NON-LINEAR SLIDING MODE CONTROL OF WHEELED MOBILE ROBOT WITH THE PRESENCE OF DYNAMIC UNCERTAINTY AND TIME-VARYING DISTURBANCE

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Abstract

This paper suggests a scheme for trajectory tracking on a two wheeled mobile robot using integral sliding mode control method in the presence of external disturbances and inertia uncertainties. In this study the modified adaptive sliding mode controller for nonholonomic wheeled mobile robot is developed. Nonlinear control used to combine the kinematic and dynamic controller to follow the desired path. Firstly, the desired path is created. Secondly, the kinematic tracking controller used linear and angular velocities form reference model and feeds posture and velocities errors as input term in the sliding controller. Finally, the dynamic control was used to follow the path. Proposed control system is verified and validated using MATLAB/SIMULINK to track the required WMR trajectory and it is shown that the suggested system has better transient efficiency on different trajectories with acceptable steady state error.

Keywords: Wheeled mobile robot, dynamic uncertainty, Kinematic and dynamic controller, Dynamic control, Transient efficiency.

I. Introduction

In general, Physical systems are nonlinear systems. Therefore, most of the control systems are nonlinear. A wheeled mobile robot (WMR) system is a nonlinear as a traditional non-holonomic system, and is full of challenges to the motion control of WMR systems (Umar et al., 2014).

Wheeled mobile robot (WMR) usually used in hazard, tired and large places that the men cannot reach. The studies of the WMR are increased in the last decade because the WMR implementation increased in wide range of industrial, military and medical applications.

The WMR studies include many fields as path planning (Raja & Pugazhenthhi, 2012), path tracking (Antonelli et al., 2007) etc. the path tracking is mean that how the WMR followed the desired path. For tracking performance of the desired path, it necessary to use the non-linear controller as a consequence of the WMR inherent...
nonlinearity. The inherent nonlinearity is caused by the non-holonomic constraints of the WMR.

There are two type of controller kinematic and dynamic controller. In the last decade used combined kinematic/dynamic controller for WMR.

The earliest work on trajectory tracking control was based on kinematic models (Samson & Ait-Abderrahim, 1991) as kinematic models are strictly related to non-holonomic constraints. Today the most studies of the WMR tracking controller are focused on the non-linear dynamic controller.

There are many types of non-linear dynamic controller as linearization-technique (Liu et al., 2008), back stepping (Saud & Hasan, 2018) and (Esmaeili et al., 2017) fuzzy (Hamoudi, 2016), (Rao et al., 2017) and (Hadi, 2005) neural (Fierro & Lewis, 1998) and (Ahmed S. Al-Araji & Ibraheem, 2019), adaptive (Fukao et al., 2000) and (Binh et al., 2019), sliding mode (Yang & Kim, 1999), (Ding et al., 2018) and (Wu et al., 2019) and etc. The nonlinear dynamic controller is used to accommodate the disturbance and uncounted dynamic in modeling system.

In linearization (Yang & Kim, 1999) technique used to linearize the input, output systems. This technique fares away from real model of the WMR because it is an approximation of the dynamic model.

Back stepping (Ahmed Sabah Al-Araji, 2014) controller used widely in WMR researches because this approach is simply and give robust control law.

They used the particle swarming to find the optimal back stepping technique. The optimum parameters of their controller founded via Lyapunov stability to grantee the stability of the WMR. The simulation and experimental results had shown the robustness of their work against external disturbance.

The adaptive controller used when the dynamic parameter is unknown or changes during the operation of the WMR. (Martins et al., 2008) presented a dynamic free controller. They at first found the references velocities and the path of the WMR depended on the kinematic controller only. After that, update online dynamic parameter to adapt the desired torque for tracking the path. The Lyapunov stability had been proved their work stability. Their work efficient with changing load at operation of the WMR.

Sliding mode is one of the most controllers used in recent years. Because of their simple structure and robustness against input uncertainties, disturbances and dynamic variation parameters (Young et al., 1999). (Yang & Kim, 1999) used kinematic and dynamic models in their SMC and proved stability of their system with Lyapunov stability theorem. (Chwa, 2004) proposed a SMC in polar coordinate. In addition, they transfer the mechanical system (dynamic\kinematic) of the WMR to the polar coordinate. The polar coordinate implemented for decupling the posture variable and makes the system to be stable on the sliding surface. Their system is discontinuous in controller so it fails in realistic case. Also, the simulation results show large tracking errors.

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Integral SMC proposed in (Bessas et al., 2016) for tacking controller in WMR. the integral SMC minimize the unmatched and release matched disturbance to come out the reaching phase problem in SMC.

SMC is a variable controller structure manner that slides along boundaries of various controller structures. In theoretical, SMC is globally stable in the slide surface but in practical, the system generates a high non-deterministic frequency along sliding surface called chattering. This phenomenon comes from signum part of the sliding math. So, the chattering is a drawback of the SMC that cause damage in the system. (Mehrjerdi & Saad, 2011) use exponential term to reduce the chattering effect in the SMC. they proved the effectiveness of their work in reducing chattering phenomena.(Xu, 2008) propose a chatter-free controller, protection against saturation at large initial gain and has smooth transient performance

In addition, SMC combined with intelligent controller to enhance the system controller performance. (Das & Kar, 2006)tuned on line the system parameter by implementation adaptive fuzzy-logic controller. The fuzzy logic implemented for uncertainty estimation of the WMR, containing nonlinearity and variation of parameters.

The gain of the PID SMC is tune by using neural network and adaptive algorithm (Li et al., 2010).

In this paper, a sliding mode controller is suggested for wheeled mobile robot. Asymptotic analysis of the trajectories is considered dependent on the system's kinematic model. Design sliding mode dynamic control for Wheeled mobile robot trajectory tracking is overcome of the variation effect of the dynamic parameters, unvalued disturbance and uncertainties but the chattering phenomena is appeared.

The remainder of this paper is structured as follows. In section 2 the wheeled mobile robot mathematical model. Kinematic controller design in section 3. In section 4 the sliding mode control design is presented and analyzed. Result simulation in section 5. The final section 6 concludes the paper.

II. Mathematical Model

The system being studied in this paper is a Differential Wheeled mobile robot non-slipping in lateral axis (non-holonomic) with a caster passive wheel to improve the stability of the WMR.

A schematic model of the WMR is shown in Figure 1. In the Figure 1 illustrates a generalized model of nonholonomic WMR, \(2r\) and \(2R\) represented the diameter od driving wheel and the distance between the two-driving wheel, Respectively. The robot configuration can be represented by three generalized coordinates that you can express

\[
q(t) = [x(t) \ y(t) \ \theta(t)]^T
\] (1)
Where \( x(t) \) \( y(t) \) denote to reference point \( c \) that the center of mass of the WMR, \( \theta(t) \) define the orientation of the WMR frame \([X,Y]\).

To achieve robot movement and orientation two separate actuators (DC motors) are used. The robot can be directed by driving one motor faster than the other.

![The WMR](image)

**Fig. 1:** The WMR

### II. Kinematic Model

The WMR descript with a generalized coordinate vector as

\[
q = [q_1, q_2, q_3, \ldots, q_n] \tag{2}
\]

And generalized velocity is given by vector

\[
\dot{q} = [\dot{q}_1, \dot{q}_2, \dot{q}_3, \ldots, \dot{q}_n] \tag{3}
\]

The WMR’s mobility is influenced by constraint. There are two kinds of constraint in mobile robot, holonomic and non-holonomic constraint.

The presented WMR is a non-holonomic WMR and the kinematic mobile is similar to the unicycle model.

The kinematic modeling of the WMR is given by

\[
\dot{q} = S(\theta)p(t) \tag{4}
\]

\[
\dot{q}(t) = \begin{bmatrix} \cos \theta(t) & 0 \\ \sin \theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix}
\]

Where \( p(\theta) \in R^{mxn} \), \([v(t) \omega(t)]\) are the linear and angular velocities of the mobile robot.
Dynamic model
The classical dynamic of non-holonomic of wheeled mobile robot come from Lagrangian formulation which is formulated as following:

\[ M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) + \tau_d = B(q)\tau - A(q)^T\lambda \]  

(5)

Where \( q = [q_1, q_2, ..., q_n]^T \) is the vector so \( q \in R^N \) is the generalized coordinate for N-DOF system , \( M(q) \) is the inertial positive definite matrix with \( N \times N \) dimension , \( V(q, \dot{q}) \) is the centrifugal and carioles force matrix with dimension \( N \times N \) , \( G(q) \) is the gravitational vector with dimension \( N \times 1 \) , \( \tau_d \in R \) is the disturbance torque vector with \( N \times 1 \) dimension , \( B(q) \in R^{N \times (N-M)} \) is the input torque transformation , \( \tau \in R^{(N-M) \times 1} \) is the control input vectors, \( A^T \in R^{N \times M} \) is the nonholonomic constraint matrix, \( \lambda \in R^M \) Lagrange constraint multiplier, respectively. The variable value in eq (5) are defined as

\[
M(q) = \begin{bmatrix}
m & 0 & 0 \\
0 & m & 0 \\
0 & 0 & I
\end{bmatrix}
\]

\[ V(q) = 0 \]

\[ B(q) = \frac{1}{r} \begin{bmatrix}
\cos \theta(t) & \cos \theta(t) \\
\sin \theta(t) & \sin \theta(t) \\
R & -R
\end{bmatrix}, \tau = \begin{bmatrix}
\tau_r \\
\tau_i
\end{bmatrix}
\]

\[ A(q) = \begin{bmatrix}
-sin \theta(t) & \cos \theta(t) & 0
\end{bmatrix}^T \]

Where \( m \) is the robot mass and \( I \) is the robot moment of inertia about the vertical axis located.

For better controlling transfer the mobile robot dynamic equation (5) into another representation, by inserting the kinematic part in equation (4) and drive the kinematic model equation (4) and transform the system to appropriate term

\[ \dot{q} = S(q)v \]  

(6)

\[ \ddot{q} = \dot{S}(q)v + S(q)v \]  

(7)

Now substitute (6) and (7) in (5) obtain

\[
M(q)[\dot{S}(q)v + S(q)v] + V(q, \dot{q})[S(q)v] + G(q) + \ddot{\tau}_d
\]

\[ = B(q)\tau - A(q)^T\lambda \]  

(8)

Next, rearranging the equation and multiplying both sides by leads to \( S^T(q) \) and \( G(q) = 0 \)

\[
S^T(q)M(q)S(q)v + S^T(q)[M(q)\dot{S}(q) + V(q, \dot{q})S(q)]v = S^T(q)B(q)\tau - S^T(q)A(q)^T\lambda
\]

(9)
Now defining the new matrices
\[
\bar{M}(q) = S^T(q) \ M(q) S(q)
\]
\[
\bar{V} = S^T(q)[M(q) \dot{S}(q) + V(q, \dot{q}) S(q)]
\]
\[
\bar{B} = S^T(q) \ B(q)
\]
The dynamic equations become
\[
\bar{M}(q) \ddot{v} + \bar{V}(q, \dot{q})v + \bar{r}_d = \bar{B}(q) \tau
\] (10)

Where:
\[
\bar{M}(q) = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}, \bar{B}(q) = \frac{1}{r} \begin{bmatrix} 1 & 1 \\ R & -R \end{bmatrix}
\]
\[
\bar{V}(q, \dot{q}) = 0
\]

By regarding the surface friction and the disturbance torque as the modeling uncertainties and disturbances, and then the dynamic equation (10) of the simple model of the nonholonomic WMR, assuming all the uncertainties and disturbances are zero, is
\[
\dot{v}(t) = E \cdot \tau(t)
\] (11)
Where the system matrix E is
\[
E = \bar{M}^{-1}(q) \bar{B}(q) = \frac{1}{m r^2} \begin{bmatrix} 1 & 1 \\ R m & -R m \end{bmatrix}
\] (12)

III. Controller

The WMR controller constructed from two parts kinematic and dynamic controllers. The kinematic part provides the controller posture for feeding dynamic controller and the dynamic controller is used to overcome the uncertainties and disturbances.

III.i. Kinematic Controller Design

In control for giving smooth reference trajectory \((x_d, y_d)\) defined in the time interval \(\in [0, t] \). The open reference model \(v_d, \omega_d\) can be derived as
\[
v_d(t) = \sqrt{\dot{x}_d(t)^2 + \dot{y}_d(t)^2}
\]
\[
\omega_d = \frac{\dot{x}_d(t) \dot{y}_d(t) - \dot{y}_d(t) \dot{x}_d(t)}{x_d(t)^2 + y_d(t)^2}
\] (13)

Where \(x_d, y_d\) are reference path
\[
\begin{bmatrix} x_d(t) \\ y_d(t) \\ \dot{\theta}_d(t) \end{bmatrix} = \begin{bmatrix} \cos \theta_d(t) & 0 \\ \sin \theta_d(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_d(t) \\ \omega_d(t) \end{bmatrix}
\] (14)
Where the $v_d$ is the desired linear velocity and $w_d$ is the angular desired WMR velocity.

The kinematic control is designed to bring the position and orientation error $e_k$ to 0 as $t \to \infty$ with the following arbitrary initial error:

$$\lim_{t \to \infty} \|q_d(t) - q(t)\| = \lim_{t \to \infty} \|e_k(t)\|$$

As in (Kanayama et al., 1990) the posture tracking error is $q_e = [x_e \, y_e \, \theta_e]^T = T e_k(t)$

Where

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos \theta(t) & \sin \theta(t) & 0 \\ -\sin \theta(t) & \cos \theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{bmatrix}$$

By differentiating equation (16) considering the kinematic model equation (4) into account and the reference

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} y_e \omega - v + v_d \cos \theta_e \\ -x_e * \omega + v_d \sin \theta_e \\ \omega_d - \omega \end{bmatrix}$$

Where $\omega, v$ are feedback linear and angular velocity

The above kinematic error model forced the wheel mobile robot to follow the reference path. This control law is asymptotically stable with $e \to 0$. The stability is shown using appropriate Lyapunov stability. The auxiliary kinematic control law is (Kanayama et al., 1990).

$$\begin{bmatrix} v_k(t) \\ \omega_k(t) \end{bmatrix} = \begin{bmatrix} v_d \cos \theta_e + k_1 x_e \\ \omega_d + K_2 v_d y_e + K_3 v_d \sin \theta_e \end{bmatrix}$$

Where $v_k, \omega_k$ is linear and angular controller velocities and $K_1, K_2, K_3 > 0$ are design parameter obtained by trial and error

**III.i. Sliding Mode Control Design**

WMR's dynamic controller is designed in this section using SMC technique. The main objective of Dynamic controller is to make the velocity tracking error to zero. The velocity tracking error can be adjusted to

$$e_c = v_k(t) - v(t) = \begin{bmatrix} v_k(t) - v(t) \\ \omega_k(t) - \omega(t) \end{bmatrix}$$

$$\dot{e}_c = \dot{v}_k(t) - \dot{v}(t) = \dot{v}_k - E \tau$$
SMC design involves two main steps: first, the selection of a stable sliding surface \( \Psi(t) \) which gives the desired dynamic characteristics as the system enters the hyperplane \( \Psi(t) = 0 \). The second step for Sliding mode control is to derive the law of control so that the dynamics of the system remain on the designed hyperplane forever. Let the sliding surface be

\[
\Psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = e_c(t) + \lambda \int_0^t e_c(\tau)d\tau
\]  \hspace{1cm} (21)

By Lyapunov theorem it’s can easily define a positive definite function as

\[
L = \frac{1}{2} \psi^2
\]  \hspace{1cm} (22)

The time derivative

\[
L = \psi \dot{\psi}
\]  \hspace{1cm} (23)

To obtain \( \dot{L} \) converges to zero, it is sufficient that

\[
L = -\gamma |\psi|
\]  \hspace{1cm} (24)

Where \( \gamma > 0 \). In other word

\[
\dot{\psi} \text{sgn}(\psi) = -\gamma
\]  \hspace{1cm} (25)

Derivatively (21) in a decentralized structure and equation (25), we can rewrite (25) as

\[
[\ddot{v}_c - E \tau + \lambda e_c(t)] \text{sgn}(\psi) = -\gamma
\]  \hspace{1cm} (26)

Then

\[
\tau(t) = E^{-1} [\ddot{v}_c(t) + \lambda e_c(t)] + \gamma \text{sgn}(\psi)
\]  \hspace{1cm} (27)

Where \( E^{-1} = -\frac{r}{2R} \begin{bmatrix} -Rm & -I \\ -Rm & I \end{bmatrix}, k = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \), and \( k_1 \) is a positive constant, \( \text{sgn}(s) = [\text{sgn}(s_1) \text{ } \text{sgn}(s_2)]^T \).

The dynamic equation (11) It becomes in the presence of uncertainty and disturbance

\[
\dot{v} = E \tau + \tau_d(t) = \dot{E}(\tau) \tau + \Delta E \cdot \tau(t) + \tau_d(t)
\]  \hspace{1cm} (28)

Where \( E \) is the nominal part of the system matrix and evaluated from the WMR physical parameters, \( m, r, I, R \) and \( \Delta E \) is the matrix of the uncertainties of \( E \). \( \tau_d \) is the external disturbances \( \tau_d \in \mathbb{R}^{n \times 1} \). \( \delta(t) \) is the upper bound of uncertain tie;

\[
\delta(t) = \begin{bmatrix} \delta_1(t) \\ \delta_2(t) \end{bmatrix} = \Delta E \cdot \tau(t) + \tau_d(t)
\]  \hspace{1cm} (29)

Then can be write the dynamic equation as

\[
\dot{v} = \dot{E}(\tau) \tau + \delta(t)
\]  \hspace{1cm} (30)

Therefore, the sliding mode dynamic control (27) can be rewritten as

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\[ \tau = \bar{E}^{-1}[v_c + \beta e_c + k \cdot \text{sgn}(\psi)] \]  

(31)

The gain of switching \( k \) should be a design parameter and chose the appropriate value to compensate of uncertainties and disturbances. The most commonly known method to decrease the chattering phenomenon is to use the saturation term \( \text{sat}(\psi, \epsilon) \). So, exchanging \( \text{sgn}(\psi_i) \) by \( \text{sat}(\psi, \epsilon) \) in (31) as:

\[ \tau = \bar{E}^{-1}[v_c + \beta e_c + k \cdot \text{sat}(\psi, \epsilon)] \]  

(32)

Where

\[ \text{sat}(\psi, \epsilon) = \begin{cases}  
\text{sgn}(\psi_i), & |\psi_i| > \epsilon > 0 \\
\frac{\psi_i}{\epsilon}, & |\psi_i| \leq 0 
\end{cases} \]

i = 1,2  

(33)

And \( \epsilon \) is a small positive constant

Desired path \arrow{qe} \text{Control law of kinematic} \arrow{v_k} \text{Sliding mode control} \arrow{w_k} \text{Dynamic controller} \arrow{Kinematic model}

**Fig. 2**: Robot Controller Architecture

### IV. Simulation Results

Both kinematic and dynamic modeling control laws are simulated using MATLAB Simulink for verification and validation. Figure (2) shows the complete control diagram. The simulation is carried out using circular shape trajectory and eight shapes. The parameter of the mobile robot is; \( M = 1.5 \text{ Kg}, \quad I = 9.36 \times 10^{-3} \text{ kg.cm}^2 \) and \( r = 3.7 \text{ cm} \). The case steady shows the mobile robot tracks the virtual reference moving circular path is given by the initial reference point are \( q_e(0) = [x_e(0) \ y_e(0) \ \theta_e(0)]^T = [2 0 \pi/2]^T \) and the initial path vector is \( q(0) = [x(0) \ y(0) \ \theta(0)]^T = [0 0 0]^T \) as shown in figure (3). The initial errors for sliding mode is indicated in figures (4) & (5) where \( q_e(0) = [x_e(0) \ y_e(0) \ \theta_e(0)]^T = [2 0 1.57]^T \) is eliminated to a very small error in both cases. Figure (6) shows the torque produce by dynamic sliding mode controller. Figure (6) is increased rapidly due to initial error and varied to constant value. Figure (7) and figure (8) shows the comparison between sign and sat of \( \psi_1, \psi_2 \). The chattering shows in figure (7) and figure (8) by blue line are come from signup part of the sliding surface. The sliding surface converges to zero as shows in figure (7) and figure (8).
Fig. 3: The real trajectories of these controllers

Fig. 4: Error in x direction

Fig. 5: Error in y direction
Fig. 6: Produced torque by driving wheels

Fig. 7: The Sliding Surface $\psi_1$

Fig. 8: The Sliding Surface $\psi_2$
The second case steady eight-shape is given by the initial reference point are\(q_r(0) = [x_r(0) \ y_r(0) \ \theta_r(0)]^T = [0 \ 0 \ \pi/2]^T\) and the WMR initial path vector is\(q(0) = [x(0) \ y(0) \ \theta(0)]^T = [2 \ 2 \ 0]^T\) as shown in figure (9). The initial errors for\(x_e\) and\(y_e\) is indicated in figures (10) and (11) where\(q_e(0) = [x_e(0) \ y_e(0) \ \theta_e(0)]^T = [2 \ 2 \ 0]^T\) is eliminated to a very small error in. Figure (12) shown the torque produce by dynamic sliding mode controller figure (13) and figure (14) shows the comparison between sign and sat of\(\psi_1, \psi_2\). The chattering shows in figure (13) and figure (14) by blue line are come from signum part of the sliding surface. The sliding surface converges to zero as shows in figure (13) and figure (14).

**Fig. 9:** The real trajectories of these controllers

**Fig. 10:** Error in x direction
Fig. 11: Error in Y direction

Fig. 12: Produced torque by driving wheels

Fig. 13: The Sliding Surface $\psi_1$
V. Conclusion

In the presented paper, a differential WMR modeled with kinematic and dynamic model (mechanical) is derived. The presented dynamic controller used to reduce the chattering effect of the sliding mode controller. The presented sliding mode controller equation used a modified compensation to decrease chattering. The kinematic controller used to produce controller posture (velocity and distance). The behavior of proposed sliding controller has been simulated in MATLAB Simulink used the mechanical derived model and illustrated the performance with reference path for two case studies circle and eight shape as show in figure (1) and figure (9). The simulation results the accuracy of the proposed controller. The result shows that the system flowed the desired path with very good performance with low number of chattering phenomena.

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