Double parton interactions in \( \gamma p, \gamma A \) collisions in the direct photon kinematics.

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We derive expressions for the differential distributions and the total cross section of double-parton interaction in direct photon interaction with proton and nuclei. We demonstrate that in this case the cross section is more directly related to the nucleon generalized parton distribution than in the case of double parton interactions in the proton - proton collisions. We focus on the production of two dijets each containing charm (anticharm) quarks and carrying \( x_1, x_2 > 0.2 \) fractions of the photon momentum. Numerical results are presented for the case of \( \gamma p \) collisions at LHeC, HERA and in the ultraperipheral \( AA \) and \( pA \) collisions at the LHC. We find that the events of this kind would be abundantly produced at the LHeC. For \( \sqrt{s} = 1.3 \) TeV the expected rate is \( 2 \cdot 10^8 \) events for the luminosity \( 10^{34} \) cm\(^{-2}\) s\(^{-1}\), the running time of \( 10^6 \) s and the transverse cutoff of \( p_t > 5 \) GeV. This would make it feasible to use these processes for the model independent determination of two parton GPDs in nucleon and in nuclei. For HERA the total accumulated number of the events is also high, but efficiency of the detection of charm seems too low to study the process. We also find that a significant number of such double parton interactions should be produced in \( p - Pb \) and \( Pb - Pb \) collisions at the LHC: \( \sim 6 \cdot 10^4 \) for \( Pb - Pb \), and \( \sim 7 \cdot 10^3 \) for \( p - Pb \) collisions for the same transverse momentum cutoff.

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I. INTRODUCTION

Multiple hard parton interactions (MPI) started to play an important role in the description of the inelastic pp collisions at the collider energies. Hence, although the studies of MPI began in eighties [1–4], they attracted a lot of theoretical and experimental attention only recently. Extensive theoretical studies were carried out in the last decade, both for pp collisions [5–23], and for pA collisions [24–26]. Attempts have been made to incorporate multi-parton collisions in the Monte Carlo event generators [27–30].

MPIs can serve as a probe for non-perturbative correlations between partons in the nucleon wave function and are crucial for determining the structure of the underlying event at the LHC energies. They constitute an important background for the new physics searches at the LHC. A number of experimental studies were performed at the Tevatron [31–33]. New experimental studies are underway at the LHC [34–37].

The analysis of the experimental data indicates [16, 17] that the rate of such collisions exceeds significantly a naive expectation based on the picture of the binary collisions of the uncorrelated partons of the nucleons (provided one uses information from HERA on the transverse distribution of gluons in nucleons).

In the parton model inspired picture MPI occur via collisions of the pairs of partons: the $2 \otimes 2$ mechanism (collision of two pairs of partons). In pQCD the picture is more complicated since the QCD evolution generates short-range correlations between the partons (splitting of one parton into two,...) –the $1 \otimes 2$ mechanism [18–20]. It was demonstrated that account of these pQCD correlations enhances the rate of MPI as compared to the parton model by a factor of up to two and may explain discrepancy of the data [31–37] with the parton model. (A much larger enhancement recently reported in the double $J/\psi$ production [38] can hardly be explained by this mechanism).

Presence of two mechanisms and limited knowledge of the nucleon multiparton structure makes a unique interpretation of the data rather difficult.

Hence here we propose to study the MPI process of $\gamma p(A)$ interaction with production of four jets in the kinematics where two jets carry most of the light cone fraction of the photon four momentum - direct photon mechanism. In this process the $2 \otimes 2$ mechanism is absent and the only process which contributes is an analog of the $1 \otimes 2$ process. Since in the proposed kinematics the contribution of the resolved photon is strongly suppressed the cross section in the leading log approximation (LLA) i.e. summing leading collinear singularities is expressed through the integral
over two particle GPDs, $2D(x_1, x_2, Q^2_1, Q^2_2, \Delta)$, introduced in [17]. This is in difference from the case of $pp, pA$ scattering where $2 \otimes 2$ contribution is proportional to a more complicated integral with the integrant proportional to the product of two double parton GPDs.

The main goal of the present paper is to show that processes with direct photon in photon proton collisions provide a golden opportunity for the model independent determination of the double parton distributions $2D$, free of the ambiguities inherent in $pp/pA$ scattering [19]. We will consider the process of the interaction of the real/quasireal photon with proton with production of two pairs of hard jets in the back to back kinematics with each dijet consisting of a heavy (charm) quark and gluon jets (see Figs. 1 and 2). We focus on the production of charm to suppress the contribution of the resolved photons.

In the discussed process a $c\bar{c}$ pair is produced in the photon fragmentation region, while two gluon jets are created predominantly in the target region, so that there is a large rapidity gap between the gluon and quark jets. The gluon and c-quark jets are approximately balanced pair vice. The
cross section of the analogous process in $pp$ collisions is influenced by parton correlations in both nucleons participating in the process, while in the case of the photon the cross section depends only on the integral over one wave function. The reason is that the process involves only one GPD from the nucleon, while the upper part of the diagram 1 is determined by the hard physics of the photon splitting to $Q\bar{Q}$ pair in an unambiguous way. It does not involve the scale $Q_0^2$ that separates perturbative and nonperturbative correlations in a nucleon. Thus the cross section of such a process is directly expressed through the nucleon double GPD. Hence the measurement of the discussed cross section would allow to perform a nearly model independent analysis of DPI in $pp$ scattering. We will demonstrate below that it would be possible also to study these processes at the future electron - proton / nucleus colliders. It maybe possible also to investigate these processes in $AA$ and $pA$ ultraperipheral collisions at the LHC.

Here we will consider the MPI rates for all three types of processes mentioned above, $\gamma p, AA, pA$. We will restrict ourselves to the kinematics $x_1, x_2 > 0.2$, thus guaranteeing the dominance of the direct photon contribution (For this cutoff the direct photons contribute 60% of the dijet cross section). For a lower $x_i$ cutoffs the relative contribution of the direct photon mechanism rapidly decreases for transverse momenta under consideration.

We will demonstrate that for the LHeC collider energies $\sqrt{s} = 1300$ GeV the rate of the discussed reaction will be very high: $2 \cdot 10^8$ events per $10^6$ s for the luminosity $10^{34}$ cm$^{-2}$s$^{-1}$ and $p_t > 5$ GeV. The relative rate of MPI to dijets is found to be 0.045%. In principle a large number of events in the discussed kinematics was produced at HERA: $\sim 1.2 \cdot 10^5$ for the total luminosity 1 fb$^{-1}$. However the efficiency of the detection of $D^*$ was pretty low [39] so it appears that it would be very difficult to study the discussed process using the HERA data.

Another way to observe the discussed process in the near future maybe possible - study of MPI in the ultraperipheral $pA, AA$ processes at the full LHC energy. For example, for $p_t > 5$ GeV, we have $\sim 6 \cdot 10^4$ events for $AA$, and $\sim 6.6 \cdot 10^5$ events for $pA$ scattering where we used luminosities $\sim 10^{27}$ (AA), $10^{29}$ cm$^{-2}$s$^{-1}$ (pA) and running time of $10^6$ s. In the discussed kinematics MPI events constitute $\sim 0.04\%$ ($\sim 0.02\%, \sim 0.0125\%$) of the dijet events for $AA, pA$ collisions respectively for the same jet cutoff. These fractions decrease rather rapidly with $p_t$ increase.

Of course, the MPI processes are contaminated by the leading twist 4 jet production the so called 2 to 4 processes. However it is possible to argue that in the back to back kinematics the contribution of these processes(see Fig.3) are parametrically small in a wide region of the phase space [18]. (Moreover for the $AA$ collisions there is an additional combinatorial $A^{1/3}$ enhancement over parasitical 2 to 4 contributions [24, 25]. ) Indeed, a detailed MC simulation analysis was done
using Pythia and Madgraph for pp collisions \cite{31-33}. These authors have demonstrated that it is possible to introduce observables that are dominated by MPI in the back to back kinematics, thus allowing to measure MPI cross sections, as distinct from 2 to 4 processes. Of course, further work is required in this direction, especially including NLO effects.

\[ \gamma + p \rightarrow c + \bar{c} + g_1 + g_2 + X \]

Note that MPI in the photon proton collisions were also studied in \cite{30}. These authors considered resolved photon kinematics, which is very different, from the one that is considered here. So there is no overlap with the present study.

The paper is organized as following. In section 2 we calculate the MPI contribution to $\gamma + p \rightarrow c + \bar{c} + g_1 + g_2 + N$ process in the back to back kinematics. In section 3 we calculate the rates of the discussed process for $ep$ collisions at LHeC and HERA, and for ultraperipheral $pA$ and $AA$ collisions at the LHC. In section 4 we carry the numerical simulations for realistic parameters corresponding to LHC and HERA runs. The results are summarized in section 5.

II. BASIC FORMULAE FOR MPI IN THE DIRECT PHOTON - PROTON SCATTERING.

A. Parton Model.

First we consider the process of production of two dijets with single charm in each pair (Fig. 4a) in the parton model. In this case the process is essentially the same as the one already considered in ref. \cite{18}. The only difference is that the parton created in the split vertex is a charmed quark – antiquark pair. The corresponding kinematics is depicted in Fig. 4a, and is analogous to the $1 \otimes 2$ transition in pp interactions. Let us parameterize the momenta of quarks and gluons using Sudakov variables ($k_1, k_2$ are momenta of virtual charm quarks and antiquark of the $qq$ pair and
\( k_3, k_4 \) are the gluon momenta. Let us analyze the lowest order amplitude shown in Fig. 4a for the double hard collision which involves parton splitting.

![Parton model and QCD diagrams](image)

**FIG. 4:** Parton model (a) and pQCD diagrams (b) for the MPI production of \( c, \bar{c} + 2 \) gluon jets.

We decompose parton momenta \( k_i \) in terms of the so called Sudakov variables using the light-like vectors \( q \) and \( p \) along the incident photon and proton momenta:

\[
\begin{align*}
    k_1 &= x_1 q + \beta p + k_{\perp}, \quad k_3 \simeq (x_3 - \beta)p; \\
    k_2 &= x_2 q - \beta p - k_{\perp}, \quad k_4 \simeq (x_4 + \beta)p; \\
    \vec{k}_{\perp} &= \vec{\delta}_{12} = -\vec{\delta}_{34} (\delta' \equiv 0); \quad k_0 \simeq (x_1 + x_2)q.
\end{align*}
\]

Here \( k_0 \) is the momentum of the quasireal photon. We can neglect the charm quark masses except while dealing with infrared singularities. The light-cone fractions \( x_i \), \((i=1,..4)\), are determined by the jet kinematics (invariant masses and rapidities of the jet pairs).

The fraction \( \beta \) that measures the difference of the longitudinal momenta of the two partons coming from the hadron, is arbitrary. The fixed values of the parton momentum fractions \( x_3 - \beta \) and \( x_4 + \beta \) correspond to the plane wave description of the scattering process in which the longitudinal distance between the two scatterings is arbitrary. This description does not correspond to the physical picture of the process we are discussing, where two partons originate from the same bound state. In order to ensure that partons 3 and 4 originate from the same hadron of a finite size, we have to introduce integration over \( \beta \) in the amplitude, in the region \( \beta = \mathcal{O}(1) \), as was explained in detail in [18].

The Feynman amplitude contains the product of two virtual propagators. The virtualities \( k_1^2 \) and \( k_2^2 \) in the denominators of the propagators can be written in terms of the Sudakov variables as

\[
\begin{align*}
    k_1^2 &= x_1 \beta s - k_{\perp}^2, \quad k_2^2 = -x_2 \beta s - k_{\perp}^2,
\end{align*}
\]
where \( s = 2(p_\alpha p_\beta) \) and \( k_\perp^2 \equiv (\vec{k}_\perp)^2 > 0 \) the square of the two-dimensional transverse momentum vector.

The singular contribution we are looking for originates from the region \( \beta \ll 1 \). Hence the precise form of the longitudinal smearing does not play role and the integral over \( \beta \) yields the amplitude \( A \)

\[
A \sim \int \frac{d\beta}{(x_1\beta - k_1^2 - m_\ell^2 + i\epsilon)(-x_2\beta s - k_2^2 - m_\ell^2 + i\epsilon)} = \frac{2\pi iN}{(x_1 + x_2) k_1^2 + m_\ell^2}.
\]

The numerator of the full amplitude is proportional to the first power of the transverse momentum \( k_\perp \). As a result, the squared amplitude (and thus the differential cross section) acquires the necessary factor \( 1/\delta^2 \) that enhances the back-to-back jet production.

The integration over \( k_t \) gives a single log contribution to the cross section \( \alpha_{em} \log(Q^2/m_\ell^2) \) where \( Q \) is the characteristic transverse scale of the hard processes. Note, that strictly speaking the answer is proportional to \( \delta(\vec{k}_{1t} + \vec{k}_{2t})/(k_{1t}^2 + m_\ell^2) \). The parton model answer is only single collinearly enhanced, while we are looking for the double collinear enhanced contributions [18]. It is well known that these contributions originate from the gluon dressing of the parton model vertex, with the \( \delta \) function becoming a new pole.

**B. Accounting for the gluon radiation.**

A typical lowest order QCD diagram which accounts for the gluon emission is presented in Fig. 4b. The compensating gluon relaxes the transverse momentum \( \delta \)– function. Note however that the gluon can not be emitted from a photon, while in the \( pp \) case such emissions contribute, since the splitting parton carries color. This eliminates the so called short split contribution, which is present in the case of hadron-hadron scattering. The rest of the calculation is completely analogous to the "long split" calculation in the \( pp \) case.

Thus using Eqs. 25,26 in [18] we can write right away the differential cross section as

\[
\pi^2 \frac{d\sigma^{(3\rightarrow4)}_{1}}{d^2\delta'_{13} d^2\delta'_{24}} = \frac{d\sigma_{\text{part}}}{d\delta'_{13} d\delta'_{24}} \cdot \frac{\partial}{\partial\delta'_{13}} \cdot \frac{\partial}{\partial\delta'_{24}} \left\{ [1]D_{a_{1}}^{1,2}(x_{1},x_{2};\delta'_{13},\delta'_{24}) \cdot [2]D_{b_{1}}^{3,4}(x_{3},x_{4};\delta'_{13},\delta'_{24}) \right\} \cdot S_{1}(Q^{2},\delta'_{13}) \cdot S_{2}(Q^{2},\delta'_{24}) \cdot S_{3}(Q^{2},\delta'_{13}) \cdot S_{4}(Q^{2},\delta'_{24}),
\]

where \( S_i \) are the quark \((S_1, S_2)\) and gluon \((S_3, S_4)\) Sudakov form factors [40, 41]:

\[
S_{q}(Q^{2},\kappa^{2}) = \exp \left\{ -\int_{\kappa^{2}}^{Q^{2}} \frac{dk_{t}^{2}}{k_{t}^{2}} \frac{\alpha_{s}(k_{t}^{2})}{2\pi} \int_{0}^{1-k/Q} dz P_{q}(z) \right\},
\]
\[ S_g(Q^2, \kappa^2) = \exp \left\{ - \int_{Q^2}^{\infty} \frac{dk^2}{k^2} \frac{A_s(k^2)}{2\pi} \int_0^{1-k/Q} dz \left[ zP^q_g(z) + n_fP^q_0(z) \right] \right\}. \] (5)

Here \( P^k_i(z) \) are the non-regularized one-loop DGLAP splitting functions (without the “+” prescription):

\[
P^q_g(z) = C_F \frac{1+z^2}{1-z}, \quad \quad P^q_0(z) = P^q_0(1-z),
\]

\[
P^q_g(z) = T_R \left[ z^2 + (1-z)^2 \right], \quad \quad P^q_0(z) = C_A \frac{1+z^4+(1-z)^4}{z(1-z)}.
\] (6)

The upper limit of the integration over \( z \) properly regularizes the soft gluon singularity, \( z \to 1 \) (in physical terms, it can be viewed as a condition that the energy of the gluon should be larger than its transverse momentum, [II]). The function \( 1D \) now corresponds to the photon split into the charm anticharm pair. Moreover, since we are looking for the production of the \( c\bar{c} \) pair in the photon fragmentation region, we can neglect all processes except a possible emission of the compensating gluon by the \( c\bar{c} \)- quark. Hence we obtain

\[
|1D(x_1, x_2; q_1^2, q_2^2; \Delta) = \int_{\Delta^2}^{\min(q_1^2, q_2^2)} \frac{dk^2}{k^2 + m_c^2} \times \int \frac{dz}{z(1-z)} R(z) G^q_0 \left( \frac{x_1}{z}; q_1^2, k^2 \right) G^q_0 \left( \frac{x_2}{1-z}; q_2^2, k^2 \right).
\] (7)

The function \( R(z) \) is the \( q\bar{q}\gamma \) vertex [12].

\[
R(z) = z^2 + (1-z)^2.
\] (8)

The \( \Delta \)-dependence of \( |1D \) is very mild as it emerges solely from the lower limit of the logarithmic transverse momentum integration \( Q_{\text{min}}^2 \). Here \( G^q_0 \) is a quark-quark evolution kernel. In the LLA for hard scale \( Q^2 \gg m_c^2 \) we can use the kernel for massless quarks.

Above we have calculated the differential MPI distributions. We now can integrate the cross section obtaining

\[
\frac{d\sigma(x_1, x_2, x_3, x_4)}{dl_1 dl_2} = \frac{d\sigma^{13}}{dl_1} \frac{d\sigma^{24}}{dl_2} \int \frac{d^2 \Delta}{(2\pi)^2} |1D_a(x_1, x_2; Q^2, Q_1^2) |2D_b(x_3, x_4; Q_1^2, Q_2^2; \Delta^2).
\] (9)

Note that we write here the dijet differential cross sections \( \frac{d\sigma}{dl_1} \) without including the corresponding PDF factors.

We see that the cross section is unambiguously determined by the integral of \( 2GPD \) over \( \Delta^2 \). The factor \( 1D \) is given by eq. [3] (with \( \Delta^2 = 0 \)) and does not pose any infrared problem, in difference from the \( pp \) case.
### III. PHYSICAL KINEMATICS.

There are three possible applications of our formalism—collisions at HERA and future ep/eA colliders and ultraperipheral AA and pA collisions at LHC.

#### A. $\Delta$ dependence of input double GPDs.

1. **The $\gamma p$ case.**

In order to estimate whether it is feasible to observe the MPI events discussed in the previous section, we have to calculate the double differential cross section and then to convolute it with the photon flux.

For the case of the proton target we have

$$
\frac{d\sigma}{dx_1 dx_2 dx_3 dx_4 dp_1^2 dp_2^2} = D(x_1, x_2, p_{1t}^2, p_{2t}^2) G(p_{1t}^2, x_3) G(p_{2t}^2, x_4) \frac{d\sigma}{dt_1 dt_2} \int \frac{d^2\Delta}{(2\pi)^2} U(\Delta). \tag{10}
$$

Here we carried the integration over the momenta $\Delta$ conjugated to the distance between partons, obtaining the last multipliers in the equations above. This integral measures the parton wave function at zero transverse separation between the partons and hence it is sensitive to short-range parton-parton correlations.

For $\gamma p$ case the factor $U(x_1, x_2, \Delta)$, in the approximation when two gluons are not correlated, is equal to a product of two gluon form factors of the proton:

$$
U(\Delta, x_3, x_4) = F_{2g}(\Delta, x_3) F_{2g}(\Delta, x_4). \tag{11}
$$

For the numerical estimates we use the following approximation for $2GPD$ of the nucleon:

$$
2D(x_3, x_4, p_{1t}^2, p_{2t}^2, \Delta) = G(x_3, p_{1t}) G(x_4, p_{2t}) F_{2g}(\Delta, x_3) F_{2g}(\Delta, x_4) \tag{12}
$$

where the two gluon form factor

$$
F_{2g}(\Delta) = \frac{1}{(1 + \Delta^2/m_g^2)^2} \tag{13}
$$

and the parameter

$$
m_g^2 = 8/\delta, \tag{14}
$$

where

$$
\delta = \max(0.28 fm^2, 0.31 fm^2 + 0.014 fm^2 \log(0.1/x)), \tag{15}
$$

are

$$
\int d^2\Delta U(\Delta). \tag{16}
$$

This integral measures the parton wave function at zero transverse separation between the partons and hence it is sensitive to short-range parton-parton correlations.
and was determined from the analysis of the exclusive $J/\Psi$ diffractive photoproduction\textsuperscript{16}. The functions $G$ are the gluon pdf of the proton, which we parameterize using \textsuperscript{43}. Then
\begin{equation}
\int \frac{d^2\Delta}{(2\pi)^2} U(\Delta) = \frac{1}{4\pi} \frac{m_2^4(x_3)m_2^4(x_4)}{(m_2^2(x_3) - m_2^2(x_4))^2} \left( \frac{1}{m_2^2(x_3)} - \frac{1}{m_2^2(x_4)} \right) + \frac{2}{m_2^2(x_3) - m_2^2(x_4)} \log \left( \frac{m_2^2(x_3)}{m_2^2(x_4)} \right). \tag{16}\end{equation}

In the limit $x_3 \sim x_4$ we recover
\begin{equation}
\int \frac{d^2\Delta}{(2\pi)^2} U(\Delta) = \frac{m_2^4}{12\pi}. \tag{17}\end{equation}

2. The $\gamma A$ case.

The general expressions for a nuclear target is
\begin{equation}
\frac{d\sigma}{dx_1x_2dx_3dx_4dp^2_1dp^2_2} = D(x_1, x_2, p^2_1, p^2_2)G(p^2_1, x_3)G(p^2_2, x_4) \frac{d\sigma}{dt_1 dt_2} \int d^2\Delta F'_A(\Delta, -\Delta) \tag{18}\end{equation}

where
\begin{equation}
F'_A(\Delta, -\Delta) = F_A(\Delta, -\Delta) + AU(\Delta). \tag{19}\end{equation}

Here $F_A(\Delta, -\Delta)$ is the nucleus body form factor, and the form factor $U$ was defined in Eq. \textsuperscript{11}. The first term in Eq. \textsuperscript{19} corresponds to the processes when two gluons originate from the different nucleons in the nucleus while the second term in Eq. \textsuperscript{19} corresponds to the case when they originate from the same nucleon. The first term is expected to dominate for heavy nuclei as it scales as $A^{4/3}$\textsuperscript{24, 25}.

For the nuclear target we have
\begin{equation}
F_A(\Delta, -\Delta) = F^2(\Delta), F(\Delta) = \int d^2b \exp(i\vec{A} \cdot \vec{b}) T(b), \tag{20}\end{equation}
and
\begin{equation}
T(b) = \int dz \rho_A(b, z) dz \tag{21}\end{equation}
is the nucleus profile function. The nuclear form factor integral is expressed through the profile function as
\begin{equation}
\int \frac{d^2\Delta}{(2\pi)^2} F(\Delta, -\Delta) = \int T^2(b) db = \pi \int T^2(b) db^2, \tag{22}\end{equation}
where $T(b)$ is calculated using the conventional mean field nuclear density \textsuperscript{44}
\begin{equation}
\rho_A(b, z) = \frac{C(A)}{A} \frac{1}{1 + \exp \left( \sqrt{b^2 + z^2} - 1.1 \cdot A^{1/3}/(0.56) \right)}. \tag{23}\end{equation}
The factor $C(A)$ is a normalization constant

$$
\int d^2 b d z \rho_A(b,z) = 1.
$$

(24)

Here the distance scales are given in fm.

There can be also the ladder splitting from the proton side. However such process corresponds to 2 to 4 process in notations of \[17\] and thus does not contribute to MPI in the LLA we consider here. Such processes constitute $\alpha_s$ corrections to conventional 2 to 4 four jet production, and it is expected they give a small contribution in the back to back kinematics. This is consistent with the results of modeling a tree level processes 2 to 4 in $p\bar{p}$ in Tevatron carried out by D0 and CMS and Atlas at LHC-see ref. \[31\]–\[37\]. Still this issue definitely deserves a further study. The relative rate of MPI and 2 to 4 processes plays an important role in accessing feasibility of observing MPI.

3. The ratio of MPI events to dijet rate for $pA$ and $AA$.

The $pA$ collisions are dominated by the $Ap$ process where a much larger flux factor is generated by projectile nuclei leading to dominance of the ultra peripheral collisions of photons with protons. Hence in such process one predominantly measures a double GPD of a proton. At the same time in the ultraperipheral $AA$ process the dominant contribution originates from the interaction of charmed pair with two gluons coming from different protons \[24\]–\[25\]. The ratio of cross section of such DPI process in $AA$ scattering to the cross section of DPI cross section in $pA$ scattering, in which both gluons belong to the same nucleon is (since the photon flux from nuclei is the same)

$$
\frac{Am_g^2}{12\pi}\int F_A(\Delta, -\Delta) \frac{d^2\Delta}{(2\pi)^2} \sim 2,
$$

(25)

where we take $A = 200$. Thus the ratio of the total number of the MPI events in $AA$ to the rate calculated in the impulse approximation is $\sim 3$. This is consistent with a numerical analysis that shows that for the same c.m. energies the ratio of number of MPI events to dijet rate in $AA$ collisions is 2.5–3 times larger than the same ratio for $pA + Ap$ process.

Note that this result is purely geometrical, we find a similar ratio \[25\] for $\gamma p$ and $\gamma A$ collisions at the LHeC.
B. Hard matrix elements.

The cross sections $d\sigma/dt$ are usual dijet cross sections calculated with $s \rightarrow s_{\gamma N} = 2k \sqrt{s}$, where $s$ is the invariant energy of the $ep(AA, pA)$. We have

$$d\sigma/dt = \frac{(4\pi \alpha_s(Q^2))^2 M^2}{(x_1 x_3 \sqrt{x_1 x_3}) 16 \pi s^{3/2} (x_1 x_3 s - 4p_t^2)},$$

(26)

where $Q^2$ is the dijet transverse scale. Here the matrix element $M$ of the $c$-quark - gluon scattering is given by

$$M^2 = (4\pi \alpha_s(Q^2))^2 (-\frac{4}{9}(\frac{\hat{u}}{\hat{s}} + \frac{\hat{t}}{\hat{s}}) + \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}),$$

(27)

where

$$\hat{s} = x_1 x_3 s, \hat{t} = -s(1 - z), \hat{u} = -\hat{s}(1 + z), z = \cos \theta = \tanh(y_1 - y_3)/2 = \sqrt{1 - 4p_t^2/(x_1 x_3 s)}.$$  

(28)

The angle $\theta$ is the scattering angle in the c.m. frame of the dijet. The region of integration is given by

$$x_1 x_3 s - 4p_t^2 > 0, x_1 > 0.2$$

(29)

The integration over the second dijet event goes in the same way, with $x_1 \rightarrow x_2, x_3 \rightarrow x_4$.

For two dijet event, in order to find the event rate, the cross section calculated above must be convoluted with photon flux determined using Weiczsacker - Williams approximation:

$$N_2 = \int dp_{1t}^2 dp_{2t}^2 du \frac{dN}{du} \int \frac{d\sigma}{dx_1 dx_2 dx_3 dx_4} dx_1 dx_2 dx_3 dx_4,$$

(30)

where the limits of integration are determined by $x_1 x_3 s - 4p_{1t}^2 > 0, x_2 x_4 s - 4p_{2t}^2 > 0, x_1, x_2 > 0.2$.

In the same way we calculate the rate for production of one pair of jets:

$$N_1 = \int dp_{1t}^2 du \frac{dN}{du} \int \frac{d\sigma}{dx_1 dx_3} dx_1 dx_3.$$  

(31)

The limits of integration are determined by $x_1 x_3 s - 4p_{1t}^2 > 0$.

C. LHeC/HERA kinematics - $ep$ collisions.

For $ep$ collisions we use the standard variable $y$

$$s_{\gamma p} / s = y$$

(32)
The photon flux is

\[ \frac{dN}{dy} = \frac{\alpha_{em}}{2\pi} \left( \frac{1 + (1 - y)^2}{y} \right) \log(\frac{Q_{\text{max}}^2}{Q_{\text{min}}^2}) - 2m_e^2y(1/Q_{\text{min}}^2 - 1/Q_{\text{max}}^2), \]

where \( Q_{\text{max}}^2 \sim 1 \text{ GeV}^2 \), and \( Q_{\text{min}}^2 = 4m_e^2y/(1 - y) \).

D. Ultraperipheral collisions at the LHC: AA and pA cases.

In ultraperipheral collisions two nuclei (proton and nucleus) scatter at large impact parameters with one of the colliding particles emitting a Weizsacker Williams photon which interacts with the second particle producing two jets in the \( \gamma + p_2 \to 4 \) jets+X reaction (for a detailed review see [45]). The corresponding total cross section is calculated by convoluting the elementary \( \gamma N \) cross section with the flux factor

\[ \frac{dN}{dk} = \frac{2Z^2\alpha_e}{\pi k} (wK_0(w)K_1(w) - w^2/2(K_1^2(w) - K_0^2(w))), \]

where \( w = 2kR_A/\gamma_L, \gamma_L = \sqrt{s_{NN}}/(2m_p), s_{\gamma N} = 2k\sqrt{s_{NN}} \). For the proton-nucleus reactions the flux is described by the same Eq. [34], the only difference is that in definition of \( w \) we substitute \( 2R_A \to R_A + r_p \) where \( r_p \) is the proton radius. In the second process the dominant contribution is the interaction of the a photon radiated by a heavy nucleus with the proton. The factor in the square brackets accounts for the full absorption at impact parameters \( b < 2R_A(b < r_p + R_A \) for \( pA \) scattering). The Bjorken fractions in the previous section where calculated relative to \( s_{\gamma p} \). In order to calculate the total inclusive cross section we must integrate over \( k \) from \( k_{\text{min}} \) corresponding to minimal \( k \) necessary to produce four jets in the discussed kinematics up to \( k_{\text{max}}, \)

\[ 2k_{\text{max}}\sqrt{s_{NN}} = 2Em_m, \text{ and } E_m = \gamma_L/R_A. \]

We must fix \( x_1 \). Then we have to calculate the cross section at \( x_1' = x_1/z \) where \( z = k/s_{NN} \). Thus to determine the total inclusive cross section for given \( x_1 \), we have to integrate over \( k \), substituting in the formulae of the previous section, \( x_1', x_2' = x_{1,2}\sqrt{s_{NN}}/k \) instead of \( x_1, x_2 \). The integration region is \( x_1\sqrt{s_{NN}} < k < k_{\text{max}}. \)

IV. NUMERICS.

Since there is no corresponding 4 to 4 process, it makes no sense to define \( \sigma_{\text{eff}} \) for these collisions as it is usually done in the studies of MPI in \( pp \) scattering. Instead we will calculate the number of MPI events as a function of jet cutoff - starting from 5 GeV, as well as the ratio of MPI events to a total number of dijet events with the same cutoffs for \( ep, Ap \) and \( AA \) collisions.
In all cases we observe a rather rapid decrease of MPI rate as a function of $p_t$. In order to calculate the rates we use the GRV structure functions for proton [43] and the GRV structure function for photon [46–48].

**A. Direct photon MPI at HERA and LHeC.**

![Graph](graph.png)

**FIG. 5:** Event rate for $ep$ MPI collisions as a function of $p_t$ cut at LHeC.

![Graph](graph2.png)

**FIG. 6:** The ratio of a number of MPI events to the of dijet rate, $\sqrt{s} = 1.3$ TeV.

To estimate the MPI event rate at LHeC at $\sqrt{s} = 1.3$ TeV we used luminosity $10^{34}$ cm$^{-2}$s$^{-1}$. The number of events and their ratio to the total number of dijet event are presented in Figs. 5,6. For cutoff $p_t > 5$ GeV we get $2 \cdot 10^8$ events for the running time of $10^6$ s. The ratio to a number of dijet events with the same cutoffs on $x$ and $p_t$ is 0.045%.

We also considered MPI event rate in the similar kinematics at HERA. To estimate the MPI event rate at HERA we use the total integrated luminosity accumulated at HERA of 1 fb$^{-1}$, at the $\sqrt{s}=300$ GeV. For cut off $p_t > 5$ GeV we get $1.2 \cdot 10^5$ events. The ratio to the number of dijet events with the same cutoffs on $x$ and $p_t$ is 0.0125%. However, it seems difficult to connect these
results with the available data on multiparton processes studied at HERA [49]. The reason is a rather low efficiency of selecting events with charm production. For example, the total number of events with charm identified in the ZEUS study was 11000 $D^*$ events corresponding to a handful of MPI events of the type we discuss [39].

B. $AA$ collisions.

![Graph](image1)

**FIG. 7:** The event rate for MPI in $AA$ collisions.

![Graph](image2)

**FIG. 8:** The ratio of MPI and the dijet event rate as a function of minimal $p_t$ for $AA$ collisions.

For $AA$ collisions we use: (i) luminosity $10^{27} cm^{-2}s^{-1}$, (ii) running time $10^6$ s, and $\sqrt{s} = 5.6$ TeV, $\gamma = E_p/m_p = 2.8 \cdot 10^3$ The radius of the lead nucleus is 6.5 fm. The exponentially decreasing Macdonald function cuts off the contribution of high photon energy. The total number of the events for the $p_t$ cut of 5 GeV is $5 \cdot 10^4$, while the ratio of MPI events to the total number of dijet events is relatively high-0.037% (cf. discussion in sec. III.3). The number of the MPI events decreases as a function of cutoff slightly faster than as $1/p_t^8$, while the ratio of MPI to total number of dijet
events scales approximately as $1/p_t^4$. The number of events and the ratio of a number of MPI and dijet events are presented in Figs. 7,8.

Recall that $x_\gamma = x_1 + x_2$ in Figs. 7,8 is 0.4. The dependence of $\log(N)$ on $x_\gamma$ is shown in Fig. 9. We see that for large $x_\gamma$ the number of events rapidly decreases, while for $x < 0.4 - 0.5$ the decrease becomes much less rapid. If we increase $x_\gamma$ to 0.8 the number of AA MPI events decreases by a factor of four only. So it may be possible to focus on the higher $x_\gamma$ regions where contribution of resolved photon is very small, while loosing relatively small fraction of events.

![Graph](attachment:image.png)

**FIG. 9:** The number of events as a function of cut off in $1/x_\gamma$.

Finally note that this ratio rapidly increases with energy. If we for example take AA energies equal to those of $pA$ scattering (i.e. a factor of 2.5 increase of $s$) the ratio increases by 30%.

C. $pA$ collisions.

![Graph](attachment:image.png)

**FIG. 10:** The event rate for MPI in $pA$ collisions
For \( pA \) collisions we use luminosity \( 10^{29} \text{cm}^{-2}\text{s}^{-1} \), running time \( 10^6 \text{s} \), and \( \sqrt{s} = 9 \text{TeV} \). The number of events for cut 5 GeV is of the order \( 6.6 \cdot 10^3 \), and ratio is of order 0.02\%, rapidly decreasing as in the \( AA \) case with increase of \( p_t \). The corresponding numbers are shown in Figs. 10,11 as a function of the jet cutoff.

From the comparison of Figs. 9 and 11 one can see that there is a factor of \( \sim 2 \) enhancement of the ratio DPI to dijet events in \( AA \) scattering, relative to \( pA \) case which is due to a combination of the geometrical enhancements we discussed above and suppression due to the smaller energy per nucleon in \( AA \) case.

\section*{V. CONCLUSIONS.}

We derived general equations for MPI processes with production of charm in direct photon hadron (nucleon,nuclei) collisions and used them to calculate the corresponding rates and compare them with dijet rates. We demonstrated that the discussed processes directly measure nucleon and nucleus \( GPDs \). We found a significant enhancement of the MPI / dijet cross section ration in the \( \gamma A \) scattering as compared to \( \gamma p \) scattering due to the scattering off two nucleons along the photon impact parameter.

The analysis was done for jet photoproduction in the realistic kinematics, of production of two pairs charm-gluon dijets with \( p_t > 5 \text{GeV} \), and cut of \( x_1, x_2 > 0.2 \), ensuring they are created mainly due to the direct photon mechanism. We considered these MPI processes for \( ep \) collisions at the LHeC and HERA and for \( AA \) and \( pA \) collisions at LHC and \( ep \) collisions at HERA. We conclude that the studies would definitely be feasible at the LHeC. In the case of the LHC the rates appear reasonable, and the key question is the efficiency of the LHC detectors. Further studies of the
feasibility of the measuring discussed processes at the LHC is highly desirable. Here we just notice
that since a larger fraction of charm in the discussed processes is produced at the central rapidities
we expect the efficiency of the detection of the discussed process would be pretty high for ATLAS
and CMS.

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