END POINT OF THE ELECTROWEAK PHASE TRANSITION

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We study the hot electroweak phase transition (EWPT) by 4-dimensional lattice simulations on lattices with symmetric and asymmetric lattice spacings and give the phase diagram. A continuum extrapolation is done. We find first order phase transition for Higgs-boson masses $m_H < 66.5 \pm 1.4$ GeV. Above this end point a rapid cross-over occurs. Our result agrees with that of the dimensional reduction approach. It also indicates that the fermionic sector of the Standard Model (SM) may be included perturbatively. We get for the SM end point $72.4 \pm 1.7$ GeV. Thus, the LEP Higgs-boson mass lower bound excludes any EWPT in the SM.

1 Introduction

The observed baryon asymmetry is finally determined at the EWPT. To understand this asymmetry a quantitative description of EWPT is needed. Since the perturbative approach breaks down for large Higgs-boson masses (e.g. $m_H > 70$ GeV) lattice MC simulations are necessary. (Another, partly analytic approach has been presented in.)

Previous works show that the strength of the EWPT gets weaker as the Higgs-boson mass increases. The line of the first order phase transitions, separating the symmetric and broken phases on the $m_H - T_c$ plane, has an end point, $m_{H,c}$. 3D results show that for $m_H > 95$ GeV no EWPT exists, moreover $m_{H,c} \approx 67$ GeV. In 4D $m_{H,c} \approx 80$ GeV was obtained. Also in 4D at $m_H \approx 80$ GeV the EWPT turned out to be extremely weak, even consistent with the no phase transition scenario on the 1.5-σ level. However, a discrepancy appeared: the 4D estimate for the end point was higher than

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Figure 1: Imaginary part of first Lee-Yang zero as a function of $\lambda$ from simulations on symmetric lattices with $L_t = 2T$. Filled symbols are obtained without $\lambda$-reweighting, while open symbols with $\lambda$-reweighting from the filled symbol with same shape.

the 3D estimates. This discrepancy has been resolved in [10,11], to be reviewed here.

2 End point analysis

We study the 4D SU(2)-Higgs model on both symmetric and asymmetric lattices, i.e. lattices with equal or different spacings in temporal ($a_t$) and spatial ($a_s$) directions. In the asymmetric case equal lattice spacings are used in the 3 spatial directions ($a_i = a_s$, $i = 1, 2, 3$). The asymmetry of the lattice spacings is given by the asymmetry factor $\xi = a_s/a_t$. The different lattice spacings can be ensured by different coupling strengths in the action for time-like and space-like directions. The action reads in standard notation

$$S[U, \varphi] = \beta_s \sum_{sp} \left( 1 - \frac{1}{2} \text{Tr} U_{pl} \right) + \beta_t \sum_{tp} \left( 1 - \frac{1}{2} \text{Tr} U_{pl} \right) + \sum_{x} \left\{ \frac{1}{2} \text{Tr} (\varphi_x^+ \varphi_x) \right\}.$$ (1)

We introduce $\kappa^2 = \kappa_s \kappa_t$ and $\beta^2 = \beta_s \beta_t$. The anisotropies $\gamma^2 = \beta_t/\beta_s$ and $\gamma^2 = \kappa_t/\kappa_s$ are functions of $\xi$. These functions have been determined perturbatively and non-perturbatively demanding the restoration of the rotational symmetry in different channels. We use $\xi = 4.052$, which corresponds to $\gamma = 4$ and $\gamma = 3.919$. Details of the simulation techniques can be found in [12].
The determination of the end point of the finite temperature EWPT is done by the use of the Lee-Yang zeros of the partition function $Z$. Near the first order phase transition point the partition function reads $Z = Z_s + Z_b \propto \exp(-V f_s) + \exp(-V f_b)$, where the indices s(b) refer to the symmetric (broken) phase and $f$ stands for the free-energy densities. We also have $f_b = f_s + \alpha (\kappa - \kappa_c)$, since the free-energy density is continuous. It follows that $Z \propto \exp[-V(f_s + f_b)/2] \cosh[-V \alpha (\kappa - \kappa_c)]$ which shows that for complex $\kappa$ $Z$ vanishes at $\text{Im}(\kappa) = 2\pi \cdot (n - 1/2)/(V \alpha)$ for integer $n$. In case a first order phase transition is present, these Lee-Yang zeros move to the real axis as the volume goes to infinity. In case a phase transition is absent the Lee-Yang zeros stay away from the real $\kappa$ axis. Thus the way the Lee-Yang zeros move in this limit is a good indicator for the presence or absence of a first order phase transition. Denoting $\kappa_0$ the lowest zero of $Z$, i.e. the position of the zero closest to the real axis, one expects in the vicinity of the end point the scaling law $\text{Im}(\kappa_0) = C(L_t, \lambda) V^{-\nu} + \kappa_0^c(L_t, \lambda)$. In order to pin down the end point we are looking for a $\lambda$ value for which $\kappa_0^c$ vanishes. In practice we analytically continue $Z$ to complex values of $\kappa$ by reweighting the available data. Also small changes in $\lambda$ have been taken into account by reweighting. The dependence of $\kappa_0^c$ on $\lambda$ from our symmetric simulations is shown in Fig. 1. To determine the critical value of $\lambda$ i.e. the largest value, where $\kappa_0^c = 0$, we have performed fits linear in $\lambda$ to the non-negative $\kappa_0^c$ values.

In the isotropic lattice simulation, we have used $L_t = 2$. The Lee-
Yang analysis gave $\lambda_c = 0.00116(16)$ for the end point. Performing $T = 0$ simulations with the same parameters this can be converted to a Higgs boson mass. The value $m_{H,c} = 73.3 \pm 6.4 \text{GeV}$ has been obtained, which is compatible with our estimate based on a study of Binder cumulants\cite{lattice4D} and the previous 4D studies\cite{4Dstudies}.

In the anisotropic lattice simulation case\cite{anisotropic} we also performed a continuum extrapolation, moving along the lines of constant physics (LCP). A schematic illustration is shown in Fig. 2.

The technical implementation of the LCP idea has been done as follows. By fixing $\beta = 8.0$ in the simulations, we have observed that $g_R$ is essentially constant within our errors. For the small differences in $g_R$ we have performed perturbative corrections. We have carried out $T \neq 0$ simulations on $L_t = 2, 3, 4, 5$ lattices (for the finite temperature case one uses $L_t \ll L_x, L_y, L_z$), and tuned $\kappa$ to the transition point. This condition fixes the lattice spacings: $a_t = a_s/\xi = 1/(T_c L_t)$ in terms of the transition temperature $T_c$ in physical units.

The third parameter $\lambda$, finally specifying the physical Higgs mass in lattice units, has been chosen (using the Lee-Yang analysis) so that the transition corresponds to the end point of the first order phase transition subspace.

Having determined the end point $\lambda_c(L_t)$ for each $L_t$, we calculated the $T = 0$ quantities ($R_{HW,c}, g_R^2$) on $V = (32L_t) \cdot (8L_t) \cdot (6L_t)^3$ lattices. Having established the correspondence between $\lambda_c(L_t)$ and $R_{HW,c}$, the $L_t$ dependence of the critical $R_{HW,c}$ is easily obtained. Fig. 3 shows the dependence of the
end point $R_{HW}$ values on $1/L_t^2$. A linear extrapolation in $1/L_t^2$ yields the continuum limit value $R_{HW,c} = 0.83 \pm 0.02$, which corresponds to $m_{H,c} = 66.5 \pm 1.4$ GeV. Note that $m_{H,c}$ decreases for increasing $L_t$. This observation resolves the discrepancy of 3D and previous $L_t = 2$ 4D results.

Comparing our result to those of the 3D analyses one observes complete agreement. Since the error bars on the end point determinations are on the few percent level, the uncertainty of the dimensional reduction procedure is also in this range. This indicates that the analogous perturbative inclusion of the fermionic sector results also in few percent error on $m_{H,c}$.

Based on our published data $^{12,13}$ and the results of $^{11}$ we draw the precise phase diagram in the $(T_c/m_H - R_{HW})$ plane of the SU(2)-Higgs model. This is shown in Fig. 4. The continuous line – representing the phase-boundary – is a quadratic fit to the data points.

Finally, we determined the end point value in the full SM. We use perturbation theory $^{16}$ to transform the SU(2)-Higgs model end point value to the full SM. We obtain $72.4 \pm 1.7$ GeV. The dominant error comes from the uncertainty on the position of the end point.

3 Conclusions

In conclusion, we have determined the end point of hot EWPT with the technique of Lee-Yang zeros from simulations in 4D SU(2)-Higgs model. The phase
diagram has been also presented. The transition is first order for Higgs masses less than $66.5 \pm 1.4$ GeV, while for larger Higgs masses only a rapid cross-over is expected. Our results show that integrating out the heavy modes perturbatively is sufficiently precise. Thus the above value can be perturbatively transformed to the full SM, yielding $72.4 \pm 1.7$ GeV for the end point Higgs mass.

The experimental lower limit of the SM Higgs-boson mass is $89.8$ GeV\(^7\). Taking into account all errors, our end point value excludes the possibility of any EWPT in the SM. Thus our work emphasizes the need of EWPT analyses based on extensions of the SM\(^8\).

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