Clues towards unified textures

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Received Day Month Year
Revised Day Month Year

The issue of texture specific mass matrices has been discussed by incorporating Weak Basis transformations and the concept of ‘naturalness’. Interestingly, we find that starting from the most general mass matrices, one can arrive at texture four zero mass matrices which can fit both quark as well as lepton mixing data and are similar to the original Fritzsch ansatze.

Keywords: Fermion mass matrices; weak basis transformations; natural mass matrices.

PACS numbers:12.15.Ff,14.60.Pq

1. Introduction

Fermion masses and mixings constitutes one of the most important aspects of Flavor Physics. Understanding the vast spectrum of fermion masses, spanning many orders of magnitude, in a unified framework is one of the biggest challenges. For a better appreciation of the problem, it is desirable to consider the present spectrum of fermion masses, e.g.,

\[ m_u, m_d \sim 10^{-3} \text{GeV}, \] \[ m_s \sim 10^{-1} \text{GeV}, \] \[ m_c, m_b \sim 1 \text{GeV}, \] \[ m_t \sim 10^2 \text{GeV}, \] \[ m_e \sim 10^{-3} \text{GeV}, \] \[ m_\mu \sim 10^{-1} \text{GeV}, \] \[ m_\tau \sim 1 \text{GeV}, \] \[ m_\nu \lesssim 10^{-11} \text{GeV}. \]

The above ranges of masses clearly span over 13 orders of magnitude. In case the theory requires the existence of right handed neutrinos, with the mass range...
10^9 \sim 10^{16} \text{ GeV}, the fermion masses would then cover almost 30 orders of magnitude.

It is well known that the quark mixing angles are hierarchical, e.g. \( s_{12} \sim 0.22, s_{23} \sim 0.04, s_{13} \sim 0.004 \), whereas the lepton mixing angles do not show any hierarchy. The non zero value of \( \theta_{13} \), on the one hand, restores the parallelism between the mixings of quarks and leptons, on the other hand, its unexpectedly ‘large’ value signifies the differences between these, the leptonic mixing angles being large as compared to the quark counterparts values. Further, the issue of neutrinos being Dirac like or Majorana particles is still an open question for physicists since Dirac neutrinos have not yet been ruled by experimental data. The problem assumes several dimensions when one finds that in the case of quarks not only the mixing angles but the quark masses also show distinct hierarchy, this being in sharp contrast to the case of neutrinos wherein neither the mixing angles nor the masses show any distinct hierarchy. Since the mixing angles are related to the corresponding mass matrices therefore formulating viable fermion mass matrices becomes all the more complicated especially when quarks and leptons have to be described in a unified framework.

The theoretical understanding of fermion masses and mixings proceeds along two approaches, i.e., ‘top-down’ and ‘bottom-up’. Despite large number of attempts from the top-down perspective, such as grand unification\(^{[5,10]}\), supersymmetry\(^{[11,13]}\), compositeness\(^{[14,18]}\), superstrings\(^{[19,21]}\), etc., we are not in a position to have a compelling theory of flavor dynamics. Bottom-up approach consists of finding the phenomenological fermion mass matrices which are in tune with the low energy data, i.e., observables like quark and lepton masses, mixing angles in both the sectors, angles of the unitarity triangle in the quark sector, etc..

Texture specific mass matrices provide a very good example of bottom-up approach. For a detailed and comprehensive review we refer the reader to Fritzsch and Xing\(^{[22]}\), along with a very recent one by Gupta and Ahuja\(^{[23]}\). A viable formulation of mass matrices which incorporates the low energy data pertaining to quark sector
as well as lepton sector is very desirable. It becomes particularly desirable in case we want to find viable, finite set of texture specific mass matrices, which then can provide vital clues for their formulation within the top-down approach.

Several attempts have been made to describe fermion mass matrices in a unified manner within the framework of grand unified theories (GUTs). Efforts have also been made to reconstruct viable fermion mass matrices keeping in mind the low energy data as well as to integrate textures within the GUT. However, the issue of finding whether a viable set of mass matrices which can accommodate the quark mixing as well as lepton mixing data within the texture framework has not been addressed yet. The purpose of present work, therefore, is to find a minimal viable set of texture specific mass matrices which are in agreement with Weak Basis (WB) transformations as well as the criterion of natural mass matrices. To this end, we first explore the possibility of finding texture specific quark mass matrices starting from the most general mass matrices. The issue of compatibility of such mass matrices with the lepton mixing data has also been discussed briefly.

The detailed plan of paper is as follows. To make the manuscript self contained, in section 2 we discuss briefly the essentials of Weak Basis transformations and in section 3 we encapsulate the idea of naturalness. In section 4 we present the analysis pertaining to the general texture specific quark mass matrices within the framework of Standard Model (SM) incorporating Weak Basis transformations and naturalness. Without going into details, in section 5 we present the status of the lepton mass matrices for the same. Finally, section 6 summarizes our conclusions.

2. Weak Basis Transformations

Within the framework of the SM, the quark mass matrices can be considered hermitian without loss of generality, encoding all the information about the quark masses and mixings. These matrices have a total of 18 real free parameters, large in number compared to only ten physical observables corresponding to six quark masses and four physical parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.
It should be noted that in the SM one has the freedom to make a unitary transformation, e.g., \( q_L \rightarrow W q_L, \quad q_R \rightarrow W q_R, \quad q'_L \rightarrow W q'_L, \quad q'_R \rightarrow W q'_R \) under which the gauge currents remain real and diagonal but the mass matrices transform as

\[ M_U \rightarrow W^\dagger M_U W, \quad M_D \rightarrow W^\dagger M_D W, \]  

(1)
such transformations are referred to as ‘Weak Basis (WB) Transformations’. It can be easily checked that such transformations preserve the hermiticity of the mass matrices. It needs to be mentioned that the CKM matrix is independent of WB transformations, e.g., noting that \((U_U, U_D)\) and \((U'_U, U'_D)\) are the respective diagonalizing transformations of \((M_U, M_D)\) and \((M'_U, M'_D)\) for either of \(M_U\) and \(M_D\) one can obtain

\[ U'_U = W^\dagger U_U; \quad U'_D = W^\dagger U_D. \]  

(2)

Using this result, the mixing matrix for the WB transformed quark mass matrices can be given as

\[ V'_{ckm} = U'_{u}^\dagger U'_{d} = (W^\dagger U_u)^\dagger (W^\dagger U_d) = (U_u)^\dagger W W^\dagger U_d = (U_u)^\dagger U_d = V_{ckm}. \]  

(3)

WB transformations have been used\(^{27-30}\) to obtain texture specific mass matrices which lead to reduction in the number of parameters defining the mass matrices without loss of generality. It can be shown that the general mass matrices can be reduced to two different types of texture specific up and down mass matrices e.g., Branco \textit{et al.}\(^{27,28}\) use the WB transformations to obtain the following texture specific mass matrices

\[
M_q = \begin{pmatrix}
0 & * & 0 \\
* & * & * \\
0 & * & 0
\end{pmatrix}, \quad M'_{q} = \begin{pmatrix}
0 & * & * \\
* & * & * \\
* & * & *
\end{pmatrix}, \quad q, q' = U, D.
\]  

(4)

In the second possibility, as observed by Fritzsch and Xing\(^{29,30}\), one ends up with
the possibility

\[ M_q = \begin{pmatrix} * & * & 0 \\ * & * \\ 0 & * & * \end{pmatrix}, \quad q = U, D. \tag{5} \]

Needless to emphasize that the two type of mass matrices are equivalent. However, we adopt the possibility given by Fritzsch and Xing as it corresponds to parallel structure for \((M_U, M_D)\) in consonance with most of the GUT models.

3. Natural mass matrices

The matrices obtained through WB transformations have large parametric space due to the extra number of parameters as well as no restriction on the elements of the mass matrices. In case such matrices are used to fit the data, it is immediately clear that one would obtain large number of viable possibilities of mass matrices, making it difficult to develop models corresponding to these. In this context, Peccei and Wang\textsuperscript{31}, in order to avoid fine tuning, have translated the hierarchy of the quark mixing matrix to the formulation of ‘natural mass matrices’ which considerably restricts the parameter space available to the elements of the mass matrices. It is interesting to note that the idea of natural mass matrices coupled with the WB transformations leads to constraints on the elements of the mass matrices. In this context, we first consider the CKM matrix in the Wolfenstein parametrization\textsuperscript{32}.

\[ V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{6} \]

where \(\lambda \sim 0.22, \quad A \sim 0.82, \quad \rho \sim 0.131\) and \(\eta \sim 0.345\). The above matrix can approximately be written as

\[ V_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & 0.3\lambda^3 \\ -\lambda & 1 & 0.8\lambda^2 \\ 0.6\lambda^3 & -0.8\lambda^2 & 1 \end{pmatrix}. \tag{7} \]
To translate the constraints of the above matrix on the mass matrices, we consider a basis in which either of the two mass matrices $M_U$ and $M_D$ is diagonal. For example, in case we choose $M_D$ to be diagonal and $M_U = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$, keeping in mind the relation $M_U = V_{ckm}^\dagger M_U^{diag} V_{ckm}$ and using eqns. (7) and (8), we get

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & 0.3\lambda^3 \\ -\lambda & 1 & 0.8\lambda^2 \\ 0.6\lambda^3 & -0.8\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} 1 & \lambda & 0.3\lambda^3 \\ -\lambda & 1 & 0.8\lambda^2 \\ 0.6\lambda^3 & -0.8\lambda^2 & 1 \end{pmatrix}.$$  \tag{9}

Since $m_u \ll m_c \ll m_t$, one obtains

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \sim \begin{pmatrix} m_c\lambda^2 & -m_c\lambda & 0.6m_t\lambda^3 \\ -m_c\lambda & m_c & -0.8m_t\lambda^3 \\ 0.6m_t\lambda^3 & -0.8m_t\lambda^3 & m_t \end{pmatrix}. \tag{10}$$

It is interesting to note that the elements $(1,3)$ and $(3,1)$ in the above matrix are larger than the $(1,2)$ and $(2,1)$ elements. However, we still have the freedom of WB transformations. Noting that WB transformations do not affect the hierarchy of elements of mass matrices, one can eliminate the $(1,3)$ and $(3,1)$ elements in the above matrix achieving the form advocated by Fritzsch and Xing. In the matrix so obtained, one can easily see that the elements of the mass matrices satisfy the natural hierarchy,

$$(1, i) \lesssim (2, j) \lesssim (3, 3); \quad i = 1, 2, 3; \quad j = 2, 3. \tag{11}$$

Further, noting that the quark mass eigen values are hierarchical, one can always choose a basis where the elements of the mass matrices satisfy the ‘natural hierarchy’
given in eqn. (11) and are of the Fritzsch-Xing form,

\[
M_{i(i=U,D)} = \begin{pmatrix}
  e^i & a_i & 0 \\
  a_i^* & d_i & b_i \\
  0 & b_i^* & c_i
\end{pmatrix}.
\] (12)

4. Naturalness and general mass matrices

To begin with, we start with the mass matrices defined in eqn. (12) obtained after imposing WB transformations on the general mass matrices as well as consider these to be natural. To study the implications of the quark mixing data on these, we underline some of the important steps used in our analysis. The elements of the matrices can be defined as \( a_i = |a_i|e^{i\alpha_i} \) and \( b_i = |b_i|e^{i\beta_i} \), further \( \phi_1 = \alpha_U - \alpha_D \), \( \phi_2 = \beta_U - \beta_D \). The hermitian matrices \( M_i \) \((i = U, D)\) can be expressed as \( M_i = P_i^\dagger M_i^r P_i \), where \( P_i = \text{diag}(e^{-i\alpha_i}, 1, e^{i\beta_i}) \) and the real matrices \( M_i^r \) are

\[
M_i^r = \begin{pmatrix}
  |e_i| & |a_i| & 0 \\
  |a_i| & d_i & |b_i| \\
  0 & |b_i| & c_i
\end{pmatrix}.
\] (13)

The matrices \( M_i^r \) can be diagonalized by the orthogonal transformation, e.g.,

\[
M_i^{\text{diag}} = O_i^T M_i^r O_i \equiv O_i^T P_i M_i P_i^\dagger O_i,
\] (14)

where

\[
M_i^{\text{diag}} = \text{diag}(m_1, -m_2, m_3),
\] (15)

the subscripts 1, 2 and 3 refer respectively to \( u, c \) and \( t \) for the \( U \) sector and \( d, s \) and \( b \) for the \( D \) sector. It may be noted that the second mass eigen value is chosen with a negative sign to facilitate the construction of the diagonalizing transformation \( O_i \), for the details in this regard we refer the reader to our earlier papers \[23\], \[33\]. The CKM matrix in terms of the diagonalizing transformations is given by

\[
V_{\text{CKM}} = O_U^T P_U P_D^\dagger O_D = V_U^\dagger V_D,
\] (16)
where the unitary matrices $V_U = (P_U^1 O_U)$ and $V_D = (P_D^1 O_D)$ are the diagonalizing matrices for the hermitian matrices $M_U$ and $M_D$ respectively.

For ready reference, the input quark masses at the $m_Z$ scale used in the analysis are

$$
\begin{align*}
  m_u &= 1.38^{+0.42}_{-0.41} \text{MeV}, \\
  m_d &= 2.82^{+0.48}_{-0.48} \text{MeV}, \\
  m_s &= 57^{+18}_{-12} \text{MeV}, \\
  m_c &= 0.638^{+0.043}_{-0.084} \text{GeV}, \\
  m_b &= 2.86^{+0.16}_{-0.6} \text{GeV}, \\
  m_t &= 172.1^{+1.2}_{-1.2} \text{GeV}.
\end{align*}
$$

(17)

The light quark masses $m_u$, $m_d$ and $m_s$ have been further constrained by using the following mass ratios

$$
\begin{align*}
  m_u/m_d &= 0.553 \pm 0.043, \\
  m_s/m_d &= 18.9 \pm 0.8.
\end{align*}
$$

(18)

For the purpose of our calculations, apart from imposing the constraints of naturalness, we further restrict the parameter space by putting the condition that the elements of the diagonalizing transformations $O_U$ and $O_D$ remain real. The phases $\phi_1$ and $\phi_2$ are given full variation, whereas the free parameters $d_U$ and $d_D$ have been given wide variation keeping in mind the above mentioned constraints.

While carrying out the analysis, we first reproduce $V_{us}$, $V_{ub}$ and $V_{cb}$ corresponding respectively to $s_{12}$, $s_{23}$ and $s_{13}$ as well as CP asymmetry parameter $\sin 2\beta$, this essentially allows us to reconstruct the corresponding CKM matrix. Interestingly,
we find that the elements $e_U$ and $e_D$ do not seem to be playing any significant role in reproducing the CKM matrix. This can be understood by considering the effect of variation of $e_U$ and $e_D$ on some of the CKM elements. To this end, in figure (1), we have plotted $|V_{us}|$ versus $e_U$ and $e_D$. As is evident from the figure, $|V_{us}|$ does not show any dependence with the variation of $e_U$ and $e_D$ indicating the redundancy of these elements. This can also be checked further by considering the variation of other CKM elements with respect to $e_U$ and $e_D$. Therefore, it seems that we do not lose any generality if we ignore the elements $e_U$ and $e_D$ and consider reducing the mass matrices given in eqn. (12) to

$$M_{i(i=U,D)} = \begin{pmatrix} 0 & a_i & 0 \\ a_i^* & d_i & b_i \\ 0 & b_i^* & c_i \end{pmatrix}. \quad (19)$$

Interestingly, this structure is very similar to the original Fritzsch ansatze for the quark mass matrices wherein the elements $d_i$ are zero. The matrices given above, in fact, can be characterized as Fritzsch-like texture four zero quark mass matrices. The analyses of such matrices have been carried out by several authors, however we briefly present the results of our present analysis. Using the inputs mentioned above and imposing the constraints given by PDG 2012 data

$$|V_{us}| = 0.22534 \pm 0.00065, |V_{ab}| = 0.00351^{+0.00015}_{-0.00014},$$

$$|V_{cb}| = 0.0412^{+0.0011}_{-0.0005}, \sin^2 \beta = 0.679 \pm 0.020, \quad (20)$$

we obtain the following CKM matrix

$$V_{ckm} = \begin{pmatrix} 0.9741 - 0.9744 & 0.2246 - 0.2259 & 0.00337 - 0.00365 \\ 0.2245 - 0.2258 & 0.9732 - 0.9736 & 0.0407 - 0.0422 \\ 0.0071 - 0.0100 & 0.0396 - 0.0417 & 0.9990 - 0.9992 \end{pmatrix}. \quad (22)$$

Interestingly, this matrix appears to be in full agreement with the CKM matrix given by PDG 2012. Similarly, one can also calculate the angles of the unitarity triangle and Jarlskog’s rephasing invariant parameter $J$ as well. Therefore, it seems
that at present the texture four zero Fritzsch-like quark mass matrices are fully compatible with the present quark mixing data.

### 4.1. Non-Fritzsch-like texture four zero quark mass matrices

Keeping in mind the WB transformations, which also include permutation symmetries, one can in fact find other texture four zero structures also which are essentially non Fritzsch-like, e.g.,

\[ a : \begin{pmatrix} d & a & 0 \\ a^* & 0 & b \\ 0 & b^* & c \end{pmatrix} \text{ and its permutations,} \tag{23} \]

\[ b : \begin{pmatrix} 0 & a & d \\ a^* & 0 & b \\ d^* & b^* & c \end{pmatrix} \text{ and its permutations,} \tag{24} \]

\[ c : \begin{pmatrix} a & 0 & 0 \\ 0 & d & b \\ 0 & b^* & c \end{pmatrix} \text{ and its permutations.} \tag{25} \]

It is immediately clear that the matrices corresponding to category ‘c’ are not viable as all the matrices in this class correspond to the scenario where one of the generations gets decoupled from the other two. Regarding categories ‘a’ and ‘b’, we can carry out an analysis similar to the one discussed earlier pertaining to Fritzsch-like texture four zero mass matrices. The corresponding CKM matrices for classes ‘a’ and ‘b’ then come out to be

\[ V^a_{\text{ckm}} = \begin{pmatrix} 0.9740 & -0.9744 & 0.2247 & -0.2260 & 0.0024 & -0.0099 \\ 0.2205 & -0.2256 & 0.9509 & -0.9727 & 0.0596 & -0.2172 \\ 0.0140 & -0.0445 & 0.0584 & -0.2127 & 0.9905 & 1.0000 \end{pmatrix}, \tag{26} \]
One finds that the CKM matrices mentioned above do not agree with the one given by PDG 2012, thereby implying that the corresponding mass matrices appear to be ruled out. Interestingly, this leads us to arrive at an important conclusion that in the case of texture four zero mass matrices, the only viable mass matrices are the Fritzsch-like ones and their permutations.

5. Lepton mass matrices

After having found that there is a set of quark mass matrices which can be obtained from the most general mass matrices within the SM, it becomes desirable to check whether similar structures can satisfy the lepton mixing data also. It needs to be emphasized that since the lepton masses and mixings are very different from those in the quark sector, it is not necessary to impose the kind of constraints considered for the quark mass matrices. However, keeping in mind the issue of unified fermion mass matrices as advocated by Smirnov\(^\text{37}\), one needs to check whether similar structure with the same constraints can satisfy the lepton mixing data also. To this end, we have examined\(^\text{33,38}\) texture four zero lepton mass matrices for Dirac as well as Majorana neutrino case using the latest lepton mixing data. The details in this regard will be published elsewhere. However, we find that in case we impose naturalness criterion, inverted hierarchy of neutrino masses is ruled out for Dirac as well as Majorana neutrinos whereas normal hierarchy and degenerate scenario are viable for both.

6. Summary and Conclusions

To summarize, in view of the recent refinements in the quark as well as lepton mixing data, the issue of formulation of fermion mass matrices which incorporate the
low energy data in both the sectors poses to be an important issue of the flavor physics. In this context, keeping in mind various broad guidelines such as Weak Basis transformations and ‘natural mass matrices’, we have made an attempt to carry out an analysis of the texture specific mass matrices pertaining to quark as well as lepton sectors. Starting with the most general mass matrices, using WB transformations, one can consider the mass matrices wherein the (1,3) and (3,1) elements in the up sector as well as down sector mass matrices are zero. Further, when these mass matrices are subjected to naturalness condition

\[(1, i) \lesssim (2, j) \lesssim (3, 3) ; \hspace{1cm} i = 1, 2, 3, \hspace{0.2cm} j = 2, 3,\]

one finds that the (1,1) element in both the mass matrices takes very small values and seems largely redundant as far as its implications on the CKM elements are concerned. In other words, starting with the most general mass matrices one essentially obtains texture four zero mass matrices, out of which only the Fritzsch-like mass matrices and their permutations seem to be the viable option.

A corresponding study in the lepton sector for texture four zero Fritzsch-like mass matrices in the Dirac as well as Majorana neutrino case has been carried out. Our present analysis indicates that these matrices are compatible with the normal hierarchy and degenerate scenario of neutrino masses whereas for inverted hierarchy such matrices are ruled out in case the naturalness conditions are imposed. In conclusion, we can perhaps say that the texture four zero Fritzsch-like mass matrices provide an almost unique class of viable fermion mass matrices giving vital clues towards unified textures for model builders.

Acknowledgments

M.G. would like to thank the organizers of ‘International Conference on Flavor Physics and Mass Generation’, NTU Singapore for providing an opportunity to present this work. S.S. would like to acknowledge UGC, Govt. of India, for finan-
cial support. G.A. would like to acknowledge DST, Government of India (Grant No: SR/FTP/PS-017/2012) for financial support. S.S., P.F., G.A. acknowledge the Chairperson, Department of Physics, P.U., for providing facilities to work.

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