Noise Induced Phase Separation in Active Systems: Creating patterns with noise

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We study the flocking and pattern formations of active particles with a Vicsek-like model that includes a configuration dependent noise term. In particular, we couple the strength of the noise with both the local density and orientation of neighboring particles. Our results show that such a configuration dependent noise can lead to the appearance of large-scale ordered and disordered patterns, without the need for any complex alignment interactions. In particular, we obtain an ordered band or line state and a disordered active cluster, similar to that seen in the case of motility induced phase separation.

I. INTRODUCTION

Understanding the collective dynamics of active systems is currently one of the most exciting areas of research in soft matter as well as biological physics. These out-of-equilibrium systems present markedly different behavior compared to their equilibrium counterparts, such as enhanced diffusion\textsuperscript{[1]}, anomalous viscosity\textsuperscript{[2]}, self-sustained turbulence\textsuperscript{[3]}, giant-number fluctuations\textsuperscript{[4]}, or motility-induced phase separation\textsuperscript{[5]}. To name but a few. Examples of these systems range from the microscopic scale, including algae, bacteria, and active janus particles, to the macroscopic scale of fish, birds, buffalo or humans. The flocking or swarming behavior typically seen in such active systems is usually explained as a consequence of some type of local velocity aligning interaction, which is the basis for all Vicsek-like descriptions\textsuperscript{[6, 7]}. Recent works have challenged this assumption\textsuperscript{[8–12]}, mainly by considering more detailed cognitive based models, and obtained drastically different flocking behavior, reminiscent of patterns seen in nature, such as the marginally opaque flocks of birds\textsuperscript{[11]}, the millinglike pattern found in fish\textsuperscript{[13]}, and the file formation seen in sheep\textsuperscript{[14]}. While these approaches have proved immensely fruitful, they can only be justified for systems where particle cognition is at play (i.e., not for active janus particles or synthetic microswimmers).

In this work, guided by the recent investigations into Motility Induced Phase separation\textsuperscript{[5]}, as well as the complex interparticle interactions observed for self-phoretic janus particles\textsuperscript{[15]} and swimmers in general\textsuperscript{[16–19]}, we propose a Vicsek-like model of flocking that maintains the velocity alignment rule, but that allows for a configuration dependent noise term. In practice, we divide the complex particle-particle interactions (which can, for example, be mediated by the solvent or an additional chemo-attractant) into an effective alignment together with an asymmetric noise term. With this simple modification, we are able to observe the emergence of complicated flocking patterns not seen within the usual Vicsek Model or its variations. In particular, we find dense polar structures or bands elongated perpendicular to the direction of propagation, as well as active disordered clusters, which maintain their size and location in space over long time scales. Thus, we show that noise can effectively be used to create large scale patterns in active systems, albeit at the cost of coupling it to the local density and orientation of particles. Of interest is the fact that the clustering tendency will increase both with density and noise, contrary to what is expected for typical Vicsek-like models.

II. MODEL

A. The Vicsek Model

The model of active particles proposed by Vicsek and collaborators\textsuperscript{[20]} describes the off-lattice motion of a set of \( N \) point particles at positions \( \mathbf{x}_i \) \((i = 1, \ldots, N)\), which are moving at constant speed \( v_0 \), in a direction \( \theta_i \), within a 2D periodic domain. For simplicity, it is customary to represent the velocities as complex numbers, i.e., \( \mathbf{v}_i = v_0 \exp(i \theta_i) \), with \( i \) the imaginary unit. Driven by the belief that the flocking observed in active systems was due to a (local) velocity alignment mechanism, Vicsek et al. proposed the following set of simplified dynamical rules governing the motion of the active particles. At each step \( t \), every particle will survey its surroundings, and attempt to align in the average direction of its neighbors, defined as all the particles within some radius \( R_V \). However, this alignment is not perfect, there is some noise in the system, and the new orientation \( \theta_i(t + \Delta t) \) will in general differ from the average alignment the particle has computed. The updated position \( \mathbf{x}_i(t + \Delta t) \) is then obtained by having each particle move along it’s new direction during the time interval \( \Delta t \). In this case, it is assumed that the noise is \textit{intrinsic}\textsuperscript{[21]}, it affects all particles equally and it reflects the fact that all of the particles make some error during their realignment process (although they are able to perfectly measure the direction of all their neighbors). Almost ten years after the original work of Vicsek and his collaborators appeared, Grégoire and Chatè\textsuperscript{[22]} devised an alternative update scheme, which proposes to reinterpret the origin.
of the noise. Instead of assuming an intrinsic noise, they adopted an extrinsic noise source [21], which represents the inability of the particles to precisely measure the orientation of their neighbors when deciding upon their new direction of motion. Following Pimentel et al. [21], we refer to the former intrinsic noise model as the Standard Vicsek Algorithm (SVA), and to the latter extrinsic noise model as the Grégoire-Chaté Algorithm (GCA). The update rules of both algorithms can be expressed as [21, 23] 

\[
\theta_i^{\text{SVA}}(t + \Delta t) = \arg \left\{ \sum_{j \in V_i} e^{i\theta_j(t)} + \eta \xi_i(t) \right\} 
\]

(1)

\[
\theta_i^{\text{GCA}}(t + \Delta t) = \arg \left\{ \sum_{j \in V_i} e^{i\theta_j(t)} + \eta e^\xi_i(t) \right\} 
\]

(2)

\[
x_i(t + \Delta t) = x_i(t) + v_i[\theta_i(t + \Delta t)]\Delta t,
\]

(3)

where \( \arg(z = z e^{i\theta}) = \theta \) is the argument function, \( V_i \) is the alignment region for particle \( i \) (of radius \( R_V \)), and \( \xi_i \) is a delta-correlated white noise random variable uniformly distributed in \([-\pi, \pi]\), with \( \eta \in [0,1] \) the noise amplitude.

### B. The Modified Vicsek Model

![Diagram of the Modified Vicsek Model](image)

FIG. 1. (color online) Schematic representation of the alignment and noise regions for the active noise model. If the distance between particles is less than \( R_V \) they will contribute to each other's realignment. In addition, if the distance is less than \( R_S \) they can contribute to the noise, depending on their relative positions and orientations. In the diagram, we consider the contributions of a particle \( j \) to the updated orientation of particles \( i \) and \( i' \), as given by Eq. (4). In the case of “back” (“front”) noise, only particles within the light green (dark blue) region would feel the noise generated by particle \( j \).

We propose an alternative variation of the Vicsek model, which again reinterprets the source of the noise, and find interesting new dynamical phases. Instead of considering the noise as inherent to the decision making process of the individual particles, we posit that the noise can be thought of as arising from the activity of the particles themselves. As a justification for this interpretation, we can point to the enhanced diffusion in suspensions of swimming particles [24], as well as the chemorepulsion in active colloidal dispersions [15]. Although we are working with a minimal model, which cannot possibly reproduce the detailed dynamics of such complicated systems, we believe it is possible to incorporate part of their dynamics within the framework of the Vicsek model. With a simple rearrangement of Eq. (3), for the GCA orientation update rule, we obtain

\[
\theta_i(t + \Delta t) = \arg \left\{ \sum_{j \in V_i} e^{i\theta_j(t)} + \eta \sum_{j \in S_i} \chi_{ij}(t)e^\xi_i(t) \right\} 
\]

(4)

where we now consider the noise amplitude \( \eta \chi_{ij} \) to be configuration dependent, as well as allow for distinct alignment and noise regions, \( V_i \) and \( S_i \). Here, \( \chi_{ij} \in [0,1] \) gives the relative noise amplitude that particle \( j \) generates on particle \( i \). It will depend on the relative positions and orientations of both particles, and will not be symmetric in general, i.e., \( \chi_{ij} \neq \chi_{ji} \). Thus, at each step, a particle \( i \) will realign in the average direction of its neighbors, located in the alignment region \( V_i \) (radius \( R_V \)); and this realignment process will exhibit random fluctuations, generated by some of the particles in the noise region \( S_i \) (radius \( R_S \)). We note that in the case where \( \chi_{ij} = 1 \) and \( R_V = R_S \) we recover the GCA (Eq. (3)).

For simplicity, we will only consider two simple noise functions \( \chi_{ij}^b \) and \( \chi_{ij}^f \), which we refer to as “back” and “front” noise

\[
\chi_{ij}^b(t) = H(\hat{x}_{ij} \cdot e^{i\theta_j(t)}) 
\]

(5)

\[
\chi_{ij}^f(t) = H(\hat{x}_{ij} \cdot e^{i\theta_j(t)}) 
\]

(6)

where \( x_{ij} = x_i - x_j \), caret (') denote unit vectors, and \( H(x) \) is the Heaviside step function. Clearly, \( \chi_{ij}^f (\chi_{ij}^b) \) is only different from zero if particle \( i \) is located in front (back) of particle \( j \). A schematic representation of the alignment and noise mechanism is given in Fig. (1) for the case where \( R_S > R_V \). Here, we consider the update of two particles \( i \) and \( i' \) in the vicinity of particle \( j \). Since \( r_{ij} > R_V \) (\( j \notin V_i \)), particle \( j \) does not affect the realignment of particle \( i' \), it only contributes to that of particle \( i \). However, both particles \( i \) and \( i' \) can in principle be affected by the noise due to the presence of particle \( j \), since \( r_{jk} \leq R_S \) (\( k = i', i \)). This will be determined by the orientation of particle \( j \), the relative positions of the particles, and the type of noise we are dealing with. In the case of “back” (“front”) noise, only particles at the back (front) of particle \( j \) would experience this noise. This coupling between the orientation of the particles and the noise will lead to non-trivial collective behavior which is not seen in either the SVA or GCA variants of the Vicsek model.

To characterize the state of our system, we identify the following five dimensionless parameters using Buck-
ingham’s II theorem \cite{25, 26}

\[ \Pi_\eta = \eta \Delta t \]
\[ \Pi_v = v_0 \Delta t / R \]
\[ \Pi_\rho = \pi R_0^2 \rho \]

where \( \Pi_\eta \) determines the strength of the noise, \( \Pi_v \) gives the ratio of the distance traveled by a particle in one time step \( v_0 \Delta t \) to the alignment radius \( R \). \( \Pi_\rho \) the average number of particles within the alignment region (\( \rho = N/L^2 \) the number density), \( \Pi_L \) the ratio of the alignment radius to the total system size, and \( \Pi_R \) the ratio of the alignment radius to the noise radius. In this, work, we are mainly interested in studying how the strength and type of noise affects the collective properties of the system; as such, unless otherwise stated, we will focus on the following region of parameter space: \( \Pi_\eta \in (0, 0.1), \Pi_v \in (0, 0.1), \Pi_\rho = 0.01, \Pi_L = 12.8228, \) and \( \Pi_R = 0.02857 \). As usual when studying swarming of active systems, we measure the amount of order using the instantaneous orientational order parameter

\[ \phi(t) = \frac{1}{N} \left\langle \sum_{i=1}^{N} e^{i \theta_i(t)} \right\rangle \]  

with \( \phi = \langle \phi(t) \rangle \) the time averaged order parameter.

III. RESULTS

A. Phenomenology

In order to identify the role played by the strength \( \Pi_\eta \) and asymmetry \( \Pi_R \) of the noise, we performed simulations over a wide range of parameters for the two basic models introduced above: the “front” and “back” noise models. For comparison purposes, we have also carried out simulations of the SVA under similar parameter ranges. For the “back” noise, we observe similar behavior to the SVA and will not discuss this system further. However, in the case of the “front” noise particles, we find two novel dynamical states, in addition to the well-known disordered and ordered/flocking states of the SVA or GCA \cite{20, 21, 23}: a disordered active clustering state and an ordered line state. As expected, for low (high) enough noise intensity \( \Pi_\eta \) we recover an ordered (disordered) state. However, for intermediate noise intensities we see the appearance of a dense ordered (\( \phi \approx 1 \)) clusters which are elongated perpendicular to the direction of motion. We refer to these structures as “lines”. For larger noise values, such ordered line states are no longer stable. Instead, we see the appearance of a large scale disordered active clusters (\( \phi \approx 0 \)). Simulation snapshots of these distinct states are given in Fig. 2 and the corresponding animated trajectories are provided as supplementary material \cite{27}. The appearance of the clusters is of particular interest, as they are able to maintain their size and center of mass over relatively large time scales, even though the constituent particles never stop moving within the cluster and the cluster itself is constantly exchanging particles with the environment. Such active clusters have been reported before for active Brownian particles, both from particle based simulations \cite{28, 29}, as well as from a continuum model with a density dependent noise term \cite{25, 30} (akin to our configuration dependent noise). In addition, we note that similar states have been observed by Barberis and Perauni using a minimal flocking model \cite{12} which closely resembles our own variation of the SVA. However, in Ref. \cite{12}, in contrast to the present work and most SVA variations, no specific velocity alignment is included. Instead, a cognitive model is proposed in which particles reorient using the instantaneous visual information at their disposal; i.e., the positions of neighboring particles within a specified visual cone, instead of their velocities. By varying the size of the vision cone, the authors report the appearance of a line type state, which they call a worm, an aggregate phase similar to our active clustering, as well as more complicated aggregates and nematic bands, which we do

FIG. 2. (color online) Simulation snapshots of the distinct dynamical states that can be observed by varying the strength and type of the noise. The colormap encodes the local density, i.e., the number of neighbor particles within the alignment region \( V \) of each particle. The top panel shows the well known flocking and disordered states of the SVA (\( \chi_0 = 1 \) and \( \Pi_R = 1.0 \)). These can be recovered within the “front” noise model (middle and bottom panels; \( \Pi_R = 0.5 \), but more interesting are the two additional states that appear at intermediate noise strength: a moving line state (\( \Pi_\eta = 0.5 \)) and an active disordered cluster state (\( \Pi_\eta = 0.8 \)).
not observe. We note however, that their worms are elongated parallel to the alignment of the particles, while we see a perpendicular alignment. Nevertheless, the same type of parallel worms can be obtained using our current model, if the “front” noise is replaced with “left/right” noise (not shown here).

B. Order Parameter

FIG. 3. (color online) Order parameter $\phi$ as a function of noise intensity $\Pi_\eta$ (top) and density $\Pi_\rho$ (bottom, $\Pi_\eta = 0.9$) for two distinct values of the alignment to noise ratio $\Pi_R = 0.5, 1.0$. For comparison purposes, we have also included the results of the SVA ($\chi_{ij} = \Pi_R = 1$).

To study the transition of the system from the low-noise flocking state to the high-noise disordered state, passing through the line and clusters, we measured the order parameter $\phi$ as a function of noise intensity $\Pi_\eta$ for two different alignment to noise size ratios $\Pi_R = 0.5$ and $1.0$ (see Figure 3a). When the alignment and noise regions coincide $\Pi_R = 1$ we obtain an ordered phase $\Phi \simeq 1$ for all noise values. However, the system is not always in a flocking state, as can be seen by inspecting the trajectories of the system. In fact, flocking is only stable for very small noise amplitudes $\Pi_\eta \lesssim 0.1$, for all higher values the stable state is that of the ordered perpendicular lines. However, if we make the alignment region smaller than the noise region ($R_V < R_S$), we observe a sharp drop in the order parameter at an intermediate noise $\Pi_\eta \simeq 0.5$. As in the previous case, flocking is only observed for $\Pi_\eta \simeq 0$; for $0.1 \lesssim \Pi_\eta \lesssim 0.5$ the system forms the ordered perpendicular lines, and for higher values we obtain active disordered clusters. The onset of the disordered clusters naturally coincides with the noise value at which the order parameter shows the abrupt drop $\Pi_\eta \simeq 0.5$. We note that for small to intermediate noise intensities, the dynamics of the system is insensitive to $\Pi_R$, the differences are only appreciable for $\Pi_\eta \gtrsim 0.5$. A similar behavior can be seen for the order parameter as a function of density (see Figure 3b). For low densities, the system is in the line state for both $\Pi_R = 0.5$ and $1.0$. As the density is increased, systems with $\Pi_R = 1.0$ show a slight decrease in order, while the stable state goes from standard flocking to the perpendicular line. For $\Pi_R = 0.5$ we observe a sharp drop at $\Pi_\rho \simeq 3$, at which point the disordered clusters start to develop. The fact the noise and the density play a similar role can seem counter-intuitive, particularly since this is not what is seen within the SVA. However, in our model, noise, alignment, and density are all coupled, and the appearance of the large scale line and cluster states is due precisely to the noise, which in turn is caused by the particles themselves. Therefore, increasing the density has the same net effect as increasing the strength of the noise.

C. Phase Diagram

FIG. 4. (color online) Phase diagram for the forward noise systems in the $\Pi_R - \Pi_\eta$ parameter subspace.

A detailed summary of the transition between the ordered and disordered phases is given by the phase diagram shown in Figure 4. We note that even for the highest noise value $\Pi_\eta = 1$, it is possible to obtain large scale ordered (lines) as well as disordered (cluster) structures. The homogeneous disordered phase expected from
the SVA only appears for small values of $\Pi_R$. In addition, we see that the cluster state is only possible over a narrow range of parameters, for $0.5 \lesssim \Pi_R < 1.0$ and relatively high noise intensities $\Pi_\eta \gtrsim 0.5$. The line state seems to be the more stable configuration, as it is observed over roughly half the parameter space $\Pi_R > \Pi_\eta > 0$. The fact that the line and cluster states are only observed for $\Pi_\eta > 0$ is a clear indication that these patterns are induced by the noise, not by the alignment; although the two are intrinsically linked thanks to our interpretation of the former (eq. (4)). As expected, the noise by itself is not enough, we require a certain degree of alignment, otherwise the system will fall back to a homogeneous disordered state.

### D. Cluster Analysis

Finally, we focus on the appearance of the large scale disordered clusters. We use a simple distance-based algorithm to identify the clusters in the system. Thus, if the distance between any two particles is less than some cutoff distance $r_c$, we consider the particles to belong to the same cluster. Since we have already established that it is the noise that is responsible for the clustering, we choose as cutoff parameter the radius of the noise region $r_c = R_\eta$. Once we have identified the distinct clusters, we can estimate their size by computing the radius of gyration

$$R_g^2 = \frac{1}{N_c} \sum_{i=1}^{N_c} (\mathbf{x}_i - \langle \mathbf{x} \rangle_c)^2$$

with $N_c$ the number of particles in a given cluster and $\langle \mathbf{x} \rangle_c$ its center of mass (average particle position). In figure [7] we show a scatter plot of the cluster’s gyration radius $R_g$, as a function of the cluster size $N_c$ (measured in number of particles), for three different noise values $\Pi_\eta = 0.5$, 0.7, and 1.0 (simulation trajectories are provided as supplemental material[21]). In the limiting case $N_c \gg 1$, we recover a power law behavior with exponent 1/2. This is equivalent to the gyration radius of an ideal polymer chain and is further evidence for the entropic origin of the clusters. In addition, we see that increasing the noise leads to larger clusters. For low noise values, we obtain relatively small but very dense clusters. As the noise is increased, the size of the clusters increases, with a concomitant decrease in the density. For the largest noise intensity, we obtain a percolating network of broad dilute clusters. Here, to facilitate the analysis of the cluster formation, we chose a set of parameters that would give us relatively small clusters (compared to the single system spanning cluster of Fig. 2): $\Pi_R = 0.3$, $\Pi_\eta = 0.05$, $\Pi_\rho = 0.5$, and $\Pi_L = 0.0057$.

### IV. CONCLUSIONS

In this work, we proposed a novel variation of the Standard Vicssek model of active particles, which reinterprets the noise as an intrinsic quantity, which is coupled to the local particle density and orientation. Specifically, we consider that in addition to an alignment in the average direction of its neighbors, each particle will also “feel” a noise which depends on the orientation of its neighbors. This is in contrast to the usual interpretation of the noise within such minimal models, which tries to represent an error in the cognitive process of the particles (i.e., the particles probe their surroundings and modify their motion accordingly). While the traditional approach makes sense when one considers the flocking of animals such as fish, birds, or humans, it is not at all clear how it can be applied to non-cognitive agents such as active colloidal particles. When one considers the dynamics of such self-propelled particles, which can move due to a wide variety of self-phoretic phenomena, such as diffusiophoresis, electrophoresis, or thermophoresis, it is obvious that the particles are coupled to their environment in a highly non-trivial manner. If the particle dynamics are to be modeled as an effective alignment to neighboring particles plus a fluctuating noise term, it then makes sense to consider the noise itself as depending on the local configuration. With this in mind, we developed a “forward” (“backward”) noise Vicsek-like system, in which the amplitude of the noise felt by any given particle depends on the number of neighboring particles pointing towards (away) from it. In spirit, this can be considered as a generalization of the pusher/puller differentiation of swimming particles[19].

Using the “forward” noise model of active particles, we found two new dynamic states, which are not seen in the standard variations of the Vicsek model (SVA or GCA): a highly ordered elongated line/filament, which appears at low to moderate noise intensities, and an active disordered cluster, which appears at high noise intensities. The latter is of particular interest, as it shows how noise can be effectively used to generate large scale steady patterns. The clusters are a striking example, as they are composed of moving particles, are constantly exchanging particles with the environment, and yet are able to maintain their size and position over large time scales, in the absence of any external field. In particular, we have shown that it is the presence of the asymmetric noise that is responsible for the formation of these active patterns. We have checked the robustness of the model by using a continuous noise function, instead of the step function of Eqs. (5-6), as well as by adding a global intrinsic noise term (as in [1]), and a short-range repulsive interaction. The same qualitative behavior is obtained: the line and cluster states are still observed, although the precise boundaries of the phase diagram will of course vary. To conclude, we have shown how noise can induce large scale patterns in active systems. We believe this observation can be useful when in-
FIG. 5. (color online) (left) Cluster size distribution for three different noise values $\Pi$, and (right) the corresponding simulation snapshots, where the color coding represents the local particle density (number of particles within the alignment region). For these simulations, we have used $\Pi_R = 0.3$, $\Pi_v = 0.05$, $\Pi_p = 0.5$, and $\Pi_L = 0.0057$.

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