Entanglement and Complexity of Measurement

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Abstract. Notion about complexity of composite systems is applied to the problem of measurement of quantum states. An expression for the determination of the value of complexity of measurement is given. Its practical meaning is illustrated in the example how this value differs when measuring complexity of authorized and non-authorized receiving of information transferred by quantum channels in protocol of direct coding.

Unconditional security of information in direct coding protocol with information masking by uniformly distributed state of carrier is shown.

1. Introduction
In the algorithm theory, the concept of computational complexity [1] that characterizes the number of typical actions that form the computational algorithm is actively used.

This value makes it possible to analyze cryptographic stability of coding algorithms with asymmetric key that are actively used today and to evaluate the key length that provides unreasonably long time for unauthorized decoding with existing computational resources.

Adaptation of the notion of complexity to physical problems, proposed recently [2], is restricted by classical mechanics. Better correspondence with the notion of complexity one finds in the case of quantum systems, with the simplest ones, like spin degree of freedom of electron, or polarization degree of freedom of photon, among those: both are characterized by two-dimensional Hilbert space and are combined by common name “qubit”. Complexity of qubit leads to impossibility of state cloning [3]: one measurement of an observable is not enough for reconstruction of state. Values needed for reconstruction can be measured as a result of long enough series of measurements only. Length of series can be used as a measure of complexity of a quantum system.

Hereinafter expression for determination of the value of complexity of measurement is proposed, and its values that characterize different processes of measurement of states of quantum systems are considered.

Measurement of state of quantum system is a component part of the problem on reconstruction of quantum state [4] that is actively studied because of practical realization of information transfer by quantum carriers.

The present article has the following structure:

In the section “Measurement of state: observables” information about observables measurement of which makes it possible to reconstruct a quantum state, pure or mixed, completely, is given. In particular, entangled states are considered.
In the section “Process of measurement of state” measurement of a state is represented as a sequence of simple events of measurement that are characterized by preparation and detection of pure states that in aggregate represent the state under consideration.

In the section “Complexity of measurement” expression for determination of this value is given, with evaluation of complexity for measurement of probability of some event. Main result that is in evaluation of complexity of measurement for the problem of reconstruction of a quantum state is formulated as well. Explicit expression for dependence of complexity of measurement on dimension of state space is obtained, and it is shown that at non-demolition measurements complexity of measurement tends to zero.

In the section “Direct coding”, as an illustration of effectiveness of evaluation of the complexity of measurement, difference between the complexities of the Eve’s and Bob’s measuring tasks in the protocol of direct coding with feedback is considered. It is shown that masking of the message transferred by a sequence with an equilibrium density matrix makes the problem of intercept of information unsolvable.

2. Measurement of state: observables

Quantum systems with $N$-dimensional Hilbert state are considered. The state is described by density matrix $\rho = \sum \rho_{n,n'} |n\rangle \langle n'|$, components of which are determined by $N^2 - 1$ real numbers. Reconstruction of state can be realized only in the case if the results of measurements make it possible to calculate the values of all those numbers.

2.1. Measuring basis of algebra of observables

Measurement of an observable is mathematically modeled by a system of projectors $D_n$, giving resolution of identity $\sum D_n = \hat{1}$. Operator of observable has representation by a weighted sum of projectors $\hat{O} = \sum O_n D_n$. Each of projectors projects a state into eigenstate $|n\rangle$ of the observable. As the result of measurements, one obtains $N - 1$ independent real numbers – probabilities $p_n = \langle n | \rho | n \rangle$ of registration of eigenstates of observable. To calculate the values of all the components of density matrix, one has to measure $N + 1$ different incompatible observables. Set of observables enough for reconstruction of state can be constructed [5] by the method of ladder operators.

Let $\{ |n\rangle, n = 1 \ldots N \}$ is an orthonormal basis. Let us define the ladder operators

$$J_- = \sum_{n=1}^{N} \sqrt{n(N-n+1)} |n-1\rangle \langle n|, \quad J_+ = J_-^\dagger, \quad J_3 = \frac{1}{2} [J_+ J_-]$$

(1)

and the measuring basis of the algebra of observables

$$B_0 = J_3, \quad B_m = \frac{e^{i\pi(m-1)/2N}}{\sqrt{2}} J_+ + \frac{e^{-i\pi(m-1)/2N}}{\sqrt{2}} J_- : \quad m = 1 \ldots N.$$  

(2)

The results of measurement of averages and moments up to $N - 1$ make it possible to calculate all the components of density matrix as a solution of a system of linear equations:

$$R_{m,p} \equiv \langle \rho B_m^p \rangle = \sum_{n,n'=1}^{N} \rho_{n,n'} \langle n' | B_m^p | n \rangle, \quad m = 0 \ldots N, \quad p = 1 \ldots N - 1.$$  

(3)

If calculated density matrix is not degenerate, i.e all its eigenvalues $p_n$ are different, one can calculate as well the basis (unique) in which the density matrix is diagonal. In the case of degenerate density matrix the basis in which the density matrix is diagonal is determined up to the group of orthogonal transformations of degeneration subspace. In the specific case of equilibrium state $\rho = \frac{1}{N} \hat{1}$ the density matrix is diagonal in any basis.
2.2. Measuring basis of a composite system

Each subsystem of a composite system is characterized by algebra of local observables \( \mathcal{A}_k \) operating in the space of local states \( \mathcal{H}_k \).

Sets of local operators \( \{ L_x^{(k)} , \ x = 3, +, - \} \), similar to (1), generate measuring bases of local observables \( \{ B_n^{(k)} , \ n_k = 0, \ldots, N_k \} \), the sum of operators

\[
J_x = \sum_{\forall k} L_x^{(k)}
\]  

(4)

generates the measuring basis \( \{ B_n, \ n = 0, \ldots, N_1 \cdot \ldots \cdot N_K \} \) of the algebra of observables of composite system.

Arbitrary state of a composite system belongs to one of three types of states:

- States of independent subsystems: density matrix of a composite state is equal to direct product of density matrices of subsystems \( \rho = \rho_A \otimes \rho_B \);
- States of a mix of independent subsystems: density matrix of a composite state is equal to weighted mix of direct products of density matrices of subsystems \( \rho = \sum p_k \rho_A^{(k)} \otimes \rho_B^{(k)} \);
- All the other states are the states of entangled subsystems (entangled states).

Measurement of the average values and moments of the measuring basis of local observables is enough for calculation of the density matrix of the simplest type of composite system states – states of independent subsystems only. More complicated state of a mix of independent subsystems can be calculated if measurement of local observables is completed by measurement of correlation of non-demolition local observables.

Reconstruction of entangled states is possible as a result of measurement of correlation of over-classical observables, in addition to solving the problem on reconstruction of the subsystem states. This correlation exceeds the correlation possible by probability theory (Bell’s inequalities).

3. Process of measurement of a state

Statistical observable is measured in sequence of events of measurement (trials). For each event a source constructed in respective way produces the measured system in a given state (prepares the state). The state prepared in each event is a pure state. Prepared sequence of states

\[
S_n = \{ |n_1 \rangle \langle n_1|, \ldots, |n_k \rangle \langle n_k|, \ldots, |n_K \rangle \langle n_K| \}
\]  

(5)

is characterized by length \( K \), by the set of states (alphabet) \( S = \{ S_n = |n \rangle \langle n|, n = 1, \ldots, N \} \) and the set of initial repetition rates \( \nu_n = \frac{K}{K} \) of each state.

Density matrix of prepared sequence is a weighted sum of density matrices of prepared states:

\[
\rho = \frac{1}{K} \sum_{k=1}^K |n_k \rangle \langle n_k| = \sum_{n=1}^N \nu_n |n \rangle \langle n|.
\]  

(6)

Sequences that have same repetition rates and differ by the order of states only, are characterized by same density matrices, and thus are physically indistinguishable. For each given set of repetition rates there exist

\[
K_I = \frac{K!}{\prod_{n=1}^N K_n!}
\]  

(7)
of different sequences that are characterized by the same density matrix, and thus the same 
average values of all the observables.

Sequences differing only by the order of states can be used for transfer of \( K_I \) different 
messages, and include amount of information equal to entropy 
\[
I = \log_2 K_I = - \sum_{n=1}^{N} \nu_n \log_2 \nu_n = - \sum_{n=1}^{N} \rho \log_2 \rho = S.
\] (8)

Sequences of states can differ only by information, if all the occupation numbers \( K_n \) are the 
same; only by physical values of observables, if sequences of the numbers of states are the same 
and alphabets are different; or by both of those. Physical and information components of the 
process of measurement are complementary.

3.1. Simple measurement, reduction of state
In each trial the state is registered by detector that consists of a set of counters that are 
mathematically represented by a set of projectors \( \mathcal{D} = \{ D_m = |m\rangle\langle m|, m = 1, \ldots, N \} \). Probability of registration of a state \( |n\rangle \) by a counter \( |m\rangle \) is given by expression:
\[
P(m|n) = \langle m | \rho_n | m \rangle = |\langle m | n \rangle|^2.
\] (9)

In each event of measurement density matrix is reduced to a pure state density matrix of which is equal to the projector of respective counter:
\[
\rho^{\{\text{reduced}\}}_k = |m\rangle \langle m|.
\] (10)

As the result of measurement one gets a sequence of the counter numbers
\[
\mathcal{R} = \{m_1, \ldots, m_k, \ldots, m_K\},
\] (11)
that can be described by repetition rates \( \mu_m = \frac{K_m}{K} \). The most probable values of repetition 
rates of counter numbers are the values of conditional probabilities weighted by repetition rates 
of the numbers of prepared states:
\[
\mu_m \approx \langle p_m \rangle = \sum_{n=1}^{N} P(m|n) \nu_n.
\] (12)

Reduced density matrix is a weighted sum of density matrices of reduced pure states:
\[
\rho^{\{\text{reduced}\}} = \frac{1}{K} \sum_{k=1}^{K} |m_k\rangle \langle m_k| = \sum_{m=1}^{N} \mu_m |m\rangle \langle m|.
\] (13)

Reduced density matrix differs from the initial one, sequences of the numbers of states differ as 
well. This difference can be revealed by comparison of the initial and resultant sequences.

3.2. Non-demolition measurements
A measurement is a non-demolition one, if the set \( \mathcal{D} \) of projectors that represent the detector is 
equal to the set \( \mathcal{S} \) of density matrices of states prepared by the detector. In this case probability 
\( P(m|n) \) is equal to unity, if the numbers of states are equal, and to zero, if not. So, the resulting 
sequence of numbers of counters is equal to the initial sequence of the numbers of prepared states
\[
\mathcal{R}_{nd} = \{m_1 = n_1, \ldots, m_k = n_k, \ldots, m_K = n_K\},
\] (14)
the message prepared is received by the detector without distortions, and reduced density matrix
is equal to the initial one.

In a more general case the measurement is a non-demolition one, if the reduced density matrix
of the system does not differ from the initial matrix. This is possible, if the projectors of counters
\( D_n \) are along with that the projectors to the eigenvectors of the density matrix \( \rho(n) = p_n |n\rangle \).

Special attention is to be paid to non-demolition measurements for the system with a
degenerate density matrix \( \rho \). In this case there exists a set of not commuting non-demolition
observables, the series of measurements for each of those gives the same reduced density matrix.
Along with that, in each specific event of measurement reduced state can differ from the initial
one. Such weakly non-demolition measurements make it possible to reconstruct the density
matrix on average, though lead to a sequence of results \( (11) \) differing from the initial sequence
\( (5) \).

Example of a weakly non-demolition measurement. The source prepares the qubit states in the
basis \( S = \{ |0\rangle \langle 0 |, \{|1\rangle \langle 1| \} \} \).

The detector consists of two counters represented in the basis of the source by matrices:
\[
D_{nd} = \left\{ |0\rangle \langle 0 | = \left( \begin{array}{cc}
\frac{1}{2} & -\frac{1}{2} \\
\frac{1}{2} & \frac{1}{2}
\end{array} \right) , |1\rangle \langle 1| = \left( \begin{array}{cc}
\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2}
\end{array} \right) \right\} .
\]

Probability \( P_{m|n} \) of operation of the detector \( m \) under condition that the state \( n \) is prepared
is equal to \( \frac{1}{2} \).

| Table 1. Density matrices for weakly non-demolition measurement. |
|---------------------------------------------------------------|
| \( S_n \) | 0 | 0 | 1 | 1 |
| \( \rho \) | \( \begin{array}{cc}
1 & 0 \\
0 & 0
\end{array} \) | \( \begin{array}{cc}
1 & 0 \\
0 & 0
\end{array} \) | \( \begin{array}{cc}
\frac{2}{3} & 0 \\
0 & \frac{1}{3}
\end{array} \) | \( \begin{array}{cc}
\frac{1}{2} & 0 \\
0 & \frac{1}{2}
\end{array} \) |
| \( \mathcal{R}_n \) | 0 | 1 | 0 | 1 |
| \( \rho^{\text{res}} \) | \( \begin{array}{cc}
1 & 0 \\
0 & 0
\end{array} \) | \( \begin{array}{cc}
\frac{1}{2} & 0 \\
0 & \frac{1}{2}
\end{array} \) | \( \begin{array}{cc}
\frac{2}{3} & 0 \\
0 & \frac{1}{3}
\end{array} \) | \( \begin{array}{cc}
\frac{1}{2} & 0 \\
0 & \frac{1}{2}
\end{array} \) | D-basis
| \( \rho^{\text{res}} \) | \( \begin{array}{cc}
\frac{1}{2} & -\frac{1}{2} \\
\frac{1}{2} & \frac{1}{2}
\end{array} \) | \( \begin{array}{cc}
\frac{1}{2} & 0 \\
0 & \frac{1}{2}
\end{array} \) | \( \begin{array}{cc}
\frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{2} & \frac{1}{2}
\end{array} \) | \( \begin{array}{cc}
\frac{1}{2} & 0 \\
0 & \frac{1}{2}
\end{array} \) | S-basis

An example of a series of measurements is given in table (1). The upper row includes the
numbers of the prepared states, in the second row current values of the initial density matrix are
given. The third row includes the numbers of the states registered by the detector, in the fourth
one the current values of the reduced density matrix in the detector basis are given, in the fifth
row – the values for the matrix in the source basis. From the table it follows that the reduced
density matrix of intermediate states in the source basis can have non-diagonal components
(the first and the third columns). This results from weak reduction of state. Degeneration of
the initial density matrix (the fourth column) is such that any measurement is a weakly non-
demolition one, and the reduced density matrix is equal to the initial matrix up to random
deviation.

3.3. Complex measurement
Complex measurement is carried out at measurement of incompatible observables. In the process
of complex measurement a set of simple detectors \( \{ D_r \} \) is applied, each of detectors represents
an observable

$$ \{ D_r = \{ D_{m:r} = |m : r \rangle \langle m : r|, m = 1, \ldots, N \} , \sum_{m=1}^{N} D_{m:r} = \hat{I} \} . $$

The observables represented by different detectors are incompatible, if at least some of projectors of one detector to not commute with the projectors of another one. Application of a measuring basis (2) as a set of detectors makes it possible to reconstruct all the components of density matrix by the results of the measurement.

In each measurement event (event number $k$) the experimenter performing the measurement chooses one of simple detectors (one of incompatible observables); let the number of this detector is $r_k$. The measured system is registered by a counter with number $m_k$, so the result of the sequence of measurements is a sequence of pairs that consists of the number of the counter $m$ and the number of the detector $r$:

$$ R = \{ [m_1 : r_1] , \ldots , [m_k : r_k] , \ldots , [m_K : r_K] \} . $$

Number of the counter is determined with probability

$$ p_{m|r;k} = \langle m : r_k || n_k \rangle \langle n_k || m : r_k \rangle $$

that depends on the number of simple detector $r_k$ chosen by experimenter and the number of state $n_k$ prepared by the source.

3.4. Measurement of the density matrix

In the process of measurement of the density matrix the average values and moments (3) of all the observables of the measuring basis are to be obtained with enough accuracy. Number of the events of measurement is to be enough for determination of distribution of probabilities of the values of each observable of the measuring basis (2). At that the measurements of $N$ observables from $N + 1$ basic observables are not non-demolition ones, so the sequence of the results (15) differs from the sequence of the source (5). Even at occasional coincidence of eigenvectors of one of the observables with the source basis, only $\frac{1}{N+1}$ part of the resultant sequence can coincide with the sequence of the source.

4. Complexity of measurement

Complexity of measurement is characterized by the number of measurements needed for reconstruction of statistical characteristics of the measured value with given accuracy. Calculation of the components of density matrix by the results of measurement of probability distributions is accompanied by accumulation of ratio error that characterizes the number of significant bits in binary notation of the results.

4.1. Definition of the value

We define the value of complexity of measurement as a logarithm of the number of events of measurement $M(\epsilon)$ needed to obtain the result with given ratio error $s = \frac{\epsilon}{\sqrt{N}}$.

$$ C_M (s) = \log_2 M (s) . $$

Logarithmic scale is convenient for evaluation of complexity of measurement of composite systems: in general case the complexities of measurement are added, in specific cases (for instant, at cloning, when one of sub-systems is a true copy of another one) complexity of measurement can be substantially smaller.
Complexity of measurement is easily calculated in the case of non-demolition measurement for a pure state. In such a case the density matrix of the measured state $|\psi\rangle \langle \psi|$ coincides with the projector into one of eigenvectors $D_N = |n\rangle \langle n|$ of the operator of observable $O = \sum O_n |n\rangle \langle n|$. The result of each event of measurement $\langle O \rangle = \text{Tr} O |\psi\rangle \langle \psi|$ is the same value $O_m$, this result is found in the first event of measurement, and all the following events can not decrease the error. So, at non-demolition measurement one event of measurement is enough, and complexity of measurement is equal to zero.

4.2. Complexity of measurement of probability

Unbiased evaluation of probability of registration of a state $|n\rangle \langle n|$ is the repetition rate $p_n \approx \nu_n = K_n$. Probability $K_n$ of repetitions in a series of $K$ events of measurement is described by Bernoulli law

$$P(K_n) = C^K_{K_n} p_n^{K_n} (1 - p_n)^{(K - K_n)}.$$ 

Error of evaluation of probability by repetition rate is calculated by means of expression for dispersion $V_n = \frac{p_n(1-p_n)}{K_n}$, so the ratio error is

$$\epsilon^2 = \frac{V_n}{p_n^2} \approx \frac{(1 - p_n)}{p_n K},$$

and the number of events of measurement needed to provide the error $s (\epsilon \leq s)$ is given by the expression $K \geq \frac{(1-p_n)}{p_n^2 s^2}$ that leads to the expression for complexity of measurement of probability of a given state:

$$C_M = \log_2 \left( \frac{1}{p_n} - 1 \right) - 2 \log_2 s.$$ 

In Fig. 1 the graphs of dependences of the complexity of measurement of probability on the expected value of probability for several values of ratio error are given. The value of complexity of measurement 10 corresponds to $2^{10} = 1024$ needed events of measurement.

4.3. Complexity of measurement of probability distribution

The events of registration of a state from a set of mutually orthogonal states are complementary, thus the complexity of measurement of probability distribution is less than the sum of complexities of measurement of each probability. All the events that do not contribute to registration of one state, contribute to registration of some other state, and provide a positive result for measurement of probability of that other state. Uncertainties of probability of registration of each state are given by the same expression (17).

Thus, the expression (18) gives the value of complexity of measurement of probability distribution, if as $p_n$ the minimal non-zero value of repetition rate is used:

$$C_M = \log_2 \left( \frac{1}{p_{\min}} - 1 \right) - 2 \log_2 s.$$ 

Numerical evaluation of the value of complexity of measurement is achieved for uniform distribution of probabilities by $N$ possible outcomes, and gives

$$C_{M,u} = \log_2 (N - 1) - 2 \log_2 s.$$
4.4. Complexity of measurement of density matrix

The measurement of density matrix is realized (see 2.1,3.4) through measurement of distributions of repetition rates \( p_{m|n} \) for not less than \( N + 1 \) incompatible detectors. Each of incompatible detectors is used in a separate series of measurements, thus the number of measurements needed for determination of density matrix is given by the sum of the numbers of measurements required for each detector, and complexity of measurement is:

\[
C_M (N, s) = \log_2 \left( \sum_{n=0}^{N} \left( \frac{1}{p_{\min|n}} - 1 \right) \right) - 2 \log_2 s. \tag{21}
\]

Numerical evaluation of the lower boundary of complexity of measurement can be found under supposition on quasi-uniform distribution for which all the probabilities being measured are of one order \( p_{m|n} \approx 1/N \):

\[
C_M (N, s) \approx \log_2 \left( N^2 - 1 \right) - 2 \log_2 s. \tag{22}
\]

The graph of dependence of complexity of measurement on dimension of state space for three values of ratio error is given in Fig. 2. One shall note that complexity of measurement is above 17 for common values of state space dimension (\( N = 8 \)) and the ratio error (\( \epsilon = 0.02 \)). This value corresponds to needed number of measurements 160000, thus practical realization of experimental determination of density matrix of an arbitrary state is not an easy problem.

Complexity of non-demolition measurements is quite different from the complexity of measurement considered above.

At exactly non-demolition measurement of density matrix \( N - 1 \) probabilities \( p_m \) are to be measured, though in each separate event conditional probability equal to zero or unity is measured, and the resultant sequence \( R \) is equal to the initial one \( S \). Thus complexity of an exactly non-demolition measurement of a pure state is zero, and complexity of non-demolition measurement of a mix is given by the same expression (19), this determines the complexity of measurement of classical probability distribution.

Measurement of density matrix of an entangled state is realized through measurement of local observables. For entangled states there is no non-demolition combination of local observables, thus the measurement is complete, and its complexity of measurement is given by (21).
5. Direct coding protocol

In this section evaluation of the value of complexity of measurement is used for a practical problem – analysis of differences in capabilities to obtain information from measurements carried out by an authorized (Bob) and unauthorized (Eve) participants of the process of information transfer by means of the direct coding protocol.

The direct coding protocol [6] is in use of the same sets of states $A = \{|1\rangle (1|, \ldots, |n\rangle (n|, \ldots, |N\rangle (N|\}$ in the Alice’s source and the Bob’s detector. Alice produces entangled states, and Bob registers covariation of special pair of local observables. Thus, complexity of measurement at the Bob’s side is zero.

The backward channel from Bob to Alice is used for compensation of errors.

5.1. Data transfer

Transferred information is masked by the state of carrier with the same repetition rates of all the symbols. For masking one uses the following procedure:

- Alice sends the next symbol of message;
- Bob sends each obtained symbol by backward channel with the same security;
- Alice resends erroneous symbol until returned symbol is correct, or sends the next symbol of the message;
- Having come by the repetition number of a symbol to a given number K, Alice accomplishes the sentence with all the other symbols up to the same number of repetitions K, and so the resultant density matrix of the sequence with length of $KN$ symbols is equivalent to density matrix of white noise $\rho = \frac{1}{N} I$.

Measurement with the set of detectors $D = \{|1\rangle (1|, \ldots, |n\rangle (n|, \ldots, |N\rangle (N|\}$ reproduces in each event of data transfer the symbol, generated by Alice.

5.2. Eavesdropping

It is supposed that the person who makes attempts of eavesdropping is restricted in capabilities by physical laws only, though is not restricted by lack of possibilities of technical realization. There exist several common strategies of eavesdropping, here we list the most typical ones:

- Non-demolition measurements. If Eve knows the set of states used for coding, she can measure a non-demolition observable. Absence of reduction of states prevents reveal of intervention to the process of data transfer.
- Intercept-resent attack. Eve completely intercepts the carrier, measures its state and resends a copy of the carrier in same state to Bob.
- Partitioning. Eve segregates a part of the beam (for instance with a semi-transparent mirror) and measures the state of segregated part. By the results of measurements Eve makes efforts for calculation of the set of carrier states used by Alice. In case of success Eve can adapt her detector like Bob’s detector and copy information.

Use of the backward channel by Alice and Bob means that Eve can attack the backward channel as well.

5.3. Making eavesdropping senseless

- Eavesdropping with non-demolition measurements is made senseless by use of entangled states of the carrier. Measurement of local observables carried out at eavesdropping is accompanied by reduction of states of the carrier, and can be revealed by Bob.
• Intercept-resent attack. Control of time of carrier propagation from Alice to Bob makes this strategy inconsistent.

• Partitioning. Eve carries out a series of measurements of incompatible observables and calculates the density matrix. Complexity of measurement for this problem is given by expression (22) and is large enough. If the density matrix is degenerate (its eigenvalues are equal within inaccuracy of measurements), it is not possible to calculate the states used by Alice and Bob as alphabet. Till Eve does not know the alphabet, the results of her measurements do not represent transferred information.

6. Conclusions
The new physical value – complexity of measurement, proposed here, makes it possible to prove unconditional security of direct coding protocol with masking information in uniform mixed state of carrier. Complexity of measurement of a quantum state depends on dimension of Hilbert space, and on entanglement of states of the sub-systems of composite quantum system.

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