Transient Flows in a Pipe System with Pump Shut-Down and the Simultaneous Closing of a Spherical Valve

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Abstract. Because of the limited value of the wave propagation speed in water the propagation of a pressure surge in transient flows can be tracked in the time series. This enables both the pressure head and the flow velocity in pipe flows to be determined as a function of both the coordinate along the pipe and the time. The propagation of the pressure surge includes both wave transmission and reflection. The latter occurs where the flow section is changed. The wave tracking method has been demonstrated as highly accurate and subsequently was applied to much more complex hydraulic systems, in which the pump is shut off and the spherical valve is simultaneously progressively closed. A combined four-quadrant characteristic of the pump and a spherical valve has been worked out, with which the computational procedure for the transient flow in the complex system could be significantly simplified. It has been demonstrated that not only the pressure surge in the hydraulic system but also the rotational speed of the pump could be satisfactorily computed. The computational algorithm has been demonstrated as quite simple, so that all calculations could be performed simply by means of the Microsoft Excel module.

1. Introduction
Transient flows, for instance, in hydro power plants are encountered at each start and shut-off of hydraulic machines as well as by regulating the operating points of the involved machines. Since such flows represent highly complex flow dynamics and always cause rapid pressure rises in the system, for safety reasons the related flow phenomena and pressure increases have to be considered already in the earlier stages of construction. The operation as well as its variation of each hydraulic system are then all subjected to the conditions, which are specified based on the transient flow analyses.

A rapid pressure rise in a transient flow caused, for instance, by closing a valve is called hydraulic shock or water hammer. For calculating transient flows, there basically exist two available computational methods which are based, respectively, on rigid and elastic water column theories. The former theory relies on the momentum equation i.e. Newton’s second law of motion in mechanics. For a complex network which, for instance, consists of \( x \) pipes of different diameters and connections, \( x \) momentum equations have to be simultaneously and dependently solved. This requires iterative numerical computations and is always associated with high computational expenses. The latter, i.e. the elastic water column theory takes the compressibility of the water into account. It is considered a special field in hydromechanics and, therefore, has widely been investigated since the initial investigations carried out by both Joukowsky and Allievi. Departing from the basic equation of
Joukowsky \( \Delta h = - \frac{a}{g} \cdot \Delta c \) for the local pressure rise in the flow, the propagation of the pressure wave is the dominant variable and, therefore, has to be followed when calculating transient flows and the related pressure surges in the system.

The compressibility of water as an additional phenomenal aspect in transient flows, actually, functions as a useful feature which contributes to simplifications of the calculations. This is simply so, because, for the limited propagation speed of the pressure surge, the flows at different locations do not simultaneously affect each other. At each time step in a time series, the flow at any given location can then be separately computed, each time by only solving one equation.

The method of computing transient flows has been progressed from the graphical approach (Schnyder-Bergero) for simple flow systems to the much more efficient and powerful computational standard based on nowadays computer technology. The most widely applied method is, obviously, the characteristic method of solving the wave equation. The method is based on dividing the pipe length into \( n \) sub-intervals of equal length, as can be found for instance in [1, 2]. In many cases, however, the sub-intervals of equal distance do not appear to be necessary when only the local pressure rise in the flow, for instance, at the regulation valve, is in focus. For this reason the method of tracking the pressure wave seems to be much more accessible.

Basically, the wave tracking method had been indicated in the early stages of investigations by Allievi [3]. It has also been explained in [4-8]. This method does not require dividing the pipe length into equal-distant sub-intervals, except for the graphical method at earlier times. The advantage and applicability of this method will be demonstrated in this paper regarding transient flows in a complex hydraulic system.

Typical transient flows in a hydraulic system of hydropower plants with pumps or turbines are often encountered at the start and stop of the machines. The hydraulic system usually comprises both low and high pressure pipe systems, each with one or more surge tanks. The related dynamic process is very complex when, for instance, the shut-off of the pump is associated with a simultaneous closing of the spherical valve on the pressure side of the pump. The calculation of the related transient flow, therefore, requires knowledge of the hydraulic characteristics of both the pump and the valve. As can be expected, it would be very convenient if the hydraulic characteristics of both the pump and the spherical valve can be combined to one characteristic. The current paper shows the computational method and applications of such a combined characteristic based on the use of four-quadrant diagrams of the pump.

As a matter of fact, transient flows and their calculations have been regarded as highly complex, so that a simple computational method still appears to be inaccessible to everyone. Nearly all available commercial software is based on long-term developments. The circumstance of using commercial software highly restricts the engineer’s activities in dealing with engineering transient flows. For this reason, the current paper provides a well validated method of simply using the tabular computations, e.g. based on the use of Microsoft’s Excel program. Due to the restriction to the paper length only few calculation results in the computational example are shown. More results will be presented in a journal paper.

2. Basic Equations of Transient Flows
From the momentum and continuity equations for viscous flows in a pipe of constant cross-section (Fig. 1), the wave equation is derived as

\[
\frac{\nabla}{\nabla} x
\]

Fig. 1 Pressure wave propagation in the pipe of constant section
The subscript 0 refers to steady state before a pressure surge is generated. For frictionless flows one concludes that the quantities \( h - h_0 \pm \frac{a}{g} (c - c_0) \) remain constant while traveling with the wave propagation speed \( a \) along the pipe in two opposite directions. Else, they change by a value (friction term) which depends on both the flow direction and the flow velocity, in terms on the r.h.s of Eq. (1).

Equation (1) will be integrated along the pipe in both the positive \( x \)-direction from \( A \) to \( x \) and the negative direction from \( B \) to \( x \) (Fig. 1). Thus, with \( L_A = x - x_A \) and \( L_B = -(x - x_B) \), both are positive, one obtains

\[
\begin{align*}
\frac{d}{dx} \left[ h - h_0 \pm \frac{a}{g} (c - c_0) \right] &= -\frac{\lambda}{2gd} \left( c|c - c_0|c_0 \right), \\
\end{align*}
\]

The resulting error is one order smaller than the friction terms and thus two orders smaller than the pressure terms in the above equations.

The above two equations indicate that from the known flows at \( A \) and \( B \) the flow at any other locations \( x \) at a certain later time can be determined. The time delays are \( \Delta t_A = L_A / a \) and \( \Delta t_B = L_B / a \), respectively.

For further calculations the following wave parameters are introduced:

\[
\begin{align*}
2F(x + at) &= h - h_0 - \frac{a}{g} (c - c_0), & 2f(x - at) &= h - h_0 + \frac{a}{g} (c - c_0), \\
2F_B &= \left[ h - h_0 - \frac{a}{g} (c - c_0) \right]_B, & 2f_A &= \left[ h - h_0 + \frac{a}{g} (c - c_0) \right]_A.
\end{align*}
\]

If the positive velocity \( c \) is defined in the other direction, then the sign related to \( c \) has to be changed.

With the above definition the wave parameters \( F \) and \( f \) travel, respectively, in the negative and the positive \( x \)-directions (Fig. 1). Much often, \( F \) is called the primary wave as it arises from the regulation organ. \( f \) is the reflected wave from the reservoir. Both \( F \) and \( f \) are independent of the definition of the \( x \)-direction. For the case of Fig. 1 and at the location of the injector valve there is a time delay \( \Delta t = 2L/a \) between \( F \) and \( f \).

Departing from the above definitions and in using the flow rate \( Q = cA \), Eq. (2) and (3), respectively, are written as

\[
\begin{align*}
f(x - at) &= f_A - \frac{1}{2} R_A \left( Q|Q - Q_A|Q_A \right), & F(x + at) &= F_B + \frac{1}{2} R_B \left( Q|Q - Q_B|Q_B \right), \\
\end{align*}
\]

The resistance constants are given by

\[
R_A = \frac{\lambda L_A}{2gdA^2} \quad \text{and} \quad R_B = \frac{\lambda L_B}{2gdA^2}.
\]

For computing transient flows one needs only to track the parameters \( F \) and \( f \) which propagate by the wave speed \( a \) in two opposite directions. At a given position in the flow, both the pressure head and the flow velocity can then be calculated from Eqs. (4) and (5) as
\[ h - h_0 = F + f, \quad c - c_0 = \frac{g}{a}(f - F). \]  \hspace{1cm} (11), (12)

### 3. Generation of the Pressure Surge

The transient flow may be caused by starting and stopping the machine or by regulating the operating point via regulation organs. For simplicity, the flow rate regulation in a Pelton turbine is considered by regulating the injector opening (Fig. 1 and 2, the Pelton wheel is not shown). By using the Bernoulli equation for the jet speed the flow rate through the injector is calculated as

\[ Q = c_n A_{\text{jet}} = \mu A_{\text{inj}} \sqrt{2gh_{\text{tot}}} \quad \text{or} \quad Q = \mu A_{\text{inj}} \sqrt{2g \left( h + \frac{1}{2g} \frac{Q^2}{A^2} \right)}. \tag{13}, (14) \]

The term with \( Q^2 \) under the square root sign represents the dynamic pressure head in the flow ahead of the injector. The regulation of the flow rate occurs by varying the needle position in the injector nozzle (Fig. 2), leading to the variation of the jet section. The ratio of the jet section to the opening area \( A_{D0} \) of the nozzle is called contraction factor (\( \mu \)). It represents the injector characteristic and can be considered as known \([11]\). From Eq. (14) the flow rate is then resolved as

\[ Q = \mu A_{D0} \sqrt{\frac{2gh}{1-\left(\mu A_{D0}/A\right)^2}}. \tag{15} \]

This equation is inserted into Eq. (5) with \( Q = cA \), leading to

\[ h + \frac{a\mu}{g} A_{D0} \sqrt{\frac{2g}{1-\left(\mu A_{D0}/A\right)^2}} \sqrt{h} - \left(h_0^* + 2f\right) = 0, \hspace{1cm} (16) \]

in which, the initial stable flow condition has been expressed as

\[ h_0^* = h_0 + \frac{a}{gA} Q_0. \tag{17} \]

Equation (16) represents a quadratic polynomial for the parameter \( \sqrt{h} \) to each given value of \( \mu \). One point to be mentioned is that one will obtain a quadratic polynomial for the flow rate \( Q \), when Eq. (15) is combined with Eq. (5) for eliminating the head \( h \).

With the solution of \( h \) from Eq. (16), thus, the primary wave \( F \) is obtained from Eq. (11) as

\[ F = h - h_0 - f. \tag{18} \]

Within the time \( t < 2L/a \), according to Fig. 1, there is \( f = 0 \) in both Eqs. (16) and (18), so that the primary wave is simply calculated by \( F = h - h_0 \). After a time delay of \( t = 2L/a \) the wave \( f \), which is the reflection of the primary wave \( F \) at the reservoir, travels back to the injector and is available because \( f(t) = -F(t - 2L/a) \).

![Fig. 2 Injector nozzle of Pelton turbines and its dimensioning](image)
In most cases of hydraulic systems, the initiation of transient flows can be simulated by a Pelton injector as a regulation organ, without having to be concerned with the fluid machines and their characteristics.

4. Verification of the method

The computational algorithm presented above has been applied to a hydraulic system of Pelton turbines (Fig. 3a) by comparing computational results with experimental measurements [9]. The pressure head measurement was accomplished while the Pelton injector was getting closed within 45 s. As can be confirmed from Fig. 3b, computational results very well agree with the measurements.

Compared to the simple hydraulic system shown in Fig. 1, the algorithm related to Fig. 3 is more complex because of the presence of the surge tank. In the realization, a T-type branch connects the surge tank with the main pipe. The wave propagation through such a T-type branch includes wave transmissions and reflections. According to the sketch in Fig. 3a, the incoming waves \( F_1, f_2 \) and \( f_3 \) are considered as known. The departing waves comprise both the transmission and the reflection of incoming waves and can be calculated as follows [10]:

\[
\begin{align*}
F_1 &= \frac{A_1 - A_2 - A_3}{A_1 + A_2 + A_3} F_3 + \frac{2A_2}{A_1 + A_2 + A_3} f_2 + \frac{2A_3}{A_1 + A_2 + A_3} f_1, \quad (19) \\
f_1 &= \frac{2A_2}{A_1 + A_2 + A_3} f_3 + \frac{A_2 - A_1 - A_3}{A_1 + A_2 + A_3} f_2 + \frac{2A_1}{A_1 + A_2 + A_3} f_3, \quad (20) \\
f_1 &= \frac{2A_3}{A_1 + A_2 + A_3} f_3 + \frac{2A_2}{A_1 + A_2 + A_3} f_2 + \frac{A_1 - A_2 - A_3}{A_1 + A_2 + A_3} f_3. \quad (21)
\end{align*}
\]

These three equations represent the fundamental equations in the wave tracking method for computing the transient flows in each complex hydraulic system. The above equations also apply to a pipe with abrupt section change. One only needs to set \( A_3 = 0 \).

5. Unification of Characteristics of the Pump and the Spherical Valve

Large pumps in pumped hydro power plants are always installed in connection with a spherical valve close to the pump on its high pressure side, as sketched in Fig. 4. While the pump, for instance, is running down, the spherical valve should be progressively closed. For computations of so resulted transient flows one must know the characteristics of both the pump and the spherical valve. It seems to be very convenient to consider the pump and the spherical valve as an integrated hydraulic unit and to use the unified characteristic. As is expected, the unified characteristic should show the flow rate as a...
function of the rotational speed of the pump and the opening degree of the spherical valve. On the other hand, the unified characteristic has to be represented in a suitable mathematical form, in order to achieve easy applications.

For the sake of demonstrating the computational method only the case of shutting down the pump by simultaneously closing the spherical valve is considered. This also includes the reverse rotation of the pump and the back flow through the pump.

To the pump characteristic only four-quadrant diagrams are applicable. Accordingly, the following dimensionless parameters for the rotational speed, the flow rate and the shaft torque are defined

\[ n_{11} = \frac{\pi d_2 n}{\sqrt{2 gH_{pu}}} \quad Q_{11} = \frac{Q}{d_2^2 \sqrt{2 gH_{pu}}} \quad M_{11} = \frac{M}{\rho d_2^3 gH_{pu}}. \]  

(22)

In these definitions, \( n \) is in \( 1/\text{s} \) and \( Q \) in \( \text{m}^3/\text{s} \). \( d_2 \) is the pump impeller diameter.

Fig. 5 shows the four-quadrant diagrams of a pump, which are measured from laboratory tests. They will be used in the present paper to compute the related transient flows. To obtain the combined characteristic, the following 4 steps, separated in Sects. 5.1-5.4, should be performed.

5.1. Pump Characteristic: Flow Rate
For later convenience of calculations the four-quadrant diagram in form of \( Q_{11} = f(n_{11}) \) in Fig. 5a should be approximated by the following quadratic equation

\[ Q_{11}^* = q + m_1(n_{11}Q_{11}) + m_2 n_{11}^2. \]  

(23)

The reason of the mathematical form in this equation will become clear below. The constants \( q, m_1 \), and \( m_2 \) can be determined by the least square curve fitting method with two independent parameters

![Fig. 4 Unification of a pump and a spherical valve](image-url)

![Fig. 5 Four-quadrant diagrams of a pump](image-url)

\( n_{1i}Q_{1i} \) and \( n_{2i} \), which can be obtained from the available data \( Q_{1i} = f(n_{1i}) \) from Fig. 5a. Because the calculation may cover through a range of rotational speed from negative to positive values, the characteristic \( Q_{1i} = f(n_{1i}Q_{1i}, n_{2i}) \), in the current case, must be approximated at least in two areas as for \( n_{1i} > 0.88 \) and \( n_{1i} < 0.88 \), respectively.

Equation (23) is then written in the explicit form

\[
H_{pu} = \frac{1}{2g} \left( \frac{Q^{2}}{d_2} - \frac{m_{1}n_{1}Q}{d_2} - \pi^{2}m_{2}n_{2}d_{2}^{2} \right). \tag{24}
\]

This form of the pump characteristic applies to a pipe system with one pump. For two or more pumps in the pump house the flow rate in the above equation must be related to one pump.

5.2. Pump Characteristic: Torque

The torque characteristic, \( M \), in the combined unit (pump and valve) remains the same as for the pump alone. For computational purposes the relation that is shown in Fig. 5b can be directly used.

5.3. Spherical Valve Characteristic

The hydraulic characteristic of a spherical valve is represented by the head drop in the flow through the valve as a function of the opening degree \( \beta \), with \( \beta = 90^\circ \) for the fully open valve. With respect to a possible back flow, the head drop at the spherical valve is given by

\[
\Delta h = c_{p} \frac{k|k|}{2g} = c_{p} \frac{|Q|Q}{2gA_{psh}}. \tag{25}
\]

The sectional area of the spherical valve is usually equal to sectional area of the pipe. For nearly all spherical valves in the hydropower plants, the pressure drop coefficient has been approximated by Zhang [10], in most appropriate form as given by

\[
c_{p} = \frac{10^{6}}{0.108 \beta^{2} + 1.35 \beta} - 1. \tag{26}
\]

5.4. Unification of Characteristics of the Pump and the Spherical Valve

According to Fig. 4 the total pressure head over the combined unit is given by

\[
H_{12} = H_{pu} - \Delta h. \tag{27}
\]

Inserting Eq. (24) and (25) into this equation yields

\[
\left( \frac{1}{d_2^2} \frac{c_{p}qQ}{A_{psh}} \frac{|Q|}{Q} \right)Q^{2} - \frac{m_{1}n_{1}Q}{d_2} - \pi^{2}m_{2}n_{2}d_{2}^{2} + 2gqH_{12} = 0. \tag{28}
\]

This describes the unified characteristic of the pump and the spherical valve. It represents a quadratic polynomial of the dependent variable i.e. the flow rate \( Q \), which can, thus, be immediately calculated. This simply justifies the reason of the mathematical expression that is proposed in Eq. (23).

In Eq. (28), \( |Q|/Q = \pm 1 \), depending on the flow direction. With respect to \( c_{p} = f(\beta) \) from Eq. (26) the unified characteristic can be represented in the general functional form as

\[
Q_{pu} = f(H_{12}, n, \beta). \tag{29}
\]

It should be reminded that the constants \( q, m_{1} \) and \( m_{2} \) are applicable each time only to an appropriate area in the four-quadrant characteristic. This has already been explained at Eq. (23).
The flow rate, according to Eq. (29), also depends on the rotational speed which by winding down the pump is not constant. The development of the rotational speed, in turn, depends on the total moment of inertia \( J \) of the rotor system. From the momentum law and with respect to the resistance torque \( M \) exerted on the rotor system the angular momentum balance, after the motor is shut down, is given by

\[
2\pi J \frac{dn}{dt} = -M ,
\]

with the aid of Eq. (22) one obtains

\[
\frac{dn}{dt} = -\frac{\pi \rho A_2}{4 J} n^2 \frac{M_1}{n_{11}} .
\]

As for \( M_{11} = f(n_{11}) \), each time when \( n_{11} \) and, thus, \( M_{11} \) have been computed, then \( dn/\!\!\!dt \) can be calculated straightforwardly. In the numerical method, the rotational speed in the current time step is then given by

\[
n_t = n_{11} + \left( \frac{dn}{dt} \right)_{n_{11}} \Delta t .
\]

5.5. Generation of the Primary Waves

The wave generation on both the suction (index 1) and pressure (index 2) sides of the pump should be considered, at which the flow rate can be assumed to be equal \((Q_1=Q_2)\). On the suction side of the pump, the pressure head increase is given, as from Eq. (5), by

\[
h_1 - h_{1,0} = 2f_1 - \frac{a}{gA_1}(Q - Q_0)
\]

Within the reflection time of the first pressure surge, the reflected wave is zero \((f_1=0)\).

On the pressure side, where the positive flow rate is in the direction of the primary wave, the pressure-head increase is followed from Eq. (5) to be

\[
h_2 - h_{2,0} = 2f_2 + \frac{a}{gA_2}(Q - Q_0) .
\]

The pressure difference on both sides of the combined unit is then obtained as

\[
h_2 - h_1 = (h_{2,0} - h_{1,0}) + 2(f_2 - f_1) + \frac{a}{g} \left( \frac{1}{A_2} + \frac{1}{A_1} \right)(Q - Q_0) .
\]

5.6. Pump Flow in Association with the Wave Parameters

Combining Eq. (34) with Eq. (28) one obtains, with \( H_{12} = h_2 - h_1 \) and \( H_{12,0} = h_{2,0} - h_{1,0} \) according to Fig. 4, the final determining equation as

\[
\left( \frac{1}{d_{\text{ph}}^2} - \frac{c_p g Q}{A_{\text{ph}} Q} \right) Q^2 = \left[ \frac{\pi m L}{d_2} + 2a q \left( \frac{1}{A_2} + \frac{1}{A_1} \right) \right] Q

+ 2 g q \left( \frac{1}{A_2} + \frac{1}{A_1} \right) \left( Q_0 - m_2 \pi n^2 d_2^2 \right) - 2 g q \left[ H_{12,0} + 2(f_2 - f_1) \right] = 0
\]

This equation remains of quadratic form for the flow rate \( Q \). From tracking the wave parameters \( f_1 \) and \( f_2 \) the flow rate at each time can immediately be calculated. In the numerical computations, the actual value \( |Q/Q| = \pm 1 \) in the above equation can be set to be equal to that of the last finite time step. The rotational speed \( n \) is obtained directly from \( dn/\!\!\!dt \) in that time step.
As soon as the flow rate is calculated, the primary waves both on the suction and the pressure sides, respectively, can again be computed from Eq. (12) as

\[ F_1 = f_1 - \frac{a}{gA_1} (Q - Q_0), \quad F_2 = f_2 + \frac{a}{gA_2} (Q - Q_0). \]  
(36), (37)

6. Application of the Method

Figure 6 shows a pumping system in the Oberhasli Hydroelectric Power Company (KWO). At this pumping system a test of emergency shut-down of the pump with simultaneous closing of the spherical valve was executed [10]. A part of the measurement results will be used to confirm the numerical calculations based on the MS Excel tool.

The basic equation for the transient flow computations is Eq. (35). For the wave parameters \( f_1 \) and \( f_2 \), one then needs to separately track the propagation of the waves \( F \) and \( f \) in both the pressure and suction sides of the pump. It is actually the same computational process as that shown in Sect 2. However, the computations must be extended to the current case with three surge tanks and eight pipes of different diameters. For calculating all relevant flow parameters and the time-dependent rotational speed of the pump, more than 40 columns in the scheme of tabular computations are necessary.

Figures 7 to 8 show the parts of computational results which are compared with measurements. Not only the pressure response in the hydraulic system but also the rotational speed of the pump could precisely be predicted by the computations. Such a high level of computational accuracy relies on the fact that highly reliable assumptions have been applied. Because the MS Excel tool is in this case powerful and, thus, appropriate for such computations, on-line investigations of all flow and operating parameters can be easily performed.

Because of the restriction of the paper length more detailed computational results will be presented in a journal paper.
7. Conclusion
Based on the investigations and applications carried out by the author the wave tracking method among diverse computational algorithms has been demonstrated to be very easily applicable, also to highly complex system with unsteady elements like the T-branches as well as with regulation organs and hydraulic machines (pumps, turbines etc.). By following the propagations of the wave parameters $F$ and $f$ (as solutions of the wave equation) in the system and through unsteady sections, the transient flow can be very easily and accurately calculated, for instance, by simply using the MS Excel-sheet. As demonstrated, the method also applies to the transient flow caused by the pump shut-down with simultaneous closing of the spherical valve. The key elements of the precondition are to work out the combined characteristics of the pump and the spherical valve on the one hand and to obtain a quadratic polynomial for easily solving the flow rate on the other hand.

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