Fluctuation depinning of the dislocation kinks under the influence of external forces

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Abstract

Within the framework of McLaughlin-Scott perturbation theory the equation-of-motion of the dislocation kink in the pinning potential is linearized, assuming the simultaneous influence of $ac$ and $dc$ forces. Based on the equations derived, the probability of kink depinning was studied. The dependencies of the depinning probability on $dc$ and $ac$ forces are analyzed in details.

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1 Introduction

In many elemental and compound semiconductors the motion of dislocations is controlled by Peierls mechanism, i.e. it occurs through formation and migration of kink and anti-kink pairs. Formation of the dislocation kink implies that various dislocation segments are located in the neighboring valleys of the periodic relief (Peierls potential) of the lattice. Motion of a dislocation in Peierls relief is typically divided into three phases [1],[2]: formation of kink pairs, migration of the kink and anti-kink in opposite directions and annihilation of kinks with different signs moving towards each other.

It is well known, that various point defects (impurity atoms, vacancies) may impede the motion of the kink along the dislocation, i.e. they can act
as pinning centers for the kink. In absence of lattice strain, the kink can be held in the potential of the pinning centre, while the influence of a stationary force can detach it with certain probability. Simultaneous effect of ac force can change (increase or decrease) the depinning probability. This effect was theoretically analyzed for the case, when a segment of the dislocation line overcomes the pinning centre [3].

The importance of the studied problem is mostly due to the issue of recombination - enhanced motion of dislocations. In semiconductors the electronic features of the dislocation kink can influence the dislocation dynamics, provided that the multiphonon capture of the charge carriers by the kink is the significant mechanism of their recombination. During the multiphonon capture the lattice fluctuations localized in proximity to the capture centre are stimulated. Subsequently, those fluctuations are dispersed through the entire crystal. Thus, if recombination of charge carriers involves the dislocation kinks and happens through the thermal capture mechanism [4], then the released energy excites additional fluctuations which may stimulate the motion of dislocation (similarly the photoplastic effect in semiconductors has been interpreted [5]). It is generally accepted that for dislocation motion by the Peierls mechanism, recombination basically contributes to the kink migration along the dislocation [2,6]. Recombination - enhanced motion of dislocations leads to extension of stacking faults in junction regions of bipolar and optoelectronic semiconductor devices and eventually results in the device degradation.

Thus, fluctuation-assisted depinning of the dislocation kink represents a single act that causes degradation of a semiconductor device. Therefore, the aim of our work is exploring stochastic dynamics of the pinned kink under the influence of stationary and alternating (harmonic) forces. During the degradation process the dislocation is influenced by constant force stemming from internal tensions of the lattice, and the lattice fluctuations resulting from thermal recombination of carriers act as the ac force.

Kinks and anti-kinks on the dislocation are non-linear objects (described by the sin-Gordon (SG) soliton equation). Therefore, the study of fluctuation-assisted depinning of the kink is a complicated task. Simple approach to solving this problem is used in Ref. [7], where dynamics of the kink is described by the equation of linear harmonic oscillator, which has the kink mass ascribed to it. This simplification allows to basically describe the process, however, for the purpose of further development of theory (to fully account for the dependence of detachment probability on the kink parameters), it is
necessary to correctly linearize the equation of motion.

Linearization of the equation of motion of the kink is possible through the use of perturbation theory developed by McLaughlin and Scott [8]. Although this theory is based on some assumptions, it is actually the only reliable approach used in many studies for exploring the effect of external forces on the kink (see Ref. [9] and references therein). In this paper, we will use a modification of above-mentioned perturbation theory developed in [10]. When linearizing the equations of the kink motion in presence of ac force, this method enables to avoid complicated calculations used by the authors [8].

2 Kink dynamics under influence of ac force

Let’s consider a smooth kink on a dislocation (directed along x axis), i.e. the width $w$ significantly exceeds the height $a$ (Fig.1). The height $a$ of the kink (equal to the inter-atomic distance) represents a period of Peierls relief. The kink is described by the transversal displacement $\varphi$ of the dislocation, which is equal to zero when $x \to -\infty$, and equal to $a$ when $x \to \infty$ (for anti-kink the reverse is true, i.e. $\varphi$ decreases from $a$ to zero). The kink width corresponds to the range where $0 < \varphi < a$. The smallness of the parameter $\tau_P/G$ is the criterion determining the smoothness of the kink, where $\tau_P$ is the Peierls tension and $G$ is the shift module. The kink width relates to its height as per following equation [1]:

$$w = a \left( \frac{\pi/2}{\tau_P/G} \right)^{1/2}.$$  \hspace{1cm} (1)

Therefore, the inequality $\tau_P/G \ll 1$ denotes that the smooth kinks are being formed, as from (1) we get the following: $w \gg a$. In this case the string model of the dislocation is applicable for description of the kink.

If the kink is located close to the pinning point, we can describe the interaction of the kink with the pinning centre (located at the point $x = 0$ on the dislocation) by the following potential [8], [10]:

$$U = -\frac{a}{2\pi \mu \delta(x)} \left( 1 - \cos \frac{2\pi \varphi}{a} \right),$$  \hspace{1cm} (2)

where parameter $\mu$ characterizes the interaction force. The multiplier $1 - \cos (2\pi \varphi/a)$ in equation (2) ensures that the pinning potential $U$ becomes...
equal to zero outside of the kink region.

As indicated in the Introduction, we are interested in the kink dynamics under influence of dc and ac forces. The constant force $\Gamma$ stemming either from the internal pressure or the lattice external strain tends to move the kink away from the pinning centre. We will consider the ac force $\gamma(t)$, which emulates the lattice fluctuations originating due to the recombination process, as harmonic process. Assuming that the energy of these forces, as well as dissipation due to friction are small compared to Peierls potential, we can consider the influence of external forces as small perturbation relative to the undisturbed system. If the periodic potential has the following form (commonly accepted for Peierls relief [1],[11]):

$$V_P = \frac{a^2 \tau_P}{2\pi} \sin^2 \left( \frac{\pi \varphi}{a} \right),$$  \hspace{1cm} (3)

then the dynamics of a string in this potential in case of existence of dissipation can be described by disturbed SG equation [8]:

$$m\varphi_{tt} - \chi \varphi_{xx} = -\frac{\pi a \chi}{2w^2} \sin \frac{2\pi \varphi}{a} + f,$$  \hspace{1cm} (4)

where $m$ and $\chi$ are the mass and the linear tension of the dislocation (per unit length) respectively. The force $f$ is the sum of all acting forces:

$$f = -\eta \varphi_t - \Gamma - \gamma(t) + \mu \delta(x) \sin \frac{2\pi \varphi}{a},$$  \hspace{1cm} (5)

where $\eta$ characterizes dislocation damping per unit length $(\eta > 0)$. For derivation of equation (4) it was taken into account that $\chi \simeq Gb^2/2$ [11], where $b$ is the module of the Burgers vector (which has the order of $a$), $s$ is the sound velocity in solids, and the $\tau_P/G$ ratio is determined by equation (1).

If the system is not perturbed, i.e. $f = 0$, then the solution of equation (4) is the following [8],[9],[10]:

$$\varphi = \frac{2a}{\pi} \arctan \left[ \exp \left( \pm \frac{\pi}{w} g(v)(x - x_0 - vt) \right) \right],$$

where $x_0$ is the initial coordinate of the kink, $v$ is the velocity of motion across the dislocation $(v < s)$, and $g(v) = 1/\sqrt{1 - (v/s)^2}$. Here, the positive sign corresponds to the kink and the negative sign – to the anti-kink. This solution
corresponds to translation invariance of the kink along the dislocation. The energy of SG system is presented by the following Hamiltonian \[8,10\]:

$$H_{SG} = \int_{-\infty}^{\infty} \left( \frac{w\tau p}{2s^2} \dot{\varphi}^2 + \frac{w\tau p}{2} \varphi^2 + \frac{\pi}{w} V_p \right) dx. \quad (6)$$

By substituting the solution of unperturbed SG equation into (6) and integrating over \(x\), we will arrive at the Hamiltonian of the unperturbed system:

$$H_{SG} = \frac{2a^2 \chi}{\omega^2} \left( 1 - \left( \frac{v}{s} \right)^2 \right)^{-1/2}. \quad (7)$$

If \(f << a\tau_{P}/2\), i.e. all forces are small compared to the amplitude of the Peierls potential, then in order to solve equation (4) we can use the perturbation technique developed in publications \[8,10\]. We assume that solution of equation (4) has the form of unperturbed SG equation, however the perturbations lead to modulation of velocity, i.e. in this case \(v\) is a function of time \(t\):

$$\varphi = \frac{2a}{\pi} \arctan \left[ \exp \left( \pm \frac{\pi}{w} (g(v)x - X(t)) \right) \right] \quad (8)$$

where \(X(t) = \int_{0}^{t} g(v(t')) v(t') \, dt'\) indicates the location of the kink centre (assuming that \(x_0 = 0\)). In absence of terms describing dissipation and \(ac\) force in equation (5), we can write the Hamiltonian of the perturbed system the following way:

$$H(\varphi) = H_{SG} + H_p, \quad (9)$$

where

$$H_p = \frac{\pi}{w} \int_{-\infty}^{\infty} \left( \Gamma \varphi - \frac{a}{2\pi} \mu \delta (x) \left( 1 - \cos \frac{2\pi \varphi}{a} \right) \right) dx \quad (10)$$

describes the energy of stationary perturbation. In this case \(H(\varphi)\) is stationary in time: \(dH/dt = 0\). When non-stationary forces are present (the friction force and \(ac\) external force), the equation of motion must be determined by the following equation [12]:

$$\frac{dH}{dt} = -\frac{\pi}{w} \int_{-\infty}^{\infty} \left( \eta \dot{\varphi}^2 + \gamma(t) \varphi \right) dx. \quad (11)$$
By inserting equation (9) into (11), we arrive at the equation of the moment-

\[ \frac{dH_{SG}}{dv} \dot{v} = \frac{\pi}{w} \int_{-\infty}^{\infty} f(\varphi) \dot{\varphi} dx, \]  

(12)

where \( \varphi = \varphi(g(v) x, X) \) is the solution of the perturbed SG equation (4) that have the time derivative \( \dot{\varphi} \simeq \varphi_X g(v) v \) [10]. The equation (12), along with the equation of time derivate of \( X \)

\[ \dot{X} = g(v) v, \]  

(13)

describes the motion of the kink. The Hamiltonian \( H_{SG} \) in equation (12) has the same form as the Hamiltonian of the unperturbed SG system (6). The string displacement \( \varphi \) is determined by (8). By inserting equation (8) into (6) and integrating, we arrive at the expression for the soliton energy (which substitutes the stationary Hamiltonian (7)):

\[ H_{SG} [\varphi(g(v(t)) x, X(t), v(t))] \simeq \frac{2a^2 \chi}{w^2} \left(1 - \left(\frac{v(t)}{s}\right)^2\right)^{-1/2}. \]  

(14)

Now, by inserting the expressions (14) and (5) into (12) and integrating, we get set of differential equations for variables \( v(t) \) and \( X(t) \)

\[ M \frac{dv}{dt} = \pm \left[\Gamma + \gamma(t)\right] (g(v))^3 - \frac{2a}{\pi w} (g(v))^{-2} - \frac{2a}{w} (g(v))^{-2} \text{sech}^2 \left(\frac{X}{w}\right) \tanh \left(\frac{X}{w}\right) \]  

\[ \frac{dX}{dt} = g(v) v, \]  

(15)

where we accounted for \( \chi = ms^2; M = 2am/\pi w \) is the effective mass of the kink. The set of equations (15) determines the dynamics of the isolated kink (i.e. the one which does not interact with other kinks) conditioned by the general force \( f \).

Let’s explore the motion of the kink in the vicinity of the pinning point. Equations (15) are typically analyzed in the phase plane \((v, X)\). The singular point, where \( \dot{v} = 0 \) and \( \dot{X} = 0 \), in presence of \( dc \) force corresponds to the kink equilibrium position [8]. When \( ac \) force is also in effect, we have to determine the equilibrium position considering only stationary force, meanwhile the \( ac \) force causes additional shift from the equilibrium. In other words, the equilibrium position is determined as the kink coordinate averaged on the
time intervals larger than the period of the ac force. As it follows from equation (15), the following conditions correspond to the position of equilibrium: \( v = 0 \) and \( X = X_0 \), where \( X_0 \) is the root of the following equation:

\[
\frac{w\Gamma}{2\mu} - \text{sech}^2\left(\frac{\pi X_0}{w}\right) \tanh\left(\frac{\pi X_0}{w}\right) = 0. \tag{16}
\]

For the purpose of analytical examination of the kink oscillations around the point \( X_0 \), we need to linearize the equation (15) with respect to variable \( v \), assuming that the motion of the kink is non-relativistic (\( v \ll s \)). In absence of ac force, the kink makes damping oscillations around the pinning point, and eventually reaches the rest [8]. If the kink is also subject to the influence of periodic force \( \gamma(t) = f_0 \sin \Omega t \), where \( f_0 \) is the amplitude of periodic force and \( \Omega \) is its frequency, then we need to analyze the following equations:

\[
M \frac{dv}{dt} = \pm \left[ \Gamma + f_0 \sin \Omega t \right] - \frac{2a\eta}{\pi w} v - \frac{2\mu}{w} \text{sech}^2\left(\frac{\pi X}{w}\right) \tanh\left(\frac{\pi X}{w}\right),
\]

\[
\frac{dX}{dt} = v, \tag{17}
\]

Further linearization of the equation of motion of the kink is possible in case of small oscillations, assuming that the displacement is small compared to the kink width (\( X - X_0 \ll w \)). Then we can expand the last term in the right-hand side of the first equation (17) (describing the interaction of the kink centre with the pinning point) into Taylor’s series around the equilibrium point \( X_0 \). Keeping only linear in \( X - X_0 \) term and using the second expression (17), we obtain the equation of linear oscillations:

\[
\ddot{X} + 2\lambda \dot{X} + \omega_0^2 (X - X_0) = \pm \frac{f_0}{M} \cos \Omega t, \tag{18}
\]

where the parameter \( \lambda = a\eta/\pi w M \) is the effective damping coefficient, and

\[
\omega_0^2 = \frac{2\pi \mu}{w^2 M} \left[ \text{sech}^4\left(\frac{\pi X_0}{w}\right) - 2 \text{sech}^2\left(\frac{\pi X_0}{w}\right) \tanh^2\left(\frac{\pi X_0}{w}\right) \right] \tag{19}
\]

assumes the role of eigenfrequency of the system.

As it follows from equation (16), when \( \Gamma = 0 \) the equilibrium position is \( X_0 = 0 \), and from equation (19) we derive \( \omega_0^2 = \frac{2\pi \mu}{w^2 M} \). With increasing constant force, \( \omega_0 \) reduces; the critical value of force \( \Gamma_c = 4\mu/\left(3\sqrt{3}w\right) \) corresponds to the condition \( \omega_0 = 0 \), which is determined from equations (16) and
In other words, when $\Gamma > \Gamma_c$ (then equation (16) haven’t roots), the kink overcomes the pinning potential and can be considered as a free kink.

Note that the constant force does not enter explicitly into equation (18). This force determines the equilibrium position $X_0$. Thus, equation (18) describes oscillations of the shifted coordinate of the kink center under the influence of the harmonic force. In case of small viscosity $\omega_0 > \lambda$, we immediately arrive at the kink displacement

$$X = X_0 + s(t),$$

where the function $s(t) = B \cos (\Omega t + \delta)$ describes harmonic displacements; $B$ is the amplitude of the kink oscillations and $\delta$ is the initial phase:

$$B = \frac{f_0}{M \sqrt{\omega_0^2 - \Omega^2}}, \tan \delta = \frac{2\lambda}{\Omega^2 - \omega_0^2}.$$  

3 Fluctuation-assisted depinning of the kink

We will use Langevin approach to describe fluctuations of the kink in the potential of the pinning centre, i.e. fluctuation force $\tilde{f}(t)$ needs to be introduced into the right hand side of the equation [13]:

$$\ddot{X} + 2\lambda\dot{X} + \omega_0^2 (X - X_0) = \pm \frac{f_0}{M} \cos \Omega t + \frac{\tilde{f}(t)}{M}.$$  

We consider the fluctuations as being white noise, i.e. the average value of the fluctuation force equals to zero, and the time correlation is determined by the delta-function:

$$\langle \tilde{f}(t) \rangle = 0,$$

$$\langle \tilde{f}(t) \tilde{f}(t') \rangle = 2D \delta(t - t'),$$

where $D$ is the intensity of the Langevin source (it also represents the coefficient of diffusion in velocity space). From now on, we will not consider quantum fluctuations (i.e. we assume that the process temperature is higher than the Debye temperature of the crystal $T >> \theta_D$); in this case $D = \lambda k_B T / M$, where $k_B$ is the Boltzmann constant.

Before proceeding to solution of the Langevin equation, let’s determine the value of the critical shift of the kink centre, which corresponds to the
kink depinning. Obviously, a parabolic potential corresponds to the linearized equation (18). However, we have not yet determined the radius of that potential corresponding to the critical shift. In order to determine that radius, let’s refer to first equation (17). We can determine the local potential of the interaction of kink and pinning centre $U(X)$ by integrating over $X$ the last term of this equation (characterizing the strength of interaction). Taking into account that this potential should become zero when $X \rightarrow \infty$, we get

$$U(X) = -\frac{\mu}{\pi} \text{sech}^2 \left( \frac{\pi X}{w} \right).$$  \hspace{1cm} (23)

Unfortunately, the potential (23) does not have the natural cutoff radius. Therefore, let’s assume that some small value of the interaction potential $U = -U_0$ corresponds to the critical shift value $X_{cr}$. The kink can be considered as detached from the pinning centre if its energy state is separated from the zero level (corresponding to the free kink) by the energy $U_0 \sim k_B T$: in this case any single-quantum thermal fluctuation will lead to final exit from the potential well. Thus, critical shift can be determined by the following expression:

$$X_{cr} = \frac{w}{\pi} \text{arcosh} \left[ \left( \frac{\mu}{\pi k_B T} \right)^{\frac{1}{2}} \right].$$  \hspace{1cm} (24)

Now we can proceed to calculation of the probability of the fluctuation-assisted shift of the kink beyond $X_{cr}$ value (24). The solution of equation (21) is the sum of the deterministic shift (18) and shift $\xi(t)$ determined by fluctuations:

$$X = X_0 + s(t) + \xi(t).$$  \hspace{1cm} (25)

By using the theory of random process overshoots [14], we can determine the depinning probability of the kink affected by the periodic force as equal to the average (during oscillation period) speed of positive overshoots of the sum of stochastic and harmonic shifts beyond the value $C = X_{cr} - X_0$:

$$\nu_\Omega = \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} dt \int_0^{\infty} dv vW \left( C - s(t) ; v - \dot{s}(t) \right)$$  \hspace{1cm} (26)

where $W$ is the joint probability density for the coordinate and velocity of the random parameter. Since we analyze influence of the white noise $\tilde{f}(t)$ on the system described by linear equation (21), then the response (25) should
be a normal process described by Gauss function:

\[
W(X, v) = \frac{1}{(2\pi \sigma_1)^{1/2}} \exp \left[ -\frac{(X - s(t))^2}{2\sigma^2} \right] \exp \left[ -\frac{(v - \dot{s}(t))^2}{2\dot{\sigma}^2} \right],
\]

(27)

where \(\sigma\) and \(\sigma_1\) are the dispersions of the coordinate and the velocity of the fluctuating parameter respectively, which are determined through the correlation function of the process.

The spectral density of the process \(S(\omega) = \langle X^2(\omega) \rangle\) is determined from equations (21) and (22):

\[
S(\omega) = \frac{D}{(\omega^2 - \omega_0^2)^2 + 4\lambda^2\omega^2},
\]

and for the correlation function \(K(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{-i\omega\tau} d\omega\) we find:

\[
K(\tau) = \frac{\pi k_B T}{2M\omega_0^2} \exp (-\lambda |\tau|) \left( \cos \sqrt{\omega_0^2 - \lambda^2 \tau} + \frac{\lambda}{\sqrt{\omega_0^2 - \lambda^2}} \sin \sqrt{\omega_0^2 - \lambda^2 |\tau|} \right).
\]

(28)

Dispersions of the coordinate and the velocity are determined by \(\sigma = K(\tau = 0)\), \(\sigma_1 = K''(\tau = 0)\):

\[
\sigma^2 = \frac{\pi k_B T}{2M\omega_0^2}, \quad \sigma_1^2 = \frac{\pi k_B T}{2M}.
\]

(29)

By using expressions (27) and (29), we can now perform integration over \(v\) in (26). By denoting by \(\Psi\) the phase of deterministic shift \(\Psi = \Omega t + \delta\) and considering that integrand is periodic as function of \(\Psi\) with the period \(2\pi\), we will get

\[
\nu_\Omega = \frac{1}{q} \sqrt{\frac{2}{2\pi}} \int_0^\pi \Phi'(c - p \cos \Psi) \; _1F_1 \left( -\frac{1}{2}; \frac{1}{2}; -\frac{1}{2} (pq \sin \Psi)^2 \right) d\Psi,
\]

(30)

where \(\Phi'(z) = \frac{1}{\sqrt{2\pi}} \exp (-z^2/2)\) is the derivative of the probability integral, \(_1F_1(a; b; z)\) is the degenerate hypergeometric function; \(p = B/\sigma\), \(q = \Omega/\omega_0\), and \(c = C/\sigma\) is the relative level.

In order to explore the dependence of the kink depinning on the stationary force, we discuss first the case when \(ac\) force is absent. In the latter case, for
the probability of the kink depinning the following expression can be easily obtained:

\[ \nu_0 = \frac{\omega_0}{2\pi} \exp \left( -\frac{c^2}{2} \right). \]  

(31)

Dependence of \( \nu_0 \) on constant force \( \Gamma \) is determined through dependence of the frequency \( \omega_0 \) and relative level \( c = \sqrt{\frac{2M\mu^2}{\pi k_B T}} (X_{cr} - X_0) \) on constant shift \( X_0 \) (while the dependence of \( X_0 \) on \( \Gamma \) is determined by the transcendental equation \( (16) \)). The analytical function \( \nu_0(\Gamma) \) can be obtained in case when constant force \( \Gamma \) is significantly smaller than the critical value of \( \Gamma_c \) \( (\Gamma << \Gamma_c) \). By denoting \( y \equiv \tanh(\pi X_0/w) \), from equation \( (16) \) we can get the cubical equation:

\[ y^3 - y + \frac{2\Gamma}{3\sqrt{3}\Gamma_c} = 0. \]

(32)

In the case \( \Gamma << \Gamma_c \) it has the following solution

\[ y = -\frac{2}{\sqrt{3}} \sin \left( \frac{\Gamma}{3\Gamma_c} \right). \]

By limiting to the first order of value \( \Gamma/\Gamma_c \), we will get the following dependence of \( X_0 \) on the force:

\[ X_0 = -\frac{w}{\pi} \text{artanh} \left( \frac{2 \Gamma}{3\sqrt{3}\Gamma_c} \right). \]

By inserting this expression into \( (19) \), we will find the relationship between the eigenfrequency of the system and \( dc \) force:

\[ \omega_0^2 = \frac{2\pi\mu}{w^2 M} \left( 1 - \frac{1}{3} \left( \frac{4\Gamma}{3\Gamma_c} \right)^2 \right). \]

(33)

Finally, using equation \( (33) \) the depinning probability is obtained:

\[ \nu_0 = \left( \frac{\mu}{2\pi w^2 M} \right)^{1/2} \left( 1 - \frac{1}{3} \left( \frac{4\Gamma}{3\Gamma_c} \right)^2 \right)^{1/2} \exp \left[ -\frac{2\mu}{\pi^2 k_B T} \left( 1 - \frac{1}{3} \left( \frac{4\Gamma}{3\Gamma_c} \right)^2 \right) \left( \text{arccosh} \left( \frac{\mu}{\pi k_B T} \right)^{1/2} + \frac{2}{3\sqrt{3}\Gamma_c} \right)^2 \right]. \]  

(34)
The dependence of (34) is presented on Fig.2. Note that for a value of $dc$ force $\tilde{\Gamma}$ ($\tilde{\Gamma} < \Gamma_c$) which is determined by the equation

$$\frac{w\tilde{\Gamma}}{2\mu} - \text{sech}^2 \left( \frac{\pi X_{cr}}{w} \right) \tanh \left( \frac{\pi X_{cr}}{w} \right) = 0,$$

the level $C = X_{cr} - X_0$ becomes zero. Then the depinning probability during the period $2\pi/\omega_0$ approaches unity, as it is shown by dashed curve on the graph.

The dependence of probability (30) on the amplitude of $ac$ force $B = p\sigma$ can also be obtained more explicitly in several limiting cases. Note that the condition of incomplete equilibrium $q = \Omega/\omega_0 << 1$ should be fulfilled for the quasi-stationary processes under discussion [15]. Therefore, depending on the force amplitude, two limiting cases can be realized: $pq << 1$ and $pq >> 1$.

a) The small amplitude of the $ac$ force, $pq << 1$. In this case we can derive from equation (30) an expression linking $\nu_\Omega$ with the probability of detachment in absence of $ac$ force $\nu_0$:

$$\nu_\Omega = \frac{\nu_0}{\pi} \int_0^\pi \exp \left( cp \cos \Psi - \frac{p^2}{2} \cos^2 \Psi \right) d\Psi$$

The dependence of $\nu_\Omega(p)$ for various values of the relative level $c$ is depicted on Fig.3. In the case $c < 1$, $\nu_\Omega$ is a monotonous decreasing function relative to signal $p$, i.e. if the level of $C$ is less than the amplitude of noise $\sigma$, then presence of external periodic force reduces the depinning probability.

If $c > 1$, $\nu_\Omega(p)$ increases reaching maximum when $p = p_m(c)$: $p_m \approx 2\sqrt{1-c^{-2}}$ for $pc < 1$, and $p_m \approx c$ for $pc > 1$. Thus, when interaction strength $\mu$ ensures high values of critical shift $X_{cr}$ (and respectively, the level $C$ exceeds the noise amplitude) and $p \approx p_m$, the probability of detachment can significantly increase. When signal continues to grow ($p > p_m$), the probability reduces as $\nu_\Omega \sim p^{-1}$.

b) The limit of large amplitudes, $pq >> 1$. In this case the probable case is $p > c$. As it follows from (26) and (27), $\nu_\Omega \to \Omega/2\pi$, i.e. the number of overshoots during the period approaches unity.

Note that the dependence of kink depinning probability on amplitude of $ac$ force is analogous to the case of dislocation line (string) detachment from the oscillating stopper [3]. The latter is true for dislocation motions in crystals with low Peierls relief (mostly in metals).
4 Conclusions

Perturbation approach allows obtaining linear equation of motion for kink in pinning potential. This equation was used for exploring fluctuation-assisted depinning of the kink affected by \textit{dc} and \textit{ac} forces. The obtained behavior of depinning probability on \textit{ac} force amplitude is rather intriguing. The fact that this force can result either in decrease or increase of the probability (depending on amplitude of the force and strength of the stopper) means that recombination enhancement, modeled in our work by the harmonic \textit{ac} force, can cause both softening and hardening of the crystal. It should be noted that both these effects were observed in ionic crystals with low Peierls potential, where dislocation segments move (under sufficiently high load) without kink formation [1]. However, in crystals with covalent and mixed covalent-ionic bonds, where the Peierls mechanism of dislocation motion is prevalent, the softening is more common phenomena (though the hardening was observed too, for example, in GaAs crystals during investigation of photoplastic effect [16]). Our analysis shows, that hardening of Peierls-type crystal due to \textit{ac} excitation can occur in the case when the depinning of migrating kinks is the bottleneck of the process.

It should be noted also that by introducing the \textit{ac} force we simulated only the oscillations of the kink, while the pinning centre was assumed to be immobile. Obviously, it is quite possible that the stopper can also be influenced by \textit{ac} excitation, particularly when this center is involved in thermal recombination process and, consequently, the heat release stimulates its oscillations. This can essentially affect the depinning probability [7]. Extension of our study to the case when both kink and stopper are subjected to \textit{ac} forces can be the issue of further development.

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