Generalize new method to determine the location of the control section in the box culvert under inlet control

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Abstract. The control section is a location at which a unique relationship between the flow rate and the water depth (critical depth) exists. When the culvert flows under inlet control, the location of the control section exists into the culvert barrel at some distance from the entrance. The exact location of the control section has not been investigated until now. The present study demonstrates the exact location of the control section experimentally based on the measured elevations along the barrel for each run undertaken. Then the correlation between the critical depth and its location is presented as proposed deterministic equations. Thus, the aim is to find the section at which the critical depth is achieved, measured from the beginning of the barrel by using the empirical equations. The application of the proposed equations is specified in terms of the geometric and hydraulic conditions that have been adopted in the present study.

Keywords:
box culvert; control section; critical depth; inlet control; submerged entrance; unsubmerged entrance; inlet transition; bevelled edge; culvert barrel

1. Introduction
The culvert is a covered channel of relatively short length where water passes through an embankment (e.g. highway). Most culverts consist of three parts: the intake (entrance), the barrel and the diffuser (outlet). The culvert operates in either inlet or outlet control. In inlet control, only entrance configuration and headwater depth determine the culvert's hydraulic capacity, thereby, the barrel characteristics and the tailwater depth are of no consequence. At inlet control, the culverts usually lie on relatively steep slopes and the flow is only partly full, however, the entrance under unsubmerged or submerged conditions [1]. In the outlet control all of the geometric and hydraulic characteristics of the culvert play a role in determining its capacity [2, 3, and 4]. A box culvert more readily lends itself to low allowable headwater situations. The height may be lowered and the span increased to satisfy hydraulic capacity with a low headwater [5]. [6] considered box culverts with nonenlarged (not tapered) inlets and constant barrel size operating under conditions of inlet control, with the objective of identifying the hydraulic performance of different leading edge geometries. For submerged entrance conditions the study refers to under conditions of inlet control, the L'/D≈0.5 where L’ is the distance from the entrance to the location of the control section and D is the entrance rise. The experiments were carried out by [7] to prove that the basic principle of the culvert is to induce critical flow conditions in the barrel in order to maximize the discharge per unit width and to reduce the barrel cross-section. The flow upstream and downstream of the culvert is typically subcritical. As the flow approaches the culvert, the constriction induces an increase in Froude number. For the design
discharge, the flow becomes near-critical in the barrel. In practice, perfect-critical flow conditions in the barrel are difficult to establish. Usually, the Froude number in the barrel is about 0.7 to 0.9. For a rectangular cross-section, the maximum discharge per unit width is achieved for critical flow conditions:

\[ Q_w^{\text{max}} = W_{\text{min}} \sqrt{g} \left( \frac{2}{3} \left( E_o + \Delta z - \Delta H \right) \right)^{3/2} \]  

The minimum barrel width for critical flow conditions is then:

\[ W_{\text{min}} = \frac{Q_w^{\text{max}}}{\sqrt{g}} \left( \frac{2}{3} \left( E_o + \Delta z - \Delta H \right) \right)^{-3/2} \]  

Where \( Q_w^{\text{max}} \) is the maximum discharge, \( E_o \) upstream specific energy, \( \Delta z \) the bed elevation difference between the upstream channel and the barrel bottom and \( \Delta H \) is the head losses. Equation (2) gives the minimum barrel width to obtain near-critical flow.

2. Experimental work

The flume available in hydraulic laboratory of Building and Construction Engineering Department with measurement facilities and devices are used for experimental program. The dimensions of flume are 30 cm width, 12 m length and 40 cm depth. The channel sides were made from toughened glass along the entire length. The centrifugal pump is used to supply different discharges with the maximum rate (102.5 m³/hr). A platform of plastic sheet 0.8 cm in thickness and 200 cm in length is installed into flume at slope 0.02 to simulate the bed of culvert model. Two sheets of the plaxi-glass are installed over the platform to simulate the side walls of culvert model. However, 30 cm x 30 cm square sheets are manufactured with different square edge openings and installed as the head wall. The opening of the head wall consisted of different aspect ratio as illustrate in table 1. For each aspect ratio, different discharges have been adopted to give a wide range of hydraulic conditions for both submerged and un-submerged entrance under inlet control. This control was achieved after adjusting the tail gate of the flume to ensure the supercritical flow along the culvert length. The velocities and depths were measured along the culvert barrel for different discharges after flow became steady. Figure 1 illustrates the view of one of the culvert model after installation into the flume.

After all runs are completed with square edged head wall, some entrance configurations are introduced to enhance the hydraulic performance of culvert. These configurations are; wingwall with 30° flare angle as shown in figure 2, beveled edge with four rates of bevel 0.04, 0.1, 0.15, 0.2 as shown in figure 3, and a new type of transition structure proposed by [8] as shown in figures 4 and 5.

![Figure 1. View of the culvert model after installation with square head wall](image1.png)

![Figure 2. Culvert of square edge with wingwall](image2.png)
Table 1: The characteristic of the selected laboratory culvert model

| Group | Sub Group | Type of Entrance | Runs No. | Range of discharges m³/s | Dimensions (cm) | Aspect ratio (B/D) |
|-------|-----------|------------------|---------|--------------------------|-----------------|-------------------|
|       |           |                  |         |                          | Span (B), Rise (D) |                  |
| A     | A1        | Square edge Headwall | 3       | 0.0055-0.0079            | 10, 6           | 1.67              |
|       | A2        | Square edge Headwall | 4       | 0.0055-0.0091            | 10, 7           | 1.429             |
|       | A3        | Square edge Headwall | 5       | 0.0055-0.0104            | 10, 8.5         | 1.176             |
|       | A4        | Square edge Headwall | 6       | 0.0055-0.0117            | 10, 10          |                  |
|       | A5        | Square edge Headwall | 11      | 0.0055-0.0181            | 10, 20          | 0.5               |
|       | A6        | Square edge Headwall | 11      | 0.0055-0.0181            | 10, 25          | 0.4               |
|       | A7        | Square edge Headwall - Wingwall | 6 | 0.0055-0.0117 | 10, 10 | 1 |
|       | A8        | Square edge Headwall-Wingwall | 11 | 0.0055-0.0181 | 10, 20 | 0.5 |
|       | A9        | Square edge Headwall-Transition | 6 | 0.0055-0.0117 | 10, 10 | 1 |
|       | A10       | Square edge Headwall-Transition | 11 | 0.0055-0.0181 | 10, 20 | 0.5 |
|       | A11       | Beveled edge (Rbv=0.04) | 6 | 0.0055-0.0181 | 10, 20 | 0.5 |
|       | A12       | Beveled edge (Rbv=0.1) | 6 | 0.0055-0.0181 | 10, 20 | 0.5 |
|       | A13       | Beveled edge (Rbv=0.15) | 6 | 0.0055-0.0181 | 10, 20 | 0.5 |
|       | A14       | Beveled edge (Rbv=0.2) | 6 | 0.0055-0.0181 | 10, 20 | 0.5 |
| B     | B1        | Square edge Headwall-Transition | 4 | 0.0055-0.0117 | 15, 6 | 2.5 |
|       | B2        | Square edge Headwall-Transition | 4 | 0.0055-0.0129 | 15, 7.5 | 2 |
|       | B3        | Square edge Headwall-Transition | 9 | 0.0055-0.0255 | 15, 20 | 0.75 |
| C     | C1        | Square edge Headwall-Transition | 4 | 0.0055-0.0129 | 20, 5 | 4 |
|       | C2        | Square edge Headwall-Transition | 5 | 0.0055-0.0142 | 20, 6 | 3.33 |
|       | C3        | Square edge Headwall-Transition | 9 | 0.0055-0.0255 | 20, 15 | 1.33 |

Figure 3. Culvert model of beveled edge

Figure 4. Culvert model of square edge with transition

Figure 5. Inlet transition configuration installed at entrance of culvert model
3. Results and Discussion

The dimensional analysis has been done and the following functional relationship is restated;

\[ \frac{HW}{D} = f \left( \frac{yc}{D}, Fr, \frac{B}{D} \right) \]  

(3)

The headwater is a dependent variable and it is a function of the critical depth, Froude number and the geometry of the entrance as the aspect ratio. Basically, as the discharge increases the headwater is accordingly increased for the same B/D. First of all, the operation has been conducted for culvert with square edge headwall. While installing the different inlets configuration for each B/D, the headwater decreases gradually for the same discharges, which means better performance for culvert model.

The aim of this study is to develop new empirical equations between the critical depth and its location based on the above dimensionless functional relationship based on multi-regression analysis. The statistical indices are calculated for each equation to indicate the accuracy and reliability [9]. These indices besides the determination coefficient $R^2$ are; the Root Mean Square Error (RMSE) and the Mean Bias Error (MBE), the zero values of these two indices indicate a perfect fit between predicted and observed data. Also the Nash–Sutcliffe Efficiency Coefficient (NSEC) was tested, based on this index the values between zero and 1.0 are generally viewed as an acceptable performance level, whereas the value less than zero indicate to unacceptable performance. The last statistical index is the Percent Bias (PBIAS). The optimal value of PBIAS is zero, whereas near zero values indicate better model simulation. On the other hand, a positive value of PBIAS indicates a tendency of the model for underestimation while negative values indicate overestimation. The determination equations of these indices are;

\[ \text{RMSE} = \left[ \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X}_0)^2 \right]^{0.5} \]  

(4)

\[ \text{MBE} = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X}_0) \]  

(5)

\[ \text{NSEC} = 1 - \frac{\sum_{i=1}^{n} (X_i - \bar{X}_0)^2}{\sum_{i=1}^{n} (X_i - \bar{X}_g)^2} \]  

(6)

\[ \text{PBIAS} = \left\{ 100 \times \frac{\sum_{i=1}^{n} X_i - \sum_{i=1}^{n} \bar{X}_0}{\sum_{i=1}^{n} X_i} \right\} \]  

(7)

Where $X_0$ and $X_i$ are the observed and simulated variables respectively, the $\bar{X}_0$ is the mean of the observed data and $n$ is the total number of observations.

This study focussed on finding the exact location of the control section and presenting equations to identify this location according to the value of the critical depth. In order to achieve this purpose, it needs to know the exact value of the critical depth. This requirement was obtained by actually using the measurement by a point gauge for each discharge. Model A5 has been taken as an example to check the conformity between the theoretical and measured critical depth. Figure 6 illustrates the consistency of the measured and theoretical critical depth for the same flow rate for this model. The same result has been conducted for all models undertaken in this study.

After checking the conformity between the theoretical and measured critical depth, the location of the control section (L) as a dimensionless term (L/D) and the dimensionless ratio of the critical depth ($yc/D$) are correlated with a functional relationship. According to the dimensional analysis the functional relationship between (HW/D) and ($yc/D$) has been extracted for each configuration with both regimes of flow at entrance (unsubmerged and submerged). Tables 2 and 3 contain a list of these relationships for unsubmerged and submerged entrances, respectively. The introduction of these relationships is to enable the calculation of the critical depth from the measuring of the headwater. Accordingly, it can calculate the exact location of the control section directly by using another
formula. As an example, figure 7 shows the relationship between \((\text{HW}/D)\) and \((\text{yc}/D)\) for the culvert models of beveled edge for unsubmerged and submerged entrance.

For submerged entrance beveled edge, all models undertaken have the same relationship between \(\text{HW}/D\) and \(\text{yc}/D\) as listed in table 3 that can be attributed to the limited range of experimental discharges available for this type of the flow.

Practically as the \(\text{yc}/D\) calculated from the equations of tables 2 and 3 based on the entrance type, it can be used to find the correlation equation between the critical depth and its location. The dimensionless ratios are also employed for this correlation. Tables 4 and 5 list the correlation equations for unsubmerged and submerged entrances, respectively based on the inlet configuration types that were adopted in this study. Figure 8 shows the relationship between \(\text{L'}/D\) and \(\text{yc}/D\) for the culvert models of square edge headwall with transition structure for both unsubmerged and submerged entrance. For submerged entrance the adverse relationship exists between critical depth and its location, while the relationship takes a similar tendency for unsubmerged entrance, where when the critical depth increased, the location of this control section will be away from the entrance accordingly. On the contrary, for submerged entrance the location of the control section approaches the entrance as the value of the critical depth increases.

![Figure 6](image1.png)

**Figure 6.** Relation between the measured and theoretical critical depth verses flow rate for model A5

![Figure 7](image2.png)

**Figure 7.** The relationship between \((\text{HW}/D)\) and \((\text{yc}/D)\) for the culvert model of beveled edge

(a) unsubmerged entrance  
(b) submerged entrance
Figure 8. The relationship between \((L/D)\) and \((yc/D)\) for the culvert model of square edge with transition structure (a) for unsubmerged entrance (b) for submerged entrance.

Table 2 Functional relationship between headwater and critical depth for unsubmerged entrance.

| Type of entrance | Equation | Eq. No | RMSE | MBE   | NSEC | PBIS |
|------------------|----------|--------|------|-------|------|------|
| A1-A6            | \(\frac{H_W}{D} = 1.633\frac{y_c}{D} + 0.012\), \(R^2 = 0.994\) | 8     | 0.029| 0.014 | 0.97 | 1.773|
| A7-A8            | \(\frac{H_W}{D} = 1.647\frac{y_c}{D} + 0.012\), \(R^2 = 0.995\) | 9     | 0.012| -0.00001| 0.995 | -0.001|
| A11              | \(\frac{H_W}{D} = 1.585\frac{y_c}{D} + 0.027\), \(R^2 = 0.989\) | 10    | 0.016| -0.00001| 0.989 | -0.001|
| A12              | \(\frac{H_W}{D} = 1.604\frac{y_c}{D} + 0.021\), \(R^2 = 0.988\) | 11    | 0.017| 0.00001 | 0.989 | -0.001|
| A13              | \(\frac{H_W}{D} = 1.587\frac{y_c}{D} + 0.032\), \(R^2 = 0.992\) | 12    | 0.013| 0.00005 | 0.992 | 0.006|
| A14              | \(\frac{H_W}{D} = 1.561\frac{y_c}{D} + 0.033\), \(R^2 = 0.988\) | 13    | 0.005| 0.0000001 | 0.988 | 0.0004|
| A9-A10, B1-B3, C1-C3 | \(\frac{H_W}{D} = 1.676\frac{y_c}{D} - 0.011\), \(R^2 = 0.994\) | 14    | 0.015| -0.00001 | 0.994 | -0.0007|
Table 3 Functional relationship between headwater and critical depth for submerged entrance.

| Type of entrance | Equation | Eq. No | RMSE | MBE  | NSEC | PBIS |
|------------------|----------|--------|------|------|------|------|
| A1-A6            | $\frac{H_W}{D} = 1.848 \frac{y_c}{D} - 0.1$, $R^2 = 0.937$ | 15     | 0.037 | 0.00004 | 0.937 | 0.003 |
| A7-A8            | $\frac{H_W}{D} = 1.543 \frac{y_c}{D} + 0.141$, $R^2 = 0.971$ | 16     | 0.017 | 0.00008 | 0.97 | 0.007 |
| A11-A14          | $\frac{H_W}{D} = 1.921 \frac{y_c}{D} - 0.156$, $R^2 = 0.995$ | 17     | 0.005 | 0.00006 | 0.995 | 0.005 |
| A9-A10, B1-B3, C1-C3 | $\frac{H_W}{D} = 2.232 \frac{y_c}{D} - 0.363$, $R^2 = 0.950$ | 18     | 0.031 | -0.0001 | 0.949 | -0.005 |

Table 4 Functional relationship between critical depth and its location for unsubmerged entrance.

| Type of entrance | Equation | Eq. No | RMSE | MBE  | NSEC | PBIS |
|------------------|----------|--------|------|------|------|------|
| A1-A6            | $\frac{L}{D} = 2.405 \frac{y_c}{D}^2 - 1.414 \frac{y_c}{D} + 0.59$, $R^2 = 0.817$ | 19     | 0.04  | 0.00004 | 0.817 | 0.009 |
| A7-A8            | $\frac{L}{D} = 9.183 \frac{y_c}{D}^2 - 7.753 \frac{y_c}{D} + 2.207$, $R^2 = 0.966$ | 20     | 0.026 | -0.00001 | 0.966 | -0.002 |
| A11              | $\frac{L}{D} = 16.69 \frac{y_c}{D}^2 - 14.48 \frac{y_c}{D} + 3.470$, $R^2 = 0.980$ | 21     | 0.022 | -0.00008 | 0.981 | -0.015 |
| A12              | $\frac{L}{D} = 37.32 \frac{y_c}{D}^2 - 33.98 \frac{y_c}{D} + 8.322$, $R^2 = 0.918$ | 22     | 0.086 | 0.00013 | 0.918 | 0.013 |
| A13              | $\frac{L}{D} = 22.57 \frac{y_c}{D}^2 - 19.07 \frac{y_c}{D} + 4.559$, $R^2 = 0.993$ | 23     | 0.022 | -0.00003 | 0.993 | -0.003 |
| A14              | $\frac{L}{D} = 27.82 \frac{y_c}{D}^2 - 23.05 \frac{y_c}{D} + 5.231$, $R^2 = 0.994$ | 24     | 0.028 | 0.00003 | 0.993 | 0.003 |
| A9-A10, B1-B3, C1-C3 | $\frac{L}{D} = 3.01 \frac{y_c}{D}^2 - 0.854 \frac{y_c}{D} + 0.505$, $R^2 = 0.948$ | 25     | 0.05  | 0.00003 | 0.948 | 0.004 |
Table 5 Functional relationship between critical depth and its location for submerged entrance

| Type of entrance | Equation | Eq. No | RMSE | MBE | NSEC | PBIS |
|------------------|----------|--------|------|-----|------|------|
| A1-A6            | $\frac{L}{D} = -383.14 \frac{y_c}{D}^3 + 869.43 \frac{y_c}{D}^2 - 652.85 \frac{y_c}{D} + 163.01$ | 26    | 0.453 | 0.161 | -0.131 | 17.915 |
|                  | $R^2 = 0.745$ |        |      |      |      |      |
| A7-A8            | $\frac{L}{D} = -18.971 \frac{y_c}{D}^2 + 24.523 \frac{y_c}{D} - 6.856$ | 27    | 0.075 | -0.0001 | 0.93 | -0.018 |
|                  | $R^2 = 0.93$ |        |      |      |      |      |
| A11              | $\frac{L}{D} = 5.593 \frac{y_c}{D} - 2.164$ | 28    | 0.114 | -0.000004 | 0.775 | 0.0002 |
| A14              | $R^2 = 0.74$ |        |      |      |      |      |
| A9               | $\frac{L}{D} = 22.162 \frac{y_c}{D}^2 - 40.332 \frac{y_c}{D} + 18.489$ | 29    | 0.164 | -0.0013 | -0.842 | -0.016 |
| A10, B1-B3, C1-C3 | $R^2 = 0.842$ |        |      |      |      |      |

4. Conclusions
The present study focused on determining the control section location based upon the determination of the critical depth directly from the measuring of the headwater, the state of the flow at the entrance and the geometry of the culvert entrance. This aim can be done through identifying the critical depth experimentally and compare it with that calculated from the theoretical equation which exists between the discharge and the critical depth for rectangular open channel cross-section. Different inlets configuration, wide range of aspect ratios and discharges, have been adopted for both unsubmerged and submerged entrance. After analysis of the experimental data, a number of new functional relationships correlating the headwater with critical depth and the critical depth with its location are proposed as deterministic equations, which can be applicable for specified boundary conditions. The reliability of the introduced equations is tested by different statistical indices. The results of these indicators were encouraging, meaning that the provided equations have a high performance when applied.

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