Loop Suppression of Dirac Neutrino Mass in the Neutrinophilic Two Higgs Doublet Model

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Abstract

We extend the scalar sector of the neutrinophilic two Higgs doublet model, where small masses of Dirac neutrinos are obtained via a small vacuum expectation value \( v_\nu \) of the neutrinophilic SU(2)\(_L\)-doublet scalar field which has a Yukawa interaction with only right-handed neutrinos. A global U(1)\(_X\) symmetry is used for the neutrinophilic nature of the second SU(2)\(_L\)-doublet scalar field and also for eliminating Majorana mass terms of neutrinos. By virtue of an appropriate assignment of the U(1)\(_X\)-charges to new particles, our model has an unbroken \( Z_2 \) symmetry, under which the lightest \( Z_2 \)-odd scalar boson can be a dark matter candidate. In our model, \( v_\nu \) is generated by the one-loop diagram to which \( Z_2 \)-odd particles contribute. We briefly discuss a possible signature of our model at the LHC.

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I. INTRODUCTION

It has been well established that neutrinos have nonzero masses as shown in the neutrino oscillation measurements [1–6] although they are massless particles in the standard model (SM) of particle physics. Since the scale of neutrino masses is much different from that of the other fermion masses, they might be generated by a different mechanism from the one for the other fermions. Usually, the possibility that neutrinos are Majorana fermions is utilized as a characteristic feature of the neutrino masses. The most popular example is the seesaw mechanism [7] where very heavy right-handed Majorana neutrinos are introduced. However, lepton number violation which is caused by masses of the Majorana neutrinos has not been discovered. Thus it is worth considering the possibility that neutrinos are not Majorana fermions but Dirac fermions similarly to charged fermions.

The neutrinophilic two Higgs doublet model ($\nu$THDM) is a new physics model where neutrinos are regarded as Dirac fermions. The second SU(2)$_L$-doublet scalar field which couples only with right-handed neutrinos $\nu_R$ was first introduced in Ref. [8] for Majorana neutrinos. Phenomenology in the model of Majorana neutrinos is discussed in Ref. [9, 10]. The neutrinophilic doublet field is also utilized for Dirac neutrinos [11] where a spontaneously broken $Z_2$ parity is introduced in order to achieve the neutrinophilic property. Smallness of neutrino masses are explained by a tiny vacuum expectation value (VEV) of the neutrinophilic scalar without extremely small Yukawa coupling constant for neutrinos. Instead of the $Z_2$ parity, the model in Ref. [12] uses a global $U(1)_X$ symmetry that is softly broken in the scalar potential. The $U(1)_X$ symmetry forbids Majorana mass terms of $\nu_R$, and then neutrinos are Dirac fermions\(^1\). We refer to the model in Ref. [12] as the $\nu$THDM.

The new particle which was discovered at the LHC [13, 14] is likely to be the SM Higgs boson [15–18]. It opens the new era of probing the origin of particle masses. Then it would be a natural desire to expect that the origin of neutrino masses are also uncovered. If the neutrinophilic scalars in the $\nu$THDM exist within the experimentally accessible energy scale (namely the TeV-scale), decays of the neutrinophilic charged scalar into leptons can provide direct information on the neutrino mass matrix because it is proportional to the matrix of new Yukawa coupling constants for the neutrinophilic scalar field [12, 19]. In such

\(^1\) Since the Majorana mass terms of $\nu_R$ can also be acceptable as soft breaking terms of the $U(1)_X$, the lepton number conservation may be imposed to the Lagrangian.
a case, the smallness of a new VEV which is relevant to Dirac neutrino masses is interpreted by the smallness of a soft-breaking parameter of the global \( U(1)_X \) symmetry. It seems then better to have a suppression mechanism for the soft-breaking parameter by extending the \( \nu \)THDM with TeV scale particles including a dark matter candidate. The existence of dark matter has also been established in cosmological observations \([20, 21]\), and it is an important guideline for constructing new physics models.

The reason why the neutrino masses are tiny can be explained by a mechanism that the interaction of neutrinos with the SM Higgs boson is generated via a loop diagram involving a dark matter candidate in the loop while the interaction is forbidden at the tree level \([22–32]\). Notice that smallness of neutrino masses in such radiative mechanisms does not require new particles to be very heavy. Similarly, if neutrino masses arise from a new VEV, smallness of neutrino masses can be explained by assuming that the VEV is generated at the loop level by utilizing a dark matter candidate \([33]\). In this paper, we extend the \( \nu \)THDM such that the new VEV is generated at the one-loop level (see also Ref. \([34]\)) where a dark matter candidate is involved in the loop.

This paper is organized as follows. We briefly introduce the \( \nu \)THDM in Sec. II. The \( \nu \)THDM is extended in Sec. III such that a small VEV is generated via the one-loop diagram which involving a dark matter candidate in the loop. Section IV is devoted to discussion on phenomenology in the extended \( \nu \)THDM. We conclude in Sec. V.

II. NEUTRINOPHILIC TWO-HIGGS-DOUBLET MODEL

In the \( \nu \)THDM, the SM is extended with the second \( SU(2)_L \)-doublet scalar field \( \Phi_\nu \) which has a hypercharge \( Y = 1/2 \) and right-handed neutrinos \( \nu_{iR} \) \((i = 1-3)\) which are singlet fields under the SM gauge group. A global \( U(1)_X \) symmetry is introduced, under which \( \Phi_\nu \) and \( \nu_{iR} \) have the same nonzero charge while the SM particles have no charge. Then, the Yukawa interaction with \( \Phi_\nu \) is only the following one:

\[
\mathcal{L}_{\nu-\text{Yukawa}} = -(y_\nu)_{\ell i} \bar{\ell}_\ell i \sigma_2 \Phi_\nu^* \nu_{iR} + \text{h.c.},
\]  

(1)

where \( \ell(= e, \mu, \tau) \) denotes the lepton flavor and \( \sigma_i \) \((i = 1-3)\) are the Pauli matrices. Since Majorana mass terms \( \overline{(\nu_{iR})^c} \nu_{iR} \) are forbidden by the \( U(1)_X \) symmetry, there appears an accidental conservation of the lepton number where lepton numbers of \( \Phi_\nu \) and \( \nu_{iR} \) are 0 and
1, respectively. When the neutral component $\phi^0_\nu$ of $\Phi_\nu$ develops its VEV $v_\nu (\equiv \sqrt{2}\langle \phi^0_\nu \rangle)$, the neutrino mass matrix arise as $(m_\nu)_{\ell i} = v_\nu(y_\nu)_{\ell i}/\sqrt{2}$. We have taken a basis where $\nu_{iR}$ are mass eigenstates. Then the mass matrix $m_\nu$ is diagonalized as $U_{\text{MNS}}\hat{M}_\nu = \text{diag}(m_1, m_2, m_3)$, where $m_i$ $(i = 1-3)$ are the neutrino mass eigenvalues and a unitary matrix $U_{\text{MNS}}$ is the so-called Maki-Nakagawa-Sakata (MNS) matrix \[35\]. Dirac neutrinos are constructed as $\nu_i = (\sum_\ell(U_{\text{MNS}}^\dagger)v_\ell, \nu_{iR})^T$. Smallness of neutrino masses is attributed to that $v_\nu$ is much smaller than $v$.

If the VEV $v_\nu$ is generated spontaneously, a CP-odd scalar $\phi^0_{\nu i}$ becomes massless as a Nambu-Goldstone boson with respect to the breaking of $\text{U}(1)_X$, where $\phi^0_{\nu i} = (v_\nu + \phi^0_{\nu r} + i\phi^0_{\nu i})/\sqrt{2}$. In addition, a CP-even neutral scalar $\phi^0_{\nu r}$ has a small mass ($\propto v_\nu \ll v$). Therefore, the scenario of the spontaneous breaking of $\text{U}(1)_X$ is not allowed by the measurement of the invisible decay of the $Z$ boson. The scalar potential in the $\nu$THDM is given by

$$V(\nu\text{THDM}) = -\mu^2_{\Phi 1}\Phi^\dagger\Phi + \mu^2_{\Phi 2}\Phi^\dagger\Phi - (\mu^2_{\Phi 12}\Phi^\dagger\Phi + \text{h.c.}) + \lambda_{\Phi 1}(\Phi^\dagger\Phi)^2 + \lambda_{\Phi 2}(\Phi^\dagger\Phi)^2 + \lambda_{\Phi 12}(\Phi^\dagger\Phi)(\Phi^\dagger\Phi) + \lambda'_{\Phi 12}(\Phi^\dagger\Phi)(\Phi^\dagger\Phi),$$

(2)

where $\mu^2_{\Phi 12}$ can be real and positive by using rephasing of $\Phi_\nu$ without loss of generality; We take $\mu^2_{\Phi 1} > 0$ and $\mu^2_{\Phi 2} > 0$. The VEV of $\phi^0_\nu$ is triggered by $\mu^2_{\Phi 12}$ which softly breaks the $\text{U}(1)_X$ symmetry. Since the term does not breaks the lepton number conservation, neutrinos are still Dirac particles. Taking $v_\nu/v \ll 1$ into account, the VEVs are calculated as

$$v \simeq \frac{\mu_{\Phi 1}}{\sqrt{\lambda_{\Phi 1}}}, \quad v_\nu \simeq \frac{2v\mu^2_{\Phi 12}}{2\mu^2_{\Phi 2} + (\lambda_{\Phi 12} + \lambda'_{\Phi 12})v^2}.$$  

(3)

If $\mu_{\Phi 2} \sim v$, we have $v_\nu \sim \mu^2_{\Phi 12}/v$. Then, $\mu_{\Phi 12}/v$ is required to be small ($\sim 10^{-6}$ for $y_\nu \sim 1$). Stability of the tiny $v_\nu$ is discussed in Refs. \[10, 36\]. In our model presented in the next section, $\mu_{\Phi 12}/v$ becomes small because $\mu^2_{\Phi 12}$ is generated at the one-loop level.

### III. AN EXTENSION OF THE $\nu$THDM

Since we try to generate $\mu^2_{\Phi 12}$ at the loop level, it does not appear in the Lagrangian. Then the $\text{U}(1)_X$ symmetry should be broken spontaneously. For the spontaneous breaking, we rely on an additional scalar $s^0_1$ which is a singlet field under the SM gauge group. Similarly to the singlet Majoron model \[37\] where a VEV of a singlet field spontaneously breaks the lepton number conservation by two units, the Nambu-Goldstone boson from $s^0_1$ is acceptable \[37\].
the Nambu-Goldstone boson couples first with only neutrinos among fermions. If $U(1)_X$-charges of $\Phi_\nu$ and $s^0_1$ are 3 and 1, respectively, a dimension-5 operator $(s^0_1)^3 \Phi^\dagger_\nu \Phi$ is allowed by the $U(1)_X$ symmetry although $\Phi^\dagger_\nu \Phi$ is forbidden. Then, $\mu_{\Phi_12}^2$ is generated from the dimension-5 operator with the VEV of $s^0_1$. In this paper, we show the simplest realization of the dimension-5 operator at the one-loop level where dark matter candidates are involved in the loop.

Table I is the list of new particles added to the SM. In the table, $\nu_{iR}$ and $\Phi_\nu$ are the particles which exist in the $\nu$THDM. The $U(1)_X$ symmetry is spontaneously broken by the VEV of $s^0_1$. We take a scenario where $\eta$ and $s^0_2$ do not have VEVs. Since their $U(1)_X$-charges are half-integers while the one for $s^0_1$ is an integer, a $Z_2$ symmetry remains unbroken after the $U(1)_X$ breaking. Here, $\eta$ and $s^0_2$ are $Z_2$-odd particles. The $Z_2$ symmetry stabilizes the lightest $Z_2$-odd particle which can be a dark matter candidate.

The Yukawa interaction in this model is identical to those in the $\nu$THDM (see Eq. (1)). The scalar potential in this model is expressed as

$$
V = -\mu_{s_1}^2 s^0_1 |s^0_2|^2 + \mu_{s_2}^2 |s^0_2|^2 - \mu_{\Phi_1}^2 \Phi^\dagger \Phi + \mu_{\Phi_2}^2 \Phi^\dagger_\nu \Phi + \mu^2 \eta^\dagger \eta
$$

$$
- (\mu_{s_1} s^0_1 (s^0_2)^* + ) h.c.
$$

$$
+ (\lambda_{s_1} s^0_1 (s^0_2)^* \Phi^\dagger \eta + h.c.) + (\lambda_{s_2} s^0_1 s^0_2 \Phi^\dagger_\nu \eta + h.c.) + \cdots.
$$

(4)

Only the relevant parts to our discussion are presented in Eq. (4). The other terms are shown in Appendix. Parameters $\mu$, $\lambda_{s_1}$, and $\lambda_{s_2}$ are taken to be real and positive values by rephasing of scalar fields without loss of generality. At the tree level, $v_\nu$, $v$, and $v_s (= \sqrt{2} (s^0_1))$ are given by

$$
v_\nu = 0, \quad \left( \frac{v^2}{v_s^2} \right) = \frac{2}{4\lambda_{s_1} \lambda_{s_1} - \lambda_{s_1}^2} \begin{pmatrix} 2\lambda_{s_1} & -\lambda_{s_1} \Phi_1 \\ -\lambda_{s_1} \Phi_1 & 2\lambda_{s_1} \end{pmatrix} \begin{pmatrix} \mu_{s_1}^2 \\ \mu_{s_1}^2 \end{pmatrix}.
$$

(5)
The $Z_2$-odd scalar fields ($\eta$ and $s^0_2$) result in the following particles: two CP-even neutral scalars ($H_1^0$ and $H_2^0$), two CP-odd neutral ones ($A_1^0$ and $A_2^0$), and a pair of charged ones ($H^\pm$). It is clear that $H^\pm = \eta^\pm$. When $H_1^0$ (or $A_1^0$) is lighter than $H^\pm$, the neutral one becomes the dark matter candidate. On the other hand, from $Z_2$-even scalar fields ($\Phi$, $\Phi^\nu$, and $s^0_1$), we have three CP-even particles ($h^0$, $H^0$, and $H^0_\nu$), two CP-odd ones ($A^0_\nu$ and a massless $z^0_2$), and a pair of charged scalars ($H^\pm_\nu$). The mixings between $\phi^0_\nu$ and others are ignored because we take $v_\nu/v \ll 1$ and $v_\nu/v_s \ll 1$. Then, $\Phi_\nu$ provides $H^0_\nu$ ($= \phi^0_\nu r$), $A^0_\nu$ ($= \phi^0_\nu i$), and $H^\pm_\nu$ ($= \phi^\pm_\nu$). It is easy to see that $z^0_2 = s^0_1$, where $s^0_1 = (v_s + s^0_1 r + is^0_1 i)/\sqrt{2}$. The formulae of scalar mixings and scalar masses are presented in Appendix. Hereafter, we assume that scalar fields in Tab. I are almost mass eigenstates just for simplicity, which is achieved when $\lambda_{s\Phi 1\eta}$ and $\lambda_{s1\Phi 1}$ are small.

By using cubic and quartic interactions shown in Eq. (4), the interaction $\Phi^\dagger_\nu \Phi$ is obtained with the one-loop diagram in Fig. 1. The coefficient $(\mu^2_{\Phi 12})_{\text{eff}}$ of the interaction is calculated as

$$
(\mu^2_{\Phi 12})_{\text{eff}} = \frac{\mu \lambda_{s\Phi 1\eta} \lambda_{s\Phi 2\eta} v^3_s}{32\sqrt{2} \pi^2 (m^2_\eta - m^2_{s_2})} \left( 1 - \frac{m^2_\eta}{m^2_\eta - m^2_{s_2}} \ln \frac{m^2_\eta}{m^2_{s_2}} \right),
$$

where

$$
m^2_\eta \equiv \mu^2_\eta + \frac{1}{2} \left( \lambda_{\Phi 1\eta} + \lambda_{\Phi 1\eta}' \right) v^2 + \lambda_{s1\eta} v^2_s,
$$

$$
m^2_{s_2} \equiv \mu^2_{s_2} + \frac{1}{2} \left( \lambda_{s2\Phi 1} v^2 + \lambda_{s12} v^2_s \right).
$$
Ignoring loop corrections to terms which exist at the tree-level, we finally arrive at

\[ v_\nu = \frac{v (\mu_{\Phi 12})_{\text{eff}}}{m_{H_\nu}^2} \tag{9} \]

where \( m_{H_\nu}^2 \equiv \mu_{\Phi 2}^2 + \frac{1}{2} (\lambda_{s12} + \lambda_{s12}) v^2 + \frac{1}{2} \lambda_{s1\Phi 2} v_s^2 \) which is the mass of \( H_\nu^0 (= \phi_{\nu r}^0) \). For example, we have \( m_{\nu} = \mathcal{O}(0.1) \text{eV} \) for \( m_s = \mathcal{O}(10) \text{GeV} \) (as the dark matter mass), \( v_s \sim m_\eta = \mathcal{O}(100) \text{GeV} \), \( \mu = \mathcal{O}(1) \text{GeV} \), \( y_\nu = \mathcal{O}(10^{-4}) \), and \( \lambda_{s1\eta} \sim \lambda_{s2\eta} = \mathcal{O}(10^{-2}) \).

### IV. PHENOMENOLOGY

Hereafter, we take the following values of parameters as an example:

\[ (y_\nu)_{\ell i} \sim 10^{-4}, \quad \lambda_{s1\eta} = \lambda_{s2\eta} = 10^{-2}, \quad \mu = 1 \text{GeV}, \quad v_s = 300 \text{GeV}, \quad \]

\[ m_{H_\nu}^0 = m_{A_\nu}^0 = m_{H^{0 \pm}} = 300 \text{GeV}, \quad m_{H_1^0} = 230 \text{GeV}, \quad m_{H_1^0} = 60 \text{GeV}. \tag{10} \]

These values can satisfy constraints from the \( \rho \) parameter, searches of lepton flavor violating processes, the relic abundance of dark matter, and direct searches for dark matter. In order to satisfy \( \rho \simeq 1 \), particles which come from an SU(2) multiplet have a common mass. If \( H_1^0 \simeq \eta_\nu^0 \) for example, we take \( m_{H^{0 \pm}} \sim m_{A_\nu}^0 \sim m_{H_1^0} \). Since \( y_\nu \) is not assumed to be very large, contributions of \( H^{0 \pm} \) to lepton flavor violating decays of charged leptons are negligible. For example, the branching ratio \( \text{BR}(\mu \to e\gamma) \) \[12\] is proportional to \( |(y_\nu y_\nu^\dagger)_{\mu e}|^2 \) and becomes about \( 10^{-22} \) which is much smaller than the current bound at the MEG experiment \[38\]: \( \text{BR}(\mu \to e\gamma) < 5.7 \times 10^{-13} \) at the 90\% confidence level.

#### A. Dark Matter

We assume that the mixing between \( s_2^0 \) and \( \eta^0 \) is negligible for simplicity, which corresponds to the case \( \lambda_{s1\eta} \ll 1 \). Then, the dark matter candidate \( H_1^0 \) is dominantly made from \( s_2^0 \) or \( \eta_\nu^0 \). We also assume that \( \lambda_{s12}|s_1^0|^2|s_2^0|^2 \) and \( \lambda_{s1\eta}|s_1^0|^2(\eta^\dagger \eta) \) are negligible in order to avoid \( H_1^0 H_1^0 \rightarrow s_2^0 s_2^0 \) which would reduce the dark matter abundance too much. Notice that these coupling constants (\( \lambda_{s12} \) and \( \lambda_{s1\eta} \)) are not used in the loop diagram in Fig. \[11\]. When \( H_1^0 \simeq s_2^0 \), the \( H_1^0 \) is similar to the real singlet dark matter in Ref. \[39\]. Experimental constraints on the singlet dark matter can be found e.g. in Ref. \[40\]. We see that 53 GeV \( \lesssim m_{H_1^0} \lesssim 64 \) GeV and 90 GeV \( \lesssim m_{H_1^0} \) are allowed. On the other hand, when \( H_1^0 \simeq \eta_\nu^0 \), the dark matter is similar to
the one in the so-called inert doublet model \cite{41,42}. See e.g. Refs. \cite{43,44} for experimental constraints on the inert doublet model. It is shown that $45 \text{ GeV} \lesssim m_{\mathcal{H}_1^0} \lesssim 80 \text{ GeV}$ is allowed.

In order to suppress the scattering of $H^0_1$ on nuclei mediated by the $Z$ boson, sufficient splitting of $m_{\mathcal{H}_1^0}$ and $m_{A^0_1}$ is required: $m_{A^0_1} - m_{\mathcal{H}_1^0} \gtrsim 100 \text{ keV}$ (See e.g. Ref. \cite{44}). Values of $m_{\mathcal{H}_1^0}$ and $m_{A^0_2}$ in Eq. (10) are obtained by using $m_{\eta} = 60 \text{ GeV}$ and $m_{s} = 231 \text{ GeV}$ in Eqs. (23) and (24) in Appendix, and then these values of $m_{\eta}$ and $m_{s}$ give $m_{A^0_1} - m_{\mathcal{H}_1^0} \simeq 400 \text{ keV}$.

Since we discuss in the next subsection a possible collider signature where $H^0_\nu$ decays into $\mathcal{H}_1^0$, a light dark matter ($m_{\mathcal{H}_1^0} \simeq m_{h^0}/2$) is interesting such that $H^0_\nu$ (and $H^\pm_\nu$) can also be light. We take $m_{\mathcal{H}_1^0} = 60 \text{ GeV}$ as an example for both cases, $\mathcal{H}_1^0 \simeq s^0_{2r}$ and $\mathcal{H}_1^0 \simeq \eta^0_r$.

**B. Collider**

In the $\nu$THDM as well as in our model, the neutrino mass matrix $m_\nu$ is simply proportional to $y_\nu$. The flavor structure of $H^+_\nu \rightarrow \ell_L \nu_R$ (summed over the neutrinos) is predicted \cite{12} by using current information on $m_\nu$ obtained by neutrino oscillation measurements. The prediction enables the $\nu$THDM to be tested at collider experiments. Since this advantage should not be spoiled, $H^+_\nu \rightarrow \mathcal{H}_1^0 \mathcal{H}_2^\pm$ ($\mathcal{H}_2^\pm \mathcal{H}_1^0$) should be forbidden for $\mathcal{H}_1^0 \simeq s^0_{2r}$ ($\mathcal{H}_1^0 \simeq \eta^0_r$). Therefore, we assume that $m_{\mathcal{H}^\pm}$ satisfies $m_{\mathcal{H}^\pm} \leq m_{\mathcal{H}_1^0} + m_{\mathcal{H}^\pm}$ for $\mathcal{H}_1^0 \simeq s^0_{2r}$ or $m_{\mathcal{H}^\pm} \leq m_{\mathcal{H}^\pm} + m_{\mathcal{H}_2^0}$ for $\mathcal{H}_1^0 \simeq \eta^0_r$, for example, $m_{\mathcal{H}^\pm} = 250 \text{ GeV}$ (100 GeV) for $\mathcal{H}_1^0 \simeq s^0_{2r}$ ($\eta^0_r$).

The process in Fig. 2 would be a characteristic collider signature of our model. Notice that the process utilizes two coupling constants ($\lambda_{s\Phi_1\eta}$ and $\lambda_{s\Phi_2\nu}$) which appear also in Fig. 1. Thus, the process indicates that $\mu^2_{\tilde{\nu}_{12}} \Phi_{\nu}^2 \Phi$ is radiatively generated with a contribution
of dark matter. In the original \( \nu \)THDM in comparison, \( H_\nu^0 \) decays into \( \nu \vec{\nu} \) for the case with \( m_{H^0} = m_{H^0} \). In order to observe the process in Fig. 2 the partial decay width \( \Gamma(H_\nu^0 \rightarrow \mathcal{H}_1^0 H_2^0) \) should be larger than \( \Gamma(H_\nu^0 \rightarrow \nu \bar{\nu}) \). Using our benchmark values, we have

\[
\Gamma(H_\nu^0 \rightarrow \nu \bar{\nu}) = \frac{\text{tr}(y_\nu^\dagger y_\nu)m_{H^0}}{16\pi} \approx 60 \text{ eV},
\]

\[
\Gamma(H_\nu^0 \rightarrow \mathcal{H}_1^0 H_2^0) = \frac{\lambda^2_{\Phi 2} v_s^2}{64\pi m_{H^0}} \left[ 1 - \frac{(m_{H_2^0} + m_{H_1^0})^2}{m_{H_2^0}} \right] \left[ 1 - \frac{(m_{H_2^0} - m_{H_1^0})^2}{m_{H_1^0}^2} \right] \approx 30 \text{ keV}.
\]

Then, \( H_\nu^0 \) decays into \( \mathcal{H}_1^0 \mathcal{H}_2^0 \) dominantly\(^2\). If \( y_\nu \) is large enough for \( \mu \rightarrow e\gamma \) to be discovered in near future, the process in Fig. 2 becomes very rare because \( H_\nu^0 \rightarrow \nu \bar{\nu} \) is the dominant channel. Next, when the mixings between \( Z_2 \)-odd particles are negligible, \( \mathcal{H}_2^0 \) can decay only into \( \mathcal{H}_1^0 h^0 \) via \( \lambda_\nu \Phi 1 \eta \) because \( H_2^0 \rightarrow \mathcal{H}_1^0 H^0 \) is kinematically forbidden for the values in Eq. (10). Thus, even if \( \lambda_\nu \Phi 1 \eta \) is rather small, the branching ratio for \( \mathcal{H}_2^0 \rightarrow \mathcal{H}_1^0 h^0 \) can be almost 100\%. As a result, the process in Fig. 2 can be free from the one-loop suppression and smallness of coupling constants \((y_\nu, \lambda_\nu \Phi 1 \eta, \text{ and } \lambda_\nu \Phi 2 \eta)\) which are used to suppress \( v_\nu \).

The cross section of \( pp \rightarrow H_\nu^+ H_\nu^0 + H_\nu^- H_\nu^0 \) for the masses in Eq. (10) is 7 fb at the LHC with \( \sqrt{s} = 14 \text{ TeV} \). The SM background events come from \( t\bar{t}, WZ, \) and \( t\bar{b} \). Cross sections for \( pp \rightarrow t\bar{t}, W^+Z+W^-Z, \) and \( t\bar{b}+t\bar{b} \) at the LHC with \( \sqrt{s} = 14 \text{ TeV} \) are 833 pb\(^{[45]}\), 55.4 pb\(^{[46]}\), and 3.91 pb\(^{[47]}\), respectively. Detailed analysis on kinematic cuts of the background events is beyond the scope of this paper.

If Nature chooses a parameter set for which the process in Fig. 2 is not possible, the deviation from the \( \nu \)THDM would be the increase of new scalar particles which might be discovered directly and/or change predictions in the \( \nu \)THDM about e.g. \( h^0 \rightarrow \gamma\gamma \).

V. CONCLUSIONS AND DISCUSSION

The \( \nu \)THDM is a new physics model where masses of Dirac neutrinos are generated by a VEV \((v_\nu)\) of the second SU(2)\(_L\)-doublet scalar field \( \Phi_\nu \) which has a Yukawa interaction with only \( \nu_R \) because of a global U(1)\(_X\) symmetry in the Lagrangian. We have presented a simple extension of the \( \nu \)THDM by introducing the third SU(2)\(_L\)-doublet scalar field \( \eta \) and two neutral SU(2)\(_L\) singlet fields \((s^0_1 \text{ and } s^0_2)\). Although the global U(1)\(_X\) is broken by a

\(^2\) Cascade decay of \( A_\nu^0 \) results in \( \mathcal{H}_1^0 \mathcal{H}_2^0 \) which is invisible similarly to \( A_\nu^0 \rightarrow \nu \bar{\nu} \).
VEV of $s^0_1$, there remains a residual $Z_2$ symmetry under which $\eta$ and $s^0_2$ are $Z_2$-odd particles. These $Z_2$-odd particles provide a dark matter candidate. The $\nu_\ell$ for neutrino masses can be suppressed without requiring very heavy particles because the VEV is generated at the one-loop level.

A possible signature of the deviation from the $\nu$THDM at the LHC is $\ell_j b_j E_T$ via $pp \to H^0_\nu H^0_\nu$ followed by $H^0_\nu \to \ell \nu$ and $H^0_\nu \to H^0_1 H^0_2 \to H^0_1 H^0_1 h^0 \to H^0_1 H^0_1 b \bar{b}$. Coupling constants which control $H^0_\nu \to H^0_1 H^0_2$ and $H^0_2 \to H^0_1 h^0$ are the ones used in the one-loop diagram which is the key to generate $\nu_\ell$.

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Appendix

1. Scalar Potential

The scalar potential $V$ is given by

$$V = V_2 + V_3 + V_4,$$

$$V_2 = -\mu s_1^0 |s_1|^2 + \mu s_2^0 |s_2|^2 - \mu_\Phi^2 \Phi \Phi + \mu_{\Phi_2}^2 \Phi_\nu^\dagger \Phi_\nu + \mu_\eta^2 \eta \eta,$$

$$V_3 = -\mu s_1^0 s_2^0 |s_2|^2 + \text{h.c.},$$

$$V_4 = \lambda_{s_1} |s_1|^4 + \lambda_{s_2} |s_2|^4 + \lambda_{s_12} |s_1|^2 |s_2|^2$$

$$+ \lambda_{\Phi_1} (\Phi \Phi^\dagger)^2 + \lambda_{\Phi_2} (\Phi_\nu \Phi_\nu^\dagger)^2 + \lambda_\eta (\eta \eta^\dagger)^2$$

$$+ \lambda_{\Phi_1 \eta} (\Phi \Phi^\dagger \eta \eta^\dagger) + \lambda_{\Phi_2 \eta} (\Phi \Phi^\dagger \eta \eta^\dagger) + \lambda_{\Phi_2 \nu} (\Phi_\nu \Phi_\nu^\dagger \eta \eta^\dagger) + \lambda_{\Phi_2 \nu} (\Phi_\nu \Phi_\nu^\dagger \eta \eta^\dagger)$$

$$+ (\lambda_{\Phi_2 \nu} (\Phi \eta \eta^\dagger) + \text{h.c.})$$

$$+ \lambda_{s_1 \Phi_1} s_1^0 (\Phi \Phi^\dagger)$$

$$+ \lambda_{s_1 \Phi_2} s_2^0 (\Phi \Phi^\dagger)$$

$$+ \lambda_{s_2 \Phi} s_1^0 (\Phi \Phi^\dagger)$$

$$+ \lambda_{s_2 \Phi} s_2^0 (\Phi \Phi^\dagger)$$

$$+ \lambda_{s_2 \Phi} s_1^0 (\Phi \Phi^\dagger) + \text{h.c.}) + (\lambda_{s \Phi_2} s_1^0 s_2^0 \Phi_\nu^\dagger \eta \eta + \text{h.c.}) .$$

$$\tag{16}$$
Actually, the following simplified $V_4$ is sufficient for our discussion:

\[
V_4(\text{simplified}) = \lambda_{\phi 1} (\Phi^\dagger \Phi)^2 + \lambda_{s2} |s_{21}|^4 + \lambda_{s2\phi 1} |s_{22}|^2 (\Phi^\dagger \Phi)
\]
\[
+ \lambda_\eta (\eta^\dagger \eta)^2 + \lambda_{\phi 1\eta} (\Phi^\dagger \Phi) (\eta^\dagger \eta) + \lambda_{\phi 2\eta} (\Phi^\dagger \Phi) (\eta^\dagger \eta)
\]
\[
+ \lambda_{s1} |s_{11}|^4 + \lambda_{s2} (\Phi^\dagger \Phi)^2
\]
\[
+ (\lambda_{s\phi 1\eta} s_{11}^0 (s_{22}^0)^* \Phi^\dagger \eta + \text{h.c.}) + (\lambda_{s\phi 2\eta} s_{12}^0 \Phi^\dagger \eta + \text{h.c.}) .
\]  

2. Masses of Scalar Bosons

Scalar fields are decomposed as follows: $\phi^0 = \frac{1}{\sqrt{2}} (v + \phi^0 + i\phi^0)$, $\phi^0_v = \frac{1}{\sqrt{2}} (v^0 + s_{11}^{0*} + is_{12}^{0*})$, $\eta^0 = \frac{1}{\sqrt{2}} (\eta^0 + i\eta^0)$, $s_{22}^0 = \frac{1}{\sqrt{2}} (s_{22}^{0*} + is_{22}^{0*})$. We ignore $v_v$ in the following formulae.

The mass matrix for $(s_{2r}, \eta_{hr})$ is obtained as

\[
M_H^2 = \begin{pmatrix}
m_{s2}^2 - \sqrt{2} \mu v_s & \frac{1}{2} \lambda_{s\phi 1\eta} v v_s \\
\frac{1}{2} \lambda_{s\phi 1\eta} v v_s & m_{\eta}^2
\end{pmatrix}
\]  

where $m_{s2}^2 \equiv \mu_{s2}^2 + \frac{1}{2} (\lambda_{s2\phi 1} v^2 + \lambda_{s12} v_s^2)$ and $m_{\eta}^2 \equiv \mu_{\eta}^2 + \frac{1}{2} \left\{ (\lambda_{\phi 1\eta} + \lambda_{\phi 1\eta}^*) v^2 + \lambda_{s1\eta} v_s^2 \right\}$. On the other hand, The mass matrix for $(s_{2i}, \eta_{hi})$ results in

\[
M_A^2 = \begin{pmatrix}
m_{s2}^2 + \sqrt{2} \mu v_s & \frac{1}{2} \lambda_{s\phi 1\eta} v v_s \\
\frac{1}{2} \lambda_{s\phi 1\eta} v v_s & m_{\eta}^2
\end{pmatrix}
\]  

Notice that the difference between $M_H^2$ and $M_A^2$ exists only in the $(1, 1)$ element as $(M_A^2)_{11} = (M_H^2)_{11} + 2\sqrt{2} \mu v_s$. Mass eigenstates ($H_1^0$ and $H_2^0$) of $Z_2$-odd CP-even scalar bosons are given by

\[
\begin{pmatrix}
H_1^0 \\
H_2^0
\end{pmatrix} = \begin{pmatrix}
\cos \theta_0 & -\sin \theta_0 \\
\sin \theta_0 & \cos \theta_0
\end{pmatrix} \begin{pmatrix}
s_{2r}^0 \\
\eta_{hr}^0
\end{pmatrix}, \quad \tan(2\theta_0') = \frac{\lambda_{s\phi 1\eta} v v_s}{m_{\eta}^2 - m_{s2}^2 + \sqrt{2} \mu v_s} .
\]  

while mass eigenstates ($A_1^0$ and $A_2^0$) of $Z_2$-odd CP-odd scalar bosons are obtained as

\[
\begin{pmatrix}
A_1^0 \\
A_2^0
\end{pmatrix} = \begin{pmatrix}
\cos \theta_A' & -\sin \theta_A' \\
\sin \theta_A' & \cos \theta_A'
\end{pmatrix} \begin{pmatrix}
s_{2i}^0 \\
\eta_{hi}^0
\end{pmatrix}, \quad \tan(2\theta_A') = \frac{\lambda_{s\phi 1\eta} v v_s}{m_{\eta}^2 - m_{s2}^2 - \sqrt{2} \mu v_s} .
\]  

The mass eigenstate $H^\pm$ of $Z_2$-odd charged scalar boson is identical to $\eta^\pm$:

\[
H^\pm = \eta^\pm .
\]
Masses of these $Z_2$-odd scalar bosons are calculated as

$$m_{\nu_1}^2 = \frac{1}{2} \left\{ m_\eta^2 + m_{s_2}^2 - \sqrt{2} \mu v_s - \sqrt{(m_\eta^2 - m_{s_2}^2 + \sqrt{2} \mu v_s)^2 + \lambda_{s\Phi_1\nu} v^2 v_s^2} \right\},$$

$$m_{\nu_2}^2 = \frac{1}{2} \left\{ m_\eta^2 + m_{s_2}^2 - \sqrt{2} \mu v_s + \sqrt{(m_\eta^2 - m_{s_2}^2 + \sqrt{2} \mu v_s)^2 + \lambda_{s\Phi_1\nu} v^2 v_s^2} \right\},$$

$$m_{A_1}^2 = \frac{1}{2} \left\{ m_\eta^2 + m_{s_2}^2 + \sqrt{2} \mu v_s - \sqrt{(m_\eta^2 - m_{s_2}^2 - \sqrt{2} \mu v_s)^2 + \lambda_{s\Phi_1\nu} v^2 v_s^2} \right\},$$

$$m_{A_2}^2 = \frac{1}{2} \left\{ m_\eta^2 + m_{s_2}^2 + \sqrt{2} \mu v_s + \sqrt{(m_\eta^2 - m_{s_2}^2 - \sqrt{2} \mu v_s)^2 + \lambda_{s\Phi_1\nu} v^2 v_s^2} \right\},$$

$$m_{H^\pm}^2 = m_\eta^2 - \frac{1}{2} \lambda_{s\Phi_1\nu} v^2.$$  

Next, the mass matrix for $(\phi_{\nu}^0, s_{1r}^0)$ is given by

$$M_H^2 = \begin{pmatrix} 2\lambda_{s\Phi_1\nu} v^2 & \lambda_{s\Phi_1\nu} v v_s \\ \lambda_{s\Phi_1\nu} v v_s & 2\lambda_{s\Phi_1\nu} v_s^2 \end{pmatrix}. \quad (28)$$

Notice that $\phi_{\nu}^0$ does not mix with them when we ignore $v_{\nu}$. Mass eigenstates ($h^0$, $H^0$, and $H_{\nu}^0$) of $Z_2$-even CP-even scalar bosons are given by

$$\begin{pmatrix} h^0 \\ H^0 \\ H_{\nu}^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \phi_{\nu}^0 \\ s_{1r}^0 \end{pmatrix}, \quad \tan(2\theta_0) = \frac{\lambda_{s\Phi_1\nu} v v_s}{\lambda_{s\Phi_1\nu} v_s^2 - \lambda_{s\Phi_1\nu} v^2},$$

$$H_{\nu}^0 = \phi_{\nu}^0.$$  

(29)  

The Nambu-Goldstone boson $z_2^0$ for the $U(1)_X$ breaking, a $Z_2$-even CP-odd scalar boson $A_{\nu}^0$, and the $Z_2$-even charged scalar boson $H_{\nu}^\pm$ are defined as follows:

$$z_2^0 = s_{1r}^0, \quad A_{\nu}^0 = \phi_{\nu r}, \quad H_{\nu}^\pm = \phi_{\nu r}^\pm.$$  

(31)  

Masses of these $Z_2$-even scalar bosons are calculated as

$$m_{h_0}^2 = \lambda_{s\Phi_1\nu} v_s^2 + \lambda_{s\Phi_1\nu} v^2 - \sqrt{\left\{ \lambda_{s\Phi_1\nu} v_s^2 - \lambda_{s\Phi_1\nu} v^2 \right\}^2 + \lambda_{s\Phi_1\Phi_1} v^2 v_s^2},$$

$$m_{H^0}^2 = \lambda_{s\Phi_1\nu} v_s^2 + \lambda_{s\Phi_1\nu} v^2 + \sqrt{\left\{ \lambda_{s\Phi_1\nu} v_s^2 - \lambda_{s\Phi_1\nu} v^2 \right\}^2 + \lambda_{s\Phi_1\Phi_1} v^2 v_s^2},$$

$$m_{s}^2 = 0,$$

$$m_{A_{\nu}^0}^2 = m_{\phi_{\nu}^0}^2 + \frac{1}{2} \left\{ \left( \lambda_{s\Phi_1\nu} + \lambda_{s\Phi_1\nu} \right) v^2 + \lambda_{s\Phi_1\Phi_1} v_s^2 \right\},$$

$$m_{H_{\nu}^\pm}^2 = m_{\phi_{\nu}^0}^2 + \frac{1}{2} \left\{ \lambda_{s\Phi_1\nu} v^2 + \lambda_{s\Phi_1\Phi_1} v_s^2 \right\}. \quad (36)$$

(32)  

(33)  

(34)  

(35)  

(36)
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