Influence of Heat Source Characteristics on Dimensionless Thermal Spreading Resistance

A L Wang and C P Yan

School of Astronautics, Beihang University, 37 Xueyuan Road, Haidian District, Beijing, 100191, China

E-mail: wanganliang@buaa.edu.cn

Abstract. Thermal spreading resistance (TSR) for eccentric circular, square and equilateral triangular heat sources on circular heat flux tubes are conducted in this paper. It is shown that the eccentricity has a great influence on TSR. Our numerical results concerning the TSR are in good agreement with Bairi & Laraqi’s analytical data. Influence of the shapes on TSR is studied utilizing a series of isosceles triangular heat sources on circular heat flux tubes and rectangular heat sources on square heat flux tubes. The numerical results show that the TSR is strongly dependent on the ratio of inscribed circle radius to circumcircle radius of the heat source. Effect of heat source with fractal boundary on dimensionless TSR is also carried out when the heat source area of every tube is kept at a constant value. The numerical results show that TSR decreases as the construction number N increases and the variation of amplitude of TSR increases as the relative contact size \( \varepsilon \) increases.

1. Introduction

Heat is mainly transferred through the discrete contact spots between the two imperfect interfaces (figure 1)[1], which results in thermal contact resistance (TCR). TCR includes thermal constriction resistance and thermal spreading resistance (TSR). The inverse of constriction resistance is spreading resistance, and they can be depicted by one dimensionless shape factor sometimes[2]. TCR is a classical research problem in the area of heat transfer field and may play an important role in microelectronics, machinery manufacture, aerospace field and sustainable energy etc. Like many other engineering problems, the TCR research includes two aspects: engineering applications and basic theories[3]. Knowing well the mechanism of heat transfer between the complex contact surfaces is essential to predict TCR[2].

Contact spots of the interface have complex shapes and random locate in the surface as shown in figure 2[4]. TCR is a complex multi scale thermal and mechanical coupling problem. Thermo-mechanical problems of surface contact are usually modeled for engineering applications, but it is known that the physical mechanisms are the interactivity and movement of atoms (or molecules) on the contact surface. The TCR problems cover all ranges, from the atomic to the macroscopic scale. It is the common key problem to investigate the material transport, energy transfer, micro structure evolution, performance variation and active time for material science, solid mechanics and condensed state physics.
Along with the development of computer technology, people gradually approach the TCR problem in two opposite simulation models: macroscopic scale and molecular scale, also called “top-down” and “bottom-up” methods.

Figure 1. Contact heat transfer in two-solid interface\textsuperscript{[8]}.

On the macroscopic scale, in the 70s of the last century, Yovanovich’s group has made comprehensive theoretical analysis and numerical simulation for influence of heat source shape on TCR by hypothesizing single spot as circular, rectangular, elliptical and polygonal point etc based on Fourier series expansions\textsuperscript{[3]}. These results are the foundation for many TCR models of surface roughness. Muzychka et al\textsuperscript{[6]} predicted TSR of rectangular flux channels using the solution for circular flux tube very well. They assessed the importance of edge cooling after comparisons with circular flux tube with edge cooling and with that adiabatic edges. Tio&Toh\textsuperscript{[7]} and Vaidya&Razani\textsuperscript{[8]} researched the TSR of a cylinder model between two similar materials and dissimilar materials respectively under steady state situation. They indicated that the temperature distribution on the cross section of the cylinder midway between the two solids has a relationship to solid material. Rostami\textsuperscript{[9]} indicated that the gap geometry, shape of contact area and certain end surface boundary conditions strongly infect thermal constriction resistance after numerical simulation of single contact point heat transfer using CFD (Fluent version 4.3.2). Garimella’s group researched influence of ratio of heat source area to contact area, gap, cone angle, conductivity of the gap and materials, coating thickness, etc on dimensionless TSR considering radiation utilizing a frustum cone model. A non-linear fitting formula and the corresponding computational domain were expressed in their paper\textsuperscript{[10],[11],[12]}. Wang & Degiovanni\textsuperscript{[13],[14]} researched transient TSR of a cylinder model between two similar materials, the analytical results agreed well with previous experiments. Bairi&Laraqi\textsuperscript{[15]} researched TSR of an eccentric spot on a circular heat flux tube. The solutions are in good agreements with experiment results. Teertstra et al\textsuperscript{[16]} researched conduction shape factor subtly utilizing hollow cylinders with nonuniform gap spacing models. The model was validated using numerical data from the literature for a variety of geometries. Recently Chaouch et al\textsuperscript{[37]} studied thermal and electrical constriction resistance under the coupled thermal and electrical phenomena using finite element method. They indicated that finite element method can simulate thermal constriction resistance exactly and combined heat and electricity in the finite tube accentuates the temperature drop.

On the microscopic scale, the solid matter is made up of atoms, ions and molecules. New methods involving the quantum molecule dynamics (QMD), molecular dynamics (MD) and Boltzmann equation (BE) were used to research the non-Fourier effect, scale and boundary effect of thermal
conductivity, and thin film conduction \cite{18}. Touzelbaev & Goodson \cite{19} investigated the thermal resistance near diamond-substrate interfaces using phonon transport theory. They found that the resistance was governed by the number of diamond nucleation sites per unit substrate area, i.e., the nucleate density, the thickness of boundary layer. Prasher & Phelan \cite{20} developed a scattering-mediated acoustic mismatch model (SMAMM) to describe the TCR from high temperature to low temperature. The parameters were defined at a crystal structure, while the prediction results were in good agreement with the experimental data on Rh/MgO interfaces. Chen \cite{21} introduced a ballistic-diffusive equation (BDE) derived from the Boltzmann equation (BE) to deal with the thin film heat conduction problems from the nano scale to the macro scale, and the contact boundary conditions were simplified. Liao & Yang \cite{22} proposed a method coupling atomic and finite element models to simulate the TCR of dissimilar materials. The results showed that the nonuniform temperature distribution was associated with the characteristic of atoms moving along the interface.

The multi-scale thermo-mechanical problems are still great challenge to us although researchers have achieved many results and found new physical mechanism on macro scale and micro scale contact heat transfer respectively. Further research on TCR either in macro scales or micro scales should be done. In this paper, from macro perspective, we study the influence of the heat source characteristics including eccentricity, shape and fractal boundary on dimensionless TSR utilizing a series of models based on the finite element method.

2. Computational Methodology

2.1. Problem Statement

TSR arises where heat flows from a smaller to a larger area. Conduction problems may involve multiple directions and time-dependent conditions. Here, the model (figure 3) assumes a circular heat source in contact with a larger cold plate which is in turn cooled with a convective heat transfer coefficient specified over the sink surface. All edges and the region outside the heat source in the source plane are assumed to be adiabatic. This model can represent accurately numerous engineering systems.

If heat is supplied to the tube for a sufficiently long time, the system will reach a steady-state situation. In the idealized system, the total thermal resistance $R_t$ is defined as \cite{2}:

$$R_t = (\bar{T}_s - \bar{T}_b) / Q$$

(1)

Where $\bar{T}_s$ and $\bar{T}_b$ are average temperature of the heat source and the bottom of the cold plate, $Q$ is the total heat flow rate through contact.
$R_s$ is composed of two terms: a one-dimensional resistance and a spreading resistance which vanishes as the source area approaches the substrate area, that is, $A_s \rightarrow A_b$. These two components are combined as follows\(^2\):

$$R_s = R_{sD} + R_s$$  \hspace{1cm} (2)

$R_{sD}$ is the material resistance, it can be calculated by the following formula:

$$R_{sD} = \frac{t}{kA_b}$$  \hspace{1cm} (3)

where $t$ is the length of the flux tube, $k$ is the thermal conductivity of the cylinder.

Nondimensionalization of $R_s$ can be made by introducing a characteristic dimension with the unit of length $\sqrt{A_s}\,[23]$, which can be defined as:

$$R'_s = k\sqrt{A_s}R_s$$  \hspace{1cm} (4)

2.2. Numerical methods

There are three classical numerical simulation methods: finite difference method, finite element method and boundary element method. They are used in different situations and have different in computational accuracy and efficiency. The finite element technique is a good method when models have complex shapes\([24],[25]\). In this paper, we research TSR of heat source with complex shapes on flux tube utilizing finite element method.

Thermal spreading resistance analysis requires the solution of Laplace’s equation in three dimensions.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$  \hspace{1cm} (5)

In most applications the following boundary conditions are applied:

$$\left.\frac{\partial T}{\partial Z}\right|_{Z=0} = 0 \quad A_s < A < A_b$$  \hspace{1cm} (6)

$$\left.\frac{\partial T}{\partial Z}\right|_{Z=0} = q/k \quad 0 < A < A_s$$  \hspace{1cm} (7)

Finally, along the lower surface $Z = t$,

$$\left.\frac{\partial T}{\partial Z}\right|_{Z=t} + \frac{h}{k}[T(x,y,t) - T_f] = 0$$  \hspace{1cm} (8)

Where $h$ is a uniform convection heat transfer coefficient or contact conductance\([6]\). According to the variation principle, the key problem to solve temperature field is to solve the minimum value of the following formula:

$$J(T) = \iint_R k\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 \, dx \, dy \, dz + \iint_\Gamma \alpha(\frac{1}{2}T^2 - T_f) \, ds$$  \hspace{1cm} (9)

where $R$ is computational domain, $\Gamma$ refers to Robin boundary condition.

It is transferred to get the sum of integral of all elements when computational domain are broken into many discrete elements using finite element method in three dimensional space, that is:

$$J(T) = \sum J^e$$  \hspace{1cm} (10)

The correlation formula of $J$ when it gains the extreme value is:
\[
\frac{\partial J}{\partial T_k} = \sum_n \frac{\partial J^e}{\partial T_k} = 0 \quad (k = 1, 2, \ldots, n)
\]  

(11)

where \( n \) is the number of nodes and \( e \) denotes the element in the computational domain.

It can be presented in the following form:

\[
\begin{bmatrix}
  k_{11} & k_{12} & \cdots & k_{1n} \\
  k_{21} & k_{22} & \cdots & k_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  k_{n1} & k_{n2} & \cdots & k_{nn}
\end{bmatrix}
\begin{bmatrix}
  T_1 \\
  T_2 \\
  \vdots \\
  T_n
\end{bmatrix}
=
\begin{bmatrix}
  p_1 \\
  p_2 \\
  \vdots \\
  p_n
\end{bmatrix}
\]

(12)

That is:

\[
[K][T] = \{P\}
\]

(13)

where \([K]\) is temperature stiffness matrix, \([T]\) is temperature matrix and \([P]\) is load matrix.

2.3. Choice of mesh

Mesh is very important to make accurate simulation of the thermal field and acquire precise TSR. A study consisting of choosing the appropriate mesh is undertaken. Here we use one classical model from Negus\(^{[23]}\) to make a validation of mesh rule. We take \( t/D = 4 \), \( D \) is the cylinder diameter. This length is sufficient for a semi-infinite cylinder assumption\(^{[12]}\). To best follow the spatial evolution of temperature fields, in the vicinity of the contact zone, we choose a very small mesh near the applied heat flux \( q \), and gradually increase it away from the contact area\(^{[17]}\). Figure 4 is part of the mesh of the circular flux tube. The commercial code ANSYS12.1 is used to solve this steady state thermal problem. The element growth rate is 0.1 and the max number of elements is about 100,000. To compare results of different cases, a dimensionless relative contact size \( \varepsilon \) is defined as the square root of the ratio of the source area to the substrate area, \( \varepsilon = \sqrt{A_s / A_b} \). In figure 5 we compare the numerical results \( R_s^* \) with the analytical solutions \( R_p \)\(^{[23]}\) and they are in good agreement. We will use the same mesh rule for the other models which are constructed in the following study.

![Figure 4. Part of finite-element mesh of circle/circle flux flux.](image1)

![Figure 5. A comparison with ANSYS results with analytic results\(^{[23]}\).](image2)

3. Results and Discussions

3.1. Influence of heat source eccentricity on TSR
In this part, the influence of heat source eccentricity on TSR is studied utilizing a series of eccentric circular, square, equilateral triangular spots on heat flux tubes (figure 6). The three groups of models have the same $\xi=0.0894$. The boundary conditions are: (1) keeping the heat source suffering a constant heat flux $q$; (2) cooling at the bottom of the cylinder; (3) the lateral and non-source top surface boundaries of the cylinder are adiabatic. To compare results of different cases, a dimensionless relative length $\xi = e/b$ is defined. Here, $e$ is the distance between the center of the heat source and the center of the circular heat flux tube. $b$ is the radius of the circular heat flux tube. $r$ and $R$ represent inscribed circle radius and circumcircle radius of the heat source respectively. Figure 7 shows the numerical results of the three groups of models when the heat source has eccentricity.

![Figure 6](image_url)

**Figure 6.** Contact area/cylinder cross-sectional configurations under consideration: (a) Circle/circle ($\xi=0$); (b) Circle/circle($\xi=0.2$); (c) Square/circle ($\xi=0$); (d) Equilateral triangle/circle ($\xi=0$).

![Figure 7](image_url)

**Figure 7.** Influence of heat source eccentricity on TSR.
We compare the results of circle/circle flux tubes with Bairi & Laraqi’s data\cite{15}. It shows that the results are in good agreement. In figure 7 we also compare the variation of TSR of the equilateral triangular spot and square spot on circular flux tubes. It shows that the growth rates of the three group models are lower to 10 percent during $\xi = 0$ to $\xi = 0.7$ due to eccentricities of contact spots. They increase quickly after $\xi = 0.7$. This phenomenon is due to the edge effect that becomes more pronounced when the spot approaches the contour of the flux tube\cite{15}. The maximum relative difference between an eccentric and a centred contact is about 35 percents when $\xi$ varies from 0 to 0.9. These results are consistent with recent works for random contacts\cite{26,27}.

3.2. Influence of heat source shape on TSR

![Figure 8](image)

**Figure 8.** Isosceles triangular heat source on circular flux tubes (a) interior angle of the two sides =30°; (b) interior angle of the two sides =80°.

3.2.1. Isosceles triangular heat source on circular flux tubes. In order to research the influence of the heat source shape on TSR, a series isosceles triangular heat source on circular flux tubes are constructed (figure 8). Here we use one shape characteristic $\eta$ to depict TSR variation, and $\eta$ is equal to $r/R$, where $r$ and $R$ represent inscribed circle radius and circumcircle radius of the isosceles triangles respectively. The models have the same heat source area and their centres of gravity are kept at the centres of the circles when the interior angles of the two sides change from 30 to 80 deg. Table 1 shows the relationship between interior angles of the two sides and $\eta$. The boundary conditions are the same with part 3.1. From figure 9 we can see TSR has the same variation trend as $\eta$. Heat will spread out when $\eta$ decreases, and this phenomenon will reduce the amount of flux-line spreading between the contact surfaces, which results in a reduction of TSR.

![Figure 9](image)

**Figure 9.** TSR of isosceles triangular heat source on circular flux tubes.
Table 1. Properties of isosceles triangular heat source.

| interior angle of the two sides | 30° | 40° | 50° | 60° | 70° | 80° |
|--------------------------------|-----|-----|-----|-----|-----|-----|
| η = r/R                        | 0.232 | 0.358 | 0.459 | 0.500 | 0.450 | 0.287 |

3.2.2. Rectangular heat source on square heat flux tubes. Another group of models (figure 10) are constructed in this part to research TSR when heat source shape changes from a square to a rectangular. The boundary conditions are the same with 3.1. Here, we also keep the same heat source area for every model. Figure 11 shows that TSR has the same changing trend as η. From results of part 3.2.1 and that of this part, we can see that η can be a good characteristic to predict variation trend of TSR.

Figure 10. Contact area/cylinder cross-sectional configurations under consideration (a) Square heat source on square flux tube; (b) Rectangle heat source on square flux tube.

Figure 11. TSR of rectangular heat source on square heat flux tubes.

3.3. Influence of heat source with fractal boundary on TSR

Fractal geometry is a mathematical language that describes the structural disorder and chaos of a number of objects found in nature (Mandelbrot, 1982). On several occasion, however, it is crucial to know the structural details of the object at the length scales relevant to a physical phenomenon. This is where fractal geometry becomes important[28].

Koch snowflakes is one kind of famous fractal geometry. Here we develop a group of models with heat source boundaries similar to Koch snowflakes (figure 12, N represents the construction number, the heat sources are kept with the same area). The following introductions are how to develop models with boundaries similar to Koch snowflakes. Figure 12(a) is an equilateral triangle heat source which
is the first model, here, \( N=1 \). The second model (the second construction, \( N=2 \)) can be constructed by starting with the first model, then recursively altering each line segment as follows: (1) divide the line segment into three segments of equal length; (2) draw an equilateral triangle that has the middle segment from step (1) as its base and points outward; (3) remove the line segment that is the base of the triangle from step (2). After one iteration of this process, the result is a shape similar to the Star of David which is the second model as shown in figure 12 (b). The same iterations can be repeated to get the other models.

From table 2, we can see \( \eta \) just increases a little from \( N=1 \) to 2, and keeps at 0.577 after \( N=2 \), although the circumference of the heat source keep increasing to infinite length as \( N \) goes to infinity when heat source area keep at a constant value.

![Figure 12](image)

**Figure 12.** Contact area with fractal boundary/cylinder cross-sectional configurations under consideration (a) \( N=1 \); (b) \( N=2 \); (c) \( N=3 \); (d) \( N=\infty \).

| construction number | \( N=1 \) | \( N=2 \) | \( N=2 \) | \( N=2 \) | \( N=2 \) | \( N=\infty \) |
|---------------------|----------|----------|----------|----------|----------|-----------|
| Circumference(m)    | 0.456    | 0.526    | 0.666    | 0.869    | ...      | \( \infty \) |
| heat source area(m²)| 0.010    | 0.010    | 0.010    | 0.010    | 0.010    | 0.010     |
| \( \eta = r/R \)    | 0.500    | 0.577    | 0.577    | 0.577    | 0.577    | 0.577     |

The boundary conditions are: (1) keeping the heat source at temperature 373K; (2) cooling at the bottom of the cylinder; (3) the lateral and non-source top surface boundaries of cylinder are adiabatic. We choose one-sixth model to simplify the calculation. Figure 13 and figure 14 are the mesh and the temperature field of the one-sixth model. From figure15 we can see that TSR of fractal geometries decreases as \( N \) increases. The variation of amplitude increases as \( \varepsilon \) increases. It means that fractal boundary has much more influence on TSR for the bigger \( \varepsilon \). The related difference is about 16% at \( N=5 \) when \( \varepsilon = 0.6 \). We can predict that TSR will converge to a constant value as their boundaries approach Koch snowflakes from figure 15.
At present, it is controversial that whether the actual contact surface has fractal characteristic \[10\]. TCR models with fractal characteristic are getting more and more attentions to researchers. Profile curve of roughness concentration may have fractal characteristic although it is hard to know whether fractal geometry can be used to predict TCR. Wavelet transformation can be used to evaluate the fractal shape curve well \[29\]. The results expressed in this paper may supply great reference value for predicting TCR.

4. Conclusions

From observations made with the above series of models, our numerical results demonstrate that eccentricities, shapes and fractal boundary of the heat source have significant influence on TSR. It is shown that TSR increases with the increase of eccentricity due to the heat source. Further study shows that the TSR also increase with the increase of shape characteristics factor, $\eta$. TSR decreases as construction number, N, increases, and finally converges to a constant value when the boundaries of the heat source approach the Koch snowflakes. Fractal boundary has much more influence on TSR for the bigger relative contact size $\varepsilon$. Hence, precise location, the shape and boundary of the heat source are important in estimating TSR.

$\eta$ can be a good characteristic to predict TSR expect for the relative contact ratio $\varepsilon$ and heat source eccentricity. The results obtained in this paper may have much reference value to the further research.
on three-dimensional TCR. For in-depth investigation of TCR, we will research molecular scale models with “bottom-up” methods.

5. Acknowledgements
The authors acknowledge the financial support of the Beihang University International Conference Fund for graduate. The authors also want to express their thanks to the group lead by Professor Yu Liu, School of Astronautics, Beihang University for their support and encouragement.

The authors further acknowledge the help of Professor Jianxin Zhong (Xiangtan University). Professor Arkadiusz Wojs (Wrocław University of Technology) and Zixiang Hu (Princeton University) were very thorough in helping to find typographical errors in many equations and the authors appreciate their contributions.

References
[1] Bahrami M, Culham J R, Yovanovich M M and Schneider G E 2004 J. Thermophys. Heat Tr. 18 218
[2] Yovanovich M M 1998 Conduction and Thermal Contact Resistances (Conductances), Handbook of Heat Transfer 3ed Rohsenow W M, Hartnett J P et al (New York: McGraw Hill )
[3] Wang A L and Zhao J F 2010 Sci.China Tech. Sci. 53 1798
[4] Sadeghi E, Hsieh S and Bahrami M 2011 Proc. Thermal Engineering Joint Conf. 8th (Honolulu, Hawaii, USA) p T10129
[5] Bahrami M, Culham J R, Yovanovich M M and Schneider G E 2004 J.Thermophys. Heat Tr. 18 209
[6] Muzychka Y S, Yovanovich M M and Culham J R 2006 J. Thermophys. Heat Tr. 20 247
[7] Tio K K and Toh K C 1998 Int. J. Heat Mass Tr. 41 2013
[8] Vaidya S Y and Razami A 1998 J. Franklin. Inst. 335 1493
[9] Rostami A A, Hassan A Y and Lim P C 2001 Heat Mass Transfer. 37 5
[10] Black A F, Singhal V, and Garimella S V 2004 J. Thermophys. Heat Tr. 18 30
[11] Olsen E L, Garimella S V and Madhusudana C V 2002 J. Thermophys. Heat Tr. 16 207
[12] Merrill C T and Garimella S V 2006 J. Thermophys. Heat Tr. 20 346
[13] Wang H, Degiovanni A and Moyne C 2002 Int. J. Therm. Sci. 41 125
[14] Wang H and Degiovanni A 2002 Int. J. Heat Mass Tr. 45 2177
[15] Bairi A and Laraqi N 2004 J. Heat Transf. 126 652
[16] Teertstra P M, Yovanovich M M and Culham J R 2009 J. Thermophys. Heat Tr. 23 28
[17] Chaouch K T, Loulou T, Rogeon P and Benkhedda Y 2011 Int. J. Therm. Sci. 50 890
[18] Liu J 2001 Micro/nano-scale Heat Transfer(in Chinese) (Beijing: Science Press)
[19] Touzelbaev M N and Goodson K E 1997 J. Thermophys. Heat Tr. 11 507
[20] Prasher R S and Phelan P E 2001 J. Heat Transf. 123 105
[21] Chen G 2002 J. Heat Transf. 124 320
[22] Liao N B and Yang P 2008 J. Thermophys. Heat Tr. 22 581
[23] Nagus K J, Yovanovich M M and Beck J V 1989 J. Heat Transf. 111 804
[24] Bhushan B and Peng W 2002 Appl. Mech. Rev. 55 435
[25] Sarzala R P and Nakwaski W 1990 J. Therm. Anal. 36 1171
[26] Das A K and Sadhal S S 1999 Heat Mass Transfer. 35 101
[27] Laraqi N and Bairi A 2002 Int. J. Heat Mass Tr. 45 4175
[28] Majumdar A and Tien C L 1991 J. Heat Transf. 113 516
[29] Wang A L, Yang C X and Yuan X G 2003 Tribol. Int. 36 517