Eleven dimensional supergravity as a constrained topological field theory

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March 27, 2022

Abstract

We describe a new first-order formulation of $D = 11$ supergravity which shows that that theory can be understood to arise from a certain topological field theory by the imposition of a set of local constraints on the fields, plus a lagrange multiplier term. The topological field theory is of interest as the algebra of its constraints realizes the $D = 11$ supersymmetry algebra with central charges.

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1 Introduction

Eleven dimensional supergravity has been an object of fascination since it was first discovered more than twenty years ago[1]. It is the highest dimension in which a supergravity theory exists, and it contains all of the lower dimensional supergravities as dimensional reductions. It has also been proposed as a description of the classical limit of a phase of $\mathcal{M}$ theory[2], which is not itself described by any consistent string theory. Anyone wanting to understand the dynamical structure of the different supergravities, and their inter-relations with each other under reductions and various kinds of duality transformations are recommended to start with the 11 dimensional theory.

It is thus very interesting to wonder whether 11 dimensional supergravity might be made into a quantum theory directly, using some non-perturbative approach. It is true that it is non-renormalizable in perturbation theory[3], but so are general relativity and the $N = 1, 2$ supergravities in four dimensions, and this has not prevented a great deal of progress being made understanding the exact structure of their canonical and path integral quantum theories[4]-[22]. A beautiful and robust structure was found that could not have been seen in perturbation theory, which leads to definite physical predictions such as the discrete spectra of the area and volume operators[6, 7]. The eigenstates of these operators provide a basis of states for background independent quantum theories of gravity, which are the spin network basis[6, 7]. These are constructed in terms of simple combinatorial and representation theory data, are closely related to the fusion algebras of conformal field theory[17, 18, 19] and exist also for supersymmetric theories[20, 21].

It is also clear now that these theories have rigorously defined hamiltonian[13] and path integral[14, 15, 16] formulations and so do exist as examples of diffeomorphism invariant quantum field theories. This existence is independent of the question of whether they have classical limits which reproduce classical general relativity or supergravity. This is presently an open question for general relativity and supergravity in $d = 4$[23]. Even if the particular dynamics of quantum general relativity in $d = 4$ leads to a theory which is sensible at the Planck scale but lacks a good classical limit, it is still reasonable to conjecture that the robust and theory independent features of the background independent formulation of quantum gravity uncovered in loop quantum gravity will play a role in a background independent
formulation of $\mathcal{M}$ theory $[18, 19, 24]$.

There is a simple reason to expect that the classical limit will exist for a background independent of 11 dimensional supergravity, but not for $d = 3 + 1$ general relativity, which is that whenever such a limit exists one expects a sensible perturbation theory to exist around any classical background that arises in a classical limit. There is no such consistent perturbation theory for pure four dimensional general relativity but there is one for many, if not all, consistent compactifications of 11 dimensional supergravity-these are of course exactly the perturbative string theories. In fact, there is a simple argument that where such a limit exists the weakly coupled excitations will be described by a string theory $[22]$.

It is then sensible to suppose that one way to uncover the structure of the states and operators that will go into the background independent formulation of $\mathcal{M}$ theory is to make an exact canonical quantization of 11 dimensional supergravity and discover what structures play the role of the spin networks and super-spin networks in $D = 3 + 1$. The first step in any such attempt must be to have a suitable form for the action. What is required to make progress in a non-perturbative approach to quantization is to have a first order polynomial form of the action, which has the property that it arises from a topological field theory by the imposition of a set of constraints. The reason is it is by now understood that all the beautiful results which have followed from the use of the Ashtekar-Sen and related variables are due ultimately to the fact that in 4 dimensions general relativity and supergravity (at least up to $N = 2$ $[21]$) can be understood as arising in this way from topological field theories. A further reason for taking this route is that in the presence of appropriate boundary conditions it leads to holographic formulations of quantum general relativity and supergravity in which the Bekenstein bound is realized naturally because the boundary theories are built from the state spaces of Chern-Simons theory $[31, 32, 33]$.

The main goal of this paper is then to present a formulation of the 11 dimensional supergravity as a constrained topological field theory. In the next section we introduce an

1Indeed, such a form of the theory was understood earlier, by Plebanski $[25]$. The significance of this form was only realized after Sen $[26]$ discovered the equivalent Hamiltonian variables, in an attempt to understand the canonical structure of supergravity. This was then formalized by Ashtekar $[27]$ for the canonical theory and in $[28, 29]$ for the lagrangian theory. Attempts to relate this form to topological field theory then led to a rediscovery of Plebanski’s action $[30]$ and its extension to supergravity.

2This method has also been used recently to construct an explicit description of the quantum geometry of the black hole horizon $[34]$.  

3
11 dimensional topological field theory which has the property that its algebra of first class constraints, which generate the gauge transformations of the theory, reproduces exactly the 11 dimensional super-Poincare algebra, including its central charges. The theory is introduced in the next section and the canonical analysis is given in section 4. In section 3 we show how the eleven dimensional supergravity action arises by imposing a certain set of constraints on the topological field theory. We then arrive at the supergravity action in the form given by Fre[35] and d’Auria and Fre[36]. A feature of that form of the theory, which plays an important role as well in our formulation, is the presence of two abelian potentials, which are a six form and a three form. Although we are not completely certain of the correct way to express it, we believe it likely that the formulation of a gravitational theory as a constrained topological field theory is closely related to the idea of formulating it in terms of free differential algebras, which was pursued by Fre and collaborators[37].

This paper represents only the first step of a program of quantization of 11 dimensional supergravity. Still to be investigated is the implications for the canonical structure of the full 11 dimensional supergravity action and the possible existence of a quantization using the methods of loop quantum gravity.

In the next section we introduce the topological quantum field theory in 11 dimensions and in section 4 we derive its canonical formalism and compute its constraint algebra. We find that it is first class and that the algebra of constraints does reproduce weakly\(^7\) the super-Poincare algebra in 11 dimensions with central charges. Along the way, in section 3, we show how constraints and lagrange multiplier terms may be added to the topological field theory to arrive at the full 11 dimensional supergravity action, in the form given by Fre[35].

\section{TQFT for the 11 dimensional super-Poincare algebra}

The first step in our construction is to find the TQFT whose algebra of constraints reproduces the supersymmetry algebra of supergravity in 11 dimensions, including the central extensions. This algebra has the form\(^5\)

\(^3\)Another approach to the relationship between topological field theory and supergravity is described in Ref.\cite{38,39}.

\(^4\)That is up to the constraints that say that the curvatures vanish in a topological field theory.

\(^5\)We list the convention used in this paper. (i) Indices \(\mu, \nu, \ldots = 0, 1, \ldots, 10\) are spacetime indices while \(i, j, \ldots = 1, 2, \ldots, 10\) are used for spatial indices; (ii) The capital \(A, B, \ldots = 1, 2, \ldots, 32\) stand for the \(Sp(32)\)
\[
\{Q^A, Q^B\} = \Gamma_a^{AB} G^a + \Gamma_{ab}^{AB} G^{ab} + \Gamma_{a5}^{AB} G^{a5},
\]
where
\[\Gamma_{ap}^{AB} := \Gamma_{[a_1 \Gamma a_2 \ldots \Gamma_{ap}]}^{AB}.
\]

Here \(G^a\) must be the generator of spacetime diffeomorphisms, so that \(G^0\) must be related to the Hamiltonian and \(G^i\) to the diffeomorphism constraints of the theory, while \(G^{ab}\) and \(G^{a5}\) are the central extensions. This can be understood to be a contraction of \(Osp(1|32)\)\(^{10}\).

One place to begin is with the fields of a gauge theory for the superalgebra \(Osp(1|32)\) in 10+1 dimensions. The generators of the superalgebra consist of the translation generator \(G_a\), Lorentz generator \(J_{ab}\), supersymmetry generators \(Q^A\) and five-index antisymmetric generator \(G^{[abcde]} \equiv G^{a5}\). One starting point would be to define a 1-form superconnection associated with those generators:
\[
\mathcal{A}_\mu := A^{ab}_\mu J_{ab} + e^a_\mu G_a + \Psi^A_\mu Q_A + A^{a5}_\mu G_{a5},
\]
where \(\Psi^A\) are 32 component Majorana spinors. We may then introduce a \(BF\) action as
\[
\mathcal{I} = \int_M dx^{11} B \wedge F,
\]
where \(B\) is a super nine form and \(F\) is the curvature of \(Osp(1|32)\) defined by
\[
F = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A}.
\]

Unfortunately this route seems not to lead to the standard 11 dimensional supergravity\(^6\). The difference seems to be that the relevant gauge group for 11 dimensional supergravity, at least at the classical level, is the Super-poincare group, which is a contraction of \(Osp(1|32)\). As a result, the central generators are realized in supergravity by functionals of a three form abelian gauge field rather than by a standard component of a connection one-form. The basic mystery of the construction of 11 dimensional supergravity (as well as many of its lower dimensional reductions) is how such a field may be seen to arise from a gauge theoretic structure as does the connection of spacetime. From the point of view in which gravitational

spinor indices; \((iii)\ a, b, \ldots \ = 0, 1, \ldots, 10\) represent \(SO(10, 1)\) indices. We sometimes use a condensed index notation in which \(a_p = a_1 a_2 \ldots a_p\).

\(^6\)But it is of interest and has been pursued in \[^{10}\] \[^{41}\].
Theories are understood as constrained topological field theories, this mystery can be solved, for topological field theories can indeed be constructed based on gauge theories of Abelian $p$-forms and they can be quantized using the methods of loop quantum gravity. Indeed as we shall see here, one can construct a topological field theory whose constraint algebra realizes perfectly the full super-Poincare algebra, with the abelian $p$ forms realizing the central charges.

In order to achieve this we have found it necessary to introduce not only the three form gauge field $a_{\mu\nu\rho}$ but its dual, which is a six form field $b_{\alpha\beta\gamma\delta\epsilon\phi}$. This leads us to a formalism which is similar to that of D’Auria and Fre. Indeed, as our results show, there is likely a close relationship between their conception of supergravity based on Cartan integrable algebras and the more recent conception of a gravitational theory as a constrained topological field theory.

We now introduce our 11 dimensional super-Poincare topological field theory.

To begin with we define the topological field theory associated to the 11 dimensional super-Poincare algebra, unextended by central charges. The gauge field is then of the form,

$$A_\mu := A_{\mu}{}^{ab} J_{ab} + e_\mu{}^{a} G_{a} + \Psi_\mu{}^{A} Q_{A}. \quad (6)$$

The components of the curvature are given by

$$F^{ab} = dA^{ab} - A^{ac} \wedge A_{c}^{b}, \quad (7)$$

$$F^{a} = de^{a} - A^{ab} \wedge e_{b} - \frac{i}{2} \Psi_{A} \wedge \Gamma^{aA} B_{b} \Psi_{B}, \quad (8)$$

$$F^{A} = D\Psi^{A} = d\Psi^{A} - \frac{1}{4} A_{ab}^{A} \Gamma_{abB} \wedge \Psi_{B}. \quad (9)$$

To represent the central charges we introduce the three form $a_{\mu3}$ and its dual which is a 6-form field $b_{\mu6}$. We introduce their curvatures,

$$F^{\otimes} = db - 15da \wedge a - \frac{i}{2} \rho^{5}, \quad (10)$$

$$F^{\Box} = da - \frac{1}{2} \rho^{2}, \quad (11)$$

\footnote{Thus, while we solve the problem of encoding the central charges in an algebra of canonical constraints, in a way that leads under suitable constraints to 11 dimensional supergravity, we do not solve the problem of what all this may have to do with $Osp(1|32)$.}
where
\[ \rho_{ap}^p := \Psi_A \wedge \Gamma^{a_p A} B \Psi_B \wedge E_{ap}^p, \]  
(12)

and
\[ E_{ap}^p = e_{a_1} \wedge e_{a_2} \ldots \wedge e_{a_p}. \]  
(13)

Using these curvatures we then write a TQFT,
\[ I^{TQFT} = -\frac{1}{g^2} \int B_{ab} \wedge F^{ab} + B_a \wedge F^a + B_A \wedge F^A + B^\Box \wedge F^\Box + B^\otimes \wedge F^\otimes. \]  
(14)

The \( B \)'s are lagrange multipliers which have the form degree indicated. The field equations are
\[ F_{ab} = F^a = F^A = F^\Box = F^\otimes = 0, \]  
(15)

and
\[ D \wedge B_{ab} - e^{[a} \wedge B^{b]} - \frac{1}{4} \Psi_A \Gamma^{a B}_A B \wedge B^B = 0, \]  
(16)

\[ D \wedge B^a - \Psi_A \Gamma^{a A}_B \Psi^B \wedge e^b \wedge B^\Box - \frac{5i}{2} \Psi_A \Gamma^{a B}_A B \Psi^B \wedge e_{b_4} \wedge B^\Box = 0, \]  
(17)

\[ D \wedge B^A - i \Gamma^A_{ab} \Psi^B \wedge B^a - \Gamma^A_{ab B} \Psi^B \wedge E_{ab} \wedge B^\Box - i \Gamma^A_{a_5 B} \Psi^B \wedge E^{a_5} \wedge B^\otimes = 0, \]  
(18)

\[ d \wedge B^\otimes = 0, \]  
(19)

\[ d \wedge B^\Box - 30 da \wedge B^\otimes + 15 a \wedge d \wedge B^\otimes = 0. \]  
(20)

In section (4) we will discuss the canonical formulation of the action (14) and show that its algebra of first class constraints replicates the superalgebra (1).

3 Constraining the TQFT to get supergravity in 11 dimensions

We obtain an action for 11 dimensional supergravity by adding constraint and lagrange multiplier terms,
\[ I^{SUGRA}_{11D} = I^{TQFT} + I^{CONST.} + I^{F_4}, \]  
(21)

where
\[ I^{CONST.} = \int \lambda_{ab} \wedge (B_{ab} - \frac{1}{9} g^2 E^{ab}) + \lambda_A \wedge (B^A - \frac{2}{l^8} \Gamma^A_{a_8 B} B \wedge E^{a_8}) \\
+ \lambda_a \wedge (B^a - \frac{7i}{30} E^{a B} \Psi_A \Gamma^A_{c_5 B} B \epsilon_{b c_5}) \\
+ \lambda_\otimes \wedge (B^\otimes - 56 da) + \lambda_\Box (B^\Box + 56 \rho^5), \]  
(22)
and
\[ I^{F_4} = \int -2F_{a_4} R^\square \wedge E_{b_7} \epsilon^{a_4 b_7} + \frac{1}{330} F_{a_4} F^{a_4} E_{11}^*. \] (23)

It is not difficult to see that the variations of the lagrange multipliers $\lambda$ reproduce the $D = 11$ supergravity action in the form given by Fre[3].

\[ I^{SG} = \frac{-1}{g^2} \int \frac{1}{9!^3} E^{*ab} \wedge F^{ab} + \frac{7t}{30} F^{ab} \Psi_A \Gamma^C_B \Psi^B \epsilon_{b_7 c_5} \wedge F^{a_4} + \frac{2}{l^8} \Gamma^A_{a_7 b} \Psi^B \wedge E^{ab} \wedge F^A \\
+ 56t \rho^5 \wedge F^\square + 56da \wedge F^\otimes \\
- 2F_{a_4} R^\square \wedge E_{b_7} \epsilon^{a_4 b_7} + \frac{1}{330} F_{a_4} F^{a_4} E_{11}^*. \] (24)

Elimination of the lagrange multipliers then leads to the theory in the original form[1].

We note that the lagrange multiplier terms $I^{F_4}$ are necessary to get the $da^2$ terms in the supergravity action. Were they absent the $a$ field would have dynamics only from the Chern-Simons like terms $a \wedge da \wedge da$. One interesting question that this approach should be able to answer is why supersymmetry requires both the Maxwell and Chern-Simons like terms in the supergravity action.

### 4 Canonical formulation of the $11D$ TQFT

We now describe to the canonical decomposition of the TQFT given by [14]. Our main goal here is to find the algebra of its constraints. We assume that the eleven dimensional spacetime $\mathcal{M}^{11}$ has the form $\mathcal{M}^{11} = \Sigma^{10} \times R$ where $\Sigma^{10}$ is a compact ten dimensional manifold. We then make a $10 + 1$ decomposition to find that\(^8\)

\[ I^{TQFT} = \int dt \int_{\Sigma^{10}} d^{10}x \left\{ \pi_{iab} \dot{A}_{iab} + \pi_{ia} \dot{\epsilon}_{ia} + \pi_{iA} \dot{\Psi}_{iA} + \frac{1}{6!} \pi_{i} \epsilon_{i} \dot{b}_{i} + \frac{1}{3!} R_{i} \dot{a}_{i3} \\
+ B_{ab} \wedge F_{ab} + B_{0}^A \wedge F^A + B_{a} \wedge F_{a} + B_{b}^\square \wedge F^\square + B_{b}^\otimes \wedge F^\otimes \\
+ A_{0ab} \dot{r}_{ab} + \Psi_{0A} Q^A + e_{0a} G^a + a_{0ij} G_{ij}^2 + b_{0ia} G_{i5}^a \right\}, \] (25)

where now all forms are in the 10 dimensional space.

The expressions for the canonical momenta are,

\[ \pi^i_{ab} = -\frac{2}{g^2} (B_{ab}^*)^i \]

--\(^8\)Note that the decomposition is much simpler than in the standard case as there is no metric and hence no lapse and shift. The decomposition is purely a matter of pulling back forms.
\[ \pi^i_a = \frac{1}{g^2} (B^*_{a})^i, \]
\[ \pi^i_A = \frac{1}{g^2} (B^*_A)^i, \]
\[ p^{i_6} = \frac{1}{g^2} (B_{A}^*)^{i_6}, \]
\[ R^{i_3} = \frac{1}{g^2} (B_{a}^*)^{i_3} - \frac{15}{3!} p^{i_3 j_3} a_{j_3}. \]

The constraints take the form
\[ Q^A = D_i \pi^{iA} - \imath \pi^{ia} \Gamma^A_{ab} \Psi^B_i - \Psi^B_i \left\{ \Gamma^A_{ab} (R^{jk} + \frac{15}{3!} p^{ijkl} a_{l_3}) E^a_{jk} - i \alpha_{a} B^A p^{ijkl} E_{k_3} \right\}, \]
\[ G^a = D_i \pi^{ia} - \Psi^a \Psi^B_j \left\{ \Gamma^a_{bA} (R^{jk} + \frac{15}{3!} p^{ijkl} a_{l_3}) e^b_k + \frac{5i}{2} \gamma^{ab} E_{k_3} \right\}, \]
\[ J^{ij} = \frac{1}{2} D_i \pi^{iab} - \pi^{[a} e^{b]} - \frac{1}{4} \Psi^A B \pi^{A B i}, \]
\[ G^{i_5}_{i_5} = \frac{1}{5!} \partial_t p^{i_5}, \]
\[ G^{i_2}_{i_2} = \frac{1}{2} [\partial_t R^{jk} - \frac{15}{3!} (\partial_k a_{l_3}) p^{ijkl}]. \]

It’s now straightforward to check the Poisson brackets of two supersymmetric constraint functional satisfies weakly the relation (1). More precisely, we find,
\[ \{Q^A(x), Q^B(y)\} = \delta^{10}(x, y) \left\{ \Gamma^A_{ab} G^a \right\} \]
\[ -\Gamma^{AB}_{ab} \left[ E^{ab}_{jk} (G_{2}^{jk} + \frac{15}{3!} G_{5}^{ijkl} a_{l_3}) + \frac{15}{3!} p^{ijkl} (F_{a_3}^{ab} E_{jk} + a_{l_3} F_{ij} e^{b}_k) \right] \]
\[ -\Gamma^{AB}_{ab} \left[ F_{a_3}^{ab} (G_{5}^{ijkl} + p^{ijkl} E_{a_3}^{b} F_{4}) \right]. \]

It is interesting to see that the constraints by which the curvatures vanish are needed to close the algebra. It is also interesting that in order to realize the central charge proportional to \( \Gamma^{AB}_{a b c d} \), it is necessary to have the canonical momenta and the constraint associated with the six form field \( b \).

Similarly, we find that
\[ \{G^a(x), G^b(y)\} = \delta^{10}(x, y) \left\{ \Gamma^{ab}_{ABC} \left[ 2 F_{a_3}^{A B} [R^{ij} + \frac{15}{3!} p^{ijkl} a_{l_3} ] + \Psi^A B (\frac{1}{4} G_{2}^{ij} + G_{5}^{ijkl} a_{l_3} + p^{ijkl} F_{a_3}^{b}) \right] + \Gamma^{ab}_{def} \left[ 2 \alpha F_{a_3}^{A B} [R^{ijkl} E_{l_3} + \frac{1}{4} G_{5}^{ijkl} E_{a_3} + p^{ijklm} F_{kl} E_{m_3}^{b_3}] \right] \right\}. \]

In both of these relations there are delicate cancellations involving two and four fermion terms. These involve careful application of the Fiertz identities for 11 dimensions. It is also
interesting that the central charges come into the commutator of the translation generators (33). This may be a clue as to how the whole structure may descend from some $Osp(1|32)$ invariant framework.

We also find that,

$$\{Q^A(x), G^{ij}_2(y)\} = \frac{5}{4}\delta^{10}_{ab} \left( -G_5^{lmnij} \Psi^B_l E^{ab}_{mn} + p^{lmnkij} [F^B_{kl} E^{ab}_{mn} + 2F^a_{kl} \Psi^B_l c^b_n] \right),$$  \hspace{1cm} (34)

$$\{G^a(x), G^{ij}_2(y)\} = \frac{5}{4}\delta^{10}_{bAB} \left( -G_5^{lmnij} \Psi^A_l \Psi^B_m c^b_n + p^{lmnkij} [2F^A_{kl} \Psi^B_m c^b_n + \Psi^A_l \Psi^B_m F^b_{kn}] \right),$$  \hspace{1cm} (35)

$$\{Q^A(x), G^{ijklm}_5(y)\} = \{G^a(x), G^{ijklm}_5(y)\} = 0.$$  \hspace{1cm} (36)

Thus, $G^{ij}_2$ and $G^{ijklm}_5$ form a supersymmetry multiplet. We find also

$$\{Q^A(x), G^a(y)\} = 0.$$  \hspace{1cm} (37)

The commutators of the $J^{ab}$ are defined by the transformation properties under the local lorentz group.

$$\{J^{ab}, J_{cd}\} = \delta^{10}(x, y) \delta^{[a}_{c} J^{b]}_{d},$$  \hspace{1cm} (38)

$$\{J^{ab}(x), G^c(y)\} = \delta^{10}(x, y) \delta^{[a}_{c} G^{b]},$$  \hspace{1cm} (39)

$$\{J^{ab}(x), Q_A(y)\} = \delta^{10}(x, y) \Gamma^B_A Q_B,$$  \hspace{1cm} (40)

$$\{J^{ab}(x), G^{ij}_2(y)\} = 0,$$  \hspace{1cm} (41)

$$\{J^{ab}(x), G^{ijklm}_5\} = 0.$$  \hspace{1cm} (42)

Finally, we find that all the other commutators vanish. Thus the constraint algebra does in fact reproduce the super-Poincare algebra with central charges.

## 5 Conclusions

We have reported here the first step of a program to construct a non-perturbative formulation of $\mathcal{M}$ theory by making a background independent quantization of 11 dimensional supergravity. The next step is to construct the quantization of the 11 dimensional topological quantum field theory, using the methods of loop quantum gravity. This can be done both canonically and through a path integral quantization using an extension of the methods of
spin foam\cite{14} or evolving spin networks\cite{16} to $p$-form gauge fields. Some work in this direction already exists\cite{12, 13, 14} and this part of the program should go through directly. It is clear from the form of the theory that this will involve extended objects whose spacetime dimensions are 2, 3 and 6. These will then give background independent objects corresponding to strings, membranes and five-branes.

In the topological quantum field theory these will have trivial dynamics and the states will be functionals only of homotopy classes of the ten dimensional spacial manifold (which is fixed in a canonical quantization). The problem will then be to reduce the gauge invariance of the topological quantum field theory so as to give rise to local degrees of freedom. There are two ways to accomplish this. The conservative, straightforward path will be to construct the canonical quantization of the eleven dimensional supergravity, by imposing the constraints in the above action classically. This will involve a lot of tedious calculation, to check the resulting algebra of constraints, but should be nonetheless straightforward. One will then impose the quantum constraints rather than the vanishing curvature conditions on the Hilbert space of states.

A second route to the theory will be to follow Barrett and Crane\cite{15} and impose the constraints directly in a path integral expression for the topological quantum field theory.

While these will involve a great deal of work, the key point is that at every stage one will be working with a background independent definition of the Hilbert space of the theory, whose degrees of freedom have the correct dimensionality and supersymmetry transformation properties to lead to strings, membranes and fivebranes in the classical limit. It is difficult to believe that something of value for the understanding of $\mathcal{M}$ theory will not come out of such an investigation.

**ACKNOWLEDGEMENT**

We are grateful to Clifford Johnson, Hermann Nicolai, Mike Reisenberger and Kelle Stelle for discussions during the course of this investigation as well as to the theoretical physics group at Imperial College for hospitality during the course of this work. This work was supported by the NSF through grant PHY95-14240 and a gift from the Jesse Phillips Foundation.
Appendix A: Conventions and notations

Here we give the conventions and notations in this paper. We adopt the convention that if \( \omega \) is a p-form, then:
\[
\omega_p := \omega_{\mu_1 \ldots \mu_p} dx^{\mu_1} \wedge dx^{\mu_2} \wedge \ldots \wedge dx^{\mu_p},
\]

(43)
or we could write it with abstract indices as,
\[
\omega_{a_1 a_2 \ldots a_p} = p! \omega_{\mu_1 \ldots \mu_p} (dx^{\mu_1})_{a_1} (dx^{\mu_2})_{a_2} \ldots (dx^{\mu_p})_{a_p},
\]

(44)

Where the antisymmetrization symbol is defined by,
\[
[a_1 \ldots a_n] = \frac{1}{n!} \sum_p (-)^{\delta_p} a_{p(1)} \ldots a_{p(n)},
\]

(45)
where \( \sum_p \) is the sum over permutations and \( \delta_p \) is the parity of the permutation. Given any two forms such that one is p-form and the other one is q-form, we can define the wedge product of these two forms as,
\[
\omega_p \wedge \omega'_q = \frac{11!}{p! q!} \omega_{[a_1 \ldots a_p} \omega'_{b_1 \ldots b_q]}.
\]

(46)

It’s straightforward to show the wedge product has the following property,
\[
\omega_p \wedge \omega'_q = (-1)^{p+q} \omega'_q \wedge \omega_p.
\]

(47)
The exterior differential \( d \) is a map from vector space of p-form to that of \((p + 1)\)-form,
\[
d\omega_p = (d\omega)_{ab_1 \ldots b_p} = (p + 1)\nabla_{[a} \omega_{b_1 \ldots b_p]}.
\]

(48)

we can also show that
\[
d(\omega_p \wedge \omega'_q) = (d\omega_p) \wedge \omega'_q + (-1)^p \omega_p \wedge d(\omega'_q).
\]

(49)
If \( \omega_{11} \) is a 11-form on the manifold \( \mathcal{M} \), we define the integral of the form on the manifold as,
\[
\int \omega_{11} = \frac{1}{11!} \int \epsilon^{\mu_1 \ldots \mu_{11}} \omega_{\mu_1 \ldots \mu_{11}} d^{11} x,
\]

(50)
where \( \epsilon^{\mu_1 \ldots \mu_{11}} \) is the volume element on \( \mathcal{M} \) such that
\[
\epsilon^{\mu_1 \ldots \mu_{11}} \epsilon_{\mu_1 \ldots \mu_{11}} = 11!,
\]

(51)
and locally if the manifold splits into space \( \Sigma_{10} \) and time \( R \) which is denoted by the coordinate 0, then an induced volume on \( \Sigma_{10} \) is given by,
\[
\epsilon^{i_1 \ldots i_{10}} = \epsilon^{0 \mu_2 \ldots \mu_{11}} = -\omega^{\mu_1 0 \ldots \mu_{11}} = \ldots = \epsilon^{\mu_1 \mu_2 \ldots \mu_{10} 0}.
\]

(52)
Appendix B: Gamma matrix

Some important features of Gamma matrices in eleven dimensional space time are derived in this part. They are essential to show the closure of the constraint algebra in present paper. It’s well known that the $\Gamma$-matrix plays an important role to describe the spinor fields in various dimensions. They form the Clifford algebra,

$$\Gamma^a \Gamma^c + \Gamma^c \Gamma^a = 2\eta^{ac},$$

we also introduce the notation

$$\Gamma^a \Gamma^c - \Gamma^c \Gamma^a := 2\Gamma^{ac},$$

Then in the case of eleven dimensional space time, we can derive the following identities

$$\Gamma_a \Gamma^a = 11, \quad (55)$$

$$\Gamma^a \Gamma^c = \Gamma^{ac} + \eta^{ac}. \quad (56)$$

(55) and (56) are very useful when we try to simplify the expression or rearrange the $\Gamma$-matrices into a new order. For instance,

$$\Gamma_a \Gamma^d \Gamma^a = \Gamma_a (2\eta^{da} - \Gamma^a \Gamma^d) = 2\Gamma^d - \Gamma_a \Gamma^a \Gamma^d = -9\Gamma^d, \quad (57)$$

$$\Gamma_a \Gamma^{d_1 \ldots d_n} \Gamma^a = (-1)^n (11 - 2n) \Gamma^{d_1 \ldots d_n}. \quad (58)$$

In this paper we also often use the following formula which are given in [45],

$$\Gamma^{a_1 \ldots a_n b} = \Gamma^{a_1 \ldots a_n} \Gamma^b - n \Gamma^{[a_1 \ldots a_{n-1} \eta^{a_n}]b}, \quad (59)$$

$$\Gamma^{b a_1 \ldots a_n} = \Gamma^b \Gamma^{a_1 \ldots a_n} - n \eta^{b[a_1} \Gamma^{a_2 \ldots a_n]}. \quad (60)$$

Next we give two important identities of Gamma matrices which are essential to show the closure of constraint algebra. Both of them involve the exchanging of spinor indices of different $\Gamma$-matrices, therefore we write down the elements of these matrices labeled by the
spinor indices explicitly. These identities are

\[ \Gamma^a_E (A \Gamma^{ab}_F) = -\frac{1}{4} (\Gamma^{AB}_a \Gamma^{ab}_F + \Gamma^{ab}_a \Gamma^{AB}_F), \]  

(61)

\[ \Gamma^a_E (\Gamma^{ab}_E \Gamma^{a\ldots b4}_F) - 3 \Gamma^{[a_E \Gamma^{ab}_F \Gamma^{a\ldots b4}_F]} = -\frac{1}{4} (\Gamma^{AB}_a \Gamma^{ab}_F \Gamma^{a\ldots b4}_F + \Gamma^{AB}_a \Gamma^{a\ldots b4}_F \Gamma^{AB}_F) + \frac{3}{2} \Gamma^{[b_E \Gamma^{ab}_F \Gamma^{b\ldots a4}_F]}. \]  

(62)

It’s not difficult to see, they go back to the ordinary Fiertz identities respectively when four spinor fields are involved.\(^9\)

\[ \Psi^A_1 \Gamma^{A\ldots B}_E \Gamma^{a\ldots a4}_F \Gamma^F = 0, \]  

(63)

\[ \Psi^A_1 \Gamma^{A\ldots B}_E \Gamma^{ab}_F \Gamma^{a\ldots b4}_F \Gamma^F = 3 \Psi^A_1 \Gamma^{[b\ldots b4}_E \Gamma^F. \]  

(64)

To prove identities (61) and (62), we need apply the Fiertz decomposition formula in eleven dimensional space time.

\[ \Gamma^a_E (\Gamma^{ab}_E \Gamma^{a\ldots b4}_F) = \frac{1}{32} \left[ \Gamma^{AB}_d \Gamma^{d\ldots a\ldots b4}_F \Gamma^F - \frac{1}{2} \Gamma^{d\ldots a\ldots b4}_d \Gamma^{d\ldots a\ldots b4}_F \Gamma^F + \frac{1}{5!} \Gamma^{AB}_d \Gamma^{d\ldots a\ldots b4}_F \Gamma^{d\ldots a\ldots b4}_F \Gamma^F \right]. \]  

(65)

Then our task is just to simplify the terms involving the multiplication of several Gamma matrices in (65). Exploiting the formula given above, it’s straightforward to derive the following results,

\[ \Gamma_a \Gamma^{d\ldots a\ldots b4}_d = -8 \Gamma^{d\ldots a\ldots b4} + (anti - symmetric terms...), \]  

(66)

\[ \Gamma_a \Gamma^{d\ldots a\ldots b4}_d = 16 \Gamma^{[d\ldots a\ldots b4}_d \eta^{b4}_d + (anti - symmetric terms...), \]  

(67)

\[ \Gamma_a \Gamma^{d\ldots a\ldots b4}_d = 0 + (anti - symmetric terms...). \]  

(68)

Substituting (66)-(68) into (65), we easily arrive at the identity (61).

As far as the second identity (62) is concerned, we just need do more complicated but straightforward calculations as in the case of first identity.

\[ \Gamma^a_E (\Gamma^{ab}_E \Gamma^{a\ldots b4}_F) = \frac{1}{32} \left[ \Gamma^{AB}_d \Gamma^{d\ldots a\ldots b4}_d \Gamma^{a\ldots b4}_F \Gamma^F - \frac{1}{2} \Gamma^{d\ldots a\ldots b4}_d \Gamma^{d\ldots a\ldots b4}_F \Gamma^{a\ldots b4}_F \Gamma^F + \frac{1}{5!} \Gamma^{AB}_d \Gamma^{d\ldots a\ldots b4}_d \Gamma^{d\ldots a\ldots b4}_F \Gamma^{a\ldots b4}_F \Gamma^F \right]. \]  

(69)

\(^9\)We ignore an important matrix in all the paper, namely, the charge conjugation matrix for a neat version of Gamma matrix. But we’d better keep it in mind and realize it appeared where it should be. In eleven dimensions it is also important to know that only \(\{\Gamma^a, \Gamma^{ab}, \Gamma^{a\ldots a5}\}\) and their dual matrices are symmetric (under the action of charge conjugation matrix) while the others are antisymmetric. Since in the Poisson bracket of supersymmetry constraints, the charge conjugation matrix is involved and any term which is anti-symmetric will vanish. Therefore in the following equations we only write down the symmetric term explicitly.
and
$$
\Gamma_{[b_1 b_2 E}^{(A \Gamma b_3 b_4)]_F} = \frac{1}{32} \left[ \Gamma^{AB}_{d} (\Gamma_{[b_1 b_2} \Gamma^{de}_{b_3 b_4]} \Gamma^{d]}_{EF} - \frac{1}{2} \Gamma^{AB}_{de} (\Gamma_{[b_1 b_2} \Gamma^{de}_{b_3 b_4]} \Gamma^{de]}_{EF} + \frac{1}{5!} \Gamma^{AB}_{d_1 \ldots d_5} (\Gamma_{[b_1 b_2} \Gamma^{d_1 \ldots d_5}_{b_3 b_4]} \Gamma^{d_1 \ldots d_5]}_{EF} \right].
$$

(70)

In (69), the terms in brackets can be simplified respectively as,

$$
\Gamma^{ab}_{-b} = -5\Gamma^{db}_{b} + \text{anti-sym. terms},
$$

(71)

$$
\Gamma^{de}_{-b} = 3\Gamma^{db_{1 \ldots b_4}} - 40\Gamma^{d_{1 \ldots b_4} \eta^{d_{1 \ldots b_4}}} + \text{anti-sym. terms},
$$

(72)

and in (70), the terms in brackets can be simplified as

$$
\Gamma_{[b_1 b_2 \Gamma^{d}_{b_3 b_4]} = \Gamma^{d}_{b_1 \ldots b_4} + \text{anti-sym. terms},
$$

(74)

$$
\Gamma_{[b_1 b_2 \Gamma^{de}_{b_3 b_4]} = \Gamma^{de}_{b_1 \ldots b_4} - 4\delta^{[d}_{b_2} \delta^{e]}_{b_1} \Gamma^{d_{1 \ldots d_5}_{b_3 b_4]} + \text{anti-sym. terms},
$$

(75)

$$
\Gamma_{[b_1 b_2 \Gamma^{d_{1 \ldots d_5}_{b_3 b_4]} = \Gamma^{d_{1 \ldots d_5}}_{b_1 \ldots b_4} - 40\delta^{[d_{1}}_{b_2} \delta^{d_2}}_{b_1} \Gamma^{d_{3 \ldots d_5}}_{b_3 b_4]} + 5!\delta^{[d_1}_{b_2} \delta^{d_2}_{b_1} \delta^{d_3}_{b_3} \delta^{d_4}_{b_4} \delta^{d_5}_{b_5]}.
$$

(76)

Substituting all the terms into (69) and (70) respectively, we will find the identity (62) holds indeed.

### Appendix C: The proof of the closure of constraint algebra

In this section we only show the closure of two Poisson brackets. One involves the Gaussian constraint, and the other is the supersymmetric constraint. The other Poisson brackets are closed trivially. First we consider the Poisson bracket of Gaussian constraint. To make the calculation clear, we divide the constraint into two parts,

$$
G^{a} =: G_{1}^{a} + G_{2}^{a},
$$

(77)

Where

$$
G_{1}^{a} = D_{k} \pi^{ka},
$$

(78)
and
\[ G_a^2 = -\Psi_i^A \Psi_j^B \left\{ \Gamma_{bAB}^a (R^{ijk} + \frac{15}{3!} p^{ijkl} a_{lk}) e^b_k + \frac{5i}{2} \Gamma_{4AB}^a \psi_{jk4} E_{k4}^{b4} \right\}. \] (79)

It’s straightforward to compute the Poisson brackets of them,
\[ \{G_i^a, G_j^b\} = 0, \] (80)
\[ \{G_i^a, G_j^b\} = \delta^{10} 30 \Psi_i^A \Gamma_{B}^{acA} \Psi_j^B \Psi_{mE}^D \Gamma_{eD}^b E_{pdp}^{mpn}, \] (81)
\[ \{G_i^a, G_j^b\} = \delta^{10} \left\{ 15 \Psi_i^A \Gamma_{B}^{acA} \Psi_j^B \Psi_{mE}^D \Gamma_{eD}^b E_{pdp}^{mpn} \right\}, \] (82)
\[ + \Gamma_{AB}^{ab} \left\{ 2 F_{A}^{ij} (R^{ij} + \frac{15}{3!} p^{ijkl} a_{lk}) + \frac{1}{4} G_{2}^{ij} + G_{2}^{ij} a_{t3} + p^{ij} F_{4}^{a} \right\} \] (83)
\[ + \frac{1}{4} G_{3}^{ij} E_{l3}^{def} + p^{ij} l_{3}^{m2} F_{kl}^{m} E_{m}^{e} \right\}. \] (84)

Now add (81) and (82) together and notice that
\[ \Gamma^{abc} = \frac{1}{3} (\Gamma^{abc} + \Gamma^{acb} + \Gamma^{bca}). \] (85)
we find the sum of three terms (81), (82) and (83) vanishes by employing the standard Fiertz identity (64). Making a collection of (80)-(84), we show the closure of Poisson bracket of Gauss constraint which corresponds to (33) in the paper.

The Poisson bracket (82) can be derived in a similar way except that we need apply the identities (61) and (62) to cancel those extra terms.

The supersymmetric constraint is
\[ Q^A = Q_1^A + Q_2^A, \] (86)
where
\[ Q_1^A = D_i^a \pi^{ia} - i \pi^{ia} \Gamma_{aB}^A \Psi_i^B, \] (87)
and
\[ Q_2^A = -\Psi_i^B \left\{ \Gamma_{abB}^A (R^{ijk} + \frac{15}{3!} p^{ijkl} a_{lk}) E_{jk}^{ab} - i \Gamma_{abB}^A \psi_{jk4} E_{k4}^{b4} \right\}. \] (88)
We find the Poisson brackets of them are
\[ \{Q_1^A, Q_1^B\} = i \delta^{10} \Gamma_{a}^{AB} D_i^a \pi^{ia}, \] (89)
\[
\{Q^A_2, Q^B_2\} = 30 \delta^{10} \Psi^C_i \Psi^D_j \Gamma^{(A}_{aC} \Gamma^{B)}_{bD} E^{abcd} p^{ijkmnp},
\]

\[
\{Q^A_1, Q^B_2\} + \{Q^A_2, Q^B_1\} = 3 \delta^{10} \left\{ D_i [\Gamma^{AB}_{ab} e^{ab}_{jk} (R^{ijk} + \frac{15}{3!} p^{ijklm} a_{lk3}) + i \Gamma^{AB}_{a5} E^{aq}_{jk} \bar{p}^{ij}] + 4i \Psi^C_i \Psi^D_j (\Gamma^{(A}_{aC} \Gamma^{B)}_{bD} e_{bk} (R^{ijk} + \frac{15}{3!} p^{ijklm} a_{lk3}) + \frac{5i}{2} \Gamma^{(A}_{aC} \Gamma^{B)}_{bD} a_{lk4} E^{bkl4}_{jk4}) \right\}
\]

In (91), it’s a little tedious to deal with the terms with covariant derivative. To express them as the sum of constraints, we use the fact that

\[ D_{[\mu} e^a_{\nu]} = \frac{1}{2} F^a_{\mu\nu} + \frac{i}{2} \Psi_A [\mu \Gamma^A B \Psi^B_{\nu}], \]

and

\[ \partial_{[\mu} q_{\nu\rho]} = \frac{1}{4} F^{\square} + \frac{3i}{2} \Psi_A [\mu \Gamma^A B \Psi^B_{\nu\rho}], \]

and then expand those terms as follows,

\[ D_i [\Gamma^{AB}_{ab} e^{a}_{jk} (R^{ijk} + \frac{15}{3!} p^{ijklm} a_{lk3})] = (...) F^a + (...) G^{ij} + (...) G^{ij} + (...) F_{imnp}^{\square} \]

\[ + i \Gamma^{AB}_{ab} \Psi^{C}_{ic} \Gamma^D e^{b}_{jk} (R^{ijk} + \frac{15}{3!} p^{ijklm} a_{lk3}) + 15 \Gamma^{AB}_{ab} \Psi^{C}_{ic} \Gamma^D e^{a}_{jk} (R^{ijk} + \frac{15}{3!} p^{ijklm} a_{lk3}) + \frac{5i}{2} \Gamma^{AB}_{ab} \Psi^{C}_{ic} \Gamma^D e^{a}_{jk} G^{ijkl4}, \]

where we ignore the explicit expressions of terms involving curvatures and constraints, but the final results are given in (32). Making use of identities (91), we add the terms containing \((R^{ijk} + \frac{15}{3!} p^{ijklm} a_{lk3})\) together in the brackets of (91) and have

\[ i \Psi^C_i \Psi^D_j e_{bk} (R^{ijk} + \frac{15}{3!} p^{ijklm} a_{lk3}) (4 \Gamma^{(A}_{aC} \Gamma^{B)}_{bD} + \Gamma^{AB}_{aC} \Gamma^{a}_{bD}) = -i \Psi^C_i \Psi^D_j e_{bk} (R^{ijk} + \frac{15}{3!} p^{ijklm} a_{lk3}) \Gamma^{AB}_{aC} \Gamma^{a}_{bD}, \]

and using (92), we pick out all the terms containing \(p^{ijklm}\) in the algebra (90) and (91), and find

\[ \Psi^C_i \Psi^D_j E^{abcd} p^{ijklm} (30 \Gamma^{(A}_{C[ab} \Gamma^{B)}_{cD]} - 10 \Gamma^{(A}_{C[ab} \Gamma^{B)}_{cD]} - \frac{5i}{2} \Gamma^{AB}_{eabcd} \Gamma^{e}_{CD} + 15 \Gamma^{AB}_{[ab} \Gamma^{e}_{cd]D}) \]

\[ = \frac{5}{2} \Psi^C_i \Psi^D_j E^{bkl4}_{jk4} \Gamma^{AB}_{a} b_{aCD}. \]
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