Constraints on galaxy halo profiles from galaxy-galaxy lensing and Tully-Fisher/fundamental plane relations

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ABSTRACT
Observations of galaxy-galaxy lensing from Sloan Digital Sky Survey (SDSS) are combined with the Tully-Fisher and fundamental plane relations to derive constraints on galactic halo profiles. We show that both for early and late type galaxies around $L_*$ the rotation velocity decreases significantly from its peak value at the optical radius to the virial radius $r_{200}$, $v_{\text{opt}}/v_{200} \sim 1.8$ with about 20% uncertainty. Such a decrease is expected in models in which the halo profile is very concentrated, so that it declines steeper than isothermal at large radii. This large decrease can be explained as a result of both a concentrated dark matter profile and a significant stellar contribution to the rotation velocity at the optical radii. We model the stellar component with a thin rotationally supported disk or a Hernquist profile and use adiabatic dark matter response model to place limits on the halo concentration as a function of the stellar mass to light ratio. For reasonable values of the latter we find concentrations $c_{200}$ consistent with CDM predictions, suggesting there is no evidence for low concentrations for the majority of halos in the universe. We also discuss the origin of Faber-Jackson relation $L \propto \sigma^4$ in light of $L \propto v_{200}^4$ relation found for early type galaxies above $L_*$ from galaxy-galaxy lensing. This leads to a decrease in $v_{\text{opt}}/v_{200}$ with luminosity above $L_*$, so that at $7L_*$ the ratio is 1.4. This is expected from the fundamental plane relation as a result of a reduction in the baryonic contribution to the total mass at the optical radius and a decrease in optical to virial rotation velocity in dark matter profile. These results imply that relations such as Tully-Fisher and Faber-Jackson are not simply those between the mass of dark matter halo and galaxy luminosity, but are also significantly influenced by the baryonic effects on the rotation velocity at optical radii.

Key words: cosmology: theory – dark matter – galaxies: haloes – galaxies.

1 INTRODUCTION
The study of dark matter profiles around galaxies has been an active area since the original discovery of dark matter from the flat rotation curves in spirals (see Sofue & Rubin 2001 for a review). These show that there must be dark matter in the outer parts of the galactic halos, but its extent is uncertain because of the limited range probed by observations. More recent observational studies of rotational velocity in dwarf and low surface brightness galaxies have suggested that the amount of dark matter in the central regions is smaller than predicted for the average galactic population by CDM models (e.g. Moore 1994, Flores & Primack 1994, Debattista & Sellwood 2000, de Blok et al. 2001). The CDM models predict very concentrated dark matter haloes. This is usually parametrized using a universal dark matter profile such as NFW (Navarro, Frenk, & White 1997). For a given virial mass of the halo, in this paper defined as the mass enclosed within a sphere of radius $r_{200}$ within which the density is 200 times critical density, the profile has a characteristic shape which depends only on a single parameter. For NFW profile, in which the slope continuously changes from the inner value of -1 to the outer value of -3, the free parameter is often defined as the concentration $c_{200}$, which is the ratio between the radius $r_{200}$ and the scale radius $r_s$ where the slope is close to -2. Higher concentration parameters imply higher densities at the scale radius $r_s$. On galactic scales CDM models predict $c_{200} \sim 7–15$ depending

† Although other profiles have been proposed that differ significantly from NFW in the inner parts of the halo, they agree well with NFW in the outer parts (Klypin et al. 2001). Since for the present work we are concerned predominantly with the outer parts we only use NFW profile.
on the halo mass, matter density, shape and amplitude of the power spectrum.

However, observational case for low concentrations is not clear and different conclusions have been reached by other studies (e.g. van den Bosch & Scatters 2001). Even if the observational results of low dark matter density in the inner regions are confirmed, they do not necessarily indicate a problem for the CDM models. The variation in the profile shapes between halos is large and it is not clear whether the dwarf or low surface brightness galaxies, which show strongest evidence for low density cores, can be associated with the average halo population. For example, if these galaxies are associated with a population that had a major merger in the recent past, this may lead to a significantly flatter and less concentrated halo structure than the average population (e.g. Wechsler et al. 2001). Another possibility is that astrophysical processes such as bar rotation redistribute the dark matter in the inner parts of the halo (Weinberg & Katz 2001). However, if the case of low concentrations is extended to the population of galaxies as a whole and to the outer parts of the halo, which cannot be affected by astrophysical processes, then the CDM crisis would become much more severe.

It would thus be useful to have some information on the halo structure for the mean population of galaxies. While many observationally determined rotation velocity profiles exist, the main uncertainty has always been the relative contribution between the baryonic component in the gas, bulge/ellipsoid and disk and the dark matter in the halo. The baryonic and dark matter components are difficult to separate because the conversion from the star light (and, to a lesser extent, gas density) to baryonic mass is still uncertain from the stellar population synthesis models and other studies. The situation is further complicated by the fact that baryonic component is not dynamically negligible and during its condensation it induces a response of the dark matter halo changing its original distribution in the inner parts. While these problems are ameliorated for low surface brightness galaxies, they are almost never negligible.

Since the modelling of relative disk or spheroid and halo contributions is rather difficult in the inner parts it is useful to concentrate on the outer parts of the halo, where the baryonic influence is less important. However, optical rotation curves typically extend only out to 10–20kpc and even the ones based on HI measurements do not extend beyond 30–50kpc. Similarly, velocity dispersion studies in early type galaxies also do not extend past a few effective radii and even X-ray studies of large ellipticals do not extend beyond 10 effective radii (Loewenstein & White 1999). Over this limited range the data on disk $L_*$ galaxies indicate that the rotation curves rise in the inner parts of the disk and then stay approximately constant or decline somewhat out to the outer limit of observations. This flatness indicates that approximating the density profile as isothermal, $\rho(r) \propto r^{-2}$, is a good approximation to the matter profile over this range. On the other hand, theoretical CDM models predict that the velocity profile of the dark matter increases out to twice the scale radius $r_\text{s}$ and then slows down beyond that as the slope gradually decreases towards $-3$. The fact that we do not observe a decrease in rotation velocity at radii below $2.15r_\text{s} \sim 30–50kpc$ is presumably due to the baryonic effects, which increase the mass in the center. The decrease at large radii is however a robust prediction of these models.

While, until recently, no accurate determination of the mass profile at large radii has been available, recent galaxy-galaxy lensing observations by the SDSS team (McKay et al. 2001) have improved the situation significantly by obtaining the morphology and luminosity dependence of the signal. Theoretical analysis of lensing data must take into account not just galactic halos, but also those from groups and clusters, which dominate on large scales above 200–300h$^{-1}$kpc (Guzik & Seljak 2002). These results show that a late type $L_*$ galaxy, with $L_\odot = 2.7 \times 10^{10}h^{-2}L_\odot$ (where we have applied a 30% internal extinction correction in $I$ band), has a mass $M_{200} \sim (3.4 \pm 2.1) \times 10^{11}h^{-1}M_\odot$, corresponding to the circular velocity at the virial radius $v_{200} \sim 115$km/s. While the error is still quite large it is gaussian distributed in mass, so an increase in $v_{200}$ by 35% to 150km/s is excluded at 95% confidence level. This number can be compared to the maximum rotation velocity for such a galaxy, which is 208km/s with a small scatter (Giovanelli et al. 1997), the well known Tully-Fisher relation. Comparing the two values shows that a decrease in rotation velocity from the optical to the virial radius indeed occurs for the average population of spiral galaxies. The decrease is large, almost a factor of 2 and even if we push the virial velocity up by 2-$\sigma$ to $v_{200} = 150$km/s the decrease is still around 40%. As shown in this paper, a similarly large decrease in rotation velocity is obtained also for early type galaxies which are not rotationally supported.

These results are interesting, since they are in the direction predicted by the CDM models and demonstrate that the halo profiles indeed become steeper than -2 in the outer parts of the halo. They set tight limits both on the structure formation models, by limiting the acceptable range of concentration parameters, and on the disk/spheroid formation models, by constraining the stellar mass to light ratio. Only for specific values of stellar mass to light ratio and halo concentration can one satisfy these constraints. The purpose of this paper is to investigate the constraints in detail using halo and disk/spheroid formation models.

Previous work on this subject has explored the constraints from the rotation velocity at optical radius given by the zero point of Tully-Fisher relation (Mo & Mao 2000, Navarro & Steinmetz 2000, Eke, Navarro, & Steinmetz 2001). In the absence of virial mass information these models must rely on additional assumptions to derive the constraints on cosmological models. The advantage of the additional information from lensing is that it provides another dynamical constraint at large radii, which can remove some of the modelling uncertainties present in previous modelling. In addition, while previous work only explored the constraints from late type galaxies, in this paper we also investigate the constraints from the early type galaxies. We find these are more robust both because the virial masses are more accurately determined and because the velocity dispersions at optical radii are obtained from the same SDSS sample.
2 LATE TYPE GALAXIES

The average rotation velocity at optical radii can be obtained from the Tully-Fisher (TF) relation, which in $I$-band is given by (Giovanelli et al. 1997)

$$L_I = 2.7 \times 10^{10} \left( \frac{v_{opt}}{208 \text{km/s}} \right)^{3.1} h^{-2} L_\odot, \tag{1}$$

where we used $I - 5 \log h = -21.7.68[\log(2v_{opt}) - 2.5]$ and $I_\odot = 4.15$. We denote with $v_{opt}$ the maximum rotation velocity typically achieved at the optical radius (roughly 3 times the scale radius of the disk $R_d$, which is not to be confused by the scale radius of the halo $r_s$). At this radius the rotation curve still has a significant contribution from the disk. Rotation curves at larger radii show that, for this range of luminosities, the rotation curve at $L_*$ are flat or decline slightly out to the largest radius observable, typically a few optical radii (Casertano & van Gorkom 1991; Persic, Salucci, & Stel 1996; Verheijen 2001).

We would like to compare the dark matter velocity in the inner parts of the halo to the SDSS galaxy-galaxy lensing results. For a late type $L_*$ galaxy with $i^* - 5 \log h = -21.26$ one finds $M_{200} = 3.4 \times 10^{11} h^{-1} M_\odot/(cz_{200}/10)^{-0.15}$ (Guzik & Seljak 2002) using an average 0.3 magnitude internal extinction correction (Verheijen 2001) and converting from the virial mass to the virial velocity using the relation $GM_{200}/r_{200} = v_{200}^2$ we find

$$L_I = 2.7 \times 10^{10} \left( \frac{v_{200}}{115 \text{km/s}} \right)^3 h^{-2} L_\odot. \tag{2}$$

The dynamical range of galaxy-galaxy lensing is still rather small and for late type galaxies luminosity dependence of the virial mass cannot be established using the present sample, so we focus on $L_*$ galaxies, which dominate the late galaxy galaxy-galaxy (g-g) lensing signal.

By combining the two equations above we can determine the best fitted ratio

$$\frac{v_{opt}}{v_{200}} \sim 1.8. \tag{3}$$

Since the virial mass of late type galaxies is consistent with 0 at 2-$\sigma$ level there is no upper limit to the velocity ratio, while the lower limit is given by $v_{opt}/v_{200} > 1.4$ at 2-$\sigma$, where the error budget includes the statistical errors on the zero point of Tully-Fisher relation (Giovanelli et al. 1997), but is dominated by the error on the virial mass from galaxy-galaxy lensing (Guzik & Seljak 2002) and we ignored the small concentration dependence. In addition to the statistical errors there are also possible systematic differences between the different Tully-Fisher zero point determinations (Giovanelli et al. 1997; Willick et al. 1996; Shanks 1997) which can be up to 0.2 magnitude. Similarly there could be systematic differences between color selected late type galaxies (McKay et al. 2001) and those selected for rotation velocity studies, although morphological studies indicate that the late type sample is dominated by Sh/Sc morphological type used also in TF studies (Strateva et al. 2001).

What constraints does equation (3) imply on the structure formation models? As discussed above, a decrease in rotation velocity implies that the mass profile is steeper than isothermal in the outer parts of the halo. This can be either because the dark matter profiles are steep or because the stellar disk has a significant contribution to the rotation velocity (or both). An NFW profile is given by $\rho(x) = \rho_s x^{-1}(1 + x)^{-2}$, where $x = r/r_s$. The rotation velocity is $v_*(r) = GM(r)/r$, where $M(r) = 4\pi r^2 \rho_s A(c)$ and $A(c) = \ln(1 + c) - c/(1 + c)$. For NFW profile the rotation velocity increases at small $x$ up to the peak at $x = 2.16$ and then declines gradually to the virial radius. The ratio between the maximum and virial velocity is $v_{\text{max}}/v_{200} = 0.46[M/A(c)]^{1/2}$ (Bullock et al. 2001). For the range of concentration parameters predicted by CDM models ($c < 20$) one finds $v_{\text{opt}}/v_{200} < 1.4$. It is thus unlikely that the dark matter halo can explain the decrease by itself, since even for very concentrated halos the velocity decrease is less than observed. We must therefore include the contribution from the disk.

We will use a model with a thin exponential disk in a dark matter halo (Mo, Mao, & White 1998; van den Bosch 2000). We model the disk as a thin exponential surface density profile,

$$\Sigma(R) = \Sigma_0 \exp(-R/R_d), \tag{4}$$

with the total disk mass given by $M_d = 2\pi \Sigma_0 R_d^2$. The disk mass is related to the disk luminosity using disk mass to light ratio $\Upsilon_I = M/L$. We will neglect the bulge contribution both to the luminosity and to the rotation curve (see Mo et al. 1998 for a discussion of this assumption). We will use observations to provide the typical scale length $R_d$ of the galactic disks. For an $L_*$ galaxy observations give $R_d \sim 3.5 h^{-1}$ kpc (Courteau 1997; de Jong 1996). The scatter for this quantity at a given luminosity is rather large, since galaxies come with a range of surface brightnesses. Theoretically, the scatter has been linked to the scatter in the spin parameter of halos (Mo et al. 1998). However, since in this paper we are primarily concerned with the average properties of galaxies and not in the scatter around the mean we will assume $R_d = 3.5 h^{-1}$ kpc in the analysis as a mean value for a typical $L_*$ galaxy.

Disk gravity contributes to the measured rotation velocity. In addition, disk gravity also induces a response of the dark matter halo in the inner regions. The standard approach to model this is to assume an adiabatic contraction of the halo, which remains spherical, so that the angular momentum of individual particles is conserved (see Barnes & White 1984; Blumenthal et al. 1986 and Navarro & Steinmetz 2000 for more details and numerical tests on the validity of this model). This leads to an implicit equation for the final radius $r_f$ of dark matter mass as a function of initial radius $r_i$:

$$M_{DM}(r_i)r_i = [M_{DM}(r_i)(1 - f_s) + M_s(r_i)]r_i, \tag{5}$$

where $M_{DM}(r)$ and $M_s(r)$ are dark matter and stellar mass, respectively and $f_s$ is the stellar mass fraction of the halo. Assuming that the baryons which do not end up in the disk have the same distribution as the dark matter allows one to solve the system completely for a given disk mass and scale length and for a given halo profile. Flattened nature of the disk is used when obtaining stellar rotation velocity from the mass profile (Binney & Tremaine 1987). As a free parameter we will use the stellar mass to light ratio $\Upsilon_I$ (expressed in solar units), which from the known virial mass and luminosity of $L_*$ galaxy (and ignoring bulge contribution to the luminosity) can be related to $f_s$ as
Typical values are \( \Upsilon_I = (1 - 2)h \) (e.g. Bottema 1997), giving \( f_* = 0.1 - 0.2 \). Note that \( f_* \) should not exceed \( \Omega_b/\Omega_m < 0.04/\Omega_m \), since only the baryons within the virial radius can condense to make stars. This implies \( \Omega_m < 0.4 \) for the fiducial value of \( M_* \) and \( \Omega_m < 1 \) for the 95% c.l. on \( M_* \). This is in agreement with other determinations that give \( \Omega_m < 0.4 \), so the fraction of baryons converted to stars does not exceed the available supply, although it comes quite close to this limit and is an argument against high values for \( \Upsilon_I \).

We can solve for \( v_{200}/v_{200}^{\text{opt}} \) for any given \( f_* \) (or \( \Upsilon_I \)) and \( c_{200} \). An example with \( \Upsilon_I = 1.7h \) and \( c_{200} = 12 \) is shown in figure 1. One can see that the velocity ratio of 1.8 can be naturally obtained in a model with concentration parameter in the range predicted by CDM models, 8 < \( c_{200} < 15 \) (Eke, Navarro, & Steinmetz 2001, Bullock et al. 2001), and with the expected stellar mass to light ratio, \( 1h < \Upsilon_I < 2h \). The rotation velocity is reasonably close to flat over the optical region, but decreases by 10-20% out to the largest range observable in HI, in agreement with the observations for this range of luminosities (Casertano & van Gorkom 1991, Verheijen 2001). The disk and dark matter contributions to the mass within the optical radius are comparable, each contributing 50% in this example. This implies that this model satisfies the requirement that the zero point of TF relation is independent of the disk surface brightness, which requires about 50% dark matter contribution to the rotation velocity at optical radius (Mo & Mao 2000). Note that the adiabatic response of dark matter is quite significant and dark matter would be subdominant if it were not compressed by baryonic condensation. Because of this the dark matter contribution to the rotation curve is never negligible, even in the inner parts of the galaxy. However, our predictions may not be reliable inside the optical radius, where bulge makes a significant contribution and adiabatic approximation may not be valid (Weinberg & Katz 2001). The example above was chosen based on the typical values for concentration and stellar mass to light ratio. More generally for any choice of one there will be a particular value for the other that satisfies the constraint in equation 3. This is shown in figure 2 for a family of stellar mass to light ratios. Low values of \( \Upsilon_I \) cannot be made compatible with the constraint in equation 3 unless the concentrations are unreasonably high (e.g. \( c > 20 \) for \( \Upsilon_I < 1h \)). Low concentrations are also not acceptable unless the observed \( v_{200}/v_{200}^{\text{opt}} \) is decreased by 2-\( \sigma \) to 1.4. In this case one must adopt either very low concentrations for reasonable \( \Upsilon_I \sim 1.5h \) or very low \( \Upsilon_I < 0.7h \) for reasonable \( c \sim 10 \). On the other hand, a positive deviation in \( v_{200}/v_{200}^{\text{opt}} \) can be explained by using a somewhat higher values for \( \Upsilon_I \). We conclude that the results are just what is expected from the CDM models with standard concentrations and standard stellar mass to light ratios. Only if the virial masses deviate by 2-\( \sigma \) in the positive direction from the mean value does one run into problems with the standard stellar mass to light ratios and one requires \( \Upsilon_I = 0.5h \). The majority of late type galaxies therefore show no evidence for shallow density profiles in the outer parts of the halo.

\[
\Upsilon_I = \frac{M_*}{L_*} = \frac{3.4 \times 10^{11} h^{-1} M_\odot f_*}{2.7 \times 10^{10} h^{-2} L_\odot} \approx 12 f_* \frac{h M_\odot}{L_\odot}.
\]
3 EARLY TYPE GALAXIES

Early type galaxies have some advantages in constraining the outer parts of the halo profiles. The main advantage is that they show a stronger g-g lensing signal, so that both $M_*$ and its scaling with luminosity are reliably determined from the current data and the errors associated with it are significantly smaller. Moreover, one can study their early type dynamics using the same spectroscopic SDSS sample also used for g-g lensing (the actual samples used in the analysis here are not completely equal since the analysis for the early type dynamics in Bernardi et al. (2001) was using a larger sample than the lensing analysis in McKay et al. (2001), but the statistical properties of the two samples should be very similar). The fundamental plane relations as derived by Bernardi et al. (2001) are $L \propto \sigma^4 \propto R_e^{5.5}$, where $\sigma$ is the central stellar velocity dispersion and $R_e$ is the effective radius of de Vaucouleurs profile. The values at $L*$ in $i'$ are $-21.26$ are $\sigma = 177 \text{ km/s}$ and $R_e = 2.7 h^{-1} \text{ kpc}$ (note that we are using $L_*$ for the luminosity function of the whole sample given in Blanton et al. 2001, not just the early type).

For early type galaxies the conversion from the stellar velocity dispersion to matter circular velocity is less straightforward. In principle one can obtain it by solving the Jeans equation, which however depends on the unknown anisotropy of velocity dispersion. If the circular velocity of the matter does not change very much over the optical region then virial theorem guarantees that $v_{\text{opt}} = 3^{1/2} \langle \sigma \rangle$, if $\langle \sigma \rangle$ is luminosity weighted line of sight velocity dispersion. In practice luminosity average is difficult to achieve and the velocity dispersion decreases with radius, so more often observers report the central velocity dispersion $\sigma_{\text{central}}$. Based on detailed kinematic analysis and on comparison between strong lensing and stellar dispersions Koachan (1999)]() argues that in this case the relation is closer to $v_{\text{opt}} = 21/2 \sigma_{\text{central}}$. In SDSS the aperture is determined by SDSS fibers and is $r_{\text{fiber}} = 1.5''$, implying that for nearby galaxies only central parts of the galaxy are detected, while for distant galaxies most of the light is observed. Bernardi et al. (2001) attempt to correct for this using an empirical fit, $\sigma_{\text{central}}/\sigma = (0.8 r_{\text{fiber}}/R_e)^{0.04}$, to obtain the central velocity dispersion. The correction is empirical and appears to be somewhat small to account for the suggested 22% difference between the central and luminosity weighted velocity dispersion, since one needs very small $R_e = 0.15''$ to achieve this. We will use an intermediate value of $v_{\text{opt}} = 1.5 \sigma$ to convert from the central velocity dispersion to the rotation velocity at the optical radius. This should have at most a 10% systematic uncertainty attached to it. This conversion agrees well with the studies of slowly rotating elliptical galaxies where both $v_{\text{opt}}$ and $\sigma_{\text{central}}$ have been measured (Gerhard et al. 2001).

Another ingredient in the modelling is the dark matter response to baryonic contraction. We model it again using the adiabatic model. In principle there is no reason why such a model would be appropriate for elliptical galaxies, where stars are not on circular orbits, but numerical simulations of galaxy formation have found that adiabatic compression model works remarkably well even for such systems (Gottbrath & Steinmetz 2001), perhaps as a consequence of conservation of radial action in such systems. We use the Hernquist profile (Hernquist 1993), which has been shown to give a light profile very close to the de Vaucouleurs profile and has an analytic 3-d radial distribution, to model the star distribution (Keeton 2001). Our canonical value for the stellar mass to light ratio in $i'$ is a factor of 2 higher than that of late type galaxies in the same band, $T_\nu = 3 h M_\odot/L_\odot$. This is based on $K - i'$ color difference between early and late type galaxies in SDSS, which is around 0.2-0.3 magnitudes (without internal extinction correction, Ivezic et al. 2001) and the fact that in K band luminosity to stellar mass conversion only depends on the age of population and differs by less than a factor of 2 between early and late type galaxies for reasonable IMF and assuming ages above 3Gyr (Drorry et al. 2002). This stellar mass to light ratio is again rather uncertain and significantly higher values have been suggested in the literature. Most of the direct studies are done in B band, but even after correcting for a factor of 2-3 difference between B and $i'$ luminosity for early type galaxies it is on the low side based on the dynamical studies of central regions using the minimal halo models (Gerhard et al. 2001). The simplest explanation is that part of the mass is actually due to dark matter, as discussed further below. Note that for $\Upsilon_* = 4 h M_\odot/L_\odot$ the stellar to virial halo mass ratio for early and late type galaxies become equal and are approaching the maximal baryon to dark matter ratio still allowed by the observations. It is thus unlikely that the average stellar mass to light ratio can significantly exceed this value if the virial masses from g-g lensing are correct.

At $L_* = 2 \times 10^{10} h^{-2} L_\odot$ the virial mass for early type galaxy is $(9.3 \pm 2.2) \times 10^{11} h^{-1} M_\odot$ (Suzik & Seljak 2002), which translates to $r_{200} = (160 \pm 15) \text{ km/s}$ at $r_{200} = 160 h^{-1} \text{ kpc}$. At the effective radius $R_e = 2.7 h^{-1} \text{ kpc}$ the rotation velocity from optical velocity dispersion is $v_{\text{opt}} \approx 1.5 \times 177 \text{ km/s} = 265 \text{ km/s}$ with a small error (Bernardi et al. 2001), leading to

$$v_{\text{opt}} = 1.68 \pm 0.2.$$ (7)

The error is dominated by stellar to dark matter velocity dispersion conversion and virial mass uncertainty. Here again we have a very large decrease from the optical to the virial radius, which is inconsistent with the flat rotation curve at more than 3-σ level. This decrease is similarly large to the one observed for the late type galaxies. Such a decrease cannot be explained by the dark matter alone unless halos are extremely concentrated. More realistic models must include baryons, which make a significant contribution to the rotation velocity at optical radii, both by direct contribution and by compressing the dark matter.

The resulting velocity profiles are shown in top of figure 3 for the canonical values $\Upsilon_* = 3 h$ and $c_{200} = 10$ (we use a somewhat lower $c_{200}$ than for late types since concentration is expected to decrease with halo mass). The maximum rotation velocity peaks very close to the optical radius and has a value of $v_{\text{max}} = 270 \text{ km/s}$. This is in a close agreement with the observed value $v_{\text{opt}} = 265 \text{ km/s}$ and is well within the estimated error of $30 \text{ km/s}$, indicating that this model has no problem explaining the observed ratio of the optical to virial rotation velocity. Note that at the optical radius the baryon and dark matter contributions are comparable, while at somewhat larger radii dark matter dominates. The resulting profile is much closer to a constant velocity SIS.
profile than if just light was contributing to the mass. This is in a good agreement with the conclusions from strong lensing (Kochanek 1995), rotation velocity studies of ellipticals (Gerhard et al. 2001) and X-ray studies of elliptical s-lensing (Kochanek 1995), rotation velocity studies of elliptical s.

A general exploration of c versus \( \Upsilon_i \) is shown in figure 4 for \( L_e \). Within 1-\( \sigma \) of the best fitted value for \( v_{\text{opt}}/v_200 \) one has \( 2h < \Upsilon_i < 4h \) assuming \( c = 10 \), whereas at 2-\( \sigma \) level this is extended to \( 1h < \Upsilon_i < 5h \). While the range of stellar mass to light ratios suggested in the literature is rather large and extends even above \( \Upsilon_i > 5h \), such high values are typically found for minimum halo models and are thus an upper limit. Our analysis suggests that \( \Upsilon_i < 5h \) both because of the dynamical constraint and because of the limited baryon supply. Very high values of stellar mass to light ratios are also not compatible with the stellar population synthesis models using the observed IMF (e.g. Kauffmann & Charlot 1998).

Since the scaling of virial velocity with luminosity \( L \propto v_{\text{opt}}^2 \) (Guzik & Seljak 2002) differs from the scaling of optical velocity with luminosity \( L \propto v_{\text{opt}}^2 \), above \( L_e \) we must also compare at a higher luminosity. We choose \( L = 7L_e \), which corresponds to the highest luminosity bin in SDSS g-g lensing analysis (McKay et al. 2001) and is dominated by early type galaxies. At this luminosity one finds \( M_{200} \sim 10^{13} h^{-1} M_{\odot} \) (Guzik & Seljak 2002), which corresponds to \( v_{200} = (310 \pm 30) \) km/s. The corresponding central velocity dispersion is \( \sigma_{\text{central}} = 290 \) km/s, implying \( v_{\text{opt}} = 435 \) km/s and \( R_e = 10h^{-1} \) kpc (Bernardi et al. 2004). Here the ratio is \( v_{\text{max}}/v_{200} = 1.4 \pm 0.2 \). The dominating error is the virial mass at this luminosity.

Results of the adiabatic model calculation are shown in figure 3 using \( \Upsilon_v = 3h \). We have used an even lower concentration value \( c = 8 \) for this higher mass halo, since numerical simulations find \( c \propto M^{-0.14} \) (Bullock et al. 2001). We again find the rotation velocity at the optical radius exceeding that at the virial radius, but the excess is significantly smaller now. The rotation curve is very flat for \( r > 5h^{-1} \) kpc with the value at the optical radius around 445 km/s, in good agreement with the observed value of 435 km/s. The peak value of the dark matter rotation curve alone is around 400 km/s and is close to the value at the optical radius given that the rotation curve is so flat. Note that at this luminosity the dark matter dominates already at the optical radius and it is only below \( 5h^{-1} \) kpc that baryon mass exceeds that of the dark matter. However, the baryons do have a significant effect on the dark matter through the adiabatic compression, so that even though the dark matter dominates at the optical radius it would have been comparable to baryons if there was no dark matter compression. Our analysis suggests that the data are consistent with brighter ellipticals being more dark matter dominated at the optical radii than the fainter ones and the stellar mass to light ratio not varying with luminosity.

It is worth noting that the model can be extended by yet another order of magnitude in virial mass, to cluster masses with an elliptical cD galaxy at the center. A typical example is Virgo cluster with M87 at the center. For M87/Virgo the rotation velocity increases from the optical radius \( r_e \sim 5h^{-1} \) kpc value \( v_{\text{opt}} \sim 450 \) km/s (Romanowsky & Kochanek 2001), to around 950 km/s deduced from the observed X-ray temperature around 3 keV. An example that can reproduce these constraints is shown in figure 4, where a halo with \( M_{200} = 2 \times 10^{14} h^{-1} M_{\odot} \), \( c = 6 \) and spheroid with
In this paper we compare average properties of rotation velocities between optical and virial radii based on optical and galaxy-galaxy lensing measurements, respectively. Our model is statistical in nature, since it is not based on analysis of individual galaxies, but on their average properties as a function of luminosity. On the other hand, the galaxies in our sample were not chosen on the basis of any selection criteria, so our results should apply to the galaxy population as a whole. Moreover, the large dynamical range between optical region (of order a few kpc) and virial region (of order a few hundred kpc) allows one to measure slow changes in rotation velocity which are not possible to detect from each of the observations individually.

Our main conclusion is that the rotation velocity in galaxies decreases significantly from optical radii to virial radii. Such a decrease is theoretically expected since the dark matter profiles in CDM models are steeper than isothermal in the central region, although for quantitative predictions baryons and their effect on dark matter must also be included.

**Figure 5.** Same as figure 3 for $7L_*$ early type galaxy.

**Figure 6.** Same as figure 3 for $2 \times 10^{11} h^{-1} M_\odot$ early type galaxy in a $2 \times 10^{13} h^{-1} M_\odot$ halo as a model for M87/Virgo.
is expected in models where halo profiles are less concentrated for higher halo masses, implying that the turnaround from an increase to a decrease in rotation velocity occurs at a larger radius relative to the virial radius. In addition, in more massive halos stars play a less important role both as a direct contribution to the rotation velocity and through their effect on the dark matter. The rotation velocity-luminosity scalings at optical radii, such as Tully-Fisher and Faber-Jackson relations, are not directly related to the properties of dark matter, but also require a proper modelling of baryons and dark matter response to baryonic contraction (see also a related discussion in [Gonzalez et al. 2000] and [Kochanek 2001] and [van den Bosch 2001]). While there are still uncertainties in the modelling of these processes, the simple models presented here reproduce well the constraints from the data both for early and late type galaxies.

How do our results compare to previous work? The zero point of TF relation problem [Eke et al. 2001] is the closest to the TF analysis done here. In the absence of virial mass information the value of rotation velocity at a given luminosity does not suffice to make any general assumptions and/or modelling. For example, the stellar mass fraction in the halo \( f_* \) obtained in previous work was lower, which lead to a higher virial mass for a given luminosity, which in turn requires lower concentrations and/or stellar mass to light ratios. By increasing \( f_* \) close to its maximum value the virial mass can be reduced and this alleviates the problem. The same solution also solves the suggested over-prediction of dark matter at the solar radius in our own galaxy [Eke, Navarro, & Steinmetz 2001], since again if the stellar fraction is higher the virial mass can be lower (there may be additional problems for CDM profiles in the inner parts of our galaxy; e.g. [Binney & Evans 2001]). For early type galaxies it has been suggested that the concentrations are low from the strong lens statistics ([Kochanek et al. 2001]), since very concentrated halos would overpredict the expected number of lenses. This is a difficult method to use since the expected number of lenses is very sensitive to the assumed luminosity function for early type galaxies as a function of redshift, which still has considerable uncertainty. Additional uncertainty arises from the adopted values for stellar mass to light ratios, which again can change the lensing statistics significantly. The lensing results are still compatible with low density CDM models, suggesting there is no discrepancy with our results, although more work is required to study this in detail.

There are other problems that have been suggested as troublesome for CDM, such as detailed shapes of velocity profiles in the optical region ([He Blok et al. 2001]), bar rotation ([Debattista & Sellwood 2000]), or the halo structure of the Milky Way ([Binney & Evans 2001]). These probe inner regions of the galaxy where complicated physical processes may be taking place, so there is considerable more uncertainty in their theoretical predictions. For example, there are processes such as bar rotation that can disrupt dark matter cusps ([Weinberg & Katz 2001]). It has recently been shown that CDM profiles can fit most of the rotation curves for normal galaxies ([Jimenez, Verde, & Oh 2002]). The galaxies that appear to be a problem for CDM belong to one of the specific subsamples, such as low surface brightness, dwarf or barred galaxies. It is possible, although not necessarily easy to arrange, that these samples are qualitatively different from the main population, for example by forming later and thus being less concentrated. Yet another possibility is that problems arise only below \( L_* \), since our analysis is only valid for galaxies around and above \( L_* \). Clearly, more work is required to resolve these issues. However, if the g-g lensing masses are correct, then for the main population of galaxies around and above \( L_* \) the CDM model predictions for the amount of dark matter outside the inner few kpc do not exceed the observations, suggesting that the problems for CDM may not be as fundamental as previously suggested.

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