Plasma microturbulence simulation of instabilities at highly disparate scales

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Abstract. This work reports on studies of the multi-scale interaction between small-scale electron turbulence and large-scale ion turbulence in tokamak plasmas. Traditionally, the long-wavelength, low-frequency turbulence driven by ion-scale instabilities (ion-temperature-gradient and trapped-electron modes) is studied separately from the short-wavelength, high-frequency turbulence driven by electron-scale instabilities (electron-temperature-gradient modes). High-resolution, massively-parallel simulations have uncovered a number of physically-important discoveries. First, we find that a popular simplified model of ion physics previously used in studies of electron-scale turbulence can lead to nonphysical runaway of electron heat transport. We have shown that this nonphysical runaway is eliminated when correct long-wavelength ion physics is self-consistently included. We have also found that under normal conditions most of the electron heat transport arises from large-scale instabilities. However, when these large-scale instabilities are suppressed by plasma rotation or other processes, the electron instabilities survive and may dominate the loss of electron heat from the plasma. A further remarkable finding is that strong turbulence at long scales can act to reduce the intensity of turbulence at short scales. Simulations were carried out on the Cray X1E computer at ORNL, with the largest runs taking about a week on 720 multi-streaming processors.

1. Introduction: Ion-scale and electron-scale instabilities

In tokamak plasmas, performance is limited by turbulent radial transport of both energy and particles driven by so-called drift-wave instabilities. These instabilities are unavoidable and will persist in reactor plasmas. The bulk of this transport arises from ion-scale instabilities, with time and space scales comparable to the $a/v_i$ and $\rho_i$ respectively (see Table 1). These relatively large-scale, slow instabilities, driven by ion-temperature-gradient (ITG) and trapped-electron (TE) modes, can nevertheless coexist with small-scale, fast instabilities driven by electron-temperature-gradient (ETG) modes. In terms of the key parameter $\mu = \sqrt{m_i/m_e}$, which is approximately 60 in a pure-Deuterium plasma, the electron space scales are smaller than the ion scales by a factor of $\mu$, while the corresponding time scales are shorter by a factor of $\mu$. In the past, transport from ETG instabilities was generally thought to be limited by this small parameter because the naïve electron-scale diffusivity (see Table 1) is a factor of $\mu$ smaller than the ion-scale diffusivity:

$$\frac{\chi_{GBe}}{\chi_{GBi}} = \frac{1}{\mu} \sim 60 .$$

(1)
In an attempt to put this result on a more rigorous footing, Jenko and Dorland carried out gyrokinetic studies of ETG turbulence [1, 2, 3] using the so-called adiabatic ion model (ETG-ai), for which the ion gyrokinetic equation is ignored and the fluctuating part of the ion distribution is set to

\[
\delta f_i = -n_0 \exp \left( -\frac{v^2}{2v_i^2} \right) \frac{e \delta \phi(x,t)}{T_i}. \tag{2}
\]

Here, we assume the equilibrium electron and ion densities are equal: \( n_e = n_i = n_0 \). The simulations reported in Refs. [1], [2] and [3] showed that, according to this model, significant transport at electron scales could occur, well in excess of levels suggested by Eq. (1). For the Cyclone base case [4] parameters, the electron energy diffusivity was found to be in the range \( \chi_e/\chi_{GBe} \sim 30 \), which is about 15 times larger than the value of \( \chi_i/\chi_{GBi} \) obtained from the analogous ion-scale simulation [5, 4]. It was further noted that the transport at moderate magnetic shear (\( s = 0.8 \)) was dramatically higher than at weak or negative shear. We remark that subsequent particle-in-cell (PIC) simulations for essentially the same case found much lower transport levels, of the order \( \chi_e/\chi_{GBe} \sim 8 \) [6]. However, it is now generally accepted that a subtle short-wavelength stabilizing effect of discrete particle noise [6, 7, 8] triggers a spurious drop in the energy flux to very low levels for this case.

2. Nominal simulation parameters and numerical resolution

Numerical results in the present paper are derived exclusively from simulations with the GYRO [9] code. For traditional long-wavelength simulations, GYRO has demonstrated weak scaling up to the full capacity of the world’s largest parallel computers (see Fig. 1).

In the present work we use, where possible, the same parameters (the Cyclone base case [4]) as in previous local studies; namely, a simple unshifted-circle equilibrium with \( q = 1.4, s = 0.786, r/a = 0.5, R/a = 2.775, a/L_Te = 2.484, a/L_{ne} = 0.8, n_e = n_i \) and \( T_e = T_i \). Unlike the original work of Jenko and Dorland, particle trapping (finite-\( r/R \)) is retained. Due to computational limitations and a cost-scaling of approximately \( \mu^{3.5} \), we present results only for \( \mu = 20 \) and \( \mu = 30 \). Note that \( \rho_i = \mu \rho_e, k_{\theta} \rho_i = \mu k_{\theta} \rho_e \) and \( v_i = v_e/\mu \).

All simulations in this paper use a 128-point velocity-space grid (8 energies, 8 pitch angles and two signs of velocity), and the parallel (to the field line) resolution is taken to be 10 points per passing particle orbit. These choices have been carefully studied and justified in previous work [11]. The details of the perpendicular resolution, characterized by \( L_x \sim 2\pi/(k_x)_{\text{min}} \) and \( L_y \sim 2\pi/(k_{\theta})_{\text{min}} \), are given in each case. We used a 4th-order explicit Runge-Kutta time-integration scheme, with a nominal simulation timestep of \( (v_e/a) \Delta t = 0.03 \) (with minor variation in some cases). Here, “min” refers to the minimum nonzero value of the respective wavenumber. All simulations use radially periodic (flux-tube) boundary conditions.
3. Adiabatic-ion and kinetic-ion models of ETG turbulence

In this work, we will go beyond the simple ETG-ai model discussed in Sec. 1 by retaining the full gyrokinetic ion dynamics. Specifically, the properties of three types of ETG models will be compared:

(i) **ETG-ai**: ions are adiabatic;
(ii) **ETG-ki**: ions are gyrokinetic with $a/L_{n_i} = a/L_{T_i} = 0.1$;
(iii) **ETG-ITG**: ions are gyrokinetic with $a/L_{n_i} = a/L_{n_e}$ and $a/L_{T_i} = a/L_{T_e}$.

Here,

$$\frac{a}{L_{n_\sigma}} = -\frac{a}{n_\sigma} \frac{\partial n_\sigma}{\partial r} \quad \text{and} \quad \frac{a}{L_{T_\sigma}} = -\frac{a}{T_\sigma} \frac{\partial T_\sigma}{\partial r},$$

where $\sigma = [i, e]$. The first model (ETG-ai) has been historically favoured in the computational community because of its simplicity. With the exception of a single axisymmetric term in the Poisson (charge neutrality) equation, it is structurally identical to the adiabatic electron model.
of ion-temperature-gradient (ITG) modes. The second and third models retain the proper long-wavelength zonal flow physics by treating both electrons and ions gyrokinetically. The difference between models (ii) and (iii) is simply one of parameter choice: the second model (ETG-ki) does not introduce ITG instability drive at long wavelengths, whereas the third model (ETG-ITG) retains long-wavelength ITG drive. In this paper, we will use line colors to consistently identify the different models and subcases: ETG-ai (black), ETG-ki at $\mu = 20$ (blue), ETG-ITG at $\mu = 20$ (magenta) and ETG-ITG at $\mu = 30$ (green).

In Fig. 2, we present the results of simple, inexpensive linear simulations which establish that the models (i)–(iii) are indistinguishable in the range $k_\theta \rho_i > 2$. At longer wavelengths, the kinetic ion physics strongly modifies the linear response. Here, $k_\theta = nq/r$ is the poloidal wavenumber at the simulation domain center, and $n$ is the toroidal mode number$^1$.

$^1$ We remark that the mode number $n$ is, generally speaking, a linear eigenvalue label. Further, in the context of the simulations reported in this paper, $k_\theta \rho_i$, such that $r$ and $q$ are evaluated at the domain center, is also a linear eigenmode label.
4. Unphysical runaway in the ETG-ai model

After numerous grid-resolution tests, it became apparent that the ETG-ai model is poorly behaved for the nominal Cyclone base case parameters (at finite \( r/R \)). To illustrate the weakness of the model, a magnetic-shear scan was carried out over the range \( 0.1 \leq s \leq 0.8 \), as shown Fig. 2. The perpendicular domain size was \( L_x \times L_y = 256 \rho_e \times 128 \rho_e \). We remark that the case \( s = 0.1 \) was the subject of a previous code comparison effort [7], for which agreement amongst four independent gyrokinetic codes (including GYRO) was obtained. The high values of \( \chi_e \) for the ETG-ai model (black curve) in the interval \( s > 0.4 \) are indicative of a state which is not physically saturated. In other words, at these operating points, the ETG-ai model does not give physically sensible results. To convince the reader that this a model rather than a code problem, we emphasize that the aforementioned code comparison effort also documented this model failure with 4-way code agreement [7].

5. Ion-scale ETG simulations with gyrokinetic ions (\( \mu = 20 \))

This more comprehensive approach has also been employed in GENE simulations of ETG transport in an edge transport barrier [12] (carried out in the limit of no particle trapping). We have increased the real-space perpendicular domain size in comparison to the simulations of Sec. 4 to give a minimal but physically correct representation of the ion-scale turbulence. It must be emphasized that simulations using gyrokinetic ions are extremely expensive, not only because of the enormous set of wavenumbers which are coupled nonlinearly, but also due to the need to carry out ion gyroaverages in real space on a tiny electron-scale grid. Furthermore, in such simulations, the sparse structure of the matrix associated with the Poisson equation is largely lost. We performed numerous resolution tests in order to develop strategies to reduce the overall computational cost.

6. ETG suppression by long-wavelength ITG instability

We begin by discussing what is perhaps the most noteworthy result of the paper. In Fig. 4, we compare the shapes of the \( \chi_e \) spectra for the ETG-ki and ETG-ITG models. Most evidently, the effect of turning on ITG drive (ETG-ITG) is to reduce the high-\( k_\perp \) ETG transport, while at the same time increasing the low-\( k_\perp \) ITG/TEM transport. To quantify the high-\( k_\perp \) decrease, we sum over the interval \( k_\theta \rho_i > 0.1 \) to find that \( \chi_e/\chi_{\text{GB}1} = 9.8 \) (ETG-ki) is reduced to \( \chi_e/\chi_{\text{GB}2} = 3.7 \) (ETG-ITG). Regarding the low-\( k_\perp \) increase, we sum over the interval \( k_\theta \rho_i < 1.0 \) to find \( \chi_e/\chi_{\text{GB}1} = 1.2 \) (ETG-ki) is increased to \( \chi_e/\chi_{\text{GB}2} = 2.3 \) (ETG-ITG). The increase in long-wavelength transport is consistent with an ITG branch that is stable in the ETG-ki model but unstable in the ETG-ITG model. Note that in the ETG-ki simulations, it is the TEM that is unstable at long wavelength. The puzzling feature of the results, however, is the significant decrease in short-wavelength transport.

Clearly, in view of the linear growth rates, the long-wavelength results are not surprising, but for \( k_\theta \rho_i > 2 \) the difference in linear growth rates is so small as to be insignificant. Thus, it seems difficult to reconcile the great disparity in heat transport in the region \( k_\theta \rho_i > 2 \) based on linear arguments only.

7. Isotropy of electron density fluctuations

First, in real space, we can compare the character of the electron density fluctuations, \( \delta n_e/n_0 \), in the \((x,y)\) plane perpendicular to the magnetic field. Here, \( x = r/a \) is the normalized minor radius, and \( y \) is the binormal distance (perpendicular to both radius and the magnetic field), as defined in Sec. 2. Figures 5a and 5b show snapshots of these fluctuations for the ETG-ki and ETG-ITG cases, respectively, at a single instant in time. The most apparent difference is the clear presence of ion-scale eddies in the ETG-ITG model. These large eddies are associated with the substantially greater long-wavelength transport. On the other hand, in Fig. 5a, the
presence of very tiny electron-scale eddies can be seen. Such eddies do exist in Fig. 5b, but they are difficult to detect in the sea of larger eddies. Animations of motion in this plane shows that the large-scale objects evolve on a very slow timescale, whereas the electron-scale eddies dart around much more rapidly.

It is of experimental interest to examine the shape of the time-averaged density powerspectrum, $\langle N \rangle_t$ at the outboard midplane, where

$$N(k_r, k_\theta) = \left( \frac{a}{\rho_i} \right)^2 \left| \frac{\delta n_e}{n_0} \right|_{k_r, k_\theta, \theta=0}^2.$$  \tag{4}

The simulation results, plotted in Fig. 6 show that $\langle N \rangle_t$ is relatively isotropic for the ETG-ki model, and remarkably so for the ETG-ITG case. The results also imply that the fluctuation spectrum is monotonically decreasing in the ETG-range $k_\theta \rho_i > 1.0$. This result runs somewhat contrary to the prior intuitive notion that a local maximum in the ETG-range of the spectrum of $\chi_e$, or perhaps $\delta n_e/n_e$, would occur in the vicinity of $k_\theta \rho_i \sim 0.2$ as it does in ITG turbulence at $k_\theta \rho_i \sim 0.2$.

8. Simulation resolution tests

We can further examine the sensitivity of the ETG-ITG results to certain changes in simulation resolution.

**Effect of reduced box size:** In Figure 7a is shown that halving the box size in each dimension has remarkably little effect on the electron-scale transport. Thus, in this case, a comparatively inexpensive $L_x \times L_y = 32 \rho_i \times 32 \rho_i$ simulation (no more than 1/4 the cost of a $64 \rho_i \times 64 \rho_i$ run) is enough to capture the total ETG contribution to the electron heat transport for $k_\theta \rho_i > 1$. This result motivates the following working definition of ETG transport: ETG energy transport, or $\chi_e^{\text{ETG}}$, can be operationally defined as that fraction of $\chi_e$ which occupies the spectral interval $k_\theta \rho_i > 1$. Thus, according to this definition, the large-box simulation predicts $\chi_e^{\text{ETG}} \approx 7.36 \chi_{GB,e}$, whereas the small-box run gives $\chi_e^{\text{ETG}} \approx 8.26 \chi_{GB,e}$. The difference is roughly within the range of the statistical uncertainty between two identical simulations sampled at different time intervals.

**Effect of perpendicular grid resolution:** A simulation to more fully resolve the ETG scales (far into the ETG range, up to $k_\theta \rho_i = 0.96$) was carried out. The result, shown in Fig. 7b, indicates that a very-high-resolution simulation flattens the spectral lip of lower-resolution run, with little
Figure 5. Temporal snapshots of the electron density fluctuations from the ETG-ki (a) ETG-ITG (b) simulations. The presence of the large ITG-driven eddies in frame (b) is associated with greater long-wavelength transport.
change in the total ETG energy transport: $\chi_{ETG} \approx 8.06\chi_{GBe}$ for the high-resolution case versus $\chi_{ETG} \approx 8.26\chi_{GBe}$ for the low resolution case. We have observed this phenomenon repeatedly, for a variety of different operating parameters (not shown) in performing perpendicular resolution tests. The noteworthy result is that even though a spectral lip always appears at the end of the spectral curve, the total integrated heat flux is approximately invariant.

9. Mass-ratio variation: $\mu = 20$ versus $\mu = 30$.

The aim of this section is to examine the robustness of previous conclusions to an increase in $\mu$ (toward more realistic values). First, we remark that the simulations for $\mu = 30$ are substantially more expensive than for $\mu = 20$. Indeed, the $\mu = 20$ case took 5 days on the ORNL Cray X1E using 192 Multi-Streaming Processors (MSPs), whereas the $\mu = 30$ case took the same 5 days using 720 MSPs. This is consistent with a cost scaling of approximately $\mu^{3.5}$.

To begin with, in Fig. 8a, we compare electron transport as measured in ion units. We remark that the drop in $\chi_e$ at ion-scales is well-known from previous experience. For the Cyclone base case [4] we know the drop in $\chi_e$ from $\mu = 20$ to $\mu = 30$ is comparable to the drop from $\mu = 30$ to $\mu = 60$. We tabulate various transport coefficients summed over selected intervals in $k_{qi}$ in Table 2. Still focusing on Fig. 8a, we further note that the drop in transport at electron scales is expected based on electron gyroBohm scaling; indeed, in the limit $\mu_i \rightarrow \infty$, we expect the high-$k_\perp$ contribution to $\chi_e/\chi_{GBe}$ to approach a universal constant. To this end, Fig. 8b measures electron transport in electron units. Here, it is clearly seen that the curve takes on a $\mu$-independent shape for $k_{qi} > 0.1$. We tentatively consider transport in this spectral interval to be the universal range for ETG-driven electron heat transport. Quantitatively, we find that in the universal range, $\chi_e/\chi_{GBe} = 3.67$ at $\mu = 20$ whereas $\chi_e/\chi_{GBe} = 3.76$ at $\mu = 30$. This
Figure 7. Plot (a) demonstrates that halving the box size in both perpendicular dimensions (increasing $(k_\theta)_{\text{min}}$) leaves the short-wavelength transport invariant. Plot (b) shows that resolving very fine scales has little effect on longer scales, with the exception of removing the spectral lip present in the lower-resolution (solid) curve.

Figure 8. (a) Transport as measured in ion units. A drop in electron energy transport is seen in moving from $\mu = 20$ (solid curve) to $\mu = 30$ (dashed curve). (b) However, when measured in electron units, the universality of the transport for $k_\theta \rho_e > 0.1$ becomes apparent.

result is also tabulated in Table 2. In practice, it turns out that the contribution to the total heat flux from the universal range tends to be small.

10. Summary
More detailed discussions of the physics results described in this paper are given in Refs. [13] and [14].
Table 2. Mass-ratio dependence of the ETG-ITG model.

| μ  | $k_{\rho_i} < 1$ | $k_{\rho_i} > 1$ | $k_{\rho_i} > 2$ | $k_{\rho_e} > 0.1$ |
|----|-----------------|-----------------|-----------------|------------------|
| X_i/| 20 | 7.378 | 0.054 | 0.011 |
| GB  | 30 | 7.754 | 0.043 | 0.009 |
| X_e/| 20 | 2.278 | 0.367 | 0.183 |
| GB  | 30 | 1.587 | 0.296 | 0.157 |
| D/| 20 | -0.81 | 0.134 | 0.009 |
| GB  | 30 | -1.60 | 0.074 | 0.010 |
| X_e/| 20 | -0.81 | 0.134 | 0.009 |
| GB_e| 30 | -1.60 | 0.074 | 0.010 |

11. Acknowledgement
This work was supported by the U.S. Department of Energy under Grants DE-FG03-95ER54309, DE-AC05-00OR22725 and DE-FG03-95ER-54301, and also by a DOE 2006 INCITE computer time award (FUS014) at the National Center for Computational Sciences at Oak Ridge National Laboratory.

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