Density Region of Low-Threshold Parametric Decay Instabilities in Electron Cyclotron Resonance Heating Experiments in Tokamak

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Abstract. The density conditions under which low-threshold parametric decay instabilities occur are analyzed by a physical model. Calculations show that low-threshold parametric decay instabilities occur in the density region. Magnetic field effects on the density region are analyzed. The maximum width of the density region is found. The plasma temperature has little effect on the density region. In order to avoid the low-threshold parametric decay instabilities, the daughter wave’s frequency must be far from the UH frequency inside magnetic island. It’s the key to control the magnetic field inside the magnetic island.

Keywords: strong scattering; density region; parametric decay instabilities.

1. Introduction
Strong scattering of millimeter waves with magnetic island was first observed at TEXTOR tokamak [1, 2]. An explanation of the anomalous scattering was suggested by Gusakov and Popov [3]. It is assumed that the pump X mode decays into backscattering X mode and ion Bernstein (IB) wave. The IB wave is trapped in radial direction accounting for non-monotonic density profile in the vicinity of magnetic island O-point. The parametric decay instabilities threshold is substantially (4 orders of magnitude) smaller than that provided by the standard theory. In order to explain the specific electron cyclotron resonance heating (ECRH) experiments in the TEXTOR tokamak, another X-wave parametric decay instability model is developed by Gusakov and Popov [4, 5]. The pump wave decays into the upper hybrid waves. Gusakov and Popov revalued the parametric decay instability power threshold in the magnetic island possessing non-monotonic density profile. The threshold power is easily overcome when the upper hybrid wave is trapped in the radial direction. The observed frequency spectrum and backscattering signal in the second harmonic X-mode ECRH experiments at TEXTOR are reproduced.

Anomalous scattering from TEXTOR plasma with rotating islands occurs in the high-density regime. The threshold of the high-density regime is strongly dependent on the magnetic field and the electron density. In order to understand these observed at TEXTOR, we conducted the calculations of a simple physical model based on Gusakov and Popov’s work. The density region of low-threshold parametric decay instabilities is obtained. We analyzed the influence of the magnetic field and plasma temperature on the density region. The maximum width of the density region is found. Calculation results are in agreement with those observed at TEXTOR.

The article is organized in the following way. In section 2 we describe the physical model used to analyze the density region. In section 3 we calculate the density region and analyze the conditions of
low-threshold parametric decay instabilities. In section 4 we analyze that the density region change with the magnetic field. In section 5 we analyze that the density region change with the plasma temperature. In section 6 we conclude the calculation results.

2. The Physical Model
According to experimental conditions [2, 6], we analyze a simple model. The density inhomogeneity direction is along x axis. The density profile is shown in figure 1. There are three density regimes in figure 1. The density profile in regime I (label I in figure 1) and regime III (label III in figure 1) are monotonous. The density profile in regime II (label II in figure 1) is non-monotonous. The regime II corresponds to the magnetic island region. The local maximum density corresponds to the O-point of the magnetic island.

\[ n(x) = \begin{cases} 
  n_0_1 \left(1 - \frac{x^2}{a^2}\right) & 0 \leq x \leq 26.5cm \\
  n_0_2 \exp\left[-\frac{(x-x_m)^2}{b^2}\right] & 26.5cm < x < 29.5cm \\
  n_0_3 (1 - \frac{x^2}{a^2}) & 29.5cm \leq x \leq 45cm 
\end{cases} \]

where \( a = 45cm \) (the minor radius of TEXTOR tokamak), \( n_0_1, n_0_2, n_0_3 \) and \( b \) are constants. \( n_0_2 = 1.08 \times n(x=26.5cm) \), the width of the magnetic island is equal to 3cm [6]. \( x_m = 28cm \) (the center of the magnetic island).

The pump wave’s polarization direction is along y axis. The pump wave propagates along the x axis in the negative direction, and it’s represented in the form

\[ E_x = E_0 \exp[i(k_0 x + w_0 t)] \]  

where \( k_0 \approx \frac{W_0}{c} \).

The external static magnetic field direction is along the z axis. The field strength is given by the expression

\[ B = B_0 \frac{R_0}{R_0 + x} \]

where \( R_0 = 175cm \) (the major radius of TEXTOR tokamak), \( B_0 \) is the magnetic field strength on the magnetic axis(\( x=0cm \)).

The pump wave is assumed to decay into a couple of the electrostatic upper hybrid (UH) waves. The two daughter waves are denoted by frequencies and wavenumbers \((w_1, k_1)\) and \((w_2, k_2)\). The two waves propagate in opposite directions. The decay resonance conditions are
\[ w_0 = w_1 + w_2 \] (3)
\[ k_2 = k_0 + k_1 \] (4)

The local dispersion relation of the UH wave is written as [7]:
\[ D = Ak^4 + Bk^2 + E = 0 \] (5)
\[ A = \frac{3w_{pe}^2 v_{ce}^2}{(w_j^2 - w_{ce}^2)(4w_{ce}^2 - w_j^2)} \] (6)
\[ B = 1 - \frac{w_{pe}^2}{w_j^2 - w_{ce}^2} \] (7)
\[ E = \frac{w_{ce}^2}{w_j^2 - w_{ce}^2} \] (8)
\[ w_{pe}^2 = \frac{ne^2}{\varepsilon_0 m_e}, \quad w_{ce} = \frac{Be}{m_e}, \quad v_e = \sqrt{\frac{k_B T_e}{m_e}}, \quad j = (1, 2), \] (9)
\[ k^2 = \frac{-B \pm \sqrt{B^2 - 4AE}}{2A} \] (10)

where \( n \) is the electron density, \( e \) is the electron charge, \( m_e \) is the electron mass, \( \varepsilon_0 \) is the permittivity constant, \( T_e \) is the electron temperature, and \( k_B \) is the Boltzmann’s constant. The contribution from the ions is neglected for the high-frequency UH wave. The electron temperature is assumed to be flat within the magnetic island according to experiment measurements [6].

The two UH waves are represented by the potentials
\[ \phi_1(x, t) = \phi_{10}(x) \exp(iw_1t) \] (11)
\[ \phi_2(x, t) = \phi_{20}(x) \exp(-iw_2t) \] (12)

The potentials of the two UH waves satisfy the following equations [4]:
\[ \hat{D}(x, w_1)\phi_1 = \frac{\rho_1}{\varepsilon_0} \] (13)
\[ \hat{D}(x, w_2)\phi_2 = \frac{\rho_2}{\varepsilon_0} \] (14)
\[ \hat{D}(x, w_j) = A\hat{k}^4 + B\hat{k}^2 + E, \quad j = (1, 2) \] (15)
\[ \hat{k}^4 = -\frac{d^4}{dx^4}, \quad \hat{k}^2 = -\frac{d^2}{dx^2} \] (16)
\[ \rho_1 = \varepsilon_0 \frac{3E_y^2}{2B} \frac{w_{pe}^2 w_{ce}^2 w_0 k_0}{(w_0^2 - w_{ce}^2)(w_1^2 - w_{ce}^2)(w_2^2 - w_{ce}^2)} k_1 k_2 \phi_2 \] (17)
where the $\hat{D}$ is obtained by identifying $k$ with $-\text{id}/\text{d}x$ in the local dispersion. $\rho_1$ and $\rho_2$ are non-linear charge densities. They describe the three-wave interactions. $E_y^*$ is the complex conjugation of $E_y$.

3. Density Region of Low-Threshold Parametric Decay Instabilities

Calculations show that the trapped region dependents on density. For example, the plasma parameters are set as follows: $T_e = 500\text{eV}$ (within the magnetic island), $B_0 = 2.25\text{T}$, $f_0 = 140\text{GHz}$, $f_1 = f_2 = 70\text{GHz}$. $w_0 = 2\pi f_0$, $w_1 = w_2 = 2\pi f_1$. The density $n(x=26\text{cm})$ of low-threshold parametric decay instabilities is in the region between $2.49\times10^{19}\text{m}^{-3}$ and $2.60\times10^{19}\text{m}^{-3}$. This density region shows that there is a density threshold for low-threshold parametric decay instabilities. At TEXTOR the anomalous scattering occurs at densities just above a threshold for the high-density region. The calculation results are in agreement with those observed at TEXTOR [2]. The dispersion curves $k_1$ and $k_1+k_0$ are shown in Fig. 2 for $n(x=26\text{cm}) = 2.5\times10^{19}\text{m}^{-3}$.

![Figure 2. Dispersion curves.](image)

It’s seen that the curve $k_1$ is closed and the curve $k_1$ intersects with the curve $k_1+k_0$ in figure 2. These characteristics are the conditions under which low-threshold parametric decay instabilities occur. The closed curve $k_1$ means that there exists a trapped region. The boundaries of the 1D trapped region correspond to $B^2-4AE=0$ defined in (6-8). When $B^2-4AE=0$, the group velocity of the corresponding UH wave along the x direction is zero. At the boundaries, the UH wave is reflected. The intersections of the curves $k_1$ and $k_1+k_0$ mean that the decay resonance condition defined in (4) is satisfied and parametric decay instabilities can happen in the trapped region. When the density $n(x=26\text{cm})$ is not in the region between $2.49\times10^{19}\text{m}^{-3}$ and $2.60\times10^{19}\text{m}^{-3}$, the curve $k_1$ is not closed or the curves $k_1$ and $k_1+k_0$ do not intersect. The conditions under which low-threshold parametric decay instabilities occur are not satisfied.

4. Magnetic Field Effects on The Density Region

The plasma parameters are set as follows: $T_e = 600\text{eV}$ (within the magnetic island), $f_0 = 140\text{GHz}$, $f_1 = f_2 = 70\text{GHz}$. The figure 3 shows that the lower and upper boundaries of the density region decrease as the magnetic field is increased. The UH frequency is defined as

$$\rho_2 = e_0 \frac{3}{2} \frac{E_i}{B} \frac{w_{ee}^2 w_0^2 k_0}{(w_0^2 - w_{ee}^2)(w_0^2 - w_{e}^2)(w_0^2 - w_{ee}^2)} k_1 k_2 \phi_i$$

(18)
Calculations show that the daughter wave’s frequency is less than the UH frequency in the density region. An inequality is written as
\[
\omega_{1,2}^2 - \omega_{ce}^2 < \omega_{pe}^2
\]
which roughly determines the lower boundary of the density region. It means that B in (7) is less than zero.

Calculations also show that the daughter wave’s frequency satisfies the inequality
\[
\omega_{uH} < 2\omega_{1,2}
\]
which roughly determines the upper boundary of the density region.

From (20) and (21) we can obtain
\[
\frac{\omega_0}{2} < \omega_{uH} < \omega_0
\]
where \(\omega_{uH}\) is the UH frequency inside the magnetic island. The density region calculated by the (22) is wider than the actual density region shown in figure 2 and figure 3. The (22) is roughly approximation of the density region’s width.

![Figure 3. The density region’s boundaries as a function of the central magnetic field.](image-url)
Figure 4. The density region’s width as a function of the central magnetic field.

The upper boundary minus the lower boundary equals the width of the density region. The figure 4 shows that the width of the density region varies with the central magnetic field. It is shown that there is a maximum density region’s width at $B_0 \approx 1.9T$. The lower boundary $n(x=26\text{cm}) \approx 3.697 \times 10^{19} \text{ m}^{-3}$ and the upper boundary $n(x=26\text{cm}) \approx 3.858 \times 10^{19} \text{ m}^{-3}$ at $B_0 \approx 1.9T$. For this case, low-threshold parametric decay instabilities will more easily take place. Calculations show that the daughter wave’s frequency is near the UH frequency inside the magnetic island. This result is in agreement with those observed at TEXTOR [1, 2]. The low-threshold parametric decay instabilities happen near the resonance condition $w_0 = 2w_{UH}$ at TEXTOR.

5. Temperature Effects on the Density Region

The plasma parameters are set as follows: $B_0 = 2.25T$, $f_0 = 140\text{GHz}$, $f_1 = f_2 = 70\text{GHz}$. The figure 5 shows that the lower and upper boundaries of the density region increase as the temperature is increased. It can be seen that the boundaries increase slowly with the temperature. The figure 6 shows that the width of the density region varies with the temperature within the magnetic island. The density region’s width decreases slowly with the temperature. It is evident that temperature has little effect on the density region. The thermal velocity of the electrons is much less than the speed of the light. In this case, the change of the thermal velocity has little effect on the propagation of the UH wave. The local dispersion relation (5) can be expressed as another form:

\[ D = A'n^4 + B'n^2 + E' = 0 \]

\[ n = \frac{ck}{w_j} \]

\[ A' = \beta^2 \frac{3w_{pe}^2w_j^2}{(w_j^2-w_{pe}^2)(4w_{ce}^2-w_j^2)} \]

\[ \beta = \frac{v_{ce}}{c} \]

\[ B' = 1 - \frac{w_{pe}^2}{w_j^2-w_{ce}^2} \]
\[ E = \frac{w_{pe}^2}{w_j} - \frac{w_{ce}^2}{w_{je}} \]

where \( n \) is the refractive index. For \( \beta \ll 1 \), the change of \( v_e \) has little effect on the coefficient \( A' \), so the dispersion relation changes little with temperature.

![Figure 5](image)

**Figure 5.** The density region’s boundaries as a function of the temperature within the magnetic island.

![Figure 6](image)

**Figure 6.** The density region’s width as a function of the temperature within the magnetic island.

6. Conclusions
Calculations show that the low-threshold parametric decay instabilities occur inside the density region. It implies that only if the density exceeds a threshold the low-threshold parametric decay instabilities will take place. The influence of the magnetic field on the density region is obvious. There is a special magnetic field value. Corresponding to this magnetic field value, the density width is the maximum. The effect of the plasma temperature on the density region is not very obvious. In order to avoid the low-threshold parametric decay instabilities, the daughter wave’s frequency must be far from the UH frequency inside magnetic island. It’s the key to control the magnetic field inside the magnetic island.

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