Investigation of dynamic behavior of Hollow Rotor Shaft System through FEM

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Abstract. Generally, there is two kinds of rotor system used for transforming the power from one source to another, one is solid rotor system, and another is the hollow rotor shaft system. In comparison to solid shaft, the hollow shaft is of less weight, for a given diameter and length. In addition, the hollow shaft has a larger Strength to weight ratio and has a greater polar moment of inertia. Thus, they can transmit more torque compared to solid shafts. The main purpose of this paper is to analyze the dynamic behavior of hollow shafts with a particular interest in estimation of damping. This study also evaluate the dynamic response of different rotor thicknesses at various speeds where hollow shafts and solid shafts modelled through ANSYS 16. The proposed analysis performed in both frequencies as well as time domains, in which the Campbell diagram, the critical speed, and the orbits are numerically determined. Further, this study also illustrates the convenience of the hollow shafts at different inner diameters for rotor dynamics applications. The analytical results (ANSYS 16) were validated through the experimental results

Keywords: Solid shaft, Hollow shaft, bearing system, Finite Element Analysis, Experimental analysis

1. INTRODUCTION

Many studies have been conducted for a hollow shaft, which presented the maximum reduction in system weight in supercritical conditions. The strength and stiffness of the hollow shaft are more than the solid shaft for the same weight. It has also been observed through literature that hollow shaft has a higher natural frequency than the solid shaft of the same weight.

The first time, the analysis of a spinning shaft was performed by Rankine [1], but due to
some distinctness, his model was not attained supercritical speeds. After that, a steam turbine was successfully running above critical rates by Laval [2]. Further progress was made in the field by Dunkley [3] through an experimental study to describe behaviour of rotating shaft with multiple discs. Further, this study was extended by Kerr [2]. He suggested in his work and shown the empirical evidence of second supercritical speed. The core of rotor instabilities and computation modelling was growing around World War II. The idea of transfer matrix method (TMM) was proposed by Myklestad et al. [4] for analysing the dynamics of the rotor. Some other essential studies were developed by Jeffcott [5] for analysing dynamics of rotating shaft rotor system, such as the Campbell diagram, which was first presented by Wilfred Campbell [6]. After that more effective methods of balancing were proposed, and the behaviour of rotors supported by hydrodynamic bearings was further investigated, resulting in the design of lighter rotating machines operating at higher speeds [7-11].

Kimball [12] studied effect of internal and interface damping through analytical studied & further performed an experimental analysis [13] on determination of speed which causes instability. However, Dimentberg [14] and Tondl [15] adopted nonlinear analysis for evaluation of effects of internal and interface damping on dynamics of rotor. Dimarogonas [16] showed through analytical analysis that rotor shows stable behavior again if its speed increases above the critical speed. Taylor [17], Billet [18], Black [19] and Ehrich [20] studied effect of dry friction forces on natural frequency of the rotor. Stodola [21] and Soderberg [22] analyzed behavior of shaft considering its stiffness variable in mutually perpendicular planes along longitudinal direction. Instability problems caused by the flow though seals in turbomachinery attempted by Thomas [23] and he introduced a stability function by considering the leakage excitation alone along with damping. This concept was later used by Alford [24]. Dimarogonas [25] modified Thomas Stability function by considering effect of circumferential flow and properties of bearings. The influence of fluid bearings on dynamics of rotor was analyzed by Stodola [26, 27], Robertson [28] extended Reynold's equation to understand effects of bearings on the motion of rotor. Work on the modelling and linearization of bearings was further extended by Haag & Sankey [29] and Sternlicht [30].

For developing the mathematical model of a rotor system, the equivalent modulus and layer-wise beam theories in association with the Rayleigh-Ritz method were applied by Singh and Gupta [31]. The first time the dynamic behaviour of the rotating disc shaft system was studied by Chatelet et al. [32] through its mode shapes along with the help of a finite element model on the bases of a multi-layered shell element to illustrate and validate the proposed model. Ren et al. [33] proposed the mathematical formulation regarding a rotating shaft embedded with shape memory alloy wires by simulations and determined the relationship between the critical speeds of the rotor-bearing system and the configuration adopted for the cables by considering the anisotropy of the composite hollow shaft. Recently the effect of internal damping through simplified homogenized beam theory was considered by Sino et al. [34] for analysing the sensitivity of the frequencies and instability thresholds regarding shear effect, stacking order, and fibre orientation. Boukhalfa et al. [35] carried out the free vibration analysis of the composite shaft by utilizing Timoshenko theory along with the p-version of the hierarchical finite element method (FEM).

The present work is dedicated to vibration analysis of hollow rotor shaft by obtaining the natural frequency of first three mode shapes and then obtained numerical results are compared with the results of experimental study to validate the hypothesis. Further results of hollow shaft rotor system are also compared with the results obtained for solid shaft of the same outer diameter (22 mm). In the present study, ANSYS16 is used for performing the computational analysis of hollow and solid shafts system. The dynamic analysis of hollow shafts and solid shafts have been carried out by studying their vibration characteristics such as natural
frequencies, vibration modes to obtain a representative model for the respective rotor system. Therefore, the proposed analysis is performed in both frequency and time domains, in which the critical speed, the Campbell diagram, and the orbits are numerically determined. This study illustrates the validation of natural frequencies at different mode shapes and further validation done with the help of the experimental analysis.

2. EXPERIMENTAL INVESTIGATION OF THE HOLLOW ROTOR SHAFT SYSTEM

For establishing the accuracy of the above hypothesis, experiments are directed on a comparable rotor-bearing system. For this particular resolution, an experimental hollow shaft of mild steel consisting of rigid foundation, supporting by two bearing housings and one rotor, driving unit with variable speed control device and different bearing blocks is invented, as shown in Fig. 1. On the above rotor-bearing conditions and dynamic response, experiments are conducted for various speeds of the rotor are measured with the support of instrumentation system.

Fig. 1: Hollow Shaft Bearing System

2.1 Method for the experiment

To find out the natural frequency of the hollow shaft bearing system, NV Gate set up with OROS software has been used in the experimental analysis. A comprehensive set of tools for vibration and noise acquisition, recording and analysis provided by the NV-Gate platform.

![Schematic diagram of experimental set-up with measuring instruments](image-url)
The schematic diagram of the experiment set up has been shown in Fig: 2. The testing system consists of a straight uniform hollow shaft of mild steel that’s of 22mm outer diameter and 16 mm inner diameter with 1000 mm length, as shown in Fig: 1, Split cylindrical journal bearings that we have been used in the experiment are made of white metal liner and brass. With the help of dynamic balancing machine and L.N keys, we have been balanced the hollow shaft and a motor, and further the same is installed on the rig for assembly. The hollow shaft that we have been used for this experimental is rigid and driven by a D.C. series motor of 3 Hp. with a speed controller device in order to vary the speed within the range of 100 to 5000 rpm. Thus the experiments are directed on the same rotor-bearing system shown in Fig; 3, varying the radial clearance of hollow shaft, bearing diameter (up to 22mm) and oil viscosity of the bearings. Then lubricants such as turbine oil of different viscosities are fed from the top of the bearings at room temperature (40 °C) in order to maintain hydrodynamic lubrication in the bearings under different conditions of the bearing.

Fig. 3: Hollow rotor shaft supporting by two bearing housings and one rotor

We attached the DC motor with the help of coupling to the hollow shaft. Whereas hollow shaft is fixed already with the bearings then motor also attached. Motor also connected with SMPS (switch motor power supply), as shown in Fig; 6 and rectifier cum speed control device. When we start to give torque to the motor with A.C power supply, motor starts running, and at that time with the help of rectifier cum speed controller as shown in Fig: 5, we can change the rpm of the motor.

Fig.4: Digital analyser
After checking the arrangement of the given rotor bearing shaft system we started the experiment from the 100 revolutions per meter and after that change up to 3000 rpm with the help of the rectifier for our experiment whereas vibrations are created which considered by proximity sensor as shown in Fig: 7 and with the help of digital analyser as represented in Fig: 4, it send the vibration reading to the computer display where we can see the graph between the natural frequency and displacement as shown in fig 8.
2.2 Computational Analysis of the hollow rotor shaft system

For a load-bearing shaft, it’s always better to choose a hollow shaft because it has higher stiffness and rigidity and can resist slightly higher bending moments.

For the simulation in the ANSYS16 software, the models were created in SolidWorks and shown in the Figure 9, given below. Finally they were saved as an IGES- file. When they were imported into the ANSYS software the IGES-geometries had to be edited in ANSYS geometry. This was necessary to use proper constraints.

![Solid model of shaft](image)

After that, modal analysis was carried out on shaft made from mild steel whose properties are as described in Table 1.

| Table 1. Simulation parameters |
|--------------------------------|
| Density                        | 7850 kg m^{-3} |
| Coefficient of Thermal Expansion| 1.2e-005 C^{-1} |
| Specific Heat                  | 434 J kg^{-1} C^{-1} |
| Thermal Conductivity           | 60.5 W m^{-1} C^{-1} |
| Resistivity                    | 1.7 × 10^{-7} ohm-m |
| Compressive Yield Strength in Pa | 2.5 × 10^8 Pa |
| Tensile Yield Strength in Pa    | 2.5 × 10^8 Pa |
| Tensile Ultimate Strength in Pa | 4.6 × 10^8 Pa |
| Reference Temperature in °C    | 22 °C |

The process of dividing the whole component into number of elements so that whenever the load is applied on the component it distributes the load uniformly called as meshing. If the load is applied on the structure or a body and the body is considered to be meshed, then the
load is distributed uniformly on the entire structure. After meshing, the entire structure is divided in number of elements and each elements having its own stiffness while loading. In our study we use sweep method as shown in figure 10 after that 24851 nodes and 5100 elements generated in the hollow shaft of 22 mm Outer diameter and 16 mm inner dia at present in table 2.

![Meshing of hollow shaft](image)

**Table 2.** Data used for simulation

|       |         |
|-------|---------|
| Nodes | 24851   |
| Elements | 5100   |
| Element size | 5.00 mm |

2.3 Method of Computational analysis

For determining the vibration characteristics, mode shapes and the natural frequencies of a machine structure or component, we used Model Analysis. As easy and an effective way of describing the resonant vibration are mode shape and majority of the structures can be made to resonant, under the proper condition, a structure can be made to vibrate with sustained, excessive and oscillatory motion. When the inertia and the elastic properties of the material interacts then the resonant vibration occurs which is the major vibration related problems in the machine components or structures. This can be accomplished by defining the structural model parameter for performing the model analysis.

1) Build the Bearing Shaft model.
2) Obtain the modal solution.
3) Developing of modes.
4) Evaluate of results

In this analysis the shaft was subjected to boundary conditions. Ball bearing constraint was applied for 180 degrees as surface contact on one side and journal bearing constraint was applied for 180 degrees on the other side as a line contact. We have been used the Cylindrical Support for boundary condition at both the bearings. As this kind of boundary condition is well suitable for a simulation of round parts that move relatively to its environmental parts. The motion can be translational as well as rotational to the surrounding parts. This boundary condition prevents cylindrical surfaces from moving and deforming in the constraint directions. For the fixed bearing the axial and the radial DOF is constraint, the tangential DOF is free. For the loose bearing, only the radial DOF is constraint, the axial and tangential DOFs are free. The entire shaft is now free to rotate. (ANSYS Inc., 2016)

Modal analysis is the process of determining the natural frequencies of a rotating system and the corresponding mode shapes of the rotating system. The natural frequencies and mode shapes are determined by finding the solutions to the equation of motion when the external forces are zero. Use the following table format in the manuscript. Note that all the tables and
figures should be cited in the paragraph as Table 1 and Fig. 1 where appropriate.

3. RESULTS

For the simulation of natural frequencies, forces and moments do not play any role. The first three mode shapes at damped frequencies of the model are showing below at the defined boundary condition.

Fig. 11: 1st natural frequency at mode shape

Fig. 12: 2nd natural frequency at mode shape

Fig. 13: 3rd natural frequency at mode shape

The solver was run as well as the simulation displays for the first, second and third mode’s frequency. This is the pretended rigid body mode and occurs for every degree of freedom of the system that is not constraint. In this case it is the revolution around the centre axis of the shaft. The fourth mode shows the value of 101.55 Hz (figure 11), which is the first natural frequency. The mode shape shows the expected bent shape between the two bearings. In the animated mode shape, the elongated shaft moves up and down. A closer look to the bearings shows, that the elements in the loose bearing move slightly along the centre axis of the shaft. When the shaft bends downwards, the upper elements in the bearing move to the left, whereas the bottom elements move to the right. In return, the elements in the fixed bearing remain in their initial position. This shows that both boundary conditions behave as they should. The 2nd natural frequency as shown in figure 12 is 296.95Hz. The mode shape and the behaviour of the elements in the bearings is the same. The 3rd natural frequency at mode shape (figure 13) was 429.51Hz.

After determining the mode shapes, we find out the critical speed of hollow shaft i.e. 6102.4 rpm with the help of the Campbell diagram for hollow shaft as shown in Fig.:14, it is one of the most important tools for understanding the dynamic behaviour of a rotating machine. It basically consists of a plot of the natural frequencies of the system as functions of the spin speed on which the frequencies of the forcing excitation functions are superimposed. All the points of the same mode are connected to each other to form a line showing how the natural frequencies change with rotor spin speed. Critical speeds are the rotor spin speeds at which the resonance of a natural frequency occurs due to an excitation force. Campbell diagrams typically plot the excitation frequency equal to the rotor spin speed (first order excitation) to determine the flexural critical speeds of the rotor. The flexural critical speeds are also important in analysing mode stability. When the rotor spin speed is above the flexural critical speed of a forward whirl mode, rotating damping is destabilizing for that mode. If this
mode is then excited by an asynchronous forcing function, the rotor itself can become unstable.

![Campbell diagram](image)

**Fig.14:** Campbell diagram

*For Solid Shaft (Diameter 22mm)*

For this particular case damped frequency for first 3 actual mode shapes for two solver points determined with the help of same process as we followed for hollow shaft in ANSYS workbench16 modal analysis. The first mode shows the value of 89.691 Hz, what is the first natural frequency. The mode shape shows the expected bent shape between the two bearings. A closer look to the bearings shows, that the elements in the loose bearing move slightly along the centre axis of the shaft. When the shaft bends downwards, the upper elements in the bearing move to the left, whereas the bottom elements move to the right. In return, the elements in the fixed bearing remain in their initial position. This shows that both boundary conditions behave as they should. The 2nd natural frequency at mode shape is 247.49Hz. The mode shape and the behaviour of the elements in the bearings is the same. The 3rd natural frequency at mode shape was 360.98Hz.

4. **COMPARISON & DISCUSSION**

With the help of ANSYS16, the natural frequency of a hollow rotating shaft system with bearing support at the outer diameter 22mm and inner diameter of 16mm and a solid shaft system were evaluated. Also, experimentation with bearing shaft system with the help of motor, proximity sensor and NV Gate software for same 16 mm inner diameter and 22 mm outer diameter shaft of mild steel with 1000mm length has been performed. For the first 3 mode shape, frequency is shown in Fig.; 15, for the first mode experimental frequency is less than the computational frequency and error % at first mode is 3.37 %. There are many reasons for this error like, misalignment of shaft, bearing lubricant, motor, damping coefficient in ansys16, material properties etc. For the second mode experimental frequency value is higher than computational frequency and %error at 2nd mode is 2.86% that is caused by the fluctuation in speed at high rpm of motor. There is a slightly bend in graph from experimental to computational frequency as shown in Fig.15.
**Fig. 15:** Comparison of experimental and computational frequency
For comparison of 3rd mode shape, experimental frequency again higher than the computational frequency and % error is 4.96% and also there is a slightly bend in graph from experimental to computational frequency as shown in Fig 6-1(b). We can also see in the graph clearly that at a same outer diameter (22 mm), the natural frequencies at different mode shapes of hollow shaft is greater than the solid shaft and it has been hands proved by the computational analysis.

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