Large \((g - 2)_\mu\) in SU(5)xU(1) Supergravity Models

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ABSTRACT

We compute the supersymmetric contribution to the anomalous magnetic moment of the muon within the context of SU(5) x U(1) supergravity models. The largest possible contributions to \(a^\text{susy}_\mu\) occur for the largest allowed values of \(\tan \beta\) and can easily exceed the present experimentally allowed range, even after the LEP lower bounds on the sparticle masses are imposed. Such \(\tan \beta\) enhancement implies that \(a^\text{susy}_\mu\) can greatly exceed both the electroweak contribution (\(\approx 1.95 \times 10^{-9}\)) and the present hadronic uncertainty (\(\approx \pm 1.75 \times 10^{-9}\)). Therefore, the new E821 Brookhaven experiment (with an expected accuracy of \(0.4 \times 10^{-9}\)) should explore a large fraction (if not all) of the parameter space of these models, corresponding to slepton, chargino, and squarks masses as high as 200, 300, and 1000 GeV respectively. Moreover, contrary to popular belief, the \(a^\text{susy}_\mu\) contribution can have either sign, depending on the sign of the Higgs mixing parameter \(\mu\): \(a^\text{susy}_\mu > 0\) (\(\mu > 0\)) for \(\mu > 0\) (\(\mu < 0\). The present \(a_\mu\) constraint excludes chargino masses in the range \(45 - 120\) GeV depending on the value of \(\tan \beta\), although there are no constraints for \(\tan \beta < \sim 8\). We also compute \(a^\text{susy}_\tau\) and find \(|a^\text{susy}_\tau| \approx (m_\tau/m_\mu)^2 |a^\text{susy}_\mu| \lesssim 10^{-5}\) and briefly comment on its possible observability.
1 Introduction

The experimental measurements of the leptonic anomalous magnetic moments have been carried out to such great accuracy that their agreement with the theoretical calculations has been one of the most spectacular successes of quantum field theory and QED in particular [1]. While efforts to go to higher experimental accuracies are being pursued mainly to test the standard model electroweak contribution to \( a_\mu \equiv \frac{1}{2}(g-2)_\mu \) [2], new physics might come into play as well. Since realistic supersymmetric models predict a sparticle mass spectrum which can be as light as \( \approx 45 \text{ GeV} \), it is possible that an experimental measurement of the muonic \( g-2 \) factor with high accuracy could put some constraints on the new and yet-to-be-found sparticles, and therefore on the parameter space on the various supersymmetric models.

The long standing experimental values of \( a_\mu \) for each sign of the muon electric charge [3] can be averaged to yield [4]

\[
a_{\mu}^{exp} = 1 165 923(8.5) \times 10^{-9}. \tag{1}
\]

The uncertainty on the last digit is indicated in parenthesis. On the other hand, the various standard model contributions to \( a_\mu \) have been estimated to be as follows [4]

\[
\begin{align*}
\text{QED} & : 1 165 846 984(17)(28) \times 10^{-12} \tag{2} \\
had.1 & : 7 068(59)(164) \times 10^{-11} \tag{3} \\
had.2 & : -90(5) \times 10^{-11} \tag{4} \\
had.3 & : 49(5) \times 10^{-11} \tag{5} \\
\text{Total hadronic} & : 7 027(175) \times 10^{-11} \tag{6} \\
\text{Electroweak} & : 195(10) \times 10^{-11} \tag{7}
\end{align*}
\]

The total standard model prediction is then [4]

\[
a_{\mu}^{SM} = 116 591 9.20(1.76) \times 10^{-9}. \tag{8}
\]

Subtracting the experimental result gives [4]

\[
a_{\mu}^{SM} - a_{\mu}^{exp} = -3.8(8.7) \times 10^{-9}, \tag{9}
\]

which is perfectly consistent with zero. The uncertainty in the theoretical prediction is dominated by the uncertainty in the lowest order hadronic contribution (had.1), which ongoing experiments at Novosibirsk hope to reduce by a factor of two in the near future. This is an important preliminary step to testing the electroweak contribution, which is of the same order. The uncertainty in the experimental determination of \( a_\mu \) is expected to be reduced significantly (down to \( 0.4 \times 10^{-9} \)) by the new E821 Brookhaven experiment [2], which is scheduled to start taking data in late 1994. Any beyond-the-standard-model contribution to \( a_\mu \) (with presumably negligible uncertainty) will simply be added to the central value in Eq. (4). Therefore, we can obtain an allowed
interval for any supersymmetric contribution, such that $a_{\mu}^{\text{susy}} + a_{\mu}^{\text{SM}} - a_{\mu}^{\exp}$ is consistent with zero at some given confidence level,

$$-4.9 \times 10^{-9} < a_{\mu}^{\text{susy}} < 12.5 \times 10^{-9}, \quad \text{at } 1\sigma;$$  \hspace{1em} (10)

$$-10.5 \times 10^{-9} < a_{\mu}^{\text{susy}} < 18.1 \times 10^{-9}, \quad \text{at } 90\%\text{CL};$$  \hspace{1em} (11)

$$-13.2 \times 10^{-9} < a_{\mu}^{\text{susy}} < 20.8 \times 10^{-9}, \quad \text{at } 95\%\text{CL}. \hspace{1em} (12)$$

The supersymmetric contributions to $a_{\mu}$ have been computed to various degrees of completeness and in the context of several models, including the minimal supersymmetric standard model (MSSM) \cite{5,6,7,8}, an $E_6$ string-inspired model \cite{10}, and a non-minimal MSSM with an additional singlet \cite{11,12}. Because of the large number of parameters appearing in the typical formula for $a_{\mu}^{\text{susy}}$, various contributions have often been neglected and numerical results are basically out of date. More importantly, a contribution which is roughly proportional to the ratio of Higgs vacuum expectation values ($\tan \beta$), even though known for a while \cite{7,8,9,12}, has to date remained greatly unappreciated. This has been the case because in the past only small values of $\tan \beta$ were usually considered and the enhancement of $a_{\mu}^{\text{susy}}$ which is the focus of this paper, was not evident. In fact, such enhancement can easily make $a_{\mu}^{\text{susy}}$ run in conflict with the bounds given in Eq. (12), even after the LEP lower bounds on the sparticle masses are imposed.

In this paper we compute $a_{\mu}^{\text{susy}}$ in the context of supergravity models based on the $SU(5) \times U(1)$ (flipped $SU(5)$) gauge group \cite{13} supplemented by two string-inspired (the so-called no-scale \cite{14} and dilaton \cite{15}) soft-supersymmetry-breaking scenarios. Among the various interesting properties these models possess, perhaps the most relevant ones here are that their sparticle mass spectra are as light as could possibly be for a supergravity model, and that large values of $\tan \beta$ are typical. Moreover, the complete parameter space can be described in terms of only three variables: the top-quark mass ($m_t$), $\tan \beta$, and the gluino mass ($m_{\tilde{g}}$). Thus, we are able to produce quite specific predictions for $a_{\mu}^{\text{susy}}$ throughout the parameter space. This paper is in line with a series of phenomenological calculations which the authors have performed recently within the context of this class of models \cite{16}.

Besides the experiments on $(g - 2)_{\mu}$, there have been suggestions \cite{17,18} that the anomalous magnetic moment of the tau lepton could be measured at hadron supercolliders to the precision of $\approx 10^{-5}$. Therefore, we also compute $a_{\tau}^{\text{susy}}$ in these models.

2 The flipped SU(5) supergravity models

The models of interest in this paper are based on the gauge group $SU(5) \times U(1)$ and have the property of gauge coupling unification at the scale $10^{18}$ GeV \cite{13}. This implies that their matter content must include additional particles beyond the supersymmetric standard model with two Higgs doublets, otherwise the unification scale would be $10^{16}$ GeV. Indeed, an extra pair of vector-like quark doublets with mass
\[ \sim 10^{12} \text{GeV} \] and a pair of charge \(-\frac{1}{3}\) quark singlets with mass \(\sim 10^6 \text{GeV} \) appear in the spectrum. This additional particles form complete \(10, \overline{10} SU(5) \times U(1)\) multiplets and are seen to occur in a string-derived version of this model [19]. The unification scale is also consistent with that expected in string models of this kind [20]. Besides contributing to the gauge coupling beta functions for scales above their masses, the new particles do not have any other noticeable effects. Nonetheless, such subtle changes in slope propagate throughout the whole system of renormalization group equations for the gauge and Yukawa couplings, as well as the scalar masses and trilinear scalar couplings. An effect of similar magnitude is a consequence of the “extra” running down from \(10^{18} \text{GeV} \) relative to a model which unifies at \(10^{16} \text{GeV} \).

This class of supergravity models can be described completely in terms of just three parameters: (i) the top-quark mass \((m_t)\), (ii) the ratio of Higgs vacuum expectation values, which satisfies \(1 \lesssim \tan \beta \lesssim 40\), and (iii) the gluino mass, which is cut off at 1 TeV. This simplification in the number of input parameters is possible because of specific scenarios for the universal soft-supersymmetry-breaking parameters \((m_0, m_{1/2}, A)\) at the unification scale. These three parameters can be computed in specific string models in terms of just one of them [21]. In the no-scale model one obtains \(m_0 = A = 0\), whereas in the dilaton model the result is \(m_0 = \frac{1}{\sqrt{3}} m_{1/2}, A = -m_{1/2}\). After running the renormalization group equations from high to low energies, at the low-energy scale the requirement of radiative electroweak symmetry breaking introduces two further constraints which among other things determine the magnitude of the Higgs mixing term \(\mu\), although its sign remains undetermined. Finally, all the known phenomenological constraints on the sparticle masses are imposed (most importantly the chargino, slepton, and Higgs mass bounds). This procedure is well documented in the literature [22] and yields the allowed parameter spaces for the no-scale [14] and dilaton [15] cases.

These allowed parameter spaces in the three defining variables \((m_t, \tan \beta, m_{\tilde{g}})\) consist of a discrete set of points for three values of \(m_t\) (\(m_t = 130, 150, 170 \text{GeV}\)), and a discrete set of allowed values for \(\tan \beta\), starting at 2 and running (in steps of two) up to 32 (46) for the no-scale (dilaton) case. The allowed values of \(m_{\tilde{g}}\) vary from a minimum value of \(\approx 200 \text{GeV}\) up to 1 TeV, depending on the value of \(\tan \beta\). For each of these points in parameter space there corresponds one set of sparticle and Higgs masses, as well as various diagonalizing matrices for the neutralino, chargino, slepton, and squark masses. In particular, all of the parameters that appear in the formula for \(a_{\mu}^{\text{susy}}\) given below can be obtained for any given point in parameter space.

In the models we consider all sparticle masses scale with the gluino mass, with a mild \(\tan \beta\) dependence. In Table 1 we give the approximate proportionality coefficient (to the gluino mass) for each sparticle mass. Note that the relation \(2 m_{\tilde{\chi}^0_1} \approx m_{\tilde{\chi}^0_2} \approx m_{\tilde{\chi}^\pm_1}\) holds to good approximation. The third-generation squark and slepton masses also scale with \(m_{\tilde{g}}\), but the relationships are smeared by a strong \(\tan \beta\) dependence. From Table 1 one can (approximately) translate any bounds on a given sparticle mass

\[ \text{Note that } \tan \beta > 1 \text{ is required by the radiative breaking mechanism, and the LEP lower bound on the lightest Higgs boson mass } (m_h > 60 \text{ GeV} [23]) \text{ is quite constraining for } 1 < \tan \beta < 2. \]
Table 1: The approximate proportionality coefficients to the gluino mass, for the various sparticle masses in the two supersymmetry breaking scenarios considered.

| Sparticle | no-scale | dilaton |
|-----------|----------|---------|
| $\tilde{e}_R, \tilde{\mu}_R$ | 0.18 | 0.33 |
| $\tilde{\nu}$ | 0.18 - 0.30 | 0.33 - 0.41 |
| $2\chi^0_1, \chi^0_2, \chi^\pm_1$ | 0.28 | 0.28 |
| $\tilde{e}_L, \tilde{\mu}_L$ | 0.30 | 0.41 |
| $\tilde{q}$ | 0.97 | 1.01 |
| $\tilde{g}$ | 1.00 | 1.00 |

on bounds on all the other sparticle masses.

3 Calculation and discussion of results

There are two sources of one-loop supersymmetric contributions to $a_\mu$: (i) with neutralinos and smuons in the loop; and (ii) with charginos and sneutrinos in the loop. In the former case it is necessary to diagonalize the smuon mass matrix to get the mass eigenstates,

$$DMD^\dagger = \text{diag}(m^2_{\tilde{\mu}_1}, m^2_{\tilde{\mu}_2}),$$  \hspace{1cm} (13)

where $M$ is the smuon mass matrix

$$M = \begin{pmatrix} m^2_{\tilde{\mu}_{LL}} & m^2_{\tilde{\mu}_{LR}} \\ m^2_{\tilde{\mu}_{LR}} & m^2_{\tilde{\mu}_{RR}} \end{pmatrix},$$  \hspace{1cm} (14)

and $D$ is the orthogonal rotation matrix. This gives the mass eigenstates

$$\tilde{\mu}_i = D_{i1}\tilde{\mu}_L + D_{i2}\tilde{\mu}_R, \quad i = 1, 2.$$  \hspace{1cm} (15)

Therefore, the rotation angle can be expressed as

$$\tan(2\theta) = \frac{2m^2_{\tilde{\mu}_{LR}}}{(m^2_{\tilde{\mu}_{LL}} - m^2_{\tilde{\mu}_{RR}})},$$  \hspace{1cm} (16)

where

$$m^2_{\tilde{\mu}_{LR}} = m_\mu(A_\mu + \mu \tan \beta).$$  \hspace{1cm} (17)

It is clear that because of the smallness of the muon mass compared with the sparticle mass scale, the mixing angle is quite small. The general formula for the lowest order supersymmetric contribution to $a_\mu$ has been given in Refs. [7, 8, 9, 12]. Here we use...
the expression in Ref. [12],

\[ a_{\mu}^{\text{susy}} = -\frac{g_2^2}{8\pi^2} \sum_{\chi^0_i} m_{\chi^0_i} \left\{ (-1)^{j+1} \sin(2\theta) B_1(\eta_{ij}) \tan \theta_W N_{i1} [\tan \theta_W N_{i1} + N_{i2}] + \frac{m_\mu}{2M_W \cos \beta} B_1(\eta_{ij}) N_{i3} [3 \tan \theta_W N_{i1} + N_{i2}] + \left( \frac{m_\mu}{m_{\tilde{\nu}}^2} \right)^2 A_1(\eta_{ij}) \left\{ \frac{1}{4} [\tan \theta_W N_{i1} + N_{i2}]^2 + [\tan \theta_W N_{i1}]^2 \right\} \right\} - \sum_{\chi^0_j} \left[ \frac{m_\mu m_{\chi^0_j}}{m_{\tilde{\nu}}^2} \frac{m_\mu}{\sqrt{2} M_W \cos \beta} B_2(\kappa_j)V_{j1}U_{j2} + \left( \frac{m_\mu}{m_{\tilde{\nu}}} \right)^2 \frac{A_1(\kappa_j)}{2} V_{j1}^2 \right\} \right\] (18)

where \( N_{ij} \) are elements of the matrix which diagonalizes the neutralino mass matrix, and \( U_{ij}, V_{ij} \) are the corresponding ones for the chargino mass matrix, in the notation of Ref. [24]. Also,

\[ \eta_{ij} = \left[ 1 - \left( \frac{m_{\tilde{\nu}}}{m_{\chi^0_j}} \right)^2 \right]^{-1}, \quad \kappa_j = \left[ 1 - \left( \frac{m_{\chi^0_j}}{m_{\tilde{\nu}}} \right)^2 \right]^{-1}, \] (19)

and

\[ B_1(x) = x^2 - \frac{1}{2} x + x^2(x - 1) \ln \left( \frac{x - 1}{x} \right), \] (20)

\[ A_1(x) = x^3 - \frac{1}{2} x^2 - \frac{1}{6} x + x^3(x - 1) \ln \left( \frac{x - 1}{x} \right), \] (21)

\[ B_2(x) = -x^2 - \frac{1}{2} x - x^3 \ln \left( \frac{x - 1}{x} \right). \] (22)

As has been pointed out, the mixing angle of the smuon eigenstates is small (although it can be enhanced for large \( \tan \beta \)) and it makes the neutralino-smuon contribution suppressed. Moreover, the various neutralino-smuon contributions (the first three lines in Eq. (18)) tend to largely cancel among themselves [3]. This means that the chargino-sneutrino contributions (on the fourth line in Eq. (18)) will likely be the dominant ones. In fact, as we stress in this paper, the first chargino-sneutrino contribution (the “gauge-Yukawa” contribution) is enhanced relative to the second one (the “pure gauge” contribution) for large values of \( \tan \beta \). This can be easily seen as follows.

Picturing the chargino-sneutrino one-loop diagram, with the photon being emitted off the chargino line, there are two ways in which the helicity of the muon can be flipped, as is necessary to obtain a non-vanishing \( a_\mu \):

\textsuperscript{2}The original Fayet formula [5] is obtained from the third neutralino-smuon contribution in the limit of a massless photino and no smuon mixing.
(i) It can be flipped by an explicit muon mass insertion on one of the external muon lines, in which case the coupling at the vertices is between a left-handed muon, a sneutrino, and the wino component of the chargino and has magnitude $g_2$. It then follows that $a_\mu$ will be proportional to $g_2^2(m_\mu/\tilde{m})^2|V_{j1}|^2$, where $\tilde{m}$ is a supersymmetric mass in the loop and the $V_{j1}$ factor picks out the wino component of the $j$-th chargino. This is the origin of the “pure gauge” contribution to $a_\mu^{\text{susy}}$.

(ii) Another possibility is to use the muon Yukawa coupling on one of the vertices, which flips the helicity and couples to the Higgsino component of the chargino. One also introduces a chargino mass insertion to switch to the wino component and couple with strength $g_2$ at the other vertex. The contribution is now proportional to $g_2\lambda_\mu(m_\mu m_\chi^\pm/\tilde{m}^2)V_{j1}U_{j2}$, where $U_{j2}$ picks out the Higgsino component of the $j$-th chargino. The muon Yukawa coupling is given by $\lambda_\mu = g_2 m_\mu / (\sqrt{2} M_W \cos \beta)$. This is the origin of the gauge-Yukawa contribution to $a_\mu^{\text{susy}}$.

The ratio of the “pure gauge” to the “gauge-Yukawa” contributions is roughly then

$$g_2^2 (m_\mu/\tilde{m})/(g_2\lambda_\mu) \sim g_2/\sqrt{1 + \tan^2 \beta}, \quad (23)$$

for $\tilde{m} \sim 100$ GeV. Thus, for small $\tan \beta$ both contributions are comparable, but for large $\tan \beta$ the “gauge-Yukawa” contribution is greatly enhanced. This phenomenon was first noticed in Ref. [7]. It is interesting to note that an analogous $\tan \beta$ enhancement also occurs in the $b \to s \gamma$ amplitude [25], although its effect is somewhat obscured by possible strong cancellations against the QCD correction factor.

The results of the calculation in the no-scale and dilaton cases are plotted in Figs. 1a,1b respectively, against the gluino mass, for the indicated values of $m_t$ [6]. As anticipated, the values of $\tan \beta$ increase as the corresponding curves move away from the zero axis. Note that $a_\mu^{\text{susy}}$ drops off faster than naively expected (i.e., $\propto 1/m_\tilde{g}$) since the $U_{12}$ mixing element decreases as the limit of pure wino and Higgsino is approached for large $m_\tilde{g}$. Note also that $a_\mu^{\text{susy}}$ can have either sign, in fact, it has the same sign as the Higgs mixing parameter $\mu$. The incorrect perception that $a_\mu^{\text{susy}}$ is generally negative appears to be based on several model analyses where either $\mu$ was chosen to be negative or only some of the neutralino-smuon pieces were kept (which are mostly negative). Interestingly, the largest allowed values of $\tan \beta$ do not exceed the $a_\mu$ constraint since consistency of the models (i.e., the radiative breaking constraint) requires larger gluino masses as $\tan \beta$ gets larger.

Comparing the results shown in Fig. 1 with the allowed ranges in Eq. (12), it is clear that some points in parameter space are already excluded, depending on the

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3A similar enhancement in the second neutralino-smuon contribution is suppressed by small Higgsino admixtures (i.e., $|N_{13}|, |N_{23}| \ll 1$).

4The choice $m_t = 170$ GeV has not been shown because it is subjected to strict constraints from the $\epsilon_1$ electroweak parameter [26].

5For comparison with earlier work, our sign convention for $\mu$ is opposite to that in Ref. [24].
confidence level one wants to use (the dashed lines in Fig. 1 represent the 95%CL limit). The corresponding excluded ranges in the other sparticle masses can be deduced from the proportionality coefficients given in Table I. To show in a more clear way which region of parameter space is excluded by the present data, in Fig. 2 we show all the allowed points in parameter space of the two models (dots and crosses) in the \((m_{\chi^\pm}, \tan \beta)\) plane for fixed values of \(m_t\). Those points marked with crosses are excluded by the \(a_\mu\) constraint at the 95%CL. Note that for not too small values of \(\tan \beta\), chargino masses in the range \(45 - 120 \text{ GeV}\) are already excluded; there are no constraints for \(\tan \beta \lesssim 8\). Using Table I this reach in chargino masses translates into \(m_{\tilde{q}, \tilde{g}} \approx 430 \text{ GeV}, m_{\tilde{e}_L, \tilde{\mu}_L} \approx 130 \text{ GeV}, m_{\tilde{e}_R, \tilde{\mu}_R} \approx 75 \text{ GeV}, m_{\chi_1^0} \approx 60 \text{ GeV},\) and \(m_{\chi_2^0} \approx 120 \text{ GeV}.\)

It is hard to tell what will happen when the E821 experiment reaches its designed accuracy limit. However, one point should be quite clear, the supersymmetric contributions to \(a_\mu\) can be so much larger than the present hadronic uncertainty (\(\approx \pm 1.76 \times 10^{-9}\)) that the latter is basically irrelevant for purposes of testing a large fraction of the allowed parameter space of the models. This is not true for the electroweak contribution and will also not hold for small values of \(\tan \beta\). Should the actual measurement agree very well with the standard model contribution, then either \(\tan \beta \sim 1\) or the sparticle spectrum would need to be in the TeV range. This situation is certainly a window of opportunity for sparticle detection before LEP II starts operating. Moreover, a significant portion of the explorable parameter space (those points with \(m_{\chi^\pm} \gtrsim 100 \text{ GeV}\) and equivalently \(m_{\tilde{g}} \gtrsim 350 \text{ GeV}\)) is in fact beyond the reach of LEP II.

Now let us turn briefly to the tau \(g - 2\) factor. As expected, one generally obtains \(a_\tau^{\text{susy}} \sim (m_\tau/m_\mu)^2 a_\mu^{\text{susy}}\) since the dominant term (the “gauge-Yukawa” chargino-sneutrino term) is now proportional to \(m_\tau^2\), everything else being the same. The largest values obtained this way do not exceed \(\approx 1.6 \times 10^{-5}\) (and these even occur for points in parameter space already excluded by the \(a_\mu^{\text{susy}}\) constraint). In comparison, the SM contribution has been estimated to be \(a_\tau^{\text{SM}} = 117.73(0.03) \times 10^{-5}\) \([18]\), and the supersymmetric contribution could easily exceed the present theoretical uncertainty in \(a_\tau^{\text{SM}}\). However, the values of \(a_\tau^{\text{susy}}\) are below the possible experimental accuracy reachable at hadron supercolliders (\(4 \times 10^{-5}\) \([7, 18]\)) and thus undetectable in the foreseeable future.

4 Conclusions

We have computed the supersymmetric contribution to the anomalous magnetic moment of the muon in the context of \(SU(5) \times U(1)\) supergravity models. The predictions are quite sharp since they depend on only three parameters, one of which is the top-quark mass. Moreover, the large values of \(\tan \beta\), which are typical in this class of models, enhance the supersymmetric contribution so much that non-negligible constraints on the parameters of the models exist even with the present data, and
in light of the LEP lower bounds on the sparticle masses. These contributions are generally much larger than the electroweak contribution and the present standard model hadronic uncertainty, and thus should be readily observable at the new E821 Brookhaven experiment. The potential for decisive exploration of the parameter space of these models is extremely bright and much greater than the direct experimental production of sparticles at present and near future collider facilities. We expect that the qualitative results in this paper will remain valid in a more general class of supersymmetric models, as long as no new light particles are introduced, and large values of \( \tan \beta \) are allowed. In contrast, in the minimal \( SU(5) \) supergravity model one would not expect large contributions to \( a_\mu^{\text{susy}} \) since the constraint from proton decay requires heavy slepton masses and \( \tan \beta \lesssim 5 \) \[27\]. Indeed, we find \( |a_\mu^{\text{susy}}| \lesssim 0.2 \times 10^{-9} \), which is unobservable even for the new Brookhaven experiment. Finally, experimental limitations indicate that the supersymmetric contribution to \((g - 2)_\mu \) is likely to remain undetected in the foreseeable future.

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Figure Captions

Figure 1: The supersymmetric contribution to the muon anomalous magnetic moment in (a) the no-scale and (b) the dilaton flipped $SU(5)$ supergravity models, plotted against the gluino mass for the indicated values of $m_t$ and $\tan \beta$ (which increase in steps of two). The dashed lines represent the 95%CL experimentally allowed range.

Figure 2: The allowed parameter space of (a) the no-scale and (b) the dilaton flipped $SU(5)$ supergravity models (in the $(m_{\chi^\pm}, \tan \beta)$ plane) for the indicated values of $m_t$. The points marked by crosses violate the present experimental constraint on $a_\mu$ at the 95%CL.