On Clustering Trend in Language Evolution Based on Dynamical Behaviors of Multi-Agent Model

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Abstract—Computational models are widely used to overcome the constraints of language evolution time and convert language theory into rules of computer models, which is very helpful to investigate the universal laws of language evolution. This paper designs a multi-agent model to simulate the evolution of human language. Language characteristics of every node should evolve under the influence of its own inherent trends and neighboring nodes in the lattice. By carrying out computational experiment and analysis, we found that the human languages in various settlements converge into several clusters with the gradual deepening of the evolution, which gave us a deeper understanding of the collective evolution in nature.

Keywords—Language Evolution, Multi-Agent Model, Clustering

I. INTRODUCTION

In the natural communication system, human language is unique: it is learned in society, and because of its recursive structure, it provides the potential for open communication [1]. The formation of the structure of this communication system originates from the evolution of biological capabilities, and is also affected by the evolution of language and culture itself [2]. Generally speaking, the research focus of language evolution is on when, where and how human language came into being, changed and died out [3], with the specific research contents containing semantics, grammar, pragmatics and other linguistic details [4-7].

Because of the shortage of language data in history and the exceedingly tardy pace of language evolution, computational model [8-13] is the most universally used means to study the evolution of language in history. One research paradigm is inspired by the perspective that language appears spontaneously in communication groups and has some adaptive characteristics [14]. The opinion has triggered a lot of research on multi-agent models that simulate this kind of language evolution, and attempts to ratiocinate the attributes of rising languages and their probable future development trends [15-16].

According to the mathematical realization method of the target language environment, the existing language models can be divided into rule-based and equation-based models [17-18]. Rule-based models formulate many nonobjective or specific regulations to express the constituent elements and relevant behaviors of language. The interaction between these regulations drives the evolution of language [19-20]. Equation-based models often integrate language and its relevant actions into mathematical equations [21]. Based on the mathematical analysis and experimental demonstration of those equations, the model based on equations can reasonably approximate the trend of language evolution or predict its future [22-23]. Many computational models of language research are designed under the guidance of the above two basic frameworks [24-27].

In this paper, we design a multi-agent model to imitate the development of language in the time dimension. Above all, the carrier of language should be a human settlement and it will act as an agent [28-29]. In order to build a framework for language exchange and mutation, we regard each human settlement as a node in the network topology. Inspired by the ecological competition model, the topology configuration between nodes is constructed. Each node in the network possesses a couple of important parameters: weight and status, which represent the degree of influence and development of language and culture respectively. Besides, the connecting edge of any two nodes has a weight value to describe the traffic flow intensity between any two settlements. Then, combined with the relevant principles of multi-disciplines [30], we proposed a language dynamics model to simulate the language and cultural development of settlements. That is to say, the state of each node in the dynamic model is modified continually.

After building the language evolution model, we study the influence of topological structure and other factors on language evolution through computer simulation experiments [31], and find that language evolution has some adaptive characteristics. Meanwhile, we analyze and judge the dynamics of language evolution through experimental verification and existing linguistic conclusion. In addition, it uncovers intrinsic common features among language, physics and biosystem, because the model and its internal mathematical principles can reflect a broader and universal
background outside the field of linguistics. In short, the current work may help enrich the methods and techniques for studying language evolution and broader complex system problems.

The rest of this article is organized as follows. In Sec. 2, the model is described in detail, and the conditions of relative stability are proved theoretically. Two computational experiments are carried out to explore the general laws of language evolution in Sec. 3. Finally, Sec. 4 draws conclusions and points out the future development direction.

II. MODEL CONSTRUCTION

The basic unit of language evolution is the settlement set in a two-dimensional plane. This conception is consistent with the framework adopted in [32]. In ancient society, limited to the single mode of transportation of the period, the communication for information between any two adjacent settlements had to rely on roads. In addition, the information exchange between non-contiguous settlements cannot be conducted directly, and multiple transit stations are needed. According to the above description, the network topology of the settlement is suitable to be designed as a planar graph [33]. We assume it is a regular lattice to make the topology as simple as possible, as shown in Fig. 1:

FIGURE 1. Network topology diagram of settlements.

Nodes and edges in network topology express settlements and roads between them respectively. As can be seen from Fig. 1, because all nodes are located in the intersection area of the vertically and horizontally distributed edges, the number of the node can be represented by a two-element vector \((i, j)\). The language evolution regulation of any node in the network can be integrated as the following equation:

\[
x_{ij}(k + 1) = x_{ij}(k) + d_{ij} + \sum_{(m,n) \in N_{ij}} w_{(i,j)(m,n)} \Delta_{(i,j)(m,n)}(k)
\]  

In above formula, \(k\) represents time, and \(x_{ij}(k) \in \mathbb{R}^p\) is \(p\)-dimensional state vector of the settlement \((i, j)\). Each term of the state vector corresponds quantitatively to the quantitative index of a certain feature of the language and the dynamic high-dimensional state vector formed by these terms can be used to describe the instantaneous portrayal of language evolution. \(d_{ij} \in \mathbb{R}^p\) denotes the self-evolution offset, which is generated randomly initially. \(A\) represents a coupling matrix. It is a parameter describing the coupling relationship among state vector terms. The neighborhood of settlement \((i, j)\) is depicted by \(N_{ij}\), which comprises each edge directly connected to the node \((i, j)\). \(a_{ij} \in \mathbb{R}^+\) denotes the degree of influence of the node \((i, j)\). \(\Delta_{(i,j)(m,n)}(k) = x_{mn}(k) - x_{ij}(k)\). \(w_{(i,j)(m,n)} \in \mathbb{R}^+\) describes the edge weight of the network, and it is about the strength of connectivity between nodes \((i, j)\) and \((m, n)\). It is important to note that the edge weight depends on the physical accessibility of the road between the two settlements. Hence, the network topology belongs to undirected graph, namely:

\[
w_{(i,j)(m,n)} = w_{(m,n)(i,j)}
\]  

Now we need to consider a special situation. If language discrepancy of any two adjacent settlements exceeds the preset threshold, even if there is a physical channel between them, they will not be able to communicate normally. In the model, if

\[
\|\Delta_{(i,j)(m,n)}(k)\|_\infty > \psi
\]  

then the value of \(\Delta_{(i,j)(m,n)}(k)\) is compulsorily modified to 0, where \(\psi\) represents the threshold.

The dynamics of each node contain two important elements: self-evolution and interaction between nodes. The driving force of interaction is positively related to the weight of nodes and the strength of connections between nodes with the driving force of self-evolution being controlled by state vector and self-evolution offset. When the language evolution model starts to run, the language of each node in the network will evolve in a certain direction in the light of the above framework. So far, the language evolution model has been designed.

A fundamental precondition for the language evolution model to operate correctly is the swarm stability [34-35], that is, to ensure that the distance between settlement states is not too far. Without losing generality, take two specific nodes as an instance, and their connection relationship is shown in the following figure:

FIGURE 2. Graph topology of a double-settlement system.

For Fig. 2, the dynamic equation of the left node is:

\[
x_{11}(k + 1) = x_{11}(k) + d_{11} + w_{(1,1)(1,2)} a_{12} \Delta_{(1,1)(1,2)}(k)
\]  

(4)

Identically, the dynamic equation of the right node is:

\[
x_{12}(k + 1) = x_{12}(k) + d_{12} + w_{(1,1)(1,2)} a_{11} \Delta_{(1,1)(1,2)}(k)
\]  

(5)

Let \(\Delta_{(1,1)(1,2)}(k) = (x_{12}(k) - x_{11}(k))\), then the dynamic equation of the relative difference \(\Delta_{(1,1)(1,2)}(k)\) can be obtained by subtracting (4) from (5) as:

\[
\Delta_{(1,1)(1,2)}(k + 1) = \Delta_{(1,1)(1,2)}(k) + \Delta d + w_{(1,1)(1,2)} A(-a_{11} - a_{12}) \Delta_{(1,1)(1,2)}(k)
\]
\[
\begin{align*}
&= \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right] \Delta_{(1,1)(1,2)}(k) \\
&\quad + \frac{\Delta d}{\text{Exponential growth}} \\
&\quad + \frac{\Delta d}{\text{Linear cumulation}} \quad (6)
\end{align*}
\]

The linear cumulative component in the above differential equation can be considered as perturbation that does not have any impact on stability, while the exponential growth component determines the stability of the system. The following theorem proves this in theory.

**THEOREM 1**

For the dynamical system (6), if \( \lambda_1, \lambda_2, \ldots, \lambda_n \) are the eigenvalues of

\[
I - w_{(1,1)(1,2)}(a_{11} + a_{12})A
\]

Then a necessary and sufficient condition for the asymptotic stability is

\[
\rho(I - w_{(1,1)(1,2)}(a_{11} + a_{12})A) = \max_{1 \leq i \leq n} |\lambda_i| < 1 \quad (8)
\]

where \( \rho(\cdot) \) denotes the spectral radius. When \( A = I \), the condition can be reduced to:

\[
w_{(1,1)(1,2)}(a_{11} + a_{12}) < 1 \quad (9)
\]

**PROOF:**

Sufficiency. Let \( \rho(I - w_{(1,1)(1,2)}(a_{11} + a_{12})A) < 1 \), then we have

\[
|1 - \lambda(w_{(1,1)(1,2)}(a_{11} + a_{12})A)| < 1 \quad (10)
\]

where \( \lambda(\cdot) \) represents any eigenvalue of the matrix. The inequality means \( \lambda(w_{(1,1)(1,2)}(a_{11} + a_{12})A) \neq 0 \), and shows that matrix \( w_{(1,1)(1,2)}(a_{11} + a_{12})A \) is nonsingular. Therefore, a conclusion can be obtained that the equation

\[
w_{(1,1)(1,2)}(a_{11} + a_{12})A \Delta_{(1,1)(1,2)} = \Delta d
\]

has a unique solution. The solution can be marked as

\[
\Delta_{(1,1)(1,2)} = \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right] \Delta_{(1,1)(1,2)} \quad (11)
\]

Then the error vector is

\[
\epsilon(k) = \Delta_{(1,1)(1,2)}(k) - \Delta_{(1,1)(1,2)}\quad (12)
\]

where \( \epsilon(0) = \Delta_{(1,1)(1,2)}(0) - \Delta_{(1,1)(1,2)} \).

Without the loss of generality, we can suppose that \( I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \) is diagonalizable [36], then we can set

\[
\begin{align*}
&= \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right] \Delta_{(1,1)(1,2)}(k) \\
&= \left[ \begin{array}{cccc}
\lambda_1 & 0 & \cdots & 0 \\
0 & \lambda_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \lambda_n
\end{array} \right] = \Lambda
\end{align*}
\]

From this, it naturally yields that

\[
\begin{align*}
&= \left[ \begin{array}{cccc}
\lambda_1^k & 0 & \cdots & 0 \\
0 & \lambda_2^k & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \lambda_n^k
\end{array} \right] = \Lambda
\end{align*}
\]

The equation

\[
\lim_{k \to \infty} \lambda_1^k = \lim_{k \to \infty} \lambda_2^k = \cdots = \lim_{k \to \infty} \lambda_n^k = 0 \quad (16)
\]

is true because Eq. (8) holds. Therefore, we can derive that

\[
\lim_{k \to \infty} \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right]^k = 0 \quad (17)
\]

Then for any \( \Delta_{(1,1)(1,2)}(0) \), we have \( \lim \epsilon(k) = 0 \), i.e.

\[
\lim_{k \to \infty} \Delta_{(1,1)(1,2)}(k) = \Delta_{(1,1)(1,2)} \quad (18)
\]

**Necessity.** If \( \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right] x = \lambda' x \)

is satisfied for certain \( x \), where \( |\lambda'| \geq 1 \) and \( x \neq 0 \). From this, one can have the conclusion that

\[
\left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right] x = \lambda' x \quad (19)
\]

is true, \( \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right]^k \) does not converge to the zero vector when \( k \to \infty \). For the inequality

\[
\left\| \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right] x \right\| \leq \left\| \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right]^k \right\| \left\| x \right\|
\]

is true, \( \left[ I - w_{(1,1)(1,2)}(a_{11} + a_{12})A \right]^k \) does not converge to the matrix zero, and

\[
\lim_{k \to \infty} \Delta_{(1,1)(1,2)}(k) \neq \Delta_{(1,1)(1,2)} \quad (22)
\]

can be concluded from Eq. (13). \( \square \)

Since the isolated double-node system is a subsystem of the whole dynamic network, the stable performance of the double-node system is a prerequisite requirement for the stability of the whole system [37-38].

**III. EXPERIMENT AND ANALYSIS**

In the language evolution model, some system variables, including self-evolution offset, edge and node weight, may affect the language evolution results. By conducting computational experiments, we will reveal the causal
relationship between these factors and the results of language evolution, so as to enrich our understanding of the dynamics of language evolution.

A. SETTING OF PARAMETERS

Before the experiment is carried out, we need to set the numerical range of relevant parameters. The setting of network topology is a 20×20 grid, and the state vector of each node will change with time. The state vector and self-evolution offset are both two-dimensional vectors. The initial value of each entry of the state vector is randomly generated within (0, 2), with that of the offset vector within (-0.02, 0.08). The weight value of the network edge is set to a random value within (0, 0.33). The node weight is divided into two levels with the weight of the high-level node set to 3 and that of the low-level node set to 1. The high/low weights denote the rank of linguistic and cultural influence of settlements. We initially set the four nodes numbered (4, 4), (4, 16), (16, 4), and (16, 16) as the high-level weighted nodes. The value of ϕ is set to 4, and the number of iterations of language evolution is set to 10000. For the convenience of description, all settings described earlier are jointly called as standard setting.

At the starting point, the distribution of language evolution states of settlements in the state space is depicted in Fig. 3. It can be seen from the figure that the distribution of language state vectors of 400 settlements is roughly uniform.

![Figure 3](image)

**FIGURE 3.** Distribution of state points in the state space at founding moment.

Each state point in the above figure has its own color, which is set according to the index of the corresponding node in the grid. For node (i, j),

\[
\begin{align*}
R(i, j) &= i/20 \\
G(i, j) &= j/20 \\
B(i, j) &= (i + j)/40
\end{align*}
\]

Defining colors in this way means that any two adjacent nodes in the grid should also have similar colors. In addition, the short distance between state points in state space means that language difference is small.

B. INFLUENCE OF SELF-EVOLUTION OFFSET

In the current subsection, we only study the impact of the magnitude of the offset on the result of language evolution, and other parameters maintain the standard initial settings. In this experiment, each entry of the offset vector is randomly generated in (-0.02, 0.08), and the magnitude of offset is revised by increasing and reducing the interval range.

![Figure 4](image)

**FIGURE 4.** The evolution results obtained from the experiment are derived from three different parameter settings. (a) is the consequence of language evolution under standard setting and is regarded as a reference experiment. (b) is the homologous experimental consequence after the offset amplitude of the control group is halved and (c) is that with the amplitude being decreased to one fourth of the standard setting.

Subsequently, we further study the space covered by the result image and the transfer of the image center position, as demonstrated in Tab. 1:

|                | (a)   | (b)   | (c)   |
|----------------|-------|-------|-------|
| Amount of space taken | 4×5   | 2×2.5 | 1×1.2 |
| Center coordinate of the image | (301,291) | (151,146) | (76,73.5) |

Note: All values in the table are estimated according to Fig. 4.

It can be seen from Figure 4 that the spatial distribution of system state points is basically the same after changing the size of the offset. To explore its internal correlation, we further analyze the data in Tab. 1. It is accessible to discover that the space covered by state points is roughly reduced to 1/k² (k > 1) of that in the control group, and the value of the center coordinate of the image also decreases to 1/k of what it was when the magnitude of the offset shrinks to 1/k of the original value.

C. INFLUENCE OF DISTRIBUTIONS OF NODES

There are two levels of node weights in our model, high and low. High-weight nodes are equivalent to relatively developed cities or settlements in the real world. According to the power-law distribution law [39], such nodes are only a few in the group. This subsection mainly explores the impact of specific high-weight node distribution structure on language evolution results. In relevant experiments, we mainly explore two special distributions of high-weight nodes, which are O-type and X-type respectively.

![Figure 5](image)

**FIGURE 5.** Distributions diagram of nodes with high weight. The red square indicates that the node corresponding to the position has high weight, and the blue square indicates that the node corresponding to the position has low weight. The distribution of high-weight nodes in (a) is O-type. This

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distribution structure of high-weight nodes in the picture is similar to an ‘O’, which is a circinate distribution. (b) is an X-type distribution. This distribution structure looks like an ‘X’, where the arrangement of core nodes radiates from the central position to the border.

When the models corresponding to these two distributions undergo 10000 evolutionary iterations, their evolution results are presented in Fig. 6:

![FIGURE 6. The language evolution consequences observed in the state space. (a) and (b) are the experimental consequences of using the o-type and x-type distribution for the high-weight node distribution in the model.](image)

At initial moment, the state points are regularly spread in a small area of the state space. However, state points have shaped a special mode which is very different from the initial time after 10000 iterations. It can be observed from Fig. 6 that two corresponding clusters may have obvious discrepancies in appearance [40], such as two clusters composed of green dots. The basic cluster configuration has not changed yet, as definitely exhibited by the four clusters along with their intertransits simultaneously appearing at the corresponding positions in two pictures.

IV. Conclusion

In order to study the general law of language evolution, this paper designs a multi-agent model, whose network topology is lattice. The motion of each node is driven by two factors, namely, the fixed evolution direction itself and the influence of adjacent nodes in the grid. The calculation experiments mentioned above uncover a crucial rule of the model: after a long time of language evolution, the distribution of state points of settlement points in the state space is no longer as uniform and random as that in Fig. 3, but presents a clustering form according to its color. The state points with similar colors are mostly adjacent, while the state points with different colors are mostly nonadjacent. In the state space, the color of any one state point is calculated according to the number of its corresponding node, so the color similarity indirectly reflects the geographical proximity. This shows that under the influence of evolution, it is common to have a small but large-scale dialect area in a language family. This kind of rule does not only exist in the language evolution system. It really mirrors the pervasiveness of the self-organization appearance of dynamic complex network system from disorder to order.

In this model, the state of language changes without interference in the continuous state space. The system must be stable in the real world, and the orbits cannot deviate too far. In order to avoid the possibility of instability of the model, we mathematically demonstrate the condition for the relative stability of the state trajectories of adjacent nodes, and ensure that there is no limitless divergence in the evolution process as a whole. This condition has been applied in the parameter setting of the experiment.

Apart from main findings of clustering phenomenon, we also noticed that several principal model parameters have noteworthy impact on evolution results, such as the offset of self-evolution and the distribution of high weight nodes in the network species. After simple data processing, we have found out the laws of some parameters affecting the evolution results.

In addition to the findings mentioned above, we found that there are some deeper contents worthy of study in the research process. For example, there is a causal relationship between the model elements, which is beneficial to grasping the evolution direction. In addition, the hierarchical clustering phenomenon and mechanism can be compared with other similar systems such as cultural gene evolution, and further research can be carried out on this basis.

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