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Investigation of X and Y Configuration Modal and Dynamic Response to Velocity Excitation of the Nanometer Resolution Linear Servo Motor Stage with Quasi-Industrial Guiding System in Quasi-Stable State

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Abstract: Research institutions and industrial enterprises demand high accuracy and precision positioning systems to fulfil cutting edge requirements of up-to-date technological processes in the field of metrology and optical fabrication. Linear motor system design with high performance mechanical guiding system and optical encoder ensures nanometer scale precision and constant static error, which can be calibrated by optical instruments. Mechanical guiding systems has its benefits in case of control theory and its stability; unfortunately, on the other hand, there exists high influence of structure geometry and tribological effects such as friction and modal response. The aforementioned effect cannot be straightforwardly identified during the assembly process. Degradation of dynamic units can be detected only after certain operating time. Single degree of freedom systems are well investigated and the effect of degradation can be predicted, but there exists a gap in the analysis of nanometer scale multi degree of freedom dynamic systems; therefore, novel diagnostic tools need to be proposed. In this particular paper, dual axes dynamic system analysis will be presented. The main idea is to decouple standard stacked XY stage and analyse X and Y configuration as two different configurations of the same object, while imitators of corresponding axes are absolutely solid and stationary. As storage and analysis of time domain data is not efficient, main attention will be concentrated on frequency domain data, while, of course, statistical and graphical representation of dynamic response will be presented. Transfer function, dynamic response, spectral analysis of dynamic response, and modal analysis will be presented and discussed. Based on the collected data and its analysis, comparison of X and Y responses to different velocity excitation will be presented. Finally, conclusions and recommendations of novel diagnostic way will be presented.

Keywords: mechanical bearings; linear motors; position control; velocity control; nanotechnologies; frequency response function; modal analysis; accelerometers; transfer function; dynamic error; robotics; CNC machines

1. Introduction

The precision of the manufacturing process, increasing efficiency and machining accuracy, determines the application of digitized precision computer control (CNC) in machine building [1,2]. It is impractical, non-economical and probably impossible to produce a perfect mechanical system. Air-bearing based systems can be a solution but, unfortunately, sharp drawbacks and disadvantages in particular designs occur; frictionless systems are more sensitive to changes of inertia, it is very hard to obtain stability of its dynamic response in a wide range of velocities, it is not compact, pneumatic feedthroughs are required; therefore, friction-based systems can be a very good solution for some of the application.
Linear motor systems are gearless where friction is found only in guiding systems; therefore, dynamic response is mostly caused by mechanical structure, geometry, characteristics of the grease, friction and preload. During the operation, because of the aforementioned reasons, mechanical adjustments (including assembly and machining), metrology and calibration must be applied. By analysing standard procedures of mechanical adjustment and metrology, it must be emphasized that is not economical and sometimes impractical to proceed with it; therefore, novel diagnostic tools are critically required. As linear motor systems have a single friction source and modern FPGA based control and drives systems have enough calculation resources, the trace of degradation must be identified by indirect methods and embedded into a real time control system [1]. Lack of scientific information related to combination of tribological effects, frequency response function, modal response and dynamic response of the motion system propose an innovative diagnostic set which may be an efficient way to predict the drift of static error and system dynamics.

Precision positioning systems, based on permanent magnet ironless linear synchronous motors, enables a wide range of industrial and laboratory applications because of their precision and dynamic capabilities. High throughput and continuous scanning or trajectory positioning modes are required for medical, microscopy, micromachining, marking, and 3D printing applications. Due to these factors, the application of the linear motor stages has increased in the past few years [2–10]. Direct drive mechanisms are mandatory if there are requirements to minimize the static and dynamic errors of the displacement [11,12] ensure nanometer reputability.

The classic linear stage based on a linear motor consists of an ironless linear motor, linear guiding system, feedback system, proximity sensors and stage housing. A “Lorentz force” model is used to generate a force in the moving part which is proportional to current added to the motor windings: \( \mathbf{F} = \mathbf{I} \times \mathbf{L} \times \mathbf{B} \). Unfortunately, force can become non-stable because of tribological and modal effects. Due to tribological effect, however, the product cycle is still limited by friction and stage structure; therefore, drift of static and dynamic errors exist. Sub-micrometre performance of linear stage is limited in time, because of guiding system degradation, deformation of mechanical structure, modal response of mechanical structure, preload changes and lack of greases.

Force fluctuations occurring in a linear motor feed system are a topical issue that affect the accuracy of system operation. Attempts are made to solve it by applying various algorithms; from the magnetic field model of the permanent magnet air gap [13], the model of permanent magnetic linear synchronous motors (PMLSM) [14], the equivalent circuit model obtained by theoretical analysis [15,16], the conformal mapping (CM) method for determining torque [17,18], PMLSM analysis method [14], multiple changes of traction harmonics of different motion parameters [19], development of compensation control strategies to improve traction oscillations as a periodic disturbance compensation strategy [20], force wave compensation strategy [21] and a comprehensive strategy for trinities, disturbances and surges [22,23]. To maintain the dynamic stability of the linear stage and its static error at its minimum, it is required to calibrate static error by optical methods and tune the transfer function in frequency domain. Any mechanical system has its eigen frequencies and resonances [24]. Frequency response function of the mechanical system can be accrued by harmonic excitation of the system and modal analysis [25]; therefore, loop shaping filters can be applied to reduce influence of resonances to dynamic response of the system and especially dynamic error [26].

The problems of both static and dynamic displacement errors of the positioning systems are under intensive research as to find a reliable way to diagnose mechanism degradation which would allow us to prevent or even predict error drift [27,28], which will allow to increase product life cycle. Velocity feedback and velocity control loop has significant impact on motion control systems, due to direct impact on system damping [2,3]; however, differentiation of the position still has it challenges.

Friction, damping forces, preload and modal response caused by linear stage structure, increase vibration magnitude during the time and changes shape of vibration dynami-
cally [29]. The characteristic of the vibrational response is that it is harmonic, but stochastic during the time. Vibrations amplify modal response in unexpected frequencies, influence the preload in the guiding system, dynamic response of the stage and finally its static error. Generally speaking, increased level of vibration and its non-linear behaviour, reduces overall positioning accuracy of the translation system. To reduce these effects, Kalman filters (KF), field oriented control (FOC) and sliding mode controller (SMC) are broadly used, while eddy current damping (ECD) can be additionally evaluated [28,30,31].

Due to vibrations, while the system is in motion and especially in start–stop operation, the dynamic error of the system increases in time. Due to vibration magnitude irregularity and frequency irregularity, it is hard to identify degradation of the robotic system by using standard control algorithms; therefore, novel control and diagnostic algorithms are required [32]. The traction harmonics of a linear motor, even at small oscillation amplitudes, cause oscillations that directly affect the mechanical system itself [16,33,34]. They have a cumulative effect that causes already large displacement fluctuations. In precision manufacturing, these substations in smooth motion are not taken into account [35], as the dynamic characteristics of the mechanical system is modeled and controlled by process controller. This is relevant given that the oscillations affect the entire mechanical system, and in particular the traction of the motor through the feedback system [36], so a detailed study of the electromechanical connection caused by the traction harmonic is necessary.

Separate studies have been performed to analyze mechanical oscillation, current circuit permeability [37], current upper limit, and control parameters in a linear motor supply system [15] but have only been defined by feedback errors without detailed analysis of traction harmonics and displacement oscillations. Such studies were performed by Yang et al. [16], where the dependences between air gap fluctuations and the harmonic error is displacement fluctuation were defined.

This paper describes the modal and dynamic response of X and Y configuration linear motor stage with quasi-industrial linear guiding system and aims to find out the margins and actual response of the single degree of freedom stage excited in two different configurations: X (while fixed constrain of stage housing is organized to have full contact area with the surface) and Y (while fixed constrain of the stage housing is organized to have limited contact area with the base surface). PID with different configurations of notch filter, low pass filter and second order bi-quad filter is used to ensure system stability, without adjusting the system mechanical construction. To obtain the frequency response functions LMS is excited by current in the limited bandwidth and fixed amplitude (which might be considered as force excitation). The bandwidth is compared with the results obtained by the accelerometers. Spectral and dynamic response is investigated to analyse important X and Y configuration information. In addition, the Fourier spectral analysis of the amplitudes of the displacement dynamic errors is performed.

The obtained results of the investigation are represented by the derived transfer functions of the plant, controllers, the whole open loop LMS. The experimentally identified frequency response functions (FRF) are compared with FRF of the theoretically derived transfer functions of the LMS.

2. Materials and Methods

2.1. The Object of Investigation

The one degree of freedom linear motor stages with quasi-industrial guiding system, hereafter denoted as LMS.

LMS is assembled on heavy granite base (pos. 2 of Figure 1) along with the load (pos. 4 of Figure 1) and imitators (pos. 5 of Figure 1) to ensure real-world application configuration and resulting conditions. Position and velocity control loops of the platform are both closed by reflective optical linear encoder. The measuring head of the encoder is attached directly to the moving platform (pos. 1 of Figure 1) to ensure the precise displacement feedback. In LMS under consideration, the guiding system is quasi-industrial linear bearings with recirculating balls. Bandwidth and frequency response of the LMS is
estimated by accelerometers (pos. 3 of Figure 1). Additionally, frequency response function is estimated by high performance servo controller and power amplifier by exciting moving platform by constant magnitude sine sweep (pos. 6 of Figure 1). The information was acquired by computer, MMI studio diagnostics software and spectrum analyser (pos. 7 of Figure 1).

![Figure 1](image_url)

**Figure 1.** The general view of linear motor stage with the quasi-industrial bearings in X configuration (a) and in Y configuration (b): 1 are moving platforms; 2 are granite bases; 3 are accelerometers; 4 are loads; 5 are configuration imitators; 6 are power amplifiers.

### 2.2. The Test Equipment

The test stand consists of: granite base by “JFA” (pos. 1 of Figure 2), LMS 8MTL165-300-Len1 by “Standa” (pos. 2 of Figure 2), diagnostics system and motion controller ACS SPii+ by “ACS Motion Control Inc.”, (pos. 3 of Figure 2), PC with control and diagnostic MMI Studio software by “ACS Motion Control Inc.” (pos. 4 of Figure 2), motion controller and amplifier module CMHP3A34N0N800WNANN by “ACS Motion Control Inc.”, (pos. 5 of Figure 2), accelerometer system by “Brüel & Kjær” (pos. 6 of Figure 2). The investigation was done in “Standa” mechatronic laboratory premises. The general view of the test equipment is shown in Figure 2.

Motor windings are directly connected to current amplifier CMHP3A34N0N800WNA NNN, which is part of the motion control setup (pos. 5 of Figure 2). Position and velocity control loops of the platform both are closed by reflective type optical encoder. The encoder interface is analog Sin/Cos which is interpolated directly through CMHP3A34N0N800WNA NNN analog encoder interface. Analog signal is interpolated x4096 times by internal multiplier to ensure encoder resolution equal to 4.8828125 nm. Sin/Cos encoder errors caused by assembly procedures is prevented by adjusting of *Lissajou* graphs after final assembly of robotic system and adding work load. Motion controller (single board computer or master ACS SPii+, which is built into CMHP3A34N0N800WNANN) is used to establish the communication with PC (pos. 4 of Figure 2) using TCP/IP interface. Further, by using PC (pos. 4 of Figure 2) and combination based on motion controller ACS SPii+ and current amplifier CMHP3A34N0N800WNANN (pos. 5 of Figure 2) real time control and diagnostics of LMS (pos. 3 of Figure 2) will be investigated. To obtain the frequency response functions LMS is excited by current in the limited bandwidth and fixed amplitude (which might be considered as force excitation as force have direct dependence on current trough motor
force constant \( K_p \)). The spectral response is compared with spectrum of dynamic response and results obtained by the accelerometers.

In the next stage, spectral and dynamic response is investigated to analyse important X and Y configuration frequency domain information. Additionally, the Fourier spectral analysis of the amplitudes of the displacement dynamic errors is performed.

2.3. Mathematical Modelling of the Linear Motor Stage with Quasi-Industrial Guiding System

The block diagram of LMS and control system is shown in Figure 3. LMS with current amplifier (including current control loop which consists of “proportional-integral” controller) is hereafter called the plant—see Figure 3. After tuning of the current loop, the plant is considered as constant, while the influence to the modal and dynamic response of the different velocity excitation and LMS configuration is investigated. As it was already mentioned, in the present article the position and velocity loops together with the velocity loop shaping filters are called controller PID1, when configuration is X; controller PID2, when configuration is Y.

The position and velocity loops together comprise the control system. The position loop corresponds to “proportional” controller, velocity loop which consists of “proportional-integral” controller and “loop shaping filters”. Further, influence on dynamic error of certain configuration and different velocity excitation will be investigated.

The mathematical modelling of LMS is based on the following assumptions:

(1) Constant friction forces appear in the moving parts of LMS; therefore, friction might be considered as minor component in mathematical model and identified frequency response function;

(2) All parts of LMS are absolutely rigid. That is, only the rigid body dynamics is considered in the present investigation;
There is no backlash in the connections of the moving LMS parts;
(4) The motion of the all moving LMS parts is straight-linear;
(5) The displacements of the all moving LMS parts are identical;
(6) The response of current amplifier can be considered as linear after tuning of PI controller in current control loop. Response of current amplifier to certain current excitation have rise time and overshoot which might be neglected. Dead time in PWM amplifier is ignored;
(7) The moving mass is constant and amplifier bus voltage is constant;
(8) Environment conditions are constant and well controlled;
(9) Ironless BLDC motor have three symmetric phases. All phases of the motor are perfectly symmetrical. The magnetic field between stator and rotor is symmetrical; therefore, cogging effect can be neglected. Effect of armature is ignored. Armature windings are continuously distributed over the inner surface of the windings.
(10) The airgap between permanent magnets of BLDC motor and motor coils is constant. Effect of slots can be ignored. Magnetic circuit cannot be saturated regardless of Eddy current and hysteresis losses.
(11) As motor commutation is done in accordance to the linear encoder feedback, harmonic effect of commutation can be neglected;

Figure 3. The block diagram of the linear motor stage with quasi-industrial guiding system (LMS).

3. Modelling and Identification
3.1. Governing Equations

In the present section, the transfer functions of the open loop system consisting of the plant and the controller are derived analytically. To assess the accuracy of the analytically obtained transfer functions the frequency response functions obtained from the transfer functions were compared with the experimentally identified frequency response functions of the open loop systems of LMS.

The system of the governing differential equations of the LMS consists of three equations that relate the displacement of the LMS platform, the electromechanical parameters of the LMS devices, the current and voltage:

\[ m_{tot} \frac{d^2x(t)}{dt^2} = F_{\text{motor}}(t) \]  
(1)

\[ F_{\text{motor}}(t) = K_I I(t) \]  
(2)

\[ L \frac{dI(t)}{dt} + RI(t) = U - K_v \frac{dx(t)}{dt} \]  
(3)

From (1)–(3), it follows that:

\[ L \frac{dI(t)}{dt} + K_f \frac{m_{tot} \frac{d^2x(t)}{dt^2}}{K_f} = U - K_v \frac{dx(t)}{dt} \]  
(4)
where, in Equations (1)-(3), \( t \) is time (s); \( x \) is displacement of the LMS platform, see Figure 1, (m); \( \dot{x} \) (m/s) and \( \ddot{x} \) (m/s\(^2\)) are the velocity and acceleration of the LMS platform or the first and second order time derivatives of the displacement \( x \) of the LMS platform, respectively; \( m_{\text{totx}} = 23.5 \) (kg), \( m_{\text{toly}} = 25 \) (kg) is the total mass of the parts of the LMS platform that move in the straight-line motion in X and Y configuration; \( I \) is current (A); \( F_{\text{motor}} \) is the motor generated force (N); \( K_f = 12.9 \) (N/A) is motor force constant; \( L = 1.24 \) (mH) is inductance of the motor winding; \( R = 6.4 \) (Ohm) is resistance of the motor winding; \( U = 320.0 \) (VDC) is nominal bus voltage; \( K_v = 15.0 \) ((Vs)/m) is motor Back EMF constant. In Equations (1)–(3), current \( i \) is time dependent given function, while quantities \( m_{\text{totx}}, K_f, K_v, L, R \) and \( U \) are time independent constant parameters. The displacement \( x(t) \), the velocity \( dx(t)/dt \) \( (t) \) and acceleration \( d^2x(t)/dt^2 \) \( (t) \) of the LMS platform are unknown time functions.

As illustrated in Figure 3, the plant consists of motor, quasi-industrial guiding system, current amplifier and current controller, which is considered linear after the current loop is tuned. Current controller might be represented as transfer function given in Equation (5), which represents the PI controller:

\[
i(s) = K_p + \frac{K_i}{s} + K_d \cdot s = \frac{K_p \cdot s + K_i + K_d \cdot s^2}{s} \tag{5}\]

where, in Equation (5) \( K_p = 25 \) is proportional coefficient of PI controller; \( K_i = 21,000 \) is integral coefficient of PI controller; \( K_d = 0 \). The current PI controller is tuned empirically by applying periodic square wave into the windings of the motor. \( K_v \) is considered as maximal after reaching the level of the reference current avoiding overshoot in whole range of current operation. The \( K_i \) is considered as maximal after reaching minimal rise time of the current signal without destabilization of the actual current.

3.2. Theoretical Transfer Functions, Their Frequency Response Functions and Comparison with the Experimentally Identified Frequency Response Functions

The transfer functions obtained in this section will be written as ratios of polynomials and will be illustrated graphically and compared with the experimental results. One degree of freedom mass-spring mechanical system will not be analysed as a set of differential equations. The \textit{Laplace} transform of differential equation of mechanical system will be considered as obvious. Additionally, ration between input force and output position (transfer function) will not be expressed to not overload the paper by standard analysis. Factorized style transfer function expressions will not be presented here in order to have more compact equations, to simplify the modelling by mathematical software tools. As mechatronic system is operated by controlling the current while current is directly proportional to the force generated by motor winding, it is mandatory to mention that transfer function is ration between force and acceleration. The structure of the controller is presented in Figure 3 while PID coefficients and filter bandwidths are represented in Table 1. The transfer function of the plants, denoted by \( H_{\text{PLANT}_X} \) and \( H_{\text{PLANT}_Y} \) that includes the motor current amplifier, see Equation (4), and the current controller see Equation (5), in the frequency domain, can be represented as the following polynomial ratio:

\[
H_{\text{PLANT}_X}(s) = \frac{7.884 \cdot 10^7 \cdot s + 6.622 \cdot 10^{10}}{6.126 \cdot 10^{-1} \cdot s^4 + 1.976 \cdot 10^4 \cdot s^3 + 1.037 \cdot 10^7 \cdot s^2} \tag{6}
\]

\[
H_{\text{PLANT}_Y}(s) = \frac{2.586 \cdot 10^7 \cdot s + 2.722 \cdot 10^8}{5.580 \cdot 10^{-1} \cdot s^4 + 1.800 \cdot 10^4 \cdot s^3 + 9.450 \cdot 10^6 \cdot s^2} \tag{7}
\]

Before starting analysis of the dynamic errors of LMS, the experimental identification of PLANT was conducted by defining the empirical FRFs \( \hat{L}_{\text{PLANT}_X} \) and \( \hat{L}_{\text{PLANT}_Y} \). Additionally, these functions \( \hat{L}_{\text{PLANT}_X} \) and \( \hat{L}_{\text{PLANT}_Y} \) were compared with the FRF of the theoretical transfer functions \( H_{\text{PLANT}_X} \) and \( H_{\text{PLANT}_Y} \) denoted by \( L_{\text{PLANT}_X}, L_{\text{PLANT}_Y} \). These frequency response functions are depicted in Figure 4.
From Figure 4, we can observe that the experimentally determined behaviour of the real PLANT has a set of the eigenfrequencies that can have an influence on the whole stability, dynamic displacement error, settling time and settling window of LMS. Usually, single degree of freedom systems are considered as modular units for multi axis configuration; therefore, it is important to investigate both configuration while other element in system is stationary. As it was mentioned above, LMS has no transmission and the only friction source is quasi-industrial linear guiding system. Therefore, eigenfrequencies can be caused by the structure of LMS, its spatial configuration, friction, cable management, etc. The more detailed mechanical analysis and design revisions can be considered to damp the parasitic resonances of the system. However, in the present investigation, the influence of the spatial configuration to PLANT modal and dynamic response will be investigated without applying any mechanical or control revisions; therefore, brief recommendations for control theory scientists and engineers will be presented.

![Figure 4](image-url)

**Figure 4.** Frequency response functions of the plant: red are experimentally identified FRF $L_{PLANT,X}$, $L_{PLANT,Y}$ and green is FRF of the theoretical transfer functions $H_{PLANT,X}$, $H_{PLANT,Y}$ given in Equations (6) and (7).

**Table 1.** Summarized parameters of open loop systems $H_{OLSYS,X}$, $H_{OLSYS,Y}$.

| Parameter                        | $H_{OLSYS,X}$ | $H_{OLSYS,Y}$ |
|----------------------------------|---------------|---------------|
| Configuration                    | X             | Y             |
| Bandwidth, Hz                    | 12.5          | 10.5          |
| Gain Margin, dB                  | 14.5          | 13.5          |
| Phase Margin, °                  | 40.5          | 30.3          |
| LPF cut off frequency, Hz        | 300           | 300           |
| NF central frequency, Hz         | 280           | 165           |
| Position loop P coefficient      | 50            | 40            |
| Velocity loop P coefficient      | 400           | 250           |
| Velocity loop I coefficient      | 225           | 200           |
Further, high order PID controllers was added for both X (6th order) and Y (8th order) cases of PLANT was added in order to stabilize the PLANT and control its dynamic response see Equations (8) and (9):

\[ H_{PID,X} = \left( \sum_{i=1}^{5} a_i s^i \right) \left( \sum_{i=2}^{6} b_i s^i \right)^{-1} \]  

(8)

where coefficients \( a_i \) and \( b_i \) are following: \( a_1 = 3.482 \cdot 10^{10}, a_2 = 6.260 \cdot 10^{10}, a_3 = 1.245 \cdot 10^9, a_4 = 1.529 \cdot 10^5, a_5 = 4.000 \cdot 10^2, b_2 = 6.810 \cdot 10^{10}, b_3 = 7.431 \cdot 10^7, b_4 = 5.859 \cdot 10^4, b_5 = 2.304 \cdot 10^1, b_6 = 6.192 \cdot 10^{-3}. \)

\[ H_{PID,Y} = \left( \sum_{i=1}^{7} a_i s^i \right) \left( \sum_{i=2}^{8} b_i s^i \right)^{-1} \]  

(9)

where coefficients \( a_i \) and \( b_i \) are following: \( a_1 = 8.085 \cdot 10^9, a_2 = 1.032 \cdot 10^{10}, a_3 = 2.631 \cdot 10^8, a_4 = 2.750 \cdot 10^5, a_5 = 5.108 \cdot 10^2, a_6 = 2.441 \cdot 10^1, a_7 = 1.954 \cdot 10^{-4}, b_2 = 1.1977 \cdot 10^{10}, b_3 = 6.944 \cdot 10^7, b_4 = 1.163 \cdot 10^3, b_5 = 1.177 \cdot 10^5, b_6 = 7.240 \cdot 10^{-2}, b_7 = 2.553 \cdot 10^{-5}, b_8 = 4.303 \cdot 10^{-9}. \)

In both cases, a relatively similar control strategy was chosen see Figure 5. The main difference between regulation strategies is central frequency of second order NF: 280 Hz for X configuration and 165 Hz for Y configuration. It should be stated that X configuration has higher parasitic frequency than Y configuration LMS system. From Figure 4, it is obvious that X configuration of the stage has less aggressive eigenfrequencies in medium frequency range (50–500 Hz); therefore, higher bandwidth and higher dynamic range can be reached by tuning the proportion and integral regulator coefficients.

Figure 5. Frequency response functions of the PID controller: red are experimentally identified FRF \( L_{PID,X}, L_{PID,Y} \) and green is FRF of the theoretical transfer functions \( H_{PID,X}, H_{PID,Y} \) given in Equations (8) and (9).
Finally, to generalize, the previously discussed PLANT functions and controllers can be integrated, while transfer functions of open loop systems can be represented by Equations (10) and (11):

\[ H_{\text{OLSYS,X}} = \left( \sum_{i=1}^{6} a_i s^i \right) \left( \sum_{j=2}^{6} b_j s^j \right)^{-1} \]  
\[ H_{\text{OLSYS,Y}} = \left( \sum_{i=1}^{7} a_i s^i \right) \left( \sum_{j=4}^{10} b_j s^j \right)^{-1} \]

where coefficients \( a_i \) and \( b_j \), are: \( a_1 = 2.306 \times 10^{21}, a_2 = 4.149 \times 10^{21}, a_3 = 8.737 \times 10^{19}, \)
\( a_4 = 1.083 \times 10^{17}, a_5 = 3.854 \times 10^{13}, a_6 = 3.154 \times 10^{10}, b_2 = 6.810 \times 10^{10}, b_3 = 7.431 \times 10^7, \)
\( b_4 = 5.859 \times 10^{4}, b_5 = 2.304 \times 10^3, b_6 = 6.192 \times 10^{-3}. \)

To sum up all that was modelled and said above, a brief illustrative summary can be represented by Table 1. Table 1 represents bandwidth and dynamic capabilities of the PLANT itself, while indicating its stability margins and controller structure. 20% higher bandwidth and <10% lower load of X configuration stage corresponds to the wider capabilities of the X stage dynamics, while practically such a load and bandwidth differences can be neglected and can be considered in the tolerance.

4. Results and Discussion

In the present section, spectral and dynamic response of the LMS stages in X and Y configurations is investigated. From Section 3 before, it might be considered that the only difference between investigated LMS systems is configuration (spatial representation) of the stage and its mounting: X configuration is mounted to the granite via full surface area of housing, Y configuration system is mounted only by limited surface area flange imitating real XY configuration. LMS stages in X and Y configurations was examined by measuring: FRF via excitation of constant harmonic force signals, dynamic error of the displacement of the stage moving platform, see detail 1 in Figure 1. The FRF was identified while moving part was moving in order to decrease the influence of stick slip friction to the measurement result. The displacement was entailed by exciting LMS by different velocities (1, 5, 10, 20, 100, 200, 400) mm/s, the mass of moving platform, the acceleration and the jerk were constant, while after the accusation of the dynamic error data and its statistical and spectral analysis, modal response of the moving platform was estimated and compared with FRF data, dynamic displacement error data transformed to frequency domain by applying Fourier transform. After converting dynamic data of dynamic process to frequency domain, 100 of the most powerful resonant frequencies were chosen and were additionally limited by 30 nm magnitude, which might be considered as non-measurable, therefore it will neglected in further analysis. After accusation of FRF and modal response, equation of resonance may be designed and actual position dynamics might be replicated by summation of harmonic signals. It is considered to be minor part; therefore, most of the analysis would be based on frequency domain data analysis, investigation of the resonance source and its modal shape.

Two configurations of the LMS were analysed: X configuration and Y configuration. Both setups were assembled from exactly same components in the same workshop. In each phase of the experiment the following sequence of activities were arranged:
- An excitation of the system with the harmonic signal of the constant amplitude to identify the transfer function;
- Identification and shaping of the open loop transfer function to reach stability criteria: gain margin not less than 6 dB and phase margin not less than 30°; particular investigation gives the information related to the stability margins, frequency domain data related to the resonances applicable to the “feed” direction.

- Excitation of moving platform in each configuration by different velocities (1, 5, 10, 20, 100, 200, 400) mm/s to collect the data of dynamic displacement error with sampling time equal to 50 µs.

- Statistical analysis of the dynamic response data in order to identify the tendency of: extremums, mean and standard deviation values \( \Delta e \), \( s \Delta e \), \( \min \{ \Delta e \} \), \( \max \{ \Delta e \} \), while LMS are excited with mentioned velocity set.

- Spectral analysis of the dynamic response by applying Fourier transform of dynamic response data in order to identify most significant modes and its influence to dynamic response;

- Simultaneously, during excitation by velocity set, modal response was collected by accelerometers to identify resonances and modal response in “feed” and “pitch” direction;

- Spectral response of the three different methods was compared.

First of all, FRF response was analysed in order to identify most significant resonance information limited by bandwidth of 1000 Hz. From Figures 6 and 7 it can be identified that total number of “feed” direction resonances in X configuration is 9, comparing to the Y configuration system, which is 12. It can be also stated that in X configuration, most significant resonances are most likely located at the higher bandwidth 108–923 Hz, while in Y configuration, it is clearly obvious that there exists a narrow sequence of low frequency resonances in the 11.7–62.1 Hz range. Low bandwidth resonances are clearly caused by the Y configuration and its mounting technique. It is more complicated to damp low frequency resonances and those resonances are directly involved in the dynamics of the moving platform.

**Figure 6.** Frequency response functions of the open loop systems: red are experimentally identified FRF \( \hat{L}_{OLSYS,X} \) and \( \hat{L}_{OLSYS,Y} \) and green is FRF of the theoretical transfer functions \( H_{OLSYS,X} \) and \( H_{OLSYS,Y} \) given in Equations (10) and (11).
Next, the direct measurement of the dynamic error of the displacement of the LMS platform has been performed. The error of the displacement, denoted by \( \Delta_e \), hereafter in the article is expressed as a difference \( \Delta_e = \Delta_f - \Delta_{ref} \), where \( \Delta_f \) is feedback displacement and \( \Delta_{ref} \) is the reference displacement set by the motion controller. The dynamic displacement errors \( \Delta_e \) of the systems in configurations X and Y and at the different velocities \( \dot{x} \in \{1,5,10,20,100,200,400\} \text{ mm/s} \), with the constant acceleration \( \dot{x} = 5000 \) while changing velocity of the moving platform are shown in Figures 8 and 9.

As we can see from Figures 8 and 9, the dependency of the displacement dynamic error \( \Delta_e \) on the time is harmonic; therefore, dynamic response can be easily replicated by applying superposition of harmonic functions. The displacement error \( \Delta_e \) is smaller and seems to be comparable in X and Y configurations at the low excitation velocities, i.e., when \( \dot{x} \in \{1,5,10,20,100\} \text{ mm/s} \) in comparison to the dynamic error \( \Delta_e \) at the higher excitation velocities, when \( \dot{x} \in \{200,400\} \text{ mm/s} \). The influence of LMS configurations and mounting technique on the dynamic errors \( \Delta_e \) is bigger at the high velocity and not so visible at low velocity range.

In Table 2, there are summary estimations of the dynamic displacement error \( \Delta_e \) of the LMS platform in different configurations X and Y at different velocities \( \dot{x} \in \{1,5,10,20,100,200,400\} \text{ mm/s} \) and at the constant acceleration \( \dot{x} = 5000 \). In this table, the estimations of mean, standard deviation, minimum and maximum are denoted by \( m_{\Delta_e}, s_{\Delta_e}, \text{min}, \text{max} \), respectively, while Figure 10 represents dependence of the standard deviation and mean values of dynamic displacement error \( \Delta_e \) on earlier defined set of velocities.
displacement errors $\Delta e$ of the systems in configurations X and Y and at the different velocities $\dot{x} \in \{1, 5, 10, 20, 100, 200, 400\}$ mm/s, with the constant acceleration $\ddot{x} = 5000$ mm/s.

Figure 8. Dynamic displacement errors $\Delta e$ of the LMS platform in X configuration at different velocities $\dot{x} \in \{1, 5, 10, 20, 100, 200, 400\}$ mm/s and at the constant acceleration $\ddot{x} = 5000$ mm/s.

Figure 9. Dynamic displacement errors $\Delta e$ of the LMS platform in Y configuration at different velocities $\dot{x} \in \{1, 5, 10, 20, 100, 200, 400\}$ mm/s and at the constant acceleration $\ddot{x} = 5000$ mm/s.
seems to be comparable in X and Y configurations at the low excitation velocities \( x\dot{} \in \{1, 5, 10, 20, 100, 200, 400\} \text{ mm/s} \) and at the constant acceleration \( x\ddot{} = 5000 \text{ mm/s} \).

From Table 3, we can observe the following minimums of the absolute value of the estimated means, \( \min\{|m_{\Delta x}| : i \in \{X, Y\}\} \), and the estimated standard deviation, \( \min\{|s_{\Delta x}| : i \in \{X, Y\}\} \), of the dynamic error \( \Delta x \) depending on the velocity \( \dot{x} \in \{1, 5, 10, 20, 100, 200, 400\} \text{ mm/s} \), at the velocity profile acceleration \( \dot{x} = 5000 \text{ m/s}^2 \):

- When the velocity \( \dot{x} = \{1, 5, 10, 20, 100\} \text{ m/s} \), LMS in X and Y configurations statistical values of dynamic response are comparable, while when velocity was increased to \( \dot{x} = \{100, 200, 400\} \text{ m/s} \), dynamic response in X and Y configuration become less predictable and some phenomena in dynamic response might be observed.
When the velocity \( x = 100 \) m/s, standard deviation \( s_{\Delta e} \) of the LMS in X configuration seem to be converging to near zero value 3.1000 \( \mu \)m, while standard deviation of the LMS in Y configuration seems to be diverging from zero with systematic increase at 100 mm/s velocity and does not significantly increases by adding additional power to the moving platform; while standard deviation in Y configuration system is more functionally predictable and is constantly increasing. Magnitude of error in Y configuration caused by excitation, diverge from stable near zero value and is >300% higher comparing to the same LMS in X configuration at maximal 400 mm/s velocity.

When the velocity \( x = 200 \) m/s, mean value \( m_{\Delta e} \) of the LMS in X configuration is slowly converging to a positive 0.1244 \( \mu \)m value, while LMS in Y configuration is rapidly converging to a negative \(-0.3412 \) \( \mu \)m; the mean value is, significantly, >270% higher in Y configuration comparing to the same system in X configuration.

| Configuration | X | Y |
|---------------|---|---|
| \( \text{Estimations of the Dynamic Displacement Error, } \Delta e \text{ in } \mu \text{m} \) | \( \text{Velocity, mm/s} \) | \( \min(\Delta e) \) | \( \max(\Delta e) \) | \( s_{\Delta e} \) | \( m_{\Delta e} \) | \( \min(\Delta e) \) | \( \max(\Delta e) \) | \( s_{\Delta e} \) | \( m_{\Delta e} \) |
| 1 | 0.645 | 0.2000 | 0.0048 | 1.300 | 1.300 | 0.4000 | 0.0100 |
| 5 | 1.400 | 0.4000 | 0.0132 | 3.100 | 2.500 | 0.9000 | 0.0170 |
| 10 | 1.800 | 0.5000 | 0.0143 | 3.100 | 2.600 | 0.9000 | 0.0394 |
| 20 | 1.800 | 0.6000 | 0.0175 | 2.900 | 3.200 | 0.9000 | 0.0557 |
| 100 | 15.200 | 5.5000 | 0.1227 | 42.600 | 42.100 | 5.5000 | 0.1227 |
| 200 | 18.100 | 2.6000 | 0.0028 | 34.900 | 35.800 | 8.5000 | 0.0202 |
| 400 | 17.000 | 3.1000 | 0.1244 | 44.700 | 41.000 | 9.9000 | 0.3412 |

In general, X configuration, which is fixed by using the whole housing surface, settles faster and dynamic error values increase is slower, while standard deviation of the dynamic error is not linear and saturates when velocity profile is systematically increased. In comparison with Y configuration behavior, system which has fixed constraints only through the flange of the limited area, dynamic error behavior increases faster, but is more predictable and regular. In connecting with these conclusions, further dynamic behavior in frequency domain will be investigated to understand the influence of X and Y configuration to dynamic error from frequency domain point of view.

Fourier transform of dynamic response data was made. After that, 100 of the harmonics with the highest amplitude was estimated, to analyse only part of the spectrum and initialize clusters of frequencies which might have highest influence to the dynamic response of the stages. In Figure 11, spectral representation of the dynamic response is presented. From the presented graphical information, it is only clear that most of the parasitic modal frequencies are located at the 10–250 Hz bandwidth in both configurations; the magnitude tendency of Y configuration stage is higher for <400% in peak response and nominally higher for >50%.

Further, more detailed spectral information was presented in Figures 12 and 13. In order to understand the frequency of each mode and its amplitude spectral response was separated for each investigated velocity velocities \( x \in \{1,5,10,20,100,200,400\} \).
presented. From the presented graphical information, it is only clear that most of the parasitic modal frequencies are located at the 10–250 Hz bandwidth in both configurations; the magnitude tendency of Y configuration stage is higher for <400% in peak response and nominally higher for >50%.

**Figure 11.** Spectral response of dynamic displacement errors $\Delta_e$ of the LMS platform in X and Y configuration at different velocities $x \in \{1, 5, 10, 20, 100, 200, 400\}$ mm/s and at the constant acceleration $x = 5000$ mm/s.

![Spectral response of dynamic displacement errors](image1)

**Figure 12.** Spectral response of dynamic displacement errors $\Delta_e$ of the LMS platform in X configuration at different velocities $x \in \{1, 5, 10, 20, 100, 200, 400\}$ mm/s and at the constant acceleration $x = 5000$ mm/s.

![Spectral response of dynamic displacement errors](image2)
Figure 13. Spectral response of dynamic displacement errors $\Delta e$ of the LMS platform in Y configuration at different velocities $\dot{x} \in \{1,5,10,20,100,200,400\}$ mm/s and at the constant acceleration $x = 5000$ mm/s.

It is important to state that vibrations of magnitude less than 6 counts of encoder (single count encoder resolution is <5 nm) will be neglected in further analysis; therefore, bandwidth of >30 nm magnitude vibration is additional introduced in Table 3. Limited amplitude bandwidth will further help to sort the vibrations modes accrued by accelerometers.

From the investigation of dynamic response earlier, it is clear that, generally speaking, in X configuration, the level of vibration magnitude is significantly lower comparing to the Y configuration. It must be emphasized that spectral response in X configuration at velocity $\dot{x} \in \{1,5,10,20\}$ mm/s is clearly influenced by higher frequency modes with bandwidth of up to 912 Hz, which is relatively easy to damp; on the other hand, at higher excitation velocity $\dot{x} \in \{100,200,400\}$ mm/s, significant influence is exerted by low frequency vibrations, which is obviously limited by bandwidth up to the 0–153 Hz, which is more complicated to damp; therefore, stability margins and overall bandwidth must be sacrificed. It must be added that if bandwidth cut at >30 nm magnitude of vibrations, X configuration spectral response bandwidth would be significantly narrowed and can be described by 0–39 Hz frequency range at all velocities. It is represented in Table 4.

The situation in Y configuration is different. Spectral response at velocities $\dot{x} \in \{1,5,10,20,100\}$ mm/s is limited by 0–1000 Hz bandwidth, while at velocities $\dot{x} \in \{200,400\}$ mm/s bandwidth is limited by 0–154 Hz. If bandwidth cut at >30 nm magnitude of vibrations, Y configuration spectral response bandwidth as well as X configuration bandwidth will be significantly narrowed and will remain: in 0–39 Hz in X configuration and 0–153.7 Hz frequency at all velocities in Y configuration.

In conclusion, it is obvious that dynamics in X configuration system are significantly influenced by lower bandwidth frequency excitation, which is most likely coming from the granite base and vibration isolation system (mechanical dampers), while Y configuration spectral response bandwidth is wider, so harmonics which are coming from above 39 Hz can...
be considered as structural resonances caused by mounting, geometry, stage configuration, preload, and friction; therefore, further, it is important to analyse the range 39–154 Hz.

Table 4. Summarized significant bandwidths of dynamic response versus velocity excitation in X and Y configurations.

| Configuration | Mode Frequency by FFT | X | Y |
|---------------|----------------------|---|---|
|               |                      | Bandwidth, Hz | Bandwidth (magnitude >30 nm), Hz | Max. Magnitude, µm | Bandwidth, Hz | Bandwidth (magnitude >30 nm), Hz | Max. Magnitude, µm |
| 1             | 0–800                | 0–14.3        | 0.0751                             | 0–800               | 0–50.0         | 0.1511                             |
| 5             | 0–912                | 0–17.9        | 0.1466                             | 0–912               | 0–20.0         | 0.2977                             |
| 10            | 0–912                | 0–20.6        | 0.2637                             | 0–912               | 0–21.4         | 0.4514                             |
| 20            | 0–912                | 0–23.7        | 0.2759                             | 0–1000              | 0–16.2         | 0.5420                             |
| 100           | 0–39                 | 0–38.7        | 0.8002                             | 0–1000              | 0–26.7         | 1.8574                             |
| 200           | 0–39                 | 0–39.0        | 1.3036                             | 0–154               | 0–153.7        | 5.8958                             |
| 400           | 0–153                | 0–38.2        | 1.2558                             | 0–154               | 0–152.4        | 7.0016                             |

Additionally, modal analysis will be further introduced to estimate the modes which would correspond to the bandwidth accrued by FRF and dynamic analysis of the plant. The modal analysis will be represented by summary in Table 5 and Figures 14 and 15.

Figure 14. Illustration of five Modal responses of X configuration stage; (a)—mode 1 or “feed” mode; (b)—mode 2 or “yaw” mode; (c)—mode 3 or “vertical” mode; (d)—mode 4 or “roll” mode; (e)—mode 5 or “pitch” mode.
Figure 15. Illustration of five Modal responses of Y configuration stage; (a)—mode 1 or “vertical” mode; (b)—mode 2 or “yaw” mode; (c)—mode 3 or “feed” mode; (d)—mode 4 or “roll” mode; (e)—mode 5 or “pitch” mode.

Table 5. Summarized modal analysis results in X and Y configuration.

| Mode | Configuration X |                      |                      | Configuration Y |                      |
|------|----------------|---------------------|---------------------|----------------|---------------------|
|      | Frequency, Hz  | Deflection Direction| Frequency, Hz       | Deflection Direction|
| 1    | 12.6           | feed                | 12.4                | vertical        |
| 2    | 18.4           | yaw                 | 15.8                | yaw             |
| 3    | 36.8           | vertical            | 36.6                | feed            |
| 4    | 89.2           | roll                | 126.4               | roll            |
| 5    | 271.6          | pitch               | 136.6               | pitch           |

In Table 5 is denoted first the five most significant modes in both X and Y configurations. To illustrate structural deformations deflection direction is added to the additional columns and illustrated by Figure 14. The feed corresponds to the deflection to the direction of the motion; the yaw corresponds to the deflection around the vertical axis perpendicular to the motion direction; the pitch corresponds to the deflection around horizontal axis perpendicular to the motion direction; the roll corresponds to the deflection around horizontal axis parallel to the motion direction; the vertical corresponds to the deflection to the direction perpendicular to motion direction in vertical plane.

It is clear that yaw mode 126.4 Hz and pitch 136.6 Hz modes in Y configuration are significant and, most likely, it is important that they be prevented and damped by controller, mechanical structure and interfaces. According to the literature, it can be stated...
that modes between 100 and 140 Hz are most likely caused by preload in guiding system, while preload is well stabilized in X configuration systems; unfortunately, because of the structural dynamics and micro vibrations, preload can vary depending on the motion profile and stage regime. Variation of preload can be a serious cause of degradation of guiding system; therefore, resonances in the range of 100–140 Hz in Y configuration must be observed and controlled during the whole life cycle of stage in order to prevent degradation of its kinematic performance.

Finally, modal response variation as a function of excitation velocity set will be presented in Figures 16 and 17 and analysed in Tables 6 and 7.

![Figure 16](image1.png)  
**Figure 16.** Modal response to velocity [1,20,100,400] mm/s excitation in X configuration.

![Figure 17](image2.png)  
**Figure 17.** Modal response to velocity (1,20,100,400) mm/s excitation in Y configuration.

It is obvious that in Y configuration there is 300% more significant harmonics than in X configuration—33 versus 11. On the other hand, it is important to take into account the bandwidth of open loop system, which is 10 Hz in Y configuration and 12.5 in X configuration. The margin frequency is the frequency which is strongly dependent on control system and its design, while frequencies above of the stability margin might be more interesting for analysis. From the summary presented in Figure 18, we could see that mounting configuration has significant influence on spectral response and, in X configuration, it is possible to estimate stability margin frequency (10 Hz), structural resonances (50 Hz) and guiding system/preload frequency (150 Hz) which have strong dependence on excitation velocity, while in Y configuration there is a number of frequencies...
which can be considered as a function of velocity, especially: 10 Hz, 90 Hz, 110 Hz, 120 Hz, 140 Hz, 150 Hz, 160 Hz, 260 Hz, 290 Hz, 310 Hz, 380 Hz, 390 Hz. Therefore, it must be stated that it is important to handle Y axes configuration with wider capabilities of bandwidth control and the “health” can be identified by observing modal response in the range of 10–150 Hz in X configuration, while in Y configuration wider bandwidth 10–390 Hz of observation must be considered. Additionally, analysing higher bandwidth in Figures 16 and 17, it can be observed that Y configuration system is less stable in frequency range of 10–150 Hz in X configuration, while in Y configuration wider bandwidth was increased to 100 Hz. 

Table 6. Summarized spectral information of the modal response at the certain velocity excitation in bandwidth (0–400) Hz in X configuration.

| Mode | Frequency, Hz | Acceleration Magnitude, mm/s² |
|------|--------------|-------------------------------|
|      |              | 1 mm/s | 5 mm/s | 10 mm/s | 20 mm/s | 100 mm/s | 200 mm/s | 400 mm/s |
| 1    | 10           | 1.9    | 1.9    | 3.5     | 10.0    | 50.0     | 150.0    | 300.0    |
| 2    | 50           | 20.0   | 10.0   | 12.0    | 20.0    | 30.0     | 20.0     | 20.0     |
| 3    | 70           | 0.2    | 0.1    | 0.1     | 0.4     | 4.0      | 8.0      |
| 4    | 100          | 2.0    | 3.0    | 2.0     | 2.0     | 3.5      | 7.0      |
| 5    | 150          | 10.0   | 10.0   | 10.0    | 10.0    | 20.0     | 50.0     | 200.0    |
| 6    | 200          | 2.5    | 4.0    | 3.5     | 3.0     | 3.5      | 8.0      | 5.0      |
| 7    | 240          | 0.5    | 0.5    | 0.4     | 1.0     | 15.0     | 20.0     | 20.0     |
| 8    | 250          | 1.5    | 4.0    | 1.5     | 1.1     | 15.0     | 20.0     | 20.0     |
| 9    | 300          | 0.4    | 0.5    | 0.4     | 0.5     | 2.0      | 3.5      | 20.0     |
| 10   | 330          | 0.4    | 0.4    | 0.4     | 3.0     | 5.0      | 5.0      |
| 11   | 340          | 4.9    | 4.9    | 4.9     | 5.0     | 7.5      | 7.5      |

Figure 18. Summarized modal response of X (a) and Y (b) configuration at the certain velocity excitation in bandwidth [0–400] Hz.
Table 7. Summarized spectral information of the modal response at the certain velocity excitation in bandwidth (0–400) Hz in Y configuration.

| Mode | Frequency, Hz | Acceleration Magnitude, mm/s² |
|------|---------------|-------------------------------|
|      | 1  | 5  | 10 | 20 | 100 | 200 | 400 |
| 1    | 10 | 2.0 | 3.5 | 3.5 | 5.0 | 50.0 | 100.0 | 200.0 |
| 2    | 40 | 5.0 | 5.0 | 5.0 | 5.0 | 5.0 | 10.0 | 10.0 |
| 3    | 50 | 20.0 | 23.0 | 25.0 | 25.0 | 10.0 | 40.0 | 20.0 |
| 4    | 60 | 1.0 | 0.8 | 0.5 | 1.5 | 8.0 | 8.0 | 10.0 |
| 5    | 70 | 0.4 | 0.4 | 0.3 | 0.2 | 7.5 | 8.0 | 16.0 |
| 6    | 80 | 0.8 | 0.8 | 0.8 | 0.8 | 5.0 | 2.0 | 20.0 |
| 7    | 90 | 0.5 | 0.8 | 1.5 | 4.0 | 17.0 | 20.0 | 40.0 |
| 8    | 100 | 3.5 | 4.0 | 4.0 | 4.0 | 10.0 | 10.0 | 10.0 |
| 9    | 110 | 0.3 | 0.4 | 0.4 | 1.0 | 6.0 | 10.0 | 15.0 |
| 10   | 120 | 0.7 | 0.6 | 0.5 | 0.5 | 4.0 | 40.0 | 40.0 |
| 11   | 130 | 0.2 | 0.15 | 0.15 | 0.5 | 4.0 | 4.0 | 10.0 |
| 12   | 140 | 0.5 | 0.4 | 0.4 | 1.0 | 15.0 | 20.0 | 75.0 |
| 13   | 150 | 7.0 | 7.0 | 7.0 | 7.0 | 10.0 | 40.0 | 75.0 |
| 14   | 160 | 0.5 | 0.2 | 0.2 | 0.4 | 8.0 | 15.0 | 15.0 |
| 15   | 180 | 0.5 | 0.5 | 0.5 | 0.5 | 2.0 | 4.0 | 4.0 |
| 16   | 190 | 0.2 | 0.3 | 0.2 | 0.2 | 4.0 | 8.0 | 8.0 |
| 17   | 200 | 0.4 | 0.5 | 0.5 | 0.5 | 2.0 | 5.0 | 5.0 |
| 18   | 210 | 0.2 | 0.2 | 0.3 | 0.2 | 2.0 | 2.0 | 5.0 |
| 19   | 220 | 0.5 | 0.5 | 0.5 | 0.5 | 2.0 | 8.0 | 8.0 |
| 20   | 230 | 0.1 | 0.2 | 0.1 | 0.2 | 2.0 | 5.0 | 5.0 |
| 21   | 240 | 0.2 | 0.2 | 0.1 | 0.2 | 2.0 | 2.0 | 2.0 |
| 22   | 250 | 0.5 | 4.0 | 0.8 | 0.8 | 4.0 | 6.0 | 8.0 |
| 23   | 260 | 1.0 | 1.5 | 1.0 | 1.0 | 10.0 | 20.0 | 40.0 |
| 24   | 280 | 0.5 | 0.5 | 0.5 | 0.5 | 3.0 | 10.0 | 8.0 |
| 25   | 290 | 0.4 | 0.2 | 0.2 | 0.4 | 5.0 | 10.0 | 18.0 |
| 26   | 300 | 0.4 | 0.8 | 0.8 | 0.8 | 8.0 | 10.0 | 30.0 |
| 27   | 310 | 0.2 | 0.2 | 0.4 | 0.4 | 4.0 | 8.0 | 15.0 |
| 28   | 320 | 0.4 | 0.4 | 0.4 | 0.4 | 2.0 | 4.0 | 5.0 |
| 29   | 340 | 0.2 | 0.5 | 0.1 | 0.1 | 2.0 | 3.0 | 5.0 |
| 30   | 350 | 5.0 | 5.0 | 5.0 | 5.0 | 5.0 | 5.0 | 5.0 |
| 31   | 360 | 0.2 | 0.2 | 0.1 | 0.1 | 2.0 | 6.0 | 8.0 |
| 32   | 380 | 0.4 | 0.4 | 0.4 | 0.5 | 8.0 | 20.0 | 40.0 |
| 33   | 390 | 0.5 | 0.5 | 0.2 | 0.5 | 8.0 | 18.0 | 30.0 |

5. Conclusions

From the analysis and investigation, the following conclusions can be presented:

- After analysis of “feed” direction resonances accrued by FRF response, it can be concluded that X configuration can be characterized by wider bandwidth 108–923 Hz, while Y configuration bandwidth is limited by 11.7–62.1 Hz range in “feed” direction.
- Further analysis of dynamic response by exciting LMS with different velocity showed that when the velocity $\dot{x} = \{1, 5, 10, 20, 100\}$ m/s, LMS in X and Y configurations statistical values of dynamic response are comparable, while when velocity excitation was increased to $\dot{x} = \{100, 200, 400\}$ m/s, dynamic response in X and Y configuration become less comparable, predictable and generally non-linear.
- It is obvious that there exists exception for velocity $\dot{x} = 100$ m/s in the investigated LMS and when the velocity $\dot{x} = 100$ m/s, standard deviation $s_{\Delta e}$ of the LMS in X configuration is converging to zero and settles at 3.1000 µm magnitude, while standard deviation of the LMS in Y configuration diverging from zero with systematic increasement in its value and become peak 9.9000 µm at maximal $\dot{x} = 400$ m/s velocity. In X configuration LMS, standard deviation $s_{\Delta e}$ saturates at 100 mm/s velocity and does not significantly increases by adding additional power to the moving platform. Standard deviation in Y configuration LMS is linear and is constantly increasing.
Magnitude of error in Y configuration caused by excitation, diverge from stable near zero value and is >300% higher comparing to the same LMS in X configuration at maximal 400 mm/s velocity as in X configuration standard error deviation settles.

- Talking about mean error value, it must be noticed that when the velocity reaches \( \dot{x} = 200 \) m/s, mean value \( m_\Delta \) of the LMS in X configuration is slowly converging to positive 0.1244 \( \mu \)m value, while LMS in Y configuration is rapidly converging to negative \(-0.3412 \) \( \mu \)m and the mean value is significantly >270% higher in Y configuration comparing to the same system in X configuration.

- X configuration, which is fixed by using whole housing surface, settles faster and dynamic error values increase is slower, but standard deviation of the dynamic error is not linear and saturates when velocity in the moving profile is systematically increased and is non-linear. In comparison with Y configuration behaviour, which has fixed constraints only through flange of the limited area, dynamic error behaviour increases faster, but is more predictable and linear.

- Spectral response in X configuration at velocity \( \dot{x} \in \{1,5,10,20\} \) mm/s is clearly influenced by higher frequency modes with bandwidth of up to 912 Hz which is technically easier to damp; on the other hand, at higher excitation velocity \( \dot{x} \in \{100,200,400\} \) mm/s significant influence to a dynamic response is done by low frequency vibrations which is obviously limited by bandwidth 0–153 Hz, which is technically more complex task to damp; therefore, stability margins and overall bandwidth must be sacrificed. It must be added that if bandwidth cut at >30 nm magnitude of vibrations, X configuration spectral response bandwidth would be significantly narrowed and can be described by 0–39 Hz frequency range at all velocities. In Y configuration spectral response at velocities \( \dot{x} \in \{1,5,10,20,100\} \) mm/s is limited by 0–1000 Hz bandwidth, while at velocities \( \dot{x} \in \{200,400\} \) mm/s bandwidth is limited by 0–154 Hz. If bandwidth cut at >30 nm magnitude of vibrations, Y configuration spectral response bandwidth as well as X configuration bandwidth will be significantly narrowed and will remain 0–153.7 Hz frequency at all velocities in Y configuration.

- Dynamics in X configuration system is significantly influenced by lower bandwidth frequency excitation which is resulted by granite base and vibration isolation system (mechanical dampers), while Y configuration spectral response bandwidth is wider, so vibrations which are caused by 39–154 Hz frequencies can be considered as structural resonances caused by mounting, geometry, stage configuration and must be investigated further.

- “Yaw” mode 126.4 Hz and “Pitch” 136.6 Hz modes in Y configuration are significant and must be prevented and damped by: controller or mechanical structure. Variation of preload can be serious cause of degradation of guiding system; therefore, resonances in the range of 100–140 Hz in Y configuration must be observed and controlled during the whole life cycle of stage in order to prevent degradation of its kinematic performance. Analysing Y configuration modal response, 300% more significant harmonics can be observed comparing it to X configuration: 33 compared to 11 in X configuration.

- X configuration can be characterized by: control loop bandwidth (10 Hz), structural resonances (50 Hz) and guiding system/preload frequency (150 Hz) which have strong dependence on excitation velocity. Y configuration is represented by: 10 Hz, 90 Hz, 110 Hz, 120 Hz, 140 Hz, 150 Hz, 160 Hz, 260 Hz, 290 Hz, 310 Hz, 380 Hz. 390 Hz frequencies which have strong dependence on excitation velocities. “Health” can be identified by observing modal response in the range of 10–150 Hz in X configuration, while in Y configuration wider bandwidth 10–390 Hz of observation must be considered.

- Higher bandwidth observations showed that Y configuration system is less stable in frequency 2000 Hz, 2900 Hz, 3000 Hz; therefore, narrowed bandwidth can be updated by individual frequency modes to observe LMS degradation.
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