Gauge hierarchy and SUSY: Think again

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Standard Model and Beyond
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1. Introduction
2. Gauge hierarchy
3. SQCD before
4. A SGUT solution
5. Quintessential axion
1. Introduction
Chirality is the theme of this talk.

Chirality ensures small scales.
Weak-Interaction Singlet and Strong CP Invariance

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Strong CP invariance is automatically preserved by a spontaneously broken chiral U(1) symmetry. A weak-interaction singlet heavy quark $Q$, a new scalar meson $\sigma^0$, and a very light axion are predicted. Phenomenological implications are also included.

Attempts\textsuperscript{1-4} to incorporate the observed made the Lagrangian CP invariant. In gen

The new scalar $\sigma^0$.—By the spontaneous symmetry breaking of U(1)$_A$, $\sigma$ will be split into a scalar boson $\sigma^0$ of mass $(2\mu)^{1/2}$ and an axion $a$. This $\sigma^0$ is not a Higgs meson, because it does not break the gauge symmetry, but the phenomenology of it is similar to the Higgs because of its coupling to quark as $m_Q/\nu'$. If this scalar mass is $\simeq 2m_Q$, we will see spectacular final state of stable particles such as $(Q\bar{u})$ and $(Q\bar{u})$. If its mass is $\ll 2m_Q$, the effective interaction through loops $(c/\nu') F_{\mu\nu} F^{\mu\nu} \sigma^0$, with numerical constant $c$, will describe the decay $\sigma^0$—ordinary hadrons. The order of magnitude of its lifetime is $\tau(\sigma^0) \approx \tau(\pi) (\nu' / f_\pi)^2 (m_Q / m_{\pi})^2 \approx 2 \times 10^{-10}$ sec for $\nu' = 10^8$ GeV and $m_{\pi} \approx 10$ GeV. This kind of particle can be identified as a jet in pp high-energy collisions,
Georgi principle: Only gauge symmetry at the ultraviolet scale: Chirality takes over the role for allowing low energy scales

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\[
Q = \frac{1}{2}: \quad \psi_i = \left( \begin{array}{c} E_i \\ N_i \end{array} \right), \\
Q = \frac{3}{2}: \quad \mathcal{L} = \left( \begin{array}{c} \mathcal{E} \\ \mathcal{F} \end{array} \right), \\
E_{i,-1}^{c} (i = 1, 2, 3). \\
\]

So, there is a good reason that these particles will be discovered at low energy. First, by kinetic mixing!!
“Invisible” axion can be a part of DM
PWW, AS, DF (1983)

Today’s concern, because we talk about scale 250 GeV
The dominant contribution is QCD anomaly term.

All pseudoscalars are massive.

Keep only the leading terms.
SUSY mu term: K-Nilles

\[ PQ : \begin{array}{cccc}
-1 & -1 & 1 & 1 \\
H_u & H_d & \sigma & \sigma \\
M & 
\end{array} \]

How to determine the VEV scale of the singlet?
Common scale for $f_a$ and source of SUSY breaking

$V_{ew}$, $5 \times 10^{10}$ GeV, $M_{Pl}$
2. Gauge hierarchy

JEK, Kyae, 1904.07371 [hep-th] PLB797, 134807 (2019): “A model for dynamical SUSY breaking”
Mass scales:

**Planck mass** $2.44 \times 10^{18}$ GeV

**Next scale defines physics disciplines**
- Particle physics $246$ GeV
- Strong Interaction $300$ MeV
- Nuclear physics $7$ MeV
- Atomic physics $1$ eV
- Condensed matter phys $10^{-3}$ eV
Mass hierarchy:

(Planck mass)/(EW scale) \(10^{16}\)

(GUT mass)/(EW scale) \(10^{14}\)

\[ V = -M^2 \sum^* \sum - v_{ew}^2 H^\dagger H + \cdots \]

In the potential \(V\), the scalar (mass)\(^2\) parameters have the ratio of \(10^{28}\).

Why is there such a large ratio of parameters? Including loop corrections?

\textbf{TeV is the cutoff.}

This was pointed out by S.Weinberg after the GUT models were proposed. The GUT models must have parameters such that the Higgs mechanism breaks both SU(5) and SU(2)xU(1) SM.
Anomaly free theories.

fundamentals: [1] one contra-variant index,
[2] two contra-variant index, etc.

SU(3): only quarks or anti-quarks
\[ \Phi[^{\alpha}] = 3, \quad \Phi[^{\alpha\beta}] = \Phi[^{\gamma}] = 3, \quad \Phi[^{\alpha\beta\gamma}] = 1, \quad \text{etc.} \]

SU(4): only quarks or anti-quarks plus [2]=self-dual (removed)

SU(5): the smallest gauge group to have a chiral representation,
[2] + [4] which is anomaly free.

\[ V = -M^2 \Sigma^* \Sigma - v_c^2 \bar{H}^\dagger H + \cdots \]

\[ \Phi[^{\alpha}] = 5, \]
\[ \Phi[^{\alpha\beta}] = 10, \]
\[ \Phi[^{\alpha\beta\gamma}] = \Phi[^{\delta\epsilon}] = \overline{10}, \]
\[ \Phi[^{\alpha\beta\gamma\delta}] = \overline{\Phi[^{\epsilon}]} = \overline{5} \]
An exponential hierarchy obtained by dimensional transmutation.
Dimensionless couplings differ by small amounts.
1st confining force:

Technicolor confines at 3 TeV: Susskind and Weinberg 1979.

exponential 3 TeV= \( M_{\text{GUT}} \times e^{-40} \).

Dimensional transmutation, e.g. 300 MeV

But it failed in flavor physics, by extended technicolor,
through S and T parameter constraints.
Yukawa couplings are definitely needed: scalars are needed definitely.

**SUSY idea:** 1981~
Supergravity phenomenology: 1983~
Supersymmetry: LSP added for DM candidate: 1984~

SUSY breaking needed: Needed for SM partners \( \sim (\text{TeV})^2 \),
Source of SUSY breaking: \( 10^{13} \) GeV, by Gaugino condensation

\[
L = \text{SUSY terms} + \text{SUSY breaking soft terms of } O(\text{TeV}^2)
\]

Gaugino condensation by R=0 singlet:
Nilles(1982),
Derendinger-Ibanes-Nilles(85),
Dine-Rohm-Seiberg-Witten(85)
Condensation of $q$ and anti-$q$ in QCD:

$$(300 \text{ MeV})^3$$

$q$ \hspace{1cm} \text{anti-}$q$

Similarly, gauginos in SUSY theory may condense: Gaugino condensation.

$$(10^{13}\text{GeV})^3$$

Then, scalar $(\text{mass})^2$ parameters in the SM feel this breaking by gravity effects:

$$(10^{13}\text{GeV})^3 / (\text{Planck mass})^2 \sim \text{TeV}$$

How come, “Is there such source of $10^{13}\text{GeV}$ confining force? Is there really “gaugino condensation”?”

DIN, DRSW.

KimJE

“Gauge hierarchy”, Corfu, 9 September 2019.
Two problems:
(1) scale problem.
(2) stability problem.

\[ \delta m_h^2 = \frac{3 G_F^2}{4 \sqrt{2} \pi^2} \left( 4 m_t^2 - 2 m_W^2 - m_Z^2 - m_h^2 \right) \Lambda^2 \]

Remove quadratic divergence. \( \rightarrow \) SUSY
\[ PQ : -1 - 1 \quad 1 \quad 1 \]
\[ \frac{H_u H_d}{M} \quad \sigma \quad \sigma \]

Planck scale

Intermediate scale

\[ \mu \text{-term} \]
3. SQCD before.

SUSY SU(N) gauge theory with L-handed q and R-handed q. 
SU(N_C) gauge group 
SU(N_f)xSU(N_f) flavor group (global) 
Introducing a vector-like representation.

Studied extensively by Seiberg and his collaborators, and many more. These focussed on duality and not obtained SUSY breaking from the gauge theory.
4. A SGUT now

SUSY GUT is defined by not introducing a vector-like representation.

SU(5) GG model is a minimal chiral example.
Anomaly free theories.

fundamentals: [1] one contra-variant index,  
[2] two contra-variant index, etc.

SU(3): only quarks or anti-quarks  
SU(4): only quarks or anti-quarks plus [2]=self-dual (removed)

SU(5): the smallest gauge group to have a chiral representation,  
[2] + [4] which is anomaly free. Due to Georgi’s criteria,  
this is the simplest example.
Meurice-Veneziano considered this SUSY one-family GG model, and suggested a possibility of dynamical SUSY breaking.

\[ [2] = \Psi^{\alpha \beta} ; \quad \Psi^{\alpha \beta} = - \Psi^{\beta \alpha} \]

In conclusion, we have found that in theories with chiral fermions the presence of several currents which do not fall into a vector/axial vector classification brings about strong constraints on SUSY vacua. These constraints, once coupled to reliable small-size instanton effects, lead in certain cases to a contradiction. Even the SQCD way out of a vacuum at infinity appears to be blocked leaving us with the only possibility of a non-supersymmetric ground state.

In the near future further calculations should not fail to provide a complete systematics of the circumstances under which spontaneous SUSY breaking take place.
Standard model (observed):

All participating here, I guess, worked on standard-like models. Not worrying about gauge symmetry breaking at the GUT scale.
Flipped SU(5) from string:

(1) Simple enough for studying flavors in detail
(2) GUTs
(3) $\sin^2(\text{weak mixing angle})=3/8$
Anomaly free theories.

fundamentals: [1] one contra-variant index,
[2] two contra-variant index, etc.

SU(3): only quarks or anti-quarks
SU(4): only quarks or anti-quarks plus [2]=self-dual (removed)

SU(5): the smallest gauge group to have a chiral representation,
[2] + [4] which is anomaly free.
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In the near future further calculations should not fail to provide a complete systematics of the circumstances under which spontaneous SUSY breaking take place.

KimJE  "Gauge hierarchy", Corfu, 9 September 2019.  29/46
Our model (took 46 years):

JEK+Kyae: 1904.07371 “A model for dynamical SUSY breaking”

SU(5) representation:

$[2] + [1] + 2[4]$ which is anomaly free.

\[
\bar{\Psi}^{\alpha\beta} \oplus \bar{\psi}_1^\alpha \oplus 2 \cdot \psi_2^\alpha \\
(\bar{10}, 1) \oplus (\bar{5}, 1) \oplus (5, 2)
\]

(SU(5)_{\text{gauge}}, SU(2)_{\text{global}})
Now we can construct superpotential terms,

\[ W_0 \equiv \frac{1}{4} \overline{\Psi}^{\alpha \beta} \psi_{2 \alpha}^i \psi_{2 \beta}^j \epsilon_{ij}, \quad \overline{\psi}_1^\alpha \psi_{2 \alpha}^i D_{1i}, \quad \frac{1}{5!} \overline{\Psi}^{\alpha \beta} \overline{\Psi}^{\gamma \delta} \psi_1^c \epsilon_{\alpha \beta \gamma \delta \epsilon}, \]

This is not possible with Meurice-Veneziano. In ours, one U(1) remaining.

\[ U(1) \overline{\psi}_1^i + 2U(1) \psi_2 = 0, \]
\[ U(1) \overline{\psi}_1^i + U(1) \psi_2 + U(1) D_1 = 0, \]
\[ 2U(1) \overline{\psi}_1^i + U(1) \overline{\psi}_1^i = 0. \]

SU(5)_{\text{gauge}}-\text{singlet chial fields},

\[ \phi = \frac{1}{5!} \overline{\Psi}^{\alpha \beta} \overline{\Psi}^{\gamma \delta} \psi_1^c \epsilon_{\alpha \beta \gamma \delta \epsilon}, \quad \Phi_i = \overline{\psi}_1^\alpha \psi_{2 \alpha}^i. \]
\[ \sim \frac{\theta}{32\pi^2} \tilde{F}^a \mu \nu \tilde{F}^a \mu \nu \]

Confinement of SU(2) leads to this anomaly, due to instanton calculus, even if we integrate out the SU(2) charged fermions. If we consider infinite spacetime, gauged SU(2) is like global SU(2). So, we satisfy

\[ U(1)_{\text{global}} \] - \[ SU(2)_{\text{gauge}} \] - \[ SU(2)_{\text{gauge}} \] anomaly
For $U(1)$, we do not have the instanton argument, and there is no need to match $U(1)_{\text{global}} - U(1)_{\text{global}} - U(1)_{\text{global}}$ anomaly.

Even if we consider it, we know that it is a total derivative.

\[
\propto \theta \frac{1}{4} \epsilon_{\mu \nu \rho \sigma} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\rho A_\sigma - \partial_\sigma A_\rho)
= \partial_\rho [\theta \epsilon_{\mu \nu \rho \sigma} (\partial_\mu A_\nu)(A_\sigma)] - [\partial_\rho (\theta \epsilon_{\mu \nu \rho \sigma} \partial_\mu A_\nu)](A_\sigma)
= \partial_\rho [\theta \epsilon_{\mu \nu \rho \sigma} (\partial_\mu A_\nu)(A_\sigma)] - [\theta \epsilon_{\mu \nu \rho \sigma} \partial_\rho \partial_\mu A_\nu](A_\sigma)
= \partial_\rho [\theta \epsilon_{\mu \nu \rho \sigma} (\partial_\mu A_\nu)(A_\sigma)]
\]
For anomaly, it is a short distance contribution. It has been used in axion physics in calculating axion-photon-photon coupling, considering the PQ charges of quarks even above the confinement scale. We did not calculate just by considering what are the composite fermions. If so, we should have done it with p and n PQ charges.
| $2\ell(R_{SU(5)})$ | $SU(2)$ | $U(1)_{\psi}$ | $U(1)_{\overline{\psi}}$ | $U(1)_{\psi_1}$ | $U(1)_{\overline{\psi}_1}$ | $U(1)_{D_4}$ | $U(1)_{AF}$ | $U(1)_R$ | dimension |
|---------------------|---------|---------------|----------------|-----------------|----------------|-------------|-------------|----------|-----------|
| $\psi$              | 0       | 0             | 0             | 0               | 0               | 0           | 0           | +1       | $\frac{1}{2}$ |
| $\overline{\psi} \sim (10, 1)$ | $-$     | 1             | $+1$          | 0               | 0               | 0           | 0           | $-1$     | 1         |
| fermion             | +3      | 0             | 0             | 0               | 0               | 0           | 0           | $-1$     | $-\frac{1}{2}$ |
| $\overline{\psi}_1 \sim (5, 1)$ | $-$     | 1             | 0             | $+1$            | 0               | 0           | 0           | $+2$     | 1         |
| fermion             | +1      | 0             | 0             | $+1$            | 0               | 0           | 0           | $+2$     | $-\frac{1}{2}$ |
| $\psi_2 \sim (5, 2)$ | $-$     | 2             | 0             | 0               | 0               | $+1$        | 0           | $+\frac{1}{2}$ | 1         |
| fermion             | $+1 \times 2$ | 2           | 0             | 0               | 0               | $+1$        | 0           | $+\frac{1}{2}$ | $-\frac{1}{2}$ |
| $D \sim (1, 2)$     | $-$     | 2             | 0             | 0               | 0               | $+1$        | 0           | $-\frac{1}{2}$ | 1         |
| fermion             | $+1 \times 2$ | 2           | 0             | 0               | 0               | $+1$        | 0           | $-\frac{1}{2}$ | $-\frac{1}{2}$ |
| $W^a \sim \lambda^a$ | 0       | $-$            | 0             | 0               | 0               | 0           | 0           | +1       | $\frac{3}{2}$ |
| $\Lambda^b$         | $-$     | $-$            | $-$           | $-$             | $-$             | $-$         | $-$         | $\frac{2b}{3}$ | $b$       |
| $\phi$              | $-$     | 1             | $-$           | $-$             | $-$             | $-$         | $-$         | $-5$     | 1         |
| fermion             | $-$     | 1             | $-$           | $-$             | $-$             | $-$         | $-$         | $-5$     | $-\frac{1}{2}$ |
| $\Phi_1$            | $-$     | 2             | $-$           | $-$             | $-$             | $-$         | $-$         | $+\frac{5}{2}$ | 1         |
| fermion             | $-$     | 2             | $-$           | $-$             | $-$             | $-$         | $-$         | $-\frac{1}{2}$ | $-\frac{1}{2}$ |
| $S$                 | $-$     | 1             | 0             | 0               | 0               | 0           | 0           | +2       | 1         |
| fermion             | $-$     | 1             | 0             | 0               | 0               | 0           | 0           | +1       | $-\frac{1}{2}$ |
| $D^i \sim (1, 2)$   | $-$     | 2             | $-$           | $-$             | $-$             | $ -$        | $+1$        | $-\frac{1}{2}$ | 1         |
| fermion             | $-$     | 2             | $-$           | $-$             | $-$             | $ -$        | $+1$        | $-\frac{1}{2}$ | $-\frac{1}{2}$ |
| \( \ell(R_{SU(5)}) \) | \( SU(2) \) | \( U(1) \varphi \) | \( U(1)\bar{\psi}_1 \) | \( U(1)\psi_2 \) | \( U(1)D_1 \) | \( U(1)A_F \) | \( U(1)R \) | dimension |
|----------------|---------|-------------|----------------|-------------|--------------|--------------|----------|----------|
| \( \vartheta \) | 0       | 0           | 0              | 0           | 0            | 0            | +1       | \( \frac{1}{2} \) |
| \( \Psi \sim (\overline{10}, 1) \) | –       | 1           | +1             | 0           | 0            | –1           | +1\( \frac{1}{2} \) | 1        |
| fermion         | +3      | 1           | +1             | 0           | 0            | –1           | +1\( \frac{1}{2} \) | –        |
| \( \bar{\psi}_1 \sim (\overline{5}, 1) \) | –       | 1           | 0              | +1          | 0            | +2           | +1\( -1 \) | 1        |
| fermion         | +1      | 1           | 0              | +1          | 0            | +2           | +1\( -1 \) | –        |
| \( \psi_2 \sim (5, 2) \) | –       | 2           | 0              | 0           | +1          | +\( \frac{1}{2} \) | +1\( -1 \) | 1        |
| fermion         | +1 \times 2 | 2         | 0              | 0           | +1          | +\( \frac{1}{2} \) | 0        | –        |
| \( D \sim (1, 2) \) | –       | 2           | 0              | 0           | +1          | –\( \frac{5}{2} \) | 0        | 1        |
| fermion         | +1 \times 2 | 2         | 0              | 0           | +1          | –\( \frac{5}{2} \) | –        | –        |
| \( W^a \sim \lambda^a \) | 0       | –           | 0              | 0           | 0            | 0            | +1       | \( \frac{3}{2} \) |
| \( \Lambda^b \) | –       | –           | –              | –           | –            | –            | \( \frac{3b}{2} \) | \( b \) |
| \( \phi \) | –       | 1           | –              | –           | –            | –5           | +2       | 1        |
| fermion         | –       | 1           | –              | –           | –            | –5           | +1       | –        |
| \( \Phi_i \) | –       | 2           | –              | –           | –            | +\( \frac{5}{2} \) | +2       | 1        |
| fermion         | –       | 2           | –              | –           | –            | +\( \frac{5}{2} \) | +1       | –        |
| \( S \) | –       | 1           | 0              | 0           | 0            | 0            | +2       | 1        |
| fermion         | –       | 1           | 0              | 0           | 0            | 0            | +1       | –        |
| \( D^i \sim (1, 2) \) | –       | 2           | –              | –           | +1          | –\( \frac{5}{2} \) | 0        | 1        |
| fermion         | –       | 2           | –              | –           | +1          | –\( \frac{5}{2} \) | –        | –        |
$U(1)_A$-SU(2)$_\text{global}$-SU(2)$_\text{global}$ anomalies above and below are matched,

Above $(+1)\times 5 = 5$ from $(5,2)$
Below 5 from $\Phi_i = \bar{\psi}_1^\alpha \psi_2^\alpha i$

Now, consider composites below the confinement scale.

SU(5)$_\text{gauge}$—singlet chial fields,

$$\phi = \frac{1}{5!} \bar{\psi}^\alpha \psi^\beta \bar{\psi}^\gamma \psi^\delta \bar{\psi}_1^\epsilon \epsilon_{\alpha\beta\gamma\delta\epsilon}, \quad \Phi_i = \bar{\psi}_1^\alpha \psi_2^\alpha i.$$
$\text{U}(1)_A - \text{SU}(2)_{\text{global}} - \text{SU}(2)_{\text{global}}$ anomalies above and below are matched,

Now, consider composites below the confinement scale.
The superpotential consistent with $SU(2)_{\text{global}} \times U(1)_{\text{global}}$ is

$$W = M^2 \phi + \frac{N_c (N_c^2 - 1)}{32 \pi^2} \mu_0^2 S \left(1 - a \log \frac{\Lambda^3}{S \mu_0^2}\right) + b M \Phi_i D^i,$$

$$\frac{\partial W}{\partial \phi} = 0 : \quad M^2 = 0,$$

$$\frac{\partial W}{\partial \Phi_i} = 0 : \quad D^i = 0,$$

$$\frac{\partial W}{\partial D_i} = 0 : \quad \Phi_i = 0,$$

$$\frac{\partial W}{\partial S} = 0 : \quad \mu_0^2 \left(1 + a - a \log \frac{\Lambda^3}{S \mu_0^2}\right) = 0,$$

SUSY is broken by the ’O Raifeartaigh mechanism!!!

This is shown here for the first time.

So, we have a solution for the gauge hierarchy problem.
At SU(5)' level

If lambda0 is nonzero, M^2 is nonzero

\[ \frac{\lambda_0}{5!} \bar{\Psi}^{\alpha\beta} \bar{\Psi}^{\gamma\delta} \bar{\psi}_1^\epsilon \epsilon_{\alpha\beta\gamma\delta\epsilon} \rightarrow \lambda_0 \mu_0^2 \phi \]
Common scale for SUSY breaking and $f_a$

So, if the hidden SU(5)' confines at $10^{13}$ GeV - $5 \times 10^{10}$ GeV, the SUSY breaking scale for SM partners is above 1 TeV.

In particular, the lower end $5 \times 10^{10}$ - $10^{11}$ GeV is particularly interesting because it is the anticipated axion scale, which is the most difficult region for axion search.

The SU(5)' confinement provides this region because of the scalar condensation, rather than gaugino condensation.
\[
\frac{M_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2
\]

TeV scale

A common view on the SUSY solution of the GHP.

With gauging condensation at \(10^{13}\) GeV
In our case, the confinement scale by singlet composite scalar is somewhere between $5 \times 10^{10}$ GeV—$10^{12}$ GeV. Not at $10^{13}$ GeV.

With this, $M_{\text{SUSY}}$ can be raised to the scale of the little hierarchy. The super partner scale at a TeV needs $\sqrt{a} \times 5 \times 10^{10}$ GeV for the confinement scale. 6 TeV needs $10^{11}$ GeV confinement scale.
Saha, Friday:
Reasonable fit of Higgs mass
’t Hooft global-anomaly matching (1979)
QCD example for global symmetry matching

Interpretation: Anomaly is related to gauge bosons. “Anomaly” is the anomaly in the process of gauge theory renormalization. How global anomaly results from this? In the confinement process?

Choose SU(2) gauge bosons with a global U(1).
5. Conclusion

1. Chirality: Low mass particles and dynamical SUSY breaking.
2. It can solve the difficult problem of gauge hierarchy
3. A guess on it came from the hidden sector of a working model of flipped SU(5) from string
Randall: To discover one, one should be an expert in model building [APS Denver, April, 2019]