MAGNETIC DIPOLE MICROWAVE EMISSION FROM DUST GRAINS

B. T. Drain and A. Lazarian

Princeton University Observatory, Peyton Hall, Princeton, NJ 08544; drain@astro.princeton.edu, lazarian@astro.princeton.edu

Received 1998 June 30; accepted 1998 September 23

ABSTRACT

Thermal fluctuations in the magnetization of interstellar grains will produce magnetic dipole emission at \( \nu \ll 100 \) GHz. We show how to calculate absorption and emission from small particles composed of material with magnetic, as well as dielectric, properties. The Kramers-Kronig relations for a dusty medium are generalized to include the possibility of magnetic grains. The magnetic permeability as a function of frequency is discussed for several candidate grain materials. Iron grains, or grains containing iron inclusions, are likely to have the magnetic analog of a Fröhlich resonance in the vicinity of \( \sim 50-100 \) GHz, which results in a large magnetic dipole absorption cross section. We calculate the emission spectra for various interstellar grain candidates. Although “ordinary” paramagnetic grains or even magnetite grains cannot account for the observed “anomalous” emission from dust in the 14–90 GHz range, stronger magnetic dipole emission will result if a fraction of the grain material is ferromagnetic, as could be the case given the high Fe content of interstellar dust. The observed emission from dust near 90 GHz implies that not more than \( \sim 5\% \) of interstellar Fe is in the form of metallic iron grains or inclusions (e.g., in “GEMS”). However, we show that if most interstellar Fe is in a moderately ferromagnetic material, with the magnetic properties suitably adjusted, it could contribute a substantial fraction of the observed 14–90 GHz emission, perhaps comparable to the contribution from spinning ultrasmall dust grains. The two emission mechanisms can be distinguished by measuring the emission from dark clouds. If ferromagnetic grains consist of a single magnetic domain and are aligned, the magnetic dipole emission will be linearly polarized, with the polarization depending strongly on frequency.

Subject headings: dust, extinction — ISM: abundances — radiation mechanisms: thermal — radio continuum: ISM

1. INTRODUCTION

Experiments to map the cosmic background radiation (CBR) have stimulated renewed interest in diffuse Galactic emission. Sensitive observations of variations in the microwave sky brightness have revealed 14–90 GHz microwave emission that is correlated with 100 \( \mu \)m thermal emission from interstellar dust (Kogut et al. 1996; de Oliveira–Costa et al. 1997; Leitch et al. 1997). The origin of this “anomalous” emission has been of great interest. Although “ordinary” paramagnetic grains or even magnetite grains cannot account for the observed “anomalous” emission from dust in the 14–90 GHz range, stronger magnetic dipole emission will result if a fraction of the grain material is ferromagnetic, as could be the case given the high Fe content of interstellar dust. The observed emission from dust near 90 GHz implies that not more than \( \sim 5\% \) of interstellar Fe is in the form of metallic iron grains or inclusions (e.g., in “GEMS”). However, we show that if most interstellar Fe is in a moderately ferromagnetic material, with the magnetic properties suitably adjusted, it could contribute a substantial fraction of the observed 14–90 GHz emission, perhaps comparable to the contribution from spinning ultrasmall dust grains. The two emission mechanisms can be distinguished by measuring the emission from dark clouds. If ferromagnetic grains consist of a single magnetic domain and are aligned, the magnetic dipole emission will be linearly polarized, with the polarization depending strongly on frequency.

At infrared and optical frequencies (\( \nu \gtrsim 10^{12} \) Hz) it has been customary to neglect the magnetic properties of the grain material when computing absorption or emission of electromagnetic radiation by dust grains; this is an excellent approximation since most materials have negligible magnetic response to oscillating magnetic fields at frequencies \( \gtrsim 10^{11} \) Hz. This is because “magnetism” is due to ordering of electron spins, and the maximum frequency for electron spins to reorient is the precession frequency of an electron in the local magnetic field (due mainly to other electron spins) in the material; these gyrofrequencies do not exceed \( \sim 20 \) GHz. In contrast, the electron charge distribution in grains can respond strongly to applied electric fields at frequencies as large as \( 10^{16} \) Hz.

At microwave frequencies materials can respond to both electric and magnetic fields, and both the dielectric constant \( \varepsilon(\omega) \) and the magnetic permeability \( \mu(\omega) \) can play a role in the absorption and emission of electromagnetic radiation. The present study is primarily directed at calculation of thermal emission from magnetic dust grains. The complex permeability \( \mu(\omega) \) obtained here is also relevant to the process of grain alignment via magnetic dissipation (Davis...
MAGNETIC DIPOLE EMISSION FROM DUST 741

and is the magnetic permeability. Where
\[ k = \text{complex permeability} \]

Kronig relations to interstellar grains to include the case of magnetic grains. The expected form of the frequency dependence of the complex permeability \( \mu(\omega) \) is discussed in \( \S \) 5. In \( \S \) 6 we review the properties of ordinary paramagnetic materials, and in \( \S \) 7 we discuss various classes of iron-rich materials, including: single-domain iron grains (\( \S \) 7.1); bulk (multi-domain) iron (\( \S \) 7.2); materials with single-domain Fe inclusions (\( \S \) 7.3), including the superparamagnetic limit where the inclusions contain less than \( \sim 10^3 \) Fe atoms each (\( \S \) 7.4); ferrimagnetic materials such as magnetite and maghemite (\( \S \) 7.5); and antiferromagnetic materials (\( \S \) 7.6).

In \( \S \) 8 we consider several ways in which iron can be present in the interstellar grain population, and for each case we calculate the expected thermal emission contributed by magnetic fluctuations. Iron grains, if present, would produce strong 50–100 GHz emission; this can be used to place an upper limit of \( \sim 5\% \) on the fraction of the interstellar Fe that can be in the form of pure iron grains or inclusions. In \( \S \) 8.2 we discuss the possibility that the Fe could be in a less strongly magnetic form (e.g., impure iron), and show that there are plausible magnetic properties for which such grains could dominate the 14–90 GHz emission. The polarization of the magnetic dipole emission from ferromagnetic grains is discussed in \( \S \) 9.

We discuss our results in \( \S \) 10 and summarize in \( \S \) 11.

2. OPTICS OF MAGNETIC GRAINS

2.1. Spherical Grains

For grains that are small compared to the wavelength, the cross section for absorption of electromagnetic waves can be written as the sum of electric dipole and magnetic dipole cross sections (Draine & Lee 1984):

\[ C_{\text{abs}} \approx C_{\text{ed}} + C_{\text{md}}. \]

(1)

For nonmagnetic materials (i.e., materials with magnetic permeability \( \mu = 1 \)), the magnetic dipole contribution arises from induced eddy currents in the grain, which give it an oscillating magnetic dipole moment that is out of phase with the applied magnetic field; the absorption cross section for this case has been given previously (Landau & Lifshitz 1960; Draine & Lee 1984). For materials with a magnetic, as well as an electric, response, the absorption cross section for small spheres of radius \( a \) can be shown to be

\[ C_{\text{ed}} = \frac{V}{c} \frac{\omega \varepsilon}{\varepsilon + 2} \left[ \frac{\mu_2}{\mu + 2} \right]^{1/2} \left[ \frac{\omega a}{c} \right]^{1/2} \left[ \frac{\varepsilon_2}{\varepsilon + 2} \right]^{1/2}, \]

(2)

\[ C_{\text{md}} = \frac{V}{c} \frac{\omega \varepsilon}{\varepsilon + 2} \left[ \frac{\mu_2}{\mu + 2} \right]^{1/2} \left[ \frac{\omega a}{c} \right]^{1/2} \left[ \frac{\mu_2}{\mu + 2} \right]^{1/2} \left[ \frac{\varepsilon_2}{\varepsilon + 2} \right]^{1/2}, \]

(3)

where \( V = 4\pi a^3/3 \), \( \varepsilon \equiv \varepsilon_1 + i\varepsilon_2 \) is the dielectric constant,\(^1\) and \( \mu \equiv 1 + 4\pi\chi \equiv \mu_1 + i\mu_2 \) is the magnetic permeability.

The dipole approximation (eqs. [1]–[3]) is valid\(^2\) provided

\[ |\varepsilon\mu|^{1/2} \left( \frac{\omega a}{c} \right) < 1. \]

(4)

In equation (2) the first term is due to the polarization in response to the applied electric field and the second term is due to the oscillating circular magnetization induced by the time-dependent displacement current \( \partial D/\partial t \). Similarly, in equation (3) the first term is due to the magnetization in response to the applied magnetic field, whereas the second term is the heating due to the “eddy currents” induced by the time-dependent magnetic field \( \partial B/\partial t \). By analogy, we will refer to the second term in equation (2) as being due to “eddy magnetization.” Under some circumstances the second-order “eddy” contributions can be important; for example, “eddy current” dissipation is important for \( a \gtrsim 0.2 \mu m \) graphite grains at \( \sim 3 \times 10^{13} \) Hz (Draine & Lee 1984). However, for \( a \lesssim 0.3 \mu m \) interstellar grains at \( \nu \lesssim 10^{15} \) Hz, \( (\omega a/c)^2 < 4 \times 10^{-5} \) and these higher order terms are generally negligible. The “eddy” terms will therefore be neglected in the remainder of this paper. Thus,

\[ \frac{C_{\text{ed}}}{C_{\text{abs}}} \approx \frac{\mu_2}{\varepsilon_2} \left[ \frac{\mu + 2}{\varepsilon + 2} \right]^{1/2}. \]

(5)

2.2. Ellipsoidal Grains

Ellipsoidal grains with semiaxes \( a_1 \leq a_2 \leq a_3 \) are characterized by “geometrical factors” \( L_1 \geq L_2 \geq L_3 \), which depend only on the grain shape (see Bohren & Huffman 1983). Table 1 contains \( L_j \) for selected ellipsoidal shapes.

For the moment, we assume \( \varepsilon \) and \( \mu \) to be isotropic; we will discuss anisotropic \( \mu \) in \( \S \) 9 below. The absorption cross section for incident radiation with the electric vector parallel to semiaxis \( e \) and magnetic vector parallel to semiaxis \( h \) is

\[ C_{\text{ed}} \approx V \frac{\varepsilon}{c} \left[ \frac{\varepsilon_2}{1 + L_2(\varepsilon - 1)} \right], \]

(6)

\[ C_{\text{md}} \approx V \frac{\varepsilon}{c} \left[ \frac{\mu_2}{1 + L_3(\mu - 1)} \right], \]

(7)

where \( V \equiv (4\pi/3)a_1a_2a_3 \), and we have neglected the “eddy current” and “eddy magnetization” terms. The thermal emission from the grain will be polarized. Interstellar grains undergoing superthermal rotation will tend to have their short axis \( a_3 \parallel J \), where \( J \) is the angular momentum. If \( J \) is in the plane of the sky, then the degree of linear polarization will be

\[ P = \frac{I_e - I_h}{I_e + I_h}, \]

(8)

\[ I_e = \frac{1}{2} \left[ \frac{\varepsilon_2}{1 + L_2(\varepsilon - 1)} + \frac{\varepsilon_2}{1 + L_3(\varepsilon - 1)} \right] \]

\[ + \left[ \frac{\mu_2}{1 + L_3(\mu - 1)} \right] \]

(9)

\(^1\) The electrical conductivity \( \sigma = \omega\varepsilon_2/4\pi \).

\(^2\) Eq. (2) neglects “shielding” of the grain interior from the applied magnetic field by induced eddy currents, and eq. (3) neglects shielding from the applied electric field by induced time-dependent magnetization. It can be shown that such shielding is negligible provided the dipole validity criterion (eq. [4]) is satisfied.
polarization arises only to the extent that both \( |\epsilon - 1| \) and/or \( |\mu - 1| \) differ from zero, and (2) the geometrical factors \( L_i \) differ from one another.

The contribution of magnetic dipole emission to the polarization from Fe grains will be discussed in §9.

3. KRAMERS-KRONIG RELATIONS

The Kramers-Kronig relations (see Landau & Lifshitz 1960) can be used to relate total grain volume to the wavelength integral of extinction (Purcell 1969) or to relate the wavelength dependence of linear and circular polarization produced by interstellar grains (Shapiro 1975; Martin 1975a, 1975b). The original discussion by Purcell (1969) assumed nonmagnetic grains; here we generalize the discussion to include magnetic grains. Following Purcell, consider an electromagnetic plane wave propagating through the dusty interstellar medium: \( E = \text{Re} \left( E_0 e^{ikx - \omega t} \right) \), \( H = \text{Re} \left( H_0 e^{ikx - \omega t} \right) \). If \( \hat{\chi}_e(\omega) \) and \( \hat{\chi}_m(\omega) \) are the electric and magnetic susceptibilities of the medium, then the plane wave dispersion relation is

\[
k^2c^2 = \omega^2(1 + 4\pi\hat{\chi}_e)(1 + 4\pi\hat{\chi}_m).
\]

(11)

For the interstellar medium we can assume \( |\hat{\chi}_e| \ll 1 \), \( |\hat{\chi}_m| \ll 1 \). Consider grains with number density \( n_{gr} \), volume \( V \), and extinction cross section \( C_{ext}(\lambda) \), where \( \lambda = 2\pi c/\omega \). Then

\[
n_{gr} C_{ext}(\lambda) = 2 \text{Re} \left[ \hat{\chi}_e(\omega) + \hat{\chi}_m(\omega) \right].
\]

(12)

Similarly, the birefringence of the interstellar medium is proportional to \( \text{Re} \left[ \hat{\chi}_e(\omega) + \hat{\chi}_m(\omega) \right] \). The linear response functions \( \hat{\chi}_e \) and \( \hat{\chi}_m \) must separately satisfy the Kramers-Kronig relation,

\[
\text{Re} \left[ \hat{\chi}_i(\omega) \right] = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{\lambda \omega d\omega}{\omega^2 - \omega_0^2} \text{Re} \left[ \hat{\chi}_i(\omega) \right] \quad \text{(for } i = e, m),
\]

(13)

where \( \hat{\chi}_i \) indicates that the principal value is to be taken. Taking \( \omega_0 = 0 \) one obtains the generalization of Purcell’s (1969) result:

\[
\text{Re} \left[ \hat{\chi}_e(0) + \hat{\chi}_m(0) \right] = \frac{1}{4\pi^2} \int_{0}^{\infty} d\lambda n_{gr} C_{ext}(\lambda).
\]

(14)

For randomly oriented ellipsoids with semiaxes \( a_1, a_2, a_3 \), we have

\[
\hat{\chi}_e(0) = \frac{3V}{4\pi} n_{gr} F \left( \epsilon_0; \frac{a_1}{a_3}, \frac{a_2}{a_3} \right),
\]

(15)

\[
\hat{\chi}_m(0) = \frac{3V}{4\pi} n_{gr} F \left( \mu_0; \frac{a_1}{a_3}, \frac{a_2}{a_3} \right),
\]

(16)

where

\[
F(x) = \frac{1}{9} \sum_{j=1}^{3} \frac{x - 1}{1 + (x - 1)L_j},
\]

(17)

and the \( L_i \) are the geometric factors discussed above (see Table 1). For spheres, \( L_1 = \frac{1}{3} \) and \( F(x) \approx 1 \) for \( x \gg 1 \) [Fig. 1 of Purcell 1969 shows \( F(x) \) for spheroidal grains (\( a_1 = a_2 = a_3 \)).

Thus

\[
n_{gr} V[F(\epsilon_0) + F(\mu_0)] = \frac{1}{3\pi^2} \int_{0}^{\infty} d\lambda n_{gr} C_{ext}(\lambda),
\]

(18)

where \( \epsilon_0 \) and \( \mu_0 \) are the zero-frequency dielectric function and magnetic permeability of the grain material. For nonmagnetic materials, \( F(\mu_0 = 1) = 0 \) and we recover Purcell’s (1969) result.

Grain materials of interest have \( \epsilon_0 \gtrsim 3 \) (conducting grains have \( \epsilon_0 \to \infty \)). For magnetic grains with \( \mu_0 \gtrsim 3 \), \( F(\mu_0) \approx F(\epsilon_0) \), and we see from equation (18) that \( \int C_{ext} d\lambda \) must be larger by a factor of \([1 + F(\mu_0)/F(\epsilon_0)]\) than it would have been had the grain been nonmagnetic.

Purcell (1969) used \( \int n_{gr} C_{ext} d\lambda \) to estimate \( n_{gr} V \). At first sight equation (18) appears to reduce the required volume of interstellar dust by about a factor of 2 [for magnetic grains with \( F(\mu_0) \approx F(\epsilon_0) \)], but it must be recognized that at the wavelengths \( \lambda < 3 \) mm that contributed to Purcell’s estimate for \( \int C_{ext} d\lambda \), magnetic effects are indeed unimportant, so that

\[
n_{gr} V F(\epsilon_0) \approx \frac{1}{3\pi^2} \int_{0}^{3 \text{ mm}} d\lambda n_{gr} C_{ext}(\lambda),
\]

(19)

and Purcell’s estimate for the grain volume is a good approximation. A second consequence, however, is that we see that if \( F(\mu_0) \approx F(\epsilon_0) \), then magnetic dipole absorption

| TABLE 1 |
| Geometrical Factors \( L_i \) for Ellipsoids with Semiaxes \( a_j \) |
| \( a_1:a_2:a_3 \) | \( L_1 \) | \( L_2 \) | \( L_3 \) | Note |
|-----------------|--------|--------|--------|-----|
| 1:1:1 ........... | 0.333333 | 0.333333 | 0.333333 | Sphere |
| 1:1:1.25 ......... | 0.3620042 | 0.3620042 | 0.2759916 | Prolate spheroid |
| 1:1:1.5 .......... | 0.385093 | 0.385093 | 0.2329815 | Prolate spheroid |
| 1:1:2 ........... | 0.4132180 | 0.4132180 | 0.1735640 | Prolate spheroid |
| 1:1:2.5 ......... | 0.3944403 | 0.3027798 | 0.3027798 | Oblate spheroid |
| 1:1:2.5:1:5 ....... | 0.4189528 | 0.3233250 | 0.2577222 | Oblate spheroid |
| 1:1:5:1:5 ........ | 0.459056 | 0.2770472 | 0.2770472 | Oblate spheroid |
| 1:1:5:2 .......... | 0.4837282 | 0.3050063 | 0.2112656 | Oblate spheroid |
| 1:2:2 ........... | 0.5272003 | 0.2363999 | 0.2363999 | Oblate spheroid |
must contribute a cross section \( C_{\text{ext}}^{(\text{md})} \) such that
\[
\int_{3 \mu m}^{\infty} C_{\text{ext}}^{(\text{md})} d\lambda \approx \int_{3 \mu m}^{\infty} C_{\text{ext}} d\lambda ;
\] (20)
hence if \( \mu_0 \gtrsim 2 \), magnetic dipole absorption and emission \textit{must} dominate the emission from (stationary) interstellar dust grains at the frequencies \( \nu \lesssim 30 \) GHz where the grain material has a strong magnetic response.

4. CANDIDATE MATERIALS

In addition to “ordinary” paramagnetism, grains may exhibit strong magnetic ordering: either ferromagnetic, ferrimagnetic, or antiferromagnetic (see Morrish 1980, and § 7 below).

Ordinary paramagnetism (§ 6.1) arises when the interaction producing alignment of electron spins is just the interaction of the electron’s magnetic moment with the applied magnetic field. Ferromagnetism, ferrimagnetism, and antiferromagnetism (§ 7) occur when the electron spins are ordered due to the “exchange interaction.” By mass, Fe is the fifth most abundant element, following H, He, O, and C, and nearly all of the interstellar Fe is in grains (Savage & Sembach 1996). Any strongly magnetic grain materials almost certainly contain Fe. Table 2 lists a number of possible Fe-containing materials that are magnetically ordered at the \( T \lesssim 20 \) K temperatures of interstellar dust.

Equations (6) and (7) show that the absorption cross sections are proportional to the grain volume, and that the volume of silicate per H atom is \( \frac{V_f}{V_0} \) times the \( \mu_0 \text{mag} \) of material \( Y \).

Table 2 shows for various candidate minerals. We see that Fe-rich materials are likely to make up a significant fraction of the grain volume, and these materials tend to be magnetically ordered.

5. THE MAGNETIC SUSCEPTIBILITY

Solids exhibit a variety of different responses to weak applied oscillating magnetic fields. When a static magnetic field is present, or the substance is spontaneously magnetized, the susceptibility \( \chi(\omega) \) has a tensor nature (see, e.g., Jones & Spitzer 1967, § IIIa). In the present discussion we will treat \( \chi \) as a scalar, but will extend the discussion to spontaneously magnetized material in § 7.1 and Appendix B.

Aside from the diamagnetism of superconductors and materials with no unpaired electron spins, at low frequencies most materials are characterized by \( \chi(0) > 0 \), which results from either “normal paramagnetism” or magnetic ordering in the form of superparamagnetism, ferrimagnetism, or ferromagnetism. We consider these cases below, but first discuss the likely behavior of the frequency-dependent susceptibility \( \chi(\omega) \equiv \chi_1 + i\chi_2 \).

5.1. Drude Susceptibility

Suppose that the magnetization \( M(t) \) obeys the equation of motion
\[
\dot{M} = \omega_0^2 [\chi(0) H - M] - M/\tau_0 ,
\] (22)
with three parameters: the static response \( \chi(0) \), a resonance frequency \( \omega_0 \), and a characteristic damping time \( \tau_0 \). Then

\[
\frac{\rho_f}{\rho_0} = \frac{V_f}{V_0} \frac{g \text{ cm}^{-3}}{9 \text{ g cm}^{-3}} = \frac{0.30 f_y \left( 4 \text{ g cm}^{-3} \right)}{P \gamma_y} .
\] (21)

Table 2 shows \( V_f/V_0 \) for various candidate minerals.
we obtain the Drude form for the susceptibility,
\[ \chi(\omega) = \frac{\chi(0)}{1 - (\omega/\omega_0)^2 - i\omega\tau}, \]
where we have defined
\[ \tau \equiv (\omega_0^2 \tau_0)^{-1}. \]

The real and imaginary parts of \( \chi(\omega) \) are
\[ \chi_1 = \chi(0) \frac{1 - (\omega/\omega_0)^2}{[1 - (\omega/\omega_0)^2]^2 + (\omega\tau)^2}, \]
\[ \chi_2 = \chi(0) \frac{\omega\tau}{[1 - (\omega/\omega_0)^2]^2 + (\omega\tau)^2}. \]

A susceptibility of the form (eq. [23]) arises, for example, as a solution to the Bloch equations (cf. Pake 1962, eqs. [6-24]) for a magnetized material.

The product \( \omega_0 \tau \equiv 1/(\omega_0 \tau_0) \) determines the shape of \( \chi(\omega) \). For \( \omega_0 \tau < 2 \), \( \chi(\omega) \) has a resonance near \( \omega_0 \). For \( \omega_0 \tau > 2 \), the system is “overdamped,” and responds to a step function change in the applied \( H \) with two distinct relaxation times.

5.2. “Critically Damped” Susceptibility

For \( \omega_0 \tau = 2 \), \( \chi(\omega) \) is “critically damped,” with a single relaxation time \( \omega_0^{-1} = 2\tau_0 \). We will consider this as a plausible form for the response function \( \chi(\omega) \) for a system that is essentially nonresonant. For this case \( \chi \) takes the simple form, shown as the heavy curves in Figure 1:
\[ \chi^{(cd)} = \frac{\chi(0)}{1 - i\omega\tau/2}, \]
\[ \chi_1^{(cd)} = \chi(0) \frac{1 - (\omega\tau/2)^2}{[1 + (\omega\tau/2)^2]^2}, \]
\[ \chi_2^{(cd)} = \chi(0) \frac{\omega\tau}{[1 + (\omega\tau/2)^2]^2}. \]

The superscript (cd) stands for critically damped. We will use \( \chi^{(cd)} \) to estimate the response associated with the magnetization of a single-domain particle, or of paramagnetic materials. We will identify the characteristic frequency \( \omega_0 = 1/2\tau_0 \) with the gyrofrequency of an electron in the internal field. From equations (27)–(29) we see that the response falls off rapidly when the frequency \( \omega > \omega_0 = 2/\tau \).

6. Paramagnetism

6.1. Ordinary Paramagnetism

For ordinary paramagnetism, the zero-frequency susceptibility is (Draine 1996)
\[ \chi(0) \approx 4 \times 10^{-2} f_p \left( \frac{1}{5.5} \right)^2 \left( \frac{15 \text{ K}}{T} \right), \]
where \( f_p \) is the fraction of the atoms that are paramagnetic, with magnetic moments \( p\mu_B \), where the Bohr magneton \( \mu_B \equiv e\hbar/2m_e \). If essentially all of the interstellar Mg, Fe, and Si are incorporated into material with approximately the composition of MgFeSiO₄, then a fraction \( f_p \approx \frac{1}{3} \) of the atoms would be Fe, presumably in the form of Fe²⁺(³D₄) or Fe³⁺(⁵S₅/₂) ions; these have \( p \approx 5.4 \) and 5.9, respectively (Morris 1980). Fayalite (Fe₂SiO₄) is antiferromagnetic (Carmichael 1989), and it appears that intermediate olivines (MgₓFeᵧSiO₄) will also be antiferromagnetic (Duff 1968). The magnetic character of amorphous olivine is uncertain. Although weak ferrimagnetism seems likely (see below), here we consider paramagnetic behavior.

The damping time \( \tau \) is the spin-spin relaxation time, which is essentially the time for electron precession in the random magnetic fields within the solid (see Caspers 1964). For amorphous olivine we estimate the rms gyrofrequency to be (Draine 1996)
\[ \omega_0/2\pi \approx 8 \text{ GHz}. \]

Draine (1996) used a different functional form⁴ to estimate the paramagnetic susceptibility, but we now consider the critically damped form \( \chi^{(cd)} \) (eq. [27]) to provide a better estimate at frequencies \( \omega \gg \omega_0 \). In Figure 2 we show \( \mu_2 = 4\pi\chi_2^{(cd)}, \) for \( \tau = 2/\omega_0 = 4 \times 10^{-11} \text{ s}. \)

At frequencies \( \nu \lesssim 200 \text{ GHz} \) “astronomical silicate” (Draine & Lee 1984) has \( \epsilon_1 \approx 11.6, \epsilon_2 \approx 1.13 \times 10^{-3}(\nu/\text{GHz}) \). From equation (5) it can then be seen that \( C_{\text{abs}} > C_{\text{eabs}} \) when \( \mu_2 > 0.05\epsilon_2 \) (assuming \( \mu_1 \approx 1, \) as is the case for ordinary paramagnetism), so for reference we also show 0.05\epsilon₄ in Figure 2. From Figure 2 it can then be seen that if FeMgSiO₄ grains are merely paramagnetic, the thermal emission from these grains at \( \nu \lesssim 30 \text{ GHz} \) will evi-

---

⁴ The form used by Draine (1996) had \( \chi_2 \propto \omega^{-1} \) for \( \omega \to \infty \).
dashed line labeled shows the level above which magnetic dipole absorption due to paramagnetism dominates electric dipole absorption (see eq. (5)).

The absorption due to paramagnetism dominates electric dipole absorption and far below what we estimate in Appendix A). At the temperatures of interstellar grains, it appears that a permeability would result. We see that within the narrow feature. At, say, 5 GHz this is only a few times larger than the value of $\mu_2$ that we estimate for ordinary paramagnetism (see Fig. 2) and hence would result in a narrow emission feature with central intensity only a few times what we estimate for the continuum due to ordinary paramagnetism, which we shall see below is itself 2 orders of magnitude weaker than the observed continuum emission. Nevertheless we cannot exclude the possibility that future sensitive measurements of the microwave background might disclose weak spectral features arising from the Stark effect.

6.3. Magnetic Materials in Interstellar Grains

As noted in § 4, the substantial fraction of Fe in interstellar grains creates the possibility that some fraction of the grains could be magnetic, with the “exchange interaction,” producing ordering of the atomic magnetic moments within a single magnetic domain (see Morrish 1980 for a review of the physics of magnetism and Dunlop & Özdemir 1997 for an excellent review of magnetic minerals).

Ferromagnetic materials as components of interstellar grains were apparently first considered by Spitzer & Tukey (1951), who discussed the possible formation of ferromagnetic materials in interstellar grains as the result of grain-grain collisions: metallic iron, Fe$_3$O$_4$, γ-Fe$_2$O$_3$, and MgFe$_2$O$_4$ were considered likely products. Jones & Spitzer (1967) suggested that interstellar grains might contain very small clusters of magnetic materials such as Fe$_3$O$_4$ or γ-Fe$_2$O$_3$. Shapiro (1975) proposed that the observed polarization of starlight could be produced by platelets of Fe$_3$O$_4$. Sorrell (1994) presented a model for the origin of small Fe$_3$O$_4$ clusters in H$_2$O ice mantles irradiated by cosmic rays. Ferromagnetic properties of mixed MgO-FeO-SiO$_2$ grains were discussed by Duley (1978).

Bradley (1994) argued that certain interplanetary dust particles (glass with embedded metals and sulfides, or GEMS) consisted of aggregates of ~0.1 µm interstellar grains, with nanometer-sized Fe-Ni metal inclusions. Martin (1995) argued that the properties of these grains were consistent with being interstellar dust, and that the Fe-Ni inclusions could be superparamagnetic and able to bring about alignment with the galactic magnetic field. Goodman & Whittet (1995) noted that the numbers of such inclusions were consistent with the requirements of Mathis’ (1986) hypothesis to account for the dependence of degree of alignment on grain size.

The S in GEMS appears to be in the form of FeS; if the excess of Fe over S is metallic, then the atomic abundances given by Bradley (1994) for three bulk GEMS indicates a volume filling factor $\phi \approx 0.03$ for metallic Fe-Ni; we will use this value when discussing the properties of silicate grains with metallic inclusions (§ 7.3).

7. FERROMAGNETIC AND FERRIMAGNETIC MATERIALS

There are three classes of magnetically ordered materials. In ferromagnetic materials the atomic spins within a domain are parallel; in ferromagnetic and antiferromagnetic materials the magnetic ions are located on two magnetic sublattices of oppositely directed spins and magnetic moments. In antiferromagnetic materials the magnetic moments of the two sublattices are exactly opposite in direction and magnitude, so the net magnetization is zero (for a perfect crystal at zero temperature). In ferrimagnetic materials the magnetic sublattices do not exactly compen-
sate, either because the magnetic moments of the two sub-lattices differ in magnitude (normal ferrimagnetism) or are not precisely opposite in direction (spin-canted ferrimagnetism).

7.1. Single-Domain Iron

Applied magnetic fields change the magnetization of a bulk sample by two processes: rotation of the magnetization within a domain and motion of domain walls. Grains smaller than a critical radius $a_c$ will contain only a single domain; for Fe $a_c \approx 3 \times 10^{-6}$ cm (Morrish 1980). Such single-domain behavior can take place for clusters of materials that are ferromagnetic (e.g., Fe) or ferrimagnetic (e.g., Fe$_3$O$_4$, or $\gamma$Fe$_2$O$_3$). Spontaneous magnetization has been observed for Fe clusters as small as $N_{cl} = 20$ atoms (Billas, Châtelain, & de Heer 1994).

If the magnetic material consists of a single domain, then there is clearly no motion of domain walls involved in the magnetization process, unlike the case for bulk samples, so susceptibilities measured for bulk samples are inapplicable.

When a ferromagnetic or ferrimagnetic material consists of a single domain, it will be spontaneously magnetized with magnetization $M$, along one of the "easy" directions. If a weak oscillating field $H$ is now applied, the single-domain sample will exhibit a susceptibility that depends on the direction of $H$. In Appendix B we show that at low temperatures only the component of $H$ perpendicular to the spontaneous magnetization induces a change in magnetization of the sample: $\Delta M = \chi_s H$. For Fe, we estimate $\chi_s(0) \approx 0.3k_{ bulk}(0)$ (see Appendix B).

Since changing the magnetization of the sample involves only small reorientations of the magnetic moments and no motions of domain walls, we assume that the only frequency characterizing the response is the gyrofrequency $\omega_g$ (e/m$_s$c)(4$\pi$M$_s^2$) of a magnetic moment with $g \approx 2$ in the internal field $4\pi M_s^2/3$ to which each of the dipoles is subject. For Fe, with $4\pi M_s = 22$ kG, we estimate $\omega_0/2\pi \approx 20$ GHz. We use $\chi^{(cd)}$ (eq. [27]) with $\tau \approx 2/\omega_0 \approx 1.6 \times 10^{-11}$ s to estimate $\chi_s(\omega)$ (see Fig. 3).

The energy dissipation rate is proportional to $\text{Im}(\mu_s)H^2$. For isotropic radiation incident on the grain, $\chi_s(\omega) = \frac{1}{2}(H_1^2)$. Assuming the higher order term in equation (3) to be negligible, we would therefore take the angle-averaged magnetic dipole absorption cross section for randomly oriented spheres to be

$$\langle C_{ab}^{(cd)} \rangle \approx \frac{\omega}{c} \frac{6\mu_{1.2}}{(\mu_{1.1} + 2)^2 + \mu_{1.2}^2},$$  \hspace{1cm} (32)

whereas for randomly oriented ellipsoids with semiaxes $a_1 \leq a_2 \leq a_3$, magnetized along the long axis,

$$\langle C_{ab}^{(cd)} \rangle \approx \frac{\omega}{3c} \sum_{j=1}^2 \frac{\mu_{1.2}}{L_j^2[(\mu_{1.1} - 1 + L_j^{-1})^2 + \mu_{1.2}^2]},$$  \hspace{1cm} (33)

with $\mu_1 = 1 + 4\pi M_s$, obtained using equation (27) with $\chi_s(0)$ obtained from either equation (B4) or equation (B6).

7.1.1. Fröhlich Resonance Condition

We now observe that if our estimate for $\chi_s(\omega)$ in Figure 3 is correct, then Fe particles will have the magnetic analog of a Fröhlich resonance (Fröhlich 1949; Buhmen & Huffinan 1983)—a peak in $C_{ab}$ where $\mu_{1.1} = 1 - L_j^{-1}$—thus minimizing the denominator in equation (7) or in equation (33).

It is apparent that for the functional forms in Figure 1 there are either zero or two frequencies where this condition is satisfied; if there are two, then $C_{ab}$ peaks at the higher of the two frequencies, which we denote $\omega_p$.

The Fröhlich resonance condition for spheres requires that $\mu_1 < -2$ for some range of frequencies, or $\chi_1 < -3/4\pi$. Whether or not Fe particles will have a Fröhlich resonance will depend on the details of the frequency dependence of $\chi$, which at this time is not experimentally determined. Recalling the Kramers-Kronig relation (Landau & Lifshitz 1960)

$$\chi_1(\omega) = \frac{2}{\pi} \int_0^\infty \frac{\chi_2(x)}{x^2 - \omega^2} \, dx$$  \hspace{1cm} (34)

(where $\mathcal{P}$ denotes principal value integral) and the fact that $\chi_2 > 0$, one sees that the integrand is negative for $\chi < \omega$. Quite generally, $\chi_2(\omega)$ will have $\chi_2(0) = 0$, will increase with increasing $\omega$ (linearly at low frequencies), will peak, and then eventually will decline to zero at high frequencies. If $\chi_2$ declines more rapidly than $\omega^{-2}$ at high frequencies, then we expect to have $\chi_1 < 0$ above some frequency. Because metallic Fe has $\chi_0 \approx 1$, it appears likely that there will be a frequency range where $4\pi \chi_1 < -3$, so that the Fröhlich resonance condition can be satisfied.

With our estimate of $\chi_1$ (see Fig. 3), we estimate that, for spheres, $\omega_p/2\pi \approx 70$ GHz. For ellipsoids with moderate axial ratios, the resonance will be shifted slightly. For example, for a $1:1.5:2$ ellipsoid, magnetized along the long axis, the Fröhlich resonances along axes 1 and 2 occur for $\mu_1 = -1.07$ and $-2.28$, or $\omega_p/2\pi \approx 100$ and 60 GHz. The functional form $\chi^{(cd)}$ used here provides what we consider a reasonable estimate for $\chi_1$, but the predicted Fröhlich frequency $\omega_p/2\pi \approx 70$ GHz is model dependent and should be regarded as quite uncertain.

\footnote{For $\chi^{(cd)}(\omega)$ (eq. [27]) it is easy to show that the Fröhlich resonance condition for spheres, $\mu_1 = -2$, can be satisfied provided $\chi(0) \geq 6/\pi = 1.91$, with $\omega_p \geq (3\chi_0)^{1/2}$. However, $\omega_p$ is associated with a distinct peak in $C_{ab}$ only for $\chi(0) \approx 3$.}
7.2. Bulk Iron

The static susceptibility of bulk, metallic Fe is \( \chi(0) = 12.1 \). The susceptibility of Fe as a function of frequency has been measured using fine wires, thin lamina, and powders (Allanson 1945; Epstein 1954) up to \( \sim 10 \text{ GHz} \). The response of bulk material will result from motion of domain walls, as well as the deflections of magnetization within single domains, as discussed above. We treat these two responses as additive: \( \chi(\text{bulk Fe}) = \frac{2}{3}\chi_L + \chi_{\text{dw}} \), where \( \chi_{\text{dw}} \) is the contribution from motion of domain walls.

The experimental results show considerable scatter, but we can reproduce the general trends if \( \chi_{\text{dw}} \) is given by equation (23) with \( \chi(0) \approx 10, \tau \approx 1 \times 10^{-9} \text{ s}^{-1} \), and \( \omega_0 \approx 10^{10} \text{ s}^{-1} \). In Figure 4 we plot \( \mu_2 \) estimated for multidomain metallic Fe using these parameters.

7.3. Fe Inclusions

Consider spherical single-domain ferromagnetic inclusions with volume filling factor \( \phi \) distributed in a nonmagnetic matrix. Suppose each randomly oriented inclusion to be spontaneously magnetized along one of its "easy" directions. We may suppose that, in effect, \( \frac{2}{3} \) of these inclusions have their magnetization \( M \) perpendicular to the direction of the applied \( H \), with the remaining \( \frac{1}{3} \) either parallel or antiparallel to \( H \) (and therefore not contributing to the susceptibility).

The effective susceptibility of the composite material may be estimated using effective medium theory (Bohren & Huffman 1983). For the present case of spherical inclusions with small volume filling factor \( \phi \ll 1 \), Maxwell-Garnett effective medium theory is appropriate, so we estimate

\[
\chi_{\text{eff}} = \frac{(2/3)\phi \chi_L}{1 + (4\pi/3)\chi_L(1 - 2\phi/3)},
\]

where \( \chi_L \) is given by \( \chi^{(\text{cd})} \) (eq. [27]). In Figure 5 we show \( \mu \) estimated for Fe inclusions with a volume filling fraction \( \phi = 0.03 \) (the estimated filling fraction of Fe-Ni inclusions in GEMS; see § 6.3) in a nonmagnetic (e.g., silicate) medium.

Note the peak in the effective \( \mu_2 \) for the composite medium at \( \omega/2\pi \approx 70 \text{ GHz} \); this arises because of the Fröhlich resonance in the individual single-domain Fe inclusions.

7.4. Superparamagnetism

If the Fe atoms are aggregated into single-domain clusters that are sufficiently small, then thermal fluctuations will cause the magnetization to fluctuate in direction, with the most probable magnetization directions being the most energetically favorable ones. If the inclusions are sufficiently small (\( N_{\text{cl}} \leq 6 \times 10^5 \) for Fe at room temperature), the energy barrier associated with reorientation of the magnetization is small enough that thermal fluctuations can reorient the magnetization of a single particle on timescales of seconds or less. Such clusters are termed superparamagnetic. If a field \( H \) is applied, the change in magnetization will be greater than would be the case for larger clusters, where only a small deflection in the direction of magnetization takes place, as discussed in § 7.1.

Suppose that the grain consists of a nonmagnetic (or weakly magnetic) material containing superparamagnetic clusters of \( N_{\text{cl}} \) Fe atoms, with volume filling factor \( \phi_{\text{sp}} \). A single grain then contains \( N \approx 3.5 \times 10^8 \phi_{\text{sp}} N_{\text{cl}}^{-1}(a/10^{-5} \text{ cm})^3 \) clusters.

The susceptibility of the medium may be approximated as

\[
\chi \approx \frac{(2/3)\phi_{\text{sp}} \chi_L(0)}{1 + (4\pi/3)\chi_L(0)(1 - (2/3)\phi_{\text{sp}})} + \chi_{\text{sp}}.
\]

The first term is what would be expected if the individual clusters have their magnetization locked into the "easy" direction, with a small reorientation of the magnetization that results from the transverse component of the applied \( H \) (see eq. [35]). The second term is what is expected from alignment of individual clusters assuming them to be free to orient their magnetization in any direction. We note that some previous discussions of the susceptibility of super-
paramagnetic materials (e.g., Jones & Spitzer 1967; Lazarian 1995; Draine 1996) have omitted the first term when considering superparamagnetism. In principle, these two estimates should not be treated as simply additive, but in general one or the other will dominate, and factor of 2 accuracy is sufficient at this time.

The superparamagnetic contribution at zero frequency is estimated to be

$$\chi_{sp}(0) \approx 0.035 \phi_{sp} N_{cl} \left(\frac{15 \text{ K}}{T}\right).$$

(37)

In general (even if spherical), a cluster must overcome an energy barrier in order to substantially reorient its magnetization. Laboratory experiments show the relaxation process to be thermally activated, with a characteristic relaxation rate

$$\tau^{-1} \approx A \exp \left( -N_{cl} \theta/T \right),$$

with $A \approx 10^9 \text{ s}^{-1}$ and $\theta \approx 0.011 \text{ K}$ for metallic Fe spheres (Bean & Livingston 1959; Jacobs & Bean 1963). We will employ equation (23) with $\omega_0 = 10^{12} \text{ s}^{-1}$ to estimate $\chi_{sp}(\omega)$.

Figure 5 shows $\mu_4$ for silicate grains containing superparamagnetic Fe clusters with volume filling factor $\phi_{sp} = 0.03$. Such inclusions, if present in all of the silicate grains, would account for $f = \phi_{sp}/0.15 = 20\%$ of the total Fe abundance (see Table 2). Permeabilities are shown for $N_{cl} = 10^2$, $10^3$, and $10^4$ Fe atoms per cluster.

The superparamagnetic response is large at low frequencies, but is small compared to normal paramagnetism at frequencies $\nu \gtrsim 1 \text{ GHz}$ because the estimated relaxation rate is too slow; at frequencies $\nu \gtrsim 1 \text{ GHz}$, the magnetic response is essentially that of the individual domains having their spontaneous magnetizations deflected slightly by the transverse component of the magnetic field.

We conclude that whereas superparamagnetism may lead to enhancements of magnetic dissipation at $\nu \lesssim 1 \text{ GHz}$, at higher frequencies it appears unlikely to make an appreciable contribution to absorption by interstellar grains. Therefore the high-frequency response of grains with Fe inclusions depends only on the volume fraction $\phi_{sp}$ of Fe inclusions, but is insensitive to the number $N_{q_{cl}}$ of Fe atoms per inclusion, provided only that $N_{cl} \gtrsim 20$ so that the inclusions are ferromagnetic.

### 7.5. Ferrimagnetic Minerals: Magnetite and Maghemite

Magnetite ($\text{Fe}_3\text{O}_4$) is a commonly occurring terrestrial mineral and is a plausible interstellar grain material. The low-temperature spontaneous magnetization $M_s \approx 480 \text{ G}$, from which we estimate a gyrofrequency $\omega_B/2\pi \approx 6 \text{ GHz}$.

At room temperature magnetite is cubic, and the “easy” direction is $\langle 111 \rangle$. At $\sim 120 \text{ K}$ it undergoes a phase transition with a slight distortion of the unit cell from cubic to monoclinic symmetry; near this temperature the magnetic anisotropy parameter $K_1$ (see Appendix B) changes sign and the “easy” direction changes to $\langle 100 \rangle$. The magnetic anisotropy parameters have been measured down to $77 \text{ K}$ (Syono & Ishikawa 1963). Extrapolation to $T \approx 18 \text{ K}$ is uncertain; we take $K_1 \approx 1.5 \times 10^5 \text{ ergs cm}^{-3}$, from which we estimate $\chi_{cl}(0) \approx 0.83$ using equation (B4).

We have not found data on the high-frequency susceptibility of magnetite. Taking $\tau = 2/\omega_B = 5 \times 10^{-11} \text{ s}$, we estimate $\mu$ for single-domain $\text{Fe}_3\text{O}_4$ using $\chi^{\text{cub}}$ (eq. [27]); the results are shown in Figure 6.

The low-temperature oxidation product of magnetite is maghemite, $\gamma\text{Fe}_2\text{O}_3$, for which we estimate $\omega_B/2\pi \approx 4 \text{ GHz}$. From $M_s$ and the crystalline magnetic anisotropy $K_1$ we would estimate $\chi_{cl}(0) \approx 0.4$ (see Appendix B). However, measurements by Valstyn, Hanton, & Morrish (1962) of the permeability as a function of frequency are better reproduced by a slightly larger value, $\chi_{cl}(0) \approx 0.6$, which we adopt. Our estimate for $\mu(\omega)$ for $\gamma\text{Fe}_2\text{O}_3$ is shown in Figure 6. Valstyn et al. (1962) measured $\mu(\omega)$ for a powder of $0.03-0.2 \mu\text{m}$ diameter $\gamma\text{Fe}_2\text{O}_3$ spheres suspended in paraffin wax with a volume filling factor 0.15; thus in effect there was a volume filling factor of 0.10 for spheres with spontaneous magnetization perpendicular to the applied oscillating field. In Figure 6 we show their experimental results for $\mu - 1$ divided by 0.10. The model that we are using appears to be in reasonably good agreement with the measured frequency-dependent susceptibility for this material.

### 7.6. Antiferromagnetic Materials

In antiferromagnetic materials the exchange interaction leads to magnetic domains with magnetic moments that are ordered, but in such a way that the net magnetization in the domain is zero. The olivine fayalite ($\text{Fe}_2\text{SiO}_4$) is an example of an antiferromagnetic material. These substances have zero-frequency susceptibilities $\chi(0)$ that are similar to (but somewhat smaller than) those of normal paramagnetic substances with similar concentrations of magnetic ions. Our estimate for “normal paramagnetism” would therefore be a reasonable guide to antiferromagnetic substances such as fayalite.

However, we do not expect interstellar grains to contain perfect crystals; defects and impurities seem likely to be common, and the magnetic materials may be amorphous (as appears to be the case for interstellar silicates, as indicated by the profile of the interstellar 10 $\mu\text{m}$ absorption feature due to the Si-O stretching mode). Furthermore, if the magnetic material is in very small inclusions, perfect pairing of spins...
will not be possible along the boundary of the inclusion. For instance, Schuele & Deetscreek (1962) find that small (\( \lesssim 10^{-6} \text{ cm} \)) particles of NiO are weakly ferromagnetic or ferrimagnetic although bulk NiO is antiferromagnetic. Antiferromagnetism therefore seems unlikely—we instead expect amorphous fayalite, for example, to be at least weakly ferrimagnetic. The high-frequency response of such small inclusions would be determined by the single-domain susceptibility (§ 7.3).

8. MICROWAVE EMISSION

For a nonrotating grain whose internal degrees of freedom are in thermal equilibrium at a temperature \( T \), the power radiated in frequency interval \( dv \) is simply

\[
P_v dv = 4\pi C_{\text{abs}}(\nu) B_v(T) dv,
\]

where \( C_{\text{abs}}(\nu) \) is the absorption cross section at frequency \( \nu \), and \( B_v(T) \) is the Planck function.

We will use the term “vibrational emission” to refer to emission arising from thermal fluctuations in the charge distribution, and therefore the electric polarization, of the grain. This will produce electric dipole emission.

We will use the term “magnetic dipole emission” to refer to emission arising from thermal fluctuations in the magnetization of the grain material. Estimation of this emission is one of the goals of the present paper.

We use the term “rotational emission” to refer to the emission arising from rotation of the grain. This is primarily due to the electric dipole moment that rotates with the spinning grain (Draine & Lazarian 1998a, 1998b). In principle, there will be rotational emission arising from the rotating magnetic dipole moment of a spinning magnetized grain. Only \( a \lesssim 10^{-7} \text{ cm} \) grains spin at frequencies \( \nu \gtrsim 10 \text{ GHz} \). The magnetic moment of a spontaneously magnetized grain is just \( M_v \nu = \frac{1}{2}(4\pi M_s a^3 G)(a/10^{-7} \text{ cm})^3 \). The largest value of \( M_v \) is for metallic Fe (see Table 2); an Fe grain would have a magnetic moment \( M_v \nu = 7.3(a/10^{-7} \text{ cm})^3 \), somewhat smaller than the electric dipole moment \( 9.3(a/10^{-7} \text{ cm})^{3/2} \) estimated for neutral grains by Draine & Lazarian (1998b). Other magnetic materials have smaller values of \( M_v \), and hence the rotational emission at \( \nu \gtrsim 10 \text{ GHz} \) is expected to be dominated by electric dipole radiation.

8.1. Candidate Materials

We consider four possible components of the interstellar grain population:

1. 100% of the Si and Fe incorporated into amorphous silicate grains with paramagnetic behavior as in Figure 2.
2. 100% of the Fe incorporated into small Fe\(_3\)O\(_4\) grains.
3. 5% of the Fe incorporated into bare 1:1:5:2 Fe ellipsoids.
4. 5% of the Fe incorporated into small spherical Fe inclusions.

For each case we assume a grain temperature \( T = 18 \text{ K} \). If \( V \) is the volume per H atom for a grain component, then its magnetic dipole contribution to the emissivity per H atom is

\[
\frac{j_v}{n_H} = \frac{n_{\text{gr}}}{n_H} \langle C_{\text{abs}}^{\text{md}} \rangle B_v(T),
\]

where we must remember that there may be additional electric dipole emission (see eqs. [1]–[3]).

The magnetic dipole emissivity for each grain component is shown in Figure 7; also shown is the estimate for the “vibrational” or electric dipole emission.

Sensitive studies of the microwave sky brightness have revealed microwave emission from interstellar matter; the emissivity per H nucleon has been deduced from the cross-correlation of the microwave sky brightness with far-infrared emission from dust grains, using measurements from the COBE DMR (Kogut et al. 1996), the ground-based Saskatoon experiment (de Oliveira–Costa et al. 1997), and Owens Valley Radio Observatory (Leitch et al. 1997). These observational results are shown in Figure 7.

We see that a population of Fe grains or grains with Fe inclusions would be predicted to produce very strong emission near 78 GHz, with a broad peak occurring near the Fröhlich resonance for Fe spheres. The 90 GHz emission reported by Kogut et al. (1996) appears to limit the amount of interstellar Fe in metallic form to perhaps \( \lessapprox 5\% \) of the total Fe. In particular, this limits the fraction of interstellar Fe that can be present in Fe-Ni inclusions such as found in “GEMS” (Bradley 1994; Martin 1995).

8.2. Hypothetical Materials

We have estimated the emission expected from pure iron and magnetite, but the true state of Fe in interstellar grains is not known. It is possible that the Fe is concentrated in a form that is magnetic—not so strongly as pure iron, but more strongly than ferrimagnetic magnetite. This could, for example, be an Fe/Ni alloy with an appreciable concentra-
tion of O, H, Si, or other impurities. We now ask whether it is possible for magnetic dipole emission to account for the observed emission in the 14–90 GHz range. We approach this question by seeking to “customize” the magnetic properties, within the range of reasonable parameters, to see whether there are possible values that would lead to the observed emission. We make the following assumptions:

1. The magnetic material is in (spontaneously magnetized) single-domain grains, with a susceptibility \( \chi_s \) perpendicular to the direction of spontaneous magnetization.

2. The frequency dependence of the susceptibility is given by equation (27).

3. The low-frequency susceptibility \( \chi_s(0) \) should be somewhat smaller than the value \( \chi_s(0) = 3.3 \) that we have estimated for pure iron.

4. The spontaneous magnetization should be appreciably smaller than the value \( 4\pi M_s = 22 \) kG for pure iron. As a result, the characteristic gyrofrequency \( \omega_0/2\pi = (e/m_c)(2M_s/3) \) should be appreciably smaller than the \( \sim 20 \) GHz value estimated for Fe.

5. We assume that nearly 100% of the interstellar Fe is in the hypothetical material \( \chi_s \) and assume the Fe within the material \( X \) to contribute a mass density of \( 4 \text{ g cm}^{-3} \) (thus the volume per H atom of the hypothetical material is \( V_X = 7.5 \times 10^{-28} \text{ cm}^3/\text{H} \); see eq. [21]). Calculations are shown for four hypothetical materials, denoted X1, X2, X3, and X4, with \( \chi_s(0) \) and \( \omega_0 \) values as shown in Figure 8. In Figure 8 we see that in order to contribute an appreciable fraction of the observed 14–90 GHz emission, the hypothetical magnetic material must have \( \chi_s(0) \approx 2 \) and \( \omega_0/2\pi \approx 6 \) GHz. Note that material X1 does not have a Fröhlich resonance, and materials X2, X3, X4, with \( \chi_s(0) = 2 \), have only a weak Fröhlich resonance (see footnote 5).

We conclude that if a large fraction of the interstellar Fe were incorporated into a moderately strong magnetic material with properties approximating those of our hypothetical material X4, then the thermal emission from these grains would approximately reproduce the observed thermal emission in the 14–90 GHz range. We stress that we are not arguing that such material exists—indeed, we have previously shown (Draine & Lazarian 1998a, 1998b, hereafter DL98a and DL98b) that the observed emission seems likely to be due to rotational emission from a population of ultrasmall grains believed to be present for independent reasons. The point of the present discussion is to show that one cannot exclude the possibility that magnetic grains make an appreciable contribution to the emission in this frequency range. This issue should be resolved through observations of dark clouds (see § 10).

9. POLARIZATION

From Figure 7 it is seen that if a small fraction of interstellar Fe is in iron grains, these could account for a substantial fraction of the diffuse emission in the 50–100 GHz region. Alternatively, we see in Figure 8 that if a large fraction of the Fe is in a magnetic material with the properties of our hypothetical material “X4,” then a substantial fraction of the 14–90 GHz emission could be magnetic dipole emission from such grains.

In recent years there has been significant progress toward understanding the alignment of interstellar grains. Analysis of the “crossover” phenomenon for grains subject to superthermal rotation now indicates that even ordinary paramagnetic relaxation may suffice to align larger (\( a > 10^{-5} \) cm) grains (Lazarian & Draine 1997), but radiative torques due to starlight appear to dominate those due to paramagnetic relaxation (Draine & Weingartner 1997). If strongly magnetic grains are present (e.g., iron or our hypothetical material X4), it appears likely that ferromagnetic relaxation would effectively align their angular momenta with the galactic magnetic field \( B_0 \), so that their “long” axes would tend to be perpendicular to \( B_0 \). If such grains are present, we then expect frequency-dependent polarization of the microwave emission from interstellar dust.

To estimate the likely polarization, we assume the grains to be ellipsoids with semiaxes \( a_1 \leq a_2 \leq a_3 \), spinning with the short axis \( a_3 \) parallel to the angular momentum \( J \) (as expected for suprathermally rotating grains; see Purcell 1979). We assume “perfect” alignment of \( J \) with the galactic magnetic field \( B_0 \), with \( B_0 \) perpendicular to the line of sight; the polarization of emitted radiation will be taken to be positive when the electric vector is perpendicular to \( B_0 \). For imperfect alignment, the polarization for perfect alignment should be multiplied by the “Rayleigh reduction factor” (Lee & Draine 1985)

\[
R = \frac{3}{2} \left( \cos^2 \theta - \frac{1}{3} \right),
\]

where \( \theta \) is the angle between \( B_0 \) and \( J \).

The predicted polarization depends on assumptions concerning the grain structure. We will consider two limiting cases.

![Figure 8: Predicted thermal emission per H nucleon of interstellar dust for hypothetical Fe-rich materials X1-X4. In each case we assume that a fraction \( f_x = 1 \) of interstellar Fe is in a substance with \( \rho_x z_x = 4 \text{ g cm}^{-3} \) (see eq. [21]), with an assumed static susceptibility \( \chi_s \) and characteristic gyrofrequency \( \omega_0 = 10^6 \omega_0 \) s\(^{-1}\). Observed emission is as described in Fig. 7. Thermal emission from hypothetical material X4 could account for a substantial fraction of the observed 14–90 GHz emission. Also shown is the estimated “vibrational” electric dipole emission from interstellar dust with \( C_{abs}^{\text{eff}} \ll v_1 \).](image)
9.1. Single-Domain Grains

Suppose the grains that dominate the 10–100 GHz emission each consist of a single magnetic domain.\(^6\) We will assume the grains to have spontaneously magnetized with \(M_S\) along the long axis \(\hat{a}_3\) (this minimizes the magnetic energy). As discussed above in § 7.1, the permeability \(\mu\) is anisotropic: \(\mu = 1\) for \(H \parallel M_S\), and \(\mu = \mu_{\perp}\) for \(H \perp M_S\). Thus the degree of polarization \(P = (I_e - I_h)/(I_e + I_h)\) where

\[
I_e = \frac{1}{2} \left[ \frac{\varepsilon_2}{|1 + L_2(\varepsilon - 1)|^2} + \frac{\varepsilon_2}{|1 + L_3(\varepsilon - 1)|^2} \right] + \frac{\mu_{\perp,2}}{|1 + L_1(\mu_{\perp} - 1)|^2},
\]

\[
I_h = \frac{\varepsilon_2}{|1 + L_2(\varepsilon - 1)|^2} + \frac{1}{2} \frac{\mu_{\perp,2}}{|1 + L_2(\mu_{\perp} - 1)|^2}.
\]

The dielectric function of metallic iron can be approximated at low frequencies by a Drude model with \(\omega_0 = 0\):

\[
\varepsilon(\omega) \approx \frac{i(\omega_0 \tau)^2}{\omega \tau - i(\omega_0 \tau)^2},
\]

where \(\tau \approx 3.8 \times 10^{-14}\) s and \(\omega_0 \tau \approx 200\). This approximately reproduces the tabulated values of \(\varepsilon\) (Palik 1991). With this dielectric constant, the microwave emission is dominated by magnetic dipole radiation (\(|\varepsilon| > 10^5\) for \(v < 100\) GHz), so that \(C_{abs}(\text{md}) \ll C_{abs}(\text{rad})\) and the precise value of \(\varepsilon\) is not critical.

The resulting frequency-dependent polarization \(P\) is shown in Figure 9 for different grain shapes. For both Fe or X4 grains, at high frequencies (\(v \gtrsim 100\) GHz for Fe, \(v \gtrsim 25\) GHz for X4) the polarization is large (\(P = \frac{1}{2}\)) and positive (i.e., \(E\) perpendicular to the “short axis” of the grain). At these high frequencies \(|\mu_{\perp} - 1|^2 \ll 1\) so that the values of the “shape factors” \(L_1 > L_2 > L_3\) are unimportant—the calculated polarization arises because the thermal fluctuations in the magnetization are perpendicular to \(\hat{a}_3\); since \(\hat{a}_3 \perp \mathbf{J}\) the magnetic dipole emission tends to have \(\mathbf{H} \parallel \mathbf{J}\), and hence \(E \perp \mathbf{a}_3\), i.e., “positive” polarization.

At lower frequencies (\(\lesssim 50\) GHz for Fe, \(\lesssim 14\) GHz for X4) \(|\mu - 1| > 1\) and shape effects now matter. There are still no magnetic fluctuations parallel to \(\hat{a}_3\), but if \(a_1\) is sufficiently small compared to \(a_2, L_1\) will be considerably larger than \(L_2\), and the magnetic dipole emission will tend to have \(\mathbf{H} \perp \hat{a}_3\), hence \(E \parallel \mathbf{a}_3\), for “negative” polarization. This explains the behavior seen in Figure 9.

9.2. Grains With Magnetic Inclusions

Suppose the grains to be ellipsoids containing isotropically oriented magnetic inclusions with filling factor \(\phi \approx 0.03\), as found for GEMS (see § 6.3). Since the grain material is now isotropic, the polarization is given by equations (8)–(10).

If the inclusions are metallic Fe, and contain \(N > 10^4\) atoms, we can use the permeability \(\mu(\omega)\) shown in Figure 5. The resulting polarization for perfectly aligned spinning grains is shown in Figure 9. Thermal emission from these grains is predominantly magnetic dipole radiation; since the grain material is isotropic,\(^7\) the emitted radiation tends to have \(\mathbf{H}\) perpendicular to the short axis \(\hat{a}_3\). However, since \(|\mu - 1| \ll 1\) (see Fig. 5), the \(L_2\)-dependent term in equation (9)–(10) is only a small correction; hence \(I_e \approx I_h \approx \varepsilon_2 + \mu_{\perp,2}\) and the polarization \(P = (I_e - I_h)/(I_e + I_h)\) is small.

10. DISCUSSION

Fe is the ninth most abundant element by number (after H, He, O, C, N, Ne, Mg, and Si), and fifth by mass (after H, He, O, C), and nearly all of the interstellar Fe is in dust grains. It is therefore inevitable that some fraction of the interstellar grain material must be quite Fe rich. We do not know what mineral form the Fe mainly resides in (a number of possibilities are given in Table 2), but the material will be at least paramagnetic, and quite possibly ferrimagnetic or ferromagnetic.

It is therefore important to consider the emission of electromagnetic radiation as a result of thermal fluctuations in the magnetization of the grain material. These fluctuations take place at microwave frequencies and below (the magnetization is constant on timescales short compared to the time for electrons to precess in the internal magnetic fields

\(^6\) Recall that Fe grains smaller than \(a_1 \approx 3 \times 10^{-6}\) cm always consist of a single domain (Morrish 1980).

\(^7\) If the number of magnetic inclusions in a grain is small, the material within each grain will not, strictly speaking, be isotropic. However, if the orientations of individual inclusions are statistically independent of the overall grain shape, it is appropriate to approximate the grain material as isotropic in order to estimate the emission from the grain ensemble.
within the material; these precession frequencies are at most \( \sim 20 \) GHz the value for metallic iron). As a result, thermal magnetic dipole emission from interstellar grains (arising from thermal fluctuations in the orientations of electron spins within the grain) may be stronger than the thermal electric dipole “vibrational” emission (arising from thermal fluctuations in the charge distribution within the grain) at frequencies \( \leq 100 \) GHz.\(^8\) We show how this magnetic dipole emission can be calculated, but the emission requires knowledge of the magnetic permeability at these frequencies. We have estimated this permeability for various materials of interest, including metallic iron and magnetite (\( \text{Fe}_3\text{O}_4 \)), but have been required to extrapolate to the high frequencies of interest. It would be of great value to have direct measurements of the magnetic properties of small Fe particles at 50–100 GHz.

According to our estimates, metallic iron would produce such strong emission near \( \sim 70 \) GHz that not more than \( \sim 5\% \) of the Fe can be in the form of pure metallic iron in order not to exceed the observed emission at these frequencies. This limits the fraction of interstellar grains with Fe-Ni inclusions such as in the GEMS found by Bradley (1994).

Magnetite, on the other hand, would radiate only a fraction of the power emitted by interstellar grains in the 14–90 GHz region. However, it is possible that interstellar Fe could be present in some Fe-rich substance (e.g., an Fe/Ni alloy with an appreciable concentration of Mg, Si, O, H impurities) with magnetic properties intermediate between those of metallic iron and magnetite, and we therefore cannot rule out the possibility that the bulk of the observed emission in the 14–90 GHz range could be thermal magnetic dipole radiation. It would be of great value to have laboratory measurements on various plausible candidate materials (e.g., amorphous olivine, or Fe-containing alloys) at these frequencies.

Rotational emission from spinning dust grains has previously been proposed as the explanation for the observed 15–90 GHz emission from dust in diffuse clouds (DL98a, DL98b). There are two ways in which this new emission mechanism—magnetic dipole radiation from thermal fluctuations in grain magnetization—can be distinguished from the hypothesized rotational emission:

1. The rotational emission requires ultrasmall grains. Since we have reason to believe that ultrasmall grains are depleted in dense regions, we would then expect the rotational emission to be relatively weak in dense gas. On the contrary, thermal magnetic dipole emission from dust should be largely unaffected by coagulation of the dust grains. Therefore, observations of (or upper limits on) the 10–60 GHz emission from dense clouds can be used to distinguish between rotational emission from ultrasmall grains or thermal magnetic dipole emission from magnetic grain materials.

2. If produced largely by single-domain grains, the magnetic dipole emission potentially has a complex and strong polarization signature, which could be very different from the polarization expected from spinning dust grains that have been partially aligned by magnetic dissipation (Lazarian & Draine 1999).

11. SUMMARY

The principal results of this paper are as follows:

1. Formulae are presented for electric and magnetic dipole absorption cross sections for homogeneous spheres or ellipsoids of material with dielectric function \( \varepsilon \) and magnetic permeability \( \mu \).

2. The Kramers-Kronig relation relating the total grain volume to \( \int_0^\infty C_{\text{ext}}(\omega)d\omega \) is generalized to include the case of magnetic grains. The resulting equation (18) shows that if an appreciable fraction of the grain material is magnetic, then magnetic dipole effects must dominate the extinction cross section at the frequencies \( \nu \lesssim 30 \) GHz where plausible grain materials have a magnetic response.

3. Because most of the Fe is in solid form in the interstellar medium, it is expected that some fraction of the interstellar grain population must be appreciably magnetic—either paramagnetic, superparamagnetic, ferrimagnetic, or ferromagnetic. The frequency-dependent magnetic properties of candidate materials are estimated.

4. The magnetic dipole emission from either paramagnetic grains or magnetite (\( \text{Fe}_3\text{O}_4 \)) grains is well below the expected electric dipole emission due to vibrational modes at \( \nu \gtrsim 100 \) GHz, or rotational emission from very small grains at \( \nu \lesssim 100 \) GHz.

5. Fe grains or inclusions will have the magnetic analog of a Fröhlich resonance at \( \sim 70 \) GHz, where the magnetic dipole absorption cross section for a sphere peaks because \( \mu_1 = -2 \). This will result in strong magnetic dipole absorption from ferromagnetic iron particles, with a broad peak near \( \sim 70 \) GHz.

6. If our estimate for the magnetic susceptibility of single-domain Fe is correct, not more than \( \sim 5\% \) of interstellar Fe can be in the form of metallic iron; otherwise the thermal magnetic dipole emission from interstellar dust at 90 GHz would greatly exceed the emission from dust measured by Kogut et al. (1996).

7. If a substantial fraction of interstellar Fe is in a moderately strong magnetic material with the properties resembling our hypothetical material “X4” in Figure 8, then thermal emission from such grains could account for a substantial fraction of the observed 14–90 GHz emission. At this time this possibility cannot be excluded, even though the 14–90 GHz emission seems most likely due to spinning ultrasmall dust grains (Draine & Lazarian 1998a, 1998b).

8. If nonspherical single-domain ferromagnetic grains are present, the magnetic dipole emission will be polarized. Such grains are expected to be spontaneously magnetized along the “long” axis. The polarization is expected to depend strongly on frequency, with “normal” polarization (electric vector perpendicular to \( B_0 \)) at frequencies \( \gtrsim 100 \) GHz, but with a decrease, and perhaps even reversal (depending upon grain shape), of the polarization for \( \lesssim 30 \) GHz.

We thank Phil Myers and Wayne Roberge for helpful discussions and Robert Lupton for the availability of the SM package. B. T. D. acknowledges the support of NSF grant AST-96 19429, and A. L. acknowledges the support of NASA grants NAG 5-2858 and NAG 5-7030.

\( ^8 \) Rotational electric dipole radiation from rapidly rotating ultrasmall grains is also important at these frequencies (Draine & Lazarian 1998a, 1998b).
APPENDIX A

MICROWAVE ABSORPTION DUE TO STARK-EFFECT SPLITTING

The electric fields within a solid—whether crystalline or amorphous—can split the magnetic sublevels of paramagnetic ions in the solid. Kittel & Luttinger (1948) noted that whereas these splittings usually correspond to frequencies \( \sim 10^{13} \text{ Hz} \), under some circumstances the splittings correspond to microwave frequencies. To estimate the magnitude of the associated absorption, suppose that the solid contain a density \( dn \) of species with energy levels split by frequencies in the interval \( d\omega \), and let \( |\mu_{ul}|^2 \) be the magnetic dipole matrix element between the upper and lower states, giving a spontaneous decay rate

\[
A_{ul} = \frac{4\omega^3}{3\hbar^3} |\mu_{ul}|^2 .
\]

If \( \omega \ll kT/\hbar \), then \( d(n_l - n_u) = (\hbar\omega/2kT)dn \). The absorption coefficient is

\[
\alpha = \frac{2\pi^2}{3} \left( \frac{\hbar\omega}{kT} \right) \frac{dn}{d\ln \omega} \frac{|\mu_{ul}|^2}{\hbar c} ,
\]

(A2)

corresponding to an imaginary component of the magnetic susceptibility

\[
\mu_2 = \frac{c}{\omega} \alpha = \frac{2\pi^2}{3} \left( \frac{|\mu_{ul}|^2}{kT} \right) \frac{dn}{d\ln \omega} ,
\]

(A3)

\[= 2.7 \times 10^{-9} \left( \frac{|\mu_{ul}|^2}{\mu_B^2} \right) \left( \frac{15 \text{ K}}{T} \right) \left( \frac{dn/d\ln \omega}{10^{22} \text{ cm}^{-3}} \right).\]

(A4)

where \( \mu_B \) is the Bohr magneton. The total atomic density \( \sim 10^{23} \text{ cm}^{-3} \) in the solid, so \( dn/d\ln \omega = 10^{22} \text{ cm}^{-3} \) would correspond to 10% of the atoms having resonances within a frequency interval \( \Delta \ln \omega = 1 \). Such a high value would probably only occur if the Stark-effect splittings are concentrated near a few frequencies.

APPENDIX B

SUSCEPTIBILITY OF SINGLE-DOMAIN FERROMAGNETIC PARTICLES

Consider a spherical particle consisting of a single domain. At temperatures \( T \ll T_c/2 \), where \( T_c \) is the Curie temperature, we may take the spontaneous magnetization to be approximately equal to the saturation magnetization \( M_s \). Suppose the crystal has cubic symmetry. The free energy is a function of the direction of the magnetization. If \( x_1, x_2, \) and \( x_3 \) are the direction cosines of the magnetization relative to the crystal axes, the “anisotropy” free energy is

\[
F_K = K_1(x_1^2 x_2^2 + x_2^2 x_3^2 + x_3^2 x_1^2) + K_2 x_1^2 x_2^2 x_3^2,
\]

(B1)

where \( K_1 \) and \( K_2 \) are the anisotropy constants (Morrish 1980).

\[\text{B1. } K_1 > 0, K_2 > 0\]

If \( K_1 > 0 \) and \( K_2 > 0 \), then the “easy” directions are along the cubic axes (e.g., \( \langle 100 \rangle \)). If we now apply a weak magnetic field \( H_{\perp} \) perpendicular to the magnetization, the magnetization will deflect by an angle \( \theta \). For weak fields, \( \theta \ll 1 \) and the anisotropy energy and magnetic energy become

\[
F_K = K_1 \theta^2 + O(\theta^4) ,
\]

(B2)

\[
F_H = -M \cdot H = M_s H_{\perp} \theta + O(\theta^2) .
\]

(B3)

Minimizing \( (F_K + F_H) \), we find \( \theta = M_s H_{\perp} / 2K_1 \). Thus, the transverse susceptibility \( \chi_{\perp} = M_s \theta / H_{\perp} \) is

\[
\chi_{\perp}(0) \approx \frac{M_s \theta}{H_{\perp}} = \frac{M_s^2}{2K_1} .
\]

(B4)

For Fe, \( M_s = 1750 \text{ G} \), and \( K_1 = 4.6 \times 10^5 \text{ ergs cm}^{-3} \) (Morrish 1980), thus \( \chi_{\perp}(0) = 3.3 \) and \( \mu_{\perp}(0) = 1 + 4\pi\chi_{\perp}(0) = 43 \). This is significantly smaller than the value \( \mu(0) \approx 150 \) characterizing bulk (multidomain) Fe.

\[\text{B2. } K_1 < 0, K_2 < 0\]

When \( K_1 < 0, K_2 < 0 \), the “easy” directions are along the diagonals (e.g., \( \langle 111 \rangle \)). The anisotropy energy is now

\[
F_K = K_1 \left( \frac{1}{2} - \frac{\theta^2}{2} \right) + K_2 \left( \frac{1}{2} - \frac{\theta^2}{2} \right) + O(\theta^4) ,
\]

(B5)
with $F_\perp$ still by equation (B3). Again minimizing $(F_K + F_H)$ we find $\theta = -9M_s H_\perp/[12K_1 + 4K_2]$ and

$$\chi_\perp(0) \approx \frac{-9M_s^2}{12K_1 + 4K_2}.$$  \hspace{1cm} (B6)

Maghemite ($\gamma$Fe$_2$O$_3$) is cubic with $4\pi M_s = 4780$ G (Dunlop & Özdemir 1997). The magnetic properties do not appear to have been measured at low temperatures; at room temperature, $K_1 = -2.5 \times 10^5$ ergs cm$^{-3}$ (Valstyn et al. 1962), from which we estimate $\chi_\perp(0) \approx 0.43$. As discussed in § 7.5 above, a slightly larger value of $\chi_\perp(0) \approx 0.6$ is in agreement with measurements by Valstyn et al. (1962) at GHz frequencies.

### B3. OSCILLATING FIELDS

The above discussion applies to static fields. For oscillating fields, we estimate $\chi_\perp(\omega)$ using equation (27), with $\omega_0 = (e/m_e c)(4\pi M_a/3)$, the precession frequency of an electron in the internal field $4\pi M_a/3$.

### REFERENCES

- Allanson, J. T. 1945, J. Inst. Elec. Engrs, 92, part III, 247
- Anders, E., & Grevesse, N. 1989, Geochim. Cosmochem. Acta, 53, 197
- Bean, C. P., & Livingston, J. D. 1959, J. Appl. Phys., 30, 1208
- Billas, I. M., Châtelain, A., & de Heer, W. A. 1994, Science, 265, 1682
- Bohren, C. F., & Huffman, D. R. 1983, Absorption and Scattering of Light by Small Particles (New York: Wiley)
- Bradley, J. P. 1994, Science, 265, 925
- Bramley, R., & Strach, S. J. 1983, Chem. Rev., 83, 49
- Carmichael, R. S. 1989, in CRC Practical Handbook of Physical Properties of Rocks and Minerals, ed. R. S. Carmichael (Boca Raton: CRC), 301
- Caspers, W. J. 1964, Theory of Spin Relaxation (New York: Interscience)
- Davis, L., & Greenstein, J. 1951, ApJ, 114, 206
- de Oliveira–Costa, A., Kogut, A., Devlin, M. J., Netterfield, C. B., Page, L. A., & Wollack, E. J. 1997, ApJ, 482, L17
- Draine, B. T. 1996, in ASP Conf. Ser. 97, Polarimetry of the Interstellar Medium, ed. W. G. Roberge & D. C. B. Whittet (San Francisco: ASP), 16
- Draine, B. T., & Lazarian, A. 1998a, ApJ, 494, L19 (DL98a)
- ______. 1998b, ApJ, 508, 157 (DL98b)
- Draine, B. T., & Lee, H. M. 1984, ApJ, 285, 89
- Draine, B. T., & Weingartner, J. C. 1997, ApJ, 480, 633
- Duff, E. J. 1968, J. Chem Soc. A, 2072
- Duley, W. W. 1978, ApJ, 219, L129
- Dunlop, D. J., & Özdemir, Ö. 1997, Rock Magnetism (Cambridge: Cambridge Univ. Press)
- Epstein, D. J. 1954, in Dielectric Materials and Applications, ed. A. von Hippel (New York: Wiley), 122
- Fröhlich, H. 1949, Theory of Dielectrics (London: Oxford Univ. Press)
- Gaustad, J. E., McCullough, P. R., & Van Buren, D. 1996, PASP, 108, 351
- Goodman, A. A., & Whittet, D. C. B. 1995, ApJ, 455, L181
- Jacobs, I. S., & Bean, C. P. 1963, in Magnetism, III, ed. G. T. Rado & H. Suhl (New York: Academic), 271
- Jones, R. V., & Spitzer, L. 1967, ApJ, 147, 943
- Kittel, C., & Luttinger, J. M. 1948, Phys. Rev., 73, N2, 162
- Kogut, A., Banday, A. J., Bennett, C. L., Gorski, K. M., Hinshaw, G., & Reach, W. T. 1996, ApJ, 460, 1
- Landau, L. D., & Lifshitz, E. M. 1960, Electrodynamics of Continuous Media (New York: Pergamon)
- Lazarian, A. 1995, ApJ, 453, 229
- Lazarian, A., & Draine, B. T. 1997, ApJ, 487, 248
- ______. 1999, in preparation
- Lee, H.-M., & Draine, B. T. 1985, ApJ, 290, 211
- Leitch, E. M., Readhead, A. C. S., Pearson, T. J., & Myers, S. T. 1997, ApJ, 486, L23
- Martin, P. G. 1975a, ApJ, 201, 373
- ______. 1975b, ApJ, 202, 389
- ______. 1995, ApJ, 445, L63
- Mathis, J. S. 1986, ApJ, 308, 281
- Morrish, A. H. 1980, The Physical Principles of Magnetism (New York: R. E. Krieger)
- Pake, G. E. 1962, Paramagnetic Resonance (New York: W. A. Benjamin)
- Palik, E. D. 1991, Handbook of Optical Constants of Solids II (New York: Academic)
- Purcell, E. M. 1969, ApJ, 158, 433
- ______. 1979, ApJ, 231, 404
- Savage, B. D., & Sembach, K. R. 1996, ARA&A, 34, 279
- Schuele, W. J., & Deetscreek, V. D. 1962, J. Appl. Phys., 33, 1136S
- Shapiro, P. R. 1975, ApJ, 201, 151
- Sorrell, W. H. 1994, MNRAS, 268, 40
- Spitzer, L., & Tukey, J. W. 1951, ApJ, 114, 187
- Syono, Y., & Ishikawa, Y. 1963, J. Phys. Soc. Japan, 18, 230
- Valstyn, E. P., Hanton, J. P., & Morrish, A. H. 1962, Phys. Rev., 128, 2078