Adaptive Robust Control Based on System Identification in Microgrid Considering Converter Controlled-Based Generator Modes

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ABSTRACT Microgrids with installation of converter controlled-based generations (CCGs), i.e. renewable energy source (RES) and battery energy storage system (BESS), may lead to a lack of system inertia. Moreover, CCG modes dominated by the CCGs may result in new control issues. Accordingly, optimal-fixed controllers designed at an operating point may not be sufficient to deal with the nonlinearity of such microgrids. This paper presents an adaptive robust control strategy of a future microgrid considering the CCG modes. Without requiring any microgrid parameters, the proposed control using a subspace-based state-space identification is used to 1) monitor microgrid changes along with moving window, 2) identify microgrid model, 3) assess stability indices, and 4) robustly design controllers of RES and BESS. Characteristic and sensitivity of the new CCG modes are analyzed. Effectiveness of the proposed control method is verified in a microgrid with 100% CCGs under various RES outputs and load patterns.

INDEX TERMS Adaptive robust control, converter controlled-based generation, low-inertia microgrid, system identification.

I. INTRODUCTION

As referred to environmental protection rules, the past few decades have witnessed a drastic growth of converter controlled-based generation (CCG), i.e. renewable energy source (RES) and battery energy storage system (BESS), especially in microgrids [1], [2]. The microgrids are able to operate in two modes, i.e. standalone or grid-connected modes [3]. In the grid-connected mode, frequency and voltage are supported by main or ideal grids, thus the problem of voltage and frequency fluctuations may not be a serious problem. However, in the standalone mode, the microgrids mainly consist of uncertain CCGs. Accordingly, the frequency and voltage fluctuations at a point of common coupling may degrade the microgrid stability. The CCGs are normally used to convert DC power from RESs and BESSs to AC loads. By means of power electronic converters, furthermore, the CCGs can regulate the voltage and maintain the microgrid stability [4]. With high penetrations of the CCGs, as they are based on power electronics, the physical inertia of the CCGs is electrically decoupled from the microgrid networks resulting in the low-inertia or inertia-less system [5], [6]. Moreover, the control capability of CCGs may be deteriorated when it is encountered with the variability, unpredictability, and climate conditions of CCGs [7]. The general CCG controls are proposed to be embedded into the generator-tied and microgrid-tied voltage source converters while the DC bus is supported by a generator, i.e. RES or BESS. The swing equation can be applied for CCGs to determine the voltage phase angle with respect to the microgrid voltage support. As a result, the phase angle of voltage can be used to determine the voltage amplitude. Thus, to regulate the microgrid stability, the active and reactive powers of CCGs can be directly supervised by the generator-tied and microgrid-tied voltage source converters. Consequently, this scenario introduces challenging problems of future microgrid stability [7].

A problem of voltage and frequency restorations in an islanded microgrid system is addressed in [8]. A distributed secondary control scheme using decentralized finite-time approach is proposed to resolve the problem of voltage and frequency fluctuations under load variations. However,
the penetration of RES is not considered. In [9], 100% CCGs using virtual synchronous generator (VSG) control are implemented in the Irish system. Two schemes, i.e. outer and inner VSGs, are considered in wind turbine generation (WTG). The results show that the frequency stability can be improved due to the fast response of the WTG. In [10], a dynamic matrix control (DMC) algorithm is applied to CCG output control in order to improve overshoot, fluctuation, and steady-state errors in the voltage and frequency responses. Nevertheless, system parameters and laborious mathematical formulations are also required to design the DMC. A frequency regulation in droop-free control for a microgrid considering electrical and communication failures is presented in [11]. It is demonstrated that the electrical and communication failures degrade the ability for confining the microgrid operation into a safe region. More detail for AC microgrid control can be found in [12], [13] and references therein. In [14], a BESS with power electronic converter is used as a VSG. A model predictive control (MPC) is applied to control BESS active and reactive outputs. The simulation and hardware-in-the-loop results show that the MPC-based BESS can support inertia during transient state and enhance the microgrid stability. Accordingly, voltage and frequency fluctuations can be suppressed. However, 1) a coordinated control with other RESs is not investigated in [11], [14], and 2) system parameters, state, input, and output matrices, are required to design the MPC proposed in [14]. In [15], the MPC and robustness optimization-based energy management system are proposed in a microgrid with RES. A supplementary Constrained Information Gap Decision Theory approach is utilized to optimize the robustness of microgrid against uncertain outputs of wind generations. In [16], an adaptive optimal MPC (AO-MPC) is applied to control a microgrid regarding weather changes, time-varying parameters, and generation unit collapse. In the simulation results, the AO-MPC provides a better control performance compared with other controllers, i.e. proportional-integral (PI), proportional–integral–derivative (PID), and optimal Fuzzy PI and PID.

Although the MPC is shown to be a better option for the microgrid control, challenges and limitations of the MPC may degrade its control performance and limit its scalability. More detail of the MPC in microgrids can be found in [17] and references therein. Besides, they require exact system parameters to design the mentioned controllers. This may not be practical in actual microgrids with uncertain CCGs, in which the microgrid parameters always change. Also, the new dominant modes introduced by CCGs are not well studied. Besides, a few works focus on the microgrids with 100% CCGs. To deal with these problems, the main contributions of this paper are highlighted as follows: 1) Analyses of the CCG modes and their impacts on the microgrid stability; 2) Applying a subspace-based state-space identification method (so-called 4SID) to design controllers of RESs and BESS without requiring any microgrid parameters; 3) Comprehensive validations in a 100% CCGs microgrid under various loading conditions, uncertain CCG outputs, and a disconnection of CCG.

The rests of this paper are proceeded as follows: Section II describes microgrid modeling. The adaptive robust control design using subspace-based state-space identification is proposed in Section III. Performances of the proposed controller are validated in Section IV. Finally, the conclusions are made in Section V.

II. MODELING OF MICROGRID

A test microgrid system consisting of CCGs is depicted in Fig. 1. This microgrid represents 100% CCG with very low inertia. The CCGs used in this study are represented by BESS, doubly-fed induction generator (DFIG), permanent magnet synchronous generator (PMSG), and solar photovoltaic (SPV). Although microgrids with 100% CCGs have never been implemented in practice, they are expected scenarios for the near future [9], [18]. This microgrid suffers from the lack of inertia problem since the SPV is inertia-less and the inertia of DFIG and PMSG is totally decoupled from the microgrid. The virtual inertia controls as demonstrated in Fig. 2 may not be enough to regulate the microgrid stability due to highly uncertain outputs of the RESs. Accordingly, additional controllers considering crucial characteristics of the microgrid should be added to improve the microgrid stability. The nonlinear equations of system in Fig. 1 are represented by,

\begin{align}
\dot{x}(t) &= g_x(x(t), \eta, u(t - \tau)), \\
y(t) &= g_y(x(t)), \quad \exists y(t) \in x(t), \\
x &= [x_{BE}, x_{DF}, x_{PM}, x_{PV}]^T, \\
u &= [u_{BE}, u_{DF}, u_{PM}, u_{PV}]^T, \\
\eta &= [\eta_{BE}, \eta_{DF}, \eta_{PM}, \eta_{PV}]^T, \\
y &= [y_{OB}, y_{CO}]^T,
\end{align}

where \(g_x\) and \(g_y\) represent the nonlinear functions of state variables and outputs, \(t\) is the moving time, superscript \(T\) is the transpose of matrix, \(x, u,\) and \(y\) are respectively the state, input, and output vectors, \(\eta \in [0, 1]\) is the event-triggered logic for input \(u, \tau \in [0, 25]\) ms is the small delay caused by local measurements, subscripts \(BE, DF, PM,\) and \(PV\) mean respectively the matrices of BESS, DFIG, PMSG, and SPV, subscript \(OB\) denotes the measured signals used for the identification of microgrid, subscript \(CO\) denotes the measured signals used as the feedback of controller.

The BESS is interfaced to the microgrid via voltage source inverter (VSI). In this paper, active and reactive powers supported by BESS are controlled by VSI. A 5th order model is considered for the BESS [19]. Accordingly, control vectors in the VSI are regarded as the state variables of BESS as given by:

\[x_{BE} = [v_{BE}, i_{BE}, v_{BE}, i_{BE}, v_{BE}, i_{BE}]^T,\]

where \(v\) and \(i\) are the voltage and current, superscripts \(d\) and \(q\) represent the direct and quadrature reference axes, and superscript \(dc\) means the DC link.

The DFIG is modelled by a 7th order model consisting of dynamics from: 1) rotor angular speed of DFIG, 2) \(d - q\)
The DFIG and PMSG are connected to the microgrid via voltage source converters. By means of the converter, the DFIG and PMSG are able to independently control the active and reactive powers injected into the microgrid. The state variables of DFIG and PMSG are given by $\mathbf{x}_{DF} = [\omega_{DF}, v_{DF}^{d}, i_{DF}^{d}, i_{DF}^{q}, a_{gr}^{d}, a_{gr}^{q}]$. 

FIGURE 1. Test microgrid system.

FIGURE 2. Control strategies of DFIG, PMSG, SPV, and BESS.
\[ x_{PM} = \begin{bmatrix} \omega_{PM} & \phi_{PM} & d_{ge} & q_{ge} & d_{gr} & q_{gr} & a_{gr} & \theta_{PM} \end{bmatrix}, \]

where \( \omega \) is the rotor angular speed, \( \theta \) is the pitch angle, superscripts \( ro \) and \( gr \) denote rotor- and grid-side converters, and superscript \( ge \) means the generator-side converter.

The dynamic behavior of SPV is dominated by the converters and associated controls, i.e. active (or power factor), and reactive power (or voltage) controls. The SPV is modelled by a 7th-order model including dynamics from: 1) SPV current, 2) inductor current of DC-DC converter, 3) DC link voltage, and 4) \( d - q \) axis currents and voltages of DC-AC converter [22]. The state variables of SPV are given by:

\[ \begin{align*}
\dot{x}_{PV} &= \begin{bmatrix} i_{PV} \; \dot{i}_{PV} \; v_{PV} \; \dot{v}_{PV} \; \phi_{PV} \; \dot{\phi}_{PV} \; \theta_{PV} \; \dot{\theta}_{PV} \end{bmatrix},
\end{align*} \]

where superscript \( l \) denotes the inductor.

It should be noted that the differential equations of these state variables are provided in Appendix. In this paper, new CCG modes. These CCGs may cause new issues regarding uncertain power flows, and iii) CCG controls interacting with such new CCG modes.

1) The CCGs may create new CCG modes completely associated with the CCG state variables. Typical control methods may be ineffective since these CCG modes may not be sensitive to conventional controllers. The CCG modes occurred in low-inertia microgrid with 100% CCGs may be more detrimental to the system with higher inertia.

2) Drastic changes in the CCG uncertain outputs may make the microgrid more unpredictable and harder to monitor and control;

3) Three major mechanisms by which they can affect the new CCG modes are i) displacing conventional machines that have well-tuned controllers, ii) impacts of uncertain power flows, and iii) CCG controls interacting with such new CCG modes.

To analyze the CCG modes, the linearization of microgrid in a small time range (\( \Delta t \)) is represented by the state equation as follows:

\[ \dot{x}(\Delta t) = Ax(\Delta t) + B\eta(u(\Delta t - \tau)), \quad (7a) \]

\[ \Delta x = A\Delta x + \sum_{n_{cg}} \left( \eta_{n_{cg}} \cdot B_{n_{cg}} \cdot u^s \right), \quad (7b) \]

\[ y(\Delta t) = Cx(\Delta t), \quad (8a) \]

\[ [\Delta y_{OB} \\Delta y_{CO}]^T = [C_{OB} \; C_{CO}]^T \Delta x, \quad (8b) \]

where \( \Delta \) means the small variation, \( u^s \) is the input vector including a small delay, \( A, B, C \) are respectively the state, input, and output matrices, \( n_{cg} = 1, \ldots, N_{cg} \) is the counter of corresponding converter controller-based generation, and \( N_{cg} \) is the total number of converter controlled-based generations.

To avoid the degradation of active power supported into the microgrid, the additional signal \( \Delta u^s \) is injected into the reactive power control loops of DFIG, PMSG, SPV, and BESS. Moreover, the inertia emulation control techniques are applied in active power control loops of CCGs in order to support an inertial response to the microgrid. However, the inertia emulation control techniques are not focused in this study since the additional signal \( \Delta u^s \) is not injected into this loop. Numerous virtual inertia control strategies, which are applied for CCGs can be found in [2]. Fig. 2 demonstrates the control strategies of DFIG, PMSG, SPV, and BESS.

Rewriting the input vector in (7b) in the form of controller \( k \), yields \( \Delta u^s = -k \Delta y_{CO} \). Next, let \( \Delta u^s = [\Delta u^s_1, \ldots, \Delta u^s_{N_{cg}}] \) substituting \( \Delta y_{CO} = C_{CO} \Delta x \) in (7c) into (7c) obtains,

\[ \Delta u^s = -k \left( C_{CO} \Delta x \right) \cdot \left( \begin{array}{c}
\Delta u^s_1 \\
\vdots \\
\Delta u^s_{N_{cg}}
\end{array} \right) = \left( -k \left( C_{CO,1} \Delta x \right) \cdot \left( \begin{array}{c}
\Delta u^s_1 \\
\vdots \\
\Delta u^s_{N_{cg}}
\end{array} \right) \right)^T. \quad (9a) \]

In this paper, the controller \( k \) is the low-order fixed structure controller. Accordingly, the controller \( k \) can be written by the transfer function,

\[ k(s) = \frac{x^k_p + x^k_{p-1}s^{-1} + \cdots + x^k_{1}s^{-p} + x^k_{0}s^{-q}}{1 + y^k_q^{-1}s^{-1} + \cdots + y^k_1s^{-q} + y^k_0s^{-q}}, \quad (10) \]

where \( s \) is the complex number, \( x^k_p, x^k_{p-1}, \ldots, x^k_0, y^k_q, y^k_{q-1}, \ldots, y^k_0 \) are the numerator parameters, and \( y^k_q, y^k_{q-1}, \ldots, y^k_0 \) are the denominator parameters. It should be noted that values of \( p \) and \( q \) can be specified by the designers so that the small order of \( k \) can be appropriately chosen.

The design of \( k \) will be given in Sections III-B and III-C. Accordingly, the closed-loop state space equation including controller \( (x_{cl}) \) is derived by substituting (9b) into (7b) as \( \Delta x_{cl} = A_{cl} \Delta x_{cl} + \Delta y_{cl} \), where \( A_{cl} = A - \sum_{n_{cg}} \left( \eta_{n_{cg}} \cdot B_{n_{cg}} \cdot k \cdot C_{CO, n_{cg}} \right) \). \( x_{cl} \in \mathbb{R}^{n_{cl} \times N_{cl}} \) is the closed-loop state matrix, and \( n_{cl} \) is the total number of closed-loop state variables. However, the matrix \( A_{cl} \) can be obtained when all of the microgrid parameters are exactly known. In uncertain microgrids, these parameters are obscured and may be varied according to the microgrid operating point and topology. Thus, it is difficult to investigate the microgrid stability by calculating \( A_{cl} \). To estimate the linearized matrix elements without requiring any microgrid parameters, an identification method is proposed in the next section.

III. ADAPTIVE ROBUST CONTROL DESIGN USING SUBSPACE-BASED STATE-SPACE IDENTIFICATION

A. SUBSPACE-BASED STATE-SPACE SYSTEM IDENTIFICATION

In this paper, the 4SID is applied to estimate the closed-loop system model without requiring any microgrid parameters [23], [24]. The 4SID requires signals from measurements or observations from the system for a black-box identification. This approach is done by formulating the identification problem as an optimization problem, in which the variables are the unknown parameters of the model. The constraints are the model equations while the objective function is a measure of the deviation between the observations and the
predictions (or simulations) obtained from the model [25]. Since the 4SID is well studied in previous research works, more details of the fitting functions related to the 4SID can be found in Appendix. Accordingly, a system including the CCG modes can be estimated by the system identification technique. The 4SID of the microgrid in Fig. 1 is written by,

\[
f_{4}(S_{l}, S_{o}) = \begin{bmatrix} \hat{A}, \hat{B}, \hat{C}, \hat{D} \end{bmatrix},
\]

Subject to: \( \sigma < \sigma^{*}, t_{c} < \epsilon \Delta t, ac > 90\% \),

\[
n_{c} \in \{ 0.5 \ \text{length} (x) \},
\]

where \( f_{4} \) represents the function of system identification using subspace-based method (for more detail, see Appendix), \( S_{l} \) is the microgrid input, which can trigger the CCG modes, \( S_{o} \) is the microgrid output, which can observe the response of the CCG modes, both \( S_{l} \) and \( S_{o} \) are obtained by measurements in microgrid, when \( \{ S_{l}, S_{o} \} \in \Delta \gamma_{OB} \), and they are used as the inputs of \( f_{4} \), and \( \hat{A}, \hat{B}, \hat{C} \) and \( \hat{D} \) are respectively the estimated state, input, output, and feed-forward matrices, length means the length of matrix, \( \sigma \) and \( \sigma^{*} \) are respectively the actual and desired singular values, \( t_{c} \) is the computational time of (11a), \( \epsilon \in (0.1, 0.5) \) is the factor, which is used to make sure that \( t_{c} \) is five to ten times lesser than \( \Delta t \), \( ac \) is the accuracy of estimated model, \( [ \cdot ] \) means the greatest integer which is less than or equal to its argument, and \( n_{c} \in \mathbb{Z} \) is the order of estimated state matrix \( \hat{A} \).

In this paper, \( ac \) is calculated by the mean of the corresponding data deviation as: \( ac = \left( \frac{1}{n_{d} \bar{n}_{d}} \sum_{n_{d}=1}^{n_{d}} \frac{S_{l} - S_{o}}{S_{l} - S_{o}} \right) \times 100 \), where subscript \( n_{d} = 1, \ldots, n_{d} \) is the data index and \( S_{l} \) and \( S_{o} \) are the data obtained from the estimated and full models, respectively.

Equation (11b) is used to guarantee 1) containing crucial features of the original model, 2) small computational time, 3) high accuracy of the estimated model, and 4) low order of the estimated model (less than half of the original model). Hence, the variable \( S_{l} \) is obtained by: \( S_{l} = \hat{\Delta} \hat{P} = \Delta P_{DF} + \Delta P_{PM} + \Delta P_{PV} + \Delta P_{BE} + \sum_{l=1}^{N_{I}} \Delta P_{L,l} \), and the variable \( S_{o} \) is obtained by: \( S_{o} = \Delta F \), where \( \Delta P_{DF}, \Delta P_{PM}, \Delta P_{PV}, \) and \( \Delta P_{BE} \) are respectively the power deviations of DFIG, PMSG, SPV, and BESS, \( \Delta P_{L,l} \) is the load variation, subscript \( l = 1, \ldots, N_{I} \) is the counter of loads, \( N_{I} \) is the total number of loads, and \( \Delta F \) is the frequency deviation. It should be noted that the signal \( S_{l} \) is the average value of the measured signals \( \Delta P_{DF}, \Delta P_{PM}, \Delta P_{PV}, \) and \( \Delta P_{L,l} \). These signals contain crucial features of the corresponding CCG modes (i.e., damping and frequency). Note that the justification of signal \( S_{l} \) will be demonstrated in Section IV-A.

The changes in such signals can trigger observable responses of the CCG modes. The characteristic of \( \Delta F \) can be represented by the form of the CCG modes as: \( \Delta F = \sum_{m=1}^{m} \Delta S_{m} e^{(2\pi f_{m} \Delta t)} e^{-\zeta_{cl,m} \Delta t} \), where \( e \) is the exponential constant, \( m = 1, \ldots, n_{d} \) is the number of corresponding CCG mode, \( S_{m} \) is the amplitude of response caused by the \( m^{th} \) CCG modes, \( f_{m} \) and \( \zeta_{cl,m} \) are the frequency and damping ratio of the \( m^{th} \) CCG mode, respectively.

During the changes in microgrid, the signal \( \Delta F \) contains characteristics of modes and it is obtained by local measurements with time stamp in the range of 40 to 100 ms. By using (11a) to estimate the microgrid model, the signal \( \Delta F \) is dominated by \( \zeta_{cl,m} \). It can be seen that the higher the value of \( \zeta_{cl,m} \), the lower the value of \( \Delta S_{m} \).

After adding the signal \( \Delta F \), the estimated closed-loop state matrix of microgrid \( \hat{A}_{cl} \) can be calculated by,

\[
\hat{A}_{cl} = \hat{A} - \hat{B} \hat{k} \hat{C}.
\]

In (12), the matrix \( \hat{A}_{cl} \) is calculated by \( \hat{A}, \hat{B}, \hat{C}, \) and \( \hat{D} \), which are estimated by (11a) with the constraints in (11b). Thus, any exact system parameters are not required to formulate the state-space model of microgrids. This is the main advantage of the proposed 4SID to conduct the steady state solution. Accordingly, the matrix \( \hat{A}_{cl} \) with high accuracy of the estimation can be used to calculate the stability indices such as eigenvalue, damping ratio, and system robustness, described as follows: Eigenvalues and damping ratios of the estimated microgrid model including the CCG modes can be calculated by: \( \det(\hat{A}_{cl} - \hat{k}_{cl} I) = 0 \), \( \hat{\lambda}_{cl} = [\hat{\lambda}_{1} \pm j2\pi \hat{f}_{1}, \ldots, \hat{\lambda}_{n_{d}} \pm j2\pi \hat{f}_{n_{d}}] \), and \( \hat{\xi}_{cl} = \frac{-\hat{\sigma}_{1}}{\sqrt{\hat{\sigma}_{1}^{2} + (2\pi \hat{f}_{1})^{2}}}, \ldots, \frac{-\hat{\sigma}_{n_{d}}}{\sqrt{\hat{\sigma}_{n_{d}}^{2} + (2\pi \hat{f}_{n_{d}})^{2}}} \), where det denotes the determinant of matrix, subscript \( cl = 1, \ldots, n_{cl} \) means the counter of the CCG modes, \( n_{cl} \) is the total number of the CCG modes, \( I \in \mathbb{R}^{n_{cl} \times n_{cl}} \) is the identity matrix, \( \hat{\lambda}_{cl} \) and \( \hat{\xi}_{cl} \) are the vectors of the estimated eigenvalue and damping ratio, \( j = \sqrt{-1} \) is the complex number, \( \hat{\sigma} \) is the estimated real part, and \( \hat{f} \) is the estimated frequency.

To observe the CCG modes, the small time range \( (\Delta t) \) in (7a) is set by,

\[
\Delta t \in \left( \max \left( \hat{f}_{1}, \ldots, \hat{f}_{n_{d}} \right) \right)^{-1} \left( \min \left( \hat{f}_{1}, \ldots, \hat{f}_{n_{d}} \right) \right)^{-1}. \tag{13}
\]

where min and max represent the minimum and maximum values.

Note that (13) is used to make sure that \( i \) the highest frequency of the CCG modes can be observed, and \( ii \) the time range \( \Delta t \) used for the estimation of the CCG modes is selected properly.

**B. CONTROLLER DESIGN**

The robustness indices calculated by the estimated matrices \( \hat{A}, \hat{B}, \hat{C}, \) and \( \hat{D} \) are described in this section. It is well known that each robust control method is mainly useful for capturing a set of design specification. For instance, the \( H_{2} \) tracking control is suitable to deal with transient performance by minimizing the linear quadratic cost of tracking error and control output, while \( H_{\infty} \) approach is more useful to maintain the closed-loop stability in the presence of model uncertainties [26]. Besides, the damping of the CCG modes is kept greater than or equal to an acceptable value so that the ability to recover microgrid responses to a steady state...
can be achieved after disturbances. Accordingly, as presented in Fig. 3, a mixed $H_2/H_\infty$ control technique [27], [28] considering damping of the CCG modes can satisfy such objectives, where $\hat{G}$ is the nominal plant, which is equivalent to $\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix}$, $w_\infty$ and $y_\infty$ are the input and output of microgrid related to $\Delta M$. $\Delta M$ is the sensitivity functions which are related to possible uncertainties in the microgrid, and $w_2$ and $y_2$ are the input and output of microgrid related to the linear quadratic cost of tracking error and control output. The proofs of $\Delta M$ are given in Appendix.

A mixed $H_2/H_\infty$ of a static output feedback control design can be achieved by minimizing the following optimization problem (J) as,

Minimize: $J = \alpha_1 \|T_{y_\infty w_\infty}\|_\infty + \alpha_2 \|T_{y_2 w_2}\|_2$, \hspace{1cm} (14a) 

Subject to: $\hat{\xi}_{cl} > \hat{\xi}_{cl}^{*}$, $T_{y_\infty w_\infty}\|_\infty < \epsilon_1$, 

$\|T_{y_2 w_2}\|_2 < \epsilon_2$, \hspace{1cm} (14b) 

when: $T_{y_\infty w_\infty}\|_\infty = \|T_{y_\infty w_\infty}\|_\infty - \|T_{y_\infty w_\infty}\|_\infty^{*}$, 

$\|T_{y_2 w_2}\|_2 = \|T_{y_2 w_2}\|_2 - \|T_{y_2 w_2}\|_2^{*}$, \hspace{1cm} (14c)

where $\| \|$ and $\| \|$ mean the $\infty$-norm and 2-norm, $\alpha_1$ and $\alpha_2$ are the weighting factors of the first and second terms of (14a), $\hat{\xi}_{cl} = \{\hat{\xi}_{cl,1}, \ldots, \hat{\xi}_{cl,n}\}$ is the vector of the desired damping ratio of $\xi_{cl}$, when $\hat{\xi}_{cl,1}, \ldots, \hat{\xi}_{cl,n}$ are respectively the specified values of $\xi_{cl,1}, \ldots, \xi_{cl,n}$, $T_{y_\infty w_\infty}$ is the transfer function between $y_\infty$ and $w_\infty$, $\|T_{y_\infty w_\infty}\|_\infty$ is the absolute difference between $\|T_{y_\infty w_\infty}\|_\infty$ and $\|T_{y_\infty w_\infty}\|_\infty^{*}$ is the desired value of $\|T_{y_\infty w_\infty}\|_\infty$, $\|T_{y_2 w_2}\|_2$ is the absolute difference between $\|T_{y_2 w_2}\|_2$ and $\|T_{y_2 w_2}\|_2^{*}$, when $\|T_{y_2 w_2}\|_2$ is the desired value of $\|T_{y_2 w_2}\|_2$ and $\epsilon_1$ and $\epsilon_2$ are the small real positive numbers set by designers.

Values of $T_{y_\infty w_\infty}$ and $T_{y_2 w_2}$ are obtained by [29]:

$T_{y_\infty w_\infty} = [\Delta M_1, \Delta M_2, \Delta M_3], \Delta M_1 = (I + \hat{G}k)^{-1}, \Delta M_2 = -(I + \hat{G}k)^{-1}\hat{G}k, \Delta M_3 = k(I + \hat{G}k)^{-1}$, and $T_{y_2 w_2} = [r - \Delta y_{CO}], \Delta y_{CO}$, where $\Delta M_1$ is the sensitivity function used to evaluate the disturbance attenuation level, $\Delta M_2$ is the sensitivity function used for noise reduction in measurements, $\Delta M_3$ is the sensitivity function used to reduce the energy effort of controller $k$, $\| \|$ means the absolute value, and $r$ is the reference signal. However, the objective function (14a) can deal with uncertainties in limited operations around the designed operating point. As a result, an adaptive robust control algorithm is required in low-inertia microgrids with CCGs.

C. PROPOSED CONTROL CONSIDERING CCG MODES

The proposed control considering the CCG modes is applied to adjust control parameters in $k$ according to the changes in microgrid operating points. Let subscript $a$ represents the $a^{th}$ operating point, the change in microgrid operating point is written by: $\bar{G}_{a+1} = \bar{G}_a \pm \Delta \bar{G}$. Accordingly, the change $\Delta \bar{G}$ leads to the changes in the microgrid stability indices as follows:

$\hat{\xi}_{cl,a+1} = \hat{\xi}_{cl,a} \pm \Delta \hat{\xi}_{cl}, \hat{\xi}_{cl,a+1} = \hat{\xi}_{cl,a} \pm \Delta \hat{\xi}_{cl}, \|T_{y_\infty w_\infty}\|_{\infty,a+1} = \|T_{y_\infty w_\infty}\|_{\infty,a} \pm \Delta \|T_{y_\infty w_\infty}\|_{\infty}$ and $\|T_{y_2 w_2}\|_{2,a+1} = \|T_{y_2 w_2}\|_{2,a} \pm \Delta \|T_{y_2 w_2}\|_{2}$.

In weak microgrids with 100% CCGs, a small change in $\Delta G$ may lead to significant changes in the microgrid stability, i.e. $\hat{\xi}_{cl}$ and $\hat{\xi}_{cl}$. If $\hat{\xi}_{cl,a+1} < \hat{\xi}_{cl,a}$ and $\hat{\xi}_{cl,a+1} < \hat{\xi}_{cl,a}$, the designed $k$ is used to move $\hat{\xi}_{cl,a+1}$ to the value which is greater than the acceptable value, i.e. $\hat{\xi}_{cl,a+1} \geq \hat{\xi}_{cl,a}$. If $\|T_{y_\infty w_\infty}\|_{\infty,a+1} < \|T_{y_\infty w_\infty}\|_{\infty,a}$ and $\|T_{y_\infty w_\infty}\|_{\infty,a+1} > \epsilon_1$, the designed $k$ is utilized to minimize the value of $\|T_{y_\infty w_\infty}\|_{\infty,a+1}$ and make sure that $\|T_{y_\infty w_\infty}\|_{\infty,a} < \epsilon_1$. If $\|T_{y_2 w_2}\|_{2,a+1} < \|T_{y_2 w_2}\|_{2,a}$ and $\|T_{y_2 w_2}\|_{2,a+1} > \epsilon_2$, in the same way, the designed $k$ is utilized to minimize the value of $\|T_{y_2 w_2}\|_{2,a+1}$ and make sure that $\|T_{y_2 w_2}\|_{2,a} < \epsilon_2$. Based on these concepts, the proposed adaptive robust control considering the CCG modes is illustrated by the flowchart in Fig. 4.
it provides the highest accuracy $ac$. Moreover, 500 different scenarios are also conducted with the same criteria when the generations of RESs and loads are randomly changed by $\pm 20\%$, for the active power, and $\pm 5\%$, for the reactive power. As a result, the values of $ac$ vary between 90.50 and 96.62%, for $SI_1 = \Delta P$, 62.72 and 85.11%, for $SI_1 = \Delta Q$, and 35.10 and 70.78%, for $SI_1 = \Delta V_1$. Accordingly, signal $\Delta P$ is the most suitable candidate for using as the input of 4SID.

The estimated model is also verified by comparison with the full model. The estimated model is obtained by (11a) while the full model is created by the state-space model of the differential equations of DFIG, PMSG, SPV, and BESS given in Appendix. Fig. 6 demonstrates eigenvalue analysis result of full and estimated models. For the full model, the participation factor is used to differentiate the association of each state variable and eigenvalue. It can be seen that the locations of eigenvalues with $5$ to $15\%$ damping ratio (dominant modes) are identical. In this microgrid, the eigenvalues with greater than $15\%$ damping ratio are well damped and they do not create any stability issues. Obviously, the proposed 4SID can capture the oscillation frequencies and
amplitudes of dominant modes. Consequently, the low-order estimated model, which is obtained by the 4SID, contains crucial features of such dominant modes. This verifies that the estimated model is sufficient to design the controller \( k \). It should be noted that, without requiring any exact microgrid parameters, the participation of eigenvalues obtained by the 4SID is justified in the next section.

**B. ANALYSIS OF CCG MODES**

Characteristics of the CCG modes (i.e., damping, frequency, and participation) are analyzed in this section. To individually analyze the CCG modes, either \( P_{DF}, P_{PM}, P_{PV}, \) or \( P_{BE} \) is varied while the others are fixed as constant values. To identify the CCG modes, the controller \( k \) is not installed in the CCGs. Fig. 7 shows the input and output pairs used for identification of CCG mode using 4SID. Accordingly, Fig. 8 demonstrates the analysis results of the CCG mode. In Figs. 7(a) and (b), the active power of DFIG is observed to conduct the CCG mode analysis. The settings of 4SID in (11a) and (11b) are decided as follows: the order of estimated model is set by \( n \) (the order of microgrid is 26), the desired singular value is set by \( \sigma^* = 10^{-5} \), and the observed time is set by \( t = 1 \) s. At the accuracy of estimated model \( ac > 90\% \), the computational time \( t_c \) is varied between 7 and 25 ms. Consequently, Figs. 8(a) and (b) show the estimated damping ratios and frequencies of the CCG modes. As can be observed, there are four dominant modes, those damping ratios are lower than 0.2 (or 20%). However, the frequencies are almost constant at approximately 1.78, 2.21, 3.32, and 5.02 Hz. To define the participation of each CCG mode, the variance of the estimated damping ratio (\( v^2 = \frac{1}{n_r} \sum_{r=1}^{n_r} (\hat{\xi}_r - \bar{\xi})^2 \)), where \( \bar{\xi} \) is the mean value of \( \hat{\xi}_r \), \( r = 1, \ldots, n_r \), and \( n_r = \frac{120}{6} \) is the total number of estimated \( \hat{\xi}_r \) data within the simulation time, is applied to track the variations of \( \hat{\xi}_r \) and \( \Delta P_{DF} \). As a result, the values of \( v^2 \) are 2.19, 0.14, 0.82, and 0.11 for the 1\(^{st}\), 2\(^{nd}\), 3\(^{rd}\), and 4\(^{th}\) CCG modes, respectively. As can be seen, the 1\(^{st}\) CCG mode (blue-dashed line) provides the greatest value of \( v^2 \). The variation of \( \Delta P_{DF} \) leads to a major change in the damping ratio of the 1\(^{st}\) CCG mode. This implies that the 1\(^{st}\) CCG mode belongs to the DFIG.

Same criteria is also applied to estimate the other CCG modes with different pairs of \( SL_i \) and \( SL_o \) (see Figs. 7 (c)-(h) and Figs. 8 (c)-(h)). It can be summarized that the 2\(^{nd}\), 3\(^{rd}\), and 4\(^{th}\) CCG modes belong to the PMSG, SPV, and BESS, respectively.

**C. TIME-DOMAIN SIMULATION**

The time-domain simulation is conducted to verify effectiveness of the proposed controller \( k \). The proposed controller is referred to as \( k_p \), where \( k_o \in k_p \). As can be seen in Fig. 6, there are four dominant CCG modes. To effectively improve these modes, the order of controller should be more than or equal to 8\(^{th}\) order. Accordingly, the 8\(^{th}\)-order controller is selected. It should be noted in (10) that, when \( p = q = 8 \), this selection represents the 8\(^{th}\)-order controller, which is used to improve the four CCG modes. Accordingly, the control parameters of \( k_p \) are varied in every \( t = 1 \) s, if the constraints in (14b) are not satisfied. The \( k_p \) is compared with a generic controller, which is referred to as \( k_c \). The \( k_c \) is designed by using objective function (14a) and (14b) with the same order as that of \( k_p \). The control parameters of \( k_c \) are optimized at the normal operating point, i.e., using \( SL_i \) and \( SL_o \) during \( t = [0 1] \) s. However, the control parameters of \( k_c \) are fixed as constant values until the end of simulation time.

Since this work considers effects of the CCG modes in which the frequencies are in the range of approximately 1.5 to 6 Hz, the observed time is set between 0.16 and 0.66 s to monitor the dynamic responses of these CCG modes. Typically, the state of charge of the BESS is normally observed from more than several minutes to hours [30]. Accordingly, the state of charge of the BESS can be neglected in this period.

**Case Study 1:** It is assumed that the loads are fluctuated between 1.5 and 4.5 MW in every 5 to 20 s. In real practice, a low-inertia microgrid such as a CIGRE microgrid benchmark [31] could experience randomly changing in loads over time. The timestamp of measurements is crucial to monitor the load variations. The lower the value of timestamp,
the more the detail of measuring load variations. The sampling rate of synchronized measurements can report the measured data between 10 and 30 samples in 1 s (or between 33 and 100 ms of timestamp) [32]. To make the microgrid conditions more sensitive to the CCG modes, the load variations at this rate are considered to verify performances of the proposed controller. Besides, the output powers of DFIG, PMSG, and SPV are managed by the active power controls in converters to meet these load demands. As a result, the total generation of such RESs is varied between 0.75 and 5 MW. Note that the DFIG and PMSG are operated according to the wind speed, which is associated to the variables \( \omega_{DF} \) and \( \omega_{PM} \), while the SPV is operated according to the solar irradiation, which is related to the variable \( i_{PV} \). Following on, the BESS is used to charge or discharge when the generation is greater or lower than the load demands, respectively. Consequently, Figs. 9 and 10 illustrate the transient response and the estimated damping of the CCG modes under such conditions, respectively.

Without \( k \), the \( \Delta F \) amplitude gradually increases and the microgrid eventually becomes unstable. As can be observed, the \( \hat{\zeta}_{cl} \) of the CCG modes moves to unstable region (negative value) under the changes in microgrid. After \( t > 20 \) s, the \( \Delta F \) is not maintained within the acceptable range, i.e. \( \Delta F \in [-0.3, 0.3] \) Hz. Therefore, this scenario highly requires the improvement of \( \hat{\zeta}_{cl} \). It should be noted that, as reported by [33], there are no specific standards defined for the frequency limits of low-inertia and isolated microgrids since these microgrids highly depend on the mixed generations and loads. Therefore, the quick changes in the generations and loads in low-inertia microgrids may lead to large frequency deviations that are beyond standard limits. From a generator point-of-view, frequency standards such as the ISO 8528-5 standard [34] could be used to provide a guideline for the frequency limits. In this case, the frequency deviation, which is increased to 1 Hz in 120 s by the variations of generations and loads, is possible in the study low-inertia microgrid with CCGs.

On the other hand, with the optimal-fixed control parameters, the \( k_c \) cannot keep the \( \Delta F \) between \(-0.3 \) and \( 0.3 \) Hz. This implies that the \( \hat{\zeta}_{cl} \) in the case of \( k_c \) is sensitive to these changes. In comparison to the \( k_p \), the \( \hat{\zeta}_{cl} \) is almost kept as constant values since the \( k_p \) can adapt its control parameters to satisfy the constraints in (14b). As can be seen, the \( k_p \) is robust to such microgrid changes. As a result, the \( k_p \)
provides higher control performance resulting in the smaller $\Delta F$ amplitude.

**Case Study 2:** The total generation is identical to that of Case Study 1. It is assumed that the loads are altered between 0.5 and 6.5 MW in every 2 to 6 s. At $t = 20$ s, the SPV is disconnected from the microgrid. Accordingly, the BESS automatically manages the loads and outputs of the DFIG and PMSG. Fig. 11 shows the frequency deviation resulted by the $k_c$ and $k_p$ and Fig. 12 illustrates the estimated damping ratio and frequency of four CCG modes. During $t = [0, 20]$ s, before the disconnection of SPV, both controllers can suppress the frequency deviation. In addition, both $k_c$ and $k_p$ can significantly improve the damping ratios of the CCG modes. After the disconnection of SPV, the $3^{rd}$ CCG mode belonging to the SPV disappears. As can be seen, all the estimated frequencies $\hat{f}$ decrease to approximately 1.5 Hz resulting in the alteration of oscillation frequency in the microgrid. Moreover, most of $\hat{\xi}_{cl}$ significantly drop to negative values, especially the $4^{th}$ mode. After the change of microgrid topology, $k_c$ with optimal-fixed control parameters cannot maintain the frequency deviation in the acceptable range, the $\Delta F$ severely fluctuates and eventually becomes unstable. By using $k_p$, the order of $k_p$ is automatically changed to 6 (i.e. $p = q = 6$) to maintain the damping of the three CCG modes. Besides, all control parameters of $k_p$ are optimally redesigned. Although the microgrid topology changes, $\hat{\xi}_{cl}$ of the three CCG modes are maintained around 5% resulting in the smaller fluctuation of $\Delta F$. Obviously, the $k_p$ can keep the frequency deviation within the acceptable range and it is robust to the disconnection of CCG.

**Case Study 3:** It is assumed that the reactive powers of loads are varied between 0.05 and 1.5 MVAR in every 5 to 10 s. To demonstrate the controller performance in terms of reactive power variations, the proposed controller $k_p$ can be modified as follows: inputs of the 4SID are changed by

- $\Delta V_1$ in Case Study 3.
$S_1 = \Delta \bar{Q}$ and $S_{10} = \Delta V_1$ while the objective function (14a), and constraints (14b) and (14c) remain the same for the adaptive robust controller design. Fig. 13 shows the voltage variation of bus 1. In comparison, the fluctuation of voltage in the case of $k_p$ is relatively small. On the other hand, the controller $k_c$ cannot resist the reactive power variations of loads, the voltage of bus 1 is not in the acceptable range, i.e between 0.95 and 1.05 pu. Fig. 14 illustrates the estimated damping ratios of CCG modes. It can be seen that the damping ratios in the case of $k_c$ are significantly degraded, especially in the 4th CCG mode. Conversely, the variations of reactive power slightly affect the performance of controller $k_p$, the damping ratios of all CCG modes are more than 0.05 or 5%.

With the modification in the 4SID, the result in this case guarantees that the proposed control algorithm can also deal with the reactive power variations of loads.

Case Study 4: In this case, the observation errors in the input and output signals of $k_c$ and $k_p$ are evaluated by using the sensitivity analysis. Possible scenarios are considered as follows:

1) For the observation error in the case of active power variations, the input of controller is the mean active power deviation of all CCGs and loads, i.e. $\Delta YCO = \Delta \bar{P}$, while the output of controller is sent to four CCGs (see Figs. 1 and 2). It is assumed that one or more signal(s) of input $\Delta YCO$ is/are missing (i.e. $\Delta P_{DF}$ or/and $\Delta P_{PM}$ or/and $\Delta P_{PV}$ or/and $\Delta P_{BE}$ or/and $\Delta P_1$). At the same time, it is also assumed that the controller fails to send the output to any CCGs. As a result, the total combination of these scenarios is equal to $2^9 = 512$.

2) For the observation error in the case of reactive power variations, all conditions are the same as that of step 1) except $\Delta YCO = \Delta \bar{Q}$, this also results in the total combination of possible scenarios ($2^9 = 512$).

3) The microgrid operating points in Case Study 1 and Case Study 3 are respectively used to evaluate the performances of both $k_c$ and $k_p$ in steps 1) and 2). In this paper, the following equations are devised to justify the observation error for each scenario, where $OE_P$ and $OE_Q$ are respectively the observation errors used for steps 1) and 2), $t_{sim}$ is the total simulation time, and $\Delta F_w$ and $\Delta V_{1,w}$ are the signals $\Delta F$ and $\Delta V_1$ without the observation errors (or $\Delta F$ and $\Delta V_1$ obtained from Case Study 1 and Case Study 3).

\[
OE_P = \int_{t=0}^{t_{sim}} |\Delta F_w| \, dt - \int_{t=0}^{t_{sim}} |\Delta F| \, dt, \quad (15a)
\]
\[
OE_Q = \int_{t=0}^{t_{sim}} |\Delta V_{1,w}| \, dt - \int_{t=0}^{t_{sim}} |\Delta V_1| \, dt. \quad (15b)
\]

In (15a) and (15b), the lower the values of $OE_P$ and $OE_Q$, the lower the effects of observation error on controller performance. After conducting steps 1) to 3), the results from (15a) and (15b) are collected to conduct the probability analysis. Accordingly, Fig. 15 shows the percentage of $OE_P$ and $OE_Q$ in the cases of $k_c$ and $k_p$. In the case of $k_c$, the value of $OE_P$ is varied between 7 and 24.5 with the probability around 16%, and between 42 and 50 with the probability between 3.5 and 15%. On the other hand, in the case of $k_p$, the value of $OE_P$ falls within the range of 2 to 12 with the highest probability of 39.8%. The same trend also occurs with the value of $OE_Q$. According to (15a) and (15b), this implies that the controller $k_p$ provides better control performances under the impacts of observation errors.

Case Study 5: In this case, the robustness of controllers is investigated by varying the converter controller gains. The simulation setup is given as follows:

1) All gains of the proportional-integral (known as PI) controllers in CCG converter control loops (see Fig. 2) are randomly varied from 0.1 to 10 pu while the generations and loads are randomly changed by ±20% from Case Study 1.

2) Consequently, the $\infty$-norms of three sensitivity functions, i.e. $|\Delta M_1|_\infty$, $|\Delta M_2|_\infty$, and $|\Delta M_3|_\infty$, are calculated to evaluate the robustness of the controllers $k_c$ and $k_p$ against such variations.

3) Repeat the steps 1) and 2) for 1,000 scenarios to conduct the probability analysis.

Fig. 16 shows the probability analysis of controller robustness under various microgrid operating points. Having in mind that the lower the values of $|\Delta M_1|_\infty$, $|\Delta M_2|_\infty$, and $|\Delta M_3|_\infty$, the higher the robustness against microgrid uncertainties. In the case of $k_p$, the occurrences of smaller values of $|\Delta M_1|_\infty$, $|\Delta M_2|_\infty$, and $|\Delta M_3|_\infty$ are higher than those of the controller $k_c$. It implies that the controller $k_p$ is more robust than the controller $k_c$ under various microgrid operating points and converter controller gains.
V. CONCLUSION

This paper presents the adaptive robust control using the signal identification of a low-inertia microgrid. The CCG modes are taken into account in the controller design process. Following on, the sensitivity functions are considered for the robustness of controller along with microgrid changes. The proposed method is more practical in actual microgrids since it does not require any exact microgrid parameters to analyze the CCG modes and design the controller. Besides, the accuracy of estimated microgrid model is very high while the order of the obtained controller is much smaller than that of the original model. As verified in the simulation results, the CCG modes are shown to be detrimental to a low-inertia microgrid. The proposed controller can be designed by identifying the signals of load patterns, output of CCGs, and frequency deviation. The proposed controller also provides a better performance than that of a conventional optimal-fixed controller under various changes in RES outputs and loads. Salient features of the proposed control are expected for future microgrids with 100% penetration of CCGs.

APPENDIX

A. DIFFERENTIAL EQUATIONS OF DFIG, PMSG, SPV, AND BESS

The differential equations of DFIG, PMSG, SPV, and BESS are given as follows:

For the DFIG model:

\[ \omega_{DF}(t) = \frac{1}{2H_{DF}} \left( -\frac{P_m(\omega_{DF}(t)) - \Gamma_{e,DF}(t)}{\omega_{DF}(t)} \right), \]  
\[ \begin{bmatrix} i_{DF}^d(t) \\ i_{DF}^q(t) \\ \theta_{DF}(t) \\ \dot{\theta}_{DF}(t) \end{bmatrix} = \begin{bmatrix} L_{DF}^d & 0 & L_{DF}^m & 0 \\ 0 & L_{DF}^d & 0 & L_{DF}^m \\ L_{DF}^m & 0 & L_{DF}^q & 0 \\ 0 & L_{DF}^m & 0 & L_{DF}^q \end{bmatrix} \begin{bmatrix} i_{DF}^d(t) \\ i_{DF}^q(t) \\ \theta_{DF}(t) \\ \dot{\theta}_{DF}(t) \end{bmatrix} + \begin{bmatrix} \frac{d}{dt} \frac{P_m(\omega_{DF}(t)) - \Gamma_{e,DF}(t)}{\omega_{DF}(t)} \end{bmatrix}, \]  
\[ M_{1,DF} = \begin{bmatrix} L_{DF}^s & 0 & L_{DF}^m & 0 \\ 0 & L_{DF}^s & 0 & L_{DF}^m \\ L_{DF}^m & 0 & L_{DF}^r & 0 \end{bmatrix}, \]  
\[ M_{2,DF} = \begin{bmatrix} R_{DF}^s & L_{DF}^s & 0 & L_{DF}^m \\ -L_{DF}^s & R_{DF}^s & -L_{DF}^m & 0 \\ 0 & 0 & \frac{1}{\omega_{DF}} & 0 \\ -1 - \frac{1}{\omega_{DF}} & \frac{1}{\omega_{DF}} & 0 & \frac{1}{\omega_{DF}} \end{bmatrix}, \]

For the PMSG model:

\[ \dot{\omega}_{PM}(t) = \frac{1}{2H_{PM}} \left( -\frac{P_m(\omega_{PM}(t)) - \Gamma_{e,PM}(t)}{\omega_{PM}(t)} \right), \]  
\[ \begin{bmatrix} i_{PM}^d(t) \\ i_{PM}^q(t) \\ \theta_{PM}(t) \\ \dot{\theta}_{PM}(t) \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} \frac{P_m(\omega_{PM}(t)) - \Gamma_{e,PM}(t)}{\omega_{PM}(t)} \end{bmatrix}, \]  
\[ M_{1,PM} = \begin{bmatrix} L_{PM}^s & 0 & L_{PM}^m & 0 \\ 0 & L_{PM}^s & 0 & L_{PM}^m \\ L_{PM}^m & 0 & L_{PM}^r & 0 \end{bmatrix}, \]  
\[ M_{2,PM} = \begin{bmatrix} \frac{R_{PM}^s}{\omega_{PM}} & L_{PM}^s & 0 & L_{PM}^m \\ 0 & \frac{R_{PM}^s}{\omega_{PM}} & -L_{PM}^m & 0 \\ \frac{1}{\omega_{PM}} & 0 & \frac{1}{\omega_{PM}} & 0 \end{bmatrix}, \]

For the SPV model:

\[ i_{PV}(t) = \frac{d}{dt} \left( \frac{P_m(t) - \Gamma_{e,SPV}(t)}{\omega_{PV}} \right), \]  
\[ \dot{i}_{PV}(t) = \frac{d}{dt} \left( \frac{P_m(t) - \Gamma_{e,SPV}(t)}{\omega_{PV}} \right), \]  
\[ \dot{v}_{PV}(t) = \frac{d}{dt} \left( \frac{P_m(t) - \Gamma_{e,SPV}(t)}{\omega_{PV}} \right), \]

For the BESS model:

\[ \dot{v}_{BE}(t) = \frac{d}{dt} \left( \frac{P_m(t) - \Gamma_{e,BESS}(t)}{\omega_{BE}} \right), \]  
\[ \dot{i}_{BE}(t) = \frac{d}{dt} \left( \frac{P_m(t) - \Gamma_{e,BESS}(t)}{\omega_{BE}} \right), \]  
\[ \dot{q}_{BE}(t) = \frac{d}{dt} \left( \frac{P_m(t) - \Gamma_{e,BESS}(t)}{\omega_{BE}} \right), \]
\[ \dot{v}_{BE}^d(t) = \frac{v_{BE}^d(t)}{C_{BE}} - \frac{R_{PV} v_{BE}^d(t)}{C_{BE}}, \quad (A-4d) \]
\[ \dot{q}_{BE}^d(t) = \frac{v_{BE}^q(t)}{C_{BE}} - \frac{R_{PV} v_{BE}^q(t)}{C_{BE}}, \quad (A-4e) \]

where superscript \( m \) represents the mutual inductance, \( R, L, \) and \( C \) are respectively resistance, inductance and capacitance, \( H \) is the aggregate generator inertia, \( \Gamma_r \) is the electrical rotor torque, and \( K_B \) and \( T_B \) are the pitch angle gain and control time constant, \( i_{ph} \) and \( i_0 \) are respectively the current produced by solar irradiation \( (\gamma_s) \) and saturation current, and \( Temp \) is the temperature at SPV arrays.

### B. COMPUTATION OF SYSTEM MATRICES USING 4SID

The computation of state matrices \( \hat{A}, \hat{B}, \hat{C}, \) and \( \hat{D} \) is given by the following steps [25].

1) Equally partition the measured signals \( S_I \) and \( S_L \). For example, \( S_I_{r} = [S_{I_1, r}, \ldots, S_{I_{n}, r}] \) and \( S_L_{r} = [S_{L_1, r}, \ldots, S_{L_{n}, r}] \).

2) Let define the oblique projection \( O \) as,
\[ O_{r-1} = S_{I_{r-1}}/S_{I_{r-1}} H_{r+1}, \]
where \( H \) is the Hankel matrix containing the past inputs and outputs, \( \hat{x} \) is the estimated state matrix, and \( \Theta \) is the extended observability matrix.

3) Calculate the oblique projections at patterns \( r \) and \( r - 1 \) by: \( O_{r} = S_{I_{r-1}}/S_{I_{r-1}} H_{r+1} \), and \( O_{r-1} \) in (A-5).

4) Calculate the singular value decomposition (known as SVD) and determine the estimated model order by inspecting the singular values in \( S_r = \begin{bmatrix} S_{11,r} & 0 \\ 0 & 0 \end{bmatrix} \). Next, partition the SVD to obtain \( U_{1,r} \) and \( S_{11,r} \), by,
\[ W_{1,r} O_{r} W_{2,r} = \begin{bmatrix} U_{1,r} & U_{2,r} \end{bmatrix} \begin{bmatrix} S_{11,r} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{1,r}^T \\ V_{2,r}^T \end{bmatrix} \]
\[ = U_{1,r} S_{11,r} V_{2,r}^T \]

where \( W_1 = [U_1 \ U_2] \) and \( W_1 = [V_1^T \ V_2^T] \) are the weighting matrices.

5) Determine \( \Theta_r \) and \( \Theta_{r-1} \) as,
\[ \Theta_r = W_{1,r}^T U_{1,r} V_{11,r} \]
\[ \Theta_{r-1} = \Theta_r \]

where \( \Theta_r \) means the matrix \( \Theta_r \) without the last row.

6) The state sequences \( \hat{x}_r \) and \( \hat{x}_{r+1} \) are calculated by,
\[ \hat{x}_r = \Theta_r^T O_r, \]
\[ \hat{x}_{r+1} = \Theta_r O_{r-1}, \]

where superscript \( \perp \) means the orthogonal complement.

7) Solve the set of estimated state matrices \( \hat{A}, \hat{B}, \hat{C}, \) and \( \hat{D} \) by,
\[ \hat{x}_{r+1} = \begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} \begin{bmatrix} \hat{x}_r \\ S_{I_{r}, r} \end{bmatrix}, \]

where \( S_{I_{r}, r} \) and \( S_{L_{r}, r} \) are the specified values of \( S_I \) and \( S_L \) at any sequence \( r \), respectively. Note that these set of matrices can be solved by a linear least squares method.

8) Repeat the steps 1) to 7) for \( r = 1, \ldots, n_r \). We get \( [\hat{A}_1, \ldots, \hat{A}_{n_r}] \), \( [\hat{B}_1, \ldots, \hat{B}_{n_r}] \), \( \{\hat{C}_1, \ldots, \hat{C}_{n_r}\} \), and \( \{\hat{D}_1, \ldots, \hat{D}_{n_r}\} \). Accordingly, the stability indices can be obtained in every moving windows to design the controller.

### C. ANALYSIS OF SYSTEM ROBUSTNESS

Fig. 17 illustrates a typical closed-loop system configuration with uncertainties [29], where \( d, n, \) and \( r \) are the sets of disturbances, communication uncertainties, and references, respectively. The following relationships can be obtained as,
\[ y = \left( I + \hat{G}k \right)^{-1} \hat{G}k r + \left( I + \hat{G}k \right)^{-1} d \]
\[ = \left( I + \hat{G}k \right)^{-1} \hat{G}k n \]
\[ \Delta u = k \left( I + \hat{G}k \right)^{-1} \cdot r - k \left( I + \hat{G}k \right)^{-1} \cdot d \]

![FIGURE 17. A typical closed-loop configuration of k, \( \hat{G} \), and uncertainties.](image)

It is known in Section III-B that we have defined the sensitivity functions \( \Delta M_1 = \left( I + \hat{G}k \right)^{-1}, \Delta M_2 = -\left( I + \hat{G}k \right)^{-1} \hat{G}k, \) and \( \Delta M_3 = k \left( I + \hat{G}k \right)^{-1} \). Substituting \( \Delta M_1, \Delta M_2, \) and \( \Delta M_3 \) into (A-10a) and (A-10b) yields,
\[ y = -\Delta M_2 \cdot r + \Delta M_1 \cdot d + \Delta M_2 \cdot n \]
\[ \Delta u = \Delta M_3 \cdot r - \Delta M_3 \cdot d - \Delta M_3 \cdot n \]

It can be observed in (A-11a) and (A-11b) that the sensitivity functions \( \Delta M_1, \Delta M_2, \) and \( \Delta M_3 \) are known and these terms are the factors of \( r, d, \) and \( n \). In this paper, the control parameters of \( k \) are optimally changed to minimize the \( \infty \)-norms of \( \Delta M_1, \Delta M_2, \) and \( \Delta M_3 \). Consequently, the disturbance attenuation, communication uncertainty rejection, and good control performance can be attended in both input and output sides of the controller \( k \).

### D. MICROGRID PARAMETERS

In this paper, the parameters of DFIG are provided as follows: DFIG size = 1.5 MW, DFIG converter rate = 25% of DFIG size, \( H_{DF} = 0.15 \) GVAs, \( R_{DF} = 0.2 \) \( \Omega \), \( L_{DF}^{d} = 0.05 \) \( \Omega \), \( L_{DF}^{q} = 2.65 \) mH, \( L_{DF}^{r} = 0.065 \) mH, and \( C_{DF} = 0.54 \) mF. The parameters of PMSG are provided as follows: PMSG size = 1.5 MW, PMSG converter rate = 25% of PMSG size (1.5 MW), \( H_{PM} = 0.12 \) GVAs, \( R_{PM} = 0.15 \) \( \Omega \),
The parameters of SPV are provided as follows: SPV size = 1 MW, SPV converter rate = 20% of SPV size (1 MW), \( R_{PV} = 0.1 \), \( L_{PV} = 0.025 \) mH, \( C_{PV} = 0.45 \) mF, \( R_{PM} = 0.02 \), \( L_{PM} = 0.0125 \) mH, and \( C_{PM} = 0.54 \) mF. The parameters of BESS are given as follows: BESS size = 1 MW, BESS converter rate = 25% of SPV size (1 MW), \( C_{BE} = 0.65 \) mF, \( R_{BE} = 0.02 \), \( L_{BE} = 0.0125 \) mH, and \( C_{BE} = 0.001 \) mF.

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