Precision measurements of $\theta_{12}$ for testing models of discrete leptonic flavour symmetries

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Abstract. Models of leptonic flavour with discrete symmetries can provide an attractive explanation of the pattern of elements found in the leptonic mixing matrix. The next generation of neutrino oscillation experiments will allow the mixing parameters to be tested to a new level of precision, crucially measuring the CP violating phase $\delta$ for the first time. In this contribution, we present results of a systematic survey of the predictions of a class of models based on residual discrete symmetries and the prospects for excluding such models at medium- and long-term oscillation experiments. We place particular emphasis on the complementary role that a future circa 50 km reactor experiment, e.g. JUNO, can play in constraining these models.

1. Introduction
The next generation of neutrino oscillation experiments has the potential to not only determine the remaining unknowns in the PMNS matrix but also to measure its parameters with unprecedented precision. This will mark the beginning of a period of high-precision neutrino physics, where the standard paradigms describing the neutrino sector will be put to proof and theoretical ideas about the origins of neutrino mass and leptonic flavour can be confronted with data.

One of the more popular beyond the standard model ideas applied to the neutrino sector is the introduction of a discrete flavour symmetry. Models based on this principle have been shown to be able to derive the observed structure of the PMNS matrix from a small set of assumptions. These models generally propose a discrete symmetry (e.g. $A_4$ or $S_4$) which is broken spontaneously, leaving residual symmetries amongst the leptonic mass terms. These symmetries reduce the degrees of freedom amongst the mixing parameters, generating a pattern of falsifiable predictions. By hypothesizing which symmetries of the leptonic mass terms are residual, this idea can be used to reconstruct the flavour group independently of many model specific assumptions. In Ref. [1], we have shown that a quite general construction of this type (first presented in Ref. [3]) leads to only 8 viable models in light of the current global oscillation data. These models fix a column of the PMNS matrix which, under the assumption of unitarity,
Table 1. The solar predictions for the 8 viable models identified in Ref. [1]. The model label denotes the flavour group and the pattern of breaking; for details, see Ref. [1].

| Model label | solar prediction | predicted $\theta_{12}$ ($r$ in 3$\sigma$) |
|-------------|------------------|------------------------------------------|
| $A_4 \, T_{\alpha}-S_2$ | $s = \sqrt{\frac{2}{2-r^2}} - 1$ | [35.62, 35.86] |
| $S_4 \, T_e-S_1$ | $s = \sqrt{1 - \frac{2r^2}{2-r^2}} - 1$ | [34.05, 34.55] |
| $S_4 \, T_{\alpha}-S_2$ | $s = \sqrt{\frac{3}{2(1-r^2)}} - 1$ | [30.29, 30.49] |
| $A_5 \, T_e-S_1$ | $s = \sqrt{3 + \frac{6}{(3-\phi)[2-r^2]} - 1}$ | [30.33, 30.90] |
| $A_5 \, T_e-S_2$ | $s = \sqrt{\frac{6}{(2+\phi)[2-r^2]} - 1}$ | [32.03, 32.24] |
| $A_5 \, T_{\alpha}-S_2$ | $s = \sqrt{\frac{3\phi}{(2\phi-1)[2-r^2]} - 1}$ | [37.56, 37.62] |

can be expressed in terms of two constraints on the PMNS parameters: the atmospheric sum rule, relating $\theta_{23}$ to $\theta_{13}$ and $\delta$, and the solar prediction, an expression for $\theta_{12}$ in terms of $\theta_{13}$ alone. In this contribution, we shall discuss the parameter correlations of these models, and how they can be constrained by the next generation of high-precision oscillation experiments. We shall employ the notation $s = \sqrt{3 \sin \theta_{12} - 1}$, $r = \sqrt{2 \sin \theta_{13}}$ and $a = \sqrt{2 \sin \theta_{23} - 1}$ [2] throughout.

2. Atmospheric sum rules
The atmospheric sum rule can be written in a linearized form by

$$a = a_0 + \lambda r \cos \delta + O(r^2),$$

where $a_0$ and $\lambda$ are constants expressible in terms of the group theoretic parameters of each model. To test these relations, we require a strong precision on the parameter $\delta$, which necessitates the consideration of the next generation of long-baseline experiments. These proposals seek to make accurate measurements of the appearance channels $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$, which are sensitive to the value of $\delta$ at a subdominant level. Although it remains a challenging measurement, two leading designs have been shown to offer significant sensitivity to $\delta$: superbeams and neutrino factories. Such facilities would be able to constrain the atmospheric sum rules over a significant fraction of the available parameter space. For example, an on-axis superbeam with a detector mass of 70 kton (35 kton) and a baseline of 2000 km would be capable of excluding models with $a_0 = 0$ and $\lambda = 1$ for over 44–88% (19–75%) of the parameter space, depending on the true value of $\theta_{23}$ [1].

3. Solar predictions and reactor experiments
In this section, we shall consider a circa 50 km reactor experiment based on the Jiangmen Underground Neutrino Observatory (JUNO) [4] and Reactor Experiment for Neutrino Oscillations (RENO-50) designs [5]. These facilities will be capable of high precision measurements of the $\nu_e$ disappearance probability. The main goal of such experiments is to observe the subdominant oscillations whose phase depends upon the mass hierarchy. However, they will also significantly increase the precision on the oscillation parameters $\theta_{12}$, $\Delta m_{21}^2$ and $\Delta m_{31}^2$, reducing their uncertainty to the sub-percent level. The 8 viable models identified in Ref. [1] make 6 distinct solar predictions, which are shown in Table 1. Such precision will have a significant impact on the viability of the correlations predicted by flavour symmetric models. To understand the impact of these high precision measurements, we have performed a simulation based on the JUNO design to determine its ability to test the correlations shown in Table 1. In
Figure 1. The $5\sigma$ allowed regions for the solar predictions shown in Table 1 after 6 years of data taking by JUNO.

In our simulation, we assume a 20 kton liquid scintillator detector with a linear energy uncertainty of $0.03/\sqrt{E}$. The JUNO facility will detect neutrinos from 10 nearby reactors; however, we model this by a single source at a baseline distance given by the power weighted average of 52.5 km and a reactor power of 36 GW [4]. We have normalised our spectrum to produce $10^5$ events, including a 5% normalisation uncertainty. In Fig. 1, we show the allowed regions at $5\sigma$ significance for the models shown in Table 1. We see that only two of the $5\sigma$ intervals overlap, which allows for a strong model discrimination. The ability for JUNO to exclude these models independently of their atmospheric sum rules provides a great complementarity between the reactor and long-baseline programmes. Furthermore, the two indistinguishable models for JUNO predict very different atmospheric sum rules,

$$a = \pm \frac{1}{6} - \frac{1}{\sqrt{6}} r \cos \delta \ (S_4 \ T_{\alpha-S_2}) \quad \text{and} \quad a = \frac{\varphi}{\sqrt{2}} r \cos \delta \ (A_5 \ T_{e-S_1}),$$

where $\varphi = \frac{1+\sqrt{5}}{2}$ is the golden ratio, and we expect these to be distinguishable with a superbeam for most of the parameter space [1].

4. Summary

The next generation of neutrino oscillation experiments, with their focus on precision measurements of the underlying parameters, will allow certain classes of models with discrete flavour symmetries to be thoroughly tested. In Ref. [1], the role of a long-baseline superbeam experiment (modelled after LBNO or LBNE) has been shown to be able to exclude these correlations for a large fraction of parameter space. In this contribution, we have highlighted the potential for experimental exclusion of these models at a circa 50 km reactor experiment based on the JUNO facility. By testing the solar predictions to high accuracy, such a facility will be able to independently distinguish between almost all models under consideration. The complementarity between reactor and long-baseline experiments will provide a stringent test of the idea that residual symmetries are responsible for the structure of the PMNS matrix.

References

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