Macroscopic quantum tunneling in multigap superconducting Josephson junctions: Enhancement of escape rate via quantum fluctuations of Josephson-Leggett mode

Yukihiro Ota,1,3 Masahiko Machida,1,3,4 and Tomio Koyama2,3

1 CCSE, Japan Atomic Energy Agency, 6-9-3 Higashi-Ueno Taito-ku, Tokyo 110-0015, Japan
2 Institute for Materials Research, Tohoku University, 2-1-1 Katahira Aoba-ku, Sendai 980-8577, Japan
3 CREST(JST), 4-1-8 Honcho, Kawaguchi, Saitama 332-0012, Japan
4 JST, TRIP, 5 Sambancho Chiyoda-ku, Tokyo 102-0075, Japan

(Dated: February 11, 2011)

We theoretically study the macroscopic quantum tunneling (MQT) in a hetero Josephson junction formed by a conventional single-gap superconductor and a multigap superconductor such as and iron-based superconductors and MgB$_2$. In such a Josephson junction more than one phase difference is defined. We clarify their phase dynamics and construct a theory for the MQT in the multigap Josephson junctions. The dynamics of the phase differences are strongly affected by the Josephson-Leggett mode, which is the out-of-phase oscillation mode of the phase differences. The escape rate is calculated in terms of the effective action renormalized by the Josephson-Leggett mode at zero-temperature limit. We successfully predict drastic enhancement of the escape rate when the frequency of the Josephson-Leggett mode is less than the Josephson-plasma frequency.

PACS numbers: 74.50.+r

Macroscopic quantum tunneling (MQT) is a counterintuitive phenomenon in quantum mechanics appearing in a macroscopic level and has been observed in various fields of physics such as condensed matters, nuclei, cosmology, etc. This phenomenon has still attracted a great interest in physics communities. In particular, the MQT in Josephson junctions, which is observed in a switching event at low temperature, has been intensively studied because it is promising for applications to a Josephson phase qubit.

In this paper we explore the physics of MQT in an unexplored type of Josephson junctions which have multiple tunneling channels. Such a Josephson junction can be fabricated by using recently discovered iron-based superconductors or MgB$_2$, because these superconductors are multiband ones having more than one disconnected Fermi surfaces and the superconducting gap can be individually well defined on each Fermi surface. In a Josephson junction made of multigap superconductors one may expect that the superconducting tunneling current has multiple channels between the two superconducting electrodes. We construct a theory for the quantum switching (i.e., MQT) in Josephson junctions with multiple tunneling channels. To our knowledge no theory has been formulated for the MQT in multigap systems. The theory predicts that the escape rate, i.e., the rate of quantum tunneling, is drastically enhanced compared with that in conventional single-channel systems.

In multigap superconductors a collective mode called the Leggett’s mode appears in the low energy region, which is an out-of-phase oscillation mode of the superconducting phases. In Ref. a theory for the Josephson effect in superconducting hetero junctions made of a single-gap superconductor and a two-gap one is formulated. In such Josephson junctions, because two kinds of gauge-invariant phase differences can be defined, there are two phase oscillation modes, i.e., the in-phase mode and the out-of-phase one, which correspond, respectively, to the Josephson-plasma and the Josephson-Leggett (JL) mode. In this paper we construct a theory for the MQT in superconducting hetero junctions, incorporating the degree of freedom of the JL mode into the quantum switching event from non-voltage to voltage states. It is shown that the zero-point motion of the JL mode significantly enhances the MQT escape rate when its frequency is less than the Josephson-plasma frequency. We also point out that the ratio $E_J/E_{in}$ in addition to $E_J/E_C$ governs the boundary between the classical and quantum regimes, where $E_C$, $E_J$ and $E_{in}$ are, respectively, the charging energy, the Josephson coupling energy between the two superconductors and the inter-band Josephson coupling energy in the two-gap superconductor.

Consider a hetero Josephson junction made of a single-gap superconductor and a two-gap one, as shown schematically in Fig. 1. Such a junction has been already fabricated, using the multigap superconductors MgB$_2$ or iron-based superconductors. In this system one can define two gauge-invariant phase differences, $\theta^{(1)}$ and $\theta^{(2)}$. Then, the Josephson current density between the two superconducting electrodes is given by the sum of the superconducting currents in the two tunneling channels $j_1 \sin \theta^{(1)} + j_2 \sin \theta^{(2)}$, where $j_i$ is the Josephson critical current density in the $i$th tunneling channel. When a voltage $v$ appears between the two superconducting electrodes, the gauge-invariant phase differences show temporal evolution satisfying the generalized Josephson relation,

$$\frac{\alpha_2}{\alpha_1 + \alpha_2} \dot{\theta}^{(1)} + \frac{\alpha_1}{\alpha_1 + \alpha_2} \dot{\theta}^{(2)} = \frac{2e \Lambda}{\hbar} v, \quad (1)$$

with $\alpha_i = \epsilon \mu_i / d$ and $\Lambda = 1 + \alpha_1 \alpha_2 / (\alpha_1 + \alpha_2)$, where $\epsilon$ is the dielectric constant of the insulator with a thick-
ness $d$ and $\mu_i$ is the charge screening length due to the
 electrons in the $i$th band. The constant $\alpha_i$ is related to
the charge compressibility in the two-gap superconduct-
ing electrode.\textsuperscript{16}

As shown in Ref.\textsuperscript{14}, the Lagrangian in the hetero
Josephson junction with a in-plane area $W$ and capac-
tance $C = \epsilon W/4\pi d$ is expressed as

$$L = \frac{1}{2} \left( \frac{\hbar^2 C}{(2e)^2} \left( \frac{\dot{\theta}^2}{\lambda} + \frac{\dot{\psi}^2}{\alpha_1 + \alpha_2} \right) - V + E_j I_{\text{ex}} I_e \right) \theta, \quad (2a)$$

$$V = -E_{j1} \cos \theta^{(1)} - E_{j2} \cos \theta^{(2)} - \kappa E_{in} \cos \psi, \quad (2b)$$

under a bias current $I_{\text{ex}}$ in the absence of an external
magnetic field, where $\theta$ and $\psi$ are the center-of-mass
phase difference and the relative phase difference defined as

$$\theta = \frac{\alpha_2}{\alpha_1 + \alpha_2} \theta^{(1)} + \frac{\alpha_1}{\alpha_1 + \alpha_2} \theta^{(2)}, \quad \psi = \theta^{(1)} - \theta^{(2)}. \quad (3)$$

The first two terms in Eq. (2a) are the Josephson-
coupling energies with the coefficients $E_{j1} = \hbar W_1 j_1/2e$ and
the third term represents the inter-band coupling ener-
y, where the coefficient $E_{in}$ is expressed as $E_{in} = \hbar W_1 |j_{in}|/2e$. Since the “inter-band current” $J_{in}$ can take
both signs, depending on the gap symmetry, we intro-
duce the sign factor $\kappa = J_{in}/|j_{in}|$. The total critical
Josephson current $I_c$ and the coefficient $E_{j}$ in the last
term in Eq. (2a) are defined as $I_{c} = W|j_{1} + \kappa j_{2}|$ and
$E_{j} = \hbar I_{c}/2e$. We note that the voltage $v$ appearing in
the junction is related to only $\theta$, as seen in Eq. (1).

From Eq. (2a) one can derive the Euler-Lagrangian
equation for the center-of-mass phase difference $\theta$ as

$$\Lambda^{-1} \dot{\theta} + \omega_{P1}^2 \sin \theta^{(1)} + \omega_{P2}^2 \sin \theta^{(2)} = \omega_{P}^2 I_{\text{ex}} I_e, \quad (3)$$

with $\hbar \omega_{P1} = \sqrt{2ECE_{j1}}$ and $\hbar \omega_{P} = \sqrt{2ECE_j}$. We note
that $\omega_{P1}$ is the Josephson-plasma frequency in the $i$th
tunneling channel. From Eq. (2a) we also have the Euler-
Lagrangian equation for $\psi$ as

$$\ddot{\psi} + \kappa \omega_{JL}^2 \sin \psi = -\alpha_1 \omega_{J1}^2 \sin \theta^{(1)} + \alpha_2 \omega_{J2} \sin \theta^{(2)}, \quad (4)$$

where $\omega_{JL}$ is the frequency of the JL mode\textsuperscript{14} given as
$\hbar \omega_{JL} = \sqrt{2(\alpha_1 + \alpha_2)E_CE_{in}}$. The above two
equations are coupled since $\theta^{(1)}$ and $\theta^{(2)}$ are functions of $\theta$ and $\psi$.

We note that the bias current is the source for the time
evolution of $\theta$ but not for $\psi$, which is consistent with
the generalized Josephson relation.\textsuperscript{11} It should be also
noted that we have two characteristic energy scales, the
Josephson-plasma frequency $\omega_{J1}$ and the JL one $\omega_{JL}$, in
this system.

Let us now study the macroscopic quantum effects in
the Josephson junction with multiple tunneling channels
on the basis of the Lagrangian (2a) and evaluate the
MQT escape rate. In the following we assume $\kappa > 0$,
since the case of $\kappa < 0$ shows qualitatively no difference.

Suppose that the switching to the voltage state is in-
duced by the quantum tunneling of the phase differences
$\theta^{(1)}$ and $\theta^{(2)}$ which are confined inside a potential well.
When both $\theta^{(1)}$ and $\theta^{(2)}$ show the tunneling at the switch-
ing, its transition probability is given by the expectation
value of the time evolution operator with respect to the
state $|\theta^{(1)} = 0, \theta^{(2)} = 0\rangle = |\theta = 0, \psi = 0\rangle$\textsuperscript{25}, which yields the
formula for the MQT escape rate as

$$\Gamma = \frac{2}{\hbar \beta} \ln K\{\{\theta\}; \{\psi\}; \beta\}, \quad (5)$$

Here, the symbol $\{\theta\}$ means $\langle \theta, \psi \rangle = (0, 0)$, and $\beta$ is
the inverse temperature, $\beta = 1/k_{B}T$. The propagator
$K(X, X'; \beta)$ in Eq. (5) is expressed in terms of the
imaginary time path-integral

$$K(X, X'; \beta) = \int_{X(0)=X'}^{X(\hbar \beta)=X} D\theta D\psi e^{-\int_{0}^{\hbar \beta} dt L_E},$$

where $X = (\theta, \psi)$ and $L_E$ is the Euclidean version of the
Lagrangian (2a). Let us assume that $\psi$ is confined in a
small region around $\psi = 0$ at the tunneling, which is
justified when the inter-band coupling is not so strong.
In this case one can utilize the expansion with respect to
$\psi$. Then, up to the order of $\psi^2$ the Euclidean Lagrangian

\[ \text{FIG. 2: (Color online) Schematic energy diagram for the fully “quantized” system with two quantum variables $\theta$ and $\psi$. In the case where $\psi$ is weakly oscillating within a potential well, the energy levels of $\psi$ coincide with those of a harmonic oscillator with frequency $\omega_{JL}$, and the energy levels of $\theta$ are corrected by the quantum oscillations of $\psi$.} \]
Here, $g_+ = (E_{11}/2E_3)[\alpha_1/(\alpha_1+\alpha_2)]^2+(E_{12}/2E_3)[\alpha_2/(\alpha_1+\alpha_2)]^2$ and $g_- = (E_{11}/E_3)[\alpha_1/(\alpha_1+\alpha_2)] - (E_{12}/E_3)[\alpha_2/(\alpha_1+\alpha_2)]$. We note that in the fully quantum case we have the discrete energy levels as schematically indicated in Fig. 2.

To calculate the escape rate $\Gamma$ in Eq. (5) we employ the mean field approximation for $\psi$, that is, $\langle \psi^2 \rangle$ and $\psi$ in Eq. (5c) are approximated with their expectation values.

Then, at zero temperature we find $\langle \psi^2 \rangle_{\text{th}} = 0$ and

$$\langle \psi^2 \rangle_{\text{th}}(T=0) = \frac{\hbar^2}{2m_{\text{th}}\omega_{\text{JL}}}, \quad m_{\text{th}} = \frac{\hbar^2}{2(\alpha_1+\alpha_2)E_C}.$$  

The finite value of $\langle \psi^2 \rangle$ originates from the zero-point motion of the “quantized” JL mode. Under this approximation we find the effective Lagrangian of single degree of freedom as follows,

$$L_{\text{cm,eff}}^E = \frac{\hbar^2}{4EC} \left( \frac{d\theta}{d\tau} \right)^2 + V_{\text{cm,eff}}, \quad (7a)$$

where $V_{\text{cm,eff}}$ is the renormalized potential

$$V_{\text{cm,eff}} = -E_2 \left[ (1-\varepsilon) \cos \theta + \frac{I_{\text{ex}}}{I_e} \theta \right], \quad (7b)$$

$$\varepsilon = g_+ \langle \psi^2 \rangle_{\text{th}} \approx \frac{g_+}{\sqrt{2}} (\alpha_1 + \alpha_2) \frac{\omega_{\text{JL}}}{\omega} \sqrt{E_C/E_1}, \quad (7c)$$

Then, in this approximation the expectation value $K(\{0\}, \{0\}; \beta)$ in Eq. (5) is to $K(\{0\}, \{0\}; \beta = \infty) \approx \int_{\theta(0)=0}^{\theta(\infty)=0} \theta \exp(-\hbar^{-1} \int_0^{\theta(\infty)} L_{\text{cm,eff}}^E d\tau)$, which can be evaluated in the standard instanton approximation. Hence, the MQT escape rate corrected by the zero-point motion of $\psi$ is

$$\Gamma = 12\omega_{\text{JL}}(I) \sqrt{\frac{3V_0}{2\pi\hbar\omega_{\text{JL}}(I)}} \exp \left( -\frac{36V_0}{5\hbar\omega_{\text{JL}}(I)} \right), \quad (8)$$

where $\omega_{\text{JL}}(I) = \omega_{\text{JL}}[(1-\varepsilon)^2 - I^2]^{1/4}$, $V_0 = \hbar^2 \omega_{\text{JL}}(I)^2 \cos^2 \theta_0/3E_C$, $(1-\varepsilon) \sin \theta_0 = I$, and $I = I_{\text{ex}}/I_e$. Figure 3 shows a contour map of the ratio $\Gamma/\Gamma_0$ in the $(I_{\text{ex}}/I_e$ vs. $\omega_{\text{JL}}/\omega_{\text{JL}}$)-plane with $\Gamma_0$ being the escape rate without correction, i.e., $\varepsilon = 0$. It is seen that the escape rate is drastically enhanced in a wide parameter region. In particular, the enhancement is pronounced in the region of large $\omega_{\text{JL}}/\omega_{\text{JL}}$. As seen in Eqs. (7a) and (7b), the Josephson coupling energy is renormalized by the zero point motion of $\psi$ and the renormalized one is decreased from the bare one since $\varepsilon > 0$. As a result, the tunneling barrier for $\theta$ is lowered as schematically shown in Fig. 4, which causes the strong enhancement of the escape rate. In fact, $R(\varepsilon) \equiv V_0/\hbar\omega_{\text{JL}}(I)$ is smaller than $R(\varepsilon = 0)$ for fixed $I$ when $0 < \varepsilon < 1$, that is, the exponent in Eq. (4b) is decreased. Thus, the renormalization increases $\Gamma$. It should be also noted that the zero-point fluctuation becomes larger as the frequency of the JL mode decreases. Thus, the considerable enhancement of $\Gamma$ occurs for the system with a lower value of $\omega_{\text{JL}}$. The MQT in the conventional systems is subject to the ratio $E_3/E_C$, which is an important parameter for designing a superconducting Josephson qubit. In the system with multiple tunneling channels the ratio $\omega_{\text{JL}}/\omega_{\text{JL}}(\propto E_1/E_{\text{in}})$ also affects the characteristics of the MQT.

In this paper we have focused on the tunneling process, $|\theta = 0, \psi = 0\rangle \rightarrow |0, 0\rangle$ and clarified the effect of the JL mode on the MQT. We mention that such a process is not the unique one that contributes to the MQT rate in this system, since a system with two degrees of freedom generally have many tunneling routes. For example, the tunneling process in which the quantum switching in the $\theta^{(1)}$ channel takes place successively after the switching in the $\theta^{(2)}$ channel will be also possible in the present system. In this case the escape rate can be calculated from the transition process $|\theta^{(1)} = 0\rangle \rightarrow |0\rangle$ with $\theta^{(2)} = f(t)$,
where $f(t)$ is a time-dependent c-number function. This tunneling process is analogous to the MQT under a periodically time-dependent perturbation. It is also noted that the relative phase difference $\psi$ might play a role of an environmental variable for $\theta$ through the term linear in $\psi$ in Eq. (6c). The MQT rate in this process can be evaluated, using the influential functional integral method. The competition between the zero-point fluctuation and the “dissipation” occurs in this case. The enhancement via the JL mode may be superior to the reduction from such dissipation when $g_+ > |g_-|$. We also mention that our theory for the MQT in the hetero Josephson junctions can be extended to the case of intrinsic Josephson junctions (IJJs) with multiple tunneling channels. The MQT in such systems will be observed in several highly-anisotropic layered iron-based superconductors recently discovered. In the IJJ's correction due to the JL mode for the corporative MQT among the junctions will be expected.

Finally, we remark that the present theoretical prediction relies on the coexistence of the Josephson-plasma and the JL modes. Since observation of Leggett’s mode in a bulk MgB$_2$ sample has been reported and in junctions with MgB$_2$, we expect that such a collective mode can be detected in a junction system and the theory will be verified experimentally.

In summary, we have constructed a theory of the MQT in hetero Josephson junctions with multiple tunneling channels. We have clarified that the zero-point fluctuation of the relative phase differences brings about the drastic enhancement of the MQT escape rate. The enhancement is gigantic when the JL mode has a lighter mass than that of the Josephson-plasma.

YO thanks M. Nakahara, S. Kawabata, and Y. Chizaki for illuminating discussions. TK was partially supported by Grant-in-Aid for Scientific Research (C) (No. 22540358) from the Japan Society for the Promotion of Science.

---

1. A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. 46, 211 (1981).
2. R. Rajaraman, Solitons and Instantons: An Introduction to Solitons and Instantons in Quantum Field Theory (Elsevier, Amsterdam, 1987).
3. E. Šimánek, Inhomogeneous Superconductors: Granular and Quantum Effects (Oxford University Press, New York, 1994) Chap. 4.
4. A. Larkin and A. Varkamov, Theory of Fluctuations in Superconductors Revised Edition (Oxford University Press, New York, 2005) Chap. 13.
5. J. Q. You and F. Nori, Phys. Today 58, 42 (2005).
6. J. Clarke and F. Wilhelm, Nature (London) 453, 1031 (2008).
7. M. Nakahara and T. Ohmi, Quantum Computing: From Linear Algebra To Physical Realizations (CRC Press, Taylor & Francis Group, Boca Raton, 2008) Chap. 15.
8. Y. Kamihara, T. Watanabe, M. Hirano, and H. Hosono, J. Am. Chem. Soc. 130, 3296 (2008).
9. M. Rotter, M. Tegel, and D. Johrendt, Phys. Rev. Lett. 101, 107006 (2008).
10. J. Paglione and R. L. Greene, Nat. Phys. 6, 645 (2010).
11. J. Nagamatsu, N. Nakagawa, T. Muranaka, Y. Zenitani, and J. Akimitsu, Nature (London) 410, 63 (2001).
12. A. Brinkman and J. M. Rowell, Physica C 456, 188 (2007).
13. X. X. Xi, Rep. Prog. Phys. 71, 116501 (2008).
14. Y. Ota, M. Machida, T. Koyama, and H. Matsumoto, Phys. Rev. Lett. 102, 237003 (2009).
15. T. Koyama, Y. Ota, M. Machida, Physica C 470, 1481 (2010).
16. Y. Ota, M. Machida, and T. Koyama, Phys. Rev. B 82, 140509(R) (2010).
17. D. F. Agterberg, E. Demler, and B. Janko, Phys. Rev. B 66, 214507 (2002).
18. A. J. Leggett, Prog. Theor. Phys. 36, 901 (1966).
19. S. G. Sharapov, V. P. Gusynin, and H. Beck, Eur. Phys. J. B 30, 45 (2002).
20. Y. Ota, M. Machida, T. Koyama, and H. Aoki, Phys. Rev. B (to be published); e-print arXiv:1008.3212.
21. H. Shimakage, K. Tsujimoto, Z. Wang, and M. Tonouchi, Supercond. Sci. Technol. 17, 1376 (2004).
22. X. Zhang, Y. S. Oh, Y. Liu, L. Yan, K. H. Kim, R. L. Greene, and I. Takeuchi, Phys. Rev. Lett. 102, 147002 (2009).
23. S. Schmidt, S. Döring, F. Schmidl, V. Grosse, P. Seidel, K. Ida, F. Kurth, S. Haindl, I. Mönch, and B. Holzapfel, Appl. Phys. Lett. 97, 172504 (2010).
24. Y. Chizaki, H. Kashiwaya, S. Kashiwaya, T. Koyama, and S. Kawabata (unpublished).
25. M. A. Fisher, Phys. Rev. B 37, 75 (1988).
26. S. Kawabata, T. Bauch, and T. Kato, Phys. Rev. B 80, 174513 (2009).
27. H. Ogino, Y. Matsumura, Y. Katsura, K. Ushiyama, S. Horii, K. Kishio, and J. Shimoyama, Supercond. Sci. Technol. 22 075008 (2009).
28. H. Nakamura, M. Machida, T. Koyama, and N. Hamada, J. Phys. Soc. Jpn. 78, 132712 (2008).
29. H. Kashiwaya, K. Shirai, T. Matsumoto, H. Shibata, H. Kambara, M. Ishikado, H. Eissaki, A. Iyo, S. Shamoto, I. Kurosawa, and S. Kashiwaya, Appl. Phys. Lett. 96, 202504 (2010).
30. M. Machida and T. Koyama, Supercond. Sci. Technol. 20, S23 (2007).
31. S. Savelev, A. O. Soboychakov, A. L. Rakhmanov, and F. Nori, Phys. Rev. B 77, 014509 (2008).
32. G. Blumberg, A. Mialitsin, B. S. Dennis, M. V. Klein, N. D. Zhigadlo, and J. Karpinski, Phys. Rev. Lett. 99, 227002 (2007).
33. Ya. G. Ponomarev, S. A. Kuzmicheva, M. G. Mikheeva, M. V. Sudakovaa, S. N. Tchesnokova, N. Z. Timergaleevaa, A. V. Yarigina, E. G. Maksimovb, S. I. Krasnoskovodtsevb, A. V. Varlashkina, M. A. Heincc, G. Müllere, H. Plece, L. G. Sevastyanovad, O. V. Kravenkod, K. P. Burdind, and B. M. Bulychevd, Solid State Commun. 129, 85 (2004).
34. A. Brinkman, S. H. W. van der Ploeg, A. A. Golubov, H. Rogalla, T. H. Kim, and J. S. Moodera, J. Phys. Chem. Solids 67, 407 (2006).