On the local well-posedness for some systems of coupled KdV equations

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Using the theory developed by Kenig, Ponce, and Vega, we prove that the Hirota-Satsuma system is locally well-posed in Sobolev spaces $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ for $\frac{3}{4} < s \leq 1$. Using the results obtained by Christ, Colliander, and Tao, we show ill-posedness for the Hirota-Satsuma system in $H^s(\mathbb{R}) \times H^{s'}(\mathbb{R})$ for $-1 \leq s < -\frac{3}{4}$ and $s' \in \mathbb{R}$. Moreover, we present some comments to scale changes performed in some previous papers concerning the Gear-Grimshaw system. We introduce some Bourgain-type spaces $X_{s,b}^a$ for $a \neq 0$, $s,b \in \mathbb{R}$, and we prove some properties for these spaces. Finally, we obtain local well-posedness for the Gear-Grimshaw system in $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ for $s > -\frac{3}{4}$, by establishing new mixed-bilinear estimates involving the two Bourgain-type spaces $X_{s,b}^{-\alpha_-}$ and $X_{s,b}^{-\alpha_+}$ adapted to $\partial_t + \alpha_- \partial_3^3$ and $\partial_t + \alpha_+ \partial_3^3$ respectively, where $|\alpha_+| = |\alpha_-| \neq 0$.

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