Is there a standard measuring rod in the Universe?

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ABSTRACT

The CaltechJodrell Bank very long baseline interferometry (VLBI) Surveys give detailed 5 GHz VLBI images of several hundred milliarcsecond (mas) radio sources, and the full width at half-maximum angular sizes of the corresponding compact cores. Using the latter, I have constructed an angular-diameter/redshift diagram comprising 271 objects, which shows clearly the expected features of such a diagram, without redshift binning. Cosmological parameters are derived which are compatible with existing consensus values, particularly when the VLBI data are combined with recent Baryon Acoustic Oscillations observations; the figures are presented as indications of what might be expected of larger samples of similar data. The importance of beaming and relativistic motion towards the observer is stressed; a model of the latter indicates that the emitting material is close to the observers line of sight and moving with a velocity which brings it close to the observers rest frame. With respect to linear size, these objects compare reasonably well in variance with the absolute luminosity of type Ia supernovae; the efficacy of the latter is improved by the brighter-slower and brighter-bluer correlations, and by the inverse-square law.

Key words: cosmological parameters – cosmology: observations – dark matter.

1 INTRODUCTION

Cosmological parameters are now known to a remarkable degree of precision, particularly those derived from Wilkinson Microwave Anisotropy Probe (WMAP) observations (Dunkley et al. 2008; Komatsu et al. 2008), in combination with observations of Type Ia supernovae (SNe Ia) (Riess et al. 2004; Astier et al. 2006; Riess et al. 2007; Wood-Vasey et al. 2007), and the imprint of Baryon Acoustic Oscillations (BAO) on the distribution of galaxies (Blake & Glazebrook 2003; Seo & Eisenstein 2003; Eisenstein et al. 2005); current values are \( \Omega_m = 0.279 \pm 0.013 \) and \( \Omega_L = 0.721 \pm 0.015 \) for the matter and vacuum parameters respectively (unless otherwise noted, all confidence limits quoted here are 68 per cent ones). Whereas the WMAP and BAO approaches are of relatively recent origin, the magnitude/redshift approach has a long and somewhat varied history, for want of a class of objects with similar absolute magnitudes (see e.g. Weinberg (1972) for a review of early work); however, over the last decade the latter approach has achieved spectacular success, with the discovery of SNe Ia as accurate standard candles (Riess et al. 1998; Schmidt et al. 1998; Perlmutter et al. 1999).

In contrast, the angular-size/redshift approach has had a surprisingly modest impact, despite significant efforts in this direction, using extra-galactic radio sources in their several guises as putative standard measuring rods. Early work considered classical double radio sources as suitable objects (Legg 1970; Miley 1971; Kellermann 1972), which approach continues to this day (Daly 1994; Buchalter et al. 1998; Daly et al. 2007). Here I will re-examine ultra-compact radio sources as standard measuring rods, with angular diameters in the milliarcsecond (mas) range, and linear sizes of order several parsecs. Their advantages in this context were first highlighted by Kellermann (1993); these objects are much smaller than their parent active galactic nuclei, so that their local environments should be similar and reasonably stable, at least over an appropriate redshift range. They are energetic and shortlived, with central engines which are reasonably standard objects (black holes with masses close to \( 1.5 \times 10^{10} M_\odot \)). In these respects they have much in common with SNe Ia, albeit with lifetimes of centuries rather than months. Kellermann (1993) presented angular sizes for a sample of 79 milliarcsecond (mas) sources, obtained using very long baseline interferometry (VLBI) at 5GHz. VLBI images typically show a compact core surrounded by debris, and Kellermann (1993) defined a characteristic angular size as the distance between the core and the most distant component having a peak brightness greater than or equal to 2 per cent of that of the core. Typical lin-
ear sizes are 20h^{-1} pc and Kellermann noted that the corresponding angular-size/redshift diagram is compatible with the once-favoured flat cold dark matter (CDM) model, \( \Omega_m = 1, \Omega_\Lambda = 0 \). However, for a critique see Pearson et al. (1994); these authors present a similar sample (with very little overlap) and use the same measure of angular size, which shows virtually no change over the same redshift range. Gurvits, Kellermann & Frey (1999) have examined a much larger 5 GHz sample (330 objects), with results which do not resolve this conflict. I suspect that the discord here is due to the definition of angular size, which is too sensitive to the details of source structure.

Gurvits (1994) presented a large VLBI compilation, based upon a 2.29 GHz survey undertaken by Preston et al. (1985); the compilation lists a rough measure of angular size based upon fringe visibility; this measure should be less sensitive to the details of source structure, and more representative of the compact core, which is usually the dominant component with respect to radio luminosity (see e.g. Henstock et al. 1992). Gurvits considered a subset comprising 258 sources divided into 12 redshift bins over the range \( 0.501 \leq z \leq 3.787 \), and found marginal support for a low-density CDM cosmological model, assuming that selection and evolutionary effects can be ignored. Gurvits considered only models with \( \Omega_\Lambda = 0 \). Using exactly the same data set (kindly supplied by Dr. Gurvits), Jackson & Dodson (1997) extended the analysis to the full \( \Omega_m - \Omega_\Lambda \) plane; the situation is very degenerate with respect to choice of \( \Omega_\Lambda \), with both 95 per cent and 68 per cent confidence regions extending well into the region \( \Omega_\Lambda < 0 \). Nevertheless, marginalizing over \( \Omega_\Lambda > 0 \) gives 0.11 \( \leq \Omega_m \leq 0.54 \), providing clear support for a low-density CDM model. Imposing spatial flatness allowed a much more definitive statement to be made: \( 0.1 \leq \Omega_m \leq 0.3 \), later refined to \( \Omega_m = 0.24 \pm 0.09/ -0.07 \) (95 per cent confidence limits) (Jackson 2004).  

2 DATA AND RESULTS

The object here is to consider a more precise definition of angular size. The Caltech-Jodrell Bank flat-spectrum (CJF) sample is a complete 5 GHz flux-density-limited sample of 293 flat-spectrum sources, complete according to the following criteria (Taylor et al. 1996):

(i) Flux density at 4850 MHz \( S_{4850} \geq 350 \) mJy.

(ii) Spectral index \( \alpha \geq -0.5 \) \( (S \propto \text{frequency}^{-\alpha}) \).

(iii) Declination \( \delta \geq 35^\circ \) (1950 coordinates).

(iv) Galactic latitude \( \geq 10^\circ \).

\( H_0 = 100 \text{ km sec}^{-1} \text{ Mpc}^{-1} \)

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\( \frac{H_0}{100} \text{ km sec}^{-1} \text{ Mpc}^{-1} \)

Tight figures are presented in Jackson & Jannetta (2000), where a larger sample is examined (613 objects), produced by updating Preston et al. (1985) with respect to redshift, and finding some of the missing flux densities from elsewhere; this work places significant constraints on \( \Omega_m \) and \( \Omega_\Lambda \) without assuming flatness. However, in retrospect we have doubts about the suitability of the extra data, and about the efficacy of redshift binning, see discussion later; the precision in Jackson & Jannetta (2000) has been overstated.

\( \Omega_m = 0.236, \Omega_\Lambda = 1 - \Omega_m = 0.764 \) and \( d = 2.94h^{-1} \) pc, see text.

Taylor et al. (1996) present a VLBI image for each successfully imaged source, and give accurate full width at half-maximum angular major and minor axes \((a \text{ and } b)\) for each component therein, with the compact core clearly identified; the major axis of the latter will be taken as the measure of angular size in this investigation. In fact I have used a somewhat larger sample than the CJF one. The latter is, in part, a subset of three earlier samples, the PR (Pearson & Readhead 1981, 1988), CJ1 (Polatidis et al. 1993, Thakkar et al. 1993, Xu et al. 1993) and CJ2 (Taylor et al. 1994, Henstock et al. 1993) samples. A composite comprising the latter three is nominally complete with respect to criteria (i), (iii) and (iv) above, but not the spectral index one. Taylor et al. (1996) selected the CJF sample from this composite by imposing the spectral index limit, and also added 18 further sources which had been missed in the earlier surveys, which additional sources meet all of the above criteria. In the interest of a modest increase in numbers I have used the full composite sample PR+CJ1+CJ2, as listed in tables 3 of Pearson & Readhead (1988) and tables 4 of Pearson & Readhead (1988) and Taylor et al. (1994), plus 17 of the 18 further sources, as listed in table 3 of Taylor et al. (1996), giving 322 objects in all. Of these 6 are assigned major axes formally equal to 0.00 milliarcsec, presumably because they are below the resolution limit of the VLBI system; the said sources have been discarded. I have updated the redshift list using the NASA/IPAC Extragalactic Database (NED), and for the remaining 316 objects I find 271 redshifts. Fig. 1 is a plot of major axis \( a \) against redshift \( z \) for these sources. Despite the spread the expected qualitative features are reasonably clear, namely a diminishing size from \( z = 0 \) to \( z \sim 1 \), followed by a gradual increase, with a minimum somewhere between \( z \sim 1 \) and \( z \sim 1.5 \), first predicted by Hovler (1950). The plot in Fig. 1 does not appear to be resolution limited.

Before turning to quantitative matters, a second diagram is instructive. Fig. 2 shows linear size plotted against radio luminosity for the 271 sources in Fig. 1 taking \( \Omega_m = 0.27, \Omega_\Lambda = 0.73 \). The cyan points correspond to \( z < 0.5 \), the

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Angular-diameter/redshift diagram for 271 sources from the composite PR+CJ1+CJ2 sample. The cyan line corresponds to \( \Omega_m = 0.236, \Omega_\Lambda = 1 - \Omega_m = 0.764 \) and \( d = 2.94h^{-1} \) pc, see text.}
\end{figure}
blue ones to $z \geq 0.5$. In a flux-limited sample sources observed at large redshifts are intrinsically the most powerful, so that a correlation between linear size and radio luminosity would introduce a selection effect. The high-redshift population shows no obvious evidence of such a correlation, and gives every indication of being statistically stable with respect to linear size. In the low-redshift case this is clearly not so. Fig. 2 gives a clear illustration of why ultra-compact radio sources with $z \leq 0.5$ are of no value in this context, first noted by Gurvits (1994), see also Jackson & Dodgson (1997) and Jackson (2004).

In what follows I will work with data points which correspond to individual sources, rather than putting the latter into redshift bins. Although binning is popular and can reveal trends in data which are not otherwise obvious to the eye, with regard to quantitative statistical analysis there is, in principle, no advantage in such a procedure; what is gained by having smaller error bars is lost by having fewer points.Appearances can be deceptive, particularly if numbers are small, and results can be overly sensitive to the choice of bins. I will concentrate on spatially flat ΛCDM models, characterized by $\Omega_m$ and a characteristic linear size $d$ associated with the source population. I will use objects in the range $0.5 \leq z \leq 3.5$; the upper limit on $z$ removes 3 points, one of which is an outlier which has an inordinate effect upon the statistical analysis.

Giving each point equal weight and taking $\log a$ as ordinate, the best-fitting flat model is $\Omega_m = 0.931$, $\Omega_A = 0.069$ and $d = 2.33h^{-1}$ pc, close to the erstwhile canonical model $\Omega_m = 1$, $\Omega_A = 0$. However, there is another parameter which I believe is an important source discriminator, namely the axial ratio $r = b/a$. This belief is based upon an astrophysical model, discussed at length in Jackson (2004). According to this model the underlying source population consists of compact symmetric objects (Wilkinson et al. 1994), comprising central low-luminosity cores straddled by two mini-lobes. The compact components which are the basis of this study are identified as cases in which the lobes are moving relativistically, and are close to the line of sight, when Doppler boosting allows just that material which is moving towards the observer to be seen. As $z$ increases a larger Doppler factor is required; it turns out that the latter approximately cancels the cosmological redshift, so that the observed component is seen in its rest frame. This is a very important effect, because the measured size of mas source components is known to increase linearly with wavelength (Marscher & Shaffer 1980, Pearson & Readhead 1984). Without the said effect mas angular-size/redshift diagrams would show something like the so-called Euclidean behaviour, angular size proportional to $1/z$ (because the emitted frequency is $(1 + z) \times$ the received one), which bedevilled early work on classical double radio sources (Legg 1971, Milev 1971, Kellermann 1972, Jackson 1973). It is quite remarkable that Fig. 1 shows no trace of such behaviour, which fact is a striking confirmation of the proposed model.

The ideal image will correspond to a head-on approach, and an axial ratio close to unity. I have examined samples which include only those objects with $r \geq r_c$, and find that the best-fitting value of $\Omega_m$ depends upon the cut-off ratio $r_c$ in a systematic fashion. The said value falls from 0.931 at $r_c = 0$ to 0.253 at $r_c = 0.35$, and thereafter remains reasonably constant until we run out of sensible numbers: $\Omega_m = 0.28 \pm 0.04$ for $0.35 \leq r_c \leq 0.6$, and $\Omega_m = 0.29 \pm 0.08$ for $0.35 \leq r_c \leq 0.7$. As a representative example I present results for the case $r_c = 0.4$, $0.5 \leq z \leq 3.5$, comprising 128 objects, for which the best figures are $\Omega_m = 0.236$ and $d = 2.94h^{-1}$ pc; the corresponding curve is shown in Fig. 1. A fixed standard deviation $\sigma$ is attached to each point, being defined by $\sigma^2 = \text{residual sum-of-squares}/(n - p)$, where $n = 128$ is the number of points and $p = 2$ is the number of fitted parameters; the appropriate value in this case is

$$\sigma = 0.252,$$

which value is used to calculate $\chi^2$ values at points in parameter space. The mid-grey lines in Fig. 3 show confidence regions in the $\Omega_m - d$ plane. Marginalizing over $d$ gives $\Omega_m = 0.24 \pm 0.40/ -0.15$.

This somewhat indeterminate result is in large part occasioned by the absence of standard objects in the redshift range $0 < z < 0.5$, which would otherwise determine a model-independent normalization of the characteristic linear size $d$. The deficiency can be remedied by combining recent BAO observations with the above data. I have in mind measures of the BAO scales at $z = 0.2$ and $z = 0.35$ (Percival et al. 2007), which constrain values of the hybrid distance $D_V(z) = [(1 + z)^2D_A(z)cz/H(z)]^{1/3}$, giving $D_V(0.35)/D_V(0.2) = 1.812 \pm 0.060$, which ratio does not refer directly to the size of the acoustic horizon at recombination. The joint mas+BAO confidence regions are represented by the filled blue/cyan areas in Fig. 3. Marginalizing over $d$ gives $\Omega_m = 0.10 + 0.09/ -0.06$. The light-grey vertical line is the right-most 95 per cent BAO confidence limit; the rest are formally at negative values of $\Omega_m$, including the best-fitting line $\Omega_m = -0.13$.

3 CODA AND CONCLUSIONS

Ultra-compact radio sources do comprise a reasonably standard unit of linear size, and in conjunction with BAO they are close to being useful for cosmological investigations. How
is equivalent to a variation Jackson & Jannetta (2006). Thus a variation signing approprate fixed values to cal parameters in the two cases: the linear size $d$ equal proportions. It is instructive to look at the basic physi-
comprise photometric and residual intrinsic errors in roughly $\delta m$ (Astier et al. 2006) and $\delta L/L\sim 1995$; Guy et al. 2005), typically to $\delta m_{SN}$ and $\delta m_{mass}$. The equivalent dispersion is thus $\delta m_{mass}$ larger than $\delta m_{SN}$ by a factor of 2? The answer is that it is the inverse-square law, implicit in equations $\delta m$ and $\delta d/d$, which puts the angular approach at a disadvantage. The numerical values are not dissimilar: $\delta d/d \sim 0.58$ and $\delta L/L \sim 0.35$; the latter is reduced by a factor of about 1.6 by the above-mentioned correlations, whereas the former is in effect increased by a factor of 2 by the inverse-square law.

I have looked at spectral index $\alpha$ as a possible size discriminator for mas sources, using the integrated values listed in Taylor et al. (1996), but find that the correlation is too weak to be useful, over the range of $\alpha$ values encompassed here. The central black hole mass $M_{bh}$ would be an interesting parameter in this context, which could be determined by the correlation between $M_{bh}$ and velocity dispersion of the the host elliptical galaxy Ferrarese & Merritt 2000; Gebhardt et al. 2000), or that between $M_{bh}$ and the Sérsic luminosity concentration index of the galaxy (Graham et al. 2001; Driver et al. 2006; Graham & Driver 2007). The appropriate information is not currently available; its acqui-
sition would be a worthwhile undertaking. The variance in measured size probably has significant contributions from both intrinsic and instrumental effects, so that an increase in relolving power would improve matters. There is certainly scope for increasing the sample size; the Caltech-Jodrell Bank surveys cover only about 20 per cent of the full sky, so that a five-fold increase would be possible without chang-
ing the flux limit. The comoving volume encompassed by a complete survey of flat-spectrum sources with flux-density limit $S_1$ increases roughly as $S_1^{-3}$, over the redshift range considered here.

The mas/BAO combination is a natural one, in that it allows cosmological parameters to be determined by data which are local ($z \lesssim 4$) and exclusively angular. Any discernible differences between parameters so determined and those determined by supernovae would be a manifestation of differential selection or evolutionary effects, or of ef-
ffects relating to the astrophysics or possibly the fundamen-
tal physics of light propagation over cosmological distances Bassett & Kunz 2004a,b; Burrage 2005). On more general grounds a new approach is always of some value, even if its weight is relatively low, because its systematic errors be-
come random ones when the new technique is added to an ensemble of existing ones.

Interested parties can obtain copies of the data set used in this investigation by sending an email request to john.jackson@unn.ac.uk.

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