Self-excited waves in complex social systems

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Abstract

A social system is considered whose agents choose between several alternatives of possible actions. The system is described by the fractions of agents preferring the corresponding alternatives. The agents interact with each other by exchanging information on their choices. Each alternative is characterized by three attributes: utility, attractiveness, and replication. The agents are heterogeneous having different initial conditions and different types of memory, which can be long-term or short-term. The agent interactions, generally, can depend on the distance between the agents, varying from short-range to long-range interactions. The emphasis in the paper is on long-range interactions. In a mixed society consisting of agents with both long-term and short-term memory, there appears the effect of spontaneous excitation of preference waves, when the fractions of agents preferring this or that alternative suddenly start strongly oscillating, either periodically or chaotically. Since the considered society forms a closed system, without any external influence, the arising waves are internally self-excited by the society.

Keywords: complex social systems, information exchange, long-term memory, short-term memory, herding effect, spontaneous preference waves
1 Introduction

Understanding the behaviour of social systems is of long standing and permanent interest [1], and it has been described employing various models based on statistical physics (see, e.g., reviews [2,3]) or on the theory of networks [5–10]. The recent review [4] summarizes both approaches. Here we keep in mind social networks consisting of agents choosing between different alternatives. Interactions between society members usually are described by given functions, similarly to the interactions between particles or spins in statistical mechanics. In that sense, society members are treated as nodes mechanically accomplishing prescribed actions. In the case of realistic biological, especially human, societies, such models, clearly, give a rather simplified picture, since the agents of these societies are not mechanical devices, but are intelligent agents. The basic feature of an intelligent agent is the ability, after evaluating the available information, to make decisions choosing between several alternative actions [11–15].

The aim of the present paper, is to suggest a model of a society composed of intelligent agents. To our understanding, such a model could provide a more realistic description of social systems consisting of intelligent agents taking decisions with respect to available alternatives. The basic points of the suggested theory are as follows.

(i) The approach is probabilistic. Each agent is characterized by the probability of choosing this or that alternative. This probability has the meaning of a frequentist measure showing the fraction of agents choosing the given alternative.

(ii) The probability measure takes into account three factors: the utility of alternatives, their attractiveness, and the attitude of agents towards replicating the actions of other members of the society. Thus, in addition to the estimation of utility of alternatives, our model includes the influence of emotions, described by the attractiveness of alternatives, and takes into account the herding effect.

Endeavors of including emotions in the process of choice have been undertaken in the frame of quantum decision theory [16–23]. However, as has been shown [24], the use of quantum theory can be avoided, and an approach can be developed incorporating emotions into decision process, without resorting to quantum techniques. Below we suggest an approach of describing the society of intelligent agents, employing only classical notions. We study a novel kind of complex society composed of agents with different types of memory, so that a fraction of the society members is endowed with long-term memory, while the others have short-term memory. In this complex society, interesting new effects appear, such as suddenly arising self-excited oscillations. These spontaneous oscillations can be either periodic or chaotic.

The layout of the paper is as follows. In Sec. 2, we formulate the model of a society of intelligent agents deciding between several alternatives. The choice is based of three attributes, utility, attractiveness, and herding. In Sec. 3, the problem is specified to the consideration of two groups of agents choosing between two alternatives. The groups are composed of agents possessing different types of memory, either long-term or short-term memory. Section 4 presents a detailed investigation of different dynamic regimes that can occur in the process of decision making. Section 5 concludes.


2 Society of intelligent agents

Consider a society of $N$ agents, enumerated by the index $j = 1, 2, \ldots, N$. Assume that each agent needs to choose between $N_A$ alternative actions $A_n$, numbered by the index $n = 1, 2, \ldots, N_A$. This can be the choice between several candidates at elections, between different goods in a shop, between several jobs, etc.

The probability that a $j$-th agent chooses an alternative $A_n$ at time $t$ is denoted as $p_j(A_n, t)$. By the meaning of probability, $p_j(A_n, t)$ has to be non-negative and normalized,

$$
\sum_{n=1}^{N_A} p_j(A_n, t) = 1, \quad 0 \leq p_j(A_n, t) \leq 1.
$$

(1)

The utility of an alternative $A_n$ defines the weight $f(A_n, t)$ ascribed to this alternative, because of which $f(A_n, t)$ can be called utility factor. Being a weight, it enjoys the properties of classical probability,

$$
\sum_{n=1}^{N_A} f_j(A_n, t) = 1, \quad 0 \leq f_j(A_n, t) \leq 1.
$$

(2)

The probability of preferring an alternative $A_n$ is the superposition of the utility, attraction, and herding factors. Taking into account the fact that the process of making a decision requires some time, say $\tau$, we have

$$
p_j(A_n, t + \tau) = f_j(A_n, t) + q_j(A_n, t) + h_j(A_n, t).
$$

(5)

This expression differentiates the probabilistic approach we follow from multi-attribute utility theory, where the expected utility functional is represented as a superposition of weighted parts associated with different attributes [26][27].

From the normalization conditions (1) and (2), it follows that the sum of the attraction factor and herding factor over all alternatives is zero:

$$
\sum_n [ q_j(A_n, t) + h_j(A_n, t) ] = 0.
$$

(6)

Assuming that attraction and herding are independent, we have

$$
\sum_n q_j(A_n, t) = 0, \quad \sum_n h_j(A_n, t) = 0.
$$

(7)
The latter equations can be called *alternation law*. Although, on a long time scale there exists time discounting [28], however the utility of alternatives varies in time slowly, which allows us to consider it constant,

\[ f_j(A_n, t) = f_j(A_n) . \] (8)

On the contrary, information exchange between the society agents is a fast process, such that the attraction factor of a \( j \)-th agent essentially depends on the amount of information \( M_j(t) \) obtained by this agent. The attraction factor can be modeled [29] by the expression

\[ q_j(A_n, t) = q_j(A_n) \exp\{-M_j(t)\} , \] (9)

where \( q(A_n) = q(A_n, 0) \) is an initial value of the attraction factor. This expression is similar to the time dependence of decoherence factor under nondestructive repeated measurements [30][31]. The amount of information obtained by a \( j \)-th agent by the time \( t \) can be written as

\[ M_j(t) = \sum_{t'=0}^{t} \sum_{i=1}^{N} J_{ji}(t, t') \mu_{ji}(t') . \] (10)

This can be called *remembered information*. Here \( J_{ji}(t, t') \) is the intensity of interactions during the information transfer from an \( i \)-th agent to the \( j \)-th agent in the period of time between \( t' \) and \( t \). The information gain, received by the \( j \)-th agent from an \( i \)-th agent at time \( t \), is given in the form of the Kullback-Leibler [32][33] relative information

\[ \mu_{ji}(t) = \sum_{n=1}^{N_A} p_j(A_n, t) \ln \frac{p_j(A_n, t)}{p_i(A_n, t)} . \] (11)

Note that the information gain enjoys the properties \( \mu_{ji} \geq 0 \) and \( \mu_{jj} = 0 \). At the initial moment of time, no information has yet been transferred, implying that

\[ M_j(0) = 0 . \] (12)

The interactions \( J_{ij}(t, t') \), in general, depend on the distance between the agents. In the case of short-range interactions the topology of the agent locations is important. In the opposite case of long-range interactions, the geometry of the agent network plays no role, with the interactions acquiring the form

\[ J_{ij}(t, t') = \frac{1}{N-1} J(t, t') . \] (13)

Our concern here is a society composed of intelligent agents, like humans who are able to interact with each other irrespectively of the distance between them. This kind of distance-independent interactions are provided nowadays by internet, mass media, and phones. In addition, the agents are not located at fixed nodes but can freely move varying their whereabouts. Keeping in mind these location-independent interactions, we accept in what follows the form (13). Then the amount of information kept in the agent memory reads as

\[ M_j(t) = \sum_{t'=0}^{t} \frac{J(t, t')}{N-1} \sum_{i=1}^{N} \mu_{ji}(t') . \] (14)
The agent memory is characterized by its duration. In two opposite situations, it can be 
long-term or short-term. The ultimate long-term memory is permanent in time,

\[ J(t, t') = J \quad (\text{long-term}) \, . \]  

(15)

Then the amount of remembered information takes the form

\[ M_j(t) = \frac{J}{N - 1} \sum_{t' = 0}^{t} \sum_{i=1}^{N} \mu_{ji}(t') \, . \]  

(16)

The opposite case is the short-term memory, when the information from only the last step is 
remembered,

\[ J(t, t') = J \delta_{tt'} \quad (\text{short-term}) \, . \]  

(17)

In that case, the available remembered information becomes

\[ M_j(t) = \frac{J}{N - 1} \sum_{i=1}^{N} \mu_{ji}(t) \, . \]  

(18)

In complex societies, there exists a collective effect, when the agents are prone to imitate 
the actions of others. This is called the herd effect. Herd behavior occurs in animal herds, 
packs, bird flocks, fish schools and so on, as well as in humans. It is well known and studied for 
many years [34–40]. In evolution equations of social and biological systems, the mathematical 
description of herding is represented by the replication term [41], which for our case takes the 
form

\[
  h_j(A_n, t) = \varepsilon_j \left\{ \frac{1}{N - 1} \sum_{i(i \neq j)}^{N} \left[ f_i(A_n) + q_i(A_n, t) \right] - \left[ f_j(A_n) + q_j(A_n, t) \right] \right\} \, .
\]

(19)

The parameters \( \varepsilon_j \) describe the intensity of the herding behaviour. Due to the normalization 
conditions [1], [2], [3], [4], and [7], these parameters satisfy the inequalities

\[ 0 \leq \varepsilon_j \leq 1 \quad (j = 1, 2, \ldots, N) \, . \]  

(20)

Without the loss of generality, time can be measured in units of \( \tau \). Substituting the herding 
term [19] into expression [5] yields the equation

\[
p_j(A_n, t + 1) = (1 - \varepsilon_j) \left[ f_j(A_n) + q_j(A_n, t) \right] + \frac{\varepsilon_j}{N - 1} \sum_{i(i \neq j)}^{N} \left[ f_i(A_n) + q_i(A_n, t) \right] \, .
\]

(21)

This it the evolution equation for the probability that the \( j \)-th agent chooses at time \( t + 1 \) the 
alternative \( A_n \). The right-hand side of this equation at time \( t = 0 \) gives the initial condition

\[
p_j(A_n, 0) = (1 - \varepsilon_j) \left[ f_j(A_n) + q_j(A_n, 0) \right] + \frac{\varepsilon_j}{N - 1} \sum_{i(i \neq j)}^{N} \left[ f_i(A_n) + q_i(A_n, 0) \right] \, .
\]

(22)
3 Agents with different types of memory

Now we need to specify what kind of agents we consider. Agents can differ by initial conditions or by the types of their memory. The simplest case is when all agents possess the same type of memory, although even then the dynamics of preferences can exhibit rather nontrivial behaviour [29]. Here we study a more interesting case, where the society consists of agents with different types of memory. A fraction of agents has long-term memory and the other part, short-term memory. Such a mixture of agents with different properties much better models the real human societies. It turns out that this more complex society exhibits unusual phenomena that are absent in homogeneous societies where all agents possess the same type of memory. For instance, there appear self-excited waves of preferences, spontaneously arising without any external influence. These waves can appear at the beginning of dynamics or may not exist at the beginning of decision processes, but appear suddenly after some time of quite smooth dynamics.

Suppose the society consists of two types of agents. One part enjoys long-term memory associated with the information function (16), while the other part has short-term memory corresponding to the information function (18). A group of similar agents can be represented by a frequentist probability showing the fraction of agents choosing an alternative \( A_n \) at time \( t \). For the case of two groups, we have the probabilities \( p_1(A_n, t) \) and \( p_2(A_n, t) \).

Let us also consider the very often met situation when the choice is between two alternatives, say \( A_1 \) and \( A_2 \). Then, keeping in mind the normalization conditions (1), (2), and (7), with \( j = 1, 2 \), we can simplify the notation for the probabilities,

\[
p_j(A_1, t) \equiv p_j(t) \quad , \quad p_j(A_2, t) = 1 - p_j(t) ,
\]

utility factors,

\[
f_j(A_1) \equiv f_j \quad , \quad f_j(A_2) = 1 - f_j ,
\]

attraction factors,

\[
q_j(A_1, t) \equiv q_j(t) \quad , \quad q_j(A_2, t) = -q_j(t) ,
\]

and herding factors,

\[
h_j(A_1, t) \equiv h_j(t) \quad , \quad h_j(A_2, t) = -h_j(t) .
\]

Then we have the probabilities

\[
p_j(t + 1) = f_j + q_j(t) + h_j(t) \quad (j = 1, 2) ,
\]

with the attraction and herding factors

\[
q_j(t) = q_j \exp\{-M_j(t)\} \quad (j = 1, 2) ,
\]

\[
h_j(t) = \varepsilon_j \left[ f_i + q_i(t) - f_j - q_j(t) \right] \quad (i \neq j) ,
\]

where \( q_j \equiv q_j(0) \).

Let us mark the group of agents with long-term memory as the group number one, and the group of agents with short-term memory, as the second group. For the corresponding available remembered information, setting \( J = 1 \), we have in the case of long-term memory

\[
M_1(t) = \sum_{t'=0}^{t} \mu_{12}(t')
\]
and for short-term memory,
\[ M_2(t) = \mu_{21}(t) . \]  (30)

The information gain reads as
\[ \mu_{ij}(t) = p_i(t) \ln \frac{p_i(t)}{p_j(t)} + [1 - p_i(t)] \ln \frac{1 - p_i(t)}{1 - p_j(t)} , \]  (31)

where \( i \neq j \). Thus we come to the evolution equations
\[ p_1(t + 1) = (1 - \varepsilon_1) [ f_1 + q_1(t) ] + \varepsilon_1 [ f_2 + q_2(t) ] , \]
\[ p_2(t + 1) = (1 - \varepsilon_2) [ f_2 + q_2(t) ] + \varepsilon_2 [ f_1 + q_1(t) ] , \]  (32)

with the initial conditions
\[ p_1(0) = (1 - \varepsilon_1) [ f_1 + q_1 ] + \varepsilon_1 [ f_2 + q_2 ] , \]
\[ p_2(0) = (1 - \varepsilon_2) [ f_2 + q_2 ] + \varepsilon_2 [ f_1 + q_1 ] \]  (33)

that are defined by the parameters \( f_1, q_1, f_2, \) and \( q_2 \).

Recall that \( p_1(t) \) is the fraction of agents with long-term memory preferring the alternative \( A_1 \) at time \( t \), while \( p_2(t) \) is the fraction of agents with short-term memory preferring the alternative \( A_1 \) at time \( t \). Because of the exchange of information and the herding effect, these fractions vary in time.

4 Dynamics of preferences

Before going to the numerical investigation of Eqs. (32), it is possible to make general conclusions on the expected role of the herding effect. To this end, it is easy to notice that, when the herding behavior is absent, hence \( \varepsilon_1 = \varepsilon_2 = 0 \), then Eqs. (32) read as
\[ p_1(t + 1) = f_1 + q_1(t) \quad (\varepsilon_1 = 0) , \]
\[ p_2(t + 1) = f_2 + q_2(t) \quad (\varepsilon_2 = 0) . \]  (34)

This does not mean that the first and second groups are independent, since they remain connected through the information exchange entering the remembered information (29) and (30). However, to some extent, the evolution of group preferences can be quite different because of so different memory properties of the groups.

Increasing the herding parameters \( \varepsilon_j > 0 \) switches on the herding behavior that, due to the replication terms \( h_j(t) \), should smooth the differences in the dynamics of the probabilities \( p_j(t) \). When the herding parameters reach the value \( \varepsilon_1 = \varepsilon_2 = 1/2 \), the behaviour of both groups becomes identical, since
\[ p_1(t + 1) = p_2(t + 1) \quad (\varepsilon_1 = \varepsilon_2 = \frac{1}{2}) . \]  (35)

With the following increase of the herding parameters, the difference in the behaviour of the groups starts growing, reaching the maximum for \( \varepsilon_1 = \varepsilon_2 = 1 \), when the probabilities become
\[ p_1(t + 1) = f_2 + q_2(t) \quad (\varepsilon_1 = 1) , \]
\[ p_2(t + 1) = f_1 + q_1(t) \quad (\varepsilon_2 = 1). \]  

Thus the expected behaviour of the group probabilities can be rather different at the point \( \varepsilon_1 + \varepsilon_2 = 0 \), when there is no herding effect. The difference smooths when approaching the line \( \varepsilon_1 + \varepsilon_2 = 1 \). An then the difference increases again when drifting to the line \( \varepsilon_1 + \varepsilon_2 = 2 \). Numerical solution of Eqs. (32) demonstrates that there occur various types of qualitatively different behaviour. In the first four Figures 1 to 4, we show dynamic regimes corresponding to what can be called moderate herding, with the herding parameters varying from the point \( \varepsilon_1 + \varepsilon_2 = 0 \) to the line \( \varepsilon_1 + \varepsilon_2 = 1 \). Figures 5 to 10 present dynamic regimes for large herding parameters between the lines \( \varepsilon_1 + \varepsilon_2 = 1 \) and \( \varepsilon_1 + \varepsilon_2 = 2 \). Overall, the following dynamic regimes can happen.

1. In the absence of herding, the functions \( p_j(t) \) tend to fixed points

\[ p_j^* = \lim_{t \to \infty} p_j(t). \]  

The appearance of herding makes the trajectories closer to each other. If there are sharp variations of the functions, they become smoothed by the herding effect, as is shown in Fig. 1.

![Figure 1: Probabilities](image)

Figure 1: Probabilities \( p_1(t) \) (solid line) and \( p_2(t) \) (dash-dotted line) for the initial conditions \( f_1 = 0.4, f_2 = 0.1, q_1 = 0.59, \) and \( q_2 = 0.6 \). (a) \( \varepsilon_1 = \varepsilon_2 = 0 \). The fixed points are \( p_1^* = 0.4 \) and \( p_2^* = 0.636 \); (b) \( \varepsilon_1 = 0.5 \) and \( \varepsilon_2 = 0 \). The consensual fixed point is \( p_1^* = p_2^* = 0.7 \); (c) \( \varepsilon_1 = 0.5 \) and \( \varepsilon_2 = 0.3 \). The consensual fixed point is \( p_1^* = p_2^* = 0.7 \).
2. Without herding, the fraction $p_1(t)$ of agents with long-term memory tends to a fixed point without sharp variations, while the fraction $p_2(t)$ of agents with short-term memory experiences at the beginning essential oscillations that attenuate and finally the function tends to a fixed point, as is demonstrated in Fig. 2.

![Graph (a)](image-a)

![Graph (b)](image-b)

![Graph (c)](image-c)

Figure 2: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 0.8$, $f_2 = 0.9$, $q_1 = 0.19$, and $q_2 = -0.8$. (a) $\varepsilon_1 = \varepsilon_2 = 0$. The fixed points are $p_1^* = 0.8$ and $p_2^* = 0.377$; (b) $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0$. The fixed points are $p_1^* = 0.778$ and $p_2^* = 0.362$; (c) $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.85$. The fixed points are $p_1^* = 0.730$ and $p_2^* = 0.695$.

3. When there is no herding, $p_1(t)$ tends to a fixed point, while $p_2(t)$ permanently oscillates. Switching on the herding parameters results in the permanent oscillation of both functions $p_1(t)$, as well as $p_2(t)$. Increasing the herding parameters further, after initial oscillations, forces both functions to tend to their fixed points. This is illustrated in Fig. 3.

4. When the herding effect is absent, $p_1(t)$ monotonically tends to a fixed point, but $p_2(t)$ exhibits an unusual behaviour. At the beginning, for sufficiently long time, $p_2(t)$ behaves smoothly. Then suddenly, it starts widely oscillating and continues oscillating for ever. Slightly increasing the herding parameters shifts the beginning of oscillations to larger times, makes the oscillation amplitude smaller, and makes the function $p_1(t)$ to also experience everlasting oscillations. The following increase of the herding parameters eliminates oscillations and leads to smooth tendency of both probabilities to different fixed points. The further growth of the herding parameters forces both functions to converge to a consensual fixed point. See Fig. 4.
Figure 3: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 0.8$, $f_2 = 1$, $q_1 = 0.1$, and $q_2 = -0.99$. (a) $\varepsilon_1 = \varepsilon_2 = 0$. The probability for the agents with long-term memory tends to the fixed point $p_1^* = 0.8$, while the function $p_2(t)$ oscillates for all times; (b) $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0$. Both functions $p_1(t)$ and $p_2(t)$ permanently oscillate; (c) $\varepsilon_1 = \varepsilon_2 = 0.1$. The functions, after experiencing at the beginning oscillations, tend to the fixed points $p_1^* = 0.75$ and $p_2^* = 0.353$; (d) $\varepsilon_1 = 0$ and $\varepsilon_2 = 0.9$. Both probabilities tend to the fixed points $p_1^* = 0.8$ and $p_2^* = 0.723$.

Figures 5 to 10 illustrate the case of large herding parameters, such that $\varepsilon_1 + \varepsilon_2 > 1$. We start with the values $\varepsilon_1 = \varepsilon_2 = 1$, so that $\varepsilon_1 + \varepsilon_2 = 2$, when the dynamics is described by Eqs. (36) and then consider diminishing parameters approaching the line $\varepsilon_1 + \varepsilon_2 = 1$, where Eq. (35) is valid. The line $\varepsilon_1 + \varepsilon_2 = 1$ corresponds to the case of the strongest herding behavior.

5. For $\varepsilon_1 + \varepsilon_2 = 2$, the probabilities tend to different fixed points. However, diminishing the herding parameters to the line $\varepsilon_1 + \varepsilon_2 = 1$ results in the convergence of both functions $p_1(t)$ to the common fixed point, as is seen from Fig. 5.

6. When $\varepsilon_1 = \varepsilon_2 = 1$, the probability $p_1(t)$ for the agents with long-term memory oscillates from the beginning, while $p_2(t)$ tends monotonically to a fixed point. This is contrary to Fig. 3, which is in agreement with Eqs. (36). Diminishing the herding parameters, first, makes both functions permanently oscillating, but then suppresses the oscillations, so that close to the line $\varepsilon_1 + \varepsilon_2 = 1$ both probabilities tend to fixed points, as is shown in Fig. 6.

7. For the case $\varepsilon_1 = \varepsilon_2 = 1$, the probability $p_1(t)$ for the agents with long-term memory
permanently oscillates, at the beginning a little, but then the oscillation amplitude increases, while \( p_2(t) \) monotonically tends to a fixed point. Diminishing the herding parameters leads to the oscillation of both probabilities and shifts the start of oscillations to larger times. Then, reducing the herding parameters to the line \( \varepsilon_1 + \varepsilon_2 = 1 \) suppresses the oscillations and forces both functions to converge, first to different fixed points and then to the same consensual fixed point, which is illustrated in Fig. 7.

8. For the herding parameters on or close to the point \( \varepsilon_1 + \varepsilon_2 = 2 \), there appear large chaotic fluctuations after rather long time from the beginning of the process. First, only the probability \( p_1(t) \) starts chaotically oscillating, while \( p_2(t) \) smoothly tends to a fixed point. Diminishing the herding parameters results in the sudden chaotic oscillation of both functions. But the further decrease of the herding parameters approaching the line \( \varepsilon_1 + \varepsilon_2 = 1 \), first, makes the oscillations periodic and then eliminates oscillations at all, so that both functions converge to the consensual fixed point (see Fig. 8).

9. At the point \( \varepsilon_1 + \varepsilon_2 = 2 \), the probability for the agents with long-term memory begins chaotically oscillating from the beginning, while that for agent with short-term memory monotonically tends to a fixed point. For smaller herding parameters, first, both functions start chaotically oscillating, then chaotic oscillations transfer to periodic fluctuations, and then both functions become monotonic, tending to fixed points. This behaviour is demonstrated in Fig. 9.

10. One more example of chaotic fluctuations for large herding parameters, which, first, become periodic and then are suppressed when moving from the point \( \varepsilon_1 + \varepsilon_2 = 2 \) to the line \( \varepsilon_1 + \varepsilon_2 = 1 \). The transformation of dynamic regimes, when reducing the herding parameters, is shown in Fig. 10.

5 Conclusion

We developed a model of a society composed of intelligent agents making decisions by choosing between several alternatives. The choice is based of three attributes, utility, attraction, and herding. By varying the herding parameters there appear different regimes of decision making dynamics. A detailed investigation of possible regimes is done for the time dependence of probabilities of choosing one of two alternatives. Two groups of decision makers are considered, one group consisting of agents with long-term memory and the other group composed of agents with short-term memory.

The most interesting observation is the sudden appearance of oscillations that can be either periodic or chaotic. They can arise either from the beginning of the decision process or after rather long time of smooth dynamics. These suddenly arising waves are self organized, since the society is a closed system having no external perturbations. The sudden growth of the self-excited waves is an interesting example of the appearance of periodic and chaotic dynamics in a closed system. The herding effect smooths the dynamics and even can suppress the self-excited waves.

The aim of the paper is the analysis of admissible dynamic regimes in a complex society. Qualitatively similar dynamics regimes can occur in real social systems. As illustrations, it is possible to adduce several dynamic trends from Google analogous to the evolution curves in our figures. Thus, Fig. 11 demonstrates the curve rising to a limiting value, with small random fluctuations. Figure 12 depicts the decreasing tendency to a limit. In Fig. 13, we see almost periodic motion. And Fig. 14 displays a kind of chaotic behavior. It looks that all types of
behavior exhibited by the model we considered can happen in realistic social systems.

**CRediT authorship contribution statement**

V.I. Yukalov and E.P. Yukalova equally contributed to the paper: Concept, Design, Analysis, Writing, or revision of the manuscript.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Figure 4: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 0.3$, $f_2 = 0$, $q_1 = 0.699$, and $q_2 = 0.98$. (a) $\varepsilon_1 = \varepsilon_2 = 0$. The function $p_1(t)$ tends to the fixed point $p_1^* = 0.3$, while $p_2(t)$, after sufficiently long time of smooth behaviour, exhibits everlasting oscillations; (b) $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0$. Both functions $p_1(t)$ and $p_2(t)$ always oscillate; (c) $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.1$. Both functions tend to different fixed points $p_1^* = 0.322$ and $p_2^* = 0.692$; (d) $\varepsilon_1 = \varepsilon_2 = 0.1$. Both probabilities tend to the consensual fixed point $p_1^* = p_2^* = 0.98$; (e) $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.3$. Both probabilities $p_1(t)$ and $p_2(t)$ monotonically tend to the consensual fixed point $p_1^* = p_2^* = 0.98$. This is an example showing the spontaneous appearance of self-excited oscillations that can be suppressed by the herding effect.
Figure 5: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 1$, $f_2 = 0.2$, $q_1 = -0.9$, and $q_2 = 0.6$. (a) $\varepsilon_1 = \varepsilon_2 = 1$. The fixed points are $p_1^* = 0.5$ and $p_2^* = 1$; (b) $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 0.7$. The consensual fixed point is $p_1^* = p_2^* = 0.8$; (c) $\varepsilon_1 = 0.6$ and $\varepsilon_2 = 0.9$. The consensual fixed point is $p_1^* = p_2^* = 0.8$; (d) $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.7$. Both functions tend to $p_1^* = p_2^* = 0.8$. 
Figure 6: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 0.6$, $f_2 = 1$, $q_1 = 0.39$, and $q_2 = -0.9$. (a) $\varepsilon_1 = \varepsilon_2 = 1$. The function $p_1(t)$ oscillates with a constant amplitude and $p_2(t)$ tends monotonically to the fixed point $p_2^* = 0.6$; (b) $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.9$. Both functions $p_1(t)$ and $p_2(t)$ permanently oscillate with constant amplitudes; (c) $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 0.8$. The functions tend to the fixed points $p_1^* = 0.265$ and $p_2^* = 0.525$, respectively; (d) $\varepsilon_1 = 0.3$ and $\varepsilon_2 = 0.9$. The fixed points are $p_1^* = 0.455$ and $p_2^* = 0.552$. 
Figure 7: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 0.1$, $f_2 = 0$, $q_1 = 0.899$, and $q_2 = 0.93$. (a) $\varepsilon_1 = \varepsilon_2 = 1$. The function $p_1(t)$ oscillates, while $p_2(t)$ tends monotonically to the fixed point $p_2^* = 0.1$; (b) $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.98$. Both functions $p_1(t)$ and $p_2(t)$ oscillate; (c) $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.9$. The probabilities tend to the fixed points $p_1^* = 0.6$ and $p_2^* = 0.15$; (d) $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.8$. Both functions converge to the same fixed point $p_1^* = p_2^* = 0.93$; (e) $\varepsilon_1 = 0.5$ and $\varepsilon_1 = 0.8$. The functions converge monotonically to the consensual fixed point $p_1^* = p_2^* = 0.93$. 
Figure 8: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 0.3$, $f_2 = 0$, $q_1 = 0.699$, and $q_2 = 0.99$. (a) $\varepsilon_1 = \varepsilon_2 = 1$. The function $p_1(t)$, after quite long time of smooth behaviour, suddenly starts chaotically oscillating, but $p_2(t)$ goes monotonically to the fixed point $p_2^* = 0.3$; (b) $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.95$. Both functions $p_1(t)$ and $p_2(t)$, after rather long time of smooth behaviour, start chaotically oscillating; (c) $\varepsilon_1 = 0.94$ and $\varepsilon_2 = 0.98$. Both functions $p_1(t)$ and $p_2(t)$, after the initial smooth behaviour, start periodic oscillations; (d) $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 0.8$. Oscillations are suppressed and the functions $p_1(t)$ and $p_2(t)$ converge to the consensual fixed point $p_1^* = p_2^* = 0.99$; (e) $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.9$. Both functions monotonically converge to the same fixed point $p_1^* = p_2^* = 0.99$. 
Figure 9: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 0.6, f_2 = 1, q_1 = 0.3,$ and $q_2 = -0.999$. (a) $\varepsilon_1 = \varepsilon_2 = 1$. The function $p_1(t)$ starts chaotically oscillating from the very beginning, while $p_2(t)$ monotonically tends to the fixed point $p_2^* = 0.6$; (b) $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.8$. Both functions $p_1(t)$ and $p_2(t)$ oscillate chaotically; (c) $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 0.8$. Both functions $p_1(t)$ and $p_2(t)$ exhibit periodic oscillations; (d) $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 0.7$. After the initial period of oscillations, both functions tend to the fixed points $p_1^* = 0.204$ and $p_2^* = 0.468$; (e) $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 1$. Both functions monotonically converge to the fixed points $p_1^* = 0.485$ and $p_2^* = 0.6$. 
Figure 10: Probabilities $p_1(t)$ (solid line) and $p_2(t)$ (dash-dotted line) for the initial conditions $f_1 = 0.2$, $f_2 = 0$, $q_1 = -0.1$, and $q_2 = 0.999$. (a) $\varepsilon_1 = \varepsilon_2 = 1$. The function $p_1(t)$ experiences chaotic oscillations, while $p_2(t)$ monotonically tends to the fixed point $p_2^* = 0.2$; (b) $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.7$. Both functions $p_1(t)$ and $p_2(t)$ chaotically oscillate; (c) $\varepsilon_1 = \varepsilon_2 = 0.9$. Both functions $p_1(t)$ and $p_2(t)$ demonstrate periodic oscillations; (d) $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 0.8$. After initial fluctuations, the functions tend to the fixed points $p_1^* = 0.685$ and $p_2^* = 0.308$; e) $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 1$. Both functions monotonically tend to the fixed points $p_1^* = 0.278$ and $p_2^* = 0.2$. 
Figure 11: Interest to USD in Canada over time from 2004 up to now. (Increases to its limit).

Figure 12: Interest to car prices worldwide over time from 2004 up to now. (Decreases to its limit).

Figure 13: Interest to football game in USA over time from 2004 up to now. (Oscillations).
Figure 14: Interest to liberalism as a political philosophy in Science category over time for 5 year up to now. (Chaotic behavior).