermionic zero modes form in the core of a string, they will not give rise to long range gauge forces outside of the string.

### IV. SUPERSYMMETRIC EXTENSION OF THE U(1) × U(1)′ - HIGGS MODEL

In order to have the possibility of describing superconducting gauge strings in a supersymmetric theory, where the gauge strings support currents that give rise to long range gauge fields, let us consider the supersymmetric extension of the \( U(1) \times U(1)’ \)-Higgs model described by the Lagrangian

\[
L = \frac{1}{4} \left[ W_A^\alpha W_{A\alpha} |\sigma^2 + \bar{W}_{A\alpha} W_A^\beta |\bar{\sigma}^2 \right] + \frac{1}{4} \left[ W_B^\beta W_{B\beta} |\sigma^2 + \bar{W}_{B\beta} \bar{W}_B^{\bar{\beta}} |\bar{\sigma}^2 \right] \\
\quad + \left( \Sigma_i e^{gQ_iB_i} \Sigma_i \right) |\sigma^2 \bar{g}^2 + (\Phi_j e^{gQ_j\Phi_j} \bar{\Phi}_j) \right) |\bar{g}^2 \bar{\sigma}^2 \\
\quad + \kappa_B D_B(x) + \kappa_A D_A(x) + W(\Sigma, \Phi) |\sigma^2 + \bar{W}(\bar{\Sigma}, \bar{\Phi}) |\bar{\sigma}^2
\]  

(16)
The chiral matter supermultiplets are described by \( \Sigma_i = (\sigma_i, \psi_i, F_i) \), with \( i = 1, \ldots, M \), and \( \Phi_j = (\phi_j, \chi_j, G_j) \), with \( j = 1, \ldots, N \), and the two vector supermultiplets are described by \( A = (A_\mu, \xi_\alpha, \bar{\xi}_{\dot{\alpha}}, D_A) \) and \( B = (B_\mu, \lambda, \bar{\lambda}, D_B) \), where \( F_i, G_j, D_A, \) and \( D_B \) are the auxiliary fields. A Fayet-Illiopoulos term associated with each vector superfield has been included in (13), and \( W_{Aa} = -\frac{1}{4} \bar{D}^2 D_a A \) is the field strength chiral superfield associated with \( A \) and \( W_{B\dot{a}} = -\frac{1}{4} \bar{D}^2 D_{\dot{a}} B \) is the corresponding field strength chiral superfield associated with the vector superfield \( B \). The chiral superfield \( \Phi_j \) carries a \( U(1) \) charge of \( q_j \) and a zero \( U(1)' \) charge \( (Q_j = 0) \), while the chiral superfield \( \Sigma_i \) carries a \( U(1)' \) charge of \( Q_i \) and a zero \( U(1) \) charge \( (q_i = 0) \). The \( U(1) \) vector gauge field is \( A_\mu \) and the \( U(1)' \) vector gauge field is \( B_\mu \). The gauge covariant derivatives for the \( U(1) \) and \( U(1)' \) gauge groups are

\[
D_i^\mu = (\nabla_\mu + \frac{i}{2} q_i A_\mu), \quad D_i'^\mu = (\nabla_\mu + \frac{ig}{2} Q_i B_\mu),
\]

respectively. The superfield \( \Phi_j \) transforms nontrivially only under the group \( U(1) \), while \( \Sigma_i \) transforms nontrivially only under the group \( U(1)' \), i.e.

\[
\Phi_j \rightarrow e^{-i\Lambda_A q_j} \Phi_j, \quad \Sigma_i \rightarrow e^{-i\Lambda_B Q_i} \Sigma_i,
\]

\[
A \rightarrow A + \frac{i}{e} (\Lambda_A - \bar{\Lambda}_A), \quad B \rightarrow B + \frac{i}{g} (\Lambda_B - \bar{\Lambda}_B),
\]

where \( \Lambda_A \) and \( \Lambda_B \) are the gauge parameter chiral superfields associated with the local \( U(1) \) and \( U(1)' \) transformations, respectively.

In terms of the component fields, let us write the Lagrangian as

\[
L = L_B + L_F + L_Y ,
\]

where

\[
L_B = (\bar{D}^i \bar{\phi}_j)(D_i^\mu \phi_j) + (\bar{D}'^i \bar{\sigma}_i)(D'^\mu_i \sigma_i) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - V,
\]

\[
L_F = -i \chi_j \sigma^\mu \bar{D}^i_j \bar{\chi}_j - i \psi_i \sigma^\mu \bar{D}^i_\mu \psi_i - i \xi \sigma^\mu \nabla_\mu \bar{\xi} - i \lambda \sigma^\mu \nabla_\mu \bar{\lambda},
\]

\[
L_Y = \frac{i}{\sqrt{2}} q_i \left[ \bar{\phi}_j \chi_j \xi - \phi_j \bar{\chi}_j \bar{\xi} \right] + \frac{i}{\sqrt{2}} Q_i \left[ \bar{\sigma}_i \psi_i \lambda - \sigma_i \bar{\psi}_i \bar{\lambda} \right] - \frac{1}{2} \left[ \sum_{k,l=1}^{M} \left( \frac{\partial W}{\partial \sigma_k \partial \sigma_l} \right) \psi_k \psi_l + \sum_{k,l=1}^{N} \left( \frac{\partial W}{\partial \phi_k \partial \phi_l} \right) \chi_k \chi_l + CC \right],
\]

with a sum over \( i \) and \( j \) implied, and where \( F_{\mu\nu} = \partial_\mu A_{\nu} - \partial_\nu A_\mu \) and \( B_{\mu\nu} = \partial_\mu B_{\nu} - \partial_\nu B_\mu \), and \( CC \) stands for complex conjugate. The scalar potential is

\[
V = \sum_i \bar{F}_i F_i + \sum_j \bar{G}_j G_j + \frac{1}{2} D_A^2 + \frac{1}{2} D_B^2 ,
\]
with
\[
\vec{F}_i = -\frac{\partial W}{\partial \sigma_i}, \quad \vec{G}_j = -\frac{\partial W}{\partial \phi_j}, \quad D_A = -\left[\kappa_A + \frac{1}{2} e \sum_j q_j \phi_j \bar{\phi}_j\right], \quad D_B = -\left[\kappa_B + \frac{1}{2} g \sum_i Q_i \bar{\sigma}_i \sigma_i\right].
\]  (24)

For a renormalizable theory (in four dimensions) the superpotential \( W \), constructed from the chiral superfields \( \Sigma_i \) and \( \Phi_j \), has the general form
\[
W = a_i X_i + b_{ij} X_i X_j + c_{ijk} X_i X_j X_k,
\]  (25)

where \( X_i \) represents either \( \Sigma_i \) or \( \Phi_i \), and \( W \) is again constrained by gauge invariance, so that each term must have a total \( U(1) \) charge of zero (\( \sum q_i = 0 \)) and a total \( U(1)' \) charge of zero (\( \sum Q_i = 0 \)).

For the simplest supersymmetric analog of the model described by (14), corresponding to the case where there is a single chiral superfield \( \Sigma \) along with a single chiral superfield \( \Phi \), we can choose \( \kappa_A \geq 0 \) and \( \kappa_B = -\frac{1}{2} g Q \eta^2 < 0 \), so that the \( U(1) \) symmetry is unbroken and the \( U(1)' \) symmetry is spontaneously broken. We then have a model containing cosmic gauge strings along with \( U(1) \) charged particles. However, the only interactions between the supermultiplets \( \Sigma \) and \( \Phi \) must come from the superpotential \( W \), given by (25). But, by gauge invariance, \( W = 0 \), so that no interactions exist between \( U(1) \) charged particles and a \( U(1)' \) gauge string. The Witten mechanism, therefore, does not exist in this simple model. Also, note that if \( |\sigma| = \eta \) in the vacuum state, and \( \kappa_A = 0 \), then \( V = 0 \) in the vacuum state and the supersymmetry is not broken in the vacuum. However, for the case where \( |\sigma| = \eta \) in the vacuum state and \( \kappa_A > 0 \), then \( V > 0 \) in the vacuum state and supersymmetry is spontaneously broken.

Evidently, from (13) - (25), the only way to generate an interaction of \( U(1) \) \( \Phi \) fields with \( U(1)' \) \( \Sigma \) fields, which can result in a theory of gauge strings possessing \( U(1) \) charge and/or current, is to build an interaction between the \( \Sigma \) and \( \Phi \) fields through a nonvanishing superpotential. The form of the superpotential, dictated by the initial requirement of a supersymmetric action, along with the constraint imposed by gauge invariance, apparently requires a more complicated supersymmetric version of the model as compared to the nonsupersymmetric version. For example, in the supersymmetric \( U(1) \times U(1)' \)-Higgs model above, it would appear that the simplest model allowing interactions between \( \Phi \) fields and \( \Sigma \) fields, due to interaction terms in the superpotential \( W \), would consist of five chiral superfields, of which two have nonzero \( U(1)' \) charges, two have nonzero \( U(1) \) charges, and one neutral chiral superfield \( Z = (Z, \psi_Z, F_Z) \) has zero \( U(1) \) charge and zero \( U(1)' \) charge (\( q_Z = Q_Z = 0 \)). Such a set of fields allows the superpotential terms
\[
W_{int} = c_1 Z \Sigma_1 \Sigma_2 + c_2 Z \Phi_1 \Phi_2,
\]  (26)

with \( Q_{\Sigma_1} = -Q_{\Sigma_2} \), \( q_{\Phi_1} = -q_{\Phi_2} \), giving rise to the scalar potential terms
\[ V_{\text{int}} = |c_1 \sigma_1 \sigma_2 + c_2 \phi_1 \phi_2|^2 \\
+ |Z|^2 \left[ c_1^2 (|\sigma_1|^2 + |\sigma_2|^2) + c_2^2 (|\phi_1|^2 + |\phi_2|^2) \right]. \]  

(27)

along with Yukawa terms of the form
\[ L_{Y, \text{int}} = -\frac{1}{2} \left( \frac{\partial^2 W}{\partial X_i \partial X_j} \right) \Psi_i \Psi_j + CC, \]  

(28)

where a sum over \( i \) and \( j \) is implied, \( CC \) stands for complex conjugate, \( X_i \) represents \( \sigma_i, \phi_i, \) or \( Z, \) and \( \Psi_i \) represents \( \psi_i, \chi_i, \) or \( \psi_Z \) (the spinor component of the \( Z \) chiral supermultiplet). Pure \( Z \) terms, such as \( aZ \bar{Z} + bZ \bar{Z}^2 + cZ \bar{Z}^3, \) can also be included in \( W, \) resulting in additional contributions to the scalar potential \( V \) and additional Yukawa terms in \( L_Y. \) The possibility may then exist for destabilizing the set of states described by \( \phi_j = 0 \) in the core of a gauge string, thereby allowing a \( U(1) \) charged bosonic condensation and/or the formation of \( U(1) \) charged fermionic zero modes in the string core.

V. A SUPERCONDUCTING COSMIC STRING SOLUTION IN THE
SUPERSYMMETRIC U(1)×U(1)'-HIGGS MODEL

Let us now consider the type of model discussed in section IV, where five chiral superfields are introduced so that a nonvanishing superpotential can be constructed which allows interactions between the \( U(1)' \) fields and the \( U(1) \) fields. Specifically, we consider a set of fields such that two of them (\( \Sigma_i \)) have nonzero \( U(1)' \) charges and zero \( U(1) \) charges, two other fields (\( \Phi_i \)) have nonzero \( U(1) \) charges and zero \( U(1)' \) charges, and one field (\( Z \)) has a zero \( U(1)' \) charge and a zero \( U(1) \) charge. The Fayet-Illiopoulos terms will be dropped, and it can be seen that there exists a supersymmetric vacuum state for the theory in which the \( U(1)' \) symmetry is spontaneously broken and the \( U(1) \) symmetry is unbroken. The spontaneous breaking of the \( U(1)' \) symmetry indicates the existence of a topologically stable gauge string. The gauge string is described by the bosonic components of the \( \Sigma_i \) fields (denoted by \( \sigma_i \)) along with the \( U(1)' \) gauge field \( B_{\mu}. \) By focusing on the bosonic sector of the theory, where the fermion fields vanish, the stability of the bosonic components of the \( \Phi_i \) and \( Z \) superfields (denoted by \( \phi_i \) and \( Z, \) respectively) in the presence of the gauge string background can be examined. Following the type of argument used by Witten [3], it is argued that a parameter range can exist such that, at least within a first approximation, the \( U(1) \) fields \( \phi_i \) form a condensate within the string core, while the neutral field \( Z \) can be associated with bound states localized within or near the core of the string.

A. Fields, Charge Assignments, and Superpotential

Let us denote the \( U(1)' \) charge of a field by \( Q_i \) and the \( U(1) \) charge of a field by \( q_i. \) Consider now a model of the type described in section IV characterized by the
five chiral superfields $\Sigma_\pm$, $\Phi_\pm$, and $Z$. The charges for the $\Sigma_\pm$ fields are taken to be $(Q_\pm, q_\pm) = (\pm 1, 0)$, those for the $\Phi_\pm$ fields are taken to be $(Q_\pm, q_\pm) = (0, \pm 1)$, and for the neutral $Z$ field we have $(Q_Z, q_Z) = (0, 0)$. The superfields can be displayed in terms of their component fields:

$$\Sigma_\pm = (\sigma_\pm, \psi_\pm, F_{\sigma_\pm}), \quad \Phi_\pm = (\phi_\pm, \chi_\pm, F_{\phi_\pm}), \quad Z = (Z, \psi_Z, F_Z).$$

We assume that the Fayet-Illiopolous terms vanish, i.e. $\kappa_A = \kappa_B = 0$. The superpotential for the model is assumed to be given by

$$W = \lambda Z (\Sigma_+ \Sigma_- - \eta^2) + c Z \Phi_+ \Phi_- + m \Phi_+ \Phi_-, \quad (30)$$

and the parameters $\lambda$, $c$, $m$, and $\eta$ are assumed to be real-valued positive quantities.

The scalar potential $V$ is then given by

$$V = \sum_i |F_i|^2 + \frac{1}{2} D_A^2 + \frac{1}{2} D_B^2, \quad (31)$$

where

$$D_A = -\frac{1}{2} e(\bar{\phi}_+ \phi_+ - \bar{\phi}_- \phi_-), \quad D_B = -\frac{1}{2} g(\bar{\sigma}_+ \phi_+ - \bar{\sigma}_- \phi_-). \quad (32)$$

The scalar potential can therefore be written fully as

$$V = \lambda^2 (\bar{\sigma}_+ \bar{\sigma}_- - \eta^2)(\sigma_+ \sigma_- - \eta^2) + \lambda c (\bar{\sigma}_+ \bar{\sigma}_- - \eta^2) \phi_+ \phi_- + \lambda c (\sigma_+ \sigma_- - \eta^2) \bar{\phi}_+ \bar{\phi}_- + c^2 \phi_+ \phi_- \bar{\phi}_- + \lambda^2 \bar{Z} Z (\bar{\sigma}_+ \sigma_+ + \bar{\sigma}_- \sigma_-) + (c \bar{Z} + m)(c Z + m)(\bar{\phi}_+ \phi_+ + \bar{\phi}_- \phi_-) + \frac{c^2}{8} (\bar{\phi}_+ \phi_- - \bar{\phi}_- \phi_+)^2 + \frac{\lambda^2}{8} (\bar{\sigma}_+ \sigma_+ - \bar{\sigma}_- \sigma_-)^2. \quad (34)$$

From (34) it can be seen that the lowest energy state of the theory is a supersymmetric vacuum state with $V = 0$, determined by the conditions $F_i = 0$ and $D_A = D_B = 0$. Specifically, the supersymmetric vacuum state that spontaneously breaks the $U(1)'$ symmetry but respects the $U(1)$ symmetry is characterized by

$$|\sigma_\pm|_0 = \eta, \quad (\phi_\pm)_0 = 0, \quad Z_0 = 0. \quad (35)$$
B. The Bosonic Sector

1. Lagrangian

Let us now focus upon the bosonic sector of the theory, where the fermion fields are assumed to vanish. The Lagrangian for the bosonic fields can be displayed as

\[ L_B = (\bar{D}^\mu \sigma_\pm) (D^\mu \sigma_\pm) + (\bar{D}^\mu \phi_\pm) (D^\mu \phi_\pm) + \partial^\mu Z \partial_\mu Z - V - \frac{1}{4} F_{\mu \nu} F_{\mu \nu} - \frac{1}{4} B_{\mu \nu} B_{\mu \nu} \]

where

\[ (D^\mu \sigma_\pm) = (\nabla^\mu \pm \frac{i g}{2} B^\mu) \sigma_\pm \quad \text{and} \quad (D^\mu \phi_\pm) = (\nabla^\mu \pm \frac{i e}{2} A^\mu) \phi_\pm \]

(36)

(37)

(Note that, due to the convention in defining the gauge transformations [see (18)], the “physical” charges are \( \bar{g} = g/2 \) and \( \bar{e} = e/2 \).)

2. Field Equations

From the Lagrangian given by (36) the equations of motion for the fields \( \sigma_+, \sigma_- \), \( \phi_+, \phi_- \), \( Z \), \( A_\mu \), and \( B_\mu \) can be obtained:

\[
\begin{align*}
(\nabla^\mu + \frac{i g}{2} B^\mu)(\nabla^\mu + \frac{i g}{2} B^\mu) \sigma_+ &+ \lambda^2 \sigma_- (\sigma_+ \sigma_- - \eta^2) \\
&+ \lambda c \bar{\sigma} \phi_+ \phi_- + 2 \bar{Z} Z \sigma_+ + \frac{i g}{2} \sigma_- (\bar{\sigma} \phi_+ \phi_-) = 0 , \\
& (\nabla^\mu - \frac{i g}{2} B^\mu)(\nabla^\mu - \frac{i g}{2} B^\mu) \sigma_+ &+ \lambda^2 \sigma_- (\sigma_+ \sigma_- - \eta^2) \\
&+ \lambda c \bar{\sigma} \phi_+ \phi_- + 2 \bar{Z} Z \sigma_- - \frac{i g}{2} \sigma_- (\bar{\sigma} \phi_+ \phi_-) = 0 , \\
& (\nabla^\mu + \frac{i e}{2} A^\mu)(\nabla^\mu + \frac{i e}{2} A^\mu) \phi_+ &+ \lambda c \bar{\phi} (\sigma_+ \sigma_- - \eta^2) + c^2 \bar{\phi} \phi_+ \phi_- \\
&+ (c \bar{Z} + m)(c \bar{Z} + m) \phi_+ + \frac{2}{4} \phi_+ (\phi_- \phi_+ - \bar{\phi} \phi_-) = 0 , \\
& (\nabla^\mu - \frac{i e}{2} A^\mu)(\nabla^\mu - \frac{i e}{2} A^\mu) \phi_- &+ \lambda c \bar{\phi} (\sigma_+ \sigma_- - \eta^2) + c^2 \bar{\phi} \phi_+ \phi_- \\
&+ (c \bar{Z} + m)(c \bar{Z} + m) \phi_- - \frac{2}{4} \phi_- (\phi_- \phi_+ - \bar{\phi} \phi_-) = 0 , \\
& \nabla^\mu \nabla^\mu Z &+ \lambda^2 Z (\bar{\sigma} \phi_+ \phi_- + \bar{\phi} \phi_+ \phi_-) + c (c \bar{Z} + m)(\bar{\phi} \phi_+ + \bar{\phi} \phi_-) = 0 , \\
& \Box B_\mu & = J_{B \mu} \\
& = \frac{i g}{2} \left\{ [\bar{\sigma}_+(D^\mu \sigma_+) - \sigma_+(\bar{D}^\mu \sigma_+)] - [\bar{\sigma}_-(D^\mu \sigma_-) - \sigma_-(\bar{D}^\mu \sigma_-)] \right\} , \\
& \Box A_\mu & = J_{A \mu} \\
& = \frac{i e}{2} \left\{ [\bar{\phi}_+(D^\mu \phi_+) - \phi_+(\bar{D}^\mu \phi_+)] - [\bar{\phi}_-(D^\mu \phi_-) - \phi_-(\bar{D}^\mu \phi_-)] \right\} ,
\end{align*}
\]

(38)

(39)

(40)

(41)

(42)

(43)

(44)

where \( \Box = \nabla^\mu \nabla_\mu = (\partial^2 - \nabla^2) \), and the Lorentz gauges \( \nabla^\mu B^\mu = 0 \), \( \nabla^\mu A^\mu = 0 \) have been used.
3. Simplifying Ansatz

In order to simplify the system considered above, let us implement a particular ansatz for the scalar fields for which

\[ \phi_+ = \phi, \quad \phi_- = \bar{\phi}, \quad \sigma_+ = \sigma, \quad \sigma_- = \bar{\sigma}, \]

i.e. \( \phi_+ \) and \( \phi_- \) are complex conjugates of one another, as are \( \sigma_+ \) and \( \sigma_- \), so that the set of five complex scalar fields \( \{ \sigma_+, \sigma_-, \phi_+, \phi_-, Z \} \) reduces to the set of three complex scalar fields \( \{ \sigma, \phi, Z \} \). The field equations (38)-(44) then reduce to

\[ (\nabla^\mu + \frac{ig}{2} B_\mu)(\nabla^\nu + \frac{ig}{2} B^\nu)\sigma + \sigma \left[ \lambda^2 (\bar{\sigma}\sigma - \eta^2) + \lambda c \bar{\phi} \phi + \lambda^2 \bar{Z} Z \right] = 0, \]  

\[ (\nabla^\mu + \frac{ie}{2} A_\mu)(\nabla^\nu + \frac{ie}{2} A^\nu)\phi + \phi \left[ \lambda c (\bar{\sigma}\sigma - \eta^2) + c^2 \bar{\phi} \phi + (c \bar{Z} + m)(c Z + m) \right] = 0, \]

\[ \nabla^\mu \nabla_\mu Z + 2\lambda^2 Z \bar{\sigma}\sigma + 2c(cZ + m) \bar{\phi} \phi = 0, \]

\[ \square B_\mu = J_{B\mu} = ig \left[ \bar{\sigma}(D'_\mu \sigma) - \sigma(D'_\mu \bar{\sigma})^* \right], \]

\[ \square A_\mu = J_{A\mu} = ie \left[ \bar{\phi}(D_\mu \phi) - \phi(D_\mu \bar{\phi})^* \right], \]

where \( D'_\mu = (\nabla^\mu + \frac{ig}{2} B_\mu) \) and \( D_\mu = (\nabla^\mu + \frac{ie}{2} A_\mu) \).

With the simplifying ansatz given by (45), the Lagrangian \( L_B \) given by (36) takes the form of an effective Lagrangian given by

\[ L_{B,\text{eff}} = 2(D'\mu \sigma)^*(D'_\mu \sigma) + 2(D'\mu \phi)^*(D_\mu \phi) + \partial^\mu Z^* \partial_\mu Z - U(\sigma, \phi, Z) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \]

where

\[ U(\sigma, \phi, Z) = \lambda^2 (\bar{\sigma}\sigma - \eta^2)^2 + 2\lambda c (\bar{\sigma}\sigma - \eta^2) \bar{\phi} \phi + \lambda^2 (\bar{\phi} \phi)^2 + 2\lambda^2 \bar{\sigma}\sigma \bar{Z} Z + 2\phi \phi (c \bar{Z} + m)(c Z + m). \]

It can be verified that the field equations (46)-(50) can be obtained directly from \( L_{B,\text{eff}} \). It can be noted that, with a rescaling of the fields \( \sigma \) and \( \phi \), \( L_{B,\text{eff}} \) contains a Lagrangian for the bosonic sector of an ordinary (nonsupersymmetric) \( U(1) \times U(1)' \)-Higgs model, described in section II, along with dynamical and interaction terms for the field \( Z \). This observation leads us to strongly suspect that the supersymmetric model considered here will admit a superconducting cosmic string solution. By examining the behavior of the fields \( \phi \) and \( Z \) in the background of the gauge string formed by the fields \( \sigma \) and \( B_\mu \), we will see that, for a suitable parameter range, such a solution does indeed exist, wherein the field \( \phi \) forms a condensate within the string core and the field \( Z \) can be associated with bound states that are localized within or near the string core.
C. The Superconducting String

In the vacuum state of the theory we have $|\sigma| = \eta$, $\phi = 0$, and $Z = 0$, so that the $U(1)'$ symmetry is spontaneously broken, allowing a topological gauge string to form. Using cylindrical coordinates $(r, \theta, z)$, let us consider the case in which the fields $\phi$, $Z$, and $A_\mu$ vanish identically. In this case, we can write $\sigma = \sigma_s(r, \theta) = s(r)e^{i\theta}$, $B_\mu = \frac{2}{g}B(r)\delta_\mu^\theta$, with the boundary conditions

$$s(r) \rightarrow \eta, \quad B(r) \rightarrow -1, \quad \text{as} \quad r \rightarrow \infty,$$

$$s(r) \rightarrow 0, \quad B(r) \rightarrow 0, \quad \text{as} \quad r \rightarrow 0.$$  \(53\)

The field equations given by (46) and (49), subject to the boundary conditions in (53), then admit an ordinary Nielsen-Olesen Abelian gauge string solution.

Now consider the field $\phi$ in the gauge string background with $Z = 0$ and $\sigma = \sigma_s(r, \theta)$. Following a similar line of reasoning as that used by Witten [5], it can be argued that there exists a parameter range for which the complex scalar field given by $\phi = 0$, with $A_\mu = 0$, is unstable in the background field of the gauge string, and must therefore relax to a lower energy state for which $\phi \neq 0$ in the string core. The first thing that can be noticed is that the potential in the string core, where $\sigma \rightarrow 0$, is minimized by a value

$$|\phi_0| = \left(\frac{\lambda c \eta^2 - m^2}{c}\right)^{\frac{1}{2}}, \quad (54)$$

provided that $(\lambda c \eta^2 - m^2) > 0$, as will be assumed to be the case. However, since $\phi \rightarrow 0$ outside of the string core in the vacuum region, there is also gradient energy to be considered. To see that the field $\phi$ does assume a nonzero value in the string core, we set $\sigma = \sigma_s$, $A_\mu = 0$, $Z = 0$, linearize the equations of motion, and examine small fluctuations of $\phi$ about $\phi = 0$. Writing $\phi(\vec{r}, t) = \varphi(\vec{r})e^{-i\omega t}$, then gives

$$- \nabla^2 \varphi + \lambda c |\sigma_s|^2 \varphi = E \varphi, \quad E \equiv \omega^2 + (\lambda c \eta^2 - m^2), \quad (55)$$

which is a Schrodinger-like equation for a particle with “mass” $\mu = \frac{1}{2}$ and “energy” $E = \omega^2 + (\lambda c \eta^2 - m^2)$ in a potential well $\lambda c |\sigma_s|^2$. For the parameter range in which $E < (\lambda c \eta^2 - m^2)$, there should exist at least one normalizable bound state for which $\omega^2 < 0$, in which case $\phi = 0$ is unstable in the string core, and therefore the field $\phi$ forms a condensate within the string, with $\phi \rightarrow \phi_0$ in the core. Furthermore, we expect, on the basis of continuity, that solutions corresponding to excitations exist for $A_\mu \neq 0$. These excitations, which can give rise to charge and current within the string, can be characterized by the parametrizations $\phi(r, z, t) = F(r)e^{i\psi(z, t)}$, $A_\mu(r, z, t) = \frac{2}{g}[P(r) - 1] \partial_\mu \psi(z, t)$. For the case in which $\psi = \kappa z \pm \omega t$, for example, the electromagnetic current density $J_{A\mu} = -\frac{2}{g} F^2(r) P(r) \partial_\mu \psi$ describes a time independent, nondissipative charge and current localized within the string core.
At the same level of approximation, let us examine the $Z$ field in the string background by setting $\sigma = \sigma_s$, $\phi = 0$, and $A_\mu = 0$, i.e., as a first approximation we examine the fields $\phi$ and $Z$ in the absence of one another in the gauge string background. Writing $Z(\vec{r},t) = Z_1(\vec{r}) e^{-i(\Omega t - k_z)}$, equation (48) gives

$$- (\partial_x^2 + \partial_y^2) Z_1 + 2\lambda^2 |\sigma_s(r)|^2 Z_1 = \mu^2 Z_1, \quad \mu^2 \equiv \Omega^2 - k^2,$$

which is a Schrodinger-like equation for a particle with “energy” $\mu^2$ in the potential well $2\lambda^2|\sigma_s|^2$. This attractive potential can accommodate one or more normalizable bound states with $0 < \mu < \sqrt{2}\lambda \eta$, allowing us to infer that $Z$ particles can be localized within or near the string core in the form of string-$Z$-particle bound states. A set of scattering states can evidently exist as well for $\mu > \sqrt{2}\lambda \eta$.

From this level of approximation it is therefore concluded that the $\phi$ condensate gives rise to a superconducting gauge string which may also accommodate $Z$ particle bound states that can drift along the string.

VI. SUMMARY

It is possible that supersymmetry was physically realized in the early universe, and that supersymmetry was broken at the same time as, or subsequent to, the formation of topological defects, such as cosmic strings. It therefore seems relevant to investigate basic models and scenarios involving defects within a supersymmetric context. Here, attention has been focused upon a relatively simple “toy” scenario involving superconducting Abelian gauge strings with long range $U(1)$ fields. Specifically, a supersymmetric $U(1) \times U(1)'$-Higgs model has been considered, where the action is constructed from scalar chiral superfields and vector superfields, with each chiral superfield transforming separately under the Abelian gauge groups. The $U(1)'$ symmetry can be spontaneously broken, giving rise to gauge strings, while the $U(1)$ symmetry remains unbroken, giving rise to long range interactions outside of the string. In the nonsupersymmetric version of the model, a suitable potential can readily be constructed from two complex scalar fields, which, by the Witten mechanism, gives rise to a model admitting as solutions bosonic superconducting strings with long range gauge fields. The basic constraints imposed upon this potential include those of renormalizability and gauge invariance. In the supersymmetric extension of this type of model, however, supersymmetry provides an additional constraint, which is partially reflected in the form of the superpotential, and therefore in the form of the scalar potential of the model. For such a model involving only one chiral superfield $\Phi$, transforming under the $U(1)$ gauge group, and one chiral superfield $\Sigma$, transforming under the $U(1)'$ gauge group, it is seen that no Witten mechanism exists for the formation of $U(1)$ charged bosonic condensates or fermionic zero modes within a string which can give rise to long range gauge fields outside of the string. Apparently, the simplest version of the model that allows interactions between the $\Sigma$ fields and the $\Phi$
fields involves at least five chiral superfields. The constraints imposed by supersymmetry and gauge invariance therefore complicate such a model of superconducting strings by requiring a proliferation of scalar and spinor fields in order to obtain the desired types of interactions. (It should be pointed out, however, that the results obtained here apply to a specific Abelian gauge model, which is not the only type of model that can describe superconducting strings with long range gauge fields. For example, certain types of realistic grand unified theories can also give rise to superconducting strings \[5, 12\], where non-Abelian gauge field potentials are involved. It could be of interest to investigate to what extent supersymmetry constrains more realistic models.)

Next, the supersymmetric version of the $U(1) \times U(1)'$-Higgs model with the five chiral superfields $\Sigma_+, \Sigma_-, \Phi_+, \Phi_-, \text{and } Z$ has been investigated, and a superpotential $W$ has been constructed which gives rise to a scalar potential $V$ describing interactions among the bosonic components $\sigma_+, \sigma_-, \phi_+, \phi_-, \text{and } Z$ of the chiral superfields. In the bosonic sector of the theory, where the fermion fields vanish, it is seen that when the bosonic fields $\phi_\pm$ and $Z$ vanish identically, and when a simplifying ansatz is used where $\sigma_- = \bar{\sigma}_+$ and $\phi_- = \bar{\phi}_+$, the model admits an ordinary Nielsen-Olesen Abelian gauge string surrounded by a supersymmetric vacuum. The two fields $\sigma_+$ and $\sigma_-$ thus conspire to form the gauge string entrapping a unit of flux of the $B_\mu$ field. The stability of the fields $\phi_\pm$ and $Z$ in the gauge string background has then been examined in a first approximation by disregarding the possible back reactions of the fields $\phi_\pm$ and $Z$ upon one another. It has been determined that, for an appropriate parameter range, the system must stabilize through the formation of a bosonic condensate of the $\phi_\pm$ fields, while the $Z$ field, rather than forming a condensate, can be associated with string-$Z$-particle bound states localized within or near the string core. One also expects string-$Z$-particle scattering states to exist as well, and transitions between states may occur due to $Z$ interactions with $\sigma_\pm$ and $\phi_\pm$ excitations. A nonvanishing current density $J_{A\mu}$, constructed from the $\phi_\pm$ and $A_\mu$ fields, can then endow the string with charge and/or current which generate the long range electromagnetic gauge fields. Thus, although the two types of solutions are generally distinct, the bosonic superconducting string solution obtained from the supersymmetric version of the $U(1) \times U(1)'$-Higgs model is seen to exhibit many of the essential qualitative features of the superconducting string solution emerging from the nonsupersymmetric version of the model.

**APPENDIX A: CONVENTIONS**

Some of the notations and conventions are briefly listed here. A metric $g_{\mu\nu}$ is used with signature $(+, -, -, -)$. Aside from the metric, the notation, conventions, and gamma matrices used conform to those of ref. \[3\]. The gamma matrices can be written in the form
\[\gamma^\mu = i \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}\] (A1)

with

\[\sigma^\mu = (1, \vec{\sigma}) , \quad \bar{\sigma}^\mu = (1, -\vec{\sigma}) ,\] (A2)

where \(\vec{\sigma}\) represents the Pauli matrices. Then

\[\gamma^0 = i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \gamma^k = i \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} , \quad k = 1, 2, 3,\] (A3)

and \(\gamma_5\) is given by

\[\gamma_5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.\] (A4)

The gamma matrices have the properties

\[\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu} , \quad \{\gamma^\mu, \gamma_5\} = 0 , \quad \gamma_5^\dagger = -\gamma_5 , \quad (\gamma_5)^2 = -1.\] (A5)

A Majorana 4-spinor \(\Psi\) is expressed in terms of the Weyl 2-spinors \(\psi\) and \(\bar{\psi}\) by \(\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}\) and we use the summation conventions for Weyl spinors [with \(\bar{\psi}^{\dot{\alpha}} = (\psi^\alpha)^*\)]

\[\xi \psi \equiv \xi^\alpha \psi_\alpha , \quad \bar{\xi} \bar{\psi} \equiv \bar{\xi}^{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} , \quad \alpha = 1, 2 , \quad \dot{\alpha} = 1, 2,\] (A6)

with \(\varepsilon\) metric tensors (for raising and lowering Weyl spinor indices)

\[(\varepsilon^{\alpha\beta}) = (\varepsilon^{\dot{\alpha}\dot{\beta}}) = i\sigma_2 , \quad (\varepsilon_{\alpha\beta}) = (\varepsilon_{\dot{\alpha}\dot{\beta}}) = -i\sigma_2 , \quad \varepsilon^{12} = 1 = \varepsilon^{\dot{1}\dot{2}}.\] (A7)
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