Signal Temporal Logic Task Decomposition via Convex Optimization

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Abstract—In this paper we focus on the problem of decomposing a global Signal Temporal Logic formula (STL) assigned to a multi-agent system to local STL tasks when the team of agents is a-priori decomposed to disjoint subteams. The predicate functions associated to the local tasks are parameterized as hypercubes depending on the states of the agents in a given sub-team. The parameters of the functions are, then, found as part of the solution of a convex program that aims implicitly at maximizing the volume of the zero superlevel set of the corresponding predicate function. Two alternative definitions of the local STL tasks are proposed and the satisfaction of the global STL formula is proven when the conjunction of the local STL tasks is satisfied.

Index Terms—Signal Temporal Logic, Convex Optimization, Multi-Agent Systems

I. INTRODUCTION

O VER the last decades, multi-agent systems have been considered in a variety of applications such as connectivity and formation control [1] or coverage [2]. The complexity of these applications has motivated the need of an expressive language, capable of describing complex task specifications for planning and control synthesis. Recent extensive interest has been shown in planning under high-level task specifications expressed by Linear Temporal Logic (LTL) [3], [4]. In these methods the temporal formula, the environment and the agent dynamics are abstracted into finite-transition systems. Then, graph-based methods are employed to find a discrete path satisfying the LTL specifications which is finally followed using continuous control laws. An important limitation of the aforementioned methods is the increasing computational complexity as the number of the agents in the team becomes larger. Towards minimizing the computational costs, large effort has been devoted to the decomposition of a global LTL formula into local LTL tasks whose satisfaction depends on subsets of agents. Existing methods, applied to heterogeneous agents [5], [6], most often employ exhausting automata-based approaches [5]–[7] or more recently, cross-entropy optimization methods limited, though, to homogeneous agents [8].

All methods presented so far consider the satisfaction of LTL tasks without explicit time constraints. On the other hand, Signal Temporal Logic (STL) [9] can express complex tasks under strict deadlines. An advantage of STL over LTL, is the robust semantics [10], [11] it offers that allow the evaluation of the satisfaction of the task over a continuous-time signal, rendering the abstractions of the agents’ dynamics obsolete.

Existing methods for planning under STL specifications consider a global STL formula and find plans as solutions to computationally prohibitive MILPs [12], [13] or to scalable convex programs [14], [15]. Other approaches compose [16] or assume the existence [14] of local STL tasks whose satisfaction involves only a small subset of agents. This facilitates the design of decentralized frameworks that are inherently more robust to agents’ failures and often cheaper in terms of communication. Towards decentralized control under global task specifications, a satisfiability modulo theories (SMT) approach has been proposed in [17] for tasks described in caSTL, in which both the global formula and the team of agents are decomposed. Here, the decomposition is based on a set of services required for each task and a set of utility functions specifying the capabilities of the agents. Nevertheless, the decomposition of a global STL formula in continuous space and time remains an open problem.

In this paper we propose a novel framework for the decomposition of a global STL formula imposed on a multi-agent system into a set of local tasks when the team of agents is a-priori divided into disjoint sub-teams. The goal of the decomposition is to make the satisfaction of every local task dependent only to a subset of agents that belong to the same sub-team. Initially, the predicate functions corresponding to STL formulas forming the local tasks are parameterized as functions of the infinity norm of the agents’ states while their parameters are found as part of the solution to a convex program that aims at maximizing the volume of their zero superlevel set. Although the choice of the parametric family of the predicate functions is not restrictive, our current choice allows us to draw conclusions on the volume of a continuous state-space set by incorporating a finite, but possibly large, number of constraints in the convex program. The number of these constraints differs per global STL task but depends solely on the number of the agents’ states involved in its satisfaction. Two definitions of the local tasks that differ on the definition of the STL tasks originating from eventually formulas are introduced. Finally, for both definitions the satisfaction of the global STL formula is proven when the conjunction of the local tasks is satisfied.
The remainder of the paper is as follows: Section II includes the preliminaries and problem formulation. Section III introduces the proposed method for STL decomposition. Simulations are shown in Section IV and conclusions are summarized in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

The set of real and non-negative real numbers are denoted by $\mathbb{R}$ and $\mathbb{R}_{\geq 0}$ respectively. True and false are denoted by $\top, \bot$ respectively. Scalars and vectors are denoted by non-bold and bold letters respectively. The infinity norm of a vector $x \in \mathbb{R}^n$ is defined as $\|x\|_\infty = \max \{|x_i|, \text{where } x = [x_1 \ldots x_n]^T\}$. Given a finite set $\mathcal{V}$, $\prod_{i \in \mathcal{V}} X_i$ denotes the Cartesian product of the sets $X_i, k \in \mathcal{V}$. Given a rectangular matrix $A \in M_{n \times m}(\mathbb{R})$ we define the set $AX$ as $AX = \{Ax : x \in \mathcal{X}\}$. Consider the vectors $x \in \mathbb{R}^n, y \in \mathbb{R}^m$ with $n \leq m$ satisfying $x = By$. The matrix $B = [b_{ij}]$ is called a selection matrix if it has the following properties: 1) $b_{ij} \in \{0, 1\}$, 2) $\sum_{j=1}^n b_{ij} = 1, \forall i = 1, \ldots, n$ and 3) $\sum_{i=1}^m b_{ij} = 1, \forall j = 1, \ldots, m$. We denote by $\theta = [\theta_{j \in \mathcal{V}} = [\theta_{a_1} \ldots \theta_{a_m}]^T$ the stacked vector of $\theta_j, j \in \mathcal{V} = \{a_k : a_1 \leq \ldots \leq a_m\}$ and $m < \infty$. The cardinality, interior and Lebesgue measure of a Lebesgue measurable set $S$ is denoted by $|S|, \text{int}(S)$ and $\lambda(S)$ respectively. The projection of the set $\mathcal{X} \subset \mathbb{R}^n$ onto $Y \subset \mathbb{R}^m$ ($m \leq n$) is denoted by $\text{pr}_Y(\mathcal{X})$.

A. Signal Temporal Logic (STL)

Signal Temporal Logic (STL) is a specification language defined over continuous time signals [9]. Let $\mu \in \{\top, \bot\}$ be a predicate defined after evaluation of a continuous predicate function $h : \mathbb{R}^n \rightarrow \mathbb{R}$ as follows:

$$\mu = \begin{cases} \top, & h(x) \geq 0 \\ \bot, & h(x) < 0 \end{cases},$$

for $x \in \mathbb{R}^n$. The basic STL formulas are given by the grammar:

$$\phi ::= \top | \bot | \neg \phi | \phi_1 \wedge \phi_2 | \mathcal{G}_{[a,b]} \phi | \mathcal{F}_{[a,b]} \phi | \phi_1 \mathcal{U}_{[0,a]} \phi_2,$$

where $\phi_1, \phi_2$ are STL formulas and $\mathcal{G}_{[a,b]}, \mathcal{F}_{[a,b]}, \mathcal{U}_{[a,b]}$ is the always, eventually and until operator defined over the interval $[a, b]$ with $0 \leq a \leq b$. Let $x \models \phi$ denote the satisfaction of the formula $\phi$ by a signal $x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$. The formula $\phi$ is satisfiable if $\exists x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$ such that $x \models \phi$. The STL semantics for a signal $x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$ are recursively given and can be found, e.g., in [9]. STL is equipped with robustness metrics determining how robustly an STL formula $\phi$ is satisfied at time $t$ by a signal $x$. These semantics are defined as follows [10], [11]: $\rho^x(x, t) = h(x(t)), \rho^x_{\neg \phi}(x, t) = \neg \rho^x(x, t), \rho^{\phi_1 \wedge \phi_2}(x, t) = \min(\rho^{\phi_1}(x, t), \rho^{\phi_2}(x, t)), \rho^{\phi_1 \mathcal{U}_{[0,a]} \phi_2}(x, t) = \max(\rho^{\phi_1}(x, t), \min_{t+\varepsilon \in [t, t+a]} \rho^{\phi_2}(x, t+\varepsilon)), \rho^{\mathcal{G}_{[a,b]} \phi}(x, t) = \max_{t+\varepsilon \in [t, t+a]} \rho^{\phi}(x, t), \rho^{\mathcal{F}_{[a,b]} \phi}(x, t) = \max_{t+\varepsilon \in [t, t+a]} \rho^{\phi}(x, t), \rho^{\mathcal{U}_{[a,b]} \phi}(x, t) = \min_{t+\varepsilon \in [t, t+a]} \rho^{\phi}(x, t)$. Finally, it should be noted that $x \models \phi$ if $\rho^x(x, 0) > 0$.

B. Problem Formulation

In this work we consider the following STL fragment:

$$\varphi ::= \mathcal{G}_{[a,b]} \mu | \mathcal{F}_{[a,b]} \mu,$$

$$\phi ::= \bigwedge_{i=1}^p \varphi_i,$$

where $0 \leq a \leq b < \infty$ and $p \geq 1$.

Remark 1. The STL fragment defined by (1a)-(1b) is expressive enough to accommodate until STL formulas of the form $\varphi = \mu_1 \mathcal{U}_{[a,b]} \mu_2$. By definition of the STL semantics in [18], for any $t^* \in [a, b]$ the until formula $\varphi = \mu_1 \mathcal{U}_{[a,b]} \mu_2$ can be written as $\varphi = \mathcal{G}_{[t^*, t^*]} \mu_1 \wedge \mathcal{F}_{[t^*, t^*]} \mu_2$. Hence, if there exists a continuous function $x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$ and a time instant $t^* \in [a, b]$ such that $x \models (\mathcal{G}_{[t^*, t^*]} \mu_1 \wedge \mathcal{F}_{[t^*, t^*]} \mu_2)$, then $x \models \varphi$.

Consider a team of $R$ agents with each agent identified by its index $k \in \mathcal{V} = \{1, \ldots, R\}$. For every agent $k$ let $x_k \in \mathcal{X}_k$ denote its state vector, where $\mathcal{X}_k \subset \mathbb{R}^n_k$ is a non-empty, compact, convex set for every $k \in \mathcal{V}$. Let $n_k = \sum_{k \in \mathcal{V}} n_k$. Finally, it should be noted that temporal operator of $\varphi_i, i \in \mathcal{I}$ satisfying (1a). Let $[a_i, b_i]$ be the interval of satisfaction associated with the signal $x \models \varphi_i, i \in \mathcal{I}$ define the sets of all STL formulas $\varphi_i, i \in \mathcal{I}$ satisfying (1a).

C. Decomposition of STL Formulas

In this Section we design a number of STL tasks the satisfaction of which depends on a known subset of agents. Consider the formula $\phi$ defined by (1b). Let the predicate function $h_i : \mathbb{Z}^i \rightarrow \mathbb{R}$ associated with the formula $\varphi_i, i \in \mathcal{I}$.
where $\bar{Z}_i = pr_{di}(X)$ and $d_i \in \{1, \ldots, n\}$. Then, the zero superlevel set of $h_i(\bar{z}_i)$ is defined as follows:

$$S_i = \{\bar{z}_i \in \bar{Z}_i : h_i(\bar{z}_i) \geq 0\}. \quad (2)$$

Here, we assume that $h_i(\bar{z}_i), i \in I$ is a continuous function whose value may depend on the states of all agents in $V$. Next, consider a set of STL tasks $\phi_l = \bigwedge_{i=1}^{n_l} \varphi_i, \ l = 1, \ldots, v$ whose satisfaction depends on the corresponding set of agents $V_l$. Here, $\varphi_i$ denotes the $q_i$-th formula of $\phi_l$ that is considered to be the result of the decomposition of the sub-formula $\varphi_i$ of (1b). Let $\bar{z}_l \in \bar{Z}_l \subset \mathbb{R}^{n_l}$ be the states of the agents in $V_l$ where $n_l = \sum_{k \in V_l} n_k$ and $\bar{Z}_l = \prod_{k \in V_l} \mathbb{R}^{n_k}$. The vector $\bar{z}_l, l = 1, \ldots, v$ can be obtained from $x$ using the following equation:

$$\bar{z}_l = E_l x, \quad (3)$$

where $E_l \in M_{n_l \times n}(\{0,1\})$ is a selection matrix. Let $d_l^i \geq 1$ be the number of states in $\bar{z}_l, l \in V_l$ contributing to $h_i(\bar{z}_i)$. Since the global formula is a-priori given, the elements of $\bar{z}_l, l \in V_l$ the predicate function $h_i(\bar{z}_i)$ depends on are known. Hence, for every $i \in I$ the following always holds:

$$\bar{z}_i = \left[\bar{z}_i\right]_{l \in V_l} \in \bar{Z}_i, \quad (4)$$

where $\bar{Z}_i = \prod_{l \in V_l} \bar{Z}_l, d_i = \sum_{l \in V_l} d_l^i, \bar{Z}_l = B_l^i \bar{Z}_l$ and $B_l^i \in M_{d_l^i \times n_l}(\{0,1\})$ is an appropriate selection matrix. Let $[a_i^l, b_i^l]$ and $h_i : \bar{Z}_l \times \Theta_l \to \mathbb{R}, q_i = 1, \ldots, p_i, l = 1, \ldots, v$ denote the interval of satisfaction and predicate function corresponding to $\varphi_i$, respectively. For every $l = 1, \ldots, v$ we assume that $h_i(\bar{z}_l; \theta_l^i), q_i = 1, \ldots, p_i$ is a continuous function on $\bar{Z}_l^i$ belonging to a known family of functions and its value depends on a set of parameters $\theta_l^i \in \Theta_l \subset \mathbb{R}^{n_l}$ to be appropriately tuned. The zero superlevel set of $h_i(\bar{z}_l; \theta_l^i)$ is then, given by:

$$S_{q_i}(\theta_l^i) = \{\bar{z}_l^i \in \bar{Z}_l^i : h_i(\bar{z}_l; \theta_l^i) \geq 0\}. \quad (5)$$

Considering (5), we pose the following assumption:

**Assumption 1.** For every $\theta_l^i \in \Theta_l, i \in I, l \in V_i$ the set $S_{q_i}(\theta_l^i)$ is Lebesgue measurable with $\int(S_{q_i}(\theta_l^i)) \neq 0$ and $\text{dim}(S_{q_i}(\theta_l^i)) = d_l^i$.

Observe that $S_{q_i}(\theta_l^i) \subseteq \bar{Z}_l^i$ and $\bar{Z}_l^i$ is compact. Hence, $\lambda(S_{q_i}(\theta_l^i)) < \infty$. Let $\Theta_l = \left[\theta_l^i\right]_{i \in V_l} \in \Theta_l$ and $\Theta_l = \prod_{l \in V_l} \Theta_l$. Based on the above we propose the following method for designing $\phi_l, l = 1, \ldots, v$.

**Theorem 1.** Consider the global STL formula $\phi$ defined by (1a)-(1b). Assume that $\int(S_l) \neq 0$, where $S_l, i \in I$ is defined in (2). Let Assumption 1 hold. Define the predicate functions $h_i(\bar{z}_l; \theta_l^i)$ with the parameters $\theta_l^i \in \Theta_l, l \in V_l, i \in I$ found as solutions to the following optimization problem:

$$\max_{\theta_l \in \Theta_l} \sum_{i \in V_l} \lambda(S_{q_i}(\theta_l^i)) \quad (6)$$

subject to:

$$\bar{z}_l^i \in S_{q_i}(\theta_l^i), \ l \in V_l \quad (6a)$$

$$\bar{z}_l \in int(S_l) \quad (6b)$$

where $\bar{z}_l$ satisfies (4). For every $l = 1, \ldots, v$ define the formulas $\varphi_l$ as follows:

$$\varphi_l = \begin{cases} \mathcal{F}_{[a_l^i, b_l^i]} \mu_l^{i_1}, & i \in I_F \\ \mathcal{G}_{[a_l^i, b_l^i]} \mu_l^{i_1}, & i \in I_G \end{cases} \quad (7)$$

with

$$[a_l^i, b_l^i] = \begin{cases} [t_i, t_i], & i \in I_F \\ [a_i, b_i], & i \in I_G \end{cases} \quad (8a)$$

where $\mu_l^{i_1} = \begin{cases} 1, & h_i^l(\bar{z}_l; \theta_l^i) \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (8b)$

where $\bar{I} = I_F \cup I_G, t_i \in [a_i, b_i]$ and $[a_i, b_i]$ is the interval of satisfaction with each $\varphi_i$ of the global formula $\phi$.

Let $\phi_l = \bigwedge_{i=1}^{n_l} \varphi_i, l = 1, \ldots, v$. If there exists a continuous function $\bar{x} : \mathbb{R} \to \mathbb{R}$ such that $\rho^{\phi_1 \land \cdots \land \phi_v}(\bar{x}, 0) > 0$, then $\rho^\phi(\bar{x}, 0) > 0$.

**Proof.** Assume that (6) is feasible for every $i \in I$. Then, by Assumption 1 it holds that $\int(S_l(\theta_l^i)) \neq 0$, implying $\lambda(S_l(\theta_l^i)) > 0$ for every $l \in V_i$. Assume that a continuous function $\bar{x} : \mathbb{R} \to \mathbb{R}$ exists satisfying $\rho^{\phi_1 \land \cdots \land \phi_v}(\bar{x}, 0) > 0$. By definition of the robust semantics and the definition of the min operator for every $l = 1, \ldots, v$ we have $\rho^{\phi_1 \land \cdots \land \phi_v}(x, 0) \leq \rho^\phi(x, 0)$. Then, if $\rho^\phi(x, 0) > 0$ then $\rho^\phi(x, 0) > 0$ for every $l = 1, \ldots, v$. By design, the satisfaction of $\phi_l$ depends on a subset of agents, thus $\rho^\phi(x, 0) = \rho^\phi(\bar{x}, 0) > 0$ where $\bar{x}$ satisfies (3). Then, by the definition of the robust semantics for every $l = 1, \ldots, v$ and $q_i = 1, \ldots, p_i$ we have $0 < \rho^\phi(\bar{x}, 0) = \min_{q_i} \rho^\phi(q_i, \bar{x}, 0) \leq \rho^\phi(q_i, \bar{x}, 0) = \rho^\phi(q_i, \bar{x}, 0)$.

In Theorem 1 a complex, possibly non-convex problem is presented that aims at maximizing the Lebesgue measure of $S_l(\theta_l^i), l \in V_i$ such that $\bar{K} \subseteq \int(S_l)$, where $\bar{K} = \prod_{l \in V_i} S_l(\theta_l^i)$. In that way, the agents in $V_l, l \in V_l$, may work towards satisfying $\mu_l^{i_1}$ without knowledge of the other agents’ actions while still guaranteeing the satisfaction of the global $\mu_l$. Based on (7), all local tasks $\varphi_l, l \in V_l$, corresponding to $i \in I$ inherit their temporal operator from the global task $\varphi_i$. While the local always formulas corresponding to $i \in I_\varphi$ share the same time interval as $\varphi_i$, a stricter time interval of satisfaction, defined by (8a), is imposed when $i \in I_\varphi$, to ensure satisfaction of $\varphi_i$. In practice, solving the proposed optimization problem (6) poses several challenges. First, (6) requires the exhaustive evaluation of the functions $h_l(\bar{z}_l; \theta_l^i)$ over the continuous set $\bar{Z}_l$, which may be intractable. Another
limitation of the proposed problem is often the lack of a known formula for computing the Lebesgue measure of a set, unless $h_{l q_i}^l(z_i^l; \theta_i^l)$ belongs to a specific class of functions such as the class of ellipsoids. The aforementioned challenges have motivated us to seek for a computationally efficient, convex problem that aims implicitly at maximizing $\lambda(S_{l q_i}^l(\theta_i^l))$ when the corresponding predicate function $h_{l q_i}^l(z_i^l; \theta_i^l)$ is defined as a function of the infinity norm of $z_i^l$. The computational benefits of the proposed approach are related to the number of points in $\mathbb{Z}_i^l, l \in V_i$ that are considered for evaluation of the satisfaction of (6b). More specifically, contrary to (6), in this approach only a finite number of points is evaluated that depends on the number of states $x_k$ of the agents $k \in V_i, l \in V_i$ involved in the satisfaction of $h_i(z_i)$. To that end, we consider a special class of concave functions of the following form:

$$h_{l q_i}^l(z_i^l; \theta_i^l) = r_{q_i}^l - \|z_i^l - c_{i q_i}^l\|_\infty, \quad q_i = 1, \ldots, p_i$$  \hspace{1cm} (9)$$

where $\theta_i^l = [r_{q_i}^l, c_{i q_i}^l]^T \in \Theta_i^l = \mathbb{R}_{\geq 0} \times \mathbb{Z}_i^l$ is a vector of parameters to be tuned. Let $J_i^l = \{1, \ldots, d_i^l\}$ denote the set of indices of the elements of $\mathbb{Z}_i^l$. Given the predicate functions defined by (9), it follows that:

$$h_{l q_i}^l(z_i^l; \theta_i^l) \geq 0 \Leftrightarrow z_i^l(\eta) \in [-r_{q_i}^l + c_{i q_i}^l(\eta), r_{q_i}^l + c_{i q_i}^l(\eta)]$$ \hspace{1cm} (10)$$

for every $\eta \in J_i^l$, where $z_i^l(\eta), c_{i q_i}^l(\eta)$ denote the $\eta$-th element of the vectors $z_i^l, c_{i q_i}^l$ respectively. For every $i \in I$ and $l \in \{1, \ldots, v\}$ consider the following set of vectors:

$$\mathcal{P}_i^l(\theta_i^l) = \{\xi \in \mathbb{Z}_i^l : \xi(\eta) = -r_{q_i}^l + c_{i q_i}^l(\eta) \text{ or } \xi(\eta) = r_{q_i}^l + c_{i q_i}^l(\eta), \eta \in J_i^l\},$$  \hspace{1cm} (11)$$

where $\xi(\eta)$ denotes the $\eta$-th element of $\xi$. If $r_{q_i}^l > 0$, the set $\mathcal{P}_i^l(\theta_i^l)$ consists of the vertices of a hypercube in $\mathbb{R}_{\geq 0}^d$ of edge length $r_{q_i}^l$ and center $c_{i q_i}^l$. Hence, its cardinality will be equal to $2^{d_i^l}$. If $r_{q_i}^l = 0$, then $\mathcal{P}_i^l(\theta_i^l) = \{c_{i q_i}^l\}$. To guarantee the convexity of the proposed problem we pose the following assumption:

**Assumption 2.** For every $i \in I$ the predicate function $h_i(z_i)$ is concave in $\mathbb{Z}_i^l$.

Concave functions can express a variety of tasks such as connectivity maintenance, formation, reachability and coverage tasks [1]. This family includes among others the linear functions, the logarithm $\log(\cdot)$ and the norms with negative sign, namely $g(\cdot) = -\|\cdot\|_p, p \geq 1$.

**Theorem 2.** Consider the global STL formula $\phi$ defined by (1a)-(1b) and the predicate functions $h_i(z_i), i \in I$ associated to $\varphi_i$. Let Assumption 2 hold. For every $i \in I$ assume that $\text{int}(S_i) \neq 0$, where $S_i$ is defined in (2). Consider the functions $h_{l q_i}^l(z_i^l; \theta_i^l), q_i = 1, \ldots, p_i, l \in V_i$ defined by (9) where $\theta_i^l = [r_{q_i}^l, c_{i q_i}^l]^T$ is the vector of parameters found as the solution to the following optimization problem:

$$\max_{\theta_i \in \Theta_i^l} \sum_{l \in V_i} r_{q_i}^l$$  \hspace{1cm} \text{subject to:}$$

$$z_i^l \in \mathcal{P}_i^l(\theta_i^l), \quad l \in V_i$$  \hspace{1cm} (12a)$$

where $z_i$ satisfies (4) and $\mathcal{P}_i^l(\theta_i^l)$ is the set defined by (11) for every $l \in V_i$. For every $l = 1, \ldots, v$ define the formula $\varphi_i^l$, based on (7) and (8a)-(8b) and consider the decomposed STL formulas $\phi_i = \bigwedge_{q_i=1}^{p_i} \varphi_{q_i}^l, l = 1, \ldots, v$. If there exists a continuous function $x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ such that $\rho_{\phi_i^1 \wedge \ldots \wedge \phi_{q_i}^l}(x, 0) > 0$, then $\rho_i^l(x, 0) > 0$.

**Proof.** For every $i \in I$, (12) finds the maximum volume sets $S_{q_i}^l(\theta_i^l), l \in V_i$ which are inner approximations of the projection sets of $S_i$ onto $\mathbb{Z}_i^l$. Since $\text{int}(S_i) \neq 0$ for every $i \in I$, (12) is always feasible, e.g., when $r_{q_i}^l = 0, \forall l \in V_i$ and $c_{i q_i}^l = [c_{i q_i}^l]_{l \in V_i}$ satisfying (12b). If there exists $l \in V_i$ such that $r_{q_i}^l = 0$, then $S_{q_i}^l(\theta_i^l) = \{c_{i q_i}^l\}$ and $h_{l q_i}^l(c_{i q_i}^l; \theta_i^l) = 0$. Hence, for every continuous $x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ we have $\rho_{\phi_i^1 \wedge \ldots \wedge \phi_{q_i}^l}(x, 0) \leq 0$.

Assume now that $r_{q_i}^l \neq 0$ for every $l \in V_i$. To simplify notation let the sets $W_i = \{z_i \in \mathbb{Z}_i^l : z_i \in S_{q_i}^l(\theta_i^l), l \in V_i\}$ and $W' = \{z_i \in \mathbb{Z}_i^l : z_i \in \mathcal{P}_i^l(\theta_i^l), l \in V_i\}$ where $z_i$ satisfies (4). Due to (9), $W$ is convex as the Cartesian product of the sets $S_{q_i}^l(\theta_i^l), l \in V_i$. Observe that $W' \subset W$ with $|W'| = 2^{d_i^l} > d_i$. By applying Caratheodory’s theorem [19, Th. 17.1], we write any point $y \in W$ as a convex combination of the form $y = \sum_{j=1}^{d_i+1} \kappa_j y_j$ where $y_j \in W', \kappa_j \geq 0, j = 1, \ldots, d_i+1$ and $\sum_{j=1}^{d_i+1} \kappa_j = 1$. By feasibility of (12) and due to Assumption 2, we can conclude that $h_i(y) > 0$ for any $y \in W$. The rest of the proof is similar to that of Theorem 1.

**Remark 2.** If for every $i \in I$ the predicate function $h_i(z_i)$ is continuously differentiable and $\text{int}(F_i) \neq 0$, where $F_i = \{\theta_i^l \in \Theta_i^l, l \in V_i : (12a) - (12b) \text{ are satisfied}\}$, Slater’s constraint qualification holds. Hence, (12) can be efficiently solved using Lagrange Multiplier Theory [20].

For $i \in I_F$ the new STL tasks, defined by (7), are expected to be satisfied at a specific time instant $t_i \in [a_i, b_i]$ which is considered a designer’s choice. However, in many cases pre-determining the time instant of satisfaction of a formula may lead to conservatism and reduced performance. An alternative would be to allow satisfaction of the local formulas over time intervals $[a_{i q_i}^l, b_{i q_i}^l] \subseteq [a_i, b_i]$. Then, in order to guarantee the satisfaction of the global formula we can define the local tasks corresponding to $\varphi_i, i \in I_F$ as STL tasks of the form $G[a_{i q_i}^l, b_{i q_i}^l] \mu$. This is depicted in the following Proposition:

**Proposition 1.** Consider the global STL formula $\phi$ defined by (1a)-(1b). Let Assumption 2 hold. For every $i \in I$ assume that $\text{int}(S_i) \neq 0$, where $S_i$ is defined by (2). For every $i \in I$ consider the functions $h_{l q_i}^l(z_i^l; \theta_i^l), l \in V_i$ defined by (9) with their parameters $\theta_i^l$ found as solutions to (12). Let the STL formula $\varphi_{q_i}^l$ be defined as:

$$\varphi_{q_i}^l = G[a_{i q_i}^l, b_{i q_i}^l] \mu_{j_i q_i}^l,$$  \hspace{1cm} (13)$$

where

$$[a_{i q_i}^l, b_{i q_i}^l] \subseteq [a_i, b_i], \quad i \in I_F$$  \hspace{1cm} (14)$$

is the interval of satisfaction of $\varphi_{q_i}^l$ chosen such that $T_i = \bigcap_{l \in V_i} [a_{i q_i}^l, b_{i q_i}^l] \neq \emptyset$ and $\mu_{j_i q_i}^l$ is the predicate defined by (8b).
Let \( \rho_l = \sum_{q=1}^{m_l} \varphi_q^l, \) \( l = 1, \ldots, v. \) If there exists a continuous function \( x : \mathbb{R} \to \mathbb{R} \) such that \( \rho_l(x, 0) > 0, \) then \( \rho_l(x, 0) > 0. \)

**Proof.** For \( i \in I_d \), the proof follows similar arguments to Theorem 1. For \( i \in I_d \), if \( \rho_l(x, 0) > 0 \) then, by (13)-(14) and the definition of the robust semantics, \( \rho^l_q(z_i(t), \theta^i) > 0 \) implies \( h_q^l(z_i(t), \theta^i) > 0 \) for every \( t \in [d^l_q, b^l_q] \) and \( l \in V_i. \) Since \( \theta^i \in V_i \) are feasible solutions of (12), we may conclude that \( h_q^l(z_i(t), \theta^i) > 0 \) for every \( t \in T_i \) and \( z_i(t) \) satisfying (4). Hence, \( \rho^l_q(z_i(t), 0) = \max_{t \in [0, b^l_q]} h_q^l(z_i(t)) \geq \max_{t \in T_i} h_q^l(z_i(t)) > 0. \) The rest of the proof is similar to that of Theorem 1.

### IV. Simulations

Consider a team of \( R = 5 \) agents. Without loss of generality the team is decomposed in 3 sub-tasks: \( \mathcal{V}_1 = \{1, 4\}, \mathcal{V}_2 = \{3, 5\}, \mathcal{V}_3 = \{2\}. \) The agents’ states \( x_k, k \in \mathcal{V} \) evolve over time based on the following equation:

\[
x_k = A_k x_k + u_k, \quad k = 1, \ldots, 5
\]

where \( A_k = \begin{bmatrix} -0.5 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \) for every \( k \in \{1, 2, 5\} \) and \( A_k = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} \) for \( k \in \{3, 4\}. \) The states and inputs of the agents are subject to constraints, i.e., \( x_k \in \mathbb{R} = B_d(0), \ u_k \in U = B_d(0) \) where \( B_d(0) \) denotes the closed ball with center the origin and radius \( \epsilon, \) \( d_1 = 1 \) and \( d_2 = 5. \) Consider the global STL formula \( \phi = \bigwedge_{i=1}^{4} \varphi^i, \) where \( \varphi^i, \) \( i \in \mathcal{I} \) are defined as: \( \varphi^1 = \mathcal{G}_{[0, 2]}(\|x_1 - x_2 - p_x\|_2 \leq 0.1), \varphi^2 = \mathcal{G}_{[2.5, 4]}(\|x_3 - x_4\|_2 \leq 0.2), \varphi^3 = \mathcal{F}_{[3, 7]}(\|x_5 - x_4\|_2 \leq 0.2) \) and \( \varphi^4 = \mathcal{F}_{[8, 10]}(\|x_5 - x_4\|_2 \leq 0.25), \) where \( p_x = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix} \) and \( P_1 = \text{diag}(4, 1), \ P_2 = \text{diag}(0.1, 0.4) \) are positive definite weight matrices. Since the predicate functions corresponding to \( \varphi^i, \) \( i \in \mathcal{I} \) are quadratic, the proposed problem (12) becomes a Quadratically Constrained Quadratic Program (QCQP) and is efficiently solved using the Opti Toolbox [21]. The average computational time of the QCQPs is 0.052sec on an Intel Core i7-8665U with 16GB RAM using MATLAB.

To verify the validity of Theorem 2 and Proposition 1 we design agents’ trajectories using the MPC scheme proposed in [22] with a sampling frequency of 10 Hz and optimization horizon length \( N = 1. \) Each team \( \mathcal{V}_i \) solves a local MPC problem without communicating with the other teams since the satisfaction of the assigned tasks depends only on \( z_i. \) Here, a single, time-varying barrier \( b_i(z_i, t), \) \( i \in \mathcal{V} \) is considered and designed offline encoding the local STL task specifications \( \phi_l \) corresponding to \( \mathcal{V}_l. \) For every subtask of \( \phi_l \) a temporal behavior is designed for the \( t \)-th team such that the satisfaction of \( \phi_l \) with a robustness value \( r = 0.005 \) is guaranteed when \( b_i(z_i, t) \geq 0 \) is true for every \( t \in [0, 10]. \) For details on the design of the barrier function \( b_i(z_i, t) \) see [14], [22]. The local STL task \( \phi_l \) assigned to each team \( l \) is defined by (7), (8a)-(8b) as follows:

\[
\phi_1 = \varphi_1 \land \varphi_2 \land \varphi_3, \\
\phi_2 = \varphi_2 \land \varphi_3 \land \varphi_4, \\
\phi_3 = \varphi_3 \land \varphi_4
\]

where \( \varphi_1 = \mathcal{G}_{[0, 2]} \mu_1, \varphi_2 = \mathcal{G}_{[2.5, 4]} \mu_2, \varphi_3 = \mathcal{F}_{[3, 7]} \mu_3, \varphi_4 = \mathcal{F}_{[8, 10]} \mu_4, \) and \( \mathcal{G}_r, \mathcal{F}_r \) are defined by (15)-(17) as before, where now \( \varphi_1 = \mathcal{G}_{[0, 2]} \mu_1, \varphi_2 = \mathcal{G}_{[2.5, 4]} \mu_2, \varphi_3 = \mathcal{F}_{[3, 7]} \mu_3, \varphi_4 = \mathcal{F}_{[8, 10]} \mu_4. \) In Figure (2a) and (2b) the agents’ trajectories are shown. Following a similar

Fig. 1: Agents’ Trajectories under the local STL tasks defined based on (7), (8a)-(8b) and Barrier Function Evolution.
procedure as before, we design a set of local barrier functions $b_l(z_i,t)$ and a function $b_r(x,t)$ with robustness $r = 0.005$. Based on Figure (2c), $\min_{t \in [0,10]} b_l(z_i,t) \geq 0$, implying $z_i \models \phi_i, i \in V$. Additionally, it holds that $\inf_{t \in [0,10]} b_r(x,t) \geq 0.0954$. Hence, $\rho^0(x,0) \geq 0.005$.

V. Conclusions

In this work a global STL formula is decomposed to a set of local STL tasks whose satisfaction depends on an a-priori chosen subset of agents. The predicate functions of the new formulas are chosen as functions of the infinity norm of the agents’ states. A convex optimization problem is then, designed for optimizing their parameters towards increasing the volume of their zero superlevel sets while the temporal operator and interval of satisfaction are chosen to ensure the satisfaction of the global formula. Future work will consider a more sophisticated framework for choosing the interval of satisfaction of the formulas aiming at increasing the total robustness of the task.

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