Determining the effective Wilson coefficient $a_2$ in terms of $BR(B_s \to \eta_c \phi)$ and evaluating $BR(B_s \to \eta_c f_0(980))$

Hong-Wei Ke$^{1*}$ and Xue-Qian Li$^{2†}$

$^1$ School of Science, Tianjin University, Tianjin 300072, China
$^2$ School of Physics, Nankai University, Tianjin 300071, China

Abstract

In this work, we investigate decays of $B_s \to \eta_c \phi$ and $B_s \to \eta_c f_0(980)$ in a theoretical framework. The calculation is based on the postulation that $f_0(980)$ and $f_0(500)$ are mixtures of pure quark states $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ and $\bar{s}s$. The hadronic matrix elements for $B_s \to \phi$ and $B_s \to f_0(980)$ are calculated in the light-front quark model and the important Wilson coefficient $a_2$ which is closely related to non-perturbative QCD is extracted. However, our numerical results indicate that no matter how to adjust the mixing parameter to reconcile contributions of $f_0(980)$ and $f_0(500)$, one cannot make the theoretical prediction on $B_s \to \eta_c + \pi^+ \pi^-$ to meet the data. Moreover, the new measurement of $BR(B_s \to J/\psi + f_0(500)) < 1.7 \times 10^{-6}$ also negates the mixture scenario. Thus, we conclude that the recent data suggest that $f_0(980)$ is a four quark state (tetraquark or $K\bar{K}$ molecule), at least the fraction of its pure quark constituents is small.

PACS numbers: 13.25.Hw, 14.40.Cs, 12.39.Ki

* khw020056@hotmail.com
† lixq@nankai.edu.cn
I. INTRODUCTION

The values of $BR(B_s \to \eta_c \phi) = (5.01\pm0.53\pm0.27\pm0.63) \times 10^{-4}$ and $BR(B_s \to \eta_c \pi^+ \pi^-) = 1.76\pm0.59\pm0.12\pm0.29) \times 10^{-4}$ recently measured by the LHCb collaboration [11] have stimulated new vigor for studying the hadron structures and the decay mechanism which is closely related to the non-perturbative QCD effects. Based on data, the Collaboration suggests that the $\pi^+ \pi^-$ pair in $B_s \to \eta_c \pi^+ \pi^-$ arises from the decay of $f_0(980)$. To understand the data and look for some hints about involved physics, corresponding theoretical calculations are needed. The traditional scheme is using the heavy quark effective theory (HQET) [2,3] and naive factorization which is an old issue, but still applicable in parallel to the fancy theories such as SCET and others.

The subprocess is $b \to c\bar{c}s$, and at the tree level, the main contribution is the internal $W-$emission while the light quark serves as a spectator. For a completeness, let us briefly retrospect the standard procedures of applying HQET. In the HQET, the corresponding lagrangian is written as

$$\mathcal{L} = c_1 \bar{c}\gamma_\mu(1-\gamma_5)b\bar{s}\gamma^\mu(1-\gamma_5)c + c_2 \bar{c}\gamma_\mu(1-\gamma_5)c\bar{s}\gamma^\mu(1-\gamma_5)b,$$

where $c_1 = \frac{1}{2}(c_+ + c_-)$ and $c_2 = \frac{1}{2}(c_+ - c_-)$ and $c_+, c_-$ are obtained by means of the renormalization group equation (RGE). Sandwiching the lagrangian between the initial and final states, we have

$$<\eta_c \phi(f_0(980))|\mathcal{L}|B_s> = a_2 (<\eta_c \phi(f_0(980))|\bar{c}\gamma_\mu(1-\gamma_5)c|0><0|\bar{s}\gamma^\mu(1-\gamma_5)b|B_s> + <\eta_c|\bar{c}\gamma_\mu(1-\gamma_5)c|0><\phi(f_0(980))|\bar{s}\gamma^\mu(1-\gamma_5)b|B_s>).$$

(2)

It is noted that the $c_1 O_1$ term contributes to the decay process via a color-re-arrangement. Naively, one can expect $a_2 = c_2 + 1/3c_1$ by the color rearrangement. However, it was pointed out by some authors [4,6] “the sub-leading order in $1/N_c$ includes not only the next-to-leading vacuum-insertion contribution but also the nonperturbative QCD correction”. Keeping the factorization form, one should replace $a_2 = c_2 + c_1/3$ by $a_2 = c_2 + c_1/3 + \epsilon_a/2$ where $\epsilon_a$ is a parameter (with Cheng’s notation [8]). Even though one can calculate $\epsilon_a$ in terms of some models [9], the result is not accurate, therefore, generally one should phenomenologically fix it by fitting the well measured data. Our work is exactly along the line. This issue was first discussed in Ref.[4]. In fact, $a_2$ includes some non-perturbative QCD effects so it is not universal for the different channels of the D or B decays [4] as shown above. Definitely, determining the value of $a_2$ based on data fitting one can obtain information about non-perturbative physics. In Ref.[7] $a_2 = 0.23 \pm 0.06$ was fixed by fitting $BR(B \to D^{(*)}\pi(\rho))$. In this work we instead use $B_s \to \eta_c \phi$ to extract the corresponding $a_2$ value. Then we evaluate $BR(B_s \to \eta_c f_0(980))$ in terms of the newly obtained $a_2$. It is worth of noticing that the derivation is based on the postulation that $f_0(980)$ is of a pure $\bar{q}q$ structure ($q$ stands as u,d and s quarks). We will come to this issue for some details in the last section.

In order to calculate the decay width under the factorization assumption one needs to evaluate the hadronic transition matrix element between two mesons. Since the transition is
governed by the non-perturbative QCD effects, so far one has to invoke certain phenomenological models. In this work, we employ the light-front quark model (LFQM). This relativistic model has been thoroughly discussed in literatures \cite{8, 9} and applied to study several hadronic transition processes \cite{10–18}. The results obtained in this framework qualitatively agree with the data for all the concerned processes.

For the transitions \( B_s \to \eta_c \phi \) and \( B_s \to \eta_c f_0(980) \) one needs to evaluate hadronic matrix elements \( B_s \to \phi \) and \( B_s \to f_0(980) \). The structure of \( f_0(980) \) is still not very clear yet, for example, Jaffe \cite{19} suggested \( f_0(980) \) to be a four-quark state, instead, since the resonance is close to the \( K \bar{K} \) threshold a \( K \bar{K} \) molecular structure was considered by Weinstein and Isgur \cite{20}. However, the regular \( s \bar{s} \) structure for \( f_0(980) \) still cannot be ruled out \cite{21–23}. In this paper the scalar meson \( f_0(980) \) is regarded as a conventional mixture of \( \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \) and \( s \bar{s} \).

In Ref. \cite{9} the authors studied the formula of \( 0^- \to 1^- \) and \( 0^- \to 0^+ \) in the LFQM. Actually, the \( 0^- \to 1^- \) hadronic matrix element can be parameterized by four form factors \( A_0, A_1, A_2 \) and \( V \) whereas for \( 0^- \to 0^+ \) transition it can be parameterized by two form factors \( F_0 \) and \( F_1 \). Their detailed expressions obtained in LFQM can be found in Ref. \cite{3}. In this work, we will calculate these form factors numerically. With the form factors one can further evaluate the transition widths of \( B_s \to \eta_c \phi \) and \( B_s \to \eta_c f_0(980) \). In this model the Gaussian-type wave functions are often used to depict the spatial distribution of the inner constituents in the hadrons. There exists a free parameter \( \beta \) in the wave-function beside the masses of the constituents. One should fix it by comparing the decay constant of the involved meson which is either theoretically calculated in LFQM with data.

This paper is organized as following: after this introduction, we list all relevant formulas in Sec.II, and then in Sec. III, we present our numerical results along with all inputs which are needed for the numerical computations. In the last section we draw our conclusion and make a brief discussion.

II. THE FORMULAS FOR THE DECAYS OF \( B_s \to \eta_c \phi \) AND \( B_s \to \eta_c f_0(980) \) IN LFQM

The leading contributions to \( B_s \to \eta_c \phi \) and \( B_s \to \eta_c f_0(980) \) are shown in Fig.1. We will discuss them respectively in the following text.

A. \( B_s \to \phi \) transition in the LFQM

The decay proceeds via \( b \to \bar{c}c\bar{s} \) at tree level which is an internal \( W \)-emission process. The hadronic matrix element is factorized as \cite{4}

\[
A = \frac{G_F V_{cs} V_{bc} a_2}{\sqrt{2}} \langle \eta_c \phi | (\bar{c}c)_{V-A}(\bar{s}b)_{V-A} | B_s \rangle = \frac{G_F V_{cs} V_{bc} a_2}{\sqrt{2}} \langle \eta_c | (\bar{c}c)_{V-A} | 0 \rangle \langle \phi | (\bar{s}b)_{V-A} | B_s \rangle, \quad (3)
\]
where \( a_2 \) is the factor introduced in the introduction. It is also noted the first term in Eq.(22) 
\[ \langle \eta \phi (f_0(980)) | \bar{c} \gamma_\mu (1 - \gamma_5) c | 0 \rangle < 0 | \bar{s} \gamma^\mu (1 - \gamma_5) b | B_s \rangle \]
can be re-organized via the crossing symmetry to a new form which indeed corresponds to a process where a \( q\bar{q} \) pair annihilates into a \( c\bar{c} \) pair. It is very suppressed, so we ignore this term in later calculations.

The transition \( B_s \to \phi \) is a typical process and the involved form factors are defined as
\[
\langle V(P'') | \bar{V}_\mu | P' \rangle = i \left\{ (M' + M'') \varepsilon''^\mu A_1^{PV} (q^2) - \frac{\varepsilon''^\mu \cdot P'}{M' + M''} p_\mu A_2^{PV} (q^2) \right\},
\]
\[
\langle V(P'') | \bar{A}_\mu | P' \rangle = -\frac{1}{M' + M''} \varepsilon''^\mu \epsilon^\nu_{\mu \rho \sigma} \varepsilon''^\nu \mathcal{P}^\rho q^\sigma V^{PV} (q^2),
\]
with
\[
A_3^{PV} (q^2) = \frac{M' + M''}{2M''} A_1^{PV} (q^2) - \frac{M' - M''}{2M''} A_2^{PV} (q^2),
\]
where \( M'(M'') \) and \( P'(P'') \) are the masses and momenta of the vector (pseudoscalar) states. We also set \( \mathcal{P} = P' + P'' \) and \( q = P' - P'' \).

In Ref.[9] the authors deduce all the expressions for the form factors \( A_0, A_1, A_2 \) and \( V \) in the covariant LFQM. For example
\[
V(q^2) = (M' + M'') \frac{N_c}{16\pi^3} \int d x_2 d^2 p'_\perp \frac{2 h'_v h''_v}{x_2 N'_1 N''_1} \left\{ x_2 m'_1 + x_1 m_2 + (m'_1 - m''_1) \frac{p'_\perp \cdot q'_\perp}{q^2} \right\},
+ \frac{2}{w''_v} \left[ p''_\perp^2 + 2 \frac{(p'_\perp \cdot q'_\perp)^2}{q^2} \right],
\]
where \( m'_1, m''_1 \) and \( m_2 \) are the corresponding quark masses, \( M' \) and \( M'' \) are the masses of the initial and final mesons respectively. The wave functions are included in \( h'_v \) and \( h''_v \) and they
are usually chosen to be Gaussian-type and the parameter $\beta$ in the Gaussian wave function is closely related to the confinement scale and is expected to be of order $\Lambda_{\text{QCD}}$. $N'_1$ and $N''_1$ come from the propagators of the inner quark or antiquark of the mesons. $N_c = 3$ is the color factor. The notations $N'_1$, $N''_1$, $h'_p$ and $h''_p$ are given in the appendix. One can refer to Eqs.(32) and (B4) of Ref.[9] for finding the explicit expressions of $A_0$, $A_1$ and $A_2$ and the corresponding derivations.

B. The transition $B_s \rightarrow f_0(980)$

The amplitude for $B_s \rightarrow \eta_c f_0(980)$ is

$$\mathcal{A} = \frac{G_F V_{cb}^* V_{bc} a_2}{\sqrt{2}} \langle \eta_c f_0(980) | (\bar{c}c)_{V-A} (\bar{s}b)_{V-A} | B_s \rangle = \frac{G_F V_{cb}^* V_{bc} a_2}{\sqrt{2}} \langle \eta_c | (\bar{c}c)_{V-A} | 0 \rangle \langle f_0(980) | (\bar{s}b)_{V-A} | B_s \rangle. \quad (7)$$

$B_s \rightarrow f_0(980)$ is a typical $P \rightarrow S$ transition process. The form factors for $P \rightarrow S$ are defined as

$$\langle S(P')|A_\mu|P(P')\rangle = i \left[ u_+(q^2) p_\mu + u_-(q^2) q_\mu \right]. \quad (8)$$

As an example, the explicit expression of $u_+$ is presented as

$$u_+(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2 p_\perp \frac{h''_s}{x_2 N'_1 N''_1} \left[ -x_1(M'^2_0 + M''_0) - x_2 q^2 + x_2 (m'_1 + m''_1)^2 \right. \left. + x_1 (m'_1 - m_2)^2 + x_1 (m''_1 + m_2)^2 \right], \quad (9)$$

where $h''_s$, $M'^2_0$ and $M''_0$ are given in the appendix. The explicit expression of $u_-(q^2)$ is formulated in Ref.[9].

As postulated, $f_0(980)$ is a pure $q\bar{q}$ state and its quark structure is a superposition state as $|f_0(980)\rangle = \sin \theta |u\bar{u} + d\bar{d}\rangle + \cos \theta |s\bar{s}\rangle$. Since strange quark $s$ in $B_s$ can directly transit into the final scalar meson as a spectator, one can notice that only $s\bar{s}$ component of $f_0(980)$ contributes to the transition $B_s \rightarrow f_0(980)$. In Ref.[24-27] the transition was studied using Covariant Light-Front Dynamics (CLFD), Dispersion Relations (DR), PQCD approach, QCD sum rules (QCDSR) and light-cone QCD sum rules (LCQCD). In those articles[24-27] the form factors of the transition are defined as

$$\langle S(P')|A_\mu|P(P')\rangle = -i \left\{ F_1(q^2) [P_\mu - \frac{m^2_{B_s} - m^2_{f_0(980)}}{q^2} q_\mu] + F_0(q^2) \frac{m^2_{B_s} - m^2_{f_0(980)}}{q^2} q_\mu \right\}. \quad (10)$$

There are two relations $F_1 = -u_+(q^2)$ and $F_0 = -[u_+(q^2) + \frac{q^2}{m^2_{f_0(980)} - m^2_{B_s}} u_-(q^2)]$ which associate the conventional form factors used in literature with that we introduced above.

C. Extension of the form factors to the physical region and the decay constant of $\eta_c$

As discussed in Ref.[9] the form factors are calculated in the space-like region with $q^+ = 0$, thus to obtain the physical amplitudes an extension to the time-like region is needed. To
make the extension one should write out an analytical expressions for these form factors, and in Ref. [9] a three-parameter form was suggested

\[ F(q^2) = \frac{F(0)}{1 - a \left( \frac{q^2}{M_{B_s}} \right) + b \left( \frac{q^2}{M_{B_s}} \right)^2} \]  

(11)

where \( F(q^2) \) denotes all \( A_1(q^2), A_2(q^2), A_3(q^2), V(q^2), F_1(q^2) \) and \( F_0(q^2) \). \( F(0) \) is the value of \( F(q^2) \) at \( q^2 = 0 \). In the scheme of LFQM one can calculate \( F(q^2) \) for the space-like region \( (q^2 < 0) \), then through Eq. (11) \( a \) and \( b \) can be solved out. When we apply that expression of \( F(q^2) \) for \( q^2 > 0 \) with the same \( a \) and \( b \), the form factors are extrapolated to the time-like physical regions. That is a natural analytical extension.

In the two processes, there is a unique matrix element \( \langle \eta_c | (\bar{c}c)_{V-A} | 0 \rangle \) which determines the decay constant of \( \eta_c \) and

\[ \langle \eta_c(p) | A_\mu | 0 \rangle = i f_{\eta_c} p_\mu. \]  

(12)

Some mesons’ decay constants can be fixed by fitting data, whereas others must be calculated in terms of phenomenological models or the lattice because no data are available so far. Here the case for \( f_{\eta_c} \) belongs to the latter.

In this scheme \( \langle \eta_c | (\bar{c}c)_{V-A} | 0 \rangle \) is factorized out from the hadronic matrix element and is independent of the matrix element \( \langle f_0(980), (\bar{s}b)_{V-A} | B_s \rangle \). Moreover, if replacing \( \langle \eta_c | (\bar{c}c)_{V-A} | 0 \rangle \) by \( \langle J/\psi | (\bar{c}c)_{V-A} | 0 \rangle \) which is related to the decay constant \( f_{J/\psi} \), one can study the transition \( B_s \to J/\psi f_0(980) \).

III. NUMERICAL RESULTS

In this work, \( m_s = 0.37 \) GeV, \( m_c = 1.4 \) GeV and \( m_b = 4.64 \) GeV are adopted according to Ref. [3]. \( V_{cs} \) and \( V_{cb} \) are taken from the databook [28]. The parameter \( \beta \) in the wave function is fixed by calculating the corresponding decay constant and comparing it with data [3]. For the vector meson \( \phi \) one can extract the decay constant \((227.7 \pm 1.2) \) MeV from the data \( BR(\phi \to e^+e^-)((2.954 \pm 0.030) \times 10^{-4}) \) [28] and then \( \beta_\phi = (0.3001 \pm 0.0010) \) GeV is achieved. For the pseudoscalar meson \( B_s \) its decay constant \((228.4 \pm 3.2) \) MeV coming from the lattice result [29] is used and we obtain \( \beta_{B_s} = (0.6165 \pm 0.0072) \) GeV.

In order to calculate the relevant form factors we need to know \( \beta_0^{f_0} \). For a scalar meson, as long as the masses of the valence quark and antiquark are equal, due to a symmetry with respect to \( x_1 \) and \( x_2 \) which are their shares of momenta in the meson, the decay constant becomes zero as it should be. It is shown by the integral over \( x_1 \) and \( x_2 \) in the framework of LFQM [9]. Following Ref. [4, 30], we set \( \beta_0^{f_0} = 0.3 \) in our numerical computations. The mixing parameter \( \theta \) takes a value of \((56 \pm 6)\) which was fixed by fitting the branching ratio of \( D_s \to f_0(980)e^+\nu_e \) [30] and then the decay constant is \( f_{\eta_c} = (387 \pm 7) \) MeV [31].

It is also noted, when the semileptonic decay of \( D_s \to f_0(980) + e^+\nu_e \) was measured by the CLEO collaboration, there were no data on \( D_s \to f_0(500) + e^+\nu_e \) available, therefore
TABLE I: the parameters \( F_1(0) \), \( a \), \( b \) are defined in Eq. (11).

| \( F(q^2) \) | \( F_1(0) \) | \( a \) | \( b \) |
|-------------|------|------|------|
| \( A_0 \)   | 0.292 | 1.590 | 1.794 |
| \( A_1 \)   | 0.247 | 1.068 | 0.310 |
| \( A_2 \)   | 0.226 | 1.764 | 1.172 |
| \( V \)     | 0.303 | 1.949 | 1.410 |
| \( F_1 \)   | 0.239\cos\theta | 1.690 | 0.917 |
| \( F_0 \)   | 0.239\cos\theta | 0.514 | 0.236 |

TABLE II: the \( B_s \rightarrow f_0(980) \) form factor \( F_0(q^2 = 0) \) with \( \cos \theta = 1 \).

|       | CLFD/DR | PQCD | QCDSR | LCQCDSR | this work |
|-------|---------|------|-------|----------|-----------|
| \( F_0 \) | 0.40/0.29 | 0.35 | 0.12  | 0.238    | 0.239     |

based on the mixing postulation, such mixing angle was obtained by fitting only the data of \( B_s \rightarrow f_0(980)e^+\nu_e \). Later in this work, we will show that the recent measurements on non-leptonic decays of \( B_s \rightarrow f_0(980) + X \) and \( B_s \rightarrow f_0(500) + X \) disagree with the mixing picture. We will give more discussions in the last section.

In Tab. I we present the parameters in those form factors when all the input parameters are taking the central values given elsewhere. In Ref. [24–27] the transition \( B_s \rightarrow f_0(980) \) were also studied and we collect the results in Tab. II. Our prediction is close to the value -0.238 obtained by the authors of [27] which includes the next-to-leading order corrections.

At first we explore whether using the value \( a_2 (0.23 \pm 0.06) \) fixed in Ref. [7] the predicted decay width can meet the present data. With all the form factors and parameters as given above, we obtain the branching ratio \( BR(B_s \rightarrow \eta_c \phi) = (2.795 \pm 1.652) \times 10^{-4} \) where the errors come from the uncertainties of \( \beta_{B_s}, \beta_\phi, f_{\eta_c} \) and \( a_2 \), but mainly from \( a_2 \). Apparently the estimate is smaller than the data \((5.01 \pm 0.53\pm 0.27\pm 0.63) \times 10^{-4}\), but as indicated above, the theoretical errors are relatively large, so within a 2\( \sigma \) tolerance, one still can count them as being consistent. If we deliberately vary the parameter \( a_2 \) within a reasonable range, as setting \( a_2 = 0.308 \pm 0.029 \) the branching ratio \( BR(B_s \rightarrow \eta_c \phi) \) becomes \((5.012 \pm 0.863) \times 10^{-4}\) which is satisfactorily consistent with data.

Using the new value of \( a_2 \) let us evaluate the branching ratio of \( B_s \rightarrow \eta_c + f_0(980) \) and we obtain \( BR(B_s \rightarrow \eta_c f_0(980)) = (1.591 \pm 0.568) \times 10^{-4}\). If one applies this result to make a theoretical prediction on the branching ratio of \( B_s \rightarrow \eta_c \pi^+\pi^- \) by assuming the \( \pi^+\pi^- \) pair fully coming from an on-shell \( f_0(980) \), he will notice that the prediction is consistent with the present measured value of \( BR(B_s \rightarrow \eta_c \pi^+\pi^-) \). It seems that the \( \pi^+\pi^- \) pair in \( B_s \rightarrow \eta_c \pi^+\pi^- \) mainly comes from \( f_0(980) \). But a discrepancy immediately emerges. In Ref. [21–23] the authors suggest that the scalar \( f_0(500)(\sigma) \) is the complemental state of \( f_0(980) \). Thus the \( s\bar{s} \) component of \( f_0(500) \) which dominantly decays into \( \pi\pi \) pairs, would play the same role as that of \( f_0(980) \). If simply setting \( \theta = 0 \), we calculate the branching ratio of \( B_s \rightarrow \eta_c 0^+(s\bar{s}) \) (
i.e. \( B_s \rightarrow \eta_c\pi^+\pi^- \) again. In that case we obtain \( BR(B_s \rightarrow \eta_c\pi^+\pi^-) = (5.089 \pm 1.022) \times 10^{-4} \) which is about three larger than the data. This would raise a conflict between theoretical prediction and experimental data.

Using the decay constant \( f_\psi = 416.3 \pm 5.3 \text{ MeV}^{[27]} \) we also estimate \( BR(B_s \rightarrow J/\psi f_0(980)) = (1.727 \pm 0.615) \times 10^{-4} \) which is slightly larger than the data \( (1.19 \pm 0.22) \times 10^{-4} \)\(^{[32]} \), it seems OK, but at the quark level, we have theoretically evaluate \( BR(B_s \rightarrow J/\psi 0^+(s\bar{s})) \) and gain it as \((5.523 \pm 1.103) \times 10^{-4} \) which leads \( BR(B_s \rightarrow J/\psi f_0(500)) \) to be much larger than the upper limit \( BR(B_s \rightarrow J/\psi f_0(500)) < 1.7 \times 10^{-4} \)\(^{[32]} \).

One possibility to pave the gap between theoretical prediction and data is to assume an exotic structure for \( f_0(980) \), namely is a \( K\bar{K} \) molecule state or tetraquark or a mixture of them. Using data of LHC whose integrated luminosity reaches 3 fb\(^{[1]} \) the structure of \( \bar{B}_s^0 \rightarrow J/\psi\pi^+\pi^- \) was studied\(^{[33]} \) and the mixing angle \( \theta < 7.7^\circ \) (at 90% C.L.) which is consistent with the prediction of the tetraquark model\(^{[34,35]} \). Apparently if the upper-limit of the mixing angle is confirmed our prediction on \( BR(B_s \rightarrow \eta_c f_0(980)) \) and \( BR(B_s \rightarrow J/\psi f_0(980)) \) will be at least twice larger than the data so the \( q\bar{q} \) structure is disfavored.

### IV. SUMMARY

In this work based on the postulation that \( f_0(980) \) and \( f_0(500) \) are mixture of \( \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \) and \( \bar{s}s \) we evaluate the decay widths of \( B_s \rightarrow \eta_c\phi \) and \( B_s \rightarrow \eta_c f_0(980) \) in LFQM. At the quark level the two transitions proceed dominantly through an internal \( W^- \) emission sub-process \( b \rightarrow \bar{c}c\bar{s} \). By the factorization assumption the hadronic matrix element can factorized into a simple transition matrix element multiplying by the decay constant of the involved pseudoscalar meson. In this scenario the effective Wilson coefficient factor \( a_2 \) plays a crucial role. By the naive factorization \( a_2 \) is just related to \( c_2 + c_1/3 \) due to the color rearrangement. However such naive combination is only a rough approximation because some nonperturbative QCD effects would get involved for a complete color rearrangement. The new contribution is not universal for \( B \) or \( D \) decays. Thus extracting the value of \( a_2 \) will provide us with information about the nonperturbative QCD effects in the corresponding decays and even more.

In order to calculate the decay widths of \( B_s \rightarrow \eta_c\phi \) and \( B_s \rightarrow \eta_c f_0(980) \) one needs to compute the transition hadronic matrix elements \( B_s \rightarrow \phi \ (0^- \rightarrow 1^-) \) and \( B_s \rightarrow f_0(980) \ (0^- \rightarrow 0^+) \) which can be parametrized by several form factors. The phenomenological model LFQM is employed to calculate these form factors in this work. With the form factors and all the input parameters we evaluate the rate of \( B_s \rightarrow \eta_c\phi \) and obtain the value as \( BR(B_s \rightarrow \eta_c\phi) = (2.795 \pm 1.652) \times 10^{-4} \) as \( a_2 \) taking value of \( 0.23 \pm 0.06 \) as an input. If one admits that \( a_2 \) is a free parameter, he can vary it to be \( 0.308 \pm 0.029 \) and the obtained result is compatible with the data.

Using the new \( a_2 \) we evaluate the branching ratio of \( B_s \rightarrow \eta_c f_0(980) \) with \( \theta = 56 \pm 6^\circ \) and obtain it as \( BR(B_s \rightarrow \eta_c f_0(980)) = (1.591 \pm 0.568) \times 10^{-4} \) which is almost consistent with the present data. It seems the \( \pi^+\pi^- \) only comes from \( f_0(980) \). However, this assumption brings up unacceptable consequence, that since \( f_0(500) \) contains a large fraction of \( s\bar{s} \) (proportional
to \sin^2 \theta), the contribution of \( B_s \to \eta_c f_0(500) \to \eta_c \pi^+ \pi^- \) becomes un-tolerably large as 
\((3.498 \pm 1.249) \times 10^{-4}\), this number would lead to the branching ratio of \( B_s \to \eta_c \pi^+ \pi^- \) to be roughly \( 5 \times 10^{-4} \) which is roughly 3 times larger than the measured value.

Moreover, the recent measurements indicate the branching ratio of \( B_s \to J/\psi + f_0(500) \) is 
\( 1.19 \times 10^{-4} \) while \( BR(B_s \to J/\psi + f_0(500) < 1.7 \times 10^{-6} \). The data imply that if the mixture scenario is correct, the mixing angle should be smaller than \( 7.7^\circ \) instead of the large \( 56^\circ \). In other words \( BR(B_s \to J/\psi + f_0(500) < 1.7 \times 10^{-6} \) implies that the fraction of \( \bar{s}s \) in \( f_0(500) \) should be very tiny.

If we accept the small mixing angle \( \theta \sim 7.7^\circ \), we obtain \( BR(B_s \to \eta_c f_0(980) \to \eta_c \pi^+ \pi^-) \) to be 
\( 4.998 \times 10^{-4} \), namely is consistent with the allegation that the final \( \pi^+ \pi^- \) pair in 
\( B_s \to \eta_c \pi^+ \pi^- \) is totally from \( f_0(980) \), however, the theoretical picture is surely disagreed by 
the data.

It is an obvious contradiction that in the mixing scenario, no matter what value the 
mixing angle is adopted, the calculated branching ratio for \( B_s \to \eta_c \pi^+ \pi^- \) is at least 3 times 
larger than the data.

A synthesis of the measured branching ratio of \( BR(B_s \to \eta_c \pi^+ \pi^-) \sim 1.76 \times 10^{-4} \) and the 
data \( BR(B_s \to J/\psi f_0(500)) < 7.7^\circ \) determines no room for a subprocess \( B_s \to \eta_c f_0(500) \to \eta_c \pi^+ \pi^- \). Namely if the mixing scenario is adopted, no matter choosing what value for the 
mixing angle, one cannot let the theoretically prediction meet the data.

Therefore, under a complete consideration, one should draw a conclusion that the main 
contents of \( f_0(980) \) are not a mixture of \((\bar{u}u + \bar{d}d)/\sqrt{2} \) and \( \bar{s}s \), but could be a four quark 
state: \( K\bar{K} \) molecule as Isgur et al. suggested or a tetraquark.

We suggest the experimentalists to carry out a more precise measurement on the 
\( B_s \to \eta_c \pi^+ \pi^- \) where the invariant mass of \( \pi^+ \pi^- \) would clearly tell us if \( \pi^+ \pi^- \) mainly come from 
\( f_0(980) \).

Acknowledgement

This work is supported by the National Natural Science Foundation of China (NNSFC) 
under the contract No. 11375128 and 11675082.

Appendix A: Notations

Here we list some variables appearing in the context. The incoming meson in Fig. [I] has 
the momentum \( P' = p'_1 + p'_2 \) where \( p'_1 \) and \( p'_2 \) are the momenta of the off-shell quark and 
antiquark and

\[
\begin{align*}
p'_{1}^+ &= x_1 P'^+, & p'_{2}^+ &= x_2 P'^+, \\
p'_{1\perp} &= x_1 P'_{\perp} + p'_{\perp}, & p'_{2\perp} &= x_2 P'_{\perp} - p'_{\perp}, \quad (A1)
\end{align*}
\]

with \( x_i \) and \( p'_{\perp} \) are internal variables and \( x_1 + x_2 = 1 \).
The variables $M', \tilde{M}', h'_p, h'_s, \hat{N}'_1$ and $\hat{N}''_1$ are defined as

\[ M'^2_0 = \frac{p'^2_1 + m'^2_1}{x_1} + \frac{p'^2_2 + m'^2_2}{x_2}, \]
\[ \tilde{M}'_0 = \sqrt{M'^2_0 - (m'_1 - m'_2)^2}, \]
\[ h'_p = (M'^2 - M'^2_0) \left( \frac{x_1x_2}{N_\epsilon} \frac{1}{\sqrt{2M'_0}} \right) \varphi', \]
\[ h'_s = (M'^2 - M'^2_0) \left( \frac{x_1x_2}{N_\epsilon} \sqrt{\tilde{M}'_0} \right) \varphi'_p, \]
\[ \hat{N}'_1 = x_1(M'^2 - M'^2_0), \]
\[ \hat{N}''_1 = x_1(M''^2 - M''^2_0). \] (A2)

where

\[ \varphi' = 4\left(\frac{\beta}{\beta^2}\right)^{3/4} \sqrt{\frac{dp'_z}{dx_2}} \exp\left(-\frac{p'^2_z + p'^2_\perp}{2\beta^2}\right), \varphi'_p = \sqrt{2/\beta}\varphi', \] (A3)

with $p'_z = \frac{x_2M'_0}{2} - \frac{m'^2_2 + p'^2_\perp}{2x_2M'_0}$. 

[1] R. Aaij *et al.* [LHCb Collaboration], arXiv:1702.08048 [hep-ex].
[2] N. Isgur and M. B. Wise, Phys. Lett. B 232, 113 (1989). doi:10.1016/0370-2693(89)90566-2
[3] H. Georgi, Phys. Lett. B 240, 447 (1990). doi:10.1016/0370-2693(90)91128-X
[4] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 34, 103 (1987). doi:10.1007/BF01561122
[5] H. Y. Cheng, Z. Phys. C 32, 237 (1986). doi:10.1007/BF01552501
[6] X. q. Li, T. Huang and Z. c. Zhang, Z. Phys. C 42, 99 (1989). doi:10.1007/BF01565132
[7] H. Y. Cheng and B. Tseng, Phys. Rev. D 51, 6259 (1995) doi:10.1103/PhysRevD.51.6259 [hep-ph/9409408].
[8] W. Jaus, Phys. Rev. D 60, 054026 (1999).
[9] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D 69, 074025 (2004).
[10] Z. T. Wei, H. W. Ke and X. F. Yang, Phys. Rev. D 80, 015022 (2009) arXiv:0905.3069 [hep-ph].
[11] H. W. Ke, T. Liu and X. Q. Li, Phys. Rev. D 89, 017501 (2014) arXiv:1307.5925 [hep-ph].
[12] H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012) arXiv:1207.3477 [hep-ph].
[13] H. M. Choi, Phys. Rev. D 75, 073016 (2007) arXiv:hep-ph/0701263; H. M. Choi, J. Korean Phys. Soc. 53, 1205 (2008) arXiv:0710.0714 [hep-ph].
[14] C. W. Hwang and Z. T. Wei, J. Phys. G 34 (2007) 687 [hep-ph/0609036].
[15] H. W. Ke, X. Q. Li, Z. T. Wei and X. Liu, Phys. Rev. D 82, 034023 (2010) arXiv:1006.1091 [hep-ph].
