Generation of Weibull distribution clutter based on correlated Gaussian sequence

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Abstract. With the continuous development of science and technology, the electromagnetic environment becomes more complex. Accurate clutter modeling is becoming increasingly difficult, which will have adverse effects in echo analysis. In this paper, in order to overcome electromagnetic interference, we use correlated Gaussian sequence to generate Weibull distribution clutter. Simulation results show that the estimated value of the proposed method is close to the theoretical value in the aspect of probability density and power spectral density. That demonstrates the validity of our method. Finally, the conclusions are given.

1. Introduction
As the increasing performance desire and new proposed system of radar, the demands for radar simulation system are increasing. Echo analysis is an important part in radar simulation, and the basis is clutter modeling. However, electromagnetic environment is more and more complex, and accurate clutter modeling is becoming increasingly difficult. So clutter modeling in complex environment is becoming hot issues of research.

Clutter simulation is the key link in the design of radar echo simulator, and many researchers have conducted a lot of work in related field. Simon Haykin proposes the concept of cognitive radar. Using all available knowledge, it can interrogate a propagation channel adaptively [1]. In [2], the authors add different clutter distributions into the framework of trackability, and significant differences are obtained. In [3], it is shown that when looking up or down wind local spectrum intensity is strongly linked to mean Doppler shift. The spectra will be given a temporal and spatial variability by related characteristics when combined with random fluctuations of spectrum width. In [4], different from traditional signal-dependent stochastic models, the authors propose a new approach of stochastic transfer function. In [5], the authors research on bistatic radar. The authors define bistatic clutter scenario and propose two land-clutter models. In [6], with the environment of Saudi Arabian urban, South African urban and Saudi Arabian date farms, the statistical analysis of three types of land clutter are presented. In [7], a new model is proposed, which is comprised from functions of radar frequency, polarization, sea state, and grazing angle. In [8], the authors describe X-band radar-based measurements of urban ground clutter, which is made from the CSIR campus in Pretoria, South-Africa. In [9], artificial neural network is used to classify four common radar clutter models. After simulating clutter models and extracting the important features, the authors make neural network with two hidden layers. In [10], the optimization of an information theoretic criterion for radar waveform design is described.

In this paper, after describing principles of monopulse radar and Zero Memory Nonlinear (ZMNL), we propose an improved mathematical modeling method for Weibull distribution clutter, which is based on correlated Gaussian sequence. In the simulation, we will compare the generated clutter with the theoretical value and verify the performance of our method.
2. Monopulse Radar
Since the invention of radar, single pulse technology has been a kind of important technology which has been greatly valued. In the early days, antenna microwave devices and other circuits do not get better treatment. Faced with such problems, cone scanning becomes an important method for automatic angle tracking. Since the advent of the sum and difference monopulse radar system, single pulse technology has more and more applications in radar system.

To determine location of the target in space, in many cases direction determination of the target is the primary task of radar, that is to say the angular coordinate of the target. Because of the advantages of strong anti-interference ability and high angular tracking precision of single pulse measuring angle system, it has been widely used.

Single pulse angle measurement is a simultaneous wave-lobe angle measurement method. In the same angular plane, if the same two beams overlap, then the equal signal axis is the overlap. Comparing the echo signals received by the two beams, the angular error signal of the target on this plane can be obtained. Then the transformed error voltage is added to the driving motor, the antenna can be controlled to move towards the direction of error reduction. As the two beams receive echo at the same time, monopulse angle measurement system will use shorter time in obtaining the target angular information. Theoretically, the determination of angular error can be analyzed by only one echo pulse. The following advantages are not available in the conical scan radar:

1. The time of obtaining angular error information is calculated in microseconds, with remarkable shortening;
2. The fluctuation of echo amplitude has no effect on it;
3. Angle measurement accuracy is improved significantly;
4. Angle measurement branch has strong ability in anti-amplitude modulation jamming.

The methods of obtaining angular error signals are different, so there is a variety kind of monopulse radars. Sum and difference patterns of amplitude are most commonly used.

3. Generation of Weibull Distribution Clutter
The simulation of clutter data should meet a certain probability distribution in amplitude and the requirements in correlation properties simultaneously. That means the first-order and second-order characteristics of the data have to be produced to meet the requirements of the clutter.

The concrete block diagram of the clutter modeling with ZMNL method is as follows:

![Figure 1. Block diagram of the clutter modeling with ZMNL method](image)

As is shown in the figure 1, white Gaussian random process changes into correlated Gaussian process after getting through the filter $H(z)$, this process can obtain the required clutter sequence $z(k)$ after the nonlinear transformation. No breaking generality, assuming that $w(k)$ is a unit power white Gaussian random process with zero mean value, and the coefficient of the filter $H(z)$ is normalized.

Specific to monopulse radar, the key problem is how to generate correlated Gaussian sequence. Correlated Gaussian clutter can be viewed as the response to Gaussian white noise with zero mean value acting on the filter. It is supposed that Gaussian white noise’s simple of a certain moment is $w(k)$ and $h(k)$ is the unit sequence response. So after getting through the filter, the response is

$$y(k) = w(k) * h(k)$$

(1)
where \( y(k) \) is the simulation of the correlated Gaussian clutter sequence. On the basis of Fourier transformation, we can obtain

\[
P_y(w) = P_w(w) |H(w)|^2
\]

(2)

where \( P_w(w) \) and \( P_y(w) \) are white noise and PSD function of correlated Gaussian noise. \( H(w) \) is the frequency response function of \( H(z) \). Assume that \( \sigma^2_w \) denotes the variance of white noise, then

\[
P_y(w) = \sigma^2_w |H(w)|^2
\]

(3)

Therefore, frequency characteristics of filter \( H(z) \) are determined by the required correlated power spectrum characteristic of clutter. So we can use the method of filter design to achieve the requirements of correlation characteristics, and then the correlated clutter can be simulated. We can use the frequency domain method and time domain method to design filter. This paper uses the latter method to generate correlated Gaussian sequence \( Y \). Principle diagram is shown in figure 2.

![Figure 2. Generation principle diagram of correlated Gaussian sequence](image)

As is shown above, to get the correlated Gaussian sequence two conditions should be known: Independent Gaussian sequence \( V \) and correlation coefficient sequence \( r(n) \) of correlated Gaussian sequence. We can easily generate independent Gaussian sequence \( V \) and determine the correlated Gaussian sequence \( Y \). Let

\[
|H(w)| = \sqrt{FT(r(n))/r(0)}
\]

(4)

An appropriate phase angle function should be selected to ensure physically feasibility of \( H(w) \). Besides, we should make the independent and non-correlated standard Gaussian sequence \( V \) change into the frequency domain simultaneously.

So the PSD is

\[
S_y(w) = S_v(w) * |H(w)|^2
\]

(5)

Change \( Y(w) \) into time domain from frequency domain, then sequence \( Y \) denotes a correlated Gaussian random sequence.

Assume there are two independent random variables \( z_1 \) and \( z_2 \), and both of them obey the Gaussian distribution \( N(0, \sigma^2) \). The nonlinear transformation is

\[
x = (z_1 + z_2)^{1/\mu}
\]

(6)
We can get the random variable \( x \) that obeys Weibull distribution with parameters \( p \) and \( q \). The relationship between \( p \) and \( q \) is

\[
q = (2\sigma^2)^{1/p}
\]  

(7)

The block diagram of generating incoherent Weibull distribution clutter is shown in figure 3.

![Block diagram of generating incoherent Weibull distribution clutter](image.png)

**Figure 3.** Generation principle diagram of Weibull distribution clutter

In figure 3, \( n_{1,i} \) and \( n_{2,j} \) \((i = 1, 2, ..., N)\) are independent Gaussian sequences which obey the distribution of \( N(0,1) \). After getting through the filter, \( y_{1,j} \) and \( y_{2,j} \) are also correlated Gaussian sequences with correlation coefficient \( \rho_y \) which obey the distribution of \( N(0,1) \). \( z_{1,j} \) and \( z_{2,j} \) obey the distribution \( N(0, \sigma^2) \) whose coefficient of correlation is also \( \rho_y \); The correlation coefficient of the generated Weibull distribution clutter \( x_i \) is \( S_y \), and the nonlinear relationship between \( \rho_y \) and \( S_y \) is

\[
S_y = \left[ \Gamma^2(1+1/p)/\Gamma(1+2/p) - \Gamma^2(1+1/p) \right] \times \left[ z_p F_1(-1/p,-1/p;1;\rho_y^2) - 1 \right]
\]  

(8)

where \( z_p F_1 \) is Gaussian hypergeometric function and \( \Gamma(\cdot) \) is Gamma function. We cannot directly solve \( \rho_y \) with the formula above, so we will have a further discussion about how to get \( \rho_y \) from \( S_y \) in the implicit function.

According to the analysis above, the steps of generating Weibull distribution clutter are as follows:

1. Select the required power spectral density \( S(w) \) for clutter, and choose appropriate sampling rate \( \Delta w \). After sampling from \( S(w) \), we can get sequence \( \{S_n\}, n = 1, 2, ..., N \).

2. Make IFFT transformation to the sequence \( \{S_n\}, i, j = 1, 2, ..., N \) of non-Gaussian random sequence.

3. We can obtain correlation coefficient sequence \( \rho_{y_j}, i, j = 1, 2, ..., N \) of correlated Gaussian random sequence \( \{y_{1,i}\}, \{y_{2,i}\}, i = 1, 2, ... N \).

4. Through \( \rho_y \) and normalized transfer function, we can generate correlated Gaussian sequence \( \{y_{1,i}\}, \{y_{2,i}\}, i = 1, 2, ...N \).
(5) We can get $\sigma^2$ in $z_1, z_2 \sim N(0, \sigma^2)$, and through the nonlinear transform $x_i = (z_{1,i} + z_{2,i})^{1/p}$ we can generate noncoherent Weibull distribution random sequence by the Gaussian random sequence $z_1, z_2$.

The next section will make Weibull distribution clutter simulation in a certain condition through the steps we have discussed above, and analyze the results of simulation.

4. Simulations

In this section, we will compare the generated clutter with the theoretical value. Simulation conditions are set up as follows. The length of the random sequence is 8000 points, power spectrum using Gaussian spectrum model. Sampling frequency $f_s = 1000Hz$, the center frequency $f_0 = 0Hz$, shape parameter $p = 1.5$, scale parameter $q = 0.5$. Simulation results are in figure 4 to figure 8.

Figure 4 is probability distribution of Weibull distribution clutter. The Rayleigh distribution model and the log normal distribution model are only applicable to the finite distribution, however, dynamic range of log normal distribution model is excessive exaggerated and Rayleigh distribution model is too conservative. Weibull distribution model lies halfway in between.

Figure 5 to figure 8 are the simulation results of Weibull distribution clutter with the proposed method. Figure 5 is independent non-correlated Gaussian random sequence. Figure 6 is Weibull distribution clutter obtained in simulation. Comparing figure 5 to figure 6, we can clearly observe the changes of sequences before and after the simulation.

![Figure 4. Probability distribution of Weibull distribution clutter](image-url)
Figure 5. Independent non-correlated Gaussian random sequence

Figure 6. Weibull distribution clutter

Figure 7 shows the comparison between actual amplitude distribution and the theoretical amplitude distribution of clutter, which is also the first-order characteristic of clutter. It can be seen that the actual amplitude distribution and the theoretical amplitude distribution have the same changing trend. As PDF feature of modulation variable is display expression, so the difference in Figure 7 which peaks about at 0.5 at clutter amplitude of 0.2 is slightly obvious.

Figure 8 shows the comparison between the estimated power spectrum and the theoretical Gaussian power distribution, which is also the second-order characteristic of clutter. It can be seen that both of them have the same changing trend. The actual spectral width is larger than the desired width, while the error of the high frequency is large. It is mainly because that the required white Gaussian noise with zero mean cannot be accurately obtained, and the simulation can only use the pseudo-random number instead whose power spectral density is not constant in the strict sense.
5. Conclusions
In this paper, take monopulse radar for example, we give an explanation to the importance of radar clutter modeling and analysis. After describing principles of monopulse radar and ZMNL, we propose an improved mathematical modeling method for Weibull distribution clutter, which is based on correlated Gaussian sequence. Simulation results show that actual amplitude distribution of clutter and the theoretical amplitude distribution of clutter are basically consistent with each other, and they have the same changing trend. Actual Gaussian power distribution and theoretical Gaussian power distribution are consistent with each other, too. The actual spectral width is larger than the desired width, while the error of the high frequency is large. So we can conclude that the generated clutter approaches the theoretical value and our method is effective.

Echo analysis is an important part in radar simulation, and the basis is clutter modeling. With the electromagnetic environment becoming more and more complex, accurate clutter modeling is becoming increasingly difficult. It is very necessary to develop more accurate and faster clutter generation methods in the future research.
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7. References
[1] S. Haykin, “Cognitive radar: a way of the future,” IEEE Signal Processing Magazine, vol. 23, no. 1, pp. 30-40, January 2006.
[2] S. Schoenecker, P. Willett and Y. Bar-Shalom, “The Effect of K-Distributed Clutter on Trackability,” IEEE Transactions on Signal Processing, vol. 64, no. 2, pp. 475-484, January 2016.
[3] S. Watts, L. Rosenberg, S. Bocquet and M. Ritchie, “Doppler spectra of medium grazing angle sea clutter: part 1: characterization,” IET Radar, Sonar & Navigation, vol. 10, no. 1, pp. 24-31, January 2016.
[4] J. R. Guerci, J. S. Bergin, R. J. Guerci, M. Khanin and M. Rangaswamy, “A new MIMO clutter model for cognitive radar,” Proc. IEEE Radar Conference, IEEE Press, 2016, pp. 1-6.
[5] M. Pola, P. Bezousek and Jan Pidanie, “Model comparison of bistatic radar clutter,” Proc. Conference on Microwave Techniques, IEEE Press, 2013, pp. 182-185.
[6] Amer Melebari, M. Y. Abdul Gaffar and J. J. Strydom, “Analysis of high resolution land clutter using an X-band radar,” Proc. IEEE Radar Conference, IEEE Press, 2015, pp. 139-144.
[7] V. Gregers-Hansen and R. Mital, “An Improved Empirical Model for Radar Sea Clutter Reflectivity,” IEEE Transactions on Aerospace and Electronic Systems, vol. 48, no. 4, pp. 3512-3524, October 2012.
[8] J. J. Strydom, J. J. de Witt and J. E. Cilliers, “High range resolution X-band urban radar clutter model for a DRFM-based hardware in the loop radar environment simulator,” Proc. International Radar Conference, IEEE Press, 2014, pp. 1-6.
[9] M. A. Darzikolaei, A. A. Ebrahimzade and E. Gholami, “Classification of radar clutters with Artificial Neural Network,” Proc. 2nd International Conference on Knowledge-Based Engineering and Innovation, IEEE Press, 2015, pp. 577-581.
[10] A. Leshem, O. Naparstek, A. Nehorai, “Information theoretic adaptive radar waveform design for multiple extended targets,” IEEE Journal of Selecte Topics in Signal Processing, vol. 1, no. 1, pp. 42-55, June 2007.