Top-Quark Mediated Effects in Hadronic Higgs-Strahlung

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Abstract

Novel contributions to the total inclusive cross section for Higgs-Strahlung in the Standard Model at hadron colliders are evaluated. Although formally of order $\alpha_s^2$, they have not been taken into account in previous NNLO predictions. The terms under consideration are induced by Higgs radiation off top-quark loops and thus proportional to the top-quark Yukawa coupling. At the Tevatron, their effects to $WH$ production are below 1\% in the relevant Higgs mass range, while for $ZH$ production, we find corrections between about 1\% and 2\%. At the LHC, the contribution of the newly evaluated terms to the cross section is typically of the order of 1\%-3\%. Based on these results, we provide updated predictions for the total inclusive Higgs-Strahlung cross section at the Tevatron and the LHC.

1 Introduction

With the LHC experiments becoming sensitive to signals for the Standard Model (SM) Higgs boson, the search for this elusive particle has entered a new and hopefully its final phase. The direct searches at LEP, Tevatron, and LHC only leave relatively small allowed windows for the Higgs mass $M_H$, the widest one between 114 GeV and about 145 GeV (see Ref. \[1\] for preliminary results).

As opposed to evidence or discovery, the exclusion limits rely heavily on theoretical predictions. The dominant cross section to compare the experimental measurements to is gluon fusion which receives large radiative corrections. Although it is probably the most-studied cross section for an unconfirmed particle, the residual theoretical uncertainty is still sizable and highly disputed (for a recent discussion, see Ref. \[3\]). A lot of this uncertainty is induced by quantum chromodynamics (QCD), specifically the strong coupling $\alpha_s$ and the parton density functions (PDFs).

For various reasons, however, gluon fusion need not be the dominant search mode. The focus of this paper is on the associated production of a Higgs boson with an electro-weak
Figure 1: (a) Leading order Feynman diagram contributing to the Higgs-Strahlung process; (b) real corrections at NLO QCD; (c) $gg$ component not covered by Drell-Yan-like corrections.

gauge boson ($pp \rightarrow VH$, $V \in \{W^\pm, Z\}$), or “Higgs-Strahlung” for short. At the Tevatron, where the $\gamma\gamma$ and $b\bar{b}$-decays of a Higgs boson produced in gluon fusion cannot be separated from the background with sufficient precision, this mode is particularly important in the low mass region. Nominally known through order $\alpha_s^2$, we identify and evaluate a previously neglected contribution which formally adds to these next-to-next-to-leading order (NNLO) terms.

For the LHC, the relevance of the Higgs-Strahlung process used to be considered marginal. This has changed with the idea of focusing on events with highly boosted Higgs bosons by analyzing the substructure of jets [4]. Even though for a proper theoretical prediction in such an analysis one needs to consider differential quantities, it is important to ensure that all effects that contribute to the total rate are under control.

Once the Higgs mass is known, precise predictions for the individual production and decay channels will be essential in order to extract the maximum information from the experiments (see, e.g., Ref. [5]).

The theoretical prediction of the total inclusive cross section due to Higgs-Strahlung at hadron colliders is under very good control[1]: the leading order contribution is of order $g^4$, where $g$ is the weak coupling constant, and is completely analogous to what used to be the main search channel at LEP, except that the initial $e^+e^-$ is replaced by a $q\bar{q}'$ pair, of course, see Fig. 1(a). The NLO QCD [7] and the bulk of the NNLO QCD corrections [8], i.e. $O(g^4\alpha_s)$ and $O(g^4\alpha_s^2)$, can be reduced to the Drell-Yan production of a virtual gauge boson [9,10]. The theoretical uncertainty due to PDFs has been estimated to be at the percent level, and the renormalization/factorization scale dependence of these terms is practically negligible [11]. For $ZH$ production at $O(\alpha_s^2)$, however, there are a few classes of diagrams that have no correspondence to the Drell-Yan process. For example, the gluon-induced virtual corrections mediated by a top-quark loop are of order $g^2\lambda_t^2\alpha_s^2$ (see Fig. 1(c)), where $\lambda_t$ is the top-quark Yukawa coupling which, in the SM, is of order one. These corrections were evaluated in Refs. [8,12] and found to be of the order of 5% at the

\[1\] For recent work on higher order differential $WH$ cross sections, see Ref. [6].
Figure 2: (a),(b) Diagrams of group $V_I$ and (c) group $R_I$ contributing to the process $q\bar{q} \to VH(g)$ at order $g^3\lambda_t\alpha_s^2$.

LHC.

In this paper we consider another class of diagrams which are formally of order $g^3\lambda_t\alpha_s^2$ and were neglected in previous analyses. For simplicity, we will refer to them as “top-mediated terms” in this paper, even though they are not the only contributions involving top-quarks, as noted above. Their numerical impact is at the percent level and therefore within the current estimated theoretical uncertainty of the NNLO result (see Ref. [11]). Note, however, that this uncertainty estimate is dominated by the effects from PDFs and $\alpha_s$; once these will be known with higher precision, the results of this paper will be required for the perturbative part to compete with this precision.

The remainder of this paper is organized as follows: Section 2 defines the effects to be calculated, briefly describes the methods applied, and presents analytical expressions for part of the results. In Section 3 we study the size of the newly evaluated effects and present updated values for the total inclusive cross section for $WH$ and $ZH$ production at the Tevatron and the LHC at collision energies of 7 and 14 TeV.

2 Calculational details

2.1 Outline of the problem

The Feynman diagrams of the top-mediated terms considered in this paper can be divided into four groups which will be described in this section.

Examples of diagrams of the first group, named $V_I$ in what follows, are shown in Fig.2(a) and (b). They are characterized by the emission of a Higgs boson off a top-quark bubble-insertion into an internal (i.e. virtual) gluon line. They contribute to the total cross section through the interference with the leading order amplitude (see Fig.1(a)).

The second group ($R_I$), see Fig.2(c), can be viewed as the real emission counterpart of group $V_I$. It is obtained by radiating the Higgs off a top-quark bubble-insertion into an
Figure 3: (a), (b) Diagrams of group $V_{II}$ and (c) group $R_{II}$ contributing to the process $q\bar{q} \rightarrow ZH(g)$ at order $g^3\lambda_t\alpha_s^2$.

The amplitude for each of these groups is separately gauge invariant and UV- as well as IR-finite, despite the fact that two of them can each be viewed as real and virtual correspondences of each other.

The diagrams of $R_I$ and $R_{II}$ can be calculated exactly, taking into account the full dependence on the top, Higgs, and vector boson mass. The amplitude arising from groups $V_I$ and $V_{II}$, on the other hand, is very challenging to compute and may be even beyond current technology. We therefore follow the established and successful method of asymptotic expansions in order to approximate the result in the limit of infinite top-quark mass $M_t$.

In a fully consistent treatment of the contributions described in this section, one needs to take into account yet another set of Feynman diagrams. They can be obtained from the previous ones by radiating the Higgs boson off the external vector boson instead of the top-quark loop, see Fig. 4 for example. If all couplings are replaced by their SM values, the resulting amplitudes are of the same perturbative order as the ones discussed in this paper; however, they receive an additional suppression factor $\sim M_V^2/(\hat{s} - M_V^2)$, where $M_V$ is the mass of the vector boson and $\sqrt{\hat{s}} \geq M_V + M_H$ the partonic center-of-mass
energy. Justifiably so, these terms have been considered Drell-Yan-like in Ref. [8]; in fact, they can be calculated by convolving the output of (a suitably modified version of) the program MASSIVE [13] with the decay $V^* \rightarrow VH$. As a check, we also calculated some of these contributions directly using the methods described below. The result confirms the statement of Refs. [8,12] that these contributions are numerically irrelevant: they are typically 2-3 orders of magnitude smaller than the other newly evaluated contributions considered in this paper, and will therefore be neglected in what follows.

2.2 Calculation of $V_I$ and $V_{II}$

A priori, an expansion of the two-loop amplitudes in the limit of large top-quark mass seems unjustified, because the partonic center of mass energy $\sqrt{\hat{s}}$ at the Tevatron and the LHC can be much larger than $M_t$ (or rather $2M_t$ which corresponds to the threshold and is thus the relevant scale). Nevertheless, there is a number of arguments that make such an approach reasonable, if $M_H$ is not too large:

- The parton luminosities at large $\hat{s}$ are strongly suppressed, and the bulk of the contribution to the total cross section indeed arises from the region below the top-quark threshold. This is similar to what happens in the case of gluon fusion, see Refs. [14,16]. For Higgs-Strahlung, however, the situation is somewhat worse, because the energy window for which the heavy-top expansion is expected to converge is much narrower than for gluon fusion: $MV + MH \leq \sqrt{\hat{s}} \leq 2Mt$. In fact, for $M_H \gtrsim 250$ GeV, this restriction cannot be obeyed at all.

- For group $V_I$ (Fig. 2(a),(b)), the typical energy scale affecting the top-quark loop is not the full center-of-mass energy, but significantly below that, because the vector boson carries off a large fraction of the momentum.

- Since group $V_{II}$ (Fig. 3(a),(b)) is closely related to the $gg$-induced contribution of Fig. 1(c), we may estimate its impact by the ratio of the $q\bar{q}$ and the $gg$ luminosity.
Figure 5: Asymptotic expansion of the diagram in Fig. 2(a). The diagrams left of $\otimes$ are evaluated after setting their external momenta to zero. The result determines the expression to be inserted into the effective $\bar{q}qVH$ or $ggH$ vertex in the diagram right of $\otimes$. For details on the general method, see Refs. [29–31], for example.

... times the $gg$-induced cross section. It should therefore not exceed a few percent of the LO cross section.

- Overall, we expect the top-mediated terms to affect the cross section at the percent level [6, 17]. The leading term of an expansion in terms of $1/M_t$ should thus be sufficiently precise.

The tools to calculate the diagrams are by now standard: asymptotic expansion of the two-loop diagrams of groups $V_I$ and $V_{II}$ leads to a factorization of scales into either two-loop massive tadpoles (i.e., vanishing external momenta) times tree-level diagrams with an effective $\bar{q}qH$ or $\bar{q}qVH$ vertex, or one-loop massive tadpoles times massless one-loop diagrams with an effective $ggZH$ or $ggH$ vertex. As an example, the graphical representation of the asymptotic expansion of the diagram in Fig. 2(a) is shown in Fig. 5.

We use the automatic setup consisting of $qgraf$ [18] for the generation, and $q2e/exp$ [19, 20] for the expansion of the diagrams, as well as MATAD [21] for the calculation of the tadpole integrals. In order to evaluate the massless one-loop box and triangle integrals corresponding to the right-most diagrams in Fig. 5, we supplement this setup by an additional routine (based on FORM [22]) which implements standard Passarino-Veltman reduction [23] in algebraic form. The scalar one-loop Feynman integrals are evaluated using the results of Ref. [24].

It turns out that for $V_I$, the sum of terms involving an effective $\bar{q}qVH$ or $\bar{q}qH$ vertex does not contribute at leading order in $1/M_t$, i.e., the corrections can be calculated by simply using an effective $ggH$ vertex and evaluating one-loop diagrams. This observation allowed us to obtain $V_I$ in a second, independent calculation by using a generalized version of the FORM program FDiag [25] and the Fortran package FF [26, 28], supplemented by a routine to facilitate the tensor reduction of rank-4 tensor 4-point functions and the subsequent numerical evaluation of the corresponding tensor coefficients.
Contracting with the LO amplitude and summing/averaging over final/initial color and spin degrees of freedom, we find for the amplitude of $V_I$, cf. Fig. 2(a), (b):

$$\frac{-d\sigma^V_I}{dt} = G_{qq'}^V \frac{G_F^2 M_V^4}{108\pi \hat{s}^2 (\hat{s} - M_V^2)} \left( \frac{\alpha_s}{\pi} \right)^2 \text{Re} \left\{ -2\hat{s} \right.$$ 

$$+ \ln \left( \frac{\hat{t}}{M_V^2} \right) \left( \hat{t} - M_V^2 + \frac{2\hat{s} M_V^2}{\hat{u} - M_V^2} \right) + \ln \left( \frac{\hat{u}}{M_V^2} \right) \left( \hat{u} - M_V^2 + \frac{2\hat{s} M_V^2}{\hat{t} - M_V^2} \right)$$ 

$$+ (\hat{s} - M_H^2 + M_V^2) \left[ \ln^2 \left( \frac{\hat{t}}{\hat{u}} \right) + 2\text{Li}_2 \left( 1 - \frac{M_H^2}{\hat{t}} \right) + 2\text{Li}_2 \left( 1 - \frac{M_H^2}{\hat{u}} \right) \right] + \mathcal{O}(1/M_t^2),$$

where $\hat{s} = (p_1 + p_2)^2$, $\hat{t} = (p_1 - p_V)^2$ and $\hat{u} = (p_1 - p_H)^2$ are the usual (partonic) Mandelstam variables, with the incoming momenta $p_1, p_2$, and $p_H, p_V$ the momenta of the Higgs and the vector boson, respectively. $M_V$ is the mass of the emitted vector boson, and $\text{Li}_2$ denotes the di-logarithm. The electro-weak couplings are given by

$$G_{qq'}^Z = (v_q^2 + a_q^2) \delta_{qq'}, \quad G_{qq'}^W = \frac{1}{2} |V_{qq'}|^2,$$

$$v_q = \pm \frac{1}{2} + \frac{1}{3} \left\{ \begin{array}{c} -4 \\ +2 \end{array} \right\} \sin^2 \theta_W, \quad a_q = \pm \frac{1}{2} \quad \text{for} \quad q \in \left\{ \begin{array}{c} u, c \\ d, s, b \end{array} \right\},$$

with the weak mixing angle $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$ and the CKM matrix elements $V_{qq'}$ (see Ref. [32] for the latest numerical values; we set $V_{qq'} = 0$ if both $q$ and $q'$ carry the same weak isospin charge $I_3$). We have expressed the top-quark Yukawa coupling by the tree-level relation

$$\lambda_t \equiv \frac{M_t}{v} = M_t \sqrt{2G_F}$$

in Eq. (3), and also in Eq. (6) below. Similar to the gluon fusion process, the factor $M_t$ cancels against its inverse from the top-loop integration.

For group $V_{II}$ (see Fig. 3(a), (b)), only the axial vector part of the $Z$ coupling to fermions contributes. We implement its Dirac structure with the help of Levi-Civita symbols $\varepsilon_{\mu\nu\rho\sigma}$ [33]

$$\gamma_{\mu} \gamma_5 \rightarrow \frac{i}{3!} \varepsilon_{\mu\alpha\beta\delta} \gamma^{\alpha} \gamma^{\beta} \gamma^{\delta},$$

and re-write their product in terms of the anti-symmetrized (denoted by square brackets) $D$-dimensional metric tensor,

$$\varepsilon_{\mu\nu\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} = -g_{[\alpha \beta} \varepsilon^{\gamma \delta]}.$$
Upon asymptotic expansion of the diagrams, the terms corresponding to an effective $ggZH$ vertex contribute only at subleading order in $1/M_t$, and the calculation reduces to massive 2-loop tadpole diagrams.

The result assumes the simple form

$$-\frac{d\Delta\sigma^{\nu}_{\Pi}}{dt} = \frac{G_F^2 a_{q} a_{t} \alpha_s}{12\pi s^2(s - M_Z^2)} \left(\frac{\alpha_s}{\pi}\right)^2 \left\{2\hat{s} - M_H^2 + \hat{t} + O\left(\frac{1}{M_t^4}\right)\right\}. \quad (6)$$

Since the expressions in Eq. (1) and (6) are free of divergences in the allowed $\hat{t}$-region, we can numerically integrate them together with the convolutions over the PDFs in order to get their contribution to the total inclusive hadronic cross section. The numerical results will be presented in the next section.

### 2.3 Calculation of $R_I$ and $R_{II}$

Being of one-loop order, the diagrams of groups $R_I$ and $R_{II}$ can be calculated including the full dependence on the top-quark, Higgs- and vector-boson mass by means of Passarino-Veltman reduction to scalar one-loop functions. In most of the phase-space, the numerical evaluation of these function can be performed with the help of the programs FDiag and FF (see Sec. 2.2), and independently with FeynArts, FormCalc, and LoopTools [34] (the latter of which relies on FF though). The contribution $R_{II}$, however, involves momentum configurations that are outside FF's capabilities. For those, we use an implementation of the one-loop integrals based on Refs. [35, 36] [3]. Again, since projection with the LO amplitude leads to a finite expression, phase-space integration and convolution with PDFs can be done fully numerically.

### 2.4 Validation of the heavy-top approximation

This section describes both a consistency check for our calculation of the real-emission contributions $R_I$ and $R_{II}$, as well as a validity check of our approach to the virtual terms $V_I$ and $V_{II}$.

In addition to the exact calculation as described in Section 2.3, we evaluated the real emission amplitudes $R_I$ and $R_{II}$ also by applying asymptotic expansions in the heavy-top limit. In these cases, it reduces to a naive Taylor expansion of the integrand before loop integration. We can again perform all phase-space integrals numerically. In the $q\bar{q}$-channel, however, we also integrated over the angular variables of the phase-space analytically in $D = 4 - 2\epsilon$ space-time dimensions, and found the cancellation of all poles at $\epsilon = 0$. The numerical integration over the remaining energy variables is straightforward and leads to the same result as the all-numerical method.

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3We thank Stefan Dittmaier for providing us with his private code.
Comparing this heavy-top result for \( R_I \) and \( R_{II} \) at the partonic level to the exact results described above, we indeed observe that heavy-top limit systematically approaches the exact result as \( \mathcal{E}/M_t \to 0 \), where \( \mathcal{E} \) is any of the external energy scales. In particular, we observe that, as opposed to the other contributions, the terms \( R_{II} \) vanish as \( 1/M_t^2 \) in the limit \( M_t \to \infty \). This agreement between the exact and the asymptotic approach provides a strong and valuable check on our results.

In order to draw conclusions for the validity of the heavy-top expansion, however, the relevant quantity to compare is the hadronic cross section, of course. For that, we find that the heavy-top result is roughly within 25%/35% of the full result at the LHC/Tevatron in the mass range considered in this paper.

We expect a similar quality of the heavy-top result for the contributions from \( V_I \) and \( V_{II} \). To be conservative, we attribute an additional uncertainty of 30%/50% (relative to the central values) to these terms for the LHC/Tevatron prediction of the hadronic cross section.

3 Numerics

3.1 Size of the top-induced terms

We present numbers for the Tevatron (\( p\bar{p} @ \sqrt{s} = 1.96 \) TeV), as well as the LHC (\( pp @ 7 \) TeV and 14 TeV). The hadronic cross section is evaluated by folding the partonic cross section with PDFs. Although the corrections evaluated in this paper are not renormalized by lower order terms, we think it is appropriate to convolve them with NNLO PDFs. We use the central MSTW2008 set, implying \( \alpha_s(M_Z) = 0.1171 \). For the physical parameters, we assume the values

\[
M_Z = 91.1876 \text{ GeV}, \quad M_W = 80.398 \text{ GeV}.
\tag{7}
\]

Since the amplitudes considered in this paper are UV- and IR-finite, they do not depend explicitly on the renormalization or factorization scale (\( \mu_R, \mu_F \)). However, these scales enter implicitly through the strong coupling and the PDFs. For the numerical analysis, we choose

\[
q^2 \equiv (p_H + p_V)^2
\]

as the central scale for \( \mu_R^2 \) and \( \mu_F^2 \), where \( p_H \) and \( p_V \) are the 4-momenta of the Higgs and the outgoing vector boson. For the Drell-Yan-like terms, \( \sqrt{q^2} \) equals the invariant mass of the intermediate gauge boson. In order to estimate the theoretical uncertainty of our results, we vary \( \mu_R \) by a factor of three around this central scale while keeping \( \mu_F \) fixed,

\[
\frac{1}{3} \sqrt{q^2} \leq \mu_R \leq 3 \sqrt{q^2}, \quad \mu_F = \sqrt{q^2},
\tag{8}
\]
then repeat the analysis after interchanging $\mu_F$ and $\mu_R$, and take the extreme values of
the cross section as uncertainty band.

Fig. 6 shows the contribution of the newly evaluated terms to the total inclusive cross sections $\sigma(pp \to WH) \equiv \sigma(pp \to W^+H) + \sigma(pp \to W^-H)$ and $\sigma(pp \to ZH)$ at 7 TeV and 14 TeV center-of-mass energy. The contributions from the various groups of diagrams (see Section 2.1) are shown separately, each of them with an error band derived from the scale variation described above. Also shown is the sum of all contributions. The corresponding plots for the Tevatron are shown in Fig. 7.

The size of the $V_1$ component amounts to about 0.5% of the LO cross section\footnote{Only central values are considered in this discussion.} both for $WH$ and $ZH$ production, independent of collider type, center-of-mass energy, and Higgs boson mass. While at the LHC the $R_1$ terms are typically a little larger than that, and increase with $M_H$ and the center-of-mass energy, they always remain below 0.5% at the Tevatron.

Since $V_1$ and $R_1$ are the only non-vanishing contributions for $WH$ production, in this case the overall effect of the top-mediated terms evaluated in this paper remains below 1% at the Tevatron, and ranges between about 1.1% (for $M_H = 100$ GeV at 7 TeV) to 2.4% (for $M_H = 300$ GeV at 14 TeV) at the LHC (see Fig. 6(a) and Fig. 7(a)).

For $ZH$ production, also the groups $V_{II}$ and $R_{II}$ have to be taken into account. Being suppressed by $1/M_t^2$ at large $M_t$, as pointed out above, the terms $R_{II}$ have only a very small numerical impact at the per-mille level. The relative contribution of $V_{II}$, on the other hand, increases with the Higgs boson mass, but remains below 0.8% at the LHC in all of the considered Higgs mass range. At the Tevatron, however, it exceeds $V_1$ and $R_1$ above $M_H \approx 130$ GeV and ranges up to about 1% at $M_H = 200$ GeV. The overall effect of $V_1$, $R_1$, $V_{II}$, and $R_{II}$ on $ZH$ production at the Tevatron is therefore between 1% and 2%, while it ranges between 1.1% (for $M_H = 100$ GeV at 7 TeV) and 2.9% (for $M_H = 300$ GeV at 14 TeV) at the LHC (see Fig. 6(b) and Fig. 7(b)).

In each case, a rough estimate of the uncertainty on the top-mediated terms due to scale variation is 20-30%. As discussed in Section 2.4 we also include an estimated uncertainty on the $V_1$ and $V_{II}$ terms of 30%/50% for the LHC/Tevatron, arising from the heavy-top limit (this is not included in Figs. 6 and 7). Considering the fact that the corrections are at the percent level, this uncertainty will affect the accuracy of the total inclusive cross section by roughly 0.5%. We do not expect the PDF uncertainties on the top-induced terms to add significantly to the one of the total cross section and will neglect it in what follows.
Figure 6: Contribution of the corrections evaluated in this work to the total inclusive cross section $\sigma(pp \to WH + X)$ (left column) and $\sigma(pp \to ZH + X)$ (right column) at the LHC with 7 TeV (upper row) and 14 TeV center-of-mass energy (lower row). The effects are shown relative to the leading order cross section $\sigma_{LO}$. The various bands show the contributions from $V_I$, $R_I$, and, in the case of $ZH$, $V_{II}$ and $R_{II}$. The upper band is the sum of all contributions. The width of the band arises from the variation of $\mu_R$ and $\mu_F$ as described in the main text.
3.2 Total inclusive cross section

As shown in the previous section, the effects of the newly evaluated terms are of the order of 1-3% of the LO cross section on $WH$ and $ZH$ production. Superficially, this lies within previous estimates of the uncertainties of the total inclusive cross section at the LHC [11]. However, most of this uncertainty is induced by the PDFs. Once systematical effects and theoretical ambiguities will have been settled (for a recent discussion, see Ref. [3]) or become irrelevant due to the inclusion of LHC data, one may expect these uncertainties to shrink significantly. Then, in order to arrive at the most precise prediction for the process, the effects of the top-mediated terms evaluated here will have to be included.

For this reason, we provide an updated prediction of the total inclusive cross section for $WH$ and $ZH$ production at the Tevatron and the LHC at 7 TeV and 14 TeV in this section. The numbers for the LHC are obtained simply by adding the top-mediated terms to the predictions of Ref. [11], which have been obtained using the program $vh@nnlo$ [37], based on the calculation of Refs. [8-10] and the program $zwprod$ [9], and an electro-weak correction factor [38]. The uncertainties on the numbers given here result from linearly adding the previous uncertainties [11] to the ones discussed in Section 3.1.

For the Tevatron, an “official” public agreement on the cross section prediction analogous to Ref. [11] is not available. We therefore follow the analysis of Ref. [11] using Tevatron parameters, and proceed as above. The electro-weak correction factor we read off from the plots and tables of Ref. [38] in this case, despite the fact that the parameters in this paper are slightly outdated. The uncertainty induced by this procedure should be negligible, however.
The final results are presented in Tables 1, 2, and 3 for the LHC at 7 TeV, 14 TeV, and
the Tevatron, respectively. The top-mediated terms slightly increase all the cross sections,
but also the theoretical uncertainty. The column labeled “Pert” in the tables contains the
estimate of the perturbative uncertainty as described in Section 3.1. The column labeled
“PDF+αs” displays the error induced by the PDFs and the input value for αs, as described
in Ref. [11].

Finally, Figs. 8 and 9 show the K-factors resulting from our compilation, including NNLO
QCD, electro-weak effects, and the newly evaluated top-mediated terms of this paper. In
the LHC case, we also show the previous numbers from Ref. [11], and for the Tevatron we
provide the analogous numbers with the same input data.

These plots clearly show that for the LHC, the top-mediated terms affect the WH pro-
duction cross section at the order of the estimated perturbative uncertainties. In fact,
note that for √s = 14 TeV and above around MH = 160 GeV, the uncertainty bands of
the predictions with and without the top-induced terms hardly overlap. Since the uncer-
tainty bands of the ZH channel are significantly larger than for WH, mostly due to the
gg-induced terms of Fig. 1 (c), the relative importance of the top-mediated terms on ZH
is much smaller than for WH.

At the Tevatron, the K-factor is typically larger than at the LHC, and the top-mediated
terms have a rather small impact. Also, the difference between WH and ZH in the K-
factor is much less pronounced than at the LHC, the reason being again the gg-induced
terms which have much smaller impact at the Tevatron.

4 Conclusions and Outlook

The effects of an as of yet neglected contribution to the Higgs-Strahlung process at hadron
colliders have been studied. The relevant Feynman diagrams contain closed top-quark
loops and are of order g3λtα2s. The one-loop real-emission contributions were evaluated
exactly, while the two-loop virtual terms were evaluated in the heavy-top limit. It was
argued that this provides a reliable approximation to the exact result. The numerical
impact of the newly evaluated terms is within the current estimate for the theoretical
uncertainty, but may well become important once the uncertainty induced by PDFs and
the strong coupling reduces.

We plan to include the newly evaluated terms in the publicly available program vh@nnlo
in order to provide a tool for the evaluation of the the full O(α2s) prediction for ZH and
WH production.

Finally, let us remark that similar contributions exist also for the weak-boson fusion pro-
cess 39 42. Parts of them were evaluated in Ref. [43].
| $M_H$ [GeV] | $\sigma(\text{WH})$ [fb] | Pert [%] | PDF+$\alpha_s$ [%] | $\sigma(\text{ZH})$ [fb] | Pert [%] | PDF+$\alpha_s$ [%] |
|------------|----------------|--------|----------------|----------------|--------|----------------|
| 100        | 1197           | +1.0  | ±3.4          | 636.8         | +1.5  | ±3.4          |
| 105        | 1027           | +0.8  | ±3.5          | 549.8         | +1.7  | ±3.7          |
| 110        | 883.7          | +0.7  | ±3.8          | 476.5         | +1.6  | ±4.1          |
| 115        | 762.0          | +0.8  | ±3.9          | 414.6         | +1.5  | ±4.2          |
| 120        | 662.7          | +0.8  | ±3.4          | 363.3         | +2.0  | ±3.5          |
| 125        | 578.8          | +0.6  | ±3.5          | 319.0         | +1.9  | ±3.5          |
| 130        | 506.1          | +0.8  | ±3.5          | 280.7         | +2.0  | ±3.7          |
| 135        | 443.7          | +1.1  | ±3.4          | 247.9         | +2.2  | ±3.6          |
| 140        | 389.9          | +1.0  | ±3.5          | 219.5         | +2.0  | ±3.7          |
| 145        | 344.4          | +0.7  | ±3.8          | 195.2         | +2.3  | ±4.0          |
| 150        | 303.5          | +0.9  | ±3.3          | 173.2         | +2.3  | ±3.6          |
| 155        | 267.7          | +1.0  | ±3.5          | 154.3         | +2.6  | ±3.6          |
| 160        | 231.9          | +1.0  | ±3.8          | 135.0         | +2.5  | ±4.0          |
| 165        | 213.2          | +1.0  | ±3.6          | 124.8         | +2.1  | ±4.1          |
| 170        | 190.7          | +1.0  | ±3.8          | 112.0         | +2.7  | ±4.2          |
| 175        | 171.0          | +0.8  | ±3.8          | 100.8         | +2.3  | ±4.1          |
| 180        | 154.0          | +1.1  | ±3.5          | 90.34         | +2.8  | ±3.8          |
| 185        | 140.5          | +0.9  | ±3.5          | 82.46         | +2.9  | ±3.8          |
| 190        | 126.9          | +1.1  | ±3.7          | 74.65         | +2.8  | ±3.9          |
| 195        | 115.2          | +1.2  | ±3.7          | 67.91         | +2.9  | ±4.0          |
| 200        | 104.6          | +0.9  | ±3.8          | 61.81         | +2.9  | ±4.1          |
| 210        | 86.70          | +1.0  | ±3.7          | 51.41         | +2.8  | ±4.2          |
| 220        | 72.38          | +0.9  | ±3.7          | 42.97         | +2.9  | ±4.2          |
| 230        | 60.87          | +1.3  | ±4.5          | 36.15         | +2.8  | ±4.8          |
| 240        | 51.45          | +1.1  | ±4.0          | 30.46         | +2.6  | ±4.4          |
| 250        | 43.68          | +1.1  | ±4.0          | 25.81         | +2.7  | ±4.2          |
| 260        | 37.25          | +1.3  | ±4.0          | 21.94         | +2.6  | ±4.5          |
| 270        | 31.91          | +1.2  | ±3.8          | 18.70         | +2.4  | ±4.3          |
| 280        | 27.39          | +0.9  | ±4.4          | 16.02         | +2.4  | ±4.9          |
| 290        | 23.65          | +1.2  | ±4.2          | 13.79         | +2.3  | ±4.5          |
| 300        | 20.47          | +1.1  | ±4.5          | 11.90         | +2.2  | ±5.0          |

Table 1: Numerical values for the total cross section at LHC at 7 TeV.
| $M_H$ [GeV] | $\sigma(WH)$ [fb] | Pert [%] | PDF+\(\alpha_s\) [%] | $\sigma(ZH)$ [fb] | Pert [%] | PDF+\(\alpha_s\) [%] |
|------------|----------------|---------|-----------------|----------------|---------|----------------|
| 100        | 3035           | +1.3\,-0.9 | ±3.7            | 1700           | +2.3\,-1.9 | ±3.8            |
| 105        | 2625           | +1.1\,-0.8 | ±3.5            | 1483           | +2.2\,-2.1  | ±3.7            |
| 110        | 2273           | +0.8\,-1.1 | ±3.8            | 1297           | +2.6\,-2.0  | ±4.0            |
| 115        | 1976           | +1.2\,-0.6 | ±3.8            | 1142           | +2.9\,-1.9  | ±3.7            |
| 120        | 1731           | +1.1\,-0.7 | ±3.8            | 1008           | +2.9\,-2.2  | ±3.6            |
| 125        | 1523           | +0.9\,-1.0 | ±3.8            | 893.2          | ±2.2\,-2.2  | ±3.7            |
| 130        | 1342           | +1.0\,-0.8 | ±3.3            | 793.9          | +3.4\,-2.2  | ±3.4            |
| 135        | 1183           | +1.2\,-0.9 | ±2.9            | 706.6          | +3.4\,-2.6  | ±3.0            |
| 140        | 1048           | +0.8\,-1.1 | ±3.1            | 633.4          | +3.4\,-2.6  | ±3.0            |
| 145        | 933.1          | +1.1\,-0.8 | ±3.3            | 567.2          | +3.8\,-2.5  | ±3.4            |
| 150        | 827.5          | +0.9\,-1.1 | ±2.7            | 508.2          | +3.8\,-2.4  | ±2.7            |
| 155        | 736.3          | +0.9\,-1.0 | ±3.1            | 457.3          | +3.8\,-2.8  | ±3.2            |
| 160        | 644.0          | +0.8\,-0.9 | ±3.1            | 404.1          | +4.1\,-2.8  | ±3.1            |
| 165        | 594.0          | +0.8\,-1.0 | ±2.4            | 375.6          | +4.3\,-2.7  | ±2.6            |
| 170        | 534.3          | +0.9\,-1.1 | ±2.8            | 340.2          | +4.1\,-2.9  | ±3.0            |
| 175        | 483.9          | +1.1\,-0.8 | ±2.9            | 308.8          | +4.0\,-3.1  | ±3.1            |
| 180        | 434.5          | +1.1\,-1.0 | ±2.8            | 278.5          | +4.3\,-3.3  | ±3.0            |
| 185        | 402.8          | +1.0\,-1.1 | ±2.5            | 256.2          | +4.1\,-3.3  | ±2.6            |
| 190        | 366.0          | +0.9\,-1.1 | ±2.8            | 233.6          | +4.1\,-3.3  | ±3.0            |
| 195        | 334.6          | +0.9\,-1.2 | ±2.7            | 214.4          | +4.1\,-3.4  | ±2.9            |
| 200        | 305.5          | +1.0\,-1.0 | ±3.0            | 196.6          | +4.2\,-3.5  | ±3.1            |
| 210        | 257.0          | +0.9\,-1.1 | ±2.6            | 165.4          | +4.5\,-3.4  | ±2.6            |
| 220        | 217.5          | +1.3\,-1.0 | ±2.8            | 140.3          | +4.1\,-3.2  | ±2.9            |
| 230        | 185.9          | +1.1\,-0.9 | ±3.5            | 119.3          | +4.1\,-3.1  | ±3.6            |
| 240        | 158.9          | +1.1\,-1.0 | ±3.3            | 101.7          | +3.9\,-3.0  | ±3.4            |
| 250        | 136.8          | +0.9\,-1.2 | ±3.0            | 86.96          | +3.7\,-2.9  | ±3.2            |
| 260        | 118.3          | +0.9\,-1.2 | ±2.8            | 74.80          | +3.8\,-2.6  | ±3.1            |
| 270        | 102.8          | +1.2\,-1.1 | ±2.6            | 64.48          | +3.3\,-2.5  | ±2.8            |
| 280        | 89.49          | +1.1\,-1.1 | ±3.0            | 55.83          | +3.3\,-2.4  | ±3.2            |
| 290        | 78.61          | +1.0\,-1.1 | ±3.2            | 48.67          | +3.0\,-2.2  | ±3.2            |
| 300        | 68.86          | +1.3\,-1.1 | ±3.3            | 42.43          | +2.9\,-2.2  | ±3.6            |

Table 2: Numerical values for the total cross section at LHC at 14 TeV.
| $M_H$ [GeV] | $\sigma(\WH)$ [fb] | Pert [%] | $\sigma(\WH)+\alpha_s$ [%] | $\sigma(\ZH)$ [fb] | Pert [%] | $\sigma(\ZH)+\alpha_s$ [%] |
|-----------|----------------|---------|-----------------------------|----------------|---------|-----------------------------|
| 100       | 278.0          | +4.0    | ±5.1                        | 161.2          | +4.1    | ±5.5                        |
| 105       | 235.7          | +4.1    | ±5.3                        | 138.2          | +4.3    | ±5.6                        |
| 110       | 200.9          | +4.0    | ±5.5                        | 119.0          | +4.2    | ±5.7                        |
| 115       | 172.1          | +4.0    | ±5.5                        | 103.0          | +4.2    | ±5.8                        |
| 120       | 148.0          | +4.1    | ±5.8                        | 89.46          | +4.3    | ±5.9                        |
| 125       | 127.5          | +4.0    | ±6.0                        | 77.86          | +4.2    | ±6.1                        |
| 130       | 110.2          | +4.0    | ±6.2                        | 67.98          | +4.3    | ±6.0                        |
| 135       | 95.61          | +4.1    | ±6.1                        | 59.55          | +4.4    | ±6.0                        |
| 140       | 83.06          | +4.2    | ±6.4                        | 52.23          | +4.5    | ±6.2                        |
| 145       | 72.36          | +4.3    | ±6.5                        | 46.01          | +4.6    | ±6.3                        |
| 150       | 63.16          | +4.4    | ±6.3                        | 40.59          | +4.7    | ±6.0                        |
| 155       | 55.28          | +4.4    | ±6.9                        | 35.91          | +4.8    | ±6.6                        |
| 160       | 48.49          | +4.6    | ±6.9                        | 31.84          | +4.9    | ±6.5                        |
| 165       | 42.54          | +4.5    | ±7.8                        | 28.23          | +4.9    | ±7.3                        |
| 170       | 37.51          | +4.6    | ±7.1                        | 25.15          | +5.0    | ±6.6                        |
| 175       | 33.26          | +4.7    | ±6.9                        | 22.42          | +5.2    | ±6.4                        |
| 180       | 29.53          | +4.9    | ±7.3                        | 20.02          | +5.3    | ±6.6                        |
| 185       | 26.29          | +4.9    | ±7.7                        | 17.92          | +5.4    | ±6.9                        |
| 190       | 23.39          | +5.0    | ±7.9                        | 16.03          | +5.4    | ±7.0                        |
| 195       | 20.86          | +5.1    | ±7.7                        | 14.38          | +5.6    | ±6.8                        |
| 200       | 18.61          | +5.0    | ±7.6                        | 12.91          | +5.5    | ±6.5                        |

Table 3: Numerical values for the total cross section at the Tevatron.
Figure 8: K-factor at the LHC at (a) 7 TeV and (b) 14 TeV center-of-mass energy, including NNLO QCD and electro-weak corrections, with and without the newly evaluated top-quark induced terms.

Figure 9: K-factor at the Tevatron for (a) $WH$ and (b) $ZH$ production, including NNLO QCD and electro-weak corrections, with and without the newly evaluated top-quark induced terms.
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