Target Training Does Adversarial Training Without Adversarial Samples

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Abstract
Neural network classifiers are vulnerable to misclassification of adversarial samples, for which the current best defense trains classifiers with adversarial samples. However, adversarial samples are not optimal for steering attack convergence, based on the minimization at the core of adversarial attacks. The minimization perturbation term can be minimized towards $0$ by replacing adversarial samples in training with duplicated original samples, labeled differently only for training. Using only original samples, Target Training eliminates the need to generate adversarial samples for training against all attacks that minimize perturbation. In low-capacity classifiers and without using adversarial samples, Target Training exceeds both default CIFAR10 accuracy ($84.3\%$) and current best defense accuracy (below $25\%$) with $84.8\%$ against CW-L$_2$($\kappa=0$) attack, and $86.6\%$ against DeepFool. Using adversarial samples against attacks that do not minimize perturbation, Target Training exceeds current best defense ($69.1\%$) with $76.4\%$ against CW-L$_2$($\kappa=40$) in CIFAR10.

1. Introduction
Neural network classifiers are vulnerable to malicious adversarial samples that appear indistinguishable from original samples (Szegedy et al., 2013), for example, an adversarial attack can make a traffic stop sign appear like a speed limit sign (Eykholt et al., 2018) to a classifier. An adversarial sample created using one classifier can also fool other classifiers (Szegedy et al., 2013; Biggio et al., 2013), even ones with different structure and parameters (Szegedy et al., 2013; Goodfellow et al., 2014; Papernot et al., 2016b; Tramèr et al., 2017b). This transferability of adversarial attacks (Papernot et al., 2016b) matters because it means that classifier access is not necessary for attacks. The increasing deployment of neural network classifiers in security and safety-critical domains such as traffic (Eykholt et al., 2018), autonomous driving (Amodei et al., 2016), healthcare (Faust et al., 2018), and malware detection (Cui et al., 2018) makes countering adversarial attacks important.

Most current attacks, including the strongest Carlini&Wagner attack (CW) (Carlini & Wagner, 2017c), are gradient-based attacks. Gradient-based attacks use the classifier gradient to generate adversarial samples from non-adversarial samples. Gradient-based attacks minimize the sum of classifier adversarial loss and perturbation (Szegedy et al., 2013), though attacks can relax the perturbation minimization to allow for bigger perturbations. The CW attack (Carlini & Wagner, 2017c) uses the $\kappa > 0$ parameter to control perturbation, while Projected Gradient Descent (PGD) (Kurakin et al., 2016; Madry et al., 2017) and Fast-GradientMethod (FGSM) (Goodfellow et al., 2014) use an $\epsilon$ parameter. Other gradient-based adversarial attacks include DeepFool (Moosavi-Dezfooli et al., 2016), Zeroth order optimization (ZOO) (Chen et al., 2017), Universal Adversarial Perturbation (UAP) (Moosavi-Dezfooli et al., 2017).

Many recent proposed defenses have been broken (Carlini & Wagner, 2016; 2017a;b; Athalye et al., 2018; Tramer et al., 2020). They fall largely into these categories: (1) adversarial sample detection, (2) gradient masking and obfuscation, (3) ensemble, (4) customized loss. Detection defenses (Meng & Chen, 2017; Ma et al., 2018; Li et al., 2019; Hu et al., 2019) aim to detect, correct or reject adversarial samples. Many detection defenses have been broken (Carlini & Wagner, 2017b;a; Tramer et al., 2020). Gradient obfuscation is aimed at preventing gradient-based attacks from access to the gradient and can be achieved by shattering gradients (Guo et al., 2018; Verma & Swami, 2019; Sen et al., 2020), randomness (Dhillon et al., 2018; Li et al., 2019) or vanishing or exploding gradients (Papernot et al., 2016a; Song et al., 2018; Samangouei et al., 2018). Many gradient obfuscation methods have also been successfully defeated (Carlini & Wagner, 2016; Athalye et al., 2018; Tramer et al., 2020). Ensemble defenses (Tramèr et al., 2017a; Verma & Swami, 2019; Pang et al., 2019; Sen et al., 2020) have also been broken (Carlini & Wagner, 2016; Tramer et al., 2020), unable to even outperform their best performing component. Customized attack losses defeat defenses (Tramer et al., 2020) with customized losses (Pang et al., 2020; Verma & Swami, 2019) but also, for example...
2.1. Adversarial Attacks

Targeted Attacks Szegedy et al. (2013) were the first to formulate the generation of adversarial samples as a constrained minimization of the perturbation under an $L_p$ norm. Because this formulation can be hard to solve, Szegedy et al. (2013) reformulated the problem as a gradient-based, two-term minimization of the weighted sum of perturbation and classifier loss. For targeted attacks, this minimization is:

$$\min_{\delta} \, c \cdot \|\delta\| + \text{loss}_f(x + \delta, l) \quad (\text{Minimization 1})$$

such that $x + \delta \in [0, 1]^d$,

where $c$ is a constant, $\delta$ is perturbation, $f$ is the classifier, $\text{loss}_f$ is classifier loss, $l$ is an adversarial label. $\|\delta\|$ in term (1) of Minimization Minimization 1 is a norm of the adversarial perturbation, while term (2) is there to utilize the classifier gradient to find adversarial samples that minimize classifier adversarial loss. By formulating the problem of finding adversarial samples this way, Szegedy et al. (2013) paved the way for adversarial attacks to utilize classifier gradients in adverserial attacks.

Minimization 1 is the foundation for many gradient-based attacks, though many tweaks can and have been applied. Some attacks follow Minimization 1 implicitly (Moosavi-Dezfooli et al., 2016), and others explicitly (Carlini & Wagner, 2017c). The type of $L_p$ norm in term (1) of the minimization also varies. For example the CW attack (Carlini & Wagner, 2017c) uses $L_0$, $L_2$ and $L_\infty$, whereas DeepFool (Moosavi-Dezfooli et al., 2016) uses the $L_2$ norm. A special perturbation case is the Pixel attack by Su et al. (2019) which changes exactly one pixel. Some attacks even exclude term (1) from the Minimization 1 and introduce an external parameter to control perturbation. The FGSM attack by Goodfellow et al. (2014), for example, uses an $\epsilon$ parameter, while the CW attack (Carlini & Wagner, 2017c) uses a $\kappa$ confidence parameter.

There are three ways (Carlini & Wagner, 2017c; Kurakin et al., 2018) to choose what the target adversarial label is: (1) Best case - try the attack with all adversarial labels and choose the label that was the easiest to attack; (2) Worst case - try the attack with all adversarial labels and choose the label that was the toughest to attack; (3) Average case - choose a target label uniformly at random from the adversarial labels.

Untargeted Attacks Untargeted attacks aim to find a nearby sample that misclassifies, without aiming for a specific adversarial label. Some untargeted attacks, such as DeepFool and UAP, have no targeted equivalent.

Stronger Attacks There are conflicting accounts of which attacks are stronger, targeted attacks or untargeted attacks. Carlini & Wagner (2017c) claim that targeted attacks are stronger. However, Kurakin et al. (2018) find targeted attacks, including worst-case targeted attacks, to be much weaker than untargeted attacks.

Fast Gradient Sign Method The Fast Gradient Sign Method by Goodfellow et al. (2014) is a simple, $L_\infty$-bounded attack that constructs adversarial samples by per-
Adversarial Training. Adversarial Training (Szegedy et al., 2013; Kurakin et al., 2016; Madry et al., 2017) is one of the first and few, undefeated defenses. It defends by populating low probability, so-called blind spots (Szegedy et al., 2013; Goodfellow et al., 2014) with adversarial samples labelled correctly, redrawing boundaries. The drawback of Adversarial Training is that it needs to know the attack in advance, and it needs to generate adversarial samples during training. The Adversarial Training Algorithm 2 in the Appendix is based on Kurakin et al. (2016). Madry et al. (2017) formulate their defense as a robust optimization problem, and use adversarial samples to augment the training. Their solution however necessitates high-capacity classifiers - bigger models with more parameters.

Detection defenses Such defenses detect adversarial samples implicitly or explicitly, then correct or reject them. So far, many detection defenses have been defeated. For example, ten diverse detection methods (other network, PCA, statistical properties) were defeated with attack loss customization by Carlini & Wagner (2017a); Tramer et al. (2020) used attack customization against Hu et al. (2019); attack transferability (Carlini & Wagner, 2017b) was used against MagNet by Meng & Chen (2017); deep feature adversaries (Sabour et al., 2016) against Roth et al. (2019).

Gradient masking and obfuscation Many defenses that mask or obfuscate the classifier gradient have been defeated (Carlini & Wagner, 2016; Athalye et al., 2018). Athalye et al. (2018) identify three types of gradient obfuscation: (1) Shattered gradients - incorrect gradients caused by non-differentiable components or numerical instability, for example with multiple input transformations by Guo et al. (2018). Athalye et al. (2018) counter such defenses with Backward Pass Differentiable Approximation. (2) Stochastic gradients in randomized defenses are overcome with Expectation Over Transformation by Athalye et al. (2017). Examples are Stochastic Activation Pruning (Dhillon et al., 2018), which drops layer neurons based on a weighted distribution, and (Xie et al., 2018) which adds a randomized layer to the classifier input. (3) Vanishing or exploding gradients are used, for example, in Defensive Distillation (DD) (Papernot et al., 2016a) which reduces the amplitude of gradients of the loss function. Other examples are PixelDefend (Song et al., 2018) and Defense-GAN (Samangouei et al., 2018). Vanishing or exploding gradients are broken with parameters that avoid vanishing or exploding gradients (Carlini & Wagner, 2016).

Complex defenses Defenses combining several approaches, for example (Li et al., 2019) which uses detection, randomization, multiple models and losses, can be defeated by focusing on the main defense components (Tramer et al., 2020). In particular, ensemble defenses do not perform better than their best components. Verma & Swami (2019); Pang et al. (2019); Sen et al. (2020) are defeated ensemble defenses combined with numerical instability (Verma & Swami, 2019), regularization (Pang et al., 2019), or mixed precision on weights and activations (Sen et al., 2020).

2.2. Adversarial Defenses

Adversarial Training. Adversarial Training (Szegedy et al., 2013; Kurakin et al., 2016; Madry et al., 2017) is one of the first and few, undefeated defenses. It defends by populating low probability, so-called blind spots (Szegedy et al., 2013; Goodfellow et al., 2014) with adversarial samples labelled correctly, redrawing boundaries. The drawback of Adversarial Training is that it needs to know the attack in advance, and it needs to generate adversarial samples during training. The Adversarial Training Algorithm 2 in the Appendix is based on Kurakin et al. (2016). Madry et al. (2017) formulate their defense as a robust optimization problem, and use adversarial samples to augment the training. Their solution however necessitates high-capacity classifiers - bigger models with more parameters.

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the classifier minimally, but focuses on steering attack convergence. Target Training is the first defense based on the minimization term of the Minimization 1 at the core of untargeted gradient-based adversarial attacks.

3. Target Training

Just as adversarial attacks have used the gradient term of Minimization 1 against defenses, the perturbation term in the same minimization can be used to steer attack convergence. By training the classifier with duplicated original samples labeled differently only in training with target labels (hence Target Training), attacks that minimize perturbation are forced to converge to benign samples because the duplicated samples minimize the perturbation towards 0.

Target Training is a form of Adversarial Training that replaces adversarial samples with original samples, leading attacks to converge to non-adversarial samples as they do in Adversarial Training defense. In Target Training, the final no-weight layer used in inference and testing essentially relabels the target labels to the original labels, which is the equivalent of labeling adversarial samples correctly in Adversarial Training. However, the fact that Target Training is a form of Adversarial Training that uses no adversarial samples against attacks that minimize perturbation presents us with a question: Might it be that Adversarial Training works not because it populates the distribution blind spots with adversarial samples, but because these adversarial samples steer attack convergence?

Target Training could also be extended to defend against more than one attack at the same time. For example, to defend simultaneously against two types of attacks that do not minimize perturbation, the batch size would be tripled and the batch would be populated with adversarial samples from both attacks. In addition, there would be two sets of target labels, one set for each attack.

3.1. Classifier Architecture

We choose low-capacity classifiers in order to investigate whether Target Training can defend such classifiers. The MNIST classifier has two convolutional layers with 32 and 64 filters respectively, followed each by batch normalization, then a $2 \times 2$ max-pooling layer, a drop-out layer, a fully connected layer with 128 units, then another drop-out layer, then a softmax layer with 20 outputs, then a summation layer without weights that adds up the softmax outputs two-by-two and has 10 outputs. The CIFAR10 classifier has 3 groups of layers, each of which has: two convolutional layers with increasing number of filters and with elu activation followed by batch normalization, then a $2 \times 2$ max-pooling layer, then a drop-out layer. Then a softmax layer with 20 outputs, and finally a no-weight summation layer that takes softmax layer outputs as inputs, sums them two-by-two, and has 10 outputs. Table 7 in the Appendix shows classifier architecture for CIFAR10 and MNIST in detail. Training uses all layers up to the softmax layer, but not the final layer. Inference and testing uses all classifier layers.

3.2. Training

Against attacks that minimize perturbation, such as CW($\kappa = 0$) and DeepFool, Target Training uses duplicates of original samples in each batch instead of adversarial samples because these samples minimize the perturbation to 0 - no other points can have smaller distance from original samples. This eliminates the overhead of calculating adversarial samples against all attacks that minimize perturbation. Figure 1 shows how Target Training trains without adversarial samples against all attacks that minimize perturbation, and with adversarial samples against attacks that do not minimize perturbation. Training in Target Training is also illustrated in Figure 2 to show that all the layers up to the softmax layer (with $2k$ class outputs) take part in training, but not the last layer. The duplicated samples are labeled as $i + k$, where $i$ is the original label and $k$ is the number of classes.

Algorithm 1 shows Target Training algorithm against all attacks that minimize perturbations. Against attacks that do not minimize perturbation, such as CW($\kappa > 0$), PGD and FGSM, Target Training uses adversarial samples in training, shown in Algorithm 3 in the Appendix. Both Target Training algorithms are based on the Adversarial Training (Kurakin et al., 2016) Algorithm 2 in the Appendix.
3.3. Inference

Figure 2 shows that inference in Target Training differs from training by using a no-weight, final layer. The final layer derives the $y_i$ output probability of the final layer, as the sum of probabilities $s_i$ and $s_i+k$ in the softmax layer output: $y_i = s_i + s_i+k$, where $k$ is the number of classes, $i \in [0 \ldots (k - 1)]$, $s$ is softmax layer output, $y$ is final layer output.

4. Experiments

Threat model We assume that the adversary goal is to generate adversarial samples that cause misclassification. We perform white-box evaluations, assuming the adversary has complete knowledge of the classifier and how the defense works. In terms of capabilities, we assume that the adversary is gradient-based, has access to the CIFAR10 and MNIST image domains and is able to manipulate pixels. Perturbations are assumed to be $L_p$-constrained. For attacks that do not minimize perturbations, we assume that the attack is of the same kind as the attack used to generate the adversarial samples used during training. We assume that the adversary can generate both targeted and untargeted attacks.

Targeted and untargeted attacks There are conflicting views (Carlini & Wagner, 2017c; Kurakin et al., 2018) whether targeted or untargeted attacks are stronger. To determine which are stronger, we conduct experiments with both untargeted and average-case targeted attacks where the target label is chosen uniformly at random from adversarial labels. Targeted attacks are not applicable for DeepFool.

Algorithm 1 Target Training of classifier $N$ against attacks that minimize perturbation based on Adversarial Training Algorithm 2 in the Appendix.

Require: $m$ batch size, $k$ classes, classifier $N$ with all layers up to softmax layer with $2k$ output classes, TRAIN trains a classifier on a batch and labels

Ensure: Classifier $N$ is Target-Trained against all attacks that minimize perturbation

while training not converged do

$B = \{x^1, \ldots, x^n\}$ \{Get random batch\}
$G = \{y^1, \ldots, y^m\}$ \{Get batch ground truth\}
$B' = \{x^1, \ldots, x^n, x^1, \ldots, x^n\}$ \{Duplicate batch\}
$G' = \{y^1, \ldots, y^m, y^1 + k, \ldots, y^m + k\}$ \{Duplicate ground truth and increase duplicates by $k$\}
TRAIN($N$, $B'$, $G'$) \{Train classifier on duplicated batch and new ground truth\}

end while

Attack parameters For CW: 9 steps, $0 - 40$ confidence values, default $1K$ iterations, but also experiments with up to $10K$ iterations in adaptive attacks. For PGD, parameters based on PGD paper (Madry et al., 2017): for CIFAR10, 7 steps of size 2 with a total $\epsilon = 8$; for MNIST, 40 steps of size 0.01 with a total $\epsilon = 0.3$. For all PGD attacks, we use 0 random initialisations within the $\epsilon$ ball, effectively starting PGD attacks from the original images. For FGSM: $\epsilon = 0.3$, as in (Madry et al., 2017).

Classifier models We purposefully do not use high-capacity models, such as ResNet (He et al., 2016), used for example by Madry et al. (2017), to show that Target Training does not necessitate high model capacity to defend against adversarial attacks. The architectures of MNIST and CIFAR datasets are described in Subsection 3.1 and shown in Table 7 in the Appendix. No data augmentation was used. Default accuracies without attack are 84.3% for CIFAR10 and 99.1% for MNIST. Adversarial Training and default classifiers have same architecture, except the softmax layer is the last layer and has 10 outputs.

Datasets The MNIST (LeCun et al., 1998) and the CIFAR10 (Krizhevsky et al., 2009) datasets are 10-class datasets that have been used throughout previous work. The MNIST (LeCun et al., 1998) dataset has 60K, $28 \times 28 \times 1$ hand-written, digit images. The CIFAR10 (Krizhevsky et al., 2009) dataset has 70K, $32 \times 32 \times 3$ images. Each dataset has 10K testing samples and all experimental evaluations
are done with testing samples.

**Tools** Adversarial samples generated with CleverHans 3.0.1 (Papernot et al., 2018) for CW-$L_2$ (Carlini & Wagner, 2017c), DeepFool (Moosavi-Dezfooli et al., 2016) attacks and IBM Adversarial Robustness 360 Toolbox (ART) toolbox 1.2 (Nicolae et al., 2018) for CW-$L_{\infty}$ (Carlini & Wagner, 2017c), FGSM (Goodfellow et al., 2014) and PGD (Kurakin et al., 2016) attacks. Target Training has been written in Python 3.7.3, using Keras 2.2.4 (Chollet et al., 2015).

**Baselines** We choose Adversarial Training as a baseline because it is the current best defense since other defenses have been defeated successfully (Carlini & Wagner, 2016; 2017b,a; Athalye et al., 2018; Tramer et al., 2020), more details in Section 2. Our Adversarial Training implementation is based on (Kurakin et al., 2016), shown in Algorithm 2 in the Appendix. We choose the Kurakin et al. (2016) implementation and not the robust optimization of Madry et al. (2017), because the Adversarial Training solution by Madry et al. (2017) necessitates high-capacity classifiers. However, we do not use high-capacity classifiers in order to show that Target Training can defend low-capacity classifiers.

4.1. Targeted Attacks Are Weaker Than Untargeted Attacks

There are conflicting views (Carlini & Wagner, 2017c; Kurakin et al., 2018) whether targeted or untargeted attacks are stronger. Carlini & Wagner (2017c) claim that targeted attacks are stronger, whereas Kurakin et al. (2018) claim that targeted attacks, even worst-case ones, are much weaker that untargeted attacks. Here, we aim to find out which type of attack is stronger, to use for the rest of the experiments. We use average-case targeted attacks, expained in Section 2.1.

Accuracy values of default classifiers in Table 1 show targeted attacks to be not strong. For example, targeted CW-$L_2(\kappa = 0)$ in CIFAR10 decreases default classifier accuracy by less than 1% point, whereas targeted CW-$L_2(\kappa = 40)$ attack even increases default classifier accuracy. Similarly, all targeted attacks against MNIST default classifier reduce accuracy by less than 3%.

By comparison, each untargeted attack is much stronger than its targeted equivalent, supporting Kurakin et al. (2018). Untargeted CW-$L_2(\kappa = 0)$ and untargeted CW-$L_2(\kappa = 40)$ in CIFAR10 reduce default classifier accuracy to below 9%, and in MNIST to below 1%.

**Table 1.** Here, we show that each untargeted attack diminishes the accuracy of default classifiers much more than its equivalent targeted attack. This indicates that untargeted attacks are stronger. DeepFool attack has no targeted equivalent.

| Attack            | CIFAR10 Default Classifier | MNIST Default Classifier |
|-------------------|----------------------------|--------------------------|
| No Attack         | 84.3%                      | 99.1%                    |
| **Targeted Attacks** |                            |                          |
| CW-$L_2(\kappa = 0)$ | 84.0%                      | 98.3%                    |
| CW-$L_{\infty}(\kappa = 0)$ | 72.7%                      | 98.6%                    |
| DEEPPFOOL         | NA                         | NA                       |
| CW-$L_2(\kappa = 40)$ | 85.7%                      | 99.0%                    |
| PGD($\epsilon = 8, \epsilon = 0.3$) | 44.9%                      | 96.4%                    |
| FGSM($\epsilon = 0.3$) | 46.4%                      | 96.4%                    |
| **Untargeted Attacks** |                           |                          |
| CW-$L_2(\kappa = 0)$ | 8.5%                       | 0.8%                     |
| CW-$L_{\infty}(\kappa = 0)$ | 23.6%                      | 94.2%                    |
| DEEPPFOOL         | 8.6%                       | 2.8%                     |
| CW-$L_2(\kappa = 40)$ | 7.9%                       | 0.8%                     |
| PGD($\epsilon = 8, \epsilon = 0.3$) | 10.9%                      | 90.7%                    |
| FGSM($\epsilon = 0.3$) | 17.6%                      | 90.7%                    |

4.2. Target Training Against Attacks That Minimize Perturbation

Table 2 shows that Target Training exceeds by far accuracies by Adversarial Training and default classifier against attacks that minimize perturbation. Without using adversarial samples in training, Target Training exceeds even default accuracy on non-adversarial samples against CW-$L_2(\kappa = 0)$ and DeepFool in CIFAR10. The only case where performances are roughly equal is against CW-$L_{\infty}(\kappa = 0)$ in CIFAR10.

4.3. Target Training Against Attacks That Do Not Minimize Perturbation

Table 3 shows that Target Training can even improve accuracy compared to Adversarial Training against attacks that do not minimize perturbation, for attacks CW-$L_2(\kappa = 40)$ and FGSM($\epsilon = 0.3$) in CIFAR10. Against such attacks, Target Training uses adversarial samples in training as Adversarial Training does. Against PGD attack, Target Training performs worse then Adversarial Training. We attribute the Target Training performance against PGD to the low capacity of the classifiers we use. Such effect of classifier capacity on performance has been previously observed by Madry et al. (2017). We anticipate Target Training performance to improve for higher-capacity classifiers.

**Table 2.** There, we show how much better Target Training performs against specific adversarial attacks compared to Adversarial Training. Target Training always performs better in CIFAR10, and sometimes in MNIST. The only case where performances are roughly equal is against attack CW-$L_{\infty}(\kappa = 0)$, and in that case Target Training is even better in MNIST. The attack DeepFool has no targeted equivalent.

| Attack            | CIFAR10 Default Classifier | MNIST Default Classifier |
|-------------------|----------------------------|--------------------------|
| No Attack         | 84.3%                      | 99.1%                    |
| **Targeted Attacks** |                            |                          |
| CW-$L_2(\kappa = 0)$ | 84.0%                      | 98.3%                    |
| CW-$L_{\infty}(\kappa = 0)$ | 72.7%                      | 98.6%                    |
| DEEPPFOOL         | NA                         | NA                       |
| CW-$L_2(\kappa = 40)$ | 85.7%                      | 99.0%                    |
| PGD($\epsilon = 8, \epsilon = 0.3$) | 44.9%                      | 96.4%                    |
| FGSM($\epsilon = 0.3$) | 46.4%                      | 96.4%                    |
| **Untargeted Attacks** |                           |                          |
| CW-$L_2(\kappa = 0)$ | 8.5%                       | 0.8%                     |
| CW-$L_{\infty}(\kappa = 0)$ | 23.6%                      | 94.2%                    |
| DEEPPFOOL         | 8.6%                       | 2.8%                     |
| CW-$L_2(\kappa = 40)$ | 7.9%                       | 0.8%                     |
| PGD($\epsilon = 8, \epsilon = 0.3$) | 10.9%                      | 90.7%                    |
| FGSM($\epsilon = 0.3$) | 17.6%                      | 90.7%                    |

**Table 3.** Here, we show that Target Training can improve accuracy compared to Adversarial Training against attacks that do not minimize perturbation in CIFAR10 and MNIST. We also show the accuracy of default classifiers which are used to train the Adversarial Training and Target Training models. The only case where performances are roughly equal is against attack CW-$L_{\infty}(\kappa = 0)$, and in that case Target Training is even better in MNIST. The attack DeepFool has no targeted equivalent.
Table 3. Using adversarial samples in training, Target Training performs better than Adversarial Training against non-$L_{\infty}$ attacks that do not minimize perturbation in CIFAR10. Against $L_{\infty}$ attacks, Target Training performs worse than Adversarial Training.

| Untargeted Attack | CIFAR10 (84.3%) | MNIST (99.1%) |
|-------------------|----------------|-------------|
|                   | Target Training | Adversarial Training | Default Classifier | Target Training | Adversarial Training | Default Classifier |
| CW-$L_2(\kappa = 0)$ | 84.8%          | 22.8%        | 8.5%                | 96.9%          | 5.0%        | 0.8%          |
| CW-$L_\infty(\kappa = 0)$ | 21.3%          | 21.4%        | 23.6%               | 96.1%          | 75.8%       | 94.2%         |
| DeepFool          | 86.6%          | 24.0%        | 8.6%                | 94.9%          | 5.2%        | 2.8%          |

Table 2. Here, we show Target Training performance against attacks that minimize perturbation, for which Target Training does not use adversarial samples. Target Training even exceeds performance of default classifier against CW-$L_2(\kappa = 0)$ and DeepFool in CIFAR10. Target Training also exceeds the performance of Adversarial Training classifier that uses adversarial samples, except for CW-$L_\infty(\kappa = 0)$ in CIFAR10 where accuracies are roughly equal.

| Untargeted Attack | CIFAR10 (84.3%) | MNIST (99.1%) |
|-------------------|----------------|-------------|
|                   | Target Training | Adversarial Training | Default Classifier | Target Training | Adversarial Training | Default Classifier |
| CW-$L_2(\kappa = 40)$ | 76.4%          | 69.1%        | 7.9%                | 95.7%          | 96.5%       | 0.8%          |
| PGD($\epsilon = 8$, $\epsilon = 0.3$) | 7.1%           | 76.2%        | 10.9%               | 57.9%          | 91.7%       | 90.7%         |
| FGSM($\epsilon = 0.3$) | 72.0%          | 71.8%        | 17.6%               | 98.2%          | 98.4%       | 90.7%         |

4.4. Target Training performance on original, non-adversarial samples

In Table 4, we show that Target Training exceeds default classifier accuracy in CIFAR10 on original, non-adversarial images when trained without adversarial samples against attacks that minimize perturbation: 86.7% (up from 84.3%). Furthermore, Table 4 shows that when using adversarial samples against attacks that do not minimize perturbation, Target Training equals Adversarial Training performance.

4.5. Transferability Analysis

For a defense to be strong, it needs to be shown to break the transferability of attacks. A good source of adversarial samples for transferability is the unsecured classifier (Carlini et al., 2019). We experiment on the transferability of attacks from the unsecured classifier to a classifier secured with Target Training. In Table 5, we show that Target Training breaks the transferability of adversarial samples generated by attacks that minimize perturbation much better than Adversarial Training in CIFAR10. Against the rest of attacks Target Training and Adversarial Training perform similarly.

5. Adaptive evaluation

Many recent defenses have failed to anticipate attacks that have defeated them (Carlini et al., 2019; Carlini & Wagner, 2017a; Athalye et al., 2018). Therefore, we perform an adaptive evaluation (Carlini et al., 2019; Tramer et al., 2020) of our Target Training defense.

Whether Target Training could be defeated by methods used to break other defenses. Target Training is a type of Adversarial Training because both use additional training samples, but there is no adaptive attack against Adversarial Training. Target Training uses none of previous unsuccessful defenses (Carlini & Wagner, 2016; 2017b;a; Athalye et al., 2018; Tramer et al., 2020) that involve adversarial sample detection, preprocessing, obfuscation, ensemble, customized loss, subcomponent, non-differentiable component. Therefore their adaptive attacks cannot be used on Target Training. In addition, we keep the loss function simple - standard softmax cross-entropy and no additional loss. Following, we discuss an adaptive attack based on the Target Training summation layer after the softmax layer.

Adaptive attack against Target Training. Based on the Target Training defense, we consider an adaptive attack that uses a copy of the Target Training classifier up to the softmax layer, without the last layer, to generate adversarial samples that are then tested on a full Target Training classifier. Table 6 shows that Target Training withstands the adaptive attack.

Iterative attacks. The multi-step PGD (Kurakin et al., 2016) attack decreases Target Training accuracy more than single-step attacks, which suggests that our defense is working correctly, according to Carlini et al. (2019).

Transferability. Our transferability analysis results in Table 5 in Subsection 4.5 show that Target Training breaks the transferability of adversarial samples much better than Adversarial Training against attacks that minimize perturbation.
Target Training Does Adversarial Training Without Adversarial Samples

Table 4. Target Training exceeds default classifier accuracy on original, non-adversarial samples, when trained without adversarial samples against attacks that minimize perturbation in CIFAR10. Adversarial Training is not applicable because it needs adversarial samples. Target Training equals Adversarial Training performance when using adversarial samples against attacks that do not minimize perturbation.

| Untargeted attack used in training | CIFAR10 (84.3%) | MNIST (99.1%) |
|-----------------------------------|-----------------|---------------|
|                                   | Target Training | Adversarial Training | Default Classifier | Target Training | Adversarial Training | Default Classifier |
| None (against no perturbation attacks) | 86.7% | NA | 84.3% | 98.6% | NA | 99.1% |
| CW-$L_2$ ($\kappa = 40$) | 77.7% | 77.4% | 84.3% | 98.0% | 98.0% | 99.1% |
| PGD($\epsilon = 8$, $\epsilon = 0.3$) | 76.3% | 76.9% | 84.3% | 98.3% | 98.4% | 99.1% |
| FGSM($\epsilon = 0.3$) | 77.6% | 76.6% | 84.3% | 98.6% | 98.6% | 99.1% |

Table 5. Target Training breaks the transferability of attacks that minimize perturbation much better than Adversarial Training in CIFAR10. Against attacks that do not minimize perturbation, Target Training and Adversarial Training have comparable performance - both Target Training and Adversarial Training break the transferability of attacks in MNIST but not in CIFAR10.

| Untargeted attack | CIFAR10 (84.3%) | MNIST (99.1%) |
|-------------------|-----------------|---------------|
|                   | Target Training | Adversarial Training | Default Classifier | Target Training | Adversarial Training | Default Classifier |
| CW-$L_2$ ($\kappa = 0$) | 84.7% | 50.8% | 8.5% | 97.0% | 92.8% | 0.8% |
| CW-$L_\infty$ ($\kappa = 0$) | 84.2% | 55.9% | 23.6% | 96.2% | 97.8% | 94.2% |
| DeepFool | 86.6% | 32.3% | 8.6% | 94.9% | 95.9% | 2.8% |
| CW-$L_2$ ($\kappa = 40$) | 35.8% | 33.8% | 7.9% | 97.8% | 97.9% | 0.8% |
| PGD($\epsilon = 8$, $\epsilon = 0.3$) | 10.8% | 10.0% | 10.9% | 72.1% | 75.2% | 90.7% |
| FGSM($\epsilon = 0.3$) | 34.1% | 45.5% | 17.6% | 72.1% | 75.2% | 90.7% |

Table 6. Target Training withstands the adaptive attack for both CIFAR10 and MNIST. Adversarial samples are generated using a Target Training classifier up to the softmax layer, without the last layer. The generated samples are tested against the original, full Target Training classifier.

| Untargeted adaptive attack | CIFAR10 (84.3%) | MNIST (99.1%) |
|---------------------------|-----------------|---------------|
|                           | Target Training | Adversarial Training |
| CW-$L_2$ ($\kappa = 0$)  | 84.7% | 97.0% |
| CW-$L_\infty$ ($\kappa = 0$) | 84.2% | 96.3% |
| DeepFool                  | 86.6% | 94.9% |
| CW-$L_2$ ($\kappa = 40$)  | 76.4% | 95.7% |
| PGD($\epsilon = 8$, $\epsilon = 0.3$) | 76.3% | 92.3% |
| FGSM($\epsilon = 0.3$)    | 72.1% | 98.2% |

in CIFAR10. Target Training performance in the rest of the attacks is comparable to Adversarial Training performance. The attacks are generated with default, unsecured classifier.

**Stronger CW attack leads to better Target Training accuracy.** Increasing iterations for CW-$L_2$ ($\kappa = 0$) 10-fold from 1$\kappa$ to 10$\kappa$ increases our defense’s accuracy. In CIFAR10, the accuracy increases from 84.76% to 84.88%, in MNIST from 96.92% to 96.96%.

6. Discussion And Conclusions

In conclusion, we show that our white-box Target Training defense counters non-$L_\infty$ attacks that minimize perturbation in low-capacity classifiers without using adversarial samples. Target Training defends classifiers by training with duplicated original samples instead of adversarial samples. This minimizes the perturbation term in attack minimization and as a result steers attacks to non-adversarial samples. Target Training exceeds default accuracy (84.3%) in CIFAR10 with 84.8% against CW-$L_2$ ($\kappa = 0$), 86.6% against DeepFool, and 86.7% on original non-adversarial samples. As a form of Adversarial Training that does not use adversarial samples against attacks that minimize perturbation, Target Training defies the common justification of why Adversarial Training works. The implication is that the reason Adversarial Training works might be the same as the reason Target Training works. Not because they populate sparse areas with samples but because they steer attack convergence based on the perturbation term in attack minimization. Target Training minimizes the perturbation further than Adversarial Training and without need for adversarial samples.
Target Training Does Adversarial Training Without Adversarial Samples

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Table 7. Architectures of Target Training classifiers for CIFAR10 and MNIST datasets. For the convolutional layers, we use $L_2$ kernel regularizer. The pre-final Dense.Softmax layers in both models have 20 output classes, twice the number of dataset classes. The default, unsecured classifiers and the classifiers used for Adversarial Training have the same architectures, except that the softmax layer is the final layer and has only 10 outputs.

| CIFAR10       | MNIST            |
|---------------|------------------|
| CONV.ELU 3x3x32 | CONV.ReLU 3x3x32 |
| BATTNorm      | BATTNorm         |
| CONV.ELU 3x3x32 | CONV.ReLU 3x3x64 |
| BATTNorm      | BATTNorm         |
| MAXPool 2x2   | MAXPool 2x2      |
| DROPOUT 0.2   | DROPOUT 0.25     |
| CONV.ELU 3x3x64 | DENSE 128       |
| BATTNorm      | BATTNorm         |
| CONV.ELU 3x3x64 | DENSE.Softmax 20 |
| BATTNorm      | LAMBDA SUMMATION 10 |
| MAXPool 2x2   | DROPOUT 0.3      |
| CONV.ELU 3x3x128 | BATTNorm        |
| BATTNorm      | BATTNorm         |
| MAXPool 2x2   | DROPOUT 0.4      |
| Dense.Softmax 20 | LAMBDA SUMMATION 10 |

Algorithm 2 Adversarial Training of classifier $N$, based on (Kurakin et al., 2016).

**Require:** $m$ batch size, $k$ classes, $N$ classifier with $k$ output classes, $\text{ADV\_ATTACK}$ adversarial attack, TRAIN trains classifier on a batch and labels

**Ensure:** Adversarially-Trained classifier $N$

while training not converged do

$B = \{x^1, ..., x^m\}$ \{Get random batch\}

$G = \{y^1, ..., y^m\}$ \{Get batch ground truth\}

$A = \text{ADV\_ATTACK}\{N, B\}$ \{Generate adv. samples from batch\}

$B' = B \bigcup A = \{x^1, ..., x^m, x^1_{\text{adv}}, ..., x^m_{\text{adv}}\}$ \{New batch\}

$G' = \{y^1, ..., y^m, y^1, ..., y^m\}$ \{Duplicate ground truth\}

TRAIN($N, B', G'$) \{Train classifier on new batch and new ground truth\}

end while

Algorithm 3 Target Training of classifier $N$ using adversarial samples against attacks that do not minimize perturbation.

**Require:** Batch size is $m$, number of dataset classes is $k$, untrained classifier $N$ with $2k$ output classes, $\text{ADV\_ATTACK}$ is an adversarial attack, TRAIN trains classifier on a batch and its ground truth

**Ensure:** Classifier $N$ is Target-Trained against $\text{ADV\_ATTACK}$

while training not converged do

$B = \{x^1, ..., x^m\}$ \{Get random batch\}

$G = \{y^1, ..., y^m\}$ \{Get batch ground truth\}

$A = \text{ADV\_ATTACK}\{N, B\}$ \{Generate adv. samples from batch\}

$B' = B \bigcup A = \{x^1, ..., x^m, x^1_{\text{adv}}, ..., x^m_{\text{adv}}\}$ \{Assemble new batch from original batch and adversarial samples\}

$G' = \{y^1, ..., y^m, y^1 + k, ..., y^m + k\}$ \{Duplicate ground truth and increase duplicates by $k$\}

TRAIN($N, B', G'$) \{Train classifier on new batch and new ground truth\}

end while
Table 8. Here, we show that targeted attacks are not strong against default classifiers, decreasing default accuracies very little. Target Training and Adversarial Training have roughly equal performance against targeted attacks, except for CW-$L_\infty$ in CIFAR10 where Target Training has better accuracy, and PGD where Adversarial Training has better accuracy. DeepFool attacks are not applicable because they cannot be targeted.

| Targeted Attack | CIFAR10 (84.3%) | MNIST (99.1%) |
|-----------------|----------------|--------------|
|                 | Target Training | Adversarial Training | Default Classifier | Target Training | Adversarial Training | Default Classifier |
| CW-$L_2(\kappa = 0)$ | 82.8% | 83.1% | 84.0% | 96.9% | 98.9% | 98.3% |
| CW-$L_\infty(\kappa = 0)$ | 69.0% | 50.1% | 72.7% | 98.2% | 98.1% | 98.6% |
| DeepFool | NA | NA | NA | NA | NA | NA |
| CW-$L_2(\kappa = 40)$ | 84.4% | 84.1% | 85.7% | 99.0% | 99.0% | 99.0% |
| PGD($\epsilon = 8, \epsilon = 0.3$) | 21.4% | 34.5% | 44.9% | 85.3% | 97.6% | 96.4% |
| FGSM($\epsilon = 0.3$) | 77.9% | 80.5% | 46.4% | 98.2% | 98.4% | 96.4% |