Adamczak, Radosław; Polaczyk, Bartłomiej; Strzelecki, Michał
Modified log-Sobolev inequalities, Becker inequalities and moment estimates. (English)
Zbl 1489.60028
J. Funct. Anal. 282, No. 7, Article ID 109349, 76 p. (2022).

In a probability space $(\mathcal{X}, \mathcal{B}, \mu)$ the Becker inequality with parameter $p \in (1, 2]$ holds if a constant $\alpha_p > 0$ exists such that
\[
\alpha_p \mu(f^p) - \mu(f) \leq \frac{p}{2} \mathcal{E}(f, f^{p-1}),
\]
where $\mathcal{E}$ is a Dirichlet form and $\mu(f)$ denotes the expectation of $f$. If the entropy is defined by
\[
\text{Ent}_\mu(f) := \mu(f \log f) - \mu(f) \log \mu(f),
\]
the modified log-Sobolev inequality is satisfied if there is a constant $\rho_0 > 0$ such that
\[
\rho_0 \text{Ent}_\mu(f) \leq \mathcal{E}(f, \log f).
\]

The authors prove the following theorem (rephrased in this reviewer’s words and in simplified form)

Theorem. The following statements are equivalent:

(a) The modified log-Sobolev inequality holds with some constant $\rho_0 > 0$;

(b) the Becker inequality holds for every $p \in (1, 2]$ with some $\alpha_p$ bounded away from zero.

Only one of the two implications is known in the literature. The authors also derive Sobolev-type moment estimations. Their results are illustrated by applications to estimates for concentration of measure.

Reviewer: Carlo Sempi (Lecce)

MSC:

60E15 Inequalities; stochastic orderings
60J25 Continuous-time Markov processes on general state spaces
60J27 Continuous-time Markov processes on discrete state spaces
46E35 Sobolev spaces and other spaces of “smooth” functions, embedding theorems, trace theorems
46E39 Sobolev (and similar kinds of) spaces of functions of discrete variables
26D10 Inequalities involving derivatives and differential and integral operators

Keywords:
modified log-Sobolev inequality; Becker inequality; moment estimates; concentration of measure

Full Text: DOI arXiv

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