The Mesonic Fluctuations and Corrections in the Chiral Symmetry Breaking Vacuum

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Abstract

The mesonic quantum fluctuations and their corrections on the chiral condensate and pion polarization function are investigated in the self-consistent scheme of SU(2) Nambu-Jona-Lasinio model by exactly calculating the next-to-leading order (NLO) Feynman diagrams in $1/N_c$ expansion. While the fluctuations and corrections depend strongly on the meson’s three-momentum cut-off $\Lambda_M$, no chiral symmetry restoration is found since the cancellation between the NLO and the one-quark-loop diagram with quark mass deviated from the mean-field value.
1 Introduction

It is generally accepted that the Nambu-Jona-Lasinio (NJL) [1] model offers a simple scheme to study the mechanism of spontaneous chiral symmetry breaking. Most calculations in this model have been based on the mean-field approximation, i.e., quark self-energy in the leading order of $1/N_c$ expansion [2].

Recently, more interests are paid on the NJL model beyond mean-field approximation. The mesonic fluctuations are variously investigated in the bosonization formalism [3]-[6] and in the Feynman diagram formalism of $1/N_c$ expansion [7]-[12]. [3] and [4] in bosonization formalism using meson-loop expansion or saddle-point expansion are equivalent to the self-consistent Feynman diagram formalism [5] in $1/N_c$ expansion, and the chiral symmetry of the NJL model can be preserved. While, [5] and [6] generalize the NJL model to the nonlinear-sigma approach for description of chiral fluctuations, and the authors in [5] claimed that at $N_c = 3$ the NJL model does not display spontaneous symmetry breaking due to chiral fluctuations.

There are two different $1/N_c$ expansion schemes which can preserve chiral symmetry of the NJL model. One is the so-called "strict" $1/N_c$ expansion scheme through iterating the quark self-energy at Hartree approximation [11] [12]. The other is the self-consistent scheme with meson-loop expansion or saddle-point expansion, in which the next-to-leading order (NLO) contributions represent the mesonic quantum fluctuations around the classical stationary chiral symmetry breaking vacuum [3] [4] [8]. In both schemes, the meson corrections to the mean-field approximation are included not only in the meson-loop contributions, but also in the one-quark-loop contributions through the corrected quark propagator.

In the above mentioned chiral symmetric schemes, emphasis is put on discussing the NLO or pure meson-loop contributions. However, to investigate whether chiral symmetry is restored in the vacuum, one should investigate the meson corrections to the mean-field approximation. In this paper we investigate not only the mesonic fluctuations around the classical vacuum, but also the mesonic corrections to the mean-field approximation in the chiral symmetric self-consistent scheme based on [8], we will exactly calculate the meson polarization function and the quark self-energy to the next-to-leading order in $1/N_c$ expansion. We will show that, by using the pole approximation for internal meson propagators, the $1/N_c$ expansion in the self-consistent
scheme is also “strict”. The momentum cut-off for quarks and mesons, i.e., $\Lambda_f$ and $\Lambda_M$, have been introduced in non-covariant three-momentum regularization. The model parameters at different values of $\Lambda_M$ have been determined by fitting the pion’s mass $m_\pi = 139$ MeV and the pion’s decay constant $f_\pi = 93.2$ MeV with an appropriate value of the current quark mass. The way to determine the parameters to the next-to-leading order here is different from that used in [11] and [12], where the parameters are fitted at leading order [12] or partially at leading order [11].

2 Mean-field approximation

The two-flavor NJL model is defined through the Lagrangian density,

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5 \vec{\tau}\psi)^2],$$

(1)

here $G$ is the effective coupling constant of dimension GeV$^{-2}$, $m_0$ is the current quark mass, assuming isospin degeneracy of $u$ and $d$ quarks, and $\psi, \bar{\psi}$ are quark fields with flavor, colour and spinor indices suppressed.

The quark mass $m_q^0$ in mean-field approximation, i.e., in Hartree approximation, can be represented by

$$m_q^0 = m_0 + \Sigma_{H}^0,$$

(2)

where $m_0$ is the current quark mass and $\Sigma_{H}^0$ is the quark self-energy of one-quark-loop contributions in mean-field approximation, which can be expressed as

$$\Sigma_{H}^0 = 8iG^0 N_c N_f m_q^0 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - (m_q^0)^2}.$$  

(3)

For quantities in mean-field approximation, we indicate them with a superscript "0".

The quark condensate calculated in mean-field approximation has a simple relation with the quark self-energy

$$- \langle \bar{q}q \rangle^0 = \frac{\Sigma_{H}^0}{4G^0}.$$  

(4)
Correspondingly, the meson polarization functions in random-phase approximation are

\[
\Pi^0_M(k) = 4iN_cN_f \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - (m_q^0)^2} - 2iN_cN_f(k^2 - 4\epsilon_M(m_q^0)^2) \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - (m_q^0)^2)((p + k)^2 - (m_q^0)^2)},
\]

with \( M \) refering to \( \pi \) or \( \sigma \), and \( \epsilon_\pi = 0 \) and \( \epsilon_\sigma = 1 \).

The NJL parameters in mean-field approximation can be fixed as \( \Lambda_f = 637.7 MeV \), and \( G^0\Lambda_f^2 = 2.16 \), by choosing an appropriate current quark mass \( m_0 = 5.5 MeV \) and through fitting the pion’s mass \( m_\pi = 139 MeV \) and decay constant \( f_\pi = 93.2 MeV \). The corresponding quark mass is \( m_q^0 = 330 MeV \) and chiral condensate \( (- < \bar{q}q >)^{1/3} = 248 MeV \).

### 3 Mesonic fluctuations and corrections on chiral condensate

The chiral symmetric self-consistent scheme is described in details in [3] [4] in effective action formalism and in [8] by using Feynman diagramatic method.

Including current quark mass \( m_0 \), the gap equation for quark mass can be expressed as

\[
m_q = m_0 + \Sigma_H + \Sigma^{fl},
\]

where \( \Sigma_H \) and \( \Sigma^{fl} \) are the leading order (LO) and next-to-leading order (NLO) of quark self-energy in \( 1/N_c \) expansion, which can be read directly from the Feynman diagrams in Fig.1.

The expression of \( \Sigma_H \) is the same as that of \( \Sigma_0^H \), except that the quark mass \( m_q^0 \) in Eq. (3) should be replaced by \( m_q \), which is the new saddle-point of the effective action including mesonic fluctuations [3] [4].

The NLO quark self-energy can be written directly from the diagrams in Fig.1

\[
\Sigma^{fl} = -8GN_cN_fm_q \int \frac{d^4p d^4q}{(2\pi)^8} \frac{1}{(-iD_\pi(q))} \frac{3}{(p^2 - m_q^2)^2}.
\]
\[
- \frac{3q^2}{(p^2 - m_q^2)^2((p + q)^2 - m_q^2)} + \left(-iD_\sigma(q)\right)\left(\frac{1}{(p^2 - m_q^2)^2}\right)
\]
\[
+ \frac{2}{(p^2 - m_q^2)((p + q)^2 - m_q^2)} - \frac{q^2 - 4m_q^2}{(p^2 - m_q^2)((p + q)^2 - m_q^2)}\right),
\] (7)

where \(-iD_M(q)\) indicate the internal meson propagator, which is in random-phase-approximation, i.e., strings of single quark loops. At this position, it is necessary to note that in [4], the NLO quark self-energy is separated into two parts: the first term of meson propagator in RPA and the remaining ring diagrams.

Evaluating quark loop and meson-loop integrals, one can get a simple relation between the quark condensate and the constituent quark mass \(m_q\),

\[-\langle \bar{q}q \rangle = -\left(\langle \bar{q}q \rangle_H + \langle \bar{q}q \rangle^{fl}\right) = \frac{\Sigma_H + \Sigma^{fl}}{4G}.
\] (8)

The magnitude of the mesonic quantum fluctuations around the classical vacuum is defined as the ratio of the pure NLO contributions over the LO contributions

\[ R^{fl}_q = \frac{\Sigma^{fl}}{\Sigma_H}, \] (9)

which had been discussed carefully in [4].

And the mesonic corrections on the chiral condensate is defined as the difference between the value in and beyond mean-field approximation, i.e.

\[ \delta \langle \bar{q}q \rangle = \left(-\langle \bar{q}q \rangle \right) - \left(-\langle \bar{q}q \rangle^0 \right) = \frac{\Sigma_H + \Sigma^{fl}}{4G} - \frac{\Sigma_H^0}{4G^0}, \] (10)

and the ratio of mesonic corrections is

\[ R^{cr}_q = \delta \langle \bar{q}q \rangle / \left(-\langle \bar{q}q \rangle^0 \right). \] (11)

4 Mesonic fluctuations and corrections on pion polarization function

Because the sigma meson is heavy, it does not play an important role in vacuum, and the NLO contributions would not affect the properties of sigma
meson. Here we only discuss the mesonic corrections on pion polarization function, and just simply choose \( m_\sigma \simeq 2m_q \).

The pion polarization function \( \Pi_\pi(k) \) to next-to-leading order is shown in Fig. 1, and can be written as

\[
\Pi_\pi(k) = \Pi^{(RPA)}_\pi(k) + \Pi^{f}_\pi(k),
\]

(12)

where \( \Pi^{(RPA)}_\pi \) and \( \Pi^{f}_\pi \) are leading and subleading contributions. The expression for \( \Pi^{(RPA)}_\pi \) is the same as \( \Pi^0_\pi \) except that the quark mass \( m_q^0 \) replaced by \( m_q \). And the contributions of sub-leading order to pion polarization function include three diagrams,

\[
\Pi^{f}_\pi = \delta \Pi^{(b)}_\pi + \delta \Pi^{(c)}_\pi + \delta \Pi^{(d)}_\pi.
\]

(13)

Each diagram can be evaluated as following:

\[
\delta \Pi^{(b)}_\pi(k) = 2N_c N_f \sum_{M=\pi,\sigma} \int \frac{d^4q d^4p}{(2\pi)^8} (-iD_M(q))
\]

\[
\times \left[ \frac{1}{(p^2 - m_q^2)((p + q - k)^2 - m_q^2)} + \frac{1}{((p + q)^2 - m_q^2)((p - k)^2 - m_q^2)} - \frac{k^2(q^2 - \epsilon M^4 m_q^2)}{(p^2 - m_q^2)((p + q)^2 - m_q^2)((p - k)^2 - m_q^2)} \right],
\]

(14)

\[
\delta \Pi^{(c)}_\pi(k) = -4N_c N_f \sum_{M=\pi,\sigma} \int \frac{d^4q d^4p}{(2\pi)^8} \lambda_M (-iD_M(q))
\]

\[
\times \left[ \frac{1}{((p + q)^2 - m_q^2)((p - k)^2 - m_q^2)} + \frac{k^2(q^2 - \epsilon M^4 m_q^2)}{(p^2 - m_q^2)((p + q)^2 - m_q^2)((p - k)^2 - m_q^2)} + \frac{1}{(p^2 - m_q^2)^2} \right.
\]

\[
\left. + \frac{1}{(p^2 - m_q^2)((p + q)^2 - m_q^2)((p - k)^2 - m_q^2)} - \frac{k^2(q^2 - \epsilon M^4 m_q^2)}{(p^2 - m_q^2)((p + q)^2 - m_q^2)(p^2 - m_q^2)} \right],
\]

(15)

with the degeneracy \( \lambda_\pi = 3, \lambda_\sigma = 1 \) and

\[
\delta \Pi^{(d)}_\pi(k) = i \int \frac{d^4q}{(2\pi)^4} (-iD_\pi(q))(-iD_\sigma(q - k))
\]

\[
\times \left[ \int \frac{d^4p}{(2\pi)^4} 8m_q N_c N_f (k \cdot q - (p^2 - m_q^2)) \right]^2.
\]

(16)
The magnitude of mesonic fluctuations on pion polarization function is defined as the ratio of the next-to-leading order contribution over the leading order ones

\[ R^\mu_\pi = \Pi^\mu_\pi / \Pi^{(RPA)}_\pi. \]  

(17)

Considering mesonic fluctuations the corrections on pion polarization function is

\[ \delta \Pi_\pi = \Pi_\pi - \Pi^0_\pi, \]  

(18)

and the magnitude of mesonic corrections on pion polarization can be defined as

\[ R^{cr}_\pi = \delta \Pi_\pi / \Pi^0_\pi. \]  

(19)

At last, we should discuss the formalism of the internal meson propagators. It has been discussed in several papers that the $1/N_c$ expansion in the self-consistent scheme is not strict [10] [11]. It has been pointed out that the exact next-to-leading order quark self-energy includes higher order contributions because the RPA internal meson propagators include any higher order $O(1/N_c^n)$ contributions. In our calculations, we choose the internal meson propagators

\[ -iD_M(q) = \frac{2iG}{1 - 2G\Pi^{(RPA)}_M(q)}, \]  

(20)

in pole approximation

\[ -iD_M(q) = -i \frac{[g^{(RPA)}_{Mqq}(q)]^2}{q^2 - m^2_M}, \]  

(21)

with the pion-quark-antiquark coupling constant

\[ g^{(RPA)}_{Mqq}(q) = (\partial \Pi^{(RPA)}_M(q) / \partial q^2)^{-2}. \]  

(22)

The internal meson propagators in pole approximation will be always in the order of $O(1/N_c)$, which ensure the $1/N_c$ expansion is "strict" in the self-consistent scheme.
5 Numerical Results

Our numerical results are based on solving three equations, the gap equation for quark Eq.(6), the equation for pion mass

\[ 1 - 2G \Pi_\pi(k^2 = m_\pi^2) = 0, \]

where the pion polarization function \( \Pi_\pi \) is calculated completely without any approximation, and the equation for pion decay constant

\[ \frac{m_\pi^2 f_\pi}{g_{\pi qq}} = \frac{m_0}{2G}, \]

where the total coupling constant \( g_{\pi qq} \) is given by the residue of the total pion polarization at the pole

\[ g_{\pi qq}^{-2} = \partial \Pi_\pi / \partial k^2 |_{k^2 = m_\pi^2}. \]

To evaluate the integrals, we introduce two three-momentum cut-offs \( \Lambda_f \) and \( \Lambda_M \) for quarks and mesons in the non-covariant regularization, in order that the fixed model parameters can also be used at finite temperature and density in the future studies. The three model parameters \( m_0, G \) and \( \Lambda_f \) are determined varying with \( \Lambda_M \) by fitting the pion mass \( m_\pi = 139MeV \) and pion decay constant \( f_\pi = 93.2MeV \) and taking an appropriate current quark mass \( m_0 = 5.5MeV \).

Different from [11] and [12], we did not use the leading order of the Gell-Mann-Oakes-Renner (GOR) relations

\[ - m_0 < \bar{q}q > = m_\pi^2 f_\pi^2 + O(m_\pi^4) + \ldots \]

as a constraint, because we did not use external momentum expansion for pions in our calculations. In addition, we did not use chiral expansion for pion mass and pion decay constant. As for the way to determine the model parameters in the calculations to the next-to-leading order, we don’t think it is self-consistent to fit the parameters at [12] or partially at [11] leading order.

Our numerical results are shown in Fig.2 and Fig.3 as a function of \( \Lambda_M/m_q \). It is noticed that our results always start at about \( \Lambda_M/m_q = 1 \). This is because for any value of \( \Lambda_M \), the solutions of \( m_q \) satisfies the relation \( \Lambda_M/m_q > 1 \).
In Fig. 2.a, the total (solid circles) and LO (stars) quark condensate, $- \langle \bar{q}q \rangle_H^{1/3}$ and $- \langle \bar{q}q \rangle_H^{1/3}$, are plotted as a function of $\Lambda_M/m_q$, the value at $\Lambda_M/m_q = 0$ corresponds to the mean-field approximation $-\langle \bar{q}q \rangle^0_H^{1/3}$. In Fig.2.b, the magnitude of mesonic fluctuations $R_q^f$ (stars) and mesonic corrections $R_q^c$ (solid circles) to quark condensate are plotted as a function of $\Lambda_M/m_q$.

In Fig. 3.a, the total (solid circles) and LO (stars) pion polarization function, $\Pi_\pi(m_\pi^2)$ and $\Pi_\pi^{(RPA)}(m_\pi^2)$, are plotted as a function of $\Lambda_M/m_q$, and the value at $\Lambda_M/m_q = 0$ is the pion polarization function in mean-field approximation $\Pi_\pi^0(m_\pi^2)$. And the magnitude of mesonic fluctuations $R_\pi^f$ (stars) and mesonic corrections $R_\pi^c$ (solid circles) on pion polarization function are plotted as a function of $\Lambda_M/m_q$ in Fig.3.b.

It is found that in the region $1 < \Lambda_M/m_q < 2.3$ with $270 MeV < m_q < 440 MeV$ and $270 MeV < \Lambda_M < 1000 MeV$, both the LO and total quark condensate and pion polarization function are smaller than their mean-field values, which is reflected in the negative meson corrections. Here the meson corrections reducing the mean-field quark condensate agree with the results in Fig.7 of [3].

The mesonic fluctuations, i.e., the pure NLO contributions of the chiral condensate and pion polarization function can be characterized by the meson-momentum cut-off $\Lambda_M$, larger $\Lambda_M$ results in larger meson-loop contributions. The NLO contributions are very small till $\Lambda_M/m_q = 1.5$ with $m_q = m_q^0 = 330 MeV$, then increase fast. When $\Lambda_M/m_q > 2.3$, corresponding to $\Lambda_M > 1000 MeV$, the mesonic fluctuations are larger than 40%, which means that the $1/N_c$ expansion around the stationary classical vacuum could not be used any more in this case. This is reasonable for only scalar and pseudo-scalar mesons involved. It is noticed that the NLO contributions enhancing the LO quark condensate is the same as that in [3] and [4].

However, the negative mesonic corrections of the chiral condensate and pion polarization function do not decrease monotonously with the increasing $\Lambda_M/m_q$, they reach the minimum $-35\%$ and $-40\%$ at about $\Lambda_M/m_q = 2$ with $m_q = 370 MeV$, then increase fast. The reason is that the meson corrections to the mean-field approximation include two parts: the pure meson-loop or NLO contributions and the deviation of the one-quark-loop from the mean-field calculation with the quark mass $m_q^0$. When $1 < \Lambda_M/m_q < 1.5$, corresponding to $m_q < m_q^0$, the pure meson-loop contributions are negative.
but very small, and the meson corrections come only from the change of quark mass in the one-quark-loop part. When $\Lambda_M/m_q > 1.5$, corresponding to $m_q > m_q^0$, the pure meson-loop contributions are contrary to the contributions from the deviation of the one-quark-loop contributions. At about $\Lambda_M/m_q = 2.3$, the two parts cancel to each other completely, and the meson corrections vanish.

It is the suppression from the LO contributions and the enhancement from the NLO contributions, that the quark condensate in Fig.2 $a$ and the pion polarization function in Fig.3 $a$ do not decrease continuously with increasing $\Lambda_M/m_q$. In [5], the authors claim that chiral symmetry will restore in vacuum by quantum fluctuations. One reason to get this strange conclusion is the lack of meson-loop contribution in the calculations in [5]. This is also pointed out in [4].

It is necessary to point out that the cancellation between the NLO contributions and the deviation of one-quark-loop contributions from the mean-field approximation is not the same as the cancellation between the exchange term and the remaining ring diagrams in [4]. In the latter case, the meson-loop contributions to quark self-energy are separated into two parts, the exchange diagram and the ring diagram. It is found that the two contributions are contrary, and the exchange term is dominant, which is very important to ensure that the NLO contribution is positive to the LO values. Our cancellation is also not the same as the cancellation between the two NLO diagrams in [12], where the cancellation induces the NLO contributions to LO are very small. The cancellation inside the NLO contributions in [4] and [12] cannot reflect the convergence of the total mesonic corrections to the mean-field approximation, because the mesonic corrections is also included in the one-quark-loop diagram due to the change of quark mass.

6 Conclusions

In conclusion, the magnitude of mesonic fluctuations and their corresponding corrections on chiral condensate and pion polarization function are investigated in a self-consistent scheme by exactly calculating the Feynman diagrams to next-to-leading order of $1/N_c$ expansion.

It is found that the magnitude of the mesonic fluctuations around the classical vacuum, i.e., the pure meson-loop contributions, can be character-
ized by the meson-momentum cut-off $\Lambda_M$, the larger $\Lambda_M$, the larger mesonic fluctuations. The NLO contributions enhance the LO values at the stationary saddle-point. However, the LO contributions reduce the values in the mean-field approximation. The two contributions to the mean-field approximation are contrary and cancel to each other.

When $\Lambda_M > 1000 MeV$, the mesonic fluctuations will be larger than 40%, which means that the meson-loop expansion or $1/N_c$ expansion could not be used any more. In the case of only scalar and pseudo-scalar mesons involved, $\Lambda_M < 1000 MeV$ is reasonable, otherwise, heavier vector mesons should be considered.

Even though we can not answer the question how much the magnitude of the mesonic fluctuations and corrections are, because the model parameters are not uniquely determined, we can conclude that the cancellation between the NLO contributions and the one-quark-loop with quark mass deviation from the mean-field value induces an appropriate mesonic corrections to the mean-field approximation, and no chiral symmetry restoration or instabilities of the vacuum is found.

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Figure 1: Quark self-energy $\Sigma$ and pion polarization function $\Pi_M$ in the quark and pion propagators. $\Sigma_H$ and $\Sigma^H$ are the leading and subleading contributions to the quark mass. $\Pi^{(RPA)}_\pi$ and $\delta\Pi^{(b,c,d)}_\pi$ are the leading and subleading order contributions to pion polarization function. The heavy solid lines indicate the constituent quark propagator, and the heavy dashed lines represent $\pi$ or $\sigma$ propagator $-iD_M(q)$ in RPA approximation.
Figure 2: Quark condensate (a) and the magnitude of mesonic fluctuations and correction on chiral condensate (b) as a function $\Lambda_M/m_q$. The solid circles and stars in (a) correspond to $-\langle \bar{q}q \rangle^{1/3}$ and $-\langle \bar{q}q \rangle_{H}^{1/3}$ respectively, and in (b) correspond to the mesonic corrections $R_q^c$ and meson fluctuations $R_q^{fl}$ on quark condensate respectively.
Figure 3: Pion polarization function (a) and the magnitude of mesonic fluctuations and correction on pion polarization function (b) as a function of $\Lambda_M/m_q$. The solid circles and stars in (a) correspond to $\Pi_{\pi}(m_\pi^2)$ and $\Pi_{\pi}^{(RPA)}(m_\pi^2)$ respectively, and in (b) correspond to the mesonic corrections $R_{\pi}^{cr}$ and meson fluctuations $R_{\pi}^{fl}$ on pion polarization function respectively.