Ultrashort Laser-pulse Diagnostics for Detection of Ordering within an Ion beam.

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Abstract

A novel diagnostic method to detect ordering within one-dimensional ion beams in a storage ring is presented. The ions are simultaneously excited by a ultrashort pulsed laser (≃ 1 ps) at two different locations along the beam and fluorescence is detected by a group of four photo-multipliers. Correlation in fluorescence signals is a firm indication that the ion beam has an ordered structure.

32.80.-t, 41.75.-i, 29.20
I. INTRODUCTION

The possibility to observe ordered structures within an ion beam has met the interest of the community of accelerator people [1]. Afterwards an even bigger effort has been put in researching on this new topic [2]. Molecular dynamics approaches [3-5] and theoretical investigations [6] have studied in some detail what one could achieve in a storage ring. A feasibility study for a storage ring dedicated to crystallization has been recently carried out [7]. As far as present knowledge is concerned no direct evidence of an ordered structure of an ion beam in a storage ring has been recorded. Some experiments [8,9] showed that, under particular circumstances, suppression of intra-beam scattering occurred; therefore the seek for a direct and unambiguous way to detect ordering within an ion beam is a crucial question.

Experiments carried out in ion traps have shown that a record of crystallization is achieved by observing the fluorescence signal of ions through a CCD camera [10]. Unfortunately this technique cannot be applied straight away to accelerators since ions are traveling at relatively high velocity. A novel method to detect ordering of a one-dimensional ion beam by resolving the single ion structure has been recently proposed [11]. The technique is based on pulsed-laser excitation of the ions in the beam. In this paper we analyze the electromagnetic-radiation–ion interaction in order to demonstrate the feasibility and the non-invasivity of the method. A specifically devoted Monte Carlo simulation will show the results of a possible application of the method to two of the existing storage rings.

II. THE DIAGNOSTIC DEVICE

We consider a pulsed laser with frequency $\nu = (E_2 - E_1)/h = \nu_{ion}$, where $\nu_{ion}$ is the frequency of an allowed transition between the energy levels of an ion in the beam. The laser beam is split into two parts, which simultaneously cross the ion beam at right angles at two nearby positions along the storage ring (see Fig.1). This laser-to-ion crossing area is followed by four photo-multipliers which detect the photons emitted by the ions that have previously been excited by the laser beams. The signals recorded by the photo-multipliers are analyzed when one laser beam is moved with respect to the other one. In the absence of ordering, no correlation in fluorescence signal should be recorded while changing the relative distance of the two laser beams. On the contrary, if a string were obtained as a result of cooling then a strong correlation between the signals should be observed. Suppose that one of the four photo-multipliers detects the fluorescence of an ion excited by one of the laser beams. If one of the other three photo-multipliers detects a simultaneous fluorescence signal, it means that the other laser has interacted with another ion in the string, in turn indicating that the distance between the two ion-to-laser crossing points is an integer multiple of the inter-particle spacing of the string. Then, by slightly moving the second laser beam, the correlation signal should vanish. A sort of periodical dependence on the distance between the laser beams should appear. The above considerations were related to a perfect string of ions. At non–zero temperature ions oscillate along both longitudinal and transversal directions. Choosing the distance between the laser beams of the order of interparticle spacing the effect of longitudinal oscillations is negligible as the distance between two nearest-neighbor particles can be regarded to be relatively uniform for a plasma parameter of the order of
100 [11]. If the laser beams are focused on the ion string through cylindrical lens also the
effect of transverse oscillations becomes negligible. The particles in the ion beam travel with
velocity \( v = \beta c \); if \( s \) denotes the interparticle distance then the time taken by an ion in
the beam to travel this distance is \( T = s/v \). Typical interparticle distance for a string in
a storage ring is \( s = 10 - 100 \, \mu m \); considering a velocity range useful for existing storage
rings, i.e. \( \beta = 0.003 - 0.05 \), \( T \) lies from 1 to 50 ps. The diagnostic method is aimed at
revealing the single-particle ion structure using laser pulses which are shorter than the time
\( T \). In this way the ions can be regarded to be at rest and all the problems connected with
analyzing a traveling structure are overcome. Thus, the laser pulse duration \( \Delta t \) must be of
the order of one picosecond or fraction of ps.

An electromagnetic radiation with this very short temporal duration is far from being
monoenergetic and this has to be taken into account in order to show the feasibility of the
method.

**III. LASER PULSE – ION BEAM INTERACTION**

The electric field of the laser is given by the following expression [12]

\[
E(t) = e(\mathbf{r})\mathcal{E}(t)\{a_L e^{-i\omega_0 t} + a_L^* e^{i\omega_0 t}\}
\]  

(1)

where the gaussian shape for \( \mathcal{E}(t) \)

\[
\mathcal{E}(t) = \frac{E}{2} e^{-\frac{(t-t_0)^2}{2\sigma^2}}
\]

(2)

is assumed to account for the finite duration of the laser pulse.

Considering the time duration and the frequency bandwidth of the intensities at FWHM

\[
\Delta t/2 = \sigma \sqrt{\ln 2}; \quad \Delta \omega/2 = \sqrt{\ln 2}/\sigma
\]

(3)

the time-bandwidth product gives

\[
\Delta t \cdot \Delta \nu = \Delta t \cdot \frac{\Delta \omega}{2\pi} = .441
\]

(4)

This means that a laser pulse of the order of one ps has a frequency bandwidth of hundreds
of GHz, i.e., is far from being monochromatic. This bandwidth is larger than hyperfine and
fine splittings of the ions that can be easily laser cooled, i.e. best candidates for reaching
crystallization. In the following we shall neglect the details of subtransitions, therefore laser-
ion interaction will be described as a two-level quantum mechanical system with an external
time-dependent perturbation [14](|g⟩, |e⟩ being ground and excited states).

The Hamiltonian of the system is:

\[
H = H_0 + V_I
\]

(5)

where the external potential is
\[ V_I = -\mathbf{d} \cdot \mathbf{E} = -d_{eg} e_z \cdot \mathbf{E}(t) \] (6)

and \( \mathbf{d} = d_{eg} \mathbf{e}_z \) is the dipole moment for the transition under consideration.

Assuming the atomic system to be initially in the ground state, one can calculate the excitation probability \( P_2(t) = |c_2(t)|^2 \) solving the following differential equation for \( c_2(t) \)

\[ \ddot{c}_2(t) + \left[ i\Delta \omega + \frac{(t-t_0)}{\sigma^2} \right] \dot{c}_2(t) + \Omega^2(t)c_2(t) = 0 \] (7)

with initial conditions

\[
\begin{align*}
    c_2(0) &= 0 \\
    \dot{c}_2(0) &= -i\Omega(0)
\end{align*}
\] (8)

where

\[ \Omega(t) = \frac{Rd_{eg}E}{2\hbar} e^{-\frac{(t-t_0)^2}{2\sigma^2}} \Delta \omega = \omega + \omega_{12}. \] (9)

and \( \omega_{21} = -\omega_{12} = (E_2 - E_1)/\hbar, R = \mathbf{e}_z \cdot \mathbf{E}(r) \).

Let us seek for an analytical estimation of the probability of excitation due to the laser pulse \( (P_2(t) \rightarrow +\infty) \) by solving Eq. (7) heuristically, i.e. neglecting the dissipation term and averaging out \( \Omega(t) \). In order to have a non negligible excitation probability the time integral of \( \Omega \) must be of the order of unity:

\[ \int \Omega(t) dt = \frac{Rd_{eg}E}{2\hbar} \sqrt{2\pi\sigma} \simeq 1. \] (10)

Taking into account that the ions are randomly oriented (the average \( \bar{R^2} \) over the solid angle gives a factor 1/3), using Eq. (3) and the relation between the dipole momentum and the spontaneous lifetime \( (\tau) \) of the ion, we have a rough estimate of the minimum value for the peak intensity of the laser to cause excitation with appreciable probability:

\[ I \simeq \frac{2\ln(2)\hbar\omega_0^3 \tau}{\pi^2 c^2 \Delta t^2} \] (11)

Introducing the saturation intensity for the transition considered

\[ I_s = \frac{\hbar\omega_0^3}{4\pi c^2 \tau} \] (12)

we obtain an approximate rule for the requested laser intensity

\[ \frac{I}{I_s} \geq k \left( \frac{\tau}{\Delta t} \right)^2 \] (13)

where \( k \) is a constant of the order of the unity.

In order to test the validity of Eq. (13), one can numerically solve the differential equation (7). Fig. 2 shows a contourplot of the excitation probability as a function of the nondimensional parameters \( \log(I/I_s) \) and \( \log(\Delta t/\tau) \). The bold line corresponds to the approximate rule of Eq. (13) with \( k = 1 \); for that value the excitation probability is close to 1/2. One can
see that the excitation probability is strongly influenced by the duration and the intensity of the laser pulse. In order to show that the excitation probability is not negligible in practical cases, we shall refer to two presently being laser-cooled ions in their respective storage rings, i.e. \( ^{24}\text{Mg}^+ \) at ASTRID (Aarhus) and \( ^{9}\text{Be}^+ \) at TSR (Heidelberg). Considering a focusing spot of \( 5 \times 200 \mu m^2 \) and a pulsed laser with some nJ/pulse with 10% jitter in intensity and pulse duration, we obtain two allowed areas for the probability of exciting ions to the upper level. Due to the focusing of the laser beam the perturbation strength depends on the transverse coordinates of the ion interacting with the electromagnetic radiation. This is the major contribution to the uncertainty over \( \log_{10}(I/I_s) \). The excitation probability averages \( P = 0.58 \) for \( ^{9}\text{Be}^+ \) and \( P = 0.51 \) for \( ^{24}\text{Mg}^+ \), i.e. the diagnostic scheme can work for either ions in their respective storage rings.

IV. MONTE CARLO SIMULATION

After having investigated about the possibility of exciting ions to the upper level by pulsed lasers with sufficiently high probability, we shall demonstrate the feasibility of the method. The rate of coincidence events detected by the group of photomultipliers depends on several parameters such as the geometry of the system, quantum efficiency of photomultipliers, filtering of fluorescence photons and the dynamical behavior of the system due to nonzero temperature of the ions in the beam \([11]\). We have developed a Monte Carlo to account for all these effects and to simulate the response of the diagnostic device when is being applied to existing storage rings. The Monte Carlo is limited to the cases of \( ^{9}\text{Be}^+ \) at TSR (Heidelberg) and of \( ^{24}\text{Mg}^+ \) at ASTRID (Aarhus); some important parameters of these rings are summarized in Table I. Also included in the table are the main parameters of a laser suited to diagnostics in each of these rings.

\[ ^{24}\text{Mg}^+ \text{ at ASTRID} \]

Typical velocity of \( ^{24}\text{Mg}^+ \) at ASTRID is about \( \beta = 0.003 \); this corresponds to a time of about 33 ps taken by an ion in the beam to travel the interparticle spacing. A 2-ps pulse duration of the laser was considered for in the simulation. Figure 3 illustrates the fluorescence response for a string of \( ^{24}\text{Mg}^+ \) ions at \( \Gamma = 100 \) with \( s = 50 \mu m \) as a function of the distance between the two laser beams. Appearance of fluorescence peaks is visible in the figure when the distance between the laser beams is an integer multiple of the string spacing. Also the pattern for a disordered beam with the same density as the string is shown; in this case a totally uncorrelated pattern is achieved and the fluorescence signal is related only to stray counting of the photomultipliers. It is remarkable that a clear firm of ordering in the ion beam can be achieved with only 1 s acquisition time, i.e. much less than the beam lifetime in the storage ring.

The same simulation was carried out with the same laser and beam parameters as for Fig. 3 but with a longer acquisition time (10 s). Correlation peaks have became more neat due to a longer interval of counting (Fig. 4).
The simulation was carried out also for the case of $^9\text{Be}^+$ ion beam at TSR (Heidelberg) at $\Gamma = 100$. Ion velocity is higher for this storage ring ($\beta = 0.05$) and therefore one should resort to still shorter pulse duration of the laser. A commercially available 200-fs pulsed laser with some nJ/pulse was considered in the Monte Carlo, the results of which are shown in Fig. 6. Correlation peaks are shorter, with respect to the case of $^{24}\text{Mg}^+$, due to the lower quantum efficiency of photomultipliers.

V. PERTURBATION TO THE ION BEAM

The proposed method of fluorescence detection makes use of absorption of a photon of the laser and its subsequent re-emission. As for any kind of diagnostics, this technique perturbs the system under consideration; here we provide an estimation of the heating rate of the ion beam owing to its interaction with the laser.

Suppose the ion suffering interaction with the light to be at rest in a given frame; when a photon is absorbed the ion receives the impulse of the photon and moves in the direction of the laser beam. Imposing impulse and energy conservations, in the ever valid approximation $\Delta E \ll mc^2$, where $\Delta E$ is the transition energy of the ion and $m$ is the ion’s mass, the increase in ion velocity holds:

$$v = \frac{\Delta E}{mc}.$$  \hspace{1cm} (14)

Then the ion emits a photon isotropically. Averaging over several absorption-emission processes, an ion gains a mean transverse velocity according to Eq. (14). Thus, each interaction with the laser enhances the transverse kinetic energy by:

$$E_K = \frac{mv^2}{2} = \frac{\Delta E^2}{2mc^2}.$$  \hspace{1cm} (15)

This energy is usually referred to as recoil energy.

The energy increase per second of the ion string is $E_K$ times the rate of excitations, $f$, to the upper level for each laser beam. Under the assumption that this energy will be shared between the other particles through intra-beam collisions, the rate of temperature increase for the ion beam is:

$$\frac{dT}{dt} = \frac{2E_Kf}{Nk_B}.$$  \hspace{1cm} (16)

For the case of $N = 8 \cdot 10^5$ $^{24}\text{Mg}^+(E_K/k_B = 2.6mK)$ ions circulating in the ASTRID storage ring and with the above parameters for the diagnostics ($f = 2.5 \cdot 10^6$ Hz), it comes out a rate:

$$\frac{dT}{dt} = 1.6 \cdot 10^{-5}K/s.$$  \hspace{1cm} (17)

From the above calculation it emerges that the perturbation to the ion beam due to the diagnostic device is negligible and should not affect the ordering within the ion beam. In
the calculation was assumed that the energy pumped by the laser to the ion beam was perfectly shared by all the ions. In reality, intrabeam scattering of highly cooled beams is expected to be considerably damped and areas with a local temperature higher than the average one should appear. However, the amount of energy given by the laser to a single ion through excitation is so small that these local temperatures should not affect significantly the ordering in the beam.

VI. CONCLUSIONS

It has been shown that the method is useful as a diagnostic tool to detect ordering in one-dimensional systems. The method enables to resolve the single-ion structure of the beam. Finally we point out that all the components of the system are commercially available equipements and that the diagnostics could be easily implemented in one of the existing storage rings.
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|                 | TSR       | ASTRID   |
|----------------|-----------|----------|
| Storage ring   |           |          |
| $\beta$       | 0.05      | 0.003    |
| ion species    | $^9Be^+$  | $^{24}Mg^+$ |
| wavelength     | 313 nm    | 280 nm   |
| upper state lifetime | 8.7 ns | 3.5 ns |
| lower state lifetime | groundstate | groundstate |
| laser repetition rate | 50 MHz | 50 MHz |
| pulse duration  | 200 fs    | 2 ps     |
| bunch energy    | 3 nJ      | 3 nJ     |
| excitation probability | 0.58  | 0.51     |

**TABLE I.** Important parameters for the simulation
FIGURES

FIG. 1. a) Layout of the experimental set-up. b) A sketch of the ion-laser interaction region.

FIG. 2. Contourplot of the excitation probability $P(t \to \infty)$ as a function of $\log_{10}[I/I_s]$ and $\log_{10}[\Delta t/\tau]$. The contourlines correspond to a probability of 0.05, 0.25, 0.5, 0.75, 0.95, respectively. The bold line corresponds to the approximate rule of Eq.13. The two boxes represent the allowed "areas" for $^{24}Mg^+$ at ASTRID (left) and for $^9Be^+$ at TSR (right).

FIG. 3. Pattern of fluorescence for $^{24}Mg^+$ ion beam at $\Gamma = 100$ for 1 s acquisition time.

FIG. 4. Pattern of fluorescence for $^{24}Mg^+$ ion beam at $\Gamma = 100$ for 10 s acquisition time.

FIG. 5. Pattern of fluorescence for $^9Be^+$ ion beam at $\Gamma = 100$ for 1 s acquisition time.
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