Design and Theoretical Analysis of High-Static-Low-Dynamic Stiffness Torsion Vibration Isolator Based on Oblique Springs

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Abstract. A torsion vibration isolator composed of oblique springs with high-static-low-dynamic stiffness (HSLDS) is proposed to attenuate the transmission of torsion vibration along the shipping shaft in this paper. It is good at low frequency vibration isolation as it can significantly reduce the resonance frequency of the system with the same load capability. Firstly, the model of HSLDS torsion vibration isolator is introduced in this paper. Secondly, the non-dimensional torsion stiffness is formulated using mechanics theory, and the HSLDS characteristic of designed torsion vibration isolator is verified. Finally, the torque transmissibility is analyzed using the Increment Harmonic Balance (IHB) method, and the effects of the system parameters on it are analyzed. The results show that the resonant frequency increases accordingly as the stiffness ratio and the excitation torque are increased. However, the peak value of the torsion transmissibility is decreased as the damper ratio increasing.

Keywords. Shipping shaft; torsion vibration isolator; high-static-low-dynamic stiffness.

1. Introduction
The traditional linear passive isolator can’t work properly in the frequency region wherein the excitation frequency is out of the isolation frequency [1]. Thus, an approach to address this problem is using the small stiffness to extend the effective band-width of isolation system, but it can also lead to large deformation result in instability. In the past decades, HSLDS vibration isolation systems have been introduced to overcome this bottleneck by connecting the negative stiffness corrector and a linear elastic element in parallel. Ibrahim [2] comprehensively assessed the recent progress of nonlinear isolators. Alabuzhev et al. [3] presented the basic theory and a few structures of quasi-zero stiffness isolation systems. Carrela et al. [4, 5] introduced a non-complicated structure of HSLDs isolator by parallelly linking a perpendicular spring with inclined springs. Moreover, other structures of negative stiffness corrector are used. Such as cam-roller-spring structure [6], bi-stable negative stiffness corrector [7], magnetic elastic element [8], and forfex-like model [9]. However, up to now, almost all of the HSLDS isolators are designed to reduce the translational vibration, few has been reported for torsion vibration attenuating.

In this paper, we propose a torsion vibration isolator with HSLDS trait using the oblique springs as torsional negative stiffness corrector, which is connected to the linear elastic element in parallel. Furthermore we analyze its torque transmissibility characteristic.

2. Model and Static Analysis of HSLDS Torsion Vibration Isolator
The model of HSLDS torsion vibration isolator is shown in figure 1. The two terminals of the isolator are the connection of propeller shafts. It is used to convey the rated torque as well as to isolate the...
torque vibration of shipping shaft. It consists of the oblique spring elements and the linear spring, while all of them arrange along the circumferential direction. The oblique spring elements and the linear springs act as the negative stiffness correctors and the positive stiffness elements respectively. The HSLD stiffness is achieved by combing the oblique spring elements with linear springs in parallel while the transmitted torque is maintained by the spring.

**Figure 1.** The model of torsion vibration isolator with HSLDS.

Figure 2 shows the schematic diagram of negative stiffness correctors. It is composed of two oblique springs hinging with two connecting rods. It is supposed that the rigidity value and original length of spring is $k_h$ and $L_{ho}$ respectively. The oblique springs are about $n_2$ groups around circumferential direction of the isolator, and the pre-compressed deformation of each oblique spring is $\delta_0$. The length of connecting rod is $b$, while the distance between the acting point of force on connecting rod to the fixed end is $a$. In addition, the rigidity value of linear positive springs is $k_z$, while they are about $n_1$ sets around the isolator. The action radius of oblique springs and positive linear springs is $r_2$ and $r_1$ respectively. The relationship between righting moment $M$ and relative torsion angle $\varphi$ can be obtained:

$$M = n_1 \cdot k_z \cdot r_2^2 \varphi - 2 \cdot n_2 \cdot k_h \cdot r_2^2 \left( \frac{L_{ho} - a}{\sqrt{b^2 - (r_2 \varphi)^2}} + 1 \right) \varphi$$

(1)

Further, the non-dimensional equation can be obtained as follows:
\[ \hat{M} = \hat{\phi} - 2 \cdot n_0 \cdot k_0 \cdot r_0^2 \left( \sqrt{\frac{1 - a_0}{b_0^2 - (r_0 \hat{\phi})^2}} + 1 \right) \cdot \hat{\phi} \]  
\[ a_0 - b_0 + h_0 = 1 \]

where \( \frac{a}{L_{ho}} = a_0, \quad \frac{b}{L_{ho}} = b_0, \quad \frac{\delta_0}{L_{ho}} = h_0, \quad \frac{r_2}{r_1} = r_0, \quad \frac{r_1 \hat{\phi}}{L_{ho}} = \hat{\phi}, \quad \frac{M}{n_1 k_z L_{ho} r_1} = \hat{M}, \quad \frac{k_h}{k_z} = k_0. \]

Figure 3 gives the relationship between the righting moment, the relative torsion angle, and the pre-compressed deformation of each oblique spring. It can be seen that they are nonlinear.

From equation (2), the torsion stiffness can be further obtained as follows:

\[ \hat{k}_e = 1 - 2 \cdot n_0 \cdot k_0 \cdot r_0^2 \left[ 1 + \frac{(1 - a_0) h_0^2}{(b_0^2 - (r_0 \hat{\phi})^2)^{3/2}} \right] \]  

The torsion stiffness characteristic curve is shown in Figure 4. It has the high-static-low-dynamic stiffness trait. By selecting the appropriate parameters \( k_0 \) and \( h_0 \), the isolator has quasi-zero stiffness feature.

Figure 4. The dimensionless stiffness of the HLSDS torsion vibration isolator with different relative torsion angle, and the pre-compressed deformation of each oblique spring.
3. Torque Transmissibility Affected by the System Parameters

Here we define the ratio of the amplitude of the oscillatory torque transmitted to the right hand shaft, to the amplitude of the input oscillatory torque as the torque transmissibility. It is used to evaluate the torque isolation effect of a vibration isolator. It can be obtained as follows:

\[ T = \frac{\hat{M}_t}{\hat{M}_r} \]  

(5)

The torque transmitted to the output shaft is given by [10]

\[ \hat{M}_t = 2\xi \hat{\phi}' + \hat{k}_1 \hat{\phi} + \hat{k}_3 \hat{\phi}^3 \]  

(6)

Then the amplitude of the transmitted torque can be obtained using IHB Methods

\[ |\hat{M}_t| = \sqrt{\left(-2\xi\Omega\hat{\phi}_f\right)^2 + \left(\frac{3k_3\hat{\phi}_f^3}{4} + \hat{k}_1\hat{\phi}_f\right)^2} \]  

(7)

The amplitude of the excitation oscillatory torque can be obtained from the reference Yang etc. [10], so the torque transmissibility can be written in the dB form:

\[ T = 20\log\left(\frac{\sqrt{\left(-2\xi\Omega\hat{\phi}_f\right)^2 + \left(\frac{3k_3\hat{\phi}_f^3}{4} + \hat{k}_1\hat{\phi}_f\right)^2}}{\hat{M}_r}\right) \]  

(8)

As shown in Figure 5, the torque transmissibility is affected by the stiffness ratio \( \hat{k}_3 \). It can be seen that the natural frequency increases accordingly as the stiffness ratio is increasing. However, the torque transmissibility remains the same in the high frequencies region when the stiffness ratio increases while the effective band-width of isolation is narrowed.

Figure 5. The torque transmissibility affected by the stiffness ratio \( \hat{k}_3 \)

Figure 6 shows the torque transmissibility is affected by the damper ratio \( \xi \). The result is that when damper ratio is increasing the peak value of the transmissibility and the resonant frequency decline. However, in the high frequency, the transmissibility increases as the damper ratio rises. It means the vibration attenuation becomes worse as the damper ratio is increased. The conclusion can be obtained that the vibration isolation property becomes deteriorate in high frequency region due to damping just like the linear isolation system.
Figure 6. The torque transmissibility affected by the damper ratio $\xi$.

As shown in figure 7, the torque transmissibility is affected by the excitation torque $\tilde{M}_e$. It can be seen that the natural frequency increases as the excitation torque is increasing and the isolation frequency bandwidth become narrowed. However, in the high frequency region, the excitation torque barely effects on the isolation performance.

Figure 7. The torque transmissibility affected by the excitation torque $\tilde{M}_e$.

4. Conclusions
A novel HSLDS torsion vibration isolator using for shipping has been proposed. The HSLDS characteristic of the vibration isolator has been verified. The torque transmissibility analysis results show that the resonant frequency increases accordingly as the stiffness ratio and the excitation torque are increased. However, the peak value of transmissibility is decreased when the damper ratio rises.

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