Abstract

We consider the minimal supersymmetric extension of the Standard Model allowing both Dirac and Majorana gauginos. The Dirac masses are obtained by pairing up extra chiral multiplets: a singlet $S$ for $U(1)_Y$, a triplet $T$ for $SU(2)_w$ and an octet $O_g$ for $SU(3)_c$ with the respective gauginos. The electroweak symmetry breaking sector is modified by the couplings of the new fields $S$ and $T$ to the Higgs doublets. We discuss two limits: i) both the adjoint scalars are decoupled with the main effect being the modification of the Higgs quartic coupling; ii) the singlet remaining light, and due to its direct coupling to sfermions, providing a new contribution to the soft masses and inducing new decay/production channels. We discuss the LSP in this scenario; after mentioning the possibility that it may be a Dirac gravitino, we focus on the case where it is identified with the lightest neutralino, and exhibit particular values of the parameter space where the relic density is in agreement with WMAP data. This is illustrated for different scenarios where the LSP is either a bino (in which case it can be a Dirac fermion) or bino-higgsino/wino mixtures. We also point out in each case the peculiarity of the model with respect to dark matter detection experiments.
1 Introduction

A remarkable fact of nature is that the light fundamental fermions appear in the smallest representations, the singlet or fundamental, of the Standard Model $SU(3)_c \times SU(2)_w \times U(1)_Y$ symmetry group. Could larger representations be present at higher energies and be discovered at the LHC? Such particles are in fact predicted by low energy supersymmetry as gauginos, superpartners of the gauge vector bosons. These gauginos can appear as fermions of Majorana (with only two degrees of freedom) or Dirac (with four degrees of freedom) type. In the Minimal Supersymmetric extension of the Standard Model (MSSM) the gauginos are Majorana. Obtaining Dirac masses would require pairing them with additional Dirac Gaugino adjoint (henceforth DG-adjoint) states: a singlet $S$ for $U(1)_Y$, a triplet $T$ for $SU(2)_w$ and an octet $O_g$ for $SU(3)_c$.

Construction of models with spontaneous breaking of supersymmetry may lead to non-minimal extensions of the Standard Model that include Dirac gauginos. The presence of the required DG-adjoints in the light spectrum is often motivated by the presence of an underlying $N = 2$ supersymmetry, that pairs them with the vector multiplets. Such a scenario was first introduced by Fayet [1] as a way to give masses for gluinos while preserving R-symmetry. The soft nature of these masses requires a modification of the interaction as was shown using $D$-term breaking by [2, 3]. More recently, Dirac gauginos have arisen in models with an extra dimension where supersymmetry is broken by a Scherk-Schwarz mechanism (see for example [4, 5]). More precisely, they are a combination of two Majorana fermions with mass given by $1/2R$ (half of the compactification scale $1/R$), one given by the (mass shifted) massless mode and the other from the (mass shifted) first Kaluza-Klein state, thus, the DG-adjoints originate as “half of the first Kaluza-Klein excitation”. It was noted there that the soft masses are UV finite and do not exhibit the usual logarithmic sensitivity to the UV cut-off [6]. This property was shown to persist in four dimensional models as being peculiar to the Dirac nature of the gaugino masses, and denoted as supersoft in [7, 8, 9, 10, 11] where the important phenomenological implications of $D$-term supersymmetry breaking were first outlined. They were further studied in constructions of non-supersymmetric intersecting brane models [12, 13, 14]. More recent examples arise from the possibility of using calculable $R$-symmetric models [15, 16, 17, 18, 19]. For instance, it was pointed out in [16] that they lessen the flavor problem in supersymmetric theories. This renewal of interest in such models has been motivated by the work of ISS [20]. Furthermore, deforming the ISS model with explicit breaking of $R$-symmetry could leave states in adjoint representations [21] and allow the simultaneous presence of both Majorana and Dirac masses for the gauginos. The generation of Dirac gaugino masses can also be included [22] in the framework of “general gauge mediation”

---

1 In [1] $R$-symmetry was later broken by Majorana masses for the DG-adjoints in order to avoid tachyonic masses for their scalar components. An issue that was recently solved in [22] and independently for an explicit model in [17].
In this work, we will study a minimal extension of the MSSM that incorporates both Majorana and Dirac gaugino masses. The field content is that of the MSSM, supplemented with the DG-adjoints. The MSSM renormalisable Lagrangian is then supplemented by i) the DG-adjoint kinetic and mass terms, ii) the Dirac gaugino masses, iii) coupling of the singlet $S$ and the triplet $T$ to the Higgs doublets with strength $\lambda_S$ and $\lambda_T$ respectively, iv) the DG-adjoint scalar soft masses and trilinear terms.

For phenomenological issues the strength of the coupling of the Higgs to the DG-adjoint is of particular importance. In general such couplings are arbitrary and subject to diverse phenomenological bounds as discussed in \[8\]. Inspired by extra dimensional models, and in order to make the role of $N = 2$ manifest, one can assume that the two MSSM Higgs doublets originate from a single hypermultiplet of an underlying $N = 2$ supersymmetry. These models with combined $N = 2$ and $N = 1$ sectors were introduced in \[14, 29\]. In this case, the couplings $\lambda_S$ and $\lambda_T$ are related to the gauge couplings by $N = 2$ supersymmetry. In this work, we will arbitrarily take the values of both couplings to go from zero to their tree level $N = 2$ value to illustrate the model.

Some particular signatures of these models at collider experiments have been studied in \[30, 31, 32, 33, 34, 35\]. They stressed that the Dirac/Majorana nature of the gluinos affects the distribution of produced squark states. It was pointed out in \[32\] that the pair creation of scalar octets at the LHC will have a peculiar signature through cascade decays giving rise to a burst of eight or more jets together with four LSPs as well as through a resonance due to the decays into gluons or a $t\bar{t}$ pair at the one-loop level.

In this work we will focus mainly on the fate of dark matter in this class of models. It is by now an important quality of the $R$-parity preserving versions of the MSSM that they provide a natural candidate for Dark Matter, the lightest supersymmetric particle (LSP). A particular case is when the LSP is identified with the lightest neutralino. While in the MSSM the latter is a linear combination of the four neutral fermions, given by the bino, the wino and the two Higgsinos, it is now a linear combination of six states, the singlet and triplet fermions adding to the previous four. In this work we shall not try to give an exhaustive discussion of such a situation but try to answer such questions as: Are there parameter regions where the neutralino is a good dark matter candidate? How does the situation compare to the MSSM? An early study \[36\] of dark matter in a related model to that considered here focussed on the bino LSP, for a very particular case with vanishing $\lambda_S$ and $\lambda_T$, and assumed dominance of the exchange of sfermions and thus neglecting, for example, the exchange of Higgs or gauge bosons. It concluded that the bino annihilation cross section can be enhanced which might help to obtain a smaller relic abundance than in the MSSM.

The present paper is organized as follows: Section 2 presents the model and defines our notations and conventions. Section 3 gives a comprehensive discussion, which we believe to be
missing in the present literature, of the electroweak breaking sector. The discussion follows the same lines as in [14] which specializes to a model with combined \( N = 2 \) and \( N = 1 \) sectors in the limit of very large soft masses for the scalar components of \( T \) and \( S \). It differs from the usual extension of the MSSM by the couplings to the Higgs of a singlet, whose vacuum expectation value (vev) is not necessarily related to the supersymmetric Higgs mass term \( \mu \), and a triplet (see [37, 38, 39, 40, 41, 42]). While the triplet scalar is required to be heavy by electroweak precision tests, the singlet can be either very heavy and integrated out or remain light with sensible mixing with the ordinary MSSM Higgs states. We discuss both limits. In section 5, we briefly discuss the gravitino LSP and then focus on the case of a neutralino LSP. The corresponding mass matrix is exhibited and the nature of the lightest eigenstates is studied for some particular limits, in particular the necessary condition for having a Dirac fermion LSP is given. Section 6 presents numerical results for the relic abundance and the corresponding signature at direct/indirect detection experiments are stressed.

2 The model

The particle content of the model is presented in table 1. The MSSM matter fields acquire masses through the Yukawa superpotential:

\[
W_{\text{Yukawa}} = y_{ui}^j u_i^c Q_j \cdot H_u - y_{dij}^j d_i^c Q_j \cdot H_d - y_{eij}^j e_i^c L_j \cdot H_d
\]

and the usual soft breaking terms:

\[
\mathcal{L}_{\text{soft}}^0 = \tilde{Q}_i^+ m_Q^2 \tilde{Q}_j + \tilde{L}_i^+ m_L^2 \tilde{L}_j + \tilde{u}^+_i m_{u_{ij}} \tilde{u}_j + \tilde{d}^+_i m_{d_{ij}} \tilde{d}_j + \tilde{e}^+_i m_{e_{ij}} \tilde{e}_j + A_{ui}^{ij} \tilde{Q}_j \cdot H_u - A_{dij}^{ij} \tilde{d}_j \cdot H_d - A_{eij}^{ij} \tilde{e}_j \cdot H_d + \text{c.c.}
\]

where the bold characters denote superfields. Here, \( i, j \) are family indices and run from 1 to 3. The \( 3 \times 3 \) \( y \) matrices are the Yukawa couplings. The “:” denotes \( SU(2) \) invariant couplings, for example: \( Q \cdot H_u = \tilde{u}_L H_u^0 - \tilde{d}_L H_u^+ \).

In order to have Dirac masses for the gauginos, additional fields in the adjoint representations, the “DG-adjoints”, are introduced. We define the superfields:

\[
S = S + \sqrt{2} \theta \chi_S + \cdots
\]

\[
T = T + \sqrt{2} \theta \chi_T + \cdots
\]

\[
O_g = O_g + \sqrt{2} \theta \chi_g + \cdots
\]
Table 1: Chiral and gauge multiplet fields in the model

| Names      | Spin 0                                                                 | Spin 1/2                                                                 | Spin 1                                                                 | SU(3), SU(2), U(1)_Y |
|------------|------------------------------------------------------------------------|-------------------------------------------------------------------------|------------------------------------------------------------------------|----------------------|
| Quarks     | $\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$                           | $(u_L, d_L)$                                                            |                                                                       | (3, 2, 1/6)          |
| (×3 families) | $\tilde{u}_L^c, \tilde{d}_L^c$                                    | $u_L^c$                                                               |                                                                       | (3, 1, -2/3)         |
| Leptons    | $e^c = (\tilde{e}_{L}, \tilde{\nu}_{L})$                           | $(e^c, \nu^c)$                                                         |                                                                       | (1, 1, 1/3)          |
| (×3 families) | $\tilde{e}_{L}^c$                                                  | $e^c_L$                                                               |                                                                       | (1, 2, -1/2)         |
| Higgs      | $H_u = (H_u^+, H_u^0)$                                              | $(H_u^+, H_u^0)$                                                        |                                                                       | (1, 2, 1/2)          |
|            | $H_d = (H_d^0, H_d^-)$                                              | $(H_d^0, H_d^-)$                                                        |                                                                       | (1, 2, -1/2)         |
| Gluons     | $W_{3\alpha}$                                                        | $\lambda_{3\alpha}$                                                  | $g$                                                                   | (8, 1, 0)            |
|            |                                                                       | $\equiv g_{\alpha}$                                                  |                                                                       |                      |
|            | $W_{2\alpha}$                                                        | $\lambda_{2\alpha}$                                                  | $W^\pm, W^0$                                                         | (1, 3, 0)            |
|            |                                                                       | $\equiv \tilde{W}^\pm, \tilde{W}^0$                                 |                                                                       |                      |
|            | $B_{1\alpha}$                                                        | $\lambda_{1\alpha}$                                                  | $B$                                                                   | (1, 1, 0)            |
|            |                                                                       | $\equiv \tilde{B}$                                                   |                                                                       |                      |
| DG-octet   | $O_g$                                                                | $\chi_g$                                                             |                                                                       | (8, 1, 0)            |
|            | $\equiv \Sigma_g$                                                   |                                                                       |                                                                       |                      |
| DG-triplet | $\{T_0, T^\pm\}$                                                    | $\{\chi_T^0, \chi_T^\pm\}$                                           |                                                                       | (1, 3, 0)            |
|            | $\equiv \{\Sigma_T^0, \Sigma_T^\pm\}$                             |                                                                       |                                                                       |                      |
| DG-singlet | $S$                                                                  | $\chi_S$                                                             |                                                                       | (1, 1, 0)            |
|            | $\equiv \Sigma_S$                                                   |                                                                       |                                                                       |                      |

where $S = \frac{1}{\sqrt{2}}(S_R + iS_I)$ is a singlet and $T = \sum_{a=1,2,3} T^{(a)}$ an SU(2) triplet parametrized as:

$$T^{(1)} = T_1 \sigma_1^1 \frac{1}{2}, \quad T^{(2)} = T_2 \sigma_2^2 \frac{1}{2}, \quad T^{(3)} = T_0 \sigma_3^3 \frac{1}{2},$$

$$T = \frac{1}{2} \left( \begin{array}{cc} T_0 & \sqrt{2}T_- \\ \sqrt{2}T_+ & -T_0 \end{array} \right),$$

$$T_0 = \frac{1}{\sqrt{2}}(T_R + iT_I), \quad T_+ = \frac{1}{\sqrt{2}}(T_{+R} + iT_{+I}), \quad T_- = \frac{1}{\sqrt{2}}(T_{-R} + iT_{-I}),$$

and $\sigma^a$ are the Pauli matrices. Their quantum numbers are presented in Table 1.
Due to the presence of these extra fields, the gauge kinetic terms are modified to become:

\[
\mathcal{L}_{\text{gauge}} = \int d^4x d^2 \theta \left[ \frac{1}{4} M_1 W^1_\alpha W^{1\alpha} + \frac{1}{2} M_2 \text{tr}(W^2_\alpha W^{2\alpha}) + \frac{1}{2} M_3 \text{tr}(W^3_\alpha W^{3\alpha}) + \sqrt{2} m^\alpha_1 W_1^\alpha + 2 \sqrt{2} m^\alpha_2 \text{tr}(W_2^\alpha T) + 2 \sqrt{2} m^\alpha_3 \text{tr}(W_3^\alpha O) \right] + \int d^4x d^2 \theta d^2 \bar{\theta} \left( \sum_{ij} \Phi_i ^\dagger e^{\theta j} \Phi_j + h.c. \right)
\]

where \( V_j \) are the vector and \( W_{j\alpha} \) the corresponding field strength superfields associated to \( U(1)_Y \), \( SU(2) \) and \( SU(3) \) for \( j = 1, 2, 3 \) respectively. Here, we have introduced spurion superfields to take into account the generation of gaugino masses:

\[
M_i = 1 + 2 \theta \theta M_i \tag{2.8}
\]

\[
m_{\alpha i D} = \theta \alpha m_{i D} \tag{2.9}
\]

The Dirac gaugino spurion superfield can be written as \( m_{\alpha i D} = -\frac{1}{4} \bar{D} \bar{D} \alpha X_i \). This mass originates as a \( D \)-term if \( X_i \) is identified as a vector field \( X_i = V^j / M_i \), or as non vanishing \( F \)-term by writing \( X_i = 2 \Sigma^\dagger \Sigma / M_i \) with \( \Sigma = \theta \theta F \), where \( M_i \) is the appropriate supersymmetry breaking mediation scale.

The DG-adjoints may also modify the Higgs superpotential, since new relevant and marginal operators are now allowed:

\[
\int d^4x d^2 \theta \left[ \mu H_u \cdot H_d + \frac{M_S}{2} S^2 + \lambda_S S H_d \cdot H_u + M_T \text{tr}(T^T) + 2 \lambda_T H_d \cdot TH_u \right] \tag{2.10}
\]

with the definition \( H_u \cdot H_d = H_u^+ H_d^- - H_u^0 H_d^0 \).

Finally, the soft supersymmetry breaking terms for the scalars are:

\[
- \Delta \mathcal{L}_{\text{soft}} = m^2_{H_u} |H_u|^2 + m^2_{H_d} |H_d|^2 + B_\mu (H_u \cdot H_d + h.c.)
+ m^2_S |S|^2 + \frac{1}{2} B_S (S^2 + h.c.) + 2 m^2_T \text{tr}(T^T) + B_T (\text{tr}(TT) + h.c.)
+ A_S \lambda_S (S H_d \cdot H_u + h.c.) + 2 A_T \lambda_T (H_d \cdot TH_u + h.c.) \tag{2.11}
\]

Note that we did not include in the superpotential a cubic term \( \text{tr}(T^T T) \) as this identically vanishes. Neither did we include linear and cubic terms in the singlet. The latter is due to the fact that we assume that the DG-adjoint appears due to some underlying \( N = 2 \) supersymmetry that forbids these terms. Of course a microscopic model explaining the origin of the soft terms...
should also address the fact that the supersymmetric $M_S$ and $M_T$ are assumed to take values of order of the electroweak scale, introducing an issue of scale hierarchy as does the Higgs $\mu$-term.

Below, we will give special attention to the scenario where the DG-states arise as a result of an $N = 2$ extension of the gauge sector. In this case, if the Higgs multiplets $H_u$ and $H_d$ are assumed to form an $N = 2$ hypermultiplet then $\lambda_S$ and $\lambda_T$ are related to the gauge couplings, at the $N = 2$ scale, by:

$$\lambda_S = \sqrt{2} g' \frac{1}{2}, \quad \lambda_T = \sqrt{2} g \frac{1}{2},$$  \hspace{1cm} (2.12)

where $g'$ and $g$ are the $U(1)_Y$ and $SU(2)$ gauge couplings respectively. The factor $1/2$ in $\lambda_S$ arises from the $U(1)_Y$ charge of the Higgs doublets.

### 3 Electroweak scalar potential

We turn now to the electroweak scalar potential. This receives contributions from three sources:

$$V_{EW} = V_{gauge} + V_W + V_{soft}$$  \hspace{1cm} (3.1)

The first is a contribution from the gauge kinetic term (2.7). Integration on the spinor coordinates, and going on-shell, leads to the $U(1)_Y$ and $SU(2)$ $D$-terms:

$$D_1 = -2m_{1D} S_R + D_Y^{(0)} \quad \text{with} \quad D_Y^{(0)} = -g' \sum_j Y_j \varphi_j^* \varphi_j$$  \hspace{1cm} (3.2)

$$D_2^a = -\sqrt{2} m_{2D} (T^a + T^a\dagger) + D_2^{a(0)} \quad \text{with} \quad D_2^{a(0)} = -g \sum_j \varphi_j^* \sigma^a \varphi_j$$  \hspace{1cm} (3.3)

where $\varphi_j$ are the scalar components of matter chiral superfields, whereas $D_Y^{(0)}$ and $D_2^{a(0)}$ are the $D$-terms in absence of Dirac masses. The resulting Lagrangian contains terms of the form:

$$L_{gauge} \rightarrow -m_1 D_1 \lambda_1^0 \chi S_\alpha - m_2 D_2 \text{tr}(\lambda_2^0 \chi T_\alpha) - \frac{1}{2} D_1^2 - \frac{1}{2} \text{tr}(D_2^a D_2^a)$$  \hspace{1cm} (3.4)

where we can identify the Dirac components of the gauginos $\lambda_1 \equiv \tilde{B}'$ and $\lambda_2^a \equiv \tilde{W}'^a$ as given in Table 1.

The contribution from the DG-triplet to $D_2^a$ is:

$$D_2 \propto \frac{1}{2} \left( \begin{array}{ccc} (|T_+|^2 - |T_-|^2) & \sqrt{2}(T_+ T_0^* - T_0 T_+^*) \\ \sqrt{2}(T_0 T_+^* - T_+ T_0^*) & -(|T_+|^2 - |T_-|^2) \end{array} \right)$$  \hspace{1cm} (3.5)
which vanishes in electrically neutral vacuum, where:
\[
< T_+ > = < T_- > = < H_+ > = < H_- > = 0. \tag{3.6}
\]
The contribution to the scalar potential of the neutral fields is then given by:
\[
V_{\text{gauge}} = 2m_{1D}^2S_R^2 - 2m_{1D}S_RD_Y^{(0)} + \frac{1}{2}D_Y^{(0)^2} + 2m_{2D}^2T_R^2 - 2m_{2D}T NDP_2^{(0)} + \frac{1}{2}D_2^{(0)^2} \tag{3.7}
\]
where we have dropped the generator label, \( D_2 = D_{2a}^a \), \( a = 3 \), for the only non-vanishing component.

The second contribution comes from the superpotential \( (2.10) \):
\[
W = (-\mu + \lambda_S S)(H_u^0 H_d^0 - H_d^+ H_u^-) + \frac{M_S}{2} S^2 + \frac{M_T}{2}(T^0 T^0 + 2T^+ T^-)
- \lambda_T(H_d^0 T^0 H_u^0 + H_d^0 T^0 H_u^-) - \sqrt{2}\lambda_T(H_d^0 T^+ H_u^0 - H_d^0 T^- H_u^+). \tag{3.8}
\]
Keeping only the neutral components, it reads:
\[
V_W = |M_S S + \lambda_S H_d^0 H_u^0|^2 + |M_T T^0 - \lambda_T H_d^0 H_u^0|^2 + |\mu - \lambda_S S + \lambda_T T^0|^2 (|H_d^0|^2 + |H_u^0|^2) \tag{3.9}
\]
The third source \( V_{\text{soft}} \) is due to soft supersymmetry breaking terms for the scalars \( (2.11) \). We define:
\[
H_u^0 = \frac{H_u^{0R} + iH_u^{0I}}{\sqrt{2}}, \quad H_d^0 = \frac{H_d^{0R} + iH_d^{0I}}{\sqrt{2}} \tag{3.10}
\]
then, all together, the scalar Lagrangian for the neutral fields is now given by:
\[
V_{\text{EW}} = (m_H^2 + \mu^2)|H_u^0|^2 + (m_H^2 + \mu^2)|H_d^0|^2 - B_\mu(H_u^0 H_d^0 + h.c.) + \frac{g^2 + g'^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2 + (\lambda_S^2 + \lambda_T^2)|H_u^0 H_d^0|^2
+ \frac{1}{2}(M_S^2 + m_S^2 + 4m_{1D}^2 + B_S)S_R^2 + \frac{1}{2}(M_T^2 + m_T^2 - B_T)T_R^2
+ \frac{1}{2}(M_S^2 + m_S^2 + 4m_{2D}^2 + B_S)S_I^2 + \frac{1}{2}(M_T^2 + m_T^2 - B_T)T_I^2
+ \left[ \frac{\lambda_S^2}{2}(S_R^2 + S_I^2) + \frac{\lambda_T^2}{2}(T_R^2 + T_I^2) - \sqrt{2}\mu(\lambda_S S_R - \lambda_T T_R)\right]
\times (|H_u^0|^2 + |H_d^0|^2) + g'm_{1D}S_R(|H_u^0|^2 - |H_d^0|^2) + gm_{2D}T_R(|H_u^0|^2 - |H_d^0|^2)
+ \frac{\lambda_S}{\sqrt{2}}(M_S + A_S)S_R(H_d^{0R} H_u^{0I} - H_d^{0I} H_u^{0R}) + \frac{\lambda_S}{\sqrt{2}}(M_S - A_S)S_I(H_d^{0R} H_u^{0I} + H_d^{0I} H_u^{0R})
- \frac{\lambda_T}{\sqrt{2}}(M_T + A_T)T_R(H_d^{0R} H_u^{0I} - H_d^{0I} H_u^{0R}) - \frac{\lambda_T}{\sqrt{2}}(M_T - A_T)T_I(H_d^{0R} H_u^{0I} + H_d^{0I} H_u^{0R})
\]
Note that the MSSM potential is given by the first line. All the parameters are chosen to be real. We are left with four neutral fields $S_R, S_I, T_R, T_I$ satisfying equations of type:

$$M_{xa}^2 x_a + X_{ST} y_a = V_{xa}, \quad a = R, I \quad \text{for } (x = S, y = T) \text{ or } (x = T, y = R)$$

(3.12)

with solutions of the form:

$$x_a = \frac{V_{xa} M_{ya}^2 - V_{ya} X_{ST}}{M_{xa}^2 M_{ya}^2 - X_{ST}^2}, \quad a = R, I \quad \text{for } (x = S, y = T) \text{ or } (x = T, y = R)$$

(3.13)

where:

$$M_{SR}^2 = M_S^2 + m_S^2 + 4m_1^2 + B_S + \lambda_S^2 (|H_u^0|^2 + |H_d^0|^2)$$

(3.14)

$$M_{TR}^2 = M_T^2 + m_T^2 + 4m_2^2 + B_T + \lambda_T^2 (|H_u^0|^2 + |H_d^0|^2)$$

(3.15)

$$M_{SI}^2 = M_S^2 + m_S^2 - B_S + \lambda_S^2 (|H_u^0|^2 + |H_d^0|^2)$$

(3.16)

$$M_{TI}^2 = M_T^2 + m_T^2 - B_T + \lambda_T^2 (|H_u^0|^2 + |H_d^0|^2)$$

(3.17)

while

$$X_{ST} = -\lambda_S \lambda_T (|H_u^0|^2 + |H_d^0|^2)$$

(3.18)

and:

$$V_{SR} = \sqrt{2} \mu \lambda_S (|H_u^0|^2 + |H_d^0|^2) - g' m_{1D} (|H_u^0|^2 - |H_d^0|^2)$$

$$- \frac{\lambda_S}{\sqrt{2}} (M_S + A_S)(H_d^0 H_u^0 R - H_d^0 H_u^0 L)$$

(3.19)

$$V_{TR} = -\sqrt{2} \mu \lambda_T (|H_u^0|^2 + |H_d^0|^2) + gm_{2D} (|H_u^0|^2 - |H_d^0|^2)$$

$$+ \frac{\lambda_T}{\sqrt{2}} (M_T + A_T)(H_d^0 H_u^0 R - H_d^0 H_u^0 L)$$

(3.20)

$$V_{SI} = -\frac{\lambda_S}{\sqrt{2}} (M_S - A_S)(H_d^0 H_u^0 L + H_u^0 H_d^0 L)$$

(3.21)

$$V_{TI} = +\frac{\lambda_T}{\sqrt{2}} (M_T - A_T)(H_d^0 H_u^0 L + H_u^0 H_d^0 L)$$

(3.22)

We are not going to pursue exact computations. Instead, we will consider the case with $m_S^2, m_T^2 \gg m_Z^2$ in which case the formulae simplify as we can neglect all terms proportional to
\(|H_u^0|^2 + |H_d^0|^2\), in particular \(X_{ST}\). This gives \(x_a \approx V_{xa}/M_{za}^2\), i.e.

\[
S_R \simeq - \frac{g'm_{1D}(|H_u^0|^2 - |H_d^0|^2) - \sqrt{2} \mu \lambda_S(|H_u^0|^2 + |H_d^0|^2) + \frac{\lambda_s}{\sqrt{2}}(M_S + A_S)(H_{dR}^0 H_{uR}^0 - H_{dI}^0 H_{uI}^0)}{M_S^2 + m_S^2 + 4m_{1D}^2 + B_S}
\]

\[
T_R \simeq - \frac{g m_{2D}(|H_d^0|^2 - |H_u^0|^2) + \sqrt{2} \mu \lambda_T(|H_u^0|^2 + |H_d^0|^2) - \frac{\lambda_T}{\sqrt{2}}(M_T + A_T)(H_{dR}^0 H_{uR}^0 - H_{dI}^0 H_{uI}^0)}{M_T^2 + m_T^2 + 4m_{2D}^2 + B_T}
\]

\[
S_I \simeq - \frac{\lambda_S (M_S - A_S)(H_{dR}^0 H_{uI}^0 + H_{dI}^0 H_{uR}^0)}{\sqrt{2}(M_S^2 + m_S^2 + 4m_{1D}^2 + B_S)}
\]

\[
T_I \simeq \frac{\lambda_T (M_T - A_T)(H_{dR}^0 H_{uI}^0 + H_{dI}^0 H_{uR}^0)}{\sqrt{2}(M_T^2 + m_T^2 + 4m_{2D}^2 + B_T)}
\]

(3.23)

We are interested by the case of CP neutral vacuum, i.e. \(H_{dI}^0 = H_{uI}^0 = 0\) which implies \(S_I = T_I = 0\). From now on, we will drop the indices 0 and define \(\bar{h_u} \equiv h_u\), \(\bar{h_d} \equiv h_d\).

\[
< H_{uR}^0 > \equiv < h_u > = v_u = v s_\beta, \quad < H_{dR}^0 > \equiv < h_d > = v_d = v c_\beta, \quad 0 \leq \beta \leq \frac{\pi}{2}
\]

\[
< S_R > = v_s, \quad < T_R > = v_t
\]

(3.24)

where we denote:

\[
c_\beta \equiv \cos \beta, \quad s_\beta \equiv \sin \beta, \quad t_\beta \equiv \tan \beta
\]

\[
c_{2\beta} \equiv \cos 2\beta, \quad s_{2\beta} \equiv \sin 2\beta
\]

(3.25)

\[
v_s \simeq \frac{v^2}{2(M_S^2 + m_S^2 + 4m_{1D}^2 + B_S)} \left[ g'm_{1D} c_{2\beta} + \sqrt{2} \mu \lambda_S - \frac{\lambda_S}{\sqrt{2}}(M_S + A_S)s_{2\beta} \right]
\]

\[
v_t \simeq \frac{v^2}{2(M_T^2 + m_T^2 + 4m_{2D}^2 + B_T)} \left[ -g m_{2D} c_{2\beta} - \sqrt{2} \mu \lambda_T + \frac{\lambda_T}{\sqrt{2}}(M_T + A_T)s_{2\beta} \right]
\]

(3.26)

Electroweak precision data give strong bounds on the expectation value of the DG-triplet as it contributes to \(\rho \simeq 1 + \alpha T = 1.0004^{+0.0008}_{-0.0004}\). Thus we require:

\[
\Delta \rho \simeq 4 \frac{v_t^2}{v^2} \lesssim 8 \cdot 10^{-4}
\]

(3.27)

which is satisfied for \(v_t \lesssim 3\) GeV. Here we will allow the Dirac and Majorana masses to vary arbitrarily and will satisfy this bound by taking \(m_T\) large enough. For instance, for \(M_S, A_S, \mu \sim 200\) GeV and all couplings of order one, this is satisfied for \(m_T \gtrsim 1\) TeV.

\(^2\)With this convention \(v \simeq 246\) GeV, \(\langle \phi^\dagger \phi \rangle^2 = M_S^2/4\).
Integrating out the DG-adjoints $S$ and $T$, leads then to the effective tree-level scalar potential at first order in $\lambda_{S,T}$

\[
V_{EW} = \left(\frac{m_{H_u}^2 + \mu^2}{2}\right)h_u^2 + \left(\frac{m_{H_d}^2 + \mu^2}{2}\right)h_d^2 - B_\mu h_u h_d + \frac{g^2 + g'^2}{32}(h_u^2 - h_d^2)^2
\]

\[
+ \frac{\lambda^2_S + \lambda^2_T}{4}h_u h_d^2
\]

\[
\frac{1}{8} \left[ g'm_{1D}(h_u^2 - h_d^2) - \sqrt{2}\mu\lambda_S(h_u^2 + h_d^2) + \sqrt{2}\lambda_S(M_S + A_S)h_d h_u \right]^2
\]

\[
\frac{1}{8} \left[ -gm_{2D}(h_u^2 - h_d^2) - \sqrt{2}\mu\lambda_T(h_u^2 + h_d^2) - \sqrt{2}\lambda_T(M_T + A_T)h_d h_u \right]^2
\]

\[
\left(\frac{M_S^2 + m_S^2 + 4m_{1D}^2 + B_S}{M_T^2 + m_T^2 + 4m_{2D}^2 + B_T}\right)
\]

This decomposes into three parts:

\[
V_{EW} = V_0 + V_1 + V_2
\]

(3.29)

The first part:

\[
V_0 = \left(\frac{m_{H_u}^2 + \mu^2}{2}\right)h_u^2 + \left(\frac{m_{H_d}^2 + \mu^2}{2}\right)h_d^2 - B_\mu h_u h_d + \frac{g^2 + g'^2}{32}(h_u^2 - h_d^2)^2
\]

is the MSSM contribution. The second,

\[
V_1 = \frac{\lambda^2_S + \lambda^2_T}{4}h_u h_d^2
\]

(3.31)

is a quartic term. The third contains the explicit dependence on the mass parameters of the DG-adjoints. We will illustrate this in taking a few limits, keeping the other parameters fixed:

- One limit is to take $M_S \to \infty$ and $M_T \to \infty$. In this case:

\[
V_2 \to -\frac{\lambda^2_S + \lambda^2_T}{4}h_u h_d^2 = -V_1
\]

(3.32)

meaning that the complete DG-adjoint supermultiplets have been decoupled and one is left with the MSSM electroweak scalar potential.

- A second limit is to take $\lambda_S \to 0$ and $\lambda_T \to 0$ switching off the superpotential couplings

\[
V_1 \to 0
\]

(3.33)

\[
V_2 \to -\left[ \frac{g^2}{32} \left( \frac{4m_{1D}^2}{4m_{1D}^2 + m_S^2 + M_S^2 + B_S} \right) + \frac{g^2}{32} \left( \frac{4m_{2D}^2}{4m_{2D}^2 + m_T^2 + M_T^2 + B_T} \right) \right] (h_u^2 - h_d^2)^2
\]
This shows that the effect of Dirac masses is to decrease the quartic coupling, and make it vanish in the absence of other masses.

- A third one is to take \( m_{1D} \to \infty \) and \( m_{2D} \to \infty \)

\[
V_1 \longrightarrow \frac{\lambda_S^2 + \lambda_T^2}{4} h_u^2 h_d^2 \\
V_2 \longrightarrow -\frac{g^2 + g'^2}{32}(h_u^2 - h_d^2)^2
\]  

(3.34)

the Higgs quartic term of the MSSM due to \( D \)-term is cancelled and all the quartic couplings are generated by superpotential coupling between the Higgses and the DG-adjoints.

- As a fourth one, we consider the case of interest in the rest of this work: \( m_S \to \infty \) and \( m_T \to \infty \)

\[
V_1 \longrightarrow \frac{\lambda_S^2 + \lambda_T^2}{4} h_u^2 h_d^2 \\
V_2 \longrightarrow 0
\]  

(3.35)

showing that the MSSM scalar potential is supplemented with a quartic term that lifts the \( D \)-term flat direction \( H_u = H_d \).

### 3.1 Integrating out the adjoints

In this last limit, the vacuum expectation values of the DG-adjoints can be neglected, allowing to write an \( SU(2) \) invariant effective scalar potential. It can be put in the usual form \[44, 45\] which parametrizes the two-doublet potential:

\[
V_{\text{eff}} = (m_{H_u}^2 + \mu^2)|H_u|^2 + (m_{H_d}^2 + \mu^2)|H_d|^2 - [m_{12}^2 H_u \cdot H_d + h.c.] \\
+ \left[ \frac{1}{2}\frac{1}{4}(g^2 + g'^2) + \lambda_1 \right]|(H_d|^2|^2) + \frac{1}{2}\left[ \frac{1}{4}(g^2 + g'^2) + \lambda_2 \right](|H_u|^2)^2 \\
+ \left[ \frac{1}{4}(g^2 - g'^2) + \lambda_3 \right]|H_d|^2|H_u|^2 + \left[ \frac{1}{2}g^2 + \lambda_4 \right](H_d \cdot H_u)(H_d^* \cdot H_u^*) \\
+ \left[ \frac{1}{2}\lambda_5 \right]|H_d \cdot H_u|^2 + \left[ \lambda_6|H_d|^2 + \lambda_7|H_u|^2 \right](H_d \cdot H_u) + h.c.
\]  

(3.36)

where now:

\[
\lambda_3 = 2\lambda_T^2 \quad \lambda_4 = \lambda_S^2 - \lambda_T^2 \\
\lambda_1 = \lambda_2 = \lambda_5 = \lambda_6 = \lambda_7 = 0.
\]  

(3.37)
As in the MSSM we expect sizeable one-loop corrections to the above potential, but we will also have new and particularly important corrections due to taking \( m_S, m_T \) large. We can approximate these new contributions by taking the leading logarithmic contributions from scalar loops involving only quartic vertices; the remaining diagrams involving scalars and sfermions will have the effect of removing the dependence on the cutoff. We can reasonably approximate this behaviour by replacing the renormalisation scale in the logarithms with \( v \) (a more precise calculation involves the full fermion mass matrices). We should then also evaluate the coupling constants at this scale. We can then write

\[
\delta^{(1)} \lambda_1 = \frac{3}{16\pi^2} v_b^4 \log \left( \frac{m_b^2}{v^2} \right) + \frac{5}{16\pi^2} \lambda^4_T \log \left( \frac{m_T^2}{v^2} \right) + \frac{1}{16\pi^2} \lambda^4_S \log \left( \frac{m_S^2}{v^2} \right) \\
- \frac{1}{16\pi^2} \frac{\lambda^2_S \lambda^2_T}{m_T^2 - m_S^2} \left\{ m_T^2 \left[ \log \left( \frac{m_T^2}{v^2} \right) - 1 \right] - m_S^2 \left[ \log \left( \frac{m_S^2}{v^2} \right) - 1 \right] \right\}
\]

\[
\delta^{(1)} \lambda_2 = \frac{3}{16\pi^2} v_t^4 \log \left( \frac{m_t^2}{m_t^2} \right) + \frac{5}{16\pi^2} \lambda^4_T \log \left( \frac{m_T^2}{v^2} \right) + \frac{1}{16\pi^2} \lambda^4_S \log \left( \frac{m_S^2}{v^2} \right) \\
- \frac{1}{16\pi^2} \frac{\lambda^2_S \lambda^2_T}{m_T^2 - m_S^2} \left\{ m_T^2 \left[ \log \left( \frac{m_T^2}{v^2} \right) - 1 \right] - m_S^2 \left[ \log \left( \frac{m_S^2}{v^2} \right) - 1 \right] \right\}
\]

\[
\delta^{(1)} \lambda_3 = \frac{5}{32\pi^2} \lambda^4_T \log \left( \frac{m_T^2}{v^2} \right) + \frac{1}{32\pi^2} \lambda^4_S \log \left( \frac{m_S^2}{v^2} \right) \\
+ \frac{1}{32\pi^2} \frac{\lambda^2_S \lambda^2_T}{m_T^2 - m_S^2} \left\{ m_T^2 \left[ \log \left( \frac{m_T^2}{v^2} \right) - 1 \right] - m_S^2 \left[ \log \left( \frac{m_S^2}{v^2} \right) - 1 \right] \right\}
\]

(3.38)

and can neglect the contributions to the other factors as subleading. For simplicity we have assumed \( B_S, B_T \ll m_S^2, m_T^2 \); this is not valid in the scenario of for example [22] where they are of equal order in magnitude, both being generated at one loop.

Note that there is also a contribution to the Higgs mass parameters proportional to

\[ \lambda^2_{S,T} m^2_{S,T} \log \left( \frac{m_{S,T}^2}{v^2} \right), \]

which is absorbed into the renormalisation of \( m^2_{H_u}, m^2_{H_d} \). If these are large then we have a large fine tuning.

### 3.2 Singlet Extension to the Higgs Sector

As can be seen from equations (3.2,3.3), the sfermions obtain a mass proportional to the expectation values of \( S_R, T_R \). Of these, we can ignore the \( T_R \) contribution due to the strong constraints upon it as discussed above, but \( \langle S_R \rangle \) may be non-negligible. Together with the dominant one loop
effect (containing gauginos or $S_R, T_R$ scalars) we find

$$m_{ii}^2 = (m_{ii}^{(0)})^2 + 2m_{1D} \langle S_R \rangle g_i Y_i + Y_i^2 (g')^2 \frac{m_{1D}^2}{4\pi^2} \log \frac{m_{SR}^2}{m_{\lambda_1}^2} + \frac{1}{2} g^2 \frac{m_{2D}^2}{4\pi^2} \log \frac{m_{T_R}^2}{m_{\lambda_2}^2}$$

(3.39)

where, since the action we are using is an effective one, we have included the soft masses induced by the supersymmetry breaking sector (via gauge mediation or otherwise) other than through the Dirac gauginos as $m_{ii}^{(0)}$. For example, in gauge mediation these are generated at two loops; the gaugino masses $m_{1D}, m_{2D}, m_{\lambda_1}, m_{\lambda_2}$ (the latter two are the total gaugino mass, including both Dirac and any Majorana effects, defined as the location of the pole in the $\langle \lambda \lambda \rangle$ propagator) are generated at one. The one loop fluctuations around our effective action then actually appear in gauge mediation at three loops.

The term proportional to $\langle S_R \rangle$ clearly gives a negative contribution to negatively charged states, so we must ensure that this does not dominate. This is particularly important for models where $m_{ii}^{(0)}$ vanishes, which can occur for certain gauge mediation models with purely Dirac gauginos (in such cases $M_S, M_T = 0$). One way to ensure positivity is to take a large $m_S, m_T$, which reduces $\langle S_R \rangle$ and increases the loop effects. We then have (where $m_{SR,T_R}^2 = m_{ST}^2 + B_{ST}$)

$$m_{1D} g_i Y_i v^2 \left( \frac{c_2 g' m_{1D} + \sqrt{2} \mu \lambda_S - 2 \sqrt{2} \lambda_S A_S S_{c\beta}}{m_S^2 + 4m_{1D}^2 + B_S} \right) + Y_i^2 (g')^2 \frac{m_{1D}^2}{4\pi^2} \log \frac{m_{SR}^2}{m_{\lambda_1}^2} + \frac{g^2}{2} \frac{m_{2D}^2}{4\pi^2} \log \frac{m_{T_R}^2}{m_{\lambda_2}^2} > 0.$$  

This is easy to arrange for large $m_{SR}$, for example for small $\lambda_S$ or large $m_{1D} c_{2\beta} \ll m_S$ we require (from SU(2) singlets, which provide the strictest constraint)

$$\log(m_S^2 + B_S)/m_{\lambda_1}^2 > \frac{2\pi^2 v^2}{m_S^2 + B_S + 4m_{1D}^2}$$

(3.40)

and therefore for $m_{1D}^2 \sim v^2$ we have $m_{SR} \gtrsim 3v + 0.94(m_{\lambda_1} - v) + \ldots \gtrsim 750 GeV$, and for $m_{1D}^2 = x m_{SR}^2$ we require $m_{SR} \gtrsim \pi v/\sqrt{2}(2x + 1/2) \log x$. If we consider for example that the masses are generated in gauge mediation, then $m_{1D} \sim \lambda_X g' \Lambda, m_S \sim B_S \sim \lambda_X^2 \Lambda^2$ for a messenger coupling $\lambda_X$ and effective supersymmetry breaking scale $\Lambda$; this gives $x = (g')^2 \sim 1/8.4$ and thus $m_{SR} \gtrsim 620 GeV$. However, this limit is only valid for $\lambda_S \ll g'/\sqrt{2}$, as the gaugino masses are smaller than $v$. For $\lambda_S \sim g'/\sqrt{2}$ we can place a bound by setting $\mu \sim v$:

$$- \frac{\sqrt{2}(g')^2 v^3}{2m_{SR}(4x + 1)} - (g')^2 \frac{x m_{SR}^2}{4\pi^2} \log x > 0$$

(3.41)
and thus $m_{S_R} > v \left(-2\pi^2/\sqrt{x}(1 + 4x) \log x\right)^{1/3} \gtrsim 650\text{GeV}$.

Now consider when $m_S$ is small, $\lambda_S(S_R) \equiv \tilde{\mu}$ is large and adds to or replaces the $\mu$ term while $A_S\lambda_S(S_R)$ contributes to $B\mu$. The loop terms are no longer significant, and we must simply impose that

$$\left(m_{\mu}^{(0)}\right)^2 + 2m_{1D} \tilde{\mu} g' Y_i > 0. \quad (3.42)$$

This will be in general quite a stringent constraint on the parameter space of the microscopic model.

Keeping $S$ in the light spectrum results in the effective potential (3.12) with $T = 0$. The MSSM spectrum is extended by one CP even Higgs and one CP odd state, corresponding to $S_R$ and $S_I$ respectively. This falls in the class of models studied by [46]. In addition to the possibility discussed there of new decays of the lightest Higgs into pairs of CP odd states (as can be seen from Eq. 3.12) we would like to stress the new feature that the Higgs has different decay/production channels due to the mixing with $S_R$ which couples to the D term as

$$\mathcal{L}_{\text{int}} \supset -2m_{1D} S_R g' \sum_j Y_j \bar{\psi}_j \psi_j. \quad (3.43)$$

The effect of this at colliders is very model dependent. It is important, for light sfermions, when $m_{S_R}$ is smaller than or comparable to $m_{1D}$. For example, if sneutrinos are arranged to be light then the Higgs may decay to them and then to neutrinos, plus the LSP.

4 Fermion mass matrix

4.1 The neutralinos

There are six neutral fermions of interest: the higgsinos $\tilde{H}_u^0$ and $\tilde{H}_d^0$, the gauginos, bino $\tilde{B}$ and wino $\tilde{W}^0$ and the DG-adjoint fermions $\tilde{B}'$ and wino $\tilde{W}'^0$. The mass terms for these fields have different origins:

- bino, wino and DG-adjoints Majorana masses:

$$-\frac{1}{2} (M_2 \tilde{W}^0 \tilde{W}^0 + M_1 \tilde{B} \tilde{B} + M'_2 \tilde{W}'^0 \tilde{W}'^0 + M'_1 \tilde{B}' \tilde{B}' + \text{h.c.}). \quad (4.1)$$

- bino and winos Dirac masses:

$$-m_{2D} \tilde{W}'^\alpha \tilde{W}'^\alpha - m_{1D} \tilde{B} \tilde{B}' + \text{h.c.} \quad (4.2)$$
• The gauge interactions between the gauginos, the higgsinos and the scalar Higgs:

\[-\frac{g'}{\sqrt{2}} \left( H_u^i \sigma^j \tilde{H}_u \tilde{B} - H_d^i \sigma^j \tilde{H}_d \tilde{B} \right) - \frac{g}{\sqrt{2}} \left( H_u^i \sigma^j \tilde{H}_u \tilde{W}^i + H_d^i \sigma^j \tilde{H}_d \tilde{W}^i \right)\]  

(4.3)

leading to

\[- m_Z \left[ \sin \theta_W (s_{\beta} \tilde{H}_u^0 \tilde{B} - c_{\beta} \tilde{H}_d^0 \tilde{B}) + \cos \theta_W (c_{\beta} \tilde{H}_d^0 \tilde{W}^0 - s_{\beta} \tilde{H}_u^0 \tilde{W}^0) + h.c. \right] \]  

(4.4)

• The superpotential (2.10) leads to couplings between the DG-adjoint fermions, Higgs and Higgsinos

\[- \lambda_S \left( H_d \cdot \tilde{H}_u \tilde{B}' - H_u \cdot \tilde{H}_d \tilde{B}' \right) - \lambda_T \left[ H_u \cdot (\sigma^j \tilde{H}_d) \tilde{W}^j + H_d \cdot (\sigma^j \tilde{H}_u) \tilde{W}^j \right]\]  

(4.5)

giving

\[- m_Z \left[ \frac{\sqrt{2} \lambda_S \sin \theta_W}{g'} (s_{\beta} \tilde{H}_d^0 \tilde{B}' + c_{\beta} \tilde{H}_u^0 \tilde{B}') - \frac{\sqrt{2} \lambda_T \cos \theta_W}{g} (c_{\beta} \tilde{H}_u^0 \tilde{W}^0 + s_{\beta} \tilde{H}_d^0 \tilde{W}^0) + h.c. \right] \]  

(4.6)

• The \( \mu \) term in the superpotential \( W \) contributes to the higgsinos masses,

\[ \mu \tilde{H}_u^0 \cdot \tilde{H}_d^0 + h.c. \]  

(4.7)

All the previous terms together describe the resulting mass matrices for both neutral gauginos and higgsinos when both Majorana and Dirac term are present. The neutralino mass matrix \( \mathcal{M}_0 \), in the \( (\tilde{B}', \tilde{B}, \tilde{W}^0, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0) \) basis is:

\[
\begin{pmatrix}
M'_1 & m_{1D} & 0 & 0 & \frac{\sqrt{2} \lambda_S}{g} m_Z s_W s_{\beta} & \frac{\sqrt{2} \lambda_S}{g} m_Z s_W c_{\beta} \\
m_{1D} & M_1 & 0 & 0 & -m_Z s_W c_{\beta} & m_Z s_W s_{\beta} \\
0 & 0 & M'_2 & m_{2D} & -\frac{\sqrt{2} \lambda_T}{g} m_Z c_W s_{\beta} & -\frac{\sqrt{2} \lambda_T}{g} m_Z c_W c_{\beta} \\
0 & 0 & m_{2D} & M_2 & m_Z c_W c_{\beta} & -m_Z c_W s_{\beta} \\
\frac{\sqrt{2} \lambda_S}{g} m_Z s_W s_{\beta} & -m_Z s_W c_{\beta} & -\frac{\sqrt{2} \lambda_T}{g} m_Z c_W s_{\beta} & m_Z c_W c_{\beta} & 0 & -\mu \\
\frac{\sqrt{2} \lambda_S}{g} m_Z s_W c_{\beta} & m_Z s_W s_{\beta} & -\frac{\sqrt{2} \lambda_T}{g} m_Z c_W c_{\beta} & -m_Z c_W s_{\beta} & -\mu & 0
\end{pmatrix}
\]  

(4.8)

where \( c_W = \cos \theta_W \), \( s_W = \sin \theta_W \). This may be diagonalised by a unitary matrix \( N \) such that \( \mathcal{M}_0^{\text{diag}} = N^* \mathcal{M}_0 N \); perturbative expansions for \( N \) in various limits are given in appendices \( \text{A} \) and \( \text{B} \).
4.2 The charginos

The chargino mass matrix describes the mixing between the charged higgsinos $\tilde{H}_u^+$, $\tilde{H}_d^-$ and the charged gauginos $\tilde{W}_+^-$ and $\tilde{W}_-^-$. The corresponding mass terms have the same origin as those presented in section 4.1. There are Dirac masses:

$$-M_2 \tilde{W}_+^- \tilde{W}_-^- - m_{2D} \tilde{W}_+^+ \tilde{W}_-^- + h.c.$$  \hspace{1cm} (4.9)

There is the usual $\mu$-term

$$-\mu \tilde{H}_u^+ \cdot \tilde{H}_d^- + h.c.$$  \hspace{1cm} (4.10)

and finally there are mixing terms of the gauginos with the Higgsinos:

$$-\sqrt{2}m_W \sin \beta \tilde{H}_u^+ \tilde{W}_-^- - \sqrt{2}m_W \cos \beta \tilde{H}_d^- \tilde{W}_+^+ + h.c.$$  \hspace{1cm} (4.11)

and with the DG-adjoints

$$-\frac{2\lambda_T}{g}m_W c_\beta \tilde{H}_u^+ \tilde{W}_-^- + \frac{2\lambda_T}{g}m_W s_\beta \tilde{H}_d^- \tilde{W}_+^+ + h.c.$$  \hspace{1cm} (4.12)

The mass terms for the charginos can be expressed in the form

$$-\frac{1}{2}((v^-)^T M_{Ch} v^+ + (v^+)^T M_{Ch}^T v^- + h.c)$$  \hspace{1cm} (4.13)

where we have adopted the basis $v^+ = (\tilde{W}_+^- , \tilde{W}_-^- , \tilde{H}_u^+ )$, $v^- = (\tilde{W}_+^- , \tilde{W}_-^- , \tilde{H}_d^- )$. Collecting all the terms presented in equations (4.9)-(4.10), (4.2) and (4.12) leads to the chargino mass matrix:

$$M_{Ch} = \begin{pmatrix} M_2' & m_{2D} & \frac{2\lambda_T}{g}m_W c_\beta \\ m_{2D} & M_2 & \sqrt{2}m_W s_\beta \\ -\frac{2\lambda_T}{g}m_W s_\beta & \sqrt{2}m_W c_\beta & \mu \end{pmatrix}.$$  \hspace{1cm} (4.14)

This nonsymmetric matrix is diagonalized by separate unitary transformations in the basis $v^+$ and $v^-$, $M_{Ch}^{diag} = U^T M_{Ch} V$, where $U$ and $V$ are unitary.

5 The LSP dark matter

The model has $R$-parity so the LSP is stable. Here, we assume the LSP to be the (lightest) neutralino and we would like to study its relic density and see how it fits with actual bounds from WMAP. However, before doing so we would like to make some comments on the fate of
gravitinos in this scenario. Gravitinos play two major roles in our model: 1) there is the issue of $R$-symmetry breaking by the gravitino mass $m_{3/2}$ and inducing Majorana masses for the gauginos 2) the gravitino may be the LSP for instance in models of gauge mediation. Depending on its mass it can also change the LSP relic abundance.

First, let us discuss the first issue. In $N = 1$ supergravity the gravitino mass needs to be non-zero to cancel the cosmological constant after supersymmetry breaking. It is proportional to the vev of the superpotential and thus breaks $R$-symmetry. If one insists on avoiding the breaking of $R$ symmetry one option is to enhance the gravitational sector to $N = 2(\equiv N = 1_1 \oplus N = 1_2)$ supersymmetry. The supersymmetry breaking preserves $R$-symmetry when the two gravitino masses are equal. The set up is then a Dirac gravitino made of two gravitinos $\psi^{\mu}_{3/2,1}$ and $\psi^{\mu}_{3/2,2}$ with the same mass $m_{3/2}$, each of them coupling to a different sector with $N = 1_1$ for the first and $N = 1_2$ for the second. The coupling strength is in both cases given by $\frac{1}{m_{3/2} M_{Pl}}$, but may couple different gravitinos to different sectors. We will not present here an explicit realisation as it goes beyond the scope of this paper.

The second issue is very model dependent. A gravitino LSP can be produced in two different ways: either through thermal production or through decays of unstable sparticles. The relic density is expected to receive contributions from the two processes by an amount that depends on the particular spectrum and in the specific thermal history of the universe (for instance depending on the value of the reheat temperature), the possible decays of the inflatons, moduli, etc. The addition of a second gravitino might help the gravitino LSP to be a dark matter candidate by increasing the number of relativistic degrees of freedom at the freezeout temperature. However, with a second sector, it will make the story even more dependent of the details. We will not pursue this scenario further here.

The neutralino sector that we shall consider differs from that of the MSSM in two aspects:

- The LSP is now a linear combination of six states. There are now two additional states compared to the MSSM case.
- In addition to the MSSM parameters, we have six additional parameters:

$$M_1', M_2', m_{1D}, m_{2D}, \lambda_S \text{ and } \lambda_T$$

In this work, our purpose is to find a region of these parameters where the relic density is compatible with the dark matter made of a thermally produced neutralino LSP and compare this with the case of MSSM. We will assume both the $S$ and $T$ scalar are very heavy and decoupled from the thermal bath.
5.1 The Dirac case

The first simplest case we will consider is the LSP to be mainly bino like.

\[
\begin{pmatrix}
0 & m_{1D} & 0 & 0 & \frac{\sqrt{2} \lambda_S}{g} m_{ZSW} s_\beta & \frac{\sqrt{2} \lambda_S}{g} m_{ZSW} c_\beta \\
m_{1D} & 0 & 0 & 0 & -m_{ZSW} c_\beta & m_{ZSW} s_\beta \\
0 & 0 & 0 & 0 & m_{2D} & -\frac{\sqrt{2} \lambda_T}{g} m_{ZCW} s_\beta & -\frac{\sqrt{2} \lambda_T}{g} m_{ZCW} c_\beta \\
0 & 0 & m_{2D} & 0 & m_{ZCW} c_\beta & -\mu & 0 \\
\frac{\sqrt{2} \lambda_S}{g} m_{ZSW} s_\beta & -m_{ZSW} c_\beta & -\frac{\sqrt{2} \lambda_T}{g} m_{ZCW} s_\beta & m_{ZCW} c_\beta & 0 & -\mu & 0 \\
\frac{\sqrt{2} \lambda_S}{g} m_{ZSW} c_\beta & m_{ZSW} s_\beta & -\frac{\sqrt{2} \lambda_T}{g} m_{ZCW} c_\beta & -m_{ZCW} s_\beta & -\mu & 0 & 0 \\
\end{pmatrix}
\]

(5.2)

With \( \mu > m_{1D} \) The DG-adjoint vevs are now

\[
v_s \approx \frac{v^2}{2(M_S^2 + M_S^2 + 4m_{1D}^2 + B_S)} \left[ g'm_{1DC_2}\beta + \sqrt{2} \mu \lambda_S \right]
\]

\[
v_t \approx \frac{v^2}{2(M'^2 + M_T^2 + 4m_{2D}^2 + B_T)} \left[ gm_{2DC_2}\beta - \sqrt{2} \mu \lambda_T + \frac{\lambda_T}{\sqrt{2}} (M_T + A_T) s_\beta \right]. \quad (5.3)
\]

To have a pure Dirac LSP, the lightest two eigenvalues of the neutralino mass matrix most form a pair of equal magnitude but opposite sign. It is straightforward to show that in the case that both \( \lambda_s, \lambda_T \) take their \( N = 2 \) values we will only have Dirac neutralinos. To do this, we note that solving for the eigenvalues of the neutralino mass matrix \( \mathcal{M} \) means solving the equation \( f(\lambda) \equiv \det(\mathcal{M} - \lambda) = 0 \). For purely Dirac states, \( f(\lambda) = \prod_{i=1}^3 (\lambda^2 - a_i^2) \) for eigenvalues \( \pm a_i \), which is true if and only if \( f(-\lambda) = f(\lambda) \). By examining the coefficients of \( \lambda^1 \) and \( \lambda^3 \) (that of \( \lambda^5 \) being automatically zero due to the tracelessness of \( \mathcal{M} \)) we find that this requires

\[
\lambda^1 : 2 \mu c_\beta s_\beta M_Z^2 \left[ c_W^2 m_{1D}^2 \left( 1 - \frac{2 \lambda_T}{g^2} \right) + s_W^2 m_{2D}^2 \left( 1 - \frac{2 \lambda_S}{(g')^2} \right) \right] = 0
\]

\[
\lambda^3 : 2 \mu c_\beta s_\beta M_Z^2 \left[ c_W^2 \left( 1 - \frac{2 \lambda_T}{g^2} \right) + s_W^2 \left( 1 - \frac{2 \lambda_S}{(g')^2} \right) \right] = 0 \quad (5.4)
\]

proving the above assertion.

If we assume a mostly bino/U(1) adjoint LSP, by assuming that \( m_{1D} < m_{2D} \ll \mu \) we can expand the LSP eigenvalues to next to leading order in \( m_{1D}^2/\mu, m_{2D}^2/\mu \) to find

\[
m_{\text{LSP}} = m_{1D} + M_Z^2 s_W^2 \left[ \sqrt{2} \lambda_S g' (s_\beta^2 - c_\beta^2) \pm (2 \lambda_S^2 - g' g^2) c_\beta s_\beta \right] + \ldots \quad (5.5)
\]
Assuming that \((2\lambda_S^2 - (g')^2)c_\beta s_\beta\) is negative, the LSP takes the upper sign, and the mass splitting is given to lowest order by

\[
\Delta m_{LSP} = -2\frac{M_Z^2 s_W^2}{\mu} \frac{(2\lambda_S^2 - (g')^2)}{(g')^2} c_\beta s_\beta. \tag{5.6}
\]

Note that this reduces to the result of [36] when \(\lambda_S \to 0\). The eigenstates at this order can be read off from the rotation matrices, given in appendix \(\Delta\).

For the case of a Higgsino LSP, where \(m_{1D} \sim m_{2D} \gg \mu\), we find

\[
m_{LSP} = \mu + \frac{\sqrt{2}\lambda_T}{gg'm_{1D}m_{2D}} \left[ c_W^2 g'\lambda_T m_{1D} + s_W^2 g\lambda_S m_{2D} \right] + \frac{M_Z^2}{\mu} \left[ \frac{4c_\beta^2 s_\beta^2 M_Z^2}{g^2(g')^2 m_{1D}^2 m_{2D}^2} (c_W^2 g'\lambda_T m_{1D} + g\lambda_S m_{2D} s_W^2)^2 \right] - \frac{m_{1D}^2}{g^2(g')^2 m_{1D}^2 m_{2D}^2} \left[ (g')^2 m_{1D}^2 c_W^2 (g^2 - 2\lambda_T^2) + g^2 m_{2D}^2 s_W^2 ((g')^2 - 2\lambda_S^2) \right] \tag{5.7}
\]

where we have had to expand to the second order to obtain a mass splitting.

If we suppose that the adjoint couplings take the \(N = 2\) values at some scale \(M_{N=2}\) and we run \(\lambda_S, \lambda_T\) down to the supersymmetry breaking scale \(M_{N=1}\) (equal to \(m_S\) or \(m_T\)) then we can generate a mass splitting since the adjoints do not couple to the matter multiplets, and the Higgs’ wavefunction renormalisation also contributes. To leading order we have

\[
\begin{align*}
2\lambda_S^2 - (g')^2 \bigg|_{M_{N=1}} &= -\frac{2(g')^2}{16\pi^2} \left[ 3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 10(g')^2 \right] \log\left(\frac{M_{N=2}}{M_{N=1}}\right) \\
2\lambda_T^2 - g^2 \bigg|_{M_{N=1}} &= -\frac{2g^2}{16\pi^2} \left[ 3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 4g^2 \right] \log\left(\frac{M_{N=2}}{M_{N=1}}\right) \tag{5.8}
\end{align*}
\]

If we assume that \(y_t \sim 1\) and the other couplings much smaller, then we find for a Bino LSP that the mass difference should be

\[
\Delta m_{LSP} \approx 2c_\beta s_\beta \frac{M_Z^2 s_W^2}{\mu} \frac{3}{8\pi^2} \log\left(\frac{M_{N=2}}{M_{N=1}}\right) \\
\approx 0.15\text{GeV} \left(\frac{TeV}{\mu}\right) \frac{t_\beta}{1 + t_\beta^2} \log\left(\frac{M_{N=2}}{M_{N=1}}\right). \tag{5.9}
\]
where, obviously, smaller splitting can be seen to correspond to larger values of $\tan \beta$ and smaller ratio $M_{N=2}/M_{N=1}$. For instance, taking $\mu = 1\text{TeV}$, $M_{N=2} \sim 10^{16}\text{GeV}$, and $t_\beta = 50$ leads to

$$\Delta m_{LSP} \approx 4.4 \frac{t_\beta}{1 + t_\beta^2} \text{GeV} \approx 90\text{MeV} \quad (5.10)$$

while for $\mu = 1\text{TeV}$, $M_{N=2} \sim 10^4\text{GeV}$, and $t_\beta = 50$, we obtain

$$\Delta m_{LSP} \approx 0.3 \frac{t_\beta}{1 + t_\beta^2} \text{GeV} \approx 6\text{MeV}. \quad (5.11)$$

The latter case could be realized in low cut-off scale models (such as for large extra-dimensions).

### 5.2 Dark matter relic abundance

In the DG model, as in the MSSM, there are several scenarios that lead to a relic abundance of the neutralino LSP, $\Omega h^2 \approx 0.11$. These include naturally the MSSM-like scenarios

- a mixed bino/Higgsino LSP that annihilates mainly into W pairs (or top pairs)
- a mixed bino/wino or bino/wino/Higgsino that annihilates mainly into W pairs
- a bino that annihilates into fermion pairs when sleptons are light, a significant region of parameter space where this process occurs has been ruled out by LEP.
- a bino that coannihilates with sfermions.
- an almost pure bino with mass $2m_\tilde{B} \approx m_{h,A}$ with efficient annihilation through Higgs exchange.

New dark matter scenarios occur as well. Firstly in the special case of a pure Dirac bino LSP (or an almost pure Dirac bino), annihilation into light fermion pairs becomes efficient. This is because the process does not have a strong p-wave suppression as in the Majorana neutralino case [36]. Secondly the MSSM-like scenarios where $\tilde{B}, \tilde{W}$ are replaced with $\tilde{B}'$ and $\tilde{W}'$ can also occur. These are found either with Dirac masses or with Majorana masses. Typically, as we will see in case studies below, these involve more coannihilation processes.

In the DG model, the dark matter detection properties can be quite different than in the usual MSSM. For one, the annihilation of a Dirac neutralino into light fermion pairs is not suppressed, even at $v/c \approx 0.001$. More generally, the content of the LSP that determines the coupling of the LSP to other particles can be different in the DG model. This is largely due to the additional $\tilde{B}'$ or $\tilde{W}'$ components. For example, the spin independent elastic scattering rate on nucleons that
is dominated by Higgs exchange unless squarks are light, depends on the higgsino content of the LSP. In the DG model, the higgsino fraction is often suppressed leading to small rates.

In the following subsections we present results for a few case studies. For each scenario, we find the parameter space that predicts a relic abundance compatible with the value determined by cosmological measurements, $\Omega h^2 = 0.113 \pm 0.0034$ [47] and examine the predictions for the direct and indirect detection rates. In all cases special attention is paid to the typical DM scenarios with either a $\tilde{B}(\tilde{B}')$ or mixed $\tilde{B}(\tilde{B}')/\tilde{h}$ or $\tilde{B}(\tilde{B}')/\tilde{W}(\tilde{W}')$ scenarios (here $\tilde{h}$ stands for $\tilde{H}^0_u$ or $\tilde{H}^0_d$). The possibility of efficient annihilation through a Higgs resonance is a generic feature of all models where s-channel resonance can occur, we will however not consider this mechanism in detail as it requires fine-tuning of the model parameters.

6 Results

All numerical results are based on micrOMEGAs2.3 for the computation of the spectrum, the relic abundance, the elastic scattering rate as well as the annihilation rate $\sigma v|_0$ relevant for indirect detection [48, 49]. We have implemented in this code the DG model described in section 3. The one-loop quark/squark corrections to the Higgs masses are computed as well as the $\lambda_S, \lambda_T$ corrections to the effective potential (3.36). The latter can increase the Higgs mass by a few GeV as compared with the MSSM. We have imposed the LEP bounds on charged sparticles as well as on the Higgs mass ($m_h > 111$ GeV), allowing a large theoretical uncertainty for $m_h$ since two-loops corrections are ignored. In all cases we fix $\tan \beta = 10$, $A_t = -1.5$ TeV and $M_{\tilde{q}} = 1$ TeV. Under these conditions the Higgs mass limit is easily satisfied. The elastic scattering rates are computed taking the micrOMEGAs default values for the quark coefficients in the nucleon [49]. Varying these coefficients could induce large corrections to the predicted rate.

6.1 Pure Dirac masses: $M_1 = M'_1 = M_2 = M'_2 = 0$

In the case of pure Dirac masses $M_{1D}, M_{2D} \neq 0$, the LSP can be a Dirac fermion provided $\lambda_S$ and $\lambda_T$ take their N=2 value, eq. (2.12). In general one expects a mass splitting generated by the N=2 breaking effect, eq. (5.19), nevertheless it is possible to tune the parameters of the model such that the two lightest Majorana states are degenerate and make a Dirac fermion. The implications for dark matter detection of a Dirac fermion are important so this case is worth consideration.
6.1.1 Dirac fermion

A Dirac neutralino, contrary to a Majorana neutralino, can annihilate into light fermion pairs with a large rate, thus offering an explanation to the excess of positrons seen by Pamela [50, 51] without spoiling the antiproton observations [52]. In particular the Dirac bino can have a much larger annihilation rate into leptons than into quarks when the mass of the right-handed sleptons are of the order of the bino mass. However, the Dirac neutralino has an effective vectorial interaction with quarks in the nucleon, this leads to potentially large rates for direct detection. For a Dirac fermion the spin-independent elastic scattering cross section receives dominant contributions from squark and Z exchange. To avoid exceeding the experimental bound [54, 55] it is enough to 1) fix the mass of the squark that couples most strongly to the bino, the one with the largest hypercharge, to \( m_{\tilde{q}_R} > 1 - 1.2 \) TeV and 2) suppress the higgsino LSP component such that the coupling of the LSP to the Z is reduced. To illustrate this we have computed the neutralino nucleon elastic scattering cross section in a scenario where \( M_{2D} = 1.5M_{1D}, \mu = 1 \) TeV, \( M_{j_L} \) TeV and \( M_{\tilde{q}_R} = 1 \) or 1.2 TeV. The value of the common mass for the right-handed sleptons, \( M_{\tilde{l}_R} \), is adjusted for each value of \( M_{1D} \) such that \( \Omega h^2 = 0.11 \). The lower value of the squark mass leads to a cross-section exceeding the CDMS limit for light neutralinos, see Fig. 1. Note that one characteristic of scenarios with a Dirac particle as DM is that the spin independent amplitude for elastic scattering on nucleons can be different for protons and neutrons, for example \( \sigma_{\tilde{\chi}_1^0 p} << \sigma_{\tilde{\chi}_1^0 n} \) if Z exchange dominates. However the experimental limits on \( \sigma_{\tilde{\chi}_1^0 p} \) are extracted the amplitudes for protons and neutrons to be equal. In order to be able to compare directly with the experimental limit from CDMS [54], we rescale the nucleon cross section and define an effective \( \sigma_{\tilde{\chi}_1^0 Ge} = (Z f_p + (A - Z)f_n)^2/A^2 \) where \( f_{p(n)} \) are the amplitudes on nucleons and \( Z = 32, A = 76 \) for Germanium. In this scenario, the rate for indirect detection is large, \( \sigma v |_{ll} \approx 2.5 \times 10^{-26} \) cm\(^3\)/sec for \( M_{1D} = 300 \) GeV and the lepton channels are almost two orders of magnitude larger than the quark channels.

For the remainder of this section we will consider the case where the LSP is a Majorana fermion, and the LSP-NLSP mass splitting in the MeV to GeV range as illustrated by eq. (5.10) and eq. (5.11). This results from shifts of the value of \( \lambda_S \) (say by only 1%) from the N=2 value in eq. (2.12). Because of the small mass splitting between the LSP and the NLSP the \( \tilde{\chi}_1^0 \tilde{\chi}_2^0 \) channels will not suffer from a Boltzmann suppression and will contribute significantly to the effective annihilation cross section that enters the standard computation of the relic abundance [48].

\footnote{Note that the recent Fermi results [63] on the total electron and positron spectrum is in agreement with the positron spectrum measured by PAMELA.}
6.1.2 Bino-LSP

The main mechanism for annihilation of a bino LSP is through exchange of sfermions, the sfermions with largest hypercharge, the right-handed (RH) sleptons, giving the dominant contribution. In general this process gives only $\Omega h^2 \approx O(1)$. This is because $\sigma \propto m_{\tilde{\chi}_1^0}^2/m_{\tilde{f}_R}^4$, and both the neutralino and the sfermion need to be near 100 GeV, that is near the LEP exclusion region, to reach $\Omega h^2 = 0.1$. Slepton coannihilation provides an alternative for reducing the relic abundance. With Dirac mass terms and nearly degenerate $\tilde{B}$ and $\tilde{B'}$, the processes $\tilde{\chi}_1^0 \tilde{\chi}_2^0 \rightarrow f \bar{f}$ provide the dominant annihilation mechanism [36]. One can obtain $\Omega h^2 \approx 0.1$ even with sleptons twice as heavy as the bino-LSP. This is displayed in Fig. 2, where we compare the slepton-LSP mass splitting that produces a relic abundance in the WMAP range in the DG model and in the MSSM. Here we show contours including a large theoretical uncertainty in the determination of $\Omega h^2 = 0.11 \pm 0.026$. In both cases we fix $\mu = 1$ TeV and take $M_{2D} = 2M_{1D}$ (DG model) or $M_2 = 2M_1$ (MSSM), so that $m_{\tilde{\chi}_1^0} \approx M_{1D}(M_1)$. For each neutralino mass we vary the RH slepton masses to find the given relic abundance contour. Because of the mixing in the stau sector, the stau turns out to be the lightest slepton. In this scenario the elastic scattering cross-section is small ($\sigma_{SI,\tilde{\chi}_1^0 p} \approx 10^{-10}$ pb) although within the reach of detectors such as Xenon [56]. The small cross sections can be linked to the LSP higgsino composition. The indirect detection cross section is also small in both models.
The prediction for the elastic scattering cross-section can be shifted significantly for a different choice of $\lambda_S$. For example for $\lambda_S = 0$, the higgsino fraction $f_H (\equiv |N_{i5}|^2 + |N_{i6}|^2, i$ denoting the index of the LSP, in this case 1) of the LSP decreases (see appendix A) thus suppressing the elastic scattering rate further by an order of magnitude. Of course $\lambda_S = 0$ will increase the $\tilde{B} - \tilde{B}'$ mass splitting, yet a splitting of only a few GeVs has little impact on the relic abundance (fig. 2b). Conversely enhanced rates can be found for $\lambda_s > g'/\sqrt{2}$.

The bino LSP up to a few hundred GeV is therefore a natural DM candidate in the DG model. The discovery of such neutralinos and sleptons of a few hundred GeV is within the reach of the LHC. Furthermore, only a rough estimate of the slepton/LSP mass difference would be sufficient to point out an inconsistency between the relic abundance of dark matter obtained from cosmological measurements and the one predicted from collider measurements of the SUSY spectrum if done in the context of the MSSM.
6.1.3 Mixed bino/higgsino LSP

A mixed bino/higgsino LSP is a more natural DM candidate than the bino because of the efficient annihilation into gauge boson pairs or top quark pairs. In particular the annihilation into gauge boson pairs proceeds through t-channel chargino exchange or Z/H exchange. For all annihilation diagrams some higgsino or $\tilde{W}$, $\tilde{W}'$ fraction of the LSP is involved, see the explicit expressions for the LSP couplings in Appendix C. Here and in the following we assume heavy sleptons $M_{\tilde{l}_L} = M_{\tilde{l}_R} = 1$ TeV as they are not needed for efficient annihilation.

As a sample scenario we take $M_{2D} = 2M_{1D}$ so that the wino fraction of the LSP is small and fix the mass of all sfermions to 1 TeV. In the DG model we find the contour of $\Omega h^2 = 0.11$ in the $\mu - M_{1D}$ plane, see fig. 3. The contour corresponds to $\mu \approx M_{1D}$ and the region below the contour gives $\Omega h^2 < 0.11$. The LSP is dominantly bino/bino' with a higgsino fraction that ranges from 2-30% along this contour as one increases the LSP mass. The small higgsino fraction of the LSP implies a small annihilation cross section into W pairs. This is however compensated by the contribution of the coannihilation channels to the relic abundance, with $\tilde{\chi}_1^0\tilde{\chi}_2^0$, $\tilde{\chi}_1^0\tilde{\chi}_2^+$, $\tilde{\chi}_2^0\tilde{\chi}_3^0$, $\tilde{\chi}_1^0\tilde{\chi}_3^0$ into gauge bosons or heavy quarks. This is to be contrasted with the MSSM, where a similar contour in the $\mu - M_1$ plane is found but which feature a Higgsino fraction $f_H \approx 30\%$. Since coannihilation channels do not enter direct/indirect detection rates, the rates are suppressed in the DG model relative to the MSSM, compare fig. 3 with fig. 4. The sharp variation in $\sigma_{\tilde{\chi}_1^0p}$ at the edge of the figure corresponds to the onset of the efficient annihilation of neutralinos through a Higgs resonance. Note that the mass spectra in the two models are rather similar, apart from the fact that there are two nearly degenerate $\tilde{B}$, $\tilde{B}'$ and an additional chargino in the DG model.

6.1.4 Mixed bino/wino/higgsino LSP

We consider finally the case of a mixed bino/wino LSP, for example we choose $M_{2D} = 1.1M_{1D}$. This choice for the gaugino masses necessarily implies a roughly 10% mass splitting between the LSP, an equal mixture of $\tilde{B}/\tilde{B}'$, and the $\tilde{\chi}_2^0(\tilde{B}/\tilde{B}')$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0(\tilde{W},\tilde{W}')$ and the $\tilde{\chi}_1^+, \tilde{\chi}_2^+$. Thus naturally one finds important contributions from a variety of coannihilation channels. The contour of $\Omega h^2 = 0.11$ is displayed in fig. 3. Along this contour the coannihilation into gauge boson pairs or fermions dominate. Note that the main difference with the bino/higgsino case occurs for a light LSP, in particular, the small wino fraction is sufficient to provide efficient annihilation through chargino exchange, thus larger values of $\mu$ are allowed. On the other hand in an MSSM model with a mixed wino LSP, here we choose $M_2 = 1.1M_1$, only about half the effective annihilation cross section comes from coannihilation with $\tilde{\chi}_1^+, \tilde{\chi}_2^0$, the dominant mode is LSP annihilation into W pairs (or top pairs). A smaller gaugino mass splitting, for example $M_2 = 1.05M_1$ requires an
Figure 3: a) Contours of $\Omega h^2 = 0.11$ in the $\mu - M_{1D}$ plane in the DG model for the mixed bino/Higgsino $M_{2D} = 2M_{1D}$ (full blue line) and for $M_{2D} = 1.1M_{1D}$ (full green line), $M_{2D} = 1.1M_{1D}, \lambda_S = 1$ (red dashed line) b) Content of the LSP, $f_h$ (full lines) $f_W$ (dashed lines), same colour code as a), c) $\sigma_{\tilde{\chi}_1^0 p}$ as a function of $m_{\tilde{\chi}_1^0}$ - same colour code as a) with the CDMS limit (full black line) d) $\sigma v_{|0}$ as a function of $m_{\tilde{\chi}_1^0}$ - same colour code as a)
Figure 4: a) Contours of $\Omega h^2 = 0.11$ in the $\mu - M_1$ plane in the MSSM for the mixed bino/higgsino, $M_2 = 2M_1$ (blue) and for $M_2 = 1.1M_1$ (green) and $M_2 = 1.05M_1$ (red) b) Content of the LSP, $f_h$ (full lines) $f_W$ (dashed lines) - same colour code as a) c) $\sigma_{\chi p}$ as a function of $m_{\tilde{\chi}^0}$ - same colour code as a) with the CDMS limit (full black line) d) $\sigma v|_0$ as a function of $m_{\tilde{\chi}^0}$ - same colour code as a)
even smaller higgsino fraction.

The direct detection rate which relies mainly on the LSP higgsino fraction is lower than for the mixed Higgsino LSP, this is especially true for light LSPs, see fig. 3c. As in other scenarios, the elastic scattering rate in the DG model is suppressed as compared to the MSSM. This statement is strongly dependent on the value of $\lambda_S$. For example for $\lambda_S = 1$ we found a rate that increases by one or two orders of magnitude, this means within the range of the next run of Xenon [56]. The indirect rate is also increased in this case.

6.2 Large Majorana masses $M_1 = M_2 = 1 \text{ TeV}$

First consider the $M'_1 = M'_2 = 0$ case. When $\mu = 1 \text{ TeV}$, the LSP is dominantly $B'$ with some $B$ admixture and with a mass $m_{LSP} \approx m_{1D}^2/M_1$. Both the wino and higgsino fraction of the LSP are very small, therefore the usually efficient annihilation channel into $W$ pairs is suppressed. A value of $\Omega h^2 = 0.11$ can only be reached because of the coannihilation channels such as $\tilde{\chi}_0^0 \tilde{\chi}_1^1, \tilde{\chi}_1^+ \tilde{\chi}_1^- \tilde{\chi}_1^\pm$ into gauge boson pairs. These channels proceed through the wino component of the heavier neutralinos and charginos. For coannihilation to work one needs roughly a 10% mass difference between the LSP and the neutral and charged NLSP, NNLSP. In that sense the weak scale chargino/neutralino sector is similar to the one of the bino/wino scenarios in the MSSM, since additional states are at the TeV scale. In fig. 5 we display the contour $\Omega h^2 = 0.11$ in the $m_{2D}-m_{1D}$ plane for different values of $\mu$. When the higgsino fraction of the LSP increases, more precisely when the LSP mass becomes comparable to $\mu$, annihilation becomes very efficient and the relic abundance is always $\Omega h^2 < 0.11$. For $\mu = 300 \text{ GeV}$, this occurs for $M_{1D} \approx 550 \text{ GeV}$, or $m_{\tilde{\chi}^0_1} \approx 260 \text{ GeV}$.

As discussed in previous scenarios, the predictions of the DG model for elastic scattering cross sections are usually suppressed when compared to an equivalent MSSM scenario. This is related to the fact that the LSP has a lower Higgsino fraction and that the relic abundance relies more heavily on coannihilation in the DG model. For example for $\mu = 1000 \text{ GeV}$ the rate is suppressed by one order of magnitude in the DG model, see fig. 5b. Here the MSSM rate corresponds to the $\Omega h^2 = 0.11$ contour for $\mu = 1000 \text{ GeV}$. As before, a large increase in the elastic scattering rate is expected when the LSP has a significant higgsino fraction. This occurs when $M_{LSP} \approx \mu$ or when $\lambda_S \neq g'/\sqrt{2}$, see for example the contour $\mu = 1 \text{ TeV}, \lambda_S = 1$ in fig. 5b. Note that when the detection rate is small, interference between the squark and Higgs exchange can lead to a further suppression of the detection rate, see the dips for the $\mu = 300,500 \text{ GeV}$ scenarios in fig. 5b.

The self-annihilation of the LSP at $v = 0$ is small for the $\tilde{B}(\tilde{B}')/\tilde{W}$ scenario, the dominant channels are $W^+W^-$ or $t \bar{t}$ when this channel becomes kinematically accessible. As usual, the rate increases significantly with the higgsino content of the LSP, see fig. 5c.

We will not discuss the more general case of the DG model where all Majorana and Dirac
mass terms are present. When these masses are at the electroweak scale, there is no difficulty in finding dark matter scenarios with $\Omega h^2 \approx 0.1$ and a LSP below the TeV scale. The LSP can be either a $\tilde{B}$ or mixed $\tilde{B}/\tilde{t}, \tilde{B}/\tilde{B}, \tilde{B}/\tilde{W}(\tilde{W}')$. All neutralinos and charginos could be within the kinematical reach of the LHC, the discovery of additional neutralino/chargino states would be the most obvious way to distinguish this model from the MSSM.

7 Conclusion

Dirac masses for gauginos can be obtained by pairing the MSSM gauginos with additional singlet, triplet and octet states in the adjoint representation. The model then contains additional neutralinos and charginos as well as new scalar particles. We have discussed both the case where the adjoint scalars decouple leaving a new quartic Higgs coupling in the effective potential and the one where the singlet remains light. This model has R-parity so the LSP is stable, the LSP could be a gravitino or a neutralino.

We have made a first exploration of the parameter space of the model in the case of the neutralino LSP to find regions where the dark matter relic abundance is in agreement with cosmological measurements (assuming the standard cosmological scenario). Among the possible scenarios the ones that have a feature that distinguish them from the MSSM include the Dirac gaugino LSP that has a non suppressed annihilation into light fermions, the pseudo-Dirac bino LSP that
(co-)annihilates into leptons via slepton exchange with sleptons much heavier than expected in the MSSM, as well as several mixed bino/wino/higgsino or bino/higgsino scenarios. For the latter scenarios the direct/indirect detection rates are often expected to be lower than in the MSSM, yet could be within the range of future detectors. In our numerical analysis we have concentrated on the DM candidates in the 100-500 GeV range, thus with several states within the range of the LHC. We have avoided a detailed discussion of scenarios where the annihilation of the LSP is made efficient by the presence of a Higgs resonance, as these scenarios require fine-tuning the masses of the LSP and Higgses. We expect that additional annihilation channels involving the new scalars in the model with a weak scale extra singlet would give new regions of parameter space with efficient DM annihilation as is found in the NMSSM. Finally we mention that the mass splitting between the two lightest neutralino states can be small although unless \( \lambda_S \) is fixed exactly to its N=2 value the splitting is expected to be larger than the 100keV needed for inelastic dark matter scattering.

Acknowledgements

K.B. and M.G. wish to thank P. Slavich for discussions. G.B. and A.P. thank A. Semenov for his help with LanHEP. We also thank Jan Kalinowski for pointing out to us a typo in the first version. This work was supported in part by the GDRI-ACPP of CNRS. The work of A.P. was supported by the Russian foundation for Basic Research, grants RFBR-08-02-00856-a and RFBR-08-02-92499.

A Rotation Matrices and Eigenvalues for Bino/Wino Neutralinos

In this appendix we give the first order rotation matrices for the neutralinos in the approximation that \( \mu \) is much larger than the other masses. Recall that the Lagrangian contains a term

\[
\mathcal{L} \ni -\frac{1}{2} \chi_i M_{ij} \chi_j = -\frac{1}{2} \chi_i^' M_{ij}^{\text{diag}} \chi_j^'
\]

we write

\[
\chi_i^' = \delta_{ii} R_{ij} \chi_j \equiv N_{ij} \chi_j
\]

where \( R_{ij} \) is a real orthogonal matrix and \( \delta_{ii} \) a unitary diagonal matrix of phases that ensures that all of the masses are positive. For the case of that the neutralino is mostly bino, we assume that \( m_{1D}, m_{2D}, M_1, M_1', M_2, M_2' \) are of the same order and much smaller than \( \mu \). Then the eigenstates prior to mixing have mass eigenvalues

\[
m_{1}^\pm = \frac{1}{2} \left[ (M_1 + M_1') \pm \sqrt{(M_1 - M_1')^2 + 4m_{1D}^2} \right]
\]
and similarly for $m_2^\pm$. We then define

\[
c_1 \equiv \cos \theta_1 \equiv \frac{m_{1D}}{\sqrt{(M'_1 - m_1^+)^2 + m_{1D}^2}}
\]

\[
s_1 \equiv \sin \theta_1 \equiv \frac{m_1^+ - M'_1}{\sqrt{(M'_1 - m_1^+)^2 + m_{1D}^2}}
\]

(A.4)

so that

\[
\begin{pmatrix}
  c_1 & s_1 \\
- s_1 & c_1
\end{pmatrix}
\begin{pmatrix}
  M'_1 & m_{1D} \\
m_{1D} & m_1
\end{pmatrix}
\begin{pmatrix}
  c_1 & -s_1 \\
s_1 & c_1
\end{pmatrix} =
\begin{pmatrix}
  m_1^+ & 0 \\
0 & m_1^-
\end{pmatrix}
\]

(A.5)

and again similarly for the Wino states. Then $\delta_{11} = \sqrt{m_1^+/|m_1^+|}$, $\delta_{22} = \sqrt{m_1^-/|m_1^-|}$ and similarly for the $m_2^\pm$ values. The matrix $R_{ij}$ is given by

\[
R_{11} = c_1 - \frac{M_Z^2 s_1 s_W^2 (2c_1 c_\beta s_1 s_\beta ((g')^2 + 2\lambda_3^2) + \sqrt{2}g'(c_1^2 - s_1^2)(c_\beta^2 - s_\beta^2)\lambda S)}{(g')^2(m_1^- - m_1^+)}
\]

\[
R_{12} = s_1 + \frac{M_Z^2 c_1 s_W^2 (2c_1 c_\beta s_1 s_\beta ((g')^2 + 2\lambda_3^2) + \sqrt{2}g'(c_1^2 - s_1^2)(c_\beta^2 - s_\beta^2)\lambda S)}{(g')^2(m_1^- - m_1^+)}
\]

\[
R_{13} = \frac{c_W s_W M_Z^2}{gg'(m_1^+ - m_2^-)(m_1^+ - m_2^+)} \left[ (m_1^+ - c_2^2 m_2^- - s_2^2 m_2^+)(\sqrt{2}c_3^2(g')s_1 - \sqrt{2}(g')s_1 s_\beta^2 - 4c_1 c_\beta s_\beta S)\lambda T 
\right.
\]

\[
- g(m_2^- - m_2^+)^2 c_2 s_2 (2c_3 g's_1 s_\beta + \sqrt{2}c_1 c_\beta^2 S - \sqrt{2}c_1 s_\beta^2 S)
\]

\[
R_{14} = \frac{c_W s_W M_Z^2}{gg'(m_1^+ - m_2^-)(m_1^+ - m_2^+)} \left[ (m_1^+ - c_2^2 m_2^- - s_2^2 m_2^+)(\sqrt{2}c_3^2(g')s_1 - \sqrt{2}(g')s_1 s_\beta^2 - 4c_1 c_\beta s_\beta S)\lambda T 
\right.
\]

\[
- \lambda T (m_2^- - m_2^+)^2 c_2 s_2 (\sqrt{2}c_3^2(g')s_1 - \sqrt{2}(g')s_1 s_\beta^2 - 4c_1 c_\beta s_\beta S)
\]

\[
R_{15} = \frac{M_Z s_W}{g'\mu} \left[ g's_1 s_\beta + \sqrt{2}c_1 c_\beta S \right]
\]

\[
R_{16} = - \frac{M_Z s_W}{g'\mu} \left[ g's_1 c_\beta - \sqrt{2}c_1 s_\beta S \right]
\]

(A.6)
\[
R_{21} = -s_1 - \frac{MZ^2 c_1 s_W^2 (2c_1 c_3 s_1 s_3 ((g')^2 + 2\lambda_3^2) + \sqrt{2}g'(c_1^2 - s_1^2)(c_3^2 - s_3^2)\lambda_T)}{(g')^2(m_1^- - m_1^+)^\mu} \\
R_{22} = c_1 - \frac{MZ^2 s_1 s_W^2 (2c_1 c_3 s_1 s_3 ((g')^2 + 2\lambda_3^2) + \sqrt{2}g'(c_1^2 - s_1^2)(c_3^2 - s_3^2)\lambda_T)}{(g')^2(m_1^- - m_1^+)^\mu} \\
R_{23} = \frac{c_W s_W M_Z^2 g(g')(m_1^- - m_2^-)(m_1^- - m_2^+)^\mu}{g(g')(m_1^- - m_2^-)(m_1^- - m_2^+)^\mu} \left[ (m_1^- - m_2^-)(\sqrt{2}c_1 c_3^2 g(g')\lambda_T - \sqrt{2}c_1 (g') s_3^2 \lambda_T + 4c_3 s_1 s_3 \lambda_S) \\
+ (m_2^- - m_2^+)(-2c_1 c_3 s_2 \beta s_2 s_3 + 4c_2 c_3 s_1 s_3 \lambda_S \lambda_T + \sqrt{2}(c_3^2 - s_3^2)(c_3 s_1 s_3 - c_2^2 (g') \lambda_T) \right] \\
R_{24} = \frac{c_W s_W M_Z^2 g(g')(m_1^- - m_2^-)(m_1^- - m_2^+)^\mu}{g(g')(m_1^- - m_2^-)(m_1^- - m_2^+)^\mu} \left[ (m_1^- - m_2^-)2c_1 c_2 g(g') s_3 + \sqrt{2}s_1 s_3 \lambda_T - 2\sqrt{2}s_3^2 s_1 s_3 \right] \\
R_{25} = \frac{M_Z s_W}{g' \mu} \left[ g' c_1 s_3 - \sqrt{2}s_1 s_3 \lambda_S \right] \\
R_{26} = -\frac{M_Z s_W}{g' \mu} \left[ g' c_1 c_3 + \sqrt{2}s_1 s_3 \lambda_S \right] \\
\begin{align*}
R_{31} &= \frac{c_W s_W M_Z^2}{g(g')(m_1^+ - m_2^-)(m_2^+ - m_1^-)^\mu} \left[ (c_1^2 m_1^- + s_1^2 m_1^+ - m_2^+)\lambda_T (\sqrt{2}g s_2 - \sqrt{2}gs_3^2 - 4c_2 c_3 s_3 \lambda_T) \\
&+ c_1(g')(m_1^- - m_1^+)(2c_2 g s_2 s_3 + \sqrt{2}c_2 c_3^2 \lambda_T - \sqrt{2}c_2 s_3^2 \lambda_T) \right] \\
R_{32} &= \frac{c_W s_W M_Z^2}{g(g')(m_1^+ - m_2^-)(m_2^+ - m_1^-)^\mu} \left[ c_1 (m_1^- - m_1^+) s_1 \lambda_T (\sqrt{2}g s_2 - \sqrt{2}gs_3^2 - 4c_2 c_3 s_3 \lambda_T) \\
&+ (m_2^+ - s_1^2 m_1^- - c_1^2 m_1^+)(-2c_2 g (g') s_3 s_2 - \sqrt{2}c_2 c_3^2 (g') \lambda_T + \sqrt{2}c_2 (g') s_3^2 \lambda_T) \right] \\
R_{33} &= c_2 - \frac{M Z^2 s_W^2 c_2^2 (2c_2 c_3 s_2 s_3 ((g)^2 + 2\lambda_T^2) + \sqrt{2}g (c_2^2 - s_2^2)(c_3^2 - s_3^2) \lambda_T)}{(g')^2(m_2^- - m_2^+)^\mu} \\
R_{34} &= s_2 + \frac{M Z^2 c_2 s_W^2 (2c_2 c_3 s_2 s_3 ((g)^2 + 2\lambda_T^2) + \sqrt{2}g (c_2^2 - s_2^2)(c_3^2 - s_3^2) \lambda_T)}{(g')^2(m_2^- - m_2^+)^\mu} \\
R_{35} &= -\frac{M Z c_W}{g \mu} \left[ g s_2 s_3 + \sqrt{2}c_2 c_3 \lambda_T \right] \\
R_{36} &= -\frac{M Z c_W}{g \mu} \left[ g s_2 c_3 - \sqrt{2}c_2 s_3 \lambda_T \right]
\end{align*}
\]
\[ R_{41} = \frac{c_{wsw} M_Z^2}{g'(m_1^+ - m_2^-)(m_2^- - m_1^-)\mu} \left[ c_1(g')(m_1^- - m_1^+)s_1(2c_2c_\beta g s_\beta + \sqrt{2}s_2(-c_\beta^2 + s_\beta^2)\lambda_T) 
+ (c_1^2 m_1^- + s_1^2 m_1^+ - m_2^-)\lambda_S(\sqrt{2}c_2 g(c_\beta - s_\beta)(c_\beta + s_\beta) + 4c_\beta s_2 s_\beta \lambda_T) \right] \]

\[ R_{42} = \frac{c_{wsw} M_Z^2}{g'(m_1^+ - m_2^-)(m_2^- - m_1^-)\mu} \left[ c_1(m_1^- - m_1^+)s_1\lambda_S(\sqrt{2}c_2 g(c_\beta - s_\beta)(c_\beta + s_\beta) + 4c_\beta s_2 s_\beta \lambda_T) 
+ (g'(c_1^2 m_1^+ + s_1^2 m_1^- - m_2^-)(2c_2 c_\beta g s_\beta + \sqrt{2}s_2(-c_\beta^2 + s_\beta^2)\lambda_T) \right] \]

\[ R_{43} = -s_2 - \frac{M Z^2 c_2 c_w (2c_2 c_\beta s_2 s_\beta)((g)^2 + 2\alpha_T^2) + \sqrt{2}g(c_2^2 - s_2^2)(c_\beta^2 - s_\beta^2)\lambda_T)}{(g)^2(m_2^- - m_1^+\mu)} \]

\[ R_{44} = c_2 - \frac{M Z^2 c_2 c_w (2c_2 c_\beta s_2 s_\beta)((g)^2 + 2\alpha_T^2) + \sqrt{2}g(c_2^2 - s_2^2)(c_\beta^2 - s_\beta^2)\lambda_T)}{(g)^2(m_2^- - m_1^+\mu)} \]

\[ R_{45} = -\frac{M Z c_w}{g'\mu} \left[ g_c s_\beta - \sqrt{2}s_2 c_\beta \lambda_T \right] \]

\[ R_{46} = \frac{M Z c_w}{g'\mu} \left[ g_c c_\beta + \sqrt{2}s_2 s_\beta \lambda_T \right] \]

\[ (A.9) \]

\[ R_{51} = -\frac{M Z s w \lambda_S(c_\beta - s_\beta)}{g'\mu} \]

\[ R_{52} = -\frac{M Z s w (c_\beta + s_\beta)}{\sqrt{2}\mu} \]

\[ R_{53} = \frac{M Z c_w \lambda_T(c_\beta - s_\beta)}{g\mu} \]

\[ R_{54} = \frac{M Z c_w (c_\beta + s_\beta)}{\sqrt{2}\mu} \]

\[ R_{55} = 1/\sqrt{2} \]

\[ R_{56} = -1/\sqrt{2} \]

\[ (A.10) \]
\begin{align*}
R_{61} &= - \frac{M_Z s_W \lambda_S (c_\beta + s_\beta)}{g' \mu} \\
R_{62} &= \frac{M_Z s_W (c_\beta - s_\beta)}{\sqrt{2} \mu} \\
R_{63} &= \frac{M_Z c_W \lambda_T (c_\beta + s_\beta)}{g' \mu} \\
R_{64} &= - \frac{M_Z c_W (c_\beta - s_\beta)}{\sqrt{2} \mu} \\
R_{65} &= 1/\sqrt{2} \\
R_{66} &= 1/\sqrt{2}
\end{align*}
(A.11)

The second order eigenvalues for the above states are given by

\begin{align*}
m_{\psi_1} &= m_1^+ - \frac{2 M_Z^2 s_W^2 \left[ c_\beta s_\beta ((g')^2 s_1^2 - 2 c_1^2 \lambda_S^2) + \sqrt{2} c_1 (g') s_1 (c_\beta - s_\beta) \lambda_S \right]}{(g')^2 \mu} \\
m_{\psi_2} &= m_1^- - \frac{2 M_Z^2 s_W^2 \left[ c_\beta s_\beta ((g')^2 s_1^2 - 2 c_1^2 \lambda_S^2) - \sqrt{2} c_1 (g') s_1 (c_\beta - s_\beta) \lambda_S \right]}{(g')^2 \mu} \\
m_{\psi_3} &= m_2^+ - \frac{2 M_Z^2 c_W^2 \left[ c_\beta s_\beta (c_2^2 g^2 - 2 s_2^2 \lambda_T^2) + \sqrt{2} c_2 g s_2 (c_\beta - s_\beta) \lambda_T \right]}{g \mu} \\
m_{\psi_4} &= m_2^- - \frac{2 M_Z^2 c_W^2 \left[ c_\beta s_\beta (c_2^2 g^2 - 2 s_2^2 \lambda_T^2) - \sqrt{2} c_2 g s_2 (c_\beta - s_\beta) \lambda_T \right]}{g \mu} \\
m_{\psi_5} &= \mu + \frac{2 M_Z^2 s_W^2 \left[ (g')^2 + 2 (c_\beta - s_\beta)^2 \lambda_S^2 \right]}{4 \mu (g')^2} + \frac{2 M_Z^2 c_W^2 \left[ g^2 + 2 (c_\beta - s_\beta)^2 \lambda_T^2 \right]}{4 \mu g^2} \\
m_{\psi_6} &= - \mu - \frac{2 M_Z^2 s_W^2 \left[ (g')^2 (c_\beta - s_\beta)^2 + 2 \lambda_S^2 \right]}{4 \mu (g')^2} - \frac{2 M_Z^2 c_W^2 \left[ g^2 (c_\beta - s_\beta)^2 + 2 \lambda_T^2 \right]}{4 \mu g^2} 
\end{align*}
(A.12)
B Rotation Matrices and Eigenvalues for Mostly Higgsino Neutralinos

In this section we give the rotation matrix to first order for the case that $\mu$ is much less than the other masses.

\[
R_{11} = c_1 \\
R_{12} = s_1 \\
R_{13} = 0 \\
R_{14} = 0 \\
R_{15} = \frac{M_{ZSW}}{g'm_1^+}(\sqrt{2}\lambda_S c_1 s_\beta - g'c_\beta s_1) \\
R_{16} = \frac{M_{ZSW}}{g'm_1^+}(\sqrt{2}\lambda_S c_\beta c_1 + g's_\beta s_1) \quad (B.1)
\]

\[
R_{21} = -s_1 \\
R_{22} = c_1 \\
R_{23} = 0 \\
R_{24} = 0 \\
R_{25} = -\frac{M_{ZSW}}{g'm_1^+}(g'c_\beta c_1 + \sqrt{2}\lambda_S s_\beta s_1) \\
R_{26} = \frac{M_{ZSW}}{g'm_1^+}(g'c_1 s_\beta - \sqrt{2}\lambda_S c_\beta s_1) \quad (B.2)
\]

\[
R_{31} = 0 \\
R_{32} = 0 \\
R_{33} = c_2 \\
R_{34} = s_2 \\
R_{35} = \frac{M_{ZCW}}{g'm_2^+}(-\sqrt{2}\lambda_T c_2 s_\beta + gc_\beta s_2) \\
R_{36} = -\frac{M_{ZCW}}{g'm_2^+}(\sqrt{2}\lambda_T c_\beta c_2 + gs_\beta s_2) \quad (B.3)
\]
\[ R_{41} = 0 \]
\[ R_{42} = 0 \]
\[ R_{43} = -s_2 \]
\[ R_{44} = c_2 \]
\[ R_{45} = \frac{M_Z c_W}{g m_2}(g c_\beta c_2 + \sqrt{2} \lambda_T s_\beta s_2) \]
\[ R_{46} = -\frac{M_Z c_W}{g m_2}(-g c_2 s_\beta + \sqrt{2} \lambda_T c_\beta s_2) \]  
(B.4)

\[ R_{51} = \frac{M_Z s_W}{2 g' m_1 m_1^+} \left[ 2 \lambda_S (m_1^- c_1^2 + m_1^+ s_1^2)(c_\beta - s_\beta) + \sqrt{2}(g')(m_1^- - m_1^+) c_1 (c_\beta + s_\beta) s_1 \right] \]
\[ R_{52} = \frac{M_Z s_W}{2 g' m_1 m_1^+} \left[ \sqrt{2}(g')(c_\beta + s_\beta)(m_1^+ c_1^2 + m_1^- s_1^2) + (m_1^- - m_1^+) \lambda_S (c_\beta - s_\beta) 2c_1 s_1 \right] \]
\[ R_{53} = -\frac{M_Z c_W}{2 g' m_2 m_2^+} \left[ 2 \lambda_T (c_\beta - s_\beta)(c_2^2 m_2^- + s_2^2 m_2^+) + \sqrt{2}(m_2^- - m_2^+)(c_\beta + s_\beta) c_2 s_2 \right] \]
\[ R_{54} = -\frac{M_Z c_W}{2 g' m_2 m_2^+} \left[ \sqrt{2}(s_2^2 m_2^- + c_2^2 m_2^+)(c_\beta + s_\beta) + 2 \lambda_T (m_2^- - m_2^+)(c_\beta - s_\beta) c_2 s_2 \right] \]
\[ R_{55} = \frac{1}{\sqrt{2}} + R_H \]
\[ R_{56} = -\frac{1}{\sqrt{2}} + R_H \]  
(B.5)

\[ R_{61} = \frac{M_Z s_W}{2 g' m_1 m_1^+} \left[ -2 (m_1^- c_1^2 + m_1^+ s_1^2) \lambda_S (c_\beta + s_\beta) + \sqrt{2}(g')(m_1^- - m_1^+) c_1 (c_\beta - s_\beta) s_1 \right] \]
\[ R_{62} = \frac{M_Z s_W}{2 g' m_1 m_1^+} \left[ -2 (m_1^- - m_1^+) \lambda_S c_1 (c_\beta + s_\beta) s_1 + \sqrt{2}(g')(c_\beta - s_\beta)(m_1^+ c_1^2 + m_1^- s_1^2) \right] \]
\[ R_{63} = \frac{M_Z c_W}{2 g' m_2 m_2^+} \left[ 2(c_2^2 m_2^- + s_2^2 m_2^+)(c_\beta + s_\beta) - \sqrt{2}(m_2^- - m_2^+)(c_\beta - s_\beta) 2c_2 s_2 \right] \]
\[ R_{64} = \frac{M_Z c_W}{2 g' m_2 m_2^+} \left[ -2 \sqrt{2}(s_2^2 m_2^- + c_2^2 m_2^+)(c_\beta - s_\beta) + 2(m_2^- - m_2^+)(c_\beta + s_\beta) 2c_2 s_2 \right] \]
\[ R_{65} = \frac{1}{\sqrt{2}} - R_H \]
\[ R_{66} = \frac{1}{\sqrt{2}} - R_H \]  
(B.6)
where

\[
R_H = \frac{M_Z^2}{8\sqrt{2}g^2(g')^2m_1^+m_2^-m_2^+\mu} \times \left[ e_{2\beta} \left\{ (g')^2m_1^-m_1^+(-m_2^+ + m_2^-)(g^2 - 2\lambda_T^2) + (m_2^- - m_2^+)(g^2 + 2\lambda_T^2)(c_1^2 - s_1^2)c_W^2 \right. \\
+ g^2m_2^-m_2^+(-m_1^- + m_1^+)((g')^2 - 2\lambda_S^2) + (m_1^- - m_1^+)((g')^2 + 2\lambda_S^2)(c_1^2 - s_1^2)s_W^2 \right\} \\
+ s_{2\beta}g(g')2\sqrt{2} \left\{ (g')m_1^-m_1^+(m_2^- - m_2^+)\lambda_Tc_W^22c_2s_2 + g(m_1^- - m_1^+)m_2^-m_2^+\lambda_S2c_1s_1s_W^2 \right\} \right]
\]

The non-LSP eigenvalues are

\[
m_{\psi_1} = m_1^+ + \frac{2M_Z^2s_W^2\left[ 2\lambda_S^2c_1^2 + (g')^2s_1^2 \right]}{(g')^2\mu} \\
m_{\psi_2} = m_1^- + \frac{2M_Z^2s_W^2\left[ c_1^2(g')^2 + 2s_1^2\lambda_S^2 \right]}{(g')^2\mu} \\
m_{\psi_3} = m_2^+ + \frac{2M_Z^2c_W^2\left[ s_2^2g^2 + 2c_2^2\lambda_T^2 \right]}{g\mu} \\
m_{\psi_4} = m_2^- + \frac{2M_Z^2c_W^2\left[ c_2^2g^2 + 2s_2^2\lambda_T^2 \right]}{g\mu}
\]
while the LSP and NLSP are given by

\[ m_{\psi_5} = \mu - \frac{M_Z^2 c_W^2 (\sqrt{2} g c_2 (c_\beta + s_\beta) + 2 \lambda_T (-c_\beta + s_\beta) s_2)^2}{4 g^2 m_2^2} \]
\[ - \frac{M_Z^2 c_W^2 (2 \lambda_T c_2 (c_\beta - s_\beta) + \sqrt{2} g (c_\beta + s_\beta) s_2)^2}{4 g^2 m_2^2} \]
\[ - \frac{M_Z^2 s_W^2 (\sqrt{2} (g') c_1 (c_\beta + s_\beta) + 2 \lambda_S (-c_\beta + s_\beta) s_1)^2}{4 (g')^2 m_1^-} \]
\[ - \frac{M_Z^2 s_W^2 (2 \lambda_S c_1 (c_\beta - s_\beta) + \sqrt{2} (g') (c_\beta + s_\beta) s_1)^2}{4 (g')^2 m_1^+} \] (B.9)

\[ m_{\psi_6} = -\mu - \frac{M_Z^2 c_W^2 (\sqrt{2} g c_2 (c_\beta - s_\beta) + 2 \lambda_T (c_\beta + s_\beta) s_2)^2}{4 g^2 m_2^2} \]
\[ - \frac{M_Z^2 c_W^2 (-2 \lambda_T c_2 (c_\beta + s_\beta) + \sqrt{2} g (c_\beta - s_\beta) s_2)^2}{4 g^2 m_2^+} \]
\[ - \frac{M_Z^2 s_W^2 (\sqrt{2} (g') c_1 (c_\beta - s_\beta) + 2 \lambda_S (c_\beta + s_\beta) s_1)^2}{4 (g')^2 m_1^-} \]
\[ - \frac{M_Z^2 s_W^2 (2 \lambda_S c_1 (c_\beta + s_\beta) + \sqrt{2} (g') (-c_\beta + s_\beta) s_1)^2}{4 (g')^2 m_1^+} \] (C.1)

**C Couplings of the LSP**

The neutralino-chargino-W interactions depend on the $\tilde{W}, \tilde{W}', \tilde{h}$ components of the neutralino,

\[ \mathcal{L} = \overline{\tilde{\chi}}^a \gamma^\mu (C_L^{ai}(1 - \gamma_5) + C_R^{ai}(1 + \gamma_5)) \tilde{\chi}^i \tilde{W}_\mu^- + h.c. \] (C.2)

where

\[ C_L^{ai} = \frac{e}{4 s_W} \left( 2 N_{i4} U_{a2} + 2 N_{i3} U_{a1} + \sqrt{2} N_{i5} U_{a3} \right) \]
\[ C_R^{ai} = \frac{e}{4 s_W} \left( 2 N_{i4} V_{a2} + 2 N_{i3} V_{a1} - \sqrt{2} N_{i6} V_{a3} \right) \] (C.3)

When only gaugino Dirac mass are presents and when $m_{1D}, m_{2D} \ll \mu$ the dominant contribution comes from the $N_{i3} U_{11}$ term.

The Majorana neutralino coupling to the Z is driven by the higgsino component, as in the MSSM,

\[ \mathcal{L} = \frac{1}{2} \overline{\tilde{\chi}}^i \gamma^\mu \gamma_5 C_Z^{ij} \tilde{\chi}^i \tilde{Z}_\mu + h.c. \] (C.3)
\[ C^{ii}_Z = \frac{g}{2c_W} \left( |N_{i6}|^2 - |N_{i5}|^2 \right) \]  

(C.4)

The neutralino couplings to the light Higgs also depend on the \( \tilde{W}, \tilde{W}', \tilde{h} \) components of the neutralino

\[ \mathcal{L} = \frac{1}{2} \overline{\chi}^0_i C^{ij}_h \chi^0_j h + h.c. \]  

(C.5)

where

\[ C^{ii}_h = \frac{-1}{s_W c_W} \left[ e \left( c_W N_{i4} - s_W N_{i2} \right) \left( Z_{h11} N_{i5} - Z_{h12} N_{i6} \right) \right. \]

\[ + \sqrt{2} s_W c_W \left( N_{i1} \lambda_S - N_{i3} \lambda_T \right) \left( Z_{h12} N_{i5} + Z_{h11} N_{i6} \right) \]  

(C.6)

Here \( Z_h \) is the scalar mixing matrix.

**References**

[1] P. Fayet, Phys. Lett. B 78 (1978) 417.

[2] J. Polchinski and L. Susskind, Phys. Rev. D 26, 3661 (1982).

[3] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, Front. Phys. 58 (1983) 1 [arXiv:hep-th/0108200].

[4] I. Antoniadis, C. Munoz and M. Quiros, Nucl. Phys. B 397 (1993) 515 [arXiv:hep-ph/9211309].

[5] A. Pomarol and M. Quiros, Phys. Lett. B 438, 255 (1998) [arXiv:hep-ph/9806263].

[6] I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, Nucl. Phys. B 544, 503 (1999) [arXiv:hep-ph/9810410].

[7] P. J. Fox, A. E. Nelson and N. Weiner, JHEP 0208, 035 (2002) [arXiv:hep-ph/0206096].

[8] A. E. Nelson, N. Rius, V. Sanz and M. Unsal, JHEP 0208 (2002) 039 [arXiv:hep-ph/0206102].

[9] Z. Chacko, P. J. Fox and H. Murayama, Nucl. Phys. B 706, 53 (2005) [arXiv:hep-ph/0406142].

[10] L. M. Carpenter, P. J. Fox and D. E. Kaplan, [arXiv:hep-ph/0503093].

[11] Y. Nomura, D. Poland and B. Tweedie, Nucl. Phys. B 745 (2006) 29 [arXiv:hep-ph/0509243].
[12] I. Antoniadis, A. Delgado, K. Benakli, M. Quiros and M. Tuckmantel, Phys. Lett. B 634, 302 (2006) [arXiv:hep-ph/0507192].

[13] I. Antoniadis, K. Benakli, A. Delgado, M. Quiros and M. Tuckmantel, Nucl. Phys. B 744, 156 (2006) [arXiv:hep-th/0601003].

[14] I. Antoniadis, K. Benakli, A. Delgado and M. Quiros, Adv. Stud. Theor. Phys. 2, 645 (2008) [arXiv:hep-ph/0610265].

[15] L. J. Hall and L. Randall, Nucl. Phys. B 352, 289 (1991).

[16] G. D. Kribs, E. Poppitz and N. Weiner, Phys. Rev. D 78, 055010 (2008) [arXiv:0712.2039 [hep-ph]].

[17] S. D. L. Amigo, A. E. Blechman, P. J. Fox and E. Poppitz, JHEP 0901, 018 (2009) [arXiv:0809.1112 [hep-ph]].

[18] D. Marques, JHEP 0903 (2009) 038 [arXiv:0901.1326 [hep-ph]].

[19] A. E. Blechman, arXiv:0903.2822 [hep-ph].

[20] K. A. Intriligator, N. Seiberg and D. Shih, JHEP 0604 (2006) 021 [arXiv:hep-th/0602239].

[21] B. K. Zur, L. Mazzucato and Y. Oz, JHEP 0810 (2008) 099 [arXiv:0807.4543 [hep-ph]].

[22] K. Benakli and M. D. Goodsell, arXiv:0811.4409 [hep-ph].

[23] P. Meade, N. Seiberg and D. Shih, arXiv:0801.3278 [hep-ph].

[24] L. M. Carpenter, M. Dine, G. Festuccia and J. D. Mason, Phys. Rev. D 79, 035002 (2009) [arXiv:0805.2944 [hep-ph]].

[25] H. Ooguri, Y. Ookouchi, C. S. Park and J. Song, Nucl. Phys. B 808, 121 (2009) [arXiv:0806.4733 [hep-th]].

[26] J. Distler and D. Robbins, arXiv:0807.2006 [hep-ph].

[27] K. A. Intriligator and M. Sudano, JHEP 0811, 008 (2008) [arXiv:0807.3942 [hep-ph]].

[28] M. Buican, P. Meade, N. Seiberg and D. Shih, JHEP 0903 (2009) 016 [arXiv:0812.3668 [hep-ph]].

[29] K. Benakli and C. Moura in M. M. Nojiri et al., arXiv:0802.3672 [hep-ph].
[30] S. Y. Choi, M. Drees, A. Freitas and P. M. Zerwas, Phys. Rev. D 78, 095007 (2008) [arXiv:0808.2410 [hep-ph]].

[31] S. Y. Choi, Eur. Phys. J. C 59 (2009) 543 [arXiv:0809.1707 [hep-ph]].

[32] S. Y. Choi, M. Drees, J. Kalinowski, J. M. Kim, E. Popenda and P. M. Zerwas, Phys. Lett. B 672 (2009) 246 [arXiv:0812.3586 [hep-ph]].

[33] M. Kramer, E. Popenda, M. Spira and P. M. Zerwas, [arXiv:0902.3795 [hep-ph]].

[34] M. M. Nojiri and M. Takeuchi, Phys. Rev. D 76, 015009 (2007) [arXiv:hep-ph/0701190].

[35] T. Plehn and T. M. P. Tait, J. Phys. G 36 (2009) 075001 [arXiv:0810.3919 [hep-ph]].

[36] K. Hsieh, Phys. Rev. D 77 (2008) 015004 [arXiv:0708.3970 [hep-ph]].

[37] J. R. Espinosa and M. Quiros, Nucl. Phys. B 384, 113 (1992).

[38] J. R. Espinosa and M. Quiros, Phys. Lett. B 279, 92 (1992).

[39] O. Felix-Beltran, “Higgs masses and coupling within an extension of the MSSM with Higgs Int. J. Mod. Phys. A 17, 465 (2002).

[40] E. Barradas-Guevara, O. Felix-Beltran and A. Rosado, Phys. Rev. D 71 (2005) 073004 [arXiv:hep-ph/0408196].

[41] J. L. Diaz-Cruz, J. Hernandez-Sanchez, S. Moretti and A. Rosado, Phys. Rev. D 77, 035007 (2008) [arXiv:0710.4169 [hep-ph]].

[42] S. Di Chiara and K. Hsieh, Phys. Rev. D 78 (2008) 055016 [arXiv:0805.2623 [hep-ph]].

[43] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667 (2008) 1.

[44] H. E. Haber and R. Hempfling, Phys. Rev. D 48, 4280 (1993) [arXiv:hep-ph/9307201].

[45] F. Boudjema and A. Semenov, Phys. Rev. D 66, 095007 (2002) [arXiv:hep-ph/0201219].

[46] S. Chang, P. J. Fox and N. Weiner, JHEP 0608, 068 (2006) [arXiv:hep-ph/0511250].

[47] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 180 (2009) 330 [arXiv:0803.0547 [astro-ph]].

[48] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, Comput. Phys. Commun. 176 (2007) 367 [arXiv:hep-ph/0607059].
[49] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, Comput. Phys. Commun. 180 (2009) 747 [arXiv:0803.2360 [hep-ph]].

[50] O. Adriani et al. [PAMELA Collaboration], GeV," Nature 458 (2009) 607 [arXiv:0810.4995 [astro-ph]].

[51] R. Harnik and G. D. Kribs, arXiv:0810.5557 [hep-ph].

[52] O. Adriani et al., Phys. Rev. Lett. 102 (2009) 051101 [arXiv:0810.4994 [astro-ph]].

[53] A. A. Abdo et al. [The Fermi LAT Collaboration], Phys. Rev. Lett. 102 (2009) 181101 [arXiv:0905.0025 [astro-ph.HE]].

[54] Z. Ahmed et al. [CDMS Collaboration], Five-Tower Phys. Rev. Lett. 102 (2009) 011301 [arXiv:0802.3530 [astro-ph]].

[55] J. Angle et al., Phys. Rev. Lett. 101 (2008) 091301 [arXiv:0805.2939 [astro-ph]].

[56] E. Aprile, L. Baudis and f. t. X. Collaboration, arXiv:0902.4253 [astro-ph.IM].